HOW SMOOTH IS THE UNIVERSE ON LARGE SCALES?

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Abstract

We review cosmological inference from optical and radio galaxy surveys, the X-Ray Background and the Cosmic Microwave Background. We focus on three topics: (i) First results from the 2dF galaxy redshift survey; (ii) Estimation of cosmological parameters by joint analysis of redshift surveys and CMB data; and (iii) The validity of the Cosmological Principle, and constraints on the fractal dimension on large scales.

1. Introduction

In summarizing a conference in memory of Yukawa nearly 10 years ago, Prof. H. Sato referred to the dark matter problem as made of three ‘islands’: (i) Astronomical facts concerning galaxies and the large scale structure of the universe, (ii) the Big Bang model and (iii) Particle Physics. Here we shall focus on the first ‘island’ and discuss briefly implications to dark matter models and the validity of the Cosmological Principle.

It is believed by most cosmologists that on the very large scales the universe is isotropic and homogeneous. However, on scales much smaller than the horizon the distribution of luminous matter is clumpy. Surveys such as CfA, SSRS, IRAS, APM and Las Campanas have yielded useful information on local structure and on the density parameter $\Omega$ from redshift distortion and from comparison with the peculiar velocity field. Together with measurements of the Cosmic Microwave Background (CMB) radiation and gravitational lensing the redshift surveys provide major probes of the world’s geometry and the dark matter.

In spite of the rapid progress two gaps remain in our understanding of the density fluctuations as a function of scale: (i) It is still unclear how to relate...
the distributions of galaxies and mass; (ii) Little is known about fluctuations on intermediate scales between these of local galaxy surveys ($\sim 100h^{-1}$ Mpc) and the scales probed by COBE ($\sim 1000h^{-1}$ Mpc).

Another related but unresolved issue is the value of the density parameter $\Omega$. Putting together different cosmological observations, the derived values seem to be inconsistent with each other. Taking into account moderate biasing, the redshift and peculiar velocity data on large scales yield $\Omega \approx 0.3 - 1.5$, with a trend towards the popular value $\approx 1$ (e.g. Dekel 1994; Strauss & Willick 1995 for summary of results). On the other hand, the high fraction of baryons in clusters, combined with the baryon density from Big Bang Nucleosynthesis suggests $\Omega \approx 0.2$ (White et al. 1993). Moreover, an $\Omega = 1$ universe is also in conflict with a high value of the Hubble constant ($H_0 \approx 70 - 80$ km/sec/Mpc), as in this model the universe turns out to be younger than globular clusters. A way out of these problems was suggested by adding a positive cosmological constant, such that $\Omega + \lambda = 1$, to satisfy inflation. Two recent observations constrain $\lambda$: the observed frequency of lensed quasars is too small, yielding an upper limit $\lambda < 0.65$ (e.g. Kochanek 1996), and the magnitude-redshift relation for Supernovae type Ia (e.g. Perlmutter et al. 1998). The next decade will see several CMB experiments (e.g. Planck, MAP, VSA) which promise to determine (in a model-dependent way) the cosmological parameters to within a few percent. We shall focus here on several issues related to clustering and cosmological parameters from new surveys.

2. The 2dF Galaxy Redshift Survey

Existing optical and IRAS redshift surveys contain $\sim 10^4$ galaxies. Multi-fibre technology now allows us to produce redshift surveys of millions of galaxies. Two major surveys have just started. The American-Japanese Sloan Digital Sky Survey (SDSS) will yield images in 5 colours for 50 million galaxies, and redshifts for about 1 million galaxies over a quarter of the sky (Gunn and Weinberg 1995). It will be carried out using a dedicated 2.5m telescope in New Mexico. The median redshift of the survey is $\bar{z} \sim 0.1$.

A complementary Anglo-Australian survey, the 2 degree Field (2dF) will produce redshifts for 250,000 galaxies brighter than $b_J = 19.5^m$ (with median redshift of $\bar{z} \sim 0.1$), selected from the APM catalogue. The survey will utilize a new 400-fibre system on the 4m AAT, covering $\sim 1,700$ sq deg of the sky. About 10,000 redshifts have been measured so far (as of June 1998). A deeper extension down to $R = 21$ for 10,000 galaxies is also planned for the 2dF survey.

The main goals of the 2dF galaxy survey are:

- Accurate measurements of the power spectrum of galaxy clustering on
scales $> 30h^{-1}$ Mpc, allowing a direct comparison with CMB anisotropy measurements such as the recently approved NASA MAP and ESA Planck Surveyor satellites. The power-spectrum derived from the projected APM galaxies (see Figure 2) gives an idea about the scales probed by the 2dF redshift survey.

