A PARTICULAR CLASS OF GUTS WITH VANISHING ONE-LOOP BETA FUNCTIONS

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Abstract
By explicit solution of the one-loop finiteness conditions for gauge and quartic scalar-boson self-interaction coupling constants, a particular class of grand unified theories with vanishing Yukawa couplings as well as vanishing one-loop renormalization-group beta functions is constructed.
1 Introduction

The most important feature of supersymmetry, which attracted in the past a lot of interest, is its particularly far-reaching ability of softening the high-energy behaviour of quantum field theories, which culminates in the possibility of constructing a certain class of perturbatively finite quantum field theories, namely, the well-known $N = 2$ supersymmetric theories satisfying a single and only one-loop “finiteness condition.” (For some review of these developments see, for instance, Ref. [1].) It soon became clear that the actual rôle of supersymmetry for finiteness may only be revealed by subjecting the most general renormalizable quantum field theory to the requirement of finiteness [2, 3, 4], in order to see whether the constraints of finiteness admit non-supersymmetric solutions too. (One has to bear in mind, however, that every eventual conclusion may depend on the chosen renormalization scheme [5] since only regularization methods which involve dimensional regularization parameters are able to take into account also quadratic divergences.)

Within this context, in Ref. [6] two classes of non-supersymmetric one-loop finite grand unified models with, in group-theoretical respect, particularly simple matter content have been introduced; the strengths of gauge, Yukawa, and scalar-boson self-couplings are determined by demanding the one-loop contributions to their renormalization-group beta functions to vanish. Nevertheless, from the investigation of their eventual quadratic divergences in Ref. [7] it may be deduced that the vector-boson masses of both classes of models as well as the scalar-boson masses of the whole class of simpler models and of all explicitly constructed representatives of the class of more sophisticated models receive already at one-loop level quadratically divergent contributions. In Ref. [8], we succeeded in constructing the complete class of simpler models while for the class of more complicated models, because of the complexity of the corresponding finiteness conditions even at one-loop level, we were only able to derive its most important general features. Here, we construct explicitly a particular subset of the class of general models, namely, that one which is characterized by vanishing Yukawa couplings (Sect. 2), by matching their behaviour for large gauge groups (Sect. 3) with the solutions [8] known for small gauge groups (Sect. 4).

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1 In general, any additional symmetry of some quantum field theory diminishes the number of uncorrelated ultraviolet divergences. Supersymmetry plays a special rôle in reducing this number eventually to zero.
2 The “Non-Yukawa” Model

The model—or, more precisely, class of models—under consideration may be deduced from the so-called “general” model proposed in Ref. [6] and discussed in Refs. [7, 8] by imposing on the latter the additional requirement of the vanishing of all Yukawa interactions. Consequently, it constitutes a completely massless gauge theory based on the special unitary group SU($N$) as gauge group. The generators of SU($N$) in its fundamental representation will be denoted by $T^a$, $a = 1, 2, \ldots, N^2 - 1$. Their normalization is determined by the freely chosen value of their second-order Dynkin index $T_f$, defined according to

$$
T_f \delta_{ab} := \text{Tr}(T^a T^b).
$$

These generators satisfy the commutation relations $[T^a, T^b] = i f_{abc} T^c$, with $f_{abc}$ the completely antisymmetric structure constants of SU($N$).

The particle content of this model comprises

- (real) gauge vector bosons $V^a_\mu$, demanded by gauge invariance and entering in both the field strength $F^a_{\mu\nu} \equiv \partial_\mu V^a_\nu - \partial_\nu V^a_\mu + g f_{abc} V^b_\mu V^c_\nu$ and the covariant derivative $D_\mu \equiv \partial_\mu - i g V^a_\mu T^a$,

- $m$ sets of Dirac fermions $\Psi(k)$, $k = 0, 1, 2, \ldots, m$, and

- real scalar bosons $\Phi$,

all of them transforming according to the adjoint representation of the gauge group, as well as

