Numerical modeling of the convective Kelvin-Helmholtz instabilities of astropauses

Sergey Korolkov$^{1,2,3}$, Dmitry Alexashov$^{2,4}$, Vladislav Izmodenov$^{1,2,3,4}$

(1) Moscow Center for Fundamental and Applied Mathematics, (2) Space Research Institute (IKI) Russian Academy of Sciences, (3) Lomonosov Moscow State University, (4) Institute for Problems in Mechanics

E-mail: izmod@iki.rssi.ru

Abstract. In this paper we present the numerical modeling results of the problem of the hypersonic stellar wind interaction with fully ionized interstellar medium that moves with respect to the star with supersonic speed. This is the classical problem that has been firstly studied in 1970 by Baranov et al. [1] under the thin layer approximation.

In this paper we present results of numerical solutions obtained with high spatial resolution of the numerical grid. The computations were performed by using GPU processors with high-performance parallel programming technology CUDA and nvcc compilers from NVIDIA. It is shown that the stationary solution depends only on two dimensionless parameters - Mach number in the interstellar medium, $M_\infty$, and adiabatic index, $\gamma$. The stationary solutions were obtained for different values of $M_\infty$ for low and medium resolution of the numerical grids. For the numerical grids of high spatial resolution the Kelvin-Helmholtz instability appears at the tangential discontinuities (both primary and secondary) in the tail part of interaction region. The instability is convective as it should be according to the linear analyses by Ruderman et al. [6]. We explore the instability evolution with increasing resolution of the numerical grid. Effects of the numerical scheme are also studied.

1. Introduction

The solar (stellar) wind is a flow of gas emitted from the Sun (star). In the case of the Sun the solar wind is a hypersonic at the Earth orbit and beyond. The solar wind speed is of the order of 300-800 km/s. Super or hyper-sonic winds exist for many stars. It is also quite common situation that the surrounding interstellar medium is moving with respect to the star. For example, the local circumsolar interstellar medium (LISM) is moving with respect to the Sun with the speed of $\sim 26$ km/s. Since the temperature of the LISM is about 7000 K, the interstellar flow is supersonic with Mach number $\sim 2$.

To study the interaction of the solar (stellar) wind with the interstellar flow one needs to consider a problem of the interaction of supersonic flow from a point source (since the radius of star is much smaller than the characteristic scale of the interaction region) with the supersonic parallel flow of interstellar medium. The flow pattern of such interaction in the nose (or upwind) region was proposed firstly by Baranov, Krasnobaev, Kulikovsky (1970) [1]. The qualitative picture of the flow is shown in Figure 1. In the head (or upwind) region of interaction, a structure consisting of two shock waves and a tangential discontinuity is formed. The inner shock is called the termination shock (TS). The supersonic stellar wind becomes subsonic at
this shock. The outer shock is called the bow shock. The supersonic interstellar flow becomes subsonic at this shock. The tangential discontinuity separates the stellar wind and interstellar gas. It is called astropause or heliopause (HP) in the case of the Sun. Note that Voyager 1 and 2 spacecraft crossed the heliopause in 2012 and 2018, respectively.

The flow pattern in the tail region has also complex structure (Figure 1) with Mach disk (MD), reflected shock wave (RS) and secondary tangential discontinuity (TD).

Current models of the solar wind interactions with the local interstellar medium are quite complex. Modern versions of the models are three-dimensional and non-stationary. They take into account multi-component nature of both interstellar medium and the solar wind, both interstellar and heliospheric magnetic field, latitudinal and time variations of the solar wind. Moreover, some components (like interstellar atoms of hydrogen and oxygen) have mean free path compared with size of the interaction region (Knudsen number ~ 1) and should be described kinetically. Modern model of the solar wind interaction with LISM has been presented, for example, in [2], [3].

In this paper we consider the simplest case of the two supersonic flows interaction. We return to the classical formulation of the problem in order to explore the Kelvin-Helmholtz (K-H) instability (Helmholtz [4]; Kelvin, [5]) that should appear at the tangential discontinuities. Ruderman et al [6] have shown using linear analyses that the for the conditions of the heliopause the K-H instability is convective. This means that at any given point the perturbations do not grow with time, and they increase while a fluid parcel moves along the tangential discontinuity from nose to the flanks.

