TWIST-FOUR DISTRIBUTIONS IN A TRANSVERSELY-POLARIZED NUCLEON
AND THE DRELL-YAN PROCESS *

Pervez Hoodbhoy †

Center for Theoretical Physics
Laboratory for Nuclear Science
and Department of Physics
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139 U.S.A.
and
Department of Physics
Quaid-e-Azam University
Islamabad, Pakistan

Xiangdong Ji

Center for Theoretical Physics
Laboratory for Nuclear Science
and Department of Physics
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139 U.S.A.

ABSTRACT

The twist-four quark and gluon distributions in a transversely polarized nucleon are identified and relations among them are discussed by using QCD equations of motion. Their contribution, at the order of $O(1/Q^2)$, to the Drell-Yan cross section in transversely polarized nucleon-nucleon collisions is calculated.

Submitted to: Phys. Rev. D

CTP#2203
April 1993

* This work is supported in part by funds provided by the U. S. Department of Energy (D.O.E.) under contract #DE-AC02-76ER03069.
† Supported by NSF Grant INT-9122027.
I. INTRODUCTION

There has been considerable interest recently in the polarization-dependent quark distributions of a transversely polarized nucleon.\textsuperscript{1−4} These distributions, which are on the same footing as those of an unpolarized or longitudinally polarized nucleon, are just as important for characterizing the nucleon’s high-energy structure. Being universal quantities — meaning that they describe the nucleon rather than a particular process — these distributions can be measured in a variety of experiments involving large momentum transfers. Although at present no parton distribution can be calculated ab-initio from QCD, nonetheless this may eventually become possible.

A very interesting distribution, $h_1(x)$, called the transversity distribution in ref. 2, was first identified by Ralston and Soper. That this involves a helicity flip of the participating quark was recognized by Artru and Mekhfi.\textsuperscript{1} The $h_1(x)$ is a twist-two quantity, meaning that it enters into the cross section of a hard process, such as Drell-Yan lepton pair production, unsuppressed by inverse powers of the hard momentum $Q$. It has been proposed that $h_1(x)$ be measured using transversely polarized beams at RHIC.\textsuperscript{4} The $\mathcal{O}\left(\frac{1}{Q^2}\right)$, or twist-four, corrections to this process involve parton distributions with four partons. The purpose of this paper is to identify and classify these distributions and to calculate their contribution to the Drell-Yan process.

Higher twist corrections, as is well-known, are notoriously complicated. Nonetheless, if one is to proceed beyond the naive parton model, it is necessary to identify all parton distributions which enter at higher twist, to study their symmetry properties, and to calculate their contributions to different hard processes. In contrast to simple distributions involving only two quark or gluon fields which occur at leading twist, higher twist distributions involve
the matrix elements of the multi quark and gluon field operators at equal light-cone times. A complete catalogue of unpolarized and longitudinally polarized distributions up to twist-four, and transverse distributions up to twist-three, can be found in ref. 3.

The results of this paper, which deals exclusively with the twist-four distributions of a transversely polarized nucleon, are summarized below:

Parton distributions are introduced through parton-hadron vertices. As is shown in Fig. 1, the twist-four distributions in the light-cone gauge are contained in the vertices up to four partons. Here the four gluon vertex is absent because it cannot mix with the chiral-odd vertices shown. The two-quark vertex, shown in fig. 1a, provides one twist-four distribution $h_3(x)$. There are four distributions, named $d_i(x,y)$ with $i = 1, 2, 3, 4$, associated with the two-quark-one-gluon vertex in fig. 1b. The two-quark-two-gluon vertex, fig. 1c, gives rise to three distributions named herein as $D_i(x, y, z)$ with $i = 1, 2, 3$. Finally, the four-quark vertex (fig. 1d) has two associated distributions, $W_i(x, y, z)$ with $i = 1, 2$. The tensor structure which gives rise to each distribution is given in the text, together with restrictions obtained from the requirement of PCT invariance. The QCD equations of motion on the light-cone impose further restrictions by specifying relations between distributions involving more light-cone momentum fractions and distributions involving fewer. Including the distributions identified in ref. 3 — i.e. $h_1(x)$, $g_T(x)$, $G_1(x, y)$, and $G_2(x, y)$ — a complete set is now available to deal with any hard process involving transversely polarized nucleons up to and including twist-four. As one application, we have calculated the twist-four part of the Drell-Yan cross section for transversely polarized nucleons. This could provide a framework to analyze corrections if an experiment to measure $h_1(x)$ is actually performed.$^4$.
II. TWIST-FOUR DISTRIBUTIONS IN A TRANSVERSELY POLARIZED NUCLEON

