The Scalar Field Effective Action for The Spontaneous Symmetry Breaking in Gravity

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Abstract

We calculate the quantum effective action for a scalar field which has been recently used for a specific kind of symmetry breaking in gravity. Our study consists of calculating the 1-loop path integral of canonical momentum and determining the renormalization conditions. We will also discuss on the new renormalization conditions to redefine the new degrees of freedom corresponding to a massive vector field.
1 Introduction

Studying the creation and formation of the universe is an important area in gravity where the spontaneous symmetry breaking can have useful application in \[1\]. Indeed, production of the particles without loss of energy and arising the gauge bosons can be regarded as the results of the symmetry breaking method. There is also a sample of symmetry breaking which gives mass to the graviton. A massive graviton leads the newtonian gravity force to fall down at large distances. In addition, studying of the massive gravity and the mutual effects between the mass and the cosmological constant and also investigating the stability and renormalizability problems are another points of interest in the massive gravity framework.

A covariant higgs mechanism method in gravity and its unitaritiy problem have also been explained by Gerard t’Hooft \[2\]. In the t’Hooft formalism, four scalar fields \(\phi^A(x)\), \(A = 1, ..., 4\), have been used to break the gauge symmetry. These fields which have the global symmetry \(so(3,1)\), enter in the action through the usual quadratic form:

\[
\sqrt{-g}g^{\mu\nu}\partial_\mu\phi^A(x)\partial_\nu\phi^A(x)
\]

In the presence of the cosmological constant, we can choose the solution in the form of \(\phi^A(x) \propto m\delta^A_\mu x^\mu\) (as a dynamical vacuum) to produce the mass terms for all gravity degrees of freedom. In this way the symmetry of diffeomorphism reduces to the global \(so(3,1)\). One of the massive degrees of freedom has a kinetic term with different sign in the Ricci scalar. Such a difference makes the classical solution unstable and also the unitaritiy failed.

There has been made some attempts to study the massive gravity solution which none of them needed to solve the unitaritiy problem \[3,4\]. However in the t’Hooft version of massive gravity, the unitaritiy problem has to be solved. In the t’Hooft method, there are six massive degrees of freedom: Five traceless \(\tilde{h}_{ij}(x)\) with definite kinetic terms. They play the role of degrees of freedom for a spin 2 particle. The remaining degree of freedom is a scalar \(u(x)\) corresponding to a scalar particle. As have pointed out above, its kinetic term is indefinite and the solution is unstable. In order to restore the unitaritiy, \(u(x)\) has to be decoupled from the other matter fields.

Although the mass of \(u(x)\) in the Fierz-Pauli approach is infinite \[4\], it is finite in the t’Hooft mechanism and therefore has to be decoupled through a different way. The proposal is to define a coupling between the matter fields and a new metric \(g^{\text{matter}}_{\mu\nu}\) which is constructed from the original metric and the Higgs scalar field. Such a coupling is defined so that \(u(x)\) doesn’t couple to any other matter field. By using this proposal, \(u(x)\) is finally decoupled from the matter fields only in the linear terms of \(g^{\text{matter}}_{\mu\nu}\).

By use of the original metric and scalar fields, Chamseddin and Mukhanov, defined a new metric as the following \[5\],

\[
H^{AB}(x) = g^{\mu\nu}(x)\partial_\mu\phi^A(x)\partial_\nu\phi^B(x)
\]
They added a new term functional of $H^{AB}$ to the Einstein and Hilbert action such that creates the Fierz and pauli mass terms. Their higgs mechanism is clean of unitarity problem and also is linearly ghost free. The action for the scalar fields that they considered is:

$$S(\phi) \propto \int (d^4x \sqrt{-g}) \left( \frac{1}{16} H^2 - 1 \right)^2 - \mathcal{T}^A_B H_A$$

Such that, $H = H^A_A$ and $\mathcal{T}^A_B = H^A_B - \frac{iA}{4} H$. In such an action, since there exist objects like $\dot{\phi}^n$, $n > 2$, we require the new canonical process to have hamiltonian and poisson brackets [6] and also to quantize the theory.

