Numerical Investigation of Blade Lean and Sweep affecting Secondary Flows in an Axial Expansion Turbine

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Abstract. Based on an axial expansion turbine used for energy recovery from working fluids an automated CFD-flow optimization was performed to increase the turbine efficiency without narrowing the operating range. The optimization results showed that even small changes in the optimization target had a significant influence on the lean and sweep of the new blade designs. The resulting blade shapes were extraordinary. The lean and sweep resembled more that of thermal turbomachinery than hydraulic machinery. It became clear that the special blade shapes were a result of the very low aspect ratio of the turbine and the resulting large influence of secondary flows. The blade lean and sweep induce secondary flow structures which are of decisive importance for the turbine performance. The simulation results showed that positive compound lean increases the efficiency and positive compound sweep improves the cavitation behavior of the investigated axial expansion turbine. However the complexity of the occurring secondary flows make an universally valid statement on the effect of blade lean and sweep on the flow behavior of an axial turbine impossible.

1. Introduction

Many industrial processes need working fluids on high pressure levels. At the end of the processes the working fluids are often released to ambient pressure through conventional throttles. The pressure energy dissipates and remains unused. An axial expansion turbine is one possibility to increase the overall system efficiency by converting the pressure energy into electrical energy. The key requirement on recovering systems is that the energy recovery has no negative impact on the plant safety and the process stability. The main advantage of an axial expansion turbine is the nearly constant flow rate even in case of power failure when the turbine reaches its runaway speed. The constant flow rate prevents a hydraulic pressure surge which could lead to damages on valves, sealings and other structures of the system. For that reason no additional safety device or auxiliary control equipment is needed.

The currently developed modular turbine concept with a single-stage version and a two-stage version has a wide operating range and a high maximum hydraulic head combined with high efficiencies. On this basis an automated CFD-flow optimization was performed to create a more flat characteristic curve on the highest possible efficiency level without narrowing the operating range. The optimization results show that an increase in efficiency comes along with worse cavitation behavior. Due to these counteracting effect small changes in the required cavitation behavior or operating range lead to significant changes in blade lean and sweep. The optimized blades have an extraordinary shape which resembles more that of thermal turbomachinery than of hydraulic machinery. The main goal is to investigate the influence of blade lean and sweep on the flow behavior of the turbine therefore all investigated rotors are designed without a blade clearance. The impact of tip-leakage flow vortices on the secondary flow structure is not taken into account in this study.
2. Optimization procedure

The automated CFD-flow optimization is performed on basis of a previously developed reference design. The primary goal of the optimization process is the development of a new turbine with a flat characteristic curve on the highest possible efficiency level without narrowing the operating range. In the optimization runs described in this paper solely the shapes of the rotor blades are varied.

The optimization procedure is divided into three levels. First, the optimizer calculates the geometry parameters of each individual in a generation and passes the data to the second level. There a Perl-interface coordinates the work flow to and from the third level consisting of the geometry generation, the meshing and the CFD-computation including the pre- and post-processing. The work flow of one generation is divided into three steps. First, the design is parameterized and the geometry generation is started. In the second step the geometries and the meshes are categorized into fit or failed designs. Fit designs are passed to the next job and failed designs are separated from the regular work flow (figure 1). The last step is the solution processing where one single quality criterion for each design is calculated and sent to the optimizer.

2.1. Optimization algorithm

The in-house optimizer uses a combination of a generic and a discrete SIMPLEX algorithm that uses a given number of individual per generation [1], [2]. Each individual represent one design variation with a specific input data configuration. When the entire output data of one generation has passed to the solution processing one single target value for each individual is calculated, e.g. the turbine efficiency. Once the optimizer got the data all bad designs are sorted out and a new set of design parameters for each individual of the following generation is calculated. This corresponds to the evolutionary principle of “surviving of the fittest”.

2.2. Blade parameterization

The blade parameterization is based on a polygon constructed from ten points. The ten corners of the polygon build the control points of a B-spline of fourth order that is used to model the blade profile. All geometric parameters of the blade, like the length, the thickness distribution as well as the inlet and outlet angle are defined by the polygon. The three-dimensional blade shape is defined by splines projected on three surfaces of revolution with different radii, the hub, the mean plane of the blade channel and the shroud. A detailed description how the polygons are constructed including the according equations is given in [3].

