Probing the Nature of Compactification with Kaluza-Klein Excitations at the Large Hadron Collider

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Abstract

It is shown that the nature of compactification of extra dimensions in theories of large radius compactification can be explored in several processes at the Large Hadron Collider (LHC). Specifically it is shown that the characteristics of the Kaluza-Klein (KK) excitations encode information on the nature of compactification, i.e., on the number of compactified dimensions as well as on the type of compactification, e.g., of the specific orbifold compactification. The most dramatic signals arise from the interference pattern involving the exchange of the Standard Model spin 1 bosons (γ and Z) and their Kaluza-Klein modes in the dilepton final state $pp \rightarrow l^+l^-X$. It is shown that LHC with 100\textit{fb}^{-1} of luminosity can discover Kaluza-Klein modes up to compactification scales of $\approx 6$ TeV as well as identify the nature of compactification. Effects of the Kaluza-Klein excitations of the W boson and of the gluon are also studied. Exhibition of these phenomena is given for the case of one extra dimension and for the case of two extra dimensions with $Z_2 \times Z_2$, $Z_3$, and $Z_6$ orbifold compactifications.
Recent analyses of the effects of extra space time dimensions on the Fermi constant and on the precision electro-weak data have led to a lower bound on the scale of the extra dimensions that well exceeds 1 TeV and could be as high as 5 TeV for the case of one extra dimension \((d = 1)\), and the lower bound is even higher for the case \(d > 1\). Given these bounds, it appears very unlikely that the Kaluza-Klein (KK) excitations associated with the Standard Model gauge bosons (\(\gamma, W, Z\) and gluon) can be observed at the upgraded Tevatron. However, the possibility exists that these modes could be directly produced at the Large Hadron Collider (LHC). In this paper we explore this possibility.

There is a long and rich history associated with Kaluza-Klein models. More recently the interest in these models has been revived because there exists the possibility that models with low scale compactifications might arise in string theory, specifically in the context of Type I string and low energy consequences of such scenarios have been investigated. Our analysis is in the spirit of Ref. 1 where we work within the context of an effective field theory of a \(p\)-brane of a Type I string where the matter and the gauge fields of the Standard Model reside while gravity propagates in all 10 dimensions. Our analysis corresponds to compactifications internal to the \(p\)-brane \((p = d + 3\) where \(d\) is the number of extra space time dimensions), and there is no influence on the analysis of compactifications which are transverse to the brane where only gravity propagates. The purpose of this analysis is to investigate the possibility of observing the Kaluza-Klein excitations of the Standard Model gauge bosons arising from the compactifications of the extra dimensions. We shall show that several channels provide clear signals for the possible observation of such states.

In the analysis below we first consider the Drell-Yan process \(pp \to l^+l^- + X\) where the Kaluza-Klein modes of the \(Z\)-boson as well as of the photon contribute. Here we find that the production cross section of a charged lepton pair in the \(d = 1\) case is about one order of magnitude larger than that for the sequential Standard Model \(Z'\) boson with the same mass, and the enhanced cross section allows one to extend the search for the Kaluza-Klein mode significantly. Thus LHC with 100\(fb^{-1}\) luminosity will be able to discover these Kaluza-Klein modes up to \(M_R \approx 6\) TeV. In the analysis we find a phenomenon specific to the Kaluza-
Klein models in that the dilepton production cross section for the process $pp \rightarrow l^+l^- + X$ as a function of the dilepton invariant mass has a dip just below the resonance peak of the Kaluza-Klein state. The dip arises from a negative interference between the exchange of the photon and for the Z boson and their Kaluza-Klein recurrences. We also study the processes $pp \rightarrow l^\pm \nu_l + X$ and $pp \rightarrow jj + X$ where the Kaluza-Klein recurrences of the W boson and those of the gluon respectively are involved, and show that they also provide interesting signals for the discovery of Kaluza-Klein excitations.

Another remarkable result seen is that the signatures at the hadron collider can be used to distinguish among different models of compactification. Specifically it is shown that for the case $d = 2$ one can distinguish via the dileptonic signal not only models with different number of compactified dimensions but also different types of compactifications, such as the $Z_2 \times Z_2$ orbifold model from the $Z_3$ and $Z_6$ orbifold models.