- $n$ sets of Dirac fermions $\psi(k)$, $k = 0, 1, 2, \ldots, n$, and

- complex scalar bosons $\varphi$,

all of them transforming according to the fundamental representation of the gauge group. The Lagrangian defining this model reads

$$
\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + i \sum_{k=0}^{m} \bar{\Psi}(k) D \Psi(k) + i \sum_{k=0}^{n} \bar{\psi}(k) D \psi(k) + \frac{1}{2} (D_\mu \Phi)^T D^\mu \Phi + (D_\mu \varphi)^\dagger D^\mu \varphi - \frac{\lambda_1}{8} (\Phi^T \Phi)^2 - \frac{\lambda_2}{8} (\Phi^a d_{abc} \Phi^b)^2 - \frac{\lambda_3}{2} (\Phi^T \Phi) (\varphi^\dagger \varphi) - \frac{\lambda_4}{2} (\Phi^a d_{abc} \Phi^b) (\varphi^\dagger T^c \varphi) - \frac{\lambda_5}{2} (\varphi^\dagger \varphi)^2,
$$

where $d_{abc}$ are the completely symmetric constants

$$
d_{abc} \equiv \frac{\text{Tr} \left( \{T^a, T^b\} T^c \right)}{T_f}.
$$
The absence of Yukawa couplings at higher orders of the perturbative loop expansion, obligatory for maintenance of renormalizability of the theory, is guaranteed by the invariance of the above Lagrangian under the reflections \( \Phi \rightarrow -\Phi \) and \( \varphi \rightarrow -\varphi \).

Once the group-theoretic affairs have been settled, the only physical parameters of this model are the gauge coupling constant, \( g \), and the five scalar-boson self-interaction coupling constants \( \lambda_1, \lambda_2, \ldots, \lambda_5 \). The (relative) magnitudes of these dimensionless coupling constants will be determined from the requirement of vanishing one-loop contributions to their renormalization-group beta functions.

For the gauge coupling constant, finiteness at the one-loop level is ensured if the “group parameter” \( N \) and the fermion multiplicities \( m \) and \( n \) are related by

\[
21N - 4(2mN + n) = 1
\]

This constraint forces the group parameter \( N \) to take one of the values \( N = 4\ell + 1 \) with \( \ell = 1, 2, \ldots \), that is, one of the values \( N = 5, 9, 13, \ldots \), and, from the (necessary) positivity of the multiplicity \( n \), i.e., \( n \geq 0 \), implies for the multiplicity \( m \) the upper bound

\[
m \leq \frac{21N-1}{8N} < 3 \quad \text{for arbitrary } N > 0
\]

which restricts \( m \) to any of the three integers \( m = 0, 1, 2 \) and, for any particular choice of \( m \), fixes \( n \) to just that value which allows to fulfil this constraint.

3 Asymptotic Behaviour for Large Gauge Groups

In order to find out by purely algebraic means the general structure to be expected for the solutions of the set of one-loop finiteness conditions for the quartic scalar-boson self-couplings \( \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5 \), we consider first the large-\( N \) limit of these finiteness conditions.

In the course of the analysis of Ref. [8] it has proven very convenient to perform the investigation of the conditions for the vanishing of the one-loop contributions [6] to the renormalization-group beta functions of the quartic scalar-boson self-couplings \( \lambda_i, i = 1, 2, \ldots, 5 \), in terms of the five real and non-negative “self-interaction-type” variables

\[
y_i \equiv \frac{\lambda_i}{g^2} \geq 0 \quad , \quad i = 1, 2, \ldots, 5
\]
The asymptotic form the set of one-loop finiteness conditions for the quartic scalar-boson self-interaction coupling constants assumes in the limit $N \to \infty$ may immediately be read off from its general form given by Eqs. (43) to (47) of Ref. [8]:

\[ N^2 y_1^2 + 24 - 12 N y_1 + 4 N y_1 y_2 + 8 y_2^2 + 2 N y_3^2 = 0 \ , \]
\[ 4 N y_2^2 - 12 N y_2 + 3 N + 12 y_1 y_2 + y_4^2 = 0 \ , \]
\[ 4 y_3^2 - 9 N y_3 + 6 + N^2 y_1 y_3 + 2 N y_2 y_3 + 2 N y_3 y_5 + 2 y_4^2 = 0 \ , \]
\[ N y_4^2 - 9 N y_4 + 3 N + 2 y_1 y_4 + 2 N y_2 y_4 + 2 y_4 y_5 + 8 y_3 y_4 = 0 \ , \]
\[ 4 y_5^2 - 12 y_5 + 2 N y_3^2 + y_4^2 + 3 = 0 \ . \]

