MF-based Dimension Reduction Signal Compression for Fronthaul-Constrained Distributed MIMO C-RAN

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Abstract—In this work we propose a fronthaul compression scheme for distributed MIMO systems with multi-antenna receivers, in which, prior to signal quantisation, dimension reduction is performed at each receiver by matching filtered the received signal with a subset of the local user channel vectors. By choosing these matched filter vectors based on global channel information, a high proportion of the potential capacity may be captured by a small number of signal components, which can then be compressed efficiently using local signal compression. We outline a greedy algorithm for selecting the matched filtering vectors for each receiver, and a local transform coding approach for quantising them, giving expressions for the resulting system sum and user capacities. We then show that the scheme is easily modified to account for imperfect CSI at the receivers. Numerical results show that with a low signal dimension the scheme is able to operate very close to the cut-set bound in the receivers. Signal dimension reduction is also an interesting observation in [7] is that at lower fronthaul rates the optimal rate allocation is sparse - only a subset of available signal components at each receiver are quantised. The scheme can thus be seen as effectively performing dimension reduction on the received signal. Signal dimension reduction is also employed in [9] using an analog beamforming stage, which is followed by a centrally performed rate allocation stage. An interesting observation in [7] is that at lower fronthaul rates the optimal rate allocation is sparse - only a subset of available signal components at each receiver are quantised. The scheme can thus be seen as effectively performing dimension reduction on the received signal. Signal dimension reduction is also employed in [9] using an analog beamforming stage, which is followed by a digitally compression stage. The dimension reduction concept has parallels to the downlink sparse beamforming approach [10], in which each transmitter only transmits to a subset of the users, reducing the number of data streams that need to be transferred over fronthaul.

Index Terms—distributed MIMO, C-RAN, fronthaul compression, dimension reduction

I. INTRODUCTION

In a distributed multiple input multiple output (MIMO) uplink system, K users are jointly served by L receivers (or remote radio heads), each equipped with M antennas and distributed geographically within the service area. This distribution of antennas provides macro-diversity and improves uniformity of service, and is facilitated by the recent shift towards a cloud radio-access-network (C-RAN) architecture, in which processing for multiple receivers is performed at a single central processor (CP). A significant practical challenge with C-RAN MIMO, however, is the transfer of data from the receivers to the CP – the large data rates associated with the transfer of raw IQ samples [1], combined with a growing interest in replacing fixed fibre with reduced capacity wireless point-to-point connections [2] resulting in a need for efficient lossy compression of the received signals.

The challenge of data compression for fronthaul constrained MIMO networks has received much research attention, see e.g. [3] and references therein. We restrict our attention to the uplink compress-and-forward architecture, in which compression is applied at the receivers before forwarding to the CP for global symbol detection. For simplicity, schemes in which each receiver independently compresses and forwards its own signal, e.g. [4], are attractive and currently implemented in practical systems, but do not exploit the inherent dependencies between signals at different receivers and therefore do not efficiently make use of the available fronthaul. On the other hand, the best performance is achieved through the use of distributed source coding techniques, in which signals at all receivers are jointly compressed and decoded, for example [5], but these have increased computational complexity. Point-to-point compression schemes in which signals at each receiver are separately compressed and decoded, but using compression codebooks that are jointly designed to exploit dependencies between receivers, present an attractive compromise.

The optimal point-to-point compression scheme for the Gaussian channel involves solving a non-convex optimization to find the quantisation noise covariances for each receiver, using, for example, a successive convex approximation approach [6]. However, this method does not scale well to large networks with rapidly changing channels. Sub-optimal approaches include [7] and [8], which apply transform coding, with a local decorrelating transform applied to the signals, followed by a centrally performed rate allocation stage. An interesting observation in [7] is that at lower fronthaul rates the optimal rate allocation is sparse - only a subset of available signal components at each receiver are quantised. The scheme can thus be seen as effectively performing dimension reduction on the received signal. Signal dimension reduction is also employed in [9] using an analog beamforming stage, which is followed by a digital compression stage. The dimension reduction concept has parallels to the downlink sparse beamforming approach [10], in which each transmitter only transmits to a subset of the users, reducing the number of data streams that need to be transferred over fronthaul.

