Abstract—Automaton based approaches have enabled robots to perform various complex tasks. However, most existing automaton based algorithms highly rely on the manually customized representation of states for the considered task, limiting its applicability in deep reinforcement learning algorithms. To address this issue, by incorporating Transformer into reinforcement learning, we develop a Double-Transformer-guided Temporal Logic framework (T2TL) that exploits the structural feature of Transformer twice, i.e., first encoding the LTL instruction via the Transformer module for efficient understanding of task instructions during the training and then encoding the context variable via the Transformer again for improved task performance. Particularly, the LTL instruction is specified by co-safe LTL. As a semantics-preserving rewriting operation, LTL progression is exploited to decompose the complex task into learnable sub-goals, which not only converts non-Markovian reward decision process to Markovian ones, but also improves the sampling efficiency by simultaneous learning of multiple sub-tasks. An environment-agnostic LTL pre-training scheme is further incorporated to facilitate the learning of the Transformer module resulting in improved representation of LTL. The simulation and experiment results demonstrate the effectiveness of the T2TL framework.

I. INTRODUCTION

One of the ultimate goals in robotic learning is to let the robot infer the key to the task completion. To enable such human-level intelligence, the capability of comprehending the semantics of instructions and evolving continuously via interactions with the environment is crucial. Among numerous learning algorithms, reinforcement learning (RL) is a sequential decision-making process that models dynamics of the interaction as a Markov decision process (MDP) and focuses on learning the optimal policy through exploration and exploitation [1]. Although RL based methods have enabled the robot to accomplish tasks from simple to complex ones (e.g., MuZero [2] and Go-Explore [3]), an important yet challenging topic is how the robot can enhance their understanding of instructions to improve task completion. In particular, there are three main challenges: 1) unlike existing works with explicit task instructions and motion constraints, how can the robot comprehend the nature of instructions by its own to improve the task completion? 2) Since many practical tasks require the robot to perform a series of logically organized sub-tasks (e.g., cleaning rooms, organizing books and washing clothes while avoiding collisions), resulting in a non-Markovian reward decision process (NMRDP), how can the NMRDP be properly handled? 3) When solving the complex task in a sparse reward environment, how can the robot facilitate learning by leveraging the potential of its representation module?

Transformer was originally presented in [4] for natural language processing and recently achieves remarkable success in many fields. In [5], a Vision Transformer (ViT) framework is developed, which proposes patch embedding for image preprocessing and performs better than state-of-the-art CNNs. The work of [6] presents an effective combination of RL and Transformer, which casts the traditional RL problem as a conditional sequence modelling by leveraging the causally masked Transformer. The structured features of Transformer are further incorporated in [7] to improve robotic manipulation by capturing the spatio-temporal relationship between the dual-arm movements. As a variant of classical ViT, Transformer is also extended for robotic grasping in [8] by developing different window attention and hierarchical skip-connected architectures to obtain the local information and model long-term connections. Despite recent progress, most of the existing methods with Transformer mainly focus on natural language processing or computer vision, lacking the guidance to drive the robots towards task completion. It is unclear how the conventional Transformer can be combined with RL to guide the agent to understand complex motion planning tasks that consist of a series of sub-goals that need to be completed logically.

Due to the rich expressivity and capability, linear temporal logic (LTL) is capable of describing a wide range of complex tasks composed of logically organized sub-tasks [9]–[11]. By converting the LTL specification into an automaton, learning algorithms are often exploited to facilitate the motion planning of robotic systems. For instance, modular deep reinforcement learning is incorporated with a limit deterministic generalized Büchi automaton (LDGBA) to enable continuous motion planning of an autonomous dynamical system [12]. Learning-based probabilistic motion planning subject to the deterministic Rabin automaton (DRA) guideline in the presence of environment and motion uncertainties is investigated in [13]. Truncated LTL is leveraged to facilitate the reward design in [14], which can be converted into a finite-state predicate automaton (FSPA) to improve the performance of reinforcement learning in robotic planning. Similar to the automaton, reward machine (RM) is proposed to offer dense rewards feedback in [15], which can be translated from a variety of temporal logic specifications to improve the sample efficiency of reinforcement learning methods. However, most of these aforementioned methods highly rely on the representation of system states in the form of either automaton or RM, which not only grows

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exponentially with respect to the task complexity, but also are not effective for neural networks to forward propagation. When considering representing the LTL as a neural network, the work of [16] exploits a compositional recurrent neural network (RNN) as an encoder to train the learning agent to understand LTL semantics. However, RNN generally suffers from high computational cost due to its inherently sequential nature precluding parallelization. In [17], the compositional syntax and the semantics of LTL are exploited by the relational graph convolutional network (R-GCN) to enable the generalization to new tasks. However, it cannot offer interpretable guidance to the learning agent due to the irregularity of the R-GCN architecture [18].

