SYMMETRIC BIDIRECTIONAL QUANTUM TELEPORTATION VIA FIVE-QUBIT CLUSTER STATE

JIN-WEI ZHAO, YUN-JING ZHOU, PAN-RU ZHAO, YUAN-HONG TAO∗

Department of Mathematics, College of Sciences, Yanbian University, Yanji 133002, China

Abstract. We propose a protocol of symmetric bidirectional quantum teleportation via five-qubit cluster state. Based on the Bell-state measurements, introduces an auxiliary particle and perform joint unitary transformation, Alice wants to transmit an arbitrary two-qubit entangled state to Bob and Bob wants to transmit an arbitrary two-qubit state to Alice.

Keywords: symmetric bidirectional quantum teleportation; five-qubit cluster state; two-qubit entangled state.

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1. Introduction

Bidirectional quantum teleportation is a new subject in the research of quantum information. Until now, many schemes have been proposed. In 2013, applying cluster state as a quantum channel, the first bidirectional quantum teleportation protocol was reported by Zha et al.[1]. In
2013, Li et al.[2] introduced a bidirectional controlled teleportation scheme by using a five-qubit composite GHZ-Bell state. Some bidirectional quantum teleportation protocols have been investigated by using different states as quantum channel[3-10], such as six-qubit cluster state, genuine six-qubit entangled state, five-qubit entangled state, maximally seven-qubit entangled state and nine-qubit entangled state. Up to now, many teleportation and controlled teleportation schemes have been reported[11-18], but symmetric bidirectional quantum teleportation has not yet been presented.

In this paper, we propose a scheme of symmetric bidirectional quantum teleportation via five-qubit cluster state. Suppose that Alice has two particles A and B in an unknown state, she wants to transmit the state of particles A and B to Bob; at the same time, Bob has two particles C and D in an unknown state, he wants to transmit the state of particles C and D to Alice.

2. Symmetric Bidirectional Quantum Teleportation

Our scheme can be described as follows. Suppose Alice has qubits A and B in an arbitrary entangled state, which is described by

\[ |\Psi\rangle_{AB} = a_0|00\rangle + a_1|11\rangle, \]  

where \( |a_0|^2 + |a_1|^2 = 1 \), and Bob has qubits C and D in the following state,

\[ |\Psi\rangle_{CD} = b_0|00\rangle + b_1|11\rangle, \]  

where \( |b_0|^2 + |b_1|^2 = 1 \).

Now Alice wants to transmit the state of qubits A and B to Bob and Bob wants to transmit the qubits C and D to Alice. Assume that Alice and Bob share a five-qubit cluster state, which has the form

\[ |C_5\rangle_{12345} = \frac{1}{2}(|00000\rangle + |00111\rangle + |11101\rangle + |11010\rangle, \]  

the qubits A, B, 1 and 5 belong to Alice and qubits C, D, 2, 3 and 4 belong to Bob, respectively. The initial state of the total system can be shown as
\[ |\Psi\rangle_{12345ABCD} = |\Psi\rangle_{AB} \otimes |\Psi\rangle_{CD} \otimes |C5\rangle_{12345} \]

\[
= \frac{1}{4} \left[ |\Phi^\pm\rangle_{A1} |\Phi^\pm\rangle_{C4} |\chi^1\rangle_{BD235} + |\Phi^\pm\rangle_{A1} |\Psi^\pm\rangle_{C4} |\chi^2\rangle_{BD235} \right.
+ |\Psi^\pm\rangle_{A1} |\Phi^\pm\rangle_{C4} |\chi^3\rangle_{BD235} + |\Psi^\pm\rangle_{A1} |\Psi^\pm\rangle_{C4} |\chi^4\rangle_{BD235} \right].
\]

where

\[ |\chi^1\rangle_{BD235} = (a_0b_0|00000\rangle \pm_1 a_0b_1|01011\rangle \pm_2 a_1b_0|10111\rangle \pm_1 \pm_2 a_1b_1|11100\rangle), \]

\[ |\chi^2\rangle_{BD235} = (a_0b_0|00011\rangle \pm_1 a_0b_1|01000\rangle \pm_2 a_1b_0|10010\rangle \pm_1 \pm_2 a_1b_1|11111\rangle), \]

