We present the results of charged current quasielastic (CCQE) scattering cross sections from free as well as bound nucleons like in $^{12}\text{C}$, $^{16}\text{O}$, $^{40}\text{Ar}$ and $^{208}\text{Pb}$ nuclear targets in $E_{\nu}(\bar{\nu}) \leq 1$ GeV energy region. The results are obtained using local Fermi gas model with and without RPA effect. The differences those may arise in the electron and muon production cross sections due to the different lepton mass, uncertainties in the axial dipole mass $M_A$ and pseudoscalar form factor, and due to the inclusion of second class currents have been highlighted for neutrino/antineutrino induced processes.

**KEYWORDS:** Lepton mass, Nuclear medium effects, Second class currents, Local density approximation.

1. **Introduction**

Precise calculations for quasielastic reactions in nuclei induced by $\nu_\mu(\bar{\nu}_\mu)$ and $\nu_e(\bar{\nu}_e)$ are required to study the CP violation and mass hierarchy in present experiments done in the sub-GeV energy region. Therefore, in this energy region, it is important to understand the differences that may arise in the electron vs muon production cross sections due to the lepton mass, the axial dipole mass $M_A$, pseudoscalar form factor and the inclusion of second class currents. Here we present the results of a study performed using a local Fermi gas model (LFG) with RPA effect to take into account nuclear medium effects, and obtained the ratio $\sigma_{\nu_e}/\sigma_{\bar{\nu}_e}$, $\sigma_{\nu_\mu}/\sigma_{\bar{\nu}_\mu}$, $\sigma_{\nu_e}/\sigma_{\nu_\mu}$ and $\sigma_{\bar{\nu}_e}/\sigma_{\bar{\nu}_\mu}$ in nuclei like $^{12}\text{C}$, $^{16}\text{O}$ and $^{40}\text{Ar}$. The uncertainties due to pseudoscalar form factor and its deviation, if any, from the PCAC and pion pole dominance value as well as the second class current form factors within the limits of present allowed constraints have been discussed. The details of the calculations are given in Ref. [1].

2. **Formalism**

For neutrino/antineutrino induced CCQE process $(\nu_l/\bar{\nu}_l)(k) + n/p(p) \to l^-/l^+(k') + p/n(p')$, the general expression of differential cross section is

$$\frac{d^2\sigma}{d\Omega_l dE_l} = \frac{|\vec{k}'|}{64\pi^2 E_{\nu} E_{n} E_{p}} \sum |M|^2 \delta[q_0 + E_n - E_p]$$

(1)

where $|M|^2$ is the matrix element square which is written as

$$|M|^2 = \frac{G_F^2}{2} \cos^2 \theta_c \ L_{\mu\nu} \ J^{\mu\nu}$$

(2)

with leptonic tensor $L_{\mu\nu} = \Sigma l_{\mu} \bar{l}_\nu$ and hadronic tensor $J^{\mu\nu} = \bar{\Sigma} f_{\mu} f_{\nu}$. 
The leptonic current is given by
\[ l_\mu = \bar{u}(k')\gamma_\mu(1 \pm \gamma_5)u(k), \] (3)
where (+ve)–ve sign is for (antineutrino)neutrino. \( J^\mu \) is the hadronic current given by [2]
\[ J^\mu = \bar{u}(p') \left[ F_1^V(Q^2)\gamma^\mu + F_2^V(Q^2)i\sigma^{\mu\nu}\frac{q_\nu}{2M} + F_A(Q^2)\gamma^5 \gamma^\nu + F_p(Q^2)\frac{q^\mu}{M}\gamma^5 \right. \]
\[ + \left. F_3^V(Q^2)\frac{q^\mu}{M} + F_A^p(Q^2)\frac{(p + p')^\mu}{M}\gamma^5 \right] u(p), \] (4)
where \( F_{1,2}^V(Q^2) \) are the isovector vector form factors and \( F_A(Q^2) \) and \( F_p(Q^2) \) are the axial and pseudoscalar form factors, respectively. \( F_3^V(Q^2) \) and \( F_A^p(Q^2) \) are respectively associated with the vector part and the axial vector part of the second class current.

