ON THE STRUCTURE OF ACCRETION DISKS WITH OUTFLOWS

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ABSTRACT

To study the outflows from accretion disks, we solve the set of hydrodynamic equations for accretion disks in spherical coordinates \((r\theta\phi)\) to obtain the explicit structure along the \(\theta\)-direction. Using self-similar assumptions in the radial direction, we change the equations to a set of ordinary differential equations about the \(\theta\)-coordinate, which are then solved with symmetrical boundary conditions in the equatorial plane; the velocity field is then obtained. The \(\alpha\) viscosity prescription is applied and an advective factor \(f\) is used to simplify the energy equation. The results display thinner, quasi-Keplerian disks for Shakura–Sunyaev disks; thicker, sub-Keplerian disks for advection-dominated accretion flows; and slim disks which are consistent with previous popular analytical models. However, an inflow region and an outflow region always exist, except when the viscosity parameter \(\alpha\) is too large, which supports the results of some recent numerical simulation works. Our results indicate that the outflows should be common in various accretion disks and may be stronger in slim disks, where both advection and radiation pressure are dominant. We also present the structure’s dependence on the input parameters and discuss their physical meanings. The caveats of this work and possible improvements for the future are discussed.

Key words: accretion, accretion disks – black hole physics – hydrodynamics

1. INTRODUCTION

Accretion disk models have been much developed in the past several decades. Many disk models have been proposed, and some of them have been widely adopted in astrophysical studies, including, but not restricted to, the Shakura–Sunyaev disk (SSD; see Shakura & Sunyaev 1973), advection-dominated accretion flow (ADAF; see Narayan & Yi 1994; Abramowicz et al. 1995), and slim disk models (Abramowicz et al. 1988). However, the outflow structure of accretion disks still remains an unsolved problem. The Navier–Stokes equations describe the hydrodynamic processes but are quite difficult to solve for accretion disks, which involve viscosity and radiation. Therefore, in most works, some kinds of simplification, such as one-zone or polytropic distribution and hydrostatic equilibrium, are usually applied in the vertical direction, and the vertical variation \((z\text{-dependence in cylindrical coordinates})\) of the velocity field is usually neglected. In this way, the equations are changed to a set of ordinary differential equations (ODEs) in the radial direction, which can be solved numerically. However, by making these assumptions, one cannot get a clear picture of the vertical structure of accretion flows; the velocity is always radially inward, and no mass will cross the disk surface, displaying no outflow structure. Among the exceptions is a work done by Narayan & Yi (1995, hereafter NY95), which used self-similar assumptions in the radial direction and solved the structure along the \(\theta\)-direction in spherical coordinates \((r\theta\phi)\). However, in their work they assumed \(v_{\theta} = 0\) and thus could not get a proper velocity field, and their solutions comprise only pure inflow. They argued that the Bernoulli parameter is positive in their solutions, so that a bipolar outflow is expected to develop near the vertical axis. Blandford & Begelman (1999, hereafter BB99) relaxed the mass conservation assumption and assumed that the mass inflow rate varies with radius, and obtained solutions with outflow called adiabatic inflow–outflow solutions. Their solutions are one-dimensional self-similar solutions that are height-averaged, and they also applied the Bernoulli parameter to argue the presence of outflow. However, Abramowicz et al. (2000) pointed out that the positive Bernoulli function (instead of the Bernoulli parameter, which is defined in context of self-similar assumptions only) is not sufficient for outflow (see also the simulation works done by Stone et al. 1999 and Yuan & Bu 2010). Blandford & Begelman (2004) furthered their work and presented some self-similar two-dimensional solutions for radiatively inefficient accretion flows with outflow. They assumed hydrostatic equilibrium in the vertical direction and that convection dominates the heat transport, which may only be applicable in certain cases. Xu & Chen (1997, hereafter XC97) relaxed \(v_{\theta} = 0\) and obtained two types of solutions with outflow: accretion and ejection solutions. However, their solutions require the net accretion rate to be 0, which is not realistic. Xue & Wang (2005, hereafter XW05) followed NY95 and solved the disk structure along the \(\theta\)-direction, considering \(v_{\phi}\). They arbitrarily set a disk surface, at which \(v_{r} = 0\), and a sound speed on the surface for their calculations. Their solutions display a field of inflow near the equatorial plane, with wind blowing out of the upper boundary. However, the boundary is set as an input parameter rather than being calculated, and they only investigated several cases of ADAFs. In Section 3.2, we will see that, according to our calculations, their assumption does not hold for accretion flows with large \(\alpha\) values. Sadowski et al. (2010) abandoned the self-similar assumptions and solved the accretion disk structure in the radial and vertical directions simultaneously. Because the Navier–Stokes equations for accretion disks cannot be decoupled intrinsically, they adopted other assumptions, e.g., the disk is not geometrically thick, to derive the equations. Because in their work they did not consider \(v_{\phi}\), while \(v_{r}\) and \(v_{\theta}\) were supposed not to vary vertically, they were not able to study outflows. In summary, in the analytical model study, the vertical or \(\theta\)-direction structure of accretion disks and outflows still cannot be dealt with satisfactorily, and many improvements still need to be made.

On the other hand, observationally there is more and more evidence of outflows in accretion systems, such as Sgr A* (Marrone et al. 2006; Xie & Yuan 2008), soft X-ray transients (Loeb et al. 2001), and quasars with blueshifted absorption...
lines (e.g., PG1115+80; Chartas et al. 2003). Many numerical simulation works have also discovered outflows in their results (e.g., Stone et al. 1999; Igumenshchev & Abramowicz 2000; Okuda et al. 2005; Ohsuga et al. 2005, 2009; Ohsuga & Mineshige 2007). The common existence of outflows in these works inspires us to explicitly investigate the vertical structure of accretion flows to find solutions that can deal with $v_0$ and positive $v_{r_+}$, get a clear velocity field, and provide more reasonable boundary conditions. As a first step, we follow the work done by NY95 and XW05, using self-similar assumptions in the radial direction and solving the ODEs along the $\theta$-direction in spherical coordinates ($r\theta \phi$). We use the $\alpha$ viscosity prescription and assume that the $r\phi$-component of the viscosity stress tensor is dominant. By neglecting other components of the viscosity stress tensor, the necessary number of boundary conditions is reduced and we only need the boundary conditions in the equatorial plane, which is obtained by symmetry and is thus quite physical. Because we did not set other arbitrary restrictions for $v_\phi$ and $v_\theta$ other than the self-similar assumptions, we can get a velocity field containing positive $v_r$ to discuss the flow structure with possible outflows. These assumptions are applicable to different kinds of accretion disk models, including the SSD, ADAF, and slim disk models, so we also perform many calculations with different sets of parameters (which is very difficult to do in a numerical simulation because it is very time consuming) in an attempt to find the flow structure’s dependence on the parameters so we can understand the physical inflow/outflow mechanism. These results can also be helpful to future numerical simulation works when they set the input parameters.

It should be noted that there is another branch of research that investigates accretion flows with an outflow (usually wind) via accretion disk models. In this research, the configuration of the outflow and accretion disk is usually preset, and the calculations are focused on either the outflow or the accretion disk; the influence of the other part is simplified or parameterized (e.g., Fukue 1989, 2004; Takahara et al. 1989; Kusunose 1991; BB99; Misra & Taam 2001; Xie & Yuan 2008). Recently, there were several works done in this way that deal with the outflow and accretion disk simultaneously (Kawabata & Mineshige 2009; Dotan & Shaviv 2011). Compared to our work, in these studies the accretion disk is usually height-integrated, and the configuration of the accretion flow is assumed rather than calculated. In our work, we solve the full hydrodynamic equations to get the configuration of the accretion flow. Our work focuses on studying the general structure of disks, outflows, and the physical mechanism behind them, and the results are complementary to one another.

In Section 2, we present the basic equations and assumptions we used in our calculations. In Section 3.1, we discuss our numerical methods and present solutions corresponding to typical parameters of the SSD, ADAF, and slim disk models. In Section 3.2, we show the disk structure’s dependence on different parameters and discuss their physical meanings. In Section 4, we present our summary and discussion.