In this section, we consider the polarization-dependent twist-four distributions in a transversely polarized nucleon. To establish notation, we consider a nucleon moving in the $z$ direction with its spin vector in the $x-y$ plane with momentum $P^\mu = p^\mu + \frac{1}{2}M^2n^\mu$ and $S_\perp^\mu = (0, \vec{S}_\perp, 0)$, where $\vec{S}_\perp \cdot \vec{S}_\perp = 1$ and $p^\mu$ and $n^\mu$ are null vectors satisfying $p^2 = n^2 = 0$ and $p \cdot n = 1$. A distribution of partons in such a nucleon is defined by the matrix element of quark and gluon field operators at equal light-cone times. We choose the light-cone gauge, $A \cdot n = 0$, Simple dimensional reasoning enables determination of the number of inverse powers of the hard momentum which accompany a given distribution, and thus its twist. Application of PCT rules out a large number of otherwise possible structures. For further details the reader is referred to refs. 2 and 3.

At the level of twist-four, the two quark vertex in fig. 1a contains only one transverse distribution,

$$h_3(x) = \frac{1}{\Lambda^2} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS_\perp | \bar{\psi}(0)\gamma_5 \vec{S}_\perp \psi(\lambda n) | PS_\perp \rangle,$$

where $\Lambda$ is a soft scale. It can also be projected out from the quark density matrix,

$$M(x) = \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS_\perp | \bar{\psi}(0)\psi(\lambda n) | PS_\perp \rangle$$

$$= \frac{1}{4} \Lambda^2 \not{\gamma}_5 S_\perp h_3(x) + ..., \tag{2}$$

where dots represent other distributions which are our concern here.

The two-quark-one-gluon vertex, fig. 1b, is more complex and involves two light-cone fractions since the quark and gluon can be removed from the hadron from different light-cone
“positions” $x^- = \lambda n^-$ or $y^- = \mu n^-$. There are four PCT allowed transverse distributions:

$$d_1(x, y) = \int [d\lambda d\mu] \langle \psi(0) iD_{\perp}(\mu n) \cdot S_{\perp} \gamma_5 \psi(\lambda n) \mid \psi(0) iD_{\perp}(\mu n) \cdot S_{\perp} \gamma_5 \psi(\lambda n) \mid PS_{\perp} \rangle,$$

$$d_2(x, y) = \int [d\lambda d\mu] \langle \psi(0) iD_{\perp}(\mu n) \cdot iS_{\perp} \gamma_5 \frac{1}{2}(\phi \gamma - \bar{\psi} \gamma) \psi(\lambda n) \mid \psi(0) iD_{\perp}(\mu n) \cdot iS_{\perp} \gamma_5 \frac{1}{2}(\phi \gamma - \bar{\psi} \gamma) \psi(\lambda n) \mid PS_{\perp} \rangle,$$

$$d_3(x, y) = \int [d\lambda d\mu] \langle \psi(0) iD_{\perp}(\mu n) \cdot iS_{\perp} \gamma_5 \frac{1}{2}(\phi \gamma - \bar{\psi} \gamma) \psi(\lambda n) \mid \psi(0) iD_{\perp}(\mu n) \cdot iS_{\perp} \gamma_5 \frac{1}{2}(\phi \gamma - \bar{\psi} \gamma) \psi(\lambda n) \mid PS_{\perp} \rangle,$$

