Optimal robust control with cooperative game theory for lower limb exoskeleton robot

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Abstract For achieving trajectory tracking issue of the lower limb exoskeleton robot, a novel optimal robust control with cooperative game theory is proposed. The uncertainties are considered (possible time-varying, bounded and fast), and the fuzzy set theory is creatively adopted to describe the boundary. From the view of analytical mechanics, the trajectory tracking is treated as the constraints control problem, including holonomic and nonholonomic constraints, which need to satisfy the conditions of human motion. Combining the robust control and optimal design, optimal robust control is formulated to satisfy both performances guaranteeing and optimal. The Pareto optimal solution is obtained to guarantee the minimum control cost. In the simulation, the adaptive robust control is chosen as a comparison. The existence of Pareto optimality and the effectiveness of optimal robust control have been verified via simulation results.

Keywords Lower limb · Robust control · Fuzzy set theory · Cooperative game theory · Pareto optimality

1 Introduction

The aging problem can no longer be ignored, which has brought a series of social issues. The strokes have struck numerous elder people, which is a predominant source of lower limb paralysis [1]. Medical theory and clinical medicine have proved that rehabilitation training process significantly effects on relieving the symptoms [2]. However, traditional rehabilitation training wastes lots of manpower and resources, and the training effect is poorer than that of lower limb exoskeleton robot (LLER) [3]. Depending on the degree of paralysis, the motion modes of LLER are primarily classified as active or passive mode [4]. In the passive mode, the LLER needs to follow a certain desired trajectory, which is usually derived from clinical gait database. Passive training is essential for further rehabilitation, and the trajectory tracking issue plays an indispensable role, which is involved in this article.

In recent years, lots of excellent efforts have been carried out, such as robust control [5], hierarchical control [6], adaptive control [7], fault-tolerant control [8], neural network control [9] and so on. Campbell et al. [10] develop the assistance-as-needed control to...
help the patient with rehabilitation which means the robot will provide auxiliary torque only when the patient deviates from the desired gait. In order to overcome the unstable phenomenon, Sun et al. [11] construct a reduced adaptive fuzzy control with compensation and verify its effectiveness on LLER system. Yang et al. [12] describe a repetitive learning control of LLER in the frame of backstepping control and demonstrate its convergence using Lyapunov method. An active disturbance rejection control is proposed to compensate for the disturbance and converge quickly with the help of fast terminal sliding mode control [13]. However, these control methods mentioned above are not available for nonholonomic constraints, which are common in reality. The optimal robust control (ORC) we propose is applicable not only to holonomic, but also to nonholonomic constraints. The reason is that we design the controller based on Udwadia–Kalaba (U-K) theory.

Professors Udwadia and Kalaba proposed a novel equation of motion from the view of analytical mechanics in 1992, called U-K theory [14, 15]. U-K theory applies to both holonomic and nonholonomic constraints. In addition, Lagrange multipliers and quasi-variables are not required in U-K theory, which can lead to less demanding computations [16]. These are the reasons why we choose U-K theory. Liu et al. [17] obtain the dynamic equation of constrained systems through U-K theory and formulate a systematic control method, which applies to not only fully actuated but also under-actuated and redundantly actuated systems. Sun et al. [18] apply the U-K theory to solve tracking control problem of mobile robots. Sun et al. [19] design an adaptive robust control using U-K theory to carry out the dual avoidance–arrival problem. Cho et al. [20] develop new continuous sliding mode controllers for multi-input multi-output systems and conduct two numerical examples to demonstrate the accuracy and robustness of them. Mylapilli and Udwadia extend the U-K theory to the control of three-dimensional incompressible hyperelastic beams [21]. Furthermore, U-K theory can also be applied to satellite formation flight control to develop a nonlinear controller without making any linearization [22]. However, the uncertainties are not considered in U-K theory, which is a limitation on the way of development.

In fact, the uncertainties cannot be negligible in trajectory tracking control. The description and management of uncertainty are similarly important, and many control methods have done related work, such as torque control [23], self-adjusting leakage-type adaptive control [24], robust bounded control with inequality constraints [25] and terminal sliding mode control [26]. Unfortunately, the bounds on the uncertainties involved in the control methods presented above are all deterministic and constant, which is not reasonable in reality. So, a better method to describe and manage the uncertainties is necessary.

Probability theory has captured the attention of many scholars on describing the uncertainties by adopting lots of observed data, and numerous achievements have been obtained [27]. Stochastic dynamic system method combined the probability theory and dynamic model [28] and has a well performance in tracking, which also needs lots of observed data. However, not all data are easily available for observation or cannot be accurately repeatable in large amount, such as flood data. In 1965, Zadeh [29] firstly proposed the fuzzy set theory, which has the ability to describe the degree to which the event occurs rather than frequency. Zhen et al. [30–32] employ fuzzy set theory to describe the uncertainties in the dynamic model and call it fuzzy dynamic system (FDS), which is not based on IF–THEN rules. In order to avoid the shortcomings of lots of probability theory, the uncertainties in LLER are described by fuzzy set theory. Then, based on U-K theory and FDS, a novel robust control with two adjustable parameters is proposed. Different parameters can cause different system’s control cost. How to select an optimal pair of two adjustable parameters is a question worth considering.

Remarkable studies have been conducted about optimal control of multi-parameters in recent years, such as ant colony [33], particle swarm optimization [34] and genetic algorithm [35]. Unfortunately, the realization of FDS is difficult and inconvenient. [36–38] have researched the optimization of FDS. But their research object is single-objective optimization. Referring to previous work of Chen et al. [39, 40], we employ the cooperative game theory to solve the problem of optimal parameter selection, which was introduced by Pareto. The cooperative game theory involves two players, whose goal is to find the Pareto optimal solution, in order to minimize both of their cost. More important, for a better interpretation about our work, it should be noticed that the FDS is quite
different with the fuzzy logic control, since the latter is based on IF–THEN rules.

In conclusion, our main contributions are listed as follows:

1. The dynamic model of LLER is formulated, and the fuzzy set theory is employed to describe the unknown boundary of uncertainties in LLER, which is within a threshold. The trajectory tracking problem is treated as constraints control problem, where the constraints can be holonomic and nonholonomic.
2. Based on the information of the uncertainties bounds and U-K theory, a robust control method is designed with adaptive law and two adjustable parameters. The control method is deterministic that the LLER system is guaranteed to meet the performance indicators for the LLER system.
3. An optimal design associated with cooperative game theory is formulated. The fuzzy performance indicators for $k_1$ and $k_2$ are constructed. Moreover, the Pareto optimal solution obtained can select the optimal solutions of $k_1$ and $k_2$ and minimize the system’s control cost.

The rest of this article is arranged as follows: Section 2 introduces the preliminary knowledge about fuzzy set theory, U-K theory and cooperative game theory. Robust control with two adjustable parameters is designed in Sect. 3. Optimal control with cooperative game theory is constructed in Sect. 4. Section 5 conducts the simulation experiments to verify the existence of Pareto optimality and the effectiveness of optimal robust control. Section 6 is the summary of the full paper.

2 Preliminary knowledge

2.1 Fuzzy set theory

Some basic knowledge about fuzzy set theory needs to be reviewed for further development (see [41, 42]).

**Fuzzy number** [41]: Let $G$ be a fuzzy set in $\mathbb{R}$, the set of real numbers. $G$ is named a fuzzy number if: (1) $G$ is normal and convex, (2) the support of $G$ is bounded and (3) all $\alpha$-cuts are closed intervals in $\mathbb{R}$. Moreover, assume the universe of discourse of a fuzzy number to be its 0-cut.

**Fuzzy arithmetic** [41]: Let $G$ and $H$ be two fuzzy numbers and $G_x = \left[ g_x^-, g_x^+ \right]$, $H_x = \left[ h_x^-, h_x^+ \right]$ be their $\alpha$-cuts, $\alpha \in [0, 1]$. Moreover, the four fundamental rules of $G$ and $H$ are listed as follows:

$$ (G + H)_x = \left[ g_x^- + h_x^-, g_x^+ + h_x^+ \right], $$

$$ (G - H)_x = \left[ \min(g_x^- - h_x^-, g_x^+ - h_x^+), \max(g_x^- - h_x^-, g_x^+ - h_x^+) \right], $$

$$ (G/H)_x = \left[ \min(g_x^- h_x^-, g_x^+ h_x^+, g_x^- h_x^+, g_x^+ h_x^-), \max(g_x^- h_x^-, g_x^+ h_x^+, g_x^- h_x^+, g_x^+ h_x^-) \right], $$

$$ (G/H)_x = \left[ \min(g_x^- h_x^-, g_x^+ h_x^+, g_x^- h_x^+, g_x^+ h_x^-), \max(g_x^- h_x^-, g_x^+ h_x^+, g_x^- h_x^+, g_x^+ h_x^-) \right]. $$

2.2 Fuzzy arithmetic

**Decomposition theorem** [41]: Define a fuzzy set $\tilde{H}_x$ in $U$ whose membership function is $\mu_{\tilde{H}_x} = 2I_{\tilde{H}_x}$, where $I_{\tilde{H}_x} = 1$ if $x \in \tilde{H}_x$ and $I_{\tilde{H}_x} = 0$ if $x \in U - \tilde{H}_x$. Then, the fuzzy set $H$ is obtained as

$$ H = \bigcup_{\alpha \in [0, 1]} \tilde{H}_x $$

where $\bigcup$ is the union of the fuzzy sets (that is, sup over $\alpha \in [0, 1]$). On the ground of decomposition theorem, we can formulate the membership function of the resulting fuzzy number with the operation between the two fuzzy numbers.

