Different atom trapping geometries with time averaged adiabatic potentials

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Received 20 April 2021 / Accepted 9 October 2021 / Published online 29 October 2021

Abstract. We have theoretically studied the time averaged adiabatic potential (TAAP) scheme for realizing different atom trapping geometries such as double-well, ring, asymmetric ring. The versatility of TAAP scheme has been shown for control and manipulation of these atom trapping geometries via variation in time orbiting potential (TOP) fields and radio frequency (rf) fields. The conversion from one trapping geometry to another is also possible.

1 Introduction

With the advent of laser cooling and trapping of atoms \cite{1}, the research in atomic physics has evolved in several dimensions from basic science \cite{2–4} to practical applications \cite{5–7} of cold atoms in quantum technologies \cite{8–10}. A magneto-optical trap (MOT) \cite{11} is a work-horse device to produce samples of cold atoms for various studies and applications. The static magnetic traps \cite{12,13} and optical dipole traps \cite{14} are widely used traps for further trapping and cooling of atoms from MOT to generate ultra-cold or degenerate atomic gas samples. In spite of a standard practice of using these magnetic and dipole traps for cold atoms, it is felt that a good handle on control and manipulation of trapping geometries can enrich our capabilities for use of cold atoms for several purposes. In this context, the rf-dressed adiabatic potential scheme, first proposed by Zobay and Garraway \cite{15}, has been implemented successfully to demonstrate atom trapping in different geometries \cite{15–19}. The rf-dressed magnetic traps \cite{12,13,16,20} are able to generate some novel trapping geometries such as double-well \cite{4}, ring trap \cite{17,19,21}, arc trap \cite{16}, which are difficult to achieve by conventional static magnetic traps or laser dipole traps. These capabilities can be further enriched by use of Time Averaged Adiabatic Potential (TAAP) technique, which is an extension of the rf-dressed adiabatic potential technique and proposed by Lesanovsky and Klitzing \cite{22}. In the TAAP scheme, a relatively lower frequency time averaging field, called time orbiting potential (TOP) \cite{23} field, is applied on atoms in the presence of rf and static magnetic fields used for rf-dressing. Gildemeister et al \cite{24} have first experimentially demonstrated the working of atom trapping using TAAP scheme.

In this work, we have theoretically investigated various atom trapping geometries using TAAP scheme. Our results show that by varying the TOP fields parameters as well as rf-dressing fields parameters, various atom trapping geometries can be realized which are not possible by conventional atom trapping techniques. In TAAP scheme, the conversion from one geometry to the other is possible. The atom trapping geometries discussed here find practical applications in splitting, guiding and focusing of matter-waves which finally can be used in developing atom-optic devices. These trapping geometries can also be used to study basic physics phenomena such as Bose-Einstein condensation \cite{25,26}, tunnelling \cite{2,4,27}, super-fluidity \cite{3,17,24}, atom interferometry \cite{6,28–30}.

The article is organized as following. The Sect. 2 presents the general theoretical framework for description of rf-dressed adiabatic potentials and Time Averaged Adiabatic Potential (TAAP). In Sect. 3, we discuss our findings on how to generate and manipulate different atom trapping geometries, such as double-wells and rings, using TAAP scheme. In Sect. 4, we present the conclusion of our studies.

2 Theory

When atoms are trapped in the static magnetic trap, the application of an rf-field results in the formation of new states, know as rf-dressed states. The energy eigen values corresponding to these dressed states are known as rf-dressed potentials (or adiabatic potentials). The rf-dressed potentials generally vary with position of the...
atom in the trap because of position-dependent rf-field coupling strength inside the trap. Thus, atoms trapped in a magnetic trap in a particular magnetic hyperfine state can be transferred to a rf-dressed potential by exposing them to a rf-field of suitable frequency and amplitude [13]. In the present theoretical work, we consider the quadrupole trap as a static magnetic trap with field variation given as

\[ B_S(r) = B'_q \begin{pmatrix} x \\ y \\ -2z \end{pmatrix}, \]

