ABSTRACT

We study the properties of branes in supergravity theory. We investigate a class of systems consisting of an M5-brane in the Kaluza-Klein monopole background with 1/4 supersymmetry in 11-dimensions. In the near core region of the KK-monopoles, the exact supergravity solution corresponding to each of these configurations is obtained. Then we argue the compactified 10-dimensional systems and suggest a way of unambiguous identification of branes in this background. Here the location of Dirac string type singularity accompanied by the D6-branes plays an important role. The method is essentially the same as that of \((p,q)\)5-branes or \((p,q)\)-strings within the 7-brane background in the IIB theory. We also argue the phenomena of D4-brane creation from D6-branes.
1 Introduction

It has been well-known that there could be various types of extended objects in string theory and 11-dimensional M-theory. D-branes \[1\] in superstring theories made us possible to study the duality relations among different theories. In particular, duality relation between gauge theory and supergravity or superstring theory, particularly AdS/CFT correspondence \[2\], has been investigated by using the D-branes as intermediates. Moreover, it has been discussed so much that moduli space structure of gauge theory can be read from the various brane configurations in background of superstring theory or M-theory \[3, 4\]. Thus, it is important to study the properties of branes in various backgrounds in order to deepen the understanding of gauge theory and string theory in the above viewpoints.

Branes in 10-dimensional type IIA string theory originate from 11-dimensional M-theory at least in the sense of low energy effective supergravity theory \[5\]. For example, D4-branes and solitonic 5-branes (NS5-branes) in IIA supergravity theory are obtained by dimensional reduction of solitonic M5-branes, and fundamental strings and D2-branes are from membranes. As for D6-branes which carry Kaluza-Klein magnetic charges, original 11-dimensional counterpart is Kaluza-Klein monopole solution \[6, 7, 8\]. It is described as the 11-dimensional background which is direct product of 7-dimensional Minkowski space \(M_{1,6}\) and the Euclidean Taub-NUT space \(M_{TN}\).

We can further consider stable M-theory backgrounds where several different types of branes live together. A family of configurations representing process of brane creation from another brane belongs to this class \[3, 4, 9, 10\]. However, since little is known about the explicit supergravity solutions corresponding to such complicated configurations, the process of brane creation in terms of supergravity is still not understood well enough.

Now, consider a supersymmetric configuration of M-theory which has \(N\) coincident KK-monopoles and an M5-brane of world-volume \(R^{1,3} \times \Sigma\) with \(\Sigma \subset M_{TN}\). Upon compactification to 10-dimensional IIA theory, this becomes a configuration of an NS5-brane, a D4-brane or some bound state of these two in the background of \(N\) D6-branes. Of such a class of systems, there is a family of configurations representing brane creation phenomenon. In a flat spacetime background, it is represented as the following process: If an NS5-brane of world-volume \(\{x_0, \ldots, x_3, x_7, x_8\}\) crosses a D6-brane of world-volume \(\{x_0, \ldots, x_3, x_4, x_5, x_6\}\), a new finite D4-brane between these two branes is created. Here \(x_i\) (\(i = 0, \ldots, 9\)) denote the spacetime coordinates. This process is U-dual to the original Hanany-Witten configuration \[3\]. An attempt of investigating such a phenomenon in 11-dimensional viewpoint was done in e.g., refs.\[4, 10\]. In particular, in ref.\[10\], one parameter family of M5-branes in the supergravity background of a KK-monopole and its compactified counterpart is investigated.

On the other hand, in another context, explicit supergravity solutions describing M2- or M5-branes localized near core of KK-monopoles were constructed by using the fact that the metric becomes flat in the vicinity of KK-monopoles \[11, 12\]. The solution of an M5-brane with world-volume \(R^{1,3} \times \Sigma_0 (\Sigma_0 \subset M_{TN})\) near core of KK-monopoles was found in ref.\[12\]. It is naturally expected that the near core version of brane creation phenomenon explained in the last paragraph can be described by dimensional reduction
of this solution. In practice, in refs. [12, 13], some discussion about brane creation in 10-dimensional viewpoint was done.

It was pointed out in ref. [12] that in the near core region of KK-monopoles, there is some ambiguity in the definition of Ramond-Ramond four-form field strength and NS-NS three-form field strength. This means that the identification of D4-branes and NS5-branes cannot be done exactly. In ref. [13], this problem was further studied and a resolution by introducing a certain non-conserved charge was suggested. Using this procedure, it was argued there that an NS5-brane is transmuted into a D4-brane in a certain limit and that this phenomenon is a near core version of the brane creation. There still remains a problem in the sense that the argument is restricted in the near core region of KK-monopoles and that the role of non-conserved charge is unclear.

Our aim of the present paper is to confirm the definition of brane current (i.e., the identification of branes) and to investigate the brane creation phenomena in 10-dimensions for systems of an M5-brane in the background of N KK-monopoles. As for the definition of current associated with branes, we adopt the natural definition of current which assigns conserved charge to branes in the background of D6-branes. The ambiguity presented above can be resolved by noticing that the location of Dirac string type singularity coming up from the D6-branes is relevant to the identification of D4-branes. This interpretation is essentially the same as the charge assignment of the systems defined by T-dual of (p, q)5-branes or (p, q)-strings [14] in the 7-brane background [14, 15, 16, 17].

We also give an explicit relation between the complex structures of $M_{TN}$ and some specific complex coordinates of the near core flat space. As a result, it becomes possible to discuss the brane creation process of ref. [10] in terms of exact supergravity solution in the near core region of KK-monopoles. By applying our definition of brane charge and the way of identifying branes, we will argue that D4-branes seem to come up from the Dirac string type singularity and go along the NS5-brane. If we take a limit such that net NS5-brane charge disappears, it can be seen that only D4-branes come up from D6-branes.

The organization of this paper is as follows. We begin in section 2 to review the Kaluza-Klein monopole solutions in 11-dimensional background. We explain the properties of Euclidean Taub-NUT space $M_{TN}$ and explicitly give three independent complex structures by defining holomorphic coordinates corresponding to them. In section 3 we see that in the near core region of KK-monopoles, the background $M_{TN}$ reduces to the flat space. Moreover, we specify the complex coordinates of this flat space so that they are connected to the complex structures of $M_{TN}$. In section 4 we introduce a family of complex one-dimensional curves $\{\Sigma\}$ in $M_{TN}$ which respectively correspond to configurations of an M5-brane of world-volume $R^{1,3} \times \Sigma$ in the KK-monopole backgrounds. Each of the curves are taken to be holomorphic with respect to one of the complex structures. In section 5, we further study such systems by restricting in the near core region of KK-monopoles and give their supergravity solutions. In section 6, we perform compactification of the systems and explain the ambiguity concerning the identification of D4-branes. In section 7 we digress from our systems for a moment and argue the configuration of M-branes in the stringy cosmic string background and its compactification. In particular, we see that the Dirac string type singularity, which appears as IIA counterpart of the branch cut related
to SL(2,\mathbb{Z}) invariance of type IIB theory, has an important role in identifying the type of branes. Then we return to the KK-monopole backgrounds in section 8 and by using the analogy with the way of identifying branes in the stringy cosmic string background, we give an unambiguous identification of D4-branes. In section 9, using our definition of brane currents, we investigate the phenomena of brane creation. In the final section 10, we conclude with some discussion.

