Lorentz violation and black-hole thermodynamics: Compton scattering process

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Abstract

A Lorentz-noninvariant modification of quantum electrodynamics (QED) is considered, which has photons described by the nonbirefringent sector of modified Maxwell theory and electrons described by the standard Dirac theory. These photons and electrons are taken to propagate and interact in a Schwarzschild spacetime background. For appropriate Lorentz-violating parameters, the photons have an effective horizon lying outside the Schwarzschild horizon. A particular type of Compton scattering event, taking place between these two horizons (in the photonic ergoregion) and ultimately decreasing the mass of the black hole, is found to have a nonzero probability. These events perhaps allow for a violation of the generalized second law of thermodynamics in the Lorentz-noninvariant theory considered.

Key words: Lorentz violation, black-hole thermodynamics, Compton scattering

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1. Introduction

Lorentz-violating theories coupled to gravity can have interesting black-hole solutions. Particles that obey Lorentz-violating dispersion relations may perceive an effective horizon different from the event horizon for standard Lorentz-invariant matter. It has been argued that such multiple-horizon structures allow for the construction of a perpetuum mobile of the second kind (involving heat transfer from a cold body to a hot body, without other change).

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This Letter considers modified Maxwell theory as a concrete realization of a Lorentz-violating theory. With an appropriate choice for the Lorentz-violating parameters, the nonstandard photons have an effective horizon lying outside the Schwarzschild event horizon for standard matter. Of interest, now, are Compton scattering events $\gamma e^- \rightarrow \gamma e^-$, which take place between these two horizons, that is, in the accessible part of the photonic ergosphere region. After the collision, the photon may carry negative Killing energy as it propagates inside the photonic ergosphere, so that the final electron carries away more Killing energy than the sum of the Killing energies of the ingoing particles. As shown in Sec. IV–B of Ref. [2], such a scattering event ultimately reduces the black-hole mass. In the following, it will be demonstrated that this particular Compton scattering event is kinematically allowed and has a nonvanishing probability to occur.

The purpose of this Letter is to give a concrete example of a Compton scattering event that can be used to reduce the black-hole mass. This requires a detailed discussion of the theory in Sec. 2, which can, however, be skipped in a first reading. The main result is presented in Sec. 3 and discussed in Sec. 4, both of which sections are reasonably self-contained.

2. Setup

2.1. Units and conventions

Natural units are used with $c = G_N = \hbar = 1$. Spacetime indices are denoted by Greek letters and correspond to $t, r, \theta, \phi$ for standard spherical Schwarzschild coordinates or to $\tau, R, \theta, \phi$ for Lemaître coordinates. Local Lorentz indices are denoted by Latin letters and run from 0 to 3. The flat-spacetime Minkowski metric is $\eta_{ab}$ and the curved-spacetime Einstein metric $g_{\mu\nu}$, both with signature $(+, -, -, -)$. The determinant of the metric is denoted by $g \equiv \det g_{\mu\nu}$. The vierbeins are introduced in the standard way by writing $g_{\mu\nu} = e_\mu^a e_\nu^b \eta_{ab}$ and obey the relations $e^\nu_a e_\mu^b = \delta^b_a$ and $e^\mu_a e_\nu^a = \delta^\mu_\nu$.

2.2. Modified QED in curved spacetime

Modified Maxwell theory is an Abelian $U(1)$ gauge theory with a Lagrange density that consists of the standard Maxwell term and an additional Lorentz-violating bilinear term. The vierbein formalism is particularly well-suited for describing Lorentz-violating theories in curved spacetime, since it allows to distinguish between local Lorentz and general coordinate transformations and to set the torsion identically to zero.
A minimal coupling procedure then yields the following Lagrange density for the photonic part of the action:

\[ L_{\text{modM}} = -\frac{1}{4} g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} - \frac{1}{4} \kappa^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}, \] (2.1a)

\[ \kappa^{\mu\nu\rho\sigma} \equiv \kappa^{abcd} e^a_\mu e^b_\nu e^\rho_c e^\sigma_d, \] (2.1b)

in terms of the standard Maxwell field strength tensor \( F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu \). The “tensor” \( \kappa^{abcd} \) has the same symmetries as the Riemann curvature tensor, as well as a double-trace condition. The numbers \( \kappa^{abcd}(x) \) are considered to be fixed parameters, with no field equations of their own.

