Homological invariants are a powerful tool in knot theory with the downside of being often hard to compute in practice. In this paper the authors are interested in Khovanov-Rozansky triply-graded link homology. They develop a new computational method that is very effective in the case of torus links. In particular this allows them to show new evidence to a conjecture relating Khovanov-Rozansky homology to algebraic geometry, namely Hilbert schemes. Also this method grew into a more general procedure developed in subsequent papers called categorical diagonalization.

Their method is not based on Khovanov and Rozansky’s original construction of the triply-graded link homology which uses matrix factorizations, but on an alternative description due to Khovanov that involves Hochschild homology and Rouquier’s faithful categorical representation of the braid group in the homotopy category of the category of Soergel bimodules. This latter bimodule category categorifies the Iwahori-Hecke algebra, quotient of the group algebra of the braid group.

The reason why their method is adapted to torus links is because these are closures of some peculiar braids called full twists. These full twists can be expressed using so-called Young-Jucys-Murphy braids and the image of those in the Hecke quotient play an important role in its representation theory. More precisely they appear in some recurrence relations involving projectors onto the sign representation of the Hecke algebra, the Jones-Wenzl projectors. These projectors turn out to be images in the Hecke quotient of braids for which the authors provide an explicit categorical representative of the form of a complex of Soergel bimodules. Indeed the recursive relations translate at the categorical level to exact triangles, which implies that the complexes involved can be constructed recursively as mapping cones. As an outcome of this procedure, they obtain a filtered complex representing the full-twist. They proceed with a description of the Hochschild cohomology complex of the full-twist complex. It turns out that it has nice properties (parity considerations) allowing to compute its homology and hence to make explicit the triply-graded homology of \((n, n)\)-torus links.

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