\[ |\chi^3\rangle_{BD235} = (a_0b_0|01111\rangle \pm_1 a_0b_1|01100\rangle \pm_2 a_1b_0|10000\rangle \pm_1 \pm_2 a_1b_1|11011\rangle), \]

\[ |\chi^4\rangle_{BD235} = (a_0b_0|00100\rangle \pm_1 a_0b_1|01111\rangle \pm_2 a_1b_0|10011\rangle \pm_1 \pm_2 a_1b_1|11000\rangle), \]

where \( |\Phi^\pm\rangle = \frac{1}{\sqrt{2}} (|00\rangle \pm |11\rangle) \) and \( |\Psi^\pm\rangle = \frac{1}{\sqrt{2}} (|01\rangle \pm |10\rangle) \) denote Bell basis. In the above equations, the notes \( \pm_1 \) correspond to the Bell-state measurements of qubit pairs \( (C, 4) \) and the notes \( \pm_2 \) correspond to the Bell-state measurements of qubit pairs \( (A, 1) \) in the basis of \( \{|\Phi^\pm\rangle, |\Psi^\pm\rangle\} \), respectively, and they mean multiplication of \( \pm \) signs.

Alice firstly perform a Bell-state measurement on qubit pairs \( (A, 1) \) and Bob perform a Bell-state measurement on qubit pairs \( (C, 4) \), respectively. Then they announce their results to each other via classical channel.

In order to simplify our descriptions, without loss of generality, we take the outcome \( |\Phi^+\rangle_{A1}, |\Phi^+\rangle_{C4} \) as an example to show the principle of this symmetric bidirectional quantum teleportation protocol, where the residual qubits system collapses into the states,

\[ |\varphi\rangle_{BD235} = (a_0b_0|00000\rangle + a_0b_1|01011\rangle + a_1b_0|10111\rangle + a_1b_1|11100\rangle)_{BD235}. \]
Alice takes X-basis measurement on qubit B and conveys her result to Bob, where X-basis is orthogonal basis including vectors,

\[
\begin{align*}
|+\rangle &= |0\rangle + |1\rangle \\
|−\rangle &= |0\rangle - |1\rangle 
\end{align*}
\]  

If Alice’s measurement result is \( |+\rangle_B \), then the state of the remaining qubits D, 2, 3 and 5 collapse into the state as

\[
|\varphi^1\rangle_{D235} = (a_0 b_0 |0000\rangle + a_0 b_1 |0111\rangle + a_1 b_0 |0111\rangle + a_1 b_1 |1100\rangle)_{D235}.
\]  

On the other hand, Bob takes X-basis measurement on qubit D and conveys his result to Alice. If Bob’s measurement result is \( |+\rangle_D \), then the state of the remaining qubits 2, 3 and 5 collapse into the state as

\[
|\varphi^2\rangle_{235} = (a_0 b_0 |000\rangle + a_0 b_1 |011\rangle + a_1 b_0 |111\rangle + a_1 b_1 |100\rangle)_{235}.
\]  

Also, Alice introduces an auxiliary particle 6 with an initial state \( |0\rangle_6 \), then the state \( |\varphi^2\rangle_{235} \) becomes the following state,

\[
|\varphi^3\rangle_{2356} = (a_0 b_0 |0000\rangle + a_0 b_1 |0110\rangle + a_1 b_0 |1110\rangle + a_1 b_1 |1000\rangle)_{2356}.
\]  

Then Alice and Bob perform joint unitary transformation on particle 2, 3, 5, 6. In order to reincarnate the original state under the basis \{ \( |0000\rangle_{2356}, |0001\rangle_{2356}, |0010\rangle_{2356}, |0011\rangle_{2356}, |0100\rangle_{2356}, |0101\rangle_{2356}, |0110\rangle_{2356}, |0111\rangle_{2356}, |1000\rangle_{2356}, |1001\rangle_{2356}, |1010\rangle_{2356}, |1011\rangle_{2356}, |1100\rangle_{2356}, |1101\rangle_{2356}, |1110\rangle_{2356}, |1111\rangle_{2356} \}, the unitary transformation (a 16 \times 16 matrix) may take the form
Having performed the joint unitary transformation, the state $|\psi_3\rangle_{2356}$ becomes the following state,

$$
|\psi_3\rangle_{2356} = (a_0 b_0 |0000\rangle + a_0 b_1 |0011\rangle + a_1 b_0 |1100\rangle + a_1 b_1 |1111\rangle)
$$