**D-Operation** [42]: Assume a fuzzy set $N = \{(\epsilon, \mu_N(\epsilon)) | \epsilon \in N\}$

For any function $f: N \to \mathbb{R}$, the D-operation $D[f(\nu)]$ is formulated by

$$ D[f(\epsilon)] = \frac{\int_{N} f(\epsilon) \mu_N(\epsilon) d\epsilon}{\int_{N} \mu_N(\epsilon) d\epsilon} $$

To some extent, the D-operation $D[f(\epsilon)]$ takes an average value of $f(\epsilon)$ over $\mu_N(\epsilon)$. In the special case that $f(\epsilon) = \epsilon$, this is simplified to the commonly known center-of-gravity defuzzification method [38]. If $N$ is crisp [i.e., $\mu_N(\epsilon) = 1$] for all $\nu \in N$, then $D[f(\epsilon)] = f(\epsilon)$.

**Lemma 1** Ref. [41]: For any crisp constant $a \in \mathbb{R}$, $D[af(\epsilon)] = aD[f(\epsilon)]$. 

$$ D[af(\epsilon)] = aD[f(\epsilon)]. $$
2.2 Udwadia–Kalaba theory

Establish the dynamic model of a mechanical system using Newtonian or Lagrangian method, which can be formulated as

\[ M \ddot{x}(t) = Q(x(t), \dot{x}(t), t) + Z\tau(t) \]  

where \( t \in \mathbb{R} \) is the time, \( x(t), \dot{x}(t), \text{ and } \ddot{x}(t) \in \mathbb{R}^n \) (\( n \) denotes dimension) stand for generalized coordinates, velocity and acceleration, separately. \( \delta(t) \in \Sigma \) denotes the uncertain parameters, which may be fast and time-varying. \( \Sigma \) represents the boundary of the uncertainties. \( M \in \mathbb{R}^{n \times n} > 0 \) denotes the inertial mass matrix; \( Q \in \mathbb{R}^n \) denotes the known force imposed on the control system. \( \tau \in \mathbb{R}^m (m \leq n) \) stands for the control input, and \( Z \) represents the corresponding coefficient and its value depends on whether the mechanical system is a fully actuated, over-actuated or under-actuated system. When the system is not constrained, \( \tau = 0 \). Furthermore, \( M(\cdot) \) and \( Q(\cdot) \) are of appropriate dimensions, continuous and Lebesgue measurable.

The coordinates \( x(t) \) can be chosen according to the specific situation, which do not necessarily to be generalized coordinates. Consider the mechanical system is subjected to the following constraints:

\[ \sum_{i=1}^{n} B_{ii}(x(t)) = d_i(x(t)) \]  

where \( B_{ii}(\cdot) \) and \( d_i(\cdot) \) are both \( C^1 \) and \( 1 \leq m \leq n \). \( i \) represents the \( i \)th element of each row of \( B_{ii}(\cdot) \) and \( d_i(\cdot) \) can denote the system constraints. Determining the first-order derivative of (10) with respect to time \( t \), we can obtain

\[ \sum_{i=1}^{n} A_{ii}(x(t)) \dot{x} = c_i(x(t)) \]  

with

\[ A_{ii}(x(t)) = \sum_{k=1}^{n} \frac{\partial B_{ii}(x(t))}{\partial x_k} - \sum_{k=1}^{n} \frac{\partial d_i(x(t))}{\partial x_k} \]

\[ c_i(x(t)) = \frac{\partial d_i(x(t))}{\partial t} - \frac{\partial B_{ii}(x(t))}{\partial t} \]

where the role of \( k \) is to iterate over \( x_n(x_1, \cdots, x_n) \).

Rewrite the constraints (11) as a matrix form:

\[ A(x(t)) \dot{x} = c(x(t)) \]  

which contains the first derivative of \( x \), where \( A = [A_{ii}]_{m \times n} \) and \( c = [c_1, c_2, \cdots, c_m]^T \).

The second derivative of (10) with respect to time is obtained:
\[
\sum_{i=1}^{n} \frac{d}{dt} A_{li}(x,t) \dot{x}_i + \sum_{i=1}^{n} A_{li}(x,t) \ddot{x}_i = \frac{d}{dt} c_l(x,t) \tag{15}
\]

where
\[
\frac{d}{dt} A_{li}(x,t) = \sum_{k=1}^{n} \frac{\partial A_{li}(x,t)}{\partial x_k} \dot{x}_k + \frac{\partial A_{li}(x,t)}{\partial x_k} \tag{16}
\]

and
\[
\frac{d}{dt} c_l(x,t) = \sum_{k=1}^{n} \frac{\partial c_l(x,t)}{\partial x_k} \dot{x}_k + \frac{\partial c_l(x,t)}{\partial x_k} \tag{17}
\]

(15) can be rewritten as the matrix form:
\[
A(x,t)\ddot{x} = b(x,\dot{x},t) \tag{18}
\]

where \( A = [A_{li}]_{m \times n} \) and \( b = [b_1, b_2, \ldots, b_m]^T \). The advantage of using the second-order constraint lies in the fact that it is linear in the acceleration. The details of (10)–(18) can be found in [44, 45]. Almost all control problems can be written in the second-order constrained form, including stability, trajectory tracking and optimal control [46].

**Assumption 1** The constraint in (18) is continuous and full rank, then \( A(x,t)\ddot{x} = b(x,\dot{x},t) \) is invertible.

**Remark 1** For a given constraint, \( A \) and \( b \) are determined. There is at least one solution for \( \ddot{x} \) in (18), then (18) can be seen as continuous.

**Theorem 1 (U-K theory):** Assumption 1 is satisfied, an unconstrained mechanical system is subjected to constraints as shown in (11), then the constraint force \( Q^c \) is shown as [47]
\[
Q^c = M^{1/2} \left( AM^{-1/2} \right)^+ (b - AM^{-1}Q) \tag{19}
\]

As mentioned above, U-K theory is applicable to both holonomic and nonholonomic constraints. For a better explanation, the flowchart of U-K theory is shown in Fig. 1.

The mechanical system considered in U-K theory is free of uncertainties or the uncertainties are known, in this case \( \tau = Q^c \). Nominal control, also called Udwa-dia control, can be designed based on U-K theory. However, the uncertainties are inevitable and undetermined in practical applications.

**2.3 Cooperative game theory**

This subsection reviews some fundamental theories of cooperative game theory as follows.

We shall consider a game with a number of players. Each player has a cost function that conforms to the rules of the game, and the cost function is affected by the decisions of all players, which is determined by the rules. Assume \( Y \) players are involved in the game, and the rules impose the following mappings:
\[
J_i(\cdot) : \prod_{i=1}^{Y} \mathcal{H}_i \to \mathbb{R} \quad i = 1, 2, \ldots, Y \tag{20}
\]

where \( J_i(\cdot) \) and \( \mathcal{H}_i \) are, respectively, the cost function and decision set for player \( i \).

Generally, for each player, the least cost in the game is his goal. Consequently, the best decision to all players is \( h^* \in \prod_{i=1}^{Y} \mathcal{H}_i \) that meets
which indicates that the Y-tuple \( h^* \) is an ideal decision and ensures that everyone’s cost is minimized. Unfortunately, the satisfactory decision is not available in the reality and how to reach the optimal decision is a thorny problem.

According to economist Pareto, if the players collaborate with each other, that is if the behavior of each player is interdependent and each player communicates with each other to obtain his minimum cost, the game is called a cooperative game. Moreover, Pareto formulates the optimal decision as Pareto optimal solution: A decision is Pareto optimal solution if and only if for every \( h \in \prod_{i=1}^{Y} \hat{\mathcal{H}}_i \) either
\[
J_i(h) = J_i(h^*) \quad \forall i \in \{1, 2, \cdots, Y\}
\]
or there is at least one \( i \in \{1, 2, \cdots, Y\} \) such that
\[
J_i(h) > J_i(h^*)
\]

Generally speaking, the Pareto optimal decision may be more than one, and the set of cost outcomes \( \{J_1, J_2, \cdots, J_Y\} \) under different Pareto optimal decisions is called the Pareto frontier.