2 Kaluza-Klein monopoles and the Taub-NUT space

We now review the 11-dimensional supergravity solution of \( N \) coincident Kaluza-Klein monopoles which represents \( N \) D6-branes after compactifying to 10-dimensions. The solution is given by taking the 11-dimensional spacetime as a product of 7-dimensional Minkowski space \( \mathcal{M}^{1,6} \) and the four dimensional Euclidean multi-centered Taub-NUT space \( M_{TN} [6, 7, 8] \). The Taub-Nut space is a Hyper-Kähler manifold which admits three independent complex structures.

For the Taub-NUT space with \( A_{N-1} \) singularity at \( r = 0 \), the metric is given by

\[
ds_{TN}^2 = V \, d\vec{r} \cdot d\vec{r} + V^{-1} (dx_{11} + \vec{A} \cdot d\vec{r})^2
\]

where \( x_{11} \) is a compact direction whose radius is \( R : x_{11} \sim x_{11} + 2\pi R \), and

\[
\vec{r} = (r_1, r_2, r_3), \quad V = 1 + \frac{NR}{2r}, \quad \vec{\nabla} \times \vec{A} = \vec{\nabla} V.
\]

We explicitly choose \( \vec{A} \) to be

\[
\vec{A} \cdot d\vec{r} = \frac{NR}{2} (\cos \theta - 1) d\psi
\]

where

\[
  r_1 = r \cos \theta, \quad r_2 = r \sin \theta \cos \psi, \quad r_3 = r \sin \theta \sin \psi
\]

and \( 0 \leq \psi \leq 2\pi \). Since the space is hyper-Kähler and has Ricci-flat metric, it solves Einstein equations and admits half of supersymmetry. We will only deal with the bosonic part of the theory.

The metric eq. (1) with eq. (3) has a coordinate singularity at \( \theta = \pi \), i.e., the negative \( r_1 \)-axis. After dimensional reduction, seen in terms of ten dimensions, this singularity is identified as the Dirac string singularity coming up from the D6-branes. This singularity can be moved to another place by carrying out a coordinate transformation. The simplest example is the transformation \((\vec{r}, x_{11}) \rightarrow (\vec{r}, y_{11})\) where \( y_{11} \equiv x_{11} - NR\psi \). Then, by the relation \( dx_{11} + \vec{A} \cdot d\vec{r} = dy_{11} + \vec{A}' \cdot d\vec{r} \) with

\[
\vec{A}' \cdot d\vec{r} = \frac{NR}{2} (\cos \theta + 1) d\psi,
\]

the coordinate singularity is moved to the positive \( r_1 \)-axis.
Now we give explicitly three independent complex structures of the space $M_{TN}$. First we present a natural choice of complex structure for the coordinate system $(\vec{r}, x_{11})$. It is specified by the holomorphic complex variables $(v_0, w_0)$ as \[ v_0 = \frac{1}{NR} (r_2 + ir_3), \] \[ w_0 = \sqrt{r + r_1} \exp \left( \frac{1}{NR} (r_1 + ix_{11}) \right). \] Here we take $v_0$ and $w_0$ to be dimensionless. Remembering $x_{11} \sim x_{11} + 2\pi R$, $w_0^N$ may be more useful than $w_0$ as a coordinate variable. Other two complex structures that are orthogonal to the above one are represented by the complex variables $(v_1, w_1)$ and $(v_2, w_2)$ which are holomorphic with respect to the complex structures respectively:

\[
\begin{align*}
  v_1 &= \frac{1}{NR} (r_1 + ir_2), \\
  w_1 &= \sqrt{r + r_3} \exp \left( \frac{1}{NR} (r_3 + ix_{11}^{(1)}) \right)
\end{align*}
\] (8)

and

\[
\begin{align*}
  v_2 &= \frac{1}{NR} (r_3 + ir_1), \\
  w_2 &= \sqrt{r + r_2} \exp \left( \frac{1}{NR} (r_2 + ix_{11}^{(2)}) \right)
\end{align*}
\] (9)

Here $x_{11}^{(1)}$ and $x_{11}^{(2)}$ are compact coordinates with period $2\pi R$ determined by the differential equations

\[
dx_{11}^{(1)} = dx_{11} - \frac{NR}{2r} \left( \frac{r_2 dr_3 - r_3 dr_2}{r + r_1} - \frac{r_1 dr_2 - r_2 dr_1}{r + r_3} \right) (= dx_{11} - NR d\chi)
\] (10)

and

\[
dx_{11}^{(2)} = dx_{11} - \frac{NR}{2r} \left( \frac{r_2 dr_3 - r_3 dr_2}{r + r_1} - \frac{r_3 dr_1 - r_1 dr_3}{r + r_2} \right).
\] (11)

These equations can be integrated explicitly except on singularity: negative $r_1$ and $r_3$ axes for $x_{11}^{(1)}$, and negative $r_1$ and $r_2$ axes for $x_{11}^{(2)}$. These singularities are due to the coordinate transformation from $(\vec{r}, x_{11})$ (which is singular on negative $r_1$ axis) to $(\vec{r}, x_{11}^{(1)})$ (which is singular on negative $r_3$ axis) or to $(\vec{r}, x_{11}^{(2)})$.

Note that the metric in the coordinate system $(\vec{r}, x_{11}^{(1)})$ is

\[
ds_{TN}^2 = V d\vec{r} \cdot d\vec{r} + V^{-1} (dx_{11}^{(1)} + \vec{A}^\nu \cdot d\vec{r})^2
\] (12)

where

\[
\vec{A}^\nu \cdot d\vec{r} = -\frac{NR}{2r} \frac{r_1 dr_2 - r_2 dr_1}{r + r_3}.
\] (13)

If we compactify along the $x_{11}^{(1)}$ direction instead of $x_{11}$ direction, then the resulting 10-dimensional D6-brane solution has Dirac string singularity on negative $r_3$ axis. The movement of the singularity corresponds to gauge transformation: $A \rightarrow \vec{A}^\nu = \vec{A} + d\chi$. In the following discussion, we fix the compactification direction as along $x_{11}$. 

4
3 Near core region of KK-monopoles

In this section, we see that the metric \( ds^2_{T_N} \) reduces to the flat metric in the near core region of KK-monopoles \(^{[18]}\). We also show that a flat complex coordinate system in this region is exactly related to one of the complex structures described in section 2. Note that by the near core region, we mean the region \( r \ll NR \) in terms of the coordinate system \((\vec{r},x_{11})\).

