Constraining CPT-even and Lorentz-violating nonminimal couplings with the electron magnetic and electric dipole moments

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We analyze some dimension-five CPT-even and Lorentz-violating nonminimal couplings between fermionic and gauge fields in the context of the Dirac equation. After evaluating the nonrelativistic Hamiltonian, we discuss the behavior of the terms under discrete symmetries and analyze the implied effects. We then use the anomalous magnetic dipole moment and electron electric dipole moment measurements to reach upper bounds of 1 part in $10^{20}$ and $10^{24}$ (eV $^{-1}$), improving the level of restriction on such couplings by at least 8 orders of magnitude. These upper bounds are also transferred to the Sun-centered frame by considering the Earth’s rotational motion.

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I. INTRODUCTION

The existence of the electric dipole of elementary particles may be seen as a consequence of asymmetric charge distribution along the spin $\mathbf{S}$ direction. Its measurement serves as an important probe for the development of several distinct theories and new interactions. The electric dipole interaction is represented by $d(\mathbf{σ} \cdot \mathbf{E})$, with $\mathbf{E}$ being the electric field, $\mathbf{σ}$ the spin operator, and $d$ the modulus of the electric dipole moment (EDM). An elementary (or not) particle can only present EDM when both parity ($P$) and time-reversal ($T$) symmetries are lost, $P(\mathbf{σ} \cdot \mathbf{E}) \rightarrow -(\mathbf{σ} \cdot \mathbf{E})$, $T(\mathbf{σ} \cdot \mathbf{E}) \rightarrow -(\mathbf{σ} \cdot \mathbf{E})$, so that the presence of EDM is associated with $CP$ and $T$ violation.

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In usual electrodynamics, the EDM stems from the Lagrangian contribution, $id(\bar{\psi}\sigma_{\mu\nu}\gamma_5F^{\mu\nu}\psi)$ [1–3], where $\psi$ stands for a Dirac spinor. Elementary particles really possess a tiny EDM, whose values can be used to constrain the magnitude of new couplings and theories that induce this kind of physical behavior [4].

Some interesting recent works have been proposed to use the EDM of particles and atoms as a key factor for constraining the theoretical possibilities of interaction. A few CPT-odd dimension-five interaction terms linear in the gauge field, $c^{\mu}\bar{\psi}\gamma^{\mu}\gamma_5F_{\mu\nu}\psi$, $d^{\mu}\bar{\psi}\gamma^{\mu}\gamma_5F_{\mu\nu}\psi$, $f^{\mu}\bar{\psi}\gamma^{\mu}\gamma_5F_{\mu\nu}\psi$, $g^{\mu}\bar{\psi}\gamma^{\mu}\gamma_5F_{\mu\nu}\psi$, were analyzed in Ref. [4], making the connection with the EDM generation and Lorentz-violating theories. The relation of these LV terms with anomalous magnetic moment (MDM) were considered in Ref. [5], which developed an analysis involving the splitting of the $g$ factors of a fermion and antifermion to constrain some of them. These CPT-odd terms constitute nonminimal couplings between fermions and photons. The term $g^{\mu}\bar{\psi}\gamma^{\mu}\gamma_5F_{\mu\nu}\psi$, for instance, was first proposed in Ref. [6] by means of the nonminimal derivative, $D_{\mu} = \partial_{\mu} + ieA_{\mu} + i\lambda\epsilon_{\mu\alpha\beta\gamma\delta}g^{\lambda}F^{\alpha\delta}$, defined in the context of the Dirac equation, $(i\gamma^{\mu}D_{\mu} - m)\Psi = 0$, where $g^{\mu}$ can be identified with the Carroll-Field-Jackiw four-vector, $(k_{AF})^{\mu} = (v_0, v)$, and $\lambda$ is the coupling constant. This particular coupling was already analyzed in several respects [7–9], including the radiative generation of CPT-violating effects [10]. See also Ref. [11] and the references therein.

