COPLANAR GAS INFLOW CAN BE HIDDEN WITHIN WARPED GALACTIC GAS DISKS

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\textbf{ABSTRACT}

Simulations suggest that galactic gas disks can be treated as “modified accretion disks”, in which coplanar gas spirals into the inner regions of the disk, while being consumed by star-formation and removed by outflows. Observationally there is little evidence for such inflows within the outer disks of galaxies. Taking realistic gas surface densities from observations, the radial velocity of the inflow is only a few km s\(^{-1}\) within two scalelengths, but gradually increases with radius to of order 50-100 km s\(^{-1}\) at the very outer disk. The effects of this inflow on the 2-d velocity field are examined and shown to be broadly similar to those produced by warped disks, with twist distortions of both the kinematic major and minor axes. By examining the twists of kinematic distortions and the spiral arms for a sample of nearby galaxies, we find that the effect of warps are likely to dominate over the effect of radial inflows. However, we then model mock HI velocity fields that combine warps with inflow velocities of the strength required in the modified accretion disks, and show that these composite systems can actually also be very well matched by pure warped disk models, with \textasciitilde85% of the mock galaxies having a mean absolute error in the residuals of less than 10 km s\(^{-1}\). This suggests that the signatures of significant radial inflows can easily be “hidden” within the warps and that this may therefore explain the apparent failure to detect radial inflows in galactic disks.

\textit{Subject headings:} galaxies: kinematics and dynamics – galaxies: spiral – galaxies: ISM – methods: model

\section{1. INTRODUCTION}

It is well known that cold gas accretion is required to sustain star formation and size growth during the evolution of star-forming (SF) galaxies. The existing cold gas in SF galaxies can only sustain the star formation for a few billion years or less (Binney et al. 2000; Bigiel et al. 2011; Wang et al. 2013; Madau & Haardt 2015). This indicates the need for gas accretion to continually replenish the fuel for star formation (Fraternali & Tomassetti 2012).

Galaxies can accrete cold gas via capture of gas-rich dwarf galaxies and also through smooth accretion (e.g. Lacey & Cole 1994; Murali et al. 2002; Sancisi et al. 2004; Veronico 2009; Bouché et al. 2010; L'Huillier et al. 2012; Conselice et al. 2013; Sánchez Almeida et al. 2014; Rodriguez-Gomez et al. 2015). The former may be linked with the presence of extraplanar gas (Chaves & Irwin 2001; Wakker et al. 2007), gas tails (Kregel & Sancisi 2001; Oosterloo et al. 2010), and lopsided HI morphologies (Shang et al. 1998; Thilker et al. 2007). However, the estimation of neutral gas accretion rate by the high velocity neutral-gas clouds is a factor of 3-5 less than the observed star formation rate (SFR) for M31/Milky Way-type galaxies (Putman 2010; Richter 2012). This suggests the importance of the smooth gas accretion, but convinces evidence of this smooth accretion from observations is still lacking, and the geometry of such accretion is still correspondingly uncertain.

Hydrodynamical simulations can be a powerful tool to understand the accretion of gas. Multi-zoom cosmological simulations suggest that smooth accretion dominates the flow of gas, while mergers may be important for the most massive high-redshift galaxies (Murali et al. 2002; Kereš et al. 2009; L'Huillier et al. 2012). Gas accretion along filamentary streams that avoid being shock-heated in the outer halo is known as “cold-mode” accretion. This is thought to occur in low mass galaxies at high-redshift (Kereš et al. 2005; Dekel & Birnboim 2006; Ocvirk et al. 2008; Brooks et al. 2009; van de Voort et al. 2011; Stern et al. 2020). From more massive disk galaxies, the accretion from the cooling of hot halo gas may be more important (Kereš et al. 2005; Ocvirk et al. 2008; Nelson et al. 2013; Stern et al. 2020).

Multiple simulations based on different hydrodynamical codes and carried out by different research groups show that the inflowing gas is almost co-planar and more or less co-rotating with the gas disk, at least at low redshifts (z \textless 0.8), regardless of its thermal history (e.g. Kereš et al. 2005; Stewart et al. 2011; Danovich et al. 2015; Stewart et al. 2017; Stern et al. 2020; Péroux et al. 2020; Trapp et al. 2021; Hafen et al. 2022; Gurvich et al. 2022). In contrast, the outflow of gas that is driven by stellar winds and/or supernova (SN) explosions is preferentially along the direction that is perpendicular to the disk (e.g. Péroux et al. 2020; Trapp et al. 2021). These results are also consistent with the structure and kinematic of circumgalactic medium (CGM) traced by the Mg II absorption from the observations (Bordoloi et al. 2011; Bouché et al. 2012; Kacprzak et al. 2012; Diamond-Stanic et al. 2016; Bielby et al. 2017; Tumlinson et al. 2017; Péroux et al. 2017; Schreutter et al. 2019).

Motivated by these simulations, we constructed in previous papers a disk formation model that simply treats the gas disk of SF galaxies as an idealized “modified” (or “leaking”) accretion disk (MAD; Wang & Lilly 2022b, hereafter Paper I). In contrast to the classical accretion disks found around compact objects, the gas inflow in the MAD galactic disk gradually decreases towards the
galactic center as gas is consumed by star formation or removed from the disk by any associated outflows.

In Paper I, we investigated the possible mechanisms of how coplanar inflow could happen within a gas disk in terms of viscous processes within the disk. We showed that magneto-rotational instability is an attractive and plausible source of viscosity for the transport of mass inwards and angular momentum outwards within the gas disk. We showed that magnetic viscosity could naturally, under certain circumstances, produce the observed exponential profiles of SFR surface density (e.g. Wang et al. 2019) in galactic disks. However, two obvious questions raised by the simple MAD model (and actually independent of the source of the viscosity in the disk) concerned whether this idea was consistent with the observed metallicity profiles and with the observed kinematic maps of galactic gas disks.

In the second paper of this series (Wang & Lilly 2022a), we therefore examined the question of metallicity and found that the MAD model could account for the radial profiles of gas-phase metallicity for nearby galaxies. Not least, a negative gradient of gas-phase metallicity with radius is a natural consequence of the progressive enrichment of the gas by in-situ star formation as it flows inwards through the disk.

In the present work, the third paper of this series, we examine the kinematic properties of the inflowing gas in the MAD framework. We find that the velocity of the required coplanar inflow gradually increases with radius, and can reach 50-100 km s$^{-1}$ in the outskirts of the gas disk. Interestingly, based on IllustrisTNG-100 simulations (Nelson et al. 2018), Wang et al. (2022) have recently found that the radial velocity of inflowing gas increases with radius for SF galaxies in the simulation, and reaches $\sim 55$ km s$^{-1}$ at a radius of 10 times half-mass radius (see also Trapp et al. 2021; Hafen et al. 2022). The good consistency between the MAD model and hydrodynamical simulations is not surprising given that the idealized MAD model was motivated by the results of such simulations.

In contrast, however, by adopting a Fourier decomposition scheme to ten THINGS (The HI Nearby Galaxy Survey; Walter et al. 2008) galaxies, Schmidt et al. (2016) found no, or at best only very weak, radial velocities ($\sim 10$ km s$^{-1}$ or less) in the neutral hydrogen, even on the outskirts of the disk. More recently, Di Teodoro & Peek (2021) extracted the radial motion for 54 local disk galaxies from 21 cm emission datacubes. They also claimed that most galaxies show only very small radial velocities of a few km s$^{-1}$, positive and negative, throughout their HI disks, with no evidence of any systematic radial inflow and without any clear indications of an increase in flow velocity in the outermost regions.

