Thermoelectrics versus thermophotovoltaics: two approaches to convert heat fluxes into electricity

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Abstract
Direct conversion of heat fluxes into electricity is usually done by thermoelectric generators (TEGs). For hot-side temperatures above 1000 K, thermal radiation carries a high energy density, comparable with the energy density extracted from TEGs and therefore a direct conversion of thermal radiation into electricity, named thermophotovoltaics (TPV), would also be an option. This paper compares both methods with respect to efficiency and extractable power density. The physical limits are estimated under simplified but realistic boundary conditions. For TPV the radiative detailed balance limit under black body radiation, which was calculated for different hot-side temperatures from 310 K to 3000 K for an optimized bandgap of the applied material was used. But, since very narrow bandgaps leading to strong non-radiative recombination mechanisms, the bandgap was limited to $E_g \geq 0.5 \text{ eV}$. The effect of suppressing sub-bandgap radiation as well as an enhanced radiation density in the nearfield (near-field TPV) were also included. The TEG efficiency and power density was calculated under thermal matching conditions with a heat transfer coefficient of $K_{cont} = 250 \text{ W m}^{-2} \text{ K}^{-1}$ and an average device $ZT = 1$. The results are compared with experimental data for TPV and TEGs from literature. It is shown, that up to 600 K hot-side temperature TEGs are superior to TPV, due to a significant higher power density. Above 1000 K TPV profits from higher efficiency by a similar power density. But above 2000 K TPV suffers from cell heating. The range 600 K to 1000 K is currently captured by high temperature thermoelectrics, but near-field TPV (NF-TPV) has good chances to compete with TEGs in this temperature range in the future.

Keywords: thermoelectrics, TEG, thermophotovoltaics, TPV, radiative detailed balance limit, efficiency, thermal energy harvesting

(Some figures may appear in colour only in the online journal)
1. Introduction

Waste heat recovery is one of the strategies implemented to improve the overall efficiency of energy processes [1, 2]. Schierning [3] pointed out recently that one third of all industrial processes within the European Union occur at temperatures above 1300 K. While thermo-mechanical engines can operate close to endoreversible conditions [4–6], they cannot operate everywhere, where waste heat occurs, for example due to missing robustness under harsh conditions, required maintenance or inefficient down-scalability. As such, direct conversion of heat fluxes into electricity becomes an interesting option. Typical application scenarios are in the steel industry [7], glass processing [8], combustion driven processes [9], automobile exhaust [10] or space applications [11].

Thermoelectric generators (TEGs) are quite often considered for such a direct conversion. There are several attempts to increase the operating range of thermoelectric materials to higher temperatures close to or even above 1000 K [12–17]. Recently, Li et al. [18] demonstrated reduced graphene oxide for thermoelectric applications up to 3000 K, which is the highest operating temperature for thermoelectric materials that has ever been reported. However, at such high temperatures, the thermal radiation carries a high energy density, which is even above to the energy density extracted from TEGs. Therefore, a direct conversion of thermal radiation into electricity would be an alternative method to the thermoelectric conversion. The question arises at which temperature a crossover between the different methods occurs and if a search for thermoelectric materials for sufficiently higher temperatures would really be meaningful. Therefore, this work compares the expectancy conversion efficiency and the extractable electric power density for both mechanisms: (a) TEG and (b) direct conversion of thermal radiation (thermophotovoltaics—TPV), within their physical limits under the same boundary conditions.

Thermoelectrics (TE) is based on the Seebeck effect, which describes the unipolar thermal diffusion of free charge carriers due to a temperature gradient within a material (see figure 1(a)).

In contrast, if thermal radiation is absorbed by a semiconductor, electron–hole pairs are generated. If these pairs are being separated by a build-in field of a p-n junction before they can recombine, electric energy can be extracted from such a semiconductor device (figure 1(b)). This effect is termed thermophotovoltaics (TPV) and it is physically the same process that occurs in any solar cell, except that the spectrum of the thermal radiation is shifted to longer wavelengths, compared to that of solar radiation. The origin of this idea is quite often referred to Aigrain [19] who presented lectures on TPV at MIT in the mid-1950s [20]. The current state of the art in TPV was recently reviewed by Daneshvar et al. [21]. The thermal radiation power scales according to the Stephan–Boltzmann law with the forth power of the absolute temperature. Therefore, at low temperatures, energy transfer by thermal radiation between two reservoirs with a given temperature difference is often negligible compared to other transport processes, but becomes dominant with increasing temperatures.

