Rare $B$ Decays in the Standard Model

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Abstract

We discuss the electromagnetic-penguin-dominated radiative $B$ decays $B \rightarrow X_s + \gamma$, $B^{\pm(0)} \rightarrow K^{*\pm(0)} + \gamma$, and $B_s \rightarrow \phi + \gamma$ in the context of the standard model (SM) and their Cabibbo-Kobayashi-Maskawa (CKM)-suppressed counterparts, $B \rightarrow X_d + \gamma$, $B^{\pm} \rightarrow \rho^{\pm} + \gamma$, $B^0 \rightarrow (\rho^0, \omega) + \gamma$, and $B_s \rightarrow K^{*0} + \gamma$, using QCD sum rules for the exclusive decays. The importance of these decays in determining the parameters of the CKM matrix is emphasized. The semileptonic decays $B \rightarrow X_s \ell^+ \ell^-$ are also discussed in the context of the SM and their role in determining the Wilson coefficients of the effective theory is stressed. Comparison with the existing measurements are made and SM-based predictions for a large number of rare $B$ decays are presented.
1 Estimates of $\mathcal{B}(B \to X_s + \gamma)$ and $|V_{ts}|$ in the Standard Model

The Standard Model (SM) of particle physics does not admit Flavour-changing-neutral-current (FCNC) transitions in the Born approximation. However, they are induced through the exchange of $W^\pm$ bosons in loop diagrams. The short-distance contribution in rare decays is dominated by the (virtual) top quark contribution. Hence the decay characteristics provide quantitative information on the top quark mass and the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements $V_{ti}$; $i = d, s, b$. We shall discuss representative examples from several such transitions involving $B$ decays, starting with the decay $B \to X_s + \gamma$, which has been measured by CLEO [2]. This was preceded by the measurement of the exclusive decay $B \to K^* + \gamma$ by the same collaboration [3]. The present measurements give [4]:

$$\mathcal{B}(B \to X_s + \gamma) = (2.32 \pm 0.57 \pm 0.35) \times 10^{-4},$$

$$\mathcal{B}(B \to K^* + \gamma) = (4.2 \pm 0.8 \pm 0.6) \times 10^{-5},$$

yielding an exclusive-to-inclusive ratio:

$$R_{K^*} = \frac{\Gamma(B \to K^* + \gamma)}{\Gamma(B \to X_s + \gamma)} = (18.1 \pm 6.8)\%.$$ (3)

These decay rates determine the ratio of the CKM matrix elements $|V_{ts}|/|V_{cb}|$ and the quantity $R_{K^*}$ provides information on the decay form factor in $B \to K^* + \gamma$. In what follows we take up these points briefly.

The leading contribution to $b \to s + \gamma$ arises at one-loop from the so-called penguin diagrams. With the help of the unitarity of the CKM matrix, the decay matrix element in the lowest order can be written as:

$$\mathcal{M}(b \to s + \gamma) = \frac{G_F}{\sqrt{2}} \frac{e}{2\pi^2} \lambda_t (F_2(x_t) - F_2(x_c)) q^\mu \epsilon^\nu \bar{s} \gamma_\mu (m_b R + m_s L) b.$$ (4)

where $x_i = m_i^2/m_W^2$, and $q_\mu$ and $\epsilon_\mu$ are, respectively, the photon four-momentum and polarization vector. The GIM mechanism [5] is manifest in this amplitude and the CKM-matrix element dependence is factorized in $\lambda_t \equiv V_{tb} V_{ts}^\ast$. The (modified) Inami-Lim function $F_2(x_i)$ derived from the (1-loop) penguin diagrams is given by [6]:

$$F_2(x) = \frac{x}{24(x-1)^4} \left[ 6x(3x-2) \log x - (x-1)(8x^2 + 5x - 7) \right].$$ (5)
The measurement of the branching ratio for $B \to X_s + \gamma$ can be readily interpreted in terms of the CKM-matrix element product $\lambda_t/|V_{cb}|$ or equivalently $|V_{ts}|/|V_{cb}|$. For a quantitative determination of $|V_{ts}|/|V_{cb}|$, however, QCD radiative corrections have to be computed and the contribution of the so-called long-distance effects estimated.

The appropriate framework to incorporate QCD corrections is that of an effective theory obtained by integrating out the heavy degrees of freedom, which in the present context are the top quark and $W^\pm$ bosons. The operator basis depends on the underlying theory and for the SM one has (keeping operators up to dimension 6),

$$
\mathcal{H}_{\text{eff}}(b \to s + \gamma) = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{8} C_i(\mu) O_i(\mu),
$$

where the operator basis, the “matching conditions” $C_i(m_W)$, and the solutions of the renormalization group equations $C_i(\mu)$ can be seen in ref. [7]. The perturbative QCD corrections to the decay rate $\Gamma(B \to X_s + \gamma)$ have two distinct contributions:

- Corrections to the Wilson coefficients $C_i(\mu)$, calculated with the help of the renormalization group equation, whose solution requires the knowledge of the anomalous dimension matrix in a given order in $\alpha_s$.

- Corrections to the matrix elements of the operators $O_i$ entering through the effective Hamiltonian at the scale $\mu = O(m_b)$.

The anomalous dimension matrix is needed in order to sum up large logarithms, i.e., terms like $\alpha_s^n(m_W) \log^m(m_b/M)$, where $M = m_t$ or $m_W$ and $m \leq n$ (with $n = 0, 1, 2, \ldots$). At present only the leading logarithmic corrections ($m = n$) have been calculated systematically and checked by several independent groups in the complete basis given in Eq. (6) [8]. First calculations of the NLO corrections to the anomalous dimension matrix have been recently reported by Misiak [9] and are found to be small. Next-to-leading order corrections to the matrix elements are now available completely. They are of two kinds:

- QCD Bremsstrahlung corrections $b \to s\gamma + g$, which are needed both to cancel the infrared divergences in the decay rate for $B \to X_s + \gamma$ and in obtaining a non-trivial QCD contribution to the photon energy spectrum in the inclusive decay $B \to X_s + \gamma$. 
Next-to-leading order virtual corrections to the matrix elements in the
decay $b \to s + \gamma$.

The Bremsstrahlung corrections were calculated in [10, 11] in the truncated
basis and last year also in the complete operator basis [12, 13]. The higher
order matching conditions, i.e., $C_i(m_W)$, are known up to the desired accu-

racy, i.e., up to $O(\alpha_s(M_W))$ terms [14]. The next-to-leading order virtual
corrections have also been calculated [15]. They reduce the scale-dependence
of the inclusive decay width. The branching ratio $\mathcal{B}(B \to X_s + \gamma)$ can be
expressed in terms of the semileptonic decay branching ratio

$$\mathcal{B}(B \to X_s\gamma) = \left[ \frac{\Gamma(B \to \gamma + X_s)}{\Gamma_{SL}} \right]^{th} \mathcal{B}(B \to X\ell\nu_\ell) ,$$

where the leading-order QCD corrected expression for $\Gamma_{SL}$ can be seen in [7].

The leading order $(1/m_b)$ power corrections in the heavy quark expansion are
identical in the inclusive decay rates for $B \to X_s + \gamma$ and $B \to X\ell\nu_\ell$, entering
in the numerator and denominator in the square bracket, respectively, and
hence drop out.

