A Modified Ant Colony Optimization Algorithm for Solving a Transportation Problem

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Authors’ contributions
This work was carried out in collaboration among all authors. Author EMUSB designed the study, managed the data collection and literature searches, performed the statistical analysis, wrote the protocol, and wrote the first draft of the manuscript. Author WBD was the first supervisor for the work. Authors SPCP and ZAMSJ were co-supervisors of the work. All authors read and approved the final manuscript.

Article Information
DOI: 10.9734/JAMCS/2020/v35i530284

Received: 20 May 2020
Accepted: 26 July 2020
Published: 10 August 2020

Abstract
Transportation of products from sources to destinations with minimal total cost plays a key role in logistics and supply chain management. The transportation problem (TP) is an extraordinary sort of Linear Programming problem where the objective is to minimize the total cost of disseminating resources from several various sources to several destinations. Initial feasible solution (IFS) acts as a foundation of an optimal cost solution technique to any TP. Better is the IFS lesser is the number of iterations to reach the final optimal solution. This paper presents a meta-heuristic algorithm, modified ant colony optimization algorithm (MACOA) to attain an IFS to a Transportation Problem. The proposed algorithm is straightforward, simple to execute, and gives us closeness optimal solutions in a finite number of iterations. The efficiency of this algorithm is likewise been advocated by solving validity and applicability examples. An extensive numerical study is carried out to see the potential significance of our modified ant colony optimization algorithm (MACOA). The comparative assessment shows that both the MACOA and the existing JHM are efficient as compared to the studied approaches of this paper in terms

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of the quality of the solution. However, in practice, when researchers and practitioners deal with large-sized transportation problems, we urge them to use our proposed MACOA due to the time-consuming computation of JHM. Therefore this finding is important in saving time and resources for minimization of transportation costs and optimizing transportation processes which could help significantly to improve the organization’s position in the market.

Keywords: Modified ant colony algorithm; transportation problem; initial feasible solution; Vogel’s approximation method; Juman and Hoque’s method.

1 Introduction

Dorigo and Maniezzo [1] developed the first foraging ant’s algorithm which is called Ant Colony Algorithm (ACA). This is based on the probabilistic technique for solving various types of optimization problems. Such as vehicle routing problem [2,3,4], traveling salesman problem, traffic assignment problem [5], shortest path problem, minimum spanning tree problem, etc. The algorithm is examining for an optimal path in the graph based on the behavior of ants.

On the other hand, the development of this algorithm was ants can find the shortest roots between food sources and their colony. Ants are social insects and walk randomly. Ants communicate to each other by setting down pheromones along their path, so where ants go inside and around their subterranean insect state is a stigmergic framework. In numerous ant species, ants strolling from or to a food source, deposit on the ground a substance called pheromone. Different ants can smell this pheromone, and its essence impacts the decision of their way, that is, they will in general follow solid pheromone fixations. The pheromone deposited on the ground shapes a pheromone trail, which permits the ants to discover great of food that have been recently recognized by different ants. Utilizing arbitrary strolls and pheromones inside a ground containing one nest and one food source, the ants will leave the nest, discover the food and return to the nest. After some time, the way being utilized by the ants will converge to the shortest path.

Generally, ACA is very easy to understand and its applications provide very good results. Very recently, Chowdhury et al. (Article-in-press) [6] proposed a modified ACA to solve a real-life dynamic traveling salesman problem.

The transportation problem (TP) deals with the distribution of a product manufactured at factories to some various warehouses. The objective of this problem is to determine the feasible amounts to be shipped from each source to each destination with a minimum total transportation cost.

The TP was first developed by Hitchcock [7] and then Koopmans [8] developed Optimum Utilization of Transportation System. After that, Charnes and Cooper [9] developed the stepping stone method and Dantzig [10,11] developed the transportation Simplex Method to this problem.

Besides, several heuristic solutions approach such as Northwest Corner Method [12], minimum cost method [12], VAM -Vogel’s approximation method [13,14,15,16], JHM –Juman and Hoque’s method [17], GVAM –Goyal’s version of VAM [18], EHA-An Efficient Heuristic Approach [19], etc were proposed to obtain an Initial Feasible Solution (IFS) to the TP. Moreover, Sabbagh [20] developed a new hybrid algorithm for balanced TP. Sharma [21] presented a new solution procedure to solve the dual of the incapacitated transportation problem, Juman and Nawarathne [22] presented an alternative approach to solving a TP.

This paper presents an overview of the concept of ACA and provides a review of its applications for solving both types of balance and unbalance TP. The proposed one is very simple, easy to understand and it always tries to reach a minimum cost solution to the concern problem. In this paper, several modifications to this ACA are made and ensured a solution that is very closer to the optimal solution with less iteration.
The remainder of this paper is as follows: Section 2 deals with the mathematical formulation of the transportation problem. In Section 3 the modified ACA is proposed and illustrated with a numerical example problem. Then, its comparative studies with the existing ones on the results of some benchmark instances are carried out to show the potential significance of the proposed approach. Finally, conclusions along with limitations and future research directions are presented in Section 5.

