Search for the stochastic gravitational-wave background induced by primordial curvature perturbations in LIGO’s second observing run

Shasvath J. Kapadia,\textsuperscript{1} Kanhaiya Lal Pandey,\textsuperscript{1} Teruki Suyama,\textsuperscript{2} Shivaraj Kandhasamy,\textsuperscript{3} and Parameswaran Ajith\textsuperscript{1,4}
\textsuperscript{1}International Centre for Theoretical Sciences, Tata Institute of Fundamental Research, Bangalore 560089, India
\textsuperscript{2}Department of Physics, Tokyo Institute of Technology, 2-12-1 Ookayama, Meguro-ku, Tokyo 152-8551, Japan
\textsuperscript{3}Inter-University Centre for Astronomy and Astrophysics, Pune 411007, India
\textsuperscript{4}Canadian Institute for Advanced Research, CIFAR Azrieli Global Scholar, MaRS Centre, West Tower, 661 University Ave, Toronto, ON M5G 1M1, Canada

Primordial density perturbations generate a stochastic background of gravitational waves (GWs) through non-linear mode couplings upon their horizon reentry. In this Letter, we report on a search for such a stochastic GW background in the data of the two LIGO detectors during their second observing run (O2). We focus on the primordial perturbations in the range of comoving wavenumbers $10^{16} \sim 10^{19} \text{Mpc}^{-1}$ for which the stochastic background falls within the detectors’ sensitivity band. We do not find any conclusive evidence of this stochastic signal in the data, and thus place the very first GW-based constraints on the amplitude of the power spectrum at these scales. We assume a lognormal shape for the power spectrum and Gaussian statistics for the primordial perturbations, and vary the width of the power spectrum to cover both narrow and broad spectra. Derived upper limits (95\%) on the amplitude of the power spectrum are $0.01 \sim 0.1$. As a byproduct, we are able to infer upper limits on the fraction of the Universe’s mass in ultralight primordial black holes ($M_{\text{PBH}} \sim 10^{-20} \sim 10^{-19} M_{\odot}$) at their formation time to be $\lesssim 10^{-25}$. If Hawking evaporation is discarded, this can be translated to constraints on the fraction of dark matter today in the form of such ultralight primordial black holes, which can be as stringent as $\sim 10^{-15} \sim 10^{-5}$.

\textbf{Introduction:—} Cosmological observations have revealed that all the structures in the present Universe originate from the primordial density perturbations (equivalently, curvature perturbations). According to the theory of inflation, which constitutes a pillar of modern cosmology, the primordial perturbations are created by the amplification of the quantum fluctuations of the scalar fields during inflation and existed over a wide range of length scales from meter size at the smallest scale up to at least the Hubble horizon on the largest scale\textsuperscript{[1]}. Knowledge of the primordial perturbations is crucial to test inflation models and physics of the early Universe.

Observations of the cosmic microwave background (CMB) and the large-scale structure have successfully measured the power spectrum of the primordial perturbations on large scales as $P_\ell \approx 2 \times 10^{-9}$\textsuperscript{[2]} with a small scale-dependence. However, much less is known of the primordial perturbations on smaller scales. At $O(0.1 \text{kpc})$, non-detection of the CMB spectral distortion places an upper limit of $P_\ell \lesssim 10^{-4}$ (see\textsuperscript{[3]} and references therein). Success of the big bang nucleosynthesis provides $P_\ell \lesssim 10^{-2}$ for a range $0.01 \text{kpc} \sim 0.1 \text{kpc}$\textsuperscript{[4–6]}. Non-detection of primordial black holes (PBHs) yields a similar level of constraints $P_\ell \lesssim 10^{-2}$ for a wide range of scales (e.g.\textsuperscript{[7]}).

Stochastic gravitational waves (GWs), which is a target of this Letter, have been attracting a lot of interest recently as a powerful probe of the primordial perturbations (e.g.\textsuperscript{[8]}). Theoretically, it has been known for a long time that at the second order in the cosmological perturbation, the mode-mode couplings of the primordial curvature perturbations induce a stochastic GW background\textsuperscript{[9, 10]}. It was suggested in\textsuperscript{[11]} that future GW detectors can be used to constrain the primordial perturbations on very small scales. In\textsuperscript{[12]}, it was pointed out that GW observations can constrain the PBHs as dark matter candidates. Ref.\textsuperscript{[8]} provides the updated summary of the expected constraints on the small-scale primordial perturbations by the current/planned GW observations. Although there are many theoretical or observational-prospect studies on such stochastic GWs, no observational tests by using real GW data have been given in the literature.

