Experimental determination of the lateral dose response functions of detectors to be applied in the measurement of narrow photon-beam dose profiles

D Poppinga¹, J Meyners¹, B Delfs¹, A Muru¹, D Harder², B Poppe¹ and HK Looe¹

¹ University Clinic for Medical Radiation Physics, Medical Campus Pius-Hospital, Carl von Ossietzky University, Oldenburg, Germany
² Medical Physics and Biophysics, Georg August University, Göttingen, Germany

E-mail: daniela.poppinga@uni-oldenburg.de

Received 3 June 2015, revised 21 September 2015
Accepted for publication 2 October 2015
Published 19 November 2015

Abstract
This study aims at the experimental determination of the detector-specific 1D lateral dose response function \( K(x) \) and of its associated rotational symmetric counterpart \( K(r) \) for a set of high-resolution detectors presently used in narrow-beam photon dosimetry. A combination of slit-beam, radiochromic film, and deconvolution techniques served to accomplish this task for four detectors with diameters of their sensitive volumes ranging from 1 to 2.2 mm. The particular aim of the experiment was to examine the existence of significant negative portions of some of these response functions predicted by a recent Monte-Carlo-simulation (Looe et al 2015 Phys. Med. Biol. 60 6585–607).

In a 6 MV photon slit beam formed by the Siemens Artiste collimation system and a 0.5 mm wide slit between 10 cm thick lead blocks serving as the tertiary collimator, the true cross-beam dose profile \( D(x) \) at 3 cm depth in a large water phantom was measured with radiochromic film EBT3, and the detector-affected cross-beam signal profiles \( M(x) \) were recorded with a silicon diode, a synthetic diamond detector, a miniaturized scintillation detector, and a small ionization chamber. For each detector, the deconvolution of the convolution integral \( M(x) = K(x) \ast D(x) \) served to obtain its specific 1D lateral dose response function \( K(x) \), and \( K(r) \) was calculated from it. Fourier transformations and back transformations were performed using function approximations by weighted sums of Gaussian functions and their analytical transformation.
The 1D lateral dose response functions $K(x)$ of the four types of detectors and their associated rotational symmetric counterparts $K(r)$ were obtained. Significant negative curve portions of $K(x)$ and $K(r)$ were observed in the case of the silicon diode and the diamond detector, confirming the Monte-Carlo-based prediction (Looe et al. 2015 Phys. Med. Biol. 60 6585–607). They are typical for the perturbation of the secondary electron field by a detector with enhanced electron density compared with the surrounding water. In the cases of the scintillation detector and the small ionization chamber, the negative curve portions of $K(x)$ practically vanish. It is planned to use the measured functions $K(x)$ and $K(r)$ to deconvolve clinical narrow-beam signal profiles and to correct the output factor values obtained with various high-resolution detectors.

Keywords: small fields, response function, volume effect, convolution kernel, dosimetry

(Some figures may appear in colour only in the online journal)

1. Introduction

The implementation of advanced radiation treatment techniques such as VMAT and step and shoot IMRT and the use of special treatment devices like Cyberknife, Tomotherapy, or microMLCs necessitate accurate dose measurements at small field sizes. Sophisticated dosimetry techniques are indispensable to safely determine reliable dose distribution characteristics. This requirement comprises the determination of output factors as well as the measurement of accurate transverse dose profiles.

The appropriate high-resolution detectors with continuous output signals are either silicon diodes, diamond detectors, miniaturized scintillation detectors, or small air-filled ionization chambers. However, the indicated values from all of these detectors are subjected to various effects that impede the accuracy of the dose measurement. All detectors show the volume-averaging effect; i.e. the detector signal associated with the point of measurement is made up of contributions from all parts of the sensitive volume of the detector (Capote et al. 2004, Crop et al. 2009, Looe et al. 2013, Beierholm et al. 2014). In addition, each detector, due to the release and transport of secondary electrons in all of its constructional parts and to its replacement of a volume of water, contributes to a perturbation of the secondary electron field compared with that at the point of measurement in the undisturbed water phantom (Scott et al. 2012, Fenwick et al. 2013, Kamio and Bouchard 2014, Looe et al. 2015). For short-hand notation, this combination of the volume averaging and the secondary electron field perturbation effects has been termed the volume effect of a detector (Herrup et al. 2005, Ketelhut and Kapsch 2015, Looe et al. 2015).

One possible approach to quantify and correct the volume effect of a small detector is the Monte Carlo simulation of the correct dose at the point of measurement and of the deviating indicated value yielded by the detector. A correction factor $k_{\text{volcorr}}$ was proposed by Alfonso et al. (2008) and has been numerically determined using appropriate accelerator and detector models (Francescon et al. 2011, 2012, Underwood et al. 2013, Benmakhlouf et al. 2014, Papaconstantopoulos et al. 2014). Alternatively, volume effect correction factors for output measurements have been derived experimentally, e.g. using reference detectors with vanishing volume effect such as alanine probes or EBT3 film (Sauer and Wilbert 2007, Pantelis et al. 2012, Bassinet et al. 2013, Ralston et al. 2012, 2014, Azangwe et al. 2014, Underwood et al. 2015).
However, such computational and experimental approaches merely yield discrete numerical values of the volume effect correction factor under defined conditions and for specified points of interest, particularly the maxima of transverse dose profiles, i.e. for the output factors. There is a demand for a more general, theoretical approach to the volume effect correction, providing the physical explanation for the deformation of dose profiles owed to the chosen kind of detector, as well as developing predictive power for a wide range of conditions as they may occur in clinical practice. In a chain of investigations, probably starting with the studies of Pychlau (1979) and Brahme (1981) on the disturbance of measured dose profiles by the finite sizes of ionization chambers, the ‘convolution model’ has stepwise evolved as an adequate theoretical description of the volume effect (Charland et al 1998, García-Vicente et al 1998, García-Vicente et al 2000, van’t Veld et al 2000, 2001, Laub and Wong 2003, Pappas et al 2006, Looe et al 2013, Looe et al 2015).