(1)

where \( B'_q \) is the quadrupole field gradient. We take the rf-field of the form given by

\[ B_{rf}(t) = \begin{pmatrix} B_{rf}^x \cos(\omega_{rf}t) \\ B_{rf}^y \cos(\omega_{rf}t + \phi_{rf}^y) \\ B_{rf}^z \cos(\omega_{rf}t + \phi_{rf}^z) \end{pmatrix}, \]

(2)

where \( B_{rf}^x, B_{rf}^y \) and \( B_{rf}^z \) are the amplitudes of the rf-field along \( x-, y- \) and \( z- \) directions, respectively, and \( \omega_{rf} \) is the angular frequency of the rf field. The parameters \( \phi_{rf}^y \) and \( \phi_{rf}^z \) are the phases of the \( y- \) and \( z- \) components of rf-field with respect to its \( x- \) component.

For an atom, in a state with hyperfine angular momentum \( \mathbf{F} \) and interacting with the fields given by Eqs. (1) and (2), the effective Hamiltonian in the absence of any kinetic energy term can be written as

\[ H_{eff}(r,t) = -\mu.\mathbf{B}_{eff}(r,t) = \frac{gF\mu_B}{\hbar} \mathbf{F}.\mathbf{B}_{eff}(r,t), \]

(3)

with

\[ \mathbf{B}_{eff}(r,t) = \mathbf{B}_S(r) + \mathbf{B}_{rf}(t) = |\mathbf{B}_S(r)| \hat{e}_S + \mathbf{B}_{rf}(t), \]

(4)

as \( U = \exp(\omega_{rf}t(F^\parallel_t)) \), then the effective Hamiltonian in the rotating frame can be written as

\[ H_{eff}^R(t) = U H_{eff} U^\dagger - i\hbar U \frac{\partial U^\dagger}{\partial t}, \]

(5)

where, \( F^\parallel \) is the angular momentum along the static field direction. The parameter \( F_R \) is the angular momentum perpendicular to the static field direction in the rotating frame. The components of \( F_R \) can be written as \( F_R^1 = UF^{1\perp}U^\dagger \) and \( F_R^{1\parallel} = UF^{1\parallel}U^\dagger \).

The first term in Eq. (5) is the Zeeman energy term, and second term involves the component of rf-field along the static field. Second term has a vanishing contribution because \( |\mu_B B_{rf}^\parallel| \ll |\hbar \omega_{rf}| \). After inserting the expression [31] for \( F_R \) and \( B_{rf}^\perp(t) \), and applying the rotating wave approximation, the expression for the effective Hamiltonian can be written as

\[ H_{eff}^R = \frac{gF\mu_B}{\hbar} \frac{\hbar B^\parallel}{gF\mu_B} \left[ B_{rf}^{1\parallel} - i B_{rf}^{1\perp} e^{i\gamma} \right], \]

(6)

where \( F_+ = F^{1\parallel} + i F^{1\perp} \) and \( F_- = F^{1\parallel} - i F^{1\perp} \) are the raising and lowering operator of hyperfine angular momentum in the rotated frame. \( \gamma = \gamma_2 - \gamma_1 \) is the relative phase separation of the rf-field components in rotated co-ordinate system.

The eigenvalues of the Hamiltonian (\( H_{eff}^R \)), when solved in the basis of \( |F, m_F\rangle \) states, lead to the expression for the rf-dressed adiabatic potential as

\[ E_{AP}(r) = m_F \hbar \sqrt{\left( \frac{gF\mu_B |\mathbf{B}_S(r)|}{\hbar} - \omega_{rf} \right)^2 + \left( \frac{gF\mu_B}{2\hbar} \right)^2 \left[ (B_{rf}^{1\parallel})^2 + (B_{rf}^{1\perp})^2 + 2B_{rf}^{1\parallel} B_{rf}^{1\perp} \sin(\gamma) \right]} . \]

(7)

where \( \hat{e}_S \) is the direction of the static magnetic field and \( \mu = -2gF\mu_B \mathbf{F} \) is the magnetic dipole moment of the atom having hyperfine angular momentum \( \mathbf{F} \).

