Magnetic field effects on the thermonuclear combustion front of Chandrasekhar mass white dwarfs

Cristian R. Ghezzi, Elisabete M. de Gouveia Dal Pino & Jorge E. Horvath

Received [date]; accepted [date]

1Instituto Astronômico e Geofísico, University of São Paulo, Av. Miguel Stéfano, 4200, São Paulo 04301-904, SP, Brasil; E-mail: cghezzi@jet.iagusp.usp.br,
ABSTRACT

The explosion of a type Ia supernova starts in a white dwarf as a laminar deflagration at the center of the star and soon several hydrodynamic instabilities, in particular, the Rayleigh-Taylor instability, begin to act. A cellular stationary combustion and a turbulent combustion regime are rapidly achieved by the flame and maintained up to the end of the so-called flamelet regime when the transition to detonation is believed to occur. The burning velocity at these regimes is well described by the fractal model of combustion. Using a semi-analytic approach, we describe the effect of magnetic fields on the fractalization of the front considering a white dwarf with a nearly dipolar magnetic field. We find an intrinsic asymmetry on the velocity field that may be maintained up to the free expansion phase of the remnant. Considering the strongest values inferred for a white dwarf’s magnetic fields with strengths up to $10^8 - 10^9$ G at the surface and assuming that the field near the centre is roughly 10 times greater, asymmetries in the velocity field higher than $10 - 20\%$ are produced between the magnetic polar and the equatorial axis of the remnant which may be related to the asymmetries found from recent spectropolarimetric observations of very young SN Ia remnants. Dependence of the asymmetry with white dwarf composition is also analyzed.

Subject headings: Thermonuclear combustion: theory, general, fractal model, turbulent combustion; - stars: white dwarfs, magnetic fields, supernovae, SN Ia
1. Introduction

The explosion of a type Ia supernova begins with the combustion, at the center of a Chandrasekhar mass white dwarf of carbon-oxygen (C+O) or oxygen-neon-magnesium (O+Ne+Mg) fuels. The heat is transported mainly by conduction due to degenerate and completely relativistic electrons as a subsonic deflagration wave propagating outwardly of the star.

The deflagration front born laminar is subject to several hydrodynamic instabilities such as Landau-Darrieus (LD) and Rayleigh Taylor (RT) instability (Arnett & Livne 1994, Khokhlov 1993) that produce an increment of the area at which the nuclear reactions take place. This causes an increase of the nuclear energy generation rate and consequently an acceleration of the front. There are two scale ranges that can be distinguished, at the smallest or “microscopic” perturbation scales the LD predominates over the RT instability, while for the greatest scales ($\gtrsim 10^5$ cm) the RT instability is more important. The combustion front is stabilized by the merging of cells, the formation of cusps, and the expansion of the exploding star, which leads to the formation of a cellular structure at microscopic scales. LD instabilities lead to an acceleration no higher than 2% near the center of the star because they are nonlinerly stabilized (Khokhlov 1995). On the other hand, at the macroscopic scales there is a critical wavelength above which the nonlinear stabilization fails Zel’dovich et al. (1985).

The bubbles that grow due to RT instability are also subject to Kelvin-Helmoltz (KH) instabilities.

---

2 This depends on the value of the Froude number $F = \frac{v_{lam}^2}{gL}$ (where $L$ and $v_{lam}$ are defined below), then if $F < 1$ RT instability predominates over LD.

3 At scales greater than this wavelength the Froude number is $F \ll 1$, and the fluid is fully turbulent.
or shear instability when nonlinear stabilization fails. The onset of the KH instability marks the transition to the fully developed turbulence regime. During this, fluid motions are characterized by the formation of a turbulent cascade in the inertial scales where viscous dissipation is not important. This turbulence can be described by the Kolmogorov’s scaling law.

