Effect of the Quark-Gluon Vertex on Dynamical Chiral Symmetry Breaking

M. Atif Sultan,1 Khépani Raya,2,3 Faisal Akram,1 Adnan Bashir,4 and Bilal Masud1

1Centre For High Energy Physics, University of the Punjab, Lahore (54590), Pakistan.
2School of Physics, Nankai University, Tianjin 300071, China
3Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, Apartado Postal 70-543, C.P. 04510, CDMX, México
4Instituto de Física y Matemáticas, Universidad Michoacana de San Nicolás de Hidalgo, Edificio C-3, Ciudad Universitaria, C.P. 58040, Morelia, Michoacán, México.

(Dated: October 16, 2023)

In this work, we investigate how the details of the quark-gluon interaction vertex affect the quantitative description of chiral symmetry breaking through the gap equation for quarks. We start from two gluon propagator models widely used in literature and constructed in direct connection with our gradually improved understanding of infrared quantum chromodynamics coupled with its exact one-loop limit. The gap equation is then solved by employing a variety of vertex Ansätze, which have been constructed in order to implement some of the key aspects of quantum chromodynamics, namely, multiplicative renormalizability of the quark propagator, gauge invariance, matching with perturbation theory in the weak coupling regime, independence from unphysical kinematic singularities as well as manifestly correct transformation properties under charge conjugation and parity operations. On general grounds, all truncation schemes exhibit the same qualitative and quantitative pattern of chiral symmetry breaking, ensuring the overall robustness of this approach and its potentially reliable description of the hadron spectrum and properties.

PACS numbers: 12.38.Aw, 12.38.-t, 12.38.Lg

I. INTRODUCTION

If quantum chromodynamics (QCD) is the underlying theory of strong interactions, we expect all hadronic observables to be calculable from the complete knowledge of the corresponding Green functions. There is an infinite set of integral field theoretic equations which describe these n-point functions in a coupled and highly non-linear manner. These are the well-known Schwinger-Dyson equations (SDEs) \([1,3]\). Their structure is such that any n-point function is related to at least one higher order Green function; the two point one-particle irreducible (1PI) Green functions (propagators) are related to the three point functions (vertices), which in turn are entangled with the four-point functions (scattering kernels), \textit{ad infinitum}. In a general formalism, not limited to the perturbative domain, this infinite set must be truncated by introducing physically reliable model(s) of some suitable set of Green functions before a solution becomes tractable. The most favorite choice, which lies on the borderline of a daunting computational complexity while still maintaining predictable exploration of hadronic physics, is to model the 3-point vertices whether they be quark-photon or quark-gluon interactions \([4,19]\). It is natural to demand any truncation of SDEs to resemble the true dynamics of quarks and gluons to the fullest extent possible, while successfully describing the observable degrees of freedom, namely, mesons and baryons. Several reviews describe the tremendous success of the SDE approach to our continually improved understanding of QCD, hadron spectrum and properties, see for example \([20,29]\). Ideally, we can impose the following restrictions on the quark-gluon vertex (QGV) which enters the gap equation directly and also constrains the kernel of the Bethe-Salpeter equation accordingly \([27–29]\):

- The QGV must satisfy the Slavnov-Taylor identity (STI) \([30,31]\). This implies that the requirement of gauge invariance fixes the longitudinal part of the quark-gluon interaction, \([19]\). Its abelian counterpart is generally known as the Ball-Chiu vertex, \([4]\). For most practical purposes and abelian-like truncations, one can start from the Ball-Chiu construction as the longitudinal one and push the remaining information in the rich transverse part of it.

- The transverse part is tightly constrained by the requirements of the generalized Landau-Khalatnikov-Fradkin transformations (LKFT) \([32,34]\) and the transverse Takahashi identities (TTI) \([9,35,40]\).

- It should reduce to its perturbation theory Feynman expansion. Note that a truncation scheme of the complete set of SDEs, which maintains multiplicative renormalizability (MR) of the quark propagator and gauge invariance at every level of approximation, is perturbation theory. Therefore, we expect physically meaningful solutions of the SDEs to agree with perturbative results in the weak coupling regime \([18,11,45]\).
the bare vertex as well. For completeness, we have included the results based upon comparison with these refined vertices and the sake of et al. Pennington (KP) [11] and Bashir Chiu (BC) [4], Curtis-Pennington (CP) [5], Kizilersu-

The article is organized as follows: in Section II we discuss the preliminaries of the gap equation, introducing the MT and the QC models. In section III we explicitly discuss all the vertex constructions we employ, highlighting, comparing and contrasting their merits. Section IV details the algebraic expressions for the kernels of the gap equation that stem from the choice of each vertex and Section V contains numerical results as well as a comparative analysis. Finally, in Section VI, we summarize our conclusions and discuss the scope and future applications of this work.

II. GAP EQUATION: PRELIMINARIES AND THE GLUON PROPAGATOR

In order to investigate how DCSB is realized, we naturally start from the renormalized SDE for the quark propagator. This equation is depicted in Fig. 1 and can be written in the following mathematical form:

$$S^{-1}(p) = Z_2(i\gamma \cdot p + m_0) + \Sigma(p),$$  \hspace{1cm} (1)

where $\Sigma(p)$ is the quark self energy defined as

$$\Sigma(p) = Z_1 C_F \int \frac{d^4k}{(2\pi)^4} g^2 D_{\mu \nu}(q) \gamma_\mu S(k) \Gamma_\nu(k, p).$$  \hspace{1cm} (2)

Here $q = k - p$, $C_F = 4/3$ and $\int k \equiv \int \frac{d^4k}{(2\pi)^4}$. $Z_1$ and $Z_2$ are the renormalization constants for the QGV and the quark propagator, respectively, which depend on the ultraviolet regulator ($\Lambda$) and the renormalization point ($\mu$). This equation, also known as the gap equation, involves not only the full quark propagator, $S(p)$, but also the full gluon propagator, $D_{\mu \nu}(q)$, and the fully dressed QGV, $\Gamma_\nu(k, P)$. Each of these Green functions also depend on the renormalization point. However, we have not explicitly displayed this dependence for notational convenience. Moreover, they obey their own SDEs. This intricate structure yields an infinite tower of coupled equations, which must be systematically truncated in order to extract the encoded physics. Regardless of the truncation scheme, the full quark propagator, representing a Dirac particle, can be defined in terms of two scalar functions, namely the mass function, $M(p^2)$, and the quark wavefunction renormalization, $Z(p^2; \mu^2)$, such that:

$$S(p; \mu) = \frac{Z(p^2; \mu^2)}{i\gamma \cdot p + M(p^2)},$$  \hspace{1cm} (3)

in analogy to its bare counterpart

$$S_0(p) = \frac{1}{i\gamma \cdot p + m_0},$$

where $m_0$ is the bare mass of the quark. Equivalent useful representations of $S(p; \mu)$ are:

$$S(p; \mu) = -i\gamma \cdot p \sigma_\nu(p^2; \mu^2) + \sigma_\nu(p^2; \mu^2),$$

