The current status of gauge unification is surveyed both in the standard model and its minimal supersymmetric extension. Implications for proton decay, Yukawa unification, and the Higgs mass are described.

1 Introduction

The precision electroweak data has spectacularly confirmed the electroweak part of the standard model and has correctly predicted \( m_t \) and \( \alpha_s \). However, the standard model has many shortcomings, most notably the large number of free parameters, the unexplained fermion families and masses, and notorious hierarchy and fine-tuning problems. Because of the many problems it is clear that there must be new physics beyond the standard model.

There are two general classes of extensions. One is the possibility of new layers of compositeness of fermions and/or dynamical symmetry breaking to replace the elementary Higgs mechanism. These ideas have run into significant difficulties which have precluded the construction of realistic model, and possibly have made the whole line of approach unlikely. The problems include the non-observation of large flavor-changing neutral current (FCNC) effects, which one generally expects in such models, and the absence of large anomalous contributions to the \( Zb\bar{b} \) vertex, new sources of \( SU_2 \)-breaking (the \( \rho_0 \) parameter), or large contributions to the parameters \( S_{\text{new}}, T_{\text{new}}, U_{\text{new}} \), which describe any new source of physics which contributes only to the gauge boson self-energies. In particular, with the probable discovery of the top quark from CDF and the direct determination of its mass, it is possible to unambiguously separate the new physics contribution to these parameters from the effects of \( m_t \), with the result that the limits on \( \rho_0 \) and on \( S_{\text{new}}, T_{\text{new}}, U_{\text{new}} \) are stringent. Another difficulty with compositeness is that one generally expects new 4-Fermi operators from constituent interchange, and these have not been observed.

The other generic class of extensions involve some form of unification at a high scale. In such schemes one does not expect much new physics at the TeV scale, except possibly for supersymmetry. In the viable supersymmetric models the new particles

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1Invited talk presented at Radiative Corrections: Status and Outlook, Gatlinburg, TN, June, 1994.
are usually heavy and decouple both from rare decays and precision experiments, so that one expects little or no deviation from the standard model predictions. In these supersymmetric extensions one might or might not also expect some form of unification of the coupling constants at a high scale.

In this talk I will survey the latter set of possibilities, including the status of the precision electroweak tests and the determination of parameters, both in the standard model and its supersymmetric extension. I will then review some of the implications for supersymmetric grand unified theories. It will be seen that the current data is in generally excellent agreement with coupling constant unification. However, the larger value of $m_t$ suggested both by the recent precision data and the CDF events favors a larger value for $\alpha_s$, in good agreement with the direct determinations at the $Z$-pole, but slightly larger than some low energy determinations. Implications for proton decay, Yukawa unification, and the Higgs mass will be briefly discussed.

## 2 The Standard Model Parameters

The high precision electroweak data from LEP and SLC, as well as $M_W$ and low energy neutral current data, is generally in excellent agreement with the predictions of the standard model [1], although there are discrepancies (at the $2 - 3\sigma$ level) in the SLD measurement of the left-right polarization asymmetry $A_{LR}$ [2], and in $R_b = \Gamma(Z \to b\bar{b})/\Gamma(Z \to \text{hadrons})$. A global fit to all indirect data yields [3]

\[
\begin{align*}
\sin^2 \hat{\theta}_W(M_Z) &= 0.2317(3)(2) \\
\alpha_s(M_Z) &= 0.127(5)(2) \quad (SM) \\
m_t &= 175 \pm 11^{+17}_{-19} \text{ GeV},
\end{align*}
\]

where $\sin^2 \hat{\theta}_W(M_Z)$ and $\alpha_s(M_Z)$ are computed in the $\overline{MS}$ scheme. The central values and first uncertainty are for a Higgs mass of $M_H = 300$ GeV, while the second uncertainty is from taking $M_H = 1000$ GeV (+) and 60 GeV (-). The predicted value of $m_t$ in (1) is in remarkable agreement with the value $174 \pm 16$ GeV suggested by the CDF candidate events [4]. The value of $\alpha_s$ is determined from the $Z$ lineshape data. It is in good agreement with the value $0.123 \pm 0.006$ obtained from LEP jet event shapes, but is higher than most low energy determinations [5], as shown in Table [4]. In particular, it is higher than a recent lattice determination from the $b\bar{b}$ spectrum [7], which claims a very small uncertainty.

