Modulation Recognition of MPSK Signals Based on Generalized Quartic Spectrum

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Abstract. Automatic modulation recognition (AMR) has become an interesting problem in communication systems with various civil and military applications. The modulation recognition method based on quartic spectrum will degrade seriously in the Alpha-stable distribution noise. To solve this problem, a generalized transform is proposed in this paper, based on which, a new modulation recognition method of MPSK is investigated. First, generalized quartic spectrums of signals are analyzed. Then the method extracts spectrum lines of two times carrier frequency and four times carrier frequency in generalized quartic spectrum as the feature parameters. Finally the method realizes the recognition of signals through judging whether spectrum lines are impact spectral lines or not. Experimental results show that the novel method has good performance in both Alpha-stable distribution noise and Gaussian noise, and it is less affected by characteristic exponent of noise ranging from 1 to 2.

1. Introduction
The aim of modulation recognition for communication signals is to determine modulation types and some parameters of the received signals in the case of certain noise, and provide the basis for subsequent signal analysis and processing. Modulation recognition for communication signals has many important applications in both civilian and military fields. For the civilian authorities, this includes signal confirmation, interference confirmation, spectrum management and making sure that the guidelines for radio communication are followed. Knowledge of which signal modulation type is used can provide valuable information and is also crucial in order to retrieve the information stored in the signal. In the military domain, signal modulation recognition can be used for electronic warfare purposes like threat detection analysis and warning. It can further assist in the decision of appropriate counter measures like signal jamming. Signal modulation recognition is also believed to play an important part in future 4G Software Radios and Cognitive Radios [1].

Most of previous researches on modulation recognition employed Gaussian distribution as the model of background noise. But in fact it often exists some significant short-time impulse noise with large amplitude that cannot be simply described by Gaussian distribution in wireless communications channels, such as car ignition noise, low frequency atmosphere noise, underwater acoustic noise, man-made noise, and electromagnetic noise [2,3]. More and more researchers confirmed that the Alpha-
stable distribution is a more effective noise model, and have done a lot of researches on signal processing in Alpha-stable distribution noise [4-9].

Even power spectrums of communication signals are effective recognition features. According to that different modulation signals have different spectrum characteristics, the effective modulation recognition of some communication signals can be realized. Quartic spectrum of signals can effectively characterize the difference among BPSK, QPSK and 8PSK signal [10-12]. Alpha-stable distribution noise does not have the second-order and higher-order moments, so quartic spectrum of communication signals will be invalid in Alpha-stable distribution noise, and the corresponding modulation recognition algorithm will be severely degraded. To solve this problem, this paper proposes the concept of generalized quartic spectrum, and applies it to the modulation recognition for MPSK signals in Alpha-stable distribution noise, which gets a good recognition result.

2. Alpha-Stable Distribution

The theoretical justification for using the stable distribution as a basic statistical modeling tool comes from the generalized central limit theorem (GCLT). It states that stable models are the only distribution that can be the limit distribution of independent and identically distributed random variables. Generally, an Alpha-stable distribution process can be described conveniently by its characteristic function [13, 14]:

$$\varphi(t) = \exp \left\{ jat - \gamma |t|^\alpha \left[ 1 + j \beta \text{sgn}(t) \omega(t, \alpha) \right] \right\}$$

(1)

Where

$$\omega(t, \alpha) = \begin{cases} \frac{\tan(\alpha \pi)}{2}, & \alpha \neq 1 \\ 2 \frac{\log|t|}{\pi}, & \alpha = 1 \end{cases}$$

(2)

$$\text{sgn}(t) = \begin{cases} 1, & t > 0 \\ 0, & t = 0 \\ -1, & t < 0 \end{cases}$$

(3)

And

$$0 < \alpha \leq 2, -1 \leq \beta \leq 1, \gamma > 0, -\infty < a < \infty.$$ 

Thus the class of Alpha-stable distribution is a simple statistical-physical model defined by only four parameters ($\alpha, \beta, \gamma, a$) that can be efficiently estimated directly from the received data. The characteristic exponent $\alpha$ controls the tails of these distributions and the smaller $\alpha$ is, the heavier the tails of the Alpha-stable distribution will be. For $\alpha = 0.5$, $\alpha = 1$ and $\alpha = 2$, these distributions become Pearson, Cauchy and Gaussian distribution, respectively. Impulse noise in wireless communication channel mainly concentrate in the Alpha-stable distribution whose characteristic exponent $\alpha$ ranges from 1 to 2. $\beta$ is the symmetry parameter. When $\beta = 0$, the distribution is symmetric and called symmetric $\alpha$-stable ($S\alpha S$) distribution. The location parameter $a$ is the mean of the distributions for all $\beta$ when $1 < \alpha \leq 2$. $\gamma$ is the dispersion parameter that determines the spread of these distributions around the location parameter $a$ [13, 14].

