A split signal polynomial as a model of an impulse noise filter for speech signal recovery

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Abstract. The synthesis of the non-linear non-recursive digital filter of impulse noise on the basis of the splitting method in time domain is described. The filter recovers speech signals, distorted by impulse noise. The filter model is constructed as the splitting polynomial of an odd degree. The splitter is the time delay line, comprising the equal number of previous and subsequent samples with respect to the current time moment. The polynomial parameters result from solving an approximation problem in the mean-square norm. It is shown that the filter with the splitting model provides more precise speech signal recovery than the median and Volterra filters.

1. Introduction

The problem of impulse noise filtration is often solved in electrical and radio engineering [1–3]. The impulse noise is emerged during switching of different electrical and electronic devices, in case of mechanical damage of the information storage device surface, in operating internal combustion engines, under the influence of various atmospheric phenomena etc. Various methods of impulse noise cancelling are applied to improve the signal recovery quality and to achieve the high signal recognizability [1–4].

The classic method of impulse noise suppression is the median filtration. However, the median filter is known for its drawback which consists in distortion of signal intervals not affected by impulse noise. The median filter is considered not to be optimal, because it does not use the information on the statistical properties of signal and noise [1–4]. As a result, the development of impulse noise filtration methods, ensuring a high quality of signal recovery, is an urgent task.

This paper represents the synthesis of impulse noise filters in time domain on the basis of the splitting method for impulse noise suppressing in speech signals. This method has the following significant advantages [4, 5]:

- the statistical properties of signals and interference are taken into account automatically in the process of filter synthesis (its training);
- in comparison with the mathematical apparat, such as the functional Volterra series [6–10], applied to filter modeling, the splitting method builds a simpler polynomial filter model, adapted to the assigned class of input signals;
- as distinct from the Volterra series the split signal polynomial is free from the convergence problem, that is why it makes possible synthesis of substantially nonlinear devices;
the split signal polynomial comprises the linearly incoming parameters, so the parameters of the filter model are defined as a globally optimal solution of the approximation problem in the uniform and mean-square norms [4].

2. A splitting method for non-linear non-recursive digital filter modeling

Digital impulse noise filters are synthesized within the framework of the “black box” principle by the split signal theory. According to this theory of the non-linear filter, operator \( F_s \) is described by the composition of two operators: the splitter operator and the operator of the nonlinear memoryless transformer [4, 5].

Splitter operator \( F_p \) transforms scalar signal \( x(n, \bar{a}) \), \( n \in I_n \), \( \bar{a} \in G_a \) into vector signal \( \bar{x}_p(n, \bar{a}) \),

\[
\bar{x}_p(n, \bar{a}) = F_p[x(n, \bar{a})] = [x_{p1}(n, \bar{a}), x_{p2}(n, \bar{a}), ..., x_{pm}(n, \bar{a})],
\]

where \( x_{pi}(n, \bar{a}) = F_{pi}[x(n, \bar{a})] \), \( x_{p2}(n, \bar{a}) = F_{p2}[x(n, \bar{a})] \), ..., \( x_{pm}(n, \bar{a}) = F_{pm}[x(n, \bar{a})] \), \( I_n \) is the length of finite or periodical input signal \( x(n, \bar{a}) \), \( n \) is the normalized discrete time variable, \( \bar{a} = [a_1, a_2, ..., a_J] \) is the vector from set \( G_a \) of the variables of signal \( x(n, \bar{a}) \) under the following conditions:

- vector signals do not vanish, i.e.

\[
\bar{x}_p(n, \bar{a}) \neq 0
\]

for every \( n \in I_n \), \( \bar{a} \in G_a \);

- the phase portraits of split signals do not cross and touch each other, and they are not self-intersecting, i.e. for any \( \bar{a}_1 \neq \bar{a}_2 \), \( \bar{a}_1 \in G_a \), \( \bar{a}_2 \in G_a \), \( n_1 \neq n_2 \), \( n_1 \in I_n \), \( n_2 \in I_n \), there is inequality written as

\[
\bar{x}_p(n_1, \bar{a}_1) \neq \bar{x}_p(n_2, \bar{a}_2).
\]

Linear, non-linear, stationary and non-stationary signal splitters can be designed [4].

Operator \( P \) of the nonlinear memoryless transformer converts vector signal \( \bar{x}_p(n, \bar{a}) \) into scalar signal \( y(n, \bar{a}) \). This operator is usually described by multidimensional polynomial

\[
y(n, \bar{a}) = P[\bar{x}_p(n, \bar{a})] = \sum_{j_1=0}^{J_1} \sum_{j_2=0}^{J_2} ... \sum_{j_m=0}^{J_m} C_{j_1, j_2, ..., j_m} [x_{p1}(n, \bar{a})]^{j_1} \times [x_{p2}(n, \bar{a})]^{j_2} ... [x_{pm}(n, \bar{a})]^{j_m},
\]

(1)

although, there are other forms of the operator model [4].