The hadronic current contains isovector form factors \( F_{1,2}^V(Q^2) \) of the nucleons, which are given as
\[ F_{1,2}^V(Q^2) = F_{1,2}^p(Q^2) - F_{1,2}^n(Q^2) \] (5)

where \( F_1^{p(n)}(Q^2) \) and \( F_2^{p(n)}(Q^2) \) are the Dirac and Pauli form factors of proton(neutron) which in turn are expressed in terms of the experimentally determined Sach’s electric \( G_E^{p,n}(Q^2) \) and magnetic \( G_M^{p,n}(Q^2) \) form factors as

\[ F_1^{p,n}(Q^2) = \left( 1 + \frac{Q^2}{4M^2} \right)^{-1} \left[ G_E^{p,n}(Q^2) + \frac{Q^2}{4M^2} G_M^{p,n}(Q^2) \right] \] (6)
\[ F_2^{p,n}(Q^2) = \left( 1 + \frac{Q^2}{4M^2} \right)^{-1} \left[ G_M^{p,n}(Q^2) - G_E^{p,n}(Q^2) \right] \] (7)

\( G_E^{p,n}(Q^2) \) and \( G_M^{p,n}(Q^2) \) are taken from BBBA05 parameterization [3].

The isovector axial form factor is obtained from the quasielastic neutrino and antineutrino scattering as well as from pion electroproduction data and is parameterized as
\[ F_A(Q^2) = F_A(0) \left[ 1 + \frac{Q^2}{M^2} \right]^{-2} ; \quad F_A(0) = -1.267. \] (8)

The pseudoscalar form factor is determined by using PCAC which gives a relation between \( F_p(Q^2) \) and pion-nucleon form factor \( g_{\pi NN}(Q^2) \) [2], and is given by
\[ F_p(Q^2) = \frac{2M^2 F_A(0)}{Q^2} \left( \frac{F_A(Q^2)}{F_A(0)} - \frac{m_\pi^2}{m_\pi^2 + Q^2} g_{\pi NN}(Q^2) \right), \] (9)

where \( m_\pi \) is the pion mass and \( g_{\pi NN}(0) \) is the pion-nucleon strong coupling constant. \( F_p(Q^2) \) is dominated by the pion pole and is given in terms of axial vector form factor \( F_A(Q^2) \) using the Goldberger-Treiman(GT) relation
\[ F_p(Q^2) = \frac{2M^2 F_A(Q^2)}{m_\pi^2 + Q^2} \] (10)

\( F_3^V(Q^2) \) which is associated with the vector part of the second class current is taken as [2]
\[ F_3^V(Q^2) = 4.4 F_1^V(Q^2) \] (11)
The form factor associated with the parity violating term of the second class current $F_3^A(Q^2)$ is taken as

$$F_3^A(Q^2) = 0.15 F_A(Q^2). \quad (12)$$

When the reaction, $\nu_l/\bar{\nu}_l + n/p \rightarrow l^-/l^+ + p/n$ takes place inside the nucleus, then due to nuclear medium effects scattering cross section gets modified and in the local Fermi gas model with RPA effects are obtained as

$$\sigma(E_\nu) = -2G_F^2 \cos^2 \theta_c \int_{r_{\text{min}}}^{r_{\text{max}}} \int_{k'_{\text{max}}}^{k_{\text{max}}} r^2 dr k' dk' \int_{Q_{\text{min}}^2}^{Q_{\text{max}}^2} dQ^2 \frac{1}{E_\nu} L_{\nu} L_{\bar{\nu}} R_{\text{RPA}} \text{Im}U_N[E_\nu - E_l - Q_l - V_c(r, \vec{q})], \quad (13)$$

where $\text{Im}U_N$ is the imaginary part of the Lindhard function, $Q_l$ is the $Q$-value of the reaction, $V_c$ is the Coulomb potential and $J_{\nu}^{RPA}$ is the modified hadronic tensor when RPA correlations are taken into account.

To observe the sensitivity of difference in lepton production cross section due to different values of axial dipole mass, we define

$$\Delta_1(E_\nu) = \frac{\sigma_{\nu_l}(M_A^{\text{modified}}) - \sigma_{\bar{\nu}_l}(M_A^{\text{modified}})}{\sigma_{\nu_l}(M_A^{\text{modified}})} \quad \Delta_2(E_\nu) = \frac{\sigma_{\nu_l}(M_A = WA) - \sigma_{\bar{\nu}_l}(M_A = WA)}{\sigma_{\nu_l}(M_A = WA)},$$

$$\Delta_{M_A} = \Delta_1(E_\nu) - \Delta_2(E_\nu). \quad (14)$$

where $M_A = WA = 1.026$ GeV and $M_A^{\text{modified}} = 0.9$ GeV & 1.2 GeV. To study the effect of second class current on the $\nu_e/\bar{\nu}_e$ and $\bar{\nu}_e/\nu_e$ cross sections we study the differences of the following ratios

$$\Delta_1(E_\nu) = \frac{\sigma_{\nu_e}(F_{3i} \neq 0) - \sigma_{\nu_e}(F_{3i} \neq 0)}{\sigma_{\nu_e}(F_{3i} \neq 0)} \quad \Delta_2(E_\nu) = \frac{\sigma_{\nu_e}(F_{3i} = 0) - \sigma_{\bar{\nu}_e}(F_{3i} = 0)}{\sigma_{\nu_e}(F_{3i} = 0)}$$

$$\Delta_{F_{3i}} = \Delta_1(E_\nu) - \Delta_2(E_\nu). \quad (15)$$

where $i = V$ or $A$. 