In its present state of development, summarized by van’t Veld et al (2001) and in the notation proposed by Looe et al (2013), Harder et al (2014), and Looe et al (2015), the ‘convolution model’ can briefly be summarized as follows: The relationship between the indicated signal profile \( M(x, y) \) of the detector and the undisturbed true dose profile \( D(x, y) \) in an \( x, y \) plane at right angles with the beam axis in a water phantom is described by the 2D convolution integral

\[
M(x, y) = K(x, y) \ast D(x, y) \tag{1}
\]

The convolution kernel \( K(x, y) \) is called the lateral dose response function of the detector. If the detector has been calibrated to indicate by its signal the absorbed dose to water at the point of measurement in a sufficiently wide homogeneous photon field, kernel \( K(x, y) \) can be shown to be area normalized (Looe et al 2015). Equation (1) is the basis for the possibility to obtain the true dose profile \( D(x, y) \) from the perturbed signal profile \( M(x, y) \) by deconvolution if the convolution kernel \( K(x, y) \) associated with the applied detector is known. If the 1D case is of interest, equation (1) can be simplified as

\[
M(x) = K(x) \ast D(x) \quad \tag{2}
\]

where \( K(x) \) is the projection of \( K(x, y) \) in the \( y \) direction,

\[
K(x) = \int_{-\infty}^{\infty} K(x, y) dy \quad \tag{3}
\]

If \( K(x, y) \) has rotational symmetry, the associated rotational symmetric 2D convolution kernel \( K(r) \) can be deduced from \( K(x) \) by the Hankel transformation (Sneddon 1955, Marchand 1964).

Equation (2) is the third one in a set of three typical convolutions (van’t Veld et al 2001). The first of these is the convolution of the photon fluence profile \( \Phi(x) \) with kernel \( K_M(x) \), yielding the detector signal profile \( M(x) \). The second is the convolution of the photon fluence profile \( \Phi(x) \) with kernel \( K_D(x) \), yielding the true dose profile in the undisturbed water phantom, \( D(x) \). A consequence of this triple of convolutions is the generally applicable relationship between the three convolution kernels,

\[
K_M(x) = K(x) \ast K_D(x) \tag{4}
\]

(Looe et al 2013, 2015). According to equation (4) \( K(x) \) can be obtained by deconvolution if \( K_M(x) \) and \( K_D(x) \) are known, e.g. from Monte-Carlo simulations or by measurement.

In a series of studies, kernel \( K(x) \) has been evaluated for ionization chambers. Various methods of deconvolution, such as Fourier deconvolution (Bednartz et al 2002, Herrup et al 2005), the use of the inverse convolution kernel (Ulmer and Kaissl 2003) and iterative
methods (Looe et al 2013) have been applied. With a focus on the volume-averaging effect, Higgins (1995), Chang (1996), García-Vicente et al (1998), and Sibata et al (1991) approximated the lateral dose response function $K(x)$ of ionization chambers as a symmetrical, downward-curved function of elliptical shape. Subsequent studies showed that the dose response functions of ionization chambers approximately resemble Gaussian functions (García-Vicente et al 2000, Bednarz et al 2002, Ulmer and Kaissl 2003, Feng 2006, Sahoo et al 2008, Yan et al 2008, Fox et al 2010, Looe et al 2012, 2013). With even higher accuracy of the Monte-Carlo simulations and measurements it was found that the lateral dose response functions of cylindrical ionization chambers have sharp peaks at the wall and central electrode due to increased photon interactions at these components (van’t Veld et al 2000, 2001, Gonzalez-Castaño et al 2012, Looe et al 2013). $K(x)$ functions for silicon diodes were experimentally determined by García-Vicente et al (1998) and Djouguela et al (2008). Since the slit-beam method is known for its difficulties (van’t Veld et al 2000, 2001), Ketelhut and Kapsch (2015) used the edge method for the experimental determination of kernel $K_M(x)$ for ionization chambers and a silicon diode. The observed over-response of semiconductor detectors in small photon fields does not concur with simple Gaussian lateral dose response functions (Scott et al 2012, Fenwick et al 2013).

The task of the present study evolved from the Monte-Carlo-based observation by Looe et al (2015), whereby, in addition to the diameter of the detector, the enhanced density of some detector materials such as silicon and diamond exceeding that of gas-filled ionization chambers by three orders of magnitude, strongly influences the curve shape of $K(x)$. Figure 1 illustrates the influence of the density of the detector material on the shape of the detector’s lateral dose response function $K(x)$ in the Monte Carlo modeled geometry of a flat circular detector surrounded by water and scanned by a photon slit beam. Kernels $K_{M}(x)$ and $K_{D}(x)$ were here obtained by Monte-Carlo simulation, and $K(x)$ was derived from them by the deconvolution of equation (4). In the modeled case of a detector filled with water of reduced density (case a), lateral disequilibrium between the inward and outward-directed secondary electron transport occurs in the sense that the inward electron transport is enhanced compared with the case of lateral equilibrium (case b). This is illustrated by the relatively large values of $K_{M}(x)$ outside the geometric borderline of the detector. Due to equation (4), convolution kernel $K(x)$ is now positive outside the borderline. However, in the opposite case of a detector filled with water of enhanced density (case c), lateral disequilibrium between the inward and outward-directed secondary electron transport occurs in the sense that the inward transport of secondary electrons is reduced, as illustrated by the relatively low values of $K_{D}(x)$ outside the geometric borderline of the detector. Due to equation (4), convolution kernel $K(x)$ is now negative outside the borderline of the detector. It is evident from the convolution relationships, equations (1) and (2), and as has been numerically shown by Looe et al (2015) that in the case of downward curved true dose profiles, positive values of $K(x)$ outside the borderline will lead to an underresponse, while negative values of $K(x)$ outside the borderline will lead to an overresponse of a detector placed in the maximum of the dose profile, e.g. for output factor measurement.

