Affine-Type Model of Variable Renewable Energy and Its Application in the Uncertain Power Flow

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Abstract. The intermittent variable renewable energy (VRE) results in uncertainty in distribution networks, and the existing affine iterative algorithm cannot consider the characteristics of VRE. To deal with such issues, an affine-type model of VRE considering its regulation characteristics was proposed in this paper. Furthermore, an alternating iterative algorithm based on Newton-Raphson iterative algorithm was proposed, which can consider both power flow equations and the VRE models. Finally, the effectiveness of the affine-type model of VRE and the affine iterative algorithm were evaluated on the modified IEEE 33-bus system and PG&E 69-bus system. The simulation results showed that the proposed method has sufficient completeness and slight conservativeness. Moreover, the proposed method with VRE models is superior over Monte Carlo method, particularly in terms of calculation burden.

1. Introduction
Variable renewable energy (VRE) can effectively reduce the pressure of environment and energy, and is clean enough to be located very close to consumers and load centers. Therefore, centralized generating facilities are giving way to VRE partially [1]. However, VRE usually causes significant uncertainty in the power system. Furthermore, the traditional deterministic power flow calculation method is not adequate to analyze an uncertain power system that has been attracting more and more attention. Generally, the uncertain power flow can be divided into three types, i.e., interval power flow, fuzzy power flow [2], and probabilistic power flow [3]. The interval power flow is more practical for engineering applications than the other two when uncertain factors lack detailed information. However, the correlation among uncertain factors cannot be considered and the obtained interval width is usually too conservative in the interval power flow. It is essential to develop the affine arithmetic to improve the interval power flow to consider the correlation among uncertain factors and reduce the conservativeness [4].

In recent years, literature about improving interval power flow was focused on the reduction of the affine conservativeness. For instance, an affine arithmetic-based forward-backward sweep power flow calculation method was proposed to reduce the conservativeness of the interval power flow for distribution network [5-6]. To assess voltage stability of power systems, a novel method based on affine arithmetic considering uncertainties associated with operating conditions was proposed [7]. A new formulation of the uncertain power flow equations was proposed to explicitly compute the Jacobian matrix and apply a traditional Newton–Raphson [8]. Huang et al. [9] proposed a solution without complex
affine matrix inversion. The above affine power flow analysis methodologies have been applied for the distribution network. However, these methodologies generally treat nodes with DGs as PQ nodes, which cannot consider DGs with special characteristics, such as energy storage and pressure-regulating equipment.

To deal with such issues, the affine-type models of VRE are designed, and an alternating iterative affine power flow algorithm is proposed in this paper. The main contributions of the paper are as follows:

1) The affine-type models based on characteristics of VRE are established to be used in the affine power flow, which considers the relationship between injected power and node voltages.

2) To improve the calculation efficiency of the affine power flow algorithm, a simplified affine algorithm is proposed in this paper.

3) Based on the Newton-Raphson iterative algorithm, an alternating iterative algorithm considering both power flow equations and the affine-type models of VRE are proposed.

The remainder of this paper is organized as follows. In Section 2, the affine power flow equations are designed based on both the basic affine algorithm and its simplified algorithm, and the affine-type models of node power-voltage are described. In Section 3, the affine-type models of VRE are introduced. Section 4 presents the alternating iterative algorithm based on Newton-Raphson iterative algorithm. In Section 5, to verify the effectiveness of the proposed model, two case studies are implemented on the modified IEEE 33-bus system and PG&E 69-bus system, and the simulation results are discussed. Finally, Section 6 concludes this paper.

2. Affine power flow equations

2.1. Elements of affine algorithm

Affine algorithm (AA) is a method for interval analysis that manipulates the error sources [10]. An affine representation of uncertain value is represented as follows:

\[
\hat{x} = x_0 + x_1\epsilon_1 + x_2\epsilon_2 + \cdots + x_n\epsilon_n
\]

where \(x_0\) is the central value, \(x_i\) is known coefficients that represent various partial deviations, the noise symbols \(\epsilon_i\) stand for independent sources of uncertainty and are assumed to lie in the interval of [-1,1].

In AA model, the same noise symbol \(\epsilon_i\) may appear in different affine variables to represent the same source of uncertainty in the given process. Therefore, the AA can fully consider the relation among different uncertain factors.

