Controlled Breaking of Phase Symmetry in a "Which-Path?" Interferometer

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Abstract. Linear response conductance of a two terminal Aharonov-Bohm (AB) interferometer is an even function of magnetic field, as dictated by Onsager-Büttiker relations. We discuss an experimental setup in which this phase symmetry can be broken in a controlled way. To this end we consider a "Which Path?" detector, which consists of interferometer with quantum dots (QDs) in each of its arms, one of the QDs being capacitively coupled to a nearby quantum point contact (QPC). This coupling results in breaking of the phase symmetry of the AB oscillations, given that a finite voltage bias is applied to the QPC. Sweeping in the same time the level in the other QD across the Fermi level allows one to observe smooth change of the phase from 0 to $\pi$, rather than an abrupt jump observed in isolated AB interferometers. We explore the possibility of using this setup in order to measure the transmission phase through a QD.

1. Introduction

Linear response conductance of a two-terminal Aharonov-Bohm (AB) interferometer is an even function of magnetic flux, as required by Onsager-Büttiker relations [1, 2]. This means that the phase of AB oscillations can take only values 0 or $\pi$. If a quantum dot (QD) is placed in one of the arms of the interferometer, the phase may jump abruptly between 0 or $\pi$ when the Fermi level passes through the level in the QD [3, 4]. This jump is extremely sharp due to the fact that the lowest harmonic of AB oscillations takes zero value, the oscillations being completely dominated by higher order harmonics [5].

Such phase symmetry precludes measuring directly the transmission phase via a QD [3]. This obstacle may be overcome by breaking the phase symmetry, e.g., by using a multiterminal interferometer [4, 6].

Other ways to break the phase symmetry include studying the interferometer in the regime of nonlinear transport [7, 8, 9, 10, 11] or coupling it to a non-equilibrium environment [12]. We address the latter case by considering a "Which Path?" detector [13, 14], which consists of an AB interferometer containing a QD in each of its arms, and a quantum point contact (QPC) capacitively coupled to one of the QDs, as shown in Fig. 1a. The QPC plays the role of an environment and causes breaking of the phase symmetry, but only when a finite bias voltage is applied to it [12]. Thus, instead of the phase jump in the linear response conductance, expected when the level in the QD, not coupled to the QPC, is swept across the Fermi level, the AB phase smoothly flows between values 0 and $\pi$. 

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2. Formulation of the problem

We describe the system in Fig. 1a by Hamiltonian $H = H_I + H_{QPC} + H_C$. $H_I = \sum_\alpha \epsilon_\alpha d_\alpha^+d_\alpha + \sum_{k,\mu} \epsilon_{k\mu}C_k^+c_{k\mu} + \sum_{\alpha, k, \mu} (t_{\alpha k}d_{\alpha}^+c_{k\mu} + h.c.)$ which includes the terms describing the QDs, the leads, and tunneling between them. Operator $d_{\alpha}$ destroys an electron in QD $\alpha = 1, 2$, $c_{k\mu}$ destroys an electron in state $k$ of lead $\mu = L, R$. The strength of coupling between the QD $\alpha$ and lead $\mu$ is characterized by $\Gamma_{\alpha \mu} = 2\pi N_\mu |t_{\alpha \mu}|^2$, where $N_\mu$ is the density-of-states in lead $\mu$. Magnetic field enters via the complex phases of tunneling matrix elements $t_{\alpha \mu}$: $t_{1L}^*t_{1R}^*t_{2R}t_{2L} = \pm |t_{1L}t_{1R}t_{2R}t_{2L}| e^{i\varphi}$, where $\varphi = 2\pi \Phi/\Phi_0$, $\Phi$ is the magnetic flux threading the interferometer, $\Phi_0 = hc/e$ is the flux quantum, and sign $\pm$ depends on parities of QD states.

$H_{QPC} = \sum_{p, p'} E_p\delta_{pp'}\alpha_p \alpha_p$ describes the QPC, the states in which are right/left-movers ($\nu = \pm$) characterized by quantum numbers $p, p'$; $\alpha_p$ is the corresponding destruction operator.