(14)

$$
= (a_0 |00\rangle + a_1 |11\rangle)_{23} \otimes (b_0 |00\rangle + b_1 |11\rangle)_{56}
$$

After doing those operations, the symmetric bidirectional quantum teleportation is successfully realized. And other cases are given in Table 1 and 2, where Unitary transformation is given in Table 3.

So the total successful probability is equal to one.
3. Conclusion and discussion

In conclusion, we have proposed a theoretical scheme for symmetric bidirectional quantum teleportation. In our scheme, five-qubit cluster state is considered as the quantum channel, while Alice and Bob are not only senders but also receivers. In this paper, Alice has two particles A and B in an unknown state, she wants to transmit the state of particles A and B to Bob; at the same time, Bob has two particles C and D in an unknown state, he wants to transmit the state of particles C and D to Alice. We hope that such a quantum teleportation scheme can be realized experimentally in the future.

Conflict of Interests
The authors declare that there is no conflict of interests.

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| Alice and Bob’s results | State of particles 2, 3, 5 and 6 | Unitary transformation |
|-------------------------|----------------------------------|------------------------|
| $|\Phi^+\rangle_A | |\Phi^+\rangle_c4 (+)\rangle a\rangle b\rangle D$ | $a_2 b_2 (0000) + a_2 b_2 (0110) + a_1 b_1 (1110) + a_1 b_1 (1000)$ | $U_{1,1}$ |
| $|\Phi^+\rangle_A | |\Phi^+\rangle_c4 (+)\rangle a\rangle b\rangle D$ | $a_2 b_2 (0000) - a_2 b_2 (0110) - a_1 b_1 (1110) + a_1 b_1 (1000)$ | $U_{1,2}$ |
| $|\Phi^+\rangle_A | |\Phi^+\rangle_c4 (-)\rangle a\rangle b\rangle D$ | $a_2 b_2 (0000) + a_2 b_2 (0110) - a_1 b_1 (1110) - a_1 b_1 (1000)$ | $U_{1,3}$ |
| $|\Phi^+\rangle_A | |\Phi^+\rangle_c4 (-)\rangle a\rangle b\rangle D$ | $a_2 b_2 (0000) - a_2 b_2 (0110) + a_1 b_1 (1110) + a_1 b_1 (1000)$ | $U_{1,4}$ |
| $|\Phi^-\rangle_A | |\Phi^-\rangle_c4 (+)\rangle a\rangle b\rangle D$ | $a_2 b_2 (0000) - a_2 b_2 (0110) + a_1 b_1 (1110) - a_1 b_1 (1000)$ | $U_{1,2}$ |
| $|\Phi^-\rangle_A | |\Phi^-\rangle_c4 (+)\rangle a\rangle b\rangle D$ | $a_2 b_2 (0000) + a_2 b_2 (0110) + a_1 b_1 (1110) + a_1 b_1 (1000)$ | $U_{1,1}$ |
| $|\Phi^-\rangle_A | |\Phi^-\rangle_c4 (-)\rangle a\rangle b\rangle D$ | $a_2 b_2 (0000) - a_2 b_2 (0110) - a_1 b_1 (1110) + a_1 b_1 (1000)$ | $U_{1,4}$ |
| $|\Phi^-\rangle_A | |\Phi^-\rangle_c4 (-)\rangle a\rangle b\rangle D$ | $a_2 b_2 (0000) + a_2 b_2 (0110) + a_1 b_1 (1110) + a_1 b_1 (1000)$ | $U_{1,1}$ |
| $|\Phi^-\rangle_A | |\Phi^-\rangle_c4 (+)\rangle a\rangle b\rangle D$ | $a_2 b_2 (0000) - a_2 b_2 (0110) - a_1 b_1 (1110) - a_1 b_1 (1000)$ | $U_{1,2}$ |
| $|\Phi^-\rangle_A | |\Phi^-\rangle_c4 (+)\rangle a\rangle b\rangle D$ | $a_2 b_2 (0000) + a_2 b_2 (0110) - a_1 b_1 (1110) - a_1 b_1 (1000)$ | $U_{1,3}$ |
| $|\Phi^-\rangle_A | |\Phi^-\rangle_c4 (-)\rangle a\rangle b\rangle D$ | $a_2 b_2 (0000) + a_2 b_2 (0110) + a_1 b_1 (1110) + a_1 b_1 (1000)$ | $U_{1,1}$ |
| $|\Phi^-\rangle_A | |\Phi^-\rangle_c4 (-)\rangle a\rangle b\rangle D$ | $a_2 b_2 (0000) - a_2 b_2 (0110) + a_1 b_1 (1110) + a_1 b_1 (1000)$ | $U_{1,2}$ |
| $|\Phi^-\rangle_A | |\Phi^-\rangle_c4 (+)\rangle a\rangle b\rangle D$ | $a_2 b_2 (0000) + a_2 b_2 (0110) - a_1 b_1 (1110) - a_1 b_1 (1000)$ | $U_{1,1}$ |
| $|\Phi^-\rangle_A | |\Phi^-\rangle_c4 (+)\rangle a\rangle b\rangle D$ | $a_2 b_2 (0000) - a_2 b_2 (0110) - a_1 b_1 (1110) + a_1 b_1 (1000)$ | $U_{1,4}$ |
| $|\Phi^-\rangle_A | |\Phi^-\rangle_c4 (-)\rangle a\rangle b\rangle D$ | $a_2 b_2 (0000) + a_2 b_2 (0110) + a_1 b_1 (1110) + a_1 b_1 (1000)$ | $U_{1,1}$ |
| $|\Phi^-\rangle_A | |\Phi^-\rangle_c4 (-)\rangle a\rangle b\rangle D$ | $a_2 b_2 (0000) - a_2 b_2 (0110) + a_1 b_1 (1110) + a_1 b_1 (1000)$ | $U_{1,2}$ |
### Table 2