In this Letter, following our previous paper\textsuperscript{[13]} that explored the detection prospects for the isotropic stochastic GWs induced by the primordial perturbations, we report the results of the very first search for this signal in LIGO data from the second (O2) observing run\textsuperscript{[14]}. This stochastic background in LIGO’s sensitive band corresponds to the primordial perturbations in the comoving wavenumber $10^{16} \text{Mpc}^{-1} \leq k \lesssim 10^{18} \text{Mpc}^{-1}$. In the following analysis, we assume that the power spectrum of the primordial curvature perturbations has a lognormal shape defined by Eq.\textsuperscript{(2)} which is characterized by three parameters: $a$ (amplitude), $k_0$ (comoving wavenumber at the peak of the power spectrum), and $\sigma$ (width). We also assume that the primordial curvature perturbations obey Gaussian statistics. Our analysis can be easily extended for other shapes of the power spectrum and the non-Gaussian primordial perturbations. We employ the cross-correlation search which is optimal for stationary and isotropic backgrounds that obey Gaussian statistics\textsuperscript{[15–18]}. Using the values of the cross-correlation statistic and its variance released by the LIGO-Virgo collaboration for the first and second observing runs\textsuperscript{[19, 20]}, we estimate signal-to-noise ratios (SNRs) on a $k_0 - \sigma$ grid pertaining to the range of the comoving wavenumbers mentioned above, and a range of widths spanning both narrow and broad power spectra ($0.01 \leq \sigma \leq 10$).

We do not find any conclusive evidence for the presence of this GW background in the data (all SNRs $\leq 2.7$). We therefore place upper limits on the amplitude of the curvature power spectrum using Bayesian parameter estimation where the likelihood is constructed from the cross-correlation and the assumed model of the stochastic background\textsuperscript{[21]}. We find that 95\% upper limits on the power spectrum amplitude span about $0.01 \sim 0.1$ for the majority of the parameter space considered.

As an important byproduct of the derived upper limits, we are able to place upper limits on the PBH abundance in the mass range $10^{-20} \sim 10^{-19} M_{\odot}$. The existing upper limits\textsuperscript{[22, 23],
and the references therein], which consider effects of radiation coming from the evaporating black holes, rely on the assumption of the Hawking radiation [24]. Since Hawking radiation has not been verified experimentally, an independent set of constraints coming from GWs might be of interest. Our upper limits on the fraction of dark matter in the form of these ultralight PBHs ($\rho_{\text{PBH}} \equiv \Omega_{\text{PBH}}/\Omega_{\text{DM}}$) can be as low as $10^{-15} - 10^{-5}$ for narrow power spectra ($0.01 \leq \sigma \leq 0.5$). Independent of the consideration of Hawking radiation, our results constrain the fraction $\beta^*$ of the Universe’s mass in the form of these ultralight PBHs at their formation time ($\beta^* \propto \rho_{\text{PBH}}/\rho$; see, e.g., [25]).

Search for the stochastic GW background:— The isotropic stochastic GW background can be described in terms of the energy density fraction $\Omega_{GW}$ per logarithmic frequency bin:

$$\Omega_{GW}(f) = \frac{1}{\rho_c} \frac{d\rho_{GW}}{d\log(f)},$$  

where $\rho_{GW}$ is the energy density of GWs and $\rho_c$, the critical energy density required for a flat Universe. If the GWs are sourced by scalar-scalar mode couplings in primordial curvature perturbations, $\Omega_{GW}(f)$ would depend on the shape of the curvature power spectrum. Here, we assume the power spectrum to be of log-normal shape, parametrized by the amplitude $A$, central wave number $k_0$ and width $\sigma^1$:

$$P_c(k) = \frac{A}{\sqrt{2\pi}\sigma} \exp\left(-\frac{\log^2(k/k_0)}{2\sigma^2}\right)$$

where $k$ is the comoving wave number that sets the spatial scale. Since $k_0$ depends on the PBH mass-scale $M_{\text{PBH}}$ [8, 27], we can also use $M_{\text{PBH}}$ to parametrize the power spectrum instead of $k_0$.

The search for a stationary, Gaussian, unpolarized, and isotropic stochastic GW background involves the calculation of the following cross-correlation statistic $\hat{C}(f)$ from the data of two detectors [20] $^2$:

$$\hat{C}(f) = \frac{2}{T} \operatorname{Re} \left\{ \mathcal{S}_\text{ref}(f) \right\}$$

where $\mathcal{S}_\text{ref}(f)$ are the Fourier transforms of the time series data $s_i(t)$ from detector $i = \{1, 2\}$, $T$ is the duration of the data used to compute the Fourier transform, $\gamma(f)$ is a geometric factor, called the overlap reduction function, that depends on the relative orientation of the detectors, while $\mathcal{S}_\text{ref}(f)$ is the spectral shape for a stochastic GW background with a flat spectrum ($\Omega_{GW} = 2/5$). The expectation value and the variance of $\hat{C}(f)$ are given by:

$$\langle \hat{C}(f) \rangle = \Omega_{GW}(f), \quad \sigma^2_{\hat{C}(f)} \approx \frac{1}{2T\Delta f} \frac{P_c(k)P_c(\tau^\prime)}{\gamma_\text{C}(f)^2 \mathcal{S}_\text{ref}^2(f)}.$$  

where $P_c(f)$ are the one-sided power spectral density of the noise in the two detectors (assumed to be Gaussian) and $\Delta f$ is the frequency resolution of the discrete Fourier transform.