The lineshape determination of $\alpha_s$ is clean theoretically assuming the validity of the standard model, but is sensitive to the presence of new physics which affects the $Zb\bar{b}$ vertex, as suggested by $R_b$ [3]. The data marginally favors a low value for $M_H$, but this is weak statistically ($M_H < 880$ GeV at 95% CL, or $M_H < 730$ GeV if the CDF $m_t$ value is included [3]). Moreover, it is driven almost entirely by the anomalous values of $A_{LR}$ and $R_b$. The data also set stringent limits on the new physics parameters $S_{new}$, $T_{new}$ (or $\rho_0 - 1$) and $U_{new}$ [3].
Table 1: Values of $\alpha_s(M_Z)$ extracted from various processes. Except for the $\tau$ decay value, the low energy determinations extrapolated to $M_Z$ are lower than the $Z$-pole values.

| Process                              | $\alpha_s(M_Z)$ |
|--------------------------------------|-----------------|
| LEP line-shape                       | $0.127 \pm 0.005$ |
| LEP event shapes                     | $0.123 \pm 0.006$ |
| hadronic $\tau$ decays (LEP)         | $0.122 \pm 0.005$ |
| deep inelastic scattering             | $0.112 \pm 0.005$ |
| $\Upsilon, J/\Psi$ decays            | $0.113 \pm 0.006$ |
| $c\bar{c}$ spectrum (lattice)        | $0.110 \pm 0.006$ |
| $b\bar{b}$ spectrum (lattice)        | $0.115 \pm 0.002$ |

In the supersymmetric extension of the standard model (SSM), most of these conclusions continue to hold. For most of the allowed parameter space, the new superpartners and extra Higgs particles are sufficiently heavy that their contributions to the radiative corrections to the precision observables are negligible. That is, one does not expect to see deviations from the standard model. However, there is a light Higgs scalar which (for most of parameter space) acts like a light standard model Higgs. One should therefore use the more restricted range $60 \text{ GeV} < M_H < 150 \text{ GeV}$, rather than $60 - 1000 \text{ GeV}$. Because of the strong $m_t - M_H$ correlations in the radiative corrections, one now finds

$$\sin^2 \hat{\theta}_W(M_Z) = 0.2316(3)(1)$$
$$\alpha_s(M_Z) = 0.126(5)(1) \text{ (SSM)}$$
$$m_t = 160^{+11+6}_{-12-5} \text{ GeV}$$

rather than (1), where the central value is for $M_H = M_Z$ and the second uncertainty is for $60 \text{ GeV} < M_H < 150 \text{ GeV}$. The $m_t$ prediction is still consistent with the CDF range, but favors the lower end.

### 3 Supersymmetry and Grand Unification

Let me now turn to supersymmetry and grand unification, which can occur independently or in combination. I will first briefly review the ideas of grand unification and of supersymmetry, and then turn to the possible unification of coupling constants.

#### 3.1 Grand Unification

Some of the shortcomings of the standard model, especially those associated with the fact that it involves three distinct gauge sectors, are addressed in grand unified theories [8, 9]. The idea is that the strong, weak, and electromagnetic interactions are unified at some large unification scale $M_X$, i.e., the interactions are embedded in
a simple gauge group $G$ which is manifest above this scale. At $M_X$ the symmetry is broken to the smaller standard model group $SU_3 \times SU_2 \times U_1$, so that at low energies the interactions appear different. If one measures the gauge coupling constants at low energies and extrapolates to large scales they should meet at the scale $M_X$ above which the symmetry breaking is irrelevant. For unification without gravity to make sense it is necessary that $M_X$ is small compared to the Planck scale, $M_p \sim G^{-1/2}_N \sim 10^{19}$GeV.

In addition to the gauge interactions, $q, \bar{q}, \ell, \bar{\ell}$ are typically unified, i.e., placed in the same multiplets. This explains charge quantization (the fact that atoms are electrically neutral), which is incorporated in, but not explained by, the standard model. Moreover, there will be new interactions between quarks and leptons and their antiparticles which will typically lead to proton decay. The simplest example is the Georgi-Glashow model, based on the gauge group $SU_5$ [8]. At a large $M_X$ the symmetry is broken to $SU_3 \times SU_2 \times U_1$, and then to the unbroken subgroup $SU_3 \times U_{1Q}$ in a second stage of breaking at the electroweak scale $M_Z$. The fermion representations are still quite complicated in this model, with each generation of fermions placed in a reducible $5^*+10$. The $5^*$ representation consists of the left-handed fields $(\nu e^{-d})_L$ while the 10 consists of $(e^+ u d\bar{u})_L$. In addition to the electroweak and QCD interactions there are two new superheavy gauge bosons $X$ and $Y$ with electric charges $\frac{4}{3}$ and $\frac{1}{3}$, respectively, which can mediate transitions between the particles in these multiplets and lead to proton decay. For example, the decay $p \rightarrow e^+ \pi^0$ proceeds by the exchange of the superheavy $X$ boson, as shown in Figure 1. The lifetime is

$$\tau_{p \rightarrow e^+ \pi^0} \sim \frac{M_X^4}{\alpha_G^2 m_p^5},$$

where $\alpha_G$ is the value of the coupling constant at the unification scale. Around 1980, when such theories were first taken seriously, the experimental limit on the lifetime was $\geq 10^{30}$ yr., corresponding to $M_X > 10^{14}$ GeV. Several new proton decay experiments were mounted to search for such decays, but they were not observed, excluding the simplest $SU_5$ and similar models.

