Scalar Quarkonium Masses

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We evaluate the valence approximation to the mass of scalar quarkonium for a range of different parameters. Our results strongly suggest that the infinite volume continuum limit of the mass of $s\bar{s}$ scalar quarkonium lies well below the mass of $f_J(1710)$. The resonance $f_0(1500)$ appears to the best candidate for $s\bar{s}$ scalar quarkonium.

For the valence approximation to the infinite volume continuum limit of the lightest scalar glueball mass, a calculation on GF11 gives $1740 \pm 71$ MeV, favoring $f_J(1710)$ as the lightest scalar glueball. Among other observed resonances which could have scalar glueball quantum numbers, only $f_0(1500)$ is near enough to $1740 \pm 71$ MeV to be a possible alternate glueball candidate. Ref. 3 suggests that $f_0(1500)$ is dominantly $s\bar{s}$ scalar quarkonium. Evidence for this identification is given in Ref. 3. As a test of the interpretation of $f_0(1500)$ as $s\bar{s}$ scalar quarkonium, we have now calculated, for Wilson quarks in the valence approximation, the mass of scalar quarkonium with two different values of lattice spacing and a range of different quark masses. Although the data we have is not sufficient to permit an extrapolation of scalar quarkonium masses to the continuum limit or to infinite volume, it is sufficient to show that the continuum infinite volume value for the valence approximation to the $s\bar{s}$ mass lies well below the mass of $f_J(1710)$. This result, combined with the discussion of Ref. 3, favors $f_0(1500)$ as $s\bar{s}$ scalar quarkonium and makes this interpretation for $f_J(1710)$ appear improbable.

The calculations reported here were done on the GF11 parallel computer and required about 5 months of operation at a sustained speed of approximation 6 Gflops.

To evaluate the mass of scalar quark-antiquark states, we define operators for these states by first fixing each gauge configuration to lattice Coulomb gauge, then convoluting, in the space direction, the Wilson quark field $\Psi_{sc}(x)$ with a gaussian $G^r(\vec{x})$ with root-mean-squared radius $r$ to produce a smeared Coulomb gauge quark field $\Psi^r_{sc}(x)$. Here $1 \leq s \leq 4$ and $1 \leq c \leq 3$ are spin and color indices, respectively. We assume only a single flavor of quark. In a gamma-matrix representation with $\gamma_5$ given by the diagonal entries 1, -1, -1, define the upper and lower quark fields

$$\Psi^r_c(x) = \sum_{s=1,2} \Psi_{sc}(x) \xi_s,$$

$$\Psi^r_d(x) = \sum_{s=3,4} \Psi_{sc}(x) \xi_s,$$

and define $\bar{\Psi}^u_c(x)$ and $\bar{\Psi}^d_c(x)$, similarly from $\bar{\Psi}_{sc}(x)$ with $\xi^*_s$ in place of $\xi_s$. Here the $\xi_s$, for each $s$, are independent random cube roots of 1. The scalar operator

$$S^r_\xi(x) = \bar{\Psi}^u_c(x) \Psi^{ru}(x) + \bar{\Psi}^d_c(x) \Psi^{rd}(x),$$

when averaged over $\xi$ becomes the more familiar

$$S^r(x) = \bar{\Psi}(x) \Psi^r(x).$$

A pseudoscalar $P^r_\xi(x)$ and its average $P^r(x)$ can be defined in analogy to $S^r_\xi(x)$ and $S^r(x)$.

For scalar correlation functions we choose

$$C^r_S(t) = \sum_{\vec{x}} < S^r(\vec{x},t) S^r_\xi(0) > - \sum_{\vec{x}} < S^r(\vec{x},t) S^r_\xi(0) >$$

where averages are over $\xi$ and gauge field configurations, with one random vector $(\xi_1, \ldots, \xi_4)$ for each field. A pseudoscalar $C^r_P(t)$ can be defined similarly. The correlations $C^r_S(t)$ and $C^r_P(t)$ together require a factor of six fewer quark matrix inversions per gauge configuration than needed for $C^r_S(t)$ and $C^r_P(t)$ defined with $S^r(x)$ and $P^r(x)$, respectively, used as both source and
sink operators. To obtain propagators with a fixed statistical uncertainty, however, some of this factor of six will be lost to the additional noise arising from $\xi$. Figure 1 shows the actual gain in arithmetic work for the scalar correlation, $6[D_S'(t)/D_S(t)]^2$, where $D_S(t)$ and $D_S'(t)$ are the statistical dispersions in $C_S^0(t)$ and $C_S^\nu(t)$ respectively. Figure 2 shows the corresponding actual gain in arithmetic work for the pseudoscalar correlation. The values in Figures 1 and 2 were found from 188 independent gauge configurations on a lattice $16^3 \times 24$ with $\beta$ of 5.70 and $\kappa$ of 0.1650. For smaller $\kappa$ we expect the gain to be greater. Figure 2 shows the corresponding actual gain in arithmetic work for the scalar correlation function given by 1733 gauge configurations on a lattice $24^3$ with $\beta$ of 5.93, $\kappa$ of 0.1567, and the fitting range for the scalar correlation function, by evaluating propagators for several different source time values on each lattice.

