The impact of the static part of the Earth’s gravity field on some tests of General Relativity with Satellite Laser Ranging

Lorenzo Iorio†

†Dipartimento di Fisica dell’ Università di Bari, via Amendola 173, 70126, Bari, Italy

Abstract

In this paper we calculate explicitly the classical secular precessions of the node $\Omega$ and the perigee $\omega$ of an Earth artificial satellite induced by the even zonal harmonics of the static part of the geopotential up to degree $l = 20$. Subsequently, their systematic errors induced by the mismodelling in the even zonal spherical harmonics coefficients $J_l$ are compared to the general relativistic secular gravitomagnetic and gravitoelectric precessions of the node and the perigee of the existing laser–ranged geodetic satellites and of the proposed LARES. The impact of the future terrestrial gravity models from CHAMP and GRACE missions is discussed as well. Preliminary estimates with the recently released EIGEN–1S gravity model including the first CHAMP data are presented.
1 Introduction

Recently, great efforts have been devoted to the investigation of the possibility of measuring some tiny general relativistic effects in the gravitational field of the Earth by analyzing the laser–ranged data to some existing or proposed geodetic laser–tracked (SLR) satellites.

The most famous experiment is that performed with LAGEOS and LAGEOS II [Ciufolini et al., 1998] and aimed to the detection of the gravitomagnetic Lense–Thirring drag of inertial frames [Lense and Thirring, 1918; Ciufolini and Wheeler, 1995] in the gravitational field of the Earth. The analysis of the orbits of the LAGEOS satellites could allow also for an alternative measurement of the gravitoelectric perigee advance [Ciufolini and Wheeler, 1995] to be performed in the gravitational field of the Earth [Iorio, 2002; Iorio et al., 2002a]. Moreover, the possibility of including also the data from other existing SLR satellites in these analysis is currently investigated [Iorio, 2002]. The proposed LAGEOS–LARES mission [Ciufolini, 1986], whose original configuration is currently being reanalyzed [Iorio et al., 2002b] in view of the inclusion of more orbital elements of various SLR satellites in the observable to be adopted, should be of great significance for both gravitomagnetic and gravitoelectric tests [Iorio et al., 2002a; Iorio et al., 2002b]. Satellite laser ranging could be the natural candidate also for the implementation of a space–based experiment aimed to the detection of the so called gravitomagnetic clock effect [Mashhoon et al., 1999; Iorio et al., 2002c], which is sensitive to the direction of motion of two counter–orbiting satellites along identical orbits in the gravitational field of a central rotating mass.

In all such performed or proposed experiments it is of the utmost importance to reliably assess the error budget. Indeed, the terrestrial space environment is rich of competing classical perturbing forces of gravitational and non–gravitational origin which in many cases are far larger than the general relativistic effects to be investigated. In particular, it is the impact of the systematic errors induced by the mismodelling in such various classical perturbations which is relevant in determining the total realistic accuracy of an experiment like those previously mentioned.

The general relativistic effects of interest here are linear trends affecting the perigee $\omega$ and the node $\Omega$ of the orbit of a satellite and amounting to $10^1$–$10^3$ milliarcseconds per year (mas/y
in the following) for the gravitomagnetic and the gravitoelectric effects, respectively.

In this context the most important source of systematic error is represented by the secular classical precessions of the node and the perigee induced by the mismodelled even \((l = 2n, \ n = 1, 2, 3, \ldots)\) zonal \((m = 0)\) harmonics \(\delta J_2, \ \delta J_4, \ \delta J_6, \ldots\) of the multipolar expansion of the terrestrial gravitational field, called geopotential. Indeed, while the time–varying orbital tidal perturbations [Iorio, 2001; Iorio and Pavlis, 2001; Pavlis and Iorio, 2002] and non–gravitational orbital perturbations [Lucchesi, 2001; 2002], according to their periods \(P\) and to the adopted observational time span \(T_{\text{obs}}\), can be viewed as empirically fitted quantity and can be removed from the signal, this is not the case of the classical even zonal secular precessions and of certain subtle non–gravitational secular effects of thermal origin [Lucchesi, 2002]. Their mismodelled linear trends act as superimposed effects which may alias the recovery of the genuine general relativistic features. Such disturbing trends cannot be removed from the signal without cancelling also the general relativistic signature, so that one can only assess as more accurately as possible their impact on the measurement. Then, the systematic error induced by the mismodelled part of the geopotential can be viewed as a sort of unavoidable, lower bound of the total systematic error.

In this paper we calculate explicitly, up to \(l = 20\), the expressions of the coefficients of the classical secular precessions on the node and the perigee due to the geopotential (section 2). Their explicit, analytic form, although rather cumbersome, may turn out to be useful in designing suitably alternative relativistic observables which are not sensitive, at least in part, to such classical aliasing effects [Iorio and Lichtenegger, 2002; Iorio and Lucchesi, 2002]. In section 3 we work out the numerical values of the mismodelled precessions for the existing SLR geodetic satellites and of the proposed LARES and compare them to the general relativistic effects. The errors for the spherical harmonics coefficients are those of EGM96 gravity model [Lemoine et al., 1998] and of the recent EIGEN–1S (see [http://op.gfz-potsdam.de/champ/results/]) which includes the first data from CHAMP mission\(^1\). These estimates can be useful in assessing the systematic errors in various possible observables, built up with such Keplerian orbital elements, which are sensitive to some relativistic effects. E.g., someone could look at the

\(^1\)It should be pointed out that the values employed in the following for the errors \(\delta J_i\) by EIGEN-1S are the formal, uncalibrated standard deviations. Moreover, EIGEN-1S is based only on the satellites tracking data.
Table 1: Orbital parameters of the existing spherical passive geodetic laser-ranged satellites and of LARES. Aj=Ajisai, Stl=Stella, Str=Starlette, WS=WESTPAC1, E1=ETALON1, E2=ETALON2, L1=LAGEOS, L2=LAGEOS II, LR=LARES. $a$ is in km, $i$ in deg and $n$ in $s^{-1}$.

|     | Aj  | Stl | Str | WS  | E1  | E2  | L1  | L2  | LR  |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $a$ | 7,870 | 7,193 | 7,331 | 7,213 | 25,498 | 25,498 | 12,270 | 12,163 | 12,270 |
| $e$ | 0.001 | 0.0204 | 0.0061 | 0.00066 | 0.0045 | 0.014 | 0.04 |
| $i$ | 50 | 98.6 | 49.8 | 98 | 64.9 | 65.5 | 110 | 52.65 | 70 |
| $n$ | 0.0009 | 0.001 | 0.001 | 0.001 | 0.00015 | 0.00015 | 0.00046 | 0.00047 | 0.00046 |

relativistic perturbations of the radial, along-track and cross-track components of the position and velocity vectors of a satellite; they can be obtained from suitable combinations of the Keplerian orbital elements. The same holds also for the range and range-rate perturbations in intersatellite tracking missions like GRACE [Cheng, 2002]. In section 4 we review the strategy of combining the orbital residuals of the nodes and the perigees of different laser-ranged satellites which allows for a reduction of the impact of the geopotential’s error and yield preliminary estimates of the errors affecting two gravitomagnetic combinations based on the first results from EIGEN–1S. They are useful in order to get an insight of the improvements which will take place when the full new gravity models will become available. Section 5 is devoted to the conclusions.

