Applications of the Reduction of Couplings*

Dedicated to Professor Wolfhart Zimmermann
on the occasion of his 70th birthday

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Abstract

Applications of the principle of reduction of couplings to the standard model and supersymmetric grand unified theories are reviewed. Phenomenological applications of renormalization group invariant sum rules for soft supersymmetry-breaking parameters are also reviewed.

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1 Application to the Standard Model

High energy physicists have been using renormalizability as the predictive tool, and also to decide whether or not a quantity is calculable. As we have learned in the previous talk by Professor Oehme, it is possible, using the method of reduction of couplings [1, 2, 3], to renormalize a theory with fewer number of counter terms then usually counted, implying that the traditional notion of renormalizability should be generalized in a certain sense [1]. Consequently, the notion of the predictability and the calculability [5] may also be generalized with the help of reduction of couplings. Of course, whether the generalizations of these notions have anything to do with nature is another question. The question can be answered if one applies the idea of reduction of couplings to realistic models, make predictions that are specific for reduction of couplings, and then wait till experimentalists find positive results [1].

In 1984 Professor Zimmermann, Klaus Sibold and myself [8] began to apply the idea of reduction of couplings to the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge model for the strong and electroweak interactions. As it is known, this theory has a lot of free parameters, and at first sight it seemed there exists no guiding principle how to reduce the couplings in this theory. There were two main problems associated with this program. The one was that it is not possible to assume a common asymptotic behavior for all couplings, and the other one is how to increase the predictive power of the model without running into the contradiction with the experimental knowledge (of that time) such as the masses of the known fermions. Professor Zimmermann suggested to use asymptotic freedom as a guiding principle, and assumed that QCD is most fundamental among the interactions of the standard model (SM). Since pure QCD is asymptotically free, we tried to switch on as many SM interactions as possible while keeping asymptotic freedom and added them to QCD. The result was almost unique: There exist two possibilities (or two asymptotically free (AF) surfaces in the space of couplings). It turned out that on the first surface, the $SU(2)_L$ gauge coupling $\alpha_2$ is bigger than the QCD

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1Earlier references related to the idea of reduction of couplings are given in [4]. Professor Shirkov who is present here was one of the authors who considered such theoretical possibilities. For reviews, see [6].

2Here I would like to restrict myself to phenomenological applications of reduction of couplings. See [7, 9] for the other applications. Professor Oehme reminded me that in his seminar talk given at the Max-Planck-Institute early 1984, professor Peccei suggested phenomenological applications of this idea.
Asymptotically free surface

\[ \alpha / \alpha_3 \]

\[ \alpha_\lambda / \alpha_3 \]

\[ \alpha_t / \alpha_3 \]

\[ 2/9 \]

\[ 0 \]

\[ 0.069.. \]

Figure 1: Reduction of \( \alpha_t \) and \( \alpha_\lambda \) in favor of \( \alpha_3 \).

coupling \( \alpha_3 \), and on the second surface, \( \alpha_2 \) has to identically vanish. We decided to choose the second possibility, because we found out that it is possible to include the \( SU(2)_L \) gauge coupling \( \alpha_2 \) as a certain kind of “perturbation” into the AF system. Since the perturbating couplings should be regarded as free parameters, the reduction of couplings in this case can be achieved only partially ("partial reduction"). For the first case, it was not possible.

Thus, the largest AF system which is phenomenologically acceptable (at that time) contains \( \alpha_3 \), the quark Yukawa couplings \( \alpha_i \) \( (i = d, s, b, u, c, t) \) and the Higgs self coupling \( \alpha_\lambda \). However, because of the hierarchy of the Yukawa couplings, we could not expect that all couplings can be expressed in terms of a power series of \( \alpha_3 \) without running into the contradiction with that hierarchy. So we decided to apply to the reduction of couplings only to the system with \( \alpha_3 , \alpha_t \) and \( \alpha_\lambda \), and to regard the other couplings as perturbations like \( \alpha_2 \).

Fig. 1 shows the AF surface in the space of \( \alpha_3, \alpha_t/\alpha_3 \) and \( \alpha_\lambda/\alpha_3 \). The reduction of the top
Yukawa and Higgs couplings in favor of the QCD coupling corresponds to the border line on the surface, i.e., the line defined by

$$
\frac{\alpha_t}{\alpha_3} = \frac{2}{9}, \quad \frac{\alpha_\lambda}{\alpha_3} = \frac{\sqrt{689} - 25}{18} \approx 0.0694 \quad (1)
$$

in the one-loop approximation. This border line was already known as the Pendleton-Ross infrared (IR) fixed point (line) [10]. Note that the existence of the AF surface (shown in Fig. 1) at least for $\alpha_3$ closed to the origin is mathematically ensured (see also [11]), while the line for large $\alpha_3$, Pendleton-Ross infrared IR line, can be an one-loop artifact which was pointed out by Professor Zimmermann. He showed explicitly in the two-loop approximation that this is indeed the case [12].

An asymptotically free renormalization group (RG) trajectory lies exactly on the surface. Fig. 2 shows trajectories projected on the $\alpha_3 - \alpha_t/\alpha_3$ plane. It may be worthwhile to mention that the branches above the Pendleton-Ross IR line (the lines left to it in Fig. 2) are used by Professor Bardeen and his collaborators [13] to interpret the Higgs particle as a bound

Figure 2: Asymptotically free surface in the $\alpha_3 - \alpha_t/\alpha_3$ space.
state of the top and anti-top quarks. From Fig. 2 one can see that the higher the energy scale where the top Yukawa coupling diverges (the horizontal dotted line in Fig. 2 will be lowered), the similar is the prediction of the top mass in two methods. However, I would like to emphasize that how to include the corrections to this lowest order system (especially those due to the non-vanishing $SU(2)_L$ and $U(1)_Y$ gauge couplings) depends on the ideas behind, so that the actual predictions are different. We included these corrections within the one-loop approximation and calculated $\alpha_t/\alpha_3$ and $\alpha_h/\alpha_3$ in terms of $\alpha_3$ and the perturbating free couplings. Then we used the formulae

$$M_t^2/M_Z^2 = 2\cos^2\theta_W \alpha_t/\alpha_2, \quad M_h^2/M_Z^2 = 2\cos^2\theta_W \alpha_h/\alpha_2,$$  \tag{2}

to calculate the top quark and Higgs masses, $M_t$ and $M_h$, from the known values of the parameters such as the $Z$ boson mass $M_Z$ and the Weinberg mixing angle $\theta_W$. We obtained

$$M_t \simeq 81 \text{ GeV}, \quad M_h \simeq 61 \text{ GeV}. \tag{3}$$

Later I included higher order corrections such as two-loop corrections and found that the earlier predictions (3) become $M_t = 98.6 \pm 9.2$ GeV and $M_h = 64.5 \pm 1.5$ GeV, which should be compared with the present knowledge [14]

$$M_t = 173.8 \pm 5.2 \text{ GeV}, \quad M_h \gtrsim 77.5 \text{ GeV}. \tag{4}$$

The failure of our prediction was disappointing in fact. However, this failure relieved Professor Zimmermann from a self-contradicting feeling. As we know he likes low-energy supersymmetry and also good wines. If our prediction would have been confirmed by an experiment, it would be very unlikely that low-energy supersymmetry is realized in nature, which would imply that he would lose again a lot of bottles of wines. So, the decision of nature was welcome at the same time. Fig. 3 summarizes.

