Complex Landau levels and related transport properties in the strained zigzag graphene nanoribbons

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The real magnetic fields (MFs) acting on the graphene can induce flat real Landau levels (LLs). As an analogy, strains in graphene can produce significant pseudo MFs, triggering the appearance of dispersive pseudo LLs. By analyzing the low-energy effective Hamiltonian, we introduce the concept of the effective orbital MFs to integrate the real MFs and pseudo MFs. Accordingly, we obtain the complex LLs which incorporate the real LLs and pseudo LLs, and calculate the related transport properties. These concepts enable us to uncover the mechanisms driving the fragility of pseudo LLs against disorders and dephasing, proving that tuning the real MFs and Fermi energy can effectively improve the robust performances. Furthermore, the tunability of the valley-polarized currents is also studied, opening up new possibilities for the design of valleytronics devices.

I. INTRODUCTION

Unique in two dimensions, graphene possesses two non-equivalent Dirac points, $K$ and $K'$, which leads to the valley degree of freedom [1, 2]. The two main edge types of graphene nanoribbons (GNRs), which are basically one-dimensional structures cut from graphene, are zigzag edges and armchair edges. The contrast between the $K$ and $K'$ valleys in the $k$-spaces is one of the primary differences between the zigzag GNRs (ZGNRs) and armchair GNRs (AGNRs). Without intervalley scattering, the $K$ and $K'$ valleys are separated and decoupled specifically for the ZGNRs in the low-energy limit, making the study of valley transport pertinent. However, the $K$ and $K'$ valleys for the AZGNRs are both projected to the $k$-space $\Gamma$ point, indicating that they are not suitable for creating valleytronics devices [3–5]. As a result, we concentrated mostly on the ZGNRs in this work. The ability of ZGNRs to generate pseudo-magnetic fields (PMFs) through strains, which in turn leads to the appearance of pseudo-Landau levels (PLLs), is another remarkable property of the material [5–11]. This phenomenon has been identified in several noteworthy investigations [12–15]. Moreover, several experimental studies have shown that graphene can resist nondestructively reversible deformations up to high values of 25%–27% [16–19], implying that it could be a promising material for building novel straintronic devices with the exceptional features associated with PMFs.

A two-terminal ZGNR with the uniaxial strain is shown in Fig. 1(a). The uniaxial strain of the ZGNR is extended along the $y$ direction. Accordingly, the hopping coefficients along the $y$ direction $t_y(n)$ are assumed to be a linear function of $n$ ($n = 1, 2, ..., N_y - 1$). Meanwhile, the hopping coefficients along the $x$ direction are considered to be constant [5, 20–22]. As a result, this strain pattern is referred to as the monotonic increasing strain (MIS) [23], which leads to the emergence of a uniform perpendicular PMF. Valley-polarized currents for the $K$ and $K'$ valleys are theoretically predicted in the ZGNRs under the influence of the PMFs [20]. Fig. 1(b) shows the $K$ and $K'$ valleys of the strained GNRs in the real magnetic fields (RMFs). As discussed in Sec. IV A, the joint effects of the PMFs and RMFs cause the $K'$ ($K$) valley to sink (raise) and get narrower (wider). Both the RMFs and PMFs can produce LLs, however, the former results in flat LLs while the latter results in dispersive ones. Additionally, it is shown by the transport characteristics research that the states associated with PLLs and RLLs have distinct robust responses to Anderson disorders and...
dephasing effects. We offer the idea of the effective orbital magnetic fields (EOMFs), which result in the creation of complex LLs (CLLs), to combine the impacts of RMFs and PMFs to explain the transport characteristics of the strained ZGNRs. We propose several mechanisms of the interivalley and intravalley to explain the distinct robust performances for the RLLs and PLLs, and point out that the valley polarization governed by the EOMFs $|B_{\text{eff}}^\pm|$ results in the distinct conductance features that are related to the $K$ and $K'$ valleys, respectively.

