Stochastic Coded Federated Learning: Theoretical Analysis and Incentive Mechanism Design

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Abstract—Federated learning (FL) has achieved great success as a privacy-preserving distributed training paradigm, where many edge devices collaboratively train a machine learning model by sharing the model updates instead of the raw data with a server. However, the heterogeneous computational and communication resources of edge devices give rise to stragglers that significantly decelerate the training process. To mitigate this issue, we propose a novel FL framework named stochastic coded federated learning (SCFL) that leverages coded computing techniques. In SCFL, before the training process starts, each edge device uploads a privacy-preserving coded dataset to the server, which is generated by adding Gaussian noise to the projected local dataset. During training, the server computes gradients on the global coded dataset to compensate for the missing model updates of the straggling devices. We design a gradient aggregation scheme to ensure that the aggregated model update is an unbiased estimate of the desired global update. Moreover, this aggregation scheme enables periodical model averaging to improve the training efficiency. We characterize the tradeoff between the convergence performance and privacy guarantee of SCFL. In particular, a more noisy coded dataset provides stronger privacy protection for edge devices but results in learning performance degradation. We further develop a contract-based incentive mechanism to coordinate such a conflict. The simulation results show that SCFL learns a better model within the given time and achieves a better privacy-performance tradeoff than the baseline methods. In addition, the proposed incentive mechanism grants better training performance than the conventional Stackelberg game approach.

Index Terms—Federated learning (FL), coded computing, straggler effect, mutual information differential privacy (MI-DP), incentive mechanism.

I. INTRODUCTION

The rapid growth of artificial intelligence (AI) technologies is boosting the development of various intelligent applications such as digital healthcare, smart transportation, and augmented/virtual reality (AR/VR). These applications generate a large volume of data at the wireless network edge, which contain valuable information for training high-quality AI models to improve user experience. Whilst the traditional solution is to directly upload the data to a cloud server for centralized training, it is prohibitive in many emerging use scenarios where the data may contain privacy-sensitive information, e.g., geographical locations and user preferences [2], [3]. Federated learning (FL) [4], which iteratively aggregates local model updates computed by the edge devices (e.g., smartphones and Internet of Things (IoT) devices) at a server, was proposed as a promising framework for privacy-preserving distributed model training.

The training process of FL is divided into multiple communication rounds. In each communication round, the edge devices perform local model training with their own data and the server aggregates their model updates to generate a new global model for the next round. Due to the heterogeneous computational and communication resources at different edge devices, e.g., computing speeds and network link rates, it may take a much longer time for edge devices with fewer resources to finish the local training [5]. Since the local data among edge devices are typically non-independent and identically distributed (non-IID), simply ignoring the updates from stragglers may lead to biased model aggregations and greatly degrade the training performance. To mitigate the straggler effect, various methods such as device scheduling [6], [7] and asynchronous model aggregation [8] have been developed. While the training efficiency can be improved to some extent, these solutions fail to exploit the valuable computational resources at the server, which can be utilized to further accelerate the training speed.

Coded federated learning (CFL) [9], [10], which exploits the server’s computational resources by constructing coded datasets at the edge devices [11], [12], has recently been proposed to alleviate the straggler issue in FL. Specifically, with the local coded datasets shared by the edge devices, the server is able to compute gradient updates to complement the missing ones of the stragglers. Unfortunately, although the server cannot decode the raw data samples, the coded datasets still carry much information that causes privacy concerns on CFL.

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In this paper, we propose a stochastic coded federated learning (SCFL) framework with both convergence and privacy guarantee, where the local coded datasets are protected by both random projection and additive noise. Also, we develop an incentive mechanism for SCFL to motivate edge devices to share less noisy coded data.

A. Related Works

To resolve the straggler issue in FL, a simple approach is to ignore the straggling edge devices in each communication round. In particular, the server can be configured to wait for a given duration in each communication round and aggregate the gradient updates received before the round deadline [5]. Nevertheless, the presence of non-IID data results in biased aggregations that deviate from the global training objective. Alternatively, asynchronous FL [8] can be adopted where the model aggregation is triggered once sufficient model updates are available at the server. Unfortunately, asynchronous FL may need more communication rounds to converge because the stale updates can be poisonous for the global model. Also, some works [6], [7] designed device scheduling and resource allocation strategies to collect more informative updates in each communication round. Besides, assigning models with heterogeneous sizes to edge devices according to their computation capabilities is also an effective approach to mitigate the straggler issue in FL [13]. In addition, a barrier control method was proposed in [14] to deal with the device heterogeneity in FL, where the server decides whether the collected local updates from edge devices should be passed to the aggregator or blocked to wait for further updates. While existing approaches can improve the training efficiency, it is still likely that some edge devices cannot upload their gradient updates in each communication round, which leads to missing information for model aggregation at the server. To combat the stragglers in conventional distributed computing systems, coding theory and develop a contract-based incentive mechanism [21], [22], [23], where a contract item is designed for each edge device so that the server reduces the noise in the coded datasets with a minimal cost.

B. Contributions

In FL, it may take a much longer time for edge devices with fewer resources to finish the local training in each communication round, which leads to the straggling effect that significantly degrades the training efficiency. Although ignoring the stragglers is a simple remedy, biased model aggregations may be incurred, which shall compromise the training performance. In this work, we aim to tackle the straggler issue in FL via an innovative framework of SCFL, where a coded dataset is constructed at the server to generate unbiased gradient estimates that compensate for the missing gradients from stragglers. Our main contributions are summarized as follows:

- We propose an SCFL framework for federated linear regression. Before training starts, each edge device generates a noisy local coded dataset via random projection, which is uploaded to the server. During training, the edge devices perform local training for multiple steps and upload the accumulated updates to the server for aggregation. Meanwhile, the server computes stochastic gradients
Table I

| Framework             | Privacy Protection of Coded Data | Mini-batch SGD | Periodical Aggregation | Convergence Analysis | Privacy Analysis | Tradeoff Characterization | Incentive Mechanism |
|-----------------------|---------------------------------|----------------|------------------------|----------------------|------------------|---------------------------|---------------------|
| CFL-PB [9]            | Low                             | ×              | ×                      | ×                    | ×                | ×                         | ×                   |
| CodedFedL [10]        | Low                             | ✓              | ✓                      | ✓                    | ✓                | ✓                         | ✓                   |
| DP-CFL [16]           | High                            | ×              | N/A                    | ×                    | ✓                | ×                         | ×                   |
| SCFL (Ours)           | High                            | ✓              | ✓                      | ✓                    | ✓                | ✓                         | ✓                   |

Based on the coded data. Notably, the added noise reduces the privacy leakage of the local data compared with existing CFL frameworks [9], [10]. Compared with our previous work [1], multiple steps of local training at both the server and edge devices in each communication round are allowed to improve the training efficiency.

- To ensure unbiased gradient estimation, we develop a novel gradient aggregation scheme at the server. We prove the convergence of SCFL and characterize the privacy guarantee of the coded datasets via the notion of mutual information differential privacy (MI-DP). Besides, by analyzing the effect of noise levels of the coded datasets, we theoretically demonstrate a tradeoff between privacy and performance of SCFL.
- We further design a contract-based incentive mechanism to motivate edge devices to share coded datasets with less noise. Specifically, it derives a set of contract items with each specifying the privacy budget and the earned reward for each edge device. Accordingly, the mutually satisfactory noise levels of local coded datasets are determined with the minimal rewards paid by the server.
- We evaluate the proposed SCFL framework on the MNIST [24] and CIFAR-10 [25] datasets. The simulation results demonstrate the benefits of SCFL in securing a higher test accuracy within the given training time. Compared with other CFL schemes, SCFL also achieves a consistently better privacy-performance tradeoff. Besides, we also investigate the impacts of various system parameters, including the number of local steps and the ratio of stragglers, on the learning performance. In addition, the proposed contract-based incentive mechanism for SCFL yields a better learned model compared with the Stackelberg game approach for a given amount of reward paid by the server.

C. Organization

The rest of this paper is organized as follows. In Section II, we describe the system model and the conventional FL algorithm for linear regression. Section III introduces the proposed SCFL framework. We analyze the convergence and privacy guarantee of SCFL in Section IV. An incentive mechanism for determining the noise levels of the coded datasets is developed in Section V. Section VI presents the experiment results and Section VII concludes this paper.

D. Notations

We use boldface upper-case letter (e.g., X) and boldface lower-case letter (e.g., x) to represent matrix and vector, respectively. For matrix $X \in \mathbb{R}^{m \times n}$, $X_{i,j}$ is the entry in the $i$-th row and $j$-th column, and $\|X\|_F \triangleq \sqrt{\sum_{i=1}^{m} \sum_{j=1}^{n} |X_{i,j}|^2}$ denotes its Frobenius norm. Besides, we denote the $i$-th diagonal entry of a diagonal matrix $Z$ as $z_i$. The set of integers $\{1, 2, \ldots, N\}$ is denoted as $[N]$. In addition, $\mathbb{1}\{\cdot\}$ is the indicator function, i.e., $\mathbb{1}\{A\} = 1$ if event $A$ happens and $\mathbb{1}\{A\} = 0$ otherwise. Table II summarizes the main notations in this paper.

