Modified Archard’s Equation to Estimate Wear Volume due to Sliding Speed

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Received: 26 September 2020, Revised: 9 January 2021, Accepted: 18 January 2021

Abstract

The objective of this research is to introduce a mathematical model of wear volume of Archard’s equation influenced by speed variable. Some researchers have studied the wear volume due to sliding speed, roughness and coefficient of friction. However, mathematical model dealing with sliding speed has never been explicitly reported. Wear analysis is oftenly expressed experimentally through charts concerning both wear volume and sliding speed instead of mathematics. This research is started by modeling mathematical representation within Buckingham Pi Theory. The mathematical parameter contains wear volume, hardness, normal loads, sliding speed, sliding distance, and density of materials. Buckingham Pi model produces three sets of equation. Two of these sets yields Archard’s equation. By combining the third set, the modified Archard’s equation is determined. Since Buckingham set requires a constant of equality, the equation is verified by experiment data. This value is called Wear-Speed Coefficient. Experiment using pin-on-disk tribometer is conducted by varying sliding speed. Furthermore, those parameters are applied to estimate wear volumes. Materials which are used for this verification are NBR Rubber Nitril, Ultra High Molecular Weight Poly Ethylene (UHMWPE), and Poly Tetra Fluoro Ethylene (PTFE). In conclusion, the modified Archard equation is determined to estimate wear volumes. Based on the experiment, the model is accurate for UHMWPE, NBR and PTFE. Moreover, ratio of density to material’s hardness is significant to control wear resistance influenced by speed.

Keywords: Sliding speed, normal load, wear volume, Buckingham PI model, wear-speed coefficient

1. Introduction

Contact surfaces frequently cause wear. The wear volume may be worsened by sliding speed. Some abrasive wear is necessarily needed, such as grinding. However, some others are strictly maintained to avoid excessive loose, such as bearing. Wear might happen together with the present of stick-slip phenomenon during wear process, such as multi-directional abrasion [1] in ball joints. Therefore, wear volume becomes very important aspect to control mechanism’s life.

Parameters influencing wear volume are surface hardness [2], normal loads [3], materials, sliding distance, sliding speed [3–5], and friction between two materials. Surface roughness is also as part of assertive variable being studied [6]. However, this cumbersome wear volume is rarely reported in mathematical presentation. The correlation among these parameters are often shown with graphical reports due to uncertainty of wear mechanism. Finally, it is important to have a mathematical model to accommodate the parameters with the intention of wear volume approximation.

This research is focused on formulating the wear volume relating to sliding speed. The formulation method uses Buckingham Pi Theory. The constant of set’s equality is drawn from experiments.

2. Method and Materials

As mention in the previous paragraph, the mathematical model of wear volume is constructed by using Buckingham Pi model. Wear parameters chosen to be repeating variables are Hardness (\(H\)), material density (\(\rho\)), and Wear Volume (\(V\)). Meanwhile, non repeating variables include normal load (\(W\)), sliding distance (\(L\)), and sliding speed (\(v\)). The general form is expressed as following:

\[ \prod = f(\rho, V, H, W, L, v) \]

with,

- \(\rho\) = Density \(\text{[ML}^{-3}\}\)
- \(V\) = Wear volume \(\text{[L}^3\}\)
- \(H\) = Hardness \(\text{[ML}^{-1}\text{T}^{-2}\}\)
- \(W\) = Normal Load \(\text{[ML}^{-1}\text{T}^{-2]}\)
- \(L\) = Sliding Distance \(\text{[L]}\)
- \(v\) = Sliding Speed \(\text{[LT}^{-1]}\)

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The first set of the dimensionless equation is below:
\[ \rho^a V^b H^c W = M^0 L^0 T^0 \]
\[ (ML^{-3})^a (L^3)^b (ML^{-1}T^{-2})^c (MLT^{-2}) = M^0 L^0 T^0 \]

By solving the equation, the first group of dimensionless parameter is determined as in Equation (1).
\[ \Pi_1 = \frac{W}{H} \cdot \frac{1}{V^{2/3}} \tag{1} \]

The second set is as in Equation (2).
\[ \rho^a V^b H^c L = M^0 L^0 T^0 \]
\[ (ML^{-3})^a (L^3)^b (ML^{-1}T^{-2})^c (L) = M^0 L^0 T^0 \]
\[ \Pi_2 = \frac{V^{1/3}}{L} \text{ or } \frac{L}{V^{1/3}} \tag{2} \]

The third set is determined as in Equation (3).
\[ \rho^a V^b H^c v = M^0 L^0 T^0 \]
\[ (ML^{-3})^a (L^3)^b (ML^{-1}T^{-2})^c (LT^{-1}) = M^0 L^0 T^0 \]
\[ \Pi_3 = \sqrt{\frac{\rho}{H}} v \tag{3} \]

Equations (1) and (2) are synced to become the following:
\[ \frac{W}{H} = \frac{1}{V^{2/3}} \sim \frac{V^{1/3}}{L} \]
\[ \frac{W}{H} k = \frac{V}{L} \tag{4} \]

Equation (4) is well known as Archard’s equation.