In the following, we explicitly choose this background tensor field to be of the form \[  \kappa^{abcd}(x) = \frac{1}{2} \left( \eta^{ac} \kappa_{bd}(x) - \eta^{ad} \kappa_{bc}(x) + \eta^{bd} \kappa_{ac}(x) - \eta^{bc} \kappa_{ad}(x) \right), \] (2.2)

in terms of a symmetric and traceless background field \( \tilde{\kappa}^{ab}(x) \). Physically, (2.2) implies the restriction to the nonbirefringent sector of modified Maxwell theory. Moreover, we employ the following decomposition of \( \tilde{\kappa}^{ab}(x) \):

\[ \tilde{\kappa}^{ab}(x) = \kappa \left( \xi^a(x) \xi^b(x) - \eta^{ab}/4 \right), \] (2.3)

relative to a normalized parameter four-vector \( \xi^a \) with \( \xi_a \xi^a = 1 \). For our purpose, we will choose the parameter \( \kappa \) in (2.3) to be spacetime independent.

The breaking of Lorentz invariance in the electromagnetic theory (2.1) is indicated by the fact that the flat-spacetime theory allows for maximal photon velocities different from \( c = 1 \) (operationally defined by the maximum attainable velocity of standard Lorentz-invariant particles to be discussed shortly). See, e.g., Refs. [4, 5, 6, 7] for further details of the simplest version of modified Maxwell theory with constant \( \kappa^{abcd} \) over Minkowski spacetime and physical bounds on its 19 parameters.

The charged particles (electrons) are described by the standard Dirac Lagrangian over curved spacetime [9] and gravity itself by the standard Einstein–Hilbert Lagrangian [10]. All in all, this particular modification of quantum electrodynamics (QED) has action

\[ S = \int_{\mathbb{R}^4} d^4x \sqrt{-g} \left( L_{\text{EH}} + L_D + L_{\text{modM}} \right), \] (2.4a)

\[ L_{\text{EH}} = R/(16\pi), \] (2.4b)

\[ L_D = \bar{\psi} \left( \frac{1}{2} \gamma^\mu e^\mu_a i\nabla_\mu - m \right) \psi, \] (2.4c)
with Ricci curvature scalar $R$ from the metric $g_{\mu\nu}$, the usual Dirac matrices $\gamma^a$, and the gauge- and Lorentz-covariant derivative of a spinor \[9\],
\[
\nabla_\mu \psi \equiv \partial_\mu \psi + \Gamma_\mu \psi - eA_\mu \psi ,
\]
(2.5a)
with spin connection
\[
\Gamma_\mu = \frac{1}{2} \Sigma^{ab} e_a^\nu \partial_\mu (e_b^\nu) , \quad \Sigma_{ab} \equiv \frac{1}{4} (\gamma_a \gamma_b - \gamma_b \gamma_a).
\]
(2.5b)

2.3. Effective background for the photons

As demonstrated in Sec. 3 of Ref. \[3\], photons described by the Lagrange density (2.1) with the Lorentz-violating parameters (2.2)–(2.3) propagate on null-geodesics of an effective metric. This effective metric is given by:
\[
\tilde{g}_{\mu\nu}(x) = g_{\mu\nu}(x) - \frac{\kappa}{1 + \kappa/2} \xi_\mu(x)\xi_\nu(x),
\]
(2.6)
with an inverse following from $\tilde{g}^{\mu\nu}\tilde{g}_{\nu\rho} = \delta^\mu_\rho$. All lowering or raising of indices is, however, understood to be performed by contraction with the original background metric $g_{\mu\nu}$ or its inverse $g^{\mu\nu}$, unless stated otherwise.

