Analysis of Rainfall Infiltration and Improvement of the Analytical Solution of Safety Factors on Unsaturated Inner Dump Slopes: A Case Study

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Abstract: Rainfall infiltration is one of the main triggers of inner dump slope failure in the process of mining coal. Changes in water content throughout the process of rainfall infiltration have rarely been studied. The reductions in soil strength due to water migration have seldomly been considered in the existing analytical solution of the safety factor (F.S) for unsaturated inner dump slopes in an open-pit mine. In this work, a new mechanical model was developed by improving the conventional analytical solutions of F.S for unsaturated inner dump slopes to accommodate water-induced degradation in the mechanical strength of waste material. Parameter analysis was carried out via a case study of the Shengli #1 open-pit coalmine. The results showed that the wetting front depth increased with increasing rainfall time, and the increasing rate was constant during the non-compressive infiltration stage, while it decreased gradually in the compressive infiltration stage. The F.S of the transition layer decreased at first and then increased with increasing infiltration depth. By considering the water migration in the inner dump slope, the calculation result of F.S by the analytical solution in the paper can more precisely represent the in situ conditions. It was larger than that of the saturated strength, but smaller than that of the natural strength. The position of the minimum F.S did not alter in the wetting front, but was close to the position of the wetting front. The depth of the potential slip surface can be calculated by the converse solution of the analytical equation when F.S = 1 for rainfall infiltration, and the most dangerous slope surface can be determined. The depth (hmin) of the potential slip surface increases with increasing wetting front (hi) by a linear function, and increases with increasing depth ratios of the saturation layer (λ). The depth ratio (i) of the minimum F.S increases with increasing λ by an exponential function. The improved analytical solution can be used to evaluate the potential sliding surface under rainfall conditions, which is helpful for evaluating slope stability and analyzing dangerous surfaces under rainfall conditions and providing guidance for reinforcement schemes.

Keywords: open-pit coalmine; inner dump slope; landslide hazard; rainfall infiltration

1. Introduction

In recent years, many different disasters have happened in coalmines, including landslides, gas explosions, roof collapses, and rock bursts, with slides being the dominant type of disaster for open-pit coalmines [1,2]. Rainfall infiltration contributes to changes in pore water pressure and reduces matric suction in soil, making it one of the main triggers of slope failure [3,4]. In particular, the unsupported inner dump slopes of open-pit coalmines have a particularly low degree of compaction, high porosity, and high looseness, making the influence of rainfall more obvious. During rainfall, the matrix suction changes, and the shear strength decreases in the unsaturated and saturated zones due
to the rising groundwater table or increasing pore pressure [5–7]. Changes in the pore pressure of slopes during rainfall are influenced by rainfall patterns and durations, slope topography, and physical properties. Meanwhile, rainfall infiltration increases the sliding force of a slope and threatens the slope’s stability. Thus, it is critical to study the process of rainfall infiltration and the safety factor of slopes.

Related research has mainly been carried out from the perspective of several typical rainfall patterns. The literature shows that many efforts have been made to study rainfall infiltration laws and the influence of rainfall on slope failure. To date, most studies have focused on the effect of rainfall amount (rainfall threshold), average intensity, and duration on unsaturated slope stability [8–10]. Research has mainly been carried out from several typical rainfall patterns (e.g., uniform, advanced, and delayed rainfall patterns) [11,12], focusing on the variation in rainfall parameters. The effect of pore water pressure on the strength of soil is also a significant factor leading to instability of slopes, attracting much scholarly attention [13,14]. During the rainfall season (from July to September), it is easy to trigger a mass gentle rock landslide. An example of landslide occurred in the east of Sichuan province, and landslide volume was millions of cubic meters. The main reasons for the landslide were the increase in pore water pressure and the weakening of mechanical properties of the soil in the sliding zone [15,16]. The technical literature proposes many approaches to compute pore water pressure changes due to rainfall, as well as the consequent changes in the displacement rate. “Physically based” and “phenomenologically based” are the most common approaches. The former is based on the hydraulic and mechanical properties of soil and attempts to reproduce the physical processes related to pore water pressure fluctuations and rainfall regimes [17–19]. Meanwhile, phenomenologically based approaches aim to establish empirical correlations between displacements and their triggers, or statistical relationships between measured groundwater pressure and weekly/monthly rainfall, without explicitly considering the physical processes occurring on the slope [20,21]. Meanwhile, the slope instability caused by rainfall is mainly caused by both mechanical and hydraulic factors. Factors such as rainfall pattern, pore water pressure, and rock mass strength, should be taken into account to study slope stability. A series of experimental studies on slope failure have been carried out using the physical slope model, with experimental results showing that rainfall pattern, pore water pressure variation, and slope body strength have significant effects on the slope failure mode and landslide probability [22–25]. Variation in the strength characteristics of the slope material with water migration has not been considered. Therefore, it is of great theoretical and practical importance to address the issue of variable rainfall parameters and complex slope characteristics.

