Constraint on the cosmological $f(R)$ model from the multipole power spectrum of the SDSS LRG sample and prospects for a future redshift survey

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A constraint on the viable $f(R)$ model is investigated by confronting theoretical predictions with the multipole power spectrum of the luminous red galaxy sample of the Sloan Digital Sky survey data release 7. We obtain a constraint on the Compton wavelength parameter of the $f(R)$ model on the scales of cosmological large-scale structure. A prospect of constraining the Compton wavelength parameter with a future redshift survey is also investigated. The usefulness of the redshift-space distortion for testing the gravity theory on cosmological scales is demonstrated.

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I. INTRODUCTION

Experimental tests of gravity on the scale of the Solar System show good agreement with predictions of general relativity (e.g., [1]). The nature of the Newtonian gravity is the attractive force, which naturally predicts a decelerated expansion of the universe. Contrary to this expectation, it has been discovered that our universe is undergoing an accelerated expansion epoch [2–4]. Though the accelerated expansion is explained by introducing a cosmological constant, its small but nonzero value cannot be explained naturally [5]. The problem might be deeply rooted in the nature of fundamental physics.

This problem has attracted many researchers, and many works have been done, both theoretically and observationally. As a generalisation of the cosmological constant, a dynamical field, called the dark energy model and its variants, are proposed to explain the accelerated expansion of the universe (see [6] and references therein). As an alternative to the dark energy model, modification of gravity may explain the accelerated expansion. General relativity is not considered to be the complete theory, because its quantum theory cannot be formulated in a well defined manner. The theory of gravity might need to be reformulated within a more general framework.

From the observational point of view, the constraint on the gravity theory on cosmological scales has not been well investigated, compared with the constraint on the scales of the Solar System. Many future projects to produce large galaxy surveys are in progress or planned [7–12], which aim to explore the nature of the dark energy. These surveys are useful for testing the theory of gravity at cosmological scales (e.g., [13]). The dynamical dark energy models may have similar expansion rates as models of modified gravity, but predict different histories for the growth of structures. The key to testing the gravity theory is the measurement of the evolution of cosmological perturbations, as many authors have concluded recently [14–25].

The cosmic microwave background anisotropies are useful for investigating the cosmological perturbations through the measurements of the integrated Sachs-Wolfe effect or the lensing effect on the angular power spectrum [26]. Imaging surveys of galaxies are also useful through the weak lensing statistics or cluster number counts [27, 28]. Similarly, redshift surveys of galaxies are helpful for testing gravity [29–34]. In the present paper, we revisit the problem of testing the gravity theory through a measurement of the multipole power spectra in the Sloan digital sky survey (SDSS) luminous red galaxy (LRG) sample [31]. Measuring the multipole power spectra is a way to estimate the redshift-space distortions, which reflects the linear growth rate of the matter density perturbations [35–37].

Many authors have investigated the clustering nature of the SDSS LRG sample [38–47]. In the references [48, 49], recent results on LRGs from the SDSS data release (DR) 7 are reported. In the reference [46], a test of gravity is considered using the observed anisotropic correlation function. Three of the authors of the present paper have shown that the SDSS LRG sample is useful to test the gravity theory by measuring the quadrupole power spectrum of galaxy
distribution, which represents the redshift-space distortions \cite{31}. In the present paper, we revisit the issue of testing the gravity theories on the cosmological scales using the SDSS LRG sample of the DR 7, especially focusing on the \( f(R) \) gravity model.

The \( f(R) \) models proposed in \cite{51,52} are viable models of modified gravity, which include some function of the Ricci scalar, \( f(R) \), added to the Einstein Hilbert action. As the modification of gravity involves the introduction of extra degree of freedom in general, one must be careful with the resulting behaviour. Furthermore, any theory must reduce to the general relativity on the scales of the Solar System. In the \( f(R) \) model, the general relativity is supposed to be recovered by the chameleon mechanism \cite{58,59}, which hides the field of the extra degree of freedom because the mass of the field becomes large for a dense region. The cosmological bounds on the \( f(R) \) model have been investigated with the cosmic microwave background anisotropies \cite{60} and also using the abundance of galaxy clusters \cite{61}. However, our approach is based on the redshift-space distortion \cite{90}.

This paper is organised as follows: In section 2, we briefly review the \( f(R) \) model and the characteristic evolution of the matter density perturbation. In section 3, we present our results for the multipole power spectrum of the SDSS LRG sample of the DR 7. In section 4, cosmological constraint is discussed by confronting the observed multipole spectra with the theoretical predictions. In section 5, a prospect of constraining the \( f(R) \) model is discussed on the basis of the Fisher matrix analysis, assuming a future large redshift survey. Section 6 is devoted to summary and conclusions. Throughout this paper, we use units in which the velocity of light equals 1, and adopt the Hubble parameter \( H_0 = 100h \text{km/s/Mpc} \) with \( h = 0.7 \).

