Application of the Mixing Theory in the Design of a High-Performance Dielectric Substrate for Microwave and Mm-Wave Systems

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ABSTRACT This paper presents the design and synthesis of a low-loss substrate with low effective permittivity ($\varepsilon_{\text{eff}}$) for microwave and mm-Wave applications. The proposed design is based on the two-phase Maxwell Garnet mixing theory, where the $\varepsilon_{\text{eff}}$ of the RF substrate can be synthesized depending on the geometry and the permittivity of mixing particles and the permittivity of the host material. A comprehensive review and error analysis of the most common mixing techniques are conducted to guarantee an accurate design for high-performance RF substrates. Several analyses based on the geometries of various particles are carried out to identify the most accurate mixing model used in the design of the proposed substrate. The effects of the direction of excitation as well as the polarization of the incident field on the $\varepsilon_{\text{eff}}$ of the anisotropic particle are analyzed and discussed. The proposed method enables the use of existing high-performance materials that do not necessarily provide a low dielectric constant and low loss tangent. For mm-Wave antenna applications, materials with a dielectric constant of 2–4, and loss tangent of less than 0.002 are desirable to maximize gain and radiation efficiency. Commercial RF substrates can satisfy those requirements, however limited thermal expansion coefficient and lamination difficulties increase the cost significantly. The proposed method enables the use of inexpensive materials that provide excellent thermal properties and great compatibility with a multi-layer fabrication process with desirable $\varepsilon_{\text{eff}}$ and loss tangent. For validation of the analysis, samples are fabricated and tested in the microwave frequency (S-band) at 3.5 GHz as well as in the mm-Wave frequency (W-band) at 77 GHz. Measured results show a reduction of 45% in the $\varepsilon_{\text{eff}}$ and 38% in the loss tangent values in the S-band, and 32% and 72% reduction in $\varepsilon_{\text{eff}}$ and $\tan\delta$, respectively, in the mm-Wave frequency band. The measured results are in excellent agreement with the simulation and calculated results.

INDEX TERMS Anisotropic, conventional, complementary, effective permittivity, isotropic, low loss-tangent, Maxwell Garnett mixing theory, polarization, dielectric constant.

I. INTRODUCTION

A complete understanding of the propagation of electromagnetic waves inside the dielectric material is of great importance in the modern applications of material design, remote sensing, aerospace, lens manufacturing, electromagnetic absorbers, carbon nanotubes, and polymers, etc. [1]–[3].

The dielectric properties of the material depend on the internal structure of the particles inside, particles shape, and the fractional volume ($f$). Several analytical models or mixing rules exist in the literature that have studied these properties and simplified modeling of the complex fields generated in the particles inside the host material. Material mixing rules are a set of algebraic formulations intended to calculate the effective permittivity ($\varepsilon_{\text{eff}}$) of the inclusions or particles inside the mixture based on the individual permittivities of
the particles as well as their fractional volumes. These mixing rules differ based on several factors such as structure or geometry of the particle (sphere, elliptical, disc, cubic, cylindrical, rod, needle-like, or any random shape) and their distribution inside the host material (aligned or randomly distributed). The geometry of the particles inside the host material determines the isotropic or anisotropic properties of the material. For instance, a spherical inclusion is considered isotropic as the \( \varepsilon_{\text{eff}} \) evaluated in \( x-, y-, \) and \( z- \) directions remains the same. This is because, the sphere is symmetric due to which the induced electric field inside the particle is uniform, thus making the polarizability (\( \alpha \)) as a scalar quantity. On the other hand, in the case of anisotropic particles, \( \varepsilon_{\text{eff}} \) is a tensor and is evaluated in each direction of the applied electric field.

Generally, for antennas or other microwave devices, \( \varepsilon_{\text{eff}} \) is evaluated in the \( z- \) direction. However, characterization of the material for its \( \varepsilon_{\text{eff}} \) in the \( x- \) or \( y- \) directions can be important in applications such as leaky wave antennas or in the devices where the surface waves are critical to the performance of the system.

Several works on controlling the permittivity of the material are presented for various applications such as multi-band arrays and integrated systems [4]–[9]. The concept of using perforations to control the effective permittivity for the substrate integrated image guide (SIIG) was first reported in [4], [5]. Cylindrical metallic particles were embedded in the host medium in [7] to change the overall permittivity of the substrate material and to obtain frequency tuning for a microstrip patch antenna. In [8], linearly tapered slot antennas were presented for 30 and 94 GHz bands on the synthesized low-permittivity of the perforated substrate. A leaky-wave antenna based on a dielectric layer with periodic perforations covering 96–108 GHz frequency band was presented in [9].

