ACCELERATION OF WIND IN OPTICALLY THIN AND THICK BLACK HOLE ACCRETION DISKS SIMULATED IN GENERAL RELATIVITY

ANTON MÖLLER
KTH Royal Institute of Technology, Brinellvägen 8, SE-11428 Stockholm, Sweden
Research Science Institute, MIT, 77 Massachusetts Ave, Cambridge, MA 02139, USA

ALEKSANDER SĄDOWSKI
MIT Kavli Institute for Astrophysics and Space Research, 77 Massachusetts Ave, Cambridge, MA 02139, USA
Einstein Fellow

ABSTRACT
We study the force balance and resulting acceleration of gas in general relativity basing on simulations of accretion on a stellar-mass, non-rotating black hole. We compare properties of acceleration in an optically thin, radiatively inefficient disk, and in an optically thick, super-critical disk accreting at 10 times the Eddington rate. We study both the average forces acting at given location and forces acting on a gas parcel along its trajectory. We show that the acceleration is not a continuous process – in most cases gas is accelerated only in short-lasting episodes. We find that in the case of optically thin disks gas is pushed out by magnetic field in the polar region and by thermal pressure and centrifugal force below the disk surface. In case of optically thick, radiative accretion, it is the radiation pressure which accelerates the gas in the polar funnel and which compensates (together with the centrifugal force) the gravity in the bulk of the disk. We also show that the Newtonian formulae for the forces are inadequate in the innermost and in the highly magnetized regions.

Keywords: accretion, accretion discs – black hole physics – relativistic processes – methods: numerical – galaxies: jets

1. INTRODUCTION
Gas accreting on compact objects liberates large amounts of its binding energy. Even small fraction of this energy converted into radiation makes accreting systems extremely luminous. But radiation is not the only product of accretion. There is a growing evidence that accretion inevitably generates outflows carrying outward significant amount of gas – sometimes much more than what reaches the compact object. Gas may be driven out of the accretion flows in a couple of ways. Large scale magnetic fields may lead to magneto-centrifugal acceleration of gas along rapidly rotating field lines (Blandford & Payne 1982; Sądowski & Sikora 2010). Radiation generated by the infalling gas may exert outward pressure that accelerates the gas (Shakura & Sunyaev 1973). Finally, illumination of the outer regions by hot radiation coming from the inner region may drive thermal outflow (Begelman et al. 1983).
Blueshifted absorption features indicating the presence of outflowing gas along the line of sight have been observed for years both in case of accreting stellar mass compact objects and active galactic nuclei (AGN) (King et al. 2013). It has been shown that winds are most prevalent in the soft state of black hole (BH) X-ray binaries (Ponti et al. 2012). Their presence is anticorrelated with the presence of radio jets (Miller et al. 2008; Neilsen & Lee 2009). They are highly variable (e.g., Neilsen et al. 2011; Neilsen & Homan 2012; Neilsen 2013) and carry matter out at significant rates (e.g., Ueda et al. 2004; Kallman et al. 2009; Miller et al. 2016).
Deriving the properties of the outflow from the observed absorption lines is not straightforward. The observed absorption occurs along the line of sight as a result of interplay between gas density and ionization, often far from the region where the outflow was generated. In addition, the absorption profiles are superimposed on the relativistically broadened reemission features coming from larger volume (Miller et al. 2015). There is no consensus on whether the observed outflows in X-ray binaries are driven magneto-centrifugally (Miller et al. 2006, 2008; Kallman et al. 2009; Neilsen et al. 2016; Miller et al. 2016), thermally (Ueda et al. 2009; Neilsen et al. 2011, 2014; Díaz Trigo et al. 2014), or whether both mechanisms work at the same time (Neilsen...
& Homan 2012). Active galactic nuclei show significant outflows as well (King et al. 2011, 2012), some of which can be mildly relativistic (King & Pounds 2015; Chartas et al. 2002; Tombesi et al. 2010), providing physical mechanism for connecting the SMBH and the properties of its host.

Analytical modeling of accretion flows usually takes strong assumptions, e.g., by neglecting the outflows completely (Narayan & Yi 1994) or prescribing their radial dependence (Blandford & Begelman 1999), and does not allow to study their generation consistently. Multi-dimensional simulations are necessary for this purpose. However, they are limited, and a single simulation is not able to cover the whole accretion flow. Woods et al. (1996), Luketic et al. (2010) and Higginbottom & Proga (2015) focused on the outer regions and studied the generation of thermal winds from the photosphere irradiated by strong X-rays produced in the innermost part. Other groups (e.g., Proga et al. 2000; Gammie et al. 2003; Ohsuga et al. 2009; Sadowski et al. 2014) studied the innermost region where magnetocentrifugal and radiative driving operates and showed that, indeed, significant outflows emerge.

In this paper we study the acceleration mechanisms acting on outflowing gas in optically thin and optically thick accretion flows by analyzing simulations of the innermost region performed with general relativistic radiation magnetohydrodynamical (GRRMHD) code KORAL. To model these phenomena, we break down the individual forces acting upon the gas and determine the main acceleration mechanisms in each case.

Similar modelling has been done previously by Yuan et al. (2015) and Takahashi & Ohsuga (2015), but this paper presents the first general relativistic modeling and directly compares driving mechanisms in optically thin and thick disks performed within the same computational framework. We study only accretion flows on non-rotating, stellar mass BHs, and limit ourselves to accretion with weak, not-saturated magnetic fields (SANE, in the formalism put forward in Narayan et al. 2012).

The paper is organized as follows. In Section 2, we decompose the forces acting on a particle. Section 3 describes the simulation environment and the initial conditions. Sections 4 presents the results and Section 5 discusses them and gives concluding remarks.