The investigation of Lorentz-symmetry violation is actually a rich line of research, much developed in the framework of the Standard Model extension (SME) [12–17], whose developments have scrutinized the Lorentz-violating effects in distinct physical systems and served to state tight upper bounds on the LV coefficients, including photon-fermion interactions [18]. Lorentz-violating scenarios are connected with the breaking of CPT symmetry, although it is known that CPT violation does not necessarily lead to the loss of Lorentz invariance and vice versa in nonlocal theories [19]. LV theories are also related to models containing nonminimal couplings with higher-order derivative terms in what is called the nonminimal extension of the SME [20]. Alternative investigations with higher derivatives [21] and higher-dimension operators [22] have also been proposed. A recent and broad investigation about LV effects on the muon MDM was performed in Ref. [23], while LV connections with the neutron EDM were reported in Ref. [24]. The LV contributions that enhance the EDM of charged leptons stemming from a CPT-even term of the SME fermion sector, $d_{\mu\nu}\bar{\psi}\gamma^{\mu}\gamma_5\psi$, was developed in Ref. [25], with the form factor one-loop evaluation.

We have studied a dimension-five CPT-even nonminimal coupling in the context of the Dirac equation [26],

$$D_{\mu} = \partial_{\mu} + ieA_{\mu} + \frac{\lambda}{2}(K_{F})_{\mu\alpha\beta\gamma\delta}g^{\lambda}F^{\alpha\delta},$$  

not contained in the broader nonminimal extension of SME. Here, $(K_{F})_{\mu\alpha\beta\gamma\delta}$ is the CPT-even tensor of the SME electrodynamics. It has the same symmetries of the Riemann tensor and a double null trace, implying 19 components [16]. Inserted in the Dirac equation,
it provides a nonrelativistic Hamiltonian endowed with contributions to the EDM and to the MDM, namely \( \lambda (\sigma \cdot \mathcal{E}) \), \( \lambda (\sigma \cdot \mathcal{B}) \), which rendered upper bounds on the Lorentz-violating parameters as good as \( \lambda (K_F)_{33} \leq 10^{-11} (eV)^{-1} \). This nonminimal coupling radiatively generates the CPT-even gauge term of the SME Lagrangian, \((K_F)_{\mu\nu\beta} F^{\mu\nu} F^{\alpha\beta}\) [27]. Related studies arguing the generation of topological phases have been reported as well [28].

In the present paper, we propose an axial version of the CPT-even nonminimal coupling considered in Eq. (1) in the context of the Dirac equation. We first access the nonrelativistic regime, evaluating the associated Hamiltonian from the Dirac’s equation. We then analyze the effects induced on the magnetic dipole and electric dipole moment, using measures of the electron anomalous MDM and the electron EDM to limit the magnitude of the nonminimal LV terms at the stringent level of 1 part in 10\(^{20}\) (eV\(^{-1}\)) and 1 part in 10\(^{24}\) (eV\(^{-1}\)), respectively.

II. AN AXIAL CPT-EVEN LORENTZ-VIOLATING NONMINIMAL COUPLING

We begin by proposing a quantum electrodynamics where the spinors interact nonminimally with the electromagnetic field. This is implemented introducing a dimension-five axial and CPT-even nonminimal coupling,

\[
D_\mu = \partial_\mu + ieA_\mu + \frac{\lambda_A}{2} (K_F)_{\mu\nu\alpha\beta} \gamma^\nu \gamma^\alpha F^{\mu\nu},
\]

in the context of the Dirac equation, \((i\gamma^\mu D_\mu - m)\Psi = 0\). Here, \((K_F)_{\mu\nu\alpha\beta}\) is the CPT-even tensor of the SME that can be written in terms of four \(3 \times 3\) matrices \(K_{DE}, K_{DB}, K_{HE}, K_{HB}\), defined in Refs. [16] as

\[
\begin{align*}
(K_{DE})_{jk} &= -2(K_F)_{0j} \delta_{0k}, \\
(K_{HB})_{jk} &= \frac{1}{2} \epsilon_{jmq} \epsilon_{kln} (K_F)_{pqlm}, \\
(K_{DB})_{jk} &= -(K_{HE})_{kj} = \epsilon_{kpq} (K_F)_{0j} \delta_{0q}.
\end{align*}
\]

The symmetric matrices \(K_{DE}, K_{HE}\) contain the parity-even components and possess together 11 independent components, while \(K_{DB}, K_{HE}\) possess no symmetry, having together 8 components, representing the parity-odd sector of the tensor \((K_F)\). This classification holds only in the context of a minimal coupling QED. In the case of a QED with nonminimal interaction involving the tensor \((K_F)\), the parameters \(K_{DE}, K_{DB}, K_{HE}, K_{HB}\) could play distinct roles concerning parity and time reversal, as it appears in Table I.

