Disembodiment of Physical Properties by Pre- and Post-Selections

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The detailed study of disembodiment of physical properties by pre- and post-selection is present. A criterion is given to disembodiment physical properties for single particle with multiple degrees of freedom. It is shown that the non-commute operators can also be well separated in different paths. We generalize the disembodiment to entangled particles, and found that the disembodiment can happen under special conditions due to the entanglement.

I. INTRODUCTION

Since the foundation of quantum physics, the controversies about the self-consistency, completeness, causality and locality in quantum physics have not stopped. It has shown that the quantum physics is wacky and hard to interpreted, but have been confirmed by experiments for nearly one century. Physicists are keeping trying to understand this wacky quantum worlds with a lot of paradoxes, such as the EPR paradox, Zeno paradox etc. These paradoxes reveal the contradiction of intuition of people and the truth of nature, and usually inspires the new ideas and new interpretations, further improve the development of quantum physics.

One of the paradoxes, the pre- and post-selection (PPS) which was firstly proposed by Aharonov, Bergmann, and Lebowitz (ABL) [1, 2], questioned about the quantum arrow of time. Further studies on PPS leads to numbers of paradoxes, such as the famous Aharonov, Alber and Vaidman (AAV) paradox which revealed that the outcome of measurement can be widely out of the range of the eigenvalues of a system through PPS [3]. The paradox about PPS is not only of theoretical interests on foundations of quantum mechanics, but also offers a powerful tool in experiments. The quantum state estimation and precision measurement based on the PPS has been proposed, and successfully demonstrated in experiments [4, 5]. Very recently, Aharonov, Popescu and Skrzypczyk (APS) proposed that the Cheshire cats, i.e. the “body” and “grin” of cat can be surprisingly separated through appropriate selections of initial and final states [6]. It is very potential for further application in theoretical and experimental studies [7, 8], but the details of the embodiment of physics properties is still waiting to be explored.

In this paper, we present a general treatment to disembodiment of physical properties by PPS and give a criterion to disembodiment physical properties for single particle with multiple degrees of freedom. It is shown that the non-commute operators can also be separated successfully. We generalize the disembodiment to entangled particles, and found that the disembodiment can happen for special conditions due to the entanglement.

II. THE CHESHIRE CATS AND WEAK MEASUREMENT

The quantum mechanics is time symmetric, that the initial state is as important as the final state. Thus, we can prepare particles in selected initial state $|\Psi\rangle$ which called the pre-selection, and then postselect the ensemble of the particles corresponding to a final state $|\Psi\rangle$ through the measurement in detectors. It has been shown in Ref. [6], in the specific ensembles with PPS, the photon number and the polarization can be separated in different paths. However, the polarization and photon number cannot be directly readout through traditional collapse measurement, where the quantum state is changed significantly. In this case, we cannot distinguish whether the Cheshire cats is found. If we resorted to the weak measurement that the ancilla measuring device weakly coupling to the system, therefore the disturbance to the state of system induced by the measurements can be neglected. The Hamiltonian of the weak measurement reads

$$H_I = ghAO,$$

where $g$ is the interaction strength, $A$ is the ancilla, and $O$ is the observer operator of the system. In the case of the PPS, the average outcome (also called Weak Value) of the observer should be

$$\langle O \rangle_w = \frac{\langle \Phi | O | \Psi \rangle}{\langle \Phi | \Psi \rangle},$$

with $\langle \Phi | \Psi \rangle \neq 0$ that the initial and final state are not orthogonal to each other. The $\langle O \rangle_w$ is amplified if the $\langle \Phi | \Psi \rangle$ is a very small, which can greatly enhance the measurement precision but with the scarifies of counts at detector.

Here we want to separate the properties described by operator $O^j$ with superscript $j = 1, 2, 3, \ldots$, for different quantum properties. Disembodiment of a physics properties requires that the expect value of operator $O^j$ is nonzero only in one output, for example, in one output path for a photon. Additionally, in this path, the expect...
value of other operator should be zero. Without loss of generality, to separate operators \( O^j \) in path \( j \), the PPS must satisfy

\[
\langle O^j \rangle_w = \frac{\langle \Phi | O^j | \Psi \rangle}{\langle \Phi | \Psi \rangle} = a_i \delta_{ij},
\]

where the subscript \( i \) denote the path, \( a_i \) are non-zero numbers and \( \delta_{ij} \) is Kronecker delta.