The acceleration of the turbulent thermonuclear flame due to the action of the several hydrodynamics instabilities can be described by the fractal model first introduced by Woosley (1990), Timmes & Woosley (1992), Niemeyer & Hillebrandt (1995) and Niemeyer & Woosley (1997). The idea of applying fractals to the combustion fronts is useful because the statistic properties of the surface change self-similarly on the spatial scales. The wrinkled surface $S$ behaves like a fractal $S \propto \bar{R}^D$, where $D$ is the fractal dimension of the surface, with $2 \leq D < 3$, and $\bar{R}$ is the mean radius of the wrinkled surface (Filyand, Sivashinsky & Frankel 1994). There is a scale range at which the similarity holds, called the similarity range, that goes from $l_{\text{min}}$ to $L$. The increase of the area at which the nuclear reactions occur induces an increase in the propagation velocity that also behaves like a fractal (see Niemeyer & Hillebrandt, 1995). This fractal model is supported by laboratory experiments involving different gas mixtures (Gostintsev, Istratov & Shulenin 1988).

The propagation of the front subject to LD instability is well described by the Sivashinsky equation (Sivashinsky 1977, 1983) that also includes thermodiffusive and acceleration effects, or by its generalization (Frankel 1990). Numerical studies of wrinkled

---

4We refer here to the fractal dimension (and not to the Hausdorff-Besicovitch dimension) of an attractor as it is commonly defined in astrophysical works (Barnsley 1988). The wrinkled surface is not a fractal in the mathematical sense, it is converging to its attractor asymptotically with time, but it is possible to treat the surface as a fractal as long as $l_{\text{min}}/L \ll 1$, for ex., $l_{\text{min}}/L \simeq 10^{-5}$ at $\rho \simeq 10^8$ g cm$^{-3}$ (see below).
surfaces utilizing these equations are able to reproduce the front on a wider range of spatial scales than direct hydrodynamic simulation and allow the study of the fractal properties of the flame (Blinnikov & Sasorov 1996). These simulations of the Sivashinsky equation show that the fractal growth of the front increases its velocity (Filyand, Sivashinsky & Frankel 1994).

Several physical effects are yet to be considered for a complete solution of the SN Ia problem. In particular, the effects of magnetic fields which are known to be present in SN Ia progenitors (believed to be white dwarfs in binary systems) have not been considered. We will incorporate here the effects of magnetic fields on the fractal growth of the turbulent combustion front and show that they can break the spherical symmetry of the explosion.

2. Formulation of the problem

Ginsburg (1964) and Woltjer (1964) proposed that the magnetic flux of a star is conserved during its evolution and collapse, so that strong magnetic fields would be generated in degenerate stars. Hence, a main sequence star with $R \sim 10^{10}$ cm and $B \sim 10 - 10^5$ G would collapse to form a white dwarf with $R \sim 10^8$ cm and $B \sim 10^5 - 10^9$ G on its surface. Inferred magnetic field strengths of known magnetic white dwarfs range from $\sim 10^6$ to $10^9$ G (Jordan 1992). There is also an important percentage of known white dwarfs in cataclysmic variables with magnetic fields ranging from $\sim 10^6$ to $10^8$ G (see Chanmugan 1992, Liebert and Stockman 1983). There are several models of white dwarfs with strong magnetic fields (Jordan 1992, Putney & Jordan 1995, Martin & Wickramasinghe 1984), which assume centered dipolar, off-centered dipolar, quadrupolar or dipolar+quadrupole magnetic field configurations. Although the origin of the fields in these stars remains unclear it is frequently assumed that the fields are primordial. In fact, Wendell, Van Horn & Sargent (1987) carried out detailed calculation of the time evolution of a white dwarf’s
magnetic field and found the decay time of the field is always longer than the typical cooling times of white dwarfs ($\tau \sim 10^{10}$ yr). We will assume in this letter that the progenitor star of an SN Ia is a magnetic white dwarf with a centered dipolar magnetic field. Considering magnetic fields with the strongest inferred values, i.e., in the range $10^8 - 10^9$ G, and using the model of Wendell, Van Horn & Sargent (1987), we find magnetic field strengths near the center of the star which are in the range $\sim 10^9 - 10^{10}$ G. We thus must expect that when the explosion begins the flame will propagate parallel to the magnetic fields lines at the magnetic poles of the star, and perpendicularly to the field lines at the equator.