Paper Overview

In this paper we focus on systems with single antenna users, and an overall excess of receive antennas, $ML \gg K$. We propose a fully-digital dimension reduction based compression scheme, in which each receiver reduces its signal dimension
by filtering its signal in the direction of a subset of $N < K$ users before applying local compression to the signals and forwarding them to the CP. The key feature of this scheme is that the lossy dimension reduction stage produces a reduced number of signal components with reduced inter-receiver dependencies, such that local signal compression can be applied efficiently.

The paper makes the follow contributions:

- a greedy algorithm for selecting the dimension reduction MF vectors is proposed.
- a transform coding compression algorithm is outlined.
- the scheme is adapted for the case of imperfect CSI.
- capacity equations are provided for both optimal and linear symbol detection.
- numerical results for Rayleigh fading channels are given, showing that:
  - a significantly reduced signal dimension can be used at each receiver whilst only losing a small proportion of the total information captured by the full dimension signal.
  - the scheme significantly outperforms local compression at all fronthaul rates, especially at high SNR.
  - with a low signal dimension, the proposed scheme can operate very close to the cut-set upper bound in the fronthaul-limited region.
  - good performance is also achieved under linear symbol detection, and with imperfect CSI at the receivers.

The paper is structured as follows: Section II outlines the system model and configurations used for numerical examples, with Section III providing a rationale for dimension reduction and outlining the overall scheme. Section IV outlines the greedy dimension reduction algorithm and gives insights into its behaviour before Section V outlines the transform coding compression scheme. Section VI gives capacity equations. Section VII adapts the dimension reduction compression scheme for the case of imperfect CSI and Section VIII recommends some modifications to the system for practical implementation. Finally, Section IX provides numerical results.

II. SYSTEM MODEL

A. System Model

We consider an uplink system in which $L$ distributed MIMO receivers, each equipped with $M$ antennas, jointly serve $K$ single antenna users, where there is an overall excess of receive antennas, $ML \gg K$. The receivers have digital processing capability and are connected via individual fronthaul links with capacity $R$ bits per channel use (bpcu) to a central processor (CP), which uses signals from all of the receivers to jointly detect and decode the transmitted user symbols.

The received uplink signal at receiver $l$ is given by

$$y_l = H_l x + \eta,$$

where $H_l \in \mathbb{C}^{M \times K}$ is the channel to receiver $l$, $\eta$ additive white Gaussian noise with unit variance

$$\eta \sim \mathcal{CN}(0, I_M),$$

and $x$ independent Gaussian uplink symbols with signal-to-noise ratio (SNR) $\rho$

$$x \sim \mathcal{CN}(0, \rho I_K).$$

Column $k$ of $H_l$ is the channel vector between user $k$ and receiver $l$

$$H_l = [h_{l,1} \ldots h_{l,K}],$$

where we assume that the $H_l$ are full rank, i.e. $t = \min(M, K)$. Each receiver has access to local CSI, and the CP has access to CSI as applicable.

B. Numerical Example Configurations

Illustrative numerical examples provided throughout this paper consider a single cell with the users and receivers positioned randomly within a $200m \times 200m$ area, with user height 1 m and receiver height 6 m. The user channels follow complex normal independent fading $h_{l,k} \sim \mathcal{CN}(0, p_k \beta_{l,k} I_M)$ where $p_k$ is the uplink power control coefficient for user $k$, and $\beta_{l,k}$ follows a log-distance path loss model with path loss exponent 2.9 and shadow fading 5.7 dB [11]. Power control is applied such that the total average received power for each user is the same

$$\frac{1}{ML} E\left[\sum_l \|h_{l,k}\|^2\right] = p_k \sum_{l=1}^L \beta_{l,k} = 1.$$

All mean quantities are averaged over both channel realisation and user & receiver locations. Note that the general methods
outlined in this paper do not rely on any channel model assumptions.