II. \( f(R) \) Gravity Model

In this section, we briefly review the \( f(R) \) model, proposed in the references \cite{50,53}. In general, higher derivative terms are expected in the low energy effective action of gravity. Inspired by this, the \( f(R) \) model introduces some function of the Ricci scalar \( f(R) \), adding to the Einstein Hilbert action. We consider the theory defined by

\[
S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} (R + f(R)) + S_m, \tag{1}
\]

where \( S_m \) is the action of the matter. Many aspects of the \( f(R) \) model have been investigated; see e.g. \cite{54,55} for a review (cf. \cite{50,57}). We assume that the chameleon mechanism is responsible for the recovery of the general relativity on the Solar-System scales. The chameleon mechanism is a nonlinear effect. Recently, the effect on the quasi-nonlinear power spectrum is investigated based on the perturbative approach or the numerical simulations \cite{62–65}. This nonlinear chameleon effect becomes influential in the nonlinear regime. In the present paper, however, we can neglect the nonlinear chameleon effect because we need to consider only rather large scales, \( k \lesssim 0.2h\text{Mpc}^{-1} \).

For the viable model, the function \( f(R) \) must satisfy some conditions. We consider the model where the asymptotic form of \( f(R) \) can be expressed by

\[
f(R) \approx -2\Lambda \left[ 1 - \left( \frac{R_c}{R} \right)^{2n} \right], \tag{2}
\]

where \( \Lambda \) is the cosmological constant, \( n \) is a constant that specifies the \( f(R) \) model, and \( R_c \) is also a constant with the same dimension as that of the Ricci scalar. The background expansion of this \( f(R) \) model is well approximated by that of the \( \Lambda \)CDM model.

It is known that the additional term \( f(R) \) involves the introduction of an extra degree of freedom. Namely, \( f_R \equiv df/dR \) corresponds to the extra degree of freedom, which behaves like a scalar field. From the above action, one can derive the equation for \( f_R \),

\[
\nabla_\mu \nabla^\mu f_R = \frac{1}{3} (R + 2f - Rf_R) + \frac{8\pi G}{3} (-\rho + 3P), \tag{3}
\]

where \( \rho \) and \( P \) are the energy density and the pressure of the matter, respectively. If we regard the right hand side of equation (2) as the derivative of the effective potential, \( df_{\text{eff}}/df_R \), the mass of \( f_R \) can be read

\[
m^2 = \frac{df_{\text{eff}}}{df_R} = \frac{1}{3} \left( 1 + \frac{f_R}{f_{RR}} \right). \tag{4}
\]

The viable \( f(R) \) theory satisfies \( f \ll R \), and \( |f_R| \ll 1 \). Assuming \( Rf_{RR} \ll 1 \), the mass of the extra degree of freedom is

\[
m^2 \approx \frac{1}{3} \frac{1}{f_{RR}}, \tag{5}
\]
where $f_{RR} = d^2 f / dR^2$. Thus, $f_{RR} > 0$ is required to avoid the extra degree of freedom to become tachyonic. This extra degree of freedom mediates an attractive force, and modifies the gravity from the range determined by the Compton wavelength $\lambda = 1/m$. From Eq. (2), we have

$$f_{RR} = \frac{d^2 f(R)}{dR^2} = 4n(2n + 1)\Lambda \frac{R^2n}{R^{2n+2}}. \quad (6)$$

In the subhorizon limit, the matter density perturbation follows (e.g., [66] and references therein),

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\delta - 4\pi G_{\text{eff}}(a,k)\rho\delta = 0, \quad (7)$$

where

$$G_{\text{eff}}(a,k) = \frac{k^2}{a^2} + \frac{1}{3f_{RR}}, \quad (8)$$

and the dot denotes the differentiation with respect to the cosmic time.

Instead of $R_c$, we introduce the parameter $k_c$ by

$$\frac{1}{3f_{RR}} = k_c^2 \left( \frac{\Omega_0/a^3 + 4(1 - \Omega_0)}{\Omega_0 + 4(1 - \Omega_0)} \right)^{2n+2}, \quad (9)$$

where $k_c$ represents the wavenumber corresponding to the Compton wavelength at the present epoch. Thus, the $f(R)$ model is specified by $n$ and $k_c$. The growth factor can be obtained by solving Eq. (7), which we denote by $D_1(a,k)$. The growth rate is given by $f = d\ln D_1(a,k)/d\ln a$.

In the Einstein de Sitter background universe, the evolution of the density perturbation can be solved analytically [68]. Two of the authors of the present paper investigated characteristic features of the evolution of the growth rate of the $f(R)$ model, both numerically and analytically in the reference [67]. In the present paper, we solve the evolution equation (7) numerically (cf. [69, 70]). Figure 1 shows the growth factor divided by the scale factor (left) and the growth rate (right), respectively, as a function of the scale factor. The solid curve is the $\Lambda$CDM model with the density parameter $\Omega_0 = 0.28$. The dashed curves are for the $f(R)$ model with different wavenumbers $k/(h\text{Mpc}^{-1}) = 0.2, 0.1, 0.05$, respectively. Here the $f(R)$ model assumes $n = 1$ and $k_c = 0.05h\text{Mpc}^{-1}$. Due to the modification of the gravity the growth factor and the growth rate are enhanced, and this enhancement is scale-dependent.
FIG. 2: The mean number density of galaxies, $\bar{n}$, as a function of the redshift $z$ of the SDSS LRG sample, where we adopted the ΛCDM model with $\Omega_0 = 0.28$ for the distance-redshift relation $s = s[z]$.

