Emergence of cooperation in a mixed population: the strength of irrationality

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How cooperation emerges in our society remains largely unknown. One reason is that most previous studies strictly follow the assumption of Homo economicus that individuals are rational and maximizing the earnings is the only incentive of their actions. We human beings, however, are complex and our behaviors are multi-motive driven. In this study, we investigate the evolution of cooperation within a population on the 2d square lattice with mixed rational and irrational individuals, or specifically, players respectively with the Fermi and Tit-for-tat updating rules. While the former rule is to imitate the strategies of those neighbors who are doing well, the latter is like emotion-driven behaviors where the players repeat what their neighbors did to them. In a structural mixing implementation, where the updating rule is fixed for each individual, we find that a moderate fraction of irrational behaviors unexpectedly boosts the overall cooperation level of the population. The boost is even more pronounced in the probabilistic mixing, where each individual randomly chooses the two rules at each step, and full cooperation is seen in a wide range. These findings also hold in complex structured populations, such as Erdős-Rényi random networks and Barabási-Albert scale-free networks. Our mean-field treatment reveals that the cooperation prevalence within the rational players linearly increases with the introduced irrational fraction and explains the non-monotonic dependence in the structural mixing. Our study indicates that the role of irrationality is highly non-trivial and is indispensable to understand the emergence of cooperation in realistic contexts.

1. INTRODUCTION

Cooperation is central to the working of our societies and can be widely observed in biological, economic and social systems [1]. Deciphering its emergence and maintenance is a fundamental scientific question, which has attracted many researchers from different fields and now becomes a highly interdisciplinary field [2–5]. The key question to be addressed is: why do individuals help each other who could potentially be in competition and incur a cost to themselves?

Important progresses have been made with the help of evolutionary game theory [6] by analyzing the stylized social dilemmas such as the prisoner’s dilemma and the public goods game. Several mechanisms are proposed [7] in the past several decades, such as reward and punishment [8], social diversity [9], direct [10] or indirect reciprocity [11], kin [12] or group selection [13, 14], spatial or network reciprocity [15]. In particular, theoretically accounting for the fact that human populations are highly organized and individuals interact repeatedly with their immediate neighbors can support cooperation [15]. The rationale behind this is that a structured neighborhood facilitates the formation of cooperator clusters, which are able to effectively resist the invasion of defectors, as opposed to the well-mixed scenario. The ensuing years have witnessed a wealth of theoretical studies that further confirm this so-called network reciprocity for various population structures, including the multilayer networks [16]. Recently, the dynamical reciprocity as the counterpart mechanism is also proposed that points out that the interaction among co-evolving games could potentially lift the cooperation preference [17].

It is worthy to note that recent human behavioral experiments do not support the network reciprocity, structured populations do not promote cooperation in general [18, 19], or at least some additional conditions are required for cooperation to survive [20]. This unsatisfactory situation implies that some essential elements could be missing in current game-theoretic models, and the experiment-driven modeling approach is needed. Note that, most previous models are developed within the paradigm of Homo economicus [21], where individuals are assumed to be fully rational, and their actions are strictly economic-driven to maximize their earnings.

The paradigm, however, has been challenged by behavioral economists. They revealed that apart from the economic incentive, the psychological and emotional ingredients are also important that influence the decision-making [22–24]. In Ref. [25], experiments show that our humans may use mixed updating strategies, e.g. imitation and spontaneous switching, and their usage frequencies generally change over time. This is in sharp contrast to most theoretical models where a single updating rule is assumed throughout the evolution. Experiment have also found that subjects respond to the cooperation in a reciprocal manner, being more likely to cooperate if, in the previous round, many of their neighbors and themselves did so [26]. This means that the humans don’t not entirely consider their neighbors’ payoffs when making decisions, but also their actions. All these observations suggest that the decision-making in the realistic world could base upon a mixture of updating rules rather than a single one, some of which stem from complex incentives beyond the paradigm of Homo economicus.

