POLRAD 2.0. FORTRAN code for the Radiative Corrections Calculation to Deep Inelastic Scattering of Polarized Particles

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Abstract

The FORTRAN code POLRAD 2.0 for radiative correction calculation in inclusive and semi-inclusive deep inelastic scattering of polarized leptons by polarized nucleons and nuclei is described. Its theoretical basis, structure and algorithms are discussed in details.

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Keywords: polarized particles, inclusive and semi-inclusive deep inelastic scattering, QED and electroweak radiative corrections, structure functions, higher order corrections, experimental data processing.

Program Library Index: Particle Physics Quantum Electrodynamics
Program Summary

Title of program: POLRAD
Version: 2.0 April 1997
Catalogue identifier:
Program obtainable from: on request from e-mail: aku@hep.by
Computer for which the program is designed and others on which it has been tested:
Computers: all
Operating systems or monitors under which the program has been tested: all
Programming language used: FORTRAN 77
Memory required to execute with typical data: 1MB
No. of bits in a word: 32
No. of processors used: 1
No. of bytes in distributed program, including test data, etc.: 300 kB
Distribution format: default ASCII else uuencoded compressed tar file
Other programs called:
PATCHY [1] — part of CERNLIB
MINUIT [2] — part of CERNLIB
Keywords: polarized particles, inclusive and semi-inclusive deep inelastic scattering, QED and electroweak radiative corrections, structure functions, higher order corrections, experimental data processing.
Nature of physical problem:
First and higher order QED and electroweak radiative corrections to the inclusive and semi-inclusive polarized deep inelastic scattering; experimental data processing.
Method of solution: Numerical integration of analytical formulae.
Restrictions on complexity of the problem:
Only selective experimental cuts are possible. For $\mathcal{O}(\alpha^2)$ order correction only leading contribution is calculated. Electroweak correction is calculated for longitudinally polarized target.
Typical running time:
The running time depends on the options used. For example: 1) calculation of the total QED correction takes about 4 seconds of the CPU time per one kinematical point; 2) calculation of electroweak plus $\mathcal{O}(\alpha^2)$ plus model for $g_2 \neq 0$ takes up to 300 seconds per one kinematical point.
References:
[1] H.J.Klein, J.Zoll, PATCHY Reference Manual, March 1988.
[2] F.James, MINUIT Reference Manual, March 1994.
1 Introduction

Data processing of the modern experiments on deep inelastic scattering (DIS) of polarized leptons on polarized nuclear target requires correct account of the radiative corrections (RC). Our program POLRAD 2.0 based theoretically on the original approach proposed in ref. [1] and developed in the ref. [2] was created to suit the demands of the present and future experiments with fixed polarized nuclear targets and at collider. Along with the possibilities of the previous versions of POLRAD [3], which calculated the QED lowest order RC to DIS of polarized leptons by polarized nuclei, the current version gives an opportunity to take into account both electroweak and higher order effects and to calculate the RC for semi-inclusive polarized experiments.

In section 2 we present the detailed description of the theoretical basis of POLRAD 2.0 along with the explicit formulae. We start with the calculation of born cross section in subsection 2.1.1. In subsections 2.1.2 and 2.1.3 we present the review of the basic formulae for each of the radiative tails: elastic, quasielastic and inelastic, — and consider the case of ultrarelativistic approximation to the lowest order QED correction. The contribution of $\alpha^2$ order correction is calculated in subsection 2.1.4 on the basis of structure functions formalism. The expressions for one-loop electroweak correction within the framework of standard theory and QCD-improved parton model are given in subsection 2.2. The POLRAD 2.0 part that calculates RC in semi-inclusive case is the modification of the code SIRAD [4] and is described in subsection 2.3. The current data processing iteration procedure discussed in subsection 2.4 is extended in comparison with the one in the previous version of POLRAD to fit the data using the CERNLIB package MINUIT.

Appendix A is devoted to structure function definition, parameterization and models used. Most cumbersome parts of explicit formulae are presented in Appendices B and C.

The current version of POLRAD gives the opportunity to choose one of five nuclear targets along with the type of its polarization, type of scattered longitudinally polarized charged leptons. It is possible to operate in standard SMC [5], HERMES [6], E142 [7] kinematics or to choose any other kinematics. Procedure of RC of experimental data can be organized with implementation of iteration procedure. The program is realized on Standard FORTRAN 77 and does not require any changes when used under different computer platforms and operation systems.

2 Theory

We consider the process of DIS of longitudinally polarized charged leptons on longitudinally and transversely polarized nuclear target

$$l + N \rightarrow l' + X,$$

(1)

and semi-inclusive DIS (SIDIS) process when a hadron is measured in coincidence with the scattered lepton. The physical interpretation of the experimental data requires the separation of the Born cross section from background contributions known as radiative corrections, which originate from loop diagrams and from processes with the emission of additional real photons. Radiative events cannot completely be removed by experimental methods and so they have to be calculated theoretically and substracted from measured cross sections.
It is well-known that there are three scattering channels of virtual boson ($\gamma, Z$) on nucleus in dependence on transfer energy $\nu = E_1 - E_2$, where $E_1(E_2)$ is initial (scattered) lepton energy: elastic, quasielastic and inelastic. Representative plot of dependence of the scattering cross section on $\nu$ and square of transfer momentum $Q^2 = -q^2$ is shown on fig.1 (only the regions that give sufficient contribution to RC calculation). Peak in the range I (for $\nu = 0$, if to neglect nuclear recoil) corresponds to elastic scattering. In this case the nucleus remains in the ground state. Range II stands for the quasielastic scattering i.e. direct collisions of leptons with nucleons inside nucleus. Wide maximum in the energy spectrum originates from the own movement of nucleons. Range III of inelastic scattering occurs when transfer energy is greater then pion threshold.

On the born level both $\nu$ and $Q^2$ are fixed by the measurements of the scattered photon momentum. Hence, the channel of scattering is fixed too. However, on a level of RC radiated real photon momentum is indefinite, hence, $\nu$ and $Q^2$ are arbitrary so each of three channels contributes to cross section. Integration over the photon phase space may be presented to that in a plane of $\nu$ and $Q^2$. Adding the virtual photon contribution $\sigma^v$, we have for the RC cross section

$$\sigma = \sigma^{in} + \sigma^{el} + \sigma^q + \sigma^v. \quad (2)$$

Here each $\sigma$ denotes the double differential cross section $d^2\sigma/dxdy$, and $x, y$ are usual scaling nucleon variables. $\sigma^{in}$, $\sigma^{el}$, $\sigma^q$ are contributions of radiative tails from continuous spectrum (IRT), of the elastic scattering radiative tail (ERT), of the radiative tail from the quasielastic scattering (QRT) respectively. Also the contribution of electroweak correction calculated in the quark-parton model is contained in $\sigma^{in}$. Both $\alpha$ and $\alpha^2$ corrections are taken into account in $\sigma^{in,el,q,v}$. To separate the contributions we introduce the lower index, f.e. $\sigma^{in} = \sigma^{in}_1 + \sigma^{in}_2$.

All above mentioned contributions are valid in the case of inclusive scattering. However, in semi-inclusive case the transfer energy is above pion threshold, so in RC calculation one have to take into account only $\sigma^{in}$ and $\sigma^v$.

### 2.1 QED formulae for inclusive case

#### 2.1.1 Born contribution

Using (A.4) for the DIS cross section on the Born level, we obtain

$$\frac{d\sigma}{dxdy} = \frac{4\pi\alpha^2 SS_x}{\lambda_s Q^4} \left\{ (Q^2 - 2m^2)3_1 + (SX - M^2Q^2)\frac{3_2}{2M^2} + mMP_L \left(2(Q^2 \xi q - qn k_2 \xi)\frac{3_3}{M^2} + (S_x k_2 \xi - 2 \xi p Q^2)q\frac{3_4}{M^2} \right) + (Q^2 - 2m^2)(Q^2 - 3(qn)^2)\frac{3_5}{M^2} + (SX - M^2Q^2)(Q^2 - 3(qn)^2)\frac{3_6}{2M^2} \right. + \left. \frac{1}{2} (Q^2 + 4m^2 + 12 \eta k_1 \eta k_2) 3_7 - \frac{3}{2} (X \eta k_1 + S \eta k_2)q\eta\frac{3_8}{M^2} \right\}. \quad (3)$$

Here $k_1(k_2), \xi, m$ are initial (final) lepton momentum, polarization vector and its mass respectively. Invariants are defined in a standard way.
\[ S = 2k_1p, \quad X = 2k_2p = (1 - y)S, \quad Q^2 = -(k_1 - k_2)^2 = xyS, \]
\[ S_x = S - X, \quad Q^2_m = Q^2 + 2m^2, \quad S_p = S + X, \quad \lambda_s = S^2 - 4m^2M^2, \]

\( P_L \) is initial lepton polarization degree. An explicit form of hadronic tensor and generalized structure functions \( \mathcal{I}_i \) are presented in Appendix A.1.

The equality (3) is true for the any direction of polarization vectors and is exact: no approximations were made yet.

The 4-vector of target polarization \( \eta \) is the covariant representation of target polarization vector \( \vec{n} \). In the lab. frame \( \eta = (\vec{n}, 0) \), and \( \vec{n} \) can be expanded in three components: parallel to initial lepton momentum \( \vec{k}_1 - \vec{n}_L \), normal to \( \vec{k}_1 \) in scattering plane \( (\vec{k}_1, \vec{k}_2) - \vec{n}_t \) and normal to scattering plane - \( \vec{n}_\perp \). If a target is polarized along \( \vec{n}_L \) (\( \vec{n}_t \)), then we speak about the longitudinal (transverse) polarization. In the third case we speak about the polarization being normal to scattering plane.

Let us build a basis in the 4-dimensional space. The process of inclusive DIS is determined by three vectors of incoming (outgoing) lepton \( k_1(k_2) \) and of incoming nuclei \( p \) defining a hyperplane in the 4-dimensional space. We can choose the orthonormal basis system in this hyperplane \((\eta_L, \eta_T, \eta_\perp)\), where

\[ \eta_L = \lambda_s^{-1/2} \left( 2Mk_1 \right), \]
\[ \eta_T = \lambda_s^{1/2} \frac{(S X + 2M^2Q^2_m))k_1 + \lambda_s k_2 - (SQ^2 + 2m^2S_\perp)p}{\lambda_s^{1/2}(S X Q^2 - m^2S_\perp^2 - M^2Q^4 - 4m^2M^2Q^2)^{1/2}}. \]

Among all possible basis system, our system differs from others in the following: two space-like vectors \( \eta_L \) and \( \eta_T \) in lab. frame have the form \((\vec{n}_L, 0)\), where \( \vec{n}_L, \vec{n}_T \) are above-considered 3-dimensional polarization vectors. The basis vector system is uniquely fixed by this requirement. Basis in the 4-dimensional space is produced by adding to the system a 4-momentum \( \eta_\perp((\vec{n}_\perp, 0) \) in the lab system) orthonormal to the hyperplane.

As a result for any 4-vector \( \eta \) we have expansion

\[ \eta = \frac{\eta p}{M} - (\eta L)\eta L - (\eta T)\eta T - (\eta \perp)\eta \perp. \]

If \( \eta \) is target polarization vector, then for three above-mentioned cases we find: \( \eta = \eta_L \) (longitudinal polarized target), \( \eta = \eta_T \) (transversely polarized target), \( \eta = \eta_\perp \) (target polarized orthogonal to the scattering plane).

Initial lepton is always longitudinally polarized (for experiments considered). Using the expansion (3) for this vector we obtain

\[ \xi = \xi_L = \lambda_s^{-1/2} \left( \frac{S}{m}k_1 - 2mp \right). \]

We note here, that calculation of real photon contribution requires to integrate over \( d^3\vec{k} \) (real photon momentum) some expressions, containing the scalar products of \( k \) and polarization vectors \( \xi \) and \( \eta \). Since scalar products \( k_\xi, k_\eta_L, k_\eta_T \) are easily expressed in terms of invariants, then our treatment allows to eliminate the intricate and tedious procedure of tensor integration used in ref.[1] and significantly simplifies results. This is the most important advantage of the considered treatment.
By applying the ultrarelativistic approximation
\[ m^2, M^2 \ll S, X, Q^2 \]  
and making transfer to scaling variables \( x \) and \( y \) we further find
\[ \frac{d\sigma}{dxdy} = \frac{4\pi\alpha^2S}{Q^4} ((F_1 - \frac{Q_N}{3}b_1)xy^2 + (F_2 - \frac{Q_N}{3}b_2)(1 - y) \]
\[ -PLP_N xy(2 - y)g_1) \]  
in the case of longitudinal polarized target and
\[ \frac{d\sigma}{dxdy} = \frac{4\pi\alpha^2S}{Q^4} ((F_1 + \frac{Q_N}{6}b_1)xy^2 + (F_2 + \frac{Q_N}{6}b_2)(1 - y) \]
\[ -2PLP_N x\sqrt{xy(1 - y)M} \sqrt{S} (yg_1 + 2g_2) \]  
for transverse one.

### 2.1.2 Exact formulae of the lowest order

The model independent RC of the lowest order can be written as the sum of bremsstrahlung and loop effects:
\[ \sigma = \sigma^{in}_1 + \sigma^{el}_1 + \sigma^q_1 + \sigma^v_1. \]  

The explicit form for these contributions was obtained in ref. [2]. For the infrared free sum of \( \sigma^v_1 \) and \( \sigma^{in}_1 \) we have
\[ \sigma^v_1 + \sigma^{in}_1 = \frac{\alpha}{\pi}\delta_v\sigma_o + \sigma_F^{in} = \frac{\alpha}{\pi}(\delta^lR + \delta_{vert} + \delta^l_{vac} + \delta^h_{vac})\sigma_o + \sigma_F^{in}. \]  

\( \sigma_F^{in} \) is the infrared free part of the IRT cross section
\[ \sigma_F^{in} = -\alpha^3 \int_{\tau_{min}}^{\tau_{max}} \frac{d\tau}{\tau} \sum_{i=1}^{8} \left\{ \frac{R_{max}}{R} \left[ \frac{3_i(R,\tau)}{(Q^2 + R\tau)^2} - \frac{3_i(0,0)}{Q^4} \right] \right. \]
\[ + \sum_{j=2}^{k_i} \frac{R_{max}}{R} \int_0^{R_{max}} \frac{dR}{(Q^2 + R^2)^2} \mathcal{I}_i(R,\tau) \right\}. \]

The integration region on variables \( R = 2pk \) and \( \tau = k(k_1 - k_2)/pk \) is sketched on fig. 2a. The limits of integration are defined as
\[ R_{max} = \frac{W^2 - (M + m_\pi)^2}{1 + \tau}, \quad \tau_{max,min} = \frac{S_x \pm \sqrt{\lambda_Q}}{2M^2}, \]
\[ \lambda_Q = S_x^2 + 4M^2Q^2, \quad W^2 = S_x - Q^2 + M^2, \]
where \( m_\pi \) is the pion mass. The explicit form of functions \( \theta_{ij}(\tau) \) is given in Appendix B.
The quantity $\delta^{IR}_{R} + \delta_{\text{vert}}$ appears when the infrared divergence is extracted in accordance with the Bardin and Shumeiko method [3] from $\sigma^{in}$. The virtual photon contribution consists of the lepton vertex correction $\delta_{\text{vert}}$ and the vacuum polarization by leptons $\delta^{\text{vac}}_{l}$ and by hadrons $\delta^{\text{vac}}_{h}$. These corrections are given by formulae (20-25) of ref. [2]. Here we give ultrarelativistic formulae ($m \to 0$) for sum of $\delta^{IR}_{R}$ and $\delta_{\text{vert}}$:

$$
\delta^{IR}_{R} + \delta_{\text{vert}} = \delta_{\text{inf}} + \frac{3}{2}l_{m} - 2 - \frac{1}{2} \ln^{2} X + \text{Li}_{2} \frac{SX - Q^{2}M^{2}}{S'X'} - \frac{\pi^{2}}{6},
$$

(15)

where $S' = X + Q^{2}$, $X' = S - Q^{2}$, $l_{m} = \ln Q^{2}/m^{2}$ and $\text{Li}_{2}$ is Spence function (dilogarithm) and

$$
\delta_{\text{inf}} = (l_{m} - 1) \ln \frac{(W^{2} - (M + m_{e})^{2})^{2}}{S'X'}.
$$

(16)

In the case of elastic scattering the nucleus remains in the ground state, so we have an additional relation

$$
R = R_{el} = (S_{xA} - Q^{2})/(1 + \tau_{A})
$$

(17)

resulting in

$$
\sigma_{1}^{el} = \frac{1}{A} d^{2}\sigma_{1}^{el} = -\frac{\alpha^{3}y}{A^{2}} \int_{\tau_{\text{min}}}^{\tau_{\text{max}}} d\tau_{A} \sum_{i=1}^{8} \sum_{j=1}^{k_{i}} \theta_{ij}(\tau_{A}) \frac{2M^{2}R_{el}^{j-2}}{(1 + \tau_{A})(Q^{2} + R_{el}\tau_{A})^{2}} \Xi_{i}^{el}(R_{el}, \tau_{A}).
$$

(18)

Here invariants with the index "A" contain the nucleus momentum $p_{A}$ instead of $p$ ($p_{A}^{2} = M_{A}^{2}$, $M_{A}$ is nucleus mass). The quantities $\Xi_{i}^{el}$ are given in Appendix A.1.

