Flavor physics and CP violation

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Abstract
Lectures on flavor physics presented at the 2012 CERN HEP Summer School.
Content: 1) flavor physics within the Standard Model, 2) phenomenology of 
$B$ and $D$ decays, 3) flavor physics beyond the Standard Model.

Introduction
According to the Standard Model (SM) of fundamental interactions the basic constituents of matter, and
their interactions, are described as excitations of fermionic fields (spin-1/2 particles) interacting with
there different sets of gauge fields (whose excitations correspond to spin-1 particles) associated to the
strong, weak, and electromagnetic interactions. The spin-1/2 particles can be grouped into three families,
or flavors, each containing two quarks (charged under strong interactions) and two leptons (neutral under
strong interactions). The four fermions within each family have different combinations of strong, weak,
and electromagnetic charges, that determine completely their fundamental interactions but for gravity.
Ordinary matter consists essentially of particles of the first family, namely the up and down quarks
(strongly bounded inside protons and neutrons), the electrons (that forms the atoms), and the electron-
neutrinos (abundantly produced by the fusion reactions occurring inside the stars). As far as we know,
quarks and leptons of the second and third family are identical copies of those in the first family but
for different, heavier, masses. The heavy quarks and charged leptons are unstable states that can be
produced in high-energy collisions and that decay very fast (via weak interactions) into particles of the
first family. Why we have three almost identical replica of quarks and leptons, and which is the origin of
their different masses, is one of the big open questions in fundamental physics.

In the limit of unbroken electroweak symmetry none of the basic constituent of matter could
have a non-vanishing mass. The problem of quark and lepton masses is therefore intimately related
to the other big open question in particle physics: which is the mechanism behind the breaking of the
electroweak symmetry, or which is the mechanism responsible for the non-vanishing masses of the weak
force carriers (the $W$ and $Z$ bosons). Within the SM these two problems are both addressed by the Higgs
mechanism: the masses of quarks and leptons, as well as the masses of $W$ and $Z$ bosons, are the result of
the interaction of these basic fields (both matter constituents and force carriers) with a new type of field,
the Higgs scalar field, whose ground state breaks spontaneously the electroweak symmetry.

The recent observation by the ATLAS and CMS experiments of a new state compatible with the
properties of the Higgs boson (or the spin-0 excitation of the Higgs field) has significantly reinforced the
evidences in favor of the Higgs mechanisms and the validity of the SM. However, we have also clear
indications that this theory is not complete: the phenomenon of neutrino oscillations and the evidence
for dark matter cannot be explained within the SM. The SM is also affected by a serious theoretical
problem because of the instability of the Higgs sector under quantum corrections. We have not yet
enough information to unambiguously determine how this theory should be extended; however, most
realistic proposals point toward the existence of new degrees of freedom in the TeV range, possibly
accessible at the high-$p_T$ experiments at the LHC.

The description of quark and lepton masses in terms of the Higgs mechanism is particularly un-
satisfactory since the corresponding interactions is not controlled by any symmetry principle, contrary to
all other known interactions, resulting in a large number of free parameters. Beside determining quark
masses, the interaction of the quarks with the Higgs is responsible for the peculiar pattern of mixing of
the various families of quarks under weak interactions, and the corresponding hierarchy in the various decay modes of the heavier quarks into the lighter ones. In particular, the interplay of weak and Higgs interactions implies that processes with a change of flavor mediated by a neutral current (FCNC processes) can occur only at higher orders in the electroweak interactions and are strongly suppressed. This strong suppression make FCNC processes natural candidates to search for physics beyond the SM: if the new degrees of freedom do not have the same flavor structure of the quark-Higgs interaction present in the SM, then they could contribute to FCNC processes comparably to the SM amplitudes even if their masses are well above the electroweak scale, resulting in sizable deviations from the SM predictions for these rare processes.

In the last few years the mechanism of quark-flavor mixing has been tested in various process (although in many interesting cases with limited accuracy), finding good agreement with the SM expectations. The situation is somehow similar to the flavor-conserving electroweak precision observables after LEP: the SM works very well and genuine one-loop electroweak effects (such as those responsible for FCNC processes) have been tested with a typical relative accuracy of about 30%. Similarly to the case of electroweak observables, also in the quark flavor sector non-standard effects can only appear as small corrections to the leading SM contribution.

Observing new sources of flavor mixing (i.e. flavor violating couplings not related to quark and lepton mass matrices) is a natural expectation for any extension of the SM with new degrees of freedom not far from the TeV scale. While direct searches of new particles at high energies provide a direct information on the mass spectrum of the possible new degrees of freedom, the indirect information from low-energy flavor-changing processes translates into unique constraints on their couplings. The present bounds on possible deviations from the SM in flavor-violating processes already set stringent limits on the flavor structure of physics beyond the SM, and this provides a key information for model-building. However, several options are still open, and the quality of this information could be substantially improved with improved studies of selected flavor-violating observables. In these lectures we focus on the interest of future measurements in the $B$- and $D$-meson systems in this perspective.

The lectures are organized as follows: in the first lecture we briefly recall the main features of flavor physics within the SM. We also address in general terms the so-called flavor problem, namely the challenge to any SM extension posed by the success of the SM in flavor physics. In the second lecture we analyse in some detail the SM predictions for some of the most interesting $B$ and $D$ physics observables to be measured in the LHC era. In the last lecture we analyse flavor physics in various realistic beyond-the-SM scenarios, discussing how they can be tested by future experiments.

These notes have a sizable overlap with a similar set of lectures I present a few years ago [1]. Independent set of lectures on the same subject can be found in Ref. [2], while more detailed presentations can be found in the review articles in Ref. [3–6].
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Chapter 1

Flavor physics within the SM and the flavor problem

1 The flavor sector of the SM

The Standard Model (SM) Lagrangian can be divided into two main parts, the gauge and the Higgs (or symmetry breaking) sector. The gauge sector is extremely simple and highly symmetric: it is completely specified by the local symmetry $G_{\text{local}} = SU(3)_C \times SU(2)_L \times U(1)_Y$ and by the fermion content,

$$L_{\text{gauge}} = \sum_{i=1,\ldots,3} \sum_{\psi=Q_L, E_R} \bar{\psi}_i D_i \psi - \frac{1}{4} \sum_{a=1,\ldots,8} G_{\mu\nu}^a G_{\mu\nu}^a - \frac{1}{4} \sum_{a=1,\ldots,3} W_{\mu\nu}^a W_{\mu\nu}^a - \frac{1}{4} B_{\mu\nu} B_{\mu\nu} \quad (1.1)$$

The fermion content consist of five fields with different quantum numbers under the gauge group $G_{\text{gauge}}$ are broken with the introduction of a $SU(2)_L$ scalar doublet $\phi$, or the Higgs field, $\langle \phi \rangle = v = (2\sqrt{2}G_F)^{-1/2} \approx 174 \text{ GeV}$, while the global flavor symmetry is explicitly broken by the Yukawa interaction of $\phi$ with the fermion fields:

$$- L_{\text{Yukawa}} = Y_d^d \bar{Q}_L^d \phi D_R^d + Y_u^u \bar{Q}_L^u \phi U_R^u + Y_e^e \bar{L}_L^e \phi E_R^e + \text{ h.c.} \quad (\tilde{\phi} = i\tau_2 \phi^\dagger) \quad (1.3)$$

The large global flavor symmetry of $L_{\text{gauge}}$, corresponding to the independent unitary rotations in flavor space of the five fermion fields in Eq. (1.2), is a $U(3)^5$ group. This can be decomposed as follows:

$$G_{\text{flavor}} = U(3)^5 \times G_q \times G_\ell \quad (1.4)$$

where

$$G_q = SU(3)_Q \times SU(3)_{U_R} \times SU(3)_{D_R}, \quad G_\ell = SU(3)_{L_L} \otimes SU(3)_{E_R} \quad (1.5)$$

Three of the five $U(1)$ subgroups can be identified with the total baryon and lepton number, which are not broken by $L_{\text{Yukawa}}$, and the weak hypercharge, which is gauged and broken only spontaneously by $\langle \phi \rangle \neq 0$. The subgroups controlling flavor-changing dynamics and flavor non-universality are the non-Abelian groups $G_q$ and $G_\ell$, which are explicitly broken by $Y_{d,u,e}$ not being proportional to the identity matrix.

The diagonalization of each Yukawa coupling requires, in general, two independent unitary matrices, $V_L Y V_R^T = \text{diag}(y_1, y_2, y_3)$. In the lepton sector the invariance of $L_{\text{gauge}}$, under $G_\ell$ allows us to freely choose the two matrices necessary to diagonalize $Y_\ell$ without breaking gauge invariance, or without observable consequences. This is not the case in the quark sector, where we can freely choose only three of the four unitary matrices necessary to diagonalize both $Y_d$ and $Y_u$. Choosing the basis where $Y_d$ is diagonal (and eliminating the right-handed diagonalization matrix of $y_u$) we can write

$$Y_d = \lambda_d, \quad Y_u = V^\dagger \lambda_u \quad (1.6)$$

\[1^\text{The notation used to indicate each field is } \psi(A, B)_Y, \text{ where } A \text{ and } B \text{ denote the representation under the } SU(3)_C \text{ and } SU(2)_L \text{ groups, respectively, and } Y \text{ is the } U(1)_Y \text{ charge.} \]
where
\[ \lambda_d = \text{diag}(y_d, y_s, y_b), \quad \lambda_u = \text{diag}(y_u, y_c, y_t), \quad y_q = \frac{m_q}{v}. \] (1.7)

Alternatively we could choose a gauge-invariant basis where \( Y_d = V \lambda_d \) and \( Y_u = \lambda_u \). Since the flavor symmetry do not allow the diagonalization from the left of both \( Y_d \) and \( Y_u \), in both cases we are left with a non-trivial unitary mixing matrix, \( V \), which is nothing but the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix \([7][8]\).

A generic unitary \( 3 \times 3 [N \times N] \) complex unitary matrix depends on three \( [N(N - 1)/2] \) real rotational angles and six \( [N(N + 1)/2] \) complex phases. Having chosen a quark basis where the \( Y_d \) and \( Y_u \) have the form in \([1.6]\) leaves us with a residual invariance under the flavor group which allows us to eliminate five of the six complex phases in \( V \) (the relative phases of the various quark fields). As a result, the physical parameters in \( V \) are four: three real angles and one complex CP-violating phase. The full set of parameters controlling the breaking of the quark flavor symmetry in the SM is composed by the six quark masses in \( \lambda_{u,d} \) and the four parameters of \( V \).

For practical purposes it is often convenient to work in the mass eigenstate basis of both up- and down-type quarks. This can be achieved rotating independently the up and down components of the quark doublet \( Q_L \), or moving the CKM matrix from the Yukawa sector to the charged weak current in \( \mathcal{L}_{\text{SM}}^\text{gauge} \):

\[ J_{W}^{\mu}\big|_{\text{quarks}} = \bar{u}_L \gamma^\mu d_L \quad \rightarrow \quad \bar{u}_L V_{ij} \gamma^\mu d_L. \] (1.8)

However, it must be stressed that \( V \) originates from the Yukawa sector (in particular by the miss-alignment of \( Y_u \) and \( Y_d \) in the \( SU(3)_Q \) subgroup of \( G_q \)): in absence of Yukawa couplings we can always set \( V_{ij} = \delta_{ij} \).

To summarize, quark flavor physics within the SM is characterized by a large flavor symmetry, \( G_q \), defined by the gauge sector, whose only breaking sources are the two Yukawa couplings \( Y_d \) and \( Y_u \). The CKM matrix arises by the miss-alignment of \( Y_u \) and \( Y_d \) in flavor space.

### 2 Some properties of the CKM matrix

The standard parametrization of the CKM matrix \([9]\) in terms of three rotational angles \( (\theta_{ij}) \) and one complex phase \( (\delta) \) is

\[
V = 
\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix} = 
\begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -s_{23}c_{12} - c_{12}s_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix},
\] (1.9)

where \( c_{ij} = \cos \theta_{ij} \) and \( s_{ij} = \sin \theta_{ij} \) (\( i,j = 1, 2, 3 \)).

The off-diagonal elements of the CKM matrix show a strongly hierarchical pattern: \(|V_{us}|\) and \(|V_{cd}|\) are close to 0.22, the elements \(|V_{cb}|\) and \(|V_{ts}|\) are of order \( 4 \times 10^{-2} \) whereas \(|V_{ub}|\) and \(|V_{td}|\) are of order \( 5 \times 10^{-3} \). The Wolfenstein parametrization, namely the expansion of the CKM matrix elements in powers of the small parameter \( \lambda = |V_{us}| \approx 0.22 \), is a convenient way to exhibit this hierarchy in a more explicit way \([10]\):

\[
V = 
\begin{pmatrix}
1 - \lambda^2 & \lambda & \lambda \lambda^3 (\varrho - i\eta) \\
-\lambda & 1 - \lambda^2 & \lambda^2 \\
\lambda^3 (1 - \varrho - i\eta) & -\lambda^3 & 1
\end{pmatrix} + \mathcal{O}(\lambda^4),
\] (1.10)

where \( A, \varrho, \) and \( \eta \) are free parameters of order 1. Because of the smallness of \( \lambda \) and the fact that for each element the expansion parameter is actually \( \lambda^2 \), this is a rapidly converging expansion.
The Wolfenstein parametrization is certainly more transparent than the standard parametrization. However, if one requires sufficient level of accuracy, the terms of $O(\lambda^4)$ and $O(\lambda^5)$ have to be included in phenomenological applications. This can be achieved in many different ways, according to the convention adopted. The simplest (and nowadays commonly adopted) choice is obtained defining the parameters $\{\lambda, A, \varrho, \eta\}$ in terms of the angles of the exact parametrization in Eq. (1.9) as follows:

$$\lambda \doteq s_{12}^\prime , \quad A \lambda^2 \doteq s_{23}^\prime , \quad A \lambda^3 (\varrho - i \eta) \doteq s_{13} e^{-i \delta} .$$

(1.11)

The change of variables $\{s_{ij}, \delta\} \rightarrow \{\lambda, A, \varrho, \eta\}$ in Eq. (1.9) leads to an exact parametrization of the CKM matrix in terms of the Wolfenstein parameters. Expanding this expression up to $O(\lambda^3)$ leads to

$$\begin{align*}
& \begin{pmatrix}
1 - \frac{1}{2} \lambda^2 - \frac{1}{8} \lambda^4 & \lambda + O(\lambda^7) & A \lambda^3 (\varrho - i \eta) \\
-\lambda + \frac{1}{2} A^2 \lambda^5 [1 - 2(\varrho + i \eta)] & 1 - \frac{1}{2} \lambda^2 - \frac{1}{8} \lambda^4 (1 + 4 A^2) & A \lambda^2 + O(\lambda^8) \\
A \lambda^3 (1 - \varrho - i \eta) & -A \lambda^2 + \frac{1}{2} A^2 \lambda^4 [1 - 2(\varrho + i \eta)] & 1 - \frac{1}{2} A^2 \lambda^4
\end{pmatrix}
\end{align*}$$

(1.12)

where

$$\varrho = \varrho (1 - \frac{\lambda^2}{2}) + O(\lambda^4), \quad \eta = \eta (1 - \frac{\lambda^2}{2}) + O(\lambda^4).$$

(1.13)

The advantage of this generalization of the Wolfenstein parametrization is the absence of relevant corrections to $V_{us}$, $V_{cd}$, $V_{ub}$ and $V_{cb}$, and a simple change in $V_{td}$, which facilitate the implementation of experimental constraints.

The unitarity of the CKM matrix implies the following relations between its elements:

$$I) \quad \sum_{k=1\ldots3} V_{ik}^* V_{ki} = 1 , \quad II) \quad \sum_{k=1\ldots3} V_{ik}^* V_{kj} \neq i .$$

(1.14)

These relations are a distinctive feature of the SM, where the CKM matrix is the only source of quark flavor mixing. Their experimental verification is therefore a useful tool to set bounds, or possibly reveal, new sources of flavor symmetry breaking. Among the relations of type II, the one obtained for $i = 1$ and $j = 3$, namely

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

(1.15)

is particularly interesting since it involves the sum of three terms all of the same order in $\lambda$ and is usually represented as a unitarity triangle in the complex plane, as shown in Fig. 1.1. It is worth to stress that Eq. (1.15) is invariant under any phase transformation of the quark fields. Under such transformations the triangle in Fig. 1.1 is rotated in the complex plane, but its angles and the sides remain unchanged. Both angles and sides of the unitary triangle are indeed observable quantities which can be measured in suitable experiments.