The structure of the paper is the following. Section 2 gives the mathematical formulation of the problem. Section 3 discusses numerical approach. The results are presented in Section 4. Section 5 gives technical details of calculations. Section 6 concludes the paper.

![Figure 1. Qualitative picture of the supersonic stellar wind interaction with the supersonic flow of the interstellar medium. TS - termination shock, HP - heliopause, BS - outer bow shock, MD - Mach disk, RS - reflected shock wave, TD - secondary tangential discontinuity.](image)

2. Model
In our model both the stellar wind and the interstellar medium are considered as ideal non-heat-conducting gases with constant heat capacity and $\gamma = 5/3$. For simplicity in this paper we also neglect influence of the magnetic fields. The model results with both external and internal
magnetic fields will be presented elsewhere. The governing Euler equations can be written in the following conservative form in Cartesian coordinates:

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{E}}{\partial x} + \frac{\partial \mathbf{F}}{\partial y} + \frac{\partial \mathbf{G}}{\partial z} = 0,$$

(1)

Where $\mathbf{U} = [\rho, \rho u, \rho v, \rho w, \rho e]^T$,  
$$\mathbf{E} = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho w \\ \rho w u \\ (e + p) u \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} \rho v \\ \rho v u \\ \rho v w \\ \rho v w u \\ (e + p) v \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} \rho w \\ \rho w u \\ \rho w v \\ \rho w w u \\ (e + p) w \end{bmatrix}.$$

To formulate boundary conditions we introduce the star-centered reference frame. Axis $x$ is directed opposite to the LISM velocity vector. The outer boundary conditions are set at $x \rightarrow \infty$ with the parameters of undisturbed (by interaction with the stellar wind) interstellar medium: $\rho_{\infty}, \rho_{\infty}, v_{\infty} = -v_{\infty}e_x$. The stellar wind is considered as hypersonic spherically symmetric flow from the point source. Therefore, the problem considered in this paper is axis-symmetric. The hypersonic stellar wind is determined by two parameters: the stellar mass loss rate, $M_\odot$, and the terminal velocity, $v_0$.

The solution depends on six dimensional parameters: $\dot{M}_\odot$, $v_0$, $\rho_\infty$, $\rho_\infty$, $v_\infty = -v_\infty e_x$, $\gamma$. According to the $\pi$ - theorem, we choose $(v_0\dot{M}_\odot/(4\pi), \rho_\infty, v_\infty)$ as three basic parameters. In this case the characteristic distance is $R_* = \left(\frac{1}{4\pi} \frac{v_0\dot{M}_\odot}{\rho_\infty v_\infty^2}\right)^{1/2}$, and the characteristic time is $t_* = R_*/v_\infty$. (To give an example, for the case of the Sun $v_0 = 450$ km / s, $\dot{M}_\odot = 1.5 \times 10^{12}$ g/s, $v_\infty = 26.4$ km/s, $n_\infty = 0.1$ cm$^{-3}$, and, therefore, $R_* = 144$ a.u.) The dimensionless boundary conditions are determined as follows ($\dot{M}_\odot = 4\pi/\chi$, $\dot{v}_0 = \chi$, $\hat{\rho}_\infty = \hat{\chi}, \hat{\rho}_\infty = 1/(\gamma M_\odot^2)$, $\hat{v}_\infty = 1, \gamma$). Therefore, the solution of the problem is determined by three dimensionless parameters: 1) the ratio of the speeds of the stellar and interstellar winds $\chi = v_0/v_\infty$, 2) the Mach number in the interstellar medium, 3) the parameter $\gamma$, which for a fully ionized plasma is 5/3.

When performing numerical calculations, instead of the parameter $M_\odot$ one must set the wind density $\rho_0$ at some specified distance $r_0$. The hypersonic source solution implies that $\rho_0 = \rho_\odot / (4\pi v_0^2)$, and the characteristic size of the problem can be written as $R_* = r_0 \sqrt{K}$, where $K = (\rho_0 v_0^2)/(\rho_\infty v_\infty^2)$ is the ratio of dynamic pressures. In the dimensionless form the boundary condition at $r_0$ for density is $\hat{\rho}_0 = 1/ (\chi r_0)^2$.