| Alice and Bob's results | State of particles 2, 3, 5 and 6 | Unitary transformation |
|-------------------------|----------------------------------|------------------------|
| \( \Psi^+ \rangle_{A1}|\Phi^+\rangle_{c4}|+\rangle_{a1} \rangle_{D} \) | \( a_0b_0|1110\rangle + a_0b_1|1000\rangle + a_1b_0|0000\rangle + a_1b_1|0110\rangle \) | \( U_{3.1} \) |
| \( \Psi^+ \rangle_{A1}|\Phi^+\rangle_{c4}|+\rangle_{a1} \rangle_{D} \) | \( a_0b_0|1110\rangle - a_0b_1|1000\rangle + a_1b_0|0000\rangle - a_1b_1|0110\rangle \) | \( U_{3.2} \) |
| \( \Psi^+ \rangle_{A1}|\Phi^+\rangle_{c4}|+\rangle_{a1} \rangle_{D} \) | \( a_0b_0|1110\rangle + a_0b_1|1000\rangle - a_1b_0|0000\rangle - a_1b_1|0110\rangle \) | \( U_{3.3} \) |
| \( \Psi^+ \rangle_{A1}|\Phi^+\rangle_{c4}|+\rangle_{a1} \rangle_{D} \) | \( a_0b_0|1110\rangle - a_0b_1|1000\rangle - a_1b_0|0000\rangle + a_1b_1|0110\rangle \) | \( U_{3.4} \) |
| \( \Psi^+ \rangle_{A1}|\Phi^+\rangle_{c4}|+\rangle_{a1} \rangle_{D} \) | \( a_0b_0|1110\rangle - a_0b_1|1000\rangle - a_1b_0|0000\rangle - a_1b_1|0110\rangle \) | \( U_{3.3} \) |
| \( \Psi^+ \rangle_{A1}|\Phi^+\rangle_{c4}|+\rangle_{a1} \rangle_{D} \) | \( a_0b_0|1110\rangle - a_0b_1|1000\rangle + a_1b_0|0000\rangle + a_1b_1|0110\rangle \) | \( U_{3.4} \) |
| \( \Psi^+ \rangle_{A1}|\Phi^+\rangle_{c4}|+\rangle_{a1} \rangle_{D} \) | \( a_0b_0|1110\rangle + a_0b_1|1000\rangle - a_1b_0|0000\rangle - a_1b_1|0110\rangle \) | \( U_{3.3} \) |
| \( \Psi^+ \rangle_{A1}|\Phi^+\rangle_{c4}|+\rangle_{a1} \rangle_{D} \) | \( a_0b_0|1110\rangle - a_0b_1|1000\rangle + a_1b_0|0000\rangle + a_1b_1|0110\rangle \) | \( U_{3.4} \) |
| \( \Psi^+ \rangle_{A1}|\Phi^+\rangle_{c4}|+\rangle_{a1} \rangle_{D} \) | \( a_0b_0|1110\rangle - a_0b_1|1000\rangle + a_1b_0|0000\rangle + a_1b_1|0110\rangle \) | \( U_{3.3} \) |
| \( \Psi^+ \rangle_{A1}|\Phi^+\rangle_{c4}|+\rangle_{a1} \rangle_{D} \) | \( a_0b_0|1110\rangle - a_0b_1|1000\rangle - a_1b_0|0000\rangle + a_1b_1|0110\rangle \) | \( U_{3.4} \) |
| \( \Psi^+ \rangle_{A1}|\Phi^+\rangle_{c4}|+\rangle_{a1} \rangle_{D} \) | \( a_0b_0|1110\rangle - a_0b_1|1000\rangle - a_1b_0|0000\rangle - a_1b_1|0110\rangle \) | \( U_{3.3} \) |
| \( \Psi^+ \rangle_{A1}|\Phi^+\rangle_{c4}|+\rangle_{a1} \rangle_{D} \) | \( a_0b_0|1110\rangle - a_0b_1|1000\rangle - a_1b_0|0000\rangle + a_1b_1|0110\rangle \) | \( U_{3.4} \) |