$$S^{-1}(p; \mu) = i\gamma \cdot p A(p^2; \mu^2) + B(p^2; \mu^2),$$  \hspace{1cm} (4)
where the dressing functions involved are interrelated as
\[
\sigma_s(k^2; \mu^2) = \frac{B(k^2; \mu^2)}{A(k^2; \mu^2) k^2 + B(k^2; \mu^2)}, \\
\sigma_v(k^2; \mu^2) = \frac{A(k^2; \mu^2)}{A(k^2; \mu^2) k^2 + B(k^2; \mu^2)},
\]
and one can easily identify
\[
M(p^2) = \frac{B(p^2; \mu^2)}{A(p^2; \mu^2)}, \quad Z(p^2) = \frac{1}{A(p^2; \mu^2)}.
\]
Notably, multiplicative renormalizability ensures that the mass function does not depend on the renormalization point. For the simplicity of notation, we will omit displaying the \( \mu \) dependence altogether. The general form of the gluon propagator is:
\[
D_{\mu\nu}(q) = \frac{D(q^2)}{q^2} \left[ \delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right] + \frac{\xi q_\mu q_\nu}{q^4},
\]
where \( D(q^2) \) is the gluon dressing function and \( \xi \) is the covariant gauge parameter. Due to the corresponding Ward identity, the longitudinal term proportional to \( \xi \) does not get corrections at any order of perturbation theory. Hence \( \xi = 0 \) is an obvious first option to work with. It corresponds to the Landau gauge, which is a convenient and natural choice for several reasons. Among others, model dependent differences between various ansätze for the QGV are least noticeable in this gauge [17]. Moreover, it is a covariant gauge which is readily implemented in lattice QCD simulations [14, 25].

The first model of the gluon propagator employed herein is the MT interaction [50], where the effective coupling, \( \alpha_s(q^2) \equiv g^2 D(q^2)/4\pi \), has the following form:
\[
\frac{\alpha_s(q^2)}{q^2} = \frac{\pi D}{\omega^6} q^2 e^{-q^2/\omega^2} + \frac{\gamma_m \pi F(q^2)}{\frac{1}{2} \ln[\tau + (1 + q^2/A_{QCD}^2)]},
\]
with \( F(q^2) = \{1 - e^{-q^2/(4m_t^2)}\}/q^2, \tau = e^2 - 1, \gamma_m = 12/(33 - 2N_f), N_f = 4, m_t = 0.5 \text{ GeV} \) and \( A_{QCD} = 0.234 \text{ GeV} \). The first term provides an infrared enhancement, controlled by the parameters \( \omega \) and \( D \), while the second term reproduces the one-loop renormalization group equation of QCD.

The other model choice is the QC interaction [28]:
\[
\frac{\alpha_s(q^2)}{q^2} = \frac{2\pi D}{\omega^4} e^{-q^2/\omega^2} + \frac{\gamma_m \pi F(q^2)}{\frac{1}{2} \ln[\tau + (1 + q^2/A_{QCD}^2)]},
\]
which differs from the MT model in the infrared enhancement term. It ensures the behavior of the effective gluon is in agreement with our modern understanding of QCD’s gauge sector; in the minimal Landau gauge in 3+1-dimensions, the gluon propagator is a bounded, regular function of spacelike momenta and is infrared enhanced [76]. It has been confirmed that the hadron properties are insensitive to small variations of \( \omega \in [0.4, 0.6] \), so long as the product \( m_G^2 = (\omega D) \) remains constant (typical values of \( m_G \sim 0.4 - 0.8 \text{ GeV} \)) [77, 78]. These models have been extensively employed to study hadron physics through the SDEs of QCD, obtaining a wide range of predictions: meson and baryon spectrum and properties [77, 79, 80], parton distribution amplitudes [77], parton distribution functions [81, 82], electromagnetic elastic [83] and transition form factors [84, 87], hadronic contribution to the anomalous electromagnetic moment of the muon [88–91], etc.

III. QUARK-GLUON VERTEX

For the 1PI QGV, the simplest choice is to replace the fully dressed fermion-boson vertex by its tree level counterpart. Along with the ladder approximation of the meson Bethe-Salpeter equation, it corresponds to the rainbow-ladder truncation. Even in the abelian case of QED, this (bare) vertex manages to satisfy the corresponding WFTI [92, 95] only in the chirally symmetric phase in the Landau gauge and in the leading log approximation for the wave-function renormalization [6]. For these particular choices and limits, it ensures \( Z(p^2; \mu^2) = 1 \) and \( M(p^2) = 0 \). The simplicity of this choice brings even further undesirable features. It obviously lacks all those six basis structures which are dynamically generated through DCSB. Moreover, the associated dressed quark anomalous chromomagnetic moment and electromagnetic distribution in the infrared, associated with DCSB, is much less than what is required from observed hadron phenomenology [96]. Some of these drawbacks can be compensated by a proper choice of parameters in the effective gluon propagator to render good description of pseudoscalar and light vector meson spectra [97]. However, this is not the case with axial vector mesons, since the bare vertex lacks a proper enhancement of the spin-orbit splitting in this channel.

In constructing a fully consistent fermion-boson Ansatz, many efforts have been made over the last few decades. We choose to explore [4, 5, 8, 11] for our numerical investigation. The general form of the vertex consists of 12 linearly independent structures, which can be obtained from three vectors \( k_\mu, p_\mu, \gamma_\mu \) and four spin scalars \( 1, \gamma \cdot k, \gamma \cdot p, \gamma \cdot k \gamma \cdot p \). A first step towards constructing a QGV is employing the STI [30, 31]. In addition to the quark propagator, it also involves the ghost propagator and the quark-quark-ghost-ghost scattering kernel. As we work with infrared enhanced effective one gluon exchange models, we can adopt the Abelian approximation...
of the STI with impunity, namely, the WGFTI which entails:
\[ i q_\mu \Gamma_\mu = S^{-1}(k) - S^{-1}(p) . \]
Following the usual arguments of Ball and Chiu, WGFTI allows this vertex to be decomposed into a longitudinal and a transverse part
\[ \Gamma_\mu(k, p) = \Gamma_\mu^{L}(k, p) + \Gamma_\mu^{T}(k, p) , \]
such that \( q_\mu \Gamma_\mu^{T} = 0 \) and the longitudinal term \( \Gamma_\mu^{L} \) is fixed by the above WGFTI. In the so-called Ball-Chiu basis, \( \Gamma_\mu^{L} \) is written as:
\[ \Gamma_\mu^{L(BC)}(k, p) = \lambda_1(k^2, p^2) \gamma_\mu - i \lambda_2(k^2, p^2) t_\mu + \lambda_3(k^2, p^2) t_\mu \gamma \cdot t/2 + \lambda_4(k^2, p^2) t_\mu \sigma_{\mu\nu} , \]
where the dressing functions are
\[ \lambda_1(k^2, p^2) = \Delta_A(k^2, p^2) , \]
\[ \lambda_2(k^2, p^2) = \Delta_B(k^2, p^2) , \]
\[ \lambda_3(k^2, p^2) = \Delta_A(k^2, p^2) , \]
\[ \lambda_4(k^2, p^2) = 0 , \]
with \( t = k + p \), \( (k^2 - p^2) \Delta_\varphi(k^2, p^2) \equiv \varphi(k^2) - \varphi(p^2) \) and \( 2\Sigma(k^2, p^2) \equiv \varphi(k^2) + \varphi(p^2) \). A generalization of the BC vertex to the non-Abelian case can be found in [19]. Note that although \( \lambda_1 = 0 \) for QED, the contribution coming from the triple gluon vertex in QCD ensures that it is non-zero for the latter case [41]. However, as we work with the abelianized version of QCD, we will stick to \( \lambda_1 = 0 \). Notice also that \( \lambda_2 \) carries an explicit dependence on the mass function. It implies that its appearance in the chiral limit owes itself entirely to DCSB. In the following, we will take \( \Gamma_\mu^{L} = \Gamma_\mu^{L(BC)} \) and discuss different choices of \( \Gamma_\mu^{T} \).

