Transmission of optical vortices through Bragg optical multihelicoidal fibers of heterogeneous type

B Lapin\textsuperscript{1}, M Yavorsky\textsuperscript{1}, E Barshak\textsuperscript{1}, D Vikulin\textsuperscript{1} and C Alexeyev\textsuperscript{1}

\textsuperscript{1}V.I. Vernadsky Crimean Federal University, Vernadsky ave., 4, Simferopol, Russia, 295007

e-mail: lapinboris@gmail.com

Abstract. In this paper we have theoretically shown that two-part and three-part multihelicoidal fibers of heterogeneous type in the presence of twist defects are able to invert the topological charge of incoming optical vortices. We have shown that three-part multihelicoidal fibers of that type can be used as compact comb filters for optical vortices. Also we have studied the emergence of topologically charged fields localized near defects in such fibers. We have established that strongly localized fields can emerge only in three-part multihelicoidal fibers.

1. Introduction

There are many living creatures in the nature with colorful external covers [1]. These covers consist of nanostructured elements that strongly reflect light in a certain spectral region and render color to the integuments. Such phenomenon of strong selective reflection have attracted worldwide attention and inspired producing and studying artificial micro- and nanostructures with similar features.

The basic feature of such artificial structures is the presence of a photonic bandgap. If an electromagnetic wave with a “forbidden” wavelength falls on such an artificial structure, it cannot propagate in it and rapidly attenuates. Due to such peculiarity of interaction with electromagnetic waves these structures are called the photonic crystals (PCs) [2].

PCs can possess not only of the bandgap but also of other features, which depend on their internal structure. For example, cholesteric liquid crystals (CLCs) can transmit light with some specific wavelength and type of circular polarization. Also, the presence of various defects in the structure of PCs generally changes their spectral characteristics [3-5]. As has been shown, defected multilayered systems consisting of CLCs demonstrate the presence of a narrow passband within the bandgap [6, 7].

Unusual features of PCs make them very useful means for creation of a number of optical devices for the rapidly developing field of informational technologies. PC-based optical filters, light flow manipulators, sensors and superprisms have currently been suggested [8-10]. Unfortunately, almost all possible suggested applications of PCs are not designed for light with screw dislocations or optical vortices (OVs) [11]. Regrettfully, only few papers are concerned with the study of PCs capable of OV control and manipulation. As has been shown, two-dimensional disordered PCs can localize OVs with a topological charge (TC) ±1 [21]. In the case of one-dimensional PCs, namely, multihelicoidal fibers (MF), it was discovered that such
crystals are able to manifest a topological activity [22, 23], that is to raise or reduce the TC of the incoming field by \( l \) units, where \( l \) is the symmetry order of the PC-fiber. The presence of defects in MFs can expand the list of their features. For instance, adding a single twist defect into the structure of MF or combining such defect with the isotropic layer defect results in arising of localized fields near defects and emerging of new spectral properties [24-26].

In this paper we continue studying defected MFs and consider the systems of a heterogeneous type, which consist of concatenated parts with opposite handedness. The aim of this paper is to establish new properties of such heterogeneous MFs, which can be useful for TC control. Also we study the possibility of emergence of defect-localized fields with TC.

2. The models of heterogeneous multihelicoidal fibers
Let us consider two simple models of heterogeneous MFs (Figure 1). For the first fiber (Figure 1a) the refractive index in cylindrical coordinates \((r, \varphi, z)\) is described by the expression:

\[
n^2(r, \varphi) = \begin{cases}n^2_{\text{cr}}(1 - 2 \Delta f(r)) - 2 n^2_{\text{cr}} \Delta \delta f' \cos l(\varphi - q z), & -d_1 \leq z \leq 0 \vspace{1em} \\
n^2_{\text{cr}}(1 - 2 \Delta f(r)) - 2 n^2_{\text{cr}} \Delta \delta f' \cos l(\varphi + q z - \theta), & 0 < z < d_1, \end{cases}
\]

where \( n_{\text{cr}} \) is the core’s refractive index, \( \Delta \delta \) is the optical contrast between the core and the cladding, \( \delta \varphi \) defines the degree of the cross section deformation, \( f' = df/2\varphi \), \( f \) is the profile function, \( q = 2\pi / H \), \( d_i \) is the length of the section of the fiber, \( \theta \) defines the rotation angle of the second part of the fiber with respect to the first one and is called the twist defect angle. The parameter \( l \) defines the symmetry order. It should be noted that in the planes \( z = \pm 0 \) fiber’s interfaces do not coincide and there exists a twist defect.