II. FEDERATED LEARNING FOR LINEAR REGRESSION

A. System Model

We consider an edge AI system consisting of a server and $N$ edge devices. Each edge device $i \in [N]$ has a local dataset denoted as $(X^{(i)}, Y^{(i)})$, where $X^{(i)} \in \mathbb{R}^{l_i \times d}$ concatenates the $d$-dimensional features of $l_i$ data samples, and $Y^{(i)} \in \mathbb{R}^{l_i \times o}$ represents the corresponding o-dimensional labels. Accordingly, the global dataset in the FL system is given as the union of all the local datasets, i.e., $X = [(X^{(1)})^T, (X^{(2)})^T, \ldots, (X^{(N)})^T]^T \in \mathbb{R}^{m \times d}$ and $Y = [(Y^{(1)})^T, (Y^{(2)})^T, \ldots, (Y^{(N)})^T]^T \in \mathbb{R}^{m \times o}$, where $m = \sum_{i=1}^{N} l_i$ is the total number of data samples in the system. We use matrix $Z^{(i)} \triangleq [z_{j,j}^{(i)}] \in \mathbb{R}^{l_i \times m}$ to indicate the data availability on edge device $i$, where $z_{j,j}^{(i)} = 1$ if the $j$-th data sample of the global dataset corresponds to the $j$-th data sample at edge device $i$ and $z_{j,j}^{(i)} = 0$ otherwise.

Thus, the local datasets can be expressed as $(X^{(i)}, Y^{(i)}) = (Z^{(i)} X, Z^{(i)} Y), i \in [N]$.

The server aims to fit a model $W \in \mathbb{R}^{d \times o}$ to the distributed data on the edge devices by solving the following linear regression problem [26]:

$$\min_{W \in \mathbb{R}^{d \times o}} \ f(W) \triangleq \frac{1}{2} \|XW - Y\|_2^2.$$  \hspace{1cm} (1)

We assume the server pays the edge devices for model training, and uses the learned model to earn profits, which is relevant to the business models of multiple industries. For example, a software developer working for a news recommendation company can deploy a server to train a machine learning model based on the subscribers’ behaviors [27], which enables high accuracy and thus increases the user viscosity. Notably, if the global dataset is available at the server, the optimal linear regression model can be obtained in closed-form by setting the gradient of $f(W)$ to zero, i.e., $\nabla f(W) \triangleq X^T (XW - Y) = 0$. However, since the local datasets typically contain privacy-sensitive information that cannot be disclosed, solving (1) in a centralized manner at the server would be infeasible.
B. Federated Learning Algorithm

Rather than pursuing the optimal solution, the server leverages FL to train a linear regression model without accessing the raw data at the edge devices, and FedAvg [4] is a classic FL algorithm that contains K communication rounds with identical duration T. In the k-th communication round of FedAvg, the edge devices first download the global model (denoted as $W_k$) from the server. The model downloading time is denoted as $t_d$. Then, each edge device performs a $\tau$-step stochastic gradient descent (SGD) to minimize the loss function defined on its local dataset, i.e., $f_i(W) = \frac{1}{T}\|X^{(i)}W - Y^{(i)}\|^2_F$. In other words, the updated local model after each SGD step can be written as $W_{k,u}^{(i)} = W_{k,u-1}^{(i)} - \eta_k g_{k,u-1}^{(i)}(W_{k,u-1}^{(i)}), \forall u \in [\tau]$, where $W_{k,0}^{(i)} = W_k$. $\eta_k$ is the learning rate, and $g_{k,u}^{(i)}(W_{k,u}^{(i)})$ denotes the stochastic gradient computed on a random subset of the local data with batch size b. Denote MACR as the Multiply-Accumulate (MAC) rate of edge device $i$ and $N_{\text{MAC}}$ as the number of MAC operations required for processing one data sample. The local model updating time in each communication round is thus given as $t_u^{(i)}(b) = \frac{h_i}{MACR_i} \cdot \frac{N_{\text{MAC}}}{b}$ [28]. After local model training, each edge device summarizes its model updates as $\hat{\Delta}_k^{(i)} = \frac{1}{\tau} \sum_{u=0}^{\tau-1} g_{k,u}^{(i)}(W_{k,u}^{(i)})$, which is uploaded to the server for aggregation. Accordingly, the uploading time is given as $t_u^{(i)}(b) = \frac{h_i}{MACR_i} \cdot \frac{\tau N_{\text{MAC}}}{b}$, where $M$ (in bits) is the size of the gradient update and $R_{U,k} = B \log_2(1 + |h_i|^2 \frac{P_t}{N_0})$ is the uplink transmission rate, with $B$, $h_i$, $P_t$ and $N_0$ denoting the channel bandwidth, channel coefficient, uplink transmit power, and receive noise power, respectively. We further denote $t_s^{(i)}(b) = t_d + t_u^{(i)}(b) + t_{U,k}$ as the minimum time required by edge device $i$ to complete model training and exchange in the k-th communication round.

Due to the time-varying wireless channel conditions, it may happen that some edge devices cannot upload their model updates before the deadline of each communication round. We use an indicator $1_k^{(i)} \triangleq 1 \{t_k^{(i)}(b) \leq T\}$ to denote the arrival status of edge device $i$ in the k-th communication round, where $1_k^{(i)} = 1$ means it successfully uploads the model update and $1_k^{(i)} = 0$ otherwise. By assuming IID block fading, the probability of successfully receiving the model update from edge device $i$ within time T is given as $p_i \triangleq \Pr[t_k^{(i)}(b) \leq T]$, and the new global model is generated at the server by aggregating the received model updates according to:


g_{k+1}^{(i)} = g_{k}^{(i)} - \eta_k \sum_{i=1}^{N} g_{k,i}^{(i)}

Similarly, the efficiency of the above federated linear regression process suffers due to device heterogeneity. On one hand, edge devices with large MAC rates and better uplink channel quality can finish their local training and model uploading earlier, and thus they have to wait for the slowest device. To avoid idling the fast devices, we adapt the batch size according to their wireless channel conditions, i.e., edge device $i$ selects the largest batch size $b_k^i$ for SGD to meet the round deadline, i.e., $b_k^i = \max_{b \in [1]} \{1 \{t_k^{(i)}(b) \leq T\} \}$.

On the other hand, the straggling devices may have low arrival probabilities, which makes it difficult to utilize their local data for model training. In the next section, we propose a new FL framework for linear regression, named stochastic coded federated learning (SCFL), which adopts the coded data at the server to compensate for the missing gradients.

III. STOCHASTIC CODED FEDERATED LEARNING

Since the straggling edge devices may fail in uploading the gradients occasionally, we introduce a coded dataset in the proposed SCFL framework for gradient compensation at the server. Before training starts, each edge device generates the local coded data and uploads them to the server. Afterward, both the server and edge devices compute the stochastic gradients, which are periodically aggregated to obtain an unbiased model update. Fig. 1 provides an overview of the SCFL process with different operations as follows.

A. Coded Data Preparation

Before the start of the first communication round, a set of coded data is generated at each edge device via random matrix projection [9], [10]. However, the privacy protection of simple random projection may be escaped via some knowledge reconstruction attacks [29]. To provide extra privacy protection, we further add noise to the projected data. To illustrate the necessity of adding noise, we launch the privacy attack in [29] that is able to reconstruct the original data from the randomly projected data with some prior knowledge. Specifically, with a small subset of the original data samples, one can estimate the projection matrix and use it to decode the original data from the projected data. The reconstruction error is used as a measure of privacy leakage. It is intuitive that a larger reconstruction error implies a higher difficulty of reconstructing the original data from the coded data, i.e., data privacy is better preserved. As shown in Fig. 2, coded data with additive noise better preserves data privacy than coded data without adding noise.

