Modulation Code for Reducing Intertrack Interference on Staggered Bit-Patterned Media Recording

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Abstract: A bit-patterned media recording (BPMR) system is a type of ultrahigh-capacity magnetic storage system that can extend to an areal density of 1 terabit per square inch or higher. However, because the space between islands in the down- and cross-track directions is reduced to extend the areal density, the effect of two-dimensional interference is increased. However, using a staggered array, which is one of the possible island distributions for BPMR, helps to decrease intertrack interference. A 7/10 modulation code for a staggered BPMR is proposed to avoid the effect of two-dimensional interference and provide distance among nonidentical codewords for improving the correcting capability.

Keywords: modulation code; bit-patterned media recording; intertrack interference

1. Introduction

For most conventional magnetic storage systems, the superparamagnetic limit is a significant obstacle to increasing the areal density (AD). To overcome the problem and extend the AD to more than 1 terabit per square inch (Tb/in²), bit-patterned media recording (BPMR) has become a candidate for the next generation of magnetic storage systems [1]. In addition, BPMR has such advantages as improved thermal stability, a decreased nonlinear transition shift, and reduced transition noise [2]. Because of these advantages, BPMR can satisfy the demand for storing a tremendous quantity of data in the information age. However, as the distances of down-track bit period $T_x$ and cross-track pitch $T_z$ for achieving high AD become closer, two-dimensional (2D) interference, which comprises intersymbol interference (ISI) and intertrack interference (ITI), is increased [3,4]. In addition, BPMR has unavoidable problems, such as track misregistration (TMR) and media noise caused by imperfect fabrication. According to the lithography method adopted, bit-patterned media (BPM) structures can be placed in a regular or staggered array BPM layout, as shown in Figure 1. When the islands are placed hexagonally in the staggered array, the bit error rate (BER) performance when the staggered array is used is better than when the regular array is used, because of reduced ITI [5,6].

To eliminate the 2D interference that degrades system performance, various schemes have been proposed for BPMR, such as signal detection methods, error control codes, and modulation codes. To address the 2D interference problem and help detect the input data, the partial response maximum likelihood (PRML) method has been employed [7]. This is applied to data storage systems where the channel response is equalized to a partial response (PR) pulse shape and a maximum likelihood (ML) sequence detector. For 2D data storage systems, a 2D equalizer and detection schemes have been proposed [6,8]. To ensure the reliability of data storage systems, error control codes, such as the low-density parity check (LDPC) code, are required. They considerably improve system performance [9,10]. In a previous
study [10], a proposed product code that consists of inner and outer code using a LDPC exhibited better BER performance than the LDPC code alone. To prevent error patterns that cause 2D interference, such modulation codes as 5/6 and 9/12 modulation have been proposed for BPMR [11, 12]. Because data storage systems cannot retransmit data, unlike typical communication systems with such retransmission schemes as automatic repeat requests, strict requirements for a low probability of decoding failure and a high code rate should be met [9].

In this work, a 7/10 modulation code for staggered BPMR is proposed. To reduce ITI effectively, the proposed code focuses on the ITI problem rather than the ISI problem, because the effect of ITI is greater than that of ISI in the BPMR [13]. Moreover, because the Hamming distance among codewords is at least 2 or more, the decoding capability is improved.

The remainder of this paper is organized as follows. In Section 2, the staggered BPMR channel model and PRML detection for staggered BPMR are explained. In Section 3, the proposed 7/10 modulation code is introduced. The simulation and results are discussed in Section 4. Section 5 provides conclusions.

2. Staggered BPMR Channel Model and PRML Detection

2.1. Staggered BPMR Channel Model

Figure 2 is a block diagram of the proposed system model. Before passing through the staggered BPMR channel, the binary user data $a_k$ ∈ {0, 1} are encoded by a modulation encoder to encode the 2D data array $c_{p,q}$, and $c_{p,q}$ is magnetized to record data $d_{p,q}$ ∈ {-1, 1}. The analytical 2D Gaussian island pulse response $P(z, x)$ without media noise and write errors is given in [14].