Contributions: In this work, by incorporating Transformer into reinforcement learning, we develop a Double-Transformer-guided Temporal Logic framework (T2TL) that exploits the structural feature of Transformer twice, i.e., first encoding the LTL instruction via the Transformer module for efficient understanding of task instructions during the training and then encoding the context variable via the Transformer again for improved task performance. In particular, the LTL instruction is specified by co-safe LTL. As a semantics-preserving rewriting operation, LTL progression is exploited to decompose the complex instruction into learnable sub-goals, which not only converts non-Markovian reward decision process to Markovian ones, but also improves the sampling efficiency by simultaneous learning of multiple sub-tasks. Inspired by [17], an environment-agnostic LTL pre-training scheme is further incorporated to facilitate the learning of the Transformer module. The simulation and experimental results demonstrate the effectiveness of the T2TL framework.

II. PRELIMINARIES

A. Co-Safe Linear Temporal Logic

Co-safe LTL (sc-LTL) is a subclass of LTL that can be satisfied by finite-horizon state trajectories [19]. Since sc-LTL is suitable to describe robotic instructions (e.g., trigger the alarm, find the extinguisher, and then put out the fire), this work focuses on sc-LTL. The syntax of an sc-LTL formula is formally defined over the set of atomic propositions \( \Pi \) as

\[
\varphi := T \mid \neg p \mid \varphi_1 \land \varphi_2 \mid \varphi_1 \lor \varphi_2 \mid \bigcirc \varphi \mid \lozenge \varphi \mid \varphi_1 \cup \varphi_2 ,
\]

where \( T \) is the boolean constant true, \( p \in \Pi \) is an atomic proposition that can be true or false, \( \land \) (conjunction), \( \lor \) (disjunction), and \( \neg \) (negation) are standard Boolean operators, \( \bigcirc \) (next), \( \lozenge \) (eventually), and \( \cup \) (until) are temporal operators, and \( \varphi_1 \) and \( \varphi_2 \) are task formulas composed of atomic propositions and operators. The semantics of an sc-LTL formula are interpreted over a word \( \sigma = \sigma_0 \sigma_1 \ldots \sigma_n \), which is a finite sequence with \( \sigma_i \in 2^{\Pi} \), \( i = 0, \ldots, n \), where \( 2^{\Pi} \) represents the power set of \( \Pi \). Denote by \( (\sigma, i) \models \varphi \) if the sc-LTL formula \( \varphi \) holds from position \( i \) of \( \sigma \). More detailed explanations and examples can be found in [10].

B. Labeled MDP and Reinforcement Learning

When performing the sc-LTL task \( \varphi \), the interaction between the robot and the environment can be modeled by a labeled MDP \( \mathcal{M}_e = (S, T, A, p_e, \Pi, L, R, \gamma, \mu) \), where \( S \subseteq \mathbb{R}^n \) is a set of states, \( T \subseteq S \) is a set of terminal states, \( A \subseteq \mathbb{R}^m \) is a set of actions, \( p_e(s', a, \sigma) \) is the transition probability from \( s \in S \) to \( s' \in S \) under action \( a \in A \), \( \Pi \) is a set of atomic propositions indicating the properties associated with the states, \( L : S \to 2^{\Pi} \) is the labeling function, \( R \) is the reward function, \( \gamma \in (0, 1) \) is the discount factor, and \( \mu \) is the initial state distribution. The labeling function \( L \) can be seen as a set of event detectors that trigger when \( p \in \Pi \) presents in the environment, allowing the robot to determine whether or not an LTL specification is satisfied. It is assumed that the transition probability \( p_e \) is unknown a priori, and the agent can only perceive its present state and the corresponding label.

For any task \( \varphi \), the robot interacts with the environment following a deterministic policy \( \pi \) over \( \mathcal{M}_e \), where \( \pi : S \to A \) maps each state to an action over the action space \( A \). Specifically, the robot starts from an initial state \( s_0 \) sampled from \( \mu \) in each episode, and transits from the current state \( s_t \) to the next state \( s_{t+1} \) sampled from \( p_e(s_{t+1} | s_t, a_t) \) under the control action \( a_t \) generated by the policy \( \pi \). The robot then receives a reward \( r_t \) from \( R \). The Q-value is \( Q(s, a) = \mathbb{E}[\gamma_0 + \gamma_1 + \ldots | s_0 = s, a_0 = a, \pi] \) and the optimal Q-value is \( Q^*(s, a) = \max_{\pi} Q(s, a) \). The optimal policy \( \pi^* \) can then be derived from the optimal Q-value [1].

