Rotating flow of viscous nanomaterial with radiation and entropy generation

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Abstract
This communication models the flow of viscous nanofluid between two heated parallel plates with radiation and uniform suction at one boundary. Two types of carbon nanotubes (CNTs) namely the single (SWCNT) and multiple (MWCNT) walls are accounted. Heat generation, radiation, and dissipation in heat expression are utilized. Entropy generation and Bejan number are examined. Formulation and analysis in rotating frame are considered. Convergent solutions for velocity and temperature are constructed and interpreted. Coefficient of skin-friction and Nusselt number are tabulated and analyzed for comparative study of SWCNT and MWCNT. Correlation for skin-friction and Nusselt number are also evaluated. An enhancement in velocity profile is seen through suction variable. A reduction occurs in axial velocity for higher Reynolds number. An opposite trend is hold for thermal field through Eckert and Prandtl numbers. An intensification in temperature is noted for radiation. An amplification in entropy rate is observed through Brinkman number. Higher Reynolds number corresponds to improve Bejan number. An improvement in radiation variable lead to rises heat transfer rate for both carbon nanotubes.

Keywords
Entropy generation, radiative flow, viscous nanofluid, magnetic field, carbon nanotubes

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Introduction
Recently, the nanomaterial is quite prominent in the engineering, technology, and bioengineering processes. A nanofluid is a mixture of various nanoparticles $Al_2O_3$, $Cu$, and $CuO$ into base liquids like oils, water, ethylene-glycol, tri-ethylene-glycol, etc. It is noticed that the base liquids have low thermal conductivity and do not meet the requirement of many materials in industry, technology, and medicine. Therefore, Choi and Eastman1 proposed the basic concept of suspension of small solid particles into ordinary materials. They examined that the small solid particles suspension significantly rises the thermal efficacy of ordinary energy transportation materials. Further the carbon nanotubes (CNTs) with superficial thermal conductivities are found to have specific thermal properties. There are various models available for evaluating the efficient thermal conductivities of nanotubes. In this regard, Xue2 suggested the theoretical approach based on Maxwell’s theory to examine the CNTs thermal conductivities. He involved the rotational elliptical nanotubes having larger axial ratio which accounted for space distribution effects on CNTs. Mechanical alloy and cold spray phenomena for the development of $CNT-Cu$ composite

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surfaces coating for heat transportation is implemented by Pialago et al. Study of melting heat transportation in SWCNTs – MWCNTs fluid flow by variable thicked stretching sheet is explored by Hayat et al. In another study, Hayat et al. analyzed the features of CNTs in thermally radiative nanofluid flow induced due to movement of cylinder. Hayat et al. examined the thixotropic nanomaterial flow subject to radiation, magnetic field, and Joule heating. Mushtaq et al. described the features of non-linear radiated nanofluid flow with respect to solar energy. The convected flow of peristaltic water-based nanofluids in presence of heat sink/source is illustrated by Shehzad et al. Sheikholeslami et al. reported thermally radiative Al\textsubscript{2}O\textsubscript{3} – water based nanoliquid flow and energy transport in uniformly heated channel. Some other activities concerning nanofluid flows can be seen in Refs. 10–24.

Optimization of entropy is attractive field of research during the past few decades. The thermodynamic irreversibility in any liquid flow procedure can be measured through entropy investigation. The second law of thermodynamics expresses that every genuine procedure is irreversible. Second law of thermodynamics has broad applications in issues including heat move and fluid flow. Entropy generation is related with thermodynamic irreversibility, which is available in all heat transfer and fluid flow forms. Bejan explored entropy generation for convective heat transport due to temperature inclination and consistency impacts in a liquid. Bejan additionally introduced different explanations behind entropy generation in applied heat designing where generation of entropy obliterates accessible work (e.g. exergy) of a framework. Hayat et al. explored radiative flow between two rotating disks with viscous dissipation, Joule heating and irreversibility. Qayyum et al. considered entropy generation in MHD flow of Walter-B nanofluid. Here heat generation and dissipation effects are present. Some studies about this topic are mentioned in Refs. 29–38.

The above studies examines that no effort has been made to discuss the irreversibility analysis in water-based carbon nanotubes between two rotating heated plates. Currently there are various scientists and researchers that scrutinize the entropy generation in water based CNTs between two heated plates. Our main interest here to examine rotating flow of viscous nanofluid between two heated plates. Single (SWCNT) and multiple (MWCNT) walled carbon nanotubes are considered. Here water is used as base as base fluid. Heat generation, radiation, and dissipation are present. Upper plate is subject to suction. Entropy generation and Bejan number are examined. Convergent series solutions by homotopic procedure are computed. Analysis for physical quantities of interest is arranged through graphs and tabulated values. Thermophysical characteristics of nanomaterial are given in Table 1. Comparative study with previous publish studies are presented in Table 2. Computational results of physical quantities are discussed in Tables 3 and 4. Correlation outcomes for skin friction and heat transfer rate are established in Tables 5 and 6.

### Table 1. Thermophysical properties of nanomaterial and base fluid

| Properties | SWCNT | MWCNT | H\textsubscript{2}O |
|------------|-------|-------|------------------|
| C\textsubscript{p} (J/kgK) | 425   | 796   | 4179             |
| k(W/mK)    | 6600  | 3000  | 0.613            |
| \(\rho\) (kg/m\textsuperscript{3}) | 2600  | 1600  | 997.1            |

Modeling

We consider steady rotating flow of water-based CNTs between two parallel plates. The plates have temperature \(T_0\) and \(T_\text{b}\). Lower plate at \(y = 0\) is stretched while the upper plate at \(y = h\) being the porous with suction. Viscous dissipation, heat absorption, and radiation are accounted. Cartesian coordinates taken in such a way that sheet is in \((xz)\) direction and \((y)\) to normal direction. Fluid rotation is through angular velocity \(\Omega\) along \(y\)-axis. Flow sketch is highlighted in Figure 1.

Governing equations for problems are:

\[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0,\]

\[\rho_\text{nf}\left(\frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} + 2\Omega_\text{w}w\right) = -\frac{\partial p}{\partial x} + \mu_\text{nf}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right),\]

\[\rho_\text{nf}\left(\frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} - 2\Omega_\text{u}w\right) = -\frac{\partial p}{\partial y} + \mu_\text{nf}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right),\]

\[\rho_\text{nf}\left(\frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + 2\Omega_\text{w}u\right) = -\frac{\partial p}{\partial z} + \mu_\text{nf}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right),\]

\[\left(\frac{\partial \theta}{\partial y} + \frac{\partial \theta}{\partial z} + \frac{\partial \theta}{\partial x}\right) = \frac{k_\text{nf}}{(\rho_\text{nf})_\text{c}_{\text{p}}} \left[\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2}\right] + \frac{\theta (T_\text{b} - T_0)}{16\sigma \rho_\text{nf} T^3} \frac{\partial \theta}{\partial x},\]

Subject to boundary condition.

\[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0,\]

\[\rho_\text{nf}\left(\frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} + 2\Omega_\text{w}w\right) = -\frac{\partial p}{\partial x} + \mu_\text{nf}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right),\]

\[\rho_\text{nf}\left(\frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} - 2\Omega_\text{u}w\right) = -\frac{\partial p}{\partial y} + \mu_\text{nf}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right),\]

\[\rho_\text{nf}\left(\frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + 2\Omega_\text{w}u\right) = -\frac{\partial p}{\partial z} + \mu_\text{nf}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right),\]

\[\left(\frac{\partial \theta}{\partial y} + \frac{\partial \theta}{\partial z} + \frac{\partial \theta}{\partial x}\right) = \frac{k_\text{nf}}{(\rho_\text{nf})_\text{c}_{\text{p}}} \left[\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2}\right] + \frac{\theta (T_\text{b} - T_0)}{16\sigma \rho_\text{nf} T^3} \frac{\partial \theta}{\partial x},\]

Subject to boundary condition.
we get
\[
\begin{align*}
&f''(0) = 1, f'(0) = 0, g(0) = 0, \theta(0) = 1, \text{ at } \xi = 0, \\
&f'(1) = 0, f(1) = S, g(1) = 0, \theta(1) = 0, \text{ as } \xi = 1.
\end{align*}
\]

Here \(K_r = \frac{\omega^2}{\nu_f^2}\) denotes rotational parameter, \(Pr = \frac{(\gamma_j + \mu_\text{nf})\nu_f}{k_\text{nf}}\) Prandtl number, \(Re = \frac{\rho_\text{nf}u^2}{\mu_\text{nf}}\) Reynolds number and \(S(= \frac{v_\text{nf}}{\nu_f}) > 0\) suction parameter. The quantities \(A_1, A_2, A_3\), and \(A_4\) are
\[
\begin{align*}
A_1 &= \frac{1}{(1-\chi)^2 + ((1-\chi) + \frac{Ec}{\rho_\text{nf}u^2})}, \\
A_2 &= \frac{1}{(1-\chi) + \frac{k_\text{nf}}{k_\text{nf} - k_f}2\ln \left(\frac{k_\text{nf} + k_f}{2k_f}\right)}, \\
A_3 &= \left(1 - \chi\right) + \frac{\rho_\text{nf}u^2}{\rho_\text{nf}u^2}f', \\
A_4 &= \frac{1}{(1-\chi)^2}.
\end{align*}
\]

Table 3. Skin friction \((C_f)\) for \((\text{SWCNT} - \text{MWCNT})\).

| \(S\) | \(K_r\) | \(\chi\) | \(R\) | \(C_f\) | \(\text{SWCNT}\) | \(\text{MWCNT}\) |
|-----|-----|-----|-----|-----|--------|--------|
| 1   | 2   | 0.4 | 2   | 7.94297 | 7.74616 |
| 1.5 | 3   | 0.5 | 2   | 8.02997 | 8.00027 |
| 2   | 4   | 0.6 | 2   | 8.15186 | 7.87598 |
| 1   | 3   | 0.4 | 3   | 12.1439 | 11.9038 |
| 1.5 | 3   | 0.6 | 4   | 20.6489 | 20.3699 |
| 2   | 4   | 0.4 | 4   | 8.61307 | 8.26179 |

(6)

Considering
\[
\begin{align*}
&u = axf'(\xi), \quad v = -ahf'(\xi), \quad w = axg(\xi), \\
&\theta(\xi) = \frac{T - T_0}{T_h - T_0}, \quad \xi = \frac{y}{h},
\end{align*}
\]
we get
\[
\begin{align*}
f'''' - ReA_1(f''f'' - f''') &= 0, \\
g'''' - ReA_1(gf'' - fg') &= 0.
\end{align*}
\]

(7)

(8)

(9)

Table 4. Nusselt number \((Nu_f)\) for \((\text{SWCNT} - \text{MWCNT})\).

| \(Pr\) | \(R\) | \(Ec\) | \(\chi\) | \(-Nu_f\) | \(\text{SWCNT}\) | \(\text{MWCNT}\) |
|-----|-----|-----|-----|-----|--------|--------|
| 6.2 | 0.3 | 0.2 | 0.2 | 0.4 | 16.8061 | 15.8924 |
| 6.6 | 0.3 | 0.2 | 0.2 | 0.4 | 17.0779 | 16.1736 |
| 7.0 | 0.3 | 0.2 | 0.2 | 0.4 | 17.3506 | 16.4556 |
| 6.2 | 0.5 | 0.3 | 0.4 | 0.4 | 15.9289 | 14.9811 |
| 6.2 | 0.5 | 0.4 | 0.3 | 0.4 | 15.4674 | 14.4984 |
| 6.2 | 0.5 | 0.4 | 0.2 | 0.3 | 15.0532 | 14.6234 |
| 6.2 | 0.5 | 0.4 | 0.2 | 0.4 | 14.2579 | 13.3543 |
| 6.2 | 0.4 | 0.3 | 0.4 | 0.3 | 17.7474 | 16.8475 |
| 6.2 | 0.4 | 0.3 | 0.4 | 0.4 | 18.7151 | 17.8307 |
| 6.2 | 0.4 | 0.2 | 0.5 | 0.5 | 22.8213 | 21.3854 |
| 6.2 | 0.4 | 0.2 | 0.6 | 0.6 | 31.4716 | 29.2352 |
Table 5. Correlation (r) for skin friction coefficient.

|   | SWCNT | MWCNT |
|---|-------|-------|
| r | 0.96414879 | 0.96497777 |
| Pr | 0.99834700 | 0.99845418 |
| R | 0.93375038 | 0.9316882 |
| Ec | 0.87182243 | 0.86883692 |
| N | 0.94372086 | 0.94529741 |
| χ | 0.99116947 | 0.98677809 |

Table 6. Correlation (r) for Nusselt number.

|   | SWCNT | MWCNT |
|---|-------|-------|
| r | −RdNu | −RdNu |
| Pr | 0.99834700 | 0.99845418 |
| R | 0.93375038 | 0.9316882 |
| Ec | 0.87182243 | 0.86883692 |
| N | 0.94372086 | 0.94529741 |
| χ | 0.99116947 | 0.98677809 |

Physical quantities

Skin friction coefficient and Nusselt number are defined as

$$Cf = \frac{\tau_w}{0.2\rho u_w v_w}, \quad Nu = \frac{h_q}{k_f (T_h - T_0)}$$

in which wall $\tau_w$ shear stress and $q_w$ heat flux at wall satisfy

$$\tau_w = \mu_w \left( \frac{du}{dy} \right)_{y=0},$$

$$q_w = -(k_f + 16\sigma T^3 / 3k_0) \partial T / \partial y_{y=0}.$$ One can found

$$Re_1^{1/2}Cf = A_1 f''(0), \quad Re_2^{1/2}Nu = -(A_2 + Rd) \theta(0).$$

Entropy optimization

Relevant expression in presence of radiation and dissipation satisfies

$$S_g = \frac{k_w}{\theta'} \left[ \frac{k_m}{\theta'} \left( \frac{Q}{\theta'} \right)^2 + \frac{16\sigma T^3}{3k_0} \left( \frac{Q}{\theta'} \right)^2 \right] + \frac{n_v}{\theta'} \left[ 2 \left( \frac{\theta}{\theta'} \right)^2 + \left( \frac{\theta}{\theta'} \right)^2 + \left( \frac{\theta}{\theta'} \right)^2 \right] + \left[ \frac{\theta}{\theta'} \right]^2 \right]$$

we get

$$Nu = \beta_1 \left( \frac{k_m}{\theta'} + R \right) \left( \frac{\theta}{\theta'} \right)^2 + \left( \frac{\theta}{\theta'} \right)^2 + \left( \frac{\theta}{\theta'} \right)^2 + \left( \frac{\theta}{\theta'} \right)^2 \right]$$

Bejan number gives

$$Be = \frac{\beta_1 \left( \frac{k_m}{\theta'} + R \right) \left( \frac{\theta}{\theta'} \right)^2 + \left( \frac{\theta}{\theta'} \right)^2 + \left( \frac{\theta}{\theta'} \right)^2 + \left( \frac{\theta}{\theta'} \right)^2 \right]$$

Series solutions

Initial guesses and linear operators satisfy:

$$f_0(\xi) = ((1 - 2\xi) \xi + (3\xi - 2) \xi^2 + \xi),$$

$$g_0(\eta) = (1 - \xi)\xi,$$

$$\theta_0(\eta) = (1 - \xi).$$

Above operators have properties

$$L_f [b_1 + b_2 \xi + b_3 \xi^2 + b_4 \xi^3] = 0,$$

$$L_0 [b_5 \theta + b_6 \theta^2] = 0,$$

$$L_v [b_7 \theta + b_8 \theta^2] = 0.$$ Convergence analysis

Convergence and approximation rate for series solution are depend on auxiliary parameter $h_f, h_g$, and $h_\theta$. Therefore the $h-$curves are plotted for 10th order approximations for SWCNTs and MWCNTs. Admirable ranges of $h-$curves are $-1.7 \geq h_f \geq -0.2,$ $-1.6 \geq h_g \geq -0.1,$ and $-0.2 \geq h_\theta \geq 1.7$ for SWCNTs case (see Figure 2). Convergence ranges for MWCNTs
are \( h_1 \approx -0.4, \quad h_2 \approx -0.3, \) and \( h_3 \approx -0.45 \) (see Figure 3).

From above table we noted that the results have an excellent agreement.

Discussion

The results of the formulated problem are developed by homotopic scheme. The behavior of involved influential parameters on fluid velocities, skin-friction, temperature, Nusselt number, irreversibility, and Bejan number are picturized and tabulated. Analysis is organized for two kinds of nanotubes known as SWCNTs and MWCNTs.

Velocity

Figures 4–9 describe the importance of various parameters including suction \( S \) and Reynolds number \( Re \) on radial \( f'(\xi) \), axial \( f(\xi) \), and tangential \( g(\xi) \) velocities. Influence of suction variable on velocity components \( (f'(\xi), f(\xi), \) and \( g(\xi)) \) is revealed in Figures 4–6. An intensification in velocity components is seen with variation in suction variable. Physical description of velocity against Reynolds number is illuminated in Figures 7–9. Clearly radial velocity has dual behavior through Reynolds number. An amplification in Reynolds number corresponds to augment axial and tangential velocities components \( (f(\xi) \) and \( g(\xi)) \). An increment in Reynolds number reduces viscous force and thus velocity boosted.

Temperature

Influence of temperature with variation in Prandtl number is illuminated in Figure 10. An increment in Prandtl number reduces thermal conductivity which decays temperature for both CNTs. A reduction in thermal field is seen through heat absorption variable for both carbon nanotubes (see Figure 11). Physical description of temperature versus radiation is portrayed in Figure 12. Higher approximation of radiation decays mean absorption coefficient, which improve heat flux. Therefore thermal field boosted for
both CNTs. It is apparent that by increasing nano-volume fraction constraint $\chi$ the temperature $\theta(\xi)$ enhances (see Figure 13). Figure 14 sketch to shows influence of thermal field versus Eckert number. An enhancement in kinetic energy against Eckert number occurs, which improves temperature for both carbon nanotubes.

**Entropy generation and Bejan number**

Aspects of temperature difference parameter ($\beta_1$), Brinkman number ($Br$), Reynold number ($Re$) and nano volume fraction parameter ($\chi$) are displayed in the Figures 15–22 for irreversibility $Ng(\xi)$ and Bejan
number ($Be$). Both SWCNTs and MWCNTs cases are considered. Figures 15 and 16 demonstrate the behavior of temperature difference parameter ($b_1$) for $Ng(\xi)$ and $Be$. For higher values of ($b_1$) the entropy generation and Bejan number are increasing for both cases.

Impact of ($Br$) on $Ng(\xi)$ and ($Be$) are portrayed in Figures 17 and 18. Irreversibility is increasing for higher ($Br$) while it decays the Bejan ($Be$) number. Physically an increment in Brinkman number rises the viscous force which improves collision amongst the liquid particles and thus irreversibility boosted. Figures 19 and 20 describe behavior of Reynold number ($Re$) for both entropy $Ng(\xi)$ and ($Be$). Same behaviors of entropy generation and Bejan number are noticed for Reynolds number. Effect of volume fraction parameter ($\chi$) versus entropy and Bejan number are studied in Figures 21 and 22. Irreversibility and Bejan number are increasing function of higher ($\chi$).

**Physical quantities**

Tables 3 and 4 are developed for skin friction ($C_{f_s}$) and Nusselt number ($Nu_x$). Here behavior of pertinent constraints can be seen through Tables 3 and 4 for SWCNTs and MWCNTs cases. Table 2 depicts that ($C_{f_s}$) is an increasing function of $S$, $Kr$, $\chi$, and $Re$. Table 4
declared that Nusselt number enhances for higher Prandtl number (Pr), heat absorption parameter (N), and volume fraction (χ) while it decreases against radiation parameter (R) and Eckert number (Ec).

**Correlation for skin friction and Nusselt number**

In Tables 3 and 4 we examined the (Cf) and (Nu) variations for both plates in presence of CNTs. The skin friction coefficient and Nusselt number with respect to parameters in Tables 3 and 4 are higher for SWCNT
than MWCNT. Tables 5 and 6 are established for correlation coefficient of skin-friction ($C_f$) and Nusselt number ($Nu_c$). It is for to evaluate the inter-dependence of constraints on drag force and rate of heat transportation. Tables 5 and 6 report the correlation coefficient ($r$). The values of coefficient of correlation are noticed between $(-1)$ to $(+1)$.

Conclusions

Main observations of this study are:

- Impacts of suction parameter and Reynolds number on axial, radial, and tangential velocities are opposite.
- Temperature enhancement occurs for Eckert number and radiation parameter.
- Temperature is an increasing function of nanoparticle volume fraction.
- Entropy production boosts up for higher Brinkman number and temperature difference parameter.
- Bejan number is higher for higher nano volume fraction, Reynolds number and temperature difference parameter while declines for Brinkman number.
- Skin-friction is higher in case of SWCNT when compared with MWCNT.
- The considered investigation has significance in polymer industry, fabrication of medicines, plastic surface stretching, architecture, and metallurgical processes. The diverse application of CNTs (carbon nanotubes) including in solar cells, cooling of nuclear reactor, heat exchangers gas storage, medical instruments, ultra-capacitors, and many others. Radiations are quite prevalent in cancer therapy, drug delivery, plasma, metallurgy, etc.

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