The paper is organized as follows. We introduce the model and numerical methods employed in this work in Sec. II. In Sec. III, we talk about the low energy effective theory and introduce the concept of the EOMFs and CLLs. Sec. IV presents the key findings of our calculations and the corresponding remarks. More specifically, in Sec. IV A, we study the effect of the Anderson disorders and reveal the mechanisms driving the fragility of PLLs against disorders. Dephasing effect, which is cov-

II. MODEL AND NUMERICAL METHODS

A two-terminal ZGNR with the MIS is illustrated in Fig. 1(a). The central region is sandwiched between the left ($L$) and right ($R$) leads. In realistic samples, Anderson disorders and dephasing effects are always present. In the following calculations, we suppose that the disorders and dephasing effects only exist in the central region. The dephasing effects are easily produced via electron-electron interactions, electron-phonon interactions, etc., and can be tuned by changing the temperature experimentally. Here, we simulate the dephasing effects by applying the Büttiker’s virtual probe assumption[24]. The tight-binding Hamiltonian of the ZGNR with MIS in the central region can be written as

$$\mathcal{H} = \sum_i \varepsilon_i a_i^\dagger a_i - \sum_{\langle ij \rangle} t e^{i\phi_{ij}} a_i^\dagger a_j,$$

(1)

where $\varepsilon_i$ represents the on-site energy, $a_i^\dagger$ and $a_i$ represent the creation and annihilation operators, and $\langle ij \rangle$ sums over the nearest neighbors. In the $L(R)$ leads, $\varepsilon_i = E_F + W$, where $W$ denotes the disorder strength. Anderson disorders are simulated by the onsite energies that are uniformly distributed in $[-W/2, W/2]$. If there exists RMFs, the hopping coefficient $t$ should have a phase $\phi_{ij} = \int_A A \cdot dl / \phi_0$ with the vector potential $A$ and the flux quantum $\phi_0 = h/e$.

As shown in Fig. 1(a), we assume that the ZGNRs are only stretched along the $y$-axis, with the hopping coefficient $t_y(n)$ being a linear function of $n$. For simplicity, $t_y(n)$ is defined as $t_0$ on the bottom edge and $t_0(1 - \eta)$ on the top edge, respectively. $t_0 = -2.75$ eV is the well-known hopping coefficient for the normal graphene and $\eta$ is an adjustable variable that reflects the strain strength. At any $n$, $t_y(n)$ can be expressed as $t_y(n) = t_0(1 - \gamma n)$, where $\gamma = \frac{n(n-1)}{(N_y - 2)n}$. Meanwhile, $t_2$ and $t_3$ are set as $t_0$. Specifically speaking, previous work stated that $t_y(n) = t_0 \exp \left[-\beta \left(t_y(n) / a_0 - 1\right)\right]$ [19], where $t_y(n)$ is the corresponding bond length along the $y$ direction, $a_0 = 0.142$ nm is the equilibrium bond length of the pristine graphene, and $\beta \approx 3.37$ is the decay rate. Consequently, $\eta = 0.5$ is corresponding to the maximum deformation strength 20% and it falls in the regime that is not destructive and reversible [16–19].

The conductance is calculated by combining the Landauer–Büttiker formula with the non-equilibrium Green function method at zero temperature [25–28]. The current in the real or virtual lead can be obtained by $I_p = \langle 2e^2 / h \sum_{qF} \mathcal{T}_{pq}(E_F) (V_p - V_q) \rangle$, where $p = L, R, 1, 2, \ldots, N$, $V_p$ is the bias in the lead $p$, and $N$ is the number of lattice sites in the central region. Here, $\mathcal{T}_{pq}(E_F) = \text{Tr}[\mathcal{G}_r(E_F)|\mathcal{G}_\ell(E_F)\mathcal{G}_p(E_F)|\mathcal{G}_p^0(E_F)]$ is the transmission function at the Fermi energy $E_F$ from lead $q$ to lead $p$, and the line width function is given by $\Gamma_p(E_F) = i(\Sigma^\ell_p(E_F) - \Sigma^r_p(E_F))$. The retarded Green function is calculated by $G^r(E_F) = [G^r]^{-1} = [E_F I - H - \sum_p \Sigma^p(E_F)]^{-1}$, where $\Sigma^p(E_F)$ denotes the retarded self-energy associated with lead $p$. For the real lead $\Sigma^\ell_p(E_F)$ can be calculated numerically [29]; for the virtual lead $p$, $\Sigma^p(E_F) = -i dp / 2$, where $dp$ describes the dephasing strength [30, 31]. To drive a current flowing along the $x$ direction, a small bias $V = V_L - V_R$ is added between the $L$ and $R$ leads. Once the current $I_L$ has been obtained, the conductance can be calculated directly by $G = (V_L - V_R) / I_L$. The average value of 500 random configurations is used to calculate the conductance.

