Kondo effect in coupled quantum dots under magnetic fields

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The Kondo effect in coupled quantum dots is investigated theoretically under magnetic fields. We show that the magnetoconductance (MC) illustrates peak structures of the Kondo resonant spectra. When the dot-dot tunneling coupling $V_C$ is smaller than the dot-lead coupling $\Delta$ (level broadening), the Kondo resonant levels appear at the Fermi level ($E_F$). The Zeeman splitting of the levels weakens the Kondo effect, which results in a negative MC. When $V_C$ is larger than $\Delta$, the Kondo resonances form bonding and anti-bonding levels, located below and above $E_F$, respectively. We observe a positive MC since the Zeeman splitting increases the overlap between the levels at $E_F$. In the presence of the antiferromagnetic spin coupling between the dots, the sign of MC can change as a function of the gate voltage.

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The microfabrication technique on semiconductors has enabled us to make quantum dots with the size of the order of the Fermi wavelength. Such dots are often referred to as “artificial atoms” due to the discreteness of their energy levels. By coupling the artificial atoms, we can fabricate “artificial molecules.” Indeed, interdot “molecular orbitals” have been observed experimentally when the tunneling coupling between the dots $V_C$ is sufficiently large. The strength of $V_C$ can be controlled by external gate voltages.

Recently the Kondo effect has been found in single quantum dot systems. The Kondo effect in coupled quantum dots under magnetic fields.

In coupled quantum dots connected in series, various Kondo phenomena have been proposed theoretically. In our previous papers, we have pointed out the importance of competition between the dot-dot coupling $V_C$ and dot-lead coupling $\Delta$. When the dot-lead coupling larger than the dot-dot coupling ($V_C < \Delta$), the Kondo resonances are created between a dot and a lead. The Kondo resonant levels appear at $E_F$ and the electron transport is determined by the hopping probability between the resonant levels. When $V_C > \Delta$, the Kondo levels are split into two, forming bonding and anti-bonding levels. They are located below and above the Fermi level, $E_F = T_K \sqrt{(V_C/\Delta)^2 - 1}$, and consequently the conductance is suppressed.

These unique characters of the Kondo resonant spectra in coupled dots can be observed directly. Aguado and Langreth have calculated the differential conductance under finite source-drain voltages. They have shown that the $dI/dV_{sd}$ curve has a single peak at $V_{sd} = 0$ when $V_C < \Delta$ and it has double peaks when $V_C > \Delta$. In this letter, we propose an alternative probe for the resonant spectra, a magnetoconductance (MC). In single quantum dot systems, the Zeeman effect lifts off the degeneracy of the spin states and hence weakens the Kondo effect. In consequence the conductance $G$ decreases with increasing magnetic field (negative MC) in coupled dots, the situation is the same when $V_C < \Delta$. The Kondo resonant levels are split into two by the Zeeman effect, below and above $E_F$ for spin down and up electrons, respectively, which reduces $G$. For $V_C > \Delta$, on the other hand, we observe a positive MC. The Zeeman splittings increase the overlap between the bonding resonant level for spin up electrons and anti-bonding resonant level for spin down electrons at $E_F$. This enhances the conductance $G$. When the Zeeman energy exceeds $T_K$, the Kondo effect is broken and $G$ drops to zero suddenly.

In the above discussion, we have disregarded the antiferromagnetic spin coupling $J$ between the dots. Since the dot-lead Kondo coupling screens the dot spin, the dot-dot spin coupling $J$ competes with the Kondo coupling. Georges and Meir have illustrated various transport properties in $J$ vs. $V_C/\Delta$ plane. We have shown that the effect of $J$ on the conductance $G$ can also be understood in terms of the resonant spectrum. In the second half of this letter, we demonstrate the calculations of MC in the presence of $J$. We obtain an interesting result when $V_C/\Delta < 1$: The sign of MC changes as a function the gate voltage $V_g$. For small negative $V_g$, the Kondo coupling is larger than the spin-spin coupling. We find a negative MC. For large negative $V_g$, the spin coupling $J$ increases the effective dot-dot coupling, which results in a positive MC.