Recently, some theoretical works have noticed that the diversity of strategy updating rules, and the mixed rules are considered to investigate their impact on the emergence of cooperation. But these works are conceived with different motivations, e.g. the influence of conformity and innovation on
the evolution of cooperation [27–30]. Till now, there is no systematic work studying the impact of irrationality on cooperation. The key question to be addressed is: what if the irrationality is engaged in the evolution of cooperation, or, what would be expected when rationality meets irrationality?

In our work, we focus on the role of irrationality in the evolution of cooperation. Specifically, we consider a population with a mixture of Fermi and Tit-for-tat (TFT) updating rules [31–33]. While the Fermi rule represents imitation learning, at the heart of many rational actions, players with the TFT rule just repeat what their neighbors did to them in the previous round, being presumably considered as emotion-driven, irrational behaviors. In the structural mixture, each player is endowed with a fixed rule, either Fermi or TFT rule; alternative, the two rules are probabilistically chosen by each player at each step. We find that in both implementations, the cooperation can be considerably enhanced compared to the pure rational population, especially, full cooperation is obtained in a wide range of parameters in the latter case. We also develop a mean-field treatment that correctly reproduces the observations for the former case.

This paper is organized as follows: in Sec. 2, we introduce a mixing model where emotion-driven irrational behaviors are incorporated into a rational population. Numerical simulation results on the 2d square lattice and complex networks are presented in Sec. 3. In Sec. 4, we provide some theoretical analysis and compare the results in the well-mixed population. Finally, we conclude and discuss our work in Sec. 5.

2. MODEL

We study the prisoner’s dilemma (PD) game with \( N \) individuals that are located on an \( L \times L \) square lattice with a periodic boundary condition. PD is a typical pairwise game for many social dilemmas, mutual cooperation brings the reward \( R \), mutual defection yields the punishment \( P \), and mixed encounter gives the cooperators the sucker’s payoff \( S \) yet the temptation \( T \) for the defectors. \( T > R > P > S \) and \( 2R > T + S \) are required for PD. It’s easy to find that defection as the Nash equilibrium is the better choice regardless of the opponent’s selection yet the mutual cooperation is optimal for their collective profit. We adopt the weak version of PD, \( R = 1 \), \( P = S = 0 \), and \( T = b \), where \( 1.0 \leq b \leq 2.0 \) [15].

In the previous practice, each individual is assigned a strategy, either cooperation (C) or defection (D) opposing to all its neighbors, which is the node-based strategy with the strategy set \( S_n = \{C, D\} \). Here, we extend this setup to the edge-based strategy that a player can use different strategies against different neighbors (i.e., along different edges), which is more commonly seen in the real world. In the edge-based setup, the state of an individual \( i \) is characterized by the fraction of cooperation strategy against its neighbors defined as \( s_i = n_i(C)/k_i \), where \( n_i(C) \) is the number of edge strategies for the player \( i \) with C and \( k_i = 4 \) on the 2d square lattice. Thus, \( s_i \in S_e = \{0, 0.25, 0.5, 0.75, 1\} \), which can be interpreted as the cooperation propensity, \( s_i = 1 \) and 0 correspond to the C and D strategies respectively in \( S_n \).

To introduce the irrationality into the evolution of cooperation, we build a model by mixing Fermi and TFT rules, which are regarded as rational and irrational behaviors, respectively. Specifically, the mixture is implemented in two ways — structural mixing (SM) and probabilistically mixing (PM). In the SM implementation, each player is randomly chosen to be an irrational player with a mixing ratio \( \omega \), who will only take the TFT rule to play the game; otherwise, she/he plays as a rational player with the Fermi rule. By contrast, in the PM implementation, players are all identical, they probabilistically adopt TFT and Fermi rules respectively with the probability \( \omega \) and \( 1 - \omega \) in every single step.