III. LOW-ENERGY EFFECTIVE THEORY

For the strained ZGNR, the effective Hamiltonian is $\mathcal{H}^\pm(k) = d \cdot \sigma$ [20], where

$$d^\pm_x = \hbar v_F \left( \mp k_x + r_\pm eB_x y / h \right),$$

$$d^\pm_y = \hbar v_F k_y \left( r_\pm - s_\pm eB_y y / h \right),$$

(2)

Here $v_F$ is the Fermi velocity of the pristine graphene, $\pm$ represent $K$ and $K'$ valleys. $B_x = \frac{\partial u_y}{\partial x}$ and $B_y = \frac{\partial u_x}{\partial y}$ are the PMF induced by the strain, $\varepsilon_y = u_y / a_0$ is the strain tensor, and $u_y$ is the in-plane displacement of carbon atoms along the $y$ direction [20, 21]. $r_\pm = 1 \pm \frac{k_x}{p^\pm}$, $s_\pm = \left( \frac{3}{2} \pm \frac{k_y}{p^\pm} \right)$, and we have set $a_0 = 1$. It should be pointed out that the Fermi velocities are modulated by the strain and should be anisotropic and momentum-dependent. By
using the same method in Ref. [22], we can obtain the Fermi velocities of the carriers in the strained graphene
\( v_{Fx}^s = \frac{3\alpha}{2} \sqrt{1 + \frac{2\gamma y^2}{3} - \frac{\gamma y^4}{3}} \) and \( v_{Fy}^s = \frac{3\alpha}{2} (1 - \gamma y) \).

In the presence of the RMFs, the canonical momentum should be replaced by the gauge invariant momentum, thus the \( d \) vector changes to

\[

d_\pm = v_F (\pm \Pi_x + r_\pm \frac{\epsilon B_y}{h}) ,
\]

\[
d_y = v_F \Pi_y \left( r_\pm - s_\pm \frac{\epsilon B_y}{h} \right) ,
\]

where \( \Pi_i = p_i + eA_{ri} (i = x, y) \). Note that \( r_\pm \) and \( s_\pm \) contain \( k_x \), thus \( p_x = \hbar k_x \) in \( r_\pm \) and \( s_\pm \) should also be replaced by \( \Pi_x \). However, the resulting additional terms in \( r_\pm \) and \( s_\pm \) can be neglected because they are smaller than other relevant terms, thus we obtain Eq. 3. For a perpendicular RMF, we choose the gauge \( A_{rx} = B_r y \) and \( A_{ry} = 0 \), then the Hamiltonian becomes

\[
\mathcal{H}_\pm = v_F \left[ \sigma_x (\pm p_x \pm eB_{cy}^\pm y) + \sigma_y p_y \left( r_\pm - s_\pm \frac{\epsilon B_y}{h} \right) \right] ,
\]

where \( B_{cy}^\pm = B_r \pm r_\pm B_y \) is the EOMF which incorporates the effects of both the PMFs and RMFs. Note that \( B_{cy}^\pm \) is not the direct addition of \( B_r \) and \( B_y \), and \( r_\pm \), the coefficient of \( B_y \), is dependent on \( k_x \). This reflects the essential differences between the RMFs and PMFs, i.e., RMFs induce flat LLs, but PMFs induce dispersive ones.

By solving the eigenvalue equation

\[
\mathcal{H}_\pm \begin{pmatrix} \psi_A(y) \\ \psi_B(y) \end{pmatrix} = \varepsilon_\pm \begin{pmatrix} \psi_A(y) \\ \psi_B(y) \end{pmatrix} ,
\]

where \( \psi_A \) and \( \psi_B \) are components for the A and B sublattices, we can obtain the bulk LLs \( \varepsilon_\pm \). Using the similar method adopted in Ref. [20], we get

\[
\varepsilon_\pm^2 = 2nehv_F^2 |B_{cy}^\pm| \left( r_\pm + k_x s_\pm \frac{B_y}{B_{cy}^\pm} \right) .
\]

Because the orientations of the RMF and PMF are opposite for the \( K' \) valley, \( B_{cy}^\pm = B_r - r_\pm B_y \) might be zero. Thus our solutions for the CLLS are invalid when \( B_{cy}^\pm = 0 \). Actually, \( B_{cy} = 0 \) is a critical point that the CLLS, as well as the edge currents for the \( K' \) valley vanish, which also can be illustrated in Fig. 2(c) and Fig. 7(b). Furthermore, the result of \( \varepsilon_\pm^2 \) in the \( K' \) valley is invalid in the vicinity of the singular point \( B_{cy}^\pm = 0 \). For more information, see the derivations and discussions in Appendix A.

