Abstract—This article provides a unifying Bayesian network view on various approaches for acoustic model adaptation, missing feature, and uncertainty decoding that are well-known in the literature of robust automatic speech recognition. The representatives of these classes can often be deduced from a Bayesian network that extends the conventional hidden Markov models used in speech recognition. These extensions, in turn, can in many cases be motivated from an underlying observation model that relates clean and distorted feature vectors. By converting the observation models into a Bayesian network representation, we formulate the corresponding compensation rules leading to a unified view on known derivations as well as to new formulations for certain approaches. The generic Bayesian perspective provided in this contribution thus highlights structural differences and similarities between the analyzed approaches.

Index Terms—robust automatic speech recognition, Bayesian network, model adaptation, missing feature, uncertainty decoding

I. INTRODUCTION

Robust Automatic Speech Recognition (ASR) still represents a challenging research topic. The main obstacle, namely the mismatch of test and training data, can be tackled by enhancing the observed speech signals or features in order to meet the training conditions or by compensating for the distorted test conditions in the acoustic model of the ASR system.

Methods that modify the acoustic model are in general termed (acoustic) model-based or model compensation approaches and comprise inter alia the following sub-categories: So-called model adaptation techniques mostly update the parameters of the acoustic model, i.e., of the Hidden Markov Models (HMMs), prior to the decoding of a set of observed feature vectors. In contrast, decoder-based approaches re-adapt the HMM parameters for each observed feature vector. The most common decoder-based approaches are missing feature and uncertainty decoding that incorporate additional time-varying uncertainty information into the evaluation of the HMMs’ probability density functions (pdfs).

Various model compensation techniques exhibit two (more or less) distinct steps that are taken interchangeably: First, the compensation parameters need to be estimated and, second, the actual compensation rule is applied to the acoustic model. The compensation rules can often be motivated based on an observation model that relates the clean and distorted feature vectors, e.g., in the logarithmic melspectral (logmelspec) or the Mel Frequency Cepstral Coefficient (MFCC) domain.

In this article, we show how the compensation rules can be deduced from the Bayesian network representations of the observation models for several uncertainty decoding [1]–[5], missing feature [6]–[9], and model adaptation techniques [10]–[19]. In addition, we give a Bayesian network description of the generic uncertainty decoding approach of [20], of the Maximum A Posteriori (MAP) adaptation technique [21], and of some alternative HMM topologies [22], [23]. While Bayesian networks have been sporadically employed in this context before [4], [9], [20], with this article, we give new formulations for certain of the considered algorithms in order to fill some gaps for a unified description.

Throughout the following, feature vectors are denoted by bold-face letters \( \mathbf{v} \), with index \( n \in \{1, \ldots, N\} \). Feature vector sequences are written as \( \mathbf{v}_{1:N} = (\mathbf{v}_1, \ldots, \mathbf{v}_N) \). Without distinguishing a random variable from its realization, a pdf over a random variable \( z_n \) is denoted by \( p(z_n) \). For a normally distributed real-valued random vector \( z_n \) with mean vector \( \mu_{zn} \) and covariance matrix \( C_{zn} \), we write \( z_n \sim \mathcal{N}(\mu_{zn}, C_{zn}) \) or

\[
p(z_n) = \mathcal{N}(z_n; \mu_{zn}, C_{zn}).
\]

To express that all random vectors of the set \( \{z_1, \ldots, z_N\} \) share the same statistics, we write \( p(z_n) = \text{const.} \), or, in the Gaussian case,

\[
p(z_n) = \mathcal{N}(z_n; \mu_z, C_z)
\]

with time-invariant mean vector \( \mu_z \) and covariance matrix \( C_z \). Finally, for a Gaussian random vector \( z_n \) conditioned on another random vector \( w_m \), we write

\[
p(z_n|w_m) = \mathcal{N}(z_n; \mu_{z|w_m}, C_{z|w_m}),
\]

if the statistics of \( z_n \) depend only on time through \( w_m \), i.e., if \( \mu_{z|w_m} = \mu_{z|m} \) and \( C_{z|w_m} = C_{z|m} \), for \( w_m = w_n \) and \( n, m \in \{1, \ldots, N\} \).

The remainder of the article is organized as follows: After summarizing the employed Bayesian network view in Section II and its difference to existing overview articles in...
Section III, this perspective is applied to uncertainty decoding, missing feature techniques, and other model-based approaches in Section IV. Finally, conclusions are drawn in Section V.

II. A BAYESIAN NETWORK VIEW

We start by reviewing the Bayesian network perspective on acoustic model-based techniques that we use in Section IV to compare different algorithms.

Given a sequence of observed feature vectors $y_{1:N}$, the acoustic score $p(y_{1:N} | W)$ of a sequence $W$ of conventional HMMs, as depicted in Figure 1(a), is given by [24]

$$p(y_{1:N} | W) = \sum_{q_{1:N}} p(y_{1:N}, q_{1:N})$$

$$= \sum_{q_{1:N}} \left\{ \prod_{n=1}^{N} p(y_n | q_n) p(q_n | q_{n-1}) \right\} ,$$

where $p(q_1 | q_0) = p(q_1)$. The summation goes over all possible state sequences $q_{1:N}$ through $W$ superseding the explicit dependency on $W$ at the right-hand side of (4) and (5). Note that the pdf $p(y_n | q_n)$ can be scaled by $p(y_n)$ without influencing the discrimination capability of the acoustic score w.r.t. changing word sequences $W$. We thus define $\tilde{p}(y_n | q_n) = p(y_n | q_n) / p(y_n)$ for later use.

The compensation rules of a wide range of model adaptation, missing feature and uncertainty decoding approaches can be expressed by modifying the Bayesian network structure of a conventional HMM and applying the inference rules of Bayesian networks [25] – potentially followed by suitable approximations to ensure mathematical tractability. While some approaches postulate a certain Bayesian network structure, others indirectly define a modified Bayesian network by assuming an observed feature vector $y_n$ to be a distorted version of an underlying clean feature vector $x_n$, which is introduced as latent variable in the HMM as, e.g., in Figure 1(b). In the latter case, the relation of $y_n$ and $x_n$ can be expressed by an analytical observation model $f(\cdot)$ that incorporates certain compensation parameters $b_n$:

$$y_n = f(x_n, b_n).$$

Note that here it is not distinguished whether $y_n$ is the output of a front-end enhancement process or a noisy or reverberant observation that is directly fed into the recognizer. By converting the observation model to a Bayesian network representation, the pdf $p(y_n | q_n)$ in (5) can be derived exploiting the inference rules of Bayesian networks [25]. For the case of Figure 1(b), the observation likelihood in (5) would, e.g., become:

$$p(y_n | q_n) = \int p(x_n, y_n | q_n) dx_n$$

$$= \int p(x_n | q_n) p(y_n | x_n) dx_n,$$

where the actual functional form of $p(y_n | x_n)$ depends on the assumptions on $f(\cdot)$ and the statistics $p(b_n)$ of $b_n$.