3 Present status of CKM fits

The values of $|V_{us}|$ and $|V_{cb}|$, or $\lambda$ and $A$ in the parametrization (1.12), are determined with good accuracy from $K \rightarrow \pi \ell \nu$ and $B \rightarrow X_c \ell \nu$ decays, respectively. According to the recent analysis of the UTfit collaboration [13] their numerical values are

$$\lambda = 0.2259 \pm 0.0006 , \quad A = 0.824 \pm 0.013 .$$

(1.16)

Using these results, all the other constraints on the elements of the CKM matrix can be expressed as constraints on $\varrho$ and $\eta$ (or constraints on the CKM unitarity triangle in Fig. 1.1). The list of the most sensitive observables used to determine $\varrho$ and $\eta$ in the SM includes:
– The rates of inclusive and exclusive charmless semileptonic $B$ decays, that depend on $|V_{ub}|$ and provide a constraint on $\bar{\rho}^2 + \bar{\eta}^2$.
– The time-dependent CP asymmetry in $B \to \psi K_S$ decays ($A_{K\psi}^{CP}$), that depends on the phase of the $B_d$--$\bar{B}_d$ mixing amplitude relative to the decay amplitude (see Sect. 2). Within the SM this translates into a constraint on $\sin 2\beta$.
– The rates of various $B \to D K$ decays constraining the angle $\gamma$ (see Sect. 3).
– The rates of various $B \to \pi\pi, \rho\pi, \rho\rho$ decays constraining the combination $\alpha = \pi - \beta - \gamma$.
– The ratio between the mass splittings in the neutral $B$ and $B_s$ systems, that depends on $|V_{td}/V_{ts}|^2 \propto [(1 - \bar{\rho})^2 + \bar{\eta}^2]$.
– The indirect CP violating parameter of the kaon system ($\epsilon_K$), that determines and hyperbola in the $\bar{\rho}$ and $\bar{\eta}$ plane (see Ref. [3] for more details).

The resulting constraints, as implemented by the CKMfitter collaboration, are shown in Fig. 1.2. As can be seen, they are all consistent with a unique value of $\bar{\rho}$ and $\bar{\eta}$ (the results obtained at present by the two most representative fitter groups, the CKMfitter and the UTfit collaboration, are in good agreement). The numerical values for the best fit values of $\bar{\rho}$ and $\bar{\eta}$ quoted in Ref. [13] are

$$\rho = 0.142 \pm 0.022, \quad \eta = 0.352 \pm 0.016.$$  \hspace{1cm} (1.17)

The consistency of different constraints on the CKM unitarity triangle is a powerful consistency test of the SM in describing flavor-changing phenomena. From the plot in Fig. 1.2 it is quite clear, at least in a qualitative way, that there is little room for non-SM contributions in flavor changing transitions. A more quantitative evaluation of this statement is presented in the next section.

4 The SM as an effective theory

As anticipated in the introduction, despite the impressive phenomenological success of the SM in flavor and electroweak physics, there are various convincing arguments which motivate us to consider this model only as the low-energy limit of a more complete theory.

Assuming that the new degrees of freedom which complete the theory are heavier than the SM particles, we can integrate them out and describe physics beyond the SM in full generality by means of an effective theory approach. The SM Lagrangian becomes the renormalizable part of a more general local Lagrangian which includes an infinite tower of operators with dimension $d > 4$, constructed in terms of SM fields and suppressed by inverse powers of an effective scale $\Lambda$. These operators are the residual effect of having integrated out the new heavy degrees of freedom, whose mass scale is parametrized by the effective scale $\Lambda > m_W$.

As we will discuss in more detail in Sect. 1.1, integrating out heavy degrees of freedom is a procedure often adopted also within the SM: it allows us to simplify the evaluation of amplitudes which
involve different energy scales. This approach is indeed a generalization of the Fermi theory of weak interactions, where the dimension-six four-fermion operators describing weak decays are the results of having integrated out the $W$ field. The only difference when applying this procedure to physics beyond the SM is that in this case, as also in the original work by Fermi, we don’t know the nature of the degrees of freedom we are integrating out. This imply we are not able to determine a priori the values of the effective couplings of the higher-dimensional operators. The advantage of this approach is that it allows us to analyse all realistic extensions of the SM in terms of a limited number of parameters (the coefficients of the higher-dimensional operators). The drawback is the impossibility to establish correlations of New Physics (NP) effects at low and high energies.

Assuming for simplicity that there is a single elementary Higgs field, responsible for the $SU(2)_L \times U(1)_Y \to U(1)_Q$ spontaneous breaking, the Lagrangian of the SM considered as an effective theory can be written as follows

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{gauge}}^{\text{SM}} + \mathcal{L}_{\text{Higgs}}^{\text{SM}} + \mathcal{L}_{\text{Yukawa}}^{\text{SM}} + \Delta \mathcal{L}_{d>4},$$  \hspace{1cm} (1.18)

where $\Delta \mathcal{L}_{d>4}$ denotes the series of higher-dimensional operators invariant under the SM gauge group:

$$\Delta \mathcal{L}_{d>4} = \sum_{d>4} \sum_{n=1}^{N_d} \frac{c_n^{(d)}}{\Lambda^{d-4}} \mathcal{O}_n^{(d)} (\text{SM fields}).$$  \hspace{1cm} (1.19)

If NP appears at the TeV scale, as we expect from the stabilization of the mechanism of electroweak symmetry breaking, the scale $\Lambda$ cannot exceed a few TeV. Moreover, if the underlying theory is natural (no fine-tuning in the coupling constants), we expect $c_n^{(d)} = O(1)$ for all the operators which are not forbidden (or suppressed) by symmetry arguments. The observation that this expectation is not fulfilled.

Fig. 1.2: Allowed region in the $\bar{\rho}, \bar{\eta}$ plane as obtained by the CKMfitter collaboration [12]. Superimposed are the individual constraints from charmless semileptonic $B$ decays ($|V_{ub}|$), mass differences in the $B_d$ ($\Delta m_d$) and $B_s$ ($\Delta m_s$) systems, CP violation in the neutral kaon ($\varepsilon_K$) and in the $B_d$ systems ($\sin 2\beta$), the combined constrains on $\alpha$ and $\gamma$ from various $B$ decays.
by several dimension-six operators contributing to flavor-changing processes is often denoted as the flavor problem.

If the SM Lagrangian were invariant under some flavor symmetry, this problem could easily be circumvented. For instance in the case of barion- or lepton-number violating processes, which are exact symmetries of the SM Lagrangian, we can avoid the tight experimental bounds promoting $B$ and $L$ to be exact symmetries of the new dynamics at the TeV scale. The peculiar aspects of flavor physics is that there is no exact flavor symmetry in the low-energy theory. In this case it is not sufficient to invoke a flavor symmetry for the underlying dynamics. We also need to specify how this symmetry is broken in order to describe the observed low-energy spectrum and, at the same time, be in agreement with the precise experimental tests of flavor-changing processes.

**4.1 Bounds on the scale of New Physics from $\Delta F = 2$ processes**

The best way to quantify the flavor problem is obtained by looking at consistency of the tree- and loop-mediated constraints on the CKM matrix discussed in Sect.3.

In first approximation we can assume that NP effects are negligible in processes which are dominated by tree-level amplitudes. Following this assumption, the values of $|V_{us}|$, $|V_{cb}|$, and $|V_{ub}|$, as well as the constraints on $\alpha$ and $\gamma$ are essentially NP free. As can be seen in Fig.1.2 this implies we can determine completely the CKM matrix assuming generic NP effects in loop-mediated amplitudes. We can then use the measurements of observables which are loop-mediated within the SM to bound the couplings of effective NP operators in Eq. (1.19) which contribute to these observables at the tree level.

The loop-mediated constraints shown in Fig.1.2 are those from the mixing of $B_d$, $B_s$, and $K^0$ with the corresponding anti-particles (generically denoted as $\Delta F = 2$ amplitudes). Within the SM, these amplitudes are generated by box amplitudes of the type in Fig.1.3 (and similarly for $B_s$, and $K^0$) and are affected by small hadronic uncertainties (with the exception of $\Delta m_K$). We will come back to the evaluation of these amplitudes in more detail in Sect.2. For the moment it is sufficient to notice that the leading contribution is obtained with the top-quark running inside the loop, giving rise to the highly suppressed result

\[
M_{\Delta F=2}^{SM} \approx \frac{G_F^2 m_t^2}{16\pi^2} V_{td}^* V_{tj} \langle \bar{M} | (\bar{d}_L^i \gamma^\mu d_L^j)^2 | M \rangle \times F \left( \frac{m_t^2}{m_W^2} \right) \quad [M = K^0, B_d, B_s],
\]

where $F$ is a loop function of order one ($i, j$ denote the flavor indexes of the meson valence quarks).

Magnitude and phase of all these mixing amplitudes have been determined with good accuracy from experiments with the exception of the CP-violating phase in $B_s - \bar{B}_s$ mixing. As shown in Fig.1.2 in all cases where the experimental information is precise, the magnitude of the new-physics amplitude cannot exceed, in size, the SM contribution.

To translate this information into bounds on the scale of new physics, let’s consider the following set of $\Delta F = 2$ dimensions-six operators

\[
\mathcal{O}^{ij}_{\Delta F=2} = (Q^i_L \gamma^\mu Q^j_L)^2, \quad Q^i_L = \left( \begin{array}{c} u^i_L \\ d^i_L \end{array} \right),
\]

\[
1.21
\]
Table 1.1: Bounds on representative dimension-six $\Delta F = 2$ operators, assuming an effective coupling $c_{\text{NP}}/\Lambda^2$. The bounds are quoted on $\Lambda$, setting $|c_{\text{NP}}| = 1$, or on $c_{\text{NP}}$, setting $\Lambda = 1$ TeV. The right column denotes the main observables used to derive these bounds (see next chapter for more details).

| Operator | Bounds on $\Lambda$ in TeV ($c_{\text{NP}} = 1$) | Bounds on $c_{\text{NP}}$ ($\Lambda = 1$ TeV) | Observables |
|----------|---------------------------------|---------------------------------|-------------|
| $(s_L \gamma^\mu d_L)^2$ | $9.8 \times 10^4$ | $9.0 \times 10^{-7}$ | $\Delta m_K; \epsilon_K$ |
| $(s_R d_L)/(s_L d_R)$ | $1.8 \times 10^4$ | $6.9 \times 10^{-9}$ | $\Delta m_K; \epsilon_K$ |
| $(e_L \gamma^\mu u_L)^2$ | $1.2 \times 10^5$ | $5.6 \times 10^{-7}$ | $\Delta m_D; |q/p|, \phi_D$ |
| $(\bar{e}_R u_L)(\bar{e}_L u_R)$ | $6.2 \times 10^3$ | $5.7 \times 10^{-8}$ | $\Delta m_D; |q/p|, \phi_D$ |
| $(b_L \gamma^\mu d_L)/(b_L d_R)$ | $6.6 \times 10^{2}$ | $2.3 \times 10^{-6}$ | $\Delta m_{B_d}; S_{eK_S}$ |
| $(b_L \gamma^\mu s_L)^2$ | $1.4 \times 10^{2}$ | $5.0 \times 10^{-8}$ | $\Delta m_{B_d}; S_{eK_S}$ |
| $(\bar{b}_R s_L)/(\bar{b}_L s_R)$ | $4.8 \times 10^{2}$ | $8.8 \times 10^{-6}$ | $\Delta m_{B_s}; S_{e\phi}$ |

where $i, j$ are flavor indexes in the basis defined by Eq. (1.6). These operators contribute at the tree-level to the meson-antimeson mixing amplitudes. Denoting $c_{ij}$ the couplings of the non-standard operators in $\Lambda$, the condition $|M_{\Delta F=2}^{\text{NP}}| < |M_{\Delta F=2}^{\text{SM}}|$ implies

$$\Lambda < \frac{3.4 \text{ TeV}}{|V_{3i}^* V_{3j}|/|c_{ij}|^{1/2}} \begin{cases} 9 \times 10^3 \text{ TeV} \times |c_{21}|^{1/2} & \text{from } K^0 - \bar{K}^0 \\ 4 \times 10^2 \text{ TeV} \times |c_{31}|^{1/2} & \text{from } B_d - \bar{B}_d \\ 7 \times 10^1 \text{ TeV} \times |c_{32}|^{1/2} & \text{from } B_s - \bar{B}_s \end{cases}$$

(1.22)

A more refined analysis, with complete statistical treatment and separate bounds for the real and the imaginary parts of the various amplitudes, considering also operators with different Dirac structure, is reported in Table 1.1. The main messages of these bounds are the following:

- New physics models with a generic flavor structure ($c_{ij}$ of order 1) at the TeV scale are ruled out. If we want to keep $\Lambda$ in the TeV range, physics beyond the SM must have a highly non-generic flavor structure.
- In the specific case of the $\Delta F = 2$ operators in (1.21), in order to keep $\Lambda$ in the TeV range, we must find a symmetry argument such that $|c_{ij}| \lesssim |V_{3i}^* V_{3j}|^2$.

The strong constraining power of $\Delta F = 2$ observables is a consequence of their strong suppression within the SM. They are suppressed not only by the typical $1/(4\pi)^2$ factor of loop amplitudes, but also by the GIM mechanism [14] and by the hierarchy of the CKM matrix ($|V_{3i}| \ll 1$, for $i \neq 3$). Reproducing a similar structure beyond the SM is a highly non-trivial task. As we will discuss in the last lecture, only in a few cases this can be implemented in a natural way.

To conclude, we stress that the good agreement of SM and experiments for $B_d$ and $K^0$ mixing does not imply that further studies of flavor physics are not interesting. On the one hand, even for $|c_{ij}| \approx |V_{3i}^* V_{3j}|$, which can be considered the most pessimistic case, as we will discuss in Sect. 1, we are presently constraining physics at the TeV scale. Therefore improving these bounds, if possible, would be extremely valuable. One the other hand, as we will discuss in the next lecture, there are various interesting observables which have not been deeply investigated yet, whose study could reveal additional key features about the flavor structure of physics beyond the SM.
Chapter 2

Phenomenology of $B$ and $D$ decays

As we have seen in the previous lecture, the exploration of the mechanism of quark-flavor mixing is entered in a new era. The precise measurements of mixing-induced CP violation and tree-level allowed semileptonic transition have provided an important consistency check of the SM, and a precise determination of the Cabibbo-Kobayashi-Maskawa matrix. The next goal is to understand if there is still room for new physics or, more precisely, if there is still room for new sources of flavor symmetry breaking close to the electroweak scale. From this perspective, the meson-antimeson mixing amplitudes, CP-violating observables, and the rates to few rare $B$ decays mediated by flavor-changing neutral-current (FCNC) represent a fundamental tool.

Beside the experimental sensitivity, the conditions which allow us to perform significant NP searches in rare decays can be summarized as follows: i) decay amplitude dominated by electroweak dynamics, and thus enhanced sensitivity to non-standard contributions; ii) small theoretical error within the SM, or good control of both perturbative and non-perturbative corrections.

In this lecture we first we introduce the main theoretical tools that allow us to evaluate at which level these two conditions are satisfied in a given observable. We then apply these tools to analyse in more detail a few selected observables: i) the determination of the CP-violating phase of the $B_s$ mixing amplitude; ii) the determination of the CKM phase $\gamma$ from charged $B \rightarrow D K$ decays; iii) the rare decays $B_{s,d} \rightarrow \ell^+ \ell^-$; iv) CP violation in $D$ decays. The selection is far from being exhaustive (for a more complete analysis we refer to the reviews in Ref. [3, 4]), but it should serve as an illustration of the interesting potential of $B$ and $D$ physics at hadron colliders.

1 Theoretical tools

1.1 Low-energy effective Lagrangians

The decays of $B$ mesons are processes which involve at least two different energy scales: the electroweak scale, characterized by the $W$ boson mass, which determines the flavor-changing transition at the quark level, and the scale of strong interactions $\Lambda_{\text{QCD}}$, related to the hadron formation. The presence of these two widely separated scales makes the calculation of the decay amplitudes starting from the full SM Lagrangian quite complicated: large logarithms of the type $\log(m_W/\Lambda_{\text{QCD}})$ may appear, leading to a breakdown of ordinary perturbation theory.

This problem can be substantially simplified by integrating out the heavy SM fields ($W$ and $Z$ bosons, as well as the top quark) at the electroweak scale, and constructing an appropriate low-energy effective theory where only the light SM fields appear. The weak effective Lagrangians thus obtained contains local operators of dimension six (and higher), written in terms of light SM fermions, photon and gluon fields, suppressed by inverse powers of the $W$ mass.