In consideration of these hitherto unknown, theoretically predicted shape details of $K(x)$, the aim of the present study has been the experimental examination of these shapes for a set of commercially available high-resolution detectors, the Si diode PTW 60017, the microdiamond PTW 60019, the Exradin W1 scintillation detector, and the ionization chamber PTW 31014. Since $K(x)$ can only be measured in an experimental arrangement warranting a significant difference between profiles $M(x)$ and $D(x)$, the core of the experiment has been the setup of a narrow photon slit beam, almost free from radiation background, across which the detector could be scanned in fine steps. Radiographic film dosimetry appeared as the technique suitable for measuring the undisturbed function $D(x)$, making use of the high spatial
resolution of radiochromic films and of the weak energy and direction dependence of their response (Suchowerska et al 2001, Rink et al 2007, van Battum et al 2008, Arjomandy et al 2010). The deconvolution technique applied to derive kernel $K(x)$ from the measured functions $M(x)$ and $D(x)$ was chosen in consideration of the known pitfalls of Fourier deconvolutions. Finally, the measured lateral dose response functions $K(x)$ of the investigated detectors had to be transformed into the radial dose response functions $K(r)$ for use with photon fields of circular symmetry. The practical implications of the observed functional shapes of the lateral dose response functions will be discussed below.
2. Materials and methods

2.1. Experimental setup of a narrow photon beam

Two lead blocks with size $10 \text{ cm} \times 5 \text{ cm} \times 15 \text{ cm}$ and serving as the tertiary collimator were positioned between the gantry head of a Siemens Artiste accelerator (Siemens, Erlangen, Germany) and a water phantom (type MP3, PTW Freiburg) as shown in figure 2. The block to water distance was 8.5 cm, and the source to block distance was 60 cm. Five sheets of printing paper fixed between the blocks formed a permanent, well-defined slit, 0.5 mm wide and 10 cm long, through which the photon beam could pass. The field size at the entrance surface of the lead blocks, achieved by the appropriate setting of the accelerator jaws and the multi-leaf collimator, was 6 cm parallel to the gap and 0.6 cm across the gap. All detectors were positioned at a depth of 3 cm in water and irradiated with a 6 MV photon beam.

![Diagram showing the experimental setup](https://example.com/diagram.png)

**Figure 2.** Two lead blocks were positioned between the gantry head of a Siemens Artiste accelerator (not displayed) and a water phantom. A 0.5 mm wide slit between the blocks was defined by placing five sheets of printing paper in it. The field size at the surface of the lead blocks, defined by the accelerator jaws and the multi-leaf collimator, was 6 cm parallel to the gap and 0.6 cm across the gap. All detectors were positioned at a depth of 3 cm in water and irradiated with a 6 MV photon beam.

Two lead blocks with size $10 \text{ cm} \times 5 \text{ cm} \times 15 \text{ cm}$ and serving as the tertiary collimator were positioned between the gantry head of a Siemens Artiste (Siemens, Erlangen, Germany) accelerator and a water phantom (type MP3, PTW Freiburg) as shown in figure 2. The block to water distance was 8.5 cm, and the source to block distance was 60 cm. Five sheets of printing paper fixed between the blocks formed a permanent, well-defined slit, 0.5 mm wide and 10 cm long, through which the photon beam could pass. The field size at the entrance surface of the lead blocks, achieved by the appropriate setting of the accelerator jaws and the multi-leaf collimator, was confined to 6 cm parallel to the gap and 6 mm across the gap in order to minimize the leakage radiation occurring under the lead blocks and hence to increase the sensitivity of the method.
Within this slit beam of 6 MV photon radiation passing into a large water phantom, the cross-beam signal profiles formed at a depth of 3 cm were measured by the radiochromic EBT3 film (Ashland Specialty Ingredients, Bridgewater, NJ), the silicon Diode E type 60 017 (PTW Freiburg, Germany), the microDiamond detector type 60 019 (PTW Freiburg, Germany), the Exradin W1 detector (Standard Imaging Inc, Middleton, USA) and the PinPoint ionization chamber type 31 014 (PTW Freiburg, Germany), see table 1. The symmetry axes of all the detectors were oriented parallel to the beam axis. The Si diode, the diamond detector, and the ionization chamber were connected to a Tandem dosimeter (PTW Freiburg, Germany). Using the water phantom software package the maximum-normalized signal profiles of these detectors were measured automatically at a step width of 0.1 mm. High reproducibility and no hysteresis effects or other accuracy problems of the detector movement were identified. Measurements were repeated with the same detectors by rotating the detectors around their axes to confirm their symmetrical design and to rule out any hysteresis effects.