At the very start, each edge strategy of every player is randomly assigned with C or D towards their neighbors with an equal chance. An elementary step of the Monte Carlo simulation for the SM implementation is as follows. A player \( i \) is randomly chosen, if player \( i \) is with TFT rule, player \( i \) will adopt the edge strategies that all its neighbor plays against to \( i \). Otherwise, the evolution of

FIG. 1. (Color online) The evolution of cooperation on 2d square lattice with SM, where \( \omega \) is the fraction of TFT players. (a) The prevalence of cooperation \( f_c \) as a function of temptation \( b \); (b) \( f_c \) as a function of \( \omega \); (c) Heat map for the cooperation prevalence \( f_c \) in the \( b-\omega \) parameter space. Other parameters: \( L = 1024 \) for (a,b) and 256 for (c).
player i’s strategy is based upon the Fermi rule as follows. First, one of i’s neighbors j is randomly selected, player i and j respectively acquire their mean payoff $\Pi_{i,j}$ (defined by their total payoffs $\Pi_{i,j}$ divided by their degrees). Next, player i adopts the cooperation propensity of player j with the probability [34, 35]

$$W(s_j \rightarrow s_i) = \frac{1}{1 + \exp[(\Pi_i - \Pi_j)/K]}.$$  \hspace{1cm} (1)

where $K$ is a temperature-like parameter, which can be interpreted as the environment uncertainties in the imitation process, and will be fixed at 0.025 throughout the study. Lastly, the strategies against the four neighbors of player i are pinned down according to the newly adopted propensity $s_j$ if player j’s strategy is successfully imitated, i.e., each of its edge strategy chooses the strategy C according to the probability $s_j$ independently. None of i’s strategy will be updated if the imitation is not successful. Notice that, the imitation in the edge-based Fermi rule is to copy the cooperation propensity not the four strategies, which means the resulting $s_i$ could be not equal to $s_j$, unless the propensity is 0 or 1.

The procedure for PM only differs at the beginning stage of every elementary step. The randomly chosen player i evolves according to the TFT rule with a probability $\omega$, and with $1 - \omega$ updates according to the edge-based Fermi rule. For the TFT case, it copies exactly what their neighbors’ edge strategies towards it; for the latter case, it tries to imitate the cooperation propensity of one random neighbor j with the same way in the above SM.

Note that, the above model simulation follows a typical asynchronous updating procedure. A complete Monte Carlo step (MCS) consists of $N$ elementary steps, meaning that every player updates its state exactly once on average. We compute the cooperation prevalence $f_C = \frac{1}{N} \sum_{i=1}^{N} s_i$ as the primary order parameter, measuring the overall preference in cooperation of the population. The total sampling time is 10000 MCSs and the equilibrium density of cooperation is obtained by averaging over the last 1000 MCSs.

3. RESULTS

A. 2d square lattice

Structural mixing (SM) – Fig. 1 reports the results on the 2d square lattice in the case of structural mixing. Let’s first see the two extreme cases ($\omega = 0, 1$), where the mixing is absent. As can be seen in Fig. 1(a), when the population is entirely consisted of rational players ($\omega = 0$), cooperation can only survive in a narrow region $b < b_c \approx 1.037$, which is exactly the same as previous studies using node-based Fermi rule [35] (see Appendix A). This means that the replacement of node-based strategy with edge-based version per se does not bring any change in $f_c$ for rational players. When all individuals use the TFT rule ($\omega = 1$), the prevalence of cooperation $f_c$ is always approximately 0.5 regardless of the value of temptation $b$, because the initial level of cooperation $f_c(t = 0) \approx 0.5$ is basically reserved according to the TFT rule. As the two types of people are mixed $0 < \omega < 1$, the cooperation levels are all lifted than the pure Fermi rule case, surprisingly there are some mixtures (e.g. $\omega = 0.6, 0.8$) that can leads to $f_c > 0.5$, higher than the expected level in the pure TFT case. Fig. 1(b) show explicitly that there exists optimal mixing ratio $\omega_o$ that leads to the highest level of cooperation $f_c$. As expected, $f_c$ is reduced as the temptation $b$ is increased, whereas the value of $\omega_o$ is shifted to be larger. The dependence of $f_c$ on the two parameters are summarized in the phase diagram shown in Fig. 1(c).

The non-monotonic dependence of cooperation prevalence on $\omega$ is confirmed by the typical time series by fixing $b = 1.2$, see Fig. 2(a). For better understanding, the cooperation prevalence for the two types is respectively shown in Fig. 2(b). As can be seen, when $\omega < \omega_o$ ($\omega_o \approx 0.52$ in this case), the values of $f_c$ for both types increase. But once $\omega > \omega_o$, the players with the Fermi rule are in almost full cooperation state (i.e., $s_i \approx 1$), whereas the cooperation prevalence of TFT players decreases from the peak value to the expected level 0.5.

