Fluid Mechanics and Heat Transfer in Fluidized Beds

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Abstract

Heat transfer between submerged surfaces and gas-fluidized beds depends on fluid mechanics and particle dynamics. Therefore, reliable prediction of the heat transfer coefficient must be based on the observation of particle motion towards and from solid surfaces. Experiments with luminescent particles reveal a rather broad particle residence time distribution at solid surfaces. This broad residence time distribution gives rise to a smearing-out effect of the two different heat transfer mechanisms, namely particle convective and gas convective, respectively. Well-defined residence times can be realized by means of a rudimentary variant of fluidized bed heat transfer in the form of moving bed heat transfer. Experimental and theoretical results obtained from moving bed heat transfer allow the sound modeling of the two "pure" mechanisms (particle convective and gas convective). A predictive equation is derived which may be seen as a safe interpolation between the two extremes. The comparison with a large number of experiments proves the reliability of the prediction with respect to any feature of fluidized bed heat transfer.

1. Introduction

Textbooks on fluid mechanics, or on heat and mass transfer, describe two different modes for the generation of dimensionless groups, namely

• listing the influencing factors and finding out the necessary number of basic units. Through Buckingham's theorem, a distinct number of dimensionless groups is evident, the actual definitions depending on the investigator's taste, or alternatively
• derivation of dimensionless groups from the inspection of the underlying partial differential equations.

Both approaches suggest a quasi automatic definition of the relevant dimensionless groups and work quite well, e.g. in the case of single-phase flow.

Particle technology, however, is a different story. In order to demonstrate the peculiarities of that discipline of chemical engineering, the heat transfer between submerged tubes and bubbling fluidized beds is discussed in detail. This variant of the heat transfer results from complex interactions between fluid and solid material properties, particle size, fluid mechanics and particle dynamics. In other words: The derivation of reliable predictive equations built up from relevant dimensionless groups is not found in any textbook, but instead is the result of an arduous step-by-step process which in retrospect can be likened to piecing together a puzzle.

2. Reynolds number and Archimedes number

Particle technology is a discipline in the broader field of chemical engineering. Stationary fluidized beds, for example, are in use with exothermic catalytic gas-phase reactions with particle sizes in the range of \( d_p \approx 50 \mu m \), or with combustion of coal in pressurized fluidized beds with particle sizes up to 5 mm. In both cases, heat removal takes place by means of submerged tubes.

For Reynolds numbers \( Re = (\frac{\rho_F v_d}{\mu}) \), a reasonable estimate for the aerodynamic resistance of a single sphere is given by the equation

\[
R^* = \frac{24}{Re} + \frac{4}{\sqrt{Re}} + 0.4.
\]

With the aerodynamic resistance defined by weight minus buoyancy \( R^* = (\pi/6)(\rho_s - \rho_l)d^2_p g \), Equation (1) rearranges to

\[
Ar = \frac{\rho_l(\rho_s - \rho_l)d^2_p g}{\mu^2} = \frac{3}{4} c_p Re^2 = 18Re + 3Re^{1/2} + 0.3Re^2.
\]

Figure 1 shows Equation (2) and indicates the regimes of the different types of flow.

Equation (2) defines a monotonic function. This means that in the case of fluidization fluid mechanics, the state of flow is defined alone by the Archimedes number, i.e. laminar flow for \( Ar \leq 10^2 \), the transitional regime for \( 10^2 \leq Ar \leq 10^5 \), and the turbulent regime...
for \( Ar \geq 10^5 \); Archimedes numbers \( Ar > 10^8 \) are attained only in pressurized systems. These drastic changes in the state of flow must also be reflected in corresponding changes in the heat transfer mechanisms. 

**Figure 2** shows measured heat transfer coefficients versus the excess gas velocity \( u - u_{mf} \) (superficial gas velocity minus minimum fluidization velocity). Obviously, the maximum heat transfer coefficient decreases significantly with increasing particle size. This behavior corresponds to that described by Botterill ([1] Fig. 5.8, p. 241).

The aim of the present paper is to elucidate the physical background of the observed phenomena. In order to limit the length of the paper, details of many experimental procedures or of the underlying theoretical considerations are not given, but instead significant graphs and plausible arguments are preferred.

