Wakes in a thermal QCD medium in presence of strong magnetic field

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Abstract

We have investigated how the wakes in the induced charge density and in the potential due to the passage of highly energetic partons through a thermal QCD medium get affected by the presence of strong magnetic field ($B$). For that purpose, we wish to analyze first the dielectric responses of the medium both in presence and absence of strong magnetic field. Therefore, we have revisited the general form for the gluon self-energy tensor at finite temperature and finite magnetic field and then calculate the relevant structure functions at finite temperature and strong magnetic field limit ($|q_f B| \gg T^2$ as well as $|q_f B| \gg m_f^2$, $q_f(m_f)$ is the electric charge (mass) of $f$-th flavour). We found that for slow moving partons, the real part of dielectric function is not affected by the magnetic field whereas for fast moving partons, for small $k$ (= $|\mathbf{k}|$), it becomes very large and approaches towards its counterpart at $B = 0$, for large $k$. On the other hand the imaginary part is decreased for both slow and fast moving partons, due to the fact that the imaginary contribution due to quark-loop vanishes. With these ingredients, we found that the oscillation in the (scaled) induced charge density, due to the very fast moving partons becomes less pronounced in the presence of strong magnetic field whereas for smaller parton velocity, no significant change is observed. For the (scaled) wake potential along the motion of fast moving partons (which is of Lennard-Jones (LJ) type), the depth of negative minimum in the backward region gets reduced drastically, resulting the decrease of amplitude of oscillation due to the strong magnetic field. On the other hand in the forward region, it remains the screened Coulomb one, except the screening now becomes much stronger for higher parton velocity. Similarly for the wake potential transverse to the motion of partons in both forward and backward regions, the depth of LJ potential for fast moving partons gets decreased severely, but still retains the forward-backward symmetry. However, for lower parton velocity, the magnetic field does not affect it significantly.

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1 Introduction

Ultra-relativistic heavy-ion collisions (URHICs) at RHIC, e-RHIC, LHC, upcoming FAIR etc. provides an enticing opportunity to identify the transition from the hadronic matter to the quark matter, known as quark-gluon plasma (QGP). The hard scatterings at the early stages of URHICs produce very high energetic partons, known as jets. The jets behaves as an external hard probe, and while ploughing through the medium, it may lose energy and momentum either through the radiation $[1-6]$ or the collision $[7,8]$ or both $[9-11]$ depending of the range of momentum, altogether known as jet quenching $[7,12,13]$ and have been manifested largely by the suppression of high $p_T$ hadronic yields $[14]$. One of the specific consequence of the jet quenching is the dihadron azimuthal correlation at RHIC, which is manifested in the hadron pair-distribution in the intermediate $p_T$ range through a double peak structure in the away side $[15,16]$. Although, how the jet-medium interaction affects the distribution is not a settled issue, nevertheless the coupling of jets to a strongly interacting medium either through the particle level or the collective level may explain the modification of the angular distribution $[17-25]$, the formation of Mach cones $[19-21]$, Cerenkov radiation $[22,23]$ etc.

In addition, the jets, while traversing through the QGP, will disturb the charge distribution around it and creates the spikes in the induced charge density, hence in the screening potential, known as wakes, which can reveal the properties of medium. The wakes in both induced charge and current densities were first investigated by Ruppert et al. $[24]$ within the framework of linear response theory for a simple minded hot QCD medium for both weakly coupled and strongly coupled descriptions in the HTL calculations and in the theory for quantum liquid, respectively. In the same spirit, the wakes in the charge density and the potential was subsequently studied by Chakraborty et al. $[26]$. The wake potential is discussed in the literature by incorporating varieties of physical effects, viz. the nonabelian effects from the resummation calculation $[27]$, the collisional effects $[28]$, the momentum anistropy $[29,30]$, the viscous effects $[31]$. Wake potential is recently studied in AdS/CFT framework too $[32]$.

All the studies discussed hereinabove are suited for the fully central collisions, where the net baryon density is negligibly small and the chance for producing strong magnetic field is very bleak, due to the symmetric configurations in central collisions. In reality, a tiny fraction of events are truly central, in fact, most collisions occur with a nonzero impact parameter or centrality. When the two highly charged ions with extreme relativistic velocities collide noncentrally, an extremely large magnetic field ranging from $m_\pi^2 \simeq 10^{18}$ Gauss to $15 m_\pi^2$ are expected to be produced at RHIC and LHC, respectively, at the very early stages of the collisions $[33,34]$. The duration of strong magnetic field is yet to be known theoretically, however, depending on the transport properties of the medium produced, mainly the electrical conductivity, the magnetic field may remain strong enough till the medium is formed $[35]$. We also know that the jets are produced in the similar time scale, therefore it becomes worthwhile to explore how the aforesaid studies in wakes get affected by the presence of strong magnetic field (SMF). For that purpose we have first analyzed the response of the medium due to the passage of moving partons through the medium in the presence of strong magnetic field by the dielectric function. Hence its evaluation is made complete by the estimation of the gluon self-energy in the similar environment. Once the tools are ready, we explore the effects of strong magnetic field on the wakes. The first noticeable observation is that the oscillation in the (scaled) induced charge density due to the faster partons becomes less
pronounced in the presence of strong magnetic field whereas for smaller parton velocity, change is minimal. The second prominent observation in the (scaled) wake potential for very fast moving partons along the parallel direction is that, in the backward region the large depth of negative minimum of the Lennard-Jones (LJ) potential is reduced drastically, leaving the forward region almost unaffected, except that the screening becomes stronger. The third observation for the same in the perpendicular direction is that in both forward and backward regions, the depth of LJ type potential gets decreased severely, but still retains the forward-backward symmetry, as in the case for no magnetic field.

