IMPERFECT INVENTORY MODEL FOR TRENDED DEMAND UNDER RADIO FREQUENCY IDENTIFICATION AND TRADE CREDIT

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Abstract: In this paper, models on concepts of radio frequency identification adoption (RFID), imperfect products reworking system, and trade credit for deteriorating inventory with / without utilizing the preservation investment technology are derived, which makes this article, a unique one. Estimation of optimal values of RFID levels of investment for ordering, operating, just in time efficiencies along with production cycle time, and preservation investment are carried out. The rate of market demand is quadratic in nature based on time and is suitable for the items for which demand rises primarily, and then after it begins to decline. This form of demand is applicable to a vast range of items like garments, fashion accessories, electronics, etc.. The model is further divided into two cases based on demand rate and product’s reworking of imperfect quality items. Further, in each case, four subcases based on credit period and time of production cycle are analysed. The main objective of the inventory problem is to calculate total manufacturing cost in each subcase. The classical optimization technique is utilized for calculating the optimal values of decision variables. For the validation of developed models in each case, numerical examples are demonstrated, then using the concept of eigen-values of a Hessian matrix, we have proved the convex nature of the system's total cost for the case which has the minimum total cost. Also the decision variable sensitivity analysis is done by altering the inventory parameters for generating fruitful managerial insights. The model derived in this article can be applied in supply chain management of packaged food products/seasonal food products/milk products like butter, cheese, etc., where the tags for RFID are applied to track eatable/milk items of during delivery and storing. Also, if the model deals with a product of improper production, then it undergoes the reworking process.
Keywords: Deterioration, Trade Credit, Radio Frequency Identification Adoption, Preservation Investment, Time Dependent Demand Rate, Reworking System for Imperfect Production.

MSC: 90B85, 90C26.

1. INTRODUCTION

An economic production quantity (EPQ) model is commonly utilized for solving a problem related to inventory. Also, we know that in a practical scenario, it is not always possible to produce a perfect item. In order to deal with these situations, where imperfect items are produced, various research work has been conducted demonstrating the effect of an imperfect production process on EPQ model.

Initially, on adopting the concept of product’s imperfect quality in EPQ/EOQ formulae, Salameh and Jaber [31] stretched the usual EPQ/EOQ inventory models. Then, many contributions dealing with rework process are carried out by; Hayek and Salameh [15], Chan et al. [5], Jamal et al. [19], Konstantaras et al. [22], Yoo et al. [47], Wahab and Jaber [46], Tsao et al. [43], Konstantaras et al. [23], Sinha [38], Jaber et al. [18], Zhou [49]. Most of the research work includes an assumption of fixed ordering cost and production cost, but it could be on considering the merits of radio frequency identification (RFID) technology. Utilizing of RFID technology, the efficiency increases, labor costs declines, inventory information accuracy improves, and manufacturing processes simplifies. Therefore, ordering cost and production costs are reduced with RFID. There is an impact of RFID on operations management, which was analyzed by various researchers, Ustundag and Tanyas [45], Shin and Eksioglu [35], Leung et al. [25] and Szmerkovsky and Zhang [39], Szmerkovsky et al. [40], Lee and Lee [24], Zhang et al. [48], Chung et al. [7], Cui et al. [10], Tsao et al. [43], Tao et al. [41], Kohli and Peng [21]. The demand rate can be supposed as a function fluctuating based on time, level of stock, and price linked with selling of items or together. Min and Zhou [28], Silver et al. [36] Afterwards various research scholars like; Chung et al. [8, 9], Bose et al. [4], Hariga [14], Silver [37], Shah et al. [33] and Shah et al. [34] assumed the nature of demand rate as fluctuating in forms of linear, quadratic exponential etc. However, considering the practical scenario, to uplift the ordered quantity, supplier grants a trade credit to the manufacturer. Firstly, Haley and Higgins [13] introduced a model with the allowable delay in payments. Then, further studies considering this concept were carried-out by Kingsman [20], Goyal [12], Aggarwal and Jaggi [2] modified Goyal’s [12] model, Mahata and Goswami [27], Mishra et al. [29], Teng et al. [42], Shah and Shah [32], Lin et al. [26], Arcelus et al. [3], Abad and Jaggi [1], Chang [6], etc.

To reduce the effect of deterioration, the preservation technology investment is utilized by various researchers, Hsu et al. [17], Dye and Yang [11], Pal et al. [30], He and Huang [16].
2. NOTATIONS AND ASSUMPTIONS

2.1. Notations

Parameters

- $P_R$: Rate of reworking of imperfect quality items in units/year (in dollars)
- $P$: Rate of production in units/year
- $PIEE$: Rate of production for imperfect quality items in units/year
- $C$: Cost of production/item (in dollars)
- $Dt(t)$: Demand rate at time $t$ in units/year
- $a$: Scale demand, where $a > 0$
- $b$: Linear variation of demand with respect to time, where $0 < b \leq 1$
- $c$: Quadratic variation of demand, where $0 < c \leq 1$
- $k$: Imperfect quality products produced percentage
- $OrE$: Efficiency associated with ordering
- $Cor$: Level of investment for efficiency associated with ordering
- $J$: Efficiency associated with JIT
- $Cj$: Level of investment for efficiency associated with JIT
- $OpE$: Efficiency associated with operating
- $Cop$: Level of investment for efficiency associated with operating
- $T$: Cycle Time (in years)
- $Q$: Total number of products produced throughout a round (in units)
- $I_R$: Level of inventory when reworking of imperfect quality done (in units)
- $I_o$: Level of inventory as soon as original production is accomplished (in units)
- $C_r$: Cost of repairing per item of imperfect quality (in dollars)
- $C_s$: Setup cost per item for each production round (in dollars)
- $h$: Annual holding cost of imperfect items/item (in dollars)
- $h_R$: Annual holding cost for imperfect products undergoes reworking (in dollars)
- $P_p$: Selling price associated with perfect quality products (in dollars)
- $\Theta_o$: Deterioration co-efficient
- $A_p$: Cost associated with material purchasing/item (in dollars)
- $I_c$: Earned rate of interest per dollar/year (in dollars)
- $I_m$: Charged rate of interest rate per dollar/year (in dollars)
- $I_m$: Rate of interest charged accumulated for items in stock (in dollars)
- $\xi$: Co-efficient of preservation investment
- $\alpha$: Mark up for efficiency associated with ordering
- $\beta$: Mark up for efficiency associated with JIT
- $\gamma$: Mark up for efficiency associated with operating
- $M$: Credit period offered by supplier to manufacturer (in years)

