Disks in the sky: A reassessment of the WMAP “cold spot”

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ABSTRACT

We reassess the evidence that WMAP temperature maps contain a statistically significant “cold spot” by repeating the analysis using simple circular top-hat (disk) weights, as well as Gaussian weights of varying width. Contrary to previous results that used Spherical Mexican Hat Wavelets, we find no significant signal at any scale when we compare the coldest spot from our sky to ones from simulated Gaussian random, isotropic maps. We trace this apparent discrepancy to the fact that WMAP cold spot’s temperature profile just happens to favor the particular profile given by the wavelet. Since randomly generated maps typically do not exhibit this coincidence, we conclude that the original cold spot significance originated at least partly due to a fortuitous choice of using a particular basis of weight functions. We also examine significance of a more general measure that returns the most significant result among several choices of the weighting function, angular scale of the spot, and the statistics applied, and again find a null result.

Key words: cosmology: cosmic microwave background

INTRODUCTION

Cosmic microwave background (CMB) maps have been studied in detail during the last few years. These studies have been motivated by the remarkable full-sky high-resolution maps obtained by WMAP [Bennett et al. 2003; Spergel et al. 2007], and led to a variety of interesting and unexpected findings. Notably, various anomalies have been claimed pertaining to the alignment of largest modes in the CMB [Tegmark et al. 2003; de Oliveira-Costa et al. 2004; Copi et al. 2004; Schwarz et al. 2004; Land & Magueijo 2005; Copi et al. 2006; Abramo et al. 2006], the missing power on large angular scales [Spergel et al. 2003; Copi et al. 2007; Copi et al. 2008], and the asymmetries in the distribution of power [Eriksen et al. 2004; Hansen et al. 2004; Bernui et al. 2006; Hajian 2007]. In the future, temperature maps obtained by the Planck experiment, and large-scale polarization information [Dvorkin et al. 2008] may be key to determining the nature of the large-scale anomalies. For a review of the anomalies and attempts to explain them, see Huterer (2006).

Several years ago, Vielva et al. (2004) reported an anomalously cold spot in the WMAP microwave signal: kurtosis of the distribution of spots (defined using Spherical Mexican Hat Wavelet weight functions) is unusually large on scales of about 5°, at < 0.5% significance. The authors also noted that the result is driven by a a cold spot in the southern hemisphere, at ($\ell,b$) = (−57°, 209°). The finding has been confirmed and further investigated in Cruz et al. (2006), who found that an equally cold or colder spot of this size is expected in < 1% of Gaussian random, isotropic skies, as well as Mukherjee & Wang (2004); Cruz et al. (2005); McEwen et al. (2005); Cavón et al. (2005); Cruz et al. (2006); Räth et al. (2007); Naselsky et al. (2007); Pietrobon et al. (2008); Rossmanith et al. (2009), some of whom also studied the spot’s morphology. The plot further thickened when Rudnick et al. (2007) claimed that there is a corresponding cold spot (underdensity in galaxy counts) — in the NVSS radio survey, and at roughly the same location as the CMB cold spot; however, this particular claim was shown by Smith & Huterer (2008) to be an artifact of the a posteriori statistics and the particular way NVSS data had been analyzed. Nevertheless, the CMB cold spot remains a much-studied topic and the source of investigations of whether exotic physics could be the cause.

Perhaps surprisingly, nearly all of the works so far considered searches for the cold spot using the same basis functions — Spherical Mexican Hat Wavelets (though with a few exceptions — Pietrobon et al. 2008) used needlets, while Räth et al. (2007) and Rossmanith et al. (2009) used the scaling indices). The only variation in the different analyses was in the choice of the statistics that was applied to the wavelet-based weights.