R. Fukuda and E. Kyriakopoulos, derived the effective potential through the path integral with a constraint on the zero mode field [7]. The effective potential is function of the zero mode field, and the extremum of the effective potential is the classical solution of the action. By using their method, the symmetry breaking could be understood more clearly [8, 9].

In this paper in section 2, we will use Fukuda and Kyriakopoulos method to define the spontaneous symmetry breaking in the t’Hooft mechanism. In section 3, the one-loop effective action will be obtained from the actions with different functionality of canonical momentum, like Chamseddine and Mukhanov sample. In section 4, firstly we will consider the normal renormalization conditions to practically calculate the effective action. Secondly In subsection 4.1, through the new renormalization conditions we will construct the New degrees of freedom. Such degrees of freedom which represent a non symmetric vector field in the IR scale.

## 2 Spontaneous symmetry breaking

In the quantum field theory, the spontaneous symmetry breaking puts the quantum modes on a solution of the classical equation of motion. In fact we exchange the quantum vacuum with a macroscopic state in which the classical field could be measured in. A quantum vacuum is a linear combination of all eigne states of the field. By expelling the classical solution from the linear combination, we can define the effective vacuum. By this definition, the symmetry in the effective vacuum state will be missed. For example, considering a lagrangian with even function of scalar field, we have:

$$\langle \Omega | \phi | \Omega \rangle \propto \int \mathcal{D}\phi \phi e^{iS(\phi^2)} = 0 \quad (2.1)$$

While by expelling the zero momentum mode from the path integral, the symmetry $\phi \rightarrow -\phi$ will be broken.

$$\langle \Omega' | \phi | \Omega' \rangle \propto \int \mathcal{D}\phi \phi e^{iS(\phi^2)} \delta\left(\frac{1}{V} \int dx \phi(x) - \phi_0\right) \neq 0 \quad (2.2)$$
As a result, $\langle \Omega' | \phi | \Omega' \rangle$ is the classical solution of the equation of motion of the scalar field, which could be measured in the macroscopic scale. In fact such a macroscopic scale also determines the renormalization conditions. These conditions define the observable effective objects from the fundamental unscreened objects. Through the averaging of all macroscopic states, the expectation value of the field in the original vacuum state will be obtained:

$$\langle \Omega | \phi | \Omega \rangle \propto \int d\phi_0 \langle \Omega' | \phi | \Omega' \rangle = 0 \quad (2.3)$$

For the diffeomorphism symmetry breaking in gravity, the scalar field lagrangian would be defined as a function of $H^{AB}$:

$$H^{AB} = g^{\mu\nu} \partial_\mu \phi^A \partial_\nu \phi^B$$

By choosing the classical solution $\phi^A \propto x^A$ as an effective vacuum, the diffeomorphism will be broken and the gravity will be massive. The Riemannian space $x^\mu$ becomes to a flat one $x^A$, and the symmetry of diffeomorphism reduces to the lorentz symmetry.

To consider the separation between the quantum modes and the classical solution, the partition function will be written like this:

$$Z = \int da^A db^B_\mu \int D\phi D\pi \delta(\frac{1}{V} \int dx \phi^A - a^A) \delta(\frac{1}{V} \int dx \partial_\mu \phi^B - b^B_\mu) \exp(i \int dx (\dot{\phi} \pi - \mathcal{H}(\phi, \pi))) \quad (2.4)$$

By considering $\phi^A(x) = a^A + b^A_\mu x^\mu + \chi^A(x)$, we can define $\chi^A(x)$ as a fourier expansion. The fourier coefficients are analytical functions of momentum like $\chi^A(p = 0) = 0$, $\chi^A(p \neq 0) < \infty$. By defining a Hilbert space constructed of the states that $a$ and $b$ are measured in, a usual quantum theory will be created for the fields $\chi^A$.

Of course, if the lagrangian has only the gravitational probes, the measurable classical values in the macroscopic scale are $\partial_\mu \phi^A$ which are coupled to the gravitational metric. And in such a situation, $\phi(x)$ is not measurable. But if the zero mode of $\phi(x)$ to be determined in an experiment, it is a result of a little term (functional of $\phi(x)$) that is attached to the real lagrangian. Such a little term, is omitted at the first order of approximation in the macroscopic scale.