2.2. Assumptions

1. Shortages are not allowed.
2. The rate of market demand is represented by a function of time $Dt(t) = a(1 + bt - ct^2)$. 
3. The imperfect quality products percentage is a known constant.
4. The rework for all imperfect product can be done with a repair cost.
5. The rate of rework for imperfect product is a predefined constant. The items undergoing repairing process are similar to the original items.
6. The manufacturer has offered a credit period $M$ by the supplier. The wholesale price per unit of the items traded throughout the credit period is deposited in an account with interest rate $I_e$. With the completion of this period, the credit is paid and manufacturers takes the payment of charged interest at rate $I_m$ for the products in stock.
7. Let $\Theta_u = \Theta_o, 0 \leq \Theta_o \leq 1$ be the deterioration co-efficient, in the situation where there is no utilization of preservation technology and let $\Theta_u = \Theta_o \exp^{-\xi u}$ is the co-efficient of deterioration, in case when there is an utilization of preservation technology.

3. MATHEMATICAL FORMULATION OF THE MODEL

On the basis of the rework and rate of market demand relationship, splitting model as in Case 1, where $P_R > \int_0^T Dtdt$ and Case 2, where $P_R < \int_0^T Dtdt$

Case 1 $P_R > \int_0^T Dtdt$ without preservation

Figure 1 and Figure 2 respectively, demonstrate the perfect and imperfect levels of inventories in case 1.

The rate of production of imperfect item is demonstrated $\text{PIEE} = kP$ (1)

Also, the rate of production of perfect quality products is always higher than or equal to the addition of the market demand rate and defective product’s rate of production,

$$P \cdot \text{PIEE} - \int_0^T Dtdt \geq 0 \Rightarrow 0 \leq k \leq (1 - \int_0^T (Dtdt)/P)$$ (2)
The production time is \( t_1 = \frac{I_o}{(P - P_{IEE} - \int_0^T Dtdt)} \) (4)

Initial Inventory level is \( I_o = (P - P_{IEE} - \int_0^T Dtdt)(Q/P) \) (5)

The rework time is \( t_2 = P_{IEE} \frac{Q}{(P_{RP})} = \frac{Qk}{P_R} \) (6)

The highest level of inventory level is given by
\[
I_R = (1 - \left(\int_0^T Dtdt(P_{IEE} + P_R)\right)/(PP_R))Q
\]
(7)

Thus, \( t_3 = \left(\frac{I_R}{(1/\int_0^T Dtdt)}\right) = Q((1/\int_0^T Dtdt) - (P_{IEE} + P_R)/PP_R) \) (8)

Therefore, \( t_a = t_1 = Q/P \) (9)

\( t_b = t_1 + t_2 = Q/P + Qk/P_R = Q/P + Qk/P_R \) (10)

\( t_c = t_1 + t_2 + t_3 = Q/P + Qk/P_R + I_R/(1/\int_0^T Dtdt) \)

\( t_c = (Q/P + Qk/P_R + Q/\int_0^T Dtdt - QP_{IEE} + QP_R/PP_R) \) (11)

Below stated differential equations demonstrate inventory level of perfect items
\[
dI_a/dt = P - P_{IEE} - \int_0^{t_a} Dtdt - \theta_u I_a, \quad 0 \leq t \leq t_a
\]
(12)

\[
dI_b/dt = P_{RP} - \int_{t_a}^{t_b} Dtdt - \theta_u I_b, \quad t_a \leq t \leq t_b
\]
(13)

\[
dI_c/dt = -\int_{t_b}^{t_c} Dtdt - \theta_u I_c, \quad t_b \leq t \leq t_c
\]
(14)

Utilizing boundary conditions, \( I_a(0) = 0; I_a(t_a) = I_b(t_a); I_b(t_b) = I_{max} = I_c(t_b); I_c(t_c) = 0; \) for solving differential equations demonstrated in equations (A1) to...
(A3) without preservation, and equations (A10) to (A12) in appendix with preservation. By using appendix equations (A2), (A3) and \( I_b(t_b) = I_{\text{max}} \) obtaining highest inventory level \( I_{\text{max}} \) given by equation (A4) in appendix. The below stated differential equations are level of inventory of imperfect items

\[
\frac{dI_d}{dt} = PIEE, \quad 0 \leq t \leq t_a
\]

\[
\frac{dI_e}{dt} = P_R, \quad t_a \leq t \leq t_b
\]

Utilizing the conditions: \( I_d(0) = 0, I_e(t_e) = 0 \), after solving we get,

\[
I_d(t) = tPIEE, \quad 0 \leq t \leq t_a
\]

To have positive inventory with no shortages then,

\[
I_e(t) = P_R(t - t_b), \quad t_a \leq t \leq t_b
\]

The below stated components plays a major role in computing total cost of the system:

- Production cost per year, \( PC = CQ \) (19)
- Repair cost per year, \( RC = C_r Q k \) (20)
- Setup cost per year, \( SC = C_s \) (21)
- Holding cost, \( HC = h(\int_0^{t_a} I_d dt + \int_{t_a}^{t_b} I_b dt + \int_{t_b}^{t_c} I_c dt) + h_r(\int_0^{t_a} I_d dt + \int_{t_a}^{t_b} I_e dt) \) (22)

### 3.1. RFID Investment cost

RFID improves the efficiency of a manufacturer including the following efficiencies described as in Lee and Lee (2010), derived as stated below

**Ordering efficiency**

\[\text{OrE} = N_1 + (G_1 - N_1)(e^{(\alpha C_{or})}), \quad 0 \leq N_1 \leq G_1 \leq 1\] (23)

- \( G_1 \) is lowermost efficiency and \( N_1 \) is uppermost efficiency associated with \( C_{or} \).

**JIT efficiency**

\[\text{JiT} = L_1 + (U_1 - L_1)(e^{(\beta C_j)}), \quad 0 \leq L_1 \leq U_1 \leq 1\] (24)

- \( U_1 \) is lowermost efficiency and \( L_1 \) is uppermost efficiency associated with \( C_j \).
Operating efficiency

$$\text{OpE}=E_1+(A_1-E_1)(e^{C_{op}})$$ \hspace{1cm} 0 \leq E_1 \leq A_1 \leq 1 \quad (25)$$

$A_1$ is lowermost efficiency and $E_1$ is uppermost efficiency associated with $C_{op}$.

