Open-Loop System Identification of MIMO Integrator Model

Ichiro Jikuya *, Masaru Kino **, and Katsuhiko Yamada ***

Abstract: This paper considers the open-loop system identification of a multi-input multi-output (MIMO) integrator model which is a multiplication of a constant matrix, called the action matrix, and the integrator. Our problem is to estimate the action matrix from the measured input and output data. The estimation algorithm is proposed by removing trends from the numerically integrated input and output so that the effects of input and output disturbances are eliminated. The proposed method is applicable to the basic scenario (constant input and output offsets), which is similar to previous studies, and the non-constant output offset; however, it is not applicable to the non-constant input offset. The discussion in this paper indicates the essential difficulties in the open-loop system identification with the integrator.

Key Words: system identification, parameter estimation, multi-input multi-output, integrator.

1. Introduction

System identification is a technique for building control models from measured input-output data. Most of the open-loop system identification techniques are formulated for an asymptotically stable system. Takeshita et al. proposed an open-loop system identification method for the single-input single-output (SISO) system which is a multiplication of an asymptotically stable system and the integrators [1]. The key idea is to restrict the class of input and output disturbances to be constant, and then, the effects of input and output disturbances can be eliminated by removing the trends from the numerically integrated input and output. Itai and Oku tested open-loop system identification experiments for the cart model which can be modeled by a multiplication of the SISO first order system and the SISO integrator [2].

In this paper, we consider open-loop system identification of the multi-input multi-output (MIMO) integrator model which is a multiplication of a constant matrix and the integrator. This work is inspired by the control system of the segmented mirrors in the SEIMEI telescope [3]–[5]. Compared with the previous studies [1], [2], our problem is simpler in one aspect because the asymptotically stable dynamics is not included in the system model, whereas our problem is more difficult in the other aspect because the MIMO system is considered. We first consider the basic scenario which is similar to the previous studies; the class of input and output disturbances is restricted to be constant, and then, the effects of input and output disturbances can be eliminated by removing the trends from the numerically integrated input and output. The estimation algorithm for estimating the action matrix is proposed by removing the trends from the numerically integrated input and output so that the effects of input and output disturbances are eliminated. A preliminary version of this paper has been presented in our conference proceeding [6], where only this basic scenario was discussed. In this paper, we further discuss additional scenarios, non-constant input offset, and non-constant output offset, and numerical examples are given to evaluate the proposed estimation method.

The paper is structured as follows: Section 2 presents the preliminaries and problem formulation. The main result, which proposes the parameter estimation method, is presented in Section 3. The numerical examples are given in Section 4. The conclusion is given in Section 5.

2. Preliminaries and Problem Formulation

Consider a MIMO integrator model given by

\[ \dot{x}(t) = Ax(t) + Bu(t), \]
\[ y(t) = Cx(t), \]

where \( t \in \mathbb{R} \) is the continuous time, \( x \in \mathbb{R}^n \) is the state, \( u \in \mathbb{R}^m \) is the input, \( y \in \mathbb{R}^p \) is the output, and \( C \in \mathbb{R}^{p \times n} \) is the constant matrix called the action matrix.

For example, the control system of the segmented main mirrors is modeled by the MIMO integrator model in the SEIMEI telescope, where the dimensions are \( n = 54 \) and \( p = 72 \) [3]–[5]. The nominal value of \( C \) is determined from the design parameters of the SEIMEI telescope. Two types of constant output feedback control algorithms have been presented and tested in simulations for the nominal value of \( C \) by the authors [3]–[5].

The action matrix \( C \) may contain errors due to manufacturing error and mounting error and may change during observation due to the bends of frames influenced by the gravity. An alternative procedure for estimating the value of \( C \) is, therefore, desirable to compensate for such error factors.

