The Cosmology of Tetradic Theory of Gravitation

H. A. Alhendi$^1$, E. I. Lashin$^{1,2}$ and G. L. Nashed$^3$

1 Department of physics and Astronomy, College of Science, King Saud University, Riyadh, Saudi Arabia
2 Department of Physics, Faculty of Science, Ain Shams University, Cairo, Egypt
3 Department of Mathematics, Faculty of Science, Ain Shams University, Cairo, Egypt

Email: alhendi@ksu.edu.sa, lashin@ksu.edu.sa

March 24, 2022

We consider a special class of the tetrad theory of gravitation which can be considered as a viable alternative gravitational theories. We investigate cosmological models based on those theories by examining the possibility of fitting the recent astronomical measurement of supernova Ia magnitude versus shift. Our investigations result in a reasonable fit for the supernova data without introducing a cosmological constant. Thus, cosmological models based on tetradic theory of gravitation can provide alternatives to dark energy models.

PACS numbers: 04.50.+h, 04.80.Cc

1 Introduction

The present observation of the distant supernovae type Ia indicates that the universe is presently accelerating[1]–[4]. The cosmic acceleration is attributed to the presence of unknown form of energy violating the strong energy conditions $\rho_X + 3p_X > 0$ where $\rho_X$ and $p_X$ are energy density and pressure of dark energy respectively. Different candidates for dark energy are attempted to yield accelerating cosmoologies at late time[5]–[14]. The cosmological constant $\Lambda$ and phantom fields [15]–[17] violating a weak energy conditions $\rho_X + p_X > 0$ are most popular ones.

Recently, different attempts [18]–[25] have been carried out to modify gravity to yield accelerating cosmologies at late times. In this paper, we exploit the possibility of modifying gravity based on absolute parallelism spaces.

The notion of absolute parallelism was first introduced in physics by Einstein[26] trying to unify gravitation and electromagnetism into 16 degrees of freedom of the tetrads. His attempt failed, however, because there was no Schwarzschild solution in his field equations.

The interest in the tetrad theory as a purely gravitational theory was revived by Møller[27] who showed that a more satisfactory treatment of energy momentum complex than that of general relativity can be achieved. In his first attempt in finding Lagrangians Møller’s was restricted by the assumption that the equations determining the metric tensor should coincide with the Einstein equations. After then, he [28] abandoned this assumption and relooked for a wider class of Lagrangians, allowing for possible deviation from the Einstein equations in the case of strong gravitational fields. Møller’s theory was generalized into scalar tetradic theory by Sáez[29]. Meyer showed that Møller’s theory is a special case of the Poincare gauge theory[30].
Quite independently, Hayashi and Nakano[31] formulated the tetrad theory of gravitation as a gauge theory of the space-time translation group. Hayashi and Shirafuji[32] studied the geometrical and observational basis of the tetrad theory, assuming that the Lagrangian be given by a quadratic form of torsion tensor. If the invariance under the parity operations is assumed, the most general Lagrangian consists of three terms with three unknown parameters to be fixed by experiment. Two of the three parameters were determined by comparison with solar-system experiments, while only an upper bound has been estimated for the third [32]–[33].

The numerical values of the two parameters found were very small, consistent with a value of zero. If these two parameters are exactly equal to zero, the theory reduces to the one proposed by Hayashi and Nakano and Møller which we shall here refer to as the HNM theory. This theory differs from general relativity only when the torsion tensor has a nonvanishing axial vector part.

Many applications of HNM theory have been done during the years. These include cosmological applications [34], investigating gravitational radiation [35]–[36] and energy momentum complex [37]–[38], and finding a general solution with spherical symmetry [39], and solution with axial symmetry in [40].

