A Fair Assignment of Drivers to Parking Lots

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Abstract
Searching for a parking spot can waste time and gasoline. This waste can be reduced by assigning drivers to parking lots based on their destination and arrival time. In such a system, drivers could request a parking spot in advance and be alerted (e.g., via their phone or vehicle) of their assignment to a specific parking lot or available spot. In this paper, a parking assignment system is described to allocate parking spaces in a fair and equitable manner. Heuristics are developed to solve the underlying large scale optimization problem. The efficacy of the system is demonstrated by applying our algorithms to real data sets.

1 Introduction
A huge societal problem arises from drivers searching for a parking space. Statistics illustrating the waste associated with this issue border on the unbelievable. For example, in the United Kingdom, the average person spends 2,549 hours looking for a parking spot over their lifetime [1]. Further, a survey conducted by Allianz Insurance estimated that 95% of British people added an extra mile to their trip searching for a parking spot [2]. Such statistics arise elsewhere and are not just in the UK. For example, it was recently reported that over one year in a small Los Angeles business district, cars cruising for parking burned 47,000 gallons of gasoline and produced 730 tons of carbon dioxide [3]. Meanwhile, the consulting firm McKinsey recently claimed that the average car owner in Paris spends four years of his or her life searching for a parking space [4]. The parking assignment problem associated with electric vehicles (EVs) becomes even more acute. Due to the limited range of these vehicles, the marginal cost of expending energy to search for spaces may, in some cities, be prohibitively high. Thus, there is a real and compelling societal and economic need to revisit parking.

Currently, parking guidance systems can alert drivers to the availability of parking spots in given areas [5]. However, this type of system may not work in a crowded area with few available spots. Moreover, if parking spaces are not specifically assigned to vehicles, then a spot may no longer be available by the time the vehicle arrives. Assigning drivers to parking lots could reduce the time and gasoline wasted by drivers searching for parking spots and could ensure that the allocation of drivers to parking spots meets specified criteria.

An assignment of drivers to parking lots can be seen as a resource allocation problem, where the resource to be distributed is parking spots. An important quality of resource allocation is

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fairness; the assignment of goods to users should be fair given a specified definition of fairness [6]. For a parking lot assignment, fairness can be measured in terms of the travel time between user destinations and assigned parking lots.

Following on from our prior work in this area [7,8], we propose an optimization-based method to assign vehicles to parking spots in a fair way. The principal conceptual difference between our previous work and the work presented here is that we consider a centralized system that solves an exact optimization problem (with some relaxation), and is built on an elaborate notion of fairness from the viewpoint of the vehicle owner. Specifically, we define a driver-centric measure to determine the fairness of a parking lot allocation, and construct an algorithm to optimize this fairness measure. We then test our method on two sets of real driving data: one from Cologne, Germany [9,10] and one from New York City taxis [11,12]. Our results show that using our method could improve the fairness of a parking assignment by around 140% when compared to basic methods with no optimization.

1.1 Related Work

Within the research community, questions concerning how to manage parking space supply-demand mismatch are actively being investigated from a variety of different angles. One important aspect concerns the delivery of up-to-date, accurate, real-time information to parking systems in order to achieve greatest system efficiency. Parking guidance and information (PGI) systems can help inform drivers of where there are available spots; for example, a system could list the number of available spots on each floor of a parking garage or in each parking lot in a city [5]. Such a system requires detecting vehicles in parking spots, and there have been a number of works that consider the technology (i.e., hardware and/or software) required to detect vehicles in parking spaces, including [13–16]. The effects of parking guidance on a city have also been analyzed [17,18]. However, these systems do not specifically assign vehicles to parking spots, they only make drivers aware of the availability in the specified areas. This type of system is not ideal for street parking, or situations when there are only a few parking spaces available, as the availability of a parking space in a crowded area can change quickly. This type of system could also result in unnecessary overcrowding of certain areas.