When applying to a large or continuous state space, the Q-value function is often parameterized with the weights function \( \theta^Q \) like \( Q(s, a; \theta^Q) \) in the Deep Q-Networks (DQN) [20]. And in the continuous action case, the parameterized policy model is often applied to the uncountable infinite problem like \( \pi_u(a; s, \theta^u) \) with weights \( \theta^u \) as in Proximal Policy Optimization (PPO) [21]. The typical reward function is often Markovian, which means that the reward acquired at \( s_{t+1} \) is only based on the transition from \( s_t \) to \( s_{t+1} \). In fact, the robot is generally rewarded depending on the completion of the assigned task \( \varphi \), i.e., \( \sigma \models \varphi \), and the episode terminates when \( \varphi \) is satisfied or falsified. Since the word \( \sigma = \sigma_0 \sigma_1 \ldots \sigma_t \) is formed from the state trajectory \( s_0 s_1 \ldots s_t \) through the labeling function \( L \), in this work we will consider the non-Markovian reward function

\[
R(s_0 s_1 \ldots s_t) = \begin{cases} 1, & \text{if } \sigma \models \varphi \\ -1, & \text{if } \sigma \models \neg \varphi \\ 0, & \text{otherwise} \end{cases}
\]

where \( \sigma_t = L(s_t) \). In the sequel, we will discuss how to deal with the challenge of NMRDP. Given a task \( \varphi \), the goal of the agent is to learn an optimal policy \( \pi^*(a|s) \) that maximizes the expected discounted return \( E \left[ \sum_{k=0}^{\infty} \gamma^k r_{t+k} | S_t = s \right] \) starting from any state \( s \in S \) at time step \( t \).

III. PROBLEM FORMULATION

To elaborate the proposed interpretable temporal logic guided reinforcement learning algorithm, the following example will be used as a running example throughout the work.

Example 1. Consider a modified safety-gym environment [22] as shown in Fig. [1] in which the robot is required to sequentially visit a set of locations
while avoiding collisions. The set of propositions $\Pi$ is $\{\text{Black Zone}, \text{White Zone}, \text{Yellow Zone}, \text{Red Zone}\}$. Using above propositions in $\Pi$, an example sc-LTL formula is $\varphi_{\text{safe}} = \lozenge(\text{Black Zone} \land \lozenge\text{White Zone}) \land \neg\lozenge\text{Red Zone} \land \neg\lozenge\text{Yellow Zone}$, which requires the robot to sequentially visit the black zone and the white zone while avoiding collisioning with red zones and yellow zones. One of the safe robot trajectories is shown by the green arrows in Fig. 1.

In this work, we are interested in encoding the task conditional states by the Transformer. By representing via Transformer we hope to take advantage of its flexibility in encoding states and provide interpretable analysis of the robot’s motion planning. Compared with automaton and RM based state representations, when using Transformer to encode the states, the gradually updated state representation can facilitate the agent’s comprehension of the sub-goal at hand as the agent interacts with the environment, resulting in a mutual improvement, in which the Transformer guides the robot’s motion and the selected actions improves the Transformer for better instructions.

Specifically, suppose the representation of an LTL task $\varphi_\theta$ can be approximated by the Transformer parameterized with weights $\theta_{\text{trans}}$, where $\theta_{\text{trans}}$ is updated by the back-propagation of the RL controller. The goal of an interpretable LTL guided RL in this work is to find the appropriate Transformer weights $\theta_{\text{trans}}$ over the LTL instruction, such that an effective representation $\varphi_\theta$ can lead to fast learning for logical motion planning. To this end, the problem can be formally presented as follows.

**Problem 1.** Given a MDP $M_c = (S, T, A, p_c, \Pi, L, \gamma, \mu)$ corresponding to task $\varphi$ with the reward function $R_\varphi(s_0, s_1, \ldots, s_t)$ to be designed, the goal of this work is to design a optimal representation $\varphi_\theta$ with $\theta_{\text{trans}}$, so that the return $E \left[ \sum_{k=0}^{\infty} \gamma^k r_{t+k} \mid S_t = s \right]$ under the policy $\pi(\alpha_t | s_0, \ldots, s_t, \varphi)$ can be maximized.

