Scale-dependent CMB asymmetry from primordial configuration

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Abstract. We demonstrate that a topological defect can explain the hemispherical power asymmetry of the CMB. The first point is that a defect configuration, which already exists prior to inflation, can source asymmetry of the CMB. The second point is that modulation mechanisms, such as the curvaton and other modulation mechanisms, can explain scale-dependence of the asymmetry. Using a simple analysis of the δN formalism, we show models in which scale-dependent hemispherical power asymmetry is explained by primordial configuration of a defect.

Keywords: alternatives to inflation, inflation, Cosmic strings, domain walls, monopoles

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1 Introduction

After the report of the detection of the hemispherical power asymmetry on the CMB [1, 2], there have been attempts to look for fingerprints of the non-standard inflationary physics in the anomaly. If the anomaly is not a statistical artifact, it strongly indicates that the single-field inflation models are not enough to explain the current observations of the Universe [3, 4]. Also, the anomaly seems to be suggesting that we are observing a fingerprint of the pre-inflationary configuration [3, 4]. If the dipolar modulation of the temperature $T(\vec{r})$ is generated by the dipolar modulation of the curvature perturbation $\zeta$, a parametrization of the bipolar asymmetry can be defined as

$$\frac{\mathcal{P}_{\zeta}^{1/2}(k, \vec{r})}{\mathcal{P}_{\zeta}^{1/2}(k, \vec{r})|_{\text{iso}}} = 1 + \frac{A(k)(\vec{p} \cdot \vec{r})}{r_{\text{CMB}}},$$

where $\mathcal{P}_{\zeta}(k, \vec{r})|_{\text{iso}}$ is the isotropic power spectrum and $A(k)$ measures the asymmetry in the direction of the unit vector $\vec{p}$. Here $r_{\text{CMB}}$ is the comoving distance to the surface of the last scattering.
The effect of a large-scale enhancement of the spectrum of the curvature perturbation \cite{5} has been investigated in ref. \cite{6} and it has been shown that the enhancement proportional to a linear function of position ($z$) is not observable because of the Doppler shift due to the induced peculiar velocity. If the enhancement has higher order corrections ($\sim z^n$), the second order term ($\sim z^2$) enhances the CMB quadrupole (Grishchuk-Zel’dovich effect), which has not been observed yet. Then in ref. \cite{3, 4}, a bound has been found for both inflationary scenario and curvaton model upon mild assumption of a function. The observed CMB asymmetry has been explained by introducing additional super-horizon size perturbation $\Delta \phi \sim \phi_0 \sin(k_A z)$ of a field. Recently, the cosmic variance has been considered in ref. \cite{7}. Another solution has been found in ref. \cite{8}, in which a contracting phase before slow-roll inflation plays the role.

One of the mysteries of those solutions would be the origin of the source perturbation $\Delta \phi$, which is supposed to have specific direction (in this paper we are specifying the direction $z$) in space. The cosmic variance \cite{7} could be a solution, but in this paper we consider topological defects that may appear because of chaotic initial conditions in the pre-inflationary Universe, and show that the defect configurations can indeed explain the asymmetry when they are combined with the curvaton or other modulation mechanisms.\footnote{See also ref. \cite{9–12}.} Here, the curvaton mechanism \cite{13, 14} uses isocurvature perturbation of a curvaton field $\sigma$. Although the curvaton is negligible during inflation, the ratio of the curvaton to the total energy density grows after inflation and finally it generates the curvature perturbation. On the other hand, other modulation mechanisms consider isocurvature perturbation of a light field (moduli), which is always a negligible fraction of the energy density. For instance, the isocurvature perturbation of a light field can make the decay rate of the inflaton \cite{15–17} spatially inhomogeneous, and it can cause generation of curvature perturbation at the time of reheating. In the same way, generation of the curvature perturbation is possible when energy density changes its scaling at phase transition \cite{18}.

Another mystery would be the significant scale-dependence of the asymmetry parameter $A(k)$. If one interprets the dipolar asymmetry in the CMB power spectrum as a spatial variation of the amplitude of primordial fluctuations, one can make predictions not just for the CMB but also for large-scale structure. Then there should be a corresponding gradient in the number density of highly biased objects. The constraint from the Quasar is first obtained in ref. \cite{19}. Using the high-redshift quasars from the Sloan Digital Sky Survey ($z \geq 2.9$), Hirata found a null result for a gradient in the number density of highly biased objects, which rules out the simple curvaton-gradient model. A tighter constraint has been obtained in a recent paper \cite{20}, in which the hemispherical power asymmetry in the CMB on small angular scales has been investigated. In ref. \cite{20}, it has been shown that the hemispherical power asymmetry must satisfy $A < 0.0045$ on the 10 Mpc scale. In this paper we will analyze this issue using a simple $\delta N$ formalism and show how to construct models in which scale-dependent asymmetry meets the criteria.

As we need a simple method for the estimation, we are going to consider approximation based on the $\delta N$ formalism. In section 2 we will review previous analysis of the CMB asymmetry in the light of the $\delta N$ formalism. Section 3 is devoted to the analysis of the asymmetry caused by a domain-wall configuration. First, we will show that a simple scenario of topological inflation cannot explain the anomaly. Then we will show why the standard modulated decay scenario, as far as the initial curvature perturbation is mainly sourced by the mechanism, cannot explain the scale-dependence of the asymmetry. The same discussion
excludes the curvaton [3, 4]. Although the domain wall configuration considered in this section is new, other discussions are basically the same as previous works [3, 4, 19]. Then in section 4, we will separate sources of the cosmological perturbations. Our idea to achieve the required scale-dependent asymmetry will be classified in this section. Then we will examine several specific models in more details. In addition, we will show that \( g_{NL} \geq 10^4 \) of the non-Gaussianity parameter could be crucial. The origin of the asymmetry parameter will be discussed in the light a domain wall; however our method is quite general and can be applied to many other models of pre-existing configurations [21].

\section{\( \delta N \) formalism for the CMB asymmetry}

During nearly exponential inflation, the vacuum fluctuation of each light scalar field \( \phi_i \) is converted at the horizon exit to a nearly Gaussian classical perturbation with a spectrum \( P_{\delta \phi_i} \simeq (H/2\pi)^2 \), where the Hubble parameter is \( H \equiv \dot{a}(t)/a(t) \). The curvature perturbation is

\[ \zeta = \delta [\ln(a(x,t)/a(t_*))] \equiv \delta N, \quad (2.1) \]

where \( t \) is time along a comoving thread of space-time and \( a(t) \) is the local scale factor. Taking \( t_* \) to be an epoch during inflation after relevant scales leave the horizon, we assume \( N(\phi_1(x,t_*), \phi_2(x,t_*), \cdots, t, t_*) \) so that

\[ \zeta(x,t) = N_i \delta \phi_i(x,t_*) + \frac{1}{2} N_{ij} \delta \phi_i(x,t_*) \delta \phi_j(x,t_*) + \cdots, \quad (2.2) \]

where a subscript \( i \) denotes \( \partial / \partial \phi_i \) evaluated on the unperturbed trajectory.

We define the fractional power asymmetry as in eq. (1.1), where \( A \sim 0.072 \pm 0.022 \) for large angular scales \( l < 64 \) is expected from the recent Planck data [2].\footnote{See also WMAP data [1], which also shows a similar result.} In this paper we will examine the possibility of \( A \sim 0.05 \).

For a single-field perturbation we can expand

\[ \frac{\Delta(\delta N)}{\delta N} = \frac{N_{\phi \phi}}{N_\phi} \Delta \phi + \frac{1}{2} \frac{N_{\phi \phi \phi}}{N_\phi} (\Delta \phi)^2 + \cdots \]

\[ = \frac{6}{5} f_{NL} N_\phi \Delta \phi + \frac{27}{25} g_{NL} (N_\phi \Delta \phi)^2 + \cdots, \quad (2.3) \]

where \( f_{NL} \) and \( g_{NL} \) are the non-linearity parameters that measure the non-Gaussianity of the curvature perturbation. Here we write

\[ \phi(\vec{x}, t) = \phi_0(t) + \Delta \phi(z, t) + \delta \phi(\vec{x}, t), \quad (2.4) \]

where \( \delta \phi \) is the conventional Gaussian perturbation and \( \Delta \phi \) is a shift of the field across the sky in the direction of \( \vec{z} \). Just for simplicity, we are assuming that \( \vec{p} \) is in the direction of \( \vec{z} \). Here we exclude \( |N_\phi \Delta \phi| > 1 \) when \( \Delta \phi \) is within the horizon, since it ruins the perturbative expansion. Note that in eq. (2.4), there are two sources for perturbation: one is the conventional Gaussian perturbation \( \delta \phi \) and the other is the shift \( \Delta \phi \).
2.1 How to introduce scale dependence in the spectrum

2.1.1 Curvaton and other modulation mechanisms

In this paper, the scalar field $\varphi$ denotes light fields other than the inflaton. We are replacing $\varphi \to \sigma$ for the curvaton and $\varphi \to \chi$ for other modulation mechanisms, if these specifications are possible.

In order to argue the scale-dependence, one must define the specific time when quantities are evaluated. Mixings between different definitions will be the source of serious confusions.