To be concrete, let’s consider the example of charged-current semileptonic weak interactions. The basic building block in the full SM Lagrangian is

$$\mathcal{L}_{\text{full SM}}^{W} = \frac{g}{\sqrt{2}} J_\mu^W(x) W^{\mu}_+(x) + \text{h.c.}, \quad (2.1)$$

where

$$J_\mu^W(x) = V_{ij} \bar{u}_L^i(x) \gamma^\mu d_L^j(x) + \bar{e}_L^j(x) \gamma^\mu \nu_L^i(x) \quad (2.2)$$

is the weak charged current already introduced in Eq. (1.8). Integrating out the $W$ field at the tree level
we contract two vertexes of this type generating the non-local transition amplitude

\[ i\mathcal{T} = -i \frac{g_2^2}{2} \int d^4x D_{\mu\nu}(x, m_W) T \left[ J^\mu_W(x), J^{\nu\dagger}_W(0) \right], \]  

(2.3)

which involves only light fields. Here \( D_{\mu\nu}(x, m_W) \) is the \( W \) propagator in coordinate space: expanding it in inverse powers of \( m_W \),

\[ D_{\mu\nu}(x, m_W) = \int \frac{d^4q}{(2\pi)^4} e^{-iq \cdot x} - i g_{\mu\nu} + \mathcal{O}(q^2/m^2_W + i\varepsilon) = \delta(x) \frac{i g_{\mu\nu}}{m^2_W} + \ldots, \]  

(2.4)

the leading contribution to \( \mathcal{T} \) can be interpreted as the tree-level contribution of the following effective local Lagrangian

\[ \mathcal{L}^{(0)}_{\text{eff}} = -4G_F \sqrt{2} \sum_i C^i(\mu) Q_i(x, j^\mu_W(x), j^{\nu\dagger}_W(x)), \]  

(2.5)

where \( G_F/\sqrt{2} = g^2/(8m^2_W) \) is the Fermi coupling. If we select in the product of the two currents one quark and one leptonic current,

\[ \mathcal{L}^{\text{semi-lept}}_{\text{eff}} = -4G_F \sqrt{2} V_{ij} \bar{u}^i_L(x) \gamma^\mu d^i_L(x) \bar{\nu}_L(x) \gamma_\mu e_L(x) + \text{h.c.}, \]  

(2.6)

we obtain an effective Lagrangian which provides an excellent description of semileptonic weak decays. The neglected terms in the expansion (2.4) correspond to corrections of \( \mathcal{O}(m^2_B/m^2_W) \) to the decay amplitudes. In principle, these corrections could be taken into account by adding appropriate dimension-eight operators in the effective Lagrangian. However, in most cases they are safely negligible.

The case of charged semileptonic decays is particularly simple since we can ignore QCD effects: the operator (2.6) is not renormalized by strong interactions. The situation is slightly more complicated in the case of non-leptonic or flavor-changing neutral-current processes, where QCD corrections and higher-order weak interactions cannot be neglected, but the basic strategy is the same. First of all we need to identify a complete basis of local operators, that includes also those generated beyond the tree level. In general, given a fixed order in the \( 1/m^2_W \) expansion of the amplitudes, we need to consider all operators of corresponding dimension (e.g. dimension six at the first order in the \( 1/m^2_W \) expansion) compatible with the symmetries of the system. Then we must introduce an artificial scale in the problem, the renormalization scale \( \mu \), which is needed to regularize QCD (or QED) corrections in the effective theory.

The effective Lagrangian for generic \( \Delta F = 1 \) processes assumes the form

\[ \mathcal{L}_{\Delta F=1} = -4G_F \sqrt{2} \sum_i C^i(\mu) Q_i, \]  

(2.7)

where the sum runs over the complete basis of operators. As explicitly indicated, the effective couplings \( C^i(\mu) \) (known as Wilson coefficients) depend, in general, on the renormalization scale. The dependence from this scale cancels when evaluating the matrix elements of the effective Lagrangian for physical processes, that we can generically indicate as

\[ \mathcal{M}(i \rightarrow f) = -4G_F \sqrt{2} \sum_i C^i(\mu) \langle f|Q_i(\mu)|i \rangle. \]  

(2.8)

The independence of \( \mathcal{M} \) from \( \mu \) holds for any initial and final state, including partonic states at high energies. This implies that the \( C^i(\mu) \) obey a series of renormalization group equations (RGE), whose structure is completely determined by the anomalous dimensions of the effective operators. These equations can be solved using standard RG techniques, allowing the resummation of all large logs of the type
The scale $\mu$ acts as a separator of short- and long-distance virtual corrections: short-distance effects are included in the $C_i(\mu)$, whereas long-distance effects are left as explicit degrees of freedom in the effective theory.[3]

In practice, the problem reduces to the following three well-defined and independent steps:

1. the evaluation of the initial conditions of the $C_i(\mu)$ at the electroweak scale ($\mu \approx m_W$);
2. the evaluation of the anomalous dimension of the effective operators, and the corresponding RGE evolution of the $C_i(\mu)$ from the electroweak scale down to the energy scale of the physical process ($\mu \approx m_B$);
3. the evaluation of the matrix elements of the effective Lagrangian for the physical hadronic processes (which involve energy scales from $m_B$ down to $\Lambda_{QCD}$).

The first step is the one where physics beyond the SM may appear: if we assume NP is heavy, it may modify the initial conditions of the Wilson coefficients at the high scale, while it cannot affect the following two steps. While the RGE evolution and the hadronic matrix elements are not directly related to NP, they may influence the sensitivity to NP of physical observables. In particular, the evaluation of hadronic matrix elements is potentially affected by non-perturbative QCD effects: these are often a large source of theoretical uncertainty which can obscure NP effects. RGE effects do not induce sizable uncertainties since they can be fully handled within perturbative QCD; however, the sizable logs generated by the RGE running may dilute the interesting short-distance information encoded in the value of the Wilson coefficients at the high scale. As we will discuss in the following, only in specific observables these two effects are small and under good theoretical control.

A deeper discussion about the construction of low-energy effective Lagrangians, with a detailed discussions of the first two steps mentioned above, can be found in Ref. [15].

### 1.1.1 Effective operators for rare processes

Let’s give a closer look to processes where the underlying parton process is $b \to s + \bar{q}q$. In this case the relevant effective Lagrangian can be written as

$$ L_{b \to s}^{non-lept} = -4 \frac{G_F}{\sqrt{2}} \sum_{q=u,c} \lambda_q^s \sum_{i=1,2} C_i(\mu)Q_i^s(\mu) - \lambda_q^{10} \sum_{i=3}^{10} C_i(\mu)Q_i(\mu) \bigg) \bigg) ,$$

where $\lambda_q^s = V_{qb}^s V_{qs}^*$, and the operator basis is

$$ Q_1^s = \frac{b}{2} \gamma^\mu q^\beta L \gamma^\mu q^\beta L, \quad Q_2^s = \frac{b}{2} \gamma^\mu q^\beta L \gamma^\mu q^\beta L, $$

$$ Q_3 = b \gamma^\mu q^\beta L \gamma^\mu q^\beta L, \quad Q_4 = b \gamma^\mu q^\beta L \gamma^\mu q^\beta L, $$

$$ Q_5 = \frac{3}{2} b \gamma^\mu q^\beta L \gamma^\mu q^\beta L, \quad Q_6 = \frac{3}{2} b \gamma^\mu q^\beta L \gamma^\mu q^\beta L, $$

$$ Q_7 = \frac{3}{2} b \gamma^\mu q^\beta L \gamma^\mu q^\beta L, \quad Q_8 = \frac{3}{2} b \gamma^\mu q^\beta L \gamma^\mu q^\beta L, $$

$$ Q_9 = \frac{3}{2} b \gamma^\mu q^\beta L \gamma^\mu q^\beta L, \quad Q_{10} = \frac{3}{2} b \gamma^\mu q^\beta L \gamma^\mu q^\beta L, $$

with $\{\alpha, \beta\}$ and $e_q$ denoting color indexes the electric charge of the quark $q$, respectively.

Out of these operators, only $Q_1^s$ and $Q_2^s$ are generated at the tree-level by the $W$ exchange. Indeed, comparing with the tree-level structure in (2.5), we find

$$ C_{1-5}^{u,c}(m_W) = 1 + \mathcal{O}(\alpha_s, \alpha), \quad C_{2-10}^{u,c}(m_W) = 0 + \mathcal{O}(\alpha_s, \alpha). $$

However, after including RGE effects and running down to $\mu \approx m_b$, both $C_{1-5}^{u,c}$ and $C_{2-10}^{u,c}$ become $\mathcal{O}(1)$, while $C_{3-6}$ become $\mathcal{O}(\alpha_s(m_b))$. In all these cases there is little hope to identify NP effects: the leading

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[1] This statement would be correct if the theory were regularized using a dimensional cut-off. It is not fully correct if $\mu$ is the scale appearing in the (often adopted) dimensional-regularization + minimal-subtraction (MS) renormalization scheme.
initial condition is the tree-level $W$ exchange, which is hardly modified by NP. In principle, the coefficients of the electroweak penguin operators, $Q_7$–$Q_{10}$, are more interesting: their initial conditions are related to electroweak penguin and box diagrams. However, it is hard to distinguish their contribution from those of the other four-quark operators in non-leptonic processes. Moreover, also for $C_{7–10}$ the relative contribution from long-distance physics (running down from $m_W$ to $m_b$) is sizable and dilute the interesting short-distance information.

For $b \rightarrow s$ transitions with a photon or a lepton pair in the final state, additional dimension-six operators must be included in the basis,

$$\mathcal{L}_{b \rightarrow s}^{\text{are}} = \mathcal{L}_{b \rightarrow s}^{\text{non-lept}} + 4 \frac{G_F}{\sqrt{2}} \lambda_i^2 \left( C_{7i} Q_{7i} + C_{8g} Q_{8g} + C_{9V} Q_{9V} + C_{10A} Q_{10A} \right),$$

where

$$Q_{7i} = \frac{e}{16 \pi^2} m_b \bar{b}_i R \sigma^{\mu \nu} F_{\mu \nu} s_L^i, \quad Q_{8g} = \frac{g_s}{16 \pi^2} m_b \bar{b}_R \sigma^{\mu \nu} G^{A}_{\mu \nu} T_A s_L^i, \quad Q_{9V} = \frac{1}{2} \bar{b}_i L T^i L \bar{l}_\mu \gamma \mu l, \quad Q_{10A} = \frac{1}{2} \bar{b}_i L T^i L \bar{l}_\mu \gamma \mu_5 l,$$

and $G^{A}_{\mu \nu}$ ($F_{\mu \nu}$) is the gluon (photon) field strength tensor. The initial conditions of these operators are particularly sensitive to NP: within the SM they are generated by one-loop penguin and box diagrams dominated by the top-quark exchange. The most theoretically clean is $C_{10A}$, which do not mix with any of the four-quark operators listed above and which has a vanishing anomalous dimension:

$$C_{10A}^{\text{SM}}(m_W) = \frac{g_s^2}{8 \pi^2} \left( \frac{4 - x_t}{1 - x_t} + \frac{3 x_t}{(1 - x_t)^2} \ln x_t \right), \quad x_t = \frac{m_t^2}{m_W^2}. \tag{2.14}$$

NP effects at the TeV scale could easily modify this result, and this deviation would directly show up in low-energy observables sensitive to $C_{10A}$, such as $A_{FB}(B \rightarrow K^* \ell^+ \ell^-)$ and $B(B \rightarrow \ell^+ \ell^-)$ (see Sects. 4.1 and 4.2). We finally note that while the operators in Eqs. (2.10) and (2.13) form a complete basis within the SM, this is not necessarily the case beyond the SM. In particular, within specific scenarios also right-handed current operators (e.g. those obtained from (2.13) for $q_{L(R)} \rightarrow q_{R(L)}$) may appear.

1.1.2 Effective operators for meson-antimeson mixing

The $\Delta F = 2$ effective weak Lagrangians are simpler than the $\Delta F = 1$ ones: the SM operator basis includes one operator only. The Lagrangian relevant for $B^0_d - \bar{B}^0_d$ and $B^0_s - \bar{B}^0_s$ mixing is conventionally written as ($q = \{d, s\}$):

$$\mathcal{L}_{\Delta B=2}^{\text{SM}} = \frac{G_F}{4 \pi^2} m_W^2 (V_{tb}^* V_{bd})^2 \eta_B(\mu) S_0(x_t) \left( \bar{b}_L \gamma_\mu q_L \bar{b}_L \gamma_\mu q_L \right), \tag{2.15}$$

where the initial condition of the Wilson coefficient is the loop function $S_0(x_t)$, corresponding to the box diagrams in Fig. 1.3. The effect of QCD correction is only a multiplicative correction factor, $\eta_B(\mu)$, which can be computed with high accuracy and turns out to be of order one. The explicit expression of the loop function, dominated by the top-quark exchange, is

$$S_0(x_t) = \frac{4 x_t - 11 x_t^2 + x_t^3}{4(1 - x_t)^2} - \frac{3 x_t^3 \ln x_t}{2(1 - x_t)^3}. \tag{2.16}$$

1.2 The gauge-less limit of FCNC amplitudes

An interesting aspect which is common to the electroweak loop functions in Eqs. (2.14) and (2.16) is the fact they diverge in the limit $m_t/m_W \rightarrow \infty$. This behavior is apparently strange: it contradicts the
Fig. 2.1: One-loop contributions $\Delta F = 2$ amplitudes in the gaugeless limit.

expectation that contributions of heavy particles at low energy decouple in the limit where their masses increase. The origin of this effect can be understand by noting that the leading contributions to both amplitudes are generated only by the Yukawa interaction. These contributions can be better isolated in the gaugeless limit of the SM, i.e. if we send to zero the gauge couplings. In this limit $m_W \to 0$ and the derivation of the effective Lagrangian discussed in Sect. 1.1 does not make sense. However, the leading contributions to the effective Lagrangians for $\Delta F = 2$ and rare decays are unaffected. Indeed, the leading contributions to these processes are generated by Yukawa interactions of the type in Fig. 2.1, where the scalar fields are the Goldstone-bosons components of the Higgs field (which are not eaten up by the $W$ in the limit $g \to 0$). Since the top is still heavy, we can integrate it out, obtaining the following result for $\mathcal{L}_{\Delta B=2}$:

$$
\mathcal{L}^{\text{SM}}_{\Delta B=2}\bigg|_{g_i \to 0} = \frac{G_F^2 m_t^2}{16\pi^2} (V^*_{tb} V_{tq})^2 (\bar{b}_L \gamma_\mu q_L)^2 = \frac{[Y_u Y_u^*]_{bq}^2}{128\pi^2 m_t^2} (\bar{b}_L \gamma_\mu q_L)^2.
$$

Taking into account that $S_0(x) \to x/4$ for $x \to \infty$, it is easy to verify that this result is equivalent to the one in Eq. (2.16) in the large $m_t$ limit. A similar structure holds for the $\Delta F = 1$ amplitude contributing to the axial operator $Q_{10A}$.

The last expression in Eq. (2.17), which holds in the limit where we neglect the charm Yukawa coupling, shows that the decoupling of the amplitude with the mass of the top is compensated by four powers of the top Yukawa coupling at the numerator. The divergence for $m_t \to \infty$ can thus be understood as the divergence of one of the fundamental couplings of the theory. Note also that in the gaugeless limit there is no GIM mechanism: the contributions of the various up-type quarks inside the loops do not cancel each other: they are directly weighted by the corresponding Yukawa couplings, and this is why the top-quark contribution is the dominant one.

This exercise illustrates the key role of the Yukawa coupling in determining the main properties flavor physics within the SM, as advertised in the first lecture. It also illustrates the interplay of flavor and electroweak symmetry breaking in determining the structure of short-distance dominated flavor-changing processes in the SM.

1.3 Hadronic matrix elements

As anticipated, all non-perturbative effects are confined in the hadronic matrix elements of the operators of the effective Lagrangians. As far as the evaluation of the matrix elements is concerned, we can divide $B$-physics observables in three main categories: i) inclusive decays, ii) one-hadron final states, iii) multi-hadron processes.

The heavy-quark expansion [16] form a solid theoretical framework to evaluate the hadronic matrix elements for inclusive processes: inclusive hadronic rates are related to those of free $b$ quarks, calculable in perturbation theory, by means of a systematic expansion in inverse powers of $\Lambda_{QCD}/m_b$. Thanks to quark-hadron duality, the lowest-order terms in this expansion are the pure partonic rates, and for sufficiently inclusive observables higher-order corrections are usually very small. This technique has been very successful in the past in the case of charged-current semileptonic decays, as well as $B \to X_s \gamma$. 

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However, it has a limited domain of applicability, due to the difficulty of selecting and reconstructing hadronic inclusive states. It cannot be used at hadronic machines, and even at $B$ factories it cannot be applied to very rare decays.