The Dirac equation can be explicitly written as

\[
\left[ i\gamma^\mu \partial_\mu - e\gamma^\mu A_\mu - \frac{\lambda_A}{2} (K_F)_{\mu\nu\alpha\beta} \gamma^\nu \gamma^\alpha F^{\mu\nu} - m \right] \Psi = 0,
\]

with the constant \(\lambda_A\) highlighting the axial character of the coupling, and

\[
\sigma^{\mu\nu} = \frac{i}{2} (\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu) = \frac{i}{2} [\gamma^\mu, \gamma^\nu].
\]

Note that \((K_F)_{\mu\nu\alpha\beta} \gamma^\gamma \sigma^{\mu\nu} F^{\alpha\beta} \Psi\) constitutes a tensor generalization of the usual dipole \(\sigma^{\mu\gamma} \gamma_5 F^{\mu\nu} \Psi\) term. Using the parametrization (3)-(5), we obtain

\[
(K_F)_{\mu\nu\alpha\beta} \gamma^\gamma \sigma^{\mu\nu} F^{\alpha\beta} = 2i \Sigma^j (E^j - B^j) + 2\alpha^j \left( \mathbf{E}^j - \mathbf{B}^j \right),
\]

where we have introduced the following rotated fields:

\[
\begin{align*}
\mathbf{E}^k &= (K_{DE})_{kj} E^j, \\
\mathbf{B}^k &= (K_{DB})_{kj} B^j.
\end{align*}
\]

\[
\begin{align*}
\mathbf{E}^k &= (K_{HE})_{kj} E^j, \\
\mathbf{B}^k &= (K_{HB})_{kj} B^j.
\end{align*}
\]

with \(K_{DE}, K_{DB}, K_{HE}, K_{HB}\), being the Lorentz-violating matrices. Here, \(F_{0j} = E^j, F_{mn} = \epsilon_{mpn} B_p\), while \(\sigma^{0j} = i\alpha^j, \sigma^{ij} = \epsilon_{ijk} \Sigma^k\), and

\[
\alpha^i = \begin{pmatrix}
0 & \sigma^i \\
\sigma^i & 0
\end{pmatrix}, \quad \Sigma^k = \begin{pmatrix}
\sigma^k & 0 \\
0 & \sigma^k
\end{pmatrix},
\]

and \(\sigma = (\sigma_x, \sigma_y, \sigma_z)\) are the Pauli matrices. In the momentum coordinates, \(i\partial_\mu \rightarrow p_\mu\), the corresponding Dirac equation is

\[
\begin{align*}
&i\partial_\mu \Psi = \begin{pmatrix}
\alpha \cdot \pi + eA_0 + m \gamma^0 - \lambda_A \gamma^0 \Sigma^k Z^k + i\lambda_A \gamma^k \Sigma^k
\end{pmatrix} \Psi, \\
&Z = (\mathbf{E} - \mathbf{B}), \quad \hat{Z} = (\mathbf{E} - \mathbf{B}).
\end{align*}
\]

We point out that the presence of the factor \(\gamma^0\), multiplying the term \(\Sigma^k Z^k\), implies an effective contribution to the energy of the system, evading the Schiff’s theorem and allowing us to use the electron EDM data to constrain the magnitude of this axial coupling.

In order to investigate the role played by this nonminimal coupling, we should evaluate the nonrelativistic limit of the Dirac equation. Writing the spinor \(\Psi\) in terms of small \((\chi)\) and large \((\phi)\) two-spinors,

\[
\Psi = \begin{pmatrix}
\phi \\
\chi
\end{pmatrix},
\]

the Dirac equation (11) leads to two 2-component equations:

\[
\begin{align*}
&[E - eA_0 - m + \lambda_A \sigma^j \tilde{Z}^j] \phi - \left[ (\mathbf{E} - eA_0 + m) + \lambda_A \sigma^j \tilde{Z}^j \right] \chi = 0, \\
&[\sigma \cdot \pi - i\lambda_A \sigma^j \tilde{Z}^j] \phi - [E - eA_0 + m - \lambda_A \sigma^j \tilde{Z}^j] \chi = 0.
\end{align*}
\]
At first order in the Lorentz violating parameters, the following nonrelativistic Hamiltonian is achieved for the case of uniform fields:

\[
H_A = \frac{1}{2m} \left[ (p - eA)^2 - e(\sigma \cdot B) \right] + eA_0 - \lambda_A(\sigma \cdot \mathcal{Z}) + \frac{\lambda_A}{m} \mathcal{Z} \cdot (\sigma \times p) - \frac{e\lambda_A}{m} \mathcal{Z} \cdot (\sigma \times A).
\]

