Stability of the orbit of a third body in binary asteroid systems

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Abstract. In this work we studied the stable regions around four binary asteroids in the main asteroid belt. The studied systems were (107) Camilla, (22) Kallipe, (45) Eugenia and (762) Pulcova. The stability was characterized with three motion indicators: relative Lyapunov indicator, maximum eccentricity, and maximum difference of eccentricities. The survey covered the P type orbits, where satellite moves around both primaries. On the basis of our work it can be decided, in which system the discovery of a third component can be expected.

1. Introduction
The first binary asteroid Ida with its moonlet Dactyl was discovered by the Galileo spacecraft in 1993 (Mason 1994). This binary asteroid orbits in the main asteroid belt which is located roughly between the orbits of Mars and Jupiter. Up to date the number of the main-belt binaries and triplets has grown to 68 and further 36 binary near-Earth objects, and 4 binary Jupiter Trojans have been observed. A list of the actually known systems can be found in http://www.johnstonsarchive.net/astro/asteroidmoons.html.

There are three main mechanisms which can form a binary asteroid system. The first is a violent collision. A proto-asteroid can break into two or more parts if it collides with a similar size body. For example the binary asteroid (90) Antiope could be the aftermath of a collision of a 100 km sized proto-Antiope with another Themis family member. This violent shock could have produced the break-up of proto-Antiope into two equisized bodies (Weidenschilling et al. 2001, Marchis et al. 2009). Another mechanism which can form a multiple asteroid system is the gravitational scattering. This mechanism can explain the origin of the Kuiper Belt binaries (Weidenschilling 2002, Goldreich et al. 2002, Funato et al. 2004, Astakhov et al. 2005). At last the so-called thermal YORP (Yarkovsky-O’Keefe-Radzievskii-Paddack) effect can explain the orbital properties of main belt and near-Earth double asteroids (Walsh et al. 2008).

According to the different formation theories, it is very probable that some of the main belt binaries (MBB) are gravitationally bound multiplets, like the Pluto-Charon system in the Kuiper Belt. It might be possible that during the formation process other smaller companions were also formed around them. Actually, two small satellites were discovered around Pluto in 2005 (Weaver et al. 2006). The purpose of this paper is to investigate this possibility. We have explored the dynamical structure of the orbital element space of some selected MBBs (listed in
Table 1. Parameters of selected main belt binaries: $a_b$ is the semi-major axis of the heliocentric orbit of the barycenter; $\mu = m_s/(m_p + m_s)$ is the mass parameter; $a_s$ and $e_s$ are the semi-major axis and the eccentricity of the relative orbit of the secondary with respect to the primary; $T_s$ is the orbital period of the secondary around the primary; $R_I$ is the radius of the sphere of influence of the binary.

| Primary   | $a_b$ [AU] | Secondary     | $\mu$ | $a_s$ [km] | $e_s$ | $T_s$ [d] | $R_I$ [km] |
|-----------|------------|---------------|-------|------------|-------|-----------|------------|
| (107) Camilla | 3.48 | S/2001 (107) 1 | 0.00035 | 1250 | 0.002 | 3.722 | 16300 |
| (45) Eugenia | 2.72 | Petit-Prince | 0.00005 | 1180 | <0.002 | 4.765 | 9900 |
| (22) Kalliope | 2.91 | Linus | | 1095 | <0.002 | 3.596 | 11600 |
| (762) Pulcova | 3.16 | S/2000 (762) 1 | 0.002 | 703 | 0.03 | 4.438 | 8340 |

Table 1) in order to find, whether there are stable regions in these systems where additional small satellites could exist. Similar investigations were made by Nagy, Süli & Érdi (2006, 2007) for Pluto-Charon system and 7 Kuiper Belt objects.

2. Model, initial conditions and methods

To study the dynamical structure of the orbital element space of the MBBs, we applied the model of the planar elliptic restricted three-body problem. We assumed that the members of a binary, a larger primary and a smaller secondary body, revolve in elliptic orbits around their common center of mass, and a third body of negligible mass, a hypothetical small satellite, moves under the gravitational forces of the two primaries. We investigated the motion of this satellite within the sphere of influence of the binary system. The radius of this sphere is

$$R_I = a_b \left(\frac{m_p + m_s}{m_{\text{Sun}}}\right)^{2/5},$$

where $a_b$ is the semi-major axis of the orbit of the barycenter of the binary system around the Sun, $m_p$, $m_s$, and $m_{\text{Sun}}$ are the masses of the primary, the secondary, and the Sun, respectively.

