Scattering of the field of a multi-element phased array by human ribs

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Abstract. The efficacy of high intensity focused ultrasound (HIFU) for the non-invasive treatment of cancer has been demonstrated for a range of different cancers including those of the liver, kidney, prostate and breast. As a non-invasive focused therapy, HIFU offers considerable advantages over other techniques such as chemotherapy and surgical resection, in terms of invasiveness and risk of harmful side effects. Despite its advantages, however, there are a number of significant challenges currently hindering its widespread clinical application. One of these challenges is the need to transmit sufficient energy through the ribcage to induce tissue necrosis at the required foci whilst minimising the formation of side lobes. Multi-element random arrays are currently showing great promise in overcoming the limitations of single-element transducers. Nevertheless, successfully treating a patient for liver tumours requires a thorough understanding of the way in which the ultrasonic pressure field from a HIFU array is scattered by the ribcage. A mesh of quadratic pressure patches was generated using CT scan data for ribs nine to twelve on the right side. A boundary element approach based on a Generalised Minimal Residual (GMRES) implementation of the Burton-Miller formulation was used, in conjunction with phase conjugation techniques to focus the field of a 256-element random HIFU array past the ribs at both intercostal and transcostal treatment locations. This method has the advantage of accounting for full effects of scattering and diffraction in three dimensions under continuous wave excitation.

1. Background
The efficacy of high-intensity focused ultrasound (HIFU) has been demonstrated for a range of cancers [1], with over 38000 prostate cancer treatments and over 50000 treatments for abdominal tumours, osteosarcoma, uterine fibroids and thyroid worldwide [2]. HIFU offers considerable advantages over techniques such as chemotherapy and surgical resection, in terms of its non-invasiveness and low risk of harmful side effects. There nevertheless are a number of significant challenges currently hindering its more widespread clinical application. In the cases of liver, kidney and pancreatic cancers, one of these challenges is the need to transmit sufficient energy through the rib cage to induce tissue necrosis at the required location whilst minimising the formation of side lobes. In
addition to this, a common side effect of focusing ultrasound in regions located behind the rib cage is the overheating of bone and surrounding tissue, which can lead to skin burns [3], [5].

Multi-element random spherical arrays are showing promise in overcoming the limitations of single-element transducers, such as side lobe formation, lack of electronic steering and lack of beam shaping capabilities [5], [6], [7], [8], [9]. Successfully treating hepatic, renal and pancreatic cancers nevertheless requires a thorough understanding of the way in which the ultrasonic pressure field generated by a HIFU array is scattered by the rib cage. Hence, an important part of the treatment planning procedure is likely to involve a model capable of predicting the pressure distribution both on the ribs and in the surrounding tissue.

Theoretical and experimental feasibility and efficacy of transcostal HIFU (THIFU) has been extensively investigated. Given the considerable computational challenges involved at Megahertz frequencies for large domain dimensions, many models have relied on simplified shadowing techniques [10], [11]. Although these techniques predict features of wave propagation during THIFU treatments, they fail to address the actual scattering and diffraction mechanisms involved in complex 3D structures. As such, they are of limited value to treatment planning applications. An experimental method based on the decomposition of the time reversal operator (DORT) goes some way towards addressing challenges in treatment planning [12]. It was implemented on human ribs immersed in water and shows promise in sparing the ribs from the incident ultrasonic field. A 3D model capable of reproducing these results theoretically would considerably further the understanding of HIFU fields in the presence of scatterers and would constitute a valuable tool for optimisation of phased array design and performance.