In Ref. [7], the present theoretical errors on the branching ratio $\mathcal{B}(B \to X_s\gamma)$ are discussed, yielding:

$$\mathcal{B}(B \to X_s\gamma) = (3.20 \pm 0.30 \pm 0.38 \pm 0.32) \times 10^{-4}$$

where the first error comes from the combined effect of $\Delta m_t$ and $\Delta \mu$ (the scale dependence), the second error arises from the extrinsic sources (such as
$\Delta(m_b)$, $\Delta(BR_{SL})$), and the third error is an estimate ($\pm 10\%$) of the NLO anomalous dimension piece in $C_7^{\text{eff}}$, the coefficient of the magnetic moment
operator. Combining the theoretical errors in quadrature gives [7]:

$$\mathcal{B}(B \to X_s\gamma) = (3.20 \pm 0.58) \times 10^{-4} ,$$

which is compatible with the present measurement $\mathcal{B}(B \to X_s\gamma) = (2.32 \pm 0.67) \times 10^{-4}$ [4]. Expressed in terms of the CKM matrix element ratio, one gets

$$\frac{|V_{ts}|}{|V_{cb}|} = 0.85 \pm 0.12(\text{expt}) \pm 0.10(\text{th}),$$

which is within errors consistent with unity, as expected from the unitarity
of the CKM matrix.
2 Inclusive radiative decays $B \rightarrow X_d + \gamma$

The theoretical interest in studying the (CKM-suppressed) inclusive radiative decays $B \rightarrow X_d + \gamma$ lies in the first place in the possibility of determining the parameters of the CKM matrix. We shall use the Wolfenstein parametrization \[16\], in which case the matrix is determined in terms of the four parameters $A, \lambda = \sin \theta_C, \rho$ and $\eta$. The quantity of interest in the decays $B \rightarrow X_d + \gamma$ is the end-point photon energy spectrum, which has to be measured requiring that the hadronic system $X_d$ recoiling against the photon does not contain strange hadrons to suppress the large-$E_\gamma$ photons from the decay $B \rightarrow X_s + \gamma$. Assuming that this is feasible, one can determine from the ratio of the decay rates $\mathcal{B}(B \rightarrow X_d + \gamma)/\mathcal{B}(B \rightarrow X_s + \gamma)$ the CKM-Wolfenstein parameters $\rho$ and $\eta$. This measurement was first proposed in \[11\], where the photon energy spectra were also worked out.

In close analogy with the $B \rightarrow X_s + \gamma$ case discussed earlier, the complete set of dimension-6 operators relevant for the processes $b \rightarrow d\gamma$ and $b \rightarrow d\gamma g$ can be written as:

$$\mathcal{H}_{\text{eff}}(b \rightarrow d) = -\frac{4G_F}{\sqrt{2}} \xi_t \sum_{j=1}^{8} C_j(\mu) \hat{O}_j(\mu),$$

(11)

where $\xi_j = V_{jb}V_{jd}^\ast$ for $j = t, c, u$. The operators $\hat{O}_j, j = 1, 2$, have implicit in them CKM factors. In the Wolfenstein parametrization \[10\], one can express these factors as:

$$\xi_u = A \lambda^3 (\rho - i\eta), \quad \xi_c = -A \lambda^3, \quad \xi_t = -\xi_u - \xi_c.$$  

(12)

We note that all three CKM-angle-dependent quantities $\xi_j$ are of the same order of magnitude, $O(\lambda^3)$. This aspect can be taken into account by defining the operators $\hat{O}_1$ and $\hat{O}_2$ entering in $\mathcal{H}_{\text{eff}}(b \rightarrow d)$ as follows \[11\]:

$$\hat{O}_1 = -\frac{\xi_c}{\xi_t} (\bar{c}_L\gamma^\mu b_{L\alpha})(\bar{d}_{L\beta}\gamma_\mu c_{L\beta}) - \frac{\xi_u}{\xi_t} (\bar{u}_{L\beta}\gamma^\mu b_{L\alpha})(\bar{d}_{L\alpha}\gamma_\mu u_{L\beta}),$$

$$\hat{O}_2 = -\frac{\xi_c}{\xi_t} (\bar{c}_{L\alpha}\gamma^\mu b_{L\alpha})(\bar{d}_{L\beta}\gamma_\mu c_{L\beta}) - \frac{\xi_u}{\xi_t} (\bar{u}_{L\alpha}\gamma^\mu b_{L\alpha})(\bar{d}_{L\beta}\gamma_\mu u_{L\beta}),$$

(13)

with the rest of the operators ($\hat{O}_j; j = 3...8$) defined like their counterparts $O_j$ in $\mathcal{H}_{\text{eff}}(b \rightarrow s)$, with the obvious replacement $s \rightarrow d$. With this choice, the
matching conditions \( C_j(m_W) \) and the solutions of the RG equations yielding \( C_j(\mu) \) become identical for the two operator bases \( O_j \) and \( \hat{O}_j \). The essential difference between \( \Gamma(B \to X_s + \gamma) \) and \( \Gamma(B \to X_d + \gamma) \) lies in the matrix elements of the first two operators \( O_1 \) and \( O_2 \) (in \( H_{eff}(b \to s) \)) and \( \hat{O}_1 \) and \( \hat{O}_2 \) (in \( H_{eff}(b \to d) \)). The branching ratio \( B(B \to X_d + \gamma) \) in the SM can be written as:

\[
B(B \to X_d + \gamma) = D_1 \lambda^2 \{ (1 - \rho)^2 + \eta^2 - (1 - \rho)D_2 - \eta D_3 + D_4 \},
\]

where the functions \( D_i \) depend on the parameters \( m_t, m_b, m_c, \mu \), as well as the others we discussed in the context of \( B(B \to X_s + \gamma) \). These functions were first calculated in [11] in the leading logarithmic approximation. Recently, these estimates have been improved in [19], making use of the NLO calculations in [15]. To get an estimate of the inclusive branching ratio, the CKM parameters \( \rho \) and \( \eta \) have to be constrained from the unitarity fits. Present data and theory restrict them to lie in the following range (at 95% C.L.) [17]:

\[
0.20 \leq \eta \leq 0.52, \\
-0.35 \leq \rho \leq 0.35,
\]

which, on using the current lower bound from LEP on the \( B^0_s - \bar{B}^0_s \) mass difference \( \Delta M_s > 9.2 \ (ps)^{(-1)} \) [20], is reduced to \(-0.25 \leq \rho \leq 0.35 \) using \( \xi_s = 1.1 \), where \( \xi_s \) is the \( SU(3) \)-breaking parameter \( \xi_s = f_{B^s}B_{B^s}/f_{B^d}B_{B^d} \). The preferred CKM-fit values are \( (\rho, \eta) = (0.05, 0.36) \), for which one gets

\[
B(B \to X_d + \gamma) = 1.63 \times 10^{-5},
\]

whereas \( B(B \to X_d + \gamma) = 8.0 \times 10^{-6} \) and \( 2.8 \times 10^{-5} \) for the choice \( \rho = 0.35, \eta = 0.40 \) and \( \rho = -\eta = -0.25 \), respectively. In conclusion, we note that the functional dependence of \( B(B \to X_d + \gamma) \) on the Wolfenstein parameters \( (\rho, \eta) \) is mathematically different than that of \( \Delta M_s \). However, qualitatively they are very similar. From the experimental point of view, the situation \( \rho < 0 \) is favourable for both the measurements as in this case one expects (relatively) smaller values for \( \Delta M_s \) and larger values for the branching ratio \( B(B \to X_d + \gamma) \), as compared to the \( \rho > 0 \) case which would yield larger \( \Delta M_s \) and smaller \( B(B \to X_d + \gamma) \).
2.1 $\mathcal{B}(B \to V + \gamma)$ and constraints on the CKM parameters

Exclusive radiative $B$ decays $B \to V + \gamma$, with $V = K^*, \rho, \omega$, are also potentially very interesting from the point of view of determining the CKM parameters \[21\]. The extraction of these parameters would, however, involve a trustworthy estimate of the SD- and LD-contributions in the decay amplitudes.

