Forward-backward asymmetry and differential cross section of top quark in flavor violating $Z'$ model at $\mathcal{O}(\alpha_s^2\alpha_X)$

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Abstract

In this paper, forward-backward asymmetry and differential cross section of top quark in flavor violating $Z'$ model up to $\mathcal{O}(\alpha_s^2\alpha_X)$ at Tevatron are calculated. In order to account for the top observed large forward-backward asymmetry, the new coupling $g_X$ among $Z'$ and quarks will be not much less than strong coupling constant $g_s$. After including the new higher order correction, the differential cross section can fit the data better than those only including the leading contributions from $Z'$, while the forward-backward asymmetry is still in agreement with the measurement.

I. INTRODUCTION

As the heaviest fermion in the standard model (SM), top quark is thought to be closely related to the mechanism of electroweak symmetry breaking and physics beyond the SM (BSM). In the last two years, D0 and CDF Collaboration have measured the forward-backward (F-B) asymmetry ($A_{FB}$) of top quark at the Tevatron [1–3]. SM predictions have been estimated in Refs. [4–6]. In the SM, the asymmetry arises from the interference among virtual box and the leading diagrams for the process $q\bar{q} \to t\bar{t}$, as well as the contributions from $q\bar{q} \to t\bar{t}g$. The present experimental measurements and SM theoretical predictions are listed in Table I in the lab ($p\bar{p}$) frame and the center-of-mass (c.m.) frame of the top quark pair ($t\bar{t}$), respectively. From the table we can see that the CDF measured $A_{FB}^{pp}$ is consistent with $A_{FB}^{ct}$, if the theoretically expected dilution of 30% is included [6]. However the SM predictions is significantly smaller than the observations.

TABLE I: A collection of experimental and theoretical results of $A_{FB}$ of top quark at the Tevatron [1–6].

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Recently some theoretical progress has been made both in the SM and the BSM, in order to explain this novel signature. In the SM, soft gluon resummation effects [7] have been scrutinized. However, the prediction involving resummation effects does not change the asymmetry at $\mathcal{O}(\alpha^3_s)$ greatly [7]. Many BSM models, for instance, supersymmetry, extra dimension and left-right model have also been considered [8–17]. New particles such as exotic gluon $G'$, extra $W'$ or $Z'$ bosons and extra scalar $S$ are introduced. All these new models should produce the required asymmetry while keep other observable qualities to be consistent with measurements. Among which, the $t\bar{t}$ invariant mass distribution is an important measurement to constrain the new models. In order to distinguish different models, as depicted in the Ref. [12], higher order effects in these new models are important.

In this paper, we are interested in a BSM model named the flavor violating $Z'$ model (FVZM) [10]. Observed asymmetry can be generated by introducing a right-handed coupling among the $Z'$, the top and up quarks $L \ni g^X_{\mu} Z'_{\mu} \bar{u} \gamma^\nu \frac{1+\gamma^5}{2} t + \text{H.c.}$ A detailed analysis based on leading order (LO) contributions has been given in Ref. [10]. For the suitable parameters, while the asymmetry can be generated, the $t\bar{t}$ invariant mass distribution does not fit the observation well. Therefore it is quite interesting to analyze the asymmetry and $t\bar{t}$ invariant mass distribution after including higher order effects.

The paper is organized as follows. In Sec. II, Born, virtual and real corrections are calculated analytically till to $\mathcal{O}(\alpha^2_s, \alpha^2_X)$. In Sec. III numerical results for differential cross sections and forward-backward asymmetry as a function of the top quark pair invariant mass $M_{t\bar{t}}$ are presented and compared to the experimental data. In Sec. IV we give a short conclusions and discussions.

**II. ANALYTICAL CALCULATION UP TO $\mathcal{O}(\alpha^2_s, \alpha_X)$**

In this section, we will present the analytical formula to calculate the top forward-backward asymmetry, as well as the differential cross section up to $\mathcal{O}(\alpha^2_s, \alpha_X)$. The corresponding Feynman diagrams of subprocesses up to $\mathcal{O}(\alpha^2_s, \alpha_X)$ in FVZM are depicted in Figs. 1-5. In order to account for the top large asymmetry, the new coupling $g_X$ will be not much less than $g_s$. Thus the relevant amplitude for $t\bar{t}$ final states can be written, in perturbation series of couplings, as

$$\mathcal{M}^{t\bar{t}} = f_s \alpha_s + f_X \alpha_X + f^1_s \alpha^2_s + f^1_X \alpha_s \alpha_X + \cdots$$

