REDUCED-RANK ADAPTIVE CONSTRAINED CONSTANT MODULUS BEAMFORMING ALGORITHMS BASED ON JOINT ITERATIVE OPTIMIZATION OF FILTERS

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ABSTRACT

This paper proposes a reduced-rank scheme for adaptive beamforming based on the constrained joint iterative optimization of filters. We employ this scheme to devise two novel reduced-rank adaptive algorithms according to the constant modulus (CM) criterion with different constraints. The first devised algorithm is formulated as a constrained joint iterative optimization of a projection matrix and a reduced-rank filter with respect to the CM criterion subject to a constraint on the output response. The constrained constant modulus (CCM) expressions for the projection matrix and the reduced-rank weight vector are derived, and a low-complexity adaptive algorithm is presented to jointly estimate them for implementation. The second proposed algorithm is extended from the first one and implemented according to the CM criterion subject to a constraint on the array response and an orthogonal constraint on the projection matrix. The Gram-Schmidt (GS) technique is employed to achieve this orthogonal constraint and improve the performance. Simulation results are given to show superior performance of the proposed algorithms in comparison with existing methods.

Index Terms—Beamforming techniques, antenna array, constrained constant modulus, reduced-rank methods.

1. INTRODUCTION

Adaptive beamforming technology is of paramount importance in numerous signal processing applications such as radar, wireless communications, and sonar [1], [2]. Among various beamforming techniques, the beamformers based on the constrained minimum variance (CMV) criterion [3] are prevalent and minimize the contribution of the total output power while maintaining the gain along the direction of the signal of interest (SOI). Another alternative beamformer design is performed according to the constrained constant modulus (CCM) [3] criterion, which is a positive measure of the beamformer output deviating from a constant modulus condition. Compared with the CMV, the CCM beamformers exhibit superior performance in many severe scenarios (e.g., steering vector mismatch).

Many adaptive algorithms [3] have been developed according to the CMV and CCM criteria for implementation. A simple and popular one is the stochastic gradient (SG) method [4], [6]. However, the performance of the SG-based algorithms is sensitive to the step size, the number of interferers and sensor elements, and the eigenvalue spread [6]. For improving the performance, reduced-rank filtering has been introduced into beamforming in order to project the received signal onto a lower dimension subspace and perform the filter optimization within this subspace. This technique shows a fast convergence rate and improves tracking ability in situations where the number of sensor elements is large [7]. The Multi-stage Wiener filter (MSWF) [8] and the auxiliary-vector filtering (AVF) [9] are two excellent approaches in this area. Employing these reduced-rank schemes, the CMV beamformers reach improved performance but suffer from the heavy computational cost and instability. A joint iterative optimization scheme [10] was presented recently with a simple adaptive implementation for reducing the complexity and improving the tracking ability.

Considering the fact that the CCM-based beamformers outperform the CMV ones for constant modulus constellations, we propose two adaptive reduced-rank algorithms according to the CCM criterion by employing a proposed reduced-rank scheme, which is based on the constrained joint iterative optimization filters. The proposed algorithms are implemented according to the constant modulus (CM) criterion with different constraints. The first one is formulated as a constrained joint iterative optimization of a projection matrix and a reduced-rank filter with respect to the CM criterion subject to a constraint on the array response. The projection matrix projects the received signal onto a lower dimension, which is then processed by the reduced-rank filter for the array output. The CCM expressions for the projection matrix and the reduced-rank filter are derived, and a simple efficient algorithm is presented to jointly estimate them for implementation. The second proposed algorithm is extended from the first one and implemented according to the CM criterion subject to a constraint on the array response and an orthogonal constraint on the projection matrix. We employ the Gram Schmidt (GS) technique [11] to achieve this orthogonal constraint for the projection matrix reformulation. The performance of the second algorithm outperforms the first one. Simulation results are given to demonstrate the superior performance and stability achieved by the proposed algorithms versus the existing algorithms in typical scenarios.

The remainder of this paper is organized as follows: we outline a system model for beamforming in Section 2. Based on this model, the problem statement is presented. The proposed scheme, optimization and filter expressions are considered in Section 3. Section 4 derives the proposed adaptive reduced-rank algorithms. The GS technique is briefly introduced in this part. Simulation results are provided and discussed in Section 5, and conclusions are drawn in Section 6.