The upper physical limit for the efficiency of any heat engine is the well-known Carnot-efficiency:

\[
\eta_{\text{CN}} = \frac{T_h - T_c}{T_h} \quad (1)
\]

with \(T_h\) the temperature on the hot-side and \(T_c\) the temperature on the cold-side. But this quantity is less helpful for real engines, because it involves the trivial case where no power is generated: The work is provided in an infinitely long time period. The more realistic limit is the Novikov–Curzon–Ahlborn (NCA) efficiency [5, 6] for an endoreversible engine:

\[
\eta_{\text{NCA}} = 1 - \sqrt{\frac{T_c}{T_h}} \quad (2)
\]

which takes entropy production of the thermal flux between the heat reservoirs and the engine into account, while the engine itself remains ‘ideal’. But the direct conversion of heat fluxes applying TE or TPV contains further irreversible processes, which have to be considered in an adequate way.

In case of TE, the thermodynamic of the conversion process is quite similar to that of a steam engine [22, 23]. The working fluid of a steam engine is in this case the electron gas of free charge carriers in the thermoelectric material. Instead of mechanical work induced in steam engines due to gas expansion, the electron gas in a TE engine directly stimulates an electric potential difference. The essential loss mechanisms are heat leakage due to thermal transport across the lattice of the thermoelectric material and the friction of the charge carriers, expressed by the electric resistivity. Both loss mechanisms are accompanied with entropy production and are therefore irreversible. Like for classical steam engines, the heat flux for a TE engine is also provided by thermal conduction from the heat reservoir. This implies entropy production as a result of feeding the generator with thermal energy. Therefore, a direct contact to the heat reservoir is required, which can become challenging if the hot reservoir is at extreme high temperatures.

In case of TPV, the thermal energy is provided by radiation and this radiation is directly coupled to the electron gas. Thus, the energy transport from the hot reservoir is free of entropy production and it is not required to be in direct contact to the reservoir, which are essential advantages of TPV. The absorbed heat radiation is first transformed into chemical energy. If the heat radiation would be monochrome and the chemical states (bandgap) would fit to this monochrome radiation, the efficiency to convert the thermal energy into chemical energy would be equal to the Carnot-efficiency [24]. But usually, thermal radiation is spectrally broad and entropy production would be expected during the relaxation of highly excited charge carriers to the band edge. Further entropy is produced, when the chemical energy is extracted as electrical energy by an electric current due to the finite electric conductance. Thus, TPV is also far from being considered as a reversible process.

But maximizing an engine just with respect to efficiency can still lead to the strange situation, where the efficiency is high, but the extracted power is low. Therefore, besides efficiency, the maximum extractable power per area (power density) is an important practical quantity too, that has to be
a p-n junction device is given by:

While $V$ describes the voltage on the p-n junction device and $J(V)$ describes its current density-voltage characteristics. The ‘minus’ sign is used in order to express extracted energy as ‘positive’ energy, when for $V$ and $J$ the convention for a passive device is used. The current density-voltage characteristics are approximated from three current density contributions. The first contribution is a recombination current density:

$$J_{\text{rec}} = e_0 \frac{2\pi}{h^3} \int_{-\infty}^{E_{\text{ph,max}}} \frac{E^2}{e^{E/k_B T} - 1} dE + e_0 \frac{\pi}{h^3} \int_{E_\text{g}}^{E_{\text{ph,max}}} \frac{E^2}{e^{E/k_B T} - 1} dE + \text{multiphonons} + J_{\text{SRH}} + J_{\text{Au}}.$$  

(4a)

