Optimization of Maintenance Schedule for Safety Instrumented Systems

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Abstract: Preventive maintenance plays an important role in the reliability of safety instrumented systems (SIS). In this paper, an approach is proposed for the optimization of maintenance schedule of SIS. The basic idea is to regard a SIS with preventive maintenance tests as a switched linear system with state jumps. Then the problem of finding the optimal time instants for maintenance tests is formulated as an optimal control problem of Bolza form. The average probability of failure on demand (PFD) is taken as the objective function. The proposed approach is able to take into account different types of maintenance tests and redundancy architecture. Examples are given to illustrate the proposed approach.

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1. INTRODUCTION

Safety instrumented systems (SIS) are an essential part of many industrial plants. The design and implementation of SIS subject to the international standard IEC 61508. The reliability of SIS is usually evaluated by the safety integrity level (SIL), which is defined based on the probability of failure on demand (PFD) of the SIS.

According to Birolini (2013), maintenance includes preventive maintenance (PM) and corrective maintenance. PM consists of regularly performed proof tests at predetermined intervals. Corrective maintenance is performed after a fault is recognized. In this paper, we focus on the planning of preventive maintenance activities for SIS.

Preventive maintenance tests can be classified into main tests and partial tests based on the test coverage. Bukowski (2001) also defined them as perfect and imperfect inspection. The difference between them is the ability of detecting dangerous failures in the SIS.

Markov models are one of the main methods for the quantitative evaluation of the PFD. Becker et al. (1994) have proposed to combine Markov models with discrete time events representing periodic inspections. In Bukowski (2001) it has been shown how to compute performance indexes such as mean time to failure for such systems. Felgner and Frey (2011) applied multi-phase Markov chains with event-triggered re-initialization to calculate systems safety-related quantities including information on maintenance actions. Mechri et al. (2015) provided the formula to calculate the average probability of failure on demand PFDavg for different architectures. Machleidt (2016) proposed a stochastic and deterministic timed automata (SDTA) to model a SIS with time-based maintenance. Two types of maintenance tests with multi-phase Markov models have been considered in Innal et al. (2016) to include the contribution of partial tests.

In this paper, we propose an approach for the optimal scheduling of PM to improve the reliability of SIS. The basic idea is to regard a multi-phase continuous Markov process as a switched linear system with state jumps, which has been studied in the control theory (see, for instance, Sun and Ge (2005)). The optimal scheduling problem is formulated as an optimal control problem, in which the switching time instants should be optimized. The optimal solution is obtained by applying the approach of Xu and Antsaklis (2003).

The rest of this paper is organized as follows. In Section 2, the Markov process based models of SIS and the PFD are briefly. In Section 3, the problem is formulated and the optimization strategy is presented. In Section 4, some application aspects are considered. The proposed algorithm is applied to two examples in Section 5. Finally, conclusions and discussions are given in Section 6.

2. PRELIMINARIES

2.1 Modeling of state transitions in SIS

For the sake of clarity, we consider an example of a SIS without redundancy, i.e. the SIS has only one channel. If this channel is unavailable, then the safety function can not be realized. From the viewpoint of reliability, the failures in the SIS can be divided into safe failures and dangerous failures. Safe failures (SF) are caused by spurious operations, for instance, false alarms. They lead to unnecessary shut down of the plant and can always be detected. If a dangerous failure occurs, then the SIS can not perform its function and becomes unavailable. A dangerous failure can be completely non-detectable (DN), detectable by a diagnostic system (DD), detectable by...
both main tests and partial tests (DUAB), or detectable only by main tests (DUA).

The state transition diagram of the SIS with six states is shown in Fig. 1 (see Machleidt (2016) for more details). The state DR is used to denote that a dangerous failure is detected and the functionality of the SIS is not yet restored. \( \lambda_s, \lambda_{dd}, \lambda_{duab}, \lambda_{dua}, \) and \( \lambda_{dn} \) are, respectively, the failure rates of safe failures, DD failures, DUAB failures, DUA failures, and DN failures. \( \mu_{cms} \) is the repair rate when a safe failure is recognized. \( \mu_{md} \) is the repair rate when a dangerous failure is recognized.