A predictive approach is proposed by Caliskan et al. [19] and further studied in [20]. The authors develop a method to predict the likelihood of a parking space being available at the estimated time that the car will arrive there. In [21], Geng et al. view the parking problem as a dynamic resource-allocation problem. In this paper, algorithms are developed to assign drivers to parking lots dynamically as they leave their destinations. The objective of their formulation is to minimize the sum of the utility functions for all the drivers. Chou et al. [22] use a different model and consider a network of parking lots with negotiable prices; parking lots are selected for drivers in a way that benefits both the drivers and the car park operators. The work of Teodorovic and Lucic [23] proposes a method to define rules for assigning drivers to parking spots using fuzzy logic and integer programming techniques. The parking assignment problem has also been solved using a stable matching algorithm [24], which yields a Nash equilibrium solution, and through differential pricing [25]. Parking lots can also be assigned to drivers through matching under preferences [26]. Griggs et al. created a distributed privacy-preserving scheme to allocate parking efficiently with quality of service guarantees [8], where fairness is one such objective. Finally, in [7] cars are assigned to lots in a balanced manner, where fairness is considered from the view point of the parking lot owner. Our work follows the prior work described in [7,8]. However, there are important differences.
We provide an advanced driver-centric notion of fairness, and give a heuristic that solves a very complicated optimization problem that goes far beyond that described in any of the aforementioned references. Importantly, our fairness notion can be extended to include other aspects of fairness (aspects associated with EV ownership) as a particular need arises.

2 Problem Statement

We propose an optimization-based method to assign parking lots to drivers in a fair way. We assume that the starting and ending times of all trips are known at the beginning of the day (i.e., a 24-hour period), as well as the starting and ending locations.

Without loss of generality, we assume that a vehicle can be collected from a parking lot as well as dropped off (i.e., a driver may pick up their vehicle from a specified parking lot and park it at a different parking lot near his/her destination). We assume that a driver will pick up a vehicle from the parking lot closest to their origin location and our method will assign the driver to a parking lot in which to drop off the vehicle that is near their destination.

2.1 Notation

Table 1 defines the terminology to be used in the remainder of the work.

| Name | Description |
|------|-------------|
| $\mathcal{R}$ | Drivers |
| $\mathcal{T}$ | Time Periods |
| $\mathcal{L}$ | Parking Lots |

| Name | Description |
|------|-------------|
| $M$ | $\mathbb{R}^{2 \times |\mathcal{L}|}$ Location of Parking Lots |
| $\bar{x}$ | $\mathbb{Z}^{||\mathcal{L}||}$ Maximum Number of Parking Spots in each Lot |
| $s, d$ | $\mathbb{R}^{2 \times |\mathcal{R}|}$ Start and End Location of Driver Trips |
| $t^s, t^d$ | $\mathbb{R}^{|\mathcal{R}|}$ Start and End Time of Driver Trips |
| $\alpha$ | $\mathbb{R}^+$ Average Walking Speed (in miles per hour) |

| Name | Description |
|------|-------------|
| $y$ | $\{0, 1\}^{||\mathcal{L}|\times |\mathcal{R}|}$ Destination Lot Assignment |
| $x(t)$ | $\mathbb{Z}^{||\mathcal{L}|}$ Number of Filled Parking Spots at time $t$ |
| $Y(t)$ | $\mathbb{Z}^{||\mathcal{L}|}$ Number of drivers arriving at each lot at time $t$ |
| $Z(t)$ | $\mathbb{Z}^{||\mathcal{L}|}$ Number of drivers leaving each lot at time $t$ |
| $D$ | $\mathbb{R}_+^{|\mathcal{R}|}$ Distance from Assigned Parking Lot to Destination (in miles) |
| $\beta$ | $\mathbb{R}_+^{|\mathcal{R}|}$ Total Overhead Time for each Driver Trip (in hours) |
For all trips $r \in R$ and parking lots $\ell \in L$, we define the binary decision variable $y \in \{0, 1\}^{L \times R}$ and the binary parameter $z \in \{0, 1\}^{L \times R}$. The parameter $z_{\ell r}$ will specify from which parking lot the vehicle will depart (its origin), and the variable $y_{\ell r}$ will specify in which parking lot the driver will park (its destination). Because we assume that each vehicle will start in the parking lot closest to the driver’s origin location, $z$ is a parameter, not a variable. However, the lot in which a vehicle will park near the destination is a decision variable to be optimized. The values of $y$ and $z$ can be defined with the equations:

$$y_{\ell r} = \begin{cases} 1 & \text{if trip } r \text{ ends up in lot } \ell \\ 0 & \text{otherwise} \end{cases}, \quad z_{\ell r} = \begin{cases} 1 & \text{if trip } r \text{ starts from lot } \ell \\ 0 & \text{otherwise} \end{cases}.$$

The values of $z$ can easily be calculated given the origin locations of the trips, $s$.