As reviewed above, much of the previous work has been completed to study the effects of rainfall infiltration on slope stability, but in the traditional theory analysis of the slope safety factor for rainfall infiltration, the effective cohesion and internal friction angle are mostly used, and the cohesion and internal friction angle of the slope are reduced by the same value. The change in water content has rarely been studied in the rainfall transition layer, and the waste strength reduction due to water migration has seldom been considered to solve safety factors (F.S) for unsaturated slopes.

To bridge this gap, in this work, we studied changes in the wetting front during the rainfall process, and a new mechanical model was developed by improving the conventional analytical solutions of F.S for unsaturated slopes to accommodate water-induced degradation in the waste strength of the inner dump. Parameter analysis was carried out via a case study of the Shengli #1 open-pit coalmine. Meanwhile, the intensity and duration of rainfall are not constant but change with time, and the process is very complex. To simplify the rainfall process, in this study, we considered the influence of rainfall intensity, rainfall time, and other parameters on slope stability as being transformed to represent the influence of the infiltration depth and water content of the slope. In the process of solving the safety factor of the slope, we quantified the relationship between the safety
factor and the depth of rainfall infiltration and clarified the influence of rainfall parameters on the F.S of a slope. Based on the broad slope criterion, the influence of water migration on soil strength reduction during rainfall is considered in this paper, which improves the accuracy of slope stability evaluation. The present work provides guidance for the stability analysis and preliminary design of slopes in an open-pit coalmine.

2. Model of Rainfall Infiltration and Solution of the Safety Factor

2.1. Model Establishment and Derivation of the Rainfall Infiltration Equation

The rainfall process can be divided into the noncompressive infiltration stage, the compressive infiltration stage, and the stable stage [26,27]. In the stage of noncompressive infiltration, the infiltration rate of the slope surface is equal to the decomposition of rainfall intensity in the vertical plane of the slope. In the stage of compressive infiltration, the slope surface reaches saturation and the slope infiltration rate decreases. The rainfall infiltration rate decreases to a certain value and tends to be stable, and this stage is called the stable stage.

The infiltration rate \( i \) in the rainfall infiltration process can be computed by Equation (1):

\[
i = \begin{cases} 
q \cos \alpha & t \leq t_0 \\
\frac{h_f \cos \alpha + h_s}{h_f} & t_0 < t < t_w \\
i_{\text{min}} & t \geq t_w
\end{cases}
\]

(1)

where \( q \) is the rainfall intensity, m/h; \( \alpha \) is the slope angle, °; \( h_f \) is the wetting front depth vertical to level surface, m; \( t \) is the rainfall time, h; \( t_0 \) is the critical time at which noncompressive infiltration shifts to compressive infiltration, h; \( t_w \) is the critical time at which compressive infiltration shifts to stable stage; \( h_s \) is the matrix suction head at the wetting front, m; \( i \) is the infiltration rate, m/h; \( k_s \) is the saturated permeability coefficient of the slope, m/h.

The slope model of the rainfall infiltration process is shown in Figure 1. In the figure, \( \alpha \) is the slope angle, °; \( h_f \) is the wetting front depth vertical to the slope surface, m. Additionally, as proposed by Or et al. [28] and validated by Peng et al. and Gavin and Xue [29,30], the soil water content in the transitional layer varies with depth as an ellipse function. Therefore, the distribution of the soil water content under infiltration (see Figure 2) assumes the following function:

\[
\theta(h_v) = \begin{cases} 
\theta_s & 0 \leq h_v \leq h_{vs} \\
\theta_0 + (\theta_s - \theta_0) \left(1 - \left(\frac{h_v - h_{vs}}{h_{vt}}\right)^2\right) & h_{vs} \leq h_v \leq h_{vs} + h_{vt}
\end{cases}
\]

(2)

where \( \theta_0 \) and \( \theta_s \) are the initial and saturated water contents, respectively; \( h_v \) is the infiltration depth, m; \( h_{vs} \) is the saturation layer depth vertical to the slope surface, m; \( h_{vt} \) is the transitional layer depth vertical to the slope surface, m.
Figure 1. Slope model of the rainfall infiltration process.

