Nonleptonic charmless two-body $B \to AT$ decays

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Abstract

In this work we have studied hadronic charmless two-body $B$ decays involving $p$-wave mesons in final state. We have calculated branching ratios of $B \to AT$ decays (where $A$ and $T$ denotes a $3P_1$ axial-vector and a tensor meson, respectively), using $B \to T$ form factors obtained in the covariant light-front (CLF) approach, and the full effective Hamiltonian. We have obtained that $B(B^0 \to a_1^+ a_2^-) = 42.47 \times 10^{-6}$, $B(B^+ \to a_1^+ a_0^0) = 22.71 \times 10^{-6}$, $B(B \to f_1 K^*_2) = (2.8 - 4) \times 10^{-6}$ (with $f_1 = f_1(1285), f_1(1420)$) for $\theta_{3P_1} = 53.2^\circ$, $B(B \to f_1(1420)K^*_2) = (5.91 - 6.42) \times 10^{-6}$ with $\theta_{3P_1} = 27.9^\circ$, $B(B \to K_1 a_2) = (1.7 - 5.7)[1 - 9.3] \times 10^{-6}$ for $\theta_{K_1} = -37^\circ [-58^\circ]$ where $K_1 = K_1(1270), K_1(1400)$. It seems that these decays can be measured in experiments at $B$ factories. Additionally, we have found that $B(B \to K_1(1270) a_2)/B(B \to K_1(1400) a_2)$ and $B(B \to f_1(1420) K^*_2)/B(B \to f_1(1285) K^*_2)$ ratios could be useful to determine numerical values of mixing angles $\theta_{K_1}$ and $\theta_{3P_1}$, respectively.

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I. INTRODUCTION

Weak nonleptonic two-body $B$ decays is a good scenario to understand the interplay of short- and long-distance QCD effects, to investigate about CP violation, to test some QCD-motivated theories such as QCD factorization, perturbative QCD and soft-collinear effective theory, to study physics beyond the Standard Model (SM), and constraint numerical values of the Cabibbo-Kobayashi-Maskawa (CKM) parameters.

Hadronic charmless two-body $B \rightarrow M_1 M_2$ decays, where $M_{1,2}$ can be a $l = 0$ or a $l = 1$ meson, have been broadly considered in the literature (for a recent review see Ref. [1]). There are comprehensive and systematic articles about meson, respectively) decays. In this work, we have studied nonleptonic charmless two-body decays. In this work, we have studied nonleptonic charmless two-body decays considering that both mesons in final state are orbitally excited $l = 1$ mesons (or $p$-wave mesons). Specifically, we have worked with $B \rightarrow AT$ decays, which could compete with $B \rightarrow VT$ modes and their branching fractions could be measured in the future LHCb experiment in the Large Hadron Collider (LHC) and at $B$-factories. Moreover, $B \rightarrow AT$ decays can also offer a good place to study polarization in a similar way to the $B \rightarrow VT$ scenario [13].

At experimental level, there are some recent measurements about the production of tensor or axial-vector mesons in $B$ decays. Recently, BABAR Collaboration reported branching fractions of nonleptonic charmless two-body $B$ decays involving tensor mesons in final state [14]. On the other hand, hadronic charmless $B^0 \rightarrow a_1(1260)\pi^\pm \pi^\mp$ decays were the first modes with axial-vector mesons in final state, measured by both $B$ factories, BABAR [15] and Belle [16]. Additionally, BABAR Collaboration reported the observation of other decays with axial-vector mesons in final state [17]. In general, these modes have branchings of the order of $10^{-6}$. So far, there is not experimental information about $B \rightarrow AT$ decays.

In this work we extend the knowledge of weak nonleptonic two-body $B$ decays, considering that both mesons in final state are $p$-wave. In principle, we have six possibilities for considering two orbitally excited (or $p$-wave) mesons in final state: $B \rightarrow S(S, A, T)$, $B \rightarrow A(A, T)$ and $B \rightarrow TT$. In this paper, we have computed branching ratios of exclusive charmless $B \rightarrow AT$ (where $A$ is a $3P_1$ axial-vector meson) decays assuming generalized factorization, considering the effective weak Hamiltonian $H_{eff}$ and taking $B \rightarrow T$ form factors from the covariant-light front (CLF) approach [18], which is one of the few models that provides the evaluation of the hadronic matrix element $\langle T | J_\mu | B \rangle$.

This paper is organized as follows: in Sec. II, we discuss briefly about the effective weak Hamiltonian, factorization scheme and $B \rightarrow T$ form factors in the CLF approach. Sec. III is dedicated to describe input parameters. In Sec. IV, we present our numerical results for branching fractions and conclusions are given in Sec. V. In Appendix, we display explicitly decay amplitudes for charmless $B \rightarrow A(1260)T$ modes.

II. THEORETICAL FRAMEWORK

A. The weak effective Hamiltonian and factorization approach

The weak effective Hamiltonian $H_{eff}$ for nonleptonic charmless two-body $B$ decays is [19]:

$$H_{eff}(\Delta B = 1) = \frac{G_F}{\sqrt{2}} \left[ V_{ub} V_{qq}^* \left( C_1(\mu)O_1^\mu(\mu) + C_2(\mu)O_2^\mu(\mu) \right) + V_{cb} V_{cq}^* \left( C_1(\mu)O_1^\mu(\mu) + C_2(\mu)O_2^\mu(\mu) \right) - V_{ub} V_{tq}^* \left( \sum_{i=3}^{10} C_i(\mu)O_i(\mu) \right) \right] + h.c.,$$

(1)

where $G_F$ denotes the Fermi constant, $C_i(\mu)$ are Wilson coefficients evaluated at renormalization scale $\mu$ and coefficients $V_{mn}$ are CKM matrix elements related to the transition. Local operators $O_i$ are given by
current-current (tree) operators

\begin{align}
O_1^c &= (\bar{q}_\alpha u_\alpha)_{V-A} \cdot (\bar{u}_\beta b_\beta)_{V-A} \\
O_2^c &= (\bar{q}_\alpha u_\beta)_{V-A} \cdot (\bar{u}_\beta b_\alpha)_{V-A} \\
O_3^c &= (\bar{q}_\alpha c_\alpha)_{V-A} \cdot (\bar{c}_\beta b_\beta)_{V-A} \\
O_4^c &= (\bar{q}_\alpha c_\beta)_{V-A} \cdot (\bar{c}_\beta b_\alpha)_{V-A}
\end{align}

\(\text{QCD penguin operators}\)

\begin{align}
O_3^{(5)} &= (\bar{q}_\alpha b_\alpha)_{V-A} \cdot \sum_q (\bar{q}'_\beta q'_\beta)_{V-A}(V+A) \\
O_4^{(6)} &= (\bar{q}_\alpha b_\beta)_{V-A} \cdot \sum_q (\bar{q}'_\beta q_\alpha)_{V-A}(V+A)
\end{align}

\(\text{electroweak penguin operators}\)

\begin{align}
O_7^{(9)} &= \frac{3}{2}(\bar{q}_\alpha b_\alpha)_{V-A} \cdot \sum_q c_{q'}(\bar{q}'_\beta q'_\beta)_{V+A}(V-A) \\
O_8^{(10)} &= \frac{3}{2}(\bar{q}_\alpha b_\beta)_{V-A} \cdot \sum_q c_{q'}(\bar{q}'_\beta q_\alpha)_{V+A}(V-A)
\end{align}

where \((\bar{q}_1 q_2)_{V=\pm A} \equiv \bar{q}_1 \gamma_\mu (1 \mp \gamma_5) q_2\), and symbols \(\alpha\) and \(\beta\) are \(SU(3)\) color indices. The sums run over active quarks at the scale \(\mu = \mathcal{O}(m_b)\), i.e. \(q'\) runs with quarks \(u, d, s\) and \(c\).