2. EQUATIONS AND ASSUMPTIONS

We consider the hydrodynamic equations of an accretion flow in the spherical coordinates ($r\theta \phi$). The flow is assumed to be steady ($\partial / \partial t = 0$) and axisymmetric ($\partial / \partial \phi = 0$). Thus, the continuity equation can be written as

$$1 \frac{\partial}{\partial r} (r^2 \rho v_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta \rho v_\theta) = 0. \quad (1)$$

We assume that for the accretion flow, only the $r\phi$-component of the viscous tensor, $t_{r\phi}$, is dominant. We use the Newtonian gravitational potential, $\Phi = -GM/r$. Then, the equations of motion can be written as (Kato et al. 2008; see also Appendix A of XW05)

$$v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \left( \frac{\partial v_r}{\partial \theta} - v_\theta \right) - \frac{v_r^2}{r} = -\frac{GM}{r^2} - \frac{1}{\rho} \frac{\partial p}{\partial r} \quad (2)$$

$$v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \left( \frac{\partial v_\theta}{\partial \theta} + v_r \right) - \frac{v_\theta^2}{r} \cot \theta = -\frac{1}{\rho r} \frac{\partial p}{\partial \theta} \quad (3)$$

$$v_r \frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\phi}{\partial \theta} + v_\phi (v_r + v_\theta \cot \theta) = \frac{1}{\rho r^3} \frac{\partial}{\partial r} (r^2 t_{r\phi}). \quad (4)$$

In our calculations, we adopt the $\alpha$ prescription of viscosity, $t_{r\phi} = -\alpha \rho$, where $p$ is the total pressure. The shearing box radiation magneto-hydrodynamic (MHD) simulations done by Hirose et al. (2009) found that vertically integrated stress is approximately proportional to vertically averaged total thermal (gas plus radiation) pressure. In our work, however, we also apply this relation locally.

Following NY95, we use the advective factor, $f \equiv Q_{adv}/Q_{vis}$, i.e., a fraction $f$ of the dissipated energy is advected as stored entropy, and a fraction $(1-f)$ is lost due to radiation. In our calculations, we assume that $f$ is constant in the accretion flow, so the energy equation can be written as

$$\rho \left( v_r \frac{\partial e}{\partial r} + \frac{v_\theta}{r} \frac{\partial e}{\partial \theta} \right) - \frac{p}{\rho} \left( v_r \frac{\partial \rho}{\partial r} + \frac{v_\theta}{r} \frac{\partial \rho}{\partial \theta} \right) = ft_{r\phi} \frac{\partial}{\partial r} \left( \frac{v_\phi}{r} \right). \quad (5)$$

where $e$ is the internal energy of the material and can be expressed as (Kato et al. 2008)

$$\rho e = \frac{p_{gas}}{\gamma - 1} + 3p_{rad}. \quad (6)$$

where $\gamma$ is the heat capacity ratio and is considered to be a constant input parameter in our calculations.

We adopt the self-similar assumptions in the radial direction; therefore, we seek a solution of the form

$$\rho = \rho(\theta) r^{-n}, \quad (7)$$

$$v_r = v_r(\theta) \sqrt{\frac{GM}{r}}, \quad (8)$$

$$v_\theta = v_\theta(\theta) \sqrt{\frac{GM}{r}}, \quad (9)$$

$$v_\phi = v_\phi(\theta) \sqrt{\frac{GM}{r}}, \quad (10)$$

$$p = p(\theta) GM r^{-n-1}. \quad (11)$$

This set of self-similar solutions is similar to that of NY95 and is actually identical to that of XW05. In contrast to those works, we use the total pressure $p$ in Equation (11) instead of the sound speed $c_s$, which is proportional to $\sqrt{p/\rho}$. As stated in NY95, the only length scale in the problem is $r$, and the only frequency is $\Omega_k$, and thus all velocities must scale with the radius as $r \Omega_k$. In NY95, Narayan & Yi set $n = 3/2$, which implies $v_\theta = 0$.
according to the continuity equation; thus, they intrinsically set no outflow for the accretion disk. Here, we relax this parameter $n$ following BB99 and XW05 to allow outflows from the disk.

If we substitute Equations (7)–(11) into Equations (1)–(4), the $r$ components can be eliminated, leaving only the dimensionless functions $\rho(\theta), \nu_r(\theta), \nu_\theta(\theta), p(\theta)$, the variable $\theta$, and some constants that are set as input parameters. The energy equation (Equation (5)) is somewhat more complex. As the internal energy depends differently on gas pressure and radiation pressure, we discuss the energy equation more carefully here. First, one may note that only the total pressure $p$ appears in Equations (1)–(4). This is not hard to understand, as the dynamical processes do not recognize whether the pressure is from gas or radiation. $p_{\text{gas}}$ and $p_{\text{rad}}$ do affect the energy equation in different ways, and to describe this effect, we define the gas pressure ratio $\beta$:

$$\beta \equiv \frac{p_{\text{gas}}}{p} = \frac{p_{\text{gas}}}{p_{\text{gas}} + p_{\text{rad}}}.$$  

When $\gamma = 4/3$, no matter what the value of $\beta$ is, the solution remains the same: in this case, Equation (6) becomes $\rho e = 3 p$. If the accretion flow is dominated by radiation pressure ($\beta \to 0$), then Equation (6) also becomes $\rho e = 3 p$, and the flow structure will not depend on $\gamma$. If the accretion flow is dominated by gas pressure, then Equation (6) becomes $\rho e = p/(\gamma - 1)$, and the result will depend on the value of $\gamma$. For a general case, Equation (6) can be written as

$$\rho e = \frac{p_{\text{gas}}}{\gamma - 1} + 3 p_{\text{rad}} = \left[\frac{\beta}{\gamma - 1} + 3(1 - \beta)\right] p.$$  

Physically, $\beta$ is a value between 0 and 1, so $\rho e$ is always between 3$p$ and $p/(\gamma - 1)$. Then, we can expect that a general solution should be somewhere between two extreme cases: the gas-pressure-dominated case and the radiation-pressure-dominated case. For these two extreme cases, we can calculate the exact solutions of the problem, as Equation (6) will be simplified to forms without $\beta$. These two cases also provide the typical solutions we emphasize in Section 3.1. However, we also want to know how the solutions change from the gas-pressure-dominated case to the radiation-pressure-dominated case. For the accretion flows in which both gas pressure and radiation pressure are important, we have to assume the gas pressure ratio $\beta$ to be constant to solve the problem. Solving the problem with variant $\beta$ is beyond the capability of this self-similar treatment, but we are planning to do that in our next work.

As we already mentioned, the radiation-pressure-dominated case has the same result as that of the gas-pressure-dominated case when $\gamma = 4/3$, so eventually we will examine how the solution changes according to $\gamma$ for the gas-pressure-dominated case. We define an equivalent $\gamma$ here:

$$\gamma_{\text{eq}} \equiv \frac{p}{\rho e} + 1 = \frac{\gamma - 1}{\beta + 3(1 - \beta)(\gamma - 1)} + 1,$$  

so that Equation (6) becomes

$$\rho e = \frac{p}{\gamma_{\text{eq}} - 1},$$  

and this $\gamma_{\text{eq}}$ represents the equivalent $\gamma$ that the accretion flow would have if the flow was treated as gas pressure dominated (even when $\beta \neq 1$). We can see that if the flow is radiation pressure dominated ($\beta = 0$), then no matter what the value of $\gamma$ is, $\gamma_{\text{eq}}$ is always $4/3$. Here, we treat $\gamma$ as a constant; for any specific case, we can get a constant $\gamma_{\text{eq}}$, and all situations can be treated as a gas-pressure-dominated flow with $\gamma_{\text{eq}}$ as an input parameter. Usually, $\gamma$ is taken as a value between 7/5 (the value of diatomic ideal gas) and 5/3 (the value of monatomic ideal gas), so that $\gamma_{\text{eq}}$ ranges between 4/3 and 5/3.