The curvaton model uses $\delta\sigma/\sigma$ to define “component perturbation” at the beginning of the sinusoidal oscillation:

$$\zeta_\sigma \equiv -H \frac{\delta \rho_\sigma}{\rho_\sigma} = \frac{\delta \rho_\sigma}{3\rho_\sigma} = \frac{2\delta\sigma}{3\sigma},$$

where quantities are defined at the beginning of the oscillation. Here the curvaton potential is assumed to be quadratic ($\rho_\sigma = \frac{1}{2}m_\sigma^2\sigma^2$). In the same way, one can define component perturbation of the radiation. Evolution of component perturbations is examined in ref. [22]. Evolution of $\delta\sigma/\sigma$ before the oscillation has been examined in ref. [23].

In the curvaton scenario, one usually puts several assumptions in advance, which makes $\delta\sigma/\sigma$ or $\zeta_\sigma$ evolves like constant until the “event” (creation of the curvature perturbation) takes place. In that case, one can calculate the curvaton perturbation using

$$\zeta_\sigma|_{\text{decay}} = \zeta_\sigma|_{\text{osc}} = \left[ \frac{2\delta \sigma}{3\sigma} \right]_{\text{osc}} \approx \left[ \frac{2\delta \sigma}{3\sigma} \right]_{\ast},$$

where “$\ast$” denotes the time when the Gaussian perturbation of the corresponding scale ($k = aH$) exits the horizon, and “decay” and “osc” denote the time of the curvaton decay and the beginning of the curvaton oscillation, respectively. Therefore, the curvature perturbation created by the curvaton can be written as

$$\zeta \approx [r\zeta_\sigma]|_{\text{decay}} = [r]|_{\text{decay}} \left[ \frac{2\delta \sigma}{3\sigma} \right]_{\ast},$$

where $r = 3\rho_\sigma/(3\rho_\sigma + 4\rho_r)$. From the last equation, one can see that the spectrum is strongly scale dependent when $\sigma_\ast$ varies significantly during inflation while $P_{\delta \sigma_\ast} \approx (H_I/2\pi)^2$ is slowly varying. Obviously, one can see no scale dependence in $r|_{\text{decay}}$. On the other hand, if one defines the curvature perturbation using $\delta\sigma/\sigma|_{\text{osc}}$, one immediately finds (by definition) that $\sigma|_{\text{osc}}$ cannot be a scale-dependent parameter. Instead, $\delta\sigma|_{\text{osc}}$ may have a scale-dependence. Confusions may arise if one mixes those different definitions.

Similar scale dependence may arise in other modulation mechanisms. For instance, modulated reheating gives $\zeta \sim \delta \Gamma/\Gamma \sim \delta \chi/\chi$, where $\Gamma(\chi)$ is the decay rate that depends on a modulus field $\chi$. For modulated reheating, “decay” does not mean the decay of the modulus field but the decay of the inflaton oscillation.

2.1.2 Scale dependent $\Delta\varphi$?

Here we define $\varphi_\ast \equiv \varphi_0|_\ast + \Delta\varphi_\ast$ for a light scalar field, so that $z$-dependence appears in $\Delta\varphi$. From the above discussion one will understand that $\Delta\varphi$ could be time dependent so that $\Delta\varphi$ is smaller (if $V(\varphi)$ is concave) when smaller scales exit horizon.

However, if $\varphi$ is the primary source of the initial curvature perturbation, one will immediately find that the spectral index of the initial curvature perturbation can put severe bound on the evolution of $\varphi$. Since the same is true for $\Delta\varphi$, the scale dependence of $\Delta\varphi$ must
be mild and therefore the scenario of the strongly scale-dependent $A$ could be excluded by using the spectral index. The situation becomes better if $\Delta\varphi$ is separated from the primary source of the initial curvature perturbation. Such models will be considered in section 4.

### 2.2 Example 1: problem in single-field inflation with a standard kinetic term

Let us apply the $\delta N$ formalism to the simplest scenario. Here we write for the inflaton field:

$$\phi(\vec{x}, t) = \phi_0(t) + \Delta\phi(z, t) + \delta\phi(\vec{x}, t),$$

where $\delta\phi$ is the Gaussian perturbation whose spectrum is $P_{\delta\phi}^{1/2} = H/2\pi$, while $\Delta\phi$ is a shift of $\phi$ across the sky in the direction of $z$. When one calculates the curvature perturbation using $\delta\phi$, one has to consider a local mean value $\bar{\phi}(z) \equiv \phi_0 + \Delta\phi(z)$. Then the $\delta N$ formalism gives,

$$\delta N = N_\phi \delta\phi + \frac{1}{2} N_{\phi\phi} \delta\phi \delta\phi + \ldots \simeq \frac{1}{\sqrt{2\epsilon_H} M_p},$$

where $M_p$ is the reduced Planck mass. Here the slow-roll parameter is $\epsilon(z) \equiv -\frac{\dot{H}}{H^2} \simeq \frac{1}{2} M_p^2 \left(\frac{V'}{V}\right)^2$, where $\epsilon_H$ is a function of $\bar{\phi}(z)$. $M_p = 2.435 \times 10^{18}$GeV is the reduced Planck mass and $H \equiv V'(\phi)/2M_p^2$ is the Hubble parameter during inflation. The shift $\Delta(\delta N)$ across the sky is evaluated as

$$\Delta(\delta N) \simeq (\delta N)_\phi \Delta\phi \simeq N_{\phi\phi} \delta\phi \Delta\phi,$$

where the scale dependence of $\delta\phi$ has been neglected. (See also footnote 3.) Then one can easily find

$$A \simeq \frac{N_{\phi\phi}}{N_\phi} |\Delta\phi|,$$

Considering the non-Gaussianity parameter

$$f_{NL} \equiv \frac{5}{6} \frac{N_{\phi\phi}}{N_\phi^2},$$

one will find

$$f_{NL} \simeq \frac{5}{6} \frac{A}{N_\phi |\Delta\phi|}.$$

For the single-field inflation scenario, a simple observation gives

$$A \sim 1.2 \times f_{NL} N_\phi |\Delta\phi|. \quad (2.14)$$

The above result suggests $\Delta N \equiv N_\phi |\Delta\phi| \sim (0.05/1.2 f_{NL}) \sim 1$ if $|f_{NL}| \sim |n_s - 1|$. Therefore, $\Delta N \sim 1$ may ruin the perturbative expansion.

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*Exact calculation gives $A \propto n_s - 1$, where $n_s$ is the spectral index. Namely, if one considers $\delta N \neq 0$ and $\epsilon_H \neq 0$, one will find additional terms that lead to $A \propto n_s - 1$. *
The above condition may be marginal, however we can see that the condition becomes more stringent when higher terms are considered. To see the constraints from the higher terms, let us examine the quadrupole and the octupole of the perturbation. First introduce a function \( F(k_A z) \), which gives \( F(k_A z) = \Delta \phi_d \) on the decoupling scale \( (z = z_d) \). For that function, we have the expansion in powers of \( (k_A z) \).

\[
F(k_A z) = F'(0)(k_A z) + \frac{1}{2!}F''(k_A z)^2 + \ldots, \tag{2.15}
\]

where the prime is for the derivative with respect to \( (k_A z) \). In addition to the above expansion, one can expand the gravitational-potential \( (\Phi = -\frac{3}{5}\zeta) \) in powers of \( F \). Here, we are temporarily considering \( \Phi \) instead of \( \zeta \), so that the reader can easily compare the result with the original calculation in ref. [3, 4]. We first expand \( \Delta N_d \) as

\[
\Delta N_d \equiv N_\phi(\Delta \phi_d) + \frac{1}{2!}N_{\phi \phi}(\Delta \phi_d)^2 + \ldots
= N_\phi(\Delta \phi_d) + \frac{6}{10}f_{NL}[N_\phi(\Delta \phi_d)]^2 + \ldots \tag{2.16}
\]

For a single-field inflation model, in which the non-Gaussianity is negligible, one can neglect terms proportional to \( N_{\phi \phi} \) and \( N_{\phi \phi \phi} \).  

For comparison, we are going to introduce a specific function \( F(k_A z) \sim \hat{\phi} \sin(k_A z + \omega_0) \), which has been used in ref. [3, 4]. This function will be replaced when we consider a topological defect. Here \( \hat{\phi}, k_A \) and \( \omega_0 \) are constants, which have the corresponding dimensions. Using eq. (2.15), one can expand \( \Delta \phi_d \) as

\[
\Delta \phi_d = (k_A z_d) \hat{\phi} \cos \omega_0 + \frac{1}{2}(k_A z_d)^2 \hat{\phi} \sin \omega_0 + \ldots \tag{2.17}
\]

Introducing \( \Phi_{\Delta \phi_d} \equiv -\frac{3}{5}\Delta N_d \) and using eq. (2.16) and (2.17), one can expand \( \Phi_{\Delta \phi_d} \). For the first order, we define \( \Phi_A \) as

\[
\Phi_{\Delta \phi_d}|_D \equiv (k_A z_d)\Phi_A \cos \omega_0, \tag{2.18}
\]

where the subscript \( D \) denotes the perturbation proportional to \( k_A z \). The terms contributing to the CMB quadrupole and octupole are [3, 4]:

\[
\Phi_{\Delta \phi_d}|_O \equiv \frac{(k_A z_d)^2}{2}|\Phi_A \sin \omega_0| \leq 2.9Q \tag{2.19}
\]

\[
\Phi_{\Delta \phi_d}|_O \equiv \frac{(k_A z_d)^3}{6}|\Phi_A \cos \omega_0| \leq 5.3\mathcal{O}, \tag{2.20}
\]

where the upper bounds are \( Q \leq 1.8 \times 10^{-5} \) and \( \mathcal{O} \leq 2.7 \times 10^{-5} \) for the quadrupole and the octupole, respectively. In ref. [3, 4], \( \omega_0 = 0 \) has been considered and thus the quadrupole vanishes.