with $\alpha_s = g^2_s/(4\pi)$, $\alpha_X = g^2_X/(4\pi)$, and $f$’s the corresponding form factors. Squaring the amplitude we obtain

$$|\mathcal{M}^{t\bar{t}}|^2 = |f_s|^2 \alpha^2_s + 2 \Re(f_s^* f_X) \alpha_s \alpha_X + |f_X|^2 \alpha^2_X + 2 \Re(f_s^* f^1_s) \alpha^3_s + 2 \Re(f^1_s f^1_X + f^1_s f^1_X) \alpha^2_s \alpha_X + \cdots$$

|\mathcal{M}^{t\bar{t}}|^2 = |f_s|^2 \alpha^2_s + 2 \Re(f_s^* f_X) \alpha_s \alpha_X + |f_X|^2 \alpha^2_X + 2 \Re(f_s^* f^1_s) \alpha^3_s + 2 \Re(f^1_s f^1_X + f^1_s f^1_X) \alpha^2_s \alpha_X + \cdots
In order to cancel the infrared divergences, the corresponding gluon radiation processes should be included. The amplitude for $t\bar{t}g$ final states can be written similarly as 

$$\mathcal{M}^{t\bar{t}g} = f_s^r \alpha_s \sqrt{\alpha_s} + f_X^r \alpha_X \sqrt{\alpha_s} + \cdots$$

(3)

Squaring this amplitude we obtain

$$|\mathcal{M}^{t\bar{t}g}|^2 = |f_s^r|^2 \alpha_s^3 + 2\mathcal{R}(f_s^{r*} f_X^r) \alpha_s^2 \alpha_X + \cdots$$

(4)

In the SM, the asymmetry arises from the $O(\alpha_s^3)$ term. In the FVZM, new contributions till to $O(\alpha_s^2 \alpha_X)$ are calculated in [10]. In this paper, the extra contributions at $O(\alpha_s^2 \alpha_X)$ will be calculated. The SM $u\bar{u} \rightarrow t\bar{t}$, $d\bar{d} \rightarrow t\bar{t}$ up to QCD NLO and $gg \rightarrow t\bar{t}$ up to QCD LO contributions are recalculated though their analytical expressions are not shown in this paper.

A. Contributions up to $O(\alpha_X^2)$

Typical Feynman diagrams, which contribute to the amplitude up to $O(\alpha_X^2)$, are shown in Fig. 1.

![Feynman diagrams](image)

FIG. 1: Typical Feynman diagrams with contributions for form factors $f_s$ and $f_X$.

The form factors of $2\mathcal{R}(f_s^r f_X^r)$ and $|f_X|^2$ with spin- and color-summed (same for the following form factors) are given by

$$2\mathcal{R}(f_s^r f_X^r) = \frac{64\pi^2 C_A C_F}{m_{Z'}^4 s (t - m_{Z'}^2)} \left[ m_t^6 + \left(2m_{Z'}^2 + s - 2t\right)m_t^4 + \left(t^2 - 2m_{Z'}^2(s + 2t)\right)m_t^2 + 2m_{Z'}^2(s + t)^2 \right],$$

(5)

$$|f_X|^2 = \frac{144\pi^2}{m_{Z'}^4 (t - m_{Z'}^2)^2} \left[ m_t^8 - 2tm_t^6 + \left(4m_{Z'}^4 + 4sm_{Z'}^2 + t^2\right)m_t^4 - 8m_{Z'}^4(s + t)m_t^2 + 4m_{Z'}^4(s + t)^2 \right],$$

(6)

where $C_A = 3, C_F = 4/3$ and $s = (p_1 + p_2)^2, t = (p_1 - k_1)^2$ are the Mandelstam variables.

B. Contributions at $O(\alpha_s^2 \alpha_X)$

The corresponding Feynman diagrams related to $O(\alpha_s^2 \alpha_X)$ are shown in Figs. 2-5. In order to regulate the divergences, dimensional regularization is adopted with $D = 4 - 2\epsilon$. 
Infrared (IR) and ultra violet (UV) divergences are represented by $1/\epsilon_{IR}$ and $1/\epsilon_{UV}$ respectively. The wave function renormalization constants are determined by the on-mass-shell scheme while the MS scheme is chosen for the strong coupling constants renormalization. The calculations are carried out with the help of FeynCalc [18], FormCalc [19] and QCD-loop [20]. At hadron collider, in order to eliminate the collinear singularity, factorization should be carried out. In this paper, the MS factorization is adopted, as shown explicitly below.

