Mathematical model of wear volumes due to sliding speed using Buckingham Pi Model

Yusuf Kaelani*, Achmad Syaifudin
Department of Mechanical Engineering, Sepuluh Nopember Institute of Technology
Campus Sukolilo, Surabaya 60111, Indonesia
*Email: y_kaelani@me.its.ac.id

Abstract. The objective of this research is to determine a mathematical model of wear volumes due to sliding speed. Some researchers have studied the influence of sliding speed, roughness and coefficient of friction. However, mathematical model dealing with sliding speed has never been explicitly reported. Wear analysis due to various speed is oftenly expressed experimentally through charts concerning both wear volume and sliding speed instead of mathematics. This research conducts variables affecting wear. The study is started by modeling mathematical representation within Buckingham Pi Theory. The mathematical model includes wear volume, hardness, normal loads, sliding speed, sliding distance, and density of materials. Since Buckingham set requires constant of equality, the equation is solved by rectifications. The model, in turn, is verified by data gathering to determine the constant. This value is called Wear-Speed Coefficient. Experiment using pin-on-disk tribometer is conducted by varying sliding speed and normal loads. Furthermore, those parameters are applied to estimate wear volumes. Furthermore, it is compared with those measured data. Moreover, the mathematical model is verified by comparing graphically to results of previous researchers having evaluated wear volumes due to sliding speed. The materials included are 2014 Aluminum Alloy, NBR Rubber Nitril, Ultra High Molecular Weight Poly Ethylene (UHMWPE), and Poly Tetra Fluoro Ethylene (PTFE). The results of discussion shows that the mathematical approach modelled using Buckingham Pi Theory is mostly similar to patterns. This study yields several constants of wear-speed coefficient relating to the materials.

Keywords: Sliding Speed, Normal Load, Wear Volume, Buckingham Pi Model, and Wear-Speed Coefficient

1. Introduction.
Abrasive contacts between surfaces yielding wear occur in many machine components. Some abrasive wear is necessarily needed, such as grinding. However, some others are strictly maintained to avoid excessive loose, such as bearing. Wear might be worsened by the present of stick-slip phenomenon during wear process, such as multi-directional abrasion [1] in ball joints. Therefore, wear volume becomes very important aspect to control mechanism’s life.

Parameters influencing wear volume are surface hardness, normal loads, materials, sliding distance, and sliding speed. Some researchers put coefficient of friction between two materials as the main aspect to generate abrasive wear. Surface roughness is also as part of assertive variable being studied. However, this cumbersome wear volume is rarely reported in mathematical
presentation. The correlation among these parameters are oftenly shown with graphical reports due to uncertainty of wear mechanism. Finally, it is important to have a mathematical model to accommodate the parameters with the intention of wear volume approximation.

This research is focused on formulating the wear volume relating to sliding speed. The formulation method uses Buckingham Pi Theory. The constant of set’s equality is drawn from experiments dan data gathering.

2. Methods and Materials.
As mentioned in the previous paragraph, the mathematical model of wear volume is constructed by using Buckingham Pi model. Wear parameters chosen to be repeating variables are Hardness ($H$), material’s density ($\rho$) and Wear Volume. Meanwhile, unrepeating variables include normal load ($W$), sliding distance ($L$) and sliding speed ($v$).

The general form is expressed following.

$$\Pi = f(r, V, H, W, L, v)$$

(1)

The first set of the dimensionless equation

$$\prod_1 = \frac{W}{H^\frac{1}{3}} \frac{1}{V^{2/3}}.$$

(2)

The second set is the following

$$\prod_2 = \frac{V^{1/3}}{L}.$$

(3)

The two equations are synced to become the following.

$$\frac{W}{H} \frac{1}{V^{2/3}} \approx \frac{V^{1/3}}{L}$$

(4)

Finally, the equation might be simplified as this below.

$$\frac{W}{H} k = \frac{V}{L}$$

(5)

Where:

- $W$ = Normal load (N)
- $H$ = Surface Hardness (Pa)
- $V$ = Wear Volume ($m^3$)
- $L$ = Sliding Distance (m)
- $k$ = Wear Rate (Dimensionless)

This equation is well known as Archard’s equation.

Furthermore, the third set of the dimensionless parameter is the equation (3). Sliding speed appeal ($v$) is to be the main subject in this research.

$$\prod_3 = \frac{\rho}{\sqrt{H}} v$$

(6)
Where:
\[ \rho = \text{Density of Material (m}^3\text{)} \]
\[ v = \text{Sliding speed (m s}^{-1}\text{)} \]
By equalizing three dimensionless sets above, it produces the following.

\[
\frac{W}{H^{4/3}} \sqrt{\frac{\rho}{H^2}} \frac{V^{1/3}}{L} \approx V \]
(7)

\[ V \approx WL \frac{\rho}{H^2} v \]
(8)

\[ V = \phi WL \frac{\rho}{H^2} v \]
(9)

where \( \phi = \text{Wear-speed Coefficient} \)

The experimentation uses pin-on-disk tribometer for measuring wear volume of Ultra High Molecular Weight Poly Ethylene (UHMWPE), Poly Tetra Flouro Ethylene (PTFE) and Nutrile Rubber (NBR), contacting on a surface of Aluminum alloy. The normal load, and sliding distance are 8 N, 700 m respectively. Hardness number is 120 Mpa.

