Characteristics of students' mathematical representation in solving algebraic thinking problems

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Abstract. Facts in the field show that there are still many students who have difficulty in understanding concepts of algebra and have a low ability in algebraic thinking. One of the factors that influence the situation is because students find it difficult to change their thinking processes from arithmetic thinking to algebraic thinking. There are three abilities in algebraic thinking and one of them is mathematical representation ability. The purpose of this study is to analyze the characteristics of students' mathematical representation in solving algebraic thinking problems. This research is a qualitative research conducted on 34 students' grade 6th elementary school and 34 students' grade 7th junior high school. Based on the analysis of the results of written tests and interviews, it was found that elementary and high school students have similarities and differences in the mathematical representations they use to solving algebraic thinking problems. In addition, the researcher found that both elementary school students and junior high school students tended to use a combination of several representations to express mathematical ideas found in the problem.

1. Introduction
Mathematics is a continuous science. The material and concepts obtained in basic education will continue to be used and become the foundation for the next concept[1]. Therefore the mathematics learning curriculum in elementary schools must prepare students to deal with higher mathematical concepts at the next level, one of which is the concept of algebra. Thus, the ability to think algebra is very important for students to have, not only to master algebraic concepts but also to other mathematical concepts[2].

But the facts on the field show that there are still many students who have difficulty in understanding the concepts of algebra [3]–[7], and has a low ability in algebraic thinking [3], [8], [9]. One of the factors that influence the situation is because students find it difficult to change their thinking processes from arithmetic thinking to algebraic thinking[10]. Students who are accustomed to operating in arithmetic terms of reference tend not to see the relational aspects of operations, but their focus is on calculations [5]. This happens because most school mathematics curriculum separate arithmetic and algebra learning [11]. In Indonesia arithmetic studied at the elementary school level, while algebra studied at junior high school level.

According to[2]there are two components in algebraic thinking, namely thinking algebra as a mathematical thinking tool and thinking algebra as a fundamental algebraic idea. The component of thinking algebra as a mathematical thinking tool itself consists of three abilities namely problem solving ability, mathematical representation ability, and quantitative reasoning abilities.
One ability that is part of algebraic thinking that is important for students to have is the ability of mathematical representation[12]. Mathematics requires representation in simplifying and solving mathematical problems because mathematics is abstract. Students need access to mathematical ideas and that can only be done through representing these ideas. Based on the previous descriptions, the researcher is interested in conducting research with the aim of obtaining an overview of students’ algebraic thinking abilities, especially in mathematical representation abilities, and described it based on school levels.

2. Methods
2.1. Research Design
This type of research is a phenomenological study. Phenomenological studies explain the meaning of one's experience of a concept or phenomenon. The main focus of this research is is the characteristics of students' mathematical representation. The subjects of this study were 34 students in grade 6th of elementary school and 34 students in grade 7th junior high schools in the province of West Java.

2.2. Instrument
The main instrument in this study is the researcher who involved in all activities, namely starting from the selection of subjects, data collection to data analysis. Supporting instruments used in this study were test and non-test instruments. The test instrument in the form of questions developed based on indicators describing the algebraic thinking skills by Kriegler. The non-test instruments used were interview guidelines. The interview guide was developed by researchers to obtain more in-depth data on students' thinking characteristics based on the results of written tests.

2.3. Procedure
To collect data, researcher provide written tests containing one mathematical problem about the concept of numerical patterns to each elementary and junior high school students to solve. After that, the students’ completion were collected. Then, based on the result of written test, researcher chose 9 students each from elementary and junior high school levels to hold some interview. The interviews conducted were semi-structured interviews. This type of interview is an in-depth interview. For this type of interview, the researcher prepares a list of general questions that will be asked to the subject, then these questions are developed by the researcher based on the interview situation in the field. The interview was audio recorded by researcher, so it can be used for data analysis process.

3. Result and Discussion
3.1. Characteristics of representation ability of elementary school students
To identify the characteristics of the mathematical representation ability of elementary school students, the mathematical problem given is a problem related to the concept of number patterns. The mathematics problem given are as follows (Table 1):

| Number | Mathematical problems |
|--------|-----------------------|
| 1      | Look at the following pictures! |
|        | ![Figure 1](image1.png) ![Figure 2](image2.png) ![Figure 3](image3.png) ![Figure 4](image4.png) ... |

Based on the picture above, answer the following questions!
(a) How many square in figure 5? How do you determine it?
(b) How many square in figure 10? How do you determine it?
There are two representations that students use in solving problems, namely numerical representation and visual representation. Initially students use visual representations to solve questions in part (a). This is clearly seen in the completion of students, which is to determine the number of squares in figure 5 students draw 5 square at the bottom and 5 square at the top so that the number is 10 square. This also shows that students understand the problem well and are able to find patterns and relationships between the images in the given mathematical problem. In addition to using visual representation, based on the existing student's completion, it can be seen that students also use numerical representations to solve this problem. This can be seen from the list made by students by including the relationship of quantity and its operation.

To confirm the students’ completion, the researcher conducted an interview with the student. The results of the interview are presented as follows:

R :  How did you understand the problem given?
S :  (while pointing to the completion)
   This is 1 + 1 = 2 (the meaning of the student statement is that the number squares at the bottom is 1 and the number squares at the top is 1, so the total is 2 square).
   This is 2 + 2 = 4 (the meaning of the student statement is that the number squares at the bottom are 2 and the number squares at the top are 2, so the total is 4 square).
   This is 3 + 3 = 6 (the meaning of the student statement is that the number squares at the bottom are 3 and the number squares at the top are 3, so the total is 6 square).
   This is 4 + 4 = 8 (the meaning of the student statement is that the number squares at the bottom are 4 and the number squares at the top are 4, so the total is 8 square).

