BRST-anti-BRST Antifield Formalism : The Example of the Freedman-Townsend Model

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Abstract

The general BRST-anti-BRST construction in the framework of the antifield-antibracket formalism is illustrated in the case of the Freedmann-Townsend model.

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1 Introduction

The most general method for quantizing gauge systems in a manifestly covariant manner is the antifield-antibracket formalism [1, 2, 3, 4]. This method allows to express the BRST symmetry in the Lagrangian context. Immediately after the discovery of this symmetry, the anti-BRST transformation was formulated for Yang-Mills models. Recently, there has been an increasing interest in a systematic formulation of the BRST-anti-BRST symmetry in both the Hamiltonian [5, 6, 7, 8] and the Lagrangian [9, 10, 11] context. Those methods rely on the construction of two nilpotent anticommuting operators \( s_1 \) and \( s_2 \) leaving the action invariant.

A model that has been intensively studied from the Lagrangian point of view is the Freedman-Townsend model. Its main interest lies in the fact that (i) it is equivalent to the non-linear \( \sigma \)-model [12], (ii) the algebraic structure of its gauge symmetries is similar to that of Witten’s string theory [13], (iii) even if its BRST structure is well understood [14, 15, 16], the attempts to incorporate the anti-BRST symmetry have not been entirely satisfactory [17]. Our purpose in this article is to illustrate the general Lagrangian BRST-anti-BRST method in the case of this representative model.

Our starting point is the classical action (where the trace over group indices is understood)

\[
S_0[B_{\mu\nu}(x), A_\mu(x)] = \int d^4x \left\{ \frac{1}{4} \epsilon^{\mu\nu\rho\sigma} B_{\mu\nu}(x) F_{\rho\sigma}(x) + \frac{1}{2} A_\mu(x) A^\mu(x) \right\},
\]

which is invariant under the gauge transformations

\[
\delta_g B_{\mu\nu}(x) = \nabla_{[\mu} \xi_{\nu]}(x), \quad \delta_g A_\mu(x) = 0.
\]

The bosonic fields \( A_\mu \) and \( B_{\mu\nu} \) take values in some semi-simple compact Lie algebra \( \mathcal{A} \). The gauge transformations (2) are not all independent; indeed, it is easy to see that if one takes for gauge parameters \( \xi_\mu(x) = \nabla_\mu \xi(x) \), then the gauge transformations reduce to an antisymmetric combination of the equations of motion:

\[
\delta_g B_{\mu\nu}(x) = -\frac{1}{2} \epsilon_{\mu\nu\rho\sigma} \left[ \frac{\delta S_0}{\delta B_{\rho\sigma}(x)}, \xi(x) \right].
\]

In the DeWitt notations, the gauge generators are written as

\[
R^a_{\mu\nu}(x, y) = \delta[\rho(\nabla_{x^\rho})]_{\mu\nu}^a \delta^4(x - y),
\]
while the first order reducibility functions are given by
\[ Z^b_{pc}(y, z) = (\nabla_y^n)^b_c \delta^4(y - z). \]  \hspace{1cm} (5)

In these notations, the reducibility relation takes the form
\[ \int d^4 y R^a_{\mu
u b}(x, y) Z^b_{pc}(y, z) = -\frac{1}{2} \epsilon_{\mu\nu\rho\sigma} \frac{\delta S_0}{\delta B_{\rho\sigma}(x)} f^a_{bc} \delta^4(x - z), \]  \hspace{1cm} (6)

where the \( f^a_{bc} \)'s are totally antisymmetric structure constants of the Lie algebra \( \mathcal{A} \). The \( Z \)'s are a complete and irreducible set of first order reducibility coefficients, that is, with this choice of \( R \) and \( Z \) the Freedman-Townsend model is a first order reducible theory. The particularity of this description is that reducibility only holds on-shell. In the usual BRST antifield-antibracket formalism this implies that the solution of the master equation contains supplementary terms in comparison with off-shell reducible gauge systems. This will also be true for the BRST-anti-BRST Lagrangian treatment.