2. SYSTEM MODEL AND PROBLEM STATEMENT

2.1. System Model

Let us suppose that q narrowband signals impinge on an uniform linear array (ULA) of m (m ≥ q) sensor elements. The sources are assumed to be in the far field with directions of arrival (DOAs) \( \theta_0, \ldots, \theta_{q-1} \). The \( i \)th snapshot’s vector of sensor array outputs
\( \mathbf{x}(i) \in C^{m \times 1} \) can be modeled as
\[
\mathbf{x}(i) = \mathbf{A}(\theta)\mathbf{s}(i) + \mathbf{n}(i), \quad i = 1, \ldots, N
\]  
(1)
where \( \theta = [\theta_0, \ldots, \theta_{q-1}]^T \in C^{q \times 1} \) is the signal DOAs, \( \mathbf{A}(\theta) = [\mathbf{a}(\theta_0), \ldots, \mathbf{a}(\theta_{q-1})] \in C^{m \times q} \) comprises the signal direction vectors \( \mathbf{a}(\theta_k) = [1, e^{-2\pi j \cos \theta_k}, \ldots, e^{-2\pi j (m-1) \cos \theta_k}]^T \in C^{m \times 1}, (k = 0, \ldots, q - 1) \), where \( \lambda_c \) is the wavelength and \( d \) is the inter-element distance of the ULA (\( d = \lambda_c/2 \) in general), and to avoid mathematical ambiguities, the direction vectors \( \mathbf{a}(\theta_k) \) are considered to be linearly independent. \( \mathbf{s}(i) \in C^{q \times 1} \) is the source data, \( \mathbf{n}(i) \in C^{m \times 1} \) is temporally white sensor noise, which is assumed to be a zero-mean spatially and Gaussian process. \( N \) is the observation size of snapshots, and \( (\cdot)^T \) stands for transpose. The output of a narrowband beamformer is given by
\[
y(i) = \mathbf{w}^H(i)\mathbf{x}(i)
\]  
(2)
where \( \mathbf{w}(i) = [w_r(i), \ldots, w_m(i)]^T \in C^{m \times 1} \) is the complex weight vector, and \( (\cdot)^H \) stands for Hermitian transpose.

2.2. Problem Statement

Let us consider the full-rank CCM optimization filter for beamforming, which can be computed by solving the following optimization problem
\[
\mathbf{w}_{opt} = \arg \min_{\mathbf{w}} \mathbb{E}\{[|y(i)|^2 - R_p]^2\}, \quad i = 1, \ldots, N
\]  
subject to \( \mathbf{w}^H(i)\mathbf{a}(\theta_0) = 1 \).
(3)
where the constant \( R_p \) is suitably chosen to guarantee that the weight solution is close to the global minimum and the constraint is set to ensure a closed-form solution. The quantity \( \theta_0 \) is the direction of the SOI, \( \mathbf{a}(\theta_0) \) denotes the normalized steering vector of the desired signal, and in general, \( p = 2 \) is selected to consider the optimization as the expected deviation of the squared modulus of the array output to a constant, say \( R_p = 1 \). The CCM beamformer minimizes the contribution of undesired interference while maintaining the gain along the look direction to be constant. Using the method of Lagrange multipliers to solve the optimization problem in (3), the weight expression is
\[
\mathbf{w} = \frac{R^{-1}\mathbf{a}(\theta_0)}{\mathbf{a}^H(\theta_0)R^{-1}\mathbf{a}(\theta_0)}
\]  
(4)
where \( R = \mathbb{E}[|y(i)|^2 - 1]|\mathbf{x}(i)|\mathbf{x}^H(i) \in C^{m \times m} \) is the expected cross correlation matrix between \( \mathbf{x}(i) \) and \( y(i) \). The complexity can be high due to the existence of the covariance matrix inverse. In practice, \( R \) is not available but has to be estimated, which may result in the poor convergence and tracking ability when \( m \) is large. Note that \( R \) depends on \( y(i) \), which is a function of current \( \mathbf{w}(i) \). By initializing \( \mathbf{w}(i) \) and estimating a prior \( y(i) \), we can estimate \( R \) and get the weight solution for each snapshot.