For simplicity of calculations, a new co-ordinate system is chosen whose one of the axes coincides with the direction of quadrupole magnetic field. In this co-ordinate system, the static field vector becomes \( (0,0,|\mathbf{B}_S(r)|)^T \), and the rf-field vector becomes

\[ (B_{rf}^{1\parallel} \cos(\omega_{rf}t), B_{rf}^{1\perp} \cos(\omega_{rf}t + \gamma_1), B_{rf}^{1\parallel} \cos(\omega_{rf}t + \gamma_2))^T, \]

where \( \gamma_1 \) and \( \gamma_2 \) are the relative phases of the field components in the new co-ordinate system. The hyperfine angular momentum is also changed to \( (F^{1\parallel}, F^{1\perp}, F^\parallel)^T \). Now, if the Schrödinger wave equation is transformed in a rotating frame using an unitary operator described in a compact form, the adiabatic potential can be expressed as

\[ E_{AP}(r) = m_F \hbar \sqrt{\left| \Delta(r) \right|^2 + \left| \Omega_R(r) \right|^2} , \]

(8)

where detuning \( (\Delta) \) and Rabi frequency \( (\Omega_R) \) for the dressing rf-field are expressed as

\[ \Delta(r) = \frac{gF\mu_B}{\hbar} |\mathbf{B}_S(r)| - \omega_{rf} \]

(9)

and

\[ \left| \Omega_R(r) \right|^2 = \left( \frac{gF\mu_B}{2\hbar} \right)^2 \left[ (B_{rf}^{1\parallel})^2 + (B_{rf}^{1\perp})^2 + 2B_{rf}^{1\parallel} B_{rf}^{1\perp} \sin(\gamma) \right] . \]

(10)
Using the reverse transformation for fields, the expression for the Rabi frequency \(\left| \Omega_B(r) \right| \) in terms of field parameters in laboratory frame is given by

\[
\left| \Omega_B(r) \right|^2 = \left( \frac{g F \mu_B}{2 \hbar} \right)^2 \left[ \frac{4 z^2}{x^2 + y^2 + 4z^2} \left( B_{rf}^x \right)^2 x^2 + \left( B_{rf}^y \right)^2 y^2 \right] + \left( B_{rf}^z \right)^2 \left( \frac{2 x^2 + y^2}{x^2 + y^2 + 4z^2} \right) - 2 \frac{B_{rf}^z B_{rf}^x y \cos(\phi_{rf}^y)}{x^2 + y^2 + 4z^2} + 4 \frac{B_{rf}^z B_{rf}^x y \sin(\phi_{rf}^y)}{x^2 + y^2 + 4z^2} + 4 \frac{B_{rf}^y B_{rf}^z x \cos(\phi_{rf}^x - \phi_{rf}^z)}{x^2 + y^2 + 4z^2} + 4 \frac{B_{rf}^x B_{rf}^z y \cos(\phi_{rf}^x)}{x^2 + y^2 + 4z^2} + 4 \frac{B_{rf}^x B_{rf}^z \sin(\phi_{rf}^x)}{x^2 + y^2 + 4z^2} + 2 \frac{B_{rf}^z B_{rf}^x y \sin(\phi_{rf}^y)}{x^2 + y^2 + 4z^2}. \tag{11}
\]

From above expression \([\text{Eq. (8)}]\) of adiabatic potential, it is evident that potential energy for atom trapping can be tailored to a large extent by choosing the phases \((\phi_{rf}^x, \phi_{rf}^y)\) and the amplitudes \((B_{rf}^x, B_{rf}^y, B_{rf}^z)\) of the rf-fields.

The time averaged adiabatic potentials (TAAP) can be achieved by time averaging the rf-dressed or adiabatic potentials. This is achieved by applying the low-frequency time orbiting potential (TOP) field in addition to dressing rf-field to perform the time averaging.