This paper considers the parameter estimation of the action matrix \( C \) from the input-output data. Equations (1) and (2) can be time-discretized by approximating the time derivative by the forward difference. The equation error, \( w \), is incorporated into the model. The input disturbance, \( e_i \), and output disturbance, \( e_o \), are also incorporated into the measured input and output. Then, the model for the parameter estimation is given by

\[ x(t) = Ax(t) + Bu(t) + e_i, \]
\[ y(t) = Cx(t) + e_o + w, \]
where \( k \in \mathbb{Z} \) is the discrete time, \( T > 0 \) is the sampling period, \( w \) is the noise, \( e_u \) is the disturbance for input measurement, \( u_{\text{obs}} \) is the measured input, \( e_y \) is the disturbance for output measurement, and \( y_{\text{obs}} \) is the measured output (see Fig. 1). This model can be classified as errors-in-variables (EIV) models, which play an important role for modeling the physical laws of the process rather than predicting its future behavior in the system identification [7].

Our problem is to estimate the action matrix, \( C \), from the pair of measured input and output, \( \{u_{\text{obs}}[k]; k = 0, \ldots, N - 1\} \) and \( \{y_{\text{obs}}[k]; k = 1, \ldots, N\} \), where \( N \) is the number of data size.

![Block diagram of the MIMO integrator model.](image)

**3. Main Result**

### 3.1 Basic Scenario

Suppose that \( u[k] \) and \( w[k] \) are independent and identically distributed (i.i.d.) random variables, persistency exciting (PE), and that their expectations are given by \( E[u[k]] = u_{\text{ave}} \) and \( E[w[k]] = 0 \). It follows that \( u[k] \) is represented as \( u[k] = u_{\text{ave}} + d[k] \), where \( d[k] \) is zero-mean, i.i.d., and PE. Suppose also that \( e_u[k] \) and \( e_y[k] \) are constant, i.e., \( e_u[k] =: e_u \) and \( e_y[k] =: e_y \) represent constant offsets. The latter assumption, i.e., constant offsets, is inspired by [1].

Let \( u_{\text{int}}[k] \) be the numerical integration of \( u_{\text{obs}}[k] \) given by

\[
u_{\text{int}}[k] := \sum_{j=0}^{k} u_{\text{obs}}[j] .
\]

Equation (7) is rewritten as

\[
u_{\text{int}}[k] = k T u_{\text{ave}} + e_u + d_{\text{int}}[k] ,
\]

where \( d_{\text{int}}[k] := T \sum_{j=0}^{k} d[j] .
\]

By direct computation, we have

\[
y_{\text{obs}}[k] = C x[0] + (k - 1) T C u_{\text{ave}} + C d_{\text{int}}[k - 1] + C w[k] + e_y
\]

By aligning the vector variables, the matrices are defined as follows:

\[
Y_{\text{obs}} := \begin{bmatrix} y_{\text{obs}}[1] & y_{\text{obs}}[2] & \cdots & y_{\text{obs}}[N] \end{bmatrix} ,
\]

\[
X_0 := \begin{bmatrix} x[0] & x[0] & \cdots & x[0] \end{bmatrix} ,
\]

\[
U_{\text{int}} := \begin{bmatrix} u_{\text{int}}[0] & u_{\text{int}}[1] & \cdots & u_{\text{int}}[N - 1] \end{bmatrix} ,
\]

\[
U_{\text{ave}} := \begin{bmatrix} 0 & u_{\text{ave}} & \cdots & (N - 1) u_{\text{ave}} \end{bmatrix} ,
\]

\[
D_{\text{int}} := \begin{bmatrix} d_{\text{int}}[0] & d_{\text{int}}[1] & \cdots & d_{\text{int}}[N - 1] \end{bmatrix} ,
\]

\[
E_u := \begin{bmatrix} 0 & e_u & \cdots & (N - 1) e_u \end{bmatrix} ,
\]

\[
W := \begin{bmatrix} w[1] & w[2] & \cdots & w[N] \end{bmatrix} ,
\]

\[
E_y := \begin{bmatrix} e_y & e_y & \cdots & e_y \end{bmatrix} .
\]

Equation (8) is matricized as follows:

\[
U_{\text{int}} = T (U_{\text{ave}} + E_u) + D_{\text{int}} .
\]