Therefore, edge device $i$ computes the coded data features and labels according to $\hat{X}^{(i)} = G_i X^{(i)} + N_i$ and $\hat{Y}^{(i)} = G_i Y^{(i)}$, respectively, where $G_i \in \mathbb{R}^{c \times L}$ is the projection matrix with each element independent and identically sampled from the standard Gaussian distribution $\mathcal{N}(0,1)$. Also, each entry in $N_i \in \mathbb{R}^{c \times d}$ is independently sampled from $\mathcal{N}(0, \sigma_i^2)$. 

| Notation | Meaning | Notation | Meaning |
|----------|---------|----------|---------|
| $(X^{(i)}, Y^{(i)})$ | Local dataset of edge device $i$ | $(X_{k,u}^{(i)}, Y_{k,u}^{(i)})$ | Sampled data in step $u$ of round $k$ on edge device $i$ |
| $(\hat{X}^{(i)}, \hat{Y}^{(i)})$ | Coded dataset of edge device $i$ | $(\hat{X}_{k,u}^{(i)}, \hat{Y}_{k,u}^{(i)})$ | Sampled data in step $u$ of round $k$ on the server |
| $(X, Y)$ | Global dataset | $G$ | Random projection matrix |
| $(\hat{X}, \hat{Y})$ | Global coded dataset | $N$ | Gaussian noise matrix |

TABLE II
MAIN NOTATIONS

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where $\sigma_i > 0$ denotes the noise level and $c$ is the number of coded data that is the same at different edge devices. The local coded data of all the edge devices are collected by the server to construct a global coded dataset as follows:

$$\tilde{\mathbf{X}} = \sum_{i=1}^{N} \tilde{\mathbf{X}}^{(i)} = G \mathbf{X} + \mathbf{N}, \quad \tilde{\mathbf{Y}} = \sum_{i=1}^{N} \tilde{\mathbf{Y}}^{(i)} = G \mathbf{Y},$$

(2)

where $G = [G_1, G_2, \ldots, G_N]$ and $\mathbf{N} = \sum_{i=1}^{N} \mathbf{N}_i$. Since the server has no access to matrices $\{G_i\}$'s and $\{\mathbf{N}_i\}$'s, it cannot directly decode the real datasets $\{\mathbf{X}^{(i)}, \mathbf{Y}^{(i)}\}$. It is worthwhile noting that the noise levels of the coded datasets have direct impacts on both the learning performance and privacy guarantee. Intuitively, adding stronger noise reinforces the privacy protection of local data but degrades the learning performance. We will characterize the trade-off between the convergence and privacy guarantee using the MI-DP metric in Section IV-B, and the noise levels at different edge devices will be determined via an incentive mechanism in Section V.

### B. Gradient Computation at the Edge Devices

In the $k$-th communication round, the local model is initialized as $\mathbf{W}_{k,u}^{(0)} = \mathbf{W}_k^{(i)}$ at the $i$-th edge device. To avoid frequent communication between the server and edge devices, we adopt the periodical averaging approach [4], [30] where the edge devices compute stochastic gradients for $\tau$ steps before uploading the model updates in each communication round. This generalizes the existing works on CFL [1], [9], [10], where the model updates need to be uploaded after every local training step. Specifically, in step $u \in [\tau]$, edge device $i$ updates local model $\mathbf{W}_k^{(i)}$ by sampling a batch of its local data samples with size $b_k$ for gradient computation. The sampling matrix is denoted as a random diagonal matrix $\mathbf{S}_{k,u}^{(i)} \in \mathbb{R}^{N \times 1}$, where each diagonal element is independently sampled from the Bernoulli distribution $\text{Bern}(\frac{b_k}{\tau})$, i.e., the $j$-th diagonal element of $\mathbf{S}_{k,u}^{(i)}$ equals 1 if the $j$-th local data sample is selected. Accordingly, the batch of randomly sampled data can be denoted as $(\hat{\mathbf{X}}_{k,u}^{(i)}, \hat{\mathbf{Y}}_{k,u}^{(i)}) \triangleq \left( \mathbf{S}_{k,u}^{(i)} \mathbf{X}^{(i)}, \mathbf{S}_{k,u}^{(i)} \mathbf{Y}^{(i)} \right)$, and the stochastic gradient is computed as follows:

$$g_k^{(i)}(\mathbf{W}_k^{(i)}) \triangleq \frac{1}{b_k} (\hat{\mathbf{X}}_{k,u}^{(i)})^T (\hat{\mathbf{X}}_{k,u}^{(i)} \mathbf{W}_k^{(i)} - \hat{\mathbf{Y}}_{k,u}^{(i)}),$$

(3)

where $\mathbf{W}_k^{(i)} = \mathbf{W}_k^{(i)} - \eta_k g_k^{(i)}(\mathbf{W}_k^{(i)})$, $u = 1, 2, \ldots, \tau$. After $\tau$ local steps, each edge device summarizes the accumulated local gradient as $\hat{g}_k^{(i)} \triangleq \sum_{u=1}^{\tau} g_k^{(i)}(\mathbf{W}_k^{(i)})$ and uploads it to the server for aggregation.

### C. Gradient Computation at the Server

To combat the stragglers, the server computes the stochastic gradients on the coded dataset in a similar way as the edge devices but with a higher MAC rate. Specifically, in the $u$-th step of the $k$-th communication round, it randomly selects $b_s$ data samples via a diagonal sampling matrix $\mathbf{S}_{k,u}^s \in \mathbb{R}^{c \times 1}$, where the $j$-th diagonal element is sampled from the Bernoulli distribution $\text{Bern}(\frac{b_s}{\tau})$ and equals 1 if the $j$-th coded data sample is selected. Accordingly, the sampled coded data at the server are denoted as $(\hat{\mathbf{X}}_{k,u}^s, \hat{\mathbf{Y}}_{k,u}^s) \triangleq (\mathbf{S}_{k,u}^s \hat{\mathbf{X}}_{k,u}, \mathbf{S}_{k,u}^s \hat{\mathbf{Y}}_{k,u})$. To make full use of its MAC rate, the server selects the largest batch size $b_s$ that meets the round deadline. To eliminate the biased gradient estimate caused by the added noise, we propose a make-up term $g_0(\mathbf{W}_k^{(i)}) \triangleq -\sigma^2 \mathbf{W}_k^{(i)}$, where $\sigma^2 = \sum_{i=1}^{N} \sigma_i^2$ and $\mathbf{W}_k^{(i)}$ denotes the model at the server, to compute the stochastic gradient as follows:

$$g_k^{(i)}(\mathbf{W}_k^{(i)}) \triangleq \frac{1}{b_s} (\hat{\mathbf{X}}_{k,u}^s)^T (\hat{\mathbf{X}}_{k,u}^s \mathbf{W}_k^{(i)} - \hat{\mathbf{Y}}_{k,u}^s),$$

(4)

where $\mathbf{W}_k^{(i)} = \mathbf{W}_k^{(i)} - \eta_k (g_k^{(i)}(\mathbf{W}_k^{(i)}))_u^{-1} + g_0(\mathbf{W}_k^{(i-1)})$ and $\mathbf{W}_{k,0} = \mathbf{W}_k$. Thus, the accumulated update computed by the server
in the $k$-th communication round is summarized as

$$\hat{g}_k = \sum_{u=0}^{s-1} g_{k,u}(W_k^{s}) + g_0(W_k).$$

### D. Gradient Aggregation Scheme

Since each communication round has a fixed duration of $T$, only a subset of the edge devices with high MAC rates and favorable channel quality can upload their model updates successfully. In this regard, if the model updates are aggregated with the same weights, it will be biased towards the local data at fast edge devices, while the training data at stragglers are rarely exploited due to the lower arrival probability. To resolve this issue, we design a new gradient aggregation scheme by attaching the aggregation weight as the reciprocal of their arrival probability $\frac{1}{p_i}$. Besides, the missing model updates from stragglers in each communication round are compensated by using the stochastic gradients over the coded data computed at the server. The global model update in the $k$-th communication round of SCFL can thus be expressed as follows:

$$g(W_k) = \frac{1}{2} \sum_{i=1}^{N} \frac{1}{p_i} \left( \hat{g}_k(i) + \hat{g}_k^s(i) \right),$$

where the gradient uploaded from edge devices and the coded gradient are given the same weight of $\frac{1}{2}$ in aggregation. This is because they are both equivalent to the gradient computed over the global training dataset in the expected sense. As will be shown in the next section, $g(W_k)$ is an unbiased gradient estimate of the model update over the global dataset. After the gradient aggregation, the server updates the global model as $W_{k+1} = W_k - \eta_k g(W_k)$ and transmits the new global model to the edge devices for the next communication round. It also maintains $W_k$ to derive the final learned model as $W_{SCFL,K} = \frac{1}{\sum_{k=0}^{K-1} \eta_k} \sum_{k=0}^{K-1} \eta_k W_k$ after $K$ communication rounds. The complete training process of SCFL is summarized in Algorithm 1.