In order to avoid obvious difficulties with causality, we restrict our considerations to a subset of theories without space-like photon trajectories (with respect to the original metric). This is ensured by the choice $0 \leq \kappa < 2$.

2.4. Schwarzschild spacetime metric

In the following, we consider a standard Schwarzschild geometry as given by the following line element:
\[
ds^2 = (1 - 2M/r) dt^2 - (1 - 2M/r)^{-1} dr^2 - r^2 d\Omega^2 ,
\]
(2.7a)
\[
d\Omega^2 \equiv d\theta^2 + \sin^2 \theta d\phi^2.
\]
(2.7b)
It will be convenient to work with Lemaître coordinates,
\[
ds^2 = d\tau^2 - \left( \frac{3(R - \tau)}{4M} \right)^{-2/3} dR^2 - \left( \frac{3}{2} (R - \tau) \right)^{4/3} (2M)^{2/3} d\Omega^2 ,
\]
(2.8)
as Lemaître coordinates describe the standard Schwarzschild solution in coordinates which are nonsingular at the horizon (corresponding to the reference frame of a free-falling observer).
The transformation to standard Schwarzschild coordinates reads

\[ d\tau = dt + \sqrt{\frac{2M}{r}} \, \frac{1}{1 - 2M/r} \, dr, \quad (2.9a) \]

\[ dR = dt + \frac{1}{(1 - 2M/r) \sqrt{2M/r}} \, dr, \quad (2.9b) \]

and the horizon is described by \((R - \tau) = (4/3) M\). A suitable choice of the vierbein \(e^a_\mu\) is given by

\[ e^0_\tau = 1, \quad e^1_R = \sqrt{|g_{RR}|}, \quad e^2_\theta = \sqrt{|g_{\theta\theta}|}, \quad e^3_\phi = \sqrt{|g_{\phi\phi}|}, \quad (2.10) \]

with all other components vanishing.

### 2.5. Effective Schwarzschild metric for the photons

For the vector field \(\xi^\mu(x) = e^\mu_a(x) \xi^a(x)\) entering the nonstandard part of the photonic action (2.1)–(2.3) and the effective Lorentz-violating parameter, we take

\[ \xi^\mu(x) = (1, 0, 0, 0), \quad \epsilon \equiv \frac{\kappa}{1 - \kappa/2}, \quad (2.11a) \]

\[ \epsilon \equiv \frac{\kappa}{1 - \kappa/2}, \quad (2.11b) \]

where the first expression (in Lemaître coordinates) makes clear that the photonic Lorentz violation is isotropic and the last expression introduces a convenient Lorentz-violating parameter for the theory considered. The particular parameter choices (2.11) correspond to Case 1 in Ref. [3]. Asymptotically \((R \to \infty\) for fixed \(\tau\)), the parameter \(\kappa\) corresponds to \(2\tilde{\kappa}_{\text{tr}}\), in terms of the parameter \(\tilde{\kappa}_{\text{tr}}\) introduced by Ref. [4] and bounded in Ref. [7].

As shown in Sec. 4.1 of Ref. [3], the effective background for the photons (2.6) is again a Schwarzschild background,

\[ d\tilde{s}^2 = d\tilde{\tau}^2 - \left(\frac{3(\tilde{R} - \tilde{\tau})}{4M}\right)^{-2/3} d\tilde{R}^2 - \left(\frac{3}{2} (\tilde{R} - \tilde{\tau})\right)^{4/3} (2\tilde{M})^{2/3} d\Omega^2, \quad (2.12) \]

with a rescaled mass \(\tilde{M} \equiv M(1 + \epsilon)\) and modified horizon coordinate \(r_{\text{hor}} = 2M(1 + \epsilon)\). The nonstandard photons perceive a horizon outside the standard Schwarzschild event horizon at \(r = r_{\text{Schw}} \equiv 2M\). The space lying between these horizons, \(2M < r < 2M(1 + \epsilon)\), will be referred to as the photonic ergoregion or ergoregion, for short.

