Optimal sliding mode tracking control of spacecraft formation flying with limited data communication

Ruixia Liu, Ming Liu, Xibin Cao and Yuan Liu

Abstract
This article deals with the optimal tracking control problem for spacecraft formation flying via a sliding mode approach in the presence of external disturbances and signal quantization, where both state quantization and input quantization are considered. First, the Gauss pseudospectral method is adopted to solve the multi-objective optimization problem, where performance optimization, thruster amplitude constraints, and collision avoidance are simultaneously taken into consideration. Second, a novel quantized sliding mode control strategy is developed by employing a dynamic logarithmic quantizer to track the obtained optimal trajectories of relative position and velocity. In this design, the quantizer parameters are input into the designed controller to compensate for the signal quantization effects. Under the proposed robust quantized sliding mode control strategy, the resulting closed-loop control system is asymptotically stable with satisfying performance multi-objective constraints. Finally, a simulation example is presented to show the effectiveness of the proposed control design scheme.

Keywords
Spacecraft formation flying, sliding mode control, signal quantization, multi-objective optimization, trajectory tracking

Date received: 7 November 2017; accepted: 16 May 2018

Handling Editor: Xiang Yu

Introduction
Spacecraft formation flying (SFF) has received extensive attention in both theoretical research and practical applications. In SFF applications, a monolithic spacecraft is replaced by multiple micro-spacecraft. The micro-spacecraft is of small quality, low cost, and high reliability. The appropriate use of spacecraft formation can improve the measurement accuracy, extend the lifetime of the on-orbit servicing, and accomplish the tasks that conventional single spacecraft cannot accomplish. The goal of tracking control for SFF is to design a control law, such that the state vectors of the nonlinear dynamics track their desired trajectories with external disturbances. It should be mentioned that, the Clohessy-Wiltshire (C-W) equations is the one of the most popular modeling method, which has been widely used to deal with the problem of linear relative motion between two neighboring spacecraft. In reality, the equations of the relative dynamic model of SFF are nonlinear. Nonlinear control theory provides a good solution to the problem of SFF. Consequently, various control strategies (including robust, optimal, adaptive, and sliding mode controls) have been presented for solving the tracking control problem. Among these control approaches, the sliding mode control method, especially the integral sliding mode control strategy, is widely applied to formation flying systems due to its...
various attractive features, such as distinguished robustness and fast response.\textsuperscript{14} It should be noted that, although the integral sliding mode can eliminate external disturbances effectively, it cannot take performance optimization or state and control input constraints into account simultaneously. In the spacecraft formation control systems design, a considerable number of methods of optimal control have been studied by several researchers.\textsuperscript{15–18} However, to date, the optimal control problem with thruster amplitude and state constraints or non-standard performance indexes have not been thoroughly investigated. Because the Gauss pseudospectral method can provide an exponentially convergent rate for the approximation of analytic functions, while offering Eulerian-like simplicity, and be utilized for a variety of nonlinear constrained optimal control problems, it has become a better method for solving optimal control problems.\textsuperscript{19}

On the other hand, networked control systems (NCSs) is a fundamental research topic that has been widely applied in underwater robot control system,\textsuperscript{20} aerospace engineering,\textsuperscript{21} power industry,\textsuperscript{22} and manipulation robot control systems.\textsuperscript{23} It should be pointed out that, in modern SFF systems, the signal information of different components or spacecraft is transmitted by wired or wireless networks. NCSs possess many advantages over traditional systems including easy installation and maintenance, decreased wiring weight and cost, and so on.\textsuperscript{24–26} However, NCSs also induce a series of network-induced phenomena such as communication delays, packet dropout, data quantization, and distortion.\textsuperscript{27–29} Generally speaking, when the signal information of spacecraft formation dynamic systems is transmitted between star sensor, orbit module, and actuator module over the digital communication network, it inevitably induces quantization errors which will bring essential difficulties and challenges to the state tracking control problems in a networked environment. Hence, it is desirable to develop new control approaches where the data quantization is taken into account. However, quantization behavior makes the analysis and design of the tracking controller difficult and complicated. When it comes to the tracking control problem involving the state and input quantization for spacecraft formation, the related results are few and the design problem is even more difficult when performance optimization, thruster amplitude constraints, and collision avoidance are simultaneously considered. Recently, the quantized control issues for spacecraft attitude control system has been investigated.\textsuperscript{21} Unfortunately, up to now, there is still few research works focused on application of the dynamic logarithmic quantizer for the SFF. Therefore, the aim of this article is to propose a tracking controller for SFF, considering performance optimization, thruster amplitude constraints, and collision avoidance by utilizing an integral sliding mode with the presence of disturbances and signal quantization.

Summarizing the aforementioned discussion, in this article, we aim to investigate a network-based multi-objective sliding mode tracking control problem for SFF with simultaneous presence of external disturbances, state quantization, and input quantization. The main contributions of this article are highlighted as follows: (1) by considering performance optimization, thruster amplitude constraints, and collision avoidance, the Gauss pseudospectral method, which can solve optimal problems with a non-standard performance index or endpoint conditions and path constraints, is employed to obtain optimal trajectories of relative position and velocity; (2) a dynamic logarithmic quantizer is employed to perform the controller design, where quantizer parameters are input into the designed controller to compensate for signal quantization effects; and (3) a robust quantized sliding mode control strategy for the nonlinear dynamics involved in SFF is developed by adopting an integral sliding mode to track the obtained optimal trajectories with external disturbances and signal quantization.

The remainder of this article is organized as follows. In section “Problem formulation,” the problem formulation is given, including nonlinear relative dynamic equations, algebraic graph theory, preliminaries, and the control objective. In section “Optimal trajectory planning,” optimal trajectory planning is accomplished by the Gauss pseudospectral method. In section “Quantizer controller design,” a novel quantized sliding mode control design strategy is designed to track the optimal trajectories. A simulation example is given in section “Simulation results” to demonstrate the effectiveness of the proposed method. Finally, some concluding comments are presented.

**Problem formulation**

**Spacecraft orbit dynamics**

The nonlinear relative motion dynamics of SFF is established, as shown in Figure 1. The SFF system comprises follower and leader spacecraft. The spacecraft is considered to be rigid-body, and a local-vertical-local-horizontal (LVLH) frame is fixed at the center of the leader spacecraft as the reference orbital coordinate.