2 Mathematical Formulation

The TP is focused on the distribution of the product from \( m \) sources having capacities \( (a_1, a_2, ..., a_m) \) to \( n \) destinations having demands \( (b_1, b_2, ..., b_n) \) to meet the demand of each destination with the least total transportation cost. Arc \((i, j)\) joining source \( i \) to destination \( j \) carries two pieces of information: the transportation cost per unit \( c_{ij} \) and the amount shipped, \( x_{ij} \), where \( i = 1, 2, ..., m \) and \( j = 1, 2, ..., n \).

Hence the above problem (sometimes called as the general, classical or Hitchcock transportation problem) can be given in the following form:

\[
\text{Min} \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}
\]

Subject to

\[
\sum_{j=1}^{n} x_{ij} = a_i \quad i = 1, 2, ..., m
\]

\[
\sum_{i=1}^{m} x_{ij} = b_j \quad j = 1, 2, ..., n
\]

Where \( x_{ij} \geq 0 \forall i, j \)

A transportation problem is said to be balanced if the total supply from all sources is equal to the total demands at all destinations. That is, \( \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \) otherwise, it is called an unbalanced transportation problem.

3 Methodology

3.1 Solution procedure

Here, we propose a meta-heuristic solution method based on Ant Colony Optimization Algorithm (ACOA) to solve the transportation problem. This strategy animated by the behavior of veritable ant colonies and it has been successfully applied to solve several combinatorial optimization problems and has achieved agreeable performances [23,24,25,26]. Following Afshar [27] the basic steps on this algorithm [1,28] can be defined as follows:

**Step 1 m ants** are randomly placed on the \( n \) decision points and the amount of pheromone trail on all arcs is initialized to some proper value at the start of the computation.

**Step 2:** A transition rule is used at each decision point \( i \) to decide which option is to be selected. Once the option at the current decision point is selected, the ants move to the next decision point and the solutions are incrementally created by ants as they move from one point to the next one. This procedure is repeated until all decision points of the problem are covered. The transition rule used in the original ant system is defined as follows (Dorigo et al. [1]):

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The ants are driven by a probability rule to pick their answer for the problem, known as a visit. The probability rule between two \( i \) hubs \( j \), called Transition Rule \([29,30]\) and it be dependent on upon two factors: the heuristic and metaheuristic. The transition rule evaluates the probability of ant \( k \), situated at city \( i \), going to city \( j \) and it is given by:

\[
P_{ij} = \frac{\varphi_{ij}^\alpha \omega_{ij}^\beta}{\sum_{j=1}^{m} \varphi_{ij}^\alpha \omega_{ij}^\beta}
\]

This equation calculates the probability of selecting a single component of the solution. In this research, \( \omega_{ij} = \frac{1}{\theta} : \alpha > 0, \beta > 0 \). Where \( \theta \) is linear and \( \theta = \min.\text{value} c_{ij}; \text{cost between node } i \text{ and node } j, \theta \neq 0 \).

For our scenario, we assumed \( \alpha = \beta = 1 \) in the transition rule,

\[
P_{ij} = \frac{\varphi_{ij}^{1} \omega_{ij}^{1}}{\sum_{j=1}^{m} \varphi_{ij}^{1} \omega_{ij}^{1}}; \text{ } ^{t}\text{th ant visits the } ^{j}\text{th city} \]

\[
0 ; \text{ Otherwise}
\]

Pheromone update rules are used to calculate the amount of pheromone level on each edge between node \( i \) and node \( j \). The solution of each ant is updated using following update function:

\[
\varphi_{ij}(t + 1) = (1 - \rho) \varphi_{ij}(t) + \sum_{k=1}^{m} \Delta \varphi_{ij}^{k}(t)
\]

Where,

\[
\Delta \varphi_{ij}^{k} = \frac{\kappa}{L^k}; \text{ if component } (i, j) \text{ was used by ant (best route)}
\]

\[
= 0 ; \text{ Otherwise}
\]

Here, \( L^k \) is the distance of the best route. \( \kappa \) is simply a parameter to adjust the amount of pheromone deposited, typically it would be set to 1. We sum \( \frac{\kappa}{L^k} \) for every solution which used component \((i, j)\), then that value becomes the amount of pheromone to be deposited on component \((i, j)\).

\[
\text{Where } \varphi_{ij}(t) = \frac{1}{\min(\text{no.demand, no.supply})} \rho \text{ represents the pheromone evaporation rate and } 0 < \rho \leq 1.
\]

### 3.2 Modified Ant Colony Optimization Algorithm (MACOA)

The steps of the proposed method can be expressed as follows:

**Step 1:** If the TP is unbalanced, convert it to a balanced TP by adding a dummy row or a dummy column accordingly.

**Step 2:** Compute the probability using the Transition Rule and form the probability matrix of order \( m \times n \).

**Step 3:** Ants in the colonies are placed at the starting nodes with the maximum value of the probability in the probability matrix to make the first allocation.

**Step 4:** Identify the min \((a_i, b_j)\) in the demand or supply cell. Allocate the particular min \((a_i, b_j)\) to cell corresponding to the maximum probability.
Step 5: If the demand in the column (or supply in the row) is satisfied, remove that column (or row) and move to the next maximum probability cell.

