$TRING THEORY AND AN ACCELERATING UNIVERSE

John ELLIS\textsuperscript{a)}, N.E. MAVROMATOS\textsuperscript{b)} and D.V. NANOPOULOS\textsuperscript{c)}

\textsuperscript{a)} Theoretical Physics Division, CERN, CH 1211 Geneva 23
\textsuperscript{b)} Department of Physics, Theoretical Physics, King’s College London, Strand, London WC2R 2LS, U.K.
\textsuperscript{c)} Department of Physics, Texas A & M University, College Station, TX 77843, USA; Astroparticle Physics Group, Houston Advanced Research Center (HARC), Mitchell Campus, Woodlands, TX 77381, USA; Chair of Theoretical Physics, Academy of Athens, Division of Natural Sciences, 28 Panepistimiou Avenue, Athens 10679, Greece

ABSTRACT

An accelerating Universe can be accommodated naturally within non-critical string theory, in which scattering is described by a superscattering matrix $\text{S}$ that does not factorize as a product of $S$- and $S^\dagger$-matrix elements and time evolution is described by a modified Liouville equation characteristic of open quantum-mechanical systems. We describe briefly alternative representations in terms of the stochastic Ito and Fokker-Planck equations. The link between the vacuum energy and the departure from criticality is stressed. We give an explicit example in which non-marginal $\text{string}$ couplings cause a departure from criticality, and the corresponding cosmological vacuum energy relaxes to zero à la quintessence.

CERN-TH/2001-134
May 2001
Conceived parthenogenetically to model the $S$ matrix of the strong interactions \cite{1}, string theory aspires to apotheosis as a quantum theory of gravity \cite{2}. To demonstrate its worthiness for this exalted status, string theory must accomplish successfully at least four Labours. The first of these to be accomplished was the cleansing of the Augean stables of perturbative quantum-gravitational divergences in the weak-field limit \cite{3}. The second was to slay the Gorgon of quantum black holes by identifying the quantum states that quantify their entropy \cite{4}. However, looking at the Gorgon’s face is still problematic: can one measure all these states in a realistic experiment? If not, the black-hole information-loss problem is reformulated rather than exorcised \cite{5}.

The third Labour is to tame the microscopic quantum fluctuations in the foamy space-time background. We have argued that the most appropriate framework for describing particles in interaction with an unmeasured (and unmeasurable) recoiling background is an effective non-critical string theory \cite{5}, which allows for information loss and entropy growth. Scattering might be describable asymptotically by a superscattering matrix $S$, that is not factorizable as a simple product of the $S$ matrix and its hermitean conjugate $S^\dagger$. In order to achieve its immortal apotheosis, string theory may need to shed its mortal $S$-matrix origins and criticality to become non-critical $\text{String theory}$. In many physical applications, a sub-asymptotic time-evolution equation is useful, and we advocate \cite{7} a generalized quantum Liouville equation for the density matrix \cite{8}:

$$\dot{\rho} = i\left[H, \rho\right] + \mathcal{H}\rho \tag{1}$$

where $\mathcal{H}$ represents the interaction of the observed system with a quantum-gravitational background ‘environment’ associated with microscopic event horizons.

The fourth Labour is to describe quantum cosmology. For this, the first requirement was to formulate string theory in a suitable Robertson-Walker-Friedman background. An early approach to this problem showed how a classical non-critical string theory could be used to accommodate a time-dependent scale factor \cite{9}. Subsequently, the issue of entropy generation during a primordial inflationary epoch was addressed, and a framework based on quantum non-critical $\text{String theory}$ was proposed \cite{10}. More recently, this approach was extended to asymptotically large times, and this $\text{String theory}$ was used to suggest that relaxation of the quantum-gravitational metric might make a time-dependent contribution to the cosmological vacuum energy \cite{11}, sufficient to accelerate the Universe even in the absence of any other contributions to the vacuum energy. This suggestion was linked intimately to the entropy growth intrinsic to $\text{String theory}$, that is generically associated with event horizons - in this case macroscopic.

The observational evidence for an accelerating Universe is becoming overwhelming, with cosmic microwave background measurements \cite{12} converging on a flat Universe containing a low baryon density consistent with predictions based on cosmological nucleosynthesis – conferring credibility on the parameter fits – and vacuum energy with a density about the two-thirds of the critical flat-space value – consistent with interferences from large-scale structure and high-redshift supernovae \cite{13}.