In the LVLH coordinates, the equation set of the nonlinear relative motion dynamics of SFF can be expressed as follows\textsuperscript{30}
where \( q_{11}(t) = [x_i, y_i, z_i]^T \) is the relative position from the \( i \)th \((i = 1, 2, \ldots, N)\) follower spacecraft to the leader spacecraft in the local coordinate frame; \( u_i(t) = [u_{ix}, u_{iy}, u_{iz}]^T \) is the control input acting on the leader spacecraft; \( u_i(t) = [u_{ix}, u_{iy}, u_{iz}]^T \) is the control input acting on the \( i \)th follower spacecraft; \( m_l \) is the mass of the leader spacecraft; \( m_f \) is the mass of the \( i \)th follower spacecraft; \( w_i(t) = [w_{ix}, w_{iy}, w_{iz}]^T \) denotes the bounded external disturbance of the \( i \)th follower spacecraft; and \( R = (0, r, 0)^T \) is the position vector from the inertial coordinate attached to the center of earth to the leader spacecraft described in the local coordinate frame. In the elliptical reference orbit, \( r \) can be described as follows

\[
   r = \frac{a_e(1 - e^2)}{1 + e \cos(\theta)}
\]

where \( a_e \) is the semi-major axis of the elliptical orbit of the leader spacecraft; \( e \) is the orbital eccentricity of the reference orbit; \( \theta \) is the true anomaly. The derivative of \( \theta \) can be expressed as follows

\[
   \dot{\theta} = \frac{n_e[1 + e \cos(\theta)]^2}{(1 - e^2)^{3/2}}
\]

and then

\[
   \dot{\theta} = \frac{2n^2e_c[1 + e \cos(\theta)]^3 \sin(\theta)}{(1 - e^2)^{5/2}}
\]

where \( n_e = \sqrt{\mu/a_e^3} \) is the mean orbital angular velocity.

Define the relative velocity vector \( q_{22} = [x_i, y_i, z_i]^T \). Then, equation (1) can be rewritten as follows

\[
   \begin{align*}
   \dot{q}_{11}(t) &= q_{22}(t) \\
   \dot{q}_{22}(t) &= A_1 q_{11}(t) + A_2 q_{22}(t) + f(q_{11}(t)) \\
   &+ \frac{1}{m} B(u_i(t) + w_i(t))
   \end{align*}
\]

where

\[
   A_1 = \begin{bmatrix} \theta^2 & 0 & 0 \\ 0 & \theta & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 2\dot{\theta} & 0 \\ -2\dot{\theta} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},
\]

\[
   B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
\]

and the nonlinear function \( f(q_{11}(t)) \in \mathbb{R}^3 \) is defined as follows

\[
   f(q_{11}(t)) = \begin{bmatrix} -\frac{\mu x_i}{\|R + q_{11}(t)\|^3} - \frac{u_{ix}}{m_l} \\ -\frac{\mu y_i + r}{\|R + q_{11}(t)\|^3} + \frac{u_{iy}}{m_l} \\ -\frac{\mu z_i}{\|R + q_{11}(t)\|^3} + \frac{u_{iz}}{m_l} \end{bmatrix}
\]

Given the desired states of \( i \)th follower spacecraft \( q_{11d} = [x_{id}, y_{id}, z_{id}]^T \) and \( q_{22d} = [x_{id}, y_{id}, z_{id}]^T \), the position tracking error vector \( e_{i1}(t) \) is defined as follows

\[
   e_{i1}(t) = q_{11}(t) - q_{11d}
\]

Then, we have the velocity tracking error vector

\[
   \dot{e}_{i1}(t) = e_{22}(t) = q_{22}(t) - q_{22d}
\]

**Algebraic graph theory**

The technology of the algebraic graph theory is employed to deal with the multi-SFF control problem with undirected communication topology. It is assumed that the topology of the information flow among \( N \) follower spacecraft is modeled by a weight undirected graph \( G(A) = (\mathbf{G}, E, A) \), where \( \mathbf{G} = [g_{ij}, \ldots, g_{Nj}] \) is a set of nodes, \( E \subseteq \mathbf{G} \times \mathbf{G} \) is a set of edges, \((g_i, g_j) \in E \) means that if, and only if, there is an information exchange between the \( i \)th and \( j \)th follower spacecraft satisfying \((g_i, g_j) \in E \). The weighted adjacency matrix of the graph \( G(A) \) is described as \( A = [a_{ij}] \in \mathbb{R}^{N \times N} \) with non-negative
elements. The weighted adjacency element $a_{ij}$ represents the communication quality between the $i$th and $j$th follower spacecraft satisfying $(\sigma_i, \sigma_j) \in E \iff a_{ij} > 0$. Throughout this article, it is assumed that $\alpha_j = \alpha_i$.

**Preliminaries**

For the synthesis of the robust quantized control scheme design, the property, assumptions, and definition are made as follows.

**Property 1.** The nonlinear function $f(q_{1i}(t))$ satisfies
\[
\| f(q_{1i}(t)) \| \leq \varepsilon + \nu \| q_{1i}(t) \|
\]
where
\[
\varepsilon = \max \left( \frac{u_n}{m_i}, \frac{\mu}{\| R \|^2} - \frac{\mu r}{\| R \|^3} + \frac{u_n}{m_i}, \frac{u_n}{m_j} \right), \quad \nu = \frac{\mu}{\| R \|^3}
\]
\[A_{\text{ss}} \]

**Assumption 1.** For spacecraft formation, the relative distances between spacecraft are short compared to their orbital radius. In this case, the external disturbances $w_i(t)$, primarily including solar radiation pressure, $J_2$ perturbations, and atmospheric drag, which are assumed to be bounded. Hence, disturbance $w_i(t)$ satisfies
\[
\| w_i(t) \| \leq d
\]
where $d > 0$ is a positive constant.

**Assumption 2.** Assume that the desired position $q_{1id}$ and velocity $q_{1vd}$ are bounded.

**Definition 1.** For any vector $y(t) = [y_1(t), y_2(t), \ldots, y_n(t)]^T$, $\nu = 1, 2, \ldots, n$ with $y_k(t) \in (\theta, \theta(1/\sigma)^{2C}]$, a dynamic logarithmic quantizer $Q(\theta, y(t)) = [Q_1(\theta, y_1(t)), \ldots, Q_n(\theta, y_n(t))]^T$ maps any $y_k(t)$ into the following set
\[
Q(\theta, \cdot) = \left\{ \frac{\theta (1/\sigma)^h + (1/\sigma)^{h+1}}{2} \right\}_{h = 0, 1, \ldots, 2^C - 1}
\]
where $C > 0$ is bit-length, $\sigma > 0$ is a quantizer density and zooming parameter $\theta$ is a piecewise constant function, which can be defined as follows
\[
Q(\theta, y_v) = \frac{\theta (1/\sigma)^h + (1/\sigma)^{h+1}}{2}
\]
where $y_v \in (\theta(1/\sigma)^h, \theta(1/\sigma)^{h+1}]$, $h = 0, 1, \ldots, 2^C - 1$.

**Lemma 1.** The equilibrium point $x = 0$ is globally finite time stable for any given initial condition $x(0) = x_0$, if a candidate Lyapunov function can be obtained as $V(x) + kV''(x) \leq 0$, $k > 0$, $0 < \nu < 1$, then the settling time can be determined as follows
\[
T(x_0) \leq \frac{V(x_0)^{1-\nu}}{k(1-\nu)}
\]
where $V(x_0)$ is the initial value of the Lyapunov function $V(x)$.