Finite difference time domain (FDTD) methods of modelling HIFU fields scattered by ribs have been implemented in 2D, in conjunction with the time reversal method [8]. These techniques have the advantage of being relatively straightforward to realise for purely acoustic problems, can be generalised to 3D and can be readily implemented on a computer cluster. They nevertheless possess a number of disadvantages. These include having to mesh the entire computational domain, dealing with artefacts due to domain truncation and relying on pulsed wave solutions. There is clearly a requirement to implement a three-dimensional continuous wave model of a multi-element HIFU array irradiating a scatterer of arbitrary shape. Exterior boundary element (BE) methods have been successfully applied to a range of acoustic scattering problems. They rely on the discretisation of the Helmholtz integral equation. BE methods have the advantage of being a frequency domain technique and are therefore suitable for simulating the quasi-continuous wave conditions encountered in HIFU. Provided the medium surrounding the scatterer can be treated as homogeneous, it is not necessary to mesh any portion of the exterior domain. Also, it is not necessary to truncate the exterior domain as in FDTD methods.

The discretisation of the Helmholtz integral equation leads to a fully populated asymmetrical system of linear equations. The conventional Helmholtz boundary integral equation (BIE) formulation for exterior wave problems yields non-unique solutions at the eigenfrequencies associated with the (interior) Dirichlet problem. When the coefficient matrix resulting from discretisation is inverted at frequencies approaching these (fictitious) eigenfrequencies, the problem becomes ill-conditioned. Burton and Miller proposed a method that guarantees uniqueness of the solution [13], which involves a formulation employing a linear combination of the conventional BIE and its derivative with respect to the outward normal. Provided the coupling coefficient has a non-zero imaginary part, this method provides unique solutions at all frequencies. Computational limitations have traditionally hampered BE methods from becoming widespread. Inverting a large, fully populated asymmetrical matrix on a stand-alone computer is impractical when considering the memory allocation necessary to store all elements simultaneously. This challenge was addressed by Oehmann et al by using an iterative...
generalised minimum residual (GMRES) based boundary element solver for acoustic scattering [14]. Shen and Liu investigated a range of scattering problems using an adaptive fast multipole BE method, also using an iterative GMRES scheme [15]. The GMRES method is amenable to cluster computing, as it performs matrix-vector multiplications at each iteration. Thus, employing the GMRES scheme offers the opportunity to significantly reduce run times.

In this paper, a feasibility study is described in which a BE approach was used to model the scattering of the field of a multi-element HIFU phased array by human ribs. The array was modelled using a linear superposition of plane circular piston sources. The ribs were assumed to be perfectly rigid. The surrounding medium was assumed to be homogeneous and inviscid. After validation on spherical scatterers, a mesh of quadratic pressure patches was generated using CT scan data for ribs nine to twelve on the right side of an adult male. A GMRES implementation of the Burton-Miller formulation based on the method proposed in [16] was used, in conjunction with phase conjugation techniques [17] to focus the field of a 256-element random HIFU array 3 cm deep behind the rib cage at locations accessible either inter- or trans-costally. The calculations were distributed over a number of cores on a dedicated computer cluster at University College London Centre for Medical Image Computing.

2. Theory

Consider an exterior domain \( V_{ext} \) bounded by a surface \( \Lambda \) and by a closed smooth surface \( S \). Let \( \Lambda \) be at a sufficiently large distance from the acoustic sources and from the surface \( S \). Let the boundary condition on \( \Lambda \) satisfy Sommerfeld’s acoustic radiation condition. The propagation of time-harmonic acoustic waves in a homogeneous isotropic inviscid medium is described by the Helmholtz equation:

\[
\nabla^2 p(\vec{r}) + k^2 p(\vec{r}) = 0, \quad \forall \vec{r} \in V_{ext} \tag{1}
\]

where \( p \) is the acoustic pressure, \( k = \omega / c \) is the acoustic wave number where \( \omega \) is the angular frequency, \( c \) the sound speed in the external medium and \( \vec{r} \) is the position vector. A source term may be included in the right hand side of (1) (incident pressure wave), but it is convenient to split the total pressure \( p(\vec{r}) \) as the sum of the incident pressure \( p_i(\vec{r}) \) and the pressure scattered by the surface \( S \), \( p_s(\vec{r}) \), \( \forall \vec{r} \in V_{ext} \).

