Calculations of total photoionization cross-section for
two-electron atomic systems

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Abstract

We outline a non-perturbative procedure for calculating the total photoionization cross-section of two-electron atomic systems. The procedure is based on the Floquet-Fourie representation of the solution of the time-dependent Schrödinger equation. The Floquet-Fourie ansatz produces a set of equations which is recast into a generalized eigenvalue problem by means of the complex rotation procedure. With the use of the Hylleraas-type basis functions, the total photoionization cross-sections are obtained within the accuracy of a fraction of a percent. The total photoionization cross-sections for neutral helium are in good agreement with the convergent close-coupling calculations of Kheifets and Bray [Phys. Rev. A 58, 4501 (1999)] but deviate notably from the experimental data of Samson et al. [J. Phys. B 27 887 (1994)].

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I. INTRODUCTION.

Photoionization of two-electron atoms has been studied theoretically by different authors starting from the pioneering paper by Wheeler. Review of early literature on this subject can be found in Ref. Subsequently, a large number of computations of helium photoionization cross-sections was reported. These calculations produced a collection of results varying typically by 5% from each other. On the experimental side, the benchmark set of data was reported by Samson et al. who measured the total photoionization cross-section of He in the photon energy range from the threshold to 120 eV. Agreement between the theoretical and experimental data was within the same margin of 5%. In the following years, the theoretical interest shifted towards calculation of differential characteristics of the photoionization process and to studies of double photoionization. Here, several approaches have been advocated including the many-body perturbation theory, convergent close-coupling method, time-dependent close-coupling method, R-matrix approach, and methods based on the computation of the dipole response function or B-spline implementations of the exterior complex scaling.

Due to this shift of focus, there have been no further attempt to produce a consistent set of photoionization cross-sections of He with an accuracy better than several percent. Yan et al. combined measurements of Samson et al. at low energies and theoretical calculations at high energies to construct a set of photoionization cross sections of He that should be reliable at all energies. However, there was no consistency check applied to the experimental data. In the meantime, accurate helium photoionization cross-sections would be highly desirable due to importance of He in astrophysics and its use as a standard gas in determination of the photoionization cross-sections of other atomic and molecular species.

In the present paper, we develop the complex rotation method (CRM) for highly accurate calculations of the total photoionization cross-section of two-electron atomic targets. One way of calculating the photoionization cross-section is to combine the CRM technique with the perturbation theory with respect to interaction of the atom with the electromagnetic field. In such a perturbation theory, the CRM provides the basis of the field-free atomic states. It was demonstrated by Johnson and Reinhardt that relying on the spectrum of
the CRM eigenvalues, one can construct a representation of the complete Green’s function of the atom. This, in turn, allows to write down a convenient representation for the projection operator corresponding to the continuous spectrum of the atom \[20\]. Using this projection operator, one can compute probabilities of transitions into continuum under the action of some perturbation, in particular, the interaction of the atom with the electromagnetic field. Calculations of total photoionization cross-sections of the helium atom based on this technique have been reported in Refs. \[21, 22\]. Similar ideas were also used to determine static and dynamic polarizabilities of helium \[23, 24\].

In the present work, we use the CRM procedure in a somewhat different, non-perturbative way by applying it to the whole system the atom plus the electromagnetic field. Thus, we are capable of going beyond the perturbation theory and considering very strong fields. In this respect, the present technique has certain features in common with Refs. \[25, 26\], where an approach based on the configuration interaction procedure was developed for atoms with more than one electron. We cast our formalism using the language of square-integrable functions with a finite norm. This approach becomes feasible in the so-called Floquet-Fourier representation of the time-dependent Schrödinger equation.

Another key ingredient of the present work is the Hylleraas basis functions which have long been used in various variational-type calculations. An excellent review of applications of the Hylleraas basis to calculations of energies of two-electron atoms is given by Drake \[27\]. A well-known trademark of the Hylleraas basis set is a very high accuracy of the atomic energies. In the present paper, we show that the same high accuracy which is achieved for field-free atomic states can also be attained when the atom is subjected to a monochromatic electromagnetic field. In particular, the total photoionization cross-sections can be calculated with an accuracy of the order of a fraction of a percent.

Thus generated cross-sections were compared with the experimental results of Samson et al. \[7\]. We discovered a systematic deviation from the experiment, especially in the region close to double ionization threshold at the photon energies of \(\sim 80\) eV. This deviation was confirmed by comparison with earlier results produced by the convergent close-coupling (CCC) method \[28\].
The true potential of the present method is realized in the strong field regime where the perturbation theory fails. As demonstrated below, the Floquet-Fourie-Hylleraas ansatz produces very accurate results in this regime as well.

