Functional renormalization group approach to the dynamics of first-order phase transitions

Yantao Li, Fan Zhong
State Key Laboratory of Optoelectronic Materials and Technologies, School of Physics and Engineering, Zhongshan University, Guangzhou 510275, People’s Republic of China

(Dated: November 9, 2011)

We apply the functional renormalization group theory to the dynamics of first-order phase transitions and show that a potential with all odd-order terms can describe spinodal decomposition phenomena. We derive a momentum-dependent dynamic flow equation which is decoupled from the static flow equation. We find the expected instability fixed points; and their associated exponents agree remarkably with the existent theoretical and numerical results. The complex renormalization group flows are found and their properties are shown. Both the exponents and the complex flows show that the spinodal decomposition possesses singularity with consequent scaling and universality.

PACS numbers: 05.10.Cc, 05.70.Fh, 64.60.ae

First-order phase transitions (FOPTs) are ubiquitous in our diverse physics world. Basically, their dynamics is described by two distinct mechanisms: one is nucleation and growth and the other is spinodal decomposition. FOPTs show discontinuity at their transition points at which the first (and higher) order derivatives of the chemical potential of the two phases involved change discontinuously. This is in sharp contrast to continuous phase transitions that have singularity at their transition points and are generally described by the elegant renormalization-group (RG) theory [3]. Consequently, there is a challenging question of whether FOPTs can also show any singularity and consequent scaling and universality. Recently, it has been found [4, 5] that the spinodal decomposition is possibly controlled by instability fixed points [6] and are generally described by the elegant functional RG (FRG) theory, which, like the perturbative Wetterich equation [11], with appropriate approximations to determine a fixed point and its exponents. We shall use the Blaizot, Méndez-Galain, and Wschebor (BMW) approximation [12], which contains full momentum dependence and has yielded excellent results for static critical exponents among others [13]. As the spinodal decomposition is kinetic in origin, we have to consider dynamics. However, dynamic flow equations have only been derived with the derivative expansion method [14]. So, we shall first derive in the BMW scheme a dynamic flow equation as well as the decoupled static BMW equation from a dynamic Wetterich equation.

Consider the Hamiltonian in the bare scale \( \Lambda \),

\[
\mathcal{H} = \int_x \left\{ \frac{1}{2} \left[ (\nabla \phi)^2 + m^2 \phi^2 \right] + \frac{1}{3!} \gamma_3 \phi^3 + \frac{1}{4!} \lambda_4 \phi^4 \right\}.
\]

The dynamic action \( S \) for Model A [15] can be expressed in the form [16],

\[
S = \int_x \left\{ \dot{\phi} \left( \partial \phi / \partial t + D \delta \mathcal{H} / \delta \phi \right) - D \dot{\phi}^2 \right\},
\]

with a response field \( \tilde{\phi} \), where \( x = (t, x) \) and \( D \) is a kinetic coefficient. To transform to the FRG scheme, we add to \( S \) a term

\[
\Delta S_k \equiv \frac{1}{2} \int_x \Phi R_k \Phi^\tau, \quad R_k = D R_k \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}
\]

described by the FRG theory does not valid in this region, it still indicates the spinodal decomposition has a potential of \( \dot{\phi}^3 \) like with a massless mode, which is just the viewpoint in [3]. In the FRG perspective, we thus expect that the potential of the spinodal decomposition contains all odd order terms of the field \( \phi \) for symmetry reason, in comparison with the critical phenomena which contain all even-order terms with \( Z_2 \) symmetry. To confirm this argument, we need to find the expected IFPs in the FRG scheme, determine the corresponding exponents, and then compare them with extant results.

In the FRG scheme, one needs to solve the exact static Wetterich equation [11] with appropriate approximations to determine a fixed point and its exponents. We shall use the Blaizot, Méndez-Galain, and Wschebor (BMW) approximation [12], which contains full momentum dependence and has yielded excellent results for static critical exponents among others [13]. As the spinodal decomposition is kinetic in origin, we have to consider dynamics. However, dynamic flow equations have only been derived with the derivative expansion method [14]. So, we shall first derive in the BMW scheme a dynamic flow equation as well as the decoupled static BMW equation from a dynamic Wetterich equation.