1.1. Definitions

We adopt the following definition of the Eddington accretion rate,

$$M_{\text{Edd}} = \frac{L_{\text{Edd}}}{\eta_{\text{B}} c^2},$$  \hspace{1cm} (1)

where

$$\eta_{\text{B}} = 1 - \sqrt{1 - \frac{2}{3R_{\text{ISCO}}}},$$  \hspace{1cm} (2)

is the efficiency of a thin disk (Shakura & Sunyaev 1973; Novikov & Thorne 1973) extending down to the marginally stable orbit located at $R_{\text{ISCO}}$. According to this definition, a thin disk accreting at $M_{\text{Edd}}$ emits the Eddington luminosity. For zero BH spin, $M_{\text{Edd}} = 2.48 \times 10^{19} M/\text{M}_\odot \text{g/s}$.

We use the term outflow to denote any consistent motion with positive radial velocity $v_r$. This definition includes both the real outflow and the turbulent outflow. While the former eventually reaches infinity, the latter eventually rejoins the accretion flow due to the turbulent gas motion in the disk. To determine whether or not a flow is part of the real outflow, we define the relativistic Bernoulli parameter $Be$ (Sadowski & Narayan 2015),

$$Be = -T^i_i + R^i_i + p u^i u^i, \hspace{1cm} (3)$$

where $T$, $R$, $\rho$, and $u^i$ are gas and radiation stress-energy tensors, gas density and four-velocity, respectively. The real outflow will satisfy $Be > 0$. However, it is important to note that the Bernoulli parameter of a flow will often fluctuate. Despite having a negative parameter when it reaches the simulation boundaries, the flow might still be part of the real outflow, if it ultimately reaches $Be \geq 0$. This can typically be seen as a steady increase in the parameter over the course of the trajectory (Yuan et al. 2015).

We define polar funnel as the conical region of low density between the disk surface and the polar axis.

Hereafter, we use the gravitational radius $r_g = G M_{\text{BH}}/c^2$ as the unit of length, and $r_g/c$ as the unit of time, and often assume $c = 1$. We also always assume that the underlying spacetime metric is fixed, i.e., is not perturbed by the accretion flow.

2. FORCE DECOMPOSITION

2.1. Conservation laws

The three general conservation laws, that is conservation of mass, energy, and angular momentum, are described by the following set of equations,

$$(\rho u^i)_{,\mu} = 0, \hspace{1cm} (4)$$

$$(T^i_v)_{,\mu} = G_{,v}, \hspace{1cm} (5)$$

$$(R^i_v)_{,\mu} = -G_{,v}, \hspace{1cm} (6)$$

where $\rho$ is the gas density in the comoving frame, $u^i = (u^t, u^i)$ is the angular four-velocity, semicolon indicates the covariant derivative, and $T^i_v$ is the magnetohydrodynamic stress-energy tensor,

$$T^i_v = w u^i u_v + \left(p_g + \frac{1}{2} b^2 \right) \delta^i_v - b^i b_v. \hspace{1cm} (7)$$

$w = \rho + u^i u_i + p_g + b^2$ denotes relativistic enthalpy, $u^i$ and $p_g = (\gamma - 1) u^t u_t$ are the internal energy and gas pressure, respectively, $\gamma = 5/3$ is the adiabatic index, $b^i$ is the magnetic field four-vector and $\delta^i_v$ is the Kronecker delta.
$R_{\mu}^{\nu}$ is the stress-energy tensor of radiation and $G_\nu$ is the radiation four-force describing the interaction between gas and radiation defined by,

$$G^\mu = G_0^\mu + G_\text{Compt}^\mu, \quad (8)$$

where $G_0^\mu$ and $G_\text{Compt}^\mu$ reflect the energy and momentum transfer due to absorption and scattering, and Comptonization, respectively. The former is given by (Sadowski et al. 2015),

$$G_0^\mu = -\rho(\kappa_a + \kappa_e)R_{\rho\sigma}u^\rho - \rho(\kappa_a R_{\rho\sigma}u_\rho + \kappa_e 4\pi B)u^\sigma, \quad (9)$$

where $\kappa_a$ and $\kappa_e$ are the absorption and scattering grey opacities respectively and $4\pi B = aT_g^4$ ($a$ is the radiation constant) is the intensity of black body radiation for a gas with temperature $T_g$. We further define an effective radiation temperature $\tilde{T}_g$ in the fluid frame as

$$\tilde{E} = aT_g^4. \quad (10)$$

The Comptonization component of Equation (8), under the “blackbody” approximation (see Sadowski & Narayan 2015) is,

$$G_\text{Compt}^\mu = -\kappa_e\hat{F}[\frac{4k(T_g - T_i)}{m_e c^2}] \times \left[1 + 3.683 \left(\frac{kT_g}{m_e c^2}\right)^2\right] \times \frac{1}{1 + \frac{kT_g}{m_e c^2}} u^\mu, \quad (11)$$

where $m_e$ is the electron mass.

In the orthonormal fluid frame, Equation (9) corresponds to the more intuitive

$$\tilde{G}_0^\mu = \kappa_a\hat{F}(E - 4\pi \tilde{B}), \quad (12)$$

$$\tilde{G}_1^\mu = (\kappa_a + \kappa_e)\rho \hat{F} u^\mu, \quad (13)$$

where $\hat{F}$ is the radiation flux. Equation (12) describes the energy transfer rate resulting from absorption and emission while Equation (13) describes the rate of change of the momentum because of absorptions and scatterings.

