The Infrared Landau Gauge Gluon Propagator from Lattice QCD

O. Oliveira* and P. J. Silva*

*Centro de Física Computacional, Departamento de Física, Universidade de Coimbra, 3004-516 Coimbra, Portugal

Abstract. The quenched Landau gauge gluon propagator is investigated in lattice QCD with large assimetric lattices, accessing momenta as low as \( q \sim 100 \text{ MeV} \) or smaller. Our investigation focus on the IR limit of the gluon dressing function, testing the compatibility with recent solutions of the Dyson-Schwinger equations. In particular, the low energy parameters \( \kappa \) and \( \alpha(0) \) are measured.

The confinement of quarks in hadrons and the chiral symmetry breaking mechanism are believed to be linked to the low energy properties of QCD. In particular, the gluon and ghost propagators can provide us information on the mechanism of confinement.

Two first principles approaches to non-perturbative problems are Dyson-Schwinger equations (DSE) and lattice methods. In lattice QCD, the gluon propagator has been revisited a number of times. However, due to the lattice sizes used in previous studies, the access to the IR region was limited. For the Dyson-Schwinger solution for the gluon propagator, recently in [4, 5] the low energy behaviour of the propagator was solved analytically. Moreover, the authors found a parametrization for the gluon dressing function that fitted the numerical solution of the DSE. The lattice being complementary to DSE, allows a check of compatibility between the two methods. In particular, we would like to check for the behaviour of small and zero momenta gluon propagator.

Our investigation uses pure gauge, Wilson action, SU(3) configurations, \( \beta = 6.0 \) \( (a^{-1} = 1.885 \text{ GeV}) \), on large assimetric lattices\(^1\): \( 16^3 \times 128 \) and \( 16^3 \times 256 \). They were generated using combinations of over-relaxation (OVR) and Cabibbo-Mariani (HB) updates. For the smaller (larger) lattice, a combined sweep of 7 OVR and 2 HB (7 OVR and 4 HB) was used and 3000 combined sweeps for thermalization. The 160 (70) configurations were saved with a separation of 3000 (1500) combined sweeps. Note that, due to the large extension on the time direction, the lowest momenta considered is about 93 MeV for the smallest lattice and 46 MeV for the largest lattice.

For notation and details see [1]. The data for the Landau gauge gluon propagator\(^2\)

\[
\langle A_\mu^a(p) A_\mu^a(p') \rangle = V \delta \left( p + p' \right) \delta^{ab} \left( \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \frac{Z(p^2)}{p^2} \tag{1}
\]

shows finite volume effects. To disentangle the finite volume effects, our data was

---

\(^1\) All configurations were generated with MILC code [http://physics.indiana.edu/~sg/milc.html](http://physics.indiana.edu/~sg/milc.html).

\(^2\) For gauge fixing we used the steepest descent Fourier accelerated method described in [2].
compared with the results reported in [3]. No essential differences were seen between our pure temporal data and the Leinweber et al data. From now on, we refer only to pure temporal data – see figure 1. Note that the data seems to support a vanishing zero momenta gluon propagator $D(p^2)$.

To test the compatibility between lattice and the DSE results [5], our data for the gluon dressing function was fitted to

$$Z(p^2) = \omega \left( \frac{p^2}{\Lambda_{QCD}^2 + p^2} \right)^{2\kappa} (\alpha(p^2))^{-\gamma}, \quad (2)$$

where $\gamma = -13/22$, using two ansatz for the running coupling [5, 6]:

$$\alpha_1(p^2) = \frac{1}{1 + \frac{e^2}{\Lambda_{QCD}^2}} \left[ \alpha(0) + \frac{p^2}{\Lambda_{QCD}^2} \frac{4\pi}{\beta_0} \left( \frac{1}{\ln(p^2/\Lambda_{QCD}^2)} - \frac{1}{p^2/\Lambda_{QCD}^2 - 1} \right) \right], \quad (3)$$

$$\alpha_2(p^2) = \frac{\alpha(0)}{\ln \left[ e + a_1(p^2/\Lambda_{QCD}^2)^{\beta_2} \right]}, \quad (4)$$

with $\beta_0 = 11$.