Each of the (even general) one-loop finiteness conditions for the five self-interaction-type variables $y_1, y_2, y_3, y_4, y_5$ involves a single negative term linear in precisely one of the variables $y_1, y_2, y_3, y_4, y_5$, which is therefore responsible for counterbalancing all positive terms. In Ref. [8] it has been demonstrated that, as a consequence of the presence of these single negative terms, the variables $y_1, y_2, \ldots, y_5$ are, for $N \geq 5$, bounded by

\[
\frac{2}{N} < y_1 < \frac{9}{N} \ , \\
\frac{1}{4} < y_2 < \frac{25}{7} \ , \\
\frac{2}{3N} < y_3 < \frac{9N}{8} \ , \\
\frac{1}{3} < y_4 < \frac{222}{13} \ , \\
\frac{1}{4} < y_5 < 3 \ .
\]

Consider a generic positive variable $y$ depending on some parameter $\lambda$, that is, $y = y(\lambda) > 0$ for all $1 < \lambda < \infty$. Let this variable be bounded from below and above by two bounds which scale like some powers $\alpha$ and $\beta$ of the parameter $\lambda$, respectively. In other words, let the variable $y$ satisfy a chain of inequalities of the form

\[ 0 < a \lambda^\alpha < y(\lambda) < b \lambda^\beta < \infty \quad \text{for all} \quad 1 < \lambda < \infty \ , \]

with some positive constants $a$ and $b$. In view of that, assume for this variable $y$ a similar power-law behaviour\footnote{This assumption precludes, of course, any oscillatory dependence of $y$ on $\lambda$.} with some exponent $\gamma$ and
some positive constant $k$:

$$y(\lambda) = k\lambda^{\gamma}, \quad 0 < k < \infty.$$  

Then the exponent $\gamma$ is necessarily confined to the range spanned by the two powers $\alpha$ and $\beta$:

$$\alpha \leq \gamma \leq \beta.$$  

Consequently, if, in particular, the two powers $\alpha$ and $\beta$ are equal, i.e., for $\alpha = \beta$, the exponent $\gamma$ has to share this common value,

$$\alpha = \gamma = \beta,$$

and, in addition, the constant $k$ is bounded by the constants $a$ and $b$,

$$a < k < b.$$  

Adhering to these lines of argumentation, we assume that our five self-interaction-type variables $y_1, y_2, y_3, y_4, y_5$ satisfy the above bounds by exhibiting, for sufficiently large values of the group parameter $N$, a power-law behaviour parametrized by five constants, $k_1, k_2, k_3, k_4, k_5$, and the only exponent not fixed by the bounds on these variables, $\gamma$:

$$y_1 = \frac{k_1}{N}, \quad 2 < k_1 < 9,$$

$$y_2 = k_2, \quad \frac{1}{4} < k_2 < \frac{25}{7},$$

$$y_3 = k_3 N^{\gamma}, \quad -1 \leq \gamma \leq +1,$$

$$y_4 = k_4, \quad \frac{1}{3} < k_4 < \frac{222}{13},$$

$$y_5 = k_5, \quad \frac{1}{4} < k_5 < 3.$$  

Inserting this ansatz into the above self-coupling finiteness conditions and recalling again the limit $N \to \infty$, the set of equations which serves to pin down the asymptotic behaviour of the five self-interaction-type variables $y_1, y_2, y_3, y_4, y_5$ becomes

$$k_1^2 + 24 - 12 k_1 + 4 k_1 k_2 + 8 k_2^2 + 2 N^{1+2\gamma} k_3^2 = 0,$$  

$$4 k_2^2 - 12 k_2 + 3 = 0,$$  

$$4 N^{2\gamma} k_3^2 + N^{1+\gamma} k_3 (k_1 + 2 k_2 + 2 k_5 - 9) + 6 + 2 k_4^2 = 0,$$  

$$k_4^2 - 9 k_4 + 3 + 2 k_2 k_4 + 8 N^{\gamma-1} k_3 k_4 = 0,$$  

$$4 k_5^2 - 12 k_5 + 2 N^{1+2\gamma} k_3^2 + k_4^2 + 3 = 0.$$  

\(1\)  

