CHARACTERIZATION OF THE KOI-94 SYSTEM WITH TRANSIT TIMING VARIATION ANALYSIS: IMPLICATION FOR THE PLANET–PLANET ECLIPSE

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ABSTRACT

The KOI-94 system is a closely packed, multi-transiting planetary system discovered by the Kepler space telescope. It is known as the first system that exhibited a rare event called a “planet–planet eclipse (PPE),” in which two planets partially overlap with each other in their double-transit phase. In this paper, we constrain the parameters of the KOI-94 system with an analysis of the transit timing variations (TTVs). Such constraints are independent of the radial velocity (RV) analysis recently performed by Weiss and coworkers, and valuable in examining the reliability of the parameter estimate using TTVs. We numerically fit the observed TTVs of KOI-94c, KOI-94d, and KOI-94e for their masses, eccentricities, and longitudes of periastrons, and obtain the best-fit parameters including $m_c = 9.4_{-2.1}^{+2.4} M_\oplus$, $m_d = 52.1_{-6.9}^{+6.5} M_\oplus$, $m_e = 13.0_{-2.1}^{+2.5} M_\oplus$, and $e < 0.1$ for all the three planets. While these values are mostly in agreement with the RV result, the mass of KOI-94d estimated from the TTV is significantly smaller than the RV value $m_d = 106 \pm 11 M_\oplus$. In addition, we find that the TTV of the outermost planet KOI-94e is not well reproduced in the current modeling. We also present analytic modeling of the PPE and derive a simple formula to reconstruct the mutual inclination of the two planets from the observed height, central time, and duration of the brightening caused by the PPE. Based on this model, the implication of the results of TTV analysis for the time of the next PPE is discussed.

Key words: planets and satellites: individual (KOI-94, KIC 6462863, Kepler-89) – techniques: photometric

Online-only material: color figures

1. INTRODUCTION

The Kepler Object of Interest (KOI) 94 system is a multi-transiting planetary system discovered by the Kepler space telescope (Borucki et al. 2011; Batalha et al. 2013), consisting of four transiting planets with periods of about 3.7 (KOI-94b), 10 (KOI-94c), 22 (KOI-94d), and 54 (KOI-94e) days (Figure 1). For the largest planet KOI-94d, Hirano et al. (2012) observed the Rossiter–McLaughlin effect (Rossiter 1924; McLaughlin 1924; Queloz et al. 2000; Ohtae et al. 2005; Winn et al. 2005; Hirano et al. 2011) for the first time in a multi-transiting system. They found $\lambda = -6_{-11}^{+13}$, showing that the orbital axis of this planet is aligned with the stellar spin axis (this result was later confirmed by Albrecht et al. (2013), who obtained $\lambda = -11 \pm 11$). Furthermore, the KOI-94 system is the first and only system in which a rare mutual event called a “planet–planet eclipse” (PPE) was identified; in this event, two planets transit simultaneously and partially overlap with each other on the stellar disk as seen from our line of sight. By analyzing the light curve of the PPE caused by KOI-94d and KOI-94e, Hirano et al. (2012) concluded that the orbital planes of these two planets are also well aligned within $2^\circ$. In this system, therefore, the stellar spin axis and the orbital axes of the two planets are all aligned. If their close-in orbits are due to planetary migration (e.g., Lubow & Ida 2011), this result suggests that they have experienced a quiescent disk migration (Goldreich & Tremaine 1980) rather than processes that include gravitational perturbations either by planets (e.g., Nagasawa et al. 2008; Wu & Lithwick 2011; Naoz et al. 2011) or stars (e.g., Wu & Murray 2003). Other processes that tilt the stellar spin axis relative to orbital axes of planets (e.g., Bate et al. 2010; Lai et al. 2011; Rogers et al. 2012; Batygin 2012) are also excluded, provided that the orbital planes of the multiple transiting planets trace the original protoplanetary disk from which they formed. For these reasons, the KOI-94 system is an important test bed that provides a clue to understand the formation process of closely packed multi-transiting planetary systems, and hence deserves to be characterized in detail.

Recently, Weiss et al. (2013) measured the radial velocities (RVs) of KOI-94 from the W. M. Keck Observatory, and estimated the masses and eccentricities of the planets by a simultaneous fit to the observed RVs and the Kepler light curve. They showed that the masses of all the planets fall into the planetary regime, and especially obtained a fairly well constraint on the mass of KOI-94d ($m_d = 106 \pm 11 M_\oplus$). However, the masses of KOI-94c ($m_c = 15.6_{-15.6}^{+35.7} M_\oplus$) and KOI-94e ($m_e = 35_{-28}^{+18} M_\oplus$) are weakly constrained because of the marginal detections of their RV signals. In addition, the best-fit eccentricity of KOI-94c ($e_c = 0.43 \pm 0.23$) is suspiciously large in light of the long-term stability of the system, as pointed out in their paper. Hence additional RV observations are definitely important, but transit timing variations (TTVs; Holman & Murray 2005; Agol et al. 2005) can also be used to improve these estimates in such a multi-transiting system like KOI-94 (TTV analysis in the Kepler-11 system by Lissauer et al. 2011). Moreover, in the KOI-94 system the orbital parameters are exceptionally well constrained by the observations of the Rossiter–McLaughlin effect and the PPE; this makes the KOI-94 system an ideal case to evaluate the reliability of the parameter estimates by TTVs in comparison to RVs.
Apart from such characterization of the KOI-94 system, the PPE itself is a unique phenomenon that is worth studying in a more general context. If this event is observed in the future transit observations, it can be used to precisely constrain the relative angular momentum of the planets, which is closely related to their orbital evolution processes. In fact, this phenomenon had been theoretically predicted before by Ragozzine & Holman (2010) (see also Rabus et al. 2009) as an “overlapping double transit,” and they emphasized its role in constraining the relative nodal angles of the planets. However, neither the analytic formulation that clarifies the physical picture of this phenomenon, nor the discussion about how gravitational interactions among the planets affect the PPE, has been presented so far.

In this paper, we investigate the constraints on masses and eccentricities of KOI-94c, KOI-94d, and KOI-94e from the phase-folded transit light curves, and estimate their masses, eccentricities, and longitudes of periastrons from their TTV signals (see Table 1). In the following analysis, we neglect the smallest and innermost planet KOI-94b, which does not affect the TTV signals of the other three, as we will see in Section 2. Here we also discuss a simple prediction of the next PPE on the basis of a two-body problem. Finally, based on the analytic model of the PPE and the result of TTV analysis, we show in Section 5 how the gravitational interaction among the planets affects the occurrence of the next PPE in the KOI-94 system, referring to the difference from the two-body prediction. Section 6 summarizes the paper. The results on the properties of the KOI-94 system are all in Section 2, and so the readers who are only interested in the TTV analysis can skip Sections 3–5, where we mainly discuss the PPE.

## 2. ANALYSIS OF THE PHOTOMETRIC LIGHT CURVES

In this section, we report the analysis of photometric light curves of KOI-94 taken by *Kepler*. We determine the orbital phases, scaled semi-major axes, scaled planetary radii, and inclinations of KOI-94c, KOI-94d, and KOI-94e from the phase-folded transit light curves, and estimate their masses, eccentricities, and longitudes ofperiastrons from their TTV signals (see Table 1). In the following analysis, we neglect the smallest and innermost planet KOI-94b, which does not affect the TTV signals of the other three, as we will see in Section 2.2.

### 2.1. Transit Times and Transit Parameters

#### 2.1.1. Data Processing

We analyze the short-cadence (~1 minute) Pre-search Data Conditioned Simple Aperture Photometry fluxes (e.g., Kinemuchi et al. 2012) from Quarters 4, 5, 8, 9, 12, and 13. We do not include the data from Quarter 1, for which only the long-cadence data is available. Since these light curves exhibit the long-term trends that affect the baseline of the transit, we remove those trends in the following manner. First, data points within ±1 day of every transit caused by KOI-94c, KOI-94d, or KOI-94e are extracted and each set of the data is fitted with a fifth-order polynomial, masking out the points during the transit. Then we calculate the standard deviation of each fit, remove outliers exceeding 5σ, and fit the data again with the fifth-order polynomial. This process is iterated until all the 5σ outliers are removed. Finally, all the data points in each chunk (including those during the transit) are divided by the best-fit polynomial.

### Table 1

| Definition |
|------------|
| $t_0$ | Time of a transit center (BJD – 2454833) |
| $P$ | Orbital period |
| $R_p/R_\star$ | Planet-to-star radius ratio |
| $a/R_\star$ | Scaled semi-major axis |
| $b$ | Impact parameter of the transit ($= a \cos i/R_\star$, $i$: orbital inclination) |
| $u_1$, $u_2$ | Coefficients for the quadratic limb-darkening law |

| Definition |
|------------|
| $\Omega$ | Longitude of the ascending node |

Note. * PPE can only constrain the difference of the nodal angles of KOI-94d and KOI-94e. We have no information on $\Omega$ of KOI-94c.
to yield a detrended and normalized transit light curve. In our analysis of the TTV, we exclude the transits whose ingress or egress is not completely observed due to the data cadence of Kepler. We also exclude the “double-transit” events, during which two planets transit the stellar disk at the same time. As an exception, the double transit of KOI-94d and KOI-94e around BJD = 2454211.5 (in which a PPE was observed) is included in our analysis; in this case the ingress and egresses of both transits are clearly seen because of their close mid-transit times. The above criteria leave us with 44, 21, and 8 transits for KOI-94c, KOI-94d, and KOI-94e, respectively.

2.1.2. Transit Parameters

Before analyzing the TTV signals, we revise the transit parameters of KOI-94c, KOI-94d, and KOI-94e obtained by the Kepler team (Table 2) so that they are consistent with the light curves obtained in Section 2.1.1. Here we first use the parameters publicized by the Kepler team to phase-fold the observed transit light curves, and then refit those phase curves to obtain the revised transit parameters.

In the first step, we fit each of the detrended light curve centered at the transit (for ~1.7 times its duration) to obtain the times of transit centers \( t_c \), using a Markov chain Monte Carlo (MCMC) algorithm. Here we use a light curve model by Ohta et al. (2009), and fix \( a/R_\star \), \( R_p/R_\star \), and \( b \) to the values obtained by the Kepler team, assuming \( e = 0 \). We model the limb-darkening using a quadratic law in Equation (7) and adopt the limb-darkening coefficients \( u_1 \) and \( u_2 \) obtained by Hirano et al. (2012) (all these parameters are summarized in Table 2). Since the detrend procedure in Section 2.1.1 can remove only the out-of-transit outliers, we also exclude in-transit \( \sigma \) outliers of this fit, if any, and fit the light curve again. Using the series of \( t_c \) obtained in this way, we construct the phase-folded transit light curve for each planet.

As the second step, we fit the resulting phase-folded transit light curve for \( a/R_\star \), \( R_p/R_\star \), \( b \), \( u_1 \), and \( u_2 \) using the same light curve model as above. In this way, we obtain the revised values of the set of parameters shown in Table 3 and the corresponding best-fit light curves (Figures 2–4). In this fit, all the parameters converge well in the case of KOI-94d. In contrast, \( a/R_\star \) and \( b \) of KOI-94c and KOI-94e do not converge well moving back and forth between several local minima in a strongly correlated fashion, because they show smaller transit depths and the ingress/egress of their transits are less clear. For this reason, we impose an additional constraint that all the planets share the same host star: we convert the well-constrained \( a/R_\star \) for KOI-94d into stellar density \( \rho_\star \) via \( \rho_\star \approx (3\pi/GP^2)a/R_\star \) (Seager & Mallén-Ornelas 2003), and calculate the corresponding values and uncertainties of \( a/R_\star \) for KOI-94c and KOI-94e. The phase curves of KOI-94c and KOI-94e are fitted with prior constraints centered on these values and with Gaussian widths of

### Table 2

| Parameter | KOI-94c | KOI-94d | KOI-94e |
|-----------|---------|---------|---------|
| \( t_0 \) (BJD – 2454833) | 138.00718 ± 0.00093 | 132.74047 ± 0.00019 | 161.23998 ± 0.00079 |
| \( P \) (days) | 10.423707 ± 0.000026 | 22.343001 ± 0.000011 | 54.31993 ± 0.00012 |
| \( a/R_\star \) | 15.70 ± 0.37 | 26.10 ± 0.62 | 47.2 ± 1.1 |
| \( R_p/R_\star \) | 0.02544 ± 0.00012 | 0.06856 ± 0.00012 | 0.04058 ± 0.00013 |
| \( b \) | 0.019 ± 0.048 | 0.305 ± 0.014 | 0.387 ± 0.014 |

### Notes.

a Data from MAST archive http://archive.stsci.edu/kepler/.

b In this paper, we define the reference direction so that the spin–orbit angle \( \lambda \) measured by the Rossiter–McLaughlin effect be equal to \( \Omega \) for the orbital inclination in the range \([0, \pi/2]\) (Fabrycky & Winn 2009). With this choice, the reference line points to the ascending node of a virtual circular orbit whose angular momentum is parallel to the stellar spin vector.