With Equations (7)–(11) and (15), Equations (1)–(5) can be reduced to a set of five ODEs:

$$2 \nu_\theta(\theta) \frac{dp(\theta)}{d\theta} + \rho(\theta) \left[ (3 - 2n) \nu_\theta(\theta) + 2 \left( \cot \theta \nu_\theta(\theta) + \frac{d\nu_\theta(\theta)}{d\theta} \right) \right] = 0,$$  

$$2(n + 1) \rho(\theta) + \rho(\theta) \left[ \nu_r(\theta)^2 + 2 \left( 1 + \nu_\theta(\theta)^2 + \nu_\phi(\theta)^2 - \nu_r(\theta) \frac{d\nu_r(\theta)}{d\theta} \right) \right] = 0,$$  

$$2 \rho(\theta) \nu_\theta(\theta) \frac{dp(\theta)}{d\theta} + \rho(\theta) \left[ \rho(\theta) [2 n \gamma_{\text{eq}} n - 1] \nu_r(\theta) + 3 \alpha f (\gamma_{\text{eq}} - 1) \nu_\phi(\theta) - 2 \gamma_{\text{eq}} \nu_\phi(\theta) \frac{dp(\theta)}{d\theta} \right] = 0,$$  

with five dimensionless functions $\rho(\theta), \nu_r(\theta), \nu_\theta(\theta), \nu_\phi(\theta), p(\theta)$; the variable $\theta$; and four input parameters ($\alpha, f, \gamma_{\text{eq}}, n$). This set of ODEs can be numerically solved with proper boundary conditions. We assume that the structure of the disk is symmetric to the equatorial plane, and thus we have

$$\theta = 90^\circ : \nu_\theta = 0 = \frac{dp}{d\theta} = \frac{d\nu_r}{d\theta} = \frac{d\nu_\phi}{d\theta}.$$

in which only four conditions are independent. For the last boundary condition, we set $\rho(90^\circ) = 1$, which can be normalized by a scale factor if the effective accretion rate at a certain radius is set (NY95; XW05; etc.). These boundary conditions are enough for our calculations, so we did not introduce other arbitrary boundary conditions.

3. NUMERICAL RESULTS

3.1. Typical Solutions

We obtained numerical solutions of Equations (16)–(20) with different sets of input parameters ($\alpha, f, \gamma_{\text{eq}}, n$). Some typical solutions are shown in Figures 1–4. The calculation starts from the equatorial plane ($\theta = 0^\circ$) and moves toward the vertical axis ($\theta = 90^\circ$). Generally, the mass density $\rho$ and total pressure...
Figure 1. Solution corresponding to the gas-pressure-dominated region of the SSD model. Here, $\alpha = 0.1$, $n = 1.3$, $f = 0.01$, and $\gamma_{\text{eq}} = 5/3$, which correspond to gas-pressure-dominated monatomic ideal gas.

Figure 2. Solution corresponding to the radiation-pressure-dominated region of the SSD model. Here, $\alpha = 0.1$, $n = 1.3$, $f = 0.01$, and $\gamma_{\text{eq}} = 4/3$, which correspond to radiation-pressure-dominated monatomic ideal gas.

$p$ will decrease as $\theta$ decreases, and at certain inclinations they will get very close to 0, as shown in the figures. We take this as the upper boundary of the flow structure. If we continue the calculation through this inclination, we will encounter numerical errors. We think the reason is that we cannot describe the flow near the vertical axis with a simple self-similar solution in the radial direction. This effect is also to be expected. If we describe the upper boundary (minimum $\theta$) that we reach in our calculation as $\theta_b$, then the effective accretion rate $\dot{M}_{\text{eff}}$ across a sphere at radius $r$ within the region calculated by us is

$$
\dot{M}_{\text{eff}} = 2 \int_{\theta_b}^{90^\circ} \rho v_r \cdot 2\pi r \sin \theta \cdot (\pi/180') d\theta = 4\pi \sqrt{GM}^{1-n} \int_{\theta_b}^{90^\circ} v_r(\theta) \rho(\theta) \sin \theta \cdot (\pi/180') d\theta,
$$

(22)
Figure 3. Solution corresponding to the ADAF model. Here, $\alpha = 0.1$, $n = 1.3$, $f = 1$, and $\gamma_{\text{equ}} = 5/3$, which correspond to gas-pressure-dominated monatomic ideal gas.

Figure 4. Solution corresponding to the slim disk model. Here, $\alpha = 0.1$, $n = 1.3$, $f = 1$, and $\gamma_{\text{equ}} = 4/3$, which correspond to radiation-pressure-dominated monatomic ideal gas.

which is a function of $r$ unless $n = 3/2$. In Equation (22) (and subsequently in this paper), negative values of $M_{\text{eff}}$ represent inflow, while positive values represent outflow. If we describe the accretion rate in the region between the vertical axis and the inclination $\theta_b$ as $M_{\text{axis}}$, then, according to the steady nature of the flow, we have

$$M_{\text{eff}} + M_{\text{axis}} = M,$$

(23)


in which $\dot{M}$ represents the total accretion rate across any sphere at a reasonable radius centered by the central accretor, and it should be a constant for a steady accretion flow. If the solution does not end at an upper boundary and instead can describe the entire flow structure in the entire space, then $\theta_0 = 0$ and $M_{\text{eff}} = M$, which should be a constant. According to Equation (22), this can only happen in the following two cases: (1) $n = 3/2$, which forces $M_{\text{eff}}$ not to change with radius $r$; and (2) when $n \neq 3/2$, the integration term in Equation (22) must be 0, in which case $M_{\text{eff}} = 0$ and is a constant along radius $r$. The first case was discussed in NY95. Because $n = 3/2$, $r^2 \rho v_r$ is independent of $r$, and the continuity equation (1) shows that $v_0 = 0$, resulting in a solution in which the flow is always radial (with rotation). The second case was discussed in XC97, and the fact that $M = 0$ requires that the outflow rate exactly equal the inflow rate at any radius. However, this is unrealistic for an accretion flow, as discussed in XW05. The reason is that, when material is accreted in the form of an accretion flow, gravitational energy is released, and part of it is changed to internal energy via viscous friction. The restriction that outflow rate equals inflow rate requires that the internal energy released from gravitational energy must be fully returned to gravitational energy, which violates the second law of thermodynamics. Therefore, the inflow rate must be larger than the outflow rate at a certain radius to compensate for the increase in entropy in the hydrodynamic process. As we have mentioned before, there are many observations of accretion systems with outflows, but they can be neither of the two cases mentioned above (case (1) has no outflow, while case (2) violates the second law of thermodynamics). So, the self-similar solutions for an accretion disk with outflow have to be truncated at some inclination. It should be noted that this effect has already been discussed in XW05, although they assumed that the self-similar solution is only valid for the inflow part. Our solutions, however, show that at least part of the outflow (with $\theta_0 < \theta < \theta_i$) can be described with self-similar assumptions.

From the above analysis, we can also see that $n$ is a very important parameter. When $n = 3/2$, the entire flow structure can be described by the same set of self-similar solutions; however, these kinds of solutions have no outflow. When $n \neq 3/2$, self-similar solutions can only describe part of the entire flow structure. According to Equation (22), when $n < 3/2$, the effective accretion rate $M_{\text{eff}}$ decreases as $r$ decreases, implying that material is lost due to outflows. This kind of solution contains outflows, which is also consistent with the results of many numerical simulation works (e.g., Okuda et al. 2005; Ohsuga et al. 2005, 2009; Ohsuga & Mineshige 2007). On the other hand, when $n > 3/2$, the solutions have an effective accretion rate $M_{\text{eff}}$ that increases toward the central accretor, which implies that there must be matter injection into the accretion flow from high latitudes. This kind of matter injection is not likely to happen in real cases. So, we would like to investigate solutions with $n \leq 3/2$. Solutions with $n = 3/2$ have been examined in NY95, and they are only for ADAFs, while solutions with $n < 3/2$ in our work are applicable to a much wider range of accretion-disc types. So, in this subsection, we take $n = 1.3$ to calculate the typical solutions. In this way, we can observe how the solutions differ from those of $n = 3/2$ when $n$ does not change much. We also calculate how the solutions change with different $n$, which will be discussed in detail in Section 3.2. The result shows that, when other parameters are the same, for a large range of $n$ with $n < 3/2$, the solutions display similar qualitative structures.

