Lower Bound for the Communication Complexity of the Russian Cards Problem

Aiswarya Cyriac, K. Murali Krishnan,
Department of Computer Science and Engineering
National Institute of Technology, Calicut 673601, India.
Email: aiswaryanitc@gmail.com, kmuralinitc@gmail.com

Abstract—In this paper it is shown that no public announcement scheme that can be modeled in Dynamic Epistemic Logic (DEL) can solve the Russian Cards Problem (RCP) in one announcement. Since DEL is a general model for any public announcement scheme [11], [3], [6], [21], [12] we conclude that there exist no single announcement solution to the RCP. The proof demonstrates the utility of DEL in proving lower bounds for communication protocols. It is also shown that a general version of RCP has no two announcement solution when the adversary has sufficiently large number of cards.

Key words: Russian cards problem, Dynamic Epistemic Logic, Communication complexity, Lower bound.

I. INTRODUCTION

In the Russian cards problem (RCP), there are three players and seven cards. The cards are randomly distributed among the players such that two players get three cards each and the third player gets one card. The problem is to find a sequence of public announcements by which players with three cards each are able to acquire complete information about all the cards, without the third player knowing about any of their cards.

The solution to the problem will imply a method to communicate information among parties in a distributed computing setting securely without using any encryption [15], [23], [20]. The analogy is that, the communicating agents and adversaries are modeled as players and the information to be communicated as the ownership of cards. It is generally believed that the above game gives unconditional security [11], [15], [23], [20].

Various solutions to the above problem can be found in the literature [9], [10], [22]. They all require at least two public announcements. Here we address the problem of formally establishing that no public announcement scheme can solve the problem in fewer announcements. The framework of Dynamic Epistemic Logic (DEL) is used to establish the lower bound. Similar bounds using other models for related problems can be found in [11], [2], [8], [14], [16].

The following sections briefly discuss dynamic epistemic logic, modeling of RCP in DEL and a proof for the lower bound. Finally a generalization of the RCP is presented and it is shown that two announcements are not sufficient to solve the general case when the adversary has sufficiently large number of cards.

II. RUSSIAN CARDS PROBLEM (RCP)

The problem was posed in 2000 [11] as the following:

From a pack of seven known cards two players each draw three cards and the third player gets the remaining card. How can the players with three cards openly (publicly) inform each other about their cards, without the third player learning from any of their cards who holds it?

Let us call the cards 0, 1, ..., 6. The players are Anne, Bill and Cath. Anne and Bill have three cards each and Cath has one. No secret communication is possible. Only announcements are allowed. The announcements are assumed to be truthful and public. Through a sequence of such announcements Anne and Bill have to learn the actual deal of the cards, i.e., for each card from the above pack, Anne and Bill should be able to say to whom that card belongs. Also for any card from the pack other than the one Cath is holding, she should not be able to say to whom that card belongs.

Various solutions to the above problem can be seen in the literature [9], [10], [22]. All these solutions require two announcements.

III. DYNAMIC EPISTEMIC LOGIC (DEL)

In this paper we express the RCP in the framework of Dynamic Epistemic Logic (DEL). This section briefly presents the syntax and semantics of Dynamic Epistemic Logic. More detailed discussion of the DEL, examples and its applications can be found in [3], [5], [13], [17], [18].

Kripke models and action models are semantical structures of dynamic epistemic logic. Given a set of agents (players) and basic propositions a Kripke model consists of the set of possible states and accessibility (or indistinguishability) relation between the states for every agent. The knowledge of the players about the state of the game in imperfect information game can be modeled by viewing game states as Kripke states and players as agents.

Action models model the actions of the players that will alter the knowledge of the players. Given an initial Kripke model modeling the knowledge of the players, the action models can be sequentially executed in the Kripke model, resulting in a new Kripke model that models the knowledge of the players after the action.