Choosing the right printed circuit board (PCB) material is significantly important for both microwave and mm-Wave frequency bands. The main losses associated with the PCB materials are: radiation losses, dielectric, conductor, and surface-wave losses. None of these losses can be ignored at mm-Wave frequencies, specifically the conductor losses, which become significant at mm-Wave frequencies. To achieve optimum performance from a PCB at any frequency of operation, several factors are evaluated: dielectric constant (\( \varepsilon_r \)), loss tangent (\( \tan \delta \)), copper roughness, coefficient of thermal expansion (CTE), moisture absorption, and material thickness [10]. However, at high frequencies, some of these factors become more critical such as \( \varepsilon_r, \tan \delta, \) copper roughness, and material thickness [11].

At microwave frequencies, high \( \varepsilon_r \) material (8–500) can be used to achieve miniaturization of the circuit [12]. However, at mm-Wave frequencies, low \( \varepsilon_r \) material (2–4) is preferred in order to reduce losses in the material (\( \tan \delta \)). The conductor losses, which are either due to the material’s surface finishing or copper roughness, become critical at mm-Wave frequencies. The losses from the copper roughness are directly related to the frequency of operation through the skin depth. At higher frequencies, these losses are significant because the value of the skin depth becomes smaller than the copper roughness. Moreover, the thickness of the PCB is also a very important factor of consideration during material selection. For instance, at higher frequencies, thick materials are prone to significant radiation losses as compared to the thin materials. But at the same time, thin materials are dominated by the conductor losses. Therefore, choosing a thin material with lower copper surface roughness is highly recommended for use at mm-Wave frequencies over a thick material with low conductor losses but higher radiation losses [11].

As applications move up in frequency, affordable and high-performance substrates are required. In military applications, polytetrafluoroethylene (PTFE) substrates had been considered as one of the best materials for RF applications. Teflon or PTFE-based substrates provide very low \( \tan \delta \) (0.0018 at 10 GHz) and offer great chemical resistant properties, small water absorption, and high-temperature resistance [13]. However, when the applications require high-performance substrates such as PTFE, those substrates present limitations in cost. For example, PTFE is ten times the price of epoxy glass (FR4), and five times the price of ceramic-based materials. In addition, PTFE requires extra preparation for good adhesion for multi-layer PCB process. Moreover, conventional PTFE substrates are soft and CTE of the conventional PTFE substrates is very high (180 to 205 ppm/°C) [13]. The proposed method enables the use of the existing inexpensive materials that provide excellent thermal properties and great compatibility with a multi-layer fabrication process to obtain high RF-performance in terms of desirable \( \varepsilon_{\text{eff}} \) and \( \tan \delta \) values for antenna applications.

In this paper, a proposed method based on Maxwell Garnett mixing theory is used to obtain a low-cost material with high RF performance, from existing high thermally stable and compatible materials. A complete set of design equations for finding desirable \( \varepsilon_{\text{eff}} \) of both isotropic and anisotropic particles is presented. For anisotropic cylindrical particle, several \( \varepsilon_{\text{eff}} \) models are analyzed and compared for its performance with the simulation results. The permittivity analysis for both isotropic and anisotropic cases is thoroughly studied including the effects of the direction of excitation: \( x-, y-, \) and \( z- \) directions. Moreover, the effect of polarization of the incident field on the \( \varepsilon_{\text{eff}} \) is also studied. Detailed material requirements and design procedures are discussed for both microwave and mm-Wave frequencies. The proposed perforated dielectric core material is fabricated and the simulation and measured results are used for validation. An excellent agreement is obtained between the simulation and measured results. For the proof of concept, Rogers 4350B material is loaded with cylindrical air particles and is tested in the microwave S-band. A reduction of 45% in the \( \varepsilon_{\text{eff}} \) and 38% in the loss tangent values were obtained. The results are also validated in the mm-Wave W-band by loading RO3003 with the complementary cylindrical air particles showing a reduction of 32% and 72% in \( \varepsilon_{\text{eff}} \) and \( \tan \delta \) values, respectively.
II. EFFECTIVE PERMITTIVITY AND MIXING TECHNIQUES

Several dielectric mixing rules have widely been used in literature such as: Maxwell Garnett mixing formula [14], Bruggeman [15], Looyenga [16], Polder–van Santen rule [17], and Lichtenecker mixing formula [18], etc. The most common of all these models is the Maxwell Garnett rule which is in the simplest form and is broadly applicable to a variety of particles. The theoretical background and analytic derivation of the Maxwell Garnett formula are not discussed here, as it is not the focus of this work. However, in this section, the required formulations that can be used for finding the $\epsilon_{\text{eff}}$ of any isotropic (sphere, cubic) and anisotropic (cylindrical, disc, elliptical) particles, are presented.

