ANGULAR DISTRIBUTIONS IN $J/\psi(\rho^0, \omega)$ STATES NEAR THRESHOLD

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A resonance $X(3872)$, first observed in the decays $B \to KX$, has been seen to decay to $J/\psi \pi^+ \pi^-$, prompting speculation that it might be a charmonium level. It is very narrow, and is not seen decaying to $D\bar{D}$ although well above threshold. To account for this, it would have to have either a very high spin $J \geq 3$ (thereby suppressing the $D\bar{D}$ by a sufficiently strong centrifugal barrier), or to belong to the unnatural spin-parity series $J^P = 0^-, 1^+, 2^-,$...

None of the $c\bar{c}$ assignments for $X(3872)$ is entirely satisfactory. An interesting alternative is that $X(3872)$ could be a “molecular charmonium” state, since it lies very close to the sum of the $D^0$ and $D^{*0}$ masses. The peaking of the $\pi^+ \pi^-$ mass spectrum near the upper kinematic limit prompted the suggestion (see, e.g.,) that the $\pi^+ \pi^-$ is in a $\rho^0$. A bound state of $D^0 \bar{D}^{*0}$ ± c.c. could be an admixture of $I = 0$ and $I = 1$ final states and thus capable of decaying to both $J/\psi \rho^0$ and to $J/\psi \omega$. The calculation of Ref. favors $J^P = 0^{-}$ and $1^{+}$, while that of Ref. favors $1^{+}$. If the state is decaying to $J/\psi \rho^0$, its charge-conjugation eigenvalue must be $C = +$. The Belle Collaboration recently reported observation of the decay $X(3872) \to J/\psi \omega$ at the expected rate for a $(D^0 \bar{D}^{*0} + \text{c.c.})$ S-wave bound state. The $J^P = 1^{+}$ assignment is highly favored since the decays to $J/\psi \rho^0$ and are seen to occur near their kinematic boundaries. Nonetheless it is of interest to confirm this assignment by the study of angular distributions in the final state.

A search for $X(3872)$ was undertaken in CLEO-III data for states produced in $\gamma \gamma$ fusion (with $C = +$ and $J \neq 1$) and those produced in the radiative return reaction.
Table I: $J^{PC}$ values for states of $J/\psi \rho^0$ with definite parity near threshold.

| $S_{J/\psi,\rho}$ | $L_{J/\psi,\rho} = 0$ | $L_{J/\psi,\rho} = 1$ |
|-------------------|---------------------|---------------------|
| 0                 | 0$^+$               | 1$^-$               |
| 1                 | 1$^{++}$            | 0$^+$, 1$^{-+}$, 2$^{--}$ |
| 2                 | 2$^{++}$            | 1$^{--}$, 2$^{--}$, 3$^{--}$ |

$e^+e^- \rightarrow X(3872) + \gamma$ (with $J^{PC} = 1^{--}$ coupling to a virtual photon). While the results of this search (and one by the BES Collaboration for states produced in radiative return [17]) were negative, their interpretation depends on the angular distribution in the final $J/\psi \pi^+\pi^-$ products. Under some circumstances the final states produced in $\gamma\gamma$ reactions could consist of $J/\psi \rho^0$. Many reactions in which pairs of photons produce pairs of vector mesons have a peak near threshold, so whether or not $\gamma\gamma \rightarrow J/\psi \rho^0$ is associated with $X(3872)$, the behavior of its partial waves near threshold is of interest [18].

The present paper gives angular distributions for states decaying to $J/\psi(\rightarrow \ell^+\ell^-)\rho^0(\rightarrow \pi^+\pi^-)$ near threshold, using the transversity analysis developed for decays of $B$ mesons to pairs of vector mesons [19, 20] and other appropriate methods. These results can help to distinguish among some hypotheses for the nature of the $X(3872)$ and to interpret $\gamma\gamma \rightarrow J/\psi \pi^+\pi^-$ reactions near threshold. Results may be applied to decays $J/\psi(\rightarrow \ell^+\ell^-)\omega(\rightarrow \pi^+\pi^-\pi^0)$ by replacing a unit vector in the $\pi^+$ direction in the $\rho$ center-of-mass system (c.m.s.) by the normal to the plane defined by $\pi^+\pi^-\pi^0$ in the $\omega$ rest frame.