III. DIMENSION REDUCTION FOR SIGNAL COMPRESSION

Consider local signal compression, where the compressed signal, \( \tilde{y}_t \), is chosen to maximise the information it provides about the user symbols

\[
\max_{\tilde{y}_t} \mathcal{I}(\tilde{y}_t; x) \quad \text{subject to} \quad \mathcal{I}(\tilde{y}_t; y_t) \leq \mathcal{R}.
\]

(6)

In [12] it is shown that this is achieved by a transform coding approach in which a local decorrelating transform is applied at each receiver to produce \( t \) signal components, which are quantised using \( t \) scalar quantisers with appropriate local rate allocation. The resulting quantisation noise power for each signal component decreases approximately exponentially (see Section V) with \( \mathcal{R}/t \)

\[
\text{quantisation noise } \sim 2^{-\mathcal{R}/t}.
\]

(7)

However, in a distributed MIMO system, the received signals are inherently correlated through their dependence on \( x \), and local compression performs poorly. If at each receiver we take a reduced number \( N < t \) of signal components, and apply local compression, the quantisation noise can be reduced,

\[
\text{quantisation noise } \sim 2^{-\mathcal{R}t/N}.
\]

(8)

Clearly, this dimension reduction causes a loss of system capacity. However, this is not inherently problematic, since any signal compression process necessarily entails a capacity loss. If the signal components are chosen at a global level to account for inter-receiver signal dependencies, the information loss due to dimension reduction can be kept small, whilst overall system capacity increased due to the reduction in quantisation noise.

The physical distribution of users and receivers means that if \( N \) signal components are chosen at one receiver that provides a lot of information about the signals of \( N \) users to which it has strong channels, the additional information about those users that receivers with weaker channels to them can provide is small. By appropriately choosing the signal component we can therefore expect that a reduced number of signal dimensions can capture a high proportion of the channel capacity.

A more in-depth look at the reduced dimension signal compression concept is provided in [13].

Proposed Scheme

Based on the insights above, a good approach is for each receiver to filter its received signal in the direction of a subset of \( N < t \) of the users. This can be achieved by matched filtering using channel vectors associated with a subset of users,

\[
z_t = F_t^\dagger y_t,
\]

(9)

where the \( N \) columns of \( F_t \) are the channel vectors for the subset of selected users, \( \mathcal{S}_t \), at receiver \( l \),

\[
F_t = [h_{l,\mathcal{S}_t(1)} \ldots h_{l,\mathcal{S}_t(N)}].
\]

(10)

We pick the MF vectors for each receiver at the CP using global CSI, to maximise the joint mutual information provided by the reduced dimension signals

\[
\max_{\mathcal{S}_1,\ldots,\mathcal{S}_L} \mathcal{I}(z_1,\ldots,z_L; x).
\]

(11)

The receivers then perform local signal compression on the reduced dimension signals for transfer to the CP,

\[
\max_{\tilde{z}_l} \mathcal{I}(\tilde{z}_l; x) \quad \text{subject to} \quad \mathcal{I}(\tilde{z}_l; z_l) \leq \mathcal{R}.
\]

(12)

The sum capacity of the distributed MIMO system is given by

\[
C_{\text{sum}} = \mathcal{I}(\tilde{z}_1,\ldots,\tilde{z}_L; x).
\]

(13)

IV. MF-BASED DIMENSION REDUCTION

Each receiver applies a dimension reduction filter to its received signal

\[
z_t = F_t^\dagger y_t.
\]

(14)

Using the QR decomposition this filter may be written

\[
F_t = Q_tR_t
\]

(15)

where \( Q_t \in \mathbb{C}^{M \times N} \) has orthonormal columns, \( q_{t,i} \), and \( R_t \in \mathbb{C}^{N \times N} \) is upper triangular. If the columns of \( F_t \) are linearly independent, \( R_t \) is invertible, and therefore by the data processing inequality

\[
\mathcal{I}(z_1,\ldots,z_L;x) = \mathcal{I}(\tilde{z}_1,\ldots,\tilde{z}_L;x)
\]

