A new way to determine the neutrino mass hierarchy at reactors

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The advances in the statistical methods may play a crucial role in the discoveries reached by the neutrino experiments, mainly due to the rear number of collected events and the limited achieved sensitivities. So that evaluating the used statistical methods and updating them is a necessary step in building a robust statistical analysis for answering the open questions in neutrino physics.
Does the neutrino spectrum follow Normal or Inverted Hierarchy?

The existing $\nu$ oscillation data allow to determine the values of the three $\nu$ mixing angles $\theta_{12}, \theta_{23},$ and $\theta_{13}$ of $\nu$ mixing matrix and the values of $\Delta m^2_{21}$ and $|\Delta m^2_{31(32)}|$. The sign of $\Delta m^2_{31(32)}$ is not determined yet: that corresponds to the neutrino mass hierarchy problem.

|             | best-fit   | $3\sigma$ region                          |
|-------------|------------|-------------------------------------------|
| $\sin^2_{12}$ | 0.2970     | 0.2500 - 0.3540                           |
| $\sin^2_{13}$(NH) | 0.02140    | 0.0185 - 0.0246                           |
| $\sin^2_{13}$(IH) | 0.02180    | 0.0186 - 0.0248                           |
| $\delta m^2_{\odot}$ | $7.37 \times 10^{-5}$ | $6.93 \times 10^{-5} - 7.97 \times 10^{-5} \text{eV}^2$ |
| $\Delta m^2$(NH)   | $2.500 \times 10^{-3}$ | $2.37 \times 10^{-3} - 2.63 \times 10^{-3} \text{eV}^2$ |
| $\Delta m^2$(IH)   | $2.460 \times 10^{-3}$ | $-2.60 \times 10^{-3}$ to $-2.33 \times 10^{-3}$ eV$^2$ |
Neutrino Mass Hierarchy Determination (MHD) Problem

\[
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix}
= 

\begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} \\
U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\
U_{\tau 1} & U_{\tau 2} & U_{\tau 3}
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix}
\]

\[
\Delta m^2_{31} (NH) \equiv | \Delta m^2_{32} (IH) | = \Delta m^2_{atm}
\]

\[\Delta m^2_{31} (NH) \rightarrow m^2_3 > m^2_2 > m^2_1 \Rightarrow \Delta m^2_{31} (NH) \text{ has positive value}
\]

\[\Delta m^2_{32} (IH) \rightarrow m^2_2 > m^2_1 > m^2_3 \Rightarrow \Delta m^2_{32} (IH) \text{ has negative value}
\]

\[\Delta m^2_{31} (NH) \equiv | \Delta m^2_{32} (IH) | = \Delta m^2_{atm}
\]

From the physics point of view, MH is ordered as follows:

- From the physics point of view, MH is ordered as follows:
- From the statistics point of view, MH is given by a test to distinguish between two hypotheses: normal (NH) and inverted hierarchies (IH).
The Ph.D project report about the issues of the standard algorithm and the alternative method that manifests itself as a solution.
JUNO is designed to determine the neutrino mass hierarchy via measurement of reactor neutrino energy spectrum with $\frac{3\%}{\sqrt{E}}$ relative energy resolution. It will start data taking from 2020 and the detector will be located about 700 meters underground.
The Reactor $\nu$ Energy Spectrum

Reactor $\bar{\nu}_e$ energy spectrum for toy JUNO-like simulated events for NH hypothesis (6 years of data taking).

$$\chi^2 = \chi^2_{para} + \chi^2_{sys} + \chi^2_{stat}.$$
\( \chi^2_{\text{para}} \) summarizes the prior knowledge on oscillation parameters. In JUNO these parameters are \( \sin^2 2\theta_{12}, \sin^2 2\theta_{13}, \Delta m^2_{12} \) and \( \Delta m^2_{13} \). Then \( \chi^2_{\text{para}} \) becomes:

\[
\chi^2_{\text{para}} = \left( \frac{(\sin^2 2\theta_{12})_{\text{fit}} - (\sin^2 2\theta_{12})_{\text{input}}}{\sigma_{\sin^2 2\theta_{12}}} \right)^2 + \left( \frac{(\sin^2 2\theta_{13})_{\text{fit}} - (\sin^2 2\theta_{13})_{\text{input}}}{\sigma_{\sin^2 2\theta_{13}}} \right)^2 + \left( \frac{(|\Delta m^2_{31}|)_{\text{fit}} - (|\Delta m^2_{31}|)_{\text{input}}}{\sigma_{|\Delta m^2_{31}|}} \right)^2 + \left( \frac{(\Delta m^2_{21})_{\text{fit}} - (\Delta m^2_{21})_{\text{input}}}{\sigma_{\Delta m^2_{21}}} \right)^2.
\] (1)
The reactor anti-neutrino flux, the IBD cross section, the fiducial volume and the weight fraction of free proton can all be combined into a single overall factor. Consequently, their contributions to the $\chi^2$ function can be represented by a single term as,

$$
\chi^2_{sys} = \left( \frac{f_{fit} - f^{input}_{sys}}{\sigma_{f_{sys}}} \right)^2,
$$

(2)

where $f^{input}_{sys} = 1$, and $\sigma_{f_{sys}} = 0.03$ (as a function of energy). The $\chi^2_{stat}$ represents the statistical fluctuation, it looks like

$$
\chi^2_{stat} = \sum_i \left( \frac{N_i^{fit} - N_i^{NH(IH)}}{\sqrt{N_i^{NH(IH)}}} \right)^2
$$

(3)
The JUNO experimental environment

The Problem: The Issues of the Standard Algorithm

- Issue I: The Limited Power of $\Delta \chi^2$
- Issue II: Non-bright Results using $\chi^2$ as a Bi-Dimensional
- Issue III: The Robustness

The Solution: The Alternative Method $\vec{F}_{MO}$

- The Construction
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BackUp

The Ph.D project report about the issues of the standard algorithm and the alternative method that manifests itself as a solution.
Issue I: The Limited Power of $\Delta \chi^2$

$$\Delta \chi^2 = \chi^2_{\text{min}(NH)} - \chi^2_{\text{min}(IH)}$$

infinite energy resolution

3% relative energy resolution

$\Delta \chi^2$ estimator for 1000 (NH) + 1000 (IH) toy JUNO-like simulated through event by event and generated at $\Delta m^2 = 2.500 \times 10^{-3} \text{eV}^2$ for NH hypothesis (blue) and $\Delta m^2 = -2.460 \times 10^{-3} \text{eV}^2$ for IH hypothesis (red) with six years of exposure and the ten near reactor cores.
The Problem: The Issues of the Standard Algorithm

The Solution: The Alternative Method

Issue 1: The Limited Power of $\Delta \chi^2$

$$\Delta \chi^2 = \chi^2_{\text{min}}(NH) - \chi^2_{\text{min}}(IH)$$

### Infinite Energy Resolution

|        | $\mu_{NH}$ | $\sigma_{NH}$ | $\mu_{IH}$ | $\sigma_{IH}$ | $Z_{\text{score}}^{(NH)}$ | $Z_{\text{score}}^{(IH)}$ |
|--------|-------------|---------------|-------------|---------------|---------------------------|---------------------------|
| $\mu_{NH}$ | $-63.02 \pm 0.74$ | $23.51 \pm 0.53$ | $89.41 \pm 0.72$ | $22.86 \pm 0.51$ | $6.484$ | $6.668$ |
| $\sigma_{NH}$ | | | | | $7.950$ | $9.456$ |

$Z_{\text{score}}^{(NH)}$: 6.484
$Z_{\text{score}}^{(IH)}$: 6.668

### 3%/$\sqrt{E}$ Energy Resolution

|        | $\mu_{NH}$ | $\sigma_{NH}$ | $\mu_{IH}$ | $\sigma_{IH}$ | $Z_{\text{score}}^{(NH)}$ | $Z_{\text{score}}^{(IH)}$ |
|--------|-------------|---------------|-------------|---------------|---------------------------|---------------------------|
| $\mu_{NH}$ | $-15.25 \pm 0.87$ | | $14.75 \pm 0.84$ | | | |
| $\sigma_{NH}$ | | $27.54 \pm 0.62$ | | | | |
| $\mu_{IH}$ | | | $14.75 \pm 0.84$ | | | |
| $\sigma_{IH}$ | | | | $26.55 \pm 0.60$ | | |

$Z_{\text{score}}^{(NH)}$: 1.089
$Z_{\text{score}}^{(IH)}$: 1.130

$\sigma = 2\sqrt{\Delta \chi^2}$
The JUNO experimental environment

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Issue I: The Limited Power of $\Delta \chi^2$

The Interpretation

The investigation of the origin of the approximation have been pursued by looking whether it is still valid in event-by-event simulations as it is in semi-analytical simulations. When the simulation is performed on a semi-analytical basis and not on a event-by-event basis, the significance drastically drops. The semi-analytical based simulation does not take into account the correlation between the bins due to systematic uncertainties. The systematic uncertainty due to the $\frac{3\%}{\sqrt{E}}$ energy resolution causes unbalanced migration between bins that manifests itself in the event-by-event simulations. That consequently creates side-bin correlations leading to a reduction in the experiment sensitivity.
Issue I: The Limited Power of $\Delta \chi^2$

To conclude the first issue

When the statistical assumptions are not valid, the limited power of the $\Delta \chi^2$ manifests itself.
Issue II: Non-bright Results using $\chi^2$ as a Bi-Dimensional

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The Problem: The Issues of the Standard Algorithm

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Issue II: Non-bright Results using $\chi^2$ as a Bi-Dimensional

infinite energy resolution

$\Delta m^2_{\text{NH}} = 2500 \times 10^{-6} \text{eV}^2$ and $|\Delta m^2_{\text{IH}}| = 2460 \times 10^{-6} \text{eV}^2$

3% relative energy resolution

$\Delta m^2_{\text{NH}} = 2500 \times 10^{-6} \text{eV}^2$ and $|\Delta m^2_{\text{IH}}| = 2460 \times 10^{-6} \text{eV}^2$

Two islands of $\chi^2$ for 1000 (NH) + 1000 (IH) toy JUNO-like simulations generated at $\Delta m^2 = 2.500 \times 10^{-3} \text{eV}^2$ for NH hypothesis (blue island) and $\Delta m^2 = -2.460 \times 10^{-3} \text{eV}^2$ for IH hypothesis (red island) with six years of exposure and the ten near reactor cores. An infinite energy resolution is assumed for left plot and 3% relative energy resolution for right plot.
The Ph.D project report about the issues of the standard algorithm and the alternative method that manifests itself as a solution.