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1The effective background (2.12) agrees with the effective metric obtained in Ref. [3] for a minimally
3. Compton scattering

3.1. Generalities

In this section, we present a concrete realization of the process proposed by Eling et al. [2], which, in an appropriate Lorentz-violating theory, corresponds to a type of Penrose-mechanism [14, 15] to extract energy from the photonic ergosphere of a nonrotating Schwarzschild black hole.

In fact, we consider a Compton scattering event [16, 17, 18, 19] from modified Maxwell theory as defined in Sec. 2. Specifically, the theory is given by the total action (2.4a) in terms of the Lagrange densities (2.1), (2.4b), and (2.4c), with Lorentz-violating parameters given by (2.2), (2.3), and (2.11a).

The scattering event is assumed to take place at

\[ r_{\text{scatter}} = 2M(1 + \epsilon \rho), \quad \theta_{\text{scatter}} = \pi/2, \quad \phi_{\text{scatter}} = 0, \]

with the Schwarzschild mass \( M \) from the metric (2.7), the effective Lorentz-violating parameter \( \epsilon \) defined by (2.11b), and a free parameter \( \rho \) taking values between 0 and 1. Using Lemaître coordinates (2.8), the transformation to a local inertial frame is given by

\[
\begin{align*}
(e_0^\tau)_{\text{scatter}} &= 1, \\
(e_1^R)_{\text{scatter}} &= 1/\sqrt{1 + \epsilon \rho}, \\
(e_2^\theta)_{\text{scatter}} &= (e_3^\phi)_{\text{scatter}} = 2M(1 + \epsilon \rho),
\end{align*}
\]

with all other components vanishing. The asymptotically time-like Killing field in local coordinates at the scattering point (3.1) reads

\[
\sigma^a_{\text{scatter}} \equiv \epsilon_{\mu}^a \sigma^\mu \bigg|_{\text{scatter}} = \left( 1, \frac{1}{\sqrt{1 + \epsilon \rho}}, 0, 0 \right).
\]

As explained in the Introduction, we are interested in a Compton scattering event (Fig. 1) where the final scattered photon carries negative Killing energy:

\[
E_{\text{Killing, } \gamma, \text{out}} = \sigma^\mu k_{\gamma, \text{out}}^\nu \tilde{g}_{\mu \nu} \equiv \sigma^\mu k_{\mu, \text{out}}^\gamma < 0,
\]

coupled scalar field interacting with the ghost condensate [11, 12, 13]. In the present article, the background field (2.11a) is introduced by hand. But it is also possible, as shown in [3], to obtain this background field \( \xi^\mu \) by spontaneous symmetry breaking from the ghost-condensate. For our purpose, though, it is more convenient to consider the background (2.11a) as coming from explicit Lorentz violation, avoiding discussion of the stability of the solution and the related flow of energy or entropy.
Figure 1: Sketch of a Compton scattering event $\gamma e^- \rightarrow \gamma e^-$ in the photonic ergoregion of a Schwarzschild black hole of mass $M$ for modified QED (2.1)–(2.4), with Lorentz-violating parameter $\epsilon > 0$ defined by (2.11b). Shown are the unit three-momenta $\hat{k}_n$ of the particles ($n = 1, \ldots, 4$) and the flow of positive charge on the electron line.