The value of the viscosity parameter $\alpha$ can be inferred both from observations and from MHD simulations of magneto-rotational instability (MRI). As reviewed by King et al. (2007), the value of $\alpha$ for ionized disks given by numerical simulations is $\sim 0.01$, while that obtained through observations is $\sim 0.1 - 0.4$. However, the observational determinations of $\alpha$ are strongly model dependent, and what the value of $\alpha$ should be for hot, ionized disks remains an open question. In this subsection, we take $\alpha = 0.1$, which is a typical value used in theoretical models of accretion disks. We also discuss how the solutions would change with different $\alpha$ values in Section 3.2.

Among the most popular analytical disk models are the SSD, ADAF, and slim disk models, so here we show the solutions with input parameters corresponding to each. For accretion flows composed mostly of fully ionized hydrogen, the material can be regarded as monatomic ideal gas, for which $\gamma = 5/3$, and that is the value we take here (accretion flows composed of non-ionized $H_2$ would have $\gamma = 7/5$). We did not discuss this situation here, but some relative results can be found in Section 3.2. Figures 1 and 2 are both for the SSD model, which has little advection ($f = 0.01$), while Figure 1 includes the gas-pressure-dominated region and Figure 2 includes the radiation-pressure-dominated region. The ADAF model is treated as advection dominated ($f = 1$) and gas pressure dominated ($\beta = 1$, so that $\gamma_{\text{eq}} = 5/3$), as shown in Figure 3. The slim disk model is treated as advection dominated ($f = 1$) and radiation pressure dominated ($\beta = 0$, so that $\gamma_{\text{eq}} = 4/3$), as shown in Figure 4. The other two parameters are all set as $\alpha = 0.1$ and $n = 1.3$. In each figure, we show the function curves of $v_r(\theta)$, $v_\phi(\theta)$, $v_\theta(\theta)$, $p(\theta)$, $\rho(\theta)$, and the Mach numbers of $v_r$ and $v_\theta$, in which the adiabatic sound speed is calculated as

$$c_s = \sqrt{\gamma \frac{p}{\rho}}. \quad (24)$$

As shown in Equations (7)–(11), all velocities scale with $\sqrt{GM/r}$. The density $\rho(\theta)$ is scaled to 1 in the equatorial plane, and it can be determined in a real case once the accretion rate at a certain radius is given. The pressure $p$ is also scaled, and it satisfies the relation that $p(\theta)/\rho(\theta) = (p/\rho)/(GM/r)$. So, once we get the actual density $\rho$ at a certain radius, we can calculate easily the corresponding pressure $p$. The Mach number curves for $v_r$, first decrease with $\theta$, reaching 0 at some inclination, and then increase. That is because they change signs along $\theta$ and we use absolute values in calculating the Mach numbers.

There are several common features among Figures 1–4. The value of $v_0$ is always non-positive. The value of $v_r$ is always negative close to the equatorial plane and becomes positive at smaller inclinations. In each of these figures, there exists a certain inclination $\theta_0$ at which $v_r(\theta) = 0$. This $\theta_0$ has similar meaning to the “$\theta_0$” in XW05, except that it is obtained through calculation rather than being preset as an input parameter. In the region $\theta_\theta \leq \theta < \theta_0$, $v_r(\theta)$ is negative, meaning that the accretion flow is moving toward the central accretor as inflow. So, this region corresponds to the “normal” accretion-disk part in basic models, and we call it the inflow region in this paper. In the region $\theta_0 < \theta < \theta_i$, $v_r(\theta)$ is positive and the accretion flow is moving away from the central accretor, and we call this region the outflow region. The region between the upper boundary and the vertical axis contains the outflow, which blows out of the upper boundary in the form of wind, as shown in Figure 5: we call this region the wind region to distinguish it from the outflow region. It should be noted that our solutions actually cannot describe the structure of the wind region. However, it is
natural to suppose that the flow structure in the wind region with $0 \leq \theta < \theta_b$ is also outflow, and the outflow region ($\theta_b < \theta < \theta_0$) can either be regarded as a transition region between inflow and outflow or as part of the outflow, which can still be described with the self-similar assumptions.

The physical properties shown in Figures 1 and 2 agree quite well with the SSD model. The rotation is very close to being Keplerian, the radial velocity is much smaller than the azimuthal velocity, and the disk is geometrically thin. However, as mentioned above, our calculation shows that even in the SSD case, there is outflow in the accretion disk. As shown in Figures 5(a) and (b), this outflow is in the form of wind blowing out from the disk surface. Both $v_r$ and $v_\theta$ are subsonic in the SSD cases.

The advection-dominated solutions, as shown in Figures 3 and 4, are geometrically much thicker than the solutions with low advection, and are able to describe the structure of the majority of the space, except a small region near the vertical axis. The rotation is sub-Keplerian, and the radial velocity is comparable to the azimuthal velocity. Both the ADAF and the slim disk cases show strong outflows compared to the SSD case, which can be seen more clearly in Figure 5. $v_\theta$ remains subsonic for both cases. However, for the slim disk case (Figure 4), $v_r$ becomes supersonic near the vertical axis, displaying a stronger outflow than the other three typical solutions.

Outflow is the result of the competition among the pressure gradient, the centrifugal force, and the gravitational force. As we have mentioned before, the key feature of the outflow in our work is positive $v_r$, so we would like to investigate the properties of these forces along the radial direction. The radial components of the pressure gradient, the centrifugal force, and the gravitational force can be written respectively as

$$\frac{1}{\rho} \frac{\partial p}{\partial r} = A(\theta) \frac{GM}{r^2},$$  \hspace{1cm} (25)

$$\frac{v_r^2}{r} = B(\theta) \frac{GM}{r^2},$$  \hspace{1cm} (26)

$$\frac{GM}{r^2} = 1 \cdot \frac{GM}{r^2},$$  \hspace{1cm} (27)

in which

$$A(\theta) = (1 + n) \frac{n(\theta)}{\rho(\theta)},$$  \hspace{1cm} (28)

$$B(\theta) = v_\theta^2(\theta).$$  \hspace{1cm} (29)

We plot the values of $A(\theta)$, $B(\theta)$, and $A(\theta) + B(\theta)$ for the four typical solutions in Figure 6. The dotted lines correspond to the gravitational force, which is scaled as 1; the dashed lines correspond to the radial component of the centrifugal force; the dot-dashed lines correspond to the radial component of the pressure gradient; and the solid lines correspond to the sum of the radial components of the centrifugal force and the pressure gradient, which drives the outflow. Figure 6(a) shows the properties of gas-pressure-dominated SSDs, while Figure 6(b) shows the properties of radiation-pressure-dominated SSDs. It can be seen that for both SSD cases, the influence of the pressure gradient is very small, and in almost the entire range of the solution, the gravitational force is balanced with the sum of
the centrifugal force and the pressure gradient in the radial direction. This is consistent with the SSD model in which both the advection and pressure gradient are small enough to be neglected. Therefore, these two solutions have similar accretion flow structure, and only some quantitative differences exist between them.

On the other hand, for the advection-dominated cases, such as the ADAF case (Figure 6(c)) and the slim disk case (Figure 6(d)), the pressure gradient plays a more important role than the centrifugal force and drives a significant outflow as θ decreases. The slim disk case has qualitative differences from the other three cases. In both SSD cases and the ADAF case, the radial component of the pressure gradient starts decreasing at smaller inclination angles, and, to balance this effect, the disk rotates faster as θ decreases to increase the centrifugal force, as shown in Figures 1–3. However, in the slim disk case, the radial component of the pressure gradient keeps increasing as θ decreases. It not only drives a much stronger outflow than the other three cases, but also reduces the required centripetal force to keep the disk rotating. Thus, the disk rotates slower as θ decreases, as shown in Figure 4.

As a comparison with NY95, we also present here a solution with parameters α = 0.1, n = 3/2, Y_{eq} = 1.6061, and f = 1 in Figure 7. This set of parameters corresponds to "c = 0.1" in NY95. In this case, the accretion flow does not contain outflow, and the three components of velocity and the sound speed remain constant in the structure. The curves of ρ(θ) and v_r(θ) do not agree well with NY95; however, this is because NY95 also sets the boundary conditions on the vertical axis, and the values of ρ(θ) and v_r(θ) are limited by these boundary conditions. Our solution only uses the boundary conditions in the equatorial plane and can still reach the vertical axis in this case; it has a reasonable distribution of physical properties, which further proves the applicability of our method.

