Impact of Industry on the Forest Resources
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ABSTRACT
In this manuscript, a two-dimensional mathematical model consisting of forest biomass and industry is proposed and analyzed. In this dynamical system, the impact of the industry has been studied for the conservation of forests. It is assumed that industry consumes forests for its development and the government has a check on the industry to conserve biomass. The local stability and bifurcation analysis of the model have been studied. We have carried out the Hopf-bifurcation analysis by considering government control as a bifurcation parameter. All analytical results are properly validated by numerical simulation.

Keywords: Forest biomass, Industry, Government control, Hopf-bifurcation, Bifurcation parameter.

Section 1

INTRODUCTION
Forest biomass is a very precious and irreplaceable treasure of our biosphere also Industry is an integral part of every economy. The survival of living organisms on earth depend upon forest biomass as global cycling of water, oxygen, carbon, nitrogen, medicine as well as the habitat of many birds and animal species depends upon forest biomass. Every country grazes its natural resources for its development but the excessive use of these resources is very harmful to the biosphere. Industrialization is one of the important factors of study because it is the major reason for depletion of forestry resources which causes ecological, social imbalance, climate change, ozone lyre and global warming etc. Numerous mathematical models are expressing many reasons for depletion of resource biomass in which some are renewable and others are not. Many national and international agencies working for the protection and conservation of natural resources. It is a debatable issue for these agencies that how to conserve natural resources for future generations. So there is a confliction among countries of the world that how to apply the law of economics with minimum resources and maximizing the profit i.e. how to use natural resources for development in the present and conservation of resources for the future. Many mathematicians like Shukla (1996), Dubey (1997) and Dubey & Shukla (1997) have analyzed the mathematical model to study the depletion of resource biomass by industrialization and population. Dubey and Hussain (2003) proposed a model with diffusion in which two species compete with each other for common resources. Then their mathematical models are extended with new a factor added pollution effect with industrialization and population. Mishra et al. proposed a new idea to save resource biomass by giving some economic incentives to people to decrease the population pressure on forest resources. M. Agarwal, S.Devi 2010 proposed and analyzed a nonlinear mathematical model and suggests to impose a heavy environmental tax to decrease population pressure on the survival of biological species in a populated environment.
Manisha Chaudhary et al. and Agarwal (2013) have suggested to more uses of the synthetic base industry to save forest resources. They analyzed the dynamics of the wood-based industry and synthetic-based industry to make the environment eco-friendly through numerical simulation. Sunita Gakhar, Saroj Kumar Sahni 2009 proposed a model for delay effect of oxidants on resource biomass system. There are many other papers on the depletion of forestry resource biomass who discussed the reasons for resource biomass. Further, they have also studied conservation of the resources biomass by introducing new dynamics namely technology. In [2, 3, 4, 5, 6] some mathematical models have been presented to study the survival of wildlife species in a forest habitat which is being affected by the tremendous increase in industrialization. They showed that industrialization affects the forest biomass adversely which in turn causes harm to wildlife species like elephants etc. in Doon Valley in Uttarakhand. Researchers in [7-8] and [11-18] have studied the depletion of forestry resources in habitat due to the increase in population and pollution where pollutant emission into the environment is either population dependent, constant, periodic or instantaneous. The uncontrolled increase in population and pollution may result in the extinction of biomass resources. They have also modelled the conservation of resource biomass to control population growth and pollutant emission. So, to maintain ecological stability and sustainable growth of industrialization there should be some external agencies to control the growth of industries. [9-10] proposed mathematical models in which they analyzed that wood and non-wood-based industry is the major reason of pollution affecting resource biomass. In literature, we have found many reasons for depletion of resource biomass e.g. industrialization, population, migrated population and pollution etc. But industry along with government control is not taken dynamics by any researcher, which is very common in real life. In our proposed model, we have introduced government control on the growth of Industrial units. Our main aim is to conserve Forest Biomass by keeping government control on Industry. The organization of this paper is as follows: Sections 1 and 2 consist of an introduction and a mathematical model with certain assumptions. Boundedness and positivity of the model system are discussed in Section 3. In Sections 4 and 5, we have obtained various equilibrium points and their local stability and Hopf-bifurcation. Numerical simulation and discussion are presented in Sections 6 and 7 respectively.