In Tab. 1 we quote the orbital parameters of the existing spherical passive geodetic laser-ranged satellites Ajisai, Stella, Starlette, WESTPAC1, ETALON1, ETALON2, LAGEOS, LAGEOS II and of the proposed LARES. In it $a$ is the semimajor axis, $e$ is the eccentricity, $i$ is the inclination and $n = \sqrt{GMA^{-3}}$, where $G$ is the Newtonian gravitational constant and $M$ is the mass of the central body, is the Keplerian mean motion. It is worth noting that the perigees of many of them, except for Starlette, cannot be employed for any relativistic tests due to the notable smallness of their eccentricities.
2 The orbital classical precessions

Here we show the explicitly calculated coefficients
\[ \dot{\Omega}_{2n} \equiv \frac{\partial \dot{\Omega}^{\text{even zonal}}_{\text{class}}}{\partial (J_{2n})} \] (1)
and
\[ \dot{\omega}_{2n} \equiv \frac{\partial \dot{\omega}^{\text{even zonal}}_{\text{class}}}{\partial (J_{2n})} \] (2)
of the satellites’ classical nodal and apsidal precessions due to the even zonal harmonics of the geopotential up to \( l = 20 \). The classical precessions of the node and the perigee due to the even zonal harmonics of geopotential can be written as
\[ \dot{\Omega}^{\text{even zonal}}_{\text{class}} = \sum_{n=1}^{\infty} \dot{\Omega}_{2n} \times J_{2n}, \] (3)
\[ \dot{\omega}^{\text{even zonal}}_{\text{class}} = \sum_{n=1}^{\infty} \dot{\omega}_{2n} \times J_{2n}. \] (4)

As we shall see later, the coefficients \( \dot{\Omega}_{2n} \) and \( \dot{\omega}_{2n} \) depend only on the orbital parameters of the satellites. Recall that \( J_l \equiv -C_{l0}, \ l = 2n, \ n = 1, 2, 3... \) where the unnormalized adimensional Stokes coefficients \( C_{lm} \) of degree \( l \) and order \( m \) can be obtained from the normalized \( \overline{C}_{lm} \) with
\[ C_{lm} = N_{lm} \overline{C}_{lm}. \] (5)

In it
\[ N_{lm} = \left[ \frac{(2l + 1)(2 - \delta_{0m})(l - m)!}{(l + m)!} \right]^{\frac{1}{2}}. \] (6)

The general expressions of the classical rates of the near Earth satellites’ Keplerian orbital elements due to the geopotential \( \dot{a}_{\text{class}}, \ \dot{e}_{\text{class}}, \ \dot{i}_{\text{class}}, \ \dot{\Omega}_{\text{class}}, \ \dot{\omega}_{\text{class}}, \ \dot{M}_{\text{class}}, \ \dot{F}_{\text{imp}}(i) \) and of the eccentricity functions \( G_{l_{pq}}(e) \) can be found in [Kaula, 1966]. The coefficients \( \dot{\Omega}_{2n} \) and \( \dot{\omega}_{2n} \) are of crucial importance in the evaluation of the systematic error due to the mismodelled even zonal harmonics of the geopotential; moreover, they enter the combined residuals’ coefficients \( c_i \) about which we speak in section 4. Since the general relativistic effects investigated are secular perturbations, we have considered only the perturbations averaged over one satellite’s orbital period. This has been accomplished with the condition \( l - 2p + q = 0 \) which allows for canceling out the rate of the mean anomaly \( \dot{M} \). Since the eccentricity functions...
$G_{tpq}$ are proportional to $e^{l \cdot q}$, for a given value of $l$ we have considered only those values of $p$ which fulfil the condition $l - 2p + q = 0$ with $q = 0$, i.e. $p = \frac{l}{2}$. This implies that in the summations

$$\sum_{p=0}^{l} \frac{dF_{l0p}}{di} \sum_{q=-\infty}^{+\infty} G_{tpq}$$

and

$$\sum_{p=0}^{l} F_{l0p} \sum_{q=-\infty}^{+\infty} \frac{dG_{tpq}}{de}$$

involved in the expressions of the classical rates we have considered only $F_{l0\frac{l}{2}}$ and $G_{l\frac{l}{2}0}$. Moreover, in working out the $G_{l\frac{l}{2}0}$ we have neglected the terms of order $O(e^k)$ with $k > 2$.

### 2.1 The nodal coefficients

The nodal coefficients, proportional to

$$\frac{1}{\sin i} \sum_{q=-\infty}^{+\infty} G_{tpq} \sum_{p=0}^{l} \frac{dF_{lmp}}{di},$$

are ($R$ is the Earth’s mean equatorial radius)