Equations (4)-(6) can be rewritten in a coordinate basis by separating the time dimension from the spatial coordinates yielding,

$$\partial_t (\rho u^\nu) + \partial_i (\sqrt{-g} \rho u^i u^\nu) = 0, \quad (14)$$

$$\partial_i (\sqrt{-g} \tilde{T}_g^{i\nu}) + \partial_j (\sqrt{-g} T_i^{j\nu}) = \sqrt{-g} \tilde{T}_g^{i\nu} \Gamma_{jk}^{i\nu} + \sqrt{-g} G_{ij}, \quad (15)$$

$$\partial_i (\sqrt{-g} R_i^{\nu}) + \partial_j (\sqrt{-g} R_j^{\nu}) = \sqrt{-g} R_i^{\nu} \Gamma_{jk}^{i\nu} - \sqrt{-g} G_{ij}, \quad (16)$$

where $\sqrt{-g}$ is the metric determinant and $\Gamma_{jk}^{i\nu}$ are the Christoffel symbols.

### 2.2. Decomposition

To decompose the forces acting on the gas, we rewrite Equation (15) assuming a stationary state ($\partial_t = 0$) and decomposing the stress-energy tensor $T_\nu^\nu$ from Equation (7), yielding,

$$\partial_i (\sqrt{-g} \rho u_i u^\nu) = -\partial_i (\sqrt{-g} p_{\mu} u_i^\mu) - \partial_i \left(\sqrt{-g} \frac{1}{2} b^2 \delta_i^\nu\right) + \partial_i (\sqrt{-g} b_i b^\nu) + \sqrt{-g} \tilde{T}_g^{i\nu} \Gamma_{jk}^{i\nu} + \sqrt{-g} G_{ij}. \quad (17)$$

The left hand side of Equation (17) equals,

$$\partial_i (\sqrt{-g} \rho u_i u^\nu) = u_i \cdot \partial_i (\sqrt{-g} \rho u^\nu) + \sqrt{-g} \rho u_i \partial_i (u^\nu)$$

$$= u_i \cdot \partial_i (\sqrt{-g} (w - \rho) u^\nu) + \sqrt{-g} \rho u_i \partial_i (u^\nu). \quad (18)$$

where $\partial_i (\sqrt{-g} \rho u^\nu) = 0$ (eq. 14) due to the assumption of a stationary state. We move the metric determinant out of the derivatives and combine this with Equations (17) and (18) to obtain,

$$u^\nu \partial_i (u_\nu) = -\frac{1}{w} \partial_\nu (\rho u_\nu) - \frac{1}{w} \partial_\nu \left(\frac{1}{2} b^2\right) + \frac{1}{w} \partial_\nu (b_i b^\nu) + \frac{1}{w} T_i^{\nu} \Gamma_{jk}^{i\nu} \text{w}$$

$$- \frac{T_i^{\nu} - \rho u_i u_\nu \partial_\nu (\sqrt{-g})}{w} - \frac{\sqrt{-g}}{w} G_{ij} - \frac{1}{w} u_i \partial_i (w - \rho u^\nu). \quad (19)$$

Here, the left hand side corresponds to the convective derivative of the gas, while the terms on the right hand side correspond to different forces acting on the gas. We define them as follows,

$$f_{\nu,\text{thermal}} = -\frac{1}{w} \partial_\nu (p_g) \quad (20)$$

$$f_{\nu,\text{magnetic}} = -\frac{1}{w} \partial_\nu \left(\frac{1}{2} b^2\right) + \frac{1}{w} \partial_\nu (b_i b^\nu) \quad (21)$$

$$f_{\nu,\text{metric}} = \frac{1}{w} T_i^{\nu} \Gamma_{jk}^{i\nu} \text{w} - \frac{T_i^{\nu} - \rho u_i u_\nu \partial_\nu (\sqrt{-g})}{w} \frac{\sqrt{-g}}{w} \quad (22)$$

$$f_{\nu,\text{radiation}} = \frac{1}{w} G_{ij} \quad (23)$$

$$f_{\nu,\text{enthalpy}} = -\frac{1}{w} u_i \partial_i (w - \rho u^\nu). \quad (24)$$

Equation (20) describes the thermal pressure force. The thermal force is dependent on the gradient of the gas pressure $\partial_\nu (p_g)$. This indicates that the force is due to the internal pressure the gas exerts on itself (or rather, the pressure the gas as a whole exerts on individual particles).

Equation (21) is composed of two terms, the first which shall be referred to as the magnetic pressure and the second as the magnetic tension. Like the thermal force, the magnetic pressure is a gradient force. This force arises from the density of the magnetic field lines. Magnetic tension, on the other hand, is the force exerted on particles as bent magnetic field lines straighten out.

Equation (22) describes the forces resulting from the space time curvature. Most importantly, one of its components reflects the gravitational acceleration. Another component, reflects the (virtual from the point of view of a stationary observer) centrifugal force. The remaining terms account
mostly for the relativistic corrections due to space-time curvature and are negligible far from the BH.

Equation (23) describes the force that radiation exerts on particles. Photons can either accelerate particles due to radiation pressure, or decelerated them because of the radiative drag, as discussed above.

Equation (24) is a relativistic correction resulting from the difference between the total system enthalpy \( w \) and the rest mass density \( \rho \).

We can furthermore decompose Equation (22) into the components driving the gravitational attraction and the (azimuthal) centrifugal force,

\[
\vec{f}_{\text{metric}} = \vec{f}_{\text{gravity}} + \vec{f}_{\text{centrifugal}} + \vec{f}_{\text{residual}},
\]

where,

\[
\begin{align*}
\vec{f}_{\text{gravity}} &= \frac{T^r}{w} \vec{\Gamma}^r \vec{\Gamma}_r^v, \\
\vec{f}_{\text{centrifugal}} &= \frac{T^\phi}{w} \vec{\Gamma}^\phi \vec{\Gamma}_\phi^v.
\end{align*}
\]

The remainder, \( \vec{f}_{\text{residual}} \), together with Equation (24) reflects the relativistic correction terms, denoted by

\[
\vec{f}_{\text{correction}} = \vec{f}_{\text{residual}} + \vec{f}_{\text{enthalpy}}.
\]

The forces can be expressed in an orthonormal basis (assuming a diagonal metric of a non-rotating BH) through \( \vec{f}_i = f_i / \sqrt{g_{ii}} \).