First-order phase transitions (FOPTs) are ubiquitous in our diverse physics world. Basically, their dynamics is described by two distinct mechanisms [1]: one is nucleation and growth and the other is spinodal decomposition. FOPTs show discontinuity at their transition points at which the first (and higher) order derivatives of the chemical potential of the two phases involved change discontinuously. This is in sharp contrast to continuous phase transitions that have singularity at their transition points [2] and are generally described by the elegant renormalization-group (RG) theory [3]. Consequently, there is a challenging question of whether FOPTs can also show any singularity and consequent scaling and universality. Recently, it has been found [4, 5] that the spinodal decomposition is possibly controlled by instability fixed points [6] and are generally described by the elegant functional RG (FRG) theory, which, like the perturbative Wetterich equation [11], with appropriate approximations to determine a fixed point and its exponents. We shall use the Blaizot, Méndez-Galain, and Wschebor (BMW) approximation [12], which contains full momentum dependence and has yielded excellent results for static critical exponents among others [13]. As the spinodal decomposition is kinetic in origin, we have to consider dynamics. However, dynamic flow equations have only been derived with the derivative expansion method [14]. So, we shall first derive in the BMW scheme a dynamic flow equation as well as the decoupled static BMW equation from a dynamic Wetterich equation.

Consider the Hamiltonian in the bare scale \( \Lambda \),

\[
\mathcal{H} = \int_x \left\{ \frac{1}{2} \left[ (\nabla \phi)^2 + m^2 \phi^2 \right] + \frac{1}{3!} \gamma_3 \phi^3 + \frac{1}{4!} \lambda_4 \phi^4 \right\}.
\]

The dynamic action \( S \) for Model A [15] can be expressed in the form [16],

\[
S = \int_x \left\{ \dot{\phi} \left( \partial \phi / \partial t + D \delta \mathcal{H} / \delta \phi \right) - D \dot{\phi}^2 \right\},
\]

with a response field \( \tilde{\phi} \), where \( x = (t, x) \) and \( D \) is a kinetic coefficient. To transform to the FRG scheme, we add to \( S \) a term

\[
\Delta S_k \equiv \frac{1}{2} \int_x \Phi R_k \Phi^\tau, \quad R_k = D R_k \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}
\]
interpolates between the mean field action with \( \Gamma_k \) and assume \( \eta \), since only \( \Gamma_k \) is the flow scale, \( \Phi = (\phi, \bar{\phi}) \), \( Z_k \) is the renormalization factor for the field \( \phi \), \( \Theta \) the Heaviside step function, and \( \tau \) denotes a transposition. \( \Delta S \) plays a role of mass that suppresses fluctuations of momentum \( |q| \lesssim k \). As a result, the scale-dependent effective action,

\[
\Gamma_k[\phi, \bar{\phi}, \psi] = -W_k[h, \bar{h}] - \frac{1}{2} \int x \psi R_k \psi^\dagger + \int \sum_{i=1}^2 h_i \phi_i ,
\]

(4)

with \( W_k[h, \bar{h}] = \int D\Phi \exp (-S - \Delta S_k) \int x \sum_{i=1}^2 h_i \phi_i , \) interpolates between the mean field action \( S[\psi] = \Gamma_k(\Lambda) \) and the effective action \( \Gamma_k[\psi] = \Gamma_k=0 \) as fluctuations are progressively taken into account as \( k \) flows from the bare microscopic scale \( \Lambda \) to the long-distance scale \( 0 \), where \( \psi = (\phi, \bar{\phi}) \) with \( \phi_i \) denoting the average of \( \phi \) and \( h, \bar{h} \) is the conjugate source. We have introduced a subscript \( i \in \{1, 2\} \) such that \( (\phi_1, \phi_2) = (\phi, \bar{\phi}) \) for convenience. Defining the \( 2 \times 2 \) matrices of the vertex functions as \( \Gamma_k^{(2)}(x_1, x_2) = \frac{\delta^2 \Gamma_k}{\delta \phi_1(x_1) \delta \phi_2(x_2)} \),

\[
\Gamma_k^{(n>2)}(x_1, \ldots, x_n) = \frac{\delta^{(n-2)} \Gamma_k^{(2)}(x_1, x_2)}{\delta \phi_i(x_3) \ldots \delta \phi_i(x_n)} ,
\]

(6)

one writes the flow of the effective average action as

\[
\partial_t \Gamma_k = \frac{1}{2} \text{Tr} \int q \hat{G}_k \partial_q \hat{R}_k ,
\]