$H_C = d_1^+d_2^+\sum_{p, p', \rho, \rho'} U_{pp'}\delta_{\rho \rho'}\alpha_p \alpha_{p'}$ is the capacitive coupling between the QPC and QD2.

We calculated the current through the interferometer using the Green’s function approach [16], taking account for the coupling to the QPC as a self-energy correction in electron Green’s function up to the first order in coupling strength $g_{\rho \rho'} = g_{\rho' \rho} = 2\pi \rho_{\rho'} |U_{\rho \rho'}|^2$ [15] ($\rho_{\rho'}$ is the density-of-states in the QPC). The current consists of three distinct contributions: (i) the one resembling the current via an interferometer uncoupled from the QPC, the coupling having effect of suppressing the visibility of AB oscillations; (ii) and (iii) the two contributions proportional to coupling strength $g_{\rho \rho'}$ which are respectively even and odd in magnetic field. The latter two contributions generally do not vanish at zero source-drain bias, $V_{sd}$, given that the bias applied to the QPC, $V_{QPC}$, is not zero. Below we describe (a) the nature of the current component odd in magnetic field, and (b) the reasons why the current does not vanish at zero source-drain bias. The detailed derivation and full mathematical expressions will be given elsewhere.

3. Processes contributing to the current odd in magnetic field

The probability of a process in which electron is transferred between the leads can be written as (up to the third order in lead-to-lead tunneling amplitude $V_{nm}$)

$$2\pi \left( |V_{fi}|^2 + 2Re \left( V_{fi}^* \frac{V_{fm}V_{mi}}{E_m - E_i + i0^+} \right) \right) \delta(E_f - E_i),$$

where indices $i, f$, and $m$ refer to the initial, final and intermediate states. When we sum over all of these states that are available the imaginary part of the denominator, treated as $1/(E + i0^+) = 1/E - i\pi \delta(E)$, projects out the states where initial, final and intermediate energies lay on the same energy shell, $E_i = E_f = E_m$ [11, 17]. The phase of the resulting contribution to AB oscillations is shifted by $\pi/2$.

One can show [11] that the only such processes that will give non-zero contribution to the odd AB oscillations are either elastic processes like the ones in Figs. 2c,e, where electron first absorbs energy from the QPC and then re-emits it, or the inelastic processes like those in Figs. 2a,b,d,f.

There are pairs of processes, such as those in Figs. 2c,d, whose contributions mutually cancel out: the processes in such a pair are composed of identical matrix elements, but effectively

Figure 1. a) Schematics of “Which path?” interferometer studied in this paper. b) The phase of AB oscillations vs the energy of the level in the reference arm, $\epsilon_1$, for different values of QPC bias.
describe electron motion around the interferometer in opposite directions.

Non-cancelling processes include the two processes in Figs. 2e,f which add up (the process in Fig. 2e describes tunneling of a hole, which implies an additional minus sign), as well as the inelastic processes shown in Figs. 2a,b (which do not have pairs). All such processes begin with creation of an electron-hole pair in one of the leads, followed by tunneling to the other lead [11].

4. Aharonov-Bohm conductance

The upper left panel of Fig. 3 shows full linear response AB conductance when the bias applied to the QPC is $V_{QPC} = 0.1$. (The other parameters are (in relative units)$\Gamma_1^L = \Gamma_2^R = 0.02$, $\Gamma_2^L = \Gamma_1^R = 0.05$, $g_{\nu\nu'} = 0.05$, $\epsilon_2 = 2$, $\epsilon_F = 0$.) The AB oscillations are nearly even in magnetic flux, their odd component (upper right of Fig. 3) being many orders of magnitude smaller than the even one. The phase jump occurs around $\epsilon_1 \approx 0$, where the oscillations change from having a minimum at zero magnetic field to having a maximum. Near the jump the lowest harmonic of the oscillations, $\sim \cos(\varphi)$, vanishes and they are dominated by the second harmonic, $\sim \cos(2\varphi)$.