### IV. Algorithm Design

To address Problem 1, this section presents a novel framework, namely Double-Transformer-guided Temporal Logic framework (T2TL), that offers interpretable LTL instruction using Transformer to guide the robot motion planning and uses Transformer again to encode context variables to further facilitate the robot learning. Section IV-A presents how LTL progression can be leveraged to convert NMRDP to MDP. Section IV-B explains how the Transformer is exploited to encode the LTL specification. Section IV-C explains in detail how Transformer can facilitate the agent’s understanding of complex tasks using simultaneous learning. Section IV-D shows how the context variable improves the agent performance and how the pre-training scheme can be further incorporated to expedite the convergence in a sparse reward environment.

#### A. LTL Progression and TL-MDP

One of the major challenges in solving Problem 1 is that the reward function $R(s_0, s_1, \ldots, s_t)$ used in the Q-value function depends on the history of the states and thus is non-Markovian. In this work, the LTL progression from [23] and [24] is applied to solve the non-Markovian issue. Let $\text{AT}(\varphi)$ denote the propositions needed to progress the current LTL specification. The LTL progression is defined formally as follows.

**Definition 1.** Given an LTL formula $\varphi$ and a word $\sigma = \sigma_0 \sigma_1 \ldots$, the LTL progression $\text{prog}(\sigma, \varphi)$ at step $i$, $\forall i = 0, 1, \ldots$, is defined as follows:

\[
\text{prog}(\sigma_i, \varphi) = \begin{cases} 
\text{True} & \text{if } p \in \sigma_i, \text{ where } p \in \Pi, \\
\text{False} & \text{if } p \notin \sigma_i, \text{ where } p \in \Pi, \\
\neg \text{prog}(\sigma_{i-1}, \varphi) & \text{if } \text{prog}(\sigma_{i-1}, \varphi_1) \land \text{prog}(\sigma_{i-1}, \varphi_2), \\
\text{prog}(\sigma_{i-1}, \varphi_1) \lor \text{prog}(\sigma_{i-1}, \varphi_2) & \text{if } \sigma_i = \sigma_{i-1}, \\
\text{prog}(\sigma_{i-1}, \varphi) & \text{if } \text{prog}(\sigma_{i-1}, \varphi_1) \land \text{prog}(\sigma_{i-1}, \varphi_2), \\
\varphi & \text{if } \text{AT}(\varphi) = p, \text{ prog}(\sigma_i, p) = \text{True}, \\
\varphi & \text{otherwise.}
\end{cases}
\]

The operator $\text{prog}$ in Def. 1 takes an LTL formula $\varphi$ and the current label $\sigma_i$ as input at each step, and outputs a formula to track which parts of the original instructions remain to be addressed.

**Theorem 1.** [23] Given any LTL formula $\varphi$ and the corresponding word $\sigma = \sigma_0 \sigma_1 \ldots$, $\langle \sigma, i \rangle \vdash \varphi$ iff $\langle \sigma, i+1 \rangle \vdash \text{prog}(\sigma_i, \varphi)$.

There are many advantages of using the LTL progression. First, since the operator $\text{prog}$ can preserve LTL semantics, applying $\text{prog}$ iteratively after each step will result in gradually diminishing LTL instructions, which indicates the progress towards task completion. For instance, consider the task $\varphi_{\text{safe}}$ in Example 1 which will progress to the subsequent sub-task $\varphi_{\text{safe}} = \lozenge\text{Red Zone} \land \neg\lozenge\text{Black Zone} \land \neg\lozenge\text{Yellow Zone}$ as soon as the robot reaches $\text{Black Zone}$. Since LTL progression can indicate the progress of tasks, the reward function can be designed by leveraging it to make the agent focus on the current progressed task rather than the original one all the time. Another benefit of utilizing $\text{prog}$ iteratively is that the complex task may be divided into a series of learnable sub-tasks that can be viewed as simultaneous sub-goals to improve the sampling efficiency. For instance, by applying the operator $\text{prog}$ over $\varphi_{\text{safe}}$, the tasks $\{\varphi_{\text{safe}}, \varphi_{\text{safe}}\}$ will be viewed as an extended training set to train the RL controller. In the following context,
we represent by $\Psi$ the extended training set for which $\varphi$ and its progressed sub-tasks are included.

Based on the LTL progression in Def. 1 and the LTL instruction $\varphi$, an augmented MDP, namely the task-driven labeled MDP (TL–MDP), is developed as follows.