(16)

Concerning the new effects induced by this Hamiltonian, we are particularly interested in the terms that lead to corrections to the anomalous magnetic moment, \(\lambda_A(\sigma \cdot \mathcal{B})\), and to the electric dipole moment of the electron, \(\lambda_A(\sigma \cdot \mathcal{E})\). We also note that the term \(\mathcal{E} \cdot (\sigma \times p)\) is a generalization of the Rashba coupling term, while \(\sigma \cdot \mathcal{E}\) also generates a kind of electric Zeeman effect, in the total absence of a magnetic field.

A parallel can now be traced to the nonminimal coupling of Ref. [26], whose Dirac equation,

\[
\left[ i\gamma^\mu \partial_\mu - e\gamma^\mu A_\mu + \frac{\lambda}{2} (K_F)^{\mu\nu\alpha\beta} \sigma^{\mu
u} F^{\alpha\beta} - m \right] \Psi = 0,
\]

(17)

\[
i\partial_t \Psi = \left[ \sigma \cdot \pi + eA_0 + i\lambda_\pi \mathcal{Z} + \lambda_\sigma \Sigma^k \mathcal{Z}_k + m_0^2 \right] \Psi,
\]

yields the following LV corrections to the nonrelativistic Hamiltonian:

\[
H_{LV} = -\lambda(\sigma \cdot \mathcal{Z}) - \frac{\lambda}{m} \mathcal{Z} \cdot (\sigma \times p) + \frac{e\lambda}{m} \mathcal{Z} \cdot (\sigma \times A),
\]

(19)

which reveals an EDM term even in the absence of the \(\gamma_5\) matrix in the nonminimal coupling of Eq. (17).

We are ready to discuss the behavior of the modified Dirac equations (11) and (18) under the discrete symmetries \(C, P, T\). While the LV parameters of Eq. (18) obey the original classification of the 3 \(\times\) 3 matrices [16] under \(P\) and \(T\), the components of the axial coupling follow reversed behavior under such operators. Table I displays the response under the \(C, P, T\) operators of the axial coupling parameters of Hamiltonian (16), \(\lambda_A(\kappa_{DE})\), \(\lambda_A(\kappa_{DB})\), \(\lambda_A(\kappa_{HE})\), \(\lambda_A(\kappa_{HB})\), and the coupling parameters of Hamiltonian (19), \(\lambda(\sigma \cdot \mathcal{E})\), \(\lambda(\kappa_{DE})\), \(\lambda(\kappa_{DB})\), \(\lambda(\kappa_{HE})\), \(\lambda(\kappa_{HB})\). We notice that the elements \(\lambda_A(\kappa_{DE}), \lambda_A(\kappa_{HE})\) and \(\lambda_A(\kappa_{DB}), \lambda_A(\kappa_{HB})\) are now \(P\) odd and \(P\) even, respectively, instead of inheriting the usual behavior of the matrices \((\kappa_{DE}), (\kappa_{DB}), (\kappa_{HE}), (\kappa_{HB})\). We can also observe that the axial term, \(\lambda_A \Sigma^k \mathcal{E}_k\), that yields the nonrelativistic interaction \(\lambda_A(\sigma \cdot \mathcal{E})\), is \(P\) odd and \(T\) odd, compatible with the EDM character. The same holds for the term \(\Sigma^k \mathcal{E}_k\) of Hamiltonian (19).

| ANM | \(\lambda_A(\kappa_{DE})\) | \(\lambda_A(\kappa_{DB})\) | \(\lambda_A(\kappa_{HE})\) | \(\lambda_A(\kappa_{HB})\) |
|-----|-----------------|-----------------|-----------------|-----------------|
| \(C\) | +               | +               | +               | +               |
| \(P\) | -               | +               | +               | +               |
| \(T\) | -               | +               | +               | +               |
| \(CPT\) | +               | +               | +               | +               |
| NM  | \(\lambda(\kappa_{DE})\) | \(\lambda(\kappa_{DB})\) | \(\lambda(\kappa_{HE})\) | \(\lambda(\kappa_{HB})\) |
| \(C\) | +               | +               | +               | +               |
| \(P\) | +               | +               | +               | +               |
| \(T\) | +               | +               | +               | +               |
| \(CPT\) | +               | +               | +               | +               |

TABLE I: Complete classification under \(C, P, T\) for the coefficients of the axial nonminimal coupling (ANM) and usual nonminimal coupling (NM).

