Study on Calculation of the Buoyancy Drag Coefficient in the High-Speed Wind Tunnel Test

Xuekong Chen, Zhi Wei * and Guangyuan Liu
High Speed Institute, China Aerodynamic R&D Center, Mianyang, China

*Corresponding author e-mail: zhiwei@cardc.cn

Abstract. In order to obtain the drag coefficient in accuracy, it needs to correct the model’s buoyancy drag coefficient in the high-speed wind tunnel test. However, there are several methods on calculating the buoyancy drag coefficient, each will generate a result differently. Three types of methods are studied in this paper, the formulas are derived, the buoyancy drag coefficients are calculated, and the results are compared in the end. It shows that the most exact method is the one based on pressure fitting function, it’s suitable for correcting the model’s drag coefficient with high accuracy requirement.

1. Introduction
In the high-speed wind tunnels, speed of the flow will decelerate and the static pressure (abbreviated as pressure below) will increase when the freestream is passing through the test section. Liking the article will generate buoyancy in the water, the increased pressure can make drags on the testing model called buoyancy drag in wind tunnel test. Generally, it will be convert the buoyancy drag into coefficient for aerodynamic usage, and buoyancy drag coefficient is a key component of the model’s drag coefficient. Accuracy of the buoyancy drag coefficient is able to affect the accuracy of the drag coefficient in the end. Hence, in some large airplanes’ wind tunnel tests, such as the civil airplanes or the large transport airplanes, it has to obtain the buoyancy drag coefficient as exact as possible to ensure the fuel economy index.

In early times, the buoyancy drag coefficient was obtained by a kind of engineering estimation method, and this method wasn’t that exact in getting the result. As the accuracy of drag coefficient becomes more and more important, the buoyancy drag coefficient in wind tunnel test must be accurately obtained. The available methodology is dividing the model into numerous cross-pieces along the axis direction, then calculating the buoyancy drag coefficient of each piece, and totally integrating these coefficients at last. Based on this, two methods of calculating the buoyancy drag coefficient are come into being. One is the method based on Mach number fitting function and the other is based on pressure fitting function. Taking the early engineering method into account, there are three methods on calculating the buoyancy drag coefficient are studied in this paper. The methodologies are learned, the formulas are derived, and the results are calculated and compared through a standard model test.

2. Model Description
The model used in the test is a kind of transport airplane model named CHN-T1, and it was designed with supercritical transonic wings [1]. The cruise Mach number is 0.78 and the designed cruise lift coefficient is 0.5. In addition, the reduction ratio is 1:19.23, the total length is 1.5917 m, the wing span
is 1.5482 m, the mean aerodynamic chord is 0.1937 m, and the wing reference area is 0.2578 m² [2]. The testing photo of CHN-T1 is shown in Figure 1.

![Figure 1. The photo of CHN-T1 standard model in the tunnel.](image)

Fixed transition was applied in this test, the wings and tails have been stuck 0.1mm height transition bands, and the bands stuck on the nose is 0.18 mm in height. The test was carried out in the full-span model test section of 2.4m transonic wind tunnel of CARDC. Testing Mach numbers were 0.7, 0.78, 0.85 with the total pressure was 130 kPa for each.

3. Principle of Calculating the Buoyancy Drag Coefficient

According to the principle of buoyancy, it needs to divide into two parts to derive the formula of calculating the buoyancy drag coefficient. The first part is deducing the formula of calculating the buoyancy drag, and the other is about the coefficient calculation.

The idea of deducing the formula of calculating the buoyancy drag is shown in equation (1), it contains 4 steps:

1. Assume there is a functional relationship \( P(x) \) of pressure distribution along the axial direction in the test section.
2. Assume there is a functional relationship \( S_c(x) \) of model’s cross-sectional area distribution along the axial direction.
3. Algorithm of the buoyancy drag is pressure times the cross-sectional area, then integrating in all.
4. The cross-sectional area distribution is not regular enough, and the function \( S_c(x) \) isn’t a smooth curve correspondingly. The integral algorithm of the buoyancy drag should be converted into a differential algorithm, which makes the calculation easy.