2 Why is Supersymmetry as Ideal Place for Application?
2.1 Naturalness and supersymmetry

Let me now come to the application of reduction of parameters to supersymmetric theories. I do not know why Professor Zimmermann likes low energy supersymmetry. But let me assume that he likes the usual argument for low energy supersymmetry, which is based on the naturalness notion of ’t Hooft [15]. I would like to spend few minutes for that. (Let me allow to do so, although for the superexperts in the audience it might be superboring.) ’t Hooft [15] said that there exist a natural scale in a given theory, and that the natural energy scale of spontaneously broken gauge theories which contain the SM is usually less than few TeV. The argument is the following. Suppose the scale at which the SM goes over to a more fundamental theory is Λ. That is, there are in the fundamental theory particles with masses of this order. Now consider the propagator Δ(p^2) of a boson field with the physical mass m_B much smaller than Λ, and suppose that it is normalized at Λ so that the propagator assumes a simple form at p^2 = −Λ^2:

$$\lim_{p^2 \to -\Lambda^2} \Delta(p^2) \rightarrow \frac{iZ(\Lambda^2)}{p^2 - m_B^2(\Lambda^2)},$$

(5)
where $Z$ is the normalization constant for the wave function. The physical mass squared $m^2_B$ can be expressed as

$$m^2_B = m^2_B(\Lambda^2) + \delta m^2_B .$$

Then we ask ourselves how accurate we have to tune the value of $m^2_B(\Lambda^2)$ to obtain a desired accuracy in the physical mass squared $m^2_B$. This depends on $\delta m^2_B$, of course. ‘t Hooft said that for a theory to be natural the ratio $m^2_B(\Lambda^2)/m^2_B$ should be of $O(1)$, which implies that $|\delta m^2_B| < m^2_B$. If quadratic divergences are involved in the theory, the correction $\delta m^2_B$ will be proportional not only to the masses of the light fields, but also to the masses of the heavy fields, and so $\delta m^2_B$ can be of the order $(\alpha/4\pi)\Lambda^2$, where $\alpha$ is some generic coupling. Since the Higgs mass should not exceed few hundred GeV in the SM, the natural scale of the fundamental theory, which contains the standard model Higgs and also involves quadratic divergences, is at best few TeV. So according ‘t Hooft, ordinary Grand Unified Theories (GUTs), for instance, are unnatural [15].

Supersymmetry, thanks to its very renormalization property known as non-renormalization theorem [16, 17], can save the situation. The cancellation of the quadratic divergences, which was first observed by Professors Wess and Zumino [16], is exact if the masses of the bosonic and fermionic superpartners are the same. However, supersymmetry is unfortunately broken in nature, so that the cancellation is not exact. The mass squared difference, $m^2_B - m^2_F$, characterizes the energy scale of supersymmetry breaking. To make compatible supersymmetry breaking with the naturalness notion of ‘t Hooft, we must impose the constraint on the supersymmetry-breaking scale $M_{\text{SUSY}}$. A simple calculation yields that $M_{\text{SUSY}}$ should be less than few TeV.

### 2.2 Soft supersymmetry-breaking parameters

Since the pioneering works by Professor Iliopolos (who could not participate in this meeting) with P. Fayet [18] and the others in late 70’s, a lot of attempts to understand supersymmetry-breaking mechanism have been done. However, unfortunately, we still do not know how supersymmetry is really broken in nature. It, therefore, may be reasonable at this moment to pick up the common feature of supersymmetry breaking which effect the SM. The so-called
minimal supersymmetric standard model (MSSM) is “defined” along this line of thought. The MSSM contains the ordinary gauge bosons and fermions together with their superpartners, and two supermultiplets for the Higgs sector. (With one supermultiplet in the Higgs sector, it is not possible to give masses to all the fermions of the MSSM.)

It is expected that the common effect of supersymmetry breaking is to add the so-called soft supersymmetry-breaking terms (SSB) to the symmetry theory. The SSB terms are defined as those which do not change the infinity structure of the parameters of the symmetric theory. So they are additional terms in the Lagrangian that do not change the RG functions such as the $\beta$- and $\gamma$-functions of the symmetric theory. (More precisely, there exists a renormalization scheme in which the RG functions are not altered by the SSB terms.)

There exist four types of such terms [19].

1. Soft scalar mass terms: $(m^2)_i^j \phi_j \phi_i^*,$
2. $B$ - terms: $B^{ij}_i \phi_j \phi_i + \text{H.C}$, (7)
3. Gaugino mass terms: $M\lambda \lambda + \text{H.C}$,
4. Trilinear scalar couplings: $h^{ijk}_i \phi_i \phi_j \phi_k + \text{H.C}$,

where $\phi_j$ and $\lambda$ denote the scalar component in a chiral supermultiplet and the gaugino (the fermionic component) in a gauge supermultiplet, respectively.

If one insists only renormalizability for the MSSM, the number of the SSB parameters amounts to about 100, which is about five times of that of the SM. The commonly made assumption to reduce this number is the assumption of universality of the SSB terms, which is often justified by saying that supersymmetry breaking occurs in a flavor blind sector [20]. That is, it is assumed that the soft scalar masses and the trilinear scalar couplings are universal or flavor blind at the scale where supersymmetry breaking takes place. The so-called constrained MSSM contains thus only four independent massive parameters. But we could easily imagine that nature might not be so universal as one wants. In fact it possible to construct a lot of models with non-universal SSB terms [21] (even in models in which supersymmetry-breaking occurs in the so-called hidden sector which does not interact directly with the observable sector), and once we deviate from the universality, there will be chaotic varieties.
The application of reduction of couplings in the SSB sector is based on the assumption that the SSB terms organize themselves into a most economic structure that is consistent with renormalizability. I will come to discuss this later. I have spent a lot of time for low energy supersymmetry, because I wanted to argue that supersymmetric theories offer an ideal place where the reduction method, especially for massive parameters, can be applied and tested experimentally. It is worthwhile to mention that the current research program of Professor Zimmermann is the reduction of massive parameters [22].

3 Supersymmetric Gauge-Yukawa Unification

Before I come to discuss the SSB sector, I would like to stay in the sector of the dimensionless couplings in realistic supersymmetric GUTs and tell about certain phenomenological successes of reduction of parameters in these theories. I would like to emphasize that in contrast to the SM, supersymmetric GUTs can be asymptotically free or even finite.

3.1 Unification of the gauge and Yukawa couplings based on the principle of reduction of couplings

Few year ago, Professor Zimmermann and I were trying to apply the reduction method in the dimensionless sector of the MSSM, but we had no success. The main reason was that the power series solution to the reduction equation seemed to diverge. So we stopped to continue. About the same time, George Zoupanos (who unfortunately could not come here today) visited the Max-Planck-Institute, and told me that he obtains a top quark mass of about 180 GeV in a finite $SU(5)$ GUT [23]. Although the top quark was not found at that time (it was end of 1993, so just before we heard the rumor from Fermilab), 180 GeV for the top quark mass was a reasonable value. Finite theories have attracted many theorists. By a finite theory we mean a theory with the vanishing $\beta$-functions and anomalous dimensions. As we know, the $N = 4$ supersymmetric Yang-Mills theory is a well-known example [24]. And there were many attempts to construct $N = 1$ supersymmetric finite theories [23, 25, 26]. Klaus Sibold and his collaborators [27] gave an elegant existence proof of finite $N = 1$ supersymmetric theories, where I would like to recall that their proof is strongly based on the Adler-Bardeen
non-renormalization theorem of chiral anomaly \cite{28} (about which Professor Bardeen talked yesterday) \cite{3}. The reduction of Yukawa couplings in favor of the gauge coupling is one of the necessary condition for a theory to be finite in perturbation theory. So in a finite theory, Gauge-Yukawa unification is achieved. Since Gauge-Yukawa unification results from the reduction of Yukawa couplings in favor of the gauge coupling, it can be achieved not only in finite theories but also in non-finite theories, as Myriam Mondragón, George Zoupanos and myself explicitly showed \cite{30}. Relations among the gauge and Yukawa couplings, which are missing in ordinary GUTs, could be a consequence of a further unification provided by a more fundamental theory. And so Gauge-Yukawa unification is a natural extension to the ordinary GUT idea. This idea of unification relies on a symmetry principle as well as on the principle of reduction of couplings. The latter principle requires the existence of RG invariant relations among couplings, which do not necessarily result from a symmetry, but nevertheless preserve perturbative renormalizability or even finiteness as I mentioned.