(7)

in the Fourier space, where \( \hat{G}_k = [\hat{\Gamma}_k^{(2)} + \hat{R}_k]^{-1} \) is the full field-dependent propagator, \( q = (\omega, \mathbf{q}) \), and \( s = k \partial_k \) is the RG time. Equation (7) is just the dynamic Wetterich equation \( \hat{\Gamma}_k \) reflecting the RG flow of \( \Gamma_k \) under an infinitesimal change of \( k \). From Eq. (7) and using Eq. (5), one can derive the flow equation of two-point vertex function in uniform field configurations \( 18 \)

\[
\partial_t \Gamma_k^{(2)}(p, q) = \text{Tr} \int q \hat{G}_k \partial_q \hat{R}_k \left[ -\frac{1}{2} \hat{G}_k^{(4)}(p, -p, q) + \hat{G}_k^{(3)}(p, q) \hat{G}_k(q) \right] \hat{G}_k(q) ,
\]

(8)

where \( \Gamma_k^{(2)} \) are the matrix elements of \( \hat{\Gamma}_k^{(2)} \). In Eqs. (5) to (8), all the \( \phi_i \) dependence are implicit.

In order to derive from Eq. (7) the specific dynamic and static flow equations for Model A, one needs considering only those vertex functions that comply with its structure. Since only \( \Gamma_k^{(1,1)} = i \omega + D(q^2 + m_\Lambda^2), \Gamma_k^{(2,1)} = D\gamma_\Lambda, \Gamma_k^{(1,2)} = D\Lambda, \Gamma_k^{(2,2)} = -2D \) exist at \( \phi_i = 0 \) in the bare scale, we set other vertex functions zero \( 19 \) in Eq. (5). To decouple the flow equations for \( \partial_t \Gamma_k^{(2)}(p) \) and the real part of \( \partial_t \Gamma_k^{(1,2)}(p) \), we drop the \( \omega \) dependence in \( \Gamma_k^{(2)} \) and assume

\[
\Gamma_k^{(1,1)}(q) = i \frac{\omega}{2D} \Gamma_k^{(2)}(q) + D\Gamma_k^{(2)}(q) ,
\]

(9)

where \( \Gamma_k^{(n)}(q) \) is the static vertex function. Equation (9) is a kind of Ward identity expressing the fluctuation-dissipation theorem. It becomes exact when the \( \omega \) dependence of \( \Gamma_k^{(0,2)}(q) \) is re-included and no mode couplings are present \( 20 \). By Eq. (9), vertex functions of higher orders such as \( \Gamma_k^{(2,1)} \) and \( \Gamma_k^{(2,2)} \) depend no longer on \( \omega \) either. In addition, the external frequency is set to zero, since we mainly focus on \( p = (0, p) \). After performing the trace and integrating over \( \omega \), one finds the static Wetterich equation about \( \Gamma_k^{(2)}(p) \) decoupled from the dynamic behavior. Then, the approximation \( \Gamma_k^{(n+1)}(q_1, \ldots, q_n, 0) = \partial_q \Gamma_k^{(n)}(q_1, \ldots, q_n) \) results in the BMW equation \( 12 \)

\[
\partial_q \Gamma_k^{(2)}(p) = \int_q \left[ \partial_q R_k(q) \right] G_k^2(q) \times \left\{ G_k(p + q) \left[ \partial_q \Gamma_k^{(2)}(p) \right]^2 - \frac{1}{2} \partial_q \Gamma_k^{(2)}(p) \right\} ,
\]

(10)

and a momentum-dependent dynamic flow equation

\[
\partial_q \Gamma_k^{(0,2)}(p) = - \int_q [\partial_q R_k(q)] G_k^2(q) G_k(p + q) \Gamma_k^{(0,2)}(q)
\]

\times \left\{ G_k(p + q) \left[ \partial_q \Gamma_k^{(2)}(p) \right]^2 \right\} (A + 2B)(A - B)^{-2} ,
\]

(11)

which is one of our main results, where \( G_k(q) = [\Gamma_k^{(2)}(q) + R_k(q)]^{-1} \), \( A \equiv \Gamma_k^{(0,2)}(p) G_k^{-1}(p + q) \), and \( B \equiv \Gamma_k^{(0,2)}(p + q) G_k^{-1}(q) \). Note that the dynamics part does not contribute to Eq. (10) and the fixed points are thus solely determined by the latter as should be.

