Nonlinear dynamic responses of FG graphene-reinforced nanocomposite plates in thermal environment

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Abstract. Nonlinear dynamic responses of functionally graded graphene-reinforced nanocomposite (FG-GRC) plates in thermal environment are studied in the present work. A modified Halpin-Tsai micromechanics model is applied to evaluate the equivalent material properties of the nanocomposite. Based on von Kármán nonlinear strains displacement relationship and the first-order shear deformation plate theory (FSDT), the nonlinear governing equations are derived by Hamilton principle. Galerkin and Runge-Kutta numerical method are employed to obtain the dynamic responses of the FG-GRC plate. Results show the GPLs distribution and temperature have remarkable effects on dynamic responses of a FG-GRC plate.

1. Introduction
The functionally graded graphene-reinforced nanocomposite (FG-GRC) is a new kind of advanced nanocomposites which combine the advantages of both functionally graded materials (FGM) and graphene nanoplatelets (GPLs). Due to its excellent mechanical, thermal and electrical properties, FG-GRC has wide potential applications in areas such as engineering, biomedical, electronic devices \cite{1}. Dynamic responses of a plate under mechanical loads in thermal environment is one of the most common and practical engineering problems. However, there is only a few studies on the topic of dynamic responses of FG-GRC plates under such combined thermal-mechanical loads. What’s more, most material parameters are sensitive to temperature, large deformations are likely to occur under thermal-mechanical loads for FG-GRC plates. Therefore, it is of great importance to research the nonlinear dynamic response of FG-GRC plates under thermal-mechanical loads.

Only a few works can be found on the dynamic responses of FG-GRCs plate under such thermal-mechanical loads. In thermal environment, Shen et al. \cite{1} studied the nonlinear vibration of FG-GRC laminated plates in a two-step perturbation method. Wu et al. \cite{2} researched the dynamic instability, buckling and vibration of FG-GRC multilayer beams subjected to a periodic axial force in thermal environments. Mirzaei et al. \cite{3} investigated the buckling characteristics of FG-GRC plates in a non-uniform rational B-spline isogeometric finite element method. Yang et al. \cite{4,5} studied the thermoelastic bending behaviors of a FG-GRC plate with different shapes by using the Mian and
Spencer method. Xu et al. [6] studied the vibration and sound radiation of a FG-GRC plate in Naiver and Rayleigh integral method under thermal-mechanical loads. Cong et al. [7] investigated the nonlinear dynamic response and vibration of FG-GRC plates rest on a viscoelastic Pasternak medium in thermal environment.

In this work, nonlinear dynamic behaviors of FG-GRC plates with simply supported boundary conditions in thermal environment are researched considering the temperature-dependent material properties. The governing equations are derived through employing Hamilton principle. Then the equations are solved by using Galerkin and Runge-Kutta numerical method.

2. Theoretical formulation

2.1. Equivalent material properties

Consider a FG-GRC rectangular plate with length a, width b and thickness h. It is assumed that GPLs are uniformly dispersed and randomly oriented in each x-y plane, and show a layer-wise variation in the thickness direction. The effective Young’s modulus of the GPLs nanocomposite (GRC) plate can be estimated by Voigt-Reuss model [8]:

\[ E = \left(\frac{3}{8}\right)E_L + \left(\frac{5}{8}\right)E_T \]  

(1)

In which \(E_L\) and \(E_T\) are the longitudinal and transverse moduli for a unidirectional plate and can be predicted by the Halpin-Tsai model [9]:

\[ E_L = \frac{1 + \xi_L \eta_L V_G}{1 - \eta_L V_G} E_m, \quad E_T = \frac{1 + \xi_T \eta_T V_G}{1 - \eta_T V_G} E_m \]  

(2)

Where parameters \(\eta_L\) and \(\eta_T\) take the following forms:

\[ \eta_L = \frac{((E_G / E_m) - 1)}{((E_G / E_m) + \xi_L)}, \quad \eta_T = \frac{((E_G / E_m) - 1)}{((E_G / E_m) + \xi_T)} \]  

(3)

In which \(E_m\) and \(E_G\) are the Young’s moduli of the matrix and GPLs, respectively; \(\xi_L\) and \(\xi_T\) are parameters related to the physical dimension of GPLs as the following forms:

\[ \xi_L = 2(a_G / t_G), \quad \xi_T = 2(b_G / t_G) \]  

(4)

where \(a_G\), \(b_G\) and \(t_G\) are the average length, width and thickness of the GPLs, respectively. The effective mass density, Poisson’s ratio and thermal expansion of GRC plates can be expressed by rule of mixture as the following:

\[ P(z) = P_m V_m + P_G V_G \]  

(5)

In which \(P_m\) and \(P_G\) denote the mass density, Poisson’s ratio, thermal expansion of matrix and GPLs, respectively. \(V_m\) and \(V_G\) represent the volume fractions of the matrix and GPLs, respectively. The volume fraction of GPLs is given by

\[ V_G = \frac{W_G}{W_G + \left(W_G / \rho_G + (\rho_m / \rho_G)(1 - W_G)\right)} \]  

(6)

Where \(W_G\) is the weight fraction of GPLs in the nanocomposite plate.