Issue II: Non-bright Results using $\chi^2$ as a Bi-Dimensional

**Table:** Two $\chi^2$ distributions for 1000 (NH) + 1000 (IH) toy JUNO-like simulations generated at $\Delta m^2 = 2.500 \times 10^{-3} \text{eV}^2$ for NH hypothesis and $\Delta m^2 = -2.460 \times 10^{-3} \text{eV}^2$ for IH hypothesis with six years of exposure and ten near reactor cores.

|          | NH          | IH          | NH          | IH          |
|----------|-------------|-------------|-------------|-------------|
| $\mu_{NH}$ | 807.6 ± 1.46 | 889.6 ± 1.61 | 862.60 ± 1.53 | 867.6 ± 1.51 |
| $\sigma_{NH}$ | 46.05 ± 1.03 | 51.05 ± 1.14 | 48.49 ± 1.08 | 47.67 ± 1.07 |
| $\mu_{IH}$  | 870.60 ± 1.53 | 800.2 ± 1.50 | 877.80 ± 1.55 | 852.90 ± 1.55 |
| $\sigma_{IH}$ | 48.34 ± 1.08 | 47.30 ± 1.06 | 49.04 ± 1.08 | 49.03 ± 1.07 |
| $Z_{score}^{(NH)}$ | 1.159 $\sigma$ | | 0.217 $\sigma$ |
| $Z_{score}^{(IH)}$  | 1.113 $\sigma$ | | 0.219 $\sigma$ |
To conclude the second issue:

The overlapping between the $\chi^2$ distributions of the two hypotheses leads to reduction of the experimental sensitivity.
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The JUNO experimental environment

The Problem: The Issues of the Standard Algorithm

The Solution: The Alternative Method

Issue III: The Robustness

| \( \Delta \chi^2 \) | vs \( \Delta m^2_{inj} \)

Infinite Energy Resolution

3% Relative Energy Resolution

\( |\Delta \chi^2| \) varies with \( |\Delta m^2|_{inj} \) in the range of \( 2.450 \times 10^{-3} \, \text{eV}^2 \leq |\Delta m^2| \leq 2.580 \times 10^{-3} \, \text{eV}^2 \) for 1000 (NH) + 1000 (IH) toy JUNO-like simulations for each point of \( |\Delta m^2|_{inj} \) with six years of exposure and the ten near reactor cores. The error bars correspond to the standard error of the \( |\Delta \chi^2| \) that is calculated as the standard deviation of the \( \Delta \chi^2 \) distribution divided by the square root of the sample size.
The JUNO experimental environment

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The significance using official procedures vs $\Delta m^2_{inj}$

Infinite Energy Resolution

3% Relative Energy Resolution

The variation of significance with $|\Delta m^2|_{inj}$ in the range of $2.450 \times 10^{-3} \text{eV}^2 \leq |\Delta m^2| \leq 2.580 \times 10^{-3} \text{eV}^2$ for 1000 (NH) + 1000 (IH) toy JUNO-like simulations for one benchmark assuming $3%/\sqrt{E}$ energy resolution where blue line is for NH sample and red line is for IH sample. The sensitivity assuming an infinite energy varies from about 6.5 $\sigma$ to 9.5 $\sigma$. The sensitivity assuming 3%/\sqrt{E} energy resolution varies from about 3.2 $\sigma$ to 4.1 $\sigma$. 

The Ph.D project report about the issues of the standard algorithm and the alternative method that manifests itself as a solution.
The significance vs $\Delta m_{inj}^2$ using event by event simulation

- **Infinite Energy Resolution**
- **3% Relative Energy Resolution**

The variation of significance with $|\Delta m^2|_{inj}$ in the range of $2.450 \times 10^{-3} \text{eV}^2 \leq |\Delta m^2| \leq 2.580 \times 10^{-3} \text{eV}^2$ for 1000 (NH) + 1000 (IH) toy JUNO-like simulations for one benchmark where blue line is for NH sample and red line is for IH sample. The sensitivity assuming an infinite energy resolution varies from about 4.5 $\sigma$ to 7.5 $\sigma$. The sensitivity assuming 3%/\sqrt{E} energy resolution oscillates from about 0.9 $\sigma$ to 1.5 $\sigma$. 

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The JUNO experimental environment

The Problem: The Issues of the Standard Algorithm

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Issue III: The Robustness

The significance using official procedures (2D) vs $\Delta m^2_{\text{inj}}$

Infinite Energy Resolution

3% Relative Energy Resolution

The oscillation of the experimental significance using $\chi^2$ as bi-dimensional distribution with $|\Delta m^2|_{\text{inj}}$ in the range of $2.450 \times 10^{-3}\text{eV}^2 \leq |\Delta m^2|_{\text{inj}} \leq 2.580 \times 10^{-3}\text{eV}^2$ for 1000 (NH) + 1000 (IH) JUNO-toy like simulations for one benchmark assuming where blue line is for NH sample and red line is for IH sample. The significance assuming an infinite energy resolution varies from about 0.8 $\sigma$ to 1.3 $\sigma$. The significance assuming 3% relative energy resolution varies from about 0.175 $\sigma$ to 0.24 $\sigma$. 

The Ph.D project report about the issues of the standard algorithm and the alternative method that manifests itself as a solution.
To summarize the problem,

1. When the statistical assumptions are not valid, the limited power of the $\Delta \chi^2$ manifests itself.
2. The overlapping between the $\chi^2$ distributions of the two hypotheses leads to a strong reduction of the experimental sensitivity.
3. The experimental sensitivity strongly depends on the input value of the neutrino atmospheric mass difference.
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The optimal weight method yields the optimal weight \( \phi_{\text{opt}} \) for the parameter \( \theta \) that has a general solution

\[
\phi_{\text{opt}} = \frac{\partial \ln \pi}{\partial \theta}.
\]  
(4)

The optimal weight is the generalized weight that minimizes the variance of the parameter, \( \sigma^2_\theta \).

\[
\sigma^2_{\theta[\phi]} = \left( \frac{\partial \langle \phi \rangle_{\text{th}}}{\partial \theta} \right)^2 \sigma^2_{\phi}.
\]  
(5)

Notice that

\[
\langle \phi_{\text{opt}} \rangle = \left\langle \frac{\partial \ln \pi}{\partial \theta} \right\rangle = \int \frac{\partial \ln \pi}{\partial \theta} \pi dN = \int \frac{\partial \pi}{\partial \theta} dN = \frac{\partial}{\partial \theta} \int \pi dN \equiv \frac{\partial}{\partial \theta} 1 = 0.
\]  
(6)

The variance of the optimal weight \( \sigma^2_{\phi_{\text{opt}}} \) is optimal because it is asymptotically equal to the minimal value established by the fundamental Cramer-Rao inequality.

\[
\sigma^2_{\phi_{\text{opt}}} = \left\langle [\phi_{\text{opt}} - \langle \phi_{\text{opt}} \rangle]^2 \right\rangle = \left\langle \left[ \frac{\partial \ln \pi}{\partial \theta} \right]^2 \right\rangle \approx \sigma^2_{\text{CRF}}
\]  
(7)
The optimal weight proposed to solve the neutrino mass ordering using reactor spectrum is the difference in number of events per each single energy bin between the observed spectrum and the model.

\[
\Delta N(E) = \left( \frac{dN}{dE} \right)_{IH} - \left( \frac{dN}{dE} \right)_{NH} \\
= T \times \sigma_{IBD} \times \phi \times \Delta p_{IH-NH} \\
= T \times \sigma_{IBD} \times \phi \times \sin^2 2\theta_{13} \times \cos(2\theta_{12}) \times \sin\left(\frac{\delta m^2 L}{4E}\right) \times \sin\left(\frac{(\Delta m^2_{atm} - \delta m^2_{sol}/2)L}{2E}\right)
\]

(8)

\[
F_\lambda = \int_{1.8}^{8.0} |\Delta N(E)| dE,
\]

(9)
The JUNO experimental environment

The Problem: The Issues of the Standard Algorithm

The Solution: The Alternative Method

The Construction

\[ \vec{F}_{MO} = F_{IH} \hat{e}_x + F_{NH} \hat{e}_y \]

\[ F_{IH} = \int_{1.8}^{8.0} |N_{obs}(E) - \mu^{IH}(E)| dE \text{ in } I^+ \text{ when } \mu^{NH}(E) > \mu^{IH}(E) \]
\[ + \int_{1.8}^{8.0} |\mu^{IH}(E) - N_{obs}(E)| dE \text{ in } I^- \text{ when } \mu^{NH}(E) < \mu^{IH}(E) \]

\[ F_{NH} = \int_{1.8}^{8.0} |N_{obs}(E) - \mu^{NH}(E)| dE \text{ in } I^- \text{ when } N^{IH}(E) > \mu^{NH}(E) \]
\[ + \int_{1.8}^{8.0} |\mu^{NH}(E) - N_{obs}(E)| dE \text{ in } I^+ \text{ when } N^{IH}(E) < \mu^{NH}(E) \]
The Ph.D project report about the issues of the standard algorithm and the alternative method that manifests itself as a solution.
Variation of the two components of $\vec{F}_{MO}$ as function of the $\Delta m^2_{atm}$. The samples were produced at infinite energy resolution at baseline of L = 52.5 km and a true $\Delta m^2_{atm} = 0.00256 \text{ eV}^2$, all the other parameters being fixed for one benchmark\(^1\) of JUNO-like data taking.

\(^1\)6 years of data taking, 10 near reactor cores, official acceptance
The Degeneracy Effect

\( F \) owns a degeneracy on \( \Delta m^2_{atm} \)!