2 The BRST-anti-BRST Lagrangian formalism for a first order reducible gauge system

From a generic point of view, the Freedman-Townsend model may be characterized by an action \( S_0[q^i] \) where the \( q^i \) denote all the fields; in our case one has the correspondence:
\[ q^i \longleftrightarrow (A_{\mu}(x), B_{\mu\nu}(x)). \]  \hspace{1cm} (7)
In particular, here the $q^i$'s are bosonic. The action $S_0[q]$ is invariant under some gauge transformations

$$\delta_g q^i = R^i_{\alpha\beta} \xi_{\alpha\beta},$$

which are abelian off-shell. If $G_i = 0$ denote the equations of motion, then the Noether identities associated to the gauge symmetries (8) are

$$G_i R^i_{\alpha\beta} = 0.$$ (9)

The first order reducibility relations are given by

$$R^i_{\alpha\beta} Z^\alpha_{\beta\gamma} = G_j T^j_{\alpha\beta}.$$ (10)

For the Freedman-Townsend model, one has the following correspondance

$$R^i_{\alpha\beta} \underset{}{\longrightarrow} \begin{pmatrix} 0 \\ R^a_{\mu\nu}(x, y) = \delta^\rho_{[\nu} (\nabla_{x^\mu})^a_b \delta^4(x - y) \end{pmatrix},$$ (11)

$$Z^\alpha_{\beta\gamma} \underset{}{\longrightarrow} \begin{pmatrix} 0 \\ (\nabla_{y^\rho})^b_c \delta^4(y - z) \end{pmatrix},$$ (12)

$$T^i_{\alpha\beta} \underset{}{\longrightarrow} \begin{pmatrix} 0 \\ -\frac{1}{2} \xi_{\mu\nu\rho\sigma} f^a_{bc} \delta^4(x - z) \end{pmatrix}. $$ (13)

### 2.1 The ghost spectrum

The main idea behind the BRST-anti-BRST algebraic structure consists in (i) the doubling of the initial gauge symmetries by introducing a double set of initial ghosts; and (ii) in the introduction of a bigrading called the new ghost bigrading and denoted by $b$ [7, 8]. Thus, to the gauge generators and the first order reducibility functions we associate the ghosts

$$R^i_{\alpha\beta} \longrightarrow (\varphi^{(0, 1)}, \varphi^{(1, 0)}),$$ (14)

$$Z^\alpha_{\beta\gamma} \longrightarrow (\varphi^{(0, 2)}, \varphi^{(2, 0)}).$$ (15)
The superscript \((a,b)\) denotes the \textit{bingh} of the corresponding generators. The total ghost spectrum is according to Batalin et al. \([10]\):

\[(1,0) (0,1) (1,1) (2,0) (1,1) (0,2) (2,1) (1,2) \]
\[\varphi^{\alpha_0}, \varphi^{\alpha_1}, \pi^{\alpha_0}, \varphi^{\alpha_1}, \phi^{\alpha_1}, \pi^{\alpha_1}, \pi^{\alpha_1}\]

It can be understood by using the extended longitudinal differential of \([18, 19]\). Instead of the two sets of ghosts of the usual BRST formalism, the additional symmetry requires eight different sets of ghosts.

### 2.2 The antifield spectrum and the boundary conditions

There are two brackets in the BRST-anti-BRST formalism \([9, 10, 11]\), respectively denoted by \((\cdot, \cdot)_1\) and \((\cdot, \cdot)_2\). This implies that to each field there will correspond two antifields, one conjugated in the first antibracket and the other conjugated in the second antibracket. This leads to the following field-antifield spectrum:

| \((0,0)\) | \((1,0)\) | \((0,1)\) |
|---|---|---|
| \(q^i\) | \(\varphi^{\alpha_0}\) | \(\varphi^{\alpha_0}\) |
| \((-1,0)*\) \(\tilde{q}_i(1)\) | \((-2,0)*\) \(\varphi^{\alpha_0}(1)\) | \((-1,1)*\) \(\varphi^{\alpha_0}(1)\) |
| \((1,1)\) \(\pi^{\alpha_0}\) | \((2,0)\) \(\varphi^{\alpha_1}\) | \((1,1)\) \(\varphi^{\alpha_1}\) |
| \((-2,1)*\) \(\tilde{\pi}_{\alpha_0}(1)\) | \((-3,0)*\) \(\varphi^{\alpha_1}(1)\) | \((-2,1)*\) \(\varphi^{\alpha_1}(1)\) |
| \((0,2)\) \(\varphi^{\alpha_1}\) | \((2,1)\) \(\pi^{\alpha_1}\) | \((1,2)\) \(\pi^{\alpha_1}\) |
| \((-1,2)*\) \(\bar{\varphi}_{\alpha_1}(1)\) | \((-3,1)*\) \(\bar{\pi}_{\alpha_1}(1)\) | \((-2,3)*\) \(\bar{\pi}_{\alpha_1}(1)\) |