As a model, we consider a symmetric quantum dot dimer connected in series as shown in Fig. 1(a). Each dot
has a single energy level $E_0$ and accommodates an electron with spin up or down. The energy level is split into $E_0 \pm g \mu_B B$ by the Zeeman effect under a magnetic field $B$. A common gate voltage $V_g$ is attached to the dots to control $E_0$. Two dots couple to each other with $V_c$, and to external leads with $V$. We assume that the intradot Coulomb interaction $U$ is sufficiently large so that (i) the double occupancy of electrons in each dot is forbidden, but (ii) the antiferromagnetic spin coupling exists between the quantum dots, through the virtual double occupancy in a dot: $JS_L \cdot S_R$ where $J = 4V_0^2/U$ and $S_\alpha$ is the spin operator in dot $\alpha = L, R$. The interdot Coulomb interaction is neglected.

Let us disregard the antiferromagnetic spin coupling $J$ for a while. The Hamiltonian reads

$$
\mathcal{H}_0 = \sum_{\alpha=L,R, k, \sigma} E(k)c_{\alpha k \sigma}^\dagger c_{\alpha k \sigma} + \sum_{\alpha=L,R \sigma} \left( E_0 + \sigma g \mu_B B \right) C_{\alpha \sigma}^\dagger C_{\alpha \sigma}^\dagger
+ \frac{V}{\sqrt{2}} \sum_{\alpha,k,\sigma} \left( c_{\alpha k \sigma}^\dagger C_{\alpha \sigma} + \text{H.c.} \right) + \frac{V_C}{2} \sum_{\sigma} \left( C_{L \sigma}^\dagger C_{R \sigma} + \text{H.c.} \right),
$$

(1)

where $c_{\alpha k \sigma}^\dagger$ creates an electron in lead $\alpha = L, R$ with energy $E(k)$ and spin $\sigma$, and $C_{\alpha \sigma}^\dagger$ creates an electron in dot $\alpha$ with spin $\sigma$. The prohibition of double occupancy in each dot is required. To treat this situation, we adopt the slave boson formalism. The annihilation operator of an electron in dot $\alpha$ is decomposed as $C_{\alpha \sigma} = b_{\alpha \sigma}^\dagger f_{\alpha \sigma}$, where a slave boson operator $b_{\alpha \sigma}^\dagger$ creates an empty state and a fermion operator $f_{\alpha \sigma}$ annihilates a singly occupied state with spin $\sigma$. The merit of this formalism is that the prohibition of double occupancy is expressed by equations:

$$
Q_\alpha = \sum_{\sigma} f_{\alpha \sigma}^\dagger f_{\alpha \sigma} + b_{\alpha \sigma}^\dagger b_{\alpha \sigma} = 1.
$$

These constraints can be taken into account by adding a term, $\sum_\alpha \lambda_\alpha \left( Q_\alpha - 1 \right)$, to the Hamiltonian. It has a single energy level $\Delta = \lambda$ in the dots and “tunneling couplings” $V_C = b^2 V_C/2, \bar{V} = bV/\sqrt{2}$. We determine $b$ and $\lambda$ by minimizing the expectation value of the Hamiltonian (3). Then the effective dot-lead coupling, $\bar{\Delta} = \pi \rho \bar{V}^2 = b^2 \Delta/2$, is equal to the Kondo temperature $T_K$ (1). Note that $V_C/\Delta = V_C/\bar{V}$ (1). The linear-conductance $G$ is written as

$$
G = \frac{\hbar^2}{e^2} \sum_{\sigma = \pm} T_\sigma(\omega = 0) \equiv \frac{2e^2}{\hbar} T(\omega = 0)
$$

(3)

with the transmission probability $T_\sigma(\omega)$ for an incident electron with spin $\sigma$ and energy $\omega$. We have chosen $\omega = 0$ at the Fermi level in the leads.

First, we study the case of $V_C/\Delta < 1$ ($\bar{V}_C/\bar{\Delta} < 1$). In Fig. (b), the conductance $G$ is plotted as a function of $B$ when $V_C/\Delta = 0.3$. As $B$ increases, $G$ decreases monotonically (negative MC). At $g \mu_B B \simeq T_K$, the Kondo coupling disappears and $G = 0$, where electrons in the dots are isolated from the leads and make a spin polarized state by the Zeeman effect. The inset in Fig. (b) shows the transmission probability $T(\omega)$. When $B = 0$, $T(\omega)$ has a single peak at $\omega = 0$ (dotted line). This is because the Kondo resonant levels are formed between a dot and a lead at $\omega = 0$. The electron transport is determined by the hopping between the resonant states, and hence the peak height of $T(\omega)$ is less than unity. When $B \neq 0$, the Zeeman effect splits the Kondo levels by $g \mu_B B$. In consequence $T(\omega)$ has double peaks at $\omega \simeq \pm g \mu_B B$ (solid line). With increasing $B$, the peaks are separated more, which results in an decrease in $T(\omega = 0)$.