The apparent inconsistency between these observational results and the expectations of both the MAD model, and more generally of the large body of hydrodynamical simulations, is rather puzzling and motivates further study of whether the signatures of radial inflow could have been missed in analyses of observational data.

The paper is organized as follows. In Section 2, we will briefly review the MAD model, and give the prediction for the radial profile of the required radial inflow velocity. We then in Section 3 explore the distortions that should be imprinted by such inflows on the 2-d projected line-of-sight (LOS) velocity fields of gas disks and compare these distortions with those expected from warping of the disks. The two distortion patterns are broadly similar, suggesting a loose degeneracy between these two effects. In Section 4, we apply a novel geometric analysis that is designed to exploit symmetry arguments to distinguish between these two possibilities. This is based on inferring the sense of rotation of the disk from the twist of the spiral arms, which are assumed to be trailing. This analysis actually suggests that, overall, the effect of warps on disk kinematics do indeed likely dominate over that of inflows. Motivated by this, in Section 5, we then construct mock datacubes and examine whether composite disks, in which there are both (dominant) warps plus inflows of the required strength, can be distinguished from systems in which warps alone are present. This allows us to address the question of whether radial flows of the expected size could in fact be “hidden” by the uncertainty in the physical parameters of the generally dominant warp. We then summarize this work in Section 6.

When quoting distance-dependent quantities, we adopt a flat cold dark matter cosmology model with $\Omega_m = 0.27$, $\Omega_{\Lambda} = 0.73$ and $H_0 = 70$ km s$^{-1}$Mpc$^{-1}$.

2. THE REQUIRED CO-PLANAR GAS INFLOW IN THE MODIFIED ACCRETION DISK MODEL

2.1. The brief recap of the MAD model

We have introduced and discussed the MAD model in Paper I and Wang & Lilly (2022a). Here we only present a brief summary of the key assumptions of this disk evolution model.

As mentioned in the Introduction, the MAD model is motivated by the gas exchange of galaxies in hydrodynamical simulations: co-planar inflow of gas dominates the gas accretion, while wind-driven outflows potentially leave perpendicular to the disk, especially for low redshift SF galaxies (e.g. Trapp et al. 2021; Hafen et al. 2022). The dominance of the radial inflow is the most important assumption in the MAD model. For simplicity, in addition to the co-planar inflow, we combine any ex-planar inflows with the expected ex-planar outflow (driven by stellar feedback) into an “effective outflow” and assume that this is always proportional to the instantaneous SFR surface density ($\Sigma_{\text{SFR}}$) multiplied by an effective mass-loading factor $\lambda$, which can be positive or mildly negative ($\lambda \gg -1$). Also for simplicity, this $\lambda$ is assumed to be the same at all radii of the disk.

We also assume that the local $\Sigma_{\text{SFR}}$ in the disk is instantaneously determined by the local gas surface density ($\Sigma_{\text{gas}}$) via some ”star formation law” expressed in terms of a star formation efficiency (SFE=$\Sigma_{\text{gas}}/\Sigma_{\text{SFR}}$). The local gas surface density is regulated by the interplay between the coplanar gas inflow, and the removal of gas by star formation and any associated outflows (e.g. Sommer-Larsen & Yoshii 1989; Thon & Meusinger 1998; Bouché et al. 2010; Schaye et al. 2010; Lilly et al. 2013). In other words, the gas disk can be treated as a “gas regulator system” (e.g. Lilly et al. 2013; Wang et al. 2019; Wang & Lilly 2021) at all galactocentric radii.

After star formation occurs, a fraction of the mass of the newly formed stars would be subsequently returned to the interstellar medium through stellar winds and su-
pernov explosion. We denote this return fraction as \( R \) and adopt the instantaneous recycling approximation. We take \( R = 0.4 \) from the Chabrier (2003) initial mass function (IMF) and stellar population models (Bruzual & Charlot 2003; Vincenzo et al. 2016).

Based on the above assumptions, the simple continuity equations for the conservation of the cold gas mass, the mass of metals, and the angular momentum are easily constructed (see equation 1 and 3 in Paper I and equation 2 in Wang & Lilly (2022a)). With the additional assumption of a steady-state of the gas disk and adopting from observations some \( \Sigma_{SFR} \) profile - e.g. an exponential profile (Bigiel et al. 2008; Wyder et al. 2009; Gonzalez-Lopeziriza et al. 2012; Gonzalez Delgado et al. 2016; Casasola et al. 2017; Wang et al. 2019, Paper I) - it is then straightforward to obtain the analytical solution of the inflow rate \( \Phi(r) \), the gas-phase metallicity \( Z(r) \), and the viscous stress required to maintain the \( \Sigma_{SFR} \).

In Paper I we found that the most plausible source of the viscosity in the accretion disk that is required in order to drive the gas inflow through the disk is magneto-rotational instability (MRI; Balbus & Hawley 1991). Therefore, we explored a disk evolution model that is driven by magnetic stresses. This successfully produces the exponential profile of \( \Sigma_{SFR} \) with reasonable scalelength, regardless of the initial conditions of the gas disk (Paper I), provided that there is a connection between the local magnetic field and the \( \Sigma_{SFR} \) of the form that has been seen observationally \( (B_{\text{tot}} \propto \Sigma_{SFR}^{0.15}) \).

The MRI-MAD combination may provide a solution for the long puzzling origin of the exponential form of star-forming disks in galaxies.

One of the striking features of this model is that the exponential form of \( \Sigma_{SFR}(r) \) does not depend on the assumed form of the star-formation law linking \( \Sigma_{SFR} \) and \( \Sigma_{gas} \), since the system will adjust to ensure the steady-state \( \Sigma_{SFR}(r) \). This is in fact a generic feature of gas-regulator models.

In addition, the resulting metallicity profile in the MAD model has a simple analytic form (see equation 12 in Wang & Lilly 2022a), which is solely determined by the \( \Sigma_{SFR} \) profile (e.g. the exponential scalelength of \( h_R \)), the mass-loading factor, and the metallicity of the gas at the outer boundary. It is, again, quite independent of the assumed SFE. The solution of \( Z(r) \) matches quite well the broad features of the extended metallicity profiles that are observed in nearby SF galaxies (Wang & Lilly 2022a). These all strengthen the idea that the simple MAD model is likely to closely approximate reality.

In this work, we focus on the radial motion of gas that is at the heart of the MAD model. We note that the detailed assumptions of chemical enrichment and rotational support of the gas disk that were adopted in Paper I and Wang & Lilly (2022a) are not required for deriving the radial velocity of coplanar inflow in this paper, and can be relaxed.

2.2. The inflow velocity required by the MAD model

The radial velocity of co-planar inflowing gas can be written as:

\[
v_r = \frac{\Phi(r)}{2 \pi r \Sigma_{gas}(r)}. \tag{1}
\]

In order to obtain the amplitude of \( v_r \), the \( \Sigma_{gas}(r) \) and the radial inflow rate \( \Phi(r) \) are needed.