An important property of Alpha-stable distributions is that only the lower moments are finite. If $X$ is subject to the Alpha-stable distribution, and its characteristic exponent ranges from 0 to 2, that is $0 < \alpha < 2$, for $0 < p < \alpha$, one obtains [13, 14]:

$$\varphi(t) = \exp \left\{ jat - \gamma |t|^\alpha \left[ 1 + j \beta \text{sgn}(t) \omega(t, \alpha) \right] \right\}$$
E\[|X|^p\] < \infty \quad (4)

And for \( p \geq \alpha \), one obtains:

E\[|X|^p\] = \infty \quad (5)

Since an Alpha-stable distribution random variable is of infinite variance when \( \alpha < 2 \), mixed signal to noise ratio(MSNR), which is the ratio of the variance of signal \( \sigma^2 \) over the noise dispersion \( \gamma \), is used to describe the relative power of signal to noise[15]

\[
\text{MSNR} = 10 \log \left( \frac{\sigma^2}{\gamma} \right)
\]

Without loss of generality, this paper employs the standard S\( \alpha \)S as channel noise model. For the received signal mixed with Alpha-stable distribution noise, its model is given by

\[ s = x + n \quad (7) \]

Where, \( s \) is the received signal, \( x \) is the original transmitted signal, \( n \) is the Alpha-stable distribution noise. Quartic spectrum (QS) is defined as the Fourier transform of signal fourth power:

\[ QS(f) = \left| F[x^4] \right| \quad (8) \]

Where \( F[\cdot] \) is Fourier transform, \( \| \) is modular arithmetic. Fourth power of the received signal can be expressed as follows:

\[ s^4 = (x + n)^4 = x^4 + 4x^3n + 6x^2n^2 + 4xn^3 + n^4 \quad (9) \]

It is can be seen from the equation [9], Fourth power of the received signal contains \( 6x^2n^2 \), \( 4xn^3 \) and \( n^4 \). The nature of Alpha-stable distribution noise shows that it does not have the \( \alpha \)-th-order and the above order statistics, which means that \( 6x^2n^2, 4xn^3, n^4 \rightarrow \infty \), so \( s^4 \rightarrow \infty \). According to constraint condition of the Fourier transform, quartic spectrum of the received signal containing Alpha-stable distribution noise does not exist, in other words, it is invalid. Fig.1 gives a more intuitive explanation on the above analysis. In the Fig.1, MSNR=15dB, \( \alpha = 1.5 \).

As it is shown in Fig.1, due to the influence of Alpha-stable distribution noise, the quartic spectrum of the BPSK signal is destroyed.
The maximum normalized amplitude

![Graph (a)](image1)

![Graph (b)](image2)

**Fig. 1** Quartic spectrum of BPSK signal. (a) The original BPSK signal; (b) BPSK signal containing Alpha-stable distribution noise

3. Generalized Transform and Generalized Quartic Spectrum

3.1. Generalized Transform

The reason for quartic spectrum becoming invalid in the Alpha-stable distribution noise, in the final analysis, is that the second-order and higher-order statistics of noise tends to infinity, then the second-order and higher order statistics of signals containing noise do not exist. Assume that the received signal containing noise can be processed through nonlinear transform, so infinite amplitude of noise can be controlled in a limited range, which makes the second-order and higher-order statistics exist. Meanwhile, this nonlinear transform retains the useful information of the interested signal. Then the second-order and higher-order statistics can still play their role in Alpha-stable distribution noise. Based on this idea, we draw on the fractional lower order thinking of the fractional lower order statistics [16-19], and use the nature of tangent function that can map any infinite range to the scope of \((-\pi/2, \pi/2)\) [20], and presents the following generalized transform (GT) equation:

\[
GT(x) = \frac{\arctan \left( |x + jH(x)| \right)}{|x + jH(x)|^x} \tag{10}
\]

Where \(H(\cdot)\) denotes the Hilbert transform, \(|\cdot|\) is modular arithmetic.

