Decentralized cooperative control with connectivity maintenance for multiagent systems

Abstract

This paper addresses the cooperative control with connectivity maintenance problem for multiagent systems. It is assumed that each agent has the same sensing capability, that is to say, each agent can access the relative measurements from its neighboring agents constrained by its sensing maximum distance. In order to fulfill the control objective, the interaction topology of multi-agent systems is modeled as the undirected graph, and a feasible set of weights is given according to the actual distances of edges established in the undirected graph. Then, the decentralized non-linear controller is provided, which guarantees the property of connectivity maintenance. Moreover, using the above obtained results, the stability analysis of cooperative control for multiagent systems is investigated by the Lyapunov theory, which shows that the control objective is achieved. To demonstrate the effectiveness of the proposed control schemes, several numerical simulation results are presented.

Keywords: multiagent systems, cooperative control, connectivity maintenance, laplacian matrix

Introduction

Multi-agent systems have received a great deal of attention by many researchers from different subjects, which can provide a highly efficient way to perform some particular tasks in extreme environments. Many potential practical applications of Multi-agent systems can range from, for instance, space-based interferometers, surveillance, and reconnaissance systems, hazardous material handling, and distributed reconfigurable sensor networks. Hence, agents are required to cooperate to achieve these tasks, which means that agents must interact with each other, in general, each agent should receive some local information needed to accomplish the tasks, which implies that he objective is to present a control approach that meet several physical constraints (see i.g. 1, 2, 3). It is assumed that agents can communicate with each other in terms of the sensing constraints, then necessary information can be obtained by using the wireless communication sensors, generally, such information can include states, relative measurements between any two agents and the distances in the Euclidean space. 4, 5 It is also pointed that the information of each individual agent is always limited due to the presence of physical communication constraints. Therefore, the information interaction plays a critical important role in the cooperation (or coordination) control of multi-agent systems. 6, 7 As a consequence; the resulting communication architecture of the agents can be abstracted by a graph, which is often called information graph. As a result, a dynamic proximity graph is developed, generally speaking, which relies on the information states of agents, the dynamic topology of the graph can be generated simultaneously according to the movements of agents, which results in different algebraic connectivity properties, therefore, it is meaningful to study the connectivity of the graph modeled by a team of agents. The valuable information can be obtained by analyzing the spectral properties of graph Laplacian, i.e., the eigenvalues and eigenvectors, thus the basic structure of graph can be clearly characterized, while the local and limited information can be obtained by each agent due to limited sensing ability and local communicate with other agents, and eigenvectors cannot be easily computed, thus the decentralized control algorithm is considered. 8

In this paper, we will investigate the nature and performance of the network of agents by analyzing its corresponding eigenvalues of the Laplacian matrix \( L(G) \). In 1973, Fielder\(^9\) defined the algebraic connectivity as the second smallest eigenvalue \( \lambda_2 \) of Laplacian matrix for undirected graph. Specifically, the connected graph can be fully characterized by using the sign of \( \lambda_2 \). It is difficult to maintain the connectivity as the connectivity is a time-varying function of movement of the vertices.\(^10\) To this purpose, several control strategies are proposed by applying the useful information from the relation between the connectivity and second smallest eigenvalue \( \lambda_2 \). In general, using the information states of the system, mathematically, \( \lambda_2 \) can be represented. In addition, each agent has local information from its neighbors defined by proximity graph described as follows or R disk graph,\(^13\) which implies that agent does not have the value of the quantity \( \lambda_2 \), or equivalently, in this way, \( \lambda_2 \) cannot be computed, thus the optimization methods are used.\(^14, 15\) In particular, if any two agents have no interaction or the local information of one agent is not available to the other agent, then the relationship between them is not established, and the corresponding graph is not connected, thus we have \( \lambda_2(L) = 0 \) and \( \lambda_2(L) = n \) if the graph is fully connected, namely, each agent have relationships with the rest of agents, from the information flow point of view, the shared information are regarded as the information states of all agents, such information is not easy to obtain in the real application. As indicated in,\(^16\) a theoretical framework is presented, and connectivity maintenance methods are discussed. In general, two main different approaches are developed to deal with connectivity maintenance problem in multi-agent systems networks, that is local and global connectivity based decentralized control. Decentralized approaches for controlling local connectivity in agent networks have been developed.\(^17, 18\) As illustrated in,\(^19\) the authors proposed a decentralized control method for maintaining the global connectivity in mobile robotic systems, both simulation and experiment results are provided to validate the effectiveness of the proposed algorithm. In,\(^20\) using the bounded control inputs, the authors first proposed control algorithm

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for maintaining the global connectivity, and a theoretical analysis of the disturbance are presented as well. In order to achieve coordinate control tasks, keeping communication among agents and inter-agent sensing costs limited are major aspects of implementing coordinated tasks in multi-agent systems, which is needed to considered, where the cardinality of the directed edges \( |e| \) is defined as communication(or sensing) cost, which is usually defined as communication complexity. This is an advantage of the directed graph.

