Situation-based Real-time Assignment of Air Combat Command and Guidance Targets

Hanghang You, Qisong Han and Haiyan Yang
Air Traffic Control And Navigation College, Air Force Engineering University, No.1 Jia zi, Changle east road, baqiao district, Xi’ an city, Shaanxi province
Email: yhang0225@163.com

Abstract. In order to solve the problem of lack of real-time allocation in traditional air combat command and guidance target allocation, this paper proposes a new model. Firstly, this paper establishes a target allocation model, discusses the situation under different situation conditions, and quantifies the situation of over-the-horizon air combat reasonably. On this basis, inspired by the algorithm of Traveling Salesman Problem (TSP), the 0-1 programming model is established to calculate the target allocation. Finally, a simulation scenario calculation is used to verify the correctness of the model and the algorithm, and the rationality of the results is analyzed from the qualitative analysis.

1. Introduction
Target assignment is an important issue in firepower application and air combat decision-making. Its core is how to assign weapons with different killing and effectiveness to different targets and establish an overall optimized firepower strike system for combat effectiveness. Foreign countries have shown great interest in this issue since the 1950s. Kuhn H W [1] proposed an algorithm for solving assignment problem-Hungarian algorithm. This algorithm is suitable for solving the balanced target assignment problem, and it is very difficult to solve the unbalanced target assignment problem. Schmaedeke [2] proposed a target allocation method based on linear programming, which has a faster solution speed. The disadvantage is that when describing complex problems, there is a big gap with real problems. A great deal of research has also been carried out in China. Luo Wenhui of Air Force Engineering University [3] proposed a target allocation method based on improved simulated annealing algorithm in order to solve the target allocation problem in air defense combat missions. Xiao Jinke and others [4] solved the problem of large-scale regional anti-missile target allocation by using artificial immune algorithm. In this paper, aiming at the problem that most researches only focus on the target allocation before the mission, the air combat situation is solved, and the real-time command and guidance target allocation is carried out based on the situation.

2. Air-To-Air Missile Attack Area

2.1 Definition of Air Combat Target Assignment
Target assignment refers to scientifically assigning enemy planes to our planes according to the known degree of mutual threat between enemy planes and enemy planes, that is, according to the results of scientific and comprehensive air combat situation assessment, so as to achieve the best effect of attacking enemy planes or to enable our planes to be better protected. According to the quantity of enemy-friend confrontation, target allocation can be divided into balanced allocation and unbalanced allocation, as shown in Figures 1 and 2.
2.2 Scientific Assumptions
(1) In order to ensure that the result of target allocation does not appear "jump", this paper only considers the result of target allocation under the current situation.
(2) A fighter plane can only attack one target at a time. When the number of our planes is less than the number of enemy planes, we need to allocate more than one enemy plane. This can be seen as the result of multiple target allocations, and the interval between target allocations can be ignored.

3. Air Combat Command and Guidance Target Assignment Modeling

3.1 The Relationship between Air Combat Command and Guidance Target Assignment Problem and Traveling Salesman Problem
Traveling Salesman Problem (TSP) \([5]\), often referred to as Traveling Saleman Problem, refers to how a salesman finds the shortest path to return to the starting point after visiting each place when visiting multiple places. This kind of problem can be represented by a weighted directed graph \(G = (N, A)\), where \(N\) is a set with \(n = |N|\) points and is a set that completely connects these points. Each edge \((i, j) \in A\) has a weight, which represents the distance between city \(i\) and city \(j\). TSP problem is to find the shortest Hamiltonian path in the graph.

For the multi-target assignment problem of over-the-horizon multi-aircraft cooperative operation, let's assume that we have \(n\) fighters and the enemy has \(m\) fighters, and take the advantage index of the \(i\) fighter against the \(j\) fighter as the weight \(d_{ij}\) in the traveling salesman problem, assuming that only one target is assigned each time the target is assigned.
The target allocation problem is that every time the target allocation calculation is carried out, our \(n\) fighter planes choose a different enemy plane from the enemy’s \(m\) fighter planes, so that the sum of the superiority index \(d_{ij}\) of our fighter planes relative to the enemy fighter planes is maximized, i.e. the benefit is maximized. If the number of our fighters is not equal to the number of enemy fighters, i.e. \(n \neq m\), then the target allocation problem of over-the-horizon multi-aircraft coordination can be regarded as TSP problem for merchants whose traversal scale is \(\max(n, m)\) cities. Among them, if some targets are not assigned to us or our fighters are not assigned to targets due to the unequal number of fighters between the enemy and ourselves, their dominance index function should be regarded as 0, i.e. the weight \(d_{ij}\) in the traveling salesman should be regarded as 0.

