Interaction-free measurement and forward scattering

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Interaction-free measurement is shown to arise from the forward-scattered wave accompanying absorption: a “quantum silhouette” of the absorber. Accordingly, the process is not free of interaction. For a perfect absorber the forward-scattered wave is locked both in amplitude and in phase. For an imperfect one it has a nontrivial phase of dynamical origin (“colored silhouette”), measurable by interferometry. Other examples of quantum silhouettes, all controlled by unitarity, are briefly discussed.

Interaction-free Measurement (IFM) is a term first used by Renninger [1], then by Dicke [2] to label the paradoxical observation that a negative-result quantum measurement, apparently without interaction, modifies the wave function of the non-detected object. Dicke’s resolution of the paradox is essentially to point out that the interaction involved in the act of measurement creates an entanglement between probe and object, thereby changing the state of their assembly irrespective of the outcome of the measurement, which is therefore not interaction-free. The present paper lends more support to the latter statement, based on a slightly different argument. Our aim is to present a clear physical picture of IFM, rather than suggesting strategies for improved performances.

The idea of IFM gained popularity with the paper of Elitzur and Vaidman [3] who amplified the argument by inventing an efficient interferometric setting for it. They suggested to place a perfect absorber in one arm of a Mach-Zehnder photon interferometer (Fig. 1). Without the absorber, on one of the exit ports (the “dark port”, in the figure: $D_a$) there is no output, because of destructive interference. Inserting the absorber changes the interference pattern; in particular, it suspends the destructive interference. Then some photons reach the previously silent detector, indicating that an absorber is there. That kind of interferometric detection of an absorber, by observing a photon that avoids absorption, is called interaction-free measurement in the growing recent literature on the subject.

Elitzur and Vaidman [3] dramatize the situation by calling the absorber a “bomb” that detects the act of absorption by exploding with 100% efficiency. The starting point of our discussion is this: the bomb plays a double role in the scheme; it is the combination of an absorber and a detector of its excited state. The act of its explosion - a quantum measurement process - should be distinguished from the preceding microscopic, unitary process of absorbing the photon. Detectors and detonators work on the final state of the unitary process, and they work under the constraint of wave-particle duality: only one term of the branching superposition can be actually detected. The absorptive branch is just one of the possible outcomes. However, even if not all terms of the superposition get detected, they are all there, leaving their imprints – silhouettes – on each other, forced by unitarity.

In what follows, we first describe that simplest situation along the line introduced by Elitzur and Vaidman [3]. We propose to use the language of scattering theory, in the sense of locating causal changes of the quantum state due to the presence of the absorber, as compared to its absence. In that context, the notion of forward scattering – an essential aspect of inelastic scattering, including absorption, as succinctly expressed by the Optical Theorem [6–8] – offers a natural framework to think

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1 That connection with wave-particle duality has been pointed out in Ref. [4]. This is a strange case however: a choice between final states and detectors of different nature.

2 An analysis from the point of view of photon - absorber entanglement, somewhat similar to ours, has been presented by Kwiat et al. [5].
about the phenomenon of IFM with all its recent versions.

“Forward scattering” is just another name for the wave-optical shadow cast by a scattering and eventually absorbing object. The image of the object formed by shadow scattering, visible against the interferometrically set featureless background, is what we propose to call a “quantum silhouette”, as discussed in more detail below.

The forthcoming analysis intends to describe what happens to a flying one-photon wave packet entering the interferometer. As usual in scattering theory, as a limiting case of long wave packets, one can think of quasi-one-dimensional plane waves; in that limiting case, however, care must be taken that the scattered wave is outgoing from the scatterer. The transverse extension of the quasi-plane wave is thought to be large with respect to the wavelength and small with respect to the size of the macroscopic absorber; in technical terms, one may think of a Gaussian or Bessel beam.

The Mach-Zehnder interferometer supports two photon modes \( a \) and \( b \), shaped by the respective mirrors indicated in Figure 1. Each of the beamsplitters causes \( a \leftrightarrow b \) transitions, for the 50% case with amplitude \( i/\sqrt{2} \), the amplitude of transmission without changing the mode being \( 1/\sqrt{2} \).

Interaction of the photon with the absorber changes the state of the latter as well. Thereby the absorber taken in the present note is that it is the main thing about IFM as mentioned by Krenn.

The whole sequence of unitary evolution is represented as follows:

\[
\begin{pmatrix}
1 \\
0
\end{pmatrix} \rightarrow \begin{pmatrix}
1/\sqrt{2} \\
i/\sqrt{2}
\end{pmatrix} \Rightarrow \begin{pmatrix}
1/\sqrt{2} \\
i/\sqrt{2}
\end{pmatrix} \rightarrow \begin{pmatrix} 
1/2 \\
0
\end{pmatrix} \Rightarrow \begin{pmatrix} 
0 \\
i
\end{pmatrix} .
\]

The plain arrows correspond to the two beamsplitters; the central, double arrow represents the absorber (other parts of the bomb being not involved yet).

