Analysis of elastic waves generated in frozen grounds by means of the electromagnetic pulse source “Yenisei”

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Abstract. Computational technology for multiprocessor computing systems of cluster architecture is developed, the ultimate goal of which is numerical modeling of wave fields generated by the electromagnetic pulse source “Yenisei” in blocky-layered geomedia. To describe the wave processes, mathematical models of the dynamics of elastic and elastic-plastic media, of porous and granular materials are applied. The algorithms of numerical realization are constructed based on the method of two-cyclic splitting with respect to spatial variables. Computational experiments showed that the proposed technology allows reproducing the system of waves near the region of excitation of seismic oscillations in 3D setting with a high degree of details and accuracy. The results of computations can be used in working out the optimal modes of functioning the source “Yenisei”, when mechanical characteristics of the layers vary in a wide range from solid and frozen grounds with inclusions of rock till granular and clayey water-saturated grounds. Numerical analysis makes also possible to obtain the averaged data, necessary for the adequate simulation of the localized pulse action from the source, using simplified mathematical models for calculating the synthetic seismograms of reflected waves over large scale and at great depth of layers bedding in complex geomedia.

1. Introduction

Electromagnetic source of seismic oscillations “Yenisei” is a non-explosive surface pulse seismic source with an electromagnetic actuator that contains one, two or four short-stroke electromagnets, working synchronously, with an autonomous power supply system from a capacitive storage of electric energy and a device for charging and discharging [1, 2, 3]. The source exists in wheeled, sledge, mobile and water variants (see Figure 1). “Yenisei” is quite competitive in comparison with sources of explosive and vibratory types by efficiency and quality of exploration works, and it has undeniable advantages in the economical and environmental aspects. The use of this source is incomparably cheaper, and it is almost the only possible means when working near buildings and constructions, in water protection zones and in areas where there are a lot of rivers and lakes. It is also applied on ice coverings of reservoirs, in shallow waters and on offshore.

In the process of creation, modification and improvement of operational and technical characteristics, the electromagnetic pulse source “Yenisei” was subjected to careful experimental analysis and testing [4, 5]. In the present work, computational technology is presented, the goal
2. Mathematical model

For numerical modeling of the processes of propagation of strain waves in rheologically complex media, we developed computational algorithms and software complexes, oriented to multiprocessor computing systems of cluster architecture. Mathematical models of elastic-plastic, granular and porous media, taking into account different resistance of materials to tension and compression, were worked out. Equations of the Cosserat continuum describing wave motion of structurally inhomogeneous media, in which along with translational degrees of freedom, independent rotations of the material particles are taking into account, were also realized numerically.

It is assumed that the structure of a medium is known and represented by a set of heterogeneous blocks with curvilinear boundaries. Each block is characterized by its homogeneous material with corresponding governing equations. In the simplest case of an elastic block, a system of equations of the linear dynamic elasticity, written in terms of velocities $v$ and stresses $\sigma$, is fulfilled:

$$\rho \frac{\partial v}{\partial t} = \nabla \cdot \sigma, \quad \frac{\partial \sigma}{\partial t} = \rho (c_1^2 - 2c_2^2)(\nabla \cdot v) I + \rho c_2^2 (\nabla v + \nabla^* v^*)$$  \hspace{1cm} (1)

Here $\rho$ is the density, $c_1$ and $c_2$ are the velocities of longitudinal and transverse elastic waves, respectively, $\nabla$ is the gradient over spatial coordinates, $I$ is the unit tensor. Asterisk denotes conjugate tensor, conventional notations of tensor analysis are used.

The initial velocities and stresses are assumed to be zero. On a part of the boundary, external stresses caused by the action of concentrated load are preset. On the planes of symmetry, if any, the symmetry conditions are formulated. A part of the boundary can be a non-reflecting surface, on which the conditions for passage of waves without appreciable reflection are simulated. At the internal interfaces, the continuity conditions are set for vectors of velocities and stresses at the contact areas of the blocks.