![Figure 1](image)

**Figure 1.** a) Pin-on-disk tribometer  b). Three different materials of specimen

Other data are also gathered to picture the work of the mathematical model. Data is from the article of Odabas D. titled “Effects of Load and Speed on Wear Rate of Abrasive Wear for 2014 Al Alloy” [2]. The wear volume resulted from this experiment due to the change of sliding speed would be compared with the above mathematical model of wear volume. The final step is to compare the theoretically approach of wear volumes to these experimental results. The wear-speed coefficient, in turn, is determined according to the materials.

3. **Results and discussion.**

Wear volumes of materials being investigated are listed in table 1, table 2 and table 3. The calculated wear volumes of every materials are also written as well. Based on the comparison between calculated wear volume and measured wear volume during experimentation, calculated wear is lower then the experiment results. However, UHMWPE has wear volumes that is almost the same between calculation and experimentation. In the other hand, for PTFE and NBR, the two wear volumes are different. The material hardness is believed to be responsible of the inequality. Furthermore, the tendency of wear caused by speed variation is similar. Wear would slowly increase while the speed increases. This phenomenon works for the three materials.
Table 1. Parameters and measured variables for UHMWPE

| Speed (rpm) | Sliding Speed (m s\(^{-1}\)) | Time (min.) | Density (kg m\(^{-3}\)) | Distance (m) | Hardness Shore D | Calculated Wear Vol. (cc) | Measured Wear Vol. (cc) |
|-------------|-----------------------------|-------------|--------------------------|--------------|------------------|--------------------------|------------------------|
| 80          | 0.173                       | 69          | 950                      | 700          | 20 (240MPa)      | 0.008                    | 0.006                  |
| 90          | 0.181                       | 65          |                          |              |                  | 0.008                    | 0.007                  |
| 100         | 0.192                       | 60          |                          |              |                  | 0.009                    | 0.008                  |

Table 2. Parameters and measured variables for PTFE

| Speed (rpm) | Sliding Speed (m s\(^{-1}\)) | Time (min.) | Density (kg m\(^{-3}\)) | Distance (m) | Hardness Shore D | Calculated Wear Vol. (cc) | Measured Wear Vol. (cc) |
|-------------|-----------------------------|-------------|--------------------------|--------------|------------------|--------------------------|------------------------|
| 80          | 0.173                       | 69          | 1030                     | 700          | 24 (300MPa)      | 0.006                    | 0.049                  |
| 90          | 0.181                       | 65          |                          |              |                  | 0.007                    | 0.052                  |
| 100         | 0.192                       | 60          |                          |              |                  | 0.007                    | 0.072                  |

Table 3. Parameters and measured variables for NBR

| Speed (rpm) | Sliding Speed (m s\(^{-1}\)) | Time (min.) | Density (kg m\(^{-3}\)) | Distance (m) | Hardness Shore D | Calculated Wear Vol. (cc) | Measured Wear Vol. (cc) |
|-------------|-----------------------------|-------------|--------------------------|--------------|------------------|--------------------------|------------------------|
| 80          | 0.173                       | 69          | 1150                     | 700          | 24 (300MPa)      | 0.006                    | 0.012                  |
| 90          | 0.181                       | 65          |                          |              |                  | 0.006                    | 0.013                  |
| 100         | 0.192                       | 60          |                          |              |                  | 0.007                    | 0.014                  |

It is discussed before that this research also compares those wear volumes reported by Odabas. The objective of the article is to examine the abrasive wear on 2014 aluminum alloy due to various load and speed. Data drawn from the article might be found in the attachment. Wear volumes relating to the changing of sliding speed are as the following.
**Figure 2.** Charts of wear volumes for: a) UHMWPE, b) PTFE, c) NBR

**Table 4.** Parameters and measured variables for aluminum alloy

| Normal Load (N) | Sliding Speed (m s\(^{-1}\)) | Density (kg m\(^{-3}\)) | Distance (m) | Hardness (kg mm\(^{-2}\)) | Calculated Wear Vol. (cc) | Measured Wear Vol. (cc) |
|-----------------|-----------------------------|--------------------------|--------------|-----------------------------|---------------------------|------------------------|
| 5               | 0.09                        | 2800                     | 11           | 161 (545 MPa)               | 0.00002                   | 0.00382                |
|                 | 0.18                        |                           |              |                             | 0.00004                   | 0.00393                |
|                 | 0.36                        |                           |              |                             | 0.00008                   | 0.00443                |
|                 | 0.72                        |                           |              |                             | 0.00012                   | 0.00464                |
|                 | 0.9                         |                           |              |                             | 0.00016                   | 0.00475                |

