1. Introduction

It is known that fundamental solutions have an essential role in studying partial differential equations. Formulation and solving of many local and non-local boundary value problems are based on these solutions. Moreover, fundamental solutions appear as potentials, for instance, as simple-layer and double-layer potentials in the theory of potentials.

The explicit form of fundamental solutions gives a possibility to study the considered equation in detail. For example, in the works of Barros-Neto and Gelfand [1], fundamental solutions for Tricomi operator, relative to an arbitrary point in the plane were explicitly calculated. We also mention Leray’s work [2], which it was described as a general method, based upon the theory of analytic functions of several complex variables, for finding...
fundamental solutions for a class of hyperbolic linear differential operators with analytic coefficients. Among other results in this direction, we note a work by Itagaki [3], where 3D high-order fundamental solutions for a modified Helmholtz equation were found. The found solutions can be applied with the boundary particle method to some 2D inhomogeneous problems, for example, see [4].

Singular partial differential equations appear at studying various problems of aerodynamics and gas dynamics [5] and irrigation problems [6]. For instance, the famous Chaplygin equation [7] describes subsonic, sonic and supersonic flows of gas. The theory of singular partial differential equations has many applications and possibilities of various theoretical generalizations. It is, in fact, one of the rapidly developing branches of the theory of partial differential equations.

In most cases boundary value problems for singular partial differential equations are based on fundamental solutions for these equations, for instance, see [8].

Let us consider the generalized bi-axially symmetric Helmholtz equation with \( p \) variables

\[
H_{\alpha,\beta}^{p,\lambda}(u) \equiv \sum_{i=1}^{p} \frac{\partial^2 u}{\partial x_i^2} + \frac{2\alpha}{x_1} \frac{\partial u}{\partial x_1} + \frac{2\beta}{x_2} \frac{\partial u}{\partial x_2} - \lambda^2 u = 0 \tag{1.1}
\]

in the domain \( R_{p}^+ = \{(x_1, \ldots, x_p) : x_1 > 0, x_2 > 0\} \), where \( p \) is a dimension of a Euclidean space \( (p \geq 2) \), \( \alpha, \beta \) and \( \lambda \) are constants and \( 0 < 2\alpha, 2\beta < 1 \).

In the article [9], the equation (1.1) was considered in two cases: (1) when \( p = 2, \alpha = 0, \beta > 0; \) (2) when \( p = 2, \lambda = 0, \beta > 0 \). In the work [10] in order to find fundamental solutions, at first two new confluent hypergeometric functions were introduced. Furthermore, by means of the introduced hypergeometric function fundamental solutions of the equation(1.1) were constructed in an explicit form. For studying the properties of the fundamental solutions, the introduced confluent hypergeometric functions are expanded in products by Gauss’s hypergeometric functions. The logarithmic singularity of the constructed fundamental solutions of equation (1.1) was explored with the help of the obtained expansion. Fundamental solutions of equation (1.1) with \( p = 3 \) and \( \lambda = 0 \) were used in the investigation of the Dirichlet problem for three-dimensional elliptic equation with two singular coefficients [11].

In the present article for the equation (1.1) in the domain \( R_{p}^+ \) at \( p > 2 \) four fundamental solutions are constructed in explicit form. Furthermore,
some properties of these solutions are shown, which will be used for solving boundary value problems for aforementioned equation.

2. About one confluent hypergeometric function of three variables

The confluent hypergeometric function of three variables which we will use in the present work looks like [10]

\[ A_2(a; b_1, b_2; c_1, c_2; x, y, z) = \sum_{m,n,k=0}^{\infty} \frac{(a)_{m+n-k}(b_1)_m(b_2)_n}{(c_1)_m(c_2)_n m! n! k!} x^m y^n z^k, \quad (2.1) \]

where \( a, b_1, b_2, c_1, c_2 \) are complex constants, \( c_1, c_2 \neq 0, -1, -2, \ldots \) and \( (a)_n = \Gamma(a + n)/\Gamma(a) \) is the Pochhammer symbol.

