Local and Nonlocal Andreev Reflections in Double Barrier Junctions

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Abstract. We study spin dependent charge transport through a junction, consisting of a superconductor connected to two normal or spin-polarized electrodes. Special attention is paid to a process known as a crossed or nonlocal Andreev reflection. The nonlocal reflection occurs when an incident electron (hole) from one electrode, injected with a subgap energy onto a superconducting layer, transforms into an Andreev hole (electron) in the second lead. When two electrons (holes), forming a Cooper pair, come from the same electrode, the process of the Andreev reflection is local. The coherent spin polarized transport in ferromagnet/superconductor/ferromagnet (F/S/F) double barrier junctions is analyzed using the Bogoliubov-de Gennes (BdG) equation with appropriate boundary conditions. We calculate and discuss probabilities of the normal, local and crossed Andreev reflections, as well as probability of the elastic cotunneling. These processes contribute to tunneling current when the distance between the magnetic electrodes is comparable to the superconducting coherence length. The dependence of the tunneling charge transport on the strength of the exchange field in the ferromagnetic electrodes, and on the height of the tunnel barriers are presented. We also discuss the local and nonlocal spin quantum entanglements of quasiparticle pairs induced by the Andreev reflections in the junction.

1. Introduction
In this paper we have addressed the problem of charge transport through nanoscale systems where normal (N) or ferrimagnetic (F) metals (M) or half metals are attached to a superconductor. In particular, we have concentrated on the role of the local and nonlocal Andreev reflections both in single particle transport and in pairwise spin entangled transport. We have considered the charge transport through a simple model of the metal-superconductor-metal (M/S/M) junction. The charge transport can be decomposed into two processes. The first type, which can be named local processes, consists of the normal and Andreev reflections from the first e.g. left M/S interface. These reflections occur in the single normal- or ferrimagnetic metal-superconductor junction. The nonlocal processes, occur in a region where there is no drive current. It is characteristic for multi-terminal junctions [1] and consists of the crossed Andreev (CAR) reflection and elastic cotunneling (EC) [2, 3, 4, 5, 6, 7, 8]. It has been shown [1, 8] that when the distance l between the electrodes is less than or comparable to the superconducting coherence length ξ, an injection of an electron (or hole) with a subgap energy leads to the additional nonlocal and coherent transport processes named the CAR and EC [2]. The experimental evidence for both CAR and EC has been given by Russo et al. [3] and also by Beckmann et al. [4]. If the thickness of S layer is large enough, a double tunnel junction
can be regarded as two independent single M/S junctions and then the tunneling processes become incoherent [6]. In addition, the total electronic current in the F/S/F junction strongly depends on the relative orientation of the magnetization of two ferromagnetic electrodes [7, 8].

The CAR is a consequence of capturing another electron (or hole) from the right electrode in order to form a Cooper pair in the superconductor. The appropriate hole (or electron) is reflected in the right electrode. The EC consists in the direct tunneling of the injected particle from one electrode to the second one, without formation of the Cooper pair [5]. The electronlike quasiparticle penetrates depth of order of the superconducting coherence length $\xi$ before its conversion into the condensate of the Cooper pairs. The charge current $2e v_F$ in the ferromagnetic metals disappears, as an appropriate quasiparticle current $J_Q$ in the SC layer, according with the formula $J_Q = 2e v_F \exp(-y/\xi)$, where the coherence length is given by $\xi = h v_F / (2\sqrt{\Delta^2 - E^2})$. Simultaneously, $J_Q$ converts into an increasing supercurrent of the Cooper pairs $J_s = 2e v_F [1 - \exp(-y/\xi)]$. In this paper, we discuss how the nonlocal effects are influenced by the energy of the incident electron and the width of the superconducting layer. These effects are important for the following reasons:

(i) The spin polarized current in metallic F/S/F heterostructures, due to the CAR, can be applied in spintronics by implementation of magnetoresistive devices, such as spin switches or memory elements;

(ii) The spin polarized subgap tunneling spectroscopy in double F/S/F junctions provides information on the pairing symmetry in ferromagnetic superconductors;

(iii) The CAR enables realization of the solid state entanglers by creation of mobile spin entangled pairs of electrons in spatially separated nanoelectronic circuits.

