Efficient LDPC Codes over GF(q) for Lossy Data Compression

Alfredo Braunstein
Politecnico di Torino
Dipartimento di Fisica
Corso Duca Degli Abruzzi 24, 10129, Torino, Italy
Email: alfredo.braunstein@polito.it

Farbod Kayhan
Politecnico di Torino
Dipartimento di Elettronica
Corso Duca Degli Abruzzi 24, 10129, Torino, Italy
Email: farbod.kayhan@polito.it

Riccardo Zecchina
Politecnico di Torino
Dipartimento di Fisica
Corso Duca Degli Abruzzi 24, 10129, Torino, Italy
Email: riccardo.zecchina@polito.it

Abstract—In this paper we consider the lossy compression of a binary symmetric source. We present a scheme that provides a low complexity lossy compressor with near optimal empirical performance. The proposed scheme is based on \( b \)-reduced ultra-sparse LDPC codes over GF\((q)\). Encoding is performed by the Reinforced Belief Propagation algorithm, a variant of Belief Propagation. The computational complexity at the encoder is \( O(< d > \cdot n \cdot q) \), where \(< d > \) is the average degree of the check nodes. For our code ensemble, decoding can be performed iteratively following the inverse steps of the leaf removal algorithm. For a sparse parity-check matrix the number of needed operations is \( O(n) \).

I. INTRODUCTION

In this paper we address lossy compression of a binary symmetric source. Given any realization \( y \in \{0,1\}^n \) of a \( \text{Ber}(\frac{1}{2}) \) source \( Y \), the goal is to compress \( y \) by mapping it to a shorter binary vector such that an approximate reconstruction of \( y \) is possible within a given fidelity criterion. More precisely, suppose \( y \) is mapped to the binary vector \( x \in \{0,1\}^k \) with \( k < n \) and \( \hat{y} \) is the reconstructed source sequence. The quantity \( R = \frac{k}{n} \) is called the compression rate. The fidelity or distortion is measured by the Hamming distance \( d_H(y, \hat{y}) = \frac{1}{n} \sum_{i=1}^{n} |y_i - \hat{y}_i| \). The goal is to minimize the average Hamming distortion \( D = \mathbb{E}[d_H(Y, \hat{Y})] \) for any given rate. The asymptotic limit, known as the rate-distortion function, is given by \( R(D) = 1 - H(D) \) for any \( D \in [0,0.5] \) where \( H(D) = -D \log_2 D - (1-D) \log_2 (1-D) \) is the binary entropy function.

Our approach in this paper is based on Low-Density Parity-Check (LDPC) codes. Let \( \mathcal{C} \) be a LDPC code with \( k \times n \) generator matrix \( G \) and \( m \times n \) parity check matrix \( H \). Encoding in lossy compression can be implemented like decoding in error correction. Given a source sequence \( y \), we look for a codeword \( \hat{y} \in \mathcal{C} \) such that \( d_H(y, \hat{y}) \) is minimized. The compressed sequence \( x \) is obtained as the \( k \) information bits that satisfies \( \hat{y} = G^T x \).

Even though LDPC codes have been successfully used for various types of lossless data compression schemes [4], and also the existence of asymptotically capacity-achieving ensembles for binary symmetric sources has been proved [21], they have not been fully explored for lossy data compression. It is partially due to the long standing problem of finding a practical source-coding algorithm for LDPC codes, and partially because Low-Density Generator Matrix (LDGM) codes, as dual of LDPC codes, seemed to be more adapted for source coding and received more attention in the few past years.

In [20], Martinian and Yedidia show that quantizing a ternary memoryless source coding with erasures is dual of the transmission problem over a binary erasure channel. They also prove that LDGM codes, as dual of LDPC codes, combined with a modified Belief Propagation (BP) algorithm can saturate the corresponding rate-distortion bound. Following their pioneering work, LDGM codes have been extensively studied for lossy compression by several researchers [8], [9], [15], [18], [19], [27]. In a series of parallel work, several researches have used techniques from statistical physics to provide non-rigorous analysis of LDGM codes [5], [12] and [24].