$$d_4(x, y) = \int [d\lambda d\mu] \langle \psi(0) iD_{\perp}(\mu n) \cdot iS_{\perp} \gamma_5 \frac{1}{2}(\phi \gamma - \bar{\psi} \gamma) \psi(\lambda n) \mid \psi(0) iD_{\perp}(\mu n) \cdot iS_{\perp} \gamma_5 \frac{1}{2}(\phi \gamma - \bar{\psi} \gamma) \psi(\lambda n) \mid PS_{\perp} \rangle,$$

where $T_1^\alpha = \epsilon^{\alpha\beta\gamma\delta} p_{\beta} n_{\gamma} S_{\perp \delta}$ is a vector orthogonal to $S_{\perp \alpha}$ with $T_{\perp \alpha} \cdot T_{\perp \alpha} = -1$ and,

$$[d\lambda d\mu] = \frac{1}{2\Lambda^2} \frac{d\lambda}{2\pi} e^{i\lambda x} e^{i\mu(y-x)}.$$

The $d_i(x, y)$ may be readily projected out of the quark-gluon density matrix,

$$M_i^\alpha(x, y) = 2\lambda^2 \int [d\lambda d\mu] \langle \psi(0) iD_{\perp}^\alpha(\mu n) \psi(\lambda n) \mid \psi(0) iD_{\perp}^\alpha(\mu n) \psi(\lambda n) \mid PS_{\perp} \rangle,$$

$$= -\frac{1}{2} \lambda^2 \left[ S_{\perp \alpha}^\gamma d_{1}(x, y) + S_{\perp \alpha}^\gamma \frac{1}{2}(\phi \gamma - \bar{\psi} \gamma) d_{2}(x, y) \right.$$

$$- iT_{\perp \alpha} d_{3}(x, y) - iT_{\perp \alpha} \frac{1}{2}(\phi \gamma - \bar{\psi} \gamma) d_{4}(x, y) \big] + \ldots \) \) (5)$$

It can be easily shown that the $d_i$'s are real and obey the symmetry relations:

$$d_1(x, y) = -d_1(y, x) \) , \ d_2(x, y) = d_2(y, x)$$

$$d_3(x, y) = -d_3(y, x) \) , \ d_4(x, y) = d_4(y, x) \) \) \) (6)$$

Note that the index $\alpha$ in eq. (5) refers to transverse polarizations ($\alpha = 1, 2$) only because $A^+ = 0$ and $A^-$ is a dependent quantity whose presence would make a distribution belong to one higher twist.

The two-quark and two-gluon vertex, fig. 1c, also with transverse gluons only, has three light-cone fractions. There are three allowed distributions:

$$D_1(x, y, z) = \int [d\lambda d\mu d\nu] \langle \psi(0) iD_{\perp}(\nu n) \cdot iD_{\perp}(\mu n) \psi(\lambda n) \mid \psi(0) iD_{\perp}(\nu n) \cdot iD_{\perp}(\mu n) \psi(\lambda n) \mid PS_{\perp} \rangle,$$

$$D_2(x, y, z) = \int [d\lambda d\mu d\nu] \langle \psi(0) iD_{\perp}(\nu n) \cdot S_{\perp} \gamma_5 \psi(\lambda n) \mid \psi(0) iD_{\perp}(\nu n) \cdot S_{\perp} \gamma_5 \psi(\lambda n) \mid PS_{\perp} \rangle,$$

$$D_3(x, y, z) = \int [d\lambda d\mu d\nu] \langle \psi(0) iD_{\perp}(\nu n) \cdot S_{\perp} \gamma_5 \psi(\lambda n) \mid \psi(0) iD_{\perp}(\nu n) \cdot S_{\perp} \gamma_5 \psi(\lambda n) \mid PS_{\perp} \rangle \) \) (7)$$
where,
\[ [d\lambda \, d\mu \, d\nu] = \frac{1}{2\Lambda^2} \frac{d\lambda}{2\pi} \frac{d\mu}{2\pi} \frac{d\nu}{2\pi} e^{i\lambda x} e^{i\mu(y-x)} e^{i\nu(z-y)}. \] (8)