$$P(z, x) = A \cdot \exp\left\{-\frac{1}{2c^2}\left[\left(\frac{z}{PW_z}\right)^2 + \left(\frac{x}{PW_x}\right)^2\right]\right\},$$

where $z$ and $x$ are the indices in the cross- and down-track directions, respectively, $A$ is the normalized peak amplitude, $c$ represents the relationship between the standard deviation of a Gaussian function and PW50 (a parameter of the pulse width at half of the peak amplitude), and $PW_z$ and $PW_x$ are the PW50 of the cross- and down-track pulses, respectively. In this study, $A = 1, c = 1/2.3548, PW_z = 24.8$ nm, and $PW_x = 19.4$ nm. The BPMR 2D channel island pulse response $h_{m,n}$ is calculated by sampling the 2D Gaussian island pulse response as follows:

$$h_{m,n} = P(mT_z + \Delta_{TMR}, nT_x),$$

where $m$ and $n$ are the indices of bit islands for the cross- and down-track directions, respectively, $T_z$ and $T_x$ denote track pitch and bit period, respectively, and $\Delta_{TMR}$ is the read head offset, which is
generated when the recording heads cannot remain at the center of the main data track. The $\Delta_{TMR}$ is expressed as follows:

$$\Delta_{TMR} = \frac{TMR_z \times T_z}{100},$$  \hspace{1cm} (3)$$

where $TMR_z$ is the percentage of the TMR. The readback signal $r_{p,q}$ corrupted by electronic noise in staggered array BPMR is given by

$$r_{p,q} = \sum_{n=-N}^{N} \sum_{m=0}^{\lfloor \frac{L}{2} \rfloor} d_p q + n \cdot h_{0,n} + \sum_{m=0}^{\lfloor \frac{L}{2} \rfloor} \sum_{n=-N+m+1}^{N} d_p q + n \cdot h_{-(2m+1),n-\frac{1}{2}} + \sum_{m=0}^{\lfloor \frac{L}{2} \rfloor} \sum_{n=-N+m}^{N} d_p q + n \cdot h_{(2m+1),n-\frac{1}{2}} + \sum_{m=1}^{\lfloor \frac{L}{2} \rfloor} \sum_{n=-N+m}^{N} d_p q + n \cdot h_{2m,n} + n_{p,q}$$  \hspace{1cm} (4)$$

where $N$ is the length of interference from neighboring islands, $\lfloor x \rfloor$ is a floor function, which is the function that takes as input a real number $x$ and gives as output the greatest integer less than or equal to $x$, and $n_{p,q}$ is electronic noise modeled as additive white Gaussian noise with variance $\sigma^2$ and zero mean. Since the interference from neighboring islands in $N = 2$ is relatively negligible, we set $N = 1$ for simplicity.

![Block diagram of the proposed system model.](image)

2.2. PRML Detection

The 2D interference that occurs when the readback signal is affected by surrounding bits is usually equalized to target the response by PRML, which is usually employed in data storage systems. The PRML detector consists of a PR equalizer and a ML channel decoder based on a Viterbi algorithm. A PR equalizer reshapes the channel response to the PR pulse shape according to the PR target. Thus, a suitable PR target for channel response is important for achieving a better performance. However, when an unsuitable PR target is used, the equalizer output can be an inaccurate value because of noise enhancement. The received data $r_{p,q}$ influenced by the BPMR channel and noise are entered into the 2D equalizer. The equalizer output $e_{p,q}$ is calculated by

$$e_{p,q} = \sum_{m=1}^{L-m+1} r_{p,q} \cdot \lfloor \frac{L}{2} \rfloor + n_{p,q} \cdot c_{m,n} + \sum_{m=1}^{L-m+1} r_{p,q} \cdot \lfloor \frac{L}{2} \rfloor + n_{p,q} \cdot c_{m,n}$$  \hspace{1cm} (5)$$

where $c_{m,n}$ is the equalizer coefficient, $L$ is the equalizer length, and $\lfloor x \rfloor$ is a ceiling function, which maps $x$ to the least integer greater than or equal to $x$. In this study, $L = 5$ was set. When $L = 5$, the equalizer coefficients are as follows:
The least mean square algorithm was used for updating equalizer coefficients.

