Thermally-Assisted Current-Driven Domain Wall Motion

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Starting from the stochastic Landau-Lifschitz-Gilbert equation, we derive Langevin equations that describe the non-zero-temperature dynamics of a rigid domain wall. We derive an expression for the average drift velocity of the domain wall $\langle \dot{r}_{\text{dw}} \rangle$ as a function of the applied current, and find qualitative agreement with recent magnetic semiconductor experiments. Our model implies that at any non-zero temperature $\langle \dot{r}_{\text{dw}} \rangle$ initially varies linearly with current, even in the absence of non-adiabatic spin torques.

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Introduction — Possibilities opened up by modern nanofabrication capabilities have motivated renewed interest in current-induced domain wall motion, a phenomenon first predicted and subsequently observed in seminal work by Berger [1, 2]. The theoretical [3, 4, 5, 6, 7, 8] and experimental [9, 10, 11, 12, 13, 14] study of domain wall motion induced by spin transfer torques [15, 16, 17, 18, 19] is currently one of the most active subfields of spintronics [20]. Recent research has highlighted a number of fundamentally interesting issues that are currently under spirited debate. One controversy concerns intrinsic pinning [4, 5, 6], i.e., domain walls that are stationary up to a critical current in the absence of spatial inhomogeneity. An intellectually distinct but phenomenologically related debate surrounds theories of nonadiabatic spin torques, which differ widely in their predictions [20, 21, 22, 23, 24, 25]. It turns out that in the presence of these torques, a domain wall is never intrinsically pinned [26, 27, 28].

This Letter is motivated primarily by the recent experiments of Yamanouchi et al. [17], in which current-induced domain wall motion was studied over five orders of magnitude of average velocity. An important conclusion of these authors is that at low temperatures the domain wall undergoes creep motion, i.e., that the domain wall does not move rigidly. Yamanouchi et al. arrive at this conclusion because the effects of non-zero temperature, when treated in a rigid domain wall approximation, seem to lead to results that are irreconcilable with experiment. In this Letter we demonstrate that a systematic theory of the influence of a thermal bath on the current-driven motion of a rigid domain wall leads to results that are in qualitative agreement with experiment. In the following sections we first explain our theory of non-zero-temperature domain wall motion and then discuss its implications for recent experiments.

Drift velocity of a rigid domain wall — Our starting point is the stochastic Landau-Lifschitz-Gilbert (LLG) equation [22, 23, 24, 25, 26, 27, 28] for the direction of magnetization $\hat{\Omega}$:

$$ \frac{\partial \hat{\Omega}}{\partial t} = \hat{\Omega} \times (\mathbf{H} + \eta) - \alpha \hat{\Omega} \times \frac{\partial \hat{\Omega}}{\partial t}, \quad (1) $$

where $\mathbf{H}$ is the effective field defined by $\mathbf{H}(\mathbf{x}) = -\delta E_{\text{MM}}[\hat{\Omega}] / (\hbar \delta \hat{\Omega}(\mathbf{x}))$ and $E_{\text{MM}}[\hat{\Omega}]$ is the micromagnetic energy functional. In Eq. (1) $\eta$ is a gaussian stochastic magnetic field with zero mean and correlations

$$ \langle \eta_\sigma(\mathbf{x}, t) \eta_\rho(\mathbf{x}', t') \rangle = \sigma \delta(t - t') \alpha^3 \delta(\mathbf{x} - \mathbf{x}') \delta_{\sigma \rho}, \quad (2) $$

where $a^3$ is the (local) volume of the finite element grid. The strength of the noise is given by $\sigma = 2ak_B T / \hbar$, proportional to the Gilbert damping parameter $\alpha$ and to the thermal energy $k_B T$. ( $\hbar$ is Planck’s constant which relates energy and frequency.) In using this expression for the strength of the fluctuations we neglected the influence of current on thermal magnetization fluctuations [29], which is higher order [30] than the spin-torque effects studied here.