The bottom panels of Fig. 3 show the situation at high bias, $V_{QPC} = 4.1$. The AB oscillations are now asymmetric in magnetic flux. Although close to the phase jump they are still dominated by the 2nd harmonic, the lowest harmonic does not disappear, but enters with a phase different from either 0 or $\pi$ ($\sim \cos(\varphi - \varphi_0)$, $\varphi_0 \neq 0, \pi$). The amplitude of the odd oscillations (lower right panel of Fig. 3) is now comparable to that of the even ones.

Fig. 1b shows AB phase $\varphi_0$ as a function of energy $\epsilon_1$ for different values of $V_{QPC}$. At low bias the phase jumps abruptly between 0, and $\pi$, but it changes smoothly between these two values as the bias increases.

Our calculation shows that in the limit $\Gamma_2 \ll |\epsilon_2 - \epsilon_F|$ the part of the differential conductance odd in magnetic flux is given by $G^{\text{odd}}(\varphi, \epsilon_F, \epsilon_1) = \sin(\varphi) \times A(\epsilon_F) \Re[G_1^r(\epsilon_F)]$, where $G_1^r(\epsilon_F)$ is the Green’s function of QD1, calculated without account for QD2 and the QPC, while factor $A(\epsilon_F)$ is independent on energy $\epsilon_1$. Transmission via QD1, when QD2 is disconnected, is given by $G_{QD1}(\epsilon_1) = G_0 \left( \Gamma_{11}^L \Gamma_{11}^R / \Gamma_{11} \right) \Im[G_1^r(\epsilon_F)]$. Thus, the transmission phase via QD1, defined by $G_1^r(\omega) = |G_1^r(\omega)| e^{i\varphi_{QD1}(\omega)}$, as a function of $\epsilon_1$ can be deduced from $\tan[\varphi_{QD1}(\epsilon_F)] \sim G^{\text{odd}}(\varphi, \epsilon_F, \epsilon_1) / G_{QD1}(\epsilon_1)$.

5. Photon-assisted tunneling

Elastic processes are forbidden at zero bias, but electrons can still be transferred between the leads by means of processes shown Figs. 2a,b,e, absorbing energy from the QPC. However, in order to have non-zero net current, the device should possess some kind of left/right asymmetry: either due to non-symmetric coupling to the leads or due to the presence of magnetic field. While the former case results in a current even in magnetic flux, the latter leads to a current odd in magnetic flux, since changing sign of the magnetic field is equivalent to interchanging the leads.
Odd signal
Low bias, \( V = .1 \)
High bias, \( V = 4.1 \)

Magnetic flux \((\Phi_0)\)

Total signal
Level energy, \( \epsilon_1 \)
Level energy, \( \epsilon_1 \)

Figure 3. Linear response conductance as a function of magnetic flux (horizontal axis) and the energy of the level in the reference arm, \( \epsilon_1 \) (vertical axis). The upper (lower) panels correspond to the QPC bias \( V_{QPC} = .1 \) (\( V_{QPC} = 4.1 \)); left (right) panels show total (odd) AB conductance.

6. Conclusion
We studied the symmetry of AB oscillations in a "Which Path?" detector, which consists of an AB interferometer with QDs in each of its arms, with one of the QDs capacitively coupled to a QPC. When the QPC is biased, the AB oscillations in response conductance of the ABI lose their symmetry in respect to the sign of the magnetic field. The effect is pronounced in the regime of phase jump, when the energy level in the other QD is swept across the Fermi level, and can level, and can be used to measure the transmission phase via a QD.

In addition, the current through the interferometer does not disappear at zero source-drain bias due to the possibility of energy transfer from the biased QPC. Existence of such a current requires asymmetry introduced either by unequal coupling to the leads or by the magnetic field, and the current itself is asymmetric in the magnetic field.

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