2.3. Bounded values and target function

The solution processing module is the last decisive interface before passing the target value of each individual in a generation to the optimizer. Individuals sorted out during the data evaluation because of a failed geometry generation or bad mesh quality and all designs where the CFD computation has terminated and no result file could be generated are treated as failed designs. The simulation results of all remaining individuals are used to generate three output variables, the turbine efficiency $\eta$, the minimum static pressure $p_{st,min}$ and the volume $v_{cav}$ in which cavitation is likely to occur. Based on predefined boundary values such as a lower bound for $p_{st,min}$ and an upper bound for $v_{cav}$ the designs are again categorized. Fit designs complying with the set bounded values are rated by a target function $f_t$. For this purpose the turbine efficiency is calculated by using equation (1), where $M$ is the torque, $\omega$
the angular velocity, \( g \) the gravity, \( \Delta h \) the hydraulic head difference between turbine in- and outlet and \( Q \) the discharge.

\[
\eta = \frac{M \cdot \omega}{\rho g \Delta h Q}
\]

The number of operating points \( n \) investigated during an optimization run is variable and can be used as an additional influencing factor on the resulting blade shapes. The goal of the optimization is to increase the average efficiency \( \eta_{ave} \) and to reduce their standard deviation \( s_{dev} \) to create a turbine design with a flat characteristic curve on a higher efficiency level. For that reason and because the optimizer searches for a global minimum the target function is defined as follows

\[
f_t = -\left(\eta_{ave} - s_{dev}\right)
\]

with

\[
\eta_{ave} = \frac{\eta_{op1} + \cdots + \eta_{opn}}{n}
\]

\[
s_{dev} = \sqrt{\frac{1}{n} \left( (\eta_{op1} - \eta_{ave})^2 + \cdots + (\eta_{opn} - \eta_{ave})^2 \right)}.
\]

3. Secondary flows in axial turbine passages

Endwall flows in blade passages are extremely complex and are dominated by the presence of secondary flows. Since the early 1950th a lot of effort has been made to understand the details of endwall flows in axial turbines and their influence on secondary flows and flow losses. In blade channels with low aspect ratios and high flow turning secondary flows are an important source of turbine losses. It is assumed that those losses also influence the current design.

3.1. Secondary flow models

The main type of secondary flow in a blade channel is induced by the flow deflection and the resulting cross flow in the endwall boundary layer. The cross flow is a result of the force equilibrium in a curvilinear motion. The momentum equation in cross flow direction is

\[
\frac{\rho u^2}{R} = \frac{\partial p}{\partial n}
\]

with \( u \) as the velocity in the boundary layer, \( p \) the pressure, \( \rho \) the density, \( R \) the radius of the streamline curvature and \( n \) the normal coordinate [4]. According to equation (4) a balanced pitch wise pressure gradient requires that a decrease of velocity comes along with a reduction of streamline curvature radius \( r \). The boundary layer is more deflected than the main flow leading to a cross flow in the endwall boundary layer from the pressure to the suction-side of the blade (figure 2 left picture). In a certain distance of the endwall a recirculation flow develops and compensates the described cross flow. This recirculation forms the passage vortex [5].

\[\text{Figure 2. Formation of passage vortex (left) [6] and horseshoe vortex (right) [5]}\]

Another important type of secondary flow is the horseshoe vortex at the leading edge of the blade. The boundary layer fluid is decelerated by an adverse pressure gradient on the leading edge and separates at a saddle point s1. The separated boundary layer flow circulates in reverse direction,
separates at a second saddle point s2 and rolls up into a vortex. The recirculation zone is transported downstream of the leading edge and extents along both sides of the blade building the pressure and the suction-side leg of the horseshoe vortex. The legs follow the two lift-off lines formed by the saddle points as shown in the right picture of figure 2. The exact position of the lift-off lines is defined by the load at the front part of the blade. The main types of secondary flow all meet at the suction surface. Langstone [7] assumed that the boundary layer crossflow and the pressure-side leg of the horseshoe vortex unite to the passage vortex. Whereas the suction-side leg builds a vortex apart from the passage vortex in counter rotating direction. In contrast to that Doerffler and Amecke [8] suggested that the pressure-side leg is located inside the passage vortex and the counter-rotating suction-side leg dissipates by interacting with the passage vortex at the contact surface.