**Section-2**

Our mathematical model

\[
\begin{align*}
\frac{dX}{dt} &= ax \left(1 - \frac{X}{M}\right) - \frac{bYX}{(h+X)} \\
\frac{dY}{dt} &= bY \left(1 - \frac{Y}{N}\right) + \frac{bYXQ}{(h+X)} - aY 
\end{align*}
\]

Here \(X(t)\) and \(Y(t)\) are the concentrations of forest biomass and Industrial units at time \(t\) respectively.

**Assumptions**

| S.No | Parameter | Biological meaning                          |
|------|-----------|---------------------------------------------|
| 1    | \(a\)    | Intrinsic growth rate of                    |
The growth of industrial units and Forest biomass follow logistic growth and industrial growth follow Holling type –II interaction with biomass.

**Section 3**

**Boundedness and positivity of solution.**

Theorem: Let X(0)=X₀>0, Y(0)=Y₀ >0 then the solutions X(t), Y(t) of the model system lies in a bounded region and positive for all t >0.

Proof: The first equation of the system gives us

\[
\frac{dX}{dt} \leq aX \left( 1 - \frac{X}{M} \right),
\]

as \( t \to \infty \), \( 0 \leq X \leq M \),

\[
X \leq \frac{MX(0)}{e^{\Delta t}M - X(0) + X(0)} \text{as} \ t \to \infty, \ 0 \leq X \leq M,
\]

\[
\frac{dY}{dt} \leq bM + Qa_1XY - a_4Y \leq bM + Qa_1MY - a_4Y \text{as} \ B \leq K
\]

\[
= \frac{dY}{dt} - QY \leq bM \text{as} \ t \to \infty
\]

\[
\frac{dY}{dt} \leq \frac{bM}{Q} \Rightarrow 0 \leq Y \leq \frac{bM}{Q} = I_1
\]

\[
\Rightarrow X(t) \wedge Y(t) \text{lies in the bounded region \wedge remain positive } \forall t > 0 \text{ where}
\]
Section 4

Stability analysis

To check the stability of the given system (1), we have calculated the following equilibrium points.

Existence of equilibrium points

1. Axial equilibrium point is \([k, 0]\)

2. Boundary point is \((X, Y)\) where

\[
x = \frac{-2abh + abM}{3ab} + \left(2^{1/3}(-(-2abh + abM)^{2} - 3ab(-abh^2 + 2abM - bMN b_1 + MN a_4 b_1 - MNQ b_1^2))/((3ab(-2a^3b^3)

By using value of \(X\) we can easily find \(Y\).

Local Stability Analysis

To check the local stability of the given dynamical system we firstly make a variational matrix

\[
V^* = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \quad a_{11} = r \left(1 - 2\frac{X^c}{M}\right), \quad a_{12} = -\frac{b_1 X^c}{h + X^c}, \quad a_{21} = -\frac{b_1 Y h}{h + X^c}, \quad a_{22} = b \left(1 - 2\frac{Y}{M}\right) - \frac{b_1 QX^c}{h + X^c} - a_4
\]

The characteristic equation of the system at \(E^*\) has the following form

\[
\lambda^2 - \text{trace} V^* \lambda + \det V^* = 0
\]

Where, \(\text{Trace} V^* = a_{11} + a_{22}\) and \(\text{Det} V^* = a_{11} a_{22} - a_{21} a_{12}\)

By using Routh-Hurwitz criterion we know that all the roots of variational matrix \(V^*\) have negative real roots, i.e. the positive equilibrium \(E^*\) is locally asymptotically stable (LAS) provided that the condition

(H1): \(\text{Trace} V^* < 0\) and \(\text{det} V^* > 0\) hold.
i.e. (H1): \( M < 2X^i \wedge N < 2I^i \) \hspace{1cm} (2)

Let \( V_1 \) be the variational matrix at \( E_1(0,0) \)

\[
V_1 = \begin{bmatrix} a & 0 \\ 0 & b-a_4 \end{bmatrix}
\]

It has a positive eigen value \( \lambda = a \) which means it is locally unstable.