Figure 2. Distribution of the soil water content under infiltration.

The cumulative rainfall infiltration in the saturation layer, $I_s$, can be expressed as:

$$ I_s = (\theta_s - \theta_0)h_{vs} $$

(3)

The cumulative rainfall infiltration in the transition layer, $I_t$, can be written as follows:

$$ I_t = \int_{h_{vs}}^{h_{vf}} \theta dh = \frac{\pi}{4}(\theta_s - \theta_0)h_{vt} $$

(4)

The total cumulative rainfall infiltration, $I$, is calculated as follows:

$$ I = I_s + I_t = (\theta_s - \theta_0)(h_{vs} + \frac{\pi}{4} h_{vt}) $$

(5)

The ratio of the depth in the saturated layer ($h_{vs}$) to the depth of the wetting front ($h_{vf}$) is $\lambda$ and we can obtain the following:

$$ h_{vs} = \lambda h_{vf} $$

(6)

$$ h_{vf} = (1 - \lambda) h_{vf} $$

(7)

The infiltration rate ($i$) of the slope surface at time $t$ is calculated as follows:

$$ i = \frac{dI}{dt} = \frac{(\theta_s - \theta_0)[\lambda + \frac{\pi}{4}(1 - \lambda)]dh_{vf}}{dt} $$

(8)

At the critical time ($t_0$), the saturated layer of the slope surface is just formed, and we can obtain the following:

$$ k_s h_{vf} \cos \alpha + S_i \frac{h_{vf}}{h_{vf}} = q \cos \alpha $$

(9)

where, $S_i$ is the average matric suction head at the wetting front.

We can obtain the following:

$$ h_0 = \frac{k_s S_i}{(q - k_s) \cos \alpha} $$

(10)

where, $h_0$ is the wetting front depth at $t_0$, m.

The critical time, $t_0$, can be expressed as follows:
\[ t_0 = \frac{I_0}{i} = \frac{(\theta_s - \theta_0)[\lambda + \frac{\pi}{4} (1 - \lambda)]k_s S_t}{(q - k_s)q \cos^2 \alpha} \]  

(11)

When \( t \leq t_0 \), the infiltration rate, \( i \), can be obtained as follows:

\[ i = \frac{dI}{dt} = \frac{(\theta_s - \theta_0)[\lambda + \frac{\pi}{4} (1 - \lambda)]dh_{vf}}{dt} = q \cos \alpha \]  

(12)

During the initial infiltration stage, the initial conditions are expressed as follows:

\[ t = 0, \quad h_{vf} = 0 \]  

(13)

\( h_{vf} \) during the non-compressive infiltration stage can be written as follows:

\[ h_{vf} = \frac{q \cos \alpha}{(\theta_s - \theta_0)[\lambda + \frac{\pi}{4} (1 - \lambda)]} t \]  

(14)

When \( t \geq t_0 \), substituting Equation (1) into Equation (6), we can obtain the following:

\[ k_s \frac{h_{vf} \cos \alpha + S_t}{h_{vf}} = \frac{(\theta_s - \theta_0)[\lambda + \frac{\pi}{4} (1 - \lambda)]dh_{vf}}{dt} \]  

(15)

The derivation of \( h_{vf} \) to \( t \) can be determined based on Equation (16):

\[ \frac{dh_{vf}}{dt} = k_s \frac{h_{vf} \cos \alpha + S_t}{h_{vf}} \left(\theta_s - \theta_0\right)\left[\lambda + \frac{\pi}{4} (1 - \lambda)\right] \]  

(16)

The initial condition at \( t_0 \) time is determined as follows:

\[ t = t_0, \quad h_{vf} = h_0 \]  

(17)

Combining Equations (16) and (17), we can obtain the following:

\[ \frac{(h_{vf} - h_0)(\theta_s - \theta_0)[\lambda + \frac{\pi}{4} (1 - \lambda)]}{k_s \cos \alpha} = (t - t_0) + \frac{S_t(\theta_s - \theta_0)[\lambda + \frac{\pi}{4} (1 - \lambda)]}{k_s \cos^2 \alpha} \ln \frac{h_{vf} \cos \alpha + S_t}{h_0 \cos \alpha + S_t} \]  