The generic Maxwell Garnett mixing rule having the depolarization information is given by (1) [19].

$$\epsilon_{\text{eff}} = \epsilon_2 \left( 1 + \frac{(\epsilon_1 - \epsilon_2)f}{\epsilon_2 + (\epsilon_1 - \epsilon_2)(1-f)\epsilon_j} \right); \quad j=x, y, z \quad (1)$$

where, $\epsilon_1$ and $\epsilon_2$ are the permittivities of the particle and the host material, respectively. $f$ is the fractional volume of the mixing particle, and $\epsilon_j$ is the depolarization factor opposing the direction of the applied field.

A. ISOTROPIC PARTICLES

For an isotropic particle, the value of $\epsilon_j = 1/3$, where $j = x, y, z$ [19]. An example geometry of the isotropic particle is shown in Fig. 1(a). By replacing the value of $N = 1/3$ in (1), it can be easily shown that the $\epsilon_{\text{eff}}$ for an isotropic particle can be calculated from a more common form of Maxwell Garnett mixing formula given by (2)

$$\epsilon_{\text{eff}} = \frac{\epsilon_1 + 2\epsilon_2 + 2f(\epsilon_1 - \epsilon_2)}{\epsilon_1 + 2\epsilon_2 - f(\epsilon_1 - \epsilon_2)} \quad (2)$$

B. ANISTROPIC PARTICLES

1) ELLIPSOID PARTICLE

The geometry of an ellipsoid inside a host medium is shown in Fig. 1(b). The depolarization factor $N_z$ can be calculated from (3) [20].

$$N_z = \frac{abc}{2} \int_0^{+\infty} \frac{ds}{(s + a^2)(s + a^2)(s + b^2)(s + c^2)} \quad (3)$$

where, $a$, $b$, and $c$ are the semi-axes of the ellipsoid. $N_x$ and $N_y$ can be calculated by interchanging $b$ and $a$, and $c$ and $a$, respectively. Closed-form expressions of (3) can be found in [21]. The depolarization factors of a general ellipsoid can also be found in [22]–[23]. Once the depolarization factors are known, (1) can be used to find the $\epsilon_{\text{eff}}$ in any direction.

2) DISC PARTICLE

The unit cell geometry of a disc particle is shown in Fig. 1(c). As the particle is symmetric in $x$- and $y$-directions, the permittivity in either $x$- or $y$- and $z$-directions need to be evaluated. Equations (4) and (5) can be used to find the $\epsilon_{\text{eff}}$ in the $z$- and $x$- or $y$-directions, respectively [24].

$$\frac{1}{\epsilon_{\text{eff}_z}} = \frac{f}{\epsilon_1} + \frac{1-f}{\epsilon_2} \quad (4)$$

$$\epsilon_{\text{eff}_{xy}} = f\epsilon_1 + (1-f)\epsilon_2 \quad (5)$$

3) CYLINDRICAL PARTICLE

The geometry of the unit cell of the cylindrical particle is shown in Fig. 1(d). The formulations for the cylindrical particle which is used in the design of our proposed material are discussed in detail. Unfortunately, the exact analytical models for finding the $\epsilon_{\text{eff}}$ of the material consisting of cylindrical particles do not exist. However, very few models discussing the approximate depolarization factor for the cylindrical particle exist, which are discussed in this section. In the following sections, we then compare the performance of these models based on their depolarization factors. We show that after choosing the right depolarization factor, the generic Maxwell Garnett formulation is then applicable to find the $\epsilon_{\text{eff}}$ using the cylindrical particle, in any direction ($x$, $y$, $z$) or any polarization ($\parallel$ or $\perp$) of the applied field with an overall error of $\leq 5\%$ in all cases.