Furthermore, Equation (4) is combined with Equation (3), this dimensionless parameter is becoming the Equation (5). Sliding speed (v) appears to be the main subject in this research.
\[ \frac{W}{H} \frac{1}{V^{2/3}} \sqrt{\frac{\rho}{H}} v \approx \frac{V^{1/3}}{L} \tag{5} \]

Finally, the wear volume is determined as in Equation (6).
\[ V \approx W L \sqrt{\frac{\rho}{H}} v \tag{6} \]
\[ V = \varphi W L \sqrt{\frac{\rho}{H}} v \]

with \( \varphi \) is the wear-speed coefficient.

Equation (5), in turn, is verified by data from experimentation using three different composite materials. The materials are Ultra High Molecular Weight Poly Ethylene (UHMWPE) [7, 8], Poly Tetra Flouro Ethylene (PTFE) [9], and Nutrile Butadene Rubber (NBR) [10].

The experiments were performed using a pin-on-disk tribometer as in [11] and shown in Figure 1. The normal Load, and sliding distance are 8 N, 700 m respectively. Hardness number and density are parameters depending on materials. Volumes of abrasion are measured.

UHMWPE has low density. It is characterized by its extremely good resistance to chemicals, high impact strength, and low weight. Furthermore, it is often used for prostheses. NBR is used in the automotive and aeronautical industries to make seals, oil handling hoses, and bushing. NBR’s density can be compounded up to 1350 kg/m^3. In this research, the density which is studied is

![Figure 1. (a) Pin-on-disk tribometer (b). Three different materials of specimen](image-url)
1150 kg/m$^3$ since of limitations. PTFE has flexural strength, low friction and high electrical resistance [6]. It is also hydrophobic. PTFE's has very wide range of density. In particular usage, it might be as 2200 kg/m$^3$. This study uses a moderate density of 1030 kg/m$^3$ [12].

The final step is to compare the theoretically approach of wear volumes to these experimental results. The wear-speed coefficient, in turn, is determined according to the materials.

3. Results and Discussion

Abrasive volumes of materials being investigated are listed in Table 1, 2 and 3. The calculated wear volumes of every materials are also written as well.

According to these experiment data comparing between calculated wear volume and measured wear volume during experimentation, calculated wear is lower then the experiment results, as shown in Figure 2. However, UHMWPE has wear volumes that is almost the same between calculation and experimentation. In the other hand, for PTFM and NBR, the two wear volumes are different. The material hardness is believed to be responsible of the inequality. Furthermore, the tendency of wear caused by speed variation is similar. Wear would slowly increase while the speed increases. This phenomenon works for the three materials.

In order to have correct approximation between calculation and experimentation, a constant of Pi model is determined. This constant figures the magnitude of abrasive resistance of materials. It may also represent wear rate in Archard’s equation. The constant is called Wear-speed coefficient. Table 4 is the list of wear-speed coefficient.

The material of UHMWPE has a wear-speed coefficient of 1. It seems to be an exact approximation. The second material of NBR has almost perfect magnitude. The last material of PTFE has quite large difference. It is obvious that Archard’s Law does not include all of wear mechanism. It ignores adhesive wear such as delamination of surfaces. Since these wear volumes are measured in general wear, it is assumed that the big difference of PTFE’s magnitude is caused by adhesion during measurement.

In other side, melting temperature of material properties may affect adhesive characteristics of materials during contact since contact surface mostly generates heat [5]. In this circumference, the UHMWPE has the lowest melting temperature (135°C). It may imply that UHMWPE produces adhesive bigger then PTFE (370°C). It is difficult either the UHMWPE or the PTFE is the correct one.

In general, the environment temperature during experimentation was kept in below 60°C. It could be assumed that wear measured is mainly from abrasion. Figure 3 shows that abrasion path on the specimen of UHMWPE is mostly caused by abrasive wear mechanism, and some areas remain delamination.