3. PROPOSED REDUCED-RANK SCHEME AND CCM FILTERS DESIGN

In this section, we employ a reduced-rank scheme to introduce two optimization problems according to the CM criteria subject to different constraints. The reduced-rank scheme is based on a constrained joint iterative optimization of a projection matrix and a reduced-rank filter. The CCM expressions of the projection matrix and the reduced-rank weight vector are derived.
where \( c(i) = |y(i)|^2 - 1 \), \( R = E[2c(i)x(i)x^H(i)] \) is the expected cross correlation matrix, and \( \bar{R}_w = E[\bar{w}(i)\bar{w}^H(i)] \) is the expected reduced-rank weight matrix. Both \( R \) and \( \bar{R}_w \) need to be estimated by sample-averaging in practice. Note that \( R \) depends on \( y(i) \), which is a function of \( T_{r}(i) \) and \( w(i) \). By initializing \( T_{r}(i) \) and \( w(i) \) and using a prior \( y(i) \), we can estimate \( R \).

Rearranging the second equation of (10) to represent \( T_{r}(i) \), which is then substituted into the constraint in (7) for solving the expected reduced-rank weight matrix. Both \( \text{rank} \) cross correlation matrix, which is estimated by sample-averaging, where the subscript "p" means the proposed and \( w(i) \) (12)

![Image](image.png)

where \( \bar{w}(i) = [\bar{a}(\theta_0)R^{-1}\bar{a}(\theta_0)]^{-1}R^{-1}\bar{a}(\theta_0) \) (13)

where \( \bar{a}(\theta_0) = T_{r}^H(i)\bar{a}(\theta_0) \in C^{r \times 1} \) is the projected steering vector of the SOI.

The update equations (11) for the projection matrix and (13) for the reduced-rank weight vector depend on each other and so are not closed-form solutions. It is necessary to iterate \( T_{r} \) and \( \bar{w} \) with initial values for implementation. Therefore, the initialization is not only for obtaining a prior \( y \) but starting the iteration of the proposed scheme. The projection matrix creates a connection between the full-rank input vectors and the reduced-rank ones, whereas the reduced-rank filter recovers the transmitted signal. They are jointly updated to solve the CCM optimization problem i), i.e., the so-called “joint iterative optimization” (JIO).

### 4. DEVELOPMENT OF ADAPTIVE ALGORITHMS

#### 4.1. Proposed Adaptive SG Algorithm for Problem i)

We describe a simple adaptive algorithm for implementation of the proposed reduced-rank scheme based on the optimization problem i). Fixing \( \bar{w}(i) \) and \( T_{r}(i) \), respectively, taking the instantaneous gradient of (10) with respect to \( T_{r}(i) \) and \( \bar{w}(i) \), and setting them equal to null, we obtain

\[
\nabla J_{p,T_{r}} = 2c(i)y^*(i)x(i)\bar{w}^H(i) + \lambda_{p,T_{r}}\bar{a}(\theta_0)\bar{w}^H(i) \tag{14}
\]

\[
\nabla J_{p,\bar{w}} = 2c(i)y^*(i)T_{r}^H(i)x(i) + \lambda_{p,\bar{w}}T_{r}^H(i)\bar{a}(\theta_0) \tag{15}
\]

where the subscript “p” means the proposed and \( (\cdot)^* \) denotes complex conjugate.

Following the gradient rules \( T_{r}(i + 1) = T_{r}(i) - \mu_{T_{r}}\nabla J_{p,T_{r}} \) and \( \bar{w}(i + 1) = \bar{w}(i) - \mu_{\bar{w}}\nabla J_{p,\bar{w}} \), substituting (14) and (15) into them, respectively, and solving the Lagrange multipliers \( \lambda_{p,T_{r}} \) and \( \lambda_{p,\bar{w}} \) by employing the constraint in (7), we obtain the iterative solutions in the form

\[
T_{r}(i + 1) = T_{r}(i) - \mu_{T_{r}}c(i)y^*(i)\left[\bar{x}(i)\bar{w}^H(i) - \bar{a}(\theta_0)\bar{w}^H(i)\bar{a}(\theta_0)\right] \tag{16}
\]