In order to obtain branching ratios of nonleptonic two-body \(B \to M_1 M_2\) decays it is necessary to evaluate the hadronic matrix element involving four-quark operators \(\langle M_1 M_2 | O_i | B \rangle\). In the framework of factorization approach, it can be approximated by the product of two matrix elements of single currents: \(\langle M_1 | J_{(1)} \mu | 0 \rangle \langle M_2 | J_{(2)} \mu | B \rangle\) or \(\langle M_2 | (J_1) \mu | 0 \rangle \langle M_1 | (J_2) \mu | B \rangle\), where \(J_\mu\) is a bilinear current. Thus, the matrix element of a four-quark operator is expressed as the product of decay constant and form factors. The hadronic matrix element is renormalization scheme and scale independent \(^\text{[20]}\) while Wilson Coefficients are renormalization scheme and scale dependent. So, the decay amplitude under this naive factorization is not truly physical.

The improved generalized factorization solves the aforementioned scale problem. For example, in Refs. \(^\text{[2, 3]}\) it is considered a method to extract the \(\mu\) dependence from the matrix element \(\langle O_i | \mu \rangle\) and combine it with the \(\mu\)-dependent Wilson coefficients \(C_i(\mu)\) to form \(\mu\)-independent effective Wilson coefficients \(c_i^{\text{eff}}\). We have taken the respective numerical values for \(c_i^{\text{eff}}\) reported in Table I of Ref. \(^\text{[6]}\). They were calculated in next to leading order Wilson coefficients for \(\Delta B = 1\) transitions obtained in the naive dimensional regularization scheme.

Effective Wilson coefficients \(c_i^{\text{eff}}\) appear in factorizable decay amplitudes as linear combinations. It allows to define effective coefficients \(a_i\), which are renormalization scale and scheme independent. \(a_i\)'s are defined as:

\begin{align}
a_i &= c_i^{\text{eff}} + \frac{1}{N_c} c_i^{\text{eff+1}} \text{ (} i \text{ odd)} \\
a_i &= c_i^{\text{eff}} + \frac{1}{N_c} c_i^{\text{eff-1}} \text{ (} i \text{ even)}
\end{align}

where the index \(i\) runs over \((1, ..., 10)\) and \(N_c = 3\) is the number of colors. Phenomenologically, nonfactorizable contributions to the hadronic matrix element are modeled by treating \(N_c\) as a free parameter and its value can be extracted from experiment. In this work we have used numerical values for \(a_i\) coefficients for \(b \to d\) and \(b \to s\) transitions displayed in Table II of Ref. \(^\text{[6]}\).
B. Form Factors in the CLF approach

In the framework of generalized factorization the hadronic matrix element \( \langle AT|O_1|B \rangle \) is approximated by \( \langle A|(J_1)_\mu|0\rangle\langle T|(J_2)_\mu|B \rangle \). So, we need to compute the hadronic matrix element \( \langle T|J_\mu|B \rangle \) in order to obtain numerical values for branching ratios of \( B \to AT \) decays. We have used the parametrization given in Ref. [21]:

\[
\begin{align*}
\langle T|V_\mu|B \rangle &= i\hbar(q^2)\epsilon^{\mu\rho\sigma\alpha}p_B^\rho(p_B + p_T)_\sigma
\langle p_B - p_T|, \\
\langle T|A_\mu|B \rangle &= k(q^2)\epsilon^{\mu\nu}(p_B)_\nu + \epsilon^{\alpha\beta}p_B^\alpha p_B^\beta
\times [b_+(q^2)(p_B + p_T)^\mu + b_-(q^2)(p_B - p_T)^\mu],
\end{align*}
\]

where \( V_\mu \) and \( A_\mu \) denote the vector and the axial-vector current, respectively; \( \epsilon_{\mu\nu} \) is the polarization of tensor meson, \( p_B \) and \( p_T \) are the momentum of \( B \) and \( T \) mesons, respectively, and \( h, k, b_\pm \) are form factors for the \( B \to T \) transition.

So far, only two models\(^3\) provide a systematical estimate of \( B \to T \) form factors: the ISGW model [21] with its improved version [23] and the CLF quark model [24]. Keeping in mind that the improved ISGW2 model [23] has difficulties in the low-\( q^2 \) region, in particular, at the maximum \( q^2 = 0 \) recoil point where the final-state meson could be highly relativistic, we have used numerical values for form factors \( h, k, b_\pm \) obtained in the CLF quark model [18]. This reference extended the covariant analysis of the light-front approach [24] to even-parity, \( p \) wave mesons.

A light-front quark model (LFQM) provides a relativistic study of the movement of the hadron and also gives a fully description of the hadron spin. The light-front wave functions do not depend on the hadron momentum and are explicitly Lorentz invariant. In the CLF quark model, the spurious contribution, which is dependent on the orientation of the light-front, is cancelled by inclusion of the zero mode contribution, and becomes irrelevant in decay constants and form factors, so that the result is guaranteed to be covariant and more self consistent.

This model has been used by different authors in the last five years, obtaining, in some cases, predictions that are favorable with available experimental information. For example, some authors worked with semileptonic decays of \( B_c \) meson including \( s \)- and \( p \)-wave mesons in final state [25] and nonleptonic \( B_c^- \to X(3872)\pi^-(K^-) \) modes [26]. Others, studied two-photon annihilation \( P \to \gamma\gamma \) and magnetic dipole transition \( V \to P\gamma \) processes for the ground-state heavy quarkonium within the CLF approach [27], and radiative \( B \to (K^*, K_1, K_2^*)\gamma \) channels in the same framework [28]. Ref. [29] examined \( B \to (K^*_1(1430), K^*_2(1430))\phi \) in the LFQM. Recently, we computed branching ratios of hadronic charmless \( B \to P(V)T \) decays in the CLF approach [11].

