Research Article

Study on Mechanical Properties of Gravelly Sand under Different Stress Paths

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1.Introduction

Many experimental studies have shown that the deformation characteristics of sand rely on the relative density and consolidation pressure [1, 2]. The stress-strain relationship and strength and dilatancy of sand samples with identical initial states are different under different stress paths [3, 4]. Though the stress paths influence the deformation characteristics and the constitutive relationship of soil [5, 6], how the stress paths put impact on them and what the extent are not clear yet [7, 8]. The effect of the stress path on the mechanical properties of soil in the field of soil mechanics has been unclear for many years, which has increased the difficulty of establishing soil constitutive models. Further examination of this problem is significant for determining the deformation mechanism of soil and solving practical engineering problems.

The deformation and strength of sand have always been a focus of research in soil mechanics [9]. Through experimental studies, researchers have examined the impacts of multiple factors on the mechanical properties of sandy soil, such as temperature [10], confining pressure [11, 12], loading rate [13, 14], initial relative density [15], moisture content [16], and stress path [17]. Due to the complex and changeable stress state of soil in practical engineering, the mechanical properties of soil under different stress paths have become a research hotspot in geotechnical mechanics earlier [18], which was regarded as a main line by many...
researchers to analyze various geotechnical engineering problems. In the early stage, to explore the effect of stress path, torsional shear apparatus, pressure meter, and triaxial apparatus were used to obtain the mechanical parameters of sand [19, 20], such as elastic modulus, Poisson’s ratio, and tangent modulus, and the mechanical parameters under different stress paths were compared to analyze the effect of stress path on the deformation characteristics of sand. Some scholars also researched the effect of stress path on the strength characteristics by establishing the relationship between stress ratio and strength parameters of sand (internal friction angle and cohesion) [21]. These methods are relatively simple and can only preliminarily explore the relationship between stress path and mechanical properties of sand. At present, the triaxial test is the main way to study the mechanical properties of sand under different stress paths. Some scholars have carried out experimental studies on various types of sand under different stress paths (the conventional triaxial compression path CTC, the triaxial compression path TC, the reduced triaxial compression path RTC, loading and unloading, isotropic compression path and cyclic loading path, and so on) and obtained some conclusions [22–26]. For example, the peak stress of the samples usually is the largest in the CTC, less in the TC, and smallest in the RTC [17, 27]. In triaxial compression tests, the volume shrinkage of test sample in the CTC tests is the minimum, the volume shrinkage in the TC tests is more than CTC, and the volume shrinkage in the RTC tests is the maximum under a certain confining pressure [28, 29]. However, some test results show that the mechanical properties of sand in different regions are affected by the composition of minerals and climatic causes, and the strength and deformation characteristics are significantly different [27, 30, 31]. A large amount of sand is extensively distributed in the Qinghai-Tibet Plateau and is widely used in road engineering in cold regions. At present, the research on the mechanical properties of sand in this region mainly focuses on the test of frozen sand under conventional triaxial paths, and there is little research on the mechanical properties of sand under different stress paths in unfrozen state. Therefore, it is of great significance for the construction and operation of projects in the Qinghai-Tibet Plateau to explore the mechanical properties of unfrozen gravelly sand under different stress paths.

The objective of doing research studies on the mechanical properties of geotechnical materials is to better solve some certain problems in geotechnical engineering [32, 33]. Based on experimental research, various constitutive models of sand were established to describe the strain hardening, strain softening, and dilatancy characteristics. Early on, the effect of the internal state of sand was ignored, and only the current stress level was considered in the process of establishing a constitutive model of sand. The same type of sand was considered different materials according to the density, and different parameters were selected to simulate the mechanical properties of sand. The model parameters in this method were more, and the simulation results were not ideal [34, 35]. To reflect the effect of the density and effective average normal stress on sand deformation, a state parameter $\psi$ that could reflect the current compact degree of sand was suggested by Been et al. based on the triaxial test results of Kogyuk sand [36]. The state parameter $\psi$ was introduced into the dilatancy equation by Dafalias and Manzari to develop an elastoplastic constitutive model with double yield surfaces, which was applied to simulate the monotonic loading characteristics of sandy soil under different drainage conditions [37]. Subsequently, a state-dependent dilatancy theory was proposed by Li and Dafalias which closely linked the dilatancy with the current state of the sand [38]. Many constitutive models of sand were established based on this theory [2, 39]. In addition, according to the modified Cambridge model, a unified hardening model was proposed by Yao et al. where the plastic volume strain was replaced with the unified hardening parameter $H$ [40, 41]. Then, a series of constitutive models of sand was established, such as the critical state constitutive model of sand and three-dimensional anisotropic UH model for sands [42, 43]. To consider the effect of the stress path, Luo et al. proposed an incremental calculation model of sand from the perspective of calculus based on the triaxial test results of Japanese Toyoura sand [44]. This model considered the effect of the stress path on the stress-strain relationship and could be used to predict the deformation and strength characteristics of sand in general stress states. Since the model could not reflect the critical state of sand, Lu et al. modified the model based on the nonassociated flow rule, and a stress path constitutive model of sand with double yield surfaces was established [45]. The stress-strain characteristics of sand under different stress paths can be described by the model, especially the stress path where the mean normal stress $p$ is reduced, and the generalized shear stress $q$ is a constant. Unlike the traditional method of establishing a constitutive model, a neural network constitutive model considering the effect of stress paths was established by Wang et al. based on the artificial neural network technology [46]. The foregoing constitutive models are widely used to reflect the dilatancy and stress-strain characteristics of sand. These models also provide ideas for other scholars to establish constitutive models of sand considering various external factors (temperature and time), complex characteristics (anisotropy and structure), and complicated loading conditions (cyclic loading and asymptotic states).