The transverse part is decomposed as a linear combination of the 8 basis vectors \( T_{i\mu} \), that is
\[ \Gamma_\mu^{T} = \sum_{i=1}^{8} \tau_i(k^2, p^2, q^2) T_{i\mu}(k, p) , \]
where \( \tau_i \) are unknown scalar functions. Rather generally, the basis vectors can be written as:
\[ T_{1\mu}(k, p) = i [p_\mu (k \cdot q) - k_\mu (p \cdot q)] , \]
\[ T_{2\mu}(k, p) = \{p_\mu (k \cdot q) - k_\mu (p \cdot q)\} \gamma \cdot t , \]
\[ T_{3\mu}(k, p) = q^2 \gamma_\mu - q_\mu \gamma \cdot q , \]
\[ T_{4\mu}(k, p) = iq^2 [\gamma_\mu \gamma \cdot t - t_\mu] + 2q_\mu p_\mu k_\lambda \sigma_{\nu\lambda} , \]
\[ T_{5\mu}(k, p) = \sigma_{\mu\nu} q_\nu , \]
\[ T_{6\mu}(k, p) = \gamma_\mu (p^2 - k^2) + t_\mu \gamma \cdot q , \]
\[ T_{7\mu}(k, p) = i/2 (k^2 - p^2) [\gamma_\mu \gamma \cdot t - t_\mu] + t_\mu p_\mu k_\lambda \sigma_{\nu\lambda} , \]
\[ T_{8\mu}(k, p) = -i \gamma_\mu p_\mu k_\lambda \sigma_{\nu\lambda} + k_\mu \gamma \cdot p - p_\mu \gamma \cdot k . \]

Note that
\[ q_\mu T_{i\mu}(k, p) = 0 \quad i = 1, \ldots , 8 . \]

This basis is not the one employed in [4]. We choose to work with a modification of this initial basis which was put forward in [98] and later employed in [41] as well. This latter, which we have adopted here, choice ensures all transverse form factors of the vertex are independent of any kinematic singularities in one-loop perturbation theory in an arbitrary covariant gauge.

The determination of the coefficients \( \tau_i \) is not arbitrary. To a reasonable extent, they are constrained by the TTI, LKFT, MR, freedom of kinematic singularities and the adequate perturbation theory limit in the weak coupling regime [4, 9, 38].

Curtis and Pennington [5] adopted a simple choice of the transverse coefficients, which ensures MR of the massless electron propagator in the quenched approximation of quantum electrodynamics (QED). This transverse part of the vertex, referred to as the CP vertex, is merely:
\[ \Gamma_\mu^{T(CP)} = \frac{\gamma_\mu (k^2 - p^2) - t_\mu \gamma \cdot t}{2 d(k, p)} [A(k^2) - A(p^2)] , \]
where \( t = k + p \) and
\[ d(k, p) = \frac{(k^2 - p^2)^2 + [M^2(k^2) + M^2(p^2)]^2}{k^2 + p^2} . \]

There is a peculiar \( [M^2(k^2) + M^2(p^2)]^2 \) factor in this Ansatz. Notice that its absence introduces an unwanted kinematic singularity. Moreover, it does not jeopardize the MR of the massless electron propagator by construction.

In a subsequent work, Kizilersu and Pennington [11] proposed two vertex constructions for the unquenched case, in the chiral limit with \( n_f = 1 \). On using any of these two Ansätze in the SDEs for the perturbative photon and massless fermion propagators simultaneously, they get the correct power law behavior for the photon dressing function and the fermion wave-function renormalization. Both proposals satisfy the same constraints and differ only beyond the leading logarithmic order, while also giving similar results in the Landau gauge [11, 99]. Thus, one could use either. We choose to work with the the following KP construction:
\[ \Gamma_\mu^{T(KP)} = \tau_2 T_{2\mu} + \tau_3 T_{3\mu} + \tau_6 T_{6\mu} + \tau_8 T_{8\mu} . \]
where the dressing functions are:

\[
\begin{align*}
\tau_2(k^2, p^2, q^2) &= -\frac{4}{3} \frac{1}{k^2 - p^2} (A(k^2) - A(p^2)) \\
&\quad - \frac{1}{3} A(k^2) + A(p^2) \ln \left[ \frac{A(k^2) A(p^2)}{A(q^2)^2} \right], \\
\tau_3(k^2, p^2, q^2) &= \frac{1}{12} k^2 - p^2 A(k^2) - A(p^2)) \\
&\quad + \frac{1}{6} \frac{1}{k^2 - p^2} (A(k^2) - A(p^2)) \, \ln \left[ \frac{A(k^2) A(p^2)}{A(q^2)^2} \right], \\
\tau_6(k^2, p^2, q^2) &= -\frac{1}{4} \frac{1}{k^2 + p^2} (A(k^2) - A(p^2)) \, , \\
\tau_8(k^2, p^2, q^2) &= 0 .
\end{align*}
\]

In 2012, Bashir et al. [8] put forward a family of fermion-boson vertices expressed solely in terms of the fermion and scalar functions appearing in the fermion propagator. Among other requirements, constraints on \( a_i \) ensure the Ansatz is consistent with one-loop perturbation theory. For the sake of computational simplicity, the coefficients of the transverse basis are chosen to be independent of the angle between the relative momenta. Strikingly, it also has no explicit dependence on the covariant-gauge parameter. Residual freedom of choice for \( a_i \) allows us to achieve the gauge-independence of the critical coupling in QED, above which chiral symmetry is dynamically broken. The set of scalar functions \( \tau_i \) for this proposal are written as:

\[
\begin{align*}
\tau_1(k, p) &= a_1 \Delta_B(k^2, p^2), \\
\tau_2(k, p) &= a_2 \Delta_A(k^2, p^2), \\
\tau_3(k, p) &= a_3 \Delta_A(k^2, p^2), \\
\tau_4(k, p) &= \frac{a_4 \Delta_B(k^2, p^2)}{[k^2 + M^2(k^2)][k^2 + M^2(p^2)]} k^2 - p^2 , \\
\tau_5(k, p) &= +a_5 \Delta_B(k^2, p^2), \\
\tau_6(k, p) &= -\frac{a_6 (k^4 - p^4)}{[k^2 - p^2]^2 + (M^2(k^2)^2 + M^2(p^2)]^2}, \\
\tau_7(k, p) &= +\left[ \frac{a_7}{k^2 + p^2} + \frac{2(k^2 - p^2)}{k^2 - p^2} \tau_i \right] \Delta_B(k^2, p^2) , \\
\tau_8(k, p) &= a_8 \Delta_A(k^2, p^2) ,
\end{align*}
\]

where \( a_i \) are momentum-independent constants whose values are listed in Table I. Such constants are interconnected by numerous constraints from perturbation theory and gauge covariance. The fixing procedure can be found in Ref. [8]. We call this proposal the BB vertex. In the next section we shall discuss the gap equation with all these vertices.