R :  Okay, How did you solve the question part (a)?
S :  There are 10 squares!
R :  How do you determine it?
S :  From 5 + 5 = 10 square
R :  Why did you use this method to solve the problem?
S :  Because the question was also drawn, sir! And to make it easier!

Based on the results of the interview, it appears that students are able to explain the results of their work properly. Students explain that there are 2 squares, which are obtained from the sum of 1 square at the top and 1 square at the bottom. Then students explain that there are 4 squares, which are obtained from the sum of 2 squares at the top and 2 squares at the bottom, and so on until students conclude that the number of squares in the fifth image is 10, which is obtained from the sum of 5 squares at the top and 5 squares at the bottom. When asked why students use these representations to solve problems, students answer that this method makes it easier to solve the problem.

3.2. Characteristics of mathematical representation of junior high school students

Had a similarity with elementary students, to identify the characteristics of mathematical representation abilities, the questions given to junior high school students are questions related to the concept of number patterns. The questions given in Table 2.

| Number | Mathematical Problem |
|--------|----------------------|
| 1      | Look at the following pictures! |

Based on the picture above, answer the following questions!
(a) How many unit squares in the figure 5? How do you determine it?
(b) How many unit squares in the figure 10? How do you determine it?
(c) How many unit squares in the figure \(n\)? How do you determine it?
There are three representations used by students to solve problems, namely visual, numerical and symbolic representations. Similar to elementary school students, a visual representation where students make a picture to find the number of squares. Numerical representations can be seen from the list made by students by including the relation of the quantity between the images listed on the question. Whereas symbolic representation can be seen from the students' answers in part (c) which have been able to generalize using variables. Students answer that to determine the number of squares in the Figure \( n \), can use formula \( 6 \times n = 6n \). The researcher conducted an interview with the students. The results of the interview are presented as follows:

R : How do you understand the mathematical problem given?
S : The first picture contains 6 square; figure 2 has 12 square; Figure 3 has 18 squares; Figure 4 has 24 squares. What is asked is how many squares in the 5th image?

R : Okay good, how do you solve this problem?
S : So, from 24 square plus 6 square to 30 square.

R : Okay, How do you solve the section (b)?
S : How many unit squares in the figure 10?
So, from picture 6 there are \( 6 \times 6 = 36 \); Figure 7 has \( 6 \times 7 = 42 \); Figure 8 has \( 6 \times 8 = 48 \); Figure 9 is \( 6 \times 9 = 54 \); So, figure 10 is \( 6 \times 10 = 60 \) square.

R : Okay well, now what about part (c)?
S : How many unit squares in picture \( n \)? The answer is \( 6n \).

R : Where did you get \( 6n \)?
S : From the first picture there are \( 6 = 6 \times 1 \); Figure 2 has \( 6 \times 2 = 12 \); Figure 3 has \( 6 \times 3 = 18 \); Figure 4 has \( 6 \times 4 = 24 \), and so on so in figure \( n \) there are \( 6n \) square.

R : Okay well, I want to ask, what exactly \( n \) is?
S : \( n \) is a number.
R : What number?
S : A number that shows many unit squares in figure \( n \)!
R : Okay good, then I want to ask, how many square in figure 100?
S : 6 times 100.
R : Ok, fine! Why do you use this method to solve the problem?
S : To make it easy!

Based on the results of the interview, it appears that students are able to explain the results of their work properly. Students are able to explain the meaning of the representation that he made. To answer part (a), students initially find a pattern that the difference in unit frequency between the pictures in the question is 6, so that to answer the questions in part (a) students simply add the number, so that 30 is obtained, but to answer the question part (b), students find a pattern that for figure 1, the number of squares is \( 6 \times 1 = 6 \); for figure 2, the number of squares are \( 6 \times 2 = 12 \); for figure 3, the number of squares are \( 6 \times 3 = 18 \); so to answer the question in part (b) student immediately conclude that the number of squares in figure 10 are \( 6 \times 10 = 60 \).Then, the list that student makes help students to generalize patterns, so to answer the question part (c), students conclude that the number squares in the figure \( n \) are \( 6 \times n = 6n \). Then, when student was asked, what exactly \( n \) is, students answer that \( n \) is an unknown number, which shows the number of squares in figure \( n \). When asked how many squares in figure 100, students answer that there are \( 6 \times 100 \). Researcher concluded that the symbolic representation which used by junior high school students was influenced by their initial knowledge of algebraic concepts, so when asked to solve questions related to algebraic symbols, students had no difficulty solve it.

4. Conclusion
Based on the results and discussion, it can be concluded that there are two representations used by elementary school students to solve the given mathematical problems, namely visual representation and symbolic representation. While for junior high school students there are three representations that are used to solve mathematical problems given, namely visual, numerical and symbolic representation.
representations. The reason students in both levels use these representations because they are considered to make it easier for them to solve problems. In addition, the researchers found that students tended not to use just one representation but several representations, this was in line with the results of the study[13] who found that in solving mathematical problems, students tended to use a combination of several representations to express mathematical ideas found in the problem.

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