The mechanical properties of geotechnical materials are significantly affected by the physical properties of the materials [47–49]. Due to strong solar radiation and long-term freeze-thaw cycles, the Qinghai-Tibet Plateau sand does not have identical particle breakage and granular constitution characteristics to sandy soils in other regions [30, 31]. Additionally, ice particles in the frozen soil melt in the warm seasons and form thawed soil with high water content, which affects the mechanical properties of sand. Considering such circumstances, further research on the mechanical properties and constitutive relationships of gravelly sand on the Qinghai-Tibet Plateau under different stress paths is very significant to reveal the subsidence mechanism of permafrost engineering on the Qinghai-Tibet Plateau. The survey results of the permafrost distribution on the Qinghai-Tibet
Plateau indicated that a large amount of gravelly sand is distributed in the Na Qu and An Duo sections from the particle composition perspective [50]. Gravelly sand, which is often used as roadbed filler in engineering, drains well and has high shear strength and low compressibility since it contains gravelly particles that are larger than 2 mm. Considering these characteristics and the prevention and control of subgrade damage in seasonal frozen soil regions, this paper takes the subgrade material of Qinghai-Tibet Highway (QTH) as the research object. The effects of the stress path on the stress-strain characteristics and volumetric deformation of gravelly sand were investigated based on triaxial shear tests under the stress paths of CTC (CD and CU), TC, and RTC. A characteristic angle $\theta$ was defined to reflect the relative movement of soil particles. The relationships between principal stress ratio $\sigma_1/\sigma_3$ and characteristic angle $\theta$ and between void ratio $e$ and characteristic stress ratio $\theta$ were derived. Subsequently, the relationship expression of stress ratio $\eta (q/p)$ and void ratio $e$ was established, and the trend of void ratio $e$ with the stress path was studied. The parameter characteristic state stress ratio $M_e$ was applied to modify the dilatancy equation of the Cambridge model to construct a constitutive model for gravelly sand, which considered the effects of the stress path. Finally, the validity of the model was verified according to triaxial test results of gravelly sand under different stress paths. The results are of great significance that they provide a scientific basis for the treatment of roadbeds and other engineering problems and offer guidance for engineering practice.

2. Laboratory Test Program

2.1. Test Materials and Soil Sample Preparation. The tested soil samples in this study consist of gravelly sand, which have been extracted from Na Qu area in the Qinghai-Tibet Plateau, representing typical soils there, as shown in Figure 1. Its composition is uneven, and the particle size varies widely. The basic physical parameters of gravelly sand are listed in Table 1. According to the particle size distribution in Table 2, which is drawn on the basis of the sand particle composition in Na Qu area, the gravelly sand is made into a cylindrical sample with a diameter of 50 mm and a height of 100 mm by the layered compaction method. The soil samples are prepared according to a standard procedure followed by the Specification of Soil Tests (GB/T50123-1999) issued by the Ministry of Water Resources, People’s Republic of China [51].

2.2. Test Device. The tests were performed using a standard stress path triaxial test system produced by the company GDS, UK, as shown in Figure 2. The apparatus equipped with an automatic control system and a data collected program contains a triaxial pressure chamber, a confining pressure controller, a back pressure controller, an axial pressure controller, and sensors. The main parameters about this system are described as follows: the maximum axial load is 25 kN; the confining pressures range from 0 to 1.3 MPa; the maximum axial displacement is 25 mm; and the back pressures range from 0 to 3 MPa.

2.3. Test Apparatus and Methods. In this paper, it is assumed that the conventional triaxial compression path CTC (drained CD) is the $T_1$. The triaxial compression path TC is the $T_2$. The reduced triaxial compression path RTC is the $T_3$. And, the conventional triaxial compression path CTC (undrained CU) is the $T_4$. A series of triaxial shearing tests was conducted with confining pressures ranging from 50 kPa to 400 kPa under $T_1$, $T_2$, $T_3$, and $T_4$, respectively.

The test conditions are illustrated in Table 3, and the loading paths are shown in Figure 3. The test process is described as follows [52]. (1) The soil sample was placed in the base of the pressure chamber, and water was then injected into the triaxial pressure chamber until the top of the sand sample was submerged. (2) Soil sample saturation process was performed (carbon dioxide saturation, hydraulic saturation, and back pressure saturation). (3) According to the test conditions, 50, 100, 200, and 400 kPa consolidation pressures were applied to consolidate the soil samples. And, the process of consolidation was considered completed when the back pressure volume change by $<5 \text{mm}^3$ in 5 min. (4) Under $T_1$ and $T_4$, the loading process of the sample was dominated by displacement and the axial loading rate was 0.1 mm/min. The loading process under $T_2$ and $T_3$ was controlled by stress, and the axial loading rate is 0.6 kPa/min. Failure was defined to occur when the axial strain reached 15%.