\(2\)  

\(3\)  

\(4\)  

\(5\)
Now, the group parameter, \( N \), enters into each of Eqs. (1) and (5) only via one and the same single term, and that with the power \( 1 + 2 \gamma \). Consequently, for non-zero \( k_3 \) as required by the above lower bound on the variable \( y_3 \), a solution to either of Eqs. (1) and (5) can only exist if the exponent \( \gamma \) is bounded from above by \( \gamma \leq -\frac{1}{2} \). Accordingly, the conceivable range of the exponent \( \gamma \) is narrowed down to \( -1 \leq \gamma \leq -\frac{1}{2} \). For \( \gamma \) within this range, the set of equations (1) to (5) simplifies, once more because of the limit \( N \to \infty \) implicitly understood, to

\[
\begin{align*}
k_1^2 + 24 - 12 k_1 + 4 k_1 k_2 + 8 k_2^2 + 2 N^{1+2\gamma} k_3^2 &= 0, \\
4 k_2^2 - 12 k_2 + 3 &= 0, \\
N^{1+\gamma} k_3 (k_1 + 2 k_2 + 2 k_5 - 9) + 6 + 2 k_4^2 &= 0, \\
k_4^2 - 9 k_4 + 3 + 2 k_2 k_4 &= 0, \\
4 k_5^2 - 12 k_5 + 2 N^{1+2\gamma} k_3^2 + k_4^2 + 3 &= 0.
\end{align*}
\]

The detailed investigation of the case \( -1 < \gamma \leq -\frac{1}{2} \), however, indicates that for \( \gamma > -1 \) there exists no solution at all to the set of equations (6) to (10). The corresponding reasoning is the following. On the one hand, Eq. (7) may immediately be solved for \( k_2 \). After that, with both possible solutions for \( k_2 \) at hand, Eq. (9) may be solved for \( k_4 \). On the other hand, for \( \gamma > -1 \) Eq. (8) reduces to \( k_1 + 2 k_2 + 2 k_5 - 9 = 0 \). Eqs. (6), (8), and (10) may then be combined to yield an expression for \( k_4 \), namely, \( k_4^2 = 4 k_2^2 + 24 k_2 - 6 \), which is, already by the mere number of possible solutions, in clear conflict with Eq. (9). Consequently, the up to now indeterminate value of the exponent \( \gamma \) is unambiguously fixed to \( \gamma = -1 \). In summary, the large-\( N \) behaviour of our self-interaction-type variables \( y_1, y_2, \ldots, y_5 \) is described by

\[
y_1 = \frac{k_1}{N}, \quad y_2 = k_2, \quad y_3 = \frac{k_3}{N}, \quad y_4 = k_4, \quad y_5 = k_5,
\]

where the five constants \( k_1, k_2, \ldots, k_5 \) are to be extracted from the set of equations

\[
\begin{align*}
k_1^2 + 24 - 12 k_1 + 4 k_1 k_2 + 8 k_2^2 &= 0, \\
4 k_2^2 - 12 k_2 + 3 &= 0, \\
k_3 (k_1 + 2 k_2 + 2 k_5 - 9) + 6 + 2 k_4^2 &= 0, \\
k_4^2 - 9 k_4 + 3 + 2 k_2 k_4 &= 0, \\
4 k_5^2 - 12 k_5 + 4 k_4^2 + 3 &= 0.
\end{align*}
\]
In this final form the set of self-coupling finiteness conditions valid in the limit \( N \to \infty \) contains, of course, no longer any reference to the group parameter \( N \). Therefore, owing to its specific internal structure, it is straightforward to derive step by step its complete set of solutions. First of all, Eq. (12) is (and has been already from the very beginning) a quadratic equation for \( k_2 \) only and may hence immediately be solved for \( k_2 \). Then, for a given \( k_2 \), Eq. (11) reduces to a quadratic equation for \( k_1 \) only and may thus be solved for \( k_1 \). Reality of \( k_1 \) eliminates one of the two solutions of Eq. (12) for \( k_2 \), leaving a unique solution for \( k_2 \). Simultaneously, again for a given \( k_2 \), Eq. (14) reduces to a quadratic equation for \( k_4 \) only and may hence be solved for \( k_4 \). After that, for a given \( k_4 \), Eq. (15) reduces to a quadratic equation for \( k_5 \) only and may thus be solved for \( k_5 \). Reality of \( k_5 \) eliminates one of the two solutions of Eq. (14) for \( k_4 \), leaving a unique solution for \( k_4 \). Finally, for a given \( k_1, k_2, k_4, \) and \( k_5 \), Eq. (13) reduces to a linear equation for \( k_3 \) only, which unambiguously entails \( k_3 \). Positivity of \( k_3 \) eliminates one of the two solutions of Eq. (13) for \( k_5 \), leaving a unique solution for \( k_5 \). All together, we end up with the following expressions for the constants \( k_1, k_2, \ldots, k_5 \):