The cost of generating gauge field configurations and gauge fixing we found could be reduced, for the scalar correlation function, by evaluating propagators for several different source time values on each lattice. Figures 3 and 4 were found from six different starting times. Figures 4 shows effective masses, the fitted mass, and the fitting range for the scalar correlation function given by 1733 gauge configurations on a lattice $24^3$ with $\beta$ of 5.93, $\kappa$ of 0.1567 and $r$ of $\sqrt{27}/2$. For each gauge configuration, propagators were found from four different starting times. In physical units, the quark mass and smearing radius $r$ of $\sqrt{6}$. Thus in physical units the lattice period at the two different values of $\beta$ we consider are nearly equal.

The $s\bar{s}$ scalar mass we found by interpolation in quark mass to the strange quark mass corresponding to hopping constants determined in Ref. 3 to be 0.16404 at $\beta$ of 5.70 and 0.15620 at $\beta$ of 5.93. The resulting masses, in units of $m_{\rho}$, are shown in Figure 6 as a function of lattice spacing in comparison to the predicted scalar glueball mass and the masses of $f_0(1710)$ and $f_0(1500)$. Figure 6 strongly suggests that for the fixed volume we are using, the continuum value of the $s\bar{s}$ scalar mass will lie well below the mass of $f_0(1710)$. Since the $s\bar{s}$ scalar has one unit of orbital angular momentum, its physics radius should be larger than that of the $s\bar{s}$ $\phi$ meson with no orbital angular momentum. The scalar mass divided by $m_\phi$ should therefore fall with volume. Volume dependence data for $m_\phi$ in Ref. 3 then implies that infinite volume values of the scalar mass divided by $m_\rho$ will be at most 2.8% above the numbers shown in Figure 6. Our qualitative conclusion about the scalar $s\bar{s}$ mass remains the same in infinite volume. Thus $f_0(1500)$ appears to be a likely candidate for the the $s\bar{s}$ scalar, while this assignment for $f_0(1710)$ appears quite unlikely.

### Table 1

| $\kappa$ | $m_{sc}$ | range | $\chi^2$ |
|---------|---------|-------|---------|
| 0.1625  | 1.299(12) | 3 - 5  | 0.320   |
| 0.16404 | 1.291(10)  |       |         |
| 0.1650  | 1.287(6)  | 2 - 4  | 0.002   |

### Table 2

| $\kappa$ | $m_{ps}$ | range | $\chi^2$ |
|---------|---------|-------|---------|
| 0.1539  | 0.856(3) | 4 - 11 | 1.39    |
| 0.1554  | 0.806(4) | 4 - 11 | 1.40    |
| 0.1562  | 0.788(17)|       |         |
| 0.1567  | 0.777(5) | 4 - 10 | 1.44    |

Masses, fitting ranges and $\chi^2$ per degree of freedom of mass fits on a $16^3 \times 24$ lattice at $\beta$ of 5.70.

Masses, fitting ranges and $\chi^2$ per degree of freedom of mass fits on a $24^3$ lattice at $\beta$ of 5.93.
Figure 1. For the scalar propagator on a $16^3 \times 24$ lattice with $\beta$ of 5.70 and $\kappa$ of 0.1650, the performance gain of a random source in comparison to a causal source.

Figure 2. For the pseudoscalar propagator on a $16^3 \times 24$ lattice with $\beta$ of 5.70 and $\kappa$ of 0.1650, the performance gain of a random source in comparison to a causal source.

Figure 3. Effective scalar mass, fitted mass and fitting range for a $16^3 \times 24$ lattice with $\beta$ of 5.70 and $\kappa$ of 0.1650.

Figure 4. Effective scalar mass, fitted mass and fitting range for a $24^4$ lattice with $\beta$ of 5.93 and $\kappa$ of 0.1567.

Figure 5. Scalar $s\bar{s}$ quarkonium masses for two different values of lattice spacing.

REFERENCES

1. H. Chen, J. Sexton, A. Vaccarino and D. Weingarten, Nucl. Phys. B (Proc. Suppl.) 34, 357 (1994).
2. J. Sexton, A. Vaccarino and D. Weingarten, Phys. Rev. Lett. 75, 4563 (1995).
3. D. Weingarten, in this proceedings.
4. F. Butler, H. Chen, J. Sexton, A. Vaccarino, and D. Weingarten, Phys. Rev. Lett. 70, 2849 (1993); Nucl. Phys. B 430, 179 (1994).