Bifurcating case 1 on credit period and replenishment cycle length into the subcases.

Subcase 1.1 $0 \leq L_1 \leq M \leq t_a$

The rate of interest charged per year is

$$T_{Ip1}=A_p I_c \int_{t_a}^M t(P-PIE_1\varepsilon-Dt)dt + \int_{t_a}^{t_b} (t_b-t)P_R dt + \int_{t_a}^{t_b} D(t_c-t)dt + \int_{t_a}^{t_c} D(t_b-t)dt + \int_{t_a}^{t_c} (t_b-t)P_R dt \quad (26)$$

The rate of interest earned per year is

$$T_{Ie1} = P_p I_c [M] tDtdt \quad (27)$$

Subcase 1.2 $t_a \leq M \leq t_b$

The rate of interest charged per year is

$$T_{Ip2} = A_p I_c \int_{t_a}^M (Dt(t_c-t_b) - (t_b-t)(P_R-Dt))dt + \int_{t_a}^{t_b} D(t_c-t)dt + \int_{t_a}^{t_b} (t_b-t)P_R dt \quad (28)$$

The rate of interest earned per year is

$$T_{Ie2} = P_p I_c [M] tDtdt \quad (29)$$

Subcase 1.3 $t_b \leq M \leq t_c$

The rate of interest charged per year is

$$T_{Ip3} = A_p I_c \int_{t_a}^{t_b} (Dt(t_c-t))dt \quad (30)$$

The rate of interest earned per year is

$$T_{Ie3} = P_p I_c [M] tDtdt \quad (31)$$

Subcase 1.4 $T < M$

The rate of interest charged per year is

$$T_{Ip4} = 0 \quad (32)$$
The rate of interest earned per year is

\[ t_{e4} = \int_0^M t \, dt + \int_T^{M-T} t \, dt \]  

(33)

Thus, annual total cost per unit time is given by equations (A5) and (A14)

Case 2 \( P_R > \int_0^T D(t) \, dt \) with preservation

Let \( \Theta_o = \Theta_o e^{-\xi u}, 0 \leq \Theta_o \leq 1 \) is the coefficient of deterioration, in case if preservation technology is utilized. The inventory level \( I(t) \) at any time, \( t \), could be calculated by equations (12) to (16) as stated in case 1. With respect to each cost in case-1, a preservation-technology investment cost \( PTI = ut \) is involved for calculating the total cost for the case, presented in appendix equation (A13).

Case 3 \( \int_0^T D(t) \, dt > P_R \) without preservation

Similarly, as in Case 1, the total cost can be stated using Cases 1.1 to 1.4.

Case 4 \( \int_0^T D(t) \, dt > P_R \) with preservation

With respect to each cost in Case-1, the cost associated with preservation-technology investment \( PTI = ut \) is inserted for calculating the total cost of the same case. So, the total cost per unit time in each case as demonstrated in equations (A6) to (A9) in Appendix. Therefore, to minimize the total cost shown in each case, Calculating the below stated partial derivatives and hence, equating them to zero;

\[ \frac{\partial TC}{\partial T} = 0 \]  

(34a)

\[ \frac{\partial TC}{\partial T} = 0 \quad \text{and} \quad \frac{\partial TC}{\partial u} = 0 \]  

(34b)

Only in case of preservation investment technology

In order to test convexity of total cost of obtained set of solutions, we implement following algorithm, Step 1 Allotting the various inventory parameters some specific hypothetical values. Step 2 Calculating the solutions by solving simultaneous equations described in Equation (34a) or (34b), utilizing the mathematical software Maple 18. Step 3 Calculating eigen values of following Hessian matrix \( H \) at the point of optimality, which is obtained from Equation (34a) or (34b),

\[ H = \begin{bmatrix} \frac{\partial^2 TC}{\partial T^2} & \frac{\partial^2 TC}{\partial T \partial u} \\ \frac{\partial^2 TC}{\partial T \partial u} & \frac{\partial^2 TC}{\partial u} \end{bmatrix} \]

- In case, if each and every eigen value of matrix \( H \) is positive, it is a positive-definite matrix. Then, the total cost is a convex down and stop.
4. NUMERICAL EXAMPLES AND SENSITIVITY ANALYSIS

4.1. Numerical Examples

Example 1: Case 3
Considering the specified values:

\[ a = 10000, b = 0.2, c = 0.5, \theta_a = 0.5, h = 20, h_R = 30, P = 20000, C = 20, C_i = 5000, C_r = 8, k = 0.05, P_{IEE} = 80, I_c = 0.15, M = 0.1, I_r = 0.2, A_p = 80, P_p = 80, G_1 = 1, N_1 = 0.3, U_1 = 1, L_1 = 0.3, A_1 = 1, E_1 = 0.5, \alpha = 0.001, \beta = 0.2, \gamma = 0.2, P_R = 1000 \]

Solution:

\[ T = 0.11231, \text{Cor} = 714.2857, \text{Cop} = 4.9807, C_j = 4.9696, Q = 127, \text{Total cost} = 48278.8005, \int_0^T Dtdt = 1133.4013, \text{Therefore}, P_R = 1000 < \int_0^T Dtdt = 1133.4013 \]

Example 2: Case 4
Considering the specified values:

\[ a = 10000, b = 0.2, c = 0.5, \theta_a = 0.5, h = 20, h_R = 30, P = 20000, C = 20, C_i = 5000, C_r = 8, k = 0.05, P_{IEE} = 80, I_c = 0.15, M = 0.1, I_r = 0.2, A_p = 80, P_p = 80, G_1 = 1, N_1 = 0.3, U_1 = 1, L_1 = 0.3, A_1 = 1, E_1 = 0.5, \alpha = 0.001, \beta = 0.2, \gamma = 0.2, P_R = 1000, \xi = 0.3 \]

Solution:

\[ T = 0.1123, u = 2.1981, \text{Cor} = 714.2857, \text{Cop} = 4.9807, C_j = 4.9988, Q = 127, \text{Total cost} = 48276.9109, \int_0^T Dtdt = 1133.6839, \text{Therefore}, P_R = 1000 < \int_0^T Dtdt = 1133.6839 \]

