Galactic halos of self-interacting dark matter

Steen Hannestad

Institute of Physics and Astronomy, University of Aarhus, DK-8000 Århus C, Denmark

(October 7, 2018)

Recent, very accurate simulations of galaxy formation have revealed that the standard cold dark matter model has great difficulty in explaining the detailed structure of galaxies. One of the major problems is that galactic halos are too centrally concentrated. Dark matter self-interactions have been proposed as a possible means of resolving this inconsistency. Here, we investigate quantitatively the effect of dark matter self interactions on formation of galactic halos. Our numerical framework is extremely simple, while still keeping the essential physics. We confirm that strongly self-interacting dark matter leads to less centrally concentrated structures. Interestingly, we find that for a range of different interaction strengths, the dark matter halos are unstable to particle ejection on a timescale comparable to the Hubble time.

PACS numbers: 95.35.+d, 98.62.Gq, 14.80.-j

I. INTRODUCTION

The concept of dark matter was originally introduced by Zwicky in 1933 [1] to explain the behavior of individual galaxy clusters. Since then the dark matter has been determined to be essential for explaining a vast number of astronomical observations. The cold dark matter model has, with modifications, been very successful in explaining how structure forms in the universe [2]. However, it was recently realized that the CDM model is apparently unable to explain the dynamics of individual galaxies. Very high resolution simulations have shown that there should be much more sub-structure in the halos of typical galaxies than is observed. Numerical simulations predict that the local group should at present contain about 1000 distinct dark matter halos. Observations yield a number which is a factor of ten lower [3].

Also, lensing measurements of clusters seem to indicate a constant density core, in strong contrast with numerical simulations [4]. Also, the central density profile in galactic halos is predicted to be much steeper than in real galaxies [5]. However, this is not a completely settled issue. For instance Kravtsov et al. [1] find that CDM simulations are consistent with observations.

If the problem with CDM halos persists, then it could possibly be remedied if structure formation is somehow suppressed on small scales. One such possibility is that perhaps the dark matter is not cold, but rather “warm”, i.e. dark matter particles have masses around 1 keV and therefore have significant thermal motion around the time of matter-radiation decoupling [4]. That would suppress structure formation on galactic scales and below. However, warm dark matter may have problems in describing properly the properties of clusters. Another possibility is that the initial power spectrum has a cut-off at some wavelength, corresponding to the substructures in galaxies [6], so that no structure below this scale will grow initially.

A quite different possibility is that the discrepancy has to do with the interaction properties of the dark matter, not just the mass of the individual particles. It was recently proposed by Spergel and Steinhardt [4] that dark matter with strong self-interactions could make the cold dark matter model consistent with observations. Strongly self-interacting dark matter will tend to produce halos with shallower core density profiles, thereby alleviating the problem of the central mass concentration.

Although the simple arguments provided in Ref. [4] are quite convincing, it seems very important to test the implications of self interacting dark matter on a more quantitative basis. That is the purpose of the present paper. With the help of a very simple numerical scheme, we solve the Boltzmann equation describing the phase-space evolution of collisional dark matter and are thus able to calculate final state density and velocity distributions.

Intriguingly, we find that there is a range of parameters for which there exist no stable equilibria, even on a relatively short timescale. In general, however, we are able to confirm the predictions of Ref. [4], namely that strongly self interacting dark matter produces halos with shallower density profiles and thus can remedy the problems that the standard cold dark matter model has in explaining galactic dynamics. It should be noted here that self interacting dark matter has been considered previously, in order to study possible effects on the initial linear power spectrum [6, 7]. However, the type of self interactions investigated in the present paper will have no discernible effects on the initial power spectrum.

II. THE PHYSICS OF SELF-INTERACTING DARK MATTER

In the standard cold dark matter model, the dark matter particles were once in thermal equilibrium in the early universe. As the temperature dropped below their rest mass, however, their abundance was exponentially suppressed. Eventually annihilation reactions were no longer

1
The fundamental equation describing the phase-space evolution of self-interacting dark matter is the Boltzmann equation. We shall in the present paper neglect the Hubble expansion of the universe. In that case, the Boltzmann equation takes on the form

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + v_r \frac{\partial f}{\partial r} + a \frac{\partial f}{\partial v} = \Lambda[f],$$

where the right-hand side is a collision operator describing possible scattering/annihilation reactions.

For normal, collisionless matter, the right-hand side is zero, but in our case it is a phase space integral of the interaction cross section for standard model particles, distributed according to the initial $f$. Then these particles are moved in phase-space according to the Boltzmann equation. Thus, the force on each phase-space “particle” is calculated using the mean gravitational field produced by all particles, $GM(r)/r$. All mean field quantities, like $M(r)$, are calculated on Eulerian grid in phase-space and interpolated using spline interpolation.