In terms of practical algorithms, lossy compression is still an active research topic. In particular, an asymptotically optimal low complexity compressor with near optimal empirical performance has not been found yet. Almost all suggested algorithms have been based on some kind of decimation of BP or SP which suffers a computational complexity of \( O(n^2) \) [3], [9] and [27]. One exception is the algorithm proposed by Murayama [24]. When the generator matrix is ultra sparse, the algorithm was empirically shown to perform very near to the associated capacity needing \( O(n) \) computations. A generalized form of this algorithm, called reinforced belief propagation (RBP) [2], was used in a dual setting, for ultra sparse LDPC codes over GF(2) for lossy compression [14]. The main drawback in both cases is the non-optimality of ultra sparse structures over GF(2) [3], [15], [24]. As we will see, this problem can be overcome by increasing the size of the finite field.

Our simulation show that \( b \)-reduced ultra sparse LDPC codes over GF\((q)\) achieve near capacity performance for \( q \geq 64 \). Moreover, we propose an efficient encoding/decoding scheme based on RBP algorithm.

The rest of this paper is organized as follows. Section II reviews the code ensemble which we use for lossy compression.
Section III describes the RBP algorithm over GF(q). We also discuss briefly the complexity and implementation of the RBP algorithm. In section IV we describe iterative encoding and decoding for our ensemble and then present the corresponding simulation results in section V. A brief discussion on further research is given in Section VI.

II. LDPC Codes over GF(q)

In this section we introduce the ultra sparse LDPC codes over GF(q). As we will see later, near capacity lossy compression is possible using these codes and BP-like iterative algorithms.

A. \((\lambda,\rho)\) Ensemble of GF(q) LDPC codes

We follow the methods and notations in [13] to construct irregular bipartite factor graphs. What distinguishes GF(q) LDPC codes from their binary counterparts is that each edge \((i,j)\) of the factor graph has a label \(h_{i,j} \in GF(q) \setminus \{0\}\). In other words, the non-zero elements of the parity-check matrix of a GF(q) LDPC code are chosen from the non-zero elements of the field GF(q). Denoting the set of variable nodes adjacent to a check node \(j\) by \(N(j)\), a word \(c\) with components in GF(q) is a codeword if at each check node \(j\) the equation \(\sum_{i \in N(j)} h_{i,j} c_i = 0\) holds.

A \((\lambda,\rho)\) GF(q) LDPC code can be constructed from a \((\lambda,\rho)\) LDPC code by random independent and identically distributed selection of the labels with uniform probability from \(GF(\lambda) \setminus \{0\}\) (for more details see [11]).

B. Code Construction for Lossy Compression

It is well known that the parity check matrix of a GF(q) LDPC code, optimized for binary input channels, is much sparser than the one of a binary LDPC code with same parameters \([11, 6]\). In particular, when \(q > 2^5\), the best error rate results on binary input channels is obtained with the lowest possible variable node degrees, i.e., when almost all variable nodes have degree two. Such codes have been called ultra sparse or cyclic LDPC codes in the literature. In the rest of this paper we call a LDPC code ultra sparse (US) if all variable nodes have degree two and the parity check’s degree distribution is concentrated for any given rate. It is straightforward to show that for a US-LDPC code defined as above check node degrees has at most two non-zero values and the maximum check node degree of the code is minimized.

Given a linear code \(C\) and an integer \(b\), a \(b\)-reduction of \(C\) is the code obtained by randomly eliminating \(b\) parity-check nodes of \(C\). For reasons to be cleared in section IV we are mainly interested in \(b\)-reduction of GF(q) US-LDPC codes for small values of \(b\) \((1 \leq b \leq 5)\). Note that by cutting out a parity check node from a code, the number of codewords is doubled. This increment of the codewords has an asymptotically negligible effect on the compression rate since it only increases by \(1/n\) while the robustness may increase.

GF(q) US-LDPC codes have been extensively studied for transmission over noisy channels \([13, 7, 6]\). The advantage of using such codes is twofold. On the one hand, by moving to sufficiently large fields, it is possible to improve the code.

