CHIRAL SYMMETRY ASPECTS OF THE SCALARS

B. MOUSSALLAM and J. STERN

Division de Physique Théorique, Institut de Physique Nucléaire
Université Paris-Sud, 91406, Orsay, France

ABSTRACT

Electromagnetic decays of the scalar mesons are shown to be constrained by chiral symmetry as a consequence of the fact that, in the chiral limit, the two and three-point functions \( <SS-PP> \) and \( <VVS> \) satisfy super-convergent dispersion relations. The QCD asymptotic behavior of the latter is canonical and it can be saturated by a finite number of resonances. The corresponding chiral lagrangian for vector and scalar resonances is constructed. Matching to the correct asymptotic structure generates non-minimal terms which have so far been ignored. It is found that the width \( a_0(980) \rightarrow 2\gamma \) can be naturally reproduced suggesting that the \( a_0(980) \) is not an exotic particle.

1- Introduction:

The chiral symmetry limit of QCD, which is expected to be a reasonably good approximation in the light quark sector, may be at the origin of some of the surprising (at first sight) properties that the scalar mesons seem to have. We will concentrate here on the question of the two photon width of the \( a_0(980) \) (since this is a gamma-gamma meeting) and it will be shown that the fact that this width is anomalously small (as compared, for instance, with the expectation from the naive quark model) has a natural explanation in this limit, and is by no means incompatible with a standard nonet status for the \( a_0 \).

Our starting point is to assume exact chiral symmetry (i.e. we set \( m_u = m_d = m_s = 0 \) in the lagrangian) and, furthermore, we will work in the leading \( N_c \) approximation. In these limits, QCD still cannot be solved in closed form, but analytic approximation methods, at least, are available. The one that we will use is similar to the boundary-layer technique used, for instance, in hydrodynamics and consists in matching three asymptotic expansions for a given correlation function: a) For large momenta, the operator product expansion (OPE) generates an expansion in inverse powers of the momenta b) for small or moderate momenta the correlation function is dominated by the contributions from the light resonances and c) at very low momenta the expansion is generated by chiral perturbation theory (\( \chi PT \)). The method is effective only for those n-point functions which transform in a non trivial way under the chiral group and thereby satisfy super-convergent dispersion relations. Only in that case is it possible to match the QCD asymptotic behavior with a finite

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2 Unité de Recherche des Universités Paris 11 et Paris 6 associée au CNRS
number of resonances. This method has been used in the past for a diversity of applications, for the first time by Weinberg[1] who obtained an estimate for the mass of the $a_1$ resonance. Das et al[2] have evaluated in this way the $\pi^+ - \pi^0$ electromagnetic mass splitting. More recently, similar ideas where discussed in the context of the effective lagrangian for resonances[3] and an interesting application to the question of whether the mass of the $u$ quark can be zero can be found in ref.[4].

### 2- Meson spectroscopy:

In the exact chiral limit, the spectrum of the lightest resonances (mesons as well as baryons) in the QCD spectrum is expected to consist of exactly degenerate octets (which, for the mesons, become nonets in the leading $N_c$ approximation). This is not what is observed in practice, of course, but already first order perturbation theory in terms of the quark mass hamiltonian predicts a pattern of splitting, ideal mixing, Gell-Mann-Okubo mass formulas, which is satisfied, in practice, to a fairly good approximation. The fact that the pion nonet does not obey ideal mixing is due to the axial anomaly and is well understood in terms of the large $N_c$ expansion and $\chi$pt techniques (e.g. [5]). The only major exception to these rules seem to be the scalar resonances. The present candidates to form a nonet that one can find in the review of particle properties[6] are:

$$
\begin{align*}
I = 1 : & \quad a_0(980), \\
I = 1/2 : & \quad K_0^*(1430) \\
I = 0 : & \quad f_0(975), \\
I = 0 : & \quad f_0(1400)
\end{align*}
$$

(1)