Definition 2. Given a labeled MDP $M_{c} = (S, T, A, p_{c}, E)$ corresponding to an LTL task $\varphi$, the TL–MDP is constructed by augmenting $M_{c}$ to $M_{\Psi} \triangleq \{ (S, T, A, \hat{p}, E, \hat{\rho}, \gamma, \mu) : \phi_{i} \in \Psi, i = 1, \ldots, |\Psi| \}$ with $|\Psi|$ indicating the number of tasks in $\Psi$, where $\hat{S} = S \times \Psi$, $\hat{T} = \{ (s, \phi) | s \in T \text{ or } \phi \in \{ \text{true, false} \}, \phi \in \Psi \}$, $\hat{\rho}((s', \phi')|(s, \phi), a) = p_{c}(s'|s, a)$ if $\phi = \text{prog}(L(s), \phi_{t})$ and $\hat{\rho}_{i}((s, \phi_{i})|(s, \phi_{i}), a) = 0$ otherwise, and $\hat{\rho}_{i}$ is the reward function associated with the task $\phi_{i} \in \Psi$ to overcome the non-Markovian reward issue which can be written as

$$
\hat{\rho}_{i}(s, \phi_{i}) = \begin{cases} 
1, & \text{if } \text{prog}(L(s), \phi_{i}) = \text{True}, \\
-1, & \text{if } \text{prog}(L(s), \phi_{i}) = \text{False}, \\
0, & \text{otherwise},
\end{cases}
$$

Thus, by defining the TL-MDP, the non-Markovian reward function can be Markovian. With LTL progression, the policy $\pi_{\Psi}(a_{t} | s_{t}, \varphi)$ that solves the LTL $\varphi$ over the TL-MDP $M_{\Psi}$ can achieve the same expected discounted return as the policy $\pi_{c}(a_{t} | s_{0}s_{1}\ldots s_{t}, \varphi)$ in the environment $M_{c}$.

B. Represent LTL via Transformer

Another challenge in solving Problem 1 is to design an appropriate parameterized encoder for the LTL specification for improved performance in a sparse reward environment. To address this challenge, inspired by the interpretable representation architecture and encoding capability for the nature language, the Transformer from [4] is exploited to represent the LTL instruction in this work. An overview of the architecture is depicted in Fig. 3(c).

Given an input $X_{\varphi} = (x_{0}, x_{1}, \ldots)$ generated by the LTL task $\varphi$ where $x_{t}, t = 0, 1, \ldots$, represents the operator or proposition, $X_{c}$ will be preprocessed by the word embedding $E$ as $X_{E} = [x_{0}E; x_{1}E; \ldots; x_{N}E] \in \mathbb{R}^{B \times (N+1) \times D}$ where $B$ is the batch size, $N + 1$ is the length of input $X_{\varphi}$, and $D$ is the model dimension of the Transformer. $X_{E}$ is then added with the frequency-based positional embedding $E_{pos}$ to make use of the order of the sequence. For instance, a task $\varphi = c \cup (a \land (\neg d \cup b))$ can be encoded as shown in Fig. 2.

The encoder is constructed by stacking identical transformer layers and each transformer layer is built with a self-attention sub-layer and a position-wise fully connected feed-forward (MLP) sub-layer. Layer norm (LN) is applied before every sub-layer and residual connections are applied after every block.

In the structure of the Transformer, the multi-head self-attention (MSA) method plays an important role in establishing the intrinsic connections between words. Specifically, given the query $Q$, key $K$, and value $V$ derived from the LTL input $X_{\varphi} = (x_{0}, x_{1}, \ldots)$, the similarity of words can be calculated by the dot-product attention as

$$
\text{Attention}(Q, K, V) = \text{softmax} \left( \frac{QK^{T}}{\sqrt{d_{k}}} \right) V,
$$

where $\sqrt{d_{k}}$ is the scaling factor. The global computation procedure of the encoder layers is represented as follows:

$X_{0} = [x_{0}E; x_{1}E; \ldots; x_{N}E] + E_{pos}$, $E_{pos} \in \mathbb{R}^{B \times (N+1) \times D}$

$X_{i} = \text{MSA}(\text{LN}(X_{i-1})) + X_{i-1}$, $l = 1, \ldots, L$

$X_{l} = \text{MLP}(\text{LN}(X_{l})) + X_{l}$, $l = 1, \ldots, L$

$Y = \text{LN}(X_{l})$

where $Y$ represents the output of the last layer from the Transformer encoder, which can be manually customized to an appropriate dimension according to the need of tasks.