3. Results and Discussion

Figure 4 illustrates deviator stress-axial strain curves of gravelly sand under stress paths of $T_1$, $T_2$, $T_3$, and $T_4$ and confining pressures of 50, 100, 200, and 400 kPa, respectively. As shown in Figure 4, all gravelly sand samples under $T_1$, $T_2$, and $T_3$ exhibited strain hardening during the process of the shearing test, while exhibited strain softening under $T_4$. Additionally, under a certain confining pressure, the hardening degree of the samples reaches the largest in the $T_1$ tests, less in the $T_2$ tests, and smallest in the $T_3$ tests. Figure 5 shows the volumetric strain-axial strain curves of gravelly sand under $T_1$, $T_2$, and $T_3$. As illustrated in Figure 5, all gravelly sand samples under $T_1$, $T_2$, and $T_3$ exhibited shear contraction during the shearing process, and samples had no volume deformation under the $T_4$. The volume shrinkage of test sample in the $T_3$ tests is the minimum, the volume shrinkage in the $T_2$ tests is more than $T_3$ tests, and the volume shrinkage in the $T_1$ tests is the maximum under a certain confining pressure. The deviator stress of gravelly sand under $T_4$ increased rapidly at the beginning of loading. The sample shows shear failure when it reaches peak stress. After that, the damage degree of the soil sample increased and the deviator stress decreased gradually. Finally, the deviator stress remained stable when the sample was completely destroyed. The pore water pressure-axial strain curves of gravelly sand under $T_4$ are shown in Figure 6. It can be seen from Figure 6 that the normalized pore pressure ($u/\sigma_3$)-axial strain curves of gravelly sand approximated hyperbolas. The pore water pressures of gravelly sand rise rapidly at the beginning of loading. Subsequently, the upward trend of pore water pressures weakened gradually and
Table 1: Basic physical parameters of gravelly sand.

| Water content | Relative density of soil particles | Natural dry density | Minimum dry density | Maximum dry density |
|---------------|-----------------------------------|---------------------|---------------------|---------------------|
| 24%           | 2.69                              | 1.79 g·cm⁻³         | 1.54 g·cm⁻³         | 2.11 g·cm⁻³         |

Table 2: Particle size distribution (%).

| Particle size   | 5–10 mm | 2–5 mm | 1–2 mm | 0.5–1 mm | 0.25–0.5 mm | 0.075–0.25 mm | <0.075 mm |
|-----------------|---------|--------|--------|----------|-------------|---------------|-----------|
|                 | 9       | 18     | 12     | 20       | 18          | 22            | 1         |

Table 3: Experimental design of gravelly sand.

| Stress path    | Confining pressure $\sigma_3$ (kPa) | Dry density (g·cm⁻³) | Saturated water content (%) |
|----------------|-------------------------------------|-----------------------|-----------------------------|
| TC (T1)        | 50, 100, 200, 400                    | 1.63                  | 21.9                        |
| RTC (T2)       | 50, 100, 200, 400                    | 1.63                  | 21.9                        |
| CTC (CD) (T3)  | 50, 100, 200, 400                    | 1.63                  | 21.9                        |
| CTC (CU) (T4)  | 50, 100, 200, 400                    | 1.63                  | 21.9                        |
Figure 3: Loading stress path.

Figure 4: Deviatoric stress-axial strain curves of gravelly sand under different stress paths: deviatoric stress-axial strain curves (a) $\sigma_3 = 50$ kPa; (b) $\sigma_3 = 100$ kPa; (c) $\sigma_3 = 200$ kPa; (d) $\sigma_3 = 400$ kPa.
Figure 5: Volumetric strain-axial strain curves of gravelly sand under different stress paths: volumetric strain-axial strain curves
(a) $\sigma_3 = 50\text{kPa}$; (b) $\sigma_3 = 100\text{kPa}$; (c) $\sigma_3 = 200\text{kPa}$; (d) $\sigma_3 = 400\text{kPa}$.

Figure 6: Normalized pore pressure ($u/\sigma_3$)-axial strain curves of gravelly sand under CTC (CU) stress path.
finally stayed stably with the increase in axial strain. Figure 7 is the test results of gravelly sand in $p$-$q$ space. It can be seen from Figure 7 that the development law of deviatoric stress under different stress paths is consistent with Figure 3. Through comparison, it was found that the test results of gravelly sand were consistent with the results of other scholars [53].

The peak stress $\sigma_p$, peak volume strain $\varepsilon_{vp}$, residual stress $\sigma_u$, and peak pore water pressure $u_f$ of gravelly sand samples under $T_1$, $T_2$, $T_3$, and $T_4$ are listed in Table 4. Figure 8 shows the relationships between stress path and peak stress $\sigma_f$ and that of stress path and peak volume strain $\varepsilon_{vp}$. It can be seen from Figure 8(a), under a certain confining pressure, the peak stress of gravelly sand in the $T_1$ tests is the maximum; the peak stress in the $T_2$ tests is lower than $T_1$ tests, the peak stress in the $T_3$ tests is lower than $T_2$ tests, and the peak stress in the $T_4$ tests is the minimum. Figure 8(b) shows that the peak volume strain of gravelly sand sample in the $T_3$ tests is the minimum; the peak volume strain in $T_2$ tests is more than $T_3$ tests, and the peak volume strain in the $T_1$ tests is the maximum under a certain confining pressure. Figure 9 shows the relationships between confining pressure $\sigma_3$ and residual stress $\sigma_u$, and the relationship between confining pressure $\sigma_3$ and peak pore water pressure $u_f$. It can be seen from Figure 9(a) and Figure 9(b) that the residual stress $\sigma_u$ and peak pore water pressure $u_f$ increase with the increase in confining pressure $\sigma_3$.