It describes the recombination of electron–hole pairs within the p-n junction device and it depends on the applied voltage $V$ and the cell-temperature $T_{\text{cell}}$. $e_0$ stands for the elementary charge and $h$ represents Planck’s constant. Equation (4a) follows from a thermodynamic consideration [30]. The first term on the right side describes the emission of photons while $c_0$ stands for the speed of light. The last two terms $J_{\text{SRH}}$ and $J_{\text{Au}}$ are standing for the Shockley–Read–Hall recombination current and the Auger-recombination current respectively. Although these two currents are present in real devices, they are classified as ‘extrinsic’ losses because they could be prevented by perfect devices. The Shockley–Read–Hall current is caused by recombination over local defect centers under emission of photons in the volume or surface of the material and would not occur in a perfect material. The Auger recombination current is related to the thickness of the active layer and can be suppressed for sufficient thin devices [31]. The second term in equation (4a) describes the direct emission of phonons, while $c_s$ stands for an effective sound velocity, ‘multiphonons’ indicates further terms for the direct emission of more than one phonon at the same time. Since $c_s \ll c_0$ phonons have a much higher density of states compared to photons and therefore the probability to emit phonons is high. But in contrast to photons, the density of states for phonons is truncated at a maximum phonon energy $E_{\text{ph,max}}$, which is usually in the order of a few tens of meV. If $E_g > E_{\text{ph,max}}$ the direct emission of single phonons is forbidden. However, due to the high phonon density of states even multi-phonon emission can still occur, at least in materials with a narrow bandgap. All these non-radiative transitions are like a ‘short circuit’ for the chemical potential of electrons and holes and prevent their energy from being extracted. Experimental experiences are indicating that for $E_g \geq 0.5$ eV the non-radiative transitions can be small (e.g. InGaAsSb: $E_g \sim 0.5...0.55$ eV [32–35], Ge: $E_g = 0.66$ eV [34], GaSb: $E_g = 0.72$ eV [32, 34, 35]), while for smaller bandgaps the non-radiative transitions are becoming dominant (e.g. InAs: $E_g = 0.345$ eV, InAs$_{0.5}$Sb$_{0.5}$: $E_g = 0.3$ eV [34, 36]). Therefore, a $E_g = 0.5$ eV is assumed to be the lower limit for the bandgap in order to keep non-radiative transitions negligible.

With the restriction that non-radiative transitions are negligible, equation (4a) reduces to:

$$P_{\text{ps}} = -J(V) \cdot V.$$  

(3)
\[ J_{\text{rec}} = \frac{2\pi}{h^3 \cdot c_0^2} \int_{E_g}^{\infty} \frac{E^2}{e^{\frac{E}{k_B T}} - 1} \, dE. \quad (4b) \]

Usually equation (4b) is further approximated by simplified or empirical analytical expressions [25, 26, 37].

The second current density contribution originates from the thermal generation of electron–hole pairs. It is independent of the applied voltage, but under thermal equilibrium it cancels out the recombination current density without external bias \( J_{\text{rec}}(V = 0) \):

\[ J_{\text{th}} = -J_{\text{rec}}(V = 0) = -\frac{2\pi}{h^3 \cdot c_0^2} \int_{E_g}^{\infty} \frac{E^2}{e^{\frac{E}{k_B T}} - 1} \, dE. \quad (5) \]

The third current density contribution is the photocurrent density, which originates from the absorption of thermal (excess) radiation:

\[ J_{\text{ph}} = -\frac{2\pi}{h^3 \cdot c_0^2} \left[ \int_{E_g}^{\infty} \frac{E^2}{e^{\frac{E}{k_B T}} - 1} \, dE - \int_{E_g}^{\infty} \frac{E^2}{e^{\frac{E - E_0}{k_B T}} - 1} \right]. \quad (6) \]

Here, the integral adds together all photons, which are generated per area from a heat source at temperature \( T_0 \), assuming black-body radiation and getting absorbed from the semiconductor, if the photon energy is above the bandgap energy \( E_g \) of the semiconductor. Photons with energy less than the semiconductor bandgap will be transmitted and absorbed at the cold side without being converted into electricity. Each photon that has been absorbed within the semiconductor will generate one charge carrier, thus the quantum yield is assumed to be one. The second term in the bracket corrects for the thermal radiation from the cell, so that only an ‘excess radiation’ is considered to contribute to the photocurrent. This photocurrent is independent of the applied voltage, but depends just on the absorbed thermal radiation. This procedure clearly shows which significant loss mechanisms exist in this transformation process. On the one hand the incoming radiation energy is lost because it cannot be absorbed, while on the other it is lost if electrons are excited high into the electronic band states, but subsequently thermalizing down to the band edge energy.