\begin{align*}
\dot{\Omega}_2 &= -\frac{3}{2} n \left(\frac{R}{a}\right)^2 \frac{\cos i}{(1 - e^2)^2}, \\
\dot{\Omega}_4 &= \dot{\Omega}_2 \left[ \frac{5}{8} \left(\frac{R}{a}\right)^2 \frac{(1 + \frac{5}{2}e^2)}{(1 - e^2)^2} \left(7 \sin^2 i - 4\right) \right], \\
\dot{\Omega}_6 &= \dot{\Omega}_2 \left[ \frac{35}{8} \left(\frac{R}{a}\right)^4 \frac{(1 + 5e^2)}{(1 - e^2)^4} \left(\frac{33}{8} \sin^4 i - \frac{9}{2} \sin^2 i + 1\right) \right], \\
\dot{\Omega}_8 &= \dot{\Omega}_2 \left[ \frac{105}{16} \left(\frac{R}{a}\right)^6 \frac{(1 + \frac{55}{2}e^2)}{(1 - e^2)^6} \left(\frac{715}{64} \sin^6 i - \frac{143}{8} \sin^4 i + \frac{33}{4} \sin^2 i - 1\right) \right], \\
\dot{\Omega}_{10} &= \dot{\Omega}_2 \left[ \frac{1,155}{128} \left(\frac{R}{a}\right)^8 \frac{(1 + 18e^2)}{(1 - e^2)^8} \left(\frac{4,199}{128} \sin^8 i - \frac{1,105}{16} \sin^6 i + \frac{195}{4} \sin^4 i - 13 \sin^2 i + 1\right) \right], \\
\dot{\Omega}_{12} &= \dot{\Omega}_2 \left[ \frac{3,003}{256} \left(\frac{R}{a}\right)^{10} \frac{(1 + \frac{55}{2}e^2)}{(1 - e^2)^{10}} \left(\frac{52,003}{512} \sin^{10} i - \frac{33,915}{128} \sin^8 i \right) \right].
\end{align*}
\[
\begin{align*}
\dot{\Omega}_{14} &= \dot{\Omega}_2 \left[ \frac{15,015}{1,024} \left( \frac{R}{a} \right)^{12} \frac{1}{(1 - e^2)^{12}} \left( \frac{334,305}{1,024} \sin^{12} i - \frac{260,015}{256} \sin^{10} i \right) \right], \\
&\quad + \frac{8,075}{32} \sin^6 i - \frac{425}{4} \sin^4 i + \frac{75}{4} \sin^2 i - 1 \right], \\
&\quad + \frac{156,009}{128} \sin^8 i - \frac{11,305}{16} \sin^6 i + \frac{1,615}{8} \sin^4 i - \frac{51}{2} \sin^2 i + 1 \right], \\
\dot{\Omega}_{16} &= \dot{\Omega}_2 \left[ \frac{36,465}{2,048} \left( \frac{R}{a} \right)^{14} \frac{1}{(1 - e^2)^{14}} \left( \frac{17,678,835}{16,384} \sin^{14} i - \right. \right. \\
&\quad - \frac{3,991,995}{1,024} \sin^{12} i + \frac{2,890,755}{512} \sin^{10} i - \frac{535,325}{128} \sin^8 i + \frac{107,065}{64} \sin^6 i \\
&\quad - \frac{2,793}{8} \sin^4 i + \frac{133}{4} \sin^2 i - 1 \right], \\
\dot{\Omega}_{18} &= \dot{\Omega}_2 \left[ \frac{692,835}{32,768} \left( \frac{R}{a} \right)^{16} \frac{1}{(1 - e^2)^{16}} \left( \frac{119,409,675}{32,768} \sin^{16} i - \right. \right. \\
&\quad - \frac{30,705,345}{2,048} \sin^{14} i + \frac{6,513,255}{256} \sin^{12} i - \frac{1,470,735}{64} \sin^{10} i + \\
&\quad + \frac{760,725}{64} \sin^8 i - \frac{28,175}{8} \sin^6 i + \\
&\quad + \frac{1,127}{2} \sin^4 i - 42 \sin^2 i + 1 \right], \\
\dot{\Omega}_{20} &= \dot{\Omega}_2 \left[ \frac{1,616,615}{65,536} \left( \frac{R}{a} \right)^{18} \frac{1}{(1 - e^2)^{18}} \left( \frac{1,641,030,105}{131,072} \sin^{18} i - \right. \right. \\
&\quad - \frac{1,893,496,275}{32,768} \sin^{16} i + \frac{460,580,175}{4,096} \sin^{14} i - \frac{30,705,345}{256} \sin^{12} i \\
&\quad + \frac{19,539,765}{256} \sin^{10} i - \frac{1,890,945}{64} \sin^8 i + \frac{108,675}{16} \sin^6 i - \\
&\quad - \frac{1,725}{2} \sin^4 i + \frac{207}{4} \sin^2 i - 1 \right].
\end{align*}
\]
2.2 The perigee coefficients

The coefficients of the classical perigee precession are much more involved because they are proportional to

\[ -\left(\frac{\cos i}{\sin i}\right) \sum_{q=-\infty}^{\infty} G_{lpq} \sum_{p=0}^{l} \frac{dF_{mp}}{dr} + \frac{(1-e^2)}{e} \sum_{q=-\infty}^{\infty} \frac{dG_{lpq}}{de} \sum_{p=0}^{l} F_{mp}. \]  

(20)

We can pose \( \dot{\omega}_{2n} = \dot{\omega}_{2n}^a + \dot{\omega}_{2n}^b. \)

The first set is given by \((R)\) is the Earth’s mean equatorial radius

\[ \dot{\omega}_{2}^a = \frac{3}{2}n \left(\frac{R}{a}\right)^2 \frac{\cos^2 i}{(1-e^2)^2}, \]  

\(21\)

\[ \dot{\omega}_{4}^a = \dot{\omega}_{2}^a \left[ \frac{5}{8} \left(\frac{R}{a}\right)^2 \frac{(1+\frac{3}{2}e^2)}{(1-e^2)^2} \left(7\sin^2 i - 4\right) \right], \]  

\(22\)

\[ \dot{\omega}_{6}^a = \dot{\omega}_{2}^a \left[ \frac{35}{8} \left(\frac{R}{a}\right)^4 \frac{(1+5e^2)}{(1-e^2)^4} \left(\frac{33}{8}\sin^4 i - \frac{9}{8}\sin^2 i + 1\right) \right], \]  

\(23\)

\[ \dot{\omega}_{8}^a = \dot{\omega}_{2}^a \left[ \frac{105}{16} \left(\frac{R}{a}\right)^6 \frac{(1+\frac{21}{2}e^2)}{(1-e^2)^6} \left(\frac{715}{64}\sin^6 i - \right. \right. \]  

\[ - \frac{143}{8}\sin^4 i + \frac{33}{4}\sin^2 i - 1\left) \right], \]  

\(24\)

\[ \dot{\omega}_{10}^a = \dot{\omega}_{2}^a \left[ \frac{1}{128} \left(\frac{R}{a}\right)^8 \frac{(1+18e^2)}{(1-e^2)^8} \left(\frac{4}{128}\sin^8 i - \frac{1,105}{16}\sin^6 i \right. \right. \]  

\[ + \frac{195}{4}\sin^4 i - 13\sin^2 i + 1\left) \right], \]  