In imperfect information games, players do not have complete information about other players’ moves.
Epistemic Language: Epistemic logic can be used to model knowledge in games of imperfect information [1]. The Epistemic language $L_{P,N}$ is the smallest closed set for which the following holds:

- $p \in P \Rightarrow p \in L_{P,N}$
- $\phi, \psi \in L_{P,N} \Rightarrow \neg \phi, (\phi \land \psi) \in L_{P,N}$
- $\phi \in L_{P,N}$ and $n \in N \Rightarrow K_n \phi \in L_{P,N}$

The sentence $K_1 \phi$ is read as: agent 1 knows $\phi$. $(\phi \land \psi)$, $\phi \land \psi$, and $\phi \land \psi$ are abbreviations for $(\neg \phi \land \neg \psi), (\neg \phi \lor \psi)$, and $(\phi \lor \psi) \lor (\neg \phi \land \neg \psi)$ respectively. The notation $\top$ stands for $\neg (p \land \neg p)$ for some $p \in P$.

Let a finite set of agents $N$ and a finite set of propositional atoms $P$ be given. A Kripke model $(M, w)$ is a tuple $(W, R, V)$ where:

- The set $W$ is a nonempty set of states $\{w_1, \ldots, w|W|\}$
- The accessibility function $R : N \rightarrow 2^{W \times W}$ assigns for each agent $n \in N$ a set of ordered pairs of states. $\forall n \in N, R(n) \subseteq W \times W$ is an equivalence relation.
- The valuation function $V : W \rightarrow 2^P$ assigns to each state $w$ a set of propositional atoms. $\forall w \in W, V(w) \subseteq P$.

$(w, w') \in R(n)$ is interpreted as state $w'$ is accessible (or indistinguishable) from state $w$ for the atom $n$. The set of propositional atoms assigned to a state by $V$ is the atoms which hold in that state.

A Kripke world is a pair consisting of a Kripke model $(M, w)$ and a state $w \in W$ and is denoted by $(M, w)$. Let a Kripke model $(M, w) = (W, R, V)$ and the epistemic language $L_{P,N}$ be given.

- The sentence $\phi$ is read as: agent $1$ knows $\phi$. $(\phi \land \psi)$, $\phi \land \psi$, and $\phi \land \psi$ are abbreviations for $(\neg \phi \land \neg \psi), (\neg \phi \lor \psi)$, and $(\phi \lor \psi) \lor (\neg \phi \land \neg \psi)$ respectively. The notation $\top$ stands for $\neg (p \land \neg p)$ for some $p \in P$.

A Player $x$ knows the fact $\phi$ in the state $w$ only if $\phi$ holds in all the states indistinguishable from $w$. Also if $\phi$ is true in all the states indistinguishable from $w$ for Player $x$, she can infer $\phi$

The action models are used to update Kripke Models. An action model is a set of actions, an accessibility (indistinguishability) relation between the actions for every agent, and a precondition function for each action.

Let a set of agents $N$ and the epistemic language $L_{P,N}$ be given. An Action model $\mu$ is a tuple $(A, R^a, \Pi)$:

- The set $A$ is the nonempty set of actions $\{a_1, \ldots, a_{|A|}\}$
- The accessibility function $R^a : N \rightarrow 2^{A \times A}$ is a function which assigns to each agent a set of ordered pairs of actions. $\forall n \in N, R^a(n) \subseteq A \times A$ is an equivalence relation.

The precondition function $\Pi : A \rightarrow L_{P,N}$ assigns to every action a precondition. $\forall a \in A, \Pi(a) \in L_{P,N}$

**Execution:** Action models are used to update Kripke model. Thus an action model is an operator on a Kripke Model. The execution of an action in a state results in a new state if and only if the precondition of the action holds in that state.

Let a Kripke model $M = (W, R, V)$ and an action model $\mu = (A, R^a, \Pi)$ be given. The execution of action model $\mu$ in Kripke model $M$ results in a Kripke model denoted by $M \otimes \mu$. $M \otimes \mu = (W', R', V')$ such that:

- The set of worlds $W'$ is $\{(w, a) \in W \times A \mid M, w \models \Pi(a)\}$
- For every Player $n \in N$, $(w, a) \otimes (w', a') \in R'(n)$ if and only if the states $w$ and $w'$ were indistinguishable for the Player $x$ in the initial Kripke model if and only if $a$ and $a'$ were indistinguishable in the action model $\mu$.