An estimate of the effective turbulent speed $u_t$ of the flame was obtained by Damköhler (1939), who proposed that $u_t \sim v(L)$, where $v(L)$ is the average of the turbulent velocity fluctuations on the largest hydrodynamical scale $L$. Later Karlovitz, Denniston, & Wells (1951) derived a statistical approach based on turbulent velocity correlations that fits better with laboratory experiments. The formula simplifies to $u_t = [2v_{lam}v(L)]^{1/2}$ in the limit $v(L) \gg v_{lam}$, where $v_{lam}$ is the laminar velocity. Niemeyer & Hillebrandt (1995) generalized those previous models and found

$$u_t(l) = v_{lam}[v(l)/v_{lam}]^n$$  \hspace{2cm} (1)

where the exponent $n$ is arbitrary, with $n = 1$ for Damköhler’s model and $n = 1/2$ for the model of Karlovitz et al.. Although numerical simulations of thermonuclear flames are in good agreement with the Karlovitz et al. model (see Khokhlov 1993), Niemeyer & Hillebrandt use their own version to derive the scaling law for the turbulent flame speed. We will give here a slightly different derivation of the scaling law for $u_t$. The turbulent motions of the fluid can be described by the Kolmogorov’s scaling law for incompressible turbulent velocity fluctuations which gives (Landau & Lifshitz 1959)

$$v(l) = v(L)\left(\frac{l}{L}\right)^{1/3}$$  \hspace{2cm} (2)

this scaling is valid in the inertial range ($\eta \ll l \ll L$) between the dissipation scale $\eta$
and the length scale $L$ at which the turbulent velocities are in equilibrium with the RT instability. So if we assume that, at the largest hydrodynamic scales which are subject to the RT instability, the turbulent velocity is in equilibrium with the velocity of RT bubbles $v_{RT}(L) \sim v(L)$, where $v_{RT}(L) \propto (gL)^{1/2}$, we can use eq. (2) to obtain the velocity of the turbulent fluctuations at the lower scales $v(l) = v_{RT}(l)(L/l)^{1/6}$. From this it is clear that $v(l) \geq v_{RT}(l)$ for $l \leq L$. Therefore, the turbulent cascade will dominate on all scales below the largest RT unstable wavelength (see also Niemeyer & Hillebrandt 1995). Using the last formula and eq. (1), we obtain $u_t(L) = u_{lam}[v_{RT}(l)/u_{lam}]^{n}(L/l)^{n/6}$. Following Timmes & Woosley (1992), we will use as a lower cutoff the minimum length scale $l_{min}$ that can deform the laminar flame front. This length is given by the condition $v_{RT}(l_{min}) = v_{lam}$. Therefore the scaling law for the turbulent flame speed is

\[ u_t(L) = v_{lam} \left( \frac{L}{l_{min}} \right)^{n/2} \]  

(3)

In a fractal description of the turbulent flame propagation, eq. (3) gives the fractal velocity of the front $v_{frac} = v_{lam}(L/l_{min})^{D-2}$ (Niemeyer & Woosley 1997, Niemeyer & Hillbrandt 1995). Comparing this with eq. (3), we see that the fractal excess is $D - 2 = 0.5$ for $n = 1$, and 0.25 for $n = 1/2$, which is in agreement with the fractal dimensions inferred from numerical studies of SN Ia explosions (Blinnikov, Sasorov & Woosley 1995).