Having these flow equations, one needs to drop their dimensions to access the fixed points. The dimensionless renormalized variables and functions are defined as

\[
\hat{p} = p/k, \quad \hat{q} = q/k, \quad \hat{\phi} = [k^{2-d}Z_k K_d^{-1}]^{1/2} \phi , \quad \hat{U}_k = k^d U_k , \quad \hat{\varphi} = [k^{-2-d}Z_k K_d^{-1}]^{1/2} \varphi , \quad \hat{D} = Z_D D , \quad \hat{\Gamma}_k = [k^{2-d}Z_k]^{1/2} \Gamma_k , \quad \hat{\Gamma}_k^{(2)} = [-2Z_D \hat{Z}_D D]^{-1} \Gamma_k^{(2)} ,
\]

(12)

where \( U_k \) is the potential, \( K_d^{-1} = d^{d-1} \pi^{d/2} (\Gamma / d) / (\Gamma / d) \) (\( \Gamma \) is the Euler gamma function), and \( Z_D \) and \( Z_k \) are the renormalization factor of \( D \) and \( \varphi \), respectively, and are related through \( Z_k^{1/2} = Z_D Z_k^{1/2} \). In the scaling regime, \( Z_D \sim k^{-\epsilon_D} \) and \( Z_k \sim k^{-\epsilon_k} \), with \( \epsilon_D \) and \( \eta_k \) being related to the dynamic and the static critical exponents by \( z = 2 - \epsilon_k = 0 \) and \( \eta = \eta_k = 0 \), respectively. Consequently, the dimensionless dynamic flow equation satisfies

\[
\partial_q \hat{\Gamma}_k^{(0,2)}(\hat{p}) = (\eta_k - \epsilon_k + \bar{p} \partial_D) \hat{\Gamma}_k^{(0,2)}(\hat{p}) + \partial_q \hat{\Gamma}_k^{(0,2)}(p) ,
\]

(13)

where the last term is Eq. (11) in its dimensionless form. The static flow equation can be separated into two dimensionless flow equations: One is the flow equation of \( \hat{U}_k' = \hat{\Gamma}_k^{(2)}(0) \) (a prime denotes a derivative with \( \hat{\phi} \)) obtained by setting \( p = 0 \) in Eq. (10), the other is the flow equation of \( \hat{\chi}_k = Z_k(\phi)/Z_k(0) \) which comes from
\[ \partial_s Z_k = [\partial (\partial \Gamma_k^{(2)}(p))/\partial p^2]_{p=0}. \] They are \ref{21}

\[ \partial_s \tilde{U}_k'' = (\eta_k - 2) \tilde{U}_k'' + \frac{1}{2} (d - 2 + \eta_k) \tilde{\phi} \tilde{U}_k'' - \left(1 - \frac{\eta_k}{d + 2}\right) \left[ \tilde{G}_k^2 \tilde{U}_k'' - 2 \tilde{G}_k^2 (\tilde{U}_k'')^2 \right], \] \tag{14}

\[ \partial_s \chi_k = \eta_k \chi_k + \frac{1}{2} (d - 2 + \eta_k) \tilde{\phi} \chi_k - (\tilde{U}_k'')^2 \tilde{G}_k^4 + \left(1 - \frac{\eta_k}{d + 2}\right) \left[ 4 \tilde{G}_k^2 \tilde{U}_k'' \chi_k - \tilde{G}_k^2 \chi_k'' \right] \] \tag{15}

for our choice of \( R_k \), where \( \tilde{G}_k = (1 + \tilde{U}_k'')^{-1} \).

From the dimensionless dynamic and static flow equations \ref{13} to \ref{15}, one can find the IFPs and their exponents in a way similar to the critical phenomena. The only difference is that we need to change to the symmetry that describes the spinodal decomposition. We use the field expansion method to find the IFPs \ref{21, 22}. The key point is how to retain terms in \( U_k \) and \( Z_k \).