In the \( k_A \rightarrow 0 \) limit with fixed \( k_A \Phi_A \), one will find a negligible bound from the quadrupole and the octupole. In that limit the size of the configuration becomes much larger than the horizon size and what we are observing in the sky is a local part of the configuration. Therefore, the configuration is approximately a linear function of \( z \).

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Footnote: These terms are not negligible in the curvaton [13, 14, 24–27] and other modulation mechanisms [15–17, 28–35].
However, if one introduces the condition “$\Phi_A \leq 1$ everywhere”, it gives $\Phi_{\Delta \phi_d} / (k_A z_d) \leq 1$.\footnote{This condition is obviously different from the condition “$|N_\phi \Delta \phi| < 1$ within the Horizon”. The difference will be crucial when we consider a defect that is expanded during inflation.} Then for $\Phi_{\Delta \phi_d}$, we have

$$ (k_A z_d) \geq \Phi_{\Delta \phi_d}. \quad (2.21) $$

Therefore, for a fixed $\Phi_{\Delta \phi_d}$ (because we need to explain $A \sim 0.05$), $k_A$ is bounded from below and finally we have

$$ |\Phi_{\Delta \phi_d}|^3 \leq 32 \Omega. \quad (2.22) $$

Here, the quadrupole is neglected assuming $\omega_0 = 0$. Using eq. (2.14), it has been concluded in ref. [3, 4] that a single-field inflation model will not produce $A \sim 0.05$.

The situation will be changed when $F(k_A z)$ is replaced by a domain wall configuration and the condition “$\Phi_A \leq 1$ everywhere” is replaced by “$\Phi_A \leq 1$ within the horizon”. Let us see more details in the next section.

### 3 Topological defects expanded during inflation

In the previous section we have introduced a planar wave perturbation for $\Delta \phi(z)$. However, it is not quite obvious why such non-spherical perturbation has been produced in the inflationary Universe. In this section we will focus on the source of $\Delta \phi(x)$, paying attention to chaotic initial conditions in the pre-inflationary epoch. The amplitude of the configuration can be as large as the Planck scale. We are choosing two of the simplest models and will show explicitly how the defect configurations can affect the asymmetry of the cosmological perturbations. More successful (but rather complicated) models will be examined in the next section so that the model can explain significant scale dependence of the asymmetry.

#### 3.1 Inflating defects

The idea of topological inflation is very old. One can find an excellent review of the cosmological defects in ref. [36, 37].

To understand the situation, consider a domain wall model for which the symmetry is broken by a real field $\phi$ and it develops a vacuum expectation value $\phi = \pm \hat{\phi}$ at a distance from the core ($\phi = 0$). Just for simplicity, we assume $V(\hat{\phi}) = 0$ and $V(0) \equiv V_0 > 0$. Before the primordial inflation, we are considering a chaotic initial condition, which is schematically shown in figure 1.

In flat spacetime the width of the domain wall ($\delta_w$) is determined by the balance of the gradient and the potential energies ($\delta_w \sim \hat{\phi} / \sqrt{V_0}$). Since the horizon radius of the Universe when the false vacuum density $V_0$ is dominating is given by $H_0^{-1} \sim M_p / \sqrt{V_0}$, one expects $\hat{\phi} \geq M_p$ for a trivial (e.g., $\sim -m^2 \phi^2 + \lambda \phi^4$) potential. See also figure 2. Numerical studies for quartic potentials suggest $\hat{\phi} \simeq 0.33 \sqrt{8 \pi} M_p$ [36, 37]. The Universe first emerged would have a chaotic initial condition $\hat{\phi}^2 \sim (\partial_i \phi)^2 \sim M_p^4$, which leads to a highly inhomogeneous Universe that have a random distribution of $\phi$ within the Hubble horizon. At this moment the width of the defects is very narrow since the chaotic initial condition is (by definition) not determined by the balance between the gradient and the energies of the potentials. After a while, the width is determined by the balance of gradient and potential energies and the topological inflation takes place.
Figure 1. The Universe is inhomogeneous because of the chaotic initial condition.

Figure 2. Topological inflation expects a broad core. What is required for inflation is $\dot{\phi} \geq M_p$.

Here, it must be noted that the curvature perturbation becomes singular at $\phi = 0$, where the slow-roll parameter vanishes ($\epsilon_H \approx 0$). In other words, topological inflation never ends deep in the core. Slightly away from $\phi = 0$, one can find a suitable region that allows conventional inflation with a safe end accompanied by the oscillation and reheating. Obviously, the inflating defect of the topological inflation scenario leads to a singular perturbation. However, in reality the singularity is not a problem since $\phi = 0$ is far away from the horizon. See also footnote 5.

The situation is almost the same for the curvaton and other modulation mechanisms, but a few trivial differences may appear. First, the potential gives $V = V(\phi) + V_I$ during inflation, where $V_I \gg V(\phi)$ is the energy density of the inflaton field. The width of the defect can become as large as the horizon radius during inflation, if the effective mass of $\phi$ is smaller than the Hubble parameter ($m_\phi^2 \leq H_I^2$). Unlike topological inflation, the Hubble radius is determined by $V_I$, which is independent of $V(\phi)$. Then the configuration is expanded during the inflationary expansion. Second, although in this paper we have been considering a specific defect configuration (domain wall), the gradient of the field ($\Delta \phi \neq 0$) can simply be created by the chaotic initial condition, even if $\Delta \phi$ is not related to a topological configuration. The gradient $\Delta \phi$ can be generated on a flat potential $V(\phi) \approx 0$, even if it is not related to a topological defect. In that case the form of the configuration is determined by the local shape of the potential $V(\phi)$. We have been using a domain wall configuration, as we are expecting that $F(k_{Az})$ is more or less akin to the local configuration of a domain wall. The initial chaotic condition may include $\varphi = 0$, where conventional perturbation will be singular. The singularity is not a problem, if it is always far away from our Universe.

We hope there will be no confusion between defects of $\phi$(inflaton) and $\varphi$(curvaton/moduli).
3.2 Example 2: problem in topological inflation

The accelerated expansion of the Universe in standard inflationary scenarios is driven by the energy of the false vacuum. Since the topological defects have false vacuum in their cores, one can expect inflating cores when a defect have (or evolves to have) a broad core \cite{36–38}.\footnote{Our idea of the topological defect is a particular case of a general super horizon-scale perturbation. See also a preceding work \cite{21}. Our method is quite general.} This gives the basic idea of the inflating defect, which can explain the initial condition of the conventional inflation model. All one needs to start inflation is a false vacuum region that is greater than the horizon.

Assume that our Universe is placed on a primordial domain wall \( \phi_w(z) = \hat{\phi}[\tanh(k_A z + \omega_0)] \). Expanding the configuration in powers of \((k_A z)\), we find

\[
\frac{\dot{\phi}_w}{\dot{\phi}} \approx \tanh(\omega_0) + (k_A z)^2 \tanh^2(\omega_0) - \frac{(k_A z)^3}{3} \left[ \cosh(2\omega_0) - 2 \right] \text{sech}^4(\omega_0) + \ldots, \tag{3.1}
\]

where we are going to define \( \phi_{CMB} \equiv \hat{\phi} \tanh(\omega_0) \). The source of the CMB asymmetry is

\[ \Delta \phi \simeq (k_A z) \hat{\phi} \text{sech}^2(\omega_0). \]

For hilltop-type inflation \cite{39}, we have \( \hat{\phi} \simeq M_p \) and

\[ \phi_{CMB} \simeq \hat{\phi} e^{-N|\eta|}, \tag{3.2} \]

where \( \eta \equiv M_p^2 V_{\phi\phi}/V \). We thus find for \( N \sim 60 \):

\[ \tanh(\omega_0) \approx e^{-60|\eta|}. \tag{3.3} \]

In the same way as in the previous section, we can calculate the coefficients of the higher terms to find the quadrupole and the octupole. Unfortunately however, since the single-field inflation with the standard kinetic term always predicts \( f_{NL} \ll 1 \), and also the exact calculation of the asymmetry parameter gives \( A \propto n_s - 1 \), which is mandatory, it is impossible to find \( A \sim 0.05 \) from the perturbative expansion.

Although the above result is disappointing, the idea of the inflating defect is interesting. Note also that primordial configuration may appear for every light field at the same time, and they could be expanded simultaneously during inflation, even though the fields themselves do not cause inflationary expansion. Just for an instance, consider a hilltop curvaton \cite{40, 41}, in which the curvaton mass is temporarily negative (i.e, the curvaton potential is initially convex) but it finally becomes concave before the decay. If we introduced the chaotic initial condition to the model, the initial configuration could be a domain wall whose shape is determined by the potential at that moment. See also some recent works in ref. \cite{42–44}, in which authors considered the evolution of both the curvaton and the modulus field in modulated reheating scenarios and explored the effects on the power spectrum and \( f_{NL} \).

