Free Vibration and Bending Behaviour of CNT Reinforced Composite Plate using Different Shear Deformation Theory

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Abstract. In the present study, the free vibration and the bending behaviour of carbon nanotube reinforced composite plate are computed using three different shear deformation theories under thermal environment. The material properties of carbon nanotube and matrix are assumed to be temperature-dependent, and the extended rule of mixture is used to compute the effective material properties of the composite plate. The convergence and validity of the present model also have been checked by computing the wide variety of the numerical example. The applicability of the proposed higher-order models has been highlighted by solving the wide variety of examples for different geometrical and material parameters under elevated thermal environment. The responses are also examined using the simulation model developed in commercial finite element package (ANSYS).

Keywords: CNT, free vibration, bending, thermal environment, FSDT, HSDT.

1. Introduction

From last few decade, many types of research have already been completed on the fibre or the laminated composite materials to show the significant improvement of the specific strength and the stiffness behaviour of the advanced structure or the structural components. Recently, carbon nanotubes (CNTs) are used as the reinforcement material in nanocomposites due to their excellent specific strength, specific stiffness, thermal, electrical, and chemical properties. The CNT has got numerous applications in the aerospace, the nuclear plant, automobile and marine structures due to their outstanding properties. The effective material properties of the carbon nanotube reinforced composite (CNTRC) are evaluated using the various available method such as molecular dynamics (MD) simulation, [1-3] representative volume element (RVE) [4-5], the rule of mixture [6-7] and Eshelby–Mori–Tanaka approach [8]. In general, the CNTs are classified based on their configurations namely, armchair, zigzag and chiral and their elastic properties vary accordingly. Ayatollahia et al. [9] estimated the nonlinear mechanical properties of the single wall nanotubes under three different set of load (tensile, bending and torsional load) using a molecular model based on the finite element method (FEM) for both armchair and zigzag configuration. Araghet al. [10] computed the free vibration behaviour of randomly oriented CNTRC cylindrical shell based on third-order shear deformation theory (TSDT). Lin and Xiang [11] investigated the free vibration response of FG-CNTRC beam using the p-Ritz method based on the first-order shear deformation theory (FSDT) and TSDT kinematic model with von Karman sense geometric nonlinearity. Shen et al. [12] investigated the vibration response of multi-wall carbon nanotube (MWCNT) based biosensor using nonlocal
Timoshenko beam theory (TBT). Keet et al. [13] computed the dynamic stability, free vibration and buckling behaviour of functionally graded carbon nanotube reinforced composite (FG-CNTRC) beam using TBT. Alva and Raja [14] examined the effects of the weight percentage and the diameter of the MWCNT on the damping characteristics of the MWCNT/epoxy composite. Jaraliet al. [15] reported the effect of agglomeration and the volume fractions on the mechanical, thermal, electrical and the moisture properties of the CNT/epoxy composite. Vodenitcharova and Zhang [16] computed the local buckling, and the bending behaviour of the SWCNT reinforced composite plates using Airy’s stress function. Formica et al. [17] numerically solved the free vibration responses of the CNTRC plate using FEM and the effective material properties of the CNTRC plate are computed based on Mori-Tanaka scheme. Zhu et al. [18] examined the free vibration and bending analysis of the FG-CNTRC plate using the FSDT kinematics. Yas and Heshmati [19] estimated the free and force vibrational behaviour of functionally graded nanocomposite beams reinforced by randomly oriented straight SWCNTs based on Timoshenko and Euler–Bernoulli beam theories using FEM. Lei et al. [20-22] computed the free vibration and the buckling behaviour of the FG-CNTRC cylindrical panels based on the FSDT shell theory using the element-free kp-Ritz method. Alibeigloo and Liew [23] examined the flexural behaviour of FG-CNTRC rectangular plate with all edges simply supported boundary conditions based on the three-dimensional theory of elasticity. Lei et al. [24] evaluated the flexural strength and free vibration behaviour of FG-CNTR cylindrical shell panels based on the FSDT. Mehrabadi and Aragh [25] computed the static response of FG-CNTRC cylindrical shell panel using TSDT.

From the above review, we concluded that considerable amount of work has already been completed on the property evaluation of the CNT and the CNT reinforced composites. In addition to that the structural analysis (free vibration, bending and buckling) of the CNT reinforced composites are computed using the FSDT kinematics with and without temperature load. Based on the authors’ knowledge no study has been reported yet on the free vibration and bending behaviour of the CNTRC plate under thermal environment using the HSDT kinematic model. The objective of the present work is to compute the free vibration and the bending responses of the CNTRC flat panel numerically using the FSDT and HSDT mid-plane kinematics in conjunction with FEM. The domain is discretized using the suitable isoparametric finite element steps through a nine noded element with nine (model 1), six (model) or ten (model 3) degrees of freedom (DOF) per node. The responses are obtained numerically through a customized computer code developed in MATLAB environment. The validity and the convergence behaviour of the present numerical model have been checked, and the responses are also computed using the simulation model developed in ANSYS through ANSYS parametric design language (APDL) code. Finally, the effect of various design parameters (aspect ratios, support conditions, thickness ratios, volume fractions and temperature load) on the free vibration, static, stress and deformation behaviour of the CNTRC plate are highlighted by solving the wide variety of examples.