\[
FIG. 2: \text{Typical Feynman diagrams with contributions to form factors } f_{sX}^1.
\]

Once the renormalization is carried out by adding appropriate counterterms, the amplitude is free of UV divergences. The counter terms are given explicitly in Appendix A. However there are still IR divergences in $f_{sX}^1$ and $f_{s}^1$. In order to eliminate the remaining infrared divergence, the gluon emission processes should be included as shown in Figs. 4 and 5. Two cutoff phase space slicing method [21] is applied for such processes. First, small parameter $\delta_s$ is introduced to separate the final phase space into soft part and hard part. Second, small parameter $\delta_c$ is introduced to divide the hard part into a hard collinear part and a hard non-collinear part. These three parts are calculated separately. Their summation should be independent of the two small parameters $\delta_s$ and $\delta_c$.

The remaining IR divergences appear in $2\Re(f_s^* f_{sX}^1 + f_{sX}^* f_s^1) \alpha_s^2 \alpha_X$ [c.f. Eq. 2] can be expressed as

\[
2 \Re(f_s^* f_{sX}^1 + f_{sX}^* f_s^1) \alpha_s^2 \alpha_X = V_f + V_1 \frac{1}{\epsilon_{IR}} + V_2 \frac{1}{\epsilon_{IR}^2},
\] (7)
FIG. 3: Typical Feynman diagrams with contributions to form factors $f_1$. 

FIG. 4: Typical Feynman diagrams with contributions to form factors $f_2$. 

where $V_f$ indicates the finite part and IR coefficients $V_1, V_2$ are

$$V_1 = [2 \mathcal{R}(f^*_s f_X) \gamma^s \alpha_X] \frac{-2\alpha_s}{3\pi e^\lambda_C F} \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)} \left\{ \frac{\alpha s}{4\pi} \log\left(\frac{1+\beta}{1-\beta}\right) ight. $$
$$+ 9 \log\left(\frac{m_t^2}{s}\right) + 10 \log\left(\frac{4\pi}{\mu^2}\right) - 16 \log\left(\frac{m_t^2 + s}{\mu^2}\right) - 2 \log\left(\frac{m_t^2 + s + t}{\mu^2}\right) \left\} \right. $$

$$\frac{16(4\pi)^3\alpha_s^2 \alpha_X}{3\pi^2 \Gamma(1-\epsilon)} \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)} \frac{3m_t^2 + 3s^2 + 5s + 8st + 3t^2 - m_t^2 (5s + 6t)}{s (t-m_t^2)^2},$$

$$V_2 = [2 \mathcal{R}(f^*_s f_X) \gamma^s \alpha_X] \frac{-16\alpha_s}{3\pi e^\lambda_C F} \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)};$$

FIG. 5: Typical Feynman diagrams with contributions to form factors $f_X$. 

where $\beta = \sqrt{1 - 4m_2^2/s}$ and $\mu$ is an energy scale introduced in dimensional regularization. Here
\[
2\bar{\mathcal{R}}(f_s^* f_X) = 128\pi^2 C_A C_F \frac{m_1^4 + (s + t)^2 - m_1^2(s + 2t)}{s(t - m_{Z'}^2)} \tag{9}
\]
is different from $2\bar{\mathcal{R}}(f_s^* f_X)$ in Eq. [5] as Goldstone boson contribution is ignored.

