Comment on ‘Systematics of radial and angular-momentum Regge trajectories of light non-strange $q\bar{q}$ states’

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Abstract

Masjuan, Arriola, and Broniowski [Phys. Rev. D85, 094006 (2012)] claim that the slope of the light-quark radial trajectories is $1.35\pm0.04$ GeV$^2$, disagreeing with the Crystal Barrel value $1.143\pm0.013$ GeV$^2$. There are defects in their choice of data. When these defects are revised, results come back close to the Crystal Barrel average for the slope. A revised average value is given here.

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1 Introduction

Masjuan, Arriola and Broniowski (referred to later as MAB for brevity) report a new analysis of the slopes of trajectories [1] [2]. They adopt a $\chi^2$ criterion which increases the errors assigned to resonance masses by up to a factor 20. They take

$$
\chi^2 = \sum_n \left( \frac{M_n^2 - M_{n,\text{exp}}^2}{\Gamma_n M_n} \right)^2,
$$

where $M_n$ and $\Gamma_n$ are masses and widths of fitted resonances. Their rationale is that the extrapolation to poles off the real $s$-axis may be inaccurate by one half-width.

The essential point of disagreement with MAB is whether this assumption is justified or not. My point is that large uncertainties in the extrapolation to the pole arise only when a strong $S$-wave threshold opens in the immediate vicinity of the resonance. An example is $f_2(1565)$ which decays strongly to the $\omega\omega$ $S$-wave, opening precisely at 1565 MeV. In this case, there is a large dispersive contribution to the real part of the amplitude, as discussed below in Section III. Clearly one should be alert to such thresholds. However, most high mass resonances have many open channels and such effects are small. After eliminating the special cases, there is no support for the assumption of Eq. (1) as the general case.

To illustrate the effect of the increased errors, it is sufficient to quote one example. The Particle Data Group (PDG) quotes a mass for the $a_4(2040)$ as $2001\pm10$ MeV [3]. For $a_4(2255)$ the PDG uses two measurements with masses $2237\pm5$ and $2255\pm40$ MeV. From these two states, MAB find a slope of $1.0\pm0.8$ GeV$^2$. This implies an error in the mass difference of 203 MeV. This is entirely inconsistent with Crystal Barrel assessment of errors for the analytic continuation to the pole.

Further disagreements arise from several sources. There do exist some missing states, and that must be realised in drawing trajectories. Another point is that it is well known that $c\bar{c}$ and $b\bar{b}$ $^3S_1$ ground states are anomalously low in mass compared with a straight trajectory through $J/\psi$ and $\Upsilon$ $n = 2, 3, 4$ radial states. There are indications that this is also true for $n\bar{n}$ states. A third point is that there is almost certainly a $J^{PC} = 0^{++}$ glueball in the mass range 1370 to 1800 MeV, but no present agreement on its identification. It will certainly mix with $q\bar{q}$ states, and this mixing makes the masses of $q\bar{q}$ components uncertain.
A primary problem is that MAB do not distinguish clearly between $^3S_1$ and $^3D_1$ states. Conventional wisdom is that $P$-state light mesons appear at masses 1200–1300 MeV, $D$ states at 1600–1700 MeV and $F$ states near 2000 MeV. However, they assign the third $^3S_1$ ω state a mass of $\sim 1970$ MeV, i.e. $\sim 300$ MeV above the $^3D_3\omega_3(1670)$ and $\rho_3(1690)$. The result is a slope for the $^3S_1\omega$ trajectory $\sim 4/3$ times larger than other trajectories.

They also replace the best determinations of trajectories for $I = 0, \ C = +1$ mesons (where there are 10 sets of data) by poorer determinations of $I = 1 \ C = +1$ mesons, where there are no polarisation data to separate $^3P_2$ and $^3F_2$ mesons, hence much larger errors for masses. They also replace the Crystal Barrel determination of the mass of the $f_3(2300)$ by including a possibly biased mass determination from data on $\bar{p}p \rightarrow \Lambda\bar{\Lambda}$. That is not a good idea, since mixing with the $s\bar{s}$ amplitude can confuse the situation.

In order to present the discrepancies with the slopes of trajectories assigned by Crystal Barrel (CB), the slopes of all trajectories are redetermined here from final CB data sets and tabulated for comparison with the slopes of MAB. The table of results makes the differences immediately apparent.

## 2 Prologue

The Crystal Barrel has produced extensive data on formation of high mass mesons in the process $\bar{p}p \rightarrow R \rightarrow A + B$, where $R$ stands for a resonance and there are 18 channels of all-neutral final states available. Ref. [4] reviews the data and technical details. The detector covers 98% of the solid angle with caesium iodide crystals which measure all-neutral final states. Quantum numbers fall into four non-interfering families $I = 0$ or 1, $C = +1$ or -1.