II. THEORY

A. General Theory.

The non-relativistic Hamiltonian of the helium atom in the presence of the external monochromatic linearly-polarized electromagnetic field can be written as:

\[ \hat{H} = \hat{T} + \hat{U} + \hat{V}, \]  

where, \( \hat{T} \) is a kinetic term:

\[ \hat{T} = \frac{\hat{p}_1^2}{2} + \frac{\hat{p}_2^2}{2}, \]  

\( \hat{U} \) potential energy term:

\[ \hat{U} = -\frac{2}{r_1} - \frac{2}{r_2} + \frac{1}{|r_1 - r_2|}, \]  

and \( \hat{V} \) describes interaction of atom and the field. In the length gauge (which will be used in the present paper), this operator assumes the form:

\[ \hat{V} = \mathbf{F} \cdot \mathbf{D} \cos \omega t, \]  

with \( \mathbf{D} = r_1 + r_2 \). Unless stated otherwise, the atomic units are used throughout the paper.

We write the solution of the time-dependent Schrödinger equation (TDSE) using the Floquet-Fourie ansatz 29:
Under this transformation, the chain of equations (6) is converted into
\[
(E - \hat{T} e^{-2i\theta} - \hat{U} e^{-i\theta} + n\omega) u_n = \frac{F \cdot D}{2} e^{i\theta}(u_{n-1} + u_{n+1}) \quad , \quad n = 0, \pm 1, \ldots, \quad (7)
\]
According to the general theory of CRM \[30, 31, 32, 33\], the set of equations (7) can be solved by means of variational techniques if the rotation angle $\theta$ is properly chosen.

We introduce a basis set of square integrable functions $|n, k\rangle$ where the index $n$ refers to the number of the Floquet block and the index $k$ denotes a particular $L^2$ function in the subspace of the $n$-th block so that $u_n = \sum_k c_{nk}|n, k\rangle$. With these notations, the set of Eqs.(7) can be rewritten in a matrix form as:
\[
\sum_k \langle n_1, k_1| E + n\omega - \hat{T} e^{-2i\theta} - \hat{U} e^{-i\theta}|n, k\rangle c_{nk} = \sum_{n_2=n\pm 1, k_2} \langle n_1, k_1| \frac{F \cdot D}{2} e^{i\theta}|n_2 k_2\rangle c_{n_2 k}, \quad (8)
\]
Notations can be further simplified by introducing obvious shorthands:
\[
\left((E + n\omega) R_{n1 k1}^{nk} - T_{n1 k1}^{nk} e^{-2i\theta} - U_{n1 k1}^{nk} e^{-i\theta}\right) c_{nk} = \sum_{n_2=n\pm 1} V_{n1 k1}^{n2 k} e^{i\theta} c_{n_2 k}, \quad (9)
\]
where it is understood that summation is carried over the repeated $k$-index. Here $\hat{V} = F \cdot D$, and $R$, $T$ and $U$ stand for the overlap, kinetic energy and potential energy matrices, respectively.

One could say here a few words about the choice of the basis allowed by the structure of the system (9). Suppose first, that in each of the subspaces corresponding to different Floquet blocks we chose some complete set of functions, such that for any $u_n$ in Eq.(7) we had: $u_n = \sum c_{nm}|n, m\rangle$. Let the set of $|n, m\rangle$’s be the same for all Floquet subspaces. Then, if we have retained $N$ Floquet blocks in the system (7) and keep $M$ terms in the expansion for each $u_n$ in Eq.(7) we have altogether $NM$ unknowns $c_{nm}$ in the system (7).

To get a correctly posed eigenvalue problem, we should have the same number of equations. This number is provided by projecting each of the equations (7) on one of the $|n, m\rangle$’s with $m = 1 \ldots M$. This way of reducing the set of equations (7) to the form of matrix eigenvalue problem is correct, but too general for our purposes. It can be seen, that one can considerably diminish the resulting dimension of the matrix eigenvalue problem by using certain symmetry properties of the system Eq.(7). It is easy to see, that this system allows the following class of solutions: $u_n$’s with even $n$ are of even parity, while $u_n$’s with odd
are of odd parity. Parity here is understood with respect to the spatial inversion. Of course, there is a class of solutions with the opposite property: \( u_n \)’s with even \( n \) are of odd parity, while \( u_n \)’s with odd \( n \) are of even parity. The solution we are looking for (which is to describe behavior of the even \( ^1S^e \) state of helium) evidently belongs to the first class. We can therefore, choose the basis set as follows.