**FIG. 2.** (a)-(e) are dispersions for the ZGNR with MIS for \( \eta = 0.5 \). The RMF is \( B_r = 0 \) in (a), \( B_r = 15 \text{ T} \) in (b), \( B_r = 35 \text{ T} \) in (c), and \( B_r = 50 \text{ T} \) in (d). \( B_r = 15 \text{ T} \) and \( \mathcal{E}_y = 0.02\alpha_0 \) are adopted in (e). \( \eta = 0.35 \) and \( B_r = 50 \text{ T} \) are adopted in (f), in which the green dashed lines labeling several Fermi energies are used to analyze the conductance in Fig. 3. The color scale represents the expectation value of the \( y \) for each eigenstate. In all cases, we take \( N_x = 200 \). Specifically, the colors in (a)-(d) and (f) predict the degenerate states at \( E_F = 0 \) are localized on both sides of the sample. The blue dashed lines label the Fermi energy \( E_F \) in (b)-(d), which are used to illustrate the valley currents in Sec. IV C.

**FIG. 3.** (a)-(f): The conductance for the ZGNRs with MIS which are one-to-one correspondence with Figs. 2(a)-(f). The green dashed lines in (f) corresponds to those in the energy bands of Fig. 2(f). In all cases, we take \( N_x = 30 \) and \( N_y = 200 \).

**IV. NUMERICAL RESULTS AND DISCUSSIONS**

**A. Anderson disorders**

Next, we examine whether the valley currents in the strained ZGNRs are robust against static disorders. Figs. 2(a)-(e) illustrate the band structures with the strain strength \( \eta = 0.5 \). As discussed in Sec. III, the electrons in both valleys encounter the EOMFs \( B_{cy}^\pm \). When \( B_r = 0 \), \( B_{cy}^\pm \) devolves to \( \pm r_\pm B_y \), producing the dispersive and symmetric PLLs in Fig. 2(a). If \( B_r \) is present, the
directions of the RMF and PMF in the $K$ ($K'$) valley are the same (different). Therefore, $|\mathcal{B}_{\text{eff}}|$ keeps decreasing as $B_r$ increases before $|\mathcal{B}_{\text{eff}}|$ reaches 0, and the CLLs become lower and narrower in the $K'$ valley as shown in Fig. 2(b). The CLLs disappear when $|\mathcal{B}_{\text{eff}}^-| = 0$ in Fig. 2(c). If $B_r$ keeps rising, $|\mathcal{B}_{\text{eff}}^-|$ gradually increases and the CLLs reappear in Fig. 2(d). Contrary to the $K'$ valley, the CLLs in the $K$ valley always become higher and wider because $|\mathcal{B}_{\text{eff}}^-|$ continues to grow as $B_r$ increase.

Fig. 3(a) shows that only the first plateau is robust against Anderson disorders when $B_r = 0$. The similar results have been obtained in previous work [32], where the authors attributed the robustness of the first plateau to the polarization of the sublattice. Figs. 3(b)-(d) illustrate the conductance with $B_r = 15$ T ($\mathcal{B}_{\text{eff}} < 0$), $B_r = 35$ T ($\mathcal{B}_{\text{eff}}^+ \approx 0$) and $B_r = 50$ T ($\mathcal{B}_{\text{eff}} > 0$), indicating that $B_r$ can improve the robust performances because the higher plateaus become more robust. Why are the edges states related to the PLLs not robust as that related to RLLs? There are two main reasons for this. First, the counter-propagating modes with spatial overlap and close energies are easily hybridized leading to the enhancement of intervalley scattering. For the quantum Hall effect (QHE) as seen from Fig. 4(a), the bulk states are localized because the group velocities are zero. The counter-propagating states are entirely separated by the bulk states on the two sides of the sample. Thus, the conductance plateaus in QHE are robust. According to Fig. 4(b), the degenerate counter-propagating states of PLLs overlap in space leading to the enhancement of the intervalley scattering, since the PMFs are opposite between the $K$ and $K'$ valleys due to the time-reversal symmetry. As a result, the conductance plateaus may not be robust due to hybridizations between the edge-edge states, the bulk-bulk states, and the edge-bulk states with spatial overlap. Additionally, since the counter-propagating modes cannot be spatially separated by the transverse electric field $E_y$, the robustness in Fig. 3(e) cannot be considerably improved. It should be noted that the second plateau becomes more robust than it is in Fig. 3(a) due to the shift of the valley degeneracy.