Based on Definition 1, the following lemma is proposed to find the Pareto optimal solution.

**Lemma 2** [48]: Decision Y-tuple \( h^* \in \prod_{i=1}^{Y} \hat{\mathcal{H}}_i \) is Pareto optimal if there exists \( \alpha \in \mathbb{R} \) with \( i > 0, i = 1, 2, \cdots, Y, \) and \( \sum_{i=1}^{Y} i = 1 \), such that
\[
J(h^*) \leq J(h) \forall h \in \prod_{i=1}^{Y} \hat{\mathcal{H}}_i
\]

where \( J(h) = \sum_{i=1}^{Y} J_i(h) \).

**Remark 2** From the perspective of cooperative game theory, we can transform the parameters and evaluation indicators to the decisions and cost functions. The optimal solution can be obtained by solving the Pareto optimal solution. That is why we choose the cooperative game theory to solve the multi-parameter optimal design problem.

### 3 Robust control design

We propose an adaptive robust control to ensure the LLER can track the desired trajectory in this section. The control design procedure is shown in Fig. 2.

Decompose the matrices/vectors \( M \) and \( Q \) of Eq. (9) into two parts which are the “nominal” parts and the uncertain parts as follows:
\[
M(x, \delta, t) = \overline{M}(x, t) + \Delta M(x, \delta, t),
\]
\[
Q(x, \dot{x}, \delta, t) = \overline{Q}(x, \dot{x}, t) + \Delta Q(x, \dot{x}, \delta, t),
\]

Assume that \( \overline{M} \) is positive definite. \( \Delta M(\cdot), \Delta Q(\cdot) \) are continuous and uninterrupted. In the ideal case, there is no uncertainty and the corresponding matrices are equal to the corresponding “nominal” parts.

**Definition 2** For the uncertain parts \( \Delta M \) and \( \Delta Q \), there exist fuzzy sets \( G_{\Delta M} \) and \( G_{\Delta Q} \) in universe of discourse \( U_{\Delta M} \in \mathbb{R} \) and \( U_{\Delta Q} \in \mathbb{R} \) characterized by membership functions \( \mu_{U_{\Delta M}} : U_{\Delta M} \rightarrow [0, 1] \) and \( \mu_{U_{\Delta Q}} : U_{\Delta Q} \rightarrow [0, 1] \). That is \( G_{\Delta M} = (\Delta M, \mu_{U_{\Delta M}}(\Delta M)) \mid \Delta M \in U_{\Delta M} \) and \( G_{\Delta Q} = (\Delta Q, \mu_{U_{\Delta Q}}(\Delta Q)) \mid \Delta Q \in U_{\Delta Q} \).

**Remark 3** The dynamic Eq. (9) with fuzzy set theory to describe the uncertainties like Definition 2 is considered as the FDS.

Let
\[
F(x, \delta, t) := \overline{M}(x, t)M^{-1}(x, \delta, t) - I,
\]
\[
E(x, t) := M^{-1}(x, t),
\]
\[
\Delta E(x, \delta, t) := M^{-1}(x, \delta, t) - M^{-1}(x, t).
\]

Then, we can obtain
\[
M^{-1}(x, \delta, t) = E(x, t) + \Delta E(x, \delta, t),
\]
\[
\Delta E(x, \delta, t) = E(x, t)F(x, \delta, t).
\]

**Assumption 2** When Assumption 1 is satisfied, for a given \( P \in \mathbb{R}^{m \times m}, P \) is positive definite. Let
\[ W(x, \delta, t) := PA(x, t)E(x, t)F(x, \delta, t)Z \times \]
\[ \left( A(x, t)M^{-1}(x, t)Z \right)^{-1} P^{-1}. \]  

(32)

There exists a constant \( \rho_E > -1 \). For all \((x, t) \in \mathbb{R}^n \times \mathbb{R},\) \( \rho_E \) satisfies the (33):

\[ \frac{1}{\Sigma} \min_{\delta \in \Sigma} (W(x, \delta, t) + W^T(x, \delta, t)) \geq \rho_E \]

(33)

where \( \lambda_\Sigma \) denotes eigenvalues of matrix.

**Remark 4** The value of \( \rho_E, W \) and \( E \) depends on \( W, E \) and the uncertainty bound \( \Sigma \) respectively. Since the bound of uncertainty \( \Sigma \subset \mathbb{R}^n \) is unknown, \( \rho_E \) is unknown. In the ideal situation, \( E = 0, W = 0 \) and \( \rho_E = 0 \).

**Assumption 3** (1) There exists an unknown \( q \)-dimensional constant vector \( \alpha \in (0, \infty)^q \) and a known function \( \Pi(\cdot) : (0, \infty)^q \times \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{R}_+ \), such that, for all \((x, \bar{x}, t) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}, \delta \in \Sigma, \)

\[ (1 + \rho_E)^{-1} \max (PA(x, t)E(x, t)\Delta Q(x, \delta, \bar{x}, \dot{x}) - PA(x, t)\Delta E(x, \delta, t)Q(x, \bar{x}, \dot{x}) + PA(x, t)\Delta E(x, \delta, t)Z \times \]
\[ (w_1(x, \bar{x}, t) + w_2(x, \bar{x}, t))) \leq \Pi(x, \bar{x}, t). \]

(34)

(2) For each \((x, \bar{x}, t) \in \Pi(x, \bar{x}, t)\) is a linearized function with respect to \( \alpha \); there exists a function \( \tilde{\Pi}(\cdot) : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{R}_+ \) such that

\[ \Pi(x, \bar{x}, t) = \alpha^T \tilde{\Pi}(x, \bar{x}, t) \]

(35)

Here, \( \Sigma \), which is unknown, determines the value of \( \alpha \).

According to U-K theory and Assumptions 1–3, the adaptive robust control \( \tau(t) \) is proposed as follows:

\[ \tau(t) = w_1(x(t), \bar{x}(t), t) + w_2(x(t), \bar{x}(t), t) \]
\[ + w_3(\dot{x}, x(t), \bar{x}(t), t) \]

where

\[ w_1(x, \bar{x}, t) := \left( A(x, t)M^{-1}(x, t)Z \right)^+ (b(x, \bar{x}, t) \]
\[ - A(x, t)M^{-1}(x, t)Q(x, \bar{x}, t) \]

(37)

\[ w_2(x, \bar{x}, t) := -k_1 \left( A(x, t)M^{-1}(x, t)Z \right)^+ \]
\[ \times P^{-1} \beta(x, \bar{x}, t) \]
\[ w_3(\dot{x}, x(t), \bar{x}(t), t) := -k_1 \left( A(x, t)M^{-1}(x, t)Z \right)^+ \]
\[ \times P^{-1} \gamma(\bar{x}, x(t), t) \]

(38)

(39)

\[ \gamma(\bar{x}, x(t), t) = \gamma(\bar{x}, x(t), t) \]
\[ \times P^{-1} \gamma(\bar{x}, x(t), t) \]

(40)

\[ \gamma(n, \bar{x}, x(t), t) = \frac{q^n-1}{q^n-1 + q^n-2k_2 + \ldots + q^n-k_2} \]

(41)

\[ \theta(\bar{x}, x(t), t) = k_2^2 \left\| \mu(\bar{x}, x(t), t) \right\|^2 \]

(42)

\[ \beta(\bar{x}, x(t), t) = A(x(t), \bar{x}) - e(x, t) \]

(43)

where \( k_1, k_2 > 0, k_1, k_2 \in \mathbb{R}, n = 1, 2, 3, \ldots \), are scalar constants, and \( \beta(x, \bar{x}, t) \) stands for the constrained first-order error.

The adaptive law can be designed to govern the parameter \( \dot{\alpha} \) as follows:

\[ \dot{\alpha} = k_1 \left( -L_1 \xi(\bar{x}, x(t), t) - L_2 e^{-\|\mu(\bar{x}, x(t), t)\|} + L_3 \right) \]

(45)

(46)

Assume that Assumptions 1–3 are satisfied; under the system of (9), the adaptive robust control (36) renders the uniform boundedness and uniform ultimate boundedness:

(1) Uniform boundedness: For any \( r > 0 \), there exists a \( d(r) < \infty \), such that if \( \|\varphi(t_0)\| \leq r \), then \( \|\varphi(t)\| \leq d(r) \) for all \( t \geq t_0 \).