The abstract perspective taken in this paper reveals a fundamental difference between model adaptation approaches on the one hand and missing feature and uncertainty decoding approaches on the other hand: Model adaptation techniques usually assume $b_n$ to have constant statistics over time [4], [26], i.e.,

$$p(b_n) = \text{const.},$$

for $n \in \{1, ..., N\}$.

or to be a deterministic parameter vector of value $b$, i.e.,

$$p(b_n) = \delta(b_n - b),$$

where $\delta(\cdot)$ denotes the Dirac distribution. In contrast, missing feature and uncertainty decoding approaches typically assume $p(b_n)$ to be a time-varying pdf [4], [26].

As exemplified in Section IV, the Bayesian network view conveniently illustrates the underlying statistical dependencies of model-based approaches. If two approaches share the same Bayesian network, their underlying joint pdfs over all involved random variables share the same decomposition properties. However, some crucial aspects are not reflected by a Bayesian network: The particular functional form of the joint pdf, potential approximations to arrive at a tractable algorithm, as well as the estimation procedure for the compensation parameters. While some approaches estimate these parameters through an acoustic front-end, others derive them from clean or distorted data. For clarity, we entirely focus in this article on the compensation rules while ignoring the parameter estimation step. We also neglect approaches that apply a modified training method to conventional HMMs without exhibiting a distinct compensation step, as it is characteristic for, e.g., the case for discriminative [27], multi-condition [28] or reverberant training [29].

III. MERIT OF THE BAYESIAN NETWORK VIEW

In the past decades, a range of survey papers and books have been published summarizing the state-of-the-art in noise and reverberation-robust ASR [26], [30]–[35]. Recently, a comprehensive review of noise-robust ASR techniques was published in [36] providing a taxonomy-oriented framework by distinguishing whether, e.g., prior knowledge, uncertainty processing or an explicit distortion model is used or not. In contrast to [36] and previous survey articles, we pursue a threefold goal with this article:

- First of all, we aim at classifying all considered techniques along the same dimension by motivating and describing them with the same Bayesian network formalism. Consequently, we do not conceptually distinguish whether a given method employs a time-varying pdf
Fig. 2. Bayesian network representation of different model compensation techniques. Detailed descriptions are given in the text. Subfigure (d) based on [4].

\[ p(b_n), \] as in uncertainty decoding, or whether a distorted vector \( y_n \) is a preprocessed or a genuinely noisy or reverberant observation. Also the distinction of implicit and explicit observation models dissolves in our formalism.

- As a second goal, we aim at closing some gaps by presenting new derivations and formulations for some of the considered techniques. For instance, the Bayesian network representations of the concepts in Subsections C, D, F, M, O, P, S, and T of Section IV have not been presented so far. Moreover, the links to the Bayesian network framework via the formulations in (33), (34), (49), (53), (59) are explicitly stated for the first time in this paper.

- The third goal of the Bayesian network description is to provide a graphical illustration that allows to easily overview a broad class of algorithms and to immediately identify their similarities and differences in terms of the underlying statistical assumptions.

By establishing new links between existing concepts, such an abstract overview should therefore also serve as a basis for revealing and exploring new directions. Note, however, that the review presented in this paper does not claim to cover all relevant acoustic model-based techniques and is rather meant as an inspiration to other researchers.

IV. EXAMPLES

In the following, we consider the compensation rules of several acoustic model-based from a Bayesian network view. The concepts in Subsections IV-A to IV-F belong to the category of uncertainty decoding and those in Subsections IV-G to IV-J to the class of missing feature approaches. Furthermore, the methods summarized in Subsections IV-K to IV-S are commonly referred to as acoustic model adaptation techniques. The concepts in Subsection IV-T are examples of model-based techniques using modified HMM topologies.

A. General Example of Uncertainty Decoding

A fundamental example of uncertainty decoding can, e.g., be extracted from [1], [37]–[42]. The underlying observation model can be identified as

\[ y_n = x_n + b_n \text{ with } b_n \sim \mathcal{N}(0, C_{b_n}), \] (10)

where \( y_n \) and \( C_{b_n} \) often play the role of an enhanced feature vector, e.g., from a Wiener filtering front-end [40], and a measure of uncertainty from the enhancement process, respectively. Thus, the point estimate \( y_n \) can be seen as being enriched by the additional reliability information \( C_{b_n} \). The observation model is representable by the Bayesian network in Figure 2(a). Exploiting the conditional independence properties of Bayesian networks [25], the compensation of the observation likelihood in (5) leads to

\[
p(y_n|q_n) = \int p(x_n|q_n)p(y_n|x_n)dx_n
= \int \mathcal{N}(x_n; \mu_{x|q_n}, C_{x|q_n})\mathcal{N}(y_n; x_n, C_{b_n})dx_n
= \mathcal{N}(y_n; \mu_{x|q_n}, C_{x|q_n} + C_{b_n}). \tag{11}\]

Without loss of generality, a single Gaussian pdf \( p(x_n|q_n) \) is assumed since, in the case of a Gaussian Mixture Model (GMM), the linear mismatch function (10) can be applied to each Gaussian component separately.