The above distributions can be projected out of the \( q\bar{q}q \) density matrix,
\[ M^{\alpha\beta}(x, y, z) = 2\Lambda^2 \int [d\lambda \, d\mu \, d\nu] \langle PS_{\perp} | \bar{\psi}(0) iD^\alpha(\nu n) iD^\beta(\mu n) \psi(\lambda n) | PS_{\perp} \rangle \]
\[ = \frac{1}{4\Lambda^2} \left[ g_{\perp}^{\alpha\beta} \gamma_5 S_{\perp} \not{p} D_1(x, y, z) \right. \]
\[ + \frac{1}{2} \left( S_{\perp}^\alpha \gamma^\beta + S_{\perp}^\beta \gamma^\alpha - g_{\perp}^{\alpha\beta} S_{\perp} \right) \not{p} \gamma_5 D_2(x, y, z) \]
\[ + i\epsilon^{\alpha\beta\gamma\delta} p_\gamma n_\delta S_{\perp} \not{p} D_3(x, y, z) \] + ...
(9)

Working from the definitions 7a-7c, certain symmetry relations can be established,
\[ D_1(x, y, z) = D_1(z, y, x), \]
\[ D_3(x, y, z) = -D_3(z, y, x). \] (10)

Finally, the four quark matrix element in fig. 1d can be similarly analyzed. The twist-four transverse spin distributions are:
\[ W_1(x, y, z) = \frac{g^2}{2} \int [d\lambda \, d\mu \, d\nu] \langle PS_{\perp} | \bar{\psi}(0) \not{\gamma_5} \not{p} \psi(\nu n) \not{\gamma_5} \psi(\mu n) \not{\gamma_5} \not{S}_{\perp} \psi(\lambda n) | PS_{\perp} \rangle, \]
\[ W_2(x, y, z) = \frac{g^2}{2} \int [d\lambda \, d\mu \, d\nu] \langle PS_{\perp} | \bar{\psi}(0) \not{\gamma_5} \psi(\nu n) \not{\gamma_5} \psi(\mu n) \not{S}_{\perp} \psi(\lambda n) | PS_{\perp} \rangle. \] (11)

The four-quark density matrix, which is a matrix in two sets of Dirac indices, from which these can be projected is,
\[ M(x, y, z) = 2g^2 \Lambda^2 \int [d\lambda \, d\mu \, d\nu] \langle PS_{\perp} | \bar{\psi}(0) \psi(\nu n) \bar{\psi}(\mu n) \psi(\lambda n) | PS_{\perp} \rangle \]
\[ = \frac{1}{4} \Lambda^2 \left[ (\not{p})(\gamma_5 \not{S}_{\perp} \not{p}) W_1(x, y, z) + (\gamma_5 \not{p})(\not{p} \not{S}_{\perp}) W_2(x, y, z) \right] + ... \] (12)
At this point one needs to take stock of the situation. So far we have, using PCT, Lorentz invariance, and dimensional counting, identified all tensor structures needed to describe a transversely polarized nucleon at the twist-four level. However, we have further constraints available to us in the form of QCD equations of motion on the light-cone,

\[ i \frac{d}{d\lambda} \psi_-(\lambda n) = -\frac{1}{2} \not\! \! \! D \psi_+(\lambda n) \]

\[ \psi_\pm = \frac{1}{2} \gamma^\mp \gamma^\pm \psi \quad . \] (13)

This can be used to establish relations between distributions with three light-cone fractions with those having two and one, etc. A particularly useful set of relations, which shall be needed for establishing the electromagnetic gauge invariance of the Drell-Yan cross is,

\[ \int dz D_2(x, y, z) = -y(d_1(x, y) + d_2(x, y) + d_3(x, y) + d_4(x, y)) \]

\[ \int dz D_2(z, x, y) = x(d_1(x, y) + d_3(x, y) - d_2(x, y) - d_4(x, y)) \]

\[ \int dy dz D_2(y, x, z) = x^2 h_3(x) \quad . \] (14)

All distributions discussed so far are implicitly dependent on the renormalization scale. Their evolution through radiative processes, however, is a complicated issue and beyond the scope of this paper.