This is a cause for concern within critical-string theory \cite{14}: ‘a challenge for string theory
as presently defined’ [13] and ‘the current sets of concepts in string theory will not be sufficient
to give a coherent description of our Universe’ (our italics) [15], or ‘quintessence, very much
like a cosmological constant, presents a serious challenge for string theory’ [16].

The purpose of this paper is to recall [5] that non-critical $tring theory provides a suitable
decohering framework for a quantum description of the Universe, outlining its observables, and
discussing the dynamical equations they obey. We extend our previous discussion of vacuum
energy within this framework. We develop an explicit example in which non-zero $\beta$ functions
for $tring \sigma$-model couplings cause a departure from criticality and cosmological vacuum energy
that relaxes to zero à la quintessence.

Critical string theory is described by a conformal two-dimensional field theory ($\sigma$ model)
on the world sheet. Non-critical $tring theory [17] is formulated in terms of non-conformal
renormalizable world-sheet field theories, described by an action

$$A[g] = A[g^*] + \int d^2z \sqrt{\gamma} g^iV_i$$

(2)

where the \{g^i\} denote the couplings of massless background field deformations \{V_i\} that are
not exactly marginal, \{g^*\} represents the equilibrium conformal background around which we
perturb, and $\gamma \equiv e^{\phi} \gamma^*$, where $\phi$ is the world-sheet Liouville field and $\gamma^*$ denotes a reference
world-sheet metric. We interpret $\phi$ as a world-sheet renormalization scale [5], so that non-
conformal scale dependence is compensated by Liouville dressing: the gravitationally-dressed
operators $V_i \to [V_i]_{\phi}$ are exactly marginal in a renormalization-group sense. We adopt the Wilson
renormalization-group-picture, according to which non-trivial scaling in the effective theory
consisting only of the massless modes \{g^i\} follows from integrating out the other degrees of
freedom. In an accelerating Universe, these would include the degrees of freedom that disappear
through the event horizon.

We have argued [5] that they also include the quantum degrees of freedom associated with
microscopic topological fluctuations in the space-time foam. In the case of a two-dimensional
space-time black hole model, it was shown explicitly that the vertex operator for a massless
‘tachyon’ field alone is not exactly marginal, but only becomes so in combination with a vertex
operator for a higher-level non-local degree of freedom associated with the black hole [18] We
further argued that the quantum state of a two-dimensional string black hole is characterized
by the quantum numbers associated with a complete set of such non-local vertex operators,
that we termed $W$ hair [18]. We conjectured that this type of characterization of a quantum
black hole could be extended to four dimensions, and just such a solitonic understanding of the
quantum states of four-dimensional black holes was subsequently attained [4]. However, in our
view, observable physics is obtained by integrating out the non-local vertex operators, leaving
behind a non-conformal effective theory for the massless mode, that is not exactly marginal [19].

What kind of animal is this effective theory? Critical string theory has a Lorentz-invariant
S matrix, but we cannot expect the same for a non-critical $tring theory [3, 20]. This obeys
a renormalization-group equation that expresses its dependence on the Liouville field $\phi$. We
identify [3] the (zero mode) of $\phi$ with the target time variable. This identification is supported
by (i) the fact that $tring theory is super-critical in general [3], giving $\phi$ a negative metric [3].
\[ (-ve) \int d^2z \; \partial \phi \bar{\partial} \phi, \]  

(ii) the recovery of the conventional Lorentz-invariant $S$-matrix description on the critical limit, and (iii) the recovery of the two-dimensional black-hole metric in an explicit soluble non-critical example \[13\]. Upon the identification of $\phi$ with time, the renormalization-group equations become the dynamical equations governing the time evolution of observables.

