Fluctuations of the transverse energy in \(Pb+Pb\) collisions and \(J/\psi\) suppression

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Abstract

The observed \(J/\psi\) suppression in \(Pb+Pb\) collisions shows a drop at those large values of the transverse energy \(E_T\) which arise from fluctuations. The validity of existing models for \(J/\psi\) suppression can be extended into this domain of \(E_T\) by introducing an \textit{ad hoc} factor proportional to \(E_T\). We propose a formalism in which the influence of \(E_T\) fluctuations on the \(J/\psi\) suppression can be calculated and discuss the conditions under which the \textit{ad hoc} factor is obtained.

The observed dependence on transverse energy of \(J/\psi\) production in \(Pb+Pb\) collisions at 158\,\textit{A} GeV is the first unambiguous signal for an anomalous mechanism for charmonium suppression, \textit{i.e.} one which goes beyond what is already observed in proton-nucleus collisions and in reactions with light ions. In order to identify the detailed nature of the anomalous mechanism, it is important to investigate each of its features using a \textit{minimum} of adjustable parameters. The past experience has shown that models succeed to reproduce the data only after several parameters are adjusted. Three years ago, the NA50 collaboration has re-measured the \(J/\psi\) suppression in \(Pb+Pb\) collisions in the region of large transverse energy \(E_T\) and has corrected its earlier result \cite{1}. Rather than being flat as a function of \(E_T\) the new data show a drop at \(E_T \gtrsim 100\) GeV. For an overview of the field, the data and their interpretation we refer to the reviews \cite{2,3} and the proceedings of the Quark Matter '99 conference \cite{4}.

Within the picture of the quark-gluon plasma the newly observed drop is interpreted as the expected onset of \(J/\psi\) melting \cite{5}. Two other groups, Capella \textit{et al.} \cite{6} and Blaizot \textit{et al.} \cite{7}, modify their previously successfully proposed expressions for the observed \(J/\psi\) suppression by introducing the factor

\[
\epsilon(E_T) = \frac{E_T}{E_T(b)}
\] (1)

at the appropriate place, without the need of a new parameter. They argue that the newly discovered drop for \(J/\psi\) suppression arises in the regime where the increase in \(E_T\) is not caused by a decrease in...
impact parameter $b$, but rather by fluctuations in $E_T$ around the mean value $\bar{E}_T(b)$, which is determined by the collision geometry. Larger values of $E_T$ lead to a correspondingly larger suppression. For the comover description by Capella et al. the modification by the factor $\epsilon$ has a small effect and fails to describe the data\footnote{In a recent paper Capella et al.\cite{8} argue, that the drop shown in the data is misleading, since the data are evaluated with respect to minimum bias events, whose $E_T$ distribution differs from the $E_T$ distribution of events in which also a $\psi$ is observed.}, while the same modification to the cut-off model by Blaizot et al. quantitatively describes the newly observed break. These partial and full successes, respectively, call for a derivation of the \textit{ad hoc} formula. This is what is attempted in this paper.

The expression for the charmonium ($\psi$) production cross section in $A + B$ collisions is usually written as

$$\frac{d\sigma_{AB}^{\psi}(E_T)}{dE_T} = \sigma_{pp}^{\psi} \int d^2b \ P_T(E_T, b) \times \int d^2s \ T_A T_B \ S_{nucl} (\vec{b}, \vec{s}) \ S_{FSI} (\vec{b}, \vec{s}),$$

where

$$S_{nucl} = \frac{1 - \exp \left( -\sigma_{abs}^{\psi N_T A} \right)}{\sigma_{abs}^{\psi N_T A}} \frac{1 - \exp \left( -\sigma_{abs}^{\psi N_T B} \right)}{\sigma_{abs}^{\psi N_T B}}$$

represents the part of the suppression (also present in $pA$ collisions) which is related to the propagation of the charmonium through both nuclei, the $\psi$ being destroyed in $\psi N$ collisions with a rate determined by the absorption cross section $\sigma_{abs}^{\psi N}$. We denote by $T_A = T_A(\vec{s})$ and $T_B = T_B(\vec{b} - \vec{s})$ the nuclear thickness functions. $P_T$ represents the probability of observation of $E_T$ in events with impact parameter $b$, and is normalized to 1 when integrated over $E_T$. The anomalous part $S_{FSI}$ accounts for final state interactions with the produced quarks and gluons (QGP) or the hadrons (comovers).