The rest of the paper is organized as follows: In section 2 we review HNM tetrad theory of gravitation. In section 3 we present the tetrad field satisfying the requirement of homogeneity and isotropy. In section 4 we discuss briefly the basic of the tetrad cosmology and derive the relation between the luminosity distance and redshift. In section 5 we present numerical investigations of the tetrad cosmological model to fit the recent observational supernovae data. The obtained results are compared with standard cosmology (with or without cosmological constant). Finally, in the last section we give our discussion.

2 HNM Tetrad Theory of Gravitation

In this paper we follow Møller construction[28] of the tetrad theory of gravitation based on the Weitzenbock space-time. In this theory the field variables are the 16 tetrad components $e_i^\mu$, from which the metric is constructed as

$$g^{\mu\nu} := \eta^{ij} e_i^\mu e_j^\nu,$$

(1)

where $\eta^{ij}$ is the Lorentz metric tensor taken as diag($-1,1,1,1$). The Latin indices ($i,j \ldots$) refer to vector numbers and Greek indices ($\mu, \nu \ldots$) to vector components, and all of them run from 0 to 3. We restrict indices $a,b, \ldots$ and $\alpha,\beta,\ldots$ (beginning of Latin and Greek alphabetic) for spatial components.

An invariant Lagrangian $L$ is constructed from $g^{\mu\nu}$ and $\gamma^{\mu\nu\rho}$, where $\gamma^{\mu\nu\rho}$ is the contorsion tensor given by:

$$\gamma^{\mu\nu\rho} := \eta^{ij} e_i^\mu e_j^\nu e^{\nu\rho},$$

(2)

where the semicolon denotes covariant differentiation using the Christoffel symbols. The most general Lagrangian density invariant under the parity operation can be constructed as a linear combination of the following expressions:

$$L^{(1)} := \Phi_\mu \Phi^\mu, \quad L^{(2)} := \gamma^{\mu\nu\sigma} \gamma_{\mu\nu\sigma}, \quad L^{(3)} := \gamma^{\mu\nu\sigma} \gamma_{\sigma\nu\mu},$$

(3)

where $\Phi_\mu$ is the basic vector defined by

$$\Phi_\mu := \gamma^{\nu}_{\phantom{\nu} \mu\nu}.$$

(4)

These expressions $L^{(i)}$ in eq. (3) are homogeneous quadratic functions in the first order derivatives of the tetrad field components.
Møller considered the simplest case, in which the Lagrangian $\mathcal{L}$ is a linear combination of the quantities $L^{(i)}$, i.e., the Lagrangian density is given by

$$\mathcal{L}_{\text{Møller}} := (-g)^{1/2} (\alpha_1 L^{(1)} + \alpha_2 L^{(2)} + \alpha_3 L^{(3)}),$$

where

$$g := \det(g_{\mu\nu}).$$

For this choice, the constants $\alpha_i$ had been chosen such that this theory gives the same results as GR in the linear approximation of weak fields. According to his calculations, one can easily see that if one chooses

$$\alpha_1 = -1, \quad \alpha_2 = \lambda, \quad \alpha_3 = 1 - 2\lambda,$$

with $\lambda$ equals to a free dimensionless parameter of order unity, the theory will be in agreement with GR to the first order of approximation. The same identification of the parameters was obtained by Hayashi and Nakano[31]

Møller applied the action principle to the Lagrangian density eq. (5) and obtained the field equations in the form[28]

$$G_{\mu\nu} + H_{\mu\nu} = -\kappa T_{\mu\nu},$$

$$F_{\mu\nu} = 0,$$

where the Einstein tensor $G_{\mu\nu}$ is defined by

$$G_{\mu\nu} := R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R.$$  \hspace{1cm} (9)