The principle of measurement with the scintillation detector is the spectrometric record of visible light in two different spectral regions. Thereby, the detector system generates two output signals, which, due to the low signal strength caused by the slit-beam setup, are not compatible with the Tandem dosimeter system. To overcome this issue the detector system was connected to two Unidos dosimeter systems (PTW Freiburg, Germany). These two channels are normally used to correct the measurement error induced by Cerenkov light within the optical fiber, and the manufacturer recommends the Cerenkov correction method as proposed by Guillot et al (2011). In our specific setup the Cerenkov light did not contribute significant signals as long as the optical fiber was shaded by the lead blocks. However, discounting the Cerenkov light was only justified if the optical fiber did not cross the beam area below the slit. Therefore, the measurement with the scintillator detector only covered one half of the profile, so that the optical fiber was always shaded by the lead block. Since we had determined, using the other detectors, that the signal profile was symmetric, the scintillator profile was obtained by mirroring its half-side measured profile. For positioning of the scintillation detector a specific holder compatible with the Trufix scanning system (PTW Freiburg, Germany) was manufactured, which fixed the detector axis parallel to the beam axis.

The EBT3 film was mounted on a 2 mm thick polystyrene plate attached to the MP3 moving system. The films were irradiated with doses not exceeding 12 Gy to ensure that the required signal resolution was also warranted in the low dose region in the distant part of the narrow beam profile. Using 4 cm × 5 cm film pieces the film calibration was performed using the same setup at the same water depth in a 10 cm × 10 cm 6 MV photon field. All films were digitized using an Epson 10000XL scanner at 600 dpi and 16 bit color depth for each color channel and analyzed using the multi-channel analysis proposed by Micke et al (2011). The uncertainty of the measured absorbed dose to water at an individual point was estimated according to Lewis et al (2012), who reported a total uncertainty of the multi-channel analysis of approximately 2.2 %. We further reduced the statistical uncertainty associated with the film

| Manufacturer       | Model | Diameter of sensitive area | Sensitive material | Mass density g cm⁻³ | Relative electron density |
|--------------------|-------|----------------------------|--------------------|---------------------|--------------------------|
| PTW                | 60017 | 1.2 mm                     | Silicon            | 2.3                 | 2.1                      |
| PTW                | 60019 | 2.2 mm                     | Diamond            | 3.5                 | 3.2                      |
| PTW                | 31 014 | 2 mm                      | Air                | 0.0012              | 0.0012                   |
| Standard imaging W1 |       | 1 mm                      | Polystyrene        | 1.05                | 1.03                     |

* Wall thickness 0.7 mm, wall material: PMMA (mass density 1.19 g cm⁻³, electron density relative to water 1.16)
dose profile by averaging all the cross-beam dose profiles over a region of 1 cm length parallel to the slit, which corresponds to 236 single profiles.

### 2.2. Determination of the detectors’ 1D dose response functions \( K(x) \)

As described by equation (1), the lateral dose response function \( K(x,y) \) plays a key role as the convolution kernel in the relation of the indicated signal profile \( M(x,y) \) of a detector with the true dose profile in water, \( D(x,y) \). In the case of a slit beam across which detectors are scanned in the x-direction, equation (1) takes the 1D form \( M(x) = K(x) * D(x) \), see equation (2), where the 1D lateral dose response function \( K(x) \) is the projection of \( K(x,y) \) in the y-direction, see equation (3). If functions \( M(x) \) and \( D(x) \) are obtained by measurement, function \( K(x) \) can be obtained from equation (2) by deconvolution. According to the Fourier convolution theorem, equation (2) corresponds to a multiplication of the associated Fourier transforms in the frequency domain:

\[
FT[M(x)] = \frac{1}{\sqrt{2\pi}} FT[K(x)] \cdot FT[D(x)]
\]

where the Fourier transform \( FT[f(x)] \) and the inverse transform \( FT^{-1}[F(\omega)] \) are defined by

\[
FT[f(x)] = F(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) \cdot \exp(i\omega x) \, dx
\]

\[
FT^{-1}[F(\omega)] = f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(\omega) \cdot \exp(-i\omega x) \, d\omega
\]

(Sneddon 1955). For brevity, we will in the following preferentially use the angular frequency \( \omega = 2\pi \nu \), where \( \nu \) means the spatial frequency. From equation (5) the lateral dose response function \( K(x) \) is obtained as the inverse Fourier transform of the quotient of \( FT[M(x)] \) and \( FT[D(x)] \):

\[
K(x) = FT^{-1}\left[ \frac{1}{\sqrt{2\pi}} \frac{FT[M(x)]}{FT[D(x)]} \right]
\]

Cross-slit signal profiles \( M(x) \) were recorded by the silicon, diamond, and scintillation detectors as well as the ionization chamber, while the true profile of the absorbed dose to water, \( D(x) \), was determined by film measurement (figure 3). In order to avoid the known problems of the Fourier deconvolution caused by the discretization and scatter of the experimental data, the recorded data of the signal profiles \( M(x) \) and of the dose profile \( D(x) \) were fitted by sums of three centered 1D Gaussian functions:

\[
M_{\text{fit}}(x) = \sum_{i=1}^{3} A_{i,M} \exp\left(-\frac{x^2}{2\sigma_{i,M}^2}\right); \quad D_{\text{fit}}(x) = \sum_{i=1}^{3} A_{i,D} \exp\left(-\frac{x^2}{2\sigma_{i,D}^2}\right)
\]

The fitted profiles \( M_{\text{fit}}(x) \) and \( D_{\text{fit}}(x) \) were then analytically Fourier transformed into

\[
FT[M_{\text{fit}}(x)] = \sum_{i=1}^{3} A_{i,M} \sigma_{i,M} \exp\left(-\frac{\sigma_{i,M}^2 \omega^2}{2}\right); \quad FT[D_{\text{fit}}(x)] = \sum_{i=1}^{3} A_{i,D} \sigma_{i,D} \exp\left(-\frac{\sigma_{i,D}^2 \omega^2}{2}\right)
\]

which are plotted in figure 4 (left). The normalization of the Fourier transforms at the value \( 1/\sqrt{2\pi} = 0.3989 \) occurring at \( \omega = 0 \) is the expression of the area normalization of the signal and dose profiles. According to equation (7), the quotient of these Fourier transforms,
FT[M(x)]/FT[D_{fit}(x)], determines the Fourier transform of \( K(x) \). The latter was calculated numerically from equations (8) and (9), and the result was fitted by a sum of five centered Gaussian functions:

\[
FT[K(x)] = \frac{1}{\sqrt{2\pi}} \frac{FT[M_{fit}(x)]}{FT[D_{fit}(x)]} = \sum_{i=1}^{5} A_{i,K} \exp\left(-\frac{\sigma_{i,K}^2}{2} \right)
\]  (10)