For a non-relativistic system where \( u_r + p_r + \frac{v^2}{2} \ll \rho \) and \( v \ll 1 \), Equations (20)-(28) reduce to the well known Newtonian formulae,

\[
\begin{align*}
\vec{f}_{\text{thermal}} &= -\frac{1}{\rho} \partial_i (p_i) \\
\vec{f}_{\text{magnetic}} &= -\frac{1}{\rho} \partial_i \left( \frac{1}{2} b^2 \right) + \frac{1}{\rho} \partial_i (b_i b_i) \\
\vec{f}_{\text{gravity}} &= -\frac{1}{r^2} \\
\vec{f}_{\text{centrifugal}} &= \sin \theta \frac{\dot{v}_\phi^2}{r \sin \theta} \\
\vec{f}_{\text{correction}} &= 0.
\end{align*}
\]

3. NUMERICAL METHOD

We analyzed two simulations performed with a general relativistic (GR) radiation magnetohydrodynamical (RMHD) code KORAL (Sadowski et al. 2013; Sadowski et al. 2014) - one purely MHD (optically thin, \texttt{hd3\#0a0}) and the other including radiation field (optically thick, \texttt{d3\#0a0}) and corresponding to the accretion rate \( \sim 10 \dot{M}_{\text{Edd}} \). Both were initialized as equilibrium torii threaded with multiple loops of weak magnetic field lines. In the case of the radiative run, local thermal equilibrium between gas and radiation was assumed. The radiation was evolved adopting the M1 closure scheme. For both runs the BH spin was zero.

The simulations assumed axisymmetry and ran in 2.5 dimensions, that is, assuming axisymmetry but allowing non-zero azimuthal components. We prevented quick dissipation of the magnetic field in axisymmetrical simulations by applying the mean-field dynamo of Sadowski et al. (2015).

The simulations were run for an exceptional amount of time, reaching \( \sim 2,000 \) orbits at the innermost stable circular orbit \( (\sim 200,000 GM_{\text{BH}}/c^3) \). This allowed for the outflow to reach inflow/outflow equilibrium (defined as the region where average properties do not change with time) in relatively large domain, especially in the polar region, where the gas moves with large velocities. Applying the criterion from Narayan et al. (2012) we obtain in both simulations the equilibrium regions extending up to \( \sim 80 r_g \) at the equatorial plane and up to the computational box boundary at \( \sim 1000 r_g \) at the axis.

For this work we adopted the following forms of the absorption and scattering opacities,

\[
\kappa_a = 6.4 \cdot 10^2 \rho T_g^{-3/2} \text{ cm}^2/\text{g}
\]

\[
\kappa_s = 0.34 \text{ cm}^2/\text{g}
\]

where \( \rho \) and \( T_g \) are the gas density and temperature, respectively. Such a form of absorption opacities reflects only the bremsstrahlung absorption and emission, and, in particular, neglects line opacities. Therefore, the radiative simulation applies only to hot accretion flows near stellar mass BHs.

4. RESULTS

4.1. Flow properties

The leftmost and center panels in Figure 1 show the averaged (top set of panels) and instantaneous (bottom set) density distribution and velocity vectors of gas in the non-radiative\(^1\) and radiative simulation respectively. The gas is clearly concentrated near the equatorial plane but the disk thickness is large – the density scaleheight in both cases is close to \( H/R = 0.3 \). In case of the hydro simulation, the gas relatively smoothly fills the polar region. For the radiative simulation, on the other hand, the polar region has significantly lower density than the bulk of the disk. In that case, the density at the axis can be as much as 6 orders of magnitude lower than at the equatorial plane. This fact results from radiation cleaning the funnel, as will be discussed below.

The streamlines in the left and center panels show the average velocity field. In the case of the hydro simulation the

\(^1\) Purely magnetohydrodynamical (non-radiative) simulations are scale-free, i.e., they correspond to optically thin flow of any density. Therefore, the values of density given above the left panels are arbitrary.
gas flows inward only deep inside the bulk of the disk. Far from the equatorial plane vertical motion is clear, leading to gas escaping from the disk. The velocity field in case of the radiative simulation is noticeably different. Very fast (much faster than in the hydro case) outflow is evident only in the polar region. At intermediate polar angles, the gas does not show clear outflowing pattern, what implies that the turbulent motion is strong and on average does not give strong outflow.

The corresponding bottom panels show instantaneous gas density distribution and velocity field. The turbulent nature of the flow, resulting from the magnetorotational instability (Balbus & Hawley 1991), is evident in both cases. It is interesting to note that in the hydro simulation gas can temporarily fall on the BH, although on average it shows positive radial velocity. These episodes feature very low density gas which does not provide significant inward mass flux.

The right panels in Figure 1 show the averaged (top) and instantaneous (bottom panel) radiation field in the radiative simulation. Colors denote magnitude of the radiative flux and streamlines show its direction. Deep inside the disk photons are trapped and advected on the BH (Sadowski & Narayan 2015). At intermediate angles radiation diffuses out of the disk and contributes to the radiation in the funnel. The efficiency of accretion in radiation emitted into the optically thin funnel is not large – radiation carries only \( \sim 1\% \) of \( Mc^2 \), what is \( \sim 5 \) times less than it would carry for a thin disk accreting on a non-rotating BH. The corresponding bottom panel shows instantaneous properties of the radiation field. Radiation escaping through the funnel is quite laminar, in contrast to turbulent disk interior, where photons are dragged with optically thick, turbulent gas, producing, on average, the pattern described above.