Our work proceeds as follows: First we have discussed the dielectric response due to the passage of the moving partons in a thermal QCD in an ambience of strong and homogeneous magnetic field in Section 2, wherein we have revisited the form of the gluon self-energy tensor at finite temperature \((T)\) and finite magnetic field \((B)\), and then calculate both real and imaginary parts of its longitudinal component at finite \(T\) and strong \(B\) in subsection 2.1. Therefore, the real and imaginary parts of complex dielectric function have been computed in subsection 2.2, which in turn facilitate the effects of strong magnetic field on the induced charged density and the resulting wake potential in the coordinate space, in subsections 3.1 and 3.2, respectively. Finally we conclude in Section 4.

## 2 Dielectric response of thermal QCD in an ambience of strong and homogeneous magnetic field

If the external disturbance is very small then the dielectric response of the medium can be envisaged by the dielectric tensor, \(\epsilon^{ij}\), whose longitudinal \((\epsilon^{L'})\) and transverse \((\epsilon^{T'})\) components can be extracted by the linear response theory [36–38]

\[
\epsilon^{L'}(k_0, k) = 1 - \frac{\Pi^{L'}(k_0, k)}{k^2},
\]

\[
\epsilon^{T'}(k_0, k) = 1 - \frac{\Pi^{T'}(k_0, k)}{k_0^2},
\]

where the respective self-energies, \(\Pi^{L'}\) and \(\Pi^{T'}\) of the medium are obtained by the self energy tensor, \(\Pi^{\mu\nu}\). However, we are interested in the former equation because the longitudinal equation is related to the density fluctuations in the medium, namely the space-charge field, which is our aim. Thus, we need to construct the gluon self-energy tensor in finite \(T\) and strong \(B\), which will then give the longitudinal component.

### 2.1 Gluon self-energy tensor in finite \(T\) and strong \(B\)

We will revisit here how to construct the general form of the gluon self-energy tensor \((\Pi^{\mu\nu})\) for an isotropic QCD medium at finite \(T\) and finite \(B\). For that, we first consider the absence of magnetic field and turn on the temperature through a heat bath to the vacuum. Since the heat bath generically defines a local rest frame, \(u^\mu\), so the Lorentz invariance is broken. Therefore,
compared to vacuum, a larger tensor basis is needed, which is conveniently constructed by the available four-vectors, \( k^\mu, u^\mu \) and the tensor, \( g^{\mu\nu} \) by the two orthogonal tensors, compatible with the physical degree of freedom, namely [39]

\[
P_L^{\mu\nu}(k) = -\frac{k^0}{k^2}(k^\mu u^\nu + u^\mu k^\nu) + \frac{1}{k^2} \left[ \frac{(k^0)^2}{k^2} k^\mu k^\nu + k^2 u^\mu u^\nu \right], \tag{3}
\]

\[
P_T^{\mu\nu}(k) = -g^{\mu\nu} + \frac{k^0}{k^2}(k^\mu u^\nu + u^\mu k^\nu) - \frac{1}{k^2} (k^\mu k^\nu + k^2 u^\mu u^\nu), \tag{4}
\]

such a way that they satisfy the 4-dimensional transversality condition - \( k_\mu P_T^{\mu\nu} = 0 \). The superscripts \( T \) and \( L \) represent the transverse (\( T \)) and longitudinal (\( L \)) with respect to the three-momentum (\( k \)), respectively. Thus the self-energy tensor at finite temperature is written in the above basis

\[
\Pi^{\mu\nu}(k_0, k) = P_T^{\mu\nu}\Pi^T(k_0, k) + P_L^{\mu\nu}\Pi^L(k_0, k), \tag{5}
\]

where the structure factors, \( \Pi^T \) and \( \Pi^L \) are obtained by evaluating the quark-loop, gluon-loops with 2-gluon and 3-gluon vertices and ghost-loop contributions in HTL approximation with the hard scale for loop momenta \(^3\) for both gluons and quarks.

Now with the presence of magnetic field in a direction, \( b^\mu \), to the same thermal medium, the remaining translation invariance is broken. Hence, much larger basis is needed and can be constructed with the vectors, \( k^\mu, u^\mu, b^\mu \) and the tensor, \( g^{\mu\nu} \). Therefore, in addition to \( P_T^{\mu\nu} \) and \( P_L^{\mu\nu} \) at finite temperature, two more tensors, \( P_{\parallel}^{\mu\nu} \) and \( P_{\perp}^{\mu\nu} \) are recently constructed in [39,40] for the leading-order perturbation theory

\[
P_{\parallel}^{\mu\nu}(k) = -\frac{k^0 k_z}{k^2_\parallel} (b^\mu u^\nu + u^\mu b^\nu) + \frac{1}{k^2_\parallel} \left[ (k^0)^2 b^\mu b^\nu + (k_z)^2 u^\mu u^\nu \right], \tag{6}
\]

\[
P_{\perp}^{\mu\nu}(k) = \frac{1}{k^2_\perp} \left[ -k^2 g^{\mu\nu} + k^0 (k^\mu u^\nu + u^\mu k^\nu) - k^z (k^\mu b^\nu + b^\mu k^\nu) + k^0 k^z (b^\mu u^\nu + u^\mu b^\nu) - k^\mu k^\nu + (k^2_\perp - (k^0)^2) u^\mu u^\nu - k^2 b^\mu b^\nu \right], \tag{7}
\]

by demanding the transversality condition for all tensors and the notations are defined as

\[
u^\mu = (1,0,0,0), \quad b^\mu = (0,0,0,-1), \quad g^{\mu\nu}_T = \text{diag}(1,0,0,1), \quad g^{\mu\nu}_L = \text{diag}(0,1,-1,0),
\]

\[
k^2_\parallel = (k_0)^2 - (k_z)^2, \quad k^2_\perp = (k_x)^2 + (k_y)^2.
\]