The polarizability ($\alpha_j$) for a cylindrical particle is calculated numerically using method of moments (MoM) [25]–[26]. For the anisotropic particle, $\alpha_j$ for $j = z$ and $j = x$ or $y$ are given by (6) and (7), respectively:

$$\alpha_z \approx 3.8662(\epsilon_r - 1)$$

$$\times \frac{\epsilon_r^3 - 0.0519\epsilon_r^2 + 0.9427\epsilon_r + 1.5840}{\epsilon_r^4 + 3.2226\epsilon_r^2 - 0.0021\epsilon_r + 5.3391\epsilon_r + 3.8662} \quad (6)$$

$$\alpha_{x,y} \approx 3.1691(\epsilon_r - 1)$$

$$\times \frac{\epsilon_r^3 + 2.0283\epsilon_r^2 + 1.9821\epsilon_r + 1.5799}{\epsilon_r^4 + 4.4453\epsilon_r^2 + 6.2134\epsilon_r^2 + 6.05\epsilon_r + 3.1691} \quad (7)$$
where, \( \varepsilon_r \) is the permittivity of the mixing particle. The depolarization factor \( N_j \) can then be found from \( \alpha_j \) using (8) [19], which is then substituted in (1) to find the \( \varepsilon_{eff} \).

The second formulation, known as Van Beek model [24], uses (1) for a prolate spheroid case with approximate \( N_j \) values and provides a final expression for calculating \( \varepsilon_{eff} \) given by (9):

\[
\varepsilon_{eff} = \frac{\varepsilon_2 (\varepsilon_1 - \varepsilon_2 f)}{\varepsilon_2 + (\varepsilon_1 - \varepsilon_2) N_j}; \quad j = x, y, z
\]

Another famous formulation, known as Rayleigh’s model for finding \( \varepsilon_{eff} \) in the direction perpendicular to the cylinder axes \( \varepsilon_{eff,y} \) is given by (10) [27]:

\[
\varepsilon_{eff,y} = \frac{\varepsilon_2 \varepsilon_1 + \varepsilon_2 + f (\varepsilon_1 - \varepsilon_2)}{\varepsilon_1 + \varepsilon_2 - f (\varepsilon_1 - \varepsilon_2)}
\]

The approximate value of \( N_z = 1/2 \) for a cylindrical particle in the \( z \)-direction is given by Kittel model [28]. To obtain the most accurate results, this value is optimized slightly. It should be noted that all the mixing rules that exist so far, have certain limitations, either in terms of the fractional volume or the particle geometry. There is no single formulation that works best for any particle’s shape or any
III. EFFECTIVE PERMITTIVITY ANALYSIS

This section shows the complete analysis for finding the $\varepsilon_{\text{eff}}$ of both isotropic (spherical) and anisotropic (cylindrical) particles. Moreover, for each particle, two cases: conventional and complementary are considered. In a conventional case, the particle’s permittivity ($\varepsilon_1 = 3.66$) is considered greater than the host permittivity ($\varepsilon_2 = 1$), i.e. $\varepsilon_1 > \varepsilon_2$. For the complementary case, the particle’s permittivity ($\varepsilon_1 = 1$) is set lower than the host permittivity ($\varepsilon_2 = 3.66$), i.e. $\varepsilon_1 < \varepsilon_2$.

A. ISOTROPIC PARTICLE

Figures. 2(a) and (b) show the unit cell geometries for the conventional and complementary isotropic particles, respectively. The dimensions of the unit cell (length ($l$), width ($w$), and height ($h$)) are chosen as: $l = w = h = \lambda_g / 10 = 4.5$ mm, at a frequency of 3.5 GHz, where $\lambda_g$ is the guided wavelength given as: $\lambda_g = \lambda_o / \sqrt{\varepsilon_r}$, and $\lambda_o$ is the free-space wavelength. $a$ is the radius of the sphere. As the particle has a full symmetry, the $\varepsilon_{\text{eff}}$ remains the same irrespective of the direction of excitation as well as irrespective of the polarization of the incident field (parallel or perpendicular). The fractional volume $f$ of the spherical particle inside the host medium can be calculated as:

$$f = \frac{4\pi a^3}{3lw}$$

Different values of $f$ are obtained by varying $a$. The range of $f$ is given as: $f_{\text{min}} = 0$ (no particle inside), $f_{\text{max}} : a = (l = w = h)/2$ (touching condition).