### 3. Particle migration at solid surfaces

In particular with fine-grained particles, heat transfer between bubbling fluidized beds and submerged tubes is drastically higher in comparison with the corresponding fixed bed situation, in other words: particle mobility largely enhances the heat transfer.

Mickley and Trilling [3] were the first to appreciate the implications of the unsteady nature of the heat transfer process between a submerged surface and a gas-fluidized bed.

A reliable theory of fluidized bed heat transfer, must therefore be based on quantitative information about the particle migration at submerged solid surfaces. The particles are swept away from the solid surfaces by rising gas bubbles. Spatial bubble distribution and rise velocities are more or less random, so only experiments will provide a useful answer. The basic principle for the detection of particle motion at solid surfaces is quite simple. Transparent side walls allow direct observation of particle migration (Fig. 3, right side). The observation of particle motion close to immersed surfaces (Fig. 3, left side), e.g. heat exchanger tubes inside a fluidized bed, is realized by using a periscopic mirror arrangement installed in a dummy tube.

Particle motion at the solid surfaces is visualized by using luminous particles. A luminescent pigment (ZnS crystals doped with copper) was available in the form of particles with a mean particle size of \( dp=50 \mu m \). Larger particles were produced in a fluidized bed by spraying clear varnish onto the bed surface. Subsequent sieving provided different fractions of luminescent lacquer particle fractions with mean particle sizes up to \( dp=300 \mu m \). After illumination by a pulse of light transmitted via fiber optics to the transparent wall area, these particles show an afterglow for several seconds. The illuminated spot shifts along the solid surface, whereby its shape deforms and its overall luminosity decreases, but the illuminated particles...
themselves stay in a close proximity. In other words, the illuminated spot remains as a single identifiable object during its lifetime. Digital image analysis can therefore be applied. In this way a cluster of particles can be passively marked for subsequent observation and tracking.

Particle migration to and from the solid surface is a random process. Simple theoretical considerations [4] yield for the probability \( W(t) \) that a particle marked at time \( t=0 \) is in contact with the wall up to the present time \( t \)

\[
W(t) = e^{-t/\tau}.
\]

In Equation (3), \( \tau \) defines the mean particle residence time at the wall. According to the underlying experimental procedure, the probability \( W \) is given by the ratio of the actual to the initial luminosity, i.e. it should follow that (compare Fig. 4).

\[
\ln \left( \frac{L}{L_0} \right) = -t/\tau
\]

According to Equation (4), the mean particle residence time is found from the slope of curve in semi-logarithmic representation. Table 1 shows typical mean residence times for two particle sizes and for different static pressures at maximum heat transfer.

4. The contact resistance

Two different experimental results described in the preceding section define the strategy for finding out the mechanisms of the heat transfer:

- The mean particle residence time at the wall is of the order of one second (Table 1)
- For given operational conditions, the distribution of the particle residence times is rather broad (Fig. 4).

The second feature largely impedes any direct evaluation of fluidized bed experiments because it introduces a smearing-out effect with respect to different mechanisms. This dilemma can be avoided by the evaluation of heat transfer experiments with a given particle residence time in the order of seconds. This situation is experimentally realized by particle beds moving along heat transfer surfaces of given length. Moving bed heat transfer has been investigated by several authors [5, 6], occasionally with the intention of understanding the fluidized bed heat transfer phenomena [7, 8]. However, instead of providing valuable insights with respect to fluidized bed heat transfer, these investigations have posed fundamental questions which still remain contentious.

With usual gas/solid systems, the particle thermal conductivity \( k_p \) is high in comparison to the gas thermal conductivity \( k_g \), i.e. \( k_p \gg k_g \). As an asymptotic limit, one can thus assume that particle bed heat transfer is governed by a contact resistance (Fig. 5). Heat transfer between a heating surface and an adjacent particle (Fig. 5, left side) as well as between two adjacent particles (Fig. 5, right side) takes place in the form of a sudden and steep temperature gradient in the gap between the two particles, which results in the case of heat transfer from left to right with a temperature difference \( T_1 > T_2 \).