On the other hand, the extreme sparseness of the factor graph is well-suited for iterative message-passing decoding algorithms. Despite the state of the art performance of moderate length GF(q) US-LDPC channel codes, they have been less studied for lossy compression. The main reason being the lack of fast suboptimal algorithms. In the next section we present RBP algorithm over GF(q) and then show that practical encoding for lossy compression is possible by using RBP as the encoding algorithm for the ensemble of \(b\)-reduced US-LDPC codes.

III. Reinforced Belief Propagation Algorithm in GF(q)

In this section first we briefly review the RBP equations over GF(q) and then discuss in some details the complexity of the algorithm following Declercq and Fossorier [7].

A. BP and RBP Equations

The GF(q) Belief Propagation (BP) algorithm is a straightforward generalization of the binary case, where the messages are \(q\)-dimensional vectors.

Let \(\mu_{v,f}^{\ell}\) denotes the message vector form variable node \(v\) to check node \(f\) at the \(\ell\)th iteration. For each symbol \(a \in GF(q)\), the \(a\)th component of \(\mu_{v,f}^{\ell}\) is the probability that variable \(v\) takes the value \(a\) and is denoted by \(\mu_{v,f}^{\ell}(a)\). Similarly, \(\mu_{f,v}^{\ell}\) denotes the message vector from check node \(f\) to variable node \(v\) at the iteration \(\ell\) and \(\mu_{f,v}^{\ell}(a)\) is its \(a\)th component. Also let \(\mathcal{N}(v) (\mathcal{M}(f))\) denote the set of check (variable) nodes adjacent to \(v (f)\) in a given factor graph.

Constants \(\mu_{v}^{0}\) are initialized according to the prior information. The BP updating rules can be expressed as follows:

**Local Function to Variable:**

\[
\mu_{f,v}^{\ell+1}(a) \propto \sum_{\text{Conf}(v,f)} \prod_{a' \in \mathcal{M}(f) \setminus \{v\}} \mu_{v,f}^{\ell}(a')
\]

**Variable to Local Function:**

\[
\mu_{v,f}^{\ell+1}(a) \propto \mu_{v}^{\ell}(a) \prod_{f' \in \mathcal{N}(v) \setminus \{f\}} \mu_{f',v}^{\ell}(a)
\]
RBP is a generalization of BP in which the messages from variable nodes to check nodes are modified as follows

$$
\mu_{u,v}^{f+1}(a) \propto (g_v^{f}(a))^{\gamma(f)}\mu_v^{f}(a) \prod_{f' \in N(v) \setminus \{f\}} \mu_{f',v}^{f}(a), \quad (4)
$$

where $g_v^{f}$ is the marginal function of variable $v$ at iteration $f$ and $\gamma(f) : [0,1] \rightarrow [0,1]$ is a non-decreasing function. Also the equation for each marginal function is changed as below

$$
\mu_u'(a) \propto (g_u'(a))^{\gamma'(u)}\mu_u^{\gamma(u)}(a) \prod_{f' \in N(u)} \mu_{f',u}^{\gamma(u)}(a). \quad (5)
$$

It is convenient to define $\gamma$ to be

$$
\gamma(f) = 1 - \gamma_0\gamma_1,
$$

where $\gamma_0, \gamma_1$ are in $[0,1]$. Note that when $\gamma_1 = 1$, RBP is the same as the algorithm presented in [24] for lossy data compression. In this case it is easy to show that the only fixed points of RBP are configurations that satisfy all the constraints.

### B. Efficient Implementation

Ignoring the normalization factor in (2), to compute all variable to check-node messages at a variable node of degree $d_v$ we need $\mathcal{O}(d_v d_f)$ computations. A naive implementation of GF($q$) BP has computational complexity of $\mathcal{O}(d_f^2 q^2)$ operations at each check node of degree $d_f$. This high complexity is mainly due to the sum in (1), that can be interpreted as a discrete convolution of probability density functions. Efficient implementations of function to variable node messages based on Discrete Fourier Transform have been proposed by several authors, see for example [25], [17], [11], [7] and the references within. The procedure consists in using the identity

$$
\bigcap_{v' \in M(f) \setminus \{v\}} \mu_{v'} = \mathcal{F}^{-1}\left( \prod_{v' \in M(f) \setminus \{v\}} \mathcal{F}(\mu_{v'}) \right),
$$

where the symbol $\mathcal{O}$ denotes convolution.

