RR Lyrae Period-Amplitude Diagrams:
From Bailey to Today

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Abstract

More than a century ago, Solon Bailey’s pioneering investigations of the variable stars in globular clusters allowed the first period-amplitude diagrams to be constructed for their RR Lyrae stars. These diagrams differ from cluster to cluster, and there has been debate as to whether these differences are correlated mainly with [Fe/H] or with Oosterhoff type. It is clear now that a cluster’s Oosterhoff type plays an important role in determining its period-amplitude relation, although the Oosterhoff dichotomy itself is correlated with metallicity. Not all clusters follow the usual patterns, however. The globular clusters NGC 6388 and NGC 6441 have period-amplitude diagrams similar to those of metal-poor Oosterhoff type II globular clusters, but they themselves are comparatively metal-rich. The period-amplitude diagrams of Oosterhoff-intermediate systems are discussed.

1. Introduction

The study of the RR Lyrae period-amplitude diagram, as with much else concerning RR Lyrae stars in globular clusters, commences with the work of Solon I. Bailey and his collaborators, who pioneered the photographic investigation of globular cluster variables. Indeed, a plot of period versus amplitude for RR Lyrae stars is now sometimes termed a Bailey diagram, although that name does not seem to have been applied to such diagrams until the 1990s.

The a, b, and c Bailey types for RR Lyrae stars were introduced in his study of
variable stars in the globular cluster ω Centauri (Bailey 1902). These are now usually condensed to just two types, the RRab stars that pulsate in the fundamental radial mode and the RRc stars that pulsate in the first overtone radial mode. More recently, RRab stars have been termed RR0 stars and RRc variables have been termed RR1 stars (Alcock et al. 2000a).

The close association of the Bailey a and b types is apparent in the period-amplitude diagrams that can be constructed even from the earliest photographic observations. In Figure 1, we plot the period versus the blue photographic amplitude of RR Lyrae stars in the globular cluster M15 based upon data from Bailey et al. (1919). There is a clear distinction between the Bailey c-type and the Bailey a- and b- type variables, but the latter form only a single sequence, though with considerable scatter. Later studies have shown that while some of this scatter can be attributed to observational error, some is intrinsic to the stars. One source of scatter, but not the only source, is the Blazhko effect. The amplitudes of RR Lyrae variables that undergo the Blazhko effect change during the secondary Blazhko cycle, complicating the determination of the correct amplitude to use in constructing the period-amplitude diagram (Smith 1995).
The Oosterhoff dichotomy is shown in this plot of the mean period of RRab stars versus [Fe/H] for Galactic globular clusters. Also shown are the two unusual clusters NGC 6388 and NGC 6441, labeled Oosterhoff III for convenience. These clusters are discussed in Section 3. See Catelan (2009) for more details.

### 2. The Period-Amplitude-[Fe/H] relation

The location of RR Lyrae stars in the period-amplitude diagram is different for different globular clusters, begging the question of what is responsible for the differences. One of the most prominent differences is the distinction between globular clusters of Oosterhoff type I and Oosterhoff type II. RRab stars in Oosterhoff I clusters have mean periods near 0.55 day, while those in Oosterhoff II clusters have mean periods near 0.64 day (Oosterhoff 1939; Smith 1995; Clement et al. 2001; Catelan 2009). Moreover, Oosterhoff I clusters tend to be less deficient in metal abundance than clusters of Oosterhoff II (Figure 2). Preston’s (1959) pioneering ∆S study of the metal abun-
Figure 3.— Period-amplitude relations for RRab stars in the Oosterhoff I cluster M3 and the Oosterhoff II cluster ω Cen, based upon Cacciari et al. (2005) and Kaluzny et al. (2004). RR Lyrae stars in ω Cen are unusual in having a significant range in [Fe/H] within a single cluster. The solid lines indicate typical locations of the RRab period-amplitude relations for Oosterhoff I and Oosterhoff II globular clusters (Clement & Shelton 1999).

dances of field RR Lyrae stars demonstrated that they, too, show a decrease in average period with increasing metallicity.