\[
\Delta D_b = \int_{x_a}^{x_b} P(x) \cdot S_c(x) \, dx
\]

\[
= \int_{x_a}^{x_b} \frac{dP}{dx} \cdot S_c(x) \cdot dx
\]

\[
= \sum_{i=1}^{n} \frac{dP}{dx} \cdot S_c(x) \cdot dx
\]

\[
\Delta C_{Db} = \frac{\Delta D_b}{S_{ref} \cdot q_x}
\]

Where \( \Delta D_b \) = buoyancy drag (N), \( P \) = static pressure (Pa), \( x \) = length of the model along the axial direction (m), \( \Delta C_{Db} \) = buoyancy drag coefficient, \( S_{ref} \) = reference area (m²), \( q_x \) = dynamic pressure.
The buoyancy drag coefficient is given by:

$$M = \sqrt{\left(\frac{P}{P_0}\right)^{2\gamma} - 1}$$  

(3)

$$P = P_0 \cdot (1 + 0.2M^2)^{-3.5}$$  

(4)

$$q_\infty = \frac{1}{2} \rho_\infty V^2 = \frac{1}{2} \gamma P_0 \cdot M_\infty^2 \cdot (1 + 0.2M_\infty^2)^{-3.5}$$  

(5)

Where \(M = \text{Mach number over model area}, M_\infty = \text{Mach number of the freestream}, P_0 = \text{total pressure (Pa)}, q_\infty = \text{dynamic pressure (kg.m}^{-1}.\text{s}^{-2}), \rho_\infty = \text{density of freestream (kg.m}^{-3}\).}

### 4. Different Methods of Calculating the Buoyancy Drag Coefficient

#### 4.1. The Early Engineering Method

The early engineering method has been applied in large transonic wind tunnel [3] to estimate the buoyancy drag coefficient. The associating formula is derived as follows:

According to formula (4), \(dP/dx\) can be easily obtained. Substituting \(dP/dx\) into formula (1), the equation can be derived into equation (6) as follows:

$$\Delta C_{Dh} = \sum_{i=1}^{n} \left[ \frac{dP}{dx} \cdot S_i(x) \cdot dx \right]$$

$$= \sum_{i=1}^{n} \left[ -3.5 \cdot P_0 \cdot (1 + 0.2M^2)^{-3.5} \cdot 0.4M \cdot dM \cdot S_i(x) \cdot dx \right]$$

(6)

$$= -1.4P_0 \sum_{i=1}^{n} \left[ M(1 + 0.2M^2)^{-4.5} \cdot dM \cdot S_i(x) \cdot dx \right]$$

Then substituting equation (5) and (6) into equation (2), the formula of calculating the buoyancy drag coefficient can be expressed as:

$$\Delta C_{Dh} = \frac{-1.4P_0 \sum_{i=1}^{n} \left[ M(1 + 0.2M^2)^{-4.5} \cdot dM \cdot S_i(x) \cdot dx \right]}{\gamma \rho_\infty \cdot M_\infty^2 \cdot (1 + 0.2M_\infty^2)^{-3.5}}$$

(7)

Here, assuming \(M_\infty = M\), then equation (7) can be written as equation (8):

$$\Delta C_{Dh} = \frac{-2}{S_{ref}} \cdot \frac{1}{M(1 + 0.2M^2)} \sum_{i=1}^{n} \frac{dM}{dx} \cdot S_i(x) \cdot dx$$

(8)

Where \(dM/dx = \text{local gradient of Machu number over the length of the model (m}^{-1}\)).
Due to \( \sum_{i=1}^{n} S_i(x)dx = V \) in equation (8), and it can be simplified \( \sum_{i=1}^{n} \frac{dM_i}{dx} = \frac{dM}{dx} \). Finally, it can obtain the engineering estimating formula of the buoyancy drag coefficient as follows:

\[
\Delta C_{Df} = \frac{V}{S_{ref}} \times \frac{2}{M \times (1 + 0.2M^2)} \times \frac{dM}{dx}
\]  

(9)

In formula (9), the model’s volume \( V \) is divided into the wings’ volume \( V_w \) and the fuselage’s volume \( V_f \). There are equations (10) below to estimate \( V_w \) and \( V_f \):

\[
V = V_w + V_f
\]

\[
V_w = 0.7 \times (t/c)_{av} \times S_{ref} \times b_A
\]

\[
V_f = 0.45 \times l_f \times d_{max}
\]

(10)

Where \( V_w = \) wing volume \( (m^3) \), \( V_f = \) fuselage volume \( (m^3) \), \( (t/c)_{av} = \) average wing thickness ratio, \( d_{max} = \) maximum diameter of the fuselage \( (m) \), \( l_f = \) length of the fuselage \( (m) \).

