The physics of kHz QPOs—strong gravity’s coupled anharmonic oscillators
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Abstract

We explain the origin of the puzzling high frequency peaks (QPOs) in the variability power spectra of accreting neutron stars and black holes as a non-linear 1:2 or 1:3 resonance between orbital and radial epicyclic motion. These resonances are present because the gravitational field deviates strongly from a Newtonian $1/r$ potential. Our theory agrees with the recently reported observations of two QPOs, at 300 Hz and 450 Hz, in the black hole candidate 1655-40.
Observations of accreting neutron stars in low-mass X-ray binaries reveal two preferred frequencies, none of which is fixed. These frequencies, called kHz QPOs (quasi-periodic oscillations), show up as peaks in the observed power spectrum (the Fourier transform of the time variation) in the X-ray flux, [1]. Their properties are a major puzzle.

The peaks typically come in pairs, at frequencies $\omega_1$ and $\omega_2$ both on the order of a kHz ($\times 2\pi$), but both varying considerably in any given source (by several hundred Hertz in intervals of hours), with their difference $\omega_1 - \omega_2$ showing markedly less variation. That the difference is not constant (as would be expected if one of the two observed frequencies were a beat between the other frequency and the constant rotation rate of the star) is a clear indication that two fundamental frequencies are present, none of which is the stellar spin frequency.

Another puzzle has been that in black hole candidates only one high frequency QPO had been reported, with properties similar to one of the variable kHz QPOs in neutron stars. Observations of the black hole candidate 1655-40, reported after the original submission date of this Letter‡, show that in fact two high frequencies show up also in black-hole candidates.

Newtonian gravity with the $1/r^2$ force law is scale-free, there is no preferred frequency (Fig. 1). If gravity were so described, only two fundamental frequencies would be expected for a quasi-spherical star with a thin accretion disk [2]: the stellar rotational frequency $\Omega_*$, and the Keplerian orbital frequency at the surface of the star $\Omega_K(R) = (GM/R^3)^{1/2}$. The most important fact about these two frequencies is that they are fixed for a star of fixed mass $M$, radius $R$, and angular momentum.

It has long been recognized [3] that for black holes general relativity predicts instead two other preferred frequencies, also fixed for a given gravitating body: the orbital frequency in the innermost (marginally) stable orbit, $\Omega(r_{ms})$, and the maximum epicyclic frequency $\omega_{max} = \max(\omega_r) \equiv \omega_r(r_{max})$, (Fig. 2). These reflect the presence of a characteristic scale, the gravitational radius $r_g = 2GM/c^2$. For example, in the Schwarzschild
metric $r_{ms} = 3r_g$, $r_{\text{max}} = 4r_g$, $\Omega = \Omega_K$, and $\omega_{\text{max}} = \Omega(4r_g)/2$. (The importance of the maximum in the epicyclic frequency was first stressed in a seminal paper by Kato and Fukue [4].) Thus, strong gravity presents us with two frequencies, but these are fixed for a given star and hence cannot be identified with the observed variable kHz QPOs in neutron stars.

However, we note that in contrast with Newtonian gravity of spherical bodies where the only frequency at a given radius is $\Omega_K(r)$, in general relativity turbulent noise may excite epicyclic motions at the different frequency $\omega_r$, so inhomogeneities in flow at radius $r$ contribute to the power spectrum at (angular) frequencies $\Omega$, and $\omega_r$, as well as at combination frequencies characteristic of coupled anharmonic oscillators (including rational fractions of eigenfrequencies), giving a rich structure to the power spectrum of X-ray variability, in agreement with observations.

We point out that in general relativity, in addition to the fixed frequencies $\Omega_s$, $\Omega(r_{ms})$, $\omega_r(r_{\text{max}})$, there are other preferred frequencies, those of 1:2, 1:3, etc., resonances between orbital and radial epicyclic frequencies. These are possible because the ratio of orbital and radial epicyclic frequencies tends to large values near the marginally stable orbit: $\Omega(r)/\omega_r(r) \to \infty$, as $r \to r_{ms}$ (Fig. 2). Frequencies in 1:2 or 1:3 ratio can be in resonance because epicyclic motion is anharmonic. As is usual for non-linear oscillators, the resonance occurs for a range of frequencies near the eigenfrequency of the oscillator, so the driving frequency (the orbital frequency here) need not be an exact multiple of the eigenfrequency of the epicyclic oscillator, nor need it be constant [5]. Thus, the resonant frequencies have just the properties which seemed puzzling in the power spectra of accreting neutron stars. We suggest that the kHz QPOs are caused by such resonances, and hence are manifestations of strong-field gravity.