### Table 3

| Parameter | KOI-94c | KOI-94d | KOI-94e |
|-----------|---------|---------|---------|
| \( a/R_\star \) | 15.7798 ± 0.0016 | 26.24 ± 0.19 | 47.4400 ± 0.0094 |
| \( R_p/R_\star \) | 0.02567 ± 0.00074 | 0.07029 ± 0.00014 | 0.04137 ± 0.00012 |
| \( b \) | 0.0214 ± 0.02 | 0.2994 ± 0.025 | 0.3840 ± 0.0048 |
| \( u_1 \) | 0.40 ± 0.06 | 0.40 ± 0.02 | 0.36 ± 0.07 |
| \( u_2 \) | 0.10 ± 0.08 | 0.14 ± 0.03 | 0.19 ± 0.10 |
| \( \chi^2/\text{dof} \) | 25843/23611 | 15473/13766 | 6822/5934 |

### Notes.

The quoted error bars denote 1σ confidence intervals obtained from the posterior distributions. The values of \( a/R_\star \) and \( b \) for KOI-94c and KOI-94e are determined with the prior information about the stellar density based on the result for KOI-94d (see the main text).

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7 We also repeated the same analysis taking account of the quarter-to-quarter flux contaminations publicized by the Kepler team and available at MAST archive. As expected, we obtained larger \( R_p \) by a factor of \( \sim c/2 \), where \( c \) is the fractional contamination (e.g., Fabrycky et al. 2012), but the other parameters were consistent within 2σ except for \( u_1 \) and \( u_2 \). These two parameters were different from those in Table 3 by ~1%, corresponding to a slight change in the value of \( a/R_\star \). In this paper, we do not apply this correction because the smaller values of \( R_p \) lead to more conservative estimates for the PPE occurrence.
their uncertainties. With this prescription, all the parameters of KOI-94c and KOI-94e converge well. Therefore, it does not make sense here to discuss the consistency of \( \rho^{\ast} \) calculated from the transit parameters to check the possible false positives. It is important to note, however, that the limb-darkening coefficients for each planet obtained individually are consistent within their 1\( \sigma \) error bars (those obtained by Hirano et al. (2012) are different from our values because they fixed smaller \( R_p/R^{\ast} \) for KOI-94d; see Table 2). This supports the notion that these three planets are indeed revolving around the same host star.

2.1.3. TTV Signals

Fixing \( a/R^{\ast}, R_p/R^{\ast}, b, u_1, \) and \( u_2 \) at the values in Table 3, we refit the transit light curves of each planet to find the values of \( t_c \), given in Tables 8–10 in Appendix A, with the values of reduced \( \chi^2 \) and 1\( \sigma \) upper/lower limits obtained from the posterior. The column labeled as \( O-C \) tabulates the residuals of a linear fit to \( t_c \) versus transit number, in which linear ephemerides in Table 4 are extracted. The values of reduced \( \chi^2 \) of the linear fits in this table indicate the significant deviations of the transit times from the linear ephemerides (i.e., TTVs) for all the three planets, as shown in Figure 5. Note that the TTV of KOI-94c shows the modulation with the period of \( (1/P_c - 2/P_d)^{-1} \approx 155 \) days, which clearly comes from near 2:1 resonance of KOI-94c and KOI-94d as pointed out by Xie et al. (2013) (see also Appendix B).

2.2. Numerical Analysis of the TTV Signals Using RV Mass of KOI-94d

We numerically analyze the TTV signals in Figure 5 to constrain the masses, eccentricities, and longitudes of periastrons of KOI-94c, KOI-94d, and KOI-94e (nine parameters in total). In this section, we fix the mass of KOI-94d at \( m_d = 106 \, M_{\oplus} \), the best-fit RV value obtained by Weiss et al. (2013). Since this value is only marginally consistent with \( m_d = 73 \pm 25 \, M_{\oplus} \) obtained by Hirano et al. (2012) based on the out-of-transit RVs taken by the Subaru telescope, we also investigate the case of \( m_d = 73 \, M_{\oplus} \) here.

2.2.1. Calculation of the Simulated TTV

Orbits of the planets are integrated using the fourth-order Hermite scheme with the shared time step (Kokubo & Makino 2004). The time step (typically \( \sim 0.025 \) days) is chosen so that the fractional energy change due to the integrator during \( \sim 1000 \) days integration should always be smaller than \( 10^{-9} \). All the simulations presented in this section integrate planetary
orbits beginning at the same epoch \( T_0(\text{BJD}) = 2455189.1703 \) (the first transit time of KOI-94d), until BJD \( = 2456133.0 \) (approximately the last transit time of KOI-94d we analyzed).

For each planet, the initial value of the orbital inclination \( i \) is fixed at the value obtained from \( a/R \), and \( b \) (see Table 3), assuming that \( e = 0 \). Here, all the \( i \) values are chosen in the range \( [0, \pi/2] \). The values of \( \Omega_d \) and \( \Omega_e \) are fixed at those in Table 2. Since we have no information on the nodal angle of KOI-94c, we assume \( \Omega_e = 0^\circ \). Initial semi-major axes are calculated via Kepler’s third law with \( M_\star \) and \( a \), orbital periods in Table 4, and planetary masses adopted in each simulation. The phases of the planets are determined from the transit ephemerides: for each planet, we convert the transit time closest to \( T_0 \) into the sum of the argument of periastron and mean anomaly \( \omega + f \), taking account of the non-zero eccentricity if any, and then move it backward in time to \( T_0 \), assuming a Keplerian orbit.

The mid-transit times of each planet are determined by minimizing the sky-plane distance \( D \) between the star and the planet, where the roots of the time derivative of \( D \) are found by the Newton–Raphson method (Fabrycky 2010). Then these transit times are fitted with a straight line and thereby the TTVs (= residuals of the linear fit), as well as the linear ephemeris \( (P \) and \( \tau_0) \), are extracted. We compute the chi squares of the simulated TTVs obtained in this way as

\[
X_j^2 = \sum_{\text{transit}} \left[ \frac{\text{TTV}_\text{obs}(i) - \text{TTV}_\text{sim}(i)}{\sigma_{\text{obs}}(i)} \right]^2, \quad (j = c, d, e),
\]

(1)

where \( \text{TTV}_\text{sim}(i) \) and \( \text{TTV}_\text{obs}(i) \) are the \( i \)-th values of simulated and observed TTVs of planet \( j \), respectively, and \( \sigma_{\text{obs}}(i) \) is the observational uncertainty of the \( i \)-th transit time of planet \( j \).

Note that we do not fit the transit times directly but only the deviations from the periodicity in our analysis, assuming that they provide sufficient information on the gravitational interaction among the planets. Indeed, although the initial values of semi-major axes are chosen to match the observed periods, periods derived from the simulations are different typically by \( \sim 0.01 \) days. This is because strong gravitational interaction among massive, closely packed planets in this system causes the oscillations of their semi-major axes with amplitudes dependent on the parameters of the planets adopted in each run. We will show that this simplified method still yields reasonable results in the last part of Section 2.4.

2.2.2. Estimates for the TTV Amplitudes

Before directly fitting the observed TTV signals, we evaluate the contribution from each planet to the TTVs of KOI-94c, KOI-94d, and KOI-94e. We divide the four planets into six pairs and integrate circular orbits for each pair using the best-fit masses by Weiss et al. (2013) listed in Table 2. Semi-amplitudes of the resulting TTVs of KOI-94c, KOI-94d, and KOI-94e are shown in Table 5. Considering the uncertainties of transit times listed in Tables 8–10 (typically 1.4 minutes, 0.3 minutes, and 0.9 minutes for KOI-94c, KOI-94d, and KOI-94e, respectively), this result indicates that KOI-94b has negligible contribution for the TTVs of the other three. In the following analysis, therefore, we integrate the orbits of the other three planets (KOI-94c, KOI-94d, and KOI-94e) only. We also find that the TTVs of KOI-94c and KOI-94e are mainly determined by the perturbation from the neighboring planet KOI-94d, while that of KOI-94d depends on both of its neighbors. Such dependence is naturally understood from the architecture of this system (see Figure 1). Consequently, each planet’s TTV mainly depends on the parameters listed in the rightmost column of Table 5, where we define \( e_j = (e_j \cos \sigma_j, e_j \sin \sigma_j) \) \( (j = c, d, e) \). Note that the TTV of each planet is insensitive to its own mass. This is why \( m_j \) \( (j = c, d, e) \) is not included in the row for planet \( j \).

2.2.3. Results

TTV of KOI-94e. Since the TTV of KOI-94e is mainly determined by \( e_d \) and \( e_e \) (and \( m_d \) of course, which we fix at the RV value), we fit it first so as to constrain these parameters. We calculate \( \chi^2_{e} \) for \( |e_d \cos \sigma_d|, |e_d \sin \sigma_d| < 0.06 \) and \( |e_e \cos \sigma_e|, |e_e \sin \sigma_e| < 0.25 \) (which well cover the 1σ regions for these parameters obtained from the RVs) at the grid-spacing of 0.01, fixing \( e_c = 0 \) and planetary masses at the best-fit values from the RVs. However, we cannot fit the observed TTV well in both \( m_d = 106 M_{\oplus} \) and 73 cases. The best fit for the former case, which gives \( \chi^2_{e} = 122 \) for 4 dof, is shown in Figure 6. For this reason, in addition to the fact that we have only eight transits observed for KOI-94e, we decide not to use the TTV of this planet to constrain the system parameters, but fit only the TTVs of KOI-94c and KOI-94d. The large discrepancy in the amplitudes of simulated and observed TTVs may suggest another source of perturbation which is not included in our model, such as a non-transiting planet or other minor bodies.

Grid-search for an Initial Parameter Set. Then we perform the grid-search fit to the TTVs of KOI-94c and KOI-94d to fit an appropriate initial parameter set for the following MCMC analysis. Based on the estimates given in Table 3, we fit these TTVs separately as follows. We first fit the TTV of KOI-94c varying \( e_c \) and \( e_d \) in \( |e_j \cos \sigma_j|, |e_j \sin \sigma_j| < 0.10 \) \( (j = c, d) \) at the grid-spacing of 0.01, and find one minimum of \( \chi^2_{e} \) for

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Table 4

| Parameter | KOI-94c | KOI-94d | KOI-94e |
|-----------|---------|---------|---------|
| \( t_0 \) (BJD − 2454833) | 138.00826 ± 0.00038 | 132.74103 ± 0.00012 | 161.23888 ± 0.00046 |
| \( P \) (days) | 10.4236888 ± 0.0000053 | 22.3429698 ± 0.0000036 | 54.319849 ± 0.000035 |
| \( x^2/\text{dof} \) | 10.4 | 13.7 | 29.2 |

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Table 5

| Pair | KOI-94b | KOI-94c | KOI-94d | KOI-94e |
|------|---------|---------|---------|---------|
| TTV | \( \lesssim 0.05 \) | \( \lesssim 0.05 \) | \( \lesssim 0.05 \) | \( \lesssim 0.05 \) |
| KOI-94c | \( m_d, e_c, e_d \) | \( m_c, e_c, e_d \) | \( m_c, e_d, e_c \) | \( m_c, e_d, (m_c, e_c) \) |
| KOI-94d | \( m_d, e_c, e_d \) | \( m_c, e_d, e_c \) | \( m_c, e_d, (m_c, e_c) \) | \( m_c, e_d, (m_c, e_c) \) |
| KOI-94e | \( m_d, e_c, e_d \) | \( m_c, e_d, e_c \) | \( m_c, e_d, (m_c, e_c) \) | \( m_c, e_d, (m_c, e_c) \) |

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As we will see in Section 4, the observed PPE light curve requires that if we (arbitrarily) choose \( i_c \) in \( [0, \pi/2] \), \( i_c \) is also in this range. There is no justification to choose \( i_c \) in this range, but this choice does not affect the result significantly because the value of \( i_c \) is very close to \( \pi/2 \), as suggested by the small value of \( b_c \) (see Table 3).
both $m_d = 106 M_\oplus$ and $m_d = 73 M_\oplus$ cases. Next, for all the sets of $(e_c, \sigma_c, e_d, \sigma_d)$ in $2\sigma$ ($m_d = 106 M_\oplus$ case) or $1\sigma$ ($m_d = 73 M_\oplus$ case) confidence regions around the minimum, we run integrations varying $e_c \cos \sigma_c$ and $e_c \sin \sigma_c$ from $-0.1$ to 1 at the grid spacing of 0.01, $m_c$ from 0 to $24 M_\oplus$ at the grid spacing of 6 $M_\oplus$, and $m_c$ from 7 to 57 $M_\oplus$ at the grid spacing of 10 $M_\oplus$ (all of these cover the 1$\sigma$ intervals from RV), to find the set of eight parameters that best fits the TTV of KOI-94d.