3.2 Mathematical Description of Air Combat Command and Guidance Target Assignment Problem
Assuming that we have \(k\) fighters and the enemy has \(l\) fighters, the over-the-horizon dominance function matrix of our fighters against the enemy can be described as:

![Figure 1. Schematic diagram of balanced target allocation](image1)

![Figure 2. Schematic diagram of unbalanced target allocation](image2)
Where $S_{ij}$ indicates the superiority index of the $i(1, 2, 3, 4, \ldots, l)$ fighter to the $j(1, 2, 3, 4, \ldots, l)$ fighter of the enemy. The greater the value of $S_{ij}$, the greater the advantage of our $J$-th fighter plane to the enemy's $J$-th fighter plane. Similarly, by analogy with the calculation method of the advantage matrix, the threat matrix $S_{ij}^T$ ($S_{ij}^T$ enemy planes to our plane can be calculated (the greater the value of FF, the greater the threat of enemy $J$-th fighter plane to our $I$-th fighter plane).

The target allocation matrix can be expressed as:

$$
X_j = \begin{bmatrix} x_{i1} & x_{i2} & \cdots & x_{iL} \\
\vdots & \vdots & & \vdots \\
x_{i1} & x_{i2} & \cdots & x_{iL} \\
\end{bmatrix}
$$

(2)

Where $x_{ij}$ is 0 or 1, 0 means that my $I$-th fighter plane will not attack the $J$-th enemy plane, and 1 means that my $I$-th fighter plane will attack the $J$-th enemy plane or specifically avoid this enemy plane. Among them, $i = 1, 2, 3, 4, \ldots, k$, $j = 1, 2, 3, 4, \ldots, l$.

The target's assigned effectiveness value is $J = \sum_{j=0}^{k} \sum_{j=0}^{L} S_{ij}^A x_{ij}$ and the enemy's threat value to us is $M = \sum_{j=0}^{k} \sum_{j=0}^{L} S_{ij}^T x_{ij}$.

When the situation is generally good or the commander's intention is to attack, the mathematical description of target allocation is as follows:

Objective function $J_{\text{max}} = \sum_{i=0}^{k} \sum_{j=0}^{L} S_{ij} x_{ij}$. Constraint $\sum_{j=0}^{L} X_{ij} \leq N_j$.

$N_j$ Takes the minimum of the number of medium-range missiles and the number of targets that our machine can track and lock at the same time.

When the situation is poor or the commander's intention is defense, the mathematical description of target allocation is as follows:

Objective function $M_{\text{max}} = \sum_{j=0}^{k} \sum_{j=0}^{L} S_{ij}^T x_{ij}$. Constraint $\sum_{j=0}^{L} X_{ij} \leq N_j$.

Then the value of $x_{ij}$ when the objective function reaches the maximum is the result of the objective allocation.

3.3 Design of Air Combat Command and Guidance Target Assignment Algorithm under Beyond Visual Range Condition

Let's assume that when we send $L$ fighters to intercept and attack the enemy $K$ fighters.

(1) The first step is to calculate the superiority index of our fighter against the enemy fighter according to the model proposed in document [6] and the actual situation, and establish the superiority index matrix $S_{ij}^A (1 \leq i \leq L, 1 \leq j \leq K)$.

(2) In the second step, if $(L \leq K)$, that is, the number of our fighters is less than the number of enemy fighters, assume that we will dispatch $K$ fighters and set the advantage index of the $(K-L)$ fighters against the enemy as 0. Establish the following dominance function matrix:
If \( L > K \), that is, when the number of our planes is larger than the number of enemy planes, there are \( L \) fighter planes in imaginary enemy, and the superiority index of our planes relative to this hypothetical \((L-K)\) fighter plane is set to 0. Establish the following advantage matrix:

\[
S_y = \begin{pmatrix}
    s_{11} & s_{12} & \cdots & s_{1K} \\
    \vdots & \vdots & \ddots & \vdots \\
    s_{L1} & s_{L2} & \cdots & s_{LK} \\
    0 & 0 & \cdots & 0 \\
    \vdots & \vdots & \ddots & \vdots \\
    s_{S1} & s_{S2} & \cdots & s_{SK}
\end{pmatrix}
\]  

(3)

(3) In the third step, using lingo or other software for calculation, the results of the first target assignment calculation can be obtained, and the situation that the enemy's \( L \) fighter planes are assigned to our planes can be obtained:

Objective function: \[
\max \quad J = \sum_{i=0}^{K} \sum_{j=0}^{K} S_{ij}x_{ij}
\]

The constraints are as follows:

I: When we only assign one target at a time: \( \sum_{j=0}^{K} x_{ij} = 1 \)

II: When enemy fighters are assigned to only one fighter at a time: \( \sum_{i=0}^{K} x_{ij} = 1 \)

III: other circumstances \( x_{ij} = 0 \) or 1   Record the value of \( x_{ij} \);

(4) The fourth step is to revise the dominance index matrix.