The total current density-voltage relation is then given by the sum of these three contributions (equations (4a)–(6)):

\[ J(V) = J_{\text{rec}}(V) + J_{\text{th}} + J_{\text{ph}}. \quad (7) \]

The efficiency is calculated from

\[ \eta_{\text{pn}} = \frac{P_{\text{pn}}}{I_{\text{net}}} = \frac{-J(V) \cdot V}{I_{\text{net}}}. \quad (8) \]

Where \( I_{\text{net}} \) describes the incoming net heat radiation power density. It is calculated from:

\[ I_{\text{net}} = \frac{2\pi}{h^3 \cdot c_0^2} \int_{E_{\text{min}}}^{\infty} \left( \frac{E^2}{e^{\frac{E}{k_B T}} - 1} - \frac{E^2}{e^{\frac{E - E_0}{k_B T}} - 1} \right) \, dE - E_0 J_{\text{rec}}(V) + J_{\text{th}}. \quad (9) \]

The first part describes the thermal radiation difference between hot-side and cell surface assuming black-body radiation, while \( E_{\text{min}} \) describes a truncation of the spectral range. For \( E_{\text{min}} = 0 \) it would yield the Stefan–Boltzmann law. The second part describes luminescence that originates from recombining electron–hole pairs under external bias with energy almost equal to the bandgap energy. As long as the device operates at the maximum power point, the recombination process is weak. The assumption of black body radiation is chosen as ‘natural’ starting point. Many oxidized metal surfaces are gray to almost black emitters. However, the effect of suppressing sub-bandgap radiation can be implemented by setting \( E_{\text{min}} = E_g \) in equation (9).

Maximizing equation (3) yields the maximum extractable power density \( P_{\text{pn,\text{max}}} \) and maximizing equation (8) yields the maximum efficiency \( \eta_{\text{pn,\text{max}}} \). But both quantities depend on the band-gap \( E_g \) of the material besides the hot-side temperature \( T_0 \) as well as the cell temperature \( T_{\text{cell}} \). The resulting optimal bandgap energy for different hot-side temperatures assuming three different cell temperatures of 0 K, 300 K and 1000 K and ignoring non-radiative transitions are calculated and presented in figure 2.

Obviously, narrow bandgap materials with a bandgap in the range of a few hundred meV would be required for an efficient direct thermal radiation conversion. As discussed before, such a small bandgap causes a high dark current due to non-radiative transition mechanisms, like Auger-recombination and eventually direct multi-phonon emission. Therefore, for hot-side temperatures below 2500 K the bandgap should be kept at 0.5 eV. In the upcoming calculations this assumption was made.

It should be pointed out that the conventional radiative detailed balance limit (also known as Shockley Queisser Limit) in combination with a black body radiation source is not a strict limit. There are also options to exceed this predicted limit if more sophisticated device structures like multi-junction cells [38] or hot carrier cells [39] are applied. Especially in the field of TPV, there are many attempts to
increase the efficiency by photonic engineering [40, 41] or engineering of the emitter material [42]. The idea is to prevent emissions below the bandgap energy or to reflect this part of the thermal radiation back to the source. As such, an essential loss mechanism described by the radiative detailed balance limit could be suppressed and efficiencies above the conventional radiative detailed balance limit are possible. In the simulation this approach was included by truncating the emission spectra below $E_g$.