One can embed the model in larger gauge groups. In the $SO_{10}$ model each fermion family is placed in an irreducible 16 dimensional representation, which decomposes under the $SU_5$ subgroup as $16 = 5^* + 10 + 1$. Each family thus has a new neutrino $\bar{N}_L$, which is a singlet under the normal electroweak gauge group but which may acquire a large Majorana mass. Even larger groups are often considered. In $E_6$ each family is placed in a 27 dimensional representation, which decomposes under $SO_{10}$ as $27 = 16 + 10 + 1$. In addition to the 16 particles already described, one has a 10 of $SO_{10}$, consisting of a new heavy down-type quark, $(D \bar{D})_L \sim (D_L D_R)$ for which both left and right-handed components are $SU_3$ singlets. The 10 also contains a vector doublet of leptons $(E^+ E^0 \bar{E}^0 E^-)_L \sim (E^+ E^0)_L (E^+ E^0)_R$, for which both the left and right components transform as weak doublets. There is also a second neutral lepton, $S_L$, which again may acquire a large Majorana mass.

4
3.2 Supersymmetry

Supersymmetry \[\text{[10]}\] is a new type of symmetry relating fermions to bosons, which can occur with or without grand unification. If unbroken, it would mean that for every fermion there is a boson with the same mass and related couplings. In the real world the symmetry is broken.

Although supersymmetry has not been observed experimentally, there are several motivations for believing that it might exist. The first is that it can stabilize the weak scale. In the absence of supersymmetry there are quadratically divergent loop corrections to the mass of the Higgs which tend to renormalize the mass up to very large scales such as the Planck scale. One has to do a fine-tuned cancellation of those corrections against the bare terms to keep the observed weak interaction scale. In the presence of supersymmetry, however, there will be cancellations between loops involving fermions and bosons. The cancellation would be exact if the supersymmetry were unbroken. In the presence of breaking there is still a sufficiently stable weak scale provided \(M_{\text{SUSY}} < O(1 \text{ TeV})\), where \(M_{\text{SUSY}}\) is a typical mass of the new particles.

The second motivation is that in gauged supersymmetry, known as supergravity, there is an automatic unification of the other interactions with gravity. This does not by itself make gravity renormalizable, but at least brings it into the game. A final motivation is that the observed coupling constants are consistent with the simplest version of supersymmetric grand unification.

There are many consequences of supersymmetry. One is that there must be a second Higgs doublet, so the spectrum of the theory will involve additional charged and neutral Higgs particles. However, there is always one light Higgs scalar satisfying

\[
M_{H^0}^2 < \cos^2 2\beta \ M_Z^2 + \text{H.O.T.} \ (O(m_t^4)) < (150 \text{ GeV})^2, \tag{4}
\]

where \(\tan \beta = v_t/v_b\) is the ratio of vacuum expectation values of the two Higgs doublets which generate masses for the \(t\) and \(b\), respectively. This is in contrast...
to the standard model, in which there is no rigorous upper-bound on the Higgs mass, though there are reasonably convincing theoretical arguments that suggest $M_{H^0} < 600 - 1000$ GeV [11]. The first term in (11) appears at tree level in the minimal supersymmetric extension of the standard model [12] (slightly higher values are allowed in non-minimal models with additional Higgs singlets [13]). If this were the only term it would bound the light Higgs scalar mass to be less than $M_Z$. However, there are large loop corrections [12] which can scale like $m_t^4$, so that one typically has an upper limit around 130 (150) GeV in the minimal (non-minimal) model. For most of parameter space the second Higgs doublet is heavier.

In addition, there are the new superpartners for every known particle: for each quark there must be a scalar quark, $\tilde{q}$, and each lepton must be associated with a new scalar lepton, $\tilde{\ell}$. Similarly, the gauge bosons have new fermionic partners; for example, the $W$ has a wino partner, $\tilde{w}$. Typically, these new particles will be in the several hundred GeV range. It is often the case that there is a lightest supersymmetric partner (LSP), and in many versions of the model this would be an excellent candidate for cold dark matter [14]. It is interesting that the most plausible mechanism for breaking electroweak symmetry in supersymmetric models requires a large $m_t$, comparable to the values needed by the precision experiments and by CDF.

### 3.3 Gauge Unification and Its Implication

#### 3.3.1 Unification of Coupling Constants

Using the observed low energy gauge couplings as boundary conditions one can compute the running couplings from the renormalization group equations

$$\frac{d\alpha_i^{-1}}{d \ln \mu} = -\frac{b_i}{2\pi} - \sum_{j=1}^{3} \frac{b_{ij} \alpha_j}{8\pi^2},$$

(5)

where $\alpha_i = g_i^2/4\pi$ and $g_i$ is the gauge coupling of the $i$th gauge group. The two terms on the right are respectively the one and two-loop contributions to the running. Equation (6) may be solved analytically to yield

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(M_X) - \frac{b_i}{2\pi} \ln \left( \frac{\mu}{M_X} \right) + \sum_{j=1}^{3} \frac{b_{ij}}{4\pi b_j} \ln \left[ \frac{\alpha_j^{-1}(\mu)}{\alpha_j^{-1}(M_X)} \right].$$

(6)

For the observed couplings the first two terms dominate and the inverse coupling runs approximately linearly in $\ln \mu$. However, the nonlinear correction from the last (two loop) term is not completely negligible.