For processes with a single hadron in the final state, the hadronic effects are often (although not always) confined to the matrix elements of a single quark current. These can be expressed in terms of the meson decay constants

$$\langle 0|b\gamma_\mu\gamma_5q|B_q(p)\rangle = ip_\mu F_{B_q},$$

(2.18)

or appropriate $B \to H$ hadronic form factors. Lattice QCD is the best tool to evaluate these non-perturbative quantities from first principles, at least in the kinematical region where the form factors are real (no re-scattering phase allowed). At present not all the form-factors relevant for $B$-physics phenomenology are computed on the lattice with good accuracy, but the field is evolving rapidly (see Ref. [17,18]). To this category belong also the so-called bag-parameters for $\Delta B = 2$ mixing, $B_{d,s}$, defined by

$$\eta_B(\mu)\langle \hat{B}_q|(bL\gamma_\mu q_L)^2|B_q\rangle = \frac{2}{3}f^2_{\hat{B}_q}m^3_{\hat{B}_q}\eta_B(\mu)B_q(\mu) = \frac{2}{3}f^2_{\hat{B}_q}m^3_{\hat{B}_q}\hat{\eta}_B\hat{B}_q,$$

(2.19)

where both $\hat{B}_q$ and $\hat{\eta}_B$ are scale-independent quantities ($\hat{\eta}_B = 0.55 \pm 0.01$). For later convenience, we report here some lattice averages for meson decay constants and bag parameters$^2$

$$F_{B_s} = 227 \pm 8 \text{ GeV}, \quad \hat{B}_s = 1.22 \pm 0.12,$$

(2.20)

$$F_{B_d} = 189 \pm 8 \text{ GeV}, \quad \frac{\hat{B}_{B_s}}{\hat{B}_{B_d}} = 1.00 \pm 0.03.$$

(2.21)

As can be seen, the meson decay constants have errors below the 5% level. For the bag parameters the absolute errors are still at the 10% level, but the error drops to 3% in the ratio, that is sensitive to $SU(3)$ breaking effects only. This is why the ratio $\Delta m_{B_d}/\Delta m_{B_s}$ gives more significant constraint in Fig. 1.1 with respect to $\Delta m_{B_d}$ only.

The last class of hadronic matrix elements is the one of multi-hadron final states, such as the two-body non-leptonic decays $B \to \pi\pi$ and $B \to K\pi$, as well as many other processes with more than one hadron in the final state. These are the most difficult ones to be estimated from first principles with high accuracy. A lot of progress in the recent pass has been achieved thanks to QCD factorization [21] and the SCET [22] approaches, which provide factorization formulae to relate these hadronic matrix elements to two-body hadronic form factors in the large $m_b$ limit. However, it is fair to say that the errors associated to the $\Lambda_{QCD}/m_b$ corrections are still quite large. This subject is quite interesting by itself, but is beyond the scope of these lectures, where we focus on clean $B$-physics observables for NP studies. To this purpose, the only interesting non-leptonic channels are those where, with suitable ratios, or using $SU(2)$ relations among hadronic matrix elements, we can eliminate completely all hadronic unknowns. Examples of this type are the $B \to DK$ channels discussed in Sect. 3.

2 Time evolution of neutral mesons

The non vanishing amplitude mixing the quasi-stable neutral pseudoscalar mesons ($M^0 \equiv B^0_s, B^0_d, D^0, \text{ or } K^0$) with the corresponding anti mesons induces a time-dependent oscillations between these states. An initially produced $M^0$ or $\bar{M}^0$ evolves in time into a superposition of $M^0$ and $\bar{M}^0$.

For the sake of simplicity, let’s concentrate on the case of $B$ mesons. Denoting by $|B^0(t)\rangle$ (or $|\bar{B}^0(t)\rangle$) the state vector of a $B$ meson which is tagged as a $B^0$ (or $\bar{B}^0$) at time $t = 0$, the time evolution

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$^2$The values for the meson decay constants are from Ref. [17], with the conservative error estimate discussed in [20]. The results for the bag parameters are from [19].
of these states is governed by the following equation:

\[
\frac{d}{dt} \begin{pmatrix} |B(t)\rangle \\ |\bar{B}(t)\rangle \end{pmatrix} = \left( M - i \frac{\Gamma}{2} \right) \begin{pmatrix} |B(t)\rangle \\ |\bar{B}(t)\rangle \end{pmatrix},
\]

(2.22)

where the mass-matrix \( M \) and the decay-matrix \( \Gamma \) are \( t \)-independent, Hermitian \( 2 \times 2 \) matrices. CPT invariance implies that \( M_{11} = M_{22} \) and \( \Gamma_{11} = \Gamma_{22} \), while the off-diagonal element \( M_{12} = M_{21} \) is the one we can compute using the effective Lagrangian \( \mathcal{L}_{\Delta B=2} \).

The mass eigenstates are the eigenvectors of \( M - i \Gamma/2 \). We express them in terms of the flavor eigenstates as:

\[
|B_L\rangle = p|B^0\rangle + q|\bar{B}^0\rangle, \quad |B_H\rangle = p|B^0\rangle - q|\bar{B}^0\rangle,
\]

(2.23)

with \( |p|^2 + |q|^2 = 1 \). Note that, in general, \( |B_L\rangle \) and \( |B_H\rangle \) are not orthogonal to each other. The time evolution of the mass eigenstates is governed by the two eigenvalues \( M_H - i \Gamma_H/2 \) and \( M_L - i \Gamma_L/2 \):

\[
|B_{H,L}(t)\rangle = e^{-i(M_{H,L} - \Gamma_{H,L}/2)t} |B_{H,L}(t = 0)\rangle.
\]

(2.24)

For later convenience it is also useful to define

\[
m = \frac{M_H + M_L}{2}, \quad \Gamma = \frac{\Gamma_L + \Gamma_H}{2}, \quad \Delta m = M_H - M_L, \quad \Delta \Gamma = \Gamma_L - \Gamma_H.
\]

(2.25)

With these conventions the time evolution of initially tagged \( B^0 \) or \( \bar{B}^0 \) states is:

\[
|B^0(t)\rangle = e^{-\im m t} e^{-\Gamma t/2} \left[ f_+(t) |B^0\rangle + \frac{q}{p} f_-(t) |\bar{B}^0\rangle \right],
\]

|\bar{B}^0(t)\rangle = e^{-\im m t} e^{-\Gamma t/2} \left[ \frac{p}{q} f_-(t) |B^0\rangle + f_+(t) |\bar{B}^0\rangle \right],
\]

(2.26)

where

\[
f_+(t) = \cosh \frac{\Delta \Gamma t}{4} \cos \frac{\Delta m t}{2} - \im \sinh \frac{\Delta \Gamma t}{4} \sin \frac{\Delta m t}{2},
\]

\[
f_-(t) = -\sinh \frac{\Delta \Gamma t}{4} \cos \frac{\Delta m t}{2} + \im \cosh \frac{\Delta \Gamma t}{4} \sin \frac{\Delta m t}{2},
\]

(2.27)

(2.28)

In both \( B_s \) and \( B_d \) systems the following hierarchies holds: \( |\Gamma_{12}| \ll |M_{12}| \) and \( \Delta \Gamma \ll \Delta m \).

They are experimentally verified and can be traced back to the fact that \( |\Gamma_{12}| \) is a genuine long-distance \( \mathcal{O}(G_F^2) \) effect (it is indeed related to the absorptive part of the box diagrams in Fig. 1.3) which do not share the large \( m_t \) enhancement of \( |M_{12}| \) (which is a short-distance dominated quantity). Taking into account this hierarchy leads to the following approximate expressions for the quantities appearing in the time-evolution formulae in terms of \( M_{12} \) and \( \Gamma_{12} \):

\[
\Delta m = 2 |M_{12}| \left[ 1 + \mathcal{O} \left( \frac{\Gamma_{12}}{M_{12}} \right)^2 \right],
\]

(2.29)

\[
\Delta \Gamma = 2 |\Gamma_{12}| \cos \phi \left[ 1 + \mathcal{O} \left( \frac{\Gamma_{12}}{M_{12}} \right)^2 \right],
\]

(2.30)

\[
\frac{q}{p} = -e^{-i\phi_B} \left[ 1 - \frac{1}{2} \frac{\Gamma_{12}}{M_{12}} \sin \phi + \mathcal{O} \left( \frac{\Gamma_{12}}{M_{12}} \right)^2 \right],
\]

(2.31)

where \( \phi = \text{arg}(-M_{12}/\Gamma_{12}) \) and \( \phi_B \) is the phase of \( M_{12} \). Note that \( \phi_B \) thus defined is not measurable and depends on the phase convention adopted, while \( \phi \) is a phase-convention quantity which can be measured in experiments.
Taking into account the above results, the time-dependent decay rates of an initially tagged $B^0$ or $\bar{B}^0$ state into some final state $f$ can be written as

$$
\Gamma[B^0(t = 0) \rightarrow f(t)] = N_0 |A_f|^2 e^{-\Gamma t} \left\{ \frac{1 + |\lambda_f|^2}{2} \cosh \frac{\Delta \Gamma t}{2} + \frac{1 - |\lambda_f|^2}{2} \cos(\Delta mt) - \text{Re} \lambda_f \sin \frac{\Delta \Gamma t}{2} - \text{Im} \lambda_f \sin(\Delta mt) \right\},
$$

$$
\Gamma[\bar{B}^0(t = 0) \rightarrow f(t)] = N_0 |A_f|^2 \left\{ 1 + \left| \frac{\Gamma_{12}}{M_{12}} \sin \phi \right| e^{-\Gamma t} \left\{ \frac{1 + |\lambda_f|^2}{2} \cosh \frac{\Delta \Gamma t}{2} - \frac{1 - |\lambda_f|^2}{2} \cos(\Delta mt) - \text{Re} \lambda_f \sin \frac{\Delta \Gamma t}{2} + \text{Im} \lambda_f \sin(\Delta mt) \right\},
$$

where $N_0$ is the flux normalization and, following the standard notation, we have defined

$$
\lambda_f = \frac{q}{p} \frac{\bar{A}_f}{A_f} \approx -e^{-i\phi_B} \frac{A_f}{\bar{A}_f} \left[ 1 - \frac{1}{2} \left| \frac{\Gamma_{12}}{M_{12}} \right| \sin \phi \right] \tag{2.32}
$$

in terms of the decay amplitudes

$$
A_f = \langle f | \mathcal{L}_{\Delta F=1} | B^0 \rangle, \quad \bar{A}_f = \langle f | \mathcal{L}_{\Delta F=-1} | \bar{B}^0 \rangle. \tag{2.33}
$$

From the above expressions it is clear that the key quantity we can access experimentally in the time-dependent study of $B$ decays is the combination $\lambda_f$. Both real and imaginary parts of $\lambda_f$ can be measured, and indeed this is a phase-convention independent quantity: the phase convention in $\phi_B$ is compensated by the phase convention in the decay amplitudes. In other words, what we can measure is the weak-phase difference between $M_{12}$ and the decay amplitudes.

For generic final states, $\lambda_f$ is a quantity that is difficult to evaluate. However, it becomes particularly simple in the case where $f$ is a CP eigenstate, $\text{CP}|f\rangle = \eta_f |f\rangle$, and the weak phase of the decaying amplitude is known. In such case $\bar{A}_f/A_f$ is a pure phase factor ($|\bar{A}_f/A_f| = 1$), determined by the weak phase of the decaying amplitude:

$$
\lambda_f |_{\text{CP-eigen.}} = \eta_f \frac{q}{p} e^{-2i\phi_A}, \quad A_f = |A_f| e^{i\phi_A}, \quad \eta_f = \pm 1. \tag{2.34}
$$

The most clean example of this type of channels is the $|\psi K_S\rangle$ final state for $B_d$ decays. In this case the final state is a CP eigenstate and the decay amplitude is real (to a very good approximation) in the standard CKM phase convention. Indeed the underlying partonic transition is dominated by the Cabibbo-allowed tree-level process $b \rightarrow c\bar{c}s$, which has a vanishing phase in the standard CKM phase convention, and also the leading one-loop corrections (top-quark penguins) have the same vanishing weak phase. Since in the $B_d$ system we can safely neglect $\Gamma_{12}/M_{12}$, this implies

$$
\lambda_{\psi K_S}^{B_d} = -e^{-i\phi_{B_d}}, \quad \text{Im} \left( \lambda_{\psi K_s}^{B_d} \right)_{\text{SM}} = \sin(2\beta), \tag{2.35}
$$

where the SM expression of $\phi_{B_d}$ is nothing but the phase of the CKM combination $(V_{tb}^* V_{td})^2$ appearing in Eq. (2.15). Given the smallness of $\Delta \Gamma_B$, this quantity is easily extracted by the ratio

$$
\frac{\Gamma[\bar{B}_d(t = 0) \rightarrow \psi K_s(t)] - \Gamma[B^0(t = 0) \rightarrow f \psi K_s(t)]}{\Gamma[B^0(t = 0) \rightarrow \psi K_s(t)] + \Gamma[B^0(t = 0) \rightarrow f \psi K_s(t)]} \approx \text{Im} \left( \lambda_{\psi K_s}^{B_d} \right) \sin(\Delta m_{B_d} t),
$$

which can be considered the golden measurement of $B$ factories.
Another class of interesting final states are CP-conjugate channels $|f\rangle$ and $|\bar{f}\rangle$ which are accessible only to $B^0$ or $B^{\ast 0}$ states, such that $|A_f\rangle = |A_{\bar{f}}\rangle$ and $A_f = \bar{A}_{\bar{f}} = 0$. Typical examples of this type are the charged semileptonic channels. In this case the asymmetry

$$\frac{\Gamma[B^0(t = 0) \to f(t)] - \Gamma[B^0(t = 0) \to \bar{f}(t)]}{\Gamma[B^0(t = 0) \to f(t)] + \Gamma[B^0(t = 0) \to \bar{f}(t)]} = \left| \frac{\Gamma_{12}}{M_{12}} \right| \sin \phi \left[ 1 + \mathcal{O} \left( \frac{\Gamma_{12}}{M_{12}} \right) \right]$$

turns out to be time-independent and a clean way to determine the indirect CP-violating phase $\phi$.

\[1\] CP violation in $B_s$ mixing

Till very recently the CP violating phase of $B_s$–$\bar{B}_s$ mixing was the last missing ingredient of down-type $\Delta F = 2$ observables. The golden channel for the measurement of this phase is the time-dependent analysis of the $B_s(\bar{B}_s) \to \psi\phi$ decay. At the quark level $B_s \to \psi\phi$ share the same virtues of $B_d \to \psi K$ (partonic amplitude of the type $b \to c\bar{c}s$), which is used to extract the phase of $B_d$–$\bar{B}_d$ mixing. However, there are a few points which makes this measurement much more challenging:

- The $B_s$ oscillations are much faster ($\Delta m_{B_s}/\Delta m_{B_d} \approx F^2_{B_s}/F^2_{B_d}|V_{ts}/V_{td}|^2 \approx 30$), making the time-dependent analysis quite difficult (and essentially inaccessible at $B$ factories).
- Contrary to $|\psi K\rangle$, which has a single angular momentum and is a pure CP eigenstate, the vector-vector state $|\psi\phi\rangle$ produced by the $B_s$ decay has different angular momenta, corresponding to different CP eigenstates. These must be disentangled with a proper angular analysis of the final four-body final state $|(\ell^+ \ell^-)\psi(K^+ K^-)\rangle$. To avoid contamination from the nearby $|\psi f_0\rangle$ state, the fit should include also a $|(\ell^+ \ell^-)\psi(K^+ K^-)s\text{-wave}\rangle$ component, for a total of ten independent (and unknown) weak amplitudes.
- Contrary to the $B_d$ system, the width difference cannot be neglected in the $B_s$ case, leading to an additional key parameter to be included in the fit.

Modulo the experimental difficulties listed above, the process is theoretically clean and a complete fit of the decay distributions should allow the extraction of

$$\chi^{B_s}_{\psi \phi} = -e^{-i\phi_{B_s}}, \quad (2.36)$$

where the SM prediction is\footnote{The quoted error takes into account possible sub-leading amplitudes contributing to the $B_s \to \psi\phi$ decay with different CKM structure.}

$$\phi^{SM}_{B_s} = -\arg \left( \frac{V_{ts}^* V_{td}}{|V_{ts}^* V_{td}|^2} \right) = -0.04 \pm 0.01. \quad (2.37)$$

The tiny value of $\phi^{SM}_{B_s}$ implies that, within the SM, no CP asymmetry should be observed in the near future. The present status of the combined fit of $\Delta \Gamma_s$ and $\phi_{B_s}$ as obtained by LHCb and other experiments is shown in Fig. 2.2 As can be noted, at present these is a good agreement with the SM prediction. However, contrary to all other $\Delta F = 2$ observables, in this case the theory error is still subleading and there is ample room for improving the precision on the experimental side.

As we will see in Sect.\footnote{Note 1} a clear evidence for $\phi_{B_s} \neq \phi^{SM}_{B_s}$ would not only signal the presence of physics beyond the SM, but would also rule out the whole class of MFV models.

\[2\] CP violation in charged $B$ decays

Among non-leptonic channels $B^{\pm} \to D K$ decays are particularly interesting since, via appropriate asymmetries, allows us to extract the CKM angle $\gamma$ in a very clean way. The extraction of $\gamma$ involves only

19
Combining fit of $\Delta \Gamma$ and $\phi_B$ from LHCb and other experiments from the time-dependent analysis of $B_s \to \psi \phi$ decays (plot from Ref. [23]).

Tree-level $B$ decay amplitudes, and is virtually free from hadronic uncertainties (which are eliminated directly by data). It is therefore an essential element for a precise determination of the SM Yukawa couplings also in presence of NP.

The main strategy is based on the following two observations:

- The partonic amplitudes for $B^- \to \bar{D} K^-$ ($b \to c\bar{u} s$) and $B^- \to \bar{D} K^-$ ($b \to u\bar{c} s$) are pure tree-level amplitudes (no penguins allowed given the four different quark flavors). As a result, their weak phase difference is completely determined and is $\gamma = \arg\left(\frac{-V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$.