Fig. 4(c) and 4(d) illustrate the edge and bulk states of quantum valley Hall effect (QVHE) when $B_r$ and $B_p$ both exist. In Fig. 4(c), $B_r = 15$ T and $\mathcal{B}_{\text{eff}} < 0$, so $|\mathcal{B}_{\text{eff}}^-| (|\mathcal{B}_{\text{eff}}^+|)$ decreases (increases) and the cyclotron radius of the electrons in the $K'$ and $K$ valleys gets larger (smaller). Furthermore, the degeneracy of energies between the $K$ and $K'$ valleys has also been lifted, as shown in Fig. 2(b). There is only $K'$ valley contributing to the transport, leading to the suppression of the intervalley scattering, especially for the low-energy regime. Even for higher energies, the states at the Fermi level are separated in real space when the Fermi energy crosses both two valleys due to the asymmetry between the $K$ and $K'$ valleys. As a result, the conductance plateaus become more robust. Fig. 4(d) show the case for a large $B_r = 50$ T and $\mathcal{B}_{\text{eff}} > 0$. In this regime, $\mathcal{B}_{\text{eff}}$ makes the electrons in the $K'$ valley counter-rotating. As a result, the directions of valley current for the two valleys coincide, further reducing hybridization and making the conductance plateaus more robust.

Second, the intravalley hybridization also has three origins: the bulk-bulk and edge-edge hybridizations are weak because they both flow in the same direction; the bulk-edge hybridization becomes important because the
directions of the bulk and edge currents are opposite at least near one edge (see Fig. 4(b)-(d)). The bulk-edge hybridization can be affected by: (i) the degeneracy of CLLs. The results of Figs. 5(a) and (b) demonstrate the valley polarization in the presence of $\mathcal{B}_r$ and are consistent with the bands in Figs. 2(b) and (d) derived from the tight-binding approach. Moreover, we plot the extent with the bands in Figs. 2(b) and (d) derived from Rashba spin-orbit coupling (RSOC) and Zeeman energy also plays a role in transport properties. Note that the Rashba spin-orbit coupling (RSOC) and Zeeman energy are not included throughout the work because it has no impact on our understanding of physics (See Appendix B for more information).

Quite interestingly, the seventh plateau is more robust than the sixth plateau in Fig. 3(d). In order to explore this phenomenon, we choose another set of parameters $\eta = 0.35$ and $\mathcal{B}_r = 50$ T in Fig. 3(f) and obtain similar results that the fifth plateau is more robust than the fourth plateau. The green dashed lines plotted in Fig. 2(f) are one-to-one correspondence with those in Fig. 3(f). The corresponding relations indicate that the fourth plateau is fragile because $E_F$ crosses the first CLL (bulk states) in the $K$ valley; however, the higher fifth plateau is still robust because only edge states are crossed by $E_F$ in the $K$ valley. The hybridization between the counter-propagating modes around $P$ (see Fig. 2(f)) with close distance in space features the fragility of the fourth plateau. The behaviors exhibit the polarization of the $K$ and $K'$ valleys, which are determined by the EOMFs $[\mathcal{B}_{\text{eff}}^+]$. In our work, the valley polarization has three major aspects: the shift of valley degeneracy, the degeneracy of CLLs, and the slopes of CLLs (group velocities) in the $K$ and $K'$ valleys. The mechanisms of the inter-valley and intravalley scattering revealed by the preceding paragraphs make the different transport behaviors of PLLs and RLLs clear. On this basis, valley polarization determined by the EOMFs $[\mathcal{B}_{\text{eff}}^+]$ uncovers the different performances of conductance which are related to the $K$ and $K'$ valleys, respectively.

Furthermore, it should be pointed out that despite the presence of RMFs in Figs. 3(b)-(f), the higher plateaus are still not as robust as the ones in the QHE. On one hand, the CLLs are broadened when the disorder is present. On the other hand, the direct energy spacing $\Delta E$ between two adjacent higher CLLs becomes smaller. Thus, the direct gap between two adjacent CLLs can be smeared out due to the broadening of these states by the disorder. In this regime, bulk-edge hybridization also plays a role in transport properties. It should be noted that only the

FIG. 6. (a) The coherent length $L_\phi$ vs the dephasing strength $dp$ for various EOMFs with $E_F = 0.2$ eV. The conductance $G$ vs (b) the Fermi energy $E_F$, (c) the strain strength $\eta$, and (d) the disorder strength $W$ for various $dp$, respectively. We set $\eta = 0.5$ in (b) and (d), $E_F = 0.15$ eV in (c) and (d), and $B_r = 50$ T in (c)-(d). $N_x = 200$ in all case; We take $N_x = 30$ in (c) and $N_x = 12$ in other cases.