If there is a grand unification then naively the three gauge couplings should have a common value at the unification scale $M_X$. More precisely, one expects

$$\alpha_i^{-1}(M_X) = \alpha_G^{-1}(M_X) + \delta_i + \Delta_i,$$

(7)
where the first term represents the common coupling and the other two are threshold corrections \[\delta \]
represents low-scale corrections due to the fact that not all of the new particles are degenerate. There are contributions from \(m_t > M_Z\), as well as from the nondegeneracy of the new superpartner and heavy Higgs particles, \(M_{\text{new}} \neq M_{\text{SUSY}}\). The last term represents the high-scale threshold corrections, due both to mass splittings at the high scale, \(M_{\text{heavy}} \neq M_X\), and to nonrenormalizable operators (NRO) which may be left over from quantum gravity.

The coefficient functions depend on the matter content of the theory. If one has only the standard model particles above the Z scale, then the one-loop coefficients are [16]

\[
b_i = \begin{pmatrix}
0 \\
-\frac{22}{3} \\
-11
\end{pmatrix} + F \begin{pmatrix}
4/3 \\
4/3 \\
4/3
\end{pmatrix} + N_H \begin{pmatrix}
1/10 \\
1/6 \\
o
\end{pmatrix}.
\] (8)

The three terms refer, respectively, to the contributions of gauge boson, fermion, and Higgs loops; \(F\) is the number of fermion generations; and \(N_H\) is the number of Higgs doublets. Similarly, in the minimal supersymmetric extension (MSSM) the new superpartners and Higgs bosons modify the equations so that

\[
b_i = \begin{pmatrix}
0 \\
-6 \\
-9
\end{pmatrix} + F \begin{pmatrix}
2 \\
2 \\
2
\end{pmatrix} + N_H \begin{pmatrix}
3/10 \\
1/2 \\
o
\end{pmatrix}.
\] (9)

The two-loop coefficients, \(b_{ij}\), may be found in [16].

To apply the renormalization group equations one uses the observed couplings at the electroweak scale as boundary conditions. The \(SU_3 \times SU_2 \times U_1\) couplings are given by

\[g_3 = g_s, \quad g_2 = g, \quad g_1 = \sqrt{\frac{5}{3}} g',\] (10)

where the coefficient in \(g_1\) is a normalization factor, needed so that all of the charges which unify are normalized in the same way. That is,

\[
\text{Tr} Q_s^2 = \frac{5}{3} \text{Tr} \left( \frac{Y}{2} \right)^2.
\] (11)

Since historically the weak hypercharge \(Y\) was renormalized differently, this must be compensated for. The observed charge of the positron and weak angle are related by

\[
e = g \sin \theta_W\] (12)

and

\[
\sin^2 \theta_W = \frac{g^2}{g^2 + g'^2} = \frac{g_1^2}{\frac{3}{2} g_2^2 + g_1^2}.
\] (13)
At the unification scale one has \( g_1 = g_2 \), implying \( \sin^2 \theta_W \rightarrow \frac{2}{3} \) \[17\]. At low energies the couplings will be different, yielding a smaller value for \( \sin^2 \theta_W \). Hence,

\[
\alpha_3 = \alpha_s, \quad \alpha_2 = \frac{\alpha}{\sin^2 \theta_W}, \quad \alpha_1 = \frac{5}{3} \frac{\alpha}{\cos^2 \theta_W}
\]  

(14)

where all quantities are to be evaluated at \( M_Z \) in the \( \overline{\text{MS}} \) scheme. I will use as input values \[18\]

\[
\alpha(M_Z)^{-1} = 127.9 \pm 0.1,
\]

where the uncertainty is due to low energy hadronic uncertainties, and \[3\]

\[
\sin^2 \hat{\theta}_W(M_Z) = 0.2316 \pm 0.0003,
\]

(16)

as determined from the precision data. I will usually use these to predict \( \alpha_s \), which is the least known. However, for those tests in which \( \alpha_s \) is an input I will use the range

\[
\alpha_s(M_Z) = 0.12 \pm 0.01
\]

(17)

which is a reasonable average of both the LEP and low energy determinations.

From these inputs one can plot the running couplings and see whether they meet. It is seen in Figure 2 that they do not meet in the ordinary standard model, but they do meet with reasonable precision in the supersymmetric extension \[19\]–\[21\]. This provides evidence, independent of the nonobservation proton decay, that the simplest versions of nonsupersymmetric grand unification are excluded, while the supersymmetric case is allowed.