2. Theory and Formulation
In this analysis, the rectangular configuration of CNTRC plate is considered for the free vibration and the bending analysis as shown in fig. 1. The dimensions of CNTRC plate are the length \( a \), width \( b \) and the thickness \( h \). The CNTs are assumed to be uniformly distributed and aligned along the length of the plate. The displacement field kinematics within the plate is assumed to be based on the HSDT/FSDT, where the in-plane displacements are expanded as cubic/linear functions of the thickness coordinate while the transverse displacement varies either linearly and/or constant through the plate thickness. In this study, three different kinematic models have been utilized for the analysis purpose and discussed in the following lines.
Fig. 1:- Carbon nanotube reinforced composite plate geometry and configuration.

**Model 1**

As a first step, the mathematical model is developed using the HSDT mid-plane kinematics with nine DOF of the CNTRCs plate as in Kishore *et al.* [26]:

\[
\begin{align*}
u(x, y, z, t) &= u_0(x, y, t) + z\varphi_x(x, y, t) + z^2\psi_x(x, y, t) + z^3\theta_x(x, y, t) \\
v(x, y, z, t) &= v_0(x, y, t) + z\varphi_y(x, y, t) + z^2\psi_y(x, y, t) + z^3\theta_y(x, y, t) \\
w(x, y, z, t) &= w_0(x, y, t) + z\varphi_z(x, y, t)
\end{align*}
\]

(1)

**Model 2**

Similarly, one another mathematical model is developed using the FSDT mid-plane kinematics with six DOF of the CNTRCs plate as in Szekrenyes[27]:

\[
\begin{align*}
\begin{bmatrix} u(x, y, z, t) \\ v(x, y, z, t) \\ w(x, y, z, t) \end{bmatrix} &= \begin{bmatrix} u_0(x, y, t) \\ v_0(x, y, t) \\ w_0(x, y, t) \end{bmatrix} + z\varphi (x, y, t) \end{align*}
\]

(2)

**Model 3**

Further, the mathematical model is developed using another HSDT mid-plane kinematics with ten DOF of the CNTRCs plate as in Kishore *et al.[26]:

\[
\begin{align*}
\begin{bmatrix} u(x, y, z, t) \\ v(x, y, z, t) \\ w(x, y, z, t) \end{bmatrix} &= \begin{bmatrix} u_0(x, y, t) \\ v_0(x, y, t) \\ w_0(x, y, t) \end{bmatrix} + z\varphi (x, y, t) + z^2\psi (x, y, t) + z^3\theta (x, y, t) \\
\end{align*}
\]

(3)

where, \(u\), \(v\), and \(w\) are the displacement of any point within the plate along X, Y and Z directions, respectively. \(u_0\), \(v_0\), and \(w_0\) are midpoint displacement along X, Y, and Z direction respectively. \(\varphi\), \(\psi\), and \(\theta\) are the rotation of normal to the mid-plane about the Y and X direction respectively. \(\varphi\) and \(\theta\) are higher-order terms of Tayler series expansion in the mid plane of the plate.

Again, Eqs.(1), (2) and (3) are rewritten in the matrix form as follows:

\[
\{\lambda\} = [f_\alpha]\{\lambda_{mn}\}
\]

(4)

where, \(\{\lambda\}\), \([f_\alpha]\) and \(\{\lambda_{mn}\}\) are the displacement field vector of any point, thickness coordinate matrix and displacement field vector within the mid-plane of \(n^{th}\) model. The details of each term are provided in the Appendix.
The mid-plane kinematics of the CNTRC plate is evaluated using the following strain-displacement relation:

\[
\{ \varepsilon \} = \begin{bmatrix}
\frac{\partial u}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial w}{\partial z} \\
\frac{\partial u}{\partial y} & \frac{\partial v}{\partial x} & \frac{\partial w}{\partial y} \\
\frac{\partial u}{\partial z} & \frac{\partial v}{\partial z} & \frac{\partial w}{\partial x}
\end{bmatrix}
\begin{bmatrix}
\xi_x \\
\xi_y \\
\xi_z
\end{bmatrix} + \begin{bmatrix}
\gamma_{xz} \\
\gamma_{xy} \\
\gamma_{yz}
\end{bmatrix} + \begin{bmatrix}
\zeta_{xx} \\
\zeta_{xy} \\
\zeta_{yy}
\end{bmatrix} + \begin{bmatrix}
\eta_{xx} \\
\eta_{xy} \\
\eta_{yy}
\end{bmatrix} (5)
\]

or

\[
\{ \varepsilon \} = [T]\{ \bar{\varepsilon} \} (6)
\]

where,

\[
\{ \bar{\varepsilon} \} = \left[\begin{array}{cccccccccccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right] (7)
\]

The effective material properties of CNTRC plate can be computed using the extended rule of mixture as[18]:
\[ E_{11} = \eta_1 V_{CNT} E_{11}^{CNT} + V_m E^{m} \]  \hspace{1cm} (9)
\[ \eta_2 = \frac{V_{CNT}}{E_{22}^{CNT}} + \frac{V_m}{E^{m}} \]  \hspace{1cm} (10)
\[ \eta_3 = \frac{V_{CNT}}{G_{12}^{CNT}} + \frac{V_m}{G^{m}} \]  \hspace{1cm} (11)
and,
\[ V_{CNT} + V_m = 1 \]  \hspace{1cm} (12)
where, \( E_{11}, E_{22} \) and \( G_{12} \) are the effective Young’s modulus in the longitudinal and lateral direction and shear modulus of CNTRC plate respectively. Similarly, \( E_{11}^{CNT}, E_{22}^{CNT} \) and \( G_{12}^{CNT} \) are the properties of the CNT in their corresponding directions. In addition, \( E^{m} \) and \( G^{m} \) are the Young’s modulus and shear modulus of the matrix material respectively and \( \eta_1, \eta_2, \eta_3 \) are the effective parameter of CNT in the material directions.