Formula (9) is the early engineering method of calculating the buoyancy drag coefficient, and formula (10) is the associating equations to estimate the model’s volume. It can get known that in the early engineering method, the model’s volume has been split into two parts and then estimating the volume of each part independently, there has been generated errors in the estimating equations because these equations are not that exact enough. Besides, the value of \( dM/dx \) will draw errors into the result as well. The parameter \( dM/dx \) is inaccurate when using a general value over the model length. The parameter \( dM/dx \) sharply changes over the model’s back section whereas its volume is small and \( dM/dx \) changes gently over the middle section of the middle section which containing the wings with the most volume. Hence, parameter \( dM/dx \) can’t stand for the real value of the whole model exactly. There is a calculating case shown in section 5 in this paper.

4.2. Method of Calculating the Buoyancy Drag Coefficient Based on Mach Number Fitting Function

In the tunnel’s flow fields calibration, centerline pipe is used to measure the pressure on each orifice along the centerline flow. Through the pressure, Mach number on each orifice can be obtained by equation (3). Mach numbers’ distribution along the axial direction can be fitted as a quadratic polynomial function as:

\[
M(x) = Ax^2 + Bx + C
\]

(11)

Substituting expression (11) into equation (8), it can get the method of calculating buoyancy drag coefficient based on Mach number fitting function as formula (12):

\[
\Delta C_{Dbh} = -\frac{1}{S_{ref}} \times \frac{2}{M(1 + 0.2M^2)} \times \sum_{i=1}^{n} \frac{(Ax + B) \times S_i(x) \times dx}{dx}
\]

(12)

Because of the assumption of \( M_\infty = M \), it will draw errors into the result of formula (12). However, the errors here are less than the early engineering method. Therefore, it can recognize that formula (12) is an exacter method for calculating the buoyancy drag coefficient.

4.3. Method of Calculating the Buoyancy Drag Coefficient Based on Pressure Fitting Function

There is a formula (13) determined by integrating \( dC_p/dx \) times \( MXC \) over the length of the model to calculating the he buoyancy drag coefficient in the Boeing Transonic Wind Tunnel [4], but it doesn’t show the details on how to process \( dC_p/dx \). Hence, another type of method of calculating the buoyancy drag coefficient based on pressure fitting function is given in this paper.
Where \( \frac{dC_p}{dx} \) = local gradient of longitudinal static pressure coefficient, \( MXC \) = model’s cross sectional area (m²).

Liking the process of Mach number above, pressure’s distribution along the axial direction can be fitted as a quadratic polynomial function as:

\[
P(x) = Ax^2 + Bx + C
\]

Substituting equation (14) into equation (1), it can get:

\[
\Delta D_b = \int_{x_0}^{x_f} \frac{dC_p}{dx} \cdot MXC \cdot dx
\]

\[\Delta = \int_{x_0}^{x_f} \left( \frac{dC_p}{dx} \cdot MXC \right) \cdot dx \]

(13)

Formulas (14) is the method of calculating the buoyancy drag based on pressure fitting function. Combining it with formula (2), the method of calculating the coefficient of the buoyancy drag based on pressure fitting function can be obtained as equation (16). In this equation, the dynamic pressure \( q_\infty \) calculated by formula (4) independently. It doesn’t draw other errors into equation (16) except the errors generated in fitting function of the pressure distribution. So, it can be known that formula (16) is the most exact method for calculating the buoyancy drag coefficient.

\[
\Delta C_{D_b} = \frac{1}{S_{ref}} \cdot \frac{1}{q_\infty} \cdot \sum_{i=1}^{n} [(2Ax + B) \cdot S_c(x) \cdot dx]
\]

(16)

5. Comparison of the Results

Taking the CHN-T1 model’s test as an example to study the different methods on calculating the buoyancy drag coefficient, calculating the result by each method and done a comparison at the end. The information of the test can be found in section 2. The values of some parameters used are given below:

(1) In the early engineering method, \((t/c)_{av} = 0.15, d_{max} = 0.2\).