For $I = 0, \ C = +1$, there are data on 6 channels: $\eta\pi^0\pi^0$, $\eta'/\pi^0\pi^0$, $3\eta$, $\pi^0\pi^0$, $\eta\eta$ and $\eta\eta'$. There are also differential cross sections and polarisation data for $\bar{p}p \rightarrow \pi^+\pi^-$ from the PS172 [5] and an earlier experiment at the CERN PS [6]. These are vital for two reasons. First, they separate $^3P_2$ and $^3F_2$ states. Secondly, polarisation is phase sensitive and reduces errors on fitted masses and widths substantially. The improvement in mass determination from polarisation data can be up to a factor 4 because of its phase sensitivity.

In the $\eta\pi\pi$ data there are prominent $f_4(2050)$ and $f_4(2300)$ signals, easily identified from their strong angular dependence. They are determined accurately in mass and width from data at 9 beam momenta from 600 to 1940 MeV/c. These states serve as interferometers for all lower spin triplet partial waves. There is also a lucky break, that two singlet states also appear prominently: an $\eta_2(2250)$ in $\eta'\pi\pi$ and $\eta(2320)$ in the $3\eta$ data in the channel $f_0(1500)\eta$. A complete set of $n\bar{n}$ states appears in two towers of resonances centred near 2000 and 2270 MeV. For $I = 1, \ C = -1$, an almost complete set of states also appears, but with poor identification of $^3S_1$ states. For $I = 1, \ C = +1$, there are actually two solutions, with one of them close to the $I = 0, \ C = +1$ solution, as one would expect for light quarks with small mass differences. For $I = 0, \ C = -1$, statistics are low for $\omega\eta$ and the $\omega\pi^0\pi^0$ data have the problem that the broad $\sigma \equiv f_0(500)$ interferes all over the Dalitz plot.

The $F$ states lie systematically $\sim 70$ MeV above the $P$ states, because high $L$ states need to overcome a centrifugal barrier in order to resonate. The $D$ states lie roughly midway; $S$ and $G$ states continue the sequence.

The $f_2(1525)$ is widely accepted as the $s\bar{s}$ partner of $f_2(1270)$. Production of $f_2(1525)$ in the
Crystal Barrel experiment is extremely weak. It is detected at the 1–2% level in $\bar{p}p \to \eta\eta\pi^0$ in flight [7]. The conclusion is that $\bar{p}p$ annihilation is dominantly to $n\bar{n}$ final states - hardly a surprise. This conclusion is supported and quantified by a combined analysis of data on $\bar{p}p \to \pi^+\pi^-, \pi^0\pi^0, \eta\eta$ and $\eta\eta'$ [8]. Amplitudes for decay to $\eta\eta$ and $\eta\eta'$ depend on the well known composition of $\eta$ and $\eta'$ in terms of singlet and octet states and the pseudoscalar mixing angle. The observed state $R$ is expressed as a linear composition $R = \cos \Phi |q\bar{q}| + \sin \Phi |s\bar{s}|$. The result is that $\Phi \leq 15^\circ$, i.e. a maximum of 25% in amplitude, for all observed states with the exception of $f_0(2105)$ (which is taken as a glueball candidate, but could possibly be due to unexpectedly strong mixing between closely spaced $n\bar{n}$ and $s\bar{s}$ states). The allocation MAB make between $n\bar{n}$ and $s\bar{s}$ states is in conflict with the fact that CB states are dominantly $n\bar{n}$.

The partial wave analysis of CB data is documented in Section 4 of Ref. [4]. This describes systematic checks which have been made on the identification of resonances, particularly their stability as the number of fitted resonances was changed. The following sections illustrate the result and discuss individual resonances and their Argand diagrams. For $I = 0, C = +1$, all states have statistical significance $> 25$ standard deviations except for the $f_2(2001)$, which is $18\sigma$ but observed clearly in four sets of data. Two states, $f_1(2310)$ and $\eta(2010)$ have rather large errors for masses. Regge trajectories are discussed in section 9. For channels $\omega\pi$ and $\omega\eta$, polarisations of $\omega$ are determined by the angular dependence of decays to $\pi^+\pi^-\pi^0$ and are very revealing. The interpretation of this polarisation is important and discussed in Sections 7.1 and 8. There is one non-standard piece of nomenclature. These polarisations are described as vector polarisation $P_y$. Strictly, the standard nomenclature is that this should be called $\text{Re}\,iT_{11}$, where $T$ is tensor polarisation.

