Direct Evidence for Proton-Neutron Out-of-Phase Coupling in Mixed-Symmetry States from Scattering Experiments on $^{92}\text{Zr}$

N Pietralla$^1$, C Walz$^1$, V Yu Ponomarev$^1$, H Fujita$^{2,3}$, A Krugmann$^1$, P von Neumann-Cosel$^1$, A Scheikh-Obeid$^1$, J Wambach$^1$

$^1$ Institut für Kernphysik, TU Darmstadt, Schloßgartenstraße 9, D-64289 Darmstadt, Germany
$^2$ Department of Physics, Osaka University, Toyonaka, Osaka 560-0043, Japan
$^3$ iThemba LABS, Post Office Box 722, Somerset West 7129, South Africa

E-mail: pietralla@ikp.tu-darmstadt.de

Abstract. The coupling of the giant quadrupole resonance to valence space configurations is shown to dominantly contribute to the formation of low-lying quadrupole collective structures in vibrational nuclei with symmetric and mixed-symmetric character with respect to the proton-neutron degree of freedom. Experimental evidence is obtained from electron- and proton scattering experiments on the nucleus $^{92}\text{Zr}$ that are directly sensitive to the relative phase of valence space amplitudes by quantum interference.

1. Introduction
A general phenomenon of the low-energy structure in heavy atomic nuclei are collective quantum states, in particular of quadrupole nature. The proton-neutron residual interaction in the valence shell is known to represent the dominant origin for the formation and the evolution of nuclear quadrupole collectivity [1]. In order to study the residual proton-neutron interaction in the valence shell of open-shell nuclei it is interesting to determine the properties of those valence shell excitations where valence protons and valence neutrons move out of phase. Such states have been modeled, for example, in terms of mixed-symmetry states in the framework of the interacting boson model [2] with proton-neutron degree of freedom (IBM-2). In the context of nuclear structure physics the Interacting Boson Model (IBM) is an example of an effective field theory (EFT), formulated consistently with the symmetries of the intrinsic shapes of the nucleus. It describes the dynamics of collective low-energy nuclear excitations. Effective field theories for relevant low-energy degrees of freedom of complex systems are a concept of fundamental importance in modern physics with broad applications in many different areas. The underlying principle is a separation of energy (respectively momentum) scales such that the high-energy degrees of freedom are integrated out leading to a low-energy theory consistent with the symmetries and their breaking pattern. The high-energy sector manifests itself in a set of parameters in the low-energy EFT which have to be determined from experiment. Thus each EFT is only capable to describe phenomena at a specific (sufficiently low) energy scale.

In the IBM the relevant low-energy degrees of freedom for the description of quadrupole-collectivity are bosons with intrinsic angular momentum $L = 0$ (s bosons) or $L = 2$ (d bosons).
These lead to an effective Hamiltonian whose parameters are adjusted to data. However, a microscopic and quantitative theory for the derivation of these parameters is still missing. Such an approach could only be hoped for being successful if the dominant origin of the quadrupole collectivity would be included in such a theory. Our recent experiments [3] have provided evidence on how the coupling to cross-shell transitions forming the giant quadrupole resonance (GQR) contributes to the formation of low-energy nuclear collectivity. This has been achieved by an analysis of transition densities in electromagnetic and hadronic scattering reactions that has enabled us for the first time to observe and interpret the interference of the valence-shell components of the lowest-energy one-quadrupole phonon wave functions with those stemming from cross-shell excitations. This lecture aimed at a presentation of our recent material and, hence, this contribution is based on the text published in Ref. [3].

2. The case of \(^{92}\text{Zr}\)

To keep the discussion transparent, we studied the formation of quadrupole collectivity in the particularly simple case of a nucleus with a low-energy structure that is dominated by one pair of valence particles each for protons and neutrons. An example is the nucleus \(^{92}\text{Zr}\) with 2 neutrons beyond the \(N = 50\) shell closure and 2 protons beyond the \(Z = 38\) sub-shell closure. The lowest 2-quasiparticle (2qp) states will therefore have \(\pi (1\text{g}_{9/2})^2\) and \(\nu (2\text{d}_{5/2})^2\) configurations. Due to the residual proton-neutron interaction two different classes of collective excitations appear at low energy in which the amplitudes of the two most important 2qp configurations are coupled in a symmetric or antisymmetric way, respectively. In \(^{92}\text{Zr}\), these are experimentally identified as the \(2^+_1\) and \(2^+_2\) states [4, 5] with some degree of configurational isospin polarization [6].