The decay properties of the $f_0(975)\) and the $f_0(1400)$ strongly suggest that the flavor transformation properties of the former is essentially like $s\bar{s}$ and the latter is $u\bar{u} + d\bar{d}$. The pattern of masses then, clearly, is a complete mystery. Furthermore, it has been argued recently that the $f_0(1400)$ should be renormalized down to a very wide $f_0(1000)$[7] which would almost certainly imply that $f_0(975)$ is an exotic. This interpretation requires that one identifies the $s\bar{s}$ member of the nonet which should, if GMO is approximately respected in this interpretation, lie somewhere in the 1.8 GeV range (note that the $\theta(1700)$ is a possible candidate). At any rate, it seems important to assess whether the $I=1$ meson $a_0(980)$ is a member of the nonet or an exotic particle. One of its properties, which was interpreted as an indication for an exotic status is the electromagnetic width[8](see [8] for a critical discussion of the determination of these widths):

$$
\Gamma(a_0(980) \to 2\gamma) = (0.24 \pm 0.08)/Rate(\pi\eta)\quad \text{KeV}
$$

(2)

which depends on the unknown $\pi\eta$ rate but, clearly, since this is the only open decay channel, one does not expect it to be much smaller than one even though some amount of $KK$ decay is certainly observed. In order to appreciate the peculiarity of this result, one may compare it with the quark model calculation[9]:

$$
\Gamma(a_0 \to 2\gamma) \simeq 1.6\quad \text{KeV}
$$

(3)
It is of theoretical interest to understand why the quark model fails by so much in this case. We will argue that the amplitude for this decay has to satisfy rather stringent chiral symmetry constraints (certainly a surprising fact, at first sight, since no pion is involved in that reaction) and this is ultimately why the naive quark model runs into trouble.

3- Chiral correlation functions:

We intend to use the asymptotic matching technique for the three-point function made out of two vector and one scalar current $<SVV>$ which is obviously related to the decay amplitude of interest. In order to determine some of the coupling constants we must discuss also the two-point function $<SS-PP>$. Let us define these objects in a more precise way. The vector, scalar and pseudoscalar currents, first, are defined as follows:

$$j_a^\mu(x) = \bar{\psi}_\lambda a \frac{\lambda^\mu}{2} \psi,$$
$$j_s^a(x) = \bar{\psi}_\lambda a \frac{\lambda^s}{2} \psi,$$
$$j_p^a(x) = i \bar{\psi}_\lambda a \frac{\lambda^5}{2} \psi.$$  

The couplings of these currents to the resonances which will play a role in what follows are then defined (in the chiral limit) as:

$$<0|j_a^\mu(0)|V^b(p,\lambda)> = \delta^{ab} M_V F_V \epsilon_\mu(\lambda),$$
$$<0|j_s^a(0)|S^b(p)> = \delta^{ab} M_S G_S,$$
$$<0|j_p^a(0)|\pi^b(p)> = \delta^{ab} B_0 F_0,$$
$$<0|j_p^a(0)|P^b(p)> = \delta^{ab} M_P G_P.$$  

Here, $F_0$ is the pion decay constant and $B_0 = -<\bar{u}u>/F_0^2$, which are standard notations in $\chi$pt. Note that in the pseudo-scalar channel we will take into account the pseudo-scalar resonance multiplet ($\pi'(1300)$) in addition to the pion multiplet, since it lies in the same mass range as the vector or scalar resonances. Finally, the definition of the two and three-point functions is as follows:

$$\Pi_{SP}^{ab}(p) = i \int \exp(ipx) <T(j_a^b(x)j_s^b(0) - j_p^b(x)j_P^b(0))>$$
$$V_{\alpha\beta}^{abc}(p, q) = \int d^4 x d^4 y \exp(ipx + iqy) <Tj_a^\alpha(x)j_b^\beta(y)j_c^c(0)>.$$  

Let us begin by considering the two-point function $\Pi_{SP}$ in three successive energy ranges:

a) $p \to \infty$:

Chiral symmetry implies that the operators which occur in the OPE transform in the same way as the operator $SS-PP$. The important point is that this operator transforms non-trivially under the chiral group (like $(3, \bar{3}) + (\bar{3}, 3) + ...$). In addition there is a discrete symmetry $\psi_L \to -\psi_L$, which implies that the operator of lowest dimensionality in the OPE must be of the kind $\bar{\psi}_L \psi_R \bar{\psi}_L \psi_R$. Ignoring anomalous dimensions here, dimensional analysis shows that for large scales:

$$\lim_{\lambda \to \infty} \Pi_{SP}^{ab}(\lambda p) \sim \frac{1}{\lambda^4}.$$  

b) $p \leq 1 - 2 \text{GeV}$

In this region, we assume that the lowest lying set of resonances dominate the two-point function. In the leading $N_c$ approximation the only singularities are resonance poles so that, using the definition of the coupling constants (5), one obtains the following representation:

$$
\Pi_{SP}^{ab}(p) = \delta^{ab} \left\{ \frac{B_0^2 F_0^2}{p^2} + \frac{M_P^2 G_P^2}{p^2 - M_P^2} - \frac{M_S^2 G_S^2}{p^2 - M_S^2} \right\}
$$

(8)

Now in the very low energy region,
c) $p << 1 \text{ GeV}$:

Chiral perturbation theory controls the expansion of the correlation functions. Up to one loop, one obtains

$$
\Pi_{SP}^{ab}(p) = \delta^{ab} F_0^2 \left\{ \frac{B_0^2}{p^2} - \frac{5B_0^2}{96\pi^2 F_0^2} \ln \frac{-p^2}{\mu^2} + A_0(\mu) + \ldots \right\}
$$

(9)

We notice that, since $F_0^2 = O(N_c)$, the second contribution which is generated by the pion loop drops out in the leading $N_c$ approximation and, in this limit, $A_0(\mu)$ is a scale independent low-energy parameter. It is not difficult to determine this parameter which controls, for instance, the expansion of the pseudo-Goldstone boson masses in powers of the quark masses. Up to and including quadratic order, the expansion reads

$$
F_\pi^2 M_\pi^2 = 2\hat{m} B_0 F_0^2 + 4\hat{m}^2 A_0 F_0^2 + \ldots
$$

$$
F_K^2 M_K^2 = (r + 1)\hat{m} B_0 F_0^2 + (r + 1)^2 \hat{m}^2 A_0 F_0^2 + \ldots
$$

(10)

where

$$
r = \frac{m_s}{\hat{m}}
$$

(11)

Note that at leading $N_c$ order loops are suppressed and consequently, the expansion is analytic. Chpt also allows one to determine the relation between $F_0$ and $F_\pi$ to the same level of accuracy:

$$
\frac{F_\pi^2}{F_0^2} = 1 + \frac{2}{r - 1} \left( \frac{F_K^2}{F_\pi^2} - 1 \right)
$$

(12)

from which one can then solve for $\hat{m} B_0$ and $\hat{m}^2 A_0$ in terms of the quark mass ratio $r$ and the well known quantities $M_\pi$, $M_K$, $F_\pi$, $F_K$. Now the value of $r$ is customarily assumed to be $r \approx 26$, an estimate which may be obtained from (10) by dropping the quadratic quark mass terms. However, an analysis of the violation of the Goldberger-Treiman relation in the baryon sector gives instead $6.3 \leq r \leq 10[10]$. Here, we will assume that $r$ is a free parameter that can vary in the whole range between 6.3 and 26 and check, eventually, the sensitivity of our results upon that.
Now, let us perform the matchings. Matching region b) with region c) gives us one relation:

$$G_S^2 - G_P^2 = A_0 F_0^2$$  \hspace{1cm} (13)

Then matching region b) with region a) gives a second relation:

$$M_S^2 G_S^2 = M_P^2 G_P^2 + B_0^2 F_0^2$$  \hspace{1cm} (14)

which is exactly similar to Weinberg’s first relation[1]. Since we can calculate $A_0$ and $B_0$ (or rather $\hat{m} B_0$ and $\hat{m}^2 A_0$ which are RG invariant combinations) as a function of $r$ from eqs.(14), we can now determine completely the two coupling constants $G_S$ and $G_P$ (note that only the RG invariant combinations $\hat{m} G$ actually occur in physical quantities) which will be needed in the sequel. Before we move to that let us show that, already, a non-trivial constraint concerning the scalar meson spectrum emerges from the two relations (13)-(14):

Relation (14) implies that one can parametrize $G_S$ and $G_P$ in terms of an angle $\theta$

$$M_S G_S = \frac{B_0 F_0}{\cos \theta} \quad M_P G_P = B_0 F_0 \tan \theta$$  \hspace{1cm} (15)