**Algorithm 1 Training Process of SCFL**

// Before Training Starts //

1. Each edge device computes the local coded dataset according to $\hat{X}^{(i)} = G_iX^{(i)} + N_i$ and $\hat{Y}^{(i)} = G_iY^{(i)}$ and uploads them to the server;
2. The server constructs the global coded dataset as $(\hat{X}, \hat{Y}) = (\sum_{i=1}^{N} \hat{X}^{(i)}, \sum_{i=1}^{N} \hat{Y}^{(i)})$;

// During Training //

1. **Training process at the server:**
2. Initialize a random global model $W_0$;
3. for $k=0,1,\ldots,K-1$ do
4. Broadcast $W_k$ to each edge device $i \in [N]$ and inform them to perform $LocalTrain(W_k)$;
5. for $u=1,2,\ldots,\tau$ do
6. Compute the stochastic gradient based on the coded data according to (4) and update the model according to $W_{k,u} = W_{k,u-1} - \eta_k \left( g_{k,u-1}(W_{k,u-1}) + g_0(W_{k,u-1}) \right)$;
7. end for
8. Aggregate the computed and received gradients according to (5) to obtain $g(W_k)$;
9. Update the global model as $W_{k+1} = W_k - \eta_k g(W_k)$ and keep a local copy of $W_{k+1}$;
10. end for
11. return $W_{SCFL,K} = \frac{1}{\sum_{k=0}^{K-1} \eta_k} \sum_{k=0}^{K-1} \eta_k W_k$ as the final learned model
12. **Training process at edge device $i$:**
13. def $LocalTrain(W_k)$:
14. Initialize $W^{(i)}_{k,0} = W_k$;
15. for $u=1,2,\ldots,\tau$ do
16. Compute the stochastic gradient based on the local data according to (3);
17. Update the local model according to $W_{k,u}^{(i)} = W_{k,u-1}^{(i)} - \eta_k g_{k,u-1}^{(i)}(W_{k,u-1}^{(i)})$;
18. end for
19. Compute the local update as $\hat{g}_k^{(i)} = \sum_{u=0}^{\tau-1} g_{k,u}^{(i)}(W_{k,u})$;
20. return $\hat{g}_k^{(i)}$

### IV. Theoretical Analysis

In this section, we characterize the convergence performance of SCFL and quantify the privacy leakage in coded data sharing. Based on these results, we derive a privacy-performance trade-off which is determined by the noise levels of the coded data.

#### A. Convergence Analysis

We first present the following assumption to facilitate the convergence analysis [10], [31].

**Assumption 1:** There exist positive constants $\\{\alpha_i\}_{i=1}^{N}$, $\\{\zeta_i\}_{i=1}^{N}$, $\\{\kappa_i\}_{i=1}^{N}$, and $\phi$ such that $\alpha_i^2 \leq \|X^{(i)}\|_F^2 \leq \zeta_i^2$, $\|X^{(i)}W - Y^{(i)}\|_F^2 \leq \kappa_i^2$, and $\|W\|_F^2 \leq \phi^2$. 

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According to the definition of matrices $G$, $N$, $\{S_{k,u}\}$'s, and $\{s_{k,u}^{(i)}\}$'s, we derive some important properties in the following lemma, which are helpful for the analysis.

Lemma 1: Matrices $G$, $N$, $\{S_{k,u}\}$, and $\{s_{k,u}^{(i)}\}$ have the following properties:
\[
\begin{align*}
&\mathbb{E}\left[\frac{1}{c}G^T G\right] = I_m \quad \text{and} \quad \mathbb{E}\left[\frac{1}{c}N^T G - G^T N\right]^2 = \frac{m^2}{c^2} \sum_{i=1}^N s_i^2, \\
&\mathbb{E}\left[\frac{1}{c}N^T N\right] = \sigma^2 I_d \quad \text{and} \quad \mathbb{E}\left[\frac{1}{c}G^T N - \sigma^2 I_d\right]^2 = \frac{d^2 + m^2}{c^2} \sum_{i=1}^N s_i^2, \\
&\mathbb{E}\left[\frac{1}{b_k} (S_{k,u}^T) S_{k,u}^T\right] = I_c \quad \text{and} \quad \mathbb{E}\left[\frac{1}{b_k} (S_{k,u}^T - I_c)^2\right] = \frac{c(c-b)}{b_k}, \\
&\mathbb{E}\left[\frac{1}{b_k} (s_{k,u}^{(i)} T s_{k,u}^{(i)} - I_i)^2\right] = \frac{l_i(l_i-b)}{b_k^2}, \quad \forall i \in [N].
\end{align*}
\]

Proof: Since each entry of $G$ is independently sampled from a Gaussian distribution $\mathcal{N}(0, 1)$, $G^T G$ follows the Wishart distribution, i.e., $G^T G \sim W_m(c, I_m)$, which leads to: $\mathbb{E}[G^T G] = I_m$ and $\mathbb{E}\left[\frac{1}{c}G^T G - I_m\right]^2 = \frac{m^2}{c^2} \sum_{i=1}^N s_i^2$. Besides, since matrix $S_{k,u}^T$ is symmetric and diagonal, each entry of $\frac{1}{b_k} (S_{k,u}^T) S_{k,u}^T$ has a unit mean. Thus, we have $\mathbb{E}\left[\frac{1}{b_k} (S_{k,u}^T) S_{k,u}^T\right] = I_c$. The proofs for $N$ and $\{s_{k,u}^{(i)}\}$'s are similar with those of $G$ and $\{S_{k,u}\}$, respectively, which are omitted for brevity. \(\square\)

We define an auxiliary model updating process initialized with global model $W_{k,0} = W_k$ at the beginning of the $k$-th communication round. Assume the server has access to the global dataset, and it performs a communication round. Assume the server has access to the global dataset, and it performs the communication round. Assume the server has access to the global dataset, and it performs the communication round. Assume the server has access to the global dataset, and it performs the communication round.

Lemma 2: The global model update in SCFL is an unbiased estimate of the virtual accumulated model update, i.e., $\mathbb{E}[g(W_k)] = u_k$, $\forall k \in [K]$.

Proof: By mathematical induction, we show that in each local step $u \in [\tau]$, the sum of the stochastic gradients over the local dataset, i.e., $\sum_{i=1}^N \frac{1}{p_i} g_{k,u}^{(i)} (W_{k,u}^{(i)}) I_k^{(i)}$, and the global coded dataset, i.e., $g_{k,u}^{(i)} (W_{k,u}^{(i)})$, is an unbiased estimate of the virtual gradient computed over the global dataset, i.e., $\nabla f(W_{k,u})$. Then we sum up the gradients over $u = 0, 1, \ldots, \tau - 1$ to complete the proof. The detailed proof is delegated to Appendix A. \(\square\)

To show the bounded variance property, we decompose the estimation error of the global update as follows:
\[
\mathbb{E}[\|g(W_k) - u_k\|^2] = \mathbb{E}\left[\frac{1}{2} \sum_{i=0}^{\tau-1} \sum_{u=1}^N \frac{1}{p_i} g_{k,u}^{(i)} (W_{k,u}^{(i)}) I_k^{(i)} - \nabla f(W_{k,u})\right]^2 + 1 + \frac{1}{2} \left( \sum_{u=0}^{\tau-1} g_{k,u} (W_{k,u}^{(i)}) - \sigma^2 W_{k,u}^{(i)} - \nabla f(W_{k,u}) \right)^2 \\
+ \frac{1}{4} \mathbb{E}\left[\sum_{u=0}^{\tau-1} \sum_{i=1}^N \frac{1}{p_i} g_{k,u}^{(i)} (W_{k,u}^{(i)}) - \nabla f(W_{k,u})\right]^2 \\
+ \frac{1}{4} \mathbb{E}\left[\sum_{u=0}^{\tau-1} g_{k,u} (W_{k,u}^{(i)}) - \sigma^2 W_{k,u}^{(i)} - \nabla f(W_{k,u})\right]^2 \\
\end{align*}
\]

where $V_1$ and $V_2$ respectively represent the gradient estimation errors attributed to the gradients computed on the local datasets and the coded dataset, and the last equality is because of their independence. Next, we upper bound $V_1$ and $V_2$ in Lemma 3 and Lemma 4, respectively.

Lemma 3: The estimation error of the aggregated gradients received from the edge devices is upper bounded as follows:
\[
V_1 \leq 2\tau \sum_{i=1}^N \frac{1}{p_i} \eta_i \xi_i^2 \zeta_i^2 + 2\tau \sum_{i=1}^N \frac{I_i (l_i - b_i)}{b_k} \xi_i^2 \zeta_i^2.
\]

Proof: Please refer to Appendix B. \(\square\)

Lemma 4: Define $\tilde{\sigma} \triangleq [\sigma_1^2, \sigma_2^2, \ldots, \sigma_N^2]$. The estimation error of the gradients computed on the global coded dataset is upper bounded as follows:
\[
V_2 \leq \frac{c}{(m + m^2) \kappa} + \frac{4c (d + d^2) \varphi^2}{c^2} \sum_{i=1}^N \sigma_i^4 + 4 \rho_1 \rho_2 \zeta_i^2.
\]

Proof: Please refer to Appendix C. \(\square\)

With the above lemmas, we characterize the training performance of SCFL in the following theorem and remark.