In Paper I it was shown that, with the assumption of an exponential \( \Sigma_{SFR} \), the steady-state solution for the radial inflow rate can be written as:

\[
\frac{\Phi(r)}{1 - R + \lambda} = \text{SFR} \cdot [1 + \eta - (x + 1) \cdot \exp(-x)], \tag{2}
\]

where \( x \) is a scaled radius defined as \( x = r/h_R \), the SFR refers to the integrated SFR of the whole gas disk, and the factor \( \eta \) accounts for any mass sink at the disk center, e.g. due to black hole accretion and associated jet-driven outflow.

Unlike the solutions for \( \Phi(r) \) and \( Z(r) \), which depend only on the \( \Sigma_{SFR}(r) \) profile and the assumed \( \lambda \) and \( \eta \), the radial velocity \( v_r \) is sensitive to the \( \Sigma_{gas} \), i.e. effectively, given the input \( \Sigma_{SFR}(r) \), on the star formation law (or SFE) for the system. In Paper I, we assumed for simplicity that the Kennicutt (1998) star formation law applied everywhere, even though this star formation law is known to fail at the outskirts of gas disks (e.g. Leroy et al. 2008; Bigiel et al. 2010; Shi et al. 2011). In order to derive a more realistic \( v_r \) profile, we here adopt a more empirical \( \Sigma_{gas}(r) \) that is obtained directly from observations. We use from Bigiel & Blitz (2012):

\[
\Sigma_{gas} = 2.1 \Sigma_{trans} \cdot e^{-\frac{r}{R_{25}}}, \tag{3}
\]

where \( \Sigma_{trans} \) is the gas surface density at the radius where the HI gas becomes dominant, and \( R_{25} \) is defined as the 25 mag arcsec\(^{-2}\) B-band isophote (Schruba et al. 2011). The \( \Sigma_{trans} \) does not vary greatly from galaxy to galaxy, and has a typical value of 14 M\(_{\odot}\) pc\(^{-2}\) (Leroy et al. 2008; Bigiel et al. 2008; Bigiel & Blitz 2012). Adopting \( R_{25} =
3h_R (Casasola et al. 2017), we can then rewrite Equation 3 as:

$$\Sigma_{\text{gas}} = 29.4 \times e^{-\frac{r}{h_R}} [M_\odot \text{pc}^{-2}]$$

We note that the h_R is the scalelength of star-formation Σ_{SFR}, rather than of the stellar mass (or gas) in the disk.

With the assumption of Σ_{gas} in Equation 4, Figure 1 shows the required v_r(r) profiles as a function of radius in the MAD model. We show a typical SF galaxy with a stellar mass of M_*= 3 \times 10^{10} M_\odot (cf. figure 3 in Paper I). The integrated SFR of this typical galaxy is set to be 3.5 M_\odot yr^{-1} adopting the star formation main sequence from Lilly & Carollo (2016) at a redshift of zero (also see Noeske et al. 2007; Speagle et al. 2014; Renzini & Peng 2015). For illustration, we consider three different values of the scalelength of the star-forming disk, h_R = 3, 4, or 5 kpc, and two different values of the effective wind load factor λ − R =0 or 1.

As shown, the v_r is very small in the inner part of the disk: a few km s^{-1} within 4h_R, corresponding to the edge of stellar disk. This is consistent with previous work that the radial velocity required by chemodynamical models are only of the order of a few km s^{-1} (Bilitewski & Schönrich 2012; Pezzulli & Fraternali 2016; Wang & Lilly 2022a). However, the v_r increases rapidly beyond 4h_R, and can be as large as 50-100 km s^{-1} at 8h_R, depending on the assumed mass-loading factor. This is because the density of gas continues to drop but the required inflow stays more or less the same, being determined by the integrated SFR and outflow within that radius, so the inflow velocities must increase. This behaviour is consistent with the results of hydrodynamical simulations that the radial velocity of gas increases with radius and can reach 30-70 km s^{-1} at 10 times the half-mass radius (Trapp et al. 2021; Wang et al. 2022; Hafen et al. 2022).

For disk galaxies, the circular velocity of the gas disk typically increases rapidly in the inner regions, and then becomes flat out to the very outer regions of the gas disk (e.g. Courteau 1997; de Blok et al. 2008a). As in Paper I, we assume for simplicity that the circular velocity can be written in a simple arctan form (Miller et al. 2011):

$$v_{\phi}(r) = V_{\text{cir}} \cdot \frac{2}{\pi} \arctan(r/R_t),$$

where V_{cir} is the maximum circular velocity, and R_t characterizes the transition radius between the rising and flat parts of the rotation curve. For the typical SF galaxy mentioned above (see Figure 1), we adopt V_{cir} = 220 km s^{-1} and R_t = 0.5h_R (Miller et al. 2011, Paper I).

The left panel of Figure 2 shows both the circular velocity (black curve) and the radial velocity (red curve) as a function of radius for our typical SF galaxy, adopting for definiteness h_R=4 kpc and λ − R=1. One can thus obtain an idea of the trajectory of the inspiraling gas elements, assuming the azimuthal velocity is always close to circular, which is shown in the red helix of the right panel of Figure 2. As shown, the inflowing gas gradually spirals in towards the disk center. This is similar to the trajectories seen in the hydrodynamical simulations (see figure 2 in Hafen et al. 2022). For the specific case shown in Figure 2, it takes ~1 Gyr for a given gas element to move from 8h_R down to 2h_R, which is of the same order of the gas depletion timescale (Shi et al. 2011; Wang et al. 2019). The inflow velocity becomes very significant with respect to the circular velocity in the outskirts of gas disk (~8h_R or larger) and the motion of gas in the disk strongly deviates from circular motion. Hydrodynamical simulations also show that the motions of gas is no longer rotationally supported far beyond the stellar disk (Trapp et al. 2021; Hafen et al. 2022), where the gas particles fall towards with the conservation of their angular momentum.
3. THE SIGNATURES OF RADIAL INFLOW IN THE 2-D PROJECTED VELOCITY FIELD

In Section 2, we showed that the inflow velocity required in the MAD picture can be very significant far beyond the stellar disk (∼50-100 km s⁻¹). Such large radial motions are likely to significantly modify the observed LOS velocity field of gas disks relative to that of pure circular motion. In this section, we investigate the basic signatures of radial motion on the observed projected velocity fields.

Figure 3 shows the 2-d velocity field (projected onto the LOS) contributed by circular motion (left column), radial inflow (middle column), and the combination of the two (right column) for the same Main Sequence galaxy as above. The gas disk is assumed to be un-warped and to be seen with a (constant) inclination (INC) of 45 degrees, and to be rotating clockwise (in other words, the upper limb of the disk in each of the panels is closer to us than the lower limb). For illustration, we take λ − R = 0.0 and λ − R = 1.0 for the top and bottom row of panels. These therefore differ by a factor of two in the required v_r(r) profile.

As expected, the velocity field along the geometric major axis is only produced by the circular velocity component and that along the minor axis only by the radial velocity component. The velocity field with only circular motion is the classic “spider diagram” (e.g. Walter et al. 2008, and references therein), which is reflection-symmetric across the major axis. The velocity field with only radial motion is reflection-symmetric across the minor axis.