### 3 Determination of slopes of trajectories

One should be aware in advance that states may deviate from straight trajectories because of dispersive effects on resonance masses. The strict form for the denominator of a Breit-Wigner amplitude is

\[
D(s) = M^2 - s - m(s) - i \sum_j g_j^2 \rho_j
\]

\[
m(s) = \frac{1}{\pi} P \int_{s \text{thr}}^\infty \sum_j g_j^2 \rho_j(s') \frac{ds'}{s' - s}.
\]

To make the Principal Value integral converge better, it is typical to make a subtraction on the resonance, although in principle this can be done at any mass; $s \text{thr}$ is the $s$ value at threshold for each channel. The $g_j^2$ are coupling constants to every decay channel, and $\rho_j(s')$ are the phase space for each final state, including centrifugal barriers and possible form factors. Near the thresholds of important decay channels, a change in the imaginary part of the amplitude is accompanied by a corresponding real part so as to obey analyticity. At sharp thresholds, the imaginary part of the phase space rises linearly from threshold, and produces a cusp in the real part of the amplitude. This acts as an attractor [9]. The $a_0(980)$ and $f_0(980)$ are attracted to the $K\bar{K}$ S-wave threshold. Likewise the $f_2(1565)$ is attracted to the sharp threshold for the $\omega\omega$ final state; this attraction is augmented by a broader threshold in the $\rho\rho$ channel, which has 3 times more events from the SU(2) relation with $\omega\omega$. The PDG mass is taken from the $\pi\pi$.
channel, but the $\omega\omega$ and $\rho\rho$ channels are respectively a factor of 8.5 and a factor of 25 stronger than $\pi\pi$ when integrated over the available mass range, see Fig. 5(b) of Baker et al [10]. In that paper, a dispersion relation was evaluated for the effect of both these channels; the pole position was determined to be $1598 \pm 11^{\text{(stat)}} \pm 9^{\text{(syst)}}$ MeV. This is lower than the $a_2(1700)$ of the PDG, but this is because of physics which is understood.

For CB data in flight above the $\bar{p}p$ threshold, there is a conspicuous $f_2(1270)\eta'$ signal fitted as an $I = 0, C = +1 J^{PC} = 2^{-+}$ resonance at 2267 MeV and clearly associated with the S-wave threshold effect, see Figs. 17 and 18 of Ref. [4]. For $I = 1, C = +1$, there are signs of similar activity near the $a_2(1320)\eta'$ threshold, but this channel cannot be reconstructed accurately enough in the very difficult final state $\eta'\eta'\pi^0$. S-wave structure could arise at thresholds for $a_2(1320)\omega$, $a_2(1320)\rho$, $f_2(1270)\omega$ and $f_2(1270)\rho$, but is blurred out by convolution of the large widths of $a_2$ and $f_2$ with the $\rho$. It would lead to structure distributed over $J^{PC} = 3^{--}, 2^{--}$ and $1^{--}$, but in the absence of polarisation data cannot be sorted out at present. For P-wave thresholds, the imaginary part of the amplitude increases as the cube of the momentum, and leads to negligible effects.

The $\bar{p}p$ and $\bar{p}n$ total cross sections follow a $1/v$ variation, where $v$ is the relativistic velocity of the incident $\bar{p}$. The result is a strong cusp at the $\bar{p}p$ threshold in both $^1S_0$ and $^3S_1$ partial waves. This $1/v$ dependence is included into the partial wave analysis for these waves. This
cusp will perturb masses of resonances near the threshold. Data from Novosibirsk on $\bar{p}p \to 6\pi$ final states have a rapid mass variation close to the $\bar{p}p$ threshold \[11\].

Returning to the question of observed slopes of trajectories, each set of quantum numbers will be examined one by one, fitting the expected states to observed masses and errors, but paying particular attention to cases where states are missing or strongly displaced by dispersive effects. Having done this, a grand average is taken of all slopes. As a guide, Fig. 1 shows the updated analysis of $I = 0, C = +1$ states.

### 4 Results

Table 1 lists slopes for all families of light mesons in GeV\(^2\). General comments are as follows.

First, $I = 0, C = +1$ states are best determined, because of the available polarisation data, which are very precise. For triplet states, decays are possible for orbital angular momentum $L = J + 1$ and $L = J - 1$. The ratio of coupling constants $r_J = g_{J+1}/g_{J-1}$ is tabulated in Ref. \[4\], but not tabulated by the Particle Data Group; it is the basic guide to whether states have $L = J + 1$ or $J - 1$.

Why should $^3F_2$ $n\bar{n}$ states decay preferentially to $\bar{p}p$ $^3F_2$? A feature of CB data is that $F$ states decay strongly to channels with high angular momentum. The origin of this is clearly a good overlap between wave functions of initial and final states. Llanes-Estrada et al. point out a formal analogy with the Frank-Condon principle of molecular physics consistent with this interpretation \[12\].

It is immediately clear from the errors in the table that the $\chi^2$ weighting used by MAB increases some errors by large amounts. From the agreement in many cases between CB and MAB, it is also clear that remaining discrepancies should be inspected closely. It is easiest to compare with results from MAB in their Section 3, taking them in the reverse order to the publication, i.e. K to A. A clear picture of disagreements then arises step by step.