3.3 Example 3: modulated reheating for a domain wall configuration

In this section we consider a modulus field \((\varphi)\), which has both \( \Delta \varphi \) and \( \delta \varphi \). We assume that \( \delta \varphi \) is primary source of the initial curvature perturbation. In contrast to the topological
inflation model considered above, $\varphi$ is not supposed to be the inflaton field. The curvaton mechanism, which has been explored in other papers [3, 4, 7], will give a similar result, although the curvaton is not mentioned explicitly in this section.

The curvaton (except for the inflating curvaton) and other modulation mechanisms (e.g., modulated reheating) do not expect $|f_{NL}| \ll 1$ even if the decaying matter component dominates the Universe [15–17, 30, 31]. In this section we consider a modulated decay scenario that gives $|f_{NL}| \sim 5$ [30, 31]. For a moduli field $\varphi$, we thus find from eq. (2.14):

$$A \sim 6N_{\varphi}|\Delta\varphi|.$$  

(3.4)

Here, we assume that the origin of $\Delta\varphi$ is the primordial domain wall that exists prior to the inflationary expansion. One can define the mean value (center of the Gaussian perturbation) of the moduli as $\bar{\varphi} \equiv \varphi_0 + \hat{\varphi}\tanh(\omega_0) + \Delta\varphi$. Here the domain-wall configuration $\varphi_w \equiv \hat{\varphi}\tanh(kAz + \omega_0)$ is centered at $\varphi_0$ and is expanded for $kAz \ll 1$ as

$$\hat{\varphi} \tanh(\omega_0) + (kz)\text{sech}^2(\omega_0) - (kz)^2\tanh(\omega_0)\text{sech}^2(\omega_0) + \frac{(kz)^3}{3}[\cosh(2\omega_0) - 2]\text{sech}^4(\omega_0) + \ldots.$$  

(3.5)

Therefore, $\Delta\varphi$ is

$$\frac{\Delta\varphi}{\hat{\varphi}} \approx (kz)\text{sech}^2(\omega_0) - (kz)^2\tanh(\omega_0)\text{sech}^2(\omega_0) \ldots$$  

(3.6)

The expansion by $\Delta\varphi$ is

$$\Delta N = N_{\varphi}\Delta\varphi + \frac{1}{2}N_{\varphi\varphi}(\Delta\varphi)^2 + \ldots$$  

(3.7)

Therefore, the dipole (not observable), the quadrupole and the octupole are

$$\frac{\Delta N|_D}{k_Azd} \equiv N_{\varphi}\hat{\varphi}\text{sech}^2(\omega_0) \equiv \Delta N_A$$  

(3.8)

$$\frac{\Delta N|_Q}{(k_Azd)^2} = \frac{1}{2}N_{\varphi\varphi}\hat{\varphi}^2\text{sech}^4(\omega_0) - N_{\varphi\varphi}\hat{\varphi}\tanh(\omega_0)\text{sech}^2(\omega_0)$$  

$$= \frac{6}{10}f_{NL}(\Delta N_A)^2 - \Delta N_A\tanh(\omega_0)$$  

(3.9)

$$\frac{\Delta N|_O}{(k_Azd)^3} = \frac{1}{6}N_{\varphi\varphi\varphi}\hat{\varphi}^3\text{sech}^6(\omega_0) - N_{\varphi\varphi\varphi}\hat{\varphi}^2\text{sech}^4(\omega_0)\tanh(\omega_0) + \frac{1}{3}N_{\varphi\varphi}\hat{\varphi}[\cosh(2\omega_0) - 2]\text{sech}^4(\omega_0)$$  

$$= \frac{9}{25}g_{NL}(\Delta N_A)^3 - \frac{6}{5}f_{NL}\tanh(\omega_0)(\Delta N_A)^2$$  

$$+ \frac{\Delta N_A}{3}[\cosh(2\omega_0) - 2]\text{sech}^2(\omega_0).$$  

(3.10)
From eq. (2.14) and eq. (3.8), we find

\[ A \sim 1.2 f_{NL} \Delta N_A(k_A z_d). \]  

(3.11)

From eq. (3.9), we find

\[ f_{NL}(\Delta N_A)^2(k_A z_d)^2 < 8 Q. \]  

(3.12)

Therefore, the asymmetry is bounded from above as

\[ A < 1.2 f_{NL} \sqrt{\frac{8 Q}{f_{NL}}} \sim 0.014 \sqrt{f_{NL}}. \]  

(3.13)

Since \( g_{NL} > 10^4 \) is not excluded in the modulated decay scenario and is giving an interesting observational possibility, an important issue is to consider a more stringent constraint that may appear from the octupole perturbation

\[ \Delta N |_O(k_A z_d)^3 \simeq \frac{9}{25} g_{NL}(\Delta N_A)^3 < 8.8 O, \]  

(3.14)

which will be significant if \( |g_{NL}|^2 \gtrsim 10^5 |f_{NL}|^3 \). We thus find that an observation of \( g_{NL} > 10^4 \) could be crucial for the models of the CMB asymmetry. This point has not been considered in previous works.

One might expect that the significant scale dependence of the asymmetry parameter \( A \) could be explained by the scale-dependent \( f_{NL} \). However, usual scenario of the curvaton and other modulation mechanisms do not expect such significant scale dependence. Moreover, since \( f_{NL} \) has the minimum value (\( f_{NL} \gtrsim 1 \)) associated with nonlinear effects in any model [30, 31], the variation stops inevitably there. On the other hand, \( f_{NL} \) has the upper bound \( |f_{NL}| \leq 10 \) on large scale, which can contradict with the required scale dependence.\(^7\)

Namely, to solve the scale dependence of the asymmetry using scale-dependent \( f_{NL} \), one needs \( 10/|f_{NL}|^{10Mpc} | \geq 0.05/0.0045 \), which gives a critical condition \( |f_{NL}^{10Mpc}| < 1 \). The inflating curvaton could be an exception, in which \( f_{NL} \sim \mathcal{O}(\epsilon_H, \eta) \ll 1 \) is possible. However, at this moment there is no concrete model that realizes strongly scale-dependent \( f_{NL} \) in the inflating curvaton model. Therefore, we conclude that all these models cannot explain the asymmetry.

## 4 Model buildings

Let us sort out cosmological models paying attention to the origin of the asymmetry and its scale dependence. Prescriptively, “multi-field models” include the curvaton and other modulation mechanisms since these models require at least two fields to achieve both the inflationary expansion and the creation of the curvature perturbations.

As we have stated in the previous section, “single-field models of inflation” are already excluded if the source of the asymmetry is \( \Delta \phi(z) \neq 0 \). The curvaton and other modulation mechanisms, which are prescriptively multi-field but one field (the curvaton \( \sigma \) or a modulus field \( \chi \)) is the primary source of the initial curvature perturbations, can be excluded by the scale dependence.

---

\(^7\)The Planck limit assumes that \( |f_{NL}| \) is constant on all CMB scales. Although we did not consider the possibility in this paper, \( |f_{NL}| \) might have a strong scale dependence and \( |f_{NL}| \) on the scale of the asymmetry could be far larger than \( |f_{NL}| \sim 10 \).
Therefore, in this section we consider separation of $\delta N$. The separation of the curvature perturbation ($\delta N = \delta N_1 + \delta N_2$) is a very old idea, which has been considered in a variety of multi-field models of inflationary cosmology. Ref. [3, 4] considers an application to explain the asymmetry, and ref. [45] calculates the scale-dependence of the parameter $A$, when the slow-roll parameter is scale-dependent. The crucial difference between previous works and our model will be explained in appendix A.

In this section we need strong scale dependence for the secondary component $\delta N_2$. Recently in ref. [46], strongly scale-dependent spectrum of the curvaton scenario has been used to explain the primordial black hole (PBH) formation. In ref. [25], it has been shown that the inflating curvaton mechanism can also explain PBH formation, in which some constraints can be relaxed because of the late-time curvaton inflation. Other modulation mechanism may also have strong scale dependence. For instance, a model of the scale-dependent modulations (tachyonic growth model) has been proposed in ref. [47].

Suppose that the potential of the field $\phi$ is given by $V(\phi) \simeq cH^2\phi^2$. The potential is familiar in supergravity inflationary models, in which F-term generates $|c| \sim O(1)$ while D-term may generate $|c| \ll 1$. It is possible to generate $V(\phi) \simeq cH^2\phi^2$ from $F(\phi)R$ in the Lagrangian, where $F(\phi)$ is a function of $\phi$ and $R$ is the Ricci scalar. In that way, $c$ is model-dependent and $c$ is determined by the component that is dominating the energy density at that moment. One will find the equation

$$\ddot{\phi} + 3H\dot{\phi} + cH^2\phi = 0,$$

whose solution is [23]

$$\phi \propto e^{-\alpha Ht} \propto k^{-\alpha},$$

where $\alpha \equiv \frac{3}{2} - \sqrt{\frac{9}{4} - c}$. If $\delta \phi$ is primary source of the initial curvature perturbation, contribution to the spectral index is $n_s - 1 = -2\epsilon_H + 2\alpha \simeq 2\epsilon_H + 2\eta_\phi$, where $\eta_\phi \equiv M_p^2 V_{\phi\phi}/V$

### 4.1 Multi-field models of separable perturbations ($N_{\phi\phi} \simeq 0$)

First, consider a separable spectrum of $\phi$ and $\varphi$ that gives

$$\delta N = \delta N_1 + \delta N_2,$$

where we defined

$$\delta N_1 = N_{\phi}\delta \phi,$$

$$\delta N_2 = N_{\varphi}\delta \varphi.$$

To be separable up to the second order, we need $N_{\phi\phi} \simeq 0$.