When both components are added, the composite velocity field shows a centrosymmetric distortion relative to the velocity field obtained with purely circular motion. It is noticeable that the kinematic major axis (white dashed line), defined to be the locus of the maximum projected velocity as a function of radius, is significantly deviated from the geometric major axis (white solid line) of the system, even though the radial velocities have, by definition, zero effect along the geometric major axis. Likewise, the kinematic minor axis, defined as the locus of zero projected velocity (relative to systemic), is similarly deviated from the geometric minor axis. The kinematic major axis and the iso-velocity contours are systematically twisted in a coherent spiral pattern. This deviation becomes more significant as the radial velocity increases towards the outer regions of the disk.

It is easy to derive the amplitude of this deviation analytically. The LOS velocity with both circular vφ(r) and radial v_r(r) motions can be written as (e.g. Begeman 1987):

\[ V_{\text{LOS}} = V_{\text{sys}} + \sin(i) \times (v_\phi \cos(\theta) + v_r \sin(\theta)), \]  (6)

where \( V_{\text{sys}} \) is the systemic velocity of the galaxy, \( i \) is the inclination angle (INC), and \( \theta \) is the azimuthal angle in the plane of the disk, with \( \theta = 0^\circ \) corresponding to the geometric major axis. Based on Equation 6, it is clear that the \( V_{\text{LOS}} \) reaches its extreme values relative to systemic velocity (i.e. the maximum and minimum observed values, defining the major kinematic axis) at

\[ \tan(\theta_{\text{major}}) = v_r / v_\phi. \]  (7)

The corresponding extremal \( V_{\text{LOS}} \) can then be written as:

\[ V_{\text{LOS}} = V_{\text{sys}} \pm \sin(i) \cdot (v_\phi^2 + v_r^2)^{1/2}. \]  (8)

Given the definition of \( \theta \), the deviation of the kinematic major axis from the geometric major axis, \( \Delta \theta_{\text{major}} \), is identical to \( \theta_{\text{major}} \) in Equation 7. Because \( \theta \) was defined in the plane of the disk, we can then convert this \( \Delta \theta_{\text{major}} \) to the deviation of the kinematic and geometric major axes seen in the plane of the sky (i.e. as observed) denoting azimuthal angles in the plane of the sky with \( \phi \):

\[ \Delta \phi_{\text{major}} = \arctan(v_r / v_\phi \cdot \cos(i)). \]  (9)

The Equation 9 is therefore the analytic solution of the white dashed line (the kinematic major axis) in the two rightmost panels of Figure 3.

In the same way, we can calculate the deviation of the kinematic minor axis from the geometric minor axis. The kinematic minor axis (denoted as \( \theta_{\text{minor}} \)) is here defined to be the locus where the LOS velocity equals the systemic velocity, \( V_{\text{sys}} \). Since the geometric minor axis lies (by definition) at \( \theta = 90^\circ \), the deviation of the kinematic minor axis from the geometric minor axis, \( \Delta \theta_{\text{minor}} \), is just given by \( \theta_{\text{minor}} - 90^\circ \).

Letting \( V_{\text{LOS}} = V_{\text{sys}} \), we can obtain the value of \( \theta_{\text{minor}} \) at the kinematic minor axis as:

\[ \tan(\theta_{\text{minor}}) = -v_r / v_\phi. \]  (10)

We can further obtain the deviation of the minor axis as:

\[ \tan(\Delta \theta_{\text{minor}}) = v_r / v_\phi. \]  (11)

In a similar way, we convert the \( \Delta \theta_{\text{minor}} \) of the disk plane to the one defined in the projected plane (or observed plane), which can be written as:

\[ \Delta \phi_{\text{minor}} = \arctan(v_r / v_\phi \cdot \cos(i)). \]  (12)

It is worth stressing that the radial motion distorts both the minor and major kinematic axes of the LOS velocity fields from the geometric ones, but with different amplitudes. The deviation of the minor axis (shown in Equation 12) is larger than the deviation of the major axis (shown in Equation 9), but this depends on the inclination of the disk.

A geometric warp in the disk can also cause radial changes of the major and minor axes, due to the straightforward change of the geometry of the disk relative to the viewing direction (e.g. Józsa et al. 2007; de Blok et al. 2008b; Kamphuis et al. 2015; Di Teodoro & Peek 2021). However, the changes in the minor and major axes due to a warp of the disk should be identical.

In principle this difference would enable these two signatures (due to radial velocities and due to a warp) to be distinguished. However, for weakly inclined disks, the \( \Delta \phi_{\text{major}} \) produced by inflows is close to \( \Delta \phi_{\text{minor}} \). This implies that the effect of radial motions in weakly inclined disks will resemble the effect of a warp in the disk, producing a degeneracy between the two.

Due to this similarity of the effects of radial motion and warped disk, it is rather difficult to decompose the two based on the velocity fields of the HI gas. In principle, the isophotes of the column density map of gas
could independently provide the geometric parameters of the disk, which potentially decomposes the above degeneracy. However, the gas distribution is usually clumpy along with spiral arms, which strongly increases the difficulty of this approach.

Recently, Di Teodoro & Peek (2021) performed a three-step fitting method to try to isolate the radial motion using datacubes of HI emission. In the first two steps, they fitted the warped geometry of the disk and the circular motion based on the location of the kinematic major axis. In the third step, they then fit the radial motion using the regions of the kinematic minor axis. This method effectively attributes the radial distortion of the kinematic major axis to only the presence of a warp and ignores the distortion of the kinematic major axis that are instead caused by radial flows. Based on the above discussion, this procedure will act to underestimate the radial motion on the disk (see also Schmidt et al. 2016) because it will explicitly ascribe the distortion of the kinematic major axis to the warp alone. As noted above, we stress again that radial flows do affect the location of the \textit{kinematic} major axis, distorting it away from the geometric axis, even though radial flows make zero contribution to the velocity field along the \textit{geometric} major axis itself.

It should also be noted that the measured circular velocity inferred from observations may also be affected by the possible presence of radial motions. According to Equation 8, the maximum/minimum observed LOS velocity is determined by both $v_\phi$ and $v_r$. Ignoring possible radial motions in modelling 2-d velocity fields could potentially overestimate the amplitude of the circular motion, especially for the outer regions of the gas disk (c.f. Jones et al. 1999; de Blok et al. 2008b; Kamphuis et al. 2015; Kam et al. 2017) where radial velocities may be high and, as shown below in Section 5.2.2, for low inclination systems.

4. A STATISTICAL TEST TO DETERMINE WHETHER WARPS OR INFLOWS ARE DOMINANT

In the previous section, we emphasized the difficulty of distinguishing the effects of radial flows from those associated with warps in disks in any individual system. In this section, we develop a statistical test that allows us to determine whether, in the population as a whole, warps or radial inflows are the dominant effect. This is based on the breaking of symmetry that is implicit in requiring the radial flow to be inflow rather than outflow.