Step 6: If the termination condition is satisfied (i.e., \( a_i = b_j = 0 \)), then go to Step 8. Otherwise go to Step 4.

Step 7: Stop and determine solution.

The MACOA is validated using a numerical example. Detailed computation of this example using the proposed method (MACOA) is provided in Appendix F. The flow chart representation of the above MACOA is also provided below:

**Chart 1. Flow chart representation of the new algorithm, MACOA**

Note that, in order to check the optimality of the initial solution obtained by the proposed algorithm, one can use the method of Stepping Stone / MODI. If it is not optimal, then proceed with the Stepping Stone / MODI
4 Comparative Assessment

This section provides performance comparisons across the various well-known methods – LCM, NWCM, ZSM, JHM, VAM, and some other popular methods by the solutions obtained from disparate problems. Comparative assessments are performed and illustrated in the immediately following sections. The detailed representation of the numerical data of Table 1 is provided in Appendix A [31].

Table 1. Comparative results of LCM, VAM, and New method for 12 benchmark instances

| Problem chosen from | TCIFS(f_{rs}) | Optimal cost(O_c) | Percentage minimal Deviation total From cost |
|---------------------|--------------|-------------------|-------------------------------------------|
| Deshmukh [32]      | 555          | 555               | 555                                       |
| Deshmukh [32]      | 114          | 112               | 112                                       |
| Ahmed (2016)       | 2,900        | 2,850             | 2,850                                     |
| Ahmed (2016)       | 3,500        | 3,320             | 3,320                                     |
| Korukoglu [13]     | 72,174       | 59,356            | 59,356                                    |
| Imam et al. [33]   | 475          | 475               | 475                                       |
| Taylor (Module B)  | 4,550        | 4,525             | 4,525                                     |
| Deshmukh [32]      | 305          | 290               | 260                                       |
| Sen et al. [34]    | 2,404,500    | 2,164,000         | 2,146,750                                 |
| Ahmed (2016)       | 9,800        | 9,200             | 9,200                                     |
| Ahmed (2016)       | 14,625       | 13,225            | 10,375                                    |
| Ahmed (2016)       | 6,450        | 6,000             | 5,600                                     |

The comparative results obtained in Table 1 are also depicted using bar graphs and the results are given in Fig. 1.

![Fig. 1. Comparative study of the result obtained by LCM, VAM with the proposed method](image-url)
Line graphs for the percentage deviation (of the LCM, VAM with the proposed method from minimal total cost solution) obtained in Table 1 are presented in Fig. 2.

It can be seen from Table 1, and Fig. 1 and Fig. 2 that the proposed method (MACA) is more efficient than LCM and VAM in every case where an improvement in efficiency was possible. Thus, proposed method of this paper performs better compared to LCM and VAM.

Performance measure of NEM over ZSM, VAM and JHM for 9 benchmark instances is shown in the Table 2. The detailed representation of the numerical data of Table 2 is provided in Appendix B.

The comparative results obtained in Table 2 are also depicted using bar graphs and the results are given in Fig. 3.
Table 2. A comparative study of ZSM, VAM, JHM and NEM for 9 benchmark instances

| Problem chosen from Juman and Hoque [17] | TCIFS (\(I_{F2}\)) | Optimal cost\((O_c)\) | Percentage minimal total from cost |
|------------------------------------------|---------------------|-----------------------|-----------------------------------|
|                                          | ZSM     | VAM     | JHM     | NEW     | ZSM     | VAM     | JHM     | NEW     |
| Srinivasan & Thomson [35]                | 910     | 955     | 880     | 880     | 3.40    | 8.52    | 0.00    | 0.00    |
| Sen et al. [34]                          | 2158500 | 2164000 | 2146750 | 2146750 | 0.55    | 0.80    | 0.00    | 0.00    |
| Deshmukh [32]                            | 798     | 779     | 743     | 743     | 7.40    | 4.84    | 0.00    | 0.00    |
| Ramadan and Ramadan [36]                  | 5,600   | 5600    | 5600    | 5600    | 0.00    | 0.00    | 0.00    | 0.00    |
| Kulkarni & Datar [37]                    | 1,200   | 880     | 840     | 840     | 42.85   | 4.76    | 0.00    | 0.00    |
| Schrenk et al. [38]                      | 59      | 60      | 59      | 59      | 0.00    | 1.69    | 0.00    | 0.00    |
| Samuel [39]                              | 28      | 28      | 28      | 28      | 0.00    | 0.00    | 0.00    | 0.00    |
| Imam et al. [33]                         | 460     | 475     | 460     | 435     | 0.06    | 3.26    | 0.00    | 0.00    |
| Adlakha and Kowalski [40]                | 400     | 390     | 390     | 390     | 2.56    | 0.00    | 0.00    | 0.00    |