The quantum number assignments to be considered are enumerated in Sec. II, while the transversity and other formalisms are described in Sec. III. Spinless cases are treated in Sec. IV, while decays of $J = 1$ and $J = 2$ positive-parity states are discussed in Secs. V and VI. Section VII concludes. Appendices are devoted to angular distributions in vector meson decays to $\pi^+\pi^-$, $\pi^+\pi^-\pi^0$, and $\ell^+\ell^-$ and to an elementary derivation of some angular correlations using Clebsch-Gordan coefficients.

II. ENUMERATION OF $J^{PC}$ ASSIGNMENTS

The states of $J/\psi$ ($J^{PC} = 1^{--}$) and $\rho^0$ ($J^{PC} = 1^{--}$) near threshold may be divided into those of positive and negative parity. If the lowest possible relative orbital angular momentum $L_{J/\psi,\rho}$ dominates, one must consider the states in Table I. Here $S_{J/\psi,\rho}$ denotes the spin of the coupled $J/\psi$ and $\rho$ before consideration of their relative orbital angular momentum.

For present purposes it is enough to consider a subset of the states in Table I. An S-wave bound state of $(D^0\bar{D}^{*0} + \text{c.c.})$ must have $J^{PC} = 1^{++}$, while P-wave bound states can have $J^{PC} = 0^{-+}$, 1$^{++}$, 2$^{-+}$. Of these only the first was proposed in Ref. [10] as possibly bound. The states accessible in $\gamma\gamma$ collisions include those with $J^{PC} = 0^{\pm+}$, 2$^{\pm+}$, and 3$^{-+}$; Yang’s Theorem [21] forbids formation of $J = 1$ states. In view of the complexity of the 2$^{-+}$ and 3$^{-+}$ states, they will be ignored. Henceforth the discussion will concentrate on the possibilities 0$^{\pm+}$, 1$^{++}$, and 2$^{++}$. 

2
III. GENERAL FORMALISM

A transversity analysis is helpful in analyzing decays to $J/\psi$ and another vector meson $V$ when the $J/\psi$ decays to a lepton pair $\ell^+\ell^-$ and the vector meson decays to a pair of pseudoscalars $P^+P^-$ [19, 20]. In the $P^+P^-$ rest frame, the $x$ axis is defined as the negative of the unit vector pointing in the direction of the $J/\psi$. The $P^+P^-$ system is assumed to lie in the $x$-$y$ plane, with $P^+$ making an angle $\psi$ with the $x$ axis ($0 \leq \psi \leq \pi$).

The $z$ axis is taken in the $J/\psi$ rest frame perpendicular to the plane containing the $P^+P^-$ pair, using a right-handed coordinate system. In this frame the unit vector $\hat{n}(\ell^+)$ along the direction of the positive lepton has coordinates $(n_x, n_y, n_z) = (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta)$, thus defining $\theta$ and $\varphi$. Results in the transversity basis will be presented for states with $J \leq 1$. For $J = 2$ positive-parity states some simpler variables will be used.

In analyzing decays involving positive-parity $J/\psi\rho^0$ states near threshold, for which $L_{J/\psi,\rho^0} = 0$ may be assumed, the rest frames of the $J/\psi$ and $\rho^0$ may be assumed to coincide. One may then consider the angle $\theta_\ell\pi$ between the $\ell^+$ in $J/\psi \rightarrow \ell^+\ell^-$ and the $\pi^+$ in $\rho^0 \rightarrow \pi^+\pi^-$. For such states produced in $\gamma\gamma$ collisions, angles $\theta_\ell$ and $\theta_{\pi^+}$ of $\ell^+$ and $\pi^+$ with respect to the photon axis (in the $\gamma\gamma$ center of mass) also are useful.