As shown in Figure 3, the radial inflow has twisted the iso-velocity lines in a direction that is counter-clockwise as we move out in radius. As noted above, the disks in Figure 3 are rotating clockwise (i.e. the upper limbs

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**Fig. 3.** — The LOS velocity map of the typical Main Sequence galaxy shown in Figure 2 as seen at an inclination (INC) of 45 degree. The upper and lower row of panels assume $\lambda - R = 0$ and $\lambda - R = 1$, respectively (the latter requiring higher inflow velocities). In each row, the leftmost panel shows the contribution of the circular motion, i.e. $v_\phi$, and the middle panel shows the contribution of the radial motion, i.e. $v_r$. The rightmost panel shows the “observed” LOS velocity obtained by adding both $v_\phi$ and $v_r$ components. In the rightmost panels, the white solid line shows the geometric major axis of this galaxy, while the white dashed line shows the kinematic major axis, defined as the locus of minimum/maximum projected velocity at each radius. In each panel, the black curves show iso-velocity contours with an interval of 40 km s$^{-1}$. The dashed elliptical lines indicate radii of $2h_R$, $4h_R$, $6h_R$ and $8h_R$.
in the panels are closer to the observer than the lower limbs). Assuming that spiral arms in galaxies are generally trailing arms (e.g. Pasha & Smirnov 1982; Iye et al. 2019), then the spiral arms in this galaxy should also be twisted in a counter-clockwise direction moving out in radius. It is easy to see that flipping the orientation of the disk, so that upper limb is now further away than the lower limb, would reverse the sense of the rotation as seen by the observer, and therefore reverse the twists of both the kinematic distortion and of the spiral arms. If radial inflow is the dominant effect in distorting the iso-velocity contours of the galaxy, then we would therefore expect that the twists of the kinematic major axis and of the spirals arms should always be in the same sense, provided only that the spiral arms are always trailing.

Warps are characterised by a migration of the rotation axis of the disk with radius. This could arise from a number of physical mechanisms. But in a symmetric universe, none of these should depend on the sense of rotation of the disk. As a result, we would expect from symmetry arguments that the twists in the kinematic distortions of the velocity field due to warps should be uncorrelated with the sense of rotation and thus the twists of the spiral arms. We should as often see these two twists in opposite senses as in the same sense.

Examining the sense of the twists of the kinematic distortions and of the spiral arms (when visible) therefore provides a statistical test of whether radial inflow or warps dominate these distortions. Provided that spiral arms are trailing, then, if warps dominate then we would expect a 50-50 split between galaxies with twists in the same sense and in the opposite sense, whereas if radial inflows dominate then these distortions should always be in the same sense. Clearly the correlation would be weakened if some galaxies have leading spiral arms.

We therefore examined the HI velocity fields and the optical images for a sample of 54 nearby galaxies taken from Di Teodoro & Peek (2021). The sample galaxies are selected from the publicly available HI surveys, including THINGS (The HI nearby Galaxy Survey; Walter et al. 2008), HALOGAS (the Hydrogen Accretion in Local GalaxieS survey; Heald et al. 2011), LVHIS (the Local Volume HI Survey; Koribalski et al. 2018), and etc (van der Hulst et al. 2001; Chung et al. 2009; Serra et al. 2012; Lutz et al. 2017). These galaxies were selected mainly to have circular velocities greater than 100 km s\(^{-1}\), an HI column-density sensitivity better than $\sim 5 \times 10^{19}$ cm\(^{-2}\), and to have inclinations INC between 30\(^{\circ}\) and 80\(^{\circ}\).

Galaxies were examined visually by both the authors independently in a blind way, i.e. all velocity fields were viewed in a random order and the distortions of the iso-velocity contours were classified to have an “S”, “Z” or “ambiguous” pattern. This classification was repeated using a second random ordering of the optical images of the galaxies in order to determine the twist of the spiral arms. For the spiral arms, the classification was generally unambiguous. Our two independent classifications of all galaxies were then compared, giving also an indication of the reliability of the classifications.

For our primary test, the 28 (50\%) of the galaxies which showed clearly classifiable distortions of their velocity fields, and also had an unambiguous sense of their spiral arm pattern, were used. Of these, 13 galaxies showed the same sense of the spiral arms and the kinematic distortion, while 15 galaxies showed opposite senses, indicating that these are uncorrelated.

With the proviso that spiral arms are assumed to be predominantly trailing, this test clearly suggests that the dominant effect in producing distortions of the velocity field is likely to be the presence of warps in the HI disks rather than the radial inflows whose signatures we had hoped to see. Neither adding less easily classifiable galaxies, nor restricting the set to only the most beautiful examples, changed this conclusion.

5. THE DEGENERACY OF RADIAL MOTION AND THE WARPED DISK ON VELOCITY FIELDS

As mentioned in the introduction, hydrodynamical simulations (and our MAD model) exhibit significant radial inflows ($\sim$30-70 km s\(^{-1}\)) of gas at large radii. Yet we concluded in the previous Section 4 that the distortions of the observed kinematic velocity fields are dominated by warps and not by inflows (see also Schmidt et al. 2016; Di Teodoro & Peek 2021).

In Section 3, we showed that the signatures of radial motion on the velocity fields resemble those of warped disks. This raises the question of whether the signatures of inflows of the required strength could be hidden amongst the similar but larger signatures of warps, especially given that the intrinsic form of the warp will generally not be well determined. Put another way, how much inflow can be missed if galaxies are assumed to be only warped?

5.1. Construction of mock velocity fields and settings of fits

We approach this question in the following way. We first construct mock datacubes of HI 21 cm emission for galaxy disks with known input radial inflow fields and with a known warped geometry of the disk. Then, we model the composite velocity fields with a pure warped disk in order to examine whether (or under which conditions) the pure warped disk model can match the composite velocity fields produced with significant inflow. These cases would represent systems where inflow was “hidden” within the warp.

We both construct the mock datacubes and model their kinematics by using the code 3D\textsuperscript{BAROLO}\(^{2}\) (Di Teodoro & Fraternali 2015; Di Teodoro & Peek 2021). The code can efficiently construct a 3-dimensional mock datacube of a gas disk based on a set of concentric rings with given column density, geometry and kinematics.

For simplicity, we input a constant column density of HI with radius in constructing the mock datacubes (Bigiel & Blitz 2012). We adopt the Equation 5 with $V_{\text{circ}} = 220$ km s\(^{-1}\) and $R_t = 0.5h_{\text{R}}$ for circular motion. We use the radial velocity predicted in MAD with $h_{\text{R}} = 4$ kpc and $\lambda - R = 0.0$ from Figure 1. This particular radial velocity field is adjusted by randomly multiplying by a factor $v_{r,\text{scale}}$, which is uniformly distributed between 0 and 2.

We then construct a large number of mock data cubes. For all, we input a constant position angle (PA, arbitrarily but unimportantly fixed to 90 degrees) for the mock datacube.

\(^{2}\) https://editeodoro.github.io/Bbarolo
galaxies within the radius of $4h_R$ (corresponding to the size of stellar disk), but then enable the PA to linearly increase or decrease beyond this radius. The change of PA at $8h_R$ ($\Delta PA$) with respect to 90 degree is set to be uniformly distributed from 0 to 40 degree across the set of mock cubes. Similarly, we input a constant INC for the mock galaxies within $4h_R$, and let it linearly decrease\(^3\) beyond this radius. The inclination INC of the mock galaxies is set to be uniformly distributed from 20 to 70 degree within the radius of $4h_R$, and the change of INC at $8h_R$ with respect to the inner disk is set to be uniformly distributed from $-10$ to 0 degree.