When \( L \leq K \), reduce the dominance index on the column where the \( L \) enemy planes have been assigned to 0.

When \( L > K \), reduce the dominance index of the assigned \( k \) planes to 0.

(5) Step 5, the modified dominance index matrix is brought into the calculation step of Step 3 to perform the second distribution calculation. That is, the third and fourth steps of the cycle will be repeated until all enemy planes have been allocated or our fighters have been allocated.

When the number of our planes is less than or equal to the number of enemy planes, the number of cycles to be counted is:

If \( \frac{K}{L} \) is an integer, \( \frac{K}{L} \) is taken; If \( \frac{K}{L} \) is not an integer, \( \left[ \frac{K}{L} \right] + 1 \) is taken.

When the number of our planes is greater than the number of enemy planes, the number of times to cycle is:

If \( \frac{L}{K} \) is an integer, \( \frac{L}{K} \) is taken; If \( \frac{L}{K} \) is not an integer, \( \left[ \frac{L}{K} \right] + 1 \) is taken.

4. Example Analysis

4.1 Overview of Simulation

The war area is over the Taiwan Strait. The climate is general and the weather conditions are sunny. Suppose the system reliability coefficient is 1.

Let's assume that our formation is a cruise formation composed of four fighters, all of which are single-seat fighters. The fighters include two A and two B fighters each. The numbers of the four fighters
are a1, a2, b1 and b2. The pilots driving the four fighters are A1, A2, B1 and B2. The task of the four fighters is to perform normal patrol tasks over the Taiwan Strait. Our fighters have multi-target attack capability, can identify and track four batches of targets at the same time, and carry four air-to-air missiles, including two near-range air-to-air missiles, two medium-range air-to-air missiles, the medium-range air-to-air missiles carried by Type A fighters are M-type, and the medium-range air-to-air missiles carried by Type B fighters are N-type.

The enemy dispatched six single-seat fighters to intercept our formation, two of which were C, D and E fighter planes, with their numbers c1, c2, d1, d2, e1 and e2. The pilots of the six fighters were C1, C2, D1, D2, E1 and E2, and also each carried four air-to-air missiles, including two close-range air-to-air missiles, two medium-range air-to-air missiles, two medium-range air-to-air missiles, the type of medium-range air-to-air missiles carried by C fighter planes was K, the type of medium-range air-to-air missiles carried by D fighter planes was L, and the type of medium-range air-to-air missiles carried by E fighter planes were L.

### 4.2 Simulation Process and Analysis

The first step is to establish the over-the-horizon dominance matrix of our fighters against enemy fighters based on the over-the-horizon air combat situation data results:

\[
S'_{ij} = \begin{bmatrix}
0.607 & 0.574 & 0.653 & 0.645 & 0.797 & 0.684 \\
0.594 & 0.562 & 0.638 & 0.630 & 0.785 & 0.752 \\
0.473 & 0.385 & 0.556 & 0.548 & 0.573 & 0.564 \\
0.494 & 0.385 & 0.556 & 0.548 & 0.573 & 0.564 \\
0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\
0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000
\end{bmatrix}
\]

(5)

The target allocation matrix is:

\[
X_y = \begin{bmatrix}
x_{11} & x_{12} & x_{13} & x_{14} & x_{15} & x_{16} \\
x_{21} & x_{22} & x_{23} & x_{24} & x_{25} & x_{26} \\
x_{31} & x_{32} & x_{33} & x_{34} & x_{35} & x_{36} \\
x_{41} & x_{42} & x_{43} & x_{44} & x_{45} & x_{46}
\end{bmatrix}
\]

(6)

Then the objective function

\[
J_{\text{max}} = \sum_{i=0}^{4} \sum_{j=0}^{6} S_{ij} x_{ij}
\]

Constraint condition \(\sum_{j=0}^{6} X_{ij} \leq 2\)