Theorem 1: Define the optimality gap after a given training time $T_{tot} \triangleq TK$ as $G(T_{tot}) \triangleq f(W_{SCFL,K}) - f(W^*)$, where $W^* = \arg\min_{W} f(W)$ is the optimal linear regression model. With Assumption 1, if the learning rates are chosen as $\eta_k L < 1$, we have:
\[
G(T_{tot}) \leq \frac{1}{2} \sum_{k=0}^{K-1} \frac{1}{\eta_k} \mathbb{E}[W_{k} - W^*]^2 + \frac{1}{2} \sum_{k=0}^{K-1} \eta_k \rho(\tilde{\sigma}).
\]

where $\alpha \triangleq \sum_{k=0}^{K-1} \alpha_i^2$ and $\rho(\tilde{\sigma}) \triangleq \frac{1}{2} \rho_1 + \frac{1}{2} \rho_2 (\tilde{\sigma})$. Proof: It is straightforward to verify that the loss function $f(\cdot)$ is $\alpha$-strongly convex. Since the aggregated model update is an unbiased estimate of $u_k$ with bound variance derived in Lemma 3 and Lemma 4, we can follow a similar proof of Lemma 7 in [32] to conclude the result. \(\square\)

Remark 1 (Convergence Performance): If the learning rates further satisfy $\lim_{K \to \infty} \sum_{k=0}^{K-1} \eta_k = \infty$ and $\lim_{K \to \infty} \sum_{k=0}^{K-1} \xi_i^2 < \infty$, the right-hand side (RHS) of (9) diminishes to zero as $K \to \infty$, which means the learned model of SCFL, i.e., $\tilde{\sigma} \triangleq \frac{1}{\sum_{k=0}^{K-1} \eta_i} \sum_{k=0}^{K-1} \eta_k W_{k}$, converges to the optimal linear regression model. In particular, by selecting $\eta_k = O\left(\frac{1}{(k+1)^L}\right)$ with some constant $\beta > 0$, the dominating
term in the RHS of (9) becomes $O\left(\frac{1}{\tau \sum_{l=1}^{\tau} (k+\beta)}\right)$. Besides, the RHS of (9) increases with the gradient estimation error $\rho(\tilde{\sigma})$, which means adding stronger noise $\tilde{\sigma}$ degrades the performance of the learned model.

With the selected learning rates $\eta_k = O\left(\frac{\tau}{(k+\beta)L}\right)$, SCFL achieves a convergence rate of $O\left(\frac{1}{\tau}\right)$, i.e., it converges geometrically fast with respect to the number of local steps $\tau$ and communication rounds $K$, which is because of the $\alpha$-strong convexity of loss function $f(\cdot)$ [33], [34]. In comparison, for general convex loss functions, the convergence rate of FL using SGD for local model updates becomes slower, which is typically given as $O\left(\frac{1}{\sqrt{\tau}}\right)$ [35].

To obtain a better model, the server prefers collecting less noisy coded datasets, which, however, degrades the privacy protection of local datasets.

### B. Privacy-Performance Tradeoff Analysis

To characterize the privacy leakage caused by sharing the local coded datasets, we adopt the $\epsilon$-mutual information differential privacy ($\epsilon$-MI-DP) [36] metric, which captures the privacy risk of individual entries in the dataset. Compared with original DP definitions [37], [38], MI-DP is often considered to be more intuitive, because it explicitly incorporates the well-established concept of mutual information. We first introduce the following assumption to meet the definition of $\epsilon$-MI-DP, which can be easily satisfied by normalizing each entry in $X$.

**Assumption 2:** The maximum absolute value of entries in $X$ is upper bounded by 1.

**Definition 1 ($\epsilon$-MI-DP) [36]:** A randomized mechanism $q(\cdot)$ that encodes local data $X^{(i)}$ to $\hat{X}^{(i)}$ satisfies the $\epsilon_i$-mutual information differential privacy if

$$
\sup_{x, P(x)} I\left(X^{(i)}[j] ; \hat{X}^{(i)}[X^{(i)}[-j]] \right) \leq \epsilon_i,
$$

where the supremum is over the entry index $j$ and distribution $P(x)$ of the local dataset $X^{(i)}$, and $X^{(i)}[-j]$ denotes the dataset $X^{(i)}$ excluding the $j$-th sample $X^{(i)}[j]$.

In particular, (10) provides an upper bound for the maximum privacy leakage of each entry in the dataset. Notably, a smaller value of the privacy budget $\epsilon_i$ offers better privacy protection. We select the maximum privacy budget of all the edge devices as the privacy loss caused by coded data sharing, i.e., $\epsilon = \max_{i \in [N]} \epsilon_i$, as stated in the following theorem.

**Theorem 2:** With Assumption 2, the privacy leakage caused by sharing the local coded datasets is given as follows:

$$
\epsilon = \max_{i \in [N]} \left\{ \epsilon_i = q_i(\sigma_i^2) \right\} = \frac{1}{2} \log_2 \left( 1 + \frac{c}{h^2(\tilde{X}^{(i)}) + \sigma_i^2} \right),
$$

where $h(\tilde{X}^{(i)}) = \min_{k_1, k_2} \sqrt{\frac{\sum_{k=1}^{k_1} |X^{(i)}_{k_1, k_2}|^2 - \max_{k \in [k_1]} |X^{(i)}_{k_1, k_2}|^2}{\sum_{k_2=1}^{k_2} |X^{(i)}_{k_1, k_2}|^2}}$ with the $X^{(i)}_{j,k}$ denoting the $(j,k)$-th entry of matrix $X^{(i)}$.

**Proof:** The proof is available at Lemma 2 of [39] and is omitted for brevity.

With the result in Theorem 2, we first compare the privacy protection of different CFL schemes and then show the privacy-performance tradeoff, as stated in the following remarks, respectively.

**Remark 2:** The coded datasets in CFL-FB [9] and CodedFedL [10] are constructed as the random linear projection of the local data, and thus their privacy leakage can be viewed as a special case of Theorem 2 with $\epsilon_i = 0, \forall i \in [N]$.

By adding Gaussian noise to the coded data, the proposed SCFL framework and DP-CFL [16] provides better privacy protection (i.e., a smaller $\epsilon$) than CFL-FB and CodedFedL.

**Remark 3 (Privacy-Performance Tradeoff):** According to Theorems 1 and 2, there is a tradeoff between privacy protection and convergence performance of SCFL. Specifically, by adding stronger noise $\sigma_i^2$ to the local coded dataset, each edge device can meet the requirement of a smaller privacy budget, which reduces the privacy leakage in coded dataset sharing. However, it also degrades the training performance, as the gradient estimation error $\rho(\tilde{\sigma})$ increases with each added noise $\sigma_i^2$.

This privacy-performance tradeoff shows a clear conflict between convergence performance and privacy leakage. In short, each edge device has its preference for privacy protection, while the server expects to receive less noisy coded datasets to improve the training performance. To solve this conflict, we next develop an incentive mechanism to determine the proper noise levels.

### V. AN INCENTIVE MECHANISM VIA CONTRACT THEORY

In this section, we propose a contract-based incentive mechanism for SCFL, which determines the mutually satisfactory noise levels for the local coded datasets.

#### A. Problem Formulation

To improve the learning performance, Theorem 1 indicates that it is beneficial to reduce the gradient variance at the server, which can be achieved by gathering less noisy local coded datasets when constructing the global coded dataset. However, it is almost impossible to obtain an exact and tractable characterization of the impacts of the added noise level on the learning performance since many parameters in the convergence bound are unknown. Therefore, we model the effect of the added noise level $\sigma_i^2$ on the learning performance as a general non-increasing concave function $\Gamma(\sigma_i^2)$ [40]. As our target is to provide a feasible mechanism to derive the mutually satisfactory noise levels for the local coded datasets, this utility function is sufficient to abstract the impacts of different noise levels. To protect the privacy of local dataset, the edge devices tend to add stronger noise to the coded samples. The server has to pay rewards to the edge devices to motivate them to share more accurate coded data. We denote the reward paid to the $i$-th edge device by the server as $r_i$.

Thus, the utility of the server can be expressed as follows:

$$
U_S(\sigma_i^2, \{r_i\}) = \sum_{i=1}^{N} \Gamma(\sigma_i^2) - \lambda \sum_{i=1}^{N} r_i,
$$

where $\lambda > 0$ is a weight parameter that can be adjusted by the server to control the total reward within an acceptable budget. Specifically, a larger value of $\lambda$ implies that the server...
pays more rewards to obtain a better learning performance. In practice, we can build a reference table (e.g., Table III in Appendix E) that shows the correspondence between the value of λ and the total reward once the optimal contract items are derived. By referring to this table, the server can choose the value of λ according to its reward budget.