For scattering problems, the integral representation of the solution to (1) is given by the Helmholtz integral equation:

\[
\int_S \left[ p(\vec{r}_q) \frac{\partial G(\vec{r} | \vec{r}_q)}{\partial n_q} - \frac{\partial p(\vec{r}_q)}{\partial n_q} G(\vec{r} | \vec{r}_q) \right] dS = \frac{p(\vec{r})}{2} - p_i(\vec{r}), \quad \forall \vec{r} \in S \tag{2}
\]

where the Green’s function in a three-dimensional space is given by:

\[
G(\vec{r} | \vec{r}_q) = \frac{1}{4\pi |\vec{r} - \vec{r}_q|} e^{-i|\vec{r} - \vec{r}_q|} \tag{3}
\]

and \( p(\vec{r}_q) \) is the pressure on the surface \( S \) at point \( \vec{r}_q \) and \( \vec{n}_q \) is the outward normal.

The problem as described by the integral equation (2) suffers from non-uniqueness at frequencies of excitation approaching an eigenvalue of one of the (fictitious) modes of the cavity inside the
scatterer. In cases where the dimensions of the scatterer are large compared with the wavelength in the propagating medium, such as in HIFU applications, this is likely to occur. The matrix formed by discretizing equation (2) then becomes close to singular. The method which appears to offer the best compromise in terms of application of the Helmholtz integral equation to exterior acoustic problems involving scatterers of arbitrary shape remains the Burton-Miller formulation [18], which solves for a linear combination of equation (2) and its derivative with respect to the outward normal vector on $S$ at $\vec{r}$.

By discretising a linear combination of equation (2) and its normal derivative, for all position vectors $\vec{r}$ on $S$ corresponding to each node on the mesh of the surface, a linear system of equations may be generated.

$$\begin{bmatrix} [H] & -[G] \frac{\partial p}{\partial n} \end{bmatrix} + \begin{bmatrix} p \end{bmatrix} + \alpha \begin{bmatrix} \frac{\partial p}{\partial n} \end{bmatrix} = \{0\}$$

(4)

For a rigid boundary, the normal derivative vector is $\{0\}$.

By knowledge of the incident pressure field and its normal derivative on $S$, together with $[H]$ and $[G]$, $\{p\}$ may be obtained through matrix inversion if the problem size is small enough. For larger problems, where storing all the elements of $[H]$ and $[G]$ simultaneously becomes challenging, the GMRES method may be used. The pressure $p(\vec{r})$ at any location in the exterior volume $V_{ext}$ may be obtained at the post-processing stage.

The HIFU transducer modelled as part of the underlying work was assumed to be spherically shaped and to contain a central aperture (used to insert an ultrasonic diagnostic treatment head). The HIFU array was assumed to be populated with $N = 256$ plane circular elements mounted onto its surface. Guidelines regarding the dimensions involved were obtained from the literature and from prior in-vivo applications [5], [11], [19]. The elements were each of $a = 3$ mm radius. Larger values for $a$ will result in an increased likelihood of scattering from the ribs whilst smaller values would be difficult to manufacture and may result in insufficient acoustic power generation to induce tissue necrosis. A radius of curvature of $D = 18$ cm was used, to ensure the applicability to deep-seated tumours. The diameter of the central aperture was chosen to be $4$ cm, allowing for an ultrasound imaging transducer to be housed. The outer diameter of the HIFU transducer was chosen as $16$ cm. The elements were randomly spatially distributed on the surface of the array. A frontal view of the array is shown in figure 1. The incident pressure field $p_i$ in equation (4), i.e. the field produced by the HIFU array in absence of scatterers, was modelled as a superposition of plane circular piston sources, assumed to be rigidly vibrating in an infinite baffle.
3. Results
The GMRES Burton-Miller boundary element approach was implemented on a dedicated computer cluster. The routines employed to generate the boundary element matrices were obtained from the PAFEC (Program for Automatic Finite Element Calculations) VibroAcoustics software with permission from PACSYS Ltd. Validation on simple scatterers was carried out prior to simulations on human ribs. The results displayed here are for a 1 MHz unit amplitude plane wave scattered by a 0.5 cm radius rigid sphere. These were compared against the analytical solution provided in [20] in the shadow zone between 1.1 cm and 5.0 cm (see figure 2).
Figure 2. GMRES implementation of Burton-Miller BE formulation on a spherical scatterer of 0.5 cm radius. Incident field: unit amplitude 1 MHz plane wave travelling in positive x direction. Comparison against analytical solution in shadow zone.

