Magnetohydrodynamic (MHD) mixed convective periodic flow through porous medium in a rotating channel

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Abstract

An attempt has been made to study heat and mass transfer effect on MHD mixed convection periodic flow in a rotating vertical porous channel with Hall effect, thermal radiation and chemical reaction. The coupled nonlinear partial differential equation are turned to ordinary differential equation by super imposing a solution with steady and time dependent transient part. Finally, the set of ordinary differential equations are solved with an analytical scheme to meet the inadequacy of boundary condition. The expressions for velocity, temperature and concentration are obtained analytically. The effects of various parameters on the velocity, temperature and concentration are discussed in detail with the aid of graphs.

Keywords: Chemical reaction; Hall current; MHD: periodic flow; porous medium
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1. Introduction

The flows of fluid through porous media are of principal interest because these are quite prevalent in nature. Such flows have attracted the attention of a number of scholars due to their applications in many branches of science and technology, viz., in the field of agriculture engineering, to study the underground water resources, seepage of water in river-beds, in petroleum technology, to study the movement of natural gas, oil and water through oil reservoirs, in chemical engineering for filtration and purification processes. The convection problem in porous medium also has important applications in geothermal reservoirs and geothermal energy extractions.

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Magneto hydrodynamics is currently undergoing a period of great enlargement and differentiation of subject matter. The interest in these new problems generates from their importance in liquid metals, electrolytes and ionized gases. MHD channel or duct flows are important from its practical point of view. The growing need for chemical reactions in chemical and hydrometallurgical industries require the study of heat and mass transfer in presence of chemical reaction. The presence of a foreign mass in a fluid causes some kind of chemical reaction. This can be presented either by itself or as mixtures with a fluid. In many chemical engineering practices, a chemical reaction occurs between a foreign mass and the fluid in which the plate is moving. These processes take place in several industrial applications, such as, polymer production, manufacturing of ceramics or glassware and food processing. A chemical reaction can be classified as either a homogenous or heterogeneous process that depends on whether it occurs on an interface or a single phase volume reaction. Unsteady MHD free convection flow past an exponentially accelerated vertical plate with mass transfer, chemical reaction and thermal radiation has been investigated by Chamkha et al.\(^1\). Raju et al.\(^2\) have gave the detailed information about the process of chemical reaction and they also studied analytically, MHD free convective, dissipative boundary layer flow past a porous vertical surface in the presence of thermal radiation, chemical reaction and constant suction. Unsteady MHD radiative and chemically reactive free convection flow near a moving vertical plate in porous medium has been addressed by Reddy et al.\(^3\). Moreover when the strength of the magnetic field is high, one cannot neglect the effects of Hall current. Raju et al.\(^4\) have investigated Hall-current effects on unsteady MHD flow between stretching sheet and an oscillating porous upper parallel plate with constant suction. Satyanarayana et al.\(^5\) have considered Hall current effects on free-convection MHD flow fast a porous plate. Takhar et al.\(^6\) have considered an unsteady free convection flow over an infinite vertical porous plate due to the combined effects of thermal and mass diffusion, magnetic field and Hall currents. Pal\(^7\) has investigated Hall current and MHD effects on heat transfer over an unsteady stretching permeable surface with thermal radiation. The rotating flow of an electrically conducting fluid in the presence of a magnetic field is encountered in geophysical and cosmosical fluid dynamics. It is also important in the solar physics involved in the sunspot development. Singh\(^8\) has derived an exact solution of MHD mixed convection periodic flow in a rotating vertical channel with heat radiation. Pal and Talukdar\(^9\) have examined the influence of Hall current and thermal radiation on MHD convective heat and mass transfer in a rotating porous channel with chemical reaction. Philip et al.\(^10\) have considered MHD rotating heat and mass transfer free convective flow past an exponentially accelerated isothermal plate with fluctuating mass diffusion. Seth et al.\(^11-14\) have studied the effects of Hall current and rotation on unsteady MHD Couette flow in the presence of an inclined magnetic field. Singh and Pathak\(^15\) have investigated the effects of radiation and Hall current on the MHD mixed convection flow through a porous medium filled in a vertical rotating channel. Numerical investigation of the effect of boundary conditions on hydro elastic stability of two parallel plates interacting with a layer of ideal flowing fluid has been done by Bochkarev and Lekomtsev\(^16\). Recently, slip effect on the magnetohydrodynamics channel flow in the presence of the across mass transfer phenomenon has been investigated by Ijaz et al.\(^17\).