The general expression of a TOP field is given as

\[
B_T(t) = B_T^x \sin(\omega_T t) \hat{e}_x + B_T^y \sin(\omega_T t + \phi_T^y) \hat{e}_y + B_T^z \sin(\omega_T t + \phi_T^z) \hat{e}_z, \tag{12}
\]

where \(B_T^x, B_T^y\) and \(B_T^z\) are the magnitudes of TOP fields along \(x, y, \) and \(z\)-directions, respectively. The parameters \(\phi_T^x, \phi_T^y,\) and \(\phi_T^z\) are the phases of the \(y\) and \(z\)-components of TOP field with respect to the \(x\)-component.

In conventionally used TOP traps, a circularly polarized TOP field in the \(x-y\) plane is used and its frequency \(\omega_T\) is kept larger than the frequency \(\omega_r\) of center of mass motion of the atom in quadrupole trap but smaller than the Larmor frequency \(\frac{2 e x B_{rf}^z}{\hbar}\) provided by the TOP field. Mathematically, the criteria can be written as

\[
\omega_r \ll \omega_T < \frac{g F \mu_B B_T}{\hbar}, \tag{13}
\]

where \(B_T\) is the magnitude of the TOP field.

Finally, the general expression [32] of time averaged adiabatic potential (TAAP) can be written as

\[
E_{TAAP}(r) = \frac{\omega_T}{2 \pi} \int_0^{2 \pi} \frac{d \phi_T}{B_T^{y}} \left( x + B_T^x \sin(\omega_T t), y + B_T^y \sin(\omega_T t + \phi_T^y), z + B_T^z \sin(\omega_T t + \phi_T^z) \right) dt. \tag{14}
\]

It is obvious that by changing the rf-field and TOP field parameters such as \((B_{rf}^x, B_{rf}^y, B_{rf}^z, \phi_{rf}^x, \phi_{rf}^y)\) and \((B_T^x, B_T^y, B_T^z, \phi_T^x, \phi_T^y)\), several interesting atom trapping geometries can be achieved for different applications. Some of these geometries are discussed in the following section.

### 3 Results and discussion

Different atom trapping geometries under TAAP scheme can be achieved by choosing the rf-fields and TOP fields parameters appropriately. Using the TAAP scheme, a vertical \((z\)-direction) double-well trap was first demonstrated by Gildemeister et al [32] by choosing a vertical direction \((z\)-direction) linearly polarized rf-field \(B_{rf}(t) = (0, 0, B_{rf}^z(t))\) and a \(x-y\) circularly polarized TOP field. Similarly, a ring type atom trap using TAAP scheme was first proposed by Lesanovsky et al [22] and experimentally demonstrated by Sherlock et al [19]. A circularly polarized rf-field in \(x-y\) plane along with a \(z\)-direction linearly polarized TOP field can be used to generate ring trap in \(xy\)-plane. The double-well trap is useful to study quantum tunnelling [2] and coherent splitting of matter waves [18], whereas a ring trap is useful to study the persistent flow [3, 17] of cold atomic gas, quantum degeneracy [19] in low dimensions, matter-wave guiding for atom interferometry [33] for developing atom-gyroscope [10], etc.

In this work, we have theoretically shown that the variation in rf-field and TOP field parameters can result in interesting TAAP traps for trapping of cold atoms. It is assumed that \(^{87}\text{Rb}\) atoms are magnetically trapped in \(F = 2, m_F = 2\) hyperfine level of the ground state \(^{5}\text{S}_1/2\). Eq. (14) is numerically solved to find the atom trapping potentials (or trapping geometries) in TAAP scheme for different combinations of rf and TOP fields. The effect of earth’s gravity is ignored here, but it may be important to consider the gravity in case of very low trap depth or very low temperature of atoms (few \(\mu\)K). The discussion on different atom trapping geometries is as follows.