3.2 The double-role of $\tan \beta$

Before I come to discuss Gauge-Yukawa unification more in details, I would like to talk about an important parameter, $\tan \beta$, in the MSSM. It is a very popular parameter among SUSY physicists, but let me allow to spend few minutes for this parameter, because it plays also an important role for Gauge-Yukawa unification. As I mentioned the MSSM contains two Higgs supermultiplets. The most general form of the Higgs potential which is consistent with renormalizability and with the softness of the SSB parameters can be written as

\begin{equation}
V = (m_{H_d}^2 + |\mu_H|^2) \hat{H}_d^\dagger \hat{H}_d + (m_{H_u}^2 + |\mu_H|^2) \hat{H}_u^\dagger \hat{H}_u + (B \hat{H}_d \hat{H}_u + \text{H.C.})
+ \frac{\pi}{2} (3\alpha_1^2/5 + \alpha_2^2)(\hat{H}_d^\dagger \hat{H}_d - \hat{H}_u^\dagger \hat{H}_u)^2,
\end{equation}

where $\mu_H$ is the only massive parameter in the supersymmetric limit, while $m_{H_u}^2$, $m_{H_d}^2$ and $B$ are the SSB parameters in this sector. ($m_{H_u}^2$, $m_{H_d}^2$ are real while $\mu_H$ and $B$ may be complex parameters.) Here $\hat{H}_{u,d}$ denote the scalar components of the two Higgs supermultiplets. There are four independent massive parameters in this sector as we can see in (8). These

\footnote{It is currently studied how to extend their theorem; for instance a non-perturbative extension has also been proposed in \cite{29}.}
parameters should give the only one independent mass parameter of the SM, for instance
the mass of \( Z \). Now instead of regarding these parameters as independent one can regard
also the ratio of the vacuum expectation values \( \tan \beta \equiv \frac{\langle \hat{H}_u \rangle}{\langle \hat{H}_d \rangle} \) \( (9) \)
as independent. \( (\tan \beta \) can be assumed to be real.\) Usually one regards \(|\mu_H|\) and \( B \) as
dependent\( [32] \). So the Higgs sector in the tree approximation is characterized by the parameters
\( \tan \beta , m^2_{H_1} , m^2_{H_2} \). \( (10) \)
The crucial point for Gauge-Yukawa unification is that \( \tan \beta \) plays a double-role. On one
hand, it is a parameter in the Higgs potential as we have seen above, and on the other hand
it it is a mixing parameter to define the standard model Higgs field out of the two Higgs
fields of the MSSM. That is, \( \tan \beta \) appears also in the dimensionless sector, and in fact it
can be fixed through Gauge-Yukawa unification with the knowledge of the tau mass \( M_\tau \), as
I would like to explain it more in detail below.

### 3.3 How to predict \( M_t \) from Gauge-Yukawa Unification

The consequence of a Gauge-Yukawa unification in a GUT is that the gauge and Yukawa
couplings are related above the GUT scale \( M_{\text{GUT}} \). In the following discussions we consider
only the Gauge-Yukawa unification in the third generation sector\[5\]:

\[
g_i = \kappa_i g \sum_{n=1}^{\infty} \left( 1 + \kappa_i^{(n)} g^{2n} \right) (i = 1, 2, 3, t, b, \tau),
\]
\( (11) \)

where \( g \) denotes the unified gauge coupling, \( g_i \) denote the gauge and Yukawa couplings of
the MSSM. Note that the constants \( \kappa_i \)'s can be explicitly calculated from the principle of
reduction of couplings. Once \( \tan \beta \) and the Yukawa couplings are known, the fermion masses
can be calculated as one can easily see from the tree level mass formulae

\[
M_t = \sqrt{2} \frac{M_Z}{g_2} \sin \beta \cos \theta_W g_t, \quad M_{b,\tau} = \sqrt{2} \frac{M_Z}{g_2} \cos \beta \cos \theta_W g_{b,\tau},
\]
\( (12) \)

\[4\]If \( \tan \beta \) is real as we assume here, \( B \) can become complex starting in one-loop order \( [31] \).

\[5\]A naive extension to include other generations into this scheme fails phenomenologically.
Table 1: The predictions for different $M_{\text{SUSY}}$ for the finite $SU(5)$ model.

| $M$ [GeV] | $\alpha_3(M_Z)$ | $\tan \beta$ | $M_{\text{GUT}}$ [GeV] | $M_b$ [GeV] | $M_t$ [GeV] |
|-----------|-----------------|---------------|-----------------|----------|----------|
| 800       | 0.118           | 48.2          | $1.3 \times 10^{16}$ | 5.4      | 173      |
| $10^3$    | 0.117           | 48.1          | $1.2 \times 10^{16}$ | 5.4      | 173      |
| $1.2 \times 10^3$ | 0.117 | 48.1          | $1.1 \times 10^{16}$ | 5.4      | 173      |

where $M_t$, $M_b$ and $M_\tau$ are the masses of the top and bottom quarks and tau, respectively. Assume that we use the tau mass $M_\tau$ as input and also that below $M_{\text{SUSY}}$ ($> M_t$) the effective theory of the GUT is the SM. At $M_{\text{SUSY}}$ the couplings of the SM and MSSM have to satisfy the matching conditions:

\[
\alpha_i^{\text{SM}} = \alpha_i \sin^2 \beta, \quad \alpha_b^{\text{SM}} = \alpha_b \cos^2 \beta, \quad \alpha_r^{\text{SM}} = \alpha_r \cos^2 \beta, \\
\alpha_\lambda = \frac{1}{4} (3 \alpha_1 + \alpha_2) \cos^2 2\beta \quad (\alpha_i = \frac{g_i^2}{4\pi}),
\]

(13)

where $\alpha_i^{\text{SM}}$ ($i = t, b, \tau$) are the SM Yukawa couplings and $\alpha_\lambda$ is the Higgs coupling. It is now easy to see that there is no longer freedom for $\tan \beta$ because with a given set of the input parameters, especially $M_\tau = 1.777$ GeV and $M_Z = 91.187$ GeV, the matching conditions (13) at $M_{\text{SUSY}}$ and the Gauge-Yukawa unification boundary condition (11) at $M_{\text{GUT}}$ can be simultaneously satisfied only if we have a specific value of $\tan \beta$. In this way Gauge-Yukawa unification enables us to predict the top and bottom masses in supersymmetric GUTs.

Table 1 shows the predictions in the case of a finite $SU(5)$ GUT [26], in which the one-loop reduction solution is given by

\[
g_t^2 = \frac{4}{5} g^2, \quad g_b^2 = g_\tau^2 = \frac{3}{5} g^2.
\]

(14)

The experimental value of $M_t$, $M_b$ and $\alpha_3(M_Z)$ are [14]

\[
\alpha_3(M_Z) = 0.119 \pm 0.002 \, , \, M_t = 173.8 \pm 5.2 \, \text{GeV} \, , \, M_b = 5.2 \pm 0.2 \, \text{GeV}.
\]

(15)

We see that the predictions of the model reasonably agree with the experimental values.