Quasielastic scattering corresponds to direct collisions of leptons with nucleons inside nucleus. Due to self movement of nucleons we have no additional relation like (17). As a result we have to integrate numerically both over $R$ and $\tau$

$$
\sigma_{1}^{q} = -\frac{\alpha^{3}y}{A} \int_{\tau_{\text{min}}}^{\tau_{\text{max}}} d\tau \sum_{i=1}^{s} \sum_{j=1}^{k_{i}} \theta_{ij}(\tau) \int_{R_{\text{min}}}^{R_{\text{max}}} dR \frac{R^{j-2}}{(Q^{2} + R\tau)^{2}} \Xi_{i}^{q}(R, \tau).
$$

(19)

The quantities $\Xi_{i}^{q}$ can be obtained in the terms of quasielastic structure functions (so-called response functions, see Appendix A.1 for explicit result), which have a form of the peak for $\omega = Q^{2}/2M$. Due to the absence of enough experimental data the fact is normally used for construction of the peak type approximation. The factors at response functions are estimated at the peak, and subsequent integration of response functions leads to results in terms of suppression factors $S_{E,M,EM}$ (or of sum rules for electron-nucleus scattering [14]):

$$
\sigma_{1}^{q} = -\frac{\alpha^{3}y}{A} \int_{\tau_{\text{min}}}^{\tau_{\text{max}}} d\tau \sum_{i=1}^{4} \sum_{j=1}^{k_{i}} \theta_{ij}(\tau) \frac{2M^{2}R_{q}^{j-2}}{(1 + \tau)(Q^{2} + R\tau)^{2}} \Xi_{i}^{q}(R_{q}, \tau).
$$

(20)

To take into account the effect of radiation of many soft photons a special procedure of exponentiation was applied [11]. In the code it is realized by the following substitutions:

$$
\sigma_{1}^{el} \to \left(\frac{y^{2}(1 - x/A)^{2}}{1 - xy/A}\right)^{t_{r}} \sigma_{1}^{el}, \quad \sigma_{1}^{q} \to \left(\frac{y^{2}(1 - x)^{2}}{1 - xy}\right)^{t_{r}} \sigma_{1}^{q},
$$

(21)

$$(1 + \frac{\alpha}{\pi} \delta_{o})\sigma_{o} \to \exp\left(\frac{\alpha}{\pi} \delta_{inf}\right) (1 + \frac{\alpha}{\pi}(\delta_{o} - \delta_{inf}))\sigma_{o},$$

where $t_{r} = \frac{\alpha}{\pi}(l_{m} - 1)$. 

7
2.1.3 Ultrarelativistic approximation

To simplify and accelerate the procedure of experimental data processing when rapid analysis is more important than accuracy, it is convenient to have the approximate formulae. In the case of RC calculation one can choose ultrarelativistic approximation:

\[ m^2, M^2 \ll S, X, Q^2, \]  

(22)

that allows to calculate exactly first two terms (corrections \( \sim \alpha m \) and \( \sim \alpha \)) of expansion over the leptonic mass \( m \) of the lowest order cross section.

The expressions derived under such an approximation are compact and have good accuracy. Also this approach allows to avoid numerical uncertainties when the value results from the difference between two large and sometimes infinite quantities, that is especially significant when quadruple polarization is considered.

A. Inelastic radiative tail

Considering \( \tau \)-dependence of quantities \( \theta_{ij}(\tau) \) in (13) one can see its peaking structure, so called \( s \)- and \( p \)-peaks \([12]\) (or \( k_1 \)- and \( k_2 \)-peaks if follow \([11]\)): \( \theta_{ij}(\tau) \sim \theta_{ij}^s(\tau) + \theta_{ij}^p(\tau) \). Using the identities

\[ \theta_{ij}^s(\tau) = \frac{\theta_{ij}^s(\tau)}{\theta_{1s}(R, \tau) - \theta_{1s}(R, \tau_s)} + \theta_{ij}^s(\tau) \theta_{1s}(R, \tau_s), \]

\[ \theta_{ij}^p(\tau) = \frac{\theta_{ij}^p(\tau)}{\theta_{1p}(R, \tau) - \theta_{1p}(R, \tau_p)} + \theta_{ij}^p(\tau) \theta_{1p}(R, \tau_p), \]

(23)

one can extract and analytically integrate the terms corresponding the mass singularities. The first terms in right-hand sides of (23) are free from mass singularities and so they contribute only to \( \sim \alpha \) correction. So one can adopt \( m^2 = M^2 = 0 \) before the integration over \( \tau \) (or photon radiation angles). SF’s do not depend on \( \tau \) in the rest terms. Hence, the last ones can be integrated analytically using the methods \([13]\) and contribute to the leading correction \( \sim \alpha l_m \).

Results for the infrared free sum of contributions from inelastic radiative tail and loop effects could be presented as following

\[ \sigma_{in}^1 + \sigma_{in}^1 = \frac{\alpha}{\pi} \delta_{in}^1 \sigma_0 + \sigma_r^V + \sigma_{in}^s + \sigma_{in}^p + \sigma_{in}^r. \]

(24)

Factorizing terms of the total correction have the form

\[ \delta_{in}^1 = \frac{1}{2} [l_m - 1] \delta_{sp} - \ln^2(1 - y) - 1], \]

(25)

where

\[ \delta_{sp} = 2 \ln((1 - z_p)(1 - z_s)) + 3, \]

(26)

The quantity \( \sigma_r^V \) is correction due to vacuum polarization effects by leptons and hadrons:

\[ \sigma_r^V = (\delta_{vac}^l + \delta_{vac}^h) \sigma_0. \]

(27)

Quantities \( \sigma_{s,p}^{in} \) are the contributions including the second terms in right-hand side of (23):

\[ \sigma_{s,p}^{in} = \frac{\alpha}{2\pi} \int_{z_s,p}^{1} \frac{dz}{1 - z} \left( [(1 + z^2 l_m - 2z] \sigma_{s,p} - 2(l_m - 1) \sigma_0 \right), \]

(28)
The quantities $\sigma_{s,p}$ are expressed in terms of born cross section $\sigma_0 = \sigma_0(S, X, Q^2)$:

$$\sigma_s = y\sigma_0(zS, X, zQ^2)/(z - 1 + y), \quad \sigma_p = y\sigma_0(S, X/z, Q^2/z)/z(z - 1 + y).$$

(29)

The low limits of integration in (28) are

$$z_s = \frac{1 - y}{1 - xy}, \quad z_p = 1 - y + xy.$$ 

(30)

The mass singularity free terms in (23) contribute to $\sigma_{r}^{in}$. After splitting the unpolarized, polarized and quadrupolarized parts, we have

$$\sigma_{r}^{in} = 2\alpha^3 y \int \frac{d\xi}{\xi^2} \left( T_u^L + P_L P_N (T_{p\parallel}^L + T_{p\perp}^x) + \frac{Q_N}{3} T_q^L \right)$$

in the case of longitudinally polarized target and

$$\sigma_{r}^{in} = 2\alpha^3 y \int \frac{d\xi}{\xi^2} \left( T_u^L + P_L P_N (T_{p\perp}^L + T_{p\perp}^x) - \frac{Q_N}{6} T_q^L \right)$$

for transversely polarized target. Quantities $T$ are

$$T_u^L = \mathcal{L}_1(F_1) + \frac{1}{\xi Q^4 \xi} \mathcal{L}_2(F_2), \quad T_q^L = \mathcal{L}_1(b_1) + \frac{1}{\xi Q^4 \xi} \mathcal{L}_2(b_2),$$

$$T_{p\parallel}^L = -\frac{1}{Q^4 \xi^2} \mathcal{L}_3(g_1), \quad T_{p\perp}^L = \frac{\xi M}{(S X Q^2)^{1/2}} (\mathcal{L}_4(g_1) + 2\mathcal{L}_5(g_2)/Q^2),$$

$$T_{p\parallel}^x = \frac{2 u (S + u_x) g_1(\xi, t_x)}{u_x X Q^2 \xi}, \quad T_{p\perp}^x = \frac{4 \xi M u (Q^2 g_1(\xi, t_x) + 2 S g_2(\xi, t_x))}{X Q^2 (S X Q^2)^{1/2}},$$

where

$$\mathcal{L}_1(\mathcal{F}) = 2 L^Y - L^L_s - L^L_x + L^L_{x_2} - L^L_t + 2 L_t,$$

$$\mathcal{L}_2(\mathcal{F}) = (T - Q^2 S x) L^Y - (Q^2 X + T) L^k_s + (T - Q^2 S) L^k_x,$$

$$\mathcal{L}_3(\mathcal{F}) = 2 S_p L^Y - (u_x + X) L^k_s - Q^2 L^k_x + (S + u_x) L^k_x - Q^2 L^k_{x_2},$$

$$\mathcal{L}_4(\mathcal{F}) = -4 X L^Y + (2 X - S) L^k_s - u_x L^k_s + (2 Q^2 - u_x) L^k_t$$

$$-(X + 2 u_x) L^k_x - u_x L^k_x - u_x L^k_{x_2} + 2 S_x L_t,$$

$$\mathcal{L}_5(\mathcal{F}) = (S x Q^2 - S^2 p^2) L^Y + (S S_p + 2 X u_x) L^k_s + u_x X L^k_t +$$

$$-(2 S u_x + S_p X) L^k_x - u_x X L^k_{x_2}.$$
The integration region on variables \( q \) and quadrupolarized contributions. For spin 1/2 nuclei where index \( A \) due to strong dependence of formfactors in the case of ultrarelativistic approximation in calculation of RC from elastic radiative tail, the limits of integration are

\[
\begin{align*}
L_t &= \frac{1}{S_x} \int_{t_1}^{t_2} \frac{d\xi}{t} F_i(\xi, t), \\
L_{s,x}^I &= \frac{F_i(\xi, t_s,x)}{u_{s,x}} \ln \frac{u^2_{s,x}}{u Q^2_\xi} + \frac{1}{u_{s,x}} \int_{t_1}^{t_2} \frac{d\xi}{t} \frac{F_i(\xi, t) - F_i(\xi, t_s,x)}{|t - t_s,x|}, \\
L_{s,x}^E &= \frac{F_i(\xi, t_s,x)}{S, X} \ln \frac{u^2_{s,x}}{u Q^2_\xi} + \frac{1}{S, X} \int_{t_1}^{t_2} \frac{d\xi}{t} \frac{t_s,x F_i(\xi, t) - t F_i(\xi, t_s,x)}{tt_s,x|t - t_s,x|}, \\
L_y &= \int_{t_1}^{t_2} \frac{d\xi}{t} \frac{u(t - t_s)(F_i(\xi, t) - F_i(\xi, t_s)) + u(t - t_s)(F_i(\xi, t) - F_i(\xi, t_x))}{(u_s|t - t_s| + u_x|t - t_x|)|t - t_s||t - t_x|}, \\
\tilde{L}_{s,x}^E &= \frac{1}{u_{s,x}} \int_{t_1}^{t_2} \frac{d\xi}{t} \frac{F_i(\xi, t)}{t^2} - \frac{1}{u_{s,x}} \int_{t_1}^{t_2} \frac{d\xi}{t} \frac{F_i(\xi, t)}{t^2}.
\end{align*}
\]

The integration region on variables \( \xi = -q^2/2pq \) and \( t = -q^2 (q = k_1 - k - k_2) \) are plotted on fig. 2b. The limits of integration are

\[
t_{s,x} = Q^2 \{S, X\}/u_{s,x}, \quad t_{2,1} = \frac{u(S_x \pm \sqrt{\alpha_Q}) + 2M^2Q^2}{2(u/\xi + M^2)}.
\]

B. Elastic and Quasielastic radiative tails

In the case of ultrarelativistic approximation in calculation of RC from elastic radiative tail, due to strong dependence of formfactors \( F_i \) on the square of transfer momenta \( Q^2 \), the leading contribution to the total cross section gives only \( t \)-peak (Compton peak) and contributions of \( s- \) and \( p- \) peaks are suppressed \cite{14}. For \( \sigma_{1,0}^A \) we have

\[
\sigma_{1,0}^A = \sigma_u^A + P_L P_N \sigma_p^A + \frac{Q_N}{6} \sigma_q^A,
\]

where index \( A \) corresponds to the considered nuclei and \( u, p, q \) define the unpolarized, polarized and quadrupolarized contributions. For spin 1/2 nuclei \( Q_N = 0 \) and for spin 0 nuclei \( P_N = Q_N = 0 \).

For proton, deuteron and carbon we obtain the results

\[
\begin{align*}
\sigma_u^p &= \frac{\alpha^2}{S} Y_+ \int_{\eta_{\min}}^{\infty} \frac{d\eta_A}{\eta_A} (\bar{X}(F_1^2 + \eta_A F_2^2) - (F_1 + F_2)^2), \\
\sigma_p^p &= \frac{\alpha^2}{S} Y_- \int_{\eta_{\min}}^{\infty} \frac{d\eta_A}{\eta_A} (F_1 + F_2)(\bar{X}F_1 - (F_1 + F_2)), \\
\sigma_u^d &= \frac{\alpha^2}{S} Y_+ \int_{\eta_{\min}}^{\infty} \frac{d\eta_A}{\eta_A} \left((F_c^2 + \frac{8}{9} F_q^2 \eta_A^2 + \frac{8}{3} F_m^2 \eta_A) \bar{X} - \frac{2}{3} (1 + \eta_A) F_m^2\right), \\
\sigma_p^d &= \frac{\alpha^2}{S} Y_- \int_{\eta_{\min}}^{\infty} \frac{d\eta_A}{\eta_A} F_m \left(\frac{1}{2} (1 + \eta_A) F_m - (F_c + \frac{1}{3} F_q \eta_A + \frac{1}{2} F_m \eta_A) \bar{X}\right).
\end{align*}
\]
σ₉ = \frac{α^3}{S} Y_+ \int_{η_{min}}^\infty \frac{dη_A}{η_A} \left\{ (1 + η_A + (\frac{3}{2}x^2 - η_A)\tilde{X})F_m^2 - \frac{X_1}{(1 + η_A)}F_q(3F_c + 3η_AF_m + η_AF_q) - 2η_A\tilde{X}F_q(4F_c - 3xF_m + \frac{4}{3}η_AF_q) \right\}.