We also assume a Gaussian perturbation and a shift given by

$$\phi(\vec{x}, t) = \phi_0(t) + \delta \phi(\vec{x}, t),$$

$$\varphi(\vec{x}, t) = \varphi_0(t) + \Delta \varphi(\vec{x}, t) + \delta \varphi(\vec{x}, t),$$

where $\delta \phi$ and $\delta \varphi$ are the Gaussian perturbation, which is expected to have the spectrum $P_{\delta \phi}^{1/2} = P_{\delta \varphi}^{1/2} = H_1/2\pi$ at the horizon exit. As before, we introduce a function $F(k_A z)$ that gives $F(k_A z_d) \equiv \Delta \varphi_d$ on the decoupling scale. Then, one can define the asymmetry as

$$A \equiv \frac{\Delta(\delta N_2)}{\delta N}$$

$$= \frac{N_{\varphi}P_{\delta \varphi}^{1/2}|\Delta \varphi|}{N_{\phi}P_{\delta \phi}^{1/2} + N_{\varphi}P_{\delta \varphi}^{1/2}}.$$





We are going to separate “multi-field models of separable perturbations” into two categories. From figure 3, “Multi A” includes the conventional curvaton and modulations, in which a field $\varphi$ is responsible for “both” the initial curvature perturbations and the asymmetry. In that case we find

$$A \simeq \frac{N_{\varphi\varphi}|\Delta \varphi|}{N_{\varphi}}. \quad (4.8)$$

The other (“Multi B”) considers one field ($\phi$) as primary source of the initial curvature perturbations and the other field ($\varphi$) as the source of the asymmetry. In that case we are expecting $\delta N_2 < \delta N_1$ but not much smaller. Since the asymmetry is expected to be $A \sim 0.05$, one can estimate $\delta N_2/\delta N_1 \geq 0.05$. Later we will show that $\delta N_2/\delta N_1 > 0.05$ could be a crucial condition. For this model, the asymmetry is defined by

$$A \simeq \frac{N_{\varphi\varphi}P_{\delta \varphi}^{1/2}|\Delta \varphi|}{N_{\phi}P_{\delta \phi}^{1/2}} = \frac{N_{\varphi\varphi}|\Delta \varphi|}{N_{\phi}} \left( \frac{P_{\delta \varphi}}{P_{\delta \phi}} \right)^{1/2}, \quad (4.9)$$

where the quantities are defined at the moment when the curvature perturbations are generated. (i.e, $P_{\delta \phi}$ is defined at the horizon exit but $P_{\delta \varphi}$ is defined when $\delta N_2$ is generated.) Then, following the discussion in section 2.1.1, significant scale-dependence may appear in the ratio $\left( \frac{P_{\delta \varphi}}{P_{\delta \phi}} \right)^{1/2}$. In order to explain the scale dependence in a familiar form, consider the function

$$\varphi \equiv g(\varphi_*), \quad (4.10)$$

where the definition first appeared in ref. [48] to explain the evolution of the curvaton perturbation. We thus find

$$A \simeq \frac{N_{\varphi\varphi}P_{\delta \varphi}^{1/2}|\Delta \varphi|}{N_{\phi}P_{\delta \phi}^{1/2}} \left( \frac{1}{g'} \right)^{1/2}, \quad (4.11)$$

where $\delta \varphi \simeq g'(\varphi_*)\delta \varphi_* \equiv P_{\delta \varphi}$. The scale-dependent asymmetry is due to the significant scale dependence of $\delta N_2$.\footnote{Our model (multi-B) does not include the running inflation scenario. Thanks to the referee of JCAP, we found that the scenario of running inflation has already been thoroughly explored by Erickcek, Hirata and Kamionkowski in the appendix A of ref. [45], where the spectrum with a discontinuity has also been considered. However, the model assumes $\varphi_* = g(\varphi_*)$ (or $\eta_\sigma \equiv m^2/3H^2 \simeq 0$ for the curvaton) and calculated the asymmetry with $1/g' \equiv 1$ (i.e, they considered the trivial evolution function and put $P_{\delta \varphi} = P_{\delta \phi}$, from the beginning). The calculation is reviewed in our appendix A, where the correspondence between these calculations will be very clear.}

\subsection{Curvaton and other modulation mechanisms (multi-B)}

Here we consider Multi-B scenario, which is shown on the right-hand side in figure 3.\footnote{Just before we accomplish the second version of this paper, we found a paper [49] in which a similar idea has been examined.} If the asymmetry of the CMB is created by a field that is NOT primary source of the initial
curvature perturbation, the role of the secondary field ($\varphi$) is to create the asymmetric part $\delta N_2 \propto z$.

To begin with, define the non-Gaussianity parameter of the component perturbation as

$$f_{NL,\varphi} \equiv \frac{5}{6} \frac{N_{\varphi \varphi}}{(N_{\varphi})^2}. \quad (4.12)$$

Then the asymmetry parameter is

$$A \equiv \frac{\Delta(\delta N_2)}{\delta N} = \frac{6 f_{NL,\varphi} N_{\varphi} |\Delta \varphi|}{5 \cdot 1 + r_N^{-1} r_P^{-1}} \approx \frac{6 f_{NL,\varphi} N_{\varphi} |\Delta \varphi| (r_N r_P)}, \quad (4.13)$$

where we defined

$$r_N^{-1} \equiv \frac{N_{\varphi}}{N_{\varphi}}, \quad (4.14)$$

$$r_P^{-1} \equiv \frac{P_{\delta \varphi}^{1/2}}{P_{\delta \varphi}^{1/2}}. \quad (4.15)$$

We used the condition $\delta N_1 > \delta N_2$ that leads to $r_N^{-1} r_P^{-1} > 1$. 

---

**Figure 3.** The left-hand side picture shows a model in which the Gaussian perturbation of a field is the primary source of the initial curvature perturbations and at the same time defect configuration of the field explains the asymmetry of the CMB. Although somewhat confusing, the model is usually dubbed “multi-field”, since one needs two fields to explain both the inflationary expansion and the curvature perturbation. The right-hand side picture shows a model in which the initial curvature perturbation is generated by a field but the source of the asymmetry is another field.
Since the secondary perturbation ($\delta N_2$) is responsible for the asymmetry, what we need to explain $A = 0.05$ is

$$f_{NL,\phi} \sim \frac{5}{6} \frac{A}{N_\phi} \Delta \varphi^{-1} r^{-1} r_P^{-1},$$

where we considered $1 > \delta N_2 / \delta N_1 > A$.

Then, the non-Gaussianity parameter of the total curvature perturbation is

$$f_{NL} = \frac{f_{NL,\phi}}{1 + r_P^2} + \frac{f_{NL,\varphi}}{1 + (r_{NTRP})^{-2}} \sim (r_{N_{TRP}})^2 f_{NL,\varphi}$$

$$\geq \frac{5}{6} A^2 \frac{A}{N_\phi} \Delta \varphi,$$

where the second line is obtained assuming $f_{NL,\phi} \equiv \frac{5 N_{\phi\phi}}{6 N_\phi^2} \simeq 0$. Note that in the above equation there is significant suppression due to an extra factor of $A \sim 0.05$. (Compare eq. (4.17) with eq. (2.13).) Therefore, the required $f_{NL}$ can be much smaller than the Multi-A scenario.

We find for the domain-wall configuration:

$$\frac{\Delta N|_D}{k_A z_d} \equiv N_{\phi\phi} \sech^2(\omega_0) \equiv \Delta N_A$$

$$\frac{\Delta N|_Q}{(k_A z_d)^2} \equiv \frac{1}{2} N_{\phi\phi\phi} \sech^4(\omega_0) - N_{\phi\phi} \tanh(\omega_0) \sech^2(\omega_0)$$

$$= \frac{6}{10} f_{NL,\varphi}(\Delta N_A)^2 - \Delta N_A \tanh(\omega_0)$$

$$\frac{\Delta N|_O}{(k_A z_d)^3} \equiv \frac{1}{6} N_{\phi\phi\phi} \sech^6(\omega_0)$$

$$- N_{\phi\phi\phi} \sech^4(\omega_0) \tanh(\omega_0)$$

$$+ \frac{1}{3} N_{\phi\phi} [\cosh(2\omega_0) - 2] \sech^4(\omega_0)$$

$$= \frac{9}{25} g_{NL,\varphi}(\Delta N_A)^3 - \frac{6}{5} f_{NL} \tanh(\omega_0)(\Delta N_A)^2$$

$$+ \frac{\Delta N_A}{3} [\cosh(2\omega_0) - 2] \sech^2(\omega_0).$$

Therefore, we find

$$f_{NL,\varphi}(\Delta N_A)^2 (k_A z_d)^2 < 8 Q.$$  (4.21)

Using eq. (4.16), we find that the asymmetry is bounded from above as

$$A < 0.014 \sqrt{f_{NL,\varphi}} (r_{NTRP}).$$  (4.22)

Then $A \sim 0.05$ gives $3.6 < \sqrt{f_{NL,\varphi}} (r_{NTRP})$.