IV. STATES WITH $J^{PC} = 0^{++}$

The decays of spinless states to two vector mesons were discussed in Ref. [20]. In terms of amplitudes $A_0$, $A_\parallel$, and $A_\perp$ describing longitudinal, parallel transverse, and perpendicular transverse polarizations of the vector mesons, the differential distribution may be written

$$\frac{d^3\Gamma[X \rightarrow (\ell^+\ell^-)_{J/\psi}(\pi^+\pi^-)_{\rho^0}]}{d\cos \theta d\varphi d\cos \psi} = \frac{9}{32\pi} [2|A_0|^2 \cos^2 \psi (1 - \sin^2 \theta \cos^2 \varphi) + \sin^2 \psi (|A_\parallel|^2(1 - \sin^2 \theta \sin^2 \varphi) + |A_\perp|^2 \sin^2 \theta - \text{Im}(A_\parallel^* A_\perp) \sin 2\theta \sin \varphi)] + \frac{1}{\sqrt{2}} \sin 2\psi \{\text{Re}(A_0^* A_\parallel) \sin^2 \theta \sin 2\varphi + \text{Im}(A_0^* A_\perp) \sin 2\theta \cos \varphi\} . \hspace{1cm} (1)$$

In terms of partial-wave amplitudes $S$, $P$, $D$ corresponding to $L = 0$, 1, 2 between the vector mesons, one has

$$A_0 = -\sqrt{\frac{1}{3}} S + \sqrt{\frac{2}{3}} D, \hspace{0.5cm} A_\parallel = \sqrt{\frac{2}{3}} S + \sqrt{\frac{1}{3}} D, \hspace{0.5cm} A_\perp = P . \hspace{1cm} (2)$$

The normalization has been chosen in such a way that the total width is given by

$$\Gamma = |A_0|^2 + |A_\parallel|^2 + |A_\perp|^2 = |S|^2 + |P|^2 + |D|^2 . \hspace{1cm} (3)$$

Specializing to the case of $0^{++}$ decaying via a pure S-wave, one finds

$$\frac{1}{\Gamma} \frac{d^3\Gamma}{d\cos \theta d\varphi d\cos \psi} = \frac{3}{16\pi} [1 - \sin^2 \theta \cos^2 (\psi - \varphi)] . \hspace{1cm} (4)$$
The dependence on $\psi - \varphi$ looks puzzling at first sight, since $\psi$ is a polar angle while $\varphi$ is an azimuthal angle. However, $\psi$ is defined with respect to the $x$ axis, while $\varphi$ is an azimuthal angle in the $x$-$y$ plane.

The angular distribution is independent of $\psi$ or $\varphi$ when $\theta = 0$ or $\pi$. In that case the $\ell^+\ell^-$ and $\pi^+\pi^-$ axes are parallel to one another, and rotational invariance for the spinless initial state guarantees that the angular distribution should not depend on overall orientation. The distribution vanishes when $\theta = \pi/2$ and $\psi = \varphi$; in that case the $\ell^+\ell^-$ and $\pi^+\pi^-$ axes are parallel to one another. The $\ell^+\ell^-$ state must always have helicity $\pm 1$ along its axis, and hence cannot couple to a collinear $\pi^+\pi^-$ state if the total angular momentum is to vanish. This last property also allows one to derive a simple expression for the correlation between the $\ell^+\ell^-$ and $\pi^+\pi^-$ axes in $0^{++}$ decay:

$$\frac{1}{\Gamma} \frac{d\Gamma}{d\cos \theta d\psi} \bigg|_{J^{PC}=0^{++}} = \frac{3}{4} \sin^2 \theta \sin^2 \psi .$$  \hspace{1cm} (5)$$

This result also follows from Eq. (4) by a simple transformation of coordinate axes and integration over two of the three variables. Let the $\pi^+$ lie along the $+x$ axis (setting $\psi = 0$) and note that $\sin \theta \cos \varphi$ is the cosine of the angle between the $\ell^+$ and $\pi^+$. An alternative derivation via Clebsch-Gordan coefficients is contained in Appendix C.