- Measurement of the distortion of the clustering pattern in redshift space providing constraints on the cosmological density parameter $\Omega$ and the spatial distribution of dark matter.
- Determination of variations in the spatial and velocity distributions of galaxies as a function of luminosity, spectral type and star-formation history, providing important constraints on models of galaxy formation.
- Investigations of the morphology of galaxy clustering and the statistical properties of the fluctuations, e.g. whether the initial fluctuations are Gaussian as predicted by inflationary models of the early universe.
- A study of clusters and groups of galaxies in the redshift survey, in particular the measurement of infall in clusters and dynamical estimates of cluster masses at large radii.
- Application of novel techniques (e.g. Principal Component Analysis and Artificial Neural Networks) to classify the uniform sample of 250,000 spectra, thereby obtaining a comprehensive inventory of galaxy types as a function of spatial position within the survey.

Figure 1 (from Folkes et al. 1998) shows cone plots of redshift-space distribution for a subset of $\sim 3000$ 2dF galaxies. The galaxies were classified according to their spectra by Principal Component Analysis (Folkes, Lahav & Maddox 1996) and then divided into two groups of nearly equal numbers. The ‘red’ (early type) galaxies do appear more clustered, with evidence for ‘finger-of-God’ effects caused by the velocity dispersion of galaxy clusters, while the ‘blue’ (late-type) galaxies show a more uniform distribution, although clustering is still evident. This is in qualitative agreement with the well-known morphology-density relation. Quantifying these differences and comparing them with the predictions of models will be a major focus of the future analysis of the 2dF galaxy survey. For more details on the 2dF galaxy survey see

http://msowww.anu.edu.au/~colless/2dF/

3. Probes at High Redshift

The big new surveys (SDSS, 2dF) will only probe a median redshift $\bar{z} \sim 0.1$. It remains crucial to probe the density fluctuations at higher $z$, and to fill in the gap between scales probed by previous local galaxy surveys and the scales probed
Fig. 1. Cone plots of the 2dF galaxies with measured redshift, split into ‘red’ (spectral Types 1 & 2) and ‘blue’ (spectral Types 3, 4 & 5) samples. The ‘red’ galaxies are more strongly clustered (from Folkes et al. 1998).
by COBE and other CMB experiments. Here we discuss the X-ray Background (XRB) and radio sources as probes of the density fluctuations at median redshift $\bar{z} \sim 1$. Other possible high-redshift traces are quasars and clusters of galaxies.

### 3.1. Radio Sources

Radio sources in surveys have typical median redshift $\bar{z} \sim 1$, and hence are useful probes of clustering at high redshift. Unfortunately, it is difficult to obtain distance information from these surveys: the radio luminosity function is very broad, and it is difficult to measure optical redshifts of distant radio sources. Earlier studies claimed that the distribution of radio sources supports the 'Cosmological Principle'. However, the wide range in intrinsic luminosities of radio sources would dilute any clustering when projected on the sky. Recent analyses of new deep radio surveys (e.g. FIRST) suggest that radio sources are actually clustered at least as strongly as local optical galaxies (e.g. Cress et al. 1996; Magliocchetti et al. 1998). Nevertheless, on the very large scales the distribution of radio sources seems nearly isotropic. Comparison of the measured quadrupole in a radio sample in the Green Bank and Parkes-MIT-NRAO 4.85 GHz surveys to the theoretically predicted ones (Baleisis et al. 1998) offers a crude estimate of the fluctuations on scales $\lambda \sim 600h^{-1}\text{Mpc}$. The derived amplitudes are shown in Figure 2 for the two assumed Cold Dark Matter (CDM) models. Given the problems of catalogue matching and shot-noise, these points should be interpreted at best as 'upper limits', not as detections.

### 3.2. The XRB

Although discovered in 1962, the origin of the X-ray Background (XRB) is still unknown, but is likely to be due to sources at high redshift (for review see Boldt 1987; Fabian & Barcons 1992). Here we shall not attempt to speculate on the nature of the XRB sources. Instead, we utilise the XRB as a probe of the density fluctuations at high redshift. The XRB sources are probably located at redshift $z < 5$, making them convenient tracers of the mass distribution on scales intermediate between those in the CMB as probed by COBE, and those probed by optical and IRAS redshift surveys (see Figure 2).