The knowledge of the players about the state of the game at the beginning of the game is modeled by a Kripke model. The knowledge actions which occur during the game are modeled by action models. The knowledge development is modeled by sequential execution of the action model in the Kripke model, resulting in a new up-to-date Kripke model modeling the knowledge of the players after the knowledge actions.

IV. PROBLEM MODELING

Given the set of players and the basic propositions, the Russian cards problem (RCP) can be modeled in Dynamic Epistemic Logic (DEL). Let $U = \{0, 1, 2, 3, 4, 5, 6\}$ be the set of cards and $N = \{a, b, c\}$ (representing Anne, Bill and Cath) be the set of players. The basic propositions are ‘card 0 is with Anne,’ ‘card 3 is with Bill’ and so on. If $i_x$ denotes ‘card $i$ is with $x$’ then the set of basic propositions $P = \{i_x \mid x \in N, i \in U\}$.

Initially Player $a$ and Player $b$ have three cards each and Player $c$ has one card. The Kripke model for the initial game state is given by $M = (W, R, V)$, where $W = \{(A, B, C) \mid |A| = |B| = 3, |C| = 1, A \cup B \cup C = U\}$ $R(a) = \{((A, B, C), (A', B', C')) \mid A = A'\}$ $R(b) = \{((A, B, C), (A', B', C')) \mid B = B'\}$ $R(c) = \{((A, B, C), (A', B', C')) \mid C = C'\}$ $V((A, B, C)) = \{i_a \mid i \in A\} \cup \{i_b \mid i \in B\} \cup \{i_c \mid i \in C\}$.

In any state $w = (A, B, C)$ the set of cards with Player $a$ is $A$, the set of cards with Player $b$ is $B$ and that with Player $c$ is $C$. $R(x, x) \in N$ contains the state pairs that are indistinguishable for Player $x$. The definition of $R(x)$ follows from the fact that initially Player $x$ knows only the cards she is holding. Hence all the states in which her hand of cards is the same will be indistinguishable for her.
For each $A \subseteq U, |A| = 3$, let $T_A = \{(A', B', C') : A' = A, |B'| = 3, |C'| = 1 \}$ and $A' \cup B' \cup C' = U$. Similarly for each $B \subseteq U, |B| = 3$, let $S_B = \{(A', B', C') : B' = B, |A'| = 3, |C'| = 1 \}$ and $A' \cup B' \cup C' = U$ and for each $C \subseteq U, |C| = 1$, let $Q_C = \{(A', B', C') : C' = C, |A'| = |B'| = 3 \}$ and $A' \cup B' \cup C' = U$.

Hence we have:

$$R(a) = \bigcup_{A \subseteq U, |A| = 3} T_A \times T_A$$

and $A \neq A' \Rightarrow T_A \cap T_A = \emptyset$

$$R(b) = \bigcup_{B \subseteq U, |B| = 3} S_B \times S_B$$

and $B \neq B' \Rightarrow S_B \cap S_B = \emptyset$

$$R(c) = \bigcup_{C \subseteq U, |C| = 1} Q_C \times Q_C$$

and $C \neq C' \Rightarrow Q_C \cap Q_C = \emptyset$

$R(a)$ is a partition of $W$ and $T_A$ will be called a component of $R(a)$. Here $W = \bigcup_{A \subseteq U, |A| = 3} T_A$ and the union is disjoint. $R(a)$ has $|\frac{3}{2}| = 35$ components each having $|\frac{3}{2}| \times |\frac{1}{2}| = 4$ states. All four states in any component $T_A$ are indistinguishable for Player $a$ by the definition of $R(a)$.

Similarly $R(b)$ is a partition of $W$ and $S_B$ is called a component of $R(b)$. $W = \bigcup_{B \subseteq U, |B| = 3} S_B$ and the union is disjoint. $R(b)$ has $|\frac{3}{2}| = 35$ components each having $|\frac{3}{2}| \times |\frac{1}{2}| = 4$ states and all four states in any component $S_B$ are indistinguishable for Player $b$.