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6There are MSSM threshold corrections to the matching conditions [33, 34], which are ignored here.

7The correction to $M_b$ coming from the MSSM superpartners can be as large as 50% for very large values of $\tan \beta$ [33, 34]. In Table 1 we have not included these corrections because they depend on the SSB parameters. The GUT threshold correction are ignored too.
This means among other things that the top-bottom hierarchy could be explained to a certain extent in this Gauge-Yukawa unified model, which should be compared with how the hierarchy of the gauge couplings of the SM can be explained if one assumes the existence of a unifying gauge symmetry at $M_{\text{GUT}}$ \cite{35}. More details on the different gauge-Yukawa unified models and their predictions can be found in \cite{7, 36, 37}.

4 Reduction of Massive Parameters: Application to the Soft Supersymmetry-Breaking Sector

To formulate reduction of massive parameters, one first has to formulate reduction of dimensionless parameters in a massive theory, which was initiated by Klaus Sibold and his collaborator Piguet \cite{38}, about ten years ago. To keep the generality of the formulation in the massive case is much more involved than in the massless case, because the RG functions now can depend on the ratios of mass parameters in a complicated way. In the massless case they are just power series in coupling constants (at least in perturbation theory). For phenomenological and also practical applications of the reduction method, it is therefore most convenient to work in a mass independent renormalization scheme, such as the dimensional renormalization scheme. There exists a transformation of one scheme to another one, which was in fact proven first by Dieter Maison in the $\phi^4$ theory as far as I am informed, but not published. As I mentioned, the current research program of Professor Zimmermann is to include into the reduction program the massive parameters. He has already succeeded to carry out the program in the most general case and is able to show the renormalization scheme independence of the reduction method \cite{22}. Consequently, there exist a transformation of a set of the reduction solutions in a mass-dependent renormalization scheme into a set of the reduction solutions in a mass-independent renormalization scheme, which generalizes the unpublished result of Dieter Maison. Thus, the naive treatment on the massive parameters (which was performed in phenomenological analyses \cite{39, 40}) can now be justified by his theorem \footnote{It is assumed in the theorem that the $\beta$-functions in a mass-dependent renormalization scheme have a sufficiently smooth behavior in the massless limit \cite{22}.}.
4.1 Application to the minimal model

Now I would like to come to the SSB sector of a supersymmetric GUT. Recall that the
Higgs potential (8) (in the tree approximation) is completely characterized by the soft scalar
masses $m_{H_u}^2$, $m_{H_d}^2$ and tan $\beta$, where tan $\beta$ is fixed through Gauge-Yukawa unification as we
have seen before. We applied the the reduction method of massive parameters to the
SSB sector of the minimal supersymmetric $SU(5)$ GUT with Gauge-Yukawa unification in
the third generation ($g_t^2 = (2533/2605)g^2$, $g_b^2 = g^2 = (1491/2605)g^2$) [30], and obtained the
reduction solution

$$h_t = g_t M, \quad h_b = g_b M,$$
$$m_{H_u}^2 = -\frac{569}{521}M^2, \quad m_{H_d}^2 = -\frac{460}{521}M^2,$$
$$m_{h_R}^2 = m_{\tau_L}^2 = m_{\nu_L}^2 = \frac{436}{521}M^2,$$
$$m_{d_R}^2 = m_{e_L}^2 = m_{\nu_e}^2 = m_{s_R}^2 = m_{\mu_L}^2 = m_{\nu_\mu}^2 = \frac{8}{5}M^2,$$
$$m_{t_L}^2 = m_{b_L}^2 = m_{t_R}^2 = m_{\tau_R}^2 = \frac{545}{521}M^2,$$
$$m_{u_L}^2 = m_{d_L}^2 = m_{u_R}^2 = m_{e_R}^2 = m_{c_L}^2 = m_{s_L}^2 = m_{c_R}^2 = m_{\mu_R}^2 = \frac{12}{5}M^2$$

in the one-loop approximation, where $h_i$’s are the trilinear scalar couplings, $m_i$’s are the
soft scalar masses, and $M$ is the unified gaugino mass. We found moreover that we can
consistently regard $\mu_H$ and $B$ as free parameters. As we can see from (16) and (17) the unified
gaugino mass parameter $M$ plays a similar role as the gravitino mass $m_{2/3}$ in supergravity
coupled to a GUT and characterizes the scale of the supersymmetry-breaking [4]. Note that
the reduction solution for the soft scalar masses (17) is not of the universal form while those
for the trilinear couplings (16) are universal in the one-loop approximation.

Regarding the reduction solutions (16) and (17) as boundary conditions at $M_{GUT}$ in the
minimal supersymmetric GUT with Gauge-Yukawa unification in the third generation, we
can compute the spectrum of the superpartners of the MSSM, which is shown in Table 2,
where we have used the unified gaugino mass $M = 0.5$ TeV. The mass values [10] in Table 2 are

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9See for instance [2].
10For the mass of the lightest Higgs, the RG improved corrections [1] are included.
Table 2: The prediction of the superpartner spectrum for $M = 0.5$ TeV in the minimal gauge-Yukawa unified model. The mass unit is TeV.

| $m_{\chi_1}$ | 0.22 | $m_{\tilde{s}_1} = m_{\tilde{d}_1}$ | 1.18 |
|--------------|------|----------------------------------|------|
| $m_{\chi_2}$ | 0.42 | $m_{\tilde{s}_2} = m_{\tilde{d}_2}$ | 1.30 |
| $m_{\chi_3}$ | 0.90 | $m_{\tilde{\tau}_1}$           | 0.42 |
| $m_{\chi_4}$ | 0.91 | $m_{\tilde{\tau}_2}$           | 0.59 |
| $m_{\chi_\pm}$ | 0.42 | $m_{\tilde{\nu}_\tau}$         | 0.54 |
| $m_{\chi_{\pm}}$ | 0.91 | $m_{\tilde{\mu}_1} = m_{\tilde{e}_1}$ | 0.72 |
| $m_{t_1}$    | 0.87 | $m_{\tilde{\mu}_2} = m_{\tilde{e}_2}$ | 0.80 |
| $m_{t_2}$    | 1.03 | $m_{\tilde{\tau}_\mu} = m_{\tilde{\nu}_e}$ | 0.72 |
| $m_{b_1}$    | 0.87 | $m_A$                           | 0.33 |
| $m_{b_2}$    | 1.01 | $m_{H^\pm}$                     | 0.34 |
| $m_{\tilde{\tau}_1} = m_{\tilde{\mu}_1}$ | 1.26 | $m_{H}$                       | 0.33 |
| $m_{\tilde{\tau}_2} = m_{\tilde{\mu}_2}$ | 1.30 | $m_H$                         | 0.124 |
| $M_3$    | 1.16 |                                  |      |
Table 3: The predictions from the dimensionless sector of the minimal model. \((M = 0.5 \text{ TeV})\)

| \(\alpha_3(M_Z)\) | \(\tan \beta\) | \(M_{\text{GUT}} \text{ [GeV]}\) | \(M_b \text{ [GeV]}\) | \(M_t \text{ [GeV]}\) |
|---------------------|----------------|----------------|----------------|----------------|
| 0.119               | 48.8           | \(1.47 \times 10^{16}\) | 5.4            | 177            |

the running masses at \(M_{\text{SUSY}}\) which is \(\sim 0.95 \text{ TeV}\) \(^\dagger\) for \(M = 0.5 \text{ TeV}\). The prediction above depends basically only on the unified gaugino mass \(M\), and so the model has an extremely strong predictive power. Note also that \(m_{H_u}^2\), \(m_{H_d}^2\) and \(\tan \beta\) (see the Higgs potential (8) and the definition (9)) are now fixed outside of the Higgs sector, so that there is no guaranty that the Higgs potential (8) yields the desired symmetry breaking of \(SU(2)_L \times U(1)_Y\) gauge symmetry. Surprisingly, in the case at hand it does! (If the sign of \(m_{H_u}^2\) in (17) were different, for instance, it would not do.) In Table 3 I give the predictions from the dimensionless sector of the model. At last but not least we would like to emphasize that the reduction solutions (16) and (17) do not lead to the flavor changing neutral current (FCNC) problem. This is not something put ad hoc by hand; it is a consequence of the principle of reduction of couplings.