### IV. GAP EQUATION

Dressing functions \( B(k^2) \) and \( A(k^2) \) can be decoupled through proper projections of Eq. [1], viz., multiplying

| Constant | \( a_1 \) | \( a_2 \) | \( a_3 \) | \( a_4 \) | \( a_5 \) | \( a_6 \) | \( a_7 \) | \( a_8 \) |
|----------|---------|---------|---------|---------|---------|---------|---------|---------|
| Value    | 0       | 3.4     | 1       | 1       | -4/3    | -1/2    | 2.167   | -3.7    |

**TABLE I.** A choice of momentum and gauge-independent coefficients of the transverse basis in the BB fermion-boson vertex [8].

Eq. (1) by \( 1 \) and \( p \), respectively, and then taking traces.

**The bare vertex:** For this approximation \( \Gamma_\mu = \gamma_\mu \), quark self-energy of Eq. (2) acquires the following simple form

\[
\Sigma(p) = Z_3 C_F \int k q^2 D_{\mu\nu}(q) \gamma_\mu S(k)(Z_2\gamma_\nu) .
\]

Using the steps suggested above, one arrives at the expressions:

\[
\begin{align*}
B(p^2) &= m_0 Z_2 + 16\pi Z_2^2 \int_k \frac{\alpha_s(q^2)}{q^2} \sigma_s(k^2) , \\
A(p^2) &= Z_2 + \frac{16\pi}{3} Z_2^2 \int_k \frac{\alpha_s(q^2)}{q^2} \\
&\quad \times \sigma_v(k^2) \left[ k \cdot p + \frac{2 k \cdot q p \cdot q}{q^2} \right] .
\end{align*}
\]

This minimal Ansatz neglects any non-Abelian contribution to the QGV Therefore, for the sake of consistency, we equate \( Z_1 = Z_2 \). In fact, we can continue to use it with impunity for the BC, CP, KP and BB vertices as they were all proposed in an Abelian set up. Note that this reasoning is no longer valid if the QGV employed is constructed form the corresponding STI because this extended identity incorporates the effects coming from the non-Abelian ghost-gluon sector. However, note that the renormalization boundary condition, independently of the truncation, entails

\[
S^{-1}(p)|_{p^2 = m^2} = i\gamma \cdot p + m(\mu) ,
\]

where \( m(\mu) = M(\mu^2) \) is the scale dependent running quark mass. The above condition implies \( A(p^2 = \mu^2) = 1 \) and \( B(p^2 = \mu^2) = m(\mu) \). One can thus define a convenient renormalization point invariant mass as follows:

\[
\hat{m} = m(\mu) \left[ \frac{1}{2} \ln \left( \frac{\mu^2}{\Lambda_{QCD}^2} \right) \right]^{\mu_m} .
\]

**The BC vertex:** Returning to the gap equation and the QGV, if one employs the BC vertex, Eq. (17) modifies as:

\[
B(p^2) = m_0 Z_2 + \frac{16\pi}{3} Z_2 \int_k \frac{\alpha_s(q^2)}{q^2} \\
&\quad \times \{ \sigma_s(k^2) | I_{BC}^{B_1} + I_{BC}^{B_2} \sigma_v(k^2) | I_{BC}^{B_3} \} .
\]
where \( I_{BC}^{B_1}, I_{BC}^{B_2} \) and \( I_{BC}^{B_3} \) are the integrands related to the BC vertex, such that

\[
I_{B_1}^{BC} = \frac{3}{2} \frac{A(k^2) + A(p^2)}{2},
\]

\[
I_{B_2}^{BC} = \Delta_A(k^2, p^2) \left( \frac{t^2 q^2 - (t \cdot q)^2}{2q^2} \right),
\]

\[
I_{B_3}^{BC} = \Delta_B(k^2, p^2) \left( \frac{q^2 t \cdot k - k \cdot q q \cdot k}{q^2} \right).
\]

Analogously, the corresponding equation for \( A(p^2) \) is:

\[
A(p^2) = Z_2 + \frac{16\pi}{3} Z_2 \int \frac{\alpha_s(q^2)}{q^2} \times \left\{ \sigma_v(k^2)[I_{A_1}^{BC} - I_{A_3}^{BC}] + \sigma_s(k^2)[I_{A_3}^{BC}] \right\},
\]

where the integrands are written as

\[
I_{A_1}^{BC} = \frac{A(k^2) + A(p^2)}{2} \frac{1}{p^2} \left\{ k \cdot p q^2 + 2 \left( [k^2 + p^2] k \cdot p - k^2 p^2 - k \cdot p^2 \right) \right\},
\]

\[
I_{A_2}^{BC} = \frac{1}{2p^2} \Delta_A(k^2, p^2),
\]

\[
I_{A_3}^{BC} = \frac{1}{2p^2} \left\{ \frac{p^2 t \cdot k - k \cdot q - k^2 t \cdot q q \cdot p}{q^2} \right\},
\]

The CP vertex: By taking into account the transverse CP vertex, one arrives at:

\[
B(p^2) = m_0 Z_2 + \frac{16\pi}{3} Z_2 \int \frac{\alpha_s(q^2)}{q^2} \times \left\{ \sigma_v(k^2)[I_{B_1}^{BC} + I_{B_2}^{BC}] + \sigma_s(k^2)[I_{B_3}^{BC}] \right\},
\]

where \( L \equiv L(k^2, p^2) \) is defined as

\[
L = \left[ A^2(k^2) A^2(p^2) \right]^2 \Delta_A(k^2, p^2)
\]

\[
= \left[ A^2(k^2) A^2(p^2) \right]^2 \Delta_A(k^2, p^2).
\]

On the other hand, the analogous expression for \( A(p^2) \) reads as:

\[
A(p^2) = Z_2 + \frac{16\pi}{3} Z_2 \int \frac{\alpha_s(q^2)}{q^2} \times \left\{ \sigma_v(k^2)[I_{A_1}^{BC} - I_{A_3}^{BC}] + \sigma_s(k^2)[I_{A_3}^{BC}] \right\} + 2 \sigma_v(k^2) \left( \frac{k^2 + p^2}{k^2 - 2p^2} [I_{C_1}^{CP} + I_{C_2}^{CP}] L(k^2, p^2) \right)
\]

\[
= \left\{ \frac{3}{2} \left( k \cdot p q^2 + 2 \left( [k^2 + p^2] k \cdot p - k^2 p^2 - k \cdot p^2 \right) \right) \right\},
\]

The integrands \( I_{A_1}^{CP} \) and \( I_{A_2}^{CP} \), related to the CP term, are

\[
I_{A_1}^{CP} = (k^2 - p^2) \left\{ \frac{3(k^2 + p^2) k \cdot p - 2k^2 p^2 - 4k \cdot p^2}{q^2} \right\},
\]

\[
I_{A_2}^{CP} = k^2 t \cdot p - p^2 t \cdot k + \frac{p^2 t \cdot q k - q t \cdot k q \cdot p}{q^2}.
\]