The effective field can be separated into magnetic energy and spin transfer torque contributions $\mathbf{H} = \mathbf{H}_0 + \mathbf{H}_j$. For a ferromagnet with an easy $z$ axis and a hard $y$ axis we have

$$ \mathbf{H}_0 = J \nabla^2 \hat{\Omega} + 2 \omega_1 \Omega z - 2 \omega_0 \Omega y + H_{\text{ext}} \hat{z}, \quad (3) $$

where $J$ is the spin stiffness, $H_{\text{ext}}$ is an external field in the easy-axis direction and $\omega_1$ and $\omega_0$ are respectively the easy axis and hard-plane anisotropy constants. Assuming only locality implied by smooth magnetization textures, the spin-transfer torque can be separated quite generally into contributions parallel and perpendicular to the spatial derivative of the magnetization:

$$ \mathbf{H}_j \times \hat{\Omega} = v_s \frac{\partial \hat{\Omega}}{\partial r} + \beta v_s \hat{\Omega} \times \frac{\partial \hat{\Omega}}{\partial r}, \quad (4) $$

where the gradient is taken in the direction of current flow. In the absence of spin-orbit coupling, it follows from total spin conservation that $\beta = 0$ and that the spin velocity $v_s \equiv (j_y - j_z) / (-e(n_1 - n_-))$, where $j_\tau$ and $n_\sigma$ are majority and minority spin contributions to the

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currents and spin-densities in the collinear limit. For realistic ferromagnets spin and orbital degrees of freedom are coupled, and the microscopic theory of $v_\sigma$ and $\beta$ is more challenging and still controversial. The term proportional to $\beta$, the nonadiabatic spin transfer torque, plays a central role in the theory of current-driven domain wall motion.

In the absence of current and noise, Eq. (11) admits time-independent solutions corresponding to domain walls. In terms of the angles $\theta$ and $\phi$ defined by $\Omega = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$, the solution corresponding to an isolated domain wall centered at $r_{dw}$ is $\phi = 0$ and $\cos (x - r_{dw}) = \tanh (x - r_{dw}) / \lambda$, where the domain wall width $\lambda = \sqrt{J/(2\omega)}$. Rigid domain wall motion is described by elevating $r_{dw} \to r_{dw}(t)$ and $\phi(x, t) \to \phi(t)$ to the role of collective dynamical variables. The Langevin equations which describe their stochastic dynamics are

$$\dot{\phi}_0 + \alpha \frac{\dot{r}_{dw}}{\lambda} = \frac{\beta v_\sigma}{\lambda} - H_{\text{ext}} + \eta_\phi;$$

$$\dot{r}_{dw} - \alpha \dot{\phi}_0 = \omega_\sigma \sin(2\phi_0) + \frac{v_\sigma}{\lambda} + \eta_r.$$  

These equations can be derived heuristically by enforcing consistency with the stochastic LLG equations at the center of the domain wall. Alternately these equations can be derived by noting that the probability distribution $P[\hat{\Omega}, t]$, generated by Eqs. (11) and (12), can be written as a path integral $P[\hat{\Omega}, t] = \int d[\hat{\Omega}] \delta[\hat{\Omega} \cdot \hat{\Omega} - 1] e^{-S[\hat{\Omega}]}$, with effective action

$$S[\hat{\Omega}] = \int dt \int dx \frac{1}{2\sigma} \left( \frac{\partial \hat{\Omega}}{\partial t} \right) \times \hat{\Omega} - H + \alpha \left( \frac{\partial \hat{\Omega}}{\partial t} \right)^2.$$  

Inserting the domain-wall solution with time-dependent $r_{dw}$ and $\phi_0$ into this action gives rigid domain wall probabilities specified by the effective action

$$S[r_{dw}, \phi_0] = \int dt \int dx \frac{N}{2\sigma} \left( \dot{\phi}_0 + \alpha \frac{\dot{r}_{dw}}{\lambda} - \frac{\beta v_\sigma}{\lambda} + H_{\text{ext}} \right)^2$$

$$+ \left( \frac{\dot{r}_{dw}}{\lambda} - \alpha \dot{\phi}_0 - \omega_\sigma \sin(2\phi_0) - \frac{v_\sigma}{\lambda} \right)^2$$

$$- \frac{4}{3} \omega_\sigma^2 \sin^4 \phi_0,$$  

where $N = 2\Lambda A / a^3$ is the number of spins in a domain wall with cross-sectional area $A$. If we ignore the last term in this effective action, which vanishes in any case in the gaussian fluctuation limit, Eqs. (5) and (6) are recovered. This minor inconsistency in the theoretical treatment can be traced to the constant azimuthal angle across the domain wall in the variational ansatz. The final term must also be dropped if the rigid domain wall approximation action is to reproduce the correct equilibrium probability distribution function. (Note that even when $\beta = 0$ the Boltzmann equilibrium distribution is approached in the steady state.) The advantage of this approach is that we can read off the strengths of the gaussian noise terms $\eta_\phi$ and $\eta_r$ which are given by

$$\langle \eta_\phi(t) \eta_\phi(t') \rangle = \langle \eta_r(t) \eta_r(t') \rangle = \frac{\sigma}{N} \delta(t - t').$$  