In the literature, two different physical reasons are given for the contact resistance observed with particle beds. For perfect spherical particles in contact with a plane wall, Schlinder [9] attributes the contact resistance to a diminished heat transfer rate in the gas in the region where the distance between the particle and the surface is less than the mean free path of the interstitial gas. Glosky et al. [10] attribute it to the

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**Table 1** Mean residence times of illuminated particles at vertical solid surfaces in a fluidized bed at maximum heat transfer

| Particle size | 1 bar (0.1 MPa) | 5 bar (0.5 MPa) | 10 bar (1.0 MPa) | 20 bar (2.0 MPa) |
|---------------|----------------|----------------|----------------|----------------|
| \( d_p = 50 \mu m \) | 1.28 s | 0.35 s | 0.52 s | 0.45 s |
| \( d_p = 250 \mu m \) | 1.54 s | 1.31 s | 0.91 s | 0.77 s |

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**Fig. 4** Decay of dimensionless computer output luminosity \( L \) in a fluidized bed

**Fig. 5** Contact resistance with particle bed heat transfer, left side: short contact time, right side: steady-state heat conduction in packed beds.
surface roughness. Both approaches, however, are not contradictory, but instead complementary.

That part of the energy which is transferred within the surface area with the radius \(0 \leq R \leq d_p/2\) (Fig. 5, left side) in terms of a Nusselt number is

\[
\text{Nu} \left( \frac{2R}{d_p} \right)^2 = \pi \left( \frac{1 + \frac{2S}{d_p}}{1 + \frac{d_p}{2S}} \right) \ln \left[ 1 + \frac{d_p}{2S} \right] \left( 1 - \sqrt{1 - \left( \frac{2R}{d_p} \right)^2} \right) \left( 1 - \sqrt{1 - \left( \frac{2R}{d_p} \right)^2} \right). \tag{5}
\]

The details of the derivation of Equation (5) can be read in [11].

The heat transfer per particle is defined by \(2R/d_p\), i.e. by the maximum Nusselt number for short contact time

\[
\text{Nu}_{\text{max}} = \frac{\text{h}_{\infty} d_p}{k_g} = \pi \left( \frac{1 + \frac{2S}{d_p}}{1 + \frac{d_p}{2S}} \right) \times \ln \left( 1 + \frac{d_p}{2S} \right) - 1. \tag{6}
\]

According to Schlünder's approach, for ideal geometries

\[
S_{\text{min}} = 2 \ell_0 \frac{2 - \gamma}{\gamma} \tag{7}
\]

where

\[
\ell_0 = \frac{16}{5} \sqrt{\frac{\bar{R} T}{2 \pi M}} \times \frac{\mu}{p} \tag{8}
\]

designates the mean free path of the gas molecules and \(\gamma\) the accommodation coefficient. The accommodation coefficient \(\gamma\) defines that fraction of the gas molecules which are not reflected when hitting a solid surface. Equation (7) reveals that the physics of gases yields a minimum effective roughness \(S_{\text{min}}\), below which particles can be considered thermodynamically smooth. Thus, Schlünder's approach provides an upper limit for the maximum Nusselt number.

As a by-product, an estimate for the effective thermal conductivity \(k_e\) of particle beds is obtained as follows (compare Fig. 5, right side): For two adjacent spheres in a cubic array in the main direction of heat flux, the relevant distance for the heat conduction is the particle diameter, so the distance in the model must simply be doubled to yield the gap between particles. The relevant cross-section for the heat flux is the particle equatorial area \((\pi d_p^2)/4\). Thus, the ratio of effective to gas thermal conductivity is also given in the form of a clear function of \(2S/d_p\) (see Fig. 6)

\[
k_e/k_g = \left( \frac{2}{\pi} \right) \text{Nu}_{\text{max}} = 2 \left[ 1 + \frac{2S}{d_p} \right] \ln \left( 1 + \frac{d_p}{2S} \right) - 1. \tag{9}
\]

Both equations (6) and (9) indicate that the heat transfer in a particle bed is exclusively a function of the length ratio \(S/d_p\). The inherent concept is that the sole physical reason for short contact time resistance between a heating surface and a particle bed as well as for steady-state heat conduction inside the bed is almost exclusively the contact resistance in the gap between the contacting solid bodies.