The scatterer was meshed using isoparametric eight-noded quadratic patches ensuring at least three elements per wavelength for a wavespeed of 1500 m s\(^{-1}\). The density of the medium was assumed to be 1000 kg m\(^{-3}\). The coupling coefficient in the Burton-Miller formulation was chosen as \( \alpha = 0.1i/k \), as this resulted in quicker convergence of the solution than using \( i/k \), as recommended in [21]. In all cases of using the GMRES scheme, the initial trial pressure vector was chosen as the incident field at nodal positions on the surface of the scatterer. 40 iterations were generally sufficient for the residual norm to fall below 1% of its initial value. Figure 2 shows agreement within 1.2% of the analytical solution.

Using CT scan data obtained from [22] for ribs nine to twelve on the right side of an adult male, a surface mesh was obtained using isoparametric eight-noded quadratic elements, ensuring at least three elements per wavelength for a wavespeed of 1500 m s\(^{-1}\). The density of the medium was assumed to be 1000 kg m\(^{-3}\). CATIA v5 Advanced Meshing Tools was used to generate the mesh, which contained 250370 nodes, corresponding to 83454 surface patches.

Two treatment locations positioned at an intercostal and transcostal space approximately 3 cm deep into the rib cage were selected. It was assumed that the HIFU array was positioned so that its geometric focus coincided with the treatment location. These arrangements are displayed in figures 3-5. To focus the beam into the rib cage, the phase conjugation method was used, which involved two stages.
**Figure 3.** Intercostal treatment between ribs 10 and 11 on right side, approximately 3 cm deep into rib cage (view 1).
Figure 4. Intercostal treatment between ribs 10 and 11 on right side, approximately 3 cm deep into rib cage (view 2).
Figure 5. Transcostal treatment approximately 3 cm from rib 10 into rib cage, on right side.

In the first stage, a calculation was carried out in which the incident pressure field was that of a unit source strength point source positioned at the treatment location. The scattered pressure on the rib cage was then computed employing the GMRES implementation of the Burton-Miller formulation and the acoustic pressures at field locations corresponding to points on the surface of the HIFU phased array elements was calculated at a post-processing stage. The total pressure was then averaged over the surface of each element, and the complex conjugate of each average was calculated. These quantities were then normalised so as to produce 256 source velocities generating a peak acoustic pressure of 10 MPa at the geometric focus in absence of the scatterers.

In the second stage, the incident field was computed from the linear combination of the pressures resulting from the 256 source velocities obtained from the point source calculation. The GMRES implementation of the Burton-Miller BE formulation was then used to obtain the scattered pressure on the ribs. Pressure values within a 10 cm³ cubic volume, centred around the treatment region field locations, were then obtained at the post-processing stage.

Figure 6 shows the total pressure field for the intercostal treatment in successive planes parallel to the x-y plane, of the phased array scattered by the ribs after applying the phase conjugation technique and scaling the source velocities so as to produce 10 MPa at the geometric focus in absence of the scatterers. Although there is a slight decrease in the peak pressure, the methodology appears to work well based these results. This particular configuration of ribs features intercostal spacing of around 2.5 cm and rib widths of around 1.5 cm, which allows the majority of the ultrasonic energy to be delivered in the vicinity of the geometric focus. Figure 7 shows the corresponding acoustic pressure distribution.
on the surface of the ribs showing maximum pressures over 2.2 MPa, which could potentially be a cause for concern in terms of potential bone and tissue heating.

http://www.npl.co.uk/acoustics/ultrasound/figure-6

**Figure 6.** Total acoustic pressure field of 1 MHz random phased HIFU array for intercostal treatment location approximately 3 cm deep into rib cage between ribs 10 and 11 on right side. Source velocity distribution obtained using phase conjugation based on point source calculation in Fig.8 (colour bar in MPa).