In the limit \( r/NR \to 0 \), the metric (1) becomes

\[
ds^2_{r \to 0} = \frac{NR}{2r}d\vec{r} \cdot d\vec{r} + \frac{2r}{NR} \left( dx_{11} + \frac{NR}{2} (\cos \theta - 1) d\psi \right)^2
\]

(14)

since \( V (= 1 + \frac{NR}{2r}) \to \frac{NR}{2r} \). By changing variables from \((r,\theta,\psi,x_{11})\) to \((\rho,\tilde{\theta},\tilde{\psi},\tilde{\phi})\) such as

\[
\rho = \sqrt{2NRr}, \quad \tilde{\theta} = \theta, \quad \tilde{\psi} = -\psi + \frac{x_{11}}{NR}, \quad \tilde{\phi} = \frac{x_{11}}{NR},
\]

(15)

the metric (14) can be rewritten:

\[
ds^2_{r \to 0} = d\rho^2 + \rho^2 d\tilde{\theta}^2 + \rho^2 (\sin^2 \tilde{\theta} d\tilde{\psi}^2 + \cos^2 \tilde{\theta} d\tilde{\phi}^2).
\]

(16)

Here the range of the variables is

\[
\rho \geq 0, \quad 0 \leq \tilde{\theta} \leq \frac{\pi}{2}, \quad 0 \leq \tilde{\phi}, \tilde{\psi} \leq 2\pi
\]

(17)

with the \( Z_N \) identification \((\tilde{\phi},\tilde{\psi}) \sim (\tilde{\phi},\tilde{\psi}) + (2\pi/N,2\pi/N)\).

Furthermore, defining the complex variables as\(^{[1]}\)

\[
W = \rho e^{i\tilde{\phi}} \cos \tilde{\theta}, \quad V = \rho e^{-i\tilde{\psi}} \sin \tilde{\theta},
\]

(18)

eq.(16) becomes

\[
ds^2_{r \to 0} = dWd\overline{W} + dVd\overline{V}
\]

(19)

where \((W,V) \sim (W e^{2\pi i/N}, V e^{-2\pi i/N})\). Thus we see that the space is a flat orbifold \(\mathbb{C}^2/Z_N\) which is considered as the ALE space with \(A_{N-1}\) singularity at the origin. For later convenience we represent \((W,V)\) by the original variables:

\[
W = \sqrt{NR} e^{i\frac{2\pi}{N} r_3} \sqrt{r + r_1}, \quad V = \sqrt{NR} e^{-i\frac{2\pi}{N} r_3} \frac{r_2 + ir_3}{\sqrt{r + r_1}} = NR W^{-1} (r_2 + ir_3).
\]

(20)

Note that there is no coordinate singularity in \((W,V)\) space except at the orbifold point. The coordinate singularity \(\theta = \pi\) in the \((\vec{r},x_{11})\) coordinate system corresponds to \(W = \)

\(^{1}\)Here we choose the complex variables \(V\) and \(W\) such that they are connected to holomorphic variables of the complex structure \((v_0,w_0)\).
0, which is completely smooth in the new coordinate system. Similarly, $\theta = 0$, which corresponds to coordinate singularity in the $(\vec{r}, y_{11})$ system, appears as $V = 0$ which is regular in $(W, V)$ space.

Next, we study the relation between $(W, V)$ and the complex structure of $M_{TN}$ represented by $(w_0, v_0)$. The behavior of $w_0$ and $v_0$ in the near core limit is obtained by extracting the lowest order terms of $r_i/NR$ in the eqs.(6) and (7):

$$w_0 \rightarrow \sqrt{\frac{r + r_1}{NR}} e^{i\frac{z_{11}}{NR}} (\equiv w_0(r \to 0)), \quad (22)$$

$$v_0 = \frac{1}{NR} (r_2 + i r_3) (\equiv v_0(r \to 0)). \quad (23)$$

Thus, we see that the complex variables $(W, V)$ are related to the $(w_0, v_0)$ of $M_{TN}$ as

$$w_0(r \to 0) = \frac{1}{NR} W, \quad (24)$$

$$v_0(r \to 0) = \frac{1}{(NR)^2} V W. \quad (25)$$

This means that in the near core limit, we can take $(W, V)$ as variables representing the complex structure.

We can also relate the variables $(W, V)$ with other complex structures of $M_{TN}$. In the near core flat space, a complex structure orthogonal to $(W, V)$ is given by a linear combination of two complex structures:

$$(z_7 + i z_8, z_9 + i z_{10}) \quad \text{and} \quad (z_9 + i z_7, z_8 + i z_{10}) \quad (26)$$

where we introduced real variables $z_i$ as

$$W = z_7 + i z_{10}, \quad V = z_8 + i z_9. \quad (27)$$

In practice, we are able to show as the same way as the above discussion that two complex structures presented in eq.(23) are considered as the limiting representations of the complex structures $(w_1, v_1)$ and $(w_2, v_2)$ of $M_{TN}$ respectively. For example, in the $r/NR \to 0$ limit, the variables $(w_1, v_1)$ become

$$w_{1(r \to 0)} = \frac{1}{NR} W^{(1)}, \quad (28)$$

$$v_1 = \frac{1}{NR} W^{(1)} V^{(1)} \quad (29)$$

where

$$W^{(1)} = \frac{1}{\sqrt{2}}((z_7 + i z_8) + (z_9 + i z_{10})), \quad (30)$$

$$V^{(1)} = \frac{1}{\sqrt{2}}((z_7 + i z_8) - (z_9 + i z_{10})). \quad (31)$$

We again have a relation $(W^{(1)}, V^{(1)}) \sim (W^{(1)} e^{2\pi i/N}, V^{(1)} e^{-2\pi i/N})$. Note that as for a choice of origin of the two periodic coordinates $x_{11}$ and $x_{11}^{(1)}$ in the flat $(W, V)$ space, we have taken $x_{11} \equiv NR \arg(z_7 + i z_{10})$ and $x_{11}^{(1)} \equiv NR \arg(W^{(1)} + V^{(1)})$.  


4 M-branes in the KK-monopole background

We consider to put an M5-brane or an M2-brane in the KK-monopole background $M_{1,6} \times M_{TN}$ such that two spatial dimensions of the M-brane are included in $M_{TN}$, i.e., the world volume of the M5-brane (or M2-brane) is $R^{1,3} \times \Sigma$ (or $R^{1,0} \times \Sigma$) where $\Sigma$ is a two-dimensional surface contained in $M_{TN}$. By the discussion in refs. [4, 19], the curve $\Sigma \subset M_{TN}$ must be holomorphic with respect to a complex structure in order to preserve a certain number, in this case 1/4 of 32, of supersymmetry. From now on, we concentrate on the case of M5-branes.

Now we consider two particular curves in $M_{TN}$ discussed in refs. [10, 20] which are holomorphic with respect to $w_0$ and $v_0$:

\[(A) \quad (w_0)^N = e^{-\frac{b+ia}{R}},\]  
\[(B) \quad (w_0)^N = e^{\frac{b+ia}{R}} (v_0)^N\]

where $b$ and $a$ are some real parameters.