$$c_{m,n}^{k+1} = c_{m,n}^{k} + \mu (e_{p,q} - \sum_{n=-1}^{1} d_{p,q-n}f_{n+2})r_{p,q},$$

where $c_{m,n}^{k+1}$ and $c_{m,n}^{k}$ are updated and current equalizer coefficients, respectively, $\mu$ is an adaptation gain, and $f_n$ is a PR target coefficient in the down-track direction. To calculate the reliability or soft value of the input data, a soft output Viterbi algorithm (SOVA) was used. The equalizer output is input to the one-dimensional (1D) SOVA detector for the down-track direction. The branch metric of 1D SOVA is calculated using the following equation:

$$\lambda_{p,q}(s_i,s_j) = \left( e_{p,q} - \left( f_1d_{p,q-1}(s_i) + f_2d_{p,q}(s_i) + f_3d_{p,q+1}(s_i) \right) \right)^2,$$

where $s_i$ and $s_j$ are the current and next state, and $d(s_i)$ and $a(s_i)$ are decisions at $s_i$ and $s_j$, respectively.

3. Proposed 7/10 Modulation Code for Mitigating ITI

3.1. Encoding Scheme

The modulation coding schemes, such as run-length limited code and maximum transition run code, make transmission that is suitable for a channel possible using constraints. Normally, modulation codes are used for preventing error patterns, timing recovery, and DC balance in data storage systems. To accommodate specific constraints, a lookup table, a finite state machine, and so on are utilized in the modulation encoder. In general, in the modulation coding schemes, the performance is excellent when the code rate is low.

In this paper, a 7/10 modulation code that prevents serious ITI is proposed. The proposed modulation code using a lookup table and one-to-one mapping encodes the 7 bits of user data sequence $a = [a_0, a_1, a_2, a_3, a_4, a_5, a_6]$ to the $5 \times 2$ (= 10 bits) array of coded data sequence $c = [c_0, c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9]$, as shown in Figure 3. To improve the performance of the proposed 7/10 modulation coding scheme, the codeword selection process was divided into Step 1 for removing error patterns causing the ITI effect, and Step 2 for providing enough Hamming distance among codewords to improve the correcting capability. The two islands on the upper track and the other two on the lower track affect one island on the main track in a staggered array BPMR. Therefore, a constraint for four neighboring islands is necessary to effectively reduce the ITI effect. However, heavy constraints on the islands cause the code rate to decrease. Thus, to increase the code rate, the proposed code restricts two pixels on the neighboring tracks not containing error patterns, which cause the ITI effect, such as $[1, 0, 1]^T$ and $[0, 1, 0]^T$.

For Step 1, Figure 4 shows the available patterns for each column ($[c_0, c_1, c_2, c_3, c_4]^T$ or $[c_5, c_6, c_7, c_8, c_9]^T$). The number of combinations in one column is 32 (=2^5). Out of 32 patterns, only 16 patterns that do not have patterns of $[1, 0, 1]^T$ and $[0, 1, 0]^T$ were selected for making codewords. Thus, 256 (=16 × 16) codewords can be included by combining 16 codewords obtained in each column.

In Step 2, codewords having a distance of at least 2, which is the Hamming distance among nonidentical codewords, were found. The distance among the codewords enables the original codeword from the received sequence in the decoding process to be recovered correctly. Table 1 shows a list of 128 codewords obtained through Steps 1 and 2. For instance, the user data sequence of $a = [0, 0, 0, 0, 0, 0, 1]$ is encoded to $c_1 = [0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1]$ by one-to-one mapping.
Figure 3. One-to-one mapping and demapping for the proposed modulation code.

Figure 4. Available patterns for each column of codeword.

Table 1. Codeword list of the proposed 7/10 modulation code.

