One-phonon excitations in $^{92}\text{Zr}$ from electron scattering

Abdulrahman Scheikh Obeid, O Burda, M Chernykh, A Krugmann, P von Neumann-Cosel, N Pietralla, I Poltoratska, V Ponomarev, and C Walz

Institut für Kernphysik, Technische Universität Darmstadt, 64289 Darmstadt, Germany
E-mail: scheikh@ikp.tu-darmstadt.de

Abstract. Low-lying collective vibrational excitations of $^{92}\text{Zr}$ have been investigated with electron scattering at the S-DALINAC. The form factors of isospin polarized one-quadrupole phonon states of $^{92}\text{Zr}$ at the $Z = 40$ proton subshell closure have been measured and the momentum-transfer dependence of the form factors for the one-quadrupole phonon states have been compared to the prediction of the Quasiparticle Phonon Model. The E2 transition strengths of the one-quadrupole phonon states and the E3 transition strength of the one-octupole phonon state have been extracted. A comparison to the data on $^{94}\text{Mo}$ and previous spectroscopic information on mixed-symmetric states (MSSs) of $^{92}\text{Zr}$ will be given.

1. Introduction
Collectivity, isospin symmetry and shell structure are generic features of many-body quantum systems that consist of two components (protons and neutrons) like heavy atomic nuclei. The study of collective valence-shell excitations is a useful way to understand how these features coexist, interplay and compete. The investigation of MSSs is an important source of information on the effective proton-neutron interaction in the valence shell [1]. MSSs have been defined in the framework of the Interacting Bosons Model IBM-2 [2]. In analogy to isospin, the symmetry of a multi-boson wave function formed by $N_\pi$ proton bosons and $N_\nu$ neutron bosons with respect to proton and neutron boson labels is quantified by the F-spin. States with $F < F_{\text{max}} = (N_\pi + N_\nu)/2$ have wave functions that contain at least one pair of proton and neutron bosons which is antisymmetric under the exchange of proton and neutron labels. They are called mixed-symmetry states. Those states with $F = F_{\text{max}}$ are symmetric under the pairwise exchange of the nucleon labels. They are fully symmetric states. The signature for MSSs are (i) strong $M1$ transition to fully symmetric states with matrix elements of about $1\mu_N$ in size and (ii) weakly-collective $E2$ transition to fully symmetric states.

The prediction of the IBM-2 with respect to a multi-phonon structure of MSSs in vibrational nuclei was well confirmed ten years ago by the observation of large $M1$ transitions between low-energy states of $^{94}\text{Mo}$ [3, 4, 5]. The mixed-symmetric $2^+$ states were also investigated with electron scattering experiments at the $130$ MeV superconducting electron accelerator S-DALINAC [6]. The evaluation of the measured form factors as a function of the momentum-transfer had supported the one-phonon interpretation of the $2^+_1$, $2^+_2$ states. In the neighbouring even-even isotope $^{92}\text{Zr}$ which is formed by $N = 52$ neutrons with two valence neutrons and
$Z = 40$ with no valence protons occupying the $\pi(g_{9/2})$ sub-shell a stronger configurational isospin polarization of the one-phonon states than in $^{94}$Mo was expected. This has recently been discussed in the literature [7, 8, 9, 10]. In order to deepen the discussions and to verify this expectation a new electron scattering experiment at the S-DALINAC has been performed on the nucleus $^{92}$Zr. A calculation of the momentum-transfer dependence of the form factors for the one-quadrupole phonon states has been done in the framework of the Quasiparticle Phonon Model.

2. Experiment and analysis

2.1. Experimental setup and data

The experiment has been carried out at the Darmstadt superconducting electron linear accelerator S-DALINAC which provides electron beams with energy up to 130 MeV and beam current up to $3 \mu A$. The $33^\circ$ - $169^\circ$ spectrometer with momentum acceptance of $\pm 2.1\%$ was used. The spectrometer has in its focal plane a detector system based on four single-strip silicon detectors, each provides 96 strips with thickness of 500 $\mu m$ and a pitch of 650 $\mu m$.