The edge devices expect to have lower privacy costs while receiving larger rewards. We model the privacy cost of edge device \(i\) as \(μ_iε_i\), where \(μ_i > 0\) denotes its privacy sensitivity \([19], [41], [42]\). Specifically, with a larger value of \(μ_i\), edge device \(i\) concerns more with the privacy risk leakage and thus aspires to receive more rewards for any given privacy budget. In practice, the values of \(\{μ_i\}\)’s are set by edge devices \([43]\), which may depend on their locations, data types, and contents. Without loss of generality, we assume that \(χ = \{μ_i : i ∈ [N]\}\) are sorted in an ascending order, i.e., \(0 < μ_1 ≤ μ_2 ≤ ⋯ ≤ μ_N\). The utility function of the \(i\)-th edge device can be written as follows:

\[
U_i(ε_i, r_i) = r_i - μ_iε_i, ∀i ∈ [N].
\]

The server needs to design a contract \(Ω(χ)\) that contains a set of contract items \(\{ε_i, r_i\}_{i=1}^N\), to maximize its utility as well as ensure the individual rationality (IR) and incentive compatibility (IC) of the edge devices, which are formally defined below.

**Definition 2 (Individual Rationality (IR)):** The edge devices upload the local coded dataset only when a non-negative utility can be achieved, i.e.,

\[
U_i(ε_i, r_i) ≥ 0, ∀i ∈ [N].
\]

**Definition 3 (Incentive Compatibility (IC)):** An edge device always adopts a contract item that can achieve the maximal utility, i.e.,

\[
U_i(ε_i, r_i) ≥ U_i(ε_i', r_i'), ∀i' ≠ i, i' ∈ [N].
\]

Note that \(σ_i^2 = q_i^2(ε_i)\) with \(q_i^2(x) = \frac{ε}{2σ_i^2} - ln(1 + \frac{ε}{2σ_i^2})\). To satisfy the IR and IC requirements, the contract design at the server can be formulated as the following optimization problem:

\[
P1 : \text{max}_i \tilde{U}_S(Ω(χ)) \quad (16)
\]

s.t. (14), (15),

\[
r_i ≥ 0, ∀i ∈ [N],
\]

\[
0 ≤ ε_i ≤ q_i(0), ∀i ∈ [N],
\]

where \(\tilde{U}_S(Ω(χ)) = \sum_{i=1}^N \Gamma(q_i^2(ε_i)) - λ \sum_{i=1}^N r_i\) and the last constraint follows the noise level is non-negative, i.e., \(σ_i^2 ≥ 0\). It is straightforward to verify Problem (P1) is a convex optimization problem. However, instead of solving (P1) via the Karush–Kuhn–Tucker (KKT) conditions, we develop a low-complexity solution in the next subsection by establishing the relationship between the optimal rewards \(\{r_i^\ast\}\) for a given set of privacy budgets \(\{ε_i\}\). Besides, while the constraint (19) indicates a minimum noise level of zero, the proposed solutions can be easily generalized to the case with a noise level constraint \(σ_{min,i}^2\), i.e., \(0 < ε_i ≤ q_i(σ_{min,i}^2), ∀i ∈ [N]\).

### B. Optimal Contract Design

To solve Problem (P1), we follow the methods in [23] and [44] to first find the optimal rewards \(\{r_i^\ast\}_{i=1}^N\) given the privacy costs of the edge devices, as stated in the following theorem.

**Theorem 3:** Given \(0 < μ_1 ≤ μ_2 ≤ ⋯ ≤ μ_N\) and \(\{ε_i\}_{i=1}^N\), the optimal rewards are given as follows:

\[
r_i^\ast = \begin{cases} μ_iε_i, & \text{if } i = N, \\ r_{i+1}^\ast - μ_iε_{i+1} + μ_iε_i, & \text{otherwise}. \end{cases}
\]

**Proof:** Please refer to Appendix D.

The above theorem simplifies the objective function in Problem (P1) to \(\tilde{U}_S(Ω(χ)) = \sum_{i=1}^N \Gamma(q_i^2(ε_i)) - λ \sum_{i=1}^N [μ_iε_i - (i-1)μ_{i-1}ε_i]\) where only \(\{ε_i\}\)’s are the optimization variables. Define \(Φ_i(ε_i) = \Gamma(q_i^2(ε_i)) - λμ_iε_i + \lambda(1-μ_i)μ_{i-1}ε_i\). Problem (P1) is equivalent to the following optimization problem:

\[
P2 : \text{max}_i \sum_{i=1}^N Φ_i(ε_i), \quad (21)
\]

s.t. \(ε_1 ≥ ε_2 ≥ ⋯ ≥ ε_N > 0\),

\[
0 < ε_i ≤ q_i(0), ∀i ∈ [N], \quad (22)
\]

where the constraint (22) indicates the privacy budget should be upper bounded by that corresponds to the case without adding noise to the coded dataset.

Notably, with the concave objective function and the monotonic property of \(\{ε_i\}\)’s, Problem (P2) can be optimally solved via the Bunching and Ironing Algorithm [44]. Specifically, we first find the maximizer \(\{ε_i^\ast\}\) of the objective function by disregarding the constraints in (22). Then, starting from \(i = N\), we check whether \(ε_i^\ast ≥ ε_{i+1}^\ast\) holds. If this inequality holds, we set \(i\) as \(i-1\) and proceed. Otherwise, there must exist some \(j ≤ i-1\) such that \(ε_j^\ast ≤ ε_{j+1}^\ast ≤ ⋯ ≤ ε_i^\ast\) (Line 4 of Algorithm 2), and we update their values by setting \(ε_j^\ast = ε_{j+1}^\ast = ⋯ = ε_i^\ast\) as \(\text{arg max}_{ε_i ∈ [0, min_i]} Q_i(0)) \sum_i Φ_i(ε)\) (Line 5 of Algorithm 2). This ensures \(ε_j^\ast ≥ ε_{j+1}^\ast ≥ ⋯ ≥ ε_i^\ast\) so that we can proceed by setting \(i = j\). In the worst case, adjustment is needed for each \(i = N, N-1, ⋯, 2\), i.e., the while loop in Algorithm 2 is repeated for at most \(N - 1\) times. Details of the Bunching and Ironing Algorithm for Problem (P2) are summarized in Algorithm 2. It is worth noting that since edge devices are privacy-sensitive, the optimal noise levels rarely attain the noise-free upper bounds in (23). Furthermore, the Bunching and Ironing Algorithm is also applicable if a minimum noise level \(σ_{min,i}^2\) is added to the coded data, i.e., \(0 < ε_i ≤ q_i(σ_{min,i}^2), ∀i ∈ [N]\).

---

**Algorithm 2** Bunching and Ironing Algorithm

1. Initialize \(ε_i^\ast = \text{arg max}_{ε_i ∈ (0,q_i^{-1}(0))} Φ_i(ε_i), ∀i ∈ [N]\) and \(i = N\);
2. while \(i > 1\) do
   3. if \(ε_{i-1}^\ast < ε_i^\ast\) then
      4. Find the smallest \(j ≤ i - 1\) such that \(ε_j^\ast ≤ ε_{j+1}^\ast ≤ ⋯ < ε_i^\ast\);
      5. Calculate \(ε = \text{arg max}_{ε_i ∈ [0,min_i]} Q_i(0)) \sum_i Φ_i(ε)\) and set \(ε_j^\ast = ε, ∀l = j, j + 1, ⋯, i\);
6. end if
7. Set \(i = j\);
8. end while
9. return \(\{ε_i^\ast\}_{i=1}^N\).
VI. PERFORMANCE EVALUATION

A. Experimental Setup

We consider an FL system with 20 edge devices. The channel bandwidth $B$ and noise power $N_0$ are set as 180 kHz and $-70$ dBm, respectively. To simulate different pathloss, the transmit powers of the edge devices are uniformly sampled in the range of $15 \sim 25$ dBm, and the channel gains $\{ |h_i|^2 \}$ follow the exponential distribution with the default mean value $\gamma = 10^{-8}$. Besides, the downloading rate and the MAC rate of the server are set as 1 Mbps and 15,360 KMAC per second, respectively. We randomly generate the MAC rates of the edge devices according to $\mu_{\text{comp},i} = \mu_{\text{comp},i} \times 1,536$ KMAC per second, where $\mu_{\text{comp},i}$ is uniformly sampled from $[0.8, 1.0]$. We evaluate the FL algorithms on two image datasets, including the MNIST [24] and CIFAR-10 [25] datasets. In addition, the default value of $\tau$ is 5, and $T$ is set as 10 and 15 seconds for experiments on the MNIST and CIFAR-10 datasets, respectively.

We compare the proposed SCFL scheme with the following three FL benchmarks:

- **FedAvg** [4]: In each communication round, the edge devices compute the stochastic gradients for $\tau$ local steps and upload the accumulated model updates to the server. The server then aggregates the received model updates from fast edge devices and generates a new global model.

- **CodedFedL** [10]: In CodedFedL, the local coded dataset generated by random projection of the local data without adding noise. Both the server and the edge devices divide their datasets into several batches. In each communication round of the training process, a random batch is selected to compute the gradient at each edge device and the server. However, the server and edge devices compute the stochastic gradients only for one step (i.e., $\tau = 1$) before aggregation.