The IR poles in Eq. [4] have two physical origins: soft & collinear divergences. The double pole $1/\epsilon_{IR}^2$ indicates an overlap between soft and collinear divergences, and the divergences can be eliminated by including contributions from the soft region for gluon emission processes and parton distribution function (PDF) redefinition [21]. Gluon emission process in soft region can be calculated by the eikonal approximation method (the details are given in Appendix B), which can be expressed as
\[
\left[2\bar{\mathcal{R}}(f_s^* f_X) \alpha_s^2 \alpha_X\right]_{\text{Soft}} = S_f + R_1 \frac{1}{\epsilon_{IR}} + R_2 \frac{1}{\epsilon_{IR}^2}, \tag{10}
\]
where $S_f$ indicates the finite part and IR coefficients $R_1, R_2$ are
\[
R_1 = [2\bar{\mathcal{R}}(f_s^* f_X)^D \alpha_s \alpha_X] \frac{\alpha_s}{3\pi C_A C_F} \frac{(4\pi)^\epsilon}{\Gamma(1 - \epsilon)} \left\{ \frac{1 + \beta^2}{\beta} \log\left(\frac{1 + \beta}{1 - \beta}\right) + 16 + 18 \log\left(\frac{m_2^2}{s}\right) - 32 \log \delta_s + 20 \log\left(\frac{\mu^2}{m_2^2}\right) - 32 \log\left(\frac{(s + t - m_2^2)}{\mu^2}\right) - 4 \log\left(\frac{(s + t - m_2^2)}{\mu^2}\right) \right\}, \tag{11}
\]
\[
R_2 = [2\bar{\mathcal{R}}(f_s^* f_X)^D \alpha_s \alpha_X] \frac{16\alpha_s}{3\pi C_A C_F} \frac{(4\pi)^\epsilon}{\Gamma(1 - \epsilon)},
\]
where $2\bar{\mathcal{R}}(f_s^* f_X)^D$ is a D-dimension version of $2\bar{\mathcal{R}}(f_s^* f_X)$,
\[
2\bar{\mathcal{R}}(f_s^* f_X)^D = 128\pi^2 C_A C_F \frac{(1 - \epsilon_{IR})(m_1^4 + (s + t)^2 - m_1^2(s + 2t) + s(t - m_{Z'}^2)\epsilon_{IR})}{s(t - m_{Z'}^2)} \tag{12}
\]
PDF redefinition in soft region can be written as [21],
\[
[2\bar{\mathcal{R}}(f_s^* f_X)^D \alpha_s \alpha_X] \frac{\alpha_s}{4\pi} \frac{(4\pi)^\epsilon}{\Gamma(1 - \epsilon)} \left[ 3C_F + 4C_F \log \delta_s + (\log\left(\frac{\mu^2}{\mu^2}\right) - \frac{1}{\epsilon_{IR}}) \right]. \tag{13}
\]
IR cancelation is realized by adding Eqs. [7] and [10], and subtract 2 times of expression in Eq. [13].

For the hard collinear part for gluon emission processes, there are collinear divergences which can be eliminated by PDF redefinition in this region. The remaining finite part is in the form of a convolution integral with a splitting function. The detail of this procedure is described in Ref. [21].

The remaining hard noncollinear part of gluon emission processes is finite and the integration is performed with a standard three body Monte Carlo code.

During the calculation, there are three scales in the hadron level cross section. $\mu$ is an energy scale introduced in dimensional regularization. $\mu_r$ is a renormalization scale introduced in the $\overline{\text{MS}}$ renormalization of the coupling constant. $\mu_f$ is a factorization scale introduced in $\overline{\text{MS}}$ factorization. The $\mu$ always comes with the divergences and is
canceled completely when the corresponding poles are subtracted. The dependence of our final results on $\mu_r$ & $\mu_f$ will be shown in the numerical results.

The independence of the total cross section with $\delta_s$ and $\delta_c$ has been checked in the situation $\delta_c << \delta_s$, as suggested by the two cut off phase space slicing method in Ref. [21].

III. NUMERICAL RESULTS

In this section, we will present the numerical results and compare them with the experimental measurements. We choose cteq6l for leading order calculation and cteq6m for higher order calculations. The scales $\mu_r$ and $\mu_f$ are set to be equal and $\alpha_S(m_Z) = 0.118$.

![Figure 6](image-url)

**FIG. 6:** Differential cross sections as a function of $M_{tt}$ with $\mu_r = \mu_f = m_t$. Here “QCD Born” and “QCD NLO” represent the results in the SM at leading order and next-to-leading order in QCD. “QCD Born + Z' Born” and “QCD NLO + Z' NLO” stand for the predictions in FVZM up to $\mathcal{O}(\alpha_X^2)$ and $\mathcal{O}(\alpha_S^2\alpha_X)$ respectively [c.f. Eqs. 2 and 4].

Differential cross sections as a function of $M_{tt}$ are shown in Fig. 6. Histograms are drawn here in order to compare conveniently with the experimental measurements. The parameters in the FVZM are taken to be $\alpha_X = 0.024$, $M_{Z'} = 160$GeV which is the best point [10] to account for the top asymmetry. From the figure it is obvious that the NLO QCD prediction is in good agreement with the data except the bin around 400 GeV. It should be noted that even the multiple soft gluon radiation effects are included, the discrepancy remains (c.f. Ref. [7]). However the top quark asymmetry at NLO...
QCD is much less than the measurement. After including the contributions from the leading diagrams from $Z'$, for the favorable parameters, the top quark asymmetry can be generated. However the differential cross section does not agree with measurement well. From the figure, we can see the prediction is lower than measurement for small $M_{t\bar{t}}$ region while higher for large $M_{t\bar{t}}$. After including the higher order effects, the differential distribution will be better while the top asymmetry can be generated. These behavior can be understood as follows. In the vicinity of $t\bar{t}$ production threshold region, the significant contributions comes from the interference among QCD and extra $Z'$ Feynman diagrams. Such contributions will decrease the cross section. In the higher $M_{t\bar{t}}$ region, the square of $Z'$ diagrams become significant and they will uplift the cross section. The deviation from NLO QCD prediction will be soften after including the higher order contributions.

