Where Have All the Submillisecond Pulsars Gone?

L. Burderi, F. D’Antona, M.T. Menna, L. Stella
Osservatorio Astronomico di Roma, Via Frascati 33, 00040 Monteporzio Catone (Roma), Italy

A. Possenti, N. d’Amico, M. Burgay
Osservatorio Astronomico di Bologna and Università di Bologna,
Dipartimento di Astronomia, via Ranzani 1, 40127 Bologna, Italy

T. Di Salvo
Astronomical Institute “Anton Pannekoek,” University of Amsterdam,
Kruislaan 403, NL 1098 SJ Amsterdam, the Netherlands

R. Iaria, N.R. Robba
Dipartimento di Scienze Fisiche ed Astronomiche, Università di
Palermo, Via Archirafi 36, 90123 Palermo, Italy

S. Campana
Osservatorio Astronomico di Milano

Abstract. The existence of pulsars with spin period below one millisecond is expected, though they have not been detected up to now. Their formation depends on the quantity of matter accreted from the companion which, in turn, is limited by the mechanism of mass ejection from the binary. Mass ejection must be efficient, at least in some cases, in order to produce the observed population of moderately fast spinning millisecond pulsars. First we demonstrate, in the framework of the widely accepted recycling scenario, using a population synthesis approach, that a significant number of pulsars with spin period below one millisecond is expected. Then we propose that significant variations in the mass–transfer rate may cause, in systems with orbital periods $\gtrsim 1$ h, the switch–on of a radio pulsar whose radiation pressure is capable of ejecting out of the system most of the matter transferred by the companion and prevent any further accretion. We show how this mechanism could dramatically alter the binary evolution since the mechanism that drives mass overflow from the inner Lagrangian point is still active while the accretion is inhibited. Moreover we demonstrate that the persistency of this “radio ejection” phase depends on the binary orbital period, demonstrating that close systems (orbital periods $P_{\text{orb}} \lesssim 1$ h) are the only possible hosts for ultra fast spinning neutron stars. This could explain why submillisecond pulsars have not been detected so far, as current radio surveys are hampered
by computational limitations with respect to the detection of very short spin period pulsars in short orbital period binaries.

1. Introduction

PSR1937+21 is, at present, the neutron star (NS) having the shortest rotational period $P_{\text{min}} = 1.558$ ms ever detected. Despite its apparent smallness, $P_{\text{min}}$ is not a critical period for NS rotation: as shown by Cook, Shapiro & Teukolsky (1994), the period $P_{\text{min}}$ is longer than the limiting period, $P_{\text{lim}}$, below which the star becomes unstable to mass shedding at its equator. Indeed $P_{\text{lim}} \sim 0.5$ ms for most equations of state of the ultra dense nuclear matter, EoS). However, the re-acceleration of a NS up to ultra short periods depends remarkably on the amount of mass (and hence of angular momentum) accreted from a binary companion. Back of the envelope calculation shows that $M_{\text{1ms}} \sim 0.2 M_\odot$ must be accreted to attain $P \sim 1$ ms. Conservation of angular momentum requires $M_{\text{1ms}} l = I \frac{2 \pi}{(\text{1ms})}$ where $I$ is the NS moment of inertia, $l = (GMR_{\text{acc}})^{1/2}$ is the specific angular momentum of the accreting matter, and $R_{\text{acc}}$ is the accretion radius. Adopting $I = 10^{45}$ g cm$^2$ and $R_{\text{acc}} \sim R_{\text{NS}} = 10^6$ cm, where $R_{\text{NS}}$ is the NS radius, one gets the above result. More accurate calculation that includes general relativistic effects and realistic EoS, have almost doubled this estimate, showing that, typically $M_{\text{1ms}} \sim 0.35 M_\odot$ must be accreted to spin up a NS to 1 ms (Burderi et al. 1999).

Most donor stars in systems hosting recycled pulsars have certainly lost, during their interacting binary evolution, a mass larger than $M_{\text{1ms}}$. They now appear as white dwarfs of mass $\sim 0.15 - 0.30 M_\odot$ (Taam, King & Ritter 2000), whose progenitors are likely to have been stars of $\sim 1.0 - 2.0 M_\odot$ (Webbink, Rappaport & Savonije 1983; Burderi, King & Wynn 1996, Tauris & Savonije 1999). Hence, a crucial parameter for attaining ultra fast rotation is the fraction of the mass lost by the companion ($M_{\text{lost}} = 0.7 - 1.85 M_\odot$) which effectively accretes onto the NS which, in turns, depends on the mechanisms driving the mass ejection from the system.

In this paper we discuss the results of population synthesis calculations for the formation of millisecond pulsars (MSPs) in the framework of the standard recycling scenario (§2). We show that, independent of the adopted EoS, the existence of a population of MSPs spinning below 1 ms (submillisecond pulsar, SMSP) is predicted, even though these systems have not been detected up to date.

In §3 we critically discuss the widely accepted recycling scenario for the formation of a MSP outlining the main difficulties of this model by a comparison of the present observations with the results obtained in §2.