(18)

The wetting front depth vertical to the slope surface, \( h_{vf} \), can be expressed as:

\[ h_{vf} = \frac{q \cos \alpha}{(\theta_s - \theta_0)[\lambda + \frac{\pi}{4} (1 - \lambda)]} t, t \leq t_0 \]

\[ \frac{(h_{vf} - h_0)(\theta_s - \theta_0)[\lambda + \frac{\pi}{4} (1 - \lambda)]}{k_s \cos \alpha} = (t - t_0) + \frac{S_t(\theta_s - \theta_0)[\lambda + \frac{\pi}{4} (1 - \lambda)]}{k_s \cos^2 \alpha} \ln \frac{h_{vf} \cos \alpha + S_t}{h_0 \cos \alpha + S_t}, t \geq t_0 \]  

(19)

2.2. Analytical Solution of the Safety Factor

In the traditional theory analysis of the slope safety factor for rainfall infiltration, the effective cohesion and internal friction angle are mostly used, and the cohesion and internal friction angle of the slope are degraded by the same value [31,32]. In inner dump slopes, waste material is affected by rainfall, and strength reduction actually takes place and thus needs to be reflected in the input parameters in the mechanical model. The improved analytical solution of the safety factor, which may be regarded as a variant of the
traditional theory analysis of the slope safety factor for rainfall infiltration, was taken and further modified to accommodate the rainfall-triggered waste strength reduction in the analysis. Meanwhile, the influence of rainfall intensity, rainfall time, and other parameters on slope stability are transformed into the influence of the infiltration depth and water content of the slope. This approach is capable of considering the influence of rainfall on the material strength properties and of solving the problem of complex and changeable parameters such as rainfall intensity and time.

The following hypotheses are proposed in the solution derivation of the safety factors for rainfall infiltration to solve the analytical solution of the safety factor for unsaturated slopes.

i. The slope material is uniform and isotropic;

ii. The main rainfall saturated layer and wetting front are parallel to the slope surface;

iii. The moisture content changes with the rainfall infiltration depth by an elliptical function.

To analyze the slope stability during rainfall infiltration, a solution model of the safety factor for rainfall infiltration and a mechanical model of the sliding surface were established, as outlined in the following sections (see Figures 3 and 4). In the model, the slope affected by rainfall can be divided into a saturated layer, a transitional layer, and a natural layer from the top of the slope to the basement surface. In the Figures 3 and 4, $h_s$ is the depth of the saturation layer, m; $q$ is the rainfall intensity, m/h; $\alpha$ is the slope angle, $^\circ$; $h_t$ is the depth of the transition layer, m; $h_f$ is the depth of the wetting front, m. Their angles are all equal to $\alpha$ (see the hypothesis above). Meanwhile, $W_s$ and $W_f$ are the slider gravities acting above the saturated layer and wetting front, respectively, kN; $N_s$ and $N_f$ are the slider normal forces acting above the saturated layer and wetting front, respectively, kN; $\sigma_{nt}$ and $\tau_{mt}$ are the normal and shear stress of the transition layer, respectively, kPa; $J_s$ is the seepage force of the saturated layer, kN; $u_a$ and $u_w$ are the air and water stress of the soil, respectively, kPa. $\phi$ is the internal friction angle of matric suction.

Figure 3. Model for calculating the safety factor in the rainfall infiltration process.
2.2.1. Calculating the Safety Factors in the Saturation Layer

The contents of the slope at different depths vary, and the corresponding strengths of the slope materials are different throughout the rainfall infiltration process. When solving the safety factor of different depths, the corresponding strength is not constant. However, it is related to the infiltration depth and water content. The functional equation between the water content ($\theta$) and cohesion ($c$) and the internal friction angle ($\phi$) of the slope can be assessed:

$$c = f(\theta)$$

$$\phi = g(\theta)$$

When $h \leq h_s$, the soil gravity ($W_s$) and normal force ($N_s$) of the sliding surface can be written as follows:

$$W_s = \gamma_h$$

$$N_s = \gamma_h \cos \alpha$$

The seepage force ($J_s$) of the saturation layer has been taken into account in the existing references [33–35] and can be expressed as follows:

$$J_s = \gamma_w h \sin \alpha$$

where $\gamma_w$ denote the unit weight of water.