We have used an incident beam energy $E_e = 63$ MeV with beam currents from 0.5 to 1 $\mu A$. A $^{92}$Zr target with an enrichment of $94.57\%$ and a thickness of $9.75$ mg/cm$^2$ was used. Data were taken at five different scattering angles $\Theta_e = 69^\circ, 81^\circ, 93^\circ, 117^\circ, 165^\circ$ covering the maximum of the $E2$ form factor. Examples of electron spectra are shown in Fig. 1. The salient peaks correspond to the elastic line, the weakly collective one-phonon $2_1^+$ state ($E_x = 0.934$ MeV), The $3_1^-$ state ($E_x = 2.339$ MeV) and the one-phonon $2_2^+$ state ($E_x = 1.847$ MeV).

![Figure 1](image_url)

The spectra taken in the $^{92}$Zr(e,e$'$) electron scattering reaction were energy calibrated with the excitation energies of the $0_1^+, 2_1^+, 2_2^+, 3_1^-$ states given in [9].

2.2. Data analysis

The experimentally extracted cross sections for the excited states were normalized to the cross section of the elastic line. The absolute values of the differential cross sections were calculated
from the corresponding peak areas taking into account the radiative corrections and the dead time of the electronics according to

$$\left( \frac{d\sigma}{d\Omega} \right) = A_n^{\text{exp}} \cdot \frac{1}{\Delta\Omega} \cdot \frac{e}{It} \cdot \frac{\mu_{\text{mol}}}{n_{\text{eff}} N_A}$$

(1)

were $\Delta\Omega$ is the spectrometer solid angle [sr], $It$ the accumulated charge [C], $\mu_{\text{mol}}$ the molecular mass, $[g/\text{mol}]$, $n_{\text{eff}}$ the effective areal density of the target $[g/cm^2]$ and $N_A$ the Avogadro number $[1/\text{mol}]$.

The squared form factor is given as:

$$|F(E_0, \theta)|^2 = \frac{(d\sigma/d\Omega)^{\text{exp}}}{(d\sigma/d\Omega)^{\text{Mott}}}$$

(2)

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{Momentum-transfer dependence of the form factor of the one-phonon FS $2_1^+$ state (top), MS $2_2^+$ state (middle) and $3_1^-$ state (bottom) of $^{92}$Zr from electron scattering. The data (full squares) are compared to the QPM (solid lines).}
\end{figure}

with the Mott cross section:

$$\left( \frac{d\sigma}{d\Omega} \right)_{\text{Mott}} = \left( \frac{Ze^2}{2E_0} \right)^2 \frac{\cos^2(\theta/2)}{\sin^4(\theta/2)}$$

(3)
Calculations of the momentum-transfer dependence of the form factor of the $2^+$ states and of the $3^{-}_1$ state have been performed within the Quasiparticle Phonon Model (QPM). Excited states of $^{92}$Zr have been described by a wave function which includes one-, two-, and three-phonon configurations. The $(e,e')$ cross sections have been calculated within the Distorted Wave Born Approximation using the code of Heisenberg and Blok [11] and employing the radial transition charge densities calculated by the QPM. The E2 and E3 strengths can be extrapolated to the photon point, $q = E_x/\hbar c$.

**Table 1.** E2 strengths from $(e,e')$ in comparison to the known values from $(n,n'\gamma)$ [5, 9] and the E3 strength from $(e,e')$ in W.u.

| This work: $^{92}$Zr$(e,e')$ E\(\lambda\) strength in W.u. | $^{92}$Zr$(n,n'\gamma)$ [9] | $^{94}$Mo$(n,n'\gamma)$ [5] |
|-----------------------------------------------------------|--------------------------|--------------------------|
| Form factor scaled to QPM | | |
| B(E2: $2^+_1 \rightarrow 0^+_1$) 6.2±0.3 | 6.4±0.5 | 16.0±0.2 |
| B(E2: $2^+_2 \rightarrow 0^+_1$) 3.3±0.2 | 3.4±0.4 | 2.2±0.2 |
| B(E3: $3^{-}_1 \rightarrow 0^+_1$) 18.3±1.1 | | |

The $B(E2; 2^+_1 \rightarrow 0^+_1)$ and $B(E2; 2^+_2 \rightarrow 0^+_1)$ values that were extracted using the form factor calculated with the QPM are in good agreement with the literature values [9]. The EM strengths are compared to the literature and to the ones of the symmetric and mixed-symmetric states of $^{94}$Mo in Tab. 1. The extracted value of the $B(E2; 3^{-}_1 \rightarrow 0^+_1)$ is 18.1 ± 1.1 W.u. and it compares well to the values obtained from proton scattering experiments (14.7, 16.2, 18.9, 21.3, 23.6) W.u in the review article by Spear [12].