(2) Uniform ultimate boundedness: For any \( r > 0 \) and \( \|\varphi(t_0)\| \leq r \), there exists a \( d > 0 \). Such that for any \( \overline{d} > d \) as \( t > t_0 + T(\overline{d}, r) \), \( \|\varphi(t)\| \leq \overline{d} \), where \( T(\overline{d}, r) < \infty \), there exists \( \|\varphi(t)\| \leq \overline{d} \).

The proof of Theorem 2 is given in Appendix A.
4 Optimal design

According to Rayleigh’s principle
\[
\lambda_{\text{min}}(P)\|\beta\|^2 \leq \beta^T P \beta \leq \lambda_{\text{max}}(P)\|\beta\|^2
\]
where \( \lambda_{\text{min}}(P) > 0 \), because \( P \) is positive definite. So
\[
-2k_1 \frac{\lambda_{\text{min}}(P)}{\lambda_{\text{max}}(P)} \|\beta\|^2 \geq -2k_1 \|\beta\|^2
\]
(46)

If we let \( \tilde{\rho} := \min\{2/\lambda_{\text{max}}(P), L_3\} \), we can obtain
\[
-2k_1 \tilde{\rho}^2 - (1 + \rho_E)L_1^{-1}L_3 \tilde{\alpha} - \alpha^2
\]
\[
\leq -2k_1 \frac{\lambda_{\text{min}}(P)}{\lambda_{\text{max}}(P)} \beta^T P \beta - k_1 L_3 (k_1 L_1)^{-1} (1 + \rho_E) \times (\tilde{\alpha} - \alpha)^T (\tilde{\alpha} - \alpha) \leq -k_1 \tilde{\rho} V
\]
where \( V \) is the Lyapunov function and the same as that in (100).

With (48) and (112),
\[
\dot{V} \leq -2k_1 \tilde{\rho}^2 - (1 + \rho_E)L_1^{-1}L_3 \tilde{\alpha} - \alpha^2 + \Omega
\]
\[
\leq -k_1 \tilde{\rho} V(t) + \Omega
\]
(49)

where \( V(t_0) = \beta^T(t_0)P\beta(t_0) + (k_1 L_1)^{-1} (1 + \rho_E) \times (\tilde{\alpha}(t_0) - \alpha)^T (\tilde{\alpha}(t_0) - \alpha) \). The analysis of (49) is presented next.

First, consider the differential equation
\[
\dot{y}(t) = -k_1 \tilde{\rho} y(t) + \Omega, y(t_0) = V_0 = V(t_0)
\]
(50)

The right-hand side of (50) meets the global Lipschitz condition with
\[
L = k_1 \tilde{\rho}
\]
(51)

Solve the differential (50) and obtain the result
\[
y(t) = \left( V_0 - \frac{\Omega}{k_1 \tilde{\rho}} \right) e^{-k_1 \tilde{\rho}(t-t_0)} + \frac{\Omega}{k_1 \tilde{\rho}}
\]
(52)

Then, based on the solution process of differential inequality, we can obtain
\[
V(t) \leq y(t)
\]
(53)
or
\[
V(t) \leq \left( V_0 - \frac{\Omega}{k_1 \tilde{\rho}} \right) e^{-k_1 \tilde{\rho}(t-t_0)} + \frac{\Omega}{k_1 \tilde{\rho}}
\]
(54)

for all \( t \geq t_0 \). Similarly, for any \( t_s \) and \( \Gamma \geq t_s \)

\[
V(\Gamma) \leq \left( V_s - \frac{\Omega}{k_1 \tilde{\rho}} \right) e^{-k_1 \tilde{\rho}(\Gamma-t_s)} + \frac{\Omega}{k_1 \tilde{\rho}}
\]
(55)

where \( V_s = V(t_s) = \beta^T(t_s)P\beta(t_s) + (k_1 L_1)^{-1} (1 + \rho_E) \times (\tilde{\alpha}(t_s) - \alpha)^T (\tilde{\alpha}(t_s) - \alpha) \). \( t_s \) is not necessary to be equal to zero; it is the time when control system starts.

To separate the variable, \( V_s \) is rewritten as
\[
V_s = V_{s1} + V_{s2}
\]
(56)

where \( V_{s1} := \beta^T(t_s)P\beta(t_s) \) and \( V_{s2} := (k_1 L_1)^{-1} (1 + \rho_E) \times (\tilde{\alpha}(t_s) - \alpha)^T (\tilde{\alpha}(t_s) - \alpha) \). With (46) and (56), we have
\[
V(\Gamma) \geq \lambda_{\text{min}}(P)\|\beta\|^2 + (k_1 L_1)^{-1} (1 + \rho_E)\|\tilde{\alpha} - \alpha\|^2
\]
and the right-hand side of (55) provides an upper bound of \( \lambda_{\text{min}}(P)\|\beta\|^2 + (k_1 L_1)^{-1} (1 + \rho_E)\|\tilde{\alpha} - \alpha\|^2 \).

This can be an upper bound of \( \|\beta\|^2 \). For each \( \Gamma \geq t_s \), let
\[
\eta(\alpha, k_1, k_2, \Gamma, t_s) = \left( V_s - \frac{\Omega}{k_1 \tilde{\rho}} \right) e^{-k_1 \tilde{\rho}(\Gamma-t_s)}
\]
(57)

\[
\eta_\infty(\alpha, k_1, k_2) = \frac{\Omega}{k_1 \tilde{\rho}}
\]
(58)

Remark 5 The uncertainty is described by fuzzy set theory, which can perfectly be solved adopting the corresponding membership function. Compared with probability theory, fuzzy set theory does not need numerous observation data. [41] has a more detailed description of fuzzy set theory.

The performance indicators for \( k_1 \) and \( k_2 \) are formulated as
\[
J_1(k_1, k_2, t_s) := D \left[ \int_{t_s}^{\infty} \eta^2(\alpha, k_1, k_2, \Gamma, t_s)d\Gamma \right] + k_1^2
\]
(59)

\[
J_2(k_1, k_2, t_s) := D\left[ \eta_\infty^2(\alpha, k_1, k_2) \right] + k_2^2
\]
(60)

It can be shown that
\[ \int_{t_i}^{\infty} \eta^2(x, k_1, k_2, \Gamma, t_i) d\Gamma 
\]
\[ = \left( V_s - \frac{\Omega}{k_1 \rho} \right)^2 \int_{t_i}^{\infty} e^{-2k_1 \rho (\Gamma - t_i)} d\Gamma 
\]
\[ = \left( V_s - \frac{\Omega}{k_1 \rho} \right)^2 \left( -\frac{1}{2k_1 \rho} \right) e^{-2k_1 \rho (\Gamma - t_i)} \bigg|_{t_i}^{\infty} 
\]
\[ = \left( V_s - \frac{\Omega}{k_1 \rho} \right)^2 \frac{1}{2k_1 \rho} 
\]
\[ = \left( V_s + \frac{V_{s2}}{k_1} - \frac{\Omega}{k_1 \rho} \right)^2 \frac{1}{2k_1 \rho} 
\]
\[ (61) \]

Considering the D-operation, we have
\[ D \left[ \int_{t_i}^{\infty} \eta^2(x, k_1, k_2, \Gamma, t_i) d\Gamma \right] 
\]
\[ = D \left[ \left( V_s + \frac{V_{s2}}{k_1} - \frac{\Omega}{k_1 \rho} \right)^2 \frac{1}{2k_1 \rho} \right] 
\]
\[ := L_1 k_1^{-7} + L_2 k_1^{-5} + L_3 k_1^{-4} + L_4 k_1^{-3} + L_5 k_1^{-2} + L_6 k_1^{-1} 
\]
\[ + L_7 k_1^{-3} k_2 + L_8 k_1^{-2} k_2 + L_9 k_1^{-2} k_2 + L_10 k_1^{-2} k_2 
\]
\[ (62) \]

Details for \( L_i, i = \{1, 2, \cdots, 10\} \) are given in Appendix B.

Similarly, by D-operation, we can obtain
\[ D\left[ \eta^2_{\infty}(x, k_1, k_2) \right] 
\]
\[ := x_1 k_1^{-6} + x_2 k_1^{-4} + x_3 k_1^{-2} + x_4 k_1^{-2} k_2 
\]
\[ + x_5 k_1^{-4} k_2 + x_6 k_1^{-2} k_2^2 \]
\[ (63) \]

Details for \( x_i, i = \{1, 2, \cdots, 6\} \) are given in Appendix B.