Two different solutions:

1. one for NH at \( \Delta m^2_{32} \)
2. one for IH at \( \Delta m^2_{31} \neq |\Delta m^2_{32}| \)

BUT the degeneracy can be overcome by the external information

\[ \bar{F}_{MO} \text{ vs } |\Delta m^2_{atm}| \text{ for ten generated JUNO-like toy experiments, in the NH (right) and the IH (left) hypotheses for 1 benchmark assuming 3\% relative energy resolution. The black (red) curves correspond to the analysis when the true (false) hypothesis is taken. The vertical lines indicate the selected } \Delta m^2_{atm}. \]
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The Robustness

\[ \Delta m_{atm}^2(\text{true}) \text{ vs } \Delta m_{atm}^2(\text{recons}) \] is drawn, \( \Delta m_{atm}^2(\text{recons}) \) being obtained by the minimum \( \vec{F}_{\text{MO}} \). The continuous lines correspond to the central values, the dashed ones to the \( \pm \sigma \) bands. Black (red) curves corresponds to the NH (IH) generation. The central circles correspond to the 68% and 95% C.L. contours of the current \( \Delta m_{atm}^2 \) uncertainties for NH and IH.
The couplings of $\Delta m_{\text{atm}}^2(\text{true})$ and $\Delta m_{\text{atm}}^2(\text{recons})$ are pointed out. The quoted sensitivities corresponds to the probability to mis-identify $(F_{\text{min}}^{\text{min}}, F_{\text{max}}^{\text{max}})$ with $(F_{\text{min}}^{\text{min}}, F_{\text{max}}^{\text{max}})$, either at the same $\Delta m_{\text{atm}}^2(\text{true})$ or the same $\Delta m_{\text{atm}}^2(\text{recons})$. An equivalent probability applies to $(F_{\text{min}}^{\text{min}}, F_{\text{max}}^{\text{max}})$. 
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Sensitivity using $\vec{F}_{MO}$ as a function of energy resolution

Evolution of the NH/IH sensitivity as function of the data taking. The different curves correspond to different energy resolution (continuous/dashed are for NH/IH hypothesis, respectively). The background has been conservatively assumed to be described by the $^{9}Li$ component. Ten near reactor cores plus two remote cores have been used, each with a $\pm 5m$ and $\pm 5km$ uniform dispersions for the relative baseline, respectively.
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In conclusion,

1. The $\Delta \chi^2$ estimator provides us with different results than the officially published due to different simulation procedures.
2. The experimental sensitivity strongly depends on the value of the neutrino atmospheric mass difference.
3. The overlapping between the $2D - \chi^2$ distributions of the two hypotheses leads to a reduction of the experimental sensitivity.
In conclusion,

1. The new alternative method overcomes the issues of the standard method.

2. The alternative method results are better than the results currently accepted by the standard method and published in [yellowBook, globalfit, deSalas:2018bym, Gariazzo:2018pei].

3. The alternative method is the one with the most robust results.

4. $\vec{F}_{MO}$ overcomes the degeneracy $\sim 12 \times 10^{-5} \text{eV}^2$ on $\Delta m^2_{atm}$ by external knowledge.

5. $\vec{F}_{MO}$ is able to distinguish the correct ordering with significance results ($\approx 5\sigma$).
THANKS TO ALL OF YOU

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The Ph.D project report about the issues of the standard algorithm and the alternative method that manifests itself as a solution.
The References
The Interpretation

Semi-Analytical simulations are generated by fluctuating the whole bin-content according to Poisson or Gaussian distributions that represent the number of events. In addition, a second fluctuation is added by applying $3\%/\sqrt{E}$ energy smearing in the whole bin content.

Event-by-Event simulations are generated by fluctuating each single event according to Poisson or Gaussian distributions that represent the number of events. In addition, a second fluctuation is added by applying $3\%/\sqrt{E}$ energy smearing in each single event in each single bin.
Strategy. Results could be:

- compatible* with both NH and IH ➔ wrong experiment
- reject* both NH and IH ➔ wrong data analysis, or new Physics
- compatible* with a MO and reject the other MO’ with some significance ➔ good experiment and data analysis,
  ➔ observe MO with that significance
- “borderline” results ➔ wait for more data
- significance at 5 σ ➔ end of the story

SENSITIVITY = probability (significance) to reject the wrong MH, once exp is compatible* with true MH

* compatible at 95% C.L.
* reject at 95% C.L.
Survival probability, \( P_{\bar{\nu}_e} \), at \( L = 52.5 \) km, 
\[ \sin^2_{12} = 0.2970, \quad \text{and} \quad \delta m^2_{\text{sol}} = 7.37 \times 10^{-5} \text{ ev}^2, \]
for NH hypothesis (blue line) 
\[ \Delta m^2 = 2.500 \times 10^{-3} \text{ ev}^2 \] and 
\[ \sin^2_{13}(\text{NH}) = 0.02140 \] and for IH hypothesis (red line) 
\[ \Delta m^2 = -2.460 \times 10^{-3} \text{ ev}^2 \] and 
\[ \sin^2_{13}(\text{IH}) = 0.02180, \] as reported in the Yellow Book.

Survival probability, \( P_{\bar{\nu}_e} \), at \( L = 52.5 \) km, 
\[ \sin^2_{12} = 0.2970, \quad \sin^2_{13} = 0.02150, \quad \text{and} \quad \delta m^2_{\text{sol}} = 7.37 \times 10^{-5} \text{ ev}^2, \]
for NH hypothesis (blue line) 
\[ \Delta m^2 = 2.5230 \times 10^{-3} \text{ ev}^2 \] and 
for IH hypothesis (red line) 
\[ \Delta m^2 = -2.5230 \times 10^{-3} \text{ ev}^2. \]
Does the neutrino spectrum follow Normal or Inverted Hierarchy?

The Neutrino Mass Hierarchy Determination ($\nu$ MHD) is one of the main goals of the major current and future neutrino experiments.

\[
P_{\text{NH}} = 1 - \frac{1}{2} \sin^2 2\theta_{13} \left( 1 - \cos \frac{\Delta m_{\text{atm}}^2 L}{2E} \right) - \frac{1}{2} \cos^4 \theta_{13} \sin^2 2\theta_{12} \left( 1 - \cos \frac{\delta m_{\text{sol}}^2 L}{2E} \right)
\]
\[
+ \frac{1}{2} \sin^2 2\theta_{13} \left[ \cos^2 \left( \theta_{12} + \frac{\pi}{2} \right) \right] \left( \cos \frac{L}{2E} \left( \Delta m_{\text{atm}}^2 - \delta m_{\text{sol}}^2 \right) - \cos \frac{L\Delta m_{\text{atm}}^2}{2E} \right)
\]

(10)

\[
P_{\text{IH}} = 1 - \frac{1}{2} \sin^2 2\theta_{13} \left( 1 - \cos \frac{\Delta m_{\text{atm}}^2 L}{2E} \right) - \frac{1}{2} \cos^4 \theta_{13} \sin^2 2\theta_{12} \left( 1 - \cos \frac{\delta m_{\text{sol}}^2 L}{2E} \right)
\]
\[
+ \frac{1}{2} \sin^2 2\theta_{13} \left[ \cos^2 (\theta_{12}) \right] \left( \cos \frac{L}{2E} \left( \Delta m_{\text{atm}}^2 - \delta m_{\text{sol}}^2 \right) - \cos \frac{L\Delta m_{\text{atm}}^2}{2E} \right).
\]
The Toy Simulations

A toy simulations were based on a single event basis and the expected systematic errors via a Gaussian distribution centered at the expected mean and with the standard deviation of the estimated uncertainty can be added. For JUNO, a global 3%/E(MeV) resolution on the energy reconstruction is expected. The oscillation parameters have been taken from the most recent global fits.

|                  | best-fit     | 3σ region                      |
|------------------|--------------|--------------------------------|
| $\sin^2_{12}$   | 0.2970       | 0.2500 - 0.3540                |
| $\sin^2_{13}(NH)$| 0.02140      | 0.0185 - 0.0246                |
| $\sin^2_{13}(IH)$| 0.02180      | 0.0186 - 0.0248                |
| $\delta m^2_{sol}$ | $7.37 \times 10^{-5}$ | $6.93 \times 10^{-5} - 7.97 \times 10^{-5}$ |
| $\Delta m^2(NH)$ | $2.500 \times 10^{-3}$ | $2.37 \times 10^{-3} - 2.63 \times 10^{-3}$ |
| $\Delta m^2(IH)$ | $2.460 \times 10^{-3}$ | $-2.60 \times 10^{-3}$ to $-2.33 \times 10^{-3}$ |

The Poisson statistical fluctuation is automatically included. Version "J17v1r1" of official JUNO Software is used for date simulations.
The Fitting Procedures

The fitting and minimization of $\chi^2$ has required to use directly the ROOT minimization libraries, in particular the TMinuit algorithm. In the minimization procedure all the parameters were fixed to the best values that are indicated in assuming a very small error on it.
The JUNO experimental environment

The Problem: The Issues of the Standard Algorithm

The Solution: The Alternative Method

Δχ² estimator for 1000 (NH) + 1000 (IH) toy JUNO-like simulations generated at 20 different values of the atmospheric mass in the range of $2.450 \times 10^{-3} \text{eV}^2 \leq |Δm^2| \leq 2.580 \times 10^{-3} \text{eV}^2$ for NH hypothesis (blue distribution in each plot) and IH hypothesis (red distribution in each plot) with six years of exposure and the ten near reactor cores, with energy resolution $3%/\sqrt{E}$.
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The comparison of the MH sensitivity using $\Delta \chi^2$ for actual distributions for NH sample and IH sample, for for 20 different values of the atmospheric mass in the range of $2.450 \times 10^{-3} \text{eV}^2 \leq |\Delta m^2| \leq 2.580 \times 10^{-3} \text{eV}^2$. The sensitivity calculations using the Z-test for $\Delta \chi^2$ in two cases. The first case is without the approximation of and the second one is using the approximation.