Using the formula of derivation

\[ \frac{\partial^{i+j+k}}{\partial x^i \partial y^j \partial z^k} A_2(a; b_1, b_2; c_1, c_2; x, y, z) = \]

\[ = \frac{(a)_{i+j-k}(b_1)_i(b_2)_j}{(c_1)_i(c_2)_j} A_2(a+i+j-k; b_1+i, b_2+j; c_1+i, c_2+j; x, y, z), \quad (2.2) \]

it is easy to show that the hypergeometric function \( A_2(a; b_1, b_2; c_1, c_2; x, y, z) \) satisfies the system of hypergeometric equations [10]

\[
\begin{aligned}
  x(1-x)\omega_{xx} - xy\omega_{xy} + xz\omega_{xz} + [c_1 - (a + b_1 + 1)x]\omega_x - \\
  -b_1 y\omega_y + b_1 z\omega_z - ab_1 \omega = 0, \\
  y(1-y)\omega_{yy} - xy\omega_{xy} + yz\omega_{yz} + [c_2 - (a + b_2 + 1)y]\omega_y - \\
  -b_2 x\omega_x + b_2 z\omega_z - ab_2 \omega = 0, \\
  z\omega_{zz} - x\omega_{xz} - y\omega_{yz} + (1-a)\omega_z + \omega = 0,
\end{aligned}
\]

where

\[ \omega(x, y, z) = A_2(a; b_1, b_2; c_1, c_2; x, y, z). \quad (2.4) \]

Really, by virtue of the derivation formula (2.2), it is easy to calculate the following expressions

\[ \omega_x = \sum_{m,n,k=0}^{\infty} \frac{(a)_{m+n-k}(b_1)_m(b_2)_n}{(c_1)_m(c_2)_n m! n! k!} \frac{(a + m + n - k)(b_1 + m)}{(c_1 + m)} x^m y^n z^k, \quad (2.5) \]

\[ x\omega_x = \sum_{m,n,k=0}^{\infty} \frac{(a)_{m+n-k}(b_1)_m(b_2)_n}{(c_1)_m(c_2)_n m! n! k!} \frac{m}{1} x^m y^n z^k, \quad (2.6) \]
the system has four linearly independent solutions

\[
y_\omega_y = \sum_{m,n,k=0}^{\infty} \frac{(a)_{m+n-k}(b_1)_m(b_2)_n}{(c_1)_m(c_2)_n m!n!k!} \frac{n}{1} x^m y^n z^k,
\]

(2.7)

\[
z_\omega_z = \sum_{m,n,k=0}^{\infty} \frac{(a)_{m+n-k}(b_1)_m(b_2)_n}{(c_1)_m(c_2)_n m!n!k!} \frac{k}{1} x^m y^n z^k,
\]

(2.8)

\[
xy_\omega_{xy} = \sum_{m,n,k=0}^{\infty} \frac{(a)_{m+n-k}(b_1)_m(b_2)_n}{(c_1)_m(c_2)_n m!n!k!} \frac{mn}{1} x^m y^n z^k,
\]

(2.9)

\[
xz_\omega_{xz} = \sum_{m,n,k=0}^{\infty} \frac{(a)_{m+n-k}(b_1)_m(b_2)_n}{(c_1)_m(c_2)_n m!n!k!} \frac{mk}{1} x^m y^n z^k,
\]

(2.10)

\[
x^2_\omega_{xx} = \sum_{m,n,k=0}^{\infty} \frac{(a)_{m+n-k}(b_1)_m(b_2)_n}{(c_1)_m(c_2)_n m!n!k!} \frac{(m-1)m}{1} x^m y^n z^k.
\]

(2.11)

Substituting (2.5)-(2.11) into the first equation of the system (2.3), we are convinced that function \(\omega(x, y, z)\) satisfies this equation. We are similarly convinced that function \(\omega(x, y, z)\) satisfies the second and third equations of the system (2.3).