B. Dephasing effects

Aside from the static impurities, dephasing is another significant impedance to robust performance. Then we look at the dephasing effects in the ZGNRs with MIS. The coherent length $L_\phi$ is a measure of coherence in experiments. Electrons can go from the left lead to the right lead directly or via the virtual leads in the presence of dephasing effects, resulting in coherent and incoherent currents, respectively. At a certain $dp$, the incoherent current grows as the length $N_x$ does as well. When the coherent and incoherent parts of the current are equal, $N_x$ is the coherent length $L_\phi$[31]. Note that $L_\phi$ is an average value of the ZGNRs with MIS. The carbon-carbon distance in our model is not uniform along the $y$ axis. The real $L_\phi$, along the edge may be less than our numerical value. $L_\phi$ changes with $dp$ for three distinct cases of EOMFs—PLLs, CLLs, and RLLs—are shown in Fig. 6(a). At a specific $dp$, the value of $L_\phi$ is the lowest for PLLs. That means dephasing effects are more sensitive to
PMFs. There are two reasons for this. First, strain modifies the bond length between carbon atoms in the ZGNRs, and the dephasing effects caused by electron-phonon and electron-electron interactions are amplified [33, 34]. However, the cases of EOMFs \((B_r ≠ 0)\) improve the value of \(L_{\phi}\) in an obvious way. Because of the weakening of the hybridization demonstrated in Sec. IV A, it can be concluded that the addition of RMFs considerably increases the sample’s robustness against dephasing effects.

In Fig. 6(b), we examine the conductance of the CLls \((B_r = 50 \, T)\) under various \(dp\). In the weak and moderate dephasing regime \((dp < 0.2 \, eV)\), the conductance related to the lowest two CLls are very robust due to the suppression of hybridization. However, the ones with higher CLls are not robust. This is due to the broadening of the higher CLls brought on by the dephasing effects. In the higher CLls, the spacing between adjacent energy levels gets smaller. Thus, the hybridization is again intensified. In the strong dephasing regime \((dp ≥ 1 \, eV)\), the conductance exhibits a quasi-quantization with \(G ≲ 1 \, eV\). This outcome is in line with earlier researches [35, 36]. According to Fig. 6(c), the plateau of conductance gradually dissipates as the strain strength \(η\) increases. The result is consistent with the performance of \(L_{\phi}\) in Fig. 6(a). Finally, we consider the combination of Anderson disorder and dephasing effects in Fig. 6(d). The findings demonstrate that when disorder strength \(W\) grows, conductance value marginally reduces under various \(dp\).

The results above show that tunable valley currents of the low CLls are robust against dephasing effects when \(B_r ≠ 0\). The edge states related to higher CLls are not robust, though. Examining Hall conductance or developing novel solutions may be required as a next step to improve the performance of tuning valley currents against dephasing effects.

C. Tunability of the valley-polarized currents

The tunability of the valley-polarized currents in our system allows for the construction of new types of useful valleytronic devices. The band structures in Figs. 2(b)-(f) clearly demonstrate that the different EOMFs between the \(K\) and \(K’\) valleys induce the valley polarization. The valley currents depicted in Figs. 7(a)-(c) correspond to the states at Fermi energies \(E_1-E_5\) in Figs. 2(b)-(d). The edge states are significantly out of balance between the \(K\) and \(K’\) valleys, which differ greatly from that of the well-known QHE, quantum spin Hall effects (QSHE), and QVHE. Therefore, our sample is a good platform for manufacturing valleytronic devices by tuning \(E_F\), RMFs, or PMFs.

We may generalize our tunability of the valley-polarized currents to the ZGNRs with symmetric strain (SS) [23]. As shown in Fig. 8(a), the ZGNRs with SS can be viewed as two reverse copies of the ZGNR with MIS, and snake states exist in the middle of the sample. Fig. 8(b) depicts the band structure of the ZGNR with SS, and previous work has demonstrated that without the RMFs, only the first plateau is robust against the Anderson disorders. If we apply a RMF \(B_r = 50 \, T\), the CLls are shown in Fig. 8(c), and as illustrated in Fig. 8(d), the higher plateaus related to the snake states are extremely robust against the Anderson disorders. In the presence of the dephasing effects, the plateaus of snake states are very robust for the first CLl and survive for the higher CLls. Therefore, the results of the ZGNRs with SS are similar to that of the ZGNRs with MIS in Sec. IV A and Sec. IV B. To sum up, snake states can survive for both Anderson disorders and dephasing effects which show excellent design potential for new quantum devices.