III. MULTIPOLe SPECTRUM OF THE SDSS LRG SAMPLE

The multipole power spectrum $P_\ell(k)$ is defined by the coefficient of the multipole expansion of the anisotropic power spectrum $P(k, \mu)$,

$$P(k, \mu) = \sum_{\ell=0,2,4,6,\ldots} P_\ell(k) L_\ell(\mu)(2\ell + 1),$$

where $L_\ell(\mu)$ are the Legendre polynomials, $\mu(= \cos \theta)$ is the directional cosine between the line of sight direction and the wavenumber vector $k$. Note that our definition of the multipole spectrum $P_\ell(k)$ is different from the conventional one by the factor $2\ell + 1$ [35, 36, 71]. Here the Legendre polynomials satisfy the normalisation condition,

$$\int_{-1}^{+1} d\mu L_\ell(\mu)L_{\ell'}(\mu) = \frac{2}{2\ell + 1}\delta_{\ell\ell'}.\quad (11)$$

The monopole $P_0(k)$ represents the angular averaged power spectrum, which is what we usually mean by the power spectrum; the quadrupole $P_2(k)$ represents the leading anisotropy in the power spectrum due to the redshift-space distortion. The hexadecapole $P_4(k)$ represents a different aspect of the redshift-space distortion. In the present paper, we focus on the monopole and quadrupole spectra. The quadrupole spectrum reflects the peculiar velocities of the galaxies [35, 36, 71]. Those peculiar motions can be used to test the gravity theory on cosmological scales.

Pioneering works on the measurement of the quadrupole spectrum was carried out by Cole, Fisher, and Weinberg [35] and Hamilton [36] using the IRAS galaxy survey catalogue. Cole et al. presented a systematic method to estimate the quadrupole power spectrum through the anisotropic power spectrum [35]. The method was applied to the Two Degree Field (2dF) galaxy survey to estimate the $\beta$ factor. Hamilton obtained the quadrupole power spectrum by a transformation of the correlation functions [36]. In the present work, however, we adopt a different method to estimate the quadrupole power spectrum [75]. Our method is in line to the widely used way to estimate the monopole power spectrum [72, 73], and allows us to obtain the multipoles of the redshift-space power spectrum without evaluating the correlation function or the anisotropic power spectrum. In Ref. [31], we applied the method to the SDSS LRG sample from DR 6 to test the general relativity on cosmological scales. In the present paper, we revisit this problem with the SDSS LRG sample of DR 7 [74].

Our LRG sample is restricted to the redshift range $z = 0.16 - 0.47$. In order to reduce the sidelobes of the survey window we remove some noncontiguous parts of the sample (e.g. three southern slices), which leads us to $\sim 7150$ deg$^2$ (=$\Delta A$) sky coverage with a total of $N = 100157$ LRGs. The data reduction procedure is the same as that described in [39]. In this power spectrum analysis, we adopted the spatially flat Lambda cold dark matter (ΛCDM) model distance-redshift relation $s = s[z]$, which is consistently chosen when comparing with theoretical prediction.

The strategy to measure the multipole power spectrum is the same as that described in [75]. We adopt the estimator of the multipole power spectrum for the discrete density field of the galaxy catalogue, as follows,

$$P_\ell(k) = \frac{1}{\Delta V_k} \int_{\Delta V_k} d^3k (R_\ell(k) - S_\ell(k)),\quad (12)$$
FIG. 3: $P_0(k)$ of the SDSS LRG sample, where we adopted the distance-redshift relation $s = s[z]$ of the $\Lambda$CDM model with $\Omega_0 = 0.28$. The dark (black) points correspond to the DR7, while the light (green) ones to the DR6. The dashed and dotted curves show the $f(R)$ model with $n = 1/2$, adopting a scale-dependent bias (case 1 in Eq. (23)) and $\sigma_v = 350\,\text{km/s}$. The dashed curve is for $k_c = 1\,h\text{Mpc}^{-1}$, while the dotted curve is $k_c = 10^{-3}\,h\text{Mpc}^{-1}$. The cosmological parameters are $\Omega_0 = 0.28$, $h = 0.7$, and $n_s = 0.96$ (primordial spectral index), and the amplitude of the perturbation is determined so as to be $\sigma_8 = 0.8$ in the limit of infinitely large $k_c$.