![Figure 6](image)

**Figure 7.** Total acoustic pressure on surface of ribs 9 to 12 on right side resulting from field of 1 MHz random phased HIFU array for intercostal treatment location between ribs 10 and 11 on right side approximately 3 cm deep into rib cage.

Figure 8 shows the total pressure field for the transcostal location in successive planes parallel to the $x$-$y$ plane, of the phased array scattered by the ribs after applying the phase conjugation technique and scaling the source velocities so as to produce 10 MPa at the geometric focus in absence of the scatterers. Again, despite a slight reduction in the peak pressure at the focus, the method appears to work well, owing to the large intercostal spacing when compared with the rib cross-section. Figure 9 shows the corresponding acoustic pressure distribution on the surface of the ribs showing maximum pressures just over 1.8 MPa.

http://www.npl.co.uk/acoustics/ultrasound/figure-8
**Figure 8.** Total acoustic pressure field of 1 MHz random phased HIFU array for transcostal treatment location approximately 3 cm deep into rib cage behind rib 10 on right side. Source velocity distribution obtained using phase conjugation based on point source calculation in Fig.13 (colour bar in MPa).

**Figure 9.** Total acoustic pressure on surface of ribs 9 to 12 on right side resulting from field of 1 MHz random phased HIFU array for transcostal treatment location approximately 3 cm deep into rib cage behind rib 10 on right side.

4. **Conclusions**
An approach based on a GMRES implementation of the Burton-Miller BE formulation was developed to model the scattering of the field of a multi-element random HIFU phased array by 3D objects of an arbitrary geometry under continuous wave excitation. The BE code was run on a dedicated computer cluster and the problem distributed over a number of cores. After validation on spherical scatterers, a mesh of quadratic pressure patches was generated using CT scan data for ribs nine to twelve on the right side of an adult male. Intercostal and transcostal treatments were simulated respectively between ribs 10 and 11, and behind rib 10, at locations approximately 3 cm deep into the rib cage. The phase conjugation method was used to focus the beam at the required treatment location. Acoustic pressure maps were obtained in 10 cm$^3$ cubic volumes centred around each treatment region. For the HIFU array and rib topology investigated, the method successfully focused the beam at the required locations, and no effects of splitting at the focus were observed. Nevertheless, maximum acoustic pressures predicted at the ribs were of the order of a quarter of those at the focus, which may need to
be addressed from the point of view of treatment planning. To further validate the model, calculations are required on periodic structures consisting of rectangular cuboids, with rounded spherical edges, where it is hoped that effects of focus splitting consistent with those reported in [11].