**III. THE TRANSVERSELY POLARIZED DRELL-YAN PROCESS**

We now consider application to the Drell-Yan production of lepton pairs in transversely polarized nucleon-nucleon collisions. Calculations beyond leading twist require considerable formal development, as in the work of Ellis, Furmanski, and Petronzio, and more recently by Qiu and Sterman, and Jaffe and Ji.\(^2\,^5\,^6\) A collinear expansion of the parton momenta is carried
Feynman diagrams are combined together to arrive at colour gauge invariance, etc. However, a simpler set of rules emerges from the formalism, which is summarized in ref. 3. These rules may be readily applied to the Drell-Yan process by first drawing the set of all diagrams which give rise to a lepton pair in the final state, and which do not contain more than four partons belonging to a single nucleon. The set of diagrams contributing up to twist-four are given in figs. 2-5. All diagrams of a given topology must be included, although only one has actually been shown in the figures.

The hadron tensor that appears in the inclusive Drell-Yan cross section can be written as,

\[ W^{\mu\nu} = \int dx dy (2\pi)^4 \delta^4 (Q - xP_A - yP_B) \Omega^{\mu\nu}(x, y), \]  

(15)

where \( x(>0) \) and \( y(>0) \) are the momentum fractions carried by quarks or antiquarks from the nucleon \( A \) and \( B \), respectively. For our purpose, we are interested in only the polarization-dependent part of the tensor. In the following discussion, we assume that a quark from \( A \) annihilates with an antiquark from \( B \). For the opposite case, an antiquark from \( A \) annihilating a quark from \( B \), is obtained by substituting \( x \to -x \) and \( -y \to x \).

To give an example of the application of the rules in ref. 3, we calculate the contribution from fig. 2a,

\[ \Omega^{\mu\nu}(x, y)|_{2a} = (-1) \text{Tr} \gamma^\nu M_A(x) \gamma^\mu M_B(-y). \]  

(16)

The factor \((-1)\) arises from the anti-commuting nature of the fermion fields, and an implicit trace over colour is understood. \( M_A(x) \) and \( M_B(-y) \) are the transverse-polarization-dependent part of the quark density matrix,

\[ M(x) = \frac{1}{2} \gamma_5 S_\perp \not p_1(x) + \frac{1}{2} \Lambda \gamma_5 S_\perp g_T(x) + \frac{1}{4} \Lambda^2 \not p_5 \gamma_5 S_\perp h_3(x). \]  

(17)
Working out the fermion and colour traces, we find

$$\Omega^{\mu\nu}(x,y)|_{2a} = \frac{1}{4} C_F \Lambda^2 (S_{\perp A} \cdot S_{\perp B}) \left[ P^\mu_B P^\nu_B h^B_1 (y) h^A_3 (x) + P^\mu_A P^\nu_A h^A_3 (-y) h^B_1 (x) \right] \frac{1}{P_A \cdot P_B}$$

$$- \frac{1}{4} C_F \Lambda^2 \left[ S^{\mu\nu} - S_{\perp A} \cdot S_{\perp B} \frac{P^\mu_A P^\nu_B + P^\nu_A P^\mu_B}{P_A \cdot P_B} \right] g^B_T (-y) g^A_T (x), \quad (18)$$

where

$$S^{\mu\nu} = S_{\perp A}^{\mu} S_{\perp B}^{\nu} + S_{\perp A}^{\nu} S_{\perp B}^{\mu} - g^{\mu\nu} S_{\perp A} \cdot S_{\perp B}, \quad (19)$$

The first term contains only the chiral-odd distributions and the second the chiral-even distributions. Quark helicity conservation for massless quarks dictates that the Drell-Yan cross section consists only of the combinations of chiral odd – chiral odd distributions or chiral even – chiral even distributions. $C_F = (N^2 - 1)/2N = 4/3$ is a colour factor for three colours.