The SD-contribution in the exclusive decays $(B^\pm, B^0) \to (K^{*\pm}, K^{*0}) + \gamma$, $(B^\pm, B^0) \to (\rho^\pm, \rho^0) + \gamma$, $B^0 \to \omega + \gamma$ and the corresponding $B_s$ decays, $B_s \to \phi + \gamma$, and $B_s \to K^{*0} + \gamma$, involve the magnetic moment operator $\mathcal{O}_7$ and the related one obtained by the obvious change $s \to d$, $\hat{\mathcal{O}}_7$. The transition form factors governing the radiative $B$ decays $B \to V + \gamma$ can be generically defined as:

$$\langle V, \lambda | \frac{1}{2} \psi_{\mu\nu} q^\nu b | B \rangle = i \epsilon_{\mu\nu\rho\sigma} e_\nu^{(\lambda)} p_B p_V F_{B \to V}^{S}(0). \tag{17}$$

Here $V$ is a vector meson with the polarization vector $e_\nu^{(\lambda)}$, $V = \rho, \omega, K^*$ or $\phi$; $B$ is a generic $B$-meson $B^\pm, B^0$ or $B_s$, and $\psi$ stands for the field of a light $u, d$ or $s$ quark. The vectors $p_B, p_V$ and $q = p_B - p_V$ correspond to the 4-momenta of the initial $B$-meson and the outgoing vector meson and photon, respectively. In (17) the QCD renormalization of the $\bar{\psi}\psi_{\mu\nu} q^\nu b$ operator is implied. Keeping only the SD-contribution leads to obvious relations among the exclusive decay rates, exemplified here by the decay rates for $(B_u, B_d) \to \rho + \gamma$ and $(B_u, B_d) \to K^* + \gamma$:

$$\frac{\Gamma((B_u^\pm, B_d^0) \to (\rho^\pm, \rho^0) + \gamma)}{\Gamma((B_u^\pm, B_d^0) \to (K^{*\pm}, K^{*0}) + \gamma)} = \frac{|\xi_u|}{|\lambda_u|^2} \frac{|F_{B \to \rho}^{S}(0)|^2}{|F_{B \to K^*}^{S}(0)|^2} \Phi_{u,d} \simeq \kappa_{u,d} \left( \frac{|V_{ud}|}{|V_{ts}|} \right)^2, \tag{18}$$

where $\Phi_{u,d}$ is a phase-space factor which in all cases is close to 1 and $\kappa_i \equiv [F_{S}(B_i \to \rho\gamma)/F_{S}(B_i \to K^*\gamma)]^2$. The transition form factors $F_S$ are model dependent. Estimates of $F_S$ in the QCD sum rule approach in the normalization of Eq. (17) range between $F_{S}(B \to K^*\gamma) = 0.31$ (Narison in \[22\]) to $F_{S}(B \to K^*\gamma) = 0.37$ (Ball in \[22\]), with a typical error of $\pm 15\%$, and hence are all consistent with each other. This, for example, gives $R_{K^*} = 0.16 \pm 0.05$, using the result from \[21\], which is in good agreement with data. The ratios of the form factors, i.e. $\kappa_i$, should therefore be reliably calculable as
they depend essentially only on the SU(3)-breaking effects which have been estimated \[21, 22\].

The LD-amplitudes in radiative B decays from the light quark intermediate states necessarily involve other CKM matrix elements. Hence, the simple factorization of the decay rates in terms of the CKM factors involving \(|V_{td}|\) and \(|V_{ts}|\) no longer holds thereby invalidating the relation \[18\] given above. In the decays \(B \to V + \gamma\) they are induced by the matrix elements of the four-Fermion operators \(\hat{O}_1\) and \(\hat{O}_2\) (likewise \(O_1\) and \(O_2\)). Estimates of these contributions require non-perturbative methods. This problem has been investigated in \[23, 24\] using a technique \[25\] which treats the photon emission from the light quarks in a theoretically consistent and model-independent way. This has been combined with the light-cone QCD sum rule approach to calculate both the SD and LD — parity conserving and parity violating — amplitudes in the decays \((B^\pm, B^0) \to (\rho^\pm, \omega) + \gamma\). To illustrate this, we concentrate on the \(B^\pm\) decays, \(B^\pm \to \rho^\pm + \gamma\) and take up the neutral B decays \(B^0 \to \rho(\omega) + \gamma\) at the end. The LD-amplitude of the four-Fermion operators \(\hat{O}_1\), \(\hat{O}_2\) is dominated by the contribution of the weak annihilation of valence quarks in the B meson and it is color-allowed for the decays of charged \(B^\pm\) mesons. Using factorization, the LD-amplitude in the decay \(B_u \to \rho^\pm + \gamma\) can be written in terms of the form factors \(F^L_1\) and \(F^L_2\),

\[
A_{\text{long}} = -\frac{eG_F}{\sqrt{2}} V_{ub} V_{ud}^* \left( C_2 + \frac{1}{N_c} C_1 \right) m_{\rho} \varepsilon_\mu^{(\gamma)} \varepsilon_\nu^{(\rho)} \\
\times \left\{ -i \left[ g^{\mu\nu} (q \cdot p) - p^\mu q^\nu \right] \cdot 2F^L_1(q^2) + e^{\nu\alpha\beta} p_\alpha q_\beta \cdot 2F^L_2(q^2) \right\}.
\]

Again, one has to invoke a model to calculate the form factors. Estimates from the light-cone QCD sum rules give \[24\]:

\[
F^L_1 / F_S = 0.0125 \pm 0.0010, \quad F^L_2 / F_S = 0.0155 \pm 0.0010,
\]

where the errors correspond to the variation of the Borel parameter in the QCD sum rules. Including other possible uncertainties, one expects an accuracy of the ratios in \(20\) of order 20\%. The parity-conserving and parity-violating amplitudes turn out to be numerically close to each other in the QCD sum rule approach, \(F^L_1 \simeq F^L_2 \equiv F_L\), hence the ratio of the LD- and the SD- contributions reduces to a number \[24\]

\[
\frac{A_{\text{long}}}{A_{\text{short}}} = R_{L/S}^{B^\pm \to \rho^\pm \gamma} \frac{V_{ub} V_{ud}^*}{V_{tb} V_{td}^*}.
\]
Using $C_2 = 1.10$, $C_1 = -0.235$, $C_{7}^\text{eff} = -0.306$ from Ref. [7] (corresponding to the scale $\mu = 5 \text{ GeV}$) gives:

$$R_{L/S}^{B^{\pm} \to \rho^{\pm} \gamma} \equiv \frac{4\pi^2 m_\rho (C_2 + C_1/N_c)}{m_b C_{7}^\text{eff}} \cdot \frac{F_L^{B^{\pm} \to \rho^{\pm} \gamma}}{F_S^{B^{\pm} \to \rho^{\pm} \gamma}} = -0.30 \pm 0.07, \quad (22)$$

which is not small. To get a ball-park estimate of the ratio $A_{\text{long}}/A_{\text{short}}$, we take the central value from the CKM fits, yielding $|V_{ub}/|V_{td}| \simeq 0.33$ [17],

$$|A_{\text{long}}/A_{\text{short}}| \equiv \left| R_{L/S}^{B^{\pm} \to \rho^{\pm} \gamma} \right| \frac{|V_{ub} V_{ud}|}{|V_{td} V_{tb}|} \simeq 10\% . \quad (23)$$

Thus, the CKM factors suppress the LD-contributions.