In Figure 3, we compare the period-amplitude diagrams of RRab stars in the Oosterhoff I cluster M3 and the Oosterhoff II cluster ω Cen. The ω Cen variables are shifted to a longer period at a given amplitude than the M3 variables. In addition, although known Blazhko effect stars have been excluded from this figure, a real scatter among the stars of each cluster remains.

Sandage (1958), employing the pulsation equation \( P \sqrt{\rho/\rho_\odot} = Q \), suggested that the RR Lyrae stars in Oosterhoff II clusters were about 0.2 magnitudes brighter in \( V \) than those in Oosterhoff I clusters. In the early 1980s, Sandage extended this result to general relationships between the location of RR Lyrae stars in the period-amplitude diagram and both absolute magnitude and [Fe/H] (Sandage, Katem, & Sandage 1981; Sandage 1982). In this work, Sandage introduced the parameter \( \Delta \text{log} P \), indicating the shift in the period of a star of a given amplitude relative to a star of the same amplitude in the mean period-amplitude relation for the globular cluster M3. Sandage
RR Lyrae period-amplitude diagrams

(1982) obtained the relation $\Delta \log P = 0.116 [\text{Fe/H}] + 0.173$.

Others have more recently adopted versions of this period-amplitude-[Fe/H] relation to determine metal abundances for RRab stars (e.g. Alcock et al. 2000b; Sandage 2004; Kinemuchi et al. 2006). These relations imply the existence of a continuous correlation between $\Delta \log P$ and both the luminosity and the metal abundance of an RRab star. However, it was noted that the mean error in the [Fe/H] values determined by this method compared to those derived from spectroscopic methods could sometimes be large, 0.3 to 0.4 in [Fe/H] (Sandage 2004; Kinemuchi et al. 2006).

The robustness of the correlation between the Bailey diagram and [Fe/H] has been questioned. Clement & Shelton (1999) found that the $V$ amplitude for a given period was not a function of metal abundance, but was instead a function of Oosterhoff type. This would mean that there was a sharp jump in the Bailey diagram at the transition between Oosterhoff I and Oosterhoff II clusters. Bono et al. (2007) also found that the Oosterhoff dichotomy rather than [Fe/H] was the critical factor in the determination of the period versus amplitude diagram for RRab stars. Kunder & Charboyer (2009) found that $\Delta \log P$ for field RRab stars in the Galactic bulge did not correlate well with [Fe/H] values determined by other methods. Despite these shortcomings, the relatively metal-rich field RRab stars of the thick disk and bulge do tend to have periods shorter than those of RRab stars in Oosterhoff I globular clusters at a given amplitude (see, for example, Figure 2 in Kunder & Charboyer (2009). Moreover, since the Oosterhoff I RRab stars have shorter periods than RRab stars in the still more metal-poor Oosterhoff II clusters, a rough correlation between the location of the Bailey diagram and [Fe/H] does exist.

The Bailey diagrams of the Oosterhoff I cluster M3 and the Oosterhoff II cluster M2 do, however, illustrate the importance of Oosterhoff type. These two globular clusters have similar metallicities. M3 is at [Fe/H] = -1.57 and M2 is at [Fe/H] = -1.62, according to Zinn & West (1984). Thus, were [Fe/H] the main determinant of the location of the cluster RRab stars in the Bailey diagram, RR Lyrae stars in M2 and M3 would be expected to fall along similar loci in the period-amplitude diagram. Instead, as shown in Figure 4, the M2 stars are shifted to longer periods on average than those in M3.