$R(c)$ is a partition of $W$ and $Q_C$ is called a component of $R(c)$. $W = \bigcup_{C \subseteq U, |C| = 1} Q_C$ and the union is disjoint. $R(c)$ has $|\frac{1}{2}| = 7$ components each having $|\frac{3}{2}| \times |\frac{1}{2}| = 20$ states. The twenty states in a component $Q_C$ are indistinguishable for Player $c$.

As an example for $A = \{0, 1, 2\}$, $T_A = \{\{0, 1, 2\}, \{3, 4, 5\}, \{6\}, \{0, 1, 2\}, \{3, 4, 6\}, \{5\}, \{0, 1, 2\}, \{3, 5, 6\}, \{4\}, \{0, 1, 2\}, \{4, 5, 6\}, \{3\}\}$. Player $a$ cannot distinguish between these four states because in all the four states Player $a$’s hand is $\{0, 1, 2\}$.

Without loss of generality let us assume that Player $a$ is having cards $\{0, 1, 2\}$, Player $b$ is having $\{3, 4, 5\}$ and Player $c$ is having $\{6\}$ initially. We denote this state by $w^*$. Thus $w^* = \{0, 1, 2\}, \{3, 4, 5\}, \{6\}\}$. Suppose that a single announcement scheme solves the RCP. In the final model we claim that there will be at least one component $T_A$ and $S_B$ of the partitions generated by $R(a)$ and $R(b)$ respectively such that $T_A = S_B = \{w^*\}$. This is because if one more state was present in the component, the Players cannot distinguish between those states. Also there will be at least one component $Q_C$ of the partition generated by $R(c)$ such that $w^* \in Q_C$ and $|Q_C| > 1$. Otherwise Player $c$ will be able to find out the actual state. We present the claim formally:

**Lemma 1:** Assume that RCP is solved in Kripke model $M = (W, R, V)$ and for all $A, B$ and $C$ such that $|A| = |B| = 3$ and $|C| = 1$, $T_A, S_B$ and $Q_C$ are components of $R(a), R(b)$ and $R(c)$ respectively. Then the following statement hold:

1. $\exists A, \exists B$ such that $T_A = S_B = \{w^*, t\}$
2. $\forall C, w^* \in Q_C \Rightarrow |Q_C| > 1$.

**Proof:** Let $w^* \in T_A$. If possible let $T_A$ contain another state — say $w_1$ such that $w^* \neq w_1$. Let $w^* = (A, B^*, C^*)$ and $w_1 = (A, B^1, C^1)$. $B^* \neq B^1$ and $C^* \neq C^1$, otherwise $w^* = w_1$. Player $a$ cannot distinguish whether Player $b$ is having $B^* \cup B^1$. Also she cannot distinguish whether Player $c$ is having $C^* \cup C^1$. This means the RCP is not yet solved contradicting the hypothesis. Therefore $T_A = \{w^*\}$. Similar argument shows $S_B = \{w^*\}$.

$\forall C, w^* \in Q_C \Rightarrow |Q_C| > 1$. If $|Q_C| = 1$, then $c$ will understand which state she is in.

There does not exist even a single card that belongs to Player $a$ in all the states of $Q_C$. If such a card exists, say 4, Player $c$ will understand that 4 is with Player $a$. (Recall the semantics for $K_a, \phi$ is discussed in section III). Similarly, there does not exist even a single card that belongs to Player $b$ in all the states of $Q_C$.

The above claim is tuned to our requirement of solving the RCP in one announcement. It is easily seen that the claim holds in any final model reached by any sequence of announcements.

**V. Lower Bound for RCP**

**Theorem 1:** There exist no single announcement solution to the RCP.

**Proof:** For the sake of contradiction assume there exists a single announcement solution to RCP. Without loss of generality it can be assumed that Player $a$ is making the announcement. Let the action model for the announcement be $\mu = (A, R^*, \pi)$.

In section III we have seen that $R(a)$ partitions $W$ into 35 components. Each component will have 4 states. Suppose $w_1, w_2, w_3$ and $w_4$ belong to one component — say $T_A$. Let $w_1$ be the actual state. Since Player $a$ will be deterministically making the announcement, she will make the same announcement, say $a_1$ for all the elements in $T_A$.