Here we set out to check the evidence for the cold spot using different, and arguably simpler, set of weight functions. We reassess evidence for the “cold spot” using circular top-hat weights (i.e. disks) of arbitrary radius $R$. We do
so in order to verify findings that relied on wavelets, and more generally to investigate the robustness of the signal. We also check results using simple Gaussian weights, finding results consistent with those with the disks. We then investigate the source of this apparent discrepancy with all of the previous work that used wavelets, and find that the cause of the discrepancy is the specific temperature profile of the cold spot which just happens to favor the profile of the spherical Mexican Hat Wavelet. In addition to the choice of the spots’ weight function, the original claim refers to the angular size of the spot of ~ 5° that is also chosen a posteriori. We investigate the effect of these choices by defining a “superstatistic” measure that combines several previously considered statistical measures of coldness and the associated choices of the spot size and weight functions, and find that the claimed spot (or any other spot in our sky) is not unusually significant using this new measure.

STATISTICS AND MAPS

Weight statistics

The top-hat weights are familiar from structure formation (where they are used in the definition of the amplitude of mass fluctuations over some scale R, for example) and effectively represent another statistic to study the cold spot. We define the disk top-hat weight of radius R as

\[ D(r) \equiv A_{\text{disk}}(R) \cdot \Theta(r) - \Theta(r - R), \]

where \( \Theta(x) \) is a Heaviside step function and \( A_{\text{disk}}(R) = (2\pi(1 - \cos(R)))^{-1/2} \) is defined so that

\[ \int_0^R D(r)^2 d\Omega = 1. \]

Note however that the normalization \( A_{\text{disk}}(R) \) is unimportant for finding the coldest spot since we only do relative comparisons of temperatures in disks on the sky. The top hat-weighted temperature coefficients are given by

\[ T_{\text{disk}}(\hat{r}; R) = \int d\Omega \Psi(\hat{r}; \theta; R) D(\alpha; R), \]

where \( \hat{r} = (\theta, \phi) \) is the location of a given spot, \( \hat{r}' = (\theta', \phi') \) is the dummy location on the sky whose temperature we integrate over, and \( \alpha = \arccos(\hat{r} \cdot \hat{r}') \) is the angle between the two directions.

The Gaussian weights that we use are defined equivalently. The weight functions are

\[ G(r) \equiv A_{\text{gauss}}(\alpha) \exp \left(-4 \ln 2 \frac{r^2}{R^2}\right), \]

so that the full width at half maximum of the distribution is equal to R. The weighted temperatures are given by

\[ T_{\text{gauss}}(\hat{r}; R) = \int d\Omega \Psi(\hat{r}; \theta; R) G(\alpha; R). \]

Finally, the corresponding procedure applied to the wavelets is as follows (Cayon et al. 2001; Martinez-Gonzalez et al. 2002). The spherical Mexican Hat wavelets are defined as

\[ \Psi(\theta; R) = A_{\text{wav}}(R) \left(1 + \left(\frac{y}{2}\right)^2\right)^2 \left(2 - \left(\frac{y}{2}\right)^2\right) \exp \left(-\frac{y^2}{2R^2}\right), \]

where \( y \equiv 2 \tan(\theta/2) \) and

\[ A_{\text{wav}}(R) = \left[\frac{2\pi R^2 \left(1 + \frac{R^2}{2} + \frac{R^4}{4}\right)}{2}\right]^{-1/2}, \]

so that \( \int d\Omega \Psi^2(\theta; R) = 1 \) over the whole sky. We can now define the continuous wavelet transform stereographically projected over the sphere with respect to \( \Psi(\theta; R) \), with T being the CMB temperature:

\[ T_{\text{wav}}(\hat{r}; R) = \int d\Omega' \Psi(\hat{r}'; \theta; R), \]

where \( \hat{r} \rightarrow (\theta, \phi) \) and \( \hat{r}' \rightarrow (\theta', \phi') \) are the stereographic projections to the sphere of the center of the spot and the dummy location, respectively, and are given by

\[ \hat{x} = 2 \tan \frac{\theta}{2} (\cos \phi, \sin \phi), \]