In this paper we focus on maintaining the connectivity of the dynamic network for a team of agents. To accomplish our control objective, we suppose that each agent can only have local and limited information, which poses theoretical and practical challenges. We investigate the algebraic connectivity of graph by using the second smallest eigenvalue \( \lambda_2 \) defined by Fielder. Using power iteration method, the estimate of \( \lambda_2 \) can be computed. Moreover, the dynamic edge weights are defined, which is usually referred to as edge (or called link) tension. In addition, a sufficient condition for a non-negative matrix is derived, which guarantees the eigenvalues are positive numbers. To maintain the connectivity of the graph, in the sequel, the energy function is defined and the stability of the system is analyzed. The remainder of the paper is organized as follows. Some basic terminologies of graph theory and problem formulation are given in Section II. A control strategy with local and limited information is proposed in Section III. Stability analysis of control law and convergence of consensus on information architecture for multiple agents are investigated in Section IV. Numerical simulation results are provided to illustrate the theoretical results in Section V. Finally, the conclusions are presented in Section VI.

Background and preliminaries

In this section, we review some of the useful terminologies of graph theory used throughout in this paper.

A graph \( G = (V, e) \) is composed of two sets of nodes (or vertices) \( V \) and links(or edges) \( e \), respectively. We can label them with integers in numerical order, namely, \( \{1, 2, 3, ..., n\} \) for future reference, thus the set of nodes \( V = \{v_1, v_2, ..., v_n\} \) is unique determined. Based on this, the edge set \( e = \{e_1, e_2, ..., e_n\} \) can be recognized if there exists an edge \( e_k \in e \) between agent i and agent j for all \( (i, j) \in \{1, 2, ..., n\} \). In general, the number of edges is no less than the number of vertices, which is considered in the sequel. An ordered pair of nodes \( (i, j) \) is called if single direction is assigned to the edges, specifically, vertices i and j are the parent point and the childhood point, respectively, the important relation \( (i, j) \neq (j, i) \) is guaranteed, and vice versa, geometrically, which is usually denoted by a single arrow, notice that, called unordered pair, if it can be denoted by double arrow or simple line, i.e., \( (i, j)\rightarrow (j, i) \in e \). In this paper, a undirected graph is only considered, for simplicity, which is referred to as a graph. The directed graph is called if all the edges in the graph are ordered pairs of nodes, and the undirected graph are created if all the edges in the graph have the property of \( (i, j)\rightarrow (j, i) \in e \). A directed path is a sequence of edges satisfies that the vertices taken from the vertices set \( V \) are different in the path, self-loops and multiple edges are neglected, i.e., \( v_i \rightarrow v_1 \rightarrow v_2 \rightarrow ...v_k \) with \( v_k \in V \), where \( v_i \) and \( v_j \) are interpreted as the start node and the end node, respectively, and \( \{v_2,v_3,...,v_{k-1}\} \) the intermediate nodes. Note that the self-loop edge \( v_i \rightarrow v_i \) consists of one vertex \( i \) such that both start vertex and end vertex are the same. \( \triangle \)

A directed graph is referred to as strongly connected if there exists a path generated by any two nodes \( v_i \rightarrow v_j \) for all \( (i, j) \in \{1, 2, ..., n\} \) with the specified direction of the edges. Using above results, the information topology of the formation for multi-agent is abstracted to the corresponding graph, where each agent \( i \) is represented the vertex \( v_i \), i.e., \( a_i \rightarrow v_j \).

To introduce the concept of algebraic connectivity, we first define a neighborhood set of agent \( i \), which is widely used in the rest of this paper. From a practical point of view, it would be interesting to consider the mobile networks, because the communication networks created by a group of agents are characterized by temporal variations and the real time movements of individual agents. \( \triangle \)

Jadbabaie et al. \( \triangle \) studied the stability properties of coupled non-linear oscillator networks under the spectral graph theory and control theory; moreover, Poonawala and Spong\( \triangle \) investigated the strong connectivity problem in directed proximity graphs, where, the decentralized control strategy is proposed, despite the presence of disturbances or additional control objectives, the strong connectivity are maintained. \( \text{[Neighborhood set].} \) For an agent \( i \), we define the neighborhood set of agent \( i \), denoted by \( N_i = \{i, (i, j) \in e\} \) with respect to agent \( i \), i.e., agent \( j \) are the neighbors of agent \( i \), implying that agent \( i \) can acquire the relative information form \( N_i \) such as relative positions, distances and angles in its local coordinate system, that is to say, the agent \( j \in N_i \) can be view as attractive goals to agent \( i \). Note that the total number of neighbors for agent \( i \) is the cardinality of neighborhood set, denoted by \( |N_i| \).

From Figure 1, three terminologies related to graphs are provided, namely, Strongly connected graph, unilateral connected graph and weakly connected graph. For a graph \( G = (V, e) \), if \( |N_i| \) exists only one path \( \pi \) consists of vertices and edges, such that \( \forall(v_i, v_j) \in V \) and \( v_i \leftrightarrow v_j \), i.e., \( v_i \rightarrow e_1 \rightarrow v_2 \rightarrow e_2 \rightarrow v_3 \rightarrow e_3 \rightarrow v_4 \rightarrow e_4 \rightarrow v_1 \), with the edge \( e_i = (v_i, v_j) \).