2.2. Two methods of nonlinear affine operation

The affine operation, introduced in [11], can be generally divided into the linear algorithm and nonlinear algorithm. Linear affine operations do not produce new noises but the nonlinear affine operations include higher-order terms of the noise element. To obtain the standard form of affine, the higher-order terms of the nonlinear algorithm are replaced by some new noise elements, which may increase the interval conservativeness. Therefore, the actual nonlinear affine operation results are approximate. If the range of the nonlinear affine operations can completely cover its real range, the nonlinear affine operation satisfies the completeness. If the range of nonlinear affine operations exceeds the real range, the affine operations are conservative. Therefore, on the premise of ensuring the affine completeness, it is necessary to reduce the affine conservativeness to improve the accuracy of affine operation.

In the existing affine multiplication and division, new noises are added to ensure the completeness of the operations. The algorithm errors are expressed as new noise elements to consider the influence of specific factors. However, the nonlinear affine equations usually need to be solved by the iterative algorithm and the number of noise elements will expand rapidly, which deteriorates the computational efficiency and convergence of the iterative affine power flow.

To solve these problems, an affine algorithm without increasing noises is proposed in this paper. The coefficients of the noises are corrected instead of adding noises in the affine operation, which can improve the efficiency of the algorithm. In this way, the completeness of calculations may be broke. In
This paper, only affine multiplication is needed in the process of affine iterative calculation, thus we consider $\hat{x}$ and $\hat{y}$. Then, the following formula is used in the affine multiplication calculation:

$$\hat{x} \times \hat{y} = x_o y_o + \sum_{i=1}^{N} (x_i y_i + y_o x_i) e_i$$  \hspace{1cm} (2)

Based on this affine operation, the affine central value and noise coefficient can be decoupled. In the process of affine power flow calculation, the noises can be ignored first. Then the affine central value is obtained by deterministic power flow calculation. Finally, the noise coefficients can be calculated by the affine power flow model.

### 2.3. Affine-type power-voltage model of VRE and load

When VRE and load change, the affine-type model of the node power can be constructed according to the fluctuation as follows:

\[
\begin{align*}
\hat{S}_i &= \hat{P}_i + j\hat{Q}_i \\
\hat{P}_i &= P_{i,0} + \sum_{j=1}^{M_i} P_{ij} e_{ij} \\
\hat{Q}_i &= Q_{i,0} + \sum_{j=1}^{M_i} Q_{ij} e_{ij}
\end{align*}
\hspace{1cm} (3)
\]

where $M_i$ is the number of noises at bus $i$. $\hat{P}_i$ and $\hat{Q}_i$ are the affine number of affine real and reactive power injections at bus $i$, respectively. The noises and the corresponding noise coefficients can be designed according to the power fluctuation of the network nodes.

The fluctuation of the injected power causes the fluctuation of the node voltage, and then the voltage complex affine expression of network nodes can be constructed as follows:

\[
\hat{U}_d = \hat{U}_{d,0} + \sum_{j=1}^{N} \sum_{i=1}^{M_i} \hat{U}_{ij} e_{ij}
\hspace{1cm} (4)
\]

where $\hat{U}_{d,0}$ is the central value of node voltage, and $\hat{U}_{ij}$ is the noise coefficients of bus voltages at bus $i$ due to changes in power injection at bus $j$. The noise symbols $e_{ij}$ represent the fluctuation of the power injection, which come from the noises in (3), and is a manifestation of the power injection fluctuation.

### 2.4. Affine-type power flow equations

We suppose that the number of network nodes in the power grid is $N$ and the affine matrix of injected power is $\hat{s}$ in this paper. When the affine voltages are known, the affine function of injected power can be obtained as follows:

\[
f_i(\hat{U}) = \hat{U}_i \sum_{j=1}^{N} Y_{ij}^* \hat{U}_j
\hspace{1cm} (5)
\]

where $Y_{ij}^*$ is the element of node admittance matrix $Y$.

The complex affine power flow of power system can be summarized as the solution of the following nonlinear affine equations:

\[
f_i(\hat{U}) = \hat{S}_i
\hspace{1cm} (6)
\]

To ensure the affine completeness, affine node voltage should satisfy the following relational expression:
\[ f_i(\hat{U}) \equiv \hat{S}_i \]  

(7)

3. Affine-type model of VRE considering its regulation characteristics

3.1. Constant power factor control mode (CPFCM)

The network nodes connected to the DGs with reactive power regulation such as doubly-fed induction generator (DFIG) can be regarded as CPFCM nodes.