\(25\)

\[ \dot{\omega}_{12}^a = \dot{\omega}_{2}^a \left[ \frac{3,003}{256} \left(\frac{R}{a}\right)^{10} \frac{(1+\frac{55}{2}e^2)}{(1-e^2)^{10}} \left(\frac{52,003}{512}\sin^{10} i - \frac{33,915}{128}\sin^8 i \right. \right. \]  

\[ + \frac{8,075}{32}\sin^6 i - \frac{425}{4}\sin^4 i + \frac{75}{4}\sin^2 i - 1\left) \right], \]  

\(26\)

\[ \dot{\omega}_{14}^a = \dot{\omega}_{2}^a \left[ \frac{15,015}{1,024} \left(\frac{R}{a}\right)^{12} \frac{(1+\frac{91}{2}e^2)}{(1-e^2)^{12}} \left(\frac{334,305}{1,024}\sin^{12} i - \frac{260,015}{256}\sin^{10} i + \right. \right. \]  

\[ + \frac{156,009}{128}\sin^8 i - \frac{11,305}{16}\sin^6 i + \frac{1,615}{8}\sin^4 i - \]
\[
\dot{\omega}_{16}^a = \dot{\omega}_2^a \left[ \frac{36,465}{2,048} \left( \frac{R}{a} \right)^{14} \left( 1 + \frac{105}{2} e^2 \right) \left( \frac{17,678,835}{16,384} \sin^{14} i - \frac{3,991,995}{1,024} \sin^{12} i \right) \right. \\
\left. + \frac{2,890,755}{512} \sin^{10} i - \frac{535,325}{128} \sin^8 i + \frac{107,065}{64} \sin^6 i \right. \\
- \frac{2,793}{8} \sin^4 i + \frac{133}{4} \sin^2 i - 1 \right], \\
\dot{\omega}_{18}^a = \dot{\omega}_2^a \left[ \frac{692,835}{32,768} \left( \frac{R}{a} \right)^{16} \left( 1 + 68 e^2 \right) \left( \frac{119,409,675}{32,768} \sin^{16} i - \right. \\
\left. \frac{30,705,345}{2,048} \sin^{14} i + \frac{6,513,255}{256} \sin^{12} i - \frac{1,470,735}{64} \sin^{10} i + \\
\right. + \frac{760,725}{64} \sin^8 i - \frac{28,175}{8} \sin^6 i + \frac{1,127}{2} \sin^4 i - \\
- \frac{42}{8} \sin^2 i + 1 \right], \\
\dot{\omega}_{20}^a = \dot{\omega}_2^a \left[ \frac{1,616,615}{65,536} \left( \frac{R}{a} \right)^{18} \left( 1 + \frac{171}{2} e^2 \right) \left( \frac{1,641,030,105}{131,072} \sin^{18} i \right) \right. \\
\left. - \frac{1,893,496,275}{32,768} \sin^{16} i + \frac{460,580,175}{4,096} \sin^{14} i - \frac{30,705,345}{256} \sin^{12} i \right. \\
\left. + \frac{19,539,765}{256} \sin^{10} i - \frac{1,890,945}{64} \sin^8 i + \frac{108,675}{16} \sin^6 i - \\
- \frac{1,725}{2} \sin^4 i + \frac{207}{4} \sin^2 i - 1 \right]. \\
\]
\[
\omega^b_{6} = w_2 \left\{ 35 (R/a)^4 \left[ \frac{10}{(1-e^2)^5} + 11 \frac{(1+5e^2)}{(1-e^2)^6} \right] \left( 33 \sin^6 i \right) + \frac{9}{8} \sin^4 i + \frac{1}{2} \sin^2 i - \frac{1}{21} \right\},
\]

\[
\omega^b_{8} = w_2 \left\{ \frac{105}{16} (R/a)^6 \left[ \frac{21}{(1-e^2)^7} + 15 \frac{(1+\frac{21}{2}e^2)}{(1-e^2)^8} \right] \left( \frac{715}{512} \sin^8 i \right) - \frac{143}{48} \sin^6 i + \frac{33}{16} \sin^4 i - \frac{1}{2} \sin^2 i + \frac{1}{36} \right\},
\]

\[
\omega^b_{10} = w_2 \left\{ \frac{1.155}{128} (R/a)^8 \left[ \frac{36}{(1-e^2)^9} + 19 \frac{(1+18e^2)}{(1-e^2)^10} \right] \left( \frac{4,199}{1,280} \sin^{10} i \right) - \frac{1,105}{128} \sin^8 i + \frac{195}{24} \sin^6 i - \frac{13}{4} \sin^4 i + \frac{1}{2} \sin^2 i - \frac{1}{55} \right\},
\]

\[
\omega^b_{12} = w_2 \left\{ \frac{3,003}{256} (R/a)^{10} \left[ \frac{55}{(1-e^2)^{11}} + 23 \frac{(1+\frac{55}{2}e^2)}{(1-e^2)^{12}} \right] \left( \frac{52,003}{6,144} \sin^{12} i \right) - \frac{6,783}{256} \sin^{10} i + \frac{8,075}{256} \sin^8 i - \frac{425}{24} \sin^6 i + \frac{75}{16} \sin^4 i 
\]

\[
- \frac{1}{2} \sin^2 i + \frac{1}{78} \right\}, \]

\[
\omega^b_{14} = w_2 \left\{ \frac{15,015}{1,024} (R/a)^{12} \left[ \frac{91}{(1-e^2)^{13}} + 27 \frac{(1+\frac{91}{2}e^2)}{(1-e^2)^{14}} \right] \times \left( \frac{334,305}{14,336} \sin^{14} i - \frac{260,015}{3,072} \sin^{12} i + \frac{156,009}{1,280} \sin^{10} i - \frac{11,305}{128} \sin^8 i + \frac{1,615}{48} \sin^6 i - \frac{51}{8} \sin^4 i + \frac{1}{2} \sin^2 i - \frac{1}{105} \right) \right\},
\]

\[
\omega^b_{16} = w_2 \left\{ \frac{36,465}{2,048} (R/a)^{14} \left[ \frac{105}{(1-e^2)^{15}} + 31 \frac{(1+\frac{105}{2}e^2)}{(1-e^2)^{16}} \right] \times \left( \frac{13,003}{6,144} \sin^{16} i - \frac{6,783}{256} \sin^{14} i + \frac{3,003}{256} \sin^{12} i - \frac{525}{24} \sin^{10} i + \frac{75}{16} \sin^8 i + \frac{1}{2} \sin^6 i + \frac{1}{36} \right) \right\},
\]
3 The mismodelled classical precessions

The results obtained in the previous section can be used in working out explicitly the contributions of the mismodelled classical nodal and apsidal precessions up to degree \( l = 20 \) of the exist-
ing spherical passive laser-ranged geodetic satellites and of the proposed LARES. They are of the form $\delta \dot{\Omega}_{(2n)} = \Omega_{2n} \times \delta J_{2n}, n = 1, 2, \ldots 10$ and $\delta \dot{\omega}_{(2n)} = \dot{\omega}_{2n} \times \delta J_{2n}, n = 1, 2, \ldots 10$. The coefficients $\dot{\Omega}_{2n}$ and $\dot{\omega}_{2n}$ are worked out in section 2 and the values employed for $\delta J_{2n} = -\sqrt{4n+1} \times \delta C_{2n} 0, n = 1, 2, \ldots 10$ are those quoted in the adopted Earth's gravity model.