Figures. 2(c) and (e) show the simulated and calculated (using Eq. 2) $\varepsilon_{\text{eff}}$ curves for the conventional and complementary cases, respectively. Figs. 2(d) and (f) show the % error for both cases. It can be seen that the results are in very good agreement with an overall error of < 2% for the whole range of $f$. Also, it can be observed that the % error starts increasing for the large values of $f$. This is one of the widely discussed limitations of the Maxwell Garnett mixing rule (1) and (2).

The other limitation of these formulations is in terms of the difference ($\Delta$) between $\varepsilon_1$ and $\varepsilon_2$. This is shown in Fig. 3, where it can be seen that by increasing the particle’s permittivity $\varepsilon_1$ from 2 to 10, while $\varepsilon_2 = 1$, the calculated results start diverging from the simulation ones. However, for the small values of $f$, a good agreement can be noticed. This analysis shows that the Maxwell Garnett mixing rule can be still be used for large $\Delta$ only by considering very small values of $f$.

B. ANISOTROPIC PARTICLE

1) $\varepsilon_{\text{eff}}$ IN THE $z$-DIRECTION

Conventional Particle: To find the $\varepsilon_{\text{eff}}$ in the $z$-direction ($\varepsilon_{\text{eff}}z$), consider a cylindrical particle having $\varepsilon_1 = 3.66$ (RO4350B), embedded in the host medium of $\varepsilon_2 = 1$ as shown in Fig. 4(a). The dimensions of the unit cell ($l$, $w$, and $h$) are chosen as: $l = w = \lambda_g / 10$, and $h = 2.97$ mm, at a frequency of 3.5 GHz. $H$ is the height of the particle, $H = h$. The particle is excited in the $z$-direction using Floquet ports with Master & Slave boundaries in HFSSTM. As the direction of propagation ($K_z$) is parallel to the axis of the cylinder, the polarization of the incident $E$-field (parallel or perpendicular) does not affect the $\varepsilon_{\text{eff}}$ due to the symmetry of the structure. The fractional volume $f$ of the particle is given as:

$$f = \frac{\pi a^2 H}{4lw} = \frac{\pi a^2}{4l}$$

where $a$ is the diameter of the cylinder. Figure 4(c) shows the comparison of the models with the simulation results for $\varepsilon_{\text{eff}}z$, considering a conventional particle, and Fig. 4(d) shows the % error. It can be seen that the value: $N_c = 1/2$ from the Kittel model is in good agreement with the simulation results with an error of < 5% for the whole range of $f$.

Complementary Particle: Fig. 4(b) shows the unit cell geometry of the complementary cylinder particle having $\varepsilon_1 = 1$ inside a host medium having $\varepsilon_2 = 3.66$. The complementary particle having $\varepsilon_1 = 1$ is represented by a perforation (a through-drill where $H = h$). Figs. 4(e) and (f) show the $\varepsilon_{\text{eff}}$ and % error, respectively, for the complementary...
particle. It can be seen that for a very small value of $f < 0.03$, the models show good agreement. However, for the large values of $f$, the MoM model presents a significant departure as compared to the other two models. This is because the $\alpha_j$ values in MoM model were calculated for $\varepsilon_1 > \varepsilon_2$. The $N_z$ value given in the kittel model is slightly tuned to $N_z = 1/1.85$ to get an error of $< 5\%$ for around $58\%$ of $f$.

2) $\varepsilon_{\text{eff}}$ IN THE $x$-, $y$-DIRECTIONS
To evaluate the $\varepsilon_{\text{eff}}$ in $x$- or $y$-direction, it is important to note that the particle is symmetric in both $x$- and $y$-directions, therefore, $\varepsilon_{\text{eff}} = \varepsilon_{\text{eff}, x}$. However, the polarization of the incident field: $E_{\parallel}$ or $E_{\perp}$ as shown in Fig. 5, has an impact on the polarization field created inside the particle, which ultimately affects the $\varepsilon_{\text{eff}}$. Hence, the $\varepsilon_{\text{eff}, x}$ and $\varepsilon_{\text{eff}, y}$ are
evaluated separately, for both conventional and complementary cases.

**Conventional Particle:** Fig. 5 shows the geometry of the conventional particle as well as the comparison of the models for both || and \( \perp \) polarizations. It can be seen that the MoM model presents a minimum of 10% error for both cases. The Rayleigh model shows a comparatively large deviation in the || case while its error is < 5% for the whole \( f \), in the \( \perp \) case. The initial value of \( N_z = 1/2 \) provided by the Kittel model is tuned to have an error of < 5% for both cases. This analysis shows the sensitivity of \( \varepsilon_{\text{eff}} \) to the polarization of the incident field.