The right-hand side depends on the specific scattering process. In the case of two-body scattering, it can be written as

$$\Lambda[f] = \frac{1}{2E} \int d^3p d^3p_2 d^3p_3 d^3p_4 \Lambda(f, f_2, f_3, f_4) \times \sum |M|^2 \delta^4(p_1 + p_2 - p_3 - p_4) (2\pi)^4.$$
velocity \( v_0 = \sqrt{GM_0/R_0} \). The interaction cross section should then be cast in units of \( \sigma_0 = mR_0^2/M_0 \), where \( m \) is the mass of the dark matter particle. Note that for \( \sigma = \sigma_0 \), the mean free path in a typical system should be \( \lambda \simeq R_0 \), so that that \( \sigma_0 \) is a natural dividing line between slow and fast interactions.

In all the simulations we start with a homogeneous sphere of radius \( R_0 \). The velocity dispersion is assumed to be Maxwellian so that

\[
    f \propto \exp\left(-v_r^2/2\sigma_0^2 - j^2/2r^2v_d^2\right).
\]

The velocity dispersion, \( \sigma_d \), is chosen to be \( \sigma_d = 0.1v_0 \). If all particles start at rest, they will pass through the center at the same time, and the simulation becomes unstable \( [24] \). However, letting the particles have a small initial velocity does not alter the final state of the whole system very much. It only results in the collisionless systems being slightly less centrally concentrated.

The system is allowed to evolve until \( t = 30\tau \), in order to test stability of the code.

A. Collisionless systems

The first simulation was done for a normal collisionless dark matter halo. Fig. 1 shows how the mass distribution \( M(r) \) evolves with time. As expected, the system very quickly approaches equilibrium, essentially in just one dynamical time. The core profile of the relaxed system approximately follows a power law, \( \rho \propto r^{-4/3} \). Note that if the initial velocity of all particles was zero, the final system would have a density profile with \( \rho \propto r^{-9/8} \) instead \( [24] \). Thus, our assumption of a small initial velocity leads to a less centrally concentrated halo structure. However, since our purpose is to compare halos of dark matter with different interaction strengths, and not so much to make exact calculations, this assumption does not matter much.

![Figure 1: The mass distribution, \( M(r) \), for the collisionless dark matter collapse at three different times. The solid line is for \( t = \tau \), the dashed for \( t = 10\tau \), and the dot-dashed for \( t = 20\tau \).](image)

B. Collisional systems

Ejection — As described above, \( \sigma = \sigma_0 \) divides the weakly and strongly interacting regimes. For \( \sigma \ll \sigma_0 \), one would expect an evolution very similar to the fully collisionless system, whereas for \( \sigma \gg \sigma_0 \), the system will behave like a collisional gas, i.e. shocks can be produced and single particles will perform random walks in phase space. The halo should therefore behave like an entirely hydrodynamical core surrounded by a collisionless system. However, at \( \sigma \simeq \sigma_0 \) the evolution is less obvious. As also noted in Ref. \( [14] \), most self gravitating systems can eject particles. In normal collisionless systems, this can happen for two reasons, either a particle experiences a single close encounter which leaves it with positive energy \( [25] \) or it suffers many weak collisions, gradually increasing its energy \( [24] \). The first process is usually referred to as ejection, whereas the second is called evaporation. This process makes the entire system unstable in the long term, since the only stable solution is a single pair of particles in a Kepler orbit, with all other particles being at infinity \( [24] \). However, for a normal galaxy of stellar objects, the instability time is vastly larger than the Hubble time, and for a collisionless dark matter halo it is for all practical purposes infinite.

However, for a collisional halo, this need not be the case. Since particle interactions are point-like, a given particle can normally only gain positive energy because of a single scattering event, not because of many weak collisions. We can estimate the instability time in the following way: In a homogeneous mass distribution with a typical velocity \( v_0 = \sqrt{GM_0/R_0} \), a particle passing through the entire system has the scattering probability

\[
    P \simeq 1 - e^{-R_0/\lambda},
\]

where

\[
    \lambda \simeq \frac{mc}{\sigma_0 \sqrt{\rho_0 R_0 G}}.
\]

If the particle scatters, and acquires a positive energy, then it should not scatter again on its way out, if it is to be ejected. Therefore the total probability for the particle to be ejected is roughly

\[
    P_{\text{ejection}} \simeq (1 - e^{-R_0/\lambda})e^{-R_0/\lambda} P(E_{\text{final}} > 0).
\]

\( P(E_{\text{final}} > 0) \) is the probability that the energy after scattering is larger than zero. Normally this probability will be \( P(E_{\text{final}} > 0) \approx 0.1 - 0.2 \), so that it is not negligible. The probability is maximal when \( \lambda \simeq R_0 \), as could be expected. The typical timescale for instability is then roughly
\[ t_{\text{instability}} \simeq \tau / P_{\text{ejection}}, \]  

(11)

From these extremely crude estimates one finds that the minimal possible instability time of the order \( t_{\text{instability}} \simeq \text{few} \times \tau \). Even if interactions are very strong, the system is unstable in the long term, because particle will escape from the system via diffusion [14]. However, the timescale for this diffusion process is much longer than the Hubble time.