In the equation above we must determine $l_{min}$ and $L$. As we have seen, the characteristic RT velocity of the growing modes must be larger than or equal to the laminar deflagration velocity $v_{lam}$, or $v_{RT}(l_{min}) = v_{lam}$. This implies that $l_{min} n_{RT}(l_{min}) = v_{lam}$, where $n_{RT}(l)$ is the inverse of the characteristic RT time $n_{RT} = (1/2\pi)\sqrt{gk\Delta \rho/2\rho}$. This results

\[ l_{min} = 8\pi \left( \frac{\frac{1}{2} \rho v_{lam}^2}{g \Delta \rho} \right) = l_{pol} \]  

(4)

where $g$ is the acceleration of the gravity, $\Delta \rho = \rho_u - \rho_b$ is the difference between the densities $\rho_u$ and $\rho_b$ of the unburned and burned fuel, respectively, and $\rho = (\rho_u + \rho_b)/2$. We
denote \( l \) as \( l_{\text{pol}} \) hereafter, where the subscript “\( \text{pol} \)” stands for polar since it gives the value of \( l_{\text{min}} \) at the magnetic poles of the star where the propagation of the flame is parallel to the magnetic field lines, and therefore unaffected by them.

The value of \( L \) has been determined from the minimum between the maximum spatial extent of the density inversion and the radius of the flame \( R_f \). The density inversion is due to electron capture in an isobaric nuclear statistical equilibrium environment that causes the number of electrons to decrease and the density to increase. We here use \( L = R_f \) because, for densities \( \lesssim 10^9 \) g cm\(^{-3} \), the density inversion scale is greater than the stellar radius (Timmes & Woosley 1992). The highest unstable mode \( L \) has the highest growing time since \( t_{RT}(l) = (4\pi l/g)^{1/2} (\rho/\Delta \rho)^{1/2} \leq t_{RT}(L) \), for \( l_{\text{pol}} \leq l \leq L \). For example, near the center of the star \( L \sim 3 \times 10^7 \) cm and the perturbation takes \( t_{RT}(L) \sim 0.5 \) s to grow to an amplitude of the order of its wavelength. On the other hand, for \( l_{\text{pol}} \simeq 10^5 \) cm \( t_{RT}(l_{\text{pol}}) = 0.03 \) s and the perturbation grows faster. Thus with a deflagration velocity \( u_{\text{lam}} \sim 10^7 \) cm s\(^{-1} \), the flame must travel only \( \sim 5 \times 10^6 \) cm for the slowest perturbations, and \( \sim 3 \times 10^5 \) cm for the fastest ones in order to the perturbations to grow to amplitudes of the order of their lengths. This suggest that all the perturbation scales grow very fast in the combustion front and therefore, they are all important.

Our analysis is applicable only for densities \( \rho \geq 10^7 \) g cm\(^{-3} \), since the turbulent motions will destroy the corrugated flamelet regime for lower densities (Niemeyer & Woosley 1997, Hillebrandt & Reinecke 2000).

3. Asymmetric velocity field

Near the center of the star the magnetic pressure \( (B^2/8\pi) \sim 10^{18} \) erg cm\(^{-3} \) is much smaller than the gas pressure behind the front \( p \sim 10^{27} \) erg cm\(^{-3} \). Therefore, it is
reasonable to assume that the scaling law for the flame speed (eq. 3) will not be modified by the presence of the magnetic field. This means that the fractal dimension can be considered independent of the magnetic field. On the other hand, the presence of magnetic fields will shift the lower unstable scales at the equator of the star with respect to the poles, as we will see, thus producing different fractal velocities at the magnetic poles and the equator of the star. We note also that the turbulent fluid motions can only scramble the field inside the “flame brush”, that is, inside the RT foam that mixes unburned and burned material, or behind the front, and therefore, they will not affect the large scale magnetic field geometry of the progenitor.

The dispersion relation for the RT instability of the front with a magnetic field transverse to the direction of the gravity and to the propagation of the flame, in a heterogeneous inviscid plasma of zero resistivity is (Chandrasekhar 1961):

\[ n_{RTB} = \frac{1}{2\pi} \sqrt{g k \left( \frac{\Delta \rho}{2 \rho} - \frac{k B^2}{4 \pi g \rho} \right)} \]  