**Control objective**

The purpose of this study is to design a sliding mode tracking control law for SFF in the presence of signal quantization and external disturbances, such that the following requirements are satisfied simultaneously:

1. The relative motion model 2 is asymptotically stable, which means that the states of the closed-loop system can converge to their desired relative position $q_{1id}$ and velocity $q_{1vd}$ when $t \to \infty$ despite state and input quantization by dynamic logarithmic quantizer $Q(\cdot)$ and external disturbances $w_i(t)$. This implies that
\[
\lim_{t \to \infty} q_{1i}(t) = q_{1id}, \quad \lim_{t \to \infty} q_{1v}(t) = q_{1vd}
\]
2. Considering performance optimization for SFF tracking control, the time-optimal performance function is chosen as one control performance index for SFF. The time-optimal performance index is defined as follows
\[
J = \min_{f_j}
\]
3. Considering the thruster amplitude constraints, the control input of $i$th follower spacecraft along each axis $u_{i, v}(t)$ satisfies
\[
|u_{i, v}(t)| < u_{v, \text{max}}, \quad v = x, y, z
\]
where $u_{v, \text{max}}$ is the maximum control force along the $v$-axis.
4. Considering flying safety requirements for multispacecraft formation, the follower spacecraft always keeps safe distances from other follower spacecraft to avoid a collision, which can be expressed as follows
\[
\forall t > 0, \| q_{1i}(t) - q_{1j}(t) \| > d_o
\]
where $d_o$ is the safe distance between the $i$th and $j$th follower spacecraft.

The relative orbit control objective for SFF will be achieved by the following two parts—guidance and tracking. During guidance, we will obtain optimal trajectories including relative position, relative velocity,
and control input trajectory using optimal trajectory planning. During tracking, considering the external disturbances, a sliding mode law $v_i(t)$ with dynamic logarithmic quantizer equations (7) and (8) is designed to track the obtained optimal trajectories of relative position and velocity.

Remark 1. In practical aerospace engineering, the output measurement states are always needed to be quantized and then transmitted to the controller module for synthesis. It is well known that the logarithmic quantization patterns are generally classified as dynamical logarithmic quantization and static logarithmic quantization. Compared to traditional static logarithmic quantization, the dynamical logarithmic quantizer has many advantages, such as sufficient accuracy and a relatively low required communication rate. In this work, the dynamical logarithmic quantizer equations (7) and (8) will be employed to perform the controller design for SFF. Although the integral sliding has strong robustness for external disturbances, it cannot solve the problem of path or thruster amplitude constraints nor satisfy the minimum performance index.

Optimal trajectory planning

In this section, the Gauss pseudospectral method is employed to solve the nonlinear constrained optimal control problem of system equation (2). Based on this method, the continuous optimization problem of equation (2) is transferred to obtain an optimal solution for the discrete nonlinear programming problem.

It is noted that the $N$ discrete moments $t \in [t_0, t_f], (n = 0, 1, \ldots, N-2, f)$ are linearly converted into $\tau \in [-1, 1]$

$$t = \frac{t_f - t_0}{2} \tau + \frac{t_f + t_0}{2}$$ (14)

In the following discussion, we can use $\tau$ to replace $t$ in performing the analysis. Based on the Lagrange interpolation polynomial, the relative position $q_i(t)$ and velocity $\dot{q}_i(t)$ can be approximated as follows

$$q_i(\tau) \approx \sum_{i=0}^{N} L_i(\tau)q_i(\tau_i), \dot{q}_i(\tau) \approx \sum_{i=0}^{N} L_i(\tau)d\dot{q}_i(\tau_i)$$ (15)

where $L_i(\tau) = \Pi_{k=0, k \neq i}^{N}(\tau - \tau_i/\tau_k - \tau_k), (i = 0, \ldots, N)$ is the zero of the Legendre orthogonal polynomials $P_n(x) = (1/2^n n!) (d^n/dx^n)[(x^2 - 1)^n]$.

Similarly, the control input $u_i(t)$ can be approximated as follows

$$u_i(\tau) \approx U_i(\tau) = \sum_{i=0}^{N} U_i(\tau_i)L_i(\tau)$$ (16)

where $L_i^*(\tau) = \Pi_{\kappa=0, \kappa \neq i}^{N}(\tau - \tau_\kappa)/(\tau_i - \tau_\kappa)$.

The derivative of equation (15) can be approximated as follows

$$\dot{q}_i(\tau) = \frac{\sum_{i=0}^{N} \dot{L}_i(\tau)q_i(\tau_i) = \sum_{i=0}^{N} D_{\kappa,i}q_i(\tau_i)}{\sum_{i=0}^{N} D_{\kappa,i}q_i(\tau_i)}$$ (17)

where $D_{\kappa,i} = \dot{L}_i(\tau), \tau = 0, 1, \ldots, N; \kappa = 0, 1, \ldots, N, D \in \mathbb{R}^{N \times (N+1)}$ is a differentiation matrix, which can be given by the following equation

$$D_{\kappa,i} = \sum_{i=0}^{N} \prod_{\kappa=0, \kappa \neq i}^{N} (\tau_k - \tau_i)$$ (18)

Thus, system equation (2) is expressed as follows

$$\begin{cases}
\sum_{i=0}^{N} D_{\kappa,i}q_i(\tau_i) = \frac{L_k}{2} \dot{q}_2(\tau_k) \\
\sum_{i=0}^{N} D_{\kappa,i}q_2(\tau_i) = \frac{L_k}{2} (A_1q_1(\tau_k) + A_2q_2(\tau_i) + f(q_1(\tau_k)) + \frac{1}{\mu}Bu(\tau_k))
\end{cases}$$ (19)

where $K = 1, 2, \ldots, N$.

After a series of transformations above, constraint conditions will be converted into corresponding approximate forms. The boundary conditions of states equation (10) can be formulated as follows

$$q_i(\tau_0) - q_i(0) = 0, \dot{q}_i(\tau_f) - \dot{q}_i(0) = 0$$ (20)

where $q_i(\tau_0)$ and $\dot{q}_i(\tau_0)$ are the initial states of the $i$th follower spacecraft, $q_i(\tau_f)$ and $\dot{q}_i(\tau_f)$ are desired states of the $i$th follower spacecraft.