Example 3: Case 1
Considering the specified values:

\[ a = 10000, b = 0.2, c = 0.5, \theta_a = 0.5, h = 20, h_R = 30, P = 20000, C = 20, C_i = 5000, C_r = 8, k = 0.05, P_{IEE} = 80, I_c = 0.15, M = 0.1, I_r = 0.2, A_p = 80, P_p = 80, G_1 = 1, N_1 = 0.3, U_1 = 1, L_1 = 0.3, A_1 = 1, E_1 = 0.5, \alpha = 0.001, \beta = 0.2, \gamma = 0.2, P_R = 1500 \]

Solution:

\[ T = 0.11231, \text{Cor} = 714.2857, \text{Cop} = 4.9807, C_j = 4.9682, Q = 127, \text{Total cost} = 48295.9764, \int_0^T Dtdt = 1132.1932, \text{Therefore}, P_R = 1500 > \int_0^T Dtdt = 1132.1932 \]

Example 4: Case 2
Considering the specified values:

\[ a = 10000, b = 0.2, c = 0.5, \theta_a = 0.5, h = 20, h_R = 30, P = 20000, C = 20, C_i = 5000, C_r = 8, k = 0.05, P_{IEE} = 80, I_c = 0.15, M = 0.1, I_r = 0.2, A_p = 80, P_p = 80, G_1 = 1, N_1 = 0.3, U_1 = 1, L_1 = 0.3, A_1 = 1, E_1 = 0.5, \alpha = 0.001, \beta = 0.2, \gamma = 0.2, P_R = 1500, \xi = 0.3 \]

Solution:

\[ T = 0.1122, u = 2.2824, \text{Cor} = 714.2857, \text{Cop} = 4.9807, C_j = 5.0005, Q = 127, \text{Total cost} = 48293.75516, \int_0^T Dtdt = 1132.5106, \text{Therefore}, P_R = 1500 > \int_0^T Dtdt = 1132.5106 \]

4.2. Convexity of Total Cost function:

It can be observed from the numerical examples that the average total cost is minimum in case 4 with preservation technology. Therefore, by utilizing algo-
rithm, we check convexity of the total cost, as shown in Figure 4, we computing the optimum solution and did going the sensitivity analysis of the decision variables by altering the inventory parameters–20 percentage to 20 percentage for this case only. Figure 3 represents the graph of total cost verses the length of replenishment cycle in case 3 without preservation. Hessian matrix in case 4 with preservation is

$$\begin{bmatrix}
\frac{\partial^2 TC}{\partial T^2} & \frac{\partial^2 TC}{\partial T \partial u} \\
\frac{\partial^2 TC}{\partial u \partial T} & \frac{\partial^2 TC}{\partial u^2}
\end{bmatrix} = \begin{bmatrix}
3.821110255 \times 10^6 & -22.59045311 \\
-22.59045311 & 0.8702763048
\end{bmatrix}$$

Eigen values of the Hessian matrix are $\lambda_1 = 0.87014 > 0$, $\lambda_2 = 3.821110 \times 10^{-6} > 0$

Figure 3: Total Cost vs Cycle length in case 3 without preservation

Figure 4: Convexity of cost function in case-4 with preservation
Figure 5: Comparative study of total cost function of four cases
4.3. Sensitivity Analysis

This section consists of the sensitivity analysis of the optimal inventory policies with respect to various inventory parameters. The values of various decision variables on fluctuating the inventory parameters from case-4 in the range -20 percentage to 20 percentage is demonstrated in Table-1, which extracts the below stated observations:

**Sensitivity analysis of the annual rate of reworking of imperfect products \( (P_R) \)**

With respect to increase in the annual rate of reworking of imperfect products, the level of investment for efficiency in JIT increases by lowering cycle time also there is an increase in the cost of preservation investment and the system's total cost hikes within the interval \([0, T]\).

**Sensitivity analysis of the annual rate of production\( (P) \)**

The variation in annual production rate results in level of investment for efficiency in operating, the level of investment for efficiency in JIT, the ordered quantity, the preservation investment cost decreases and cycle length shorten. Also, with the declination in total demand rate within the interval \([0, T]\), which is a desirable virtue for the system.

**Sensitivity analysis of annual rate of production of imperfect quality \( (PIEE) \)**

With the variation in the annual rate of production of imperfect quality, with the declination in total demand rate within the interval \([0, T]\), the system’s total cost hikes slightly.

**Sensitivity analysis of the production cost per product \( (C) \)**

The variation in production cost per item results in the level of investment for efficiency in operating, increases initially and then start to decrease. The level of investment for efficiency in JIT, ordered quantity, preservation investment cost declines, by shortening the replenishment cycle length. But due to the declination in total demand rate within the interval \([0, T]\), the total cost increases initially and then decrease.

**Sensitivity analysis of Scale demand \( (a) \)**

When scale demand is altered, investment level for JIT efficiency, preservation investment cost decreases. The length of replenishment cycle cut-shorts due to increment in total demand rate within interval \([0, T]\) and total cost rises rapidly with the variation of scale demand.

**Sensitivity analysis of Linear variation of demand with respect to time \( (b) \)**

With the fluctuation in the linear variation of demand with respect to time, the cycle length shortens. The level of investment for efficiency in JIT increases. The preservation investment cost decreases initially and then increase but the system's total cost uplifts in this case with rise of total demand rate within interval \([0, T]\).

**Sensitivity analysis of quadratic variation of demand \( (c) \)**

There is a lengthening of the replenishment cycle length which occurs due to increase in demand rate within the interval \([0, T]\) and in preservation investment cost which decreases the total cost of the system.

There is an increase in level of investment for efficiency in operating, the level of investment for efficiency in JIT increases then decreases slightly, and shortens
cycle length. There is a decrease in preservation investment cost and the total demand rate within the interval and the total cost rises rapidly with the variation of imperfect product’s production percentage.

**Sensitivity analysis of cost of repairing of imperfect quality per item** ($C_r$)

When the cost associated with repairing of imperfect quality is varied the ordered quantity, preservation investment cost, demand rate within the interval decreases, along with the shortening of the length of cycle and So, system's total cost rises.

**Sensitivity analysis of setup cost per item for each production run** ($C_s$)

There is an increment in level of investment for efficiency in ordering, level of investment for efficiency in operating, level of investment for efficiency in JIT, ordered quantity, preservation investment cost, and demand rate within the interval along with the increment in length of cycle and hence, system's total cost rises.