The $f_1$ and $f_3$ states are considered in MAB Sec. 3K. The CB approach is to fit $f_1(1285)$, $f_1(1910)$ and $f_1(2310)$ using their errors. The first two have small errors, with the result that the fit misses the weak $f_1(2310)$ by just over one standard deviation. Neither $f_1(1420)$ nor $f_1(1510)$ is fitted well by the CB approach or that of MAB. In the CB fit, a state is expected at 1660 MeV, close to its isospin partner $a_1(1640)$. The $f_1(1510)$ is naturally assigned as the $s\bar{s}$ analogue of $f_1(1285)$; the mass difference is close to that between $f_2(1270)$ and $f_3(1525)$. Longacre proposes that $f_1(1420)$ is a molecule where an $L = 1$ pion circles a $K\bar{K}$ core \[13\]. For $f_3$, the MAB slope has an error a factor 9 larger than the CB value. Sections J and I agree well on slopes between CB and MAB for $b_3$, $b_1$, $h_3$ and $h_1$ states.

For $f_3$, it is clear from the slope of MAB that they use a mass well above the CB determination of the mass of $f_3(2300)$, $2303 \pm 15$ MeV; it seems likely that it is replaced by the value $2334 \pm 25$ MeV from $\bar{p}p \to \Lambda\bar{\Lambda}$ data. This is dangerous, since it may introduce mixing with $s\bar{s}$. If one takes the weighted mean of the two masses quoted by the PDG \[3\] for $f_3(2300)$, namely $2311 \pm 13$ MeV, the slope is $1.15 \pm 0.08$. 

Sections 3J and 3I of Ref. [1] agree between CB and MAB for $\omega(1650)$ on a trajectory with an $\omega(1420)$ as a standard state. This could be an $s\bar{s}$ state. The consequence is that MAB ignore the well known $\omega(1650)$ as a standard $^3D_1$ state.

The scheme listed in the PDG meson summary tables takes the $\omega(1420)$ as a $^3S_1$ state and $\omega(1650)$ as $^3D_1$. The $\rho$ states run parallel, with $\rho(1450)$ as a $^3S_1$ state and the well established $\rho(1700)$ as $^3D_1$. The $\phi(1680)$ fits in as $^3S_1$ and $\phi(1850)$ as $^3D_1$; the $\phi$ states lie higher than $n\bar{n}$ states by a similar mass difference to that between $f_2(1270)$ and $f_2(1525)$ (which is well established as $s\bar{s}$). Fitting the $\omega(1660)$ together with $\omega(1650)$ gives a slope of $1.091 \pm 0.068$,

| Family | Nominal masses(MeV) | CB slope | MAB slope | Comments |
|--------|---------------------|----------|-----------|----------|
| $f_4$  | 2050,2300           | $1.140 \pm 0.093$ | -        |          |
| $f_3$  | 2050,2300           | $1.147 \pm 0.071$ | $1.27 \pm 0.64$ | +        |
| $f_2(^3F_2)$ | 2000,2295 | $1.254 \pm 0.075$ | -        |          |
| $f_3(^3P_2)$ | 1270,1565,1910,2240 | $1.113 \pm 0.025$ | -        | +        |
| $f_1$  | 1285,−,1970,2310    | $1.130 \pm 0.064$ | $1.19 \pm 0.15$ | +        |
| $f_0$  | 1370,−,2330         | $1.24 \pm 0.045$ | $1.24 \pm 0.18$ | +        |
| $h_3$  | 2025,2275           | $1.075 \pm 0.147$ | $1.08 \pm 0.54$ |          |
| $h_1$  | 1170,1595,1965,2215  | $1.195 \pm 0.059$ | $1.20 \pm 0.25$ |          |
| $b_3$  | 2025,2275           | $0.95 \pm 0.191$ | $1.08 \pm 0.54$ |          |
| $b_1$  | 1235,−,1960,2240    | $1.169 \pm 0.058$ | $1.17 \pm 0.18$ |          |
| $\omega_3$ | 1670,1945,2255  | $1.059 \pm 0.163$ | $1.16 \pm 0.26$ |          |
| $\omega_2$ | 1975,2195          | $0.917 \pm 0.160$ | -        |          |
| $\omega_1(^3D_1)$ | 1650,1960,2295  | $1.091 \pm 0.068$ | $1.27 \pm 0.47$ | +        |
| $\omega_1(^3S_1)$ | 782,1420,1650,−,2.205 | $1.080 \pm 0.029$ | $1.50 \pm 0.12$ | +        |
| $a_4$  | 2040,2255           | $1.000 \pm 0.045$ | $1.00 \pm 0.8$ |           |
| $a_3$  | 2030,2275           | $1.051 \pm 0.170$ | $1.5 \pm 0.11$ |           |
| $a_2(^3F_2)$ | 2030,2255        | $0.964 \pm 0.128$ | $1.00 \pm 0.7$ |           |
| $a_2(^3P_2)$ | 1320,1700,1950,2175 | $1.000 \pm 0.060$ | $1.39 \pm 0.26$ | +        |
| $a_1$  | 1260,1640,1930,2270 | $1.084 \pm 0.063$ | $1.36 \pm 0.49$ | +        |
| $a_0$  | 1450,2025           | $0.964 \pm 0.073$ | $1.42 \pm 0.26$ | +        |
| $\pi_2$ | 1670,2005,2245      | $1.218 \pm 0.062$ | $1.21 \pm 0.36$ |           |
| $\pi_0$ | 1300,1800?,2070,2360 | $1.29 \pm 0.200$ | $1.27 \pm 0.27$ |           |
| $\rho_3$ | 1690,1990,2265     | $1.094 \pm 0.050$ | $1.19 \pm 0.32$ | +        |
| $\rho_2$ | 1940,2225          | $1.206 \pm 0.085$ | -        |          |
| $\rho_1(^3D_1)$ | 1700,2000,2270  | $1.203 \pm 0.052$ | $1.08 \pm 0.47$ | +        |
| $\rho_1(^3S_1)$ | 770,1450,−,1900,2150 | $1.365 \pm 0.108$ | $1.43 \pm 0.13$ | +        |
| $\eta_2$ | 1645,2030,2265     | $1.188 \pm 0.038$ | $1.32 \pm 0.32$ |           |
| $\eta_0$ | 1295,2320          | $1.241 \pm 0.030$ | $1.33 \pm 0.11$ | +        |