The analogous LD-contributions to the neutral $B$ decays $B^0 \rightarrow \rho \gamma$ and $B^0 \rightarrow \omega \gamma$ are expected to be much smaller. The corresponding form factors for the decays $B^0 \rightarrow \rho^0(\omega) \gamma$ are obtained from the ones for the decay $B^{\pm} \rightarrow \rho^{\pm} \gamma$ discussed above by the replacement of the light quark charges $e_u \rightarrow e_d$, which gives the factor $-1/2$; in addition, and more importantly, the LD-contribution to the neutral $B$ decays is colour-suppressed, which reflects itself through the replacement of the factor $a_1$ by $a_2$. This yields for the ratio

$$R_{L/S}^{B^0 \rightarrow \rho^0 \gamma} \equiv \frac{e_d a_2}{e_u a_1} \simeq -0.13 \pm 0.05, \quad (24)$$

where the numbers are based on using $a_2/a_1 = 0.27 \pm 0.10$ [28]. This would then yield $R_{L/S}^{B^0 \rightarrow \rho^0 \gamma} \simeq R_{L/S}^{B^0 \rightarrow \omega \gamma} = 0.05$, which in turn gives

$$\left| A_{\text{long}}^{B^0 \rightarrow \rho^0 \gamma} / A_{\text{short}}^{B^0 \rightarrow \rho^0 \gamma} \right| \leq 0.02. \quad (25)$$

This, as well as the estimate in eq. 23, should be taken only as indicative in view of the approximations made in [23, 24]. That the LD-effects remain small in $B^0 \rightarrow \rho \gamma$ has been supported in a recent analysis based on the soft-scattering of on-shell hadronic decay products $B^0 \rightarrow \rho^0 \rho^0 \rightarrow \rho \gamma$ [24], though this paper estimates them somewhat higher (between 4 – 8%).

Restricting to the colour-allowed LD-contributions, the relations, which obtains ignoring such contributions (and isospin invariance),

$$\Gamma(B^{\pm} \rightarrow \rho^{\pm} \gamma) = 2 \Gamma(B^0 \rightarrow \rho^0 \gamma) = 2 \Gamma(B^0 \rightarrow \omega \gamma), \quad (26)$$
get modified to

\[
\frac{\Gamma(B^\pm \to \rho^\pm \gamma)}{2\Gamma(B^0 \to \rho \gamma)} = \frac{\Gamma(B^\pm \to \rho^\pm \gamma)}{2\Gamma(B^0 \to \omega \gamma)} = \left| 1 + R_{L/S}^{B^\pm \to \rho^\pm \gamma} \frac{V_{ud} V_{ud}^*}{V_{tb} V_{tb}^*} \right|^2 = \\
1 + 2 \cdot R_{L/S} V_{ud} \frac{\rho(1 - \rho) - \eta^2}{(1 - \rho)^2 + \eta^2} + (R_{L/S})^2 V_{ud}^2 \frac{\rho^2 + \eta^2}{(1 - \rho)^2 + \eta^2}.
\]

where \( R_{L/S} \equiv R_{L/S}^{B^\pm \to \rho^\pm \gamma} \). The ratio \( \Gamma(B^\pm \to \rho^\pm \gamma)/2\Gamma(B^0 \to \rho \gamma) = \Gamma(B^\pm \to \rho^\pm \gamma)/2\Gamma(B^0 \to \omega \gamma) \) is shown in Fig. 1 as a function of the parameter \( \rho \), with \( \eta = 0.2, 0.3 \) and 0.4. This suggests that a measurement of this ratio would constrain the Wolfenstein parameters \( (\rho, \eta) \), with the dependence on \( \rho \) more marked than on \( \eta \). In particular, a negative value of \( \rho \) leads to a constructive interference in \( B_u \to \rho \gamma \) decays, while large positive values of \( \rho \) give a destructive interference.

The ratio of the CKM-suppressed and CKM-allowed decay rates for charged \( B \) mesons gets modified due to the LD contributions. Following earlier discussion, we ignore the LD-contributions in \( \Gamma(B \to K^{*\gamma}) \). The ratio of the decay rates in question can therefore be written as:

\[
\frac{\Gamma(B^\pm \to \rho^\pm \gamma)}{\Gamma(B^\pm \to K^{*\gamma})} = \kappa_u \lambda^2[(1 - \rho)^2 + \eta^2]
\]

\[
\times \left\{ 1 + 2 \cdot R_{L/S} V_{ud} \frac{\rho(1 - \rho) - \eta^2}{(1 - \rho)^2 + \eta^2} + (R_{L/S})^2 V_{ud}^2 \frac{\rho^2 + \eta^2}{(1 - \rho)^2 + \eta^2} \right\},
\]

Using the central value from the estimates of the ratio of the form factors squared \( \kappa_u = 0.59 \pm 0.08 \) [21], we show the ratio (28) in Fig. 2 as a function of \( \rho \) for \( \eta = 0.2, 0.3 \), and 0.4. It is seen that the dependence of this ratio is rather weak on \( \eta \) but it depends on \( \rho \) rather sensitively. The effect of the LD-contributions is modest but not negligible, introducing an uncertainty comparable to the \( \sim 15\% \) uncertainty in the overall normalization due to the \( SU(3) \)-breaking effects in the quantity \( \kappa_u \).

Neutral \( B \)-meson radiative decays are less-prone to the LD-effects, as argued above, and hence one expects that to a good approximation (say, better than 10\%) the ratio of the decay rates for neutral \( B \) meson obtained in the approximation of SD-dominance remains valid [21]:

\[
\frac{\Gamma(B_d \to \rho \gamma, \omega \gamma)}{\Gamma(B \to K^{*\gamma})} = \kappa_d \lambda^2[(1 - \rho)^2 + \eta^2],
\]

(29)
Figure 1: Ratio of the neutral and charged $B$-decay rates $\Gamma(B^{\pm} \rightarrow \rho^{\pm}\gamma)/2\Gamma(B^{0} \rightarrow \rho\gamma)$ as a function of the Wolfenstein parameter $\rho$, with $\eta = 0.2$ (short-dashed curve), $\eta = 0.3$ (solid curve), and $\eta = 0.4$ (long-dashed curve). (Figure taken from [24].)

where this relation holds for each of the two decay modes separately.

Finally, combining the estimates for the LD- and SD-form factors in [24] and [21], respectively, and restricting the Wolfenstein parameters in the range $-0.25 \leq \rho \leq 0.35$ and $0.2 \leq \eta \leq 0.4$, as suggested by the CKM-fits [17], we give the following ranges for the absolute branching ratios:

$$B(B^{\pm} \rightarrow \rho^{\pm}\gamma) = (1.5 \pm 1.1) \times 10^{-6},$$

$$B(B^{0} \rightarrow \rho\gamma) \simeq B(B^{0} \rightarrow \omega\gamma) = (0.65 \pm 0.35) \times 10^{-6},$$

(30)

where we have used the experimental value for the branching ratio $B(B \rightarrow K^{*} + \gamma)$ [3], adding the errors in quadrature. The large error reflects the poor knowledge of the CKM matrix elements and hence experimental determination of these branching ratios will put rather stringent constraints on
Figure 2: Ratio of the CKM-suppressed and CKM-allowed radiative $B$-decay rates $\Gamma(B_u \to \rho \gamma)/\Gamma(B \to K^* \gamma)$ (with $B = B_u$ or $B_d$) as a function of the Wolfenstein parameter $\rho$, a) with $\eta = 0.2$ (short-dashed curve), $\eta = 0.3$ (solid curve), and $\eta = 0.4$ (long-dashed curve). (Figure taken from [24].)

the Wolfenstein parameter $\rho$.

In addition to studying the radiative penguin decays of the $B^\pm_u$ and $B^0_d$ mesons discussed above, hadron machines such as HERA-B will be in a position to study the corresponding decays of the $B^0_s$ meson and $\Lambda_b$ baryon, such as $B^0_s \to \phi + \gamma$ and $\Lambda_b \to \Lambda + \gamma$, which have not been measured so far. We list below the branching ratios in a number of interesting decay modes calculated in the QCD sum rule approach in [21].

\[
\begin{align*}
\mathcal{B}(B_s \to \phi \gamma) &= \mathcal{B}(B_d \to K^* \gamma) = (4.2 \pm 2.0) \times 10^{-5}, \\
\frac{\mathcal{B}(B_s \to K^* \gamma)}{\mathcal{B}(B_d \to K^* \gamma)} &\approx (0.36 \pm 0.14) \left| \frac{V_{td}}{V_{ts}} \right|^2 \\
&\Rightarrow \mathcal{B}(B_s \to K^* \gamma) = (0.75 \pm 0.5) \times 10^{-6}. \quad (31)
\end{align*}
\]

11
The estimated branching ratios in a number of inclusive and exclusive radiative $B$ decay modes are given in Table 1, where we have also listed the branching ratios for $B_s \to \gamma\gamma$ and $B_d \to \gamma\gamma$.