B. Dynamic Variance Compensation

The concept of dynamic variance compensation [2] is based on a reformulation of the log-sum observation model [39]:

\[ y_n = x_n + \log(1 + \exp(\hat{r}_n - x_n)) + b_n \tag{12}\]

with \( \hat{r}_n \) being a noise estimate of any noise tracking algorithm and \( b_n \sim \mathcal{N}(0, C_{b_n}) \) a residual error term. Since the analytical derivation of \( p(y_n|q_n) \) is intractable, an approximate pdf is evaluated based on the assumption of \( p(x_n|y_n) \) being Gaussian and that the compensation can be applied to each Gaussian component of the GMM separately [2] such that the observation likelihood in (5) becomes according to Figure 2(a):

\[
\tilde{p}(y_n|q_n) = \int p(x_n|q_n)\frac{p(x_n|y_n)}{p(x_n)}dx_n
\approx \int p(x_n|q_n)p(x_n|y_n)dx_n
\approx \int \mathcal{N}(x_n; \mu_{x|q_n}, C_{x|q_n})\mathcal{N}(x_n; \mu_{x|y_n}, C_{x|y_n})dx_n
= \mathcal{N}(\mu_{x|y_n}; \mu_{x|y_n}, C_{x|y_n} + C_{x|y_n}), \tag{13}\]

where the first approximation can be justified if \( p(x_n) \) is assumed to be significantly “flatter”, i.e., of larger variance, than \( p(x_n|y_n) \). The estimation of the moments \( \mu_{x|y_n}, C_{x|y_n} \) of \( p(x_n|y_n) \) represents the core of [2].

C. Uncertainty Decoding with SPLICE

The Stereo Piecewise Linear Compensation for Environment (SPLICE) approach, first introduced in [43] and further developed in [44], [45], is a popular method for cepstral feature enhancement based on a mapping learned from stereo (i.e., clean and noisy) data [36]. While SPLICE can be used to derive an Minimum Mean Square Error (MMSE) [44] or MAP
[43] estimate that is fed into the recognizer, it is also applicable in the context of uncertainty decoding [3], which we focus on in the following. In order to derive a Bayesian network representation of the uncertainty decoding version of SPLICE [3], we first note from [3] that one fundamental assumption is:

$$p(x_n|y_n, s_n) = \mathcal{N}(x_n; y_n + r_{s_n}, \Gamma_{s_n}),$$  \hspace{1cm} (14)$$

where \(s_n\) denotes a discrete region index. Exploiting the symmetry of the Gaussian pdf

$$p(x_n|y_n, s_n) = \mathcal{N}(x_n; y_n + r_{s_n}, \Gamma_{s_n})$$

and defining \(b_n = y_n - x_n\), we identify the observation model to be

$$y_n = x_n + b_n$$  \hspace{1cm} (16)$$

given a certain region index \(s_n\). In the general case of \(s_n\) depending on \(x_n\), the observation model can be expressed by the Bayesian network in Figure 2(b) with

$$p(b_n|s_n) = \mathcal{N}(b_n; -r_{s_n}, \Gamma_{s_n}).$$  \hspace{1cm} (17)$$

By introducing a separate prior model

$$p(y_n) = \sum_{s_n} p(s_n) \cdot p(y_n|s_n)$$

for the distorted speech \(y_n\), the likelihood in (5) can be adapted according to

$$p(y_n|q_n) = \int p(x_n|q_n)p(y_n|x_n)dx_n$$

$$= \int p(x_n|q_n)p(x_n, y_n)p(y_n|s_n)p(s_n)dx_n$$

$$= \int p(x_n|q_n)\sum_{s_n} p(x_n|y_n, s_n)p(y_n|s_n)p(s_n)d\Gamma_n.d\mu_n.$$  \hspace{1cm} (19)$$

Although analytically tractable, both the numerator and the denominator in (19) are typically approximated for the sake of runtime efficiency [3].

**D. Joint Uncertainty Decoding**

Model-based joint uncertainty decoding [4] assumes an affine observation model in the cepstral domain

$$y_n = A_{k_n}x_n + b_n$$  \hspace{1cm} (20)$$

with the deterministic matrix \(A_{k_n}\) and \(p(b_n|k_n) = \mathcal{N}(b_n; \mu_{b_n|k_n}, C_{b_n|k_n})\) depending on the considered Gaussian component \(k_n\) of the GMM of the current HMM state \(q_n\):

$$p(x_n|q_n) = \sum_{k_n} p(k_n)p(x_n|k_n).$$  \hspace{1cm} (21)$$

The Bayesian network is depicted in Figure 2(c) implying the following compensation rule:

$$p(y_n|q_n) = \int p(x_n|q_n)p(y_n|x_n, k_n)dx_n,$$  \hspace{1cm} (22)$$

which can be analytically derived analogously to (11). In practice, the compensation parameters \(A_{k_n}, \mu_{b_n|k_n}, C_{b_n|k_n}\) are not estimated for each Gaussian component \(k_n\) but for each regression class comprising a set of Gaussian components [4].

**E. REMOS**

As many other techniques, the Reverberation Modeling for Speech Recognition (REMS) concept [5], [46] assumes the environmental distortion to be additive in the melspectral domain. However, REMOS also considers the influence of the \(L\) previous clean speech feature vectors \(x_{n-L:n-1}\) in order to model the dispersive effect of reverberation and to relax the conditional independence assumption of conventional HMMs. The observation model reads in the logmelspec domain:

$$y_n = \log \left( \exp(c_n) + \exp(h_n + x_n) \right)$$

$$+ \exp(a_n) \sum_{l=1}^{L} \exp(\mu_l + x_{n-l})$$,  \hspace{1cm} (23)$$

where the normally distributed random variables \(c_n, h_n, a_n\) model the additive noise components, the early part of the room impulse response (RIR), and the weighting of the late part of the RIR, respectively, and the parameters \(\mu_1:L\) represent a deterministic description of the late part of the RIR. The Bayesian network is depicted in Figure 3 with \(b_n = [c_n, a_n, h_n]\). In contrast to most of the other compensation rules discussed in this article, the REMOS concept necessitates a modification of the Viterbi decoder due to the introduced cross-connections in Figure 3. In order to arrive at a computationally feasible decoder, the marginalization over the previous clean speech components \(x_{n-L:n-1}\) is circumvented by employing estimates \(\hat{x}_{n-L:n-1}(q_{n-1})\) that depend on the best partial path, i.e., on the previous HMM state \(q_{n-1}\). The resulting analytically intractable integral is then approximated by the maximum of its integrand:

$$p(y_n|q_n, \hat{x}_{n-L:n-1}(q_{n-1})) =$$

$$= \int p(y_n|x_n, \hat{x}_{n-L:n-1}(q_{n-1}))p(x_n|q_n)dx_n$$

$$\geq \max_{x_n} p(y_n|x_n, \hat{x}_{n-L:n-1}(q_{n-1}))p(x_n|q_n).$$  \hspace{1cm} (24)$$
The determination of a global solution to (24) represents the core of the REMOS concept. The estimates \( \hat{x}_{n-L:n-1}(q_{n-1}) \) in turn are the solutions to (24) at previous time steps. We refer to [5] for a detailed derivation of the corresponding decoding routine.