In general, communication signals can be expressed in the form of \(x = r \cos \theta\), where \(r\) is the amplitude, and \(\theta\) represents the phase. Its generalized transform is as follows:

\[
GT(x) = \frac{\arctan \left( r \cos \theta + jH(r \cos \theta) \right)}{r \cos \theta + jH(r \cos \theta)} \tag{11}
\]

For the narrowband communication signal in this study, it is well known that the following equation is valid:

\[
r \cos \theta + jH(r \cos \theta) = r \exp(j \theta) \tag{12}
\]

Equation [10] may be derived as follows:
\[ GT(x) = \frac{\arctan(\sqrt{|r\exp(j\theta)|})}{r\cos\theta} = \arctan\left(\frac{r}{|r|}\cos\theta\right) = \arctan|r| \cdot \text{sgn}(r) \cdot \cos\theta = \arctan r \cdot \cos\theta \] (13)

Where \(\text{sgn}(\cdot)\) is symbolic function.

It can be found from the above analysis that, the generalized transform defined by equation [10] just map the amplitude of signal to a finite interval, for signal containing Alpha-stable distribution noise, the amplitude of noise is also mapped to this interval. Meanwhile, this transformation does not change phase and cycle information of signal, which is particularly important for the phase-shift keying signals. Some more intuitive descriptions of this nature are given by Fig.2-Fig.5. Fig.4 and Fig.5 show the mode \(2\pi\) instantaneous phase of signal. In Fig.3 and Fig.5, characteristic exponent of standard \(S_{\alpha}\) noise is 1.5, MSNR=5dB.

Fig.2 shows that generalized transform can lower signal amplitude, and keep changes of signal waveform envelope effectively at the same time. Fig.3 illustrates that noise amplitude can be suppressed in a finite interval by generalized transform, so that the signal is less affected by noise. Fig.4 and Fig.5 show that, allowing a certain error, generalized transform can keep the most of phase information for signal or signal with noise.

![Fig. 2 Waveforms of BPSK signal before and after generalized transform](image)

From the above analysis, we can conclude that the amplitude of signal containing Alpha-stable distribution noise is in a limited range after generalized transform, so it has second-order and higher-order statistics. Based on this analysis, we can improve quartic spectrum by embedding generalized transform into original concept, and get a novel generalized concept, so that its scope of application can be expand to the Alpha-stable distribution noise background. In fact, many of the concepts that become invalid in Alpha-stable distribution noise can follow the above improvement idea.
Fig. 3 Waveforms of BPSK signal containing noise before and after generalized transform

(a) Original signal containing noise  
(b) The signal containing noise after generalized transform

Fig. 4 Phases of BPSK signal before and after generalized transform. (a) Original signal; (b) Signal after generalized transform

(a) Original signal; (b) Signal after generalized transform

Fig. 5 Phases of BPSK signal containing noise before and after generalized transform. (a) Original signal; (b) Signal after generalized transform
3.2. Generalized Quartic Spectrum

Generalized quartic spectrum (GQS) can be defined as:

$$GQS(f) = |F[GT^4(x)]|$$  \hspace{1cm} (14)

Where $F[\cdot]$ is Fourier transform, $GT(\cdot)$ is defined in equation [10], $|$ is modular arithmetic.

The previous section has illustrated that the main role of the generalized transform is to keep signal phase and restrain amplitude of signal with noise. So the fourth power of received signal is no longer tends to infinity, Its Fourier transform is generalized quartic spectrum.

Fig. 6 shows the generalized quartic spectrum of BPSK signal under the same conditions with Fig.1 (b), the spectrum characteristic has been greatly improved. Therefore, generalized quartic spectrum can be used as recognition feature in Alpha-stable distribution noise.