Remember that $M_{TN}$ is spanned by the coordinate system $(\vec{r}, x_{11})$ except on the negative $r_1$ axis. By representing above two curves in this coordinate system, we can investigate the shape of curves as embedding objects in $(r_1, r_2, r_3)$ and the behavior of the parameter $x_{11}$ on these curves. It can easily be seen that the two curves with the same $b$ have exactly the same shape in $(r_1, r_2, r_3)$. The curve (A) is

$$\sqrt{\frac{r + r_1}{NR}} = e^{-\frac{r_1}{NR}}$$

and the curve (B) is represented as the same equation but with $r_1 \to -r_1$. The behavior of the parameter $x_{11} = x_{11}(\vec{r})$ is as follows: The curve (A) is at $x_{11} = -\alpha$, while (B) is winding around $x_{11}$ along the $\psi$ [= $\arg(r_2 + ir_3)$] direction as $x_{11} = NR\psi + \alpha$. The important point is that the locations of coordinate singularity, which will be interpreted as the Dirac string singularity in 10-dimensions, relative to the curves (A) and (B) are different from each other. (See Fig.[4])

In the case of $N=1$, it was argued in refs.[10, 20] that each of these two curves corresponds to a single NS5-brane or a configuration consisting of a finite D4-brane between an NS5-brane and a D6-brane in the compactified 10-dimensions. We postpone the analysis of compactification until section 6. We notice that there are many other possible curves which leave the same number of supersymmetry and have the same shape as the above two curves: We can take the curves holomorphic to any linear combination of all three complex structures of $M_{TN}$. They all have the same shape in $(r_1, r_2, r_3)$ space and are represented by the equations rotating eq.(34) around the $r = 0$ point suitably. The location of the singularity is still on the negative $r_1$ axis. The value of $x_{11}$ on each of the curves is represented as $x_{11} = f(\vec{r}) - \alpha$. The form of the function $f = f(\vec{r})$ is generically not a simple form except for the curve based on the complex structure $(w_0, v_0)$. For example, the curves holomorphic with respect to $(v_1, w_1)$ are given as

\[(A)' \quad (w_1)^N = e^{-\frac{b+ia}{R}},\]  
\[(B)' \quad (w_1)^N = e^{\frac{b+ia}{R}} (v_1)^N.$$
Figure 1: Three representative curves (A), (B) and (A)' are depicted in \((r_1, r_2, r_3)\) space. Wavy lines represent the string like coordinate singularity. The values of \(x_{11}\) are given respectively as (A) \(x_{11} = -\alpha\), (B) \(x_{11} = NR\psi + \alpha\) and (A)' \(x_{11} = NR\chi - \alpha\).

For the curve (A)', \(f(\vec{r}) = NR\chi\) where \(\chi\) is given by eq. (10).

In the following discussion, we only deal with a class of M5-branes specified by the curves we have given above. In particular, we often proceed with discussion by choosing the curves (A), (B) and (A)' as examples.

5 M5-branes in the near core region of KK-monopoles and the supergravity solutions

In the last section, we considered a family of curves which correspond to configurations of an M5-brane in the background of KK-monopoles. Supergravity solutions representing such systems have not been constructed. However, if we restrict ourselves to the near core region of KK-monopoles, solutions can be obtained by using the method of [11, 12]. We know that the near core region of KK-monopole solution is represented by the flat complex coordinates \((W, V)\). Thus, by considering some two-dimensional flat plane \(\Sigma_0\) in this region, supergravity solution of an M5-brane with world-volume \(R_1^3 \times \Sigma_0\) is obtained. By transforming back the coordinates to \((\vec{r}, x_{11})\), we know the behavior of the M5-brane in the near core region of KK-monopoles [12].

Now, from the analysis in section 3, we have the relation between the near core coordinates \((W, V)\) and the complex structures of the whole Taub-NUT space. Using this knowledge, we study the behavior of the class of holomorphic curves considered in the last section in the near core region. In order to do this, parts of the curves must be in the near core region, \textit{i.e.}, the distance between the curve and the core of the monopoles \(r = 0\) has to be much smaller than \(NR\). This is equivalent to the condition \(e^{-b/NR} \ll 1\).

Assuming this, by the relations (24) and (25), the two curves (A) and (B) given in eqs. (32) and (33) can be approximated in the \(r/NR \to 0\) limit as

\[
\begin{align*}
(A) & \quad W^N = e^N, \\
(B) & \quad V^N = e^N
\end{align*}
\]
where
\[ c = NR e^{\frac{b+i\alpha}{NR}}. \]  
(39)

Also, the curves \((A)'\) and \((B)'\) in (B3) and (36) can be rewritten in this region as
\[
\begin{align*}
(\tilde{A})' &\quad (W^{(1)})^N = c^N, \\
(\tilde{B})' &\quad (V^{(1)})^N = c^N.
\end{align*}
\]  
(40, 41)

These are \(N\) copies of flat planes in the flat \((W, V)\) space as is expected. Similarly, all the other curves in the class considered in the previous section are reduced to flat curves in this region. Note that each of these curves is a paraboloid if seen in \((r_1, r_2, r_3)\). For example, the curve \((\tilde{A})\) is
\[
\begin{align*}
r_1 &= -\frac{r_2^2 + r_3^2}{2a} + \frac{a}{2}
\end{align*}
\]  
(42)

where \(a = |c|^2/NR\).

The supergravity solution realizing one of these curves \(\Sigma_0\) as an M5-brane of world-volume \(R^{1,3} \times \Sigma_0\) can be obtained as the same way as in ref. [12]. Also, the explicit forms of four-form field strength \(F_4\) and the current \(J^{(11)}\) of M5-brane can be given [13].

Note that although for any flat plane in the space \((W, V)\) we can obtain the corresponding supergravity solution, we limit ourselves to the class of flat curves such that they are represented as the \(r/NR \to 0\) limit of the curves described in the last section. For example, curves like \((aW + bV)^N = c^N\) or \((aW + bV)^N = c^N\) are excluded unless \(a = 0\) or \(b = 0\).

Now, we give the solution of M5-brane corresponding to the curve \((\tilde{A})\), \(W^N = c^N\), explicitly. The metric is given by
\[
d s_{11}^2 = f_5^{\frac{1}{4}} (\eta_{\mu \nu} dx^\mu dx^\nu + dV d\overline{V}) + f_5^{\frac{2}{3}} (dy^m dy^m + dW d\overline{W}) \]  
(43)

where \(\mu, \nu = 0, \cdots, 3\), \(m = 4, 5, 6\), and
\[
f_5 = 1 + \sum_{l=1}^N k \left( s^2 + |W - c e^{\frac{2\pi i}{N} l}|^2 \right)^{3/2}, \quad s^2 = y_4^2 + y_5^2 + y_6^2. \]  
(44)

Four-form field strength \(F_4\) is
\[
F_{p_1p_2p_3p_4} = 3k \epsilon_{p_1p_2p_3p_4q} \sum_{l=1}^N \left( s^2 + |W - c e^{\frac{2\pi i}{N} l}|^2 \right)^{5/2} \]  
\[ \tilde{y}^q \]  
(45)

where \(\tilde{y}^p = (x_4, x_5, x_6, z_7, z_{10})\). The Hodge dual of current associated with the M5-brane is given by
\[
d F_4 \equiv \tilde{\ast} J_{[5]}^{(11)}
= \sum_{l=1}^N 3k \Omega_4 \delta(y_4) \delta(y_5) \delta(y_6) \delta(W - c e^{\frac{2\pi i}{N} l}) dy_4 \wedge dy_5 \wedge dy_6 \wedge dz_7 \wedge dz_{10}. \]  
(46)
where $\ast$ denotes the 11-dimensional Hodge dual. Similarly, we can give the same information as above for other M5-brane systems.