**F. Ion and Haeb-Umbach**

Similar to REMOS, the generic uncertainty decoding approach proposed by [20] considers cross-connections in the Bayesian network in order to relax the conditional independence assumption of HMMs. The concept in [20] is an example of uncertainty decoding, where the compensation rule can be defined by a modified Bayesian network structure – given in Figure 4(a) – without fixing a particular functional form of the involved pdfs via an analytical observation model. In order to derive the compensation rule, we start by introducing the sequence \( x_{1:N} \) of latent clean speech vectors in each summund of (4):

\[
p(y_{1:N}, q_{1:N}) = \int p(y_{1:N}, x_{1:N}, q_{1:N})dx_{1:N}
\]

\[
= \int p(y_{1:N} | x_{1:N}) \prod_{n=1}^{N} p(x_n | q_n)p(q_n | q_{n-1})dx_{1:N}
\]

\[
\approx \int \frac{p(x_{1:N} | y_{1:N})}{p(x_{1:N})} \prod_{n=1}^{N} p(x_n | q_n)p(q_n | q_{n-1})dx_{1:N},
\]

where we exploited the conditional independence properties defined by Figure 4(a) (respecting the dashed links) and dropped \( p(y_{1:N}) \) in the last line of (25) as it represents a constant factor with respect to a varying state sequence \( q_{1:N} \). The pdf in the numerator of (25) is next turned into

\[
p(x_{1:N} | y_{1:N}) = p(x_1 | y_{1:N}) \prod_{n=2}^{N} p(x_n | y_{1:N}, x_{1:n-1})
\]

\[
\approx \prod_{n=1}^{N} p(x_n | y_{1:N}),
\]

where the conditional dependence (due to the head-to-head relation) of \( x_n \) and \( x_{1:n-1} \) is neglected. This corresponds to omitting the respective dashed links in Figure 4(a) for each factor in (26) separately. The denominator in (25) can also be further decomposed if the dashed links in Figure 4(b), i.e., the head-to-tail relations in \( q_n \), are disregarded:

\[
p(x_{1:N}) \approx \prod_{n=1}^{N} p(x_n).
\]

With (26) and (27), the update rule (25) is finally turned into the following simplified form:

\[
p(y_{1:N}, q_{1:N}) \approx \prod_{n=1}^{N} \int \frac{p(x_n | y_{1:N})}{p(x_n)} p(x_n | q_n)dx_n p(q_n | q_{n-1})
\]

that is given in [20]. Due to the approximations in Figure 4(a) and (b), the compensation rule defined by (28) exhibits the same decoupling as (5) and can thus be carried out without modifying the underlying decoder. In practice, \( p(x_n) \) may, e.g., be modeled as a separate Gaussian density and \( p(x_n | y_{1:N}) \) as a separate Markov process [24].

**G. Feature Vector Imputation**

We next turn to missing feature techniques, which can be used to model feature distortion due to a front-end enhancement process [7], noise [47] or reverberation [48]. A major subcategory of missing feature approaches is called feature vector imputation [6], [7], [26] where each feature vector component \( y_n^{(d)} \) is either classified as reliable \( (d \in \mathcal{R}_n) \) or unreliable \( (d \in \mathcal{U}_n) \), where \( \mathcal{R}_n \) and \( \mathcal{U}_n \) denote the set of reliable and unreliable components of the \( n \)-th feature vector, respectively [24]. While unreliable components are withdrawn and replaced by an estimate \( \hat{x}_n^{(d)} \) of the original observation \( y_n^{(d)} \), reliable components are directly “plugged” into the pdf. The score calculation in (5) therefore becomes

\[
\hat{p}(y_n | q_n) = \int p(x_n | q_n) p(x_n | y_{1:n})dx_n
\]

\[
\approx \int p(x_n | q_n)p(x_n | y_{1:n})dx_n
\]

with

\[
p(x_n | y_{1:n}) = \prod_{d=1}^{D} p(x_n^{(d)} | y_n^{(d)})
\]

and [24]

\[
p(x_n^{(d)} | y_n^{(d)}) = \begin{cases} 
\delta(x_n^{(d)} - y_n^{(d)}) & d \in \mathcal{R} \\
\delta(x_n^{(d)} - \hat{x}_n^{(d)}) & d \in \mathcal{U}
\end{cases}
\]

with the general Bayesian network in Figure 5(a).

**H. Marginalization**

The second major subcategory of missing feature techniques is called marginalization [6], [7], [26], where unreliable components are “replaced” by marginalizing over a clean-speech distribution \( p(x_n^{(d)}) \) that is usually not derived from the HMM but separately modeled. The posterior likelihood in (31) thus becomes [24]

\[
p(x_n^{(d)} | y_n^{(d)}) = \begin{cases} 
\delta(x_n^{(d)} - y_n^{(d)}) & d \in \mathcal{R} \\
p(x_n^{(d)}) & d \in \mathcal{U}
\end{cases}
\]

with the general Bayesian network in Figure 5(a).
the underlying Bayesian network corresponds to Figure 2(a). This explains the name of PMC as (35) combines two independent parallel models: the clean-speech HMM and the distortion model \( p(b_n) \). Since the resulting adapted pdf

\[
p(y_n|q_n) = \int p(x_n|q_n)p(y_n|x_n, b_n)p(b_n)d(x_n, b_n)
\]

(37)
cannot be derived in an analytical closed form, a variety of approximations to the true pdf \( p(y_n|q_n) \) have been investigated [10]. For nonstationary distortions, [10] proposes to employ a separate HMM for the distortion \( b_n \), leading to the Bayesian network representation of Figure 2(d). Marginalizing over the distortion state sequence \( \tilde{q}_{1:N} \) as in (5) reveals the acoustic score to become

\[
p(y_{1:N}|W) = \sum_{q_{1:N}} p(y_{1:N}, q_{1:N}, \tilde{q}_{1:N})
\]

(38)

where

\[
p(y_n|q_n, \tilde{q}_n) = \int p(x_n|q_n)p(y_n|x_n, b_n)p(b_n|\tilde{q}_n)d(x_n, b_n).
\]

(39)

The overall acoustic score can be approximated by a 3D Viterbi decoder, which can in turn be mapped onto a conventional 2D Viterbi decoder [10].