Within the sphere of influence, solar perturbations or tides can be neglected, and the members of the binary essentially move around their barycenter in elliptic orbits. Note, that $R_I$ is about 3 times smaller than the Hill radius within which solar perturbations still can be neglected.

The orbital plane of the binary system served as reference plane, in which the direction of the pericenter of the secondary’s relative orbit with respect to the primary was used as reference direction, from which the angular orbital elements of the satellite were measured. The semi-major axis of the secondary’s relative orbit was taken as distance unit, which is denoted with A.

We changed the initial orbital elements of the hypothetical satellite. We determined the stability character of the resulting orbit of the satellite by integrating the equations of motion using a Bulirsch-Stoer integrator and computing three indicators that characterize the dynamical behaviour of the orbit. One quantity was the relative Lyapunov indicator (RLI) (Sándor, Érdi & Efthymiopoulos 2000; Sándor et al. 2004). The RLI measures the difference between the convergence of the finite time Lyapunov indicators to the maximal Lyapunov characteristic exponent of two initially very close orbits. This method is extremely fast in determining the ordered or chaotic nature of individual orbits, as well as to distinguish between ordered and chaotic regions of the phase space. This powerful method has several applications in different problems of planetary dynamics (Érdi et al. 2004; Sándor et al. 2004; Nagy et al. 2006; Sándor et al. 2007).

The second indicator was the maximum eccentricity (ME), reached by an orbit during a given integration time. Orbits with higher eccentricity may become unstable more likely, due to close encounters, thus the ME is a good indicator of stability (Dvorak et al. 2003; Nagy et al. 2006).
The third indicator was the maximum difference in the values of the eccentricity (MDE) of two initially very close orbits. This method, introduced by Nagy et al. (2006), is based on the notice that in a chaotic region the eccentricities of two initially close orbits develop quite differently and their momentary differences can be very large even if the average value of the eccentricity of each orbit remains small. The three methods are not equivalent, however, they complete each other.

The integrations were made for $10^3$ orbital periods of the secondary ($T_s$), in order to determine the time span that is long enough to have a trusted value of the indicators. The test runs, performed for all the investigated MBBs, showed that the orbital element space can be surveyed in a reliable way by using a time span of $10^3 T_s$ for the 4 systems, listed in Table 1.

We plotted the logarithm of the computed indicator on the plane $(a, e)$. The indicators give information about the dynamical character of the orbits, thus in this way maps of the dynamical properties are obtained.
3. Results

We investigated the stability structure of the parameter space of 4 MBBs listed in Table 1. The results are shown in Figs 1-4. In each panel, the top, middle, and bottom figures show the indicators RLI, ME-$e_0$, where $e_0$ is the initial eccentricity and MDE, respectively. Initial points in the light regions correspond to stable orbits, while in the dark domains they result in chaotic motion. These are not necessarily unstable, however, chaotic regions are the birthplaces of unstable motion. In Figs 1-4 a dashed and a solid line are plotted. The orbits right from the dashed line come out from the sphere of influence. If a particle moves in orbit right from the solid line, then it resides outside the sphere of influence in half of its period. In these cases our model assumptions become less and less valid because of the perturbations of the Sun. In these regions the model of the four-body problem should be applied.

(107) Camilla. The parameter space is unstable until $a = 1.5$ A. The boundary line between the stable and unstable regions reaches $e = 0.8$ at $a = 5.5$ A (Fig. 1). The largest stable semi-major axis belong to circular orbit. This is equal to the value of the radius of the sphere of influence, which is 9.3 A.

(45) Eugenia. The stable region begins at $a = 1.3$ A and extends up to $e = 0.73$ at $a = 4.8$ A (Fig. 2). The stable circular orbits extend up to $a = 8.3$ A.

(22) Kalliope. There is instability up to $a = 1.3$ A, and the stable region extends only to $e = 0.68$ at $a = 6.5$ A (Fig. 3). The stable circular orbits extend up to $a = 11$ A.

(762) Pulcova. The stable region begins at $a = 1.3$ A and at $a = 5.6$ A it reaches to $e = 0.65$ (Fig. 4). Circular orbit is stable up to $a = 9.3$ A.

4. Conclusions

The orbits with high eccentricity and large semi-major axes become unstable in every investigated systems within a short time interval. Outside the sphere of influence but inside the Hill sphere orbits can exist with high eccentricity. However, in this region the effect of the Sun becomes significant, consequently a model of the four-body problem should be used stability investigation. Gravitational capturing take place at high eccentric primary orbit, accordingly this mechanism cannot establish a long time stable bounded triplet. However the stable region extends to remarkably large values of the semi-major axes, therefore it is possible that a third body exists in these systems. These bodies might have been born by thermal YORP effect or a major impact.

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