The effective Poisson’s ratio \((v_{12})\) and the density of each constituent material \((\rho)\) are obtained using the formulas as [18]:
\[ v_{12} = V_{CNT} v^{CNT} + V_m v^{m} \]  \hspace{1cm} (13)
\[ \rho = V_{CNT} \rho^{CNT} + V_m \rho^{m} \]  \hspace{1cm} (14)
where, \( v^{CNT} \) and \( v^{m} \) are Poisson’s ratio of the SWCNT and the matrix material, respectively. Similarly, \( \rho^{CNT} \) and \( \rho^{m} \) are the densities of the CNT and the matrix, respectively.

2.1. Finite Element Method
FEM has been widely appreciated numerical tool for the structural analysis with various geometrical and material complexities. In this present analysis, the displacement fields for different assumed kinematic models are expressed in terms of desired field variables and the models are discretized using suitable FEM steps. The displacement vector \( \{ \lambda_0 \} \) at any point within the mid-surface is given by:
\[ \{ \lambda_0 \} = \sum_{i=1}^{9} N_i \{ \lambda_{0i} \} \]  \hspace{1cm} (15)
where, \( \{ \lambda_{0i} \} = [u_0 \ v_0 \ w_0 \ \varphi_x \ \psi_x \ \psi_y \ \theta_y \ \theta_y]^T \), \( \{ \lambda_{0i} \} = [u_0 \ v_0 \ w_0 \ \varphi_x \ \varphi_y \ \psi_x \ \psi_y \ \theta_y \ \theta_y]^T \) and \( \{ \lambda_{0i} \} = [u_0 \ v_0 \ w_0 \ \varphi_x \ \varphi_y \ \psi_x \ \psi_y \ \theta_y \ \theta_y]^T \) are the nodal displacement vector for model 1, model 2 and model 3 respectively and \( N_i \) is the corresponding interpolating function associated with node ‘i’.

The total strain energy of the CNTRC plate can be expressed as:
\[ U = \frac{1}{2} \iint \left[ \int_{\frac{v}{h}} \{ \varepsilon \}^T [D] \{ \varepsilon \} \right] \, dx \, dy \]  \hspace{1cm} (16)
Now, Eq. (16) can be rewritten by substituting the strain from Eqs. (6) and conceded as:
\[ U = \frac{1}{2} \iint \{ \varepsilon \}^T [D] \{ \varepsilon \} \, dx \, dy \]  \hspace{1cm} (17)
where, $[D] = \int_{-h/2}^{+h/2} [T]\{Q\}T \ dz$.

The total work done by externally applied load ($p$) and in-plane thermal load is given by

$$W = \iint \{\lambda_0\}^T \{p\} \ dxdy + \iint \{\epsilon_0\}^T \{Q\} \{\epsilon_{th}\} \ dxdy$$

$$W = \{\lambda_0\}^T \{F_m\} + \{\lambda_0\}^T \{F_{th}\}$$  \hspace{1cm} (18)

where, $\{F_m\}$ and $\{F_{th}\}$ are the mechanical and thermal load vectors, respectively.

The kinetic energy of the CNTRC plate can be expressed as:

$$T = \frac{1}{2} \int V \rho \{\dot{x}\}^T \{\dot{x}\} \ dV$$ \hspace{1cm} (19)

where, $\rho$ and $\{x\}$ are the mass density and the global velocity vector.

Using, Eqs. (3) and (19), the kinetic energy expression of the CNTRC plate with thickness $h$ can be written as:

$$T = \frac{1}{2} \int_A \int_{-h/2}^{+h/2} \{\dot{x}\}^T \{f\} \{\dot{x}\} \ dA = \frac{1}{2} \int_A \{\dot{x}\}^T [m] \{\dot{x}\} \ dA$$ \hspace{1cm} (20)

where, $[m] = \int_{-h/2}^{+h/2} [f]^T \rho \{f\} \ dz$ is the inertia matrix.

### 2.2. ANSYS model and solution steps

As discussed earlier, a simulation model has been developed in ANSYS using APDL code to compute the desired bending and the free vibration responses of the CNTRC plate under thermal environment. The present model is discretized using eight node serendipity element (SHELL281) with six degrees of freedom at each node from the ANSYS element library. The SHELL281 element is capable of solving the thick/thin panel under combined thermo-mechanical load. The solution procedure used in the ANSYS are as follows:

(i) The first step is to create the desired geometry of the plate with sides $a$ and $b$.

(ii) Assign the material properties.

(iii) Discretize the plate model using SHELL281 element, from ANSYS element library, to obtain the required mesh.

(iv) Then, apply the boundary condition and the load to get the required responses.

(v) The linear eigenvalue vibration problem is solved using Block Lanczos method.

### 2.3. Bending analysis

The final form of the governing equation of the CNTRC plate is obtained using variational principle and conceded as:

$$\delta \Pi = \delta U - \delta W = 0$$ \hspace{1cm} (21)

where, $\delta$ is the variational symbol and $\Pi$ is the total potential energy.
Now, the desired governing equation for the static analysis of the CNT RC plate is obtained by substituting the values of the strain energy and total work done from the Eqs. (17) and (18) in the Eq. (21) and rewritten as:

\[
\begin{bmatrix} K_S \end{bmatrix} \{ \lambda_0 \} = \{ p \}
\]

(22)

where, \( \begin{bmatrix} K_S \end{bmatrix} \) is the global stiffness matrix.