\[
\begin{align*}
k_1 &= 3 + \sqrt{6} \pm \sqrt{3 \left(6 \sqrt{6} - 13\right)} = \begin{cases} 7.7057\ldots, \\ 3.1932\ldots \end{cases}, \\
k_2 &= 1/2 \left(3 - \sqrt{6}\right) = 0.2752\ldots, \\
k_3 &= 2\sqrt{3} \frac{7 + 2\sqrt{6} - (\sqrt{6} + 1) \sqrt{5 + 2\sqrt{6}}}{\sqrt{(\sqrt{6} + 1) \sqrt{5 + 2\sqrt{6} - 4 - 2\sqrt{6}}} = 6 \sqrt{6} - 13} \\
&= \begin{cases} 38.0607\ldots, \\ 1.3417\ldots \end{cases}, \\
k_4 &= \frac{3}{2} \left(\sqrt{6} + 1 - \sqrt{5 + 2\sqrt{6}}\right) = 0.3713\ldots, \\
k_5 &= \frac{1}{2} \left(3 - \sqrt{3 \left[\left(\sqrt{6} + 1\right) \sqrt{5 + 2\sqrt{6} - 4 - 2\sqrt{6}}\right]}\right) \\
&= 0.2894\ldots.
\end{align*}
\]

The asymptotic set of self-interaction finiteness conditions, Eqs. (11) to (13), thus possesses precisely two sets of solutions with all members real and non-negative, and respecting, of course, their above bounds.
4 Summary and Conclusions

The present note has been devoted to the complete construction of a particular class of grand unified theories; the members of this class are characterized by the two requirements of vanishing Yukawa couplings and vanishing one-loop beta functions for the gauge and scalar-boson self-interaction coupling constants. Quite generally, one-loop finiteness of the gauge coupling constant may be trivially satisfied by a suitable choice of the matter content of the theory under consideration. Hence, in order to get an idea of the spectrum of solutions to be expected for the whole set of finiteness conditions, we investigated in the preceding section the relations entailed by one-loop finiteness of the scalar-boson self-interaction coupling constants in the limit of infinitely large gauge groups. We were able to identify precisely two distinct sets of solutions for the scalar-boson self-couplings.

Now, the actual object of our interest is, of course, the spectrum of solutions for gauge groups of finite size. The respective set of one-loop finiteness conditions for the self-interaction-type variables $y_1, y_2, \ldots, y_5$ is given by Eqs. (43) through (47) of Ref. [8]. Furthermore, according to Ref. [8], the smallest possible gauge group allowing for a solution of these general finiteness conditions is SU(9). In Ref. [8], two and only two sets of solutions for the self-interaction-type variables $y_1, y_2, \ldots, y_5$ have been found for the gauge group SU(9); we reproduce them here, for the sake of completeness, in Table 1.