L. Vector Taylor Series

The concept of Vector Taylor Series (VTS) is frequently employed in practice yielding promising results [36]. Its fundamental idea is to linearize a nonlinear distortion model by a Taylor series [11], [49], [50]. The standard VTS approach [49] is based on the log-sum observation model:

\[
y_n = \log(\exp(h_n + x_n) + \exp(c_n)),
\]

(40)

where \( p(h_n) = \mathcal{N}(h_n; \mu_h, C_h) \) captures short convolutive distortion and \( p(c_n) = \mathcal{N}(c_n; \mu_c, C_c) \) models additive noise components. The Bayesian network is represented by Figure 2(a) with \( b_n = [h_n, c_n] \). Note that in contrast to uncertainty decoding, \( p(b_n) \) is constant over time. As the adapted pdf is again of the form of (37) and thus analytically intractable, it is assumed that (40) can, firstly, be applied to each Gaussian component \( p(x_n|k_n) \) of the GMM

\[
p(x_n|q_n) = \sum_{k_n} p(k_n)p(x_n|k_n)
\]

(41)

individually and, secondly, be approximated by a Taylor series around \( [\mu_{x|k_n}, \mu_h, \mu_c] \), where \( \mu_{x|k_n} \) denotes the mean of the component \( p(x_n|k_n) \). There are various extensions to the VTS concept that are neglected here. For a more comprehensive review of VTS, we refer to [36].


M. CMLLR

Constrained Maximum Likelihood Linear Regression (CMLLR) [12], [51] can be seen as the deterministic counterpart of joint uncertainty decoding (Subsection IV-D) with the observation model

\[ y_n = A_{k_n} x_n + b_{k_n} \]  

(42)

and deterministic parameters \( A_{k_n}, b_{k_n} \). The adaptation rule of \( p(y_n|x_n, k_n) \) has the same form as (22) with

\[ p(y_n|x_n, k_n) = \delta(y_n - A_{k_n} x_n - b_{k_n}). \]  

(43)

The Bayesian network corresponds to Figure 5(b), where the use of regression classes is again reflected by the dependency of the observation model parameters on the Gaussian component \( k_n \) (cf. Subsection IV-D). The affine observation model in (42) is equivalent to transforming the mean vector \( \mu_{x|k_n} \) and covariance matrix \( C_{x|k_n} \) of each Gaussian component of \( p(x_n|k_n) \):

\[ \begin{align*}
\mu_{y|k_n} &= A_{k_n} \mu_{x|k_n} + b_{k_n}, \\
C_{y|k_n} &= A_{k_n} C_{x|k_n} A_{k_n}^T.
\end{align*} \]  

(44)

CMLLR represents a very popular adaptation technique in practice due to its promising results and versatile fields of application, such as speaker adaptation [51], adaptive training [52] as well as noise [53] and reverberation-robust [54] ASR.

N. MLLR

The Maximum Likelihood Linear Regression (MLLR) concept [13] can be considered as a generalization of CMLLR as it allows for a separate transform matrix \( B_{k_n} \) in (45):

\[ \begin{align*}
\mu_{y|k_n} &= A_{k_n} \mu_{x|k_n} + b_{k_n}, \\
C_{y|k_n} &= B_{k_n} C_{x|k_n} B_{k_n}^T.
\end{align*} \]  

(45)

In practice, however, MLLR is frequently applied to the mean vectors only [55]–[58] while neglecting the adaptation of the covariance matrix:

\[ C_{y|k_n} = C_{x|k_n}, \]  

(48)

This principle is also known from other approaches that are applicable to both means and variances but are often only carried out on the former (e.g., for the sake of robustness) [10], [59].

If applied to the mean vectors only, MLLR can in turn be considered as a simplified version of CMLLR, where the observation model (42) and the Bayesian network in Figure 5(b) is assumed while the compensation of the variances is omitted.

The Bayesian network representation in Figure 5(b) also underlies the general MLLR adaptation rule (46) and (47). In this case, however, it seems impossible to identify a corresponding analytic observation model representation.

\[ \begin{align*}
\mu_{x|q_n} &= A \mu_{x|q_n} + c \]  

(51)

O. MAP Adaptation

We next describe the MAP adaptation applied to any parameters \( \theta \) of the pdfs of an HMM. In MAP, these parameters are considered as Bayesian parameters, i.e., random variables that are drawn once for all times as depicted in Figure 5(c) [25]. As a direct consequence, any two observation vectors \( y_i, y_j \) are conditionally dependent given the state sequence. The predictive pdf in (4) therefore explicitly depends on the adaptation data that we denote as \( y_{M:0}, M < 0, \) and becomes

\[ \begin{align*}
p(y_{1:N}, q_{1:N}|y_{M:0}) &= \int p(y_{1:N}, q_{1:N}|\theta, y_{M:0}) d\theta \\
&= \int p(y_{1:N}, q_{1:N}|\theta, y_{M:0}) p(\theta|y_{M:0}) d\theta \\
&\approx p(y_{1:N}, q_{1:N}|\theta_{\text{MAP}}, y_{M:0}),
\end{align*} \]  

(49)

where the posterior \( p(\theta|y_{M:0}) \) is approximated as Dirac distribution \( \delta(\theta - \theta_{\text{MAP}}) \) at the mode \( \theta_{\text{MAP}} \):

\[ \theta_{\text{MAP}} = \arg \max_\theta p(\theta|y_{M:0}) = \arg \max_\theta p(y_{M:0} | \theta) p(\theta). \]  

(50)

An iterative (local) solution to (50) is obtained by the Expectation Maximization (EM) algorithm. Note that due to the MAP approximation of the posterior \( p(\theta|y_{M:0}) \), the conditional independence assumption is again fulfilled such that a conventional decoder can be employed.

P. Bayesian MLLR

As mentioned before, uncertainty decoding techniques allow for a time-varying pdf \( p(b_n) \), while model adaptation approaches, such as in Subsections IV-K, IV-L, and IV-Q, mostly set \( p(b_n) \) to be constant over time. In both cases, however, the “randomized” model parameter \( b_n \) is assumed to be redrawn in each time step \( n \) as in Figure 6(a). In contrast, Bayesian estimation – as mentioned before – usually refers to inference problems, where the random model parameters are drawn once for all times [25] as in Figure 6(b).