As mentioned above, the potential should be of the form

\[ \tilde{U}_k = \lambda_2 \tilde{\phi}^2 + \lambda_3 \tilde{\phi}^3 + \lambda_5 \tilde{\phi}^5 + \cdots + \lambda_n \tilde{\phi}^n + \cdots. \] \tag{16}

The expansion of \( Z_k \) (or \( \chi_k \)) is delicate. Any of its \( \tilde{\phi} \)-dependent term will bring even besides odd order terms to the unrenormalized potential. In this potential, however, the even-order terms that arise from the even-order terms of \( Z_k \) are generated by the coupling of the latter terms with \( \lambda_2 \) only, which vanishes at the mean-field or bare-scale spinodal point. Consequently, in order to have a potential of odd-order couplings at the bare scale, we retain only the even-order terms in \( Z_k \) and write

\[ \chi_k = 1 + \chi_2 \tilde{\phi}^2 + \chi_4 \tilde{\phi}^4 + \cdots + \chi_{2n} \tilde{\phi}^{2n} + \cdots. \] \tag{17}

Substituting Eqs. \ref{17} and \ref{17} into Eqs. \ref{14} and \ref{15}, one gets a set of nonlinear flow equations for the coefficients \( \lambda_{n_1} \) and \( \chi_{n_2} \), which reach their fixed points at

\[ \partial_s \lambda_{n_1} = \partial_s \chi_{n_2} = 0. \]  

To obtain the exponents, we use \( \tilde{\phi} = 0 \) and \( \tilde{\phi} = 0 \) as the renormalization point and let \( \tilde{G}_k^{(0,2)}(\tilde{\phi}) = 1 \) in Eq. \ref{11}. With this approximation, Eq. \ref{13} recovers the dynamic flow equation in Ref. \ref{14} if the \( \tilde{\phi} \) dependence of the latter equation is neglected, which confirms ours. From Eqs. \ref{13} and \ref{13}, one finds

\[ \eta_k = \frac{\tilde{G}_k^2 \tilde{U}_k'' - 4 \tilde{G}_k^2 \tilde{U}_k'' \chi_k + (\tilde{U}_k'')^2 \tilde{G}_k^4}{\chi_k + \frac{1}{2} \tilde{G}_k^2 \tilde{U}_k'' \chi_k - \frac{1}{2} \tilde{G}_k^2 \tilde{U}_k'' \chi_k}, \] \tag{18}

\[ z_k = 2 - \eta_k + \frac{3}{2} \left(1 - \frac{\eta_k}{d + 2}\right) (\tilde{U}_k'')^2 \tilde{G}_k^4 \] \tag{19}

respectively. In order to compare with direct numerical results, we also compute the hysteresis exponents \ref{1}, \ref{15},

\[ n_H = \frac{d + 2 - \eta}{d + 2 - \eta + 2z}, \quad n_m = \frac{d - 2 + \eta}{d + 2 - \eta + 2z}, \] \tag{20}

which reflect the hysteresis of the coercivity and the remnant magnetization, respectively.

![FIG. 1.](https://example.com/fig1.png) (Color online) (a) Stability of \( \text{Im}(\lambda_3) \) with increasing \( N \) in \( d = 3, 4, 5 \). RG flows of (b) \( \text{Re}(\eta_k) \) and \( \text{Im}(\lambda_3) \) and (c) \( \text{Im}(\eta_k) \) and \( \text{Re}(\lambda_3) \) from \( s = 0 \) to \( s = -12 \) with four different values of \( \text{Re}(\lambda_3) \): \( 1 \times 10^{-4} \), \( 0.8 \times 10^{-5} \), \( 0.6 \times 10^{-5} \), \( 0.4 \times 10^{-5} \) (top to bottom). Whereas the flows in (b) do not depend on \( \text{Re}(\lambda_3) \), those in (c) are proportional to \( \text{Re}(\lambda_3) \) and can thus overlap when scaled by the proportionality. (d) The solid line and the dashed line are the maximum of \( \text{Im}(\eta_k) \) and \( \text{Re}(\lambda_3) \), respectively. Both of them are linear with \( \text{Re}(\lambda_3) \). This relation is broken with larger \( \text{Re}(\lambda_3) \). In (b), (c), and (d), \( d = 3 \) and \( N = 9 \).