\[\text{Figure 1. Models of heterogeneous multihelicoidal fibers consisting of sections with different handedness; adjacent sections have opposite handedness. In a two-part fiber (a) the blue part is rotated through an angle } \theta \text{ with respect to its red one. In a three-part fiber (b) its middle red and rightmost blue parts are rotated through angles } \tau \text{ and } \theta \text{, correspondingly, with respect to the leftmost part. Here the order of rotational symmetry } l = 5.\]

In the second model the MF consists of three uniformly twisted parts (Figure 1b). The adjacent parts have differing handedness and the second and the third ones are rotated through angles \( \tau \) and \( \theta \), correspondingly, with respect to the first fiber’s section. The refractive index of such system can be described as

\[
n^2(r, \varphi) = \begin{cases}n^2_{\text{cr}}(1 - 2 \Delta f(r)) - 2 n^2_{\text{cr}} \Delta \delta f' \cos l(\varphi - q z), & -d_1 \leq z \leq 0 \vspace{1em} \\
n^2_{\text{cr}}(1 - 2 \Delta f(r)) - 2 n^2_{\text{cr}} \Delta \delta f' \cos l(\varphi + q z - \theta), & 0 < z < d_1 \vspace{1em} \\
n^2_{\text{cr}}(1 - 2 \Delta f(r)) - 2 n^2_{\text{cr}} \Delta \delta f' \cos l(\varphi + q z - \theta - \tau), & d_1 \leq z \leq d_1 + d_2, \end{cases}
\]

where \( d_0 \) is the length of the second part of the fiber and \( d_1 \) defines lengths of the external fiber’s parts. The symmetry order \( l \) for two- and three-part MFs is the same. Also in the following we suppose
that the profile function \( f \) given by the Heaviside function \( \Xi(\frac{r}{r_0} - 1) \), where \( r_0 \) is the radius of the fiber’s core.

The chosen approximation \( (\Delta \ll 1) \) allows one to study the transmission-reflection problem for the heterogeneous MF in the paraxial approximation. Also we will treat the problem in the scalar case. To this end, one should, first, to write the wave equation \([27]\) in these approximations for the homogeneous uniformly twisted MF:

\[
\left[ \nabla^2 + k^2 \frac{n^2}{\rho^2} - 2k^2 n_{e0}^2 \Delta \delta f_j \cos l(\varphi + \kappa q) \right] E_j = 0 ,
\]

where \( \nabla^2 \) is the Laplace operator, \( \rho^2 = n_{e0}^2 (1 - 2 \Delta f(\rho)) \), \( k = 2\pi / \lambda \), \( \lambda \) is the wavelength, \( \kappa = \pm 1 \) and \( E_j \) is the transverse component of the electric field \( E \). It is clear that the operator in (3) has no translational invariance with respect to the longitudinal coordinate \( z \). To restore this invariance one should make the substitution \( \tilde{r} = r, \tilde{\varphi} = \varphi + \kappa q, \tilde{z} = z \). Then the standard ansatz \( E_j = e_j \exp(i\beta \tilde{z}) \) allows one to write (3) in the following form:

\[
\left( \frac{\partial^2}{\partial \tilde{r}^2} + \frac{\partial}{\partial \tilde{r}} + \frac{\partial^2}{\partial \tilde{\varphi}^2} + \left( i\beta + \kappa \rho \frac{\partial}{\partial \tilde{\varphi}} \right)^2 + k^2 \frac{n^2}{\rho^2} - 2k^2 n_{e0}^2 \Delta \delta f_j \cos l(\varphi + \kappa q) \right) e_j = 0 .
\]