**Figure 1** Connected graph. (A) Strongly connected graph; (B) Unilateral connected graph; (C) Weakly connected graph.

To proceed, we now introduce the important concept of the proximity graphs with the quantity \( \Delta \), where \( \Delta \) can be view as the feasible distance or maximum distance; hence the resulting proximity graphs are referred to as \( \Delta \) disk proximity graphs. Our purpose here is how to define the relative edge between agents \( i \) and \( j \), to this end, the standard procedure is applied and the definition is provided as follows (Figure 2). \( \text{[Proximity Graphs]} \) Let the positions of agent \( i \) and \( j \) be \( p_i(x_i(t), y_i(t)) \) and \( p_j(x_j(t), y_j(t)) \) in 2-Dimensional plane, respectively, \( \Delta \) the Euclidean distance, i.e.,

Citation: Yang X, Fan X, Li G. Decentralized cooperative control with connectivity maintenance for multiagent systems. Int Rob Auto J. 2019;5(1):1–10.
DOI: 10.15406/iraJ.2019.05.00163
\[ d_{ij} = \sqrt{(x_i(t)-x_j(t))^2+(y_i(t)-y_j(t))^2} \] (1)

Which results in dynamic proximity graphs characterize by the time-varying states \( p_i(t) \) and \( p_j(t) \). We here assume that \( d_{ij} = d_{ji} \) by neglecting model errors, a parameter sensor noise and sensor bias. For undirected proximity graph, if \( d_{ij} \leq \Delta \), agent j has entered the measurable area of agent i, or equivalently, the information of agent j is available to agent i, and vice versa, as a result, the dynamic edge (or link) \( e_{ij} \) is generated between two vertices \( v_i \) and \( v_j \), and if agents are out of range of each other, namely, \( d_{ij} > \Delta \), no information about each other is acquired by agents i and j, thus no edge is established between two agents \( v_i \) and \( v_j \). For brevity, we use symbol to denote the edge relation as

\[ \ell (e_k) = \begin{cases} 1, & d_{ij} \leq \Delta, \\ 0, & \text{otherwise}. \end{cases} \] (2)

\[ \text{Figure 2 Three cases of balanced graphs.} \]

With above results in hand, a proximity graph is obtained, which can provide a rigorous formalization of communication graph of agents.

As indicated, from the dynamic nature of the proximity graphs, one can readily observe that an edge should be appeared and disappeared simultaneously. As mentioned above, to achieve the desired formation, information interaction plays a critical role to the topology of agents; agent must interact with its neighbors to obtain necessary knowledge used to change its interior state by selecting appropriate controller. Using the definition of neighborhood set, we present following assumptions.

As before, suppose all agents have the same measure range, denoted by \( \Delta \), thus the sensing area of agent i is the sphere \( \epsilon \times \Delta \) with radius \( \Delta \). If any agent \( j \in N_i \) travels into this area and agent i can see it, then the relative measurements can be obtained by agent i, note that not all agents are within this area can be sensed by agent i, for instance, three agents are located at the same line. Moreover, the relative displacements between agents j and i can be obtained by agent j, this requirement is readily satisfied if we assume agent j can acquire knowledge from agent i.

To proceed, let the cardinality of edges be \( m \), together with the total number of vertices \( n \) mentioned above, then the incidence matrix of the graphs, denoted by \( M(V,E)_{\text{inc}} = [m_{ij}] \) are obtained, with each entry \( m_{ij} = 1 \) if i is the start vertex of edge \( e_k \), \( k = [1,2,3,...n] \), \( m_{ij} = -1 \) if i is the end vertex of \( e_k \), \( k = [1,2,3,...n] \), otherwise, \( m_{ij} = 0 \).

In view of \( M(V,E)_{\text{inc}} \), the Laplacian matrix \( L(V,\epsilon) \) is given

\[ M(V,E)_{\text{inc}} = [m_{ij}] \] (3)

Note that Laplacian matrix can be defined by

\[ L(V,\epsilon) = D(V,\epsilon) - A(V,\epsilon) \in R^{n \times n}, \] where \( D(V,\epsilon) \) and \( A(V,\epsilon) \) denote the diagonal matrix and adjacency matrix, respectively.

We below state important properties of Laplacian matrix \( L(V,\epsilon) \) (3).

One can readily verify that \( L(V,\epsilon) \) is positive semi-definite and symmetric.

Each row sum of \( L(V,\epsilon) \) is equal to zero, the rest eigenvalues are positive numbers, i.e., \( \{\lambda_2,\lambda_3,\ldots,\lambda_n\} > 0 \), moreover, we assume that \( \lambda_2 < \lambda_3 < \ldots < \lambda_n \) apparently, the total number of different positive eigenvalues is \( n-1 \).

The smallest eigenvalue of (3) is always 0, whose corresponding eigenvector is typically \( v = \sqrt{n} \) if \( v \) is the null space of G. The strongly connected graph is also defined if a weighted undirected graph has Laplacian matrix (3) and its rank equals to \( n-1 \), i.e., rank \( (L) = 1 \).

In the case of undirected graphs G, we introduce the following well-known property:

\[ \lambda_2 (L) = \min \frac{p^T LP}{\|p\|^2} \] (4)

With superscript T representing the transpose, \( P \) taking from (10) defined in the section III.

We next introduced the important concept of algebraic connectivity. As mentioned above, for undirected graphs, the first positive eigenvalue \( \lambda_2 \) taken from \( \{\lambda_2,\lambda_3,\ldots,\lambda_n\} \) of its associated Laplacian (3) is called the Fiedler eigenvalue, therefore, from \( \lambda_2 \) is also called algebraic connectivity. Let \( X \in R^{n \times n} \) be an appropriate permutation matrix, \( \Psi \in R^{n \times n} \) the upper block triangular, if \( X^{-1} M X \in \Psi \), then \( M \) is referred to as the reducible matrix. If \( M \) is not reducible, then \( M \) is irreducible.