The differences in the secondary flow models show the constant process of getting a deeper understanding of the endwall flows in course of time. But it is also a sign for the complexity of secondary flow phenomena and that the blade geometry has a significant influence on intensity and structure of secondary flows. At the blade channel exit the velocity difference between the suction and the pressure-side leads to the formation of a trailing edge vortex that interacts with the passage vortex [5]. However this paper primarily focuses on the secondary flows in the blade channel.

3.2. Vorticity and secondary losses

According to the classical secondary flow models the turning of the vorticity vector in the blade channel leads to a streamwise vorticity component generating transverse velocities. The vorticity \( \vec{\omega} \) is a pseudo-vector field defined as the curl of the velocity vector \( \vec{u} \). In a cylindrical coordinate system with \( r, \phi \) and \( z \) as the radial, the circumferential and the axial flow direction the vorticity is defined as

\[
\vec{\omega} = \nabla \times \vec{u} = \begin{pmatrix}
\frac{1}{r} \frac{\partial u_z}{\partial \phi} - \frac{\partial u_\phi}{\partial z} \\
\frac{1}{r} \frac{\partial u_\phi}{\partial z} - \frac{\partial u_z}{\partial r} \\
\frac{1}{r} \frac{\partial}{\partial r} (ru_\phi) - \frac{1}{r} \frac{\partial u_r}{\partial \phi}
\end{pmatrix}
\]

(5)

In secondary flow theory based on linear blade cascades the streamwise vorticity is often taken as the vorticity component in primary flow direction. During this study the secondary flow is calculated and visualized in different axial planes along the blade channel. For that reason the streamwise vorticity is defined as the component of vorticity in the local flow direction and is then calculated from

\[
\bar{\omega}_s = \bar{\omega}_\phi \sin \bar{\alpha}(z) + \bar{\omega}_r \cos \bar{\alpha}(z)
\]

(6)

where \( \bar{\alpha}(z) \) is the mass flow averaged local flow angle. The losses in this paper are mainly expressed in terms of the total pressure loss coefficient \( \bar{Y}_t \). The mass flow averaged total pressure loss coefficient in the rotor blade channel is calculated as follows

\[
\bar{Y}_t(z) = \frac{\bar{p}_{t,\text{in}} - \bar{p}_t(z)}{\frac{1}{2} \rho \bar{u}_{\text{out}}^2}
\]

(7)

with \( \bar{p}_{t,\text{in}} \) as the mass flow averaged total pressure at rotor inlet, \( \rho \) as the density and \( \bar{u}_{\text{out}} \) as the mass flow averaged velocity at rotor outlet.

4. Optimization runs and rotor blade geometries

In this section two different optimization runs and their resulting rotor blade geometries will be introduced. The differences between the two optimization runs are the number of variable input values used for the blade parameterization and the number of simulated operating points. The variable blade parameters are the inlet angle, the blade length and the thickness distribution along the chord. The optimization results show that the number of simulated operating points and thus, the required operating range or cavitation behavior have a significant influence on the blade shapes.
The reference turbine has a linear leaned and backward swept blade (figure 3). The optimized blades have a more three-dimensional shape (figure 4 and 5) and differ from the reference design mainly in their lean and sweep as well as in their inlet angles. In this paper the main focus lay on the lean and sweep. There are different ways to define blade lean and blade sweep. During this work the lean and the sweep are defined relative to the axial and circumferential direction. Blade lean describes the movement of the blade profile in circumferential direction and blade sweep in axial direction over the radial extension of the blade.