where \( \beta \) is the propagation constant. Equation (4) can be solved with the help of perturbation theory with degeneracy if the term proportional to \( \cos l(\varphi + \kappa q) \) is taken as a small perturbation \([28, 29]\). Almost at all values of \( q \) the modes of homogeneous MFs are presented by OVs, which in the basis of linear polarizations have the form:

\[
|\sigma, m\rangle = \left( \frac{1}{i\sigma} \right) e^{im\tilde{\varphi}} F_{m}|\tilde{r}\rangle ,
\]

where \( \sigma = \pm 1 \) defines the type of circular polarization, \( m \) is the topological charge (TC) of the OV, \( F_{m}|\tilde{r}\rangle \) is the radial function \([27]\). The OV with zero TC coincides with the fundamental mode (FM) \(|1, 0\rangle\). If the absolute value of the reciprocal lattice vector \( q \) satisfies the condition

\[
q = q, (\lambda_0) = (\tilde{\beta}_1 (\lambda_0) + \tilde{\beta}_0 (\lambda_0)) / l ,
\]

where \( \lambda_0 \) is the resonance wavelength, which is determined by the fiber parameters, and \( \tilde{\beta}_{0,1} \) is the scalar propagation constant \([27]\), then the modes of the fiber become the sum of contra-propagating fields, whose TDCs differ by \( l \) units. In the following we will study the case of coupling of the FM \(|1, 0\rangle\) and OVs \(|1, \pm 1\rangle\). Then for the leftward twisted MFs the modes are:

\[
|\psi_1\rangle = (c_1 |1, 0\rangle + |1, l\rangle e^{-i\lambda_0} e^{-i\beta \tilde{z}} ,
|\psi_2\rangle = (-c_2 |1, 0\rangle + |1, l\rangle e^{i\lambda_0} e^{i\beta \tilde{z}} ,
|\psi_3\rangle = (c_1 |1, 0\rangle + |1, -l\rangle e^{i\lambda_0} e^{-i\beta \tilde{z}} ,
|\psi_4\rangle = (-c_2 |1, 0\rangle + |1, -l\rangle e^{-i\lambda_0} e^{i\beta \tilde{z}} ,
\]

where \( c_{1,2} = (\mp i\epsilon + R) / Q \), \( \epsilon = q (\lambda - q (\lambda_0)) \ll q (\lambda_0) \), \( R = (l^2 e^2 - Q^2)^{1/2} \), \( Q \) is the coupling constant, \( \beta_3 = \tilde{\beta}_0 + i\epsilon / 2 \pm R \). For the chosen profile function \( f \) the coupling constant \( Q = A^2 / \tilde{\beta}_1^2 \), where \( A = -k^2 n_{e0}^2 \Delta / N_0 N_f \) and \( N_f^2 = \int_0^\infty x F_0^2(x) dx .

For the rightward twisted MFs the modes are:

\[
|\xi_1\rangle = (c_2 |1, 0\rangle + |1, l\rangle e^{i\lambda_0} e^{-i\beta \tilde{z}} ,
|\xi_2\rangle = (-c_1 |1, 0\rangle + |1, l\rangle e^{-i\lambda_0} e^{i\beta \tilde{z}} ,
|\xi_3\rangle = (c_2 |1, 0\rangle + |1, -l\rangle e^{i\lambda_0} e^{i\beta \tilde{z}} ,
|\xi_4\rangle = (-c_1 |1, 0\rangle + |1, -l\rangle e^{-i\lambda_0} e^{-i\beta \tilde{z}} ,
\]
It should be noted, that the modes (7) and (8) contain the uncoupled fields, namely four OVs: two forward propagating and two backward propagating ones. Now we can study the features of heterogeneous defected MF.