(16)

where

\[
z_t = Q_t^\dagger y_t = Q_t^\dagger H_t x + \eta
\]

(17)

i.e. the information in the filtered signal depends only on the \( N \) dimensional-subspace spanned by the selected user vectors. The columns of \( Q_t \) may be calculated iteratively using the Gram-Schmidt procedure

\[
q_{t,i} = \frac{P_{t,i}h_{t,\mathcal{S}_t(i)}}{\|P_{t,i}h_{t,\mathcal{S}_t(i)}\|}
\]

(18)

where

\[
P_{t,i} = I_M - \sum_{j<i} q_{t,j}q_{t,j}^\dagger
\]

(19)

The joint mutual information is

\[
\mathcal{I}(\tilde{z}_1,\ldots,\tilde{z}_L;x) = \log_2 \det (I_K + \rho \sum_{l=1}^L \sum_{i=1}^N H_{l,i}q_{t,i}q_{t,i}^\dagger H_{l,i}^\dagger)
\]

(20)

The problem of selecting the optimal set of users vectors for all receivers is combinatorial with \( \binom{K}{N}^L \) possible combinations, and hence an exhaustive search is prohibitive. A more tractable approach is to use a greedy algorithm to select the user vectors one at a time, such that each selection stage maximises the mutual information.
A. Greedy Algorithm

If after \( n \) stages the set of selected MF vectors at receiver \( l \) is \( S_l^{(n)} \), the joint mutual information is

\[
\log_2 \det \left( I_K + \rho \sum_{i=1}^{L} S_l^{(n)} H_i^* q_{l,i} q_{l,i}^* H_i \right),
\]

which may be written using the matrix determinant lemma

\[
\log_2 \det \left( A_n^{-1} \right) + \log_2 \left( 1 + \rho q_{l,i}^* H_i A_n^{-1} H_i^* q_{l,i} \right)
\]

where

\[
A_n^{-1} = (I_K + \rho \sum_{i=1}^{L} S_l^{(n-1)} H_i^* q_{l,i} q_{l,i}^* H_i)^{-1}
\]

Substituting (18), the information at stage \( n \) is maximised by choosing the user vector at receiver \( l \) that maximises

\[
\max_{k \notin S_l^{(n-1)}} \frac{h_{l,k}^* P_l^H A_n^{-1} H_i^* P_l h_{l,k}}{\|P_l h_{l,k}\|^2}.
\]

The \( A_n^{-1} \) matrix can then be updated using a rank-1 update

\[
A_n = A_n^{-1} - \frac{A_n^{-1} h_{l,k}^* q_{l,k} q_{l,k}^* H_i A_n^{-1}}{1 + q_{l,k}^* H_i A_n^{-1} H_i q_{l,k}}.
\]

This greedy selection can be carried out in a round-robin manner, selecting a MF vector for each receiver in turn, as shown in Algorithm 1. We refer to this as the matched-filter Gram-Schmidt (MF-GS) algorithm. For good performance, the

### Algorithm 1 MF-GS Algorithm

**inputs:** \( H_i \) \( \forall l \)

**A** ← \( I_K \)

**P_l** ← \( I_M \) \( \forall l \)

\( S_l[1 : N] \) ← 0 \( \forall l \) **sets of select user vectors**

for \( n = 1 : N \) do

for \( l = 1 : L \) do

\[
k^l' \leftarrow \arg\max_{k \notin S_l} \frac{h_{l,k}^* P_l^H A_n^{-1} h_{l,k}^* P_l h_{l,k}}{\|P_l h_{l,k}\|^2}
\]

\( q^l \leftarrow \frac{P_l h_{l,k'}}{\|P_l h_{l,k}\|} \) **select vector**

\( S_l[n] \leftarrow k^l' \) **index of selected user vector**

**A** ← **A** - \( \frac{A_n^{-1} H_i q_{l,i} q_{l,i}^* H_i A_n^{-1}}{1 + q_{l,i}^* H_i A_n^{-1} H_i q_{l,i}} \) **rank-1 inverse update**

**P_l** ← **P_l** - \( q_{l,i} q_{l,i}^* H_i A_n^{-1} H_i q_{l,i} \) **update projection matrix**

end for

end for

**outputs:** \( S_l \)

number of signal components available to the CP must be at least the number of users, i.e. \( N \geq K/L \).