5. Heat transfer in moving beds with a stagnant interstitial gas

The most striking manifestation of the role of the length ratio \(S/d_p\) is observed with the heat transfer between a heated solid surface and a particle bed sliding over it. With a short contact time, a maximum Nusselt number according to Equation (6) is observed, whereas with progression of the temperature front inside the bulk of the bed, a long contact time variant of the heat transfer is attained, characterized by gradual heating up of the bed. Figure 7 shows \(\text{Nu}/\text{Nu}_{\text{max}}\)
According to Equations (5) and (6) for different values of $S/d_p$.

Due to the cohesiveness of extremely fine-grained particles, efficient fluidization is restricted to particle sizes $d_p > 20 \mu m$. According to Figure 7, $d_p > 20 \mu m$ means length ratios $S/d_p < 10^{-3}$ and, hence, concentration of the heat transfer to a region close around the contact point. This fact influences the gradual means length ratios $S/d_p$.

The contact point. This fact influences the gradual means length ratios $S/d_p$.

The capacity of a particle to store thermal energy on the other hand is less contact time (due to the mean free path effect) or real (due to surface roughness) distance. The capacity of a particle to store thermal energy is thus reasonably defined by the nominal particle size $d_p$. A dimensional scale to the gain of thermal energy per particle up to the time $t$.

$$\frac{C(k_c/S_{min})S_{min}}{d_p c_p d_p^2/t} = C_k C_k S_{min} t = \frac{\rho_p C_p d_p^2}{\rho_p C_p d_p^2}.$$

(10)

In Equation (10), a factor $C > 1$ defines the significance of surface roughness for the heat transfer ($S > S_{max}$). In practice, the term $C x S_{min}$ must be seen as a constant which has to be determined from experiments. Experiments throughout evidence the insignificance of the particle size for the heat transfer, when the heating up proceeds into the bulk of bed. The insignificance of the particle size requires for the actual Nusselt number at time $t$.

$$Nu \times \sqrt{Co} = \frac{h \times d_p}{k_x} \sqrt{\frac{k_x C \times S_{min} t}{\rho_p C_p d_p^2}} = \text{const},$$

(11)

i.e. $Nu \sim Co^{-1/3}$ (compare Fig. 8).

6. Particle convective heat transfer at Archimdes numbers $Ar \leq 10^2$

According to the two-phase theory of fluidization, the emulsion phase remains more or less in the minimum fluidization state, i.e. the excess gas velocity $u - u_{mf}$ is that part of the total gas throughput which passes the bed in the form of bubbles. With fine-grained particles, viscous resistance balances weight minus buoyancy, i.e.

$$\mu d_p u_{mf} \sim (\rho_p - \rho_g) d_p^2 g.$$  

(12)

Equation (12) rearranges to

$$\frac{u_{mf}}{d_p g} \sim \left(\frac{d_p}{\xi_1}\right)^{3/2}$$

(13)

with a laminar flow length scale

$$\xi_1 = \left[\frac{\mu}{\sqrt{g (\rho_p - \rho_g)}}\right]^{2/3}.$$  

(14)

Further rearrangement yields the final result

$$\frac{u_{mf}}{\sqrt{\xi_1 g}} \sim \left(\frac{d_p}{\xi_1}\right)^2.$$  

(15)

Equation (15) provides a clear message: according to its definition, Equation (14), the laminar flow length scale $\xi_1$ is constant for a given combination of gas and solid material.

Thus, Equation (15) predicts a sharp decrease in the gas velocity in the emulsion phase with decreasing particle size.

From Figure 2, for example, one reads for fine-grained particles, i.e. curve 1, maximum heat transfer at $u - u_{mf} = 0.7 m/s$ and $u_{mf} = 2.8 \times 10^{-3} m/s$, i.e. $u_{mf}/(u - u_{mf}) = 4 \times 10^{-3}$.

This means that with fine-grained particles, the emulsion-phase permeability becomes so small that the system can be seen as a homogeneous particle/gas mixture percolated by rising bubbles. In other words: the particle size becomes irrelevant for the fluid mechanics, and the emulsion phase is close to moving bed conditions with a stagnant interstitial gas.