**Sensitivity analysis of the annual holding cost of imperfect products per item** ($h$)

With the variation of cost associated with holding the imperfect products is varied, there is a decrement in investment level for operating efficiency, ordered quantity. The level of investment for efficiency in JIT increases initially and then decrease with shortening of cycle length. The rate of demand within $[0, T]$ interval decreases and So, system's total cost rises.

**Sensitivity analysis of annual holding cost of imperfect items reworked/year** ($h_R$)

When the annual holding cost of imperfect products undergoing reworking process per item is varied, there is a fluctuation in the rate of demand within $[0, T]$ interval. So, system's total cost oscillates.

**Sensitivity analysis of selling price of perfect quality items** ($P_p$)

A reduction is seen with respect to the variation of selling price of perfect quality items in the various inventory parameters like; investment level of operating efficiency, ordered quantity, preservation investment cost, demand rate within the interval $[0, T]$ along with the shrinking of cycle length resulting in the drop of total cost of the system. The level of investment for efficiency in JIT increases.

**Sensitivity analysis of deterioration co-efficient** ($\theta_o$)

The level of investment for efficiency in JIT efficiency, rate of demand within the interval $[0, T]$ decreases with the variation of the deterioration coefficient. The preservation investment cost increases. But the system's total cost hikes.

**Sensitivity analysis of credit period offered by supplier to manufacturer** ($M$)

A reduction is seen with respect to variation of credit period offered by supplier to manufacturer in various inventory parameters like; investment level of operating efficiency, ordered quantity, the investment level for JIT efficiency, preservation investment cost, rate of demand within $[0, T]$ interval along with the shrinking of the length of replenishment cycle resulting in drop of system's total cost.

**Sensitivity analysis of cost associated with purchasing of Material per item** ($A_p$)

A decrement is seen in level of investment for efficiency in operating, level of investment for efficiency in JIT, ordered quantity, preservation-investment cost, the demand rate within the interval $[0, T]$, along with the shortening of length
cle and hence, system’s total cost rises with respect to variation of cost associated with purchasing of material per item.

**Sensitivity analysis of annual rate of interest earned per dollar** ($I_e$)

When the annual rate of interest earned per dollar is varied, there is a decrement in various inventory parameters like; the level of investment for efficiency in operating efficiency, the preservation investment cost, the investment for JIT efficiency, ordered quantity, along with the rate of demand within the $[0, T]$ interval. Hence, the total cost of the system reduces.

**Sensitivity analysis of annual rate of interest charged per dollar** ($I_c$)

All inventory parameters decrease with respect to the variation in annual rate of interest charged per dollar but system’s total cost rises.

**Sensitivity analysis of markup for ordering efficiency** ($\alpha$)

There is a decrease in level of investment for efficiency in ordering, level of investment for efficiency in operating, level of investment for efficiency in JIT, ordered quantity, length of cycle, rate of demand within the interval $[0, T]$. Also, system’s the preservation investment cost decreases then increase total cost drops with markup for ordering efficiency.

**Sensitivity analysis of markup for JIT efficiency** ($\beta$)

When the markup for JIT efficiency is varied, there is a decrement in various inventory parameters like; the investment for JIT efficiency, preservation investment cost along with the rate of market demand within $[0, T]$ interval. Hence, the total cost of the system reduces.

**Sensitivity analysis of markup for operating efficiency** ($\gamma$)

There is a decrease in investment level for operating efficiency, preservation investment cost, the demand rate within the interval $[0, T]$, system’s total cost drops by variation of markup for operating efficiency.

**Sensitivity analysis of markup for preservation investment cost** ($\xi$)

There is a decrease in preservation investment cost, the demand rate within the interval $[0, T]$ and hence, the system’s total cost drops by variation of markup for preservation investment cost.
| Page | Column 1 | Column 2 | Column 3 | Column 4 | Column 5 | Column 6 | Column 7 | Column 8 | Column 9 | Column 10 | Column 11 | Column 12 | Column 13 | Column 14 | Column 15 | Column 16 | Column 17 | Column 18 | Column 19 | Column 20 | Column 21 | Column 22 | Column 23 | Column 24 | Column 25 | Column 26 | Column 27 | Column 28 | Column 29 | Column 30 | Column 31 | Column 32 | Column 33 | Column 34 | Column 35 | Column 36 | Column 37 | Column 38 | Column 39 | Column 40 | Column 41 | Column 42 | Column 43 | Column 44 | Column 45 | Column 46 | Column 47 | Column 48 | Column 49 | Column 50 | Column 51 | Column 52 | Column 53 | Column 54 | Column 55 | Column 56 | Column 57 | Column 58 | Column 59 | Column 60 | Column 61 | Column 62 | Column 63 | Column 64 | Column 65 | Column 66 | Column 67 | Column 68 | Column 69 | Column 70 | Column 71 | Column 72 | Column 73 | Column 74 | Column 75 | Column 76 | Column 77 | Column 78 | Column 79 | Column 80 | Column 81 | Column 82 | Column 83 | Column 84 | Column 85 | Column 86 | Column 87 | Column 88 | Column 89 | Column 90 | Column 91 | Column 92 | Column 93 | Column 94 | Column 95 | Column 96 | Column 97 | Column 98 | Column 99 | Column 100 |
| Inv Par. Decision Var. | h | Cor | 714.2857 | 714.2857 | 714.2857 | 714.2857 | 714.2857 |
|------------------------|---|------|-----------|-----------|-----------|-----------|-----------|
| g | Cop | 4.9890 | 4.9890 | 4.9890 | 4.9890 | 4.9890 |
| 120 | 120 | 120 | 120 | 120 |
| 1 | 0.1128 | 0.1128 | 0.1128 | 0.1128 | 0.1128 |
| 0 | 3.2618 | 2.0794 | 2.1983 | 2.3663 | 2.5758 |
| TC | 48136.1150 | 48206.4178 | 48276.9042 | 48347.4320 | 48416.8089 |

| op | Cop | 4.9890 | 4.9890 | 4.9890 | 4.9890 | 4.9890 |
| 120 | 120 | 120 | 120 | 120 |
| 1 | 0.1128 | 0.1128 | 0.1128 | 0.1128 | 0.1128 |
| 0 | 3.2618 | 2.0794 | 2.1983 | 2.3663 | 2.5758 |
| TC | 48136.1150 | 48206.4178 | 48276.9042 | 48347.4320 | 48416.8089 |