Fig. 2b contains one gluon coming from the nucleon $A$, and the corresponding expression for $\Omega^{\mu\nu}$ is,

$$\Omega^{\mu\nu}(x,y)|_{2b} = (-1) \int dz \text{Tr} \left[ \gamma^\nu M^\alpha_A (z, x) \gamma^\mu M_B (-y) i\gamma_\alpha \frac{i}{(z-x)} \right] \left[ \frac{1}{P_A \cdot P_B} \right] \cdot (20)$$

where the twist-four part of the matrix $M_A(z, x)$ is given in eq. (5), and the twist-three part in eq. (43) in ref. 3. After a lengthy calculation, we find the contributions from all four diagrams having the fig. 2b topology,

$$\Omega^{\mu\nu}(x,y)|_{2b} = - \frac{1}{4} C_F \Lambda^2 (S_{\perp A} \cdot S_{\perp B}) \left( \frac{P^\mu_A P^\nu_B + P^\nu_A P^\mu_B}{P_A \cdot P_B} \right)$$

$$\times \left[ \frac{h^B_1 (-y)}{y} x h^A_3 (x) + \frac{h^A_1 (x)}{x} y h^B_3 (-y) \right]$$

$$+ \frac{1}{4} C_F \Lambda^2 (S_{\perp A} \cdot S_{\perp B}) g^{\mu\nu} \left[ \frac{h^B_1 (-y)}{y} \right] \int dw \frac{dz}{z} D^A_2 (x, z, w)$$

$$+ \frac{h^A_1 (x)}{x} \int dw \frac{dz}{z} D^B_2 (-y, -z, -w)$$

$$- \frac{1}{2} C_F \Lambda^2 (S_{\perp A} \cdot S_{\perp B}) \left[ \frac{x^2 P^\mu_A P^\nu_A + y^2 P^\nu_A P^\mu_B}{x y P_A \cdot P_B} \right] g^B_T (-y) g^A_T (x)$$

$$+ \frac{1}{2} C_F \Lambda^2 (S_{\perp A} \cdot S_{\perp B}) g^{\mu\nu} \left[ \frac{g^B_T (-y)}{x} \right] \int dz \frac{G^A_1 (x, z) - G^A_2 (x, z)}{x - z}$$

$$- \frac{g^A_1 (x)}{y - z} \int dz \frac{G^B_1 (-y, -z) - G^B_2 (-y, -z)}{y - z} \quad (21)$$
The first two terms in the above formula are chiral odd, and the second two are chiral even.

The transverse metric tensor has only $g_{11}^{T} = g_{22}^{T} = -1$ as non-zero components.

For the diagrams with two gluons from the same nucleon shown in figs. 3a and 3b, there are only chiral-odd contributions. Using the density matrix in eq. (9), we find,

\begin{equation}
\Omega^{\mu\nu}(x,y) \big|_{3a} = \frac{1}{4} C_F \Lambda^2 (S_{\perp A} \cdot S_{\perp B}) \left[ x^4 P_A^{\mu} P_B^{\nu} h_3^{B}(-y) h_1^{A}(x) \right.
\end{equation}

\begin{equation}
+ y^4 P_B^{\mu} P_A^{\nu} h_3^{B}(-y) h_1^{A}(x) \left. \right] \frac{1}{x^2 y^2 P_A \cdot P_B},
\end{equation}

\begin{equation}
\Omega^{\mu\nu}(x,y) \big|_{3b} = -\frac{1}{4} C_F \Lambda^2 S^{\mu\nu} \left[ \frac{h_1^B(-y)}{y} \int dx_1 dx_2 \frac{D_1^A(x,x_2,x_1) + D_3^A(x,x_2,x_1)}{x-x_1} \right.
\end{equation}

\begin{equation}
+ \frac{h_1^A(x)}{x} \int dx_1 dx_2 \frac{D_1^B(-y,-x_2,-x_1) + D_3^B(-y,-x_2,-x_1)}{y-x_1} \bigg].
\end{equation}

In eq. (22), we have made use of eq. (14c). Clearly, the chiral-odd longitudinal part from eq. (22), combined with these from eqs. (18) and (21) is electromagnetically gauge invariant.