We now present the results for the fixed points. For \( d \geq 6 \), the only infrared-stable fixed point found is a Gaussian fixed point with \( \lambda_1 = 0 \), confirming the mean-field behavior in these dimensions. For \( d < 6 \) on the other hand, a lot of new fixed points appear. To find the nontrivial IFPs, we start with \( n_1 = 3 \) and \( n_2 = 0 \). There are only three fixed points: Besides the Gaussian one, the others are just the IFPs of a pair of two purely imaginary conjugate values of \( \lambda_3 \). This can be checked by their stability against increasing \( n_1 \) and \( n_2 \). When \( n_1 \) is fixed, the IFPs become stable with increasing \( n_2 \), whose maximum value is \( n_1 - 3 \) for closing the set of equations. So, the stability can be checked by \( n_1 = 2N + 1 = 3 \), \( 5 \) \cdots and \( n_2 = n_1 - 3 \). One sees from Fig. \ref{fig1} \ref{a} that the imaginary part of \( \lambda_3 \), denoted by \( \text{Im}(\lambda_3) \), indeed stabilizes quickly as \( N \) increases.

Having determined the IFPs, we can then extract their exponents. We note first that if we neglect the dependence of \( \chi_k \) on \( \tilde{\phi} \) from the beginning, we find \( \eta = -0.176 \), \( -0.427 \), and \( -0.669 \) for \( d = 5, 4 \), and \( 3 \), respectively, which are already quite good compared with those obtained from a perturbative RG theory \ref{22} shown in Table \ref{tab1}. Taking into account the field expansion of \( \chi_k \) spoils, however, the results. Yet, if we set \( \chi_2 = 0 \) in Eqs. \ref{18} and \ref{18}, the resultant exponents given in Table \ref{tab1} agree remarkably with the extant ones. Two remarks are in order here. First, the perturbative results were derived only to two- and three-loop orders and thus may not be
accurate. Second, our zs are a bit smaller than the others, this may arise either from our setting \( \Gamma_k(0,2)(\hat{q}) = 1 \) in Eq. (11), or our omitting the \( \hat{\phi} \) dependence in Eq. (13). Still, the remarkable agreement in all the three dimensions considered indicates that the condition of \( \chi_2 = 0 \) is not accident. A consequence is that it leads to a relation between \( \lambda_2 \) and \( \lambda_3 \) via \( \eta \), from Eqs. (11) and (13), the relation which may be pertinent to the fact that the \( \varphi^3 \) theory has only one independent static exponent. Although further studies are clearly needed, the good agreements still confirm our identification of the imaginary fixed points with the IFPs.

To show how to reach the imaginary IFPs from the bare scale, we fix the value of \( \lambda_2 \) to its fixed point value at any scale, and set \( \text{Im}(\lambda_{n1>3}) = 10^{-16} \), \( \text{Re}(\lambda_{n1>3}) = 0 \), and \( \chi_{n2} = 0 \) at the bare scale indicated by the superscripts, while varying \( \text{Re}(\lambda_{s}) \). Then, we solve the set of nonlinear flow equations of \( \lambda_{n1} \) and \( \chi_{n2} \) up to \( N = 9 \) in \( d = 3 \) to 5. The flows start from the bare scale \( \Lambda \) (s = 0) to the zero scale (s \( \to \) \(-\infty \)), see Fig. 1(b) and (c). The flows of \( \text{Re}(\eta) \) and \( \text{Im}(\lambda_3) \) do not depend on \( \text{Re}(\lambda_{s}) \); they overlap completely for different \( \text{Re}(\lambda_{s}) \).s. The flows of \( \text{Im}(\eta) \) and \( \text{Re}(\lambda_3) \), on the other hand, depend on \( \text{Re}(\lambda_{s}) \). Nevertheless, they are proportional to \( \text{Re}(\lambda_{s}) \). In other words, if \( \text{Re}(\lambda_{s}) \) is multiplied by a constant, then the flows can still overlap after multiplying all values of the flows by the same constant. If the value of \( \text{Re}(\lambda_{s}) \) is too large, however, the proportional relation is broken [see Fig. 1(d)], and the IFP becomes unreachable. Although this appears to indicate that the IFP has a controlling region, it becomes larger as \( \text{Im}(\lambda_{s}) \) increases. In addition, flows of purely real \( \lambda_{s} \) cannot reach the fixed points in our numerical analysis, though those from analytical solutions may. Therefore, we see that the IFPs are indeed infrared stable and reachable. A tiny initial complex coupling appears sufficient for the flows to reach the fixed point in the FRG theory. This confirms a previous perturbative analysis [3]. As the complex flows connecting the physics at the bare scale with the physics at the zero scale are a process of adding fluctuations to the mean-field theory, which is controlled by the Gaussian fixed point and describes the spinodal decomposition well for \( d \geq 6 \) [4], the FRG theory thus shows that the same phenomena in \( d < 6 \) should be controlled by the IFP, though the initial purely imaginary coupling may have to be identified.