| \( \Psi^+ \rangle_{A1}|\Phi^+\rangle_{c4}|+\rangle_{a1} \rangle_{D} \) | \( a_0b_0|1110\rangle - a_0b_1|1000\rangle + a_1b_0|0000\rangle + a_1b_1|0110\rangle \) | \( U_{3.3} \) |
| \( \Psi^+ \rangle_{A1}|\Phi^+\rangle_{c4}|+\rangle_{a1} \rangle_{D} \) | \( a_0b_0|1110\rangle - a_0b_1|1000\rangle + a_1b_0|0000\rangle + a_1b_1|0110\rangle \) | \( U_{3.4} \) |
| \( \Psi^+ \rangle_{A1}|\Phi^+\rangle_{c4}|+\rangle_{a1} \rangle_{D} \) | \( a_0b_0|1110\rangle - a_0b_1|1000\rangle - a_1b_0|0000\rangle + a_1b_1|0110\rangle \) | \( U_{3.3} \) |
| \( \Psi^+ \rangle_{A1}|\Phi^+\rangle_{c4}|+\rangle_{a1} \rangle_{D} \) | \( a_0b_0|1110\rangle - a_0b_1|1000\rangle - a_1b_0|0000\rangle - a_1b_1|0110\rangle \) | \( U_{3.4} \) |
| \( \Psi^+ \rangle_{A1}|\Phi^+\rangle_{c4}|+\rangle_{a1} \rangle_{D} \) | \( a_0b_0|1110\rangle - a_0b_1|1000\rangle - a_1b_0|0000\rangle + a_1b_1|0110\rangle \) | \( U_{3.3} \) |
| \( \Psi^+ \rangle_{A1}|\Phi^+\rangle_{c4}|+\rangle_{a1} \rangle_{D} \) | \( a_0b_0|1110\rangle - a_0b_1|1000\rangle + a_1b_0|0000\rangle + a_1b_1|0110\rangle \) | \( U_{3.4} \) |
| \( \Psi^+ \rangle_{A1}|\Phi^+\rangle_{c4}|+\rangle_{a1} \rangle_{D} \) | \( a_0b_0|1110\rangle - a_0b_1|1000\rangle + a_1b_0|0000\rangle + a_1b_1|0110\rangle \) | \( U_{3.3} \) |
| \( \Psi^+ \rangle_{A1}|\Phi^+\rangle_{c4}|+\rangle_{a1} \rangle_{D} \) | \( a_0b_0|1110\rangle - a_0b_1|1000\rangle - a_1b_0|0000\rangle + a_1b_1|0110\rangle \) | \( U_{3.4} \) |
| \( \Psi^+ \rangle_{A1}|\Phi^+\rangle_{c4}|+\rangle_{a1} \rangle_{D} \) | \( a_0b_0|1110\rangle - a_0b_1|1000\rangle - a_1b_0|0000\rangle - a_1b_1|0110\rangle \) | \( U_{3.3} \) |
| \( \Psi^+ \rangle_{A1}|\Phi^+\rangle_{c4}|+\rangle_{a1} \rangle_{D} \) | \( a_0b_0|1110\rangle - a_0b_1|1000\rangle - a_1b_0|0000\rangle - a_1b_1|0110\rangle \) | \( U_{3.4} \) |
| \( \Psi^+ \rangle_{A1}|\Phi^+\rangle_{c4}|+\rangle_{a1} \rangle_{D} \) | \( a_0b_0|1110\rangle - a_0b_1|1000\rangle - a_1b_0|0000\rangle + a_1b_1|0110\rangle \) | \( U_{3.3} \) |
| \( \Psi^+ \rangle_{A1}|\Phi^+\rangle_{c4}|+\rangle_{a1} \rangle_{D} \) | \( a_0b_0|1110\rangle - a_0b_1|1000\rangle + a_1b_0|0000\rangle + a_1b_1|0110\rangle \) | \( U_{3.4} \) |
| \( \Psi^+ \rangle_{A1}|\Phi^+\rangle_{c4}|+\rangle_{a1} \rangle_{D} \) | \( a_0b_0|1110\rangle - a_0b_1|1000\rangle + a_1b_0|0000\rangle + a_1b_1|0110\rangle \) | \( U_{3.3} \) |
### Table 3