In the comover approach\footnote{In a recent paper Capella et al.\cite{8} argue, that the drop shown in the data is misleading, since the data are evaluated with respect to minimum bias events, whose $E_T$ distribution differs from the $E_T$ distribution of events in which also a $\psi$ is observed.} one writes

$$S_{FSI}^{co} (\vec{b}, \vec{s}) = \exp \{ -\sigma_{co N_T}^{\psi y} (\vec{b}, \vec{s}) \ \ln (N_{co y}^{\psi} (\vec{b}, \vec{s})/N_f) \}.$$ \hspace{1cm} (4)

The suppression function depends on the density of comovers $N_{co y}$ at the rapidity $y$ of the $\psi$, while $\sigma_{co}$ describes the $\psi$ absorption by a comoving meson and is usually taken as the adjustable parameter of the theory (of the order of 1 mb). The corresponding density of comovers in $pp$ collisions is denoted by $N_f$.

In the cut-off model\footnote{In a recent paper Capella et al.\cite{8} argue, that the drop shown in the data is misleading, since the data are evaluated with respect to minimum bias events, whose $E_T$ distribution differs from the $E_T$ distribution of events in which also a $\psi$ is observed.} one assumes

$$S_{FSI}^{cut} (\vec{b}, \vec{s}) = \Theta (n_p^c - n_p (\vec{b}, \vec{s})),$$ \hspace{1cm} (5)

where the (mean) density of participant nucleons in impact parameter space is

$$n_p (\vec{b}, \vec{s}) = T_A [1 - \exp (-\sigma_{pp} T_B)] + (T_A \leftrightarrow T_B).$$ \hspace{1cm} (6)
The “critical participant density” \( n^c_p \), a parameter adjusted to \( n^c_p = 3.7 \text{ fm}^{-2} \), represents the density above which \( \psi \) absorption is 100% effective. The QGP phase transition may (but must not) be the mechanism for the critical transition. In ref. [7] the \( \Theta \)-function in eq. (5) is smeared at the expense of a further parameter \( \lambda \), which is obtained from a fit to the data:

\[
S_{\text{cut}}^{\text{FSI}}(b, s) = \frac{1 + \tanh[\lambda(n^c_p - n_p(b, s))]}{2}. \tag{7}
\]

In order to describe the endpoint behavior of charmonium suppression at large \( E_T \), the factor \( \epsilon \) of eq. (1) is introduced in front of the factor \( N^\text{co}_y \) in eq. (4), with the argument that the number of comovers fluctuates proportionally to the observed \( E_T \), a very plausible assumption. In eq. (5) for the cut-off model, the factor \( \epsilon \) multiplies \( n_p \), the mean number of participants. This modification is not obvious. Why should the number of participants in the tube, where a \( \psi \) is produced, fluctuate proportionally to the global value of \( E_T \)? For instance, in a central collision practically all nucleons participate. Their number equals \( A + B \) and cannot fluctuate. However, the number of produced particles or the produced energy density does fluctuate. Therefore, if the introduction of the factor \( \epsilon \) in front of \( n_p \) has to make sense, we must interpret \( n_p \) and \( n^c_p \) as being proportional to the energy or particle density in the tube where \( J/\psi \) is produced. This is indeed the point of view of ref. [7]. A proportionality factor is irrelevant in the \( \Theta \)-function, but has bearing on the parameter \( \lambda \).

The actual situation is somewhat more complicated: the hadrons which are observed as transverse energy are measured in a pseudo-rapidity interval \( 1 \leq \eta \leq 2.3 \), while the \( \psi \) is measured in the rapidity interval \( 3 \leq y \leq 4 \). (Recall that at the SPS mid-rapidity corresponds to \( y \simeq 3 \).) It is not obvious to which degree fluctuations in \( E_T \) in one rapidity interval should have a bearing on the comovers or the energy density which suppress the charmonium in another rapidity interval. There must be a mechanism for “cross talk”.

The paper addresses the following two issues:

(i) To establish a framework in which the influence of fluctuations in \( E_T \) on \( \psi \) suppression can be calculated, to derive a correction factor containing \( E_T \) and to investigate the limits under which it reduces to the \textit{ad hoc} expression of eq.(4).