Another example of Bayesian model adaptation, besides MAP, is Bayesian MLLR [14] applied to the mean vector \( \mu_{x|q_n} \) of each pdf \( p(x_n|q_n) \):

\[ \mu_{y|q_n} = A \mu_{x|q_n} + c \]  

(51)
with \( b = [A, \varepsilon] \) being usually drawn from a Gaussian distribution [14]. Here, we do not consider different regression classes and assume \( p(x_n|q_n) \) to be a single Gaussian pdf since, in the case of a GMM, the linear mismatch function (51) can be applied to each Gaussian component separately. The likelihood score in (4) thus becomes\(^1\):

\[
\sum_{q_1:N} p(y_{1:N}, q_{1:N}) = \sum_{q_1:N} \int p(y_{1:N}, q_{1:N}, b) db = \sum_{q_1:N} \int p(y_{1:N}, q_{1:N}|b)p(b) db.
\]

This score can, e.g., be approximated by a frame-synchronous Viterbi search [60]. Another approach is to apply the Bayesian integral in a frame-wise manner and use a standard decoder [61]. In this case, the integral in (52) becomes

\[
\int p(y_{1:N}|q_{1:N}|b)p(b) db = \int \prod_{n=1}^N p(y_n|q_n, b) p(q_n|q_{n-1}) p(b) db = \prod_{n=1}^N \int p(y_n|q_n, b) p(q_n|q_{n-1}) p(b) db,
\]

where the original assumption of \( w \) being identical for all time steps \( n \) was relaxed to the case of \( w \) being identically distributed for all times steps \( n \). The approximation in (53) is representable by the conversion of the Bayesian network in Figure 6(b) to the one in (a) with constant probability density function (pdf) \( p(b_n) = p(b) \) for all \( n \).

**Q. Reverberant VTS**

Reverberant VTS [15] is an extension of conventional VTS (Subsection IV-L) to capture the dispersive effect of reverberation. Its observation model reads for static features in the logmelspec domain:

\[
y_n = \log \left( \sum_{l=0}^L \exp(x_{n-l} + \mu_l) + \exp(b_n) \right)
\]

with \( b_n \) being an additive noise component modeled as normally distributed random variable and \( \mu_{0:L} \) being a deterministic description of the reverberant distortion. For the sake of tractability, the observation model is approximated in a similar manner as in the VTS approach. This concept can be seen as an alternative to REMOS (Subsection IV-E): While REMOS tailors the Viterbi decoder to the modified Bayesian network, reverberant VTS avoids the computationally expensive marginalization over all previous clean-speech vectors by averaging – and thus smoothing – the clean-speech statistics over all possible previous states and Gaussian components. Thus, \( y_n \) is assumed to depend on the extended clean-speech vector \( x_n = [x_{n-L}, \ldots, x_n] \), cf. Figure 7(a) vs. (b).

**R. Convolutional Model Adaptation**

Besides the previously mentioned REMOS, reverberant VTS, and reverberant CMLLR concepts, there are three related approaches employing a convolutive observation model in order to describe the dispersive effect of reverberation [16]–[18]. All three approaches assume the following model in the logmelspec domain:

\[
y_n = \log \left( \sum_{l=0}^L \exp(x_{n-l} + \mu_l) \right),
\]

where \( \mu_{0:L} \) denotes a deterministic description of the reverberant distortion that is differently determined by the three approaches. The observation model (55) can be represented by the Bayesian network in Figure 3 without the random component \( b_n \). Both [16] and [18] use the “log-add approximation” [10] to derive \( p(y_n|q_n) \), i.e.,

\[
\mu_{y|q_n} = \log \left( \exp(\mu_{x|q_n} + \mu_0) + \sum_{l=1}^L \exp(\mu_{x|q_n-l} + \mu_l) \right),
\]

where \( \mu_{x|q_n} \) and \( \mu_{x|q_n-l} \) denote the mean of the \( k_n \)-th Gaussian component of \( p(y_n|q_n) \) and \( p(x_n|q_n-l) \), respectively. The previous means \( \mu_{x|q_n-l}, l > 0 \), are averaged over all means of the corresponding GMM \( p(x_{n-l}|q_{n-l}) \). On the other hand, [17] employs the “log-normal approximation” [10] to adapt \( p(y_n|q_n) \) according to (55). While [16] and [17] perform the adaptation once prior to recognition and then use a standard decoder, the concept proposed in [18] performs an online adaptation based on the best partial path [15].

It should be pointed out here that there is a variety of other approximations to the statistics of the log-sum of (mixtures of) Gaussian random variables (as seen in Subsections B, E, K, L, Q of Section IV), ranging from different PMC methods [10] to maximum [62], piecewise linear [63], and other analytical approximations [64]–[68].

**S. Takiguchi et al.**

In contrast to the approaches of Subsections IV-Q and IV-R, the concept proposed in [19] assumes the reverberant

---

\( ^1 \)In contrast to (49), we do not explicitly mention the dependency on the adaptation data \( y_{3:T0} \) for notational convenience.
observation vector $y_{n-1}$ at time $n-1$ to be an approximation to the reverberation tail at time $n$ in the logmelspec domain:

$$y_n = \log \left( \exp(h + x_n) + \exp(\alpha + y_{n-1}) \right). \quad (57)$$

where $h$ and $\alpha$ are deterministic parameters modeling short convolutive distortion and the weighting of the reverberation tail, respectively. Thus, each summand in (4) becomes

$$p(y_1:N, q_1:N) = \prod_{n=1}^{N} p(y_n|y_{n-1}, \cdots, y_1, q_n|q_{n-1}) \cdot p(q_n|q_{n-1})$$

with the Bayesian network of Figure 8. It seems interesting to note that (57) can be analytically evaluated as a nonlinear mapping $f(\cdot)$ of one random vector $x_n$: $y_n = f(x_n)$ with

$$p(y_n|q_n, y_{n-1}) = \frac{p(x_n|q_n)}{\det(J_{y_n}(f^{-1}(y_n)))} \quad (59)$$

where $x_n = f^{-1}(y_n)$ and $J_{y_n}$ denotes the Jacobian w.r.t. $y_n$.

**T. Conditional HMMs [22] and Combined-Order HMMs [23]**

We close this section by turning to two model-based approaches that cannot be classified as “model adaptation” as they postulate different HMM topologies rather than adapting a conventional HMM. Both approaches aim at relaxing the conditional independence assumption of conventional HMMs in order to improve the modeling of the inter-frame correlation.