4.1. First optimization run

In the first optimization run the CFD simulation is executed for three different operating points, such as part load, best point and full load. Previous flow simulations showed that the reference turbine can run over the whole operating range almost without cavitation. Only in full load a small cavitation bubble develops on the suction-side near the trailing edge. For that reason the maximum cavitation volume for the first two operating points is set to zero. For full load the cavitation is restricted to a defined maximum value $v_{cav, max}$ with a specified minimum static pressure $p_{st, min}$. The two boundary values are more restrictive than the values calculated for the reference design at full load. The underlying idea is to create a flat characteristic curve on a higher efficiency level with a better cavitation behavior. In this run the blade parameterization has 15 degrees of freedom, five for each B-spline in the three radial cutting planes. In each of the three respective planes three variables specify the thickness distribution of the profile along the chord whereas the remaining two input variables are used to define the inlet angle and the chord length. Figure 4 shows the resulting blade geometry.

4.2. Second optimization run

The second optimization run is an approach to accelerate the optimization. The previous design shows that the deviation of the thickness distribution in the three radial cutting planes is small. For that purpose the thickness distribution of the second optimized design remains constant in radial direction and the degrees of freedom shrink from 15 to 9. The inlet angles and the blade lengths stay variable in all three planes. For an additional simplification of the optimization process only the best point of the turbine is simulated. The cavitation volume $v_{cav, max}$ stays restricted to zero. Both modifications lead to a significantly faster convergence of the optimizer. But due to the defined target function $f_i$ the
optimization goal of a flat characteristic curve can no longer taken into account because there is only one efficiency value and no standard deviation $s_{dev}$ can be calculated. In addition the maintenance of a wide operating range cannot be considered because the cavitation and the minimum static pressure at part and full load are not restricted anymore. Figure 5 shows the resulting changes in blade design.

The resulting rotor blade designs show that the optimizer is very sensitive to changes in the optimization procedure. The differences in lean and sweep of the blade geometries are summarized in table 1. The plus and the minus signs give an indication how obtuse or acute the angles between the blade and the suction-side or the endwalls are. Two plus signs e.g. indicate a very obtuse angle whereas one minus sign defines a moderate acute angle.

| Table 1. Lean and sweep of the different rotor blade geometries. |
|---------------------------------------------------------------|
| ref_design         | 1st_design       | 2nd_design      |
| lean | hub | blade tip | lean     | hub | blade tip | lean | hub | blade tip |
| linear | - | + | compound | + | ++ | compound | + | ++ |
| backward | + | - | compound | ++ | ++ | compound | - | - |

5. Simulation results

In this section the influence of the newly developed rotor geometries on the flow behavior of the turbines shall be investigated. For that reason CFD-flow simulations for several operating points from part load to over load are performed. The goal is to show the influence of the rotor blade shape on the characteristic curves and the cavitation behavior of the turbines. Special attention is given to the relationship between lean and efficiency increase and blade sweep affecting the cavitation behavior.

The left graph in figure 6 shows the characteristic curves over the hydraulic head of the two optimized turbines in relation to the reference turbine. It can be seen that for both newly developed
rotor blades the efficiency increases in an operating range between 60 to 110 meters hydraulic head. Below 60 meters at part load the characteristic curves of all three turbines are nearly identical. The differences in curve shape are very small. Therefore the different optimization procedures have nearly no influence on the first optimization target of a flat characteristic curve. The second rotor design reaches the highest efficiency level. The maximum efficiency increases by 0.4 %. The first optimized blade design leads to a smaller increase of 0.2 %. By focusing on the geometry data of the different blade designs listed in table 1 it can be assumed that compound lean has a positive effect on the turbine efficiency.

The difference in the cavitation behavior of the three turbine designs can be derived from the right graph in figure 6 which shows the cavitation volume $v_{\text{cav}}$ over the hydraulic head $h$. The turbine with the first rotor blade design reaches the highest maximum hydraulic head of $h_{\text{max}} = 95 \text{ m}$. In contrast to that the second optimization run leads to a deterioration of the cavitation behavior compared to the reference design. The improved cavitation behavior of the first design results from the restrictive bounded values of the optimization run. By comparing the simulation results and the geometry data a relationship between blade sweep and cavitation behavior can be observed. It is assumed that the rotor blade sweep is the decisive influence factor on the cavitation behavior of the turbine.