![Figure 1: Contour of integration for a proper definition of the path integration for the Liouville field, where the quantity $A$ denotes the (complex) world-sheet area. This is known in the literature as the Saalschutz contour, and has been used in conventional quantum field theory to relate dimensional regularization to the Bogoliubov-Parasiuk-Hepp-Zimmermann renormalization method. Upon the interpretation of the Liouville field with target time, this curve resembles closed-time-paths in non-equilibrium field theories.](image)

We recall briefly our formal arguments \[20\] supporting the point of view that it is impossible to define a unitary $S$ matrix in Liouville strings, with the only well-defined object being the $S$ matrix. This follows from the fact that an $N$-point world-sheet correlation function of vertex operators in Liouville strings, $\mathcal{F}_N \equiv \langle V_{i_1} \ldots V_{i_N} \rangle$, where $\langle \ldots \rangle$ signifies a world-sheet expectation value in the standard Polyakov treatment, transforms under infinitesimal Weyl shifts of the world-sheet metric $\gamma$ in the following way \[20\]:

\[
\delta_{\text{weyl}} \mathcal{F}_N = \left[ \delta_0 + \mathcal{O} \left( \frac{s}{A} \right) \right] \mathcal{F}_N
\]

where the standard part $\delta_0$ of the variation involves a sum over the conformal dimensions $h_i$ of the operators $V_i$ that is independent of the world-sheet area $A$ (whose logarithm is the world-sheet zero mode of the Liouville field). The quantity $s$ is the sum of the gravitational anomalous dimensions \[17\]:

\[
s = -\sum_{i=1}^{N} \frac{\alpha_i}{\alpha} - \frac{Q}{\alpha}, \quad \alpha_i = -\frac{Q}{2} + \frac{1}{2} \sqrt{Q^2 + 4(h_i - 2)}, \quad \alpha = -\frac{Q}{2} + \frac{1}{2} \sqrt{Q^2 + 8}
\]

where $Q$ is the central-charge deficit, that is non-zero for non-critical string.

From \[3\], and upon the identification of the logarithm of the world-sheet area $\ln A$ with the target time $t$ \[5\], one observes that such a correlator cannot represent a unitary $S$-matrix element in target space, as a result of the explicit $A- \text{ (i.e. time } t-\text{) dependence. Nevertheless, according to the analysis in }[5, 20], \text{ it is clear that one can construct a well-defined quantity free from such}
world-sheet area ambiguities. However, this is a $ matrix that is not factorizable into a product of $ and $'. This results from the way one performs the world-sheet path integration over the Liouville mode $ over a first-quantized approach to string theory. This integration can be done via a steepest-descent method [21,5,20], along the curve in Fig. [3], which is reminiscent of the closed time-like path used in non-equilibrium field theories [22]. Close to the ultraviolet fixed point on the world sheet, i.e., in the limit when $ → 0, there are divergences. These are responsible for the lack of factorization of the $ matrix in this case. The definition of the Liouville path integral over such a curve is responsible, in this formalism, for the appearance of a density-matrix rather than a wave-function interpretation of the world-sheet partition function of the Liouville string, in full agreement with its non-equilibrium (open-system) nature.

This impossibility of defining a consistent unitary $-matrix element in Liouville string, only a $ matrix, provides an argument that non-critical string theory may be the key [11] to a resolution of the accelerating Universe puzzle, where the presence of a cosmological (particle) horizon seems to prevent a consistent definition of a unitary $ matrix [14,15,16].

We now return to our discussion of qualitative features of Liouville (effective) string theory. The state of the observable system may be characterized by a density matrix $.$ The full density matrix of string theory might in principle be a pure-state density matrix $ ρ = |Ψ><Ψ|$. However, in our$-picture $|Ψ> = |ψ,ψ⟩$, where $|ψ⟩$ denotes a state of the observable system and $|ψ⟩$ the unobserved degrees of freedom that are integrated out in the renormalization-group approach. The reduced observable density matrix $ ρ ≡ f dψ ρ = f dψ|ψ,ψ⟩<ψ,ψ|$ will in general be mixed as a result of entanglement with the unobserved degrees of freedom $|ψ⟩$, e.g., those that disappear across the macroscopic event horizon in an accelerating Universe, or across a microscopic event horizon in a model of space-time foam.

The renormalization-group equation for the reduced density matrix $ ρ$ is easy to derive and can be cast in the form (1), with an explicit form for the non-Hamiltonian operator $δH$ in terms of the non-conformal field couplings $g^i$: $δH = β^i G_{ij} g^j$ (5)

where the $β^i$ are the non-trivial renormalization functions of the $g^i$, and $G_{ij}$ is a suitable metric in the space of these couplings [3]. An equation of the form (1) is familiar from the theory of quantum-mechanical systems in interaction with an environment, that is provided in our case by the modes $|ψ⟩$ that disappear across the event horizon (5).