Anyhow in the measured states with values $b^A_\mu$, we can use a generating function for obtaining the quantum quantities:

$$e^{iF(j)} = \int D\phi D\pi \delta(\frac{1}{V} \int dx \partial_\mu \phi^A - b^A_\mu) \exp(i \int dx (\dot{\phi} \pi - \mathcal{H} + iJ\phi)) \quad (2.5)$$
3 Effective action

In all lagrangians that are quadratic functions of time derivative of fields, it can be shown that:

$$\int D\pi e^{i\int dx(\dot{\phi}\pi - \mathcal{H}(\phi, \pi))} = e^{iS(\phi, \dot{\phi})}$$  \hspace{1cm} (3.1)

While the lagrangians are used to break the gauge symmetry in the gravity are mainly functions of the higher power of time derivative, therefore equation (3.1) loses its efficiency. In such lagrangians to use the path integral method, the hamiltonian and the canonical momentum have to be obtained and the integration over the canonical momentum has to be calculated. In the general form, we can calculate the path integral via the perturbation method around the extremum point of the term inside the exponent:

$$\dot{\phi}(x, \pi(x)) = -\frac{\delta}{\delta \pi_A(x)} \int dx \mathcal{H}(\phi, \pi) = 0$$  \hspace{1cm} (3.2)

That the extremum point of $\pi_A(x)$ is arrived from the time derivative of the scalar fields.

Through expansion of $\pi_A(x)$ around the extremum point, we can obtain the generating function at the one-loop order:

$$e^{i F(J)} = \int D\phi e^{S(\phi, \dot{\phi}) + i \int dx J(x) \phi(x)} \int D\pi \exp -i \int dx dy (\pi_A(x) - \pi_A(\phi, \dot{\phi})) \delta^2 \int dz \mathcal{H}(\phi, \pi) \delta \pi_A(x) \delta \pi_B(x) \big|_{\pi(\phi, \dot{\phi})} \big|_{\pi_B(x) - \pi_B(\phi, \dot{\phi})}$$  \hspace{1cm} (3.3)

It has to be attended to the first integral that has been written only in the space of the fourier modes of $\phi(x)$. In the equation (3.3), the term $\text{Ln} \text{det} \left( \frac{\delta^2 \int dz \mathcal{H}(\phi, \pi)}{\delta \pi_A(x) \delta \pi_B(y)} \right)$ will be added to the action. We can derive this term, through the action and its functionality of the fields:

$$\delta^2 \int dx \mathcal{H}(\phi, \pi) \big|_{\pi(\phi, \dot{\phi})} = \frac{\delta}{\delta \pi_A(x)} \phi^B(\phi(y), \pi(y)) \big|_{\pi(\phi, \dot{\phi})}$$  \hspace{1cm} (3.4)

Take attention to $\dot{\phi}^B(x)$ in the right hand side that is defined independent of $\phi^A(x)$ and is not time derivative. And also we have the definition of the independent canonical momentum:

$$\pi_A(\phi, \dot{\phi}) = \frac{\delta S(\phi, \dot{\phi})}{\delta \dot{\phi}^A}$$  \hspace{1cm} (3.5)

As regards, $\dot{\phi}^A(x)$ and $\pi_A(x)$ are supposed to be independent of the scalar fields, therefore by use of the chain rule of the functional derivatives we can write:

$$\int dz \frac{\delta \phi^B(\phi(x), \pi(x))}{\delta \pi_A(z)} \big|_{\pi(\phi, \dot{\phi})} \times \frac{\delta \pi_A(\phi(z), \dot{\pi}(z))}{\delta \dot{\phi}^C(y)} = \delta^B \delta(x - y)$$  \hspace{1cm} (3.6)
L_{\text{det}}(\frac{\delta^2}{\delta\pi_A(x)\delta\pi_B(y)}) = -L_{\text{det}}(\frac{\delta^2 S(\phi, \dot{\phi})}{\delta\phi^A(x)\delta\phi^B(y)}) \quad (3.7)