Table 2: Mismodelled classical nodal precessions $\delta \dot{\Omega}_{(2n)}$ and predicted Lense-Thirring nodal precessions $\dot{\Omega}_{LT}$ of the existing geodetic laser-ranged satellites and of LARES. L1=LAGEOS, L2=LAGEOS II, LR=LARES, Aj=Ajisai, Stl=Stella, Str=Starlette, WS=WESTPAC1, E1=ETALON1, E2=ETALON2. All the values are in mas/y. For the ETALON satellites, when the values are less than $10^{-4}$ mas/y a – has been inserted. EGM96 gravity model has been adopted.

| $2n$ | L1  | L2  | LR  | Aj  | Stl | Str | WS  | E1  | E2  |
|------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 2    | -33.4 | 61  | 33.4 | 296.8 | -94.6 | 382.3 | -87.1 | 3.2  | 3.1 |
| 4    | -48.3 | 17.4 | 48.7 | 51.5 | -519.2 | 59.5  | -479.2 | 0.8  | 0.8 |
| 6    | -17  | -26.1 | 17.3 | -809.7 | -912.2 | -1,397.7 | -847.9 | 0.03 | 0.03 |
| 8    | -1.9  | -10.3 | 2    | -366.3 | -1,487.2 | -674.4 | -1,399.7 | -0.005 | -0.004 |
| 10   | 2.1  | 3.1  | -2.2 | 823.5 | 1,855  | 1,933.4 | -1,781.8 | -0.001 | - |
| 12   | 1.6  | 2.5  | -1.7 | 647.5  | -2,144.6 | 1,636.4 | -2,126.6 | - |
| 14   | 0.6  | -0.007 | -0.6 | 542.6  | -1,963.4 | 1,780.9 | -2,049.4 | - |
| 16   | 0.09 | -0.2  | -0.1 | 517.2  | -1,204.6 | -1,787.9 | -1,376.8 | - |
| 18   | -0.007 | -0.03 | 0.008 | 117.9  | -512.4  | 580 | -717 | - |
| 20   | -0.01 | 0.01  | 0.01 | 247.6  | -79.5  | 1,177 | -309 | - |

$\dot{\Omega}_{LT}$ 30.7 31.6 30.8 116.7 152.8 144.4 151.5 3.4 3.4

From Tab. 2 it is interesting to note that for the satellites orbiting at lower altitudes than the LAGEOS satellites the impact of the mismodelled part of the geopotential does not reduce to the first two or three even zonal harmonics. This feature is very important in calculating the error budget, especially if the nodes of low–orbiting satellites are to be considered. Moreover, while for the LAGEOS family a calculation up to $l = 20$ is rather adequate, this is not the case for the other satellites for which the even zonal harmonics of degree $l > 20$ should be considered as well. In regard to this topic, the choice of the Earth gravity model becomes crucial because EGM96, for example, does not seem to be particularly reliable at degrees higher than 20. The same considerations hold also for the perigee whose mismodelled classical precessions are quoted in Tab. 3. We have considered only LAGEOS II, Starlette and the LARES due to the extreme
smallness of the eccentricity of the other satellites. From both Tab. 2 and Tab. 3 the relevant

Table 3: Mismodelled classical perigee precessions $\delta \dot{\omega}_{(2n)}$ and predicted Lense-Thirring and gravitoelectric perigee precessions $\dot{\omega}_{LT}$ and $\dot{\omega}_{GE}$ of the existing spherical passive geodetic laser-ranged satellites and of LARES. L1=LAGEOS, L2=LAGEOS II, LR=LARES, Aj=Ajisai, Stl=Stella, Str=Starlette, WS=WESTPAC1, E1=ETALON1, E2=ETALON2. All the values are in mas/y. EGM96 gravity model has been adopted.

| $2n$ | L1 | L2 | LR | Aj | Stl | Str | WS | E1 | E2 |
|------|----|----|----|----|-----|-----|----|----|----|
| 2    | -42.3 | 20.3 | -   | -   | -320.7 | -   | -   | -   | -   |
| 4    | -122.7 | -17.6 | -   | -   | -1,924.4 | -   | -   | -   | -   |
| 6    | -18.2 | -49.2 | -   | -   | 429.1 | -   | -   | -   | -   |
| 8    | 43.1 | -42.6 | -   | -   | 6,355.8 | -   | -   | -   | -   |
| 10   | 19.5 | -18 | -   | -   | 2,805.1 | -   | -   | -   | -   |
| 12   | -5.3 | -3 | -   | -   | -10,862.2 | -   | -   | -   | -   |
| 14   | -6.2 | 2 | -   | -   | -10,774.7 | -   | -   | -   | -   |
| 16   | -0.2 | 1.3 | -   | -   | 8,395.8 | -   | -   | -   | -   |
| 18   | 0.4 | 0.4 | -   | -   | 9,086.4 | -   | -   | -   | -   |
| 20   | 0.1 | 0.08 | -   | -   | -3,043.3 | -   | -   | -   | -   |

$\dot{\omega}_{LT}$ -57.5 -31.6 - - 68.5 - - -

$\dot{\omega}_{GE}$ - 3.348 3.2786 - - 11,804.7 - - -

impact of the first two or three even zonal harmonics, at least for the LAGEOS satellites, is quite apparent. The situation for the lower orbiting satellites is far more unfavorable.

In Tab. 4 and Tab. 5 we repeat the analysis with the very recently released EIGEN–1S gravity model which includes the first data from CHAMP.

From an inspection of Tab. 2–Tab. 5 it turns out very clearly that, if the orbital elements of satellites other than those of LAGEOS family are to be considered, the observable which would account for them must cope with the problem of reducing the impact also of the degrees higher than 4.