The details of the models we consider can be found in Ref. 1 and we discuss here some of its salient features. For the case $D=5$ ($d = 1$), the model is described by a Lagrangian whose first few terms are exhibited below

$$L_5 = -\frac{1}{4} F_{MN} F^{MN} - (D^M H)^\dagger (D^M H) - \bar{\psi} \frac{1}{i} \Gamma^\mu D_\mu \psi - V(H) + .. \quad (1)$$

where $A_M$ ($M=0,1,2,3,4$) is the vector potential in 5 dimensions, $D_M = \partial_M - ig^{(5)}A_M$ is the gauge covariant derivative, and $H$ is the Higgs doublet. The potential $V(H)$ is arranged to allow spontaneous breaking of the electro-weak symmetry which allows the $W$ and $Z$ bosons in 5D to gain electro-weak masses. We make the assumption that the gauge and the Higgs fields lie in the bulk while the quark-lepton generations lie on the 4D wall. We compactify the model on $S^1/Z_2$ with a radius of compactification $R$ (and compactification mass scale $M_R = \frac{1}{R}$) and with the quarks and leptons residing on one of the orbifold points. After compactification the zero modes of the model correspond exactly to the spectrum of the Standard Model (SM). The relative strength of the gauge couplings of the 4D gauge fields and their Kaluza-Klein modes is given by

$$ L_{int} = g_i j_i^\mu (A_{\mu i} + \sqrt{2} \sum_{n=1}^{\infty} A_{\mu i}^n) \quad (2)$$
where $A_{\mu i}$ are the zero modes and $A_{\mu i}^n$ are the Kaluza-Klein modes. We note that the Kaluza-Klein modes couple more strongly by a factor of $\sqrt{2}$ than their zero mode counterparts.

Throughout this paper, we assume that the Kaluza-Klein modes decay only into fermion pairs. In supersymmetric theories, they may also decay into sfermions, which will make the decay width $3/2$ times larger. The main results of our analysis, however, will not be significantly affected by the inclusion of these effects.

With these preliminaries we discuss now the main results of the analysis. We consider first the Drell-Yan processes $pp \rightarrow l^+ l^- + X$. (The relevant formulae can be found in Ref. 11). In our computations, we used CTEQ5L parton distribution functions. In Fig. 1, a plot of the production cross section of a charged lepton pair from Kaluza-Klein excitations of $Z$ and $\gamma$ as a function of the compactification scale $M_R$ is given. The plot is for one generation of leptons, and no summation over generations is taken. For comparison, we also plot the same production cross section for the sequential Standard Model (SSM) $Z'$ boson which has exactly the same couplings with the quarks and leptons as the $Z$ boson does. We find that the cross section in the Kaluza-Klein case is about one order of magnitude larger than that for the case of the SSM $Z'$ boson. This result arises in part because as noted earlier the coupling of the Kaluza-Klein gauge boson is enhanced by $\sqrt{2}$ and in part because there is a constructive interference between the photonic Kaluza-Klein mode and the $Z$ boson Kaluza-Klein mode which lie close to each other and essentially overlap. In Ref. 13, the discovery reach for the sequential Standard Model $Z'$ boson at the LHC with a luminosity of $100 \ fb^{-1}$ (5 events for one-generation lepton pair) was found to be 5 TeV. We find that the corresponding reach for the Kaluza-Klein mode will give $M_R$ around 6 TeV. We note that this mass region is not yet excluded by the constraint from the electroweak precision data.

In Fig. 2 the cross section $d\sigma/dm_{ll}$ is given as a function of the dilepton invariant mass $m_{ll}$ for the cases where $M_R = 2, 5, 8$ TeV. For comparison a plot of the contribution from only the Standard Model $Z$ boson and the photon exchange is also given. The plot exhibits clear resonance peaks corresponding to the masses of the Kaluza-Klein states. We note that each
peak is a superposition of the contribution from the exchange of a Kaluza-Klein excitation of the photon and from the exchange of a Kaluza-Klein excitation of the $Z$ boson. A close scrutiny shows that the resonances are not of the typical Breit-Wigner form due to the fact as mentioned above that each resonance is a superposition of two resonances one from the Kaluza-Klein excitation of the photon and the other from the Kaluza-Klein excitation of the $Z$ boson. The superposition of the two resonances which have significantly different widths gives a composite resonance shape which is a distorted Breit-Wigner form. However, it remains to be seen if the LHC detectors will be sensitive enough to observe such distortions. A more interesting phenomenon arises from the negative interference between the exchange contributions of the gauge bosons and of their Kaluza-Klein excited states below the peak of the Kaluza-Klein excitation. The negative interference leads to a sharp dip below the peak. This signal is specific to the Kaluza-Klein excitations and would not arise for any other type of excitation which are not recurrences of the Standard Model gauge bosons. One may worry if the loop corrections to the gauge coupling constants of the gauge boson and their Kaluza-Klein modes would diminish or even eliminate the negative interference effect. However, the loop corrections are expected to be small and we have checked that the appearance of the dip is stable against small perturbations of the gauge couplings.