(ii) To investigate the importance of “cross talk”, i.e. to calculate the correlation function for the coincidence of hadrons in different rapidity intervals.

To deal with issue (i) we consider a nucleus-nucleus collision at fixed impact parameter \( b \). The individual \( NN \) collisions produce many particles, possibly first as strings or partons which then convert into the observed hadrons. In what follows we will speak of particles, leaving open whether

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\[^3\]After the first version of our paper had appeared on the web, Chaudhuri [8] posted a paper addressing similar issues.
we deal with partons or hadrons. We consider the 6-dimensional phase-space for the particles and divide it into cells numbered as $i = 0, \ldots, k$, where the index 0 is reserved for the cell in which $\psi$ is found together with its comoving particles. We denote by $m_i$ the number of particles in the $i$-th cell and by $p_i(m_i)$ their probability distribution. It will be characterized by a mean value $\bar{m}_i$ and a variance $\sigma_i^2$. The probability to find $m_0$ particles in cell 0 in an event with a total of $M$ produced particles is then

$$\varphi(M, m_0) = \sum_{m_1, \ldots, m_k} p_0(m_0) \cdots p_k(m_k) \delta(M - \sum_{i=0}^k m_i).$$

(8)

Note that we do not sum over $m_0$. We also introduce the probability distribution

$$\phi_T(E_T, M) = \frac{e_T}{\sqrt{2\pi\sigma_{E_T}^2}} \exp\left[-\frac{(E_T - E_T M)^2}{2\sigma_{E_T}^2}\right],$$

(9)

which describes the correlation between the observed transverse energy $E_T$ and the total number $M$ of particles, $e_T$ being the mean transverse energy which each particle contributes to the total $E_T$. The correlation between the observed transverse energy $E_T$ and the number $m_0$ of particles, which suppress $\psi$ is then given by

$$P(E_T, m_0) = \sum_M \phi_T(E_T, M) \varphi(M, m_0).$$

(10)

If Gaussian distributions are used for the correlation functions $\phi_T$ and $\varphi$ and if the sums over $M$ and $m_0$ are replaced by integrals, eq. (10) can be evaluated exactly, but a rather cumbersome expression results. For the case of many cells, $k \gg 1$, the result, can be factorized as

$$P(E_T, m_0) = P_T(E_T, b) P_c(m_0, \bar{m}_0, E_T).$$

(11)

Here the distribution of the fluctuations in $E_T$ is

$$P_T(E_T, b) = \frac{1}{\sqrt{2\pi\sigma_{E_T}^2}} \exp\left[-\frac{(E_T - \bar{E}_T(b))^2}{2\sigma_{E_T}^2}\right],$$

(12)

where $\bar{E}_T$ is the mean value of the produced transverse energy for a given value of the impact parameter. In eq. (12) $P_T$ does not depend on the number $m_0$ of particles in cell 0 because they contribute a negligible amount. However, the distribution function $P_c$ of particle number $m_0$ does depend on $E_T$ via

$$P_S(m_0, \bar{m}_0, E_T) = \frac{1}{\sqrt{2\pi\sigma_{E_T}^2}} \exp\left[-(m_0 - \bar{m}_0(b, \hat{s})\hat{e}(E_T))^2/2\sigma_{0}^2\right],$$