In the CLF approach, form factors are explicit functions of \( q^2 \) in the space-like region and then analytically extend them to the time-like region in order to compute physical form factors at \( q^2 \geq 0 \). They are parametrized and reproduced in the three-parameter form [18]:

\[
F(q^2) = \frac{F(0)}{1 - aX + bX^2}, \tag{7}
\]

with \( X = q^2/m_B^2 \). In Tables VI and VII of Ref. [18] it is displayed the parameters \( a, b \) and \( F(0) \) (form factor at the zero momentum transfer) for \( B \to a_2(1320) \) and \( B \to K_2^*(1430) \) transitions, which are \( B \to T \) transitions required in this work. In Table I, we have summarized these numerical values.

Table I. Form factors for \( B \to a_2(1320) \) and \( B \to K_2^*(1430) \) transitions obtained in the CLF model [18] are fitted to the 3-parameter form in Eq.(7). \( k \) is dimensionless and \( h, b_+, b_- \) have dimensions of \( \text{GeV}^{-2} \).

\(^3\) Recently, Ref. [22] calculated \( B \to K_2^* \) form factors using large energy effective theory (LEET) techniques.
[In terms of the spectroscopic notation distinctive mixing angles between $K$ or $f$ terms of the respective decay constants 1 words, for the $\langle B$ scalar mesons is not easy experimentally and the underlying structure is not well understood at theoretical level [1].

In this section we present numerical inputs that are necessary to obtain our predictions. We also discuss about the situation with hadronic $B$ decays such as $B \rightarrow J^{(+)}P^{(-)}$ processes if we compare them with those for charmless two-body $B$ decays such as $B \rightarrow PP, PV$, and $VV$ [2] and $B \rightarrow AP, AV$, and $AA$ [3, 6].

The matrix element of the current between the vacuum and final $^3P^1$ axial-vector meson ($A$) can be expressed in terms of the respective decay constants $f_A$, in the form

$$\langle A(p_A, \epsilon)|A_\mu(0)\rangle = f_A m_A \epsilon_\mu,$$  \hspace{1cm} (8)

where $\epsilon_\mu$ is the vector polarization of the $^3P^1$ axial-vector meson. On the other hand, the polarization $\epsilon_{\mu\nu}$ of the $^3P^2$ tensor meson satisfies the relations

$$\epsilon_{\mu\nu} = \epsilon_{\nu\mu}, \hspace{1cm} \epsilon_\mu^\mu = 0, \hspace{1cm} p_\mu \epsilon^{\mu\nu} = p_\nu \epsilon^{\mu\nu} = 0.$$  \hspace{1cm} (9)

Therefore,

$$\langle 0|(V-A)_\mu(T) = a\epsilon_{\alpha\mu}\epsilon_\nu^\nu + b\epsilon_\nu^\nu p_\mu = 0,$$  \hspace{1cm} (10)

and hence the decay constant of the tensor meson vanishes, i.e., the tensor meson can not be produced from the vacuum and we can not approximate the hadronic matrix element $\langle AT|O_1|B\rangle$ by $\langle T|(J_1)_\mu(0)\langle A|(J_2)_\mu(0)|B\rangle$. This fact simplifies considerably decay amplitudes for $B \rightarrow A(^3P^1)T$ processes if we compare them with those for charmless two-body $B$ decays such as $B \rightarrow PP, PV$, and $VV$ [2, 3] and $B \rightarrow AP, AV$, and $AA$ [3, 6].

On the other hand, we do not have considered $B \rightarrow MT$ decays where $M$ can be a $^1P^1$ axial-vector or a scalar meson. If we assume factorization hypothesis, the amplitude decay of these modes is $\langle M|(J_1)_\mu(0)\langle J_2|T_\mu(0)|B\rangle$. There is not a contribution of the form $\langle T|(J_1)_\mu(0)\langle J_2|T_\mu(0)|B\rangle$ because $\langle T|J_\mu(0)\rangle$ is zero. Thus, $B \rightarrow MT$ decays, in general, imply the evaluation of $\langle M|J_\mu(0)\rangle$.

$^1P^1$ axial-vector mesons with $J^{PC} = J^{+-}$ ($b_1$ and $h_1$) have even $G$-parity and the axial current which produces a $b_1$ or a $h_1$ meson has odd $G$-parity. So, $\langle 0|\bar{u}\gamma_\mu\gamma_5d|h_1(b_1)\rangle = 0$ by $G$-parity conservation and hence $f_{h_1} = f_{b_1} = 0$. In other words, for the $^1P^1$ axial-vector meson its decay constant is small, vanishing in the SU(3) limit. So, we do not consider in this work $B \rightarrow A(^1P^1)T$ decays because their factorizable amplitude is proportional to decay constant $f_{A(^1P^1)}$.

The situation with hadronic $B \rightarrow ST$ decays is similar. The vector decay constant of scalar mesons, defined as $\langle S(p)|q_i^\gamma\gamma_\mu q_j(0)\rangle = f_{Sp_\mu}$, is either zero or small (of order of $m_d - m_u, m_s - m_d,u$). Moreover, the identification of light scalar mesons is not easy experimentally and the underlying structure is not well understood at theoretical level [1]. For these reasons, we have not studied neither $B \rightarrow ST$ decays in this work.

### III. Input Parameters

In this section we present numerical inputs that are necessary to obtain our predictions. We also discuss about mixing angles between $K_{1A}$ and $K_{1B}$ mesons and $^3P^1$ states $f_1(1285)$ and $f_1(1420)$.

In the quark model, there are two nonets of $J^P = 1^+$ axial-vector mesons as the orbital excitation of the $q\bar{q}$ system. In terms of the spectroscopic notation $^{2S+1}L_J$, these two types of axial-vector mesons are $^3P^1$ and $^1P^1$. They have distinctive $C$ quantum numbers, $C = +$ and $C = -$, respectively. Experimentally, the $J^{PC} = 1^{++}$ nonet consists of $a_1(1260)$, $f_1(1285)$, $f_1(1420)$, and $K_{1A}$, while the $1^{+-}$ nonet is conformed by $b_1(1235)$, $h_1(1170)$, $h_1(1380)$, and $K_{1B}$ [30]. However, the physical strange axial-vector mesons $K_1(1270)$ and $K_1(1400)$ are a mixture of $K_{1A}$ and $K_{1B}$:

$$\begin{align*}
K_1(1270) &= K_{1A} \sin \theta_{K_1} + K_{1B} \cos \theta_{K_1}, \\
K_1(1400) &= K_{1A} \cos \theta_{K_1} - K_{1B} \sin \theta_{K_1},
\end{align*}$$  \hspace{1cm} (11)
where $\theta_{K_1}$ is the $K_{1A} - K_{1B}$ mixing angle.