Therefore, the gluon self-energy tensor in finite \( T \) and \( B \) can be written as the superposition of finite \( T \) and finite \( B \) basis,

\[
\Pi^{\mu\nu}(k) = P_L^{\mu\nu}(k)\Pi^L + P_T^{\mu\nu}(k)\Pi^T + P_{\parallel}^{\mu\nu}(k)\Pi^\parallel + P_{\perp}^{\mu\nu}(k)\Pi^\perp, \tag{8}
\]

with the two new structure factors, \( \Pi^\parallel \) and \( \Pi^\perp \) for finite \( B \). However, the evaluations of \( \Pi_i \)'s \((i = T, L, \parallel, \perp)\) in finite \( T \) and strong \( B \) need more care because the hard scale for the gluons is

\(^3\)In pure thermal medium, the gluon loop-momenta may be hard, \( \mathcal{O}(T) \) or soft, \( \mathcal{O}(g'T) \) because the Matsubara frequencies \( (2m\pi T) \) are integral multiples of \( 2\pi T \). Thus, the hard (soft)-momentum regime requires \( m \neq 0 \) \((m = 0)\) bosonic matsubara mode. On the other hand the quark loop-momenta will always be hard, even for \( m = 0 \) Matsubara fermionic mode.
still given by the temperature only whereas for quarks the dominant scale is given by the strong magnetic field ($\sqrt{eB}$).

Thus a comparative strength of the structure factors in finite $T$ and strong $B$ are in order: Let us start for the structure factors, $\Pi_\parallel$ and $\Pi_\perp$, whose evaluations are made only from the quark-loop. Since the magnetic field is strong, the quarks are constrained to be in the lowest Landau levels and the dispersion relation ensures the perpendicular component (with respect to the magnetic field), $p_{\perp} \approx 0$, which manifests by the vanishing structure factor, $\Pi_\parallel(\approx 0)$ and (real part of) $\Pi_\parallel$ is of the order $O(g^2|q_fB|)$. Last but not the least, the finite temperature structure factors, $\Pi_T$ and $\Pi_L$, for which both quark and gluon-loops contribute, are of the order $O(g^2T^2)$. Therefore, the longitudinal component ($\mu = \nu = 0$ with the negative sign, denoted by $L'$) of the gluon self-energy tensor at finite T and strong magnetic field ($B$) in (8) is written in terms of quark (q) and gluon (g) loop contributions

$$\Pi'(k) = \Pi_{q,\parallel}(k) + \Pi_{q,L}(k) + \Pi_{g,L}(k),$$

(9)

because $P_{L0}, P_{00} = 1, P_{T0}, P_{00} = 0, \Pi_\parallel = 0$.

Let us first calculate the $\Pi_{q,\parallel}$ for the quark-loop only in the real-time formalism. Using the “11”-component of the quark-propagator matrix in thermal QCD in presence of strong magnetic field

$$iS_{\parallel1}(p) = \frac{1}{p_{\parallel}^2 - m_{f}^2 + i\epsilon + 2\pi i n(p_0)\delta(p_{\parallel}^2 - m_{f}^2)}(1 + \gamma^0\gamma^3\gamma^5)(\gamma^0 p_0 - \gamma^3 p_z + m_f)e\frac{-p_{\parallel}^2}{4\pi T^2},$$

(10)

the “11”-component of the gluon self-energy matrix due to quark-loop diagram up to one-loop is given by (omitting the label “11”-component)

$$\Pi_{q,\mu\nu}(k) = \frac{ig^2}{2} \sum_f \int \frac{dp}{(2\pi)^3} Tr[\gamma^\mu S_{\parallel1}(p)\gamma^\nu S_{\parallel1}(q)]$$

$$= \frac{ig^2}{2} \sum_f \int \frac{dp}{(2\pi)^3} Tr[\gamma^\mu(1 + \gamma^0\gamma^3\gamma^5)(\gamma^0 p_0 - \gamma^3 p_z + m_f)\gamma^\nu(1 + \gamma^0\gamma^3\gamma^5)(\gamma^0 q_0 - \gamma^3 q_z + m_f)]$$

$$\left\{ \frac{1}{p_{\parallel}^2 - m_{f}^2 + i\epsilon} + 2\pi i n(p_0)\delta(p_{\parallel}^2 - m_{f}^2) \right\} \left\{ \frac{1}{q_{\parallel}^2 - m_{f}^2 + i\epsilon} + 2\pi i n(q_0)\delta(q_{\parallel}^2 - m_{f}^2) \right\} e\frac{-p_{\parallel}^2 - q_{\parallel}^2}{4\pi T^2},$$