FIG. 4: $P_2(k)/P_0(k)$ of the SDSS LRG sample. The meaning of the points corresponds to those of Fig. 3. The dashed (dotted) curve is the theoretical prediction of the $f(R)$ model with $n = 1/2$, $k_c = 1h\text{Mpc}^{-1}$ ($10^{-3}h\text{Mpc}^{-1}$). The parameters of the bias model and $\sigma_v$ are the same as those of Fig. 3. The other cosmological parameters and the amplitude of the primordial perturbation of the $f(R)$ model are also the same as those of Fig. 3.
where $\Delta V_k$ is the shell in the Fourier space and
\[
R_\ell(k) = A^{-1} \left[ \sum_{i_1}^N \psi(s_{i_1}, k)e^{ik\cdot s_{i_1}}L_\ell(\hat{s}_{i_1} \cdot \hat{k}) - \alpha \sum_{j_1}^N \psi(s_{j_1}, k)e^{ik\cdot s_{j_1}}L_\ell(\hat{s}_{j_1} \cdot \hat{k}) \right] \times \left[ \sum_{i_2}^N \psi(s_{i_2}, k)e^{-ik\cdot s_{i_2}} - \alpha \sum_{j_2}^N \psi(s_{j_2}, k)e^{ik\cdot s_{j_2}} \right],
\]
(13)
\[
S_\ell(k) = A^{-1}(1 + \alpha) \sum_{i}^N \psi(s_{i}, k)L_\ell(\hat{s}_{i} \cdot \hat{k}),
\]
(14)
where $s_{i_1}$ ($s_{j_1}$) is the position of galaxies (random sample), $\psi$ is the weight factor, which we take $\psi = 1$, $\mu = \hat{s} \cdot \hat{k}$ is the directional cosine between $\hat{s}(=s/|s|)$ and $\hat{k}(=k/|k|)$, $\alpha \equiv N/N_{\text{rand}}$ in our case is 0.05, and $A$ is determined by
\[
A = \int_{s(z_{\text{min}})}^{s(z_{\text{max}})} ds n^2(z)\psi^2(s, k).
\]
(15)
Here the integral in the expression for $A$ means the integration over the whole survey volume, and $\bar{n}(z)$ is the mean (comoving) number density of the galaxies. The error for the estimator $P_\ell(k)$ is given by the variance [75],
\[
\langle \Delta P_\ell(k)^2 \rangle \approx \frac{2(2\pi)^3}{\Delta V_k}Q^2_\ell(k)
\]
(16)
with
\[
Q^2_\ell(k) = \frac{1}{\Delta V_k} \int_{\Delta V_k} dkd\zeta^{-2} \int_{s(z_{\text{min}})}^{s(z_{\text{max}})} ds n^4(z)\psi^4(s, k)[P_l(k, s) + 1/\bar{n}(s)]^2L_\ell^2(\hat{s} \cdot \hat{k}).
\]
(17)
Here we have assumed $\alpha \ll 1$. The covariance between the errors of different multipole spectra $\langle \Delta P_\ell(k)\Delta P_{\ell'}(k) \rangle$ can be evaluated with the same formulae (16) and (17), but only replacing $L_\ell(\hat{s} \cdot \hat{k})$ by $L_{\ell'}(\hat{s} \cdot \hat{k})L_\ell(\hat{\kappa} \cdot \hat{k})$ in (17) [91]. In our analysis we adopt $\psi(s, k) = 1$. Figure 2 shows the mean number density as a function of $z$, when assuming the $\Lambda$CDM with $\Omega_0 = 0.28$ for the distance-redshift relation $s = s(z)$. Figure 3 compares the observed monopole power spectrum and our theoretical model. The dark (black) points with error bars in figure 3 show the monopole power spectrum of the DR7. The light (green) points are the previous results for the DR6 [31]. The dashed and the dotted curves represent the $f(R)$ model with $n = 1/2$, with the scale-dependent bias model of case 1 (see the next section for details). The dashed curve is for $k_c = 1h$Mpc$^{-1}$, while the dotted one for $10^{-3}h$Mpc$^{-1}$. The cosmological parameters are $\Omega_0 = 0.28$, $h = 0.7$, $n_s = 0.96$ (primordial spectral index). The amplitude of the primordial perturbation is chosen to be $\sigma_8 = 0.8$ in the limit of infinitely large $k_c$. The Smith’s nonlinear fitting formula [82] is adopted. One can see that $P_0(k)$ can be fitted with our theoretical model, by choosing suitable bias parameters.

Figure 4 plots $P_2(k)/P_0(k)$. The meaning of the points and the parameters of the curves corresponds to those of Fig. 3. This figure shows that the quadrupole power spectrum can be used to constrain the $f(R)$ model. Also it is clear that the long Compton wavelength model doesn’t fit the data.

**IV. COSMOLOGICAL CONSTRAINT**

In order to investigate the cosmological constraint on the $f(R)$ model from the multipole spectra, our theoretical model needs to include nonlinear effects. In the present paper, for simplicity, we adopt the following model for the galaxy power spectrum [76, 77],
\[
P_{\text{gal}}(k, \mu, z) = (b + f\mu^2)^2P_{\text{nl}}(k, z)D[\sigma_\chi^2k^2]\]
(18)
where $P_{\text{nl}}(k, z)$ denotes a nonlinear matter power spectrum, $D[k\mu\sigma_\chi]$ is the damping factor due to the Finger of God effect, and $\sigma_\chi^2$ is the pairwise velocity dispersion. Assuming an exponential distribution function for the pairwise velocity, $e^{-\sqrt{2v_{12}/\sigma_\chi}}/\sqrt{2\sigma_\chi}$, where $v_{12}$ is the pairwise peculiar velocity projected along the separation of a pair, the damping function is [80] (cf. 78, 79),
\[
D[\sigma_\chi^2k_2] \equiv \frac{1}{1 + \sigma_\chi^2k_2^2/2}
\]
(19)
FIG. 5: $\Delta \chi^2$ on the $k_c - \sigma_v$ plane. Here we adopted the model $n = 1/2$. The other parameters are $\Omega_0 = 0.28$, $h = 0.7$, $\alpha_s = 0.96$. The normalisation is fixed $\sigma_8 = 0.8$ in the limit of large $k_c$. The Peacock and Dodds's nonlinear fitting formula is used for the thin curves, while the Smith formula is used for the thick curves. Solid (dashed) contours correspond to $\Delta \chi^2 = 6.2$ (2.3). The left panel adopted Eq. (25), while the right panel the covariance matrix from the mock catalogues.