In section III we have seen that the states $(w_1, a_1)$ and $(w_2, a_2)$ will be indistinguishable for Player $a$ if $(w_1, w_2) \in R(a)$ and $(a_1, a_2) \in R^*(a)$. So in the final model $(w_1, a_1), (w_2, a_1), (w_3, a_1)$ and $(w_4, a_1)$ will belong to the same component of $R(a)$. This contradicts Claim III. Hence there cannot be a single announcement solution to the RCP.

**VI. A Generalization**

In this section we will consider a natural generalization of the RCP in which Anne and Bill are holding $k$ cards each and Cath is holding $l$ cards from a pack of $2k+l$ cards. We denote this version of the RCP as RCP$(k; l)$. Hence the original RCP discussed before is RCP$(3; 1)$ in the new notation.

It can be easily seen that for any $k \geq 1$ and $l \geq 1$ there does not exist a one announcement solution for RCP$(k; l)$ as the Theorem III and the proof extends to this case as well. We
will now examine the impossibility of a two announcement solution for $\text{RCP}(k; l)$ using similar strategies.

The set of cards $U = \{1, 2, \ldots, 2k + l\}$. The initial Kripke model $M = (W, R, V)$ where,

$W = \{(A, B, C) \mid |A| = |B| = k, |C| = l, A \cup B \cup C = U\}$

$R(a) = \{((A, B, C), (A', B', C')) \mid A = A'\}$

$R(b) = \{((A, B, C), (A', B', C')) \mid B = B'\}$

$R(c) = \{((A, B, C), (A', B', C')) \mid C = C'\}$

$V((A, B, C)) = \{i_a \mid i \in A\} \cup \{i_b \mid i \in B\} \cup \{i_c \mid i \in C\}$

$T_A, S_B$ and $Q_C$ are also defined similarly as in Section[V].

It follows that $R(a)$ and $R(b)$ will have $(\binom{2k + l}{k})$ components each with $(\binom{k + l}{k})$ elements each.

Suppose Player $a$ is making an announcement $\alpha$ for a set of components $T_A$. Since Player $c$ should not learn about a single card other than his own hand, we get the following lemma.

**Lemma 2:**

\[
\bigcup_{T_A \in \mathcal{T}_a} A = U
\]

\[
\bigcap_{T_A \in \mathcal{T}_a} A = \emptyset
\]

**Proof:** Assume that this was not the case. i.e., $\bigcap_{T_A \in \mathcal{T}_a} A = Q \neq \emptyset$. Player $c$ can infer that the cards in $U \setminus Q$ are not with Player $a$. Hence $U \setminus Q$ has to be $\emptyset$.

Similarly it can be seen that $\bigcap_{T_A \in \mathcal{T}_a} A = \emptyset$ since if $\bigcap_{T_A \in \mathcal{T}_a} A = Q \subseteq U$, $Q \neq \emptyset$, then Player $c$ can infer that the set of cards $Q$ is with Player $a$.

Before proving the impossibility of a two announcement solution for $\text{RCP}(k; l)$ we need to prove the following technical lemma.

**Lemma 3:** For $k \geq 2$, $l \geq \frac{2k^2}{\ln k}$

\[
\left\lfloor \frac{2k + l}{k} \right\rfloor \times \binom{k + l}{k} > \left(\frac{2k + l}{k}\right)^k
\]

**Proof:** Enough to have

\[
\frac{2k + l}{k} \times \binom{k + l}{k} > \left(\frac{2k + l}{k}\right)^k
\]

i.e.,

\[
\frac{2k + l}{k} \times \frac{(k + l)!}{k! \times (k + l)!} > \left(\frac{2k + l}{k}\right)! = \left(\frac{2k + l}{k}\right)!
\]

\[
\frac{2k + l}{k} \prod_{i=1}^{k} (i + l) > \prod_{i=1}^{k} (i + k + l)
\]

i.e.,

\[
\frac{2k + l}{k} > \prod_{i=1}^{k} \left(1 + \frac{k}{l + i}\right)
\]

Since $(1 + x) \leq e^x$

\[
\text{RHS} \leq e^{k \sum_{i=1}^{l+k} \frac{1}{i+1}}
\]

Bounding the summation by integral we get,

\[
\text{RHS} \leq e^{k \cdot \ln(l + k)} = \left(\frac{l + k}{l}\right)^k.
\]

Enough to have $\frac{2k + l}{k} > \left(\frac{l + k}{l}\right)^k$.