### 2.4. Free vibration analysis

The governing equation of free vibrated CNTRC plate is derived using Hamilton’s principle and expressed as:

\[
\delta J \left( T - U \right) dt = 0
\]

(23)

The equilibrium equation for vibrated CNTRC plate can be obtained by substituting the values of the kinetic energy \( T \) and the strain energy \( U \) in Eq. (23) and modified as follows:

\[
\begin{bmatrix} M \end{bmatrix} \{ \ddot{\lambda} \} + \begin{bmatrix} K \end{bmatrix} \{ \lambda \} = 0
\]

(24)

where, \( \begin{bmatrix} M \end{bmatrix} = \begin{bmatrix} N \end{bmatrix}^T \begin{bmatrix} m \end{bmatrix} \begin{bmatrix} N \end{bmatrix} \) is the system mass matrix and \( \begin{bmatrix} K \end{bmatrix} = \begin{bmatrix} B \end{bmatrix}^T \begin{bmatrix} D \end{bmatrix} \begin{bmatrix} B \end{bmatrix} \) is the system stiffness matrix.

Again, Eq. (24) is rewritten in the eigenvalue equation form and represented as:

\[
\left( \begin{bmatrix} K \end{bmatrix} - \omega^2 \begin{bmatrix} M \end{bmatrix} \right) \Delta = 0
\]

(25)

where, \( \omega \) is the natural frequency and \( \Delta \) is the corresponding eigenvector.

### Table 1. Material properties of (10, 10) SWCNT \((L= 9.26 \text{ nm}, R=0.68 \text{ nm}, h=0.067 \text{ nm}, v_{12}^{CNT} = 0.175)\). [28]

| Temperature (K) | \( E_{11}^{CNT} \) (TPa) | \( E_{22}^{CNT} \) (TPa) | \( G_{12}^{CNT} \) (TPa) | \( \alpha_{11}^{CNT} \) \(10^6\)/K | \( \alpha_{22}^{CNT} \) \(10^6\)/K |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 300             | 5.6466          | 7.0800          | 1.9445          | 3.4584          | 5.1682          |
| 400             | 5.5679          | 6.9814          | 1.9703          | 4.1496          | 5.0905          |
| 500             | 5.5308          | 6.9348          | 1.9643          | 4.5361          | 5.0189          |
| 700             | 5.4744          | 6.8641          | 1.9644          | 4.6677          | 4.8943          |

### Table 2. Effective parameter of CNT. [28]

| \( V_{CNT} \) | \( \eta_1 \) | \( \eta_2 \) | \( \eta_3 \) |
|----------------|----------------|----------------|----------------|
| 0.12           | 0.137          | 1.022          | 0.7185         |
| 0.17           | 0.142          | 1.626          | 1.138          |
| 0.28           | 0.141          | 1.585          | 1.109          |

### 3. Results and Discussions

The free vibration and bending behaviour of the CNTRC plate are investigated using suitable finite element model developed in MATLAB environment. For the present analysis, PMMA is considered as the matrix phase and the SWCNT as the fibre phase. The material properties of both matrix and fibre are assumed to be temperature-dependent. The properties of PMMA are taken same as [28]\( \rho = 1150 \text{kg/m}^3, v^m = 0.34, E^m = (3.52-0.0034T) \text{ GPa. and } \alpha^m = 45(1+0.0005 \Delta T) \times 10^{-6} \text{ in which } \Delta T = T – \)
and \( T_0 = 300\text{K} \) (ambient temperature). Similarly, the properties of the armchair (10, 10) configuration of the SWCNTs is taken same as the reference [28] and presented in Table 1. The effectiveness parameters of CNT is also provided in Table 2 and the effective material properties of the composite plate are obtained using the extended rule of mixture. It is true that the extended rule of mixture is one of the appropriate methods for the analysis of the short fibre reinforced composite structure. Table 3 shows comparison study of the elastic properties computed using the extended rule of mixture and compared with MD simulation results. It is clear from the table that the present results are showing very good agreement and MD simulation is well accepted in materials related analysis due to its accuracy. In the present analysis, the thickness of the composite plate is taken as 2mm throughout if not stated otherwise.

| \( V_{CNT} \) | MD [2] | Extended rule of mixture |
|-------------|--------|-------------------------|
|             | \( E_{11} \) (GPa) | \( E_{22} \) (GPa) | \( E_{11} \) (GPa) | \( E_{22} \) (GPa) |
| 0.12        | 94.6   | 2.9                     | 95.021              | 2.8917              |
| 0.17        | 138.9  | 4.9                     | 138.38              | 4.8776              |
| 0.28        | 224.2  | 5.5                     | 224.72              | 5.4807              |

The following sets of support conditions are employed to compute the desired responses:

(a) All edges simply supported condition (SSSS):
\[ v = w = \phi_{y} = \psi_{y} = \theta_{x} = 0 \text{ at } x = 0, a \]
\[ u = v = \phi_{x} = \psi_{x} = \theta_{y} = 0 \text{ at } y = 0, b \]

(b) All edges clamped condition (CCCC):
\[ u = v = w = \phi_{x} = \phi_{y} = \psi_{x} = \psi_{y} = \theta_{x} = \theta_{y} = 0 \text{ for both end i.e., } x = 0, a \text{ and } y = 0, b. \]

(c) Combine simply support and clamped condition (SCSC)
\[ v = w = \phi_{y} = \psi_{y} = \theta_{x} = 0 \text{ at } x = 0, a \text{ and } y = 0, b. \]
\[ u = v = w = \phi_{x} = \phi_{y} = \psi_{x} = \psi_{y} = \theta_{x} = \theta_{y} = 0 \text{ at } y = 0, b. \]
Fig. 2:- Convergence behavior of the nondimensional central deflection: (a) Model 1 (b) Model 2 and (c) Model 3.