![FIG. 2. (Color online) (a) Time series on 2d square lattice with SM. (b) The prevalence of cooperation $f_c$ versus $\omega$ computed separately for TFT and Fermi players, together with the whole population for comparison. The black dashed line is the fitting line. Parameters: $L = 1024, b = 1.2$.](image-url)
Once $s_i = 1$ for Fermi players, the strategies between Fermi and TFT players are also all cooperation, the defection then only comes from the TFT players. By estimation, about half TFT-TFT interaction edges finally will be in the deadlock of mutual defection D-D state, while the other half in C-C state. Therefore,

$$f_c \approx 1 - \omega^2/2, \tag{2}$$

the approximation for the overall cooperation prevalence when $\omega > \omega_0$, which fits very well with the numerical results in Fig. 2(b).

To develop the intuition of why mixing promotes cooperation, some typical spatial patterns are provided in Fig. 3, where the states for all players together with TFT- and Fermi-players are respectively shown in different columns. A critical observation is the difference in cooperation prevalence between TFT- and Fermi-players. By combination, all interactions within the mixing populations can be classified into three types:

i) **Fermi-Fermi interactions**: when only Fermi-players are present, they compute, compare and imitate, finally the rationality drives the population to the Nash equilibrium point, the full defection solution, only the network reciprocity may help (but not the case for $b = 1.2$ here).

ii) **TFT-TFT interactions**: since TFT players just repeat what their opponents have done to them, the edge strategies will be frozen as either C-C or D-D pairs in the asynchronous updating, irrespective of the parameter $b$. The value of $f_c \approx f_c(t = 0)$ for random initial conditions, and $f_c(t = 0) \approx 0.5$ in our study.

iii) **Fermi-TFT interactions**: due to the random initialization, TFT-players have diverse payoffs, and those of high cooperation propensity $s_i$ generally have higher payoffs than those of small $s_i$, which are more likely to be imitated by their Fermi-neighbors (see e.g. white sites in Fig. 3(c)). This in turn increases their neighbors’ payoff and improve their values in $s_i$ through both type i) and ii) interactions, and drives the overall cooperation prevalence to rise in the end.

This means that if the fraction of Fermi-players is too large (i.e., $\omega \to 0$), Fermi-Fermi interactions deteriorate the cooperation; likewise, if TFT-players are too much ($\omega \to 1$), $f_c \to 0.5$ due to the outcome of TFT-TFT interactions. Only when the two fractions are comparable, Fermi-TFT interactions then come into play that improve cooperation; especially $\omega \approx \omega_0$, a balance point is reached to reduce both type i) and ii) interactions effectively. Otherwise, when $\omega < \omega_0$, Fermi-players still form sizeable clusters that Fermi-Fermi interactions deteriorate the cooperation (Fig. 3(a)). On the contrary, when $\omega > \omega_0$, even though all Fermi-TFT edge strategies are still largely within the C-C pairs (Fig. 3(i)), more TFT-TFT interactions yield more defection (Fig. 3(h)), reducing the overall cooperation. This explains the non-monotonic dependence of $f_c$ on the mixing ratio $\omega$ and existence of the optimal value, shown in Fig. 1(b).

**Probabilistic mixing (PM)** – The results of probabilistic mixing are shown in Fig. 4, where $\omega$ is now interpreted as the probability to behave irrationally (with the TFT rule) at every single step. Fig. 4(a) provides the cooperation prevalence $f_c$ as the function of the temptation $b$ for a couple of $\omega$, where $f_c$ increases and $f_c = 1$ for $\omega = 0.4, 0.6, 0.8$, irrespective
gives the time needed when starting from random to the TFT rule and to the Fermi rule otherwise at every single step. (a) The prevalence of cooperation (b); The inset are several time series for \( \omega \) being very close to 1, with fixed \( b = 1.2 \). (c) Phase diagram for \( f_c \) in \( b - \omega \) parameter space. Other parameters: \( L = 1024 \) for (a,b) and 256 for (c).

