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Nonphononic spectrum of two-dimensional structural glasses

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Extensive numerical work from recent years has established that the low-frequency nonphononic vibrational spectrum of structural glasses follows an $\sim \omega^5$ scaling with angular frequency $\omega$. This universal quartic ($\sim \omega^4$) law featured by the nonphononic spectrum has been shown to be independent of the details of the interparticle interaction potential,\textsuperscript{1} glass formation protocol,\textsuperscript{1} and spatial dimensions $d \geq 2$.\textsuperscript{1,4} Furthermore, indirect evidence for the quartic scaling of the nonphononic spectrum of two-dimensional structural glasses was presented in Refs. 1 and 6–8. In Refs. 4 and 7, it was shown that in two-dimensional structural glasses, the prefactor $\omega_{\text{g}}^4$ of the quartic scaling law is $N$-dependent, scaling as $\omega_{\text{g}}^4 \sim (\log N)^5$, where $N$ denotes the number of particles in a glass. At the same time, deviations from the quartic scaling of the nonphononic spectrum were reported for three-dimensional (3D) structural glasses in Refs. 9 and 10 depending on the system size and formation protocol of the glasses studied. These previous works associated these deviations with a glassy length scale $\xi_\text{g}$—on the order of a few interparticle distances—and established that the nonphononic spectrum of glasses whose linear size sufficiently exceeds $\xi_\text{g}$ features the universal quartic law $\sim \omega^4$.\textsuperscript{1,4} The length $\xi_\text{g}$ has been shown to be glass-formation-protocol dependent—decreasing for lower-energy, more stable glasses—fully consistent with previously observed deviations\textsuperscript{1,4} in the exponent $\beta$ of the nonphononic spectrum $\sim \omega^\beta$ from the apparently universal value $\beta = 4$.\textsuperscript{1,4}

Recently, in Refs. 11 and 12, it has been argued that the nonphononic spectrum of two-dimensional (2D) structural glasses rather follows $\sim \omega^\beta$ with $\beta < 4$, presumably casting doubt on the validity of previous observations and claims.\textsuperscript{1,11} Here, we study the nonphononic spectrum of two-dimensional structural glasses and show that (i) the exponent $\beta$ is glass-formation-protocol and system-size dependent, as seen previously for structural glasses in three dimensions,\textsuperscript{9,10} and (ii) the scaling $\omega_{\text{g}} \sim 1/(\sqrt{\log N})$ put forward in Refs. 4 and 7 is consistent with numerical observations presented below.

We simulate a generic glass-forming model in 2D—also studied in Refs. 11 and 12—in which point-like particles interact via an inverse-power-law potential that is smoothed at a cutoff distance up to two derivatives; see, e.g., Ref. 7 for the model’s details. We prepare ensembles of glasses by (i) instantaneously quenching high-temperature liquid states to zero temperature (as done in Refs. 11 and 12) and (ii) by annealing liquids for $10^4$ simulational time units (described, e.g., in Ref. 7) at temperature $T = 0.5$, which is approximately the (computer) glass transition temperature of our studied glass-forming model. Each of these annealing runs are followed by an instantaneous quench to zero temperature to form a glass (termed “well-annealed” in Fig. 1). The ensemble sizes can be found in Ref. 13.

In Fig. 1, we present our results; Figs. 1(a) and 1(c) show the raw vibrational spectra $D(\omega; N)$ of our pair of glass ensembles, respectively, as indicated in the legends. A clear system-size dependence is apparent in both ensembles. We further find that the exponent featured by the low-frequency power-law regime of $D(\omega; N)$ drops below 4 for smaller, quickly quenched glasses, cf. Fig. 1(c), consistent with observations in 3D.\textsuperscript{11} That is, we find that $\beta(N) \approx 4$ increases with the system size $N$ from $\beta \approx 3.2$ for $N = 36$ to $\beta \approx 3.6$ for $N = 1600$. In Figs. 1(b) and 1(d), we show that rescaling both axes according to the $N$ dependence of the nonphononic-excitations’ characteristic scale $\sim 1/(\sqrt{\log N})$ as put forward in Refs. 4 and 7 leads to a convincing data collapse, confirming that the nonphononic...
VDoS of 2D glasses is, indeed, $N$ dependent. In Fig. 2, we present the cumulative distributions $C(\omega) \equiv \int_{\omega_0}^{\omega_0/\omega} D(\omega')d\omega'$, showing again that $\beta$ features both glass-formation protocol and system-size dependencies. We note that the $N$ dependence of the exponent $\beta(N)$ also appears to be consistent with the results presented in the supplementary material file of Refs. 11 and 12 (which can be found in Ref. 14). For example, in Fig. 2(b), therein, the cumulative density of states normalized by $\omega^{4.5}$ is presented for various system sizes.
The data that support the findings of this study are available from the corresponding author upon reasonable request.

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