Having substituted \(\omega = x^\tau y^\nu z^\mu \psi(x, y, z)\) in the system (2.3), it is possible to be convinced that for the values

\[
\tau : \quad 0, \quad 1 - c_1, \quad 0, \quad 1 - c_1,
\]

\[
\nu : \quad 0, \quad 0, \quad 1 - c_2, \quad 1 - c_2,
\]

\[
\mu : \quad 0, \quad 0, \quad 0, \quad 0,
\]

the system has four linearly independent solutions

\[
\omega_1(x, y, z) = A_2(a; b_1, b_2; c_1, c_2; x, y, z),
\]

(2.12)

\[
\omega_2(x, y, z) = x^{1-c_1} A_2(a + 1 - c_1; b_1 + 1 - c_1, b_2; 2 - c_1, c_2; x, y, z),
\]

(2.13)

\[
\omega_3(x, y, z) = y^{1-c_2} A_2(a + 1 - c_2; b_1, b_2 + 1 - c_2; c_1, 2 - c_2; x, y, z),
\]

(2.14)

\[
\omega_4(x, y, z) = x^{1-c_1} y^{1-c_2} \times
\]

\[
x A_2(a + 2 - c_1 - c_2; b_1 + 1 - c_1, b_2 + 1 - c_2; 2 - c_1, 2 - c_2; x, y, z).
\]

(2.15)

In order to further study the decomposition properties of the products by Gauss’s hypergeometric functions, we need to know the same expansions of the function \(A_2(a; b_1, b_2; c_1, c_2; x, y, z)\). For this purpose we shall consider the expression

\[
A_2(a; b_1, b_2; c_1, c_2; x, y, z) =
\]
\[ F_2(a; b_1, b_2; c_1, c_2; x, y) = \sum_{m,n=0}^{\infty} \frac{(a)_{m+n}(b_1)_m(b_2)_n}{(c_1)_m(c_2)_n m! n!} x^m y^n. \]

In [12] for Appell’s hypergeometric function \( F_2(a; b_1, b_2; c_1, c_2; x, y) \) following expansion was found

\[ F_2(a; b_1, b_2; c_1, c_2; x, y) = \sum_{i=0}^{\infty} \frac{(a)_i(b_1)_i(b_2)_i}{(c_1)_i(c_2)_i i!} x^i y^i \times \]

\[ \times F(a + i; b_1 + i; c_1 + i; x) F(a + i; b_2 + i; c_2 + i; y), \quad (2.17) \]

where \( F(a, b; c; z) = \sum_{n=0}^{\infty} \frac{(a)_n(b)_n}{(c)_n n!} z^n \) is a hypergeometric function of Gauss.

Considering expansion (2.17), from the identity (2.16) we find [10]

\[ A_2(a; b_1, b_2; c_1, c_2; x, y, z) = \sum_{i,j=0}^{\infty} \frac{(a)_{i-j}(b_1)_i(b_2)_i}{(c_1)_i(c_2)_i i! j!} x^i y^i z^j \times \]

\[ \times F(a + i - j, b_1 + i; c_1 + i; x) F(a + i - j, b_2 + i; c_2 + i; y). \quad (2.18) \]

By virtue of the formula

\[ F(a, b; c; x) = (1 - x)^{-b} F\left(c - a, b; c; \frac{x}{x - 1}\right), \]

we get from expansion (2.18)

\[ A_2(a; b_1, b_2; c_1, c_2; x, y, z) = (1 - x)^{-b_1} (1 - y)^{-b_2} \times \]

\[ \times \sum_{i,j=0}^{\infty} \frac{(a)_{i-j}(b_1)_i(b_2)_i}{(c_1)_i(c_2)_i i! j!} \left( \frac{x}{1 - x} \right)^i \left( \frac{y}{1 - y} \right)^j \times \]

\[ \times F\left(c_1 - a + j, b_1 + i; c_1 + i; \frac{x}{x - 1}\right) \times \]

\[ \times F\left(c_2 - a + j, b_2 + i; c_2 + i; \frac{y}{y - 1}\right). \quad (2.19) \]
Expansion (2.19) will be used for studying properties of the fundamental solutions.

We note, that the expansions for the hypergeometric function of Lauricella $F_A^{(s)}$ were found in [13].