We also define the state variables $Y, Z \in \mathbb{Z}^{L \times |T|}$ that keep track of the number of vehicles arriving and leaving each parking lot over time:

$$Y_\ell(t) = \sum_{r \in R: t_d^r = t} y_{\ell r}, \quad Z_\ell(t) = \sum_{r \in R: t_s^r = t} z_{\ell r},$$

where $t_s^r, t_d^r$ are the origin and destination locations of driver $r$ respectively. The variable $x_\ell(t)$ represents the number of filled (or parked-in) spots in lot $\ell$ at time $t$, and $\bar{x}_\ell$ is the maximum number of available spots in parking lot $\ell$. A feasible set of lot assignments $y_{\ell r}$ and parking lot states $x_\ell(t)$ will be in the set:

$$\Omega = \{ (x, y) : \begin{array}{ll} x_\ell(t) = x_\ell(t-1) + Y_\ell(t) - Z_\ell(t) & \forall \ell \in L, t \in T \\ x_\ell(t) \leq \bar{x}_\ell & \forall \ell \in L, t \in T \\ \sum_{\ell \in L} y_{\ell r} = 1 & \forall r \in R \\ y_{\ell r} \in \{0, 1\} & \forall \ell \in L, r \in R \end{array} \}. \tag{FEAS}$$

The first equation in (FEAS) updates the state of each parking lot over time: the number of occupied parking spots is equal to the number of occupied spots in the previous time period, plus the arriving vehicles and minus the departing vehicles in the previous period. The second equation in (FEAS) ensures the number of vehicles in each lot is less than the capacity, the third equation makes sure that each trip ends in only one parking lot, and the last equation constrains $y$ to be binary.

### 3 Optimizing Fairness

Assigning drivers to parking lots leads us to the following questions: Is there a fair way to assign drivers to parking lots? And what does it mean for an assignment of parking lots to be fair?

In our problem, a fair assignment will be one in which the distance between drivers’ destinations and their assigned parking lots are similar. Let $\beta_r$ be the amount of time driver $r \in R$ spends traveling from the assigned parking lot to his/her destination:

$$\beta_r = \alpha \left\| d_r - \sum_{\ell \in L} (y_{\ell r} \cdot M_\ell) \right\|_1. \tag{1}$$
We define fairness as a lack of envy among drivers, where the envy between two drivers \( r_1, r_2 \in \mathcal{R} \) is defined as:

\[
E_{r_1, r_2} := |\beta_{r_1} - \beta_{r_2}|, \tag{2}
\]

where \( \beta_r \) is the walking time for driver \( r \in \mathcal{R} \) from his/her parking lot to destination.

We define fairness as the mean value of \( E_{r_1, r_2} \) over all pairs of drivers \( r_1, r_2 \in \mathcal{R} \), where a smaller value is better. To attain an assignment of drivers to parking lots that optimizes this fairness measure, we can minimize the mean of the envy values over all pairs of drivers with the objective function:

\[
F(\beta) = \text{mean}_{r_1, r_2 \in \mathcal{R}}(E_{r_1, r_2}) = \frac{1}{|\mathcal{R}|^2} \sum_{r_1, r_2 \in \mathcal{R}} |\beta_{r_1} - \beta_{r_2}|. \tag{3}
\]

**Definition 1** A fair assignment of destination lots to drivers results in a value of \( F(\beta) \) that is as small as possible.