**Unitary transformation**

| $U_{11} = (u_{i,j})$ | $u_{1,1} = u_{1,2} = u_{3,3} = u_{4,4} = u_{5,5} = u_{6,6} = u_{7,7} = u_{8,8} = u_{9,16} = u_{10,10}$ |
|----------------------|-----------------------------------------------------------------------------------------------------|
|                      | $= u_{11,11} = u_{12,12} = u_{13,13} = u_{14,14} = u_{15,13} = u_{16,9} = 1$ |
| $U_{12} = (u_{i,j})$ | $u_{1,1} = u_{1,2} = u_{3,3} = u_{4,4} = u_{5,5} = u_{6,6} = u_{7,4} = u_{8,8} = u_{9,16} = u_{10,10}$ |
|                      | $= u_{12,12} = u_{13,15} = u_{14,14} = u_{15,13} = u_{16,9} = -1$ |
| $U_{13} = (u_{i,j})$ | $u_{1,1} = u_{1,2} = u_{3,3} = u_{4,7} = u_{5,5} = u_{6,6} = u_{7,4} = u_{8,8} = u_{9,16} = u_{10,10}$ |
|                      | $= u_{11,11} = u_{12,12} = u_{14,14} = u_{15,13} = u_{16,9} = -1$ |
| $U_{14} = (u_{i,j})$ | $u_{1,1} = u_{1,2} = u_{3,3} = u_{4,7} = u_{5,5} = u_{6,6} = u_{7,4} = u_{8,8} = u_{9,16} = u_{10,10}$ |
|                      | $= u_{12,12} = u_{13,15} = u_{14,14} = u_{15,13} = u_{16,9} = -1$ |
| $U_{21} = (u_{i,j})$ | $u_{1,1} = u_{1,2} = u_{3,3} = u_{4,1} = u_{5,5} = u_{6,6} = u_{7,4} = u_{8,8} = u_{9,13} = u_{10,10}$ |
|                      | $= u_{11,11} = u_{12,12} = u_{13,9} = u_{14,14} = u_{15,16} = u_{16,15} = 1$ |
| $U_{22} = (u_{i,j})$ | $u_{1,1} = u_{1,2} = u_{3,3} = u_{4,7} = u_{5,5} = u_{6,6} = u_{7,4} = u_{8,8} = u_{9,13} = u_{10,10}$ |
|                      | $= u_{12,12} = u_{13,9} = u_{14,14} = u_{15,16} = u_{16,15} = -1$ |
| $U_{23} = (u_{i,j})$ | $u_{1,1} = u_{1,2} = u_{3,3} = u_{4,1} = u_{5,5} = u_{6,6} = u_{7,4} = u_{8,8} = u_{9,13} = u_{10,10}$ |
|                      | $= u_{11,11} = u_{12,12} = u_{14,14} = u_{15,16} = u_{16,15} = -1$ |
| $U_{24} = (u_{i,j})$ | $u_{1,1} = u_{1,2} = u_{3,3} = u_{4,7} = u_{5,5} = u_{6,6} = u_{7,4} = u_{8,8} = u_{9,13} = u_{10,10}$ |
