The study of the unstably-stratified marine atmospheric boundary layer by direct numerical simulation

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Abstract. Prediction of the properties of the atmospheric flows over sea is important for local and regional weather forecast. This prediction typically relies on the performance of large-scale meteorological models based on bulk formulae for air velocity, temperature and humidity. The bulk formulas relate turbulent fluxes of momentum, heat and vapor to bulk air velocity, humidity and air-sea temperature difference. The most widely used parameterizations used to compute the coefficients of proportionality in the bulk formulae are formulated on the basis of the Monin-Obukhov similarity theory (MOST) for different types of air stratification (stable, neutral and unstable). In our previous studies we showed that MOST quite accurately predicts the properties of the air-flow over waved water surface under neutral and stable stratification conditions provided that stratification effects are relatively weak and flow is in a statistically stationary state. In the present work, we perform direct numerical simulation and study the air flow over a waved water surface under unstable stratification conditions where the sea surface temperature is larger than the bulk air temperature. Such situation occurs in the tropical cyclone conditions as well as in polar lows at high latitudes. Our results show that in this case, the air flow dynamics is dominated by the development of large-scale cylindrical coherent vortex structures elongated in the direction of the mean wind. These structures cause a notable deviations from the MOST predictions for the air velocity and temperature profiles.

1. Introduction

Detailed knowledge of the interaction of turbulent wind with surface water waves is necessary for correct parameterization of turbulent exchange at the air-sea interface in prognostic models. This interaction may crucially depend on the temperature difference between air and water surface. Under the conditions of relatively small (about several degrees of K) air-water temperature difference and sufficiently high wind speed (typically, about several m/s) the air flow in the boundary layer (BL) is weakly stratified and turbulent, and its properties are well predicted by the Monin-Obukhov similarity theory (MOST) [1], regardless of the sign of the air-water temperature difference (i.e. for both stable and unstable stratification of the air). If
Fig. 1. Schematic of numerical experiment: $L_x, L_y, L_z$ are the domain sizes in the horizontal ($x$), spanwise ($y$), and vertical ($z$) directions; $a$ and $\lambda$ are the surface water wave amplitude and length; $T_0$ and $T_1$ are the bulk temperatures of water and air; $U_0$ the bulk velocity of the air-flow. The acceleration due to gravity is directed downwards.

This temperature difference increases to tens of Ks, with fixed bulk air velocity, stratification effects may become important.

The case of a sufficiently strong stable stratification (i.e. when the water is cold and the air is warm) was studied recently by performing direct numerical simulation [2]. The results show that strong stratification effectively suppresses turbulence and the flow becomes laminar if the surface wave amplitude is negligible. However, finite air fluctuations in the near-surface layer still survive for sufficiently large wave amplitudes. This regime, where turbulence is induced by surface waves, is poorly described by MOST.

A different situation is realized when the water surface is warm and the air is cold. In this case, stratification is unstable and buoyancy forces induce convective air motions. It is well known that in the absence of shear flow, turbulence can be supported by sufficiently strong (as compared to molecular viscosity) buoyancy forces [1]. Moreover, large-scale convective motions develop into coherent structures forming cells and/or rollers depending on the geometry of the physical domain in the laboratory experiment (or, by topography and planetary boundary layer height in field conditions). These coherent vortical structures are not taken into account by known classical closures. However, recent numerical and laboratory studies indicate that coherent structure significantly influence turbulence statistics and production and dissipation balance [4,5].

In the present paper, we investigate how coherent structures influence the mean velocity and temperature fields in a turbulent Couette flow over a waved water surface. This flow is widely used in direct numerical simulations of atmospheric boundary layer since it is characterized by a constant total momentum flux since it is periodic in the streamwise direction and driven by viscous shear stress at the upper boundary, not by pressure gradient [3]. Section 2 below presents the governing equations and numerical method. Numerical results are discussed in Section 3, and final conclusions and discussion are provided in Section 4.
2. Numerical method and results

We perform direct numerical simulation of turbulent, droplet-laden Couette flow above waved water surface. The schematic of the numerical experiment is similar to that used in our previous DNS study of turbulent air-flow over waved surface [2,3] (Fig. 1). DNS is performed in a reference frame moving with the wave phase velocity, $c$, so that the lower boundary, representing the wave surface, is stationary in the moving reference frame. The no-slip boundary condition is prescribed at the lower boundary, so that the wind velocity at the boundary coincides with the velocity in the water wave. The no-slip boundary condition is also prescribed at the upper horizontal plane moving in $x$-direction with bulk velocity $U_0$. This condition provides an external source of momentum due to the viscous shear stress, which compensates viscous dissipation in the boundary layer and makes the flow statistically
stationary. Air and water surface temperatures is prescribed at the top and bottom boundary planes, respectively, as \( T_0 \) at \( z = 0 \) and \( T_1 \) at \( z = L_z \); where \( T_0 > T_1 \).