The KP vertex: The KP vertex Ansatz yields the following equation for \( B(p^2) \):

\[
B(p^2) = m_0 Z_2 + \frac{16\pi}{3} Z_2 \int \frac{\alpha_s(q^2)}{q^2} \times \left\{ \sigma_v(k^2)[I_{B_1}^{BC} + I_{B_2}^{BC}] + \sigma_s(k^2)[I_{B_3}^{BC}] \right\} + \langle A(k^2) \rangle \left( \frac{A(k^2)}{A(q^2)} \right)^2 \}
\]

\[
= \left\{ \frac{1}{3} \left( k \cdot p^2 - k^2 p^2 \right) \left( \frac{4 A(k^2) - A(p^2)}{3} \right) \right\}
\]

\[
+ \left\{ \frac{1}{3} \left( k \cdot p^2 - k^2 p^2 \right) \left( \frac{A(k^2)}{A(q^2)} \right)^2 \right\}
\]

\[
= \left\{ \frac{5}{4} \Delta_A(k^2, p^2) \right\}
\]

\[
= \left\{ \frac{5}{4} \Delta_A(k^2, p^2) \right\}
\]

\[
= \left\{ \frac{5}{4} \Delta_A(k^2, p^2) \right\}
\]

\[
= \left\{ \frac{5}{4} \Delta_A(k^2, p^2) \right\}
\]

The corresponding equation for \( A(p^2) \), for KP vertex, is:

\[
A(p^2) = Z_2 + \frac{16\pi}{3} Z_2 \int \frac{\alpha_s(q^2)}{q^2} \times \left\{ \sigma_v(k^2)[I_{A_1}^{BC} - I_{A_3}^{BC}] + \sigma_s(k^2)[I_{A_3}^{BC}] \right\} + \sigma_v(k^2) \left( \frac{I_{A_1}^{CP} + I_{A_2}^{CP} - I_{A_3}^{KP}}{2} \right)
\]

\[
= \left\{ \frac{3}{4} (k^2 - p^2) \right\}
\]

\[
= \left\{ \frac{3}{4} (k^2 - p^2) \right\}
\]

\[
= \left\{ \frac{3}{4} (k^2 - p^2) \right\}
\]

\[
= \left\{ \frac{3}{4} (k^2 - p^2) \right\}
\]

The integrands related specifically to the KP vertex, \( I_{A_1}^{KP}, I_{A_2}^{KP} \) and \( I_{A_3}^{KP} \), can be expressed as

\[
I_{A_1}^{KP} = \left( \frac{k \cdot p^2 - k^2 p^2}{k^2 - 2p^2} \right) \left( \frac{4 A(k^2) - A(p^2)}{3} \right)
\]

\[
+ \left\{ \frac{1}{3} \left( k \cdot p^2 - k^2 p^2 \right) \left( \frac{A(k^2)}{A(q^2)} \right)^2 \right\}
\]

\[
I_{A_2}^{KP} = \left( \frac{k \cdot p^2 - k^2 p^2}{k^2 - 2p^2} \right) \left( \frac{A(k^2)}{A(q^2)} \right)^2
\]

\[
I_{A_3}^{KP} = \left( \frac{3}{4} \right) \left( k \cdot p^2 - k^2 p^2 \right)
\]

\[
= \left\{ \frac{3}{4} \right) \left( k \cdot p^2 - k^2 p^2 \right)
\]

\[
= \left\{ \frac{3}{4} \right) \left( k \cdot p^2 - k^2 p^2 \right)
\]

Unlike the other vertex Ansätze, the transverse part of the KP vertex introduces a non-trivial angular dependence, in connection to the logarithmic terms which contain \( A(q^2) \). Thus, the numerical evaluation of such integrals is considerably more complicated.

The BB vertex: Finally, the integral equations for the scalar functions \( B(p^2) \) and \( A(p^2) \), using the BB vertex, [8]
are written as:

\[B(p^2) = \text{r.h.s. of Eq. (20)}\]

\[- \frac{16\pi}{3} Z^2 \int_k \frac{\alpha_s(q^2)}{q^2} k^2 A^2(k^2) \frac{1}{k^2 A^2(k^2) + B^2(k^2)} \times \left\{ A(k^2)(k \cdot p)^2 - k^2 p^2 \right\} \tau_1 + 2B(k^2)((k \cdot p)^2 - k^2 p^2) \tau_2 - 3B(k^2)(k^2 + p^2 - 2k \cdot p) \tau_3 + A(k^2)[k^2(p^2 - 3k \cdot p) + k \cdot p(3p^2 - 4k \cdot p)] \tau_4 + 3A(k^2)(k^2 - k \cdot p) \tau_5 + 3B(k^2)(k^2 - p^2) \tau_6 + \frac{A(k^2)}{2} [k^2(3k \cdot p - p^2) - k \cdot p(2k \cdot p + 3p^2) + 3k^4] \tau_7 + 3\alpha_8 B(k^2) k \cdot p \Delta_A(k^2, p^2) \right\}, \tag{26}\]

\[A(p^2) = \text{r.h.s. of Eq. (21)}\]

\[- \frac{16\pi}{3} Z^2 \int_k \frac{\alpha_s(q^2)}{q^2} k^2 A^2(k^2) \frac{1}{k^2 A^2(k^2) + B^2(k^2)} \times \left\{ A(k^2)(k \cdot p)^2 - k^2 p^2 \right\} \tau_1 + 2B(k^2)((k \cdot p)^2 - k^2 p^2) \tau_2 - 3B(k^2)(k^2 + p^2 - 2k \cdot p) \tau_3 + A(k^2)[k^2(p^2 - 3k \cdot p) + k \cdot p(3p^2 - 4k \cdot p)] \tau_4 + 3A(k^2)(k^2 - k \cdot p) \tau_5 + 3B(k^2)(k^2 - p^2) \tau_6 + \frac{A(k^2)}{2} [2(k \cdot p)^2 - 3p^2(k \cdot p) + k^2(3k \cdot p + p^2) - 3p^4] \tau_7 - 2A(k^2)(k \cdot p)^2 - k^2 p^2 \tau_8 \right\}. \tag{27}\]

In the next section, we present and discuss the numerical results obtained from employing different vertex Ansätze we chose to study the gap equation with.

V. RESULTS

We solve the gap equation for a few largely employed quark-gluon vertices suggested in literature over the last three decades. This exercise is carried out in conjunction with effective MT and QC models for the gluon propagator. The renormalization point is set to \(\mu = \mu_3 = 2.86\) GeV. The infrared strength of the gluon models is fixed from the chiral quark condensate \([100]\), as we now explain. Note that the chiral limit is defined when \(m = 0\) \((\mu \to \infty)\) in Eq. (19) and we label it as \(m_q\). In this limit, we can express the chiral quark condensate as follows:

\[-\langle \bar{q}q \rangle^0_m = Z_4 N_c \text{Tr} \int_k S_m(k; \mu) . \tag{28}\]

We can thus define a renormalization point invariant condensate \(\langle \bar{q}q \rangle\) as

\[m(\mu)\langle \bar{q}q \rangle^0_m = \hat{m}\langle \bar{q}q \rangle . \tag{29}\]

![FIG. 2. [MT model] Mass functions for different current quark masses and QGV Ansätze. The boundaries of the bands are given by the lowest and highest produced values of \(M(p^2)\), for the different vertices: BC, CP, KP, BB and the bare vertex. Purple band with dotted boundaries corresponds to the chiral limit. Blue (dot dashed boundary), green (dashed boundary) and red (solid boundary) bands correspond to \(m_q = 0.1\) GeV, \(m_q = 1\) GeV and \(m_q = 4.1\) GeV, respectively. Mass functions or each quark flavor have been normalized such that \(M_{\text{max}}(0) = 1\). Bare vertex results are highlighted with a thicker line which forms the top edge of each band.](image)