Instead of choosing the same set \( |n, m\rangle \) for each Floquet block, we choose two sets: a set \( |n_{\text{even}}, m\rangle \), consisting of basis functions of even parity, is used as a basis to represent \( u_n \)’s with even \( n \)’s. Another set \( |n_{\text{odd}}, m\rangle \), composed of odd parity functions is used as a basis to represent \( u_n \)’s with odd \( n \)’s. Suppose that in the expansions of \( u_n \)’s with even \( n \)’s we retain \( M_{\text{even}} \) terms, and in the expansions of \( u_n \)’s with odd \( n \)’s - \( M_{\text{odd}} \) terms. Let the number of Floquet blocks with even and odd \( n \)’s be respectively \( N_{\text{even}} \) and \( N_{\text{odd}} \). Than we have \( N_{\text{even}}M_{\text{even}} + N_{\text{odd}}M_{\text{odd}} \) unknown coefficients \( c_{nm} \). We obtain the same number of equations by projecting equations (7) on \( |n_{\text{even}}, m\rangle, m = 1 \ldots M_{\text{even}} \) for even \( n \) and on \( |n_{\text{odd}}, m\rangle, m = 1 \ldots M_{\text{odd}} \) for odd \( n \). Projection of equations with even \( n \) on the \( |n_{\text{odd}}, m\rangle \) and of equations with odd \( n \) on the \( |n_{\text{even}}, m\rangle \) gives identically zero and does not add new equations. More details about the basis functions \( |n_{\text{even}}, m\rangle \) and \( |n_{\text{odd}}, m\rangle \) is given below.

According to the general theory of CRM, some of the energy values (generally complex) for which system (9) has a solution are related to the position and width of the resonance state via \( E = E_r - i\Gamma/2 \), where \( E_r \) is position of the resonance and \( \Gamma \) its width. This leads one to solving a generalized eigenvalue problem. Effectiveness of finding eigenvalues of such a problem depends crucially on the choice of the basis used to represent the matrices in Eq.(9).

B. Basis set.

The basis set used in the present paper was constructed from the Hylleraas type functions:

\[
g_{n_1, n_2, N}(r_1, r_2) = r_1^{n_1} r_2^{n_2} |r_1 - r_2|^N e^{-ar_1 - br_2} |l_1(1) l_2(2) L\rangle, \tag{10}
\]
where $a, b$ are some constants (to be specified below), $n_1, n_2, N$ are integers and the angular part

$$|l_1(1)l_2(2)L⟩ = \sum_{m_1 m_2} C_{l_1 m_1 l_2 m_2}^L Y_{l_1 m_1}(n_1) Y_{l_2 m_2}(n_2),$$

represents two angular momenta $l_1, l_2$ coupled to a state with a total angular momentum $L$. The basis functions (10) must also be properly symmetrized with respect to exchange of the electron coordinates. When choosing parameters in Eq. (10), we followed the following empirical rules [27, 36]. All the basis functions with the parameters satisfying:

$$n_1 + n_2 + N < N_{\text{max}}$$

were included in the calculation. The parameter $N_{\text{max}}$ determines the overall size of the basis. There is also a semiempirical rule for choosing angular momenta $l_1, l_2$ in the Eq. (11). Thus, for states of the natural parity $l_1, l_2$ are best chosen so that $l_1 + l_2 = L$. Both these criteria help to avoid the numerical problems due to near-degeneracy of the basis set when its dimension becomes large.

III. NUMERICAL RESULTS

A. Field-free case

In the present work, our main goal is to obtain accurate photoionization cross-sections from the ground state of neutral helium for not very large electromagnetic field intensities. Accordingly, our main interest is focused on the states of $S$ and $P$ symmetries. Therefore, our first goal is to choose such a basis that solution of the eigenvalue problem (9) yields accurate energies for the ground $^1S$ and first excited $^1P^o$ state of the helium atom in the absence of the field.