[Balanced Graphs]: If the total numbers of out-degree \( D_{out}(v) \) and in-degree \( D_{in}(v) \) of a vertex \( v \) are equal, namely, \( D_{out}(v) = D_{in}(v) \),
then the vertex $v_i$ of a directed graph $G$ is balanced. Moreover, for all $i, j \in V$, if all vertices of a directed graph $G$ are balanced, then the directed graph $G$ is called balanced, which is shown in Figure 2. Apparently, each node $(v_i, i \in \{1, 2, 3, ... n\})$ of a bi-directed graph is balanced, thus a bi-directed graph is balanced.

Using (3), for an undirected graph $G (V, \varepsilon)$, we can define a Lyapunov candidate function (or called Laplacian potential) as

$$V_G = \frac{1}{2} \sum_{i,j}^{n} \alpha_{ij} \Phi(p_i, p_j)$$

(5)

Where $\alpha_{ij}$ is the entry of the adjacency matrix $A (V, \varepsilon)$, i.e., $\alpha_{ij} \in \mathcal{A}(V, \varepsilon)$, to be specific, $\alpha_{ij} = 1$ if $e_{ij} = (v_i, v_j) \in \varepsilon$, otherwise, $\alpha_{ij} = 0$. Physically, the agents i and j are within the $\Delta -$ proximity graphs as time evolves. Note that, if the edge $e_{ij} = (v_i, v_j) \in \varepsilon$ is established at a special time $t=t_o$, which will be contained in the networks of a group of agents in future $t > t_o$. The undirected graph is called if $\alpha_{ij} \equiv \alpha_{ji}, (v_i, v_j) \in V, (v_i, v_j) \in \varepsilon$

$\Phi(p_i, p_j)$ is a function that depends on both states $p_i, p_j$, the following properties should be satisfied.

$\Phi(p_i, p_j)$ is a continuous function, and $\Phi(p_i, p_j) < I$, where I denotes a bounded constant number, i.e., $I < \infty \in R$, since the feasible distance domain defined as above.

$\Phi(p_i, p_j)$ is a non-negative scalar function, $\Phi(p_i, p_j) = 0$ if and only if $p_i = p_j$, i.e., $p_i - p_j$ is the relative measurement between agents (or nodes) i and j.

To obtain the gradient flow of (5), we first introduce the gradient operator $\nabla p_i$ with respect to $p_i$, namely,

$$\nabla p_i = \frac{\partial}{\partial p_i} {i \in \{1, 2, ..., n\}}$$

(6)

thus the gradient flow of (5) is defined as

$$\dot{p}_i = -\nabla p_i V$$

(7)

(7) Can be viewed as the control input or velocity of $p_i$ to be determined, thus the controller is obtained, which is usually called as the gradient descent approach. In order to find the value of algebraic connectivity $\lambda_{min}(L)$ of the weighted Laplacian matrix, we first define a matrix as

$$H = I_n - \zeta L_{min}$$

(8)

Where $I$ denotes the identity matrix; & is a sufficiently positive number. To this end, we can guarantee that each entry $h_{ij}, i, j \in \{1, 2, 3, ..., n\}$ of $H$ is a zero or positive number by selecting a proper value of $\zeta$ which is readily achieved, to be specific, for instance, we can choose $\zeta \leq \frac{1}{\lambda_{min}}$. With the help of $H$, the estimate of $\lambda_{min}$ can be calculated. For this purpose, the basic procedures of power iteration method are briefly introduced as follows.

For a given matrix $A$, selecting the initial vector $x^{(0)}$ randomly, we then have for $i=1, 2, ..., n$

$$y^{(i+1)} = Ax^{(i)}$$

$$\alpha_k = \max \{y_i^{(i)}, \alpha = \max \{y_i^{(0)} \}$$

and $\lambda^{(i+1)} = \alpha \lambda^{(i)}$

(9)

If $\|\nu^{(i+1)} - \nu^{(i)}\| < \epsilon$ where $\epsilon > 0$ is sufficiently small number end.

Note that all eigenvalues and corresponding eigenvectors will be calculated. Moreover, we can readily verify that all eigenvalues $\lambda (H)$ have following important property, namely, $\lambda (H) = 1$, as the property of spectral norm of $H$. $H$ is right stochastic matrix or transition matrix, moreover, which can be rewritten as

$$H_{ij} = \begin{cases} \zeta, & (i, j) \in \varepsilon \\ 1 - \zeta d_{ij}, & i = j \\ 0, & \text{otherwise} \end{cases}$$

(9)

With $d_i$ denoting the degree of vertex i. In order to guarantee the non-negative entry $H_{ij} \geq 0$ of $H$ and the convergence of the system, the sufficient condition for $\zeta$ can be deduced by $\zeta \in (0, \frac{1}{d_{max}})$ with $d_{max}$ representing the maximum value of degree $d_{max}$ vector, i.e., $d_{max} = \{d_1, d_2, ..., d_n\}$ which can be readily obtained if the maximum number of neighbors defined in the proximity graph is determined. Note that the system is unstable as the negative eigenvalue is computed if $\zeta = d_{max}$. This case here is avoided.