In the light of the above studies, in this paper, we studied the effects of thermal radiation and Hall current on MHD mixed convection periodic flow in a rotating vertical porous channel with chemical reaction. The expressions for velocity, temperature and concentration are obtained analytically. The effects of various parameters on the velocity, temperature and concentration are discussed in detail with the aid of graphs.

2. Formulation of the problem

We have consider an unsteady MHD free convective flow of an electrically conducting, viscous incompressible fluid through a porous medium bounded between two insulated infinite vertical plates in the presence of Hall current and thermal radiation. The plates are at a distance’d’ apart. A Cartesian coordinate system with \(X^+\)-axis oriented vertically upward along the centerline of the channel is introduced. The \(Z^+\)-axis
taken perpendicular to the planes of the plates is the axis of the rotation and the entire system rotates about the
axis with uniform angular velocity $\Omega^*$. Since the plates of the channel are of definite extent, all the physical
quantities depend only on $Z^*$ and $t^*$. The temperature and concentration $T^*_w \cos \omega^* t^*$ and $C^*_w \cos \omega^* t^*$ of
the right plate at $Z^* = \frac{d}{2}$ is considered to be varying periodically with time and the temperature
$T^* = T_0 = 0, C^* = C_0 = 0$ of the left plate at $Z^* = \frac{d}{2}$ is taken to be zero. Let $(u^*, v^*, w^*)$ be the components
of velocity in the directions $(x^*, y^*, z^*)$ respectively. A strong transverse magnetic field of uniform strength $B_0$
is applied along the $Z^*$ axis. Under the usual assumptions that the electron pressure (for a weekly ionized
gas), the thermoelectric pressure, ion slip and the external field arising due to polarization of charges are
negligible. It is assumed that no applied and polarization voltage exists. This corresponds to the case where no
energy is being added or extracted from the fluid by electrical means. Under the foregoing assumptions, the set
of equations governing the slow in Cartesian components are given below:

$$\frac{\partial u^*}{\partial t^*} = -\frac{1}{\rho} \frac{\partial p^*}{\partial x^*} + \nu \frac{\partial^2 u^*}{\partial z^*^2} + 2\Omega^* v^* + \frac{\sigma B_0^2}{\rho (1 + m^2)} (m v^* - u^*) - \frac{\nu}{K^*} u^* + g \beta T^* + g \beta_e C^* \quad (1)$$

$$\frac{\partial v^*}{\partial t^*} = -\frac{1}{\rho} \frac{\partial p^*}{\partial y^*} + \nu \frac{\partial^2 v^*}{\partial z^*^2} - 2\Omega^* u^* - \frac{\sigma B_0^2}{\rho (1 + m^2)} (m u^* + v^*) - \frac{\nu}{K^*} v^* \quad (2)$$

$$\rho C_p \frac{\partial T^*}{\partial t^*} = k \frac{\partial^2 T^*}{\partial z^*^2} - \frac{\partial q^*}{\partial z^*} \quad (3)$$

$$\frac{\partial C^*}{\partial t^*} = D \frac{\partial^2 C^*}{\partial z^*^2} - k^* C^* \quad (4)$$

The boundary conditions for the flow problem are:

$$u^* = v^* = T^* = C^* = 0 \quad \text{at} \quad Z^* = -\frac{d}{2} \quad (5)$$

$$u^* = v^* = 0 \quad T^* = T_w \cos \omega^* t^* \quad C^* = C_w \cos \omega^* t^* \quad \text{at} \quad Z^* = \frac{d}{2} \quad (6)$$

It is assumed that the fluid is optically thin with relatively low density and the radiative heat flux is given by

$$\frac{\partial q^*}{\partial z^*} = 4\alpha^2 T^* \quad (7)$$

Where $\alpha$ is the mean radiation absorption coefficient.