![Fig. 6 Generalized quartic spectrum of BPSK signal containing Alpha-stable distribution noise](image)

4. Modulation Recognition Algorithm

In Alpha-stable distribution noise, quartic spectrum obviously can not be used as recognition feature. This study employs generalized quartic spectrum that can be seen as an improved type of quartic spectrum to recognize BPSK, QPSK and 8PSK signal.

4.1. Quartic Spectrums of Signals

BPSK, QPSK and 8PSK can be modeled as:

$$x_{psk}(t) = \sqrt{E} \left[ \sum_{n=-\infty}^{\infty} g(t - nT) \right] \cos(2\pi f_c t + \phi_n)$$ \hspace{1cm} (15)

Where $E$ is the average power, $g(\cdot)$ represents rectangular pulse function, $T$ is the symbol period, $f_c$ is carrier frequency. $\phi_n$ Represents the phase parameters controlled by information code, $\phi_n \in \left\{ \frac{2\pi (m - 1)}{M}, m = 1, 2, \ldots, M \right\}$, $M = 2, 4, 8$ corresponds to BPSK, QPSK and 8PSK signals respectively.

According to the definition of the generalized transform, one obtains:
\[ GT(x_{nak}) = \arctan\left(\sqrt{E} \left[ \sum_{n=-\infty}^{\infty} g(t-nT) \right] \right) \times \cos(2\pi f t + \varphi_n) \] (16)

Then

\[ GT^4(x_{nak}) = \arctan^4\left(\sqrt{E} \left[ \sum_{n=-\infty}^{\infty} g^4(t-nT) \right] \right) \times \cos^4(2\pi f t + \varphi_n) \]

\[ = \arctan^4\left(\sqrt{E} \left[ \sum_{n=-\infty}^{\infty} g^4(t-nT) \right] \right) \times \left[ \cos\left(8\pi f t + 4\varphi_n\right) + \cos\left(4\pi f t + 2\varphi_n\right) + \frac{3}{8} \right] \] (17)

For

\[ \left[ \sum_{n=-\infty}^{\infty} g^4(t-nT) \right] = \left[ \sum_{n=-\infty}^{\infty} g(t-nT) \right] \] (18)

So

\[ GT^4(x_{nak}) = \arctan^4\left(\sqrt{E} \left[ \frac{1}{8} \cos(8\pi f t + 4\varphi_n) \right] \times \right. \]

\[ \sum_{n=-\infty}^{\infty} g(t-nT) + \frac{1}{2} \cos(4\pi f t + 2\varphi_n) \times \]

\[ \sum_{n=-\infty}^{\infty} g(t-nT) + \frac{3}{8} \]

\[ = \arctan^4\left(\sqrt{E} \left[ \frac{1}{8} \cos(8\pi f t) \cos(4\varphi_n) \times \right. \right. \]

\[ \sum_{n=-\infty}^{\infty} g(t-nT) - \sin(8\pi f t) \sin(4\varphi_n) \times \]

\[ \sum_{n=-\infty}^{\infty} g(t-nT) \] + \frac{1}{2} \cos(4\pi f t) \cos(2\varphi_n) \times \]

\[ \sum_{n=-\infty}^{\infty} g(t-nT) - \sin(4\pi f t) \sin(2\varphi_n) \times \]

\[ \sum_{n=-\infty}^{\infty} g(t-nT) \] + \frac{3}{8} \] \] (19)

1) For BPSK signal, \( \varphi_n \in \{0, \pi\} \), \( 4\varphi_n \in \{0, 4\pi\} \), \( 2\varphi_n \in \{0, 2\pi\} \), therefore, some equations can be derived as follows:

\[ \cos(4\varphi_n) \sum_{n=-\infty}^{\infty} g(t-nT) = 1 \] (20)
\begin{equation}
\cos(2\varphi) \sum_{n=-\infty}^{\infty} g(t-nT) = 1
\end{equation}

\begin{equation}
\sin(4\varphi) \sum_{n=-\infty}^{\infty} g(t-nT) = 0
\end{equation}

\begin{equation}
\sin(2\varphi) \sum_{n=-\infty}^{\infty} g(t-nT) = 0
\end{equation}

Considering these four equations and equation [19], one obtains:

\begin{equation}
\begin{split}
GT^4(x_{\text{BPSK}}) &= \frac{1}{8} \arctan^4(\sqrt{E}) \cos(8\pi f_t) + \\
&\quad + \frac{1}{2} \arctan^4(\sqrt{E}) \cos(4\pi f_t) + \\
&\quad + \frac{3}{8} \arctan^4(\sqrt{E}) 
\end{split}
\end{equation}