**Control strategy**

In this section, using the limited information, we study the control strategy for multi-agent system. Consider a team of agents, as before, the positions of agents are given in the following form

$$P = \begin{bmatrix} ...p_i' \end{bmatrix}, i = \{1, 2, ..., n\}$$

(10)

with $p_i = [x_i, y_i]^T \in R^2$ denoting the positions of agent i. As such, the configurations of the team of agents can be compacted $P \in R^{2n}$. In what follows, for notational simplicity, we use symbol $p_i$ to denote the $p_i = [x_i, y_i]$. Suppose each individual agent is governed by a single-integrator, we obtain

$$\dot{p}_i = u_i, \quad i = \{1, 2, ..., n\}$$

(11)

Where $p_i$ remains the same as above, $u_i$ denotes the control input that is needed to determined, which is also referred to as the velocity of agent i.
To proceed, in light of the dynamic proximity, the useful notion is then presented as follow. Let \( \omega_{ij} \) be weights that assigned to edges, actually, \( \omega_{ij} \) depends on the states \( i \) and \( j \), mathematically, which can be stated as

\[
\omega_{ij} = \phi(d_{ij})
\]  

(12)

Where \( d_{ij} \) is the actual distance between agents \( i \) and agent \( j \). Note that 12 is a smooth function. Moreover, \( \omega_{ij} = \Delta \text{ if } d_{ij} > \Delta \). As indicated, in terms of \( \omega_{ij} \in (0, \Delta) \), the edge \( e(v_i, v_j) \) between agent \( i \) and agent \( j \) is established. More specifically, we define the edge-weights by using the bump function

\[
\phi(d_{ij}) = \begin{cases} 
\frac{d_{ij} - d_0}{d_{max} - d_0} & \text{if } d_{ij} < d_{max} \\
0 & \text{otherwise}
\end{cases}
\]  

(13)

Where \( d_{ij} \) denotes the maximum sensing distance, to be specific, \( d_{ij} \) is equal to \( d_{max} \), which will be discussed in the sequel. It is evident that (13) is a smooth function, which implies that (13) is a continuously differentiable, and (13) can be viewed as the variant of the Gaussian function, which is illustrated in Figure 3. Note that the quantity value of \( \omega_{ij} \) takes on \([0, 1]\), i.e., \( \omega_{ij} \in (0,1) \), and \( \omega_{ij} = \omega_{ij} \) because of \( d_{ij} = d_{max} \), which can greatly simplify the computation complexity.

It is also noted that \( \omega_{ij} = 0, i \in \{1, \ldots, n\} \), by convention, the self-loops in the graph is precluded. Moreover, the degree of vertex \( i \) is computed by

\[
d_i = \sum_{j=1}^{n} \omega_{ij}, \{i,j\} \in e
\]  

(14)

Figure 3 The Edge weights \( \omega_{ij} \) vary with the actual distances \( d_{ij} \) between any two agents.

Since \( \omega_{ij} \) denotes the actual relation between any two vertices, which means that the dynamic properties of multi-agent system can be revealed by \( \omega_{ij} \), it is instructive to investigate the quantity \( \omega_{ij} \), moreover, the interaction topology of agents can be dynamically determined, it is noted that any two agents \( i, j \in \{1, 2, \ldots\} \) are expected to interact positively. Therefore, we consider a smooth function \( \omega_{ij} \) that depends on the distance of agent \( i \) and \( j \). To obtain this function, we then provide several important concepts as follows, in order to facilitate our analysis, we provided the limited information generated from a given valid sensing area of agent \( i, j \in \{1, 2, \ldots, n\} \), which is illustrated as Figure 4 is an arbitrarily sufficiently small positive number, \( d_{ij} > 0 \) or \( d_{max} \) is a proper positive number to be chosen according to the constraint condition, physically, to avoid the collision between any two agents \( i \) and \( j \), the minimum distance \( d_{ij} \) must be required, equivalently, the collision avoidance between any two agents \( i \) and \( j \) can be then guaranteed; \( d_{ij} \) is feasible control area, which implies that two agent \( i \) and \( j \) can interact with each other, namely, the knowledge of agent \( j \) is available to agent \( i \), thus the communication link \( e(v_i, v_j), e_k \in e, v_i, v_j \in e \). Note that agent \( j \) may not need to acquire the information from agent \( i \) under the assumption on the directed graph, for instance, the relationship between two agents \( i \) and \( j \) is denoted by a single arrow; as a result, agent \( i \) has a limited information from the agents (or called the neighbors) that enter into its valid sensing area at time \( t \), using this limited information, the control objective will be accomplished, and the dynamic nature of multi-agent system can be revealed. For this purpose, the following procedures are now presented. With above results in hand, from a physical point of view, three constraint conditions are provided as follows.

Figure 4 (A) The feasible sensing area of agent \( i \); (B) Geometrical constraints.

We use solid red point denote agent \( i, d_{max} \) denotes the maximum sensing capability, as indicated, \( d_{max} = \Delta \). From a theoretical point of view, using \( \epsilon_1 \) defined as follows, the boundary conditions can be defined, which is also referred to as \( \epsilon_1 \) approximation approach.

Collision avoidance. Collision avoidance is a critical issue for multi-agent system, there are a great number of control strategies that deal with collision problem. In this paper, we do not intend to propose a control method to solve this problem, for brevity, we here assume that the relationship between agents \( i \) and \( j \) is invalid if the actual distance \( d_{ij} < d_{min} \), thus the collision issue is solved, it would be interesting to address this problem in our future works.