It is also noteworthy, however, that the period-amplitude diagrams of RRab stars in the most metal-poor Oosterhoff II clusters, such as M15 and M68, show considerable scatter, with many stars falling into the region between the mean trends of the two Oosterhoff groups (e.g. figure 4 in Cacciari et al. 2005 and figures 5 and 6 in Corwin et al. 2008). Thus, Oosterhoff type alone does completely describe the location of the RR Lyrae stars in the Bailey diagram.
Figure 4.— Period-amplitude diagrams for RR Lyrae stars in the Oosterhoff I cluster M3 (from Cacciari et al. 2005) and the Oosterhoff II cluster M2 (from Lee & Carney 1999). These two clusters have similar values of [Fe/H]. The solid lines are as in Figure 3.

3. NGC 6388 and NGC 6441: Oddballs

In Figure 2, two clusters are plotted that have large values of $\langle P_{ab} \rangle$ but also relatively high metal abundances. These are the unusual globular clusters NGC 6388 and NGC 6441. For convenience, these clusters have been denoted as Oosterhoff III clusters, to mark their distinction from the other clusters in Figure 2. Studies of the RR Lyrae stars in these two clusters (Layden et al. 1999; Pritzl et al. 2001, 2002, 2003; Corwin et al. 2006) show that they have period-amplitude diagrams similar to those in Oosterhoff II systems (Figure 5). Yet the metal abundances of the clusters are relatively high, about [Fe/H] = -0.6 (Armandroff & Zinn 1988; Clementini et al. 2005), comparable to the metal abundances of the shorter-period RRab stars of the thick disk and bulge.

Color-magnitude diagrams of NGC 6388 and NGC 6441 (Rich et al. 1997; Pritzl et al. 2001, 2003; Yoon et al. 2008) show that these two clusters have not only the stubby red horizontal branches usually expected in metal-rich globular clusters, but also blue horizontal branch components that extend through the instability strip. The long periods of the RR Lyrae stars imply that they are more luminous than field RR
Figure 5.— Period-amplitude diagram for RR Lyrae stars in NGC 6441 (Pritzl et al. 2001, 2003). The solid lines are the same as in Figure 3.

Lyrae stars of comparable metallicity which have shorter periods. It has been suggested that these globular clusters contain more than a single stellar population (Catelan et al. 2006; Yoon et al. 2008; Moretti et al. 2009), with the smaller population responsible for the blue horizontal branch component possibly being enhanced in helium. Though NGC 6388 and NGC 6441 are globular clusters that are far from ordinary, they again show that factors other than [Fe/H] can determine the location of the RR Lyrae period-amplitude diagram.

4. Oosterhoff-Intermediate Clusters

As shown in Figure 2, globular clusters in the Milky Way avoid the Oosterhoff gap, the region of $\langle P_{ab} \rangle$ around 0.60 days. On the other hand, globular clusters and dwarf galaxies in the satellite systems of the Milky Way do not show the Oosterhoff dichotomy but populate the Osterhoff gap, as shown in Figure 6 (see also Catelan 2009; Smith et al. 2009). There are at least two alternative ways by which nature might produce a value of $\langle P_{ab} \rangle$ intermediate between the two Oosterhoff groups. The intermediate value of $\langle P_{ab} \rangle$ might result from a mixture of stars of Oosterhoff types I and II, or
Figure 6.— The Milky Way’s dwarf companion galaxies and their globular clusters have been added to the Galactic globular clusters shown in Figure 2. The Oosterhoff gap is erased by these additions. See Catelan (2009) and Smith et al. (2009) for more details.

The intermediate mean period might indicate the existence of a Bailey diagram falling between those of the two Oosterhoff groups. A possible third way of producing an Oosterhoff-intermediate cluster is discussed below.

Figure 7 shows the Bailey diagram for one of these Oosterhoff-intermediate systems, the Draco dwarf spheroidal galaxy (Kinemuchi et al. 2008), which has a value of $\langle P_{ab}\rangle = 0.615$ days. It is clear from this figure that the intermediate period in this case does not arise from an even split of the periods between the two Oosterhoff groups. Instead, the individual RRab stars scatter along a mean line that falls slightly to the long period side of the Bailey diagram of an Oosterhoff I cluster.

