Detecting Gravitational Waves by Twisted Light - Dipole Interaction of Photons and Gravitational Waves

Feng Longlong
School of Physics and Astronomy,
Sun Yat-Sen University
Motivations

The Detection of Gravitational Wave: the Advanced LIGO Interferometer
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The Detection of Gravitational Wave: the Advanced LIGO Interferometer

Transformation Optics: Optical Activity from Helicity-Rotation Interaction
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- The Detection of Gravitational Wave: the Advanced LIGO Interferometer

- Transformation Optics: Optical Activity from Helicity-Rotation Interaction

- The Next-Generation of Laser-Interferometer GW Detectors
Maxwell - Schrodinger Equation

Vector Wavefunction

$$|\Psi\rangle = D + iB$$

Schrodinger Equation

$$i\frac{\partial}{\partial t}|\Psi\rangle = \nabla \times (|\Psi\rangle/\sqrt{g^{00}} + ig \times |\Psi\rangle)$$

Transverse Condition

$$\nabla \cdot |\Psi\rangle = 0$$
Electromagnetism in a Rotating Frame of Reference

Schrodinger Equation

\[ i \frac{\partial}{\partial t} |\Psi\rangle = (k \cdot s + \Omega \cdot (L + s)) |\Psi\rangle \]

\[ k \cdot |\Psi\rangle = 0 \]

\[ k = -i \nabla \quad L = x \times k \]

Time Evolution

\[ |\Psi(t)\rangle = e^{it\Omega \cdot J} |\Psi_i\rangle \]
Light Propagating in Gravitational Waves

Maxwell Equation in a linear Medium

\[ i \frac{\partial}{\partial t} \Psi^h = \nabla \times (U \Psi^h) \quad U = \begin{pmatrix} I + Q & q \\ q^\dagger & 1 + q_0 \end{pmatrix} \]

Multipole Components

\[ q_0 = h_{33} \]
\[ q = \{ Q_{+1}, Q_{-1} \}^T, \quad Q_{\pm 1} = \frac{1}{\sqrt{2}} (h_{13} \mp i h_{23}) \]
\[ Q_{11} = Q_{22} = \frac{1}{2} (h_{11} + h_{22}) \]
\[ Q_{12} = \frac{1}{2} (h_{11} - h_{22}) - i h_{12}, \quad Q_{21} = Q_{12}^* \]
Paraxial Equation for Twisted Light

Positive Helicity State:

\[ |\Psi\rangle^h_\perp = \Psi_0^+ e^{-i\omega t + ikz} e_+ \]

\[ i \frac{\partial \Psi_0^+}{\partial z} = - \frac{1}{k'} \nabla_+ \nabla_- \Psi_0^+ - (q \cdot \frac{1}{i} \nabla_\perp) \Psi_0^+ \]
Paraxial Equation for Twisted Light

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Paraxial Equation in the Flat Space
Paraxial Equation for Twisted Light

Positive Helicity State: \[ |\Psi\rangle^h \perp = \Psi_0^+ e^{-i\omega t + ikz} e_+ \]

\[ i \frac{\partial \Psi_0^+}{\partial z} = - \frac{1}{k'} \nabla_+ + \nabla_- \Psi_0^+ - (\mathbf{q} \cdot \frac{1}{i} \nabla_\perp) \Psi_0^+ \]

Paraxial Equation in the Flat Space

\[ LG_n^l = \frac{a_n^l}{w(z)} \left( \frac{\sqrt{2\rho}}{w(z)} \right)^{|l|} L_n^{|l|} \left( \frac{2\rho^2}{w(z)^2} \right) e^{-\rho^2/w(z)^2} e^{ik\rho^2/2R} e^{il\phi} e^{-i\varphi(z)} \]

Laguerre-Gaussian Modes

\[ a_n^l = \frac{(2n!/\pi (n + |l|)!)^{1/2}}{z_R = \frac{1}{2} kw_0^2} \]

\[ \varphi(z) = (2n + |l| + 1) \tan^{-1}(z/z_R) \]
Dipole Interaction of Photons and Gravitational Waves I

\[ H_I = -q \cdot k_\perp \quad k_\perp = -i \nabla_\perp \]

\[ q = h^+[e_k - (e_g \cdot e_k)e_g] + h^\times (e_g \times e_k) \]
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Dipole Transition of Photons

Selection Rule

\[\nabla_{\pm}|n, \pm l\rangle_{LG} = k_w \left[ \sqrt{n + l} |n, \pm (l - 1)\rangle_{LG} + \sqrt{n + 1} |n + 1, \pm (l - 1)\rangle_{LG} \right] \]

\[\nabla_{\mp}|n, \pm l\rangle_{LG} = -k_w \left[ \sqrt{n + l + 1} |n, \pm (l + 1)\rangle_{LG} + \sqrt{n} |n - 1, \pm (l + 1)\rangle_{LG} \right] \]

Selection Rule

\[\Psi_{0+} = \sum_{m,k} \xi_{m,k}(z) |m, k\rangle_{LG}\]

\[\frac{d\xi_{n,l}(z)}{dz} = \sum_{m,k} \langle n, l|Q_{-1} \nabla_+ + Q_{+1} \nabla_- |m, k\rangle\]
Dipole in the Intensity Pattern

Input Light - Gaussian Mode

\[ |\text{in}\rangle = |0, 0\rangle_{LG} \]

\[ |\text{out}\rangle = |\text{in}\rangle - Q_1 k_w L \left[ e^{i\alpha} |0, +1\rangle_{LG} + e^{-i\alpha} |0, -1\rangle_{LG} \right] \]

Rotation

\[ Q_{\pm 1} = \sin \theta_g h(\theta_g) e^{\pm i\alpha} \]

\[ \alpha + \phi_g = \frac{\pi}{2} + \beta \]
\[ I \approx I_0 \left[ 1 - 4\sqrt{2}\hbar k_w L \frac{\rho}{w(L)} \sin \theta_g \cos \varphi \cos(\phi + \alpha) \right] \]
\[ |\text{in}\rangle = HG_{10} = \frac{1}{\sqrt{2}} (|0, +1\rangle_{LG} + |0, -1\rangle)_{LG} \]

\[ |\text{out}\rangle = |\text{in}\rangle + Q_{1} k_{w} L \cos \alpha \left[ |0, 0\rangle_{LG} + |1, 0\rangle_{LG} \right] \]

\[ - \frac{1}{\sqrt{2}} Q_{1} k_{w} L \left[ e^{i\alpha} |0, +2\rangle_{LG} + e^{-i\alpha} |0, -2\rangle_{LG} \right] \]
Michelson Interferometer
Summary

- Induced Dipole Structure for the Gaussian Beams
- Macroscopic Rotation of the Intensity Pattern
- Central Intensity Brighten

Detecting Gravitational Waves in 2D Intensity Space