Using the first relation we can solve for $\theta$ in the following form:

$$\cos^2 \theta = \frac{M_P^2}{M_S^2} - 1 \quad \text{where} \quad M_0^2 = \frac{\hat{m}^2 B_0^2}{\hat{m}^2 A_0}$$  \hspace{1cm} (16)

Since $\cos^2 \theta$ must be positive and smaller than one, $M_0, M_P$ and $M_S$ must satisfy one of the two inequalities:

$$M_P \geq M_S \geq M_0 \quad \text{or} \quad M_0 \geq M_S \geq M_P$$  \hspace{1cm} (17)

Now $M_0$ is a function of $r$ which is easily seen to be increasing and reaches $M_0 = 980$ MeV for $r = 29.5$. This means that for any reasonable value of $r$ the first inequality must hold i.e $M_P \geq M_S$. Since $M_P \simeq M_{\pi'} = 1.2 - 1.3$ GeV this inequality is compatible with the assumption that $M_S = M_{a_0} = 980$ MeV. On the contrary, it would be violated if $a_0$ is assumed to be an exotic and the true nonet mass were assumed to be 1.4 GeV.

Let us now turn to the VVS correlation function and, again, go through the three energy regimes.

a) $p, q \to \infty$:

It is a simple matter to check that for $\lambda \to \infty$, $V_{\alpha \beta}(\lambda p, \lambda q)$ scales as $1/\lambda^2$. In addition, the leading term of the OPE is the scalar current $\bar{\psi} \psi$ and, consequently, the corresponding Wilson coefficient does not carry any anomalous dimension. This
means that the asymptotic behavior is exactly $1/\lambda^2$, and one can as well impose matching to the full Lorentz structure of the three-point function $V_{\alpha\beta}$. This Lorentz structure can be found from the QCD tree graphs to be:

$$V_{\alpha\beta}^{abc}(p, q)_{p, q \to \infty} \sim d_{abc} \frac{B_0 F_0^2}{4p^2q^2r^2} \{ (p^2 - q^2 + r^2)(p^2 - q^2 - r^2)\eta_{\alpha\beta} + 4q^2p_{\alpha}p_{\beta} + 4p^2q_{\alpha}q_{\beta} + 2(p^2 + q^2 + r^2)p_{\alpha}p_{\beta} \} (1 + O(\alpha_s))$$

where $r = -(p + q)$. Now in the resonance region:

b) $p, q \leq 1 - 2$ GeV

We will include the poles corresponding to the lowest lying vector and scalar nonets. The $VVS$ Green’s function is transverse $p^\alpha V_{\alpha\beta} = q^\beta V_{\alpha\beta} = 0$. It can be expressed in terms of the two independent tensors with this property:

$$P_{\alpha\beta}^{(2)} = p_{\beta}q_{\alpha} - p.q\eta_{\alpha\beta} \quad \text{and} \quad P_{\alpha\beta}^{(4)} = q^2p_{\alpha}p_{\beta} + p^2q_{\alpha}q_{\beta} - p.q p_{\alpha}q_{\beta} - p^2q^2\eta_{\alpha\beta}$$

The most general form of the amplitude which matches with the asymptotic behavior (18) is the following:

$$V_{\alpha\beta}^{abc} = \frac{d_{abc}}{(p^2 - M_1^2)(q^2 - M_1^2)(r^2 - M_1^2)} \left\{ B_0 F_0^2 \left( \frac{1}{2}(p^2 + q^2 + r^2)P_{\alpha\beta}^{(2)} + P_{\alpha\beta}^{(4)} \right) + a P_{\alpha\beta}^{(2)} \right\}$$

One observes that the expression contains a single arbitrary parameter: $a$. As before, it can be determined by matching with the low-energy regime:

c) $p, q \to 0$

where (forgetting about subleading pion loops) $VVS$ has the following expansion:

$$V_{\alpha\beta}^{abcd}(p, q) = -\frac{9c}{5\hat{m}} d_{abc}(p_{\beta}q_{\alpha} - p.q\eta_{\alpha\beta}) + \ldots$$

One may recognize here the low-energy parameter $c$ which contributes to the reaction $\gamma\gamma \to \pi^0\pi^0$ at chiral order larger than four, which was recently studied in [11] (in this reference the parameter is called $d_3$) and in [12]. A sum-rule was established for $c$ in [12] which yields the following estimate

$$c = \frac{-(4.6 \pm 2.3)10^{-3}}{r - 1}$$

The relation of the parameter $a$ to the low-energy constant $c$ is easily determined by matching the low-energy region:

$$\hat{m}a = \frac{9}{5} M_V^4 M_S^2 c$$

(23)
The preceding discussion has a bearing on the effective lagrangian description of the vector and scalar resonances, which is interesting as such, and will provide an alternative derivation of the estimate (22) for $c$.