σ^C = \frac{α^3}{S} Z^2 Y_- \int_{η_{min}}^\infty \frac{dη_A}{η_A} \tilde{X}F^2,

for the case of longitudinally polarized target, where

\begin{align*}
X_1 &= x^2 + 4xη_A - 4η_A, \quad \tilde{X} = \frac{X_1}{2η_Ax^2}, \quad Y_\pm = \frac{1 \pm (1 - y)^2}{1 - y},
\end{align*}

(39)

\begin{align*}
η_A &= \frac{t}{4M_A^2}, \quad η_{min} = \frac{x^2}{4(1 - x)},
\end{align*}

and formfactors

\begin{align*}
F_1 &= \frac{G_E + η_AG_M}{1 + η_A}, \quad F_2 = \frac{G_M - G_E}{1 + η_A}.
\end{align*}

(40)

Polarized contribution in the case of transversely polarized target is proportional to nuclear mass and therefore equals to zero in the case of ultrarelativistic approximation (22). Nevertheless we present an explicit formulae for the first nonzero order correction:

\begin{align*}
σ_p &= \frac{α^3xy^2}{S(1 - y)^{3/2}} M \int_{η_{min}}^\infty \frac{dη_A}{η_A} \left\{ G^A_1 \left( \bar{y}_1(2 + \frac{x}{yη_A}) - \frac{(2 - y)x}{y}\tilde{X} \right) + G^A_2 \left( (x + 2η_A - 2x\bar{y}_1^2 + 6\frac{η_A\bar{y}_1}{y})\tilde{X} - \frac{4\bar{y}_1}{xy}(1 + η_A) \right) \right\}.
\end{align*}

(41)

where \( \bar{y}_1 = 1/y - 1 \). The cross section dependence on nuclei formfactors are contained in quantities \( G \). For protons and deuterons we take

\begin{align*}
G^p_1 &= (F_1 + F_2)^2, \quad G^p_2 = F_2(F_1 + F_2),
\end{align*}

\begin{align*}
G^d_1 &= -\frac{1}{2}(1 + η_A)F_m^2, \quad G^d_2 = F_m(F_c + \frac{1}{3}η_AF_q - \frac{1}{2}F_m).
\end{align*}

(42)

Also we present relations between quadrupolization parts for the cases of longitudinally and transversely polarized targets:

\begin{align*}
σ_q^d = \sigma_q^d = -\frac{1}{2}σ_q^d = \frac{1}{2}σ_q^d.
\end{align*}

(43)

For the calculation of the contribution to RC from the quasielastic radiative tail in the case of ultrarelativistic approximation we use the results for the proton target for elastic radiative tail replacing

\begin{align*}
F^2_e(Q^2) \rightarrow S_e(Q^2)F^2_e(Q^2), \quad F^2_m(Q^2) \rightarrow S_m(Q^2)F^2_m(Q^2),
\end{align*}

\begin{align*}
F_e(Q^2)F_m(Q^2) \rightarrow S_{em}(Q^2)F_e(Q^2)F_m(Q^2).
\end{align*}

(44)
2.1.4 Higher order effects

There are no known reasons to consider the $O(\alpha^2)$ corrections to be negligible. We follow the method of structure functions used in \cite{15} for the calculation of the higher order electromagnetic radiative corrections to neutral current unpolarized lepton-proton DIS and generalize it for the case of polarized leptons and polarized nuclei target in the current version of POLRAD. For the case of $s$- and $p$-peaks the formulae could be obtained in the terms of the Born cross section and practically coincide with the expressions for unpolarized particles, however the contribution of the $t$-peak which is extremely important in the cases of elastic and quasielastic radiative tails has to be obtained for polarized DIS.

The sum of second order inelastic correction and correction due to loop effects has the form

$$
\sigma_2^{in} + \sigma_2^V = \delta_2^{in}\sigma^0 + \sigma_{Vs}^{in} + \sigma_{Sp}^{in} + \sigma_{\sigma}^{in} + \sigma_{l_{sp}}^{in} + \sigma_{\sigma_{l_{sp}}}^{in} + \sigma_{f_{sp}}^{in} + \sigma_{f_{ps}}^{in},
$$

where factorized part is

$$
\delta_2^{in} = \frac{\alpha^2}{4\pi^2} \left( 3\delta^4(Q^2) + 2l_m\delta_{sp}\delta(Q^2) + \frac{1}{2}l_m^2[\delta_{sp}^2 + 4L_2(1 - z_p) + 12L_2(1 - z_s) - \frac{8}{3}\pi^2] \right),
$$

and $\delta(Q^2) = \delta_{vac}^l + \delta_{vac}^t$.

The contribution from vacuum polarization if coincide with real photon radiation has the form

$$
\sigma_{Vs,Vp}^{in} = \frac{\alpha^2}{2\pi^2} l_m \int_{z_{s,p}}^{1} \frac{dz}{1 - z} \left( (1 + z^2)\delta(t_{x,s})\sigma_{s,p} - 2\delta(Q^2)\sigma_0 \right],
$$

where $t_x = zQ^2$ and $t_s = Q^2/z$.

Next three terms correspond to the cases when two radiated photons are collinear to incident electron ($\sigma_{ss}$), outgoing electron ($\sigma_{pp}$)

$$
\sigma_{pp,ss}^{in} = \frac{\alpha^2}{8\pi^2} l_m \int_{z_{p,s}}^{1} \frac{dz}{1 - z} \left( \frac{2}{1 - z}(2\ln(1 - z)(1 - z_{p,s}(z)) - \ln z + 3)(1 + z^2)\sigma_{p,s} - 2\sigma_0 \right]
$$

$$
+ ((1 + z)\ln z - 2(1 - z))\sigma_{p,s},
$$

or when one photon is radiated in incident electron direction and the other in the outgoing electron direction ($\sigma_{sp}$):

$$
\sigma_{sp}^{in} = \frac{\alpha^2}{4\pi^2} l_m \int_{z_s}^{1} \frac{dz_1}{1 - z_1} \int_{z_{p}(z)}^{1} \frac{dz_2}{1 - z_2} \left[ (1 + z_1^2)(1 + z_2^2)\sigma_{sp} - 2(1 + z_1^2)\sigma_s - 2(1 + z_2^2)\sigma_p + 4\sigma_0 \right].
$$

Here

$$
z_s(z) = (1 - y)/(z - xy), \quad z_p(z) = (1 - y + xy)/z,
$$

and $\sigma_{s,p}$ depend on $z_{1,2}$ and are given by \cite{29} with $z \to z_{1,2}$ respectively. The quantity $\sigma_{sp}$ depends on both $z_1$ and $z_2$ and also is calculated in terms of born cross section $\sigma_0$:

$$
\sigma_{sp} = y\sigma_0(z_1S, X/z_2, z_1Q^2/z_2)/(z_2(z_2z_1 - 1 + y)).
$$
There are two channels (singlet and non-singlet) of fermion pair production that give a contribution to $\alpha^2$ order RC. The singlet channel

$$\sigma_{ip,ls}^{in} = \frac{\alpha^2}{8\pi^2} l_m^2 \int_{z_{p,s}}^{1} dz (2(1 + z) \ln z + 1 - z + \frac{4}{9}(1 - z^3)z) \sigma_{p,s}$$

(52)

corresponds to the case when incident and outgoing lepton as well as leptons of the unregistered pair belong to different leptonic lines connected by additional virtual photon.

The rest non-singlet part

$$\sigma_{fp,fs}^{in} = \frac{\alpha^2}{12\pi^2} \sum_f \ln^2 \frac{Q^2}{m_f^2} \int_{z_{p,s}}^{1} dz \frac{(1 + z^2)}{(1 - z)} \sigma_{p,s}$$

(53)

arises from the two-lepton decay of additional virtual photon.

The main contribution to second order elastic and quasielastic radiative tail arises when additionally radiated photon is collinear to incident or outgoing fermion line:

$$\sigma_{el}^{2} = \frac{\alpha}{2\pi} l_m \delta_{sp} \sigma_{el}^{1} + \sigma_{el}^{V} + \sigma_{el}^{pt} + \sigma_{el}^{st},$$

$$\sigma_{el}^{s,p t} = \frac{\alpha}{2\pi} l_m \int_{z_{s,p}}^{1} dz \frac{(1 + z^2)(\sigma_{s,p} - 2\sigma_{1}^{el})}{1 - z},$$

(54)

where the quantities $\sigma_{el}^{s,p}$ are obtained in terms of approximate elastic radiative tail (37) $\sigma_{el}^{1} = \sigma_{el}^{1}(x, y, S)$:

$$\sigma_{s}^{el} = y\sigma_{el}^{1}(xyz/(z + y - 1), (z + y - 1)/z, zS)/(z - 1 + y),$$

$$\sigma_{p}^{el} = y\sigma_{el}^{1}(xy/(z + y - 1), (z + y - 1)/z, S)/z(z - 1 + y).$$

(55)

The integral over $z$ in (54) can be calculated explicitly. For longitudinally polarized target the results have the form of eq.(38)

$$\sigma_{u,p}^{A} = \frac{\alpha^4 l_m}{2\pi S} \int_{\eta_{min}}^{\infty} \frac{d\eta_A}{\eta_A} \left( F_{u,p}^{A1} R_{1}^{u,p} + F_{u,p}^{A2} R_{2}^{u,p} \right),$$

(56)

where the quantities $R_{1,2}^{u,p}$ are given in Appendix C. The functions $F_{u,p}^{A1,2}$ are quadratic combinations of nuclear formfactors and could be found by comparison with (38) which can be written in the common form

$$\sigma_{u,p}^{A} = \frac{\alpha^3}{S} Y_{\pm} \int_{\eta_{min}}^{\infty} \frac{d\eta_A}{\eta_A} \left( \{\hat{X}, x\hat{X}\} F_{u,p}^{A1} + F_{u,p}^{A2} \right).$$

(57)

The correction due to vacuum polarization $\sigma_{el}^{V}$ is defined by formulae (57) and (38) with additional factor $\frac{\alpha}{\pi} \delta(4M_A^2\eta_A)$ under integral.
2.2 Electroweak radiative correction

The next evident step both from the theoretical and experimental points of view is the treatment of electroweak effects contribution. So we included in POLRAD 2.0 the results of ref.[16] for one-loop electroweak correction within the framework of standard theory and the on-shell renormalization scheme in t’Hooft-Feynman gauge. The result for the correction is obtained as the sum of loop and radiative effects

\[
\sigma_{1\text{ew}}^{in} + \sigma_{1\text{ew}}^{v} = \sigma_{B}^{S} + \sigma_{Vl} + \sigma_{Vq} + \sigma_{\text{box}} + \frac{\alpha}{\pi} \sum_{q} \delta q \sigma_{q}^{0} + \sum q \sigma_{R}^{q}, \tag{58}
\]

where \(\sigma_{B}^{S}\) is the correction to boson propagator, \(\sigma_{Vl,q}\), \(\sigma_{\text{box}}\) are infrared free parts of lepton and quark vertex functions and box graphs. The loop correction is calculated on the basis of ref.[17]. The quantity \(\sigma_{R}^{q}\) is an infrared free part of the real photon emission cross section. The correction \(\delta q\) is obtained after infrared divergence cancelation. It is factorized front of the born cross section on a quark \(\sigma_{q}^{0}\) and is an analog of quantity \(\delta_{IR}^{R}\) in (12). For radiative effect the methods developed in ref.[8] are applied. The quantity \(\sum q \sigma_{R}^{q}\) can be derived in terms of leptonic \((\sigma_{ij}^{l}, \hat{\sigma}_{ij}^{l})\), hadronic \((\sigma_{ij}^{h}, \hat{\sigma}_{ij}^{h})\) radiation and their interference \((\sigma_{ij}^{lh})\):

\[
\sigma_{R}^{q} = \sum_{ij=\gamma,Z} \left\{ \sigma_{ij}^{l} + \hat{\sigma}_{ij}^{l} + e_{q} \sigma_{lh}^{q} + e_{q}^{2} \left( \sigma_{h}^{l} + \hat{\sigma}_{h}^{l} \right) \right\}, \tag{59}
\]

The quantities \(\sigma_{lh,h}^{ij}\) have the form of one-dimensional integrals over \(\xi\)

\[
\sigma_{b}^{ij} = \frac{\alpha^{3} y}{4} \int_{x}^{1} \frac{d\xi}{\xi} \left\{ R_{V}^{ij} \left[ T_{ij}^{0} + F_{ij}^{0} \right] (\xi) - T_{ij}^{0} + F_{ij}^{0} (x) \right\} + R_{A}^{ij} \left[ T_{ij}^{0} - F_{ij}^{0} \right] (\xi) - T_{ij}^{0} - F_{ij}^{0} (x) \right\}, \tag{60}
\]

where \(T_{ij}^{0}\) are the combinations of kinematical invariants, \(R_{V,A}^{ij}\) and \(F_{V,A}^{0ij}\) are functions of electroweak coupling constants and parton distributions [16]. The hat-quantities in (59) are not large corrections arising from the non-leading terms of the expansion of polarization vectors (see 2.1).

2.2.1 Correction to leptonic current in QCD-improved model

An implementation of QCD-improved parton model for the most important case of leptonic current correction requires an additional generalization of (59, 60) valid for the simple parton model and cannot be generated directly because an analytical integration over \(Q_{h}^{2}\) been already done.

As a result correction takes the same form as eqn.(24):

\[
\sigma_{1\text{lept}}^{in} + \sigma_{1\text{lept}}^{v} = \sigma_{in}^{l} + \sigma_{in}^{l} = \frac{\alpha}{\pi} \delta_{in}^{l} \sigma_{0} + \sigma_{r}^{V} + \sigma_{s}^{in} + \sigma_{p}^{in} + \sigma_{r}^{in}. \tag{61}
\]

The quantities \(\sigma_{in}^{l,p}\) and \(\delta_{in}^{l}\) are defined by (28) and (25). The Born contribution can be written as

\[
\sigma_{0} = \sum_{i,j=\gamma,Z} \sigma_{0}^{ij} = \sum_{i,j=\gamma,Z} \frac{\pi \alpha^{2}}{2 Q^{4}} X \left\{ Y_{+} R_{+}^{ij} F_{+}^{ij} (x, Q^{2}) + Y_{-} R_{-}^{ij} F_{-}^{ij} (x, Q^{2}) \right\}. \tag{62}
\]

Here the following notation is introduced in (A.10).
The contribution of electroweak loops can be found in the form
\[ \sigma_r^V = \sigma_0(R^{ij}_\pm \rightarrow \delta R^{ij}_\pm), \] (63)
where
\[
\delta R^{\gamma\gamma}_\pm = -2\Pi^{\gamma\gamma}_\pm R^{\gamma\gamma}_\pm - 2\Pi^{Z\gamma}_\pm \chi R^{Z\gamma}_\pm
+ \frac{\alpha}{4\pi} \left[ \lambda^{\gamma\gamma}_V R^{Z\gamma\gamma}_\pm \Lambda_2(-Q^2, M_Z) + v_i^\gamma(1 - P_L) \frac{3}{s^2_w} \Lambda_3(-Q^2, M_W) \right],
\]
\[
\delta R^{Z\gamma}_\pm = \delta R^{Z\gamma}_\pm = -2(\Pi^{\gamma}_\pm + \Pi^{Z}_\pm)(R^{\gamma\gamma}_\pm + R^{Z\gamma}_\pm) - \Pi^{Z\gamma}_\pm(R^{\gamma\gamma}_\pm + \chi R^{Z\gamma}_\pm)
+ \frac{\alpha}{4\pi} \left[ (\lambda^{\gammaZ}_V R^{Z\gamma\gamma}_\pm + \lambda^{\gammaZ}_A R^{ZZ\gamma}_\pm) \Lambda_2(-Q^2, M_Z) - 2\Pi^{\gamma}_{\pm} \Lambda_2(\frac{Q^2}{4s^2_w} + \frac{3}{2s^2_w} (v_i^\gamma + a_i^\gamma - \frac{c_w}{s_w} v_i^\gamma) \Lambda_3(-Q^2, M_W) \right],
\]
\[
\delta R^{ZZ}_\pm = -2\Pi^{Z\gamma}_\pm R^{Z\gamma\gamma}_\pm - 2\Pi^{Z\gamma}_\pm R^{Z\gamma}_\pm
+ \frac{\alpha}{4\pi} \left[ (\lambda^{\gammaZ}_V R^{Z\gamma\gamma}_\pm + \lambda^{\gammaZ}_A R^{ZZ\gamma}_\pm) \Lambda_2(-Q^2, M_Z) - 2\Pi^{\gamma}_{\pm} \Lambda_2(\frac{Q^2}{4s^2_w} + \frac{3}{2s^2_w} (v_i^\gamma + a_i^\gamma) \Lambda_3(-Q^2, M_W) \right].
\] (64)
(65)

Here \( M_{Z,W} \) are masses of \( Z \) and \( W \) bosons and
\[
\Pi^{\gamma} = -\frac{\hat{\Sigma}^\gamma(-Q^2)}{Q^2}, \quad \Pi^{Z} = -\frac{\hat{\Sigma}^Z(-Q^2)}{Q^2 + M^2_Z}, \quad \Pi^{\gamma Z} = -\frac{\hat{\Sigma}^{\gamma Z}(-Q^2)}{Q^2}. \] (66)

Quantities \( \hat{\Sigma}^{\gamma,\gamma Z,Z} \) are defined by formulae (A.2,3.17,B.2-5) of \[18\] and \( \Lambda_2,3 \) by (B.4,B.6) of \[17\].