We used two different set of mixing angle predictions given in Ref. [9]: $\theta_{K_1} = -37^\circ, -58^\circ$, where $\theta_{K_1}$ is favored to be negative as implied by the experimental measurement of the $B(B \to K_1(1270)\gamma)/B(B \to K_1(1400)\gamma)$ ratio in $B$ decays. In Table II, we present numerical values of decay constants depending of the value of $\theta_{K_1}$. Additionally, Ref. [31] predicted that the mixing angle $\theta_{K_1}$ must be negative, $\theta_{K_1} = -34^\circ$ and obtained $f_{K_1(1270)}$ and $f_{K_1(1400)}$ (see Table II), from the combining analysis for $B \to K_1\gamma$ and $\tau^- \to K_1(1270)^-\nu_\tau$ decays. In this work, the $K_{1A} - K_{1B}$ mixing is introduced through decay constants.

Table II. Numerical values (in MeV) of decay constant $f_{K_1}$.

| $\theta_{K_1}$ | $f_{K_1(1270)}$ | $f_{K_1(1400)}$ |
|----------------|----------------|----------------|
| $-37^\circ$    | $(-184 \pm 11)$ | $(177 \pm 12)$ |
| $-58^\circ$    | $(-234 \pm 11)$ | $(100 \pm 12)$ |
| $-34^\circ$    | $(-169 \pm 25)$ | $(179 \pm 13)$ |

Analogous to the $\eta - \eta'$ mixing in the pseudoscalar nonet, $3P_1$ states $f_1(1285)$ and $f_1(1420)$ are mixed in terms of the pure octet $|f_8\rangle$ and singlet $|f_1\rangle$ due to $SU(3)$ breaking effects, and can be parameterized as [8, 32]:

$$|f_1(1285)\rangle = |f_1\rangle \cos \theta_{f_1} + |f_8\rangle \sin \theta_{f_1},$$

$$|f_1(1420)\rangle = -|f_1\rangle \sin \theta_{f_1} + |f_8\rangle \cos \theta_{f_1}.$$  \hspace{1cm} (12)

Decay constants $f_{f_1(1285)}^q$ and $f_{f_1(1420)}^q$ are defined by

$$\langle 0|q\mu \gamma_5 q|f_1(1285)\rangle = -im_{f_1(1285)} f_{f_1(1285)}^q \epsilon_\mu,$$

$$\langle 0|q\mu \gamma_5 q|f_1(1420)\rangle = -im_{f_1(1420)} f_{f_1(1420)}^q \epsilon_\mu.$$  \hspace{1cm} (13)

Thus, it is obtained

$$f_{f_1(1285)}^q = \frac{f_{f_1}}{\sqrt{3} m_{f_1(1285)}} \cos \theta_{f_1} + \frac{f_{f_8}}{\sqrt{6} m_{f_1(1285)}} \sin \theta_{f_1},$$

$$f_{f_1(1420)}^q = -\frac{f_{f_1}}{\sqrt{3} m_{f_1(1420)}} \sin \theta_{f_1} + \frac{f_{f_8}}{\sqrt{6} m_{f_1(1420)}} \cos \theta_{f_1},$$  \hspace{1cm} (14)

and

$$f_{f_1(1285)}^s = \frac{f_{f_1}}{\sqrt{3} m_{f_1(1285)}} \cos \theta_{f_1} - \frac{2 f_{f_8}}{\sqrt{6} m_{f_1(1285)}} \sin \theta_{f_1},$$

$$f_{f_1(1420)}^s = -\frac{f_{f_1}}{\sqrt{3} m_{f_1(1420)}} \sin \theta_{f_1} - \frac{2 f_{f_8}}{\sqrt{6} m_{f_1(1420)}} \cos \theta_{f_1}.$$  \hspace{1cm} (15)

The mixing angle $\theta_{P_{f_1}}$ has been calculated theoretically in some references (see for example [8, 32]). The Ref. [32] found that this mixing angle has two values: $\theta_{P_{f_1}} = 38^\circ, 50^\circ$. The previous phenomenological analysis did in Ref. [33] suggests that $\theta_{P_{f_1}} \simeq 50^\circ$.

In this work we use the predictions of Ref. [9] for decay constants of $f_1(1285)$ and $f_1(1420)$ mesons (see Table III). These values were calculated using the Gell-Mann-Okubo mass formula and the value $\theta_{K_1} = -37^\circ \pm 58^\circ$ for the $K_{1A} - K_{1B}$ mixing angle. It was found that the mixing angle for $3P_1$ states is $\theta_{P_{f_1}} = 27.9^\circ \pm 53.2^\circ$. So, we can see that the mixing angle $\theta_{P_{f_1}}$ depends on the angle $\theta_{K_1}$. If the mixing were ideal, the $f_1(1285)$ meson will be made up of $(u\bar{u} + d\bar{d})/2$ while $f_1(1420)$ is composed of $s\bar{s}$.

On the other hand, for the $a_1(1260)$ decay constant we have taken $f_{a_1} = 238$ MeV obtained using the QCD sum rule method [32].
Table III. Decay constants $f_{1/1}^f(1285)$ and $f_{1/1}^f(1420)$

| MeV | 172 | 178 |
|-----|-----|-----|
| $f_{1/1}^f(1285)$ | -55 | 23 |
| $f_{1/1}^f(1420)$ | -72 | 29 |
| $f_{1/1}^f(1420)$ | -219 | -230 |

We use the parametrization of the CKM matrix in terms of Wolfenstein parameters $\lambda$, $A$, $\bar{\rho}$ and $\bar{\eta}$

$$V_{\text{CKM}} = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4),$$

with $\rho = \bar{\rho}(1 - \lambda^2/2)^{-1}$ and $\eta = \bar{\eta}(1 - \lambda^2/2)^{-1}$. We take central values from the global fit for Wolfenstein parameters: $\lambda = 0.2257$, $A = 0.814$, $\bar{\rho} = 0.135$ and $\bar{\eta} = 0.349$.

Masses and average lifetimes of neutral and charged $B$ mesons were taken from [30]. The running quark masses are given at the scale $\mu \approx m_b$, since the energy released in $B$ decays is of the order of $m_b$. We use $m_u(m_b) = 3.2$ MeV, $m_d(m_b) = 6.4$ MeV, $m_s(m_b) = 127$ MeV, $m_c(m_b) = 0.95$ GeV and $m_b(m_b) = 4.34$ GeV (see Ref. [35]).

IV. NUMERICAL RESULTS AND DISCUSSION

In this section, we present our numerical values for branching ratios of nonleptonic charmless $B \to A(3P_1)/T$ decays, using $B \to T$ form factors obtained in the CLF approach [18]. Also, we establish a comparison between $B \to AT$ and $B \to VT$ modes.