|                      | $= u_{12,12} = u_{13,15} = u_{14,14} = u_{15,16} = u_{16,15} = 1$ |
| $U_{31} = (u_{i,j})$ | $u_{1,15} = u_{2,2} = u_{3,3} = u_{4,9} = u_{5,5} = u_{6,6} = u_{7,16} = u_{8,8} = u_{9,4} = u_{10,10}$ |
|                      | $= u_{11,11} = u_{12,12} = u_{13,1} = u_{14,14} = u_{15,13} = u_{16,7} = 1$ |
| $U_{32} = (u_{i,j})$ | $u_{1,15} = u_{2,2} = u_{3,3} = u_{4,9} = u_{5,5} = u_{6,6} = u_{7,16} = u_{8,8} = u_{9,4} = u_{10,10}$ |
|                      | $= u_{12,12} = u_{13,1} = u_{14,14} = u_{15,13} = u_{16,7} = -1$ |
| $U_{33} = (u_{i,j})$ | $u_{1,15} = u_{2,2} = u_{3,3} = u_{4,9} = u_{5,5} = u_{6,6} = u_{7,16} = u_{8,8} = u_{9,4} = u_{10,10}$ |
|                      | $= u_{11,11} = u_{12,12} = u_{14,14} = u_{15,13} = u_{16,7} = -1$ |
| $U_{34} = (u_{i,j})$ | $u_{1,15} = u_{2,2} = u_{3,3} = u_{4,9} = u_{5,5} = u_{6,6} = u_{7,16} = u_{8,8} = u_{9,4} = u_{10,10}$ |
|                      | $= u_{12,12} = u_{14,14} = u_{15,13} = u_{16,7} = 1$ |
| $U_{41} = (u_{i,j})$ | $u_{1,9} = u_{2,2} = u_{3,3} = u_{4,15} = u_{5,5} = u_{6,6} = u_{7,13} = u_{8,8} = u_{9,4} = u_{10,10}$ |
|                      | $= u_{11,11} = u_{12,12} = u_{13,7} = u_{14,14} = u_{15,1} = u_{16,16} = 1$ |
| $U_{42} = (u_{i,j})$ | $u_{1,9} = u_{2,2} = u_{3,3} = u_{4,15} = u_{5,5} = u_{6,6} = u_{7,13} = u_{8,8} = u_{9,4} = u_{10,10}$ |
|                      | $= u_{12,12} = u_{13,7} = u_{14,14} = u_{15,1} = u_{16,16} = 1$ |
| $U_{43} = (u_{i,j})$ | $u_{1,9} = u_{2,2} = u_{3,3} = u_{4,15} = u_{5,5} = u_{6,6} = u_{7,13} = u_{8,8} = u_{9,4} = u_{10,10}$ |
|                      | $= u_{11,11} = u_{12,12} = u_{14,14} = u_{15,1} = u_{16,16} = 1$ |
| $U_{44} = (u_{i,j})$ | $u_{1,9} = u_{2,2} = u_{3,3} = u_{4,15} = u_{5,5} = u_{6,6} = u_{7,13} = u_{8,8} = u_{9,4} = u_{10,10}$ |
|                      | $= u_{12,12} = u_{14,14} = u_{15,1} = u_{16,16} = 1$ |