In this present analysis, the nondimensional form of the central deflection and the frequencies are obtained using the formulae: 
\[ \bar{W} = \frac{W_{\text{max}}}{h} \ \text{and} \ \bar{\omega} = \omega \left( \frac{a^2}{h} \right) \sqrt{\frac{\rho}{E_0}} \] respectively, where \( E_0 \), \( \rho_0 \) and \( W_{\text{max}} \) are the Young’s modulus and the density of the matrix material at ambient temperature (300K) and the maximum central deflection of the plate, respectively.

3.1. Convergence and validation study

In this section, the convergence behaviour of the present numerical models of the CNTRC plate is computed for different support conditions and volume fractions. The convergence behaviour of the nondimensional central deflection parameter and the nondimensional fundamental frequencies of the CNTRC plate is presented in fig. 2(a)-(c) and fig. 3(a)-(c), respectively. In this example, the responses are computed for two support conditions (SSSS and CCCC) and three volume fraction (\( V_{\text{CNT}} = 0.12, 0.17 \) and 0.28) at room temperature (300K). The nondimensional central deflections are obtained under point load of 10KN at the center of the plate. It is clearly observed from the figures that the present results are converging well with mesh refinement for all the volume fractions and the support conditions. Therefore, a (6x6) mesh is utilized to compute the further analysis.

Now the proposed finite element model is extended to show the validation behaviour by comparing the responses with the available published literature and the simulation model. The nondimensional bending and vibration responses of the clamped CNTRC plate is computed and presented in Table 4 and Table 5, respectively. From results, it is clear that the presently developed models are good agreement with the previously published results. For the comparison purpose all the geometrical and material properties are taken same as to the Zhu et al.[18].
(c) Model 3

Fig. 3:- Convergence behavior of the nondimensional fundamental frequency: (a) Model 1 (b) Model 2 and (c) Model 3.

| Table 4. Comparison study of nondimensional central deflection of CNTRC plate ($T = 300K$ and uniformly distributed load = 0.1MPa). |
|---|---|---|---|---|---|---|
| $a/h$ | $V_{CNT}$ | Model 1 | Model 2 | Model 3 | ANSYS | Zhu et al. [18] |
| 10 | 0.11 | 0.0021 | 0.0021 | 0.0022 | 0.0022 |
| 10 | 0.14 | 0.00020 | 0.0020 | 0.00210 | 0.0021 |
| 17 | 0.17 | 0.00014 | 0.0011 | 0.000199 | 0.0014 |

| Table 5. Comparison study of nondimensional fundamental frequencies of square CNTRC plate ($T = 300K$). |
|---|---|---|---|---|---|---|
| $a/h$ | $V_{CNT}$ | Model 1 | Model 1 | Model 2 | ANSYS | Zhu et al. [18] |
| 10 | 0.11 | 18.0904 | 18.5970 | 16.5539 | 17.6385 | 17.625 |
| 10 | 0.14 | 18.6661 | 19.2038 | 17.0819 | 18.1493 | 18.127 |
| 17 | 0.17 | 22.5748 | 23.2008 | 20.6588 | 22.0240 | 22.011 |

| Table 6. Effect of volume fraction on the nondimensional central deflection. |
|---|---|---|---|---|---|---|
| Support conditions | Load (KN) | $V_{CNT}$= 0.12 | $V_{CNT}$= 0.17 | $V_{CNT}$= 0.28 |
| | | Model 1 | Model 2 | Model 3 | Model 1 | Model 2 | Model 3 | Model 1 | Model 2 | Model 3 |
| SSSS | 2 | 8.511 | 8.430 | 8.511 | 5.598 | 5.550 | 5.598 | 3.859 | 3.482 | 3.859 |
| 4 | 17.021 | 16.859 | 17.021 | 11.195 | 11.101 | 11.195 | 7.17 | 6.963 | 7.171 |
| 6 | 25.532 | 25.289 | 25.532 | 16.793 | 16.651 | 16.793 | 11.576 | 10.445 | 11.576 |
| 8 | 34.042 | 33.719 | 34.042 | 22.391 | 22.202 | 22.391 | 15.435 | 13.927 | 15.435 |
| 10 | 42.553 | 42.148 | 42.553 | 27.989 | 27.752 | 27.989 | 19.293 | 17.409 | 19.293 |
| CCCC | 2 | 3.568 | 3.482 | 3.568 | 2.308 | 2.258 | 2.308 | 1.661 | 1.614 | 1.662 |
| 4 | 7.137 | 6.963 | 7.137 | 4.616 | 4.515 | 4.616 | 3.323 | 3.229 | 3.323 |
| 6 | 10.705 | 10.445 | 10.705 | 6.924 | 6.773 | 6.924 | 4.984 | 4.843 | 4.985 |
| 8 | 14.274 | 13.927 | 14.274 | 9.232 | 9.031 | 9.232 | 6.646 | 6.457 | 6.646 |
| 10 | 17.842 | 17.409 | 17.842 | 11.540 | 11.289 | 11.540 | 8.307 | 8.072 | 8.308 |
| SCSC | 2 | 8.394 | 8.317 | 8.394 | 5.481 | 5.436 | 5.481 | 3.835 | 3.793 | 3.835 |
| 4 | 16.789 | 16.633 | 16.789 | 10.963 | 10.873 | 10.963 | 7.670 | 7.585 | 7.670 |
| 6 | 25.183 | 24.950 | 25.183 | 16.444 | 16.309 | 16.444 | 11.505 | 11.378 | 11.505 |
| 8 | 33.578 | 33.266 | 33.578 | 21.926 | 21.745 | 21.926 | 15.341 | 15.170 | 15.341 |
| 10 | 41.972 | 41.583 | 41.972 | 27.407 | 27.182 | 27.407 | 19.176 | 18.963 | 19.176 |