The thruster amplitude constraints equation (13) and path constraints equation (12) can be formulated as follows

$$P(q(\tau_K), u(\tau_K)) \leq 0, \ K = 1, 2, \ldots, N$$ (21)

Now, the original multi-objective optimal control problem is converted to a nonlinear programming problem. This is used to determine the states $q_i(\tau)$ and $\dot{q}_i(\tau)$, control input $u_i(\tau)$, terminal time $t_f$, and initial time $t_0$, which minimize the objective performance function equation (11), subject to the system equation (19), boundary conditions equation (20), and constraints of path and thruster equation (21). Then, a numerical algorithm is employed to calculate the discretized optimal solutions of the relative position, relative velocity, and control input. When the optimal points are obtained, the approximate expressions for the optimal trajectories (including $\dot{q}_{id}$ and $\dot{q}_{id}$) and the corresponding optimal control input $\dot{u}_i(\tau)$ can be
Formed by the Lagrange interpolation polynomial method.

**Remark 2.** Recently, θ - D optimal technique was presented for spacecraft orbit maneuver.\(^3^4\) Since θ - D method does not require to solve Riccati equation repetitively at every instant, it shows a great advantage in saving calculation compared with State Dependent Riccati Equation (SDRE) method. However, the θ - D method is only applicable for the optimal control problems with standard cost functions. Comparing to the θ - D method, the Gauss pseudospectral method can solve optimal problems with a non-standard performance index or endpoint conditions and path constraints. Therefore, by considering performance optimization, thruster amplitude constraints, and collision avoidance, the Gauss pseudospectral method is employed to obtain optimal trajectories of relative position and velocity.

**Remark 3.** The optimal control input \(\hat{u}_i(t)\), obtained in this section, is an open-loop controller and does not have robustness for external disturbances. Therefore, it is necessary to design a closed-loop controller for system equation (2) to precisely track the optimal trajectories (including \(\dot{q}_{1d} \) and \(\dot{q}_{2d}\)) and effectively reduce the influences of the external disturbance.

**Quantizer controller design**

In this section, a quantized sliding mode tracking control strategy for SFF is proposed for tracking the obtained optimal trajectories with external disturbances and dynamic logarithmic quantizer \(Q(\cdot)\). The structure of the NCSs for the SFF is illustrated in Figure 2.

**Remark 4.** As shown in Figure 2, the state error vectors, \(e_1(t), e_2(t), e_{10}(t), e_{20}(t)\), and, sliding surface vector, \(s_i(t)\), are required to be quantized before transmitting to the controller module over the digital network links. Thus, the exact value of the state error vectors, \(e_1(t), e_2(t), e_{10}(t), e_{20}(t)\), and the proposed sliding surface, \(s_i(t)\), are indeed not available for the control scheme design. As discussed in the following, \(e_1(t), e_2(t), e_{10}(t), e_{20}(t)\), and \(s_i(t)\) will be replaced by the quantized information of \(e_1(t), e_2(t), e_{10}(t), e_{20}(t)\), and \(s_i(t)\), respectively, to perform the sliding mode control design work.

In this work, the integral sliding surface function \(s_i(t)\) is designed as follows:\(^3^5\)

\[
s_i(t) = e_{12}(t) + k_pe_{11}(t) + k_I\int_0^te_1(\tau)d\tau
\] (22)

where \(e_{12}(t) = q_{1a}(t) - \dot{q}_{11d}, e_{11}(t) = e_{22}(t) = q_{2a}(t) - \dot{q}_{22d}\). \(k_p \in \mathbb{R}^{3 \times 3}\) is defined as \(k_p = diag(k_{p1}, k_{p2}, k_{p3})\), where \(k_{p1} > 0, n = 1, 2, 3; k_I \in \mathbb{R}^{3 \times 3}\) is defined as \(k_I = diag(k_{I1}, k_{I2}, k_{I3})\) where \(k_{I1} > 0, n = 1, 2, 3\).

The following theorem gives the proof for the asymptotic stability of the sliding dynamic system.

**Theorem 1.** Considering the spacecraft relative dynamic control system equation (2), if on the sliding surface \(s_i(t) = 0\), then the system equation (2) is asymptotically stable.

**Proof.** For \(s_i(t) = 0\), we can further obtain...
\[ \dot{s}_i(t) = \dot{e}_{i2}(t) + k_p e_{i2}(t) + k_i e_{i1}(t) = 0 \quad (23) \]

Let the Lyapunov function candidate for the system be chosen as follows

\[ V_c(t) = \frac{1}{2} e_{i2}^T(t) e_{i2}(t) + \frac{k_i}{2} e_{i1}^T(t) e_{i1}(t) \quad (24) \]

Taking the first derivative of \( V_c(t) \) and using equation (23) yields

\[ \dot{V}_c(t) = e_{i2}^T(t) \dot{e}_{i2}(t) + k_p e_{i2}^T(t) e_{i2}(t) \\
= e_{i2}^T(t)(-k_p e_{i2}(t) - k_i e_{i1}(t)) + k_i e_{i1}^T(t) e_{i2}(t) \\
= -k_p e_{i2}^T(t) e_{i2}(t) \leq 0 \quad (25) \]

the above inequality implies that \( V_c(t) = 0 \) holds for \( e_{i2} = 0 \). Applying the LaSalle invariance principle, it can be concluded that on the sliding mode surface \( s_i(t) = 0 \)

\[ \lim_{t \to \infty} e_{i2}(t) = 0, \quad \lim_{t \to \infty} e_{i1}(t) = 0 \quad (26) \]

Before giving the next theorem, we first introduce the following lemma to present the quantization error of the dynamic logarithmic quantizer \( Q(\cdot) \).

**Lemma 2.** Consider the dynamic logarithmic quantizer equations (7) and (8), and define the quantization error \( E_{q_i}(t) = Q(q_{i1}(t)) - q_{i1}(t), \) \( E_{q_2}(t) = Q(q_{i2}(t)) - q_{i2}(t), \) \( E_{e_1}(t) = Q(e_{i1}(t)) - e_{i1}(t), \) \( E_{e_2}(t) = Q(e_{i2}(t)) - e_{i2}(t), \) \( E_s(t) = Q(s_i(t)) - s_i(t), \) \( E_{v_i}(t) = Q(v_i(t)) - v_i(t). \) If the quantizer density satisfies \( \sigma > 0.71 \), then \( E_{q_i}(t), E_{q_2}(t), E_{e_1}(t), E_{e_2}(t), E_{s}(t), E_{v_i}(t), E_{e_{i1}}(t), E_{e_{i2}}(t), \) and \( E_{v_i}(t) \) satisfies the following constraints

\[
\begin{align*}
\| E_{q_1}(t) \| & \leq \frac{(1 - \sigma)}{\sigma(1 + \sigma)} \| Q(q_{i1}(t)) - q_{i1}(t) \| \\
\| E_{q_2}(t) \| & \leq \frac{(1 - \sigma)}{\sigma(1 + \sigma)} \| Q(q_{i2}(t)) - q_{i2}(t) \| \\
\| E_{e_1}(t) \| & \leq \frac{(1 - \sigma)}{\sigma(1 + \sigma)} \| Q(e_{i1}(t)) - e_{i1}(t) \| \\
\| E_{e_2}(t) \| & \leq \frac{(1 - \sigma)}{\sigma(1 + \sigma)} \| Q(e_{i2}(t)) - e_{i2}(t) \| \\
\| E_s(t) \| & \leq \frac{(1 - \sigma)}{\sigma(1 + \sigma)} \| Q(s_i(t)) - s_i(t) \| \\
\| E_{v_i}(t) \| & \leq \frac{(1 - \sigma)}{\sigma(1 + \sigma)} \| Q(v_i(t)) - v_i(t) \| \\
\| E_{e_{i1}}(t) \| & \leq \frac{(1 - \sigma)}{\sigma(1 + \sigma)} \| Q(e_{i1}(t)) - e_{i1}(t) \| \\
\| E_{e_{i2}}(t) \| & \leq \frac{(1 - \sigma)}{\sigma(1 + \sigma)} \| Q(e_{i2}(t)) - e_{i2}(t) \| \\
\end{align*}
\]