### Relative Energy Resolution 3%/\sqrt{E}

| $|\Delta m^2|_{\text{NH/IH}} \times 10^{-3}$ | 2.450   | 2.455   | 2.460   | 2.465   |
|------------------------------------------|---------|---------|---------|---------|
| $\mu_{\text{NH}}$                       | $-16.91 \pm 0.880$ | $-15.19 \pm 0.834$ | $-15.58 \pm 0.848$ | $-15.48 \pm 0.85$ |
| $\sigma_{\text{NH}}$                    | $27.82 \pm 0.622$  | $26.38 \pm 0.590$  | $26.83 \pm 0.599$  | $26.88 \pm 0.601$  |
| $\mu_{\text{IH}}$                       | $15.72 \pm 0.871$  | $14.29 \pm 0.856$  | $14.75 \pm 0.839$  | $15.22 \pm 0.847$  |
| $\sigma_{\text{IH}}$                    | $27.55 \pm 0.616$  | $27.66 \pm 0.605$  | $26.55 \pm 0.593$  | $26.65 \pm 0.595$  |
| $z_{\text{score}}^{(\text{NH})}$        | $1.173$  | $4.112$ (app.) | $1.118$  | $3.897$ (app.) |
| $z_{\text{score}}^{(\text{IH})}$        | $1.184$  | $3.965$ (app.) | $1.089$  | $3.780$ (app.) |
| $|\Delta m^2|_{\text{NH/IH}} \times 10^{-3}$ | 2.470   | 2.475   | 2.485   | 2.485   |
| $\mu_{\text{NH}}$                       | $-17.10 \pm 0.8709$ | $-15.55 \pm 0.826$ | $-17.21 \pm 0.8646$ | $-16.76 \pm 0.9159$ |
| $\sigma_{\text{NH}}$                    | $27.54 \pm 0.6158$ | $25.70 \pm 0.5746$ | $27.34 \pm 0.6114$ | $28.96 \pm 0.6477$ |
| $\mu_{\text{IH}}$                       | $15.07 \pm 0.8645$ | $12.54 \pm 0.8437$ | $14.49 \pm 0.8539$ | $12.99 \pm 0.856$  |
| $\sigma_{\text{IH}}$                    | $27.34 \pm 0.6113$ | $26.68 \pm 0.5996$ | $27.00 \pm 0.6038$ | $27.07 \pm 0.6053$ |
| $z_{\text{score}}^{(\text{NH})}$        | $1.168$  | $4.135$ (app.) | $1.093$  | $3.943$ (app.) |
| $z_{\text{score}}^{(\text{IH})}$        | $1.177$  | $3.882$ (app.) | $1.053$  | $3.541$ (app.) |
| $|\Delta m^2|_{\text{NH/IH}} \times 10^{-3}$ | 2.490   | 2.495   | 2.500   | 2.510   |
| $\mu_{\text{NH}}$                       | $-13.86 \pm 0.8974$ | $-13.89 \pm 0.874$ | $-15.25 \pm 0.8709$ | $14.52 \pm 0.871$ |
| $\sigma_{\text{NH}}$                    | $28.38 \pm 0.6345$ | $26.80 \pm 0.5994$ | $27.54 \pm 0.6158$ | $27.55 \pm 0.616$ |
| $\mu_{\text{IH}}$                       | $13.58 \pm 0.8955$ | $13.50 \pm 0.8372$ | $12.83 \pm 0.8681$ | $11.87 \pm 0.853$ |
| $\sigma_{\text{IH}}$                    | $28.39 \pm 0.6332$ | $26.47 \pm 0.5990$ | $27.45 \pm 0.6138$ | $26.97 \pm 0.663$ |
| $z_{\text{score}}^{(\text{NH})}$        | $0.967$  | $3.723$ (app.) | $1.025$  | $3.727$ (app.) |
| $z_{\text{score}}^{(\text{IH})}$        | $0.969$  | $3.685$ (app.) | $1.038$  | $3.686$ (app.) |
| $|\Delta m^2|_{\text{NH/IH}} \times 10^{-3}$ | 2.520   | 2.523   | 2.530   | 2.540   |
| $\mu_{\text{NH}}$                       | $-16.15 \pm 0.870$ | $-16.52 \pm 0.872$ | $-16.25 \pm 0.891$ | $-13.91 \pm 0.856$ |
| $\sigma_{\text{NH}}$                    | $27.52 \pm 0.615$  | $27.57 \pm 0.616$  | $27.24 \pm 0.600$  | $27.07 \pm 0.605$  |
| $\mu_{\text{IH}}$                       | $13.55 \pm 0.857$  | $13.72 \pm 0.858$  | $13.26 \pm 0.856$  | $12.91 \pm 0.888$  |
| $\sigma_{\text{IH}}$                    | $27.11 \pm 0.606$  | $27.14 \pm 0.607$  | $27.63 \pm 0.605$  | $28.98 \pm 0.628$  |
| $z_{\text{score}}^{(\text{NH})}$        | $1.079$  | $4.019$ (app.) | $1.097$  | $4.064$ (app.) |
| $z_{\text{score}}^{(\text{IH})}$        | $1.096$  | $3.681$ (app.) | $1.114$  | $3.704$ (app.) |
| $|\Delta m^2|_{\text{NH/IH}} \times 10^{-3}$ | 2.550   | 2.560   | 2.570   | 2.580   |
| $\mu_{\text{NH}}$                       | $-16.32 \pm 0.848$ | $-15.69 \pm 0.881$ | $-12.82 \pm 0.880$ | $-14.04 \pm 0.834$ |
| $\sigma_{\text{NH}}$                    | $26.83 \pm 0.600$  | $27.24 \pm 0.609$  | $27.84 \pm 0.623$  | $26.37 \pm 0.590$  |
| $\mu_{\text{IH}}$                       | $11.97 \pm 0.922$  | $10.54 \pm 0.860$  | $12.00 \pm 0.861$  | $11.58 \pm 0.876$  |
| $\sigma_{\text{IH}}$                    | $29.14 \pm 0.652$  | $27.20 \pm 0.608$  | $27.24 \pm 0.600$  | $27.70 \pm 0.619$  |
| $z_{\text{score}}^{(\text{NH})}$        | $1.054$  | $4.040$ (app.) | $0.963$  | $3.961$ (app.) |
| $z_{\text{score}}^{(\text{IH})}$        | $0.971$  | $3.460$ (app.) | $0.964$  | $3.247$ (app.) |

The Ph.D project report about the issues of the standard algorithm and the alternative method that manifests itself as a solution.
The JUNO experimental environment

The Problem: The Issues of the Standard Algorithm

The Solution: The Alternative Method

The Ph.D project report about the issues of the standard algorithm and the alternative method that manifests itself as a solution.

$\Delta \chi^2$ estimator for 1000 (NH) + 1000 (IH) toy JUNO-like simulations generated at 20 different values of the atmospheric mass in the range of $2.450 \times 10^{-3} \text{eV}^2 \leq |\Delta m^2| \leq 2.580 \times 10^{-3} \text{eV}^2$ for NH hypothesis (blue distribution in each plot) and IH hypothesis (red distribution in each plot) with six years of exposure and the ten near reactor cores. An infinite energy resolution is assumed.
The JUNO experimental environment

The Problem: The Issues of the Standard Algorithm

The Solution: The Alternative Method

The comparison of the MH sensitivity using $\Delta \chi^2$ assuming infinite energy resolution for NH sample and IH sample, for 20 different values of the atmospheric mass in the range of $2.450 \times 10^{-3} \text{eV}^2 \leq |\Delta m^2| \leq 2.580 \times 10^{-3} \text{eV}^2$. The sensitivity calculations using the Z-test for $\Delta \chi^2$ in two cases. The first case is without the approximation and the second one is obtained using the approximation.

| $|\Delta m^2|_{NH/IH} \times 10^{-3}$ | 2.450 | 2.455 | 2.460 | 2.465 |
|---------------------------------|-------|-------|-------|-------|
| $\mu_{NH}$                      | -51.00 ± 0.735 | -53.72 ± 0.732 | -59.20 ± 0.788 | -69.43 ± 0.7681 |
| $\sigma_{NH}$                   | 23.24 ± 0.520  | 23.14 ± 0.518  | 24.91 ± 0.557  | 24.29 ± 0.5431 |
| $\mu_{IH}$                      | 78.03 ± 0.752  | 85.41 ± 0.720  | 89.41 ± 0.723  | 90.69 ± 0.7482 |
| $\sigma_{IH}$                   | 23.77 ± 0.532  | 22.76 ± 0.520  | 22.86 ± 0.511  | 23.65 ± 0.5201 |
| $z(\text{core})_{NH}$           | 5.590 ± 0.720(app.) | 6.013 ± 0.720(app.) | 5.966 ± 0.720(app.) | 6.567 ± 8.332(app.) |
| $z(\text{core})_{IH}$           | 5.466 ± 8.833(app.) | 6.113 ± 9.242(app.) | 6.501 ± 9.456(app.) | 6.745 ± 9.456(app.) |

| $|\Delta m^2|_{NH/IH} \times 10^{-3}$ | 2.470 | 2.475 | 2.480 | 2.485 |
|---------------------------------|-------|-------|-------|-------|
| $\mu_{NH}$                      | -76.04 ± 0.7854 | -82.90 ± 0.7452 | -55.70 ± 0.7471 | -85.54 ± 0.7959 |
| $\sigma_{NH}$                   | 24.77 ± 0.554  | 23.55 ± 0.5269  | 23.62 ± 0.5283  | 24.29 ± 0.5431 |
| $\mu_{IH}$                      | 86.13 ± 0.762  | 78.36 ± 0.7904  | 66.17 ± 0.7649  | 90.09 ± 0.7482 |
| $\sigma_{IH}$                   | 24.07 ± 0.5388 | 24.99 ± 0.5589  | 24.19 ± 0.5409  | 23.65 ± 0.5291 |
| $z(\text{core})_{NH}$           | 6.547 ± 8.720(app.) | 6.848 ± 9.105(app.) | 5.160 ± 7.463(app.) | 7.231 ± 9.249(app.) |
| $z(\text{core})_{IH}$           | 6.737 ± 9.281(app.) | 6.453 ± 8.852(app.) | 5.038 ± 8.134(app.) | 7.425 ± 9.492(app.) |