The expectation of \( D_{\text{int}} \) is zero. Because \( T (U_{\text{ave}} + E_u) \) is affine with respect to the column indexes, \( D_{\text{int}} \) can be approximated by removing the first order trend in \( Y_{\text{obs}} \) as follows:

\[
D_{\text{int}} \approx U_{\text{int}} - \text{Aff}(U_{\text{int}}) .
\]

\[
\text{Aff}(X) \text{ denotes the affine interpolation of the matrix } X \in \mathbb{R}^{n \times n}
\]

by

\[
\text{Aff}(X) := \begin{bmatrix} \beta & T a + \beta & \cdots & (N - 1) T a + \beta \end{bmatrix} ,
\]

where \( a \) and \( \beta \) minimize the approximation error of each row component in terms of the Euclidean norm as follows:

\[
\begin{bmatrix} a_j, \beta \end{bmatrix} := \arg \min_{a, \beta} \| X_j - [\beta \ T a_j + \beta \cdots (N - 1) T a_j + \beta \] \|
\]

where the subscript \( j \) denotes the \( j \)-th row component. This linear interpolation calculation can be performed, for example, using the detrend command in MATLAB.

Equation (9) is matricized as follows:

\[
Y_{\text{obs}} = C X_0 + C T U_{\text{ave}} + C D_{\text{int}} + C W + E_y
\]

The expectation of \( C D_{\text{int}} + C W \) is zero. Because \( C X_0 + C T U_{\text{ave}} + E_y \) is affine with respect to the column indexes, \( C D_{\text{int}} + C W \) can be approximated by removing the first order trend in \( Y_{\text{obs}} \) as follows:

\[
C D_{\text{int}} + C W \approx Y_{\text{obs}} - \text{Aff}(Y_{\text{obs}}) .
\]

By assumption, \( d[k] \) is PE and \( d[k] \) and \( w[k] \) are independent, and therefore, we have

\[
\det E[D_{\text{int}} D_{\text{int}}^T] \neq 0, \quad E[W D_{\text{int}}^T] = 0 .
\]

for sufficiently large \( N \), where \( E[\cdot] \) is the expectation. It follows from Eqs. (19) and (24) that the Moore-Penrose pseudo-inverse \( (U_{\text{int}} - \text{Aff}(U_{\text{int}}))^\dagger \) is available, where the Moore-Penrose pseudo-inverse of the column full rank matrix \( X \in \mathbb{R}^{n \times n} \) is given by \( X^\dagger := X^T (XX^T)^{-1} \). It also follows from Eqs. (19) and (24) that

\[
E[W (U_{\text{int}} - \text{Aff}(U_{\text{int}}))^\dagger] \approx E[W D_{\text{int}}^\dagger]
\]

\[
= E[W D_{\text{int}}^\dagger (D_{\text{int}} D_{\text{int}}^T)^{-1}] \approx 0 .
\]

By combining Eqs. (19) and (23), we have

\[
Y_{\text{obs}} - \text{Aff}(Y_{\text{obs}}) \simeq C (U_{\text{int}} - \text{Aff}(U_{\text{int}})) + C W
\]

Hence, by multiplying \( (U_{\text{int}} - \text{Aff}(U_{\text{int}}))^\dagger \) from the right of Eq. (26) and by taking the expectation, we have

\[
E[Y_{\text{obs}} - \text{Aff}(Y_{\text{obs}}) (U_{\text{int}} - \text{Aff}(U_{\text{int}}))^\dagger] \simeq C .
\]

The equation for estimating the action matrix \( C \) is obtained as follows:

\[
C_{\text{est}} = (Y_{\text{obs}} - \text{Aff}(Y_{\text{obs}}) (U_{\text{int}} - \text{Aff}(U_{\text{int}}))^\dagger .
\]
3.2 Non-Constant Input Offset

Suppose that \( e_u[k] \) is not constant, i.e., \( e_u[k] = e_{u,\text{ave}} + e_{u,r}[k] \). Suppose also that \( e_{u,r}[k] \) is zero mean, i.i.d., and independent from \( d[k] \) and \( \nu[k] \). The other assumptions are supposed to be the same as the basic scenario in the previous subsection.