(11)

where the strong coupling ($g$) runs with the magnetic field [42], and the trace due to $\gamma$ matrices is

$$L^\mu_\nu = 8 \left[ p^\mu q^\nu - q^\mu p^\nu - g^\mu_\nu ((p.q)_\parallel - m_{f}^2) \right].$$

(12)

Due to the magnetic field, the quark-loop momentum ($p$) becomes factorizable into parallel and perpendicular with respect to the magnetic field. As a result, the quark-loop contribution also becomes factorizable

$$\Pi_q^{\mu\nu}(k) = \sum_f \Pi_q^{\mu\nu}(k_\parallel)A(k_\perp),$$

(13)

where the perpendicular component of loop momentum is integrated out to give

$$A(k_\perp) = \frac{\pi|q_f|B}{2} e^{-\frac{k^2}{4\pi T^2}}.$$
which, in SMF approximation \(e^{-\frac{k^2}{m_{f}B}} \approx 1\), becomes

\[A(k_{\perp}) = \frac{\pi |q_{f}| B}{2}.\] (15)

The parallel component of momentum, \(\Pi^\mu{}_{\nu}(k_{\parallel})\) is decomposed into vacuum and thermal parts

\[\Pi^\mu{}_{\nu}(k_{\parallel}) = \Pi^\mu{}_{\nu}(k_{\parallel})^{\text{vac}} + \Pi^\mu{}_{\nu}(k_{\parallel})^{\text{Th}},\] (16)

where the vacuum and thermal contributions due to single and double distribution functions, respectively, are

\[
\Pi^\mu{}_{\nu}(k_{\parallel})^{\text{vac}} = \frac{i g^2}{2(2\pi)^4} \int dp_0 dp_z L^{\mu\nu} \left\{ \frac{1}{(q^2_0 - m_f^2 + i\epsilon)} \frac{1}{(p^2_{\parallel} - m_f^2 + i\epsilon)} \right\}, \tag{17}
\]

\[
\Pi^\mu{}_{\nu}(k_{\parallel})^{\text{Th}} = \frac{i g^2(2\pi i)}{2(2\pi)^4} \int dp_0 dp_z L^{\mu\nu} \left\{ n(p_0) \frac{\delta(p^2_\parallel - m_f^2)}{(q^2_0 - m_f^2 + i\epsilon)} + n(q_0) \frac{\delta(q^2_0 - m_f^2)}{(p^2_{\parallel} - m_f^2 + i\epsilon)} \right\}, \tag{18}
\]

\[
\Pi^\mu{}_{\nu}(k_{\parallel})^{\text{Th}} = \frac{i g^2}{2(2\pi)^4} \int dp_0 dp_z L^{\mu\nu} \{ (-4\pi^2) n(p_0) n(q_0) \delta(p^2_\parallel - m_f^2) \delta(q^2_0 - m_f^2) \}. \tag{19}
\]

Now we obtain the real part of the vacuum term (17) [41]

\[
\Re \Pi^\mu{}_{\nu}(k_{\parallel})^{\text{vac}}(k_{0}, k_{z}) = \left( g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2} \right) \frac{g^2}{4\pi^3} \left[ \frac{2m_f^2}{k_{\parallel}^2} \left( 1 - \frac{4m_f^2}{k_{\parallel}^2} \right) \right]^{-\frac{3}{2}} \left\{ \ln \left( \frac{1 - \frac{4m_f^2}{k_{\parallel}^2}}{1 + \frac{4m_f^2}{k_{\parallel}^2}} \right) \right\} + 1. \tag{20}
\]

Thus after multiplying the transverse momentum component (15), the real part of the longitudinal component ("00" component, labelled as \(\parallel\)) in the limit of massless quarks \(m_{f} = 0\) is simplified

\[
\Re \Pi^\parallel_{\mu\nu}(k_{0}, k_{z}) = \frac{g^2}{2\pi^2} \sum_{f} |q_{f}| B \left| \frac{k^2}{k_{\parallel}^2} \right|, \tag{21}
\]

and the imaginary part vanishes

\[
\Im \Pi^\parallel_{\mu\nu}(k_{0}, k_{z}) = 0. \tag{22}
\]

The real part of medium contribution due to single distribution function (18) is now calculated as

\[
\Re \Pi^\parallel_{\mu\nu}(k_{\parallel}) = -\frac{g^2}{2(2\pi)^4} \int dp_0 dp_z L^{\mu\nu} \left\{ n(p_0) \frac{\delta(p_0 - \omega_{\parallel}) + \delta(p_0 + \omega_{\parallel})}{(q^2_0 - q^2_z - m_{f}^2)(2\omega_{\parallel})} \right\} + n(q_0) \left\{ \delta(q_0 - \omega_{\parallel}) + \delta(q_0 + \omega_{\parallel}) \right\} \left( \frac{p^2_0 - p^2_z - m_{f}^2}{(2\omega_{q})} \right), \tag{23}
\]

whose \(\"\parallel\"\) component becomes

\[
\Re \Pi^\parallel_{\mu\nu}(k_{\parallel}) = \frac{g^2}{2(2\pi)^4} \int dp_0 dp_z L^{00} \left\{ n(p_0) \frac{\delta(p_0 - \omega_{\parallel}) + \delta(p_0 + \omega_{\parallel})}{(q^2_0 - q^2_{\parallel})(2\omega_{\parallel})} \right\} + n(q_0) \left\{ \delta(q_0 - \omega_{\parallel}) + \delta(q_0 + \omega_{\parallel}) \right\} \left( \frac{p^2_0 - \omega^2_{\parallel}}{(2\omega_{q})} \right), \tag{24}
\]
with the following notations

\begin{align}
L^{00} &= 8[p_0 q_0 + p_z q_z + m_f^2], \\
\omega_p &= \sqrt{p_z^2 + m_f^2}, \\
\omega_q &= \sqrt{(p_z - k_z)^2 + m_f^2}.
\end{align}