3.2. Solution Dependence on Input Parameters

In this section, we show how the solutions change according to different input parameters and discuss their physical meanings. As shown in Section 3.1, the entire space is generally divided into three regions: an inflow region, an outflow region, and a wind region. As the accretion flow is symmetrical both axially and under reflection in the equatorial plane, we only need to investigate the flow structure in a quadrant of the rθ plane. In this case, the inflow region starts from the equatorial plane (θ = 90°) and extends to the inclination θ = θ_0, where v_r(θ) = 0; the outflow region starts from the inclination θ = θ_0 and extends to the upper boundary of the flow structure at θ = θ_b, and beyond that is the wind region (0 < θ < θ_b), in which the motion of matter does not satisfy the self-similar assumptions. Here, we use θ_0 and θ_b as indicators of disk size in the θ-direction, which directly reflect the size of the inflow region and the entire inflow/outflow region, respectively. In the figures showing how the properties of solutions change according to input parameters, each line represents a set of solutions, and we assign a unique label to each solution set for clarity. Table 1 lists all solution set labels, their input parameters, and corresponding lines.

3.2.1. Solution Dependence on α

The dependence of θ_0 and θ_b on α is shown in Figure 8. First, we can clearly see that over a large range of α, the advection-dominated solutions have both a larger inflow region and a larger inflow/outflow region. That is because in these solutions, most of the viscous heating is stored in the accretion flow as internal
energy, resulting in larger pressure at a given density, and thus a larger inflow/outflow region.

For solutions with the same advective factor $f$, gas-pressure-dominated solutions generally have a larger inflow region. The inflow region covers the equatorial plane to the inclination $\theta_0$, where $v_r$ reaches 0, so its size depends on the behavior of $v_r$. In the equatorial plane, the reflection symmetry gives Equation (21), and when it is substituted into Equations (17), (19), and (20), we obtain

\begin{equation}
2(n + 1) \frac{p(\theta)}{\rho(\theta)} + v_r^2(\theta) + 2(v_r^2(\theta) - 1) = 0, \tag{30}
\end{equation}

\begin{equation}
-2(n - 2) \frac{p(\theta)}{\rho(\theta)} + v_r(\theta)v_\phi(\theta) = 0, \tag{31}
\end{equation}

\begin{equation}
2(n_{\text{equ}} - n - 1)v_r(\theta) - 3\alpha f(\gamma_{\text{equ}} - 1)v_\phi(\theta) = 0. \tag{32}
\end{equation}

Equation (30) describes the hydrodynamical balance in the radial direction among the pressure gradient, centrifugal force, and gravitational force, which determines the acceleration of $v_r$ (the term $v_r^2(\theta)$ represents the acceleration of $v_r$ under the self-similar assumptions). Equation (31) describes how the viscosity affects the angular momentum transfer rate in the equatorial plane. Equation (32) describes the energy mechanism of the accretion flow that a fraction $f$ of the dissipated energy is advected as stored entropy. It should be noted that $v_r$ is generally negative in the equatorial plane, so the absolute value of $v_r$ can be written as (according to Equation (32))

\begin{equation}
|v_r(90^\circ)| = -v_r(90^\circ) = \frac{3\alpha f}{2n} v_\phi(90^\circ) \left[ \frac{1}{1 - n(\gamma_{\text{equ}} - 1)} - 1 \right]. \tag{33}
\end{equation}

If $v_\phi(90^\circ)$ remains the same, then in the range of parameters we study in this paper, $|v_r(90^\circ)|$ increases with $\gamma_{\text{equ}}$. However, $v_\phi(90^\circ)$ also changes with input parameters, and the exact value of $v_r(90^\circ)$ should be calculated via Equations (30)–(32). Figure 9 shows the profile of $v_r(90^\circ)$ versus $\gamma_{\text{equ}}$ when other parameters are taken as $n = 1.3$ and $\alpha = 0.1$. It can be seen that for both the advection-dominated solutions and the solutions with little advection, $|v_r(90^\circ)|$ increases with $\gamma_{\text{equ}}$. With a larger value of $v_r$ in the equatorial plane, the accretion flow is inclined to undergo a larger range of $\theta$ to change $v_r$ to 0, resulting in a larger inflow region. For the two SSD cases, because their flow structures are quite similar, larger values of $v_r$ in the equatorial plane give the gas-pressure-dominated solution set $a_1$ larger inflow regions than $a_2$. For the advection-dominated cases, not only is the equatorial value of $v_r$ smaller in the slim disk case than that in the ADAF case, but its radial direction pressure gradient also increases as $\theta$ decreases, giving a larger outward acceleration to $v_r$. Thus the gas-pressure-dominated solution $a_3$
has larger inflow regions than the radiation-pressure-dominated solution $a_4$.

The solution set $a_4$, which corresponds to the slim disk model, has a large outflow over a broad range of $\alpha$, displaying a large outflow area. The solutions corresponding to the SSD model, $a_1$ and $a_2$, have very small outflow regions, and their outflows are mainly in the form of wind blowing out of the disk surface instead of escaping in the outflow region. The solution set $a_3$, which corresponds to the ADAF model, has a moderate outflow region with small $\alpha$. However, as $\alpha$ increases, this outflow region quickly disappears, and the outflow takes the form of wind blowing out of the upper surface. The velocity field displayed in Figure 5 is an example for $\alpha = 0.1$.

$\alpha$ is connected with both the viscous heating and the angular momentum transport of the accretion flow. Larger $\alpha$ will increase the viscous heating for a fixed surface density. At the same time, it also increases the angular momentum transfer rate, thus increasing $v_r$ and decreasing surface density, which decreases the total viscous heating rate. Therefore, the dependence of the viscous heating rate on $\alpha$ is somewhat complex.
Viscous heating represents how much gravitational energy is converted to internal energy. But some of it is lost due to radiation, and eventually there is only an $f$ fraction of the viscous heating, which is converted to internal energy. The increase in internal energy will raise the temperature and subsequently the pressure, thus increasing the size of the accretion flow. However, for the SSD cases shown in $\alpha$-presented solutions, the increase in internal energy is negligible and the accretion flow is dominated dynamically by the gravity and the disk structure is dominated by the gravity and the rotation of the material, rather than by the pressure gradient. As a result, the sizes of accretion flows change very little with different $\alpha$ for $a_1$ and $a_2$.

On the other hand, for the advection-dominated solutions $a_3$ and $a_4$, a large fraction of the gravitational energy is eventually stored as internal energy, so the pressure gradient plays a more important role in determining the flow structure, as mentioned in Section 3.1. Therefore, the sizes of accretion flows are closely connected with $\alpha$. In Figure 8, we can see that for $a_3$ and $a_4$, when $\alpha$ is very small, the viscous process is not quite effective, and therefore the inflow and outflow region sizes are small (it is easy to understand if we consider the situation $\alpha = 0$, in which case no accretion flow will be formed). From this small value, along with the increasing value of $\alpha$, the viscous process starts to take effect, and the entire inflow/outflow region size starts to increase, until it reaches a maximum size at $\sim 0.05$ for line $a_3'$ and $\sim 0.12$ for line $a_4'$ (the exact value of $\alpha$ for the maximum size also depends on other parameters). From then on, the larger values of $\alpha$, start to dominate the total effect, and the corresponding accretion flow size starts to decrease. This property can also be seen in the top left panel of Figure 17, which displays the ratio of the outflow rate to the inflow rate.

It should be noted that, in Figure 8, the size change based on $\alpha$ is different in the entire inflow/outflow region and in the pure inflow region. The size of the entire inflow/outflow region decreases after a peak at a certain $\alpha$, while the size of the pure inflow region keeps increasing. For solution sets $a_1$, $a_3$, and $a_4$, this difference is enough to generate a critical $\alpha_c$ at which point the outflow region totally disappears. So, when $\alpha > \alpha_c$, the accretion flow that we can resolve with self-similar assumptions is composed of pure inflow, and we did not find a surface at which $\nu_r(\theta_0) = 0$. That is why we say that the assumption used in XW05 may not be true for large $\alpha$. Kluzniak & Kita (2000) also found this critical $\alpha$, which they calculated to be $\sqrt{15}/32 \approx 0.685$. In our work, however, $\alpha_c$ depends on other parameters and can change significantly, as shown in Figure 8 ($\sim 0.96$ for $a_1$, $\sim 0.24$ for $a_3$, and $\sim 0.74$ for $a_4$). For $a_2$, the $\alpha_c$ is above 1 and is not displayed.