To better understand the mechanism behind, we estimate the changes in the cooperation propensity per step, respectively for Fermi- and TFT-rule, shown in Fig. 6. Specifically, \( \delta f_c \) is the average change in \( s_i \) for every single update over an MCS. It shows that the main contribution for the cooperation promotion comes from the actions based on the Fermi-rule, while the changes from TFT-rule are much less pronounced. When \( \omega \) is small (see Fig. 6(a)), \( \delta f_c < 0 \), that the cooperation prevalence \( f_c \) is low in this case; but if \( \omega \) becomes larger, Fermi-players actively entrain the cooperation level to a pretty high level (Fig. 6(b-d)).

### B. Complex networks

To check the robustness of our observations, we also carry out numerical experiments on two complex networks. Specifically, we adopt Erdős-Rényi (ER) random networks\cite{36} and Barabási-Albert (BA) scale-free networks\cite{37}, respectively represent homogeneous and heterogeneous networks in the real world. As is well-known, the degrees of ER networks satisfy poisson distribution, whereas the degree distribution of BA networks is a power-law with an exponent of -3. For the ease of comparison, both network sizes are \( N = 2^{20} \) with the same average degree \( \langle k \rangle = 4 \).

The results of structural mixing are shown in Fig. 7, where the TFT-players are randomly chosen. Fig. 7(a,b) show the cooperation prevalence \( f_c \) as a function of temptation \( b \) for different \( \omega \) values in ER and BA networks, respectively. When \( \omega = 0 \) the results are consistent with the previous study. Note that, because the evolution is based upon the average payoffs rather than the total payoffs, the value of \( f_c \) for BA networks is not higher than that of ER case as might be expected \cite{38,39}. In both cases, the inclusion of TFT-players promotes cooperation, and increasing the temptation \( b \) generally decreases \( f_c \) except for the case of \( \omega \rightarrow 1 \). Fig 7(c,d) show the dependence on the probability \( \omega \), and the existence of optimal \( \omega_o \) is clearly seen that yields the best cooperation. Compare to the lattice case (Fig. 1(a,b)), complex network topologies do not alter the
FIG. 6. (Color online) The average value of $\delta f_c = \langle s_i(\text{new}) - s_i(\text{old}) \rangle$ per MCS with PM for different $\omega$: (a) $w = 0.1$, (b) $w = 0.3$, (c) $w = 0.5$, and (d) $w = 0.7$. Parameters: $L = 1024$ and $b = 1.2$.

FIG. 7. (Color online) The dependence of cooperation prevalence $f_c$ on two parameters with SM on ER networks (a,c) and BA networks (b,d). Parameters: $N = 2^{20}$, the average degree $\langle k \rangle = 4$.

dependence on the mixing qualitatively.

To further investigate the impact of the network heterogeneity, the following three ways are adopted to select TFT-players:

i) *Neutral correlation* — $\omega N$ players are randomly chosen irrespective of their degrees; this is the way we used in Fig. 7.

ii) *Positive correlation* — nodes with larger degrees are chosen to use the TFT rule;

iii) *Negative correlation* — nodes with smaller degrees are selected.
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For type i) interactions, which correspond to the shown in Fig. 8. We see that in both cases the positive correlation shifts the optimal ratio \( \omega_n \) to be smaller, and in the opposite direction for the negative correlations. Besides, this shift is more pronounced in BA networks. The reason lies in the approximate correspondence between the cooperation prevalence and the number of interactions of Fermi-TFT type. When hub nodes are occupied by TFT-players, a smaller fraction is just able to have the most Fermi-TFT interactions. By contrast, when the TFT-players are periphery nodes, a more fraction is needed.

The probabilistic mixing is also implemented on both networks, the results are quite similar to the lattice case (Fig. 4), and are not shown here anymore.

4. THEORETICAL ANALYSIS

To understand why the mixture of TFT- and Fermi-players is able to promote the cooperation prevalence, we provide a mean-field analysis for the structural mixing implementation. For simplicity, let’s consider a well-mixed population, numerical results are shown in Fig. 9, where qualitatively the same observations are seen compared to the above findings (e.g. Fig. 1(b) and Fig. 7(c,d)).