Table 1: Nominal resonances masses and slopes (in GeV$^2$) from CB and MAB analyses; comments are marked by a + in the final column and discussed in the text; a dash in column 2 indicates a missing state (or unused for reasons discussed in the text).
close to other CB slopes. Fitting the remaining ω(3S1) trajectory with states at 782, 1425 and 2205 MeV, but with two missing states gives a slope of 1.080 ± 0.029. The fit to the second state however gives it a mass of 1329 MeV. Many authors have noticed this point. It could well be due to the fact that the ground state is abnormally low in mass, just as the J/ψ and Υ(1S) lie significantly below a straight line through n = 2, 3 and 4 radial excitations.

Section 3G of Ref. [1] considers f0 states. Here physics remarks are required. One of the fundamentals of Particle Physics is chiral symmetry breaking. This was first proposed by Gell-Mann and Lévy in 1960 [14]. It became well known to theorists that the attractive NN interaction requires exchange of two correlated pions with a broad peak at 450–650 MeV, denoted by the σ. Bicudo and Ribiero provided a detailed account of how this arises [15]. The mechanism today accounts for the low mass σ ≡ f0(500), κ, a0(980) and f0(980). It is now well understood [16] how a crossover arises between these exceptional states and regular qq̅ states near 1 GeV. The surviving mixing above 1 GeV is likely to push the q̅q 0+ states up in mass. This can explain the anomalously high masses of f0(1370) and a0(1450). A further complication for f0 is the likely existence of a glueball in the mass range 1500–1800 MeV, still obscure. A further point is that there is evidence [17] that the f2(1810) claimed by GAMS has been confused with the f0(1790) candidate for the radial excitation of f0(1370); the f0(1790) is consistent with the BES II 0+ peak observed at 1812 MeV in ωφ decays[18]. So, in summary, f0 states are complex. Conclusions about the f0 slope are therefore ambiguous. Using only the ππ mass of f0(1370) and the mass of f0(2330), the slope is 1.24 ± 0.045 GeV².

The f0(1370) decays dominantly to 4π; this introduces large dispersive effects on the mass. Crystal Barrel data at rest on ¯p p → 3π0 contain 600,000 precisely measured events and interference effects between the three ππ components determine phase variations very precisely. The mass fitted to the ππ channel is 1309 ± 1(stat) ± 15(syst) MeV. However rapidly increasing phase space for its dominant ρρ decay channel moves the peak in 4π data up by ~ 75 MeV. Pole positions on different sheets are given in Table 4 of [19] for f0(1370) and f0(1500) and are quite revealing. This strong threshold shifts the pole of f0(1370) by at most 17 MeV from the peak in ππ; the fitted 2π full-width is 325 MeV. So in this extreme case, the shift in pole position is ≤ 10.5% of the half-width. For the f0(1500), the shift in pole position is 8 MeV from the nominal mass compared with a half-width of 54.4 ± 3.5 MeV. These shifts are a factor at least 6 less than MAB assume.