**MCMC Fit to the TTVs of KOI-94c and KOI-94d.** Choosing the above set as initial parameters, we then simultaneously fit the TTVs of KOI-94c and KOI-94d using an MCMC algorithm. In this fit, we use $\chi^2$ as the $\chi^2$ statistic. The resulting best-fit parameters and their 1$\sigma$ uncertainties are summarized in Table 6 for the two choices of $m_d$ (the second and third columns). The best-fit simulated TTVs are plotted in Figures 7 and 8 for KOI-94c and KOI-94d, respectively.

Note that uncertainties of $e_c \cos \sigma_c$, $e_d \cos \sigma_d$, and $e_c \cos \sigma_c$ are relatively large for $m_d = 73 M_\oplus$ case. This is because the posterior distributions of these parameters have two peaks, the smaller of which lies close to the best-fit value for $m_d = 106 M_\oplus$ case. Considering this fact, the two results are roughly consistent with each other. Nevertheless, a total $\chi^2$ in $m_d = 73 M_\oplus$ case is smaller by 40 for 57 dof than in $m_d = 106 M_\oplus$ case. This suggests that the TTV alone favors $m_d$ smaller than the RV best-fit value, as will be confirmed in the next subsection.

### 2.3. Solution Based Only on TTV

In order to obtain a solution independent of RVs, we perform the same MCMC analysis of TTVs of KOI-94c and KOI-94d, this time also allowing $m_d$ to float. Since the above analyses suggest that the eccentricities of all the planets are small, we choose circular orbits with the best-fit RV masses as an initial parameter set. The resulting best-fit parameters are summarized in Table 6, and the corresponding best-fit simulated TTVs are shown in Figures 9 and 10. As expected, we find results with small eccentricities and with $m_d$ smaller than the RV best-fit value. This solution is similar to that for $m_d = 73 M_\oplus$ case, except that $m_d$ is even smaller.

### 2.4. Discussion: Comparison with the RV Results

While the values of $e_d$ and $e_c$ obtained in our TTV analysis are consistent with the RV values in Table 2, the best-fit $e_c$ obtained from the TTV is $\sim 1.8\sigma$ smaller than the RV best fit ($e_c = 0.43 \pm 0.23$). Considering the marginal detection of KOI-94c's RV and the dynamical stability of the system, however, the TTV value seems to be preferred. In fact, this value is robustly constrained by the clear TTV signal of KOI-94c; in the grid-search performed in Section 2.2.3, we searched the region where $e_c \lesssim 0.14$ to fit the TTV of this planet, but the resulting $\chi^2$ strongly disfavored large $e_c$ regions in both $m_d = 106 M_\oplus$ and $m_d = 73 M_\oplus$ cases.

The TTV values of $m_c$ and $m_e$ are consistent with the RV results, but $m_e$ is constrained to a rather lower value than the
RV best fit. Using this value, along with the photometric values of $R_p/R_\star$ and $\rho_\star$, and spectroscopic value of $M_\star$, the density of KOI-94e is given by $\rho_e \sim 0.3 \text{ g cm}^{-3}$. This implies that it is one of the lowest-density planets ever discovered.

The largest discrepancy arises in the value of $m_d$, mass of KOI-94d, for which the RV best-fit value is smaller than the RV value by $\sim 4\sigma$. The worse fit in the case of $m_d = 106 \, M_\oplus$ is mainly due to the fact that the observed TTV amplitude of KOI-94c is smaller than expected from this value of $m_d$. As shown in Table 5, the TTV of this planet is completely dominated by the perturbation from KOI-94d, and so the parameters relevant to this TTV are $m_d$, $e_c$, and $e_d$. Table 5 also shows that $m_d = 106 \, M_\oplus$ leads to the TTV semi-amplitude of $\sim 11$ minutes for $e_c = e_d = 0$, which is much larger than the observed TTV amplitude of KOI-94c (see Figure 5). As a result, the values of $e_c$ and $e_d$ are fine-tuned to fit the observed signal, resulting in strict constraints on these parameters. The problem is that these values of $e_c$ and $e_d$ do not fit the TTV of KOI-94d well. In the grid-search, we first fit the TTV of KOI-94c alone, and the best fit gives $\chi_c^2 = 66$ for $m_d = 106 \, M_\oplus$. However, this value is largely increased in the simultaneous MCMC fit to the TTVs of KOI-94c and KOI-94d (Table 6), which means that these two TTVs cannot be explained with the same set of $e_c$ and $e_d$. On the other hand, this tension does not exist in $m_d = 73 \, M_\oplus$ case, in which both of the grid-search and MCMC return the same values for the best-fit $\chi_c^2$. In fact, the analysis using the analytic formulae of TTVs from two coplanar planets lying near $j:j - 1$ resonance (Lithwick et al. 2012) also supports the above reasoning, suggesting that $m_d$ expected from the TTV of KOI-94c is rather small (see Appendix B).

For these reasons, it is clear that the TTV favors the solution with $m_d$ smaller than the RV best-fit value. It is also true, however, that the RV of KOI-94d is detected with high significance, in contrast to those of the other planets. Indeed, we calculate the RVs using the best-fit TTV parameters and find that the

| Parameter | Value ($m_d = 106 \, M_\oplus$) | Value ($m_d = 73 \, M_\oplus$) | Value (TTV Only) |
|-----------|-------------------------------|-------------------------------|------------------|
| $m_c$ ($M_\oplus$) | $11.8^{+3.6}_{-1.5}$ | $13.9^{+2.7}_{-2.1}$ | $9.4^{+2.4}_{-2.1}$ |
| $e_c \cos \mu_c$ | $0.0329^{+0.0047}_{-0.0055}$ | $0.0092^{+0.0026}_{-0.0050}$ | $0.0143^{+0.0009}_{-0.0039}$ |
| $e_c \sin \mu_c$ | $-0.0104^{+0.0038}_{-0.0042}$ | $-0.0031^{+0.0067}_{-0.0061}$ | $0.0045^{+0.0091}_{-0.0079}$ |
| $\chi_c^2$ | $84$ | $62$ | $56$ |

KOI-94d

| Parameter | Value ($m_d = 106 \, M_\oplus$) | Value ($m_d = 73 \, M_\oplus$) | Value (TTV Only) |
|-----------|-------------------------------|-------------------------------|------------------|
| $m_d$ ($M_\oplus$) | $106$ (fixed) | $73$ (fixed) | $52.1^{+6.9}_{-7.1}$ |
| $e_d \cos \mu_d$ | $0.055^{+0.011}_{-0.014}$ | $-0.0169^{+0.0064}_{-0.011}$ | $-0.022^{+0.014}_{-0.011}$ |
| $e_d \sin \mu_d$ | $0.013^{+0.011}_{-0.012}$ | $0.009^{+0.018}_{-0.018}$ | $0.008^{+0.021}_{-0.018}$ |
| $\chi_d^2$ | $66$ | $48$ | $43$ |

KOI-94e

| Parameter | Value ($m_d = 106 \, M_\oplus$) | Value ($m_d = 73 \, M_\oplus$) | Value (TTV Only) |
|-----------|-------------------------------|-------------------------------|------------------|
| $m_e$ ($M_\oplus$) | $15.9^{+2.4}_{-3.2}$ | $12.9^{+2.0}_{-2.3}$ | $13.0^{+2.5}_{-2.1}$ |
| $e_e \cos \mu_e$ | $0.067^{+0.014}_{-0.015}$ | $-0.069^{+0.120}_{-0.018}$ | $-0.078^{+0.021}_{-0.014}$ |
| $e_e \sin \mu_e$ | $0.043^{+0.012}_{-0.017}$ | $-0.022^{+0.032}_{-0.016}$ | $-0.025^{+0.017}_{-0.014}$ |
| $(\chi_e^2 + \chi_d^2)/\text{dof}$ | $150/57$ | $110/57$ | $99/56$ |
the TTV best-fit parameters (rightmost column of Table 6). We run two MCMC chains starting from (1) circular orbits with et al. (2013). Since its origin is not yet clear, this discrepancy resulting amplitude is much smaller than that observed by Weiss et al. (2013). The Astrophysical Journal, 778:185 (16pp), 2013 December 1

Finally, we note again that in the above analysis we just fit the TTVs rather than the transit times of KOI-94c and KOI-94d. For this reason, our solution corresponds to the planetary orbits whose periods are slightly different from the actually observed values. One may argue that such an approximate method leads to an incorrect solution. In order to assure that this is not the case, we fit the transit times of the two planets, choosing m, e, cos σ, e sin σ, and α + f of KOI-94c, KOI-94d, and KOI-94e at time Tp as free parameters (fifteen parameters in total). We run two MCMC chains starting from (1) circular orbits with RV best-fit masses and (2) a local minimum reached by the Levenberg–Marquardt method (Markwardt 2009) starting from the TTV best-fit parameters (rightmost column of Table 6). We find that the best-fit values of (m, e, σ) obtained in these ways show similar trends as the TTV best fit in Section 2.3, giving comparable reduced χ² values. Namely, the mass of KOI-94d is much smaller than the RV best fit and eccentricities of the three planets are close to zero.

Incidentally, as in the case of the fit to TTVs, the transit times of KOI-94d are less well reproduced than those of KOI-94c. In fact, the fit to transit times gives a better χ² than that to TTVs because the small linear trend apparent in the lower panel of Figure 9 is removed; on the other hand, χ² values are not so different in both cases. The difficulty in completely reproducing the orbit of KOI-94d may also indicate the presence of the additional perturber discussed in Section 2.2.3.

3. ANALYTIC FORMULATION OF THE PPE

In this section, we present a general analytic model of the PPE caused by two planets 1 and 2 on circular orbits (see Appendix C for the formulation taking account of O(ε) terms). In what follows, we use the stellar radius R*, as the unit length because all the observables are only related to the lengths normalized to this value.

3.1. Flux Variation Due to a PPE

A PPE is observed as an increase in the relative flux of a star (or a “bump” in the light curve) during the double transit of two planets. Assume that the two overlapping planets are spherical and neglecting the effect of limb-darkening, the increase in the relative flux S is given by the area of overlapping region of the two planets divided by that of the stellar disk (Figure 11):

$$S = \begin{cases} 0 & \text{if } R_{p1} + R_{p2} < d, \\ \frac{1}{2\pi} R_{p1}^2 (2\alpha - \sin 2\alpha) + \frac{1}{2\pi} R_{p2}^2 (2\beta - \sin 2\beta) & \text{for } |R_{p1} - R_{p2}| < d < R_{p1} + R_{p2}, \\ \text{(min}(R_{p1}, R_{p2}))^2 & \text{for } d < |R_{p1} - R_{p2}|, \end{cases}$$

(2)

where d is the distance between the centers of the two planets in the plane of the sky, and the angles α and β are defined as

$$\cos \alpha = \frac{R_{p1}^2 + d^2 - R_{p2}^2}{2 R_{p1} d}, \quad \cos \beta = \frac{R_{p2}^2 + d^2 - R_{p1}^2}{2 R_{p2} d}.$$  

An alternative expression of S for |R_{p1} - R_{p2}| < d < R_{p1} + R_{p2} can be obtained from the derivative of S with respect to d. Using $d\alpha/dd$ and $d\beta/dd$ obtained from Equation (3) and $R_{p1} \sin \alpha = R_{p2} \sin \beta$, we obtain

$$\frac{dS}{dd} = -\frac{1}{\pi} \left[ -d^2 + 2(R_{p1}^2 + R_{p2}^2) - \left( \frac{R_{p1}^2 - R_{p2}^2}{d} \right)^2 \right]$$

(4)

for $|R_{p1} - R_{p2}| < d < R_{p1} + R_{p2}$. Equation (4) can be integrated from $R_{p1} + R_{p2}$ to d by changing the variable from d to $\gamma = \alpha + \beta = \pi - \arccos \left( \frac{R_{p2}^2 + R_{p1}^2 - d^2}{2 R_{p1} R_{p2}} \right)$.

$$S = \frac{1}{\pi} \left[ \frac{R_{p1}^2 + R_{p2}^2}{2} \gamma - R_{p1} R_{p2} \sin \gamma \right.$$  

$$- \left. \left( \frac{R_{p1}^2 - R_{p2}^2}{2 R_{p1} R_{p2}} \right) \arctan \left( \frac{R_{p1} - R_{p2}}{R_{p1} + R_{p2}} \tan \frac{\gamma}{2} \right) \right].$$

(6)

The result is

$$S = \frac{1}{\pi} \left[ \frac{R_{p1}^2 + R_{p2}^2}{2} \gamma - R_{p1} R_{p2} \sin \gamma \right.$$  

$$- \left. \left( \frac{R_{p1}^2 - R_{p2}^2}{2 R_{p1} R_{p2}} \right) \arctan \left( \frac{R_{p1} - R_{p2}}{R_{p1} + R_{p2}} \tan \frac{\gamma}{2} \right) \right].$$

(6)

Here, S is given as a function of a single angle γ. These two expressions of S show that the shape of a bump due to a PPE is solely determined by γ as a function of time, which will be derived in the following subsection.