**Complementary Particle:** Fig. 6 shows the geometry of the complementary particle and the \( \varepsilon_{\text{eff}} \) analysis for both || and \( \perp \) polarizations. It can be seen that, as expected the MoM models show significant deviation from the simulation results for both || and \( \perp \) cases. The Rayleigh model shows a maximum of 12% error for the || case while it converges well
for the $\perp$ case with an error of < 5% for the whole $f$. The values of $N_z$ in the Kittel model are tuned for both cases to have an error of < 5%.

IV. MATERIAL DESIGN AND SYNTHESIS
A. MATERIAL REQUIREMENTS
Among the several critical parameters discussed in the introductory section for choosing the right material for both microwave and mm-Wave applications, the $\varepsilon_r$ and the material’s dielectric loss or tan$\delta$ are of significant importance. As compared to the microwave frequencies, where a high $\varepsilon_r$ can be used, at mm-Wave frequencies, a low $\varepsilon_r$ material is highly recommended, because using a high $\varepsilon_r$ material would further reduce the size of the structure thus creating more challenges in the fabrication [29]. Also, the losses in the material are often related to the value of $\varepsilon_r$. Higher the value of $\varepsilon_r$, higher are the losses in the material [29]. Therefore, at mm-Wave frequencies where the losses are significant,
a low $\varepsilon_r$ material is used to reduce further losses in the material. Moreover, at high frequencies, the speed of signal propagation in the high-speed circuits is very important. The signal propagation delay ($t_d$) depends on both the guiding structure as well as on the $\varepsilon_r$ of the material and is given by Eq. (12):

$$t_d = \sqrt{\varepsilon_r \times L \times C}$$  \hspace{1cm} (13)

where $L$ and $C$ are the inductance and capacitance of the line. Eq. (13) shows that by using a low $\varepsilon_r$ material or different transmission line structures (microstrip, stripline or coplanar waveguide, etc.), $t_d$ can be directly reduced. Cross-talk is also of utmost consideration when referring to mm-Wave frequencies. Using a low $\varepsilon_r$ material in the high-frequency circuits can decrease the capacitive coupling between the conductors and thus cross-talk can be reduced [12]. Aside from the values of $\varepsilon_r$, more important is the consistency of the $\varepsilon_r$ at mm-Wave frequencies. A slight variation in material $\varepsilon_r$ can cause a change in the impedance of the transmission line which can cause unexpected variations in the phase angles resulting in errors in radar signal detection [11].

Another important property of the material is the tan$\delta$. At both microwave and mm-Wave frequencies, dielectrics with low losses are used to reduce attenuation and heating in the circuit [12]. Current substrate materials have the tan$\delta$ values between $0.002 < \tan\delta < 0.02$. However, at the mm-Wave frequencies, the requirements become more stringent and tan$\delta$ values of $< 0.002$ are desired [29].

**B. DESIGN TRADE-OFFS**

As compared to other particles discussed in Sec. III, the complementary cylindrical particles inside a host medium which are commonly known as air perforations, offer several advantages in terms of easiness in fabrication (can be easily drilled) and measurement, reduce complexity in modeling the depolarization field due to its perfect alignment inside the structure, and help in reduction of the loss tangent of the material. The air perforations in the host material with $\varepsilon_1 \approx 1$ and a tan$\delta \approx 0$, has a very useful feature of reducing the overall tan$\delta$. Fig. 7 shows the reduction in the loss tangent of the host material by using a complementary cylindrical particle. It can be seen that when there is no particle inside i.e. $f = 0$, the original value of tan$\delta$ = 0.004 (RO4350B). However, for the maximum value of $f = 0.78$, the value of tan$\delta = 0.0012$, which shows a reduction of around 70% in the tan$\delta$ of the core-material.

**C. DESIGN PROCEDURE**

Figure 8 shows the simulation setup for the unit cell geometry of the proposed material structure using an infinite array approach in HFSS™. The required Floquet ports along with the Master and Slave boundaries are used. In this case, the material is excited in the $z$-direction with two de-embedded ports. The real and imaginary parts of the S-parameters are used to extract the constitutive parameters ($\varepsilon_{eff}$, $\mu_{eff}$, tan$\delta$) by using the Smith algorithm [30].