4. Elastoplastic Constitutive Model of Gravelly Sand

Strength and deformation characteristics of sand are determined by void ratio $e$ and effective mean normal stress $p'$ [54]. In the early establishment of sand constitutive models, only the current stress level was considered, and the density was ignored, which led to models with too many parameters and poor applicability [34, 35]. Later, Jefferies et al. defined a state parameter $\psi$ to describe the current dense state of sand under the framework of critical state soil mechanics [36]. A state-dependent dilatancy theory was proposed by introducing state parameters into a dilatancy equation [38]. On the basis of the state-dependent dilatancy theory, a new dilatancy equation by introducing a characteristic state stress ratio $M_c$ into the modified Cambridge model dilatancy equation was proposed. The elastoplastic constitutive model of gravelly sand has been established when the nonassociated flow rule was applied. Finally, the proposed model is used to predict stresses and strains of gravelly sand under different stress paths.

The average normal stress $p$, generalized shear stress $q$, and stress ratio $\eta$ in this paper are expressed as follows:

$$p = \frac{(\sigma_1 + \sigma_2 + \sigma_3)}{3},$$

$$q = \frac{\sqrt{2}}{2} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2},$$

$$\eta = \frac{q}{p}.$$  

4.1. Dilatancy Equation and Yield Function. With the recommendation of Roscoe and Burland, the dilatancy equation of the modified Cam-Clay model satisfies the following equation [40]:

$$\frac{d\varepsilon_v^p}{d\varepsilon_v^p} = \frac{M^2 - \eta^2}{2\eta}.$$  

Here, $M$ is the critical stress ratio [55], $d\varepsilon_v^p$ is the plastic volume strain increment, and $d\varepsilon_v^p$ is the plastic shear strain increment.

The dilatancy of sand depends on its current stress level and density [56]. Equation (2) shows that the dilatancy equation of the modified Cambridge model only considers the relationship between dilatancy and stress level, without considering effects of densities. Therefore, the dilatancy equation of the modified Cambridge model is not applicable for sand. This paper introduces a characteristic state stress ratio $M_c$ into the dilatancy equation of the modified Cambridge model, and we obtain a dilatancy equation of gravelly sand as follows:

$$\frac{d\varepsilon_v^p}{d\varepsilon_v^p} = \frac{M_c^2 - \eta^2}{2\eta}.$$  

Here, the parameter critical state stress ratio $M_c$ reflects the current compact degree of sand [38]. This paper uses a characteristic state stress ratio $M_c$ of sand proposed by Liu and Luo [57].

Based on the normal flow rule and combining equation (3), we can get

$$\frac{dq}{dp} + \frac{M_c^2 p^3 - q^2}{2pq} = 0.$$  

By solving the ordinary differential equation of equation (4), we can obtain the plastic potential function $g$ as follows:

$$g = \ln \frac{p}{p_0} + \ln \left(1 + \frac{q^2}{M_c^2 p^3}\right) = 0.$$  

Combining with the nonassociated flow rule, the yield function expression of sand proposed by Liu and Luo is used [57] as follows:

$$f = \frac{\eta^2}{M^2 - x\eta^2} + 1 - \frac{p^2}{p_0} = 0,$$

where parameter $x$ is a critical state parameter [42, 58], which is used to correct the deviation between the critical state line CSL calculated by the model and the critical state line obtained from test results.

4.2. Hardening Parameter. As shown in Figure 10, the isotropic compression lines of gravelly sand with initial void ratios of 0.68, 0.59, 0.5, and 0.39 all appear as curves in the $e$-$lnp$ space. The slope of the isotropic compression line is small at relatively low stress levels. The slope of the isotropic compression line increases gradually as the stress level increases. The isotropic compression line of gravelly sands
with different initial void ratios eventually tends to the asymptote of the isotropic compression line at high stress levels, and the expression of the isotropic compression line for gravelly sand is \( e = Z - \lambda \ln\left(\frac{p + p_s}{1 + p_s}\right) \) [58]. The test results of Cambria sand conducted by Bopp and Lade also show similar trends [59]. As shown in Figure 11, \( \lambda \) is the slope of the asymptotic line of the isotropic compression line in the \( e \)-\( \ln p \) space. \( Z \) is the pore ratio corresponding to the

\[ e = Z - \lambda \ln\left(\frac{p + p_s}{1 + p_s}\right) \]
isotropic compression line (NCL) of gravelly sand when the average normal stress is \(1\) kPa, and \(p_s\) is a compressibility parameter that can reflect the degree of curvature of the isotropic compression line.

In the loading and unloading test, when the load increased from \(p_{x0}\) to \(p_x\), the pore ratio changed by

\[
\Delta e = e - e_0 = -\lambda \ln \left( \frac{p_x + p_s}{p_{x0} + p_s} \right).
\]  

(7)

Here, \(\lambda\) is the slope of the asymptotic line of the isotropic compression line in the \(e-\ln p\) space and \(p_s\) is a compressibility parameter that can reflect the degree of curvature of the isotropic compression line [54].

From equation (7), we have \(p_x = (p_{x0} + p_s) \exp (\frac{\epsilon_f}{C_p}) - p_s\), where parameter \(C_p = (\lambda - k' (1 + e_0))\); the parameter \(k\) is the slope of the rebound curve. By substituting \(p_x\) into equation (6), the yield function can be obtained:

![Figure 8](image_url)

**Figure 8**: Relationship between \(\sigma_f\) and stress path and that of stress path and \(\epsilon_f\) of gravelly sand under \(T1, T2, T3,\) and \(T4\). (a) Peak stress under \(T1, T2, T3,\) and \(T4\) paths. (b) Peak volume strain under \(T1, T2,\) and \(T3\) paths.