The resolution bidegree \(bires = (\text{res}_{(1)}, \text{res}_{(2)})\) is given by \(-\text{bingh}\) and is non-zero only for the antifields. The extended master equation will be decomposed according to the resolution degree \(\text{res} = \text{res}_{(1)} + \text{res}_{(2)}\). Following \([10, 11]\), we introduce supplementary antifields, referred to as the "bar-variables":

\[-(1,-1) -(2,-1) -(1,-2) -(2,-2) -(3,-1) -(2,-2) -(1,-3) -(3,-2) -(2,-3) -(1,-3)\]
\[\bar{q}_i, \bar{\varphi}_{\alpha_0}, \bar{\varphi}_{\alpha_0}, \bar{\pi}_{\alpha_0}, \bar{\varphi}_{\alpha_1}, \bar{\varphi}_{\alpha_1}, \bar{\pi}_{\alpha_1}, \bar{\pi}_{\alpha_1}\quad (16)\]
Having the complete fields-antifields spectrum, we can now give the boundary conditions of the solution of the extended master equation:

\[
S^{(0)} = S_0, \\
S^{(1)} = (-1,0)^* q^{(1)}_i R_{\alpha_0} (1,0)^* \phi^{(0)}_{\alpha_0} + q^{(1)}_i R_{\alpha_0} (0,1)^* \phi^{(0)}_{\alpha_0}, \\
S^{(2)} =_{\varphi_{\alpha_0}(1)} Z_{\alpha_1}^{(2,0)} \phi^{(2,0)}_{\alpha_1} +_{\varphi_{\alpha_0}(2)} Z_{\alpha_1}^{(0,2)} \phi^{(0,2)}_{\alpha_1} + \frac{1}{2} \left( _{-\varphi_{\alpha_0}(1)} (1,1)^* + _{-\varphi_{\alpha_0}(2)} (1,1)^* \right) Z_{\alpha_1}^{(1,1)} \phi^{(1,1)}_{\alpha_1} \\
+ \left( (-1,1)^* q^{(1)}_i R_{\alpha_0} (1,1)^* \phi^{(1)}_{\alpha_1} \right) \pi^{(1,0)}_{\alpha_0} + \ldots, \\
S^{(3)} = \left( (1,2)^* + \frac{1}{2} \left( _{\varphi_{\alpha_0}(1)} (2,1)^* + _{\varphi_{\alpha_0}(2)} (2,1)^* \right) \pi^{(2,1)}_{\alpha_1} \\
+ \left( (1,2)^* - \frac{1}{2} \pi_{\alpha_0(1)} (1,2)^* + (1,2)^* \phi^{(1)}_{\alpha_1} \phi^{(1)}_{\alpha_1} \right) \pi^{(1,2)}_{\alpha_1} + \ldots \right) 
\]

The total field-antifield spectrum as well as the boundary conditions of the master equation can be understood through homological arguments \cite{7, 8, 11}.

### 2.3 Resolution of the master equation

In this section we will explicitly solve the classical extended master equation

\[
\frac{1}{2} (S,S) + V S = 0, 
\]

for a first order on-shell reducible, off-shell abelian gauge system\footnote{The fact that the Freedman-Townsend model is abelian greatly simplifies the forthcoming computations. Nevertheless, the case of higher order reducible gauge systems with non-abelian open algebras can be treated exactly along the same lines, the only difference being the appearance of supplementary terms in the solution of the master equation.}. We introduce at this stage the generic notation \( \Phi^A \) for all the fields, \( \Phi^{\ast}_{A(1)} \) and \( \Phi^{\ast}_{A(2)} \) for the antifields respectively conjugated in the first and the second
antibracket and also $\bar{\Phi}_A$ for the bar-variables. The antibracket in equation (21) is defined by

\[
(F, G) = (F, G)_1 + (F, G)_2 = \frac{\delta F}{\delta \Phi^A} \frac{\delta G}{\delta \Phi^*_{A(1)}} - \frac{\delta F}{\delta \Phi^*_{A(1)}} \frac{\delta G}{\delta \Phi^A} + \frac{\delta F}{\delta \Phi^A} \frac{\delta G}{\delta \Phi^*_{A(2)}} - \frac{\delta F}{\delta \Phi^*_{A(2)}} \frac{\delta G}{\delta \Phi^A}.
\]  

(22)

Here $V$ acts only on the bar-variables and is defined as

\[
V = V_1 + V_2 = \Phi^*_{A(2)} \frac{\delta}{\delta \Phi^A} - \Phi^*_{A(1)} \frac{\delta}{\delta \Phi^A}.
\]  

(23)

The requirement $b\text{ing}h(S) = (0, 0)$ implies that the equation (21) splits into two parts:

\[
\frac{1}{2} (S, S)_1 + V_1 S = 0 = \frac{1}{2} (S, S)_2 + V_2 S.
\]  

(24)

The resolution of (24) is performed along the lines of homological perturbation theory; one develops $S$ with respect to the resolution degree

\[
S = \sum_{k=0}^{\infty} (k) S,
\]  