### 3. Fundamental solutions

We consider the generalized bi-axially symmetric multivariable Helmholtz equation in the domain $\mathbb{R}^p_+$. The equation (1.1) has the following constructive formulas

$$H^{p,\lambda}_{\alpha,\beta} (x_1^{1-2\alpha} u) \equiv x_1^{1-2\alpha} H^{p,\lambda}_{1-\alpha,\beta}(u), \quad (3.1)$$

$$H^{p,\lambda}_{\alpha,\beta} (x_2^{1-2\beta} u) \equiv x_2^{1-2\beta} H^{p,\lambda}_{\alpha,1-\beta}(u). \quad (3.2)$$

The constructive formulas (3.1), (3.2) give the possibility to solve boundary value problems for equation (1.1) for various values of the parameters $\alpha, \beta$.

We search the solution of equation (1.1) in the form

$$u(x) = P(r) \omega(\xi, \eta, \zeta), \quad (3.3)$$

where

$$r^2 = \sum_{i=1}^{p} (x_i - x_{0i})^2, \quad r_1^2 = (x_1 + x_{01})^2 + \sum_{i=2}^{p} (x_i - x_{0i})^2,$$

$$r_2^2 = (x_1 - x_{01})^2 + (x_2 + x_{02})^2 + \sum_{i=3}^{p} (x_i - x_{0i})^2, \quad \xi = \frac{r^2 - r_1^2}{r^2} = -\frac{4x_1x_{01}}{r^2}, \quad \eta = \frac{r^2 - r_2^2}{r^2} = -\frac{4x_2x_{02}}{r^2},$$

$$\zeta = -\frac{\alpha \beta}{4} r^2, \quad P(r) = (r^2)^{1-\alpha-\beta-\frac{p}{2}}.$$

Substituting (3.3) into equation (1.1), we have

$$A_1 \omega_\xi + A_2 \omega_\eta + A_3 \omega_\zeta + B_1 \omega_\xi_\eta + B_2 \omega_\xi_\zeta + B_3 \omega_\eta_\zeta +$$

$$+ C_1 \omega_\xi + C_2 \omega_\eta + C_3 \omega_\zeta + D \omega = 0, \quad (3.5)$$

where

$$A_1 = P \sum_{i=1}^{p} \left( \frac{\partial \xi}{\partial x_i} \right)^2, \quad A_2 = P \sum_{i=1}^{p} \left( \frac{\partial \eta}{\partial x_i} \right)^2, \quad A_3 = P \sum_{i=1}^{p} \left( \frac{\partial \zeta}{\partial x_i} \right)^2,$$
$B_1 = 2P \sum_{i=1}^{p} \frac{\partial \xi_i}{\partial x_i} \frac{\partial \eta_i}{\partial x_i}$, $B_2 = 2P \sum_{i=1}^{p} \frac{\partial \xi_i}{\partial x_i} \frac{\partial \zeta_i}{\partial x_i}$, $B_3 = 2P \sum_{i=1}^{p} \frac{\partial \eta_i}{\partial x_i} \frac{\partial \zeta_i}{\partial x_i}$,

$C_1 = 2 \sum_{i=1}^{p} \frac{\partial P}{\partial x_i} \frac{\partial \xi_i}{\partial x_i} + P \sum_{i=1}^{p} \frac{\partial^2 \xi_i}{\partial x_i^2} + P \left( \frac{2\alpha}{x_1} \frac{\partial \xi_i}{\partial x_1} + \frac{2\beta}{x_2} \frac{\partial \xi_i}{\partial x_2} \right)$,

$C_2 = 2 \sum_{i=1}^{p} \frac{\partial P}{\partial x_i} \frac{\partial \eta_i}{\partial x_i} + P \sum_{i=1}^{p} \frac{\partial^2 \eta_i}{\partial x_i^2} + P \left( \frac{2\alpha}{x_1} \frac{\partial \eta_i}{\partial x_1} + \frac{2\beta}{x_2} \frac{\partial \eta_i}{\partial x_2} \right)$,

$C_3 = 2 \sum_{i=1}^{p} \frac{\partial P}{\partial x_i} \frac{\partial \zeta_i}{\partial x_i} + P \sum_{i=1}^{p} \frac{\partial^2 \zeta_i}{\partial x_i^2} + P \left( \frac{2\alpha}{x_1} \frac{\partial \zeta_i}{\partial x_1} + \frac{2\beta}{x_2} \frac{\partial \zeta_i}{\partial x_2} \right)$,

$D = \sum_{i=1}^{p} \frac{\partial^2 P}{\partial x_i^2} + 2\alpha \frac{\partial P}{\partial x_1} + 2\beta \frac{\partial P}{\partial x_2} - \lambda^2 P$.