According to the definition of the dimensionless contact time $Co$, Equation (10), decreasing particle size means increasing numerical values of $Co$. From Figure 8 together with the definition of $Co$, Equation (11), we thus conclude that with fine-grained particles, the particle size is not only insignificant for the fluid mechanics, but also for the heat transfer mechanism.

The slope of curve 1 in Figure 2 suggests the following procedure: at first an equation for the maxi-
The normalized equation is deduced, which fits the slope of curve 1. The maximum heat transfer coefficient must be a clear function of only the relevant material properties and the particle size. As dimensionless groups, we thus have the length ratio

\[ \frac{d_p}{\xi_1} \]  

which defines the fluid mechanics, a ratio which combines the relevant thermal properties \((c_p, k_g)\)

\[ \frac{c_p \xi_1}{k_g}, \]  

the maximum heat transfer coefficient in terms of a Nusselt number

\[ \text{Nu}_{\text{max}} = \frac{h_{\text{max}} d_p}{k_g} \]  

and the solids volume concentration \(1 - \varepsilon_{\text{mf}}\) in the emulsion phase.

The insignificance of the particle size \(d_p\) for the fluid mechanics and the heat transfer reduces the number of dimensionless groups, i.e.

\[ \frac{h_{\text{max}} \xi_1}{k_g} = f \left( \frac{c_p \xi_1}{k_g}, 1 - \varepsilon_{\text{mf}} \right). \]  

Evaluation of measurements [12] yields to

\[ \frac{h_{\text{max}} \xi_1}{k_g} = f \left( \frac{c_p \xi_1}{k_g}, 1 - \varepsilon_{\text{mf}} \right). \]  

(19)

(20)

with the void fraction \(\varepsilon_{\text{mf}}\) at minimum fluidization. Equation (20) predicts \(h_{\text{max}} \to 0\) for \(k_g \to 0\) as well as for \(c_p \to 0\), i.e. for the so-called particle convective heat transfer, the gas thermal conductivity \(k_g\) as well as the solid material specific heat \(c_p\) are relevant. The dependence of the heat transfer on the excess gas velocity is taken into account as follows: the heat transfer from the heating surface takes place in the form of heated particles being swept away from the heating surface by rising bubbles. Swept-away particles come to rest again in the gravitational field after a distance

\[ \xi \sim \frac{(u - u_{\text{mf}})^2}{g}. \]  

The term \(\rho_g c_p (u - u_{\text{mf}})\) (units \(\text{Wm}^{-3} \text{K}^{-1}\)) defines a heat transfer by particle migration. The product of these two terms yields an effective thermal conductivity

\[ k_{\text{eff}} \sim \frac{\rho_g c_p (u - u_{\text{mf}})^3}{g} \]  

\((\text{Wm}^{-1} \text{K}^{-1})\)

(21)

The dimensionless excess gas velocity

\[ \frac{1}{1 + P} \left( \frac{u - u_{\text{mf}}}{u_{\text{mf}}} \right) \frac{1}{\frac{\rho_g c_p (u - u_{\text{mf}})^3}{k_g g}} \]  

(22)

(23)

with a constant \(P\) to be determined from experiments. Obviously, Equation (23) predicts \(\text{Nu}_{\text{pc}} \to 0\) for \(u - u_{\text{mf}} \to 0\) and \(\text{Nu}_{\text{pc}} \to 1\) for \(u - u_{\text{mf}} \to \infty\). (Compare curve 1, Fig. 2.)

The combination of Equations (20) and (23) together with evaluation of experiments yields the final result for the particle convective heat transfer

\[ \text{Nu}_{\text{pc}} (u - u_{\text{mf}}) = 0.19 (1 - \varepsilon_{\text{mf}}) \times \frac{1}{1 + \frac{k_g}{2c_p \mu} + 25 \left( \frac{u - u_{\text{mf}}}{u_{\text{mf}}} \right)^2 \frac{\rho_g c_p (u - u_{\text{mf}})^3}{k_g g}}. \]  

(24)

The comparison of Equation (24) with the experimental results in Figure 9 confirms that the influence of the relevant material data (gas thermal conductivity: air and helium; particle specific heat: glass and bronze) is correctly taken into account.