The SNRs don’t exceed $\approx 2.7$; we therefore do not find any conclusive evidence of a signal consistent with the $\Omega_{GW}$s pertaining to the model parameter grid considered here.

FIG. 1: Search SNR evaluated on a grid of $\log_{10}(k_0/\text{Mpc}^{-1})$--$\log_{10}(\sigma)$. The SNRs don’t exceed $\approx 2.7$; we therefore do not find any conclusive evidence of a signal consistent with the $\Omega_{GW}$s pertaining to the model parameter grid considered here.

$^1$ The relation between the log-normal curvature power spectrum and the GW background is complicated and does not in general have a closed form. See, for example, [13], for a summary, and [26, 27] for details.

$^2$ Strictly speaking, Eq. (3) should be interpreted as an average over multiple frequency bins $\Delta f$, where the cross-correlator in each bin is given by:

$$\hat{C}(f) = \frac{1}{2T\Delta f} \frac{d\mathcal{S}(f)}{d\log(f)} \frac{d\mathcal{S}_\text{ref}(f)}{d\log(f)}.$$

For the O2 stochastic search, $\Delta f = 0.03125$ Hz, $T = 192$ sec., and the total livetime after removing non-stationarities, was 99 days [20].
We find no conclusive evidence of a signal: The maximum with existing ones from other experiments, including from GWs pertaining to compact binary coalescences. A the upper limits on certain mass-scales and amplitude to values as low as ∼10−16. We are able to constrain the in log A mentioned above, from the likelihood described in Eq. (6), and power spectra, does not produce any unexpected biases. We then estimate Bayesian posterior distributions on the parameters are varied to span both narrow and broad noise. With increasing number of noise realizations, the mean estimated from several independent realizations of Gaussian background over the same grid of parameters yields SNRs that are consistent with that reported in Fig. 1. We also evaluate the mean and standard deviation of the distributions of SNRs estimated from several independent realizations of Gaussian noise. With increasing number of noise realizations, the mean (standard deviation) of the distributions approach zero (unity), as expected. This confirms that applying the cross-correlation method described in the previous section to searching for the stochastic backgrounds, pertaining to both narrow and broad power spectra, does not produce any unexpected biases.

We test our analysis on simulated values of Ĉ(fj) for stationary Gaussian noise and find that searching for the stochastic background across the same grid of parameters yields SNRs that are consistent with that reported in Fig. 1. We also evaluate the mean and standard deviation of the distributions of SNRs estimated from several independent realizations of Gaussian noise. With increasing number of noise realizations, the mean (standard deviation) of the distributions approach zero (unity), as expected. This confirms that applying the cross-correlation method described in the previous section to searching for the stochastic backgrounds, pertaining to both narrow and broad power spectra, does not produce any unexpected biases.

We then estimate Bayesian posterior distributions on the amplitude, p(A | Ĉ, σ, k0) over the model parameter grid mentioned above, from the likelihood described in Eq. (6), and two choices of prior, one uniform in A, and the other uniform in log10(A). The upper limits derived from these posteriors are presented in Fig. 2 (left plot). We are able to constrain the amplitude to values as low as ∼0.01 at 95% confidence for certain mass-scales and σ values. Fig. 3 (top panel) compares the upper limits on A (for certain fiducial values of σ = 0.01, 5) with existing ones from other experiments, including from GWs pertaining to compact binary coalescences.

From the upper limits on the amplitude A, upper limits on fPBH can be estimated, neglecting Hawking radiation (see, for e.g., [13] for details). As shown in [13, 26, 27], fPBH is highly sensitive to changes in the amplitude; a change of a factor of 2 could result in a change of many orders of magnitude in fPBH. The results of the conversion from upper limits on A to upper limits on fPBH on the model parameter grid MPBH = σ are summarized in Fig. 2 (right plot). The 95% upper limits on fPBH are rather weak for a large portion of the parameter space considered. Nevertheless, for certain mass-scales between 10−20 − 10−19M⊙ and narrow spectra, upper limits can be as stringent as 10−15 − 10−5.