The result for $0^{-+}$ decaying by a pure P-wave is quite simple:

$$\frac{1}{\Gamma} \frac{d\Gamma}{d\cos \theta d\varphi d\cos \psi} = \frac{9}{32\pi} \sin^2 \theta \sin^2 \psi .$$  \hspace{1cm} (6)$$

The proportionality of this distribution to $\sin^2 \theta$ is a general feature of transversity amplitudes for decay of a CP-odd spinless state (see Ref. [20]). The vanishing of the distribution when $\psi = 0$ or $\pi$ is a feature of the coupling for a pseudoscalar decaying to two vectors; the dipion axis (in the $\pi\pi$ rest frame) cannot be parallel to the $J/\psi$ recoil momentum in that frame. One cannot write a simple relation corresponding to Eq. (5) since the direction of the recoil momentum is important. The matrix element actually vanishes for zero recoil momentum, as it does for all negative-parity states decaying to $J/\psi\rho^0$. This, in fact, is an argument against negative parity for the $X(3872)$, since a matrix element proportional to recoil momentum would inevitably suppress both the highest $\pi^+\pi^-$ masses in the decay $X(3872) \to J/\psi\rho^0$ and the observed process $X(3872) \to J/\psi\omega$ [15].

For both $0^{++}$ and $0^{-+}$ states produced in $\gamma\gamma$ collisions and decaying to $J/\psi(\to \ell^+\ell^-)\rho^0(\to \pi^+\pi^-)$, individual distributions of lepton or pion pairs do not show any dependence on the angles $\theta_\ell$ or $\theta_\pi$ with respect to the $\gamma\gamma$ axis. This is a feature of the spinless nature of the initial state.

V. STATES WITH $J^{PC} = 1^{++}$

The angular distributions in a decay $X(1^{++}) \to J/\psi\rho$ are best worked out using a Cartesian basis for the polarizations of all three vector mesons. The interaction Lagrangian is of the form (in a nonrelativistic basis, which is satisfactory for an S-wave decay with negligible recoil momentum)

$$\mathcal{L}_{\text{int}} \propto \vec{e}_X \cdot \vec{e}_{J/\psi} \times \vec{e}_\rho + \text{c.c.}$$  \hspace{1cm} (7)$$
This simplifies calculations considerably.

It is convenient to neglect the relative momentum between the $J/\psi$ and the $\rho^0$ and to assume they (and the $X$) are in the same c.m.s. Let the direction in which the $X$ was boosted to reach its c.m.s. define the $x$-axis, and let the $\pi^+$ lie in the $x$-$y$ plane, making an angle $\chi$ with the $x$-axis. Thus a unit vector along the direction of the $\pi^+$ is $\hat{n}(\pi^+) = \hat{x} \cos \chi + \hat{y} \sin \chi$, where $\hat{x}$ and $\hat{y}$ are unit vectors along the $x$ and $y$ axes. The polarization vector of the $\rho$ must have this same form: $\vec{\epsilon}_\rho = \hat{x} \cos \chi + \hat{y} \sin \chi$. The $z$ axis is defined with respect to $\hat{x}$ and $\hat{y}$ by a right-handed coordinate system. Angles in polar coordinates are defined so that a unit vector $\hat{n}$ with arbitrary direction has coordinates

$$\hat{n} = \hat{x} \sin \theta \cos \phi + \hat{y} \sin \theta \sin \phi + \hat{z} \cos \theta \quad .$$

For $J(X) = 1$, only two polarization states need be considered unless one is interested in parity-violating processes. These two polarizations in the frame of interest are either along the $X$ boost direction, or perpendicular to that direction, where one must sum over both such perpendicular directions after squaring matrix elements. The corresponding amplitudes may be denoted $B_0$ and $B_T$, respectively. The sum over two transverse polarizations is equivalent to summing over two amplitudes corresponding to helicity $\pm 1$, which must be equal if $X$ is produced in parity-conserving processes. Although $\bar{B}$ decays do not necessarily conserve parity, the effect of summing over $B$ and $\bar{B}$ decays is to populate states of helicity $\pm 1$ equally.