| Input Sequence | Codeword | Input Sequence | Codeword | Input Sequence | Codeword | Input Sequence | Codeword |
|----------------|----------|----------------|----------|----------------|----------|----------------|----------|
| 0000000        | 0000000000 | 0100000       | 001100001 | 1000000       | 100000001 | 1100000       | 1100100001 |
| 0000001        | 0000000001 | 0100001       | 001110011 | 1000001       | 100001011 | 1100010       | 110010111 |
| 0000010        | 0000000110 | 0100010       | 0011110110 | 1000010       | 100001110 | 1100010       | 110011011 |
| 0000011        | 0000001000 | 0100101       | 0011110001 | 1000101       | 1001001001 | 1100101       | 110100111 |
| 0000100        | 0000010111 | 0101010       | 0011111001 | 1001010       | 1001010101 | 1101010       | 110110111 |
| 0000110        | 0000010010 | 0101100       | 0011111100 | 1001101       | 1001110100 | 1101101       | 110111011 |
| 0000111        | 0000011000 | 0101111       | 0011111110 | 1001110       | 1001111010 | 1101111       | 110111111 |
| 0001000        | 0000100000 | 0110000       | 0100000000 | 1010000       | 1001001001 | 1110000       | 111000001 |
| 0001010        | 0000100100 | 0110100       | 0101000001 | 1010010       | 1000101010 | 1110101       | 111100001 |
| 0001011        | 0000101000 | 0110101       | 0101100001 | 1010101       | 1000110101 | 1110110       | 111101000 |
| 0001100        | 0000101001 | 0110110       | 0110000001 | 1010110       | 1000111101 | 1110111       | 111101000 |
| 0001110        | 0000101001 | 0110100       | 0111000001 | 1010111       | 1000111111 | 1110111       | 111110100 |
| 0001111        | 0000101001 | 0110100       | 0111000001 | 1010111       | 1000111111 | 1110111       | 111111000 |
| 0001111        | 0000101001 | 0110100       | 0111000001 | 1010111       | 1000111111 | 1110111       | 111111100 |
| 0001111        | 0000101001 | 0110100       | 0111000001 | 1010111       | 1000111111 | 1110111       | 111111110 |
| 0001111        | 0000101001 | 0110100       | 0111000001 | 1010111       | 1000111111 | 1110111       | 111111111 |
3.2. Decoding Scheme

To decode the received sequence \( \hat{c} = [c_0, c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9] \), a minimum Euclidean distance was implemented in the demodulation process. The Euclidean distance \( d^l \) between the received sequence and codeword is calculated by

\[
d^l(\hat{c}, c^l) = \sum_{i=0}^{9} (\hat{c}_i - c^l_i)^2
\]

where \( c^l = [c_0^l, c_1^l, c_2^l, c_3^l, c_4^l, c_5^l, c_6^l, c_7^l, c_8^l, c_9^l] \) is the \( l \)-th codeword. In the proposed code, there are 128 Euclidean distances. The smallest of all Euclidean distances, which is the minimum Euclidean distance, was selected. Finally, the codeword corresponding to the minimum Euclidean distance was determined by demapping.

4. Simulation and Results

In this work, 100 pages with a page size of 900 \( \times \) 900 islands per page were simulated. The 1D PR target for the 2D equalizer was (0.1, 1.0, 0.1). The 1D SOVA was used for channel detection. The channel signal-to-noise ratio (SNR) was defined as \( 10 \log_{10}(1/\sigma^2) \), where \( \sigma^2 \) is additive white Gaussian noise. Table 2 shows the read head and pulse response parameters [14].

| Parameters                                      |       |
|-------------------------------------------------|-------|
| Square island with length                       | 11 nm |
| Square island with thickness                    | 10 nm |
| Read head element thickness                     | 4 nm  |
| Read head element width                         | 15 nm |
| Read head gap distance                          | 6 nm  |
| Read head fly height                            | 10 nm |

Figure 5 displays the BER performance of the PRML and the proposed 7/10 modulation code at the same AD. At BER = \( 10^{-6} \), when the AD was 2.0 Tb/in\(^2\), the performance of the proposed code was 2 dB better than that of the PRML. In addition, when the AD was 3.0 Tb/in\(^2\), the BER curve of the proposed code exhibited better performance than that of PRML detection. Since the proposed 7/10 modulation code eliminated ITI error patterns and provided enough distance among the nonidentical codewords, the performance of the proposed codes showed a better performance.

To verify the performance in accordance with the modulation coding scheme, the PRML (uncoded system) and the 4/6 [15], 8/10 [16], and the proposed 7/10 modulation codes were compared. For a fair comparison, the user density (UD), which is defined by UD = AD \( \times \) code rate, should be considered. The code rate of the PRML was 1 because of the uncoded system. The code rates of the 4/6, 8/10, and 7/10 modulation codes were 0.66, 0.8, and 0.7, respectively.