Assuming $q = b^p$, the Fourier transform of each message $\mu_{v'}$ needs $\mathcal{O}(q,p)$ computations and hence the total computational complexity at check node $f$ can be reduced into $\mathcal{O}(d_f^2 q^2 p)$. This number can be further reduced to $\mathcal{O}(d_f q p)$ by using the fact that $\prod_{v' \in M(f) \setminus \{v\}} \mathcal{F}(\mu_{v'}) = \prod_{v' \in M(f)} \mathcal{F}(\mu_{v'}) / \mathcal{F}(\mu_{v})$, or alternatively by using the summation strategy described in [3] which has the same complexity but is numerically more stable. Therefore, the total number of computations per iteration is $\mathcal{O}(d f > .q.p.n)$

where $< d >$ is the average degree.

### IV. Iterative Lossy Compression

In the following three subsections we first describe a simple method for identifying information bits of a $b$-reduced US-LDPC code and then present a near capacity scheme for iterative compression (encoding) and linear decompression (decoding).

#### A. Identifying a Set of Information Bits

For $b$-reduced US-LDPC codes, one can use the leaf removal (LR) algorithm to find the information bits in a linear time. In the rest of this section we briefly review the LR algorithm and show that $1$-reduction (removal of a sole check node) of a US-LDPC code significantly changes the intrinsic structure of the factor graph of the original code.

The main idea behind LR algorithm is that a variable on a leaf of a factor graph can be fixed in such a way that the check node to which it is connected is satisfied [22]. Given a factor graph, LR starts from a leaf and removes it as well as the check node it is connected to. LR continues this process until no leaf remains. The residual sub-graph is called the core. Note that the core is independent of the order in which leaves (and hence the corresponding check nodes) are removed from the factor graph. This implies that also the number of steps needed to find the core does not depend on the order on which leaves are chosen.

While US-LDPC codes have a complete core, i.e. there is no leaf in their factor graph, the $b$-reduction of these codes have empty core. Our simulations also indicate that even $1$-reduction of a code largely improves the encoding under RBP algorithm (see section [V]). How RBP exploits this property is the subject of ongoing research. It is straightforward to show that a code has empty core if and only if there exists a permutation of columns of the corresponding parity-check matrix $H$ such that $h_{ij} \neq 0$ for $i = j$ and $h_{ij} = 0$ for all $i > j$.

As we have mentioned, LR algorithm can be also used to find a set of information bits of a given US-LDPC code. At any step $t$ of LR algorithm, if the chosen leaf is the only leaf of the check node $f_t$ into which it is connected, then its value is determined uniquely as a function of non-leaf variables of check node $f_t$. If the number of leaves $d_i$ is greater than 1, there are $2^{d_i-1}$ configurations which satisfy the check node after fixing the values of non-leaf variables. At each step of LR we choose a subset of $d_i - 1$ leaves. This set is denoted by $F_t^{LR}$ and we call it the free subset at $t^{th}$ step. Note that there are $d_i$ free subsets among which we choose only one at each step. It is straightforward to show that the union of all free subsets $F = \cup_t F_t^{LR}$ is a set of information bits for a given US-LDPC code.

#### B. Iterative Encoding

Suppose a code of rate $R$ and a source sequence $y$ is given. In order to find the codeword $\hat{y}$ that minimizes $d_H(\hat{y},y)$, we will employ the RBP algorithm with a strong prior $\mu_v^{\gamma(u)}(a) = \exp(-Ld_H(y_v,a))$ centered around $y$. The sequence of information bits of $\hat{y}$ is the compressed sequence and is denoted by $x$. In order to process the encoding in GF($q$), we first need to map $y$ into a sequence in GF($q$). This can be simply done by grouping $b$ bits together and use the binary representation of the symbols in GF($q$).
C. Linear Decoding

Given the sequence of information bits $x$, the goal of the decoder is to find the corresponding codeword $y$. This can be done by calculating the $G^T x$ which in general needs $O(n^2)$ computations. One of the advantages of our scheme is that it allows for a low complexity iterative decoding. The decoding can be performed by iteratively fixing variables following the inverse steps of the LR algorithm; at each step $t$ only one non-information bit is unknown and its value can be determined from the parity check $f_i$. For a sparse parity-check matrix, the number of needed operations is $O(n)$.