Let $l = \frac{2k^2}{\ln k}$.

Enough to have $2 + \frac{2k}{\ln k} > \left(1 + \frac{\ln k}{2k}\right)^k$

since $(1 + x) \leq e^x$ we get

\[
\left(1 + \frac{\ln k}{2k}\right)^k \leq e^\frac{\ln k}{2k} = \sqrt{k}
\]

\[
\therefore \ 2 + \frac{2k}{\ln k} > \sqrt{k}.
\]

We can see that the above equation holds for all $k \geq 2$.

Now we will prove that for sufficiently large $l$, there exist at least two states in $\bigcup_{T_A \in \mathcal{T}_a} T_A$ which are indistinguishable for player $b$ for any announcement $\alpha$ satisfying Lemma 2.

**Lemma 4:** For $k \geq 2, l \geq \frac{2k^2}{\ln k}$, for any announcement $\alpha$ satisfying Lemma 2, $\exists s_1, s_2 \in \bigcup_{T_A \in \mathcal{T}_a} T_A$ such that $s_1 \neq s_2$ and $(s_1, s_2) \in R(b)$

**Proof:** Assume that there do not exist two indistinguishable states for Player $b$ in $\bigcup_{T_A \in \mathcal{T}_a} T_A$. From Lemma 2, we will get $|\mathcal{T}_a| \geq \left\lceil \frac{2k^2}{\ln k} \right\rceil$ since $|A| = k$ and $|U| = 2k + l$ (as the $2k + l$ elements need to be distributed among sets of size $k$). Now the number of different hands possible for Player $b$ should be at least $\left\lceil \frac{2k^2}{\ln k} \right\rceil \times \binom{k + l}{k}$. But the different number of combinations possible is $(\frac{2k + l}{k})$. By Lemma 3, $\left(\frac{2k + l}{k}\right) \times \binom{k + l}{k} > \left(\frac{2k + l}{k}\right)^k$ for $l \geq \frac{2k^2}{\ln k}$ and $k \geq 2$.

Now we will prove that there does not exist a two-announcement solution for $\text{RCP}(k; l)$ if $k \geq 2$ and $l \geq \frac{2k^2}{\ln k}$.

**Theorem 2:** For $k \geq 2, l \geq \frac{2k^2}{\ln k}$, there exists no two announcement solution to the $\text{RCP}(k; l)$.
VII. CONCLUSION

We have analyzed the Russian Cards Problem and the generalization $\text{RCP}(k; l)$ in the framework of Dynamic Epistemic Logic. It is shown that there does not exist a single announcement solution for the Russian Cards Problem within the framework of Dynamic Epistemic Logic. Since the framework is considered sufficiently general [11], [3], [6], [21], [12] we claim that there can be no one-announcement solution to RCP in general and no two announcement solution to $\text{RCP}(k; l)$ for $l \geq 2\ln k$, $k \geq 2$.

The problem of deriving upper and lower bounds for $\text{RCP}(k; l)$ in general in terms of $k$ and $l$ remains open for further investigation.

Acknowledgements

The authors would like to thank Dr. L. Sunil Chandran for helpful discussions and suggestions. The first author would like to thank Prof. R. Ramanujam for introducing her to the Russian cards problem.