The four types of edge strategies are denoted as \( C^F, D^F, C^T, D^T \), where \( F \) (Fermi) and \( T \) (TFT) represent the type of player holding that strategy. All interaction pairs are summarized in the following fraction matrix:

\[
\begin{bmatrix}
C^T & D^T & C^F & D^F \\
C^T & f_{11} & f_{12} & f_{13} & f_{14} \\
D^T & f_{21} & f_{22} & f_{23} & f_{24} \\
C^F & f_{31} & f_{32} & f_{33} & f_{34} \\
D^F & f_{41} & f_{42} & f_{43} & f_{44}
\end{bmatrix}.
\]

Each item represents the fraction of the associated two edge strategies in the population, which can also be interpreted as the probability of finding that edge. For example, \( f_{14} \) is the probability of finding the links that connect a TFT-player with a Fermi-player holding the strategy D. Since these probabilities are irrespective of the pair order, therefore

\[
f_{ij} = f_{ji}, \quad (i, j = 1, 2, 3, 4).
\]

The sum of each row is the overall strategy density respectively for TFT and Fermi rule, defined as below,

\[
f_{11} + f_{12} + f_{13} + f_{14} = f_{CT},
\]

\[
f_{21} + f_{22} + f_{23} + f_{24} = f_{DT},
\]

\[
f_{31} + f_{32} + f_{33} + f_{34} = f_{CF},
\]

\[
f_{41} + f_{42} + f_{43} + f_{44} = f_{DF}.
\]

The four fractions satisfy the following relations in the structural mixing,

\[
f_{CT} + f_{DF} = \omega,
\]

\[
f_{CF} + f_{DF} = 1 - \omega.
\]

Now let’s consider all three type of interactions listed in Sec. 3A. For type i) interactions, which correspond to the Fermi-Fermi pairs, i.e., the lower right corner of the matrix, the four items can be expressed in the mean-field sense as

\[
f_{33} = f_{CF}^2, f_{44} = f_{DF}^2,
\]

\[
f_{34} = f_{43} = f_{CF} f_{DF}.
\]
For type ii) interactions, where TFT-player encounters TFT-player, these edge strategies will finally evolve into C-C and D-D pairs with an equal chance, meaning

\[ f_{11} = f_{22} = \omega^2/2, \quad (13) \]

\[ f_{12} = f_{21} = 0. \quad (14) \]

The cooperation of TFT-players \( f_{CF} \) comes from two contributions, one is from the TFT-TFT interactions \( f_{11} \); The other is from type iii) interactions, where TFT-players copy exactly what Fermi-players did to them. This then leads to

\[ f_{CF} = \omega f_{CF} + \omega^2/2. \quad (15) \]

Inserting Eq. (11-15) into Eq. (5, 7), we have

\[ f_{13} + f_{14} = \omega f_{CF}, \quad (16) \]

\[ f_{31} + f_{32} = \omega f_{CF}. \quad (17) \]

With Eq. (4), we find

\[ f_{14} = f_{32} = f_{41} = f_{23} = \omega f_{CF} - f_{31}. \quad (18) \]

With these relationships, the matrix (3) can be rewritten as follows

\[
\begin{pmatrix}
\frac{\omega^2}{2} & 0 & f_{31} & f_{31} \\
0 & \frac{\omega^2}{2} & \omega f_{CF} - f_{31} & \omega f_{CF} - f_{31} \\
f_{31} & \omega f_{CF} - f_{31} & f_{DF} - \omega f_{CF} + f_{31} & f_{DF} - \omega f_{CF} + f_{31} \\
\omega f_{CF} - f_{31} & \omega f_{CF} - f_{31} & f_{DF} - \omega f_{CF} + f_{31} & f_{DF} - \omega f_{CF} + f_{31}
\end{pmatrix}
\]

where \( f_{DF} = 1 - \omega - f_{CF} \) from Eq. (10).