To shed light on the microscopic origin of the effective coupling strength in the valence shell we consider the quasiparticle-phonon model (QPM) [7]. The QPM starts with interacting particle-hole excitations from which ‘phonons’ are generated by the Random-Phase Approximation (RPA) approach. These form elementary degrees of freedom from which a more general Hamiltonian can be derived which allows for multi-phonon states. The QPM covers a sufficiently

![Figure 1](image-url). Transition strengths as a function of the maximum 2qp energy included in the QPM calculation for (a) g.s. excitation of the \(2^+_1\) and \(2^+_2\) states, and (b) \(M1\) transition between both states. The latter are dominated by valence-shell components while the (moderately) collective \(E2\) transitions are dominated by admixtures of cross-shell excitations. For easier interpretation the situation can be simplified to a three-state mixing scenario as sketched on the right.
large single-particle space to satisfy the energy-weighted sum-rules. The QPM approach has proven to account very successfully for the low-energy properties in a large number of vibrational nuclei [8, 9] including \(^{92}\text{Zr}\). In the spirit of an interpretation of the IBM-2 as an EFT, the QPM can be viewed as 'complete theory' including the high-energy degrees of freedom. The QPM wave functions are dominated by the lowest \(\pi\) and \(\nu\) 2qp components, that show the expected in-phase and out-of-phase behavior for the \(2^{+}_{fs}\) and \(2^{+}_{ms}\) states. The electromagnetic properties and excitation energies are in excellent agreement with the data [9]. The magnetic moments of these states and the strong \(M1\) transition between them originate almost entirely from the valence-shell configurations as it is shown in Fig. 1. However, the \(B(E2)\) strengths are generated to about 80% from many components beyond the valence shell albeit their total contribution to the wave function norm is small.

The role of the GQR is demonstrated in Fig. 1, which shows the running sum of the \(B(M1)\) and \(B(E2)\) transition strengths as a function of the maximum 2qp energy included in the calculation. The \(M1\) strength saturates at about 6 MeV, i.e., only valence-shell configurations are relevant, while the largest part of the \(E2\) strengths exciting the \(2^{+}_{fs}\) and \(2^{+}_{ms}\) phonons is generated by states in the energy region of the GQR around 15 – 20 MeV. This observation motivates a simple three-state mixing scenario between the proton-valence shell configuration, the neutron-valence shell configuration, and the GQR for a deeper insight in the formation of the one-quadrupole phonon states with symmetric and mixed-symmetry character even on a semi-quantitative level [3]. For the nucleus \(^{92}\text{Zr}\) with higher energy for the proton valence-shell component than the neutron valence-shell component at the \(Z=40\) sub-shell closure, this scheme inevitably requires that the neutron valence-shell component flips its phase with respect to the GQR component when going from the proton-neutron symmetric \(2^{+}_{1}\) state to the \(2^{+}_{2}\) state with predominant mixed symmetry. Can we find experimental evidence for this simple scheme, \(i.e.,\) is it possible to find a signature for the different interference of the dominant valence-shell components with the high-energy mode as predicted in the three-state scheme?

3. Experiment

Apparently, two probes with different sensitivity to protons and neutrons are needed to study this quantum interference experimentally which has not been done so far. Electron scattering at low momentum transfer provides a measure of the charge transition radius. An \((e,e')\) experiment was performed at the high-energy-resolution spectrometer [10] of the Darmstadt superconducting electron linear accelerator (S-DALINAC). An enriched (94.6 %) self-supporting \(^{92}\text{Zr}\) target of 9.8 mg/cm\(^2\) areal density was used. Data were taken covering a momentum transfer range

![Figure 2](image)

**Figure 2.** Momentum-transfer dependence of the cross sections of the \(2^{+}_{fs}\) and \(2^{+}_{ms}\) in proton scattering (left) and electron scattering (right). The data are compared to QPM calculations.
between \( q \approx 0.3 - 0.6 \text{ fm}^{-1} \) indicating no difference between the charge transition radii of the \( 2^+_{fs} \) and \( 2^+_{ms} \) states within experimental uncertainties (Fig. 2, right). Information about the neutron transition radii can be derived from the proton scattering data of Ref. [11]. At the incident energy of 800 MeV protons interact predominantly via the isoscalar central piece of the effective projectile-nucleus interaction [12]. Clearly, the refraction pattern of the \((p,p')\) cross section for the \( 2^+_{ms} \) state are shifted to higher \( q \) values as compared to those for the \( 2^+_{fs} \) state (Fig. 2, left) corresponding to a smaller transition radius.

4. Analysis

Using the transition densities from Fig. 3, cross sections were calculated in distorted wave Born approximation. The \( T \)-matrix parametrization of Franey and Love [12] was used to describe the effective proton-nucleus interaction. The QPM calculation reproduces well both the absolute values of cross sections for both probes and the shift of the refraction pattern to higher \( q \) values for the \( 2^+_{ms} \) state in the \((p,p')\) reaction as displayed in Fig. 2.