(25)

where the boundary terms of $S, S$ and $S$ have been given in the preceding subsection. Note that the resolution of the master equation will also fix all the remaining terms of the two Koszul-Tate operators in such a way that they become nilpotent and anticommuting off-shell (this follows from the generalized Jacobi identity). The equation in resolution degree 0 is satisfied if $(1) S$ contains only the already given boundary terms:

\[
S = (1) q_{\alpha(1)} R^{i\alpha}_\alpha \varphi^{\alpha_0} + (0, -1) q_{\alpha(1)} R^{i\alpha}_\alpha \varphi^{\alpha_0}.
\]  

(26)
The equation in resolution degree 1 is written as (see [4])

\[(S, S)^{(0)} + (S, S)^{(1)} + VS + \frac{1}{2} (S, S)^{(0)} = 0 \quad (27)\]

where (i) \((\cdot, \cdot)^{(k)}\) stands for the pieces of the antibrackets containing only the antifields of resolution degree \((k + 1)\) and (ii) the last term vanishes due to the abelian structure of the model. Note that the equation (27) also splits into two pieces according to the new ghost bigrading. One can check that in addition to the boundary terms given in (19) the only supplementary terms contained in \((S)\) are

\[-\frac{1}{2} (-1, 0) \ast (-1, 0) \ast T^{ij}_{\alpha_1} \varphi^{(2, 0)}_{\alpha_1} - \frac{1}{2} (-1, 0) \ast (0, -1) \ast q_i(1) q_j(2) T^{ij}_{\alpha_1} \varphi^{(1, 1)}_{\alpha_1} - \frac{1}{2} (0, -1) \ast (0, -1) \ast T^{ij}_{\alpha_1} \varphi^{(0, 2)}_{\alpha_1}. \quad (28)\]

These terms are due to the fact that we only have reducibility on-shell. The equation at resolution degree 2 reads

\[\sum_{k=0}^{2} (S, S)^{(k)} + V S + (S, S)^{(0)} + \frac{1}{2} (S, S)^{(1)} = 0. \quad (29)\]

Again the last two terms vanish because the model is abelian and because the \(T^{ij}_{\alpha_1}\) do not depend on the fields. A close inspection of the two resulting equations shows that out of the 50 \textit{a priori} possible supplementary terms, only the following are needed to satisfy the master equations at this level:

\[\frac{1}{2} (-1, 0) \ast (-1, -1) q_i(1) \tilde{q}_j(1) T^{ij}_{\alpha_1} \pi^{(2, 1)}_{\alpha_1} + \frac{1}{2} (0, -1) \ast (0, -1) q_i(2) \tilde{q}_j(2) T^{ij}_{\alpha_1} \pi^{(1, 2)}_{\alpha_1}. \quad (30)\]

One can then verify that it is possible to choose \(S = 0\) for \(k > 3\). This completes our derivation of the solution of the extended master equation. Using the identifications (7),(11) and (12), we get for the Freedman-Townsend model
The following change of variables will be useful:

3.1 Equivalence with the ordinary BRST formalism

Gauge fixing

Here we make the choice $\hat{T}$ and anticommuting transformations:

$$S = S_0 + \int d^4x \left\{ (-1,0)^* B_{\mu (1)}^{\nu} \nabla_\mu \varphi^{(1)\nu} + B_{\mu (2)}^{\nu} \nabla_\mu \varphi^{(0,1)\nu} \right\}$$

This solution is by construction invariant under the two following nilpotent and anticommuting transformations:

$$s_1 = (S, \cdot)_1 + V_1 \quad \text{and} \quad s_2 = (S, \cdot)_2 + V_2. \quad (32)$$

as well as under any linear combinations of these transformations.

3 Gauge fixing

3.1 Equivalence with the ordinary BRST formalism

The following change of variables will be useful: $u^*_A = \frac{1}{2}(\Phi^*_A + \Phi^*_A(1))$ and $v^*_A = \frac{1}{2}(\Phi^*_A - \Phi^*_A(1))$. According to reference [11], the gauge fixed action is given by

$$\exp iS_{gf} = \exp([\hat{K}, \Delta]) \exp iS|_{\text{anti fields}=0}. \quad (33)$$