### 3.3.2 Implications of Coupling Unification

The coupling constants unify in the supersymmetric extension of the standard model at a scale

\[
M_X \sim 3 \times 10^{16}\text{GeV},
\]

(18)
but not in the ordinary standard model. Does this constitute proof of supersymmetry? No, of course not. The apparent unification could be an accident, having nothing to do with grand unification. Similarly, ordinary models without supersymmetry could be modified to force unification, for example, by introducing intermediate scales or new multiplets of particles split into light and heavy sectors. Of course, such models are ad hoc and have no predictive power for the gauge couplings. Such new ingredients could also disturb the agreement within the supersymmetric extension.

The predictions are largely independent of the specific GUT group, \( SU_5, SO_{10}, E_6 \) etc., provided the charge normalizations are preserved. Also, the predictions do not depend on the number of families at 1-loop order, although weak dependence does enter at 2-loops. This is because a family is a complete multiplet of the underlying GUT, which affects the slopes of all of the gauge couplings equally. In contrast, there is a strong dependence on the number of Higgs doublets, because the latter involves multiplets splits into light and superheavy components. Only the light components (the Higgs doublets) affect the running, and these contribute to the running of the electroweak but not the strong couplings. One could improve the unification of couplings in the ordinary grand unified theory by adding additional Higgs doublets, but only at the expense of having a lower unification scale and therefore a more rapid (and unacceptable) proton decay rate.

So far, the predictions are idealized and ignore the threshold corrections. A number of authors \([22]-[24]\) have pointed out that there are irreducible theoretical uncertainties in the predictions. One source is from the low scale thresholds. It was emphasized by Roberts and Ross \([22]\) that the approximation made in the early papers of treating all of the new particles as degenerate with a common mass \( M_{\text{SUSY}} \) is not adequate, and that the splittings between the sparticles are more important. For example, the colored particles are usually heavier than the uncolored ones, and this is more important than the average mass because it discriminates between the gauge couplings. There are also uncertainties associated with the splittings of the superheavy particles about the unification scale \([23, 24]\). Also, since the unification scale is two or three orders of magnitude below the Planck scale there may be significant nonrenormalizable operators (NRO), typically of \( O(M_X/M_P) \sim 10^{-1} - 10^{-3} \), which may contribute to the relative normalization of the gauge couplings \([23, 24]\).

It is not convenient to display these uncertainties on the plot of the three couplings. It is more useful to use two of the couplings to predict the third, allowing one to show the theoretical uncertainties explicitly. Traditionally, people have used \( \alpha \) and \( \alpha_s \) to predict \( \sin^2 \theta_W \). However, that is not the optimum approach because the largest uncertainties are then in the input quantity, \( \alpha_s \). Furthermore, different authors have used different values of \( \alpha_s \), leading to considerable confusion. It is more enlightening to use \( \alpha \) and \( \sin^2 \theta_W \) to predict \( \alpha_s \), leading to the prediction shown in Figure 3. The standard model prediction \([23]\)

\[
\alpha_s = 0.073 \pm 0.001 \pm 0.001 \tag{19}
\]
Figure 3: Predictions for $\alpha_s$ using $\alpha$ and $\sin^2 \theta_W$ as input quantities in both the standard model and in the MSSM. The large theoretical error bar includes a reasonable (but not rigorous) estimate of the low and high scale threshold uncertainties and of NRO’s. The smaller error bars assume that the new particles are all degenerate at $M_Z$ or 1 TeV and ignore the high scale uncertainties. From [23].

is far from the experimental data. In (19) the first uncertainty is from the inputs and $m_t$, while the second is from the high-scale thresholds and NRO. The prediction in the supersymmetric extension is

$$\alpha_s = 0.129 \pm 0.002 \pm 0.005 \pm 0.002 \pm 0.006,$$

where the uncertainties refer respectively to the inputs at $m_t$; the low scale (SUSY) thresholds; the high scale thresholds; and NRO. It must be emphasized that the uncertainties are only estimates, based on reasonable ranges for the magnitudes of the mass splittings and NRO coefficients. They should be interpreted as typical values, not absolute error bars. The central value in (20) is in good agreement with the higher values of $\alpha_s$ determined from the $Z$ line shape and JET event topologies but is somewhat higher than some of the low energy determinations. If the latter turn out to be true then supersymmetric unification would require large but not unreasonable threshold corrections. In Figure 3 the predicted point includes the full uncertainties, combined in quadrature. The theoretical uncertainties would be much smaller if one ignored the high scale threshold and NRO’s and assumed that all of the new particles are degenerate with a mass either $M_Z$ or 1 TeV. However, the larger error bar indicated in the figure is much more reasonable. One can also adopt the more traditional approach of using $\alpha$ and $\alpha_s$ to predict $\sin^2 \theta_W$ as shown in Figure 4.