REFERENCES

[1] M.H. Albert, R.E.L. Aldred, M.D. Atkinson, H.P. van Ditmarsch, C.C. Handley, “Safe communication for card players by combinatorial design for two-step protocols,” Australasian journal of Combinatorics, vol. 33, pp. 33-45, 2005.
[2] A. Ambainis, “Upper bound on the communication complexity of private information retrieval,” LNCS, vol. 1256, pp. 401-407, Springer, 1997.
[3] A. Baltag, L. S. Moss, and S. Solecki, “The Logic of Public Announcements, Common Knowledge, and Private Suspicions,” Technical Report, UMI Order Number: SEN-R9922, CWI (Centre for Mathematics and Computer Science), 1999.
[4] A. Baltag, “A Logic for Suspicious Players: Epistemic Actions and Belief-Updates in Games,” Bulletin of Economic Research, vol. 54, no. 1, pp. 1-45, 2002.
[5] Johan van Benthem, “Games in Dynamic Epistemic Logic,” Bulletin of Economic Research, vol. 53, no. 4, pp. 219-248, October 2001.
[6] Johan van Benthem, “One is a lonely number: on the logic of communication,” Logic Colloquium, 2002.
[7] Johan van Benthem, “Logic and Reasoning: do the facts matter?,” Studia Logica, Special Issue: Psychology in Logic?, January 2008.
[8] J. Cai, R. Lipton, L. Longpre, M. Oghara, K. Regan, and D. Sivakumar, “Communication complexity of key agreement on limited ranges,” In Proc. of the Annual Symposium on Theoretical Aspects of Computer Science, LNCS, vol. 834, pp. 56–65, Springer-Verlag, 1995.
[9] H. P. van Ditmarsch, “Killing cluedo,” Natuur & Techniek, vol. 69, no. 11, pp. 32-40, 2001.
[10] H. P. van Ditmarsch, “Oplossing van het mysterie (solution of the murder mystery),” Natuur & Techniek, vol. 70, no. 2, pp. 17, 2002.
[11] H. P. van Ditmarsch, “The Russian cards problem,” Studia Logica, vol. 71, no. 1, pp. 1-32, 2003.
[12] H. P. van Ditmarsch and B. P. Kooi, “The secret of my success,” Synthese, vol. 153, no. 2, pp.339, Springer, November 2006.
[13] Jjord Druven, “Knowledge Development in Games of Imperfect Information,” Master’s thesis, University of Groningen, Groningen, the Netherlands, 2002.
[14] R. Fagin, J.Y. Halpern, Y. Moses, and M.Y. Vardi, Reasoning about Knowledge, MIT Press, Cambridge MA, 1995.
[15] Michael J. Fischer and Rebecca N. Wright, “Bounds on secret key exchange using a random deal of cards,” Journal of Cryptology, vol. 9, no. 2, pp. 71-99, March 1996.
[16] M. Franklin, M. Yung, “Communication complexity of secure computation,” In Proc. 24th annual ACM symposium on Theory of computing, pp. 699-710, 1992.
[17] Jelle Gerbrandy, “Dynamic Epistemic Logic,” in Logic, Language and Information, vol. 2, pp.67-84, CSLI, 1999.
[18] Jelle Gerbrandy, “The Surprise Examination in Dynamic Epistemic Logic,” Synthese, vol. 155. no. 1, pp. 21-23, March 1992.
[19] Jaakko Hintikka, “Knowledge Acknowledged: Knowledge of Propositions vs. Knowledge of Objects,” Philosophy and Phenomenological Research, vol. 56, no. 2, pp.251-275, June 1996
[20] Koichi Koizumi, Takaaki Mizuki and Takao Nishizeki, “Necessary and Sufficient Numbers of Cards for the Transformation Protocol,” Lecture Notes in Computer Science, vol. 3106, pp:92-101, 2004.
[21] C. Lutz, “Complexity and succinctness of public announcement logic,” In Proceedings of the Fifth international Joint Conference on Autonomous Agents and Multiagent Systems, AAMAS ’06, ACM, pp.137-143, May 2006. http://doi.acm.org/10.1145/1160633.1160657
[22] K. S. Makarychev and Yu. S. Makarychev, “The importance of being formal,” Mathematical Intelligencer, vol.23, no. 1, pp. 41-42, 2001.
[23] R. Ramanujam and S. P. Suresh, “Information based reasoning about security protocols,” Electronic Notes in Theoretical Computer Science, vol. 55, no. 1, pp. 89-104, 2001.
[24] F Voorbraak, “Generalized Kripke models for epistemic logic,” In Proc. 4th Conference on Theoretical Aspects of Reasoning About Knowledge, pp.214-228, March 1992.