Here we make the choice $\hat{K} = -\Psi(\Phi^A)$ in order to make the comparison with the ordinary BRST treatment. The gauge fixed action in (33) then simply becomes (see [11]):

$$S_{gf} = S(\Phi^A, u_A^* = \frac{\delta}{\delta \Phi^A}, v_A^* = 0, \bar{\Phi}_A = 0). \quad (34)$$
When expliciting this last expression, one obtains

\[
S_{gf} = S_0 + \frac{\delta}{\delta q^i} R_{\alpha_0}^i \phi^{\alpha_0} - \frac{\delta}{\delta \phi^{\alpha_0}} \pi^{\alpha_0} \\
+ \frac{\delta}{\delta \phi^{\alpha_0}} Z_{\alpha_1}^0 \phi^{\alpha_1} + \frac{1}{2} \frac{\delta}{\delta \phi^{\alpha_0}} Z_{\alpha_1}^0 \bar{\phi}^{\alpha_1} - \frac{1}{2} \frac{\delta}{\delta q^i} \frac{\delta}{\delta q^j} T_{\alpha_1}^{ij} \phi^{\alpha_1} \\
- \frac{1}{2} \frac{\delta}{\delta \pi^{\alpha_0}} Z_{\alpha_1}^0 \bar{\pi}^{\alpha_1} - \frac{\delta}{\delta \phi^{\alpha_1}} \bar{\pi}^{\alpha_1} - \frac{\delta}{\delta \bar{\phi}^{\alpha_1}} \bar{\pi}^{\alpha_1},
\]

(35)

where we have made the following change of variables

\[
(\varphi^{\alpha_0} + \varphi^{\alpha_0}) = \phi^{\alpha_0} \quad \text{and} \quad (\varphi^{\alpha_0} - \varphi^{\alpha_0}) = \bar{\phi}^{\alpha_0}, \quad (1.0)
\]

(36)

\[
(\varphi^{\alpha_0} + \varphi^{\alpha_0} + \varphi^{\alpha_0}) = \phi^{\alpha_1}, \quad (2.0)
\]

(37)

\[
(\varphi^{\alpha_0} - \varphi^{\alpha_0} + \varphi^{\alpha_0}) = \bar{\phi}^{\alpha_1}, \quad (2.0)
\]

(38)

\[
(\varphi^{\alpha_1} - \varphi^{\alpha_1}) = \bar{\phi}^{\alpha_1}, \quad (2.0)
\]

(39)

\[
\pi^{\alpha_0} = -\frac{1}{2} \pi^{\alpha_0}, \quad (1.1)
\]

(40)

\[
\pi^{\alpha_1} - \pi^{\alpha_1} = -\frac{1}{2} \pi^{\alpha_1} \quad \text{and} \quad \pi^{\alpha_1} + \pi^{\alpha_1} = -\frac{1}{2} \pi^{\alpha_1}. \quad (2.1)
\]

(41)

From the transformations (32), one can define their gauge fixed counterparts, respectively denoted by \( s_{1 \, gf} \) and \( s_{2 \, gf} \) and obtained by first calculating \( s_1 \) and \( s_2 \) and fixing the gauge afterwards. Let us define \( s = s_{1 \, gf} + s_{2 \, gf} \). The gauge fixed action \( S_{gf} \) is then invariant under \( s \) explicitly defined by :

\[
sq^i = R_{\alpha_0}^i \phi^{\alpha_0} - \frac{\delta}{\delta \pi^{\alpha_0}} T_{\alpha_1}^{ij} \phi^{\alpha_1} \quad \quad s\phi^{\alpha_0} = Z_{\alpha_1}^0 \phi^{\alpha_1} \\
\]

\[
s^{\bar{\phi}^{\alpha_0}} = \frac{1}{2} Z_{\alpha_1}^0 \bar{\phi}^{\alpha_1} - \pi^{\alpha_0} \quad \quad s\bar{\pi}^{\alpha_0} = -\frac{1}{2} Z_{\alpha_1}^0 \bar{\pi}^{\alpha_1} \\
\]

\[
s^{\phi^{\alpha_1}} = -\bar{\pi}^{\alpha_1} \quad \quad s^{\phi^{\alpha_1}} = -\bar{\pi}^{\alpha_1} \\
\]

\[
s^{\phi^{\alpha_1}} = s\bar{\pi}^{\alpha_1} = s\bar{\pi}^{\alpha_1} = 0. \quad (42)
\]
As expected on general grounds, one can check that \( s \) is nilpotent on the stationary surface defined by the equations of motion associated to the gauge fixed action. In fact \( s \) is nilpotent off-shell for all generators with the exception of \( s^2 q^i = - \delta S_{gf}/\delta q^i T^{ij}_{\alpha_1} \phi^{\alpha_1} \). However, the choice \( \hat{K} = -\Psi(\Phi^4) \) spoils the invariance of \( S_{gf} \) under \( s_{1gf} \) and \( s_{2gf} \) separately.