Here $H_{\mu\nu}$ and $F_{\mu\nu}$ are given by

$$H_{\mu\nu} := \lambda \left[ \gamma_{\alpha\beta\mu} \gamma_{\alpha\beta\nu} + 2 \gamma_{\alpha\beta\mu} \gamma_{\alpha\beta\nu} + \gamma_{\alpha\beta\mu} \gamma_{\alpha\beta\nu} + g_{\mu\nu} \left( \gamma_{\alpha\beta\gamma} \gamma_{\alpha\beta\gamma} - \frac{1}{2} \gamma_{\alpha\beta\gamma} \gamma_{\alpha\beta\gamma} \right) \right]$$  \hspace{1cm} (10)

and

$$F_{\mu\nu} := \lambda \left[ \Phi_{\mu,\nu} - \Phi_{\nu,\mu} - \Phi_{\alpha} \left( \gamma_{\alpha\mu\nu} - \gamma_{\alpha\nu\mu} \right) + g_{\mu\nu} \gamma_{\alpha} \right].$$  \hspace{1cm} (11)

The term $H_{\mu\nu}$ by which equations eq. (8) deviate from Einstein’s field equations increases with $\lambda$, which can be taken of order unity without destroying the first order agreement with Einstein’s theory in case of weak fields.

Møller assumed that the energy-momentum tensor of matter fields is symmetric. In the Hayashi-Nakano theory, however, the energy-momentum tensor of spin-1/2 fundamental particles has a nonvanishing antisymmetric part arising from the effects due to the intrinsic spin.

3 Tetrad fields for applications to cosmology

The tetrad fields satisfying the symmetry requirements of homogeneity and isotropy have been given by Robberson[41]. It was found that there are two possible teterads, which in Cartesian coordinate can be written in the form

$$e_0^0 = 1, \quad e_a^0 = 0, \quad e_0^\alpha = 0,$$

$$R(t) e_a^\alpha = \delta_a^\alpha h - \frac{k}{2} x^\alpha x^\beta \pm k \epsilon_{\alpha\beta\gamma} x^\gamma,$$  \hspace{1cm} (12)
\[ e_0^0 = \frac{h_-}{h_+}, \quad e_a^0 = \pm \left(\frac{-k}{h_+}\right)^\frac{3}{2} x^\alpha, \quad R(t) e_0^\alpha = \pm \left(\frac{-k}{h_+}\right)^\frac{1}{2} x^\alpha, \]
\[ R(t) e_a^\alpha = \delta_a^\alpha h_+ - \frac{k}{2} x^\alpha x^a, \]  
where the constant \( k \) takes the values \( +1, -1, \) or zero. while \( h_\pm \) and \( r^2 \) are defined by
\[ h_\pm = 1 \pm \frac{k}{4} r^2, \quad r^2 = x^2 + y^2 + z^2. \]  
Here, \( \epsilon_{\alpha\beta\gamma} = \pm 1 \) when \((\alpha\beta\gamma)\) is an even or odd permutation of \((123)\) and \(0\) otherwise.

Both of these two tetrads through eq. (1) lead to Roberston-Walker metric given by
\[ ds^2 = -dt^2 + R(t)^2 h_+^2 (dx^2 + dy^2 + dz^2), \]
Explicit calculations based on eqs. (2)–(4) and the Christoffel symbols of the metric defined by eq. (1) result in the following nonvanishing components of the tensor \( \gamma \) and \( \Phi \) for the first tetrad in eq. (12).
\[ \gamma_{0ab}^0 = -\delta_{ab} R \dot{R} h_+^{-2}, \quad \gamma_a^0 = -\delta_{ab} \frac{\dot{R}}{R}, \]
\[ \gamma_{bc}^a = -\epsilon_{abc} \frac{k}{4} h_+^{-1}, \quad \Phi_0 = -3 \frac{\dot{R}}{R}, \quad \Phi_a = 0, \]
where \( \dot{R} = \frac{dR}{dt} \).

Concerning the second tetrad, because the physically relevant second rank tensors derived from it, could be complex as pointed out in [42]. Thus, it will be dropped out in the present work.