III. TREE-LEVEL MAGNETIC AND ELECTRIC DIPOLE MOMENTS

An interesting feature of the Hamiltonian (16) is that the term \(\lambda_A(\sigma \cdot (\mathcal{E} - \mathcal{B}))\) is able to generate a magnetic moment and an electric dipole moment. Investigations about LV effects on the electron anomalous MDM were developed in Refs. [29].

The electron magnetic moment is \(\mu = -g_B S\), where \(g_B = e/2m\), \(S\) is the spin operator and \(g = 2\) is the gyromagnetic factor. The anomalous magnetic moment of the electron is given by \(g = 2(1 + a)\), with \(a = \alpha/2\pi \approx 0.00116\) representing the deviation in relation to the usual case. Its most precise calculation is found in Ref. [30]. The magnetic interaction is \(H' = \mu_B (1 + a)(\sigma \cdot B)\). In accordance with very precise measurements [31], the experimental error on the electron MDM is at the level of 2.8 parts in \(10^{13}\), that is, \(\Delta a \leq 2.8 \times 10^{-13}\). In our case, the Hamiltonian (16) provides tree-level LV contributions to the usual \(g = 2\) gyromagnetic factor, which cannot be larger than \(\Delta a\). The total magnetic interaction in Eq. (16) is \(\mu_B (\sigma \cdot B) + \lambda_A(\sigma \cdot \mathcal{B})\). For the magnetic field along the \(z\) axis, \(B = B_0 z\), and a spin-polarized configuration in the \(z\) axis, this interaction assumes the form

\[
\mu_B \left[ 1 + \frac{2m}{e} \lambda_A(\kappa_{DB})_{33}\right] (\sigma_z B_0),
\]

(20)

where \(\lambda_A(\kappa_{DB})_{33}\) stands for the tree-level LV correction that should be smaller than \(\Delta a\), so that

\[
|\lambda_A(\kappa_{DB})_{33}| \leq 2.3 \times 10^{-20} (\text{eV})^{-1},
\]

(21)

represents an improvement by a factor \(\approx 10^{10}\) on the strength of the corresponding bound of Ref. [26]. Alternatively, if we use the analysis of Ref. [5], based on the splitting of the \(g\) factor of electron and positron, we conclude that \(\lambda_A(\kappa_{DB})_{33} \leq 2.3 \times 10^{-12} \mu_B\), leading to

\[
|\lambda_A(\kappa_{DB})_{33}| \leq 1.9 \times 10^{-19} (\text{eV})^{-1}.
\]

(22)

We should now discuss the EDM term, \(\lambda_A(\sigma \cdot \mathcal{E})\), which can be written as

\[
\lambda_A(\sigma \cdot \mathcal{E}) = \lambda_A(\kappa_{DE}) \frac{2m}{3} (\sigma \cdot \mathcal{E}).
\]

(23)
The strongest limitations to be achieved involve the electron EDM, $d_e$, that is the minor known one. The magnitude of $d_e$ has been constrained with increasing precision [1, 32, 33], reaching the level $|d_e| \leq 1.1 \times 10^{-29}$e.m or $|d_e| \leq 4.7 \times 10^{-24}$ (eV)$^{-1}$. Very recent experiments [34] improved this limit as $|d_e| \leq 8.7 \times 10^{-31}$e.m, or

$$|d_e| \leq 3.8 \times 10^{-25} \text{ (eV)}^{-1}.$$  \hspace{1cm} (24)

Considering this experimental measure, we attain the following upper bound:

$$|\lambda_A(\kappa_{DE})_{ii}| \leq 1.1 \times 10^{-24} \text{ (eV)}^{-1},$$  \hspace{1cm} (25)

surpassing the best magnetic bound (21) by the factor $10^4$. We remark that this limit is at least 8 orders of magnitude better than the bounds first attained in the analogue nonminimal coupling of Ref. [26].

