Four-body Faddeev-type equations for $\bar{K}NNN$ quasi-bound state calculations

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Abstract. The paper is devoted to the $\bar{K}NNN$ system, which is an exotic system consisting of an antikaon and three nucleons. Four-body Faddeev-type AGS equations, which are being used for evaluation of the possible quasi-bound state in the system are described.

Keywords: few-body physics, antikaon-nucleon systems, four-body Faddeev equations

1 Introduction

The attractive nature of $\bar{K}N$ interaction has stimulated theoretical and experimental searches for bound states of $K^-$ with different number of nucleons. The interest in few-body antikaonic systems was stimulated by calculations, which predicted deep and relatively narrow quasi-bound $K^-$-nuclear states. Many theoretical calculations devoted to the lightest possible system $\bar{K}NN$ have been performed since then using different methods, see e.g. a review [1]. All of them agree that a quasi-bound state in the $K^-pp$ system exists, but they yield quite diverse binding energies and widths.

Some theoretical works were devoted to the question of the quasi-bound state in the four-body $\bar{K}NNN$ system, but more accurate calculations within Faddeev-type equations are needed. Indeed, only these dynamically exact equations in momentum representation can treat energy dependent $\bar{K}N$ potentials, necessary for the this system, exactly.

The paper contains description of the four-body Faddeev-type AGS equations [2], written down for the $\bar{K}NNN$ system. We will solve the equations using our programs written for the three-body AGS calculations of the $\bar{K}NN$ system, described in [1], and our two-body potentials constructed for them.

2 Four-body Faddeev-type AGS equations

The four-body AGS equations contain three-body transition operators, obtained from the three-body AGS equations. The three-body Faddeev-type equations in AGS form [3] written for separable potentials $V_\alpha = \lambda_\alpha |g_\alpha\rangle\langle g_\alpha|$ have a form

$$X_{\alpha\beta}(z) = Z_{\alpha\beta}(z) + \sum_{\gamma=1}^{3} Z_{\alpha\gamma}(z) \tau_\gamma(z) X_{\gamma\beta}(z)$$  \hspace{1cm} (1)
with transition $X_{\alpha\beta}$ and kernel $Z_{\alpha\beta}$ operators

$$X_{\alpha\beta}(z) = \langle g_\alpha | G_0(z) U_{\alpha\beta}(z) G_0(z) | g_\beta \rangle, \quad (2)$$

$$Z_{\alpha\beta}(z) = (1 - \delta_{\alpha\beta}) \langle g_\alpha | G_0(z) | g_\beta \rangle. \quad (3)$$

Here $U_{\alpha\beta}(z)$ is the three-body transition operator, which describes process $\beta + (\alpha\gamma) \rightarrow \alpha + (\beta\gamma)$, while $G_0(z)$ is the three-body Green function. Faddeev partition indices $\alpha, \beta = 1, 2, 3$ simultaneously define a particle ($\alpha$) and the remained pair ($\beta\gamma$). The operator $\tau_\alpha(z)$ in Eq.\((1)\) is an energy-dependent part of a separable two-body $T$-matrix $T_\alpha(z) = \langle g_\alpha | \tau_\alpha(z) | g_\alpha \rangle$, describing interaction in the $(\beta\gamma)$ pair: $|g_\alpha\rangle$ is a form-factor.

The four-body Faddeev-type AGS equations \([2]\), written for separable potentials, have a form

$$\bar{U}^{\sigma\rho}_{\alpha\beta}(z) = (1 - \delta_{\sigma\rho})(\bar{G}_0^{-1})_{\alpha\beta}(z) + \sum_{\tau,\gamma,\delta} (1 - \delta_{\sigma\tau}) T^\tau_{\alpha\gamma}(z)(\bar{G}_0)_{\gamma\delta}(z) \bar{U}^{\tau\rho}_{\delta\beta}(z), \quad (4)$$

$$\bar{T}^\tau_{\alpha\beta}(z) = \langle g_\alpha | G_0(z) U^\tau_{\alpha\beta}(z) G_0(z) | g_\beta \rangle, \quad (5)$$

$$\bar{U}^\tau_{\alpha\beta}(z) = (g_\alpha | G_0(z) U^\tau_{\alpha\beta}(z) G_0(z) | g_\beta) , \quad \bar{G}_0(z)_{\alpha\beta} = \delta_{\alpha\beta} \tau_\alpha(z). \quad (6)$$