6. Effect of blade lean and blade sweep on secondary flows

Both the change in efficiency due to blade lean as well as the effect of blade sweep on the cavitation behavior of the different turbine designs is attributed to endwall and secondary flow phenomena. In the first step the influence of blade lean on the turbine efficiency is investigated. As mentioned before both new turbine designs have characteristic curves on higher efficiency levels (figure 6) and both have blades with compound lean. In terms of lean the two optimized designs look very similar (table 1). The second design has the highest maximum efficiency. For that reason the influence of lean on the efficiency is investigated by comparing the second and the reference design.

In contrast to the lean the rotor blades of the two optimized designs differ significantly in their sweep (table 1). The difference in sweep comes along with a big difference in cavitation behavior. The first design reaches by far the highest maximum hydraulic head and the second design reaches the lowest. For that reason the influence of blade sweep on the cavitation behavior shall be investigated by comparing the first and the second optimized rotor blade design.

6.1. Blade lean

The examination of blade lean influencing the secondary flows and flow losses are all based on simulation results in the best point of both turbines. The two investigated rotor geometries have the same small aspect ratio and nearly identical flow turning. For that reason it is assumed that the different secondary flow structures in the blade channel are mainly influenced by the shape and the strength of the horseshoe vortices developing at the leading edge. The differences in the development of the horseshoe vortices result from the different angles between the blade and the endwalls. Figure 7 shows the contour plots of the absolute streamwise vorticity $\omega_{s,\text{abs}}$ at the leading edges of the two investigated rotor blade designs. The rotational direction is neglected to allow a better visualization of the vortex shape and intensity. It can be seen that differences between the horseshoe vortex legs on the pressure-side (PS) of the two blade designs are very small whereas the suction-side (SS) legs clearly differ from each other. The different angles between the pressure-side of the blade and the endwalls have no significant influence on the development of the pressure-side legs of the horseshoe vortex. For that reason only the influence of the angles on the suction-side are investigated in more detail.

On the blade tip the second design is more positively leaned than the reference design and thus it has a more obtuse angle between blade and casing. The right picture in figure 7 shows that the very obtuse angle leads to a weaker suction-side leg of the horseshoe vortex. At the same time an obtuse angle on the hub has the opposite effect. The obtuse angle on the hub of the second design leads to a higher intensity of the vortex leg compared to the reference design where the angle between suction-side and hub is acute. The vortex visualization shows that the angles between blade and endwalls and thus the lean of the blades have a great influence on the secondary flow at the leading edge.
The simulation results further show that the horseshoe vortices influence the secondary flow in the entire blade channel. The pressure-side legs of the horseshoe vortices separate from the pressure surface shortly after the leading edge and move towards the suction surface. There, the pressure-side legs and the boundary layer crossflow unite to the passage vortex as Langstone [7] assumed. Further downstream the different vortices start to interact with each other. Figure 8 shows the vortex structure at the blade tip for the two rotor designs in several cutting planes along the blade channel. The passage vortex (positive vorticity) moves towards the suction surface and presses the suction-side leg of the horseshoe vortex (negative vorticity) towards the blade wall. The suction-side leg warps but does not dissipate. Even for the second design in the lower picture where the suction-side leg is very small and is not visual anymore there is still a small area between the passage vortex and the suction surface with counter-rotating vorticity. Further downstream at the end of the blade channel the suction-side leg of the horseshoe vortex appears below the passage vortex for both designs.

A similar vortex structure is observed on the hub. The passage vortex presses the suction-side leg of the horseshoe vortex towards the blade wall and deforms it. In contrast to the blade tip the higher flow turning on the hub leads to a stronger boundary layer crossflow and thus to a higher intensity of the passage vortex. Due to this the suction-side leg on the hub stays locked between the blade wall and the passage vortex. The two pictures on the left in figure 9 show the contour-plots of the different vortex structure and the total pressure loss coefficient at the end of the blade channel. The reference design is shown in the upper picture and the second design in the lower picture. It can be seen that the total pressure losses result from the interaction of the upper and lower passage vortex with the upper suction-side leg of the horseshoe vortex. For that reason most of the total pressure losses occur in the rear half of the blade channel above midspan. The differences in the lower suction-side legs of the horseshoe vortices have no significant influence on the flow losses.