In summary, we have applied the FRG theory to the dynamics of FOPTs. The momentum-dependent dynamic flow equation in the BMW scheme has been derived. We have found the IFPs with their exponents in good agreement with extant results albeit with some subtleties. This gives a physical meaning of spinodal decomposition for a potential with all the odd-order terms as compared with the critical phenomena for a potential with all the even-order terms. The complex flows and their properties have also been shown in the FRG scheme. Both the exponents and the complex flows show that the spinodal decomposition has a behavior of singularity and consequent scaling and universality.

This work was supported by the NSF of PRC (No.10625420).

---

TABLE I. Instability Exponents: \(^{a}\) This work. \(^{b}\) Perturbative RG Results. \(^{c}\) Numerical results [3].

| \(d\) | 6  | 5  | 4  | 3  |
|------|----|----|----|----|
| \(\eta^h\) | 0  | -0.147 | -0.328 | -0.517 |
| \(\eta^t\) | 0  | -0.147 | -0.328 | -0.524 |
| \(\eta^s\) | 2  | 1.922  | 1.809  | 1.661  |
| \(\eta^b\) | 2  | 1.938  | 1.874  | 1.809  |
| \(n_{n1}^H\) | \(2/3\) | 0.650  | 0.636  | 0.624  |
| \(n_{n2}^H\) | \(0.655(3)\) | \(0.645(3)\) | \(0.625(12)\) | \(0.595(30)\) |
| \(n_{n1}^0\) | \(1/3\) | 0.260  | 0.168  | 0.0546 |
| \(n_{n2}^0\) | \(0.33(1)\) | 0.27(4) | 0.17(8) |

---

\(^{a}\) s.tszf@mail.sysu.edu.cn

[1] For a recent review, see K. Binder et al, in Phase Transformsations in Materials, edited by G. Kosterz, (Wiley, Weinhein, 2001).

[2] M. E. Fisher, Rep. Prog. Phys. 30, 615 (1967).

[3] K. G. Wilson et al., Phys. Rep. C 12, 75 (1974).

[4] F. Zhong et al., Phys. Rev. Lett. 95, 175701 (2005).

[5] F. Zhong, submitted for publication (2011).

[6] M. E. Fisher, Phys. Rev. Lett. 40, 1610 (1978).

[7] S. Fan et al., J. Stat. Phys. 143, 1136 (2011).

[8] For a review, see e.g., J. Berges et al., Phys. Rep. 363, 223 (2002).

[9] J. Langer, Ann. Phys. 41, 108 (1967).

[10] A. Strumia et al., Nucl. Phys. B 542, 719 (1999); A. Strumia et al., Phys. Lett. B 467, 279 (1999); N. Tetrakis, Int. J. Mod. Phys. A 16, 1927 (2001).

[11] C. Wetterich, Phys. Lett. B 301, 90 (1993).

[12] J.-P. Blaizot et al., Phys. Lett. B 632, 571 (2006).

[13] F. Benitez et al., Phys. Rev. E 80, R030103 (2009).

[14] L. Canet et al., J. Phys. A: Math. Theor. 40 1937 (2007); L. Canet, et al., arXiv:1106.4129v1.

[15] P. C. Hohenberg et al., Rev. Mod. Phys. 49, 435 (1977).

[16] H. K. Janssen, Z. Phys. B 23, 377 (1976).

[17] D. F. Litim, Phys. Rev. D 64, 105007 (2001).

[18] L. Canet et al., Phys. Rev. Lett. 104, 150601 (2010).

[19] Keeping \( \xi^{(2)}_k = \partial_k \xi^{(0)}_k \) and \( \xi^{(2,2)}_k = \partial^2_k \xi^{(0,2)}_k \), i.e., the \( \varphi \) dependence, may improve the result for \( z \). This was considered in [14] in critical phenomena using the derivative expansion method.

[20] R. Folk et al., J. Phys. A: Math. Gen. 36 R207 (2006).

[21] D. Guerra et al., Eur. Phys. J. B 59, 357 (2007).

[22] L. Canet, et al., Phys. Rev. D 67, 065004 (2003).

[23] O. F. de Alcântara Bonfim et al., J. Phys. A 13, L247 (1980); 14, 2391 (1981).

[24] N. Breuer et al., Z. Phys. B 41, 55 (1981).