Representation of dynamic edge. To investigate the dynamic nature of system, the edge effect is vital to the dynamic nature of system, the edge effect is characterized by a edge function (12), as mentioned above, (12) is a time vary and continuously weight function, i.e \( \phi(d_{ij}) \in C' \) with \( C' \) denoting the \( \gamma \) order continuous time derivatives.
No interaction. Agents i and j has no limited information from each other in terms of different tasks, for example, to achieve the connectivity, or equivalently, agent i need to follow agent j at a certain instant, if the actual distance \( d_{ij} > d_{\text{max}} \), to be specific, in view of \( d_{ij} = d_{ij}' \), agent j is not within the feasible sensing area of agent i. To proceed, to facilitate our analysis and deduce the control laws, we can use the mathematical expression to denote the constraint conditions, from (12), thus its associated piece-wise function can be rewritten as follows:

\[
\phi(d_{ij}) = \begin{cases} 
1, & d_{ij} \in [0, d_{\text{min}}) \\
 f(x), & d_{ij} \in [d_{\text{min}}, d_{ij} + \epsilon_1) \\
0, & d_{ij} \in [d_{ij} + 2\epsilon_1, \infty)
\end{cases}
\]

(15)

\( \phi(d_{ij}) \) is defined with the geometrical constraint set (or feasible set), which is shown in Figure 3(A). From (15), if third case \( \phi(d_{ij}) = 0 \) happens, it is hard to establish the edge between agent i and j, as two agents move freely without any control input, thus we cannot maintain the connectivity, to overcome this issue, the external control input \( u \) are needed, note that the \( u \) can be properly chosen as the continuous and bounded signals produced by the external force. Note that \( f(x) \) is a class of bump function, and the coefficient can be properly chosen such that \( 0 < f(x) < 1 \) as time goes to infinity, which is omitted here.

Hence, substituting \( u_j \) into (11), we have

\[
\dot{p}_i = u_{i_{Lj}} + u_{i_{ej}}
\]

(16)

where \( u_{i_{Lj}} \) and \( u_{i_{ej}} \) denote the Laplacian control input and external control input of \( i - \text{th} \) agent i, respectively. Compared with (11), one can obtain \( u_i = u_{i_{Lj}} + u_{i_{ej}} \). The control objective of this paper is to maintain the algebraic connectivity \( L(\lambda_2) \) and its sufficient condition is guaranteed if \( \lambda_2(L) > 0 \), \( \lambda_2(L) \) is a function of states of agents as edges vary with time. As indicated, agent i has the knowledge about positions \( p_i \in \mathbb{R}^n \), relative measurements and the magnitudes (or distances) of relative positions. To proceed, we suppose the minimum value of quantity \( \lambda_2 = \epsilon_2 \) with \( \epsilon_2 \) denoting the arbitrary small positive number. As is customary, we cannot obtain the exact value of the quantity, but we can get the estimates \( \hat{\lambda}_2 \) of second eigenvalues using following rules:

\[
\hat{\lambda}_2 - \lambda_2 = \pm \epsilon_2
\]

(17)

Where \( \epsilon_2 \) is sufficiently small positive number, the interested reader can consult the material\(^\ddagger\) for more details. In light of (5), which is also referred to as potential function (or energy function), we obtain

\[
V(\hat{\lambda}_2) = V(\hat{\lambda}_2, \pm \frac{\epsilon_2}{2})
\]

(18)

Note that \( V(\hat{\lambda}_2) \in \mathbb{R}^n \cup \{0\} \) is continuously differentiable.

To guarantee the algebraic connectivity maintenance of the graph, the control strategy is derived using the negative gradient descent of \( V(\hat{\lambda}_2) \). Using the chain rule, we then take first time derivative of \( \dot{V}(\hat{\lambda}_2) \) with respect to \( \dot{p}_i \) agent \( p_i \), one obtains

\[
\dot{V}(\hat{\lambda}_2) = \frac{\partial V(\hat{\lambda}_2)}{\partial \dot{p}_i} = \frac{\partial V(\hat{\lambda}_2)}{\partial \dot{p}_i}
\]

(19)

Where \( \dot{p}_i \) is \( k \)th component of \( p_i \). Thus, the control strategy of agent i is provided as follows

\[
\dot{u}_i = \dot{V}(\hat{\lambda}_2)
\]

(20)

**Stability analysis**

In this section, we investigate the stability properties of the system. As discussed, the control law for agent i is proposed as follows

\[
\dot{p}_i = u_{i_{Lj}} + u_{i_{ej}}
\]

(21)

Where \( u_{i_{Lj}} \) and \( u_{i_{ej}} \) are the control law and external control input, respectively. The external control input \( u_{i_{ej}} \) can be viewed as the additional task imposed on agent i. As before, notice that agent i will travel freely if \( u_{i_{ej}} = 0 \), physically, no information interaction among agents, any connection between two agents may not be established, in other words, the results of connectivity preservation is not obtained, the proof is omitted due to page constraints. Moreover, since \( u_{i_{ej}} \) is bounded, \( p_i \) is bounded for any initial positions \( p(0) \). To this end, the control law is therefore considered. In what follows, using \( \dot{p}_i \), we can analyze the stability of the system. To proceed, the following procedures are first stated.