There are four diagrams in which one gluon comes from $A$ and the other comes from $B$. These diagrams contribute only to chiral-even part of the cross section. Our calculation shows,

\begin{equation}
\Omega^{\mu\nu}(x,y) \big|_{4a} = \frac{1}{4} C_F \Lambda^2 (S_{\perp A} \cdot S_{\perp B}) \frac{P_A^{\mu} P_B^{\nu} + P_B^{\mu} P_A^{\nu}}{P_A \cdot P_B} g_T^{B}(-y) g_T^{A}(x)
\end{equation}

\begin{equation}
\Omega^{\mu\nu}(x,y) \big|_{4b} = C_F \Lambda^2 (S_{\perp A} \cdot S_{\perp B}) g_T^{\mu\nu} \int \frac{dz \, dw}{z(w-y)} \left[ G_1^A(x,w) - G_2^A(x,w) \right]
\end{equation}

\begin{equation}
\times \left[ G_1^B(-z,-y) - G_2^B(-z,-y) \right] + (x \leftrightarrow -y, A \leftrightarrow B)
\end{equation}

\begin{equation}
- \frac{1}{2} C_A \Lambda^2 (S_{\perp A} \cdot S_{\perp B}) g_T^{\mu\nu} \int \frac{dz \, dw}{z(w-x)} \left[ \tilde{G}_1^A(x,w) - \tilde{G}_2^A(x,w) \right]
\end{equation}

\begin{equation}
\times \left[ \tilde{G}_1^B(-z,-y) - \tilde{G}_2^B(-z,-y) \right] + (x \leftrightarrow -y, A \leftrightarrow B)
\end{equation}

where $C_A = 3$. The second term in eq. (25) requires some explanation: $\tilde{G}_i(x,w)$ is defined exactly as $G_i(x,w)$ as in eq. 38 of ref. 3, except that the covariant derivative has only the gluon potential. This is because the partial derivative coming from transverse momentum
expansion of fig. 2a does not contribute to terms multiplied by $C_A$. Since the final result has to be gauge invariant, we replace the $A^\perp$ by $\imath n^- F^+ / (w - x)$, and thus, $\tilde{G}_1(x, w)$ is manifestly gauge invariant. And,

$$\Omega^{\mu\nu}(x, y)|_{4c} = C_F \Lambda^2 S^{\mu\nu} \int \frac{dz}{z} \frac{dw}{w} [G^A_1(x, w) + G^A_2(x, w)]$$

$$\times \left[ G^B_1(-y, -z) + G^B_2(-y, -z) \right]$$

$$- \frac{1}{2} C_A \Lambda^2 S^{\mu\nu} \int \frac{dz}{z} \frac{dw}{w} \left[ \tilde{G}_1^A(x, w) + \tilde{G}_2^A(x, w) \right]$$

$$\times \left[ \tilde{G}_1^B(-y, -z) + \tilde{G}_2^B(-y, -z) \right]$$

$$\Omega^{\mu\nu}(x, y)|_{4d} = - C_A \Lambda^2 (S_{\perp A} \cdot S_{\perp B}) g_T^{\mu\nu} \int \frac{dz}{z - y} \frac{dw}{w - x} \left[ \tilde{G}_1^A(x, w) \tilde{G}_1^B(-y, -z) \right.$$}

$$- \tilde{G}_2^A(x, w) \tilde{G}_2^B(-y, -z) \right] \quad (26)$$

Again, it is simple to see that the chiral-even longitudinal part in eq. (24), combining with these from eqs. (18) and (21) is electromagnetically gauge invariant.