It is an order parameter of DCSB and, as explained elsewhere \([101, 102]\), it is also the chiral limit value of the in-meson condensate. Therefore, it is natural to fix the effective gluon strength to produce a reasonable value of the chiral quark condensate and study its impact on other quantities. We fix MT and QC gluon
model parameters, \( \omega \) and the product \( \omega D \), to obtain
\(-<\bar{q}q>_\mu_0^0 = (0.256 \text{ GeV})^3. \) Along with Eq. (29), this
value yields: \(-<\bar{q}q>^0_{\mu_2} = (0.250 \text{ GeV})^3 \) and
\(-<\bar{q}q>^0_{\mu_19} = (0.280 \text{ GeV})^3 \) (where \( \mu_{2,19} = 2, 19 \text{ GeV} \), in agreement
with modern estimates [87]. The specific choice of parameters is displayed in Table [H] The resulting chiral
limit mass functions are shown in Figs. [2,3] along
with those obtained for different non-zero current quark
masses: \( m(\mu) = 0.004, 0.1, 1.0, 4.1 \text{ GeV} \), labeled as
\( m_{u/d}, m_s, m_c \) and \( m_b \), respectively (the \( m_{u/d} \) results
are not displayed, in order to avoid overlap with the chiral
limit results). In an intermediate range of momenta,
Figs. [4,5] show a more pronounced comparison of chiral
limit results for different Ansätze of the QGV.

The mass functions exhibit the expected features,
namely: saturation at a finite value as \( p^2 \to 0 \) and
a monotonic decrease as \( p^2 \) increases. The saturation value,
\( M(0) \), in comparison with the current quark mass, is ex-
pectedly much larger in the case of the light quarks and it
decreases sharply with increasing \( p^2 \), whereas it exhibits
far less steep running for the heavy quarks. It is clear that
dynamical mass generation via strong-interaction processes
(DCSB) is the dominant mass generating mechanism in
the light sector, while the heavy sector is largely
overshadowed by its predominant coupling to the Higgs
field.

Also readily observed is the fact that bare vertex
calculations tend to produce larger values of \( M(0) \) (although
the asymptotic behavior of \( M(p^2) \) for \( p^2 \to \infty \) is reached
faster). This is a consequence of the artificial enhancement
of the effective coupling in order to produce suf-
ficient amount of phenomenologically required DCSB.
In fact, if the infrared gluon model were enhanced just as
much as with the other vertices, chiral condensate and
\( M(0) \) would decrease 40 – 60%. Moreover, had we omitted
the bare vertex results, the bands in Figs. [2,3] would
have become much narrower. This is exactly what is dis-
played in Figs. [3,7]. It clearly indicates that the results
obtained from properly constructed quark-gluon vertices
are more robust and less sensitive to the gluon models
parameters.

Another important feature of the mass function is its
asymptotic behavior. In the chiral limit: [100, 103]:

\[
M(p^2 \to \infty) \sim \frac{\ln \left( \frac{p^2}{\Lambda_{QCD}^2} \right)}{p^2} \gamma^{-1}.
\]  \hspace{1cm} (30)

Naturally, since bare vertex is the leading order term in
the perturbative expansion, mass function reaches this
behavior faster in such case. It is followed (consistently
with both MT and QC interactions) by CP, BC, KP and
BB vertices, respectively. Beyond the chiral limit, Eq.

\footnote{The strength needed to simultaneously produce reasonable values
of vacuum quark condensate, mass spectrum and decay constants.}
Dynamical Mass

\[ p^2 [\text{GeV}^2] \]

\[ M(p^2) \]

\[ 10^{-3} 10^{-2} 10^{-1} 10^0 10^1 10^2 10^3 \]

\[ 0.0 0.2 0.4 0.6 0.8 1.0 \]

\[ p^2 \]

\[ M^2 = \{ p^2 \mid p^2 = M^2(p^2) \} \]

\[ m_{\text{crit}} \equiv \{ m(\mu) \mid \tilde{M}(m) = 1/2 \} \]

Therefore, one can interpret the vicinity around \( m_{\text{crit}} \), as the explicit mass generation becomes dominant.

Similarly, \( \bar{M}^2 \) is modified by including an extra term, which is proportional to the current quark mass [100, 103], but the overall pattern persists.

To further understand the interplay between explicit and dynamical mass generation, let us define the following quantity:

\[ \bar{M}(m) = \left| \frac{M_E - m(\mu)}{M_E} \right|, \quad (31) \]

where \( M_E \), the constituent Euclidian mass, is defined through

\[ M_E^2 \equiv \{ p^2 \mid p^2 = M^2(p^2) \} \].

\[ f^2 = \frac{3}{4\pi^2} \int dp^2 \frac{p^2 Z(p^2)M(p^2)}{p^2 + M^2(p^2)} \left[ M(p^2) - \frac{p^2}{2} \bar{M}'(p^2) \right] \]

and from the improved Pagels-Stokar-Cornwall formula derived in [105]:

\[ f^2 = \frac{3}{8\pi^2} \int dp^2 p^2 B^2(p^2) \left( \sigma^2_{\mu} - 2[\sigma^2_{\mu} + p^2\sigma^2_{\nu}] \right) \]

\[ - p^2[\sigma^2_{\mu} - \sigma^2_{\nu}] - p^4[\sigma^2_{\mu} - \sigma^2_{\nu}] \]

\[ \sigma_{\mu, \nu}(p^2) \]

where the dependence of \( \sigma_{\mu, \nu} \) on \( p^2 \) has been omitted for notational convenience. We denote Eqs. (32)-(33) as F.1 and F.2, respectively. The obtained values are shown in Table V. Unsurprisingly, for all dressed vertices employed
\begin{table}[h]
\centering
\begin{tabular}{lcccccc}
\hline
Vertex & $m_q$ & $m_{u/d}$ & $m_s$ & $m_c$ & $m_b$ \\
\hline
MT Model & \cite{50} & & & & \\
Bare & 0.484 & 0.492 & 0.649 & 1.464 & 4.223 \\
BC & 0.331 & 0.337 & 0.468 & 1.284 & 4.186 \\
CP & 0.315 & 0.323 & 0.468 & 1.279 & 4.184 \\
KP & 0.306 & 0.315 & 0.471 & 1.297 & 4.186 \\
BB & 0.353 & 0.356 & 0.483 & 1.186 & 4.072 \\
QC Model & \cite{28} & & & & \\
Bare & 0.573 & 0.581 & 0.730 & 1.520 & 4.240 \\
BC & 0.360 & 0.366 & 0.485 & 1.295 & 4.188 \\
CP & 0.399 & 0.403 & 0.502 & 1.284 & 4.185 \\
KP & 0.330 & 0.339 & 0.479 & 1.296 & 4.187 \\
BB & 0.435 & 0.438 & 0.512 & 1.186 & 4.055 \\
\hline
\end{tabular}
\caption{Calculated constituent quark masses $M(0)$ for different current quark masses and QGV Ansätze. Dimensioned quantities are expressed in GeV. Gluon model parameters are shown in Table \textbf{II}.}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{lcccc}
\hline
Vertex & $m_q$ & $m_{u/d}$ & $m_s$ & $m_b$ \\
\hline
MT Model & \cite{50} & & & \\
Bare & 0.388 & 0.395 & 0.523 & 1.274 & 4.016 \\
BC & 0.301 & 0.307 & 0.417 & 1.184 & 4.037 \\
CP & 0.290 & 0.297 & 0.419 & 1.181 & 4.038 \\
KP & 0.280 & 0.287 & 0.430 & 1.198 & 4.036 \\
BB & 0.344 & 0.347 & 0.445 & 1.186 & 4.072 \\
QC Model & \cite{50} & & & \\
Bare & 0.442 & 0.449 & 0.574 & 1.303 & 4.009 \\
BC & 0.319 & 0.324 & 0.429 & 1.191 & 4.036 \\
CP & 0.347 & 0.352 & 0.439 & 1.183 & 4.038 \\
KP & 0.302 & 0.310 & 0.435 & 1.199 & 4.036 \\
BB & 0.390 & 0.391 & 0.449 & 1.173 & 4.055 \\
\hline
\end{tabular}
\caption{Calculated Euclidean constituent quark masses $M_E$ for different current quark masses and QGV Ansätze. Dimensioned quantities are expressed in GeV. Gluon model parameters are shown in Table \textbf{II}}
\end{table}