FIG. 6: The correlation matrix, Eq. (26), for $\ell = 0$ (left) and $\ell = 2$ (right), respectively, from 1000 mock catalogues.

with $\bar{\sigma}_v = \sigma_v / H_0$. In this case, we have

$$P_0(k, z) = \frac{1}{3k^5 \bar{\sigma}_v^5} \left[ 2f k \bar{\sigma}_v (-6f + (6b + f)k^2 \bar{\sigma}_v^2) + 3\sqrt{2} (-2f + bk^2 \bar{\sigma}_v^2) \tan^{-1} \frac{k \bar{\sigma}_v}{\sqrt{2}} \right] P_{nl}(k, z),$$  \hspace{1cm} (20)

$$P_2(k, z) = \frac{1}{30k^7 \bar{\sigma}_v^7} \left[ -360bfk^3 \bar{\sigma}_v^3 + 90b^2k^5 \bar{\sigma}_v^5 + 8f^2k \bar{\sigma}_v (45 + k^4 \bar{\sigma}_v^4) \right. $$

$$\left. -15\sqrt{2} (6 + k^2 \bar{\sigma}_v^2) (-2f + bk^2 \bar{\sigma}_v^2) \tan^{-1} \frac{k \bar{\sigma}_v}{\sqrt{2}} \right] P_{nl}(k, z),$$ \hspace{1cm} (21)

$$P_4(k, z) = \frac{(-2f + bk^2 \bar{\sigma}_v^2)^2}{24k^9 \bar{\sigma}_v^9} \left[ -10k \bar{\sigma}_v (42 + 11k^2 \bar{\sigma}_v^2) + 3\sqrt{2} (140 + 60k^2 \bar{\sigma}_v^2 + 3k^4 \bar{\sigma}_v^4) \tan^{-1} \frac{k \bar{\sigma}_v}{\sqrt{2}} \right] P_{nl}(k, z),$$ \hspace{1cm} (22)
FIG. 7: $\Delta \chi^2$ on the $k_c - n$ plane, which we evaluated with Eqs. (24) and (25). For each pair of $k_c$ and $n$, the minimum value of $\chi^2$ is computed by fitting the bias parameter and $\sigma_v$. Other parameters are fixed $\Omega_0 = 0.28$, $h = 0.7$, and $\sigma_8 = 0.96$. The normalisation of the primordial perturbation is chosen so as to be $\sigma_8 = 0.8$ (a) $\sigma_8 = 0.82$ (b), and $\sigma_8 = 0.78$ (c), in the limit of large $k_c$. The panels (a)-(c) adopt the bias model of case 1. The panel (d) is the same as (a) but with bias model of case 2. Solid (dotted) contours correspond to $\Delta \chi^2 = 6.2$ (2.3). Almost overlapping thin and thick curves assume the Peacock and Dodds's formula and the Smith's formula, respectively.

from Eqs. (18) and (19). For the nonlinear matter power spectrum, $P_{nl}(k,z)$, we adopt the fitting formulas by Peacock and Dodds [81] or by Smith et al. [82]. For the bias, we consider the following scale-dependent forms,

$$b(k) = \begin{cases} b_0 + b_1 \left( \frac{k}{0.1 h \text{Mpc}^{-1}} \right)^\alpha & \text{(case 1)} \\ b_0 + b_1 \left( \frac{k}{0.1 h \text{Mpc}^{-1}} \right) + b_2 \left( \frac{k}{0.1 h \text{Mpc}^{-1}} \right)^2 & \text{(case 2)} \end{cases},$$

(23)

where $b_0$, $b_1$, $b_2$, and $\alpha$ are the fitting parameters.

Our strategy is the following. We use the monopole and quadrupole spectra in the wavenumber range $0.02 h \text{Mpc}^{-1} \leq k_i \leq 0.2 h \text{Mpc}^{-1}$, and compute the chi squared

$$\chi^2 = \sum_{\ell,\ell'} \sum_{i,j}(P_\ell(k_i) - P_{\ell \ell'}^{\text{obs}}(k_i))C_{\ell \ell'}^{-1}(k_i, k_j)(P_{\ell'}(k_j) - P_{\ell \ell'}^{\text{obs}}(k_j)),$$

(24)

where $P_{\ell \ell'}^{\text{obs}}(k_i)$ is the observed power spectrum and $C_{\ell \ell'}(k_i, k_j) = \langle \Delta P_\ell(k_i) \Delta P_{\ell'}(k_j) \rangle$ is the covariance matrix. Here the covariance of the errors of the monopole and quadrupole spectra is taken into account, however it does not affect our results quantitatively.

The left panel of Fig. 7 shows the contours of $\Delta \chi^2$ on the $k_c - \sigma_v$ plane, where we used the covariance matrix from section 3,

$$C_{\ell \ell'}(k_i, k_j) = \langle \Delta P_\ell(k_i) \Delta P_{\ell'}(k_j) \rangle \delta_{ij}.$$

(25)

The one-sigma (dashed curve) and two-sigma (solid curve) contour-levels are given, respectively. Here the chi squared is computed to minimise (24) by fitting the bias parameters $b_0$, $b_1$, $b_2$, or $\alpha$, for each value of $k_c$ and $\sigma_v$. The other
FIG. 8: The same as the Fig. 7 but with the covariance matrix from the mock catalogues.