Through numerical simulations, we identify the following relationships when \( \omega \leq \omega_o \) (before the absorbing state is reached \( s_i < 1 \) for the Fermi-players)

\[ \Pi_{CF} = \Pi_{DF} = \Pi_F \quad (20) \]

where the payoff of \( DT - D^T \) is excluded in \( \Pi_F \). Specifically,

\[
\frac{f_{31}R + f_{32}S + f_{33}R + f_{34}S}{f_{CF}} = \frac{f_{41}T + f_{42}P + f_{43}T + f_{44}P}{f_{DF}}, \quad (22)
\]

\[
\frac{f_{31}R + f_{32}S + f_{33}R + f_{34}S + f_{41}T + f_{42}P + f_{43}T + f_{44}P}{1 - \omega - f_{22}} = \frac{f_{11}R + f_{12}S + f_{13}R + f_{14}S + f_{21}T + f_{23}T + f_{24}P}{\omega}. \quad (23)
\]

By combining the elements in matrix (19) and solving Eq. (22), Eq. (23), we get

\[ f_{CF} = \frac{\omega}{\sqrt{2(b-1)}}. \quad (24) \]

Insert the above expression into Eq. (15), we obtain \( f_{CT} \). Finally, we add them up \( f_C = f_{CT} + f_{CF} \), the expression of overall cooperation prevalence is

\[ f_C = \frac{(1 + \omega)\omega}{\sqrt{2(b-1)}} + \omega^2. \quad (25) \]

Together with Eq. (24), our analysis show that the fraction of cooperation for Fermi-players \( f_{CF} \) increases linearly with the fraction of TFT-players. Without TFT-players (\( \omega = 0 \)), the cooperation disappears, as is well-known in previous studies [2, 40]. The overall cooperation prevalence \( f_C \) is quadratic function of \( \omega \).

Since \( f_{CF} \leq 1 - \omega \) given by the Eq. (10), which leads to

\[ \omega \leq \frac{\sqrt{2(b-1)}}{\sqrt{2(b-1)}} = \omega_o. \quad (26) \]

When \( \omega = \omega_o \), full cooperation is reached for Fermi-players, also with the least D-D pairs for type ii) interactions. When \( \omega > \omega_o \), the overall cooperation prevalence \( f_C \) satisfies Eq. (2).

The comparison between theoretical analysis results (Eq. (25) for \( \omega \leq \omega_o \), and Eq. (2) for \( \omega > \omega_o \)) and numerical results is shown in Fig. 9, confirming the correctness of the above derivation. The inset shows the dependence of \( \omega_o \) on the parameter \( b \), the value shifts to be larger with increasing \( b \), in line with the numerical observations.

5. CONCLUSION AND DISCUSSION

In summary, motivated by the diversity of human incentives in the realistic evolution of human cooperation, we relax the assumption of Homo economicus, and allow for irrational behaviors. We build a model with a mixture of Fermi and TFT rules, respectively corresponding to rational and irrational actions. In the first implementation where the individuals use
one fixed rule, we find the mixing can promote cooperation, and there exists an optimal amount of TFT individuals that bring the highest level of cooperation. In the second implementation, individuals probabilistically behave in the either way, the full cooperation is always achieved if the probability of using the TFT rule is beyond a critical value in the mixing population. These findings are verified on two complex networks, where the degree heterogeneity only changes the results quantitatively. Finally, we derive a semi-analytic mean-field treatment for the first implementation, give the dependence of cooperation prevalence on the mixing ratio and the game parameters, explicitly revealing how the irrationality promotes cooperation.

One might argue that the Fermi-rule cannot be completely regarded as rational due to the presence of uncertainty $K$, which sometimes is interpreted as the bounded irrationality by some researchers. For the robustness check, we minimize the uncertainty by taking $K \rightarrow 0$ in Eq. (1) in the Fermi rule to maximize the degree of rationality and our results remain quantitatively changed. Actually, when the Fermi rule is replaced with the deterministic follow-the-best rule [15], the above findings remain unchanged qualitatively. Other model variants like the replacement of asynchronous updating with the synchronous scheme also show similar observations. An equally arguable point is the interpretation of the TFT rule as being irrational. As the winning strategy in Robert Axelrod’ two tournaments, TFT is also generally considered as being strategic for its clear, nice, and provocable properties [33] and is suggested to be the cooperation mechanism in some animal communities. Our interpretation here is to emphasize that the incentive behind the action is not to maximize its profit as the economic man assumes, but to replicate its opponent’s previous action. From the perspective of the economic man, this is irrational. Note that, the TFT rule used in our work does not follow the strict version of “tit-for-tat” strategy [33] that a cooperative action is taken in the first round, but a random initial condition. The subsequent reciprocation/retaliation is kept.