The effect of limb-darkening can be included in our model by multiplying S by a factor that corresponds to the limb-darkening at the position on the stellar disk over which a PPE occurs. We adopt the quadratic limb-darkening law

$$I(\mu)/I(0) = 1 - u_1(1 - \mu) - u_2(1 - \mu)^2,$$

(7)

where $\mu = (1 - r^2)^{1/2}$ and r is the radial coordinate on the stellar disk. For a PPE that occurs totally within the stellar disk, the approximation by Mandel & Agol (2002), which is valid for a small planet whose radius is less than about 0.1R*, yields the modified relative increase S’ as

$$S' = S \cdot \int_0^1 I(r_u) dr_u \approx S \cdot \frac{1 - u_1 - u_2 - u_3}{1 - u_1/3 - u_2/6},$$

(8)

where $r_u$ is the distance to the overlapping region. During the whole PPE, $r_u$ is given in terms of d, $\Delta R_{12}^2 \equiv R_{p1}^2 - R_{p2}^2$, and

Figure 11. Two overlapping planets 1 and 2. The definitions of angles α, β, and γ are shown. Area of the shaded region corresponds to S in Equation (2) times the area of the stellar disk.

(A color version of this figure is available in the online journal.)
Suppose that the two planets 1 and 2 are on Keplerian orbits whose angular momentum vector is parallel to the vector from the star, the position of a planet is expressed as a two-dimensional distance between the planet and the star, the position of a planet is expressed as

\[ X = R \left( \cos \Omega \cos(\omega + f) - \sin \Omega \sin(\omega + f) \cos i \right), \quad (9) \]

\[ Y = R \left[ \sin \Omega \cos(\omega + f) + \cos \Omega \sin(\omega + f) \cos i \right], \quad (10) \]

\[ Z = R \sin(\omega + f) \sin i. \quad (11) \]

Suppose that the two planets 1 and 2 are on Keplerian orbits whose semi-major axes are \( a_1 \) and \( a_2 \), respectively. Neglecting the corrections arising from the non-zero eccentricity, the two-dimensional position vectors \( r_j \) \((j = 1, 2)\) of these planets in the plane of the sky can be written as

\[ r_j = \left( a_j \cos \Omega_j \cos(\omega_j + f_j) - b_j \sin \Omega_j \sin(\omega_j + f_j) \right) \sin i_j + \left( a_j \sin \Omega_j \cos(\omega_j + f_j) + b_j \cos \Omega_j \sin(\omega_j + f_j) \right) \cos i_j. \quad (12) \]

If the transits of the two planets are observed, \( a_j, b_j, R_{pj}, \) and the periods \( P_j \) are obtained as in Section 2. In this case, the relative motion of the planets is completely described except for the dependence on the relative nodal angle defined as

\[ \Omega_{21} = -\Omega_{12} = \Omega_2 - \Omega_1. \quad (13) \]

Note that photometric surveys determine the absolute value of \( b \), but not its sign. For a single transiting planet, \( b \) is conventionally defined to be positive (or equivalently, \( i \) is chosen to be in the range \([0, \pi/2]\)), because the choice of its sign does not affect the transit signals. For multiple transiting planets, however, a different choice of the relative signs of \( b \) corresponds to a different orbital configuration. In this paper, we choose \( b_1 \) to be positive (i.e., \( 0 \leq i_1 \leq \pi/2 \)), but allow \( b_2 \) to be either positive or negative (i.e., \( 0 \leq i_2 \leq \pi \)). The sign of \( b_2 \), as well as the relative nodal angle \( \Omega_{21} \), is determined from the observed data of a PPE event.

During a double transit by planets \( j = 1 \) and \( 2 \), their phases at time \( t \) are given by

\[ \omega_j + f_j = \frac{\pi}{2} + n_j(t - t_c^{(j)}), \quad (14) \]

where \( n_j = 2\pi/P_j \) is the mean motion and \( t_c^{(j)} \) is the central transit time of planet \( j \) in this double transit. We then expand Equation (12) to the first order of \( n(t - t_c) \sim \mathcal{O}(a^{-1}) \) to obtain

\[ r_j = v_j(t - t_c^{(j)}) + r_0^{(j)}, \quad (15) \]

where

\[ v_j = -a_j n_j \left( \frac{\cos \Omega_j}{\sin \Omega_j} \right), \quad r_0^{(j)} = -b_j \left( \frac{\sin \Omega_j}{-\cos \Omega_j} \right). \quad (16) \]

### 3.2. Distance between the Planets During a Double Transit

Hereafter, we adopt the Cartesian coordinate system \((X, Y, Z)\) centered on the star, where the \( +Z \)-axis is chosen in the direction of our line of sight, and \( X \)- and \( Y \)-axes are in the plane of the sky, forming a right-handed triad. As stated in the note of Table 2, we align the \( +X \) axis with the ascending node of a virtual circular orbit whose angular momentum vector is parallel to the stellar spin vector, without loss of generality. Using \( R \) for the three-dimensional distance between the planet and the star, the position of a planet is expressed as

\[ r = \sqrt{(r_1^2 + r_2^2)/2 - d^2/4 + (r_2^2 - r_1^2 + \Delta R_{12}^2/2)\Delta R_{12}^2/2d^2}. \]

These give the distance between the two planets \( d \) as\(^9\)

\[ d^2 = |r_2 - r_1|^2 = v^2 \left( t + \frac{r_0 \cos \theta_0}{v} \right)^2 + r_0^2 \sin^2 \theta_0, \quad (17) \]

where \( v \equiv v_2 - v_1, r_0 = r_2(0) - v_2 t(0) - r_1(0) + v_1 t(1), v \equiv |v|, r_0 \equiv |r_0|, \) and \( \theta_0 \) is the angle between \( v \) and \( r_0 \). Thus, the minimum value of \( d \) in this double transit

\[ d_{\text{min}} \equiv r_0 \sin \theta_0, \quad (18) \]

is reached at the time

\[ t = t_{\text{min}} \equiv \frac{r_0 \cos \theta_0}{v}. \quad (19) \]

If \( d_{\text{min}} < R_{p1} + R_{p2} \), a PPE occurs during this double transit for a duration of

\[ \Delta t = \frac{2}{v} \sqrt{(R_{p1} + R_{p2})^2 - d_{\text{min}}^2}. \quad (20) \]

Equations (18)–(20) can be readily understood by considering the geometry of the PPE shown in Figure 12. The explicit expressions of \( v, r_0, \) and \( \cos \theta_0 \) are

\[ v^2 = a_1^2 i_1^2 + a_2^2 i_2^2 - 2a_1 a_2 i_1 i_2 \cos \Omega_{21}, \quad (21) \]

\[ r_0^2 = (a_1 n_1 t_c^{(1)})^2 + (a_2 n_2 t_c^{(2)})^2 + b_1^2 + b_2^2 + 2(a_1 n_1 b_1 t_c^{(1)} b_2) \sin \Omega_{21} - 2(a_1 n_1 b_1 t_c^{(1)} a_2 n_2 t_c^{(2)} b_2) \cos \Omega_{21}, \quad (22) \]

\[ \cos \theta_0 = \frac{1}{r_0 \sqrt{v}} \left[ - (a_1 n_1 t_c^{(1)} - a_2 n_2 t_c^{(2)})^2 + (a_2 n_2 b_1 - a_1 n_1 b_2) \sin \Omega_{21} + a_1 n_1 a_2 n_2 (t_c^{(1)} + t_c^{(2)}) \cos \Omega_{21} \right]. \quad (23) \]

### 3.3. Reconstruction of the Mutual Inclination \( \Omega_{21} \)

The observed shape of a bump is characterized by its maximum height \( S_{\text{max}} \), central time \( t_{\text{min}} \), and duration \( \Delta t \). If the bump is not saturated, i.e., \( d_{\text{min}} > |R_{p1} - R_{p2}| \), \( S_{\text{max}} \) is uniquely translated into \( d_{\text{min}} \) via Equation (2). In this case, we can use

\[^9\] Strictly speaking, this expression of \( d \) contains \( \mathcal{O}(n(t - t_c)^2) \) terms, and so we should use the second-order expansion in Equation (15). Nevertheless, this only introduces the correction of order \((b/a)^2 \sim a^{-2}\), which can be safely neglected in our treatment below.
Equations (18)–(20), the expressions for the three observables of a bump \((d_{\text{min}}, t_{\text{min}}, \Delta t)\), to calculate

\[
v = \frac{2}{\Delta t} \sqrt{(R_p l + R_p e)^2 - d_{\text{min}}^2},
\]

\[
r_0^2 = d_{\text{min}}^2 + (vt_{\text{min}})^2
\]

\[
= d_{\text{min}}^2 + \frac{2t_{\text{min}}}{\Delta t} \left[(R_p l + R_p e)^2 - d_{\text{min}}^2\right],
\]

\[
\tan \theta_0 = \frac{d_{\text{min}}}{vt_{\text{min}}} = -\frac{\Delta t}{2t_{\text{min}}} \frac{d_{\text{min}}}{\sqrt{(R_p l + R_p e)^2 - d_{\text{min}}^2}}.
\]

Furthermore, Equations (21) and (22), the explicit expressions for \(v\) and \(r_0\), are rewritten as

\[
\cos \Omega_{21} = \frac{a_1^2 n_1^2 + a_2^2 n_2^2 - v^2}{2a_1 a_2 n_1 n_2},
\]

\[
\sin \Omega_{21} = \frac{r_0^2 - x_1^2 - x_2^2 + 2(x_1 \times x_2) \cos \Omega_{21}}{2|x_1 \times x_2|},
\]

where we define \(x_1 = (a_1 n_1 t^{(1)}_1, b_1)\) and \(x_2 = (a_2 n_2 t^{(2)}_2, b_2)\). In this way, the relative nodal angle \(\Omega_{21}\) can be specified explicitly from \((d_{\text{min}}, t_{\text{min}}, \Delta t)\) along with the photometrically obtained parameters \((a_j, b_j, n_j, t^{(j)}_e, R_p e)\), provided that the sign of \(b_2\) is determined. Although we do not present any general procedure to determine the sign of \(b\) here, it is possible to do so at least empirically, as described in Section 4. Equation (28) shows that \(\sin \Omega_{21}\) is only weakly constrained when \(a_1 n_1 / b_1 \approx a_2 n_2 / b_2\), in which case the coefficients in front of \(\sin \Omega_{21}\) in Equations (22) and (23) are close to zero because \(t^{(j)}_e \approx t^{(2)}_e\).

In the case of \(d_{\text{min}} < |R_p l - R_p e|\), only the upper limit on \(d_{\text{min}}\) can be obtained, and so the entire shape of the bump is required to determine \(\Omega_{21}\).

Note that the formulation in this section is also valid even in the presence of non-Keplerian effects. In such a case, the Keplerian orbital elements in this formulation should be interpreted as the osculating orbital elements around a certain double transit.