### 4.2. Average forces

We begin by calculating the average forces acting on gas at a given location. We averaged the simulations’ output on the go, i.e., accounting for states after every single step of time iteration. Each product appearing in Equations 20-24 was averaged independently, e.g.,

\[
\left\langle \frac{T^i_j - \rho u^i u_j}{w} \right\rangle = \frac{\left\langle T^i_j \right\rangle - \left\langle \rho u^i u_j \right\rangle}{\left\langle w \right\rangle}.
\]

In such a way we are able to obtain the average forces acting at a given location, i.e., the forces that observer sitting at fixed coordinates would see acting on gas flowing by. There is no guarantee that every single parcel of gas will feel exactly these forces when following its own trajectory. However, one can hope that forces obtained in this way will give qualitatively right picture.

The results of this analysis for the optically thin (hd300a0) and thick (d300a0) simulations are shown in Figures 2 and 3, respectively. The background color denotes the logarithm of the magnetic to rest-mass energy densities ratio. This quantity measures how strong the magnetic field is, and in particular, how much the relativistic enthalpy \( w = \rho + p_g + u_g + b^2 \) departs from the non-relativistic value \( w = \rho \).

Using Equations (20)-(24) we calculated radial and polar components of each force and transformed them to the orthonormal frame. Acceleration induced by each force component are plotted with vectors. Blue arrows correspond to the gravitational force (Eq. 26), green to the thermal pressure gradient force (Eq. 20), purple to the magnetic forces (Eq. 21), red to the centrifugal force (Eq. 27), and cyan to the relativistic correction (Eq. 28). In the case of the optically thick, radiative run (Fig. 3), an additional set of yellow arrows denote the radiative force (Eq. 23). In both plots the black arrows correspond to the sum of all the forces, i.e., to the total acceleration acting on gas. The larger is the length of this vector, the faster gas particles gain velocity. All the vectors are scaled in the same way, with the scaling factor chosen arbitrarily to fit the arrows in the panels.

#### 4.2.1. Optically thin disk

Figure 2 shows the average forces in the optically thin, purely magnetohydrodynamical simulation. The two forces which are most obvious are the gravitational pull (blue) and the centrifugal force (red arrows). Gravitational attraction always works towards the BH and its magnitude decreases (up to relativistic correction) with the square of radius. The centrifugal force acts only in the direction perpendicular to the axis of rotation. Its strength depends on the value of azimuthal velocity and the distance from the axis. The largest centrifugal acceleration acts on particles close to the BH and located near the equatorial plane, inside the almost Keplerian disk. It is much less significant in case of slowly rotating gas in the polar region.

Outside of the polar region gas moves in a turbulent way with relatively low velocities and the velocity changes it undergoes are much less violent then for a free-falling particle. It is therefore reasonable to expect that the net force acting on the gas in the bulk of disk would be much smaller in magnitude than the gravitational and centrifugal forces. To satisfy this condition one has to provide another force, or forces, which will balance the two. In case of optically thin disk it is the thermal pressure force (green vectors) which acts this way. It provides the necessary vertical component which balances the vertical component of gravitational pull. As a result, the total acceleration (black arrows) is small in the bulk of the disk.

Situation is different in the polar region where the gas is accelerated rapidly and reaches significant radial velocities. In this case, the magnitude of the net acceleration may be even
gas density and velocity (hd300a0)

gas density and velocity (d300a0)

radiation field (d300a0)

Figure 1. Leftmost and center panels show the logarithm of the gas density distribution (in CGS units, g/cm$^3$) and the gas velocity field for the non-radiative and radiative simulations respectively. The width of the vectors are proportional to the velocity magnitude. The rightmost panels shows the logarithm of the magnitude of the radiative flux (erg/s/cm$^2$) and the radiative flux vector field. The top panels correspond to averaged data. The bottom panels represent instantaneous snapshot data at an arbitrary chosen time in the simulation.

comparable with the gravitational pull. In contrast to particles inside the disk, the centrifugal force is no longer substantial. Instead, the magnetic force (purple arrows) affects significantly the acceleration. Together with the thermal force, it provides the vertical force that balances and overcomes the gravitational pull. For the gas in the innermost polar region, where the magnetic energy density becomes comparable with the rest-mass energy density, one should not neglect the relativistic corrections (cyan arrows), which may reach magnitudes comparable with the other forces.

4.2.2. Optically thick disk

Figure 3 shows average forces acting on gas in the optically thick simulation. The colors once again denote the strength of the magnetic field. Because of much lower density of gas in the polar region of this simulation than in the case of optically thin disk, the magnetic field now dominates over rest-mass energy and satisfies $b^2/\rho > 1$ in most of the funnel region.

The properties of the gravitational pull (blue arrows) and the centrifugal force (red arrows) are quite similar to the described above - the former points always towards the BH, while the latter is perpendicular to the axis. The centrifugal force points outward everywhere but for the points located
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Figure 2. Averaged data from the non-radiative simulation. The figure shows the ratio \( \log(b^2/\rho) \) (shades of grey) and the vectorial force distribution at arbitrary points where the arrows indicate both direction and magnitude of the forces.

Figure 3. Similar to Figure 2 but for the averaged data from the radiative simulation.

in the funnel region. This surprising effect comes from the fact that the \( T_\phi \) component of equation (27) changes sign in that region because of predominant magnetic field component \( b^\phi b_\phi \) (if we had defined this force as proportional to \( \rho u^\phi u_\phi \), instead of \( T_\phi \), the force would always point outward, as in the non-relativistic limit). Similar effect was not present in the optically thin case because magnetic field in the polar region was much weaker than in the case described here.