- Thanks to $D-\bar{D}$ mixing, there are several final states $f$ accessible to both $D$ and $\bar{D}$, where the two tree-level amplitudes can interfere. By combining the four final states $B^\pm \to fK^\pm$ and $B^\pm \to \bar{f}K^\mp$, we can extract $\gamma$ and all the relevant hadronic unknowns of the system.

The first strategy, proposed by Gronau, London, and Wyler [24] was based on the selection of $D(\bar{D})$ decays to two-body $CP$ eigenstates. But it has later been realized that any final state accessible to both $D$ and $\bar{D}$ (such as the $K^\mp \pi^\pm$ channels [25], or multibody final states [26]) may work as well.

Let’s start from the case of $D(\bar{D})$ decays to $CP$ eigenstates, where the formalism is particularly transparent. The key quantity is the ratio

$$r_{BE} e^{i\delta_B} = \frac{A(B^+ \to D^0 K^+)}{A(B^+ \to D^0 K^+)} ,$$

where $\delta$ is a strong phase. Denoting CP-even and CP-odd final states $f_+$ and $f_-$, we then have

$$
\begin{align*}
A(B^- \to f_+ K^-) &= A_0 \times \left[ 1 + r_{BE} e^{i(\delta_B - \gamma)} \right] \\
A(B^- \to f_- K^-) &= A_0 \times \left[ 1 - r_{BE} e^{i(\delta_B - \gamma)} \right] \\
A(B^+ \to f_+ K^+) &= A_0 \times \left[ 1 + r_{BE} e^{i(\delta_B + \gamma)} \right] \\
A(B^+ \to f_- K^+) &= A_0 \times \left[ 1 - r_{BE} e^{i(\delta_B + \gamma)} \right]
\end{align*}
$$
It is clear that combining the four rates we can extract the three hadronic unknowns ($A_B$, $r_B$, and $\delta$) as well as $\gamma$. It is also clear that the sensitivity to $\gamma$ vanishes in the limit $r_B \to 0$, and indeed the main limitation of this method is that $r_B$ turns out to be very small.

The formalism is essentially unchanged if we consider final states that are not CP eigenstates, such as the $K^\pm \pi^\mp$ states. These have the advantage that the suppression of $r_B$ is partially compensated by the CKM suppression of the corresponding $D(\bar{D}) \to K^\pm \pi^\mp$ decays. Indeed the effective relevant ratio becomes

$$ r_{\text{eff}} e^{i\delta_{\text{eff}}} = \frac{A(B^+ \to D^0 K^+)}{A(B^+ \to D^0 K^+)} \times \frac{A(D^0 \to K^- \pi^+)}{A(D^0 \to K^- \pi^+)} $$

(2.40)

which is substantially larger than $r_B$.

Once $r_B$ and $\delta_B$ (or $r_{\text{eff}}$ and $\delta_{\text{eff}}$) are determined from data, the extraction of $\gamma$ has essentially no theoretical uncertainty. In principle a theoretical error could be induced by the neglected CP-violating effects in charm mixing. In practice, the experimental bounds on charm mixing make this effect totally negligible. The key issue is only collecting high statistics on this highly-suppressed decay modes: a clear target for $B$ physics at hadron machines.

4 Rare FCNC $B$ decays

On general grounds, theoretical predictions for exclusive FCNC decays are not easy: non-perturbative effects are difficult to be kept under good theoretical control. Even if the final state involve only one hadron, in most of the kinematical region re-scattering effects of the type $B \to K^* H \bar{H} \to K^* \ell^+ \ell^-$ are possible, making difficult to estimate precisely the decay rate.

However, there are a few exceptions. In the $B \to K^* \ell^+ \ell^-$ case the largest source of uncertainty is the normalization of the hadronic form factors. The theoretical error can be substantially reduced in appropriate ratios or differential distributions. A clean example of this type is the normalized forward-backward asymmetry in $B \to K^\pm \ell^\mp \ell^\mp$.

An even cleaner case is the pure leptonic decays, $B \to \ell^+ \ell^-$, where re-scattering effects are negligible (due to the peculiar choice of initial and final state) and all relevant non-perturbative effects are encoded into the meson decay constant (that is easily accessible on the Lattice).

4.1 The forward-backward asymmetry in $B \to K^* \ell^+ \ell^-$. The observable is defined as

$$ A_{FB}(s) = \frac{1}{d\Gamma(B \to K^* \mu^+ \mu^-)/ds} \int_{-1}^{1} d\cos \theta \frac{d^2\Gamma(B \to K^* \mu^+ \mu^-)}{ds \; d\cos \theta} \sgn(\cos \theta), $$

(2.41)

where $\theta$ is the angle between the momenta of $\mu^+$ and $\bar{B}$ in the dilepton center-of-mass frame. Assuming that the leptonic current has only a vector ($V$) or axial-vector ($A$) structure (as in the SM), the FB asymmetry provides a direct measure of the $A-V$ interference. Indeed, at the lowest-order one can write

$$ A_{FB}(q^2) \propto \Re \left\{ C_{10}^A \left[ \frac{q^2}{m_b^2} C_{g\ell}^{\text{eff}} + r(q^2) \frac{m_b C_7}{m_B} \right] \right\}, $$

where $r(q^2)$ is an appropriate ratio of $B \to K^*$ vector and tensor form factors [27]. There are three main features of this observable that provide a clear and independent short-distance information:

1. The position of the zero ($q^2_0$) of $A_{FB}(q^2)$ in the low-$q^2$ region (see Fig. 2.3 [27]): as shown by the detailed analyses in Ref. [28,29], the experimental measurement of $q^2_0$ could allow a determination of $C_7/C_9$ at the 10% level.
2. The sign of $A_{FB}(q^2)$ around the zero. This is fixed unambiguously in terms of the relative sign of $C_{10}$ and $C_9$: within the SM one expects $A_{FB}(q^2 > q^2_0) > 0$ for $|\bar{B}| \equiv |b\bar{d}|$ mesons.
3. The relation $A[\bar{B}]_{FB}(q^2) = -A[B]_{FB}(q^2)$. This follows from the CP-odd structure of $A_{FB}$ and holds at the $10^{-3}$ level within the SM [30], where $C_{10}$ has a negligible CP-violating phase.

The FB asymmetry has been measured recently with high statistics by LHCb [31] (see Fig. 2.3) and, contrary to previous low-statistics results, the LHCb data are in good agreement with the SM prediction. As can be seen in Fig. 2.3, the experimental errors are not far from the level of the theoretical uncertainty in the CP-averaged FB asymmetry. However, there is still room for more precise test of the theory in other type of (normalized) differential distributions related to CP asymmetries [32, 33].

4.2 $B \rightarrow \ell^+\ell^-$

The purely leptonic decays constitute a special case among exclusive transitions. Within the SM only the axial-current operator, $Q_{10A}$, induces a non-vanishing contribution to these decays. As a result, the short-distance contribution is not diluted by the mixing with four-quark operators. Moreover, the hadronic matrix element involved is the simplest we can consider, namely the $B$-meson decay constant in Eq. (2.18). As we have seen, present Lattice errors on $F_{B_d}$ and $F_{B_s}$ from lattice QCD are already below 5%, and could further improve in the future.

The price to pay for this theoretically-clean amplitude is a strong helicity suppression for $\ell = \mu$ (and $\ell = e$), or the channels with the best experimental signature. Following the recent theoretical analysis in [20], the theoretical branching ratio of the flavor averaged state (equal mixture of $B_s$ and $\bar{B}_s$) into a muon pair (fully inclusive of soft-photon emission) can be written as

$$B(B_s \rightarrow \mu^+\mu^-)_{SM} = 3.235 \times 10^{-9} \times \left( \frac{M_t}{173.2 \text{ GeV}} \right)^{3.07} \left( \frac{F_{B_s}}{227 \text{ MeV}} \right)^2 \left| \frac{V_{tb}V_{ts}^*}{4.05 \times 10^{-2}} \right|^2$$

where in the second line we have explicitly separated the present contribution to the error due to $F_{B_s}$. As far as the other leptons are concerned, we get

$$\frac{B(B_s \rightarrow \tau^+\tau^-)}{B(B_s \rightarrow \mu^+\mu^-)}_{SM} = 215 , \quad \frac{B(B_s \rightarrow e^+e^-)}{B(B_s \rightarrow \mu^+\mu^-)}_{SM} = 2.4 \times 10^{-5} .$$

The corresponding $B_d$ modes are both suppressed by an additional factor $|V_{td}/V_{ts}|^2 F_{B_d}^2/F_{B_s}^2 \approx 1/30$.

As recently pointed out in [34], an important point when comparing the above predictions with experiments is the observation that, at present, experiments extract the $B_s$ decay rates from a time-integrated distribution. As a result, we cannot access the decay rate of a flavor averaged state (that
is what is produced at initial time), but its time-integrated evolution. Due to the non-vanishing width difference $\Delta \Gamma_s$, this imply a nontrivial correction factor of $O(10\%)$.

What is presently measured by the LHC experiments is the flavor-averaged time-integrated distribution,

$$\langle B(B_s \rightarrow f) \rangle_{[t]} = \frac{1}{2} \int_0^t dt' \left[ \Gamma(B_s(t') \rightarrow f) + \Gamma(B_s(t') \rightarrow \bar{f}) \right],$$  \hspace{1cm} (2.44)

where $\Gamma(B_s(t') \rightarrow f)$ denotes the decay distribution, as a function of the proper time ($t'$), of a $B_s$ flavor eigenstate at initial time (and correspondingly for $\bar{B}_s$). Following the discussion in sect. 2, let’s define

$$\Gamma_s = \frac{1}{\tau_{B_s}} = \frac{1}{2} \left[ \Gamma_s^H + \Gamma_s^L \right], \hspace{1cm} y_s = \frac{\Gamma_s^L - \Gamma_s^H}{2\Gamma_s} = 0.088 \pm 0.014.$$  \hspace{1cm} (2.45)

The time-integrated distribution is related to the flavor-averaged rate at $t = 0$ [that is what is predicted in Eq. (2.42)], by

$$\langle B(B_s \rightarrow f) \rangle_{[t]} = \kappa_f(t, y_s) \langle B(B_s \rightarrow f) \rangle_{[t=0]} \equiv \kappa_f(t, y_s) \frac{\Gamma(B_s \rightarrow f) + \Gamma(\bar{B}_s \rightarrow f)}{2\Gamma_s},$$  \hspace{1cm} (2.46)

where $\kappa_f(t, y_s)$ is a model- and channel-dependent correction factor.

For the $\mu^+\mu^-$ final state (inclusive of bremsstrahlung radiation) the SM expression of the $\kappa_f(t, y_s)$ factor is $[35]$

$$\kappa_{SM}^{\mu\mu}(t, y_s) = \frac{1}{1 - y_s} \left[ 1 - e^{-t/\tau_{B_s}} \sinh \left( \frac{y_s t}{\tau_{B_s}} \right) - e^{-t/\tau_{B_s}} \cosh \left( \frac{y_s t}{\tau_{B_s}} \right) \right] \left[ t \gg \tau_{B_s} \right] \frac{1}{1 - y_s},$$

$$\langle B(B_s \rightarrow \mu\mu) \rangle_{[t=\infty]}^{SM} = (3.54 \pm 0.30) \times 10^{-9},$$  \hspace{1cm} (2.47)

where on the second line we have given the SM prediction for the fully integrated branching ratio, that is what we should compare with present experimental data.

The LHCb collaboration has recently reported an evidence of the $B_s \rightarrow \mu^+\mu^-$ decays, reporting the following first measurement of the branching ration:

$$\langle B(B_s \rightarrow \mu\mu) \rangle_{[t=\infty]}^{exp} = (3.2^{+1.5}_{-1.2}) \times 10^{-9}.$$  \hspace{1cm} (2.48)

As can be seen, this result is in good agreement with the SM prediction in Eq. (2.47). However, the error is still large and correspondingly there is still a sizable region of possible new-physics contributions still to be explored.

The strong helicity suppression and the theoretical cleanliness make these modes excellent probes of several new-physics models and, particularly, of scalar FCNC amplitudes. Scalar FCNC operators, such as $b_{RS} \mu_{RH} L$, are present within the SM but are negligible because of the smallness of down-type Yukawa couplings. On the other hand, these amplitudes could be non-negligible in models with an extended Higgs sector (see Sect. 1.2). In particular, within the MSSM, where two Higgs doublets are coupled separately to up- and down-type quarks, a sizable enhancement of scalar FCNCs can occur at large $\tan \beta = v_u/v_d$. This effect is very small in non-helicity-suppressed $B$ decays (because of the small Yukawa couplings), but could easily enhance $B \rightarrow \ell^+\ell^-$ rates by one order of magnitude. The latter possibility is ruled out by the present bounds on $B(B_s \rightarrow \mu\mu)$, resulting in a significant constraint on such class of models.

An illustration of the possible deviations from the SM predictions in a constrained version of the MSSM (that will be discussed in Sect. 2.2) is shown in Fig. 2.4. This figure shows that the present search for $B_s \rightarrow \mu^+\mu^-$ is complementary to the strong limits already sets on such class of models by the direct searches at ATLAS and CMS. In a long-term perspective, the discovery and the precise measurement of all the accessible $B \rightarrow \ell^+\ell^-$ channels is one of the most interesting items of the $B$-physics program at hadron colliders.
5 CP violation in the charm system

5.1 General considerations

On general grounds, long-distance contributions are usually largely dominant with respect to the short-distance ones in charm mixing and decay amplitudes. This happens is because SM short-distance contributions are not top-mass enhanced as in the $B$ and $K$ systems, and are strongly disfavored by the CKM hierarchy with respect to the dominant transition amplitudes into light quarks. Within the SM the genuine short-distance contributions are suppressed by five powers of the Cabibbo angle. Other contrary, long-distance amplitudes into light quarks can be Cabibbo allowed (i.e. not suppressed by any power of $\lambda$) for partonic transitions of the type $c \to us\bar{d}$, Cabibbo suppressed ($c \to uu\bar{d}(s\bar{s})$), or at most doubly Cabibbo suppressed ($c \to ud\bar{s}$).

Given this hierarchy of amplitudes, within the SM charm physics does not provide interesting precision tests of the CKM mechanism. However, the charm system offers a unique opportunity to explore up-type FCNC amplitudes that maybe significantly enhanced over the SM level in possible extensions of the SM. For instance, as shown in Table 1.1, very stringent constrains on generic $|\Delta C| = 2$ operators can be derived by the experimental constraints on $D-\bar{D}$ mixing. The neutral $D$ system is the latest system of neutral mesons where mixing between the particles and anti-particles has been established. The observation of a non-vanishing amplitude at more than 5$\sigma$ has been reported a few months ago by LHCb [38] collaboration and turns out to be consistent with the (long-distance dominated) SM expectation.

While CP-conserving observables in $D$ decays are largely dominated by long-distance effects, CP-violating observables are typically strongly suppressed within the SM and offer a potentially deeper probe of short-distance dynamics. One of the most interesting recent developments in flavor physics has been the experimental evidence of direct CP violation in two-body Cabibbo-suppressed $D$ decays. An asymmetry close to the 1% level has been announced first by LHCb [39] and soon after confirmed both by CDF [40] and by Belle, although none of the experiments has reached the 5$\sigma$ level. Such a large direct CP asymmetry was not expected within the SM according to pre-LHCb theoretical predictions, and the theoretical interpretation [41] of this result has open an interesting debate that is still in progress.

5.2 Standard Model vs. New Physics in $\Delta a_{\text{dir}}^{\text{CP}}$

The current experimental world average for the direct CP-violating asymmetry in two-body Cabibbo-suppressed $D$ decays can be summarized as follows

$$\Delta a_{\text{CP}}^{\text{dir}} \equiv a_{\text{CP}}^{\text{dir}}(D \to K^+K^-) - a_{\text{CP}}^{\text{dir}}(D \to \pi^+\pi^-) = (-0.67 \pm 0.16)\%,$$

Fig. 2.4: Predictions for $B(B_s \to \mu\mu)$ in the $M_A$–$\tan\beta$ plane in the CMSSM (left panel) and in the CMSSM with non-universal Higgs masses (right panel) [37]. The red and blue contours denote the allowed region of parameter space at 68% and 98% C.L. taking into account all available data (see Sect. 2.2 for more details).
where

$$a_{CP}^{dir}(D \rightarrow f) \equiv \frac{\Gamma(D^0 \rightarrow f) - \Gamma(D^0 \rightarrow \bar{f})}{\Gamma(D^0 \rightarrow f) + \Gamma(D^0 \rightarrow \bar{f})}. \quad (2.50)$$

The separate determinations of $a_{CP}^{dir}(D \rightarrow K^+K^-)$ and $a_{CP}^{dir}(D \rightarrow \pi^+\pi^-)$ are affected by larger relative uncertainties and, at present, do not allow to establish a clear evidence of CP-violation in one of the two channels.