### 3.1 Double-well TAAP trap

A double-well trap is useful for coherent splitting of Bose condensate or cold atom cloud. In TAAP scheme,
it can be generated by using a linearly polarized rf-field and a circularly polarized TOP field [32]. The polarization direction of rf-field is to be kept perpendicular to the plane of TOP field polarization. In this arrangement, the double-well gets oriented along the direction of applied rf-field. For example, a double-well along x-axis can be generated by using a x-polarized rf-field and a y – z circularly polarized TOP field. From Eq. (14), the TAAP double-well trap oriented along x-axis with this choice of rf- and TOP fields can be written as,

\[
E_{TAAP}^{DW}(r) = \frac{\omega_T}{2\pi} \int_0^{2\pi} E_{AP}\left(x, y + \frac{B_T}{B_q} \sin(\omega_T t), z + \frac{B_T}{2B_q} \cos(\omega_T t)\right) dt.
\]

Using Eq. (15), it can be shown that separation between the wells is given as,

\[
d_{DW} = \frac{2\hbar\omega_{rf}}{gf\mu_B B_q} \sqrt{1 - \left(\frac{gf\mu_B B_T}{\hbar\omega_{rf}}\right)^2}.
\]  

The results of the calculation performed using Eq. (15) and Eq. (16) are shown in Fig. 1 and Fig. 2, respectively. Fig. 1 shows the three-dimensional (3D) and two-dimensional (2D) iso-potential surfaces of TAAP which clearly implies the formation of double-well along x-axis.

The ultracold atom clouds or Bose condensates are nowadays prepared in varied sizes ranging from few tens of micron to few hundreds of micron. In order to split such clouds adiabatically, the wells separation \(d_{DW}\) should be comparable to the size of the cloud. It is evident from Eq. (16) that the separation between the wells \(d_{DW}\) can be changed by changing the parameters \(B_q', B_T\) and \(\omega_{rf}\). For the formation of TAAP trap with double-well, the condition \(\omega_{rf} > |g\mu_B B_T|\) should be satisfied. Fig. 2 shows the contours of \(d_{DW}\) for different values of \(B_q', B_T\) and \(\omega_{rf}\) parameters satisfying Eq. (16). For a desired value of separation \(d_{DW}\), the dressing-field frequency \(\omega_{rf}\) should be chosen such that it corresponds to workable values of quadrupole field gradient \(B_q\) and TOP field amplitude \(B_T\). A very low value of \(B_T\) \((B_T < 0.4 \text{ G})\) or a very high value of \(B_q\) affects the TAAP trap life-time adversely due to increase in the Landau–Zener losses in the trap [32]. At the same time, \(B_q'\) should be large enough to enable trap to hold atoms against gravity. For example, as shown by a contour (red) in Fig. 2a, the wells separation of \(10 \mu m\) can be achieved with a field gradient \(B_q' > 150 \text{ G/cm}\) and \(B_T < 0.7 \text{ G}\) (for \(\omega_{rf} = 2\pi \times 0.5 \text{ MHz}\)).

Next, we investigated the possibility of changing orientation of a double-well by changing the TAAP parameters. We find that a x-directional double-well trap can be changed to a y-directional double-well trap by changing only the TOP field configuration to x-z polarized. These results are shown in plots (b) and (c) in Fig. 3. The double-well in y-direction becomes clear at a higher value of \(B_q'\) as shown in plot Fig. 3 (c). Thus, in TAAP scheme, the orientation of double-well can be changed by changing the TOP field without any change in rf-field.

3.2 Ring TAAP trap and its manipulation with TOP field modulations

The ring traps using TAAP scheme offer a very smooth waveguide for ultracold atoms and Bose condensate. Recently, Pandey et al [34] have demonstrated the control of momentum spread of a condensate by applying a gravito-magnetic potential [33] inside a TAAP ring waveguide. It is known that in TAAP scheme, a circularly polarized rf-field and a linearly polarized TOP field in the presence of a dc quadrupole magnetic trap result in a ring trap [19,22]. For example, a x – y circularly polarized rf-field and z-polarized TOP field in the presence of quadrupole trap oriented along z-axis result in a ring atom trap in x – y plane. The expression for the ring TAAP with this choice of rf- and TOP fields can be written as,

\[
E_{TAAP}^{Ring}(r) = \frac{\omega_T}{2\pi} \int_0^{2\pi} E_{AP}\left(x, y, z + \frac{B_T}{2B_q} \sin(\omega_T t)\right) dt.
\]  