![Figure 9](image_url)

**Figure 9**: (a) Relationship between \(\sigma_u\) and \(\sigma_3\) and (b) relationship between \(u_f\) and \(\sigma_3\) under \(T4\).
In order to overcome the inability of the modified Cambridgemodel to describe the strain softening of gravelly sand, replace plastic volumetric strain with hardening parameter $H$. From equation (8), we have

$$f = \ln\left[1 + \frac{q^2}{M^2} \left( \frac{p}{p_s} \right) + p_s \right] - \ln(p_{x0} + p_s) - \frac{\varepsilon_p^0}{C_p}$$

(8)

Here, the hardening parameter $H = \int dH = \int \left[ (M_x^4 - \eta^4)/(M_x^4 - \eta^4) \right] \varepsilon_p^0$, $M_f$ is the potential strength of the gravelly sand, and the determination method of $M$ can be seen in Section 4.4.

4.3. Constitutive Relationship. According to the elastoplastic theory, we can obtain the incremental form of the stress-strain relationship as follows:

$$\begin{align*}
\begin{bmatrix}
\frac{dp}{dq} \\
\frac{d\varepsilon}{d\varepsilon_d}
\end{bmatrix} &= \left[ C^P \right] \begin{bmatrix}
\frac{d\varepsilon}{d\varepsilon_d}
\end{bmatrix} = \left[ \left[ C^P \right] - \left[ C^P \right] \right] \begin{bmatrix}
\frac{d\varepsilon}{d\varepsilon_d}
\end{bmatrix} \\
\begin{bmatrix}
\frac{d\varepsilon}{d\varepsilon_d}
\end{bmatrix} &= \left[ C^P \right] \begin{bmatrix}
\frac{d\varepsilon}{d\varepsilon_d}
\end{bmatrix} = \left[ \left[ C^P \right] - \left[ M \right] \right] \begin{bmatrix}
\frac{d\varepsilon}{d\varepsilon_d}
\end{bmatrix} \\
\begin{bmatrix}
\frac{d\varepsilon}{d\varepsilon_d}
\end{bmatrix} &= \left[ C^P \right] \begin{bmatrix}
\frac{d\varepsilon}{d\varepsilon_d}
\end{bmatrix} = \left[ \left[ C^P \right] - \left[ C^P \right] \right] \begin{bmatrix}
\frac{d\varepsilon}{d\varepsilon_d}
\end{bmatrix}
\end{align*}$$

(10)

The parameter $M$ and $N$ can be expressed as follows:
\[
[M] = [C^c] \begin{bmatrix} \frac{\partial g}{\partial \sigma} \\ \frac{\partial f}{\partial \sigma} \end{bmatrix}^T [C^c],
\]
\[
[N] = -\frac{\partial f}{\partial H} \begin{bmatrix} \frac{\partial g}{\partial \sigma} \\ \frac{\partial g}{\partial \sigma} \end{bmatrix}^T \begin{bmatrix} \frac{\partial f}{\partial \sigma} \\ \frac{\partial f}{\partial \sigma} \end{bmatrix} + \begin{bmatrix} \frac{\partial g}{\partial \sigma} \\ \frac{\partial g}{\partial \sigma} \end{bmatrix}^T [C^c] \begin{bmatrix} \frac{\partial g}{\partial \sigma} \\ \frac{\partial g}{\partial \sigma} \end{bmatrix}.
\] (11)

From equations (6) and (9), we obtain the following:

\[
\begin{align*}
\left\{ \begin{array}{c}
\frac{\partial g}{\partial \sigma} \\
\frac{\partial g}{\partial q}
\end{array} \right\} &= \begin{bmatrix}
\frac{1}{p} \frac{M^2 p^2 - q^2}{M^2 p^2 + q^2} \\
2q
\end{bmatrix}, \\
\left\{ \begin{array}{c}
\frac{\partial f}{\partial \sigma} \\
\frac{\partial f}{\partial q}
\end{array} \right\} &= \left\{ \begin{array}{c}
\frac{-2M^2 p^3 q^2 + (M^2 p^2 - x q^2)^2 \left[(px0 + ps)\exp\left(\frac{e^p}{C_p}\right) - p_s\right]}{(M^2 p^2(1+p_s) - p_s q^2)(q^2 - M^2 p^2)} \\
\frac{2q M^2 p^3}{(M^2 p^2 - x q^2)^2}
\end{array} \right\}, \\
\frac{\partial f}{\partial H} &= -1,
\end{align*}
\] (12)

By substituting equation (12) into equation (11), we can determine parameter \(M\) and \(N\) as follows:

\[
[M] = c_p \begin{bmatrix}
K^2(M_i^4 - \eta^4)B_3 & 6KGM^2\eta(M_i^4 - \eta^4) \\
6KGM^2\eta(M_i^4 + \eta^4)B_3 & 3GB_2
\end{bmatrix}B_3,
\]
\[
N = B_1 + B_2 + Kc_p(M^4 - \eta^4)B_3,
\] (13)

where \(B_1, B_2,\) and \(B_3\) can be expressed as follows:

\[
B_1 = p(M_i^4 - \eta^4)(M^2 - x \eta^2) \left[M^2 + (1-x)\eta^2 + (M^2 - x \eta^2)\frac{P_s}{p}\right],
\]
\[
B_2 = 12c_p GM^2\eta^2(M_i^2 + \eta^2),
\]
\[
B_3 = M^4 - (1+2x)M^2\eta^2 - x(1-x)\eta^4.
\] (14)
Combining equations (13) and (14), we have the following:

\[
\frac{[M]}{N} = \begin{bmatrix}
K^2(M_i^4 - \eta^4)B_3 & 6KGM^2\eta(M_i^4 - \eta^4) \\
6KGM\eta(M_i^2 + \eta^2)B_3 & 3GB_2
\end{bmatrix}
\]