![Graphs showing differential cross sections](image)

FIG. 7: Differential cross sections $d\sigma/dM_{t\bar{t}}$ as a function of $M_{t\bar{t}}$ with $\mu_r = \mu_f = 0.5m_t, m_t, 2m_t$ respectively, for 4 sets of parameters. Other conventions are the same with Fig. 6. The factor $k = \sigma^{NLO}/\sigma^{LO}$ is also indicated.

In Fig. 7, we show the differential cross sections $d\sigma/dM_{t\bar{t}}$ with $\mu_r = \mu_f = 0.5m_t, m_t, 2m_t$ respectively, for four sets of typical parameters, namely different $M_{Z'}$ and $\alpha_X$. From the histograms, we can see that the above-mentioned improvement after including the higher order effects is universal. We calculated the k factor, which is defined as $k = \sigma^{NLO}/\sigma^{LO}$. Here $\sigma^{LO}$ and $\sigma^{NLO}$ are the cross sections up to $\mathcal{O}(\alpha_X^2)$ and $\mathcal{O}(\alpha_S^2\alpha_X)$ respectively. For four sets of parameters, the k-factor is equal to 1.03, 0.996, 0.998, 0.992, respectively.
for $\mu_r = \mu_f = m_t$. It is obvious that the NLO contributions mainly change the shape of distribution. As for the scale dependence, the results of “QCD NLO” and “QCD NLO+$Z'$ NLO” are about the same size. “QCD NLO+$Z'$ Born” is significantly smaller than them. “QCD NLO + $Z'$ Born” scale dependence is small because QCD NLO and $Z'$ Born have opposite $\mu_r/\mu_f$ dependence.

![Graph showing forward-backward asymmetry distributions as a function of $M_{t\bar{t}}$](image)

**FIG. 8:** Forward-backward asymmetry distributions as a function of $M_{t\bar{t}}$ with $\mu_r = \mu_f = m_t$.

Histograms of the forward backward asymmetry $A_{FB}^{t\bar{t}}$ as a function of $M_{t\bar{t}}$ in $t\bar{t}$ rest frame are drawn in Fig. 8 where $A_{FB}^{t\bar{t}}$ in each bin is defined as

$$A_{FB}^{t\bar{t}} = \frac{N(\Delta Y > 0) - N(\Delta Y < 0)}{N(\Delta Y > 0) + N(\Delta Y < 0)}$$  \hspace{1cm} (14)$$

where $\Delta Y = Y_t - Y_{\bar{t}}$ denotes the difference between the $t$ and $\bar{t}$ rapidities. The total $A_{FB}^{t\bar{t}}$ is calculated to be 15.8%, by summing $A_{FB}^{t\bar{t}}$ in each bin multiplied by their corresponding weights. To compare directly with experimental data [22], we also draw the so called
“above” and “below” $A_{FB}$ distribution at the bottom of Fig. 8 in which $A_{FB}$ is measured or calculated for $M_{\bar{t}t}$ above or below a certain value. It should be noted that the experimental data are measured in the $p\bar{p}$ lab frame. As the simplest approximation, we utilize the relation $A_{FB}^{p\bar{p}} \approx 0.7A_{FB}^t$. Obviously more measurements are needed to decrease the experimental uncertainties in order to confirm/exclude the FVZM.

Total cross sections and total $A_{FB}^t$ and are shown together in Table II. For total cross section, the $Z'$-born contribution decreases the NLO QCD cross section. Including the $Z'$-NLO corrections makes the cross section even smaller although these corrections are not significant. On the contrary $A_{FB}^t$ is sensitive to $Z'$-NLO correction and can drop about 30% from the $Z'$-Born value.

| QCD NLO | QCD NLO+Z' Born | QCD NLO+Z' NLO |
|---------|----------------|----------------|
| $A_{FB}^t$ (%) | 6.8 | 22.2 | 15.8 |
| Total cross section(pb) | 6.29 | 5.52 | 5.13 |

IV. CONCLUSIONS AND DISCUSSIONS

In this paper, we calculate the top quark differential cross section and asymmetry up to $\mathcal{O}(\alpha_s^2\alpha_X)$ in a flavor violating $Z'$ model (FVZM). In the FVZM, the leading $Z'$ contribution can induce the measured top asymmetry, while the differential distribution of $M_{\bar{t}t}$ does not fit measurement well. After including the higher order contribution, the differential distribution can be improved while the top asymmetry is still in agreement with the observed value.