3. Conversion of optical vortices in heterogeneous multihelicoidal fibers

3.1. Principles of obtaining transmission coefficients

Let the field $|1.s\rangle$ be incident at the input end of the fiber (Figure 1a), where $s = 0, \pm 1$. Then on the left of the fiber the field is:

$$
|z < -d_1\rangle = |1.s\rangle e^{i\beta l z} + (X_1 |1.0\rangle + X_2 |1.1\rangle + X_3 |1.-1\rangle) e^{-i\beta l z}.
$$

(9)

It should be noted that in the scalar approximation MF does not alter the polarization state of the field, therefore the reflected fields must be of the same circular polarization as the incoming field $|1.s\rangle$. In the first section of the fiber the incoming field is presented as the sum of modes (7):

$$
|z < -d_1 < z < 0\rangle = X_3 |\psi_1\rangle + X_5 |\psi_2\rangle + X_6 |\psi_3\rangle + X_7 |\psi_4\rangle + X_8 |\psi_5\rangle + X_9 |\psi_6\rangle.
$$

(10)

In the second section of the fiber the field should be decomposed in modes (8), where the substitution $|\sigma, m\rangle \rightarrow |\sigma, m\rangle e^{i(\sigma + m)\theta}$ must be done to take into account the Pancharatnam-Berry phase shift due to rotation of the second part of the fiber [30]. Then one has:

$$
|z > 0 < z < d_1\rangle = X_{10} |\xi_1\rangle + X_{11} |\xi_2\rangle + X_{12} |\xi_3\rangle + X_{13} |\xi_4\rangle + X_{14} |\xi_5\rangle + X_{15} |\xi_6\rangle.
$$

(11)

Here the primes indicate that the modes $|\xi\rangle$ are the modified modes $|\xi\rangle$ in the above mentioned way. On the right of the fiber we have:

$$
|z > d_1\rangle = (X_{16} |1.0\rangle + X_{17} |1.1\rangle + X_{18} |1.-1\rangle) e^{i\beta l (z-d_1)}.
$$

(12)

Matching the fields (9)-(12) and their derivatives at the boundaries $z = 0$ and $z = \pm d_1$ one can obtain the set in the unknown coefficients $X_j$, which is solved by standard methods of linear algebra. The coefficients $X_j$ carry information about transmitted and reflected fields. For instance, $|X_{17}|^2$ is the reflection coefficient for OV $|1.1\rangle$ and the transmission coefficient for this OV is $|X_{17}|^2$. In numerical simulations we assume that $q = 7.436 \times 10^{-6} \text{ m}^{-1}$, $n_{eq} = 1.5$, $\Delta = 5 \times 10^{-3}$, $\delta = 0.05$, $H = 8.45 \times 10^{-7} \text{ m}$, $r_0 = 10 \lambda_0$, $\lambda_0 = 632.8 \text{ n m}$. It should be noted that the same scheme is valid for the three-part MF. We also assume that all the fiber parts have the fourth order of axial symmetry: $l = 4$.

3.2. Comb filter for OV

Let the defect-free ($\theta, \varpi = 0$) three-part MF be excited with the OV $|1.-1\rangle$. If the wavelength of this OV belongs to the bandgap, then upon reflection from the first leftward twisted part of the fiber (Figure 1b) the OV $|1.-1\rangle$ becomes the FM $|1.0\rangle$. In the presence of the twist defect ($\theta \neq 0, \varpi = 0$) the OV $|1.-1\rangle$ is able to pass in a narrow band within the bandgap (Figure 2a,c). If the length $d_o$ of the middle section is sufficiently large, the system’s parts act independently and that the passband within the bandgap vanishes (Figure 2b,d).

Outside the bandgap the transmission of $|1.-1\rangle$ OV is very sensitive to the wavelength variation. As is seen from Figure 2c,d transmission and reflection curves behave as typical curves for a comb filter. For instance, at the small length $d_o = 0.17$ mm the number of peaks in the transmitted light for the OV $|1.-1\rangle$ is roughly 30 peaks per nm (Figure 2c). As $d_o$ increases up to 17 mm, then the number
of peaks grows (Figure 2d) and becomes near 200 peaks per nm. It should be noted that these curves are almost insensitive to the variation of twist defect angles $\tau$ and $\theta$.