### Table 1. Parameters and measured variables for UHMWPE

| Speed (rpm) | Sliding Speed (m/s) | Time (minute) | Density (kg/m$^3$) | Hardness Shore D | Calculated Wear Vol. (cc) | Measured Wear Vol. (cc) |
|-------------|---------------------|---------------|--------------------|------------------|--------------------------|------------------------|
| 80          | 0.173               | 69            | 950                | 20 (240MPa)      | 0.008                    | 0.006                  |
| 90          | 0.181               | 65            | 1030               | 24 (300MPa)      | 0.007                    | 0.007                  |
| 100         | 0.192               | 60            | 1150               | 24 (300MPa)      | 0.006                    | 0.012                  |

### Table 2. Parameters and measured variables for PTFE

| Speed (rpm) | Sliding Speed (m/s) | Time (minute) | Density (kg/m$^3$) | Hardness Shore D | Calculated Wear Vol. (cc) | Measured Wear Vol. (cc) |
|-------------|---------------------|---------------|--------------------|------------------|--------------------------|------------------------|
| 80          | 0.173               | 69            | 950                | 20 (240MPa)      | 0.008                    | 0.006                  |
| 90          | 0.181               | 65            | 1030               | 24 (300MPa)      | 0.007                    | 0.007                  |
| 100         | 0.192               | 60            | 1150               | 24 (300MPa)      | 0.006                    | 0.012                  |

### Table 3. Parameters and measured variables for NBR

| Speed (rpm) | Sliding Speed (m/s) | Time (minute) | Density (kg/m$^3$) | Hardness Shore D | Calculated Wear Vol. (cc) | Measured Wear Vol. (cc) |
|-------------|---------------------|---------------|--------------------|------------------|--------------------------|------------------------|
| 80          | 0.173               | 69            | 950                | 20 (240MPa)      | 0.008                    | 0.006                  |
| 90          | 0.181               | 65            | 1030               | 24 (300MPa)      | 0.007                    | 0.007                  |
| 100         | 0.192               | 60            | 1150               | 24 (300MPa)      | 0.006                    | 0.012                  |
Table 4. Wear-speed coefficient

| Materials | Hardness (Mpa) | Density (kg/m$^3$) | Wear-speed Coefficient $\phi$ |
|-----------|----------------|--------------------|-----------------------------|
| NBR       | 300            | 1030               | 1.9                         |
| PTFE      | 300            | 1150               | 8                           |
| UHMWPE    | 240            | 900                | 1                           |
Table 4 also indicates that the hardness and density, such as grain size, play the important control of wear mechanism [13]. The effects of speed seem to have similar tendency in every material. Since hardness and density are considered the major reason, the mathematical model need to find the wear-speed coefficient according to hardness and density, as in Equation (7).

\[
V = \varphi WL \sqrt{\frac{\rho}{H}} V
\]

\[
\frac{V}{L} = \frac{W}{H} \varphi \sqrt{\frac{\rho}{H}} V
\]

\[
\frac{V}{L} = \frac{W}{H} \left[ \varphi \sqrt{\frac{\rho}{H}} V \right]
\]  

(7)

The modified Archard equation appears in Equation (7). Assuming that wear rate is not constant during contact due to heat generated, it becomes Equation (8).

\[
k(t) = \left[ \varphi \sqrt{\frac{\rho}{H}} \frac{dL}{dt} \right]
\]

Moreover, since wear rate is not constant, consequently wear-speed coefficient is not in constant magnitude. Therefore, predicting wear volume is getting cumbersome when the material wear coefficient start changing. Acceleration and deceleration during contact may worsen the abrasion process. Assuming that wear-speed coefficient changes insignificantly, the wear rate turns into Equations (9) and (10).

\[
\frac{d}{dt} k(t) = \sqrt{\frac{\rho}{H}} \left[ \frac{d\varphi}{dt} + \varphi \frac{dv}{dt} \right]
\]

\[
\frac{d}{dt} k(t) = \sqrt{\frac{\rho}{H}} \varphi \frac{dv}{dt}
\]

(9)

(10)

Equation (8) shows that the wear rate in Archard’s Law depends on the operational speed. Even though the material is the same, wear rate may grow when speed increases [4,5]. It indicates that the phenomenon of Equation (9) does occur. Consequently, wear rate degradation in Equation (9) may happen in this local circumferences. Abrasive wear degradation in Equation (10) is linear to the sliding speed. Slopes of the degradation is a constant which is root square of density-hardness ratio. This parameter comes from material properties. Finally, this degradation will remain constant as far as these surface hardness or densities are stable.

### 4. Conclusion

In conclusion, the equation which has been formed through Buckingham Pi model mostly works on non metal materials, such as NBR, PTFE and UHMWPE. Sliding speed greatly influence wear volumes. The more speed may result the more wear volume. Wear-speed coefficient as product of equality process in Buckingham Pi model is influenced by hardness and density of material.

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