![Fig. 1 The upper plot shows the 3D view of iso-potential surfaces in a double-well TAAP geometry for different potential values 25 µK (intense red) and 35 µK (light red). The lower plot shows the 2D contours of TAAP potentials for this double-well geometry. The rf-field is linearly polarized in x-direction and TOP field is y-z circularly polarized (\(B_q' = B_T = 1.3 \text{ G})\. The other parameters are \(B_{rf} = 700 \text{ mG}, \omega_{rf} = 2\pi \times 1.5 \text{ MHz}, \omega_T = 2\pi \times 7 \text{ kHz}\) and \(B_q' = 100 \text{ G/cm}\).](image-url)
Fig. 2  Contour plots for $d_{DW}$ showing dependence of $d_{DW}$ on TOP field ($B_T$) and quadrupole field gradient ($B'_q$) values. The value of $d_{DW}$ in $\mu$m is indicated near the curve. The plots (a), (b) and (c) correspond to different values of $\omega_{rf}$.

Fig. 3  The plots (a) to (c) show the conversion of a x-direction double-well into a y-direction double-well due to change in TOP field. The upper plots in (a), (b) and (c) show the 3D view of iso-potential surfaces for different potential values 25 $\mu$K (intense red) and 35 $\mu$K (light red). The rf-field is linearly polarized in x-direction in all the plots. In plot (a), the TOP field is y-z circularly polarized ($B_y^T = B_z^T = 1.3$ G ). In plots (b) and (c), the TOP field is x-z polarized with $\pi/2$ phase difference between x- and z- components. For plot (b), $B_y^T = 1.3$ G and $B_z^T = 650$ mG, and for plot (c), $B_y^T = 1.3$ G and $B_z^T = 850$ mG. The other common parameters in plots (a) to (c) are $B_{rf}^x = 700$ mG, $\omega_{rf} = 2\pi \times 1.5$ MHz, $\omega_T = 2\pi \times 7$ kHz and $B'_q = 100$ G/cm.

Fig. 4  The iso-potential contours of TAAP showing the conversion of a ring type trapping potential into a double-well along y-direction. The circularly polarized rf-field has $B_{rf}^x = B_{rf}^y = 700$ mG. The value of z- TOP field ($B_z^T$) is 1.3 G for plot (a). For plot (b), the values of TOP field components are $B_{rf}^x = 1.3$ G and $B_{rf}^y = 650$ mG, and for plot (c), $B_{rf}^x = 1.3$ G and $B_{rf}^y = 850$ mG. The other parameters in plots (a), (b) and (c) are $\omega_{rf} = 2\pi \times 1.5$ MHz, $\omega_T = 2\pi \times 7$ kHz and $B'_q = 100$ G/cm.
A three-dimensional (3D) plot of iso-potential surface showing formation of tilted TAAP ring trap. Intense red colour shows potential surface of 25\,\mu K, and the light red colour shows that of 45\,\mu K temperature. Circularly polarized rf-field values are \(B_{x,f}^\alpha = B_{y,f}^\alpha = 700\,\text{mG}\), and the magnitude of \(z\)-TOP field (\(B_{z,T}^\alpha\)) before tilt is 1.3\,G. The other parameters are \(\omega_{rf} = 2\pi \times 1.5\,\text{MHz}, \omega_{T} = 2\pi \times 7\,\text{kHz}\) and \(B_{q}^\prime = 100\,\text{G/cm}\).

The radius of the above ring TAAP is given by,

\[
R_{\text{ring}} = \frac{\hbar \omega_{rf}}{g_F\mu_B B_{q}^\prime} \sqrt{1 - \frac{1}{2} \left(\frac{g_F\mu_B B_{T}}{\hbar \omega_{rf}}\right)^2}. \tag{18}
\]

From the above Eq. (18), it clearly turns out that radius of the ring TAAP trap can be changed by changing the parameters \(B_{q}^\prime, B_{T}\) and \(\omega_{rf}\). The maximum and minimum values of radii achievable experimentally will be governed by the parameters restrictions as discussed in the case of double-well TAAP trap.