### 3.1 Formulation

We assume that we are given the origin and destination locations and times for each driver, \((s, d, t^s, t^d)\) at the start of the day (i.e., a 24-hour period). Let \( g : \mathcal{R}^{|\mathcal{R}|} \rightarrow \mathbb{R} \) be a function that takes the waiting time of each driver \( r \in \mathcal{R} \) and assesses the fairness, where a smaller value implies a more fair assignment. Then we can formulate the problem that maximizes fairness as:

\[
\begin{align*}
& \text{minimize} \quad g(\beta) \\
& \text{subject to} \quad (x, y) \in \Omega \\
& \hspace{1cm} \beta_r = \alpha \left\| d_r - \sum_{\ell \in \mathcal{L}} (y_{\ell r} \cdot M_\ell) \right\|_1 \quad \forall r \in \mathcal{R} \hspace{1cm} (4)
\end{align*}
\]

The first constraint ensures that the lot assignments defined by \( x \) and \( y \) are feasible for the given drivers and parking lots. Constraint (4) calculates the distance from the driver’s destination to the assigned parking lot and multiplies it by the average walking speed, \( \alpha \), to get the total travel time for driver \( r \). The formulation \((\text{OPT-FAIR})\) is a mixed-integer linear program (MILP) that can be solved in CPLEX [27].

This optimization problem can be solved for a chosen time period (e.g., a single day); our aim is to choose an objective function \( g(\beta) \) for \((\text{OPT-FAIR})\), so that the average overhead waiting time over a set of drivers is fair.

### 3.2 Intractability of \( F(\beta) \) as the objective

Using the definition of fairness based on the function \( F(\beta) \) in (3), an optimally fair lot assignment can be found by setting the objective function in \((\text{OPT-FAIR})\) to \( g(\beta) = F(\beta) \). That is, ideally we could minimize the mean of all the envy values, or differences between driver walking times. However, setting \( g(\beta) = F(\beta) \) as the objective in \((\text{OPT-FAIR})\) results in an intractable optimization problem; taking the absolute value of each pairwise difference in travel times results in a large
problem that cannot be solved in reasonable time. For example, we used CPLEX to try to solve the problem (OPT-FAIR) with the objective \( g(\beta) = F(\beta) \), for \(|\mathcal{L}| = 10\) parking lots, \(|\mathcal{R}| = 100\) drivers, and \(|\mathcal{T}| = 24\) time periods, and the solver did not even find a feasible solution after 12 hours.

### 3.3 Choosing an Objective Function

Given the intractability of using \( F(\beta) \) as the objective, we would like to construct a method that achieves a similar result, but that is computationally tractable. The idea of minimizing \( F(\beta) \) in (OPT-FAIR) is to obtain a lot assignment in which all drivers’ walking times are similar. We propose a method to find a solution of (OPT-FAIR) that achieves this same goal, but with an objective that makes this problem tractable.

Given a feasible set of lot assignments \( y \in \{0, 1\}^{|\mathcal{L}| \times |\mathcal{R}|} \) with corresponding walking times \( \beta \in \mathbb{R}^{|\mathcal{R}|} \), the mean walking time is defined as:

\[
H(\beta) := \frac{1}{|\mathcal{R}|} \sum_{r \in \mathcal{R}} \beta_r. \tag{5}
\]

A fair assignment of drivers to parking lots can be attained by minimizing the absolute value of the differences between each driver’s walking time and the mean walking time, \( H(\beta) \). This will provide a solution where the walking times of the drivers are similar and as close to the mean value as possible.

Given a feasible solution \((\hat{x}, \hat{y}, \hat{\beta})\) to (OPT-FAIR), the mean walking time \( H(\hat{\beta}) \) can be calculated and the problem (OPT-FAIR) can be re-solved with the objective:

\[
G(\beta) = \sum_{r \in \mathcal{R}} |\beta_r - H(\hat{\beta})|. \tag{6}
\]

Our method is based on solving (OPT-FAIR) with this objective, which will yield a tractable problem and a fair solution.

To further reduce the size of the problem, we can consider only a subset of the lot assignments as decision variables. That is, because some of the walking times \( \beta_r \) may be sufficiently close to the mean value \( H(\hat{\beta}) \) already, we can fix the lot assignments within a chosen range of \( H(\hat{\beta}) \) and only change those with a large or small travel time. Define the set \( S \subseteq \mathcal{R} \):

\[
S := \{ r \in \mathcal{R} : \hat{\beta}_r \in [(1 - \epsilon)H(\hat{\beta}), (1 + \epsilon)H(\hat{\beta})] \}, \tag{7}
\]

where \( \epsilon > 0 \) is a chosen value that determines the desired range of walking times from the mean.