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The JUNO experimental environment

The Problem: The Issues of the Standard Algorithm

The Solution: The Alternative Method

The Conclusions

\[ z_{\text{score}}^{(NH)} = \frac{\Delta \chi^2(IH) - \Delta \chi^2(NH)}{\sigma_{NH}} \quad z_{\text{score}}^{(IH)} = \frac{\Delta \chi^2(NH) - \Delta \chi^2(IH)}{\sigma_{IH}} \]

The \( \Delta \chi^2(NH) \), \( \sigma_{NH} \), \( \Delta \chi^2(IH) \) and \( \sigma_{IH} \) are the mean value and standard deviation of the \( \Delta \chi^2 \) distribution assuming NH and IH be the true model, respectively. There an approximation is usually used [yellowBook, globalfit, deSalas:2018bym, Gariazzo:2018pei]:

\[ \sigma_{\Delta \chi^2} = 2\sqrt{\Delta \chi^2}, \]

leading to:

\[ z_{\text{score}}^{(NH)}(\text{approximated}) = \sqrt{\Delta \chi^2(NH)} \quad z_{\text{score}}^{(IH)}(\text{approximated}) = \sqrt{\Delta \chi^2(IH)}. \]
The Sensitivity using $\chi^2$ as a Bi-Dimensional

Using Z-test for 2D, the MH sensitivity can be calculated as

$$z_{\text{score}}^{(\text{NH})} = \frac{\sqrt{\overline{\chi^2_{\text{IH}}} - \overline{\chi^2_{\text{IH}}})^2 + (\overline{\chi^2_{\text{NH}}} - \overline{\chi^2_{\text{NH}}})^2}}{\sqrt{(\sigma_{\text{IH}}^2)^{\text{NH}} + (\sigma_{\text{NH}}^2)^{\text{NH}}}}$$

$$z_{\text{score}}^{(\text{IH})} = \frac{\sqrt{\overline{\chi^2_{\text{IH}}} - \overline{\chi^2_{\text{IH}}})^2 + (\overline{\chi^2_{\text{NH}}} - \overline{\chi^2_{\text{NH}}})^2}}{\sqrt{(\sigma_{\text{IH}}^2)^{\text{IH}} + (\sigma_{\text{NH}}^2)^{\text{IH}}}}$$

where $\overline{\chi^2_{\text{IH}}}^{\text{NH}}$ and $(\sigma_{\text{IH}}^2)^{\text{NH}}$ are the mean and the standard derivation of $\chi^2$ distribution of the NH sample assuming IH hypothesis is the true hypothesis. $\overline{\chi^2_{\text{NH}}}^{\text{NH}}$ and $(\sigma_{\text{NH}}^2)^{\text{NH}}$ are the mean and the standard derivation of $\chi^2$ distribution of the NH sample assuming NH hypothesis is the true hypothesis. $\overline{\chi^2_{\text{IH}}}^{\text{IH}}$ and $(\sigma_{\text{IH}}^2)^{\text{IH}}$ are the mean and the standard deviation of $\chi^2$ distribution of the IH sample assuming IH hypothesis is the true hypothesis. $\overline{\chi^2_{\text{IH}}}^{\text{IH}}$ and $(\sigma_{\text{IH}}^2)^{\text{IH}}$ are the mean and the standard deviation of $\chi^2$ distribution of the IH sample assuming NH hypothesis is the true hypothesis.
Two $\chi^2$ distributions for 1000 (NH) + 1000 (IH) toy JUNO-like simulations generated at 20 different values of the atmospheric mass in the range of $2.450 \times 10^{-3} \text{eV}^2 \leq |\Delta m^2| \leq 2.580 \times 10^{-3} \text{eV}^2$ for NH hypothesis (blue distribution in each plot) and IH hypothesis (blue distribution in each plot) with six years of exposure and the ten near reactor cores, with energy resolution $3%/\sqrt{E}$. 

The Ph.D project report about the issues of the standard algorithm and the alternative method that manifests itself as a solution.
The Ph.D project report about the issues of the standard algorithm and the alternative method that manifests itself as a solution.

| $\Delta m^2_{\text{NH/IH}} \times 10^{-3}$ | 2.450 | 2.451 | 2.460 | 2.465 | 2.470 |
|-----------------------------------------|-------|-------|-------|-------|-------|
| $\mu_{\text{NH}}$                       | 861.7 ± 5.32  | 869.1 ± 1.601  | 858.7 ± 1.519  | 867.1 ± 1.596  | 866.1 ± 1.542  | 876.6 ± 1.508  |
| $\sigma_{\text{NH}}$                     | 48.45 ± 1.083 | 50.61 ± 1.127  | 54.06 ± 1.128  | 49.39 ± 1.104  | 49.76 ± 1.066  | 50.90 ± 1.138  |
| $\mu_{\text{IH}}$                        | 878.6 ± 1.514 | 853.4 ± 1.594  | 873.8 ± 1.501  | 852.8 ± 1.570  | 875.8 ± 1.542  | 852.9 ± 1.557  |
| $\sigma_{\text{IH}}$                     | 48.45 ± 1.083 | 50.54 ± 1.132  | 47.48 ± 1.062  | 49.64 ± 1.110  | 48.77 ± 1.091  | 49.03 ± 1.096  |
| $z_{\text{score}}$                       | 0.2397 | 0.2184 | 0.2194 | 0.2190 | 0.2218 | 0.2273 |
| $z_{\text{score}}$                       | 0.2286 | 0.2084 | 0.2227 | 0.2220 | 0.2193 | 0.2193 |

Relative energy resolution $3\%/\sqrt{E}$

|$\Delta m^2_{\text{NH/IH}} \times 10^{-3}$ | 2.475 | 2.480 | 2.485 | 2.490 | 2.495 |
|-----------------------------------------|-------|-------|-------|-------|-------|
| $\mu_{\text{NH}}$                       | 858.5 ± 5.11 | 869.2 ± 1.509 | 859.4 ± 1.566 | 866.7 ± 1.551 | 866.3 ± 1.604 | 881.4 ± 1.561 | 873 ± 1.566 | 864.4 ± 1.546 | 873 ± 1.497 |
| $\sigma_{\text{NH}}$                     | 47.77 ± 1.068 | 47.73 ± 1.067 | 49.51 ± 1.097 | 50.71 ± 1.134 | 48.50 ± 1.107 | 50.9 ± 1.138 | 49.54 ± 1.108 | 48.89 ± 1.063 | 47.35 ± 1.059 |
| $\mu_{\text{IH}}$                        | 874.1 ± 5.78 | 856.7 ± 1.502 | 876.6 ± 1.596 | 855.2 ± 1.55 | 877.1 ± 1.631 | 855.7 ± 1.54 | 878.2 ± 1.623 | 859.4 ± 1.554 | 878.7 ± 1.553 | 859.4 ± 1.473 |
| $\sigma_{\text{IH}}$                     | 49.89 ± 1.116 | 47.49 ± 1.062 | 50.46 ± 1.128 | 49.05 ± 1.097 | 51.57 ± 1.153 | 48.71 ± 1.089 | 51.31 ± 1.147 | 49.14 ± 1.099 | 46.6 ± 1.059 |
| $z_{\text{score}}$                       | 0.2045 | 0.2290 | 0.2073 | 0.1898 | 0.1938 |
| $z_{\text{score}}$                       | 0.2098 | 0.2284 | 0.2159 | 0.1907 | 0.2068 |

|$\Delta m^2_{\text{NH/IH}} \times 10^{-3}$ | 2.500 | 2.510 | 2.520 | 2.523 | 2.530 |
|-----------------------------------------|-------|-------|-------|-------|-------|
| $\mu_{\text{NH}}$                       | 862.6 ± 5.83 | 870.2 ± 1.599 | 862.4 ± 1.526 | 876.0 ± 1.621 | 864.4 ± 1.585 | 872.9 ± 1.561 | 861.8 ± 1.505 | 871.9 ± 1.574 | 862.9 ± 1.538 | 873.8 ± 1.552 |
| $\sigma_{\text{NH}}$                     | 48.49 ± 1.084 | 50.58 ± 1.131 | 48.24 ± 1.079 | 51.68 ± 1.156 | 50.12 ± 1.121 | 49.37 ± 1.104 | 47.59 ± 1.064 | 49.78 ± 1.13 | 48.64 ± 1.088 | 49.07 ± 1.097 |
| $\mu_{\text{IH}}$                        | 877.8 ± 1.531 | 857.4 ± 1.578 | 877 ± 1.52 | 858.8 ± 1.634 | 861.5 ± 1.616 | 859.3 ± 1.534 | 878.3 ± 1.508 | 858.2 ± 1.566 | 879.1 ± 1.556 | 860.5 ± 1.54 |
| $\sigma_{\text{IH}}$                     | 49.64 ± 1.066 | 49.90 ± 1.116 | 48.14 ± 1.08 | 51.35 ± 1.146 | 51.09 ± 1.142 | 48.50 ± 1.065 | 47.7 ± 1.067 | 49.31 ± 1.07 | 49.21 ± 1.10 | 860.3 ± 1.54 |
| $z_{\text{score}}$                       | 0.2040 | 0.1925 | 0.2083 | 0.2253 | 0.2154 |
| $z_{\text{score}}$                       | 0.1980 | 0.1829 | 0.2153 | 0.2164 | 0.2146 |