It can be noted that this ring TAAP trap can be converted into a double-well TAAP trap, which can be used for coherent splitting of atom cloud in the ring. The ring trap in \(x-y\) plane can be converted into a \(y\)-direction double-well by changing the \(z\)-polarized TOP field to a \(x-z\) polarized TOP field with a phase difference of \(\frac{\pi}{2}\) between \(x-\) and \(z\)-components. This conversion is shown in Fig. 4. The orientation of double well can be controlled by changing the value of phase between \(x\)- and \(z\)-components. Similarly, if \(y-z\) polarized TOP field is used, double-well can be oriented along \(x\)-axis.

In the above ring TAAP arrangement, if the direction of \(z\)-polarized TOP field is tilted with respect to \(z\)-axis, the plane of the ring gets tilted. The tilted ring TAAP is shown in Fig. 5 for a TOP field tilt angle \(\delta = 20^\circ\) from \(z\)-axis in the azimuthal direction \(\phi_0 = 45^\circ\) from the \(x\)-axis. In the results shown here, the earth gravitational potential has been recently exploited for control of spread of a Bose condensate and termed as magneto-gravitational matter-wave lensing [34]. As demonstrated earlier, a tilted ring TAAP can also be generated by applying \(z\)-polarized rf-field and a tilted \(z\)-TOP field [35].

Rotation of a bucket TAAP around a ring circumference by modulating the phase \(\phi_{rf}^\alpha\) of the \(z\)-rf field. The potential contours (a)–(i) are shown for different values of \(\phi_{rf}^\alpha\). The values of \(\phi_{rf}^\alpha\) from figure (a) to (i) are \(0, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{3\pi}{2}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{4}\), respectively. The amplitudes of \(x-y\) circularly polarized rf-fields are \(B_{x,f}^z = B_{y,f}^z = 700\,\text{mG}\). The linearly polarized rf-field along \(z\)-direction is \(B_{z,f}^z = 500\,\text{mG}\), and value of \(z\)-TOP field (\(B_{T}^z\)) is 1.3\,G. The other parameters are \(\omega_{rf} = 2\pi \times 1.5\,\text{MHz}, \omega_{T} = 2\pi \times 7\,\text{kHz}\) and \(B_{q}^\prime = 100\,\text{G/cm}\).
3.3 Asymmetric ring TAAP and atom cloud rotation on ring

Asymmetric TAAP rings can be generated by using multiple rf-fields and TOP fields. For example, if a $z$-direction rf-field is added with the $x$-$y$ circularly polarized rf-field, in the presence of a $z$-TOP field and static quadrupole field, the generated potential has a minimum at one position on the ring. This bucket kind of trap can be rotated on the ring circumference by changing the phase of $z$-direction rf-field. The results are shown in Fig. 6. The rotation of the cloud can be useful to study super-fluidity in condensate [33, 36] and atom interferometry [33].

4 Conclusion

Different atom trapping geometries have been investigated in time averaged adiabatic potential (TAAP) scheme. The versatility of TAAP scheme to modify and manipulate these trapping geometries has been brought out for applications in atom-optic devices. Some of the discussed geometries can be useful for studying fundamental physics and atomtronic matter-wave optics.

Acknowledgements

Sourabh Sarkar acknowledges the financial support by Raja Ramanna Centre for Advanced Technology, Indore under Homi Bhabha National Institute (HBNI) programme.

Author contributions

Sourabh Sarkar and S. P. Ram have understood the theoretical problem and performed the numerical calculations. V. B. Tiwari has provided necessary inputs for the work. S. R. Mishra has conceptualized the problem. All the authors have contributed in preparation of the manuscript.

Data Availability Statement

This manuscript has no associated data or the data will not be deposited. [Authors' comment: This manuscript has no additional data other than shown in figures. The numerical data related to the figures can be deposited as per the requirement of the journal.]

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