Figure 3 shows that the calculated wear volume is far below the results. It also indicates that the hardness and density play the important control of wear mechanism. Effects of speed seem to have similar tendency in every material. Since hardness and density are considered the major reason, the mathematical model need to find the wear-speed coefficient according to hardness and density.
From equation (3), it could be rearranged density-hardness factor by multiplying with material density.

\[ m = \varphi WL \sqrt[3]{\frac{\rho^3}{H^3}} v \]  

(10)

And finally, the mass which is worn can be written as below.

\[ m = \varphi WL \left( \frac{\rho}{H} \right)^{3/2} v \]  

(11)

The magnitudes of wear-speed coefficients are listed the following.

| Materials | Hardness (Mpa) | Density (kg m\(^{-3}\)) | Wear-speed Coefficient \(\varphi\) |
|-----------|----------------|---------------------------|-------------------------------|
| NBR       | 300            | 1030                      | 1.9                           |
| PTFE      | 300            | 1150                      | 8                             |
| UHMWPE    | 240            | 900                       | 1                             |
| Al Alloy  | 545            | 2800                      | 45                            |

Density Hardness Factor influence significantly to the wear-speed coefficient. This particular research has been trying to determine the relation among wear parameters such as hardness, density, velocity to wear volumes. The corelation value \(R^2\) equals to 0.9642.
4. Conclusion
In conclusion, the equation which has been formed through Buckingham Pi model mostly works on non metal materials, such as NBR, PTFE and UHMWPE. However, for metal materials, 2014 Aluminum Alloy, has big differences between calculated wear and real wear volumes. Sliding speed does influence wear volumes. The more speed may result the more wear volume. Wear-speed coefficient as product of equality process in Buckingham Pi model is influenced by Hardness and density of material.

5. References
[1] Yusuf Kaelani and Roy Yamsi, 2018, Identification method for stick slip contact within multi directional contact friction AIP Conference Proceedings, page 050012.
[2] Odabas D. 2018, Effects of Load and Speed on Wear Rate of Abrasive Wear for 2014 Al Alloy, 9th International Conference on Tribology IOP Conf. Series: Materials Science and Engineering 295, page 012008.
[3] Riyadh A. Al-Samarai, Haftirman1, Khairel Rafezi Ahmad, Y. Al-Douri, 2012, Effect of Load and Sliding Speed on Wear and Friction of Aluminum-Silicon Casting Alloy International Journal of Scientific and Research Publications, Volume 2, Issue 3, March 2012 1 ISSN 2250-3153
[4] Bisma A. Permana and Y Kaelani, 2016 Studi Eksperimen dan Analisa Keausan Material Alternatif Bearing pada Poros Propeller Kapal. Undergraduate Thesis Institut Teknologi Sepuluh Nopember, Surabaya, Indonesia
[5] Fadhel A. Abdulla, Nader M. Moustafa, Ehsan S. Al-Ameen, 2018, Calculation of Wear Rate by Weight and Volume for Aluminum Samples, Journal of University of Babylon for Engineering Sciences, Vol. (26), No. (7)
[6] Sachin Salunkhe, 2019, Friction and Wear Analysis of PTFE Composite Materials: Volume 2 at: https://www.researchgate.net/publication/329842683
[7] A. El-Domiaty, M. El-Fadaly, and A.Es. Nassef, 2002, Wear Characteristics of Ultrahigh Molecular Weight Polyethylene (UHMWPE), Journal of Materials Engineering and Performanc Volume 11, (5):577-583 · October 2002
[8] Venkateswarlu G., Sharada R. and Bhagvanth Rao M., 2014, Polytetrafluoroethylene (PTFE) based composites, *Journal of Chemical and Pharmaceutical Research*, 2014, 6(10): page 508-517.

[9] Rooplal, R. C. Singh, Ranganath M. S., Ankit Kumar Saxena, 2015, Investigation of Wear Behavior of Aluminium Alloy and Comparison with Pure Aluminium, (on line) at : https://www.researchgate.net/publication/274249759_Investigation_of_Wear_Behavior_of_Aluminium_Alloy_and_Comparison_with_Pure_Aluminium

[10] Shuyuan Song, Rui Nie, Shijie Wang, and Yunlong Li, 2019, Tribological Properties of Swollen Nitrile Rubber Under Dry and Wet Sliding Conditions, *IOP Publishing Mater. Res. Express* 7 (2020) 015311.