(5)

here \( k = \frac{2\pi}{l} \), and \( B \) is the magnetic field strength. It is, therefore, possible to define an effective surface tension, \( T_{\text{eff}} = \frac{B}{2\pi k} \), for which the magnetic field will have a stabilizing effect over perturbations with wavelengths below \( \sim \frac{B^2}{(g \Delta \rho)} \). However, if the velocity of the flame is taken into account another cut-off length appears, \( l_{eq} \), which is given by the condition \( l_{eq} n_{RTB}(l_{eq}) = v_{\text{lam}} \). From this equation the minimum scale of the self-similar range in presence of magnetic field is

\[ l_{eq} = \frac{8\pi \left( \frac{B^2}{8\pi} + \frac{1}{2} \rho v_{\text{lam}}^2 \right)}{g \Delta \rho} \]  

(6)

here \( eq \) stands for “equator” because \( l_{eq} \) gives the minimum instability scale at the magnetic equator of the star where the propagation of the front is perpendicular to the field lines. Inserting eq. (6) in the definition of the fractal velocity (eq. 3) we have

\[ v_{eq} = v_{\text{lam}} \left( \frac{L}{l_{eq}} \right)^{D-2} \]  

(7)
for the fractal velocity in the magnetic equator of the star, while, as we have seen before, eq. (3) gives \( v_{pol} = u_t \) for the magnetic poles,

\[
v_{pol} = v_{lam} \left( \frac{L}{L_{pol}} \right)^{D-2}
\]

with \( L_{pol} \) given by eq. (4).

Taking the ratio between the velocities \( v_{pol} \) and \( v_{eq} \), we obtain

\[
\frac{v_{pol}}{v_{eq}} = \left( \frac{B^2/8\pi + \rho v_{lam}^2/2}{\rho v_{lam}^2/2} \right)^{D-2}.
\]

The relevant data for do the calculations can be found in Timmes & Woosley (1992). We will use a fractal dimension \( D = 2.5 \) as in Blinnikov, Sasorov & Woosley (1995) and references therein.

Figure 1 displays the percentage of asymmetry in the velocity field for two white dwarfs with different compositions: (a) \( X(^{12}C) = 0.5 \ X(^{16}O) = 0.5 \), \( \Delta \rho/\rho = 0.426 \), \( v_{lam} = 2.33 \times 10^5 \text{ cm s}^{-1} \) and (b) \( X(^{12}C) = 0.2 \ X(^{16}O) = 0.8 \), \( \Delta \rho/\rho = 0.415 \), \( v_{lam} = 0.415 \times 10^5 \text{ cm s}^{-1} \), and fuel densities \( \rho \sim 10^8 \text{ g cm}^{-3} \) which are encountered by the front at a middle radius distance \( \sim 8 \times 10^7 \text{ cm} \) from the center of the star. We see that the asymmetry is very sensitive to the composition of the progenitor and is quite insignificant for type (a) progenitor. If at these densities, magnetic fields as high as \( \gtrsim 6 \times 10^8 \text{ G} \) are present at this radius, the velocity field has an asymmetry \( \gtrsim 10 \% \) for progenitors of type (b). With surface magnetic fields of the order of \( 10^9 \text{ G} \), and fields \( \gtrsim 6 \times 10^9 \text{ G} \) at the interior, these progenitors could suffer huge asymmetries \( \gtrsim 120 \% \) (see Fig. 1). Progenitors with compositions \( ^{16}O^{20}Ne \) show even higher asymmetries (see Ghezzi, de Gouveia Dal Pino & Horvath 2001) because the laminar velocity in \( ^{16}O^{20}Ne \) is smaller than in \( ^{12}C^{16}O \). Also, we note that for regions very close to the center of the star (at higher densities) the asymmetry percentages are smaller than 5%.

When the flame reaches the latter stages of the flamelet regime, at a density of
∼ 5 × 10⁷ g cm⁻³, the magnetic field intensity drops to ∼ 10⁸ − 10⁹ G, however our calculations indicate that the percentage of asymmetry is maintained or increased (Ghezzi, de Gouveia Dal Pino & Horvath 2000), leading to an asymmetry higher than ≥ 10% for a progenitor with \( X(^{12}C) = 0.2 \) \( X(^{16}O) = 0.8 \) with fields higher than 10⁸ G at the surface of the star.