Based on the above settings, we construct 200 mock datacubes. However, given the handedness discussed earlier, it is clear that the effect of the radial inflow can either be to enhance, or to suppress, the effect of the warp, depending on the sense of the disk rotation as seen by the observer. We therefore construct two versions of each mock datacube: one where the two effects are in the same sense, i.e. the radial flow increases the kinematic distortion from the warp, and one where the effect of the inflow reduces the distortion from the warp. This doubles the number of mock galaxies.

After constructing these mock datacubes, we then fit each of them with a model that contains only a warped disk (i.e. without any radial flow of gas) by adopting the 3DFIT task under the code 3DFIT BAROLO, which is recommended to use (Di Teodoro & Peek 2021). This software fits 3-dimensional tilted-ring models to HI datacubes, resulting in the circular velocity, velocity dispersion, PA and INC for a given set of radial bins. It should be noted that while this code can simultaneously fit for the radial motion, we do not enable this option for the purpose of our experiment.

5.2. Nine galaxies for illustration

In this subsection, we show the fitting results and the recovered parameters from the above test. For illustration, we show the fitting results for a representative sample of nine galaxies, which span a large range of both $vr_{scale}$ and INC. The selected nine galaxies can be separated into three groups according to the values of $vr_{scale}$: $vr_{scale} \sim 0$, $vr_{scale} \sim 1$ and $vr_{scale} \sim 2$. For orientation, $vr_{scale} \sim 1$ (or $vr_{scale} \sim 2$) corresponds to $v_r \sim 45$ km s\(^{-1}\) (or $90$ km s\(^{-1}\)) at $8h_R$ (see the blue solid line in Figure 1). For each group, three mock galaxies are selected to have very different INC (within $4h_R$). As will be shown later, the INC and $vr_{scale}$ are the two most important parameters in determining the goodness of the pure-warp fits. The $\Delta PA$, INC and $vr_{scale}$ for the selected nine galaxies are indicated later in Figure 10.

5.2.1. The fitting of the velocity fields with pure-warp models

Figure 4 shows the fitting of the velocity fields for three selected galaxies with low $vr_{scale}$. For each galaxy, we show the two cases in which the radial inflow has either reduced (−) or enhanced (+) the distortions caused by the warp, with all other parameters the same. For each of the three galaxies, the panels show from left to right the velocity field of circular motion (above) or radial motion (below), and then for both reduced and enhanced cases, the composite velocity field, the velocity field fitted with a pure warped disk model, and the residual map, respectively. These three galaxies are displayed from top to bottom with increasing INC. In the velocity field of $v_o$, the white solid line shows the input geometric major axis of the mock galaxy, the same as the white solid line in the modeled velocity field, where the recovered major axis is also indicated by the white dashed line.

As shown in Figure 4, the modeled velocity fields using the pure-warp model are overall in very good agreement with the constructed composite inflow+warp velocity fields for these three galaxies. The fitted kinematic major axes closely follow the actual ones, and the residuals are extremely small (only a few km s\(^{-1}\)). This is not surprising, because these three galaxies were constructed with small radial motions. In other words, Figure 4 primarily indicates the robustness of the fitting code we used.

Figure 5 and Figure 6 are similar to Figure 4, but are for additional mock galaxies with intermediate and high $vr_{scale}$, respectively. As we increase the strength of the radial motion, the modeled kinematic major axes show clear deviations from the input geometric major axis, because of the extra contribution of the radial flow. But the overall 2-d velocity fields are still very well recovered by the pure-warp model, especially for galaxies of low and intermediate INC. In other words, the effects of the inputted radial motions are being treated without difficulty as a contribution from the warped disk in the fitting, with reasonably small residuals (we will quantify the residuals in later Section 5.2.2). This result is consistent with our theoretical expectation from Section 3 that radial motions cannot be easily separated from the effects of a warped disk based on kinematic features alone, at least for reasonably weakly inclined disks. If a warp-plus-inflow system is equally well fittable as a pure-warp system, then given the difficulty of independently knowing a priori the parameters of the warp, it is clear that radial flows can be effectively “hidden” within the uncertain warps.

For highly inclined galaxies with significant radial inflows, there are significant bi-polar symmetric residuals near the minor axis when the composite velocity field is fit with a pure-warp model. The maximum value of the residuals depend strongly on the INC of galaxies with increasing from 10% or 20% to as large as 70% of input inflow velocity for the mock galaxies. Such residuals could be taken as a signature of radial motion in observations. However, one should realize that the recovered kinematic major axes still show significant deviations from the input geometric ones, indicating the degeneracy of radial motion and warped disk even for these highly inclined galaxies. This further suggests that even if signatures of radial motion are found in the residual maps, the extraction of accurate estimates of the radial motion may still be challenging due to this degeneracy.

5.2.2. The residuals and the recovered parameters

We introduce the mean absolute error (MAE) to quantify the residuals. This is defined as the mean value of the absolute deviations of the fitted pure-warp velocity field

\(^3\) Here we only consider the case that INC decreases with radius, because we want to reduce the cases that the motions at different radii are overlapped on some regions of the LOS velocity fields.
The modeling of the velocity fields for three selected three mock galaxies with low radial velocities (i.e. low $v_r$ scale). For each galaxy, we show the two cases in which the inflow reduces the distortion of the dominant warp (marked with “−”) and in which it augments it (marked with “+”). For each row from left to right, the panels show the velocity field of circular motion (and the radial motion immediately below it), the composite velocity field in the two warp-enhancing and warp-suppressing cases, the fitted velocity field with pure warped disk model for both cases, and the residual map, respectively. The black ellipticals in the second column of panels from the left indicate the fitted regions, corresponds to the radius of $8h_R$ for the modeled field. In the two rightmost panels, the four black ellipticals indicate the radii of $2h_R$, $4h_R$, $6h_R$ and $8h_R$ for the modeled velocity field. In the velocity field of $v_φ$, the white solid line shows the input geometric major axis of the corresponding mock galaxy, the same as the white solid line in the modeled velocity field, while the recovered major axis is shown in the white dashed line.
Fig. 5.— The same as Figure 4 but for three galaxies of intermediate $v_r$ scale.
Fig. 6.— The same as Figure 4 but for the selected three galaxies with high \( v_r \).
Fig. 7.— The MAE as a function of radius for the nine selected mock galaxies. The panels from left to right show galaxies with increasing $v_{r\_scale}$. In each panel, the galaxies are then shown with increasing INC from the blue, green and red. For each galaxy, the solid line shows the case where the flow suppresses the effect of the warp, and the dash line shows the case in which it enhances the warp.

Fig. 8.— The recovered PA, INC and $v_\phi$ (left to right in each row) as a function of radius for the nine selected mock galaxies when their composite velocity fields are fitted with pure warped disk models. The IDs of the mock galaxies are indicated in the left column of panels only. The solid lines show the input parameters, while the dashed and dotted lines show the fitted parameters when the flows suppress and enhance the warps, respectively. From the top to the bottom the galaxies are shown with increasing $v_{r\_scale}$. Within each row, the galaxies are shown with increasing INC from blue, green to red, as in Figure 7.
from the actual input warp+inflow composite velocity field at a particular radius. Figure 7 shows the MAE as a function of radius for the nine selected galaxies shown in Figure 4-6. The galaxies are shown with increasing \( vr_{\text{scale}} \) from left to right. In each panel, galaxies are then color-coded by inclination INC. For each galaxy, the solid line shows the case where the inflow suppresses the effect of the warp and the dashed line shows the case where the inflow enhances it.