Figure 6 shows the BER comparison with respect to the modulation coding scheme at UD = 1.4 Tb/in\(^2\). The AD of the PRML and the 4/6, 8/10, and 7/10 modulation codes were 1.4, 2.1, 1.75, and 2.0 Tb/in\(^2\), respectively. The track pitch \( T_b \) and bit period \( T_s \) for each AD are shown in Table 3. For example, when the UD was 1.4 Tb/in\(^2\) and 7/10 modulation code was used, the AD was 2.0 Tb/in\(^2\). To achieve an AD of 2.0 Tb/in\(^2\), \( T_b \) and \( T_s \) were 18 nm. In this simulation, the 8/10 modulation code had the worst performance. At a BER of \( 10^{-6} \), the proposed 7/10 modulation code provided performance gains of approximately 0.4 and 1.6 dB over the 4/6 modulation code and the PRML detector, respectively. The proposed scheme has a higher code rate and better performance than the 4/6 modulation code, because the patterns that generate ITI are effectively removed. The 4/6 modulation code has the advantage of providing enough distance among the nonidentical codewords, and the 8/10 modulation code has the advantage of removing the ITI pattern. The proposed code showed good performance, because it combines these two advantages.
Suitable for BPMR. Most importantly, however, the PRML detector provided the best performance in all cases because the ITI effect dramatically increased, because of the fair comparison. For a fair comparison, the user density (UD), which is defined by UD = AD × code rate, should be considered. The code rate of the PRML was 1 because of the uncoded system. The code distance, was selected. Finally, the codeword corresponding to the minimum Euclidean distance between nonidentical codewords, the performance of the proposed codes showed a better performance. Since the proposed 7/10 modulation code eliminated ITI error patterns and provided enough distance among the codewords, the performance of the proposed code exhibited better performance than that of PRML detection. Since the proposed 7/10 modulation code was the poorest. The reason is that the 4/6 modulation code proposed for holographic data storage is not suitable for BPMR. Most importantly, however, the PRML detector provided the best performance in all cases because the ITI effect dramatically increased, because of the fair comparison. Therefore, the PR target for the 2D equalizer was (0.1, 1.0, 0.1). The 1D SOVA was used for channel detection. The channel signal-to-noise ratio (SNR) was defined as 10log10(1/

Table 2. Square island with length 11 nm

Table 3. Square island with thickness 10 nm

Figure 5 displays the BER performance of the PRML and the proposed 7/10 modulation code at the same areal densities (AD). Figure 6 shows the BER comparison with respect to the modulation coding scheme at UD = 1.4 Tb/in².

Figure 7 illustrates the BER performance according to the modulation coding scheme at UD = 2.1 Tb/in². For a fair comparison, the suitable Tx and Tz for each AD are as presented in Table 2. At a BER of 10⁻³, the proposed modulation code performed approximately 0.8 and 2.3 dB better than the 8/10 and 4/6 modulation codes, respectively. In this simulation, the performance of the 4/6 modulation code was the poorest. The reason is that the 4/6 modulation code proposed for holographic data storage is not suitable for BPMR. Most importantly, however, the PRML detector provided the best performance in all cases because the ITI effect dramatically increased, because of the fair comparison. Therefore,
when the AD is high and interference is severe, one must consider whether a modulation code should be used.

![Figure 7. BER performance according to modulation coding scheme at UD = 2.1 Tb/in².](image)

Figure 7. BER performance according to modulation coding scheme at UD = 2.1 Tb/in².

Figure 8 shows the BER performance depending on TMR from 0 to 30% when the UD was 1.4 Tb/in². In a situation with some reasonable error factors, the proposed 7/10 modulation code showed the best performance at UD = 1.4 Tb/in².

![Figure 8. BER performance depending on TMR from 0% to 30% when UD = 1.4 Tb/in².](image)

Figure 8. BER performance depending on TMR from 0% to 30% when UD = 1.4 Tb/in².

5. Conclusions

A 7/10 modulation code is proposed for staggered BPMR. To improve the performance of the proposed 7/10 modulation code, error patterns that cause ITI are eliminated, and enough distance among the nonidentical codewords is provided for improving the correcting capability. At a UD of 1.4 Tb/in², the proposed 7/10 modulation code has the best performance. However, at a UD of 2.1 Tb/in², the PRML detector achieves the best performance because increasing the AD for fair comparison causes a significant ITI problem. Thus, if the UD is high, one must consider whether a modulation code should be used.
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