### 3. Discussion

The $(e,e')$ electron scattering reaction is predominantly a one-step process. Fig. 1 shows that the $2^+_2$ state of $^{92}$Zr is almost as strongly populated in $(e,e')$ as the one-phonon $2^+_1$ state suggesting a large one-phonon component being present in the wave function of the $2^+_2$ state either. Figure 2 presents the form factors of these $2^+$ states and of the $3^{-}_1$ state as a function of the momentum transfer $q$. The similarity of the momentum-transfer dependence of the form factors between the $2^+$ states further supports the one-phonon nature of the $2^+_2$ state of $^{92}$Zr.

The theoretical results shown in Fig. 2 are absolute and not scaled to the experimental data. The QPM results provide a good description of the $(e,e')$ form factor in this interval of the momentum transfer. The main configurations of the wave functions used to calculate the form factors are given as following:

$$
\psi_{2^+_1} = \sqrt{0.59} \nu \left(2d_{5/2}^2\right) + \sqrt{0.14} \pi \left(1g_{9/2}^2\right) \quad (4)
$$

$$
\psi_{2^+_2} = -\sqrt{0.38} \nu \left(2d_{5/2}^2\right) + \sqrt{0.27} \pi \left(1g_{9/2}^2\right)
$$

The QPM calculations predict the main proton and neutron components in the wave functions of the one-phonon $2^+$ states to have the opposite relative signs which confirms the picture of symmetric and mixed-symmetric character in the valence shell. The $2^+_1$ state, because of the large neutron component, is a predominantly symmetric albeit isospin polarized state and the $2^+_2$ state is a mixed-symmetric one with a weak isospin polarization.

The E2 strengths extracted in this work are close to the values, 6.4±0.6 and 3.4(4) for the $2^+_1$ and $2^+_2$ respectively, that were obtained from a QPM calculation performed in Ref. [10]. However,
the wave functions published for the $2^+$ states in Ref. [10] show stronger isospin polarisation of
the $2^+_1$ and $2^+_2$ states than what we have found from our calculation.

Acknowledgments
We thank Ch Stoyanov, N Lo Iudice, V Werner, C Fransen, G Rainovski and D Tarpanov for
discussions. Support by the DFG under Grant No. SFB 634, by the Helmholtz International
Center for FAIR and by the DAAD within the German-Bulgarian exchange program under grant
Nos. D/06/05918 and DAAD-09 is gratefully acknowledged.

References
[1] Pietralla N, von Brentano P and Lisetskiy A F 2008 Prog. Part. Nucl. Phys. 60 225
[2] Iachello F and Arima A 1987 The Interacting Boson Model (Cambridge University Press.)
[3] Pietralla N et al. 1999 Phys. Rev. Lett. 83 1303
[4] Pietralla N, Fransen C, von Brentano P, Dewald A, Fitzer A, Frießner C and Gableske J 2000 Phys. Rev.
Lett. 84 3775
[5] Fransen C et al. 2003 Phys. Rev C 67 024307
[6] Burda O et al. 2007 Phys. Rev. Lett 99 029503
[7] Werner V et al. 2002 Phys. Lett. A 550 140
[8] Holt J D, Pietralla N, Holt J W, Kuo T T S and Rainovski G 2007 Phys. Rev. C 76 034325
[9] Fransen C et al. 2005 Phys. Rev C 71 054304
[10] Loliudice N and Stoyanv C 2006 Phys. Rev. C 73 037305
[11] Heisenberg J and Blok H P 1983 Annu. Rev. Nucl. Part. Sci. C 33 0569
[12] Spear R H 1989 Atom. Dat. And Nuc. Dat. Tab 42 55