Thus:

\begin{equation}
\begin{split}
GQS_{\text{BPSK}}(f) &= \left| F\left[ GT^4(x_{\text{BPSK}}) \right] \right| \\
&= \frac{\pi}{8} \arctan^4(\sqrt{E}) \left[ \delta(2\pi f + 8\pi f_c) + \delta(2\pi f - 8\pi f_c) \right] + \\
&\quad + \frac{\pi}{2} \arctan^4(\sqrt{E}) \left[ \delta(2\pi f + 4\pi f_c) + \delta(2\pi f - 4\pi f_c) \right] + \\
&\quad + \frac{3\pi}{4} \arctan^4(\sqrt{E}) \delta(2\pi f)
\end{split}
\end{equation}

The above equation indicates that generalized quartic spectrum of BPSK signal consists of impact spectral lines at \( f = 0 \), \( f = \pm 2f_c \), \( f = \pm 4f_c \). It is shown in Fig. 7, the carrier frequency of the signal is 150 kHz.

![Fig. 7 Generalized quartic spectrum of BPSK signal](image)
2) For QPSK signal, \( \varphi_n \in \{0, \pi/2, \pi, 3\pi/2\} \), \( 4\varphi_n \in \{0, 2\pi, 4\pi, 6\pi\} \), \( 2\varphi_n \in \{0, \pi, 3\pi\} \), therefore, two equations can be derived as follows:

\[
\cos(4\varphi_n) \sum_{n=-\infty}^{\infty} g(t-nT) = 1
\]

\[
\sin(4\varphi_n) \sum_{n=-\infty}^{\infty} g(t-nT) = 0
\]

Considering these two equations and equation [19], one obtains:

\[
GT^4(x_{QPSK}) = \frac{1}{8} \arctan^4(\sqrt{E}) \cos(8\pi f,t) + \\
\frac{1}{2} \arctan^4(\sqrt{E}) \cos(4\pi f,t + 2\varphi) \times \\
\sum_{n=-\infty}^{\infty} g(t-nT) + \frac{3}{8} \arctan^4(\sqrt{E})
\]

Thus:

\[
GQS_{QPSK}(f) = F[GT^4(x_{QPSK})] = \frac{\pi}{8} \arctan^4(\sqrt{E}) \left[ \delta(2\pi f + 8\pi f_n) + \delta(2\pi f - 8\pi f_n) + \frac{1}{4} \arctan^4(\sqrt{E}) \times \\
\left\{ \begin{array}{c} \\
\frac{\sin[\pi(f - 2f_n)T]}{\pi(f - 2f_n)T} + \\
\frac{\sin[\pi(f + 2f_n)T]}{\pi(f + 2f_n)T} \\
\end{array} \right\} + \\
\frac{3\pi}{4} \arctan^4(\sqrt{E}) \delta(2\pi f)
\right]
\]

Equation [29] indicates that generalized quartic spectrum of QPSK signal consists of impact spectral lines at \( f = 0 \), \( f = \pm 4f_n \) and sin functions whose center are located in \( f = \pm 2f_n \). Fig.8 shows the quartic spectrum of QPSK signal, its carrier frequency is also 150 kHz.

![Fig. 8 Generalized quartic spectrum of QPSK signal](image-url)
3) For 8PSK signal, the fourth power of generalized transform may be directly expressed as follows:

\[
GT^4(x_{\text{8PSK}}) = \frac{1}{8} \arctan^4(\sqrt{E}) \cos(8\pi f_c t + 4\phi_c) \times \\
\sum_{n=-\infty}^{\infty} g(t-nT) + \frac{1}{2} \arctan^4(\sqrt{E}) \times \\
\cos(4\pi f_c t + 2\phi_c) \sum_{n=-\infty}^{\infty} g(t-nT) + \\
\frac{3}{8} \arctan^4(\sqrt{E})
\]