In the electroweak case the term \( \sigma_r^{in} \) has form
\[
\sigma_r^{in} = \sum_{i,j=\gamma,Z} \alpha^3 y \frac{1}{x} \int \frac{d\xi}{\xi^2} \left[ R^{ij}_+ T^{ij}_+ + R^{ij}_- T^{ij}_- + P_L(\lambda^{ij}_A \hat{T}^{ij}_+ + \lambda^{ij}_V \hat{T}^{ij}_-) \right],
\] (67)
where
\[
T^{ij}_+ = \frac{1}{4\xi Q^2_\xi} [Q^2_\xi L_1(F^{ij}_+) + 2L_2(F^{ij}_+)],
\] \[
T^{ij}_- = \frac{1}{4\xi Q^2_\xi} L_3(F^{ij}_-)
\] \[
\hat{T}^{ij}_\pm = -\frac{u(S^2 + u_x^2)}{2\xi u_x \lambda Q^2_\xi} F^{ij}_\pm.
\] (68)

Functions \( L_{1,2,3} \) are defined in \([14]\).
2.2.2 Correction to hadronic current

Exact formulae for correction to hadronic current could be obtained from (60) when \( b = h \) and could be easily generated for the case of QCD-improved model (see also ref.\[19\]).

Result for leading log approximation could be presented in standard form \[20\]:

\[
\sum_q e_q^2 \sigma_{ij}^q = \sigma_0^{ij} \left( f_q^+(x) \to f_q^{+\text{rad}}(x) \right),
\]

where

\[
f_q^{+\text{rad}}(x) = e_q^2 \frac{\alpha}{2\pi} \ln \frac{Q^2}{m_q^2} \left\{ -f_q^+(x) \ln \frac{Q^2}{(1-x)^2m_q^2} \right. \\
\left. + \int_x^1 \frac{dz}{z} \left( \frac{1+z^2}{1-z} f_q^+(x/z) - \frac{2}{1-z} f_q^+(x) \right) \right\}.
\]

Sometimes fits for partonic distributions are constructed from the data extracted without taking into account the of hadronic current correction. Therefore if such a fit is used in the calculation the correction does not have to be taken into account.

2.3 Semi-inclusive physics

We calculate the radiative corrections to data of semi-inclusive polarized experiments when a hadron is detected in coincidence with the outgoing lepton. In this case the cross section depends additionally on variable \( z \) defined as

\[
z = \frac{p_1 p_2}{p_1 q},
\]

where \( p_1, p_2 \) \((p_2^2 = m_h^2)\) and \( q \) are 4-momenta of initial nucleus, coincident hadron and virtual photon. This variable corresponds to the amount of virtual photon energy transmitted to measured hadron in lab frame.

For the Born cross section of semi-inclusive DIS we use the formula

\[
\sigma_0^s \equiv \frac{d^3 \sigma_0}{dxdydz} = \frac{2\pi \alpha^2}{S_{xy}} \left[ F_0^u \Sigma^+(x, z) + P_L P_N F_0^p \Sigma^-(x, z) \right]
\]

where

\[
F_0^u = 2(1/y - 1 - \mu_N x) + y, \quad F_0^p = y - 2, \quad \mu_N = M^2/S
\]

and \( \Sigma^{+(-)}(x, z) \) are defined in A.2.

The lowest order QED correction was calculated in ref.\[4, 21\] and can be written as the sum of factorizing and non-factorizing parts

\[
\sigma_{EM} \equiv \frac{d^3 \sigma_{EM}}{dxdydz} = \sigma_R^F + \frac{\alpha}{\pi} \delta_R \sigma_0^s + \sigma_r^V,
\]

where

\[
\delta_R = \delta_V + \delta^{IR}_R = \delta^s_{inf} - \frac{1}{2} \ln^2 \frac{r_3}{r_4} + \frac{3}{2} l_m - 2 + \text{Li}_2(r_2/r_1) - \frac{\pi^2}{6},
\]

\[
\delta^s_{inf} = (l_m - 1) \ln(t_{i1}^2/r_1), \quad t_{i1} = \min\{t_{1m}, t_{2m}/r_+\}
\]
and
\[
t_{1m} = y(1-x) + \mu_N - (M + m_h)^2/S, \quad r_\pm = [y + 2\mu_N \pm (y^2 + 4\mu_N xy)^{1/2}]/2r_9,
\]
\[
t_{2m} = y(1-z), \quad r_1 = 1 - x^2y^2 - yr_4, \quad r_2 = r_5 - yr_8, \quad r_3 = 1 - y(1-x),
\]
\[
r_4 = 1 - xy, \quad r_5 = 1 - y, \quad r_8 = \mu_Nx, \quad r_9 = \mu_N + y - xy.
\]

The finite part of (72) has the form
\[
\sigma_R^F = \frac{2\alpha^3}{S} \int_{t_{1d}}^{t_{2d}} \int_{t_{1u}}^{t_{2u}} \frac{y^2}{xy_{t}}[F_R^{\mu}(\bar{x}, \bar{z}) + PLP_N F_R^p\Sigma^{-}(\bar{x}, \bar{z}) - \theta(t_{1u} - t_{1})[F_R^{\mu}\Sigma^{+}(x, z) + PLP_N F_R^p\Sigma^{-}(x, z)]]
\]
where the integration region is defined as (see fig.2)
\[
t_{1d} = 0, \quad t_{1u} = \min(t_{1m}, t_{2m}/r_-),
\]
\[
t_{2d} = t_{1r_-}, \quad t_{2u} = \min(t_{2m}, t_{1r_+})
\]
and we select the unpolarized \(F_R^{\mu}\) and polarized \(F_R^p\) terms:
\[
F_R^{\mu} = (x_1/y_1)[\{ -2\mu I^2 - I^1 + xy\bar{I}\}(r_5(1-t_2) - \mu_N x_t + y_t^2/2) +
\]
\[
[-2\mu\bar{I}^2 + \bar{I}^1 + xy\bar{I}](r_5 + t_2 - \mu_N x_t + y_t^2/2) -
\]
\[
-(1 + yr_8)I^1 + (r_5^2 + yr_8)\bar{I}^1 - 2\mu_N I^0]\]
\[
+ \frac{1}{2}xy(I^1 - \bar{I}^1) + I^0,
\]
and
\[
F_R^p = (1/y_t)[Gxy_{t} - 2x_1] - y_{t}(t_1 - t_2)\bar{I}^2 + x^2 y^2 x_t^2 I^2 +
\]
\[
[xy(x + y_t - 1) + (t_1 - t_2)(y_t/2 - 1)]I^1-
\]
\[
-xy(x^2 - 1) + (t_1 - t_2)(y_t^2/2 - 1)]I^1\}
\]
In displayed above formulae
\[
\mu = m^2/S, \quad y_{t} = y + t_2, \quad x_t = xy + t_1 - t_2, \quad y_t = y - t_2,
\]
\[
F_R^{(u,p)} = F_0^{(u,p)}G, \quad G = -\mu(I^2 + \bar{I}^2) + xy\bar{I}.
\]
Here we use the formulae
\[
I^0 = \bar{I}^0 = \Delta^{1/2}, \quad I^1 = \{A^2 + C\}^{-1/2}, \quad \bar{I}^1 = \{B^2 + C\}^{-1/2},
\]
\[
I^2 = y[(1 + 2r_8)t_1 - (r_7 + xr_6)t_2](I^1)^3,
\]
\[
\bar{I}^2 = y[(r_5 - 2r_8)t_1 - (r_5 r_7 - 2r_6)r_2](\bar{I}^1)^3.
\]
where
\[ A = t_1 - r_4t_2, \quad B = r_5t_1 - r_3t_2, \quad C = 4\mu(r_6t_1t_2 - \mu_N t_1^2 - r_9t_2^2), \]
and
\[ r_6 = y + 2\mu_N, \quad r_7 = 1 - 2x. \] (83)

To take into account the higher order contribution of soft photons the standard exponentiation procedure [11] is used
\[ \delta_{EM} \to \delta_{exp} \equiv \exp[(\alpha/\pi)\delta_{inf}]\delta_{EM}, \] (85)
so that
\[ \sigma^H \equiv \sigma^S_0(1 + \delta_{exp}). \] (86)

Since the experimental analysis is often performed for all pions with \( z > z_0 \) we have to integrate the expression (86) over \( z \). As \( z \)-dependence is contained only in quantities \( D^H_q \) and integration limits (78), then one can use the following identity
\[ \int_{z_0}^{1} dz \int_{\Omega} dt_1 dt_2 \sum_q \varphi_q D^H_q(\tilde{z}) = \int_{\Omega_0} dt_1 dt_2 \sum_q \varphi_q \tilde{D}^H_q(z_0, \tilde{y}_t/y), \] (87)
where
\[ \tilde{D}^H_q(z_0, \zeta) = \zeta \int_{z_0/\zeta}^{1} dz D^H_q(z), \] (88)
and \( \varphi_q \equiv \varphi_q(S, x, y, t_1, t_2) \) is an arbitrary function. Hence, substituting some new effective fragmentation function \( \tilde{D}^H_q(z_0, \tilde{y}_t/y) \) instead of \( D^H_q(\tilde{z}) \) and \( \tilde{D}^H_q(z_0, 1) \) instead of \( D^H_q(z) \) one can represent the integration over \( z \) in \( \sigma^H \) in the form of eq. (86) and get
\[ \sigma^H(x, y) \equiv \int_{z_0}^{1} dz \sigma^H = \sigma^S_0(1 + \delta_{exp}) \left\{ \begin{array}{l} D^H_q(\tilde{z}) \to \tilde{D}^H_q(z_0, \tilde{y}_t/y) \quad \Omega \to \Omega_0 \\ D^H_q(z) \to \tilde{D}^H_q(z_0, 1) \end{array} \right\}. \] (89)

2.3.1 Experimental cuts

In the previous section the integrated in the whole kinematical region over hadron variables \( \phi_H \) and \( p_\perp \) cross section was considered. For the real situation the region of integration is limited by the experimental cuts. To deal with the ones on the angles of registered hadron a special procedure was developed. For cross section we have
\[ \frac{d\sigma_R}{dx dy dz} = \int \frac{1}{2\pi} \frac{d^3k}{2k_0 dz} dp_\perp d\phi_H \theta(\sin^2 \vartheta_{max} - \sin^2 \vartheta) \theta(\sin^2 \vartheta - \sin^2 \vartheta_{min}) \frac{d\sigma}{dx dy dz dp_\perp d\phi_H}, \] (90)
where \( \vartheta \) is the angle between the beam direction (\( \vec{k}_1 \)) and hadron momentum in the lab frame. One can obtain (77) straightway if \( \theta \)-functions in (90) are removed. We note, that (90) is valid when asimutal symmetry of detector is supposed. As the distribution on \( p_\perp \) is unknown it was approximated by \( \delta \)-function
\[ D(\tilde{z}, p_\perp) = D(\tilde{z}) \delta(p_\perp) \frac{1}{2\pi}, \] (91)
according to the normalization condition
\[ \int D(\tilde{z}, p_\perp) dp_\perp d\phi_H = D(\tilde{z}). \] (92)
The presence of δ-function in (91) takes off the integration over $p_\perp$, so the integration over $\phi_H$ becomes trivial as arguments of $\theta-$functions are now not dependent on $p_2$ (and on $\phi_H$), but acquires dependence on photon momenta.

Implying peaking approximation in (90) we preserve only $t_1$ dependence for $\theta-$functions, that allow to carry out analytical integration over one of photon variables. As a result we have the same formula as (90), but we replace

\[ I_{1,2}^0 \longrightarrow \frac{1}{2} I_{1,2}^0 \left\{ \theta(t_1 - r_4(y - d)) + \theta \left( t_1 - \frac{r_3(y - d)}{1 - (y - d)} \right) \right\} , \]

where we take into account, that in nonradiative case the approximation (91) is equivalent to the assumption, that only those hadrons are registered, for which

\[ \beta_{\text{min}} \leq \Delta_3 \leq \beta_{\text{max}} , \]

where

\[ \beta_{\text{min, max}} = \frac{\sin^2 \vartheta_{\text{min, max}}}{2\mu_N} , \quad d = \left\{ \frac{\Delta_3}{\beta_{\text{max}}} \right\}^{1/2} , \]

\[ \Delta_3 = y(-4x\mu_N x - xy - \mu y + x - \mu_N x^2 y) . \]

### 2.4 Iteration procedure of data processing

From the POLRAD beginning particular emphasis has been placed on the procedure of RC of experimental data. In the current version the iteration procedure, which allows to extract Born data sets for cross sections, SF or asymmetries from observables ones taking into account the radiative effects is realized both for the cases of inclusive experiments.

As an example we consider the procedure of radiative correction to extract the structure function $g_1(x)$ from measured spin asymmetry $A^{m}_{1i}$ with error $\epsilon_i$. The spin average structure functions are considered to be constant and $g_2(x)$ equals to 0. The measured asymmetry is defined as

\[ A^{m}_{1i} = \frac{g_1}{F_1} + \Delta A_1(g_1) , \]

where the radiative correction to asymmetry $\Delta A_1$ can be written in terms of spin-average and spin-dependent parts ($\sigma^{u,p}$) of cross sections (2)

\[ \Delta A_1 = \frac{\sigma^u_{0}(g_1) + \sigma^q_{p} + \sigma^{el}_{p} - \sigma^0_{p}(g_1)(\sigma^u_{u} + \sigma^q_{u} + \sigma^{el}_{u})}{\sigma^u_{0}(1 + \delta_v)\sigma^u_{u} + \sigma^q_{u} + \sigma^{el}_{u}} . \]

where $\delta_v = \sigma^v_p/\sigma^0_p = \sigma^v_u/\sigma^0_u$. The Born and inelastic radiative tail polarized parts of cross sections depend on SF $g_1$, and in the last case the dependence is non-trivial. So the equation (96) becomes functional one in $g_1$. This functional equation transforms into a system considering the extraction of $g_1$ in concrete binning over $x$ in $n$ kinematical points $x_i$ ($i = 1, ..., n$):

\[ A^{m}_{1i} = \frac{g_{1i}}{F_1} + \Delta A_1(g_{1j}; j = 1, ..., n) . \]
Usually the iteration methods are used to solve such a system of equations. The variant of iteration formula is ambiguous, but in practice only two types are used. The first and most evident one is to take for \( k \)-th step:

\[
g_{1i}^{(k)} = F_1(A_{1i}^m - \Delta A_1(g_{1j}^{(k-1)}; j = 1, ..., n)).
\]  

(99)

Another possibility to obtain the formulae for iteration procedure arises when both born and radiative correction cross section are separated into spin-averaged and spin-dependent parts. Then for the measured asymmetry we have

\[
A_{1i}^m = \frac{1}{D} \frac{\sigma_0^p + \sigma_1^p}{\sigma_0^u + \sigma_1^u} = \frac{g_1/F_1 + \sigma_1^p/D\sigma_0^u}{1 + \sigma_1^u/\sigma_0^u}.
\]

(100)

Thus we obtain the iteration formulae

\[
g_{1i}^{(k)} = F_1\left[A_{1i}^m\left(1 + \frac{\sigma_1^p}{\sigma_0^u}\right) - \frac{\sigma_1^p(g_{1j}^{(k-1)}; j = 1, ..., n)}{D\sigma_0^u}\right],
\]

(101)

where in right-hand side the dependence on \( g_1 \) is contained only on the level of RC, but not on the Born level.

On the each step of iteration procedure \( g_{1i}^{(k)} \) is fitted with the help of CERNLIB package MINUIT taking into account an experimental uncertainty \( \epsilon_i(g_1) = \epsilon_i(A_{1i}^m)F_1 \). On the first step we adopt \( g_{1i}^{(0)} = A_{1i}^m F_1 \). The procedure converges within 4-5 steps. As a result we extract values of \( g_{1i} \) and parameters of its fit.