4. APPLICATION TO THE PPE OBSERVED IN THE KOI-94 SYSTEM

Hirano et al. (2012) fit the whole light curve of the PPE for \(u_1, u_2, t^{(1)}_e, t^{(3)}_e, \) and \(\Omega_{ed}\) using an MCMC algorithm, and estimated the relative nodal angle between KOI-94d (planet 1 in Section 3) and KOI-94e (planet 2 in Section 3) to be \(\Omega_{ed} = 1.15 \pm 0.55\)\(^{10}\). In their analysis, the light curve is modeled as a sum of two single transit light curves (Ohta et al. 2009) and the bump function, which is calculated essentially in the same way as described in Section 3, but neglecting the effect of limb-darkening. In this section, we confirm that this is a unique solution expected from the observed features of the bump, based on analytic expressions obtained in the previous section.

The top panel of Figure 13 plots \(d_{\text{min}}\) as a function of \(\Omega_{ed}\) in the double transit during which the PPE was observed, calculated by Equations (18), (22), and (23). In Figure 13, we fix \(a_j, b_j, n_j, \) and \(R_p e\) at the values publicized by the Kepler team (Table 2), and \(t^{(j)}_e\) at the best-fit values obtained by Hirano et al. (2012). The red and blue lines correspond to the cases of \(b_e > 0\) and \(b_e < 0\), respectively, and the solid parts of the lines show the range of \(\Omega_{ed}\) for which the PPE occurs, i.e., \(d_{\text{min}} < R_p d + R_p e\). This panel implies that the PPE itself could have occurred for a wide range of \(\Omega_{ed}\) except for those around \(\pm 50^\circ\). The central time \(t_{\text{min}}\) and duration \(\Delta t\) for a specific value of \(\Omega_{ed}\) can be obtained from the middle and bottom panels. These also indicate that a wide variety of bumps could have resulted.

Indeed, these three observables have enough information to reconstruct the value of \(\Omega_{ed}\), in addition to the sign of \(b_e\). As for the observed eclipse, the best-fit light curve yields \(S_{\text{max}} \approx 3.88 \times 10^{-4}, t_{\text{min}} \approx 378.508\) day (BJD − 2454833), and \(\Delta t \approx 0.076\) days. Equation (2) shows that the above value of \(S_{\text{max}}\) uniquely translates into \(d_{\text{min}} \approx 0.0829\). The values of these \(d_{\text{min}}, t_{\text{min}}, \) and \(\Delta t\) are plotted in Figure 13 in horizontal dashed lines.\(^{11}\) For the observed value of \(d_{\text{min}}\), Figure 13 allows

\(^{10}\) Note that the “mutual inclination” \(\delta\) defined in Hirano et al. (2012) corresponds to \(\Omega_{ed} = -\Omega_{eq}\) in our definition.

\(^{11}\) The analysis that includes the effect of limb-darkening by Equation (8) returned the same values of \(d_{\text{min}}, t_{\text{min}}, \) and \(\Delta t\) with a slightly different \(\Omega_{ed} \sim 1:21\), in which case the following discussion is also valid.
eight solutions, four for each of \( b_2 > 0 \) and \( b_2 < 0 \). However, the asymmetry of \( t_{\text{min}} \) curve in the middle panel of Figure 13 shows that only the solutions around \( \Omega_{\text{ed}} \sim 0^\circ \) (slightly positive, \( b_e > 0 \)) or \( \Omega_{\text{ed}} \sim 180^\circ \) (\( b_e < 0 \)) are possible. These correspond to the nearly parallel and anti-parallel planetary orbits, respectively. This degeneracy can be broken with the value of \( \Delta t \): the retrograde (anti-parallel) orbit results in a much shorter bump due to the larger relative velocity between the planets than the prograde (parallel) case. The bottom panel of Figure 13 shows that the observed duration allows only the prograde orbit with \( b_e > 0 \). In this way, \( \Omega_{\text{ed}} = 1:15 \) (and \( b_e > 0 \)) proves to be the unique solution that reproduces the observed features of the bump. In fact, one can show that any set of \( (d_{\text{min}}, t_{\text{min}}, \Delta t) \) allows the unique determination of \( \Omega_{\text{ed}} \) in the case discussed here. Mathematically, this comes from the fact that the curve \( (d_{\text{min}}, t_{\text{min}}, \Delta t) \) parameterized by \( \Omega_{\text{ed}} \) has no self-intersection.

Combining \( \Omega_{\text{ed}} \) with the result of the spin–orbit angle measurement, both \( \Omega_{d} \) and \( \Omega_{c} \) can be constrained. Since we have assumed that \( b_d > 0 \) or \( 0 < i_d < \pi/2 \), the observed spin–orbit angle \( \lambda \) is equal to \( \Omega_{d} \) in our definition, and so \( \Omega_{d} = -6^\circ \pm 1^\circ \). Thus, \( \Omega_{\text{ed}} = \Omega_{d} - \Omega_{c} = 1:15 \pm 0.55 \) (Hirano et al. 2012) gives \( \Omega_{e} = -5^\circ \pm 1^\circ \) (Table 2). Using these two parameters along with the transit parameters in Table 2, we trace the orbits of KOI-94d and KOI-94e for 100 yr, assuming that their orbits never change over time. The result of this calculation indicates that the next PPE will occur in the double transit around BJD = 2461132.4 (date in UT 2026 April 1/2), which is the third double transit after the one discussed here. The same conclusion is obtained even when we vary \( \Omega_{\text{ed}} \) within its 3σ interval. In a real system, however, it is not at all obvious that the next PPE will occur in this double transit, because orbital elements do change over time; in the next section, we discuss how the mutual gravitational interaction among the planets affects this result.

5. IMPLICATION FOR THE NEXT PPE: THE EFFECT OF MULTI-BODY INTERACTION

In Section 2, we obtained a set of system parameters by analyzing photometric light curves of the three planets. Based on this result, we now address the question whether the PPE will occur in the same double transit as predicted in Section 4, even in the presence of the mutual gravitational interaction among the planets.

5.1. Fixing Double-transit Parameters

In order to determine the relative nodal angle crucial in predicting the next PPE, we refit the transit light curve around BJD = 2454211.5, in which the PPE was observed. We model the light curve as described in Section 4 including the effect of limb-darkening to obtain \( R_{pd}, R_{pc}, a_d, a_e, b_d, b_e, t_{c(d)} \), \( t_{c(d)} \), and \( \Omega_{ed} \). Here, the same prior as used in fitting the phase-folded light curve is assumed to fix the value of \( a_e \). We adopt the limb-darkening coefficients obtained from the light curve of KOI-94d, \( u_1 = 0.40 \) and \( u_2 = 0.14 \), which are determined with the best precision among the three planets. The result is summarized in Table 7.

In this fit, \( R_{pd}/R_{*} \) is different from our revised mean value \( (R_{pd}/R_{*} = 0.070288 \pm 0.0014) \) by 1.7σ. We suspect that this difference comes from the systematics introduced in the detrending procedure. When an artifact or some other astrophysical processes (e.g., star spots) accidentally increase the relative flux just before (or after) the transit, the result of detrending is biased toward such features in setting the baseline of the transit light curve. If we use a wider range of data points, the effect of such a small feature is averaged out and does not change the result so significantly. In contrast, if a narrow region around a transit is used, the baseline is somewhat distorted and the resulting detrended light curve becomes either deeper or shallower. Such systematics are averaged in the phase-folded light curves, but may be significant in an individual transit. In the case of the double transit light curve analyzed here, the relative flux begins to increase just before the ingress, and the depth of the transit is shallower in the first half of the transit than in the latter, making the light curve slightly asymmetric. This feature, along with the fact that the revised \( R_{pd} \) gives better \( \chi^2 \) in all the other transit light curves than \( R_{pd} \) given by the Kepler team, suggests that the discrepancy in \( R_{pd} \) is caused by such an incidental brightening.

To test this scenario, we repeat the analysis above changing the span of detrending from \( \sim 1 \) day to \( \sim 0.5 \) days, and find that the resulting mean transit parameters are consistent with those above, but the depth of the double-transit light curve becomes deeper, consistently with our revised parameters.

5.2. Evaluation of the Multi-body Effect

The occurrence of the PPE in the multi-body context can be assessed in a similar way as in Section 4; we compare \( R_{pd} + R_{pc} \) with \( d_{\text{min}} \) calculated from Equations (18), (22), and (23), but this time the variation of orbital elements must be taken into account. Specifically, we need to evaluate the variations of the parameters relevant to \( d_{\text{min}} \), i.e., \( n \), \( \nu \), \( a \), \( R_{pc} \), mean motion \( n \), transit center of the double transit \( t_e \), impact parameter \( b \), and nodal angle \( \Omega \) (as long as the eccentricities are small). In order to give an estimate for these variations, we integrate the orbits of the three planets using the best-fit \( (m, e, \nu) \) in the rightmost column of Table 6 up to BJD = 2461132.4, the double transit in which the PPE is expected from the two-body analysis in Section 4. The results are the following:

1. The oscillation amplitudes of \( a_d \) and \( a_e \) are less than \( 5 \times 10^{-5} \) AU (\( \sim 0.03% \)) and \( 4 \times 10^{-4} \) AU (\( \sim 0.13% \)), respectively. Since these are much smaller than the observed uncertainties of \( a_d/R_{*} \) and \( a_e/R_{*} \) (\( \sim 1\% \)), the multi-body effect can be neglected for these two parameters.
2. Corresponding to the oscillations in semi-major axes above, \( n_d \) and \( n_e \) also show the modulations whose peak-to-peak amplitudes are \( \sim 2\pi \) (0.01 day)\(^{-1}\) and \( \sim 2\pi \) (0.1 day)\(^{-1}\), respectively. These are much larger than the uncertainties in \( n_d \) and \( n_e \) that come from those in \( P_d \) and \( P_e \), and so the multi-body effect is important for these parameters.
3. The differences of the periods calculated from the transit centers in the first \( \sim 1000 \) days (the range in which we

| Parameter | Best-fit Value |
|-----------|----------------|
| \( R_{pd}/R_{*} \) | 0.06954 ± 0.00043 |
| \( R_{pc}/R_{*} \) | 0.04123 ± 0.00070 |
| \( a_d/R_{*} \) | 26.21 ± 0.082 |
| \( a_e/R_{*} \) | 47.44 ± 0.027 |
| \( b_d \) | 0.2951 ± 0.0092 |
| \( b_e \) | 0.3693 ± 0.0095 |
| \( \Delta \theta \) (BJD = 2454833) | 378.51372 ± 0.00023 |
| \( \Delta \eta \) (BJD = 2454833) | 378.51785 ± 0.00060 |
| \( \Omega_{ed} \) (deg) | 1.13 ± 0.52 |
| \( \chi^2/\text{df} \) | 746/687 |
analyzed the TTVs of integration and from the whole orbit are at most comparable to the observed uncertainties of these parameters in Table 4. Thus the uncertainties of $t_e^{(0)}$ and $t_e^{(0)}$ can be evaluated using those of $P$ and $m_2$ in the table. Note that the effect of TTV is taken into account in obtaining these errors.

4. Monotonic increase in $i_4$ and decrease in $i_5$ lead to at most $\sim 30\%$ variations of $b_3$ and $b_4$, larger than the observed errors. The multi-body effect is dominant for these parameters.

5. $\Omega_d/\Omega_e$ also monotonically increases/decreases, but only by $<0.03$. This means that the uncertainty in the relative nodal angle $\Omega_{ed}$ is completely dominated by the error in Table 7, and the multi-body effect is negligible.

The above results indicate that the multi-body effect is the most significant for $b_3$ and $b_4$. In order to relate the values of these parameters to the occurrence of the PPE during the double transit around $BJD = 2461132.4$, we use Equation (18), (22), and (23) to calculate the maximum value of $d_{min}$ during this double transit in terms of $b_3$ and $b_4$, varying (1) $a_d$, $a_e$, $t_e^{(0)}$, $t_e^{(0)}$, and $\Omega_{ed}$ within $1\sigma$ intervals calculated from the photometric errors, and (2) $m_d$ and $m_e$ by the amplitudes of modulations estimated above. The region of $(b_3, b_4)$ plane in which $d_{min} < R_p + R_\star$, i.e., the PPE occurs in this double transit, is shown in Figure 14 with light-gray shade. When we vary the parameters in the set (1) within their $2\sigma$ and $3\sigma$ intervals, the edges of the shaded region can be as narrow as black solid and dashed lines. In fact, the gray-shaded region is mainly determined by the difference between $b_3$ and $b_4$, as seen from this figure, and the edge of this region is found to be most sensitive to the uncertainties in $t_e^{(0)}$ and $t_e^{(0)}$. The former fact originates from the well-aligned orbital planes of KOI-94d and KOI-94e: since their orbital planes are nearly parallel in the plane of the sky, the minimum separation during the double transit in which KOI-94d overtakes KOI-94e is nearly the same as the difference between $b_3$ and $b_4$. However, if their transit times are far away from each other, such closest encounter may occur out of the double transit. This explains the latter feature.