| Inv Par. Decision Var. | h | Cop | 4.9980 | 4.9980 | 4.9980 | 4.9980 | 4.9980 |
|------------------------|---|-----|-----------|-----------|-----------|-----------|-----------|
| g | Cop | 4.9890 | 4.9890 | 4.9890 | 4.9890 | 4.9890 |
| 120 | 120 | 120 | 120 | 120 |
| 1 | 0.1128 | 0.1128 | 0.1128 | 0.1128 | 0.1128 |
| 0 | 3.2618 | 2.0794 | 2.1983 | 2.3663 | 2.5758 |
| TC | 48136.1150 | 48206.4178 | 48276.9042 | 48347.4320 | 48416.8089 |

| op | Cop | 4.9980 | 4.9980 | 4.9980 | 4.9980 | 4.9980 |
| 120 | 120 | 120 | 120 | 120 |
| 1 | 0.1128 | 0.1128 | 0.1128 | 0.1128 | 0.1128 |
| 0 | 3.2618 | 2.0794 | 2.1983 | 2.3663 | 2.5758 |
| TC | 48136.1150 | 48206.4178 | 48276.9042 | 48347.4320 | 48416.8089 |

| Inv Par. Decision Var. | h | Cop | 4.9980 | 4.9980 | 4.9980 | 4.9980 | 4.9980 |
|------------------------|---|-----|-----------|-----------|-----------|-----------|-----------|
| g | Cop | 4.9890 | 4.9890 | 4.9890 | 4.9890 | 4.9890 |
| 120 | 120 | 120 | 120 | 120 |
| 1 | 0.1128 | 0.1128 | 0.1128 | 0.1128 | 0.1128 |
| 0 | 3.2618 | 2.0794 | 2.1983 | 2.3663 | 2.5758 |
| TC | 48136.1150 | 48206.4178 | 48276.9042 | 48347.4320 | 48416.8089 |
Table 1: Sensitivity analysis of optimal variables with respect to various inventory parameters
5. CONCLUSION AND FUTURE SCOPE

This article proposes an inventory model based on radio frequency identification adoption (RFID), reworking of imperfect products, and trade-credit for deteriorating inventory with / without utilizing the preservation investment technology. Estimation of the optimal values of RFID levels of investment for efficiencies in ordering, operating, just-in-time along with production cycle time and preservation investment. The demand function fluctuates with respect to time. The classical optimization technique is utilized for calculating the optimal values. We demonstrated validity of the developed models on numerical examples. Then using the concept of eigen-values of a Hessian matrix, convexity of the system's total cost for the case 4: $\int_{0}^{T} Dtdt > P_{R}$ with preservation, which has the system's minimum total cost. Also, the sensitivity analysis of optimal variables is done by fluctuating the inventory parameters for generating fruitful managerial insights for this case. Also, some possible future directions for research related to this model are: 1. To reduce the system's total cost efforts for investments in advertisement and/or servicing can be utilized. 2. Learning-effects and/or some discounts on purchasing price may be considered. 3. Shortages can be considered.

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6. APPENDICES

Appendix 1: The levels of inventory and system’s total-cost in Case 1 and Case 3 (Without Preservation)
Let
\[ I_4(t) = \begin{cases} \frac{2 acta^{3} - 3 tbca^{2} + 6 cota - 6 P - 6 P R E}{-6} & 0 \leq t \leq T \text{ (A1)} \\ \frac{2acta^{3} - 3 tbca^{2} + 6 cota + 6 P - 6 P R E}{-6} & 0 \leq t \leq t_2 \text{ (A2)} \\ \frac{2acta^{3} - 3 tbca^{2} + 6 cota + 6 P - 6 P R E}{-6} & 0 \leq t \leq t_3 \text{ (A3)} \\ \frac{2acta^{3} - 3 tbca^{2} + 6 cota + 6 P - 6 P R E}{-6} & 0 \leq t \leq t_4 \text{ (A4)} \\ \end{cases} \]

\[ I_5(t) = \begin{cases} \frac{2acta^{3} - 3 tbca^{2} + 6 cota + 6 P - 6 P R E}{-6} & 0 \leq t \leq T \text{ (A1)} \\ \frac{2acta^{3} - 3 tbca^{2} + 6 cota + 6 P - 6 P R E}{-6} & 0 \leq t \leq t_2 \text{ (A2)} \\ \frac{2acta^{3} - 3 tbca^{2} + 6 cota + 6 P - 6 P R E}{-6} & 0 \leq t \leq t_3 \text{ (A3)} \\ \frac{2acta^{3} - 3 tbca^{2} + 6 cota + 6 P - 6 P R E}{-6} & 0 \leq t \leq t_4 \text{ (A4)} \\ \end{cases} \]

For \( i = 1, 2, 3, 4 \)

\[ T C_{C21} = \begin{cases} \frac{1}{2} \left( \frac{1}{2} \int_{0}^{M} (P - P R E - D i) d i + \frac{1}{2} \int_{0}^{M} (P + P R E - D i) d i \right) & \text{ (A5)} \\ \frac{1}{2} \left( \int_{0}^{M} (P - P R E - D i) d i + \int_{0}^{M} (P + P R E - D i) d i \right) & \text{ (A6)} \\ \frac{1}{2} \left( \int_{0}^{M} (P - P R E - D i) d i + \int_{0}^{M} (P + P R E - D i) d i \right) & \text{ (A7)} \\ \frac{1}{2} \left( \int_{0}^{M} (P - P R E - D i) d i + \int_{0}^{M} (P + P R E - D i) d i \right) & \text{ (A8)} \\ \frac{1}{2} \left( \int_{0}^{M} (P - P R E - D i) d i + \int_{0}^{M} (P + P R E - D i) d i \right) & \text{ (A9)} \\ \end{cases} \]