We next consider $pp \rightarrow l^{\pm} \nu_l + X$ through the exchange of the Kaluza-Klein excitations of the $W$ boson. Plot of the production cross section is given in Fig. 3 as a function of $M_R$ for the case $d = 1$. Compared to the case of the sequential Standard Model $W'$ boson exchange which is also shown in Fig. 3 one finds an enhancement of the cross section for the Kaluza-Klein case. Another interesting process concerns the dijet production $pp \rightarrow jj + X$ ($j \neq t$) via the exchange of the gluon and its Kaluza-Klein excitation. In Fig. 4 a plot of $d\sigma/dm_{jj}$ is given as a function of the dijet invariant mass $m_{jj}$. In this process t-channel

* It is interesting to mention that the flavor universal coloron model shares similar properties with the KK modes of the gluon.
as well as s-channel exchanges contribute. Further, the resonance widths are much broader than for the dilepton case of Fig. 2. Because of the combination of these two effects the resonance peaks are not so manifest for this process as is evident from Fig. 4. However, a signal for the Kaluza-Klein excitations still exists since one finds an excess of the dijet events with large invariant mass when compared to the Standard Model case. This is evident in Fig. 5 where the production cross section of the two jets with invariant mass larger than 1, 3, and 5 TeV is shown and the cross section for the last two cases exceed that of the SM case (corresponding to $M_R \to \infty$). Thus we conclude that dijet production is also a promising channel to probe the extra dimensions. We note that all Standard Model gauge bosons have common Kaluza-Klein mass spectra.

Finally we consider the case of more than one extra dimension ($d > 1$). Here the information regarding the number of extra dimensions may appear directly in the decay pattern of the Kaluza-Klein excitations. For illustration we consider the case of $d$ extra dimensions with the compactification $S^1/Z_2 \times S^1/Z_2 \times \cdots \times S^1/Z_2$, i.e., each extra dimension is compactified on a $S^1/Z_2$ and we assume that each circle has a common radius $R$. In this case there are $d$ degenerate states at the first Kaluza-Klein excitations, each of which has a mass $M_R$ and a coupling enhancement factor of $\sqrt{2}$ just as in the $d = 1$ case. Although the decay width of each Kaluza-Klein mode at the lowest level is the same as the one in $d = 1$, the multiplicity enhances the cross section around the resonance peak by $d^2$ compared to the $d = 1$ case. Thus the height of the resonance peak at the first Kaluza-Klein excitation, for example, provides a direct count of the number of the extra dimensions. For the second Kaluza-Klein excitation, there are $d(d - 1)/2$ states with mass $\sqrt{2}M_R$ and a coupling enhancement factor of 2. Similar analyses can be done for the higher Kaluza-Klein states and the dependence on the number of extra dimensions identified.

Unlike the $d = 1$ case the compactifications for $d > 1$ are much more model dependent. To illustrate this point we consider the $d = 2$ case in detail and compare the following models of compactification: (1) a $Z_2 \times Z_2$ orbifold model as above where the compactified space is $S^1/Z_2 \times S^1/Z_2$ and the two $S^1$ are assumed to have the common radius $R$, and (2)
$Z_3$ and $Z_6$ compactifications with a two-dimensional torus of periodicity $2\pi R$. The analysis of case (1) is given in Table 1 where we list the masses, the multiplicities, and the coupling enhancement of the Kaluza-Klein vector bosons to the boundary fermions relative to their zero modes. The $Z_3$ and $Z_6$ orbifold compactifications have an interesting relationship. For the $Z_3$ orbifold compactification of 2d the mass formula for Kaluza-Klein states is

$$M_{Z_3}^2 = \frac{4}{3R^2}(m_1^2 + m_1m_2 + m_2^2)$$

where $m_1, m_2$ take on positive and negative integer values, while for $Z_6$ orbifold compactification of 2d the analogous mass formula is