The decay width for the $B \to AT$ process is given by

$$\Gamma(B \to AT) = \frac{G_F^2}{32\pi m_B^3} f_A^2 |V_{ub}^* V_{ud} |^2 |a_{1.2} - V_{tb}^* V_{td} | ^2 \left( \alpha(m_A^2)\lambda^{7/2} + \beta(m_A^2)\lambda^{5/2} + \gamma(m_A^2)\lambda^{3/2} \right),$$

where we have summed over polarizations of the tensor meson $T$. The $\Phi_{\text{penguin}}$ factor is a linear combination of penguin coefficients $a_3,...,a_{10}$, $\lambda \equiv \lambda(m_B^2, m_T^2, m_A^2) = (m_B^2 + m_T^2 - m_A^2)^2 - 4m_B^2 m_T^2$ is the triangle function, and $\alpha$, $\beta$ and $\gamma$ are quadratic functions of form factors $k$, $b_+$ and $h$, evaluated at $q^2 = m_A^2$. They are expressed by

$$\alpha(m_A^2) = \frac{b_+^2}{24m_T^2},$$

$$\beta(m_A^2) = \frac{1}{24m_T^2} \left[ k^2 + 6m_T^2 m_A^2 h^2 + 2(m_B^2 - m_T^2 - m_A^2)kb_+ \right],$$

$$\gamma(m_A^2) = \frac{5m_T^2 k^2}{12m_T^2}.$$

The ratio between decay widths of $B \to AT$ (see Eq. (17)) and $B \to VT$ (see Ref. [36]) channels, where $A$ and $V$ mesons have the same quark content, is
\[ \mathcal{R}_{AT/VT} = \frac{\Gamma(B \to AT)}{\Gamma(B \to VT)} = \left( \frac{f_A}{f_V} \right)^2 \left[ \frac{Z^{QCD}_A}{Z^{QCD}_V} \right]^2 \frac{\alpha(m_B^2) \lambda_{1/2} + \beta(m_B^2) \lambda_{3/2} + \gamma(m_B^2) \lambda_{5/2}}{\alpha(m_B^2) \lambda_{1/2} + \beta(m_B^2) \lambda_{3/2} + \gamma(m_B^2) \lambda_{5/2}} \right] \]

where \( \lambda_{A(V)} = \lambda(m_B^2, m_T^2, m_{A(V)}^2) \) and \( Z^{QCD}_{A(V)} \) is a sum of products of CKM elements with QCD coefficients \( a_i \) \((i = 1, \ldots, 10)\). We can see that \( \mathcal{R}_{AT/VT} \) is conformed by the product of three terms: the first one is the ratio between decay constants \( f_A \) and \( f_V \); the second factor corresponds to the ratio between QCD contributions; and the third term comes from form factors and kinematical \( \lambda_{A(V)} \) function. This ratio can be considered as a test of the factorization approximation. If \( a_i \) coefficients and form factors are known, decay constants can be determined from \( \mathcal{R}_{AT/VT} \). On the other hand, \( \mathcal{R}_{AT/VT} \) is a test of form factors if decay constants and \( a_i \) coefficients are known.

The QCD contributions for \( B^{+,0} \to a_2^{+,0} \) and \( B^{+,0} \to \rho^+ a_2^{0,-} \) modes are the same, i.e., the ratio \( Z^{QCD}_A/Z^{QCD}_V = 1 \). A similar situation occurs with decays \( B^{+,0} \to K_1^0 K_2^{+,0} \) and \( B^{+,0} \to \bar{K}^{0} K_2^{*,+,0} \), and \( B \to K_1 a_2 \) and \( B \to K^* a_2 \). In these cases, \( \mathcal{R}_{AT/VT} \) gives a better test of factorization scheme. If decay constants are known \( \mathcal{R}_{AT/VT} \) can give a test of form factors.

For obtaining branching ratios of exclusive charmless \( B \to A(3P)T \) decays, we have taken expressions for decay amplitudes given in Appendix. These expressions include all contributions of \( H_{eff} \). Our numerical results are listed in Tables IV and V. Specifically, branching fractions of \( B \to K_1 T \) (with \( K_1 = K_1(1270), K_1(1400) \)) modes are shown in Table IV, and predictions for branchings of \( B \to A(3P)T \), where \( A \) is a nonstrange axial-vector meson, are collected in Table V.

| Process          | \(-37^\circ\) | \(-58^\circ\) |
|------------------|---------------|---------------|
| \( B^+ \to K_1^{(1270)} a_2^+ \) | 1.88          | 3.04          |
| \( B^+ \to K_1^{(1400)} a_2^+ \) | 1.70          | 5.43          |
| \( B^0 \to K_1^{(1270)} a_2^- \) | 3.51          | 5.68          |
| \( B^0 \to K_1^{(1400)} a_2^- \) | 3.18          | 1.02          |

| Penguin process  | \(-37^\circ\) | \(-58^\circ\) |
|------------------|---------------|---------------|
| \( B^+ \to K_1^{(1270)} a_2^+ \) | 0.37          | 0.60          |
| \( B^+ \to K_1^{(1400)} a_2^+ \) | 0.33          | 0.11          |
| \( B^+ \to K_1^{(1270)} a_2^+ \) | 5.77          | 9.34          |
| \( B^+ \to K_1^{(1400)} a_2^+ \) | 5.24          | 1.67          |
| \( B_0 \to K_1^{(1270)} a_2^+ \) | 0.34          | 0.55          |
| \( B_0 \to K_1^{(1400)} a_2^+ \) | 0.30          | 0.09          |
| \( B_0 \to K_1^{(1270)} a_2^0 \) | 2.70          | 4.38          |
| \( B_0 \to K_1^{(1400)} a_2^0 \) | 2.48          | 0.78          |

Branching ratios of \( B^0 \to a_2^+ a_2^- \) and \( B^+ \to a_1^+ a_2^0 \) modes are the biggest. They are \( 42.47 \times 10^{-6} \) and \( 22.71 \times 10^{-6} \), respectively (see Table V). These decays receive contributions of the \( a_1 \) coefficient and the linear combination \( a_4 + a_10 \) (see Appendix). \( B^+ \to K_1^+ a_2^0 \) and \( B^0 \to K_1^+ a_2^0 \) modes, with \( K_1 = K_1(1270), K_1(1400) \), also have sizable branching ratios of \( (1.7 - 3.5) \times 10^{-6} \) and \( (1 - 5.6) \times 10^{-6} \) with \( \theta_{K_1} = -37^\circ \) and \( \theta_{K_1} = -58^\circ \), and receive contribution of same QCD coefficients with different CKM matrix elements, respectively (see Table IV). Another feature is that branching fractions of \( B \to K_1(1270) a_2 \) and \( B \to K_1(1400) a_2 \) are almost equal for \( \theta_{K_1} = -37^\circ \) while are different for \( \theta_{K_1} = -58^\circ \). Thus, the measurement of the ratio \( B(B \to K_1(1270) a_2)/B(B \to K_1(1400) a_2) \) will be a test of the value of the mixing angle \( \theta_{K_1} \).