FIG. 9: The same as the Fig. 7 but with the covariance matrix from the mock catalogues and the redshift-space power spectrum [27]. Only the curves with Peacock Dodds's formula for the nonlinear matter power spectrum are plotted.
parameters are fixed \( n = 1/2, \Omega_0 = 0.28, \Omega_b = 0.044, n_s = 0.96, \) and \( h = 0.7. \) For \( P_{m}(k, z), \) we adopted the Peacock and Dodds’s formula \( ^{81} \) (thin curve) and the Smith formula \( ^{82} \) (thick curve), respectively. The redshift is fixed to \( z = 0.3, \) which is typical for the LRG sample. The amplitude of the matter power spectrum is fixed so as to be \( \sigma_8 = 0.8 \) in the limit of infinitely large \( k, \) i.e., in the limit of the \( \Lambda \) CDM model.

For comparison, the right panel of Fig. 5 shows the contours of \( \Delta \chi^2, \) which take the correlation of the errors of different wavenumbers into account by evaluating Eq. \( \ref{eq:24} \), with the covariance matrix obtained from mock catalogues. Due to the inclusion of the correlation of errors of different wavenumbers, the constraint becomes weaker compared with the left panel.

In the right panel of Fig. 5, we obtain the covariance matrix by using mock catalogues, which were built by following the procedure described in the reference \(^{39} \). First, we generate density field using a second order Lagrangian perturbation calculation. Then, we perform Poisson sampling of the generated density field so as to end up with a galaxy sample that has a clustering strength enhanced by a bias and a number density equal to the observed LRG sample density. We then extract the catalogue by applying the radial and angular selection function. We have checked that the mock catalogues have the amplitude of the monopole and quadrupole power spectra consistent with the observed LRG power spectra, and also that the diagonal components of the covariance matrix from the mock catalogues give almost the same error as those of Eq. \( \ref{eq:10} \) in the range of \( 0.02 h \text{Mpc}^{-1} \leq k_i \leq 0.2 h \text{Mpc}^{-1} \) \(^{39, 83} \). Figure \( \ref{fig:6} \) shows the two dimensional map of the correlation matrix,

\[
r_{ij}(k_i, k_j) = \frac{C_{\ell}(k_i, k_j)}{\sqrt{C_{\ell}(k_i, k_i) C_{\ell}(k_j, k_j)}}
\]

for \( \ell = 0 \) and 2 from 1000 mock catalogues. The binning of the covariance matrix is \( \Delta k = 0.01 h \text{Mpc}^{-1}. \) One can see from Fig. 5 that the off diagonal part is suppressed.

The normalisation of the cosmological perturbations should be determined by the cosmic microwave background anisotropies, depending on the parameters \( n \) an \( k_c \) of the \( f(R) \) model. However, the background expansion of the viable \( f(R) \) model is almost the same as that of the \( \Lambda \) CDM model, and the evolution of the matter density perturbations is only altered at late time, if compared with the \( \Lambda \) CDM model. This alteration will raise an additional integrated Sachs Wolfe effect on the CMB anisotropies due to the modified evolution of the matter density perturbation at late time. We neglect this effect on the normalisation of the perturbation, for simplicity. Then, we simply fixed the amplitude of the primordial cosmological perturbation by \( \sigma_8 \) in the limit of large \( k_c, \) i.e., the \( \sigma_8 \) of the \( \Lambda \) CDM model.

Figure 5 shows that the shorter Compton wavelength model with \( \sigma_c \approx 350 \text{km/s} \) gives the best fit to the data. Figure 7 shows the contours of \( \Delta \chi^2 \) on the \( k_c - n \) plane. Here \( \chi^2 \) is computed with Eq. \( \ref{eq:24} \) with \( \ref{eq:25} \) by fitting the bias parameters and \( \sigma_v. \) The panels (a), (b), and (c) fix the normalisation of the perturbation to be \( \sigma_8 = 0.8, 0.82 \) and 0.78, in the limit of large \( k_c, \) respectively. The contour levels of \( \Delta \chi^2 = 2.3 \) (dotted curve) and 6.2 (solid curve), correspond to 1\( \sigma \) and 2\( \sigma \) confidence, respectively. In figure 4 we used the Peacock and Dodds’s formula (thin curve) and the Smith formula \( ^{82} \) (thick curve), respectively, though the two curves almost overlap. The panels (a), (b), and (c) adopt the bias model of case 1. The panel (d) is the same as (a), but adopted the bias model of case 2. The left lower region in each panel is excluded.

Figure 8 is the same as Fig. 7 but adopted the covariance matrix from the mock catalogues for the chi squared. The constraint of Fig. 8 is weaker compared with that of Fig. 7. Especially, the constraint for the model with larger \( n \) becomes weaker. However, Fig. 8 indicates that the long Compton wavelength case of the \( f(R) \) model with the smaller value of \( n \) is excluded.