4- Effective lagrangian for vector and scalar resonances:

We will denote the vector and scalar resonances by $V_\mu$ and $S$ respectively and will assume that they both transform according to the non-linear representation of the chiral group (see e.g. [3]) as $R \rightarrow h(\pi)Rh^\dagger(\pi)$. The effective lagrangian can be used to compute the three-point function $VVV$ and it has to reproduce the same one as found above (20). This uniquely determines the interaction part of the lagrangian with two vectors and one scalar fields. Writing $L_{\text{eff}} = L_S + L_V + L_{VVS}$, one has:

\[
L_S = \langle \nabla_\mu S \nabla^\mu S - M_S^2 S^2 - 2 M_S G SS \rangle \\
L_V = \langle -\frac{1}{2} V_{\mu\nu} V^{\mu\nu} + M_V^2 V_\mu V^\mu + f_V V_{\mu\nu} v^{\mu\nu} \rangle
\] (24)

and

\[
L_{VVS} = \frac{1}{F_V^2 M_S G S} \left( \frac{a}{M_V} (V_{\mu\nu} - f_V v_{\mu\nu})^2 S \\
- B_0 F_0^2 \left[ 2 M_V^2 V_\mu V^\mu S - V_{\mu\nu} (V^{\mu\nu} - f_V v^{\mu\nu}) S + \frac{1}{2 M_V^2} (V_{\mu\nu} - f_V v_{\mu\nu})^2 \nabla^2 S \right] \right) (25)
\]

where $f_V = F_V/M_V$. (A form which is manifestly covariant is obtained by replacing the vector source $v_{\mu\nu}$ by $1/2 f_+^{\mu\nu}$ and the scalar source $s$ by $1/2 \chi^+$. We will use this effective lagrangian for two purposes. Firstly, if one is interested in the decay $S \rightarrow 2\gamma$ then there are two terms only which contribute (in this formalism, the coupling of a photon to a vector meson vanishes when the photon is on-shell):

\[
L_{S\gamma\gamma} = \frac{1}{M_V^4 M_S G S} \left( a S v_{\mu\nu} v^{\mu\nu} - \frac{B_0 F_0^2}{2} S v_{\mu\nu} v^{\mu\nu} \right)
\] (26)

This lagrangian differs from the one which was used e.g. in [11] by the presence of the second term, proportional to $B_0$. This term is needed to enforce the QCD short distance behavior even though it is non-leading at low energies. In a sense this is an unusual situation: up to now it seemed possible to achieve compatibility of effective lagrangians for resonances with the short distance properties of QCD by retaining only terms with a minimal number of derivatives [3]. Here, we face a counterexample to this situation. Both terms contribute to the decay $S \rightarrow 2\gamma$ but only the minimal term contributes to the low-energy parameter $c$ (or $d_3$). Notice, however, that this remark has no practical bearing on the evaluation of the $\gamma\gamma \rightarrow \pi^0\pi^0$ cross-section in ref. [11].

As a second application, the form of $L_{\text{eff}}$ suggests an alternative way to determine the parameter $a$. Let us set the scalar source equal to the quark mass matrix:
s = M = diag(\hat{m}, \hat{m}, m_s). The equation of motion allows us to eliminate the scalar resonance field:

\[ S = -\frac{G_S}{M_S} M \]  

(27)

Clearly, the lagrangian (25) then describes propagation of vector mesons in the presence of a vector source and the sole effect of the scalar source is to induce ideal-mixing and mass-splitting in the vector nonet. One observes that the quark masses act on the kinetic energy as well as on the vector mass term. As a consequence, the actual meson masses get expressed as a function of the chiral limit mass \( M_V \) and the quark masses in the following way

\[ M_\phi^2 = \frac{M_V^2}{1 + 2\hat{m}a/F_V^2 M_S^2 + O(m_q^2)} \]  