Then, \( J_1 \) and \( J_2 \) can be rewritten as
\[ J_1(k_1, k_2) = L_1 k_1^{-7} + L_2 k_1^{-5} + L_3 k_1^{-4} + L_4 k_1^{-3} 
\]
\[ + L_5 k_1^{-2} + L_6 k_1^{-1} + L_7 k_1^{-3} k_2 + L_8 k_1^{-2} k_2^2 
\]
\[ + L_9 k_1^{-2} k_2 + L_10 k_1^{-2} k_2 + k_1^2 \]
\[ (64) \]
\[ J_2(k_1, k_2) = x_1 k_1^{-6} + x_2 k_1^{-4} + x_3 k_1^{-2} + x_4 k_1^{-2} k_2 
\]
\[ + x_5 k_1^{-4} k_2 + x_6 k_1^{-2} k_2^2 + k_2^{-2} \]
\[ (65) \]

The optimal design is obtained by the following cooperative game.

Find \( (k_1^*, k_2^*) \) such that for every \( (k_1, k_2) \in \prod_{i=1}^{2} \mathcal{H}_i \) either
\[ J_i(k_1, k_2) = J_i(k_1^*, k_2^*) \quad \forall \{1, 2, \cdots, Y\} \]
\[ (66) \]

or there is at least one \( i \in \{1, 2\} \) such that
\[ J_i(k_1, k_2) > J_i(k_1^*, k_2^*) \]
\[ (67) \]

To find \( (k_1^*, k_2^*) \), we adopt Lemma 3: The decision pair \( (k_1^*, k_2^*) \) is Pareto optimal if there exist weighting factors \( l_1 > 0 \) and \( l_2 > 0 \) with \( l_1 + l_2 = 1 \) such that
\[ J(k_1^*, k_2^*) \leq J(k_1, k_2) \forall k_1 \in \mathcal{H}_1, k_2 \in \mathcal{H}_2 \]
\[ (68) \]

where \( J(k_1, k_2) = \ell_1 J_1(k_1, k_2) + \ell_2 J_2(k_1, k_2) \),

Consequently, we can solve the following constrained optimization problem.

For any \( t_i \) and given \( \mathcal{H}_{1,2} \)
\[ \min_{k_1, k_2} : J(k_1, k_2) \text{ subject to } : k_1 \in \mathcal{H}_1, k_2 \in \mathcal{H}_2 \]
\[ (69) \]

To obtain the solution, we apply a partial differential operator to \( J(k_1, k_2) \) with respect to \( k_1 \)
\[ \frac{\partial J}{\partial k_1} = l_1 (-7 L_1 k_1^{-8} - 5 L_2 k_1^{-6} - 4 L_3 k_1^{-5} - 3 L_4 k_1^{-4} 
\]
\[ - 2 L_5 k_1^{-3} - L_6 k_1^{-2} - 3 L_7 k_1^{-4} k_2 - 2 L_8 k_1^{-3} k_2 + 2 k_1) 
\]
\[ + \ell_2 (-6 x_1 k_1^{-7} - 4 x_2 k_1^{-5} - 2 x_3 k_1^{-3} - 2 x_4 k_1^{-3} k_2 
\]
\[ - 4 x_5 k_1^{-5} k_2 - 2 x_6 k_1^{-2} k_2^2) \]
\[ (70) \]

Similarly, applying partial differential operator to \( J(k_1, k_2) \) with respect to \( k_2 \) gives
\[ \frac{\partial J}{\partial k_2} = \ell_1 (L_7 k_1^{-3} + 2 L_8 k_1^{-3} k_2 + L_9 k_1^{-5} + L_{10} k_1^{-2}) 
\]
\[ + \ell_2 (x_4 k_1^{-2} - x_5 k_1^{-4} + 2 x_6 k_1^{-2} k_2 - 2 k_2^{-3} k_2) \]
\[ (71) \]

Let
\[
\frac{\partial J}{\partial k_1} = 0 \tag{72}
\]

\[
\frac{\partial J}{\partial k_2} = 0 \tag{73}
\]

with \( k_1 \in \mathcal{H}_1, k_2 \in \mathcal{H}_2 \).

By analyzing the second-order derivative of \( J \), we have
\[
\Delta(k_1, k_2) = \begin{bmatrix}
\frac{\partial^2 J}{\partial k_1^2} & \frac{\partial^2 J}{\partial k_1 \partial k_2} \\
\frac{\partial^2 J}{\partial k_1 \partial k_2} & \frac{\partial^2 J}{\partial k_2^2}
\end{bmatrix}
\tag{74}
\]

Suppose the candidate solutions \( k_1^*, k_2^* \) can meet the sufficient condition
\[
\Delta(k_1, k_2) > 0 \tag{75}
\]

Then, (68) can be minimized by the solutions of (72) and (73) and the solutions are Pareto optimal.

**Remark 6** The \( k_1^* \) and \( k_2^* \) can be determined with suitable choices of \( P, \rho_E \) and \( L_i (i = 1, 2, 3) \).

**Theorem 3** Suppose the solutions \( k_1^* > 0 \) and \( k_2^* > 0 \) which satisfy (72), (73) and (75) exist. Then, they are the Pareto optimality.

**Proof** The proof of Theorem 3 has already been presented in (59)-(75).

Based on the above analysis, the resulting minimal performance index is acquired by
\[
J_{\text{min}} = l_1J_1(k_1^*, k_2^*) + l_2J_2(k_1^*, k_2^*) \tag{76}
\]

The design procedure of optimal design is displayed in Fig. 3.

### 5 Numerical simulation

#### 5.1 Description of LLER

Figure 4 demonstrates the 3D prototype and simplified 2D model of LLER. We assume the LLER has two active degrees of freedom (DOFs) of one leg, containing the flexion and extension DOFs of hip and knee joints. This idea is inspired by [9, 11, 50]. \( x = [\theta_1, \theta_2]^T \) is chosen as the generalized coordinates, which denotes the rotation angle of hip and knee, respectively.

By employing the Lagrangian dynamics, the dynamic model of LLER is formulated
\[
M\ddot{x} = Q + \tau_{wr} + Z\sigma \tag{77}
\]

with
\[
M = M(x(t), \delta(t), t) = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \tag{78}
\]
\[
Q = Q(x(t), \dot{x}(t), \delta(t), t) = [Q_{11} \quad Q_{21}]^T \tag{79}
\]
\[
\tau_{wr} = k_p(x - x_d) + k_d(\dot{x} - \dot{x}_d) \tag{80}
\]
\[
Z = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{81}
\]
\[
M_{11} = m_1d_1^2 + I_1 + m_2l_1^2 \tag{82}
\]
\[
M_{12} = M_{21} = -m_2l_1d_2\cos(\theta_1 + \theta_2) \tag{83}
\]
\[
M_{22} = m_2d_2^2 + I_2 \tag{84}
\]
\[
Q_{11} = -m_2l_1d_2\sin(\theta_1 + \theta_2)\dot{\theta}_2\dot{\theta}_1 + m_2l_1d_2\sin(\theta_1 + \theta_2)
\times (\dot{\theta}_1 + \dot{\theta}_2)\dot{\theta}_2 + m_1gd_1\sin\theta_1 + m_2gl_1\sin\theta_1 \tag{85}
\]
\[ Q_{21} = m_2 l_1 d_2 \sin(\theta_1 + \theta_2) \left( \dot{\theta}_1 + \dot{\theta}_2 \right) \hat{\theta}_1 - m_2 l_1 d_2 \times \sin(\theta_1 + \theta_2) \hat{\theta}_1 \hat{\theta}_2 + m_2 g d_2 \sin \theta_2 \]  

where \( \tau_{wr} \) is the interaction torque between the wearer and robot [23, 49]. \( x_d \) and \( \dot{x}_d \) are the desired trajectory and velocity, respectively. \( k_p \) and \( k_d \) are constants that amplify the differences of trajectories and velocities between the wearer and robot. Moreover, \( m_1 \) stands for the sum of the masses of the thigh of wearer and robot, and \( m_2 \) stands for that of the shank, similarly. \( g \) is the gravitational acceleration. \( l_i = 2d_i (i = 1, 2) \) denotes the length of thigh and shank. \( I_i (i = 1, 2) \) represents the moment of inertia of thigh and shank, respectively.