Appendix 3: The inventory levels in Case 2 and Case 4.
\[ \text{(Without Preservation)} \]

\[ \text{C}_{\text{R}} = \frac{1}{\alpha} \left( A \right) \]

\[ \text{C}_{\text{RP}} = \frac{1}{\alpha \gamma} \left( \alpha \gamma \right) \]

\[ \text{Let} \]

\[ (\text{−} \beta_1 + \alpha_1 + 1) \]

\[ \text{S}_1 = \begin{pmatrix} 2aT^2 Y P & 2aT^2 Y P & 2aT^2 Y P & 2aT^2 Y P \\ 2aT^2 Y P & 2aT^2 Y P & 2aT^2 Y P & 2aT^2 Y P \\ 2aT^2 Y P & 2aT^2 Y P & 2aT^2 Y P & 2aT^2 Y P \end{pmatrix} \]

\[ \text{Let} \]

\[ (\text{−} \beta_1 + \alpha_1 + 1) \]

\[ \text{S}_2 = \begin{pmatrix} (2T^3 a_T - 3T^2 a_T - 3a_T) & 2aT^4 Y P & 2aT^4 Y P & 2aT^4 Y P \\ 2aT^4 Y P & 2aT^4 Y P & 2aT^4 Y P & 2aT^4 Y P \\ 2aT^4 Y P & 2aT^4 Y P & 2aT^4 Y P & 2aT^4 Y P \end{pmatrix} \]

\[ \text{Let} \]

\[ (\text{−} \beta_1 + \alpha_1 + 1) \]

\[ \text{S}_3 = \begin{pmatrix} (2T^3 a_T - 3T^2 a_T - 3a_T) & 2aT^4 Y P & 2aT^4 Y P & 2aT^4 Y P \\ 2aT^4 Y P & 2aT^4 Y P & 2aT^4 Y P & 2aT^4 Y P \\ 2aT^4 Y P & 2aT^4 Y P & 2aT^4 Y P & 2aT^4 Y P \end{pmatrix} \]

\[ \text{Let} \]

\[ (\text{−} \beta_1 + \alpha_1 + 1) \]

\[ \text{S}_4 = \begin{pmatrix} (2T^3 a_T - 3T^2 a_T - 3a_T) & 2aT^4 Y P & 2aT^4 Y P & 2aT^4 Y P \\ 2aT^4 Y P & 2aT^4 Y P & 2aT^4 Y P & 2aT^4 Y P \\ 2aT^4 Y P & 2aT^4 Y P & 2aT^4 Y P & 2aT^4 Y P \end{pmatrix} \]
\[
S_5 = \begin{pmatrix}
+6e & \frac{1}{2} \left( \frac{1}{p} \right)^2 \frac{12}{T_2} \frac{1}{y} \frac{1}{v} \left( \frac{1}{p} \frac{1}{h} \right)^2 \frac{1}{v} \\
\frac{1}{2} \left( \frac{1}{p} \right)^2 \frac{1}{y} \frac{1}{v} \left( \frac{1}{p} \frac{1}{h} \right)^2 \frac{1}{v} & 6 \frac{1}{T_2} \frac{1}{y} \frac{1}{v} \left( \frac{1}{p} \frac{1}{h} \right)^2 \frac{1}{v}
\end{pmatrix}
\]

Appendix 1: The investment level for JIT efficiency in Case 2/Case 4

(With Preservation)

\[ C = \frac{1}{p} - \left( \frac{1}{p} \right)^2 \frac{12}{T_2} \frac{1}{y} \frac{1}{v} \left( \frac{1}{p} \frac{1}{h} \right)^2 \frac{1}{v}\]

Where, \( R = b(1/2) \left( \frac{1}{h} \right) \left( \frac{1}{h} \right)^2 - 2 u \xi U_1 + U_2 \)

\[ U_2 = U_{21} + U_{22} + U_{23} + U_{24} \]

\[ U_1 = \begin{pmatrix}
\frac{4b(T^2)}{p^3} \frac{1}{2} \left( \frac{1}{p} \right)^2 \frac{1}{y} \frac{1}{v} \left( \frac{1}{p} \frac{1}{h} \right)^2 \frac{1}{v} & \frac{4b(T^2)}{p^2} \frac{1}{2} \left( \frac{1}{p} \right)^2 \frac{1}{y} \frac{1}{v} \left( \frac{1}{p} \frac{1}{h} \right)^2 \frac{1}{v}
\end{pmatrix}
\]

\[ U_{21} = \begin{pmatrix}
\frac{u_2^2}{p^3} \frac{1}{2} \left( \frac{1}{p} \right)^2 \frac{1}{y} \frac{1}{v} \left( \frac{1}{p} \frac{1}{h} \right)^2 \frac{1}{v} & \frac{u_2^2}{p^2} \frac{1}{2} \left( \frac{1}{p} \right)^2 \frac{1}{y} \frac{1}{v} \left( \frac{1}{p} \frac{1}{h} \right)^2 \frac{1}{v}
\end{pmatrix}
\]