Fig. 9 Comparison of own measurements (symbols) with predictions of Eq. (24) (lines); experiments carried out under standard conditions.

7. Gas convective heat transfer at Archimedes numbers \(10^5 \leq \text{Ar} \leq 10^8\)

In comparison with particle convective heat transfer, gas convective heat transfer in the regime of \(10^3 \leq \text{Ar} \leq 10^8\) is a simple affair. Starting again from
the rudimentary variant of fluidized bed heat transfer, the moving bed, Figure 8 tells us that for a given contact time $t$, an increasing particle size $d_p$ shifts the dimensionless contact time $C_t$ towards smaller values. Finally the regime $Nu = \text{const.}$ is attained, where the contact time is far too short for significant heating up of the particles at the heating surface. On the other hand, one reads from Figure 1 that $Re > 5 \times 10^3$ for $Ar > 10^5$. This means that boundary layer effects dominate the heat transfer. Further valuable information can be deduced from Figure 2. Gas convective heat transfer is represented by curve 4, which indicates the maximum heat transfer attained at $u-u_{mf} = 0.5 \text{ m/s}$, whereas the minimum fluidization velocity is 1.15 m/s. From these two figures, it follows that the gas flow rate is higher in the emulsion than in the bubble phase.

All these facts together indicate a single-phase-type of heat transfer in the form of $Nu = Nu(Re, Pr)$. Figure 1 shows a single valued function $Ar = Ar(Re)$. This fact legitimates the replacement of the Reynolds number by the Archimedes number. According to Baskakov and Filippovski [13], the maximum heat transfer coefficient is given by

$$Nu_{max} \sim (Ar Pr)^{1/3} \quad \text{for} \quad 10^3 \leq Ar \leq 10^8. \quad (25)$$

According to the definitions of $Nu$ and $Ar$, Equation (25) predicts that the particle size does not affect the heat transfer. The insignificance of the particle size is easy to understand from the following considerations: at higher Reynolds numbers, the heat transfer is defined by the laminar boundary layer thickness $\delta_l$ with $\delta_l/d_p \sim Re^{-1/2}$ as indicated by the Ranz equation [14] for the heat transfer from a single particle

$$Nu \sim Re^{1/2} Pr^{-1/3}. \quad (26)$$

From Equation (2), for the regime of intermediate Reynolds numbers, $Ar \sim Re^{3/2}$ and, hence, $Re^{1/2} \sim Ar^{1/3}$. Substitution of this relation into Equation (26) yields to Baskakov's statement, Equation (25).

The irrelevance of the particle size $d_p$, however, requires its elimination from Equation (25). A turbulent flow length scale $l_t$ follows from

$$Ar^{1/3} \equiv \frac{d_p}{\sqrt{\frac{\mu^2}{\rho_e (\rho_e-\rho_g) g}}}$$

i.e.

$$l_t \equiv \sqrt{\frac{\mu^2}{\rho_e (\rho_e-\rho_g) g}}. \quad (27)$$

Substitution of $l_t$ into Equation (25) yields

$$\frac{h_{max} \delta_l}{k_g} Pr^{-1/3} = \text{const.} \quad (28)$$

It is worth noting that the independence of the particle size in the regime of $10^5 \leq Ar \leq 10^8$ has a completely different physical background in comparison with that observed at $Ar \leq 10^2$. The dependence on the excess gas velocity according to curve 4 of Figure 2 is given by the normalized function

$$n_{gc} = \frac{1}{1 + G (u-u_{mf})} \quad (29)$$

with a constant $G$ to be determined from experiments. Equation (29) predicts $n_{gc} \rightarrow 0$ for $u-u_{mf} \rightarrow 0$ and $n_{gc} \rightarrow 1$ for $u-u_{mf} \rightarrow \infty$.

The combination of Equations (28) and (29) together with evaluation of experiments yields the final result for gas convective heat transfer (compare Fig. 10)

$$\frac{h_{gc} \delta_l}{k_g} = \frac{0.165 Pr^{1/3}}{1 + 0.05 (u-u_{mf})^{-1}} \quad \text{for} \quad 10^5 \leq Ar \leq 10^8 \quad (30)$$

Fig. 10 Comparison of measurements by Wunder [2] (symbols) with predictions of Eq. (30) (lines). pressure varied.