Apply the servo constraints to the LLER, which are obtained from Clinical Gait Analysis data [9]

\[ \theta_1 = 0.8 + 0.2 \sin(2\pi t) \]  

(87)

\[ \theta_2 = 1.2 - 0.2 \cos(2\pi t) \]  

(88)

Rewriting Eqs. (87)-(88) in the form of (10)

\[ B(x, t) = d(x, t) \]  

(89)

where

\[ B(x, t) = [ \theta_1 \quad \theta_2 ]^T \]  

(90)

\[ d(x, t) = \begin{bmatrix} 0.8 + 0.2 \sin(2\pi t) \\ 1.2 - 0.2 \cos(2\pi t) \end{bmatrix} \]  

(91)

We differentiate Eq. (89) and rewrite the result in the form of Eqs. (14) and (18):

\[ A(x, t) \ddot{x} = b(x, \dot{x}, t) \]  

(92)

where

\[ A(x, t) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \]  

(93)

\[ c(x, t) = \begin{bmatrix} 0.4 \pi \cos(2\pi t) \\ 0.4 \pi \sin(2\pi t) \end{bmatrix} \]  

(94)

\[ b(x, \dot{x}, t) = \begin{bmatrix} -0.8 \pi^2 \sin(2\pi t) \\ 0.8 \pi^2 \cos(2\pi t) \end{bmatrix} \]  

(95)

here Eqs. (92–93) denote velocity and acceleration constraints, respectively. (92) can be treated as nonholonomic constraints.

**Remark 7** Considering that our main purpose is to realize the trajectory tracking control of LLER and verify the effectiveness of ORC, only one human motion actuation scenario is considered in this paper. The desired constraints (87)-(88) are obtained from Clinical Gait Analysis data and holonomic. The proposed control method is also applicable to other actuation scenarios and nonholonomic constraints. People can calculate \( A \) and \( b \) according to Fig. 1 and then design the controller. As shown in Assumption 1, the scenarios and constraints should be continuous.

Due to the imperfect knowledge of the LLER system parameters and/or external disturbance, uncertainties are inevitable in the system model. As we know, different wearers and even the same wearer may have different weights, so the masses \( m_i (i = 1, 2) \) are chosen as the uncertain parameters: \( m_1 = \bar{m}_1 + \Delta m_1 \), \( m_2 = \bar{m}_2 + \Delta m_2 \). When there are no uncertainties, \( \Delta m_i (i = 1, 2) = 0 \). The parameters are...
Table 1 Parameters of the LLER

| Parameter | Value | Unit |
|-----------|-------|------|
| $m_1$     | 28    | kg   |
| $m_2$     | 23    | kg   |
| $l_1$     | 0.45  | m    |
| $l_2$     | 0.35  | m    |
| $l_1$     | 2.5   | kgm$^2$ |
| $l_2$     | 1.0   | kgm$^2$ |
| $g$       | 9.8   | m/s$^2$ |

Fig. 5 Relationship between $\partial J(k_1,k_2)/\partial k_1$, $\partial J(k_1,k_2)/\partial k_2$ and $k_1$, $k_2$ ($l_1 = l_2 = 0.5$)

5.2 Optimal robust control

Choose the initial conditions as $\theta_1(0) = 1\text{rad}$, $\theta_2(0) = 0.5\text{rad}$, $\dot{\theta}_1(0) = 0.5\text{rad/s}$, $\dot{\theta}_2(0) = 0.5\text{rad/s}$ which do not satisfy the initial conditions in the ideal condition. Furthermore, the parameters $P = I_{2 \times 2}$, $n = 5$, $L_1(i = 1, 2, 3) = 1$, $\rho_E = -0.1$, $\tilde{z}(0) = 0.2$, $k_P = 1.5$ and $k_d = 2$ are chosen. Then, Assumptions 1-2 can be easily satisfied. In order to meet Assumption 3, $\Pi(\alpha, x, \dot{x}, t)$ can be chosen as follows:

$$\Pi(\alpha, x, \dot{x}, t) = \alpha_1 \|\dot{x}\|^2 + \alpha_2 \|x\| + \alpha_3$$

(99)

where $\alpha_i > 0 (i = 1, 2, 3)$. $\Pi(\alpha, x, \dot{x}, t)$ can also be chosen as follows to satisfy Assumption 3

$$\alpha_1 \|\dot{x}\|^2 + \alpha_2 \|x\| + \alpha_3 \leq \alpha (\|\dot{x}\| + 1)^2 = \alpha^T \Pi(x, \dot{x}, t)$$

(100)

where $\alpha = \max \left\{ \alpha_1, \frac{\alpha_2}{2}, \alpha_3 \right\}$.

The task we should consider is to seek the optimal solution of $k_1$ and $k_2$. Cooperative game theory is adopted, and we calculate that $L_1 = 0.1013$, $L_2 = -0.2403$, $L_3 = -0.1534$, $L_4 = 0.1362$, $L_5 = 0.1780$, $L_6 = 0.0581$, $L_7 = -0.9396$, $L_8 = 1.6200$, $L_9 = 0.2025$, $L_{10} = -0.6137$, $\kappa_3 = 0.2025$, $\kappa_2 = 0.0486$, $\kappa_3 = 0.0029$, $\kappa_4 = 0.1944$, $\kappa_5 = 1.6200$ and $\kappa_6 = 3.2400$.

According to Eqs. (72–73), the Pareto optimal solution can be solved. Figure 5 also verifies the
existence of that by showing the plots of \( \partial J(k_1, k_2) / \partial k_1 \), \( \partial J(k_1, k_2) / \partial k_2 \) and \( \partial J(k_1, k_2) / \partial k_i \) \((i = 1, 2)\) in the case of \( l_i = 0.5 \). After verification, the yellow point is the Pareto optimal solution. Figure 6 shows the relationship between \( J \) and \( k_1, k_2 \) under \( l_i = 0.5 \), and the yellow point is the solution of minimizing the cost performance \( J \), named the Pareto optimality. For a better interpretation of cooperative game theory, Table 2 gives five groups of \( k_i \) \((i = 1, 2)\) and corresponding minimal cost performance \( J_{\text{min}} \) under different \( l_i \) \((i = 1, 2)\).

### 5.3 Simulation results

The software is MATLAB R2019b. Figures 6, 7, 8, 9, 10, 11, 12, 13, 14, 15 and 16 display the simulation results by using ode15i algorithm. The uncertainties are represented in the simulation as \( \Delta m_1(t) = 0.05|\cos(t)|kg, \Delta m_2(t) = 0.01|\sin(t)|kg \).

Figures 7, 8, 9, 10 and 11 demonstrate the trajectories of \( \theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2 \) and \( \ddot{z} \), when \( l_1 = l_2 = 0.5 \). The actual trajectories of ORC proposed by us can suppress the impact of uncertainties and have an excellent tracking performance.
Figure 11 reveals the relationship between \( \hat{x} \) and time. Firstly, the value of \( \hat{x} \) increases from initial value 0.2 to the maximum 0.34 in order to suppress the impact of uncertainties. As the deviation is compensated (about 1 s), the value of \( \hat{x} \) decreases rapidly and gradually stabilized. This phenomenon also implies that ORC method possesses well robustness in trajectory tracking of LLER.

To better illustrate the effect of \( l_i (i = 1, 2) \) in Table 2, Figs. 12, 13, 14 and 15 compare the trajectories of \( \tau_1 \), \( \tau_2 \), error1 and error2 under five groups of \( k_i (i = 1, 2) \) to explore its impact for system performance. A significant event is discovered by us: The smaller the peak value of \( \tau_i (i = 1, 2) \), the slower the error1 and error2’s speed converges to zero. We need to endure the larger torque cost to achieve the faster error convergence, which means more expensive motors. In conclusion, the value of \( l_i (i = 1, 2) \) should be chosen according to the specific situation.

Adaptive robust control (ARC) of [18] is chosen as a comparative control method, and the corresponding simulation results are also shown in Figs. 12, 13, 14 and 15. The trajectories tracking errors are larger than that of ORC, which means the \( \hat{\delta}_i (i = 1, 2) \) tracking effect of [18] is worse. And the absolute torque value of hip is larger than that of ORC, although that of knee...
is smaller than that of ORC. Considering the overall system, the performance of ORC is better than that of ARC of Sun.

Figures 16 and 17 illustrate the absolute cumulative error (ACE) of $\theta_i (i = 1, 2)$ under five groups of $k_i (i = 1, 2)$ and ARC of Sun. ACE of $\theta_i (i = 1, 2)$ with ARC of [18] is significantly larger than that with ORC under five groups of $k_i (i = 1, 2)$. In summary, the tracking performance of ORC proposed by us is better than that of ARC of [18], and people can choose the appropriate pair of $l_i (i = 1, 2)$ to meet the practical requirements of LLER.

6 Conclusion

In this paper, the optimal robust control with cooperative game theory is designed to solve the trajectory tracking problem of LLER, including the uncertainties. The uncertainties are time-varying and bounded. For a better description and control of uncertainties, fuzzy set theory is employed, which is essentially different from the traditional fuzzy logic theory. Afterward, the robust control with two adjustable parameters is proposed in the frame of U-K theory, which guarantees the uniform boundedness and uniform ultimate boundedness. Furthermore, the optimal design with cooperative game theory is formulated to find the Pareto optimal solution of two adjustable parameters. Finally, the simulation results verify the existence of Pareto optimal solution and the
effectiveness of the optimal robust control proposed in this paper.