We assume three ideal detectors, each covering one component on the above basis. Then the sequence of unitary events is concluded by a quantum measurement stage in which the system is forced to randomly choose one of the respective mutually exclusive possibilities: to detect a photon on mode \( a \) (detector \( D_a \)), a photon on mode \( b \) (detector \( D_b \)), or the excited state of the absorber (its macroscopic detector being the famous bomb or a more friendly laboratory version of it).

Let us turn to the scattering theory language. If we want to locate causal changes caused by the absorber, we have to define a reference sequence of unitary events – the “incoming wave” – with no absorber (i.e. \( U_{AB} \) replaced by unity):

\[
\begin{pmatrix}
1 \\
0
\end{pmatrix} \rightarrow \begin{pmatrix}
1/\sqrt{2} \\
i/\sqrt{2}
\end{pmatrix} \Rightarrow \begin{pmatrix}
1/\sqrt{2} \\
i/\sqrt{2}
\end{pmatrix} \rightarrow \begin{pmatrix} 
0 \\
i
\end{pmatrix} .
\]

As expected, there is full destructive interference on mode \( a \).

The difference between Equations (1) and (4) is the “scattering amplitude”

\[
\begin{pmatrix} 
0 \\
0
\end{pmatrix} \rightarrow \begin{pmatrix} 
0 \\
0
\end{pmatrix} \Rightarrow \begin{pmatrix} 
0 \\
i/\sqrt{2}
\end{pmatrix} \rightarrow \begin{pmatrix} 
1/2 \\
i/\sqrt{2}
\end{pmatrix} .
\]

More specifically: the third line of the final column vector is the amplitude of photon absorption (as part of the microscopic, unitary evolution), not yet detected, but detectable by the bomb if the subsequent measurement process chooses that branch of the superposition, whereas the first two lines represent the “forward-scattered wave” added to the one-photon components by the absorber: unitary imprints of the same interaction process, carrying information about it, much the way a person can be recognized from her or his silhouette.

What one usually calls IFM is the non-zero output on the first line of the final column vector of Equation (1) or (4): neglecting detector background noise, each non-absorbed photon counted by detector \( D_a \) indicates the presence of an absorber. However, the same non-absorbed photons inform about the same absorber also...
by reducing the number of counts on $D_b$ by a factor of 4, which is therefore an important part of the scenario. That reduction, apparent in Equation (3), comes about by destructive interference between the incoming wave $\Psi^a$ and the forward-scattered one $\Psi^b$. The same feature is present in the case of an imperfect absorber, see below.

The above discussion explains why we think that “interaction-free” is a misleading adjective, unless we use it just as a label. As long as we follow the initial, microscopic stage, the evolution is unitary, and the appearance of a non-zero probability amplitude on the absorptive state must be accompanied by a corresponding change somewhere on the non-absorptive components, everything being controlled by the same interaction. Of course, the Optical Theorem focuses just on that aspect of unitarity: any reaction on scattering, including absorption, must be accompanied by elastic forward scattering. The final state is a superposition of three orthogonal components, with their respective detectors, that of the third line being – if you insist – the bomb. Macroscopic quantum measurements are forced to choose one of the detectors.

One can also notice that there is nothing specific to an absorber in the above equations: the object can be a perfect mirror as well, sending the incoming photons to some outgoing mode $c$ irreversibly leaving the interferometer and occasionally caught by some detector $D_c$, as done actually in Ref. 4 (as a matter of fact, “outgoing” and “irreversible” can be used as synonyms in the present case). Then line 3 of the state vector is the amplitude along basis vector $|0_c0_b1_e\rangle$, all the rest of the formulas remaining unchanged.

From the above discussion we see that with the bomb present, what appears at $D_a$ is the forward-scattered wave, produced by the object-photon interaction on mode $b$ and partially transferred to mode $a$ by the second beamsplitter. If that component is detected, we have IFM in the now standard sense.

For a perfect absorber, the forward-scattered wave is locked both in amplitude and phase, since it has to cancel the incoming wave, therefore the information conveyed by IFM is just about the presence or absence of the absorber (bomb): a black-and-white silhouette of it. In that particular limiting case the insertion of the absorber can be regarded as a modification of the boundary conditions obeyed by the wave field, limited by reflecting and absorbing diaphragms and walls anyway.