4 The systematic zonal error

A possible strategy for reducing the impact of the geopotential’s error consists of suitable combinations of the orbital residuals of the rates of the nodes and the perigees of different SLR
Table 4: Mismodelled classical nodal precessions $\delta\dot{\Omega}_{(2n)}$ and predicted Lense-Thirring nodal precessions $\dot{\Omega}_{LT}$ of the existing geodetic laser-ranged satellites and of LARES. L1=LAGEOS, L2=LAGEOS II, LR=LARES, Aj=Ajisai, Stl=Stella, Str=Starlette, WS=WESTPAC1, E1=ETALON1, E2=ETALON2. The errors $\delta J_{2n}$ are those of the preliminary EIGEN–1S Earth gravity model from 88 days of CHAMP data. All the values are in mas/y. For the ETALON satellites, since all the values are less than $10^{-1}$ mas/y a – has been inserted.

| $2n$ | L1  | L2  | LR  | Aj  | Stl | Str | WS  | E1 | E2 |
|------|-----|-----|-----|-----|-----|-----|-----|----|----|
| 2    | -3.9 | 7.2 | 3.9 | 35  | -11.1 | 45.1 | -10.2 | –  | –  |
| 4    | -3.8 | -5.8 | 3.8 | -181.5 | -204.5 | -313.4 | -190.1 | –  | –  |
| 6    | -0.4 | -2.3 | 0.4 | -83.8 | -340.3 | -154.3 | -320.3 | –  | –  |
| 8    | 0.4  | 0.6  | -0.4 | 168.5 | -379.6 | 395.6 | -364.6 | –  | –  |
| 10   | 0.3  | 0.4  | -0.3 | 125.9 | -417  | 318.2 | -413.5 | –  | –  |
| 12   | 0.1  | -0.001 | -0.1 | -122.1 | -441.9 | -400.8 | -461.3 | –  | –  |
| 14   | 0.02 | -0.07 | -0.03 | -152  | -354.1 | -525.6 | -404.7 | –  | –  |
| 16   | -0.003 | -0.01 | 0.003 | 49.2  | -214  | 242.2 | -299.4 | –  | –  |
| 18   | -0.005 | 0.007 | 0.006 | 131.3 | -42.2  | 624.3 | -163.9 | –  | –  |
| 20   | 30.7 | 31.6 | 30.8 | 30.8 | 116.7 | 152.8 | 144.4 | 151.5 | 3.4 | 3.4 |

satellites [Ciufolini, 1996; Iorio, 2002]. Such combinations can be written in the form

$$\sum_{i=1}^{N} c_i f_i = X_{GR} \mu_{GR},$$

(42)

in which the coefficients $c_i$ are, in general, suitably built up with the orbital parameters of the satellites entering the combinations, the $f_i$ are the residuals of the rates of the nodes and the perigees of the satellites entering the combination, $X_{GR}$ is the slope, in mas/y, of the general relativistic trend of interest and $\mu_{GR}$ is the solve–for parameter, to be determined by means of usual least–square procedures, which accounts for the general relativistic effect. For example, in the case of the Lense–Thirring–LAGEOS experiment [Ciufolini, 1996] $X_{LT} = 60.2$ mas/y, while for the gravitoelectric perigee advance [Iorio, 2002] $X_{GE} = 3,348$ mas/y. More precisely, the combinations of eq. (42) are obtained in the following way. The equations for the residuals of the rates of the $N$ chosen orbital elements are written down, so to obtain a non homogeneous algebraic linear system of $N$ equations in $N$ unknowns. They are $\mu_{GR}$ and the first $N - 1$ mismodelled spherical harmonics coefficients $\delta J_l$ in terms of which the residual rates are expressed. The coefficients $c_i$ and, consequently, $X_{GR}$ are obtained by solving for $\mu_{GR}$ the
Table 5: Mismodelled classical perigee precessions $\delta \dot{\omega}_{(2n)}$ and predicted Lense-Thirring and gravitoelectric perigee precessions $\dot{\omega}_{LT}$ and $\dot{\omega}_{GE}$ of the existing spherical passive geodetic laser-ranged satellites and of LARES. L1=LAGEOS, L2=LAGEOS II, LR=LARES, Aj=Ajisai, Stl=Stella, Str=Starlette, WS=WESTPAC1, E1=ETALON1, E2=ETALON2. All the values are in mas/y. The errors $\delta J_{2n}$ are those of the preliminary EIGEN–1S Earth gravity model from 88 days of CHAMP data.

| $2n$ | L1 | L2 | LR | Aj | Stl | Str | WS | E1 | E2 |
|------|----|----|----|----|-----|-----|----|----|----|
| 2    | -  | -4.9 | 2.4 | -  | -   | -37.8 | -  | -  | -  |
| 4    | -  | -18.3 | -2.6 | -  | -   | -288.2 | -  | -  | -  |
| 6    | -  | -4   | -11 | -  | -   | 96.2  | -  | -  | -  |
| 8    | -  | 9.8  | -9.7 | -  | -   | 1,454.6 | -  | -  | -  |
| 10   | -  | 4    | -3.6 | -  | -   | 574   | -  | -  | -  |
| 12   | -  | -1   | -0.6 | -  | -   | -2,112.2 | -  | -  | -  |
| 14   | -  | -1.4 | 0.4  | -  | -   | -2,425.3 | -  | -  | -  |
| 16   | -  | -0.07 | 0.3  | -  | -   | 2,468.3 | -  | -  | -  |
| 18   | -  | 0.2  | 0.1  | -  | -   | 3,794.6 | -  | -  | -  |
| 20   | -  | 0.05 | 0.04 | -  | -   | -1,614.6 | -  | -  | -  |

$\dot{\omega}_{LT}$ | -57.5 | -31.6 | -   | -   | 68.5 | -  | -  | -  |
$\dot{\omega}_{GE}$ | 3,348 | 3,278.6 | -   | -   | 11,804.7 | -  | -  | -  |

system of equations. So, the coefficients $c_i$ are calculated in order to cancel out the contributions of the first $N - 1$ even zonal mismodelled harmonics which, as we have seen in the previous section, represent the major source of uncertainty in the Lense–Thirring and gravitoelectric precessions [Ciufolini, 1996; Iorio, 2002]. The coefficients $c_i$ can be either constant or depend on the orbital elements of the satellites entering the combinations through the coefficients $\dot{\Omega}_{2n}$ and $\dot{\omega}_{2n}$ worked out in section 2.

Now we expose how to calculate the systematic error due to the mismodelled even zonal harmonics of the geopotential for the combinations involving the residuals of the nodes and the perigees of various satellites.