Let \( d_k(I, j) \) be \( k \), \( k \in \{1, 2, ..., m\} \) distance between agent i and j at time t, for brevity, we drop the quantity of time t, then \( d_k(I, j) \) has the properties characterized as before, which can be viewed as a function of states \( p_i \) and \( p_j \), namely

\[
d_k(I, j) = \left\| p_i - p_j \right\|_2
\]

(22)

\[
\tilde{d}_k(i, j) = \frac{\tilde{d}(p_i - p_j)}{p_i} \frac{(p_i - p_j)^T}{p_i} = (p_i - p_j^T)\frac{\tilde{d}(i, j)}{p_i}
\]

(23)

Where \( \tilde{d}(i, j) = \frac{\tilde{d}_k(i, j)}{p_i} \) represents the gradient information, which is called the limited information as \( d_k(i, j) \) is valid on the feasible sensing area defined by proximity information graph, proximity graph for short.

iii) Since \( p_i \) is a column vector, with a slight abuse of notation, denoted by \( p_i = [p_{i_1}, p_{i_2}, ..., p_{i_k}]^T \), where \( p_{i_k} \) is the \( k \)th component of \( p_i \), and the superscript T is the transpose, with the aid of \( \tilde{d}_k(i, j) \), we have

\[
\tilde{d}_k(i, j) = \left\| p_i - p_j \right\|_2
\]
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\[ \frac{\partial d_{ik}}{\partial p_{ij}} = (p_{ik} - p_{kj}) \]  

where \( \frac{\partial d_{ik}}{\partial p_{ij}} \) indicates the derivative of \( \partial d_{ik} \) with respect to \( p_{ij} \). From (20), the first part of right hand side of 21 is obtained by using the negative of \( \frac{\partial d_{ik}}{\partial p_{ij}} \), i.e., \(-\dot{\phi}(\lambda)\). Similarly, using above results, both derivatives with respect to \( p_{ij} \) and \( p_{ik} \) of \( \partial d_{ik} \) can be computed, respectively. To proceed, with above results in hand, we next define two sets of control laws and external controls as

\[ \mu_e = \{\mu_1^e, ..., \mu_n^e\} \in \mathbb{R}^{n \times n} \]

and

\[ \mu_e = \{\mu_1^e, ..., \mu_n^e\} \in \mathbb{R}^{n \times n} \]

respectively.

Note that \( \mu \) are unknown and bounded external control tasks.

In light of \( \mathcal{P} \), denoted by \( \mathcal{P} = \{p_1, ..., p_n\} \in \mathbb{R}^{n \times n} \) thus the overall control laws can be given as

\[ \dot{p} = \mu^e + \mu_e \]  

From (3), for simplicity, we omit the symbols \((u, e)\), the control laws are given

\[ \mu_e = -((L \otimes I_{n \times n}) \mathcal{P}) \]

Where \( I_{n \times n} \) is a \( n \times n \) identity matrix.

As indicated, we define the energy function as \( V(p, p_j) \), \( j \in \mathbb{N} \),\( > 0 \) it is noted that \( V(p, p_j) \), \( j \in \mathbb{N} \), \( > 0 \). Taking the first time derivative of \( V(p, p_j) \), w.r.t. \( p_i \), one has

\[ V(p, p_j) = \frac{\partial V(p, p_j)}{\partial p_i} + \sum_{i=1}^{n} p_i L \dot{p}_i \]

\[ = \phi(\mathcal{P}) + \phi_e(\mathcal{P}) \]

\[ \phi(\mathcal{P}) = -\sum_{i=1}^{n} p_i \dot{L} \mathcal{P}_i \]

\[ \phi_e(\mathcal{P}) = \sum_{i=1}^{n} p_i \dot{L} \mu_e^i \]

where the capital letter \( T \) remains the same as before. To proceed, the valuable property is introduced, which is stated as follows.

Let \( A_{n \times n} \in \mathbb{R}^{n \times n} \) and \( B_{n \times n} \in \mathbb{R}^{n \times n} \) be two vectors, we have

\[ \|A\| \times \|B\| \leq AB \] the proof is straightforward.

Using above results, we have

\[ \phi(\mathcal{P}) \leq -\sum_{i=1}^{n} \|p_i\| \]

and similarly,

\[ \phi_e(\mathcal{P}) \leq -\sum_{i=1}^{n} \|p_i\| \]

Thus

\[ \phi(\mathcal{P}) + \phi_e(\mathcal{P}) \leq -\sum_{i=1}^{n} \|p_i\| \]

For simplicity, it is noticed that we have dropped the dependence on time for each term in the above. Let \( \phi_{\{u,e\}}(i, t) \) be expressed by

\[ \phi_{\{u,e\}}(i, t) = -\|p_i(t)\|\|p_i(t)\| \]

By factoring out the term \( \|p_i(t)\| \), we obtain

\[ \phi_{\{u,e\}}(i, t) = -\|p_i(t)\||\|p_i(t)\| - \|p_i(t)\| \]

As a result,

\[ V(p_i, p_j) \leq \sum_{i=1}^{n} \phi_{\{u,e\}}(i, t) \]