QCD soft gluon resummation effects for the top quark pair production in the SM have been considered in Ref. [7]. Such effects do not change the $M_{\bar{t}t}$ distribution significantly. It is expected that resummation effect in the FVZM is similar to that in the SM because the internal $Z'$ contributions has nothing to do with the soft gluon radiations from external quark legs. Such effects are under investigation [23].

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Appendix A. Renormalization Constants

Renormalization constants are needed when calculating Fig. 2 and Fig. 3. $\delta Z_u, \delta Z_t, \delta Z_A$ corresponds to up quark, top quark and gluon on-mass-shell renormalization constants respectively. $\delta Z_g$ is the coupling renormalization constants.
\[
\delta Z_u = \frac{\alpha_s}{2\pi \Gamma(1-\epsilon)} \left( -\frac{1}{2} C_F \frac{1}{\epsilon_{UV}} + \frac{1}{2} C_F \frac{1}{\epsilon_{IR}} \right), \\
\delta Z_t = \frac{\alpha_s}{2\pi \Gamma(1-\epsilon)} \left( -\frac{1}{2} C_F \frac{1}{\epsilon_{UV}} - C_F \frac{1}{\epsilon_{IR}} - 2C_F + \frac{3}{2} C_F \log\left(\frac{m_t^2}{\mu^2}\right) \right), \\
\delta Z_A = \frac{\alpha_s}{2\pi \Gamma(1-\epsilon)} \left( \left( \frac{5}{6} C_A - 2T_F n_{lf} - \frac{2}{3} T_F n_{hf} \right) \frac{1}{\epsilon_{UV}} - \left( \frac{5}{6} C_A - \frac{2}{3} T_F n_{lf} \right) \frac{1}{\epsilon_{IR}} \right) \\
+ \frac{2}{3} T_F \left( \log\left(\frac{m_t^2}{\mu^2}\right) + \log\left(\frac{m_t^2}{\mu^2}\right) + \log\left(\frac{m_t^2}{\mu^2}\right) \right),
\]
\[
\delta Z_g = \frac{\alpha_s}{2\pi \Gamma(1-\epsilon)} \left( \left( \frac{1}{\epsilon_{UV}} - \log\left(\frac{\mu^2}{\mu^2}\right) \right) \left( -\frac{11}{12} C_A + \frac{1}{3} T_F \right) + \frac{1}{3} T_F n_{hf} \right),
\]
where \(C_F = 4/3, C_A = 3, T_F = 1/2\), \(n_{lf} = 3\) is the number of light quark flavors, \(n_{hf} = 3\) is the number of heavy quark flavors, \(\mu\) is the energy scale introduced in dimensional regularization, \(\mu_r\) is the renormalization scale.

There are new contributions to top and up quark field renormalization constants, when we calculate Fig. 2. Top quark field renormalization constants \(\delta Z^V_t, \delta Z^A_t\) are calculated from \(Z^\prime\) induced top Self-energy \(-i\Sigma_t(y)\)\(\text{[24]}\), as showed in Fig. 9.

![FIG. 9: Self energy diagrams for counterterm calculation.](image)

\[
\Sigma_t(y) = y'(\Sigma_t^V(p^2) + \Sigma_t^A(p^2)\gamma^5) + m_t\Sigma_t^S(p^2),
\]
\[
\delta Z_t^V = \Sigma_t(p^2)|_{p^2=m_t^2} + 2m_t^2 \frac{\partial}{\partial p^2} (\Sigma_t^V(p^2)|_{p^2=m_t^2} + \Sigma_t^A(p^2)|_{p^2=m_t^2}),
\]
\[
\delta Z_t^A = \Sigma_t^A(p^2)|_{p^2=m_t^2}.
\]