**D. SIZE OF THE UNIT CELL**

Determining the size of the unit cell of the periodic structure in terms of the wavelength is critical for extracting the correct values of the constitutive parameters [31]. For this the particle should meet the effective-homogeneity condition which is when the size of the unit cell is $< \frac{\lambda_g}{4}$ [31]. Moreover, a slight change in the value of $\varepsilon_r$ over the range of frequencies can degrade the performance specifically at mm-Wave frequencies. Fig. 9 shows the effect of the size of the unit cell on the $\varepsilon_{eff}$ and tan$\delta$ over the range of frequencies, for $z$- and $x$-, $y$-direction. For the purpose of illustration, a cubic unit cell of a complementary cylindrical particle excited in the $z$- and $x$- or $y$-directions is considered with $f = 56\%$. Different dimensions of the unit cell are considered in terms of the $\lambda_g$ at the highest frequency of operation i.e. 40 GHz. The cases considered have the dimensions $\leq 0.5\lambda_g$, because, for the size of the unit cell greater than $0.5\lambda_g$, no convergence was observed. For comparison, $f$ should remain the same for all the cases. Therefore, for each case, the diameter of the cylinder is adjusted to get the same values of $f$.

Figures. 9(a) and (b) show the $\varepsilon_{eff}$ and the tan$\delta$ in the $z$-direction. Fig. 9(a) shows the difference in the values of $\varepsilon_{eff}$.
FIGURE 9. $\varepsilon_{\text{eff}}$ and $\tan \delta$ as function of the unit cell dimensions for the complementary cylindrical particle using 4350B with perforation. (a) $\varepsilon_{\text{eff}}$ in the $z$-direction. (b) $\tan \delta$ in the $z$-direction. (c) $\varepsilon_{\text{eff}}$ in the $x$, $y$-directions. (d) $\tan \delta$ in the $x$, $y$-directions.

FIGURE 10. Material measurement setups for low and high frequencies. (a) Photo of the measurement setup for S-band, based on the waveguide method. (b) Material samples used for the S-band test. (c) Photo of the measurement setup for W-band, based on the free-space Gaussian beam method. (d) Material samples used for W-band test.

at lower frequencies as well as an increasing trend at higher frequencies for different dimensions of the unit cell. However, for the case of $0.1\lambda_g$, the values of $\varepsilon_{\text{eff}}$ are stable over the entire frequency range. Figs. 9(c) and (d) show this analysis for the $x$, $y$-directions. It can be seen that for the unit cell size of $0.1\lambda_g$, stable performance is obtained. However, when the
size gets larger, a decreasing trend can be observed. From this, it can be concluded that in order to have a steady performance in terms of $\varepsilon_{\text{eff}}$ or $\tan \delta$, the z- or x-, y-dimensions of the unit cell should be at least $\leq 0.1 \lambda_g$ at the higher frequency of operation. Smaller the size of the unit cell, the more stable is the performance. However, fabrication tolerances should be considered while determining the size of the unit cell.

The results from this analysis are summarized in Table 1. The dimensions of the unit cell are $0.1 \lambda_g$ at 40 GHz. As the reference RO4350B sample is isotropic, the values of $\varepsilon_{\text{eff}}$ and $\tan \delta$ for any direction are the same given in the data sheet. However, for the proposed sample loaded with anisotropic complementary cylindrical particle, the values of $\varepsilon_{\text{eff}}$ and $\tan \delta$ slightly differ between the x-, y-, and z-directions. The reason for this has been explained in detail in Sec. III.

For determining the size of the unit cell for the W-band, the same unit cell size analysis was carried out at 77 GHz, using solid RO3003 as the reference sample. The dimensions of the unit cell were chosen based on the stable performance as well as keeping the fabrication tolerances in consideration. As the permittivity is measured in the z-direction, for sample-3 the z-dimension or thickness $H$ of the material, which is more sensitive to the performance, was chosen to be small as $0.226 \lambda_g$ at 77 GHz in order to achieve accurate and stable results. The other dimensions of the unit cell were kept comparatively larger as $1 \lambda_g$ due to fabrication restrictions.