After elementary evaluations we find

$A_1 = -\frac{4P}{r^2} \frac{x_0^1}{x_1} \xi(1 - \xi)$, $A_2 = -\frac{4P}{r^2} \frac{x_0^2}{x_2} \eta(1 - \eta)$, (3.6)

$A_3 = -\lambda^2 P \zeta$, $B_1 = \frac{4P}{r^2} \frac{x_0^1}{x_1} \xi \eta + \frac{4P}{r^2} \frac{x_0^2}{x_2} \xi \eta$, (3.7)

$B_2 = -\frac{4P}{r^2} \frac{x_0^1}{x_1} \xi \zeta + \lambda^2 P \xi$, $B_3 = -\frac{4P}{r^2} \frac{x_0^2}{x_2} \eta \zeta + \lambda^2 P \eta$, (3.8)

$C_1 = -\frac{4P}{r^2} \frac{x_0^1}{x_1} \left[ 2\alpha - \left( 2\alpha + \beta + \frac{p}{2} \right) \xi \right] + \frac{4P}{r^2} \frac{x_0^2}{x_2} \beta \xi$, (3.9)

$C_2 = \frac{4P}{r^2} \frac{x_0^1}{x_1} \alpha \eta - \frac{4P}{r^2} \frac{x_0^2}{x_2} \left[ 2\beta - \left( \alpha + 2\beta + \frac{p}{2} \right) \eta \right]$, (3.10)

$C_3 = -\frac{4P}{r^2} \frac{x_0^1}{x_1} \alpha \zeta - \frac{4P}{r^2} \frac{x_0^2}{x_2} \beta \zeta - \lambda^2 P \left( \frac{p}{2} - \alpha - \beta \right)$, (3.11)

$D = \frac{4P}{r^2} \left[ \frac{x_0^1}{x_1} \alpha + \frac{x_0^2}{x_2} \beta \right] \left( \alpha + \beta - 1 + \frac{p}{2} \right) - \lambda^2 P$. (3.12)

Substituting equalities (3.6)-(3.12) into equation (3.5), we get the system of hypergeometric equations
\[
\begin{aligned}
&\begin{cases}
\xi(1-\xi)\omega_{\xi\xi} - \xi\eta\omega_{\xi\eta} + \xi\zeta\omega_{\xi\zeta} + [2\alpha - (2\alpha + \beta + \frac{p}{2})]\xi\omega_{\xi} \\
-\alpha\eta\omega_{\eta} + \alpha\zeta\omega_{\zeta} - \alpha (\alpha + \beta - 1 + \frac{p}{2}) \omega = 0,
\end{cases} \\
&\begin{cases}
\eta(1-\eta)\omega_{\eta\eta} - \xi\eta\omega_{\eta\xi} + \eta\zeta\omega_{\eta\zeta} + [2\beta - (\alpha + 2\beta + \frac{p}{2})]\eta\omega_{\eta} \\
-\beta\xi\omega_{\xi} + \beta\zeta\omega_{\zeta} - \beta (\alpha + \beta - 1 + \frac{p}{2}) \omega = 0,
\end{cases} \\
&\zeta\omega_{\zeta\zeta} - \xi\omega_{\xi\zeta} - \eta\omega_{\eta\zeta} + (2 - \alpha - \beta - \frac{p}{2}) \omega_{\zeta} + \omega = 0.
\end{aligned}
\]