For $X$ polarized along its boost direction, $\vec{\epsilon}_X = \hat{x}$. In view of the coupling \[\text{(7)},\] only the $\hat{y} \sin \chi$ component of $\vec{\epsilon}_\rho$ can contribute, and the $J/\psi$ must be polarized along $\hat{z}$ (leading to a lepton distribution proportional to $\sin^2 \theta$). Then the full angular distribution of lepton pairs for this polarization state must be proportional to $\sin^2 \theta \sin^2 \chi$.

For $X$ polarized transversely there are two contributions. When $\vec{\epsilon}_X = \hat{y}$, only the $\hat{x} \cos \chi$ component of the $\rho$ polarization contributes. Again, the $J/\psi$ is polarized along $\hat{z}$ leading to a lepton distribution $\sim \sin^2 \theta$. Thus the angular distribution for this polarization state is proportional to $\sin^2 \theta \cos^2 \chi$.

When $\vec{\epsilon}_X = \hat{z}$, the $J/\psi$ polarization vector is perpendicular to it and to $\vec{\epsilon}_\rho$, so $\vec{\epsilon}_{J/\psi} = -\hat{x} \sin \chi + \hat{y} \cos \chi$. The lepton distribution for such a state (see Appendix B) is proportional to $1 - \sin^2 \theta \sin^2 (\chi - \phi)$.

The overall differential distribution, with longitudinal (0) and transverse (T) amplitudes normalized so that $\Gamma = |B_0|^2 + 2|B_T|^2$, is

$$\frac{d^3 \Gamma}{d \cos \theta \, d \phi \, d \cos \chi} = \frac{9}{32 \pi} \left[ |B_0|^2 \sin^2 \theta \sin^2 \chi + |B_T|^2 [\sin^2 \theta \cos^2 \chi + 1 - \sin^2 \theta \sin^2 (\chi - \phi)] \right] \quad .$$ \hspace{1cm} (9)

If $X$ is unpolarized, corresponding to $|B_0| = |B_T|$, the result becomes

$$\frac{d^3 \Gamma}{d \cos \theta \, d \phi \, d \cos \chi} = \frac{9}{32 \pi} |B_0|^2 [1 + \sin^2 \theta \cos^2 (\chi - \phi)] \quad .$$ \hspace{1cm} (10)

This can be expressed as the simpler form

$$\frac{1}{\Gamma} \frac{d \Gamma}{d \cos \theta_{\ell\pi}} \bigg| \_{JPC=1^{++}} = \frac{3}{8} (1 + \cos^2 \theta_{\ell\pi}) \quad ,$$ \hspace{1cm} (11)
VI. STATES WITH $J^{PC} = 2^{++}$

This assignment is probably not so relevant for the $X(3872)$, since there are several arguments against it, including the absence of the decay $X(3872) \rightarrow D\bar{D}$ which would be possible for a $2^{++}$ particle. However, it is relevant for states of $J/\psi \rho^0$ such as can be produced near threshold in $\gamma \gamma$ collisions, as studied in Ref. [16] and discussed in Ref. [18].

Two independent helicity states of a $2^{++}$ particle produced in $\gamma \gamma$ collisions are possible: $J_z = \pm 2$ and $J_z = 0$. Assuming the decay $2^{++} \rightarrow J/\psi \rho^0$ takes place with zero orbital angular momentum and that the rest frames of $J/\psi$ and $\rho^0$ coincide, it is a simple matter of Clebsch-Gordan coefficients to calculate the individual distributions of leptons and pions with respect to the $\gamma \gamma$ axis. For instance, when $J_z = \pm 2$, the individual vector mesons must have $J_z = \pm 1$, and one should see the characteristic angular distributions

$$\frac{1}{\Gamma} \frac{d\Gamma}{d \cos \theta_{\ell}} = \frac{3}{8} \left(1 + \cos^2 \theta_{\ell}\right), \quad \frac{1}{\Gamma} \frac{d\Gamma}{d \cos \theta_{\pi}} = \frac{3}{4} \sin^2 \theta_{\pi} \quad .$$