In general, if we have an observable $q$ which is a function $q = q(x_j)$, $j = 1, 2,...M$ of $M$
correlated parameters \( x_j \) the error in it is given by

\[
\delta q = \left[ \sum_{j=1}^{M} \left( \frac{\partial q}{\partial x_j} \right)^2 \sigma_j^2 + 2 \sum_{h \neq k=1}^{M} \left( \frac{\partial q}{\partial x_h} \right) \left( \frac{\partial q}{\partial x_k} \right) \sigma_{hk}^2 \right]^{\frac{1}{2}}
\]  

(43)

in which \( \sigma_j^2 \equiv C_{jj} \) and \( \sigma_{hk}^2 \equiv C_{hk} \) where \( \{C_{hk}\} \) is the square matrix of covariance of the parameters \( x_j \).

In our case the observable \( q \) is any residuals’ combination

\[
q = \sum_{i=1}^{N} c_i f_i(x_j), \quad j = 1, 2...10,
\]  

(44)

where \( x_j, \ j = 1, 2...10 \) are the even zonal geopotential’s coefficients \( J_2, J_4...J_{20} \). Since

\[
\frac{\partial q}{\partial x_j} = \sum_{i=1}^{N} c_i \frac{\partial f_i}{\partial x_j}, \quad j = 1, 2...10,
\]  

(45)

by putting eq. (45) in eq. (43) one obtains, in mas/y

\[
\delta q = \left[ \sum_{j=1}^{10} \left( \sum_{i=1}^{N} c_i \frac{\partial f_i}{\partial x_j} \right)^2 \sigma_j^2 + 2 \sum_{h \neq k=1}^{10} \left( \sum_{i=1}^{N} c_i \frac{\partial f_i}{\partial x_h} \right) \left( \sum_{i=1}^{N} c_i \frac{\partial f_i}{\partial x_k} \right) \sigma_{hk}^2 \right]^{\frac{1}{2}}.
\]  

(46)

The percent error, for a given general relativistic trend and for a given combination, is obtained by taking the ratio of eq. (46) to the slope in mas/y of the general relativistic trend for the residual combination considered.

The validity of eq. (43) has been checked by calculating with it and the covariance matrix of EGM96 gravity model the systematic error due to the even zonal harmonics of the geopotential of the gravitomagnetic LAGEOS experiment; indeed the result

\[
\delta \mu_{LT} = 12.9\% \ \mu_{LT}
\]  

(47)
In order to get a preliminary insight of what the improvement due to the new Earth gravity models might be, let us consider, for example, the usual observable by Ciufolini [Ciufolini, 1996] for the detection of the Lense–Thirring drag

$$\delta \dot{\Omega}^{L1} + 0.295 \times \delta \dot{\Omega}^{L2} - 0.35 \times \delta \dot{\omega}^{L2} \sim 60.2 \mu_{LT}. \quad (48)$$

The root–sum–square error due to geopotential, according to the diagonal part only of the covariance matrix of EGM96 model, amounts to 46.5% \(^3\); according to the diagonal part only of the covariance matrix of EIGEN–1S (at present, its full covariance matrix is not yet publicly available), it reduces to 10.5%.

If we consider the combination proposed in [Iorio, 2002]

$$\delta \dot{\Omega}^{L1} + 0.444 \times \delta \dot{\Omega}^{L2} - 0.027 \times \delta \dot{\Omega}^{A1} - 0.341 \times \delta \dot{\omega}^{L2} \sim 61.2 \mu_{LT} \quad (49)$$

in this case the root–sum–square error due to geopotential, according to the diagonal part only of the covariance matrix of EGM96 model, amounts to 64.2% \(^3\); according to the diagonal part only of the covariance matrix of EIGEN–1S, it reduces to 15.5%.

It should be noted that, according to [Ries et al., 1998], it would not be entirely correct to automatically extend the validity of the full covariance matrix of EGM96, which is based on a multi–year average that spans the 1970, 1980 and early 1990 decades, to any particular time span like that, e.g., of the LAGEOS–LAGEOS II Lense–Thirring analysis which extends from the middle to the end of the 1990 decade. Indeed, there would not be assurance that the errors in the even zonal harmonics of the geopotential during the time of the LAGEOS–LAGEOS II Lense–Thirring experiment remained correlated exactly as in the EGM96 covariance matrix, in view of the various secular, seasonal and stochastic variations that we know occur in the terrestrial gravitational field and that have been neglected in the EGM96 solution. Then, the use of the diagonal part only of the covariance matrix of EGM96 should yield more conservative results. However, since it turns out that such seasonal effects would mainly affect just the first even zonal harmonic coefficients of the geopotential, the uncertainty related to them should be

\(^3\)It reduces to 12.9% by considering also the correlation among the spherical harmonics coefficients, according to EGM96, as stated before.

\(^4\)It reduces to 10.8% by considering also the correlation among the spherical harmonics coefficients, according to EGM96 [Iorio, 2002].
very small for residual combinations which, by construction, cancel out just the first even zonal harmonic coefficients of the geopotential. On the other hand, if we cancel out as many even zonal harmonics as possible, the uncertainties in the evaluation of the systematic error based on the remaining correlated even zonal harmonics of higher degree should be greatly reduced, irrespectively of the chosen time span. This would have a relevant importance, e.g., for those even zonal harmonics like $J_6$ and $J_8$ whose favorable correlation in the covariance matrix of EGM96 seems to be the source of the perhaps optimistic evaluation of the systematic error due to the even zonal harmonics of the geopotential in the case of the LAGEOS–LAGEOS II Lense–Thirring experiment [Ries et al., 1998]. The LAGEOS–LAGEOS II–LARES proposed combination of [Iorio et al., 2002b] would cancel out, apart from $\delta J_2$ and $\delta J_4$, just $\delta J_6$ and $\delta J_8$.

5 Conclusions

The systematic error induced by the mismodelled static part of the geopotential is the major source of uncertainty in many proposed or performed tests of General Relativity in the gravitational field of the Earth via Satellite Laser Ranging.

In this paper we have explicitly calculated the expressions of the coefficients of the classical secular precessions of the node and the perigee induced by the even zonal harmonics of the geopotential up to degree $l = 20$. The explicit expressions of the classical precessions may be useful, e.g., in getting insights for designing suitably new relativistic observables which are not too sensitive to such disturbing effects.