Relatively slow gas velocities in the bulk of the disk results from weak acceleration. This is once again the reason why the forces deep in the disk balance each other and the resulting average net force (and acceleration) is very weak (in most cases the corresponding arrow is not visible in the plot). However, on the contrary to the optically thin case, it is not the thermal pressure gradient which balances the vertical component of the gravitational force. In case of optically thick, radiation-pressure dominated disk like the one simulated, it is the radiation pressure force (equation 23, yellow vectors in Figure 3) which provides necessary vertical force. In the optically thick disk it follows the gradient of radiative energy density and therefore points away of the equatorial plane towards the axis. In other words, the radiative force has replaced the thermal pressure force and is now supporting the disk against gravity.

The properties of the forces change significantly in the polar region. As shown in the left panels of Fig. 1, strong flux of radiation coming from the innermost region is streaming out of the system along the axis. Gas which enters this region is immediately swept up and with mildly-relativistic velocities moves away from the BH through the funnel. This radiative acceleration is clearly seen in Fig. 3 – radiative force in the polar region has significantly larger magnitude than the gravitational force at a given location and always points away from the BH. However, the radiative force is not the only one pushing the gas away from the BH. The magnetic force (equation 21) and the relativistic correction (equation 24) also point upward and reflect the magnetic acceleration of gas in the magnetic tower that develops when magnetic field lines brought into the innermost region are wound up by disk rotation and expand vertically in the polar region dragging the gas behind. This dragging effect, and the resulting magnetic acceleration, is more effective in the optically thick case than it was in the other simulation because of lower density of the gas and larger magnetic to rest-mass energy ratio.

4.3. Trajectory approach

The averaged data that we based on in the previous section reflects average forces acting at a single point over the course of the simulation, and does not reflect forces affecting a particular moving particle that happens to pass by the point in question at the specific time. What is more, not all the gas crossing given location, that on average belongs to the outflow region, is guaranteed to escape to infinity. There is also gas which either moves (probably temporarily) inward or is not energetic enough (is bound) to escape the BH gravitational pull. It is therefore not straightforward to extend the analysis done in the previous section, which was based on averaged disk properties, to every single gas parcel. The force balance may be significantly different for outflowing
and inflowing gas crossing the same location.

We therefore traced a series of particle trajectories for each simulation. We chose the trajectories to represent the jet, wind and disk region of the outflow. We injected virtual test particles and tracked their trajectories by interpolating the velocity from snapshot data at time $t$, and thus obtaining the positions of particles at time $t + \Delta t$. This method holds for sufficiently small $\Delta t$ intervals. We used snapshot data saved every $\Delta t = 1GM/c^3$ what gave accuracy good enough to track even the fastest gas moving with mildly-relativistic velocities $v \sim 0.3c$. For each trajectory, we then calculated the decomposed real-time forces along that trajectory, which then were used to determine the acceleration mechanisms of the outflow.

4.3.1. Optically thin disk

The top panel in Figure 4 shows the four representative trajectories that we chose for the optically thin model. We will refer to these trajectories as H1 to H4 (numbered left to right). First two (H1 and H2) correspond to gas entering the funnel and escaping along the axis. The other two (H3 and H4) reflect typical trajectories of gas escaping in the wind region of the simulation. The colors along the trajectories reflect the velocity of the gas. As expected, gas in the funnel undergoes the strongest acceleration and reaches velocities of the order of $0.1c$. The wind trajectories (H3 and H4) reflect much slower gas, moving on average with velocities $\sim 0.01c$.

The four panels below the top one show the forces acting on gas along each of the trajectories. The arrows have the same meaning as in the previous plots. The numbers on the plots reflect the Bernoulli number of the gas (equation 3) at each location for which we calculated the forces. Positive values correspond to unbound gas.

The jet trajectories (H1 and H2) both accelerate and then decelerate while maintaining a growing Bernoulli number making them part of the real outflow. However, the main initial acceleration mechanism of these two trajectories differ, as trajectory H1 is initially accelerated by the magnetic force (magenta arrows) and trajectory H2 is initially accelerated by the thermal pressure force (green arrows). It is evident that for these trajectories the acceleration is not a continuous process – gas is rapidly accelerated in the innermost region, it enters the funnel with large velocity, but then the further acceleration ceases and the gravitational pull slows down the gas. There is no unique force which is responsible for this initial acceleration – it could be either magnetic or thermal force.

Wind trajectories H3 and H4 both exhibit relatively low positive radial velocities and show negative but growing Bernoulli numbers. They are part of the turbulent outflow within our simulation boundaries, but the growing number indicates that they could be part of the real outflow. We do not observe a sudden acceleration, but rather a series of minor acceleration events giving the trajectories on average positive radial velocities. The main acceleration mechanisms for the trajectory closest to the funnel region (H3) and the one deeper in the disk (H4) are the magnetic and thermal forces, respectively, with the centrifugal force also providing significant component along the trajectory.

4.3.2. Optically thick disk

Figure 5 presents in a similar way four outflowing trajectories in the optically thick simulation. We will refer to them as trajectories R1 (leftmost) to R4 (rightmost). The top panel shows the trajectories on top of the average gas density distribution. The colors along them denote the gas velocity. Trajectory R1 stands out as the one closest to the axis – the gas following this trajectory reaches the highest velocity, $v \sim 0.4c$. The second leftmost trajectory, R2, follows roughly the boundary between the polar funnel and the disk. The corresponding gas is not accelerated so efficiently but still reaches velocity of the order of $\sim 0.1c$. The other two trajectories are deep inside the optically thick region and are much slower with gas not exceeding $0.02c$.