V. CONCLUSIONS

In this work, we investigate the strained ZGNRs in the presence of the RMFs. The essential distinction between the RLLs and PLLs—which are produced by the RMFs and PMFs, respectively—is that the former are flat while the latter are dispersive. Because of their dispersive nature, the PLLs are susceptible to disorders because of the hybridization of their bulk and edge states. In order to incorporate the effects of the RMFs and PMFs, the concept of the EOMFs is introduced. Accordingly, we
obtain the CLLs which combine the RLLs and PLLs. Then we examine the robust behaviors of the valley-polarized currents by calculating the conductance and discover that the RMF can improve the robust performances. Our transport calculations of the K and K’ valleys demonstrate distinctive robust behaviors against the Anderson disorders and dephasing effect. The several mechanisms of the intervalley and intravalley scattering make it clear how PLLs and RLLs behave differently on transport properties. Moreover, the valley polarization induced by the EOMFs’s [B_{\text{eff}}] reveals the distinct conductance performances that are related to the K and K’ valleys, respectively. The behaviors of the conductance G show that the RMF B_r is a valid tool for tuning the valley currents. Furthermore, we investigate the tunability of the valley-polarized currents in the ZGNRs with valley currents. Furthermore, we investigate the tunability of the valley-polarized currents in the ZGNRs with MIS and SS, respectively. We discover that our polarized currents by calculating the conductance and show that the RMF \( B_r \) makes it clear how PLLs and RLLs behave differently on transport properties. Moreover, the valley polarization makes it clear how PLLs and RLLs behave differently on transport properties. Moreover, the valley polarization makes it clear how PLLs and RLLs behave differently on transport properties. Moreover, the valley polarization makes it clear how PLLs and RLLs behave differently on transport properties.

Thus the eigenvalue equations for the K valley (Eq. (5) in the main text) become

\[
\left[ k_x + \frac{eB_{\text{eff}}}{\hbar} \left( y + \frac{h r_+}{e B_p s_+} \right) + s_+ \frac{eB_p}{\hbar} \left( y \partial_y + \frac{1}{2} \right) \right] \psi_B^0 = \pm \psi_A^0,
\]

\[
\left[ k_x + \frac{eB_{\text{eff}}}{\hbar} \left( y + \frac{h r_+}{e B_p s_+} \right) - s_+ \frac{eB_p}{\hbar} \left( y \partial_y + \frac{1}{2} \right) \right] \psi_B = \pm \psi_A.
\]

(A4)

Eliminating \( \psi_A \), we can obtain

\[
\left[ \left( \frac{eB_{\text{eff}}}{\hbar} y \right)^2 + \frac{eB_{\text{eff}}}{\hbar s_+} \left( 2k_x s_+ + 2r_+ \frac{B_{\text{eff}}}{B_p} - \frac{eB_p}{\hbar} s_+ \right) y \right. \\
\left. + \frac{\Delta}{s_+^2} - \frac{e^2 B_p^2 s_+^2}{4\hbar^2} \right] \psi_B^0 - \frac{e^2 B_p^2 s_+^2}{2\hbar^2 s_+^2} \left( 2y\psi_B^0 + y^2\psi_B^{0*} \right) = 0,
\]

(A5)

where

\[
\Delta = \left( k_x s_+ + \frac{r_+ B_{\text{eff}}}{B_p} \right)^2 - s_+^2 e^2.
\]

(A6)

Two singularities of Eq. A5 are 0 and \( \infty \). Toward 0, Eq. A5 can be asymptotically expressed as

\[
\left( \frac{\Delta}{s_+^2} - \frac{e^2 B_p^2 s_+^2}{4\hbar^2} \right) \psi_B - \frac{e^2 B_p^2 s_+^2}{2\hbar^2 s_+^2} \left( 2y\psi_B^0 + y^2\psi_B^{0*} \right) = 0.
\]

(A7)