In order to be non zero, $\Delta a_{CP}^{dir}$ requires the interference of two amplitudes with different weak and strong phases. Within the SM, taking into account that one of the two amplitudes is necessarily generated at the one-loop level, this implies the following naive expectation $\Delta a_{CP}^{dir} = O(|V_{cb}^*V_{ub}/V_{cs}^*V_{us}|\alpha_s/\pi) \sim 10^{-3}$ [41], well below the experimental result in Eq. (2.49). This has led to extensive speculations in the literature that the measurement of $\Delta a_{CP}^{dir}$ is a signal of NP. This is a particularly exciting possibility, given that reasonable NP models can be constructed in which all related flavor changing neutral current constraints from $D$ meson mixing are satisfied.

The naive expectation for the SM value of $\Delta a_{CP}^{dir}$ is based on a perturbative (short-distance) estimate of the loop amplitude with suppressed CKM factors. In fact, there is consensus that a SM explanation for $\Delta a_{CP}^{dir}$ would have to proceed via a dynamical (long-distance) enhancement of specific hadronic matrix elements, the so-called penguin contractions. The latter are nothing but penguin-type matrix elements that vanish at the tree level, with internal light-quark loops ($s$ and $d$): they cannot be estimated reliably in perturbation theory [42]. The enhancement necessary to explain the observed result is quite large compared to the typical size of non-perturbative effects at the charm scale (the naively suppressed penguin contractions should exceed by a factor 3 to 5 the naively dominant tree-level contractions of the same operators [45]). However, such possibility cannot be excluded from first principles and could even lead to a more coherent picture of available data on two-body Cabibbo-suppressed $D$ decays [43].

On the other hand, a value of $\Delta a_{CP}^{dir}$ of $O(1\%)$ can naturally be accommodated in well-motivated extensions of the SM. In particular, it fits well in models generating at short distances a sizable CP violating phase for the effective $\Delta C = 1$ chromomagnetic operators (see e.g. [41,44,45]). Given this situation, it is important to identify possible future experimental tests able to distinguish standard vs. non-standard explanations of $\Delta a_{CP}^{dir}$.

A general prediction of this class of models, that could be used to test this hypothesis from data, are enhanced direct CP violating (DCPV) asymmetries in radiative decay modes [46] (see also [47,48]). The first key observation to estimate DCPV asymmetries in radiative decay modes is the strong link between the $\Delta C = 1$ chromomagnetic operator, $Q_{8g}$, and the $\Delta C = 1$ electromagnetic-dipole operator, $Q_{7\gamma}$ (these operators are defined as in (2.13), with the proper replacement of quark fields: $\{b,s\} \rightarrow \{c,u\}$). In most explicit NP models, the short-distance Wilson coefficients of these two operators are expected to be similar. Moreover, the two operators undergo a strong model-independent mixing (from QCD) in running down from the electroweak scale to the charm scale. Thus if $\Delta a_{CP}$ is dominated by NP contributions generated by $Q_{8g}$, we can infer that sizable CP asymmetries should occur also in radiative decays, given the presence of a CP-violating electromagnetic-dipole operator.

The second important ingredient is the observation that in the Cabibbo-suppressed $D \rightarrow V_\gamma$ decays, where $V$ is a light vector meson ($V = \phi, \rho, \omega$), $Q_{7\gamma}$ has a sizable hadronic matrix element. More explicitly, the short-distance contribution induced by $Q_{7\gamma}$, relative to the total (long-distance) amplitude, is substantially larger with respect to the corresponding relative weight of $Q_{8g}$ in $D \rightarrow P^+P^-$ decays. As a result, DCPV asymmetries in these modes could easily reach the few $\times\%$ level in presence of NP. An observation of $|a_{V_\gamma}| \gtrsim 3\%$ would be a clear signal of physics beyond the SM, and a clean indication of new CP-violating dynamics associated to dipole operators.
Chapter 3

Flavor physics beyond the SM: models and predictions

If the physics beyond the SM respects the SM gauge symmetry, as we expect from general arguments, the corrections to low-energy flavor-violating amplitudes can be written in the following general form

$$ A(f_i \to f_j + X) = A_0 \left[ \frac{c_{\text{SM}}}{M_W^2} + \frac{c_{\text{NP}}}{\Lambda^2} \right], $$

(3.1)

where $\Lambda$ is the energy scale of the new degrees of freedom. This structure is completely general: the coefficients $c_{\text{SM(NP)}}$ may include appropriate CKM factors and eventually a $\sim 1/(16\pi^2)$ suppression if the amplitude is loop-mediated. Given our ignorance about the $c_{\text{NP}}$, the values of the scale $\Lambda$ probed by present experiments vary over a wide range. However, the general result in Eq. (3.1) allows us to predict how these bounds will improve with future experiments: the sensitivity on $\Lambda$ scale as $N^{1/4}$, where $N$ is the number of events used to measure the observable. This implies that it is not easy to increase substantially the energy reach with indirect NP searches only. Moreover, from Eq. (3.1) it is also clear that indirect searches can probe NP scales well above the TeV for models where $(c_{\text{SM}} \ll c_{\text{NP}})$, namely models which do not respect the symmetries and the symmetry-breaking pattern of the SM.

The bound on representative $\Delta F = 2$ operators have already been shown in Table 1.1. As can be seen, for $c_{\text{NP}} = 1$ present data probes very high scales. On the other hand, if we insist with the theoretical prejudice that NP must show up not far from the TeV scale in order to stabilize the Higgs sector, then the new degrees of freedom must have a peculiar flavor structure able to justify the smallness of the effective couplings $c_{\text{NP}}$ for $\Lambda = 1$ TeV.

1 The Minimal Flavor Violation hypothesis

The main idea of MFV is that flavor-violating interactions are linked to the known structure of Yukawa couplings also beyond the SM. In a more quantitative way, the MFV construction consists in identifying the flavor symmetry and symmetry-breaking structure of the SM and enforce it also beyond the SM.

The MFV hypothesis consists of two ingredients [49]: (1) a flavor symmetry and (ii) a set of symmetry-breaking terms. The symmetry is noting but the large global symmetry $G_{\text{flavor}}$ of the SM Lagrangian in absence of Yukawa couplings shown in Eq. (1.4). Since this global symmetry, and particularly the $SU(3)$ subgroups controlling quark flavor-changing transitions, is already broken within the SM, we cannot promote it to be an exact symmetry of the NP model. Some breaking would appear at the quantum level because of the SM Yukawa interactions. The most restrictive assumption we can make to protect in a consistent way quark-flavor mixing beyond the SM is to assume that $Y_u$ and $Y_d$ are the only sources of flavor symmetry breaking also in the NP model. To implement and interpret this hypothesis in a consistent way, we can assume that $G_q$ is a good symmetry and promote $Y_{u,d}$ to be non-dynamical fields (spurions) with non-trivial transformation properties under $G_q$:

$$ Y_u \sim (3, 3, 1), \quad Y_d \sim (3, 1, 3). $$

(3.2)

If the breaking of the symmetry occurs at very high energy scales, at low-energies we would only be sensitive to the background values of the $Y$, i.e. to the ordinary SM Yukawa couplings. The role of the Yukawa in breaking the flavor symmetry becomes similar to the role of the Higgs in the breaking of the gauge symmetry. However, in the case of the Yukawa we don’t know (and we do not attempt to construct) a dynamical model which give rise to this symmetry breaking.
Within the effective-theory approach to physics beyond the SM introduced in Sect. 4, we can say that an effective theory satisfies the criterion of Minimal Flavor Violation in the quark sector if all higher-dimensional operators, constructed from SM and $Y$ fields, are invariant under CP and (formally) under the flavor group $G_q$ [49].

According to this criterion one should in principle consider operators with arbitrary powers of the (dimensionless) Yukawa fields. However, a strong simplification arises by the observation that all the eigenvalues of the Yukawa matrices are small, but for the top one, and that the off-diagonal elements of the CKM matrix are very suppressed. Working in the basis in Eq. (1.6) we have

$$[Y_u(Y_u)^\dagger]_i^n \approx y_t^n V_{ti}^* V_{tj},$$  \hspace{1cm} (3.3)

As a consequence, in the limit where we neglect light quark masses, the leading $\Delta F = 2$ and $\Delta F = 1$ FCNC amplitudes get exactly the same CKM suppression as in the SM:

$$A(d^i \rightarrow d^j)_{\text{MFV}} = (V_{ti}^* V_{tj}) A_{\text{SM}}^{(\Delta F = 1)} \left[ 1 + a_1 \frac{16\pi^2 M_W^2}{\Lambda^2} \right],$$  \hspace{1cm} (3.4)

$$A(M_{ij} - \bar{M}_{ij})_{\text{MFV}} = (V_{ti}^* V_{tj})^2 A_{\text{SM}}^{(\Delta F = 2)} \left[ 1 + a_2 \frac{16\pi^2 M_W^2}{\Lambda^2} \right].$$  \hspace{1cm} (3.5)

where the $A_{\text{SM}}^{(i)}$ are the SM loop amplitudes and the $a_i$ are $O(1)$ real parameters. The $a_i$ depend on the specific operator considered but are flavor independent. This implies the same relative correction in $s \rightarrow d$, $b \rightarrow d$, and $b \rightarrow s$ transitions of the same type: a key prediction which can be tested in experiment.

As pointed out in Ref. [50], within the MFV framework several of the constraints used to determine the CKM matrix (and in particular the unitarity triangle) are not affected by NP. In this framework, NP effects are negligible not only in tree-level processes but also in a few clean observables sensitive to loop effects, such as the time-dependent CPV asymmetry in $B_d \rightarrow \psi K_{L,S}$. Indeed the structure of the basic flavor-changing coupling in Eq. (3.5) implies that the weak CPV phase of $B_d \bar{B}_d$ mixing is $\arg[(V_{td}V_{tb})^2]$, exactly as in the SM. This construction provides a natural (a posteriori) justification of why no NP effects have been observed in the quark sector: by construction, most of the clean observables measured at $B$ factories are insensitive to NP effects in the MFV framework. A comparison of the CKM fits in the SM and in generic MFV models is shown in Fig. 3.1. Essentially only $\epsilon_K$ and $\Delta m_{B_d}$ (but not the ratio $\Delta m_{B_s}/\Delta m_{B_d}$) are sensitive to non-standard effects within MFV models.

Fig. 3.1: Fit of the CKM unitarity triangle (in 2008) within the SM (left) and in generic extensions of the SM satisfying the MFV hypothesis (right) [13].
phenomenology can still be described using the general MFV criterion discussed above. With small
within weakly coupled theories at the TeV scale with only one light Higgs doublet, such as the MSSM.
the only relevant ones also beyond the SM. This condition is realized in
MFV, or CMFV) contains the additional requirement that only the effective FCNC operators which play
degrees of freedom of the theory (identified with the SM fields in the minimal case).
The only two assumptions are: i) the flavor symmetry and its breaking sources; ii) the number of light
implemented independently of any specific hypothesis about the dynamics of the new-physics framework.
violations of flavor symmetry), since the full structure of Yukawa matrices is preserved. At the same
time, respecting the MFV criterion illustrated above provides the maximal protection of FCNCs (or the minimal
mining the structure of FCNCs is also played by quark masses, or by the Yukawa eigenvalues. In this
It is worth stressing that the CKM matrix represents only one part of the problem: a key role in deter-
mental precision on the clean FCNC observables required to obtain bounds more stringent than those
that a deeper study of rare decays is definitely needed in order to clarify the flavor problem: the exper-
rconserving operators derived by precision electroweak tests. This observation reinforces the conclusion
bound on higher-dimensional operators in the MFV framework turns out to be in the TeV range. This can easily be understood by the discussion in Sect. 4.1: the MFV bounds on operators contributing to $\epsilon_K$ and $\Delta m_{B_d}$ are obtained from Eq. (1.22) setting $|c_{ij}| = |y_3^2 V_{3i}^* V_{3j}|^2$. In Table 3.1, we report a few representative examples of the bounds on the higher-
dimensional operators in the MFV framework.1 These bounds are very similar to the bounds on flavor-
conserving operators derived by precision electroweak tests. This observation reinforces the conclusion
that a deeper study of rare decays is definitely needed in order to clarify the flavor problem: the exper-
amental precision on the clean FCNC observables required to obtain bounds more stringent than those
derived from precision electroweak tests (and possibly discover new physics) is typically in the 1%–10% range.

| Operator                                                                 | Bound on $\Lambda$ | Observables          |
|-------------------------------------------------------------------------|---------------------|----------------------|
| $\phi^3 \left( \overline{Q}_L Y_u \gamma_\mu Q_L \right)^2 (\epsilon F_{\mu\nu})$ | 6.1 TeV             | $B \to X_s \gamma$, $B \to X_s \ell^+ \ell^-$ |
| $\frac{1}{2} \left( \overline{Q}_L Y_u \gamma_\mu Q_L \right)^2 (g s G_{\mu\nu})$ | 5.9 TeV             | $\epsilon_K$, $\Delta m_{B_d}$, $\Delta m_{B_s}$ |
| $\phi^3 \left( \overline{Q}_L Y_u \gamma_\mu Q_L \right)^2 (\epsilon F_{\mu\nu})$ | 3.4 TeV             | $B \to X_s \gamma$, $B \to X_s \ell^+ \ell^-$ |
| $\left( \overline{Q}_L Y_u \gamma_\mu Q_L \right)^2 (\epsilon F_{\mu\nu})$ | 5.7 TeV             | $B_s \to \mu^+ \mu^-$, $B \to K^* \mu^+ \mu^-$ |
| $\left( \overline{Q}_L Y_u \gamma_\mu Q_L \right)^2 (\epsilon F_{\mu\nu})$ | 4.1 TeV             | $B_s \to \mu^+ \mu^-$, $B \to K^* \mu^+ \mu^-$ |
| $\left( \overline{Q}_L Y_u \gamma_\mu Q_L \right)^2 (\epsilon F_{\mu\nu})$ | 1.7 TeV             | $B \to K^* \mu^+ \mu^-$ |

Table 3.1: Bounds on the scale of new physics (at 95% C.L.) for some representative MFV operators (assuming
effective coupling $\pm 1/\Lambda^2$, and considering only one operator at a time), with the corresponding observables used
to set the bounds.

Given the built-in CKM suppression, the bounds on higher-dimensional operators in the MFV framework turns out to be in the TeV range. This can easily be understood by the discussion in Sect. 4.1: the MFV bounds on operators contributing to $\epsilon_K$ and $\Delta m_{B_d}$ are obtained from Eq. (1.22) setting $|c_{ij}| = |y_3^2 V_{3i}^* V_{3j}|^2$. In Table 3.1, we report a few representative examples of the bounds on the higher-dimensional operators in the MFV framework. These bounds are very similar to the bounds on flavor-conserving operators derived by precision electroweak tests. This observation reinforces the conclusion that a deeper study of rare decays is definitely needed in order to clarify the flavor problem: the experimental precision on the clean FCNC observables required to obtain bounds more stringent than those derived from precision electroweak tests (and possibly discover new physics) is typically in the 1%–10% range.

1.1 General considerations

The idea that the CKM matrix rules the strength of FCNC transitions also beyond the SM has become a
very popular concept in the recent literature and has been implemented and discussed by several authors. It is worth stressing that the CKM matrix represents only one part of the problem: a key role in determining the structure of FCNCs is also played by quark masses, or by the Yukawa eigenvalues. In this respect, the MFV criterion illustrated above provides the maximal protection of FCNCs (or the minimal violation of flavor symmetry), since the full structure of Yukawa matrices is preserved. At the same time, this criterion is based on a renormalization-group-invariant symmetry argument. Therefore, it can be implemented independently of any specific hypothesis about the dynamics of the new-physics framework. The only two assumptions are: i) the flavor symmetry and its breaking sources; ii) the number of light degrees of freedom of the theory (identified with the SM fields in the minimal case).

This model-independent structure does not hold in most of the alternative definitions of MFV models that can be found in the literature. For instance, the definition of Ref. 51 (denoted constrained MFV, or CMFV) contains the additional requirement that only the effective FCNC operators which play a significant role within the SM are the only relevant ones also beyond the SM. This condition is realized within weakly coupled theories at the TeV scale with only one light Higgs doublet, such as the MSSM with small $\tan \beta$ and small $\mu$ term. However, it does not hold in other frameworks, such as composite-Higgs models (see e.g. [52,54]) or the MSSM with large $\tan \beta$ and/or large $\mu$ term, whose low-energy phenomenology can still be described using the general MFV criterion discussed above.

1Table 3.1 updates the corresponding table of Ref. 5 taking into account the recent measurements of $B \to K^* \mu^+ \mu^-$ and $B \to \mu^+ \mu^-$ from LHCb, as analysed in Ref. 33.
Fig. 3.2: Correlation between $B_s \rightarrow \mu^+ \mu^-$ and $B_d \rightarrow \mu^+ \mu^-$ in presence of non-standard amplitudes respecting the MFV hypothesis. The continuous red line indicates the central value of the correlation, while the green points take into account the uncertainties in $|V_{ts}|$ and $|V_{td}|$ [56].

