A STANDARD KINETIC ENERGY RESERVOIR IN GAMMA-RAY BURST AFTERGLOWS

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ABSTRACT

We present a comprehensive sample of X-ray observations of 41 γ-ray burst (GRB) afterglows, as well as jet opening angles, θζ, for a subset with measured jet breaks. We show that there is a significant dispersion in the X-ray fluxes, and hence isotropic X-ray luminosities (LXiso), normalized to t = 10 hr. However, there is a strong correlation between LXiso and the beaming fractions, fj = [1 − cos(θζ)]. As a result, the true X-ray luminosity of GRB afterglows, LX = fjLXiso, is approximately constant, with a dispersion of only a factor of 2. Since ejEb ∝ LX, the strong clustering of LX directly implies that the adiabatic blast wave kinetic energy in the afterglow phase, Eb, is tightly clustered. The narrow distribution of LX also suggests that p ≈ 2, that inverse Compton emission does not in general dominate the observed X-ray luminosity, and that radiative losses at t < 10 hr are relatively small. Thus, despite the large diversity in the observed properties of GRBs and their afterglows, the energy imparted by the GRB central engine to the relativistic ejecta is approximately constant.

Subject headings: gamma rays: bursts — ISM: jets and outflows — shock waves

1. INTRODUCTION

Gamma-ray bursts (GRBs) exhibit a remarkable diversity: fluences range from 10−7 to 10−3 ergs cm−2, peak energies range from 50 keV to an MeV, and possibly from the X-ray to the GeV band (Fishman & Meegan 1995), and durations extend from about 2 to 103 s (for the long-duration GRBs). This diversity presumably reflects a dispersion in the progenitors and the properties of the central engine. Perhaps the most impressive features of GRBs are their brilliant luminosities and isotropic energy releases approaching the rest mass of a neutron star, Eiso ∼ 1054 ergs (Kulkarni et al. 1999; Andersen et al. 2000).

The quantity of energy imparted to the relativistic ejecta, Erel, and the quality parameterized by the bulk Lorentz factor, Γ, are the two fundamental properties of GRB explosions. In particular, extremely high energies push the boundaries of current progenitor and engine models, while low energies could point to a population of sources that is intermediate between GRBs and core-collapse supernovae.

The true energy release depends sensitively on the geometry of the ejecta. If GRB explosions are conical (as opposed to spherical), then the true energy release is significantly below that inferred by assuming isotropy. Starting with GRB 970508 (Waxman, Kulkarni, & Frail 1998; Rhoads 1999) there has been growing observational evidence for collimated outflows, coming mainly from achromatic breaks in the afterglow light curves.

In the conventional interpretation, the epoch at which the afterglow light curves steepen (“break”) corresponds to the time at which Γ decreases below θζ−1, the inverse opening angle of the collimated outflow or “jet” (Rhoads 1999). The break happens for two reasons: an edge effect, and lateral spreading of the jet, which results in a significant increase of the swept-up mass. Many afterglows have tj ∼ 1 to a few days, which are best measured from optical/near-IR light curves (e.g., Harrison et al. 1999; Kulkarni et al. 1999; Stanek et al. 1999), while wider opening angles are easily measured from radio light curves (e.g., Waxman et al. 1998; Berger et al. 2001).

Recently, Frail et al. (2001) inferred θζ for 15 GRB afterglows from measurements of tj and found the surprising result that Eiso is strongly correlated with the beaming factor, fj−1; here, fj = [1 − cos(θζ)] is the beaming fraction and Eiso is the γ-ray energy release inferred by assuming isotropy. In effect, the true γ-ray energy release, Eγ = fjEiso, is approximately the same for all the GRBs in their sample, with a value of about 5 × 1050 ergs (assuming a constant circumburst density, n0 = 0.1 cm−3). In the same vein, broadband modeling of several GRB afterglows indicates that the typical blast wave kinetic energy in the adiabatic afterglow phase is Eb ∼ 5 × 1050 ergs, with a spread of about 1.5 orders of magnitude (Panaitescu & Kumar 2002). However, the general lack of high-quality afterglow data severely limits the application of the broadband modeling method.