Eq. A7 has two independent solutions \( y^{-\frac{1}{2}} e^{-\frac{1}{2}\frac{B_{\text{eff}}}{|B_p|} |y|^{\frac{1}{2}}} \), however, because we consider the asymptotic solution toward 0, only \( y^{-\frac{1}{2}} e^{-\frac{1}{2}\frac{B_{\text{eff}}}{|B_p|} |y|^{\frac{1}{2}}} \) is acceptable. Toward \( \infty \), Eq. A5 is asymptotically expressed as

\[
\left( \frac{eB_{\text{eff}}}{\hbar} y \right)^2 \psi_B - \left( \frac{eB_p^2}{\hbar} y \right)^2 \psi_B^{0*} = 0,
\]

(A8)

and the asymptotic solution can be written as \( e^{-\frac{2}{h} B_{\text{eff}} |y|} \), where

\[
z = -\text{sgn}(B_p) \frac{2}{s_+} \frac{B_{\text{eff}}}{B_p} |y|.
\]

(A9)

Note that Eq. A9 differs from the asymptotic solution in Ref. [20] which contains solely PMF’s. In conclusion, the general solution of Eq. A5 can be written as

\[
\psi_B(y) = e^{-\frac{2}{\hbar} B_{\text{eff}} |y|} u(y).
\]

(A10)
Substitute Eq. A10 into Eq. A5, and change the variable from $y$ to $z$, we can obtain the equation:

$$zu''(z) + (\xi - z) u'(z) - \alpha u(z) = 0, \quad (A11)$$

where

$$\xi = 1 + \frac{2\hbar \sqrt{\Delta}}{e|B_p|s^+_z},$$

$$\alpha = \frac{\sqrt{\Delta} - k_z s_+ \text{sgn}(B_p B^-_p) - r_+ |B^+_p|}{e|B_p|s^+_z / \hbar} + \frac{(B^+_p + |B^-_p|) s^+_z}{2|B^-_p| s^+_z}. \quad (A12)$$

Eq. A11 is a confluent hypergeometric equation, and $\alpha$ must be a nonpositive integer to guarantee that the solution is convergent. Thus $\alpha = -n$ leads to the CLLs

$$\varepsilon^2_+ = nehv_F^2 |B^+_p| - 2k_z s_+ \frac{B^-_p}{B^-_p} + 2r_+ - \frac{eB^2_p}{h|B^-_p|} s^2_z. \quad (A13)$$

Because the third term is small, we can neglect it and obtain

$$\varepsilon^2_+ = 2nehv_F^2 |B^+_p| \left( r_+ + k_z s_+ \frac{B^-_p}{B^-_p} \right). \quad (A14)$$

Similarly, the CLLs for the $K'$ valley are

$$\varepsilon^2_- = 2nehv_F^2 |B^-_p| \left( r_- + k_z s_- \frac{B^-_p}{B^-_p} \right). \quad (A15)$$

It should be pointed out that $B^-_p = B_r - r_+ B_p$ can be zero because the directions of the RMF and PMF are opposite for the $K'$ valley. Actually, $\alpha$ for the $K'$ valley has the singular point $B^-_p = 0$, as a result, our solutions for the CLLs are invalid when the EOMF $B^-_p = 0$. Furthermore, $\varepsilon^2_-$ in Eq. A15 may be negative if the absolute value of $B^-_p$ is a small value. Therefore, our result for $\varepsilon_-$ is invalid in the vicinity of the singular point $B^-_p = 0$.

### Appendix B: Rashba spin-orbit coupling

We investigate the ZGNRs with MIS for various RSOC strengths $V_R$. It should be pointed out that since the RSOC breaks the spin degeneracy, we take into account the spin degree of freedom in this part. The band structures and conductance for $V_R = 0.02t$ and $V_R = 0.05t$ are shown in Fig. 9, demonstrating that the conductance are still quantized even if the RSOC causes the extended states. Additionally, odd plateaus appear as a result of the RSOC lifting the spin degeneration. Due to the existence of the RMFs, both the odd and even plateaus are robust against Anderson disorder. As a result, the RSOC does not invalidate the primary findings in the main paper. Additionally, we ignore the Zeeman energy in our calculations because it is negligibly small and only splits the energy band, having no impact on the physics discussed in this paper.

![Figure 9](image_url)  
FIG. 9. (a) and (b) show the band structures with the RSOC strength $V_R = 0.02t$ and $V_R = 0.05t$, respectively. (c) and (d) are the conductance corresponding to (a) and (b). In all cases, we set $B_r = 50$ T and $\eta = 0.5$. In all cases, $N_x = 30$ and $N_y = 200$.

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