2.2 Inclusive rare decays $B \to X_s \ell^+\ell^-$ in the SM

The decays $B \to X_s \ell^+\ell^-$, with $\ell = e, \mu, \tau$, provide a more sensitive search strategy for finding new physics in rare $B$ decays than for example the decay $B \to X_s \gamma$, which constrains the magnitude of $C_7^{\text{eff}}$. The sign of $C_7^{\text{eff}}$, which depends on the underlying physics, is not determined by the measurement of $B(B \to X_s + \gamma)$. This sign, which in our convention is negative in the SM, is in general model dependent. It is known (see for example [28]) that in supersymmetric (SUSY) models, both the negative and positive signs are allowed as one scans over the allowed SUSY parameter space. We recall that for low dilepton masses, the differential decay rate for $B \to X_s \ell^+\ell^-$ is dominated by the contribution of the virtual photon to the charged lepton pair, which in turn depends on the effective Wilson coefficient $C_7^{\text{eff}}$. However, as is well known, the $B \to X_s \ell^+\ell^-$ amplitude in the standard model has two additional terms, arising from the two FCNC four-Fermi operators, which are not constrained by the $B \to X_s + \gamma$ data. Calling their coefficients $C_9$ and $C_{10}$, it has been argued in [28] that the signs and magnitudes of all three coefficients $C_7^{\text{eff}}$, $C_9$ and $C_{10}$ can, in principle, be determined from the decays $B \to X_s + \gamma$ and $B \to X_s \ell^+\ell^-$. The SM-based rates for the decay $b \to s\ell^+\ell^-$, calculated in the free quark decay approximation, have been known in the LO approximation for some time [29]. The LO calculations have the unpleasant feature that the decay distributions and rates are scheme-dependent. The required NLO calculation is in the meanwhile available, which reduces the scheme-dependence of the LO effects in these decays [30]. In addition, long-distance (LD) effects, which are expected to be very important in the decay $B \to X_s \ell^+\ell^-$, have also been estimated from data on the assumption that they arise dominantly due to the charmonium resonances $J/\psi$ and $\psi'$ through the decay chains $B \to X_s J/\psi(\psi',... \to X_s \ell^+\ell^-$. Likewise, the leading $(1/m_b^2)$ power corrections to the partonic decay rate and the dilepton invariant mass distribution

Footnote: This also holds for a large class of models such as MSSM and the two-Higgs doublet models but not for all SM-extensions. In LR symmetric models, for example, there are additional FCNC four-Fermi operators involved [24].
have been calculated with the help of the operator product expansion in the effective heavy quark theory \[31\]. The results of \[31\] have, however, not been confirmed in a recent independent calculation \[32\], which finds that the power corrections in the branching ratio \[B(B \to X_s \ell^+ \ell^-)\] are small (typically \(-1.5\%\)). The corrections in the dilepton mass spectrum and the FB asymmetry are also small over a good part of this spectrum. However, the end-point dilepton invariant mass spectrum is not calculable in the heavy quark expansion and will have to be modeled. Non-perturbative effects in \[B \to X_s \ell^+ \ell^-\] have been estimated using the Fermi motion model in \[33\]. These effects are found to be small except for the end-point dilepton mass spectrum where they change the underlying parton model distributions significantly and have to be taken into account in the analysis of data \[32\].

The amplitude for \[B \to X_s \ell^+ \ell^-\] is calculated in the effective theory approach, which we have discussed earlier, by extending the operator basis of the effective Hamiltonian introduced in Eq. (6):

\[
H_{\text{eff}}(b \to s + \gamma; b \to s + \ell^+ \ell^-) = H_{\text{eff}}(b \to s + \gamma) - \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} [C_9 O_9 + C_{10} O_{10}],
\]

where the two additional operators are:

\[
O_9 = \frac{\alpha}{4\pi} \bar{s} \gamma^\mu P_L b \bar{\ell} \gamma_\mu \ell,
\]

\[
O_{10} = \frac{\alpha}{4\pi} \bar{s} \gamma^\mu P_L b \bar{\ell} \gamma_\mu \gamma_5 \ell.
\]

The analytic expressions for \(C_9(m_W)\) and \(C_{10}(m_W)\) can be seen in \[30\] and will not be given here. We recall that the coefficient \(C_9\) in LO is scheme-dependent. However, this is compensated by an additional scheme-dependent part in the (one loop) matrix element of \(O_9\). We call the sum \(C_9^{\text{eff}}\), which is scheme-independent and enters in the physical decay amplitude given below,

\[
\mathcal{M}(b \to s + \ell^+ \ell^-) = \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \frac{\alpha}{\pi}
\times \left[ C_9^{\text{eff}} \bar{s} \gamma^\mu P_L b \bar{\ell} \gamma_\mu \ell + C_{10} \bar{s} \gamma^\mu P_L b \bar{\ell} \gamma_\mu \gamma_5 \ell - 2C_7^{\text{eff}} \bar{s} i \sigma_{\mu\nu} \frac{q^\nu}{q^2} (m_b P_R + m_s P_L) b \bar{\ell} \gamma^\mu \ell \right],
\]

\[
(34)
\]

13
with
\[ C_{9}^{\text{eff}}(\hat{s}) \equiv C_{9}\eta(\hat{s}) + Y(\hat{s}). \] (35)
The function \( Y(\hat{s}) \) is the one-loop matrix element of \( O_{9} \) and can be seen in literature \[30, 7\]. A useful quantity is the differential FB asymmetry in the c.m.s. of the dilepton defined in refs. \[35\]:
\[ \frac{dA(\hat{s})}{d\hat{s}} = \int_{0}^{1} \frac{dB}{dz} - \int_{-1}^{0} \frac{dB}{dz}, \] (36)
where \( z = \cos \theta \), with \( \theta \) being the angle between the lepton \( \ell^{+} \) and the \( b \)-quark. This can be expressed as:
\[ \frac{dA(\hat{s})}{d\hat{s}} = -B_{sl} \frac{3\alpha^{2}}{4\pi^{2}} \frac{1}{f(\hat{m}_{c})} u^{2}(\hat{s})C_{10} \left[ \Re(C_{9}^{\text{eff}}(\hat{s})) + 2C_{7}^{\text{eff}}(1 + \hat{m}_{s}^{2}) \right]. \] (37)
The Wilson coefficients \( C_{7}^{\text{eff}}, C_{9}^{\text{eff}} \) and \( C_{10} \) appearing in the above equation and the dilepton spectrum (see, for example \[32\]) can be determined from data by solving the partial branching ratio \( B(\Delta\hat{s}) \) and partial FB asymmetry \( A(\Delta\hat{s}) \), where \( \Delta\hat{s} \) defines an interval in the dilepton invariant mass \[28\].