It is natural to ask whether one can write down a wave equation for the observable subsystem $|ψ⟩$, and an answer is provided by the theory of open quantum-mechanical systems. One may represent (1,2) as a stochastic Ito process for $|ψ⟩ [23]:$ $Q|dψ⟩ = -iH|ψ⟩ + ∑_e (<B_e^†>ψ B_e - 1/2 B_e^†B_e) |ψ⟩dt + (∑_e (<B_e^-<B_e>ψ)|ψ⟩dξ_e)|ψ⟩dξ_e$ (6)

where $H$ is the Hamiltonian of the subsystem $|ψ⟩$, and the $B_e, B_e^†$ are ‘environment’ operators that may be defined as appropriate ‘square roots’ of the various partitions of the operator.
$\beta^i G_{ij} g_j$. The $d\xi_e$ are complex differential random variables associated with stochastic Wiener or Brownian processes, which represent the environmental effects that are averaged out by a low-energy local observer.

In the cosmological context, we have proposed applying this formalism to the problem of inflation [10]. The key issue there may be regarded as the generation of a large amount of entropy. Whenever one has entanglement with an environment over which one integrates, as in the non-critical String formalism, one necessarily encounters entropy growth. In the renormalization-group formalism [1, 2], there is a simple formula for the rate of growth of the entropy:

$$\dot{S} = G_{ij} \beta^i \beta^j$$

which is positive semi-definite. If any of the renormalization functions $\beta^i \neq 0$, entropy will necessarily grow. Whether the entropy might have grown during a primordial epoch sufficiently rapidly to have generated the observed entropy in the Universe is a dynamical question.

The most appropriate equation for addressing this dynamical question may be the Fokker-Planck equation corresponding to (1) and (6) [24, 10]:

$$\frac{\partial}{\partial t} P(g, t) = \frac{1}{8\pi^2} \delta g^i \delta^3 g^j \left[ Q^2 P(g, t) \right] + \frac{\delta}{\delta g^i} \left[ \beta^i P(g, t) \right]$$

modulo quantum ordering ambiguities for the $g$-dependent diffusion coefficients $Q$. The quantity $P(g, t)$ denotes the probability distribution in the theory space of strings, i.e., the probability of finding the system in a configuration $\{g^i\}$ at time $t$. The quantity $Q^2$ denotes the central charge deficit of the non-critical Liouville theory, which is a measure of the deviation of the theory from criticality. In our context such deviations arise by, e.g., splitting the field modes into those inside the particle horizon in a cosmological context, and those that lie outside and hence are essentially unobservable by low-energy local observers. A stochastic equation like (8) arises formally when one splits a field mode $g^i$ at a time instant $t + \delta t$ as follows:

$$g^i(t + \delta t) = g^i_c(t) + \dot{g}^i_c(t) \delta t + \delta q^i(\delta t)$$

The first two terms in (9) describe the conventional classical motion of an inflaton field determined by the gradient of an effective potential, and the remaining term in (9) corresponds to quantum fluctuations in its motion, which play a key role in chaotic inflationary models [24], for example. The average of the quantum fluctuations is connected to the diffusion coefficient $Q^6$ as follows: $< (\delta q)^2 > = \frac{Q^6}{4\pi^2} \delta t$.

The inflationary paradigm is now being subjected to observational tests of ever-increasing precision [12]. Not only has the first acoustic peak been measured accurately, but the second peak has been discovered and there is good evidence for a third [12]. In addition to the mass density and other cosmological parameters, the spectral index of the inflationary perturbations is highly constrained and increasingly sophisticated tests of Gaussianity are being applied. The conventional field theories derived from string theory provide no convincing candidate for an inflaton, and the inflationary scale may not be many decades away from the scale of quantum gravity. There is a need for a String rethink of the inflationary paradigm, and measurements
of the cosmic microwave background radiation might open an observational window on string theory.