The IR propagator was investigated fitting the lattice data to

$$Z_{IR1}(p^2) = \omega \left( \frac{p^2}{\Lambda_{QCD}^2 + p^2} \right)^{2\kappa} \quad Z_{IR2}(p^2) = \omega \left( \frac{p^2}{\Lambda_{QCD}^2 + p^2} \right)^{2\kappa} \quad (5)$$

Only when the first three lowest momenta of the larger lattice ($|q| \leq 139$ MeV) were considered, we were able to fit the IR analytical solution of DSE $Z_{IR1}$ with an acceptable $\chi^2/d.o.f. = 0.39$, meaning that the solution $Z(q^2) \sim (q^2)^{2\kappa}$ seems to be valid only for momenta below 150 MeV. The measured $^3 \kappa = 0.5003^{+0.032}_{-0.029}$ does not provide a clear

---

3 Statistical errors for fit parameters were computed with 2000 bootstrap samples.
answer about the zero momenta gluon propagator. The results for the IR2 approximation are shown in table 1. For both lattices, the fit to the largest range of momenta supports an infrared finite gluon propagator.

| Lattice | Range               | $\kappa$ | $\Lambda_{QCD}$ (MeV) | $\chi^2/d.o.f.$ |
|---------|---------------------|----------|------------------------|-----------------|
| $16^3 \times 128$ | $||q|| \leq 461\text{MeV}$ | $0.5020^{+10}_{-9}$ | $429^{+46}_{-49}$ | 0.50          |
| $16^3 \times 128$ | $||q|| \leq 553\text{MeV}$ | $0.5122^{+42}_{-46}$ | $403^{+8}_{-7}$ | 1.41          |
| $16^3 \times 256$ | $||q|| \leq 644\text{MeV}$ | $0.5199^{+25}_{-31}$ | $377^{+5}_{-5}$ | 1.19          |

The fits to all data distinguishes the functional forms $\alpha_1(p^2)$ and $\alpha_2(p^2)$ for the running coupling constant. For $\alpha_1(p^2)$, the $\chi^2/d.o.f. > 2$. Moreover, $\alpha(0) \sim 10$ is much larger than the DSE result: $\alpha(0) = 2.972 \pm 0.0063$ [5, 4]. However, if one uses $\alpha_2(p^2)$, the lattice data is well described by 2 - see table 2. $\alpha(0)$ can be computed from the asymptotic behaviour of the QCD $\beta$ function and $\alpha_2(p^2)$. Then, $\alpha(0) = (4\pi/\beta_0)a_2$ giving $\alpha(0) = 2.78^{+2}_{-2}$ and $2.74^{+1}_{-2}$ for the smaller and larger lattices, respectively. The analysis of all momenta favours a finite zero momenta gluon propagator. Our figures for $\kappa$ are smaller than those reported in the DSE studies, $\kappa \sim 0.595$ [4]. However, our $\kappa$ values reported in table 2 are within the figures discussed in [7, 8].

| Lattice | $\kappa$ | $\Lambda_{QCD}$ (MeV) | $a_1$ | $a_2$ | $\chi^2/d.o.f.$ |
|---------|----------|------------------------|-------|-------|-----------------|
| $16^3 \times 128$ | $0.5439^{+36}_{-41}$ | $352^{+4}_{-3}$ | $0.0063^{+4}_{-3}$ | $2.43^{+2}_{-2}$ | 1.74 |
| $16^3 \times 256$ | $0.5314^{+13}_{-24}$ | $354^{+3}_{-3}$ | $0.0065^{+4}_{-3}$ | $2.40^{+1}_{-2}$ | 1.56 |

We are currently involved in improving the statistics of our analysis and, simultaneously, trying to understand the role of Gribov copies relying on the method discussed in [9]. We plan also to compute the ghost propagator and running coupling constant directly from the lattice.

P.J.Silva acknowledges financial support from FCT via grant SFRH/BD/10740/2002.

REFERENCES

1. P.J.Silva, O.Oliveira, Nucl. Phys. B690 (2004) 177 [hep-lat/0403026].
2. C.T.Davies et al, Phys. Rev. D37(1988)1581.
3. D. B. Leinweber et al Phys. Rev. D60 (1999) 094507; erratum-ibid Phys. Rev. D61 (2000) 079901 [hep-lat/9811027].
4. Ch. Lerche, L. von Smekal, Phys. Rev. D65 (2002), 125006 [hep-ph/0202194].
5. R. Alkofer et al, hep-ph/0309078, references therein and these proceedings.
6. C. S. Fischer, R. Alkofer, Phys. Rev. D67(2003)094020 [hep-ph/0301094].
7. J. M. Pawlowski et al, Phys. Rev. Lett. 93(2004)152002 [hep-th/0312324] and these proceedings.
8. C. S. Fischer, H. Gies, hep-ph/0408089.
9. O.Oliveira, P.J.Silva, Comp. Phys. Comm. 158 (2004) 73 [hep-lat/0309184].

---

4 In [1], the effect of the Gribov copies was estimated as a 2 to 3$\sigma$ effect. This does not change our predictions for the zero momenta gluon propagator.