To explain the scale-dependence of the asymmetry, we consider (again) a variation of $r_P$. Here, what we need is $(r_P^{\text{small}} / r_P^{\text{CMB}}) \lesssim 0.1$, which is quite conceivable if the factor is obtained.
from the variation caused by a quadratic potential: $e^{-\eta \varphi^N} \sim 10$ for $N = 6.1$ (Quasar) or $N \sim 4$ (CMB small scale). Here the slow-roll parameter required for the Quasar is

$$\eta \varphi \equiv \frac{m^2}{3H^2} \sim -0.38.$$ (4.23)

For the small-scale CMB, it becomes $\eta \varphi \sim -0.58$.\(^{10}\)

In this scenario, there will be a possible tension between the spectral index and the scale-dependence in $\delta N_2$, where the latter is needed to explain the asymmetry of the small-scale perturbations while the former is strictly constrained by the Planck observation. In the above scenario, we can estimate the additional contribution to the spectral index as (for the Quasar):

$$\Delta(n_s - 1) \sim \frac{\delta N_2}{\delta N_1} (n_s - 1) \delta N_2 \geq 2A\eta \varphi \sim -0.038,$$ (4.24)

where $(n_s - 1)\delta N_2$ is the spectral index of the component $\delta N_2$. The small-scale CMB constraint [20] requires $n_s \sim (n_s)\delta N_1 + \Delta(n_s) \sim 0.96$ for $\Delta(n_s - 1) \sim 2A\eta \varphi \sim -0.058$, which cannot be satisfied without fine-tuning. Therefore, in reality $\Delta(n_s - 1)$ is larger than $2A\eta \varphi$ and the issue of the fine-tuning is serious.

### 4.2 Multi-field models of mixed perturbations ($N_{\phi \varphi} \neq 0$)

To start with, assume a primary field ($\phi$) and a mechanism (inflation, the curvaton or other modulation mechanisms) that creates curvature perturbation $\zeta = \delta N \sim N_{\phi \varphi}\delta \phi$. Then, one can introduce a modulation sourced by a secondary field $\varphi$ (moduli). We write

$$\phi(\vec{x}, t) = \phi_0(t) + \delta \phi(\vec{x}, t)$$
$$\varphi(\vec{x}, t) = \varphi_0(t) + \Delta \varphi(\vec{x}, t) + \delta \varphi(\vec{x}, t),$$ (4.25)

where $\delta \phi$ is the Gaussian perturbation, while $\Delta \varphi$ has specific direction ($z$) and it is supposed to be the source of the CMB asymmetry $A \neq 0$.

A shift $\Delta(\delta N)$ appears because of $\Delta \varphi$:

$$\Delta(\delta N) \simeq (\delta N)\varphi \Delta \varphi \simeq N_{\phi \varphi} \delta \phi \Delta \varphi.$$ (4.26)

Then the asymmetry parameter is

$$A \simeq \frac{N_{\phi \varphi}}{N_{\varphi}} |\Delta \varphi|.$$ (4.27)

Recall that the usual definition of the non-Gaussianity parameter is [48]

$$f_{NL} = \frac{5}{6} \frac{N_a N_b N_{ab}}{(N_c N^c)^2},$$ (4.28)

which is obtained assuming $r_P = 1$. Therefore we have

$$f_{NL} \simeq \frac{5}{3} \frac{N_{\phi \varphi} N_{\phi \varphi}}{(N_{\phi}^2 + N_{\varphi}^2)^2},$$ (4.29)

\(^{10}\)If $\eta \varphi > 3/2$ then the field will not be perturbed during inflation [50].
where $N^{\phi\phi} = N^{\varphi\varphi}$. From eq. (4.27) and (4.29), the asymmetry will be estimated as

$$A \sim \frac{N^{\phi\phi} |\Delta \varphi|}{N^{\phi}} \sim \frac{3}{5} f_{NL} \left( 1 + r^2 N^{\varphi \varphi} \right)^2 \frac{|\Delta \varphi|}{r^2 N^{\phi}} \sim \frac{3}{5} \frac{1}{r^2 N^{\phi}} f_{NL} |\Delta \varphi|. \quad (4.30)$$

The mixed perturbation $N^{\phi\varphi} \neq 0$ may appear when one considers modulated decay for the curvaton [30–33]. Just for an instance, introduce a moduli-dependent mass $m(\phi, \varphi)$ for the decaying matter component and consider modulated curvaton decay [30–33]. Then one will find $N^{\phi}$ and $N^{\phi\varphi}$ in terms of $m(\phi, \varphi)$. In that case, choosing a point on the landscape of $m(\phi, \varphi)$, one may find a significant $N^{\phi\varphi}$ that determines the asymmetry. A simple modulation of an inflation parameter may lead to the asymmetry [28, 29], which will be considered below.\textsuperscript{11}

### 4.2.1 Two-field inflation with a non-standard kinetic term ($c_s$-modulation)

First we review the preceding model considered in ref. [51]. Although the model is slightly complicated compared with our model in section 4.2.2, the kinetic term is well motivated in the string theory.

In section 2.2 we showed that “the single-field inflation model with the standard kinetic term” is severely constrained and cannot explain the asymmetry. On the other hand, a non-standard kinetic term may change the sound speed (usually labeled by $c_s$), which causes deviation from the conventional model of single-field inflation. The most obvious character of the model appears in the equilateral non-Gaussianity parameter $f_{NL}^{eq}$, which can be enhanced keeping the spectral index small. The reason has been clearly explained in ref. [52], which shows that the shrinking sound horizon during inflation cancels the deviation from a de Sitter background to retain a scale invariant spectrum. In that way the enhancement of the non-Gaussianity parameter is possible without violating the conditions for the spectral index.

Unfortunately, the model with $c_s \ll 1$ has been excluded by the Planck observation; that is the reason why we did not consider the model in section 2.2. However, this model may be applied to explain the hemispherical asymmetry if the value of sound speed varies from one side of the sky to the other (i.e, when $c_s(k, z)$ is a non-trivial function of both $k$ (scale) and $z$ (place).) The example considered in ref. [51] uses the Lagrangian for the two-field model:

$$\mathcal{L} = \frac{1}{f(\phi, \chi)} \left( 1 - \sqrt{1 + f \partial_{\mu} \phi \partial^{\mu} \phi} \right) - \frac{1}{2} \partial_{\mu} \chi \partial^{\mu} \chi - V(\phi, \chi). \quad (4.31)$$

Here, $\phi$ is the inflaton and $\chi$ is an extra light field (moduli), which does not contribute to the background evolution. $\chi$ is introduced just to explain $k$- and $z$-dependent sound speed $c_s(k, z)$. The kinetic term of the inflaton $\phi$ involves a function $f(\phi, \chi)$. Since we do not have new result for this model, we will not examine this model in this paper. See ref. [51] for further discussions.

\textsuperscript{11}A stringent condition has been found in ref. [11], in which they argued that $N_\varphi \geq N_\varphi \dot{\phi} H^{-1}$ can break a constraint [2]. However, true inequality is $N_\varphi \geq N_\varphi \dot{\phi} H^{-1} + N_{\varphi\phi} \dot{\phi} H^{-1}$, which allows cancellation between terms. Therefore, the mixed perturbation scenario is not excluded but it may require fine-tuning.
4.2.2 Modulated inflation with a non-standard kinetic term

We introduce a moduli field ($\chi$) and consider the non-standard kinetic term for the inflaton \([53]\):\(^{12}\)

$$
L_{\text{kin}} = \frac{1}{2} \omega(\chi) \partial_{\mu} \phi \partial^{\mu} \phi. 
$$

(4.32)

Because of $\omega$, the equation of motion becomes

$$
\ddot{\phi} + 3H \dot{\phi} + \frac{V_{\phi}}{\omega} \dot{\chi} = 0, 
$$

(4.33)

where we can assume $\dot{\chi} \simeq 0$.

Let us examine if this simplest version of the non-standard kinetic term could work to explain the CMB asymmetry.

The curvature perturbation generated during inflation is

$$
\delta N = N_{\phi} \delta \phi + \frac{1}{2} N_{\phi \phi} \delta \phi \delta \phi + \ldots
\simeq H \frac{\delta \phi}{\dot{\phi}} = \frac{\omega}{\sqrt{2 \epsilon H}} \frac{\delta \phi}{M_p}.
$$

(4.34)

Since the kinetic term is modulated by $\chi$, one can expect an inhomogeneous $\dot{\phi}$ in the direction of $z$. If $\chi$ has a domain-wall configuration ($\Delta \chi(z)$), one can expect asymmetry caused by the shift. The asymmetry is

$$
A \simeq \frac{N_{\phi \chi}}{N_{\phi}} |\Delta \chi| = \frac{\omega \chi}{\omega} |\Delta \chi|. 
$$

(4.35)

For a specific function, eq. (B.7) suggests

$$
A \simeq 2 \frac{\Delta \chi}{\chi}. 
$$

(4.36)

This is a convincing way of obtaining asymmetry from a very simple extension of the single-field inflation model.