2. Model and calculations

In this paper, we consider a double junction, consisting of two metallic (normal or ferromagnetic) electrodes and a superconducting layer separated from the electrodes by thin insulating interfaces. Here, to avoid unnecessary complications, we consider a model of F/S/F junction with a clean superconductor. In the clean limit one can easily solve the BdG equations within the approach proposed in [9]. The wave function, describing the quasiparticle propagation across the F/S/F junction, can be obtained from the extended BdG equations. In the ferromagnetic regions, these equations are reduced to the appropriate Schrödinger equations. The extended BdG equations take the following form:

$$
\left( \begin{array}{c} H_0 - E - \sigma h_{ex} \\ \Delta \end{array} \right) \left( \begin{array}{c} f_{\sigma}(\vec{r}) \\ g_{-\sigma}(\vec{r}) \end{array} \right) = \hat{O},
$$

where $H_0 = -\hbar^2/2m\nabla^2 - E_F + W(\vec{r})$ is the single-particle Hamiltonian, $E$ is the quasiparticle energy measured from the Fermi energy $E_F$, and $\sigma = +(-1)$ denotes the up (down) spin subband. $W(\vec{r})$ stands for the interface potential which is measured by the dimensionless parameter $z = 2mW/\hbar^2 k_F$. For the ferromagnetic electrodes, we adopt the Stoner model. Thus, the exchange field in both the left ($y < -l/2$) and the right electrodes ($y > l/2$) can be defined as follows

$$
h_{ex} = \begin{cases} h_0 & y < -l/2 \\ -l/2 < y < +l/2 \\ \pm h_0 & y > l/2 \end{cases}
$$

where $+h_0$ and $-h_0$ define the exchange fields for the parallel and antiparallel configuration, respectively. The solution of eq.(1) corresponding to the injection of an electron with spin $\sigma$
Figure 1. The probability of the four transport processes as a function of $E/\Delta$ for $l k_F = 10^3$ and for several values of $x$ and $z$.

Figure 2. The probability of the four transport processes as a function of $E/\Delta$ for $l k_F = 10^4$ and for several values of $x = h_0/E_F$ and $z$. 

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Figure 3. The probability of the ordinary ($|a|^2$) and crossed Andreev ($|d|^2$) reflections as a function of $l k_F$ for $E/\Delta = 0.0$ (left panel) and $E/\Delta = 4.0$ (right panel) and for several values of $x$ and $z$: ($x = 0.0$, $z = 0.0$)- full line, ($x = 0.5$, $z = 0.5$)- dotted line, ($x = 0.5$, $z = 0.0$)- dashed line.

(from one, e.g. the left ferromagnetic electrode) and with energy $E > 0$, and angle of incidence $\Theta$ can be written in the following form:

$$\Psi_{\sigma}^{(y<-l/2)}(y) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{ik_{\sigma}^- y} + a_{-\sigma} \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{ik_{\sigma}^- y} + b_{\sigma} \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-ik_{\sigma}^- y},$$ (3)

$$\Psi_{\sigma}^{(y>l/2)}(y) = c_{\sigma} \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{ik_{\sigma}^+ (-\sigma) y} + d_{\sigma} \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{-ik_{\sigma}^+ \sigma y},$$ (4)

where the $y$ component of the momentum for both an electron (+) and hole(−) takes the form: $k_{\sigma}^\pm = \sqrt{(2m/\hbar^2)(E_F + \sigma h_0 \pm E) - k_{||,\sigma}^2}$. The momentum parallel to the interface has the form: $|k_{||,\sigma}| = \sqrt{(2m/\hbar^2)(E_F + \sigma h_0 + E) \sin(\Theta)}$. The probability amplitudes of the transport processes $a_{\sigma}(E, \Theta)$, $b_{\sigma}(E, \Theta)$, $c_{\sigma}(E, \Theta)$, $d_{\sigma}(E, \Theta)$ determine the ordinary Andreev and normal reflections, elastic cotunneling, and crossed Andreev reflection, respectively[5, 6, 7, 8, 10].

For $E/\Delta > 1$ the interference of incident and reflected quasiparticles in the superconducting layer leads to oscillations of all the transport coefficients and, in addition, both $a_{\sigma}$ and $d_{\sigma}$ vanish if the following resonance condition is fulfilled i.e. $q_{\sigma}^+ - q_{\sigma}^- = 2n\pi/l$. The moment $q_{\sigma}^\pm$ in the superconducting layer $S$ is given by $q_{\sigma}^\pm = \sqrt{(2m/\hbar^2)(E_F \pm \sqrt{E^2 - \Delta^2}) - k_{||,\sigma}^2}$.

