Non-Markovian Stochastic Processes
and the Wave-like Properties of Matter

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A non-markovian stochastic model is shown to lead to a universal relationship between particle’s energy, driven frequency and a frequency of interaction with the medium. It is briefly discussed the possible relevance of this general structure to various phenomena in the context of the formation of patterns in granular media, computation in a Brownian-type computer and the Haisch-Rueda-Puthoff inertial mass theory.

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I. INTRODUCTION

The study of physical systems with non-markovian statistical properties has provided a natural basis for the understanding of the role played by memory effects in such different fields as anomalous transport in turbulent plasmas, Brownian motion of macroparticles in complex fluids, in the vortex solid phase of twinned YBa$_2$Cu$_3$O$_7$ single crystals. Recently, experimental evidence were reported of the quantum jumps of particles in the Earth’s gravitational field giving a strong evidence that jumping process is quite ubiquitous in natural processes. Thus, it becomes interesting to inquire: how the medium interaction, perturbing the free motion of a particle, leaves its own signature?

It is the purpose of this paper to provide the main lines of a derivation of what we believe to be a non-trivial property of the jumping process particles undergo, whenever they move in a perturbing medium. The idea is to put a particle moving straight in a discrete planar geometry, jumping from one site to another and, in the meanwhile, subject to a random interaction.

It is found that a universal and structurally very simple expression of the particle’s energy prevails, intrinsically linked to the generation of such properties as the mass of a particle and the build-up of regular geometrical patterns.

This paper is organized as follows: In Section 2 we introduce, for completeness, the basic features of a non-markovian stochastic model. It is then shown that the classical particle’s energy must be proportional to the squared driven frequency over a frequency of dissipation in the medium; in particular, whenever the particle’s dynamics is described by a planar wave, the de Broglie relationship is retrieved. In Section 3, the evidence of the referred general structure of the particle’s energy is intuitively discussed in several branches of physics. We believe that the relationship between driving frequency of a given process and its corresponding frequency of interaction with a medium provides a method for a general approach to dissipative systems and is able to predict a class of new relations.

II. INFINITE MEMORY MODEL

In a non-markovian model the prediction about the next link \((x_{n+1})\) are defined in terms of mutually dependent random variables in the chain \((x_1, x_2, \ldots, x_n)\). Consider a particle jumping from one site to another in Euclidean space. We will address here the much simple situation of a deterministic jump process along a given direction. The jumping sites are assumed to be equidistantly distributed along the axis. Now, add to this jumping process an oscillatory motion due to interaction with a medium and characterized by stochasticity. The frequency of oscillation around an equilibrium position between two jumps is denoted by \(\nu\) and \(\beta\) is the probability that each oscillation in the past has to trigger a new oscillation in the present. The simplicity of the described geometry is to same extent well justified by the recent experiments done by Nesvizhovsky and collaborators. Ultracold neutrons in a vertical fall subject to a constant acceleration due to gravity, were shown do not move continuously, but rather jump from one height to another in quantum leaps.

Let \(Q_m(q(t))\) be the probability that one oscillation from the \(M = m_0 + \ldots + m_{q-1}\) which occurred in the past generates \(m\) oscillations at the \(q\)th step. Since we assume \(\beta\) is constant, this is an infinite memory model, meaning that an oscillation which has occurred long time ago produces the same effect as an oscillation which has occurred in the near past. Let’s introduce the probability density, \(Q_n(t)dt\), that the \(n\)th oscillation takes place in the interval of time \((t, t + dt)\) at \(q\)th step. Then

\[
Q_{n+1}[q(t)] = \int_0^{q(t)} Q_n[q(t)]p_0[(q(t) - q(t'))dq(t'),
\]

where \(p_0(t - t')\) is the probability per unit time that the \((n + 1)st\) oscillation takes place in the time interval \((t, t + dt)\) and \(p_0\) is the probability density for an oscillation at \(q\)th step.
or, in complete form, the probability that \( M \) previous oscillations generate \( m_q \) oscillations at \( q \)th step. The Bose-Einstein distribution is favored since many oscillations can pertain to the same step:

\[
g_M(m_q) = \frac{(M + m_q - 1)!}{m_q!(M - 1)!} \beta^m \alpha^M. \tag{12}\]

Introducing the conditional probability \( \varphi_q(m_q|m_{q-1},...,m_0) \) that at \( q \)th step there are \( m_q \) oscillations provided that at the previous steps \( m_{q-1},...,m_0 \) oscillations have occurred, subject to the normalization condition

\[
\sum_{m_q} \varphi_q(m_q|m_{q-1},...,m_0) = 1, \tag{13}\]

then, it can be shown \( \footnote{Footnote:} \) that

\[
\xi_q(n) = \sum_{m} \xi_0(m)(1 - \beta)^m |1 - (1 - \beta)^{n - m} \]

\[
= \frac{(n - 1)!}{(m - 1)!(n - m)!}. \tag{14}\]