(13)
where the dependence on transverse energy is contained in the expression
\[ \hat{\epsilon}(E_T) = 1 + \alpha_0 \frac{(E_T - E_T(b))}{E_T(b)}, \]
which is close in shape to the one of eq. (1). The factor
\[ \alpha_0 = \frac{\sigma_0^2}{\bar{m}_0} \frac{e_T\bar{E}_T}{\sigma_{E_T}^2} \]
depends on the mean value \( \bar{m}_0 \) and the variance \( \sigma_0 \) of the probability distributions \( p_0 \) for the number of particles in cell 0 and on the corresponding quantities \( \bar{E}_0(b) \) and \( \sigma_{E_T}(b) \) for the distribution of the observed transverse energy. When both distributions are normal, i.e. \( \bar{m}_0 = \sigma_0^2, \sigma_{E_T}^2 = e_T\bar{E}_T \), each of the two factors in eq. (15) equals 1. Then \( \alpha_0 = 1 \) and
\[ \hat{\epsilon}(E_T) = \epsilon(E_T) = \frac{E_T}{E_T} \]
as assumed in refs. [3, 7]. A value for the second factor \( e_T\bar{E}_T/\sigma_{E_T}^2 \) in \( \alpha_0 \) can be deduced from a fit of calculated \( E_T \) distributions (using eq. (12)) to the measured one. One finds values between 1 and 0.7 [3, 7]. Although we have no direct information on the ratio \( \sigma_0^2/\bar{m}_0 \), the multiplicity distribution of produced particles in \( pp \) collisions are known to be negative binomials [10] for which \( \sigma_0^2/\bar{m}_0 = 1 + \frac{m_0}{k} \), where \( k \), the order of the binomial, is found to be of order 3 or 4 for \( pp \) collisions at ISR energies. Therefore \( \alpha_0 \) is a product of two factors, one larger than 1, the other smaller than 1. Although they may compensate each other to a large extent, we are not in a position to give a reliable value for \( \alpha_0 \). The derivation of the expression \( \hat{\epsilon}(E_T) \), eq. (14) is the first result of our paper.

Now we address issue (ii), the effect of incomplete cross talk. In the above derivation it has been tacitly assumed that produced particles in cell 0, which contribute to \( \psi \) suppression, also contribute to the observed \( E_T \). As explained in the introduction, the rapidity intervals for particles which suppress \( \psi \) and for those which produce the observed \( E_T \) do not overlap. Therefore the observables \( m_0 \) and \( E_T \) should not be correlated at all, unless there is cross talk between rapidity intervals. String formation and decay is a possible mechanism which leads to correlations between produced particles in different rapidity intervals.

The following calculation of cross talk is based on string formation and their decay within the dual parton model [11] for particle production (Fig. 1): Two protons, \( P_1 \) and \( P_2 \), with rapidities \( y = 0 \) and \( y = Y \), respectively, interact via color exchange. After the interaction, two strings form: string 1 between the quark with \( y_1 \) and the diquark of \( P_2 \) (assumed to have rapidity \( Y \)) and string 2 between the quark with \( y_2 \) from \( P_2 \) and the diquark from \( P_1 \). The probability distribution for the occurrence of string \( i \) is denoted by \( w_i(y_i), i = 1, 2 \) and is normalized to 1. The strings fragment into hadrons where the number of produced hadrons per rapidity interval is roughly independent of \( y \). In
a $NN$ collision, a hadron with rapidity $y_a$ can arise from each string, provided it covers the rapidity $y_a$. Therefore the probability to find a hadron with rapidity $y_a$ in the event shown in Fig. 1 is

$$P(y_a) = \int dy_1dy_2 \ w_1(y_1) \ w_2(y_2) \ [\theta(y_2 - y_a) + \theta(y_a - y_1)],$$

(17)

where each $\theta$-function refers to the contribution of one string. Furthermore, the probability to observe two hadrons, one at rapidity $y_a$ and another of $y_b$ is

$$P(y_a, y_b) = \int dy_1dy_2 \ w_1(y_1) \ w_2(y_2)$$

$$\times [\theta(y_2 - y_a) \theta(y_2 - y_b) + \theta(y_a - y_1) \theta(y_b - y_1)].$$

(18)

Let us identify $y_a$ with the rapidity of the charmonium $y_\psi$ and $y_b$ with the rapidity $y_{E_T}$ of a hadron contributing to the observed $E_T$. The probability $\alpha_x$ for cross talk ($\alpha_x \leq 1$) is then given by the ratio

$$\alpha_x(y_\psi, y_{E_T}) = \frac{P(y_\psi, y_{E_T})}{P(y_\psi)},$$

(19)

where $P(y_\psi, y_{E_T})$ gives the probability that a hadron at $y_\psi$ (comoving with the $\psi$) and a hadron at $y_{E_T}$ (contributing to $E_T$) are correlated and where $P(y_\psi)$ gives the probability to find a comoving hadron at $y_\psi$ without any further condition.