\[
\bar{w}(i + 1) = \bar{w}(i) - \mu_{\bar{w}}c(i)y^*(i)\left[I - \frac{\bar{a}(\theta_0)\bar{a}^H(\theta_0)}{\bar{a}^H(\theta_0)\bar{a}(\theta_0)}\right]\bar{x}(i) \tag{17}
\]

where \( \mu_{T_{r}} \) and \( \mu_{\bar{w}} \) are the corresponding step sizes, which are small positive values. The projection matrix \( T_{r}(i) \) and the reduced-rank weight vector \( \bar{w}(i) \) are jointly updated. The output \( y(i) \) at time instant \( i \) can be estimated after each joint optimization procedure with respect to the CCM criterion. We denote this proposed algorithm (16) and (17) as JIO-CCM.

#### 4.2. Extended Algorithm for Problem ii)

Now, we consider the optimization problem ii). As explained before, the constraint is added to orthogonalize a set of vectors \( t_{l}(i) \) for the performance improvement. We employ the Gram-Schmidt (GS) technique (11) to realize this constraint. Specifically, the adaptive SG algorithm in (15) is implemented to obtain \( T_{r}(i + 1) \). Then, the GS process is performed to reorthogonalize the projection matrix, which is (11)

\[
t_{l,ort}(i) = t_{l}(i) - \sum_{j=1}^{l-1}\text{proj}_{t_{j,ort}(i)}t_{l}(i) \tag{18}
\]

where \( t_{l,ort}(i) \) is the normalized orthogonal vector after the GS process and \( \text{proj}_{t_{j,ort}(i)}t_{l}(i) = t_{j,ort}(i)\bar{t}_{j,ort}(i)\bar{t}_{j,ort}(i)^{*}t_{l}(i) \) is a projection operator.

The reformulated projection matrix \( T_{r,ort}(i) \) is constructed when we obtain a set of orthogonal \( t_{l,ort}(i) \), \( l = 1, \ldots, r \). By employing \( T_{r,ort}(i) \) to get \( \bar{x}(i, \bar{a}(\theta_0)) \), and jointly update with \( \bar{w}(i + 1) \) in (17), the performance can be further improved. Simulation results will be given for showing this result. We denote this GS version algorithm as JIO-CCM-GS, which is performed by computing (16), (18), and (17).

#### 4.3. Computational Complexity

The computational complexity with respect to the existing and proposed algorithms is evaluated according to additions and multiplications. The complexity comparison is listed in Table 1. The complexity of the proposed JIO-CCM and JIO-CCM-GS algorithms increases with the multiplication of \( rm \), specifically \( m \) since the rank \( r \) is selected around a small range that is much less than \( rm \) in large arrays’ conditions without performance degradation. This fact will be shown in the simulation. This complexity is about \( r \) times higher than the full-rank algorithms [5, 6], slightly higher than the recent JIO-CMV based algorithm [10], but much lower than the MSWF-based [8, 13], and AVF [29] methods.

| Algorithm       | Additions | Multiplications |
|-----------------|-----------|-----------------|
| Full-Rank-CMV   | \( 3m - 1 \) | \( 4m + 1 \) |
| Full-Rank-CCM   | \( 3m \)   | \( 4m + 3 \) |
| MSWF-CMV        | \( rm^2 + rm + m \) | \( rm^2 + m^2 + 2rm + 2r + 4 \) |
| MSWF-CCM        | \( rm^2 + rm + m \) | \( rm^2 + m^2 + 2rm + 2r + 4 \) |
| AVF             | \( r(4m^2 + m - 2) \) | \( r(5m^2 + 3m) + 5m - 1 + 8m + 2m \) |
| JIO-CMV         | \( 4rm + m + 2r - 3 \) | \( 4rm + m + 7r + 3 \) |
| JIO-CMV-GS      | \( 7rm - m - 1 \) | \( 7rm - 2m + 8r + 2 \) |
| JIO-CCM         | \( 4rm + m + 2r - 2 \) | \( 4rm + m + 7r + 6 \) |
| JIO-CCM-GS      | \( 7rm - m \) | \( 7rm - 2m + 8r + 5 \) |
5. SIMULATIONS

Simulations are performed by an ULA containing $m = 32$ sensor elements with half-wavelength interelement spacing. We compare the proposed JIO-CCM and JIO-CCM-GS algorithms with the full-rank [5], MSWF [8, 13], and AVF [7] methods and in each method, the CMV and CCM criterions are considered with SG algorithms for implementation. A total of $K = 1000$ runs are used to get the curves. In all experiments, the BPSK source power (including the desired user and interferers) is $\sigma_i^2 = \sigma_d^2 = 1$ and the input SNR $= 10$ dB with spatially and temporally white Gaussian noise.