In this paper we firstly aim to obtain a stationary solution (if such exists) and do not aim to explore time-evolution of the astrosphere. It is easy to prove that for stationary solutions ($\partial/\partial t = 0$) the geometrical pattern (i.e. locations of the discontinuities) does not depend on the parameter $\chi$. The gasdynamic parameters (density, velocity, pressure) can be easily recalculated from one value of $\chi$ to another. Indeed, if $p = p_1(\chi)$, $p = p_1(\chi)$, $v = v_1(\chi)$ is the solution of the problem for dimensionless parameters ($\chi_1, M_\infty$), then the following functions form the solution in the stellar wind region: $p_2 = \left(\frac{\chi_1}{\chi_2}\right)^2 p_1(\chi), v_2 = \frac{\chi_1}{\chi_2} v_1(\chi), p_2 = p_1(\chi)$. In the case of a hypersonic flow, the shock wave is strong. The pressure behind the shock wave is determined by the value of the dynamic pressure before the shock wave, which has not changed in the new solution. The equations and boundary conditions on the contact discontinuity are also satisfied. Thus, the solution for the set of dimensionless parameters ($\chi_2, M_\infty$) can be obtained.

Therefore, in order to perform a complete parametric study of the stationary solutions of the problem, we need to conduct a parametric calculations by varying only two parameters ($M_\infty, \gamma$). Unfortunately, this conclusion is not valid for unstable time-dependent solutions.
3. Numerical approach

To get the stationary solution of the formulated above problem we solve non-stationary equations (1) with formulated above boundary conditions. The stationary solution can (ideally) be obtained at $t \to \infty$.

For initial conditions we assume that at $t < 0$ the entire computation region is filled with a moving ideal interstellar gas with constant gasdynamic parameters $p_\infty$, $\rho_\infty$, and $v_\infty$ in the entire computational domain. At the initial time $t_0 = 0$, a supersonic stellar wind begins to flow out of the origin. The stellar wind is determined by the terminal speed of the stellar wind $v_0 = v_0 e_r$, and the mass loss rate of the star, $M = 4\pi \rho v r^2$. The parameters do not change with time.

The developed numerical code is based on the finite volume Godunov type method. The space is divided into cells. The mass, momentum and energy balance equations are employed for each of the cells. To calculate the fluxes of mass, momentum and energy at the cell surfaces we employed either exact Riemann solution (as in the classical Godunov method) or HLL and HLLC solvers [7]. The approximate solutions (obtained using HLL and HLLC solvers) give less accurate solutions at discontinuities. The minmod limiter (e.g. [8]) is used to obtained linear interpolation of the gas parameters within a cell.

The described above method has been implemented using the specific shock-capturing grids (see, e.g. [2]) for the modelling of heliosphere as well as on unstructured Dirichlet grid [9]. However, for the purposes of this paper we used a simple Cartesian grid. Such a grid allows to perform high resolution calculations by using GPU processors with high-performance parallel programming technology CUDA (nvcc compilers by NVIDIA).

4. Results

Firstly, we present the stationary solutions obtained for different values of $M_\infty$ by using the exact Riemann solver and minmod limiter (Figures 2 and 3). All calculations presented in this paper were obtained for $\chi = 1.628$. As it was pointed out above, the stationary solutions for other values of $\chi$ can be obtained by simple renormalization of gasdynamical parameters.

It is seen in Figure 2 that for the upwind region the positions of the inner shock (TS) and the tangential discontinuity (HP) do not depend strongly on the Mach number. The bow shock (BS) approaches the star when the Mach number increases. The solution tends to reach limiting solution for $M_\infty \to \infty$. Indeed, for the large Mach numbers the external shock wave becomes strong and does not depend on the pressure in the interstellar medium. The numerical solutions show that for the Mach numbers larger than 5 the solution is very close to the limiting one.

The case is different in the tail region. As it is seen on the right panel of Figure 2, the triplet point moves out when the Mach number increases.

Figure 3 presents isolines of the Mach number and the flow streamlines. As it is seen from the Figure, the obtained numerical solution is not fully stationary. Indeed, the Kelvin-Helmholtz (K-H) instability is clearly seen in the tail region at the secondary tangential discontinuity, which is originated at the triple point. The other interesting feature seen in Figure 3 is that the Mach disk becomes smaller when $M_\infty$ increases.

To explore the K-H instability in the tail, we performed specific calculations with extended computational domain toward the tail. Figure 4 presents results for $M_\infty = 3$ (upper panel) and $M_\infty = 2$ (bottom panel). The computations were performed with higher spatial resolution as compared with previously presented results. These computations show that the K-H instability appears not only at the secondary tangential discontinuity but also at the astropause (heliopause) that is primary tangential discontinuity separating stellar and interstellar flows. Interestingly, the reflected shock wave that originated at the triple point passes through the astropause. The reflection or refraction of the shock is not observed in the figure.