### III. SINGLE PARTICLE

For any single particle system, the quantum states can be present by a vector in the Hilbert space with the basis \( \{|h^i_k\rangle\} \), where \( i = 1, \ldots, p \) and \( j = 1, \ldots, q \) stand for different paths and physical properties, and \( k = 1, 2, \ldots, d \) show the degrees of freedom of the \( j \)th physical properties. For path \( i \), the PPS can be present as \( \langle \Phi_i | = \{x^1_i, x^2_i, \ldots, x^n_i\} \) and \( |\Psi_i \rangle = \{y^1_i, y^2_i, \ldots, y^n_i\}^T \), where the dimension \( n = q \times d \). The condition for disembodiment in path \( i \) becomes

\[
\langle \Phi_i | O^j_i | \Psi_i \rangle = a_i \delta_{ij},
\]

with the weak value unnormalized. Or we can write the equation in the matrix form as

\[
\sum_{kl} (O^j_k)_{kl} x^k_i y^l_i = a_i \delta_{ij},
\]

where \( (O^j_k)_{kl} = \langle h^k_l | O^j | h^i_l \rangle \). Combining all operators \( j = 1, \ldots, q \), we can write the tensor in the matrix form as

\[
M_i \{x^1_i y^1_i, x^2_i y^2_i, \ldots, x^n_i y^n_i\} = \{0, \ldots, 0, x^1_i y^1_i, \ldots, x^n_i y^n_i\}^T,
\]

where the dimension of \( M_i \) is \( q \times n^2 \). In the present case, the physical properties and operators are the same for all paths, thus we have \( M_i = M \) independent on the path. Suppose there are \( m \) operators to separate \( (m \leq p) \), then we should solve the linearized equations

\[
M_1 \vec{v}_1 = \{a_1, 0, \ldots, 0\}^T,
\]

\[
M_2 \vec{v}_2 = \{0, a_2, 0, \ldots, 0\}^T.
\]

The criterion of the existence of solutions to above equations is

\[
\text{rank}(M) = m.
\]

Applying this criteria, we study two examples of photon with polarization degree of freedom.

**Example I**: Single photon with the polarization \(|+\rangle\) or \(|-\rangle\) in two paths \(|1\rangle\) or \(|2\rangle\). The operator of the which photon number operator is

\[
O^1_i = I_i = \text{diag}\{1, 1\},
\]

and the polarization operator is

\[
O^2_i = \sigma^z_i = \text{diag}\{1, -1\}.
\]

Then, we have the matrix \( M = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \), with \( \text{rank}(M) = 2 \). Thus, we can separate the photon number and polarization in two different paths. We need to solve the equations

\[
M \begin{pmatrix} x^1_1 y^1_1 \\ x^2_1 y^1_1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix},
\]

\[
M \begin{pmatrix} x^2_1 y^2_1 \\ x^2_2 y^2_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix},
\]

and we have \( x^1_1 y^1_1 = x^2_1 y^2_1 = \frac{1}{2} \) and \( x^1_2 y^2_1 = -x^2_2 y^2_2 = \frac{1}{2} \). There are infinite choices of pre-selections and post-selections. Let the pre-selection state be \( |\Phi \rangle = \{x^1_1, x^2_1, x^1_2, x^2_2\} = \{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\} \), then \( |\Phi \rangle = \{y^1_1, y^2_1, y^1_2, y^2_2\}^T = \{(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2})^T \}. \) This is exactly the case in the original Cheshire Cats paper by Aharonov et al. [6].