Table 3. A comparative results obtained by ZSM, VAM, JHM and New method for the seven benchmark instances

| Problem chosen from Juman and Hoque [17] | TCIFS (\(I_{F2}\)) | Optimal cost\((O_c)\) | Percentage minimal | Deviation total | From cost |
|------------------------------------------|---------------------|-----------------------|--------------------|-----------------|-----------|
|                                          | ZSM     | VAM     | JHM     | NEW     | ZSM     | VAM     | JHM     | NEW     |
| Problem 1.                               | 4525    | 5125    | 4525    | 4525    | 4525    | 3.40    | 8.52    | 0.00    |
| Problem 2.                               | 3460    | 4525    | 4525    | 4525    | 3460    | 0.55    | 0.80    | 0.00    |
| Problem 3.                               | 920     | 960     | 920     | 920     | 920     | 7.40    | 4.84    | 0.00    |
| Problem 4.                               | 864     | 859     | 809     | 809     | 809     | 0.00    | 0.00    | 0.00    |
| Problem 5.                               | 475     | 475     | 475     | 475     | 417     | 42.85   | 4.76    | 0.00    |
| Problem 6.                               | 3598    | 3458    | 3458    | 3458    | 3458    | 4.04    | 9.25    | 0.00    |
| Problem 7.                               | 136     | 112     | 109     | 109     | 109     | 0.00    | 0.00    | 0.00    |
Line graphs for the percentage deviation (of the ZSM, VAM, JHM and the proposed method) from minimal total cost solution obtained in Table 2 are presented in Fig. 4.

![Deviation Chart](image)

**Fig. 4. Percentage of deviation of the results obtained by ZSM, VAM, JHM and the proposed method**

According to the simulation results (Table 2, Fig. 3 and Fig. 4), the proposed method yields better results to all problems in Table 2 compared with ZSM and VAM. It provides the same results as JHM.

Comparative results obtained by ZSM, VAM, JHM, and the proposed method for the seven benchmark instances are shown in the following Table 3. Detailed data representation of these seven problems is provided in Appendix C.

The comparative results obtained in Table 3 are also depicted using bar graphs and the results are given in Fig. 5.

![Bar Graphs](image)

**Fig. 5. Comparative study of the result obtained by ZSM, VAM, JHM with NEW method**
Line graphs for the percentage deviation (of the ZSM, VAM, JHM and New method) from minimal total cost solution obtained in Table 3 are depicted in Fig. 6.

![Deviation Chart](image)

**Fig. 6. Percentage of deviation of the results obtained by ZSM, VAM, JHM and the proposed method**

Based on the above results (Table 3, Fig. 5 Fig. 6), the proposed method outperforms ZSM and VAM. It provides the same results as JHM. However, this cannot deny the value of our proposed method, modified ant colony algorithm. Note that, here, the formula is \( D = \frac{|y_{PS} - o_c|}{o_c} \times 100 \) used to obtain the percentage deviation from the optimal result. This calculation is carried out to evaluate how much nearer the \( y_{PS} \) is to \( o_c \).

In addition to the above benchmark instances, we have also studied some other numerical example problems chosen from Ahmed [41] in order to determine the performance of our new method over the available 14 approaches. The obtained results are presented in Table 4. Detailed data representation of these four example problems is provided in Appendix D.

**Table 4. Comparative results of new method along with the 14 available approaches for 4 benchmark instances**

| Methods                          | Total initial cost feasible | for solution Ex.3 | the Ex.4  |
|----------------------------------|-----------------------------|-------------------|-----------|
| North West Corner Method (NWCM)  | 4,400                       | 4,160             | 540       | 1,500     |
| Row Minimum Method (RMM)         | 2,850                       | 4,120             | 470       | 1,450     |
| Column Minimum Method (CMM)      | 3,600                       | 3,320             | 435       | 1,500     |
| Least Cost Method (LCM)          | 2,900                       | 3,500             | 435       | 1,450     |
| Vogel’s Approximation Method (VAM) | 2,850           | 3,320             | 470       | 1,500     |
| Extremum Difference Method (EDM) | 2,900                       | 3,620             | 415       | 1,390     |
| Highest Cost Difference Method (HCDM) | 2,900           | 3,620             | 435       | 1,450     |
| Average Cost Method (ACM)        | 2,900                       | 3,320             | 455       | 1,440     |
| TOCM-MMM Approach                | 2,900                       | 3,620             | 435       | 1,450     |
| TOCM-VAM Approach                | 2,850                       | 3,620             | 430       | 1,450     |
| TOCM-EDM Approach                | 2,850                       | 3,620             | 435       | 1,450     |
| TOCM-HCDM Approach               | 2,900                       | 3,620             | 435       | 1,450     |
| TOCM-SUM Approach                | 2,850                       | 3,320             | 455       | 1,440     |
| ATM Approach ATM                 | 2,850                       | 3,320             | 415       | 1,390     |
| Proposed New Method              | **2,850**                   | **3,320**         | **410**   | **1,390** |
| Optimal Solution                 | 2,850                       | 3,320             | 410       | 1,390     |
Fig. 7. Comparative study of the result obtained by the proposed method against the existing 14 approaches