There are other quantities which one can measure in the decays \( B \to X_{s}\ell^{+}\ell^{-} \) to disentangle the underlying dynamics. We mention here the longitudinal polarization of the lepton in \( B \to X_{s}\ell^{+}\ell^{-} \), in particular in \( B \to X_{s}\tau^{+}\tau^{-} \), proposed by Hewett \[36\]. In a recent paper, Krüger and Sehgal \[37\] have stressed that complementary information is contained in the two orthogonal components of polarization (\( P_{T} \), the component in the decay plane, and \( P_{N} \), the component normal to the decay plane), both of which are proportional to \( m_{\ell}/m_{b} \), and therefore significant for the \( \tau^{+}\tau^{-} \) channel. A third quantity, called energy asymmetry, proposed by Cho, Misiak and Wyler \[38\], defined as
\[ A = \frac{N(E_{\ell^{-}} > E_{\ell^{+}}) - N(E_{\ell^{+}} > E_{\ell^{-}})}{N(E_{\ell^{-}} > E_{\ell^{+}}) + N(E_{\ell^{+}} > E_{\ell^{-}})}, \] (38)
where \( N(E_{\ell^{-}} > E_{\ell^{+}}) \) denotes the number of lepton pairs where \( \ell^{+} \) is more energetic than \( \ell^{-} \) in the \( B \)-rest frame, is, however, not an independent measure, as it is directly proportional to the FB asymmetry discussed above. The relation is \[32\]:
\[ \int A(\hat{s}) = B \times A. \] (39)
This is easy to notice if one writes the Mandelstam variable $u(\hat{s})$ in the dilepton c.m. and the $B$-hadron rest systems.

Next, we discuss the effects of LD contributions in the processes $B \rightarrow X_s \ell^+ \ell^-$. Note that the LD contributions due to the vector mesons such as $J/\psi$ and $\psi'$, as well as the continuum $c \bar{c}$ contribution already discussed, appear as an effective $(\hat{s}_L \gamma_{\mu} b_L)(\ell \gamma^\mu \ell)$ interaction term only, i.e., in the operator $O_9$. This implies that the LD-contributions should change $C_9$ effectively, as discussed earlier is dominated by the SD-contribution, and $C_{10}$ has no LD-contribution. In accordance with this, the function $Y(\hat{s})$ is replaced by,

$$Y(\hat{s}) \rightarrow Y'(\hat{s}) \equiv Y(\hat{s}) + Y_{\text{res}}(\hat{s}),$$

where $Y_{\text{res}}(\hat{s})$ is given as [35],

$$Y_{\text{res}}(\hat{s}) = \frac{3}{\alpha_s^2} \kappa (3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6) \sum_{V_i = J/\psi, \psi', \ldots} \frac{\pi \Gamma(V_i \rightarrow l^+ l^-) M_{V_i}}{M_{V_i}^2 - \hat{s} m_b^2 - i M_{V_i} \Gamma_{V_i}},$$

where $\kappa$ is a fudge factor, which appears due to the inadequacy of the factorization framework in describing data on $B \rightarrow J/\psi X_s$. The long-distance effects lead to significant interference effects in the dilepton invariant mass distribution and the FB asymmetry in $B \rightarrow X_s \ell^+ \ell^-$ shown in Figs. 3 and 4, respectively. This can be used to test the SM, as the signs of the Wilson coefficients in general are model dependent. For further discussions we refer to Ref. [32] where also theoretical dispersion on the decay distributions due to various input parameters is worked out. Taking into account the spread in the values of the input parameters, $\mu$, $\Lambda$, $m_t$, and $B_{SL}$ discussed in the previous section in the context of $B(B \rightarrow X_s + \gamma)$, we estimate the following branching ratios for the SD-piece only (i.e., from the intermediate top quark contribution only) [32]:

$$B(B \rightarrow X_s e^+ e^-) = (8.4 \pm 2.3) \times 10^{-6},$$

$$B(B \rightarrow X_s \mu^+ \mu^-) = (5.7 \pm 1.2) \times 10^{-6},$$

$$B(B \rightarrow X_s \tau^+ \tau^-) = (2.6 \pm 0.5) \times 10^{-7},$$

where theoretical errors and the error on $B_{SL}$ have been added in quadrature. The present experimental limit for the inclusive branching ratio in $B \rightarrow X_s \ell^+ \ell^-$ is actually still the one set by the UA1 collaboration some time ago.
Figure 3: Dilepton invariant mass distribution in $B \to X_s \ell^+ \ell^-$ in the SM with the next-to-leading order QCD corrections and non-perturbative effects calculated in the Fermi motion model (solid curve), and including the LD-contributions (dashed curve). The model parameters ($p_F, m_q$) are indicated in the figure. Note that the height of the $J/\psi$ peak is suppressed due to the linear scale. (Figure taken from [32].)

As far as we know, there are no interesting limits on the other two modes, involving $X_s e^+e^-$ and $X_s \tau^+\tau^-$. It is obvious from Fig. 3 that only in the dilepton mass region far away from the resonances is there a hope of extracting the Wilson coefficients governing the short-distance physics. The region below the $J/\psi$ resonance is well suited for that purpose as the dilepton invariant mass distribution there is dominated by the SD-piece. Including the LD-contributions, following branching ratio has been estimated for the dilepton mass range $0.2 \leq \hat{s} \leq 0.36$ in [32]:

$$B(B \to X_s \ell^+ \ell^-) = (1.3 \pm 0.3) \times 10^{-6},$$

(43)
with $\mathcal{B}(B \to X_s e^+ e^-) \simeq \mathcal{B}(B \to X_s \mu^+ \mu^-)$. The FB-asymmetry is estimated to be in the range 10% - 27%, as can be seen in Fig. 4.

The experimental limits on the decay rates of the exclusive decays $B \to (K, K^*) \ell^+ \ell^-$ [26, 40], while arguably closer to the SM-based estimates, can only be interpreted in specific models of form factors, which hinders somewhat their transcription in terms of the information on the underlying Wilson coefficients. Using the exclusive-to-inclusive ratios

$$R_{K\ell\ell} \equiv \frac{\Gamma(B \to K \ell^+ \ell^-)}{\Gamma(B \to X_s \ell^+ \ell^-)} = 0.07 \pm 0.02,$$

and

$$R_{K^*\ell\ell} \equiv \frac{\Gamma(B \to K^* \ell^+ \ell^-)}{\Gamma(B \to X_s \ell^+ \ell^-)} = 0.27 \pm 0.07,$$
which were estimated in [42], the results are presented in Table 1. In conclusion, the semileptonic FCNC decays $B \to X_s \ell^+ \ell^-$ (and also the exclusive decays) will provide very precise tests of the SM, as they will determine the signs and magnitudes of the three Wilson coefficients, $C_7$, $C_9^{\text{eff}}$, and $C_{10}$. This, perhaps, may also reveal physics beyond-the-SM if it is associated with not too high a scale. The MSSM model is a good case study where measurable deviations from the SM are anticipated and worked out [28, 38].

2.3 Summary and overview of rare $B$ decays in the SM

The rare $B$ decay mode $B \to X_s \nu \bar{\nu}$, and some of the exclusive channels associated with it, have comparatively larger branching ratios. The estimated inclusive branching ratio in the SM is [42] - [44]:

$$\mathcal{B}(B \to X_s \nu \bar{\nu}) = (4.0 \pm 1.0) \times 10^{-5},$$ (44)

where the main uncertainty in the rates is due to the top quark mass. The scale-dependence, which enters indirectly through the top quark mass, has been brought under control through the NLL corrections, calculated in [45]. The corresponding CKM-suppressed decay $B \to X_d \nu \bar{\nu}$ is related by the ratio of the CKM matrix element squared [42]:

$$\frac{\mathcal{B}(B \to X_d \nu \bar{\nu})}{\mathcal{B}(B \to X_s \nu \bar{\nu})} = \left[ \frac{|V_{td}|}{|V_{ts}|} \right]^2.$$ (45)

Similar relations hold for the ratios of the exclusive decay rates which depend additionally on the ratios of the form factors squared, which deviate from unity through $SU(3)$-breaking terms, in close analogy with the exclusive radiative decays discussed earlier. These decays are particularly attractive probes of the short-distance physics, as the long-distance contributions are practically absent in such decays. Hence, relations such as the one in (45) provide, in principle, one of the best methods for the determination of the CKM matrix element ratio $|V_{td}|/|V_{ts}|$ [42]. From the practical point of view, however, these decay modes are rather difficult to measure, in particular at the hadron colliders and probably also at the $B$ factories. The best chances are in the $Z^0$-decays at LEP, from where the present best upper limit stems [46]:

$$\mathcal{B}(B \to X \nu \bar{\nu}) < 7.7 \times 10^{-4}.$$ (46)
The estimated branching ratios in a number of inclusive and exclusive decay modes are given in Table 1, updating the estimates in [7].