(28)

and

\[ M_{K^*}^2 = \frac{M_V^2}{1 + (\hat{m} + m_s)a/F_V^2 M_S^2 + O(m_q^2)} \]  

(29)

Note that the terms proportional to \( B_0 \) in the lagrangian only contribute at order \( O(m_q^2) \). From (28) one obtains an estimate of the parameter \( a \) in terms of the \( \phi \) and \( \rho \) meson masses.

\[ \hat{m}a = M_V^4 M_S^2 \frac{F_V^2}{2(r - 1)} \left( \frac{1}{m_\phi^2} - \frac{1}{m_\rho^2} \right) \]  

(30)

If one uses the \( K^* \) and \( \rho \) masses instead one obtains the same result with a 10% accuracy. It turns out, in fact, that eq.(30) reproduces nearly exactly the same expression for the constant \( c \) (via (23)) as the one found in [12] by saturating the sum-rule with the contribution of the vector mesons (in the approximation where \( F_V^2 = F_\rho^2 = F_\omega^2 = F_\phi^2 \) which can be shown to hold, using (23) to first order in the quark masses). A remark is in order here. At first order in the quark masses, an equally valid way to determine \( a \) would be to expand first the denominators in (28) and determine \( a \) from the mass difference \( M_\phi^2 - M_\rho^2 \). The result is easily seen to differ from the preceeding one by a factor \( \simeq M_\phi^2/M_\rho^2 \). This essentially confirms the size of the error bar in the determination of the constant \( c \) in (22).

5- Results and conclusions:

Now that the unique parameter \( a \) is determined it is not difficult, using either directly the three-point function (20) or the lagrangian form (26), to evaluate the amplitude for the decay \( S \rightarrow 2\gamma \) and to calculate the width. The result is shown in the figure as a function of the quark mass ratio \( r \) and for three values of the constant \( c \), the central one and the two extreme values allowed by the error bar in (22). One observes that for the central value the width is in surprizingly good agreement with experiment and depends very weakly on the value of the quark mass ratio. Now if \( c \) is allowed to be larger (in magnitude) then the width is predicted to be also larger and
becomes more strongly dependent on $r$. The width can become as large as 1 KeV if $r$ is large, $r = 26$, and $c$ is at its upper bound. In that case, the width can still be in qualitative agreement with experiment provided the value of the quark mass ratio is of the order $r \approx 10$.

At this point we should also mention that there is a further uncertainty in the calculation, which is associated with the value of $M_p \simeq M_{\pi'}$ (in the figure we have taken $M_p = 1300$ GeV). Clearly, since the $\pi'$ is a rather wide resonance it is not a very precise approximation to treat it in the narrow width limit. Despite all these uncertainties, we feel that this calculation strongly suggests that there seems to be nothing surprising about the small value of the electromagnetic width of the $a_0$, which has been reproduced rather naturally. Chiral symmetry was an essential ingredient which allowed us to construct the correct form of the three-point function (or the equivalent effective lagrangian) and to determine all the parameters. The important role of chiral symmetry in this application is at high energy rather than, as usual, at low energy. This approach can be extended to discuss the main decay mode of the $a_0$, $a_0 \to \pi \eta$ (see [13]). In that case, the dependence upon $r$ is rather strong but it is not difficult to reproduce the experimental result. All this seem to indicate that there could be nothing exotic about the $a_0(980)$ (as e.g. in the suggestion of ref [14] ) which could well be an ordinary nonet member. An interesting question at this point is whether the $f_0(975)$ is also a nonet member with an exotic mixing angle or a completely exotic state. As has been pointed out and discussed in [15] this question could be answered at Da\phi\ne by measuring the decay rates $\phi \to a_0\gamma$ and $\phi \to f_0\gamma$. These rates can also be evaluated in our approach as a function of the mixing angle, the results can be found in [13].

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**Figure caption:** Result for the width \(a_0 \rightarrow 2\gamma\) as a function of the quark mass ratio for three values of the low-energy parameter \(c (c = -10^{-3} \hat{c}/(r - 1)).\)
This figure "fig1-1.png" is available in "png" format from:

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