But even more important is to point out that for energy harvesting application, the efficiency is a less important quantity compared to the extractable power density. The harvestable power density is restricted to the radiative power density and here, the black body radiation is the upper limit in the far field. Any spectral modification on the emitter material or due to filters or photonic structures that might increase the efficiency, will not increase the power density. In order to overcome this physical limitation, the concept of near-field TPV (NF-TPV) has been developed [43, 44]. If the absorber is in the near-field of the emitter, thus, if the spatial gap between emitter and absorber is smaller than the wavelength of emitted radiation, the emitted radiation density can much exceed the energy density of black body radiation in the far field. According to recent experimental studies [36], a gap of 60 nm yields a 40-fold enhancement of the radiative power density. Therefore, in order to implement the concept of NF-TPV in the simulation, an enhancement factor of 40 is inserted in equations (6) and (9).

The thermalization energy, which originates from electrons that are excited high into the electronic band states, but subsequently thermalizing down to the band edge energy will heat-up the TPV cell. Also, photons with energy below the bandgap transmit the active semiconducting region and will contribute 0 to the thermal radiation. The emitted radiation density can much exceed the energy density of black body radiation in the far field. For a temperature (hot-side temperature) assuming a thermal transmission coefficient of 500 W m$^{-2}$ K$^{-1}$ to the ambient of 300 K with and without sub-bandgap radiation absorption. Far field is calculated with black body radiation and near-field assumes a 40-fold enhanced black body radiation.

![Figure 3. TPV cell temperature as a function of the heat source temperature (hot-side temperature) assuming a thermal transmission coefficient of 500 W m$^{-2}$ K$^{-1}$ to the ambient of 300 K with and without sub-bandgap radiation absorption. Far field is calculated with black body radiation and near-field assumes a 40-fold enhanced black body radiation.](image)

start to increase significantly above a hot-side temperature of 1000 K if also sub-bandgap radiation is absorbed by the electrodes and contributes to the cell heating. Above 2000 K hot-side temperature, the cell temperature would exceed 1000 K, which probably would destroy a real cell by thermal stress. If the absorption of sub-bandgap radiation is suppressed, either by filters, photonic structures or selected emitters, the thermal stress is significantly less. But even there, for hot-side temperature above 2000 K a significant cell heating occurs. In the case of near-field radiation, cell heating is even more an issue due to the enhanced radiation density. Further, since the bandgap of the cell is limited to $E_g = 0.5$ eV the fraction of sub-bandgap radiation increases below hot-side temperatures of 2500 K. Therefore, especially for near-field TPV the suppression of sub-bandgap radiation is important.

### 2.2. Modelling thermoelectric generators

In case of thermoelectric energy conversion, heat is first transported from the heat source into the TEG and then rejected in the cold reservoir. This heat transport process produces entropy and lowers the efficiency. The thermal coupling of the TEG with these reservoirs is expressed by the total heat transfer coefficient $K_{cont}$. It summarizes both, the heat transfer coefficient on the hot-side $K_h$ as well as that on the cold-side $K_c$ (figure 4) and can be well approximated by [46, 47]:

$$K_{cont} \approx \frac{K_h \cdot K_c}{K_h + K_c}. \quad (11)$$

A maximum thermoelectric efficiency is obtained when the heat transfer coefficient to the environment exceeds the effective thermal conductance $K_{TEG}$ of the device. The ideal case is therefore attained for:

$$\frac{K_{cont}}{K_{TEG}} \to \infty. \quad (12)$$
Practically, since $K_{cont}$ will always be a finite quantity, the limit can be reached only for

$$K_{T_{\text{TEG}}} \to 0.$$  \hspace{1cm} (13)

But such a device is not able to produce any power anymore. The maximum TE efficiency that could be obtained within this limit would be \cite{46, 48, 49}:

$$\eta_{\text{TE,max}} = \frac{T_h - T_c}{T_h} \cdot \frac{\sqrt{1 + ZT} - 1}{\sqrt{1 + ZT} + \frac{T_c}{T_h}},$$  \hspace{1cm} (14)

where $T$ stands for the average temperature and $Z$ is:

$$Z = \frac{\alpha^2_{\text{TEG}}}{K_{\text{TEG},0} \cdot R_{\text{TEG}}}.$$  \hspace{1cm} (15)