Figure 2. Reflection (a,b) and transmission (c,d) coefficients for $|1,0\rangle$ FM and $|1,-l\rangle$ OV, correspondingly, vs wavelength of the incident OV $|1,-l\rangle$ for the three-part MF. At the wavelengths outside the bandgap the three-part MF serve as a comb filter for the OV $|1,-l\rangle$. Within the bandgap it converts the incident OV $|1,-l\rangle$ into the FM $|1,0\rangle$, save for a narrow zone right in the middle of the bandgap (a) at the small $d_0$. Fiber parameters: $2d_1 = 1$ cm, $\theta = \pi / l$, $\tau = 0$ ; a,c) $d_0 = 0.17$ mm; b,d) $d_0 = 1.17$ mm.

3.3. Topological charge conversion in heterogeneous MFs

Consider the case where the input end of the two-part fiber (Figure 1a) is excited with the OV $|1,i\rangle$. Then this OV becomes the OV $|1,-l\rangle$ in the transmitted light if the wavelength of the incoming field belongs to the bandgap (Figure 3 a,b). From the qualitative point of view, the process of conversion of this OV can be described as follows. The OV $|1,i\rangle$ freely passes the first leftward twisted part of the fiber. Upon reflection from the second rightward twisted part of the fiber it becomes the backward propagating FM $|1,0\rangle$. The FM upon reflection from the first part of the fiber becomes forward propagating OV $|1,-l\rangle$. This vortex almost freely passes through the second part of the fiber. The conversion process takes place at any twist defect angles $\tau$ and $\theta$. The scheme of this process is shown in Figure 3c.

Finally, consider the case of excitation of three-part MF by the OV $|1,i\rangle$. If its wavelength belongs to the bandgap and the twist defect between external parts of the fiber is maximal ($\theta = \pi / l$), then the energy of this incoming OV distributes almost equally between energies of the transmitted OV $|1,-l\rangle$.
and the reflected FM $|1,0\rangle$ (Figure 4). Therefore, the three-part MFs can serve as TC-invertors like two-part MFs. It should be noted that the just studied and other [31, 32] all-fiber methods of TC inversion have advantage over the ones using crystals, mirrors, and photonic lattices [33-35] due to its better compatibility with fiber optical lines.

Figure 3. Transmission coefficients $T$ for $|1,l\rangle$ OV (a) and $|1,-l\rangle$ OV (b) vs wavelength of the incident OV $|1,l\rangle$ for the two-part MF; twist defect angle $\theta = 0$. c) The scheme of the conversion process of the incident OV $|1,l\rangle$ into $|1,-l\rangle$ OV; SI and SII indicate the fiber parts; $2d_1 = 0.01$ m, $4l = 0.001$ m, $0.38d = 0.0038$ mm, $\equiv \equiv \equiv \equiv \equiv \equiv \equiv \equiv \equiv \equiv \equiv (1,\pi)$.

The above studied examples allow one to conclude that MFs of heterogeneous type can serve as compact TC invertors and tunable optical comb filters for OVs in the transmitted light. It should be noted that the crucial parameter of the three-part fiber is the angle $\theta$, while the angle $\tau$ almost does not affect the spectral properties of the system.

Figure 4. Transmission and reflection coefficients vs wavelength of the incident $|1,l\rangle$ OV for the three-part MF; green dotted line corresponds to the transmitted $|1,l\rangle$ OV, red dashed line – to the reflected $|1,0\rangle$ FM, solid blue line – to the transmitted $|1,-l\rangle$ OV; the type of the field is indicated near each curve; $2d_1 = 0.01$ m, $d_0 = 0.38$ mm, $\tau = 0$, $\theta = \pi / l$, $l = 4$. 
4. Localized topological states

If the MF is excited with the incoming field \( |i,s\rangle \), then the field within the fiber presents the superposition of the modes (7) or (8), which consist of contra-propagating OVs and FMs or the propagating OVs. From the instrumental point of view [36, 37], one can extract a separate field \( |i,m\rangle \) in spite of the fact that the field’s constituents \( |i,m\rangle \) “belong” to different modes. For example, in the case of the two-part fiber the amplitude for the OV \( E_{|i,l\rangle}(z) \) in the leftward twisted part reads

\[
E_{|i,l\rangle}(z) = X_5 e^{i\mu_5 z} - X_6 e^{i\mu_6 z} + X_8 e^{i\mu_8 z}. \tag{13}
\]

The expression (13) is obtained by selecting the coefficients in the expression (10) at the partial field \( |i,l\rangle \). One should remember that the unknown coefficients \( X_i \) have been already obtained on the previous steps, where transmission characteristics were studied.