with $k_{\gamma,\text{out}}^\nu$ the tangent vector to the path of the final photon. [Here, and in the following, the label ‘in’ or ‘out’ on a particle momentum refers only to the scattering point and the label ‘out,’ in particular, does not foretell the ultimate destiny of the particle.] Such processes are allowed, since the asymptotically time-like Killing field for the photon becomes space-like for $r < 2M(1 + \epsilon)$. The final electron should, however, be able to leave to infinity, carrying more Killing energy than the sum of the initial Killing energies. [The physical interpretation is that energy is extracted from the black hole. Thus, it is clear that the complete process is not just an isolated 2–2 scattering, but that the black hole itself should be considered as a participant, making this essentially a 3–3 scattering process. However, the treatment as a 2–2 scattering process in a fixed spacetime background is justified for a black-hole mass $M$ very much larger than all Killing energies involved.] Moreover, we demand that such a Gedankenexperiment can be prepared in the asymptotically flat region of spacetime, i.e., that the two initial particles come in from spatial infinity.

These conditions impose several constraints on the initial and final four-vectors of the particles. For the sake of brevity, these constraints are omitted, but it has been checked that the example of Sec. 3.3 fulfills all requirements.
3.2. Parametrization

For our purpose, a useful parametrization of the Compton-scattering wave vectors (in the local inertial frame with Cartesian coordinates) is given by

\[
\begin{align*}
(k_a)^{\gamma,\text{out}} &= E_{\gamma,\text{out}} \left(1, -\zeta \omega_1, 0, \zeta \sqrt{1 - \omega_1^2}\right), & (3.5a) \\
(k_a)^{e,\text{out}} &= p_{e,\text{out}} \left(\sqrt{m^2/(p_{e,\text{out}})^2 + 1}, \hat{p}_{e,\text{out}}\right), & (3.5b) \\
(k_a)^{\gamma,\text{in}} &= \tilde{E}_{\gamma,\text{in}} \left(1, -\zeta \beta_1, -\zeta \beta_2, s_1 \zeta \sqrt{1 - \beta_1^2 - \beta_2^2}\right), & (3.5c) \\
k_a^{e,\text{in}} &= k_a^{e,\text{out}} + k_a^{\gamma,\text{out}} - k_a^{\gamma,\text{in}}, & (3.5d)
\end{align*}
\]

with arbitrary photon energy \(E_{\gamma,\text{out}} > 0\), electron three-momentum \(\vec{p} \equiv (p_1, p_2, p_3) \equiv p_{e,\text{out}} \hat{p}_{e,\text{out}}\) for modulus \(p_{e,\text{out}} \equiv |\vec{p}_{e,\text{out}}| > 0\), Lorentz-violating parameter \(\zeta \equiv \sqrt{1 + \epsilon} > 1\), and energy \(\tilde{E}_{\gamma,\text{in}} > 0\) to be determined from the dispersion relation of the incoming electron. The parameters \(\omega_1, \beta_1, \beta_2\) vary between \(-1\) and \(1\), with the additional constraint \(\beta_1^2 + \beta_2^2 \leq 1\). The parameter \(s_1\) takes the value \(+1\) or \(-1\).

The Ansatz (3.5) ensures that the dispersion relations for massless Lorentz-violating photons and massive electrons are fulfilled.

3.3. Concrete example

Since the experimental bounds on isotropic Lorentz violation are tight [7], very small Lorentz violation \((0 < \kappa \ll 1)\) would be physically more interesting than large Lorentz violation \((\kappa \sim 1)\). However, the Compton scattering process with negative Killing energy of the final photon appears to be kinematically forbidden for a small Lorentz-violating parameter \(\kappa\) (see Sec. 3.6).