Figure 10 displays the $\nu_r$ distribution along the $\theta$-direction for solutions with different $\alpha$. Panels (a)–(d) correspond to the four sets of solutions in Figure 8, respectively, and the solid, dashed, and dotted lines correspond to different $\alpha$ as indicated in the legends. The range of the axes is adjusted to show more details in comparison. We can see that in the solutions corresponding to the SSD model, i.e., Figures 10(a) and (b), $\nu_r$ is almost proportional to $\alpha$. That is because when alpha increases, the angular momentum transfer rate also increases. For the SSD case, the accretion flow structure is mostly determined by gravity and rotation, as discussed in Section 3.1; the rotation is quasi-Keplerian, so the increase in angular momentum transfer rate leads to near-proportional increases in $\nu_r$. A special case in the equatorial plane is shown via Equation (31): when $\nu_\phi$ is Keplerian, the value of $\nu_r$ is proportional to $\alpha$. For the ADAF and slim disk cases, because the pressure gradient plays an important role in the accretion flow structure, $\nu_r$ is generally not proportional to $\alpha$ but is still positively correlated with $\alpha$, as shown in Figures 10(c) and (d). We can also see that some lines in Figure 10 do not contain positive $\nu_r$. That is because the $\alpha$ values for these solutions are greater than the critical value $\alpha_c$ mentioned above.
3.2.2. Solution Dependence on $n$

The parameter $n$ describes how the density changes along the radius in our self-similar assumptions. As discussed in Section 3.1, when $n = 3/2$, the entire flow structure can be described by the same set of self-similar solutions but does not contain outflow; when $n < 3/2$, the solution contains outflow but the self-similar assumptions cannot be applied to the region near the vertical axis. Solutions with $n > 3/2$ are unlikely to happen in real cases.

The dependence of $\theta_{b1}$ and $\theta_{b2}$ on $n$ is shown in Figure 11. Figures 12 and 13 display the $v_r$ and $v_\theta$ distributions along the $\theta$-direction for solutions with different $n$. We can see that the solutions all achieve a maximum size at $n = 3/2$ for both the entire inflow/outflow region and the inflow-only region. This is because when $n = 3/2$, the entire space can be described by the same set of self-similar equations, as discussed in Section 3.1. In this case, $v_\theta = 0$, and mass is conserved on the streamlines pointing exactly at the central accretor; thus, no outflow is required. Before reaching the maximum value at $n = 3/2$, the flow size is positively correlated with $n$ as a general trend. As $n$ decreases, according to Equation (22), $M_{\text{eff}}$ decreases faster inward, which means a larger mass-loss rate in the area beyond $\theta_b$. This results in the increase in the size of the wind region and the decrease in the size of the entire inflow/outflow region (Figure 11), as well as the increase in $v_\theta$ (Figure 13).

In Figure 11, line $b_4'$ has two peaks. According to Equation (25), the radial pressure gradient is proportional to $(1 + n)p/\rho$, so when $p/\rho$ does not change much, the radial pressure gradient increases with $n$. In advection-dominated solutions $b_3$ and $b_4$, the radial pressure gradient is more important than centrifugal force in accelerating $v_r$, so for larger $n$, which is inclined to have a larger radial pressure gradient, one would expect to see a faster increase in $v_r$ and stronger outflow in the outflow region. These features of $v_r$ can be seen in Figure 12.

The radial pressure gradient is dominant in the slim disk case, as mentioned in Section 3.1, so the acceleration of $v_r$ increases significantly with increasing $n$ in the slim disk case, which can be seen in Figure 12(d), with a much steeper profile of $v_r$ when $n = 1.3$. This increased outflow velocity in the outflow region can cause a steeper density drop along the $\theta$-direction toward the upper boundary and therefore make the outflow region become smaller, creating a valley in line $b_4'$ around $n = 1.4$.

The top right panel in Figure 17 shows how the ratio of the outflow rate to the inflow rate changes depending on $n$. We can see that when $n$ becomes small enough, the ratio changes very little with $n$. In this range of $n$, the size of different regions also does not change much in Figure 9, and flow structures are similar to each other.

For $\gamma_{\text{eq}} = 5/3$, we cannot get a reasonable set of equatorial values for physical properties when $n$ is larger than $3/2$. For $\gamma_{\text{eq}} = 4/3$, i.e., solutions $b_2$ and $b_3$, we can still get reasonable values of equatorial physical properties when $n$ is larger than $3/2$, so we can calculate solutions with $n > 3/2$ in this case. Solutions with $n > 3/2$ have an effective accretion rate $M_{\text{eff}}$ that increases toward the central accretor, which implies that there must be matter injection into the accretion flow from high latitudes. This kind of matter injection is not likely to happen in real cases, so we will not devote more attention to the solutions with $n > 3/2$ here. Solutions with $n = 3/2$ and $n < 3/2$ both exist and have quite different structures. However, as in NY95, solutions with $n = 3/2$ are only for ADAFs, while solutions with $n < 3/2$ in our work are applicable to a much wider range of accretion disks. In this sense, we believe that outflows should be common in various accretion disks.

3.2.3. Solution Dependence on $f$

The dependence of $\theta_{b1}$ and $\theta_{b2}$ on $f$ is shown in Figure 14. It has already been discussed that as the advective factor $f$ increases,
the size of the entire inflow/outflow region also increases, which is shown more clearly here in Figure 14. The gas-pressure-dominated solutions have larger inflow regions, in agreement with Figures 8 and 11, and the reason for this has been discussed in Section 3.2.1.

As $f$ increases, the size of the outflow region increases faster for radiation-pressure-dominated flows, displaying a larger outflow size for solutions corresponding to the slim disk model. The reason is that, as $f$ increases, a larger fraction of viscous heating is converted to internal energy instead of being lost.
via radiation, raising the temperature of the accretion flow. The gas pressure is proportional to $T$, while the radiation pressure is proportional to $T^4$, so in the radiation-pressure-dominated solutions, $p$ increases faster as $f$ increases. The increased pressure inflates the outflow region and causes a stronger outflow. On the other hand, as $f$ gets close to 1, eventually the outflow velocity at small inclinations becomes so large that it will reduce the size of the outflow region, because the material lost in the outflow region is limited. As shown in the bottom left panel of Figure 17, when $f \sim 0.9$, the fraction of material lost in the outflow region reaches an upper limit; with larger values of $f$, this fraction almost remains unchanged. In this range of $f$, the larger outflow velocity will shrink the size of the outflow region. This causes line $c_2'$ in Figure 14 to reach a peak of $\sim 90^\circ$ at $f \sim 0.9$ (corresponding to Figure 17(c)), and start decreasing as $f$ becomes larger.

3.2.4. Solution Dependence on $\gamma_{\text{equ}}$ (and Consequently $\beta$)

The dependence of $\theta_0$ and $\theta_b$ on $\gamma_{\text{equ}}$ is shown in Figure 15. The top axis displays the corresponding values of $\beta$ when $\gamma$ takes the value of 5/3. If the intrinsic heat capacity ratio is not 5/3 (e.g., when dealing with proto-stellar accretion disks with non-ionized gas), the corresponding $\beta$ in Figure 15 may not be correct, but one can still calculate the correct $\beta$ with $\gamma_{\text{equ}}$ and $\gamma$ according to Equation (14). Figure 16 displays the $v_r$ and $v_\theta$ distributions along the $\theta$-direction for solutions with different $\gamma_{\text{equ}}$. The bottom right panel of Figure 17 shows how the ratio of the outflow rate to the inflow rate changes depending on $\gamma_{\text{equ}}$ (and consequently $\beta$).