Another window is perhaps being provided by the increasing evidence for cosmological vacuum energy \([12]\). As an explicit example how this may arise in non-critical string theory, we consider the model proposed in \([11]\), in which our four-dimensional world is viewed as a three-brane ‘punctured’ with heavy, quantum-fluctuating \(D\) particles. As discussed in \([11]\), in the context of string theory, these induce a background physical metric of the following Friedmann-Robertson-Walker form:

\[
G_{00} = -a^2, \quad G_{ij} = t^4 \delta_{ij}
\]  

(10)

There is also a non-trivial dilaton field, which at times larger than any spatial scale in the problem assumes the form \([11]\):

\[
\Phi(t) = -2\ln t
\]  

(11)

In general, for subasymptotic times, the dilaton also exhibits a complicated spatial dependence, but for cosmological models the time dependence (11) is sufficient.

It can be readily shown that (10),(11) are solutions to Einstein and dilaton equations of motion obtained from the following effective action (in Einstein frame) with a non-trivial vacuum-energy term

\[
S = \int d^4x \sqrt{G} \left( -R + (\nabla_\mu \Phi)^2 + \xi e^{\zeta \Phi} \right)
\]  

(12)

where \(\zeta = 1\), and

\[
\xi e^{\zeta \Phi} = \frac{20}{a^2 t^2} M_s^4, \quad \xi = \frac{20}{a^2 M_s^4}.
\]  

(13)

Here \(M_s\) is the string scale and, in our sign conventions, the positive sign of the vacuum energy corresponds to a de Sitter-type Universe. The normalization constant \(a^2\) cannot be determined in this framework, and is considered as a free parameter \([6]\).

There is an alternative, more stringy way, however, to interpret the results (10),(11), which is appropriate for a non-critical string framework. One may consider the metric and dilaton functions (10), (11) as couplings in a non-critical stringy \(\sigma\)-model with central charge deficit \(Q^2\). As we shall see below, this allows the normalization constant \(a^2\) to be determined by consistency with the generalized conformal invariance conditions of Liouville strings \([17, 25, 10]\):

\[
\ddot{g}_i + Q \dot{g}_i = G^{00} \beta_i
\]  

(14)

where \(Q\) is the square root (with either sign) of the central charge deficit. In the (toy) case of bosonic string, \(Q^2 = \frac{1}{3}(c_{\text{total}}[g] - 25)\), where \(c_{\text{total}}[g]\) is the total central charge of the non-conformal ‘matter’ theory, the \(g_i\) are the non-conformal \(\sigma\)-model couplings (omitting the dilaton), and the \(\beta^i\) are the appropriate renormalization-group \(\beta\) functions. The renormalization-group \(\beta\) functions in string theory are replaced by the so-called Weyl anomaly coefficients, \(\tilde{\beta}_i\), taking into account the general coordinate diffeomorphism invariance of the target space of

---

\(^1\)The constancy of \(\xi\) is consistent with Lorentz invariance: all the time dependence of the vacuum energy being due to the dilaton field \(\Phi(t)\).
the σ model, which is assumed to be respected by the non-critical string. The dilaton Weyl coefficient \( \tilde{\beta}_\Phi \) is given by the difference of the overall central charge deficit of the matter theory plus the Liouville contributions and the ghost contributions. In a Liouville framework, as a result of the restoration of conformal invariance, this difference vanishes. Hence for the dilaton one has the condition:

\[
\tilde{\beta}_\Phi = c_{\text{total}} + c_{\text{Liouville}} - 26 = 0
\]

(15)

On the other hand, the dilaton β function is determined in terms of the rest of the β functions by certain consistency relations stemming from world-sheet renormalizability. In our interpretation of the Liouville field as a local world-sheet scale [5], these consistency relations are assumed to be valid for the Liouville-dressed couplings as well.

The Minkowski sign of \( G^{00} < 0 \) on the right-hand side of (14) is that found for the relevant supercritical-string case [9]. The overdot denotes differentiation with respect to the target time, which is identified in our approach with the Liouville mode. This is an important difference of our approach [10] from apparently similar standard Liouville string approaches [17, 25], where the Liouville mode is simply viewed as an extra dimension of the Liouville-dressed string. For us, the non-conformal deformations are dressed by the Liouville mode, but the overall target-space dimension remains the same under the identification of the Liouville mode with the target time. For consistency, this can only happen for supercritical strings, where the Liouville mode is timelike [9]. Previously, we have verified that the \( t \leftrightarrow \phi \) identification reproduces correctly the metric of the two-dimensional string black hole [5]. In this case, the fact that there are solutions of the generalized conditions (14, 15) is a non-trivial consistency check on our approach.