Considering the solutions of the system of hypergeometric equations (2.12)-(2.15), we define

\[
\omega_1(\xi, \eta, \zeta) = A_2 \left( \alpha + \beta - 1 + \frac{p}{2}; \alpha, \beta; 2\alpha, 2\beta; \xi, \eta, \zeta \right),
\]

\[
\omega_2(\xi, \eta, \zeta) = \xi^{1-2\alpha} A_2 \left( -\alpha + \beta + \frac{p}{2}; 1 - \alpha, \beta; 2 - 2\alpha, 2\beta; \xi, \eta, \zeta \right),
\]

\[
\omega_3(\xi, \eta, \zeta) = \eta^{1-2\beta} A_2 \left( \alpha - \beta + \frac{p}{2}; \alpha, 1 - \beta; 2\alpha, 2 - 2\beta; \xi, \eta, \zeta \right),
\]

\[
\omega_4(\xi, \eta, \zeta) = \xi^{1-2\alpha}\eta^{1-2\beta} \times \\
\quad \quad \times A_2 \left( 1 - \alpha - \beta + \frac{p}{2}; 1 - \alpha, 1 - \beta; 2 - 2\alpha, 2 - 2\beta; \xi, \eta, \zeta \right).
\]

Substituting the equalities (3.14)-(3.17) into the expression (3.3), we get some solutions of the equation (1.1)

\[
q_1(x, x_0) = k_1 (r^2)^{1-\alpha-\beta-\frac{p}{2}} \times \\
\quad \quad \times A_2 \left( \alpha + \beta - 1 + \frac{p}{2}; \alpha, \beta; 2\alpha, 2\beta; \xi, \eta, \zeta \right),
\]

\[
q_2(x, x_0) = k_2 (r^2)^{\alpha-\beta-\frac{p}{2}} x_1^{1-2\alpha} x_0^{1-2\alpha} \times \\
\quad \quad \times A_2 \left( -\alpha + \beta + \frac{p}{2}; 1 - \alpha, \beta; 2 - 2\alpha, 2\beta; \xi, \eta, \zeta \right),
\]

\[
q_3(x, x_0) = k_3 (r^2)^{-\alpha+\beta-\frac{p}{2}} x_2^{1-2\beta} x_0^{1-2\beta} \times \\
\quad \quad \times A_2 \left( \alpha - \beta + \frac{p}{2}; \alpha, 1 - \beta; 2\alpha, 2 - 2\beta; \xi, \eta, \zeta \right),
\]

\[
q_4(x, x_0) = k_4 (r^2)^{-1+\alpha+\beta-\frac{p}{2}} x_1^{1-2\alpha} x_0^{1-2\alpha} x_2^{1-2\beta} x_0^{1-2\beta} \times \\
\quad \quad \times A_2 \left( 1 - \alpha - \beta + \frac{p}{2}; 1 - \alpha, 1 - \beta; 2 - 2\alpha, 2 - 2\beta; \xi, \eta, \zeta \right),
\]
where \( k_1, \ldots k_4 \) are constants which will be determined at solving boundary value problems for equation (1.1). It is easy to notice that the considered functions (3.18)-(3.21) possess the properties

\[
\frac{\partial q_1(x, x_0)}{\partial x_1} \bigg|_{x_1=0} = 0, \quad \frac{\partial q_1(x, x_0)}{\partial x_2} \bigg|_{x_2=0} = 0, \quad (3.22)
\]

\[
q_2(x, x_0) \bigg|_{x_1=0} = 0, \quad \frac{\partial q_2(x, x_0)}{\partial x_2} \bigg|_{x_2=0} = 0, \quad (3.23)
\]

\[
\frac{\partial q_3(x, x_0)}{\partial x_1} \bigg|_{x_1=0} = 0, \quad q_3(x, x_0) \bigg|_{x_2=0} = 0, \quad (3.24)
\]

\[
q_4(x, x_0) \bigg|_{x_1=0} = 0, \quad q_4(x, x_0) \bigg|_{x_2=0} = 0. \quad (3.25)
\]

From the expansion (2.19) follows that the fundamental solutions (3.18)-(3.21) at \( r \to 0 \) possess a singularity of the order \( \frac{1}{r^{p-2}} \), where \( p > 2 \).

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Ко’п о’згарувчилни умумлашган ики о’қли симметрик Гельмгольц tenglamasining fundamental yechimlari

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Фундаментальные решения обобщенного двуосесимметрического многомерного уравнения Гельмгольца

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