The counterterm for \(t\bar{t}g\) vertex is written as
\[
(-ig_s T^a \gamma^\rho)(\delta Z_t^V + \delta Z_t^A \gamma^5),
\]
where the vector and axial vector parts are

\[
\delta Z^V_t = \frac{\left(4\pi\right)^2}{\Gamma(1-\epsilon)} \frac{g_X^2}{32m_t^2 \pi^2} \left\{ \frac{m_t^2}{c_{UV}} + \left[ 2(m_t^2 + m_u^2 - m_z^2) \frac{\partial}{\partial p^2} B_0(p^2, m_u^2, m_z^2) \right]_{p^2 = m_t^2} - 1 \right\} \left[ m_t^2 + (m_t^2 - m_z^2)B_0(0, m_u^2, m_z^2) + \left( -m_t^2 + m_z^2 \right)B_0(m_t^2, m_u^2, m_z^2) \right],
\]

\[
\delta Z^A_t = \frac{\left(4\pi\right)^2}{\Gamma(1-\epsilon)} \frac{g_X^2}{32m_t^2 \pi^2} \left\{ \frac{m_t^2}{c_{UV}} + \left[ -m_t^2 + (m_z^2 - m_u^2)B_0(0, m_u^2, m_z^2) \right] \right\}.
\]

Field renormalization constants of the up quark can be calculated similarly and the counterterm for \( u \bar{u} g \) vertex is written as

\[
\left( -ig_s T^a \gamma^\rho \right) \left( \delta Z^V_u + \delta Z^A_u \gamma^5 \right),
\]

where

\[
\delta Z^V_u = \frac{\left(4\pi\right)^2}{\Gamma(1-\epsilon)} \frac{g_X^2}{32m_u^2 \pi^2} \left\{ \frac{m_u^2}{c_{UV}} + \left[ 2(m_t^2 + m_u^2 - m_z^2) \frac{\partial}{\partial p^2} B_0(p^2, m_u^2, m_z^2) \right]_{p^2 = m_u^2} - 1 \right\} \left[ m_t^2 + (m_t^2 - m_z^2)B_0(0, m_u^2, m_z^2) + \left( -m_t^2 + m_z^2 \right)B_0(m_t^2, m_u^2, m_z^2) \right],
\]

\[
\delta Z^A_u = \frac{\left(4\pi\right)^2}{\Gamma(1-\epsilon)} \frac{g_X^2}{32m_u^2 \pi^2} \left\{ \frac{m_t^2}{c_{UV}} + \left[ -m_t^2 + (m_z^2 - m_u^2)B_0(0, m_u^2, m_z^2) \right] \right\}.
\]

**Appendix B. Soft part of the real gluon emission cross section**

Soft real squared amplitude \( |\mathcal{M}^{soft}_{q\bar{q} \to t\bar{t}g}|^2 \), which is expressed as \( 2\Re \left( f_s^* f_X \right) \alpha_s^2 \alpha_X \), can be obtained by the interference of diagrams in Figs. 4 and 5 with requirement that the gluon’s energy is smaller than \( \delta_s \sqrt{3}/2 \). \( |\mathcal{M}^{soft}_{q\bar{q} \to t\bar{t}g}|^2 \) can be expressed as

\[
|\mathcal{M}^{soft}_{q\bar{q} \to t\bar{t}g}|^2 = |\mathcal{M}_{q\bar{q} \to t\bar{t}}|^2 \sum_{i,j=1}^4 \frac{C_{ij}}{C_0} s_{ij},
\]

where \( |\mathcal{M}_{q\bar{q} \to t\bar{t}}|^2 = 2\Re \left( f_s^* f_X \right) \alpha_s \alpha_X \) is the interference term of the two born diagrams in Fig. 4. \( C_{ij} = C_{ji} \) is the color factor of interference terms with one gluon emitting from
external leg $i$ of one diagram and from external leg $j$ of another diagram.

$$C_{12} = C_{14} = C_{23} = C_{34} = -C_F/2,$$

$$C_{11} = C_{22} = C_{13} = C_{24} = C_{33} = C_{44} = C_A C_F^2.$$  \hspace{1cm} (22)

$C_0 = C_A C_F$ is the color factor of the interference of the two diagrams in Fig.1. $S_{ij} = S_{ji}$ are the soft factors of the corresponding interference terms. They are calculated by using eikonal approximation method. According to Ref. [21],

$$S_{ij} = \frac{a_s}{4\pi} \left(\frac{4\pi^*}{\Gamma(1-\epsilon)}\right) \left(\frac{\mu^2}{\pi}\right)^\epsilon$$

$$\times \frac{1}{\pi} \left(\frac{A}{\pi}\right)^\epsilon \int_0^{\delta_s} dE_d \int_0^{\delta_s} d\theta_1 \int_0^{\delta_s} d\theta_2 \left(\eta_i \eta_j \frac{p_i^\mu}{p_i^\nu} \frac{p_j^\nu}{p_j^\mu} (-g^{\mu\nu})\right) E_d^{1-2\epsilon} \sin^{1-2\epsilon} \theta_1 \sin^{1-2\epsilon} \theta_2,$$

where $\eta_i$ is a sign which is positive for outgoing fermion or incoming antifermion, and is negative for incoming fermion or outgoing antifermion.