The concept of conditional HMMs [22] models the observation $y_n$ as depending on the previous observations at time shifts $\psi = (\psi_1, \ldots, \psi_p) \in \mathbb{N}^p$. Each summand in (4) therefore becomes

$$p(y_1:N, q_1:N) = \prod_{n=1}^{N} p(y_n|y_{n-\psi_1}, \ldots, y_{n-\psi_p}, q_n) \cdot p(q_n|q_{n-1}) \quad (60)$$

according to Figure 9(a). Such HMMs are also known as autoregressive HMMs [25].

In contrast to conditional HMMs, combined-order HMMs [23] assume the current observation $y_n$ to depend on the previous HMM state $q_{n-1}$ in addition to state $q_n$:

$$p(y_1:N, q_1:N) = \prod_{n=1}^{N} p(y_n|q_n, q_{n-1}) \cdot p(q_n|q_{n-1}) \quad (61)$$

according to Figure 9(b).

V. SUMMARY

In this article, we described the compensation rules of several acoustic model-based techniques employing Bayesian network representations. Some of the presented Bayesian network descriptions are already given in the original papers and others can be easily derived based on the original papers (cf. Subsections IV-C, IV-D, IV-F, and IV-T). In contrast, the link of the concepts of modified imputation (Subsection IV-I), significance decoding (Subsection IV-J), CM-LLR/MLLR (Subsections IV-M and IV-N), MAP (Subsection IV-O), Bayesian MLLR (Subsection IV-P), and Takiguchi et al. [19] (Subsection IV-S) to the Bayesian network framework via the formulations in (33), (34), (43), (49), (53), and (59), respectively, are explicitly stated for the first time in this paper.

Clearly, the graphical model description neglects various crucial aspects related to the considered concepts: Most importantly, neither the particular functional form of the joint pdf, nor potential approximations to arrive at a tractable algorithm nor the provenance of (i.e., the estimation procedure for) the compensation parameters are reflected.

On the other hand, the Bayesian network description provides a convenient language to immediately and clearly identify some major properties:

- The cross-connections depicted in Figures 3, 4, 7(a), 8, 9 show that the underlying concept aims at improving the modeling of the inter-frame correlation, e.g., to increase the robustness of the acoustic model against reverberation. If applied in a straightforward way, such cross-connections would entail a costly modification of the Viterbi decoder. In this paper, we summarized some important approximations that allow for a more efficient decoding of the extended Bayesian network, cf. Subsections IV-E, IV-F, IV-Q, IV-R. Some of these typically empirically motivated or just intuitive approximations, especially neglected statistical dependencies, become obvious from a Bayesian network, as shown in Figures 4 and 7.

- The approaches introducing instantaneous (here: purely vertical) extensions to the Bayesian network, as in Figures 2(a)-(c) and 5(c), usually aim at compensating for nondispersive distortions, such as additive or short convolutive noise.

- The arcs in Figures 2(c) and 5(b) illustrate that the observed vector $y_n$ does not only depend on the state $q_n$ (or mixture component $k_n$) through $x_n$. As a consequence, one can deduce that the compensation parameters do
depend on the phonetic content, as in Subsections IV-D, IV-M, and IV-N.

- The graphical model representation also succinctly highlights whether a Bayesian modeling paradigm is applied, as in Figures 5(c) and 6(b), or not, as in Figures 5(a) and (b).
- The existence of the additional latent variable \( x_n \) in most of the presented Bayesian network representations expresses that an explicit observation model or an implicit statistical model between the clean and the corrupted features is employed. In contrast, the graphical representations in Figures 5(c) and 9 show that – instead of a distinct compensation step – a modified HMM topology is used.

In summary, the condensed description of the various concepts from the same Bayesian network perspective shall offer other researchers the possibility to more easily exploit or combine existing techniques and to link their own algorithms to the presented ones. This seems all the more important as the recent acoustic modeling approaches based on Deep Neural Networks (DNNs) raise new challenges for the conventional robustness techniques [36]. As pointed out in [36], a possible solution for exploiting the traditional GMM-based robustness approaches – such as the ones reviewed here – within the deep learning paradigm could be the use of DNN-derived bottleneck features in a GMM-HMM, which offers various possibilities for further research.