Thus far, we have used the redshift-space power spectrum \( \mathcal{P}(k, \mu) \). In order to check the reliability of our result, we next consider the other possible model for the redshift-space power spectrum,

\[
P_p(k, \mu) = (b^2(k) \delta_{\delta}(k) + 2fb(k)P_{\delta}(k) \mu^2 + f^2P_{\mu}(k) \mu^4) e^{-(f k \mu \sigma_c)^2},
\]

where \( \delta_{\delta}(k) \) is the nonlinear matter power spectrum, \( P_{\delta}(k) \) is the power spectrum of the velocity divergence, and \( P_{\mu}(k) \) is the cross power spectrum of matter and the velocity divergence. This model is obtained from the model proposed by Scoccimarro \(^{85} \) and assumes a linear bias relation. Very recently, Jenning et al. proposed a fitting formula for the redshift-space power spectrum of the form \( \ref{eq:27} \), assuming \( b(k) = 1. \) The fitting formula relates the nonlinear matter power spectrum \( P_{\delta}(k) \) to \( P_{\delta}(k) \) and \( P_{\mu}(k) \). By using the N-body simulations it was demonstrated that the fitting formula is accurate to better than 10% for the \( \Lambda \) CDM model and quintessence dark energy models for \( k \leq 0.2 h \text{Mpc}^{-1}. \) Although the accuracy of the fitting formula for the \( f(R) \) model has not been explicitly demonstrated, we assume its validity, and use it in the following \( \Delta \chi^2 \) calculations.

Figure 9 shows the contours of \( \Delta \chi^2 \) on the \( k_c - n \) plane, the same as Fig. 8, but with the covariance matrix from the mock catalogues and the redshift space power spectrum \( \ref{eq:27} \). In the original formula, \( \sigma_v \) is obtained from \( P_{\delta}(k), \) however, we assumed \( \sigma_v \) to be a fitting parameter, as is done in Fig. 8. This figure shows that the constraint becomes weaker when compared to the previous model \( \mathcal{P}(k, \mu) \). The models with large value of \( n \) are not constrained. However,
the long Compton wavelength case of the \( f(R) \) model with the smaller value of \( n \) is excluded. This new model predicts that \( P_{\delta 0}(k) \) is smaller than \( P_{\delta 0}(k) \) for values of \( k \lesssim 0.1 \text{Mpc}^{-1} \), which reduces the quadrupole power spectrum and thus weakens the constraint.

Let us compare our result with the other constraints on the \( f(R) \) model. Refs. \[60, 61\] have investigated the constraints on the \( f(R) \) model for the case \( n = 1/2 \). In Ref. \[60\], the constraint from the CMB anisotropies through the integrated Sachs Wolfe effect is investigated. However, the constraint is weak. Only the horizon-scale Compton wavelength model is excluded. In Ref. \[61\], the constraint from the cluster number count is investigated. Though it is restricted to the case \( n = 1/2 \), they obtained \( |f_{R0}| \lesssim 10^{-3} \), where \( f_{R0} \) is the value of \( f_R \) at the present epoch. In the case \( n = 1/2 \), \( |f_{R0}| \) is related to \( k_c \) by

\[
k_c \approx 0.04 \left( \frac{10^{-4}}{|f_{R0}|} \right)^{1/2} \text{hMpc}^{-1}. \tag{28}\]

Ref. \[61\] reports that \( k_c \lesssim 0.04 \text{hMpc}^{-1} \) is excluded. The constraint is similar to our result, when the redshift-space power spectrum \[18\] is used (See Figure 8). When arguably more accurate model \[27\] is used, the constraint becomes slightly weaker than that of \[18\] (See Figure 9).

V. FUTURE PROSPECT OF MEASURING COMPTON SCALE

In this section, we estimate future prospects of constraining the Compton scale with the use of the Fisher matrix technique, which is frequently used for estimating minimal attainable constraint on model parameters. We focus on the error of the Compton wavenumber \( k_c \). We adopt the Fisher matrix of the form (e.g., \[87\]),

\[
F_{ij} = \frac{1}{4\pi^2} \int_{k_{\text{min}}}^{k_{\text{max}}} dk d^3k \int_{-1}^{+1} d\mu \frac{\partial P_{\text{gal}}(k, \mu)}{\partial \theta} \frac{\partial P_{\text{gal}}(k, \mu)}{\partial \theta} \frac{V}{(P_{\text{gal}}(k, \mu) + 1/n)^2}, \tag{29}\]

where \( \theta_i \) denotes a model parameter, \( V \) is a survey volume, \( n \) is a mean number density of galaxies.

In the Fisher matrix analysis, for simplicity, we consider the 6 parameters \( k_c, n, \sigma_c, b_0, b_1 \) and \( \alpha \), adopting the bias model of case 1. The panel (a) of Fig. 10 shows the 1\( \sigma \) error \( \Delta k_c \), in determining the Compton wavenumber \( k_c \) as a function of the target value of \( k_c \), assuming a redshift survey like the SUMIRE (SUBaru Measurement of Imaging and REDshift of the universe) \[9\], which assumes the survey parameters like those of the WFMOS survey \[88\], the range of the redshift \( 0.9 < z < 1.6 \), the survey area 2000 square degrees, and the mean number density \( n = 4 \times 10^{-4} (\text{h}^{-1} \text{Mpc})^{-3} \). Here we adopted the target values \( \sigma_c = 400 \text{km/s} \), \( b_0 = 2.5, b_1 = 0.5, \alpha = 0.5 \), and \( n = 1/2, 2, 1, 2, \) and 4, from the bottom to the top, respectively. The other parameters are fixed \( \Omega_0 = 0.28, h = 0.7, n_s = 0.96 \), and the normalisation so as to be \( \sigma_8 = 0.8 \) in the limit of the \( \Lambda \)CDM model. We obtained \( \Delta k_c \) by marginalizing the Fisher matrix over the 5 parameters \( n, \sigma_c, b_0, b_1 \) and \( \alpha \). The panel (b) of Fig. 10 shows the relative error \( \Delta k_c/k_c \).

In the Fisher matrix we used the power spectrum in the range of wavenumbers \( k < 0.3 \text{hMpc}^{-1} \). This immediately implies that the redshift survey cannot be very sensitive to the models with the short Compton wavelength, as seen from figure. The error becomes very large for \( k_c \gtrsim 0.1 \text{hMpc}^{-1} \), but it will be possible to obtain a useful constraint on the Compton scale, in principle, for models with \( k_c \lesssim 0.1 \text{hMpc}^{-1} \). However, the constraint becomes weak for the case of large \( n \).

The panel (a) assumes the power spectrum analysis without dividing the full galaxy sample, which spans the redshift range \( 0.9 \lesssim z \lesssim 1.6 \), into redshift subsamples. The panel (c) assumes the case when the galaxy sample is divided into the three subsample in redshift bins and that the power spectra are obtained from each subsample. In this case, the parameters \( \sigma_c, b_0, b_1 \) and \( \alpha \) should be fitted in each redshift bin, and the total number of parameters in the Fisher matrix analysis is 14. The panel (d) is the relative error, corresponding to (c). The cosmological parameters are the same as those of (a). The possible advantage of this method is that the additional information of the redshift evolution might improve the constraint. One can see that the constraint is improved in comparison with the panel (a) or (b). The degree of the improvement is small for \( n = 1/2 \), but is not negligible for the case \( n = 4 \). This is understood because the redshift evolution of the Compton scale is faster for larger \( n \).

VI. SUMMARY AND CONCLUSIONS

In this paper, we determined a cosmological constraint on the viable \( f(R) \) model based on the redshift-space distortion by measuring the monopole and quadrupole spectra of the SDSS LRG sample of DR7. The monopole and
FIG. 10: (a) $1\sigma$ error $\Delta k_c$ as a function of the target value of $k_c$. The result is based on the Fisher matrix analysis with the 6 parameters, $k_c$, $n$, $\sigma_v$, and $b_0$, $b_1$ and $\alpha$ for the bias model 1, and marginalized over the 5 parameters other than $k_c$. The target parameters are $b_0 = 2.5$, $b_1 = 0.5$, $\alpha = 1/2$, and $n$ is chosen $n = 1/2, 1, 2, 4$ from the bottom to the top, respectively. The other parameters are fixed $\Omega_0 = 0.28$, $h = 0.7$, $n_s = 0.96$, and the normalisation $\sigma_8 = 0.8$ in the limit of the $\Lambda$CDM model. Eq. (18) with the Peacock and Dodds nonlinear fitting formula is adopted. (b) the relative error $\Delta k_c/k_c$ corresponding to (a). (c) and (d) are the same as (a) and (b), respectively, but assumed the analysis where the full sample is divided into 3 redshift bins.

The quadrupole spectra are used to fit the bias parameters and to constrain the growth factor and the growth rate of the density perturbations, which depend on the Compton scale of the $f(R)$ model. Our results show that short Compton wavelength model fits the data better, while the long Compton wavelength model is excluded, though the constraint depends on the evolution parameter $n$. For the case $n = 1/2$, our constraint is similar to that from the cluster number counts reported in [61]. When we adopt more accurate model for the redshift-space power spectrum [84], the constraint becomes slightly weaker. However, the long Compton wavelength case of the $f(R)$ model with the smaller value of $n$ is excluded. Our results exemplify that the redshift-space distortion is quite useful in testing gravity theory. We also demonstrated that a future redshift survey like the WFMOS/SUMIRE is potentially useful in obtaining a constraint on the Compton wavelength scale.

We acknowledge that the widely used theoretical model of the anisotropic power spectrum adopted in the present paper might need careful improvements. We adopted the Peacock and Dodds formula and the Smith formula for the nonlinear modelling of the mass power spectrum. Our results do not significantly depend on the choice. However, there might be a need to adopt a more sophisticated formula for the precise nonlinear modelling within the framework of the modified gravity, as demonstrated by Koyama, Taruya, Hiramatsu [86]. The treatment of the Finger of God effect in our paper was simple, which assumed the exponential distribution function for the pairwise velocity and introduced one free parameter – the pairwise velocity dispersion. In reality it might not be an adequate model to describe the nonlinear region of the redshift-space power spectrum [83]. We checked the reliability of our results by adopting the other possible model proposed in Ref. [84], extensively applying the fitting formula to the $f(R)$ model, whose accuracy in this case, however, has not been demonstrated. We found that there is a non-negligible effect on the constraint on the $f(R)$ model. Therefore, a more precise modelling of the redshift-space power spectrum should arguably be needed in the future. Concerning the modelling of the clustering bias, we adopted a simple scale-dependent bias. Here too there is potentially a lot of room for improvement. These issues are out of scope for the present paper, but need to
be elaborated for a precise test of gravity with the future redshift surveys.

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