Thus:

\[
GQS_{\text{8PSK}}(f) = \mathcal{F}\left[GT^4(x_{\text{8PSK}})\right]
\]

\[
= \frac{1}{16} \arctan^4(\sqrt{E}) \left\{ \sin\left[\pi(f - 4f_c)T\right] \right\} + \\
\sin\left[\pi(f + 4f_c)T\right] \right\} + \frac{1}{4} \arctan^4(\sqrt{E}) \times \\
\left\{ \sin\left[\pi(f - 2f_c)T\right] \right\} + \frac{3\pi}{4} \arctan^4(\sqrt{E}) \delta(2f)
\]

Equation [31] indicates that generalized quartic spectrum of 8PSK signal consists of impact spectral lines at \( f = 0 \) and sin functions whose center are located in \( f = \pm 2f_c, f = \pm 4f_c \). Fig.9 shows the quartic spectrum of 8PSK signal, its carrier frequency is also 150 kHz.

![Fig. 9](image)

**Fig. 9** Generalized quartic spectrum of 8PSK signal

4.2. **Feature Extraction and Method Description**

It can be seen from the previous subsection that spectrum lines of both two times carrier frequency and four times carrier frequency in generalized quartic spectrum are effective recognition feature, which can be used for modulation recognition of three signals. But how to distinguish between the impact spectral line and the sin function is a difficulty. To solve this problem, two variables are defined by:
\[
\begin{align*}
\text{th}_1 &= \frac{2|GQS_{11}|}{|GQS_{11-1}| + |GQS_{11+1}|} \\
\text{th}_2 &= \frac{2|GQS_{11}|}{|GQS_{11-1}| + |GQS_{11+1}|}
\end{align*}
\]

(32) (33)

Where \( k1 \) the sampling is point of frequency corresponding to \( 2f_c \), \( k1 - 1 \) and \( k1 + 1 \) are adjacent sampling points. \( k2 \) is the sampling point of frequency corresponding to \( 4f_c \), \( k2 - 1 \) and \( k2 + 1 \) are adjacent sampling points. For the symmetry of spectrum characteristics, we only consider the positive axle of generalized quartic spectrum.

According to the nature of impact spectral lines and sin function, if the impact spectral line exists in \( f = 2f_c \) or \( f = 4f_c \), \( \text{th}_1 >> 1 \) and \( \text{th}_2 >> 1 \). While if \( f = 2f_c \) or \( f = 4f_c \) is the center of sin function, \( \text{th}_1 \approx 1 \) and \( \text{th}_2 \approx 1 \). Based on experience, this paper sets thresholds \( \text{th}_1 = \text{th}_2 = 3 \) to distinguish the impact spectral lines from sin function.

5. Simulation Results

In this subsection, we perform the different simulation experiments to verify and evaluate our method performance. The simulations were performed using MATLAB. In the experiments, signal parameters: the carrier frequency is 150 kHz, the sampling frequency is 2400 kHz, the symbol rate is 24 kb/s. Assuming that the received signal has been preprocessed, it means that carrier frequency of the received signal has been estimated, which is not the research content of this paper, so we do not cover them.

Experiment 1. This experiment evaluates the performance of the novel method based on generalized quartic spectrum and the traditional method based on quartic spectrum in the standard \( S_\alpha \) noise background. Generalized transform does not change the signal phase and cycle information, so the two methods have same feature extraction process. Characteristic exponent of noise is set to 1.5. Signal points are 8192.100 tests for each MSNR, the recognition results are shown in Fig. 10.

![Fig. 10 Recognition results in Alpha-stable distribution noise](image)

It can be seen from Fig.10 that, for the method based on generalized quartic spectrum, spectrum characteristics for recognizing both BPSK and 8PSK are obvious, so these two signals have high recognition rate at all MSNRs we set. In the generalized quartic spectrum of QPSK signal, the amplitude of impact spectral line at \( 4f_c \) is relatively small and vulnerable to the influence of noise, so
its recognition effect is poor at low MSNRs, but recognition rates are greater than 80% when MSNR \( \geq 10 \) dB.

Fig.10 also shows that, for the method based on quartic spectrum, the recognition rates of both BPSK and QPSK are close to zero, but the recognition rates of 8PSK are high at all MSNRs we set. Investigate its reason, is mainly that the quartic spectrum becomes invalid in Alpha-stable distribution noise. As shown in Fig.1, two feature parameters \( th1 \) and \( th2 \) are close to 1, resulting the method will recognize all signals as 8PSK, not BPSK and QPSK.