\[ U_{22} = \begin{pmatrix}
\frac{u_2^2}{p^3} \frac{1}{2} \left( \frac{1}{p} \right)^2 \frac{1}{y} \frac{1}{v} \left( \frac{1}{p} \frac{1}{h} \right)^2 \frac{1}{v} & \frac{u_2^2}{p^2} \frac{1}{2} \left( \frac{1}{p} \right)^2 \frac{1}{y} \frac{1}{v} \left( \frac{1}{p} \frac{1}{h} \right)^2 \frac{1}{v}
\end{pmatrix}
\]
\[
\begin{align*}
&\left[\frac{u^2 a^2 b^2 \text{TY}}{2PR} + \frac{1}{2} \text{TY} \left(\frac{1}{k} + \frac{k}{PR}\right) \right] \frac{2}{2} \frac{2}{2} \left[\right. \\
&+ 3 PR \left(\frac{1}{k} - \frac{1}{PR}\right) \left[-\frac{1}{2} \left(\frac{1}{k} + \frac{k}{PR}\right) \right] \left[\frac{1}{PR} \right] ^3 \\
&\left. + \frac{12 a^2 b^2 \text{TY} \left(\frac{1}{k} + \frac{k}{PR}\right) \text{TY} \left(\frac{1}{k} + \frac{k}{PR}\right)}{2PR} \right] \frac{2}{2} \frac{2}{2} \left[\right. \\
&+ 3 PR \left(\frac{1}{k} - \frac{1}{PR}\right) \left[-\frac{1}{2} \left(\frac{1}{k} + \frac{k}{PR}\right) \right] \left[\frac{1}{PR} \right] ^3 \\
&\left. + \frac{12 a^2 b^2 \text{TY} \left(\frac{1}{k} + \frac{k}{PR}\right) \text{TY} \left(\frac{1}{k} + \frac{k}{PR}\right)}{2PR} \right] \frac{2}{2} \frac{2}{2} \left[\right. \\
&+ 3 PR \left(\frac{1}{k} - \frac{1}{PR}\right) \left[-\frac{1}{2} \left(\frac{1}{k} + \frac{k}{PR}\right) \right] \left[\frac{1}{PR} \right] ^3 \\
&\left. + \frac{12 a^2 b^2 \text{TY} \left(\frac{1}{k} + \frac{k}{PR}\right) \text{TY} \left(\frac{1}{k} + \frac{k}{PR}\right)}{2PR} \right] \frac{2}{2} \frac{2}{2} \left[\right. \\
&+ 3 PR \left(\frac{1}{k} - \frac{1}{PR}\right) \left[-\frac{1}{2} \left(\frac{1}{k} + \frac{k}{PR}\right) \right] \left[\frac{1}{PR} \right] ^3 \\
&\left. + \frac{12 a^2 b^2 \text{TY} \left(\frac{1}{k} + \frac{k}{PR}\right) \text{TY} \left(\frac{1}{k} + \frac{k}{PR}\right)}{2PR} \right] \frac{2}{2} \frac{2}{2} \left[\right. \\
&+ 3 PR \left(\frac{1}{k} - \frac{1}{PR}\right) \left[-\frac{1}{2} \left(\frac{1}{k} + \frac{k}{PR}\right) \right] \left[\frac{1}{PR} \right] ^3 \\
&\left. + \frac{12 a^2 b^2 \text{TY} \left(\frac{1}{k} + \frac{k}{PR}\right) \text{TY} \left(\frac{1}{k} + \frac{k}{PR}\right)}{2PR} \right] \frac{2}{2} \frac{2}{2} \left[\right. \\
&+ 3 PR \left(\frac{1}{k} - \frac{1}{PR}\right) \left[-\frac{1}{2} \left(\frac{1}{k} + \frac{k}{PR}\right) \right] \left[\frac{1}{PR} \right] ^3 \\
&\left. + \frac{12 a^2 b^2 \text{TY} \left(\frac{1}{k} + \frac{k}{PR}\right) \text{TY} \left(\frac{1}{k} + \frac{k}{PR}\right)}{2PR} \right] \frac{2}{2} \frac{2}{2} \left[\right. \\
&+ 3 PR \left(\frac{1}{k} - \frac{1}{PR}\right) \left[-\frac{1}{2} \left(\frac{1}{k} + \frac{k}{PR}\right) \right] \left[\frac{1}{PR} \right] ^3 \\
&\left. + \frac{12 a^2 b^2 \text{TY} \left(\frac{1}{k} + \frac{k}{PR}\right) \text{TY} \left(\frac{1}{k} + \frac{k}{PR}\right)}{2PR} \right] \frac{2}{2} \frac{2}{2} \left[\right. \\
&+ 3 PR \left(\frac{1}{k} - \frac{1}{PR}\right) \left[-\frac{1}{2} \left(\frac{1}{k} + \frac{k}{PR}\right) \right] \left[\frac{1}{PR} \right] ^3 \\
&\left. + \frac{12 a^2 b^2 \text{TY} \left(\frac{1}{k} + \frac{k}{PR}\right) \text{TY} \left(\frac{1}{k} + \frac{k}{PR}\right)}{2PR} \right] \frac{2}{2} \frac{2}{2} \left[\right. \\
&+ 3 PR \left(\frac{1}{k} - \frac{1}{PR}\right) \left[-\frac{1}{2} \left(\frac{1}{k} + \frac{k}{PR}\right) \right] \left[\frac{1}{PR} \right] ^3 \\
&\left. + \frac{12 a^2 b^2 \text{TY} \left(\frac{1}{k} + \frac{k}{PR}\right) \text{TY} \left(\frac{1}{k} + \frac{k}{PR}\right)}{2PR} \right] \frac{2}{2} \frac{2}{2} \left[\right. \\
&+ 3 PR \left(\frac{1}{k} - \frac{1}{PR}\right) \left[-\frac{1}{2} \left(\frac{1}{k} + \frac{k}{PR}\right) \right] \left[\frac{1}{PR} \right] ^3 \\
&\left. + \frac{12 a^2 b^2 \text{TY} \left(\frac{1}{k} + \frac{k}{PR}\right) \text{TY} \left(\frac{1}{k} + \frac{k}{PR}\right)}{2PR} \right] \frac{2}{2} \frac{2}{2} \left[\right. \\
&+ 3 PR \left(\frac{1}{k} - \frac{1}{PR}\right) \left[-\frac{1}{2} \left(\frac{1}{k} + \frac{k}{PR}\right) \right] \left[\frac{1}{PR} \right] ^3 \\
&\left. + \frac{12 a^2 b^2 \text{TY} \left(\frac{1}{k} + \frac{k}{PR}\right) \text{TY} \left(\frac{1}{k} + \frac{k}{PR}\right)}{2PR} \right] \frac{2}{2} \frac{2}{2} \left[\right. \\
&+ 3 PR \left(\frac{1}{k} - \frac{1}{PR}\right) \left[-\frac{1}{2} \left(\frac{1}{k} + \frac{k}{PR}\right) \right] \left[\frac{1}{PR} \right] ^3 \\
&\left. + \frac{12 a^2 b^2 \text{TY} \left(\frac{1}{k} + \frac{k}{PR}\right) \text{TY} \left(\frac{1}{k} + \frac{k}{PR}\right)}{2PR} \right] \frac{2}{2} \frac{2}{2} \left[\right. \\
&+ 3 PR \left(\frac{1}{k} - \frac{1}{PR}\right) \left[-\frac{1}{2} \left(\frac{1}{k} + \frac{k}{PR}\right) \right] \left[\frac{1}{PR} \right] ^3 \\
&\left. + \frac{12 a^2 b^2 \text{TY} \left(\frac{1}{k} + \frac{k}{PR}\right) \text{TY} \left(\frac{1}{k} + \frac{k}{PR}\right)}{2PR} \right] \frac{2}{2} \frac{2}{2} \left[\right. \\
&+ 3 PR \left(\frac{1}{k} - \frac{1}{PR}\right) \left[-\frac{1}{2} \left(\frac{1}{k} + \frac{k}{PR}\right) \right] \left[\frac{1}{PR} \right] ^3 \\
&\left. + \frac{12 a^2 b^2 \text{TY} \left(\frac{1}{k} + \frac{k}{PR}\right) \text{TY} \left(\frac{1}{k} + \frac{k}{PR}\right)}{2PR} \right] \frac{2}{2} \frac{2}{2} \left[\right."