In the problem at hand, \( g_i \equiv \{ G_{\mu \nu}, \mu, \nu = 0, \ldots 3 \} \). For the graviton and dilaton backgrounds that we are considering here, the Weyl coefficients \( \tilde{\beta}^i \) are given to \( \mathcal{O}(\alpha') \) by

\[
\tilde{\beta}_\mu^\nu = R_{\mu \nu} + 2\gamma \nabla_\mu \partial_\nu \Phi, \quad \tilde{\beta}_\Phi = -R + 4\gamma^2 (\nabla_\mu \Phi)(\nabla_\nu \Phi)G^{\mu \nu} - 4\gamma \nabla^2 \Phi + Q^2
\]

(16)

where \( \gamma \) is an appropriate normalization factor to be determined by consistency with the form (14). Using the explicit expression for the dilaton Weyl anomaly coefficient \( \tilde{\beta}_\Phi \), one can easily check the consistency of our approach, in which the Liouville mode \( \rho \) is identified with the target time \( t \), respecting (15). The condition (15) is satisfied by the \( \tilde{\beta}_\Phi \) appropriate for a σ model in \( d + 1 \) dimensions, the extra dimension being provided by the Liouville mode \( \rho \), with \( Q_{\text{total}}^2 = 0 \), as dictated by the restoration of the conformal invariance by the Liouville field [17]. In such a σ model, one has a dilaton coupling of the generic form dictated by Liouville dynamics [17, 1, 3]:

\[
\Phi(t, \vec{x}, \rho) = \Phi(t, \vec{x}) + \frac{1}{2\gamma} Q \rho
\]

where \( Q \) is the central-charge deficit of the matter theory, e.g., that caused by the quantum fluctuations (‘recoil’) of the \( D \) particles in the specific example [14] considered here. Interpreting \( \rho \) as a local world-sheet scale, one has \( \frac{d}{d\rho} Q = 0 \) on account of world-sheet renormalizability. In this case, then, one may split the \( d + 1 \)-dimensional tensor quantities appearing in \( \tilde{\beta}_\Phi \) into Liouville (\( \rho \)) components and the rest, taking into account the above properties of the dilaton. It is then seen immediately that upon such a splitting, taking into account the cosmological backgrounds of interest to us here and the fact that the metric component in the \( \rho \rho \) direction is a constant, and, finally, identifying the Liouville mode \( \rho \) with the target time \( t \), one arrives easily at the expression for the \( \tilde{\beta}_\Phi \) given in (16), where now the central-charge deficit \( Q^2 \) of the ‘matter’ theory has appeared explicitly. It is this last expression, then, which should vanish according to (15).
It can be checked that there are two non-trivial solutions to the above system of equations \((14, 15, 16)\):
\[
\gamma = 3/2, \quad a^2 = 0.72, 14.41, \quad -Q = \frac{1}{4} a^2 t \left( \frac{22}{a^2} + 12a^2 \right) \approx 13.61 / t, \frac{3.03}{t}.
\]
\[17\]

We recall that the induced central charge deficit, \(Q^2\), in our approach is defined with the appropriate dilaton exponential factors \([5]\) stemming from the \(\sigma\)-model formalism. It, therefore, plays the rôele of a vacuum energy \(\Lambda\) in our non-critical string theory \([9, 5]\), and thus, in the above example, it relaxes to zero asymptotically as \(t^{-2}\) for large times, where the analysis is valid \([11]\):
\[
\Lambda = Q^2 \sim 185 \frac{M_4^4}{t^2} \quad \text{for} \quad a^2 = 0.72,
\]
\[
\Lambda = Q^2 \sim 9 \frac{M_4^4}{t^2} \quad \text{for} \quad a^2 = 14.41.
\]
\[18\]

The same scaling has been found in the Einstein frame approach above \([13]\), and this justifies this identification of \(Q^2\) with \(\Lambda\). Moreover, although the equations \((14),(15)\) are formally different from the conventional equations encountered in Einsteinian gravity, the \(\sigma\)-model metric \((10)\) we find is of Robertson-Walker form, and may be identified with the physical metric \([4]\). For \(M_4 \sim \frac{1}{10} M_P \sim 10^{18} \text{ GeV},\) and \(t \sim 10^{60}\) in Planck time units, the results \((13),(18)\) are consistent with the current phenomenology \([12]\).