8. Heat transfer in the regime $10^2 \leq Ar \leq 10^5$

With two well-understood end points, namely purely particle convective heat transfer at $Ar \leq 10^2$ and purely gas convective type at $Ar \geq 10^5$, a predictive equation for the intermediate regime can be derived in the form of an interpolation between the two extremes. A successful strategy can be derived from Figures 4 and 2. Figure 4 evidences a rather wide particle residence time distribution. This means that in the intermediate regime of Archimedes numbers, short residence times result in a "gas convective" behavior, whereas long residence times favor the "particle convective" behavior. An increase in the
Excess gas velocity \( u - u_{mf} \) increases the bubble frequency which, for its part, reduces the mean particle residence time at the heating surface. All these features are evident from Figure 2: an increase in particle size \( dp \), and, hence, in the minimum fluidization velocity \( u_{mf} \), shifts the behavior from particle convective towards gas convective (compare curves 1, 2, 3 and 4). The above-mentioned considerations also explain the humps observed in curves 2 and 3. These curves start more "particle convective" and end more "gas convective". In particular, curve 3 approaches the purely gas convective curve 4 from top. All these considerations taken together, a damping function must be implemented in the formula for the particle convective heat transfer which reflects two features: the particle convective heat transfer must decrease with increasing \( U_{mf} \) as well as with increasing \( u - U_{mf} \). The final result of a prolonged trial-and-error procedure in the form of the comparison with all available well-documented experiments resulted in the formula [15]

\[
\frac{h}{k} = \frac{0.125 (1 - \epsilon_{mf}) (1 + 3.3 (\frac{(u - u_{mf})/u_{mf}}{\sqrt{\frac{P_{ref}}{P_{ref}}}}))}{1 + \frac{U_{mf}}{2 U_{mf}} (1 + 0.28 (\frac{1}{\epsilon_{mf}}))^{1/3} \sqrt{\frac{P_{ref}}{P_{ref}}}} \left(1 + 0.28 (\frac{1}{\epsilon_{mf}})\right)^{1/3} \sqrt{\frac{P_{ref}}{P_{ref}}}
\]

(31)

which fits all situations for \( Ar \leq 10^8 \). Its complexity is justified by comparison with experimental data in Figures 11 to 15, which reflect all variations in the relevant data (Fig. 11: particle size together with particle shape in the form of \( \epsilon_{mf} \); Figure 12: particle density and particle specific heat; Figure 13: gas thermal conductivity; Figure 14: gas density; Figure 15: bed temperature).

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**Fig. 11** Measurements (symbols) with mullite-air at ambient conditions, heat transfer surface: single vertical immersed tube, data from Wunder [2], prediction (lines) according to Eq. (30).

**Fig. 12** Measurements (symbols) with different solids - air at ambient conditions, heat transfer surface: single vertical immersed tube, data from Wunder [2], prediction (lines) according to Eq. (30).

**Fig. 13** Measurements (symbols) with sand - different gases at ambient conditions, heat transfer surface: single vertical immersed tube, data from Wicke and Fetting [16], prediction (lines) according to Eq. (30).

**Fig. 14** Own measurements (symbols) with glass beads - air, pressure varied, heat transfer surface: single vertical immersed tube, prediction (lines) according to Eq. (30).
From the viewpoint of chemical engineering practice, the most important result is depicted in Figure 15. At Archimedes numbers $\text{Ar} < 10^5$, the cooling down of hot particles at cooling surfaces results in an effective shielding against radiative effects - even at elevated temperatures up to 780°C. On the other hand, with purely gas convective heat transfer at $\text{Ar} \geq 10^5$, only marginal cooling down or heating up of the particles at the cooling or heating surface is observed. Therefore, at $\text{Ar} \geq 10^5$, the radiative component of the heat transfer gives a significant contribution for temperatures $> 500°C$.

9. Additional remarks

Fluidization ceases with the dominance of adhesion forces in comparison with aerodynamic resistance and weight minus buoyancy [18] usually observed at particle sizes of $d_p \leq 10 \mu m$. The then observed particle immobility manifests itself in a sharp breakdown of the heat transfer.