Specifically, we suggest that one of the observed high QPO frequencies could be one of the variable orbital frequencies driving the 1:2, or 1:3 epicyclic resonance. All other things being equal, the most prominent peaks are expected where most of the luminosity
is generated, and that is between the marginally stable orbit, at \( r = r_{ms} \), where the disk terminates and the X-ray flux vanishes, and the radius \( r_{max} \) where \( \omega_r \) peaks. For the Schwarzschild metric the position (radius) of the 1:2 resonance, \( r_2 \), coincides with the epicyclic maximum, \( r_2 = r_{max} \), and for rotating bodies \( r_2 > r_{max} \), typically. For realistic metrics of rapidly rotating neutron stars, the radius of the 1:3 resonance, \( r_3 \), is close to \( r_{max} \). We expect the prominent resonance in the power spectrum to be the one closest to \( r_{max} \) on the side of the star, so for rapidly rotating neutron stars the most prominent resonance should be 1:3, i.e., the one with eigenfrequency \( \omega_r(r_3) = \Omega(r_3)/3 \).

For maximally rotating Kerr metric, \( r_3 \) and \( r_{max} \) nearly coincide, but in view of the relatively low accretion rate (per mass) of most Galactic black-hole candidates, such as 1655-40, it would not be surprising if their metrics were not maximally rotating Kerr. For accretion in the Schwarzschild metric not much luminosity is released at \( r_3 = (9/8)r_{ms} \). Thus, there should be much less power in the black hole QPO, than is the case for neutron stars, as indeed is observed.

We might remark that observations reveal similar electromagnetic spectra of the X-ray emissions of black hole candidates and of neutron stars, at least in some states. This strongly suggests that accretion disks in neutron stars are similar to those in black holes, as they would be if \( R < r_{ms} \), as preferred by our model. The fact that only one high frequency QPO is observed in black hole candidates has been puzzling. After this Letter was submitted, the discovery of a second high frequency peak in the black-hole candidate 1655-40 has been reported [6]. If the two frequencies are the orbital frequency and its beat with the epicyclic frequency in 1:3 resonance, as suggested by us, they should be (approximately) in 3:2 ratio. The reported frequencies in 1655-40 are about 450 Hz and 300 Hz.

In summary, strong-field effects of general relativity, and in particular metric properties of space-time around rapidly rotating neutron stars, make natural the excitation of a 1:3 or 1:2 anharmonic epicyclic resonance, driven by orbital motion whose variable (orbital)
frequency may be imprinted on the X-ray flux as a fairly prominent kHz QPO. A second QPO may be present at a frequency differing from the first by the epicyclic frequency of the same resonance, with the difference frequency varying to a lesser degree than the QPO frequency. The same high frequency QPOs may appear in black hole systems, and indeed they now have been reported in the black hole candidate 1655-40 ([6]).

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REFERENCES AND NOTES

† This manuscript was submitted to Physical Review Letters in December 2000, before the observational discovery of a second QPO in a black hole candidate has been reported. We have added on May 3, 2001 a few sentences explaining that the discovery confirms our theory.

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[2] It is thought that fluid in accretion disks of small vertical extent (“thin disks”) approximately follows circular geodesic trajectories. Observations of water masers in some thin disks in active galactic nuclei (AGNs) do indeed reveal a Keplerian distribution of velocity for the water masers, $v_K = (GM/r)^{1/2}$; Miyoshi, M., et al., Nature 373, 127 (1995).

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FIGURE CAPTIONS

Figure 1. The 1/r potential of a Newtonian point mass is scale-free. The orbital frequency in circular orbit and the frequency of epicyclic motions coincide (a small perturbation of a test particle originally in circular motion gives rise to periodic motions with the same period as in the original orbit).

Figure 2. In general relativity, the frequency of radial epicyclic motions, \( \omega_r \), goes to zero in the marginally stable orbit. The epicyclic frequency, \( \omega_r \), and the orbital frequency in circular orbits, \( \Omega \), both in units of \( c/r_g \), are shown as a function of the circumferential radius in units of the gravitational radius \( r_g = 2GM/c^2 \) (after Kato et al., 1998 [3]). The numerical values are for the Schwarzschild metric, where the marginally stable orbit is at \( r = 3r_g \), but the qualitative features of the frequencies are general.
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\[ \Omega, \omega \]