For a feasible set of lot assignments, \( \hat{y} \in \{0, 1\}^{|\mathcal{L}| \times |\mathcal{R}|} \) with corresponding walking times \( \hat{\beta} \in \mathbb{R}^{|\mathcal{R}|} \), we define the set of constraints based on \( S \) as:

\[
\hat{\Omega}(S) = \left\{ (x, y, \beta) \mid \begin{array}{l}
(x, y) \in \Omega \\
\beta_r = \alpha \cdot \| d_r - \sum_{\ell \in \mathcal{L}} (y_{\ell r} \cdot M_{\ell}) \|_1 \quad \forall r \in \mathcal{R} \\
y_{\ell r} = \hat{y}_{\ell r} \\
\beta_r = \hat{\beta}_r \\
\forall r \in S, \ell \in \mathcal{L} \\
\forall r \in S
\end{array} \right\}, \tag{8}
\]

and we define the new objective function:

\[
G(\beta, S) = \sum_{r \in S} |\beta_r - H(\hat{\beta})|. 
\]
The following optimization problem will then find new lot assignments that optimizes fairness and reduces the envies among drivers’ walking times:

\[
\begin{align*}
\text{minimize} & \quad G(\beta, S) \\
\text{subject to} & \quad (x, y, \beta) \in \hat{\Omega}(S)
\end{align*}
\] (OPT-MEAN)

Notice that after solving the MILP (OPT-MEAN) and obtaining a new assignment \((y^*, x^*, \beta^*)\), the mean walking time will change according to the new lot assignments and walking times \(\beta^*\). That is, the mean walking of the new solution, \(H(\beta^*)\), will differ from \(H(\hat{\beta})\), and thus (OPT-MEAN) can be re-solved with the new mean walking time, \(H(\beta^*)\). This process of re-solving can continue until the mean walking value converges (i.e., stops changing). Iteratively re-solving (OPT-MEAN) with the updated mean value \(H(\beta)\), and the corresponding updated set \(S\) will reduce the envies of all the drivers until their values cannot be any closer.

Our full method is described in Algorithm 1. This method iteratively minimizes the difference between a subset of the drivers’ walking times and the mean walking time of the previous iteration, \(H(\hat{\beta})\), until the solution has converged. The algorithm takes three parameters: \(\epsilon\) is the desired range from the mean walking time, \(\delta\) is the tolerance used to determine if the algorithm has converged, and \(\text{maxiter}\) is the maximum number of iterations.

**Algorithm 1 Minimizing the envy**

1: Choose values for \(\epsilon, \delta, \text{maxiter}\)
2: Find a feasible lot assignment \(y^0 \in \{0, 1\}^{|L| \times |R|}\) with corresponding walking times \(\beta^0 \in \mathbb{R}^{|R|}\)
3: \(i \leftarrow 1, \text{converged} \leftarrow 0\)
4: while not converged do
5:     \(S^i \leftarrow \{r \in R : \beta^{i-1}_r \in [(1 - \epsilon)H(\beta^{i-1}), (1 + \epsilon)H(\beta^{i-1})]\}\)
6:     \((x^i, y^i, \beta^i) \leftarrow \arg\min_{(x, y, \beta) \in \hat{\Omega}(S^i)} G(\beta, S^i)\)
7:     if \(|H(\beta^i) - H(\beta^{i-1})| < \delta\) or \(i > \text{maxiter}\) then
8:         converged \(\leftarrow 1\)
9:     end if
10:    \(i \leftarrow (i + 1)\)
11: end while

4 Results

We tested Algorithm 1 on two sets of real driving data. A description of the data and the results are below. We also compared our method to the parking assignment formulation considered by Geng and Cassandras in [21,28]. All of our implementation was done in Matlab [29] and used IBM ILOG CPLEX [27] to solve the optimization problems.