As seen in Figure 7, the MAE increases significantly with the inclination, INC. This is only partly due to the fact that the amplitude of all LOS velocities increase with INC. As we shown later in Section 5.3, the fits depend on the INC even with considering the above effect. This is in accord with the theoretical expectation in Section 3 that the effects of radial motion become increasingly different from those of the warped disk as INC increases.

For galaxies with significant radial motion (see the middle and right panels of Figure 7), the MAE increases with radius (or inflow velocity), and with \( vr_{\text{scale}} \). For the most inclined galaxies, the MAE is as large as 10 km s\(^{-1}\) at \( 8R_h \) when the radial velocity reaches \( \sim 41 \) km s\(^{-1}\) (see the mock galaxy of ID: 198), and the MAE is 14 km s\(^{-1}\) at \( 8R_h \) when the radial velocity reaching \( \sim 86 \) km s\(^{-1}\) (see the mock galaxy of ID: 011). We can conclude that, at least for our mock galaxies, the radial motion can to a very large extent hidden behind the warped disk, with MAE of the residuals at the level of 15 km s\(^{-1}\).

Force-fitting the composite velocity fields with warp-only models also returns perturbed values of PA, INC and circular velocity as a function of radius. Figure 8 shows the comparisons of the returned values of these parameters with the input ones for the same nine galaxies. The solid lines show the input parameters as a function of radius, while the dashed and dotted lines show the returned perturbed ones for the two cases in which the radial inflow suppresses and enhances the effect of the wars, respectively. From the top to bottom row, the galaxies are shown with increasing \( vr_{\text{scale}} \). In each panel, the galaxies are shown with increasing INC from blue, green to red, as above.

As can be seen in Figure 8, the returned circular velocity and INC at different radii are generally not strongly perturbed by the force-fit of the pure-warp model. Weakly-inclined galaxies show a slightly larger deviation of the returned \( v_R \) (blue lines) from the true input value. This increase is mostly due to the correction of velocities for the inclination of the galaxies. However, the recovered PA (as a function of radius) shows significant deviations when the inflow velocity is significant. This again illustrate the point made earlier in Section 3 that the kinematic major axis is noticeably distorted from the geometric major axis in the presence of significant radial motion (they should be the same in a pure-warp model), even though radial motions have no contribution to the LOS velocity map along the geometric major axis.

5.3. The MAE of all 200 mock galaxies

Figure 9 shows the MAE (defined as above) as a function of radius for all the 200 mock galaxies. We show the case of warp-suppression in the top row of panels and warp-enhancement in the bottom row of panels. In each row, the MAE of 200 mock galaxies are displayed separated into three equal subsamples, according to \( \Delta \text{PA} \) (left panel), INC (middle panel) and \( vr_{\text{scale}} \) (right panel).

As shown, the MAE of mock galaxies remain rather low for the vast majority of the population (<10 km s\(^{-1}\)), with only a few outliers. Strikingly, the outliers with catastrophic MAE tend to have simultaneously large \( \Delta \text{PA} \), large INC and high \( vr_{\text{scale}} \) (as marked in red in all three panels in each row). We have examined the velocity fields of these outliers, and find that large \( \Delta \text{PA} \) and INC lead to situations in which motions of gas at different radii are superposed at the same location in the LOS velocity fields. This is probably the reason for the catastrophic modeling of these mock galaxies.

Since the MAE typically increases with radius, we chose the MAE calculated at \( 8R_h \) as a metric for the goodness of fit. We find that 92% (or 87%) of mock galaxies have MAE(\( 8R_h \)) less than 20 km s\(^{-1}\) (or 10 km s\(^{-1}\)) for the case of warp-suppressing flows, and 94% (or 84.5%) of mock galaxies have MAE(\( 8R_h \)) less than 20 km s\(^{-1}\) (or 10 km s\(^{-1}\)) for the case of warp-enhancing flows.

Figure 10 shows the MAE(\( 8R_h \)) of all the mock galaxies on the plane of input parameter space. In this figure, the nine representative galaxies illustrated in Section 5.2 are indicated by red circles. Consistent with the results of Figure 9, galaxies with extremely large MAE (in red) have both large INC and \( \Delta \text{PA} \). It is interesting that the MAE for the remainder of the galaxies does not depend much on the \( \Delta \text{PA} \), but depends mostly on the INC and \( vr_{\text{scale}} \) (see the middle panel). The pure-warped disk model is not a good fit only for galaxies with high INC and high \( vr_{\text{scale}} \). This is true for both cases of suppression and enhancement of the warp-induced distortions.

As was pointed out in Section 5.2, galaxies with higher INC tend to have higher LOS velocities (since all motions are planar), and this may contribute to the dependence of MAE(\( 8R_h \)) on INC. We therefore eliminate this effect by normalizing the MAE(\( 8R_h \)) with the maximum LOS velocity (with respect to the systematic velocity) and re-plot the data from Figure 10 in Figure 11. The normalized MAE(\( 8R_h \)) indeed still depends on the INC, as suggested from Section 3.

As shown in Figure 10, almost all the mock galaxies with \( v_R(8R_h) < 45 \) km s\(^{-1}\), and almost all mock galaxies with \( v_R(8R_h) < 90 \) km s\(^{-1}\) and INC<50\(^\circ\), have MAE(\( 8R_h \)) less than 10 km s\(^{-1}\). We can conclude that, at least for these mock galaxies, significant radial flows can be hidden within the kinematic signatures of warps of the disks, at least at the level of AME ~ 10 km s\(^{-1}\). We note that the residuals from modeling actual observed HI velocity fields are usually of this same order of magnitude (e.g. de Blok et al. 2008a; Kamphuis et al. 2015).

One might worry that the parameters of the mock galaxies used in this study may not be representative of real galaxies. We therefore show in the left panel of Figure 12 the actually observed \( \Delta \text{PA} \) and INC for the galaxy sample of Di Teodoro & Peek (2021). Di Teodoro & Peek (2021) fit the kinematic PA as a function of radius for these galaxies and we extract \( \Delta \text{PA} \) from their fits, defined as the range of returned PA across the full range of radius. The blue box in the left panel of Figure 12 shows the parameter space of our mock galaxies. The black small dots show the distribution of the “observed” \( \Delta \text{PA} \) and INC for one individual system with a intrin-
Fig. 9.—The MAE as a function of radius for all 200 mock galaxies. The galaxies are roughly evenly separated into three subsamples, according to ∆PA (left panel), INC (middle panel) and $v_r$ scale (right panel). The top row is for the case of inflows suppressing the effect of the warps, and the bottom row is for the case of enhancing the effects of the warps.

The intrinsic ∆PA of 30 degree but “observed” from all different directions. As shown, the ∆PA and INC of our mock galaxies are largely overlapped with those of real observed galaxies used in observational studies. The maximum ∆PA for the real galaxies is ∼30 degree with two outliers, M83 and NGC 2685. NGC 2685 is a polar ring galaxy (e.g. Józsa et al. 2009), and M83 shows outer disc streamers, an asymmetric tidal arm and a thoroughly twisted velocity field in HI map, which may be due to the accretion of dwarf galaxies in the past (Koribalski et al. 2018).