If in the left hand side of the present equation, \pi_A(x) goes to the extremum point which is defined in the equation (3.2), thus \dot{\phi}_A(x) in the right hand side presents as the time derivative of the scalar field. Then by defining the effective action:

\[ \Gamma(\varphi(x)) = F(J(x)) - \int dx J(x)\varphi(x) \quad (3.8) \]

and using the equations (3.7) and (3.3), we will find the effective action at one-loop order:

\[ \Gamma^{(1)}(\varphi) = S(\varphi) + \frac{i}{2} tr Ln(\frac{\delta^2 S(\phi)}{\delta\phi^2}) |_{\varphi} - \frac{i}{2} tr Ln(\frac{\delta^2 S(\phi, \dot{\phi})}{\delta\dot{\phi}^2}) |_{\dot{\varphi}} + \delta S^{(1)}(\varphi) \quad (3.9) \]

The first trace is defined in the fourier space as already reminded. And \delta S^{(1)}(\varphi) is the one-loop ordered counterterm added to the renormalized action.

### 4 Renormalization conditions

Usually, for calculation of the effective action, people have used the assumption that the effective fields are constant. And as a result the continuum matrix inside the trace has been diagonalized. While in our paper the scalar fields cannot be constant values and the derivation of the effective action seems to be difficult.

By the way, as an important property, the effective action is a generating function. And we can derive the n-point functions through the functional derivatives of the effective action. An important problem that we encounter in calculating the effective action and n-point functions is renormalization, particularly in the Lagrangians which are functions of \( H^{AB} \) and have the nonrenormalizable monomials.

The first renormalization condition could be assumed is accepting the common equation of motion between the effective action and the renormalized action:

\[ \frac{\delta \Gamma(\varphi(x))}{\delta \varphi(x)} |_{\varphi=b} = 0 \quad (4.1) \]

Inserting (3.9) in (4.1), the present equation will be produced:

\[ \frac{i}{2} tr[(\delta^2 S/\delta\varphi^2)^{-1} \frac{\delta}{\delta\varphi(x)} (\delta^2 S/\delta\dot{\varphi}^2)] |_{\varphi=b} = \frac{i}{2} tr[(\delta^2 S/\delta\varphi^2)^{-1} \frac{\delta}{\delta\varphi(x)} (\delta^2 S/\delta\dot{\varphi}^2)] |_{\varphi=b} + \frac{\delta}{\delta\varphi(x)} \delta S^{(1)} |_{\varphi=b} = 0 \quad (4.2) \]
Since the renormalized action is a local function, standing continuum matrices like
\[ G_{xy}^{AB} = \left( \frac{\partial^2 S}{\partial \phi^A(x) \partial \phi^B(y)} \right)^{-1} \] on the point \( \partial \phi = b \), leads them to be diagonalized in the
fourier space. For example:
\[ \alpha(b)p^2 G(p) = 1 \quad (4.3) \]
And the term \( \frac{\Delta}{\delta \phi} \delta S^{(1)} |_{\partial \phi = b} \), simply will be calculated in the fourier space. And it
seems that for the actions with even function of fields, this term is trivially zero,
\[ \beta(b) \delta^{\mu}_A \int dp \, p^\mu = 0. \] But there are many different monomials that are not considered
in the condition (4.1), and to determine them we need more conditions like:
\[ \int dx_1 \cdots dx_{n-1} \frac{\delta^n \Gamma(\phi)}{\delta \phi(x_n) \cdots \delta \phi(x_1)} |_{\partial \phi = b} = \int dx_1 \cdots dx_{n-1} \frac{\delta^n S(\phi)}{\delta \phi(x_n) \cdots \delta \phi(x_1)} |_{\partial \phi = b} \quad (4.4) \]
And the written equations are trivial for odd \( n \). These equations can be solved in
the fourier space because the continuum matrices on the point \( \partial \phi = b \) are diagonal
as before.

And the point \( x_n \) has remained outside integrals because of the translational
symmetry of the connected and 1PI n-point functions. It means the same as the
momentum conservation in the fourier space.