For color-suppressed decays, \( B \to f_1 K_2^\ast \) modes, with \( f_1 = f_1(1285), f_1(1420) \), have the biggest branching ratios (see Table V). They are \( (2.8 - 4) \times 10^{-6} \) with the mixing angle \( \theta_{f_1} = 53.2^\circ \). On the other hand, if \( \theta_{f_1} = 27.9^\circ \),
Table V. Branching ratios (in units of $10^{-6}$) for charmless $B \to A(3P_i)^T$ decays, where $A$ is a nonstrange axial-vector meson. Processes including $f_1(1285)$ and $f_1(1420)$ mesons are considered with two different values of mixing angle $\theta_{s,P_1} = 53.2^\circ$ [27.9$^\circ$].

| Process                             | $B$       |
|-------------------------------------|-----------|
| $B^+ \to a_1^+ a_2^0$              | 22.71     |
| $B^+ \to a_1^0 a_2^+$               | 0.085     |
| $B^+ \to f_1(1285) a_2^0$          | 0.17 [0.12] |
| $B^+ \to f_1(1420) a_2^0$          | 0.02 [0.06] |
| $B^+ \to a_1^0 K_2^{*+}$            | 0.77      |
| $B^+ \to f_1(1285) K_2^{*+}$        | 3.12 [0.61] |
| $B^+ \to f_1(1420) K_2^{*+}$        | 4.02 [6.42] |
| $B^0 \to a_1^+ a_2^0$              | 42.47     |
| $B^0 \to a_1^0 a_2^+$               | 0.04      |
| $B^0 \to f_1(1285) a_2^0$          | 0.08 [0.06] |
| $B^0 \to f_1(1420) a_2^0$          | 0.009 [0.03] |
| $B^0 \to a_1^0 K_2^0$               | 0.71      |
| $B^0 \to f_1(1285) K_2^{*0}$        | 2.87 [0.56] |
| $B^0 \to f_1(1420) K_2^{*0}$        | 3.70 [5.91] |

only $B \to f_1(1420) K_2^{*0}$ decays have branching ratios of $10^{-6}$. We can see that $B(B \to f_1(1420) K_2^{*0}) \approx (1.3)B(B \to f_1(1285) K_2^{*0})$ and $B(B \to f_1(1420) K_2^{*0}) \approx (10.5)B(B \to f_1(1285) K_2^{*0})$ with $\theta_{s,P_1} = 53.2^\circ$ and $\theta_{s,P_1} = 27.9^\circ$, respectively. Thus, the measurement of the ratio $B(B \to f_1(1420) K_2^{*0})/B(B \to f_1(1285) K_2^{*0})$ will help to determine the mixing angle $\theta_{s,P_1}$.

$B \to K_1^0 K_2^*(a_2)$ decays (see Table IV) are penguin-dominated and receive contribution of the linear combination $(a_4 - a_{10}/2)$. $B \to K_1^0 a_2$ processes, with $K_1 = K_1(1270), K_1(1400)$, have branching ratios of $(2.4 - 5.7) \times 10^{-6}$ with $\theta_{K_1} = 37^\circ$, while their branching fractions are $(0.7 - 9.34) \times 10^{-6}$ if $\theta_{K_1} = 58^\circ$. Another interesting relation is that

$$B(B \to K_1^0(1270) K_2^*(a_2))|_{\theta_{K_1} = 37^\circ} < B(B \to K_1^0(1270) K_2^*(a_2))|_{\theta_{K_1} = 58^\circ}$$

while

$$B(B \to K_1^0(1400) K_2^*(a_2))|_{\theta_{K_1} = 37^\circ} > B(B \to K_1^0(1400) K_2^*(a_2))|_{\theta_{K_1} = 58^\circ}.$$

Moreover, branching ratios of $B \to K_1^0 a_2$ decays are insensitive to the mixing angle $\theta_{K_1} = 37^\circ$. This behavior is opposite for $\theta_{K_1} = 58^\circ$. Hence, the ratio $B(B \to K_1(1270) a_2)/B(B \to K_1(1400) a_2)$ can offer a better determination for $\theta_{K_1}$. Decay rates of $B \to K_1^0 K_2^*$ channels are small because they arise from $b \to d$ penguin transition and are suppressed by the smallness of respective Wilson coefficients.

We have compared branching ratios of $B \to AT$ decays (obtained in this work) and $B \to VT$ channels (obtained recently for us using the CLF approach[11]), where $A$ and $V$ mesons have the same quark content. In general, the ratio $\mathcal{R}_{AT/VT} \gtrsim 1$ when $A$, $V$ and $T$ mesons are non-strange. It means that the product of the second and the third factors in Eq. (21) is approximately 1 and that $B(B \to a_1 a_2)/B(B \to \rho a_2) \approx (f_{a_1}/f_{\rho})^2$. This ratio can be at the reach of $B$ factories and LHC-b experiment.
V. CONCLUSIONS

In this work we study the production of excited orbitally ($p$-wave) mesons in nonleptonic charmless two-body B decays. We compute branching ratios of $B \to A(3P_1)T$ decays within the framework of generalized factorization, using form factors from CLF approach for $B \to T$ transitions. Respective factorized amplitudes of these decays are explicitly showed in Appendix. We obtained that $\mathcal{B}(B^0 \to a_1^0 a_2^0) = 42.47 \times 10^{-6}$, $\mathcal{B}(B^+ \to a_1^+ a_2^0) = 22.71 \times 10^{-6}$, $\mathcal{B}(B\to f_1 K_2^+) = (2.8 - 4) \times 10^{-6}$ (with $f_1 = f_1(1285), f_1(1420)$) for $\theta_{P_1} = 53.2^\circ$, $\mathcal{B}(B \to f_1(1420)K_2^+) = (5.91 - 6.42) \times 10^{-6}$ with $\theta_{P_1} = 27.9^\circ$, $\mathcal{B}(B \to K_1 a_2) = (1.7 - 5.7)[1 - 9.3] \times 10^{-6}$ for $\theta_{K_1} = 37^\circ[58^\circ]$ where $K_1 = K_1(1270), K_1(1400)$. It seems that the majority of these modes could be measured at the present asymmetric B factories, BABAR and Belle, as well as at future hadronic B experiments such as BTeV and LHC-b. Additionally, we have found that $\mathcal{B}(B \to K_1(1270) a_2)/\mathcal{B}(B \to K_1(1400) a_2)$ and $\mathcal{B}(B \to f_1(1420) K_2^+)/\mathcal{B}(B \to f_1(1285) K_2^+)$ ratios will help to determine mixing angles $\theta_{K_1}$ and $\theta_{P_1}$, respectively.

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APPENDIX: DECAY AMPLITUDES

In this appendix, we present expressions for the factorizable decay amplitudes of charmless $B \to A(3P_1)T$ decays. They must be multiplied by $G_F e^{\mu\nu}/\sqrt{2}$.