The affine-type model is as follows:

\[
\begin{align*}
\hat{P}_i &= P_{i,0} + \sum_{j=1}^{M} p_{ij} \epsilon_j \\
\hat{Q}_i &= \hat{P}_i \tan \phi
\end{align*}
\]  

(8)

where \( \hat{P}_i \) uses the model of (3) and \( \epsilon \) is the power factor angle.

3.2. Constant voltage control mode (CVCM)

The photovoltaic power generation system, some wind turbines, micro gas turbines, and fuel cells are generally connected to the grid through inverters. Therefore, the network nodes with sufficient reactive power compensation and operated under the constant voltage control mode can usually be regarded as CVCM nodes.

The rectangular coordinate system is difficult to deal with CVCM nodes. So, it is necessary to use some simplified methods. To consider the characteristics of CVCM nodes, the affine voltage of the nodes is modified after the iteration.

In the affine power flow, the reactive power controller maintains a constant voltage magnitude of the CVCM nodes. So, the expression of affine voltage is as follows:

\[
\hat{U}_i = V_i \angle (\hat{\theta})
\]  

(9)

where \( V_i \) is the node voltage magnitude and \( \hat{\theta} \) is the affine node voltage angle.

Through Taylor expansion, (9) can be converted to the following expression:

\[
\hat{U}_i = V_i \cos \hat{\theta}_i + j V_i \sin \hat{\theta}_i \\
= V_i (1 - \frac{\hat{\theta}_i^2}{2}) + j V_i \hat{\theta}_i
\]  

(10)

The affine node voltage angle can be deduced from (10) as follows:

\[
\hat{\theta}_i = \text{imag}(\hat{U}_i^{(k-1)}) / V_i
\]  

(11)

where \( \hat{U}_i^{(k-1)} \) is the affine-type voltage of node \( i \). After obtaining the affine angle, the modified affine node voltage can be recalculated by (10).

The affine-type model of the constant voltage control nodes can be summarized as follows:

\[
\begin{align*}
\hat{P}_i &= P_{i,0} + \sum_{j=1}^{M} p_{ij} \epsilon_j \\
\hat{Q}_i^{(k)} &= g(U_i^{(k-1)})
\end{align*}
\]  

(12)

where \( g(U_i^{(k-1)}) \) is the corrective calculation function of the reactive power.
3.3. Constant current control mode (CCCM)

The photovoltaic power generation system and energy storage system can be treated as nodes with controllable injection current through the current inverter. Furthermore, it is considered that the frequency and phase of the output current are consistent with the node voltage. Therefore, the following model can be constructed as follows:

\[
\begin{align*}
\hat{P}_i &= P_{i,0} + \sum_{j=1}^{M} p_{ij} e_j \\
Q_i &= 0
\end{align*}
\]  

(13)

3.4. Wind induction generator (WIG)

Wind induction generator that absorbs reactive power from the grid to establish a magnetic field has no voltage regulation capability. Generally, the value of reactive power is:

\[
Q = -\frac{U^2}{x_m} + \frac{-U^2 + \sqrt{U^4 - 4P^2x_{d}}}{2x_{d}}
\]

(14)

where \(x_m\) is the excitation reactance of generator, \(x\) is the sum of stator reactance and rotor reactance of generator.

To simplify the calculation, the part of (14) can be ignored since \(U^4\) is usually much larger than \(4P^2x\). The simplified model is as follows:

\[
\begin{align*}
\hat{P}_i &= P_{i,0} + \sum_{j=1}^{M} p_{ij} e_j \\
\hat{Q}_i &= -\frac{U_i^2}{x_m}
\end{align*}
\]

(15)

4. Alternating iterative affine power flow algorithm

4.1. Affine-type power flow solution by Newton-Raphson

Existing iterative affine power flow calculations usually use the forward-backward sweep method. However, this method is usually only applicable to the radial distribution network. Therefore, the Newton-Raphson power flow algorithm with more applications is used in this paper.