4 The basic equations of cosmology

The skew symmetric part of the field equations in eq. (8) is satisfied identically by the tetrad defined by eq. (12), which can be verified by explicit calculations. Assuming the energy-momentum tensor for a perfect fluid, then the symmetric part of the field equations in eq. (8) reduces to
\[ \left(2 \ddot{R}/R\right) + \left(\ddot{R}/R\right)^2 + D/R^2 = -p, \]
\[ 3 \left(\ddot{R}/R\right)^2 + 3 D/R^2 = \rho, \]
where \( D = k (1 - 3 \lambda) \), \( p \) is the pressure, and \( \rho \) is the density of energy associated with the fluids. Throughout this work we employ units in which \( G = 1/8\pi \) and \( c = 1 \), and in this section we use the notations and equations (with suitable modification) presented in [43].

As in the standard cosmology, the Hubble parameter \( H \) is defined by \( H = \dot{R}/R \). Then critical density is given as
\[ \rho_c = 3 \left(\ddot{R}/R\right)^2 = 3H^2 \]
Many useful parameters can be defined, among them the density \( \Omega \) and the deceleration parameters defined respectively as:
\[ \Omega = \rho/\rho_c, \quad q = -\ddot{R}R/\dot{R}^2, \]
In this manner, eq. (17) and eq. (18) take the form
\[ q = \Omega/2 + p/2 H^2, \quad \Omega + \Omega_D = 1, \]
where $\Omega_D = -D/R^2 H^2$. It is worthy to mention that the metric in eq. (15) and the field equations eqs. (17)–(18) involve two different constants, namely $k$ and $D$ respectively. Since $\lambda$ is completely free, then the two constants are independent. Thus, there is no unique relation between the geometry of the universe and its fate, in contrast to the case of general relativity (without cosmological constant), where such a relation exists.

For matter dominated universe eq. (18) can be written in the form

$$\dot{R}^2 - \frac{\rho_0 R_0^3}{R} = -D,$$

(22)

here $D$ can take any real value in contrast to the case of general relativity where $k$ is restricted to $\pm 1, 0$ values.

Let us define the parameters

$$a = \frac{1}{1 + z} = \frac{R}{R_0}, \quad \tau = H_0 t$$

(23)

where $z$ is the red shift. In terms of these parameters eq. (22) can be transformed into

$$\left(\frac{da}{d\tau}\right)^2 = 1 + \Omega_0 \left(\frac{1}{a} - 1\right),$$

(24)

which can be converted, in terms of $z$ into

$$\frac{dz}{d\tau} = (1 + z)^2 \left(1 + \Omega_m z\right)^{-\frac{1}{2}}.$$

(25)

The time lapse between the present time and the emission time of light which suffers redshift $z_1$ can be obtained from eq. (25) leading to

$$t_0 - t_1 = \int_{z_0}^{z_1} \frac{1}{(1 + z)^2} \left(1 + \Omega_m z\right)^{-\frac{1}{2}} dz.$$

(26)

Light signal moves along null geodesic whose equation is

$$\frac{dr}{dt} = \frac{1}{(1 - k r^2)^{1/2}},$$

(27)

and using eqs. (23)–(25) we obtain

$$\frac{dr}{(1 - k r^2)^{1/2}} = \frac{(1 + z) R_0}{R_0 H_0 (1 + z)} dz.$$

(28)

Integrating eq. (28) for a light traveling through the universe from $r_1$ at time $t_1$, and reaching at $r_1 = 0$ at the present time $t_0$ we obtain

$$\int_0^{r_1} \frac{dr}{(1 - k r^2)^{1/2}} = \int_{z_0}^{z_1} \frac{(1 + \Omega_m z)^{-\frac{1}{2}}}{R_0 H_0 (1 + z)} dz$$