To overcome these difficulties we propose, in §4, that, in some cases, most of the transferred mass is ejected by the radiation pressure generated by the rapidly spinning NS in a radio pulsar regime. Moreover we investigate how the duration of this process, and hence the amount of mass ejected, depends on the orbital period of the binary system. The mechanism proposed (some aspects of which were originally suggested by Ruderman, Shaham, & Tavani 1989) naturally provides an explanation for the observed values of masses and spin periods.
of MSPs (Thorsett & Chakrabarty 1999; Tauris & Savonije 1999). Our analysis indicates that only close systems ($P_{\text{orb}} \lesssim 1$ h) could harbour SMSPs. In §5 we discuss how this results in an observational bias that has prevented the detection of SMSPs up to now.

2. On the existence of submillisecond pulsars: a population synthesis approach

Although $P_{\text{lim}}$ depends only on the EoS adopted for the NSs, the effective possibility to accelerate a NS to periods below 1 ms is determined by the modalities of the mass transfer from the binary companion and the evolution (decay) of the NS magnetic moment $\mu$ (Burderi et al. 1999). In §3 we discuss in some detail the possible scenarios for the mass transfer in these systems. No unique theory exists for the origin and evolution of the NS magnetic moment. The most credited theories suggest either that the field is a remnant from the progenitor star or that it has been generated soon after the formation of the NS. In the first case, in which the electric currents generating the magnetic field are buried in the NS core, the evolution of $\mu$ is driven by secular variations in the rotational period of the NS during its evolution (e.g. Srinivasan et al. 1990; Miri & Bhattacharya 1994; Ruderman 1998). In the other case, the electric currents are generated and confined in the thin crust of the NS and the magnetic field decay is driven by the ohmic diffusion (Sang & Chanmugam 1987). In this latter case magnetic evolution is not univocal: as crustal matter is assimilated into the core during accretion, the magnetic field at the crust–core boundary can either be expelled (boundary condition I, BC I: Urpin, Geppert & Koenkov 1998) or advected into the superconductive core where it no longer decays (BC II: Konar & Bhattacharya 1999). Thus, at the endpoint of evolution, the NS can either become almost non magnetic, or preserve a significant relic field. In this class of models, the magnetic field decay is connected with the mass accretion history which determines both the amount of accreted mass (which drives the advection process) and the thermal evolution of the crust (which modulates the ohmic dissipation process by influencing the electron and ion resistivity in the crust).

To explore all the possibilities mentioned above within the recycling scenario for the formation of MSPs (Alpar et al. 1982; Bhattacharya 1995) Possenti et al. (1998, 1999) carried on a statistical analysis of the NS spin period distributions in the millisecond and submillisecond range, using a population synthesis model\footnote{The population synthesis code calculate the mass transfer during the Roche lobe overflow considering a range of mass–transfer rates which is close to the one observed in low mass X-ray binaries. Eventually the population synthesis code follows the radio pulsar phase. The model incorporates the detailed physics of the evolution of a crustal magnetic field (using the two boundary conditions described in the text to mimic expulsion or assimilation of the field by the NS core) and includes the relativistic corrections necessary to describe the spin–up process (Burderi et al. 1999). A Monte Carlo code (with 3,000 particles) is used to compute the sample population.}.

There is observational evidence (see Tanaka and Shibazaki 1996 for a review) that NSs in low mass X-ray binaries (LMXBs) may undergo phases of transient accretion (perhaps due to thermal viscous instabilities in an irradiated
disc e.g. van Paradijs 1996; King, Kolb, Burderi 1996). This in turn can start one or more phases of propeller which have been invoked to effectively spin down the NS and eject most of the mass from the system, thus mitigating the effects of the previous phases of spin–up. We will critically discuss the propeller mechanism in §3 jeopardizing the whole propeller scenario. In view of our discussion in §3, we investigate here the effects of a propeller phase on the population of LMXBs by comparing two possibilities: a persistent accretion for the whole duration of Roche Lobe Overflow (RLO) phase, or a persistent accretion for half the previous time followed by a quick stop of the accretion modeled as a mass–transfer rate that varies with time $\dot{M} \propto t^\Gamma$ with a power law index $\Gamma = 8$ representing a sudden drop of the $M$. We assume that all the transferred mass is ejected from the system in this phase.

| Statistical Distribution | Values | Units |
|--------------------------|--------|-------|
| NS period at $t_0$       | Flat   | from 1 to 100 s |
| $\mu$ at $t_0$          | Gaussian in Log | $\log \mu_0 = 28.50; \sigma = 0.32$ G cm$^3$ |
| $\dot{m}$               | Gaussian in Log | $\log \dot{m} = -1.00; \sigma = 0.5$ |
| Minimum accreted mass    | One-value | 0.01 M$_E$ |
| Maximum accreted mass    | One-value | 0.5 M$_E$ |
| duration of RLO phase    | Flat in Log | from $10^6$ to $5 \times 10^8$ year |
| duration of MSP phase    | Flat in Log | from $10^8$ to $3 \times 10^9$ year |

Table 1. Population synthesis parameters. $t_0$ is initial time of the RLO phase and $M_E$ is the Eddington accretion rate.