The operators $\bar{U}^{\sigma\rho}_{\alpha\beta}$ and $\bar{T}^\tau_{\alpha\beta}$ contain four-body $U^\sigma_{\alpha\beta}(z)$ and three-body $U^\sigma_{\alpha\beta}(z)$ transition operators, correspondingly. The last one $(U^\sigma_{\alpha\beta}(z))$ differs from the three-body operator in Eq.\((2)\) by additional upper index $\tau$ $(\sigma, \rho)$, which defines a three-body subsystem of the four-body system. The free Green function $G_0(z)$ now acts in four-body space. If, in addition, the "effective three-body potentials" $\bar{T}^\tau_{\alpha\beta}(z)$ in Eq.\((4)\) are presented in a separable form: $\bar{T}^\tau_{\alpha\beta}(z) = |\bar{g}_\alpha^\tau\rangle \tau^\tau_{\alpha\beta}(\bar{g}_\alpha^\tau)$, the four-body equations can be written as \([4]\)

$$\bar{X}^\sigma_{\alpha\beta}(z) = \bar{Z}^\sigma_{\alpha\beta}(z) + \sum_{\tau,\gamma,\delta} \bar{Z}^\sigma_{\alpha\gamma}(z) \tau^\tau_{\gamma\delta}(z) \bar{X}^\tau_{\delta\beta}(z) \quad (7)$$

with new transition $\bar{X}^\sigma_{\alpha\beta}$ and kernel $\bar{Z}^\sigma_{\alpha\beta}$ operators defined by

$$\bar{X}^\sigma_{\alpha\beta}(z) = \langle \bar{g}_\alpha^\sigma | \bar{G}_0(z)_{\alpha\alpha} \bar{U}^\sigma_{\alpha\beta}(z) G_0(z)_{\beta\beta} | \bar{g}_\beta^\sigma \rangle, \quad (8)$$

$$\bar{Z}^\sigma_{\alpha\beta}(z) = (1 - \delta_{\sigma\rho}) \langle \bar{g}_\alpha^\rho | \bar{G}_0(z)_{\alpha\beta} | \bar{g}_\beta^\rho \rangle. \quad (9)$$

The separabilization of the "effective three-body potentials" $\bar{T}_\alpha(z)$ can be performed using e.g. the Hilbert-Schmidt expansion of the three-body AGS equations with separable potentials Eq.\((1)\).

### 3 Four-body equations for the $\bar{K}NNN$ system

There are two types of partitions for a four-body system: $3 + 1$ and $2 + 2$. For the $\bar{K}NNN$ system they are: $|\bar{K} + (NNN)\rangle$, $|N + (KNN)\rangle$ and $|(\bar{K}N) + (NN)\rangle$. At the begin we considered all three nucleons as different particles, so we started
by writing down the four-body system of equations Eq. (7) for the following 18 channels $\sigma_\alpha$ (with $\alpha = NN$ or $KN$):

\begin{align}
1_{NN} : & |K + (N_1 + N_2 N_3)>, |K + (N_2 + N_3 N_1)>, |K + (N_3 + N_1 N_2)>,
2_{NN} : & |N_1 + (K + N_2 N_3)>, |N_2 + (K + N_3 N_1)>, |N_3 + (K + N_1 N_2)>,
2_{KN} : & |N_1 + (N_2 + K N_3)>, |N_2 + (N_3 + K N_1)>, |N_3 + (N_1 + K N_2)>,
3_{NN} : & |(N_2 N_3) + (K + N_1)>, |(N_3 N_1) + (K + N_2)>, |(N_1 N_2) + (K + N_3)>,
3_{KN} : & |(K N_1) + (N_2 + N_3)>, |(K N_2) + (N_3 + N_1)>, |(K N_3) + (N_1 + N_2)>
\end{align}

(10)

After this the operators and equations were antisymmetrized, and the system of operator equations was written in a form:

$$\hat{X} = \hat{Z} \hat{\tau} \hat{X},$$

(11)

were $\hat{Z}$ and $\hat{\tau}$ are the $5 \times 5$ matrices containing the kernel operators $\hat{Z}_{\alpha}^{\sigma \rho}$ and $\hat{\tau}_{\alpha \beta}^{\sigma \rho}$, correspondingly. Since the initial state is assumed to be fixed, only one column of the $5 \times 5$ matrix $\hat{X}$, containing transition operators $\hat{X}_{\alpha \beta}^{\sigma \rho}$, is necessary:

$$\hat{X}_{\alpha}^{\rho} = \begin{pmatrix}
\hat{X}_{1 \alpha}^{\rho} \\
\hat{X}_{2 \alpha}^{\rho} \\
\hat{X}_{3 \alpha}^{\rho} \\
\hat{X}_{4 \alpha}^{\rho} \\
\hat{X}_{5 \alpha}^{\rho}
\end{pmatrix}, \quad \hat{Z}_{\alpha}^{\sigma \rho} = \begin{pmatrix}
0 & \hat{Z}_{11}^{12} & 0 & \hat{Z}_{11}^{13} & 0 \\
\hat{Z}_{22}^{12} & 0 & \hat{Z}_{22}^{13} & 0 & 0 \\
0 & \hat{Z}_{31}^{22} & 0 & \hat{Z}_{31}^{23} & 0 \\
\hat{Z}_{41}^{22} & 0 & \hat{Z}_{41}^{23} & 0 & 0 \\
0 & \hat{Z}_{51}^{22} & 0 & \hat{Z}_{51}^{23} & 0
\end{pmatrix}, \quad (12)
$$

$$\hat{\tau}_{\alpha \beta}^{\sigma \rho} = \begin{pmatrix}
\tau_{1 \alpha \beta}^{11} \\
\tau_{2 \alpha \beta}^{11} \\
\tau_{3 \alpha \beta}^{11} \\
\tau_{4 \alpha \beta}^{11} \\
\tau_{5 \alpha \beta}^{11}
\end{pmatrix}, \quad (13)
$$