The interpretation of the results depends somewhat on the nature of the X-ray sources and their evolution. The rms dipole and higher moments of spherical harmonics can be predicted (Lahav et al. 1997) in the framework of growth of structure by gravitational instability from initial density fluctuations. By comparing the predicted multipoles to those observed by HEAO1 (Treiver et al. 1998) we estimate the amplitude of fluctuations for an assumed shape of the density fluctuations (e.g. CDM models). Figure 2 shows the amplitude of fluctuations
Fig. 2. A compilation of density fluctuations on different scales from various observations: a galaxy survey, deep radio surveys, the X-ray Background and Cosmic Microwave Background experiments. The measurements are compared with two popular Cold Dark Matter models (with normalization $\sigma_8 = 1$ and shape parameters $\Gamma = 0.2$ and $0.5$). The Figure shows mean-square density fluctuations $(\frac{\delta \rho}{\rho})^2 \propto k^3 P(k)$, where $k = 1/\lambda$ is the wavenumber and $P(k)$ is the power-spectrum of fluctuations. The open squares at small scales are estimates from the APM galaxy catalogue (Baugh & Efstathiou 1994). The elongated 'boxes' at large scales represent the COBE 4-yr (on the right) and Tenerife (on the left) CMB measurements (Gawiser & Silk 1998). The solid triangles and crosses represent amplitudes derived from the quadrupole of radio sources (Baleisis et al. 1998) and the quadrupole of the XRB (Lahav et al. 1997; Treyer et al. 1998). Each pair of estimates corresponds to assumed shape of the two CDM models. (A compilation from Wu, Lahav & Rees 1998).
derived at the effective scale $\lambda \sim 600 h^{-1}$ Mpc probed by the XRB. The observed fluctuations in the XRB are roughly as expected from interpolating between the local galaxy surveys and the COBE CMB experiment. The rms fluctuations $\delta \rho / \rho$ on a scale of $\sim 600 h^{-1}$Mpc are less than 0.2%.

4. Is the FRW Metric Valid on Large Scales?

The Cosmological Principle was first adopted when observational cosmology was in its infancy; it was then little more than a conjecture. Observations could not then probe to significant redshifts, the ‘dark matter’ problem was not well-established and the Cosmic Microwave CMB and the XRB were still unknown. If the Cosmological Principle turned out to be invalid then the consequences to our understanding of cosmology would be dramatic, for example the conventional way of interpreting the age of the universe, its geometry and matter content would have to be revised. Therefore it is important to revisit this underlying assumption in the light of new galaxy surveys and measurements of the background radiations. The question of whether the universe is isotropic and homogeneous on large scales can also be phrased in terms of the fractal structure of the universe. A fractal is a geometric shape that is not homogeneous, yet preserves the property that each part is a reduced-scale version of the whole. If the matter in the universe were actually distributed like a pure fractal on all scales then the Cosmological Principle would be invalid, and the standard model in trouble. As shown in Figure 2 current data already strongly constrain any non-uniformities in the galaxy distribution (as well as the overall mass distribution) on scales $> 300 h^{-1}$ Mpc.

If we count, for each galaxy, the number of galaxies within a distance $R$ from it, and call the average number obtained $N(< R)$, then the distribution is said to be a fractal of correlation dimension $D_2$ if $N(< R) \propto R^{D_2}$. Of course $D_2$ may be 3, in which case the distribution is homogeneous rather than fractal. In the pure fractal model this power law holds for all scales of $R$.

The fractal proponents (Pietronero et al. 1997) have estimated $D_2 \approx 2$ for all scales up to $\sim 1000 h^{-1}$ Mpc, whereas other groups have obtained scale-dependent values (for review see Wu et al. and references therein).

These measurements can be directly compared with the popular Cold Dark Matter models of density fluctuations, which predict the increase of $D_2$ with $R$ for the hybrid fractal model. If we now assume homogeneity on large scales, then we have a direct mapping between correlation function $\xi(r)$ (or the Power-spectrum) and $D_2$. For $\xi(r) \propto r^{-\gamma}$ it follows that $D_2 = 3 - \gamma$ if $\xi \gg 1$, while if $\xi(r) = 0$ then $D_2 = 3$. The predicted behaviour of $D_2$ with $R$ from three different CDM models
is shown Figure 3. Above $100 h^{-1}\text{Mpc}$ $D_2$ is indistinguishably close to 3. We also see that it is inappropriate to quote a single crossover scale to homogeneity, for the transition is gradual.