At this point, we comment on the definition of current with an M5-brane in the background of KK-monopoles besides the near core region. In the generic $r$-finite region, we do not have the supergravity solution and the definite form of the current cannot be obtained. However, only from the knowledge of the location of the M5-brane in a certain coordinate system of $M_{TN}$, e.g., $(\vec{r}, x_{11})$, an approximate form of the Hodge dual of current associated with the M5-brane is determined. That is, for a two dimensional curve described by $g_i(\vec{r}, x_{11}) = 0$ ($i = 1, 2$), Hodge dual of the current of the M5-brane at $g_i = 0$, $y_4 = y_5 = y_6 = 0$ is denoted by the 5-form which has the nonzero value only on the M5-brane:

$$\tilde{\ast} J^{(11)}_{[5]} \sim \delta(y_4)\delta(y_5)\delta(y_6)\delta(g_1)\delta(g_2)dy_4 \land dy_5 \land dy_6 \land dg_1 \land dg_2.$$ (47)

We will analyze the brane identification in 10 dimensions approximately from the knowledge of the form of the wedge product $dg_1 \land dg_2$, in particular, the behavior of the term including $dx_{11}$.

6 Ten-dimensional analysis

We investigate the compactification of the systems of an M5-brane in the KK-monopole background. We fix the compactification direction along $x_{11}$. It is known that an M5-brane becomes an NS5-brane, a D4-brane or a bound state of these two types of branes upon compactification. However it is argued in refs. [12, 13] that there is a sort of subtlety in identification of branes when the original 11-dimensional system has M5-branes and KK-monopoles together. We clarify the problem by using the exact form of metric and four-form field strength in the $r \to 0$ region and by performing the compactification explicitly.

Assuming the isometry along $x_{11}$ direction, 11-dimensional theory is related to 10-dimensions as follows:

$$ds^2_{11} = e^{-\frac{2}{\Phi}} ds^2_{10} + e^{\frac{4}{\Phi}} (dx_{11} + A_{[1]})^2$$

$$F^{(11)}_{[4]} = G_{[4]} + G_{[3]} \land dx_{11}.$$ (49)

Here $ds^2_{10}$ is 10-dimensional string metric, $\Phi$ is 10-dimensional dilaton and $A_{[1]}$ is Ramond-Ramond 1-form potential. As for the four-form field strength, we used the notation $G_{[4]} \equiv dB_{[3]}$ and $G_{[3]} \equiv dB_{[2]}$ where $B_{[3]}$ and $B_{[2]}$ are Ramond-Ramond 3-form potential and NS-NS 2-form potential respectively. There is another description of 10-dimensional four-form field strength $\tilde{G}_{[4]}$ given by

$$F^{(11)}_{[4]} = \tilde{G}_{[4]} + G_{[3]} \land (dx_{11} + A_{[1]})$$ (50)

where $\tilde{G}_{[4]} = G_{[4]} - G_{[3]} \land A_{[1]}$. This definition may be more convenient since $A_{[1]}$ and $G_{[4]}$ couple with each other and the related term in 10-dimensional supergravity action.
is collected by the form $\sim \tilde{G}_2$ after all. There is another advantage in using the latter definition which is related to the ‘gauge invariance.’ In terms of 11-dimensions, the gauge symmetry is represented as a coordinate transformation $(\vec{r}, x_{11}) \rightarrow (\vec{r}, x'_{11})$ where $x'_{11} = x_{11} + \gamma(\vec{r})$. As a metric of the form eq. (48), it is written as gauge transformation of $A[1]$:

$$\begin{align*}
  dx_{11} & \rightarrow dx'_{11} = dx_{11} + d\gamma \\
  A[1] & \rightarrow A'[1] = A[1] - d\gamma.
\end{align*}$$

(51)

It is easily seen that under this gauge transformation, $\tilde{G}_4$ is invariant but $G_4$ is not. We will argue this point later again.

We will apply the general formula of compactification to our configurations given in the last section. In order to compactify the system along $x_{11}$, there should be an isometry along the direction. Thus for any of our M5-brane systems with parameter $c \neq 0$, we have to put infinite number of M5-branes uniformly along the $x_{11}$ direction, i.e., we consider a family of curves with $\alpha = 2\pi R \cdot k/n$ ($k = 1, 2, \cdots n$), and take the limit $n \rightarrow \infty$. Note that if $c = 0$, there is already an isometry along $x_{11}$. For example, for the M5-brane system given by the curve $W^N = c^N$ in eq. (13), we should put infinite M5-branes on the circle of radius $|W| = |c|$, which corresponds to take

$$f_5 = 1 + k' \int_0^{2\pi} (s^2 + (W - |c|e^{i\xi})^2)^{-\frac{3}{2}} d\xi$$

(52)

where $k'$ is some regularized parameter. Then, the compactification can be performed, and $ds^2_{10}$, $\Phi$ and $A[1]$ are calculated according to eq.(15).

Now we define the currents associated with D4-branes and with NS-NS 5-branes in 10-dimensions. The most straightforward definition is

$$dG[3] = *j[5], \quad dG[4] = *j[4]$$

(53)

where $*j[5]$ and $*j[4]$ are the 10-dimensional Hodge dual of the currents of NS-NS 5-branes and D4-branes respectively. The relation of these currents with 11-dimensional current $J^{(11)}$ is obtained by differentiating eq.(49) as

$$\tilde{*j}^{(11)} = *j[4] + *j[5] \wedge dx_{11}.$$  

(54)

In ref.[13], another definition of D4-brane current $\tilde{j}[4]$ is proposed as

$$* \tilde{j}[4] = d\tilde{G}[4] - F[2] \wedge G[3]$$

(55)

where $F[2] = dA[1]$. Two different definitions of D4-brane current coincide with each other when there is no R-R 1-form potential. The main difference is that the latter definition, $\tilde{j}[4]$, respects the gauge invariance eq.(14) of $A[1]$ but the former does not, and that the former assures the conservation of charge but the latter does not. This means that the D4-brane charge defined by the current $\tilde{j}[4]$ remains unchanged if we change the compactification direction as $x_{11} \rightarrow x'_{11}$. In ref.[13], by taking the current $\tilde{j}[4]$ as physically relevant definition of D4-branes, charge non-conservation and brane transmutation are argued.
On the contrary, here we choose a standpoint of interpreting the naive definition of D4-brane current \( j_{[4]} \) as physically relevant one. This viewpoint gives the physical meaning to the Dirac string type singularity associated with \( A_{[1]} \). It seems strange at first sight, however, we can offer a reasonable explanation. First of all, we remember that the gauge invariance of \( A_{[1]} \), whose determination specifies the location of the singularity, is not a symmetry of the compactified 10-dimensional theory in itself, but a symmetry in 11-dimensions as in eqs.\((\text{II})\). In compactifying to 10-dimensions, we have to specify \( x_{11} \) or \( x'_{11} \) definitely as an eleventh direction along which the reduction is executed. It is nothing but fixing of a gauge in terms of (\text{II}). The Dirac string singularity in 10-dimensions cannot be moved to another place without changing the compactification direction. Thus, looking in 10-dimensions, this singularity is not necessary to be unphysical at least concerning the identification of branes. In practice, in our generic configurations, we will see that D4-branes come up from the singularity and go to infinity along an NS5-brane.