The values of fitting parameters \( A_{i,K} \) and \( \sigma_{i,K} \) are collected in table 2, and figure 4 (right) shows the functions \( FT[K(x)] \) thereby calculated. In order to obtain \( K(x) \), the inverse Fourier transformation of this sum of Gaussian functions was analytically performed:

\[
K(x) = \sum_{i=1}^{5} \frac{A_{i,K}}{\sigma_{i,K}} \exp\left(-\frac{x^2}{2\sigma_{i,K}^2}\right)
\]  (11)
Table 2. Parameters of functions $FT[K(x)], K(x)$ and $K(r)$, experimentally determined for four detectors at 6 MV.

| Detector       | $A_{1,K}$ | $A_{2,K}$ | $A_{3,K}$ | $A_{4,K}$ | $A_{5,K}$ | $\sigma_{1,K}$ mm$^{-1}$ | $\sigma_{2,K}$ mm$^{-1}$ | $\sigma_{3,K}$ mm$^{-1}$ | $\sigma_{4,K}$ mm$^{-1}$ | $\sigma_{5,K}$ mm$^{-1}$ |
|----------------|-----------|-----------|-----------|-----------|-----------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
| Si diode 60017| -0.2843   | -1.0694   | 0.226     | 0.9303    | 0.5966    | 4.2409                   | 1.0085                   | 4.5310                   | 0.4692                   | 1.3450                   |
| Diamond 60019 | 1.2327    | 0.1359    | -0.6025   | 0.6400    | -1.0071   | 0.8191                   | 2.8555                   | 2.1594                   | 1.8195                   | 1.1141                   |
| Scintillator W1| 0.3652    | 0.4442    | 0.0587    | -0.3703   | -0.0983   | 1.5200                   | 0.2114                   | 3.2949                   | 1.7452                   | 0.8249                   |
| PinPoint 31014 | 0.6031    | 0.0966    | 0.1321    | -0.3133   | -0.1226   | 0.8121                   | 0.8114                   | 2.1218                   | 1.5569                   | 0.6957                   |
The maximum-normalized result is shown in figure 5 (left). Since equation (11) represents the 1D projection in the $y$-direction of $K(r)$, a function with rotational symmetry, the latter is:

$$K(r) = \frac{1}{\sqrt{2\pi}} \sum_{i=1}^{5} A_{i,K} \exp\left(-\frac{r^2}{2\sigma_{i,K}^2}\right)$$

with $r^2 = x^2 + y^2$ (12)

as can be proved by integration of equation (12) along the $y$-axis. Figure 5 (right) shows a maximum-normalized plot of $K(r)$. The rotational symmetry of the 2D lateral dose response function corresponds to the geometry of silicon diodes, synthetic diamond detectors, scintillation detectors, and micro-ionization chambers when their axes are adjusted parallel to the direction of the beam axis. Equation (12) can also be obtained from equation (10) by the Hankel transformation of $FT[K(x)]$ (Sneddon 1955, Marchand 1964).

All fits by Gaussian functions were performed using Matlab Version 2013 and the Curve Fitting Toolbox (The MathWorks Inc, Natick, USA).

### 3. Results

The cross-slit signal profiles $M(x)$ measured with the Si diode E 60017, the microDiamond 60019, the scintillator W1 Exradin, and the PinPoint ionization chamber 31014 are shown in figure 3 in comparison with the true cross-slit dose profile $D(x)$ measured with radiochromic EBT3 film. In the axis distance region $x > 10$ cm (not shown in figure 3) all the detector signals reached their zero-dose levels. Each profile was fitted by a weighted sum of three centered Gaussian functions as described in equation (8).

Using equation (9) the Fourier transforms of the slit-beam signal and dose profiles displayed in figure 3 were calculated and are shown figure 4 (left). $FT[K(x)]$, the Fourier transforms of the sought lateral dose response functions $K(x)$ of the investigated detectors as addressed in equation (10), are displayed in figure 4 (right). According to equation (10) each of the functions $FT[K(x)]$ was closely fitted by a weighted sum of five centered Gaussian functions for all frequencies where the amplitude-normalized values of $FT[K(x)]$ exceeded 3 %. The fitting parameters are summarized in table 2.
The 1D lateral dose response functions $K(x)$ for the silicon, diamond, and scintillation detectors as well as the small ionization chamber, obtained from the sum of the five Gaussians of equation (10) by analytical inverse Fourier transformation, are plotted in figure 5 (left). The rotational symmetric lateral response functions $K(r)$ deduced therefrom by equation (12) are displayed in figure 5 (right). To simplify the comparison between the shapes of the curves, they are plotted in maximum-normalized form.

4. Discussion

The different shapes of the measured lateral dose response functions $K(x)$ shown in figure 5 reflect the entity of effects making up the volume effect. The volume averaging is reflected in the lateral widths of these curves, but in addition the curve shapes are influenced by the disturbance of the secondary electron field by the presence of the detector. The curve characteristics in figure 5 are described in more detail below for the different types of detectors investigated.