If the apparent coupling unification is not just an accident, then, barring fortuitous cancellations, there are very few types of new physics other than supersymmetry which would be allowed and not mess up the unification. These include new gauge structures which commute with the standard model, such as new heavy $Z'$ bosons; new complete multiplets of fermions and their superpartners, including sequential, mirror, or exotic families; and new gauge singlets.
The connection of all of this with superstring theories is somewhat obscure. The basic ideas of supersymmetric coupling unification are consistent with the ideas of superstrings. However, the unification scale $M_X \sim 3 \times 10^{16}$ GeV is smaller than what one would expect in a naive string theory, where typically $M_X^{\text{string}} \sim g \times 5 \times 10^{17}$ GeV \cite{26}. On the one hand, the fact that $M_X$ is lower than the gravity scale implies that the unification of the microscopic interactions without gravity is consistent. It is not at all clear whether the observed pattern can emerge consistently from a superstring theory. It is conceivable that it does and that there are large string-scale threshold corrections \cite{27}, but so far no realistic explicit models. It is hard, but not impossible, to imagine a real grand unified theory emerging below the string scale, but so far nobody has found a successful compactification. Attempts have yielded a large number of unwanted new particles and no mechanism for breaking the grand unification symmetry. For these reasons I will concentrate on true supersymmetric grand unified theories, and the connections with the more elegant possibility of string theories must wait.

There are a number of implications, predictions, and problems for a true grand unified theory. One possible difficulty (which is not necessarily shared by superstring theories) is proton decay. Proton decay is especially problematic for ordinary (non-supersymmetric) grand unified theories \cite{3}, where the $SU_3$ and $U_1$ couplings unify at the relatively low scale $M_X \sim (2 - 7) \times 10^{14}$ GeV. (The $\sin^2 \theta_W$ prediction does not work in such theories.) From diagrams such as shown in Figure 1a the predicted lifetime is of order $\tau_{p \to e^+ \pi^0} \sim 10^{31+1}$, which is excluded by the experimental limit $\tau_{p \to e^+ \pi^0} > 10^{33}$ yr \cite{28}.

In the supersymmetric grand unified theories the unification scale is much higher, typically $M_X \sim (1 - 5) \times 10^{16}$ GeV. This leads to a safely unobservable decay rate by the ordinary superheavy boson exchange, $\tau_{p \to e^+ \pi^0} \sim 3 \times 10^{38+1}$ yr. However, the supersymmetric models have a new mechanism, the dimension-5 operators mediated by (fermionic) superheavy higgsinos, as shown in Figure 1b. Since the exchanged
Figure 5: Regions in the $\tan \beta - m_t$ plane allowed by Yukawa unification. $m_b$ is too large in between the small and large $\tan \beta$ branches while outside one of the Yukawa couplings diverges below the unification scale. Also shown is the band predicted by the unification of all three Yukawa couplings, which occurs in some specific models. From [32].

Heavy particle is a fermion one has a much more rapid decay with lifetime $\tau_p \sim M_X^2$, implying $\tau_{p \rightarrow \nu K^+} \sim 10^{29\pm 4} \text{yr}$ [29]. Much of this range is excluded by the experimental limit $\tau_{p \rightarrow \nu K^+} > 10^{32} \text{yr}$. Many specific models are either pressed or eliminated. Again, I stress that such mechanisms may not be present in superstring theories, which do not have the full structure of grand unification.

Many of the simpler grand unified theories (those in which the fermion masses are generated by Higgs bosons in the fundamental representation) predict Yukawa unification between the $b$ and $\tau$ couplings [30], i.e., $m_b(M_X) = m_{\tau}(M_X)$ at the unification scale. At low energies, due to different renormalizations of the $b$ and $\tau$ vertices, one has $m_b \sim (2 - 3)m_\tau$. A number of authors have studied whether this actually holds [31]-[33]. The difficulty is that the predicted $m_b$ tends to be large due to gluonic vertex corrections, which are large if $\alpha_s$ is. They are partially cancelled by Higgs mediated corrections involving the top quark Yukawa. We have seen that the $\alpha_s$ predicted by grand unification is on the high side of the data, and this in turn implies relatively large predictions for $m_b$. Consistency suggests that one should use the prediction for $\alpha_s$ from gauge unification rather than a fixed input value[2]. The result is that two narrow bands in the ratio $\tan \beta \equiv v_t/v_b$ of the two Higgs vacuum expectation value are consistent with the data, as shown in Figure 5.