The left panels in figure [1] give the percentage distribution of the recycled NSs synthesized using the set of parameters reported in table [1]. Adopting as references the values of $P_{\text{min}}$ and $\mu_{\text{min}}$ (the shortest rotational period observed in PSR 1937+21 and the weakest magnetic moment observed in PSRJ2317+1439), the NS are divided into four groups:

a) $P \geq P_{\text{min}}$ and $\mu \geq \mu_{\text{min}}$ these resemble the MSPs that have been observed so far;

b) $P < P_{\text{min}}$ and $\mu \geq \mu_{\text{min}}$ these objects represent a population of ultra fast radio pulsars (see Burderi & D’Amico 1997);

c) $P < P_{\text{min}}$ and $\mu < \mu_{\text{min}}$ ultra fast radio pulsars with a weak magnetic field. Indeed most of these objects will be above the Chen and Ruderman (1993) “death–line”, and might have a bolometric luminosity comparable to that of the known MSPs. For simplicity, hereafter we refer to all the objects having $P < P_{\text{min}}$ and $\mu$ above the “death–line” as SMSPs;

d) $P \geq P_{\text{min}}$ and $\mu < \mu_{\text{min}}$ these are probably radio quiet NSs because they are close or below the theoretical “death–line” (actually this period range was already searched with negative results by present, good sensitivity radio surveys).
Figure 1. Left: percentage distributions of the synthesized MSPs. The four panels indicate, from upper right and counterclockwise: a) objects having $P > P_{\text{min}}$ and $\mu > \mu_{\text{min}}$; b) $P < P_{\text{min}}$ and $\mu > \mu_{\text{min}}$; c) $P < P_{\text{min}}$ and $\mu < \mu_{\text{min}}$; d) $P > P_{\text{min}}$ and $\mu < \mu_{\text{min}}$; (the typical variance is about 1%). Right: spin period distributions of the simulated NS ($\mu$ varies over the whole range). The solid lines indicate the distributions in absence of propeller, the dashed areas represent the distributions with a strong propeller effect. The total number of objects is in arbitrary units (a.u.).

The four panels represent two EoSs (a very stiff one corresponding to a minimum spin period $P_{\text{lim}} \simeq 1.4$ ms and a mildly soft one, with $P_{\text{lim}} \simeq 0.7$ ms) and the two different boundary conditions discussed above for the magnetic field at the crust–core interface (BC I & BC II).

It appears that objects with periods $P < P_{\text{min}}$ are present in a statistically significant number. In effect, a tail of SMSPs is always present for any reasonable choice of the parameters in table 1. The right panels of figure 1 show the distributions of the number of objects vs the spin period for the NS of our synthetic sample.

We first discuss the results in absence of a propeller phase (solid lines). For the mildly soft EoS, the “barrier” at $P_{\text{lim}} \simeq 0.7$ ms is clearly visible as the mildly soft EoS produces distributions that increase rather steeply towards periods smaller than 2 ms, irrespective of the boundary condition adopted for $\mu$. On the other hand, the boundary conditions on $\mu$ affect the fraction of objects with short periods since the magnetospheric radius (that determines the equilibrium spin period) depends on the magnetic moment (see §3). BC II produces a smaller number of objects with a low field as the field initially decays but, when the currents are advected towards the crust–core boundary, the decay is halted and the field reaches a bottom value. Even the very stiff EoS permits periods $P < P_{\text{min}}$, but the “barrier” of mass shedding (at $P_{\text{lim}} \simeq 1.4$ ms) is so close to $P_{\text{min}} = 1.558$ ms that only few NSs reach these extreme rotational rates.

The effects of a strong propeller phase (lasting $\sim 50\%$ of the RLO phase with an efficiency in ejecting matter from the system close to 100% – although
this assumption will be strongly questioned in §3) are shown by the dashed areas. We note that significant mass losses from the system can threaten the formation of NSs with \( P < P_{\text{min}} \) only in the (unlikely) case of the very stiff EoS.

Possenti et al. (1998, 1999) computed the distributions of the number of objects \( v s \) the final masses for the different spin periods attained at the end of the evolution. The distributions steepens towards high mass values (approaching \( M \sim 1.7 \div 1.8 M_\odot \)) for final spin periods below \( \sim P_{\text{min}} \). All this results are in line with a general conclusion, namely: an accretion of about 0.35 \( M_\odot \) is a general necessary condition to spin a NS to ultra short periods (see Burderi et al. 1999 for a more technical discussion that takes into account general relativistic effects).

3. Recycling at sub–Eddington rates

In line with a widely used convention let us define primary the NS and secondary its companion.

It is well known that an upper limit to the accretion rate is given by the Eddington limit, but for typical companion initial masses \( M_{\text{ini}} \lesssim 1.6 M_\odot \) and initial \( P_{\text{orb}} \lesssim 50 \) days, the donor transfers mass at sub–Eddington rates, making, in principle, the whole mass lost by the secondary available for the recycling of the NS.

| System n. | \( M_{NS}(M_\odot) \) | \( M_{\text{donor}}(M_\odot) \) | \( P_{\text{orb}}(h) \) |
|----------|----------------|-----------------|-----------|
| Initial 1 | 1.40           | 1.20            | 10.46     |
| Final    | 2.58           | 0.02            | 1.42      |
| Initial 2 | 1.40           | 1.20            | 18.64     |
| Final    | 2.39           | 0.21            | 92.15     |
| Initial 3 | 1.40           | 1.20            | 30.55     |
| Final    | 2.35           | 0.25            | 338.1     |
| Initial 4 | 1.40           | 1.60            | 34.00     |
| Final    | 2.67           | 0.33            | 346.0     |

Table 2. Mass and period evolution in some LMXBs

We explored this scenario (i.e. conservative mass transfer) computing the system evolution (with initial parameters as in table 2) with the ATON1.2 code (D’Antona, Mazzitelli & Ritter 1989). The mass loss rate by the companion vs. the orbital period is shown in figure 2. The mass loss rate was computed, following the formulation by Ritter (1988), as an exponential function of the distance between the stellar radius and the secondary Roche lobe in units of the pressure scale height. This method also allows to compute the first phases of mass transfer, during which the rate reaches values which can be much larger than the stationary values due to the thermal response of the star to mass loss.