The state with $J_z = 0$ is composed 1/3 of the time of vector mesons with $J_z = \pm 1$ and 2/3 of the time of vector mesons with $J_z = 0$ (see Appendix C). One then finds for the lepton angular distribution with respect to the beam

$$\frac{1}{\Gamma} \frac{d\Gamma}{d \cos \theta_{\ell}} = \frac{1}{3} \left[\frac{3}{8} \left(1 + \cos^2 \theta_{\ell}\right)\right] + \frac{2}{3} \left[\frac{3}{4} \sin^2 \theta_{\ell}\right]$$

$$= \frac{1}{8} \left(5 - 3 \cos^2 \theta_{\ell}\right) \quad .$$

and for the pion angular distribution

$$\frac{1}{\Gamma} \frac{d\Gamma}{d \cos \theta_{\ell}} = \frac{1}{3} \left[\frac{3}{4} \sin^2 \theta_{\pi}\right] + \frac{2}{3} \left[\frac{3}{2} \cos^2 \theta_{\pi}\right]$$

$$= \frac{1}{4} \left(1 + 3 \cos^2 \theta_{\pi}\right) \quad .$$

Putting these results together and defining normalized helicity amplitudes $A_{\pm 2}$ and $A_0$ in such a way that $|A_0|^2 + 2|A_{\pm 2}|^2 = 1$, one has

$$\frac{d\Gamma}{d \cos \theta_{\ell}} = \frac{1}{8} \left[6|A_{\pm 2}|^2 \left(1 + \cos^2 \theta_{\ell}\right) + |A_0|^2 \left(5 - 3 \cos^2 \theta_{\ell}\right)\right] \quad ,$$

$$\frac{d\Gamma}{d \cos \theta_{\pi}} = \frac{1}{4} \left[6|A_{\pm 2}|^2 \left(1 - \cos^2 \theta_{\pi}\right) + |A_0|^2 \left(1 + 3 \cos^2 \theta_{\pi}\right)\right] \quad .$$

The correlation between $\ell$ and $\pi$ directions is independent of the polarization of the $2^{++}$ state. It is worked out in Appendix C and is given by

$$\frac{1}{\Gamma} \left. \frac{d\Gamma}{d \cos \theta_{\ell\pi}} \right|_{J^{PC}=2^{++}} = \frac{3}{40} \left(7 - \cos^2 \theta_{\ell\pi}\right) \quad (17)$$
VII. CONCLUSIONS

Several angular distributions for decays of $J/\psi \rho^0$ and $J/\psi \omega$ states near threshold have been given. These apply both to the $X(3872)$, whose favored quantum numbers of $J^{PC} = 1^{++}$ if it is indeed a molecule of $D^0$ and $\bar{D}^{*0}$ [10, 11, 15], and to states formed near threshold by photon-photon collisions. A number of such states for lighter-quark systems have turned out to have $J^{PC} = 2^{++}$ [18]. It will be interesting to apply the present methods to the few events reported in Ref. [16] and to potentially larger samples available from asymmetric $e^+e^-$ colliders.

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APPENDIX A: DECAYS OF POLARIZED $\rho$, $\omega$

In the c.m.s. of the $\rho$, its coupling to two pions is of the form

$$L_{\text{int}} \propto \vec{\epsilon}_\rho \cdot [\vec{p}(\pi^+) - \vec{p}(\pi^-)] .$$

Thus, a $\rho$ linearly polarized along the $\hat{z}$ direction ($\vec{\epsilon}_\rho = \hat{z}$ or $J^z_\rho = 0$) will give rise to a pion angular distribution $\sim \cos^2 \theta$, where $\theta$ is the polar angle of the direction of either pion, while a $\rho$ with $\vec{\epsilon}_\rho = \mp(\hat{x} \pm i\hat{y})/\sqrt{2}$ ($J^z_\rho = \pm 1$) will have an angular distribution $\sim \sin^2 \theta/2$ (in the same normalization). Adopting a normalization for which a distribution $W(\cos \theta_\pi)$ obeys

$$\int_{-1}^{1} d \cos \theta_\pi W(\cos \theta_\pi) = 1 ,$$

one has

$$W_{J^z=0}(\cos \theta_\pi) = \frac{3}{2} \sin^2 \theta_\pi \quad , \quad W_{J^z=\pm 1}(\cos \theta_\pi) = \frac{3}{4} \sin^2 \theta_\pi .$$