When the time in each state is exponentially distributed, the SIS can be modeled by a continuous-time Markov process (CMP) with \( N \) states, where \( N \) denotes the total number of states. See Birolini (2013). Let \( p_j(t) \) \( (j = 1, 2, \ldots, N) \) denote the probability of the SIS in the \( j \)-th state at the time \( t \). Then \( p_j(t + dt) \) satisfies the Chapman-Kolmogorov equation as follows

\[
p_j(t + dt) = \sum_{k=1}^{N} p_k(t) q_{kj} dt + p_j(t) (1 - \sum_{k=1}^{N} q_{kj} dt)
\]

for \( j = 1, 2, \ldots, N \). Here \( q_{kj} \) is the probability per time unit that the system makes a transition from state \( k \) to state \( j \)

\[
q_{kj} = \lim_{\Delta t \to 0} \frac{Pr(X_t + \Delta t = j | X_t = k)}{\Delta t}, \quad k \neq j,
\]

where \( X_t = j \) denotes that the system is in the \( j \)-th state at time \( t \). Note that \( q_{kj} \) is also called transition rates (Birolini (2013)).

In matrix notation (1) can be re-written into

\[
\frac{dp(t)}{dt} = Qp(t)
\]

where \( p(t) = [p_1(t) \cdots p_N(t)]^T \) and \( Q = [q_{ij}] \).

As an example, the matrix \( Q \) of the SIS with only one channel (see Fig. 1) is

\[
Q = \begin{bmatrix}
q_{11} & \mu_{cms} & \mu_{md} & 0 & 0 & 0 \\
\lambda_s & -\mu_{cms} & 0 & 0 & 0 & 0 \\
\lambda_{dd} & 0 & -\mu_{md} & 0 & 0 & 0 \\
\lambda_{duab} & 0 & 0 & 0 & 0 & 0 \\
\lambda_{dua} & 0 & 0 & 0 & 0 & 0 \\
\lambda_{dn} & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

(3)

2.2 Modeling of PM activities

The tests in the PM can be divided into main tests and partial tests according to the depth of the tests. For instance, a complete functional test is the main test. In comparison, a partial stroke test is a partial test.

As shown in Fig. 1, the main test will cause jumps from both State 4 and State 5 to State 3, while a partial test will only change the system from State 4 to State 3.

Therefore, a PM test can be described by a jump in the state as

\[
p(t_j^+) = Mp(t_j^-), \quad j = 1, \ldots, K
\]

(4)

where \( M \) is a matrix containing the information of changes caused by the tests, \( K \) denotes the total number of tests, \( t_j^- \) is the time instant just before the \( j \)-th test and \( t_j^+ \) is the time instant just after the \( j \)-th test.

Note that for different types of tests \( M \) will be different. In this work, the main test is called as type A test and corresponds to the matrix \( M_A \), the partial test is a type B test and corresponds to the matrix \( M_B \).

\[
M_A = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}, \quad M_B = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

(5)

2.3 Probability of failure on demand (PFD)

The probability of failure on demand \( \text{PFD}(t) \) is defined as SIS unavailability \( U(t) \), i.e.

\[
U(t) = \text{PFD}(t) = Pr\{\text{system is unavailable at time } t\}
\]

In the example shown in Fig. 1, the SIS is not available in states 3, 4, 5, and 6. Hence, the PFD is related to the probability distribution \( p(t) \) by

\[
\text{PFD}(t) = p_3(t) + p_4(t) + p_5(t) + p_6(t) = [0 \ 0 \ 1 \ 1 \ 1 \ 1]p(t)
\]

In general, PFD can be written as

\[
\text{PFD}(t) = cp(t)
\]

(6)

where \( c \) is a row vector that selects unavailable states.

In practice, the average value of the PFD over the life cycle is often used a performance index for the SIS, which is defined as

\[
\text{PFD}_{\text{avg}} = \frac{1}{t_f - t_0} \int_{t_0}^{t_f} \text{PFD}(t)dt
\]

(7)

where \( t_0 \) and \( t_f \) denote, respectively, the beginning time and the end time of the life cycle of the SIS.
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