The soft limit of the energy spectra in QCD jets

SERGIO LUPIA

Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)
Föhringer Ring 6, D-80805 München, Germany
E-mail: lupia@mppmu.mpg.de

Abstract

A scaling law for the one-particle invariant density $E \frac{dn}{d^3p}$ at small momenta is observed in experimental data. We show that these results are consistent with the predictions of the analytical QCD approach, based on Modified Leading Log Approximation plus Local Parton Hadron Duality, which includes colour coherence in soft gluon bremsstrahlung.

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Abstract

A scaling law for the one-particle invariant density \( E \frac{\Delta n}{\Delta p} \) at small momenta is observed in experimental data. We show that these results are consistent with the predictions of the analytical QCD approach, based on Modified Leading Log Approximation plus Local Parton Hadron Duality, which includes colour coherence in soft gluon bremsstrahlung.

1 Introduction

The aim of our research project is to test the validity of the perturbative QCD approach to multiparticle production in the semihard and soft region; we also wish to study the sensitivity of the physical observables to different aspects included in the theory, like the running of the coupling \( \alpha_s \) (see in this respect\(^1\)); in this paper we mainly concentrate on the sensitivity of soft particle production to QCD coherence.

The theoretical framework for the description of inclusive observables in jet physics at parton level is provided by the Modified Leading Log Approximation (MLLA) of QCD (for a review see\(^2\)), where coherence, running of \( \alpha_s \) and energy-momentum conservation are taken into account. Predictions depend on two free parameters only, i.e., the infrared cut-off at which the parton evolution is stopped, \( Q_0 \), and the effective QCD scale \( \Lambda \) which appears in the one-loop expression for the running coupling. An interesting limiting case is obtained by pushing the parton cascade down to the very end, i.e., by choosing \( Q_0 = \Lambda \); in this case, one obtains
a simple closed expression for the spectrum, called Limiting Spectrum. To connect predictions at parton level with experimental data, Local Parton Hadron Duality (LPHD) is taken as hadronization prescription, i.e., the inclusive hadron spectra are required to be proportional to the corresponding inclusive parton spectra. The whole hadronization is then parametrized in terms of only one parameter, which gives the overall normalization of the distribution, but does not affect its moments of order greater or equal than one.

2 Phenomenology of inclusive energy spectra: an update

The description of the inclusive energy spectrum for charged particles is one of the main successes of the analytical QCD approach, this approach turned out to be valid at LEP-1.5 \textit{cns} energy too. Let us just list a few points of interest (see \cite{2} for details).

\textit{Analysis of the shape.} The inclusive energy distribution is well described by the Limiting Spectrum with $Q_0 = \Lambda = 270$ MeV and 3 active flavours. After a rescaling, which allows to have the same kinematics both at parton and hadron level, the very soft tail of the distribution is well described as well. The overall normalization factor at LEP-1.5 turns out to be consistent with the LEP-1 value, in agreement with the predictions of the perturbative approach.

\textit{The position of the maximum.} The energy dependence of the position of the maximum of the spectrum is well described by the Limiting Spectrum predictions with $Q_0 = \Lambda = 270$ MeV and 3 active flavours. Notice that the popular three terms formula for the energy dependence of the maximum underestimates by an almost constant value 0.1 the true position of the maximum of the Limiting Spectrum; this gives rise to a difference of the order of 20-30 MeV in the determination of the best value of the cut-off $Q_0$. Let us also point out that the number of flavours enters in the expressions for the cumulant moments through the running coupling $\alpha_s(y, n_f)$; since the MLLA is defined at one-loop level, a scale ambiguity in the expression of $\alpha_s$ is present; kinematical reasons suggest to push the heavy quark thresholds to larger scales and keep 3 active
flavours only.

Moment analysis. This study is particularly interesting, because it does not depend on the overall normalization parameter and, since theoretical predictions are in this case absolutely normalized at threshold, it allows to test the perturbative picture down to low $cms$ energies. Theoretical predictions of the Limiting Spectrum with $Q_0 = 270$ MeV and 3 active flavours are in good agreement with experimental results. Switching off the running of the coupling, one cannot reproduce the behavior of high order cumulants. By relaxing the absolute normalization, thus building an effective model with one more parameter for each cumulant to reproduce the high energy region, one can describe the experimental data only in a small energy region. These results show the sensitivity of this analysis to the running of the coupling. In this respect, the moment analysis of spectra in high-$p_T$ jets in $p\bar{p}$ collisions, where larger values of jet energy can be reached, is eagerly awaited.