$$S_{11} = S_{22} = 0,$$

$$S_{12} = \frac{a_s}{2\pi} \left(\frac{4\pi^*}{\Gamma(1-\epsilon)}\right) \left(\frac{1}{\epsilon i_R}\right) - 2 \log\left(\frac{\delta_{\text{IR}}}{\mu}\right) \frac{1}{\epsilon i_R} + 2 \log^2\left(\frac{\delta_{\text{IR}}}{\mu}\right) - \frac{\pi^2}{6},$$

$$S_{13} = S_{24} = \frac{a_s}{2\pi} \left(\frac{4\pi^*}{\Gamma(1-\epsilon)}\right) \left\{ \frac{1}{\epsilon i_R} - \log\left(\frac{\delta_{\text{IR}}}{\mu}\right) \frac{1}{\epsilon i_R} + \frac{1}{2} \log\left(\frac{(1-\beta \cos \theta)^2}{1-\beta^2}\right) \right\}$$

$$-\frac{1}{4} \log^2\left(\frac{1}{1-\beta}\right) + \log^2\left(\frac{\sqrt{\delta_{\text{IR}}}}{\mu}\right) + \frac{1}{2} \log^2\left(\frac{1}{1-\beta \cos \theta}\right)$$

$$+ \log\left(\frac{\delta_{\text{IR}}}{\mu}\right) \log\left(\frac{(1-\beta \cos \theta)^2}{1-\beta^2}\right) + li_2\left(\frac{-\beta (\cos \theta - 1)}{1-\beta}\right) - li_2\left(\frac{-\beta \cos \theta + 1}{1-\beta}\right) - \frac{\pi^2}{12},$$

$$S_{14} = S_{23} = \frac{a_s}{2\pi} \left(\frac{4\pi^*}{\Gamma(1-\epsilon)}\right) \left\{ -\frac{1}{\epsilon i_R} + \log\left(\frac{\sqrt{\delta_{\text{IR}}}}{\mu}\right) + \frac{1}{2} \log\left(\frac{(1-\beta \cos \theta)^2}{1-\beta^2}\right) \right\}$$

$$+\frac{1}{4} \log^2\left(\frac{1}{1-\beta}\right) - \log\left(\frac{\delta_{\text{IR}}}{\mu}\right) \frac{1}{\epsilon i_R} + \log^2\left(\frac{1}{1-\beta \cos \theta}\right)$$

$$- \log\left(\frac{\delta_{\text{IR}}}{\mu}\right) \log\left(\frac{(1-\beta \cos \theta)^2}{1-\beta^2}\right) - li_2\left(\frac{-\beta (\cos \theta - 1)}{1-\beta}\right) + li_2\left(\frac{-\beta \cos \theta + 1}{1-\beta}\right) + \frac{\pi^2}{12},$$

$$S_{33} = S_{44} = \frac{a_s}{2\pi} \left(\frac{4\pi^*}{\Gamma(1-\epsilon)}\right) \left\{ \frac{1}{\epsilon i_R} + \frac{1}{\beta} \log\left(\frac{\beta + 1}{1-\beta}\right) - 2 \log\left(\frac{\sqrt{\delta_{\text{IR}}}}{\mu}\right), \right\},$$

$$S_{34} = \frac{a_s}{2\pi} \left(\frac{4\pi^*}{\Gamma(1-\epsilon)}\right) \left\{ -\frac{(\beta + 1)}{\beta} \log\left(\frac{\beta + 1}{1-\beta}\right) \frac{1}{\epsilon i_R} \right\}$$

$$- \frac{(\beta + 1)}{\beta} \left\{ \frac{1}{4} \log^2\left(\frac{\beta + 1}{1-\beta}\right) - \log\left(\frac{\delta_{\text{IR}}}{\mu}\right) \log\left(\frac{\beta + 1}{1-\beta}\right) + li_2\left(\frac{2\beta}{\beta + 1}\right) \right\}.$$
in which $\beta = \sqrt{1 - 4m^2/s}$ and $\theta$ is the angle between the incoming $u$ and outgoing $t$ quark in $t\bar{t}$ rest frame.

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