Translating these results to the Freedman-Townsend model with the following choice of the gauge fixing fermion

\[
\Psi = - \int d^4 x \left\{ \alpha(\nabla^\mu \bar{\phi}^\nu) B_{\mu\nu} + \beta(\phi_\mu \nabla^\mu \bar{\phi} - \bar{\phi}^\mu (\pi_\mu - \frac{1}{2} \nabla_\mu \tilde{\phi}) + \gamma \bar{\pi} \bar{\phi} + \delta \bar{\phi} \right\}
\]

(43)

This choice of \( \Psi \) gives a well defined path integral as can be seen by elimination of the auxiliary fields \( \pi_\mu, \bar{\pi}, \tilde{\pi}, \), and will allow us the comparison with the BRST-anti-BRST gauge fixed action given below. The gauge fixed BRST symmetry is given by:

\[
\begin{align*}
    s B^a_{\mu\nu} &= \nabla_{[\mu \phi_{\nu]} + \frac{1}{2} (\nabla^\rho \bar{\phi})^b f^{abc} \epsilon_{\mu
u\rho\sigma} \phi^a \\
    s \phi_\mu &= \nabla_\mu \phi \\
    s \bar{\phi}_\mu &= \frac{1}{2} \nabla_\mu \bar{\phi} - \pi_\mu \\
    s \pi_\mu &= -\frac{1}{2} \nabla_\mu \bar{\pi} \\
    s \bar{\phi} &= -\bar{\pi} \\
    s \tilde{\phi} &= -\bar{\pi} \\
    s A_\mu &= s \phi = s \bar{\pi} = s \tilde{\pi} = 0.
\end{align*}
\]

(45)

Apart from the cohomologically trivial variables \( \tilde{\phi}^{\alpha_0}, B^{\alpha_0}, \bar{\phi}^{\alpha_1}, \bar{\phi}^{\alpha_1}, \tilde{B}^{\alpha_1}, \tilde{B}^{\alpha_1} \) and their antifields, this result, with the identifications \((7), (11) \) and \((12)\), coincides with the result obtained from the usual BRST formalism \([14, 15, 16]\) where some of the above variables are reintroduced as a non-minimal sector for gauge fixing purposes.
3.2 BRST-anti-BRST gauge fixed action

According to references [9, 10], the BRST-anti-BRST invariant gauge fixed action $S_{gf}$ is given by:

$$S_{gf} = S(\Phi^A, \Phi^*_A, \bar{\Phi}^A)$$

$$+ \int d^4x \left\{ \Phi^*_A \xi^{Ai} + \left( \bar{\Phi}^A - \frac{\delta}{\delta \Phi^A} F \right) \lambda^A - \frac{1}{2} \epsilon_{ij} \xi^{Ai} \frac{\epsilon^{-2}}{\delta \Phi^A \delta \Phi^B \delta P_i A} \xi^{Bj} \right\} \tag{46}$$

where $S(\Phi^A, \Phi^*_A, \bar{\Phi}^A)$ is the solution of the extended master equation (21), $\epsilon_{ij} i, j = 1, 2$ is completely antisymmetric with $\epsilon_{12} = -1$ and $F$ is a bosonic gauge fixing functional depending on the $\Phi^A$’s only. The $\Phi^*_A, \Phi^A, \xi^{Ai}$ and $\lambda^A$’s play the role of ordinary variables in the gauge fixed action and must be integrated over in the path integral just like the $\Phi^A$’s. The gauge fixed action is invariant under the following two transformations:

$$s^i \Phi^A = \xi^{Ai} \quad s^i \Phi^*_A = \delta^i_j \frac{\delta S}{\delta \Phi^A} (-)^{\epsilon_A} \quad s^i \lambda^A = 0 \tag{47}$$

$$s^i \bar{\Phi}^A = \epsilon^{ij} \Phi^*_A (-)^{\epsilon_A + 1} \quad s^i \xi^{Aj} = \lambda^A \epsilon^{ij} \tag{48}$$

where $s^i$ acts as an odd right derivative.

For the Freedmann-Townsend model, the final form of the gauge fixed action, with the choice

$$F = \int d^4x \left\{ \frac{\alpha}{2} B_{\mu \nu} B^{\mu \nu} + \beta \phi_\mu \bar{\phi}^\mu + \gamma \phi \bar{\phi} + \frac{\delta}{2} \phi \bar{\phi} \right\} \tag{49}$$

and after elimination of some auxiliary fields (and the corresponding modifi-

\[\text{An alternative derivation of this gauge fixed action in the framework of the standard BRST formalism and a constructive approach to the on-shell nilpotent and anticommuting gauge fixed BRST-anti-BRST symmetries from which the above transformations differ by antisymmetric combinations of the equations of motion obtained from $S_{gf}$ will be given in a forthcoming paper [8].}\]
cations of the symmetries (see [20])) is given by \(^3\):