C. T1TL and Simultaneous Learning

As shown in Table 4, traditional product-MDP algorithms usually represent the states of automaton or RM with one-hot encoding or sorted index, whose representation needs to be customized manually and the dimensions are dependent on the complexity of the LTL task. Unlike these works, we encode the LTL specification as normalized vectors using transformer, which is not only appropriate for the forward propagation of the neural network, but also can be continuously updated as Transformer evolves. In addition, its dimension can be customized with appropriate designs of transformer, leading to improved agent’s performance. Compared with automatic-based methods, the product-MDP based Transformer can be constructed on-the-fly without concern of exponential explosion of algorithm complexity with LTL tasks. Furthermore, with the update of the Transformer, the weights or heads in self-attention can offer reasonable interpretability for the agent’s motion planning.

The interpretable LTL representation encoded via the Transformer is illustrated in Fig. 4. Initially, the weights of Transformer are set randomly. As the agent interacts with the environment, the RL module is updated when a proposition is encountered by the agent, which leads to an indirect update of the Transformer module, i.e., the agent has new knowledge of the pros and cons about the currently encountered proposition for completing the task. Thus, as the RL module converges, the Transformer module achieves a better representation of
The Transformer module will be initialized from pretrained weights. During each episode, with the help of LTL progression, the Q-values of the state process of the self-attention out-projection weights in Transformer from between the agent and the environment. The heatmap depicts the update of Q-values of task \( \phi \in \Psi \) with the LTL representation encoded by Transformer module, and a series of episodes over the tasks in \( \Psi \) is progressed to the extended tasks \( \Psi^\prime \) by LTL progression to generate an extended training set \( \Psi^\prime \) augmented the original MDP \( \mathcal{M}_e \) with the LTL representation encoded by Transformer module, and use Q-learning to simultaneously learn these sub-tasks.

However, conventional off-policy DRL algorithm usually performs a random exploration in the early stage. If an action effective for other tasks is performed rather than the current task, such an action is often ignored and will not be utilized to update the Q-value for associated tasks, resulting in low sampling efficiency and delayed convergence to the optimal policy. Note that the on-policy DRL algorithms, such as PPO, usually train the agent with parallel environments to improve sampling efficiency by reducing the correlation of transition data. However, this trick usually can’t be applied to off-policy DRL algorithm due to the existence of experience replay buffer.

Compared with vanilla DQN, the idea of simultaneous learning is to extract sub-tasks from \( \varphi \) via LTL progression as described in Sec. [V-A] augment the original MDP \( \mathcal{M}_e \) with the LTL representation encoded by Transformer module, and use Q-learning to simultaneously learn these sub-tasks.

Particularly, the simultaneous learning begins with extracting sub-tasks from \( \varphi \) by LTL progression to generate an extended training set \( \Psi \). All tasks \( \phi \in \Psi \) are associated with a Q-value function \( Q_{\phi^\prime} (s, a) \) where the LTL instruction is encoded by the Transformer module, and a series of episodes over the tasks in \( \Psi \) is performed using the off-policy learning method. For each \( \phi \in \Psi \), the robot updates the Q-value functions as if it is currently trying to solve \( \phi \). Specially, given the current state \( s \), the formula \( \phi \) will be the progressed LTL task if \( \phi^\prime = \text{prog}(L(s), \phi) \). The robot selects an action \( a \) following a behavior policy (e.g., the \( \varepsilon \)-greedy one) based on the Q-value \( Q_{\phi^\prime} \) and then transits to the next state with rewards received from the extended MDP \( \mathcal{M}_e \). Let \( Q_{\phi^\prime} (s, a) \) and \( Q_{\phi^\prime} (s, a) \) be the Q-value function of task \( \phi \) and \( \phi^\prime \), respectively. Thus under the simultaneous learning, \( Q_{\phi^\prime} \) is updated following a modified double DQN as

\[
Q_{\phi^\prime} \leftarrow Q_{\phi^\prime} + \alpha \left( R_{\phi^\prime} + \gamma \max_{a^\prime} Q_{\phi^\prime} (s^\prime, a^\prime) - Q_{\phi^\prime} \right) \quad \text{(3)}
\]

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\]
By this way, the Q-value of φ will be propagated backwards from its sub-tasks φ′ and the weights of Transformer will also be updated over the state representation. Thus by developing TL-MDP $\mathcal{M}_φ$, it will not only convert the non-Markovian reward process to Markovian ones, but also provide simultaneous update for sub-task’s Q-value. Such a method enables the update of the current $Q_{φ}$ and its sub-task $Q_{φ′}$, resulting in an effective representation of LTL for improved convergence.