Although the extension of the Fermi updating rule [34, 35] from the node-based strategy to the edge-based version does not alter the cooperation prevalence at all, this extension is necessary in our study for the need of mixture since the TFT is also via an edge-based updating scheme. Furthermore, the edge-based strategy scheme seems more reasonable in most realistic scenarios, since individuals treat their different neighbors potentially in different strategies, not a uniform strategy against all their neighbors as most current game-theoretical models assume.

Although there have been many critics of the paradigm of Homo economicus, the idea that an economic man behaving in his own self-interest still remains a reasonable approximation for theoretical research. Our work indicates that the incorporation of irrationality seems indispensable when aiming to understand the cooperation evolution of realistic humankind. But, how the irrationality impacts the evolution of cooperation depends very much on the specific scenario under study and the incentives behind, the TFT rule studied in our work is only one of the many possible choices. Our work calls for experiment-driven studies beyond the paradigm of Homo economicus that finally help understand the emergence of cooperation in real scenarios.

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Appendix A: The comparison between node-based and edge-based Fermi rule

When the Fermi rule is extended from node-based strategy to edge-based version, it’s natural to ask whether this change per se brings any alteration in the cooperation prevalence. With this aim, we make some numerical simulations on the 2d square lattice for populations with the pure edge-base Fermi updating rule, and compare the results to the case when they use pure node-based Fermi rule.

The time evolution of the spatial patterns with the edge-based Fermi rule at different times are shown in the Fig. 10, where the cooperation propensity $s_i$ is color-coded. An interesting observation is that, all partial cooperation propensity states (i.e., $s_i = 0.25, 0.5, 0.75$) disappear, they are unstable in the long run, leaving only pure cooperation ($s_i = 1$) or pure defection state ($s_i = 0$). The reason lie in the imitation process that even the partial cooperation propensity is imitated, the resulting state still could be different, while the pure cooperation/defection state is imitated with a hundred percent probability. Thus, only the two propensities $s_i = 0$ and 1 survives in the long run.

In addition, we compare the cooperation prevalence between edge-based Fermi rule and node-based Fermi rule as the function of temptations $b$ (Fig. 11), find that the two cooperation prevalences $f_i$ are exactly the same, with $b_i = 1.038$ [35]. Put together, these observations imply that the evolution with the edge-based Fermi rule is almost the same as the node-based version, since the partial cooperation states all vanish in the end.

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[1] J. Maynard Smith and E. Szathm´ ary , The Major Transitions in Evolution (Oxford University Press, 1995).
[2] J. Maynard Smith, Evolution and the Theory of Games (Cambridge University Press, 1982).
[3] H. Gintis, Game Theory Evolving (Princeton University Press, 2000).
FIG. 10. (Color online) The spatial pattern evolution on 2d square lattice with the edge-based Fermi updating rule at different moments: (a) $t = 1$, (b) $t = 10$, (c) $t = 20$, (d) $t = 100$. The colors (black, crimson, orange, yellow, white) represent the cooperation propensity of players $s_i = (0, 0.25, 0.5, 0.75, 1)$. Other parameters: $N = 128 \times 128$, $b = 1.02$.

FIG. 11. (Color online) The comparison of cooperation prevalence $f_c$ between the edge-based and node-based Fermi updating rule on the 2d square lattice. Parameter: $L = 1024$.

[4] D. G. Rand and M. A. Nowak, Trends in cognitive sciences 17, 413 (2013).
[5] M. Perc, J. J. Jordan, D. G. Rand, Z. Wang, S. Boccaletti, and A. Szolnoki, Physics Reports 687, 1 (2017).
[6] M. A. Nowak and S. Karl, Science 303, 793 (2004).
[7] M. A. Nowak, Science 314, 1560 (2006).