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References
[1] Crum L, Bailey M, Hwang J H, Khokhlova V and Sapozhnikov 2010 O Therapeutic ultrasound: Recent trends and future perspectives Physics Procedia 3 25
[2] Communication with Gail ter Haar, Therapeutic Ultrasound Group, Physics Department, Institute of Cancer Research, Sutton, UK. March 2011
[3] Wu F, Zhi-Biao W, Wen-Zhi C, Hui Z, Jin B, Jian-Zhong Z, Ke-Quan L, Cheng-Bing J, Fang-Lin X, Hai-Bing S 2004 Extracorporeal high intensity focused ultrasound ablation in the treatment of patients with large hepatocellular carcinoma Ann. Surg. Oncol. 11 1061
[4] Li J-L, X-Zh Liu, Zhang D, Gong X-F 2007 Influence of ribs on nonlinear sound field of therapeutic ultrasound. Ultrasound Med. Biol. 33 1413
[5] Gavrilov L R, Hand J W 2000 A theoretical assessment of the relative performance of spherical phased arrays for ultrasound surgery IEEE Trans. Ultrason. Ferroelectr. Freq. Control 47 125
[6] Pernot M, Aubry J F, Tanter M, Thomas J L, Fink M 2003 High power transcranial beam steering for ultrasonic brain therapy Phys. Med. Biol. 48 2577
[7] Tanter M, Pernot M, Aubry J F, Montaldo G, Marquet F, Fink M 2007 Compensating for bone interfaces and respiratory motion in high intensity focused ultrasound Int. J. Hyperthermia 23 141.
[8] Aubry J-F, Pernot M, Marquet F, Tanter M, Fink M 2008 Transcostal high-intensity-focused ultrasound: ex vivo adaptive focusing feasibility study Phys. Med. Biol. 53 2937
[9] Hand J W, Shaw A, Sadhoo N, Dickinson R J, Gavrilov L R 2009 Initial testing of a prototype phased array device for delivery of high intensity focused ultrasound (HIFU) Phys. Med. Biol. 54 5675
[10] Botros Y Y, Ebbini E S, Volakis J L 1998 Two-step hybrid virtual array-ray (VAR) technique for focusing through the ribcage IEEE Trans. Ultrason. Ferroelectr. Freq. Control 45 989
[11] Bobkova S, Gavrilov L, Khokhlova, Shaw A and Hand J. Focusing of high-intensity ultrasound through the rib cage using a therapeutic random phased array 2010 Ultrasound Med. Biol. 36 888
[12] Cochard E, Prada C, Aubry J F, Fink M. Ultrasonic focusing through the ribs using the DORT method 2009 Med. Phys. 36 3495
[13] Burton A J and Miller G F 1971 The application of integral equation methods to the numerical solution of some exterior boundary-value problems Proc. R. Soc. London Ser. A323 201
[14] Oehmann M, Homm A, Makarov S and Semenov S 2003 An iterative GMRES-based boundary element solver for acoustic scattering Eng. Anal. Bound. Elem. 27 717
[15] Shen L and Liu Y J 2007 An adaptive fast multipole boundary element method for three-dimensional acoustic wave problems based on the Burton-Miller formulation Comput. Mech. 40 461
[16] Liu Y and Rizzo F J 1992 A weakly singular form of the hypersingular boundary integral equation applied to 3-D acoustic wave problems Comput. Method Appl. M. 96 271
[17] Zel’dovich B Y, Pilipetsky N F, and Shkunov V V 1985 Principles of Phase Conjugation. Berlin. CO: Springer-Verlag

[18] Chien C C, Rjiyah H and Atluri S N 1990 An effective method for solving the hypersingular integral equations in 3-D acoustics J. Acoust. Soc. Amer. 88 343

[19] Visioli A G, Rivens I H, ter Haar G R, Horwich A, Huddart R A, Moskovic E, Padhani A, Glees J 1999 Preliminary results of a phase I dose escalation clinical trial using focused ultrasound in the treatment of of localized tumours Eur. J. Ultrasound 9 11

[20] Morse P M and Ingard K U, Theoretical Acoustics 1968 New York. CO: McGraw-Hill 419

[21] Kress R 1985 Minimizing the condition number of boundary integral operators in acoustic and electromagnetic scattering Q. J. Mech. Appl. Math. 38 323

[22] Christ A, Kainz W, Hahn E G, Honegger K, Zefferer M, Neufeld E, Rascher W, Janka R, Bautz W, Chen J, Kiefer B, Schmitt P, Hollenbach H-P, Shen J, Oberle M, Szczerba D, Kam A, Guag J W and Kuster N 2010 The Virtual Family—development of surface-based anatomical models of two adults and two children for dosimetric simulations Phys. Med. Biol. 55 N23