\section{Algorithm 1 T2TL with Pre-training Scheme}

1: **procedure** $\text{INPUT}:(A \text{ LTL} \text{ instruction } \varphi \text{ and the MDP } \mathcal{M}_s \text{ corresponding to } \varphi)$
2: **Output: An approximately optimal stationary policy** $\pi^* \in \mathcal{P}$ for the TL-MDP $\mathcal{M}_φ$
3: **Initialization:** All neural network weights
4: 2: Load the pre-trained weights to the Transformer module, extract sub-tasks as $\Psi$, and initialize $Q_{φ}$ and $Q_{φ'}$ for $φ$ and its sub-task $φ'$
5: 3: while $T < T_{\text{max}}$ do
6: 4: Augment the state $s$ with $φ$ encoded by Transformer, and set the context variable to zero
7: 5: while $t < t_{\text{max}}$ do
8: 6: if $φ' \in \{\text{True, False}\}$ or $s \in T$ then
9: 7: Break
10: 8: end if
11: 9: Gather data from $φ$ and encode the context variable through Transformer
12: 10: for $Q_{φ} \in Q$ do
13: 11: $φ' \leftarrow \text{prog}(L(s), φ)$
14: 12: Determine $R_{φ}$ by (3) and update $Q_{φ}$ following (4)
15: 13: end for
16: 14: end while
17: 15: $T \leftarrow T + 1$
18: 16: end while
19: end procedure

By this way, the robot is able to comprehend the LTL task by considering the context information and expedite the learning in a sparse reward environment.

Inspired by the competitive performance on downstream tasks when using the pre-training method in [17], an environment-agnostic module is further incorporated as the pre-training scheme in this work. First, a single-state MDP $\mathcal{M}_s = (S, T, A, p_s, \Pi, L, γ, µ)$ is built, where $S = \{s_o\}$, $T = \emptyset$, $A = p \in \Pi$, $p_s(s_0 | s_0, \cdot) = 1$, $µ(s_0) = 1$ and $L(s_0) = \{p\}$. Then the single-state MDP $\mathcal{M}_s$ can be augmented to TL-MDP $\mathcal{M}_φ$, with the LTL instruction $φ$. Second, the agent tries to complete the LTL task in each episode until the Transformer module converges. The learned Transformer weights are then set as the initial LTL module in the downstream MDP (e.g., the TL-MDP $\mathcal{M}_φ$ in the modified safety-gym). With the pre-training scheme, the LTL presentation from $φ$ can help the agent infer which part of the information should be emphasized to increase the probability of achieving sub-goals. The overall method is illustrated in Fig.3(b) and the pseudo-code is outlined in Alg.1.

\section{V. CASE STUDIES}

In this section, the developed T2TL framework is evaluated against the state-of-the-art algorithms in simulation and real-world experiments. Specifically, we consider the following aspects. 1) **Performance:** how well does our approach out-perform the state-of-the-art algorithms in the discrete and continuous environments? 2) **Representation:** What is the role of the representation dimensions for LTL specifications? 3) **Interpretability:** How well can the agent understand LTL specifications via Transformer?

\subsection{A. Simulation Results}

To show the effectiveness of the T2TL framework, denoted by T2TL$_{\text{pre}}$, it is empirically compared with four baselines. The first baseline is DLA from [28] which is used to construct the product MDP for the LTL task over a finite horizon. The second baseline is RM from [29] which has automaton-based representations that exploit the reward function’s internal structure to learn optimal policies. The third baseline is GNN$_{\text{pre}}$, which uses a pre-training scheme from [17] and exploits the compositional syntax and semantics of LTL by GNN to solve complex multiple tasks. Note that the simultaneous learning is incorporated in GNN$_{\text{pre}}$ for fair comparisons with our method. The fourth baseline is our T2TL$_{\text{pre}}$, which focuses more on the architecture of the Transformer compared than GNN$_{\text{pre}}$.

To evaluate the performance in a sparse reward environment, our framework is verified in three different environments covering both discrete and continuous spaces. In discrete cases, we set performance $= \hat{R}_φ + γ^{t_{\text{com}}/t_{\text{opt}}} \times 1$ for each episode where $t_{\text{com}}$ represents steps at the end of current episode and $t_{\text{opt}}$ is the optimal number of steps for the LTL task. In continuous cases, performance $= \frac{1}{t_{\text{frame}}} \text{sum}_{t_{\text{frame}}=1}^{t_{\text{frame}}=1} γ^{-1} \times \hat{R}_φ$, where $t_{\text{frame}}$ represents a fixed number of steps to train the agent. The RL algorithms applied to discrete and continuous cases are double DQN [30] and PPO [21], respectively.