This goal was achieved as follows. We chose parameters $N_{\text{max}} = 18$, $a = b = 2$ for the $S$-states and $N_{\text{max}} = 13$, $a, b = 1, 2$ for the $P$-states. The reason for enlarging the basis set for the excited $P$-states is that the electrons in such states are generally on different distances from the nucleus. This choice combined with restriction on angular momenta (12) resulted in $N_S = 372$ basis functions for the $S$-states and $N_P = 660$ basis functions for the $P$-states.
The next step was to solve the generalized eigenvalue problem for the field-free case. In Eq. (9) we put $F = 0$, $\omega = 0$, and limited ourselves to the blocks with $n = 0$, $n = \pm 1$, the $n = 0$ block being composed of the states of $^{1}S^e$ symmetry, and $n = \pm 1$ blocks composed of the states of $^{1}P^o$ symmetry. All the numerical results reported below were obtained using the quadruple precision arithmetics.

We note, that in the presence of the weak electromagnetic field account of the blocks with $n = \pm 1$ corresponds to absorption and emission of one photon. We shall use this fact below to extract the photoionization cross-section from our calculation. For the moment, we are concerned with testing the accuracy of our basis. Diagonalization of the eigenvalue problem (9) with $F = 0$, $\omega = 0$ in the basis described above produced the following results for the complex energies:

- $E = -2.903724384 + i 1.3 \times 10^{-8}$ (the ground state)
- $E = -2.123843094 + i 7.6 \times 10^{-9}$ ($1s2p^1P^o$ state).

A small imaginary part which, in the absence of the field, should be zero could be taken as an indication of an accuracy of our basis set. Either this criteria or a direct comparison with the well-known results of highly accurate calculations [27] shows that we have achieved an accuracy of the order of $10^{-8}$ a.u. This accuracy, as will be demonstrated below, is sufficient to obtain the photoionization cross-sections with at least three significant figures.

B. Total photoionization cross sections

To calculate the total photoionization cross sections we adopted the following strategy. The eigenvalue problem (9) was solved with the Floquet blocks $n = 0, \pm 1$ retained, the composition of each block was the same as described above for the field-free case. Diagonalization of the eigenvalue problem (9) produced energy shift and total width for the ground state. By definition, the photoionization cross-section from this state is related to the total width $\Gamma$ via

$$
\sigma = \lim_{F \to 0} 8\pi \alpha \Gamma \omega / F^2,
$$

where $F$ is field strength, $\omega$ its frequency, $\alpha$ is the fine structure constant. We need therefore to extract from our calculation the coefficient with $F^2$ in the asymptotic law defining the
TABLE I: Results for the ground state eigenvalue of problem (9) as functions of parameters $N_{\text{max}}$ in Eq.(12), $\omega = 80$ eV, $F = 0.1$ a.u.

| $N_{\text{max}}^S$ | $N_{\text{max}}^P$ | Total dimension of the eigenvalue problem | ReE(a.u.) | $\Gamma$ (a.u.) |
|-------------------|------------------|------------------------------------------|---------|--------------|
| 17                | 11               | 1300                                     | -2.90307660 | 0.000487738  |
| 18                | 12               | 1692                                     | -2.90307661 | 0.000487698  |
| 19                | 13               | 2204                                     | -2.90307659 | 0.000487689  |

The weak-field behavior of the width:

$$\Gamma(F) = \Gamma_0 F^2 + \Gamma_1 F^3 + \ldots$$  \hspace{1cm} (14)

To implement this strategy, we need an extrapolation procedure since the calculation based on the system (2) is performed for a non-zero field strength. Although finite, this field strength should not be too small to compute $\Gamma$ with sufficient accuracy.

The issue of accuracy can be addressed as usual in variational-type calculations, by merely increasing the basis size and verifying that the results do not change appreciably. Such a test was performed for a photon energy $\omega = 80$ eV and a field strength $F = 0.1$ a.u. by varying the parameter $N_{\text{max}}$ in Eq.(12) for the $S$ and $P$ states. The diagonalization of the problem (9) was performed with the Floquet blocks $n = 0, \pm 1$ retained. All the remaining details of the basis (nonlinear parameters etc.) were the same as in the field-free case reported above. The calculation was performed for the value of the rotation angle $\theta = 0.3$.

The test results are presented in Table I. One can observe that, just as in the field-free case, the accuracy is on the level of $10^{-8}$ a.u., which implies that $\Gamma$ has at least four significant digits in this interval of field strengths.