The simplest supersymmetric grand unified theories predict Yukawa unification, and are only viable for two bands in $\tan \beta$. In particular, one of the solutions implies that $\tan \beta$ is close to unity. These predictions need not hold in superstring-inspired imitators, but only in true grand unified theories. However, the same prediction for the Yukawa couplings fails for the first two families, and in practice one must invoke

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2Threshold corrections affect both $\alpha_s$ and the $m_b$ prediction, and should be treated consistently [32].
some sort of perturbation \[3\] on the theory to correct the masses of the first two families, which presumably would not be large for the third family. Some specific models, in particular \(SO_{10}\) models in which the fermion masses are generated by a single complex ten-dimensional Higgs multiplet, make the stronger prediction of three Yukawa unification, \(m_b = m_\tau = m_t\) at the Planck scale \[32\]. This is consistent with the observed value of \(m_t\) only for the large \(\tan \beta\) branch. This is a much more model dependent assumption.

Supersymmetric models may have strong implications for the mass of the lightest Higgs scalars. Almost all supersymmetric models imply the upper bound \(M_{H^0} < 150\) GeV on the lightest Higgs scalar. However, the models with Yukawa unification on the small \(\tan \beta\) branch have a stronger prediction \[33\]-\[36\]. For \(\tan \beta \sim 1\) the tree-level contribution to the mass approximately vanishes, so that the masses are generated mainly by the loop corrections. One finds the more stringent upper limit \[36\]

\[
M_{H^0} < 110 \text{ GeV. (21)}
\]

Although there are many assumptions, namely supersymmetric unification, Yukawa unification, and the small \(\tan \beta\) branch, this is a striking prediction. If true, it may be possible to observe the Higgs at LEP-II or possibly even LEP-I. For most allowed value of the supersymmetry breaking parameters the mass is much less than the upper limit, as is shown in Figure 3. The various limits on the Higgs mass as a function of \(m_t\) are shown in Figure 4.

From Figure 6 one sees that there are upper bounds on the Higgs mass in the supersymmetric models, and there is also a lower bound from vacuum stability in the standard model \[37, 38\]. There is actually a forbidden gap, at least in the restricted model with Yukawa unification.

Another implication of grand unification is that typically one generates small masses for the ordinary neutrinos by a seesaw mechanism \[39\]. If the large scale in the seesaw is a typical grand unification mass, or a few order of magnitude smaller, then one expects masses that are in the range relevant to solar neutrino oscillations and perhaps a component of hot dark matter associated with \(m_\nu\) \[10\]. This connection is very exciting, but the details are extremely model dependent \[11, 12\].

Another aspect of supersymmetric theories is that typically the superpartners and additional Higgs particles have masses much greater than \(M_Z\). In that case they decouple from precision experiments and one does not expect to see large deviations from the standard model predictions.

The only successful scheme for supersymmetry breaking is the hidden sector scenario \[10\], in which the supersymmetry is broken in a hidden sector which has no interactions with the ordinary particles except gravity. The breaking is transmitted to the observable world only by very weak gravitational interactions. Typically, this implies that all of the scalars in the theory will have a common mass evaluated at \(M_p/\sqrt{8\pi}\), and that the gauginos will also have a common mass at that scale. At lower energies the masses diverge due to running effects, and if \(m_t\) is sufficiently large
Figure 6: Predictions for the lightest Higgs mass for various ranges of $m_t$ as the supersymmetry breaking parameters are chosen randomly. The Higgs mass is typically well below the upper limit of 110 GeV. From [36].
one of the Higgs masses will be driven negative at low energies, leading to radiative $SU_2 \times U_1$ breaking [10]. This elegant connection between the supersymmetry and $SU_2 \times U_1$ breaking requires a large $m_t$. The fact that the scalars start out degenerate at a high scale means that flavor changing neutral current (FCNC) effects at low energy are relatively small. The latter are generated by box diagrams involving the superpartners. They are proportional to mass splittings and are sufficiently suppressed in the hidden sector models, at least for transitions involving the first two families. There are several new sources of CP violation in supersymmetric models, in addition to the usual CKM phases [13]. In the hidden sector scenario the neutron electric dipole moment $d_n < 10^{-24}e\text{cm}$ places severe constraints on the phases of the supersymmetry breaking terms. The most plausible solution is that the supersymmetry breaking parameters are real, which is not very surprising if they are generated by gravity. However, if they are real there will usually be no observable new sources of CP violation in the MSSM.

Although the hidden sector models present a nice general picture, the detailed mechanism for the supersymmetry breaking in the hidden sector is unclear.

There are many implications for the sparticles and the second Higgs doublet, but the predictions for their spectra are model dependent [44]. In most such models there is a lightest supersymmetric particle (LSP). If this is neutral it will be a candidate for cold dark matter [14]. Recently, it has been emphasized that the usual assumption that the hidden sector models imply a common scalar mass at the unification scale $M_X$ will not necessarily hold [15]. It might be more plausible to assume that the
common scalar mass is at $M_p/\sqrt{8\pi}$. In that case, the running between $M_p/\sqrt{8\pi}$ and $M_X$ can have a nontrivial effect on the low energy theory.