(29)

where

$$\int_0^{r_1} \frac{dr}{(1 - k r^2)^{1/2}} = \left\{\begin{array}{ll}
\frac{1}{(\sqrt{|k|})^{1/2}} \sin^{-1} (|k|^{1/2} r_1) & k = -1 \\
\frac{1}{(\sqrt{|k|})^{1/2}} \sin^{-1} (|k|^{1/2} r_1) & k = 1 \\
r_1 & k = 0
\end{array}\right.$$  

(30)

From $\Omega_{D0} = -D/R_0^2 H_0^2$, we can express $|k|$ as

$$|k| = \frac{|\Omega_{D0}| R_0^2 H_0^2}{|b\lambda|}$$

(31)
where $b_\lambda = 1 - 3\lambda$. Using eq. (29), for the case $b_\lambda > 0$ and $k = -1$, we obtain

$$
\left(\frac{|b\lambda|}{\Omega_{D0}}\right)^{1/2} \frac{1}{R_0 H_0} \sinh^{-1}\left(\left(\frac{|\Omega_{D0}|}{|b\lambda|}\right)^{1/2} R_0 H_0 r_1\right) = \int_0^{z_1} \frac{(1 + \Omega_m z)^{-1/2}}{R_0 H_0 (1 + z)} dz
$$

(32)

then solving for $r_1$ one gets

$$
r_1 = \left(\frac{|b\lambda|}{\Omega_{D0}}\right)^{1/2} \frac{1}{R_0 H_0} \sinh\left(\left(\frac{|\Omega_{D0}|}{|b\lambda|}\right)^{1/2} \int_0^{z_1} \frac{(1 + \Omega_m z)^{-1/2}}{(1 + z)} dz\right)
$$

(33)

The luminosity distance can be shown to be

$$
d_L = R_0^2 r_1 / R_1,
$$

(34)

and with the help of eq. (33) we get

$$
d_L = \left(\frac{|b\lambda|}{\Omega_{D0}}\right)^{1/2} \frac{1}{H_0} (1 + z) \sinh\left(\left(\frac{|\Omega_{D0}|}{|b\lambda|}\right)^{1/2} \int_0^z \frac{(1 + \Omega_m u)^{-1/2}}{(1 + u)} du\right)
$$

(35)

5 Testing the model against the Supernovae data

The supernovae of type Ia (SNe Ia) serve as excellent cosmological standard candles. The apparent magnitude of a "standard candle" is related to its luminosity distance $d_L$ through

$$
m(z) = M + 5 \log_{10} \left[ \frac{d_L}{Mpc} \right] + 25,
$$

(36)

where $M$ is the absolute magnitude and is assumed to be constant for a standard candle like Sne Ia. The apparent magnitude also can be expressed in terms of the dimensionless luminosity distance $D_L(z)$ as

$$
m(z) = M + 5 \log_{10} D_L(z),
$$

(37)

with

$$
D_L(z) = \frac{H_0}{c} d_L
$$

(38)

and

$$
M = M + 5 \log_{10} \left( \frac{c/H_0}{1Mpc} \right) + 25 = M - 5 \log_{10} h + 42.38.
$$

(39)

For our present analysis we use "gold" sample compiled by Reiss et al.[4]. The sample consists of 157 data points which in terms of distance modulus are

$$
\mu_{\text{obs}} = m(z) - M,
$$

(40)

$$
= 5 \log_{10} D_L(z) - 5 \log_{10} h + 42.38.
$$

The best fit model to the observation is obtained by using $\chi^2$ statistics, i.e.,

$$
\chi^2 = \sum_{i=1}^{157} \left[ \frac{\mu_{\text{th}}^i - \mu_{\text{obs}}^i}{\sigma_i} \right]^2,
$$