Note that, although our proposed method yields the optimal solution to the above four benchmark instances, the available fourteen approaches do not. We have also studied a further set of benchmarks in determining the performance measure of the proposed method (PM) over NWCM, LCM, VAM, and EDM. Detailed data representation of the problems is given in Appendix E [42]

Table 5. Performance measure of new method (NM) over NWCM, LCM, VAM and EDM

| Solution methods | Total cost for the initial feasible solution |
|------------------|---------------------------------------------|
|                  | Ex.1            | Ex.2            |
| NWCM             | 107             | 19,700          |
| LCM              | 83              | 13,100          |
| VAM              | 80              | 12,250          |
| EDM              | 83              | 12,250          |
| PM               | **76**          | **11,500**      |
| Optimal Solution | 76              | 11,500          |

Fig. 8. Comparative study of the result obtained by PM against the existing NWCM, LCM, VAM, EDM

Note that, although PM yields the optimal solution to both benchmark instances above, the available four approaches (NWCM, LCM, VAM, and EDM) do not. Hence, the comparative assessments of the above different cases show that both the modified ant colony algorithm and JHM are efficient as compared to the studied approaches of this paper in terms of the accuracy of the solution
The comparative results obtained in Table 5 are also depicted using bar graphs and the results are given in Fig. 8.

5 Conclusion

The transportation problem is one of the significant problems in the field of operation research for its wide application, in reality. It is related to finding the minimum cost transportation plan for moving various origins to different destinations. Transportation problem (TP) is one of the special class of Linear Programming problems in which the objective is transportation problem solution methods to reach the optimal solution. Many researchers have paid attention to solve this problem using different approaches. Out of all the existing methods in the literature, Northwest, Least Cost, Vogel’s Approximation, and Juman and Hoque’s methods are the most prominent and renowned methods in finding an initial feasible solution to a TP. Modified Distribution (MODI) Method and Stepping Stone Method are the most acceptable methods in finding the minimal total cost solution to the transportation problem. These well-known minimal total cost solution techniques start with an Initial Feasible Solution (IFS). Thus an IFS acts as a foundation of an optimal cost solution technique to any TP. Better is the IFS lesser is the number of iterations to reach the final optimal solution. However, in this research paper, we discuss a new alternative method, a modified ant colony optimization algorithm which gives often an optimal solution to the transportation problem.

In this research paper, we first examine different initial solutions providing methods for attaining initial feasible solutions to balanced and unbalanced transportation problems. This research paper presents an overview of the concept of an Ant colony algorithm and provides a review of its applications to solve transportation problems. The PM is very simple, easy to understand, and easy to implement. Several modifications to the ant colony algorithm are made and ensured a solution which is very closer to the optimal solution. This method requires a minimum number of steps to reach the optimality as compared to the existing methods. An extensive numerical study is carried out to see the potential significance of our modified ant colony algorithm (MACOA). The comparative assessment shows that both the MACOA and JHM are efficient as compared to the studied approaches of this paper in terms of the quality of the solution. Also, A comparative study shows that the new method gives the minimal total cost solution to 34 out of 34 benchmark instances. However, in practice, when researchers and practitioners deal with large-sized transportation problems, we urge them to use our proposed MACOA due to the time-consuming computation of JHM.

Although our proposed Modified Ant Colony Optimization algorithm provides better IFS that is often optimal, it does not always guarantee the exact optimal cost solution. Since the existing well-known exact optimal cost solution technique (SSM) deals fully with the path tracing technique, it becomes very difficult in solving large-scaled transportation problems. Thus, we intend to devote ourselves in near future in proposing an alternate exact optimal approach that gets rid of this difficulty.

Acknowledgements

The authors are thankful to the three referees for their valuable comments and suggestions. The first author thank his supervisor and co-supervisors for their direct support and important time offered him to do this investigation a pleasurable achievement. Finally, I might likewise want to offer my most profound thanks to my wife, daughters, and son for their consolation and important help on this occasion work out.

Competing Interests

Authors have declared that no competing interests exist.
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### Appendix A