- **DP-CFL** [16]: The coded dataset is generated with the same method as the proposed SCFL scheme. However, only the server performs centralized model training using the global coded dataset without further cooperation with the edge devices.

The CodedFedL scheme obtains the optimal batch sizes of the data at the server and edge devices by solving the optimization problem stated in (23) of [10]. In all CFL methods, the number of coded data samples $c$ is set as 10,000, and the batch sizes of the server $b_s$ are 499 and 502 for the MNIST and CIFAR-10 datasets, respectively.

To simulate the non-IID data on edge devices, we sort the images in each dataset by their labels, divide them into 20 shards with identical size, and assign one random shard to each edge device [45]. Following [10], the classification task on the MNIST dataset is transformed into a linear regression problem by using the random Fourier feature mapping (RFFM) [46]. Accordingly, each transformed vector has a dimension of 2,000. As for the CIFAR-10 dataset, we use a 4,096-dimensional feature vector extracted by a pretrained-VGG model [47] to represent each image. Therefore, we can train linear regression models for classifying both image datasets.

B. Results

1) Training Performance Comparison: We compare the model accuracy achieved by different FL frameworks with respect to the training latency in Fig. 3, where the noise levels in all the CFL schemes are set as $\sigma_i^2 = 0.29, \forall i \in [N]$, implying the same privacy budgets. It is observed that CodedFedL achieves the lowest test accuracy due to the frequent communication and model update bias introduced by the noisy coded datasets. Even without the added noise (i.e., $\sigma_i^2 = 0$), the performance of CodedFedL is still worse than other schemes, which further demonstrates the benefit of periodical averaging in accelerating the convergence speed. DP-CFL achieves a similar test accuracy with FedAvg, since the server continuously computes the stochastic gradients without any communication overhead. Compared with the baseline methods, our proposed SCFL achieves the highest test accuracy within the given training time, and the batch adaptation scheme further accelerates the model convergence by exploiting more data samples for training. This is because the proposed aggregation scheme in SCFL can effectively utilize the global coded dataset to mitigate the stragglers by fully exploiting the computational resources at both the server and edge devices.

2) Privacy-Performance Tradeoff: We change the noise levels $\sigma_i^2$ of the local coded datasets to simulate different privacy budgets and investigate the privacy-performance tradeoff of different CFL schemes in Fig. 4. It can be observed that the learned model achieves a higher test accuracy by increasing the privacy budget of MI-DP since the coded data are less noisy. This demonstrates the privacy-performance tradeoff highlighted in Remark 3. Among the three CFL schemes, SCFL secures the highest test accuracy under any given privacy budget, i.e., it achieves the best privacy-performance tradeoff. This is again attributed to the proposed gradient aggregation scheme of SCFL that effectively mitigates the negative effects of the added noise. Besides, the model obtained by CodedFedL has the worst test accuracy since its efficiency is degraded by the frequent communications. Comparing Fig. 4(a) and Fig. 4(b), while the test accuracies on the MNIST dataset achieved by different CFL schemes keep increasing with the privacy budget, the test accuracies on the CIFAR-10 dataset plateau when the privacy budget is larger than 8.0. This is because the MNIST dataset is much easier to
classify and therefore more sensitive to the added noise. It is also interesting to note that under a very low noise level, i.e., \( \sigma^2 \leq 5 \), the proposed SCFL scheme still outperforms DP-CFL by notable margins since it improves the training efficiency by exploiting the computational resources on both the server and edge devices.

3) Effect of Local Steps \( \tau \): In Fig. 5, we evaluate the model accuracy of SCFL with different local training steps \( \tau \) in each communication round. In particular, we consider two cases with different average channel gains \( \gamma \), where a larger value of \( \gamma \) corresponds to the case with better channel quality. In both cases, slightly increasing the local steps, e.g., from 1 to 3, allows the edge devices to perform more local training before gradient uploading, which improves the training efficiency by saving the number of communication rounds to achieve a target accuracy. However, when \( \tau \) further increases, e.g., \( \tau > 3 \), the model accuracy drops drastically due to more uplink transmission failures, and thus fewer gradients can be received at the server. Also, with unfavorable communication quality, i.e., with a smaller value of \( \gamma \), the model accuracy degrades more significantly since more edge devices struggle in completing the local training and model uploading within a communication round. Therefore, the number of local steps should be selected properly to avoid too frequent communication or too many uplink failures.

4) Effect of Straggler Ratio: To simulate different levels of straggler effect, we vary the communication bandwidth to achieve a certain straggler ratio, i.e., the expected proportion of edge devices that fail to upload their local gradients to the server within a communication round. We see from Fig. 6 that all the methods suffer from accuracy drop with an increased number of stragglers. While SCFL achieves the best test accuracy due to the use of coded data, its accuracy still decreases slightly as more stragglers cause larger gradient estimation errors in aggregation. In comparison, FedAvg has no mechanism to handle the straggling edge devices and thus experiences more severe accuracy degradation. Besides, CodedFedL achieves the lowest test accuracy since its efficiency is heavily degraded by frequent communications. We notice that uploading \( c = 10,000 \) coded data samples causes a communication overhead of 76 MB in the MNIST training task and 156 MB in the CIFAR-10 training task. In comparison, the communication overhead in the training stage is 123 MB and 208 MB, respectively. Despite with a non-negligible communication cost, the above results show that the coded data effectively mitigates the straggling effect and improves the learned model accuracy.

5) Effect of the Number of Coded Data Samples: We evaluate the test accuracy of SCFL on the MNIST training task with different numbers of coded data samples \( c \), as shown in Fig. 7. It can be observed that as the number of coded data increases, the learned model achieves a higher test accuracy, which verifies the theoretical result in Theorem 1. Besides, a larger amount of coded data samples are needed to achieve a target accuracy when the straggler ratio becomes higher.
6) Incentive Mechanism Evaluation: We adopt $\Gamma(\sigma_i^2) = -\sigma_i^2$ as an example to characterize the negative effects of the added noise on the training performance. The privacy sensitivity of each edge device is generated according to $\mu_i = 1 + 0.02i, \forall i \in \mathbb{N}$. In Fig. 8(a), we show the utility of four of the edge devices with respect to different contract items, in which, the maximal utility of edge device $i$ is achieved by accepting the $i$-th contract item instead of others, which verifies the IC requirement in (15). It is clear that the IR requirement in (14) is also satisfied since the maximal utility of the edge devices are all positive.

Now we validate the benefits of the proposed contract-based incentive mechanism in [20] by showing the test accuracy of the learned models with respect to the total reward paid by the server in Fig. 8(b)-(8(c)). Without an incentive scheme, edge devices do not upload coded data to avoid privacy leakage, which can be viewed as FedAvg. In the Stackelberg game approach, the server computes the Nash equilibrium to determine the total reward which is allocated to the edge devices proportionally to their privacy budgets. To achieve a fair comparison, we vary the values of $\lambda$ in (12) such that two incentive mechanisms yield the same total reward. In particular, a small value of $\lambda$ indicates that the server concerns more on the training performance and thus provides more rewards as incentives for the edge devices, as shown in Table III in Appendix E. In return, the edge devices generate less noisy coded data that helps to improve the model accuracy. By implementing an incentive mechanism, we observe severe leakage, which can be viewed as FedAvg. In the Stackelberg game approach, the server computes the Nash equilibrium to determine the total reward which is allocated to the edge devices proportionally to their privacy budgets. As such, those edge devices with higher privacy sensitivity may receive a small reward and thus prefer to add much stronger noise. On the contrary, the contract-based mechanism judiciously designs a contract item for each edge device, so that it is able to yield a less noisy coded dataset and thus improve the training performance.

VII. Conclusion

In this paper, we proposed a stochastic coded federated learning (SCFL) framework, where a coded dataset is constructed at the server to mitigate the straggler effect in FL. We designed an unbiased aggregation scheme for SCFL which enables periodical averaging that significantly improves the training performance. We characterized the tradeoff between the training performance and privacy guarantee of SCFL, which is determined by the noise levels of the coded datasets. The conflict between convergence performance and the privacy protection of local coded datasets was resolved by developing a contract-based incentive mechanism. The simulation results demonstrated the benefits of SCFL and corroborated the privacy-performance tradeoff. We also verified the feasibility of the proposed incentive mechanism and showed it is more cost-effective than the Stackelberg game approach.

Limitations and Future Works: While SCFL is able to solve classification problems by resorting to feature mapping methods, it is worth investigating how to extend it for non-linear ML problems in future works. It is also important to adapt the number of coded data samples at each edge device to strike a good balance between their communication cost and privacy budget. Furthermore, the duration of each communication round needs further optimization to achieve better training efficiency, and exploring SCFL in decentralized setups is also interesting.