The four panels below the top one show forces acting along each of the trajectories. Gas which managed to enter the funnel region (reflected in path R1) has very low density and is strongly accelerated by the radiation force (yellow arrows) which is responsible for effective and continuous acceleration. This acceleration results in a mildly-relativistic outflow of unbound gas along the axis. This outflow is characterized by lowest densities but many carry significant amount of kinetic energy (Sadowski & Narayan 2015).

The gas following the boundary of the funnel and the disk (trajectory R2) is also pushed forward predominantly by the radiation force. This time, however, the force is weaker and is pointing more towards the axis, not along it. This results from the fact that at this location the radiation diffuses out of optically thick disk following the gradient of radiative energy density. Radiative flux, together with the centrifugal force, is strong enough, however, to accelerate the gas and make it unbound.

The wind trajectories (R3 and R4) correspond to gas flowing through surface layers of the disk. The radiative flux in the comoving frame of the gas, which results from photon diffusion, points towards the disk surface and so does the radiative force. The radiative and centrifugal forces are not strong enough, however, to efficiently accelerate the gas – it is just enough to move the gas slowly outward. The corresponding Bernoulli numbers are negative but growing and it is possible that ultimately the radiation will manage to make the gas unbound. The properties of the force balance along these two trajectories very well agree with the results of the analysis based on averaged data.

5. DISCUSSION

The outflow in the optically thin, non-radiative simulation is mainly driven by the magnetic and thermal forces. The
Figure 4. Trajectories for optically thin model hd300a0. The top panel shows two jet and two wind/disk particle trajectories (H1-H4, left to right), where the color is proportional to the speed of the particle in fractions of the speed of light plotted over the logarithm of gas density. The four bottom panels show the force decomposition at arbitrarily chosen points for each individual trajectory. The length of the force vectors is proportional to the force magnitude, scaled by $r^2$, and the width of the trajectory is proportional to the particle velocity (for exact value, see top panel). Bernoulli number, $Be$, along the trajectories is shown.

Figure 5. Similar to Figure 4 but for the optically thick model d300a0 and trajectories R1-R4 (left to right). The additional radiative force is denoted by yellow arrows.

initial acceleration mechanism for the radiative simulation is the radiative force. In highly magnetized and the innermost regions in both models the relativistic correction (Eq. 28) is not negligible, and the standard Newtonian decomposition of forces would not be adequate.

The jet trajectories (H1 and H2, see Section 4.3.1) left the
much larger velocities, approaching 0.10c. The radiative accretion flow revealed much larger velocities, approaching 0.40c for low-density gas in the jet region accelerated vertically by the uniform radiation flux. Both radiative jet trajectories we studied (R1 and R2, see Section 4.3.2) left the simulation bounds with positive and growing Bernoulli parameters. As in the non-radiative model, the gas contained in the wind was bound inside the domain, but the outward forces were still increasing its Bernoulli number.

In case of the optically thin, non-radiative disks, the initial acceleration seems to play the major part in generating real outflow, as the gas is accelerated rapidly in the inner region and then propagates with forces near equilibrium. The trajectories in the jet region of the radiative simulation, on the contrary, are continuously exposed to radiation force and thus undergo continuous acceleration.

The average forces acting on the gas at a given location reflect dominating forces acting on a gas parcel along its trajectory. However, the average forces do not allow to catch the exact timing and location of the short-lived but significant acceleration episodes. They also do not reflect the turbulent nature of the flow. The trajectory approach provides full information about the acceleration mechanism for traced trajectories, making it ideal for studying this phenomena. While it is not possible to trace every single gas trajectory, the averaged data serves as a reference of what one should expect given a large enough set of traced trajectories.

We have compared an optically thin disk, corresponding to the radiatively inefficient regime and to the lowest accretion rates, with a model of optically thick disk accreting at a super-critical rate of 10\(\dot{M}_{\text{Edd}}\). To get full understanding of processes driving outflows in accretion disks, one would have to extend our work with a similar study covering wider range of accretion rates, including geometrically thin, sub-Eddington disks, and taking rotating BHs into account. Especially in the case of spinning BHs one may expect that the extra energy extracted from the BH spin will significantly alter the acceleration mechanisms in the jet region.

Similar modelling of forces acting on gas in accretion flows was done previously in Yuan et al. (2015) and Takahashi & Ohsuga (2015). The former work focused solely on optically thin disks, while the latter analyzed only a super-critical disk similar to our radiative model. Our conclusions are in general agreement with both these works. However, we calculated and decomposed the forces in fully relativistic way and directly compared driving mechanisms in optically thin and thick disks.

6. SUMMARY

We have analyzed the force balance and the resulting acceleration of the outflow in two simulations of a BH accretion flow. Both were performed in general relativity with a state-of-the-art numerical methods and assumed zero BH spin and weak, non-saturated magnetic field. One model corresponded to radiatively inefficient, optically thin disk, while the other to a super-critical disk with accretion rate of 10\(\dot{M}_{\text{Edd}}\). We found that:

1. **Gas acceleration:** Driving gas out of the disk is not, in most cases, a continuous and simple process. There are usually only a couple of short-lived episodes which result in significant outward acceleration of the gas. But for them, the gas velocity fluctuates, as does the net force acting on gas. An exception is the polar region in radiative disks, where gas is constantly pushed away by radiation pressure.

2. **Outflows in optically thin disks:** Radiatively inefficient accretion produces outflows at large range of angles. The gas in the polar region is accelerated mostly by the magnetic forces and reaches velocities of the order of 0.1c. At larger distance from the axis, it is the thermal pressure and centrifugal forces which compensate for gravity and push the gas out of the disk. The magnetic forces are not significant but for the most polar region. Therefore, the outflow outside of the polar region is not driven by the magnetocentrifugal acceleration.