\[ \hat{\mu}' = 2 \tan \frac{\phi'}{2} (\cos \phi', \sin \phi'); \]

see Martinez-Gonzalez et al. (2002) for details. To work in terms of purely spherical coordinates, we center the spot location to the north pole of the sphere, and rewrite the above as

\[ T_{\text{wav}}(\hat{r}; R) = \int d\Omega' M(\hat{r}'; \theta; R), \]

where \( M(\hat{r}') \) is the mask, defined to be 1 for pixels within the mask and 0 for those outside of it. As the wavelet is effectively zero for \( \alpha \) values greater than ~ 4 times the radius, we can carry out the integral by using the Healpix command query_disc to find all pixels within a circle of that radius from the wavelet center.

To account for the masked parts of the sky, at each spot location \( \hat{r} \) we first calculate the “occupancy fraction”

\[ N(\hat{r}; R) = \int d\Omega' M(\hat{r}'; \theta; R). \]

We only include results for spot locations \( \hat{r} \) for which \( N(\hat{r}; R) > 0.95 \). Additionally, we do not include individual pixels that have \( M(\hat{r}') < 0.9 \) in order to limit biases due to masking (partially masked pixels come about after degrading maps to a lower resolution). As discussed further below, we tested our procedures by using a higher occupancy fraction and found consistent results.

Maps

We use WMAP’s five year maps in our analysis (Hinshaw et al. 2009). Following Vielva et al. (2004), the fiducial map we use is the coadded foreground-cleaned map

\[ T = \frac{\sum_{i=1}^{10} T_{\text{wav}}(i) \omega_i(i) \omega_i(i)}{\sum_{i=1}^{10} \omega_i(i)}, \]

where \( T \) is the coadded temperature, determined from the weighted sum of temperatures \( T_r \) of each individual radiometer \( r \in \{Q1, Q2, V1, V2, W1, W2, W3, W4\} \), divided by the total weight. The weights at each pixel for each radiometer are \( \omega_i(i) = N_r(i)/\sigma^2 \), where \( N_r(i) \) is the number of effective observations at the pixel, and \( \sigma_r \) is the noise dispersion for the given receiver.

This coadding was performed on maps at resolution of \( N_{\text{side}} = 512 \) (~ 8’), then the KQ75 mask was applied. As
mentioned earlier, spots with more than 5% of the weighted area masked ($N(\hat{r}; R) > 0.95$) were not used.

The locations of centroids of spots are chosen to be centers of pixels in $N_{\text{side}} = 32$ resolution; therefore, we examine $N_{\text{disk}} = 12 N_{\text{side}}^2 \approx 12,000$ spots on the sky. In order to calculate the spots’ weighted temperatures, however, we analyze the coadded map at the $N_{\text{side}} = 128$ ($\sim 0.5^\circ$) resolution, which is sufficiently high to lead to converged results for $R \geq 2^\circ$ spots, yet sufficiently low to be numerically feasible.

The results of our analysis were then compared to 10,000 randomly generated Gaussian full sky maps, with the same methodology applied. The skies have been generated using the Healpix facility $\text{synfast}$, and used as input the power spectrum determined in the WMAP 5-year analysis ($\text{Nolta et al.}[2009]$). The maps were then smoothed by a Gaussian with FWHM $= 1^\circ$ to match the WMAP procedure.

**Significance statistics**

The principal statistic that we use is the temperature of the coldest spot divided by the standard deviation of the distribution of all spots

$$S_{\text{disk}}(\hat{r}; R) \equiv \frac{T_{\text{coldest}}(\hat{r}, R)}{\sigma_{\text{disk}}(R)}$$

and equivalently for the Gaussian weights and the wavelets. Here $\sigma_{\text{disk}}(R)$ is the standard deviation of the distribution of all spots in a given map, while $T_{\text{coldest}}(R)$ is the coldest spot in the distribution. Note that the distribution of spot temperatures is not Gaussian as we noted earlier, but this is irrelevant for us; we scale $T$ by $\sigma$ in Eq. (14) in order to account for small ($\sim 10\%$) differences in the overall level of power in spots of characteristic size $R$ in the different maps — in effect, $\sigma_{\text{disk}}(R)$ provides units in which to best report the coldest temperature.