Hence, the probability density of occurrence of \( q \)-jumps is written in the form

\[
\Psi_q(t)dt = \frac{\alpha}{\nu} \exp(-\alpha t) \sum_{m} \xi_0(m)\frac{(\alpha t)^{m-1}}{(m - 1)!} dt, \tag{15}\]

where we put \( \alpha(q) \equiv (1 - \beta)^q \nu \). It must be assumed we know \( \xi_0(m) \), that is the probability to occur \( m \) oscillations from \( t = 0 \) up to the first jump.

With the assumption of a Poisson distribution for \( \xi_0(m) \), the summation gives

\[
\sum_{m=0}^{\infty} \xi_0(m)\frac{(\alpha t)^{m-1}}{(m - 1)!} = \frac{1}{\sqrt{\lambda \alpha t}} I_1(\sqrt{\lambda \alpha t}), \tag{16}\]

where \( I_1(x) \) is the first class modified Bessel function of order 1. Hence, the final result for the probability of occurrence of \( q \)-jumps between \( t \) and \( t + dt \) is given by

\[
\Psi_q(t)dt = \frac{\alpha}{\lambda \nu^2 t} \exp(-\alpha t)I_1(\sqrt{\lambda \alpha t}) dt. \tag{17}\]

Eq. \( 17 \) is characterized by a temporal argument and, in particular, for a sufficient number of steps, the limit \( \sqrt{\lambda x} \rightarrow 0 \) is satisfied and Eq. \( 17 \) reduces to

\[
\Psi_q(t)dt \approx \frac{\alpha}{2} \exp(-\alpha t) dt. \tag{18}\]

We have in view a deterministic particle system evolving according to a local mapping in a space of equidistant sites. This idealization lies in the Ehrenfest’s equation describing the quantum mechanical mean values of position, and thus avoids the solution of a much more
the third term on the right-hand side of Eq. 20 with the

According to this representation we are lead to identify
get the following universal and structurally very simple
The integer

by means of the relativistic formula for energy, yielding

The expansion allowed the identification of some me-

chanical properties of the particle in the medium. In
fact, in analogy with a transversal wave in a vibrant
string, we can define a group velocity \( V \equiv \frac{2\pi}{\nu l} \), with \( n = 1, 2, 3, \ldots \) (assuming a non-dispersive medium) as the

The medium perturbation is characterized by \( \nu \) shaping

the non-markovian character of the stochastic process is
related to the nature of the medium rather than the past
history of the particle.

III. APPLICATION IN VARIOUS BRANCHES OF PHYSICS

Having presented the mainframe of the infinite mem-

ory model, we give now some illustrations upon our point
of view that it is possible to achieve an understanding of
some physical phenomena in, possibly, a wide range. In
the examples given below the general structure described
by Eq. 22 seems to govern geometrical patterns in gran-

ular media, inertial effects in stochastic electrodynamics
and minimal energy expenditure in computational ther-

modynamics.

All phenomena hereby referred share the same imprint
of a stochastic interaction with a medium, building-up a
given physical structure.

A. Spheres packing and gravitational surface waves

Granular matter consists of macroscopic particles of
different size, shape and surface properties and those
characteristics lead to specific packing behavior. Particle
clustering results from an energy loss associated with
particle-particle interactions. This interesting and fasci-
nating behavior can be described by the infinite memory
model and the constitutive equation embodied in 22. To
probe it, we start by rearranging Eq. 22 in the form
shown the display of repetitive geometric patterns, similar to the instabilities reported by Faraday.

Actually, it was observed a dependence of the wavelength of those geometrical patterns and the frequency of excitation \( f \) imposed vertically on a thin layer of granular matter. Both are related through the equation

\[
\lambda = \lambda_{\text{min}} + \frac{g_{\text{eff}}}{f^2}, \tag{24}
\]

where \( \lambda_{\text{min}} \) represents a threshold near \( 11d \), with \( d \) denoting the particles diameter. Incidentally, gravitational waves in the surface of a fluid have the same dependency.

The two main mechanisms governing the phenomena are the direct excitation of surface waves and a mechanism of successive bifurcations resulting from the excitations due to the vibrations of granular matter. In fact, memory-effects have been experimentally shown to occur in granular materials.

### B. Computation in a Brownian-type computer

The work on classical, reversible computation has laid the foundation for the development of quantum mechanical computers.