The traditional Newton-Raphson power flow algorithm will cause two problems when it is applied to affine power flow: the Jacobian matrix needs to be updated before every iteration, which decreases the efficiency of calculation; the inversion of the affine Jacobian matrix increases the difficulty of solving the affine power flow calculation.

Therefore, the Newton-Raphson complex affine iterative calculation method of the constant Jacobian matrix is proposed in this paper. Since the affine operation in this paper can be decoupled, the affine central value and the Jacobian matrix \(J_0\) can be calculated by deterministic power flow. The matrix \(J_0\) not only avoids the problem of complex affine matrix inversion but also does not need to be updated in subsequent iterations.

The Newton-Raphson iteration of the complex affine power flow is as follows:

\[
\begin{align*}
J_0 \Delta \hat{X}^{(k)} &= \hat{Y}_r - Y(\hat{X}^{(k)}) \\
\hat{X}^{(k+1)} &= \hat{X}^{(k)} + \Delta \hat{X}^{(k)}
\end{align*}
\]

(16)

where the affine vector \(\hat{X}^{(k)}\) is composed of affine node voltage, and the affine vector \(\hat{Y}_r\) is composed of affine node power and affine voltage.
In addition to the traditional node types, Newton-Raphson complex affine power flow can also deal with the DG model that is independent of the node voltage. The DG operated under CPFCM and CCCM are all such nodes.

4.2. Calculation of VRE affine-type model
Newton-Raphson complex affine iterative can only consider some types of VRE affine-type models such as CPFCM and CCCM. Therefore, it is necessary to further process the special network nodes.

According to the affine-type models of VRE, the node voltage obtained by iterative calculation of Newton-Raphson complex affine can be used as the known quantity and substituted into affine-type models to calculate the affine reactive power of nodes with VRE. Then the injected active and reactive power is substituted into (16) and step to the next to continue iteration. The above process is repeated until convergence.

The results obtained by the mentioned alternating iteration process can sufficiently consider both power flow balance equations and the characteristics of different VRE models, so it can well apply to the power flow calculation in complex networks.

4.3. Alternating iterative algorithm
In this paper, since the affine operation method ignore the noise, the completeness of the iterative algorithm may be challenged. Therefore, it is necessary to judge whether the calculation result satisfies the completeness of (7). The iteration convergence condition is shown in (17), i.e., whether or not the Euclidean distance between two iterations noise coefficient is less than the limit value. is a very small positive number.

$$\left| \hat{U}_i^{(k)} - \hat{U}_i^{(k-1)} \right| < \zeta$$

(17)

The iterative steps of alternating iteration affine power flow are as follows:

Step 1: Input the network information and data to form affine-type models of node power.

Step 2: Ignoring the noise, the deterministic power flow is calculated with the known central value of affine. The calculation result is taken as the central value of the affine node voltage and power.

Step 3: According to the Newton-Raphson affine iterative algorithm, (16), calculate the affine noise coefficients of each node voltage.

Step 4: Calculate and update the reactive power of network nodes by the affine-type model of VRE.

Step 5: Determine whether the result satisfies the completeness requirement of (7). If not, then return to step 3.

Step 6: Determine whether the convergence is satisfied. If not, then return Step 3.

5. Case study
The proposed affine-type models of VRE and an alternating iterative affine power flow algorithm are implemented using the platform of MATLAB equipped with the MATPOWER toolbox on an Inter 4-Core i5-8259U CPU 2.3GHz personal computer. The proposed method is evaluated by the expanded DG power grid based on the IEEE 33-bus system and the PG&E 69-bus system.

To verify the completeness and conservativeness of the proposed algorithm, the Newton-Raphson method based on the Monte-Carlo simulation (MC method) is used to evaluate the proposed algorithm. Random sampling is performed within the load and power data values, and the power flow is calculated by the Newton-Raphson method. The random sampling number is set to 10^4. It is generally considered that the interval range obtained by the MC method is approximately a precise interval. The proposed method without VRE models just treats all network nodes as PQ nodes.

5.1. Case study of modified IEEE 33-bus system
A modified IEEE 33-bus system with DGs is adopted, as shown in figure 1.
The power output range of each distributed generation is from 300 kW to 500 kW, and the load variation range of each node is ±20% of the rated power. The parameters of different distributed power are shown in Table 1. The calculated intervals of the node voltage magnitude and angle of the modified IEEE 33-bus system are shown in Figure 2 and Figure 3, respectively. The iteration times and calculation time of the two algorithms are shown in Table 2.