Eqs. (5), (6) and (7) generalize the variational approach to current-induced domain-wall motion of Ref. 3 to finite temperature. Tatara and Kohno derive their equations from an energy functional, an approach that has been criticized recently by Barnes and Maekawa. Our derivation does not appeal to an energy functional. Therefore, in addition to deriving the correct form of the noise terms required for a consistent treatment of thermal fluctuations, it also provides an alternative justification of the zero-temperature approach of Tatara and Kohno.

These equations can be explored numerically without difficulty. However, it turns out to be possible to make some analytic progress. Solving for $\phi_0$ alone specializing to purely current-driven motion ($H_{\text{ext}} = 0$) we find that

$$(1 + \alpha^2) \dot{\phi}_0 = -\alpha \omega_\sigma \sin(2\phi_0) + \frac{(\beta - \alpha) v_\sigma}{\lambda} + \eta_\phi - \alpha \eta_r.$$  

This is the equation of motion for an overdamped Brownian particle which has been studied extensively with a large number of different physical motivations. Our theory for the equation of motion of the magnetization tilt should be contrasted with the treatment in Ref. 21 which ultimately arrives at an underdamped Brownian particle equation to describe nonzero temperature domain wall motion.

The average $\langle \dot{\phi}_0 \rangle$ can be calculated exactly for overdamped Brownian motion in a tilted periodic potential. Inserting this well known result in Eq. (6) we find that

$$\langle \dot{r}_{dw} \rangle = \frac{\beta v_\sigma}{\alpha}.$$

$$2\pi k_B T \left[ 1 - e^{2\pi h N (\alpha - \beta) v_\sigma / (\alpha k_B T)} \right]$$

$$- \int_0^{2\pi} e^{V(\phi)/(k_B T)} d\phi \int_0^{2\pi} e^{-V(\phi)/(k_B T)} d\phi' - \left[ 1 - e^{2\pi h N (\alpha - \beta) v_\sigma / (\alpha k_B T)} \right] \int_0^{2\pi} e^{-V(\phi)/(k_B T)} d\phi \int_0^{2\pi} e^{V(\phi)/(k_B T)} d\phi'$$

$$= \frac{\beta v_\sigma}{\alpha}.$$  

(11)
\[ V(\phi_0) = -\hbar N \omega_o \cos(2\phi_0)/2 + \hbar N(\alpha - \beta)v_s \phi_0/(\alpha \lambda) \]  

is the tilted-washboard potential experienced by the domain wall. It can be expressed as

\[ \langle \dot{r}_{dw} \rangle = \frac{\beta v_s}{\alpha} + 2\sqrt{\langle \lambda \omega_o \rangle^2 - \langle v_s \rangle^2} \exp \left\{ \frac{\beta}{\alpha - 1} \frac{N\hbar \omega_o}{k_B T} \left[ \sqrt{1 - \left( \frac{v_s}{v_{sc}} \right)^2} + \left( \frac{v_s}{v_{sc}} \right) \sin^{-1} \left( \frac{v_s}{v_{sc}} \right) \right] \right\} \sinh \left( \frac{\pi \hbar N(\alpha - \beta) v_s}{\alpha \lambda k_B T} \right). \]

This difference in estimated domain wall velocities is closely analogous to the difference between the Langer-Ambegaokar and Halperin-McCumber estimates of phase slip resistance in thin superconducting wires. It follows from Eq. (13) that the observation of linear dependence of domain wall velocity on current does not necessarily imply that \( \beta \neq 0 \). A careful analysis of the temperature dependence of \( \langle \dot{r}_{dw} \rangle \) will be necessary to determine the value of \( \beta \) from low current experiments, especially so if \( N\hbar \omega_o \) is not very large compared to \( k_B T \). Because of thermal noise, even when \( \beta = 0 \) the domain is not intrinsically pinned at any nonzero temperature.