**Proof:** We only prove the first inequality in equation (27) holds, and the proof for the other inequalities is similar. First, we assume \( \sigma > 0 \), from the quantizers (7) and (8). Then, it is easy to show that

\[
\vartheta \left( \frac{1}{\sigma} \right)^h \left( \frac{1}{\sigma} + 1 \right) - \vartheta \left( \frac{1}{\sigma} \right)^{h+1} \leq E_{q_i}(t) \quad (28)
\]

which implies that

\[
|E_{q_i}(t)| \leq \frac{(1 - \sigma)}{2\sigma} |q_{i1}(t)| \quad (29)
\]

On the other hand, it is not difficult to see from equations (7) and (8) that

\[
\frac{1}{2} \sigma + 1 \leq \frac{Q(\vartheta, q_{i1}(t))}{q_{i1}(t)} \leq \frac{1}{2\sigma} + \frac{1}{2} \quad (30)
\]

Hence, we obtain

\[
|E_{q_i}(t)| \leq \frac{(1 - \sigma)}{2\sigma} |q_{i1}(t)| \leq \frac{(1 - \sigma)}{2\sigma} \frac{2}{(\sigma + 1)} |Q(\vartheta, q_{i1}(t))| \leq \frac{(1 - \sigma)}{\sigma(1 + \sigma)} |Q(\vartheta, q_{i1}(t))| \quad (31)
\]

If the quantizer density satisfies \( \sigma > 0.71 \), then it is shown that

\[
|E_{q_i}(t)| \leq \frac{(1 - \sigma)}{\sigma(1 + \sigma)} |Q(\vartheta, q_{i1}(t))| < \frac{1}{4} |Q(\vartheta, q_{i1}(t))| \quad (32)
\]

From equation (32), it can be derived that

\[
|E_{q_i}(t)| \leq \sqrt{\sum_{i=1}^{3} E_{q_i}(t)^2} \leq \frac{(1 - \sigma)}{\sigma(1 + \sigma)} \| Q(\vartheta, q_{i1}(t)) \| < \frac{1}{4} \| Q(\vartheta, q_{i1}(t)) \| \quad (33)
\]

For the case when \( \vartheta < 0 \), the proof is similar and we can omit here.

In order to ensure the reachability of the integral sliding surface equation (22), a robust quantized sliding mode tracking controller will be constructed. To design this control law and illustrate that this control scheme can ensure the system equation (2) is asymptotically stable, we require the following theorem.
Consider the following Lyapunov function:

$$V(t) = \frac{1}{2} s_i^T(t) s_i(t)$$  \hspace{1cm} (35)$$

where $\rho = \sigma(1 + \sigma)/(1 - \sigma) > 4$. $e_{i0}(t) = e_i(t) - e_j(t)$, $e_{20}(t) = e_{j1}(t) - e_{j2}(t)$, the external disturbances satisfy $\| w_i(t) \| \leq d$, and $q > 0$ is the small scalar to be determined. Then, under the robust quantized sliding mode control law $v_i(t)$, the trajectory of the closed-loop system equation (2) will arrive on the sliding surface equation (22) in finite time.

Proof. Consider the following Lyapunov function:

$$V(t) = \frac{1}{2} s_i^T(t) s_i(t)$$  \hspace{1cm} (35)$$

the differentiation of the Lyapunov function $V(t)$ with respect to time yields

$$\dot{V}(t) = Q^T(s_i(t)) \left[ A_1 q_1(t) + \frac{1}{m_i} B v_i(t) + \frac{1}{m_i} B E u(t) \right. $$

$$\left. - \dot{\hat{q}}_{12d} + f(q_{i1}) + k_p e_{i2}(t) + k_f e_{i1}(t) + w_i(t) \right]$$

$$- \xi^T(t) \left[ A_1 q_1(t) + \frac{1}{m_i} B v_i(t) + \frac{1}{m_i} B E u(t) \right. $$

$$\left. - \dot{\hat{q}}_{12d} + f(q_{i1}) + k_p e_{i2}(t) + k_f e_{i1}(t) + w_i(t) \right]$$

$$\dot{v}_i(t) = - \frac{\rho - 1}{\rho - 3} \frac{1}{\rho - 1} m_i B^{-1} \frac{\rho + 1}{\rho} (v_i + A_1)$$

$$- \| Q(q_{i1}(t)) \| + \frac{\rho + 1}{\rho} (\| A_2 \| \| Q(q_{i2}(t)) \|$$

$$+ \| Q(q_{i2d}) \| + k_p \| Q(e_{i2}(t)) \|$$

$$+ d + e + q + \sum_{j=1}^{N} \alpha_{ij} \| Q(e_{i1}(t)) \|$$

$$+ \sum_{j=1}^{N} \alpha_{ij} \| Q(e_{i2}(t)) \| \times \frac{1}{\| Q(s_i(t)) \|}$$

$$\leq \frac{\rho - 1}{\rho - 3} \frac{1}{\rho - 1} \| Q(s_i(t)) \| B \| \frac{\rho - 1}{\rho - 1} \| v_i(t) \|$$

$$\leq \frac{\rho - 1}{\rho - 1} \| Q(s_i(t)) \| \times \frac{\rho - 1}{\rho - 3} \| d + e$$

$$+ \frac{\rho + 1}{\rho} (v_i + A_1) \| Q(q_{i1}(t)) \| + \| \| A_2 \| \| Q(q_{i2}(t)) \|$$

$$\leq \| A_2 \| \| Q(q_{i2}(t)) \| + \| Q(q_{i2d}) \| + k_p \| Q(e_{i2}(t)) \|$$

$$+ k_l \| Q(e_{i1}(t)) \| + q + \sum_{j=1}^{N} \alpha_{ij} \| Q(e_{i1}(t)) \|$$

$$+ \sum_{j=1}^{N} \alpha_{ij} \| Q(e_{i2}(t)) \| \leq \frac{\rho - 1}{\rho - 1} \| Q(s_i(t)) \| \cdot d$$