Next, we study the case of $V_C/\Delta > 1$ ($\bar{V}_C/\bar{\Delta} > 1$). In Fig. (b) $G$ is plotted as a function of $B$ when $V_C/\Delta = 1.6$ (solid line). As $B$ increases from zero, $G$ increases (positive MC). This result is in contrast to that in the case of $V_C/\Delta < 1$. At $g \mu_B B \sim T_K$, the Zeeman effect destroys the Kondo coupling and $G$ drops to zero suddenly.
inset in Fig. 2 presents $T(\omega)$. When $B = 0$, $T(\omega)$ has double peaks at $\omega = \pm \sqrt{V_C^2 - \Delta^2}$ (dotted line). These peaks correspond to the molecular levels between the Kondo states. When $B \neq 0$, the Zeeman effect splits both of these molecular levels. There are totally four peaks (solid line). As $B$ increases, the middle two peaks are overlapped more at $E_F$, which leads to an increase in $T(\omega = 0)$.

Now we consider the antiferromagnetic spin coupling between the dots, $J \mathbf{S}_L \cdot \mathbf{S}_R = (J/2) \sum_{\sigma, \sigma'} \langle f_{R\sigma}^\dagger f_{L\sigma} f_{L\sigma'}^\dagger f_{R\sigma'} + \text{H.c.} \rangle$. We introduce an order parameter for the spin-spin coupling, $\kappa = (J/2) \sum_{\sigma} \langle f_{R\sigma}^\dagger f_{L\sigma} \rangle$, to decouple the spin-spin interaction.

\[
J \mathbf{S}_L \cdot \mathbf{S}_R \rightarrow \frac{\kappa^2}{J} + \sum_{m} \kappa \left( f_{Lm}^\dagger f_{Rm} + \text{H.c.} \right). 
\]

Thus $\tilde{V}_C$ in Eq. (3) is replaced by $\kappa + \tilde{V}_C$. The spin-spin coupling $\kappa$, therefore, increases the dot-dot coupling effectively. $T(\omega)$ has a single peak at $\omega = 0$ when $\kappa + \tilde{V}_C < \Delta$, and double peaks below and above $\omega = 0$ when $\kappa + \tilde{V}_C > \Delta$.

Let us discuss the case of $V_C/\Delta < \frac{1}{2}$. In Fig. 3(a), $G$ is plotted as a function of the gate voltage $V_g$ when $B = 0$. $V_C/\Delta = 0.3$ and $J/\Delta = 9.0 \times 10^{-4}$. The gate voltage changes the dot level $E_0$ (we define $V_g = E_0$). As $V_g$ is smaller, the Kondo coupling $\Delta (= T_K)$ is weaker. (i) At sufficiently large negative $V_g$, $\kappa = J/2$ and $\Delta = 0$. This indicates that a spin-singlet state appears between the dot spins and the Kondo coupling between a spin and lead completely disappears. The conductance $G = 0$. With increasing $V_g$, $\Delta$ begins to increase whereas $\kappa$ becomes smaller. (ii) At sufficiently small negative $V_g$, the electronic state is dominated by the Kondo coupling and spin-spin coupling is ineffective ($\kappa \ll T_K$). Then $\kappa + \tilde{V}_C < \Delta$ and the transmission probability $T(\omega)$ has a single peak at $\omega = 0$. The peak height is less than unity and hence $G$ takes a finite value ($< 2e^2/h$). (iii) In the intermediate region of $V_g$, $G$ has a sharp peak of $2e^2/h$ in height. This is due to the coexistence of the coherence between a dot and lead (Kondo coupling) and that between the dots (spin-spin coupling). At the peak of $G$, $\kappa + \tilde{V}_C = \Delta$. $T(\omega)$ has a single peak at $\omega = 0$ with the height of unity, which reflects a coherent transport channel connecting the left and right leads via the two dots. Just on the left side of the peak, $\kappa + \tilde{V}_C > \Delta$ and $T(\omega)$ has double peaks. On the right side of the peak, $\kappa + \tilde{V}_C < \Delta$.

In Fig. 3(b), the MC is shown for three values of $V_g$ around the peak of $G$. On the right side of the peak, we observe a negative MC (solid line). On the left side of the peak, the MC is positive (broken line). They are explained by the above-mentioned peak structures of the transmission spectrum $T(\omega)$. At the peak of $G$, the conductance is almost constant at $2e^2/h$ for $g\mu_B B < T_K/2$ and decreases considerably with $B$ for $g\mu_B B > T_K/2$. This behavior of MC reflects a flat-topped single peak structure of $T(\omega)$ with $\kappa + \tilde{V}_C = \Delta$.