TABLE V. Chiral limit decay constants computed from Eqs. \textbf{32}--\textbf{33} (F.1 and F.2, respectively). Dimensioned quantities are expressed in GeV.

\begin{table}[h]
\centering
\begin{tabular}{lcccccc}
\hline
Vertex & F.1 & F.2 & F.1 & F.2 \\
\hline
MT Model & \cite{50} & QC Model & \cite{28} & & & \\
Bare & 0.088 & 0.101 & 0.088 & 0.106 \\
BC & 0.083 & 0.093 & 0.079 & 0.093 \\
CP & 0.082 & 0.091 & 0.082 & 0.094 \\
KP & 0.080 & 0.090 & 0.078 & 0.090 \\
BB & 0.085 & 0.100 & 0.088 & 0.103 \\
\hline
\end{tabular}
\caption{Calculated Euclidean constituent quark masses $M_E$ for different current quark masses and QGV Ansätze. Dimensioned quantities are expressed in GeV. Gluon model parameters are shown in Table \textbf{II}}
\end{table}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig9}
\caption{QC model Effective coupling parameterized as in Eq. \textbf{34}. The black line corresponds to the effective coupling associated with the bare vertex. Results for the other vertices lie within the band whose height is considerably diminished.}
\end{figure}

Finally, motivated by the GellMann-Oakes-Renner relationship (see \textbf{102}, for example) and our values of chiral condensate and decay constants, one could argue that $m_{sg}$ can be accurately obtained from realistic solutions of the Bethe-Salpeter equation, with a fully-consistent symmetry-preserving kernel. This is an outstanding challenge that we shall address elsewhere.
VI. CONCLUSIONS AND SCOPE

We have investigated the features of the dressed QGV and their impact on DCSB through the SDE for the quark propagator. Within a small phenomenologically sensible variation of the MT and QC model parameters, fixed solely by the chiral quark condensate, the results obtained from the refined vertex Ansätze exhibit very similar quantitative behavior. The robustness of the momentum-dependent mass function and the pion decay constant suggests that hadron observables could be accurately reproduced. Though the bare vertex results for the condensate and the decay constant compare well with other truncations, a notorious infrared enhancement (in the gluon models) is required. Firstly, recall that half of the structures which define the QGV can only contribute if chiral symmetry is dynamically broken. Secondly, there is a natural interplay between the role of the gluon propagator and the QGV. In order to generate required amount of DCSB, the bare vertex result depends on large infrared enhancement of the gluon propagator as it receives no such contribution from the dynamically generated vertex structures which are left out in this truncation scheme. A realistic and currently converging understanding of the gluon propagator can generate an acceptable running quark mass, via QCD’s gap equation, only as long as the QGV exhibits material infrared enhancement itself. Thus an intimate connection between the QGV and the DCSB is established. In this article, employing the MT and QC gluon models, we solve the gap equation using the following vertex Ansätze: bare, BC, CP, KP and BB. All truncations described herein point towards the same qualitative pattern of DCSB. Expectedly, apart from the bare vertex, the infrared enhancement band of the mass function for all the other Ansätze is rather narrow. Its width is what we expect to introduce error bars when we predict hadron observables using this formalism.

The light quarks, weakly coupled to the Higgs field, owe their mass primarily to the infrared QCD dynamics. As one moves towards the heavy sector, weak mass generation commensurate with that coming from QCD’s strong interactions; it is between the strange and charm quark masses (but closer to the former) that emergent and explicit mass generation have equal strength. These are qualitatively robust features of the SDE studies [87,104], independent of the details of the truncation.

To enhance the connection with hadron physics, it would be worth investigating if the vertex Ansätze studied in this work are suitable for use in the non-perturbative studies of sophisticated hadron physics phenomenology in its fine details, the electromagnetic and transition form factors.

An immediate task would be writing a consistent Bethe-Salpeter kernel for all those vertices. It is known that, along with the bare QGV, a ladder-like kernel is sufficient for many needs, providing an accurate description of light pseudo-scalars and vector mesons (see for example, [77,84,86,107]). Nevertheless, for a fully-dressed QGV, the construction of a consistent Bethe-Salpeter kernel could be the next challenge [27,29]. Moreover, DCSB generates a momentum-dependent dressed-quark anomalous chromomagnetic moment, which is large at infrared momenta and has an impact on the mass splitting between parity partners [8,96,108]. Thus, we strongly believe that the truncations which go beyond RL should be relevant for a variety of hadron properties, including the spectrum of the excited states, and the nucleon electromagnetic elastic and transition form factors such as [47,48,109,110]. Some of those aspects are currently being investigated and will be reported elsewhere.

VII. ACKNOWLEDGEMENTS

This research was partly supported by Coordinación de la Investigación Científica (CIC) of the University of Michoacan and CONACYT-Mexico through Grants No. 4.10 and CB2014-22117, respectively. KR acknowledges support from CONACYT-Mexico. FA acknowledges the financial support of HEC of Pakistan through Project No. 20-4500/NRPU/R&D/HEC/14/727.

[1] Julian S. Schwinger. On the Green’s functions of quantized fields. 1. Proc. Nat. Acad. Sci., 37:452–455, 1951.

[2] Julian S. Schwinger. On the Green’s functions of quantized fields. 2. Proc. Nat. Acad. Sci., 37:455–459, 1951.
[39] Yi-Da Li and Qing Wang. Beyond Symmetries: Anomalies in Transverse Ward-Takahashi Identities. *Phys. Rev. D*, 102(5):056008, 2020.

[40] Cui-Bai Luo and Hong-Shi Zong. Transverse Ward-Takahashi identities and full vertex functions in different representations of QED$_3$. *Chin. Phys. C*, 44(7):073105, 2020.

[41] Andrei I. Davydychev, P. Osland, and L. Saks. Quark gluon vertex in arbitrary gauge and dimension. *Phys. Rev.*, D63:014022, 2001.

[42] A. Bashir, A. Kizilersu, and M. R. Pennington. Analytic form of the one loop vertex and of the two loop fermion propagator in three-dimensional massless QED. 1999.