Introducing the following non-dimensional quantities
in the set of equations (1) - (4), we get the following set of equations

\[ \begin{align*}
\frac{\partial u}{\partial t} &= -\frac{\partial P}{\partial x} + \frac{1}{\text{Re}} \frac{\partial^2 u}{\partial \eta^2} + \frac{2 \Omega}{\text{Re}} \frac{M^2 (mv - u)}{\text{Re}(1 + m^2)} - \frac{1}{\text{Re}} \frac{u + Gr}{\text{Re}} T + \frac{Gm \eta}{\text{Re}} C \\
\frac{\partial v}{\partial t} &= -\frac{\partial P}{\partial y} \frac{1}{\text{Re} \partial \eta^2} - \frac{2 \Omega}{\text{Re}} \frac{u - M^2 (mu + v)}{\text{Re}(1 + m^2)} - \frac{1}{\text{Re}} \frac{v}{\text{Re}}
\end{align*} \]  

(8)

\[ \frac{\partial T}{\partial t} = \frac{1}{\text{Pe}} \frac{\partial^2 T}{\partial \eta^2} - \frac{N^2}{\text{Pe}} T \]  

(9)

\[ \frac{\partial C}{\partial t} = \frac{1}{\text{Sc}} \frac{\partial^2 C}{\partial \eta^2} - kr C \]  

(10)

Where \( U \) is the mean axial velocity

\[ R_c = \frac{U d}{\nu} \]

\[ \Omega = \frac{\Omega d^2}{\nu}, \quad K = \frac{K^*}{d^2}, \quad \beta = \frac{\nu^2}{\nu U}, \quad P_e = \frac{\rho \nu u}{k}, \quad N = \frac{2 \alpha d}{\sqrt{k}}. \]

\[ M = B_d \frac{\sigma}{\mu}, \quad S_c = \frac{\nu}{D}, \quad k_r = \frac{k_i d^2}{\nu} \]

The corresponding transformed boundary conditions are:

\[ u = v = T = C = 0 \quad \quad \text{at} \quad \eta = -\frac{1}{2} \]  

(12)

\[ u = v = 0, \quad T = C = \cos \omega t \quad \text{at} \quad \eta = \frac{1}{2} \]  

(13)

For the oscillatory internal flow, we shall assume that the fluid flows only under the influence of a non-dimensional pressure gradient oscillating in the direction of x-axis which is of the form:

\[ -\frac{\partial P}{\partial x} = P \cos \omega t \]  

(14)

3. Solution of the problem:

In order to combine Eqs. (8) and (9) into single equation, let us introduce a complex function \( F = u + iv \) and now we get:
\[
\frac{\partial^2 F}{\partial \eta^2} - \text{Re} \frac{\partial F}{\partial t} - \left[ 2\Omega i + \text{Re} K^{-1} + M^2 \frac{(1 + im)}{(1 + m^2)} \right] F = - P \cos \omega t - GrT - GmC
\]  
(15)

The corresponding boundary conditions in complex form can be written as

\[ F = T = C = 0 \quad \text{at} \quad \eta = - \frac{1}{2} \]  
(16)

\[ F = 0, T = C = e^{i \omega t} \quad \text{at} \quad \eta = \frac{1}{2} \]  
(17)

In order to solve Eqs. (10), (11) and (15) under the boundary conditions given in Eqs (16) and (17), the solution of the problem is assumed in complex form as

\[ F(\eta, t) = F_0(\eta)e^{i \omega t}, T(\eta, t) = \theta_0(\eta)e^{i \omega t}, C(\eta, t) = \phi_0(\eta)e^{i \omega t}, - \frac{\partial P}{\partial x} = P e^{i \omega t} \]  
(18)

The boundary conditions given in Eqs. (16) and (17) become:

\[ F_0 = \theta_0 = \phi_0 = 0 \quad \text{at} \quad \eta = - \frac{1}{2} \]  
(19)

\[ F_0 = 0, \theta_0 = 1, \phi_0 = 1 \quad \text{at} \quad \eta = \frac{1}{2} \]  
(20)