In the case of the Si-diode 60 017 and the diamond detector 60 019, the inward-directed secondary electron transport at the outer boundary is reduced due to the enhanced electron density of the detector material, as expected in accordance with figure 1. This disturbance of the lateral secondary electron equilibrium leads to significant negative portions of the lateral dose response function $K(x)$ outside the boundaries of the sensitive zones of these detectors (figure 5). Although the relative electron density of the microDiamond detector exceeds that of the silicon diode (see table 1), a larger disturbance is observed for the silicon diode. This may be explained by its thicker sensitive volume (30 μm) compared to the diamond detector (1 μm).

For the air-filled ionization chamber PinPoint 31 014 the inward-directed secondary electrons dominate the energy deposition in the sensitive volume, leading to an extension of the lateral dose response function $K(x)$ beyond the chamber boundaries in accordance with the Monte Carlo prediction in figure 1. Negative curve portions of very low magnitude are observed in the region outside the sensitive volume of the chamber; these may be caused by the enhanced electron density of the 0.7 mm thick chamber wall consisting of PMMA with a relative electron density of 1.16.

For the nearly water equivalent scintillation detector Exradin W1, a lateral dose response function with very low values outside the detector boundaries has been found. This is in accordance with figure 1, since the secondary electron fluence is only minimally perturbed, and the volume effect almost solely arises from the averaging effect within the sensitive volume. The lateral dose response function shown in figure 5 has slightly negative values beyond the radius of the sensitive volume. This may be due to the slightly enhanced relative electron densities of the scintillator (see table 1), the plastic light guide fiber, and the encapsulation of the fiber.

In the rotational symmetric kernels shown in figure 5 (right) the same effects are expressed, but now in a cut plane through the symmetry axis of the detectors, whereas figure 5 (left) shows the projections of the kernels in the $y$-direction.

The occurrence of negative portions of the side wings of the 1D lateral dose response functions $K(x)$ in figure 5 (left) and also of $K(r)$ in figure 5 (right) may appear somewhat surprising, because in the underlying convolution relationship presented in equation (2) a negative value of a lateral dose response function at a distance from the center of the convolution kernel means numerically a negative contribution of the true dose value at this distance to the measured value of the detector. However, this effect is real and is due to the contribution of the secondary electrons, generated within the detector, to the detector’s signal in a photon beam;

9432
it agrees with the theoretical prediction by Monte Carlo simulation (figure 1). At this point it is helpful to consider that the true dose profile in water only exists in the absence of the detector, while in its presence the secondary electron field is perturbed.

In order to examine these theoretically expected, but surprising results on the shapes of kernels \(K(x)\), we convolved the film-measured true dose profile \(D(x)\) of the slit from figure 3 with the \(K(x)\) functions shown in figure 5 (left). The convolution results were then compared with the detector specific measured signal profiles \(M(x)\) shown in figure 3. It was found for all the detectors that the convolution results were indistinguishable from the measured signal beam profiles. This verification included the negative curve portions of the lateral dose response functions appearing in figure 5.

Furthermore, we explored the possibility that the negative curve portions of \(K(x)\) in figure 5 might have been deconvolution artefacts arising when \(K(x)\) was obtained by use of equation (2). This examination was performed by repeating the deconvolutions using an alternative technique, the van Cittert iterative deconvolution (see Looe et al 2015). The resulting functions \(K(x)\) closely reproduced those in figure 5, including the negative curve portions, thereby confirming the absence of deconvolution artefacts.

Another possible source of uncertainty leading to the negative curve portions of \(K(x)\) in figure 5 has to be sought in the uncertainties of the measured data of \(M(x)\) and \(D(x)\) needed to deconvolve equation (2) in order to obtain \(K(x)\). The critical data are those of the \(D(x)\) curve measured by radiochromic film dosimetry. In order to investigate the impact of the measurement uncertainties on the deconvolution results, the values of \(D(x)\) were tentatively varied to represent the estimated maximum uncertainties of the measured data. The result was that the negative portions of \(K(x)\) for the silicon diode and the diamond persisted, but for the scintillator and the PinPoint chamber decreased to insignificant levels.

As the spatial frequency spectrum is concerned, the function \(\text{FT}[K(x)]\) shown in 4 (right) has been fitted by the weighted sum of 5 centered Gaussian functions according to equation (10). A close fit within 1% deviation was achieved over the whole frequency region, in which the values of \(\text{FT}[K(x)]\) exceeded 3% of the maximum value. This means that the spatial frequency spectra of the finally resulting 1D dose response functions \(K(x)\) can be regarded as having been completely considered with a small residual uncertainty associated with the frequency region where the \(\text{FT}[K(x)]\) values have declined below 3%. This is acceptable in clinical photon dosimetry because high-energy photon dose profiles practically do not reach into the frequency region beyond about \(\nu \approx 1 \text{ mm}^{-1}\) respectively \(\omega \approx 2\pi \text{ mm}^{-1}\) due to the broadening of lateral dose profiles by the geometrical penumbra of the photon fluence profiles and the ranges of the secondary electrons.

This experimental determination of the shapes of convolution kernels \(K(x)\) and \(K(r)\) is regarded as a first step of a wider program which aims at performing the deconvolutions involved in determinations of the true dose profiles \(D(x,y)\) from the recorded signal profiles \(M(x,y)\) of clinical photon beams in accordance with equation (1). This approach will provide the corrections to be applied to the output factors obtained with different detectors as well as the corrections of the shapes of the dose profiles. In particular, the negative portions of the lateral response functions of silicon and diamond detectors are expected to be manifested as overresponses of these detectors in small-field output measurements.