Finally we look at the case of $V_C/\Delta > 1$. In Fig. 4, the MC is shown by a dotted line when $V_C/\Delta = 1.6$ and $J/\Delta = 1.0$. In this case, $\kappa + \tilde{V}_C$ is always larger than $\Delta$ and thus $T(\omega)$ has double peaks. We observe a positive MC as in the case of $J = 0$ (solid line). Note that the Kondo state survives in a larger region of the magnetic field, in the presence of $J$ than in the absence of $J$, although $G$ is smaller at small $B$. This is because the antiferromagnetic spin coupling prevents the formation of the spin polarized state, and as a result, keeps the Kondo couplings until larger values of $B$.

In conclusions, we have investigated the magnetoconductance in coupled quantum dots in the Kondo region. The MC illustrates peak structures of the Kondo resonant spectra, single peak or double peaks, depending on a ratio of $V_C/\Delta$ and value of $J$. We expect that the observation of the Kondo resonances by the MC is easier than that by the differential conductance under finite source-drain voltages because the dephasing processes should influence the Kondo states in the latter case.

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34 When \( J = 0 \), \( T_K = (\Delta/\pi) \exp(\pi E_0/\Delta) f(V_C/\Delta) \) with \( f(x) = \exp(x \arctan x)/\sqrt{1 + x^2} \). The dot level \( E_0 \) is located below the Fermi level (\( E_0 < 0 \)). For larger negative \( E_0 \), \( T_K \) becomes smaller.
35 We set \( J = 4V_C^2/U \) where \( U/\Delta = 4 \times 10^2 \) is fixed.
36 The positive MC is more pronounced than in Fig. 2. In this case, the magnetic field \( B \) suppresses the antiferromagnetic spin coupling \( \kappa \) and consequently enhances the Kondo effect. With increasing \( B \), \( \kappa \) decreases and hence the bonding and anti-bonding peaks of \( T(\omega) \) become closer to each other. This enhances MC besides the Zeeman effect. At a certain value of \( B, G = 2e^2/h. \) As \( B \) increases further, \( \kappa + V_C \) becomes less than \( \Delta; \) the molecular levels disappear and MC is negative.
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FIG. 1. (a) Coupled quantum dots in the presence of the Zeeman effect, $g\mu_B B$, under a magnetic field $B$. The antiferromagnetic spin-spin coupling is denoted by $J$. (b) Conductance $G$ as a function of the Zeeman splitting $g\mu_B B$ when $V_C/\Delta < 1$ ($V_C/\Delta = 0.3$), in the absence of the antiferromagnetic spin coupling $J$. $E_0/\Delta = -2$. $g\mu_B B$ is normalized by the Kondo temperature $T_K$ at $B = 0$. Inset: Transmission probability $T(\omega)$ with $\omega$ being the energy of an incident electron. The dotted and solid lines represent the cases of $B = 0$ and $g\mu_B B/T_K = 0.71$, respectively. The arrow indicates the direction of an increase in magnetic field $B$.

FIG. 2. Conductance $G$ as a function of the Zeeman splitting $g\mu_B B$ when $V_C/\Delta > 1$ ($V_C/\Delta = 1.6$). The solid and dotted lines represent the cases of $J = 0$ and $J/\Delta = 1.0$, respectively. $E_0/\Delta = -1.5$. $g\mu_B B$ is normalized by the Kondo temperature $T_K$ at $B = 0$. Inset: Transmission probability $T(\omega)$ with $\omega$ being the energy of an incident electron, in the absence of $J$. The dotted and solid lines represent the cases of $B = 0$ and $g\mu_B B/T_K = 0.63$, respectively. The arrow indicates the direction of an increase in magnetic field $B$. 
FIG. 3. (a) The gate voltage dependence of the conductance $G$ when $V_C/\Delta = 0.3$ and $J/\Delta = 9.0 \times 10^{-4}$, $B = 0$. We define $V_g = E_0$. (b) Conductance $G$ as a function of the Zeeman splitting $g\mu_B B$. $V_g/\Delta = -1.0$ (solid line), $-2.16$ (dotted line), and $-2.23$ (broken line). $g\mu_B B$ is normalized by the Kondo temperature $T_K$ at $B = 0$. 