The metric \((10)\) has a Friedman-Robertson-Walker form with scale factor \(a(t)^2 = t^4\) and spatial curvature \(k = 0\), and, as we have shown, satisfies Einstein’s equations with a scalar (dilaton) field \(\phi\) responsible for the appearance of a cosmological constant. The proper horizon distance in such a spatially-flat universe is given by:
\[
\delta H(t) = a(t)c \int_0^\infty \frac{dt'}{a(t')} = ct^2/t_0^2
\]
\[19\]

where \(c\) is the speed of light \textit{in vacuo}. This is constant in the specific model considered here, but we recall that there are Liouville-string based models in which there is a time-dependent \(c(t)\) \[29\]. We observe from \((13)\) that the expression for \(\delta H(t)\) is finite, and hence there is a future horizon. This implies that the \(S\)-matrix description of particle scattering breaks down in such a Universe, which is consistent with the generic Liouville framework, as discussed previously.

It is interesting to estimate the possible magnitude of non-Hamiltonian terms in the evolution equation \((1)\) today. We have previously suggested a maximum possible magnitude \([3, 20]\)
\[
|\phi H| \sim \frac{E^2}{m_P}
\]
\[20\]

for such terms originating from microscopic space-time foam effects. The experimental sensitivity to tests of quantum mechanics in the neutral-kaon system approaches the possible magnitude.

\footnote{We note that the set of equations \((14)\) satisfies the Helmholtz conditions \([10]\), and so can be derived from an \textit{off-shell} action subject to canonical quantization. However, they characterize an out-of-equilibrium system that does not lead to a well-defined \(S\)-matrix.}
In the case of effects associated with the macroscopic event horizon, we note that an event horizon of the scale of the observable Universe would emit De Sitter radiation with a temperature

\[ T \sim H \sim 10^{-35}\text{eV} \sim 10^{-31}\text{K} \]

which appears somewhat distant from experimental possibilities, since the microwave background radiation itself has a temperature \( \simeq 2.7\,\text{K} \), and the background neutrino radiation is expected to have \( T \sim 1.9\,\text{K} \). Thus, the possibility of a macroscopic event horizon today, whilst a profound theoretical challenge, may not be of much experimental significance.

As explained in this paper, our response to this theoretical challenge is rooted in non-critical string theory, or $\text{string theory}$ [3]. We had argued previously that this was the most appropriate dynamical framework for the effective low-energy theory obtained by integrating out higher string modes in a renormalization-group approach, and had applied it to cosmology [10, 11]. In the $\text{string}$ framework, the basic dynamical object is the density matrix, the $S$ matrix is abandoned in favour of the superscattering operator $\$ \neq SS^\dagger$ [6], and canonical Hamiltonian evolution is supplemented by a non-Hamiltonian term in the quantum Liouville equation reminiscent of open quantum-mechanical systems [8]. As we have shown in this paper, string theory can certainly accommodate non-zero vacuum energy, and might even generate a time-varying value à la quintessence.

Acknowledgements

J.E. thanks Farhad Ardalan for an encouraging discussion. The work of D.V.N. is partially supported by DOE grant DE-F-G03-95-ER-40917. N.E.M. and D.V.N. also thank H. Hofer for his interest and support.

References

[1] G. Veneziano, G. Veneziano, Nuovo Cim. A57, 190 (1968).
[2] J. Scherk and J. H. Schwarz, Phys. Lett. B52, 347 (1974).
[3] M. B. Green and J. H. Schwarz, Phys. Lett. B149, 117 (1984).
[4] For a recent review and references, see: S. R. Das and S. D. Mathur, gr-qc/0105063.
[5] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Erice Subnuclear Series Vol. 31 (World Scientific, Singapore, 1993), p. 1.

\footnote{It has been suggested [27] that models with a time-varying speed of light [28] may remove the cosmological horizon in an accelerating Universes, obviating the concerns [13, 16]. We recall that such a time-varying effective speed of light arises quite naturally in Liouville $\text{string}$ theory, where it was proposed as a way to avoid the horizon problem in inflationary models [24], and later revived by [31]. However, in our Liouville context, a time-varying speed of light cannot save the $S$ matrix.}
[6] S. W. Hawking, Commun. Math. Phys. 87, 395 (1982).