Here $K_{\text{TEG},0}$ describes the thermal conductance of the TEG if no electric current flows through the TEG, $R_{\text{TEG}}$, the internal electric resistance of the TEG and $\alpha_{\text{TEG}}$ the average Seebeck coefficient of the TEG, which is the ratio of the provided open circuit voltage to the applied temperature difference on the TEG. The dimensionless quantity $ZT$ is the thermoelectric device figure of merit that contains besides the thermoelectric material properties also parasitic device losses such as contact resistances or thermal leakages. Consequently, it will always be lower than the pure materials figure of merit $\varepsilon T$:

$$ZT < \varepsilon T = \frac{\alpha^2 \sigma}{\kappa} T,$$  \hspace{1cm} (16)

where $\alpha$ stands for the Seebeck coefficient of the material, $\sigma$ its electrical conductivity and $\kappa$ its thermal conductivity. Since $Z$ is a function of temperature, $ZT$ peaks at a specific temperature while at other temperatures $ZT$ will be lower. A large average $ZT$ value is more important for a device than a solitary peak value.

As opposed to the maximum efficiency, the maximum power density that can be obtained from a TEG is \cite{46}:

$$P_{\text{TEG,max}} = K_{\text{cont}} \cdot \frac{ZT}{1 + \sqrt{1 + ZT}} \cdot \frac{(\Delta T)^2}{4T}.$$  \hspace{1cm} (17)

It is obviously directly proportional to the total heat transfer coefficient $K_{\text{cont}}$, which becomes an important parameter besides $ZT$ estimating the extractable electric power density. But to obtain this maximum power density, a thermal matching condition has to be fulfilled \cite{46}:

$$K_{\text{cont}} = K_{\text{TEG}}.$$  \hspace{1cm} (18)

Due to this matching condition, just half of the available temperature difference drops over the thermoelectric active material. The efficiency that can be obtained under maximum power generation is:

$$\eta_{\text{TEG},P_{\text{max}}} = \frac{2}{K_{\text{cont}}} \cdot \frac{P_{\text{TEG,max}}}{\Delta T},$$  \hspace{1cm} (19)

which is almost half of the maximum efficiency.
3. Results and discussion

Figure 5 shows different efficiency limits as a function of the hot-side temperature at a fixed cold-side temperature of $T_c = 300\,\text{K}$ for the different conversion processes. The Novikov–Curzon–Ahlborn (NCA) efficiency is added as reference (curve (1) in figure 5). For the TE-efficiency limit a device $Z\mathcal{T} = 1$ was assumed. As mentioned before, a solitary peak in $Z\mathcal{T}$ is less important than a large average value. While in the past material $Z\mathcal{T}$ values were less than 1 and the device $Z\mathcal{T}$ were in the order of $Z\mathcal{T} \approx 0.5$, today, materials with figure of merits close to or even above $Z\mathcal{T} = 2$ are reported [16, 17], and even average $Z\mathcal{T}$ values of 1.4 to 1.5 over the temperature range of 300...900K have been demonstrated [50, 51].

Furthermore, segmented modules are developed in order to optimize TEGs with respect to the temperature dependent $Z\mathcal{T}$ values [52–54], whereby different materials for different temperature ranges are applied. Therefore, a device $Z\mathcal{T} = 1$ is assumed to be an approachable value and therefore a good estimate for the comparison done in this paper. Nevertheless, other $Z\mathcal{T}$ values will be discussed below.

But besides the TE-efficiency limit, the TE-efficiency under maximum power transfer is also included. A real and optimized state-of-the-art device is expected to operate somewhere in the range limited by both curves (curve (2) and (3) in figure 5). Experimental data from literature are added in order to indicate that the proposed range can be reached.