APPENDIX A

PROOF OF LEMMA 2

We begin with an important lemma that characterizes the relationships between the stochastic gradients in the $u$-th step of the $k$-th communication round.
Lemma 5: For any \( u \in [\tau] \), \( k \in [K] \), \( i \in [N] \), the following equality holds:

\[
E[g_k^{(i)}(W_{k,u}^i)] = Z^{(i)} \nabla f(W_{k,u}) \text{ and } E[g_{k,u}^s(W_{k,u}^s)] = \nabla f(W_{k,u}).
\]

Proof: We prove Lemma 5 by mathematical induction. Recall \( \nabla f(W_{k,u}) = X^T(W_{k,u} - Y) \). It is straightforward that (24) holds for \( u = 0 \) since the models have the same initialisation, i.e., \( W_{0,0} = W_{0,0} = W_k \). Suppose \( E[W_{k,u}^i] = E[W_{k,u}^s] = E[W_{k,u}] \), \( \forall u \), \( 0, \ldots, \tau - 1 \), and thus we have:

\[
E[g_{k,u}^s(W_{k,u}^s)] = E\left[X^{(i)}^T \left( \frac{1}{b_k} S_{k,u}^s S_{k,u}^s \right) (X^{(i)} W_{k,u} - Y) \right] = E \left[ (Z^{(i)})^T Z^{(i)} \right] X^T W_{k,u} - Y
\]

\[
= Z^{(i)} X^T W_{k,u} - Y,
\]

(25)

and

\[
E[g_{k,u}^s(W_{k,u}^s)] = E\left[ \tilde{X}^T \left( \frac{1}{b_k} S_{k,u}^s S_{k,u}^s \right) (\tilde{X} W_{k,u} - \tilde{Y}) - \sigma^2 W_{k,u} \right]
\]

where (a), (b), and (c) adopt the properties in Lemma 1. Then for \( u + 1 \) we have

\[
E[W_{k,u+1}] = E[W_{k,u}] - \frac{1}{P_i} \left( \sum_{i=1}^{N} 1 \frac{1}{P_i} g_k^{(i)}(W_{k,u}^i) \right) = E[W_{k,u}] - \frac{1}{P_i} g_k^{(i)}(W_{k,u}^i) = E[W_{k,u+1}].
\]

Therefore, Lemma 5 holds.

Since \( \tau \) steps of SGD are independent, we can decompose the aggregated gradient \( g(W_k) \) into \( \tau \) parts, i.e., \( g(W_k) = \sum_{u=0}^{\tau-1} \frac{1}{P_i} (\sum_{i=1}^{N} 1 \frac{1}{P_i} g_k^{(i)}(W_{k,u}^i) \right) + g_{k,u}^s(W_{k,u}^s) - \sigma^2 W_{k,u} \). According to Lemma 5, we show the gradient obtained in step \( u \) is an unbiased estimate of \( \nabla f(W_{k,u}) \) in (27), shown at the bottom of the next page, where (d) and (e) follow the properties derived in Lemma 1. Since the variance in each step is independent, the result in Lemma 2 is completed by summing up both sides of (27) over \( u = 0, 1, \ldots, \tau - 1 \).

APPENDIX B

PROOF OF LEMMA 3

We decompose the gradient variance into each SGD step \( u \) as follows and provide an upper bound for them respectively.

\[
E\left[ \sum_{u=0}^{\tau-1} \frac{1}{P_i} g_k^{(i)}(W_{k,u}^i) \nabla f(W_{k,u}) \right]^2
\]

\[
\leq \tau \sum_{u=0}^{\tau-1} E \left[ \sum_{i=1}^{N} 1 \frac{1}{P_i} g_k^{(i)}(W_{k,u}^i) \nabla f(W_{k,u}) \right]^2.
\]

Define the full-batch gradient on the coded dataset \( (\tilde{X}, \tilde{Y}) \) as \( \nabla f_s(W_{k,u}^s) \). We decompose the gradient variance in (28) as

\[
E\left[ \sum_{u=0}^{\tau-1} \frac{1}{P_i} g_k^{(i)}(W_{k,u}^i) \nabla f(W_{k,u}) \right]^2
\]

\[
\leq \tau \sum_{u=0}^{\tau-1} E \left[ g_k^{(i)}(W_{k,u}^i) \right]^2.
\]

Defining the full-batch gradient on the coded dataset \( (\tilde{X}, \tilde{Y}) \) as \( \nabla f_s(W_{k,u}^s) \), we decompose the gradient variance in (28) as

\[
E\left[ \sum_{u=0}^{\tau-1} \frac{1}{P_i} g_k^{(i)}(W_{k,u}^i) \nabla f(W_{k,u}) \right]^2
\]

\[
\leq \tau \sum_{u=0}^{\tau-1} E \left[ g_k^{(i)}(W_{k,u}^i) \right]^2.
\]

(30)

where (a) holds since the error caused by two parts is independent. The RHS of (31) can be further upper
bounded as follows:

\[
\mathbb{E} \left[ \left\| g_{k,u}^t(W_{k,u}) - \nabla f_s(W_{k,u}) \right\|_F^2 \right] \\
\leq \left( b \right) \mathbb{E} \left[ \frac{1}{c^2} \left\| C \right\|_{F}^2 \right] \left\| W_{k,u}^t - \bar{Y} \right\|_F^2 \\
\leq \left( e \right) \left\| W_{k,u}^t - \bar{Y} \right\|_F^2 \\
\leq \left( c \right) \left\| W_{k,u}^t - \bar{Y} \right\|_F^2 \\
\leq \frac{c - b_k}{cb_k} \left\| X \right\|_F^2 \\
\leq \frac{c - b_k}{cb_k} \kappa, \\
\leq (32)
\]

and

\[
\mathbb{E} \left[ \left\| f_s(W_{k,u}) - \sigma^2 W_{k,u} - \nabla f(W_{k,u}) \right\|_F^2 \right] \\
\leq (d) \left\| X \right\|_F^2 \mathbb{E} \left[ \left\| \frac{1}{c} G^T G - I \right\|_F^2 \right] \left\| W_{k,u} = Y \right\|_F^2 \\
+ (d) \mathbb{E} \left[ \left\| \frac{1}{c} N^T N - \sigma^2 I \right\|_F^2 \right] \left\| W_{k,u} = Y \right\|_F^2 \\
+ (d) \mathbb{E} \left[ \left\| N^T G \right\|_F^2 \right] \left\| W_{k,u} = Y \right\|_F^2 \\
+ (d) \mathbb{E} \left[ \left\| N^T G \right\|_F^2 \right] \left\| W_{k,u} = Y \right\|_F^2 \\
\leq \left( e \right) \left\| X \right\|_F^2 \left( \frac{m + m^2}{c} \kappa + \frac{4}{c} (d + d^2) \phi^2 \right) \left\| A \right\|_F^2 \left\| B \right\|_F^2 \left\| X \right\|_F^2 \\
+ \left( e \right) \left( m + m^2 \right) \kappa + \frac{4}{c} (d + d^2) \phi^2 \left\| X \right\|_F^2 \\
+ \left( e \right) \left\| A \right\|_F^2 \left\| B \right\|_F^2 \left\| X \right\|_F^2 \\
\leq (33)
\]

where (b) and (d) follow the inequality \( \| ABx \|_F^2 \leq \| A \|_F^2 \| B \|_F^2 \| x \|_2 \) for any compatible matrices \( A, B \) and vector \( x \). (c) and (e) hold due to Lemma 1.

**Appendix D**

**Proof of Theorem 3**

We first present the necessary and sufficient conditions to ensure the feasibility of Problem (P1), which can be shown by following similar lines of the proofs of Lemmas 1-3 in [44].

**Lemma 6:** A feasible contract \( \Omega(\alpha) \) for Problem (P1) should satisfy: 1) \( \epsilon_1 \geq \epsilon_2 \geq \cdots \geq \epsilon_N > 0 \) and \( \epsilon_N \geq \mu_{NCN} > 0 \); and 2) \( r_{i+1} = \mu_i + \epsilon_{i+1} - \mu_{i+1} \).

Then Theorem 3 can be proved by contradiction. Assume there exist \( \{ r_1, r_2, \ldots, r_N \} \) with at least one \( i \in \{ 1, 2, \ldots, N \} \) satisfying \( r_i < r_i^* \) that achieves a larger utility at the server. Since \( r_i^* = r_{i-1}^* - \mu_i + \epsilon_i \) and according to the IC constraint, i.e., \( r_i \geq r_{i-1} - \mu_i - \epsilon_i \), we have \( r_i \leq r_{i-1} \). We iterate this...
inequality to show $\tilde{r}_N \leq r_N \leq \mu N \epsilon_N$, which violates the IR constraint of edge device $i$. Hence, Theorem 3 is verified.

**APPENDIX E**

Table III shows the relationship between the total reward and values of $\lambda$ and $\sigma^2$.

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