5 Sum Rules for the Soft Supersymmetry-Breaking Parameters

5.1 Renormalization group invariant sum rules

Now I would like to come the next topic. To proceed I recall the result of the reduction of the SSB parameters in favor of the unified gaugino mass \(M\) in the minimal SUSY \(SU(5)\) model which I have discussed just above. As we have seen, the reduction solutions for the trilinear couplings are universal while those for the soft scalar masses are not (see (18) and (19)). However, if one adds the soft scalar mass squared in an appropriate way, one finds

\[^{11}M_{\text{SUSY}}\] is no longer an independent parameter and we use \(M_{\text{SUSY}}^2 = (m_{t_1}^2 + m_{t_2}^2)/2\), where \(m_{t_1,2}\) are the masses of the superpartners of the top quark.
something interesting \[12\]. For instance,

\[ M^2 = m_{t_L}^2 + m_{t_R}^2 + m_{H_u}^2 = m_{b_L}^2 + m_{b_R}^2 + m_{H_d}^2. \]  

(18)

This is not an accidental coincidence. One can in fact show that the sum rules in this form are RG invariant at one-loop \[12\].

In last years there have been continues developments \[13\]–\[17\] in computing the RG functions in softly broken supersymmetric Yang-Mills theories, and the well-known result on the QCD $\beta$-function obtained by Professor Zakharov and his collaborators \[18\] \[17\] has been generalized so as to include to the SSB sector \[13\]–\[17\], which is based on a clever spurion superfield technique along with power counting \[13\]. Using this result, it is possible to find a closed form of the sum rules that are RG invariant to all orders in perturbation theory \[14\]–\[17\].

To be specific, we consider a softly broken supersymmetric theory described by the superpotential

\[ W = \frac{1}{6} Y^{ijk} \Phi_i \Phi_j \Phi_k + \frac{1}{2} \mu^{ij} \Phi_i \Phi_j, \]  

(19)

along with the Lagrangian for the SSB terms,

\[ - \mathcal{L}_{SB} = \frac{1}{6} h^{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} b^{ij} \phi_i \phi_j + \frac{1}{2} (m^2)^i_j \phi^* i \phi_j + \frac{1}{2} M \lambda \lambda + \text{H.c.}, \]  

(20)

where $\Phi_i$ stands for a chiral superfield with its scalar component $\phi_i$, and $\lambda$ is the gaugino field. It has been found \[45\] that the expressions \[13\]

\[ b^{ij} = -M \mu^{ij} \frac{d \ln \mu^{ij}(g)}{d \ln g}, \]

\[ h^{ijk} = -M \frac{d Y^{ijk}(g)}{d \ln g}, \]  

(21)

\[ m_i^2 = \frac{1}{2} |M|^2 (g/\beta_g) \frac{d \gamma_i(g)}{d \ln g} \]  

(22)

\[ ^{12}\text{Klaus Sibold pointed out that there is some correction to this } \beta \text{-function. See} \[27\] \text{for the argument.} \]

\[ ^{13}\text{It is not clear at the moment in which class of renormalization schemes exactly the result is valid; a renormalization scheme independent investigation of this result is certainly desirable.} \]

\[ ^{14}\text{The Yukawa couplings } Y^{ijk} \text{ and } \mu^{ij} \text{ are assumed to be functions of the gauge coupling } g. \]
are RG invariant to all orders in perturbation theory in a certain class of renormalization schemes, which are the higher order results for the one-loop reduction solutions (13) and (17). Similarly, the sum rule (18) in higher orders becomes

\[ m_i^2 + m_j^2 + m_k^2 = |M|^2 \left\{ \frac{1}{1 - g^2 C(G)/(8\pi^2)} \frac{d\ln Y_{ijk}}{d\ln g} + \frac{1}{2} \frac{d^2 \ln Y_{ijk}}{d(\ln g)^2} \right\} + \sum_l m_l^2 T(R_l) \frac{d\ln Y_{ijk}}{d\ln g}, \]

(23)
in the renormalization scheme which corresponds to that of [48]. Here \( C(G) \) is the quadratic Casimir in the adjoint representation, \( T(R) \) stands for the Dynkin index of the representation \( R \), \( \beta_g \) is the \( \beta \)-function of the gauge coupling \( g \), and \( \gamma_i \) is the anomalous dimension of \( \Phi_i \). These expressions look slightly complicated. But if one uses the freedom of reparametrization [3] (as discussed in the previous talk of Professor Oehme), they can be transformed into a more simple form \((d\ln Y_{ijk}/d\ln g = 1)\):

\[ h_{ijk} = -Y_{ijk}(g)M, \]

(24)

\[ m_i^2 + m_j^2 + m_k^2 = |M|^2 \frac{1}{1 - g^2 C(G)/(8\pi^2)} + \sum_l \frac{m_l^2 T(R_l)}{C(G) - 8\pi^2/g^2}, \]

(25)

It is exactly this form which coincides with the results obtained in certain orbifold models of superstrings [26]. I believe that this coincidence is not accidental, and I also believe that target-space duality invariance [51], which is supposed to be an exact symmetry of compactified superstring theories [19] is most responsible for the coincidence. In fact there exist already some indications for that. I hope I can report on the true reason of this interesting coincidence in near future.

5.2 Finiteness and sum rules

At this stage it may be worthwhile to mention that the reduction solution (21) and the sum rules (23) ensure the finiteness of the SSB sector in a finite theory [17]. For the \( N = 4 \) supersymmetric Yang Mills theory written in terms of \( N = 1 \) superfields, for instance, we

[15] Tree-level sum rules (like (18) in string theories are found in [42], [49]-[50]

[16] See [52], for instance, for target-space duality.

[17] There exists a fine difference in the opinions about this point. See, for instance, [44, 45].
have $\sum_i m_i^2 T(R_i) = (m_i^2 + m_j^2 + m_k^2) C(G)$ so that the all order sum rule (25) assumes the tree level form $m_i^2 + m_j^2 + m_k^2 = |M|^2$. Applied to the finite $SU(5)$ model [26] which I discussed in the previous section (Table 1 presents the prediction from the dimensionless sector), it means that the sum rules [26]

$$m_{10}^2 + 2m_{10}^2 = M^2, \ m_{H_u}^2 - 2m_{10}^2 = -\frac{M^2}{3}, \ m_{\tilde{f}}^2 + 3m_{10}^2 = \frac{4M^2}{3}$$ (26)

should be satisfied at and above $M_{GUT}$ for the two-loop finiteness of the SSB sector requires that, where

$$m_{10} = m_{t_L} = m_{b_L} = m_{t_R} = m_{\tau_R}, \ m_{\tilde{f}} = m_{b_R} = m_{\tau_L} = m_{\nu_R}.$$ (27)

In this case we have an additional free parameter, $m_{10}$, in the SSB sector. It turned out that the mass of a superpartner of the tau (s-tau) tends to become very light in this model. Consequently, in order to obtain a neutral lightest superparticle (LSP) (because we assume that $R$-parity is intact), we have to have a large unified gravitino mass $M > 0.8$ TeV. For $M = 1$ TeV, only the window $0.62$ TeV < $m_{10}$ < $0.66$ TeV is allowed. In Table 4 we give the prediction of the superpartner spectrum of the model for $m_{10} = 0.62/0.66$ TeV and $M = 1$ TeV. We have assumed the universal soft masses for the first two generations. But this assumption does not change practically our prediction of the spectrum expect for those that are directly of the first two generations.