Nonetheless, the Bailey diagrams of all Oosterhoff-intermediate systems are not identical. In Figure 8, we compare the Bailey diagram for the globular cluster...
Figure 7.—The Bailey diagram of the Draco dwarf spheroidal galaxy, an Oosterhoff-intermediate system (Kinemuchi et al. 2008). The solid lines are the same as in Figure 3. Double-mode RRd stars are plotted at the periods of their dominant first overtone periods.

NGC 1466, located in the outskirts of the Large Magellanic Cloud (Kuehn et al. 2011), to that of Draco. NGC 1466 has a mean period of $\langle P_{ab} \rangle = 0.59$ days, slightly shorter than that of Draco. The NGC 1466 RRab stars in Figure 8 fall in a location similar to those in Draco. However, the shortest period RRab star in NGC 1466 has a period significantly shorter than the shortest period RRab star in Draco. Draco contains many more RRab stars than does NGC 1466, so that this cannot be a consequence solely of small-number statistics.

In Figure 8, we also note that the bulk of the RRc stars in NGC 1466 are displaced to shorter periods than those in Draco. In part, this may reflect a different distribution of stars along the horizontal branch. However, the longest-period, first overtone mode pulsators in Draco (actually double-mode RR Lyrae with a dominant first overtone mode, as noted below) clearly occur at longer periods than those in NGC 1466. Taken together with the shorter period cutoff for the RRab stars in NGC 1466, this would be consistent with the transition between RRc and RRab stars in NGC 1466 occurring at a shorter fundamental mode period than for the Draco RR Lyrae. Note that such a change in the transition period might be a third way of producing clusters with
Figure 8.— The Bailey diagram of NGC 1466 is compared to that of the Draco dwarf spheroidal galaxy, an Oosterhoff-intermediate system (Kinemuchi et al. 2008). The solid lines are the same as in Figure 4. Double-mode RRd stars are plotted at the periods of their dominant first overtone periods.

Oosterhoff-intermediate values of $\langle P_{ab} \rangle$.

Both NGC 1466 and Draco contain double-mode RR Lyrae stars (RRd stars), pulsating simultaneously in the first overtone and fundamental radial modes (Kuehn et al. 2011; Nemec 1985; Kinemuchi et al. 2008). When these stars are plotted in the Petersen diagram in Figure 9, we see another distinction between the two systems. All except one of the Draco RRd stars have first overtone mode periods near 0.55 day and period ratios near 0.746, similar to the values usually seen among RRd stars in Oosterhoff type II globular clusters (Popielski et al. 2000). On the other hand, the NGC 1466 RRd stars have first overtone periods near 0.48 day and period ratios near 0.744, comparable to those seen among Oosterhoff type I globular clusters. In neither system do the RRd stars show properties intermediate between those of RRd stars in Oosterhoff I and II systems.
5. Discussion

While there is generally a rough correlation of the location of the period-amplitude diagram and [Fe/H], there are other factors, particularly Oosterhoff type, that determine the location of the period-amplitude relation. Thus, one cannot rely upon period and amplitude to obtain more than an approximate value of [Fe/H] for an RR Lyrae star. The method is, however, not entirely useless, and can often give an indication of metal abundance that can be valuable in the absence of other information.

The period-amplitude diagrams for RRab stars in the Oosterhoff-intermediate dwarf galaxies around the Milky Way appear to be genuinely intermediate, rather than a mixture of the period-amplitude relations of Oosterhoff I and Oosterhoff II stars. However, as the cases of NGC 1466 and Draco illustrate, a general similarity in the location of stars in the RRab period-amplitude relation does not imply an equal similarity among the RRcd stars. The location of the transition period between the RRab and RRcd stars, as well as the distribution of stars in color across the horizontal branch, may play a role in producing these differences.
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References