Last but not least, the fuzzy set theory describes the uncertainties first, which are embedded into the robust control. Then, the cooperative game theory is used to optimal design. Our work extends the application of U-K, and cooperative game theory has a great significance on trajectory tracking control of LLER with uncertainties. The experiment will be another important work in the next step.

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Code availability Custom code is available upon request at Liang Yuan email address.

Declarations

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Appendix A: Proof of Theorem 2

Proof Choose a legitimate Lyapunov function candidate as follows [51]:

\[ V(\beta, \bar{x} - x) = \beta^T P \beta + k_1^{-1} L_1^{-1}(1 + \rho_E) \]

\[ (\bar{x} - x)^T (\bar{x} - x). \] (101)

For a specific mechanical system with uncertainty and desired constraints, \( \dot{V} \) is shown as

\[ \dot{V} = 2 \beta^T P \dot{\beta} + 2k_1^{-1} L_1^{-1}(1 + \rho_E) (\bar{x} - x)^T \dot{\bar{x}}. \] (102)

The first term of (102) can be analyzed as

\[ 2 \beta^T P \dot{\beta} = 2 \beta^T P (A\bar{x} - b) \]

\[ = 2 \beta^T P (AM^{-1} (Q + \tau) - b) \]

\[ = 2 \beta^T P (AM^{-1}(Q + Zw_1 + Zw_2 + Zw_3) - b) \] (103)

Based on (25)—(26),

\[ AM^{-1}(Q + Zw_1 +ZW_2 + Zw_3) - b \]

\[ = A(E + \Delta E)(\bar{Q} + \Delta Q + Zw_1 + Zw_2 + Zw_3) - b \]

\[ = AE\bar{Q} + AE\Delta Q + A\Delta E(\bar{Q} + \Delta Q) + AEZw_1 + AEZw_2 \]

\[ + A\Delta E(Zw_1 + Zw_2) + A(E + \Delta E)Zw_3 - b. \] (104)

According to (37),

\[ AE\bar{Q} + AEZw_1 - b = 0. \] (105)

According to Assumption 3–1,

\[ 2 \beta^T P(AE\Delta Q + A\Delta E(Q + Zw_1 + Zw_2)) \]

\[ \leq 2 \beta \|E\| \|PAE\Delta Q + PA\Delta E(Q + Zw_1 + Zw_2)\| \]

\[ \leq 2(1 + \rho_E) \|E\| \Pi(\bar{x}, x, \dot{x}, t) \] (106)

Based on (38),

\[ 2 \beta^T PAEZw_2 = 2 \beta^T PAE \left( -k_1 \left( AM^{-1} B \right)^+ P^{-1} \beta \right) \]

\[ = -2k_1 \beta^T \beta - 2k_1 \|\beta\|^2. \] (107)

Based on (31), (39) and (43),

\[ 2 \beta^T PA(E + \Delta E)Zw_3 \]

\[ = 2 \beta^T PAEZw_3 + 2 \beta^T PAEFZw_3 \]

\[ = 2 \beta^T PAE \left( -k_1 \left( AM^{-1} Z \right)^+ P^{-1} \chi(\bar{x}, x, \dot{x}, t) \right) \]

\[ + 2 \beta^T PAEFZw_3 \]

\[ = -2k_1 \beta^T \chi(\bar{x}, x, \dot{x}, t) + 2 \beta^T PAEFZw_3 \]
\[
= -2k_1^2 \gamma |\mu|^2 + 2\beta^T PAEFZw_3
\]
\[
-2k_1^2 \gamma |\mu|^2 + 2\beta^T PAEFZw_3 \tag{108}
\]
According to Assumption 3 and Rayleigh's principle,
\[
2\beta^T PAEFZw_3
\]
\[
= 2\beta^T PAEFZ \left( -k_1^2 (A^{-1}Z)^e P^{-1}(x, x, x, t) \right)
\]
\[
-2k_1^2 \frac{1}{2} \mu^T \left( PAEFZ \left( A^{-1}Z \right)^e P^{-1} + P^{-1} \left( A^{-1}Z \right)^e \right)^T \mu
\]
\[
Z^T F^T PA^T E \mu = -2k_1^2 \frac{1}{2} \mu^T (W + W^T) \mu
\]
\[
\leq -2k_1^2 \frac{1}{2} \lambda_{\min}(W + W^T) |\mu|^2 \leq -2k_1^2 \gamma |\mu|^2. \tag{109}
\]
Combining (108) and (109),
\[
2\beta^T PA(E + \Delta E)Zw_3 \leq -2k_1^2 \gamma (1 + \rho_E) |\mu|^2. \tag{110}
\]

By (41) and (43), we have
\[
2\beta^T P\beta
\]
\[
\leq -2k_1 |\beta|^2 + 2(1 + \rho_E) |\beta| \Pi(x, x, x, t)
\]
\[
-2k_1^2 \gamma (1 + \rho_E) |\mu|^2
\]
\[
= -2k_1 |\beta|^2 + 2(1 + \rho_E) |\beta| \Pi(x, x, x, t) - 2(1 + \rho_E)
\]
\[
\times \frac{\gamma^n}{\gamma^{n-1} + \gamma^{n-2}k_2 + \cdots + \gamma k_2^n + k_2^n}
\]
\[
\leq -2k_1 |\beta|^2 + 2(1 + \rho_E) |\beta| \Pi(x, x, x, t) - 2(1 + \rho_E)
\]
\[
\times \frac{\gamma^n - k_2^n}{\gamma^{n-1} + \gamma^{n-2}k_2 + \cdots + \gamma k_2^n + k_2^n}
\]
\[
= -2k_1 |\beta|^2 + 2(1 + \rho_E) |\beta| \Pi(x, x, x, t) - 2(1 + \rho_E) \#
\]
\[
\times \left( \frac{\gamma^n}{\gamma^{n-1} + \gamma^{n-2}k_2 + \cdots + \gamma k_2^n + k_2^n} \right)
\]
\[
= -2k_1 |\beta|^2 + 2(1 + \rho_E) |\beta| \Pi(x, x, x, t) + 2(1 + \rho_E)^2 k_2
\]
\[
-2(1 + \rho_E)^2 k_2 |\beta|^2 \Pi^2(x, x, x, t)
\]
\[
= -2k_1 |\beta|^2 + 2(1 + \rho_E) |\beta| \Pi(x, x, x, t)
\]
\[
+ 2(1 + \rho_E) k_2
\]
\[
+ 2(1 + \rho_E)^2 k_2 |\beta|^2 \Pi^2(x, x, x, t)
\]
\[
\leq -2k_1 |\beta|^2 + 2(1 + \rho_E) |\beta| (x - \hat{x})^T \Pi (x, x, t)
\]
\[
\leq 2(1 + \rho_E) |\beta|^2 + 2(1 + \rho_E) |\beta| (x - \hat{x})^T \Pi (x, x, t)
\]
\[
\leq 2(1 + \rho_E) |\beta|^2 + 2(1 + \rho_E) |\beta| (x - \hat{x})^T \Pi (x, x, t)
\]
\[
+ 2(1 + \rho_E) k_2 + \frac{1}{2} (1 + \rho_E) k_1^2. \tag{111}
\]