3.2. Parametric Study

Based on the convergence and comparison study, the developed higher-order CNTRC plate model is extended to compute the responses for the different geometrical and material parameters. The effect of various parameters on the flexural and free vibration behaviours discussed in detailed. In general the responses are computed for $a/h = 50$, $V_{CNT} = 0.12$ and $a/b = 1$ at 300K throughout the analysis if not
stated otherwise. For the computational purpose, the thickness of the composite panel is taken as $h = 0.002 \text{ m}$ and discussed in detail.

### 3.2.1. Flexural behaviour

In this section, flexural behaviour of CNTRC panel is investigated for different geometrical and material parameters. The effect of the CNT volume fractions on the bending responses of the square CNTRC plate is examined for three volume fractions ($V_{\text{CNT}} = 0.12, 0.17$ and $0.28$), five point loads ($p = 2, 4, 6, 8, \text{ and } 10 \text{KN}$) at the center with three support conditions (SSSS, CCCC and SCSC) and presented in Table 6. It is clearly observed from the figures that the nondimensional central deflections are increasing with externally applied load and decreasing with the volume fractions of the CNT.

The thickness ratio of any structural component plays an important role on the stiffness behavior of the structure. The effect of five thickness ratios ($a/h = 10, 20, 30, 40 \text{ and } 50$) on the nondimensional central deflections of the square CNTRC plate are examined for two volume fraction ($V_{\text{CNT}} = 0.12$ and $0.17$) and three support conditions (SSSS, CCCC and SCSC) with point load ($p = 10 \text{KN}$) at the center point of the composite plate and presented in Table 7. It can be seen that the nondimensional central deflections are increasing with thickness ratios of CNTRC plate.

| $a/h$ | Model 1     | Model 2     | Model 3     | Model 1     | Model 2     | Model 3     |
|-------|-------------|-------------|-------------|-------------|-------------|-------------|
| 10    | 4.9587      | 4.6417      | 4.9587      | 3.0398      | 2.8504      | 3.0398      |
| 20    | 10.0051     | 9.6305      | 10.0051     | 6.3644      | 6.1432      | 6.3644      |
| SSSS  | 17.9343     | 17.5405     | 17.9344     | 11.6228     | 11.3918     | 11.6229     |
| 40    | 28.7761     | 28.3745     | 28.7761     | 18.8273     | 18.5925     | 18.8273     |
| 50    | 42.5531     | 42.1482     | 42.5531     | 27.9887     | 27.7522     | 27.9887     |
| 10    | 4.0394      | 3.7729      | 4.0394      | 2.4268      | 2.2644      | 2.4269      |
| 20    | 6.2431      | 5.8627      | 6.2431      | 3.8452      | 3.6192      | 3.8453      |
| CCCC  | 9.2030      | 8.7840      | 9.2030      | 5.7963      | 5.5505      | 5.7963      |
| 40    | 13.0553     | 12.6239     | 13.0553     | 8.3543      | 8.1031      | 8.3543      |
| 50    | 17.8422     | 17.4087     | 17.8423     | 11.5402     | 11.2887     | 11.5403     |
| 10    | 4.7813      | 4.4959      | 4.7814      | 2.9159      | 2.7464      | 2.9160      |
| 20    | 9.8152      | 9.4615      | 9.8152      | 6.2033      | 5.9967      | 6.2033      |
| SCSC  | 17.6576     | 17.2812     | 17.6577     | 11.3652     | 11.1467     | 11.3652     |
| 40    | 28.3673     | 27.9817     | 28.3674     | 18.4284     | 18.2053     | 18.4284     |
| 50    | 41.9723     | 41.5829     | 41.9724     | 27.4070     | 27.1818     | 27.4071     |

The CNT is well known for its good thermal property, and it becomes more significant when the structure is exposed to the elevated temperature field. In this present example, the CNT properties are assumed to be temperature-dependent, and, therefore, environmental temperature plays a significant role on the flexural strength of the CNTRC plate. The nondimensional central deflections of the square CNTRC plate are analyzed for three different temperature loading ($T = 300K, 500K \text{ and } 700K$), three support conditions (SSSS, CCCC and SCSC) by varying the value of point load ($p = 10 \text{KN}$) at the center point of the composite plate and presented in Table 7. It is observed that the nondimensional central deflections of the CNTRC plate are increasing as the temperature load increase.