By substituting equation (15) into equation (10), the matrix elements \(C_{pp}, C_{pq}, C_{qp}, \) and \(C_{qq}\) in equation (10) are expressed as follows

\[
C_{pp} = \frac{K(B_1 + B_2)}{B_1 + B_2 + KC_p(M_e^4 - \eta^4)B_3},
\]

\[
C_{pq} = \frac{-6KGC_pM^2\eta(M_e^4 - \eta^4)}{B_1 + B_2 + KC_p(M_e^4 - \eta^4)B_3},
\]

\[
C_{qp} = \frac{-6KGC_p\eta(M_i^2 + \eta^2)B_3}{B_1 + B_2 + KC_p(M_e^4 - \eta^4)B_3},
\]

\[
C_{qq} = \frac{3G(B_1 + KC_p(M_i^4 - \eta^4)B_3)}{B_1 + B_2 + KC_p(M_e^4 - \eta^4)B_3}.
\]

4.4. Model Parameter Determination, Preliminary Verification, and Sensitivity Analyses of Model Parameters. An elastoplastic constitutive model of gravelly sand was established in the above section. The parameters in the model were determined firstly.

1. The parameter critical state ratio \(M\) is the slope of the critical state line.

2. \(M_f\) is the potential strength of gravelly sand, and \(M_f = 6[(k+1+(k/R))/R]^{1/2} - (k/R)\) [58]. Here, the parameter \(k = M_f^{1/2}/(3-M)\) and \(R = \exp[-\xi/(\lambda-k)]\), where \(\xi\) is the density parameter of sand, which is the vertical distance from point \(b\) to point \(d\) in Figure 12. The parameter \(\xi = e_\eta - e\). From equation (8), we can obtain the expression of parameter \(e_\eta\) as follows:

\[
e_\eta = Z - \lambda \ln[(p + p_\eta)/(1 + p_\eta)] - (\lambda - k)\ln[(1 + \eta^2/(M^2 - \eta^2))p + p_\eta]/(p + p_\eta).
\]

All the parameters in the model can be obtained through triaxial shear tests. The model parameters for gravelly sand are listed in Table 5.

To compare fitting effects of the proposed model, the simulation results were compared with test results for stress paths T1, T2, T3, and T4, and the comparison results are shown in Figure 13. As shown in Figure 13, the proposed constitutive model could simulate the strain hardening, strain softening, and shear contraction characteristics of gravelly sand samples under the T1, T2, T3, and T4 paths.

The model only uses a set of parameters to simulate the test results of gravelly sand under different stress paths and exhibits a strong applicability. Additionally, as the model introduces a parameter of characteristic state stress ratio \(M_c\), which reflects the density of sand, theoretically, it is believed that the model can be used to predict the stress-strain and volume deformation characteristics of sand with different densities. This greatly improves the applicability of the model. To apply the model to numerical calculations in future, it is necessary to further reduce model parameters or simplify the method for determining model parameters. In addition, this paper studies the applicability of the model only under a small consolidation pressure. With increasing consolidation pressures, the particle breakage phenomenon becomes increasingly obvious. It is necessary to consider particle breakage as an influencing factor in the proposed model. These problems need to be solved in the future.

Taking the test results for the situation where the stress path was T1 and confining pressure was 50 kPa, respectively, as an example, the stress-strain relationship and volumetric strain-axial strain relationship of gravelly sand were simulated using different model parameters \(M\), and the results are shown in Figures 14(a) and 14(b). With increasing \(M\), the hardening degree of gravelly sand sample increased significantly, and the volume shrinkage increased slightly. Take the test results for the situation where the stress path was T1 and confining pressure was 100 kPa, respectively, as an example. By using different model parameters \(p_v\), the stress-strain relationship and volumetric strain-axial strain relationship of gravelly sand were simulated. The results are illustrated in Figures 14(c) and 14(d). With increasing parameter \(p_v\), the hardening degree of gravelly sand sample increased gradually, and the peak volume strain decreased rapidly. In a word, the parameter \(M\) has a significant effect on the hardening degree of the gravelly sand sample, and the shrinkage is affected by \(p_v\).
Figure 13: Continued.
Figure 13: Comparisons between experimental data and the calculated results for the stress-strain curves and volumetric strain-axial strain curves. (a) Conventional triaxial compression path CTC (CD). (b) Conventional triaxial compression path CTC (CU). (c) Triaxial compression path TC. (d) Reduced triaxial compression path RTC.

Figure 14: Continued.
5. The Effect of Stress Path on Deformation Characteristics of Gravelly Sand

To explain the microscopic deformation mechanism of gravelly sand samples under different stress paths, the following assumptions are made about the motion process of soil particles under an external force. (1) The soil particles are simplified into a rigid cylinder with radius $r$ and a unit length of 1, as shown in Figure 15(a). (2) The motion state of the soil particle objects follows Newton’s law. (3) The soil particles experience no energy loss during movement. (4) Particle breakage does not occur under contact and collision between soil particles.