From the above analysis and Fig.10, it can be found the novel method is better than the method based on quartic spectrum in Alpha-stable distribution noise.

**Experiment 2.** This experiment evaluates the performance of the method in the Gaussian noise. Signal points are 8192. 100 tests for each SNR, the results of the experiment are shown in Fig.11.

It can be seen from Fig.11 that two methods have similar performance at all SNRs we set, and both of methods can achieve effective recognition when SNR>8dB. Results imply that our novel method also has effective recognition property in Gaussian noise.

![Recognition results in Gaussian noise](image)

**Fig. 11** Recognition results in Gaussian noise

**Experiment 3.** In addition to the MSNR, the characteristic exponent of the noise is also a factor that affects the performance of the method. This experiment evaluates the performance of the method in different \( \alpha \) values. For standard S\( \alpha \)S noise, MSNR=15dB. Characteristic exponent \( \alpha \) changes with the step 0.1 in the interval \([0.1, 1.9]\). Signal points are 8192. 100 tests for each \( \alpha \) value, the recognition results are shown in Fig.12.

Fig.12 shows that the proposed method can effectively recognize BPSK and QPSK signal when \( \alpha > 1 \), and recognition results of 8PSK signal are good at all \( \alpha \) values. This is mainly because of the fact that the pulse characteristic of noise becomes more obvious with \( \alpha \) value decreasing, and this strong pulse characteristics results in degradation of the spectral lines characteristic at \( f = 2f_c \) hand \( f = 4f_c \). So the smaller \( \alpha \) values have a great impact on the recognition results of BPSK and QPSK signal, but almost do not affect the 8PSK signal. Impulse noise in wireless communication channel mainly concentrate in the Alpha-stable distribution with characteristic exponent \( \alpha > 1 \), strong impulse noise with \( \alpha \leq 1 \) rarely occurs, so the proposed method is suitable for application in practice.
Experiment 4. It should be mentioned that method performance may be affected by data point, so this experiment evaluates the performance of method in situation of different data points. Background noise is still the standard $\alpha$-stable noise, MSNR=15dB, $\alpha=1.5$. The signal points are respectively 512, 1024, 2048, 4096, 8192. 100 tests for each data points, Table 1 shows the recognition results.

From these results it can be seen that correct recognition rate of both BPSK and QPSK signal increase with the data points increasing, but recognition results of 8PSK signal are almost independent of the data points. This is mainly because of the fact that, in practice, we always use a limited number of data points to approximately substitute the infinite points in Fourier transform. If as many as data points are used to calculate, then the calculation results would be closer to the theoretical value. Fewer data points will affect the spectral line characteristics of generalized quartic spectrum, thus affect the recognition results of BPSK and QPSK signal. The feature parameter used to recognize 8PSK.

| Signal modulation type | Data points |
|------------------------|-------------|
|                        | 512 | 1024 | 2048 | 4096 | 8192 |
| BPSK                   | 0   | 100% | 100% | 100% | 100% |
| QPSK                   | 39% | 54%  | 84%  | 90%  | 92%  |
| 8PSK                   | 95% | 94%  | 95%  | 96%  | 95%  |

The signal does not depend on the impact spectral line, so its recognition results will not be affected.

6. Conclusion
Modulation recognition is a key step of many new technologies, such as software radio and cognitive radio. Most of previous researches on modulation recognition employed Gaussian distribution as the model of background noise, but some of concepts and methods involved in these researches become invalid in Alpha-stable distribution noise, such as the quartic spectrum often used to recognize MPSK signals. In this paper, we have proposed a new concept of generalized quartic spectrum firstly, the new concept has good spectrum characteristics in Alpha-stable distribution noise and effective properties in signal representation. Then we have proposed a novel recognition method for MPSK signals based on generalized quartic spectrum. Finally, we have evaluated method performance from many aspects. Experimental results indicate that the proposed method has good recognition performance in
Alpha stable distribution noise as well as Gaussian noise. The disadvantage of the method is that how to distinguish the impact spectral line from sin function depends on experience, so future investigations will focus on developing the effective intelligent method to distinguish between the impact spectral lines and sin function.

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