MAB construct two trajectories for what they take to be n̅n states and s̅s. The n̅n trajectory starts with f0(980) and finishes with f0(2200). However, there is almost universal agreement today that f0(980) and a0(980) are not qq̅ states but have dominant 4-quark composition [20]. The f0(980) decays dominantly to KK̅, not ππ. BES II quote a KK/ππ branching ratio 4.21 ± 0.2(stat) ± 0.21(syst) [21]; this is one of the very few experiments which has data on both KK̅ and ππ. A further important source of information is the decay of χ0 → K+K−. There is a conspicuous f0(1710) in the data and a further peak at f0(2200) [22]. This suggests that they both have substantial ss̅ components. So the MAB n̅n trajectory looks unlikely.

On their s̅s trajectory, MAB start with f0(1370) and finish with f0(2330). The f0(1370) is observed dominantly in decays to 4π, largely ρρ. It has a branching ratio to KK̅ of 0.12 ± 0.06 in CB data, so it does not look like an s̅s state. The last member of this trajectory is f0(2330). This has been observed in Crystal Barrel data in decays to ππ and ηη [8] with a flavour angle of 15.1°, so it is certainly not a dominantly s̅s state.

Section E of MAB discusses a0, a2 and a4 states. This is again a complex story. They use
the mass of $a_0(980)$, which is unwise in view of its association with chiral symmetry breaking. The $a_0(1450)$ is well determined [23], but the signal for $a_0(2025)$ is very weak. An additional $a_0$ is to be expected somewhere between these two, but there are no data adequate to detect it; finding spin 0 states is difficult. Assuming this state has been missed, the slope from $a_0(1450)$ and $a_0(2025)$ is $0.96 \pm 0.08 \text{ GeV}^2$, but is probably affected by chiral symmetry breaking and is not used in the overall CB average for the slope. MAB make the opposite assumption that this is the first radial excitation of $a_0(1450)$ and hence find a slope of $1.42 \pm 0.26$.

Moving on to $a_2$ states, MAB consider as an upper trajectory ($^3F_2$) $a_2(2030)$ and $a_2(2255)$ and arrive at a similar slope to CB. For the $^3P_2$ trajectory, they take $a_2(1320)$, $a_2(1700)$ and $a_2(2175)$. This is to be compared with the well established $f_2$ trajectory $f_2(1270)$, $f_2(1565)$, $f_2(1910)$, $f_2(2240)$. They miss an $a_2$ state to be identified with the $a_2(1950)$ of Anisovich et al. [24]. They find a slope $1.39 \pm 0.26$ compared with the CB determination $1.00 \pm 0.06$.

There is agreement between CB and MAB for the $a_4$ slope. However, one comment is needed on the PDG determination of the mass. It is determined largely by the data of Uman et al. [25]. If one looks at their Fig. 6, the difference in $\chi^2$ between $a_2$ and $a_4$ is small. They do not consider the possibility that both $a_2$ and $a_4$ are present; that would not be at all surprising. Therefore the CB determination of the mass is preferred here.

Consider next $f_2$ states. The problem here is that MAB do not discriminate between the $^3P_2$ states and $^3F_2$, which are well separated in CB data. MAB launch into four alternative scenarios, all of which have problems.

Their $f_2^q$ trajectory is made from $f_2(1370)$, $f_2(1750)$ and $f_2(2150)$. The $f_2(2150)$ is not seen in CB data. It is observed only in decays to $KK$ and $\eta\eta$. The $f_2(2010)$ of the PDG [3], observed by Etkin et al. in $\bar{p}p \rightarrow \phi\phi$ actually peaks at $2150 \text{ MeV}$. The mass quoted by Etkin et al. [26] is the K-matrix mass, and can differ from the T-matrix mass; the K-matrix formalism assumes that all decay channels are known, but that is unlikely. The obvious interpretation of the $f_2(2150)$ is the $s\bar{s}$ partner of $f_2(1910)$, i.e. a $^3P_2$ state. Etkin et al. also report an $f_2(2300)$ in the $\phi\phi$ S-wave and $f_2(2340)$ in the $\phi\phi$ D-wave. This is naturally to be interpreted as an $s\bar{s}$ $^3F_2$ state. The $f_2(1750)$ of Schegelsky et al. [27] is observed in $\gamma\gamma \rightarrow KK$, and is interpreted by them as an $s\bar{s}$ state - the radial excitation of $f_2(1525)$, though rather low in mass.

The MAB $f_2^q$ trajectory uses $f_2(1430)$. That entry in PDG tables has a straightforward interpretation. The $\omega\omega$ channel (and therefore $\rho\rho$) couples strongly to $f_2(1565)$. When analysing data on Dalitz plots, it is necessary to continue the Flatté formula for $f_2(1565)$ below the $\omega\omega$ threshold, rather than just cutting it off. This is the way in which Crystal Barrel analyses Dalitz plots. The result is a cusp in the $\pi\pi$ channel at $1430 \text{ MeV}$, see Fig. 7 of Adomeit et al. [28]. It is likely that the data listed by the PDG under $f_0(1430)$ were due to this phenomenon.