In Figure 15, the size of the pure inflow region generally increases with $\gamma_{\text{equ}}$ (and consequently $\beta$). As shown in Figures 9 and 16 and discussed in Section 3.2.1, solutions with larger $\gamma_{\text{equ}}$ (and consequently a larger gas-pressure ratio $\beta$) have larger values of $v_r$ in the equatorial plane. This effect, combined with the acceleration properties of $v_r$, causes solutions with larger $\gamma_{\text{equ}}$ (and consequently larger $\beta$) to have larger inflow regions. For solutions with low advection, because the outflow region is very small, the size of the entire inflow/outflow region also displays a similar profile with increasing $\gamma_{\text{equ}}$ (and consequently $\beta$). That is why lines $d_1$, $d_1'$, and $d_2$ all have positive slopes in Figure 15.

In Figure 17(d), there exists a transition point for the advection-dominated solutions. For solutions with $\gamma_{\text{equ}} \lesssim 3/2$ (corresponding to $\beta \lesssim 2/3$), the ratio of the outflow rate to the inflow rate remains large. As discussed above, for the advection-dominated cases, radiation-pressure-dominated solutions have larger pressure gradients, shown as steeper velocity profiles in Figure 16, which drive stronger outflow. As shown in Figure 17(d), the outflow to inflow ratio increases as $\gamma_{\text{equ}}$ decreases from 5/3 (which indicates the decreasing gas-pressure ratio $\beta$ from 1). However, the outflow rate cannot exceed the inflow rate, and eventually the increase will cease, reaching a transition point at $\gamma_{\text{equ}} \sim 3/2$ (corresponding to $\beta \sim 2/3$). For $\gamma_{\text{equ}} \lesssim 3/2$ (corresponding to $\beta \lesssim 2/3$), the outflow to inflow ratio remains almost unchanged. Further decreases in $\gamma_{\text{equ}}$ will continue to increase the outflow velocity, while the fraction of the inflow lost in the outflow region will remain almost unchanged, so the size of the outflow region will shrink. This feature can also be seen in Figure 15, which shows that the size of the outflow region for advection-dominated solutions first increases with decreasing $\gamma_{\text{equ}}$, reaching an upper limit at $\gamma_{\text{equ}} \sim 3/2$ (corresponding to $\beta \sim 2/3$), and then decreases with decreasing $\gamma_{\text{equ}}$. It should be noted that this transition value of $\gamma_{\text{equ}}$ depends on other input parameters, and this value of $\sim 3/2$ corresponds to $\alpha = 0.1, \beta = 1.3$, and $f = 1$. For the solutions with low advection, the ratio of the outflow rate to the inflow rate does not change much and is always very small, as shown in Figure 17(d).

3.2.5. Bends in Solutions

We have shown the solution dependence on different input parameters. The solutions sometimes show bends, which we will discuss more specifically here.
Figure 15. Solution dependence on $\gamma_{\text{equ}}$. Solid lines $d_1$ and $d_2$ represent the upper boundary of the inflow region, and dotted lines $d_1'$ and $d_2'$ represent the upper boundary of the entire inflow/outflow region. All lines correspond to $\alpha = 0.1$ and $n = 1.3$. Lines $d_1$ and $d_1'$ correspond to $f = 0.01$, and lines $d_2$ and $d_2'$ correspond to $f = 1$. $\beta$ on the top axis corresponds to $\gamma_{\text{equ}}$ on the bottom axis and is calculated based on $\gamma = 5/3$.

Figure 16. $v_r$ and $v_\theta$ distributions along the $\theta$-direction for solutions with different $\gamma_{\text{equ}}$. Panels (a) and (c) correspond to solutions with $n = 1.3$, $\alpha = 0.1$, and $f = 0.01$, while panels (b) and (d) correspond to solutions with $n = 1.3$, $\alpha = 0.1$, and $f = 1$. The solid, dashed, and dotted lines correspond to $\gamma_{\text{equ}} = 4/3$, $3/2$, and $5/3$, respectively.

The bend about $\alpha$ is closely connected with the energy mechanism. As discussed in Section 3.2.1, the dependence of viscous heating on $\alpha$ is somewhat complex. It has a peak at a certain value of $\alpha$ (which depends on other input parameters) and decreases on both sides. For the advection-dominated solutions $a_3$ and $a_4$, most of the viscous heating is stored as internal energy in the accretion flow, which raises the temperature and subsequently the pressure, thus increasing the size of the accretion flow. On the other hand, the increase in $\alpha$ also increases the absolute value of $v_r$ of the inflow near the equatorial plane,
in which case more work needs to be done to drive an outflow, decreasing the outflow region as \( \alpha \) increases. So, naturally, the size of the entire inflow/outflow region also has a peak at some value of \( \alpha \). As \( \alpha \) increases from a very small value, at first the increase in viscous heating dominates the total effect, and the size of the accretion flow increases until it reaches a maximum value; from then on, the increase in \( v_r \) starts to dominate the total effect, which makes the outflow more difficult to produce and also reduces viscous heating when \( \alpha \) becomes large enough, thus shrinking the size of the entire accretion flow. To show the energy profile for solutions with different \( \alpha \), here we calculate the \( \theta \)-averaged Bernoulli function \( \overline{Be} \). The Bernoulli function represents the specific total energy in the accretion flow. A positive value of the Bernoulli function is only a necessary, not sufficient, condition for outflow formation (Abramowicz et al. 2000). However, because our solutions have already showed outflow structure in the velocity fields, a larger value of the Bernoulli function indicates that the accretion flow is more likely to contain stronger outflow. Locally, the Bernoulli function can be written as

\[
\overline{Be} = \frac{\int_{\theta_0}^{\theta_0} \rho \cdot \Phi \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \· \]

(34)

in which \( W \) is the specific enthalpy, \( V \) is the velocity (all three components included), and \( \Phi \) is the gravitational potential. So, the \( \theta \)-averaged Bernoulli function can be written as

\[
\overline{Be} = \frac{\int_{\theta_0}^{\theta_0} \rho \cdot \Phi \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \· \]

Figure 17. Ratio of the outflow rate to the inflow rate. The vertical axes show the absolute values of the ratio, and the horizontal axes correspond to different input parameters. The lines in the top left panel correspond to the lines in Figure 8. The lines in the top right panel correspond to the lines in Figure 11. The lines in the bottom left panel correspond to the lines in Figure 14. The lines in the bottom right panel correspond to the lines in Figure 15. Certain parts of some lines are missing because the solutions in those parts do not contain an outflow region.

Figure 18 displays how the \( \theta \)-averaged Bernoulli function \( \overline{Be} \) changes with variant \( \alpha \) for the advection-dominated cases. It can be seen that the positions of the peaks agree quite well with the bends of lines \( a'_3 \) and \( a'_4 \) in Figure 8.

Figure 19 displays the velocity fields of solutions around the bend for variant \( \alpha \). The values of \( \alpha \) are chosen so that the solutions on both sides of the peaks have similar upper boundaries (\( \sim 10^5 \)). It can be seen that the velocities in the outflow region are more aligned with the \( \theta \)-direction in the right panel of each row than with those in the left panel. That is because larger \( \alpha \) increases the angular momentum transfer rate and the absolute value of \( v_r \) near the equatorial plane, where most of the material is accreted. The value of \( v_r \) is negative in the equatorial plane and changes gradually to a positive value in the outflow region under the effect of the pressure gradient and centrifugal force. The absolute value of \( v_r \) in the equatorial
Figure 18. $\theta$-averaged Bernoulli function for variant $\alpha$. Line $a_3$ corresponds to solutions with $f = 1$, $\gamma_{\text{equ}} = 5/3$, and $n = 1.3$. Line $a_4$ corresponds to solutions with $f = 1$, $\gamma_{\text{equ}} = 4/3$, and $n = 1.3$.