After performing the \( p_0 \) integration in Eq. (24)

\[
\Re \Pi_n^{q\|}(k_\parallel) = \frac{g^2}{2(2\pi)^3} \int dp_z \left[ \frac{L^{00} n_p}{\omega_p[(\omega_p - k_0)^2 - \omega_q^2]} \bigg|_{p_0 = \omega_p}^{p_0 = -\omega_p} + \frac{L^{00} n_q}{\omega_q[(\omega_q + k_0)^2 - \omega_p^2]} \bigg|_{p_0 = \omega_q + k_0}^{p_0 = -\omega_q + k_0} \right].
\]

Thus multiplying the transverse momentum dependent part, the contribution to the real part having single distribution function for massless quarks \((m_f = 0)\) becomes

\[
\Re \Pi_n^{q\|}(k_0, k_z) = \frac{g^2}{2\pi^2} \sum_f |q_f| B \left[ -\frac{k_\perp^2}{k_\parallel^2} + \frac{k_\perp}{k_\parallel} T \ln(2) - \frac{k_\perp}{k_\parallel} T \ln(1 + e^{-k_\perp T}) \right],
\]

which, in the HTL approximation (external momentum, \( |k_z| < T \)) gives the final form, where the \( T \)-dependence is gone

\[
\Re \Pi_n^{q\|}(k_0, k_z) = \frac{g^2}{2\pi^2} \sum_f |q_f| B \left[ -\frac{k_\perp^2}{k_\parallel^2} + \frac{k_\perp}{2k_\parallel^2} \right],
\]

and the imaginary part vanishes

\[
\Im \Pi_n^{q\|}(k_0, k_z) = 0.
\]

However, the medium contribution involving the product of two distribution functions (19) does not contribute to the real part, i.e.

\[
\Re \Pi_n^{q\|}(k_0, k_z) = 0,
\]

and the imaginary part vanishes for massless flavours [43]

\[
\Im \Pi_n^{q\|}(k_0, k_z) = 0.
\]

We thus add the vacuum in Eq.(21) as well as the medium contributions in Eq.(30) and Eq.(32) together to obtain the real part due to quark loop contribution

\[
\Re \Pi_n^{q\|}(k_0, k_z) = \Re \Pi_V^{q\|}(k_0, k_z) + \Re \Pi_n^{q\|}(k_0, k_z) + \Re \Pi_n^{q\|}(k_0, k_z) = \frac{g^2}{4\pi^2} \sum_f |q_f| B \frac{k_\perp^2}{k_\parallel^2},
\]
and the imaginary-part due to quark loop contribution vanishes

\[
\Im \Pi^{q,\parallel}(k_0, k_z) = \Im \Pi^{q,\parallel}(k_0, k_z) + \Im \Pi^{q,\parallel}_w(k_0, k_z) + \Im \Pi^{q,\parallel}_n(k_0, k_z),
\]

\[
= 0.
\] (35)

Now we need to calculate the finite temperature structure factors, \(\Pi^{q,L}\) and \(\Pi^{g,L}\) for quark and gluon loop, respectively, which are obtained long time ago by the HTL approximation \[44–46\]

\[
\Pi^{q,L}(k_0, k) = g^2 T^2 \left( \frac{N_f}{6} \right) \left( \frac{k_0 + k}{2k} \ln \frac{k_0 + k}{k_0 - k} - 1 \right),
\]

\[
\Pi^{g,L}(k_0, k) = g^2 T^2 \left( \frac{k_0 + k + i\epsilon}{2k} \ln \frac{k_0 + k + i\epsilon}{k_0 - k + i\epsilon} - 1 \right).
\] (36, 37)

Using the following identity

\[
\ln \left( \frac{k_0 + k \pm i\epsilon}{k_0 - k \pm i\epsilon} \right) = \ln \left| \frac{k_0 + k}{k_0 - k} \right| \mp i\pi \theta(k^2 - k_0^2),
\]

\[
= \ln \left| \frac{k_0 + k}{k_0 - k} \right| \mp i\pi \theta(k^2 - k_0^2),
\] (38)

the real parts of \(\Pi^{q,L}\) and \(\Pi^{g,L}\) can be written as

\[
\Re \Pi^{q,L}(k_0, k) = g^2 T^2 \left( \frac{N_f}{6} \right) \left( \frac{k_0 + k}{2k} \ln \frac{k_0 + k}{k_0 - k} - 1 \right),
\]

\[
\Re \Pi^{g,L}(k_0, k) = g^2 T^2 \left( \frac{k_0 + k}{2k} \ln \frac{k_0 + k}{k_0 - k} - 1 \right),
\] (39, 40)

and the imaginary parts can be extracted as

\[
\Im \Pi^{q,L}(k_0, k) = -g^2 T^2 \left( \frac{N_f}{6} \right) \frac{\pi k_0}{2k},
\]

\[
\Im \Pi^{g,L}(k_0, k) = -g^2 T^2 \frac{\pi k_0}{2k},
\] (42, 43)

where \(N_f\) is the number of flavour and the coupling \((g')\) now runs with the temperature.