Figure 3 displays the proton and neutron transition densities of the \( 2^+_{fs} \) (top) and \( 2^+_{ms} \) (bottom) states calculated in the full QPM approach. The full transition densities (solid curves) are decomposed in a collective part stemming from the GQR (dotted curves) and the predominant \( 2qp \) \( \nu(2d_{5/2})^2 \) or \( \pi(1g_{9/2})^2 \) contributions (dashed curves). The key point is the different

![Figure 3](image_url)

**Figure 3.** Neutron transition densities of the \( 2^+_{fs} \) (top) and \( 2^+_{ms} \) (bottom) states of \(^{92}\text{Zr}\) from QPM calculations. The full transition densities (solid lines) are decomposed in parts stemming from the GQR (dotted lines) and from the main \( 2qp \) configurations (dashed lines). The arrows indicate the maxima of the corresponding full transition densities.
radial behaviour of both parts. An out-of-phase coupling between the neutron valence shell contribution and the contribution from the GQR leads to a destructive quantum interference that predominantly reduces the neutron transition density at large radii (due to the larger radius of the $\nu(2d_{5/2})^2$ orbital) and consequently shifts the maximum of the total neutron transition density to the interior with respect to that one for the $2^+_fs$ state, as indicated by the arrows in Fig. 3. This effect reduces the neutron transition radius of the $2^+_ms$ with respect to the $2^+_fs$ state of $^{92}$Zr. In contrast, the proton transition radius remains essentially unchanged since the $\pi(1g_{9/2})$ part couples in-phase to the GQR contribution in both states. The combination of both data sets unambiguously demonstrates that the phase of the neutron valence-shell configurations changes its sign between the $2^+_fs$ and the $2^+_ms$ state for the first time [3].

5. Summary
We have presented a simple mechanism to explain the formation of the collective symmetric and mixed-symmetric $2^+$ states in vibrational nuclei. The proton-neutron quadrupole interaction responsible for the mixing of the valence space configurations is mediated by their coupling to the GQR, whose contribution dominates the $E2$ strengths of these states. In contrast, magnetic properties can be understood to a large extent within the scope of effective valence-space models. A combination of proton and electron scattering data provides direct evidence for the interference of high-energy configurations from the GQR with leading valence-space configurations. The data are sensitive to the relative signs of the leading valence shell configurations with respect to the GQR-contribution and confirm the symmetric and mixed-symmetric nature of the low-energy $2^+$ one-phonon states. In particular, a flip of phase of the neutron valence configuration between the $2^+_fs$ and the $2^+_ms$ state was observed and is understood as a consequence of the lower neutron $2qp$ energy at the $Z = 40$ proton sub-shell closure.

Since typical excitation energies of the low-lying collective states are $1–2$ MeV and that of the GQR $10–20$ MeV, a clean separation of scales is given, leading to an interpretation of valence-space approaches like the IBM-2 and shell model as EFTs of microscopic models capable to consistently treat valence-shell excitations and giant resonances. The present findings highlight the peculiar role played by mixed-symmetry states in such efforts and in our understanding of the formation of nuclear collectivity and of collective phenomena in any strongly coupled multi-component quantum system.

Acknowledgments
Discussions with B. Friman, G. Rainovski, R.V. Jolos, T. Otsuka, Ch. Stoyanov, and V. Werner are greatly appreciated. We thank the S-DALINAC crew for preparation of electron beams. This work was supported by the Deutsche Forschungsgemeinschaft under grant No. SFB 634.

[1] Casten R F 1990 *Nuclear Structure from a Simple Perspective* (Oxford: Oxford University Press).
[2] Iachello F and Arima A 1987 *The Interacting Boson Model* (Cambridge: Cambridge University Press).
[3] Walz C, Fujita H, Krugmann A, von Neumann-Cosel P, Pietralla N, Ponomarev V Yu, Scheikh-Obeid A and Wambach J 2011 *Phys. Rev. Lett.* **106** 062501
[4] Werner V et al. 2002 *Phys. Lett. B* **550** 140; Fransen C et al. 2005 *Phys. Rev. C* **71** 054304
[5] Pietralla N, von Brentano P and Lisetskiy A F 2008 *Prog. Part. Nucl. Phys.* **60** 225
[6] Holt J D et al. 2007 *Phys. Rev. C* **76** 034325
[7] Soloviev V 1992 *Theory of Atomic Nuclei: Quasiparticles and Phonons* (Bristol: Institute of Physics Publishing)
[8] Lo Iuduce N and Stoyanov Ch 2000 *Phys. Rev. C* **62** 047302; Lo Iuduce N, Stoyanov Ch and Tarpanov D 2008 *Phys. Rev. C* **77** 044310
[9] Lo Iuduce N and Stoyanov Ch 2006 *Phys. Rev. C* **73** 037305
[10] Lenhardt A W et al. 2006 *Nucl. Instrum. Methods Phys. Res. A* **562** 320
[11] Baker F T et al. 1983 *Nucl. Phys. A* **393** 283
[12] Franey M A and Love W G 1985 *Phys. Rev. C* **31** 488