Separately, Kumar (2000) and Freedman & Waxman (2001) noted that the afterglow flux at frequencies above the synchrotron cooling frequency, νc, is proportional to eνc dEνc/dΩ, where eνc is the fraction of the shock energy carried by electrons and dEνc/dΩ is the energy of the blast wave per unit solid angle. The principal attraction is that the flux above νc does not depend on the circumburst density and depends only weakly on the fraction of shock energy in magnetic fields, EB. For reasonable conditions (which have been verified by broadband afterglow modeling, e.g., Panaitescu & Kumar 2002), the X-ray band (2–10 keV) lies above νc starting a few hours after the burst. Thus, this technique offers a significant observational advantage; namely, the X-ray luminosity can be used as a surrogate for the isotropic-equivalent afterglow kinetic energy.

Piran et al. (2001) find that the X-ray flux, estimated at a common epoch (t = 11 hr), exhibits a narrow distribution of log(FX), (FX = 0.43 ± 0.12, here, σ2(X) is the variance of log(x). Taken at face value, the narrow distribution of FX.
implies a narrow distribution of $c dE_b/d\Omega$. This result, if true, is quite surprising since if the result of Frail et al. (2001) is accepted, then $dE_b/d\Omega$ should show a wide dispersion comparable to that of $f_b^{-1}$.

Still, Piran et al. (2001) extend their statistical analysis with the following argument. The relation between $dE_b/d\Omega$ and $E_b$ can be restated as $\log(dE_b/d\Omega) = \log(E_b) + \log(f_b^{-1})$. Thus, $\sigma^2(E_b/d\Omega) = \sigma^2(E_b) + \sigma^2(f_b^{-1})$. Since $dE_b/d\Omega \propto L_{X,iso}$ (for a constant $c$), they express, $\sigma^2(E_b) = \sigma^2(L_{X,iso}) - \sigma^2(f_b^{-1})$. Given the diversity in $\theta$ (Frail et al. 2001) and the apparent narrowness in $F_X$ (above), it would then follow that $E_b$ should be very tightly clustered.

However, the approach of Piran et al. (2001) makes a key assumption, namely, that $E_b$ and $f_b^{-1}$ are uncorrelated. This is certainly true when $E_b$ is constant, but the assumption then presupposes the answer! In reality, a correlation between $E_b$ and $f_b$ can either increase or decrease $\sigma^2(E_b)$, and this must be addressed directly. In addition, as appears to be the case (see § 2), $\sigma^2(f_b^{-1})$ is dominated by bursts with the smallest opening angles, which results in a distinctly different value than the one used by Piran et al. (2001) based only on the observed values of $\theta$.

In this paper, we avoid these concerns by taking a direct approach: we measure the variance in $E_b \propto f_b L_{X,iso}$ rather than bounding it through a statistical relation. We show, with a larger sample, that $L_{X,iso}$ is not as narrowly distributed as claimed by Piran et al. (2001) and in fact shows a dispersion comparable to that of $F_X$. This is a result that is not the claimed clustering of $L_X$ and provides a physical basis for the strong clustering of $L_X$ and hence the blast wave kinetic energy, $E_b$.

2. X-RAY DATA

In Table 1 we provide a comprehensive list of X-ray observations for 41 GRB afterglows, as well as temporal decay indices, $\alpha_X$ ($F_\nu \propto t^{-\alpha_X}$), when available. In addition, for a subset of the afterglows for which jet breaks have been measured from the radio, optical, and/or X-ray emission, we also include the inferred $\theta$ (Frail et al. 2001; Panaitescu & Kumar 2002). We calculate $\theta$ from $t_j$ using the circum-burst densities inferred from broadband modeling, when available, or a fiducial value of 10 cm$^{-2}$, as indicated by the best-studied afterglows (e.g., Yost et al. 2002). This normalization for $n_0$ is different from that of Frail et al. (2001), who used $n_0 = 0.1$ cm$^{-3}$.

For all but one burst we interpolate the measured $F_X$ to a fiducial epoch of 10 hr (hereafter $F_{X,10}$), using the measured $\alpha_X$ when available, and the median of the distribution, $\langle \alpha_X \rangle = -1.33 \pm 0.38$, when a measurement is not available. The single exception is GRB 020405 for which the first measurement was obtained $t \approx 41$ hr, while the inferred jet break time is about 23 hr (Berger et al. 2003). In this case, we extrapolate to $t = 10$ hr using $\alpha_X = -1.69$ for $t > 23$ hr and $\alpha_X = -0.78$ for $t < 23$ hr. We list the values of $F_{X,10}$ in Table 2.

In Figure 1 we plot the resulting distribution of $F_{X,10}$. For comparison we also show the distribution of $\gamma$-ray fluences from the sample presented by Bloom, Frail, & Sari (2001) and updated from the literature. Clearly, while the distribution of X-ray fluxes is narrower than that of the $\gamma$-ray fluences, $\sigma(F_{\gamma}) = 0.79 \pm 0.10$, it still spans $\sim 2.5$ orders of magnitude, i.e., $\sigma(F_{X,10}) = 0.57 \pm 0.07$. The value of $\sigma(F_{X,10})$ and all variances quoted below are calculated by summing the Gaussian distribution for each measurement and then fitting the combined distribution with a Gaussian profile.

We translate the observed X-ray fluxes to isotropic luminosities using

$$L_{X,iso}(t = 10 \text{ hr}) = 4\pi d_L^2 F_{X,10}(1+z)^{\alpha_X-\beta_X-1}. \quad (1)$$

We use $\beta_X \approx -1.05$, the weighted mean value for $X$-ray afterglows (De Pasquale et al. 2002), and the median redshift, $\langle z \rangle = 1.1$, for bursts that do not have a measured redshift. The resulting distribution of $L_{X,iso}$, $\sigma(L_{X,iso}) = 0.68 \pm 0.17$, is wider than that of $F_X$ due to the dispersion in redshifts. We note that this is wider than the value quoted by Piran et al. (2001) of $\sigma(L_{X,iso}) \approx 0.43$ based on a smaller sample and ignoring the dispersion in redshift. Using the same method we find $\sigma(L_{E,iso}) = 0.92 \pm 0.15$.

In the absence of a strong correlation between $f_b$ and $L_{X,iso}$, the above results indicate that the distribution of the true X-ray luminosities, $L_X \equiv f_b^{-1} L_{X,iso}$, should have a wider dispersion than either $L_{X,iso}$ or $f_b$, for which we find $\sigma(f_b) = 0.52 \pm 0.13$ (Frail et al. 2001). Instead, when we apply the individual beaming corrections for those bursts that have a measured $\theta$ and redshift$^3$ (see Table 2), we find a significantly narrower distribution, $\sigma(L_X) = 0.32 \pm 0.10$.

$^3$ These do not include GRB 990705, which is poorly characterized; see § 3.
| GRB    | z   | Epoch (hr) | Flux ($10^{-11}$ ergs cm$^{-2}$ s$^{-1}$) | $\alpha_X$ | $\theta_{jet}$ | Reference |
|--------|-----|------------|----------------------------------------|------------|---------------|-----------|
| 970111 | ... | 24.0       | 1.05 ± 0.46                            | −0.4 ± 3.2a| ...           | 1, 2      |
|        | ... | 30.7       | 0.95 ± 0.34                            | ...        | ...           | 2         |
| 970228 | 0.695 | 8.5        | 33.8 ± 3.3                            | −1.27 ± 0.14| ...           | 2, 3      |
|        | ... | 12.7       | 28 ± 4                                | ...        | ...           | 2         |
|        | ... | 92.4       | 1.5 ± 0.4                             | ...        | ...           | 2         |
| 970402 | ... | 9.9        | 2.9 ± 0.4                             | −1.35 ± 0.55| ...           | 2         |
|        | ... | 16.8       | 1.5 ± 0.4                             | ...        | ...           | 2         |
| 970508 | 0.835 | 13.1       | 7.13                                  | −1.1 ± 0.1 | 0.391         | 4, 5      |
|        | ... | 72.3       | 4.3 ± 0.5                             | ...        | ...           | 2         |
|        | ... | 104        | 2.3 ± 0.7                             | ...        | ...           | 2         |
| 970815 | ... | 89.6       | <1                                    | ...        | ...           | 6         |
| 970828 | 0.958 | 4.0        | 118                                   | −1.42      | 0.128         | 5, 7      |
|        | ... | 42.6       | 4.1                                   | ...        | ...           | 7         |
| 971214 | ... | 3.418      | 9.0 ± 0.9                             | −1 ± 0.2   | >0.100        | 2, 5      |
|        | ... | 28.9       | 2.1 ± 0.4                             | ...        | ...           | 2         |
| 971227 | ... | 16.5       | 2.5 ± 0.7                             | −1.12 ± 0.06| ...           | 8         |
| 980326 | ~1h | 8.5        | <16                                   | 110 ± 0.10 | <0.110        | 9         |
| 980329 | ... | 8.4        | $14 \pm 2.1$                         | −1.55 ± 0.3| 0.081         | 10, 11    |
|        | ... | 11.8       | $6.2 \pm 1.2$                        | ...        | ...           | 10        |
|        | ... | 16.4       | $3.4 \pm 1.0$                        | ...        | ...           | 10        |
|        | ... | 23.7       | $2.7 \pm 0.7$                        | ...        | ...           | 10        |
|        | ... | 43.6       | $1.1 \pm 0.4$                        | ...        | ...           | 10        |
| 980515 | ... | 11         | $2.0^{+0.3}_{-0.9}$                    | ...        | ...           | 12        |
| 980519 | <2h | 10.9       | $5.3 \pm 1.0$                        | −1.7 ± 0.7 | 0.040         | 13, 14    |
|        | ... | 15.3       | $2.0 \pm 0.4$                        | ...        | ...           | 13        |
|        | ... | 21.5       | $1.6 \pm 0.5$                        | ...        | ...           | 13        |
|        | ... | 27.2       | $0.8 \pm 0.4$                        | ...        | ...           | 13        |
| 980613 | ... | 1.096      | 9.9                                   | −0.92 ± 0.62| >0.226        | 2         |
|        | ... | 22.4       | $4.0 \pm 0.8$                        | ...        | ...           | 2         |
| 980703 | 0.966 | 34.0       | $4.0 \pm 1$                         | −1.24 ± 0.18| 0.200         | 2, 15     |
| 981226 | ... | 14.0       | $4.0$                                 | −1.3 ± 0.4 | ...           | 16        |
| 990123 | 1.600 | 6.4        | $124 \pm 1.1$                       | −1.41 ± 0.05| 0.089         | 2, 5      |
|        | ... | 23.4       | $19.1 \pm 2.2$                      | ...        | ...           | 2         |
| 990217 | ... | 11         | <1.1                                  | ...        | ...           | 12        |
| 990510 | 1.619 | 8.7        | $47.8 \pm 3.1$                      | −1.41 ± 0.18| 0.054         | 5, 14, 17 |
|        | ... | 10.1       | $40.5 \pm 2.6$                      | ...        | ...           | 17        |
|        | ... | 11.7       | $32.8 \pm 3.7$                      | ...        | ...           | 17        |
|        | ... | 13.4       | $22.8 \pm 2.8$                      | ...        | ...           | 17        |
|        | ... | 15.3       | $24.1 \pm 2.7$                      | ...        | ...           | 17        |
|        | ... | 17.1       | $18.5 \pm 3.1$                      | ...        | ...           | 17        |
|        | ... | 19.1       | $20.9 \pm 2.3$                      | ...        | ...           | 17        |
|        | ... | 24.0       | $12.1 \pm 1.4$                      | ...        | ...           | 17        |
|        | ... | 26.3       | $9.9 \pm 1.1$                       | ...        | ...           | 17        |
|        | ... | 29.4       | $7.8 \pm 1.1$                       | ...        | ...           | 17        |
| 990627 | ... | 11.9       | 3.5                                   | ...        | ...           | 18        |
| 990704 | ... | 10.1       | $10.1 \pm 2.9$                      | −1.3 ± 0.3 | ...           | 19        |
|        | ... | 13.4       | $8.9 \pm 2.2$                       | ...        | ...           | 19        |
|        | ... | 23.3       | $3.1 \pm 2.0$                       | ...        | ...           | 19        |
|        | ... | 26.8       | $2.9 \pm 1.6$                       | ...        | ...           | 19        |
| 990705 | 0.840 | 14.5       | $1.9 \pm 0.6$                      | ...        | 0.096         | 5, 20     |
| 990806 | ... | 13.6       | $5.5 \pm 1.5$                      | −1.4 ± 0.7 | ...           | 21        |
|        | ... | 34.3       | $1.5 \pm 0.6$                       | ...        | ...           | 21        |
| 990907 | ... | 11         | $10.2 \pm 5.6$                      | ...        | ...           | 12        |
| 991014 | ... | 11         | $4.0^{+1.1}_{-1.2}$                   | ...        | ...           | 12        |
| 991216 | 1.020 | 4.0        | $1240 \pm 40$                     | −1.61 ± 0.07| 0.051         | 5, 14, 22 |
|        | ... | 10.9       | $250 \pm 10$                       | ...        | ...           | 22        |
| 000115 | ... | 2.9        | 270                                  | <1         | ...           | 23        |
| 000210 | 0.846 | 11         | $4.0 \pm 1.0$                      | −1.38 ± 0.03| ...           | 24        |
| 000214 | ... | 14.9       | 5                                    | −1.8       | ...           | 25        |
|        | ... | 22.1       | 2.5                                  | ...        | ...           | 25        |
| 000528 | ... | 11         | $2.3 \pm 1.0$                      | ...        | ...           | 12        |
| 000529 | ... | 9.0        | 2.8                                  | ...        | ...           | 26        |
3. BEAMING CORRECTIONS AND KINETIC ENERGIES

The reduced variance of $L_X$ compared with that of $L_{X,iso}$ requires a strong correlation between $L_{X,iso}$ and $f_b$, such that bursts with a brighter isotropic X-ray luminosity are also more strongly collimated. Indeed, as can be seen from Figure 2, the data exhibit such a correlation. Ignoring the two bursts, which are obvious outliers (980326 and 990705), as well as GRBs 980329 and 980519, which do not have a measured redshift, we find $L_{X,iso} \propto f_b^{-0.80}$. The linear correlation coefficient between $\log(L_{X,iso})$ and $\log(f_b^{-1})$ indicates a probability of only $4.6 \times 10^{-4}$ that the two quantities are not correlated. For $\log(E_{iso})$ and $\log(f_b^{-1})$ we find a similar probability of $4.2 \times 10^{-4}$ that the two quantities are not correlated.

Thus, as with the $\gamma$-ray emission, the observed afterglow emission also exhibits strong luminosity diversity due to strong variations in $f_b$. Therefore, the mystery of GRBs is no longer the energy release but understanding what aspect of the central engine drives the wide diversity of $f_b$.

We note that there are four possible outliers in the correlation between $L_{X,iso}$ and $f_b^{-1}$. The afterglows of GRBs 980326 and 980519 exhibit rapid fading (Groot et al. 1998; Vrba et al. 2000), which has been interpreted as the signature of an early jet break. However, it is possible that the rapid fading is instead due to a $\rho \propto r^{-2}$ density profile, and in fact, for GRB 980519 such a model indicates $\theta_j \approx 0.12$, 3 times wider than in the constant density model. This is sufficient to bring GRB 980519 into agreement with the observed correlation. The redshift of GRB 980329 is not known, but with $z = 2$ it easily agrees with the correlation. Finally, the X-ray flux and jet opening angle for GRB 990705 are poorly characterized because of contamination from a nearby source (De Pasquale et al. 2002) and a poor optical light curve (Masetti et al. 2000).

4. DISCUSSION AND CONCLUSIONS

We have presented a comprehensive compilation of early X-ray observations of 41 GRBs, from which we infer $F_{X,10}$, the flux in the 2–10 keV band at 10 hr. As first pointed by Kumar (2000) and Freedman & Waxman (2001), the afterglow luminosity above the cooling frequency is $L_{X,iso} \propto \varepsilon_e E_{iso}$, where $E_{iso}$ is the isotropic-equivalent explosion kinetic energy. More importantly, the flux is independent of the ambient density and weakly dependent on $\varepsilon_B$. For all well-modeled afterglows, the cooling frequency at 10 hr is below the X-ray band. Thus, $F_{X,10}$ can be utilized to yield information about the kinetic energy of GRBs.

Earlier work (Piran et al. 2001) focused on statistical studies of $F_{X,10}$ and found the very surprising result that it is narrowly clustered. By assuming that the true kinetic

| GRB       | $z$ | Epoch (hr) | Flux ($10^{-11}$ ergs cm$^{-2}$ s$^{-1}$) | $\alpha_X$ | $\theta_j$ | Reference |
|-----------|-----|------------|----------------------------------------|----------|---------|-----------|
| 000926.... | 2.037 | 54.9       | 2.23 ± 0.77                           | −3.7 ± 1.5 | 0.140   | 14, 27    |
| ....       | 66.5  | 0.94 ± 0.14 | ...                                   | ...      | ...     | 27        |
| 001025.... | 50.4  | 0.53 ± 0.10 | ...                                   | ...      | ...     | 28        |
| 001109.... | 19.3  | 7.1 ± 0.5   | ...                                   | ...      | ...     | 29        |
| 010214.... | 7.7   | <0.5        | ...                                   | ...      | ...     | 30        |
| 010220.... | 24.1  | 0.33        | ...                                   | ...      | ...     | 28        |
| 010222.... | 1.477 | 8.9         | 101 ± 11                              | −1.33 ± 0.04 | 0.080 | 14, 31    |
| ....       | 32.7  | 18.7 ± 1.8  | ...                                   | ...      | ...     | 31        |
| 011211.... | 2.14  | 11.0        | 1.9                                   | −1.7 ± 0.2 | 0.32    | ...       |
| 020322.... | 18.8  | 3.5 ± 0.2   | ...                                   | −1.26 ± 0.23 | ... | 33        |
| 020405.... | 0.698 | 41.0        | 13.6 ± 2.5                           | −1.15 ± 0.95 | 0.285 | 34, 35, 36 |
| 020813.... | 1.254 | 31.9        | 22                                    | −1.42 ± 0.05 | 0.066 | 37, 38    |
| 021004.... | 2.323 | 31.4        | 4.3 ± 0.7                             | −1.0 ± 0.2  | 0.240  | 39, 40    |

**Note.**—Col. (1): GRB name. Col. (2): Redshift. Col. (3): Midpoint epoch of X-ray observation. Col. (4): X-ray flux. Col. (5): Temporal decay index ($F_X \propto r^n$). Col. (6): Jet opening angle. Col. (7): References for the X-ray flux and jet opening angle.

a Due to the large uncertainty in the value of $\alpha_X$ we use the median value for the sample, $\langle \alpha_X \rangle = −1.33 ± 0.38$.
b The redshift is based on matching the optical light curve of SN 1998bw to the red excess reported by Bloom et al. 1999.

d The inferred jet break is at $t = 0.95$, prior to the X-ray observation—we use the model fit to extrapolate the flux to $t = 10$ hr (see E. Berger et al. 2003, in preparation).

References.—(1) Feroci et al. 1998; (2) Piro 2001; (3) Frontera et al. 1998; (4) Piro et al. 1998; (5) Frail et al. 2001; (6) Murakami et al. 1997; (7) Smith et al. 2002; (8) Antonelli et al. 1999; (9) Marshall & Takashima 1998; (10) in’t Zand et al. 1998; (11) Yost et al. 2002; (12) De Pasquale et al. 2002; (13) Nicastro et al. 1999b; (14) Panaitescu & Kumar 2002; (15) Vreeswijk et al. 1998; (16) Frontera et al. 2000; (17) Pian et al. 2001; (18) Nicastro et al. 1999a; (19) Feroci et al. 2001; (20) Amati et al. 2000a; (21) Frontera et al. 1999; (22) Takashima et al. 1999; (23) Marshall et al. 2000; (24) Piro et al. 2002; (25) Antonelli et al. 2000; (26) Feroci et al. 2000; (27) Harrison et al. 2001; (28) Watson et al. 2002; (29) Amati et al. 2000b; (30) Frontera et al. 2001; (31) in’t Zand et al. 2001; (32) Reeves et al. 2002; (33) Watson et al. 2002; (34) Price & et al. 2002; (35) Mirabal, Paerels, & Halpern 2003; (36) E. Berger et al. 2003, in preparation; (37) Price et al. 2003; (38) Vanderspek et al. 2002; (39) D. W. Fox et al. 2003, in preparation; (40) D. A. Frail et al. 2003, in preparation.
energy, $E_b = E_{b, \text{iso}} f_b \propto L_x = L_{X, \text{iso}} f_b$, and $f_b$ (the beaming factor) are uncorrelated, the authors deduced that $L_x$ and thus $E_b$ are even more strongly clustered. However, this approach is weakened by assuming (in effect) the answer. Furthermore, the approach of Piran et al. (2001), which relies on subtracting variances, is very sensitive to measurement errors. To illustrate this point, we note $\sigma^2(L_{X, \text{iso}}) = 0.68^{+0.17}_{-0.07}$ for the entire sample presented here, whereas $\sigma^2(f_b) = 0.52^{+0.13}_{-0.12}$. Thus, $\sigma^2(L_x) = 0.16^{+0.30}_{-0.21}$ may be negative using the statistical approach.

In contrast to the statistical approach, we take the direct approach and estimate the true kinetic energy, $E_b \propto L_{X, \text{iso}} f_b$, by using the measured $L_{X, \text{iso}}$ and inferred $f_b$. The advantage of our approach is that we do not make assumptions of correlations (or lack thereof), and more importantly, we do not subtract variances. We directly compute the variance of the desired physical quantity, namely $L_x$, and find that it is strongly clustered.

Even more importantly, with our direct approach we have uncovered the physical reason for the wide dispersion in $L_{X, \text{iso}}$ and the clustering of $L_x$, namely, the dispersion in jet opening angles.

The true X-ray luminosity is related to the physical quantities as follows (Freedman & Waxman 2001):

$$\epsilon c E_b \propto A L_x Y^\epsilon,$$  

where

$$Y \equiv B c^3 \epsilon^{-1} E_b^{-1} L_{X, \text{iso}}^{-1}.$$

Here $\epsilon = (p - 2)/(p - 1)$, as well as $A$ and $B$, depend to some extent on the details of the electron distribution.
First, the afterglow X-ray emission on timescales of 10 hr must be primarily dominated by synchrotron emission (which is the basis of eq. [2]). The contribution from inverse Compton (IC) emission, which depends strongly on $n_0$ and $\epsilon_B$ (Sari & Esin 2001), is apparently not significant. A possible exception is GRB 000926 (Harrison et al. 2001), but even there the IC contribution is similar to that from synchrotron emission.