Interferometry, as a tool of detecting the presence of objects, is a technique of contrast enhancement; in the case discussed so far, that happens by means of producing a dark-field image on port $D_a$ where the background due to the incoming wave has been extinguished. However, interferometric detection has better chances with at least partially transparent objects, where more amplitude contrast can be produced out of phase shifts. In that case, IFM would mean rejecting the phase information available in the statistics of $D_a$ and $D_b$ counts.

Even if $D_a$ and $D_b$ are imperfect detectors, they are detectors of the same nature, and therefore can be calibrated together to the no-absorber case. Then, using the statistics of repeated measurements, one can evaluate the respective calibrated detection probabilities $P_1$ and $P_2$.

The simplicity of the above three-line state vector is lost in the general case: besides absorption with a well-defined final state and transmission with some phase shift, all kinds of scattering events are possible. What can be determined from an extended IFM scheme is the total probability $W$ of absorption and non-forward scattering:

$$W = 2(1 - P_1 - P_2), \quad (6)$$

the factor 2 (to be modified for asymmetric beamsplitters) representing the presence of the object-free interferometric path. Besides the above, the detector counts contain information about the phase shift $\chi$ caused by the object to the transmitted mode-$b$ photons: that can be extracted as

$$\cos \chi = (P_2 - P_1)/\sqrt{1-W}. \quad (7)$$

The latter formulas are trivial adaptations of those used to evaluate the simplest case of interferometric IFM (not called IFM that time): the so-called “stochastic absorption” of neutrons. 4 To make contact with the above formalism, let us introduce the object transmission amplitude $te^{i\chi}$; the sequence of events generated by the object sandwiched between two beamsplitters is now

$$\begin{pmatrix} 1 \\ 0 \\ \vdots \end{pmatrix} \rightarrow \begin{pmatrix} 1/\sqrt{2} \\ i/\sqrt{2} \\ \vdots \end{pmatrix} \Rightarrow \begin{pmatrix} \frac{\sqrt{2}}{2} e^{i\chi} \\ \frac{\sqrt{2}}{2} e^{i\chi} \\ \vdots \end{pmatrix} \rightarrow \begin{pmatrix} \frac{1}{2} (1 - t e^{i\chi}) \\ \frac{1}{2} (1 + t e^{i\chi}) \\ \vdots \end{pmatrix}, \quad (8)$$

where the dots represent all those channels - absorption, reaction or scattering modes, excluding forward-scattering - which get populated by the action of the object. Subtracting Equation (3) from Equation (5), the

$^4$The situation is somewhat similar, although not identical, to the so-called no-cloning theorem of quantum computation. There, one encounters the impossibility of having a unitary projector (apart from unity). The present situation is rather the impossibility for a quantum thief to take his pickings away without leaving his footprint. Bad news: if the footprint is detected, that happens interaction-free; the thief is not caught. Bad news: if the footprint is detected, that happens interaction-free; the thief is not caught.

$^5$That is not the case for neutrons, since mode $b$, undergoing two more beamsplitter reflections, is significantly damped with respect to mode $a$. 5
first two lines of the difference are the forward-scattered wave – a colored silhouette of the object, carrying phase information as well – which can be fully reconstructed from the interferometric measurement through formulas (6) and (7). Let us observe that here, like in the perfect absorber case, the output on $D_a$ is just forward-scattered photons processed by beamsplitter $BS_2$, whereas $D_b$ output is resulting from interference between the incoming and forward-scattered waves.

Quantum silhouettes – even if not called that way – are much more widespread than that. In quantum jump phenomena, the unitary silhouette of a weak transition is observed in the fluorescence signal of a strong one [17]. In particle physics, unitarity can be used to draw inference about the contributions of non-detected processes [18]. Last, not least: according to Mott’s explanation of track formation in a cloud chamber [19], individual ionization events are strung into a track by their accompanying forward-scattered waves, taking the shape of a geometrical shadow – a well-drawn silhouette – for sufficiently high energies.

In conclusion we note that interaction-free measurement is but an unusual combination of familiar features of quantum mechanics. Our description in terms of forward scattering follows in a way unavoidably from the principles of superposition and causality: nothing can extinguish an incoming wave except a forward scattered one, destructively interfering with it. The forward scattered wave is indeed there, being a by-product of absorption.

The remaining ambiguity is of philosophical nature. State vector reduction – or, if you prefer to call it that way, wave-particle duality – makes absorption and any accompanying phenomena not-happened in those individual cases when a non-absorbed particle has been detected. In the statistics of many observations, the non-happened (“counterfactual” [20]) events do appear through their unitary silhouette. From that point on, it is rather a matter of taste whether you consider unitary evolution a true physical process – that view is implicit in the present paper – or a bookkeeping for measurements. Anyway, the label “interaction-free” is certainly correct in the technical sense of supporting the observation of fragile objects [21].

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