The scale-dependence can appear from the evolution of $\chi_{\ast} \equiv \chi^{(0)}_{\ast} + \Delta \chi_{\ast}$, where $\delta \chi$ is neglected for simplicity. In this model, scale dependence of the asymmetric perturbation $\Delta(\delta N) \sim N_{\phi \chi} \Delta \chi$ can be independent of the dynamics of $\phi$. This extension can be applied to many kinds of single-field inflationary models without changing the predictions of the original model, as far as the model predicts negligible $N_{\chi}$. On the other hand, the predictions of the model are very weak, since the effective action is quite ambiguous.

5 Conclusion and discussion

In this paper, we considered a chaotic initial condition and a domain wall configuration that may exist prior to the inflationary expansion. We showed that such configuration can explain the CMB asymmetry. Remembering that topological inflation may usually include a

\(^{12}\)See also appendix B.
point where the curvature perturbation diverges ($\epsilon_H \simeq 0$), we have to reconsider the conventional assumption “$\Phi_A \leq 1$ everywhere”. In this paper, the condition has been replaced by “$N_o \Delta \phi \leq 1$ (or $N_e \Delta \phi \leq 1$) within the horizon”. Although the new condition is looser than the former, we found that topological inflation cannot explain the asymmetry. Then we explored the curvaton and other modulation mechanisms using a primordial defect configuration as the source of $\Delta \varphi(z)$. The scale-dependence of the asymmetry is examined in various scenarios.

Our result shows that Multi-A model, in which $\varphi$ is primary source of the initial curvature perturbation, cannot explain the asymmetry (and its scale dependence).

On the other hand, Multi-B model, in which $\Delta \varphi$ is separated from the primary source of the initial curvature perturbation, can explain both the asymmetry and its scale dependence but requires fine-tuning. For the spectral index, the reason is very clear. Since the asymmetric part is entirely due to $\delta N_2$, the fraction $\delta N_2/\delta N_1$ cannot be smaller than $A \sim 0.05$. Since the scale dependence of $A$ is due to the significant scale dependence of $\delta N_2$, it can contribute to the spectral index of the total curvature perturbation ($\delta N = \delta N_1 + \delta N_2$). If $\delta N_2$ has the spectral index $(n_{s2} - 1) \sim \mathcal{O}(1)$, its contribution to the total $(n_s - 1)$ is more than $A \times (n_{s2} - 1) \sim 0.05$, which is not acceptable without fine-tuning. Large $g_{NL}$, which might be observed in future observations, may put a stringent bound on the octupole perturbation.

We also examined mixed perturbation models. (In this case we don’t have to assume $\delta N_2/\delta N_1 > A$.) $c_s$-modulation has been considered in ref. [51]. In this paper we considered a modulated inflation model [28, 29] and showed that the model does not require fine-tuning in the spectral index because (unlike the scenario of separable perturbation) evolution of $\Delta \chi$ can be separated from the spectral index of the total.

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A Running inflation model in ref. [45]

In contrast to our Multi-B model, ref. [45] considers the inflaton as a source of the scale-dependent asymmetry.

In ref. [45], the fraction of the total power that comes from the curvaton has been defined as

$$\xi(k) = \frac{\mathcal{P}_{\zeta_\phi}(k)}{\mathcal{P}_{\zeta_\phi}(k) + \mathcal{P}_{\phi}(k)},$$

(A.1)

where

$$\mathcal{P}_{\zeta_\phi}(k) = \frac{R^2}{9} \frac{H^2}{\pi^2 \sigma_s^2},$$

(A.2)

$$\mathcal{P}_{\phi}(k) = \frac{G H^2}{\pi \epsilon_H}.$$  

(A.3)

Note that their definitions are different from the component perturbations used in the conventional curvaton scenario [13, 14, 54]. Here we have followed the notations in ref. [45] and
have used $\epsilon_H \equiv -\ddot{H}/H^2$ and $G \equiv 8\pi M_p^{-2}$. $R$ is defined at the curvaton decay as

$$R \equiv \frac{3\Omega_\sigma}{4\Omega_\gamma + 3\Omega_\sigma + 3\Omega_{\text{CDM}}}. \quad (A.4)$$

Their primary observation is that any scale-dependence in $\xi$ must originate from variation in the slow-roll parameter $\epsilon_H(\phi_*)$ during inflation, which is correct only when the evolution before the curvaton oscillation is trivial (i.e., when $\delta \sigma/\sigma$ is constant until the beginning of the sinusoidal oscillation).

If the variation of $\epsilon_H$ is smooth, one inevitably find

$$\frac{d}{d\ln k} \ln \xi = \frac{d}{d\ln k} \ln \mathcal{P}_\zeta + \xi \frac{d}{d\ln k} \ln \mathcal{P}_{\zeta \sigma} - (1 - \xi) \frac{d}{d\ln k} \ln \mathcal{P}_{\zeta \sigma}$$

$$= (2\eta_\sigma - 2\epsilon_H) - \xi (2\eta_\sigma - 2\epsilon_H) - (1 - \xi)(-4\epsilon_H + 2\eta_H)$$

$$= -(1 - \xi)(2\eta_H - 2\epsilon_H - 2\eta_\sigma), \quad (A.5)$$

where ref. [45] uses the definition $\eta_H \equiv -\ddot{\phi}/(\dot{\phi}H) \simeq M_p^2 [V''(\phi)/V(\phi)] - \epsilon_H$. Here the result is for a simple quadratic curvaton potential.

The index for the total power spectrum is

$$n_s - 1 = \frac{d}{d\ln k} \ln \left[ \mathcal{P}_{\zeta \sigma} + \mathcal{P}_{\zeta \sigma} \right]$$

$$= \xi (2\eta_\sigma - 2\epsilon_H) + (1 - \xi)(2\eta_H - 4\epsilon_H)$$

$$= -2\epsilon_H + 2\xi \eta_\sigma + (1 - \xi)(2\eta_H - 2\epsilon_H), \quad (A.6)$$

and the tensor to the scalar ratio is

$$r = 16\epsilon_H (1 - \xi). \quad (A.7)$$

Therefore, we find

$$-\frac{d}{d\ln k} \ln \xi = n_s - 1 + \frac{r}{8(1 - \xi)} - 2\eta_\sigma. \quad (A.8)$$

Ref. [45] ignores $\eta_\sigma$ to find

$$-\frac{d}{d\ln k} \ln \xi = n_s - 1 + \frac{r}{8(1 - \xi)}. \quad (A.9)$$

which is wrong when $\eta_\sigma$ is significant. Indeed, our Multi-B model considers the opposite case:

$$-\frac{d}{d\ln k} \ln \xi \approx -2\eta_\sigma. \quad (A.10)$$

They also considered a discontinuity in $\xi(k)$, which can be used to avoid the blue spectrum. In that case the small-scale perturbations are simply disconnected from the large-scale anomaly.
B Modulated inflation

We introduce moduli $\chi$ and consider the action:

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{4\pi G} R - \frac{1}{2} \omega(\chi) \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} \partial_\mu \chi \partial^\mu \chi \right] - V(\phi) - W(\chi),$$  

(B.1)

where $\omega(\chi)$ and the potential $W(\chi)$ are functions of the moduli. The inflaton potential is $V(\phi) \simeq V_0$ during inflation. The specific form of the coefficient $\omega(\chi)$ could be $\omega(\chi) = \beta \frac{\chi^2}{M_*^2}$, where $M_*$ is a cutoff scale. Variation of the action leads to the equations

$$\ddot{\phi} + 3H \dot{\phi} + \frac{V'}{\omega} + \frac{\omega'}{\omega} \dot{\chi} = 0$$

(B.2)

$$\ddot{\chi} + 3H \dot{\chi} + W' - \frac{\omega'}{2} \dot{\phi}^2 = 0.$$  

(B.3)

The slow-roll inflation gives

$$\dot{\phi} \simeq - \frac{V'}{3H + (\omega'/\omega)\dot{\chi}} \frac{\omega}{\omega'}$$

(B.4)

where $3H \gg (\omega'/\omega)\dot{\chi}$ is assumed.

The $\delta N$ formalism gives

$$N_\phi = - \frac{H}{\dot{\phi}} \simeq \frac{3H^2}{V'} \omega,$$

(B.5)

which leads to the mixed perturbation

$$N_{\phi\chi} \simeq \frac{\omega'}{\omega} N_\phi \neq 0.$$  

(B.6)

For $\omega(\chi) = \beta \frac{\chi^2}{M_*^2}$, one will find

$$\frac{N_{\phi\chi}}{N_\phi} \simeq \frac{2}{\chi}.$$  

(B.7)

References

[1] J. Hoftuft, H.K. Eriksen, A.J. Banday, K.M. Gorski, F.K. Hansen et al., Increasing evidence for hemispherical power asymmetry in the five-year WMAP data, Astrophys. J. 699 (2009) 985 [arXiv:0903.1229] [insPIRE].

[2] Planck collaboration, P.A.R. Ade et al., Planck 2013 results. XXIII. Isotropy and statistics of the CMB, arXiv:1303.5083 [insPIRE].

[3] A.L. Erickcek, M. Kamionkowski and S.M. Carroll, A Hemispherical Power Asymmetry from Inflation, Phys. Rev. D 78 (2008) 123520 [arXiv:0806.0377] [insPIRE].