Interestingly, the ∆PA of an individual system has a maximum value, when seen at high INC, which is close to the actual intrinsic ∆PA. This suggests that the intrinsic ∆PA for the observed galaxies is often to 30 degree. Combining with the result from Section 4 that the radial change of kinematic major axis is dominated by the warps, we can roughly provide an upper limit of inflow velocity from the observations based on Equation 9 and 12:

$$v_r/v_\phi < \tan(30^\circ) \simeq 0.58.$$  

Based on our mock galaxies, we select a subsample that matches both the ∆PA and INC with the galaxies (that are located within the blue box in the left panel of Figure 12) from observations. We then show the MAE as a function of radius for this subsample in the right panel of Figure 12. As shown, the MAE($\delta hR$) are less than 15 km s$^{-1}$ for almost all the mock galaxies in this matched subsample. This further suggests that radial motions as high as 90 km s$^{-1}$ can be hidden behind the warp-induced features in the kinematic velocity fields in the observations, within the level of MAE ∼ 15 km s$^{-1}$.

A possible approach to break the degeneracy between radial motion and warped disk is to use the column density map of gas that independently provides the geometric parameters of the disk. However, the gas distribution is usually clumpy along with spiral arms, which strongly increases the difficulty for this method. In addition, in the present work we ignore the possible cases that the gas at different radius orbits to slightly different centers, the possible elliptical orbits of gas motion, and any variations with azimuth angles. Including these would make the extraction of radial motion of gas to be more difficult.

### 6. SUMMARY AND CONCLUSION

Hydrodynamical simulations suggest that the inflow of gas onto galaxies is almost co-planar and broadly co-rotating with the gas disk, regardless of its thermal history, and the outflowing gas potentially leaves the disk from two sides of the gas disk (Kereš et al. 2005; Stewart et al. 2011; Danovich et al. 2015; Stewart et al. 2017; Stern et al. 2020; Péroux et al. 2020; Trapp et al. 2021; Hafen et al. 2022; Gurvich et al. 2022). Motivated by this, we constructed a modified accretion disk (MAD) model to describe the formation of gas disks for SF galaxies (Wang & Lilly 2022b,a). Similar to but different from the accretion disk of compact objects, the accretion disk of galactic gas disk is “leaky” because of the removal of mass from the inflow due to star formation on the disk and the associated outflows. In earlier papers of this series, we found that the MAD model can very well recover the observed profiles of gas-phase metallicity in the disk, and also proposed that MRI is the most plausible source of viscosity for driving the co-planar inflow and, under certain plausible circumstances, will produce an exponential profile of the SF disk.

Observationally, however, there is a noticeable lack of evidence for significant co-planar inflows in disks. In the present work, we therefore firm up the prediction of the inflow velocity in the MAD model, investigate the effects of the required radial motion on the observed 2-d LOS velocity field, and explore the possible origin for this inconsistency, and in particular address the question
whether the required flows could be hidden within the kinematic signatures of warped disks.

The main results of this analysis are as follows.

- After adopting a realistic gas surface density from observations, we find that the velocity of the co-planar inflow is likely to be only a few km s\(^{-1}\) within two disk scalelengths. It gradually increases with radius and can reach to around 50-100 km s\(^{-1}\) at the very outer disk. This is consistent with the results from the hydrodynamical simulations (Trapp et al. 2021; Wang et al. 2022; Hafen et al. 2022).

- The radial inflow will distort the observed 2-d LOS velocity field from that expected for pure rotation. Both the kinematic major and minor axes are distorted from the geometric ones, even though the effect of the inflow is by definition zero along the major geometric axis. This signature of inflow resembles the effect of a spatial warp of the disk (see Figure 3) indicating a loose degeneracy between these two effects.

- In detail, it is found that the distortion of the velocity field by the radial motion in the region of the minor axis is larger than that in the region of the major axis, whereas for a warped disk these are in principle the same. But this difference depends quite strongly on the inclination of the disk in the sense that the differences between major and minor axis become more significant with increasing INC and vanishes in face-on disks (see Equation 9 and 12).

- The twisting of the iso-velocity contours that characterises the distortion of the 2-d velocity field by gas inflow depends on the rotation sense of the galaxy but the similar pattern that arises due to warps should not (at least in a symmetric universe). Assuming that the spiral arms in galaxies are “trailing”, the twists of the kinematic distortion due to inflow should therefore be in the same sense (“S” or “Z”) as that of the spiral arms. In contrast, the broadly similar kinematic distortions arising from spatial warps of the disks should be uncorrelated with the sense of the spiral arms. By visually inspecting the HI velocity fields and visible-light images for a sample of nearby galaxies, no correlation is found. This suggests that the distortions of the kinematic major axis and the iso-velocity con-

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**Fig. 10.** The parameter space of 200 mock galaxies color-coded with the MAE measured at 8h\(_R\). The top row is for the case of suppressing the effect of the warps, and the bottom row is for the case of enhancing the effect of the warps. In each panel, the red circles indicate the nine galaxies illustrated in Section 5.2. We note that \(v_{r, scale} \sim 1\) (or \(v_{r, scale} \sim 2\)) corresponds to \(v_r \sim 45\) km s\(^{-1}\) (or 90 km s\(^{-1}\)) at 8h\(_R\).
Fig. 11.— The same as Figure 10 but normalizing the MAE($8h_R$) with the amplitude of the LOS velocity at $8h_R$.

Fig. 12.— Left panel: the $\Delta$PA and INC for the observed galaxy sample of Di Teodoro & Peek (2021). The blue box shows the parameter space of our mock galaxies. The black small dots show the distribution of “observed” $\Delta$PA and INC for one individual system with an intrinsic $\Delta$PA of 30 degree but “observed” from all different directions. Right panel: the MAE as a function of radius for a subsample of mock galaxies that is selected to match the $\Delta$PA and INC of the Di Teodoro & Peek (2021) galaxies (located in the blue box in the left panel).
However, when we model mock HI velocity fields for galaxies with both inflows and warped disks, and then force fit these velocity fields with pure-warped disk models, we find that the latter can very well match the composite velocity fields of mock galaxies in most cases. The goodness of fits mainly depends on the inclination and on the amplitude of inflow velocity. By examining the map of velocity residuals, we find that ∼85% of mock galaxies have MAE in the outer parts (at 8σR) less than 10 km s⁻¹ for radial velocities of order 90 km s⁻¹.

This indicates that the signatures of substantial radial inflow can be hidden behind the warped disks. We suggest that this may have contributed to the absence of observational proof of the significant radial inflows that are expected from hydrodynamical simulations and our own MAD model.

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We stress that radial motions can distort both the kinematic major and minor axes in 2-d projected velocity field, resulting in a strong degeneracy between warped disks and radial motion, especially for weakly inclined disks. This makes the extraction of inflow velocity to be extremely difficult based on the LOS velocity fields. Previous studies have usually attributed the radial change of the kinematic major axis only to the warped disk (e.g. Schmidt et al. 2016; Di Teodoro & Peek 2021). We have shown that this would significantly underestimate the amplitude of the inflow velocity. We also stress that due to the strong degeneracy for weakly inclined disks discussed above, the signatures of radial motion would potentially be found in the more highly inclined disks.

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