Second, let us handle the term $Q^T(s_i(t))w_i(t)$ in equation (36). Note that the following holds

$$Q^T(s_i(t))w_i(t) \leq \| Q(s_i(t)) \| \cdot d$$

Subsequently, considering the third term $Q^T(s_i(t))f(q_{i1}(t))$ in equation (36), it is true the following inequalities hold

$$\| q_{i1}(t) \| = \| Q(q_{i1}(t)) - E_{q_{i1}}(t) \| \leq \frac{\rho + 1}{\rho} \| Q(q_{i1}(t)) \|$$

$$Q^T(s_i(t))f(q_{i1}(t)) \leq \| Q(s_i(t)) \| \| f(q_{i1}(t)) \|$$

$$\leq \| Q(s_i(t)) \| \left( e + \nu \frac{\rho + 1}{\rho} \| Q(q_{i1}(t)) \| \right)$$

$$\leq \| Q(s_i(t)) \| \left( e + \nu \frac{\rho + 1}{\rho} \| Q(q_{i1}(t)) \| \right)$$

Therefore, the term $Q^T(s_i(t))(A_1 q_{i1}(t) - \dot{\hat{q}}_{i2d} + k_p e_{i2}(t) + k_f e_{i1}(t))$ in equation (36) can be enlarged as follows

$$Q^T(s_i(t))(A_1 q_{i1}(t) - \dot{\hat{q}}_{i2d} + k_p e_{i2}(t) + k_f e_{i1}(t))$$

$$\leq \| Q(s_i(t)) \| \frac{\rho + 1}{\rho} (\| A_1 \| \| Q(q_{i1}(t)) \| + \| Q(q_{i2d}) \|$$

$$+ k_p \| Q(e_{i2}(t)) \| + k_f \| Q(e_{i1}(t)) \|)$$

Then, the first row in equation (36) can be calculated as follows (see equation (41))
\[
Q^T(s_i(t)) \left[ A_1 q_{i1}(t) + \frac{1}{m} B v(t) + \frac{1}{m} B e_u(t) - \dot{q}_{12d} + f(q_{i1}(t)) + k_p e_2(t) + k_t e_1(t) + w_i(t) \right]
\leq Q^T(s_i(t)) \frac{1}{m} B v(t) + \| Q(s_i(t)) \| \frac{\rho + 1}{\rho} (A_1 \| Q(q_{i1}(t)) \| + \| \dot{q}_{12d} \|)
+ k_p \| Q(e_2(t)) \| + k_t \| Q(e_1(t)) \| + \| \dot{q}_{12d} \| + \frac{1}{m} \| B \| \frac{1}{\rho} \| Q(s_i(t)) \|.v_i(t) \|
\leq Q^T(s_i(t)) \frac{1}{m} B v(t) + \| Q(s_i(t)) \| \frac{\rho + 1}{\rho} (A_1 \| Q(q_{i1}(t)) \| + \| \dot{q}_{12d} \| + k_p \| Q(e_2(t)) \|
+ k_t \| Q(e_1(t)) \| + \| \dot{q}_{12d} \| + \frac{1}{m} \| B \| \frac{1}{\rho} \| Q(s_i(t)) \|.d + \| Q(s_i(t)) \| \cdot d
\]

Considering the term \(-E^T_i(t)(A_1 q_{i1}(t) - \dot{q}_{12d} + k_p e_2(t) + k_t e_1(t))\) in equation (36), note that

\[
\begin{align*}
&-E^T_i(t)(A_1 q_{i1}(t) - \dot{q}_{12d} + k_p e_2(t) + k_t e_1(t))
\leq \frac{1 + \rho}{\rho} \| Q(s_i(t)) \| (A_1 \| Q(q_{i1}(t)) \| + \| \dot{q}_{12d} \| + k_p \| Q(e_2(t)) \| + k_t \| Q(e_1(t)) \|)
\end{align*}
\]

On the other hand, the second row in equation (36) can be derived as follows (see equation (43))

\[
\begin{align*}
&-E^T_i(t) \left[ A_1 q_{i1}(t) + \frac{1}{m} B v(t) + \frac{1}{m} B e_u(t) - \dot{q}_{12d} + f(q_{i1}(t)) + k_p e_2(t) + k_t e_1(t) + w_i(t) \right]
\leq \| E_i(t) \| \left[ A_1 q_{i1}(t) + \frac{1}{m} B v(t) + \frac{1}{m} B e_u(t) - \dot{q}_{12d} + f(q_{i1}(t)) + k_p e_2(t) + k_t e_1(t) + w_i(t) \right]
\leq \frac{1}{\rho} \| Q(s_i(t)) \| \left[ \frac{1}{m_{ij}} B \| v_i(t) \| + \frac{1}{\rho} \| B \| \frac{1}{m_{ij}} B \| v_i(t) \| + d + \varepsilon \right] + \frac{1}{\rho} \| Q(s_i(t)) \| \left[ \frac{\rho + 1}{\rho} \right]
\leq \frac{1}{\rho} \| Q(s_i(t)) \| \left[ \left( 1 + \frac{\rho + 2}{\rho} \right) \frac{\rho - 1}{\rho - 1} \frac{\rho + 1}{\rho} \right] \times \left( d + \varepsilon + \frac{\rho + 1}{\rho} (v + A_1 \| Q(q_{i1}(t)) \| + k_p \| Q(e_2(t)) \| + k_t \| Q(e_1(t)) \| + \| \dot{q}_{12d} \| + \frac{1}{m} \| B \| \frac{1}{\rho} \| Q(s_i(t)) \|.d + \| Q(s_i(t)) \| \cdot d \right)
\end{align*}
\]

Thus, it follows from equations (41) and (43) that
Note that the term $Q^T(s_i(t))v_i(t)$ in equation (44) can be decomposed as the following version

$$Q^T(s_i(t))v_i(t) = \left(\frac{\rho^2 - 1}{\rho^2} + \frac{2}{\rho^2}\right)Q^T(s_i(t))v_i(t)$$

and substituting equation (45) into equation (44) yields equation (46)

$$\dot{V}(t) \leq \frac{2}{\rho - 1} \frac{1}{m_f} B Q^T(s_i(t))v_i(t) + \frac{2}{\rho - 1} \frac{1}{\rho - 3} \rho \times \left[ d + e + \frac{\rho + 1}{\rho} (v + || A_1 ||) || Q(q_1(t)) || \right. \\
+ \frac{\rho + 1}{\rho} \left. \cdot \left( || A_2 || || Q(q_2(t)) || + || Q(q_{12a}) || + k_p || Q(e_{12a}) || + k_f || Q(e_{12a}) || + \rho \sum_{j=1}^{N} a_{ij} || Q(e_{12a}) || + \sum_{j=1}^{N} a_{ij} || Q(e_{12a}) || \right) \right]$$ \hspace{1cm} (46)

Finally, substituting the control law equation (34) into equation (46) yields

$$\dot{V}(t) \leq -\varepsilon ||Q(s_i(t))|| - \sum_{j=1}^{N} a_{ij} || Q(e_{1j}(t)) || \times ||Q(s_i(t))||$$ \\
- \sum_{j=1}^{N} a_{ij} || Q(e_{2j}(t)) || \times ||Q(s_i(t))|| \\
\leq -\varepsilon ||Q(s_i(t))|| \leq -\frac{\rho \varepsilon}{\rho + 1} |s_i(t)|$$ \hspace{1cm} (47)

Using Lemma 1, the trajectory of the closed-loop system equation (2) will arrive on the sliding surface equation (22) in finite time. We complete the proof.