(2) In the methods of calculating the buoyancy drag coefficient based on either Mach number or pressure fitting function, \( n = 50 \).

| \( M \) | \( 0.7 \) | \( 0.78 \) | \( 0.85 \) |
|--------|--------|--------|--------|
| Early engineering method | -0.00095 | -0.00086 | -0.00072 |
| Method based on Mach number fitting function | -0.00055 | -0.00039 | -0.00028 |
| Method based on pressure fitting function | -0.00038 | -0.00027 | -0.00011 |

The values of each method for calculating the buoyancy drag coefficient for CHN-T1 model are shown in Table 1. Figure 2 shows the CHN-T1 model’s cross-sectional area distribution along the axial direction and Figure 3 shows the buoyancy drag coefficient distribution at different Mach numbers in polygonal line form. It shows that the early engineering method owns the greatest absolute values, each value calculated by the method based on Mach number fitting function becomes smaller in absolute form, and the method based on pressure fitting function owns the smallest absolute value at each testing
Mach number point. According to the previous derivation, we have known that the most exact method is the one based on pressure fitting function. Based on this point, setting the values calculated by this method as the standard to evaluate the other two methods. Then it reveals that there are 5.4 ~ 6.1 drag counts more than the standard values when calculating the results by the early engineering method, and there are 1.2 ~ 1.7 drag counts more when calculating the results by the method based on Mach number fitting function. Through this comparison, it shows that the buoyancy drag coefficient is much different when calculating by different methods. In addition, under the requirement of high accuracy of drag coefficient in modern large aircraft models’ wind tunnel tests, applying the method based on pressure fitting function can obtain the most exact results on buoyancy drag coefficient.

Figure 2. The CHN-T1 mode’s cross-sectional area distribution along the axial direction.

![Cross-sectional area distribution](image)

Figure 3. The buoyancy drag coefficient distribution at different Mach numbers in polygonal line form.

6. Conclusion
Through the high-speed wind tunnel test of CHN-T1 standard model, three types of methods on calculating the buoyancy drag coefficient were studied in this paper. The main conclusions are as follows:

1) The early engineering method is the easiest one to use but owns the greatest errors. The method based on Mach number fitting function makes a significant progress on improving the errors but owns errors as well.
(2) The most exact method for calculating the buoyancy drag coefficient is the one based on pressure fitting function. This method is suitable for large aircraft models’ wind tunnel tests with high accuracy of drag coefficient requirement.

Acknowledgments
The author of this article thanks everyone for their hard work on the wind tunnel test, and the senior engineer Zhi Wei had provided a significant guidance in deriving the formulas of the buoyancy drag coefficient. In addition, this work was financially supported by a fund and the code is 11802328.

References
[1] Yuntao Wang, Gang Liu and Zuobing Chen, Summary of the first aeronautical computational fluid dynamics credibility workshop, Acta Aerodynamica Sinica, 2019, 37(2):248.
[2] Qiang Li, Dawei Liu and Xin Xu, et al, Experimental Study of Aerodynamic Characteristics of CHN-T1 Standard Model in 2.4 m Transonic Wind Tunnel, Acta Aerodynamica Sinica, 2019, 37(2):338.
[3] Venkit Iyer, A Wall Correction Program Based on Classical Methods for the National Transonic Facility (Solid Wall or Slotted Wall) and the 14x22-Ft Subsonic Tunnel at NASA LaRC, Analytical Services & Materials Inc., Virginia, 2004, pp.3-6.
[4] Mark S. Hudgins and Dennis W. Hergert, Methodology and Results from a Recent Calibration of the Boeing Transonic Wind Tunnel, AIAA 2005-4280, 41st AIAA/ASME/SAE/ASEE Joint Propulsion Conference & Exhibit, Tucson, 2005.