Computing the significance of our statistic $S_{\text{disk}}(\hat{r}; R)$ is then in principle straightforward: we compare it to values obtained from simulated Gaussian random maps and rank-order it; the rank gives the probability.

In addition to the cold spot significance, we follow $\text{Vielva et al.}[2004]$ and $\text{Cruz et al.}[2005]$ and consider the kurtosis of spots in a given map. The kurtosis is simply related to the fourth moment of the distribution of the spots

$$K_{\text{disk}}(R) \equiv \frac{1}{N_{\text{spots}}} \sum_{i=1}^{N_{\text{spots}}} \frac{T_{\text{disk}}(\hat{r}_i, R)^4}{\sigma_{\text{disk}}(R)^4} - 3$$

and equivalently for the Gaussian weights and the wavelets.

**RESULTS**

**Wavelet weighted spot**

We first make sure that we reproduce the cold spot results of $\text{Vielva et al.}[2004]$ and $\text{Cruz et al.}[2005]$. For Spherical Mexican Hat Wavelets with $R = 5^\circ$, we find the center of the coldest spot in the five-year combined cleaned map, is at coordinates $(\ell, b) = (-57.7^\circ, 209.3^\circ)$ (corresponding to spherical coordinates $(\theta, \phi) = (147.7^\circ, 209.3^\circ)$). In general agreement with $\text{Cruz et al.}[2007]$ results, we find that only $0.99 \pm 0.10\%$ of simulated statistically isotropic, Gaussian random maps exhibit a more significant cold spot (i.e. a more negative value of $S_{\text{coldest}}^w(R)$ for this value of $R$). Here and throughout, the error bars account for the finite number of $(N = 10,000)$ simulated maps; we quote the standard margin of error which, for a fraction $p$ of a total of $N$ events, is given as $\sigma(p) = \sqrt{p(1-p)/N}$. Moreover, we confirm that while the variance and skewness of the distribution of $S_{\text{coldest}}^w(R)$ from synthetic maps, the kurtosis at $R = 5^\circ$ is high at the $(1.44 \pm 0.12\%)$ confidence.

Reporting the significance only for the $R = 5^\circ$ may be unfair, however. To address this, we consider the range $2^\circ \leq R \leq 8^\circ$ in steps of $0.5^\circ$ (the lower bound is set to correspond to spots significantly larger than smoothing of the maps of
Cold spot contribution is most significant at \( R \) and significant; see again Fig. 1. In particular, the coldest spot while the kurtosis is significant in the range \( 3^\circ \) (about 94\%) of the weight is applied within the radius \( R \).

Compare to simulated maps at \( N_1 \) or small at any scale. \( -\)\( R \) equal to the scale maximum (FWHM) of the Gaussian weight has been set for tests with Gaussian weighting, where the full-width half-maximum within the expectation on all scales we examined.

As shown in the right panel of Fig. 1 these distributions fall between the two. This figure shows the case of \( R = 5^\circ \) which approximately maximizes significance of the wavelet-based cold spot.

The density profile of the cold spot

The question is obvious: Why was the cold spot so significant for wavelets, but not so much for disks? To address this, we show the disk and wavelet weights, together with the azimuthally-averaged CMB temperature profile, as a function of radial distance from the (wavelet-found) center of the cold spot in the left panel of Fig. 2. In the right panel, we show contribution to the weighted temperature using disks and wavelets, as well as the cumulative difference between the two. This figure shows the case of \( R = 5^\circ \) which approximately maximizes significance of the wavelet-based cold spot.