Finally, we consider the four-quark diagram shown in fig. 5. Evaluation requires a double trace as there are two quark loops. This is straightforwardly done, and using eq. (12), we get:

$$\Omega^{\mu\nu}(x, y)|_5 = - \frac{1}{4} C_F \Lambda^2 S^{\mu\nu} h^B_1(-y) \int \frac{dz}{z - x} \frac{dw}{w - x} \sum_{i=1}^{2} \left[ W^A_i(x, z, w) \right.$$}

$$+ W^A_i(x - z, -z, w - z) - W^A_i(x, z, -w) - W^A_i(x - z, -z, -w) \right)$$

$$- \frac{1}{4} C_F \Lambda^2 S^{\mu\nu} h^A_1(x) \int \frac{dz}{z - y} \frac{dw}{w - y} \sum_{i=1}^{2} \left[ W^A_i(-y, -z, -z + w) \right.$$}

$$+ W^A_i(x - z, -z, w - z) - W^A_i(-y, -z, -w) \right.$$}

$$- W^A_i(-y + z, z, -w + z) \right] \quad (28)$$

The different terms in eq. (28) comes from eight different diagrams having the same topology as fig. 5.
It may be readily verified that our final result for $W^{\mu\nu}$, which is given by the sum of all contributions from diagrams 2-5, is electromagnetically gauge invariant: $q^\mu W_{\mu\nu} = W_{\mu\nu} q^\nu = 0$. This is an important partial check on the correctness of the calculation.

IV. SUMMARY

In this paper, we introduce the polarization-dependent twist-four distributions in a transversely polarized nucleon. The most general ones contain three light-cone momentum fractions and are $D_i$ and $W_i$ defined in eqs. (7) and (11). These distributions, together with the twist-two and twist-three distributions defined for the same nucleon state, form a complete set for describing any hard scattering processes involving traverse polarization.

As an example, we have calculated the twist-four correction to the Drell-Yan process. The final result is gauge invariant in both electromagnetic and colour interactions. For lack of information on these distributions, we cannot access numerical significance of the correction other than simple dimensional analysis. However, our formulas can be coupled with model calculations of the distributions to give a detailed prediction.
REFERENCES

1) J. Ralston and D.E. Soper, *Nucl. Phys.* **B152**, 109, (1979).

   X. Artu and M. Mekhfi, *Z. Phys.* **C45**, 669, (1990).

2) R.L. Jaffe and X. Ji, *Phys. Rev. Letts.* **67**, 552, (1991).

   R.L. Jaffe and X. Ji, *Nucl. Phys.* **B375**, 527, (1992).

3) X. Ji, MIT-CTP preprint #2141, (1992).

4) G. Bunce et al, *Part. World* **3**, 1, (1992);

   X. Ji, *Phys. Lett.* **284**, 137, (1992).

5) R.K. Ellis, W. Furmanski, and R. Petronzio, *Nucl. Phys.* **B212**, 29, (1983).

6) J. Qiu and G. Sterman, *Nucl. Phys.* **B353**, 105, (1991).
FIGURE CAPTIONS

For a hard copy of the figures, please send email to ereidell@marie.mit.edu

Fig. 1: Parton density matrices which can contribute to a hard process at the twist-four level.

Fig. 2: A quark from hadron A annihilates with an antiquark from hadron B leading to a large mass virtual photon. Other diagrams of this type are obtained by allowing the gluons in (b) to interact with the quark line on the right, and interchanging A and B.

Fig. 3: Representative diagrams which give chiral odd contributions to the twist-four Drell-Yan cross section.

Fig. 4: Diagrams involving one gluon from each nucleon which give chiral even contribution to the Drell-Yan cross section.

Fig. 5: Four quark diagram contributing to the Drell-Yan twist-four cross section.