4.1 Data

Real trip data was collected from New York City taxis in 2013 [11,12]. From this dataset, we extracted information on the origin and destination locations and times for over 50,000 trips in January 2013.
We also used real driving data from Cologne, Germany, collected for the Simulation of Urban Mobility (SUMO) project [9, 10]. This data is known as the TAPAS Cologne data and provides us with driver behaviors in Cologne for a 2-hour period.

In order to model a system with known parking lot locations, for each data set we created 10 parking lots in the given area. For each set of drivers and their known destinations \( d \in \mathbb{R}^{2 \times |R|} \), we used k-means clustering [30] on the locations \( d \) to find 10 clusters of destinations; we then set the locations of the 10 parking lots to the centroids of each of the clusters.

Let \( w \sim \bar{U}[a, b] \) signify that \( w \) is a pseudorandom integer from a discrete uniform distribution between \( a \) and \( b \). For each of the parking lots, we assigned a capacity \( \bar{x}_\ell \) and initial occupancy \( x(0) \) of:

\[
\bar{x}_\ell \sim \bar{U}[\lceil |R|/|L| \rceil + 1, \lceil |R|/|L| \rceil + 2] \\
x(0) \sim \bar{U}[\bar{x}_\ell/4, (3 \cdot \bar{x}_\ell/4)].
\]

In other words, the capacity of each parking lot will be the average number of parking spots needed in each lot \((|R|/|L|)\) plus either 1 or 2, and each lot will initially be between one-quarter and three-quarters full. The average walking speed \( \alpha \) was assumed to be 5 km/hour [31].

### 4.2 Fairness Results

To determine the relative fairness of our method, we also implemented the two methods described in the algorithms below. Algorithm 2 (Minimum Sum) solves the optimization problem (OPT-FAIR) once with the objective of minimizing the sum of the travel times between the parking lot and destination. Algorithm 3 (No Scheme) simulates what might happen with no scheme, if drivers were to park in the closest available lot to their destination. (Note that this algorithm will perform slightly better than no scheme, because it assumes drivers know exactly where the best available spot is.)

**Algorithm 2 Minimizing the sum**

1. Find the lot assignments by solving the optimization problem:

\[
(x, y, \beta) \longleftarrow \text{argmin}_{(x, y) \in \Omega} \sum_{r \in R} \beta_r \\
\text{subject to } \beta_r = \alpha \cdot \left\| d_r - \sum_{\ell \in L} (y_{r, \ell} \cdot M_{\ell}) \right\|_1 \quad \forall r \in R
\]

**Algorithm 3 No scheme**

1. for \( t = 1, \ldots, |T| \) do
2. for all drivers \( r \in R \) arriving at their destination at time \( t \) do
3. Find the closest lot to the destination of driver \( r \) that has spaces available and assign the driver to this parking lot
4. end for
5. end for

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The figures below show a comparison of the fairness of our algorithm to the fairness of Algorithms 2 and 3. For the data for New York City, we ran 500 tests on the data, where each test randomly extracted a different set of 100 drivers over a 24-hour period from the data set described in Section 4.3. For each run, we found the mean envy value for that run (i.e., we calculated the value of $F(\beta)$ in [3] for the solution to each method).

Our Algorithm Iteration

| Mean Envy (Minutes) |
|---------------------|
| 4                  |
| 5                  |
| 6                  |
| 7                  |
| 8                  |
| 9                  |
| 10                 |
| 11                 |
| 12                 |

Minimum Sum
No Scheme
Our Algorithm

Figure 1a shows a progression of the mean envy values, $F(\beta)$, over iterations of Algorithm 1 in comparison to Algorithm 2 (Minimum Sum) and Algorithm 3 (No Scheme). Figure 1b shows the probability that the mean envy $F(\beta)$ will be greater than a chosen value. That is, the x-axis in Figure 1b corresponds to a given mean envy value $\gamma$, and the y-axis is the probability that the mean envy $F(\beta)$ is greater than $\gamma$ for each method. For example, Figure 1b shows that 60% of envy values for Algorithm 3 (No Scheme) are greater than 4.79 minutes, and 60% of envy values for Algorithm 1 (Minimum Envy) are greater than 3.89 minutes. Moreover, the mean improvement of Algorithm 1 over Algorithm 2 is 28.4%, and the mean improvement of Algorithm 1 over Algorithm 3 is 25.6%.