The following example of allowed kinematics is, therefore, of purely theoretical interest. Specifically, the parameters are chosen to be

\[
\begin{align*}
\epsilon &= 1/2, & \rho &= 99/100, & (3.6a) \\
E_{\gamma,\text{out}} &= 5 \, m, & \omega_1 &= 9984/10000, & (3.6b) \\
p_{e,\text{out}} &= 20 \, E_{\gamma,\text{out}}, & \hat{p}_{e,\text{out}} &= \left(-41, 0, 3\sqrt{91}\right)/50, & (3.6c) \\
\beta_1 &= 74/100, & \beta_2 &= 0, & s_1 = 1, & (3.6d)
\end{align*}
\]

with corresponding Lorentz-violating parameter \(\kappa = \epsilon/(1 + \epsilon/2) = 2/5\). It has taken considerable effort to find this single example. Apparently, the allowed domain of the multi-
dimensional parameter space is very small, which is confirmed by preliminary numerical calculations.

The above parameters allow for a Compton scattering event that ultimately reduces the black-hole mass, because the Killing energy of the final photon is negative: \( \sigma^a k^\gamma_{\text{out}} < 0 \) using (3.3) and the above numbers [the actual value of this energy will be given in Sec. 3.5].

3.4. Squared matrix element

To ensure that the Compton scattering event discussed above has a nonvanishing probability to occur, the corresponding matrix element must be nonzero. The squared matrix element for the Compton scattering process at tree level (calculated with flat-spacetime electron propagators) reads

\[
\frac{1}{4} \sum_{s_1, s_2 = \pm 1/2} \sum_{\lambda_1, \lambda_2 = \pm 1} |\mathcal{M}|^2 =
\]

\[
\Pi_{ac} \Pi_{bd} \frac{e^4}{4} \operatorname{tr} \left\{ \left( k^\gamma_{\text{out}} + m \right) \left[ \frac{\gamma^a k^\gamma_{\text{in}} \gamma^b + 2 \gamma^a k^b_{\text{in}}}{2k^\gamma_{\text{in}} \cdot k^\gamma_{\text{in}} + k^2_{\text{in}}} + \frac{\gamma^b k^\gamma_{\text{out}} \gamma^a - 2 \gamma^b k^a_{\text{in}}}{2k^\gamma_{\text{in}} \cdot k^\gamma_{\text{out}} - k^2_{\text{out}}} \right] \right\}
\]

with Feynman slash \( k \equiv k_a \gamma^a \) and photon polarization sum

\[
\Pi_{ab} = \sum_{\lambda = \pm 1} (\varepsilon^{(\lambda)})_a (\varepsilon^{(\lambda)})_b .
\]

The Ward identities ensure that, in gauge-invariant expressions like the one leading up to (3.7), the polarization sum can be replaced by the following expression

\[
\Pi_{ab} \mapsto \frac{1}{1 + \kappa/2} \left( -\eta_{ab} + \frac{\kappa}{1 + \kappa/2} \xi_a \xi_b \right) .
\]

For \( k^\gamma_{\text{in}} = k^\gamma_{\text{out}} = 0 \) and standard photon polarization sums, (3.7) reproduces the standard squared matrix element of Compton scattering; see, for example, Eq. (5.81) in Ref. [19].

It has now been checked by explicit calculation that the average squared amplitude (3.7) is nonzero for the large Lorentz-violating parameter and kinematics defined by (3.6). This particular Compton scattering event has, therefore, a nonvanishing probability to occur [it has also been verified that the same holds for final photon energies \( E_{\gamma_{\text{out}}} \geq m \), while keeping the other values in (3.6) unchanged].
3.5. Gedankenexperiment

At this moment, it may be instructive to give the numerical values of the four-vectors of the Compton scattering event (3.5)–(3.6):

\[
\begin{align*}
(k_a)^{e,\text{in}} &\approx m \left(17.0968, -8.44173, 0, -14.833628\right), \\
(k_a)^{\gamma,\text{in}} &\approx m \left(87.9082, -79.6722, 0, +72.4163\right), \\
(k_a)^{e,\text{out}} &\approx m \left(100.005, -82.0000, 0, +57.2364\right), \\
(k_a)^{\gamma,\text{out}} &\approx m \left(5.00000, -6.11393, 0, +0.346272\right),
\end{align*}
\]