While the notion of these ultralight PBHs constituting even a fraction of the dark matter in the current cosmological epoch needs to neglect the effect of Hawking evaporation, our results can constrain the fraction β′ of the Universe’s mass in the form of these PBHs at their formation time, independent of the existence/nonexistence of Hawking radiation. Fig. 3 (lower panel) compares our constraints on β′ with existing ones from other experiments. Note that our constraints fall in a mass-range where existing constraints assume Hawking radiation. For a narrow range of PBH masses, our constraints assuming σ = 0.01, 0.1 are comparable, and sometimes marginally stronger than the non-GW ones. These constraints can get significantly stronger for other σ values (Ref. Fig. 2), but again limited to a narrow mass-range.

Summary and outlook:— In this Letter, we report the results from a search for the stochastic background of GWs induced by scalar-scalar mode couplings of primordial curvature perturbations, in the data from the two LIGO detectors collected during their O2 run. We assume a log-normal ansatz for the shape of the curvature power spectrum. The model parameters are varied to span both narrow and broad spectra for which the GW background fall in the LIGO sensitivity band. We find no conclusive evidence of the signals we searched for in the data. The SNRs are all within about 2.7 (excluding trials
We were therefore able to place upper limits on the amplitude $A$ of the curvature power spectrum using Bayesian parameter estimation. At 95% confidence, the upper limits span $\sim 0.01 - 0.1$ for a significant fraction of the wavenumber-scales considered. This limits the fraction $\beta'$ of the Universe’s mass in ultralight PBHs ($M_{\text{PBH}} \approx 10^{-20} - 10^{-19} M_\odot$) at their formation time to be less than $\sim 10^{-25}$, assuming narrow power spectra ($\sigma \leq 0.5$). This means that, even if these black holes exist in the current cosmological epoch (i.e., neglecting Hawking evaporation), they would constitute only a very small fraction of the dark matter ($f_{\text{PBH}} \lesssim 10^{-15} - 10^{-13}$).

To the best of our knowledge, this is the first work that conducts a search for the stochastic GW background induced by scalar-scalar mode couplings in primordial curvature perturbations. In addition, it presents the first (GW data-driven) constraints on the amplitude of the curvature power spectrum, and the corresponding upper limits on the PBH abundance.

Our upper limits in terms of the PBH abundance are stronger than the existing ones (derived from the non-observations of the effects caused by the Hawking radiation from PBHs) only for a narrow parameter region. Nevertheless, our GW-based constraints demonstrate that we have finally entered a new era where GW astronomy brings us meaningful information about the extremely small-scale primordial perturbations and ultralight PBHs, hitherto probed exclusively by experiments which assume the still-unconfirmed mechanism of BH-evaporation. In this sense, our results represent a milestone in bridging early-universe cosmology and GW astronomy.

It is almost certain that non-detection of the stochastic GWs originating from the scalar perturbations by future detectors will tighten the upper limits on the primordial power spectrum and abundance of PBHs by many orders of magnitude [8, 13], thus becoming the most powerful probe of the small-scale perturbations. Since PBH abundance depends quite sensitively on the amplitude of the primordial power spectrum, non-detection of such stochastic GWs will completely exclude PBHs in the

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**FIG. 3:** Top Panel: Upper limits (95%) on the amplitude $A$ of the curvature power spectrum (assuming a log-uniform prior and two fiducial values 0.01 and 5 for $\sigma$), along with upper limits from other experiments [25]. The dashed vertical line corresponds to PBH masses below which Hawking radiation, if exists, should have caused PBHs to evaporate by the current cosmic age. For $\sigma = 0.01$, the constraints from this work are marginally better than existing constraints, for a narrow mass range. For $\sigma = 5$, the constraints are marginally worse. Note however, that our constraints are Hawking-radiation independent, as opposed to other constraints in the same mass-range. Bottom Panel: Upper limits on the abundance of PBHs at the time of their formation, $\beta'$, for $\sigma = 0.01, 0.1$, along with constraints from other experiments [25]. While non-trivial limits, corresponding to $f_{\text{PBH}} < 1$, can only be placed for a narrow range of masses, they are comparable and can even be stronger than existing constraints, which are Hawking radiation-based. (Abbreviations: LSP: Lightest supersymmetric particle, BBN: Big bang nucleosynthesis, CMB: Cosmic microwave background, GGB: Galactic gamma-ray background, EGB: Extra-galactic photon background, CR: Cosmic rays, $f_{\text{PBH}} = 1, \Omega$: Gravitational lensing, GW: GWs from compact binary coalescence events, XB: X-ray background, DF: Dynamical friction, LSS: Large scale structure; see [25] for details. $\mu$ distort; $\mu$ distortion, $y$ distort; $y$ distortion; (deviations of CMB energy spectrum from blackbody spectrum); see [29, 30] for details.)
corresponding PBH mass range irrespective of whether they undergo Hawking evaporation or not. A caveat is that the GW-based constraints on PBHs are indirect and the exclusion of PBHs may be circumvented if the primordial curvature perturbations are strongly non-Gaussian [31].

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