The PPE occurrence for different choices of $(m, e, \varpi)$ can be judged by evaluating the variation of $b_3$ and $b_4$ in this plot. Since the variations of $a$, $e$, $c$, and $\Omega_{ed}$ would be of the same order as long as the resulting TTVs are consistent with the observation, we fix the shaded area in Figure 14 determined by these parameters. We perform a similar MCMC calculation as in Section 2.3 to obtain the distribution of $(b_3, b_4)$ in the double transit at issue; we fit the observed TTVs again using the same $\chi^2$, but this time extend the orbit integration up to $BJD = 2461132.4$ and record the final values of $b_3$ and $b_4$ calculated via $b = a \cos i/R_\star$. The resulting distribution of $(b_3, b_4)$ is plotted with red points in Figure 14. We also repeat the same procedures fixing $m_d = 106 M_\odot$ and $m_d = 73 M_\odot$, and the distributions for these cases are plotted with blue points for comparison. In these calculations, we choose the initial values of $i_4$ and $i_5$ based on $(b_3, b_4) = (0.2951, 0.3693)$ in Table 7, a red point with error bars in Figure 14, rather than the mean values obtained from the phase-folded light curves in Table 3. This is because the mean parameters do not take account of the actual occurrence of the PPE. The black, dark-gray, and light-gray points around the double-transit value in Figure 14 respectively show its $1\sigma$, $2\sigma$, and $3\sigma$ confidence regions based on the posterior distribution of the double-transit fit. Here, the difference between $b_3$ and $b_4$ is rather sharply constrained by the minimum separation between the planets, namely, the height of the bump caused by the PPE. Even considering this uncertainty in the initial $(b_3, b_4)$, as well as the significant variation of impact parameters (or inclinations) due to the multi-body effect, $(b_3, b_4)$ around $BJD = 2461132.4$ are well inside the region where the PPE occurs, at least within $1\sigma$ of transit and TTV parameters for all the three solutions.

For the best-fit $(m, e, \varpi)$ obtained from the TTV alone, the expected height of the bump is much larger than in the last PPE (Figure 15), and so the detection of this PPE is highly feasible. In contrast, for $(m, e, \varpi)$ based on the RV values of $m_d$, the bump height is comparable to the last PPE. This difference may be used to settle the difference of $m_d$ values in RV and TTV analyses. For the RV-based solutions, the blue distributions in Figure 14 indicate that the PPE may not even happen when the variation of $b_3$ is too large (corresponding to the large $m_e$ values), and/or $\gtrsim 2\sigma$ deviations of $t_e^{(0)}$ and $t_e^{(0)}$ from the linear ephemerides make the shaded region too narrow for the PPE to occur (see solid and dashed lines).

We also check the other two double transits before the one discussed above (around $BJD = 2457982$ and $BJD = 24583612$), in case that the variations of $b_3$ and $b_4$ lead to the PPE which never happens without the interaction among the planets. In both of them, we find that the PPE does not happen for any possible values of $b_3$ and $b_4$ (from 0 to 1). Therefore, we can safely conclude that the next PPE will still occur during the same double transit as predicted by the two-body calculation, even when we include the mutual gravitational interaction among the planets.

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12 Here, the small uncertainties in $R_p$ and $R_\star$ are neglected.

13 Here we adopt $R_\star = (1.37 \pm 0.02) R_\odot$ obtained from photometric $a_d/R_\star, P_d$, and spectroscopic $M_\star$. This value is slightly smaller than $R_\star = (1.52 \pm 0.14) R_\odot$ obtained by Weiss et al. (2013) using the Spectroscopy Made Easy (Valentini & Piskunov 1996), but this difference is consistent with the conclusion of Torres et al. (2012) that the log g value based on the Spectroscopy Made Easy is systematically underestimated for stars with $T_\text{eff} \gtrsim 6000\, K$. 
6. SUMMARY AND DISCUSSION

We have performed an intensive TTV analysis in KOI-94, the first multi-planetary system exhibiting the PPE (Hirano et al. 2012). Comparison of the resulting system parameters with those estimated independently from the RV data (Weiss et al. 2013) works as a valuable test to examine the reliability with those estimated independently from the RV data (Weiss et al. 2012). Comparison of the resulting system parameters here a few possible interesting points.

Among the four planets reported so far, we considered the TTVs of KOI-94c, KOI-94d, and KOI-94e; we made sure that the contribution from the innermost and smallest planet KOI-94b is negligible at the current level of observational uncertainties.

We numerically integrated the orbits of the three planets that are directly incorporated in the MCMC search for the best-fit values of their masses, eccentricities, and longitudes of periastrons; our best-fit values include $m_c = 9.4^{+2.4}_{-2.1} M_\oplus$, $m_d = 52.1^{+6.9}_{-7.1} M_\oplus$, $m_e = 13.0^{+2.5}_{-2.1} M_\oplus$, and $e \lesssim 0.1$ for all the three planets. Those results are in reasonable agreement with the RV results (Weiss et al. 2013), but we would like to note here a few possible interesting points.

1. Although the RV analysis results in a fairly large eccentricity for KOI-94c ($e_c = 0.43 \pm 0.23$), the TTVs indicate a significantly smaller value. In fact, the stability analysis of the system favors the RV result.

2. The TTV best-fit value of $m_d$ differs from the RV result $m_d = 106 \pm 11 M_\oplus$ by $\sim 4\sigma$ level. If the TTV value is correct, KOI-94d may be inflated, in contrast to the conclusion obtained by Weiss et al. (2013).

3. The TTV of the outermost planet KOI-94e is not well reproduced in the current modeling with the three planets. This might suggest the presence of additional planets and/or minor bodies that have evaded the detection so far.

It is definitely premature to draw any decisive conclusions at this point. Nevertheless, the above possible discrepancies between the TTV and RV analyses point to the importance of future follow-up observations of the KOI-94 system.

In addition, we constructed an analytic model of the PPE. We derived a practical approximate formula that explicitly yields the difference between the longitudes of ascending nodes (mutual inclination in the plane of the sky) of the two planets in terms of the observed height, central time, and duration of the brightening caused by the PPE. We showed that the PPE light curve observed in the KOI-94 system indeed gives a unique solution for the mutual inclination. The effect of the non-zero eccentricities is taken into account in the formulation described in Appendix C, though it is safely neglected for the KOI-94 system. Combining the TTV best-fit parameters and our analytic PPE model, the next PPE in this system is predicted to occur in the double transit around BJD $= 2461132.4$ (date in UT 2026 April 1/2). The occurrence of the next PPE is robust against the 1σ uncertainties of the parameters. Since the predicted height of the bump is much larger than the last one, the detection of this PPE is highly feasible.

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APPENDIX A

OBSERVED TRANSIT TIMES OF KOI-94c, KOI-94d, AND KOI-94e

In Section 2, we fit each transit of KOI-94c, KOI-94d, and KOI-94e for the time of transit center, using the transit parameters in Table 3. The resulting transit times of the three planets, as well as their errors, $\chi^2$ values, and deviations from the linear ephemerides in Table 4 are shown in Tables 8–10 in this Appendix.

APPENDIX B

ANALYSIS OF THE TTV OF KOI-94c USING ANALYTIC FORMULAE

Lithwick et al. (2012) derived analytic formulae for the TTV signals from two coplanar planets near a $j:j - 1$ mean motion resonance. Here we present a brief outline of their formulation and report the analysis of the TTVs of KOI-94c and KOI-94d using these formulae.

We let unprimed and primed symbols stand for the quantities associated with inner and outer planets, respectively. Then $\delta t \equiv (\text{observed } t_c) - (\text{calculated from linear ephemeris})$ for the inner and outer planets are given by

$$\delta t = |V| \sin(\lambda^j + \arg V), \quad \delta t' = |V'| \sin(\lambda^j + \arg V').$$  \hspace{1cm} (B1)

where $\lambda^j$, $V$, and $V'$ are defined as follows.
The longitude of conjunction \( \lambda_j \) is defined as

\[
\lambda_j = j \lambda' - (j - 1) \lambda,
\]

where \( \lambda' = 2 \pi (t - T')/P' \) and \( \lambda = 2 \pi (t - T)/P \). If we measure angles with respect to the line of sight, \( T \) and \( T' \) are the times of any particular transits of the inner and outer planet, respectively. Here we choose \( T(T') \) to be \( t_0 (t_0') \) in Table 4. Defining the super-period \( P' \) and the normalized distance to resonance by

\[
P_j \equiv \frac{1}{|j/P' - (j - 1)/P|},
\]

and

\[
\Delta \equiv \frac{P'j - j}{P} - 1,
\]

\( \lambda_j \) can be written as

\[
\lambda_j = -2 \pi \left( \frac{P - j}{P'} \right) t + 2 \pi \left( \frac{j - 1 - \pi}{P} - \frac{P'j}{P'} \right)
\]

\[
= - \Delta \frac{2 \pi}{|\Delta|} \left( t - \left( 1 + \Delta T - T' \right) \right).
\]

Thus, if \( \Delta > 0 \), \( \lambda_j \) is retrograde with respect to the orbital motion and is prograde for \( \Delta < 0 \).

The complex TTV amplitudes \( V \) and \( V' \) are given by

\[
V = P \frac{\mu'}{\pi j^{2/3}(j - 1)^{1/3} |\Delta|} \left( -f - \frac{3 Z_{\text{free}}}{2 |\Delta|} \right)
\]

and

\[
V' = P' \frac{\mu}{\pi j |\Delta|} \left( -g + \frac{3 Z_{\text{free}}^*}{2 |\Delta|} \right),
\]

where \( f \) and \( g \) are the sums of the Laplace coefficients given by

\[
f = -1.190 + 2.20 \Delta - 1.032 < 0,
\]

\[
g = 0.4284 - 3.69 \Delta = 0.1637 > 0
\]
tricity
V

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KOI-94c based on Equation (B1) by Lithwick et al. (2012). Points with error bars are the observed TTVs of KOI-94c calculated with \( t_0 \) and \( P \) obtained from the fit including TTVs (see Equation (B14)). Vertical arrows show the times at which \( \lambda_j = 0 \), i.e., the longitude of conjunction points to the observer. The observed phase of the TTV is slightly shifted from these points, suggesting small but nonzero eccentricities.

(A color version of this figure is available in the online journal.)

For \( j = 2 \), \( \Delta = 0.07174 \), and \( \mu \) (\( \mu' \)) is the mass ratio of the inner (outer) planet to that of the star. They also introduce \( Z_{\text{free}} \) as a linear combination of the free complex eccentricities of the two planets

\[
Z_{\text{free}} = f z_{\text{free}} + g z'_{\text{free}}, \quad (B9)
\]

where \( z_{\text{free}} \) is defined as the “free” part of the complex eccentricity

\[
z = e \exp(i \sigma), \quad (B10)
\]

and obtained by subtracting \( z_{\text{forced}} \), the forced eccentricity due to the planet’s proximity to resonance, from \( z \). The forced eccentricities for the inner and outer planets are

\[
\begin{align*}
\frac{z_{\text{forced}}}{\Delta} &= -\frac{1}{\sqrt{\mu}} \left( \frac{\mu'}{\mu} \right)^{1/3} \sin(\sigma) e^{i \sigma}. \quad (B11)
\end{align*}
\]

Since \( \Delta \gtrsim 0.01 \) and \( \mu \lesssim 10^{-4} \) typically, \( |z_{\text{forced}}| \lesssim 10^{-2} \), in which case

\[
Z_{\text{free}} \simeq f e^{i \sigma} + g e^{i \sigma'} = (f e \cos \sigma + g e' \cos \sigma') + i (f e \sin \sigma + g e' \sin \sigma'). \quad (B12)
\]

(Xie 2013). Note that in either the limit that \( |Z_{\text{free}}| \ll |\Delta| \) or \( |Z_{\text{free}}| \gg |\Delta| \), phases of the two planets’ TTVs are anticorrelated, as can be seen from the expressions for \( V \) and \( V' \). In this case, TTV signals of the two planets provide only three independent quantities, making it impossible to uniquely determine \( |V|, |V'|, \text{Re}(Z_{\text{free}}) \), and \( \text{Im}(Z_{\text{free}}) \).