V. MEASUREMENT RESULTS AND VALIDATION
To validate the analysis presented before, two different samples were fabricated and measured in the S-band at 3.5 GHz and a third sample in the mm-Wave W-band at 77 GHz. The proposed samples were designed using the analysis presented in Sec. II-IV. These samples were fabricated using LPKF Protomat S103 machine and were measured using the setups shown in Fig. 10. The $\varepsilon_{\text{eff}}$ of the S-band samples: sample-1 and sample-2 were measured using the waveguide setup of Fig. 10(a), and the W-band sample-3 was characterized using the mm-Wave RF scanner shown in Fig. 10(c).
The measurement results were compared with the reference samples. The simulated, calculated, and measured results in terms of $\varepsilon_{\text{eff}}$ and $\tan\delta$ are shown in Fig. 11 for the S-band samples, and in Fig. 12 for the W-band sample-3. Fig. 10(b) and (d) shows the reference and the proposed samples for the S-band and the W-band, respectively. For
the S-band samples: sample-1 and sample-2, the dimensions of the unit cell are kept \( \leq \frac{\lambda}{10} \) at 3.5 GHz based on the analysis presented in Sec. IV. The dimensions of the unit cell in both cases are varied to obtain different values of \( f \). Figs. 11 and 12 show an excellent agreement between the simulated, calculated, and measured results. For sample-3, the thickness \( H \) of the material is chosen to be small as 0.226\( \times \varepsilon \), at 77 GHz in order to achieve accurate and stable results. The other dimensions of the unit cell are kept comparatively large due to fabrication challenges at 77 GHz. However, all the dimensions are tuned to get convergence in the results.

All the dimensions as well as the results for the S-band and the W-band samples are also summarized in Table 2. It should be noted that the measured value of the \( \tan\delta \) for the reference RO4350B sample given in the Table 2 is 0.008 and not 0.004 (from data sheet). Similarly, the values of \( \varepsilon_r \) and \( \tan\delta \) for the reference RO3003 sample are 2.91 and 0.0072, respectively, measured at 77 GHz. This variation in the results can be attributed to the measurement setup, frequency of operation, external environmental conditions, and slight inaccuracy in the measurement setup. It should also be noted that each measured \( \varepsilon_{\text{eff}} \) curve in Fig. 11 or 12 represents the statistical average of several repeated measurements to ensure stability in the results and to reduce errors associated with the measurement. It can be seen that for sample-1 having \( f = 37\% \), the reduction in \( \varepsilon_{\text{eff}} \), and \( \tan\delta \) is 33% and 25%, respectively. For sample-2 with \( f = 55\% \), a reduction of 45% and 38% is obtained in the \( \varepsilon_{\text{eff}} \), and \( \tan\delta \), respectively, as compared to the solid reference sample RO4350B shown in Fig. 11(a). Similarly, for sample-3 with \( f = 50\% \), a reduction of 32% and 72% is achieved in the values of \( \varepsilon_{\text{eff}} \) and \( \tan\delta \), respectively, as compared to the reference RO3003 sample shown in Fig. 12(a).

Table 3 shows the performance comparison of the proposed work with other related works. It can be noticed that as compared to other works, the proposed work validates the analytical models based on the error analysis and also discusses anisotropy of the material based on the direction of excitation and polarization analysis. Moreover, the proposed design procedures can be applied in the design of a high-performance material at any frequency between 1-110 GHz, for many applications.

VI. CONCLUSION

In this paper, a proposed method based on Maxwell Garnett mixing theory is used to obtain a high-performance RF material, from an existing high thermally stable and compatible material. A complete set of design equations for finding desirable \( \varepsilon_{\text{eff}} \) of both isotropic and anisotropic particles is presented. The permittivity analysis for both isotropic and anisotropic cases is thoroughly studied including the effects of the direction of excitation as well as polarization of the incident field on the \( \varepsilon_{\text{eff}} \). Detailed design procedures and requirements are discussed for both microwave and mm-Wave applications. Several analytical models are discussed and evaluated for their performance. The most accurate model is identified and used to find the \( \varepsilon_{\text{eff}} \) of the anisotropic cylindrical particle in any direction of excitation as well as any polarization of the incident field. Building on all the advantages of the existing substrate materials, the proposed method can be used to obtain the desired lower \( \varepsilon_r \) and \( \tan\delta \) values. Thus, enabling the use of inexpensive materials to be used as high-performance RF substrates for applications that require high antenna efficiency, multi-layer PCB compatibility, and high thermal stability. For the proof of concept, samples were fabricated and tested in the S-band at 3.5 GHz as well as in the W-band at 77 GHz. A reduction of 45% in the \( \varepsilon_{\text{eff}} \) and 38% in the loss tangent values was obtained in the S-band, and 32% and 72% reduction in \( \varepsilon_{\text{eff}} \) and \( \tan\delta \), respectively, in the W-band. Excellent agreement was obtained between the simulation, calculated, and measured results.

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