Although the MFV seems to be a natural solution to the flavor problem, it should be stressed that we are still very far from having proved the validity of this hypothesis from data. A proof of the MFV hypothesis can be achieved only with a positive evidence of physics beyond the SM exhibiting the flavor-universality pattern (same relative correction in $s \rightarrow d$, $b \rightarrow d$, and $b \rightarrow s$ transitions of the same type) predicted by the MFV assumption. While this goal is quite difficult to be achieved, the MFV framework is quite predictive and thus could easily be falsified. Some of the most interesting predictions which could be tested in the near future are the following:

- No new CPV phases in $B_s$ mixing, hence $|\phi_{B_s}| < 0.05$ from $A_{\text{CP}}(B_s \rightarrow \psi \phi)$.
- Ratio of $B_s$ and $B_d$ decays into $\ell^+ \ell^-$ pairs determined by the CKM matrix: $B(B_d \rightarrow \ell^+ \ell^-)/B(B_s \rightarrow \ell^+ \ell^-) \approx |V_{td}/V_{ts}|^2$ (see Fig. 3.2).
- No new CPV phases in $b \rightarrow s\gamma$, hence vanishingly small CP asymmetries in $B \rightarrow K^* \gamma$ and $B \rightarrow K^* \ell^+ \ell^-$. Violations of these bounds would not only imply physics beyond the SM, but also a clear signal of new sources of flavor symmetry breaking beyond the Yukawa couplings.

### 1.2 MFV at large $\tan \beta$.

If the Yukawa Lagrangian contains more than one Higgs field, we can still assume that the Yukawa couplings are the only irreducible breaking sources of $G_q$, but we can change their overall normalization. A particularly interesting scenario is the two-Higgs-doublet model where the two Higgses are coupled separately to up- and down-type quarks:

$$L^{2\text{HDM}}_Y = \bar{Q}_L Y_d D_R \phi_D + \bar{Q}_L Y_u U_R \phi_U + \bar{L}_L Y_e E_R \phi_D + \text{h.c.}$$

(3.6)

This Lagrangian is invariant under an extra $U(1)$ symmetry with respect to the one-Higgs Lagrangian in Eq. (1.3): a symmetry under which the only charged fields are $D_R$ and $E_R$ (charge $+1$) and $\phi_D$ (charge $-1$). This symmetry, denoted $U_{\text{PQ}}$, prevents tree-level FCNCs and implies that $Y_{u,d}$ are the only sources of $G_q$ breaking appearing in the Yukawa interaction (similar to the one-Higgs-doublet scenario). Coherently with the MFV hypothesis, we can then assume that $Y_{u,d}$ are the only relevant sources of $G_q$ breaking appearing in all the low-energy effective operators. This is sufficient to ensure that flavor-mixing is still governed by the CKM matrix, and naturally guarantees a good agreement with present data.
in the $\Delta F = 2$ sector. However, the extra symmetry of the Yukawa interaction allows us to change the overall normalization of $Y_{u,d}$ with interesting phenomenological consequences in specific rare modes.

The normalization of the Yukawa couplings is controlled by the ratio of the vacuum expectation values of the two Higgs fields, or by the parameter $\tan \beta = \langle \phi_U \rangle / \langle \phi_D \rangle = v_u / v_d$. Defining the eigenvalues $\lambda_{u,d}$ as in Eq. (1.6),

$$
\lambda_u = \frac{1}{v_u} \text{diag}(m_u, m_c, m_t), \\
\lambda_d = \frac{1}{v_d} \text{diag}(m_d, m_s, m_b) = \frac{\tan \beta}{v_u} \text{diag}(m_d, m_s, m_b).
$$

(3.7)

For $\tan \beta \gg 1$ the smallness of the $b$ quark can be attributed to the smallness of $v_d$, with respect to $v_u \approx v$, rather than to the smallness of the corresponding Yukawa coupling. As a result, for $\tan \beta \gg 1$ we cannot anymore neglect down-type Yukawa couplings. Since the $b$-quark Yukawa coupling becomes $O(1)$, the large-$\tan \beta$ regime is particularly interesting for all the helicity-suppressed observables in $B$ physics (i.e. the observables suppressed within the SM by the smallness of the $b$-quark Yukawa coupling).

Another important aspect of this scenario is that the the $U(1)_{PQ}$ symmetry cannot be exact: it has to be broken at least in the scalar potential in order to avoid the presence of a massless pseudoscalar Higgs boson. Even if the breaking of $U(1)_{PQ}$ and $G_q$ are decoupled, the presence of $U(1)_{PQ}$ breaking sources can have important implications on the structure of the Yukawa interaction, especially if $\tan \beta$ is large [57,58]. We can indeed consider new dimension-four operators such as

$$
\epsilon Q_L \lambda_d D_R \tilde{\phi}_U \quad \text{or} \quad \epsilon Q_L \lambda_u \lambda_d^\dagger D_R \tilde{\phi}_U,
$$

(3.8)

where $\epsilon$ denotes a generic MFV-invariant $U(1)_{PQ}$-breaking source. Even if $\epsilon \ll 1$, the product $\epsilon \times \tan \beta$ can be $O(1)$, inducing large corrections to the down-type Yukawa sector:

$$
\epsilon Q_L \lambda_d D_R \tilde{\phi}_U \xrightarrow{\text{ew}} \epsilon Q_L \lambda_d D_R \langle \phi_U \rangle = (\epsilon \times \tan \beta) Q_L \lambda_d D_R \langle \phi_D \rangle.
$$

(3.9)

This is what happens in supersymmetry, where the operators in Eq. (3.8) are generated at the one-loop level [$\epsilon \sim 1/(16\pi^2)$], and the large $\tan \beta$ solution is particularly welcome in the contest of Grand Unified models [59].

One of the clearest phenomenological consequences is a suppression (typically in the $10 - 50\%$ range) of the $B \to \ell \nu$ decay rate with respect to its SM expectation [60]. But the most striking signature could arise from the rare decays $B_{s,d} \to \ell^+ \ell^-$ whose rates could still be significantly different from the corresponding SM expectations. A deviation of both $B_s \to \ell^+ \ell^-$ and $B_d \to \ell^+ \ell^-$ respecting the MFV relation $\Gamma(B_s \to \ell^+ \ell^-)/\Gamma(B_d \to \ell^+ \ell^-)$ illustrated in Fig. 3.2 would be an unambiguous signature of MFV at large $\tan \beta$ [55,62].

1.3 Beyond the minimal set-up

The breaking of the flavor group $G_q$ and the breaking of the discrete CP symmetry are not necessarily related: generic MFV models can contain flavor-blind (or flavor-universal) phases [53,63,64]. Because of the experimental constraints on electric dipole moments (EDMs), which are generally sensitive to such flavor-blind phases [64,65], in this more general case the bounds on the scale of new physics are substantially higher with respect to the “minimal” case, where the Yukawa couplings are assumed to be the only breaking sources of both symmetries [49].

The correlation of CP-violating effects in the three down-type $\Delta F = 2$ mixing amplitudes ($B_{d,s}$ and $K$ meson mixing) is a powerful test of possible flavor-blind phases in a MFV framework. At small $\tan \beta$ there is only one relevant $\Delta F = 2$ operator:

$$
(Q_L Y_u Y_d^\dagger \gamma_\mu Q_L)^2.
$$

(3.10)
Since this operator is Hermitian, its coupling must be real and no deviations are expected in the $\Delta F = 2$ sector compared to the case without flavor-blind phases. The situation changes if $\tan \beta$ is large. In this case, thanks to the large bottom Yukawa coupling, additional operators with the insertion of $Y_d$ break the universality between $K$ and $B$ systems. In the limit where we can neglect the strange-quark Yukawa coupling, the extra CPV induced by flavor blind phases is equal in $B_s - \bar{B}_s$ and $B_d - \bar{B}_d$ mixing and does not enter $K^0 - \bar{K}^0$ mixing [53].

As stressed above, the MFV expansion relies on the smallness of the off-diagonal elements of the CKM matrix and the hierarchies between the Yukawa eigenvalues. It does not suffer from the fact of $y_t$ (and possibly $y_b$, at large $\tan \beta$) being sizable. As explicitly shown in Eq. (3.3), the effect of considering high powers in $y_t$ only modify the overall strength of the basic flavor-violating spurion

$$ (V^\dagger \lambda^2_u V)_{i \neq j}. $$

An elegant implementation of the MFV hypothesis, taking into account explicitly the special role the diagonal third-generation Yukawa couplings is obtained with a non-linear realization of the flavor symmetry [53,66]. Particularly interesting is the so-called GMFV case, where both $y_t$ and $y_b$ are assumed to be of order one and their effects are re-summed to all orders [53]. As shown in [53], the flavor symmetry group surviving after this resummation and linearly realised (with small breaking terms) is a $U(2)^3 \times U(1)$ group.

Given the smallness of $y_c,u/y_t$ and $y_s,d/y_t$, as well as the smallness of the off-diagonal elements of the CKM matrix, the phenomenological predictions derived in the GMFV framework are not different from those obtained with the standard MFV expansion in $Y_u$ and $Y_d$, provided the expansion is carried out up to the first non trivial terms. Indeed the difference between the GMFV predictions derived in [53] with respect to those obtained in [49], employing the standard MFV expansion at large $\tan \beta$, can all be attributed to the presence of flavor-blind phases in the GMFV set-up.

2 Flavor breaking in the Minimal Supersymmetric extension of the SM

The Minimal Supersymmetric extension of the SM (MSSM) is one of the most well-motivated and definitely the most studied extension of the SM at the TeV scale. For a detailed discussion of this model we refer to the review in Ref. [67] and to the lectures by D. Kazakov at this school. Here we limit our self to analyse some properties of this model relevant to flavor physics.

The particle content of the MSSM consists of the SM gauge and fermion fields plus a scalar partner for each quark and lepton (squarks and sleptons) and a spin-1/2 partner for each gauge field (gauginos). The Higgs sector has two Higgs doublets with the corresponding spin-1/2 partners (higgsinos) and a Yukawa coupling of the type in Eq. (3.6). While gauge and Yukawa interactions of the model are completely specified in terms of the corresponding SM couplings, the so-called soft-breaking sector of the theory contains several new free parameters, most of which are related to flavor-violating observables. For instance the $6 \times 6$ mass matrix of the up-type squarks, after the up-type Higgs field gets a vev $(\phi_U \rightarrow \langle \phi_U \rangle)$, has the following structure

$$ \bar{M}_U^2 = \begin{pmatrix} \bar{m}_{Q,L}^2 & A_U \langle \phi_U \rangle \\ \bar{A}_U^\dagger \langle \phi_U \rangle & \bar{m}_{U,R}^2 \end{pmatrix} + \mathcal{O}(m_Z,m_{\text{top}}), $$

(3.12)

where $\bar{m}_{Q,L}^2$, $\bar{m}_{U,R}^2$, and $A_U$ are $3 \times 3$ unknown matrices. Indeed the adjective minimal in the MSSM acronyms refers to the particle content of the model but does not specify its flavor structure.

\[\text{Supersymmetry must be broken in order to be consistent with observations (we do not observe degenerate spin partners in nature). The soft breaking terms are the most general supersymmetry-breaking terms which preserve the nice ultraviolet properties of the model. They can be divided into two main classes: 1) mass terms which break the mass degeneracy of the spin partners (e.g. sfermion or gaugino mass terms); ii) trilinear couplings among the scalar fields of the theory (e.g. sfermion-sfermion-Higgs couplings).}\]
Because of this large number of free parameters, we cannot discuss the implications of the MSSM in flavor physics without specifying in more detail the flavor structure of the model. The versions of the MSSM analysed in the literature range from the so-called Constrained MSSM (CMSSM), where the complete model is specified in terms of only four free parameters (in addition to the SM couplings), to the MSSM without $R$ parity and generic flavor structure, which contains a few hundreds of new free parameters.

Throughout the large amount of work in the past decades it has became clear that the MSSM with generic flavor structure and squarks in the TeV range is not compatible with precision tests in flavor physics. This is true even if we impose $R$ parity, the discrete symmetry which forbids single s-particle production, usually advocated to prevent a too fast proton decay. In this case we have no tree-level FCNC amplitudes, but the loop-induced contributions are still too large compared to the SM ones unless the squarks are highly degenerate or have very small intra-generation mixing angles. This is nothing but a manifestation in the MSSM context of the general flavor problem illustrated in the first lecture.

The flavor problem of the MSSM is an important clue about the underlying mechanism of supersymmetry breaking. On general grounds, mechanisms of SUSY breaking with flavor universality (such as gauge mediation) or with heavy squarks (especially in the case of the first two generations) tends to be favored. However, several options are still open. These range from the very restrictive CMSSM case, which is a special case of MSSM with MFV, to more general scenarios with new small but non-negligible sources of flavor symmetry breaking.

2.1 Flavor Universality, MFV, and RGE in the MSSM.

Since the squark fields have well-defined transformation properties under the SM quark-flavor group $G_q$, the MFV hypothesis can easily be implemented in the MSSM framework following the general rules outlined in Sect. 1.

We need to consider all possible interactions compatible with i) softly-broken supersymmetry; ii) the breaking of $G_q$ via the spurion fields $Y_{u,d}$. This allows to express the squark mass terms and the trilinear quark-squark-Higgs couplings as follows [49, 68]:

\[
\tilde{m}^2_{Q_L} = \tilde{m}^2 \left( a_1 \mathbb{I} + b_1 Y_u Y_u^\dagger + b_2 Y_d Y_d^\dagger + b_3 Y_d Y_d^\dagger Y_u Y_u^\dagger + \ldots \right), \\
\tilde{m}^2_{U_R} = \tilde{m}^2 \left( a_2 \mathbb{I} + b_5 Y_d^\dagger Y_u + \ldots \right), \\
A_u = A \left( a_3 \mathbb{I} + b_6 Y_d Y_d^\dagger + \ldots \right) Y_d, \\
\]

and similarly for the down-type terms. The dimensional parameters $\tilde{m}$ and $A$, expected to be in the range few 100 GeV – 1 TeV, set the overall scale of the soft-breaking terms. In Eq. (3.13) we have explicitly shown all independent flavor structures which cannot be absorbed into a redefinition of the leading terms (up to tiny contributions quadratic in the Yukawas of the first two families), when $\tan \beta$ is not too large and the bottom Yukawa coupling is small, the terms quadratic in $Y_d$ can be dropped.

In a bottom-up approach, the dimensionless coefficients $a_i$ and $b_i$ should be considered as free parameters of the model. Note that this structure is renormalization-group invariant: the values of $a_i$ and $b_i$ change according to the Renormalization Group (RG) flow, but the general structure of Eq. (3.13) is unchanged. This is not the case if the $b_i$ are set to zero, corresponding to the so-called hypothesis of flavor universality. In several explicit mechanism of supersymmetry breaking, the condition of flavor universality holds at some high scale $M$, such as the scale of Grand Unification in the CMSSM (see below) or the mass-scale of the messenger particles in gauge mediation (see Ref. [69]). In this case non-vanishing $b_i \sim (1/4\pi)^2 \ln M^2/M^2$ are generated by the RG evolution. As recently pointed out in Ref. [70] the RG flow in the MSSM-MFV framework exhibit quasi infra-red fixed points: even if we start with all the $b_i = O(1)$ at some high scale, the only non-negligible terms at the TeV scale are those associated to the $Y_a Y_a^\dagger$ structures.
If we are interested only in low-energy processes we can integrate out the supersymmetric particles at one loop and project this theory into the general MFV effective theory approach discussed before. In this case the coefficients of the dimension-six effective operators written in terms of SM and Higgs fields are computable in terms of the supersymmetric soft-breaking parameters. The typical effective scale suppressing these operators (assuming an overall coefficient $1/\Lambda^2$) is $\Lambda \sim 4\pi\tilde{m}$. Since the bounds on $\Lambda$ within MFV are in the few TeV range, we then conclude that if MFV holds, the present bounds on FCNCs do not exclude squarks in the few hundred GeV mass range, i.e. well within the LHC reach.

2.2 The CMSSM framework.

The CMSSM, also known as mSUGRA, is the supersymmetric extension of the SM with the minimal particle content and the maximal number of universality conditions on the soft-breaking terms. At the scale of Grand Unification ($M_{\text{GUT}} \sim 10^{16}$ GeV) it is assumed that there are only three independent soft-breaking terms: the universal gaugino mass ($\tilde{m}_{1/2}$), the universal trilinear term ($A$), and the universal sfermion mass ($\tilde{m}_0$). The model has two additional free parameters in the Higgs sector (the so-called $\mu$ and $B$ terms), which control the vacuum expectation values of the two Higgs fields (determined also by the RG running from the unification scale down to the electroweak scale). Imposing the correct $W$- and $Z$-boson masses allow us to eliminate one of these Higgs-sector parameters, the remaining one is usually chosen to be $\tan \beta$. As a result, the model is fully specified in terms of the three high-energy parameters $\{\tilde{m}_{1/2}, \tilde{m}_0, A\}$, and the low-energy parameter $\tan \beta$. This constrained version of the MSSM is an example of a SUSY model with MFV. Note, however, that the model is much more constrained than the general MSSM with MFV: in addition to be flavor universal, the soft-breaking terms at the unification scale obey various additional constraints (e.g. in Eq. (3.13) we have $a_1 = a_2$ and $b_i = 0$).