### 4. Conclusions and Discussion

We have found that an asymmetric field velocity caused by the presence of a dipolar magnetic field during the fractal growth of the deflagration front of a type Ia supernova can lead to the formation of a prolate remnant. The magnetic field introduces an effective surface tension in the equator of the white dwarf progenitor that reduces the velocity of the combustion front, \( v_{eq} \), with respect to the velocity at the poles, \( v_{pol} \), so that \( v_{pol} > v_{eq} \) by a percentage of 10% to 20% for progenitors with a composition \( X(^{12}C) = 0.2 \) \( X(^{16}O) = 0.8 \), \( \Delta \rho/\rho = 0.415 \), and surface fields ∼ 10⁸ G (type (b), Fig. 1). This leads to prolate explosions along the magnetic poles. Lower magnetic field strengths have no detectable effects on the explosion. As only a small fraction of the observed white dwarfs must have magnetic fields as high as 10⁸ G, asymmetric explosions are not expected to occur very frequently. If there is no transition to detonation the asymmetry will be probably maintained during the free expansion phase of the supernova remnant, because the expansion velocity is constant at this phase. Detonation in an SN Ia is still controversial, but if a transition to detonation occurs the remnant can be symmetrized very fast (Ghezzi, de Gouveia Dal Pino & Horvath 2001).

Recent spectropolarimetric observations have revealed a linear polarization component in the radiation of very young SN Ia remnants, which suggests that prolate atmospheres with asymmetries ∼ 20% are producing it (see Leonard, Filippenko, & Matheson 1999,
The model presented here offers a plausible explanation for such observations and provides new motivation for theoretical studies of supernovae involving magnetic fields. However, its predictions must still be confirmed through fully numerical simulations of the explosion in the presence of magnetic fields.

This paper has benefited from many valuable comments by the referee F. X. Timmes, and discussions with M. Diaz. C.R.G., E.M.G.D.P. and J.E.H. have been partially supported by grants of the Brazilian Agencies FAPESP and CNPq.
REFERENCES

Arnett W. D., & Livne E. 1994a, ApJ, 427, 315

Livne E., & Arnett W. D., Preprints of the Steward Observatory, No. 1137

Arnett, W. D., & Livne, E., ApJ, 427:330-341, 1994 May 20

Barnsley M., “Fractals Everywhere ”, 1988, Academic Press

Blinnikov S. I., Sasorov P. V., Phys. Rev. E, Vol. 53, Nro. 5, 5/96

Blinnikov S. I., Sasorov P. V. & Woosley S. E., Space Science Review, v 74, p. 299-311, 1995

Chandrasekhar, S., “Hydrodynamic and Hydromagnetic stability”, Dover (1961)

Chanmugam G., Annu. Rev. Astron. Astrophys. 1992, 30:143-84

Filyand L., Sivashinsky G. I. & Frankel M. L., Physica D, Vol. 72, 1994, 110-118

Frankel N., Sivashinsky G., Physica 30D (1988) 28-42

García-Senz D., Bravo E. & Serichol N., ApJ Supplement Series, 115:000, 1998 March

García-Senz D., Bravo E. & Woosley S. E., A&A. 349, 177-188 (1999)

Ghezzi C. R., de Gouveia Dal Pino M. E., Horvath J. E., 2001, in preparation

Ginzburg V. L., 1964, Sov. Phys. Dokl. 9: 329 32

Gostintsev Yu. A., Istratov A. G. & Shulenin Yu. V., Combustion Explosion and Shock Waves, Vol. 24, pp. 70, 1988

Höflich P., Wheeler J. C. & Wang L., ApJ, 521: 179-189, 1999 August 10
Istratov A. G., Librovich V. B., *J. Appl. Math.*, Vol. 30, No. 3, 1966, pp. 541-556