Equation (8) is rewritten as

\[
u_{\text{int}}[k] = kT(u_{\text{ave}} + e_{u,\text{ave}}) + d_{\text{int}}[k] + e_{u,r,\text{int}}[k],\tag{29}
\]

where

\[
e_{u,r,\text{int}}[k] := \sum_{j=0}^{k} e_{u,r}[j].\tag{30}
\]

Equation (29) is matricized as follows:

\[
U_{\text{int}} = T(U_{\text{ave}} + E_{u,\text{ave}}) + D_{\text{int}} + E_{u,r,\text{int}},\tag{31}
\]

where

\[
E_{u,\text{ave}} := [0 \quad e_{u,\text{ave}} \ldots (N-1)e_{u,\text{ave}}],
\]

\[
E_{u,r,\text{int}} := [e_{u,r,\text{int}}[0] \quad e_{u,r,\text{int}}[1] \ldots e_{u,r,\text{int}}[N-1]].\tag{33}
\]

Because \( T(U_{\text{ave}} + E_{u,\text{ave}}) \) is affine with respect to the column indexes, \( D_{\text{int}} + E_{u,r,\text{int}} \) can be approximated by removing the first order trend in \( U_{\text{int}} \) as follows:

\[
D_{\text{int}} + E_{u,r,\text{int}} \approx U_{\text{int}} - \text{Aff}[U_{\text{int}}].\tag{34}
\]

By combining Eqs. (34) and (23), we have

\[
Y_{\text{obs}} - \text{Aff}[Y_{\text{obs}}] = C(U_{\text{int}} - \text{Aff}[U_{\text{int}}]) - CE_{u,r,\text{int}} + CW.\tag{35}
\]

By multiplying \( (U_{\text{int}} - \text{Aff}[U_{\text{int}}])^\dagger \) from the right of Eq. (35) and by taking the expected value, we have

\[
E[(Y_{\text{obs}} - \text{Aff}[Y_{\text{obs}}])(U_{\text{int}} - \text{Aff}[U_{\text{int}}])^\dagger] = C \cdot E[-E_{u,r,\text{int}} + W(D_{\text{int}} + E_{u,r,\text{int}})]
\]

\[
\ne C.\tag{36}
\]

We note that \( E[-E_{u,r,\text{int}} + W(D_{\text{int}} + E_{u,r,\text{int}})] \neq 0 \) because \( -E_{u,r,\text{int}} + W \) and \( D_{\text{int}} + E_{u,r,\text{int}} \) are not independent.

Hence, the equation for estimating \( C \) in Eq. (28) is not valid when \( e_u[k] \) is not constant.

3.3 Non-Constant Output Offset

Suppose that \( e_y[k] \) is not constant, i.e., \( e_y[k] = e_{y,\text{ave}} + e_{y,r}[k] \). Suppose also that \( e_{y,r}[k] \) is zero mean, i.i.d., and independent from \( d[k] \) and \( \nu[k] \). The other assumptions are supposed to be the same as the basic scenario.

Equation (9) is rewritten as

\[
y_{\text{obs}}[k] = Cx[0] + (k - 1)TCu_{\text{ave}} + Cd_{\text{int}}[k - 1] + CW[k] + e_{y,\text{ave}} + e_{y,r}[k].\tag{37}
\]

Equation (37) is matricized as follows:

\[
Y_{\text{obs}} = CX_0 + CTU_{\text{ave}} + CD_{\text{int}} + CW + E_{y,\text{ave}} + E_{y,r}.\tag{38}
\]

where

\[
E_y,\text{ave} := [e_{y,\text{ave}} \quad e_{y,\text{ave}} \cdots e_{y,\text{ave}}],
\]

\[
E_y := [e_y[1] \quad e_y[2] \cdots e_y[N]].\tag{40}
\]