V. SIMULATION RESULTS

A. Approximating the Weight Enumeration Function by BP

Given an initial vector $y$, and a probability distribution $P(c)$ over all configurations, the $P$-average distance from $y$ can be computed by

$$D_P(y) = \sum_{c} \sum_{y_i} P(c_i) d_H(c_i, y_i)$$

(6)

where $P(c_i)$ is the set of marginals of $P$. On the other hand, the entropy of the distribution $P$ is defined by

$$S(P) = -\sum_{c} P(c) \log P(c).$$

(7)

Even though it is a hard problem to calculate analytically both marginals and $S(P)$ of a given code, one may approximate them using messages of the BP algorithm at a fixed point [26]. Assuming the normalized distance is asymptotically a self-averaging quantity for our ensemble, $S(P)$ represents the logarithm of the number of codeword at distance $D_P(y) + O(1)$ from $y$. By applying a prior distribution on codewords given by $\exp(-Ld_H(c, y))$ one is able to sample the sub-space of codewords at different distances from $y$.

Fig. 1 demonstrates the WEF of random GF(q) US-LDPC codes for rates 0.3, 0.5, and 0.7 and field orders 2, 4, 16, 64 and 256. The blocklength is normalized so that it corresponds to $n = 12000$ binary digits.

Though BP is not exact over loopy graphs, we conjecture that the WEF calculated for US-LDPC codes is asymptotically exact. This hypothesis can be corroborated by comparing the plot in Fig. 1 with the simulation results we obtained by using RBP algorithm (Fig. 3).

B. Performance

In all our simulations the parameter $\gamma_1$ of RBP algorithm is fixed to one and therefore the function $\gamma$ is constant and does not depend on the iterations. We also fix the maximum number of iterations into $t_{\text{max}} = 300$. If RBP does not converge after 300 iterations, we simply restart RBP with a new random scheduling. The maximum number of trials allowed in our simulations is $T_{\text{max}} = 5$. The encoding performance depends on several parameters such as $\gamma_0$, $L$, the field order $q$, and the blocklength $n$. In the following we first fix $n$, $q$ and $L$, in order to see how the performance changes as a function of $\gamma_0$.

VI. DISCUSSION AND FURTHER RESEARCH

Our results indicate that the scheme proposed in this paper outperforms the existing methods for lossy compression by low-density structures in both performance and complexity. The main open problem is to understand and analyze the behavior of RBP over $b$-reduced US-LDPC codes.

As we have mentioned, $b$-reduction of a US-LDPC code not only provides us with simple practical algorithms for finding information bits and decoding, but also largely improves the convergence of RBP. It is interesting to study the ultra sparse
even though average number of iterations show a steep increase as a function of $\gamma_0$. For $\gamma_0 > 0.96$ the RBP does not converge within 300 iterations. The averages are taken over 50 samples. (a) Average distortion as a function of $\gamma_0$. For $\gamma_0 > 0.96$ the RBP does not converge within 300 iterations. The averages are taken over 50 samples. (b) The average number of trials. (d) The average number of iterations needed for each trial. Note that even though average number of iterations show a steep increase as a function of $\gamma_0$, the average number of iterations needed per trial increases only linearly.

Fig. 2. Performance as a function of $\gamma_0$ for a PEG graph with n=1600 and R=0.33. The averages are taken over 50 samples. (a) Average distortion as a function of $\gamma_0$. For $\gamma_0 > 0.96$ the RBP does not converge within 300 iterations. (b) The average number of iterations. (c) The average number of trials. (d) The average number of iterations needed for each trial. Note that even though average number of iterations show a steep increase as a function of $\gamma_0$, the average number of iterations needed per trial increases only linearly.

Fig. 3. The rate-distortion performance of $GF(q)$ LDPC codes encoded with RBP algorithm for $q = 2, 16$ and 256. The blocklength is 12000 binary digits and each point is the average distortion over 50 samples.

ensembles where a certain fraction of variable nodes of degree one is allowed.

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