The probability distribution $w_i(y_i)$ for the occurrence of the string $i$ is taken proportional to the quark distribution function $f_i(x_i) \ dx_i = dy_iw_i(y_i)$, where $x_i$ is the fractional momentum of the quark. If we confine ourselves to the low values of $x_i$ then $f_i(x_i) \sim x_i^{-\frac{1}{2}}$ and

$$w_1(y_1) = ce^{-\frac{1}{2}y_1},$$

$$w_2(y_2) = ce^{-\frac{1}{2}(y-y_2)},$$

(20)

where the normalization drops in the ratio eq. (18). In the experiment under consideration, we are interested in the cross talk between the comoving hadrons ($3 \leq y_\psi \leq 4$) and the hadrons in transverse energy $E_T (1 \leq y_{E_T} \leq 2.3)$. Instead of integrating $\alpha_x$ over these intervals, we evaluate eq. (19) at the mean values $y_\psi = 3.5, \ y_{E_T} = 1.65$. A straightforward calculation then leads to

$$\alpha_x(3.5, 1.65) = 0.82.$$

(21)

The high value of the coefficient for cross talk ($\alpha_x = 1$ corresponds to complete cross talk) reflects the property of the probability distributions $w_i$ in that they are largest for those strings which span the full rapidity range. In our calculation, we have neglected the strings involving sea quarks. Since these strings are shorter, their inclusion would reduce the value of $\alpha_x$.

With this result, the factor $\hat{\epsilon}(E_T)$ from eq. (14) has to be modified to

$$\hat{\epsilon}(E_T) = 1 + \alpha_0 \cdot \alpha_x \left( \frac{E_T - \bar{E}_T(b)}{E_T(b)} \right),$$

(22)

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with $\alpha_0$ from eq. (15) and $\alpha_x$ from eq. (19). Eq. (22) is the final result of our paper.

We now look at the experimental $J/\psi$ suppression as a function of $E_T$ for $Pb+Pb$ collisions at 158 $A$ GeV and compare with it the results of several calculations in order to show the importance of the effects discussed in this paper (Fig. 2). We have repeated the calculation of [7] with the form given in eq. (7) for the anomalous suppression. However, the expression $\epsilon = E_T/\bar{E}_T$ in front of $n_p(\vec{b}, \vec{s})$ is replaced by the expression $\hat{\epsilon}$ derived in eq. (22) with $\alpha = \alpha_0 \cdot \alpha_x$ which describes the influence of the shape of the probability distributions and of the incomplete cross talk. Then $\alpha = 0$ means that no fluctuations are taken into account while $\alpha = 1$ gives the maximal influence of fluctuations as in [6, 7]. A reasonable estimate may be $\alpha = 0.8$, but we also give a curve for $\alpha = 0.4$. While $\alpha = 1$ and $\alpha = 0.8$ are compatible with the data, $\alpha = 0.4$ and smaller values are definitely ruled out, provided the effect discussed in ref. [8] is not too important.

In this paper we have derived and discussed the phenomenological prescriptions by Capella et al. [6] and Blaizot et al. [7] to calculate $J/\psi$ suppression in the region of large values of $E_T$ where the transverse energy fluctuates. The derivation has shown which physical parameters determine the strength with which fluctuations in $(E_T - \bar{E}_T)/\bar{E}_T$ influence $J/\psi$ suppression. These are the width parameters for the multiplicity distributions of comovers and for the $E_T$ distributions and the correlation function for incomplete cross talk. We conclude that the ad hoc factor, eq. (1), holds to a good approximation.

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Figure 1: Particle production for proton-proton collisions within the dual parton model [11]. Two protons, $P_1$ and $P_2$, with rapidities $y = 0$ and $y = Y$ collide and exchange color after which two strings form. We study production of hadrons at the rapidities $y_\psi$ (which comove with the $\psi$) and $y_{E_T}$ (which contribute to the transverse energy $E_T$).
Figure 2: The experimental values for the ratio $J/\psi/DY$ in $Pb + Pb$ collisions at 158 $A$ GeV [1]. The curves are calculated like in the paper by Blaizot et al. [7] and differ by the degree $\alpha$ to which fluctuations are taken into account $\alpha = \alpha_0 \cdot \alpha_x$, eq. (21). $\alpha = 0$ : Fluctuations are not accounted for. $\alpha = 1$ : Fluctuations influence suppression fully. $\alpha = 0.8$ and $\alpha = 0.4$ : Partial reduction of the importance of fluctuations.