The GPLs distribution patterns can be expressed by general formulas based on the same total weight fraction of GPLs, thus forming different functionally graded forms.
2.2. Governing equations

The Cartesian coordinate system is assigned to the middle surface of the plate. The origin of coordinates is located at one corner of the middle surface. According to the first-order shear deformation plate theory (FSDT), the displacement of the plate can be described as [10]

\[
\begin{align*}
\mathbf{u}(x, y, z, t) &= u_0 + z\phi_x, \\
\mathbf{v}(x, y, z, t) &= v_0 + z\phi_y, \\
\mathbf{w}(x, y, z, t) &= w_0 + z\phi_z
\end{align*}
\]

(8)

Where \((u, v, w)\) are the displacement components of any point of the plate along \((x, y, z)\) coordinates directions, \((u_0, v_0, w_0)\) are the displacement components of a point on the middle surface of the plate along \((x, y, z)\) coordinates directions, respectively. \(\phi_x\) and \(\phi_y\) denote rotations about y-axis and x-axis, respectively. The nonlinear strain-displacement relations take the form as Eq. (9). When removing the quadratic terms, the nonlinear form is changed into the linear form.

\[
\begin{align*}
\varepsilon_{xx} &= u_{0,x} + 0.5(w_{0,x})^2 + z\phi_{x,x}, \\
\varepsilon_{yy} &= v_{0,y} + 0.5(w_{0,y})^2 + z\phi_{y,y}, \\
\gamma_{xy} &= u_{0,y} + v_{0,x} + w_{0,x}w_{0,y} + \phi_{x,y} + \phi_{y,x}.
\end{align*}
\]

(9)

The constitutive relations for the FG-GRC plate considering the thermal effect are

\[
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy}
\end{bmatrix} =
\begin{bmatrix}
Q_{11} & Q_{12} \\
Q_{21} & Q_{22}
\end{bmatrix}
\times
\begin{bmatrix}
\varepsilon_{xx} - \alpha_{xx}\Delta T \\
\varepsilon_{yy} - \alpha_{yy}\Delta T
\end{bmatrix},
\begin{bmatrix}
\tau_{yz} & \tau_{xz} & \tau_{xy}
\end{bmatrix} =
\begin{bmatrix}
Q_{44} & Q_{55} & Q_{66} & Q_{45} & Q_{56}
\end{bmatrix}
\]

(10)

Where \(Q_{ij}\) are functions of \(z\) axis and temperature \(T\), they are expressed as

\[
Q_{11} = Q_{22} = E/(1 - v^2), Q_{12} = Q_{21} = vE/(1 - v^2), Q_{44} = Q_{55} = Q_{66} = 0.5Ev/(1 + v)
\]

(11)

Based on Hamilton principle, the dynamic governing equations can be derived by

\[
\int_0^t (\delta U + \delta V - \delta T) dt = 0
\]

(12)

Where \(t\) is the time value, \(\delta U\) is the virtual strain energy, \(\delta V\) is the virtual work done by external force and \(\delta T\) is the virtual kinetic energy. They are given by

\[
\delta U = \int_A \int_0^h \left(\sigma_{xx}\delta \varepsilon_{xx} + \sigma_{yy}\delta \varepsilon_{yy} + \tau_{xy}\delta \gamma_{xy} + \tau_{yz}\delta \gamma_{yz} + \tau_{xz}\delta \gamma_{xz}\right)dz dA,
\]

\[
\delta V = -\int_A q\delta w dA,
\]

(13)

\[
\delta T = \int_A \rho(z)\left(\dot{u}\delta u + \dot{v}\delta v + \dot{w}\delta w\right) dA.
\]