| Problem chosen from. | Size | Data of the problem |
|---------------------|------|---------------------|
| 1. BTP-1 (12)       | 3 × 3| $c_{ij} = [6,4,1;3,8,7;4,42], s_i = [50,40,60], d_j = [20,95,35]$ |
| 2. BTP-2 (12)       | 4 × 6| $c_{ij} = [9,12,9,6;9,10;7,3,7,7,5,5;6,5,9,11,3,11;6,8,11,2,2,10], s_i = [5,6,2,9], d_j = [4,6,2,4,2]$ |
| 3. BTP-3(4)         | 3 × 4| $c_{ij} = [3,1,7,4;2,6,5,9;8,3,3,2], s_i = [300,40,500], d_j = [250,350,400,200]$ |
| 4. BTP-4(4)         | 3 × 4| $c_{ij} = [50,60,100,50;80,40,70,50;90,70,30,50], s_i = [20,38,16], d_j = [10,18,22,24]$ |
| 5. BTP-24           | 5 × 5| $c_{ij} = [46,74,9,28,99;12,7,5,6,3,64,8;35;199,4,5,71;61,81,44,88,9;85,60,14,25,79], s_i = [4,61,270,356,488,393], d_j = [278,60,461,116,1060]$ |
| 6. BTP-18           | 3 × 4| $c_{ij} = [10,2,20,11;12,7,9,20;4,14,16,18], s_i = [15,25,10], d_j = [5,15,15,15]$ |
| 7. BTP-7 (21)       | 3 × 3| $c_{ij} = [6,8,10;7,11,11;4,5,12], s_i = [150,175,275], d_j = [200,100,300]$ |
| 8. UTP-1 (12)       | 3 × 5| $c_{ij} = [4,1,2,4,4;2,3,2,3;3,5,2,4,4], s_i = [60,35,40], d_j = [22,45,20,18,30]$ |
| 9. UTP-2 (35)       | 5 × 4| $c_{ij} = [60,120,75,180;58,100,60,165;62,110,65,170;65,115,80,175;70,135,85,195], s_i = [80000,9200,6250,4900,6100], d_j = [50000,2000,10000,6000]$ |
| 10. UTP-3 (5)       | 3 × 5| $c_{ij} = [5,8,6,6,3;4,7,7,6,5;8,4,6,6,4,4], s_i = [800,500,900], d_j = [400,400,500,400,800]$ |
| 11. UTP-4 (5)       | 5 × 4| $c_{ij} = [12,10,6,13;19,8,16,25;17,15,15,20;23,22,26,12], s_i = [150,500,600,225], d_j = [300,500,75,100]$ |
| 12. UTP-5 (5)       | 4 × 4| $c_{ij} = [5,4,8,6,5;4,5,4,3,2,3,6,5,8,4], s_i = [600,400,1000], d_j = [450,400,200,250,300]$ |

### Appendix B

| Problem chosen from Juman & Hoque [23] | Data of the problem |
|---------------------------------------|---------------------|
| Problem 1 [39]                       | $c_{ij} = [3,6,3,4;6,5,11,15;1,3,10,5], s_i = [80,90,55], d_j = [70,603,560]$ |
| Problem 2 [36]                       | $c_{ij} = [60,120,75,180;58,100,60,165;62,110,65,170;65,115,80,175;70,135,85,195], s_i = [80000,9200,6250,4900,6100], d_j = [50000,2000,10000,6000]$ |
| Problem 3 [12]                       | $c_{ij} = [19,30,50,10;70,30,40,60;40,87,20,2], s_i = [7,9,18], d_j = [5,8,7,14]$ |
| Problem 4 [30]                       | $c_{ij} = [32,40,120;60,88,104;200,80,60], s_i = [20,20,45], d_j = [30,35,30]$ |
| Problem 5 [27]                       | $c_{ij} = [3,4,6;7,3,8;6,4,5;7,5,], s_i = [100,80,90,120], d_j = [110,90,60]$ |
| Problem 6 [35]                       | $c_{ij} = [3,6,1,5;7,9,27;2,4,2,1], s_i = [6,6,6], d_j = [4,5,4,5]$ |
| Problem 7 [34]                       | $c_{ij} = [1,2,3,4;4,3,2,0;0,2,2,1], s_i = [6,8,10], d_j = [4,6,8,6]$ |
| Problem 8 [20]                       | $c_{ij} = [10,2,20,11;12,7,9,20;4,14,16,18], s_i = [15,25,10], d_j = [5,15,15,15]$ |
| Problem 9 [2]                        | $c_{ij} = [2,1,3,2,2;3,2,1,1,1;5,4,2,1,3;7,5,5,3,1], s_i = [20,70,30,60], d_j = [50,30,50,20]$ |

### Appendix C

| Problem chosen from Juman & Hoque [23] | Data of the problem |
|---------------------------------------|---------------------|
| Problem 1                             | $c_{ij} = [6,8,10;7,11,11;4,5,12], s_i = [150,175,275], d_j = [200,100,300]$ |
| Problem 2                             | $c_{ij} = [20,22,17,4;24,37,9,7;32,37,20,15], s_i = [120,70,50], d_j = [160,40,30,110]$ |
| Problem 3                             | $c_{ij} = [4,6,8,8;6,8,6,7,5;7,6,8], s_i = [40,60,50], d_j = [20,30,50,50]$ |
| Problem 4                             | $c_{ij} = [19,30,50,12;70,30,40,60;40,10,60,20], s_i = [7,10,18], d_j = [5,7,8,15]$ |
| Problem 5 [29]                        | $c_{ij} = [13,18,30,8;55,20,25,40;30,65,50,10], s_i = [8,10,11,1], d_j = [4,6,7,12]$ |
| Problem 6                             | $c_{ij} = [25,14,34,46,45;10,47,14,20,41;22,42,38,21,46;36,20,41,38,44], s_i = [27,35,37,45], d_j = [22,27,28,33,34]$ |
| Problem 7                             | $c_{ij} = [9,12,9,6,9,10;7,3,7,7,5,5;6,5,9,11,3,11;6,8,11,22,10], s_i = [150,175,275], d_j = [200,100,300]$ |
Appendix D

| Problem chosen from Ahmed et al. [5] | Data of the problem |
|-------------------------------------|---------------------|
| Problem 1                           | \(c_{ij} = [3,1,7,4; 2,6,5,9; 8,3,3,2], s_i = [300,400,500], d_j = [250,350,400,200] \) |
| Problem 2                           | \(c_{ij} = [50,60,100,50; 80,40,70,50; 90,70,30,50], s_i = [20,38,16], d_j = [10,18,22,24] \) |
| Problem 3                           | \(c_{ij} = [7,5,9,11; 4,3,8,6; 3,8,10,5; 2,6,7,3], s_i = [30,25,20,15], d_j = [30,30,20,10] \) |
| Problem 4                           | \(c_{ij} = [4,3,5; 6,5,4; 8,10,7], s_i = [90,80,100], d_j = [70,120,80] \) |