In fact these conditions are written for the 1PI n-point functions with zero mo-
mentum legs in the fourier space. Therefore these conditions determine the Feynman
vertices between interacting particles which has been probed by an observer in the
IR scale.

If the bare action has the linear symmetry \( \phi^A(x) \rightarrow \phi^A(x) + a^A \), it is trivial that
the effective action has the same symmetry too. And also we can find the effective
action as a functional of \( \partial^\mu \phi^A \), and the observed object is \( \partial^\mu \phi^A \) which is coupled to
the metric.

### 4.1 Broken gauge symmetry action for the new degrees of
freedom

Assume a bare action that the renormalization conditions cause the counterterm
monomials which are constructed by both \( \phi^A \) and \( \partial^\mu \phi^A \). In this case, in the
points other than the classical solution, the effective action loses the symmetry
\( \phi(x) \rightarrow \phi(x) + a \). And even if we have \( \partial^\mu \phi^A \) as classical measurable values, but the
quantum degrees of freedom of the effective action are values of the scalar fields in
all points.

And now if we want to have the quantum degrees of freedom analogous to the
classical measurable values in the IR scale, it has been required that the effective
action to be functional of only \( \partial^\mu \phi \) and also to have the symmetry \( \phi \rightarrow \phi + a \). For
this purpose, the renormalization conditions have to be redefined. By adding an integral in the free coordinate $x_n$ to the equations (4.4) and choosing a $\varphi$ other than the classical solution, we will have these conditions:

$$\int dx_1 \cdots dx_n \frac{\delta^n \Gamma(\varphi)}{\delta \varphi(x_n) \cdots \delta \varphi(x_1)} = \int dx_1 \cdots dx_n \frac{\delta^n S(\varphi)}{\delta \varphi(x_n) \cdots \delta \varphi(x_1)} = 0$$

(4.5)

In which are derived from the Taylor series of $\Gamma(\varphi + a)$ around $a = 0$. Indeed we can use the equation $\int dx \frac{\delta \Gamma(\varphi)}{\delta \varphi(x)} = 0$ which applied for the all possible $\varphi(x)$, instead of using the equations (4.5) that are used for a selected $\varphi(x)$. On this approach, the symmetry $\varphi \rightarrow \varphi + a$ will be appear in the effective action again. Now we can write $\Gamma(\varphi)$ as a functional of $\nu^A_{\mu}(x) = \partial_{\mu} \varphi^A(x)$ and define $\nu^A_{\mu}(x)$ as the effective degrees of freedom. By this definition, we can have:

$$\frac{\delta^n \Gamma(\varphi)}{\delta \varphi(x_1) \cdots \delta \varphi(x_n)} = \partial_{\mu_1} \cdots \partial_{\mu_n} \frac{\delta^n \Gamma(\nu)}{\delta \nu_{\mu_1}(x_1) \cdots \delta \nu_{\nu}(x_n)}$$

(4.6)

In which are consistent to the equations (4.5). It has to be attended to the renormalization conditions (4.5) that eliminate only the nonsymmetric monomials. To renormalize the remain monomials, we need more conditions. Assume that an observer probes the effective vector fields $\nu^A_{\mu}(x)$ as the experimental objects. By considering $\nu^A_{\mu}(x) = 0$ as the expectation values in the effective vacuum state, we can define the new action for such vector fields. At first we define the mass $m$ for such vector fields (without gauge symmetry):

$$\int dx e^{ip(x-y)} \frac{\delta^2 \Gamma(\nu)}{\delta \nu_{\mu}(x) \delta \nu_{\nu}(y)} |_{p^2=m^2} = 0$$

(4.7)

At second, the coupling constants $g_n$ in the new action (interaction amplitudes between recent experimental objects) will be obtained through the following equations:

$$g_n = \int dx_1 \cdots dx_{n-1} e^{ip_{1}x_1} \cdots e^{ip_{n-1}x_{n-1}} e^{ip_n x_n} \frac{\delta^n \Gamma(\nu)}{\delta \nu_{\mu}(x_1) \cdots \delta \nu_{\nu}(x_n)} |_{\text{on-shell}} \quad n \geq 3$$

(4.8)

Therefore we have the scalar fields that behave like the massive vector fields in the IR scale.