In Fig. 2 we consider the presence of $q = 7$ users (one desired) in the system. The projection matrix and the reduced-rank weight vector are initialized with $T_r(0) = [T_r^r \ 0_r^{T \times (m-r)}]$ and $\bar{w}(0) = (T_r^H(0)a(\theta_0))/(||T_r^H(0)a(\theta_0)||^2)$ to ensure the constraint in 7. The rank is $r = r_{gs} = 5$ for the proposed JIO-CCM and JIO-CCM-GS algorithms. The expected matrix $R$ used in the MSWF and AVF is estimated by sample-averaging. Fig. 3 shows that all output SINR curves increase to steady-state as increase of the snapshots. The joint optimization based algorithms have superior performance as compared with the full-rank, MSWF, and AVF methods. Their GS version algorithms enjoy further developed performance comparing with corresponding JIO-CMV and JIO-CCM methods. The proposed JIO-CCM and JIO-CCM-GS algorithms outperform the existing methods in the output performance. Checking the convergence, the proposed algorithms are slightly slower than the AVF, which is least squares (LS)-based, and much faster than the other methods.

![Fig. 2. Output SINR versus the number of snapshots with $m = 32$, $q = 7$, SNR= 10 dB, $\mu_{T_r} = 0.002$, $\mu_w = 0.001$, $\mu_{T_r,gs} = 0.003$, $\mu_{w,gs} = 0.0007$.](image)

In Fig. 4 we keep the same scenario as that in Fig. 2 and check the rank selection for the existing and proposed algorithms. The number of snapshots is fixed to $N = 500$. The optimum choices for the proposed algorithms are $r = r_{gs} = 5$, which are comparatively lower than most existing algorithms, but reach superior performance. We also checked the change of these values for different array sizes and data records, and verified that they are nearly invariant, which saves computation cost.

Finally, the mismatch (steering vector error) condition is analyzed in Fig. 4. Here, the number of users is $q = 10$, including one desired user. In Fig. 4(a), the exact DOA of the SOI is used in the algorithms. The output performance of the proposed algorithms is better than those of the existing algorithms, and the convergence is a little slower than that of the AVF algorithm, but higher than the others. In Fig. 4(b), we set the DOA of the SOI estimated by the receiver to be 2° away from the actual direction. This indicates that the mismatch problem induces performance degradation to all the analyzed algorithms. The CCM-based methods are more robust to this scenario than the CMV-based ones. The proposed algorithms still retain outstanding performance compared with other techniques.

![Fig. 3. Output SINR versus rank ($r$) with $m = 32$, $q = 7$, SNR= 10 dB, $N = 500$, $\mu_{T_r} = 0.002$, $\mu_w = 0.001$, $\mu_{T_r,gs} = 0.003$, $\mu_{w,gs} = 0.0007$.](image)

![Fig. 4. Output SINR versus the number of snapshots with $m = 32$, $q = 10$, SNR= 10 dB, $\mu_{T_r} = 0.002$, $\mu_w = 0.001$, $\mu_{T_r,gs} = 0.003$, $\mu_{w,gs} = 0.0007$ for (a) ideal steering vector condition; (b) steering vector mismatch 2°.](image)
6. CONCLUDING REMARKS

We proposed a reduced-rank scheme based on the joint iterative optimization filters for beamforming and devised two adaptive reduced-rank algorithms according to the CCM criteria, namely, JIO-CCM and JIO-CCM-GS. They are implemented by employing a low-complexity adaptive algorithm to jointly update the projection matrices and reduced-rank filters. The JIO-CCM-GS algorithm, by reformulating the projection matrix, achieves faster convergence and better performance than the JIO-CCM. The GS technique is employed to realize the reformulation. The devised algorithms, compared with the existing methods, show preferable performance and fast convergence in the studied scenarios.

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