The expectation of \( CD_{\text{int}} + CW + E_{y,r} \) is zero. Because \( CX_0 + CTU_{\text{ave}} + E_{y,\text{ave}} \) is affine with respect to the column indexes, \( CD_{\text{int}} + CW + E_{y,r} \) can be approximated by removing the first order trend in \( Y_{\text{obs}} \) as follows:

\[
CD_{\text{int}} + CW + E_{y,r} \approx Y_{\text{obs}} - \text{Aff}[Y_{\text{obs}}].\tag{41}
\]

By assumption, \( e_{y,r}[k] \) and \( d[k] \) are independent and, therefore, we have

\[
E[1r_{\text{int}}D_{\text{int}}^\dagger] = 0.\tag{42}
\]

Hence, by multiplying \( (U_{\text{int}} - \text{Aff}[U_{\text{int}}])^\dagger \) from the right of Eq. (38) and by taking the expected value, we have

\[
E[(Y_{\text{obs}} - \text{Aff}[Y_{\text{obs}}])(U_{\text{int}} - \text{Aff}[U_{\text{int}}])^\dagger] = C.\tag{43}
\]

Hence, the equation for estimating \( C \) in Eq. (28) is valid when \( e_y[k] \) is not constant.

4. Examples

In this section, the effectiveness of the proposed estimation method and the appropriateness of the scenario are discussed for a toy example. We first consider the basic scenario in Section 3.1, where \( T = 1, C, x[0], u_{\text{ave}}, e_u, \) and \( e_y \) are selected as

\[
C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}, \quad x[0] = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}, \quad u_{\text{ave}} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}, \quad e_u = \begin{bmatrix} 6 \\ 7 \end{bmatrix}, \quad e_y = \begin{bmatrix} 3 \\ 5 \\ 6 \end{bmatrix},
\]

and \( d[k] \) and \( \nu[k] \) are independently sampled from the Gaussian distribution \( N(0, 10I) \). The proposed equation for estimating \( C \) in Eq. (28) is applied to 1000 samples of input-output data for \( N = 10, 100, 1000, \) and 10000. The effectiveness of the proposed estimation method is evaluated by the mean values and standard deviations of the error rate

\[
\rho := \frac{||C - C_{\text{est}}||_F}{||C||_F},
\]

where \( ||X||_F \) is the Frobenius norm of the matrix \( X \). Figures 2 and 3 show the mean values and the standard deviation of the error rate \( \rho \) for \( N = 10, 100, 1000, \) and 10000. These figures indicate that the proposed estimation method is successful in the basic scenario.

We then consider the non-constant input offset in Section 3.2, where \( e_{u,r}[k] \) is independently sampled from the Gaussian distribution \( N(0, I) \) and the others are the same as the basic scenario. Figures 4 and 5 show the mean values and the standard deviation of the error rate \( \rho \) for \( N = 10, 100, 1000, \) and 10000. These figures indicate that the proposed estimation method is not successful in the non-constant input offset.

We finally consider the non-constant output offset in Section 3.3, where \( e_{y,r}[k] \) is independently sampled from the Gaussian distribution \( N(0, I) \) and the others are the same as the basic scenario. Figures 6 and 7 show the mean values and the standard deviation of the error rate \( \rho \) for \( N = 10, 100, 1000, \) and 10000. These figures indicate that the proposed estimation method is successful in the non-constant output offset.
5. Conclusion

This paper has presented the open-loop system identification method for the MIMO integrator model. The estimation algorithm for estimating the action matrix is proposed by removing the trends from the numerically integrated input and output so that the effects of input and output disturbances are eliminated. The proposed estimation algorithm is applicable for the basic scenario and the non-constant output offset. However, the proposed algorithm is not applicable for the non-constant input offset. The non-constant input offset has not been discussed in the previous works for the open-loop identification of the SISO system with the integrator. The discussion in this paper indicates the essential difficulties in the open-loop system identification with the integrator. Further research on the open-loop system identification with the integrator would be expected by taking account of the noise effects.

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