Besides the cases of systems with long \( P_{\text{orb}} \) in which the donor fills the Roche lobe while it is evolving towards the red giant branch (case B of mass transfer, systems n.2,3,4), we consider the case of a system with short \( P_{\text{orb}} \) and a secondary of 1.2 \( M_\odot \) that fills the Roche lobe during the core–hydrogen burning.
Where Have All the Submillisecond Pulsars Gone?

Figure 2. $\dot{M}$ vs $P_{\text{orb}}$ for the systems of table 4. The temporary stop of $\dot{M}$ in track 1 is due to the switch-off of the magnetic braking once the star becomes fully convective. The sharp drop of $\dot{M}$ in track 3 is due to a chemical readjustment of the star structure. The arrows show the $P_{\text{orb}}$ that separates cases A and B of mass transfer for a 1.2 $M_\odot$ secondary.

phase (case A of mass transfer, system n.1). In systems with long $P_{\text{orb}}$, mass transfer is driven by the thermal and nuclear evolution of the secondary; in systems with short $P_{\text{orb}}$ systems mass transfer is driven by magnetic braking acting until $P \approx 2.5$ h, when the star becomes fully convective. The mass transfer then stops and resumes at a shorter period driven by losses of angular momentum by gravitational radiation, like in cataclysmic variables. In this case the minimum period attainable is $\sim 1.03$ h. On the other hand in the systems with long $P_{\text{orb}}$ the mass transfer terminates with periods in the 90 – 400 h range.

In the systems with long $P_{\text{orb}}$ the final masses of the donor stars are in the 0.21 – 0.33 $M_\odot$ range, in agreement with the masses measured for the companions of MSP, as inferred from accurate timing campaigns (see Burderi, King, & Wynn 1996). However, the predicted final masses of the NSs are in the 2.35 – 2.67 $M_\odot$ range, very close to (or even bigger than) the maximum mass allowed for a NS in most of the proposed EoSs (Cook, Shapiro & Teukolsky 1994). This suggests either that mass transfer cannot be conservative – and most of the donor mass must be expelled from the system – or that the final NS must be very massive implying that accretion–induced collapse to a black hole is a likely outcome for LMXBs.
The propeller effect (Illarionov & Sunyaev 1975) has often been invoked as the process capable of expelling most of the mass transferred by the companion star. The accretion disc is truncated at the magnetospheric radius $R_m$, at which the magnetic field pressure equals the pressure of the matter in the disc (see e.g. Hayakawa 1985 for a review). For a Shakura–Sunyaev accretion disc an expression of $R_m$ can be derived (e.g. Burderi et al. 1998)

$$R_m = 4.70 \times 10^5 \alpha^{4/15} \epsilon^{34/135} n_0^{8/27} L_{37}^{-34/135} m^{-1/135} R_6^{-34/135} f^{-136/135} \mu_{26}^{16/27} \text{ cm}$$

(1)

where $\alpha$ is the Shakura–Sunyaev viscosity parameter, $\epsilon \sim 1$ is the ratio between the observed luminosity and the total gravitational potential energy released per second by the accreting matter, $n_0 = n/0.615 \sim 1$ for a gas with solar abundances (where $n$ is the mean particle mass in units of the proton mass $m_p$), $L_{37}$ is the accretion luminosity in units of $10^{37}$ erg/s that measures $\dot{M}$ in the hypothesis that all the transferred mass accretes onto the NS radiating a specific energy $\epsilon \times GM/R_{\text{NS}}$, $m$ is the NS mass $M$ in solar masses, $R_6$ is the NS radius in units of $10^6$ cm, $f = [1 - (R_6/R_m)^{1/2}]^{1/4} \sim 1$, $\mu_{26}$ is the magnetic moment of the NS in units of $10^{26}$ G cm$^3$ ($\mu = B_s R^3$ where $R$ and $B_s$ are the NS radius and surface magnetic field along the magnetic axis respectively). At radii $r < R_m$ the accreting matter is forced by the magnetic field to corotate with the NS and accretes along the field lines. As $R_m \propto L_{37}^{-34/135}$, a decrease in the mass–transfer rate results in an expansion of the magnetosphere. Accretion onto a spinning magnetized NS is centrifugally inhibited once $R_m$ lies outside the corotation radius $R_{\text{CO}}$, the radius at which the Keplerian angular frequency of the orbiting matter is equal to the NS spin:

$$R_{\text{CO}} = 1.50 \times 10^6 m^{1/3} P_{-3}^{2/3} \text{ cm}$$

(2)

where $P_{-3}$ is the spin period in milliseconds. In this case a significant fraction of the accreting matter might be centrifugally ejected from the system: this is the so called propeller phase.