Multidimensional polynomial (1) of degree \( p \) \( (p = J_1 + J_2 + ... + J_m) \) for all the \( n \in I_n \), \( \bar{a} \in G_a \) satisfies the following condition:

\[
\| y^o(n, \bar{a}) - y(n, \bar{a}) \| \leq \varepsilon,
\]

where \( \varepsilon \) \( (\varepsilon > 0) \) is the assigned error of the approximation of ideal non-linear filter operator \( F_s \), \( y^o(n, \bar{a}) \) is the output signal of the ideal non-linear filter:

\[
y^o(n, \bar{a}) = F_s[x(n, \bar{a})].
\]

The structure of the non-linear non-recursive digital filter (NNDF) model, described by equation (1), is shown in Figure 1.
It should be noted that the number of coefficients in polynomial (1) is calculated by formula

\[ \sum_{i=1}^{p} R_{m+i-1}^{i} = \sum_{i=1}^{p} \frac{(m+i-1)!}{i!(m-1)!}. \]

If number \( m \) of splitting channels in polynomial (1) is large, the task of operator \( F_s \) approximation has a high dimension. As a result, the solution of this task causes ill-conditioning and large computational cost.

Let us consider the synthesis of NNDF, a restoring speech signal from the mixture of it with impulse noise, on the basis of the splitting method.

3. Impulse noise cancellation in speech signals by non-linear filtration in time domain
The speech signal, used for training the NNDF model, had the time length of 35 seconds (280 000 samples at an 8 kHz sampling rate). It comprised various phrases of four speakers (two men and two women). These phrases differed in loudness.

The speech signal, applied for estimating the NNDF model quality, had the time length of 20 seconds (\( Q = 160000 \) samples at an 8 kHz sampling rate). It differed from the training speech signal.

The impulse noise was formed as a random process with the uniform distribution in the range of \([-0.5; 0.5]\). The moments of the impulse noise appearance were selected according to the following rule. If at the \( n \)-th moment, the generator of random numbers with the uniform distribution in the range of \([0; 1]\) has given a number less than assigned threshold \( \alpha \) (in research \( \alpha = 0.01 \)), then the impulse interference appeared at the \( n \)-th time. Thus, the probability of the impulse interference appearance at the current \( n \)-th moment is \( \alpha \), and the absence probability is \( (1-\alpha) \). There was an additional restriction: the distance between adjacent interference samples is no less than five samples of the speech signal.

The filter quality was evaluated by means of the root-mean-square error written as

\[ \varepsilon = \frac{1}{Q-3} \sqrt{\sum_{n=4}^{Q} (y^0(n,\bar{a}) - y(n,\bar{a}))^2}, \]

where \( y^0(n,\bar{a}) \) is the desirable filter output signal (undistorted speech signal) with \( Q \) samples, \( y(n,\bar{a}) \) is the output signal of non-linear filter model (1).

The splitter was built in the form of the time delay line. Every unit of this line delayed signal with one sample. The delay line length varied. The causal polynomial filter models, which take into account signal samples at the \( n \)-th, \( (n-1) \)-th, \( (n-2) \)-th and etc. time moments, as well as the non-causal polynomial models, which operate with samples at previous and subsequent time moments relative to the current \( n \)-th moment, were synthesized.

The dependencies of error \( \varepsilon \) on variable \( \xi \) are depicted in Figure 2. Variable \( \xi \) is the number of subsequent samples relevant to the \( n \)-th moment for the splitter with memory length \( m \) equal to five. The dependencies are represented for various degrees of the polynomial model. It was found, that the rise of splitter memory length \( m \) does not decrease the approximation error, so it is recommended to build the NNDF splitter with the least memory length (the results, represented below, were obtained for \( m = 5 \)).
The analysis of curves in Figure 2 shows the following:
– the way of signal splitting affects the filtration precision, namely, error $\varepsilon$ is minimum under the condition of equal number ($\xi = 2$) of previous and subsequent samples with respect to the $n$-th time moment;
– the members of the even degree in the split signal polynomial do not influence root-mean-square error $\varepsilon$, so they can be eliminated from filter model (1).