Fig. 1. Total scattering cross section per interacting nucleon for neutrino/antineutrino induced CCQE process for $^{12}C$, $^{16}O$ and $^{40}Ar$ nuclear target. The cross sections are evaluated using local Fermi gas model(LFG) and LFG with RPA effect(RPA).
3. Results and discussions

In Fig. 1, we have presented the results of total scattering cross section for neutrino/antineutrino induced CCQE process in $^{12}C$, $^{16}O$ and $^{40}Ar$ using local Fermi gas model (LFG) with and without RPA effect. As compared to the free nucleon cross section, Pauli blocking and Fermi motion, reduce the total scattering cross section significantly, particularly at low energies. Inclusion of RPA effect further reduces the cross section considerably and the reduction is more in heavier nuclear targets. The suppression due to nuclear medium effects is larger in the case of antineutrinos as compared to the neutrino induced processes.

In Fig. 2, we have shown the ratio of total scattering cross sections for electron and muon type neutrinos/antineutrinos i.e. $\sigma(\nu_e) / \sigma(\bar{\nu}_e)$, $\sigma(\nu_\mu) / \sigma(\bar{\nu}_\mu)$, $\sigma(\bar{\nu}_e) / \sigma(\nu_\mu)$ and $\sigma(\bar{\nu}_\mu) / \sigma(\nu_\mu)$ for CCQE scattering process in free nucleon and for $^{12}C$, $^{16}O$ and $^{40}Ar$ nuclear targets using local Fermi gas model (LFG) with RPA effect. Q-value of the reaction for $\nu_\mu - ^{40}Ar$ reaction is much smaller than in $^{12}C$ and $^{16}O$ nuclei. Furthermore, Coulomb energy correction is large for $^{40}Ar$. Thus the results in $^{40}Ar$ is different in nature than in $^{12}C$ and $^{16}O$ nuclei at low energies.

Fig. 3. Effect of axial dipole mass on the cross section (from left to right): on free nucleon; LFG, with and without RPA effect on $^{40}Ar$ target.
We have also studied the sensitivity of the difference in electron and muon production cross sections due to the uncertainty in the choice of axial dipole mass $M_A$. For this we define $\Delta M_A$ in Eq. 14 and the results for free nucleon and $^{40}$Ar nuclei are shown in Fig. 3. The percentage difference in electron and muon production cross sections due to uncertainty in axial dipole mass is more in the case of nuclear targets as compared to free nucleon target but always remains less than 1%. The difference increases with the increase in mass number.

The fractional difference in the cross sections due to the presence of pseudoscalar form factor is more in the case of $\bar{\nu}_\mu$ induced CCQE process than $\nu_\mu$ induced process for the free nucleon case as well as in nuclear targets. This difference vanishes with the increase in energy (not shown here).

We have also studied the individual sensitivity due to $F^3_V$ and $F^3_A$ in the electron and muon production cross sections for free nucleon as well as for $^{12}$C, $^{40}$Ar and $^{208}$Pb nuclear targets and results are presented in Fig. 4. We find that the sensitivity is non-negligible at low energies and becomes almost negligible beyond $E_\nu = 0.5$ GeV. For example, for neutrino induced reactions, $\Delta F^3_V$ is 3% for free nucleon, ~ 4% for $^{12}$C & $^{40}$Ar and ~ 2% for $^{208}$Pb at $E_\nu = 0.2$ GeV. For antineutrino induced reactions at $E_\nu = 0.2$ GeV, $\Delta F^3_V$ is ~ 9% for free nucleon, ~ 12% for $^{12}$C, $^{40}$Ar and $^{208}$Pb targets. The sensitivity in the difference between the electron and muon production cross sections due to $F^A_3$

![Fig. 4. The difference of fractional changes $\Delta F^+_3$ and $\Delta F^-_3$ for free nucleon case, and for bound nucleons using LFG with RPA effect in $^{12}$C, $^{40}$Ar and $^{208}$Pb nuclear targets.](image)

is very small as compared to sensitivity due to $F^V_3$ for free nucleon as well as for nuclear targets. It is also non-zero at low energies and becomes almost negligible beyond $E_\nu = 0.5$ GeV.

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