1. Process $|\Delta S| = 0$

\begin{align*}
\mathcal{A}(B^+ \to a_1^0 a_2^0) &= \frac{1}{\sqrt{2}}m_{a_1} f_{a_1} F_{\mu\nu}^{B \to a_2} (m_{a_1}^2) \left\{ V_{ub}^* V_{ud} a_1 - V_{tb}^* V_{td} (a_4 + a_{10}) \right\} \tag{A.1} \\
\mathcal{A}(B^+ \to a_1^0 a_2^+) &= \frac{1}{\sqrt{2}}m_{a_1} f_{a_1} F_{\mu\nu}^{B \to a_2} (m_{a_1}^2) \left\{ V_{ub}^* V_{ud} a_2 - V_{tb}^* V_{td} \left[ -a_4 + \frac{1}{2}(-3a_7 + 3a_9 + a_{10}) \right] \right\} \tag{A.2} \\
\mathcal{A}(B^+ \to f_1 a_2^+) &= m_{f_1} f_{f_1} F_{\mu\nu}^{B \to a_2} (m_{f_1}^2) \left\{ V_{ub}^* V_{ud} a_2 - V_{tb}^* V_{td} \left[ 2(a_3 - a_5) + a_4 - \frac{1}{2}(a_7 - a_9 + a_{10}) \right] \\
&\quad + \left( a_3 - a_5 \right) \left( \frac{f_{f_1}^*}{f_{f_1}} \right) \right\} \tag{A.3} \\
\mathcal{A}(B^+ \to K_1^0 K_2^{*+}) &= m_{K_1} f_{K_1} F_{\mu\nu}^{B \to K_2^*} (m_{K_1}^2) \left\{ -V_{tb}^* V_{td} \left[ a_4 - \frac{1}{2}a_{10} \right] \right\} \tag{A.4} \\
\mathcal{A}(B^0 \to a_1^0 a_2^0) &= m_{a_1} f_{a_1} F_{\mu\nu}^{B \to a_2} (m_{a_1}^2) \left\{ V_{ub}^* V_{ud} a_1 - V_{tb}^* V_{td} (a_4 + a_{10}) \right\} \tag{A.5} \\
\mathcal{A}(B^0 \to a_1^+ a_2^0) &= 0 \tag{A.6} \\
\mathcal{A}(B^0 \to a_1^0 a_2^0) &= \frac{1}{2}m_{a_1} f_{a_1} F_{\mu\nu}^{B \to a_2} (m_{a_1}^2) \left\{ V_{ub}^* V_{ud} a_2 - V_{tb}^* V_{td} \left[ -a_4 + \frac{1}{2}(-3a_7 + 3a_9 + a_{10}) \right] \right\} \tag{A.7}
\end{align*}
\[ A(B^0 \to f_1 a_2^0) = \frac{1}{\sqrt{2}} m_{f_1} f_{f_1} F^{\mu \nu}_{\mu \nu} (m_{f_1}) \left\{ V_{ub}^* V_{ud} a_2 - V_{tb}^* V_{td} \left[ 2(a_3 - a_5) + a_4 - \frac{1}{2} (a_7 - a_9 + a_{10}) \right] + \left( a_3 - a_5 \right) + \frac{1}{2} (a_7 - a_9) \left( \frac{f_{f_1}}{f_{f_1}} \right) \right\} \]  
(A.8)

\[ A(B^0 \to K_1^0 K_2^0) = m_{K_1} f_{K_1} F^{B \to K_1^0} (m_{K_1}^2) \left\{ - V_{tb}^* V_{td} \left[ a_4 - \frac{1}{2} a_{10} \right] \right\} \]  
(A.9)

2. Process \( |\Delta S| = 1 \)

\[ A(B^+ \to K_1^+ a_2^0) = \frac{1}{\sqrt{2}} m_{K_1} f_{K_1} F^{B \to a_2} (m_{K_1}^2) \left\{ V_{ub}^* V_{us} a_1 - V_{tb}^* V_{ts} (a_4 + a_{10}) \right\} \]  
(A.10)

\[ A(B^+ \to a_1^0 K_2^+ ) = \frac{1}{\sqrt{2}} m_{a_1} f_{a_1} F^{B \to K_1^0} (m_{a_1}^2) \left\{ V_{ub}^* V_{us} a_2 - V_{tb}^* V_{ts} \left[ \frac{3}{2} (a_3 - a_5) - \frac{1}{2} (a_7 - a_9) \right] + \left( a_3 + a_4 - a_5 \right) + \frac{1}{2} (a_7 - a_9 - a_{10}) \right\} \left( \frac{f_{f_1}}{f_{f_1}} \right) \right\} \]  
(A.11)

\[ A(B^+ \to f_1 K_2^+) = m_{f_1} f_{f_1} F^{B \to K_1^0} (m_{f_1}^2) \left\{ V_{ub}^* V_{us} a_2 - V_{tb}^* V_{ts} \left[ 2(a_3 - a_5) - \frac{1}{2} (a_7 - a_9) \right] + \left( a_3 + a_4 - a_5 \right) + \frac{1}{2} (a_7 - a_9 - a_{10}) \left( \frac{f_{f_1}}{f_{f_1}} \right) \right\} \]  
(A.12)

\[ A(B^+ \to K_1^+ a_2^0) = m_{K_1} f_{K_1} F^{B \to a_2} (m_{K_1}^2) \left\{ V_{ub}^* V_{us} a_1 - V_{tb}^* V_{ts} (a_4 + a_{10}) \right\} \]  
(A.13)

\[ A(B^0 \to K_1^+ a_2^-) = m_{K_1} f_{K_1} e^{\mu \nu} F^{B \to a_2} (m_{K_1}^2) \left\{ V_{ub}^* V_{us} a_1 - V_{tb}^* V_{ts} (a_4 + a_{10}) \right\} \]  
(A.14)

\[ A(B^0 \to a_1 K_2^0) = \frac{1}{\sqrt{2}} m_{a_1} f_{a_1} F^{B \to K_1^0} (m_{a_1}^2) \left\{ V_{ub}^* V_{us} a_2 - V_{tb}^* V_{ts} \left[ \frac{3}{2} (a_3 - a_5) - \frac{1}{2} (a_7 + a_9) \right] \right\} \]  
(A.15)

\[ A(B^0 \to f_1 K_2^0) = m_{f_1} f_{f_1} F^{B \to K_1^0} (m_{f_1}^2) \left\{ V_{ub}^* V_{us} a_2 - V_{tb}^* V_{ts} \left[ 2(a_3 - a_5) - \frac{1}{2} (a_7 - a_9) \right] + \left( a_3 + a_4 - a_5 \right) + \frac{1}{2} (a_7 - a_9 - a_{10}) \right\} \right\} \]  
(A.16)

\[ A(B^0 \to K_1^0 a_2^0) = \frac{1}{\sqrt{2}} m_{K_1} f_{K_1} F^{B \to a_2} (m_{K_1}^2) \left\{ - V_{tb}^* V_{ts} \left[ a_4 - \frac{1}{2} a_{10} \right] \right\} \]  
(A.17)
with

\[ F^{\mu \nu}_{\mu' \nu'}(m_A^2) \equiv c_\alpha^{(p_B + p_T)} \left[ i h(m_A^2) \cdot c_{\alpha' \beta \rho} g_{\mu \beta}(p_A)_{\nu}(p_A)_{\sigma} + k(m_A^2) \cdot \delta_{\mu \nu} + b_{+}(m_A^2) \cdot (p_A)_{\mu}(p_A)_{\nu} g^{\alpha \rho} \right], \]

(A.18)

where \( T \) stands for \( a_2 \) and \( K^*_2 \).