Table 1. Parameters of distributed generation.

| Device | Node type | Voltage magnitude/p.u. |
|--------|-----------|------------------------|
| DG9    | DFIG      | CVCM                   | 0.98 |
| DG18   | PV        | CCCM                   | N/A  |
| DG25   | DFIG      | CVCM                   | 0.99 |
| DG33   | WIG       | WIG                    | N/A  |

Figure 2. Comparison of voltage magnitude among different methods in the modified IEEE 33-bus system.

Figure 3. Comparison of voltage angle among different methods in the modified IEEE 33-bus system.

Table 2. Comparison of IEEE 33-bus system computing efficiency.

|                      | Iteration times | Calculation time/s |
|----------------------|-----------------|--------------------|
| Proposed method      | 4               | 0.04               |
| MC method            | 10000           | 114.23             |
It is shown from figure 2 and figure 3 that the voltage magnitude and angle intervals obtained by the proposed method can completely cover the results of the MC method, indicating that the proposed method can satisfy the requirements of completeness. Besides, the interval obtained by the proposed method is only slightly more conservative than that of the MC method, but its iteration times and calculation time are both much less than those of the MC method, as shown in table 2. Furthermore, the proposed method without VRE models cannot consider the characteristics of VRE and its intervals are quite different from other methods, which verified the effectiveness and correctness of the affine-type model of VRE. Therefore, it can be seen that the proposed method with VRE models can accurately handle uncertain power flow calculations with distributed generation in the modified IEEE 33-bus system efficiently.

5.2. Case study of modified PG&E 69-bus system

In this case, five DGs are connected to the PG&E 69-bus system, as shown in figure 4.

![Figure 4. The modified PG&E 69-bus system.](image)

The power output range of each distributed generation is from 300 kW to 500 kW, and the load variation range of each node is ±20% of the rated power. The parameters of different DGs are shown in table 3. The calculated intervals of the node voltage magnitude and angle of the modified PG&E 69-bus system are shown in figure 5 and figure 6, respectively. The iteration times and calculation time of the two algorithms are shown in table 4.

| Device | Node type | Voltage magnitude/p.u. | Power factor |
|-------|-----------|------------------------|--------------|
| DG9   | DFIG      | CVCM                   | 0.99         | N/A          |
| DG17  | PV        | CCCM                   | N/A          | N/A          |
| DG27  | WIG       | WIG                    | N/A          | N/A          |
| DG32  | DFIG      | CPFCM                  | N/A          | 0.9          |
| DG41  | DFIG      | CPFCM                  | N/A          | 0.9          |
Figure 5. Comparison of voltage magnitude between the proposed method and MC method in the modified IEEE 33-bus system.

Figure 6. Comparison of voltage angle between the proposed method and MC method in the modified PG&E 69-bus system.

Table 4. Comparison of IEEE 33-bus system computing efficiency.

|                | Iteration times | Calculation time/s |
|----------------|-----------------|--------------------|
| Proposed method| 4               | 0.04               |
| MC method      | 10000           | 114.23             |

The results of the intervals in figure 5 and figure 6 show that the proposed method can handle large-scale distribution networks with DGs. Compared with the results obtained in IEEE 33-bus system shown in table 4, the iteration times and calculation time of PG&E 69-bus system do not increase, which shows that the computational efficiency of the proposed method is not affected by the size of the network.

6. Conclusion
This paper proposes an affine-type model of VRE considering its regulation characteristics, as well as an alternating iterative power flow affine algorithm based on the Newton-Raphson method to consider both power flow equations and the affine-type models of VRE.

Furthermore, the proposed affine-type model of VRE can sufficiently consider the characteristics of VRE and be used in the affine algorithm. Besides, the proposed affine algorithm will not increase noise, which can effectively reduce the calculation burden. It also avoids the problem of interval expansion caused by the accumulation of noise. Compared with the Monte Carlo method that can only obtain the interval value of the node voltage magnitude and angle, the proposed method can quantitatively analyze the influence of various uncertain factors, and provide support for subsequent applications.

Besides, compared with the Monte Carlo method, the proposed method is slightly conservative, but its calculation efficiency is much higher.

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