The deformation behaviour of the CNTRC plate is computed for model 1 and model 2 with three support condition (SSSS, CCCC and SCSC) and presented in fig. 4 (a)-(f). The responses are computed by setting $a/h = 50$, $V_{\text{CNT}} = 0.12$ and $p = 10 \text{KN}$ at room temperature ($T = 300K$). It is clearly observed that the deformation of the plate is maximum and minimum for the SSSS and the CCCC support conditions, respectively and the responses are within the expected line.
Table 8. Effect of temperature on the nondimensional central deflection.

| Support Conditions | Load (KN) | Temperature (K) | |
|--------------------|-----------|-----------------|-----|
|                    | 300       | 500             | 700 |
|                    | Model 1   | Model 2         | Model 3 |
|                    | Model 1   | Model 2         | Model 3 |
|                    | Model 1   | Model 2         | Model 3 |
|                    | Model 1   | Model 2         | Model 3 |
| SSSS               | 2         | 8.5106          | 8.4296          | 8.5106          | 9.0049          | 9.0049          | 9.5673          | 9.5673          |
|                    | 4         | 17.0212         | 16.8593         | 17.0212         | 18.0097         | 17.8197         | 18.0097         | 19.1346         | 19.1346         |
|                    | 6         | 25.5318         | 25.2889         | 25.5319         | 27.0146         | 26.7296         | 27.0146         | 28.7018         | 28.3579         | 28.7019         |
|                    | 8         | 34.0425         | 33.7185         | 34.0425         | 36.0194         | 35.6395         | 36.0195         | 38.2691         | 37.8106         | 38.2691         |
|                    | 10        | 42.5531         | 42.1482         | 42.5531         | 45.0243         | 44.5493         | 45.0243         | 47.8364         | 47.2632         | 47.8364         |
| CCCC               | 2         | 3.5685          | 3.4817          | 3.5685          | 3.7331          | 3.8354          | 3.8354          | 4.1621          | 4.0381          | 4.1621          |
|                    | 4         | 7.1369          | 6.9635          | 7.1369          | 7.4663          | 7.6708          | 7.6708          | 8.3243          | 8.0761          | 8.3243          |
|                    | 6         | 10.7054         | 10.4452         | 10.7054         | 11.5062         | 11.1994         | 11.5062         | 12.4864         | 12.1142         | 12.4864         |
|                    | 8         | 14.2738         | 13.9270         | 14.2738         | 15.3416         | 14.9326         | 15.3416         | 16.6486         | 16.1522         | 16.6486         |
|                    | 10        | 17.8423         | 17.4087         | 17.8423         | 18.6657         | 19.1770         | 18.6657         | 20.8107         | 20.1903         | 20.8107         |
| SCSC               | 2         | 8.3945          | 8.3166          | 8.3945          | 9.2555          | 9.8334          | 9.2555          | 9.5240          | 9.4118          | 9.5240          |
|                    | 4         | 16.7889         | 16.6332         | 16.7889         | 17.8510         | 17.6668         | 17.8510         | 19.0480         | 18.8236         | 19.0480         |
|                    | 6         | 25.1834         | 24.9497         | 25.1834         | 26.7765         | 26.5001         | 26.7765         | 28.5720         | 28.2354         | 28.5720         |
|                    | 8         | 33.5779         | 33.2663         | 33.5779         | 35.7020         | 35.3335         | 35.7020         | 38.0960         | 37.6472         | 38.0960         |
|                    | 10        | 41.9723         | 41.5829         | 41.9724         | 44.6275         | 44.1669         | 44.6275         | 47.6200         | 47.0590         | 47.6200         |

Fig. 4:- Deformed configuration of CNTRC panel: (a) Model 1, SSSS, (b) Model 2, SSSS, (c) Model 1, CCCC and (d) Model 2, CCCC.
3.2.2. Vibration and modal analysis

In this section, the effect of different geometrical and material parameters on the free vibration responses are examined using the developed three micromechanical finite element model of the CNTRC plate. In general, the responses are examined for $T = 300K$, $V_{CNT} = 0.12$ and $a/h = 50$ throughout the analysis if not stated otherwise.

It is well known that the volume fraction of the CNT plays a significant role in stiffness response of the CNTRC plate. The effect of volume fractions ($V_{CNT} = 0.12$, $0.17$ and $0.28$) of the CNT on the nondimensional fundamental frequencies of the square CNTRC plate is analyzed using $a/h = 50$ under two temperature ($T = 300K$ and $400K$) load. The nondimensional fundamental frequency of the CNTRC plate is computed and presented in Table 9 for three support conditions (SSSS, CCCC and SCSC). It is clearly observed that the nondimensional fundamental frequencies are increasing with the volume fractions of the CNT and decreasing with the temperature.

| $V_{CNT}$ | $T = 300K$ | $T = 400K$ |
|-----------|------------|------------|
|           | Model 1 | Model 2 | Model 3 | Model 1 | Model 2 | Model 3 |
| SSSS      | 0.12    | 17.6564 | 17.7262 | 17.6563 | 17.3654 | 17.4427 | 17.3651 |
|           | 0.17    | 21.3910 | 21.4649 | 21.3910 | 21.0425 | 21.1245 | 21.0425 |
|           | 0.28    | 26.3164 | 26.4439 | 26.3163 | 25.9105 | 26.0522 | 25.9104 |
| CCCC      | 0.12    | 36.4265 | 36.9554 | 36.4264 | 35.5105 | 36.0831 | 35.5101 |
|           | 0.17    | 44.5219 | 45.0966 | 44.5245 | 43.4643 | 44.0877 | 43.4651 |
|           | 0.28    | 53.4669 | 54.3944 | 53.4666 | 52.0619 | 53.0620 | 52.0617 |
| SCSC      | 0.12    | 18.9113 | 18.9813 | 18.9125 | 18.4697 | 18.5471 | 18.4698 |
|           | 0.17    | 23.1087 | 23.1871 | 23.1118 | 22.5583 | 22.6404 | 22.5601 |
|           | 0.28    | 27.8613 | 27.9886 | 27.8616 | 27.2702 | 27.4090 | 27.2683 |