Second, the energy radiated by the afterglow from the time of the explosion to $t = 10$ hr cannot be significant. This constrains the radiative losses at an early time to at most a factor of a few.

Third, $p$ must be relatively constant (as one may expect in any case from insisting that the microphysics should not be different for different bursts). For example, changing $p$ from a value of 1.5 to 3 results in $Y^*$ ranging from 0.003 to 117, a factor of 39,000! Even small changes in $p$, e.g., from $p = 1.75$ to 2.25, result in a factor of 8 change in $Y^*$. In contrast, some afterglow models yield values of $p$ significantly below 2 (e.g., Panaitescu & Kumar 2002), while others have $p$ approaching 3 (Chevalier & Li 2000). Our results, on the other hand, indicate that one should set $p \approx 2$ and attribute apparent deviant values of $p$ to the external environment or energy injection from the central source.

Finally, since both the prompt and afterglow emission exhibit a strong correlation with $f_b$, which is determined from late-time observations (hours to weeks after the burst), the resulting constancy of both $E_b$ and $E_0$ indicates that GRB jets must be relatively homogeneous and maintain a simple conical geometry all the way from internal shocks ($\sim 10^{13}-10^{14}$ cm) to the epoch of jet break ($\sim 10^{17}$ cm). This rules out the idea that brighter bursts are due to bright spots along specific lines of sight (Kumar & Piran 2000). At the same time, the possible deviation from a linear relation between $\log(L_{X,iso})$ and $\log(f_b^{-1})$ may hold a clue to the structure of the jet.

With the result that GRB afterglows have a standard kinetic energy firmly established, the next step is to closely investigate bursts that deviate from this relation; such sources may be a clue to subclasses of GRBs (e.g., Bloom, Frail, & Kulkarni 2003). Fortunately, while the statistical study of afterglow energetics used previously misses this point completely, the direct method employed in this paper can easily uncover these sources. More importantly, this method provides a framework for understanding the underlying physical processes that may give rise to such a diversity.

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Fig. 2.—Top: Isotopic-equivalent X-ray luminosity. Bottom: isotopic-equivalent $\gamma$-ray energy as a function of the beaming factor, $|1 - \cos(\theta_f)|^{-1}$. There is a strong positive correlation between $L_{X,iso}$ and $f_b^{-1}$, as well as between $E_{\gamma,iso}$ and $f_b^{-1}$, resulting in an approximately constant true X-ray luminosity and $\gamma$-ray energy release. In fact, while the distributions of all three parameters span about 3 orders of magnitude, the distributions of the beaming-corrected parameters span about 1 order of magnitude.

(power-law vs. relativistic Maxwellian; the value of the power-law index, $p$).

There is no reason to expect that $L_X$ should be clustered. However, one can argue that the microphysics should be the same for each GRB afterglow, in particular $\epsilon_e$ and $p$. The best-studied afterglows appear to favor $p = 2.2$ (e.g., Frail, Waxman, & Kulkarni 2000; Galama et al. 1998), a value also favored by our current theoretical knowledge of shock acceleration (see Ostrowski & Bednarz 2002 and references therein). In addition, as already indicated by the $\gamma$-ray observations, there is evidence supporting strong clustering of explosion energies in GRBs (Frail et al. 2001). Given these reasonable assumptions, a strong clustering of $L_X$ makes sense if the physical quantities that are responsible for $L_X$ are clustered. As can be seen from equation (2), this would require that $L_X$ be linearly related to $E_0$. Such a relation is possible if four conditions are met.
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