Further down the entries in Table 1 are listed some two-body rare decays, such as \((B_s^0, B_d^0) \to \gamma\gamma\), studied in [48], where only the lowest order contributions are calculated, i.e., without any QCD corrections, and the LD-effects, which could contribute significantly, are neglected. The decays \((B_s^0, B_d^0) \to \ell^+\ell^-\) have been studied in the next-to-leading order QCD in [45]. Some of them, in particular, the decays \(B_s^0 \to \mu^+\mu^-\) and perhaps also the radiative decay \(B_s^0 \to \gamma\gamma\), have a fighting chance to be measured at LHC. The estimated decay rates, which depend on the pseudoscalar coupling constant \(f_{B_s}\) (for \(B_s\)-decays) and \(f_{B_d}\) (for \(B_d\)-decays), together with the present experimental bounds are listed in Table 1. Since no QCD corrections have been included in the rate estimates of \((B_s, B_d) \to \gamma\gamma\), the branching ratios are rather uncertain. The constraints on beyond-the-SM physics that will eventually follow from these decays are qualitatively similar to the ones that (would) follow from the decays \(B \to X_s + \gamma\) and \(B \to X_s\ell^+\ell^-\), which we have discussed at length earlier.

3 Acknowledgements

I would like to thank Hrachia Asatrian, Vladimir Braun, Christoph Greub and Tak Morozumi for helpful discussions. The warm hospitality extended by Fernando Ferroni and his collaborators in Rome during the Beauty 96 workshop is thankfully acknowledged. Finally, I acknowledge the editorial assistance of Peter Schlein in the preparation of this manuscript.
References

[1] N. Cabibbo, Phys. Rev. Lett. 10 (1963) 531; M. Kobayashi and K. Maskawa, Prog. Theor. Phys. 49 (1973) 652.

[2] M.S. Alam et al. (CLEO Collaboration), Phys. Rev. Lett. 74 (1995) 2885.

[3] R. Ammar et al. (CLEO Collaboration), Phys. Rev. Lett. 71 (1993) 674.

[4] R. Ammar et al. (CLEO Collaboration), contributed paper to the International Conference on High Energy Physics, Warsaw, 25 - 31 July 1996, CLEO CONF 96-05.

[5] S.L. Glashow, J. Iliopoulos and L. Maiani, Phys. Rev. D2 (1970) 1285.

[6] T. Inami and C.S. Lim, Prog. Theor. Phys. 65 (1981) 297.

[7] A. Ali, preprint DESY 96-106 [hep-ph/9606324]; to appear in the Proceedings of the XX International Nathiagali Conference on Physics and Contemporary Needs, Bhurban, Pakistan, June 24-July 13, 1995 (Nova Science Publishers, New York, 1996).

[8] M. Ciuchini et al., Phys. Lett. B316 (1993) 127; Nucl. Phys. B415 (1994) 403; G. Cella et al., Phys. Lett. B325 (1994) 227; M. Misiak, Nucl. Phys. B393 (1993) 23; [E. B439 (1995) 461].

[9] M. Misiak, contribution to the International Conference on High Energy Physics, Warsaw, 25 - 31 July 1996.

[10] A. Ali and C. Greub, Z. Phys. C49 (1991) 431; Phys. Lett. B259 (1991) 182; Z. Phys. C60 (1993) 433.

[11] A. Ali and C. Greub, Phys. Lett. B287 (1992) 191.

[12] A. Ali and C. Greub, Phys. Lett. B361 (1995) 146.

[13] N. Pott, Phys. Rev. D54 (1996) 938.
[14] K. Adel and Y.-p. Yao, Phys. Rev. D49 (1994) 4945.

[15] C. Greub, T. Hurth and D. Wyler, Phys. Lett. B380 (1996) 385; Phys. Rev. D54 (1996) 3350.

[16] L. Wolfenstein, Phys. Rev. Lett. 51 (1983)

[17] A. Ali and D. London, preprint DESY 96-140, UdeM-GPP-TH-96-45, [hep-ph/9607392], to appear in the Proc. of QCD Euroconference 96, Montpellier, July 4-12, 1996.

[18] R.M. Barnett et al. (Particle Data Group), Phys. Rev. D54 (1996) 1.

[19] A. Ali, H.M. Asatrian, and C. Greub, to be published.

[20] L. Gibbons, Plenary talk at the International Conference on High Energy Physics, Warsaw, ICHEP96 (1996).

[21] A. Ali, V.M. Braun and H. Simma, Z. Phys. C63 (1994) 437.

[22] P. Ball, TU-München Report TUM-T31-43/93 (1993); P. Colangelo et al., Phys. Lett. B317 (1993) 183; S. Narison, Phys. Lett. B327 (1994) 354; J. M. Soares, Phys. Rev. D49 (1994) 283.

[23] A. Khodzhamirian, G. Stoll, and D. Wyler, Phys. Lett. B358 (1995) 129.

[24] A. Ali and V.M. Braun, Phys. Lett. B359 (1995) 223.

[25] I.I. Balitsky, V.M. Braun, and A.V. Kolesnichenko, Nucl. Phys. 312 (1989) 509.

[26] T.E. Browder and K. Honscheid, Prog. Part. Nucl. Phys. 35 (1995) 81.

[27] J.F. Donoghue, E. Golowich and A.A. Petrov, preprint UMHEP-433 (1996) [hep-ph/9609530].

[28] A. Ali, G. F. Giudice and T. Mannel, Z. Phys. C67 (1995) 417.
[29] S. Bertolini, F. Borzumati and A. Masiero, Phys. Rev. Lett. 59 (1987) 180; R. Grigjanis et al., Phys. Lett. B213 (1988) 355; B. Grinstein, R. Springer, and M.B. Wise, Phys. Lett. 202 (1988) 138; Nucl Phys. B339 (1990) 269; G. Cella et al., Phys. Lett. B248 (1990) 181.

[30] M. Misiak in ref. [8]; A.J. Buras and M. Münz, Phys. Rev. D52 (1995) 186.

[31] A. F. Falk, M. Luke and M. J. Savage, Phys. Rev. D49 (1994) 3367.

[32] A. Ali, G. Hiller, L.T. Handoko, and T. Morozumi, preprint DESY 96-206; Hiroshima univ. report HUPD-9615 [hep-ph/9609449].

[33] A. Ali and E. Pietarinen, Nucl. Phys. B154 (1979) 519; G. Altarelli et al., Nucl. Phys. B208 (1982) 365.

[34] K. Fujikawa and A. Yamada, Phys. Rev. D49 (1994) 5890; P. Cho and M. Misiak, Phys. Rev. D49 (1994) 5894.

[35] A. Ali, T. Mannel and T. Morozumi, Phys. Lett. B273 (1991) 505.

[36] J. Hewett, Phys. Rev. D53 (1996) 4964.

[37] F. Krüger and L.M. Sehgal, Phys. Lett. B380 (1996) 199.

[38] P. Cho, M. Misiak, and D. Wyler, Phys. Rev. D54 (1996) 3329.

[39] C. Albajar et al. (UA1), Phys. Lett. B262 (1991) 163.

[40] T. Skwarnicki, in Proc. of the 17th Int. Symp. on Lepton Photon Interactions, Beijing, P.R. China, 10 - 15 August 1995; Editors: Zheng Zhi-Peng and Chen He-Sheng.