The bottom panel also shows the artificial reflected wave. The wave appears due to interaction...
Figure 2. Left panel: The positions of discontinuities obtained for different values of Mach number $M_\infty$. The distances are scaled in dimensionless units as introduced in Section 2. Right panel: Dimensionless density shown in the upwind direction (i.e. along of positive x axis). The distance is shown in dimensionless units multiplied by a factor of 184. It corresponds to the astronomical units in the case of the heliosphere.

of the bow shock with the outer boundary. This artificial feature can be removed by using non-reflected boundary conditions but it does not effect the flow in the area of interest and can be observed only at small values of the Mach number (for large Mach numbers, the shock wave has a more acute angle of inclination and its interaction with the upper wall does not occur), so we simply ignore it.

It should be clear that the presented results on the K-H instabilities depend on both (1) the resolution of the numerical grid and (2) the numerical scheme. Figure 5 illustrates how the numerical solution depends on the numerical grid. It is clearly seen that the higher the grid resolution is the earlier K-H instability appears at the tangential discontinuities. The absolute values of the perturbations do not increase with time. Thus, the instability is stationary in this sense. This is typical convective K-H instability [6]. The size of the perturbations increases along the astropause towards the tail. To illustrate how the perturbations evolve along the astropause we present the density distribution along the discontinuity (figure 6) as well as 2D plot of vorticity ($\omega = \text{rot} v$) isolines. It is seen that perturbations increase along the heliopause but the amplitude amplifications are quite moderate and do not influence global structure of the interaction region. The series of vortexes are also observed along of both tangential discontinuities.

The results that are shown in Panel e) of Figure 5 are similar the results shown in panel d). This is despite the fact that we double the resolution in panel e) as compared to d). Therefore, we can make a tentative conclusion about the grid convergence in our numerical results.

Figure 8 demonstrates the dependence of the results on the numerical scheme and solver. Panels a) and b) show the results of the first order schemes with HLL and HLLC solvers. The first order methods have large numerical viscosity, so the obtained solutions are smooth and stationary. Some small effects of instability are seen in panel b) for HLLC solver. Panels c), d) and e) present results of the “second order” schemes with minmod TVD limiter for gradient reconstructions within a cell. For the “second-order” schemes the K-H instability is clearly seen. The solutions obtained with different solvers are quite (but not completely!) close to each order.
Figure 3. Isolines of the Mach number are shown for various parameters $M_\infty$. Distances are shown in dimensionless units. The number of grid-cells is given in the captions to the individual panels.

The difference is connected with smearing of the tangential discontinuity in a particular scheme. The HLL method has a strong viscosity and "smears" tangential discontinuities into $\sim 10$ cells and shock waves into $\sim 2-3$ cells. The HLLC "smears" the discontinuities into 2-3 cells, and Godunov’s solvers "smears" into 1-2 cells. It is important to note that computations are much faster with HLLC solver as compared with Godunov solver. Since the smearing is not large, the HLLC solver is computationally preferable effective.

5. Effectiveness of gpu calculations
In this section we describe technical details of our numerical calculations. The calculations were performed by using GeForce GTX 1080 Ti and Nvidia CUDA compiler (nvcc). Tables 1-3 provide a summary of computational times to perform one hundred time steps for various resolutions of numerical grid and different numerical solvers. Table 1 gives a summary for GPU CUDA calculations. Table 2 gives a computational times for calculations performed on Intel Core i9 9900K processor by using 1 thread. Table 3 presents times for computations performed on the same processor with Open MP parallel compiler and 16 threads.

In it seen that GPU CUDA computations are in about 30 times faster than the CPU computations with one thread, and in about 5 times faster than Open MP parallel calculations with 16 threads. So the GPU CUDA computations are very effective.
You can see that the HLLC + minmod method works twice as fast as the Godunov’s + minmod method, however, this difference decreases with increasing mesh resolution, which indicates that most of the time is spent working with memory for a large number of cells. From the numerical calculations presented earlier, it can be seen that these two methods show almost similar results. Therefore, the HLLC method is preferable in this sense. It is also seen that the use of the minmod limiter slows down the work of the HLL method by 2 times, the HLLC method by 3 - 3.5 times. However, it is interesting that the work of Godunov’s method is almost not slowed down by the limiter, perhaps this is due to the fact that this method itself is much more resource-intensive than the minmod limiter. From similar considerations, minmod is much more resource-consuming than the HLL and HLLC methods, which makes them even more attractive for problems where the first-order approximation is sufficient.