Besides, package MINUIT is used to fit the data with the account of experimental uncertainties that gives the opportunity to theoretically calculate the error propagation of statistical uncertainty of fitted experimental data to the value of \( \Delta A \):

\[
\epsilon_{i}^{ext} = \left[ \epsilon_i^2(1 + \bar{\delta}_{iu} - \bar{\delta}_{ip})^2 + \sum_{j \neq i} \bar{\delta}_{jp}^2 \epsilon_j^2 \right]^{1/2},
\]

\[
\bar{\delta}_{iu} = \frac{\sigma_{iu}^u + \sigma_{iu}^p + \sigma_{il}^u}{\sigma_{iu}^u + \sigma_{iu}^1} \sum_{k=1}^{N_p} \frac{\partial \sigma_0^p}{\partial p_k} \frac{\partial \sigma_0^p}{\partial A_{j}^m} p_k,
\]

\[
\bar{\delta}_{ip} = \frac{1}{\sigma_{iu}^0 + \sigma_{iu}^1} \sum_{k=1}^{N_p} \frac{\partial \sigma_{in}^p}{\partial p_k} \frac{\partial \sigma_{in}^p}{\partial A_{j}^m} p_k.
\]

(102)

The sum runs over parameters \( p_k \) of fitting function \( f(x) \). The first derivative is calculated by direct calculation of \( \sigma_{in}^p \) by POLRAD with using \( \frac{\partial f(x)}{\partial p_k} \) instead of model \( A_1 \) (or \( g_1 \)). The second derivative is obtained from the system of linear equations

\[
\sum_{k=1}^{N_p} \frac{\partial p_k}{\partial A_{j}^m} \sum_{i=1}^{N} \frac{1}{\epsilon_i^2} (A_{1i}^m f''_{kn}(x_i) - f''_{kn}(x_i) - f'_{n}(x_i)f'_{k}(x_i)) = -\frac{1}{\epsilon_j^2} f'_{n}(x_j).
\]

(103)

Let us present an explicit formulae for three most important cases, when proton structure function \( g_1 \) is extracted from the data with hydrogen target and neutron structure function \( g_1 \) is extracted from the data with deuteron and \( ^3 \)He targets. Also in the last subsection we consider the other target possibilities without taking into account experimental uncertainties (not using CERNLIB package MINUIT).
2.4.1 Proton, Deuteron and Helium-3 targets

The formulae of previous section could be applied directly for experimental data on hydrogen and deuteron. Three-parameter fit is used for fitting of the spin asymmetry [5]:

\[ A_p^a(x) = A + x^B(1 - e^{-Cx}) , \]
\[ A_d^a(x) = (1 - e^{-Ax})(B - xC) . \]

For proton asymmetry the one- and two parameter fits of ref.[22] are also used:

\[ A_p^1(x) = x^B , \quad A_p^2(x) = Ax^B . \]

For the neutron asymmetry extracted from the deuteron and helium-3 data the nuclear corrections have to be taken into account:

\[ A_n^1(x) = A_d^3He , \quad A_n^2(x) = (1 - f_d(x))P_pA_p^1(x) . \]

Dilution factors for both cases are given by the formulae

\[ f_d(x) = \frac{1}{F_2^p + 1} , \quad f_d(x) = \frac{1}{2F_2^p + 1} , \]

respectively. The numbers \( P_p \) and \( P_n \) are effective nucleon polarizations in nucleus. For deuteron target \( P_p = P_n = 0.5 - 0.75\omega_d \), where \( \omega_d \) is the D-state probability (\( \sim 5\% \)), and \( P_p = -0.028 \) and \( P_n = 0.86 \) for helium-3.

In both cases we construct the fit for neutron asymmetry using Schaefer’s model [24] (see Appendix A.4).

2.4.2 Other schemes

In many experimental cases when the model for structure function extracted is unknown it is convenient to use simple spline approximation of experimental data. Below we consider two examples, first when \( g_1 \) and \( g_2 \) are extracted simultaneously from data on spin asymmetries \( A_1, A_2 \) and the second, when \( b_1 \) is extracted from the data on quadruple asymmetry \( A_4 \).

\[ A_1^m = A_1^B(x) + \Delta A_1(g_1, g_2) = \frac{g_1 - \gamma^2 g_2}{F_1} + \Delta A_1(g_1, g_2) , \]
\[ A_2^m = A_2^B(x) + \Delta A_2(g_1, g_2) = \frac{\gamma(g_1 + g_2)}{F_1} + \Delta A_2(g_1, g_2) , \]

where \( \gamma = \sqrt{Q^2}/\nu \). SF \( g_1(x) \) and \( g_2(x) \) are calculated on the each step:

\[ g_1^{(n)} = \frac{F_1}{(1 + \gamma^2)} \left( A_1^m - \Delta A_1(g_1^{(n-1)}, g_2^{(n-1)}) + \gamma(A_2^m - \Delta A_2(g_1^{(n-1)}, g_2^{(n-1)})) \right) , \]
\[ g_2^{(n)} = \frac{F_1}{\gamma(1 + \gamma^2)} \left( A_2^m - \Delta A_2(g_1^{(n-1)}, g_2^{(n-1)}) - \gamma(A_1^m - \Delta A_1(g_1^{(n-1)}, g_2^{(n-1)})) \right) . \]
For quadruple deuteron spin-dependent SF \( b_1(x) \) we have:

\[
A_q^m = \frac{b_1}{F_1} + \Delta A_q(b_1) \tag{110}
\]

and

\[
b_1^{(n)} = F_1 \left( A_q^m - \Delta A_q(b_1^{(n-1)}) \right). \tag{111}
\]

Also the spline method, described above can be used for the cases of previous section to obtain the model independent data.

3 User manual

3.1 Program structure

In this section we present the common structure of POLRAD 2.0 along with the short description of subroutines used.

3.1.1 Main program body

The code POLRAD 2.0 operates under PATCHY from CERNLIB \[25\]. It means that there are two files POLRAD20.CAR and POLRAD20.CRA. The second file contains a set of switches for user. The first one is text of code for all possible combinations of the switches. FORTRAN code is obtained by calling like \textit{pat polrad20}, where \textit{pat} is a simple executable file. For example for SUN station it can have form:

```
ytobin $1 $1 $1 $1 .go
ypatchy $1 $1 $1 $1 .go
rm y.lis
rm $1.pam
rm $1.lis
```

POLRAD 2.0 allows user to conduct the calculations and to obtain the results for the any set of the following positions (in brackets we refer to the corresponding theoretical description).

1) Exact calculation of the total RC to DIS of polarized particles (see subsection \[2.1\]);
2) Electroweak RC calculation along with 1) (see subsection \[2.2\]);
3) The contribution of \( \alpha^2 \) order correction calculation along with 1) (see subsection \[2.1.4\]);
4) Approximate calculation of the total RC (see subsection \[2.1\]);
5) Exact calculation of the total RC to SIDIS of polarized particles.

Hence, the main file POLRAD20.CAR consists from the parts (patches) POLRAD; EXACT; POLRAD_ADD; SIRAD corresponding an above mentioned positions, parts (patches) STRFFUN; FITS2; INTEGRAT of the common use. The correlation between the patches is presented on the figs.\[3,7\].

**Patch POLRAD.** POLRAD guides the calculation and sends the obtained data in output files. On the first step of program run subroutine \[^1\] CONKIN sets up the value of the invariants in dependence on the given lepton, target and kinematics. Subroutines FSPENS and FSPEN

\[^1\]Usually one subroutine corresponds to one DECK from PATCHY with the same name.
calculate Spence function. Subroutine DELTAS calculates the factorizing part of virtual and real leptonic bremsstrahlung. Subroutine BORNIN calculates the Born cross section on the basis of known formulae (3).

**Patch EXACT.** In TAILS and FFU the τ-dependent part of an integrand ($\theta_{ij}(\tau)$ in (1)) is calculated. The integration procedure has the following steps:

1) QQT and QQTTINT set up the integration parameters and call the integrators;
2) Integrators use the integrands which are calculated in subroutines RV2DI, RV2 and PODINL.

**Patch SIRAD** deals with semi-inclusive RC. Main program MAINPRG guides the calculation of Asymmetry or $r(z)$. Subroutine COMVAR defines some often used variables and the limits for the phase space of the emitted photon. FSP and FSP1 give the Spence function. Subroutines EXHH, QXT, INTEG and NII carry out integration over photon variables. Models for parton distributions and fragmentation functions are given in DZ, FITDZ, SIGMA, QS QP. Subroutine DOS and DOP defines unpolarized and polarized Born cross section.

Functions FYS(F0YS) and FYP(F0YP) define the sum and difference of cross sections with the target polarization vector parallel and antiparallel to the momentum vector of the incident lepton. Radiative effects are (are not) included.

Functions FCS(FCP) and FRS(FRP) are the integrands of the unpolarized (polarized) part of bremsstrahlung cross section over invariant variable corresponding to the energy and to the polar angle of the emitted photon.

Subroutines VPQRK and Z0_EXH calculates effects contribution of the quarks to the vacuum polarization and effects of $Z_0$-exchange at the Born level.

Subroutines INPUT and OUTPUT create input arrays of kinematics variables and form output files.

**Patch STRFFUN.** Subroutine STRF carries out the calculation of inelastic, elastic quasielastic SF ($\mathcal{I}_i(R, \tau)$), defined for different cases in [Appendix A] as the combinations of usual unpolarized ($F_2$, $F_1$ or $R$), polarized ($g_1$, $g_2$), quadruple ($b_1$) structure functions and formfactors.

SF $F_2$ is calculated in dependence on the user defined model for the whole kinematical range in the subroutines:

- COMFST and F2BRAS calculate $F_2$ outside and inside resonance region on the basis of experimental data from refs. [26, 27, 28];
- PGRV corresponds to the calculation of SF $F_2$ in according to the ref. [29] model.
- Subroutines DF2H8 and DF2D8 calculate $F_2$ for proton and deuteron within the ref. [30] model.
- RANUCL gives the relation between $F_2$ for deuteron and $F_2$ for carbon and oxygen;
- F2SFUN is subroutine managing the calculation of $F_2$.
- Subroutine R1990 calculates R from ref. [31].
- $F_1$ is calculated in subroutine F1SFUN.
- For the calculation of $g_1$ in different models, the following subroutines can be used:
  - PARPOL calculates $g_1$ in ref. [32] model;
  - SCHAF corresponds the calculation of $g_1$ using ref. [24] model;
  - G1SFUN is subroutine managing the calculation of $g_1$.
- $g_2$ equals to zero or is calculated in subroutine G2SFUN within the model of ref. [34].
- Quadruple structure functions are calculated in the subroutine B14SF.
- Subroutines FFPRO, FFDEU, FFHE3 and FFCO correspond the calculation of proton, deuteron, $^3$He, carbon and oxygen formfactors.
Subroutine FFQUAS calculates formfactors taking into account quasielastic suppression.
SUPST calculates suppression factors as in ref. [24];
PORTN joins together models for SF calculations in different kinematical regions. As a result the continuous fit for all SF in whole kinematical region is obtained.

**Patch FITS2** contains subroutines for fit modeling based on the different methods:
AMNK, GRAM, GAUSS, FI, BASS — on the method of the minimal squared deviations;
ADIDI, DIVDIF — on the Newton interpolation method;
COEFSP — on the cubic spline method;
MINSTA — on the method of minimal squared deviation by MINUIT (requires CERNLIB).
Subroutine REMNK2 reads a data file, calls fitting subroutines and stores the parameters of constructed fits. The constructed fit can be called by subroutine FITFUN in any place of POLRAD.

**Patch INTEGRAT.** This file contains standard integrators D01FCE, from FORTRAN library NAGLIB which are used for the double integration over the square region. Single integration is carried out by QUNC8 by the Newton-Cotes method of 8th order, DQG32 by the Gauss method or SIMPS by Simpson method.

**Patch POLRAD_ADD** corresponds to the calculation of electroweak RC calculation, $\alpha^2$ order calculation and approximate RC calculation. It consists on the set of the subroutines (decks), that can be divided in three parts: 1) subroutines managing the calculation, 2) subprocesses that are integrands and 3) auxiliary subroutines.

1) APPTAI manages approximate and electroweak (only if partonic distributions are $Q^2$ dependent) calculations;
TARGWS and GWS manage electroweak corrections calculation when partonic distributions are $Q^2$ independent;
AL2LL manages the calculation of $\alpha^2$ corrections contribution;
2) Here we present the list of subroutines, which calculate the integrands of the corresponding formulae: PEAK1 (23), PEAK2 (28), UPRE (31,32), ELU (38), ELP (39,11), ELQ (38), RA21PP (43), RA2ISS (43), RA2ISP (49), ELUAL2 (50), ELPAL2 (50), RA2LSS (52), RA2LPP (72), RA2FSS (53), RA2FPP (52), RA2FSS (53), FXI (60).
3) SIGMAB and BOURSC calculates the quantities (29,51) and (9,10). VERCON define electroweak coupling constants; SIGALL, VERTS and TT5 calculates one loop effects: polarization operators (50), vertex functions and boxes (eqs.(10) and (12) of ref. [16]) respectively. FHOLL and DLAMABC are auxiliary function defined in (B.1,B.4,B.6) of ref.[17] VCONEW calculates born cross section with and without taking into account loop effects. FFVAPM calculates functions (A.10). DISTR calculates partonic distributions. STRFP2 is auxiliary subroutine corresponding to the calculation of structure functions.

### 3.1.2 Input, Output files

User can set input parameters for POLRAD 2.0 run in files POLRAD20.CRA, INPUT.DAT and ITDAT*.DAT.

**Input file POLRAD20.CRA** contains switches to get necessary case of a calculation. Below switches are given with short comments.

```
polrad - switch on inclusive DIS run of POLRAD ;
```
polrad_add - gives the opportunity to calculate the electroweak RC, $\alpha^2$ order effects and QED RC by approximate methods;  
sirad - switch on SIDIS run of POLRAD; 
strfun - calculates SFs (always on);  
fits2 - launches the fitting procedure;  
integrat - adds subroutines for integrations;  
exact - adds subroutines for exact calculation of RC to DIS.

The next switches have to be used to choose the way of calculations, type of leptons and target polarization:

approx - gives opportunity to use approximate methods together or instead of exact calculation of RC to DIS;  
alpha2ll - calculates the $\alpha^2$ order correction in leading log approximation;  
elect - switches on the electron as an input lepton;  
muons - switches on the muon as an input lepton;  
long - switches on the longitudinal polarization of a nuclear target;  
tran - switches on the transverse polarization of a nuclear target.

The next switches are necessary for selection of kinematics within iteration procedure. In this case the input data are taken from ITDAT*.DAT.

iter_pr - launches the iteration procedure for $g_1$ or $b_1$;  
minuit - sets the fitting of experimental data by MINUIT (only in the case of iteration procedure run for $g_1$);  
err_prop - calculates error propagation factor in according to eq.(102).  
iter_pr_g2 - launches the iteration procedure for $g_2$.

The next switches are necessary for selection of kinematics beyond iteration procedure. The only one has to be set on. It should be noted that user can also specify kinematics in the beginning of main program body.

kin_net - switches on the kinematical net over $x, y$.  
kin_smc - switches on the SMC kinematical set;  
kin_hermes - switches on the HERMES kinematical set;  
kin_e142 - switches on the E142 kinematical set;  
kin_own - switches on the user defined kinematical set (see file INPUT.DAT description).

The following switches allow to select the type or nuclear target. The only one has to be set on.

targ_h - switches on proton target;  
targ_d - switches on deuteron target;  
targ_he3 - switches on Helium-3 target;  
targ_c - switches on carbon target;  
targ_o - switches on oxygen target.
The next keys allow to select models for SF, partonic distribution and fragmentation function calculation (see A.3 also). They correspond to

- $f2nm_{c,d8}$ - the model for $F_2$ from ref. [31];
- $f2comfst$ - the model for $F_2$ from ref. [27, 26, 28];
- $f2g1sch$ - the model for $F_2$ and $g_1$ from ref. [24];
- $f2g1grsv96$ - the model for $F_2$ and $g_1$ from ref. [32];
- $f1qpm$ - the Callan-Gross relation for $F_1 = F_2/2x$;
- $r_{eq,0}$ - $R = 0$;
- $g1asym$ - the calculation of $g_1$ when iteration procedure is off and switch $f2nm_{c,d8}$ or $f2comfst$ is set on;
- $qdstr_{gu}$ - the model of ref. [33] for partonic distributions;
- $g2_{eq,0}$ - $g_2 = 0$;
- $g2_{ww}$ - the model for $g_2$ from ref. [34];
- $ffrg_{aub}$ - the model for fragmentation function from ref. [35];
- $ffrg_{cmb}$ - the model for fragmentation function from ref. [36];
- $ffrg_{arn}$ - the model for fragmentation function from ref. [37].