**Numerical simulations** — We have performed simulations for different values of \( \sigma / \sigma_0 \), specifically \( \sigma / \sigma_0 = 10^{-3}, 10^{-2}, 10^{-1}, 1, 10, 100, 1000 \). Fig. 2 shows the evolution of \( M(r) \) for \( \sigma / \sigma_0 = 10^{-3}, 1 \) and \( 10^3 \). For \( \sigma / \sigma_0 = 10^{-3} \), it is practically indistinguishable from the collisionless case, as would be expected. For the very strongly interacting case \( \sigma / \sigma_0 = 1000 \), we find a very different behaviour. The system again settles into equilibrium very fast, roughly at \( t \sim \text{few} \times \tau \), but we also see that the core is much less dense because low entropy material is ejected. This is the result predicted by Ref. [14].

The intermediate case, \( \sigma / \sigma_0 = 1 \), never settles into a true long-term equilibrium. It starts out by approaching the collisionless equilibrium, but scattering interaction drives the mass distribution towards the equilibrium distribution for the strongly interacting particles. However, on the same timescale the system loses particles due to the ejection mechanism described above, so that no true equilibrium is ever reached. The timescale for particle loss indeed seems to be \( \sim 10 \times \tau \).

**Observations** — We cannot expect our simulations to fit observational rotation curves because of the approximations we have made, i.e. Hubble expansion was neglected and spherical symmetry was assumed. Nevertheless, it is interesting to compare our results to observations. In Fig. 3 we show the predicted rotation curves at \( t = 15\tau \) for the three different values, \( \sigma = 10^{-3}, 1 \) and \( 10^3 \). The rotation curve for the strongly interacting halo is much less centrally peaked than the collisionless one.

The crosses in Fig. 3 show the measured rotation curve for the typical low surface brightness galaxy UGC128 [9]. The observational rotation curve has an asymptotic circular velocity of \( v = 200 \text{ km/s} \) at \( R = 100 \text{ kpc} \). Taking \( v_0 = 200 \text{ km/s} \) and \( R_0 = 100 \text{ kpc} \) gives a mass of \( M_0 = 1.35 \times 10^{12} M_\odot \) and a dynamical timescale of \( \tau = 4.7 \times 10^8 \text{ y} \). These numbers are typical for such galaxies. Notice that for this choice of mass and radius, \( \sigma_0 = 8.5 \times 10^{-23} \text{ cm}^2 \text{ m}^{-1} \text{ GeV} \), i.e. corresponding to the \( \sigma_0 \) quoted in Eq. (2).

**FIG. 2.** The mass distribution as a function of \( r \) at three different times and three different values of \( \sigma \). The solid lines are for \( t = \tau \), the long-dashed for \( t = 15\tau \), the dashed for \( t = 20\tau \) and the dotted for \( t = 25\tau \).

**FIG. 3.** The rotational velocity at \( t = 20\tau \) for three different values of \( \sigma \). The crosses are the observational rotation curve for the low surface brightness galaxy UGC128 [9].
The rotation curve for the strongly interacting system provides a better fit to the observational rotation curve than that for the collisionless system. However, in all cases, the fits are quite poor. As mentioned, this should not be taken too seriously since our model involves some essential approximations.

V. DISCUSSION

We have performed quantitative calculations of how dark matter halos form in models with self interacting dark matter. For simplicity we assumed spherical symmetry, neglected the Hubble expansion, and used a very simple prescription for the dark matter self-interaction. Our results have essentially confirmed the estimates of Ref. [14], in that dark matter halos with sufficient self interaction will have much shallower density profiles that normal collisionless halos. Thus, they can provide better fits to observational galactic rotation curves, while still being consistent with all other known data.

A very curious feature is that halos with intermediate self interaction ($\lambda \approx R_0$) are unstable on a rather short timescale because they eject particles continuously. This effect was estimated to be unimportant in Ref. [14], but our numerical simulations indicate that there exist regions of parameter space where the instability would probably have shown up in present day halos.

As mentioned, our calculations assume spherical symmetry and neglect the Hubble expansion. It will be very interesting to investigate the effect of self interacting dark matter using detailed N-body calculations [28]. Most likely, such simulations will confirm the general statements made by Spergel and Steinhardt [14] as well as in the present paper.

ACKNOWLEDGMENTS

Support from the Carlsberg foundation is gratefully acknowledged.