The azimuthally averaged density profile of the temperature is about equally distributed between zero and \( 5^\circ \) (that is, the blue area in the right panel is about the same as the red one up to \( 5^\circ \)). However, at distance beyond the edge of the disk of \( 5^\circ \), the wavelet accumulates more weight as seen in the right panel. The reason is shown with the curve labeled “averaged CMB temperature” in the left panel: the CMB profile goes from negative to positive with increasing distance from the center of the (wavelet-based) cold spot, precisely favoring the wavelet profile that has roughly the opposite behavior.

What is the likelihood of this conspiracy that the temperature profile of the coldest spot mimics the shape of the wavelet? Using Gaussian random maps, we estimate the likelihood that a given map has the wavelet-determined cold spot is more significant than the disk-determined cold spot by \textit{at least} as much as in WMAP where \( S_{\text{wav}}^{\text{coldest}}(R) - S_{\text{circ}}^{\text{coldest}} = 1.5 \). We find that the wavelets are more significant that the disks by at least this margin in only \( 1.89 \pm 0.13\% \) of the random maps (while the disks are as or more significant in only \( 1.96 \pm 0.14\% \) cases). From this, we conclude that typical Gaussian random CMB maps do not show increased significance of the wavelet determined cold spot, relative to the disk-determined one, to the same extent as our sky does.

\[ S_{\text{wav}}^{\text{coldest}}(R) - S_{\text{circ}}^{\text{coldest}} = 1.5 \]

\[ \sigma = 0.2 \]

\[ -10 \]

\[ -5 \]

\[ 0 \]

\[ 5 \]

\[ 10 \]

\( \sigma = 0.2 \) level, while the kurtosis is not significantly large at all scales.

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Robustness

We have tried varying a number of details, with the following results:

- In addition to the coadded foreground cleaned Q+V+W map, we also used the coadded Q+V+W map, the coadded V+W map, and the coadded foreground cleaned V+W maps available from WMAP. All of the significances are very close to the foreground QVW map value; we have checked that smaller scales (1-1.5'), which were not in the range we presented in the final analysis, would be somewhat discrepant; this is not surprising given that the maps are smoothed to a posteriori.

- To test the effects of the resolution of the map, we vary the resolution from $N_{\text{side}} = 128$ (≈ 0.5') pixels to $N_{\text{side}} = 32$ (≈ 2') pixels, with 16 times fewer pixels. We again find consistent results except at small scales, $R < 2'$, which makes sense since pixelization is expected to play a role only when pixel size becomes comparable to the spot scale $R$.

- To ensure that our finite step size has not accidentally overlooked a cold spot, we refine the resolution in our search for the coldest spot in WMAP by querying at every pixel in an $N_{\text{side}} = 512$ map within 3' of the center of the reported cold spot; this stepping size is effectively $\sim 256$ times higher than before. While an increase in the temperature of the cold spot is entirely expected, we find that this increase is small enough not to appreciably change the significance results for all three choices of the weight function.

- To test our prescription for dealing with partially masked spots, where we only analyze spots that have the “occupancy fraction” $N(r; R) > 0.95$ (see Eq. (12)), we repeat the analysis with the minimum occupancy number of 0.98. While the resulting number of spots retained in the analysis is now much smaller, decreasing by between tens of percent (for spots at $R = 2'$) to about a factor of 10 (for $R = 8'$ disks), we find results generally consistent with our fiducial case: the statistic $S$ and kurtosis calculated using the wavelet weights are significant, while the same statistics calculated using the disk and Gaussian weights are not.

More general tests

It is clear that a posteriori choices were made in the original claims for the existence of the cold spot — in addition to the choice of the weighting function (which is the principal subject of this paper), the moment of the distribution of spots (kurtosis) and spot scale (5') have been called out after noticing that they are unusual. In contrast, variance and skewness of the spot distribution, or kurtosis and scales larger or smaller than $\sim 5'$, do not show departures from expectations based on Gaussian random isotropic maps, as we have checked as well.