It should also be noted that there are already next generation video cards: the GeForce RTX 2080 Ti. We expect a performance gain of at least 20 percent when using it. Also in the fall of 2020, NVIDIA is releasing a new line of video cards.

6. Conclusions

In this paper we present the results of numerical solution of classical problem of the ideal gasdynamic interaction of the hypersonic source with parallel supersonic flow.

The novelty of the presented results is in the high resolution of the numerical grid that

---

**Figure 4.** Illustration of K-H instability in the tail region of the flow. The results are obtained for the following physical and numerical parameters: a) \(M_\infty = 3\), Godunov solver and minmod. b) \(M_\infty = 2\), HLLC and minmod. For both panels, number of grid-cells: 3584 \times 2200, the size of a cell is 0.78 AU \times 1 AU.
Table 1. GPU CUDA calculations: Times (in seconds) to calculate one hundred time steps for different cell resolutions and computational scheme

| Number of cells | HLL    | HLLC   | Godunov | HLL+minmod | HLLC+minmod | Godunov+minmod |
|----------------|--------|--------|---------|------------|-------------|----------------|
| 448 × 275      | 0.04   | 0.056  | 0.7     | 0.097      | 0.2         | 0.72           |
| 896 × 550      | 0.16   | 0.2    | 2.13    | 0.35       | 0.74        | 2.3            |
| 1792 × 1100    | 0.61   | 0.81   | 7.01    | 1.37       | 2.8         | 7.48           |
| 3584 × 2200    | 2.47   | 3.37   | 24.37   | 5.38       | 11.05       | 26.02          |
| 7168 × 4400    | 9.9    | 13.94  | 87.2    | 21.44      | 44          | 91.67          |

Table 2. Intel Core i9 (one thread) calculations: Times (in seconds) to calculate one hundred time steps for different cell resolutions and computational scheme

| Number of cells | HLL    | HLLC   | Godunov | HLL+minmod | HLLC+minmod | Godunov+minmod |
|----------------|--------|--------|---------|------------|-------------|----------------|
| 448 × 275      | 1.07   | 1.27   | 7.85    | 2.27       | 5.35        | 10.87          |
| 896 × 550      | 4.86   | 5.51   | 29.61   | 9.66       | 24.14       | 39.16          |

Table 3. Intel Core i9 Open MP parallel calculations (16 threads): Times (in seconds) to calculate one hundred time steps for different cell resolutions and computational scheme

| Number of cells | HLL    | HLLC   | Godunov | HLL+minmod | HLLC+minmod | Godunov+minmod |
|----------------|--------|--------|---------|------------|-------------|----------------|
| 896 × 550      | 0.72   | 0.84   | 9.8     | 1.74       | 3.29        | 10.46          |
| 1792 × 1100    | 3.22   | 3.79   | 33.58   | 7.09       | 13.74       | 36.19          |
| 7168 × 4400    | 49.17  | 61.231 | 406.55  | 106.39     | 211.16      | 445.55         |

becomes possible due through the use of high-performance parallel programming technology CUDA and nvcc compilers from NVIDIA.

We investigated the flow in the interaction region as a function of the incoming flow Mach number, numerical methods, and grid resolutions. The positions of the discontinuities coincide with the results of previous numerical studies performed with low grid resolution [10, 11].

We explored how the position and the size of the Mach disk depends on $M_\infty$. It is shown that the Mach disk moves out and the size of Mach disk reduces with increasing $M_\infty$.

The high grid resolution calculations show existence of the K-H instability at the astropause and at the secondary tangential discontinuity. The instability is observed for small wavelengths, as evidenced by the absence of perturbations on a coarse grid. Our calculation demonstrate that the K-H instability at the primary and secondary tangential discontinuities are convective.

Finally, it is important to note that although the stationary solution of the problem does not depend on the parameter $\chi$, the unstable flow is time-dependent and, therefore, it will depend on the parameter $\chi$. The study of the K-H instability change with $\chi$ parameter variation is the subject of future studies, as well as the study on the influence of the magnetic fields, which are supposed to have a strong stabilizing effect.