The tilt length of the slider (m), $l$, can be expressed as:

$$l = \frac{1}{\cos \alpha}$$

The anti-sliding force is:

$$F_a = c l + N_s \tan \varphi$$

The down-sliding force is:
\[ F_d = W_s \sin \alpha + J_s \]  

As proposed by the literatures [36,37], the safety factor of the sliding surface in the saturation layer is as follows:

\[
F = \frac{F_d}{F_s} = \frac{cl + N_s \tan \phi}{W_s \sin \alpha + J_s} = \frac{cl + \gamma'h \cos \alpha \tan \phi}{\gamma' h \sin \alpha + \gamma_w h \sin \alpha} = \frac{\tan \phi}{(1 + \frac{\gamma_w}{\gamma'}) \tan \alpha} + \frac{2c}{(\gamma' + \gamma_w) h \sin 2\alpha}
\]

\[
= \frac{\tan[g_s(\theta)]}{(1 + \frac{\gamma_w}{\gamma'}) \tan \alpha} + \frac{2J_s(\theta)}{(\gamma' + \gamma_w) h \sin 2\alpha}
\]  

2.2.2. Calculating the Safety Factors in the Transition Layer

The unit weight of the slope (\(\gamma_s\)) in the transitional layer can be expressed as follows:

\[ \gamma_s = \gamma_d (1 + \theta) \]  

where \(\gamma_d\) is the natural weight of the slope, kPa.

The normal and shear stress of the sliding surface in the transitional layer are as follows:

\[ \sigma_n = (\gamma'h_s + \int_{h_s}^{h} \gamma'dh) \cos \alpha + (\gamma'h_s + A) \cos \alpha \sin \alpha \]

\[ \tau_n = (\gamma'h_s + \int_{h_s}^{h} \gamma'dh) \cos \alpha \sin \alpha = (\gamma'h_s + A) \cos \alpha \sin \alpha \]

where

\[ A = \int_{h_s}^{h} \gamma'dh = \gamma_d ((1 + \theta_0)(h - h_s) + \frac{\theta - \theta_0}{h_s} \frac{h}{2} \arcsin \frac{h - h_s}{h_s} + \frac{h - h_s}{h} \sqrt{\frac{h^2}{h_s^2} - (h - h_s)^2}) \]

As proposed in the literature [36–38], air compression \((u_a)\) can usually be neglected in the process of rainfall infiltration. Water compression can be expressed using the compressive head \((h)\) in the transition layer and the water unit weight \((\gamma_w)\). Equation (33) can be obtained by:

\[ u_a = 0 \]  

\[ u_w = \gamma_w h \]

where \(u_a\) and \(u_w\) are the air and water compression, respectively, Pa.

Similar to the approach proposed by Lu et al. [39], the additional friction angle \(\phi^b\) can be related to the soil water retention curve and the friction angle \(\phi\) [40,41]:

\[ \tan \phi^b = \frac{\theta_0 - \theta}{\theta - \theta_0} \tan \phi \]

The safety factor of the slope in the transition layer can be expressed as:

\[ F_{sl} = \frac{c + (\gamma'h_s + \gamma_d (h - h_s)(1 + \theta_0 + (\theta - \theta_0) \frac{\pi}{4})) \cos \alpha \tan \phi - \gamma_w (h - h_s) \frac{\theta_0 - \theta}{\theta - \theta_0} \tan \phi}{(\gamma'h_s + \gamma_d (h - h_s)(1 + \theta_0 + (\theta - \theta_0) \frac{\pi}{4})) \cos \alpha \sin \alpha + \gamma_w h_s \sin \alpha \cos \alpha} \]

By simplifying Equation (34), we can obtain the following:
Equation (37) can be used as an improved analytical solution of the F.S of the unsaturated slope in the transition layer for the rainfall infiltration process. The waste strength reduction due to the influence of water was considered in the improved analytical solution of F.S for unsaturated slopes, which is closer to the actual situation in the field compared to the traditional analytical solution. According to Equations (2) and (37), the safety factor of the sliding surface in the transition layer is only related to the depth of the transition layer for certain rainfall conditions. Meanwhile, the critical failure depth and the potential sliding surface of the slope can be conversely solved by Equations (2) and (37) when $F_{st} = 1$.