The coupling of an $\omega$ in its rest frame to $\pi^+\pi^-\pi^0$ is of the form $L_{\text{int}} \propto \vec{\epsilon}_\omega \cdot \hat{n}$, where $\hat{n}$ is the normal to the plane of decay defined by the three pions in the $\omega$ rest frame. Thus all results for $\rho^0 \to \pi^+\pi^-$ may be transcribed for $\omega$ decays by replacing a unit vector in the direction of the $\pi^+$ in $\rho$ decay by the normal to the $\omega$ decay plane.

APPENDIX B: DECAYS OF POLARIZED $J/\psi$

The decay of a vector meson such as $J/\psi$ to $\ell^+(p_+)\ell^-(p_-)$ involves an interaction proportional to $\epsilon_\mu \bar{u}(p_-)\gamma^\mu v(p_+)$, where $u$ and $v$ are Dirac spinors. Performing the sum
over lepton spins, and denoting the initial $J/\psi$ spin configuration by a density matrix $\rho_{J/\psi}$, one finds that a general angular distribution in the $J/\psi$ c.m.s. for any $\rho_{J/\psi}$ is proportional to $\text{Tr}(\rho_{J/\psi} L)$, where $L$ is a $3 \times 3$ matrix: $L_{ij} = \delta_{ij} - n^i n^j$, with $n^i$ denoting the Cartesian coordinates of the $\ell^+$ momentum. In spherical polar coordinates, one has

$$L = \begin{pmatrix}
1 - \sin^2 \theta \cos^2 \phi & -\sin \theta \sin \phi \cos \phi & -\sin \theta \cos \theta \cos \phi \\
-\sin^2 \theta \sin \phi \cos \phi & 1 - \sin^2 \theta \sin^2 \phi & -\sin \theta \cos \theta \sin \phi \\
-\sin \theta \cos \theta \cos \phi & -\sin \theta \cos \theta \sin \phi & 1 - \cos^2 \theta
\end{pmatrix}.$$  \hfill (21)

The spin density matrix for a pure $J/\psi$ polarization state $\epsilon_i$ is $\rho_{ij} = \epsilon_i \epsilon_j^\ast$. Thus, for example, $\rho(J_z = 0) = \text{diag}(0, 0, 1)$ and the corresponding angular distribution for lepton pairs is proportional to $1 - \cos^2 \theta = \sin^2 \theta$. A $J/\psi$ is produced in $e^+ e^-$ collisions in a mixed state, half $J_z = 1$ and half $J_z = -1$, corresponding to the density matrix $\rho = (1/2)\text{diag}(1, 1, 0)$, so the corresponding angular distribution for final leptons is proportional to $(1/2)(2 - \sin^2 \theta) = (1/2)(1 + \cos^2 \theta)$. The suitably normalized forms are

$$W_{J_z^\psi = 0} = (3/4) \sin^2 \theta, \quad W_{J_z^\psi = \pm 1 \text{ (mixed)}} = (3/8)(1 + \cos^2 \theta) \quad .$$ \hfill (22)

APPENDIX C: CORRELATIONS IN $\theta_{\ell\pi}$

The decays discussed here are of the form $X \to J/\psi(\to \ell^+ \ell^-)\rho^0(\to \pi^+ \pi^-)$ when the $J/\psi$ and $\rho$ are in a relative S-wave and when their relative momentum may be neglected. The angle $\theta_{\ell\pi}$ is defined as that between the $\ell^+$ and $\pi^+$ in the common rest frame of $J/\psi$ and $\rho^0$.