With coarse-grained particles and elevated pressures, Archimedes numbers $\text{Ar} > 10^6$ are attained. As can be read from Figure 1, Archimedes numbers $\text{Ar} > 10^6$ result in Reynolds numbers $\text{Re} > 10^4$, which means an effective heat transfer even in fixed beds. The by-pass effect of gas bubbles in fluidized beds then results in even a decrease of the heat transfer in fluidized beds in comparison with fixed beds (for details see [19]).

10. Conclusions

Heat transfer in gas-fluidized beds is the result of complex mutual interactions of fluid mechanics, particle dynamics and the thermal properties of the involved media. The derivation of a predictive equation for the reliable prediction of the heat transfer in bubbling fluidized beds thus involves a step-by-step procedure which takes into account all relevant aspects of the problem in question. Due to the large number of influencing factors (altogether eleven) with four basic units ($K, \text{kg, m, s}$), the resulting equation, Equation (30) must be complex, namely built up from seven different dimensionless groups.

List of Symbols

- $C$: constant
- $c_p$: specific heat of particle material [Ws kg$^{-1}$ K$^{-1}$]
- $d_p$: particle diameter [m]
- $G$: constant
- $g$: gravitational acceleration [ms$^{-2}$]
- $h_{gc}$: gas convective heat transfer coefficient [Wm$^{-2}$ K$^{-1}$]
- $h_{pc}$: particle convective heat transfer coefficient [Wm$^{-2}$ K$^{-1}$]
- $h, h_{\text{max}}$: heat transfer coefficient, maximum value [Wm$^{-2}$ K$^{-1}$]
- $k_e$: effective thermal conductivity [Wm$^{-1}$ K$^{-1}$]
- $k_g$: gas thermal conductivity [Wm$^{-1}$ K$^{-1}$]
- $k_s$: solid material thermal conductivity [Wm$^{-1}$ K$^{-1}$]
- $L, L_0$: digital luminosity, initial value
- $\ell$: length [m]
- $\ell_0$: mean free path of gas molecules [m]
- $M$: molar mass [kg mol$^{-1}$]
- $\ell_l$: laminar flow length scale [m]
- $\ell_t$: turbulent flow length scale [m]
- $n_{gc}$: normalized gas convective heat transfer function
- $n_{pc}$: normalized particle convective heat transfer function
- $p$: pressure [Nm$^{-2}$]
- $P$: constant
- $R$: radius [m]
- $R'$: aerodynamic resistance force [N]
- $\bar{R}$: gas constant [Nm mol$^{-1}$ K$^{-1}$]
- $r$: integration variable [m]
- $S$: size of surface asperities [m]
- $S_{\text{min}}$: minimum effective surface roughness [m]
- $T$: absolute temperature [K]
- $t$: time [s]
- $u$: superficial gas velocity [ms$^{-1}$]
- $u_{\text{inf}}$: superficial gas velocity at minimum fluidization condition [ms$^{-1}$]
- $v$: fluid velocity [ms$^{-1}$]
- $W$: probability
Greek letters

\( \epsilon_{mf} \): minimum fluidization void fraction
\( \gamma \): accommodation coefficient
\( \varphi \): angle
\( \rho_g \): gas density \([\text{kg} \text{m}^{-3}]\)
\( \rho_s \): solid density \([\text{kg} \text{m}^{-3}]\)
\( \tau \): mean residence time \([\text{s}]\)

Dimensionless groups

\[ \text{Ar} = \frac{d^3 g (\rho_s - \rho_g) \rho_g}{\mu^2} \] Archimedes number

\[ \text{Co} = \frac{k_s C_{s,m}}{\rho_s c_p d_p^2} \] Dimensionless contact time

\[ c_D = \frac{R_e}{(\pi d_p^2/4) (\rho_d/2) u^2} \] Drag coefficient

\[ \text{Nu}, \text{Nu}_{\text{max}} = \frac{h d_p}{k_g} \] Nusselt number

\[ \text{Pr} = \frac{c_p \mu}{k_g} \] Prandtl number

\[ \text{Re} = \frac{\rho_g v d_p}{\mu} \] Reynolds number

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Author's short biography

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