The next switches allow to specify the quantities to be calculated:

- $born$ - the only Born DIS cross section is calculated;
- $pol_{asym}$ - polarized and unpolarized parts of the total cross section are calculated separately;
- $qua_{asym}$ - quadruple and unpolarized parts of the total cross section are calculated separately;
- $cr_{sec}$ - total cross section is calculated for all polarized state;
- $onlyin$ - switch to exclude the contribution of elastic and quasielastic tails to RC;
- $output_{a}$ - sets asymmetry as a SIRAD output;
- $output_{r}$ - sets quantity $r(z)$ as a SIRAD output;
- $intdy$ - sets the integration of cross section over $y$;
- $intdz$ - sets the integration of cross section over $z$;
- $cuts$ - applies the kinematical cuts for RC in semi-inclusive case.

The next keys guides the calculation of electroweak effects:

- $electroweak$ - calculates the total radiative correction including electroweak RC using partonic distributions;
- $ew_{onlyqed}$ - calculates the electromagnetic RC using partonic distributions by eq. (60);
- $ew_{onlylep}$ - calculates the RC to leptonic current using partonic distributions by eq. (59);
- $eweak$ - calculates the BORN electroweak RC to SIDIS;

The last group of keys serves for selection of final hadron type in the case of SIDIS. They switch on the following particles as registered hadron for SIDIS:
proton - proton;
\( a_- \)- antiproton;
k\(_{-}\) - \( K^- \);
k\(_{0}\bar{b} \)- \( \bar{K}^0 \);
k\(_{+}\) - \( K^+ \);
k\(_{0}\) - \( K^0 \);
\( \pi_\text{min} \)- \( \pi^- \);
\( \pi_\text{pl} \)- \( \pi^+ \);
\( \pi_\text{zer} \)- \( \pi^0 \);
\( \pi_\text{dif} \)- the difference between \( \pi^+ \) and \( \pi^- \) production to be the measured observable for SIDIS.

**Input file** INPUT.DAT contains eight lines for DIS and nine lines for SIDIS cases. They correspond to lepton momentum, target momentum, lepton and nucleon polarization degrees, quadrupolarization degree (for deuteron target). The rest lines are used only for *kin own* switch: number of \( (x, y) \) pairs for DIS \((x, y, z)\) for SIDIS and arrays of these pairs (or triads).

**Input file** BRASSM.DAT gives coefficients of Ba treason ref.\[27\] for construction of SF fits in resonance region.

**Input file** BB1FIT.DAT gives coefficients for construction of quadruple SF \( b_1(x) \) fit [38].

**Input file** STDLOA1.GRI is used for the calculation by model of polarized partonic distributions of ref.\[12\].

**Input-output files** ITDAT1.DAT, ITDAT2.DAT, ITDAT3.DAT, ITQUAD.DAT, ITASM2.DAT contain input-output information for iteration procedure for the cases of extraction of \( g_1^p, g_1^d, g_3^{3\text{He}}, b_1 \) and \( g_2 \) respectively. All of them are organized in the same way. Each line if it is not a comment (a symbol (not '0') in the first position) gives the information on the one kinematical point to be processed: \( x, y \) (or \(-Q^2\) if negative), measured asymmetry, last step extracted asymmetry, previous step extracted asymmetry, measured error, output factor for error recalculation given by \( (102) \).

It should be noted that a kinematical point can be removed from analysis if to comment this line typing a symbol in the first position. If the symbol is zero correspondent point is processed but does not participate in fitting procedure.

**Output files** ALL.DAT, ASM.DAT, TAILS.DAT have a title lines with information about version, switches used for the calculation in files POLRAD20.CRA and INPUT.DAT.

The file ASM.DAT gives for each calculated kinematical point quantities \( x, W^2, Q^2 \), Born, observed asymmetries and difference between them.

The file ALL.DAT gives a technical information for each kinematical point about quality of numerical integration of all tails and their parts separately for polarization and unpolarization parts of cross section.

The file TAILS.DAT gives eight terms contributed to radiative correction to asymmetry \( (107) \).

**Output files** ALLP.DAT, ALLU.DAT are usually used to plot a different output quantities. Each line of these files gives the quantities \( x, y, s, \sigma_0^u,p, \sigma_{1}^u,p, \sigma_{2}^u,p, \sigma_{1}^{in u,p}, \sigma_{2}^{in u,p}, \sigma_{1}^{in u,p}, \sigma_{2}^{in u,p}, \sigma_{1}^{el u,p}, \sigma_{2}^{el u,p}, \sigma_{1}^{q u,p}, \sigma_{2}^{q u,p}, \sigma_{1}^{q u,p}, \sigma_{2}^{q u,p}, \sigma_{1}^{q u,p} \) respectively.

It should be noted when the electroweak effects are not calculated the approximate results (if approx is set on) for \( \sigma_{1}^{u,p} \) are printed instead of \( \sigma_{1}^{u,p} \).
Output files DATA.DAT, FIT.DAT are intended for the control of fitting quality. First file contains base points for the fit and the second contains points of the fitting curve.

3.2 Some examples

In this subsection we give three basic examples to illustrate the POLRAD 2.0 run: iteration procedure of data processing with MINUIT in the case of helium-3 target; the calculation of the radiative correction factor for collider DIS; semi-inclusive RC with and without applying kinematical cuts.

3.2.1 Iteration procedure with random input

Here we demonstrate the RC procedure run within experiments on DIS with polarized Helium-3 target with HERMES beam energy $E = 27.5 \text{ GeV}$. Values of measured asymmetry as well as averaged $x$ and $Q^2$ were obtained randomly. The following switches were active: polrad, strffun, integrat, fits2, exact, elect, long, iter_pr, minuit, targ_he3, f2nmec_d8, g2_eq_0, pol_asym.

The output file ITDAT3 after one step of iteration procedure is given in Appendix D.1. Fig.8 shows these results together with constructed fit of neutron asymmetry (see Appendix A.4).

3.2.2 Radiative correction in experiments at collider

Here we give results for unpolarized and polarized radiative correction factors

$$\delta_u = \frac{\sigma_u^0 + \sigma_u^1 + \sigma_u^2}{\sigma_u^0}, \quad \delta_p = \frac{\sigma_p^0 + \sigma_p^1 + \sigma_p^2}{\sigma_p^0}$$

(112)

within kinematics close to future polarized experiments at HERA collider (lepton and proton beam energies equal to 27.5 GeV and 810 GeV respectively). The following switches were active: polrad, strffun, integrat, polrad_add, fits2, alpha2ll, exact, elect, long, kin_net, targ_h, f2g1grs96, g2_eq_0, pol_asym, onlyin, electroweak. The quantities (112) are shown in fig.9. The output file ASM.DAT is given in Appendix D.2 (the only $x = .001$ case is kept).

3.2.3 Semi-inclusive radiative correction with and without cuts

Here we illustrate the run of the code for semi-inclusive DIS. The following switches were active: sirad, strffun, integrat, elect, kinOwn, targ_h, qdstrgu, splineff, ffrg_aub, outfun_a, intdz, cuts, psi_diff.

The fig.10 shows Born $A^{\text{born}}$ and observed $A^{\text{obs}}$ asymmetry as well as relative correction

$$\delta = (A^{\text{obs}} - A^{\text{born}})/A^{\text{born}}$$

with and without taking into account kinematical cuts. The output files ASM.DAT with and without applying of experimental cuts are given in Appendices D.3 and D.4.

4 Tests and implementation of POLRAD

POLRAD passed a number of both analytical and numerical tests. It was shown that the combinations of coefficients $\Theta(\tau)$ coincide with the corresponding combinations of coefficients
from ref. [12]. Also with the help of the algebraic programming system REDUCE 3.5 we proved that the formulae for electroweak inelastic unpolarized radiative correction are the same as in Appendix C of ref. [39]. The spin-independent part of POLRAD was tested numerically in comparison with FORTRAN codes TERADS6, FERRAD35 and HECTOR [40] and revealed an excellent agreement practically in all kinematical regions when all input models and parameters were the same. The spin-dependent part of POLRAD was compared with the program of Kukhto and Shumeiko [1] and with E143 radiative correction program kindly placed in our disposal by Linda Stuart [41]. We found the satisfactory agreement between all three programs when choosing the same models and parameters again. It was impossible to test the part of POLRAD corresponding to the calculation of spin one particle quadruple polarization due to the total absence of the results in this region.

Also POLRAD is self-tested program: the part corresponding to approximate calculation (see Sect. 2.1.3) have excellent agreement with the exact results; the QED part of electroweak correction (Sect. 2.2) coincides with the calculations by the corresponding exact formulae for the QED lowest order correction [12]; under the simple modifications of the semi-inclusive formulae, the corresponding results coincide with the inclusive ones.

Now POLRAD is used as the basic and official program for the procedure of radiative corrections in SMC (CERN) and HERMES (DESY) and together with the above mentioned E143 program in SLAC experiments with polarized particles.

Appendix A Structure functions

A.1 Inclusive structure functions

For hadronic tensor we use

\[ W_{\mu\nu} = -\tilde{g}_{\mu\nu} \mathcal{S}_1 + \frac{\tilde{p}_\mu \tilde{p}_\nu}{M^2} \mathcal{S}_2 + \frac{i \epsilon_{\mu\nu\alpha\beta} q_\alpha q_\beta}{M} \mathcal{S}_3 - \frac{i \epsilon_{\mu\nu\alpha\beta} g_\alpha p_\beta (q\eta)}{M^3} \mathcal{S}_4 \]

\[ + \tilde{g}_{\mu\nu} k_n \mathcal{S}_5 - \frac{\tilde{p}_\mu \tilde{p}_\nu}{M^2} k_n \mathcal{S}_6 - (\tilde{g}_{\mu\nu} + 3 \tilde{\eta}_\mu \tilde{\eta}_\nu) \mathcal{S}_7 - \frac{3}{2} \tilde{\Omega}_{\mu\nu} \mathcal{S}_8, \]

where

\[ k_n = \frac{3(q\eta)^2 - Q^2}{M^2}, \quad \tilde{\Omega}_{\mu\nu} = \frac{(\tilde{p}_\mu \tilde{\eta}_\nu + \tilde{\eta}_\mu \tilde{p}_\nu) q\eta}{M^2}, \]

\[ \tilde{g}_{\mu\nu} = g_{\mu\nu} + \frac{q_\mu q_\nu}{Q^2}, \quad \tilde{p}_\mu(\tilde{\eta}_\nu) = p_\mu(\eta_\nu) + \frac{pq(\eta q)}{Q^2} q_\mu. \]

The quantities \( \mathcal{S}_i \) are defined as some combinations of SF and formfactors, and the exact expressions in dependence of the variant of calculation are considered below in the correspondent sections. The dependence of the hadronic tensor on \( pq \) and polarization degrees is included in \( \mathcal{S}_i \) too.

We note, that the hadronic tensor for spin-1/2 particle is derived from (A.1) by putting \( Q_N = 0 \). The formulae for the covariant representation of polarization vector are also valid in this case. The hadronic tensor for scalar particles is derived by \( P_N = Q_N = 0 \).

Defining \( \epsilon = M^2/pq \) we have for various \( \mathcal{S}_i \) in the case of IRT
\[ \mathfrak{I}_1 = F_1 + \frac{Q_N}{6} b_1, \quad \mathfrak{I}_2 = \epsilon ( F_2 + \frac{Q_N}{6} b_2 ), \]
\[ \mathfrak{I}_3 = P_N \epsilon ( g_1 + g_2 ), \quad \mathfrak{I}_4 = P_N \epsilon^2 g_2, \]
\[ \mathfrak{I}_5 = \frac{Q_N}{6} \epsilon^2 b_1, \quad \mathfrak{I}_6 = \frac{Q_N}{6} \epsilon^3 ( b_2/3 + b_3 + b_4 ), \]
\[ \mathfrak{I}_7 = \frac{Q_N}{6} \epsilon ( b_2/3 - b_3 ), \quad \mathfrak{I}_8 = \frac{Q_N}{6} \epsilon^2 ( b_2/3 - b_4 ). \]

Definitions of SF \( F_i \), \( g_i \) and \( b_i \) are the same as in ref. [42].

Explicit form of the expression for elastic nuclear formfactors depends on a target spin. For deuteron we have
\[ \mathfrak{I}_{el}^1 = \frac{1}{6} \eta_A F_m^2 (4(1 + \eta_A) + \eta_A Q_N), \]
\[ \mathfrak{I}_{el}^2 = ( F_c^2 + \frac{2}{3} \eta_A F_m^2 + \frac{2}{3} \eta_A F_q^2 ) \]
\[ + \frac{Q_N}{6} \left( \eta_A F_m^2 + \frac{4\eta_A^2}{1+\eta_A} ( \frac{2}{3} F_q + F_c - F_m ) F_q \right), \]
\[ \mathfrak{I}_{el}^3 = - \frac{P_N}{2} (1 + \eta_A) F_m ( \frac{4}{3} F_q + F_c ), \]
\[ \mathfrak{I}_{el}^4 = \frac{P_N}{4} F_m ( \frac{1}{2} F_m - F_c - \frac{4}{3} F_q ), \]
\[ \mathfrak{I}_{el}^5 = \frac{Q_N}{24} F_m^2, \]
\[ \mathfrak{I}_{el}^6 = \frac{Q_N}{24} ( F_m^2 + \frac{4}{1+\eta_A} ( \frac{2}{3} F_q + F_c + \eta_A F_m ) F_q ), \]
\[ \mathfrak{I}_{el}^7 = \frac{Q_N}{6} \eta_A (1 + \eta_A) F_m^2, \]
\[ \mathfrak{I}_{el}^8 = - \frac{Q_N}{6} \eta_A F_m ( F_m + 2 F_q ). \]

Here \( \eta_A = t/4M_A^2 = (Q^2 + R_{et}\tau)/4M_A^2 \), and \( F_c, F_m, F_q \) - charge, quadruple formfactors of deuteron.

We also give the expressions for the case of arbitrary spin-1/2 nuclei
\[ \mathfrak{I}_{el}^1 = Z^2 \eta_A G_m^2, \quad \mathfrak{I}_{el}^2 = Z^2 G_e^2 + \eta_A G_m^2 \]
\[ \frac{1}{1 + \eta_A}, \]
\[ \mathfrak{I}_{el}^3 = \frac{P_N Z^2}{2} G_m G_e, \quad \mathfrak{I}_{el}^4 = \frac{P_N Z^2}{4} G_m G_e - G_m \]
\[ \frac{1 + \eta_A}, \]

and for scalar nucleus
\[ \mathfrak{I}_{el}^2 = Z^2 F^2, \]

where \( Z \) is the nucleus charge. All but indicated SF must be set equal to zero.

The quantities \( \mathfrak{I}_i \) can be obtained in the terms of quasielastic response functions, which have a form of peak for \( \omega = Q^2/2M \). The fact is normally used for construction of the peak type approximation. All quantities at response functions are estimated in peak, and subsequent
Let us define the fermion vertexes and boson propagators. We introduce

\[ \mathcal{V}_1^q = \eta (\mu_n^2 + 2\mu_p^2) S_M, \]
\[ \mathcal{V}_2^q = \frac{\eta (\mu_n^2 + 2\mu_p^2) S_M + (e_n^2 + 2e_p^2) S_E}{1 + \eta}, \]
\[ \mathcal{V}_3^q = \frac{P_N (P_n e_n \mu_n + 2P_p e_p \mu_p) S_{EM}}{2}, \]
\[ \mathcal{V}_4^q = \frac{P_N (P_n e_n \mu_n + 2P_p e_p \mu_p) S_{EM} - (P_n \mu_n^2 + 2P_p \mu_p^2) S_M}{1 + \eta}, \]

where \( \eta \) is \( \eta_A \) for nucleon. \( P_n \) and \( P_p \) are effective proton and neutron polarization in \(^3\)He and \( e_{p,n}, \mu_{p,n} \) are electric and magnetic formfactors of proton and neutron.