(41)
where $\mu_{\text{th}}$ is the predicted distance modulus for a supernova at redshift $z$ and $\sigma_i$ is the dispersion of the measured distance modulus due to intrinsic and observational uncertainties in SNe Ia peak luminosity. In our model we obtain the best fit for $\Omega_m$ and $b_\lambda$, from the minimization of $\chi^2$ by scanning the whole relevant parameter space, the values we obtain are $\Omega_m = 0.3$ and $b_\lambda = 0.26$ for $k = -1$ and $h = 65 \text{ km s}^{-1}\text{Mpc}^{-1}$. The other values of $k$ don’t lead to a best fit. In fig. 1 we illustrate the variation of $\chi^2$ versus $b_\lambda$ for fixed $\Omega_m = 0.3$ and $k = -1$. For a one parameter fit ($b_\lambda$), the SNe Ia data provides the following ranges: $0.234 \leq b_\lambda \leq 0.288$ at 69% confidence level (CL) and $0.207 \leq b_\lambda \leq 0.314$ at 95% CL. For a quantitative comparison we use

![Figure 1: Variation of $\chi^2$ with $b_\lambda$.]

| Model | $\chi^2$ |
|-------|----------|
| $\Omega_M = 0.30$, $\Omega_\Lambda = 0.70$ | 178 |
| $\Omega_M = 0.30$, $\Omega_\Lambda = 0.00$ | 273 |
| $\Omega_M = 0.30$, $b_\lambda = 0.26$ | 212 |

Table 1: Values of $\chi^2$

the $\chi^2$ fit. In table 5 we present the $\chi^2$ values corresponding to best fit for the general relativity with and without cosmological constant [4], and present work. The $\chi^2$ value for the present work seems to give a reasonable fit of the SNe Ia data without the necessity of introducing the cosmological constant. In Fig. 2, we also make a comparison between these theories using the effective luminosity and the redshift. Qualitatively, as can be seen from Fig. 2 the tetrad theory fits the data better than general relativity without cosmological constant and it is almost close to the general relativity with cosmological constant for the current available data.

6 Discussion

In this paper we consider a special class of the tetrad theory of gravitation as a viable alternative gravitational theories. We have made use of recent measurements of supernovae Ia to compare the tetrad theory with general relativity with and without cosmological constant. As has been shown in the previous section the tetrad theory leads to a reasonable fit of the supernovae data without introducing cosmological constant. However, to make a best of the data we find that the
The density parameter $\Omega_M = 0.30$ of our work is the same as that of general relativity with $\Omega_\Lambda = 0.70$, however in our case the cosmological constant is already absent. Concerning the age of the universe, our model gives the same value as that of general relativity ($\Omega_M = 0.30, \Omega_\Lambda = 0$) which is $0.81H_0^{-1}$. In the presence of cosmological constant ($\Omega_M = 0.30, \Omega_\Lambda = 0.70$), the age of the universe turns out to be little more around $0.96H_0^{-1}$.

Acknowledgement

This work was supported by Research Center at College of Science, King Saud University under project number Phys/1423/02.