B. Algorithm Behaviour

We now provide some insights into the behaviour of the MF-GS algorithm. Dropping subscripts for clarity, at each selection stage, the mutual information is increased by

\[
\log_2 \left( 1 + \rho q^* H A H^* q \right)
\]

where \( A \) can be written using the eigendecomposition

\[
A = U (I_K + \rho Y)^{-1} U^T
\]

with \( Y \) a diagonal matrix containing the \( K \) ordered eigenvalues, \( v_i \), of the equivalent channel \( (\sum_j H_j^* q_j q_j^* H_j) \), and \( U = [u_1 \ldots u_K] \) the corresponding eigenvectors. Defining the normalised signal power, \( \gamma \), and normalised vector, \( c \),

\[
\gamma = q^* H H^* q, \quad c = \frac{H^* q}{\|H^* q\|}
\]

the mutual information increase can be written

\[
\log_2 \left( 1 + \rho q^* H A H^* q \right) = \log_2 \left( 1 + \gamma \sum_{i=1}^{K} \rho |u_i|^2 \frac{1}{1 + \rho v_i} \right)
\]

where \( u_i^* c \) is the projection of \( c \) onto eigenvector \( i \), with

\[
\sum_{i=1}^{K} |u_i|^2 = 1.
\]

From (29) we can observe that:

1. simply selecting the candidate vector that contains the most signal power (large \( \gamma \)) is a sub-optimal strategy. If the eigenvalue spread of the equivalent channel is large, then signals that project mainly onto the weaker eigenvectors may be selected, despite having lower power. Hence it is not generally optimal to just select the MF vectors corresponding to the \( N \) strongest user channels at each receiver.

2. for a given signal power, \( \gamma \), the best possible candidate signal lies parallel to the eigenvector associated with the smallest eigenvalue, i.e. \( c = u_K \). This optimal vector can be shown to increase the smallest eigenvalue of the equivalent channel from \( v_K \) to \( v_K + \gamma \). Similarly, the worst possible candidate signal lies parallel to the largest eigenvector, \( c = u_1 \), increasing the largest eigenvalue from \( v_1 \) to \( v_1 + \gamma \). Furthermore, it can be shown that any choice of \( c \) gives an updated equivalent channel with all \( v_i' \geq v_i \).

3. when the \( v_i \) are large, the information provided by additional signal components reduces, and for \( \rho v_i \gg 1 \) is independent of \( \rho \).

From these observations we can expect that at each stage the MF-GS algorithm will generally act to make selections that increase the smaller eigenvalues of the equivalent channel, and tend to produce a full rank equivalent channel matrix (all \( v_i > 0 \)) when \( LN \geq K \). As more selections are made and the channel eigenvalues increase, the capacity will grow more slowly with each selection, and hence there are diminishing
returns from increasing $N$. For large $\rho$ (and all $\nu_j > 0$) the information loss due to dimension reduction is independent of $\rho$, and hence the proportion of information lost vanishes as $\rho \to \infty$, as shown in Figure 2 (where $t = 8$). We see that even with small $N$, a high proportion of the available information can be captured.