We have investigated how results change with more general tests as follows. We have formed a “superstatistic” defined as maximum significance of either variance, skewness, kurtosis, or coldness (the last two being defined earlier in this section) over any scale $R$ or weight function set $W$.

\[
S_{\text{super}} \equiv \max_{R, W, \text{Stat}} \{P(\text{Stat}_W(R))\}
\]

where

\[
R \in \{2', 2.5', \ldots, 8'\}
\]

\[
W \in \{\text{wavelet, disk, Gaussian}\}
\]

\[
\text{Stat} \in \{\text{Variance, Skewness, Kurtosis, S}\}
\]

and where each probability $P$ was individually calculated relative to Gaussian random isotropic maps as described earlier. [The $S$ and kurtosis statistics have been defined in Eqs. (14) and (15), while the variance and skewness are defined analogously to kurtosis.] Note that we define $P$ to capture the possibility that the statistics in question is either small or large relative to expectation; in other words, we adopt the minimum of $r$ and $(100\% - r)$, where $r$ is the rank of the statistics relative to simulated maps.

We find that the value of the $S_{\text{super}}$ statistic for the WMAP cleaned QVW map is 0.54%, and this value is attained by the kurtosis statistic $S_{\text{wavelet}}$ at scale $R = 3.5'$.

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However, this value is not too unusual: we find that 23% of the Gaussian random skies have a smaller value of $S_{\text{super}}$; see Fig. 3.

To test the robustness of this result, we consider an alternative, more restricted, definition of the superstatistic where only the wavelet weights are considered, but where we still varying scale $R$ and statistic Stat; see Eq. (15). Here we effectively assume that, for whatever reason, wavelets are the preferred weight functions to be used, but we still seek to avoid the \textit{a posteriori} choices of the scale and statistics. The new superstatistic is again not statistically significant; we find that 15% of Gaussian random skies show greater significance.

Thus, superstatistic results confirm our earlier conclusion that less \textit{a posteriori} tests do not indicate a statistically significant cold spot in the WMAP data.

CONCLUSIONS

The “cold spot”, together with low power at large angles, multipole alignments, north-south power asymmetry, has been one of the most studied anomalies in WMAP CMB temperature maps. So far there have been no compelling proposals, cosmological or systematic, that would explain the existence of the claimed cold spot, which is perhaps not surprising given that neither its radius ($\sim 5^\circ$) nor its direction in the sky are particularly special.

In this paper, we have investigated evidence for the cold spot. While we confirmed its high statistical significance using the wavelet basis of weight functions, we did not confirm the existence using the disk top-hat, or Gaussian, weights. The cold spot is indeed at the same location in WMAP maps with the latter two bases, but it is not significant when compared to expectation based on Gaussian random, isotropic skies.

We traced the apparent inconsistency to the fact that the radial temperature profile around the cold spot center is such that it favors the wavelet profile; see Fig. 2. This is a chance event, since only 5% of the Gaussian random, isotropic skies exhibit equal or more significant discrepancy in favor of the wavelets. Moreover, we found that the result is insensitive to the choice of the map or the statistic used for the cold spot.

Motivated by these findings, we also examined significance of a more general measure – which we called the “superstatistic” – that combines the various choices of the weighting function, spot size, and statistics, and returns the most significant choice consistently for each map. We again find a null result; the WMAP superstatistic is low only at $\sim 20\%$ level relative to Gaussian random and isotropic simulated maps.

Therefore, we find no compelling evidence for the anomalously cold spot in WMAP at scales between 2 and 8 degrees. The existing evidence apparently hinges on the particular choice of the weight functions to define the spot (Spherical Mexican Hat Wavelets) and their scale ($R \sim 5^\circ$). While our conclusion may sound like a depressing null result, we are upbeat about future tests with WMAP (and soon, Planck) to uncover and test unexpected features and anomalies.

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