Appendix E

Example 1 (Balanced TP)

A company manufactures motor tyres and it has four factories \(F_1, F_2, F_3\) and \(F_4\) whose weekly production capacities are 5,8,7 and 14 thousand pieces of tyres respectively. The company supplies tyres to its three showrooms located at \(D_1, D_2\) and \(D_3\) whose weekly demand are 7, 9 and 18 thousand pieces respectively. The transportation cost per thousand pieces of tyre is given below in the TT:

| Factory | Showrooms |
|---------|-----------|
|         | \(D_1\)  | \(D_2\)  | \(D_3\)  |
| \(F_1\) | 2        | 7        | 4        |
| \(F_2\) | 3        | 3        | 1        |
| \(F_3\) | 5        | 4        | 7        |
| \(F_4\) | 1        | 6        | 2        |

We want to schedule the shifting of tyres from factories to showrooms with a minimum cost.

Example 2 (Unbalanced TP)

A company has four plants located at A,B,C and D, which supply to warehouses located at E,F,G,H and I. Monthly plant capacities are 300,500,825 and 375 units respectively and monthly warehouses requirements are 350,400,250,150 and 400 units respectively. Unit transportation costs are given below.

| Plants | Warehouses | Capacities |
|--------|------------|------------|
|        | E          | F          | G          | H          | I          |
| A      | 10         | 2          | 16         | 14         | 10         | 300        |
| B      | 6          | 18         | 12         | 13         | 16         | 500        |
| C      | 8          | 4          | 14         | 12         | 10         | 825        |
| D      | 14         | 22         | 20         | 8          | 18         | 375        |

Determine a distribution plan for the company in order to minimize the total transportation cost.

Appendix F

Validation of the new Algorithm via numerical example

Consider the following Transportation problem:

|      | \(D_1\) | \(D_2\) | \(D_3\) | \(D_4\) | Supply |
|------|---------|---------|---------|---------|--------|
| \(S_1\) | 60      | 120     | 75      | 180     | 8,000  |
| \(S_2\) | 58      | 100     | 60      | 165     | 9,200  |
| \(S_3\) | 62      | 110     | 65      | 170     | 6,250  |
| \(S_4\) | 65      | 115     | 80      | 175     | 4,900  |
| \(S_5\) | 70      | 135     | 85      | 195     | 6,100  |

Demand | 5,000 | 2,000 | 10,000 | 6,000 |
Step 1: Formulate the Transportation Cost Matrix. The problem is unbalanced, make it a balanced problem by introducing a dummy destination.

|       | $D_1$ | $D_2$ | $D_3$ | $D_4$ | $D_5$ | Supply |
|-------|-------|-------|-------|-------|-------|--------|
| $S_1$ | 60    | 120   | 75    | 180   | 0     | 8,000  |
| $S_2$ | 58    | 100   | 60    | 165   | 0     | 9,200  |
| $S_3$ | 62    | 110   | 65    | 170   | 0     | 6,250  |
| $S_4$ | 65    | 115   | 80    | 175   | 0     | 4,900  |
| $S_5$ | 70    | 135   | 85    | 195   | 0     | 6,100  |
| Demand| 5,000 | 2,000 | 10,000| 6,000 | 11,450|

Step 2: Compute the path according to the probability matrix (using eq.(2)).

|       | $D_1$ | $D_2$ | $D_3$ | $D_4$ | $D_5$ | Supply |
|-------|-------|-------|-------|-------|-------|--------|
| $S_1$ | .204  | .194  | .195  | .197  | 0     | 8,000  |
| $S_2$ | .208  | .219  | .220  | .210  | 0     | 9,200  |
| $S_3$ | .201  | .205  | .212  | .204  | 0     | 6,250  |
| $S_4$ | .196  | .200  | .189  | .201  | 0     | 4,900  |
| $S_5$ | .188  | .179  | .182  | .185  | 0     | 6,100  |
| Demand| 5,000 | 2,000 | 10,000| 6,000 | 11,450|

Step 3, 4, 5, and 6:

|       | $D_1$ | $D_2$ | $D_3$ | $D_4$ | $D_5$ | Supply |
|-------|-------|-------|-------|-------|-------|--------|
| .204  | .194  | .195  | .197  | 0     | 8,000  |
| .208  | .219  | .220  | .210  | 0     | 7,200  |
| .201  | .205  | .212  | .204  | 0     | 6,250  |
| .196  | .200  | .189  | .201  | 0     | 4,900  |
| .188  | .179  | .182  | .185  | 0     | 6,100  |
| 5,000 | 2,000 | 10,000| 6,000 | 11,450|