### 5.3 Sum rules in the superpartner spectrum

The sum rules (18) or (25) can be translated into the sum rules of the superpartner spectrum of the MSSM [53] as I will show now. To be specific we assume an $SU(5)$ type Gauge-Yukawa unification in the third generation of the form (11). For a given model, the constants $\kappa$’s are fixed, but here we consider them as free parameters. As before we use the tau mass $M_\tau$ as an input parameter, and we go from the parameter space ($\kappa_t, \kappa_b$) to another one ($\kappa_t, \tan \beta$), because in this analysis we use the physical top quark $M_t$, too, as an input parameter. Then the unification conditions of the gauge and Yukawa couplings of the MSSM (i.e., $g = g_1 = g_2 = g_3, \ g_b = g_\tau$) fixes the allowed region (line) in the $\kappa_t - \tan \beta$ space for a given value of the unified gaugino mass $M$. The parameter space in the SSB sector at $M_{GUT}$
Table 4: The predictions of the superpartner spectrum for the finite SU(5) model. $M = 1$ TeV and $m_{10} = 0.62/0.66$ TeV.

| $m_{\chi_1}$ | 0.45/0.45 | $m_{\tilde{\chi}_1} = m_{\tilde{d}_1}$ | 1.95/1.95 |
|--------------|-----------|----------------------------------|-----------|
| $m_{\chi_2}$ | 0.84/0.84 | $m_{\tilde{\chi}_2} = m_{\tilde{d}_2}$ | 2.06/2.05 |
| $m_{\chi_3}$ | 1.29/1.32 | $m_{\tilde{\tau}_1}$ | 0.46/0.46 |
| $m_{\chi_4}$ | 1.29/1.32 | $m_{\tilde{\tau}_2}$ | 0.73/0.66 |
| $m_{\chi_1^\pm}$ | 0.84/0.84 | $m_{\tilde{\nu}_e}$ | 0.70/0.57 |
| $m_{\chi_2^\pm}$ | 1.29/1.32 | $m_{\tilde{\nu}_1} = m_{\tilde{\nu}_1}$ | 0.70/0.71 |
| $m_{\tilde{\chi}_1}$ | 1.50/1.51 | $m_{\tilde{\nu}_2} = m_{\tilde{\nu}_2}$ | 0.89/0.89 |
| $m_{\tilde{\chi}_2}$ | 1.72/1.74 | $m_{\tilde{\nu}_\mu} = m_{\tilde{\nu}_e}$ | 0.89/0.88 |
| $m_{\tilde{b}_1}$ | 1.51/1.46 | $m_A$ | 0.63/0.77 |
| $m_{\tilde{b}_2}$ | 1.70/1.71 | $m_{H^\pm}$ | 0.63/0.77 |
| $m_{\tilde{c}_1} = m_{\tilde{u}_1}$ | 1.96/1.96 | $m_H$ | 0.63/0.77 |
| $m_{\tilde{c}_2} = m_{\tilde{u}_2}$ | 2.05/2.05 | $m_h$ | 0.127/0.127 |
| $M_3$ | 2.21/2.21 | | |
is constrained due to unification:

\[
M = M_1 = M_2 = M_3, \\
m_{t_R}^2 = m_{t_L}^2 = m_{b_L}^2 = m_{\tau_R}^2, \\
m_{b_R}^2 = m_{\tau_L}^2 = m_{\nu_{\tau}}^2,
\]

where \(M_i (i = 1, 2, 3)\) are the gaugino masses for \(U(1)_Y\) (bino), \(SU(2)_L\) (wino) and \(SU(3)_C\) (gluino). And the one-loop sum rules at \(M_{\text{GUT}}\) yield

\[
h_t = -M, h_b = h_\tau = -M g_b , M^2 = m_{\Sigma(t)}^2 = m_{\Sigma(b)}^2 = m_{\Sigma(\tau)}^2,
\]

where

\[
m_{\Sigma(i)}^2 \equiv m_{t_R}^2 + m_{t_L}^2 + m_{H_u}^2, \\
m_{\Sigma(b,\tau)}^2 \equiv m_{b_{R,\tau}}^2 + m_{b_{L,\tau}}^2 + m_{H_u}^2.
\]

(The above equations are the same as (16) and (18), respectively.) I would like to emphasize that in the one-loop RG evolution of \(m_{\Sigma}^2\)'s in the MSSM only the same combinations of the sum of \(m_i^2\)'s enter. Therefore, as far as we are interested in the evolution of \(m_{\Sigma}^2\)'s, we have only one additional parameter \(M_{\text{SUSY}}\). To derive the announced sum rules for the superpartner spectrum, we define

\[
s_i \equiv m_{\Sigma(i)}^2/M_3^2 \quad (i = t, b, \tau) \quad \text{at} \quad Q = M_{\text{SUSY}}.
\]

The parameters \(s_i\)'s do not depend on the value of the unified gaugino mass \(M\), but they do on \(\tan\beta\). This dependence is shown in Fig. 4. We then express the masses of the superpartners in terms of the soft scalar masses and the masses of the ordinary particles to obtain the sum rules

\[
-cos 2\beta \ m_A^2 = (s_b - s_t)M_3^2 + 2(\hat{m}_t^2 - m_t^2) - 2(\hat{m}_b^2 - m_b^2) \\
= (s_\tau - s_t)M_3^2 + 2(\hat{m}_t^2 - m_t^2) - 2(\hat{m}_\tau^2 - m_\tau^2),
\]

where \(m_A^2\) is the neutral pseudoscalar Higgs mass squared, and \(\hat{m}_i^2\) stands for the arithmetic mean of the two corresponding scalar superparticle mass squared.
Since we have assumed an $SU(5)$-type supersymmetric GUT with a gauge-Yukawa unification in the third generation, the result (32) is not a direct consequence of a superstring model, although the form of the sum rules in both kinds of unification schemes might coincide with each other as I mentioned (see footnote 11). However, under the following circumstances (only rough), the sum rules (32) could be a consequence of a superstring model: (i) The Yukawa coupling of the third generation is field-independent in the corresponding effective $N = 1$ supergravity. (ii) Below the string scale an $SU(5)$-type gauge-Yukawa unification is realized so that the sum rules are RG invariant below the string scale and are satisfied down to $M_{\text{GUT}}$. (iii) Below $M_{\text{GUT}}$ the effective theory is the MSSM.

The sum rules (32) could be experimentally tested if the superpartners are found in future experiments, e.g., at LHC. In any event, an experimental verification of the sum rules of the SSB parameters would give an interesting information on physics beyond the GUT scale.

6 Conclusion

Now I will come to conclusion. Professor Zimmermann, obviously an interesting feature is coming. So please keep staying in physics and experience new developments in physics with us.
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References

[1] W. Zimmermann, Com. Math. Phys. 97 (1985) 211; R. Oehme and W. Zimmermann, Com. Math. Phys. 97 (1985) 569.

[2] R. Oehme, K. Sibold and W. Zimmermann, Phys. Lett. B147 (1984) 117; B153 (1985) 142.