The graph in figure 9 shows the spanwise total pressure loss distribution at the blade channel inlet and outlet. The difference in total pressure loss at the blade channel inlet is small except of an area \( A \) near the blade casing resulting from the greater intensity of the suction-side leg of the horseshoe vortex. The losses at the outlet have the same cause, but the impact is significantly higher (see \( B \)). The stronger horseshoe vortex leads to greater interaction of the different vortices at the outlet and due to
this to higher total pressure losses. For that reason the greater positive lean at the blade tip and the more obtuse angle between suction surface and casing leads to a higher efficiency of the new design.

![Contour plots](image)

**Figure 9.** Contour plots $\omega_s / Y_t$ at blade end (left) and $Y_t$ at blade channel inlet and outlet (right)

### 6.2. Blade sweep

The investigation of blade sweep on the flow behavior is based on simulation results at full load. The operating point is specially chosen that the first design can be run without cavitation whereas for the second design cavitation is likely to occur. The comparison of the secondary flow behavior at the leading edge of the optimized blade designs shows that the positive sweep at both endwalls of the first design weakens the horseshoe vortices. Nevertheless the vortex structure at the blade tip looks nearly identical for both designs whereas significant differences can be observed on the hub. The weak suction-side leg of the horseshoe vortex at the hub of the first blade design leads to an accelerated growth and a higher intensity of the passage vortex. In comparison to the second design the passage vortex of the first design moves faster towards midspan. The upper and lower vortex structures start to interact further upstream and the total pressure losses increase. For that reason the turbine with the first rotor design does not reach the same efficiency level as the second design (figure 6).

![Static pressure distribution](image)

**Figure 10.** Static pressure over normalized chord length (left 1st blade design / right 2nd design)

At the same time the more intense secondary flow in the blade channel of the first design has a positive effect on the pressure distribution. Figure 10 shows the pressure distribution along the first and the second blade profile in three different spanwise planes, near the hub $h(r)/h = 0.1$, at midspan $h(r)/h = 0.5$ and near the blade tip $h(r)/h = 0.9$. The comparison of the two graphs in figure 10 shows that the positive compound sweep of the first blade design leads to a shift in blade load. The secondary flows in the blade channel of the first design move the blade load upstream and towards midspan. In figure 11 the pressure distribution of the two optimized design at $h(r)/h = 0.25$ can be seen. The comparison shows that the movement of the blade load upstream and away from the endwalls results in a weaker pressure drop at the suction-side of the blade. The right graph in figure 11 clearly shows that the static pressure on the suction-side of the second blade drops below the vapor pressure. In this area cavitation is likely to occur. It can thus be concluded that the positive compound sweep and the resulting change in blade load leads to an improved cavitation behavior.

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7. Conclusion

The extraordinary blade shapes arisen form the flow optimization are a result of the very low aspect ratio of the turbine and the resulting large effect of secondary flows on the ideal primary flow. The simulation results show that the development of the horseshoe vortices at the leading edge of the blades is the determining factor for the secondary flow structure in the blade channel. To a large extent the developing secondary flow is responsible for the turbine efficiency and the cavitation behavior. For that reason the small changes in the optimization targets had a significant influence on the shape of the leading edge. By comparing the different blade designs it becomes clear that the positive compound lean of the optimized rotor blades lead to an efficiency increase due to a change in shape and intensity of the horseshoe vortices at the leading edge. In addition to the blade lean the positive compound sweep of the first optimized rotor blade induces secondary flows moving the blade load upstream and towards midspan. This shift in blade load leads to a changed static pressure distribution and as a result to an improved cavitation behavior of the turbine.

The interactions of secondary flows are of decisive importance for the turbine performance. In conclusion it has to be stated that the complexity of secondary flows makes it impossible to transfer the results described above to any blade design.

8. References

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Figure 11. Static pressure distribution at $h(r)/h = 0.25$ of first and second blade design