The MAB $f_2^q$ trajectory uses $f_2(1525)$, which is a well known $s\bar{s}$ state and is obviously invalid. Their $f_2^d$ trajectory uses $f_2(1565)$, $f_2(2000)$ and $f_2(2295)$, hence mixing $^3P_2$ and $^3F_2$ states. This is also invalid.

They continue with three further trajectories, the first based on $f_2(1640)$ together with $f_2(2150)$. The $f_2(1640)$ has been explained by Baker et al. [10] as the $\omega\omega$ decay of $f_2(1565)$; the rapidly rising $\omega\omega$ phase space shifts the peak in $\omega\omega$ up to $1640 \text{ MeV}$ [10]. The second is based on the questionable $f_2(1810)$ and $f_2(2220)$, which is a very narrow peak, $23 \text{ MeV}$ wide, claimed in BES II data. If such a narrow state contributes to non-strange $q\bar{q}$ states, it is a mystery why it is not observed very conspicuously in CB data. Their final trajectory is made of $f_2(2010)$ and $f_2(2340)$, which are observed in $\phi\phi$ and $KK$ and finds a slope of $1.43 \pm 0.83 \text{ GeV}^2$, they are
obvious candidates for $s\bar{s}$ states.

Section 3D of MAB [11] concerns $\pi$ and $\pi_2$ trajectories. These agree with CB. The $\pi(1800)$ is usually considered as a hybrid candidate; it has little effect on the fitted slope.

Section C discusses $\rho_1$ and $\rho_3$ states. Their slope for the latter is close to the CB value but with much larger error from their $\chi^2$ criterion. The physics situation concerning $\rho_1$ states is a mess, for physics reasons. The $\rho(1450)$ couples weakly to $2\pi$ and there are large dispersive effects in the $4\pi$ channel, which have not yet been taken into account. The natural interpretation of it is the $^3S_1$ radial excitation of $\rho(770)$, but the large slope may arise simply from the fact that the ground state is abnormally low, like the $J/\psi$ and $Y(1S)$. The $\rho(1570)$ of Babar has a larger error in mass: $\pm 36(stat) \pm 62(syst)$ MeV and is marginally consistent with $\rho(1450)$, which actually has a mass of $1465 \pm 25$ MeV.

The $\rho(1900)$ can be identified with a recent Novosibirsk observation of a $6\pi$ peak almost exactly at the $\bar{p}p$ threshold [11]. This is likely to be a $^3S_1$ state captured by the very strong $\bar{p}p$ S-wave, but could be a non-resonant cusp.

CB data list ratios $r_J$ of coupling constants to orbital angular momentum $L = J + 1$ and $L = J - 1$. The $\rho(2000)$ has a sizable $r$ value $0.70 \pm 0.23$ requiring at least some $^3D_1$ contribution. The $\rho(2150)$ in CB data has an $r$ value $-0.05 \pm 0.42$, consistent with $^3S_1$. The $\rho(2270)$ in CB data has $r = -0.55 \pm 0.66$ which is ambiguous. However, the $\rho(2000), (2150), (2270)$ make a natural sequence of $^3D_1, ^3S_1, ^3D_1$. If this solution to the puzzle is accepted, the slope of the CB $^3S_1$ trajectory is $1.34 \pm 0.03$, with a rather poor fit to the mass of $\rho(1450)$. In view of the problems with $\rho(1450)$, this is not included in the grand average. The trajectory of $\rho(1700), \rho(2000)$ and $\rho(2270)$ gives a slope of $1.17 \pm 0.03$; MAB quote $1.08 \pm 0.47$.

Section 3B of MAB [11] discusses $\eta$ and $\eta_2$ states. The $\eta(548)$ is believed to be a Goldstone boson and should not be included in the assessment of $q\bar{q}$ states. The $\eta(1760)$ and $\eta(2100)$ were claimed by DM2, but later identified in Mark III data [29] as having $J^{PC} = 0^{++}$, though they sit on a large non-interfering $0^{-+}$ background; $0^{++}$ and $0^{-+}$ do not interfere in $J/\psi$ radiative decays after summing over relevant spin states of the $J/\psi$. They are also identified in E760 data [30] in the $\eta\eta$ channel, where $J^{PC} = 0^{-+}$ is forbidden by Bose statistics. The remaining trajectory, $\eta(1295)$ and $\eta(2320)$ gives a CB slope of $1.24 \pm 0.03$; MAB quote $1.33 \pm 0.11$.

For $\eta_2$ states, the averaged slope agrees with the global average within errors but the $\chi^2$ of the fit to the $\eta_2(2030)$ in the middle is high. There is a likely explanation. There is an extra state $\eta_2(1870)$ which is naturally explained as a hybrid partner to $\pi_1(1600)$ predicted near this mass. By the usual level repulsion, this pushes the $\eta_2(1645)$ down and the $\eta_2(2030)$ up, though the overall effect on the average slope is small.