[41] CDF Collaboration, Fermilab Conf. 95/201-E(1995).

[42] A. Ali, C. Greub and T. Mannel, DESY Report 93-016 (1993), and in B-Physics Working Group Report, ECFA Workshop on a European B-Meson Factory, ECFA 93/151, DESY 93-053 (1993); Editors: R. Aleksan and A. Ali.
[43] G. Buchalla, A.J. Buras, and M.E. Lautenbacher, MPI-Ph/95-104; TUM-T31-100/95; FERMILAB-PUB-95/305-T; SLAC-PUB 7009; [hep-ph/9512380].

[44] Y. Grossman, Z. Ligeti, and E. Nardi, Nucl. Phys. B465 (1996) 369.

[45] G. Buchalla, and A.J. Buras, Nucl. Phys. B400 (1993) 225.

[46] Contributed paper by the ALEPH collaboration to the International Conference on High Energy Physics, Warsaw, ICHEP96 PA10-019 (1996).

[47] CDF Collaboration, Fermilab Conf. 95/201-E(1995); P. Sphicas (CDF), private communication.

[48] G.-L. Lin, J. Liu, and Y.-P. Yao, Phys. Rev. D42 (1990) 2314; H. Simma and D. Wyler, Nucl. Phys. B344 (1990) 283; S. Herrlich and J. Kalinowski, Nucl. Phys. 381 (1992) 501; T.M. Aliev and G. Turan, Phys. Rev. D48 (1993) 1176; G.G. Devidze, G.R. Jibuti, and A.G. Liparteliani, preprint HEP Institute, Univ. of Tbilisi (revised version) (1995).

[49] S. Abachi et al. (D0 Collaboration), contributed paper to the International Conference on High Energy Physics, Warsaw, ICHEP96 (1996).

[50] L3 Collaboration, contributed paper to the EPS Conference, (EPS-0093-2) Brussels, 1995.
| Decay Modes          | $B$ (SM)                         | Measurements and 90% C.L. Upper Limits |
|----------------------|----------------------------------|---------------------------------------|
| $(B^+, B^0) \rightarrow X_s \gamma$ | $(3.2 \pm 0.58) \times 10^{-4}$ | $(2.32 \pm 0.67) \times 10^{-4}$ [4] |
| $(B^+, B^0) \rightarrow K^* \gamma$ | $(4.0 \pm 2.0) \times 10^{-5}$ | $(4.2 \pm 1.0) \times 10^{-5}$ [4] |
| $(B^+, B^0) \rightarrow X_d \gamma$ | $(1.6 \pm 1.2) \times 10^{-5}$ | – |
| $B^\pm \rightarrow \rho^\pm + \gamma$ | $(1.5 \pm 1.1) \times 10^{-6}$ | $< 1.1 \times 10^{-5}$ [4] |
| $B^0 \rightarrow \rho^0 + \gamma$ | $(0.65 \pm 0.35) \times 10^{-6}$ | $< 3.9 \times 10^{-5}$ [4] |
| $B^0 \rightarrow \omega + \gamma$ | $(0.65 \pm 0.35) \times 10^{-6}$ | $< 1.3 \times 10^{-5}$ [4] |
| $B_s \rightarrow \phi + \gamma$ | $(4.2 \pm 2.0) \times 10^{-5}$ | – |
| $B_s \rightarrow K^* + \gamma$ | $(0.8 \pm 0.5) \times 10^{-6}$ | – |
| $(B_d, B_u) \rightarrow X_s e^+ e^-$ | $(8.4 \pm 2.2) \times 10^{-7}$ | – |
| $(B_d, B_u) \rightarrow X_d \mu^+ \mu^-$ | $(5.7 \pm 1.2) \times 10^{-6}$ | $< 3.6 \times 10^{-5}$ [49] |
| $(B_d, B_u) \rightarrow X_d \tau^+ \tau^-$ | $(3.3 \pm 1.9) \times 10^{-7}$ | – |
| $(B_d, B_u) \rightarrow X_d \tau^+ \tau^-$ | $(2.6 \pm 0.5) \times 10^{-7}$ | – |
| $(B_d, B_u) \rightarrow X_d \tau^+ \tau^-$ | $(1.5 \pm 0.8) \times 10^{-8}$ | – |
| $(B_d, B_u) \rightarrow K e^+ e^-$ | $(5.9 \pm 2.3) \times 10^{-7}$ | $< 1.2 \times 10^{-5}$ [40] |
| $(B_d, B_u) \rightarrow K \mu^+ \mu^-$ | $(4.0 \pm 1.5) \times 10^{-7}$ | $< 0.9 \times 10^{-5}$ [40] |
| $(B_d, B_u) \rightarrow K \tau^+ \tau^-$ | $(4.0 \pm 1.5) \times 10^{-7}$ | $< 0.9 \times 10^{-5}$ [40] |
| $(B_d, B_u) \rightarrow K^* e^+ e^-$ | $(2.3 \pm 0.9) \times 10^{-6}$ | $< 1.6 \times 10^{-5}$ [40] |
| $(B_d, B_u) \rightarrow K^* \mu^+ \mu^-$ | $(1.5 \pm 0.6) \times 10^{-6}$ | $< 2.5 \times 10^{-5}$ [47] |
| $(B_d, B_u) \rightarrow X_s \nu \bar{\nu}$ | $(4.0 \pm 1.0) \times 10^{-5}$ | $< 7.7 \times 10^{-4}$ [46] |
| $(B_d, B_u) \rightarrow X_d \nu \bar{\nu}$ | $(2.3 \pm 1.5) \times 10^{-6}$ | – |
| $(B_d, B_u) \rightarrow K \nu \bar{\nu}$ | $(3.2 \pm 1.6) \times 10^{-6}$ | – |
| $(B_d, B_u) \rightarrow K^* \nu \bar{\nu}$ | $(1.1 \pm 0.55) \times 10^{-5}$ | – |
| $B_s \rightarrow \gamma \gamma$ | $(3.0 \pm 1.0) \times 10^{-7}$ | $< 1.1 \times 10^{-4}$ [50] |
| $B_d \rightarrow \gamma \gamma$ | $(1.2 \pm 0.8) \times 10^{-8}$ | $< 3.8 \times 10^{-5}$ [50] |
| $B_s \rightarrow \tau^+ \tau^-$ | $(7.4 \pm 1.9) \times 10^{-7}$ | – |
| $B_d \rightarrow \tau^+ \tau^-$ | $(3.1 \pm 1.9) \times 10^{-8}$ | – |
| $B_s \rightarrow \mu^+ \mu^-$ | $(3.5 \pm 1.0) \times 10^{-9}$ | $< 8.4 \times 10^{-6}$ [44] |
| $B_d \rightarrow \mu^+ \mu^-$ | $(1.5 \pm 0.9) \times 10^{-10}$ | $< 1.6 \times 10^{-6}$ [47] |
| $B_s \rightarrow e^+ e^-$ | $(8.0 \pm 3.5) \times 10^{-14}$ | – |
| $B_d \rightarrow e^+ e^-$ | $(3.4 \pm 2.3) \times 10^{-15}$ | – |

Table 1: Estimates of the branching fractions for FCNC $B$-decays in the standard model taking into account the uncertainties in the input parameters as discussed in [4]. The entries in the second column correspond to the short-distance contributions only, except for the radiative decays $B^\pm \rightarrow \rho^\pm + \gamma$ and $B^0 \rightarrow (\rho^0, \omega) + \gamma$, where long-distance effects have also been included. For the two-body branching ratios, we have used $f_{B_d} = 200$ MeV and $f_{B_s}/f_{B_d} = 1.16$. Experimental measurements and upper limits are also listed. In the second row, the statistical and systematic uncertainties have been combined to give the quoted experimental uncertainty.