Numerical study of non-linear deformation processes of reinforced concrete structures elements interacting with soils

I S Balafendieva*, A A Notfullina and I A Galimullin
Kazan Federal University, 18 Kremlyovskaya Street, Kazan, 420008, Russian Federation

* e_xo@mail.ru

Abstract. In the framework of an approach based on the mechanics of a transforming structure, a method is proposed for calculating the deformation processes of elastoplastic reinforced structures interacting with soils of complex physical nature. The technique is based on the use of the defining relations, connecting the increments of the true stresses and strains. As an illustration of the performance of the proposed methodology, some results of the calculation of the stress-strain state of the walls of a metro tunnel station at different stages of its construction are given.

1. Introduction
The paper is devoted to the problem of modeling the processes of geometrically and physically non-linear deformation of interacting reinforced concrete structures and environments of complex physical nature [1-10]. The literature describes various variants of three-dimensional non-linear equations, which are constructed taking into account geometric non-linearity [11-14]. In most of them, the differences in the components of the metric tensor before and after deformations of the body are taken as a measure of strain. However, as shown in recent studies [15–16], such physical relationships of this type in solving some specific problems lead to the appearance of so-called “false” bifurcation solutions.

2. Statement of the problem
The state of static body balance is described by a variation equation of the principle of possible displacements:

\[ \delta II = \int_\Omega \left( \tau_{ij} \delta \varepsilon_{ij} + \tau_{i} \delta \varepsilon_{i} + \tau_{j} \delta \varepsilon_{j} + \tau_{ij} \delta \sin \gamma_{ij} + \tau_{i} \delta \sin \gamma_{i} + \tau_{j} \delta \sin \gamma_{j} \right) (1 + \varepsilon_{i})(1 + \varepsilon_{j})dV', \]

where

\[ \tau_{ij} = \sigma_{ij} \cos \gamma_{ij}, \quad \tau_{i} = \sigma_{i} \cos \gamma_{i}, \quad \tau_{j} = \sigma_{j} \cos \gamma_{j}, \quad \sigma_{i}, \sigma_{j} - \text{true stresses} \}

\[ \sin \gamma_{ij} - \text{true deformations:} \]

\[ \varepsilon_{ij} = \sqrt{1 + 2e_{ij} - 1}, \quad \varepsilon_{i} = \sqrt{1 + 2e_{i} - 1}, \quad \varepsilon_{j} = \sqrt{1 + 2e_{j} - 1}, \quad (1 + \varepsilon_{i})(1 + \varepsilon_{j})^{-1}. \]

The plasticity condition is used in the form of Huber-Mises
\[ \Phi \equiv \sigma - H(\sigma_T, \chi) = 0, \]

where \( \sigma \) – stress intensity, \( \sigma_T \) – yield strength, \( \chi \) – Odquist form hardening parameter. The constitutive equations connecting the components of the increments of true stresses \( \sigma_{ij} \) and true deformations \( \varepsilon_{ij}^\sigma \) are chosen in the form

\[
\Delta \sigma_{ij} = \frac{E}{1-2\mu} \delta_{ij} \Delta \varepsilon_{ij}^\sigma + 2G \Delta \varepsilon_{ij}^\sigma \alpha \left( \frac{2}{H_x} \right) \sigma_{ij}.
\]  

(1)

If the soil limit state is described by the condition of the strength of the Mises-Botkin

\[ \tau_{ij} + \sigma_{ij} \varphi_{oct} + c_{oct}^* = 0, \]

which is written through \( \varphi_{oct}^* \) - the angle of internal friction on octahedral sites and \( c_{oct}^* \) - the limiting resistance to pure shear, then relations (1) for the soil take the form

\[
\Delta \sigma_{ij} = 2G \left( \Delta \varepsilon_{ij}^\sigma + \delta_{ij} \frac{3\mu}{1-2\mu} \Delta \varepsilon_{ij}^\sigma \right) - \alpha \left( \frac{G}{\tau_{ij}} + Ktg \varphi_{oct}^* \delta_{ij} \right) \sum \left( \frac{G}{\tau_{ij}} + Ktg \varphi_{oct}^* \delta_{ij} \right) \Delta \varepsilon_{ij}^\sigma.
\]