3. Parameter Analysis and Discussion

In this work, an inner dump in an open-pit coalmine was studied and the change laws of the slope parameters were discussed. The slope parameters were as follows: $\alpha = 25^\circ$, $\theta_b = 0.13$, $\theta_s = 0.45$, $\theta_r = 0.08$, $S_f = 10$ cm, $q = 0.02$ m/h, and $k_s = 0.016$ m/h. Based on Equations (10) and (11), the critical wetting front depth ($h_0$) and the critical time ($t_0$) from noncompressive infiltration into compressive infiltration were $h_0 = 19.62$ cm and $t_0 = 2.4345$ h. The change of the wetting front with time under rainfall conditions is shown in Figure 5.

![Figure 5. The wetting front depth versus time with different slope angles.](image)

Figure 5 shows that the wetting front depth increased linearly with the increasing rainfall time, and the rate of increase was constant during the noncompressive infiltration stage. However, the wetting front depth increased with increasing rainfall time and the increasing rate gradually decreased during the compressive infiltration stage. This is because the infiltration rate gradually decreased with continuous rainfall during the compressive infiltration stage, which caused the increasing rate of the wetting front depth to decrease. The increasing rate of the wetting front depth decreases with increasing slope angle from $10^\circ$ to $45^\circ$ (see Figure 5) in the Line. This is because the larger the slope angle,
the lower the rainfall infiltration rate and the more difficult the rainfall infiltration into the slope. The critical time ($t_0$) for slope angles of $10^\circ$, $25^\circ$, and $45^\circ$ from noncompressive infiltration to compressive infiltration were 2.0618, 2.4345, and 3.9993 h, respectively. That is, the critical time ($t_0$) from noncompressive infiltration to compressive infiltration gradually increased with increasing slope angles.

According to Li et al. and Liu et al. [42,43], shear strength can be described by various functional equations considering the changes in water content. Cohesion ($c$) and the internal friction angle ($\varphi$) of the open-pit slope showed power function with the moisture content ($\theta$) in the direct shear experiment. The power function equations can be written as follows:

$$c = a \theta^n$$

$$\varphi = m \theta^o$$

A direct shear test was carried out to obtain the shear strength of waste under different water contents, and the experimental results were fitted (see Figure 6). The fitting curve showed that the internal friction angle and the cohesion decreased with an increasing water content, and the fitting function is consistent with the results obtained in the literature [42,43]. Based on the fitting results, we obtained $a = -3.38$, $b = 11.319$, $m = -2.522$, and $n = 11.905$.

![Figure 6. Cohesion and internal friction angle versus water content.](image)

Equations (40) and (41) can be obtained by fitting:

$$c = 11.655 \theta^{-0.25}$$

$$\varphi = 18.138 \theta^{0.118}$$

The safety factor of the sliding surface in the transition layer can be computed by Equation (42):
The experimental and theoretical analyses indicated that the depth of the saturated layer is approximately half of the depth of the wetting front [36,44]. Thus, the depth of the saturated zone can be expressed as follows:

\[ h_s = 0.5h_t \]  

(43)

The change of the safety factor in the transition layer for different depths of the transition layer can be obtained by Equations (42) and (43), which are shown in Figures 7 and 8.

Figure 7. Safety factor in the transition layer versus the depth of transition layer \((h_s = 1 \text{ m and } h_t = 2 \text{ m})\).
Figures 7 and 8 show that the safety factor in the saturation layer decreased with increasing depth in the saturation layer, and the safety factor decreased at first and then increased with increasing depth. The depth of the potential slip surface existed not in the wetting front but rather in the position close to the wetting front. Meanwhile, the safety factor considering the strength reduction in the transition layer, \( w \), as larger than that of the saturated strength, but less than that of the natural strength without considering the strength reduction. By considering the water migration in the inner dump slope, the calculation result of F.S by the analytical solution in the paper can be more precisely represent the in situ conditions. Thus, it makes it better than the traditional theoretical calculation of F.S, which takes soil strength as constant (the saturated strength or the natural strength) ignoring the water migration in the inner dump slope.

Meanwhile, we assumed that the depth of the potential slip surface is proportional to the wetting front depth, \( i = \frac{h_{\text{min}}}{h_t} \). The depth of the potential slip surface with the wetting front depth \( (h_t) \) and \( i \) with \( \lambda \) were calculated, and the results were fitted (see Figures 9 and 10). Figure 10 shows that the fitting degree was high between the calculation data and the fitting function, and \( R^2 > 0.99 \). The depth \( (h_{\text{min}}) \) of the potential slip surface increased by linear function with increasing wetting front depth \((h_t)\) and increased with \( \lambda \). The ratio \((i)\) increased by the exponential function with \( \lambda \).
Calculation data when $\lambda=0.9$
$h_{\text{min}}=0.93322h_{f}+0.0023,R^2=0.99958$

Calculation data when $\lambda=0.7$
$h_{\text{min}}=0.83846h_{f}+0.12868,R^2=0.99961$

Calculation data when $\lambda=0.5$
$h_{\text{min}}=0.76695h_{f}+0.225613,R^2=0.99836$

Calculation data when $\lambda=0.3$
$h_{\text{min}}=0.70305h_{f}+0.32805,R^2=0.99617$

Figure 9. Depth ($h_{\text{min}}$) of the potential slip surface versus the wetting front depth.