Under numerical implementation, independent computational grids are constructed in the blocks. An algebraic method, consisting in the finding a one-to-one mapping of computational domain in the form of a unit cube with uniform grid onto the physical domain, is used.
The system of equations (1) during the construction of finite-difference scheme is represented in the matrix form:

$$\frac{\partial U}{\partial t} = \sum_{i=1}^{3} A_i \frac{\partial U}{\partial x_i},$$

where $U$ is the vector–function composed of the components of velocity vector and stress tensor, $A_i$ are the matrices–coefficients of the equations. To solve this system, the scheme of integration based on the two-cyclic splitting method is applied [6]. The splitting is performed not in physical space, but in parametric one. The system is transformed to the following form:

$$\frac{\partial U}{\partial t} = \sum_{i=1}^{3} \tilde{A}_i \frac{\partial U}{\partial \xi_i}, \quad \tilde{A}_i = \sum_{j=1}^{3} \frac{\partial \xi_i}{\partial x_j} A_j.$$  

(3)

The splitting method leads to a series of 1D problems, which are solved based on the explicit monotone ENO–scheme of the “predictor–corrector” type with the limit reconstruction of solution. To preserve the conservativity of scheme on curvilinear grids, when the matrices–coefficients $\tilde{A}_i$ depend on coordinates, the approximation of 1D systems of equations on the “corrector” step is performed by means of the integro-interpolation method. As a result of integrating the system (2) over a curvilinear cell in the physical area with the subsequent application of the Green formula, we obtain the following equality:

$$\frac{\partial \bar{U}}{\partial t} = \frac{1}{\omega} \sum_{k=1}^{6} \gamma^k \left(n_1^k A_1 + n_2^k A_2 + n_3^k A_3\right)U^k,$$  

(4)

where $n_i$ are the directing cosines of the external normal, $\omega$ is the cell volume, $\bar{U}$ is the average integral value of the solution vector, and the index $k$ is related to the faces of the cell, in particular $\gamma^k$ is the area of corresponding face. The sum in the right-hand side of (4) is divided into three pairs of terms on opposite faces, each of which corresponds to the approximation of spatial derivatives in 1D systems.

Described algorithm was generalized for the cases of a granular medium [7], a porous material [8], a Cosserat continuum and a multiblocky medium [7, 9]. The developed method of solution was implemented as a software package in the Fortran language by means of the MPI (Message Passing Interface) library. Technology of parallelization is based on the uniform distribution of computational domain between the nodes of a cluster. Verification of the software was performed on exact solutions – formulas of geometric seismics for the hodographs of reflected and refracted waves. Parallel program system was registered in Rospatent [10].

### 3. Results of computations

Preliminary series of computations was fulfilled with the goal of validating the software package by basic parameters of the source – the frequencies and amplitudes of oscillations. Comparison of the numerical results with the available experimental data showed a satisfactory quantitative correspondence.

The problem was solved for a two-layered massif of an elastic medium of $60 \text{ m} \times 40 \text{ m} \times 40 \text{ m}$, in two variants, when the upper 10-meter layer is more compliant and, conversely, more rigid as compared with the lower 50-meter basic layer. Similar problem was considered for the case, when the upper layer is water, and for a three-layered medium consisting of the upper 3.5-meter layer from ice, the middle 10-meter layer of water and the lower thick layer of ground. To demonstrate the capabilities of the program, the interface between layers of ground with different properties (or between water and ground) was curved.

Figure 2 demonstrates the uniform distribution of two-layered computational domain between 96 computational nodes: 16 nodes in the upper layer and 80 nodes in the lower one. Each node...
of a cluster performs computations in parallel mode. The difference grid in the upper layer of a massif is $50 \times 200 \times 200$ cells, and in the lower layer – $250 \times 200 \times 200$ cells, that is, each cluster node performs computations on a grid of $50 \times 50 \times 50$ cells. For visibility, the difference grid in Figure 2 is thinned 5 times in each direction. In the case of a three-layered medium, computational domain is distributed uniformly between 16 nodes in the upper layer (ice), 16 nodes in the middle layer (water) and 64 nodes in the lower layer (ground).

At the upper boundary of computational domain a localized action from the source with four electromagnets is set. Taking into account the symmetry, computations are made for a quarter of the whole massif bounded by vertical coordinate planes. On the left and right boundaries of computational domain (see Figure 2) the symmetry conditions are given. The back boundaries and the lower base are considered as non-reflecting surfaces. At the stages of solving 1D systems of the splitting method, the boundary values of Riemann invariants corresponding to outgoing characteristics are assumed to be zero on these surfaces, that is equivalent to the absence of reflected waves in 1D problems.