Istratov A. G., Librovich V. B., *Astronautica Acta*, Vol. 14, pp. 453-467, 1969

Jordan S., *Astron. Astrophys.*, 265, pp. 570-576, 1992

Frankel M. L., *Phys. Fluids A* 2 (10), October 1990, pags. 1879-1883

Khokhlov A., *ApJ*, 419:L77-L80, 1993 December 20

Khokhlov A., *ApJ*, 424:L115-L117, 1994 April 1

Khokhlov A., *ApJ*, 449:695-713, 1995 August 20

Khokhlov A., [astro-ph/9910453](http://arxiv.org/abs/astro-ph/9910453), 25 Oct. 1999

Khokhlov, A. M., Oran, S., & Wheeler, J. C., 1997, *ApJ*, 478, 678

Kriminski, S., A., Bychkov, V., V., & Liberman, M., N., 1998, *New Astronomy*, 3, 363

Landau, L. D., & Lifshitz, E. M. 1959, Fluid Mechanics (Oxford: Pergamon Press)

Leonard D. C., Filippenko A. V. & Matheson T., [astro-ph/9912337](http://arxiv.org/abs/astro-ph/9912337). 16 Dec. 1999

Lisewski A. M., Hillebrandt W., Woosley S. E., Niemeyer J. C. & Kerstein A. R., [astro-ph/9909508](http://arxiv.org/abs/astro-ph/9909508) v2, 24 Jan. 2000

Livne E. & Arnett D., “On the instability of deflagration fronts in white dwarfs”, preprint nro. 1137 of the Steward Observatory, Arizona, USA

Mandelbrot, B. B., “The Fractal Geometry of Nature”, *New York: Freeman.*, 1983

Niemeyer J. C., [astro-ph/9906142](http://arxiv.org/abs/astro-ph/9906142) v2, 19 Jul 1999.

Niemeyer J. C. & Hillebrandt W., *ApJ*, 452:769-778, 1995a October 20

—- *ApJ*, 452:779-784, 1995b October 20
Niemeyer J. C. & Woosley, S. E., 1997, ApJ, 475, 74
Niemeyer J.C., Woosley S. E., ApJ, 475:740-753, 2/97
Niemeyer J.C. & Kerstein A. R., astro-ph/9707108, 9 Jul. 1997
Putney A. & Jordan S., ApJ, 449:863-873, 1995 August 20
Sharp D. H., Physica 12D (1984) 3-18
Sivashinsky G. I., Acta Astronautica. Vol. 4, 2/77
Sivashinsky G. I., Ann. Rev. Fluid Mech., 1983, 15:179-99
Timmes F. X. & Woosley S. E., ApJ, 396:649-667, 1992 September 10
Wang L., Wheeler J. C. & Höflich P., ApJ, 476: L27-L30, 1997 February 10
Wang L., Wheeler J. C. & Höflich P., American Astronomical Society Meeting, 193, pp. 1325, 47.15
Wendell C. E., Van Horn H. M. & Sargent D., ApJ, 313:284-297, 1987 February 1
Wheeler J. C., Höflich P., Wang L., astro-ph/9904047, 4/99
Woltjer L., ApJ, 140:1309-13, 1964
Zeldovich Ya. B., Istratov A. G., Kidin N. I. & Librovich V. B., Combustion Science Technology, 1980, Vol. 24, pp. 1-13
Zeldovich Ya. B., Librovich V. B., Makhviladze G. M. & Sivashinsky G. I., Astronautica Acta. Vol. 15, pp. 313-321
Zeldovich Ya. B., Barenblatt G. I., Librovich V. B., Makhviladze G. M., 1985, “The Mathematical Theory of Combustion and Explosions.” New York: Plenum
Fig. 1.— Asymmetry percentage in the field velocity for two progenitors with initial compositions $X(^{12}C) = 0.5$ $X(^{16}O) = 0.5$ and $X(^{12}C) = 0.2$ $X(^{16}O) = 0.8$, at $\rho \simeq 10^8$ g cm$^{-3}$, as a function of the magnetic field strength at a radius $\sim 8 \times 10^7$ cm. The asymmetry percentage is given by: $100 \times \left( \frac{v_{pol}}{v_{eq}} - 1 \right)$ where $v_{pol}/v_{eq}$ is obtained by eq. (9).