The numerical algorithm is based on the integration of full, 3D Navier-Stokes equations for the carrier air-flow velocity and temperature written under the Boussinesq approximation [3]. The air-flow bulk Reynolds and Richardson numbers in DNS are defined as:

\[
\text{Re} = \frac{U_0 \lambda}{\nu_a}, \quad \text{Ri} = \frac{g (T_0 - T_1) \lambda}{(T_0 U_0^2)}
\]

and set equal to \( \text{Re} = 15000 \) and \( \text{Ri} = 0.02 \) and 0.2 for weak and strong stratification cases. Two different wave-slopes, \( \kappa a = 0.1 \) and \( \kappa a = 0.2 \), are considered. We also prescribe the wave celerity to be sufficiently small, \( c/U_0 = 0.05 \), which corresponds to “slow” waves and strong wind-forcing conditions [3].

Figures 2 and 3 present instantaneous and ensemble-averaged flow fields obtained in DNS. The figures show that the flow field is dominated by large-scale coherent cylindrical counter-rotating rollers elongated in the streamwise direction. Figures 4 and 5 present mean velocity and temperature profiles obtained in DNS and compare them with the MOST prediction [4]:

Fig. 3. Ensemble-averaged velocity and temperature fields. Top panel shows mean velocity vector field in \((x,z)\)-plane, and middle and bottom panels present mean \( x\)-velocity component and temperature in \((y,z)\) plane, respectively.
Fig. 4. Mean velocity profiles in DNS (red line) and MOST prediction (black line) obtained for Re = 15000, and Ri = 0, 0.02 and 0.2 (top, middle and bottom panels, respectively) for wave slope $ka = 0.1$ (left) and $ka = 0.2$ (right). Wave celerity $c/U_0 = 0.05$.

Fig. 5. Mean temperature profiles in DNS (red line) and MOST prediction (black line) obtained for Re = 15000, and Ri = 0.02 and 0.2 (top and bottom panels, respectively) for wave slope $ka = 0.1$ (left) and $ka = 0.2$ (right). Wave celerity $c/U_0 = 0.05$.

\[ \frac{<U_z>}{u_*} = \frac{1}{\kappa} \left[ \ln \frac{z}{z_{0U}} - \psi_U \left( \frac{z}{L} \right) \right] , \quad \frac{z}{L} < \frac{z_m}{L} = 1.574 \]  \hspace{1cm} (2)

\[ \frac{<U_z>}{u_*} = \frac{1}{\kappa} \left[ \ln \frac{z_m}{z_{0U}} - \psi_U \left( \frac{z_m}{L} \right) + 1.14 \left( \frac{z}{L} \right)^{1/3} - \left( \frac{z_m}{L} \right)^{1/3} \right] , \quad z > z_m \]  \hspace{1cm} (3)

\[ \frac{<T_0 - T>}{T_*} = \frac{1}{\kappa} \left[ \ln \frac{z_{T}}{z_{0T}} - \psi_T \left( \frac{z}{L} \right) \right] , \quad \frac{z}{L} < \frac{z_T}{L} = 0.465 \]  \hspace{1cm} (4)

\[ \frac{<T_0 - T>}{T_*} = \frac{1}{\kappa} \left[ \ln \frac{z_{T}}{z_{0T}} - \psi_T \left( \frac{z_T}{L} \right) + 0.8 \left( \frac{z_T}{L} \right)^{-1/3} - \left( \frac{z}{L} \right)^{-1/3} \right] , \quad z > z_T \]  \hspace{1cm} (5)
where \( L = \frac{\nu_s^2}{RT_s} \) is the Obukhov length scale, defined by the fricition velocity and temperature, \( z_{0U,T} \) are velocity and temperature roughness lengths and

\[
\psi_U = 2 \ln \left( \frac{1 + \chi}{2} \right) + \ln \left( \frac{1 + \chi^2}{2} \right) - 2 \tan^{-1} \chi + \frac{\pi}{2}, \quad \psi_T = 2 \ln \left( \frac{1 + \chi^2}{2} \right), \quad \chi = \left( 1 + 16 \frac{z}{L} \right)^{1/4}.
\]  

The figures show that MOST prediction, Eqs. (1-6), works well throughout the domain in the non-stratified case and in close vicinity of the water surface for stratified cases. However, in the stratified cases, the similarity theory significantly overestimates both mean velocity and temperature as compared to DNS results. The reason for this can be gleaned from the structure of coherent rollers and their effects on ensemble-averaged velocity and temperature fields in Fig. 3. This figure shows that rotating rollers transport slow-moving warmer air upwards and replace it with relatively colder and faster moving air from the upper layers. This transport due to coherent rollers promotes an enhanced mixing which affects mean velocity and temperature profiles, thus causing the deviation of these profiles from MOST prediction which does not take into account the presence of coherent rollers.

4. Conclusions
We have performed direct numerical simulation (DNS) of a turbulent atmospheric boundary layer over waved water surface and investigated the effects of unstable stratification of the air (i.e., the situation where the relatively cold air flows over warm water). The DNS results reveal the presence of large-scale counter-rotating coherent rollers elongated in the streamwise direction. Under the influence of these rollers, an enhanced mixing of the temperature and velocity fields occurs. This additional mixing causes substantial deviations of the mean velocity and temperature profiles from the MOST prediction. Thus, common closure procedure needs improvement under sufficiently strong unstable stratification in order to account for the effects of these coherent vortical structures. This is a subject for future research.

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