### A.2 Electroweak and semi-inclusive structure functions

Let us define the fermion vertexes and boson propagators. We introduce

\[ v_f^0 = -e_f, \quad a_f^0 = 0, \]
\[ v_f^Z = \frac{I_f^3 - 2s_W e_f}{2s_W c_W}, \quad a_f^Z = \frac{I_f^3}{2s_W c_W}, \]
\[ v_f^W = a_f^W = \frac{1}{2\sqrt{2}s_W}, \]

where \( e_f \) and \( I_f^3 \) — electric charge and the third component of the fermions weak isospin, \( s_W \) and \( c_W \) — Weinberg angle sine and cosine respectively. Hence, the fermion vertexes takes the form

\[ -ie\gamma_\mu (v_f^0 - a_f^0 \gamma_5). \] 

The next couple constants combinations are contained in observables

\[ \lambda_V^{ij} = 2(v_f^i v_f^j + a_f^i a_f^j), \quad \lambda_A^{ij} = 2c_f (v_f^i a_f^j + a_f^i v_f^j), \]
\[ R_+ = \lambda_V^{ij} - P_L \lambda_A^{ij}, \quad R_- = \lambda_A^{ij} - P_L \lambda_V^{ij}, \]
\[ F_+(x, Q^2) = \chi_{nZ}^{xz} \sum_q \left[ \lambda_V^{qij} x f_q^{(+)}(x, Q^2) + P_N \lambda_A^{qij} x f_q^{(-)}(x, Q^2) \right], \]
\[ F_-(x, Q^2) = \chi_{nZ}^{xz} \sum_q \left[ \lambda_A^{qij} x f_q^{(+)}(x, Q^2) + P_N \lambda_V^{qij} x f_q^{(-)}(x, Q^2) \right], \]

where \( c_f = 1(-1) \) for fermions (antifermions), \( \chi = Q^2/(Q^2 + M_Z^2) \) and \( n_z = (0, 1, 2) \) for \( ij = \gamma_\gamma, \gamma Z \) or \( Z \gamma, ZZ \). In QCD-improved model the parton distributions \( f_q^{(\pm)}(x, Q^2) \) depend on \( Q^2 \):

\[ f_q^{(\pm)}(x, Q^2) = f_q^{+\downarrow}(x, Q^2) \pm f_q^{\downarrow+}(x, Q^2), \]
where \( f^\uparrow\downarrow_q(x, Q^2) \) and \( f^\uparrow\uparrow_q(x, Q^2) \) — densities of type \( q \) partons with helicities, (anti)parallel to nucleon helicity respectively.

The quantities

\[
\Sigma^{+(-)}(x, z) = \sum_q e_q^2 [f^+_q(x) \pm f^-_q(x)] D^H_q(z)
\]

is the ordinary for QPM combination of the distribution functions \( f^{\uparrow\downarrow}_q(x) \) for the quark of flower \( q \) polarized (anti)parallel to the nucleon polarization, and of the fragmentation functions \( D^H_q(z) \) of the quark \( q \) into the hadron \( H \), \( e_q \) being the quark charge in units of elementary charge.

### A.3 Fits and models for structure functions

RC calculation requires fits or models for SF, elastic formfactors, quasielastic suppression factors, fragmentation functions and partonic densities to be known in a whole region of varying of integration variables.

POLRAD 2.0 gives the opportunity to choose between three models of spin-average and spin-dependent proton and neutron (deuteron) SF \( F^{p,d}_2(x, Q^2) \) and \( g^{p,d}_1(x, Q^2) \).

For the explanation of the first model for unpolarized SF see fig.\[11\]. In the small \( Q^2 \) region parametrizations of the ref.[27](in resonance region) and ref.[26] are used, and for all the rest kinematics 15-parameter NA-47 fit [28] is adopted. The advantage of this model is the implementation of the modern experimental data in small \( x \) regions. For the calculation of \( R(x, Q^2) \) and \( F^d_2/F^p_2 \) the fits shown on the corresponding graf are used.

The second model is based on the fit of [30] with the modern parameters and has the simple analytical form, same for the all kinematical region. However, it does not give good description of the modern data when \( x > 0.01 \). In this model for \( R \) we adopt the Whitlow fit \( R^{1990} [31] \).

For both models \( g_1(x, Q^2) = F_1(x, Q^2) A_1(x, Q^2) \) and \( A_1(x, Q^2) \) could be obtained either from the iteration procedure data or from the asymmetry fit \( [10][106] \).

In the third model partonic distributions (with [29], or without [24] taking into account \( Q^2 \) dependence) are used for the construction of \( F^{p,d}_2(x, Q^2) \) and \( g^{p,d}_1(x, Q^2) \). This fit is usually used in experiments at collider.

\( F^A_2(x, Q^2) \) for other nuclei is calculated in accordance with ref. [41] and \( R \) is considered to be A-independent (see review [13], for example). Convolution expressions for \( g^A_1, g^{\alphaHe}_1 \) are obtained via \( g^p_1(x) \) and \( g^n_1(x) [15] \).

For \( g_2 \) one can choose two possible variants: simple partonic approximation \( g_2 = 0 \) and the Wandzura-Wilczek relation [34]

\[
g_2(x, Q^2) = -g_1(x, Q^2) + \int_x^1 \frac{dz}{z} g_1(z, Q^2). \tag{A.13}
\]

Quadruple SF \( b_1, b_2 \) should be taken into account for deuterons as spin-1 particles. They are related by Callan-Gross equality

\[
b_2(x) = 2xb_1(x) \tag{A.14}
\]

and conform to a sum rule obtained in ref.[16]. Model of ref. [35] is used for them.

Deuteron formfactors are calculated in accordance with the model of ref. [17], which provides the right asymptotic behavior. For \( ^3\text{He} \) form-factors we use fit from ref. [18]. Nucleon formfactors
are taken from ref. [49]. Charge formfactor for scalar nuclei can be found in ref. [50] (see sect. 3.8).

The suppression factors for QRT for DIS on deuteron target are calculated as the same in ref. [26]. For other nuclei we used the Fermi gas model [51, 52].

There are three possible models for the fragmentation functions [35, 36, 37] in SIDIS calculation.

### A.4 Parameterization of neutron spin asymmetry

The following function taken from Schaefer’s parameterization [24] is used for fitting of the neutron spin asymmetry:

$$A_n^q(x) = \frac{1}{a_0 + 3a_1} \left( a_0 f_d^0 + \frac{a_1}{9}(-16f_u^1 + 8f_u^0 - 2f_d^1 + f_d^0) \right), \quad (A.15)$$

where

$$a_0 = \frac{2x^{\alpha_u}(1-x)^{\beta_u}}{B(\alpha_u, \beta_u + 1)} - \frac{x^{\alpha_d}(1-x)^{\beta_d}}{2B(\alpha_d, \beta_d + 1)}, \quad (A.16)$$

and

$$f_u^0 = \frac{1}{1 + a_{u0}x^{\alpha_u}(1-x)^2}, \quad f_u^1 = \frac{1}{1 + a_{u0}a_{10}x^{\alpha_u}(1-x)^2},$$

$$f_d^0 = \frac{1}{1 + a_{d0}x^{\alpha_d}(1-x)^2}, \quad f_d^1 = \frac{1}{1 + a_{d0}a_{10}x^{\alpha_d}(1-x)^2}. \quad (A.17)$$

Only parameters $a_7, a_{u0}$ and $a_{10}$ are fitted, and $\alpha_u, \alpha_d, \beta_u$ and $\beta_d$ are considered to be constant

$$\alpha_u = 0.588, \quad \alpha_d = 1.03, \quad \beta_u = 2.69, \quad \beta_d = 6.89. \quad (A.18)$$

### Appendix B  Quantities $\Theta_{ij}(\tau)$

In this Appendix we give the explicit form for the functions $\theta_{ij}(\tau)$, which are contained in the final formulae for the radiative tails. $i$ runs from 1 to 8. This fact corresponds to the contributions of eight SF or formfactor combinations, and $j$ runs from 1 to $k_i$ which are defined in (B.2). The function $\theta_{ij}(\tau)$ can be found as a sum over $k$

$$\theta_{ij}(\tau) = \sum_k a_{ik} T_{ijk}(\tau), \quad (B.1)$$

which is calculated from $max(1, j + l_i - k_i)$ to $min(j, l_i)$, where

$$k_i = (3, 3, 4, 5, 5, 3), \quad l_i = (1, 1, 1, 2, 3, 3, 1, 2), \quad (B.2)$$

and

$$a_{ik} = \begin{cases} 1 & \text{for } k = 1, \quad i = 1, 2, 3, 7; \\ \{\eta q, -1\} & \text{for } k = \{1, 2\}, \quad i = 4, 8; \\ \{Q^2 - 3(\eta q)^2, 6\eta q, -3\} & \text{for } k = \{1, 2, 3\}, \quad i = 5, 6. \end{cases} \quad (B.3)$$
The quantities $T_{ijk}(\tau)$ for $k = 1$ take the form

\[
T_{111}(\tau) = 4(Q^2 - 2m^2)F_{IR},
\]

\[
T_{121}(\tau) = 4\tau F_{IR},
\]

\[
T_{131}(\tau) = -4F - 2\tau^2 F_d,
\]

\[
T_{211}(\tau) = 2(SX - M^2 Q^2)F_{IR}/M^2,
\]

\[
T_{221}(\tau) = (2m^2S_p F_{d-} + S_p S_x F_{1+} + 2(S_x - 2M^2\tau)F_{IR} - \tau S_p F_d) / 2M^2,
\]

\[
T_{231}(\tau) = (4M^2 + (4m^2 + 2M^2\tau - S_x\tau)F_d - S_p F_{1+}) / 2M^2,
\]

\[
T_{311}(\tau) = \frac{-8P_L m}{M} (\eta_2 k_2 - Q^2 \xi_2) F_{IR},
\]

\[
T_{321}(\tau) = \frac{2P_L m}{M} (\eta_2 (4m^2 F_{d-}^\xi - 4m^2 F_{2+}^\xi + 2F_{IR}^\xi - Q^2 F_{2-}^\xi + Q_m^2 F_{2+}^\xi) +
+k_2(\tau - 8m^2 F_d^\eta + 4m^2 F_{2+}^\eta + 2 \eta_2 \tau F_d) - 4 \eta_2 F_{IR}^\xi + 4 \xi_2 \eta F_{IR}),
\]

\[
T_{331}(\tau) = \frac{2P_L m}{M} (\eta_2 ((F_{2+}^\xi - F_{2-}^\xi) +
2 k_2 \tau F_d^\eta + 4m^2 F_{2+}^\xi + 6F_{IR}^\eta + Q^2 F_{2-}^\xi - Q_m^2 F_{2+}^\xi),
\]

\[
T_{341}(\tau) = \frac{-2P_L m\tau}{M} (2F_d^\xi + F_{2+}^\xi - F_{2-}^\xi),
\]

\[
T_{411}(\tau) = \frac{4mP_L}{M^2} (S_x \xi k_2 - 2 \xi p Q^2) F_{IR},
\]

\[
T_{421}(\tau) = \frac{mP_L}{M^2} (4m^2(2 k_2 F_d - k_2 F_{2-} - S_p F_d + S_p F_{2+}) - 2 k_2 \tau S_x F_d

- 8 \xi \tau F_{IR} + 2 (2S_x - S_p) F_{IR}^\xi + S_p (Q^2 F_{2-}^\xi - Q_m^2 F_{2+}^\xi)),
\]

\[
T_{431}(\tau) = \frac{-mP_L}{M^2} (2F_d^\xi (2m^2 - \tau S_p) + 2 k_2 \tau F_d + 6F_{IR}^\xi

+ (Q^2 - \tau S_p) F_{2-}^\xi - (Q_m^2 - \tau S_p) F_{2+}^\xi),
\]

\[
T_{441}(\tau) = \frac{mP_L\tau}{M^2} (2F_d^\xi - F_{2-}^\xi + F_{2+}^\xi),
\]

\[
T_{711}(\tau) = -2 (Q^2 + 4m^2 + 12 \eta k_1 \eta k_2) F_{IR},
\]

\[
T_{721}(\tau) = -2 (3 \eta \zeta (2m^2 F_{d-}^\eta - (2\eta_2 F_{d+} + \eta_2 F_{1+}) + 6 \eta_2 F_{IR}^\eta + \zeta F_{IR}),
\]

\[
T_{731}(\tau) = 2F + \tau^2 F_d + 6 (\eta \zeta F_{1+}^\eta + \eta_2 \tau F_d^\eta - 4m^2 F_d^\eta),
\]
\[
T_{811}(\tau) = -\frac{6}{M}(S\,\eta k_2 + X\,\eta k_1)\,F_{IR},
\]
\[
T_{821}(\tau) = -\frac{3}{M}(\eta K(m^2F_{2-} - \tau S_pF_d) + \eta q\,F_{IR} + m^2S_pF_{2-}^n + (S\,\eta k_1 + X\,\eta k_2)\,F_{1+} + s_xF_{IR}^n),
\]
\[
T_{831}(\tau) = -\frac{3}{2M}(8m^2F_d^n - \eta K F_{1+} - \eta q\,\tau F_d - S_x\tau F_d^n - S_pF_{1+}^n).
\]

For \(i = 5\) and \(i = 6\) we have
\[
T_{(5,6)j1}(\tau) = T_{(1,2)j1}(\tau).
\]
(B.5)

The quantities \(T_{ijk}(\tau)\) for \(k = 2, 3\) are calculated as
\[
T_{ijk}(\tau) = T_{ij-1k-1}(\tau)\left\{F_{all} \rightarrow F_{all}^n, F'^{\xi}_{all} \rightarrow F'^{\xi}_{all}, F'^{\eta}_{all} \rightarrow F'^{\eta}_{all}\right\} + q_{ik}T_{ij-1k-1}(\tau).
\]
(B.6)

The second term appears only for \(i = 5, 6\) and \(k = 2\):
\[
q_{ik} = \delta k_2(\delta_5 + \delta_6)\frac{\tau}{M}.
\]
(B.7)

The substitution in the first term of (B.6) has to be applied for all \(F\) contained in \(T_{ij-1k-1}(\tau)\). The quantities \(F\) with an upper index are obtained in terms of \(F\) without the index:
\[
2F'^{\xi}_{2+} = (2F_{1+} + \tau F_{2-})s^{\xi}_{\eta} + F_{2-}r^{\xi}_{\eta};
\]
\[
2F'^{\xi}_{2-} = (2F_d + F_{2+})\tau s^{\xi}_{\eta} + F_{2-}r^{\xi}_{\eta};
\]
\[
2F'^{\xi}_{d} = F_{1+}s^{\xi}_{\eta} + F_{2-}r^{\xi}_{\eta};
\]
\[
4F'^{\xi}_{2+} = (2F_{1+} + \tau F_{2-})(r_{\eta}s^{\xi}_{\eta} + s_{\eta}r^{\xi}_{\eta}) + F_{2+}(r_{\eta}s^{\xi}_{\eta} + \tau^2s_{\eta}s^{\xi}_{\eta}) + 4(2F + F_d\tau^2)s_{\eta}s^{\xi}_{\eta};
\]
\[
4F'^{\xi}_{2-} = (2F_d + F_{2+})(r_{\eta}s^{\xi}_{\eta} + s_{\eta}r^{\xi}_{\eta}) + F_{2-}(r_{\eta}s^{\xi}_{\eta} + \tau^2s_{\eta}s^{\xi}_{\eta}) + 4\tau F_{1+}s_{\eta}s^{\xi}_{\eta};
\]
\[
4F'^{\xi}_{d} = F_{1+}(r_{\eta}s^{\xi}_{\eta} + s_{\eta}r^{\xi}_{\eta}) + F_{2+}(r_{\eta}s^{\xi}_{\eta} + \tau^2s_{\eta}s^{\xi}_{\eta}) + 4F_{1+}s_{\eta}s^{\xi}_{\eta};
\]
\[
2F'^{\eta}_{1+} = (4F + \tau^2 F_d)s_{\eta} + F_{1+}r_{\eta};
\]
\[
4F'^{\eta}_{1+} = 2(4F + \tau^2 F_d)r_{\eta}s_{\eta} + F_{1+}(r_{\eta}^2 + \tau^2s_{\eta}^2) + 4(2F - \tau F)s_{\eta}^2;
\]
\[
2F'^{\eta}_{d} = (F_{2-} - \tau s_{\eta}) + 2F_{1+}r_{\eta};
\]
\[
4F'^{\eta}_{d} = F(r_{\eta} - \tau s_{\eta})^2 + 4F_{1+}(r_{\eta} - \tau s_{\eta}) + 4F_{1+}r_{\eta}s_{\eta}^2.
\]
(B.8)