Numerical simulations show that the localization of topologically charged fields within the two-part MFs is impossible at the resonance wavelength \( \lambda_0 \). Typical intensity distributions for the case of incoming fields \(|i,0\rangle\) and \(|i,l\rangle\) at this wavelength are shown in Figure 5a,b. If the wavelength changes, the weak localization becomes possible. As is seen from Figure 5c,d weakly localized states appear within the fiber, which is excited with the FM \(|i,0\rangle\) or the OV \(|i,l\rangle\). Their maximal intensities almost do not depend on the twist defect angle \( \theta \). The same is also true for the previous case.

![Figure 5](image-url)

**Figure 5.** The logarithm of relative intensity distribution within the two-part MF vs longitudinal coordinate \( z \). a, c) The incoming field is the FM \(|i,0\rangle\); b, d) the incoming field is the OV \(|i,l\rangle\). The type of the field is indicated near each curve: the dashed green line corresponds to the FM \(|i,0\rangle\), the solid red line – to the OV \(|i,l\rangle\), the dashed-dot blue line – to the OV \(|i,-l\rangle\). Parameters: a, b) \( 2d_1 = 2 \text{ cm} \), \( \lambda = 632.8 \text{ nm} \); c, d) \( 2d_1 = 6 \text{ cm} \), \( \lambda \approx 632.824 \text{ nm} \); \( \theta = 0 \).
Let us consider the case of the three-part MF the input end of which is excited with the field $|1,s\rangle$, where $s = 0, 1$. Then at the maximal twist defect between external parts of the fiber ($\theta = \pi / l$) topologically charged states arise near the defects (Figure 6a,b). As is seen from this Figure, the intensities of the localized fields near defects are much greater than the one of the incoming field. If the twist defect angle deviates from its maximal value, then the degree of localization decreases for all the fields (Figure 6c,d).

It is convenient to describe the localized fields with the help of the average relative linear energy density. Let the FM $|1,0\rangle$ be incident at the input end of the three-part MF. Then the average relative linear energy density of the localized states decreases as the length $d_0$ increases (Figure 7a). The situation is different if the fiber is excited with the OV $|1,l\rangle$. The energy of this localized vortex firstly increases as the length $d_0$ grows and after reaching some maximal value rapidly decreases (Figure 7b). It should be noted that such behavior of linear energy density for the OV $|1,l\rangle$ is quite different from the one for homogeneous MFs with the twist defect, which cannot nestle any localized state in the case of excitation with OV that is uncoupled within the bandgap [26].

![Figure 6](image_url)

**Figure 6.** The logarithm of relative intensity distribution within the three-part MF vs longitudinal coordinate z. a,c) The incoming field is the FM $|1,0\rangle$; b,d) the incoming field is the OV $|1,l\rangle$. The type of the field is indicated near each curve: the dashed green line corresponds to the FM $|1,0\rangle$, the solid red line – to the OV $|1,l\rangle$, the dashed-dot blue line – to the OV $|1,-l\rangle$. Parameters: a,b), $\lambda = 632.8$ nm, $\theta = \pi / l$; c,d) $\lambda = 632.817$ nm, $\theta = \pi / l$; $\tau = 0$, $2d_1 = 1.5$ cm, $d_0 / H \approx 20.25$.

It should be noted that until quite recently the interest in Bragg MFs was mainly theoretical. Nevertheless, the progress in the study of helical gratings makes fabrication of their Bragg version more probable in the near future [38-40]. Moreover, we believe that the results obtained in this paper could be useful for a rapidly evolving field of terahertz singular optics [41-43]. Indeed, the scaling
principle for chiral fibers established in [44] shows that our theory could be applied for chiral fibers operating at other spectral ranges.

![Figure 7](image-url) Figure 7. The logarithm of the relative linear energy density distribution within the three-part MF vs the reduced length $d_0 / H$. a) The incoming field is the FM $|I,0\rangle$; b) the incoming field is the OV $|I,l\rangle$ at $\lambda = 632.8$ nm. The type of the field is indicated near the each curve. Parameters: $\tau = 0$, $\theta = \pi / l$, $2d_1 = 1.5$ cm.