Step 3, 4, 5, and 6:

|       | $D_1$ | $D_2$ | $D_3$ | $D_4$ | $D_5$ | Supply |
|-------|-------|-------|-------|-------|-------|--------|
| .204  | .194  | .195  | .197  | 0     | 8,000  |
| .208  | .219  | .220  | .210  | 0     | 7,200  |
| .201  | .205  | .212  | .204  | 0     | 6,250  |
| .196  | .200  | .189  | .201  | 0     | 4,900  |
| .188  | .179  | .182  | .185  | 0     | 6,100  |
| 5,000 | 2,000 | 10,000| 6,000 | 11,450|

Step 3, 4, 5, and 6:

|       | $D_1$ | $D_2$ | $D_3$ | $D_4$ | $D_5$ | Supply |
|-------|-------|-------|-------|-------|-------|--------|
| .204  | .194  | .195  | .197  | 0     | 8,000  |
| .208  | .219  | .220  | .210  | 0     | 7,200  |
| .201  | .205  | .212  | .204  | 0     | 6,250  |
| .196  | .200  | .189  | .201  | 0     | 4,900  |
| .188  | .179  | .182  | .185  | 0     | 6,100  |
| 5,000 | 2,000 | 10,000| 6,000 | 11,450|

Step 3, 4, 5, and 6:

|       | $D_1$ | $D_2$ | $D_3$ | $D_4$ | $D_5$ | Supply |
|-------|-------|-------|-------|-------|-------|--------|
| .204  | .194  | .195  | .197  | 0     | 8,000  |
| .208  | .219  | .220  | .210  | 0     | 7,200  |
| .201  | .205  | .212  | .204  | 0     | 6,250  |
| .196  | .200  | .189  | .201  | 0     | 4,900  |
| .188  | .179  | .182  | .185  | 0     | 6,100  |
| 5,000 | 2,000 | 10,000| 6,000 | 11,450|

Step 3, 4, 5, and 6:
Step 3, 4, 5 and 6:

\[
\begin{array}{cccccc}
& 0.204 & 0.194 & 0.195 & 0.197 & 0 & 8,000 \\
& 0.208 & 0.219*2,000 & 0.220*7,200 & 0.240 & \varnothing & 0 \\
& 0.201 & 0.205 & 0.212*2,800 & 0.204*3,450 & \varnothing & 0 \\
& 0.196 & 0.200 & 0.189 & 0.201*2,550 & 0 & 2,350 \\
& 0.188 & 0.170 & 0.182 & 0.185 & 0 & 6,100 \\
5,000 & 0 & 0 & 2,550 & 11,450 & \\
\end{array}
\]

Step 3, 4, 5 and 6:

\[
\begin{array}{cccccc}
& 0.204*5000 & 0.194 & 0.195 & 0.197 & 0 & 3,000 \\
& 0.208 & 0.219*2,000 & 0.220*7,200 & 0.240 & \varnothing & 0 \\
& 0.201 & 0.205 & 0.212*2,800 & 0.204*3,450 & \varnothing & 0 \\
& 0.196 & 0.200 & 0.189 & 0.201*2,550 & 0 & 2,350 \\
& 0.188 & 0.170 & 0.182 & 0.185 & 0 & 6,100 \\
5,000 & 0 & 0 & 2,550 & 11,450 & \\
\end{array}
\]

Step 3, 4, 5 and 6:

\[
\begin{array}{cccccc}
& 0.204*5000 & 0.194 & 0.195 & 0.197 & 0*3,000 & 3,000 \\
& 0.208 & 0.219*2,000 & 0.220*7,200 & 0.240 & \varnothing & 0 \\
& 0.201 & 0.205 & 0.212*2,800 & 0.204*3,450 & \varnothing & 0 \\
& 0.196 & 0.200 & 0.189 & 0.201*2,550 & 0*2,350 & 2,350 \\
& 0.188 & 0.170 & 0.182 & 0.185 & 0*6,100 & 6,100 \\
0 & 0 & 0 & 0 & 11,450 & \\
\end{array}
\]

Step 7: Stop and determine solution.

\[
\begin{array}{ccccc}
\hline
 & D_1 & D_2 & D_3 & D_4 & \text{Supply} \\
\hline
S_1 & 60*5000 & 120 & 75 & 180 & 8,000 \\
S_2 & 58 & 100*2000 & 60*7200 & 165 & 9,200 \\
S_3 & 62 & 110 & 65*2800 & 170*3450 & 6,250 \\
S_4 & 65 & 115 & 80 & 175*2550 & 4,900 \\
S_5 & 70 & 135 & 85 & 195 & 6,100 \\
\hline
\text{Demand} & 5,000 & 2,000 & 10,000 & 6,000 & \\
\hline
\end{array}
\]

Total cost = 60x5000 + 100x2000 + 60x7200 + 65x2800 + 170x3450 + 175x2550 = 2,146,750

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