3. **Outflows in optically thick disks:** The fastest outflows in radiative disks take place near the axis, in the optically thin funnel region. Gas can reach there mildly-relativistic velocities exceeding at times 0.4c. Such significant acceleration results mostly from the radiation pressure force pushing the gas along the axis. Magnetic acceleration is also important in the polar region. In the optically thick disk it is the radiation pressure, not the thermal pressure as in the other case, which balances gravity and pushes gas out of the disks, either into the funnel or radially outward. The magnetic acceleration is once again not significant but for the most polar region.

4. **Relativistic corrections:** We found that the relativistic correction that we have singled out from other forces (Eq. 24) contributes significantly in the regions close to the BH and where magnetic field energy is comparable or exceeds the rest-mass energy density. Therefore, force decomposition based on non-relativistic formulae would be inadequate in these regions.

5. **Average vs trajectory approach:** Only the study of balance of forces based on following particular gas trajectories is able to catch the non-uniform nature of gas acceleration.
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REFERENCES

Balbus S. A., Hawley J. F., 1991, The Astrophysical Journal, 376, 214
Begelman, M. C., McKee, C. F., & Shields, G. A. 1983, ApJ, 271, 70
Blandford, R. D., & Begelman, M. C. 1999, MNRAS, 303, L1
Blandford R. D., Payne D. G., 1982, MNRAS, 199, 883
Chartas, G., Brandt, W. N., Gallagher, S. C., & Garmire, G. P. 2002, ApJ, 579, 169
Díaz Trigo, M., Migliari, S., Miller-Jones, J. C. A., & Guainazzi, M. 2014, A&A, 571, A76
Gammie, C. F., McKinney, J. C., & Tóth, G. 2003, ApJ, 589, 444
Higginbottom, N., & Proga, D. 2015, ApJ, 807, 107
Kallman, T. R., Bautista, M. A., Goriley, S., et al. 2009, ApJ, 701, 865
King, A. R., Zubovas, K., & Power, C. 2011, MNRAS, 415, L6
King, A., & Pounds, K. 2015, ARA&A, 53, 115
King, A. L., Miller, J. M., & Raymond, J. 2012, ApJ, 746, 2
King, A. L., Miller, J. M., Raymond, J., et al. 2013, ApJ, 762, 103
Luketic, S., Proga, D., Kallman, T. R., Raymond, J. C., & Miller, J. M. 2010, ApJ, 719, 515
Miller, J. M., Raymond, J., Fabian, A., et al. 2006, Nature, 441, 953
Miller, J. M., Raymond, J., Reynolds, C. S., et al. 2008, ApJ, 680, 1359-1377
Miller, J. M., Fabian, A. C., Kaastra, J., et al. 2015, ApJ, 814, 87
Miller, J. M., Raymond, J., Fabian, A. C., et al. 2016, ApJL, 821, L9
Narayan, R., & Yi, I. 1994, ApJL, 428, L13
Narayan R., Sadowski A., Penna R. F., Kulkarni A. K., 2012, Monthly Notices of the Royal Astronomical Society, 426, 3241
Neilsen, J., & Lee, J. C. 2009, Nature, 458, 481
Neilsen, J., Remillard, R. A., & Lee, J. C. 2011, ApJ, 737, 69
Neilsen, J., & Homan, J. 2012, ApJ, 750, 27
Neilsen, J. 2013, Advances in Space Research, 52, 732
Neilsen, J., Coriat, M., Fender, R., et al. 2014, ApJL, 784, L5
Neilsen, J., Rahoui, F., Homan, J., & Buxton, M. 2016, ApJ, 822, 20
Novikov I. D., Thorne K. S., 1973, in Dewitt C., Dewitt B. S., eds, Black Holes (Les Astres Occlus). pp 343–450
Ohsuga, K., Mineshige, S., Mori, M., & Yoshiaki, K. 2009, PASJ, 61, L7
Ponti, G., Fender, R. P., Begelman, M. C., et al. 2012, MNRAS, 422, L11
Proga, D., Stone, J. M., & Kallman, T. R. 2000, ApJ, 543, 686
Sadowski, A., Narayan, R., Tchekhovskoy, A., & Zhu, Y. 2013, MNRAS, 429, 3533
Sadowski A., Narayan R., McKinney J. C., Tchekhovskoy A., 2014, Monthly Notices of the Royal Astronomical Society, 439, 503
Sadowski A., Narayan R., Tchekhovskoy A., Abarca D., Zhu Y., McKinney J. C., 2015, Monthly Notices of the Royal Astronomical Society, 447, 49
Sadowski, A., & Narayan, R. 2015, MNRAS, 453, 3213
Sadowski A., & Narayan, R. 2015, arXiv:1508.04980
Sadowski A., Narayan R., Monthly Notices of the Royal Astronomical Society, submitted
Sadowski, A., & Sikora, M. 2010, A&A, 517, A18
Shakura N. I., Sunyaev R. A., 1973, A&A, 24, 337
Takahashi H. R., Ohsuga K., 2015, Publications of the Astronomical Society of Japan, p. psu145
Tombesi, F., Sambruna, R. M., Reeves, J. N., et al. 2010, ApJ, 719, 700
Ueda, Y., Murakami, H., Yamaoka, K., Dotani, T., & Ebisawa, K. 2004, ApJ, 609, 325
Ueda, Y., Yamaoka, K., & Remillard, R. 2009, ApJ, 695, 888
Woods, D. T., Klein, R. I., Castor, J. I., McKee, C. F., & Bell, J. B. 1996, ApJ, 461, 767
Yuan F., Gian Z., Narayan R., Sadowski A., Bu D., Bai X.-N., 2015, The Astrophysical Journal, 804, 101