Before demonstrating the consequence of the above interpretation of 10-dimensional currents, in the next section we deal with similar configurations in which there exists Dirac string type singularity. Using the models, we examine the relation between the singularity and the brane identification.

7 M-branes in the stringy cosmic string backgrounds

We take the stringy cosmic string solution as 11-dimensional supergravity background. The solution is known to represent a configuration with some parallel D7-branes ([1, 0]-branes) or their SL(2, \( \mathbb{Z} \)) dual \([p, q]\)-7-branes if it is taken as a IIB background \([\text{I}]\). It is also interpreted as the compactification of 12-dimensional F-theory on K3, or non-compact K3, which admits elliptic fibration. We will only deal with non-compact K3 manifolds, especially with at most one point-like singularity of \( A_N \) type as a total space. We first explain the corresponding IIB background, and then, by using the conjectured duality between \( \text{F}/(K3 \times S^1) \) and \( \text{M}/K3 \), or IIB/\( S^1 \) and \( \text{M}/T^2 \), we construct the corresponding background in terms of M-theory.

It is important that the complex modulus \( \tau \) of the fiber torus of the non-compact K3 is only determined up to \( \text{SL}(2, \mathbb{Z}) \) as a function on the base manifold which is isomorphic to some orbifold of \( \mathbb{C} \). This \( \text{SL}(2, \mathbb{Z}) \) is the IIB S-duality group and parametrized as \( \tau = \chi + i e^{-\phi} \) where \( \chi \) is axion and \( \phi \) is dilaton field. If we try to assign some definite value of \( \tau \) at each point in the base manifold, there should exist branch cut coming up from each 7-brane to infinity \([\text{I}]\). The behavior of the \((p, q)\)-string, or some extended configuration including three string junctions, in the 7-brane background has been investigated in various situations. Here a \((p, q)\)-string is a bound state of \( p \) fundamental strings and \( q \) D-strings and can end on a \([p, q]\)-7-brane.

Here we focus on the role of branch cut and take some duality transformation to obtain the M-theory counterpart. Moreover, since we want M5-branes in the dual 11-dimensional theory, we apply the argument to the case of \((p, q)\)5-branes of world-volume \( R^{1,4} \times l \) where \( l \) is a one dimensional object in the base space. Here a bound state of \( p \) D5-branes and \( q \) NS5-branes is denoted by a \((p, q)\)5-brane. If a \((p, q)\)5-brane goes around
N D7-branes counterclockwise, the brane is converted into the \((p + Nq, q)\)5-brane with the shift \(\tau \rightarrow \tau + N\) as in Fig.2. This means that if the 5-brane crosses the branch cut singularity, then the charge assignment of the brane changes as in Fig.2 and if it crosses \(N\) 7-branes, \(Nq\) new D5-branes are created from the 7-branes to form a three 5-brane junction [21]. This is regarded as U-dual of original Hanany-Witten effect where a D3-brane is created if D5-brane crosses an NS5-brane [3].

![Diagram](image)

Figure 2: \((p, q)\)5-branes in the background of \(N\) coincident D7-branes seen in the plane transverse to the D7-branes. The charge assignment of 5-branes changes if the branes cross the branch cut. Brane creation from D7-branes are explained in terms of charge conservation of (a) and (b). T-dualized IIA theory picture of this configuration is depicted in the right hand side of each figure.

Using the duality conjecture between F-theory and M-theory, we can obtain the M-theory background from this 7-brane background. Consequently, the fiber torus turns up as a geometrical object in 11-dimensions, i.e., the background becomes elliptic (non-compact) K3 manifold times 7-dimensional Minkowski space. There is another method to reach the M-theory background; beginning with the explicit background of 7-branes, we perform T-duality along the appropriate world component of 7-branes as well as \((p, q)\)5-branes, and then go up to 11-dimensions [22]. The \((p, q)\)5-brane turns into a bound state of \(p\) D4-branes and \(q\) NS5-branes in IIA theory, and then becomes an M5-brane winding around \((p, q)\)-cycle of the torus. In particular, if we begin with \(N\) D7-branes, we obtain the IIA background with \(N\) D6-branes and thus we see that this background resembles the compactification of KK-monopole background in the sense that both deal with D6-branes. However, unlike the KK-monopole case, the number \(N\) must be less than 24 for geometrical reason.

Since the supergravity solutions including both 7-branes and \((p, q)\)5-branes are not known, we study the configuration by putting 5-branes as probes in the background of 7-branes. Since this configuration reserves some supersymmetry, the M5-brane transformed from IIB \((p, q)\)5-brane must be holomorphically embedded in the corresponding M-theory background [19]. We use the same embedding as in the case of M2-branes given in ref. [22].

For the background with \(N\) D7-branes at the origin of the base space spanned by
(z, \bar{z}), the metric in terms of dual 11-dimensional theory is
\[
d s_{\text{scs}}^2 = \eta_{\mu\nu} dx^\mu dx^\nu + e^{\Phi(z, \bar{z})} dz d\bar{z} + \tau_2^{-1} d\zeta d\bar{\zeta}
\]
where
\[
e^{\Phi(z, \bar{z})} = \tau_2 \eta^2 |z|^{-N},
\]
\[\tau = \tau_1 + i\tau_2 \text{ and } \zeta = \bar{u} + \tau \bar{v}.\]
The torus is spanned by the periodic coordinates \((\bar{u}, \bar{v}) \sim (\bar{u} + 2\pi R, \bar{v} + 2\pi R)\). And the curve wound along \((p, q)\)-cycle of the torus is represented as \(q\bar{u} - p\bar{v} = \text{const.}\). The modulus \(\tau\) is some holomorphic function of \(z\), and the behavior in the \(z \to 0\) limit is
\[
\tau(z) \sim \frac{N}{2\pi i} \log z,
\]
which requires a branch cut from the origin. As we mentioned earlier, this branch cut is coordinate singularity in 11-dimensions. Other notations are the same as ref.\([16, 17]\). By compactifying this metric along \(\bar{u}\) direction, we obtain the IIA background of D6-branes with
\[
e^{\frac{2}{3}\Phi} = \tau_2^{-1},
\]
\[A_{[1]} = \tau_1 d\bar{v} \sim 0 \frac{N}{2\pi} \arg z d\bar{v}.
\]
We see that the coordinate singularity is now interpreted as the Dirac string singularity with respect to one-form potential \(A_{[1]}\) in IIA theory. In the three-dimensional space spanned by \((z, \bar{z}, \bar{v})\), the singularity is two-dimensional, which is different from the case of KK-monopole solution.