where the three-momenta are seen to lie in a plane \((k_2 = 0)\). The resulting (conserved) Killing energies of the particles are

\[
\begin{align*}
(E_{\text{Killing}})^{e,\text{in}} &\approx 10.19264 \, m, \\
(E_{\text{Killing}})^{\gamma,\text{in}} &\approx 22.74743 \, m, \\
(E_{\text{Killing}})^{e,\text{out}} &\approx 32.94041 \, m, \\
(E_{\text{Killing}})^{\gamma,\text{out}} &\approx -0.00034 \, m,
\end{align*}
\]

where the energy \((3.11c)\) of the escaping electron is seen to be larger than the total energy of the two incoming particles, \(E_{\text{Killing}}^{\text{in}} \approx 32.94007 \, m\).

A possible Gedankenexperiment (in the Gedankenwelt of this Letter) consists of three steps. First, prepare electron and photon beams to give momenta (3.10a)–(3.10b) at the scattering point (3.1). Second, count the number of electrons scattered in the direction corresponding to (3.10c) and measure their energy. Third, determine the change of black-hole mass (for example, by measuring the change in the orbit of a test particle encircling the black hole).

3.6. Small Lorentz violation

A straightforward but tedious analysis for the case of vanishing electron mass, \(m = 0\), shows that the above Compton scattering process is not allowed for small (but finite) Lorentz-violating parameter \(\epsilon\). Very briefly, the argument consists of two steps. First, the dispersion relation for the initial electron can be solved in terms of the energy of the initial photon. Second, this initial photon energy can be expanded in \(\epsilon\). For any configuration of the parameters discussed in Sec. 3.2 it can be shown that this photon energy becomes negative or imaginary for sufficiently small \(\epsilon\), if the constraints mentioned in the last paragraph of
Sec. 3.1 are taken into account. It is not easy to get the explicit analytic bound, but a conservative bound can be found and is given by \( \epsilon < 1/10 \). That is, it can be shown rigorously that the Compton scattering process with negative Killing energy of the final photon is kinematically forbidden for \( \epsilon < 1/10 \).

For the case of nonvanishing electron mass, \( m > 0 \), numerical investigations show that the process is, once more, kinematically forbidden for small enough \( \epsilon \). A conservative bound is, again, given by \( \epsilon < 1/10 \) (corresponding to \( \kappa < 2/21 \)).

The surprising result, then, is that the reduction of the black-hole mass by the specific Compton scattering process appears to be separated from the standard situation of non-decreasing black-hole mass \([20]\) by a finite gap of the Lorentz-violating parameter \( \kappa \). At the moment, it is not clear if this is just an artefact of the specific process considered (to be overcome by a more complicated setup) or if it indicates the existence of a mechanism that protects the Hawking area theorem \([20]\) for the case of “small enough Lorentz violation.” This interesting question deserves further study.

4. Discussion

This Letter investigated the kinematics of Compton scattering in the accessible part of the photonic ergoregion of a Schwarzschild black hole for nonbirefringent modified Maxwell theory \((2.1)–(2.4)\). More specifically, a Compton scattering event (Fig. 1) was considered, for which the scattered photon carries away negative Killing energy \((3.4)\) and ultimately reduces the mass of the black hole. By giving a concrete example, it has been shown that such an event is kinematically allowed and has a nonzero matrix element.

This particular type of Compton scattering event has, therefore, a nonvanishing probability to occur, at least, for a relatively large Lorentz-violating parameter \( \kappa \). In a Gedanken-experiment starting with a large number \( N_{BH} \) of Schwarzschild black holes of identical mass \( M \) and having a large number \( N_{scat} \) of repeated Compton scattering events on each of these black holes, it is then possible to find certain black holes for which the initial mass \( M \) has been reduced by a macroscopic amount. In this way, the Hawking area theorem \([20]\) is circumvented by the presence of negative-energy states outside the Schwarzschild radius, whose existence is due to the Lorentz violation of the photonic theory considered.