REFERENCES

[1] J. A. Arrowood and M. A. Clements, “Using observation uncertainty in HMM decoding,” in Proc. ICSLP, 2002, pp. 1561–1564.
[2] L. Deng, J. Droppo, and A. Acero, “Dynamic compensation of HMM variances using the feature enhancement uncertainty computed from a parametric model of speech distortion,” IEEE Trans. Speech Audio Process., vol. 13, no. 3, pp. 412–421, 2005.
[3] J. Droppo, A. Acero, and L. Deng, “Uncertainty decoding with SPLICE for noise robust speech recognition,” in Proc. ICASSP, vol. 1, 2002, pp. 57–60.
[4] H. Liao, “Uncertainty decoding for noise robust speech recognition,” Ph.D. dissertation, Univ. of Cambridge, 2007.
[5] R. Maas, W. Kellermann, A. Sehr, T. Yoshioka, M. Delcroix, K. Kinoshita, and T. Nakatani, “Formulation of the REMOS concept from an uncertainty decoding perspective,” in Proc. Int. Conf. on Digital Signal Process., 2013.
[6] M. Cooke, P. Green, L. Josifovski, and A. Vizinho, “Robust automatic speech recognition with missing and unreliable acoustic data,” Speech Commun., vol. 34, no. 3, pp. 267–285, 2001.
[7] B. Raj and R. M. Stern, “Missing-feature approaches in speech recognition,” IEEE Signal Process. Mag., vol. 22, no. 5, pp. 101–116, 2005.
[8] D. Kolossa, A. Klimas, and R. Orglmeister, “Separation and robust recognition of noisy, convolutive speech mixtures using time-frequency masking and missing data techniques,” in Proc. WASPAA, 2005, pp. 82–85.
[9] A. H. Abdelaziz and D. Kolossa, “Decoding of uncertain features using the posterior distribution of the clean data for robust speech recognition,” in Proc. Interspeech, 2012.
[10] M. J. F. Gales, “Model-based techniques for noise robust speech recognition,” Ph.D. dissertation, Univ. of Cambridge, 1995.
[11] A. Acero, L. Deng, T. Kristjansson, and J. Zhang, “HMM adaptation using vector Taylor series for noisy speech recognition,” in Proc. ICSLP, vol. 3, 2000, pp. 869–872.
[12] V. V. Digalakis, D. Ritschel, and L. G. Neumeyer, “Speaker adaptation using constrained estimation of Gaussian mixtures,” IEEE Trans. Speech Audio Process., vol. 3, no. 5, pp. 357–366, 1995.
[13] C. J. Leggetter and P. C. Woodland, “Maximum likelihood linear regression for speaker adaptation of continuous density hidden Markov models,” Comput. Speech Lang., vol. 9, no. 2, pp. 171–185, 1995.
[14] J.-T. Chien, “Linear regression based Bayesian predictive classification for speech recognition,” IEEE Trans. Speech Audio Process., vol. 11, no. 1, pp. 70–79, 2003.
[15] Y. Q. Wang and M. J. F. Gales, “Improving reverberant VTS for hands-free robust speech recognition,” in Proc. ASRU, 2011, pp. 113–118.
[16] H. G. Hirsch and H. Finster, “A new approach for the adaptation of HMMs to reverberation and background noise,” Speech Commun., vol. 50, no. 3, pp. 244–263, 2008.
[17] C. K. Raut, T. Nishimoto, and S. Sagayama, “Model adaptation for long convolutional distortion by maximum likelihood based state filtering approach,” in Proc. ICASSP, vol. 1, 2006, pp. 1133–1136.
[18] A. Sehr, R. Maas, and W. Kellermann, “Frame-wise HMM adaptation using state-dependent reverberation estimates,” in Proc. ICASSP, 2011, pp. 5484–5487.
[19] T. Takiguchi, M. Nishimura, and Y. Arikii, “Acoustic model adaptation using first-order linear prediction for reverberant speech,” IEICE Trans. Inform. and Systems, vol. E89-D, no. 3, pp. 908–914, 2006.
[20] V. Ion and R. Haeb-Umbach, “A novel uncertainty decoding rule with applications to transmission error robust speech recognition,” IEEE Trans. Audio, Speech, Lang. Process., vol. 16, no. 5, pp. 1047–1060, 2008.
[21] J. L. Gauvain and C. H. Lee, “MAP estimation of continuous density HMM: theory and applications,” in Proc. Workshop on Speech and Natural Lang., 1992, pp. 185–190.
[22] J. Ming and F. J. Smith, “Modelling of the interface dependence in an HMM using conditional Gaussian mixtures,” Comput. Speech Lang., vol. 10, pp. 229–247, 1996.
[23] R. Maas, S. R. Kotha, A. Sehr, and W. Kellermann, “Combined-order hidden Markov models for reverberation-robust speech recognition,” in Proc. Int. Workshop on Cognitive Inform. Process., 2012, pp. 167–171.
[24] R. Haeb-Umbach, “Uncertainty decoding and conditional Bayesian estimation,” in Robust speech recognition of uncertain or missing data, B. Kolossa and R. Haeb-Umbach, Eds. Springer, 2011, pp. 9–33.
[25] C. M. Bishop, Pattern Recognition and Machine Learning. Springer, 2006.
[26] D. Kolossa and R. Haeb-Umbach, Robust speech recognition of uncertain or missing data. Springer, 2011.
[27] G. Heigold, H. Ney, R. Schütter, and S. Wieseler, “Discriminative training for automatic speech recognition: modeling, criteria, optimization, implementation, and performance,” IEEE Signal Process. Mag., vol. 29, no. 6, pp. 58–69, 2012.
[28] M. Matassoni, M. Omologo, D. Giuliani, and P. Svaizer, “Hidden Markov model training with contaminated speech material for distant-talking speech recognition,” Comput. Speech Lang., vol. 16, no. 2, pp. 205–223, 2002.
[29] A. Sehr, C. Hofmann, R. Maas, and W. Kellermann, “A novel approach for matched reverberant training of HMMs using data pairs,” in Proc. Interspeech, 2010, pp. 566–569.
[30] Y. Gong, “Speech recognition in noisy environments: A survey,” Speech Commun., vol. 16, no. 3, pp. 261–291, 1995.
[31] C.-H. Lee, “On stochastic feature and model compensation approaches to robust speech recognition,” Speech Commun., vol. 25, no. 13, pp. 29–47, 1998.
[32] Q. Huo and C.-H. Lee, “Robust speech recognition based on adaptive classification and decision strategies,” Speech Commun., vol. 34, no. 12, pp. 175–194, 2001.
[33] J. Droppo, “Environmental robustness,” in Springer Handbook of Speech Processing. Springer, 2008, pp. 653–680.
[34] T. Yoshioka, A. Sehr, M. Delcroix, K. Kinoshita, R. Maas, T. Nakatani, and W. Kellermann, “Making machines understand us in reverberant rooms: robustness against reverberation for automatic speech recognition,” IEEE Signal Process. Mag., vol. 29, no. 6, pp. 114–126, 2012.
[35] T. Virtanen, R. Singh, and B. Raj, Techniques for noise robustness in automatic speech recognition. John Wiley & Sons, 2013.
[36] J. Li, L. Deng, Y. Gong, and R. Haeb-Umbach, “An overview of noise-robust automatic speech recognition,” IEEE/ACM Trans. Audio, Speech, Lang. Process., vol. 22, no. 4, pp. 745–777, 2014.
[37] J. N. Holmes, W. J. Holmes, and P. N. Garner, “Using formant frequencies in speech recognition,” in Proc. Eurospeech, vol. 97, 1997, pp. 2083–2087.
[38] T. T. Kristjansson and B. J. Frey, “Accounting for uncertainty in observations: A new paradigm for robust speech recognition,” in Proc. ICASSP, 2002, pp. 1–6.
[39] L. Deng, J. Droppo, and A. Acero, “Exploiting variances in robust feature extraction based on a parametric model of speech distortion,” in Proc. ICSLP, vol. 4, 2002, pp. 2449–2452.

[3]  [4]  [5]  [6]  [7]  [8]  [9]  [10]  [11]  [12]  [13]  [14]  [15]  [16]  [17]  [18]  [19]  [20]  [21]  [22]  [23]  [24]  [25]  [26]  [27]  [28]  [29]  [30]  [31]  [32]  [33]  [34]  [35]  [36]  [37]  [38]  [39]