Alcock, C. et al. 2000a, ApJ, 542, 257
Alcock, C. et al. 2000b, AJ, 119, 2194
Armandroff, T. E., & Zinn, R. 1988, AJ, 96, 92
Bailey, S. I. 1902, Annals of Harvard College Observatory, 38, 1
Bailey, S. I., Leland, E. F., Woods, I. E., & Pickering, E. C. 1919, Annals of Harvard College Observatory, 78, 195
Bono, G., Caputo, F., & Di Criscienzo, M. 2007, A&A, 476, 779
Catelan, M., Stetson, P. B., Pritzl, B. J., Smith, H. A., Kinemuchi, K., Layden, A. C., Sweigart, A. V., & Rich, R. M. 2006, ApJ, 651, L133
Catelan, M. 2009, Ap&SS, 320, 261
Cacciari, C., Corwin, T. M., & Carney, B. W. 2005, AJ, 129, 267
Clement, C. M., & Shelton, I. 1999, ApJ, 515, L85
Clement, C. M., et al. 2001, AJ, 122, 2587
Clementini, G., Gratton, R. G., Bragaglia, A., Ripepi, V., Martinez Fiorenzano, A. F., Held, E. V., & Carretta, E. 2005, ApJ, 630, L145
Corwin, T. M., Sumerel, A. N., Pritzl, B. J., Smith, H. A., Catelan, M., Sweigart, A. V., & Stetson, P. B. 2006, AJ, 132, 1014
Corwin, T. M., Borissova, J., Stetson, P. B., Catelan, M., Smith, H. A., Kurtev, R., & Stephens, A. W. 2008, AJ, 135, 1459
Kaluzny, J., Olech, A., Thompson, I. B., Pych, W., Krzemiński, W., & Schwarzenberg-Czerny, A. 2004, A&A, 424, 1101
Kinemuchi, K., Smith, H. A., Woźniak, P. R., & McKay, T. A. 2006, AJ, 132, 1202
Kinemuchi, K., Harris, H. C., Smith, H. A., Silbermann, N. A., Snyder, L. A., LaCluyzé, A. P., & Clark, C. L. 2008, AJ, 136, 1921
Kuehn, C., et al. 2011, AJ, (submitted)
Kunder, A., & Chaboyer, B. 2009, AJ, 138, 1284
Layden, A. C., Ritter, L. A., Welch, D. L., & Webb, T. M. A. 1999, AJ, 117, 1313
Lee, J.-W., & Carney, B. W. 1999, AJ, 117, 2868
Moretti, A., et al. 2009, A&A, 493, 539
Nemec, J. M. 1985, AJ, 90, 204
Oosterhoff, P. T. 1939, The Observatory, 62, 104
Popielski, B. L., Dziembowski, W. A., & Cassisi, S. 2000, AcA, 50, 491
Preston, G. W. 1959, ApJ, 130, 507
Pritzl, B. J., Smith, H. A., Catelan, M., & Sweigart, A. V. 2001, AJ, 122, 2600
Pritzl, B. J., Smith, H. A., Catelan, M., & Sweigart, A. V. 2002, AJ, 124, 949
Pritzl, B. J., Smith, H. A., Stetson, P. B., Catelan, M., Sweigart, A. V., Layden, A. C., & Rich, R. M. 2003, AJ, 126, 1381
Rich, R. M., et al. 1997, ApJ, 484, L25
Sandage, A. 1958, Ricerche Astronomiche, 5, 41
Sandage, A., Katem, B., & Sandage, M. 1981, ApJS, 46, 41
Sandage, A. 1982, ApJ, 252, 553
Sandage, A. 2004, AJ, 128, 858
Smith, H. A. 1995, Cambridge Astrophysics Series, Cambridge, New York: Cambridge University Press, 1995
Smith, H. A., Catelan, M., & Clementini, G. 2009, in American Institute of Physics Conference Series, 1170, 179
Yoon, S.-J., Joo, S.-J., Ree, C. H., Han, S.-I., Kim, D.-G., & Lee, Y.-W. 2008, ApJ, 677, 1080
Zinn, R., & West, M. J. 1984, ApJS, 55, 45