Figure 10. $i$ versus $\lambda$.

4. Case Study

The Shengli #1 Surface Coal Mine is located in the center of the Shengli Coal Field, Xilinhaote, Inner Mongolia autonomous region, China. The field view of the inner dump slopes are shown in Figure 11. It covers a total area of 37.14 km$^2$, which extends 6.84 km from east to west and 5.43 km from south to north. The main coal seam is 5# coal seam and 6# coal seam. This inner dump slope started was formed in 2010; currently it is discharged to the level of +975 m and its height is about 165 m.
Figure 11. Field view of the inner dump slope.

The landslide disaster of Shengli 1# open-pit mine in 2014 was taken as a case study in the paper. The sliding range is continuously extended outwards from a sliding area on the slope surface of 960–975 bench. The posterior border of the sliding body sank 10 m, and the anterior border of the sliding body slid 140 m. The spindle direction of the sliding body is 308°. The length and width of the sliding body are 600 m and 302 m respectively. The sliding area is 134,000 m², the volume of the sliding body is 1.7 million m³ (see Figure 12). The landslide blocked the transport channel in the inner dump and greatly increased the transport distance of the waste. It had a great impact on coal production and caused great economical loss. Before the landslide disaster occurred, the open-pit mine slope had sustained meteoric precipitation by investigation. A large amount of rainwater pours into the slope through the cracks, resulting in the reduction of the soil strength of the slope.

Figure 12. Site image of the landslide.

Statistics show that slope failure due to rainfall in the open-pit mine often occurs during the rainy season (see Figure 13). The rainfall amount was counted during the rainy season in June 2014. To evaluate the slope stability by considering the analytical solution of the safety factor in the above work, the F.S of the dump slope was calculated and the
displacement variation was monitored by radar during the rainy season (see Figures 14 and 15). Since rainfall lasted for half a month, the safety factor was calculated using the cumulative rainfall.

![Figure 13. Slope failure due to rainfall in the open pit mine.](image)

![Figure 14. Theoretical F.S during rainfall in June 2014.](image)
The safety factor began to decline on 6 June 2014. Half a month of rainfall and a large number of cracks at the top of the slope led to the accumulation of rainfall in the slope and an upward trending groundwater level. Therefore, the continuous rainfall from 16 June replenished the groundwater, and the safety factor was close to 1 on 21 June. Based on the results of the theoretical analysis, the safety factor was less than 1 on 25 June, and slope failure occurred.

The radar monitoring results showed that the increase rate of displacement became larger after 16 June, especially on 21 June; the deformation velocity of the slope was larger and a field warning appeared, and then local slope failure occurred. The results of the radar monitoring are consistent with the results of the slope safety factor calculated by theory, and the rationality of the model was verified.

Previously, the open pit mine used the traditional safety factor evaluation standard. It did not consider the influence of water migration on the soil strength during rainfall, and the obtained F.S results were inaccurate. However, the open pit mine uses the analytical solution proposed in this paper to evaluate the slope stability, which reflects the field situation more accurately and greatly reduces the loss caused by rainfall. For other slope geometry, as long as the parameters of geometry are changed, the F.S can be calculated and the slope stability can be predicted by the analytical solution of F.S in this paper. The above research results can provide guidance for the stability evaluation of field dumps in later years and have good application value in the field.