[1] H. Y. Cheng and J. G. Smith, arXiv:0901.4396v1 (to appear in Ann. of Nucl. and Part. Sci.).
[2] A. Ali, G. Kramer, and C. D. Li, Phys. Rev. D 58, 094009 (1998).
[3] Y. H. Chen, H. Y. Cheng, B. Tseng, and K. C. Yang, Phys. Rev. D 60, 094014 (1999).
[4] D. S. Du et al., Phys. Rev. D 65, 074001 (2002); 65, 094025 (2002); 66, 079904 (E) (2002); M. Beneke and M. Neubert, Nucl. Phys. B675, 333 (2003); M. Z. Yang and Y. D. Yang, Phys. Rev. D 62, 114019 (2000); C. D. Lü, Int. J. Mod. Phys. A 23, 3250 (2008); C. D. Lü and M. Z. Yang, Eur. Phys. J. C 23, 275 (2002); X. Liu et al., Phys. Rev. D 73, 074002 (2006); L. Guo, Q. G. Xu, and Z. J. Xiao, Phys. Rev. D 75, 014019 (2007); D. Q. Guo, X. F. Chen and Z. J. Xiao, Phys. Rev. D 75, 054033 (2007); J. Chay and C. Kim, Phys. Rev. D 75, 054033 (2007).
[5] K. C. Yang, Phys. Rev. D 72, 034009 (2005); 72, 059901(E) (2005); C. H. Chen, C. Q. Geng, Y. K. Hsiao, and Z. T. Wei, Phys. Rev. D 72, 054011 (2005); G. Nardulli and T. N. Pham, Phys. Lett. B 623, 65 (2005); V. Laporta, G. Nardulli, and T. N. Pham, Phys. Rev. D 74, 054035 (2006); 76, 079903(E) (2007).
[6] G. Calderón, J. H. Muñoz and C. E. Vera, Phys. Rev. D 76, 094019 (2007).
[7] K. C. Yang, Phys. Rev. D 76, 094002 (2007).
[8] H. Y. Cheng and K. C. Yang, Phys. Rev. D 76, 114020 (2007).
[9] H. Y. Cheng and K. C. Yang, Phys. Rev. D 78, 094001 (2008).
[10] A.C. Katoch, R.C. Verma, Int. J. Mod. Phys. A 11, 129 (1996); H. Y. Cheng, C. K. Chua and K. C. Yang, Phys. Rev. D 73, 014017 (2006); 77, 014034 (2008).
[11] J. H. Muñoz and N. Quintero, arXiv:0903.3701 and references therein.
[12] C. S. Kim, J. P. Lee, and Sechul Oh, Phys. Rev. D 67, 014002 (2003).
[13] C. H. Chen and C. Q. Geng, Phys. Rev. D 75, 054010 (2007); A. Datta et al., Phys. Rev. D 77, 114025 (2008).
[14] B. Aubert et al. (BABAR Collaboration), Phys. Rev. Lett. 98, 051801 (2007); 101, 161801 (2008); arXiv:0901.3703.
[15] B. Aubert et al. (BABAR Collaboration), Phys. Rev. Lett. 97, 051802 (2006).
[16] K. Abe et al. (Belle Collaboration), arXiv:0706.3279.
[17] B. Aubert et al. (BABAR Collaboration), Phys. Rev. Lett. 99, 261801 (2007); 99, 241803 (2007); 100, 051803 (2008); 101, 091801(2008); 101, 161801 (2008); Phys. Rev. D 78, 011104 (2008); arXiv:0807.4760; arXiv:0808.0579.
[18] H. Y. Cheng, C. K. Chua, and C. W. Hwang, Phys. Rev. D 69, 074025 (2004).
[19] G. Buchalla, A. J. Buras, and M. E. Lautenbacher, Rev. Mod. Phys. 68, 1125 (1996).
[20] A. J. Buras, Nucl. Phys. B434, 606 (1995); arXiv:hep-ph/9806471.
[21] N. Isgur, D. Scora, B. Grinstein, and M. B. Wise, Phys. Rev. D 39, 799 (1989).
[22] H. Hatanaka and K.-C. Yang, arXiv:0903.1917.
[23] D. Scora and N. Isgur, Phys. Rev. D 52, 2783 (1995).
[24] W. Jaus, Phys. Rev. D 60, 054026 (1999).
[25] C. H. Chen and C. Q. Geng, Phys. Rev. D 77, 054034 (2008).
[26] W. Wang, Y. L. Shen and C. D. Lü, arXiv:0811.3748.
[27] W. Wang, Y. L. Shen and C. D. Lü, Eur. Phys. J. C 51, 841 (2007).
[28] C. W. Hwang and Z. T. Wei, J. Phys. G 34, 687 (2007).
[29] H. Y. Cheng and C. K. Chua, Phys. Rev. D 69, 094007 (2004).
[30] C. H. Chen and C. Q. Geng, Phys. Rev. D 75, 054010 (2007).
[31] C. Amsler et al. (Particle Data Group), Phys. Lett. B 667, 1 (2008).
[32] H. Hatanaka and K. C. Yang, Phys. Rev. D 77, 094023 (2008); 78, 059902(E) (2008).
[33] K. C. Yang, Nucl. Phys. B776, 187 (2007).
[34] E. Close and A. Kirk, Z. Phys. C 76, 469 (1997).
[35] L. Wolfenstein, Phys. Rev. Lett. 51, 1945 (1983); A. J. Buras, M. E. Lautenbacher, and G. Ostermaier, Phys. Rev. D 50, 3433 (1994).
[36] F. E. Close and A. Kirk, Z. Phys. C 76, 469 (1997).
[37] L. Wolfenstein, Phys. Rev. D 51, 1945 (1983); A. J. Buras, M. E. Lautenbacher, and G. Ostermaier, Phys. Rev. D 50, 3433 (1994).
[38] H. Fusaoku and Y. Koide, Phys. Rev. D 57, 3986 (1998).
[39] G. López Castro and J. H. Muñoz, Phys. Rev. D 55, 5581 (1997).