In Figure 3 one can see the dependence on time of pressure from the source in the localization zone, a circle of area $1 \text{m}^2$, which is located at the distance of 2.5 and 1.25 m from the left and right boundaries of symmetry, respectively. The pressure in this form was determined on the basis of experimental measurements of the acceleration of reactive mass of the electromagnet. Densities and velocities of elastic waves in the layers from different materials are given in Table 1.

![Figure 2. Uniform distribution of computational load between 96 cluster nodes.](image2)

![Figure 3. Dependence on time of pressure from the source in the localization zone.](image3)

|                | $\rho$ [kg/m$^3$] | $c_1$ [m/s] | $c_2$ [m/s] |
|----------------|-------------------|-------------|-------------|
| ice            | 900               | 3000        | 1800        |
| water          | 1000              | 1450        | 0           |
| clay           | 2100              | 1800        | 1100        |
| ground         | 2400              | 4500        | 2700        |
| rigid ground   | 2600              | 6000        | 3500        |
Figure 4. Level surfaces of the normal stress in the vertical direction. Lower layer is ground, upper layer is: clay (a), rigid ground (b), water (c), upper layer is ice, middle layer is water (d).

Figure 4 shows the characteristic level surfaces of the normal stress $\sigma_{11}$ in the cases of: compliant upper layer of clay (Figure 4a), rigid upper layer of rigid ground (Figure 4b), upper layer of water (Figure 4c), upper layer of ice and middle layer of water (Figure 4d) at the time moment $t = 18$ ms. The rest part of massif has the parameters of ground from the Table 1.

It can be seen that perceptible reflection of waves from the back surfaces does not occur. On seismograms of reflected waves the errors in modeling the boundary conditions also appear weakly. In computations for a three-layered massif with a layer of water covered by ice, the thickness of the ice cover was taken as 3.5 m, which is typical for the Arctic shelf.
Figure 5. Amplitude-frequency dependencies for vertical velocity at the depth of 30 m. Lower layer is ground, upper layer is clay (a), upper layer is rigid ground (b), upper layer is water (c), middle and upper layers – water covered by ice (d).
The amplitude–frequency dependences for the velocity \( v_1 \) of the particles of a medium at the depth of 30 m in a system of points under the trace passing near the symmetry plane in the direction of \( x_2 \) axis are represented in Figure 5. Red line refers to the point under the source, green line – to the middle point to the left, and blue one – to the edge left point. Fourier analysis of the seismograms of velocities shows that the main frequency, which depends weakly on the point of registration, is about 50 – 100 Hz. At the same time, the received signal contains high-frequency oscillations, the degree of damping of which with the passage of waves due to the dissipation of mechanical energy must be analyzed using more general equations of viscoelastic media.

Due to the properties of the Fourier transform, the amplitude–frequency dependences for acceleration \( a_1 \) can be obtained from Figure 5 by means of multiplying the amplitude of the velocity by the frequency, and for displacement \( u_1 \) – by means of dividing by the frequency. The main frequencies of displacement turn out to be close to zero, and the main acceleration frequencies exceed hundreds of Hertz. This circumstance can serve as a basic information for choosing the new types of geophones (see, e.g., [11]) for exploration by means of the electromagnetic pulse sources “Yenisei”.

4. Conclusions
The Arctic territory of Russian Federation is characterized by a permafrost structure of the surface layer of soil and the ice covered shelf structure, which reduces the efficiency of geological exploration using traditional seismic sources of explosive and vibratory types. Therefore, Geotech Holding Company developed a special eco-friendly electromagnetic pulse source “Yenisei”, which acoustic waves will be the subject of analysis using high-performance computing in our further research. Now we worked out the computational technology and software complex for a detailed study of wave fields excited by the source in blocky-layered massifs of geosphere with different properties of layers. The conducted computational experiments showed that the main frequency of vertical velocity, regardless of the material of the surface layer (clay, rigid frozen ground, water, ice-covered water), lies in the characteristic range for seismic exploration from 30 to 100 Hz.

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