Now in this background we put an M5-brane of world-volume \(R^{1,3} \times \Sigma\) where \(\Sigma\) is a holomorphic curve winding around the \((p, q)\)-cycle of the torus. We choose the complex structure such that it is orthogonal to that defined by \((z, \zeta)\) \([22]\). If we use the discussion in the end of section 5, the Hodge dual of the current associated with this M5-brane is given as
\[
\tilde{*}J^{(11)}_{\Sigma} \sim \delta(y_4)\delta(y_5)\delta(y_6)\delta(q\bar{u} - p\bar{v})\delta(g) dy_4 \wedge dy_5 \wedge dy_6 \wedge d(q\bar{u} - p\bar{v}) \wedge dg.
\]
Here \(g = g(z, \bar{z})\) is defined as a real one-dimensional line in the \(z\)-plane representing the location of branes in the plane \([\Sigma]\). If the brane is located over the both sides of the singularity as in Fig.2(a), coordinates \((\bar{u}, \bar{v})\) are discontinuous at the singularity, and the definition of \((p, q)\)-cycle changes at the singularity: If we cross the singularity counterclockwise, \((p, q)\)-cycle changes to \((p + Nq, q)\)-cycle.

In the flat spacetime \(M^{1,8} \times T^2\), an M5-brane winding along \((p, q)\)-cycle of the torus leads to the bound state of \(p\) D4-branes and \(q\) NS5-branes upon compactification along the \(p\)-cycle (\(\bar{u}\) direction). This formula is also applicable to our case and the current associated with D4-brane charge is represented by the NS5-brane current if \(q \neq 0\) as
\[
* j_{\Sigma} = \frac{p}{q} * j_{[5]} \wedge \frac{p}{q} d\bar{v}
\]
where we used eq.\([54]\).
If the M5-brane is put such that it crosses the singularity, the interpretation of D4-brane charge after compactification depends on the place of the brane, left or right of the singularity. Note that the current associated with NS5-brane is determined independently of the location of the singularity. In the case of the brane depicted in Fig.2(a), D4-brane current is given as

\[
\begin{align*}
  j^l_{[4]} &= - * j_{[5]} \wedge \frac{p_0}{q} d\tilde{v}, \\
  j^r_{[4]} &= - * j_{[5]} \wedge (\frac{p_0}{q} + N) d\tilde{v}
\end{align*}
\]

where the superscript \(l\) and \(r\) denote left and right hand side of the singularity respectively as in Fig.2(a).

From this assignment of currents, it can be seen as if new D4-brane charges are created from the string-like singularity, though there is no discontinuity in the original 11-dimensional viewpoint. This is not so peculiar since if we perform T-duality transformation along \(q\)-cycle (\(\tilde{v}\) direction), the determination of current is consistent with that of \((p,q)\)5-branes in terms of the \(\text{SL}(2,\mathbb{Z})\) symmetry of IIB background with 7-branes.

8 Definition of D4-brane charges : Identification of D4-branes

We know from the discussion of M5-branes in the stringy cosmic string background where the location of the Dirac string type singularity plays an important role in identifying the brane as an NS5-brane, a D4-brane, or a bound state of them in the compactified 10-dimensions. Now we return to the case of KK-monopole background and consider the identification of 10-dimensional brane dimensional reduced from an M5-brane.

We explicitly argue three types of M5-branes specified by the following curves in \(M_{TN}\) among others:

\[
\begin{align*}
  (i) \quad f_1 &\equiv |w_0| - e^{-\frac{b}{NR}} = 0 \quad [\Leftrightarrow (A)] \\
  (ii) \quad f_2 &\equiv \left| \frac{v_0}{w_0} \right| - e^{-\frac{b}{NR}} = 0 \quad [\Leftrightarrow (B)] \\
  (iii) \quad f_3 &\equiv |w_1| - e^{-\frac{b}{NR}} = 0 \quad [\Leftrightarrow (A)']
\end{align*}
\]

They are the typical curves concerning the location of the singularity relative to the curve seen in \((r_1, r_2, r_3)\) space (see Fig.1 or [3]). The curve (i) does not intersect the singularity for any value of \(b\), (ii) intersects the singularity at the vertex of the curve for any \(b\) and (iii) intersects the singularity asymmetrically. Note that all other curves belonging to our family are classified into the same class as (iii) in the sense that they intersect the singularity asymmetrically. Also notice that this class is further divided into two classes: One class \([(iii)A]\) consists of curves which cross the singularity only one time for all values of \(b\), and the other \([(iii)-2]\) is the set of curves that do not cross the singularity if we take \(b\) small enough. The curve (iii) in \([(iii)]\) itself belongs to the former class (iii)-1. A curve in the class (iii)-2 can be realized by choosing a complex structure, e.g., given by the holomorphic coordinates \((w_0 + w_1, v_0 + v_1)\). In the limit \(b/NR \to \infty\), the point of
The appearance of D4-branes on the NS5-brane is depicted by the striped pattern for three representative cases eqs. (64) \sim (66). D4-branes arise at the singularity depicted by the wavy lines and go to infinity along the NS5-brane.

intersection becomes closer to the D6-branes for all curves. Note that the curves in the class (iii)-2 has another intersection point at large \( r \) in the limit.

We analyze the configuration of D4-branes and NS5-branes by the currents \( j^{[4]} \) and \( j^{[5]} \) in a similar way as in the previous section. First we consider the 11-dimensional current \( J^{(11)}_{[5]} \) for each of three curves. They are obtained by applying the general formula eq.(47) to the above cases:

\[
\begin{align*}
(i) \quad & \ast J^{(11)}_{[5]} \sim \delta(y_4)\delta(y_5)\delta(y_6)\delta(f_1(\vec{r}))dy_4 \wedge dy_5 \wedge dy_6 \wedge df_1 \wedge dx_{11}, \\
(ii) \quad & \ast J^{(11)}_{[5]} \sim \delta(y_4)\delta(y_5)\delta(y_6)\delta(f_2(\vec{r}))dy_4 \wedge dy_5 \wedge dy_6 \wedge df_2 \wedge (dx_{11} - NRd\psi), \\
(iii) \quad & \ast J^{(11)}_{[5]} \sim \delta(y_4)\delta(y_5)\delta(y_6)\delta(f_2(\vec{r}))dy_4 \wedge dy_5 \wedge dy_6 \wedge df_2 \wedge (dx_{11} - NRd\chi)
\end{align*}
\]

where \( f_i \) is given in eqs.(64) \sim (66) and \( \chi \) is in eq.(10). Note that in the near core limit, we can show the definite form of the 11-dimensional current as in eq.(46). In the reduced 10-dimensional theory, the relation between D4-brane current \( j^{[4]} \) and NS5-brane current \( j^{[5]} \) for each brane is determined independent of the scale of \( r \) as

\[
\begin{align*}
(i) \quad & \ast j^{[4]} = 0, \\
(ii) \quad & \ast j^{[4]} = \ast j^{[5]} \wedge (-NRd\psi), \\
(iii) \quad & \ast j^{[4]} = \ast j^{[5]} \wedge (-NRd\chi)
\end{align*}
\]

This means that there is no D4-brane current in the case (i). In other cases, D4-branes can be present only on an NS5-brane. By investigating the form of \( \ast j^{[4]} \) on the NS5-brane, we see how D4-branes are stretched on the NS5-brane. If the NS5-brane intersects with the singularity only once, then it is interpreted that the smeared D4-branes come up from the Dirac singularity, i.e., the D4-branes are created from the singularity, and go to infinity. For the curves in the class (iii)-2, there are two points on the corresponding NS5-brane where the D4-branes can be created or absorbed. Note that since \( \ast j^{[4]} \) is proportional to the number \( N \), the number of D4-branes smeared on the NS5-brane is also proportional to \( N \). The results are depicted in Fig.3.
9 On the brane creation phenomena

Brane creation was first discussed in the flat IIB background \cite{3} as the Hanany-Witten effect representing a phenomenon that a new brane is created by crossing of certain two types of branes. This effect is confirmed by charge conservation, although the process of brane creation has not yet been clarified in the framework of supergravity theory. One reason for this is that it is difficult to describe the situation of a brane ending on another brane as a solution of supergravity. Now, our discussion given in the preceding sections enables us to study the process of brane creation in the vicinity of D6-branes in terms of the corresponding exact solution of supergravity.

Using the argument, we now discuss how the brane creation is explained in the near core region of KK-monopoles and also consider the extension to the \( r: \text{finite region of } M_{TN} \). In particular, we again deal with three types of M5-branes (i) \( \sim (iii) \) and their compactification.

First, consider the curves (ii) and (iii). They intersect the Dirac string singularity if the parameter \( b \) is taken to be large enough. As was indicated in the last section, it is interpreted in 10-dimensions that D4-branes come up from the point of intersection with the singularity and go to infinity or another intersection point along the NS5-brane. In the near core limit \( r/NR \rightarrow 0 \), each curve is a paraboloid and the shape of the paraboloid changes with the parameter \( |c| \) as in eq.(42). In the limit \( c/NR \rightarrow 0 \) (i.e., \( b/NR \rightarrow \infty \)), the paraboloid becomes like a thin tube and degenerates to an object like a half-string. As \( c \) approaches 0, the NS5-brane bends so as to wrap the D6-branes and gives multipole moment if measured far away from the brane \cite{13}. Finally in the limit \( c \rightarrow 0 \), the total NS5-brane charge vanishes. On the other hand, in this limit, all the D4-branes spread over the NS5-brane gather to form a bundle of coincident D4-branes coming up from D6-branes. This process of disappearing the NS5-brane and the assembling of D4-branes into one place may be regarded as the brane creation in the near core limit.

![Figure 4:](image)

Figure 4: In the limit \( b/NR \rightarrow \infty \) (\( c/NR \rightarrow 0 \)), the NS5-brane charge disappears and only the coincident D4-branes coming up from D6-branes remain in the near core region. This is the brane creation process of D4-branes for the M5-branes of the classes (ii) and (iii).

Extending this process of brane creation to the outer region, \( r: \text{finite region} \), we see a transition of branes with respect to the value \( b \). For the curves (ii) and (iii)-1, the transition process is essentially the same as in the near core region, since these curves already intersect with the singularity in the \( b/NR \rightarrow -\infty \). A curve in the class (iii)-2 begins with a pure NS5-brane if \( b \) is small enough. Then as \( b/NR \) becomes larger, the
NS5-brane begins to bend and at some value of $b$ the curve comes in contact with the singularity, and after that we have two points of intersection and the D4-brane charges turn out. Note that in the limit $b/NR \rightarrow \infty$, NS5-brane charge disappears and only D4-brane charge remains roughly in the region $r$ smaller than $b$ for these cases.

Figure 5: (a) The $b \rightarrow \infty$ limit of the NS5-brane specified by the curve (i). There is no D4-brane charges, though a D4-brane like object appears in the limit. (b) Similar phenomenon occurs in the case of stringy cosmic string background: When an NS5-brane crosses $N$ D6-branes, D4-branes which should be created from D6-branes cannot be seen if the branch cut lies on the D4-branes. If we shift the cut, D4-branes can be observed.

On the other hand, consider the curve (i) for which there is no D4-brane charge on the NS5-brane. In the limit $b/NR \rightarrow \infty$, the brane is bent as the same way as other cases, however, we have to take another explanation as others. In particular, in the region near the D6-branes, this curve reduces to an object which has an appearance of a D4-brane ending on D6-branes, but has neither NS5-brane charges nor D4-brane charges. We have a problem how to interpret the object. The resolution of this can be done by noticing the singularity again: The singularity is placed as overlapped on the D4-brane-like object in the limit $b/NR \rightarrow \infty$. Thus the situation is exceptional and the identification of the brane-like object cannot be performed.

The analogous structure is found in the IIB stringy cosmic string background where some 5-branes ending on $N$ coincident D7-branes are exactly on the branch cut. In terms of IIA theory, the branes must be identified as D4-branes, while it cannot be seen from charge conservation. The identification can be done by shifting the branes away from the branch cut singularity as in Fig.5(b).

In our case of KK-monopole background, shifting the D4-brane away from the singularity corresponds to taking another curves holomorphic with respect to different complex structure, or shifting the compactification direction as in eq.(54). By using either method, the identification of the branes can be done. This resolves the puzzle we stated.
10 Summary and Discussion

We have proposed a method of identifying the branes in the IIA background especially in the presence of D6-branes obtained by compactification of 11-dimensional Kaluza-Klein monopole solution. It is essentially the same as the identification of branes in the IIB background with 7-branes. We have also discussed the brane creation from D6-branes. In particular, since we know exact supergravity solutions of M5-branes in the near core region of KK-monopoles in 11-dimensions, we have clarified the mechanism of brane creation in this region as a compactified 10-dimensional theory.

Now we explain the relation between the brane creation based on our supergravity argument and the original Hanany-Witten argument based on flat branes in the flat background. In the flat space argument, created branes between two branes have finite length. On the other hand, in our argument the created branes are interpreted as half-infinite branes: The D4-branes coming up from the D6-branes are not cut on the NS5-brane, but continue along the NS5-brane. The difference may be related to the fact that there exist Dirac string singularities in the background with D6-branes. If we could obtain supergravity solutions of the original Hanany-Witten configurations, we would be able to clarify whether a finite brane can exist.

We comment on the case of M2-branes instead of M5-branes. In this case, we have to take care of the definition of currents after compactification: In order to proceed with the argument on identification of F-strings or D2-branes as the same method as in the case of M5-branes, it seems to be necessary to use the definition of 10-dimensional field strength as $\hat{F}_{[4]}^{(11)} = \hat{G}_{[3]} + \hat{G}_{[4]} \wedge dx^{11}$ instead of $G_{[3]}$ and $G_{[4]}$ in eq. (49). Although the two definitions coincide with each other if there are no Kaluza-Klein charges, in general $G_{[n]} \neq \hat{G}_{[n]}$ in the presence of KK-charges. Nevertheless we do not know direct reason to change the definition of 10-dimensional field strengths as above depending on the objects we deal with. Even if we decide to use such a definition, the problem still remains in the situations where both M5-branes and M2-branes exist simultaneously. (In such a situation, Chern-Simons term $F \wedge F \wedge A$ in 11-dimensional supergravity action may play an important role.) Note that a same kind of confusion in defining the field strengths in 10-dimensions was pointed out in the previous argument [12, 13]. We do not have definite explanation of this puzzle at the present point.

Moreover, note that our 10-dimensional configuration with Dirac string singularity is based on singular compactification from the 11-dimensional background. We do not know if such dimensional reduction is consistently described as a compactification of string theory.

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