[3] R. Oehme, Prog. Theor. Phys. Suppl. 86 (1986) 215.

[4] N.-P. Chang, Phys. Rev. D10 (1974) 2706; N.-P. Chang, A. Das and J. Perez-Mercader, Phys. Rev. D22 (1980) 1829; E.S. Fradkin and O.K. Kalashnikov, J. Phys. A8 (1975) 1814; Phys. Lett. 59B (1975) 159; 64B (1976) 177; E. Ma, Phys. Rev. D11 (1975) 322; D17 (1978) 623; D31 (1985) 1143; Prog. Theor. Phys. 54 (1975) 1828; Phys. Lett. 62B (1976) 347; Nucl. Phys. B116 (1976) 195; D.I. Kazakov and D.V. Shirkov, Singular Solutions of Renormalization Group Equations and Symmetry of the Lagrangian, in Proc. of the 1975 Smolence Conference on High Energy Particle Interactions, eds. D. Krupa and J. Pisut (VEDA, Publishing House of the Slovak Academy of Sciences, Bratislava 1976).

[5] H. Georgi and A. Pais, Phys. Rev. D10 (1974) 539.

[6] R. Oehme, Reduction in Coupling Parameter Space, in Proc. of Anomalies, Geometry and Topology, ed. A White (World Scientific, Singapore, 1985) pp. 443; Reduction of Coupling Parameters, hep-th/9511006 in Proc. of the XVIIIth Int. Workshop on
High Energy Physics and Field Theory, June 1995, Moscow-Protvino; W. Zimmermann, Renormalization Group and Symmetries in Quantum Field Theory, in Proc. of the 14th ICGTMP, ed. Y.M. Cho (World Scientific, Singapore, 1985) pp. 145; K. Sibold, Reduction of Couplings, Acta Physica Polonica, 19 (1989) 295; J. Kubo, Is there any relation between dynamical symmetry breaking and reduction of couplings?, in Proc. of the 1989 Workshop on Dynamical Symmetry Breaking, eds. T. Muta and K. Yamawaki, Nagoya 1989, pp. 48.

[7] J. Kubo, M. Mondragón and G. Zoupanos, Acta Phys. Polon. B27 (1997) 3911.

[8] J. Kubo, K. Sibold and W. Zimmermann, Nucl. Phys. B259 (1985) 331; Phys. Lett. B220 (1989) 185.

[9] J. Avan and H.J. De Vega, Nucl. Phys. B269 (1986) 621; H. Meyer-Ortmanns, Phys. Lett. B186 (1987) 195; G. Grunberg, Phys. Rev. Lett. 58 (1987) 1180; K. Sibold and W. Zimmermann, Phys. Lett. B191 (1987) 427; T.E. Clark and S.T. Love, Mod. Phys. Lett. A3 (1988) 661; K.S. Babu and S. Nandi, Oklahoma State University Preprint, OSU-RN-202 (1988); W.J. Marciano, Phys. Rev. Lett. 62 (1989) 2793; J. Kubo, K. Sibold and W. Zimmermann, Phys. Lett. B220 (1989); F.M. Renard and D. Schildknecht, Phys. Lett. B219 (1989) 481; M. Bastero-Gil and J. Perez-Mercader, Phys. Lett. B247 (1990) 346; A. Denner, Nucl. Phys. B347 (1990) 184; E. Kraus, Nucl. Phys. B349 (1991) 563; B354 (1991) 245; F. Cooper, Phys. Rev. D43 (1991) 4129, [Erratum] D45 (1992) 3012; L.-N. Chang and N.-P. Chang, Phys. Rev. D45 (1992) 2988; L.A. Wills Toro, Z. Phys. C56 (1992) 635; R.J. Perry and K.G. Wilson, Nucl. Phys. B403 (1993) 587; K.G. Wilson, T.S. Wallhout, A. Harindranath, W.-M. Zhang and R.J. Perry and S. D. Glazek, Phys. Rev. D49 (1994) 6720-6766; A.B. Lahanas and V.C. Spanos, Phys. Lett. B334 (1994) 378; H. Skarke, Phys. Lett. B336 (1994) 32; E. A. Ammons, Phys. Rev. D50 (1994) 980; M. Harada, Y. Kikukawa, T. Kugo and H. Nakano, Prog. Theor. Phys. 92 (1994) 1161; B. Schrempp and F. Schrempp, Phys. Lett. B299 (1993) 321; B. Schrempp, Phys. Lett. B344 (1995) 193; A.A. Andrianov and N.V. Romanenko, Phys. Lett. B343 (1995) 295; N. Krasnikov, G. Kreyerhoff and R. Rodenberg, Nuovo Cim. 108A (1995) 565; A.A. Andrianov and R. Rodenberg, Nuovo Cim. 108A (1995) 577; J. Kubo, Phys. Rev. D52 (1995) 6475; M. Atance and J.L. Cortes, Phys. Lett. B387 (1996) 697; Phys. Rev. D54 (1996) 4973; D56 (1997) 3611; M.V. Chizhov, hep-ph/9610220; R. Oehme, Phys. Lett. B399 (1997) 67; [hep-th/9808054]; A.Karch and D. Lust and G.Zoupanos, Nucl. Phys. B529 (1998) 96; E. Umezawa, Prog. Theor. Phys. 100 (1998) 375; J. Erdmenger, C. Rupp and K. Sibold, Nucl. Phys. B530 (1998) 501; Y. Kawamura, T. Kobayashi and H. Shimabukuro, Phys. Lett. B436 (1998) 108; A. Karch, T. Kobayashi, J. Kubo and G. Zoupanos, Phys. Lett. B441 (1998) 235; R. J. Perry, To be published in the proceedings of APCTP - RCNP Joint International School on Physics of Hadrons and QCD, Osaka, Japan, 12-13 Oct 1998, nucl-th/9901080, and references therein.

[10] B. Pendleton and G.G Ross, Phys. Lett. B98 (1981) 291.

[11] W. Zimmermann, Lett. Math. Phys. 30 (1993) 61.

[12] W. Zimmermann, Phys. Lett. B308 (1993) 117.

[13] W.A. Bardeen, C.T. Hill and M. Lindner, Phys. Rev. D41 (1990) 1647.
[14] Particle Data Group, C. Caso et al., Eur. Phys. J. C3 (1998) 1.

[15] G. ’t Hooft, “Naturalness, chiral symmetry, and spontaneous chiral symmetry breaking”, in Recent developments in gauge theories, Cargèse, 1979.

[16] J. Wess and B. Zumino, Phys. Phys. B49 52.

[17] J. Iliopoulos and B. Zumino, Nucl. Phys. B76 (1974) 310; S. Ferrara, J. Iliopoulos and B. Zumino, Nucl. Phys. B77 (1974) 413; K. Fujikawa and W. Lang, Nucl. Phys. B88 (1975) 61.

[18] P. Fayet and J. Iliopoulos, Phys. Lett. B51 (1974) 461.

[19] S. Ferrara, L. Girardello and F. Palumbo, Phys. Rev. D20 (1979) 403; K. Harada and N. Sakai, Prog. Theor. Phys. 67 (1982) 1887; L. Girardello ans M.T. Grisaru, Nucl. Phys. B194 (1982) 65.

[20] H.P. Nilles, Phys. Rep. 110 (1984) 1; H.E Haber and G.L. Kane, Phys. Rep. 117 (1985) 75.

[21] L.J. Dixon, V.S. Kaplunovsky and J. Louis, Nucl. Phys. B382 (1992) 305; L. Ibáñez and D. Lüst, Nucl. Phys. B382 (1992) 305; Y. Kawamura, H. Murayama and M. Yamaguchi, Phys. Rev. D51 (1995) 1337.