We know that the nonminimal coupling of Ref. [26] yields an EDM term even without containing a $\gamma_5$ matrix. Therefore, we can also improve the upper bounds on some of its components by the same procedure. We begin with the term $\lambda(\sigma \cdot \hat{E})$ of Eq. (19) using the electron EDM. In this case, as the matrix $\kappa_{HE}$ is traceless, we should choose a particular direction for the electric field, $\vec{E} = E_0\hat{x}$, which yields

$$\lambda(\sigma \cdot \hat{E}) = \lambda(\kappa_{HE})_{11} (\sigma_x E_0),$$  \hspace{1cm} (26)

implying

$$\lambda(\kappa_{HE})_{11} \leq 3.8 \times 10^{-25} \text{ (eV)}^{-1},$$  \hspace{1cm} (27)

which represents an improvement of the corresponding previous bound of [26] by a factor $\sim 10^8$.

As for the term $\lambda(\sigma \cdot \hat{B})$ of Eq. (19), we use the anomalous electron EDM measures to improve the previous bound by a factor $\sim 10^{10}$, that is,

$$\lambda(\kappa_{HB})_{33} \leq 2.3 \times 10^{-20} \text{ (eV)}^{-1}.$$  \hspace{1cm} (28)

In these evaluations, we have used natural units: $m_e = 5.11 \times 10^5$ eV, $e = \sqrt{1/137}$, and $\hbar = 5.06 \times 10^{-6}$ (eV)$^{-1}$.

**IV. SIDEREAL VARIATIONS**

Strictly speaking, neither the Earth nor the Sun is an ideal inertial reference frame (RF). Nevertheless, the latter is closer to being one, as its rotation period, around the center of the galaxy, is about 230 million years. Since the Lorentz-violating (LV) tensors are constant for an inertial RF, the time dependence of their components is expected in experiments performed on the Earth, exhibiting a periodic variation associated with the Earth’s rotation time ($1/\Omega$). A reasonable choice for an inertial RF is the Sun, with the $z$ axis matching the direction of the Earth’s rotation axis and the $x$ axis pointing from the Earth’s center to the Sun on the vernal equinox in 2000. For more details, see Refs. [35]. In experiments up to a few weeks long, it is possible to neglect the Earth’s motion around the Sun, so that the transformation on the LV tensors is a mere rotation, due to the Earth’s rotation around its own axis. According to this, a 3-component rank-1 tensor, on the spinning Earth’s RF, transforms as

$$V_{i}^{\text{Lab}} = R_{ik} V_{k}^{\text{Sun}},$$  \hspace{1cm} (29)

where

$$R_{ij} = \begin{pmatrix} \cos \chi \cos \Omega t & -
\sin \Omega t & \sin \chi \\ 
\sin \Omega t & \cos \chi \cos \Omega t & -\sin \chi \\ -
\sin \chi \cos \Omega t & \sin \chi \sin \Omega t & 0 \end{pmatrix},$$  \hspace{1cm} (30)

in which $\chi$ is the colatitude of the lab and $\Omega = 2\pi/23h56$ min is the Earth’s rotation angular velocity. A rank-2 tensor transforms according to the rule

$$A_{ij}^{\text{Lab}} = R_{ik} R_{jl} A_{kl}^{\text{Sun}}.$$  \hspace{1cm} (31)

In the literature [35], the Earth-based frame, where the laboratory is located, has axis $x, y, z$, while the Sun-centered frame has $X, Y, Z$ as axis. Hence, $A_{ij}^{\text{Lab}} = A_{ij}^{X,Y,Z}$, $A_{ij}^{\text{Lab}} = A_{ij}^{(x,y,z)}$. Applying this transformation to the 33-component of the matrix, we obtain

$$(A)_{33}^{\text{Lab}} = \begin{pmatrix} \sin^2 \chi \cos^2 \Omega t \lambda_{11}^{\text{Sun}} + \sin^2 \chi \cos \Omega t \sin \Omega t \lambda_{22}^{\text{Sun}} + \sin^2 \chi \sin \Omega t \cos \Omega t \lambda_{33}^{\text{Sun}} \\
\sin^2 \chi \sin^2 \Omega t \lambda_{22}^{\text{Sun}} + \sin^2 \Omega t \sin \chi \cos \chi \lambda_{33}^{\text{Sun}} \\
\cos \Omega t \cos \chi \sin \chi \lambda_{33}^{\text{Sun}} + \sin \Omega t \cos \chi \sin \chi \lambda_{33}^{\text{Sun}} + \cos^2 \chi \lambda_{33}^{\text{Sun}} \end{pmatrix},$$  \hspace{1cm} (32)

whose time average leads simply to

$$\langle A_{zz}^{\text{Lab}} \rangle = \frac{1}{2} \sin^2 \chi A_{11}^{\text{Sun}} + \frac{1}{2} \sin^2 \chi A_{22}^{\text{Sun}} + \cos^2 \chi A_{33}^{\text{Sun}}.$$  \hspace{1cm} (33)