### Simulation results

In this section, we present a numerical example to show the effectiveness of the proposed tracking control law for the SFF. For simplicity, the leader spacecraft is assumed to be in a circular reference orbit of radius 6728 km, $u(t) = [0, 0, 0]^T$; certain symbols used in the numerical simulations are summarized in Table 1, and the numerical simulation parameters are given in Table 2. We chose the safe distance, $d_s = 10$ m, to ensure that no collision occurs. In addition, the maximum thruster amplitude satisfies $u_{c, \text{max}} = 0.0561$. 

To demonstrate the performance of the proposed strategy, it, respectively, carries out two simulations of different working conditions with different formation sizes:

**Working condition 1.** The initial state of $S_1$ is $(-15$ m, $-140$ m, $5$ m, $1$ m/s, $1$ m/s, $1$ m/s)$^T$, the initial state of $S_2$ is $(10$ m, $-140$ m, $10$ m, $1$ m/s, $1$ m/s, $1$ m/s)$^T$; the desired state of $S_1$ is $(0$ m, $30$ m, $0$, $0$, $0$, $0)$, the desired state of $S_2$ is $(-15$ m, $-20$ m, $0$, $0$, $0$, $0)^T$.

**Working condition 2.** The initial state of $S_1$ is $(-10$ m, $20$ m, $20$ m, $5$ m/s, $5$ m/s, $5$ m/s)$^T$, the initial state of $S_2$ is $(6$ m, $25$ m, $20$ m, $5$ m/s, $5$ m/s, $5$ m/s)$^T$; the desired state of $S_1$ is $(50$ m, $87$ m, $0$, $0$, $0$, $0)$, the desired state of $S_2$ is $(-50$ m, $87$ m, $0$, $0$, $0$, $0)^T$.

---

**Table 1.** Symbols used in the numerical simulations.

| Symbols | Meaning |
|---------|---------|
| $S_1$   | The first follower spacecraft in the formation |
| $S_2$   | The second follower spacecraft in the formation |
| $T_1$   | The desired position of $S_1$ |
| $T_2$   | The desired position of $S_2$ |

**Table 2.** Numerical simulation parameters.

| Parameter name | Value |
|----------------|-------|
| Mass of the leader and follower spacecraft | $m_l = 1$ kg, $m_f = 1$ kg, $m_{2f} = 1$ kg |
| Earth's gravitational constant | $\mu = 39.86, 000 \text{ km}^3/\text{s}^2$ |
| External disturbance | $\omega(t) = 2\times(\cos(2\pi t), \sin(2\pi t), \cos(2\pi t))^T \text{ mN}$ |
| Parameters of the controller | $d = 2$, $k_p = \text{diag}(0.0001, 0.0001, 0.0001)$, $k_i = \text{diag}(0.0001, 0.0001, 0.0001)$ |
| Quantizer density | $\sigma = 0.9$ |
| Weighted adjacency element | $a_{12} = 2$, $a_{21} = 2$ |
By the Gauss pseudospectral method for working condition 1, the optimal trajectory planning is completed by about 99.80 s, which implies $t_f = 99.80$ s. For working condition 1, the simulation results of robust quantized control law equation (34) with optimal trajectory planning are given in Figures 3–8. The relative position and velocity tracking errors of the follower spacecraft $S_1$ and $S_2$ are given in Figures 3 and 4. It is clearly shown that the follower spacecraft converge to their desired states quickly when they completely track the optimal planning trajectories $\hat{q}_{1d}$ and $\hat{q}_{2d}$ at around 95 s, and the relative position and velocity tracking errors converge to near zero within 100 s by the robust quantized control scheme (equation (34)).

The trajectories of the relative distances between the two follower spacecraft are compared in Figure 5. As we can see, the relative distance of the two follower spacecraft $S_1$ and $S_2$ is always greater than the safe distance with optimal trajectory planning, and the minimum distance is about 25.495 m. In view of these simulation results, collision avoidance is guaranteed by trajectory planning.

The comparisons of follower spacecraft states and their quantized values are shown in Figures 6 and 7. It
can be observed that the validity of the dynamic logarithmic quantizer is verified. The thruster amplitudes of the follower spacecraft S1 and S2 are shown in Figure 8. As we can see, the maximum value of the control input force is $0.0561 \text{ N}$, which satisfies the input force constraint.

By the Gauss pseudospectral method for working condition 2, the optimal trajectory planning is completed by $269.81 \text{ s}$, which implies $t_f = 269.81 \text{ s}$. The simulation figures of working condition 2 are shown in Figures 9–14. The trajectories of the relative distances between the two follower spacecraft are compared in

---

**Figure 6.** States and their quantized values of $S_1$ (condition 1).

**Figure 7.** States and their quantized values of $S_2$ (condition 1).
Figure 8. Control forces trajectories of $S_1$ and $S_2$ (condition 1): (a) $S_1$ and (b) $S_2$.

Figure 9. The relative position tracking errors of $S_1$ and $S_2$ (condition 2): (a) $S_1$ and (b) $S_2$.

Figure 10. The relative velocity tracking errors of $S_1$ and $S_2$ (condition 2): (a) $S_1$ and (b) $S_2$. 
Conclusion

In this article, we have addressed the robust quantized sliding mode optimal tracking control problem for SFF with external disturbances and signal quantization. First, by taking into account performance optimization, thruster amplitude constraints, and collision...
avoidance, the Gauss pseudospectral method has been employed to complete the optimal trajectory planning of the relative position and velocity. Then, in the presence of the state quantization and input quantization, a quantized sliding mode control design strategy has been designed by employing a dynamic quantizer density design approach to track the obtained optimal trajectories. The proposed tracking controller can guarantee the stability of the closed-loop system and ensure multi-objectives are satisfied with the digital data transmission constraints. Finally, an illustrative example has been utilized to demonstrate the effectiveness of the robust quantized sliding mode controller presented in this article. A future research topic would be to investigate the problems of nonlinear tracking control for SFF with more network-induced limitations, such as communication delay and data packet losses.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: This work is supported by the National Natural Science Foundation of China (grant numbers 91438202, 61473096, 61690212, and 61333003) and the Open Fund of National Defense Key Discipline Laboratory of Micro-Spacecraft Technology (grant number HIT.KLOF.MST.201701).