5. Conclusion
In conclusion, in this paper we have theoretically studied the processes of topological charge inversion of the incoming fields by multihelicoidal fibers of heterogeneous type with twist defects. We have demonstrated that such two-part and three-part defected fibers can be used as compact topological charge invertors in transmitted light and vortex comb filters. We have also studied the emergence of the topologically charged defect-localized fields in such fibers. We have shown that the topologically charged fields may arise in two-part and three-part multihelicoidal fibers at certain conditions. We believe that the obtained results can be scaled to the terahertz range.

References
[1] Fudouzi H 2011 Sci. Technol. Adv. Mater. 12 064704 DOI: 10.1088/1468-6996/12/6/064704
[2] Joannopoulos J D, Johnson S G, Winn J N and Meade R D 2011 Photonic Crystals: Molding the Flow of Light (Princeton, NJ: Princeton University Press) p 304
[3] Vetrov S Y, Pyatnov M V and Timofeev I V 2014 Phys. Rev. E 90 032505 DOI: 10.1103/PhysRevE.90.032505
[4] Gevorgyan A H, Kocharian A N and Vardanyan G A 2016 Liq. Cryst. 43 448 DOI: 10.1080/02678292.2015.1118768
[5] Yeh H-C and Wun K-S 2017 Laser Phys. Lett. 14 086202 DOI: 10.1088/1612-202X/aa7879
[6] Wang H-T, Timofeev I V, Chang K, Zyryanov V Y and Lee W 2014 Opt. Express 22 15097 DOI: 10.1364/OE.22.015097
[7] Serra F, Matranga M A, Ji Y and Terentjev E M 2010 Opt. Express 18 575 DOI: 10.1364/OE.18.000575
[8] Gumus M, Giden I H, Akcaalan O, Turduev M and Kurt H 2018 Appl. Phys. Lett. 113 131103 DOI: 10.1063/1.5032197
[9] Makwana M, Craster R and Guenneau S 2019 Opt. Express 27 16088 DOI: 10.1364/OE.27.016088
[10] Shokri A A and Jamshidi R 2019 AIP Adv. 9 055318 DOI: 10.1063/1.5089413
[11] Nye J F and Berry M V 1974 Proc. R. Soc. Lond. A. 336 165 DOI: 10.1098/rspa.1974.0012
[12] Karpeev S V and Khonina S N 2007 Optical Memory and Neural Networks 16 295-300 DOI: 10.3103/S1060992X07040133
[13] Kotlyar V V, Soifer V A and Khonina S N 1998 Optics and Lasers in Engineering 29 343-350 DOI: 10.1016/S0143-8166(97)00121-8
[14] Khonina S N and Karpeev S V 2004 Excitation and detection of angular harmonics in a fiber waveguide using DOE Computer Optics 26 16-26
[15] Sokolenco B V and Poletaev D A 2017 Proc. SPIE 10350 1035012 DOI: 10.1117/12.2273395
[16] Sokolenco B, Poletaev D and Halilov S 2017 J. Phys. Conf. Ser. 917 062047 DOI: 10.1088/1742-6596/917/6/062047
[17] Yao A M and Padjett M 2011 Adv Opt Photonics 3(2) 161-204 DOI: 10.1364/AOP.3.000161
[18] Sokolenco B V, Rubass A F, Lapaeva S N, Glumova M V and Volyar A V 2013 Proc. SPIE 9066 90660E DOI: 10.1117/12.2052904
[19] Kozlova E S 2018 Modeling of the optical vortex generation using a silver spiral zone plate Computer Optics 42(6) 977-984 DOI: 10.18287/1464-8978/13/9/095701