Thus the real part of the longitudinal component of total quark-loop contribution, in the SMF limit \(|q_f B| >> T^2\), becomes

\[
\Re \Pi^{q,\parallel}(k_0, k_z) + \Re \Pi^{q,L}(k_0, k) \approx \Pi^{q,\parallel}(k_0, k_z).
\] (44)

Thus Eq.(9) finally becomes

\[
\Pi'(k_0, k) = \Pi^{q,L}(k_0, k) + \Pi^{q,\parallel}(k_0, k_z),
\]

\[
= \Re \Pi^{q,\parallel}(k_0, k_z) + \Re \Pi^{q,L}(k_0, k) \approx \Pi^{q,\parallel}(k_0, k_z) + \Re \Pi^{q,L}(k_0, k).
\] (45)

whose real and imaginary parts will give the corresponding real and imaginary parts of dielectric response function, that we will obtain in the next subsection.
2.2 Dielectric function in presence of strong magnetic field

The real and imaginary parts of longitudinal dielectric function for thermal medium in presence of strong magnetic field is given by

\[
\Re \epsilon^L(k_0, k) = 1 + \frac{g'^2 T^2}{k^2} \left( 1 - \frac{k_0}{2k} \ln \left| \frac{k_0 + k}{k_0 - k} \right| \right) - \frac{g'^2}{4\pi^2} \sum_f |q_f| B \frac{1}{k^2} \langle k \parallel \rangle, \quad (46)
\]

\[
\Im \epsilon^L(k_0, k) = \frac{g'^2 T^2 \pi k_0}{2k}. \quad (47)
\]

To see the effects of strong magnetic field alone, the real and imaginary parts of the same in absence of magnetic field needs to be mentioned here as a baseline for comparison. These are obtained from the HTL techniques \[44–46\]

\[
\Re \epsilon^L_{B=0}(k_0, k) = 1 + \frac{g'^2 T^2}{k^2} \left( 1 + \frac{N_f}{6} \right) \left( 1 - \frac{k_0}{2k} \ln \left| \frac{k_0 + k}{k_0 - k} \right| \right), \quad (48)
\]

\[
\Im \epsilon^L_{B=0}(k_0, k) = \frac{g'^2 T^2 \pi k_0}{2k}. \quad (49)
\]

Figure 1: Real part of dielectric response function in absence and presence of strong magnetic field for \(v = 0.55\) (left panel) and \(v = 0.99\) (right panel), respectively.
To see the effects of magnetic field on the dielectric function, we have computed numerically its real and imaginary parts in absence and presence of strong magnetic for slow ($v = 0.55$) and fast ($v = 0.99$) moving partons in Fig.(1) and Fig.(2), respectively. For $v = 0.55$, the change in the real part in the presence of strong magnetic field is meagre as compared to the case of no magnetic field. On the other hand, for $v = 0.99$, its magnitude becomes much larger and the variation also becomes opposite compared its counterpart in the absence of magnetic field. More specifically, for small $k$ ($|k|$), the real part becomes very large and approaches towards its counterpart at $B = 0$, for large $k$. On the other hand, the imaginary part is decreased for both slow and fast moving partons, due to the fact that the imaginary contribution due to quark-loop vanishes. However, for larger $v$, its magnitude in both absence and presence of magnetic field increases. In brief, only the real part of dielectric function for higher value of parton velocity is largely affected in presence of strong magnetic field, whose ramifications will now be explored in the induced color charge density and in the wake potential in the next section.

3 Wakes in the presence of strong magnetic field

When a moving test charge particle is traversing through the plasma, the charge density is induced locally, as a result a wake is observed in the potential due to the induced charge density. Both of them depend on the velocity of the moving test charge as well as the distribution of background particles, which are going to be discussed in coming subsections.
3.1 Induced color charge density

Now we are going to discuss the wakes created in the induced charge density, which can be expressed in terms of the dielectric response function $\epsilon^{L'}(k_0, k)$ and the external color charge density as

$$\rho_\text{ind}^a(k_0, k) = -\left\{1 - \frac{1}{\epsilon^{L'}(k_0, k)}\right\} \rho_\text{ext}^a(k_0, k),$$  \hspace{1cm} (50)

where the external color $(a = 1, 2, \ldots)$ charge density of charge particle $Q^a$ with velocity $v$ is given as

$$\rho_\text{ext}^a(k_0, k) = 2\pi Q^a \delta(k_0 - k \cdot v).$$  \hspace{1cm} (51)

Combining Eq.(50) and Eq.(51) and taking the inverse Fourier transform, we obtain the induced charge density in the coordinate space $(r, t)$ as

$$\rho_\text{ind}^a(r, t) = 2\pi Q^a \int \frac{d^3k}{(2\pi)^3} \int \frac{dk_0}{2\pi} e^{i(k \cdot r - k_0 t)} \left\{\frac{1}{\epsilon^{L'}(k_0, k)} - 1\right\} \delta(k_0 - k \cdot v).$$  \hspace{1cm} (52)