[7] J. Ellis, N. E. Mavromatos and D. V. Nanopoulos, hep-th/9305117 and Mod. Phys. Lett. A10, 425 (1995).

[8] J. Ellis, J. S. Hagelin, D. V. Nanopoulos and M. Srednicki, Nucl. Phys. B241, 381 (1984).

[9] I. Antoniadis, C. Bachas, J. Ellis and D. V. Nanopoulos, Phys. Lett. B211, 393 (1988) and Nucl. Phys. B328, 117 (1989).

[10] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Mod. Phys. Lett. A10 (1995) 1685.

[11] J. Ellis, N. E. Mavromatos and D. V. Nanopoulos, Gen. Rel. Grav. 32, 943 (2000).

[12] A. T. Lee et al., MAXIMA Collaboration, astro-ph/0104453; C. B. Netterfield et al., BOOMERANG Collaboration, astro-ph/0104460; N. W. Halverson et al., DASI Collaboration, astro-ph/0104489; P. de Bernardis et al., astro-ph/0105290.

[13] S. Perlmutter et al., Supernova Cosmology Project Collaboration, Astrophys. J. 517, 565 (1999); A. G. Riess et al., Supernova Search Team Collaboration, Astron. J. 116, 1009 (1998); A. G. Riess et al., astro-ph/0104457; N. Bahcall, J. P. Ostriker, S. Perlmutter and P. J. Steinhardt, Science 284, 1481 (1999).

[14] T. Banks and W. Fischler, hep-th/0102077.

[15] S. Hellerman, N. Kaloper and L. Susskind, hep-th/0104180.

[16] W. Fischler, A. Kashani-Poor, R. McNees and S. Paban, hep-th/0104181.

[17] F. David, Mod. Phys. Lett. A3, 1651 (1988); J. Distler and H. Kawai, Nucl. Phys. B321, 509 (1989).

[18] J. Ellis, N. E. Mavromatos and D. V. Nanopoulos, Phys. Lett. B267, 465 (1991); Phys. Lett. B284, 27; (1992) and ibid. 43 (1992).

[19] J. Ellis, N. E. Mavromatos and D. V. Nanopoulos, Phys. Lett. B 293, 37 (1992).

[20] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Chaos, Solitons and Fractals, Vol 10, 345 (1999) hep-th/9805120, and references therein; Phys. Rev. D 63, 024024 (2001).

[21] I. Kogan, Phys. Lett. B265, 269 (1991).

[22] The CTP formalism is due to: J. Schwinger, J. Math. Phys. 2, 407 (1961). For reviews, see: E. Calzetta and B.L. Hu, Phys. Rev. D37, 2878 (1988); E. Calzetta, S. Habib and B.L. Hu, Phys. Rev. D37, 2901 (1988); H. Umezawa, Advanced Field Theory: micro, macro and thermal concepts (American Inst. of Physics, N.Y. 1993).

[23] N. Gisin and I. C. Percival, J. Phys. A26, 2233 (1993) and ibid. 2245 (1993).

[24] A.D. Linde and A. Mezhlumian, Phys. Rev. D49, 1783 (1994).
[25] C. Schmidhuber and A. A. Tseytlin, Nucl. Phys. B 426, 187 (1994).

[26] R. Adler et al., CPLEAR collaboration, and J. Ellis, J. Lopez, N.E. Mavromatos, D.V. Nanopoulos, Phys. Lett. B 364, 239 (1995).

[27] J. W. Moffat, hep-th/0105017.

[28] J. W. Moffat, Int. J. Mod. Phys. D 2, 351 (1993) [gr-qc/9211020];

[29] J. Ellis, N. E. Mavromatos and D. V. Nanopoulos, Recent Advances in the Superworld, Proc. of HARC meeting, 14-16 April 1993, (World Sci., 1994), p. 1 [hep-th/9311148].

[30] A. Albrecht and J. Magueijo, Phys. Rev. D 59, 043516 (1999); J. D. Barrow, Phys. Rev. D 59, 043515 (1999); B. A. Bassett, S. Liberati, C. Molina-Paris and M. Visser, Phys. Rev. D 62, 103518 (2000).