Comparison with experiment — The results presented in Fig. 1 look qualitatively similar to the experimental results of Yamanouchi et al., in particular to the inset in Fig. 3 of Ref. 15. A direct comparison with these experimental results is complicated by the fact that they are performed close to the critical temperature, with the result that the magnetic anisotropy energy-density and the polarization of the current depend on temperature. Moreover, the Gilbert-damping parameter \( \alpha \) may also depend on temperature. The fact that curves at different temperatures in Fig. 1 are not as strongly offset vertically from each other at \( v_s/v_{sc} > 1 \) as the experimental results is most likely due to these additional temperature-dependent effects.

One of the main results of Yamanouchi et al. is the empirical finding that for small currents \( \ln(\dot{r}_{dw}) \propto \sqrt{v_s} \). From this the authors conclude that the domain wall undergoes current-induced creep motion at small currents. In Fig. 2 we plot the average domain wall velocity as a function of \( \sqrt{v_s} \) for various temperatures and values of \( \beta \). These curves were calculated for Gilbert damping parameter \( \alpha = 0.02 \). We consider only \( \beta < \alpha \) since for \( \beta > \alpha \) the domain wall velocity would decrease with increasing temperature in clear disagreement with experiment 15. One of the main conclusions of Ref. 21 is that for \( \beta = 0 \) and small currents the average domain wall velocity \( \langle \dot{r}_{dw} \rangle \propto \exp(Cv_s) \) where \( C \) is a constant. In the low-temperature limit we find that for \( v_s/v_{sc} < 1 \) the average domain wall velocity is given by

\[ \langle \dot{r}_{dw} \rangle \propto \sqrt{v_s} \]

Brownian “\( \phi_0 \)-particle” 3. In the above equations we have assumed that \( \alpha^2 < 1 \), for notational convenience.

It is well-known 33 that at zero temperature the equation for an overdamped particle in the tilted periodic potential of Eq. (10) has solutions with \( \langle \dot{\phi} \rangle \neq 0 \) only if \( |(\alpha - \beta)v_s| > \alpha \lambda \omega_o \). For \( \beta = 0 \) the zero temperature result for the average velocity of the domain wall is

\[ \langle \dot{r}_{dw} \rangle = \sqrt{(v_s/v_{sc})^2 - 1} \]

where the critical current for depinning the domain wall is \( v_{sc} = \lambda \omega_o \). For \( \beta \neq 0 \), the average domain wall velocity is nonzero at any finite value of \( v_s \) even at zero temperature.

In Fig. 2 we plot the average domain wall velocity calculated from the full expression [Eq. (13)] for various temperatures and values of \( \beta \). The results are performed close to the critical temperature, with the result that the magnetic anisotropy energy-density and the polarization of the current depend on temperature. Moreover, the Gilbert-damping parameter \( \alpha \) may also depend on temperature. The fact that curves at different

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FIG. 2: Test of the scaling $\ln(\langle \dot{r}_{dw} \rangle) \propto \sqrt{\omega_o}$, for various temperatures and values of $\beta$. We take $\alpha = 0.02$.

theory is therefore in agreement with the experimental results of Yamanouchi et al. [15]. Moreover, from Fig. 2 we also conclude that for $\beta = 0.01$ this scaling does not hold. Hence, an additional conclusion from the experimental data is that $\beta$ is much smaller than $\alpha$ in these ferromagnetic semiconductors.

A similar estimate for typical parameters of the metallic nanowires used in studies of current-driven domain wall motion [10] leads to the conclusion that $k_B T/(\hbar \omega_o) \sim 0.0002$ and therefore that temperature effects on rigid domain wall motion are likely much less important in these systems. This difference arises mainly because the density of moments is roughly 40 times higher in metallic systems.

In conclusion, we have presented a theory of the influence of nonzero temperatures on current-driven motion of a rigid domain wall, and found qualitative agreement with ferromagnetic semiconductor experiments. An estimate of the bending energy of the domain wall shows that these degrees of freedom are also thermally accessible. It is, however, difficult to assess how they influence translation of the domain wall. The qualitative agreement of our results with the experiments of Yamanouchi et al. [15] indicates that thermal activation of rigid domain wall motion could play an important role in these experiments. We expect that accounting for thermal fluctuations will also be important in assessing the impact of intended and unintended extrinsic domain wall pinning. This work was supported by the National Science Foundation under grants DMR-0115947 and DMR-0210383, by a grant from Seagate Corporation, and by the Welch Foundation.

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