\[
S_{gf} = S_0 + \int d^4x \left\{ B^{\mu
u*}_{(1)} \nabla_\mu \phi_\nu + B^{\mu
u*}_{(2)} \nabla_\mu \bar{\phi}_\nu + \alpha B^{\mu\nu} \nabla_\mu \bar{\pi}_\nu \\
- \frac{1}{4} [B^{\mu
u*}_{(1)} , B^{\rho\sigma*}_{(1)}] \epsilon_{\mu\nu\rho\sigma} \phi + \frac{1}{4} [B^{\mu
u*}_{(2)} , B^{\rho\sigma*}_{(2)}] \epsilon_{\mu\nu\rho\sigma} \bar{\phi} \\
+ \frac{\alpha}{4} [B^{\mu
u*}_{(1)} , B^{\rho\sigma*}_{(2)}] \epsilon_{\mu\nu\rho\sigma} + \frac{\alpha}{4} [B^{\mu
u*}_{(2)} , B^{\rho\sigma*}_{(1)}] \epsilon_{\mu\nu\rho\sigma} + B^{\mu
u*}_{(1)} \xi^{(1)}_{\mu\nu} + B^{\mu
u*}_{(2)} \xi^{(2)}_{\mu\nu} + \alpha \xi^{(1\mu) \nu} \xi^{(2)}_{\mu\nu} \\
+ \beta (-\pi^\mu \pi_\mu + \frac{1}{4} (\nabla^\mu \bar{\phi}) \nabla_\mu \bar{\phi} - (\nabla^\mu \phi) \nabla_\mu \phi + (\nabla^\mu \bar{\phi}) \bar{\pi} - (\nabla^\mu \phi) \bar{\pi}) \\
+ (\gamma - \delta) \bar{\pi} \bar{\pi}\right\}. \tag{50}
\]

In a perturbative approach to quantization, this action is gauge fixed because one treats the cubic terms as interactions and one can then eliminate the fields \(B^{\mu\nu*}_{(i)}\) and \(\xi^{(i)}_{\mu\nu}\) getting the usual kinetic term for the ghosts and antighosts of first order. The integration over \(\pi_\mu, \bar{\pi}\) and \(\bar{\pi}\) then leads to well-defined propagators for the remaining fields. The action (50) is invariant under the transformations:

\[
\begin{align*}
& s^i A_\mu = 0 \quad s^i B^{\mu\nu} = \xi^{(i)}_{\mu\nu} \quad s^i \bar{\pi} = s^i \bar{\pi} = 0 \\
& s^1 \phi_\mu = -\nabla_\mu \phi \quad s^2 \phi_\mu = -\bar{\pi}_\mu - \frac{1}{2} \nabla_\mu \bar{\phi} \quad s^2 \bar{\phi}_\mu = -\nabla_\mu \bar{\phi} \\
& s^1 \phi = 0 \quad s^2 \phi = -\bar{\pi} \quad s^2 \bar{\phi} = 0 \\
& s^1 \bar{\pi}_\mu = \frac{1}{2} \nabla_\mu \bar{\pi} \quad s^2 \bar{\pi}_\mu = \frac{1}{2} \nabla_\mu \bar{\pi} \quad s^2 \bar{\phi} = 0 \\
& s^i \xi^{(j)a}_{\mu\nu} = -\left((\nabla_\mu \bar{\pi}_\nu)^a + \frac{1}{4} B^{\rho\sigma*}_{(1)b} f^{\rho\sigma\xi} \epsilon_{\mu\nu\rho\sigma} \bar{\pi}_c + \frac{1}{4} B^{\rho\sigma*}_{(2)b} f^{\rho\sigma\xi} \epsilon_{\mu\nu\rho\sigma} \bar{\pi}_c\right) \epsilon^{ij} \\
& s^i B^{\mu\nu*}_{(j)} = \delta^{ij}_3 \frac{1}{4} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}. \tag{51}
\end{align*}
\]

4 Conclusion

Even though in this model the fields \(B^{\mu\nu*}_{i}\) and \(\xi^{i}_{\mu\nu}\) are auxiliary, their elimination is cumbersome because the matrix of their quadratic part depends

\(^3\)Here an obvious renaming of the fields has been performed in order to make the comparison to the previous result more transparent: the new ghost bigrading has been dropped and the numerotation of the ghosts has been replaced by the superscript. The bar now means antighost; no confusion with the previous bar-variables can arise, because those variables have been eliminated.
on the second generation ghosts and the structure constants. In the abelian case however, this elimination can easily be performed and the resulting gauge fixed action is:

\[ S_{gf} = S_0 + \int d^4x \left\{ -\alpha (\partial^{[\mu} \phi^{\nu]}) \partial_{\mu} \phi_{\nu} + \alpha B^{\mu \nu} \partial_{\mu} \pi_{\nu} + \beta (-\pi^{\mu} \pi_{\mu} + \frac{1}{4} (\partial^\mu \phi) \partial_\mu \phi - (\partial^\mu \phi_{\mu}) \pi) + (\gamma - \delta) \pi \bar{\pi} \right\} \]  