Let the $\pi^+$ direction define the $z$-axis. States $|J, J_z\rangle$ of definite total $J = 0, 1, 2$ and definite $J_z$ may now be decomposed into linear combinations of substates $|J_{z^\psi}, J_{z^\rho}\rangle$. Since the $\pi^+$ has been taken along the $z$ axis, one is only concerned with cases with $J_{z^\rho} = 0$. As shown in Appendix B, states with definite $J_{z^\psi}$ lead to distributions

$$W_{J_{z^\psi} = \pm 1}(\cos \theta_{\ell\pi}) = \frac{3}{8} \left(1 + \cos^2 \theta_{\ell\pi}\right), \quad W_{J_{z^\psi} = 0}(\cos \theta_{\ell\pi}) = \frac{3}{4} \sin^2 \theta_{\ell\pi} \quad ,$$ \hfill (23)

where these distributions are normalized according to

$$\int_{-1}^{1} d \cos \theta_{\ell\pi} W(\cos \theta_{\ell\pi}) = 1 \quad .$$ \hfill (24)

One must sum over all the $J_z$ values and divide by $2J + 1$ (the number of $J_z$ values) to obtain the final result. One must also multiply by 3 to take account of the fact that a specific polarization state $J_z = 0$ of the $\rho^0$ has been chosen. As an example, a state with $J = J_z = 0$ has the decomposition

$$|0, 0\rangle = \frac{1}{\sqrt{3}} \left(|1, -1\rangle - |0, 0\rangle + | -1, 1\rangle\right) \quad .$$ \hfill (25)
Table II: Normalized distributions \((1/\Gamma)d\Gamma/d\cos\theta_{\pi}\) in cosine of the angle between the \(\ell^+\ell^-\) and \(\pi^+\pi^-\) axes, for decays to S-wave states of \(J/\psi\rho^0\) near threshold.

| \(J\) | Distribution |
|-------|--------------|
| 0     | \((3/4)\sin^2\theta_{\ell\pi}\) |
| 1     | \((3/8)(1 + \cos^2\theta_{\ell\pi})\) |
| 2     | \((3/40)(7 - \cos^2\theta_{\ell\pi})\) |

Only the substate \(|0,0\rangle\) contributes, with weight (the square of the Clebsch-Gordan coefficient) equal to 1/3. One thus finds

\[
\frac{1}{\Gamma} \frac{d\Gamma}{d\cos\theta_{\ell\pi}} \bigg|_{J=0} = 3 \cdot \frac{1}{3} \cdot \frac{3}{4} \sin^2\theta_{\ell\pi} = \frac{3}{4} \sin^2\theta_{\ell\pi} .
\]  

(26)

For \(J = 1\) one uses the decomposition

\[
|1 \pm 1\rangle = \frac{1}{\sqrt{2}} \pm [|\pm 1, 0\rangle - |0, \pm 1\rangle] ,
\]  

(27)

noting that \(|1,0\rangle\) has no contribution from \(|0,0\rangle\). Then one finds

\[
\frac{1}{\Gamma} \frac{d\Gamma}{d\cos\theta_{\ell\pi}} \bigg|_{J=1} = 3 \cdot \frac{1}{3} \cdot \frac{1}{2} \cdot 2 \cdot \frac{3}{8}(1 + \cos^2\theta_{\ell\pi}) = \frac{3}{8}(1 + \cos^2\theta_{\ell\pi}) ,
\]  

(28)

where the factor of 1/3 is \(1/(2J+1)\), the factor of 1/2 is the square of the Clebsch-Gordan coefficient, and the factor of 2 corresponds to the two substates \(J_z = \pm 1\). Finally, for \(J = 2\), one uses the decompositions

\[
|2, \pm 1\rangle = \frac{1}{\sqrt{2}} [|\pm 1, 0\rangle + |0, \pm 1\rangle] ,
\]  

(29)

\[
|2, 0\rangle = \frac{1}{\sqrt{6}} [|1, -1\rangle + 2|0, 0\rangle + |-1, 1\rangle]
\]  

(30)

to obtain

\[
\frac{1}{\Gamma} \frac{d\Gamma}{d\cos\theta_{\ell\pi}} \bigg|_{J=2} = \frac{3}{40}(7 - \cos^2\theta_{\ell\pi})
\]  

(31)
after a similar calculation. (Note that the substates with \(J_z = \pm 2\) do not involve \(J_\rho^0 = 0\) and hence do not contribute.) The results are summarized in Table II.

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