As indicated, the external control term \( \mu_e \) is bounded, let the maximum magnitude of \( \mu_e \) be \( \bar{\sigma} \), then we have \( \bar{\sigma} < \infty \), which means that \( \phi_{\{u,e\}}(i, t) \leq 2\bar{\sigma}^2 \) for \( i \in \{1, ..., n\} \) as time approaches to infinity. Using the stability theory of Lyapunov, we obtain

\[ V(p_i, p_j) > 0 \]

\[ V(p_i, p_j) < 0 \]

implying that \( \|Lp_i(t)\| > \bar{\sigma} \) for any \( i \in \{1, ..., n\} \), therefore, \( \phi_{\{u,e\}}(i, t) < 0 \). Suppose the graph is connected, which means that the second smallest eigenvalue is bigger than zero, i.e., \( \lambda_2 > 0 \). Moreover, the quantity \( \lambda L > 0 \) depends on the states of agents, specifically, which is a non-decreasing function of the edge weights. Then we have

\[ \|Lp_i(t)\| > \lambda_2 \|Lp_i(t)\| \]

where \( p \) is any vector, \( X \) the any matrix such that \( X^T X = 0 \) and \( X^T X = 1 \) \( \lambda \in \mathbb{R}^{n \times n} \) is the symmetric graph Laplacian matrix. Since the sensing distance for each agent \( i \) is limited, we have that \( d_j \rightarrow d_{max} \) as \( t \) tends to the some instant i.e., \( t \rightarrow t^* \) which means that each agent \( i \) can converge to the specified position, denoted by \( p_i \rightarrow p_i^d \). See [5] for more detail treatment.

Using above results, the extension to the consensus control of multiagent systems is also discussed here. In this setting, the external control for each agent is neglected, and the initial positions of agents are generated randomly, in particular, which characterizes the stable equilibrium of a team of agents. As a consequence, the overall control law for the system is given as

**Citation:** Yang X, Fan X, Li G. Decentralized cooperative control with connectivity maintenance for multiagent systems. *Int Rob Auto J.* 2019;5(1):1–10.

DOI: 10.15406/irajt.2019.05.00163
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\[ \mu_i = -\nabla V_i (p_i - p_j), j \in N_i \]

Where \( V \) is a Lyapunov candidate function, \( \nabla V \) is the gradient of \( V \).

**Simulation**

In this section, illustrative examples are provided to validate the theoretical results. We first assume that the multi-agent system consists of 7 agents, where no any control law that imposed on the agent 7, then the agent 7 move freely with any given arbitrary trajectory, that is

\[ \begin{align*}
    \mu_{inc, i} &= (1 + i \cdot inc)\cos(t) \\
    &= (1 + i \cdot inc)\sin(t)
\end{align*} \]

Where \( inc \) is an increment quantity, we select \( inc = 0.25 \), for all \( i = [1, 2, 3, 4, 5, 6, 7] \). To implement the control laws for agents, the initial positions are given randomly.

\[
\begin{align*}
    p_1(0) &= [-2.50; 0.58] \\
    p_2(0) &= [1.35; 0.78] \\
    p_3(0) &= [-1.50;-2.90] \\
    p_4(0) &= [0.15;-0.48] \\
    p_5(0) &= [-2.60; 5.60] \\
    p_6(0) &= [5.70;-2.30] \\
    p_7(0) &= [0.80;-3.12]
\end{align*}
\]

From Figure 5, except the seventh agent, other agents can converge to stable point under the control laws. Any two agents in six agents can be connected with each other, while the seventh agent does not have any connection with other agents if only external control is considered. To achieve the overall connectivity maintenance, it is necessary to show that the control law is needed, which is illustrated in Figure 6. The connected network of seven agents is given, which is shown in Figure 7. The extension to consensus control problem is also considered, which is illustrated in Figure 8, which shows that all agents can converge to the stable equilibrium with the proposed control law.

**Figure 5** Trajectories of seven agents in the plane. Assume no controller is designed only for the seventh agent, while other agents move under the proposed control law, and the seventh agent can not have any interaction among the rest of agents, thus the connectivity cannot be maintained, which is illustrated by two thick solid curves.

**Figure 6** Trajectories for the seven agents in the plane. From this figure, we can see that the seven agents can converge to stable equilibrium.

**Figure 7** Final configurations of seven agents. The final positions of agents are represented as different colour circles. The connected network of seven agents generated at the stable point.

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In general, the algebraic connectivity can be viewed as the second largest eigenvalue of Laplacian matrix $\lambda_2(L)$, and $\lambda_2(L)$ is a continuously differentiable function, which consists of all information states of all agents, as a result, each agent cannot compute the $\lambda_2(L)$ directly, then $\lambda_2(L)$ can be calculated by using the optimization algorithm for each agent. In addition, the time varying weight that imposed on each edge $e_{ij} \in \mathcal{E}$ between any two agents $i$ and $j$ is defined as a piecewise function under the proximity graph mentioned above. Based on above results, we conclude that the algebraic connectivity can be maintained by implementing the proposed control law.

**Conclusion**

In this paper, we investigate the connectivity maintenance problem for multi-agent system; each agent is described with a single integrator. Using local and limited communication information, we establish the Lyapunov function, and then the decentralized control method is derived. To achieve the group objective, each information state must be provided and a common control protocol should be obeyed, which implies that each agent must communicate with the rest of agents as time evolves, then the connectivity can be maintained and connected network of a team of agents can be established.

**Acknowledgments**

This work is supported by the National Natural Science Foundation of China under Grant No. 61402540, No. 60903222, No. 61672538, and No. 61272024, Hunan Provincial Science and Technology Foundation No. 2014GK3049.

**Conflicts of interest**

The author declares there are no conflicts of interest.

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Citation: Yang X, Fan X, Li G. Decentralized cooperative control with connectivity maintenance for multiagent systems. *Int Rob Auto J*. 2019;5(1):1–10.
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