5. Conclusions

The aim of this study has been to examine, using experimental methods, the Monte-Carlo based prediction by Looe et al (2015) concerning the curve shapes of the lateral dose response
functions $K(x)$ of various high-resolution detectors, particularly the negative curve portions of these functions. We investigated the lateral dose response functions of the silicon diode E 60017 and the microDiamond 60019, of the scintillation detector Exradin W1, and of the air-filled ionization chamber PinPoint 31014. The Monte-Carlo-predicted characteristics of their curve shapes, including the negative portions of the lateral curve contours, have been experimentally confirmed in the cases of the silicon and diamond detectors. For the scintillation detector and the PinPoint ionization chamber, significant negative portions of $K(x)$ have not been observed. The deconvolution procedure described in this study, using sums of weighted Gaussian functions and their analytically calculated transforms, is useful to avoid the noise and data discretization related artifacts known in the Fourier transformation processing of experimental data. Based on the knowledge of the convolution kernels $K(x)$ determined in this investigation, the lateral signal profiles of narrow clinical photon beams of any size as well as the output factors indicated by various high-resolution detectors can be corrected.

References

Alfonso R et al 2008 A new formalism for reference dosimetry of small and nonstandard fields Med. Phys. 35 5179–86

Arjomandy B, Tailor R, Anand A, Sahoo N, Gillin M, Prado K and Vicic M 2010 Energy dependence and dose response of gafchromic EBT2 film over a wide range of photon, electron, and proton beam energies Med. Phys. 37 1942–47

Azangwe G et al 2014 Detector to detector corrections: a comprehensive experimental study of detector specific correction factors for beam output measurements for small radiotherapy beams Med. Phys. 41 072103–1–16

Bassinet C et al 2013 Small fields output factors measurements and correction factors determination for several detectors for a cyberknife® and linear accelerators equipped with microMLC and circular cones Med. Phys. 40 071725–1–13

Bednarz G, Huq M S and Rosenow U F 2002 Deconvolution of detector size effect for output factor measurement for narrow gamma knife radiosurgery beams Phys. Med. Biol. 47 3643–49

Beierholm A R, Behrens C F and Andersen C E 2014 Dosimetric characterization of the Exradin W1 plastic scintillator detector through comparison with an in-house developed scintillator system Radiat. Meas. 69 50–6

Bennmakhlouf H, Sempau J and Andreo P 2014 Output correction factors for nine small field detectors in 6 MV radiation therapy photon beams: a PENEOPE Monte Carlo study Med. Phys. 41 041711–13

Brahme A 1981 Correction of a measured distribution for the finite extension of the detector Strahlentherapie 157 258–9 (PMID: 7245267)

Capote R, Sánchez-Doblado F, Leal A, Lagares J I, Arrans R and Hartmann G H 2004 An EGSnrc Monte Carlo study of the microionization chamber for reference dosimetry of narrow irregular IMRT beamlets Med. Phys. 31 2416–22

Chang K-S, Yin F-F and Nie K 1996 The effect of detector size to the broadening of the penumbra—a computer simulated study Med. Phys. 23 1407–11

Charland P, El-Khatib E and Wolters J 1998 The use of deconvolution and total least squares in recovering a radiation detector line spread function Med. Phys. 25 152–10

Crop F, Reynaert N, Pittonvils G, Paelinck L, De Wagner C, Vakaet L and Thierens H 2009 The influence of small field sizes, penumbra, spot size and measurement depth on perturbation factors for microionization chambers Phys. Med. Biol. 54 2951–69

Djouguela A, Irmgard G, Harder D, Kollhoff R, Chofof N, Ruehmann A, Willborn K and Poppe B 2008 Dosimetric characteristics of an unshielded P-type Si diode: linearity, photon energy dependence and spatial resolution Z. Med. Phys. 18 301–6

Feng Y 2006 Deconvolution of detector response function PhD thesis

Fenwick J D, Kumar S, Scott A J D and Nahum A E 2013 Using cavity theory to describe the dependence on detector density of dosimeter response in non-equilibrium small fields Phys. Med. Biol. 58 2901–23
for several small detectors and for correction factors for microchamber and diode detectors

Francescon P, Cora S and Satariano N 2011 Calculation of $k_{\text{iso}}/k_{\text{c}}$ for several small detectors and for two linear accelerators using Monte Carlo simulations Med. Phys. 38 6513–27

Francescon P, Kilby W, Satariano N and Cora S 2012 Monte Carlo simulated correction factors for machine specific reference field dose calibration and output factor measurement using fixed and iris collimators on the CyberKnife system Phys. Med. Biol. 57 3741–58

Garcia-Vicente F, Delgado JM and Peraza C 1998 Experimental determination of the convolution kernel for the study of the spatial response of a detector Med. Phys. 25 202–7

Garcia-Vicente F, Delgado JM and Rodriguez C 2000 Exact analytical solution of the convolution integral equation for a general profile fitting function and Gaussian detector kernel Phys. Med. Biol. 45 645–50

Gonzalez-Castaño D M, González L B, Gago-Arias M A, Pardo-Montero J, Gómez F, Luna-Vega V, Sánchez M and Lobato R 2012 A convolution model for obtaining the response of an ionization chamber in static non standard fields Med. Phys. 39 482–11

Guillot M, Gingras L, Archambault L, Beddar S and Beaulieu L 2011 Spectral method for the correction of the Cerenkov light effect in plastic scintillation detectors: a comparison study of calibration procedures and validation in Cerenkov light-dominated situations Med. Phys. 38 2140–12

Harder D, Loee H K and Poppe B 2014 Convolutions and deconvolutions in radiation dosimetry Comprehensive Biomedical Physics (Amsterdam: Elsevier) pp 249–69