The same tests were run on the data from Cologne, described in Section 4.3. We ran 500 tests on the data, where each test used randomly extracted driving data for 100 drivers over a 2-hour period and the data was extrapolated to cover a 24-hour period.

Figures 2a and 2b show the results for the Cologne data that are analogous to Figures 1a and 1b for the New York City data. Figure 2b shows that 60% of envy values for Algorithm 3 (No Scheme) are greater than 2.89 minutes, and 60% of envy values for Algorithm 1 (Minimum Envy) are greater than 1.23 minutes. Moreover, the mean improvement of Algorithm 1 over Algorithm 2 is 139.6%, and the mean improvement of Algorithm 1 over Algorithm 3 is 100.2%.
The Jain’s fairness measure is commonly used in congestion control protocols to measure fairness of a resource allocation [6,32,33]. The value of the Jain’s measure is between 0 and 1, where a more fair assignment results in a higher value. For a set of travel times $\beta_r$, the Jain’s fairness measure is defined as:

$$J(\beta) = \frac{(\sum_{r \in R} \beta_r)^2}{|R| \cdot \sum_{r \in R} \beta_r^2}$$ (9)

We computed the Jain’s fairness measure for each dataset for each method; Figure 3 shows the probability that the Jain’s measure $J(\beta)$ will be greater than a chosen value.

Note that, as opposed to in Figures (2b) and (1b), for the Jain’s fairness measure, it is better for the values to be higher.

For Figure 3 the $x$-axis corresponds to a given value $\gamma$, and the $y$-axis is the probability that the Jain’s measure $J(\beta)$ is greater than $\gamma$ for each method. For example, using the Cologne data, Figure 11 shows that 60% of Jain’s measure values for Algorithm 3 (No Scheme) are greater than 0.89, and 60% of Jain’s measure values for Algorithm 1 (Minimum Envy) are greater than 0.97. Using the data from NYC taxi trips, 60% of Jain’s measure values for Algorithm 3 (No Scheme) are greater than 0.86, and 60% of Jain’s measure values for Algorithm 1 (Minimum Envy) are greater than 0.91.

Moreover, using the Cologne data, the mean improvement of Algorithm 1 over Algorithm 2 is 8.95%, and the mean improvement of Algorithm 1 over Algorithm 3 is 7.67%. Using the NYC taxi trips, the mean improvement of Algorithm 1 over Algorithm 2 is 6%, and the mean improvement of Algorithm 1 over Algorithm 3 is 4.61%.
4.3 Comparison to Other Methods

We compared our algorithm to the method used by Geng and Cassandras in [21, 28]. In this work, the authors assign parking lots to drivers dynamically as they leave their destinations (that is, the driving patterns are not known ahead of time). Geng et al. do not consider fairness of lot assignments; their method minimizes the sum of the utility functions of each driver requesting a parking spot at a given time.

We implemented the method of Geng et al. as described in the paper [21] and compared their method to ours. We generated random data using the same methods they described, according to the specified distributions and parameters meant to simulate parking around Boston University. We also implemented our method on a dynamic basis, so that our algorithm was implemented to assign parking lots to the drivers that were currently requesting parking spots.

Geng et al. define the utility function of each driver $r \in \mathcal{R}$ as

$$J_{\ell r} = \lambda_r \frac{M_{\ell r}}{M_r} + (1 - \lambda_r) \frac{D_{\ell r}}{D_r}, \quad (10)$$

where $\lambda_r \in [0, 1]$ is a weight defined by the user, $M_{\ell r}$ is the monetary cost for driver $r$ to park in parking lot $\ell$, and $D_{\ell r}$ is the distance from driver $r$'s destination to parking lot $\ell$. The values $M_r$ and $D_r$ are also provided by the user, and these are the maximum values of $M_{\ell r}$ and $D_{\ell r}$ that user $r$ is willing to accept.