|$\Delta m^2_{\text{NH/IH}} \times 10^{-3}$ | 2.540 | 2.550 | 2.560 | 2.570 | 2.580 |
|-----------------------------------------|-------|-------|-------|-------|-------|
| $\mu_{\text{NH}}$                       | 866.0 ± 5.15 | 873.5 ± 1.587 | 874.6 ± 1.50 | 875.1 ± 1.566 | 863.3 ± 1.568 | 873.0 ± 1.540 | 867.3 ± 1.538 | 872.2 ± 1.571 | 876.6 ± 1.613 | 877.9 ± 1.601 |
| $\sigma_{\text{NH}}$                     | 47.73 ± 1.067 | 50.19 ± 1.122 | 47.44 ± 1.061 | 49.53 ± 1.108 | 49.59 ± 1.109 | 48.70 ± 1.089 | 48.64 ± 1.088 | 49.68 ± 1.11 | 51.01 ± 1.141 | 50.64 ± 1.132 |
| $\mu_{\text{IH}}$                        | 879.9 ± 1.555 | 860.9 ± 1.533 | 880.9 ± 1.539 | 863.1 ± 1.5 | 879.0 ± 1.554 | 862.5 ± 1.493 | 880.1 ± 1.541 | 860.2 ± 1.579 | 881.6 ± 1.606 | 866.3 ± 1.551 |
| $\sigma_{\text{IH}}$                     | 49.17 ± 1.067 | 48.47 ± 1.084 | 48.67 ± 1.061 | 47.32 ± 1.06 | 49.16 ± 1.099 | 47.2 ± 1.055 | 48.73 ± 1.09 | 49.94 ± 1.117 | 50.79 ± 1.136 | 49.06 ± 1.097 |
| $z_{\text{score}}$                       | 0.1936 | 0.2164 | 0.1909 | 0.1803 | 0.1793 |
| $z_{\text{score}}$                       | 0.1902 | 0.2086 | 0.1966 | 0.1763 | 0.1831 |
Two $\chi^2$ distributions for 1000 (NH) + 1000 (IH) toy JUNO-like simulations that generated at 20 different values of the atmospheric mass in the range of $2.450 \times 10^{-3} \text{eV}^2 \leq |\Delta m^2| \leq 2.580 \times 10^{-3} \text{eV}^2$ for NH hypothesis (blue distribution in each plot) and IH hypothesis (blue distribution in each plot) with six years of exposure and the ten near reactor cores with infinite energy resolution.
The Ph.D project report about the issues of the standard algorithm and the alternative method that manifests itself as a solution.

### Infinite Energy Resolution

| $|\Delta m^2|_{\text{NH}}/|\Delta m^2|_{\text{IH}} \times 10^{-3}$ | 2.450 | 2.455 | 2.460 | 2.465 | 2.470 |
|-----------------|-------|-------|-------|-------|-------|
| $\mu_{\text{NH}}$ | 810.7 ± 1.245 | 874.6 ± 1.528 | 808.1 ± 1.523 | 879.4 ± 1.602 | 810.7 ± 1.533 | 889.6 ± 1.614 | 811.3 ± 1.494 | 889.7 ± 1.542 | 811.7 ± 1.593 | 890.1 ± 1.441 |
| $\sigma_{\text{NH}}$ | 45.07 ± 1.085 | 48.31 ± 1.080 | 48.17 ± 1.077 | 50.67 ± 1.133 | 48.48 ± 1.084 | 51.64 ± 1.141 | 47.23 ± 1.056 | 48.76 ± 1.090 | 47.7 ± 1.067 | 49.39 ± 1.104 |
| $\mu_{\text{IH}}$ | 862.6 ± 1.510 | 796.6 ± 1.432 | 861.7 ± 1.619 | 794.1 ± 1.507 | 899.8 ± 1.631 | 800.2 ± 1.496 | 880.7 ± 1.601 | 799.5 ± 1.445 | 887.7 ± 1.593 | 803.8 ± 1.441 |
| $\sigma_{\text{IH}}$ | 47.75 ± 1.068 | 45.27 ± 1.012 | 51.2 ± 1.145 | 47.67 ± 1.066 | 51.57 ± 1.153 | 47.3 ± 1.058 | 50.62 ± 1.132 | 45.69 ± 1.022 | 50.39 ± 1.127 | 45.57 ± 1.019 |
| $z_{\text{score}}$ | 1.0095 | 1.0134 | 1.0717 | 1.163 | 1.1728 |
| $z_{\text{ratio}}$ | 1.0013 | 1.0261 | 1.0891 | 1.2050 | 1.2113 |
| $\Delta m^2_{\text{NH}}/\Delta m^2_{\text{IH}} \times 10^{-3}$ | 2.475 | 2.480 | 2.485 | 2.490 | 2.495 |

### Conclusions

- Back Up

The JUNO experimental environment
The Theoretical Derivation of $\vec{F}_{MO}$

The Optimal Weights Method [Alexey, Tkachov3, Lokhov] is derived from Pearson’s generalized weights. Assuming that there is a set of events $N_i$ binned in bin number $i$ and their assumed probability $\pi^i(N_{obs})$ that follows Poisson distribution with the mean $\mu_i$.

$$\pi^i(N_{obs}) = \frac{\mu_i^{N_{obs}}}{N_{obs}!} e^{-\mu_i}. \quad (13)$$

The average number of events per each energy bin $i$ under $\lambda$ hypothesis is $\mu_i(E)$:

$$\mu_i(E) \sim \sigma_E \times \phi_E \times p\bar{\nu}_e \rightarrow \bar{\nu}_e \quad (14)$$

The optimal weight $\phi_{opt}^i(N_i)$ whose variance $\sigma_{\phi_{opt}}^2$ is optimal because it is asymptotically equal to the minimal value established by the fundamental Cramer-Rao inequality:

$$N\phi_{opt}({\{N_{obs}\}_i}) = \frac{\partial \ln \pi({\{N_{obs}\}_i})}{\partial \lambda} = \sum_i \frac{\partial \ln \pi^i(N_{obs})}{\partial \lambda} = \sum_i \phi_{opt}^i(N_{obs}). \quad (15)$$

$$\phi_{opt}^i(N_{obs}) = \frac{\partial \ln \mu_i}{\partial \lambda} = \frac{\partial}{\partial \lambda} (\ln \mu_i - \mu_i) = \left( \frac{\partial \ln \mu_i}{\partial \lambda} \right) (N_{obs} - \mu_i). \quad (16)$$
\[
F_\lambda = \langle \phi(N_{obs}) \rangle_{\text{exp}} \\
= N \phi_{opt}(\{N_{obs}\}_i) \\
= \sum_i \phi_{opt}^i(N_{obs}) \\
= \sum_i \left( \frac{\partial \ln \mu_i^\lambda}{\partial \lambda} \right) (N_{obs} - \mu_i^\lambda) \\
= \sum_i \left[ \begin{cases} +1, & \text{in } I^+ \text{ intervals when } \mu_i^{NH} > \mu_i^{IH} \\ -1, & \text{in } I^- \text{ intervals when } \mu_i^{NH} < \mu_i^{IH} \end{cases} \right] (N_{obs} - \mu_i^\lambda) \\
= \sum_{i \in I^+} (N_{obs} - \mu_i^\lambda) + \sum_{i \in I^-} (\mu_i^\lambda - N_{obs}) \\
= \sum_{i \in I^+} \Delta_i^+ + \sum_{i \in I^-} \Delta_i^-
\]

where

\[
\Delta_i^+ = (N_{obs} - \mu_i^\lambda) \text{ in } I^+ \text{ intervals when } \mu_i^{NH} > \mu_i^{IH} \\
\Delta_i^- = (\mu_i^\lambda - N_{obs}) \text{ in } I^- \text{ intervals when } \mu_i^{IH} < \mu_i^{NH}
\]
Methodology for p-value Calculation for Bi-Dimensional estimator

\[ p_{val}(IH = \text{null hypothesis})_{\text{weighted}} = p_{val}(IH) \otimes \text{Weight} \]

\[ = \int_{\Omega_{NH}} d\vec{x} \left[ \int_{\Omega_{NH}(\vec{x})} d\vec{x} f_{IH}(\vec{x}) \cdot f_{NH}(\vec{x}) \right] \]

\[ \Omega_{NH} \perp \int_{\Omega_{NH}} d\vec{x} f_{NH}(\vec{x}) = C.L. \quad (18) \]

C.L. = 99.7% is the expected, alternative, hypothesis confidence level.

\[ \Omega_{NH}(\vec{x}) \ni f_{IH}(\vec{x}') \leq f_{IH}(\vec{x}); \vec{x}' \in \Omega_{NH} \quad (19) \]
After 2 years of data taking

| NH true | 4%         | 3.5%        | 3%          | 2.5%        |
|---------|------------|-------------|-------------|-------------|
| \( \mu_{NH} \) | 2834.2 ± 3.9 | 2807.4 ± 3.8 | 2770.5 ± 3.9 | 2726.6 ± 3.7 |
| \( \sigma_{NH} \) | 115.6 ± 1.5  | 117.2 ± 2.4  | 121.1 ± 2.5  | 115.5 ± 1.5  |
| \( \mu_{IH} \) | 3125.2 ± 3.9 | 3145.5 ± 4.1 | 3179.8 ± 4.0 | 3220.9 ± 4.1 |
| \( \sigma_{IH} \) | 123.5 ± 1.6  | 126.8 ± 2.7  | 125.7 ± 2.5  | 125.6 ± 1.6  |
| \( r_{NH} \) | -0.585 ± 0.018 | -0.606 ± 0.013 | -0.620 ± 0.014 | -0.644 ± 0.012 |
| IH true | 4%         | 3.5%        | 3%          | 2.5%        |
| \( \mu_{NH} \) | 3138.9 ± 4.2 | 3177.9 ± 4.3 | 3225.0 ± 4.1 | 3264.5 ± 3.9 |
| \( \sigma_{NH} \) | 131.7 ± 2.6  | 132.3 ± 1.8  | 126.0 ± 2.6  | 130.2 ± 2.8  |
| \( \mu_{IH} \) | 2831.9 ± 4.0 | 2785.5 ± 4.1 | 2732.2 ± 3.8 | 2691.4 ± 3.8 |
| \( \sigma_{IH} \) | 124.6 ± 2.5  | 125.4 ± 1.5  | 118.9 ± 0.8  | 118.1 ± 2.3  |
| \( r_{IH} \) | -0.632 ± 0.016 | -0.628 ± 0.010 | -0.610 ± 0.016 | -0.618 ± 0.018 |
| \( p \)-value (IH) | 3.21 \( \times \) 10^{-2} | 1.14 \( \times \) 10^{-2} | 1.92 \( \times \) 10^{-3} | 2.20 \( \times \) 10^{-4} |
| \( n \sigma \) (IH) | 2.14 | 2.53 | 3.10 | 3.69 |
| \( p \)-value (NH) | 4.23 \( \times \) 10^{-2} | 1.46 \( \times \) 10^{-2} | 1.85 \( \times \) 10^{-3} | 2.59 \( \times \) 10^{-4} |
| \( n \sigma \) (NH) | 2.03 | 2.44 | 3.11 | 3.65 |
The Ph.D project report about the issues of the standard algorithm and the alternative method that manifests itself as a solution.