Herrup D, Chu J, Cheung H and Pankuch M 2005 Determination of penumbral widths from ion chamber measurements Med. Phys. 32

Higgins P D 1995 Deconvolution of detector size effect for small field measurement Med. Phys. 22 1663–5

Kamio Y and Bouchard H 2014 Correction-less dosimetry of nonstandard photon fields: a new criterion to determine the usability of radiation detectors Phys. Med. Biol. 59 4973–5002

Ketelhut S and Kapsch R-P 2015 Measurement of spatial response functions of dosimetric detectors Phys. Med. Biol. 60 6177–94

Laub W U and Wong T 2003 The volume effect of detectors in the dosimetry of small fields used in IMRT Med. Phys. 30 341–8

Lewis D, Micke A, Wu X and Chan M F 2012 An efficient protocol for radiochromic film dosimetry combining calibration and measurement in a single scan Med. Phys. 39 6339–50

Loee H K and et al 2012 Correction of the limited spatial resolution of ionization chambers by iterative deconvolution Jahrestagung der Deutschen Gesellschaft für Medizinische Physik pp 218–20

Loee H K, Harder D and Poppe B 2015 Understanding the lateral dose response functions of high-resolution photon detectors by reverse Monte Carlo and deconvolution analysis Phys. Med. Biol. 60 6585–607

Loee H K, Stelljes T S, Foschepoth S, Harder D, Willborn K, Poppe B and Willborn K C 2013 The dose response functions of ionization chambers in phantom dosimetry—gaussian or non-gaussian? Z. Med. Phys. 23 129–43

Marchand E W 1964 Derivation of the point spread function from the line spread function J. Opt. Soc. Am. 54 915–9

Micke A, Lewis D F and Wu X 2011 Multichannel film dosimetry with nonuniformity correction Med. Phys. 38 2523–34

Pan telis E and et al 2012 On the output factor measurements of the CyberKnife iris collimator small fields: experimental determination of the $k_{\text{iso}}/k_{\text{c}}$ correction factors for microchannel and diode detectors Med. Phys. 39 4875–12

Papaconstadopoulos P, Tessier F and Seuntjens J 2014 On the correction, perturbation and modification of small field detectors in relative dosimetry Phys. Med. Biol. 59 5937–52

Pappas E, Maris T G, Papadakis A, Zacharopoulos F, Damlakas J, Papanikolaou N and Gourtsoyiannis N 2006 Experimental determination of the effect of detector size on profile measurements in narrow photon beams Med. Phys. 33 3700–12

Pychlau P J 1979 Beitrag zur ortsauflösung bei messungen mit detektoren mit zylindrischem querschnitt Strahlentherapie 155

Ralston A, Liu P, Warrener K, McKenzie D and Suchowerska N 2012 Small field diode correction factors derived using an air core fibre optic scintillation dosimeter and EBT2 film Phys. Med. Biol. 57 2587–602
Ralston A, Tyler M, Liu P, McKenzie D and Suchowerska N 2014 Over-response of synthetic microdiamond detectors in small radiation fields Phys. Med. Biol. 59 5873–81
Rink A, Vitkin I A and Jaffray D A 2007 Energy dependence (75 kVp to 18 MV) of radiochromic films assessed using a real-time optical dosimeter Med. Phys. 34 458
Sahoo N, Kazi A M and Hoffman M 2008 Semi-empirical procedures for correcting detector size effect on clinical MV x-ray beam profiles Med. Phys. 35 5124–11
Sauer O A and Wilbert J 2007 Measurement of output factors for small photon beams Med. Phys. 34 1983–7
Scott A J D, Kumar S, Nahum A E and Fenwick J D 2012 Characterizing the influence of detector density on dosimeter response in non-equilibrium small photon fields Phys. Med. Biol. 57 4461–76
Sibata C H, Mora H C, Beddar A S, Higgins P D and Shin K H 1991 Influence of detector size in photon beam profile measurements Phys. Med. Biol. 36 621–31
Sneddon I N 1955 Functional analysis Encyclopedia of Physics/Handbuch der Physik. vol 11 ed S Flügge (Berlin: Springer)
Suchowerska N, Hoban P, Butson M, Davison A and Metcalfe P 2001 Directional dependence in film dosimetry: radiographic and radiochromic film Phys. Med. Biol. 46 1391–7
Ulmer W and Kaissl W 2003 The inverse problem of a Gaussian convolution and its application to the finite size of the measurement chambers Phys. Med. Biol. 48 707–27
Underwood T S A, Rowland B C, Ferrand R and Vieillelagne L 2015 Application of the exradin W1 scintillator to determine ecldiode 60017 and microdiamond 60019 correction factors for relative dosimetry within small MV and FFF fields Phys. Med. Biol. 60 6669–83
Underwood T S A, Winter H C, Hill M A and Fenwick J D 2013 Detector density and small field dosimetry: integral versus point dose measurement schemes Med. Phys. 40 082102–17
van Battum L J, Hoffmans D, Piersma H and Heukelom S 2008 Accurate dosimetry with gafchromic™ EBT film of a 6 MV photon beam in water: what level is achievable? Med. Phys. 35 704–16
van’t Veld A A, van Luijk P, Praamstra F and van der Hulst P C 2000 Slit x-ray beam primary dose profiles determined by analytical transport of compton recoil electrons Med. Phys. 27 923–13
vant Veld A A, van Luijk P, Praamstra F and van der Hulst P C 2001 Detector line spread functions determined analytically by transport of compton recoil electrons Med. Phys. 28 738–15
Yan G, Fox C, Liu C, and Li J G 2008 The extraction of true profiles for TPS commissioning and its impact on IMRT patient-specific QA Med. Phys. 35 3661–11