![Figure 2](image)

**V. SIGNAL COMPRESSION**

The reduced dimension signals are compressed at each receiver separately using locally optimal compression. We may equivalently compress either $z_l$ or $\tilde{z}_l$, choosing $\tilde{z}_l$ for simplicity of analysis.

\[
\begin{align*}
\text{maximise} & \quad I(\tilde{z}_l; x) \\
\text{subject to} & \quad I(\tilde{z}_l; z_l) \leq R,
\end{align*}
\]

As discussed above, this is achieved by applying a linear decorrelating transform to $\tilde{z}_l$ to produce a set of independent variables which are then independently quantised using $N$ scalar quantisers [12],

\[
\tilde{z}_l = V_l^T \tilde{z}_l + \delta_l,
\]

where $\delta_l \sim CN(0, \Phi_{l})$ is the resulting quantisation noise, with diagonal covariance

\[
[\Phi_{l}]_{i,i} = \frac{[V_l^T E[\tilde{z}_l \tilde{z}_l^T] V_l]_{i,i}}{2^{\eta_l/2} - 1}
\]

\[
= \frac{\rho \lambda_{l,i} + 1}{2^{\eta_l/2} - 1}
\]

where $V_l A_l V_l^T = Q_l H_l^T H_l Q_l$, with $A_l = \text{diag} (\lambda_{l,i})$. The optimal rate allocation is given by the waterfilling solution

\[
\rho_{l,i} = \left[ \frac{R + \log_2 (1 + \eta_l)}{N_l} - \frac{1}{N_l} \sum_{j=1}^{N_l} \log_2 (1 + \eta_j) \right]^{+}
\]

where $[a]^{+} = \max(0, a)$ and $N_l$ is the corresponding number of $r_{l,i} > 0$. Note that for the full dimension case ($N = t$) the $\lambda_{l,i}$ are the eigenvalues of $\Phi_{l}$. Assuming $R$ is sufficiently large that all $N$ dimensions are quantised ($N_l = N$) substituting (35) into (34) the quantisation noise power is approximately

\[
[\Phi_{l}]_{i,i} \approx \rho \left( \prod_{j=1}^{N} \lambda_{l,j} \right)^{1/N} 2^{-R/N},
\]

which is tight for $\rho \lambda_{l,i} \gg 1$, $2^{R/N} \gg 1$.

**VI. ACHIEVABLE RATES**

The combined action of the propagation channel, dimension reduction filter and decorrelating transform can be described by an equivalent channel

\[
\tilde{z}_l = V_l^T \Phi_{l} + \eta + \delta_l
\]

\[
= G_l x + \eta + \delta_l
\]

The sum capacity is then given by

\[
C_{\text{sum}} = \log_2 \det \left( \Phi_{l} + I_{N_l} \right)^{-1},
\]

where $\text{SQINR}_{k}$ is the signal-to-quantisation-plus-interference-plus-noise ratio of user $k$:

\[
\text{SQINR}_{k} = \frac{\rho}{\left[ \left( I_{K} + \rho \sum_{i=1}^{L} G_l^T (\Phi_{l} + I_{N_l})^{-1} G_l \right) \right]_{k,k}^{-1}}.
\]

**VII. IMPERFECT CSI**

Assuming MMSE channel estimation, the dimension reduction method can be readily adapted for the case of imperfect CSI at the receivers. The channel may be written,

\[
\bar{H}_l = H_l + E_l
\]

where $\bar{H}_l$ is the channel estimate and $E_l$ the channel estimation error. For a given channel realisation and estimate, $E_l$ is fixed.
we assume that these variances are also not perfectly known. For analytical tractability, here be lower bounded small 15.

A small mismatch in input variance the performance loss is This is reasonable since it is known that for quantisers with equivalent noise term, unknown channel may be absorbed into an uncorrelated equivalent noise calculated using the eigenvectors and eigenvalues of ...

Using the method outlined in 14 the signal through the whitened signal, y_l, using equivalent whitened channel vectors, H_l,k = \Omega l^{-1/2} h_l,k.

Transform coding compression is then applied as described in Section V, with decorrelating transform and rate allocation calculated using the eigenvectors and eigenvalues of Q_l H_l H_l^T Q_l. However, since for a given channel realisation E_l is unknown, the variance of the scalars being quantised are also not perfectly known. For analytical tractability, here we assume that these variances are perfectly known, and accordingly calculate the quantisation noise covariance as

\begin{equation}
[\Phi_l]_{k,d} = \frac{\left( V_l Q_l \Omega_l^{-1/2} (\rho H_l H_l^T + I_M) \Omega_l^{-1/2} Q_l V_l \right)_{k,d}}{2^{l_1} - 1}
\end{equation}

This is reasonable since it is known that for quantisers with a small mismatch in input variance the performance loss is small 15.