The efficiency limit for TPV shall be discussed in several steps. Curve (4) in figure 5 presents the conventional radiative detailed balance (RDB) limit, assuming always an optimized bandgap and ignoring non-radiative transitions. Since, as discussed in section 2.1, non-radiative transitions are becoming dominant for narrow bandgaps, $E_g$ was kept $\gtrapprox 0.5\,\text{eV}$. This assumption leads to curve (5) in figure 5, which shows a strong drop in efficiency for lower hot-side temperatures. Curve (6) in figure 5 takes cell heating into account and shows obviously a further drop of efficiency, especially for higher hot-side temperatures. But, since most of the losses are originating from sub-bandgap radiation, it is a very important approach to suppress this sub-bandgap radiation. This is usually done by photonic structures [40, 41] or selected emitters [42]. Curve (7) in figure 5 represent the efficiency limit, if sub-bandgap radiation would be suppressed completely. While at higher hot-side temperatures cell heating is still an issue, causing a decrease in efficiency, a huge gain in efficiency is obtained for lower hot-side temperatures. Since common TPV systems are always working with some spectral shaping of the emission, real systems are expected to be between curve (6) and (7) in figure 5. The added experimental data from literature in figure 5 confirm this. While curve (7) represents a more hypothetical limit, since any further loss mechanisms are ignored, it demonstrates also the potential compared to thermoelectrics. It is remarkable that curve (7) exceeds partially the NCA efficiency (curve (1)). This is possible, because the transfer of heat from the hot side to the cell is free of entropy production. Even more, if the radiation of the heat source would be monochrome and close to the bandgap of the cell material, Carnot-efficiency (equation (1)) would be possible (not shown in figure 5). A theoretical simulation by DeSutter et al [55] taking several loss mechanism into account predicted for a hot-side temperature of 2000K a maximum efficiency of 38.8% with an almost monochrome radiation (open star data point in figure 5). But on the other side it is remarkable, that cell heating becomes crucial, if the hot-side temperature exceeds 2000K, even if sub-bandgap emission is suppressed. A heat transfer coefficient of about 5000 W m\(^{-2}\) K\(^{-1}\) to the ambient would be required in order to prevent cell heating. Alternatively, the radiation density could by lowered by geometrical aspects, for example in a cylindrical symmetry.

Curve (8) in figure 5 shows the calculated efficiency for a NF-TPV cell with sub-bandgap radiation. The strong heating of the cell due to the high radiation density would make such a cell complete inefficient. But if sub-bandgap radiation would be suppressed completely, again, a huge gain in efficiency is obtained (curve (9)). This indicates the critical role of cell-cooling and spectral shaping especially for NF-TPV. Francouet et al [56] pointed out already the crucial role of thermal management for near-field TPV.

Coming back to thermoelectrics. To aim at the radiative detailed balance efficiency limit by a TEG, a device $Z\mathcal{T}$ of $Z\mathcal{T} \approx 2.2$ would be required (see figure 6). To reach this efficiency even under maximum power output conditions, the device figure of merit would have to be increased even further up to $Z\mathcal{T} \approx 8$ (figure 6). It is also remarkable to mention that the efficiency of TE is significantly less than the Novikov–Curzon–Ahlborn (NCA) efficiency.

As mentioned above, efficiency is just one part of the story. For energy harvesting devices, the question for the maximal extractable power density is even more important. Figure 7 compares these quantities for different scenarios of these different conversion methods.

In case of a TEG, the extractable power density is closely related to the heat transfer coefficient $K_{\text{conv}}$ (equation (17)). Therefore, assumptions should be made for this quantity. It should be noted that the effective heat transfer coefficient is determined from both, the thermal coupling on the hot-side as
For TPV, the hot-side temperature has to exceed 1000 K in order to obtain a power density which would be comparable to the power density of TEGs. Astonishingly, at hot-side temperatures above 2000 K the TPV power density starts to decrease again, due to cell heating. The difference between the cases where sub-bandgap radiation is emitted or not is not as strong pronounced as on the efficiency. This is because the sub-bandgap radiation does not directly influence the output power density, but it effects it only indirect via the cell temperature. A more sophisticated study on the impact of spectral shaping on the output power density by DeSutter [55] leads to the conclusion that even the high energy range of the emission spectrum should be cut-off. Again, the impact is just indirect, because radiation much above the bandgap energy produces heat due to thermalization, which lowers the efficiency of the cell. Experimental data from literature are added in figure 7 and they are not so far away from this predictions, indicating that these cells obviously are operating already close to the limit.