Above expressions for \( V \) and \( V' \) imply that the phases as well as the amplitudes of the two TTV signals contain important information about their eccentricities. For ease of discussion, they define

\[
\phi_{ttv} \equiv \arg \left( V \times \frac{\Delta}{|\Delta|} \right), \quad \phi'_{ttv} \equiv \arg \left( V' \times \frac{\Delta}{|\Delta|} \right). \quad (B13)
\]

With these definitions, \( Z_{\text{free}} = 0 \) leads to \( \phi_{ttv} = 0^\circ \) and \( \phi'_{ttv} = 180^\circ \), independently of the sign of \( \Delta \). In this case, since \( \lambda_j \) (decreases (increases) with time for \( \Delta > 0 (\Delta < 0) \), \( \delta \) crosses zero from above (below) whenever \( \lambda_j = 0 \). If the observed TTVs have a phase shift with respect to \( \phi_{ttv} = 0^\circ \) and \( \phi'_{ttv} = 180^\circ \), this implies that non-zero \( Z_{\text{free}} \) exists. On the other hand, no phase shift does not necessarily mean \( Z_{\text{free}} = 0 \), for the phase of \( Z_{\text{free}} \) may vanish by chance. Although it is impossible to judge whether \( Z_{\text{free}} \) is really zero or not in a single resonant pair with no phase shift, important conclusions can be obtained by statistical analyses (Wu & Lithwick 2013).

Based on the formulation above, the transit times \( t_{\text{trans}} \) for the inner planet are written as

\[
t_{\text{trans}} = t_0 + P_{\text{trans}} + \text{Re}(V) \sin \lambda_j + \text{Im}(V) \cos \lambda_j, \quad (B14)
\]

where \( t_{\text{trans}} = 0, 1, \ldots \) is the transit number. For each observed \( t_{\text{trans}} \), we calculate \( \lambda_j \) using \( P \) and \( t_0 \) obtained by a linear fit (Table 4), and fit for the four parameters \( t_0, P, \text{Re}(V), \) and \( \text{Im}(V) \) by a least-square fit. We also repeat the same procedure for the outer planet, and obtain the results in Table 11. The best-fit theoretical curve in Figure 16 shows that the TTV of KOI-94c is well explained only by the effect from KOI-94d, having the same period as expected from their proximity to 2:1 resonance. In contrast, the TTV of KOI-94d is poorly explained by the contribution from KOI-94c alone (Figure 17). These results are consistent with our estimates in Table 5.

TTV amplitudes listed in Table 11 give estimates for the masses of KOI-94d and KOI-94c. If we assume \( Z_{\text{free}} = 0 \), i.e., that both of the planets have zero eccentricities, Equation (B6) translates the amplitude of KOI-94c’s TTV \( |V| \) into the nominal mass \( m_d = 63 M_{\oplus} \). However, the accuracy of this estimate is rather limited, because the slight phase shift in KOI-94c’s TTV suggests that KOI-94d and/or KOI-94c have small but nonzero eccentricities. If this nominal mass is actually close to...
the true one, the density of KOI-94d is \( \sim 0.3 \, \text{g/cm}^3 \), which is comparable to that of the lowest-density exoplanet ever discovered.

**APPENDIX C**

\( O(e) \) FORMULATION OF THE PPE

In Section 3, we modeled the PPE caused by two planets on circular orbits. Here we summarize how the \( O(e) \) correction modifies those results.

In the presence of a nonzero eccentricity, the sky-plane distance between the star and planet \( r_{\text{sky}} \) is given by

\[
r_{\text{sky}} = \frac{a(1-e^2)}{1+e \cos f} \sqrt{1 - \sin^2(\omega + f) \sin^2 i}. \tag{C1}
\]

Defining \( \Delta \equiv \pi/2 - (\omega + f) \), \( \Delta \) that minimizes this \( r_{\text{sky}} \) satisfies

\[
\Delta = \frac{1}{2} \arcsin \left[ 2e \cos(\omega + \Delta) \left( \frac{1}{\sin i} - \cos^2 \Delta \right) \right. \\
\left. - e \sin(\omega + \Delta) \sin 2\Delta \right], \tag{C2}
\]

which can be solved to the leading orders of \( e \) and \( \pi/2 - i \) to give

\[
\Delta \approx e \cos \omega \left( \frac{\pi}{2} - i \right)^2 \sim e \cos \pi \left( \frac{a}{R_*} \right)^2. \tag{C3}
\]

Thus, the impact parameter \( b \) can be approximated as

\[
b \simeq \frac{a \cos i}{R_*} \cdot \frac{1 - e^2}{1 + e \sin \omega} \simeq \frac{a \cos i}{R_*} (1 - e \sin \omega) \tag{C4}
\]

(Winn 2010). This alters the expression (12) as

\[
r_j \simeq (1 - e_j \cos f_j) \times \\
\left( \frac{(a_j/R_*) \cos \Omega_j \cos(\omega_j + f_j)}{-b_j(1 + e_j \sin \omega_j) \sin \Omega_j \sin(\omega_j + f_j)} \right. \\
\left. + a_j/R_*) \sin \Omega_j \cos(\omega_j + f_j) + b_j(1 + e_j \sin \omega_j) \cos \Omega_j \sin(\omega_j + f_j) \right). \tag{C5}
\]

In addition, the expansion of \( \omega + f \) around \( t_c \) is modified as

\[
\omega + f \simeq \frac{\pi}{2} + n(1 + 2e \sin \omega)(t - t_c). \tag{C6}
\]

Using these expressions, \( r_j \) (\( j = 1, 2 \)) can be expanded as

\[
r_j = v_j \left[ (1 + e_j \sin \omega_j)(t - t_c^{(j)}) \right] + r_0^{(j)}, \tag{C7}
\]

where \( v_j \) and \( r_0^{(j)} \) are the same as defined in Equation (16).

Accordingly, the expression for \( d \) with \( O(e) \) terms included (but \( O((a/R_*)^{-2}) \) terms neglected) is obtained by replacing \( b_j \) and \( n_i \) in the circular case with \( b_j(1 - e_i \sin \omega_i) \) and \( n_i(1 + e_i \sin \omega_i) \), respectively.

*Note added in proof.* The Kepler number 89 was assigned to KOI-94, with the same planet letters as assigned in this paper.

**REFERENCES**

Agol, E., Steffen, J., Sari, R., & Clarkson, W. 2005, *MNRA*, 359, 567

Albrecht, S., Winn, J. N., Marcy, G. W., et al. 2013, *ApJ*, 771, 11

Batalha, N. M., Rowe, J. F., Bryson, S. T., et al. 2013, *ApJS*, 204, 24

Bate, M. R., Lodato, G., & Pringle, J. E. 2010, *MNRA*, 401, 1505

Batygin, K. 2012, *Natur*, 491, 418

Bonucki, W. J., Koch, D. G., Basri, G., et al. 2011, *ApJ*, 736, 19

Fabrycky, D. C. 2010, arXiv:1006.3834

Fabrycky, D. C., Ford, E. B., Steffen, J. H., et al. 2012, *ApJ*, 750, 114

Fabrycky, D. C., & Winn, J. N. 2009, *ApJ*, 696, 1230

Goldreich, P., & Tremaine, S. 1980, *ApJ*, 241, 425

Hirano, T., Narita, N., Sato, B., et al. 2012, *ApJL*, 759, L36

Hirano, T., Sato, Y., Winn, J. N., et al. 2011, *ApJ*, 742, 69

Holman, M. J., & Murray, N. W. 2005, *Sci*, 307, 1288

Kinemuchi, K., Barclay, T., Fanelli, M., et al. 2012, *PASP*, 124, 963

Kokubo, E., & Makino, J. 2004, *PASJ*, 56, 861

Lal, D., Foucart, F., & Lin, D. N. C. 2011, *MNRA*, 412, 2790

Lissauer, J. J., Fabrycky, D. C., Ford, E. B., et al. 2011, *Natur*, 470, 53

Lithwick, Y., Xie, J., & Wu, Y. 2012, *ApJL*, 767, 122

Lubow, S. H., & Ida, S. 2011, in Planet Migration, ed. S. Seager (Tucson, AZ: Univ. Arizona Press), 347

Mandel, K., & Agol, E. 2002, *ApJL*, 580, L171

Markwardt, C. B. 2009, in ASP Conf. Ser. 411, Astronomical Data Analysis Software and Systems XVIII, ed. D. A. Bohliender, D. Durand, & P. Dowler (San Francisco, CA: ASP), 251

McLaughlin, D. B. 1924, *ApJL*, 60, 22

Nagasawa, M., Ida, S., & Bessho, T. 2008, *ApJ*, 678, 498

Naoz, S., Farr, W. M., Lithwick, Y., Rasio, F. A., & Teyssandier, J. 2011, *Natur*, 473, 187

Nesvorný, D., Kipping, D. M., Buchhave, L. A., et al. 2012, *Sci*, 336, 1133

Ohta, Y., Taruya, A., & Suto, Y. 2005, *ApJ*, 622, 1118

Ohta, Y., Taruya, A., & Suto, Y. 2009, *ApJ*, 690, 1

Queloz, D., Eggenberger, A., Mayor, M., et al. 2000, *A&A*, 359, L13

Rabus, M., Alonso, R., Belmonte, J. A., et al. 2009, *A&A*, 494, 391

Ragozzine, D., & Holman, M. J. 2010, arXiv:1006.3727

Rogers, T. M., Lin, D. N. C., & Lau, H. H. B. 2012, *ApJL*, 758, L6

Rossiter, R. A. 1924, *ApJL*, 60, 15

Seager, S., & Mallén-Ornelas, G. 2003, *ApJL*, 585, 1038

Torres, G., Fischer, D. A., Sozzetti, A., et al. 2012, *ApL*, 757, 161

Valenti, J. A., & Piskunov, N. 1996, *A&A*, 118, 595

Weiss, L. M., Marcy, G. W., Rowe, J. F., et al. 2013, *ApJL*, 768, 14

Winn, J. N. 2010, arXiv:1001.2010

Winn, J. N., Noyes, R. W., Holman, M. J., et al. 2005, *ApJL*, 631, 1215

Wu, Y., & Lithwick, Y. 2011, *ApJL*, 735, 109

Wu, Y., & Lithwick, Y. 2013, *ApJ*, 772, 74

Wu, Y., & Murray, N. 2003, *ApJL*, 589, 605

Xie, J.-W. 2013, *ApJL*, 708, 22

Xie, J.-W., Wu, Y., & Lithwick, Y. 2013, arXiv:1308.3751
ERRATUM: “CHARACTERIZATION OF THE KOI-94 SYSTEM WITH TRANSIT TIMING VARIATION ANALYSIS: IMPLICATION FOR THE PLANET–PLANET ECLIPSE” (2013, ApJ, 778, 185)

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In Table 3 of the published version of this article, the uncertainties in $a/R_*$ and $b$ of KOI-94c and KOI-94e were unreasonably small due to a coding error. The correct best-fit parameters and uncertainties are given in Table 3 below. Since we used the best-fit transit parameters in this table to determine individual transit times, the transit times of KOI-94c and KOI-94e, and that of the 11th transit of KOI-94d (fitted simultaneously with the 4th transit of KOI-94e) have also changed. The revised transit times as well as the new ephemerides derived from them are given in Tables 8–10 and Table 4, respectively. The shifts in the transit times are only $\sim 0.1\sigma$ for KOI-94c and $\sim 0.006\sigma$ for KOI-94e on average, and $< 1\sigma$ for the 11th transit of KOI-94d. Therefore, the conclusion of the following transit timing variation (TTV) analysis does not change significantly.

Because of the same error in $a/R_*$ and $b$, we revise Table 7 as well. Corresponding to the different constraints on $b_d$ and $b_c$, Figures 14 and 15 are also modified (the same solutions as in Table 6 of the published version are used in these figures). Even with the enlarged error bars for $b_d$ and $b_c$ during the last planet–planet eclipse (PPE), however, Figure 14 still allows us to conclude that the occurrence of the next PPE is robust against $1\sigma$ uncertainties of the transit and TTV parameters. In addition, the signal of the next PPE (Figure 15) is still larger than the previous one and changes sensitively with the value of $m_d$. Thus, these corrections do not alter the conclusion of the paper, either.