The coefficients $a_{\sigma}(E, \Theta)$, $b_{\sigma}(E, \Theta)$, $c_{\sigma}(E, \Theta)$, $d_{\sigma}(E, \Theta)$ are determined by the appropriate boundary conditions for the wave functions and their derivatives at both the interfaces are placed at $y = \pm l/2$. Although it is true that in experiments one does not measure directly the coefficients $a, b, c, d$, however, these coefficients determine the charge transport through $F/S/F$ junctions and the pairing symmetry of the superconductor. For example, the sign of the voltage measured across
the second \( S/F \) tunnel barrier of the junction \( F/S/F \) depends upon whether \( EC \) or \( CAR \) occurs with larger probability \[3\]. Similarly, in the case of half-metallic ferromagnet electrodes the process of \( CAR \) can be completely suppressed while \( EC \) will be allowed, or vice versa, depending on the mutual alignments (parallel or antiparallel) of the electrodes magnetization.

3. The Andreev entangler: spin entanglement in the \( F/S/F \) junction

As shown in the previous section, the coherent quasiparticle charge transport through the double \( F/S/F \) junction exhibits interesting characteristic features. In particular, for \( E > \Delta \), all probabilities of the transport processes (the AR, CAR, NR, and EC) oscillate with energy and the thickness of the superconducting layer as a result of the interference of incoming and outgoing particles in this layer. This coherent transport is particularly interesting because of its promising spintronic applications by implementation of magnetoresistive devices, such as spin switches, memory elements, magnetic read-out heads for computer hard drives, spin transistors etc. Additionally, as it was proved, the Andreev and normal reflected electrons (or holes) transported from the superconductor into the conducting leads can be mutually spin entangled. Pairing interaction in a superconductor is the source of spin entanglement. The pair of electrons can tunnel coherently, due to the Andreev reflections processes, from the superconductor into the same or different leads. In the case of the singlet superconductors, non-local processes, i.e. the processes which lead to tunneling of the electrons to the different electrodes, can be enhanced for the antiparallel configuration, or can be suppressed for the parallel configuration of the ferromagnetic electrodes. Entanglement of either electrons or holes depends on the sign of the voltage applied to the left lead, for instance. For negative voltage an incident spin up (down) hole in the left conductor can be Andreev reflected as spin down (up) electron either at the left (ordinary AR) or at the right (CAR) lead.

Figure 4. Sketch of the reflection processes responsible for local and non-local entanglement in the \( M/S/M \) junction.

In our analysis, we consider the concept of the local and nonlocal spin-entanglement between electrons or holes, in a double barrier junction consisting of two conductors connected to a superconductor. At most, the separation between the both metallic electrodes should be comparable to the superconducting coherence length \( \xi \) in order to enable the crossed Andreev reflection. An application of a bias voltage \( eV = \mu_S - \mu_L \), between the superconductor and the normal or ferromagnetic conductor, leads to a possible stationary tunneling transport of spin entangled pairs of electrons or holes. \( \mu_S \) and \( \mu_L \) stand for chemical potentials of the s-wave superconductor and the left metallic leads, respectively. Depending on the sign of the voltage applied to one of the conductors we get a stream of entangled fermions. If a negative voltage is applied to the left conductor, then an incident spin up (down) hole in the left conductor can
be Andreev reflected as a spin down (up) electron either at the left conductor or at the right one. In such a way, the holes incident on the superconducting layer e.g. from the left electrode convert, due to the pairing interaction in the superconductor, their charge and spin into the opposite ones. In order to create entangled pairs of electrons, let us apply a negative voltage to the left electrode (see Fig.4). Then, the input state forming a stream of pairs of holes with opposite spins and the same energies $E$ injected to the superconductor can be written in the form [11, 12]:

$$ |\Psi_{in} > = \prod_{0 < E < eV} a_{L1}^{\dagger}(E)a_{L1}^{\dagger}(E) | G > = \frac{i}{2} \prod_{0 < E < eV} a_{L}^{\dagger}(E)\sigma_y a_{L}^{\dagger T}(E) | G > , $$