In this present analysis, the material properties of the CNT and the matrix are assumed to be temperature-dependent, therefore, the temperature field effect on the vibration behaviour of the CNTRC plate is investigated. Fig 5(a) and (b) shows the nondimensional fundamental frequency of the square CNTRC plate under four different temperature loads ($T = 300K$, $400K$, $500K$ and $700K$), two support conditions (SSSS and CCCC), and three volume fractions of CNT ($V_{CNT}$). It is observed that the nondimensional fundamental frequency responses of the CNTRC plate decrease as the temperature load increases for all type support conditions.

Fig. 5: Effect of temperature on the nondimensional fundamental frequency: (a) SSSS and (b) CCCC.
In this example, the effect of thickness ratios on the nondimensional fundamental frequency of CNTRC square plate is examined. The responses are computed for five thickness ratios ($a/h = 10, 20, 30, 40$ and $50$) and two support conditions (SSSS and CCCC) under two uniform thermal loads ($T = 300K$ and $700K$) with $V_{CNT} = 0.12$ and plotted in fig.6 (a) and (b). It is observed that the nondimensional fundamental frequency is increased with the volume fractions of the CNT. Moreover, the nondimensional fundamental frequency increase sharply with increase the thickness ratio but the rate of increase reduces for the higher value of thickness ratio.

Fig. 6: Effect of thickness ratio ($a/h$) on the nondimensional fundamental frequency: (a) SSSS and (b) CCCC.
Fig. 7:- Free vibration mode shapes of a simply supported uniformly distributed CNTRC square plate (Model-1): (a) 1st Mode (b) 2nd Mode (c) 3rd Mode (d) 4th Mode.

Fig. 7(a)-(d) represent the four mode shapes of the square clamped CNTRC plate (Model 1) at ambient temperature \( T = 300K \) by setting the geometrical parameters as \( a/h = 50 \) and \( V_{\text{CNT}} = 0.12 \). It is well known that the mode shapes are not giving any numerical value rather it provides the direction of vibration and the present responses are following the expected line.

4. Conclusions

In the present article, the free vibration and bending responses of the SWCNT reinforced composite plate are computed using the FSDT and HSDT mid-plane kinematics in conjunction with the FEM under uniform thermal environment. To achieve the realistic behaviour, the material properties of both (CNT and matrix) are assumed to be temperature dependent. The CNTs are uniformly distributed with respect to the dimension of the plate, and the effective material properties of the composite are computed using the extended rule of mixture. The desired governing equation of bending and free vibration is obtained using the suitable FEM steps. The responses are computed numerically using the suitable FEM steps with the help of ninenodedisoparametric elements with nine (model 1), six (model 2) and ten (model 3) DOFseachnode. The present models are showing good convergence rate with mesh refinement. The comparison study also indicates the validity of the present first and higher order models. The models are also validated by comparing the responses of the simulation model developed in ANSYS using APDL code. The effect of the different geometrical and material parameters on the free vibration and bending under thermal environment are computed. The following conclusions are drawn from the detailed parametric study.

(i) The convergence and comparison study shows the accuracy of the present FSDT and HSDT models with and without temperature load.

(ii) The nondimensional central deflections are increasing with the rise of the mechanical load, temperature load and the thickness ratio. However, the deflections are showing a reverse trend for the volume fraction.

(iii) The nondimensional fundamental frequency is increasing with the increase in the volume fraction and the thickness ratios. However, the nondimensional frequency is showing a reverse trend for the temperature load.

(iv) The results indicated that the FSDT model (Model 2) is the stiffest configuration as compared to the HSDT models (Model 1 and Model 3).

Appendix

\[ \{\vec{\lambda}\} = [u \ v \ w]^T, \{\vec{\lambda}_{01}\} = [u_0 \ v_0 \ w_0 \ \varphi \ \psi \ \psi_0 \ \Theta_y \ \Theta_z]^T, \{\vec{\lambda}_{02}\} = [u_0 \ v_0 \ w_0 \ \varphi \ \varphi \ \psi \ \psi_0 \ \Theta_y \ \Theta_z]^T \]

\[ \{f_1\} = \begin{bmatrix} 1 & 0 & 0 & z & 0 & z^2 & 0 & z^3 & 0 \\ 0 & 1 & 0 & 0 & z & 0 & z^2 & 0 & z^3 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \{f_2\} = \begin{bmatrix} 1 & 0 & 0 & z & 0 & 0 \\ 0 & 1 & 0 & 0 & z & 0 \\ 0 & 0 & 1 & 0 & 0 & z \end{bmatrix} \]

\[ \{f_3\} = \begin{bmatrix} 1 & 0 & 0 & z & 0 & z^2 & 0 & z^3 & 0 \\ 0 & 1 & 0 & 0 & z & 0 & z^2 & 0 & z^3 \\ 0 & 0 & 1 & 0 & 0 & z & 0 & 0 & 0 \end{bmatrix} \]
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