Subsequently, we have compared the mismodelled precessions, according to EGM96 gravity model and EIGEN–1S preliminary gravity model which includes 88 days of data from CHAMP, to the general relativistic gravitomagnetic and gravitoelectric secular trends affecting the same orbital elements of the existing or proposed laser–ranged geodetic satellites. Since such satellites are the natural candidates for a number of relativistic tests in the gravitational field of the Earth, the presented calculations are useful in order to get an idea of the level of aliasing induced by the geopotential on the relativistic signatures of possible observables which may be built up with the orbital elements of such satellites. Of course, the obtained results could turn out to be useful also for other nonrelativistic investigations. While for the LAGEOS satellites a
calculation up to \( l = 20 \) is well adequate, for the other satellites orbiting at lower altitudes also the other harmonics of higher degrees should be carefully considered. The need for reducing the impact of the mismodelled classical precessions on the relativistic signals is quite apparent.

Finally, we have shown how to calculate explicitly the static gravitational systematic error on suitably designed combinations involving the orbital residuals of different satellites. Preliminary estimates of the errors affecting such combinations with EGM96 and EIGEN–1S suggest that the future, more accurate terrestrial global gravity models from CHAMP and GRACE missions will have a notable impact on the improvement of, among other things, the precision of many general relativistic tests.

**Acknowledgements**

L. Iorio is grateful to L. Guerriero for his support while at Bari. Special thanks also to the CHAMP and GRACE team of GFZ at Potsdam for their kind collaboration.

**References**

[Cheng, 2002] Cheng, M.K., Gravitational perturbation theory for intersatellite tracking, *J. of Geodesy*, 76, 169–185, 2002.

[Ciufolini, 1986] Ciufolini, I., Measurement of Lense–Thirring drag on high-altitude, laser ranged artificial satellites, *Phys. Rev. Lett.*, 56, 278-281, 1986.

[Ciufolini and Wheeler, 1995] Ciufolini, I., and J. A. Wheeler, *Gravitation and Inertia*, 498 pp., Princeton University Press, New York, 1995.

[Ciufolini, 1996] Ciufolini, I., On a new method to measure the gravitomagnetic field using two orbiting satellites, *Il Nuovo Cimento, 109A*, 12, 1709-1720, 1996.

[Ciufolini et al., 1998] Ciufolini, I., E. Pavlis, F. Chieppa, E. Fernandes-Vieira, and J. Pérez-Mercader, Test of General Relativity and Measurement of the Lense-Thirring Effect with Two Earth Satellites, *Science, 279*, 2100-2103, 1998.
[Iorio, 2001] Iorio, L., Earth tides and Lense-Thirring effect, *Celest. Mech. & Dyn. Astron.*, 79, 201-230, 2001b.

[Iorio and Pavlis, 2001] Iorio, L. and E.C. Pavlis, Tidal Satellite Perturbations and the Lense-Thirring Effect, *J. of the Geod. Soc. of Japan*, 47, 169-173, 2001.

[Iorio et al., 2002a] Iorio, L., I. Ciufolini and E.C. Pavlis, Measuring the Relativistic Pericenter Advance with Satellite Laser Ranging, *Class. and Quantum Grav.*, 19, 4301-4309, 2002a.

[Iorio et al., 2002b] Iorio, L., D.M. Lucchesi, and I. Ciufolini, The LARES mission revisited: an alternative scenario, *Class. and Quantum Grav.*, 19, 4311-4325, 2002b.

[Iorio et al., 2002c] Iorio, L., H.I.M. Lichtenegger, and B. Mashhoon, An alternative derivation of the gravitomagnetic clock effect, *Class. and Quantum Grav.*, 19, 39-49, 2002c.

[Iorio and Lichtenegger, 2002] Iorio, L., and H.I.M. Lichtenegger, On the gravitomagnetic effects on the orbits of two counter-orbiting satellites, submitted to *Class. and Quantum Grav.*, preprint gr-qc/0210030, 2002.

[Iorio and Lucchesi, 2002] Iorio, L., and D.M. Lucchesi, LAGEOS–type Satellites in Critical Supplementary Orbit Configuration and the Lense–Thirring Effect Determination, submitted to *Class. and Quantum Grav.*, preprint gr-qc/0209027, 2002.

[Iorio, 2002] Iorio, L., Is it possible to improve the present LAGEOS–LAGEOS II Lense–Thirring experiment?, *Class. and Quantum Grav.*, 19, 5473–5480, 2002.

[Kaula, 1966] Kaula, W. M., *Theory of Satellite Geodesy*, 124 pp., Blaisdell Publishing Company, Waltham, 1966

[Lemoine et al., 1998] Lemoine, F. G., et al., The Development of the Joint NASA GSFC and the National Imagery Mapping Agency (NIMA) Geopotential Model EGM96, NASA/TP-1998-206861, 1998.

[Lense and Thirring, 1918] Lense, J., and H. Thirring, Über den Einfluss der Eigenrotation der Zentralkörper auf die Bewegung der Planeten und Monde nach der Einsteinschen
Gravitationstheorie, *Phys. Z.*, 19, 156-163, 1918, translated by Mashhoon, B., F. W. Hehl, and D. S. Theiss, On the Gravitational Effects of Rotating Masses: The Thirring-Lense Papers, *Gen. Rel. Grav.*, 16, 711-750, 1984.

*[Lucchesi, 2001]* Lucchesi, D., Reassessment of the error modelling of non-gravitational perturbations on LAGEOS II and their impact in the Lense-Thirring determination. Part I, *Plan. and Space Sci.*, 49, 447-463, 2001.

*[Lucchesi, 2002]* Lucchesi, D.M., Reassessment of the error modelling of non-gravitational perturbations on LAGEOS II and their impact in the Lense–Thirring determination. Part II, to be published in *Plan. and Space Sci.*, 2002.

*[Mashhoon et al., 1999]* Mashhoon, B., F. Gronwald, and D.S. Theiss, On Measuring Gravitomagnetism via Spaceborne Clocks: A Gravitomagnetic Clock Effect, *Annalen Phys.*, 8, 135-152, 1999.

*[Pavlis and Iorio, 2002]* Pavlis, E. C., and L. Iorio, The impact of tidal errors on the determination of the Lense-Thirring effect from satellite laser ranging, *Int. J. of Mod. Phys. D*, 11, 599-618, 2002.

*[Ries et al., 1998]* Ries, J.C., R.J. Eanes, and B.D. Tapley, Lense-Thirring Precession Determination from Laser Ranging to Artificial Satellites, *Proceedings of The First ICRA Network Workshop and Third William Fairbank Meeting*, Rome and Pescara, Italy, June 28-July 4, 1998, to appear in: Ruffini, R., and C. Sigismondi (Eds.), *Nonlinear Gravitodynamics. The Lense–Thirring Effect*, World Scientific, Singapore, 2002.