References

[1] S. Perlmutter, et al., Nature 391, (1998) 51.
[2] S. Perlmutter, et al., Astrophys. J 517, (1999) 565.
[3] A.G. Reiss, et al., Astron. J 116, (1998) 1009.
[4] A.G. Reiss, et al., Astrophys. J 607, (2004) 665.
[5] V. Sahni, Class. Quantum Grav. 19, (2002) 3435.
[6] O. Lahav, A. R. Liddle, Phys. Lett. B 592, (2004) 1.
[7] A. Y. Kamenshchik, U. Moschella, V. Pasquier, Phys. Lett. B 511, (2001) 265.
[8] Z.-H. Zhu, M.-K. Fujimoto, Astrophys. J 585, (2003) 52.
[9] S. Sen, A. Sen, *A. Astrophys. J* **588**, (2003) 1 .
[10] W. Godlowski, M. Szydlowski, A. Krawiee, *Astrophys. J* **605**, (2004) 599 .
[11] W. Godlowski, M. Szydlowski, A. Krawiee, *Gen. Relativ. Gravit.* **36**, (2004) 767 .
[12] W. Godlowski, J. Stelmach, M. Szydlowski, *Class. Quantum Grav.* **21**, (2004) 3953 .
[13] D. Puetezfeld, X. Chen, *Class. Quantum Grav.* **21**, (2004) 2703 .
[14] M. Biesiada, W. Godlowski, M. Szydlowski, *Astrophys. J* **622**, (2005) 28 .
[15] R. R. Caldwell, *Phys. Lett.* B **554**, (2002) 23 .
[16] S. M. Carroll, M. Hofman, M. Trodden, *Phys. Rev.* D **68**, (2003) 023509 .
[17] S. D. H. Hsu, A. Jenkins, M. B. Wise, *Phys. Lett.* B **597**, (2004) 270 .
[18] S. M. Carroll, V. Duvvuri, M. Trodden and M. S. Turner, *Phys. Rev.* D **70**, (2004) 043528 .
[19] S. Capozziello, S. Carloni and A. Troisi, arXiv:astro-ph/0303041.
[20] C. Deffayet, G. R. Dvali and G. Gabadadze, *Phys. Rev.* D **65**, (2002) 044023 .
[21] K. Freese, M. Lewis, Phys. Lett. B *Phys. Lett.* B **540**, (2002) 1 .
[22] N. Arkani-Hamed, S. Dimopoulos, G. Dvali, G. Gabadadze, arXiv:hep-th/0209227.
[23] G. Dvali, M. S. Turner, arXiv:astro-ph/0301510.
[24] S. Nojiri and S. D. Odintsov, *Mod. Phys. Lett.* A **19**, (2004) 627 .
[25] N. Arkani-Hamed, H. C. Cheng, M. A. Luty, S. Mukohyama, arXiv:hep-th/0312099.
[26] A. Einstein, *Sitzungser Preuss. Akad. Wiss.* , (1928) 217 .
[27] C. Møller, *Ann. of Phys.* **12**, (1961) 118 .
[28] C. Møller, Mat. Fys. Skr. Dan. Vid. Selsk. **39**(1978),13.
[29] D. Sáez, *Phys. Rev.* D **27**, (1983) 2839 .
[30] H. Meyer, *Gen. Relativ. Gravit.* **14**, (1982) 531 .
[31] K. Hayashi, T. Nakano *Prog. Theor. Phys.* **38**, (1967) 491 .
[32] K. Hayashi, T. Shirafuji *Phys. Rev.* D **19**, (1979) 3524 .
[33] S. Miyamoto, T. Nakano *Prog. Theor. Phys.* **45**, (1971) 295 .
[34] D. Sáez, *Gen. Relativ. Gravit.* **16**, (1984) 501 .
[35] M. Schweizer, N. Straumann, *Phys. Lett.* A **71**, (1979) 493 .
[36] M. Schweizer, N. Straumann, A. Wipf, *Gen. Relativ. Gravit.* **12**, (1980) 951 .
[37] F. I. Mikhail, M. I. Wanas, A. Hindawi, E. I. Lashin, *Int. J. of Theor. Phys.* **32**, (1993) 1627 .
[38] T. Shirafuji, G. L. Nashed, K. Hayashi, *Prog. Theor. Phys.* **95**, (1996) 665 .
[39] F. I. Mikhail, M. I. Wanas, A. Hindawi, E. I. Lashin, *Gen. Relativ. Gravit.* **26**, (1994) 869 .
[40] D. Sáez, *Phys. Lett.* A **106**, (1984) 293 .
[41] H. P. Robertson, *Ann. of Phys.* **33**, (1932) 496 .
[42] W. H. McCrea, F. I. Mikhail  *Proc. Roy. Soc. London A* **253**, (1956) 11 .

[43] S. M. Carroll, W. H. Press, E. L. Turner  *Annu. Rev. Astron. Astrophys.* **30**, (1992) 499 .

[44] A. Linde,  *Phys. Rev.* D **59**, (1999) 023503 .