For the second term of the right-hand side of (102), by using adaptive law (45) and \(e^{-|\beta|} \leq 1\), we can obtain
\[
2k_1^{-1} L_1^{-1} (1 + \rho_E) (\hat{x} - x)^T \hat{x}
\]
\[
= 2(1 + \rho_E) (\hat{x} - x)^T \left( \Pi - L_1^{-1} \left( L_2 e^{-|\beta|} + L_3 \right) \hat{x} \right)
\]
\[
= 2(1 + \rho_E) (\hat{x} - x)^T \left( \Pi - 2(1 + \rho_E) L_1^{-1} \left( L_2 e^{-|\beta|} + L_3 \right) \hat{x} \right)
\]
\[
\leq 2(1 + \rho_E) (\hat{x} - x)^T \left( \Pi - 2(1 + \rho_E) L_1^{-1} \left( L_2 e^{-|\beta|} + L_3 \right) \hat{x} \right)
\]
\[
\leq 2(1 + \rho_E) (\hat{x} - x)^T \left( \Pi - 2(1 + \rho_E) L_1^{-1} \left( L_2 e^{-|\beta|} + L_3 \right) \hat{x} \right)
\]
\[
\leq 2(1 + \rho_E) (\hat{x} - x)^T \left( \Pi - 2(1 + \rho_E) L_1^{-1} \left( L_2 e^{-|\beta|} + L_3 \right) \hat{x} \right)
\]
\[
\leq 2(1 + \rho_E) (\hat{x} - x)^T \left( \Pi - 2(1 + \rho_E) L_1^{-1} \left( L_2 e^{-|\beta|} + L_3 \right) \hat{x} \right)
\]
\[
\leq 2(1 + \rho_E) (\hat{x} - x)^T \left( \Pi - 2(1 + \rho_E) L_1^{-1} \left( L_2 e^{-|\beta|} + L_3 \right) \hat{x} \right)
\]
\[
\leq 2(1 + \rho_E) (\hat{x} - x)^T \left( \Pi - 2(1 + \rho_E) L_1^{-1} \left( L_2 e^{-|\beta|} + L_3 \right) \hat{x} \right)
\]
\[
\begin{align*}
&\leq 2(1 + \rho_E)(\hat{\mathbf{z}} - \mathbf{z})^T \Pi \| \mathbf{b} \| + \frac{1}{2}(1 + \rho_E) L_1^{-1} L_2 \times \\
&\| \mathbf{z} \|^2 - 2(1 + \rho_E) L_1^{-1} L_3 (\| \hat{\mathbf{z}} - \mathbf{z} \|^2 + \| \hat{\mathbf{z}} - \mathbf{z} \| \| \mathbf{z} \|) \\
&= 2(1 + \rho_E)(\hat{\mathbf{z}} - \mathbf{z})^T \Pi \| \mathbf{b} \| + \frac{1}{2}(1 + \rho_E) L_1^{-1} L_2 \| \mathbf{z} \|^2 - \\
&2(1 + \rho_E) L_1^{-1} L_3 \| \hat{\mathbf{z}} - \mathbf{z} \|^2 - 2(1 + \rho_E) L_1^{-1} L_3 \| \hat{\mathbf{z}} - \mathbf{z} \| \| \mathbf{z} \| \\
&\leq 2(1 + \rho_E)(\hat{\mathbf{z}} - \mathbf{z})^T \Pi \| \mathbf{b} \| + \frac{1}{2}(1 + \rho_E) L_1^{-1} L_2 \| \mathbf{z} \|^2 \\
&- 2(1 + \rho_E) L_1^{-1} L_3 \| \hat{\mathbf{z}} - \mathbf{z} \|^2 - (1 + \rho_E) L_1^{-1} L_3 \| \mathbf{z} \|^2 \\
&-(1 + \rho_E) L_1^{-1} L_3 \| \hat{\mathbf{z}} - \mathbf{z} \|^2 \\
&= 2(1 + \rho_E)(\hat{\mathbf{z}} - \mathbf{z})^T \Pi \| \mathbf{b} \| + \frac{1}{2}(1 + \rho_E) L_1^{-1} L_2 \| \mathbf{z} \|^2 \\
&- 3(1 + \rho_E) L_1^{-1} L_3 \| \hat{\mathbf{z}} - \mathbf{z} \|^2 - (1 + \rho_E) L_1^{-1} L_3 \| \mathbf{z} \|^2 \\
&\leq -\rho \| \mathbf{z} \|^2 + \Omega
\end{align*}
\]

where \( \rho := \min \{2k_1, 3(1 + \rho_E) L_1^{-1} L_3\} \), \( \Omega := (1/2) \times (1 + \rho_E) k_1^{-2} + (1 + \rho_E) k_2 + (1/2) (1 + \rho_E) L_1^{-1} L_2 \mathbf{x}^2 + (1 + \rho_E) L_1^{-1} L_3 \mathbf{x}^2 \).

The function \( d(r) \) in Theorem 2(1) is shown as
\[
d(r) = \begin{cases} 
\sqrt{\frac{\Psi_2}{\Psi_1}} & \text{if } r \leq \bar{R} \\
\sqrt{\frac{\Psi_1}{\Psi_2}} & \text{if } r > \bar{R}
\end{cases}
\]
\[
\bar{R} = \sqrt{\frac{\Omega}{\rho}}
\]  

where \( \Psi_1 = \min \{ \lambda_{\min}(\mathbf{P}), (k_1 L_1)^{-1}(1 + \rho_E) \} \) and \( \Psi_2 = \max \{ \lambda_{\max}(\mathbf{P}), (k_1 L_1)^{-1}(1 + \rho_E) \} \).

In addition, uniform ultimate boundedness in Theorem 2(2) is also obeyed to
\[
d = \sqrt{\frac{\Psi_2}{\Psi_1}}
\]
\[
T(\tilde{d}, r) = \begin{cases} 
0, & \text{if } r \leq \tilde{d} \sqrt{\frac{\Psi_2}{\Psi_1}} \\
\frac{\Psi_2 r^2 - (\Psi_1^2 / \Psi_2) \tilde{d}^2}{\rho d (\Psi_1 / \Psi_2) - \Omega}, & \text{otherwise}
\end{cases}
\]

Appendix B
\[
\mathcal{L}_1 = \frac{(1 + \rho_E)^2}{8 \tilde{\rho}^3}
\]
\[
\mathcal{L}_2 = \frac{(1 + \rho_E)^2 L_1^{-1} L_2 D[\| \mathbf{z} \|^2]}{4 \tilde{\rho}^3} + \frac{(1 + \rho_E)^2 L_1^{-1} L_3 D[\| \mathbf{z} \|^2]}{2 \tilde{\rho}^3} - \frac{(1 + \rho_E)^2 D[V_{s2}]}{2 \tilde{\rho}^2}
\]
\[
\mathcal{L}_3 = -\frac{(1 + \rho_E)^2 D[V_{s1}]}{2 \tilde{\rho}^2}
\]
\[
\mathcal{L}_4 = \frac{D[V_s]}{2\rho} + \frac{(1 + \rho_E)^2 L_1^{-2} L_2^2}{8\rho^3} D\left[\|z\|^4\right] \\
+ \frac{(1 + \rho_E)^2 L_1^{-2} L_2^2}{2\rho^3} D\left[\|z\|^4\right] \\
+ \frac{(1 + \rho_E)^2 L_1^{-2} L_2 L_3^2}{2\rho^3} D\left[\|z\|^4\right] \\
- \frac{(1 + \rho_E) L_1^{-1} L_2}{2\rho^2} D\left[V_{s2}\|z\|^2\right] - \frac{(1 + \rho_E) L_1^{-1} L_3}{\rho^3} D\left[V_{s3}\|z\|^2\right] \\
\mathcal{L}_5 = \frac{D[V_{s1}]}{\rho} - \frac{(1 + \rho_E) L_1^{-1} L_2}{2\rho^2} D\left[V_{s1}\|z\|^2\right] \\
- \frac{(1 + \rho_E) L_1^{-1} L_3}{\rho^2} D\left[V_{s1}\|z\|^2\right] \\
\mathcal{L}_6 = \frac{D[V_{s1}]}{2\rho} \\
\mathcal{L}_7 = \frac{(1 + \rho_E)^2 L_1^{-1} L_2}{\rho^3} D\left[\|z\|^2\right] + \frac{2(1 + \rho_E)^2 L_1^{-1} L_3}{\rho^3} D\left[\|z\|^2\right] \\
D\left[\|z\|^2\right] - \frac{2(1 + \rho_E)}{\rho^2} D\left[V_{s2}\right] \\
\mathcal{L}_8 = \frac{2(1 + \rho_E)^2}{\rho^3} \\
\mathcal{L}_9 = \frac{(1 + \rho_E)^2}{\rho^3} \\
\mathcal{L}_{10} = -\frac{2(1 + \rho_E)}{\rho^2} D\left[V_{s1}\right] \\
\chi_1 = \frac{(1 + \rho_E)^2}{4\rho^2} \\
\chi_2 = \frac{(1 + \rho_E)^2 L_1^{-1} L_2}{2\rho^2} D\left[\|z\|^2\right] \\
+ \frac{(1 + \rho_E)^2 L_1^{-1} L_3}{\rho^2} D\left[\|z\|^2\right] \\
\chi_3 = \frac{(1 + \rho_E)^2 L_1^{-1} L_2^2}{4\rho^2} D\left[\|z\|^4\right] \\
+ \frac{(1 + \rho_E)^2 L_1^{-1} L_3^2}{\rho^2} D\left[\|z\|^4\right] \\
+ \frac{(1 + \rho_E)^2 L_1 L_2 L_3}{\rho^2} D\left[\|z\|^4\right] \\
\chi_4 = \frac{2(1 + \rho_E)^2 L_1^{-1} L_2}{\rho^2} D\left[\|z\|^2\right] + \frac{4(1 + \rho_E)^2 L_1^{-1} L_3}{\rho^2} D\left[\|z\|^2\right] \\
\chi_5 = \frac{2(1 + \rho_E)^2}{\rho^2}, \chi_6 = \frac{4(1 + \rho_E)^2}{\rho^2}
\]

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