Angle $O_3O_4$, connected by the circle centre, is defined as the characteristic angle $\theta$, and the prism linked by the axis lines is the researched unit body. The movement processes of soil particles $O_1$, $O_2$, $O_3$, and $O_4$ under external load are illustrated in Figure 15. As shown in Figure 15, the blue and red particles represent the initial and final motion states of the soil particles, respectively. Figure 15(b) shows that particles $O_1$ and $O_2$ approach each other with increasing $\theta$ and that the pore volume between the particles shrinks. In Figure 16(a), the green projection plane is the horizontal projection plane of the unit body under external forces $F_1$ and $F_2$, and the pink projection plane is the vertical projection plane of the unit body under external forces $F_3$ and $F_4$. Figure 16(b) shows that the horizontal stress area increases and the vertical stress area decreases as $\theta$ gradually increases. Finally, the principal stress ratio $\sigma_1/\sigma_3$ decreases. Since both the void ratio $e$ and the principal stress ratio $\sigma_1/\sigma_3$ are affected by the characteristic angle $\theta$, this paper takes the characteristic angle $\theta$ as an intermediate variable and derives the relationships between the principal stress ratio $\sigma_1/\sigma_3$ and the characteristic angle $\theta$ and relationships between the void ratio $e$ and the characteristic angle $\theta$. Finally, the relationship between the stress ratio $\eta (q/p)$ and the void ratio $e$ is established, and the effect of the stress path on the deformation trend of sand is analyzed.

Based on the volume calculation formula of the prism, it can be seen that the total volume $v$ of the unit body from Figure 16(a) is as follows:

$$v = \frac{1}{2} \times (2r \sin \frac{\theta}{2} \times 2) \times \left(2r \cos \frac{\theta}{2} \times 2\right) \times 2 \times 1 = 4r^2 \sin \theta.$$  \hspace{1cm} (17)

According to the theorem of polygon’s internal angle, the volume $v_s$ of rigid-rod in the unity body from Figure 17 can be obtained:

$$v_s = \frac{(4 - 2)\pi}{2\pi}nr^2 = nr^2.$$  \hspace{1cm} (18)

By combining equation (17) with (18), we can determine the void ratio $e$ of unity body from Figure 17 as follows:

$$e = \frac{\nu_s}{\nu} = \frac{\nu - \nu_s}{\nu_s} = \frac{4 \sin \frac{\theta}{n} - 1}. \hspace{1cm} (19)$$

From equation (19), the following expression can be derived:

$$\theta = \arcsin \frac{\pi(1 + e)}{4}.$$  \hspace{1cm} (20)

Figure 18 is the force analysis diagram of particles $O_2$ and $O_4$ under triaxial compression condition. From Figure 18(a), the external force $F_2$ can be expressed as follows:
Figure 15: The movement among soil particles. (a) The movement among sand particles. (b) The change of the pore among soil particles.

Figure 16: The force analysis of the unit body. (a) The projection plane of the unit body. (b) The change of the projection plane area under axial compression.

Figure 17: The cross-section of the unit body.
\[
F_2 = f_{23} \sin \frac{\theta}{2} + N_{23} \cos \frac{\theta}{2} + f_{23} \sin \frac{\theta}{2} + N_{24} \cos \frac{\theta}{2} 
\]

(21)

Assuming the soil particles are in the same size, we can obtain \(N_{23} = N_{24}\) and \(f_{23} = f_{24} = N_{24} \tan \varphi\) based on the friction law and the force balance. The tangent is the friction coefficient of the particle surface which is determined by the material properties; it is a constant for the same kind of sand. Equation (21) is simplified as follows:

\[
F_2 = 2N_{24} \tan \varphi \sin \frac{\theta}{2} + 2N_{24} \cos \frac{\theta}{2} 
\]

(22)

From Figure 18(b), the expression of the external force \(F_4\) can be derived:

\[
F_4 = N_{24}' \sin \frac{\theta}{2} + N_{14}' \sin \frac{\theta}{2} - f_{41} \cos \frac{\theta}{2} - f_{42} \cos \frac{\theta}{2} 
\]

(23)

According to the friction law and the force balance, we can obtain \(N_{14}' = N_{24}'\) and \(f_{41} = f_{42} = N_{24}' \tan \varphi\). Equation (23) is simplified as follows:

\[
F_4 = 2N_{24}' \sin \frac{\theta}{2} - 2N_{24}' \tan \varphi \cos \frac{\theta}{2} 
\]

(24)

Based on Newton’s third law, we have \(N_{24} = N_{24}'\). Dividing Formula (22) by Formula (24), the ratio of \(F_2\) to \(F_4\) can be derived:

\[
\frac{F_2}{F_4} = \frac{F_1}{F_3} = \frac{1 + \tan (\theta/2) \tan \varphi}{\tan (\theta/2) - \tan \varphi} 
\]

(25)

From Figure 16(a), it can be seen that the external forces \(F_1, F_2\), and \(F_3\) are uniformly distributed to the vertical and horizontal projection surfaces, respectively, and the relationship between the principal stress \(\sigma_1\) and \(\sigma_3\) to the external forces can be expressed as follows:

\[
\sigma_1 = \frac{F_1}{4r \sin (\theta/2)} = \frac{F_2}{4r \sin (\theta/2)} 
\]

(26)

\[
\sigma_3 = \frac{F_3}{4r \cos (\theta/2)} = \frac{F_4}{4r \cos (\theta/2)} 
\]

(27)