$$M_{Z_6}^2 = \frac{4}{3R^2}(m_1^2 - m_1m_2 + m_2^2)$$

Thus one finds that the transposition $(m_1, m_2) \rightarrow (m_1, -m_2)$ takes one from the $Z_3$ mass relation to the $Z_6$ mass relation. In Table 2 we list the masses, the multiplicities, and the coupling enhancement of the first few Kaluza-Klein vector bosons to the boundary fermions relative to the zero modes for the $Z_3$ and $Z_6$ case. The analysis of Table 2 shows that for the case of couplings to the boundary fermions the cross sections for the production of Kaluza-Klein states for $Z_3$ and $Z_6$ are the same. Thus we shall discuss only $Z_3$ in detail and similar results hold for the $Z_6$ compactification. A comparison of Table 1 and Table 2 shows that the masses of the Kaluza-Klein excitations, their multiplicities and the strength of their couplings to the boundary fermions depend on the nature of compactification. These attributes will manifest in the production cross-section and in the resonance structure of these states at the LHC. An exhibition of this phenomena is given in Fig. 6 where the cross section for the process $pp \rightarrow e^+e^- + X$ is given for the case $d=1$ and for the case $d=2$ for the two orbifold compactifications, $Z_2 \times Z_2$ and $Z_3$ where the mass of the first Kaluza-Klein excitation is taken to be 3 TeV for each case. The analysis of Fig. 6 shows that the three cases can be distinguished by a detailed study of the dileptonic cross section as a function of the dilepton invariant mass. We note that a study of this channel can allow one to differentiate the case when the radii of compactification are unequal. In this case
the degeneracy of the Kaluza-Klein states (as in case (2) for d=2) will be lifted. However, the pattern of the resonance peaks will be more complex resulting in a richer structure of the dileptonic cross section as a function of the dilepton invariant mass. We thus conclude that a close study of the resonance structure of the Kaluza-Klein states will allow one to determine the dimensionality of the compactified space as well as the detailed nature of compactification.

Table 1: \(Z_2 \times Z_2\) orbifold model for d=2 with common radius \(R\)

| \(\frac{1}{R}\) unit | 1st KK | 2nd KK | 3rd KK | 4th KK | 5th KK |
|----------------------|--------|--------|--------|--------|--------|
| mass                 | 1      | \(\sqrt{2}\) | 2      | \(\sqrt{5}\) | \(2\sqrt{2}\) |
| multiplicity         | 2      | 1      | 2      | 2      | 1      |

Table 2: \(Z_3 \& Z_6\) orbifold models for d=2

| \(\frac{2}{\sqrt{3}R}\) unit | 1st KK | 2nd KK | 3rd KK | 4th KK | 5th KK |
|-------------------------------|--------|--------|--------|--------|--------|
| mass                          | 1      | \(\sqrt{3}\) | 2      | \(\sqrt{7}\) | 3      |
| multiplicity                  | 1      | 1      | 1      | 2      | 1      |

In conclusion we have discussed the signatures of the Kaluza-Klein excitations at the LHC associated with extra dimensions with large radius compactifications consistent with the precision electro-weak data. It is shown that the dileptons from the Drell-Yan process \(pp \to l^+l^- + X\) exhibit a remarkable dip from an interference and provide an important signal for the discovery of such states up to \(M_R \approx 6\) TeV. It is found that the processes \(pp \to l^\pm \nu_l + X\) and \(pp \to jj + X\) provide further signals for the observation of Kaluza-Klein modes. We also discussed the d=2 case and showed that the dilepton signal can distinguish between the d=1 and the d=2 cases. Further, for the d=2 case we considered the orbifold compactifications, \(Z_2 \times Z_2, Z_3,\) and \(Z_6\) and found that the \(Z_2 \times Z_2\) case can be distinguished from the \(Z_3\) and \(Z_6\) cases. Thus if the low scale Kaluza-Klein dimensions exist at the scale accessible to LHC one can not only determine the compactification scale \(M_R\) but also the
number of compactified dimensions and the nature of compactification itself from a detailed study of the dileptonic and other signatures in pp collisions.

Note added: While this paper was in preparation there appeared a paper by I. Antoniadis, K. Benakli and M. Quirós, which has an overlap with some of the topics discussed here.

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REFERENCES

1. P. Nath and M. Yamaguchi, hep-ph/9902323.

2. M. Masip and A. Pomarol, hep-ph/9902467.

3. P. Nath and M. Yamaguchi, hep-ph/9903298.

4. W.J. Marciano, hep-ph/9903451.

5. Th. Kaluza, Sitzungsber. Preuss.Akad.Wiss.Berlin, p.966 (1921); O. Klein, Z. Phys. 37 (1926) 895. For the early history of Kaluza-Klein theories see, L. O’Raifeartaigh and N. Straussman, hep-ph/9810524. See also, T. Appelquist, A. Chodos, and P.G.O. Freund, Modern Kaluza-Klein Theories, Frontiers in Physics, Addison Wesley, 1987.