Substituting Eq. (18) in Eqs. (8), (9) & (15), we get:

\[ \frac{d^2 F_0}{d \eta^2} - S^2 F_0 = - P \text{Re} - Gr\theta_0 - Gm\phi_0 \]  
(21)

\[ \frac{d^2 \theta_0}{d \eta^2} - r^2 \theta_0 = 0 \]  
(22)

\[ \frac{d^2 \phi_0}{d \eta^2} - m^2 \phi_0 = 0 \]  
(23)

The ordinary differential Eqs. (21)- (23), are solved under the boundary conditions given in Eqs. (19) and (20), for the velocity, temperature and concentration fields as follows.

\[ F(\eta, t) = \left[ \frac{P \text{Re}}{S^2} \left[ 1 - \frac{\cosh S\eta}{\cosh \frac{S}{2}} \right] + \frac{Gr}{r^2 - S^2} \left[ \frac{\sinh s(\eta + \frac{1}{2})}{\sinh S} - \frac{\sinh r(\eta + \frac{1}{2})}{\sinh r} \right] \right] e^{i \omega t} \]  
(24)
\[ T(\eta, t) = \left[ \frac{\sinh r(\eta + \frac{1}{2})}{\sinh r} \right] e^{i\omega t} \tag{25} \]

\[ C(\eta, t) = \left[ \frac{\sinh m(\eta + \frac{1}{2})}{\sinh m} \right] e^{i\omega t} \tag{26} \]

From the velocity field obtained in equation (24), we can find skin-friction \( \tau_L \) at the lower plate as

\[ \tau_L = \left( \frac{\partial F}{\partial \eta} \right)_{\eta = \frac{1}{2}} - \left( \frac{\partial F_0}{\partial \eta} \right)_{\eta = \frac{1}{2}} = \left| F \right| \cos(\omega t + \phi) \]

The amplitude \( |F| \) and the phase angle \( \phi \) of the skin-friction can be calculated with the help of the following expressions

\[ |F| = \sqrt{F_r^2 + F_i^2} \quad \text{and} \quad \phi = \tan^{-1} \left( \frac{F_i}{F_r} \right) \]

\[ F_r + iF_i = \left\{ \frac{P \text{Re}}{S^2} \left[ \tanh \frac{S}{2} \right] + \frac{Gr}{r^2 - S^2} \left[ \frac{S}{\sinh S} - \frac{r}{\sinh r} \right] + \frac{Gm}{m^2 - S^2} \left[ \frac{S}{\sinh S} - \frac{m}{\sinh m} \right] \right\} \tag{27} \]

From the temperature field obtained in equation (25), we can find Nusselt-number \( Nu \) at the lower plate as

\[ Nu = \left( \frac{\partial T}{\partial \eta} \right)_{\eta = \frac{1}{2}} - \left( \frac{\partial \theta_0}{\partial \eta} \right)_{\eta = \frac{1}{2}} = \left| H \right| \cos(\omega t + \psi) \tag{28} \]

The amplitude \( |H| \) and the phase angle \( \psi \) of the Nusselt-number can be calculated with the help of the following expressions

\[ |H| = \sqrt{H_r^2 + H_i^2} \quad \psi = \tan^{-1} \left( \frac{H_i}{H_r} \right) \quad H_r + iH_i = \frac{r}{\sinh r} \tag{29} \]

From the concentration field obtained in equation (26), we can find Sherwood number \( Sh \) at the lower plate as

\[ Sh = \left( \frac{\partial C}{\partial \eta} \right)_{\eta = \frac{1}{2}} - \left( \frac{\partial \phi_0}{\partial \eta} \right)_{\eta = \frac{1}{2}} = \left| G \right| \cos(\omega t + \theta) \tag{30} \]
The amplitude $|G|$ and the phase angle $\theta$ of the Sherwood number can be calculated with the help of the following expressions

$$|G| = \sqrt{G_r^2 + G_i^2}, \quad \theta = \tan^{-1}\left(\frac{G_i}{G_r}\right),$$

where $G_r + iG_i = \frac{m}{\sinh m}$ (31)