(52)

This result coincides with the corresponding result of equation (44) and the anti-BRST symmetry can be implemented on the same set of fields than the one used for the solution of the standard master equation (with an appropriate non-minimal sector). Furthermore, because of the linear dependence of the solution of the master equation (and the extended master equation) in the antifields, the BRST- anti-BRST gauge fixed action is given by

\[ S_{gf} = S_0 - s^1 s^2 F \]  

(53)

and coincides with the result given in reference [17]. Consequently, it is only for the abelian case that the anti-BRST symmetry can be implemented in the same way than for Yang-Mills theory [9, 17]. In the non-abelian case, the incorporation of the anti-BRST symmetry needs the supplementary auxiliary fields \( B_i^{\mu \nu} \) and \( \xi_{\mu \nu}^i \). This is yet another illustration of the general idea underlying the BRST construction, namely that a more symmetric description requires in general more variables.

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References

[1] I.A. Batalin and G.A. Vilkovisky, ”Gauge algebra and quantization”, Phys. Lett. B102 (1980) 27.

[2] I.A. Batalin and G.A. Vilkovisky, ”Quantization of gauge theories with linearly dependent generators”, Phys. Rev D28 (1983) 2567.

[3] I.A. Batalin and G.A. Vilkovisky, ”Existence theorem for gauge algebra”, J. Math. Phys. 26 172.

[4] J.M.L. Fisch and M. Henneaux, ”Homological perturbation theory and the algebraic structure of the antifield-antibracket formalism for gauge theories”, Commun. Math. Phys. 128 (1990) 627.

[5] I.A. Batalin, P.M. Lavrov and I.V. Tyutin, ”Extended BRST quantization of gauge theories in the generalized canonical formalism”, J. Math. Phys. 31 (1990) 6.

[6] I.A. Batalin, P.M. Lavrov and I.V. Tyutin, ”An Sp(2)-covariant version of generalized canonical quantization of dynamical systems with linearly dependent constraints”, J. Math. Phys. 31 (1990) 2708.

[7] P. Grgoire and M. Henneaux, ”Hamiltonian formulation of the anti-BRST transformation”, Phys. Lett. B277 (1992) 459.

[8] P. Grgoire and M. Henneaux, in preparation.

[9] I.A. Batalin, P.M. Lavrov and I.V. Tyutin, ”Covariant quantization of gauge theories in the framework of extended BRST symmetry”, J. Math. Phys. 31 (1990) 1487.

[10] I.A. Batalin, P.M. Lavrov and I.V. Tyutin, ”An Sp(2)-covariant quantization of gauge theories with linearly dependent generators”, J. Math. Phys. 32 (1991) 532.

[11] M. Henneaux, ”Geometric interpretation of the quantum master equation in the BRST-anti-BRST formalism”, Phys. Lett. B282 (1992) 372.

[12] D.Z. Freedman and P.K. Townsend, ”Antisymmetric tensor gauge theories and non-linear σ-models”, Nucl. Phys. B177 (1981) 282.
[13] E. Witten, ”Non-commutative geometry and string field theory”, Nucl. Phys. B268 (1986) 253.

[14] S.P. De Alwis, M.T. Grisaru and L. Mezincescu, ”Quantization and unitarity in antisymmetric tensor gauge theories”, Nucl. Phys. B303 (1988) 57.

[15] C. Battle and J. Gomis, ”Lagrangian and Hamiltonian BRST structures of the antisymmetric tensor gauge theory”, Phys. Rev D38 (1988) 1169.

[16] L. Baulieu, E. Bergshoeff and E. Sezgin, ”Open BRST algebras, ghost unification and string field theory” Nucl. Phys. B307 (1988) 348.

[17] J. Thierry-Mieg and L. Baulieu, ”Covariant Quantization of non-abelian antisymmetric tensor gauge theories”, Nucl. Phys. B228 (1983) 259.

[18] M. Henneaux and C. Teitelboim, ”Quantization of gauge systems”, Princeton University Press (Princeton:1992).

[19] J. Fisch, M. Henneaux, J. Stasheff and C. Teitelboim, ”Existence, uniqueness and cohomology of the classical BRST charge with ghosts of ghosts”, Commun. Math; Phys. (1989) 379.

[20] M. Henneaux, ”Elimination of the auxiliary fields in the antifield formalism”, Phys. Lett. B238 (1990) 299.