For each time step $k = 1, 2, \ldots$, let $P(k) \subseteq \mathcal{R}$ be the set of drivers that have requested a parking spot after time $k$ and have not yet arrived at their destination by time $k$. Using notation consistent with Table 1 and adding the time step parameter $k$, Geng et al. find a lot assignment
\( y(k) \in \{0, 1\}^{|L| \times |P(k)|} \) by solving the problem

\[
\begin{align*}
\text{minimize} & \quad \sum_{\ell \in L, r \in P(k)} y_{\ell r}(k) \cdot J_{\ell r} + \sum_{r \in P(k)} \left( 1 - \sum_{\ell \in L} y_{\ell r}(k) \right) \\
\text{subject to} & \quad x_{\ell}(t) = x_{\ell}(t-1) + Y_{\ell}(t) - Z_{\ell}(t) \quad \forall \ell \in L, t \in T \\
& \quad x_{\ell}(t) \leq \bar{x}_{\ell} \quad \forall \ell \in L, t \in T \\
& \quad \sum_{\ell \in L} y_{\ell r}(k) \leq 1 \quad \forall r \in P(k) \quad (11) \\
& \quad y_{\ell r} \in \{0, 1\} \quad \forall \ell \in L, r \in P(k) \\
& \quad \sum_{\ell \in L} (y_{\ell r}(k) \cdot J_{\ell r}) \leq \sum_{\ell \in L} (y_{\ell r}(k-1) \cdot J_{\ell r}) \quad (12)
\end{align*}
\]

Notice that the only difference between the first four constraints in SMART-PARK and the set defined by \( \Omega \) in FEAS is the equation (11), where this equation is an inequality in SMART-PARK and an equality in FEAS. This implies that each driver may not necessarily be assigned to a parking spot in SMART-PARK, but is guaranteed a parking spot assignment with the constraints in FEAS. The last constraint (12) ensures that any new assignment to driver \( r \) is no worse than their assignment from the previous period, \( k - 1 \).

To compare our method, we used the formulation SMART-PARK, but changed the objective function to:

\[
\begin{align*}
\text{minimize} & \quad \sum_{\ell \in L, r \in P(k)} (y_{\ell r}(k) \cdot J_{\ell r} - \bar{J}(k)) + \sum_{r \in P(k)} \left( 1 - \sum_{\ell \in L} y_{\ell r}(k) \right) \\
\text{subject to} & \quad \sum_{\ell \in L, r \in P(k)} (y_{\ell r}(k) \cdot J_{\ell r}) \leq \sum_{\ell \in L} (y_{\ell r}(k-1) \cdot J_{\ell r}) \quad \forall \ell \in L, r \in P(k)
\end{align*}
\]

where

\[
\bar{J}(k) := \frac{1}{|P(k)|} \left( \sum_{\ell \in L, r \in P(k)} y_{\ell r}(k-1) \cdot J_{\ell r} \right).
\]

This is analogous to the objective defined in (3), where the only difference is that (13) substitutes the travel times \( \beta \) with the utility function values \( J \). We then use the iterative method defined in Algorithm 1 with the optimization problem defined by the objective (13) and the constraints to SMART-PARK. This allows us to compare the fairness of our algorithm with that of Geng et al.

Table 2 compares the relative improvement in the mean of the envy values \( F(\beta) \) as defined in (3), over 500 runs. Each run uses different randomly generated data using the guidelines provided in [21], and compares the fairness of Geng et al.’s method to Algorithm 1 using the formulation SMART-PARK with the objective (13). The values in Table 2 are the mean of the relative improvement in the fairness measures \( F(\beta) \) and \( J(\beta) \) for the solutions of each of the 500 runs.

| Mean Envy | Jain’s Fairness Measure |
|-----------|-------------------------|
| 14.51%    | 4.56%                   |

Table 2: Mean Relative Improvement in Fairness using Algorithm 1 over SMART-PARK
5 Conclusion

In this work, we constructed a measure to determine the fairness of a given assignment of drivers to parking lots. We used this measure to construct an optimization-based algorithm that iteratively reduces the envy among drivers (or iteratively increases fairness) and finds a fair parking lot allocation. Our method was tested on real driving data and compared to a number of similar methods to show that our algorithm provides fair parking lot assignments.

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