where $a_{L}^{\dagger} = (a_{L1}^{\dagger}(E)a_{L1}^{\dagger}(E))^T$ is a matrix of the creation operators of a pair of holes with the opposite spins at the left conductor and $| G >$ denotes the Fermi sea filled up to the superconducting chemical potential in the left conductor; $\sigma_y$ is the Pauli matrix. $T$ stands for the matrix transpose. The evolution of the input state $|\Psi_{in} >$ into the output state $|\Psi_{out} >$ can be connected through unitary transformation, which can be expressed in terms of the scattering matrix $S$. The scattering matrix matches the wave functions at both $FS$ and $SF$ interfaces in a double barrier junction [13]. In our further analysis we take into account only the Andreev reflections and completely neglect the elastic cotunneling process since such process does not lead to the entanglement of the particles. Thus, the relation between the input $\hat{b}$ and output $\hat{b}$ states can be written in a form similar to that given in [11] i.e.

$$ \hat{b} = \hat{S} \hat{a}, $$

where $\hat{b}$ and $\hat{a}$ are the $8 \times 1$ matrices. $\hat{S}$ is an $8 \times 8$ scattering matrix, which describes all reflections processes in left ($L$) and right ($R$) interfaces:

$$ \hat{S} = \begin{pmatrix}
S_{ee}^{ee} & S_{ee}^{OE} & 0 & 0 & S_{he} & S_{he} & S_{he} & S_{he} \\
S_{LL}^{ee} & S_{LL}^{OE} & 0 & 0 & S_{LL}^{OE} & S_{LL}^{OE} & S_{LL}^{OE} & S_{LL}^{OE} \\
0 & 0 & S_{RR}^{ee} & S_{RR}^{OE} & S_{RR}^{ee} & S_{RR}^{ee} & S_{RR}^{ee} & S_{RR}^{ee} \\
0 & 0 & S_{RR}^{ee} & S_{RR}^{OE} & S_{RR}^{ee} & S_{RR}^{ee} & S_{RR}^{ee} & S_{RR}^{ee} \\
S_{he} & S_{he} & S_{he} & S_{he} & S_{he} & S_{he} & S_{he} & S_{he} \\
S_{he} & S_{he} & S_{he} & S_{he} & S_{he} & S_{he} & S_{he} & S_{he} \\
S_{LR}^{ee} & S_{LR}^{oe} & S_{LR}^{ee} & S_{LR}^{ee} & S_{LR}^{ee} & S_{LR}^{ee} & S_{LR}^{ee} & S_{LR}^{ee} \\
S_{LR}^{ee} & S_{LR}^{oe} & S_{LR}^{ee} & S_{LR}^{ee} & S_{LR}^{ee} & S_{LR}^{ee} & S_{LR}^{ee} & S_{LR}^{ee} \\
S_{RR}^{ee} & S_{RR}^{oe} & S_{RR}^{ee} & S_{RR}^{ee} & S_{RR}^{ee} & S_{RR}^{ee} & S_{RR}^{ee} & S_{RR}^{ee} \\
S_{RR}^{ee} & S_{RR}^{oe} & S_{RR}^{ee} & S_{RR}^{ee} & S_{RR}^{ee} & S_{RR}^{ee} & S_{RR}^{ee} & S_{RR}^{ee}
\end{pmatrix} $$

All the elements of the scattering matrix $\hat{S}$ describe appropriate Andreev reflection processes. For example, $S_{LL}^{ee}$ denotes the ordinary Andreev reflection amplitude for a hole with, spin up injected into the left lead ($L$) and reflected as an electron with spin up in the same ($L$) lead.

In the tunneling limit i.e when the transparencies of the tunnel barriers ($T_{L(R)}$) fulfil the relation $T_{L(R)} << 1$, the outgoing state, to the first order of the scattering matrix elements $S_{\alpha\beta}$ can be written as a superposition of the vacuum state $|0 >$ and the local and non-local spin singlet entangled states ([11]):

$$ |\Psi_{out} > = \sqrt{1 - T_{L}^{2} - T_{L}T_{R}G(l)}|0 > + T_{L}|\Psi_{loc} > + \sqrt{T_{L}T_{R}G(l)}|\Psi_{non-loc} > , $$