ORCID iD

Ruixia Liu https://orcid.org/0000-0001-6947-3891

References

1. Kristiansen R and Nicklasson PJ. Spacecraft formation flying: a review and new results on state feedback control. Acta Astronaut 2009; 65: 1537–1552.
2. Yang X, Yu J and Gao H. An impulse control approach to spacecraft autonomous rendezvous based on genetic algorithms. Neurocomputing 2012; 77: 189–196.
3. Yu X, Liu Z and Zhang Y. Fault-tolerant formation control of multiple UAVs in the presence of actuator faults. Int J Robust Nonlin 2016; 26: 2668–2685.
4. Nair RR and Behera L. Robust adaptive gain higher order sliding mode observer based control-constrained nonlinear model predictive control for spacecraft formation flying. IEEE/CAA J Autom Sin 2018; 5: 367–381.
5. Liu X and Kumar K. Network-based tracking control of spacecraft formation flying with communication delays. IEEE T Aero Elec Sys 2012; 48: 2302–2314.
10. Ma X, Sun F, Li H, et al. Neural-network-based sliding-control of spacecraft with fault-tolerant capability. *IEEE T Contr Syst T* 2015; 23: 1338–1350.

7. Yang X, Cao X and Gao H. Sampled-data control for relative position holding of spacecraft rendezvous with thrust nonlinearity. *IEEE T Ind Electron* 2012; 59: 1146–1153.

8. Hu Q, Dong H, Zhang Y, et al. Tracking control of spacecraft formation flying with collision avoidance. *Aerosp Sci Technol* 2015; 42: 353–364.

9. Nair RR and Behera L. Robust adaptive gain nonsingular fast terminal sliding mode control for spacecraft formation flying. In: *Proceedings of the 54th annual conference on decision and control*, Osaka, Japan, 15–18 December 2015, pp.5314–5319. New York: IEEE.

10. Ma X, Sun F, Li H, et al. Neural-network-based sliding-mode control for multiple rigid-body attitude tracking with inertial information completely unknown. *Inform Sciences* 2017; 400–401: 91–104.

11. Yang H, You X, Xia Y, et al. Adaptive control for attitude synchronisation of spacecraft formation via extended state observer. *IET Control Theory A* 2014; 8: 2171–2185.

12. Bae J and Kim Y. Adaptive controller design for spacecraft formation flying using sliding mode controller and neural networks. *J Frankl Inst* 2012; 349: 578–603.

13. Liu X and Kumar K. Input-to-state stability of model-based spacecraft formation control systems with communication constraints. *Acta Astronaut* 2011; 68: 1847–1859.

14. Xiao B, Hu Q and Zhang Y. Adaptive sliding mode fault tolerant attitude tracking control for flexible spacecraft under actuator saturation. *IEEE T Contr Syst T* 2012; 20: 1605–1612.

15. Pakdeboon C and Zinober A. Control Lyapunov function optimal sliding mode controllers for attitude tracking of spacecraft. *J Frankl Inst* 2012; 349: 456–475.

16. Ni Q, Huang Y and Chen X. Nonlinear control of spacecraft formation flying with disturbance rejection and collision avoidance. *Chinese Phys B* 2017; 26: 014502.

17. Yu X and Jiang J. Hybrid fault-tolerant flight control system design against partial actuator failures. *IEEE T Contr Syst T* 2012; 20: 871–886.

18. Wu J, Han D, Liu K, et al. Nonlinear suboptimal synchronized control for relative position and relative attitude tracking of spacecraft formation flying. *J Frankl Inst* 2015; 352: 1495–1520.

19. Liu R and Li S. Optimal integral sliding mode control scheme based on pseudospectral method for robotic manipulators. *Int J Control* 2014; 87: 1131–1140.

20. Zheng H, Wang N and Wu J. Minimizing deep sea data collection delay with autonomous underwater vehicles. *J Parallel Distr Com* 2017; 104: 99–113.

21. Wu B. Spacecraft attitude control with input quantization. *J Guid Control Dynam* 2016; 39: 176–181.

22. Sun Q, Chen S, Chen L, et al. Quasi-Z-source network-based hybrid power supply system for aluminum electrolysis industry. *IEEE T Ind Inform* 2017; 13: 1141–1151.

23. Katic D and Vukobratovic M. A neural network-based classification of environment dynamics models for compliant control of manipulation robots. *IEEE T Syst Man Cy B* 1998; 28: 58–69.

24. Li H, Gao Y, Shi P, et al. Observer-based fault detection for nonlinear systems with sensor fault and limited communication capacity. *IEEE T Automat Contr* 2016; 61: 2745–2751.

25. Liu M, Zhang L, Shi P, et al. Sliding mode control of continuous-time Markovian jump systems with digital data transmission. *Automatica* 2017; 80: 200–209.

26. Gong Q, Ross I and Kang W. A unified pseudospectral framework for nonlinear controller and observer design. In: *Proceedings of the American control conference*, New York, 9–13 July 2007, pp.1943–1949. New York: IEEE.

27. Niu Y and Ho D. Control strategy with adaptive quantizer's parameters under digital communication channels. *Automatica* 2014; 50: 2665–2671.

28. Li H, Chen Z, Wu L, et al. Event-triggered control for nonlinear systems under unreliable communication links. *IEEE T Fuzzy Syst* 2017; 25: 813–824.

29. Shi P, Wang H and Lim C. Network-based event-triggered control for singular systems with quantizations. *IEEE T Ind Electron* 2016; 63: 1230–1238.

30. Wong H, Pan H, De Queiroz MS, et al. Adaptive learning control for spacecraft formation flying. In: *Proceedings of the 40th conference on decision and control*, Orlando, FL, 4–7 December 2001, pp.1089–1094. New York: IEEE.

31. Ran D, Chen X and Misra AK. Finite time coordinated formation control for spacecraft formation flying under directed communication topology. *Acta Astronaut* 2017; 136: 125–136.

32. Yu S and Wang X. Quantization scheme design in distributed event-triggered networked control systems. *IFAC Proc Vol* 2011; 44: 13257–13262.

33. Lu Y, Huang P, Meng Z, et al. Finite time attitude takeover control for combination via tethered space robot. *Acta Astronaut* 2017; 136: 9–21.

34. Xin M and Pan H. Integrated nonlinear optimal control of spacecraft in proximity operations. *Int J Control* 2010; 83: 347–363.

35. Wang B and Zhang Y. Adaptive sliding mode fault-tolerant control for an unmanned aerial vehicle. *Un Sys* 2017; 5: 209–221.