Section 3A of MAB [11] discusses the $a_1$ trajectory. The $a_1(2095)$ has a large error in mass of $\pm 121$ MeV. The $a_1(2270)$ completes the trajectory. There is an obvious problem that the $a_1(1260)$ has a large width; the PDG quotes it as $250–660$ MeV. A recent Babar estimate is $410 \pm 31 \pm 30$ MeV. The CB slope is $1.08 \pm 0.06$ GeV$^2$; MAB find $1.43 \pm 0.26$ GeV$^2$.

Finally, MAB discard all slope determinations which have only two points. This removes all the determinations from $^3F_2, ^3F_3$ and $^3F_4$ states which are amongst the best. As one sees from Table 1 and Fig. 1, these determinations have slopes consistent with other CB values. MAB also assign $a_2(2030)$ and $a_2(2255)$ the radial quantum numbers $n = 2$ and 3, while the nearby $f_2(2000)$ and $f_4(2295)$ obviously have $n = 3$ and 4.

In summary, the MAB classification of slopes unfortunately contains a number of problems, and there is no significant case for the large slopes they claim for some cases.
5 Epilogue

Values of CB and MAB differ significantly only where there are clear problems in their selection of states in the fit. The weighted mean of CB slopes is revised slightly. On close inspection, the $\chi^2$ contributions from $^3S_1\rho_1$ results are high by a factor 4. Warnings about the problem in this case have already been given. Likewise contributions to $\chi^2$ from $a_0$ are high by a factor 5. Again the text has pointed out problems for these states. Finally, $\chi^2$ contributions from $a_2\ ^3P_2$ states are high by a factor 4. This is no surprise, since there are no polarisation data to provide clear identifications of these states.

MAB remark that Anisovich, Anisovich and Sarantsev (AAS) proposed a scheme in the year 2000 where the lowest $J^{PC} = 0^{++}$ states were taken as $a_0(980)$ and $f_0(980)$ [31]. Since then, there have been many studies of the effects of chiral symmetry breaking. It is now widely believed that the $\sigma$, $\kappa$, $a_0(980)$ and $f_0(980)$ are meson-meson states, and that there is a crossover to $q\bar{q}$ states near 1 GeV, where chiral symmetry breaking decreases rapidly.

Summarising, the mean slope without any corrections is $1.130 \pm 0.011$. Reducing the weights of the three troublesome cases to 1 modifies this to the final value $1.135 \pm 0.012$, compared with the old value of $1.143 \pm 0.013$ GeV$^2$. There is no clear case for the large error assessment of MAB. In fact, states with large widths already enlarge the errors for masses appropriately.

A comment on the MAB approach is that they adopt their assumption that meson masses can move by $\Gamma/2$ from theoretical predictions; those are that for large-$N_c$ the strong coupling constant scales as $g \sim 1/\sqrt{N_c}$ with the result that meson masses change by $\Gamma/2$ when evolved from $N_c = 3$ to $N_c = \infty$, see the references 16-18 given in their paper. The conclusion from the present analysis is that the $N_c$ world is different from $N_c \rightarrow \infty$.

The PDG lists CB data under ‘Other Light Mesons, further states’ on the grounds that they need confirmation. Perhaps, but $I = 0, C = +1$ does contains a complete spectrum of expected states. For other isospin and $C$ values, it is desirable to improve the data base. That cannot be done in production experiments, because the exchanged meson is not usually known, i.e. no polarisation information is available. The $\rho$ and $\omega$ states can be improved at VEPP 2 in Novosibirsk by using transversely polarised electrons. Two measurements are readily made of asymmetries normal to the plane of polarisation and in the plane of polarisation. Electron polarisation of 70% is already achieved and two detectors CMD and SMD are available and running. The presence of $^3D_1$ states is then revealed by distinctive azimuthal angular dependence in the polarisation and can measure whether these are pure $^3D_1$ states or linear combinations with $^3S_1$, and if so how big the contributions are. Longitudinal polarisation does not help much because it depends only on the difference of intensities of the two helicities available.

In order to trace the missing states above 1910 MeV, polarisation measurements are needed for $I = 1, C = +1$ ($\eta\pi^0$, $\eta\pi^0$ and $3\pi^0$), $I = 1, C = -1$ ($\omega\pi^0$ and $\eta\omega\pi^0$), and $I = 0, C = -1$ ($\omega\eta$ and $\omega\pi^0\pi^0$). Polarisation data also introduce interference between singlet and triplet states, hence determining the singlet states much better. Data are required down to $\bar{p}$ momenta of $\sim 360$ MeV/$c$, the lowest momentum reached in the PS172 experiment [5]. The PANDA experiment cannot do this measurement because their lowest available beam momentum will be 1.5 GeV/$c$. \[10\]
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