Table 1: Numerical solutions for the five self-interaction-type variables $y_1, y_2, y_3, y_4, y_5$ of the “non-Yukawa” model based on the gauge group SU(9) (taken from Ref. [8]) as well as the corresponding quantities $N y_1$ and $N y_3$.

| Variable  | Solution I         | Solution II        |
|-----------|--------------------|--------------------|
| $y_1$     | 0.4017...          | 0.5054...          |
| $y_2$     | 0.2864...          | 0.2907...          |
| $y_3$     | 0.1862...          | 0.3676...          |
| $y_4$     | 0.3860...          | 0.4008...          |
| $y_5$     | 0.4167...          | 0.7812...          |
| $N y_1$   | 3.6161...          | 4.5491...          |
| $N y_3$   | 1.6762...          | 3.3092...          |
Our findings at the above two opposite extremes of possible values of the group parameter $N$, that is, $N = 9$ and $N \to \infty$, may easily be reconciled by tracing numerically, for $N$ covering its allowed range, the variation of the solutions of the general one-loop finiteness conditions. Qualitatively, the following behaviour of our five self-interaction-type variables $y_1, y_2, y_3, y_4, y_5$ with increasing group parameter $N$ emerges:

- For the quantity $N y_1$, the larger solution increases monotonously from the value $N y_1 = 4.5491\ldots$ at $N = 9$ towards its asymptotic value $k_1 = 7.7057\ldots$ for the limit $N \to \infty$ whereas the smaller solution decreases monotonously from the value $N y_1 = 3.6161\ldots$ at $N = 9$ towards its asymptotic value $k_1 = 3.1932\ldots$ for the limit $N \to \infty$.

- For the variable $y_2$, both of the solutions decrease monotonously from their values $y_2 = 0.2907\ldots$ and $y_2 = 0.2864\ldots$, respectively, at $N = 9$ towards their common asymptotic value $k_2 = 0.2752\ldots$ for the limit $N \to \infty$.

- For the quantity $N y_3$, the larger solution increases monotonously from the value $N y_3 = 3.3092\ldots$ at $N = 9$ towards its asymptotic value $k_3 = 38.0607\ldots$ for the limit $N \to \infty$ whereas the smaller solution decreases monotonously from the value $N y_3 = 1.6762\ldots$ at $N = 9$ towards its asymptotic value $k_3 = 1.3417\ldots$ for the limit $N \to \infty$.

- For the variable $y_4$, both of the solutions decrease monotonously from their values $y_4 = 0.4008\ldots$ and $y_4 = 0.3860\ldots$, respectively, at $N = 9$ towards their common asymptotic value $k_4 = 0.3713\ldots$ for the limit $N \to \infty$.

- For the variable $y_5$, interestingly, the larger solution first starts to increase from the value $y_5 = 0.7812\ldots$ at $N = 9$ and then, after passing its maximum value $y_5 = 1.2742\ldots$ at $N = 29$, continues to decrease in order to approach finally the common asymptotic value $k_5 = 0.2894\ldots$ for the limit $N \to \infty$ whereas the smaller solution decreases monotonously from the value $y_5 = 0.4167\ldots$ at $N = 9$ towards this common asymptotic value $k_5 = 0.2894\ldots$ for the limit $N \to \infty$.

In summary, by the above findings we are led to conclude that for each gauge group allowed by one-loop finiteness of the gauge coupling
constant there will exist exactly two “non-Yukawa” models of the kind described in Sect. 2: the numerical values of the quartic scalar-boson self-interaction coupling constants (relative to the square of the gauge coupling constant) are fixed to be precisely those which guarantee the vanishing of the one-loop contributions to their renormalization-group beta functions.

Finally, concerning the question of eventual quadratic divergences, within the present class of models both vector-boson and scalar-boson masses will be plagued by quadratic divergences already from one-loop level on: On the one hand, Eqs. (17) and (25) of Ref. [7] state that for both classes of models proposed in Ref. [6] the quadratically divergent contribution to the one-loop renormalization of the vector-boson mass is definitely non-vanishing. On the other hand, since the present class of models is a subset of the class of general models considered in Ref. [4], we infer from Eqs. (18) and (19) of Ref. [4] that the quadratically divergent contributions to the one-loop renormalization of the masses of both kinds of scalar bosons, Φ and ϕ, must be non-vanishing for the whole class of non-Yukawa models; the trivial reason for this being the fact that a non-vanishing Yukawa interaction is a necessary ingredient for compensating the contributions of gauge and quartic scalar-boson self-interactions to these renormalizations.
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