Figure 19. Velocity field patterns around the bends for variant $\alpha$. The first row of figures corresponds to the bend of line $a_3'$ in Figure 8, with $\alpha = 0.043, 0.05$, and 0.12 from left to right, respectively, and $f = 1$, $\gamma_{\text{equ}} = 5/3$, and $n = 1.3$. The second row of figures corresponds to the bend of line $a_4'$ in Figure 8, with $\alpha = 0.106, 0.12$, and 0.165 from left to right, respectively, and $f = 1$, $\gamma_{\text{equ}} = 4/3$, and $n = 1.3$. The lengths of arrows indicate the absolute values of the vector $\vec{v}_r(\theta) + \vec{v}_\theta(\theta)$, and are scaled logarithmically. The solid lines correspond to the inclination $\theta_b$, while the dashed lines correspond to the inclination $\theta_0$.

plane in the right panel of each row is significantly larger than that in the left panel, so when the pressure gradient does not differ much, the positive values of $v_r$ in the outflow region will be smaller for larger $\alpha$, and the velocities will be more aligned with the $\theta$-direction. The solutions corresponding to slim disks (the second row) show velocities more aligned with the radial
direction in the outflow region compared to those for ADAFs (the first row), due to a larger pressure gradient and thus larger values of $v_r$ in the outflow region.

The bends in solution dependence on $n$, $f$, and $\gamma_{\text{eq}}$ have different physical mechanisms from those of $\alpha$. It has been mentioned that, as $n$ increases, $f$ increases, or $\gamma_{\text{eq}}$ decreases, the pressure gradient increases and drives stronger outflow. However, the outflow rate cannot exceed the inflow rate. As seen in Figure 17, the ratio of the outflow rate to the inflow rate for lines $b_1$, $c_2$, and $d_2$ will eventually reach an upper limit at certain values of corresponding parameters. For solution sets $c_2$ and $d_2$, further increases in $f$ or further decreases in $\gamma_{\text{eq}}$ will continue to increase the radial outflow velocity, while the outflow rate in the outflow region will remain almost unchanged, so the size of the outflow region will shrink. For the solution dependence on $n$, the bend near $n = 1.25$ has similar physical mechanism to the bends for $f$ and $\gamma_{\text{eq}}$. However, the size of the accretion flow has another peak at $n = 3/2$, where the flow is always radial (with rotation) and no outflow is needed. So, line $b_4$ in Figure 17 starts to drop as $n$ gets close to $3/2$, and line $b'_4$ in Figure 11 reaches another peak at $n = 3/2$.

Figure 20 displays the velocity fields of solutions around the bend for variants $n$, $f$, and $\gamma_{\text{eq}}$. It can be seen that, as $n$ increases (the first row), $f$ increases (the second row), or $\gamma_{\text{eq}}$ decreases (the third row), the flow patterns display similar evolution with these parameters: at first, both the accretion flow size and the radial outflow velocity increase; after the corresponding parameter eclipses the respective value at the transition point (the middle panels in all three rows of Figure 20), the radial outflow velocity continues to increase, while the size of the accretion flow decreases. The velocities in the outflow regions become more aligned with the radial direction from left to right in all three rows, due to increased radial outflow velocities.

The labels for solution sets and lines that appear in the text and figures are summarized in Table 1, along with their corresponding input parameters.

4. SUMMARY AND DISCUSSION

With the self-similar assumptions along the radial direction and boundary conditions obtained from the reflection symmetry in the equatorial plane, we are able to explicitly solve the Navier–Stokes equations along the $\theta$-direction and obtain the velocity field. The result shows that outflows are common in all kinds of accretion disk models, which is consistent with some numerical simulation works (e.g., Ohsuga et al. 2005, 2009; Ohsuga & Mineshige 2007). Generally, the accretion flow consists of three different regions: an inflow region near the equatorial plane that contains the largest portion of mass, an outflow region above the inflow region in which matter starts escaping the central accretor in the $r$-direction, and a wind region...
that contains the material blowing out from the boundary of the outflow region. The structure of the inflow and outflow regions can be resolved in our solutions, while the wind region does not obey the self-similar assumptions and cannot be resolved by our calculation. The inflow region and the wind region are essential for ADAFs, which are missing in the cases with large viscosity parameter $\alpha > \alpha_c$. The critical value $\alpha_c$ depends on other parameters and is $\sim 0.24$ for ADAFs, $\sim 0.74$ for slim disks, and $\sim 0.96$ for gas-pressure-dominated SSDs when $n = 1.3$.

To compare with the popular analytical models, we calculated solutions with parameters corresponding to the SSD, ADAF, and slim disk models. The solutions corresponding to the SSD model have relatively thin, quasi-Keplerian disks with very small outflow regions, and the outflows mostly take the form of wind from the disk surface. The solutions corresponding to the ADAF and slim disk models have thick, sub-Keplerian disks. The solutions corresponding to the slim disk model have a large outflow region in which most of the outflow takes place, and near the upper boundary of the outflow region the material escapes with supersonic velocity.

Our calculation displays very small outflow regions for input parameters corresponding to the SSD model, and for these solutions the inflow region has a similar structure to the one-dimensional SSD model. However, the accretion rate inside the disk may not be conserved along the radius due to mass lost via wind. For the ADAF and slim disk cases, our result displays an outflow region too large to be neglected, and we think it is necessary to consider the explicit structure in the $\theta$- or $z$-direction to get more realistic results than those from the traditional analytical models. It was proposed in some works (Gu & Lu 2007; Jiao et al. 2009) that the slim disk model has an upper limit of accretion rates, above which outflows seem to be inevitable. Later, in another paper (Jiao & Lu 2009), one-dimensional, steady transonic global solutions for slim disks with a presumed amount of outflows were calculated, and in this case the proposed upper limit of accretion rates for slim disks can be exceeded. Another paper written by Takeuchi et al. (2009) used the outflow data from two-dimensional radiation-hydrodynamic (RHD) simulations to construct a one-dimensional steady solution, and the result shows that the emergent spectra do not sensitively depend on the amount of mass outflow. However, these models are still one-dimensional models, and the authors only incorporated the mass-loss effect of outflow into their models, so the results still remain debatable. We think that two-dimensional solutions considering the hydrodynamical processes and radiative transfer along both the radial and $\theta$-directions are required to examine the topic, which we are planning to do in our future work.

We also investigated the dependence of the accretion flow structure on different input parameters. $f$ and $\gamma_{eq}$ (and consequently $\beta$) both have some transition values, as discussed in Sections 3.2.3 and 3.2.4, at which the outflow region reaches a maximum size. Generally, when $f \gtrsim 0.9$ and $\beta \lesssim 2/3$ (for intrinsic $\gamma = 5/3$), the accretion flow will have a large outflow region and strong outflow, just like the aforementioned solutions corresponding to the slim disk model. However, in what conditions the transition happens depends on other input parameters, and it will not happen in the case of large $\alpha$ because of the existence of the critical value $\alpha_c$. Solutions with $n = 3/2$ do not contain outflow, and they correspond to the solutions in NY95, which is only applicable to ADAFs. Solutions with $n < 3/2$ have the aforementioned three-region (or two-region for $\alpha > \alpha_c$) structure. Solutions with $n > 3/2$ are not likely to happen in real cases.

Finally, we would like to mention some caveats to our work. The solutions obtained in this paper are steady solutions, and thus it is not guaranteed that they will be stable solutions. These solutions do not cover the entire space, so for the unresolved region near the axis, we know nothing about the detailed structure other than that it contains the material blowing out of the upper boundary in the form of wind. Our work is based on the simple $\alpha \rho$ prescription of viscosity, which is acceptable according to recent MHD simulation works (Hirose et al. 2009; Ohsuga et al., 2009), especially when the flow is steady. However, it may be necessary to consider other components of the viscous stress tensor for solutions with large $\gamma$. We also used the advective factor $f$ to simplify the energy equation and assumed $f$ to be a constant. Realistically, $f$ should vary with both $\theta$ and $r$, which could be achieved by considering the details of radiative transfer. We used Newtonian gravitational potential in our work, but it needs to be corrected when studying the structure close to the innermost stable circular orbit. In our calculation, we did not consider the effects of convection (e.g., Stone et al. 1999; Igumenshchev & Abramowicz 2000; Yuan & Bu 2010) or the magnetic field (e.g., Blandford & Payne 1982; Naso & Miller 2010), which may be important in studying the accretion flow and the generation of outflows. Finally, it is necessary to abandon the self-similar assumptions if we want to investigate the disk and outflow structures more quantitatively and precisely. These caveats will be mitigated in our future work.

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