One of the most important problems in the standard model is an explanation of the fermion masses and mixing. Unfortunately, neither ordinary nor supersymmetric grand unified theories yield much information on this. (The one thing that they do predict is the ratio $m_b/m_\tau$ in those models with Yukawa unification.) If there is really an underlying superstring theory then the fermion mass spectrum will ultimately be determined by the compactification of the extra dimensions. However, nobody has concrete and realistic models. Recently, there has been considerable activity in texture models, in which one postulates a form of the fermion mass matrices at high energies and then makes predictions for low energies [10]. Much of this work has been quite successful phenomenologically. However, the form of the initial textures of the mass matrices are model dependent, and to implement them requires the addition of extra symmetries. Usually, they require higher-dimension Higgs multiplets, which are not compatible with the simplest superstring theories.

4 Conclusions

- The precision electroweak experiments have confirmed the electroweak standard model spectacularly even though there are $2-3\sigma$ deviations in $R_b$, and $A_{LR}$.
- Currently the weak angle is determined to be [3]
  \[
  \begin{align*}
  \overline{MS} & \quad \hat{s}_Z^2 = 0.2317(3)(2) \\
  \text{on-shell} & \quad s_W^2 = 0.2243(12) = 1 - \frac{M_W^2}{M_Z^2} \\
  \text{effective} & \quad \hat{s}_t^2 = 0.2320(3)(2) = \kappa_t \hat{s}_Z^2
  \end{align*}
  \]
  in various renormalization schemes, where the central value and first uncertainty are for $M_H = 300$ GeV, and the second uncertainty is for $60$ GeV $< M_H < 1000$ GeV.
- One predicts
  \[
  m_t = 175 \pm 11^{+17}_{-19} \text{GeV}
  \]
  in the standard model, where the second uncertainty is from $M_H$. This is in spectacular agreement with the CDF direct candidate event masses $174 \pm 16$ GeV.
- The predictions change slightly in the supersymmetric extension due to the fact that the Higgs mass is lower. There one predicts
  \[
  m_t = 160^{+11+6}_{-12-5} \text{GeV},
  \]
  which is on the lower side of, but still consistent with, the CDF range. Also, $\hat{s}_Z^2 \rightarrow 0.2316(3)(1)$. 

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• One can also determine the value of the strong coupling at the Z-pole, namely

\[ \alpha_s(M_Z) = 0.127(5)(2) \]  

from the Z line shape. This is in good agreement with the value obtained from jet event shapes at LEP, 0.123 ± 0.006, and also with the predictions of supersymmetric grand unification. It is somewhat higher, however, than some low energy determinations based on deep inelastic scattering or lattice calculations of the c\bar{c} or b\bar{b} spectra. Although the value in (25) is reliable assuming the standard model, it is sensitive to new physics in the Z → b\bar{b} vertex, which is experimentally high. If new physics contributes to that vertex there can be a smaller \( \alpha_s \), in better agreement with the low energy determinations.

• The data exhibits some preference for a light \( M_H \), consistent with the expectations of supersymmetry. However, this evidence is weak statistically, with values up to \( \sim 800 \text{ GeV} \) still allowed. Furthermore, most of the sensitivity to \( M_H \) is due to the input values of \( R_b \) and \( A_{LR} \), both of which are high compared to the standard model predictions. For these reasons, one should not take the \( M_H \) limit too seriously at present.

• The combination of the precision data with the CDF direct determination of \( m_t \) allows one, for the first time, to separate the contributions of certain types of new physics from the dependence of \( m_t \). In particular, the parameters \( \rho_0 - 1 \), \( S_{\text{new}} \), \( T_{\text{new}} \), and \( U_{\text{new}} \) which describe various types of \( SU_2 \) breaking beyond the standard model are all consistent with zero and stringently constrained [3].

• Most theories involving compositeness or dynamical symmetry breaking are strongly disfavored by the precision experiments as well as the nonobservation of FCNC.

• In supersymmetry, on the other hand, for most of the allowed parameter space the heavy particles decouple from FCNC, precision experiments, and CP violation, consistent with the nonobservation of deviations. Another implication is that \( M_H < 150 \text{ GeV} \), and in many cases the limit is still lower. This implies that the light Higgs may be observable in the immediate future at accelerators. The coupling constant unification predicts \( \alpha_s = 0.129(8) \), where the uncertainty is a typical (and non-rigorous) range of theoretical uncertainties associated with the low and high scale thresholds [23]. The central value is in good agreement with the LEP determinations from the line shape and jet event shapes. However, given the theoretical uncertainties, lower values, as suggested by low energy data, are possible. If one has a true supersymmetric grand unification, then depending on assumptions about the Higgs sector there may be interesting predictions for the ratio \( \tan \beta \) of the two Higgs vacuum expectation values and for the Higgs mass \( M_H \). The models often predict an observable proton decay...
rate, and there are model dependent but nevertheless interesting predictions for cold dark matter and for neutrino mass. The ideas of supersymmetric unification are, in many ways, suggestive of superstring theories. However, the precise connection between the two sets of ideas still escapes us.

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