### Table 3
Revised Transit Parameters Obtained from Phase-folded Light Curves

| Parameter | KOI-94c | KOI-94d | KOI-94e |
|-----------|---------|---------|---------|
| $a/R_*$   | 15.64^{+0.070}_{-0.151} | 26.24^{+0.20}_{-0.19} | 47.69^{+0.67}_{-0.64} |
| $R_p/R_*$ | 0.025618^{+0.000093}_{-0.000097} | 0.0702^{+0.00014}_{-0.00015} | 0.04132^{+0.00016}_{-0.00018} |
| $b$       | 0.089^{+0.079}_{-0.064} | 0.299^{+0.022}_{-0.025} | 0.371^{+0.031}_{-0.036} |
| $u_1$     | 0.35 ± 0.06 | 0.40 ± 0.02 | 0.36 ± 0.07 |
| $u_2$     | 0.21 ± 0.11 | 0.14 ± 0.03 | 0.19 ± 0.10 |
| $\chi^2$/dof | 25843/23611 | 15473/13766 | 6827/5934 |

**Notes.** The quoted error bars denote $1\sigma$ confidence intervals obtained from the posterior distributions. The values of $a/R_*$ and $b$ for KOI-94c and KOI-94e are determined with the prior information about the stellar density based on the result for KOI-94d (see the main text).

### Table 4
Linear Ephemerides of KOI-94c, KOI-94d, and KOI-94e

| Parameter | KOI-94c | KOI-94d | KOI-94e |
|-----------|---------|---------|---------|
| $t_0$ (BJD − 2454833) | 138.00826 ± 0.00039 | 132.74101 ± 0.00012 | 161.23915 ± 0.00046 |
| $P$ (days) | 10.4236892 ± 0.0000054 | 22.342704 ± 0.0000036 | 54.319832 ± 0.000035 |
| $\chi^2$/dof | 9.9 | 13.6 | 28.4 |
### Table 7

Best-fit Parameters for the PPE Light Curve

| Parameter          | Best-fit Value |
|--------------------|----------------|
| $R_{pl}/R_*$       | 0.06942 ± 0.00046 |
| $R_{pe}/R_*$       | 0.04118 ± 0.00069 |
| $a_{d}/R_*$        | 26.39 ± 0.29     |
| $a_{e}/R_*$        | 47.87 ± 0.68     |
| $b_0$              | 0.273 ± 0.036    |
| $b_2$              | 0.347 ± 0.041    |
| $\Omega_{d}$ (deg) | 1.19 ± 0.55      |
| $\chi^2$/dof       | 746/687          |

### Table 8

Transit Times of KOI-94c

| Transit Number | $t_0$ (BJD − 2454833) | $1\sigma_{\text{lower}}$ | $1\sigma_{\text{upper}}$ | $\chi^2$/dof | $O - C$ (days) |
|----------------|------------------------|---------------------------|---------------------------|--------------|----------------|
| 21             | 356.90813              | 0.00102                   | 0.00092                   | 1.10         | 0.00240        |
| 22             | 367.33153              | 0.00114                   | 0.00142                   | 1.17         | 0.00211        |
| 23             | 377.75172              | 0.00122                   | 0.00130                   | 0.97         | −0.00139       |
| 24             | 388.17505              | 0.00098                   | 0.00099                   | 1.15         | −0.00175       |
| 26             | 409.02133              | 0.00237                   | 0.00199                   | 1.08         | −0.00285       |
| 27             | 419.44216              | 0.00092                   | 0.00084                   | 1.02         | −0.00571       |
| 28             | 420.86634              | 0.00237                   | 0.00170                   | 1.02         | −0.00521       |
| 29             | 440.29075              | 0.00081                   | 0.00084                   | 1.24         | −0.00449       |
| 30             | 450.71586              | 0.00120                   | 0.00125                   | 0.99         | −0.00308       |
| 31             | 461.13918              | 0.00240                   | 0.00167                   | 1.10         | −0.00345       |
| 32             | 471.56721              | 0.00130                   | 0.00111                   | 1.16         | 0.00089        |
| 33             | 481.99312              | 0.00077                   | 0.00078                   | 1.05         | 0.00312        |
| 34             | 492.41694              | 0.00134                   | 0.00156                   | 1.18         | 0.00325        |
| 35             | 502.84210              | 0.00106                   | 0.00101                   | 1.13         | 0.00472        |
| 36             | 513.26446              | 0.00128                   | 0.00156                   | 1.14         | 0.00339        |
| 37             | 523.68723              | 0.00104                   | 0.00093                   | 1.05         | 0.00247        |
| 38             | 534.10888              | 0.00105                   | 0.00097                   | 1.22         | 0.00043        |
| 39             | 742.57681              | 0.00100                   | 0.00100                   | 1.11         | −0.00542       |
| 40             | 773.85397              | 0.00140                   | 0.00107                   | 1.17         | 0.000006       |
| 41             | 784.27823              | 0.00156                   | 0.00118                   | 1.17         | 0.00124        |
| 42             | 794.70493              | 0.00079                   | 0.00088                   | 1.13         | 0.00425        |
| 43             | 815.55090              | 0.00126                   | 0.00124                   | 1.10         | 0.00284        |
| 44             | 825.97554              | 0.00070                   | 0.00072                   | 0.93         | 0.00380        |
| 45             | 836.39892              | 0.00104                   | 0.00120                   | 1.01         | 0.00348        |
| 46             | 846.81857              | 0.00104                   | 0.00121                   | 1.13         | −0.00056       |
| 49             | 857.24116              | 0.00098                   | 0.00090                   | 1.08         | −0.00166       |
| 71             | 878.08526              | 0.00094                   | 0.00099                   | 1.09         | −0.00493       |
| 72             | 888.51112              | 0.00134                   | 0.00212                   | 1.08         | −0.00277       |
| 73             | 898.93561              | 0.00094                   | 0.00086                   | 1.16         | −0.00197       |
| 93             | 1107.41512             | 0.00093                   | 0.00106                   | 1.10         | 0.00376        |
| 95             | 1128.26374             | 0.00135                   | 0.00208                   | 0.98         | 0.00050        |
| 96             | 1138.68565             | 0.00106                   | 0.00090                   | 1.36         | 0.00323        |
| 97             | 1149.10767             | 0.00131                   | 0.00115                   | 0.86         | 0.00155        |
| 98             | 1159.52817             | 0.00142                   | 0.00123                   | 0.93         | −0.00163       |
| 99             | 1169.95299             | 0.00092                   | 0.00104                   | 1.03         | −0.00051       |
| 100            | 1180.37320             | 0.00138                   | 0.00149                   | 1.10         | −0.00398       |
| 101            | 1190.79831             | 0.00125                   | 0.00118                   | 1.00         | −0.00256       |
| 102            | 1201.21883             | 0.00103                   | 0.00123                   | 1.14         | −0.00573       |
| 103            | 1211.64371             | 0.00121                   | 0.00119                   | 1.19         | −0.00454       |
| 104            | 1222.06920             | 0.00076                   | 0.00075                   | 1.06         | −0.00274       |
| 105            | 1232.49521             | 0.00162                   | 0.00154                   | 1.17         | −0.00042       |
| 106            | 1242.92078             | 0.00122                   | 0.00114                   | 1.29         | 0.00146        |
| 107            | 1253.34406             | 0.00087                   | 0.00086                   | 0.95         | 0.00106        |
| 108            | 1263.77103             | 0.00123                   | 0.00093                   | 1.18         | 0.00433        |
Figure 14. Relation between the occurrence of the PPE by KOI-94d and KOI-94e and the values of their impact parameters in the double transit around BJD = 246132.4. If \((b_d, b_e)\) in this double transit is inside the gray shaded region, PPE occurs for all values of photometrically derived parameters within 1\(\sigma\) of their best-fit values. When we vary them within their 2\(\sigma\) and 3\(\sigma\) intervals, the corresponding boundaries become black solid and dashed lines, respectively. The values of \((b_d, b_e)\) at the last PPE are shown by the red point with error bars, and the black, dark gray, and light gray points around it show, respectively, their 1\(\sigma\), 2\(\sigma\), and 3\(\sigma\) confidence regions. Red and blue points are the distributions of \((b_d, b_e)\) in the double transit around BJD = 2461132.4 for the three different solutions in Table 6. The scattering of each set of points reflects the uncertainties in \((m, e, \varpi)\).

(A color version of this figure is available in the online journal.)

Table 9
Transit Times of KOI-94d

| Transit Number | \(t_c\) (BJD – 2454833) | \(1\sigma_{\text{lower}}\) | \(1\sigma_{\text{upper}}\) | \(\chi^2/\text{dof}\) | \(O – C\) (days) |
|----------------|-------------------------|----------------|----------------|-----------------|-----------------|
| 10             | 356.17032               | 0.00023        | 0.00023        | 1.10            | −0.00040        |
| 11             | 378.51379\(^a\)         | 0.00022        | 0.00022        | 1.12            | 0.00011         |
| 13             | 423.19948               | 0.00023        | 0.00023        | 1.17            | −0.00015        |
| 14             | 445.54263               | 0.00022        | 0.00023        | 1.14            | 0.00003         |
| 15             | 467.88668               | 0.00021        | 0.00022        | 1.06            | 0.00111         |
| 16             | 490.22897               | 0.00022        | 0.00022        | 1.07            | 0.00043         |
| 17             | 512.70501               | 0.00021        | 0.00021        | 1.17            | −0.00099        |
| 18             | 534.91389               | 0.00022        | 0.00022        | 1.08            | −0.00058        |
| 19             | 736.00250               | 0.00023        | 0.00023        | 1.00            | 0.00129         |
| 20             | 758.34519               | 0.00023        | 0.00023        | 1.10            | 0.00101         |
| 21             | 780.68635               | 0.00023        | 0.00023        | 1.13            | −0.00080        |
| 22             | 825.37220               | 0.00022        | 0.00022        | 1.06            | −0.00089        |
| 23             | 847.71579               | 0.00022        | 0.00022        | 1.14            | −0.00027        |
| 24             | 870.05926               | 0.00026        | 0.00026        | 1.03            | 0.00022         |
| 25             | 892.40212               | 0.00022        | 0.00022        | 1.11            | 0.00012         |
| 26             | 1115.83180              | 0.00024        | 0.00024        | 2.02            | 0.00009         |
| 27             | 1138.17339              | 0.00023        | 0.00023        | 1.10            | −0.00129        |
| 28             | 1205.20344              | 0.00023        | 0.00023        | 1.11            | −0.00015        |
| 29             | 1227.54827              | 0.00022        | 0.00023        | 1.01            | 0.00171         |
| 30             | 1249.89001              | 0.00022        | 0.00022        | 1.08            | 0.00048         |
| 31             | 1272.23152              | 0.00022        | 0.00023        | 0.94            | −0.00099        |

Note. \(^a\) Double transit with KOI-94e: obtained simultaneously with the relative nodal angle and \(t_c\) for KOI-94e.

Table 10
Transit Times of KOI-94e

| Transit Number | \(t_c\) (BJD – 2454833) | \(1\sigma_{\text{lower}}\) | \(1\sigma_{\text{upper}}\) | \(\chi^2/\text{dof}\) | \(O – C\) (days) |
|----------------|-------------------------|----------------|----------------|-----------------|-----------------|
| 4              | 378.51729\(^a\)         | 0.00057        | 0.00056        | 1.12            | −0.00119        |
| 5              | 432.83743               | 0.00063        | 0.00063        | 1.22            | −0.00088        |
| 6              | 487.15505               | 0.00061        | 0.00062        | 1.11            | −0.00309        |
| 11             | 758.76221               | 0.00060        | 0.00061        | 0.98            | 0.00491         |
| 12             | 813.08159               | 0.00061        | 0.00061        | 1.10            | 0.00446         |
| 13             | 1138.99255              | 0.00061        | 0.00062        | 1.21            | −0.00087        |
| 14             | 1193.31416              | 0.00062        | 0.00063        | 1.19            | −0.00179        |
| 15             | 1247.63414              | 0.00061        | 0.00062        | 1.24            | −0.00165        |

Note. \(^a\) Double transit with KOI-94d: obtained simultaneously with the relative nodal angle and \(t_c\) for KOI-94d.
Figure 15. Expected light curves of the PPE in the double transit around BJD = 2461132.4 (date in UT 2026 April 1/2). The black dotted curve (no interaction) shows the result for the parameters in Table 7. The other curves use the median values of $(b_d, b_e)$ for the three distributions in Figure 14, with other parameters fixed at the same values as above.

(A color version of this figure is available in the online journal.)