These area-reducing events are believed \([2]\) to contradict the generalized second law of thermodynamics \([21]\), since they may allow for a construction of a perpetuum mobile of the second kind. The basic idea is that such events decrease the mass of the black hole and with
it the associated entropy. If the scattering were classical \cite{2}, the outgoing electron would not carry entropy. The whole process would, then, globally \textit{decrease} entropy, in contradiction with the generalized second law \cite{21}.

However, the Compton scattering process discussed above \textit{is} a quantum process. Certainly, a particular type of Compton scattering event has been shown to have a nonvanishing probability to decrease the black-hole mass and reduce the black-hole entropy. But there is also the possibility that both particle trajectories after the scattering head towards the black hole and that the black-hole entropy increases.

An analogous classical process with reduced black-hole entropy would surely be able to violate the generalized second law, since it would be possible to conceive of a deterministic experiment that would result in a decrease of entropy. But the possibility of an entropy-decreasing quantum process need not imply, by itself, the violation of the generalized second law. For example, already in a system with two types of molecules, there is a nonvanishing probability that a slow-moving (“cold”) molecule transfers energy to a fast-moving (“hot”) molecule. In fact, it is only the application of statistical mechanics to a system with a large number of molecules that recovers the second law of thermodynamics \cite{23}.

A quantitative analysis would be needed to see whether or not the Compton scattering process of Sec. \ref{sec3} would be able to violate the generalized second law. This would require phase-space integrations with nontrivial cuts to determine the probabilities for the interesting Compton scattering events to occur. Perhaps one might, then, be able to show a violation of the fluctuation theorem \cite{24,25}, which might, in turn, imply the breakdown of the generalized second law.

A more speculative idea expands on the \textit{Gedankenexperiment} discussed in the last paragraph of Sec. \ref{sec3.5}. Perhaps it is possible to arrange for a cloud of electrically charged particles and a pulse of light coming in from infinity to scatter elastically at point (3.1) with average momenta \ref{3.10a}--\ref{3.10b} and to have a final cloud and pulse taking off with average momenta \ref{3.10c}--\ref{3.10d}. If that arrangement were possible (admittedly a big ‘if’), the discussion of Sec. IV–B of Ref. \cite{2} could be taken over literally, with the consequent violation of the generalized second law (the incoming and outgoing charged clouds would have the same velocity dispersion and other characteristics, the scattering being elastic by assumption).

\footnote{This would precisely be the difference with the mining technique of Ref. \cite{22}, for which the black-hole mass is also reduced but the outgoing box (with the mined energy) does carry entropy, namely, that of the trapped “acceleration radiation.”}
Whether or not a violation of the generalized second law of thermodynamics occurs in Lorentz-violating theories remains, therefore, an open question. The present Letter tried to find a concrete realization of the promising idea of exploiting a Penrose-mechanism-type process. However, as discussed above, we did not succeed in obtaining an entirely convincing and totally explicit Gedankenexperiment that is able to violate the generalized second law. Still, the presented Compton scattering events, being able to reduce the black-hole mass, may provide a step towards demonstrating the violation of the generalized second law in the Lorentz-noninvariant theory considered, if at all possible.

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\[3\] A different mechanism to violate the generalized second law was suggested in Ref. 13. A quasi-stationary solution was constructed from the ghost condensate to describe the flow of negative energy into a black hole. Just as the process described in the present article, this flow of negative energy appears to be able to reduce the black-hole mass. Potential problems with the stability of this solution have been discussed and seem to be under control. But, whether or not this reduction of the black-hole mass results in a violation of the generalized second law remains, in our opinion, an open question, since the entropy of the effective ghost-condensate fluid has not been considered and the issue of turning the flow on and off requires further analysis. Similar and other reservations regarding the claim of generalized-second-law violation by Ref. 13 have been presented in Ref. 26.
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