In the MSSM with $R$ parity we can distinguish five main classes of one-loop diagrams contributing to FCNC and CP violating processes with external down-type quarks. They are distinguished according to the virtual particles running inside the loops: $W$ and up-quarks (i.e. the leading SM amplitudes), charged-Higgs and up-quarks, charginos and up-squarks, neutralinos and down-squarks, gluinos and down-squarks. Within the CMSSM, the charged-Higgs and chargino exchanges yield the dominant non-standard contributions.

Given the low number of free parameters, the CMSSM is very predictive and phenomenologically constrained by the precision measurements in flavor physics. The most powerful low-energy constraints come from $B \to Xs\gamma$, $B_s \to \mu^+\mu^-$, and $B^+ \to \tau^+\nu$. In particular, as illustrated in Fig. 2.4 (left), $B_s \to \mu^+\mu^-$ does provide a very significant constraint for large values of $\tan \beta$.

It is worth to stress that as long as we relax the strong universality assumptions of the CMSSM, the phenomenology of the model can vary substantially. An illustration of this statement is provided by the the two panels in Fig. 2.4 where we compare the predictions for $B_s \to \mu^+\mu^-$ in the CMSSM and in its minimal variation, the so-called Non-Universal Higgs Mass (NUHM) scenario. In the latter case only the condition of universality for the soft breaking terms in the Higgs sectors is relaxed, increasing by one unit the number of free parameters of the model. As can be noted, the difference is substantial (in both cases all existing constraints are satisfied). This also illustrate how precise data from flavor physics are essential to discriminate different versions of the MSSM.

2.3 The Mass Insertion Approximation in the general MSSM.

Flavor universality at the GUT scale is not a general property of the MSSM, even if the model is embedded in a Grand Unified Theory. If this assumption is relaxed, new interesting phenomena can occur in flavor physics. The most general one is the appearance of gluino-mediated one-loop contributions to FCNC amplitudes [71].

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3 More precisely, for each choice of $\{\tilde{m}_{1/2}, \tilde{m}_0, A, \tan \beta\}$ there is a discrete ambiguity related to the sign of the $\mu$ term.
The main problem when going beyond simplifying assumptions, such as flavor universality or MFV, is the proliferation in the number of free parameters. A useful model-independent parametrization to describe the new phenomena occurring in the general MSSM with R parity conservation is the so-called mass insertion (MI) approximation \[72\]. Selecting a flavor basis for fermion and sfermion states where all the couplings of these particles to neutral gauginos are flavor diagonal, the new flavor-violating effects are parametrized in terms of the non-diagonal entries of the sfermion mass matrices. More precisely, denoting by \( \Delta \) the off-diagonal terms in the sfermion mass matrices (i.e. the mass terms relating sfermions of the same electric charge, but different flavor), the sfermion propagators can be expanded in terms of \( \delta = \Delta / \tilde{m}^2 \), where \( \tilde{m} \) is the average sfermion mass. As long as \( \Delta \) is significantly smaller than \( \tilde{m}^2 \) (as suggested by the absence of sizable deviations form the SM), one can truncate the series to the first term of this expansion and the experimental information concerning FCNC and CP violating phenomena translates into upper bounds on these \( \delta \)'s \[73\].

The major advantage of the MI method is that it is not necessary to perform a full diagonalization of the sfermion mass matrices, obtaining a substantial simplification in the comparison of flavor-violating effects in different processes. There exist four type of mass insertions connecting flavors \( i \) and \( j \) along a sfermion propagator: \( (\Delta_{ij})_{LL} \), \( (\Delta_{ij})_{RR} \), \( (\Delta_{ij})_{LR} \), and \( (\Delta_{ij})_{RL} \). The indexes \( L \) and \( R \) refer to the helicity of the fermion partners.

In most cases the leading non-standard amplitude is the gluino-exchange one, which is enhanced by one or two powers of the ratio \( (\alpha_{\text{strong}} / \alpha_{\text{weak}}) \) with respect to neutralino- or chargino-mediated amplitudes. When analysing the bounds, it is customary to consider one non-vanishing MI at a time, barring accidental cancellations. This procedure is justified a posteriori by observing that the MI bounds have typically a strong hierarchy, making the destructive interference among different MIs rather unlikely. The bound thus obtained from recent measurements in \( B \) and \( K \) physics are reported in Tab. 3.2. The bounds mainly depend on the gluino and on the average squark mass, scaling as the inverse mass (the inverse mass square) for bounds derived from \( \Delta F = 2 \) (\( \Delta F = 1 \)) observables.

The only clear pattern emerging from these bounds is that there is no room for sizable new sources of flavor-symmetry breaking around the TeV scale. However, it is too early to draw definite conclusions, especially given we have no positive evidences of supersymmetry so far: the smallness of the bounds could be due to some approximate symmetry, if the scale of the soft-breaking terms is not far from the TeV, or it could simply be an indirect indication of a heavy scale for the superpartners.

### 3 Flavor protection in models with partial compositeness

So far we have assumed that the suppression of flavor-changing transitions beyond the SM can be attributed to a flavor symmetry, and a specific form of the symmetry-breaking terms. An interesting alternative is the possibility of a generic dynamical suppression of flavor-changing interactions, related to the weak mixing of the light SM fermions with the new dynamics at the TeV scale. A mechanism of

| \( q \) | \( i \) | \( (\delta_{ij})_{MM} \) | \( q \) | \( i \) | \( (\delta_{ij})_{LR} \) |
|---|---|---|---|---|---|
| \( d \) | 12 | 0.03 | \( d \) | 12 | \( 2 \times 10^{-4} \) |
| \( d \) | 13 | 0.2 | \( d \) | 13 | 0.08 |
| \( d \) | 23 | 0.6 | \( d \) | 23 | 0.01 |
| \( u \) | 12 | 0.1 | \( u \) | 12 | 0.02 |

Table 3.2: The phenomenological upper bounds on \( (\delta_{ij})_{MM} \) (left) and \( (\delta_{ij})_{LR} \) (right), where \( q = u, d \) and \( M = L, R \). The constraints are given for \( m_{\tilde{q}} = 1 \) TeV and \( \tilde{m}^2 / m_{\tilde{q}}^2 = 1 \). The bounds are obtained assuming that the phases suppress the imaginary parts by a factor \( \sim 0.3 \) (see Ref. \[5\] for more details).
this type is the so-called RS-GIM mechanism occurring in models with a warped extra dimension. In this framework the hierarchy of fermion masses, which is attributed to the different localization of the fermions in the bulk [74], implies that the lightest fermions are those localized far from the infra-red (SM) brane. As a result, the suppression of FCNCs involving light quarks is a consequence of the small overlap of the light fermions with the lightest Kaluza-Klein excitations [75].

As shown in [76] (see also [77]), also the general features of this class of models can be described by means of an effective theory approach. The two main assumptions of this approach are the following:

– There exists a (non-canonical) basis for the SM fermions where their kinetic terms exhibit a rather hierarchical form:

\[ L^\text{quarks}_\text{kin} = \sum_{\Psi = Q_L, U_R, D_R} \Psi Z^{-2}_\Psi \bar{p} \Psi, \]

\[ Z_\Psi = \text{diag}(z^{(1)}_\Psi, z^{(2)}_\Psi, z^{(3)}_\Psi), \quad z^{(1)}_\Psi \ll z^{(2)}_\Psi \ll z^{(3)}_\Psi \lesssim 1. \]  

(3.14)

– In such basis there is no flavor symmetry and all the flavor-violating interactions, including the Yukawa couplings, are \( \mathcal{O}(1) \).

Once the fields are transformed into the canonical basis, the hierarchical kinetic terms act as a distorting lens, through which all interactions are seen as approximately aligned on the magnification axes of the lens. The hierarchical \( z^{(i)}_\Psi \) can be interpreted as the effect of the mixing of an elementary SM-like sector of massless fermions with a corresponding set of heavy composite fermions: the elementary fermions feel the breaking of the electroweak (and flavor) symmetry only via this mixing.

The values of the \( z^{(i)}_\Psi \) can be deduced, up to an overall normalization, from the known structure of the Yukawa couplings, that can be decomposed as follows

\[ Y_{ij}^u \propto z^{(i)}_Q z^{(j)}_U, \quad Y_{ij}^d \propto z^{(i)}_Q z^{(j)}_D. \]  

(3.15)

Inverting such relations we can express the \( z^{(i)}_\Psi \) combinations appearing in the effective couplings of dimension-six operators involving SM fields [e.g. the combination \( (z^{(1)}_Q z^{(2)}_Q)^2 \) for the operator \( (\bar{s}_L \gamma_\mu d_L)^2 \), etc...] into appropriate powers of quark masses and CKM angles. The resulting suppression of FCNC amplitudes turns out to be quite effective being linked to the hierarchical structure of the SM Yukawa couplings.

As anticipated, this construction provide an effective description of a wide class of models with a warped extra dimension or, equivalently, four-dimensional models with the mixing between a composite and an elementary sector. However, it should be stressed that this mechanism is not a general feature of such models: as shown for instance in [54], also in extra-dimensional (partial-composite) models is possible to postulate the existence of additional symmetries and, for instance, recover a MFV structure.

The dynamical mechanism of hierarchical fermion profiles is quite effective in suppressing FCNCs beyond the SM. In particular, it can be shown that all the dimensions-six FCNC left-left operators, such as the \( \Delta F = 2 \) terms in Eq. (1.21), have the same parametric suppression as in MFV [76]. However, a residual problem is present in the left-right operators contributing to CP-violating observables in the kaon or charm system. On the one hand, some tuning is need to avoid the bounds from \( \epsilon_K [78] \) and \( \epsilon'/\epsilon_K [79] \). On the other hand, in such class of is not difficult to generate a sizable contribution to \( \Delta a_{\text{CP}} \) able to saturate the present experimental result [77].

Contrary to most of the models discussed before, in this framework no significant NP effects in the \( B \) system are expected. Sizable non-standard contributions in the \( K \) and \( D \) systems could be hidden by the present theoretical uncertainties. As a result, improving our theoretical description of low-energy flavor dynamics could be the tool to reveal the presence of physics beyond the SM.
Chapter 4

Conclusions

The absence of significant deviations from the SM in quark flavor physics is a key information about any extension of the SM. Only models with a highly non generic flavor structure can both stabilize the electroweak sector and, at the same time, be compatible with flavor observables. In such models we expect new particles within the LHC reach; however, the structure of the new theory cannot be determined using only the high-$p_T$ data from LHC. As illustrated in these lectures, there are still various open questions about the flavor structure of the model that can be addressed only at low energies, and in particular via $B$, $D$ and $K$ decays.

The set of flavor-physics observables to be measured with higher precision, and the rare transitions to be searched for is limited, if we are interested only on physics beyond the SM. But is far from being a small set. As discussed in these lectures, we still have a limited knowledge about CP violation in the $B_s$ and $D$ systems. Despite significant recent progress, new-physics effects could still be hidden in the helicity suppressed $B_{s,d} \rightarrow \ell^+\ell^-$ decays. Last but not least, a systematic reduction in the determination of the SM Yukawa couplings, such as the determination of $\gamma$ from $B \rightarrow D K$ decays, could possibly reveal non-standard effects in observables that we have already measured well but we are not able yet to predict with corresponding accuracy, such as $\epsilon_K$ or the $B_d$ mixing phase.

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References

[1] G. Isidori, B Physics in the LHC Era, arXiv:1001.3431 [hep-ph].
[2] A. J. Buras, arXiv:hep-ph/0505175. M. Neubert, arXiv:hep-ph/0512222. Y. Nir, arXiv:0708.1872 [hep-ph]; O. Gedalia and G. Perez, arXiv:1005.3106 [hep-ph]; Y. Grossman, arXiv:1006.3534 [hep-ph].
[3] M. Antonelli et al., arXiv:0907.5386 [hep-ph].
[4] M. Artuso et al., Eur. Phys. J. C 57 (2008) 309 [arXiv:0801.1833 [hep-ph]].
[5] G. Isidori, Y. Nir and G. Perez, Ann. Rev. Nucl. Part. Sci. 60 (2010) 355 [arXiv:1002.0900 [hep-ph]].
[6] R. Aaij et al. [LHCb Collaboration], arXiv:1208.3355 [hep-ex].
[7] N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963).
[8] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).
[9] L. L. Chau and W. Y. Keung, Phys. Rev. Lett. 53 (1984) 1802.
[10] L. Wolfenstein, Phys. Rev. Lett. 51, 1945 (1983).
[11] A. J. Buras, M. E. Lautenbacher, and G. Ostermaier, Phys. Rev. D 50, 3433 (1994) [arXiv:hep-ph/9403384].
[12] J. Charles et al. [CKMfitter Collaboration], Eur. Phys. J. C 41, 1 (2005) [hep-ph/0406184], online update at http://www.slac.stanford.edu/xorg/ckmfitter/
[13] M. Bona et al. [UTfit Collaboration], JHEP 0803 (2008) 049 [arXiv:0707.0636], online update at http://www.utfit.org/
[14] S. L. Glashow, J. Iliopoulos and L. Maiani, Phys. Rev. D 2, 1285 (1970).
[15] G. Buchalla, A. J. Buras and M. E. Lautenbacher, Rev. Mod. Phys. 68 (1996) 1125 [arXiv:hep-ph/9512380].
[16] H. Georgi, Phys. Lett. B 240 (1990) 447.
[17] C. Davies, PoS LATTICE 2011 (2011) 019 [arXiv:1203.3862 [hep-lat]].
[18] B. Blossier et al. [ALPHA Collaboration], JHEP 1209 (2012) 132 [arXiv:1203.6516 [hep-lat]].
[19] V. Lubicz and C. Tarantino, Nuovo Cim. 123B (2008) 674 [arXiv:0807.4605 [hep-lat]].
[20] A. J. Buras, J. Girrbach, D. Guadagnoli and G. Isidori, Eur. Phys. J. C 72 (2012) 2172 [arXiv:1208.0934 [hep-ph]].
[67] S. P. Martin, arXiv:hep-ph/9709356.
[68] L. J. Hall and L. Randall, Phys. Rev. Lett. 65, 2939 (1990).
[69] G. F. Giudice and R. Rattazzi, Phys. Rept. 322 (1999) 419 [arXiv:hep-ph/9801271].
[70] P. Paradisi, M. Ratz, R. Schieren and C. Simonetto, Phys. Lett. B 668 (2008) 202 [arXiv:0805.3989 [hep-ph]]; G. Colangelo, E. Nikolaidakis and C. Smith, Eur. Phys. J. C 59 (2009) 75 [arXiv:0807.0801 [hep-ph]].
[71] J. R. Ellis and D. V. Nanopoulos, Phys. Lett. B 110, 44 (1982); R. Barbieri and R. Gatto, Phys. Lett. B 110, 211 (1982).
[72] L. J. Hall, V. A. Kostelecky and S. Raby, Nucl. Phys. B 267, 415 (1986).
[73] F. Gabbiani, E. Gabrielli, A. Masiero and L. Silvestrini, Nucl. Phys. B 477, 321 (1996) [arXiv:hep-ph/9604387].
[74] N. Arkani-Hamed and M. Schmaltz, Phys. Rev. D 61, 033005 (2000) [arXiv:hep-ph/9903417]; T. Gherghetta and A. Pomarol, Nucl. Phys. B 586, 141 (2000) [arXiv:hep-ph/0003129]; S. J. Huber and Q. Shafi, Phys. Lett. B 498, 256 (2001) [arXiv:hep-ph/0010195].
[75] K. Agashe, G. Perez and A. Soni, Phys. Rev. D 71, 016002 (2005) [arXiv:hep-ph/0408134], Phys. Rev. Lett. 93, 201804 (2004) [arXiv:hep-ph/0406101]; R. Contino, T. Kramer, M. Son and R. Sundrum, JHEP 0705 (2007) 074 [arXiv:hep-ph/0612180].
[76] S. Davidson, G. Isidori and S. Uhlig, Phys. Lett. B 663, 73 (2008) [arXiv:0711.3376 [hep-ph]].
[77] B. Keren-Zur, P. Lodone, M. Nardecchia, D. Pappadopulo, R. Rattazzi and L. Vecchi, Nucl. Phys. B 867 (2013) 429 [arXiv:1205.5803 [hep-ph]].
[78] C. Csaki, A. Falkowski and A. Weiler, JHEP 0809, 008 (2008) [arXiv:0804.1954 [hep-ph]]; M. Blanke et al., JHEP 0903, 001 (2009) [arXiv:0809.1073 [hep-ph]]; M. Bauer et al., arXiv:0811.3678 [hep-ph].
[79] O. Gedalia, G. Isidori and G. Perez, arXiv:0905.3264 [hep-ph].