![Figure 2. Dependencies $\varepsilon(\xi)$ for various degrees of the NNDF polynomial model.](image)

As a result, the NNDF model is described by expression

$$
y(n, \tilde{a}) = \sum_{r=2k-1}^{p} \sum_{j_1=0}^{J_1} \sum_{j_2=0}^{J_2} \cdots \sum_{j_5=0}^{J_5} C_{j_1, j_2, \ldots, j_5} x^{j_1}(n-2, \tilde{a}) x^{j_2}(n-1, \tilde{a}) x^{j_3}(n, \tilde{a}) x^{j_4}(n+1, \tilde{a}) x^{j_5}(n+2, \tilde{a}),$$

where $r = j_1 + j_2 + \ldots + j_5$, $k = 1, 2, \ldots, (p + 1)/2$.

Let us compare the impulse noise suppression results, obtained by NNDF, the median filter and the Volterra filter, which has the following model:

$$
y(n) = \sum_{r=2k-1}^{p} \sum_{j_1=0}^{J_1} \sum_{j_2=0}^{J_2} \cdots \sum_{j_5=0}^{J_5} h_{j_1, j_2, \ldots, j_5} x^{j_1}(n) x^{j_2}(n-1) x^{j_3}(n-2) x^{j_4}(n-3) x^{j_5}(n-4),$$

where $r = j_1 + j_2 + \ldots + j_5$, $k = 1, 2, \ldots, (p + 1)/2$.

The dependencies of error $\varepsilon$ on polynomial model degree $p$ are shown in Figure 3. Curves 1–3 are obtained by the median filtration with the aperture length of 3, 5, 7 samples respectively, curve 4 – by Volterra filtration (3), curve 5 – by NNDF with model (2).

![Figure 3. Dependencies $\varepsilon(p)$ for different filtration methods.](image)

As it is seen from Figure 3, NNDF, synthesized by the splitting method, provides a more accurate signal recovery as compared to the median and Volterra filtrations. The signal recovery accuracy is increasing at the rise of the NNDF polynomial model degree.
4. Conclusion
The proposed method of impulse noise filter synthesis is based on the splitting theory [4, 5] and the principle of “supervised learning” [4–10]. Filter model parameters are determined by solving the nonlinear operator approximation problem. Eventually, the input-output mapping of the filter is created. The statistical properties of signals and noises are taken into account in the process of “learning” the filter model automatically. Since the filter model in the form of the split signal polynomial is linear with respect to incoming parameters, these parameters, resulted from approximation problem solving, are globally optimal.

An essential feature of the filter, synthesized by the splitting method, is its input signal invariance under condition of keeping the statistical signal and noise characteristics. Thus, the filter, synthesized at test signals, acceptably works at other signals with similar statistical characteristics. The rise of splitting polynomial degree enables to achieve the high filtration accuracy.

At the NNDF synthesis for cancelling impulse noise from speech signals it was revealed the following facts:
- the NNTSF model is a multi-dimensional polynomial of an odd degree;
- the splitter should be built as the time delay line with the length equal to 5;
- the least error is achieved, if the split signals contain an equal number of previous and subsequent samples with respect to the current time moment;
- NNDF, synthesized on the basis of the splitting method, provides the least root-mean-square error in comparison with the median and Volterra filters.

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References
[1] Gonzalez R C and Woods R E 2008 Digital Image Processing (New York: Prentice Hall)
[2] Gonzalez R C, Woods R E and Eddins S L 2009 Digital Image Processing Using MATLAB (Knoxville: Gatesmark Publishing)
[3] Trussell H J and Vrhel M J 2008 Fundamentals of Digital Imaging (United Kingdom: Cambridge University Press) p 556
[4] Lanne A A and Solovyeva E B 2000 Radioelectronics and Communications Systems 43(3) 3–10
[5] Solovyeva E B and Degtyarev S A 2008 Radioelectronics and Communications Systems 51(12) 661–668
[6] Schetzen M 2006 The Volterra and Wiener Theories of Nonlinear Systems (Melbourne: Krieger Publishing Co. Inc.) p 618
[7] Nelles O 2001 Nonlinear System Identification. From Classical Approaches to Neural Networks and Fuzzy Models (Berlin: Springer-Verlag Berlin Heidelberg) p 785
[8] Ogunfunmi T 2007 Adaptive Nonlinear System Identification. The Volterra and Wiener Model Approaches (New York: Springer Science+Business Media, LLC) p 229
[9] Block-oriented Nonlinear System Identification 2010 ed F Giri and E-W Bai (Berlin: Springer-Verlag Berlin Heidelberg) p 423
[10] Advances in Nonlinear Signal and Image Processing 2006 ed S Marshall and G L Sicuranza (New York: Hindawi Publishing Corporation) p 361