7. Acknowledgments
This work was supported by the Foundation for the Advancement of Theoretical Physics and Mathematics “BASIS” 18-1-1-22-1.
a) Number of cells: $448 \times 275$. Cell size: $7.17$ a.u. $\times 8.18$ a.u.

b) Number of cells: $896 \times 550$. Cell size: $3.58$ a.u. $\times 4$ a.u.

c) Number of cells: $1792 \times 1100$. Cell size: $1.79$ a.u. $\times 2$ a.u.

d) Number of cells: $3584 \times 2200$. Cell size: $0.89$ a.u. $\times 1$ a.u.

e) Number of cells: $7168 \times 4400$. Cell size: $0.45$ a.u. $\times 0.5$ a.u.

Figure 5. Isolines of Mach number obtained numerically with Godunov solver and minmod TVD procedure for different resolutions of numerical grid. The number of cells and the size of cells is given in the captures to the individual panels. The Mach number of the incoming flow is $3, \chi = 1.628$.

Figure 6. Density distribution along the contact surface at different distances from it in the direction of the star. Number of cells: $7168 \times 4400$. Size of a cell is $0.45$ a.u. $\times 0.5$ a.u.; $M_\infty = 3, \chi = 1.628$. 
Figure 7. Isolines of the vorticity. Number of cells: 7168 × 4400. Size of a cell is 0.45 a.u. × 0.5 a.u.; M∞ = 3, χ = 1.628.

a) First order HLL.  

b) First order HLLC.  
c) HLL solver + minmod.  
d) HLLC solver + minmod.  
e) Godunov’s solver + minmod.

Figure 8. Isolines of the Mach number and streamlines obtained with different numerical schemes. Number of cells: 3584 × 2200. Size of a cell is 0.89 a.u. × 1 a.u.; M∞ = 3, χ = 1.628.

References
[1] Baranov, V. B., Krasnobaev, K. V., Kulikovski, A. G. A model of the interaction of the solar wind with the interstellar medium 1970, Sov. Phys. Dokl. 15, 791–793. solar wind with the interstellar medium. Sov. Phys. Dokl. 15, 791–793.
[2] Izmodenov V. V., Alexashov D. B. Three-dimensional kinetic-mhd model of the global heliosphere with the heliopause-surface fitting Astrophysical Journal, Supplement Series. — 2015. — Vol. 220, no. 2. — P. 32. [10.1088/0067-0049/220/2/32]
[3] Izmodenov V. V., Alexashov D. B.Magnitude and direction of the local interstellar magnetic field inferred from voyager 1 and 2 interstellar data and global heliospheric model Astronomy and Astrophysics. — 2020. — Vol. 633. — P. L12. [10.1051/0004-6361/201937058]
[4] Hermann von Helmholtz On the discontinuous movements of fluids — 1868, Monthly reports of the Royal Prussian Academy of Sciences in Berlin. 23: 215-228 (in German)
[5] Lord Kelvin (William Thomson) Hydrokinetic solutions and observations — 1871, Philosophical Magazine. 42: 362-377.
[6] M. S. Ruderman, L. Brevo, R. Erdelyi, Kelvin-Helmholtz absolute and convective instabilities of, and signalling in, an inviscid fluid–viscous fluid configuration — 2004 Physics Proceedings of the Royal Society
of London. Series A: Mathematical, Physical and Engineering Sciences.

[7] T. Miyoshi, K. Kusano A multi-state HLL approximate Riemann solver for ideal magnetohydrodynamics, J. of Computational Physics 208 (2005) 315-344

[8] Hirsch, C. Numerical Computation of internal and external flows — 1990, John Willey & Sons, 691

[9] Korolkov S. D., Alexashov D. B., Izmodenov V. V. Numerical 3D modeling of the interaction of the solar wind with the interstellar medium on the Dirichle grid — 2020, 15th Annual Conference "Plasma Physics in the Solar System" (in russian)

[10] Myasnikov, — 2004 Doctoral dissertation (in russian)

[11] Lebedev M. G., Sandomirskaya I. D., Counter interaction of inviscid supersonic gas flows – Computational methods and programming. Issue 34.M.: Moscow State University, 1981, S. 70-81 (in russian)

[12] Godunov S. K. Numerical solution of multidimensional problems of gas dynamics — Moscow, 1976 (in russian)