where the local and non-local spin singlet states have the form

$$ |\Psi_{loc} > = \frac{1}{\sqrt{2}}(|\uparrow (E) >_{L}\downarrow (E) >_{L} - |\downarrow (E) >_{L}\uparrow (E) >_{L}| \quad (9)$$
\[ |\Psi_{\text{non-loc}}\rangle = \frac{1}{\sqrt{2}} [\uparrow (-E) >_L \downarrow (E) >_R - \downarrow (-E) >_L \uparrow (E) >_R]. \]  
\( \gamma(l) = \exp\left(-\frac{2l}{\xi}\right) \) describes distance at which a Cooper pair can still exist. As seen from the above relations, due to the superconducting layer, the charge current of independent particles can be transformed into the (locally) \( (T_L) \) and nonlocally \( (\sqrt{T_LT_R}) \) Andreev reflected current of spin entangled pairs.

4. Results and conclusions

The coherent charge transport through the \( F/S/F \) junction exhibits many peculiar features both for the subgap transmission and for energy greater than superconducting gap. The important role, in the transport processes through the double junction, play both the paring symmetry of a superconducting state and the magnetic orientation of the ferromagnetic electrodes [14]. For example, in the case of the spin singlet pairing and the parallel magnetic configuration of the electrodes, the ordinary and crossed Andreev reflections are suppressed and for half-metals \( (x = h_o/E_F = 1) \) these processes are completely blocked. In the case of antiparallel alignment, the crossed Andreev reflection is favoured while the normal Andreev reflection is suppressed (or blocked for half-metals). In the paper, we discuss the probabilities of all the components of the coherent transport processes, i.e.: the ordinary Andreev, crossed Andreev and normal reflections \( (|a|^2, |d|^2, |b|^2) \), as well as the direct tunneling through the double junction \( (|c|^2) \).

The sum of these probabilities is normalized to 1. The numerical calculations were performed for a wide range of the model parameters i.e. for the strength of the exchange field \( x \), the height of the interfacial barriers \( z \), and the thickness of the superconducting layer \( l \). If the energy of the incident electron is greater than the superconducting gap \( \Delta \), all these probabilities oscillate as the functions of \( E/\Delta \) and \( l \) due to the interference of incoming and outgoing particles in the superconducting layer (see Figs. 1-3). If the thickness of the superconductor is smaller than the superconducting coherence length the nonlocal processes \( (EC \text{ and } CAR) \) play an important role in the charge transport for the subgap energy (see Fig.1). In the opposite limit, i.e. if the thickness changes in the tunneling current in double junctions for different widths of the superconducting layer were reported in [3]. The maxima (minima) of the elastic cotunneling correspond one to one to minima (maxima) of the ordinary Andreev, crossed Andreev and normal reflections. If the thickness \( l \geq \xi \), then the non-local processes decay. When the \( EC \) dominates i.e. when the charge carriers pass from the left to the right electrode without conversion into the supercurrent, the ordinary Andreev and crossed Andreev reflections decay at the both interfaces. It is worthy mentioning that typical ranges of the coherence length are the order of \( \xi = 10\mu m \) for high-\( T_c \) and low-temperature superconductors, respectively. The electron-beam lithography technique allows one to fabricate devices which have sizes of the order of tens \( nm \).

In the last years, the transport phenomena in multiterminal junctions have attracted a great interest where the local and non-local entanglements, caused by the Andreev reflections, play an important role. Multiterminal junctions e.g. \( N/S/N \) and \( F/S/F \) are the promising examples of solid state entanglers. The controlled creation and detection of mobile entangled pairs of charge carriers, in spatially separated nanoelectronic leads, is crucial in quantum information. The spin of these mobile particles can be used as a qubit. Besides, the nonlocal spin-entangled electron pairs can be detected e.g. via the current-noise measurements in transport experiments.

Finally, it can be said that various aspects of the tunneling quasiparticle transport, especially the coherent one, through the double barrier junctions exhibits interesting features which can be used and applied in nanoelectronic physics. First, the single particle coherent transport in \( F/S/F \) and \( N/S/N \) junctions can be applied in spintronics. Second, the coherent pairwise
tunneling transport of spin entangled pairs in spatially separated leads can be applied in quantum information. Third, it is worth mentioning that the spectroscopy of the subgap and spin polarized transport determines the pairing symmetry of Cooper pairs in the ferromagnet superconductors.

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