If the degrees of freedom like $g_{\mu\nu} : \eta_{AB} \partial_{\mu} \varphi^A \partial_{\nu} \varphi^B$ (one of the metric $g_{\mu\nu}^{\text{matter}}$ introduced by t Hooft) can be defined in a more advanced way, we can have an arbitrary massive gravity action. Through choosing the correct mass terms, we can conserve the unitarity. In fact the one particle state of such an effective gravity theory will be occurred in the bounding states of the defined vector field theory.

5 Conclusion

The first purpose in this letter, was studing the quantum theory of the symmetry breaking in the t’Hooft method [2]. In the path integrals, we separated the usual
fourier modes from the classical solution. By solving the new form of the path integral of the canonical momentum, we obtained the effective action. Then the renormalization conditions were implemented on the effective action.

The next purpose, was finding the new degrees of freedom corresponded to the coordinate derivative of the fields. Such new degrees of freedom define a quantum massive vector field theory in the IR scale. Through studding the new renormalization conditions, we discussed on the recent purpose.

From now on, one can select a real sample in the actions to continue more practically these present computations. And also, the effective action of the gravity could be calculated and added to the present effective action of the scalar fields. As well, one can obtain the new degrees of freedom from the original scalar fields to offers them as the quantum massive gravity in the IR scale.

Acknowledgments
I would like to thank dear Navid Abbasi for his useful discussions and comments.

References

[1] R.Brout, F.Englert, E.Gunzig, The creation of the universe as a quantum phenomenon, Annals of Physics. Vol 115, page 78.

[2] Gerard t’Hooft, Unitarity in the Brout-Englert-Higgs mechanism for gravity, arXiv:0708.3184.

[3] H. van Dam, M. J. G. Veltman, Massive and massless Yang-Mills and gravitational fields, Nucl. Phys. B Vol 22, page 397.

[4] M. Fierz and W. Pauli, On relativistic wave equations for particles of arbitrary spin in an electromagnetic field, Proc. Roy. Soc. Lond. A 173, 211 (1939).

[5] A.H.Chamseddine, V.Mukhanov, Higgs for graviton: simple and elegant, JHEP 1008, 011 (2010).

[6] J.Kluson, Hamiltonian analysis of the higgs mechanism for graviton, Class.Quant.Grav.28:155014,2011

[7] R. Fukuda, E. Kyriakopoulos, Derivation of the effective potential, Nuclear Physics B85 (1975) 354-364.

[8] L. O’Raifeartaigh, A. Wipf and H. Yoneyama, The constraint effective potential, Nuclear Physics B271 (0986) 653-680.
[9] A. Ringwald, C. Wetterich, Average action for the N-component $\varphi^4$ theory, Nuclear Physics B334 (1990) 506-526.

[10] R. Percacci, The higgs phenomenon in quantum gravity, Nuclear Physics B. Vol 353, page 271.

[11] S. Weinberg, The quantum theory of fields, Cambridge University Press, 1995.

[12] A. I. Vainshtein, To the problem of nonvanishing gravitation mass, Phys. Lett. B 39, 393 (1972).

[13] C. Deffayet, G. R. Dvali, G. Gabadadze and A. I. Vainshtein, Nonperturbative continuity in graviton mass versus perturbative discontinuity, Phys. Rev. D 65, 044026 (2002).

[14] D. G. Boulware and S. Deser, Can gravitation have a finite range?, Phys. Rev. D 6, 3368 (1972).

[15] C. J. Isham, A. Salam and J. A. Strathdee, F-dominance of gravity, Phys. Rev. D 3, 867 (1971).

[16] C. de Rham, A manifestly gauge-invariant realization of massive gravity, Phys. Lett. B 688, 137 (2010).

[17] N. Arkani-Hamed, H. Georgi and M. D. Schwartz, Effective field theory for massive gravitons and gravity in theory space, Annals Phys. 305 (2003) 96.

[18] Kurt Hinterbichler, Theoretical aspects of massive gravity, arXiv:1105.3735 [hep-th] 2 Oct 2011.