In 1961 Landauer analyzed the physical limitations on computation due to dissipative processes. He showed that energy loss could be made as small as you want, provided the device works infinitesimally slowly. In fact, it is shown that the first condition for any deterministic device to be reversible is that its input and output be uniquely recoverable from each other - this is the condition of logical reversibility. If, besides this condition, a device can actually run backwards, then it is called physically reversible and it dissipates no heat, according to the second law of thermodynamics.

An example of reversible computing involves a copying machine, for example. Feynman derived a formula estimating the amount of free energy it takes to realize a computation in a given interval of time. Envisioning a computer designed to run by a diffusion process, characterized by a slightly higher probability to run forward than backwards, Feynman proposed the relationship to hold

\[
E = k_B T \frac{t_m}{t_a}. \tag{25}
\]

Here, \( E \) is the energy loss per step, \( k_B T \) is thermal energy, \( t_m \) and \( t_a \) are, resp., the minimum time taken per step and the time per step actually taken. It is easily seen that the \( k_B T = h \omega \) holds true since thermal energy is the driven process and \( \omega = \frac{2\pi}{t_a} \) and \( \nu = \frac{2\pi}{t_m} \). The forward transition rate to a new configuration of available states (say from \( \{ n_i \} \) to \( \{ n_j \} > \{ n_i \} \)) span in a time scale which has a non-null correlation factor and thus generating a memory effect. Although, as stressed by Feynman, Eq. 25 is only approximative, we view in it a particular manifestation of Eq. 22. In fact, Eq. 25 represents the minimum energy that must be expended per computational step in a given process. We have actually a new deep insight to our Eq. 22; it results from the best match between energy cost versus speed. It is a by-product of minimum principles.

### C. Haisch-Rueda-Puthoff inertial mass theory

Based on stochastic electrodynamics, Haisch, Rueda and Puthoff put in evidence the relationship between the zero-point field (ZPF) and inertia. ZPF is uniform and isotropic in inertial frames, while showing asymmetries when viewed in accelerated frames. Applying a technique developed formerly by Einstein and Hopf and which is at the foundation of stochastic electrodynamics, the charged particles constituent of matter (partons or quarks) were driven to oscillate at velocity \( v_{osc} \) by the electric component of the ZPF, \( E_{ZPF} \), thereby accelerating in a direction perpendicular to the oscillations induced by the ZPF. The action of the magnetic component of the ZPF generate a Lorentz force whose average value is given by

\[
F_L = \left( \frac{q}{c} \right) \{ v_{osc} \times B_{ZPF} \} = -\frac{\Gamma \omega_c^2}{2 \pi c^2} a. \tag{26}
\]

They interpreted this result as an account for inertia and the inertial mass, \( m_i \), is shown to be a function of the damping constant for the oscillations and \( \omega_c \), the characteristic frequency of particle-ZPF interactions:

\[
m_i = \frac{\Gamma \omega^2}{2 \pi c^2}. \tag{27}
\]

In the above expression, \( \Gamma = \frac{q^2}{6 \pi \epsilon_0 m_0 c} \) is the Abraham-Lorentz radiation damping constant appearing on the nonrelativistic equation of motion for a particle of mass \( m_0 \) and charge \( q \) when submitted to the zero-point radiation electric field; \( \omega_c = \sqrt{\frac{\pi \epsilon_0}{\epsilon_0}} \) is the effective Planck cut-off frequency of the vacuum zero-point-fluctuation spectrum. This idea is rooted in a former publication by Sakharov envisioning gravitation as resulting from a small disturbance of the metrical elasticity of space.

Comparing Eq. 27 with Eq. 22, it is clear that those expressions have the same structure. Interestingly, Puthoff and collaborators conjectured that their interpretation of mass as resulting from a resonance with the ZPF leads directly to the de Broglie relation. In their interpretation, inertia is a kind of electromagnetic drag that affects charged particles undergoing acceleration through the electromagnetic zero-point field. In fact, according to the de Broglie perspective, the inertial mass of a particle is the vibrational energy divided by \( c^2 \) of a localized oscillating field. It is not an intrinsic property but instead a measure of the degree of coupling with a localized field (already, in his own view, of electromagnetic origin).
However, it will be noted that our analysis gives evidence of a more complex structure although, when the resonant condition is verified, it collapses to the well-known the Broglie relation.

IV. CONCLUSION

Exploring the underlying transport mechanism of a test particle with infinite memory induces us to attribute a universal and structurally simple property to the particle’s energy, embodied in Eq. 22. In our interpretation, in a perturbative medium the particle’s energy results from a balance between the driven frequency \( \omega \) and a frequency of interaction in the medium, \( \nu \). In the particular case of planar waves this result is consistent with de Broglie wavelength relationship.

Our approach incorporates the fundamental properties of dynamics and how deeply rooted in natural phenomena is this general structure, it is illustrated in various objects and different kind of fields.

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