After 2 years of data taking

|                    | NH true | IH true |      | 4%   | 3.5%  | 3%   | 2.5%  |
|--------------------|---------|---------|------|------|-------|------|-------|
|                    | \(\mu_{NH}\) | \(\sigma_{NH}\) | \(\mu_{IH}\) | \(\sigma_{IH}\) | \(r_{NH}\) | \(\mu_{IH}\) | \(\sigma_{IH}\) | \(r_{IH}\) |
|                    | 2413.7 ± 3.8 | 2382.1 ± 3.7 | 2339.4 ± 3.8 | 2290.3 ± 3.8 | -0.557 ± 0.014 | 2730.0 ± 4.3 | 2771.6 ± 4.3 | 2819.5 ± 4.2 | 2864.8 ± 4.2 | -0.618 ± 0.017 | -5.91 ± 0.017 | -5.85 ± 0.013 | -5.94 ± 0.012 |
|                    | 117.4 ± 1.6  | 116.9 ± 2.4  | 120.2 ± 2.5  | 117.3 ± 1.6  | -0.581 ± 0.018 | 135.6 ± 2.7  | 133.3 ± 2.7  | 130.5 ± 1.8  | 132.4 ± 2.4  | -0.581 ± 0.017 | -0.591 ± 0.017 | -0.585 ± 0.013 | -0.594 ± 0.012 |
|                    | 2680.9 ± 4.0 | 2705.4 ± 4.1 | 2742.3 ± 4.1 | 2784.7 ± 4.2 | -0.585 ± 0.018 | 2377.4 ± 3.9 | 2327.5 ± 4.0 | 2268.3 ± 3.9 | 2226.2 ± 3.7 | -0.585 ± 0.018 | -0.591 ± 0.017 | -0.585 ± 0.013 | -0.594 ± 0.012 |
| \(p\)-value \(\text{IH}\) | 2.73 \times 10^{-2} | 7.66 \times 10^{-3} | 1.07 \times 10^{-3} | 1.07 \times 10^{-4} | 2.21 | 2.67 | 3.27 | 3.87 |
| \(p\)-value \(\text{NH}\) | 3.64 \times 10^{-2} | 1.00 \times 10^{-2} | 1.12 \times 10^{-3} | 1.11 \times 10^{-4} | 2.09 | 2.58 | 3.26 | 3.86 |
After 2 years of data taking

|                      | 4%       | 3.5%     | 3%       | 2.5%     |
|----------------------|----------|----------|----------|----------|
| NH true              |          |          |          |          |
| $\mu_{NH}$           | 4192.6 ± 6.9 | 4149.4 ± 6.8 | 4083.4 ± 6.9 | 4016.1 ± 6.6 |
| $\sigma_{NH}$        | 212.4 ± 5.0  | 214.6 ± 4.4  | 215.2 ± 4.5  | 209.2 ± 4.5  |
| $\mu_{IH}$           | 4606.3 ± 7.2 | 4639.1 ± 7.4 | 4699.3 ± 7.2 | 4747.1 ± 7.2 |
| $\sigma_{IH}$        | 225.2 ± 4.7  | 232.1 ± 4.8  | 226.6 ± 4.7  | 228.1 ± 4.7  |
| $r_{NH}$             | -0.570 ± 0.012 | -0.555 ± 0.019 | -0.545 ± 0.020 | -0.566 ± 0.013 |
| IH true              |          |          |          |          |
| $\mu_{NH}$           | 4637.6 ± 6.7 | 4711.0 ± 7.6 | 4775.1 ± 7.0 | 4841.5 ± 7.5 |
| $\sigma_{NH}$        | 239.8 ± 3.7  | 236.9 ± 4.8  | 219.9 ± 4.5  | 232.2 ± 4.9  |
| $\mu_{IH}$           | 4167.6 ± 5.2 | 4095.3 ± 7.2 | 4018.7 ± 6.8 | 3951.7 ± 6.8 |
| $\sigma_{IH}$        | 224.3 ± 3.7  | 225.1 ± 4.6  | 212.6 ± 4.4  | 211.5 ± 4.4  |
| $r_{IH}$             | -0.588 ± 0.019 | -0.597 ± 0.017 | -0.552 ± 0.019 | -0.601 ± 0.013 |
| $p$−value (IH)       | $7.60 \times 10^{-2}$ | $3.14 \times 10^{-2}$ | $6.71 \times 10^{-3}$ | $1.57 \times 10^{-3}$ |
| $n \sigma$ (IH)      | 1.77      | 2.15      | 2.71      | 3.16      |
| $p$−value (NH)       | $9.44 \times 10^{-2}$ | $3.59 \times 10^{-2}$ | $6.08 \times 10^{-3}$ | $1.72 \times 8^{-3}$ |
| $n \sigma$ (NH)      | 1.67      | 2.10      | 2.74      | 3.13      |
The JUNO experimental environment

The Problem: The Issues of the Standard Algorithm

The Solution: The Alternative Method

The Conclusions

After 2 years of data taking

8 reactor cores without the two remote cores

|           | 4%                | 3.5%               | 3%                | 2.5%               |
|-----------|-------------------|--------------------|-------------------|--------------------|
| NH true   |                   |                    |                   |                    |
| $\mu_{NH}$| 4026.9 ± 6.5      | 3979.9 ± 4.1       | 3912.9 ± 6.6      | 3845.6 ± 5.5       |
| $\sigma_{NH}$| 203.9 ± 4.2     | 204.7 ± 3.2        | 203.6 ± 3.2       | 198.4 ± 2.9        |
| $\mu_{IH}$ | 4438.1 ± 6.9      | 4474.3 ± 3.6       | 4537.4 ± 7.1      | 4590.8 ± 4.7       |
| $\sigma_{IH}$| 215.8 ± 4.4       | 221.9 ± 3.2        | 219.3 ± 2.7       | 220.9 ± 4.8        |
| $r_{NH}$   | -0.575 ± 0.018    | -0.564 ± 0.015     | -0.553 ± 0.014    | -0.575 ± 0.013     |
| IH true    |                   |                    |                   |                    |
| $\mu_{NH}$ | 4474.2 ± 7.3      | 4546.0 ± 7.1       | 4614.9 ± 6.8      | 4680.5 ± 7.2       |
| $\sigma_{NH}$| 225.7 ± 4.8       | 223.7 ± 4.6        | 212.9 ± 4.4       | 224.5 ± 3.9        |
| $\mu_{IH}$ | 3998.8 ± 6.8      | 3928.5 ± 6.9       | 3849.7 ± 6.4      | 3782.6 ± 5.6       |
| $\sigma_{IH}$| 210.1 ± 4.5       | 215.4 ± 4.4        | 200.9 ± 4.1       | 202.1 ± 4.1        |
| $r_{IH}$   | -0.597 ± 0.012    | -0.572 ± 0.018     | -0.571 ± 0.018    | -0.580 ± 0.018     |
| $p$-value (IH) | $6.48 \times 10^{-2}$ | $2.33 \times 10^{-2}$ | $4.33 \times 10^{-3}$ | $8.34 \times 10^{-4}$ |
| $n \sigma$ (IH) | 1.85 | 2.27 | 2.85 | 3.34 |
| $p$-value (NH) | $7.41 \times 10^{-2}$ | $2.65 \times 10^{-2}$ | $3.93 \times 10^{-3}$ | $9.26 \times 10^{-4}$ |
| $n \sigma$ (NH) | 1.79 | 2.22 | 2.88 | 3.31 |
The JUNO experimental environment

The Problem: The Issues of the Standard Algorithm

The Solution: The Alternative Method

The Ph.D project report about the issues of the standard algorithm and the alternative method that manifests itself as a solution.

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The results after 4 years of data taking

|                 | 4%          | 3.5%         | 3%           | 2.5%         |
|-----------------|-------------|--------------|--------------|--------------|
| **NH true**     |             |              |              |              |
| $\mu_{NH}$      | 4784.8 ± 3.1| 4712.3 ± 5.5 | 4638.2 ± 5.9 | 4545.7 ± 5.2 |
| $\sigma_{NH}$   | 170.2 ± 3.0 | 170.6 ± 2.3  | 174.4 ± 3.6  | 162.5 ± 2.0  |
| $\mu_{IH}$      | 5357.8 ± 4.3| 5411.3 ± 6.0 | 5483.2 ± 6.8 | 5560.4 ± 5.9 |
| $\sigma_{IH}$   | 178.8 ± 3.2 | 184.9 ± 2.1  | 190.9 ± 4.2  | 181.8 ± 2.3  |
| $r_{NH}$        | -0.656 ± 0.009| -0.687 ± 0.010| -0.696 ± 0.014| -0.698 ± 0.009|
| **IH true**     |             |              |              |              |
| $\mu_{NH}$      | 5399.6 ± 6.0| 5475.5 ± 5.2 | 5557.6 ± 6.1 | 5653.7 ± 6.1 |
| $\sigma_{NH}$   | 184.6 ± 3.8 | 181.4 ± 3.1  | 187.9 ± 2.3  | 189.3 ± 2.3  |
| $\mu_{IH}$      | 4754.5 ± 5.4| 4668.3 ± 4.9 | 4576.9 ± 5.4 | 4478.2 ± 5.4 |
| $\sigma_{IH}$   | 167.6 ± 3.5 | 170.4 ± 3.3  | 168.6 ± 2.0  | 167.0 ± 2.1  |
| $r_{IH}$        | -0.650 ± 0.010| -0.670 ± 0.014| -0.707 ± 0.010| -0.687 ± 0.010|
| $p$—value (IH)  | $3.50 \times 10^{-3}$| $4.14 \times 10^{-4}$| $3.06 \times 10^{-5}$| $1.23 \times 10^{-7}$|
| $n \sigma$ (IH) | 2.92         | 3.53         | 4.17         | 5.29         |
| $p$—value (NH)  | $3.64 \times 10^{-3}$| $3.92 \times 10^{-4}$| $2.60 \times 10^{-5}$| $1.85 \times 10^{-7}$|
| $n \sigma$ (NH) | 2.91         | 3.55         | 4.21         | 5.21         |
The JUNO experimental environment