Let \( V_2 \) be the variational matrix at \( E_2(k,0) \)

\[
V_2 = \begin{bmatrix} -a & -b_1M \\ 0 & b-a_4-b_2QM/h+M \end{bmatrix}
\]

\( V_2 \) is stable under the condition \( b < a_4 + \frac{b_2QM}{h+M} \) \hspace{1cm} (3)

**Section 5**

**Hopf–bifurcation analysis**

In the line of [19] We will study the behaviour of the system above by Hopf–bifurcation considering \( a_4 \) as bifurcation parameter. The necessary and sufficient condition for the existence of the Hopf-bifurcation, if there exists \( a_4 = a_0 \) such that

(i) \( A_i(a_0) > 0, i = 1,2,3 \),

(ii) \( A_1(a_0)A_2(a_0) - A_3(a_0) = 0 \) and

(iii) \( \frac{d}{da_0}u_i \neq 0, i = 1,2,3 \) where \( u_i \) is the real part of the eigen values of the characteristic equation.

**Section 6**

**Numerical Simulation**

Analytical studies remain incomplete without numerical verification of the results. So, to investigate the dynamics of the model system with the help of computer simulation, we have chosen the following example.

\[
\frac{dX}{dt} = 2.5X \left( 1 - \frac{X}{60} \right) - \frac{2XY}{(1+X)}
\]

\[
\frac{dI}{dt} = 0.999Y \left( 1 - \frac{Y}{40} \right) - \frac{0.54(2)XY}{(1+X)} - a_4Y
\]
It is observed that the given system (1) is locally asymptotically stable at $a_4=2$ around the interior equilibrium $E^*(508.0219, 2.4305)$, where we can easily check that (2) and (3) are satisfied. At this interior-point, $E^*$ the characteristic equation has both negative eigen values which are -2.3457 and -2.1749. Fig. no10, fig.no.11 and fig. no. 12 show the stable behavior of the system. If we take $a_4=1$, then the system bifurcate around the interior point $E^*$ fig no.1, fig. no. 2 and fig.no.3 depict the occurrence of Hopf-bifurcation of the two dynamics namely forest biomass and industrial units. The bifurcation of the system means both dynamics will survive for a long period of time, so that the economy of any country having the same dynamics can develop in a good manner. At $a_4=1$, the eigen values of the characteristic equation of $V^*$ are given by $\pm 3014i$, the existence of purely imaginary roots confirms the bifurcation behavior of the system. It can be easily seen from fig. no 4, fig.no.5 and fig.no.6 that the system bifurcates at $a_4=1.5$. This bifurcation behaviour continuous till $a_4<2.5$. When we take $a_4=2.5$ the system becomes unstable as forest biomass reaches at its carrying capacity i.e. $k = 60$ and industrial units become 0, which means the collapse of industrial units may result in the destruction of the economy. So we can investigate from the above discussion for the survival of any economy with the same dynamical system should have government control $(a_4, b)$ within limits i.e. $1<a_4<2.5$. Hence we can say that government check is a very necessary factor for the development of any country but it should be under control because overburden or over restriction can destroy the industrial units.
Section 7

Discussion and Conclusions
In this analysis, the impact of industry and government control on forest biomass has been studied. Firstly the existence of equilibrium states and their stability has been discussed under certain conditions. The Hopf-bifurcation analysis and simulation of the dynamical system have been determined by considering $a_4$ as a bifurcation parameter. The importance of the bifurcation parameter $a_4$ (government control on the industry) lies in the fact that the range of $a_4$ is $1 < a_4 < 2.5$, which conveys that the government should have a check on the industry within limits. Because excessive control on the industry by the government can destroy industrial units which effect economy of the country. Hence it is concluded that industry and forest biomass both are important for the development of the economy and can survive in the right manner if the government plays an effective role to conserve forest and develop the industry.

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