The issue of the stability of the results with respect to the number of the Floquet blocks included in diagonalization of (9) is addressed in the next section where we consider effects of going beyond the first order perturbation theory. We shall say in advance that including the Floquet blocks with $n = \pm 2$ in diagonalization of (9) does not alter the numerical accuracy appreciably.
TABLE II: Extrapolation of the Γ’s to the zero-field limit.

| ω (eV) | $F = 0.07$ a.u. | $F = 0.1$ a.u. | $F = 0.13$ a.u. | $F = 0$ (Extrapolation) |
|-------|----------------|----------------|----------------|------------------------|
| 40    | 0.4208622      | 0.4201601      | 0.4192063      | 0.4215215              |
| 80    | 0.0488002      | 0.0487698      | 0.0487239      | 0.0488112              |
| 85    | 0.0392854      | 0.0392618      | 0.0392330      | 0.0393202              |
| 91    | 0.0306858      | 0.0306720      | 0.0306524      | 0.0306961              |
| 95    | 0.0262180      | 0.0262082      | 0.0261936      | 0.0262224              |
| 111   | 0.0147116      | 0.0147084      | 0.0147033      | 0.0147116              |
| 205   | 0.0013719      | 0.0013726      | 0.0013729      | 0.0013687              |

As to the extrapolation procedure needed to extract the coefficient $Γ_0$ in Eq. (14), we chose a scheme based on the three-point Lagrange formula. For each frequency reported below, we performed calculations for the field strengths $F = 0.07, 0.1, 0.13$ a.u. We also used a mid size basis set with $N_{\text{max}}^S = 18, N_{\text{max}}^P = 12$, Floquet blocks with $n = 0, \pm 1$, all other details of the basis are the same as in the field-free case above. Results of this calculation and extrapolation are shown in Table II.

Using an estimate for the remainder of the series (14), it is a simple matter to verify that for the field strengths considered the possible relative error introduced by the extrapolation of $Γ/F^2$ is of the order of 0.1%. Hence, at least three digits in our result for the extrapolated ratio $Γ/F^2$ and the cross-sections reported below must be reliable. This level of accuracy can easily be improved by merely going to extrapolation schemes of higher order and computing $Γ$ for more field values.

In Table III we present our results for the cross-sections based on formula (13) in which we fed the extrapolated ratios from the last column of Table II. Along with our data, we present the benchmark experimental results of Samson et al. [7] as well as earlier theoretical results from Ref. [28]. The experimental setup of Samson et al. [7] was such that the measured cross-section was summed over all final states of the remaining ion including the doubly ionized states. It is exactly the cross-section that is calculated presently and therefore comparison
between the theory and experiment should be straightforward. Our theoretical results agree with the data of Samson et al. [7] within the postulated experimental accuracy of few percents. The strongest deviation is for $\omega = 80$ eV where the difference between the present result and the experimental value is 6%. This is deviation is clearly seen in the Figure where we plot the present Floquet calculation along with the CCC calculation in three gauges of the electromagnetic interaction and the experiment. Agreement between the present calculation and that of the CCC is much better, difference of the results of two approaches not exceeding 1%. The accuracy of the CCC result is hard to estimate directly as this method relies on the numerical solution of a set of close-coupling equations. The only implicit indication is the difference between the cross-sections calculated in the three gauges of the electromagnetic interaction, the length (L), velocity (V) and acceleration (A). This difference is typically 1-2%. Thus, the deviation of the present calculation with the CCC is more likely to be the problem of the latter as the former is believed to be much more accurate.

As a by-product of the calculation described above, we also obtained the shift of the ground state of helium due to the interaction of atom with the linearly polarized monochromatic field (Table IV).
FIG. 1: The total photoionization cross-section (sum of the single $\sigma^+$ and double $\sigma^{2+}$ cross-sections) as a function of the photon energy. The present calculation for selected photon energies is denoted by dots. The CCC calculation in the three gauges of the electromagnetic interaction (Length, Velocity and Acceleration) is exhibited by different colors / line styles. The experiment of Samson et al. [7] is presented by a dotted line.

C. Extended calculation

We now turn to extended calculations with inclusion of a larger number of the Floquet blocks $n = 0, \pm 1, \pm 2$ in Eq. (9). The aim of these calculations is two-fold. First, we shall confirm the stated accuracy of the present weak field results which is not effected by the number of the Floquet blocks retained in the calculation. Second, we report some preliminary results concerning behavior of the widths parameter in stronger fields where inclusion of a
TABLE IV: Real part of the energy of the ground state of helium in the presence of the linearly polarized monochromatic field.

| ω (eV) | $F = 0.07$ a.u. | $F = 0.1$ a.u. | $F = 0.13$ a.u. |
|--------|----------------|----------------|-----------------|
| 40     | -2.90281954    | -2.90187690    | -2.90060016     |
| 80     | -2.90340686    | -2.90307658    | -2.90263014     |
| 85     | -2.90344158    | -2.90314741    | -2.90274973     |
| 91     | -2.90347684    | -2.90321933    | -2.90287117     |
| 95     | -2.90349708    | -2.90326062    | -2.90294089     |
| 111    | -2.90355845    | -2.90338582    | -2.90315235     |
| 205    | -2.90367802    | -2.90362977    | -2.90356450     |

The larger number of the Floquet blocks becomes essential due to a non-perturbative nature of the processes involved.