With these transformations, it is possible to write the upper bounds here attained in terms of combinations of the Sun-based frame ones. The bound (21) yields

$$|\langle \lambda_A(\kappa_{DE})_{ii}^{\text{Lab}} \rangle| = |\langle \lambda_A(\kappa_{DE})_{ii}^{\text{Sun}} \rangle| \leq 1.1 \times 10^{-24} \text{ (eV)}^{-1}.$$  \hspace{1cm} (35)

As the trace is invariant under rotation, the bound (25) reads as

$$|\langle \lambda_A(\kappa_{DE})_{ii}^{\text{Lab}} \rangle| = |\langle \lambda_A(\kappa_{DE})_{ii}^{\text{Sun}} \rangle| \leq 1.1 \times 10^{-24} \text{ (eV)}^{-1}.$$  \hspace{1cm} (35)

Finally, the bounds (27) and (28) then become

$$|\langle \lambda_A(\kappa_{HE})_{ii}^{\text{Lab}} \rangle| = |\langle \lambda_A(\kappa_{HE})_{ii}^{\text{Sun}} \rangle| \leq 3.8 \times 10^{-25} \text{ (eV)}^{-1}.$$  \hspace{1cm} (36)
\[
\left| \frac{1}{2} (\lambda_{\kappa HB})_{XX}^\text{Sun} \sin^2 \chi + \frac{1}{2} (\lambda_{\kappa HB})_{YY}^\text{Sun} \sin^2 \chi \right| + (\lambda_{\kappa HB})_{ZZ}^\text{Sun} \cos^2 \chi \leq 2.3 \times 10^{-20} \text{ (eV)}^{-1}.
\]

For attaining a clearer scenario of constraining on the Sun-based components, it is necessary to know the co-latitude \( \chi \) and details of the experimental device, as the electric alignment.

V. CONCLUSIONS

We have proposed an axial CPT-even, dimension-five, and Lorentz-violating nonminimal coupling between fermionic and gauge fields, involving the tensor \( (K_F)_{\alpha\beta} \) of the gauge sector of the SME, in the context of the Dirac equation. The nonrelativistic Hamiltonian was carried out, revealing corrections to the anomalous magnetic moment and to electron EDM. Effects of these terms on the electron EDM and on the anomalous magnetic moment of electrons and positrons have yielded upper bounds as tight as \( |\lambda_A (\kappa_{DE})_{ii}| \leq 1.1 \times 10^{-24} \text{ (eV)}^{-1} \) and \( |\lambda_A (\kappa_{DB})_{33}| \leq 2.3 \times 10^{-20} \text{ (eV)}^{-1} \), respectively, which can be also expressed in terms of the Sun-based frame coefficients. Using the same procedure, we have shown that it is possible to improve the previous bounds on the nonminimal coupling of Ref. [26] by the factors \( 10^8 \) and \( 10^{10} \).

We remark that the CPT-even nonminimal coupling proposed in Eq. (6) avoids the Schiff’s theorem, implying physical effects in the energy of the system, \( \Delta U = -d \cdot E \), as explained in Ref. [3], which allows to use the electron EDM to attain the tightest upper bounds on the nonminimal LV parameters. Such evasion, however, is not fulfilled by Dirac particles interacting only via electromagnetic CPT-odd nonminimal couplings, requiring the use of the EDM of composite particles, as discussed in Ref. [4]. As composite particles have larger EDM, it leads to weaker upper bounds on the quark sector by at least a factor \( 10^4 \) (when compared with the ones attained with electron EDM).

The bounds found on the axial coupling coefficients, \( \lambda_A (\kappa_{DB}) \), should not be confused with the upper bounds on the parameters \( \lambda (\kappa_{HB}) \), \( \lambda (\kappa_{HE}) \) of the first CPT-even nonminimal coupling [26]. They constitute restrictions on different but analogue interactions, both restrictions being distinct from the known upper bounds on the CPT-even parameters of the SME [16].

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