The virial theorem sets stringent limits on the fraction of matter that can be ejected in this phase, in fact, it states that, at any radius in the disc, the virialized matter has already liberated (via electromagnetic radiation) half of its available energy, corresponding to the variation of its potential energy. Considering that $R_m \sim R_{\text{NS}}$ for MSPs, the matter at the magnetosphere has irradiated $\sim 50\%$ of the whole specific energy obtainable from accretion $\eta_{\text{acc}} = GM/R_{\text{NS}}$. To eject the same matter (that is close to the NS surface) $\sim 1/2 \eta_{\text{acc}}$ must be given back to it. As the only source of energy is the accretion process itself, the maximum ejection efficiency is $\leq 50\%$. For instance, with a fine tuned alternation of accretion and propeller phases of the right duration, the kinetic energy stored by the NS in the form of rotational energy during an accretion phase, can be used in a subsequent propeller phase to eject almost the same amount of matter previously accreted. The major difficulty with this scenario is that, during the accretion phase, once the system has reached the spin–equilibrium, no further spin up takes place, and thus the storage of accretion energy in a form that allows its subsequent re–usage for ejection is impossible. This is the reason why
the duration of the accretion and propeller phases has to be \textit{ad hoc} chosen in order to reach ejection efficiencies close to 50\%. It follows that, \textit{in any case, the accreted mass is no less than half of the transferred mass, i.e. } \geq 0.4 - 0.8 M_\odot. This has two main consequences:

1) even taking a propeller phase into account, NSs in MSPs will either be very massive or even collapse into black holes;

2) as the amount of mass accreted is considerably bigger than the minimum required to spin up the NSs to ultra short periods (Burderi et al. 1999), one has to invoke an \textit{ad hoc} final, long lasting propeller phase with an highly effective spin down to form the observed population of moderately fast spinning MSPs.

The only way to overcome these difficulties is to obtain ejection efficiencies close to unity. This is indeed possible if matter is ejected far from the NS surface through a large expansion of the magnetospheric radius \((R_m \rightarrow r >> R_{NS})\), at a distance where the orbiting matter has an almost negligible binding energy \(GM/r << \eta_{acc}\). As the NS is spinning very fast, the switch on of a radio pulsar is unavoidable once \(R_m \rightarrow r\) (see §4). In the next section we demonstrate that, under particular circumstances, the pressure exerted by the radiation field of the radio pulsar overcomes the pressure of the accretion disc, thus determining the ejection of the matter from the system. We note that, once the disc has been swept away, the radiation pressure stops the infall of matter far away from the NS surface, \textit{i.e.} in a region where \(\eta_{acc} \sim 0\), thus overcoming the 50\% efficiency limit imposed by the virial theorem. This means that the pressure of the radiation emitted by pulsar is a viable mechanism to sweep most of the matter away from the system. We discuss this possibility in the next section.

4. The effects of the pulsar energy outflow

The interaction of the accreting matter with the (assumed dipolar) magnetic field of the NS can be described in terms of an outward pressure (we use the expressions \textit{outward} or \textit{inward pressures} to indicate the versus of the force with respect to the radial direction) exerted by the rotating magnetic field on the accretion flow (see \textit{e.g.} Hayakawa, 1985).

The light–cylinder radius, \(R_{LC} = cP/2\pi\) (where an object corotating with the NS reaches the speed of light \(c\)), separates an inner region, where the magnetic dipole pressure term is relevant, from an outer region where the radiation pressure of the rotating dipole is acting.

For \(r < R_{LC}\), the outward pressure, due to the magnetic field, is

\[
P_{MAG} = \frac{B^2}{8\pi} = 7.96 \times 10^{14} \mu_{26}^2 r_6^{-6} \text{ dy/cm}^2
\]

(3)

where \(r_6\) is the distance from the NS center in units of \(10^6\) cm. For \(r > R_{LC}\) the outward pressure is given by the radiation pressure which, assuming isotropic emission, is given by:

\[
P_{RAD} = 2.04 \times 10^{12} P_{-3}^{-4} \mu_{26}^2 r_6^{-2} \text{ dy/cm}^2.
\]

(4)
In figure 3 the two outward pressures defined above are shown as bold lines for typical values of the parameters (see figure caption).

The accretion flow, in turns, exerts a inward pressure on the field that is the sum of the thermal gas pressure \( P_{\text{gas}} = \rho kT/nm_p \), and the light radiation pressure \( P_{\text{light}} = \sigma T^4/c \), where \( \rho \) and \( T \) are the flow density and temperature, \( k \) is the Boltzmann constant, and \( \sigma \) is the Stefan–Boltzmann constant. In the case of a Shakura–Sunyaev accretion disc orbiting a compact object of \( \sim 1 \, M_\odot \) we have \( P_{\text{light}} \lesssim P_{\text{gas}} \) for \( \dot{M} < \dot{M}_\text{E} \) and \( r > R_{\text{NS}} \) (see e.g. Frank, King & Raine 1992). Adopting for the inward disc pressure \( P_{\text{DISC}} \approx P_{\text{gas}} \) we have

\[
P_{\text{DISC}} = 1.02 \times 10^{16} \alpha^{-9/10} \epsilon^{-17/20} n_{0.615}^{1/6} L_{\odot}^{17/20} m^{1/40} \]
\[
R_6^{17/20} f^{17/5} r_6^{-21/8} \text{ dy/cm}^2 \tag{5}
\]

where \( f = [1 - (R_6/r_6)^{1/2}]^{1/4} \leq 1 \) and \( r_6 \) is the generic radial distance in units of \( 10^6 \) cm. As before we measure \( \dot{M} \) in units of \( L_{\odot} \) in the hypothesis that all the transferred mass accretes onto the NS radiating a specific energy \( \epsilon \times GM/R_{\text{NS}} \).