6. I. Antoniadis, Phys. Lett. B246 (1990) 377;

V.A. Kostelecký and S. Samuel, Phys. Lett. B270 (1991) 21;

I. Antoniadis, K. Benakli and M. Quirós, Phys. Lett. B326 (1994) 69.

7. P. Hořava and E. Witten, Nucl. Phys. B460 (1996) 506;

E. Witten, Nucl. Phys. B 471 (1996) 135;

J. Lykken, Phys. Rev. D54 (1996) 3693;

I. Antoniadis and M. Quirós, Phys. Lett. B392 (1997) 61.

8. N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B429 (1998) 263; Phys. Rev. D59 (1999) 086004;

I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B436 (1998) 257.

9. K.R. Dienes, E. Dudas and T. Gherghetta, Phys. Lett. B436 (1998) 55; Nucl. Phys. B537 (1999) 47.

10. G. Shiu and S.H.H. Tye, Phys. Rev. D58 (1998) 106007;

Z. Kakushadze and S.H.H. Tye, hep-th/9809147.
D. Ghilencea and G.G. Ross, Phys. Lett. B442 (1998) 165;
Z. Kakushadze, hep-th/9811193; hep-th/9812163;
S. Nussinov and R. Shrock, Phys. Rev. D59 (1999) 105002;
L. Ibáñez, C. Muños and C. Rigolin, hep-ph/9812397;
C.D. Carone, hep-ph/9902407;
A. Donini and S. Rigolin, hep-ph/9901443;
A. Delgado and M. Quirós, hep-ph/9903400;
P. Frampton and A. Rašin, hep-ph/9903479;
A. Pérez-Lorenzana and R.N. Mohapatra, hep-ph/9904504;
G. Shiu, R. Shrock and S.H.H. Tye, hep-ph/9904262

11. F. del Aguila, M. Quíros and F. Zwirner, Nucl. Phys. B287 (1987) 419.

12. H.L. Lai, J. Huston, S. Kuhlmann, J. Morfin, F. Olness, J.F. Owens, J. Pumplin, and
W.K. Tung, hep-ph/9903282.

13. T. Rizzo, SLAC-PUB-7365, hep-ph/9612440, in Proceedings of the 1996 DPF/DPB
Summer Study on New Directions for High Energy Physics-Snowmass96, Snowmass,
Colorado, 1996, edited by D.G. Cassel, L. Trindle Gennari and R.H. Siemass (SLAC,
Stanford, 1997), and references therein.

14. R.S. Chivukula, A.G. Cohen, E.H. Simmons, Phys. Lett. B380 (1996) 92;
E.H. Simmons, Phys. Rev. D55 (1997) 1678.

15. I. Antoniadis, K. Benakli and M. Quirós, hep-ph/9905311.
Fig. 1. Plot of the production cross section of a charged lepton pair in the process $pp \rightarrow l^+l^- + X$ (solid line) as a function of the mass scale $M_R$ via exchange of photonic and Z Kaluza-Klein modes of the compactified dimension. The same analysis for the sequential Standard Model Z’ boson is shown by the dashed line.
Fig. 2. Differential cross section $d\sigma/dm_{ll}$ as a function of the invariant mass $m_{ll}$ of the charged lepton pair for three different values of the compactified dimension $M_R$. For comparison the analysis for the SM case is also shown.
Fig. 3. Plot of the production cross section $pp \rightarrow l^\pm \nu_l + X$ via exchange of the Kaluza-Klein excitations of $W$ as a function of $M_R$ (solid). For comparison the cross section for the sequential Standard Model $W'$ boson is also given (dashed).
Fig. 4. The differential cross section $d\sigma/dm_{jj}$ ($j \neq t$) for dijet production in the process $pp \rightarrow jj + X$ including Kaluza-Klein gluon exchange as a function of the dijet invariant mass $m_{jj}$ for the cases when the mass scale $M_R$ of the compactified dimension is 2 TeV, 5 TeV, 8 TeV, and for the SM. The cross section is evaluated at the leading order. A rapidity cut $\eta < 0.5$ is imposed for both jets.
Fig. 5. Plot of the production cross section $pp \rightarrow jj + X \ (j \neq t)$ as a function of the compactified dimension with $jj$ invariant mass cut of $m_{jj} > 1$ TeV, $m_{jj} > 3$ TeV, and $m_{jj} > 5$ TeV. A rapidity cut $\eta < 0.5$ is imposed for both jets.
Fig. 6. Plot of the cross section for dilepton production as a function of the dilepton invariant mass $m_{ll}$ for the case $d=1$ (solid), $d=2$ with $Z_2 \times Z_2$ orbifolding (dashed) and $d=2$ with $Z_3$ orbifolding (dot-dashed) when the mass of the first Kaluza-Klein excitation is taken to be 3 TeV.