$^1$Our codes are available at https://github.com/Charlie0257/T2TL
(1) Case 1: Office Gridworld. We first evaluate the developed T2TL framework in an office gridworld. As shown in Fig. 5(a), the triangle agent can move in the four cardinal directions and pick the corresponding object at its location. The set of propositions $\Pi$ in this environment is $\{\text{Coffee, Office, Email, Decoration, A, B, C, D}\}$. Note that the snowflake symbol represents the Decoration, and the agent breaks if it steps on it. The letters A, B, C, D, o represent different areas and the office room, respectively. In this environment, we consider an sc-LTL task $\varphi_{\text{deliver}} = \lozenge(\text{Coffee} \land \lozenge \text{Office}) \land \neg \square \text{Decoration}$, which requires the agent to visit the region Coffee and Office in order while avoiding Decoration.

Fig. 5(b) shows the performances of all baselines against ours over the task $\varphi_{\text{deliver}}$ with 3 random seeds in an office gridworld. Clearly, the method of simultaneous learning shows improved convergence than DFA. By encoding the LTL representation using by Transformer, T1TL$_{\text{pre}}$, outperforms RM but has a similar tendency as GNN$_{\text{pre}}$. Compared with T1TL$_{\text{pre}}$, by incorporating context variable T2TL$_{\text{pre}}$ shows a rapid convergence after a learning of 200 episodes.

(2) Case 2: MiniCraft-like gridworld. We further test our developed framework in a MiniGrid-like [31] environment. To consider a more challenging environment, we increased the traditional MiniGrid map size to $45 \times 45$. The set of propositions $\Pi$ is $\{\text{Wood, Workshop, Trap, Marsh}\}$. And the map contains 5 of each sub-goal but 10 of each dangerous area, and unknown dangers are distributed in front of each sub-goal as shown in Fig. 6(a). Consider a complex task as in [17] requiring the robot to visit the Wood area and Workshop area sequentially while avoiding to encounter Trap areas and Marsh areas in order, which can be written as $\varphi_{\text{live}} = \neg \text{Trap} \cup (\text{Wood} \land (\neg \text{Marsh} \lor \text{Workshop}))$. As shown in Fig. 6(b), DFA can’t converge to the optimal policy due to its lower sampling efficiency, the representation encoded by the Transformer helps T1TL$_{\text{pre}}$ to outperform GNN$_{\text{pre}}$ and RM, and T2TL$_{\text{pre}}$ shows improved convergence using the context variable.

(3) Case 3: ZoneEnv. We finally evaluate our framework in a modified Safety-gym [22] environment as shown in Fig. 7(a). Consider a sequential task requiring the robot to visit the red zone, black zone, and yellow zone in order, which can be written as $\varphi_{\text{zone}} = \lozenge(\text{Red Zone} \land \lozenge(\text{Black Zone} \land \lozenge \text{Yellow Zone})).$ Fig. 7(b) shows the performance of GNN$_{\text{pre}}$ and T1TL$_{\text{pre}}$, which outperform DFA clearly, reflecting the effect of the LTL representation encoded by neural networks. T2TL$_{\text{pre}}$ shows a competitive performance compared with T2TL and GNN$_{\text{pre}}$.

(4) Dimension Comparison. To emphasize the influence of the representation dimension of the LTL instruction on agent performance in a high-dimensional state space or complex environment, Fig. 8 shows the results between T1TL$_{\text{pre}}$ and RM with different dimensions in the MiniCraft-like gridworld and the ZoneEnv case. It is clear in Fig. 8(a) that an appropriate increase for the representational dimension of the LTL instruction is beneficial in providing agents with more comprehensive information. In Fig. 8(b), an appropriate representational dimension further improves the performance of DFA, and the agent achieves a good performance when the dimension is 8 or 16 in T1TL.

(5) Interpretability via Attention. To further visualize how well the agent understand the LTL task when the Transformer module converges, Fig. 9 shows a view from heads in attention to interpret which tokens the agent would be more interested in. In Fig. 9 different color bars represent different heads in the zero layer of attention and its length indicates the weights of the head on this token. As shown in Fig. 9(a), all heads are...
In this work, we present a T2TL framework that incorporates Transformer to represent the LTL formula for improved performance and interpretability. Future work will consider extensions to multi-task learning.

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Figure 9. The heads concentration from attention view of Transformer on the instruction ϕWood. (a) and (b) reflect the process of the comprehension of the agent to the LTL instruction.

B. Experiment Results

Experiments are also performed on a mobile robot Turtlebot3 burger for the office gridworld considered in Sec. V-A to verify the effectiveness of our approach. The experimental video and more experiment details are provided.2

VI. CONCLUSIONS

In this work, we present a T2TL framework that incorporates Transformer to represent the LTL formula for improved performance and interpretability. Future work will consider extensions to multi-task learning.