[43] A. Bashir, A. Kizilersu, and M. R. Pennington. Does the weak coupling limit of the Burden-Tjiang deconstruction of the massless quenched three-dimensional QED vertex agree with perturbation theory? *Phys. Rev.*, D62:085002, 2000.

[44] A. Bashir, Y. Concha-Sanchez, and Robert Delbourgo. 3-point off-shell vertex in scalar QED in arbitrary gauge and dimension. *Phys. Rev. D*, 76:065009, 2007.

[45] Adnan Bashir, Alfredo Raya, and Saul Sanchez-Madrigal. Chiral Symmetry Breaking and Confinement Beyond Rainbow-Ladder Truncation. *Phys. Rev.*, D84:036013, 2011.

[46] Lei Chang, Craig D. Roberts, and Peter C. Tandy. Selected highlights from the study of mesons. *Chin. J. Phys.*, 49:955–1004, 2011.

[47] Gernot Eichmann, Helios Sanchis-Alepuz, Richard Williams, Reinhard Alkofer, and Christian S. Fischer. Baryons as relativistic three-quark bound states. *Prog. Part. Nucl. Phys.*, 91:1–100, 2016.

[48] Gernot Eichmann, Christian S. Fischer, and Helios Sanchis-Alepuz. Light baryons and their excitations. *Phys. Rev.*, D94(9):094033, 2016.

[49] Si-Xue Qin, Craig D. Roberts, and Sebastian M. Schmidt. Ward–Green–Takahashi identities and full vertex functions in different representations of vector meson masses and decay constants. *Phys. Rev.*, C60:055214, 1999.

[50] A. C. Aguilar and A. A. Natale. A Dynamical gluon mass solution in a coupled system of the Schwinger-Dyson equations. *JHEP*, 08:057, 2004.

[51] Attilio Cucchieri and Tereza Mendes. What’s up with the Gribov-Zwanziger approach in full QCD? *Phys. Rev.*, D86:074512, 2012.

[52] David Dudal, John A. Gracey, Silvio Paolo Sorella, Nele Vandersickel, and Henri Verschelde. A Refinement of the Gribov-Zwanziger approach in the Landau gauge: Infrared propagators in harmony with the lattice results. *Phys. Rev.*, D78:065047, 2008.

[53] D. Dudal, O. Oliveira, and N. Vandersickel. Indirect lattice evidence for the Refined Gribov-Zwanziger formalism and the gluon condensate ($A^2$) in the Landau gauge. *Phys. Rev.*, D81:074505, 2010.

[54] John M. Cornwall. Dynamical Mass Generation in Continuum QCD. *Phys. Rev.*, D26:1453, 1982.

[55] Patrick O. Bowman, Urs M. Heller, Derek B. Leinweber, Maria B. Parappilly, Andre Sternebeck, Lorenz von Smekal, Anthony G. Williams, and Jian-bo Zhang. Scaling behavior and positivity violation of the gluon propagator in full QCD. *Phys. Rev.*, D76:094505, 2007.

[56] A. Ayala, A. Bashir, D. Binosi, M. Cristoforetti, and J. Rodriguez-Quintero. Quark flavour effects on gluon and ghost propagators. *Phys. Rev.*, D86:074512, 2012.

[57] Philippe Boucaud, J.P. Leroy, A. Le Yaouanc, J. Micheli, O. Pene, et al. On the IR behaviour of the Landau-gauge ghost propagator. *JHEP*, 0806:099, 2008.

[58] Christian S. Fischer, Axel Maas, and Jan M. Pawlowski. On the infrared behavior of Landau gauge Yang-Mills theory. *Annals Phys.*, 324:2408–2437, 2009.

[59] A. C. Aguilar, D. Binosi, J. Papavassiliou, and J. Rodriguez-Quintero. Non-perturbative comparison of QCD effective charges. *Phys. Rev. D*, 80:085018, 2009.

[60] M.R. Pennington and D.J. Wilson. Are the Dressed Gluon and Ghost Propagators in the Landau Gauge presently determined in the confinement regime of QCD? *Phys. Rev.*, D84:119901, 2011.

[61] Adrian Blum, Markus Q. Huber, Mario Mitter, and Lorenz von Smekal. Gluonic three-point correlations in pure Landau gauge QCD. *Phys. Rev.*, D89:061703, 2014.

[62] Anton K. Cyrol, Leonard Fister, Mario Mitter, Jan M. Pawlowski, and Nils Strodthoff. Landau gauge Yang-Mills correlation functions. *Phys. Rev.*, D94(5):054005, 2016.

[63] Markus Q. Huber. On non-primitively divergent vertices of Yang–Mills theory. *Eur. Phys. J.*, C77(11):733, 2017.

[64] D. Dudal, S. P. Sorella, N. Vandersickel, and H. Verschelde. New features of the gluon and ghost propagator in the infrared region from the Gribov-Zwanziger approach. *Phys. Rev.*, D77:071501, 2008.

[65] David Dudal, John A. Gracey, Silvio Paolo Sorella, Nele Vandersickel, and Henri Verschelde. New features of the Gribov-Zwanziger approach in the Landau gauge: Infrared propagators in harmony with the lattice results. *Phys. Rev.*, D78:065047, 2008.

[66] D. Dudal, O. Oliveira, and N. Vandersickel. Indirect lattice evidence for the Refined Gribov-Zwanziger formalism and the gluon condensate ($A^2$) in the Landau gauge. *Phys. Rev.*, D81:074505, 2010.

[67] John M. Cornwall. Dynamical Mass Generation in Continuum QCD. *Phys. Rev.*, D26:1453, 1982.

[68] Patrick O. Bowman, Urs M. Heller, Derek B. Leinweber, Maria B. Parappilly, Andre Sternebeck, Lorenz von Smekal, Anthony G. Williams, and Jian-bo Zhang. Scaling behavior and positivity violation of the gluon propagator in full QCD. *Phys. Rev.*, D76:094505, 2007.

[69] A. Ayala, A. Bashir, D. Binosi, M. Cristoforetti, and J. Rodriguez-Quintero. Quark flavour effects on gluon and ghost propagators. *Phys. Rev.*, D86:074512, 2012.

[70] A. C. Aguilar, D. Binosi, and J. Papavassiliou. Unquenching the gluon propagator with Schwinger-Dyson equations. *Phys. Rev.*, D86:014032, 2012.

[71] A. Bashir, A. Raya, and J. Rodriguez-Quintero. QCD: Restoration of Chiral Symmetry and Deconfinement for Large $N_f$. *Phys. Rev.*, D88:054003, 2013.

[72] Daniele Binosi, Cedric Mezrag, Ioannis Papavassiliou, Craig D. Roberts, and Jose Rodriguez-Quintero. Process-independent strong running coupling. *Phys. Rev.*, D96(5):054026, 2017.

[73] Zhu-Fang Cui, Jin-Li Zhang, Daniele Binosi, Feliciano de Soto, Cedric Mezrag, Ioannis Papavassiliou, Craig D. Roberts, Jose Rodriguez-Quintero, Jorge Segovia, and Sarvas Zafeiropoulos. Effective charge from lattice QCD. 12 2019.

[74] Attilio Cucchieri, Tereza Mendes, and Elton M. S. Santos. Covariant gauge on the lattice: A New implementation. *Phys. Rev. Lett.*, 103:141602, 2009.