Assuming that the partons are moving along the z-direction, i.e. $v(0, 0, v)$ and using the cylindrical coordinate system for $r(s, 0, z)$, spherical polar coordinate system for $k(k \cos \phi \sin \theta, k \sin \phi \sin \theta, k \cos \theta)$, we get the induced charge density in terms of the spherical Bessel function of first kind $(J_0)$

$$\rho_\text{ind}^a(r, t) = \frac{Q^a}{(2\pi)^2} \int_0^\infty k^2 dk \int_{-1}^1 d\chi J_0(k s \sqrt{1 - \chi^2}) e^{i\Gamma} \left\{\frac{1}{\epsilon^{L'}(k \chi, k)} - 1\right\},$$  \hspace{1cm} (53)

where the notations are defined as

$$\chi = \cos \theta, \quad \Gamma = k \chi(z - vt),$$
$$J_0(x) = \frac{1}{2\pi} \int_0^{2\pi} d\theta e^{ix \cos \theta}.$$

By decomposing the dielectric function $(\epsilon^{L'})$ into the real and imaginary parts, the scaled induced charge density, $\rho_\text{ind}^0(r, t) \equiv \rho_\text{ind}^a(r, t) \frac{Q^a}{(2\pi)^2 m_D}$ is recast in the following form

$$\rho_\text{ind}^0(r, t) = \int_0^\infty k^2 dk \int_{-1}^1 d\chi J_0(k s \sqrt{1 - \chi^2}) \left[\cos \Gamma \left(\frac{\Re \epsilon^{L'}(k \chi, k)}{\Delta} - 1\right) + \sin \Gamma \frac{\Im \epsilon^{L'}(k \chi, k)}{\Delta}\right],$$  \hspace{1cm} (54)

with $\Delta = [\Re \epsilon^{L'}(k \chi, k)]^2 + [\Im \epsilon^{L'}(k \chi, k)]^2$.

We are now going to investigate the effects of magnetic field on the induced charge density quantitatively, due to the passage of moving partons through a thermal QCD medium, characterized by the dielectric function through the three-dimensional plots in Fig.3 and Fig.4 for higher and smaller parton velocities, respectively.
For faster moving partons (i.e. faster than the average phase speed), the induced charge density is found to carry the oscillations along the direction of moving charge (as seen in Fig.3), which generate the Cerenkov like radiation and Mach shock waves. This oscillatory behaviour is found in both absence and presence of strong magnetic field. However, in presence of magnetic field the oscillation is less pronounced. This observation can be understood in terms of real part of dielectric response function, whose magnitude in presence of strong magnetic field for higher parton velocity is enhanced (as illustrated in the right panel of Fig.1).

On the other hand, for smaller parton velocities ($v = 0.55$), magnetic field does not alter the induced charge density significantly, because the real part of the dielectric function in the similar case (seen in the left panel of Fig.1) does not change much in the presence of magnetic field.

Figure 3: Scaled induced charge density ($\rho_{ind}^0$) in absence (Left panel) and presence (Right panel) of strong magnetic field for higher parton velocity $v = 0.99$, respectively.
3.2 Wake potential

In this subsection we will study the wakes in the potential due to the induced charge density as discussed in the previous subsection. The wake potential in the momentum space is obtained from the Poisson equation as

$$\Phi^a(k_0, k) = \frac{\rho^a_{\text{ext}}(k_0, k)}{k^2 \epsilon L'(k_0, k)}.$$  \hfill (55)

Substituting the external color charge density from Eq. (51), the inverse Fourier transform gives the wake potential in the coordinate space

$$\Phi^a(r, t) = Q^a \int \frac{d^3k}{(2\pi)^3} \int e^{i(k \cdot r - k \cdot v t)} \frac{1}{k^2 \epsilon L'(k, v, k)}.$$  \hfill (56)

Similar to the coordinate transformations used for obtaining the induced charge density, we obtain the wake potential in the coordinate space

$$\Phi^a(r, t) = \frac{Q^a}{2\pi^2} \int_0^\infty dk \int_0^1 d\chi J_0(ks\sqrt{1 - \chi^2}) \left[ \frac{\cos \Gamma \Re \epsilon L'(kv\chi, k)}{\Delta} + \frac{\sin \Gamma \Im \epsilon L'(kv\chi, k)}{\Delta} \right],$$  \hfill (57)

which will be computed for two directions, one is along the direction of motion of the partons ($r \parallel v$) and other is perpendicular to the direction of motion of the partons ($r \perp v$). Therefore,
the scaled wake potential along the direction of motion (we presumed that the partons are moving along the z-direction, i.e. \( s = 0 \)) is thus obtained as

\[
\Phi_0^\parallel(r, t) = \int_0^\infty dk \int_0^{1} d\chi \left[ \frac{\cos \Gamma \Re e L^I(k v \chi, k)}{\Delta} + \frac{\sin \Gamma \Im e L^I(k v \chi, k)}{\Delta} \right],
\]

and the potential perpendicular to the direction of motion is (\( \Gamma' = k \chi v t \))

\[
\Phi_0^\perp(r, t) = \int_0^\infty dk \int_0^{1} d\chi J_0(k s \sqrt{1 - \chi^2}) \left[ \frac{\cos \Gamma' \Re e L^I(k v \chi, k)}{\Delta} - \frac{\sin \Gamma' \Im e L^I(k v \chi, k)}{\Delta} \right].
\]

Figure 5: Left panel: Scaled wake potential (\( \Phi_0^\parallel \)) along the motion of the partons for smaller and higher parton velocities in the absence of magnetic field. Right panel: Same as the left panel but in the presence of magnetic field.