**Example II**: Single photon with polarization \(|+\rangle\) or \(|-\rangle\) in four path \(|1\rangle\), \(|2\rangle\), \(|3\rangle\) or \(|4\rangle\). We want to separate the photon number operator \( I \) and the polarization operators \( \sigma^x \), \( \sigma^y \) and \( \sigma^z \) in different paths. Thus, we have

\[
I_i = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},
\]

\[
\sigma^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},
\]

\[
\sigma^y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},
\]

\[
\sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},
\]

The matrix form is

\[
M = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & -i & i & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix}.
\]

In path \( j \), the equation should be satisfied
IV. ENTANGLED PARTICLES

Now, we turn to consider the two-particle system. If there is no entanglement, the two-particle system is only the simple extension of single particle systems with more degrees of freedom. Take the two photons in four path (photon A in path |1⟩ or |2⟩; photon B in path |3⟩ or |4⟩) with the the polarization is entangled \( (|H⟩_A |V⟩_B + |H⟩_B |V⟩_A) \). Thus, the Hilbert space of state can be represent in the basis

\[
|1⟩, |2⟩, |3⟩, |4⟩, |1⟩, |2⟩, |3⟩, |4⟩, |1⟩, |2⟩, |3⟩, |4⟩, |1⟩, |2⟩, |3⟩, |4⟩.
\]

Since the polarization is entangled, the dimensional of the state space is 8. The operators of the photon number are

\[
I^A_1 = \text{diag} \{1, 1, 1, 0, 0, 0, 0, 0\},
I^A_2 = \text{diag} \{0, 0, 0, 0, 1, 1, 1, 1\},
I^B_1 = \text{diag} \{0, 1, 0, 1, 1, 0, 0, 0\},
I^B_2 = \text{diag} \{0, 1, 0, 1, 0, 1, 0, 1\}.
\]

and operators of photon polarization are

\[
s^A_1 = \text{diag} \{1, 1, -1, -1, 0, 0, 0, 0\},
s^A_2 = \text{diag} \{0, 0, 0, 0, 1, 1, -1, -1\},
s^B_1 = \text{diag} \{-1, 0, 1, 0, -1, 0, 1, 0\},
s^B_2 = \text{diag} \{0, -1, 0, 1, 0, -1, 0, 1\}.
\]

The matrix form is

\[
M = \begin{pmatrix}
1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\
1 & 1 & -1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\
-1 & 0 & 1 & 0 & -1 & 0 & 1 & 0 \\
0 & -1 & 0 & 1 & 0 & -1 & 0 & 1 \\
\end{pmatrix}.
\]

We find that, for the expecting value for operator as

\[
\Phi^T = \{1, 0, 0, 1, 1, 0, 0, 1\}^T,
\]

the solution to

\[
M \Phi^T = \Phi^T
\]

does not exist. However, for

\[
\Phi^T = \{1, 0, 0, 1, 1, 0, 0, -1\}^T,
\]

the disembodiment can happen. This is due to the polarization entanglement between the two photons, where the expecting values should be 1 and −1 respectively. So, we can separate the physics properties even for entangled particles.

V. DISCUSSION

(1) From the analysis for single particle above, the disembodiment of physical properties is not restricted to the system with two degrees of freedom. The physics properties can always be separated through particular PPS ensemble according to the criterion. In addition, the selected initial and final states for disembodiment are not sole.

(2) The disembodiment is not restricted to separation physical properties in different paths. It can be extended to any other degree of freedom that we can address in experiment, such as internal degree of atoms.

(3) Potential application of disembodiment would be selectively measure the parameters with different operators. For instance, for a system interact with ancilla \( H_I = \hbar A g_1 O^1 + g_2 O^2 \), where \( O^1 \) and \( O^2 \) are independent to each other, the observer \( O^1 \) and \( O^2 \) can be selectively measured through disembodiment by only one ancilla.

(4) In all above analysis, the PPS is perfect regardless the actual preparation and detection process. The real experimental situation, the imperfect devices give errors in state preparation and detection. This type of error is stationary and can be modified through transformations when the imperfection of devices are calibrated. Noting that, the effect of noise is random that can not be estimated through transformation.
VI. CONCLUSIONS

In summary, we have studied the disembodiment of physical properties by pre- and post-selection in detail. We give a criterion to disembodify physical properties for single particle with multiple degrees of freedom. It is shown that the non-commute operator can also be separated in different paths. We generalize the disembodiment to entangled particles and find that the disembodiment can happen for special conditions due to the entanglement.

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