Direct estimates of $D_2$ are not possible for much larger scales, but we can calculate values of $D_2$ at the scales probed by the XRB and CMB by using CDM models normalised with the XRB and CMB as described above. The resulting values are consistent with $D_2 = 3$ to within $10^{-4}$ on the very large scales (Peebles 1993; Wu et al. 1998). Isotropy does not imply homogeneity, but the near-isotropy of the CMB can be combined with the Copernican principle that we are not in a preferred position. All observers would then measure the same near-isotropy, and an important result has been proven that the universe must then be very well approximated by the FRW metric (Maartens et al. 1996).

While we reject the pure fractal model in this review, the performance of CDM-like models of fluctuations on large scales have yet to be tested without assuming homogeneity a priori. On scales below, say, $30 h^{-1}\text{Mpc}$, the fractal nature of clustering implies that one has to exercise caution when using statistical methods which assume homogeneity (e.g. in deriving cosmological parameters).

As a final note, we emphasize that we only considered one ‘alternative’ here, which is the pure fractal model where $D_2$ is a constant on all scales.

5. A ‘Best Fit Universe’

Observations of anisotropies in the Cosmic Microwave Background (CMB) provide one of the key constraints on cosmological models and a significant quantity of experimental data already exists (e.g. Figure 2 and Gawiser & Silk 1998).

On the other hand, galaxy redshift surveys, mapping large scale structure (LSS), provide another cosmologically important set of observations. The clustering of galaxies in redshift-space is systematically different from that in real-space (Kaiser 1987, Hamilton 1997 for review). The mapping between the two is a function of the underlying mass distribution, in which the galaxies are not only mass tracers, but also velocity test particles. Estimates derived separately from each of the CMB and LSS data sets have problems with parameter degeneracy. Webster et al. (1998) combined results from a range of CMB experiments, with a likelihood analysis of the IRAS 1.2Jy survey, performed in spherical harmonics. This method expresses the effects of the underlying mass distribution on both the CMB potential fluctuations and the IRAS redshift distortion. This breaks the degeneracy inherent in an isolated analysis of either data set, and places tight constraints on several cosmological parameters.

The family of CDM models analysed corresponds to a spatially-flat uni-
Fig. 3. The fractal correlation dimension $D_2$ versus length scale $R$ assuming three Cold Dark Matter models of power-spectra with shape and normalization parameters ($\Gamma = 0.5; \sigma_8 = 0.6$), ($\Gamma = 0.5; \sigma_8 = 1.0$) and ($\Gamma = 0.2; \sigma_8 = 1.0$). They all exhibit the same qualitative behaviour of increasing $D_2$ with $R$, becoming vanishingly close to 3 for $R > 100\, h^{-1}\, \text{Mpc}$ (from Wu, Lahav & Rees 1998).
verse with with an initially scale-invariant spectrum and a cosmological constant \( \lambda \). Free parameters in the joint model are the mass density due to all matter \((\Omega)\), Hubble’s parameter \((h = H_0/100 \text{ km/sec})\), IRAS light-to-mass bias \((b_{\text{iras}})\) and the variance in the mass density field measured in an \(8h^{-1}\text{Mpc} \) radius sphere \((\sigma_8)\). For fixed baryon density \(\Omega_b = 0.024/h^2\) the joint optimum lies at (Webster et al. 1998; Bridle et al., in preparation) \(\Omega = 1 - \lambda = 0.41 \pm 0.13\), \(h = 0.52 \pm 0.10\), \(\sigma_8 = 0.63 \pm 0.15\), \(b_{\text{iras}} = 1.28 \pm 0.40\) (marginalised 1-sigma error bars). For these values of \(\Omega, \lambda\) and \(H_0\) the age of the universe is \(\sim 16.6\) Gyr.

6. Discussion

We have shown some recent studies of galaxy surveys, and their cosmological implications. New measurements of galaxy clustering and background radiations can provide improved constraints on the isotropy and homogeneity of the Universe on large scales. In particular, the angular distribution of radio sources and the X-Ray Background probe density fluctuations on scales intermediate between those explored by galaxy surveys and CMB experiments. On scales larger than \(300h^{-1}\text{Mpc} \) the distribution of both mass and luminous sources satisfies well the ‘Cosmological Principle’ of isotropy and homogeneity. Cosmological parameters such as \(\Omega\) therefore have a well defined meaning. With the dramatic increase of data, we should soon be able to map the fluctuations with scale and epoch, and to analyze jointly LSS (2dF, SDSS) and CMB (MAP, Planck) data.

Acknowledgments I thank my collaborators for their contribution to the work presented here. I also acknowledge JSPS and Tokyo and Kyoto Universities for the hospitality.

7. References

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