4 Three-body subsystems and two-body input

We are studying the $\bar{K}NN$ system with the lowest value of the four-body isospin $I^{(4)} = 0$, which can be denoted as $K^- ppn$. Its total spin $S^{(4)}$ is equal to one half, while the orbital momentum is zero, since all two-body interactions are chosen to be zero. For the $\bar{K}NN$ system with these quantum numbers the following three-body subsystems contribute:

- $\bar{K}NN$ with $I^{(3)} = 1/2$, $S^{(3)} = 0$ ($K^- pp$) or $S^{(3)} = 1$ ($K^- d$).
- $NNN$ with $I^{(3)} = 1/2$, $S^{(3)} = 1/2$ ($^3$H or $^3$He),

were $I^{(3)}$ and $S^{(3)}$ are the three-body isospin and spin.

The three-body antikaon-nucleon system $\bar{K}NN$ with different quantum numbers was studied in our previous works, see Ref. [11]. In particular, quasi-bound
state pole positions in the $K^-pp$ system ($\bar{K}NN$ with $I^{(3)} = 1/2$, $S^{(3)} = 0$) and near-threshold $K^-d$ scattering amplitudes ($\bar{K}NN$ with $I^{(3)} = 1/2$, $S^{(3)} = 1$) were calculated (no quasi-bound states were found in the $K^-d$ system). It was done using the three-body AGS equations with separable potentials Eq. (1) with three models of the $\bar{K}N$ interaction: two phenomenological potentials having one- or two-pole structure of the $\Lambda(1405)$ resonance and a chirally motivated model. All three potentials describe low-energy $K^-p$ scattering and $1s$ level shift of kaonic hydrogen with equally high accuracy. We also used a two-term separable $N\bar{N}$ potential, which reproduces Argonne v18 $NN$ phase shifts and scattering lengths. The same potentials are used in our four-body calculations.

The programs of numerical solution of the three-body AGS equations for the $\bar{K}NN$ systems can be used for separabilization of the "effective three-body potentials" (after some modifications). However, it is far not enough since the three-body system Eq. (1) was solved with the initial channel fixed by $\beta = 1$, which corresponds to the $\bar{K}+NN$ partition. It allows to calculate the three-body transition amplitudes, which in the four-body formalism are denoted as $\bar{T}_{NN,NN}^{2i}$ and $\bar{T}_{\bar{K}NN,NN}^{2i}$. The other two three-body amplitudes, necessary for the four-body calculations: $\bar{T}_{NN,\bar{K}N}^{2i}$ and $\bar{T}_{\bar{K}NN,\bar{K}N}^{2i}$, - were calculated additionally by solving the AGS equations Eq. (1) with other two initial channels $\beta = 2, 3$, corresponding to the $N + \bar{K}N$ initial state (which were properly antisymmetrized).

The three-nucleon system $NNN$ was treated by solving the system of AGS equations Eq. (1) with our separable two-term $NN$ potential. The calculated binding energy was found to be 9.95 MeV for both $^3\text{H}$ and $^3\text{He}$ nuclei since Coulomb interaction was not taken into account.

5 Summary

The four-body Faddeev-type calculations of the $\bar{K}NN$ system is very complicated and time-consuming task. Up to now the four-body AGS equations were written down for the $\bar{K}NN$ system and antisymmetrized. All necessary calculations of the three-body (sub)systems were performed, their separable forms were numerically evaluated. The spin-isospin re-coupling parts of the four-body kernel functions $\bar{Z}_{\alpha\rho}^2$ were also calculated. The work is in progress.

References

1. Shevchenko, N.V.: Three-body antikaon-nucleon systems. Few Body Syst. 58, 6 (2017). doi:10.1007/s00601-016-1170-5
2. Grassberger, P., Sandhas, W.: Systematical treatment of the non-relativistic n-particle scattering problem. Nucl. Phys. B 2, 181-206 (1967).
3. Alt, E.O., Grassberger, P., Sandhas, W.: Reduction of the three-particle collision problem to multi-channel two-particle Lippmann-Schwinger equations. Nucl. Phys. B 2, 167-180 (1967).
4. Casel, A., Haberzettl, H., Sandhas, W.: Unitary pole approximations and expansions in few-body systems. Phys. Rev. C 25, 1738 (1982). doi:10.1103/PhysRevC.25.1738