4. Results and Discussion

To discuss the physical significance of various parameters involved in the problem, the numerical calculations have been carried out and only the real parts of the results obtained have been considered. Our results are found in good agreement with the results of Singh and Reena Pathak$^{8,15}$ in the absence of the mass transfer parameters. The effects of the various parameters involved in the governing equations on velocity, temperature, concentration, skin friction, Nusselt number and Sherwood number are studied by Singh$^{8}$ and Singh and Pathak$^{15}$ are not repeating here. However the effects of more prominent parameters which were not reported by them are shown and discussed through graphs.

When an electrical current passes through a sample placed in a magnetic field, a potential proportional to the current and to the magnetic field is developed across the material in a direction perpendicular to both the current and to the magnetic field. This effect is known as the Hall effect, and is the basis of many practical applications and devices such as magnetic field measurements, and position and motion detectors. From Fig. 1, velocity profiles are displayed for different values of Hall parameter $m$. From this figure, it is noticed that the velocity increases with an increase in $m$. These results are in good agreement with the results of Raju et al.$^{5}$. The variations of the amplitude of skin friction with the influence chemical reaction parameter are shown in the Fig. 2. It is observed that the amplitude of skin friction decreases with increase in chemical reaction parameter. The variations of the phase angle of skin friction with the chemical reaction parameter $kr$ are shown in the Fig. 3, it is observed that the phase angle of skin friction initially decreases and then increases with increase in $kr$. Fig. 4 shows that the value of amplitude of Nusselt number decreases with increase of $N$ and $Pe$. Phase angle of the Nusselt number increases with the increasing values of $N$ which can be seen from Fig. 5. From Fig. 6, it is observed that the amplitude of Sherwood number decreases with an increase in $kr$. From Fig. 7, it is observed that the phase angle of the Sherwood number initially increases and then decreases with increase of $kr$.
Fig. 3. Phase angle of the skin-friction for different $K_r$ with $\Omega = 5$, $N = 1$, $Re = 1$, $Pe = 0.7$, $Gr = 1$, $Gm = 1$, $K = 1$, $k = 0.1$, $P = 5$, $m = 1$, $t = 0$ and $Sc = 1$

Fig. 4. Amplitude of the Nusselt number when $t = 0$

Fig. 5. Phase angle of the Nusselt number for different $N$ with $Pe = 0.7$ and $t = 0$

Fig. 6. Amplitude of the Sherwood number for different $K_r$ with $Sc = 1$, $Re = 1$ and $t = 0$

Fig. 7. Phase angle of the Sherwood number for different $K_r$ $Sc = 1$, $Re = 1$ and $t = 0$

Fig. 8. Velocity profiles for different $\Omega$ with $m = 1$, $N = 1$, $M = 1$, $Pe = 0.7$, $\omega = 5$, $Gr = 1$, $Re = 1$, $K = 1$, $Gm = 0$, $P = 5$, $k = 0$, $t = 0$ and $Sc = 0$
5. Validation of the results:

In the absence of Hall current, Porous permeability and mass transfer velocity profiles are obtained and presented in Fig. 8. These results are found to be in good agreement with the results of Singh and Pathak. From this figure, it is observed that the velocity decreases with an increase in the values of rotation parameter.

6. Conclusions

In this manuscript, we studied the effects on chemical reaction, Hall current and radiative periodic flow through porous medium in a rotating vertical channel. The governing equations are solved by the usual analytical method. The findings of this study can be summarized as follows.

- Velocity increases with increasing values of modified Grashof number and Hall parameter while decreases with increasing values of chemical reaction and Schmidt number.
- Temperature decreases with increasing values of Peclet number Pe.
- Concentration decreases with increasing values of chemical reaction parameter and Schmidt number.
- Amplitude of the skin friction decreases with increasing values of chemical reaction parameter.
- Amplitude of the Sherwood number decreases with increasing values of chemical reaction parameter.

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