[22] W. Zimmermann, “Reduction of Couplings in Massive Models of Quantum Field Theory”, talks given at Kanazawa university, March 1998, at the 12th Max Born Symposium, Wroclaw, September 1998, Max-Planck-Institute preprint MPI/PhT/98-97, and at the Ringberg Conference on Trends in Theoretical Particle Physics, Tegernsee October, 1998.

[23] D. Kapetanakis, M. Mondragón and G. Zoupanos, Zeit. f. Phys. C60 (1993) 181; M. Mondragón and G. Zoupanos, Nucl.Phys. B (Proc. Suppl) 37C (1995) 98.

[24] S. Mandelstam, Nucl. Phys. B182 (1981) 125.

[25] A. Parkes and P. West, Nucl. Phys. B222 (1983) 269; D.R.T. Jones, L. Mezincescu and Y.-P. Yao, Phys. Lett. B148 (1984) 317; A.J. Parkes and P.C. West, Phys. Lett. B138 (1984) 99; Nucl. Phys. B256 (1985) 340; P. West, Phys. Lett. B137 (1984) 371; D.R.T. Jones and A.J. Parkes, Phys. Lett. B160 (1985) 267; D.R.T. Jones and L. Mezinescu, Phys. Lett. B136 (1984) 242; B138 (1984) 293; A.J. Parkes, Phys. Lett. B156 (1985) 73; S. Hamidi, J. Patera and J.H. Schwarz, Phys. Lett. B141 (1984) 349; X.D. Jiang and X.J. Zhou, Phys. Lett. B197 (1987) 156; B216 (1985) 160; S. Hamidi and J.H. Schwarz, Phys. Lett. B147 (1984) 301; D.R.T. Jones and S. Raby, Phys. Lett. B143 (1984) 137; J.E. Bjorkman, D.R.T. Jones and S. Raby, Nucl. Phys. B259 (1985) 503; J. León et al., Phys. Lett. B156 (1985) 66; A.V. Ermushev, D.I. Kazakov and O.V. Tarasov, Nucl. Phys. B281 (1987) 72; D.I. Kazakov, Mod. Phys. Lett. A2 (1987) 663; Phys. Lett. B179 (1986) 352; D.I. Kazakov and I.N. Kondrashuk, Int. J. Mod. Phys. A7 (1992) 3869; K. Yoshioka, Kyoto University preprint KUNS-1444, hep-ph/9705443; L.E. Ibáñez, hep-ph/9801230; S. Kachru and E. Silverstein, Phys. Rev. Lett. 80 (1998) 4855; A. Hanany, M.J. Strassler and A. Uranga, Princeton University preprint IASSNS-HEP-23; hep-ph/9803080; A. Hanany and Y.-H. He, MIT preprint MIT-CTP-2803.
[26] T. Kobayashi, J. Kubo, M. Mondragón and G. Zoupanos, Nucl. Phys. B511 (1998) 45.

[27] C. Lucchesi, O. Piquet and K. Sibold, Helv. Phys. Acta 61 (1988) 321; O. Piquet and K. Sibold, Int. J. Mod. Phys. A1 (1986) 913; Phys. Lett. B177 (1986) 373.

[28] S.L. Adler and W.A. Bardeen, Phys. Rev. 182 (1969) 1517.

[29] R.G. Leigh and M.J. Strassler, Nucl. Phys. B447 (1995) 95; M.J. Strassler; Prog. Theor. Suppl. 123 (1996) 373.

[30] J. Kubo, M. Mondragón and G. Zoupanos, Nucl. Phys. B424 (1994) 291.

[31] A. Pilaftsis, Phys. Lett. B435 (1998) 88.

[32] K. Inoue, A. Kakuto, H. Komastu and S. Takeshita, Prog. Theor. Phys. 67 (1982) 1889; 68 (1983) 927.

[33] L. Hall, R. Rattazzi and U. Sarid, Phys. Rev. D50 (1994) 7048; M. Carena, M. Olechowski, S. Pokorski and C.E.M. Wagner, Nucl. Phys. B426 (1994) 269.

[34] B.D. Wright, Yukawa Coupling Thresholds: Application to the MSSM and the Minimal Supersymmetric SU(5) GUT, University of Wisconsin-Madison report, MAD/PH/812, hep-ph/9404217.

[35] H. Georgi, H. Quinn, S. Weinberg, Phys. Rev. Lett. 33 (1974) 451.

[36] J. Kubo, M. Mondragón, N.D. Tracas and G. Zoupanos, Phys. Lett. B342 (1995) 155; J. Kubo, M. Mondragón, S. Shoda and G. Zoupanos, Nucl. Phys. B469 (1996) 3.

[37] See J. Kubo, M. Mondragón, M. Olechowski and G. Zoupanos, Nucl. Phys. B479 (1996) 25.

[38] O. Piguet and K. Sibold, Phys. Lett. 229B (1989) 83.

[39] I. Jack and D.R.T. Jones, Phys. Lett. B349 (1995) 294; I. Jack, D.R.T. Jones and K.L. Roberts, Nucl. Phys. B455 (1995) 83; D.I. Kazakov, M.Yu. Kalmykov, I.N. Kondrashuk and A.V. Gladyshev, Nucl. Phys. B471 (1996) 387.

[40] J. Kubo, M. Mondragón and G. Zoupanos, Phys. Lett. B389 (1996) 523.

[41] H.E. Haber, R. Hempfling and A. Hoang, Z. Phys. C75 (1997) 539, and references therein.

[42] Y. Kawamura, T. Kobayashi and J. Kubo, Phys. Lett. B405 (1997) 64.

[43] Y. Yamada, Phys. Rev. D50 (1994) 3537; J. Hisano and M. Shifman, Phys. Rev. D56 (1997) 5475; I. Jack and D.R.T. Jones, Phys. Lett. B415 (1997) 383.; L.V. Avdeev, D.I. Kazakov and I.N. Kondrashuk, Nucl. Phys. B510 (1998) 289.

[44] D.I. Kazakov, Phys. Lett. B421 (1998) 211.
[45] I. Jack, D.R.T. Jones and A. Pickering, Phys. Lett. B426 (1998) 73.

[46] T. Kobayashi, J. Kubo and G. Zoupanos, Phys. Lett. B427 (1998) 291.

[47] I. Jack, D.R.T. Jones and A. Pickering, Phys. Lett. B432 (1998) 114.

[48] V. Novikov, M. Shifman, A. Vainstein and V. Zakharov, Nucl. Phys. B229 (1983) 381; Phys. Lett. B166 (1986) 329; M. Shifman, Int. J. Mod. Phys. A11 (1996) 5761 and references therein.

[49] A. Brignole, L.E. Ibáñez, C. Muñoz and C. Scheich, Z. f. Phys. C74 (1997) 157; A. Brignole, L.E. Ibáñez and C. Muñoz, Soft supersymmetry breaking terms from supergravity and superstring models, hep-ph/9707209.

[50] L.E. Ibáñez, A chiral $D = 4$, $N = 1$ string vacuum with a finite low-energy effective field theory, hep-th/9802103; New perspectives in string phenomenology from dualities, hep-ph/9804236.

[51] K. Kikkawa and M. Yamasaki, Phys. Lett. B149 (1984) 357; N. Sakai and I. Senda, Prog. Theor. Phys. 75 (1998) 692.

[52] J. Polchinski, “STRING THEORY”, vol. I and II, Cambridge University Press (1998).

[53] T. Kawamura, T. Kobayashi and J. Kubo, Phys. Lett. B432 (1998) 108.