Finally, to visualize the effects of magnetic field, we have computed the wake potential both in absence and presence of strong magnetic field along the parallel and perpendicular directions in Fig. (5) and Fig. (6), respectively. In the absence of magnetic field (left panel of Fig. 5), for the static case, the wake potential is by default forward backward symmetric but for finite \( v \) it falls off very fast compared to the static one and loses the forward backward symmetry. In the backward region, the wake potential decreases with the distance and shows a minimum when partons move with the velocity \( v = 0.55 \) (less than the phase velocity, \( v_p \)). However for parton velocity greater than \( v_p \), i.e. \( v = 0.99 \), the wake potential shows an oscillatory behaviour and behaves differently.
from \( v = 0.55 \) case and the width of the negative minimum is increased and shifted towards the origin compared to \( v = 0.55 \). Thus in the backward region, the wake potential is a Lennard-Jones potential type which shows a short-range repulsive part and a long-range attractive part. On the other hand, in the forward region, the wake potential behaves more like a screened Coulomb potential and attains the Coulombic form on increasing the value of parton velocity. Thus the forward part is not much affected by the motion of charged particles. Now, in the presence of magnetic field (right panel of Fig.(5)), in the backward region the wake potential decreases on increasing the value of \((z - vt)m_D\), and shows a minimum but the depth of the minimum is in general decreased compared to the absence of magnetic field. For \( v = 0.55 \) the width has been reduced slightly, however for \( v = 0.99 \), the reduction is very large. Moreover, in the backward region for \( v = 0.99 \), the potential also shows oscillatory behavior but with lesser amplitude. This can be understood again in terms of enhancement of real part of dielectric response function in presence of magnetic field (seen in Fig.(1)). Thus in the backward region, the wake potential is still found to be of Lennard-Jones type potential. On the other hand in the forward region, the wake potential is screened Coulomb potential. However, for higher parton velocity, it does not attain the Coulombic form as found in the case when there is no magnetic field, rather it becomes more screened on increasing the value of parton velocity. Overall the magnetic field is found to affect the higher velocity partons only.

![Scaled wake potential (Φ_0^⊥) perpendicular to the direction of motion of the partons for smaller and higher parton velocities in the absence of magnetic field.](image)

![Scaled wake potential (Φ_0^⊥) perpendicular to the direction of motion of the partons for smaller and higher parton velocities in the presence of magnetic field.](image)

Figure 6: Left panel: Scaled wake potential (\( Φ^0_⊥ \)) perpendicular to the direction of motion of the partons for smaller and higher parton velocities in the absence of magnetic field. Right panel: Same as the left panel but in the presence of magnetic field.
In absence of magnetic field, the wake potential, perpendicular to the moving partons (left panel of Fig.(6)), is symmetric in backward and forward regions, independent of the speed of the moving partons and the depth of negative minimum increases with the velocity of parton. The nature of wake potential is Lennard-Jones type for negative as well as positive value of $smD$. On the other hand, in the presence of strong magnetic field (right panel of Fig.(6)), the potential has the forward backward symmetry and of Lennard-Jones nature. However, the magnetic field decreases the depth of negative minimum for higher parton velocity, which is opposite to the absence of magnetic field.

4 Conclusions

In the present study we have calculated the wakes in the induced charge density as well as in the potential generated due to the passage of highly energetic partons through a thermal QCD medium in the presence of strong magnetic field, believed to be created in the off central events of heavy-ion collision experiments. For that purpose we have calculated the responses of the medium both in presence and absence of strong magnetic field through the dielectric function. To calculate the response function, first we have revisited the general covariant form for the one-loop gluon self-energy tensor at finite temperature and finite magnetic field and then approximated the relevant structure functions at finite temperature in the strong magnetic field limit. Thus we have obtained the real and imaginary parts of complex dielectric response function both in absence and presence of strong magnetic field for slow ($v = 0.55$) and fast ($v = 0.99$) moving partons. For slow moving partons, we have found that the real part of dielectric response function is not much affected by the magnetic field whereas for fast moving partons, it becomes very large compared to its counterpart in absence of magnetic field for small $k$ ($|k|$), however, it approaches towards the value in absence of magnetic field for large $k$. On the other hand the magnitude of the imaginary part of dielectric response function is slightly decreased for both slow and fast moving partons in presence of strong magnetic field. Finally, we have computed the scaled induced charge density ($\rho_{\text{ind}}^{0}$) and the scaled wake potential for parallel ($\Phi^{0}_{\parallel}$) and perpendicular ($\Phi^{0}_{\perp}$) directions to the motion of moving partons. The oscillation found in $\rho_{\text{ind}}^{0}$ due to the very fast partons becomes less pronounced in the presence of strong magnetic field whereas for smaller parton velocity, no significant change is observed. The wake potential, $\Phi^{0}_{\parallel}$ for very fast moving partons is found to be of Lennard-Jones (LJ) type and the depth of negative minimum in the backward region is reduced drastically, resulting in the decrease of amplitude of oscillation due to the strong magnetic field. On the other hand in the forward region, $\Phi^{0}_{\parallel}$ remains the screened Coulomb one, except the screening now becomes relatively stronger for higher parton velocity. For $\Phi^{0}_{\perp}$ in both forward and backward region, the depth of negative minimum in LJ potential gets decreased severely for fast moving partons, but still retains the forward-backward symmetry. However, for lower parton velocity, the magnetic field does not affect the nature of $\Phi^{0}_{\perp}$ significantly.
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