By substituting equation (25) into equation (26), we can determine the relationship between principal stress ratio \(\sigma_1/\sigma_3\) and \(\theta\) under triaxial compression as follows:

\[
\frac{\sigma_1}{\sigma_3} = \frac{\cot (\theta/2)(1 + \tan (\theta/2) \tan \varphi)}{\tan (\theta/2) - \tan \varphi} = \frac{\cot (\theta/2) + \tan \varphi}{\tan (\theta/2) - \tan \varphi} 
\]

(28)

Similarly, Figure 19 is the force analysis diagram of soil particles \(O_2\) and \(O_4\) in the triaxial extension condition. Based on the force balance, friction law, and Newton’s law, the relationship between the principal stress ratio \(\sigma_1/\sigma_3\) and the characteristic angle \(\theta\) can be expressed as follows:

\[
\frac{\sigma_1}{\sigma_3} = \frac{\cot (\theta/2)(1 - \tan (\theta/2) \tan \varphi)}{\tan (\theta/2) + \tan \varphi} = \frac{\cot (\theta/2) - \tan \varphi}{\tan (\theta/2) + \tan \varphi} 
\]

(29)

In the above study, we derived the relationship between the principal stress ratio \(\sigma_1/\sigma_3\) and the characteristic angle \(\theta\) and the relationship between the void ratio \(e\) and the characteristic angle \(\theta\). By substituting equation (20) into
From equation (27), we can obtain the relationship between the principal stress ratio $\sigma_1/\sigma_3$ and the void ratio $e$ in the triaxial compression condition as follows:

$$\sigma_1 = \frac{\cot (\arcsin(\pi(1+e)/4))/2 + \tan \phi}{\tan (\arcsin(\pi(1+e)/4))/2 - \tan \phi} \quad (29)$$

The relationship between stress ratio $\eta$ and principal stress ratio $\sigma_1/\sigma_3$ can be obtained in the triaxial test condition:

$$\eta = \frac{q}{p} = \frac{\sigma_1 - \sigma_3}{(1/3)(\sigma_1 + 2\sigma_3)} = \frac{3((\sigma_1/\sigma_3) - 1)}{(\sigma_1/\sigma_3) + 2} \quad (30)$$

By substituting equation (29) into equation (30), the relationship between stress ratio $\eta$ and void ratio $e$ can be expressed as follows:

$$\eta = 3 - \frac{9}{(\sigma_1/\sigma_3) + 2} = 3 \left[ 1 - \frac{3}{2} \frac{\tan (\arcsin(\pi(1+e)/4))/2 - \tan \phi}{2 \tan (\arcsin(\pi(1+e)/4))/2 + \cot (\arcsin((\pi(1+e))/4)/2) - \tan \phi} \right] \quad (31)$$

To explore the effect of stress path on the void ratio $e$, the principal stress ratio $\sigma_1/\sigma_3$ under the $T_1$, $T_2$, and $T_3$ is substituted into equation (31) to calculate the corresponding void ratio $e$. The calculation results are shown in Table 6 (in this paper, the internal friction angle of gravelly sand is 11.86° and $\tan \phi = 0.21$). Figure 20 shows the relationship between void ratio $e$ and axial strain $\varepsilon_1$ under $T_1$, $T_2$, and $T_3$. It can be seen from Figure 20 that the void ratio $e$ under the $T_1$, $T_2$, and $T_3$ paths is almost equal under the same axial strain at the beginning of loading. In the process of loading, the void ratio $e$ of the unit body under $T_1$ is the minimum, the void ratio $e$ under $T_2$ is bigger than $T_1$, and the void ratio $e$ under $T_3$ is the maximum under a same axial strain, which are consistent with the experimental results shown in Figure 5 and the conclusions in the literature [60]. Establishing relationships between microscopic mechanisms and macrophenomena is a complicated task. This section describes some exploration on the task as a reference for researchers.
6. Conclusions

(1) All gravelly sand samples presented strain hardening and shear contraction during the shearing tests under the CTC (CD, TC, and RTC) while exhibited strain softening under the CTC (CU). Under a certain confining pressure, the test sample in the T1 tests has the largest hardening degree, the hardening degree in the T2 tests is less than that in the T1 tests, and the sample in the T3 tests has the smallest hardening degree. The gravelly sand sample in the T3 tests has the smallest volume shrinkage, the sample in the T2 tests has more volume shrinkage than that in the T3 tests, and the sample in the T1 tests has the largest volume shrinkage under a certain confining pressure.

(2) In the framework of critical-state soil mechanics, a characteristic state stress ratio \( M_c \) was introduced into the dilatancy equation of the modified Cam-bridge model based on the state-dependent dilatancy theory, and a new dilatancy equation was obtained.

(3) A comparison of the calculation results and tests shows that the proposed model in this paper can well describe the strain hardening, strain softening, and volume shrinkage characteristics of gravelly sand under different stress paths. In addition, the parameter sensitivity analysis shows that when parameter \( M \) increases, the hardening degree of gravelly sands significantly increases, and the volume shrinkage of gravelly sand slightly increases. With increasing parameter \( p_o \), the hardening degree of the test sample gradually increases, and the peak volume strain rapidly decreases. In a word, parameter \( M \) significantly affects the hardening degree of the gravelly sand sample, and the shrinkage is affected by \( p_o \).

Data Availability

Since the experiment was completed with the support of Sichuan Agricultural University, the data used to support the results of this study are available from the responsible person and the author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

Authors’ Contributions

Dongjie Zhang and Fei Luo contributed equally to this work.

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