The quantities
\[
s^{\xi}_{\eta} = a^{\xi}_{\eta} + b^{\xi}_{\eta}, r^{\xi}_{\eta} = \tau (a^{\xi}_{\eta} - b^{\xi}_{\eta}) + 2c^{\xi}_{\eta}
\]
(B.9)
are combinations of coefficients of polarization vectors $\xi$ and $\eta$ expansion over basis (see sect.2.1.1)

$$\xi, \eta = 2(a_{(\xi,\eta)}k_1 + b_{(\xi,\eta)}k_2 + c_{(\xi,\eta)}p).$$  
(B.10)

We note that the scalar products from (B.5) and (3) are also calculated in terms of the coefficients

$$\eta q = -Q^2(a_n - b_n) + S_x c_n, \quad \eta K = (Q^2 + 4m^2)(a_n + b_n) + S_p c_n,$$

$$2\eta k_1 = \eta K + \eta q, \quad 2\eta k_2 = \eta K - \eta q,$$

$$k_2 \xi = Q^2 m a_\xi + 2m^2 b_\xi + X c_\xi, \quad \xi p = S a_\xi + X b_\xi + 2M^2 c_\xi,$$

$$\frac{1}{2} \xi \eta = 2m^2(a_\xi a_n + b_\xi b_n) + 2M^2 c_\xi c_n + Q^2 m(a_\xi b_n + b_\xi a_n) + S(a_\xi c_n + c_\xi a_n) + X(b_\xi c_n + c_\xi b_n).$$  
(B.11)

The following equalities define the functions $F$:

$$F = \lambda_Q^{-1/2}, \quad F_{IR} = m^2 F_{2+} - Q_m^2 F_d,$$

$$F_d = \tau^{-1}(C_2^{-1/2}(\tau) - C_1^{-1/2}(\tau)) \quad F_{1+} = C_2^{-1/2}(\tau) + C_1^{-1/2}(\tau),$$

$$F_{2+} = B_2(\tau)C_2^{-3/2}(\tau) - B_1(\tau)C_1^{-3/2}(\tau), \quad F_{2-} = B_2(\tau)C_2^{-3/2}(\tau) + B_1(\tau)C_1^{-3/2}(\tau),$$

$$F_i = -\lambda_Q^{-3/2}B_1(\tau), \quad F_{ii} = \frac{1}{2} \lambda_Q^{-5/2}(3B_1^2(\tau) - \lambda_Q C_1(\tau)),$$

where

$$B_{1,2}(\tau) = -\frac{1}{2} \left( \lambda_Q \tau \pm S_p(S_x \tau + 2Q^2) \right),$$

$$C_1(\tau) = (S\tau + Q^2)^2 + 4m^2(Q^2 + \tau S_x - \tau^2 M^2),$$

$$C_2(\tau) = (X\tau - Q^2)^2 + 4m^2(Q^2 + \tau S_x - \tau^2 M^2).$$  
(B.13)

We note that $F_d$ has a uncertainty like $0/0$ for $\tau = 0$ (inside of integration region). It leads to difficulties for numerical integration, so the another form is used also

$$F_d = \frac{S_p(\tau S_x + 2Q^2)}{C_1^{1/2}(\tau)C_2^{1/2}(\tau)(C_1^{1/2}(\tau) + C_2^{1/2}(\tau))}. $$  
(B.14)

Appendix C  Quantities $R_{1,2}^{\mu,p}$

The functions $R_{1,2}^{\mu,p}$ contributed to [50] are listed here:

$$R_1^\mu = -2Y_+ \tilde{X} \tilde{L} + 2Y_+ (\frac{x}{3} - \frac{3}{x}) f(\eta) - 2 \frac{x}{x} + 14 \frac{2}{3 x^2} - \frac{2}{3} (2 x f(\eta) + \frac{1}{x} - \frac{6}{x})$$

$$+ \frac{x}{3} + \frac{4 x}{3} f(\eta) - 3 y^2 \frac{2 y_1 \eta}{2 y_1 \eta} - \frac{2}{x^2} (1 - x) \left( \frac{1}{y_1} + Y_+ \right) L_1 + \frac{1}{2 \eta} (2 y_1 + Y_+) L_{y_1}$$

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\[ + \frac{1}{2\eta} \left( \frac{2}{y_1} + Y_+ \right) L_1 - \frac{2}{\eta} Y_+ L_y - \frac{2}{x^2} (1 - x) (y_1 + Y_+) L_y + \frac{2y_1}{x^2 y} (xy + y_1) D_1 \]
\[ - \frac{y}{2\eta} (D_1 + D_{y_1}) + \frac{2}{yx} (1 - xy) D_{y_1}, \]

\[ \mathcal{R}_2^u = -2Y_+ \tilde{L} + \frac{y^2}{y_1} (2xf(\eta) - 3) + \left( \frac{2}{y_1} + Y_+ \right) L_1 + (2y_1 + Y_+) L_{y_1} - 4Y_+ L_y \]
\[ - y(D_1 + D_{y_1}), \]

\[ \mathcal{R}_2^v = 2Y_+ \tilde{L} - 2Y_-(1 - \frac{3}{x} + 2f(\eta)) + 2(1 - \frac{1}{x}) L_1 + \]
\[ 2(1 - \frac{1}{x}) \frac{y}{x} L_{y_1} + \frac{x}{2\eta} (Y_+ + \frac{2y_1}{y_1}) L_1 - \frac{x}{\eta} Y_+ L_{y_1} - \frac{x}{2\eta} (2Y_+ - \frac{y}{y_1}) L_{y_1} \]
\[ + (2y + \frac{2y_1}{x} - \frac{y^2 x}{2y_1 \eta}) D_1 + \left( \frac{1}{x} - 2y - \frac{y^2 x}{2y_1 \eta} \right) D_{y_1}, \]

\[ \mathcal{R}_2^r = 2Y \tilde{L} + Y_-(L_1 - 1 + L_{y_1}) + y(D_1 - D_{y_1}), \]  \text{ (C.1)}

where

\[ L_y = \ln(xf(\eta)), \quad L_1 = \ln(y_1 + xfyf(\eta)), \quad L_{y_1} = \ln(\frac{1 - xyf(\eta)}{y_1}), \]

\[ D_1 = \frac{1}{y_1 + xfyf(\eta)}, \quad D_{y_1} = \frac{-1}{1 - xfyf(\eta)}, \]  \text{ (C.2)}

\[ \tilde{L} = \ln \left( \frac{(1 - x)^2}{(1 - xfyf(\eta))^2(1 - xy)(y_1 + xy)} \right) + L_1 + L_{y_1} \]

and \( 2f(\eta) = 1 + \sqrt{1/\eta} + 1. \)

**Appendix D  Test Run Output**

**D.1 Example 1. Output file ITDAT3.DAT**

```
*************** Helium-3 data ***********************
   .010  .830  -.03500  -.04566  -.03500  .02000
   .020  .790  -.05500  -.06745  -.05500  .02100
   .035  .740  -.07500  -.08794  -.07500  .02300
   .050  .660  -.08000  -.09195  -.08000  .02500
   .070  .620  -.10000  -.11211  -.10000  .02700
   .095  .570  -.12800  -.13951  -.12800  .03000
   .115  .530  -.14000  -.15128  -.14000  .03200
   .140  .480  -.16000  -.17111  -.16000  .03400
   .180  .450  -.12500  -.13951  -.12500  .03500
   .230  .425  -.17000  -.18141  -.17000  .03600
   .285  .400  -.14500  -.15666  -.14500  .03800
```
D.2 Example 2. Output file ASM.DAT

program polrad20 version from 10.04.1997

the file gives born asymmetry, observed asymmetry
and radiative correction

the following switches are active
   polrad strffun integrat polrad_add fits2 alpha2ll exact elect long
   kin_net targ_h f2g1grsv96 glasyr g2_eq_0 pol_asym onlyin
   electroweak

leptons are electrons
target is proton
target is longitudinally polarized
bmom = 27.5
tmom = 830.0
   pl = 1.00     pn = 1.00    qn = 0.00

a is in %

| x   | w2   | q2   | a(born) | a(obs) | del(%) |
|-----|------|------|---------|--------|--------|
| 0.001 | 913.0 | 0.9  | 0.378   | 0.334  | -0.043 |
| 0.001 | 1369.0 | 1.4  | 0.378   | 0.337  | -0.040 |
| 0.001 | 1825.1 | 1.8  | 0.378   | 0.339  | -0.038 |
| 0.001 | 2737.1 | 2.7  | 0.378   | 0.342  | -0.035 |
| 0.001 | 3649.2 | 3.7  | 0.378   | 0.345  | -0.032 |
| 0.001 | 4561.3 | 4.6  | 0.378   | 0.347  | -0.030 |
| 0.001 | 5473.4 | 5.5  | 0.378   | 0.349  | -0.029 |
| 0.001 | 6385.5 | 6.4  | 0.378   | 0.349  | -0.028 |
| 0.001 | 7297.6 | 7.3  | 0.378   | 0.350  | -0.027 |
| 0.001 | 8209.7 | 8.2  | 0.378   | 0.351  | -0.027 |
D.3 Example 3. Output file ASM.DAT (with CUTS)

program polrad20 version from 10.04.1997

the file gives born asymmetry, observed asymmetry
and radiative correction

the following switches are active
  sirad strffun integrat elect long kin_net targ_h f2g1grsv96 glasym
  qdstr_gu g2_eq_0 ffrg_aub pol_asym onlyin outfun_a intdz cuts eweak
  pi_diff

leptons are electrons
target is proton
target is longitudinally polarized
bmom = 27.5
tmom = .0
  pl = 1.00    pn = 1.00    qn = .00

0.001  9121.8  9.1  0.378  0.351 -0.027
0.001 11402.0 11.4  0.378  0.352 -0.026
0.001 13682.2 13.7  0.378  0.352 -0.025
0.001 15962.4 16.0  0.378  0.353 -0.025
0.001 18242.6 18.3  0.378  0.353 -0.024
0.001 20522.8 20.6  0.378  0.353 -0.023
0.001 21802.9 21.9  0.378  0.353 -0.023
0.001 23083.1 23.2  0.378  0.353 -0.022
0.001 24363.3 24.5  0.378  0.353 -0.021
0.001 25643.5 25.8  0.378  0.353 -0.021
0.001 26923.7 27.1  0.378  0.353 -0.020
0.001 28203.9 28.4  0.378  0.353 -0.020
0.001 29484.1 29.7  0.378  0.353 -0.019
0.001 30764.3 30.9  0.378  0.353 -0.018
0.001 32044.5 32.2  0.378  0.353 -0.017
0.001 33324.7 33.5  0.378  0.353 -0.017
0.001 34604.9 34.8  0.378  0.352 -0.016
0.001 35885.1 36.1  0.378  0.352 -0.015
0.001 37165.3 37.4  0.378  0.352 -0.015
0.001 38445.5 38.7  0.378  0.352 -0.014
0.001 39725.8 39.9  0.378  0.352 -0.014
0.001 40906.0 41.2  0.378  0.351 -0.013
0.001 42186.2 42.5  0.378  0.351 -0.013
0.001 43466.4 43.8  0.378  0.351 -0.012
0.001 44746.6 45.1  0.378  0.351 -0.012
0.001 46026.9 46.4  0.378  0.351 -0.011
0.001 47307.1 47.7  0.378  0.351 -0.011
0.001 48587.3 49.0  0.378  0.351 -0.010
0.001 49867.5 50.3  0.378  0.351 -0.010
0.001 51147.7 51.6  0.378  0.351 -0.009
0.001 52427.9 52.9  0.378  0.351 -0.009
0.001 53708.1 54.2  0.378  0.351 -0.008
0.001 54988.3 55.5  0.378  0.351 -0.008
0.001 56268.5 56.8  0.378  0.351 -0.007
0.001 57548.7 58.1  0.378  0.351 -0.007
0.001 58828.9 59.4  0.378  0.351 -0.006
0.001 60109.1 60.7  0.378  0.351 -0.006
0.001 61389.3 62.0  0.378  0.351 -0.005
0.001 62669.5 63.3  0.378  0.351 -0.005
0.001 63949.7 64.6  0.378  0.351 -0.005
0.001 65229.9 65.9  0.378  0.351 -0.004
0.001 66500.1 67.2  0.378  0.351 -0.004
0.001 67779.3 68.5  0.378  0.351 -0.003
0.001 69059.5 69.8  0.378  0.351 -0.003
0.001 70339.7 71.1  0.378  0.351 -0.003
0.001 71619.9 72.4  0.378  0.351 -0.002
0.001 72899.1 73.7  0.378  0.351 -0.002
0.001 74179.3 75.0  0.378  0.351 -0.002
0.001 75459.5 76.3  0.378  0.351 -0.001
0.001 76739.8 77.6  0.378  0.351 -0.001
0.001 78019.9 78.9  0.378  0.351 -0.001
0.001 79299.1 80.2  0.378  0.351 -0.001
0.001 80579.3 81.5  0.378  0.351 -0.000
0.001 81859.5 82.8  0.378  0.351 -0.000
0.001 83139.8 84.1  0.378  0.351 -0.000
0.001 84419.9 85.4  0.378  0.351 -0.000
0.001 85699.1 86.7  0.378  0.351 -0.000
0.001 86979.3 88.0  0.378  0.351 -0.000
0.001 88259.5 89.3  0.378  0.351 -0.000
0.001 89539.7 90.6  0.378  0.351 -0.000
0.001 90819.9 91.9  0.378  0.351 -0.000
0.001 92099.1 93.2  0.378  0.351 -0.000
### D.4 Example 3. Output file ASM.DAT (without CUTS)

program polrad20 version from 10.04.1997

the file gives born asymmetry, observed asymmetry and radiative correction

the following switches are active

- sirad
- strffun
- integrat
- elect
- long
- kin_net
- targ_h
- f2g1grsv96
- g1asym
- qdstr_gu
- g2_eq_0
- ffrg_aub
- pol_asym
- onlyin
- outfun_a
- intdz
- eweak
- pi_diff

leptons are electrons

target is proton

target is longitudinally polarized

bmom = 27.5

tmom = .0

pl = 1.00  pn = 1.00  qn = .00

| x    | y    | q**2  | A1     | rc   | A1 born | meas   |
|------|------|-------|--------|------|---------|--------|
| .115 | .421 | 2.490 | 58.912 | -2.828 | 30.237  | 29.382 |
| .183 | .421 | 3.980 | 61.646 | -1.666 | 31.289  | 30.768 |
| .252 | .421 | 5.470 | 63.000 | -.986  | 31.780  | 31.467 |
| .320 | .421 | 6.960 | 63.683 | -.574  | 32.015  | 31.831 |
| .389 | .421 | 8.449 | 63.999 | -.322  | 32.116  | 32.013 |
| .457 | .421 | 9.939 | 64.005 | -.198  | 32.103  | 32.040 |
| .526 | .421 | 11.429| 63.979 | -.171  | 32.106  | 32.051 |
| .594 | .421 | 12.919| 63.938 | -.298  | 32.150  | 32.054 |

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FIGURE CAPTIONS

1. The double differential cross section for inclusive lepton-nuclei scattering sketched at a certain value of $Q^2$ as function of $\nu$ (arbitrary scale). The three basic channels are the elastic (I), the quasielastic (II), and the inelastic (III) one.

2. The limits of integration for variables a) $R$ and $\tau$ [13]; b) $\xi$ and $t$ [32,35]; b) $t_1$ and $t_2$ [77].

3. Scheme of program POLRAD. The subroutine APPTAI, AL2LL and TARGWS are presented on a separate figures.

4. Scheme of program SIRAD.

5. Scheme of program AL2LL.

6. Scheme of program APPTAI.

7. Scheme of program TARGWS.

8. The results of iteration procedure for spin asymmetry (see sect. 3.2.1) along with the fit constructed.

9. The unpolarization a) and polarization b) radiative correction factors defined by eqn. (112). The curves 1,2 and 3 correspond to different values of $x = 0.001, 0.01, 0.1$ respectively.

10. The born (dash) and observed (solid) SIDIS asymmetries (a) and relative correction with (dash) and without (solid) kinematical cuts apllied (b).

11. The spin-independent proton and neutron SF. The citations denote that fit from the cited references is used in the range. The fits are extrapolated into hatched region. The procedure of joining together of two-dimensional surfaces gives continuous fits in the whole kinematical region.
Figure 1:
Figure 2:
Figure 3:
Figure 4:
Figure 5:
Figure 6:
Figure 7:
Figure 8:
Figure 9:
Figure 10:
\[ R(Q^2, W^2) \]

\[ F^p_2(Q^2, W^2) \]

Figure 11: