Implement the particle finite element method in ABAQUS

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ABSTRACT

We implement the particle finite element method (PFEM) in the general numerical simulation framework ABAQUS to investigate the quasi-static collapse of two-dimensional granular columns and landslides. As a recently proposed continuum approach, the PFEM inherits both the solid mathematical foundation of the particle of the traditional finite element method and the flexibility of particle methods in simulating ultra-large deformation problems. In our work, the governing equations of the PFEM are solved by the ABQUS iteratively to improve the accuracy and efficiency of simulation results. The typical collapse patterns of granular columns are reproduced in the PFEM simulation, and the physical mechanism behind the collapse phenomenon is evaluated. We compare the simulated collapse processes with the experimental observations, where a satisfactory agreement is achieved.

Keywords: particle finite element method, large deformation, ABAQUS

1 INTRODUCTION

In nature, many important phenomena such as avalanche, landslide, soil fluidization, and blood flow can be related to granular flow, understanding the behavior of granular materials under different flow conditions is of great importance in practice. The large granular systems normally are comprised of billions of particles, using the discrete element method (DEM) (Cundall and Strack 1979), to conduct related numerical simulation will be time-costing to achieve reliable results. On the other hand, in dense granular flow, granular materials may exhibit both fluid-like and solid-like behaviors to resist shear stress and contain the potential of flowing as fluids. Treating granular materials as a continuum is particularly attractive in numerical simulation.

Recently, a novel continuum approach called the particle finite element method (PFEM) has been widely adopted in the geotechnical numerical simulations and proven to be particularly suitable for simulation granular materials (Oñate, Idelsohn et al. 2004, Zhang, Krabbenhof et al. 2015, Salazar, Irazábal et al. 2016, Zhang, Ding et al. 2016). The PFEM makes used of particles to represent materials, as in meshfree particle methods, but solves the governing equations via a standard finite element procedure, such as the general numerical simulation framework ABAQUS. As a result, this approach inherits the solid mathematical foundation of the traditional finite element method (FEM) and is able to cope with arbitrary changes in geometry.

The quasi-static spreading of granular columns has been widely studied both in experimental tests and numerical simulations due to its importance in the industry. Meriaus (2006) performed the quasi-static collapse experiments, and Owen et al. (2009) documented additional results to investigate the effect of the microscopic characteristics and properties of the granules. X Zhang et al. (2016) reproduced the experiments by using the PFEM, the effects of the macro density and friction angle of the granular matter, as well as the roughness of the wall surfaces on the quasi-static collapse are studied. Kermani and Qiu (2018) developed a three-dimensional smoothed particle hydrodynamics (SPH) model to investigate the collapse of rectangular sand columns with aspect ratios ranging from 0.26 to 11. The SPH results reveal that both quasi-static and dynamic collapse show qualitative similarities and primarily depend on the initial aspect ratio of the column.

The PFEM can also be used to predict the velocity and runout distance of a sliding mass, and the simulation results can be used to assess and manage the risks posed by landslides. Salazar et al. (2015) modeled the
landslide-generated impulse waves using the PFEM. X Zhang et al. (2015) applied the PFEM in the whole process simulation of the landslide and suggested that both the geometry and the roughness of the slip surface play essential roles in the on the sliding process. Furthermore, X Zhang et al. (2019) developed a unified Lagrangian formulation for solid and fluid dynamics to model the submarine landslides.

In this paper, the PFEM was implemented in the ABQSU to improve the simulation accuracy and efficiency by solving the governing equations with the embedded routines. The proposed approach was verified by comparing the predicted quasi-static collapse of sand columns with the experimental results. The proposed approach also models the 2011 deep-seated Akatani landslide in central Japan (Yamada et al. 2016), and the predicted sliding pattern is a good agreement with the field observations.

2 Numerical implementation

2.1 PFEM Approach

The PFEM is a numerical method that uses a finite-element mesh to discretize the physical domain and to integrate the differential equations of motion in an updated Lagrangian fashion. The mesh nodes can move freely and may be separated from their original computational domain due to large deformation, thus behaving as particles. At the end of each time step, the mesh has to be rebuilt as the nodes have been moved to their new time step position.

The α-shape method, as illustrated in Fig.1, is used to identify the computational domain, and the Delaunay Tessellation is chosen to connect all the particles at the new time position giving as a result a new mesh. The physical properties are transformed from the old mesh to the new mesh with the Unique Element Method (UEM), as suggested by X Zhang et al. (2014).

The main disadvantage of the PFEM applied in Solid Mechanics problems is that the external surface generated using α-shape may affect the mass conservation of the computational domain, and it can be overcome by using a constrained Delaunay algorithm (Rodriguez et al. 2016). In our work, the mesh is generated with the constrained conforming Delaunay triangulation (CCDT) through the open-source Triangle (Shewchuk et al. 1996), and the quadratic triangle plane strain element employed in ABAQUS/Standard is adopted to conduct the calculation.

2.2 Governing Equations

Consider a constitutive medium in the domain Ω, which is bounded by its surface Γ. The surface Γ is divided into two potions Ωu, subjected to prescribed displacement, and Ωf, subjected to tractions. The portions of the surface obey the constraints that Ωu ∩ Ωf = Φ and Ωu ∪ Ωf = Γ+, where Φ is a null set. The governing equations of motion in the framework of PFEM are summarized as follows:

\[ \mathbf{B}^T \mathbf{σ} + \mathbf{b} = \rho \ddot{\mathbf{u}} \quad \text{in} \quad \Omega, \]

\[ \mathbf{u} = \ddot{\mathbf{u}} \quad \text{on} \quad \Omega_u, \]

\[ \mathbf{N}^T \mathbf{σ} = \mathbf{f} \quad \text{on} \quad \Gamma_f \]

Where \( \mathbf{σ} \) is Cauchy stress, \( \mathbf{b} \) is the body force, \( \mathbf{u} \) is the displacement, \( \ddot{\mathbf{u}} \) and \( \mathbf{f} \) are the prescribed displacement and external tractions. \( \mathbf{N} \) consists of components of the outward normal to the boundary \( \Gamma_f \), and \( \mathbf{B} \) is the transposed gradient operation. The standard weak form of equilibrium equation is obtained as

\[ \int_\Omega \mathbf{N}^T \rho \mathbf{N} \ddot{\mathbf{u}} d\Omega + \int_\Omega \mathbf{B}^T \mathbf{σ} d\Omega - \int_\Omega \mathbf{N}^T \mathbf{f} d\Omega = 0 \] (2)

Performing the sum over all elements, we obtain a semi-discrete problem given by the set of ordinary differential equations

\[ \mathbf{M} \ddot{\mathbf{u}} + \mathbf{p}(\mathbf{σ}) = \mathbf{f} \] (3)

Where

\[ \mathbf{M} = \sum_\epsilon \int_{\Omega_\epsilon} \mathbf{N}^T \rho \mathbf{N} d\Omega \] (4)

\[ \mathbf{p}(\mathbf{σ}) = \sum_\epsilon \int_{\Omega_\epsilon} \mathbf{B}^T \mathbf{σ} d\Omega \] (5)

\[ \mathbf{f} = \sum_\epsilon \left( \int_{\Omega_\epsilon} \mathbf{N}^T \mathbf{f} d\Omega + \int_{\Gamma e} \mathbf{N}^T \mathbf{f} d\Gamma_e \right) \] (6)

We can use the implicit dynamic procedure in ABAQUS/standard to solve the governing equation. The implicit operator used for time integration of the dynamic problem is the operator defined by Hilber, Hughes and Taylor (1978). The Newmark formulae complete the operator definition for displacement and velocity integration:

\[ \mathbf{u}_{\epsilon+\Delta t} = \mathbf{u}_{\epsilon} + \Delta t \dot{\mathbf{u}}_{\epsilon} + \Delta t^2 \left( \frac{1}{2} - \beta \right) \ddot{\mathbf{u}}_{\epsilon} + \beta \ddot{\mathbf{u}}_{\epsilon+\Delta t} \] (7)

and

\[ \dot{\mathbf{u}}_{\epsilon+\Delta t} = \dot{\mathbf{u}}_{\epsilon} + \Delta t \left( 1 - \gamma \right) \ddot{\mathbf{u}}_{\epsilon} + \gamma \ddot{\mathbf{u}}_{\epsilon+\Delta t} \] (8)

with

\[ \beta = \frac{1}{4}, \quad \gamma = \frac{1}{2} \]
\[
\beta = \frac{1}{4}(1 - \alpha)^2, \quad \gamma = \frac{1}{2} - \alpha, \quad -\frac{1}{3} \leq \alpha \leq 0 \quad (9)
\]

This operator is a single parameter operator with controllable numerical damping, which is essential in the automatic time-stepping scheme for the reason that a small amount of numerical damping will significantly reduce the high-frequency numerical noises. The parameter \(\alpha\) provides control over the amount of numerical damping: with \(\alpha = 0\), there is no damping, and the operator is the trapezoidal rule, while with \(\alpha = -1/3\), significant damping is available. In this paper, \(\alpha\) is given as -0.05 to remove the high-frequency numerical noises without having any significant effect on the lower frequency response. Thereafter, we can use the employed automatic time incrementation procedure to speed up the simulation; the time increment size is adjusted automatically according to the convergence behavior of the Newton iterations and the accuracy of the time integration.

Since the boundary of the computational domain may be merged due to large deformation, it is difficult to track the boundary conditions using the element index. Similar with the work by Zhang et al. (2014), the rigid component is served as the boundary, and the traditional node-to-surface discretization is introduced to capture the interaction behavior between the particles and the boundary, while the particles are not allowed to penetrate the boundary surface.

It should be noted that the governing equation of the PFEM is solved via a standard finite element procedure, implementing the PFEM in Abaqus will inherit some advanced features, as mentioned above, leading to a more robust and efficient approach.

### 2.3 Constitutive Model

The non-associative linear Drucker-Prager (DP) model is chosen in this study to determine the plastic flow regime of granular materials, which was employed in Abaqus/Standard. The yield function and the potential plastic function of the DP model are expressed through the following equations:

\[
F = t - p\tan\beta - d = 0 \quad (10)
\]

\[
G = t - p\tan\psi
\]

Where \(p\) and \(t\) are, respectively, the mean stress and deviator stress. \(\beta\) and \(d\) are the DP model’s constants, which related to the Coulomb’s material constants \(c\) (cohesion), \(\phi\) (internal friction angle), and \(\psi\) (dilation angle). In plane strain condition, their relationship is obtained as

\[
\tan\beta = \frac{\sin\phi}{\sqrt{3(9 - \tan^2\psi) + \sin\phi\tan\psi}} \quad (12)
\]

\[
d = \frac{c\cos\phi(9 - \sin\phi\tan\psi)}{\sqrt{3(9 - \tan^2\psi)}} \quad (13)
\]

As illustrated by Kermani and Qiu (2018), during the process of granular collapse, once the critical state in the material is reached after large deformation, no further plastic volume change is occurred. Thus the dilation angle should be zero. That conclusion is supported by Bui et al. (2011) and Chen and Qiu (2012), and it will be adopted in current work. Besides, the Abaqus/Standard requires that the value of constant \(d\) should not be zero, in order to represent the behavior of non-cohesion granular materials, Coulomb’s material constants \(c\) is chosen to be 15 Pa for convergence.

### 3 Two-dimensional PFEM simulations

#### 3.1 Quasi-static collapse of sand columns

To verify the proposed approach, we simulated the experiment of Meriaus (2006) on the quasi-static collapse of rectangle sand columns with an initial aspect ratio is 0.5. Referred to Meriaus (2006), the compacted dry sand had a bulk density of 1,570 kg/m\(^3\) and a friction angle of 28°. Further, the bulk modulus and Poisson's ratio of the sand is assumed to be 10 MPa and 0.3, respectively, in the PFEM simulation. Fig. 2 shows the schematic of the experimental rectangular column. The wall on the left side is fixed during the experiments, while the wall on the right side was prescribed move slowly at a horizontal velocity, leading to the quasi-static collapse of the sand column.

![Fig. 2 Schematic of the experimental rectangular sand column.](image)

Fig. 3 shows the collapse evolution of the sand column, and the colors are proportional to the norm of the velocity, \(||v||\) (mm/s). As illustrated, during the collapse process, the material near the fixed wall remains static while a triangle block of material moves down towards the moving wall. The collapse pattern obtained from the current simulation coincides with the experimental observations, and the capability in dealing with the large deformation issues of the proposed approach is initially verified.
Fig. 3 Comparison of the internal structure of the deposit between the PFEM simulations and experimental tests. Colors are proportional to the norm of the velocity, $|v|$ (mm/s).

Fig. 4 and 5 show the height of both sides of the granular bed against its length from the simulations. The experimental and numerical simulation data available in the literature are provided as well for comparison. As illustrated, the PFEM simulation results are slightly different with the experimental data, but correspond well with the results from three-dimensional SPH simulations, which are performed by Kermani and Qiu, 2018).

Our simulation shows that the proposed approach can account for fluid-like and solid-like behaviors of granular materials. In the following section, the proposed approach will be applied to simulate the landslide, which contains more complex boundary conditions.

3.2 Akatani landslide simulation

On 2011 September 3-4, extensive bedrock landslides occurred across a vast region of the Kii Peninsula as Typhoon Talas produced heavy rainfalls across western Japan (Yama da et al. 2012; Chigira et al. 2013). The elevation changes at the Akatani landslide estimated from airborne LiDAR topographic surveys are presented in Fig. 6, which is obtained from Yamada et al., 2016. In the PFEM simulation, the mass is assumed to have a bulk density of 1,670 \(\text{kg/m}^3\), a friction angle of 25°, and cohesion of 20 kPa. Further, the bulk modulus and Poisson's ratio of the sand is assumed to be 10 MPa and 0.3, respectively.

The whole procedure of the landslide, from initiation, sliding to deposition, is presented in Fig. 7, where the colors are proportional to the norm of the velocity. At the beginning of sliding in 10 s, the mass will enter the yielding state due to the gravity loading and produce elastoplastic deformation. The tip of the deposit will reach the bottom of the valley, and the mass begins decelerating at approximately 12 s. After that stage, velocity quickly increases, and the sliding mass will produce a tremendous impact force on the bottom of the valley. At approximately 25 s, the velocity of the sliding will be reduced, which will be kept still after 30 s.
Fig. 7 Norm velocity distribution of the sliding mass during the whole procedure.

Fig. 8 shows the Mises stress distribution of the sliding mass during the whole procedure. As illustrated, the stress distribution is influenced by the boundary geometry condition.

Fig. 8 Misses stress distribution of the sliding mass during the whole procedure.

The predicted and filed observed deposits are given in Fig. 9. As indicated, the predicted result is a good agreement with the observed result.

4 CONCLUSIONS

Since a standard finite element procedure solves the governing equations of PFEM, the PFEM is implemented in the ABAQUS in current work to ensure the accuracy and efficiency of the simulation results. We simulated the quasi-static collapse of a sand column and the 2011 dee-seated Akatani landslide in central Japan with the proposed model. The main conclusions are summarized as follows:

(1) The proposed method takes advantage of the mutual finite element procedure in the ABAQUS and can deal with complex geometry boundary conditions;

(2) The proposed model can capture both the solid-like and fluid-like behaviors of the granular material, and the simulated quasi-static collapse is a well agreement with the experimental observations;

(3) The proposed approach is particularly suitable to model the landslide, which contains complex geometry boundary conditions. The whole procedure of the landslide is reproduced, and the simulation results can be used for risk assessment and management.

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