To solve problems of contact interaction of structures, a special contact element with specific properties is used [18-21]. The resolving equation is written based on the principle of virtual displacements in a variational form:

\[
\sum \int \{ \sigma \}^T \{ \delta \varepsilon \} d\Omega + \sum \int \{ \sigma_{ii} \}^T \{ \delta \varepsilon_{ii} \} d\Omega = \sum \int \int \rho \{ g \}^T \{ \delta V \} d\Omega + \sum \int \int \{ P \}^T \{ \delta V \} dS.
\]

3. Numerical results

For example, some results of the calculation of the stress-strain state of the retaining walls of the excavation of a subway station, as well as the station building itself, are presented during a phased implementation. Preparation for the construction and construction of metro stations is associated with a number of technical difficulties. A metro station is an underground structure erected in an open way in the immediate vicinity of city buildings and communications. Therefore, much attention is paid to ensuring the sustainability, strength and low mobility of the surrounding soil massifs during the excavation works.

Before work in the construction area of the metro station, the soil is dried. Before excavating the soil from the excavation, its edges are reinforced in advance with retaining walls (these walls are driven into the ground at the boundary of the excavation). As the soil is excavated, the walls of the excavation are reinforced with spacers that can be installed at different levels. When the foundation pit is ready, its bottom is concreted, the station building is erected, and the struts are simultaneously removed. After this, the station body is filled up with earth, and the bilge pumps are turned off, which leads to the water saturation of the soil surrounding the station. All these works lead to a gradual change in the stress-strain state of the retaining walls of the pit that surrounds the station of the soil massif and the erected body of the station. Since the pit has the shape of a parallelepiped, its length is large compared to its width, the spacers between the walls of the pit for each level are set at almost equal intervals, to identify the basic laws of deformation, the calculation can be carried out in a two-dimensional formulation, under conditions of plane strain. The mechanical characteristics of discretely located objects, in particular struts or columns located along the station axis, were converted to average values during calculations.

The calculation of the stress-strain state of the retaining walls of the pit of the metro station during the phased work related to the construction of the station. The calculation is carried out for the case of plane deformation. The lateral and lower boundaries of the region are specified by straight lines, and on them the conditions for the absence of point displacement in the direction perpendicular to the
rectilinear boundaries are specified. The distances from the retaining walls to the borders of the area are chosen from the condition that the retaining walls have a small effect on the displacement field and the stress-strain state of the soil and are determined during the computational experiment. Figure 1 schematically shows all the structural states that are realized during the phased construction of a metro station, which determine the discreteness of the transformation of the design schemes.

![Figure 1. Structural states.](image)

Figure 2 shows all the main geometric parameters of the problem: HV = 13 m, HN = 4.5 m, HR = 0.7 m, HNR = 1.0 m, HMR = 3.9 m, T1 = T2 = 0.5 m, LMS = 20.2 m.

Conducted three main options for the calculation. The first option is for the thickness of the retaining walls of the pit T1 = 0.607 m, T2 = 0.589 m. The second option is one retaining wall of the excavation has thicknesses T1 = 0.607 m, T2 = 0.589 m, the second has thicknesses T1 = T2 = 0.130 m. The third option is one retaining wall of the pit has a thickness of T1 = 0.607 m, T2 = 0.589 m, the second one has a thickness of T1 = T2 = 0.130 m, the ground is reinforced on the side of a thinner wall. Figure 3 illustrate the stresses in the concrete substructures of a metro station for the final (12th) structural state for the 1st calculation options, respectively.

![Figure 2. Diagram of substructures and subdomains of the computational domain.](image)

![Figure 3. Normal stresses in the substructures of a metro station for the 12th structural state of the 1st design case.](image)
4. Conclusion
The proposed method of solving problems of mechanics refers to the modern technology of scientific support for the design and construction of complex objects. Its use allows us to trace the change in the stress-strain state and the field of displacements of the structurally changing computational domain from the beginning to the end of construction. The results of numerical studies have allowed to identify the most dangerous stages of the technological process of building tunnels and metro stations, to establish factors affecting the stress-strain state of structures, to determine the zones of maximum deformation and stress.

Acknowledgments
The reported study was funded by RFBR according to the research project №18-31-00419.

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