5. Conclusions

In this work, wetting front functions and improved analytic solutions of the safety factor were established, considering the elliptical variation in the water content and the strength reduction during the rainfall process, and the change laws of the slope safety factor under rainfall conditions were analyzed. The conclusions can be summarized as follows:

1. The function of the wetting front was established considering the elliptical variation of the water content. The results of the parameter analysis indicated that the wetting front depth increased by a linear function with increasing rainfall time and that the increasing rate was constant during the noncompressive infiltration stage. However, the wetting front depth increased with increasing rainfall time and the increasing rate gradually decreased during the compressive infiltration stage.
The relationship between the safety factor and the depth of rainfall infiltration were quantified, and improved analytic solutions of the safety factor were obtained considering the strength reduction with water content during the rainfall process. The form of the analytical solution is simple, easy to apply, and efficient. The main application values are as follows. Aimed at the existing geometry of open-pit mine slopes, the change results of the safety factor can be calculated quantitatively with the rainfall changing based on the improved analytic solutions of F.S and the stability of open-pit slopes can be evaluated.

The change law of the slope safety factor under rainfall conditions was clarified using the improved analytic solutions of the safety factor. The results showed that the safety factor in the saturation layer decreased with increasing saturation layer depth, and the safety factor in the transition zone decreased at first and then increased with the increasing depth in the transition layer. Meanwhile, the safety factor considering strength reduction in the transition zone was larger than that of the saturated strength but less than that of the natural strength. By considering the water migration in the inner dump slope, the calculation result of F.S by the analytical solution in the paper can be more precisely represent the in situ conditions.

The depth of the potential slip surface existed not in the wetting front but rather in a position close to the wetting front. The depth \( \left( h_{\text{min}} \right) \) of the potential slip surface increased by a linear function with the increase in the wetting front \( (h_f) \) and increased with \( \lambda \). The ratio \( (i) \) increased in an exponential manner with \( \lambda \).

The critical failure depth and the potential sliding surface can be conversely solved by the improved analytic solutions of F.S when \( F_s = 1 \). The improved analytic solutions of the safety factor were used in a case study of the Shengli #1 open-pit coalmine and the field application results showed that the method provides guidance for the stability evaluation of field dumps in later years and has good field application value.

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Abbreviations

| Symbol | Description                                      | Unit  |
|--------|--------------------------------------------------|-------|
| \( i \) | Infiltration rate                                 | 1 m/h |
| \( S_t \) | Matrix suction head at the wetting front          | m     |
| \( h_{\text{vs}} \) | Rainfall saturation depth vertical to the slope surface | m     |
| \( \theta_0 \) | Initial water content                            | 1     |
| \( h_{\text{vt}} \) | Transitional layer depth vertical to the slope surface | m     |
| \( q \) | Rainfall intensity                                | m/h   |
| \( k_s \) | Saturated permeability coefficient of the slope   | m/h   |
| \( h_f \) | Wetting front depth vertical to the slope surface | m     |
| \( \theta_s \) | Saturated water content                          | 1     |
| \( t_0 \) | Critical time of non-compressive infiltration into compressive infiltration | h     |
\[ t \text{ Rainfall time} \quad h \quad \gamma_u \quad \text{Natural weight} \quad kN/m^3 \]
\[ I_c \text{ Cumulative rainfall infiltration in the saturation layer} \quad m \quad l_h \quad \text{Cumulative rainfall infiltration in the transition layer} \quad m \]
\[ I \text{ Total cumulative rainfall infiltration} \quad m \quad l_h \quad \text{Wetting front depth at } t \text{ time} \quad m \]
\[ l \text{ Tilt length of the slider} \quad m \quad l_h \quad \text{Saturation layer depth} \quad ^o \]
\[ h_s \text{ Wetting front depth} \quad m \quad l_h \quad \text{Rainfall transition layer depth} \quad m \]
\[ W_s \text{ Slider gravities above rainfall saturated layer} \quad kN \quad W_l \quad \text{Slider gravities above the wetting front} \quad kN \]
\[ N_s \text{ Slider normal force above the rainfall saturated layer} \quad kN \quad N_l \quad \text{Slider normal force above the wetting front} \quad kN \]
\[ \sigma_{ns} \text{ Normal stress in the transition layer} \quad kPa \quad \tau_{ns} \quad \text{Shear stress in the transition layer} \quad kPa \]
\[ J_s \text{ Seepage force of the rainfall saturated layer} \quad kN \quad u_s \quad \text{Air stress of the soil} \quad kPa \]
\[ t_w \text{ Water stress of the soil} \quad kPa \quad \theta \quad \text{Water content} \quad 1 \]
\[ c \quad \text{Cohesion} \quad kPa \quad \phi \quad \text{Internal friction angle} \quad ^o \]
\[ \gamma' \text{ Buoyant weight} \quad kN/m^3 \quad \gamma_w \quad \text{Weight of water} \quad kN/m^3 \]

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