The intrinsic drawback of TPV to have only a low power density at lower hot-side temperatures due to Stefan–Boltzmann law is attempted to overcome by using near-field radiation, as already mentioned. In the calculation, we assumed a 40-fold enhanced black body radiation, as it was recently experimentally determined over a 60 nm gap [36]. As shown in figure 7, the NF-TPV power density is increased at lower hot-side temperatures and becomes compatible with TE power densities already at hot-side temperatures above 600 K. But NF-TPV suffers even stronger than TPV under cell heating, which causes a decrease of power densities already at hot-side temperatures above 1000 K. In contrast to TPV, the power density of NF-TPV is quite strong effected by the circumstance if sub-bandgap radiation occurs or not, despite the influence is again only indirect via the cell temperature. But the experimental realization of NF-TPV is still challenging. An experimental value for the power density of about 6 W m$^{-2}$ at 655 K is extracted from a recent work [36] and added in figure 7, while the main intention of this work was not an optimized NF-TPV cell, but the experimental investigation of the enhanced radiation density in the near-field.

4. Summary and conclusion

Two ways to convert heat fluxes directly into electricity, either by thermoelectric effect or by thermophotovoltaic effect within a p-n junction device have been compared with respect to efficiency and obtainable electric power density in a temperature range between 310 K to 3000 K hot-side temperature and 300 K cold-side temperature. Although at high temperatures the thermal radiation carries a high energy density, that even exceeds the energy density of thermoelectric generators, it is not straight forward to convert this energy into electricity. It turned out, that cell heating is a critical aspect that lowers the efficiency of TPV cells for hot-side temperatures above 2000 K, if no great effort is made for cell cooling.

But, concerning efficiency, the best reported TPV cells around hot-side temperatures of 1300 K have about two times higher efficiencies than the best reported TEGs close...
to 1000 K, which underlines the potential of TPV. Currently, all measured efficiencies of both conversion processes remain significantly below the endoreversible thermodynamic efficiency limit (NCA efficiency limit). But, while for TE there is currently no concept how to increase the efficiency significantly further, TPV profits from the fact, that radiation provides a heat flux without entropy production and is therefore not restricted to the NCA efficiency limit. The calculations have shown that with a sufficient suppression of sub-bandgap radiation, for example by photonic structures, the NCA efficiency limit could be approachable for TPV.

But for energy harvesting, the obtained energy density is quite often more important than the efficiency. Here TEGs are leading. For hot-side temperatures up to 600 K there is no indication how TPV could reach the power densities of TEGs. The energy density of TPV is limited by the Stefan–Boltzmann law. But even modern concepts like the NF-TPV would require hot-side temperatures of at least 600 K in order obtain energy densities comparable with TEGs. While the near-field concept enhances the energy density of TPV at lower hot-side temperatures, it limits also the energy density at higher hot-side temperatures due to enhanced cell heating. It turns out, that cell heating becomes an even more critical aspect for NF-TPV due to the enhanced radiation density.

Thus, up to 600 K hot-side temperature TEGs are superior to TPV, which is mainly due to the significant higher power density of TEGs. Above 1000 K TPV will be preferred, because it becomes challenging to make stable electric contacts for TEGs at such high temperatures, while TPV provides comparable energy densities and even higher efficiencies. Astonishingly, above 2000 K both concept are reaching their limit. While it is difficult to imagine stable thermoelectric materials and contacts at such high temperatures, also TPV suffers from cell heating, which lowers the efficiency and extractable power density. But for TPV, a too high radiation power density could be lowered at least by geometrical spreading of the radiation. The range 600 to 1000 K is currently captured by high temperature thermoelectrics, but could be in future also used by NF-TPV. While NF-TPV is currently a challenging technology, it has the potential to reach or even to exceed the performance of TEGs in this temperature range.

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