The Problem: The Issues of the Standard Algorithm

The Solution: The Alternative Method

The Conclusions

The results after 4 years of data taking

ten reactor cores, no background

|                  | 4%          | 3.5%         | 3%            | 2.5%          |
|------------------|-------------|--------------|---------------|---------------|
| NH true          |             |              |               |               |
| \(\mu_{\text{NH}}\) | 3763.2 ± 5.6 | 3674.5 ± 5.7 | 3586.8 ± 5.8  | 3470.8 ± 5.4  |
| \(\sigma_{\text{NH}}\) | 176.8 ± 3.6 | 177.6 ± 3.8  | 180.7 ± 3.6   | 175.0 ± 2.9   |
| \(\mu_{\text{IH}}\) | 4313.9 ± 6.0 | 4367.7 ± 6.1 | 4449.0 ± 6.4  | 4538.6 ± 6.0  |
| \(\sigma_{\text{IH}}\) | 186.5 ± 3.8 | 181.4 ± 3.9  | 201.5 ± 4.0   | 194.3 ± 2.6   |
| \(r_{\text{NH}}\) | -0.650 ± 0.016 | -0.651 ± 0.010 | -0.668 ± 0.015 | -0.648 ± 0.010 |
| IH true          |             |              |               |               |
| \(\mu_{\text{IH}}\) | 4428.6 ± 6.2 | 4510.6 ± 6.1 | 4591.0 ± 6.2  | 4717.0 ± 6.3  |
| \(\sigma_{\text{IH}}\) | 195.0 ± 4.0 | 191.4 ± 3.7  | 190.1 ± 2.5   | 173.5 ± 3.2   |
| \(\mu_{\text{IH}}\) | 3666.2 ± 5.8 | 3563.0 ± 5.7 | 3460.9 ± 5.9  | 3342.8 ± 5.3  |
| \(\sigma_{\text{IH}}\) | 181.6 ± 3.6 | 177.4 ± 0.6  | 178.8 ± 2.4   | 173.5 ± 3.2   |
| \(r_{\text{IH}}\) | -0.629 ± 0.016 | -0.644 ± 0.013 | -0.663 ± 0.010 | -0.633 ± 0.011 |
| \(p\)-value (IH) | 2.58 \times 10^{-3} | 1.99 \times 10^{-4} | 9.69 \times 10^{-6} | 1.94 \times 10^{-8} |
| \(n\sigma\) (IH) | 3.01 | 3.73 | 4.42 | 5.21 |
| \(p\)-value (NH) | 3.07 \times 10^{-3} | 1.95 \times 10^{-4} | 7.55 \times 10^{-6} | 2.49 \times 10^{-8} |
| \(n\sigma\) (NH) | 2.96 | 3.73 | 4.48 | 5.16 |
The JUNO experimental environment

The Problem: The Issues of the Standard Algorithm

The Solution: The Alternative Method

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The results after 6 years of data taking

| 10 reactor cores plus the 2 remote cores | 6 years |
|-----------------------------------------|---------|
|                                         | 4%      | 3.5%    | 3%      | 2.5%    |
| NH true                                 |         |         |         |         |
| $\mu_{NH}$                              | 6650.4 ± 6.8 | 6538.2 ± 6.8 | 6427.2 ± 6.6 | 6292.0 ± 6.5 |
| $\sigma_{NH}$                           | 213.1 ± 4.1  | 212.1 ± 2.2  | 203.8 ± 4.0  | 201.1 ± 2.5  |
| $\mu_{IH}$                              | 7504.1 ± 7.2 | 7600.8 ± 7.1 | 7693.3 ± 7.3 | 7817.2 ± 7.5 |
| $\sigma_{IH}$                           | 224.7 ± 4.3  | 220.4 ± 2.8  | 226.4 ± 4.4  | 232.1 ± 2.9  |
| $r_{NH}$                                | -0.709 ± 0.013 | -0.726 ± 0.010 | -0.736 ± 0.009 | -0.719 ± 0.009 |
| IH true                                 |         |         |         |         |
| $\mu_{IH}$                              | 7583.7 ± 7.1 | 7685.1 ± 7.3 | 7801.1 ± 7.3 | 7964.8 ± 7.2 |
| $\sigma_{IH}$                           | 222.0 ± 2.7  | 226.7 ± 2.7  | 227.3 ± 4.6  | 222.5 ± 4.3  |
| $\mu_{IH}$                              | 6585.6 ± 6.7 | 6472.1 ± 6.5 | 6349.9 ± 6.4 | 6179.9 ± 6.2 |
| $\sigma_{IH}$                           | 208.9 ± 2.5  | 202.8 ± 2.4  | 198.6 ± 4.0  | 193.5 ± 3.8  |
| $r_{IH}$                                | -0.704 ± 0.009 | -0.731 ± 0.008 | -0.713 ± 0.010 | -0.708 ± 0.013 |

$p$-value (IH)

| 4.67 × 10$^{-4}$ | 1.46 × 10$^{-5}$ | 1.36 × 10$^{-7}$ | 4.13 × 10$^{-11}$ |
| 3.50            | 4.34            | 5.27            | 6.50            |

$p$-value (NH)

| 4.27 × 10$^{-4}$ | 1.35 × 10$^{-5}$ | 1.13 × 10$^{-7}$ | 1.96 × 10$^{-11}$ |
| 3.52            | 4.35            | 5.30            | 6.71            |
The results after 6 years of data taking

|                | 4%            | 3.5%           | 3%             | 2.5%           |
|----------------|---------------|----------------|----------------|----------------|
| **NH true**    |               |                |                |                |
| $\mu_{NH}$     | 4956.5 $\pm$ 7.3 | 4815.3 $\pm$ 7.1 | 4667.2 $\pm$ 7.0 | 4486.2 $\pm$ 7.1 |
| $\sigma_{NH}$  | 230.6 $\pm$ 4.3 | 222.3 $\pm$ 4.4 | 227.9 $\pm$ 4.3 | 219.9 $\pm$ 3.1 |
| $\mu_{IH}$     | 5802.5 $\pm$ 7.9 | 5906.1 $\pm$ 7.7 | 6006.1 $\pm$ 7.4 | 6163.2 $\pm$ 8.3 |
| $\sigma_{IH}$  | 248.8 $\pm$ 0.9 | 239.7 $\pm$ 4.8 | 252.0 $\pm$ 4.6 | 255.9 $\pm$ 3.1 |
| $r_{NH}$       | -0.674 $\pm$ 0.012 | -0.684 $\pm$ 0.010 | -0.682 $\pm$ 0.015 | -0.706 $\pm$ 0.009 |
| **IH true**    |               |                |                |                |
| $\mu_{NH}$     | 6001.9 $\pm$ 7.9 | 6110.2 $\pm$ 7.8 | 6242.9 $\pm$ 7.9 | 6440.7 $\pm$ 7.4 |
| $\sigma_{NH}$  | 246.4 $\pm$ 4.8 | 241.9 $\pm$ 5.0 | 248.4 $\pm$ 4.9 | 233.5 $\pm$ 2.9 |
| $\mu_{IH}$     | 4791.1 $\pm$ 7.2 | 4644.1 $\pm$ 7.0 | 4491.3 $\pm$ 7.0 | 4280.0 $\pm$ 6.5 |
| $\sigma_{IH}$  | 226.8 $\pm$ 4.4 | 216.6 $\pm$ 4.5 | 219.2 $\pm$ 4.3 | 205.9 $\pm$ 2.7 |
| $r_{IH}$       | -0.692 $\pm$ 0.014 | -0.673 $\pm$ 0.011 | -0.686 $\pm$ 0.14 | -0.664 $\pm$ 0.010 |
| $p$-value (IH) | $3.48 \times 10^{-4}$ | $3.51 \times 10^{-6}$ | $3.98 \times 10^{-8}$ | $7.96 \times 10^{-11}$ |
| $n \sigma$ (IH)| 3.58            | 4.64            | 5.49            | 6.50            |
| $p$-value (NH) | $3.30 \times 10^{-4}$ | $3.21 \times 10^{-6}$ | $3.12 \times 10^{-8}$ | $1.08 \times 10^{-11}$ |
| $n \sigma$ (NH)| 3.59            | 4.66            | 5.53            | 6.79            |
The Background and systematic studies

Figure: The cosmogenic background distribution due to $^9$Li for a six year long JUNO-like experiment. This is the distribution, conservatively scaled to the total amount of expected incoherent background, used to extract the sensitivity on NH/IH with this kind of background.
\( F_{NH} \) vs \( F_{IH} \) distributions when all the parameters but \( \Delta m^2_{atm} \) are letting free within their uncertainties. The two populations correspond to the NH (bottom region) and the IH (top region) generations. There is no observed change on the dispersions nor in the relative distance. Then, the result is that the separation between two \( F_{IH} \) and \( F_{NH} \) remains constant.
The Flux Effect
The Bump Effect

The Ph.D project report about the issues of the standard algorithm and the alternative method that manifests itself as a solution.
The Baseline Effect

\[ \vec{F}_{MO} \text{ vs } \Delta m_{atm}^2 \] for a couple of generated JUNO-like toy experiments. The blue (red) curves correspond to the NH (IH). Different baselines have been considered increasingly from 10-90 KM.
The Baseline Effect

$F_{MO}$ vs $\Delta m^2_{atm}$ for a couple of generated JUNO-like toy experiments, in the IH hypothesis. The black (red) curves correspond to the analysis when the true (false) hypothesis is taken. Different baselines have been considered, as