In the second step, the number of our fighters is 4 and the number of the enemy is 6. At this moment, assuming that we have dispatched 6 fighters, we will set the hypothetical superiority index of these two fighters to the enemy as 0 and establish the following superiority matrix:

\[
X_y = \begin{bmatrix}
0.607 & 0.574 & 0.653 & 0.645 & 0.797 & 0.684 \\
0.594 & 0.562 & 0.638 & 0.630 & 0.785 & 0.752 \\
0.473 & 0.385 & 0.556 & 0.548 & 0.573 & 0.564 \\
0.494 & 0.385 & 0.556 & 0.548 & 0.573 & 0.564 \\
0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\
0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000
\end{bmatrix}
\]

(7)

Similarly, the target allocation matrix is extended to a 6*6 0-1 matrix.

The third step is to use lingo software to carry out calculation:

The objective function: \(\max J = \sum_{i=0}^{6} \sum_{j=0}^{6} S_{ij} x_{ij}\), Constraint condition:

\[
\begin{align*}
\sum_{i=0}^{6} x_{ij} &= 1 \\
\sum_{j=0}^{6} x_{ij} &= 1 \\
x_{ij} &= 0 \text{or } 1
\end{align*}
\]

Record the \(x_{ij}\) value of our four fighter planes on the line, and the result is:

\(x_{15} = x_{26} = x_{34} = x_{43} = 1\)
The fourth step is to revise the dominance matrix. The advantage in the column of enemy fighters assigned to our planes will only drop to 0.

In the fifth step, the modified dominance matrix will be brought into the calculation process in the third step, and the second distribution calculation will be carried out, which requires two cycles. Record \( x \) on enemy's remaining C1 and C2 columns: \( x_{11} = x_{22} = 1 \)

### 4.3 Performance Analysis and Comparison of Algorithms

From the perspective of modeling ideas, advantages and disadvantages, and single calculation time, the model in this paper is compared with the models in documents 7 and 8. The comparison is shown in Table 1.

### Table 1. Comparative analysis of models

| Model source          | Modeling ideas                                                                 | Advantages and disadvantages                                                                 | The time (in seconds) to calculate once |
|-----------------------|--------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------|----------------------------------------|
| This model            | Based on the real-time situation between the enemy and ourselves, the principle of TSO algorithm is applied | The model is comprehensive and real-time                                                        | 0.657                                  |
| The model in document 7 | The algorithm is affected by the allocation scale. The scale boundary is not clear | The dynamic processing of the number of fighter planes is good. Lack of enemy threat analysis  | 0.752                                  |
| The model in document 8 | In order to optimize the expectation of damaging the enemy, Lack of consideration of threat from one's own side | The model does not fit well with actual combat.                                              | 0.689                                  |

### 5. Conclusion

1. The air combat command and guidance target allocation model based on TSP algorithm is based on the air combat real-time enemy-friend situation, which has good real-time performance and is more in line with the actual operational requirements.
2. Compared with the traditional model, this model has lower computational complexity and better engineering application value.

### 6. References

[1] Kuhn H W. The Hungarian method for the assignment problem [J]. Naval research logistics quarterly, 1955, 2(1-2): 83-97.
[2] Schmaedeke W. Information based Sensor Management[C]. Proc, SPIE.1955, signal Proceeing, Sensor Fusion, and Target Reclgnition, 1993.
[3] LUO Wenhui, LIU Shaowei, ZHOU Jianmei, et al Research on simulated annealing algorithm to solving target assignment problem[J].Aerospace shanghai, 2008,1(17):61-64.
[4] XIAO Jinke, WANG Gang, LI Weimin, et al. Optimization on target assignment model in theater anti-missile system[J]. Systems Engineering Theory & Practice, 2015, 35(04):1027-1034.
[5] ZHANG Hongda, ZHNENG Quandi. Simulation and Research of TSP Based on Ant Colony Algorithm [J]. Aeronautical Computer Technique, 2005(04):103-106.
[6] SHI Zhenqing, LIANG Xiaolong, ZHANG Jiaqiang, LIU Liu. Situation Assessment for Air Combat Based on Missile Attack Zone [J]. Control & Command Control, 2018, 43(09):89-93+98.
[7] GAO Yangyang, YU Minjian, YANG Jie. Research on the method of unbalanced target allocation
in multi-aircraft air combat command and Guidance [J]. Advances in aeronautical science and engineering, 2018, 9(04):523-529.

[8] ZHU Defa, SHAN Lianping, GUAN Yingying. Multi-fighters cooperative target assignment based on improved PSO [J]. Control & Command Control, 2015, 40(08):38-41.