3 The invariant density in the soft limit

3.1 A new scaling law in experimental data

Let us consider the behaviour of the charged particle invariant density, $Edn/d^3p$, at small particle energy $E$. Figure (1a) shows data at different $cms$ energies ranging from 3 GeV up to LEP-1.8. The value of 270 MeV has been used for the effective mass $Q_0$ which enters in the kinematical relation, $E^2 = p^2 + Q_0^2$. It is remarkable that all data scale with $cms$ energies within 20% at particle energy of the order of few hundreds MeV; at larger particle energies, a violation of the scaling-law is visible. LEP data seem to tend to a larger limiting value; it is not yet clear whether this is a signal of a different physics at LEP energies, like for instance an enhanced contribution of weak decays to particle production in the soft region, or a systematic effect in the overall normalization of the different experiments.
3.2 Predictions of QCD coherence

QCD colour coherence forbids branchings of very soft gluons; therefore, the emission of soft partons should be proportional to the colour charge of the initial system only and the invariant density $Edn/d^3p$ should be independent of cms energy at low particle energy. Recently, predictions for the invariant density have been analytically computed by solving explicitly in the limit of small particle energy the evolution equation for the energy spectrum in a jet of type $A$, $D_A(\xi, Y, \lambda)$. The relation $E_h dn/d^3p_h \equiv K_h D_A(\xi_E, Y, \lambda)/[4\pi(E_h^2 - Q_0^2)]$, with $E_h = \sqrt{p_h^2 + Q_0^2} = E_p$, $\xi_E \equiv \log 1/x_E = \log s/(2E)$ and $K_h$ the overall normalization parameter, has been then used to obtain the invariant density $Edn/d^3p$. 

Figure 1: a) Invariant density $Edn/d^3p$ for charged particle as a function of the particle energy $E$ with $Q_0 = 270$ MeV. b) same as in a), with theoretical predictions from MLLA with the same $Q_0$ and $\Lambda = 257$ MeV ($K_h = 0.45$).
Within DLA, one gets an iterative solution for a gluon-jet

\[ D_g(\xi, Y, \lambda) = \delta(\xi) + \frac{4N_C}{b} \log \left( 1 + \frac{Y - \xi}{\lambda} \right) [1 + f(\xi, Y, \lambda)] + \ldots \] (1)

where \( Y = \log \sqrt{s}/(2Q_0) \), \( \lambda = \log Q_0/\Lambda \), \( b = \frac{11}{3}N_C - \frac{2n_f}{3} \) and \( f(\xi, Y, \lambda) \) is a known function. The leading term at small momenta of order \( \beta^2 \) corresponding to the emission of a single gluon is indeed proportional to the colour charge factor of the primary parton and does not depend on the cms energy. The DLA prediction for the invariant density exhibits therefore an approximate scaling law and tends at small particle energy to a finite soft limit.

The MLLA solution differs from the DLA limit by a simple exponential damping factor

\[ D(\xi, Y, \lambda)\big|_{\text{MLLA}} = D(\xi, Y, \lambda)\big|_{\text{DLA}} \exp \left[ -a \int_{\xi}^{Y} \frac{\alpha_s(y)}{2\pi} dy \right] \] (2)

with \( a = 11/(3N_C) + 2n_f/(3N_C^2) \). This solution satisfies the MLLA evolution equation, except for a small term proportional to \( a[\alpha_s(\xi) - \alpha_s(Y)]/(2\pi) \), which then vanishes in the soft limit, where \( \xi \to Y \). The MLLA still satisfies an approximate scaling law and tends to a finite soft limit, but the additional damping factor modifies the dependence on the cms energy which appears at moderate particle energy. Figure (1b) compares the data shown in Figure (1a) with the theoretical predictions from MLLA with \( Q_0 = 270 \text{ MeV} \) and \( \Lambda = 257 \text{ MeV} \) (and \( K_h = 0.45 \) fixed a posteriori); a rather good description of data is obtained within this approach.

4 Conclusions

The invariant density \( E dn/d^3p \) has been shown to satisfy an approximate scaling law at low particle energy of few hundreds MeV. In the framework of the perturbative QCD approach, which has been shown to successfully describe the invariant energy spectra in the whole available cms energy range, predictions for the invariant
density $Edn/d^3p$ have been explicitly computed. The MLLA has been found to describe well the data down to very low particle energies, thus suggesting the validity of this picture in the very soft region and a possible link of the observed scaling law with QCD coherence.

Further tests of the universality of soft particle production in different reactions and alternative methods to point out the sensitivity of soft particle production to QCD coherence are presented in 8.

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