[4] A.L. Erickcek, S.M. Carroll and M. Kamionkowski, Superhorizon Perturbations and the Cosmic Microwave Background, Phys. Rev. D 78 (2008) 083012 [arXiv:0808.1570] [insPIRE].

[5] D.H. Lyth, A.R. Liddle, The primordial density perturbation: cosmology, inflation and the origin of structure, Cambridge, U.K., Cambridge Univ. Pr., 2009, pg. 497.
[6] L.P. Grishchuk and I.B. Zel’dovich, *Long-wavelength perturbations of a Friedmann world and anisotropy of the RELICT radiation*, Sov. Astron. **22** (1978) 125.

[7] D.H. Lyth, *The CMB modulation from inflation*, JCAP **08** (2013) 007 [arXiv:1304.1270] [SPIRE].

[8] Z.-G. Liu, Z.-K. Guo and Y.-S. Piao, *Obtaining the CMB anomalies with a bounce from the contracting phase to inflation*, Phys. Rev. **D 88** (2013) 063539 [arXiv:1304.6527] [SPIRE].

[9] L. Wang and A. Mazumdar, *Small non-Gaussianity and dipole asymmetry in the cosmic microwave background*, Phys. Rev. **D 88** (2013) 023512 [arXiv:1304.6399] [SPIRE].

[10] M.H. Namjoo, S. Baghram and H. Firouzjahi, *Hemispherical Asymmetry and Local non-Gaussianity: a Consistency Condition*, Phys. Rev. **D 88** (2013) 083527 [arXiv:1305.0813] [SPIRE].

[11] S. Kanno, M. Sasaki and T. Tanaka, *A viable explanation of the CMB dipolar statistical anisotropy*, PTEP **2013** (2013) 111E01 [arXiv:1309.1350] [SPIRE].

[12] J.F. Donoghue, K. Dutta and A. Ross, *Generating the curvature perturbation without an inflaton*, Phys. Rev. **D 80** (2009) 023526 [astro-ph/0703455] [SPIRE].

[13] T. Moroi and T. Takahashi, *Effects of cosmological moduli fields on cosmic microwave background*, Phys. Lett. **B 522** (2001) 215 [Erratum ibid. **B 539** (2002) 303] [hep-ph/0110096] [SPIRE].

[14] L. Kofman, *Probing string theory with modulated cosmological fluctuations*, astro-ph/0303614 [SPIRE].

[15] G. Dvali, A. Gruzinov and M. Zaldarriaga, *Cosmological perturbations from inhomogeneous reheating, freezeout and mass domination*, Phys. Rev. **D 69** (2004) 083505 [astro-ph/0305548] [SPIRE].

[16] T. Matsuda, *Cosmological perturbations from an inhomogeneous phase transition*, Class. Quant. Grav. **26** (2009) 145011 [arXiv:0902.4283] [SPIRE].

[17] C.M. Hirata, *Constraints on cosmic hemispherical power anomalies from quasars*, JCAP **09** (2009) 011 [arXiv:0907.0703] [SPIRE].

[18] S. Flender and S. Hotchkiss, *The small scale power asymmetry in the cosmic microwave background*, JCAP **09** (2013) 033 [arXiv:1307.6069] [SPIRE].

[19] M.S. Turner, *A Tilted Universe (and Other Remnants of the Preinflationary Universe)*, Phys. Rev. **D 44** (1991) 3737 [SPIRE].

[20] D. Wands, K.A. Malik, D.H. Lyth and A.R. Liddle, *A New approach to the evolution of cosmological perturbations on large scales*, Phys. Rev. **D 62** (2000) 043527 [astro-ph/0003278] [SPIRE].

[21] K. Dimopoulos, C.-M. Lin and T. Matsuda, *Primordial black holes from the inflating curvaton*, Phys. Rev. **D 87** (2013) 103527 [arXiv:1211.2374] [SPIRE].
[26] S. Enomoto, K. Kohri and T. Matsuda, Non-Gaussianity in the inflating curvaton, *Phys. Rev. D* **87** (2013) 123520 [arXiv:1210.7118] [iSPIRE].

[27] K. Dimopoulos, K. Kohri and T. Matsuda, The hybrid curvaton, *Phys. Rev. D* **85** (2012) 123541 [arXiv:1201.6037] [iSPIRE].

[28] T. Matsuda, Free light fields can change the predictions of hybrid inflation, *JCAP* **04** (2012) 020 [arXiv:1204.0303] [iSPIRE].

[29] T. Matsuda, Modulated Inflation, *Phys. Lett. B* **665** (2008) 338 [arXiv:0810.2648] [iSPIRE].

[30] S. Enomoto, K. Kohri and T. Matsuda, Modulated decay in the multi-component Universe, *JCAP* **08** (2013) 020 [arXiv:1301.3787] [iSPIRE].

[31] K. Kohri, C.-M. Lin and T. Matsuda, Delta-N Formalism for Curvaton with Modulated Decay, *JCAP* **06** (2013) 009 [arXiv:1303.2750] [iSPIRE].

[32] D. Langlois and T. Takahashi, Density Perturbations from Modulated Decay of the Curvaton, *JCAP* **04** (2013) 014 [arXiv:1301.3319] [iSPIRE].

[33] H. Assadullahi, H. Firouzjahi, M.H. Namjoo and D. Wands, Modulated curvaton decay, *JCAP* **03** (2013) 041 [arXiv:1301.3439] [iSPIRE].

[34] T. Matsuda, Generating the curvature perturbation with instant preheating, *JCAP* **03** (2007) 003 [hep-th/0610232] [iSPIRE].

[35] T. Matsuda, Cosmological perturbations from inhomogeneous preheating and multi-field trapping, *JHEP* **07** (2007) 035 [arXiv:0707.0543] [iSPIRE].

[36] A. Vilenkin and E.P.S. Shellard, *Cosmic Strings And Other Topological Defects*, Cambridge, U.K., Cambridge Univ. Pr., 1994.

[37] A. Vilenkin, Topological inflation, *Phys. Rev. Lett.* **72** (1994) 3137 [hep-th/9402085] [iSPIRE].

[38] T. Matsuda, Q ball inflation, *Phys. Rev. D* **68** (2003) 127302 [hep-ph/0309339] [iSPIRE].

[39] L. Boubekeur and D. Lyth, Hilltop inflation, *JCAP* **07** (2005) 010 [hep-ph/0502047] [iSPIRE].

[40] T. Matsuda, Hilltop curvaton, *Phys. Lett. B* **659** (2008) 783 [arXiv:0712.2103] [iSPIRE].

[41] T. Matsuda, Topological curvaton, *Phys. Rev. D* **72** (2005) 123508 [hep-ph/0509063] [iSPIRE].

[42] M. Kawasaki, T. Kobayashi and F. Takahashi, Non-Gaussianity from Curvatons Revisited, *Phys. Rev. D* **84** (2011) 123506 [arXiv:1107.6011] [iSPIRE].

[43] N. Kobayashi, T. Kobayashi and A.L. Erickcek, Rolling in the Modulated Reheating Scenario, arXiv:1308.4154 [iSPIRE].

[44] T. Kobayashi and T. Takahashi, Runnings in the Curvaton, *JCAP* **06** (2012) 004 [arXiv:1203.3011] [iSPIRE].

[45] A.L. Erickcek, C.M. Hirata and M. Kamionkowski, A Scale-Dependent Power Asymmetry from Isocurvature Perturbations, *Phys. Rev. D* **80** (2009) 083507 [arXiv:0907.0705] [iSPIRE].

[46] M. Kawasaki, N. Kitajima and T.T. Yanagida, Primordial black hole formation from an axionlike curvaton model, *Phys. Rev. D* **87** (2013) 063519 [arXiv:1207.2550] [iSPIRE].

[47] J. McDonald, Isocurvature and Curvaton Perturbations with Red Power Spectrum and Large Hemispherical Asymmetry, *JCAP* **07** (2013) 043 [arXiv:1305.0525] [iSPIRE].

[48] D.H. Lyth, Can the curvaton paradigm accommodate a low inflation scale?, *Phys. Lett. B* **579** (2004) 239 [hep-th/0308110] [iSPIRE].

[49] J. McDonald, Hemispherical Power Asymmetry from Scale-Dependent Modulated Reheating, *JCAP* **11** (2013) 041 [arXiv:1309.1122] [iSPIRE].
[50] M. Mijic, *Particle production and classical condensates in de Sitter space*, Phys. Rev. D 57 (1998) 2138 [gr-qc/9801094] [inSPIRE].

[51] Y.-F. Cai, W. Zhao and Y. Zhang, *CMB Power Asymmetry from Primordial Sound Speed Parameter*, Phys. Rev. D 89 (2014) 023005 [arXiv:1307.4090] [inSPIRE].

[52] M. Alishahiha, E. Silverstein and D. Tong, *DBI in the sky*, Phys. Rev. D 70 (2004) 123505 [hep-th/0404084] [inSPIRE].

[53] T. Matsuda, *Modulated inflation from kinetic term*, JCAP 05 (2008) 022 [arXiv:0804.3268] [inSPIRE].

[54] M. Sasaki, J. Valiviita and D. Wands, *Non-Gaussianity of the primordial perturbation in the curvaton model*, Phys. Rev. D 74 (2006) 103003 [astro-ph/0607627] [inSPIRE].