The disc pressure line tangent to the intersection point of (3) and (4) defines a critical mass–transfer rate \( M_{\text{switch}} \) (i.e. a critical luminosity \( L_{\text{switch}} \)) at which the radio pulsar switches–on. This depends on the position of the light–cylinder radius, and hence on the spin period of the NS: the faster is the rotation of the NS, the smaller is \( R_{\text{LC}} \), and the larger is the critical mass–transfer rate at which the pulsar switches–on.

In figure 3 the inward disc pressures (equation 5) for two different \( \dot{M} \) (measured in units of \( L_{\odot} \) as discussed above) are shown as solid lines. The disc pressure corresponding to \( L_{\text{switch}} \) is shown as a dotted line.

For a given mass–transfer rate \( \dot{M}_{\text{high}} \) (corresponding to a luminosity \( L = 2 \times 10^{37} \) erg/s in figure 3), the intersections of the relative \( P_{\text{DISC}} \) line with each of the inward pressure lines (points S and U in the upper panel of the same figure) define equilibria points between the inward and outward pressures. However, the equilibrium is stable at S (i.e. at \( r = R_m \), as defined by equation 1), and unstable at U (i.e. at \( r = R_{\text{STOP}} \)).

In fact, as \( P_{\text{MAG}} \) is steeper than \( P_{\text{DISC}} \) at S, if a small fluctuation forces the inner rim of the disc inward (outward), in a region where the magnetic pressure is greater (smaller) than the disc pressure, this results in a net force that pushes the disc back to its original location \( R_m \). On the other hand, as \( P_{\text{RAD}} \) is flatter than \( P_{\text{DISC}} \) at U, no stable equilibrium is possible at \( R_{\text{STOP}} \). In fact, if a small fluctuation forces the inner rim of the disc inward, \( P_{\text{DISC}} \) dominates over \( P_{\text{RAD}} \) and the disc moves inward until the stable equilibrium point S at \( R_m \) is reached. But, conversely, if a small fluctuation forces the inner rim of the disc outward, \( P_{\text{RAD}} \) dominates over \( P_{\text{DISC}} \) and the disc is swept away by the radiation pressure. This means that, for \( r > R_{\text{STOP}} \) no disc can exist for any mass–transfer rate \( \leq \dot{M}_{\text{high}} \).

\[\text{An explicit expression for } R_{\text{STOP}} \text{ can be derived equating (4) and (5).}
\]

\[
R_{\text{STOP}} = 8.26 \times 10^{11} \alpha^{-36/25} \epsilon^{-34/25} n_{0.615}^{-8/5} L_{\odot}^{34/25} m^{1/25} R_6^{44/25} f^{136/25} \mu_{26}^{-16/5} P_{-3}^{32/5} \text{ cm.} \tag{6}
\]
It is convenient to define *compact* systems those in which the primary Roche lobe radius \((R_{L1} \propto P_{\text{orb}}^{2/3})\) lies inward \(R_{\text{STOP}}\) (figure 3, upper panel) and *wide* those in which the primary Roche lobe radius lies outward \(R_{\text{STOP}}\) (figure 3, lower panel), as these systems behave very differently in response to significant and abrupt variations of the mass–transfer rate.

Let us consider, indeed, the effects of such a sudden variation in the mass–transfer rate, as is the case e.g. of a transient system alternating between outbursts \((\dot{M}_{\text{high}})\) and quiescence \((\dot{M}_{\text{low}})\) phases. The corresponding luminosities are \(2 \times 10^{37}\) and \(3 \times 10^{33}\) erg/sec, respectively.

For simplicity, in the transition from outburst to quiescence (quiescence to outburst), let us assume that the timescale on which the drop (rise) in the disc pressure occurs is much shorter than the timescale on which the magnetosphere expands (contracts), as this does not affect the general conclusions.

The behaviour of a compact system (short orbital period, e.g. \(P_{\text{orb}} = 0.5\) h) is cyclic, as shown in figure 3, upper panel.

**Ic Accretor.** During the outburst, the magnetospheric radius (delimiting the inner radius of the disc) is smaller than both the corotation and the light–cylinder radius and the NS will normally accrete matter and angular momentum, thus increasing its spin.

**IIc Radio ejector.** The sudden drop in the mass–transfer rate (quiescence) initiates a phase, that we termed “radio ejection”, in which the mechanism that drives mass overflow from the inner Lagrangian point \(L_1\) is still active, while the pulsar radiation pressure at \(L_1\) prevents mass accretion. In fact, as soon as the new disc pressure (at the same outburst magnetospheric radius, because of our assumption on the drop and rise timescales) drops below \(P_{\text{switch}}\), the magnetosphere expands beyond the light cylinder radius and the radio pulsar switches–on pushing the disc away from the system until \(R_{L1}\) is reached starting the radio ejection phase (track A).

As the overflowing matter cannot accrete, it is now ejected as soon as it enters the Roche lobe of the primary, where its binding energy is negligible, thus circumventing the limit imposed by the virial theorem discussed in the previous section and allowing ejection efficiencies close to 100%. The ejected matter presumably leaves the system with a specific angular momentum \(l\) between those of the inner and outer Lagrangian points \((2\pi/P_{\text{orb}})d_{L1}^2 < l < (2\pi/P_{\text{orb}})d_{L2}^2\), where \(d_{L1}\) and \(d_{L2}\) are the distances between the centre of mass of the system and the inner and outer Lagrangian points respectively.

**Ic Accretor.** When a new outburst occurs, the mass–transfer rate rises back to its original value \((L = 2 \times 10^{37}\) ergs/s), the disc follows track B, and the accretion resumes. In the subsequent quiescence, the system goes back to a radio ejection phase IIc, alternating between Ic and IIc in response to the variations of the mass–transfer rate.

The response of a wide system (long orbital period, e.g. 47 h) with the same transient behaviour is quite different, as shown in figure 3, lower panel.

**Iw Accretor.** For the same initial mass–transfer rates as in Ic, the NS accretes matter and increases its spin.
IIw *Radio ejector.* The same drop in mass–transfer as in case IIc triggers the switching–on of the radio pulsar and the subsequent radio ejection of the matter overflowing $R_{L1}$ (track A).

IIIw *Radio ejector.* Contrary to the compact system case, when the mass–transfer rate recovers its previous value, the cyclic behaviour is lost. Indeed, since for wide systems $R_{L1}$ (that is the endpoint of track A for both wide and compact systems) is located beyond $R_{STOP}$, $P_{DISC} < P_{RAD}$ even for $L = 2 \times 10^{37}$ ergs/s and the accretion can not resume. The growth of pressure indicated by track B is not sufficient to allow disc formation. This means that for a wide system once a drop of the mass–transfer rate has started the radio ejection, a subsequent restoration of the original mass–transfer rate is unable to quench the ejection process (as already pointed out by Ruderman, Shaham, & Tavani 1989).

We note that the radio ejection mechanism proposed here can also work if the variations of the mass–transfer rate are determined by the secular evolution rather than by a transient behaviour. Especially in this case, once the situation described in cases Iw, IIw, & IIIw occurs, the subsequent orbital evolution is very sensitive to the exact value of $l$ and will be discussed elsewhere (Burderi et al., 2001, in preparation) leading in some cases to the disruption of the companion. This disruption mechanism (let us call it “Roche lobe disruption”) is in many respects similar to the effect of the “evaporation” of the companion irradiated by the wind (composed of electromagnetic radiation and relativistic particles) of a MSP (Tavani 1991). However several differences exists that will be discussed elsewhere (Burderi et al., 2001, in preparation).

In conclusion, while the evolution without a radio ejection phase implies that a large fraction of the transferred mass is accreted onto the NS (because of the constraints imposed by the virial theorem), in this paragraph we have demonstrated that the switch–on of a radio pulsar (associated to a significant drop in mass transfer) could determine ejection efficiencies close to 100% as the matter is ejected before it falls into the deep gravitational potential well of the primary.

For compact systems, the evolution outlined in Ic, IIc implies that radio ejection is swiftly quenched by a resume of the original mass–transfer rate, leading to the prediction that the amount of mass accreted is substantial. On the other hand, if a radio ejection starts in a wide system (IIw), IIIw implies that the accretion is inhibited in the subsequent evolution. This suggests that wide systems are the progenitors of the observed population of moderately massive and moderately fast spinning MSPs.

5. Where to search for ultra fast spinning NSs

The population synthesis calculations, described in §2, which include significant mass ejection and different assumptions for the evolution of the magnetic field, showed that the process of recycling in LMXBs can produce a significant amount of ultra fast spinning objects. In line with this, a statistical analysis based on the current samples of detected MSPs, using different hypothesis for the period
Figure 3. The radial dependence of the pressures relevant for the evolution of accreting NSs and recycled pulsars for: a compact system ($P_{\text{orb}} = 0.5$ h), upper panel, and a wide systems ($P_{\text{orb}} = 47$ h), lower panel. The parameters adopted are: $\mu_{26} = 7.7$, $P_{-3} = 1.5$, $\alpha = 1$, $\epsilon = 1$, $n_{0.615} = 1$, $R_6 = 1$, $m = 1.65$. 
distributions of these sources, proved that there is always a non negligible probability for MSPs with $P < P_{\text{min}}$ (Cordes & Chernoff 1997). However, despite the large observational efforts over the recent years (D’Amico 2000, Edwards 2000, Crawford, Kaspi & Bell 2000), no pulsar with $P < P_{\text{min}}$ has been discovered so far.

In §4 we noted that if the NS is spinning very fast, a drop in the mass transfer rate causes the switch-on of a radio pulsar and starts a radio ejection phase. Moreover, a new phase of accretion is only possible if the orbital period is short enough. Thus, our model predicts that the spin up of a NS to $P \lesssim 1$ ms requires very close ($P_{\text{orb}} \lesssim 1$ h) binary systems.

A consequence of the shortness of $P_{\text{orb}}$ is that the strong Doppler modulation of the pulsar period induced by the orbital motion provide a natural observational bias against the detection of ultra fast spinning pulsars. All the algorithms for the detection of periodicities from a source in a close binary system are, unavoidably, a trade-off between computational capability and sensitivity: on each data set, they must perform a two dimensional search in the space of the dispersion measure DM (related to the unknown distance of the object), and acceleration (resulting from binary motion). Very short data sets reduce the Doppler modulations during the observation and reduce the CPU demands, at the price of limiting the sensitivity (see e.g. Camilo et al. 2000). As a consequence, $P_{\text{orb}}$ shorter than $\sim 90$ min have been poorly searched up to now, even in the most favorable case of searches on a definite target (such as those in globular clusters), in which one of the parameters (DM) is known.

The two MSPs with the shortest orbital periods, 96 min and 102 min, have been just detected in the globular clusters 47 Tuc (Camilo et al. 2000) and NGC 6544 (D’Amico et al. 2000), whereas in the galactic field the best case is that of PSR J2051–0827 ($P_{\text{orb}} = 2.37$ h). This observational bias would be worse in the presence of eclipses (favoured in very close binary pulsar systems, Nice 2000), in case of large duty cycles of the pulsed signal, in case of low radio luminosity, and strong interstellar scintillation, phenomena which have already been suggested to explain the elusiveness of SMSPs (Possenti 2000).

Aknowledgments

This work was supported by a grant by the Italian Ministero dell’ Università e della Ricerca Scientifica (Cofin-99-02-02).

References

Alpar, M.A., Cheng, A.F., Ruderman, M.A., & Shaham, J. 1982, Nature, 300, 728
Bhattacharya, D. 1995, in X–ray Binaries, ed. W.H.G. Lewin, J. van Paradijs & E.P.J. van den Heuvel (Cambridge Univ. Press), 5
Burderi, L., D’ Amico, N. 1997, ApJ, 490, 343
Burderi, L., Di Salvo, T., Robba, R., Del Sordo, S., Santangelo, A., Segreto, A. 1998, ApJ, 498, 831
Burderi, L., King, A.R. & Wynn, G.A. 1996, ApJ, 457, 348
Burderi, L., Possenti, A., Colpi, M., Di Salvo, T., & D’Amico, N. 1999, ApJ, 519, 285
Where Have All the Submillisecond Pulsars Gone?

Camilo, F., Lorimer, D.R., Freier, P., Lyne, A.G., Manchester, R.N., 2000, ApJ, 535, 975
Chen, K. & Ruderman, M. 1993, ApJ, 402, 264
Cook, G.B., Shapiro S.L., & Teukolsky, S.A. 1994, ApJL, 423, L117
Cordes, J. & Chernoff, D.F. 1997, ApJ, 482, 971
Crawford, F., Kaspi, V.M. & Bell, J.F. 2000, in Pulsar Astronomy - 2000 and beyond, ASP Conf.Series, 202, 31
D’Amico, N. 2000, in Pulsar Astronomy - 2000 and beyond, ASP Conf.Series, 202, 27
D’Amico, N., Lyne, A.G., Manchester, R.N., Possenti, A., Camilo, F. 2000, Nature, in press
D’Antona, F., Mazzitelli, I., Ritter, H. 1989, A&A, 225, 391
Edwards, R.T., 2000, in Pulsar Astronomy - 2000 and beyond, ASP Conf.Series, 202, 33
Frank, J., King, A.R., Raine, D.J. 1992, Accretion power in astrophysics, second edition, Cambridge Astrophysics Series
Hayakawa, S., 1985, Phys. Rep., 121, 317
Illarionov, A., & Sunyaev, R. 1975, A&A, 39, 185
King, A.R., Kolb, U., & Burderi, L. 1996, ApJ, 464, L127
Konar, S., & Bhattacharya, D., 1999, MNRAS, 303, 588
Miri, M.J., & Bhattacharya, D. 1994, MNRAS, 269, 455
Nice, D.J., Arzoumanian, Z., & Thorsett, S.E. 2000, in Pulsar Astronomy - 2000 and beyond, ASP Conf.Series, 202, 67
Possenti, A., 2000, Ph.D. Thesis, Bologna, 159
Possenti, A., Colpi, M., D’Amico, N., & Burderi, L. 1998, ApJL, 497, L97
Possenti, A., Colpi, M., Geppert, U., Burderi, L., & D’Amico, N. 1999, ApJS, 125, 463
Ritter, H. 1988, A&A, 202, 93
Ruderman, M., Shaham, J., & Tavani, M. 1989, ApJ, 336, 507
Ruderman, M.A., Zhu, T., & Chen, K 1998, ApJ, 492, 267
Sang, Y., & Chunmugam, G. 1987, ApJ, 323, L61
Shakura, N.I., & Sunyaev, R.A. 1973, A&A, 24, 337
Srinivasan, G., et al. 1990, Curr.Sci. 59, 31
Taam, R.E., King, A.R., & Ritter, H. 2000, ApJ, submitted
Tanaka, Y., Shibazaki, N., 1996, ARA&A, 34, 607
Tauris, T.M., & Savonije, G.J. 1999, A&A, 350, 928
Tavani M. 1991, Nature, 351, 39
Thorsett, S.E., & Chakrabarty, D. 1999, ApJ, 512, 288
Urpin, V.A., Geppert U., & Konenko, D. 1998, MNRAS, 295, 907
van der Klis, M. 1998, in The Many Faces of Neutron Stars, eds. Alpar, Buccheri, van Paradijs, Dordrecht: Kluwer, 337
van Paradijs, J. 1996, ApJ, 464, L139
Webbink, R.F., Rappaport, S.A., & Savonije, G.J. 1983, ApJ, 270, 678