By the reasoning in 14 the mean sum capacity can then be lower bounded

\begin{equation}
E \left[ C_{\text{sum}}^\text{CSS} \right] \geq E \left[ \log_2 \det \left( I_K + \rho \sum_{l=1}^L \tilde{G}_l (\Phi_l + I_N)^{-1} \tilde{G}_l^T \right) \right],
\end{equation}

where

\begin{equation}
\tilde{G}_l = V_l Q_l^{-1/2} H_l.
\end{equation}

Capacity bounds under linear detection are similarly found by replacing G_l with \tilde{G}_l in (42).

B. Fixed-rate Scalar quantisation

The analysis in Section V assumes the use of optimal Gaussian scalar compression, requiring long block lengths and complex encoders and decoders. Fixed rate Lloyd-Max scalar quantiser achieves the same quantisation noise using an additional 1.4 bits per scalar 16, but with unit block length, and represents an attractive alternative for practical implementation.

IX. NUMERICAL RESULTS

Figure shows the rate-capacity curves for the reduced compression scheme for different signal dimensions, N. For comparison, the cut set bound,

\begin{equation}
C_{\text{sum}} \leq \min \left( RL, I(y_1, \ldots, y_L; x) \right),
\end{equation}

is shown, which represents an upper bound for all compression schemes. Using N = 2 gives the highest capacity in the rate limited region due to the lower quantisation noise, and operates close to the cut-set bound. At higher fronthaul rates, the capacity is limited due to the reduced dimension, and N must be increased to increase capacity.

A. Sum Capacity

Figure 4 shows the overall rate-capacity performance of the scheme, where for each value of R, N is chosen to maximise sum capacity. In practice this involves computing the equivalent channels for different values of N, which is simplified by noting that the MF-GS algorithm selects the same first n' MF vectors for any N \geq n', and for a given R we need only evaluate for a small range of N.

We see that the scheme significantly outperforms standard local signal compression at all rates, and operates close to the cut-set bound in the rate limited region. The relative performance improvement of the scheme at high SNR can be understood with reference to Figure 2, since at high SNR a small value of N is able to capture an increased proportion of the total capacity.
Fig. 3. Rate-capacity performance for varying signal dimensions with $K = 8$, $L = 4$, $M = 8$, $\rho = 15$ dB.

Fig. 4. Rate-capacity performance for varying SNRs with $K = 8$, $L = 4$, $M = 8$.

B. User Rates

Figure 5 shows the mean and 5% outage user capacities under dimension reduction compression. We see that the scheme offers a significant gain in both mean and outage capacity compared to local compression, for example an improvement of around 1.5 bpcu per user is achieved at $\mathcal{R}L = 100$ bpcu.

C. Imperfect CSI

Figure 6 shows the lower bound on sum capacity when MMSE channel estimation is performed using orthogonal uplink pilots with signal-to-noise ratio $\rho_{pl}$. When the CSI is good, the rate-capacity curve shows a similar shape to the perfect CSI case. With lower quality CSI, a fronthaul rate penalty is incurred due to the increased proportion of channel estimation error noise in the quantised signal.

X. Conclusion

In this work we have outlined a signal compression scheme for fronthaul-constrained distributed MIMO systems, based on applying dimension reduction prior to signal quantisation. Numerical examples demonstrate that the proposed dimension reduction algorithm is able to significantly reduce the number of signal components required at each receiver, and therefore significantly increase the rate-capacity performance of the scheme relative to local compression schemes - operating close to the cut-set capacity bound when the signal dimension is small. We further show that the scheme can be readily adapted for the case of imperfect CSI, and provide some practical suggestions for ways in which the signalling overheads and complexity of the quantisers can be reduced for implementation in practical systems.
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