The basis for the extended calculations was constructed as follows. As we discussed above the basis subset spanning each Floquet block in the system can be chosen to consist of the functions of a given parity, two adjacent blocks having opposite parities. Thus, in the low-field calculations described above, the block $n = 0$ was composed of even basis functions while two blocks with $n = \pm 1$ contained odd basis functions. Inclusion of the blocks with $n = \pm 2$ is, therefore, equivalent to adding more even basis functions. We did it in the following way. In addition to the $^1S_e$ states we previously had in the $n = 0$ block, the states of the symmetries $^1D_e$ and $^1P_e$ were included in the calculation. Thus the blocks with $n = 0$ and $n = \pm 2$ had the following composition: $N_{\text{max}} = 18$ for the $^1S_e$-basis functions, $N_{\text{max}} = 8$ for the $^1P_e$ and $^1D_e$-basis functions. As before, the blocks with $n = \pm 1$ were composed of basis functions of $^1P_o$-symmetry with $N_{\text{max}} = 13$. Thus, the basis set is considerably enlarged comparing to the one used in the previous section. With this choice of parameters $N_{\text{max}}$, the overall dimension of the eigenvalue problem was 2676. Results produced for the ground state of He by diagonising this eigenvalue problem are shown in Table.

Comparison of the results given in Table supports the assertion we made in the previous
TABLE V: Results of the calculation with Floquet blocks \( n = 0, \pm 1, \pm 2 \) included in the system \( \omega \).

| \( F \) (a.u.) | \( \Re E \) (a.u.) | \( \Gamma/F^2 \) (a.u.) | \( \omega = 111 \text{ eV} \) | \( \Re E \) (a.u.) | \( \Gamma/F^2 \) (a.u.) | \( \omega = 205 \text{ eV} \) |
|---------------|---------------------|------------------------|-----------------------------|---------------------|------------------------|-----------------------------|
| 0.10          | -2.90338569         | 0.014714               | -2.90362976                 | 0.0013734           |
| 0.13          | -2.90315198         | 0.014715               | -2.90356447                 | 0.0013739           |
| 0.20          | -2.90236955         | 0.014715               | -2.90334589                 | 0.0013743           |
| 0.50          | -2.89525524         | 0.014706               | -2.90135842                 | 0.0013745           |
| 1.0           | -2.86985102         | 0.014665               | -2.89426254                 | 0.0013701           |

section as to the accuracy of our results for the widths. As one can see, for the field strengths \( F \approx 0.1 \text{ a.u.} \), inclusion of the additional Floquet blocks and basis states of symmetries other than \( S \) and \( P \) produces relative variations in the widths on the order of 0.01 percent. This means that for such field values we are still within the domain of the validity of the perturbation expansion. For the frequencies presented in the Table the domain of the perturbation theory actually extends quite far in the region of large field strengths. As one can see from the Table \( \text{V} \), the ratio \( \Gamma/F^2 \) starts changing in a more or less appreciable manner only for field strengths as large as \( F \approx 1 \text{ a.u.} \).

IV. CONCLUSION

We performed a calculation of the total photoionization cross-sections from the ground state of helium. We employed a theoretical procedure based on the Floquet-Fourie representation of the solutions of the TDSE describing the helium atom in the presence of the linearly polarized monochromatic electromagnetic field. The resulting set of Floquet equations was cast into a generalized eigenvalue problem by the complex rotation method. Our approach is essentially non-perturbative. This is in contrast with other works \[20, 22\] where the CRM was used to produce an accurate description of the field-free helium atom, thus giving the ground for application of the perturbation theory. In our approach, we do not rely
on any perturbation expansion to describe interaction of the atom and the electromagnetic field. This interaction is included into the theory from the beginning. We would like to emphasize the accuracy of the present results for the photoionization cross-sections which, we believe, is on the level of a fraction of a percent. Although only few selected photon energies were reported in the paper, far wider and denser energy grid was covered by the present calculation. These results might serve as an accurate database and find their use in various astrophysics and atomic physics applications. The authors shall gladly communicate these data on request.

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