Substituting Eqs. (13) into Eq. (12) and the Euler-Lagrange equations are obtained by setting the coefficients of \(\delta u_0, \delta v_0, \delta w_0, \delta \phi_x, \delta \phi_y\) into zero separately.
\[ \delta u_0 : N_{xx,x} + N_{xy,y} = I_0 \ddot{u}_0 + I_1 \dddot{\phi}_x \]
\[ \delta v_0 : N_{yy,y} + N_{yx,x} = I_0 \ddot{v}_0 + I_1 \dddot{\phi}_y \]
\[ \delta w_0 : N_{xx,xx} + 2N_{xy,xy} + N_{yy,yy} + Q_{xx,xx} + Q_{yy,yy} + q = I_0 \ddot{w}_0 \]  \hspace{1cm} (14)
\[ \delta \phi_x : M_{xx,x} + M_{xy,y} - Q_x = I_1 \dddot{u}_0 + I_2 \dddot{\phi}_x \]
\[ \delta \phi_y : M_{yy,y} + M_{yx,x} - Q_y = I_1 \dddot{v}_0 + I_2 \dddot{\phi}_y \]

All the quantities mentioned above are given by
\[ (N_q, M_q, Q_q, I_0, I_1, I_2) = \int_{-h/2}^{h/2} (\sigma_q, z, \tau_{xz}, k, \tau_{yz}, \rho, \rho z, \rho z^2) dz \]  \hspace{1cm} (15)

Where \( k \) is the shear correction factor, \( i \) and \( j \) represent \( x \) and \( y \), respectively. It is assumed boundary conditions are simply supported, then the five variables are taken as the following
\[ u = \sum_{m,n=1}^{M,N} u_{mn}(t) \cos(m\pi x) \sin(n\pi y), \]
\[ v = \sum_{m,n=1}^{M,N} v_{mn}(t) \sin(m\pi x) \cos(n\pi y), \]
\[ w = \sum_{m,n=1}^{M,N} w_{mn}(t) \cos(m\pi x) \sin(n\pi y), \]
\[ \phi_x = \sum_{m,n=1}^{M,N} \phi_{xmn}(t) \cos(m\pi x) \sin(n\pi y), \]
\[ \phi_y = \sum_{m,n=1}^{M,N} \phi_{ymn}(t) \sin(m\pi x) \cos(n\pi y). \]  \hspace{1cm} (16)

By substituting Eqs. (15-16) into Eq. (14), the governing equations can be derived. Then the dynamic responses are solved by Galerkin and variable-step size Runge-Kutta method.

3. Results and discussion
Epoxy/ GPLs nanocomposite is chosen in this work, temperature-dependent material properties are the same with the open literature [6]. A sinusoidal force is loaded at the center of the plate (0.16m*0.16m*0.002m). The amplitude of the force is 5N and the frequency is 25Hz.

![Figure 1](image-url). Central dynamic responses of a FG-GRC plate under different m*n combinations.
Figure 2. Central dynamic responses of a FG-GRC plate in present method and FEM simulation.

Figure 3. Central dynamic responses of a FG-GRC plate calculated in linear and nonlinear method.

Figure 4. Central dynamic responses of FG-GRC plates with different distribution patterns.
Figure 5. Central dynamic responses of a FG-GRC plate under different temperature (b/h = 20).

Figure 1 compares the central dynamic responses of FG-GRC plates under different m*n combinations. It verifies the convergence of the numerical method. It can be seen that the results of 3*3 and 4*4 are very close to each other, therefore, it is stable and accurate enough to use 4*4 to calculate.

Figure 2 compares the dynamic responses results calculated in the present method and FEM simulation. It can be observed that the analytical approximation solution in this paper is in good agreements with the FEM simulation result.

Figure 3 compares the linear and nonlinear dynamic responses. It is revealed that there is nearly no difference between the two methods when the external force is small. However, the difference becomes larger with the increase of the force. It is can be explained by that the plate may probably occur nonlinear deformations as the force arises. Thus, it is necessary to use nonlinear method.

Figure 4 illustrates the dynamic responses of FG-GRC plates with four GPLs distribution forms. As can be seen the FG-O pattern has the biggest amplitude and the FG-C has the lowest amplitude. Distribution patterns of GPLs have significant effects on dynamic responses of FG-GRC plates.

Figure 5 presents the dynamic responses of FG-GRC plates under different temperature. It is revealed that with the increase of temperature, the vibration amplitude of the FG-GRC plate becomes larger gradually. This is because temperature-dependent material properties and thermal stresses decrease the stiffness of the plate in the process of temperature rising, which leads to an amplitude increase.

4. Conclusion
The nonlinear dynamic responses of the FG-GRC plate in thermal environment have been studied based on FSDT plate theory in this paper. The governing equations are derived by Hamilton principle and nonlinear strain-displacement relations. The dynamic responses in time-domain are solved out by Galerkin and variable-step size Runge-Kutta numerical iterative method. A good agreement has been achieved between the present method and FEM simulation. Results show that the present method is suitable for the calculation of both linear and nonlinear dynamic responses. The parametric study shows that the distribution pattern and temperature have significant influence on material properties and dynamic responses of FG-GRC plate.
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