[20] Alexeyev C N, Alexeyev A N, Fadeyeva T A, Lapin B P and Yavorsky M A 2011 J. Opt. 13 095701 DOI: 10.1088/2040-8978/13/9/095701
[21] Alexeyev C N, Lapin B P and Yavorsky M A 2013 Opt. Spectrosc. 114 778-783 DOI: 10.1134/S0030400X13040048
[22] Alexeyev C N, Lapin B P and Yavorsky M A 2018 J. Opt. 20 025603 DOI: 10.1088/2040-8986/aa9e0d
[23] Alexeyev C N, Lapin B P and Yavorsky M A 2017 J. Opt. 19 045604 DOI: 10.1088/2040-8986/aa60d9
[24] Alexeyev C N, Lapin B P, Milione G and Yavorsky M A 2016 Phys. Rev. A 93 063829 DOI: 10.1103/PhysRevA.93.063829
[25] Snyder A W and Love J D 1985 Optical Waveguide Theory (London, New York: Chapman and Hall) p 750
[26] Alexeyev C N, Volyar A V and Yavorsky M A 2006 J. Opt. A: Pure Appl. Opt. 8 L5 DOI: 10.1088/1464-4258/8/11/L01
[27] Alexeyev C N, Volyar A V and Yavorsky M A 2008 J. Opt. A: Pure Appl. Opt. 10 095007 DOI: 10.1088/1464-4258/10/9/095007
[28] Milione G, Szutil H I, Nolan D A and Alfano R R 2011 Phys. Rev. Lett. 107 053601 DOI: 10.1103/PhysRevLett.107.053601
[29] Alexeyev C N, Alexeyev A N, Lapin B P and Yavorsky M A 2008 J. Opt. A: Pure Appl. Opt. 10 055009 DOI: 10.1088/1464-4258/10/5/055009
[30] Alexeyev C N, Alexeyev A N, Lapin B P and Yavorsky M A 2009 J. Opt. A: Pure Appl. Opt. 11 105406 DOI: 10.1088/1464-4258/11/10/105406
[31] Roychowdhuury S, Jaiswal V K and Singh R P 2004 Opt. Commun 236(4-6) 419-424 DOI: 10.1016/j.optcom.2004.03.036
[32] Berz’anskis A, Matijos’iūs A, Piskarskas A, Smilgevič’ius V and Stabinis A 1997 Opt. Commun 140(4-6) 273-276 DOI: 10.1016/S0030-4018(97)00178-8
[33] Bezryadin A, Neshev D N, Desyatnikov A S, Young J, Chen Z and Kivshar Y S 2006 Opt. Express 14 8317 DOI: 10.1364/OE.14.08317
[34] Baghdady J, Miller K, Morgan K, Byrd M, Osler S, Ragusa R, Li W, Cochenour B M and Johnson E G 2016 Opt. Express 24 9794 DOI: 10.1364/OE.24.009794
[35] Lyubopytov V S 2017 Opt. Express 25 9634 DOI: 10.1364/OE.25.009634
[36] Shi Z, Preece D, Zhang C, Xiang Y and Chen Z 2019 Opt. Express 27(1) 121-131 DOI: 10.1364/OE.27.000121
[37] Liao C, Yang K, Wang J, Bai J, Gan Z and Wang Y 2019 IEEE Photonic Tech L 31(12) 971-974 DOI: 10.1109/LPT.2019.2912634
[38] Liu F, Lin H F, Liu Y, Zhou A and Dai Y T 2018 Opt. Express 26 17388 DOI: 10.1364/OE.26.017388
[39] Minasyan A, Trovato C, Degert J, Freysz E, Brasselet E and Abrahm E 2017 Opt. Lett. 42(1) 41-44 DOI: 10.1364/OL.42.00041
[42] Dhaybi A A, Degert J, Brasselet E, Abraham E and Freysz E 2019 J. Opt. Soc. Am. B 36(1) 12-18 DOI: 10.1364/JOSAB.36.000012
[43] Kulya M, Semenova V, Gorodetsky A, Bespalov V G and Petrov N V 2018 Appl. Opt. 58 A90 DOI: 10.1364/AO.58.000A90
[44] Napiorkowski M and Urbanczyk W 2018 Opt. Express 26 12131 DOI: 10.1364/OE.26.012131

Acknowledgments
This work was supported by the V.I. Vernadsky Crimean Federal University Development Program for 2015-2024 (Grant № VG10/2018).