Next-to-leading order Calculation of a Fragmentation Function in a Light-Cone Gauge

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Abstract

The short-distance coefficients for the color-octet $^3S_1$ term in the fragmentation function for a gluon to split into polarized heavy quarkonium states are re-calculated to order $\alpha_s^2$. The light-cone gauge remarkably simplifies the calculation by eliminating many Feynman diagrams at the expense of introducing spurious poles in loop integrals. We do not use any conventional prescriptions for spurious pole. Instead, we only use gauge invariance with the aid of Collins-Soper definition of the fragmentation function. Our result agrees with a previous calculation of Braaten and Lee in the Feynman gauge, but disagrees with another previous calculation.

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I. INTRODUCTION

Heavy quarkonium state is one of the simplest hadron with which we can probe both perturbative and nonperturbative nature of quantum chromodynamics. Among various quarkonia, $S$-wave spin-triplet states are especially interesting because of their clean leptonic decay modes. Based on factorization theorems for inclusive single-hadron production \cite{1,2}, one can deduce that dominant production mechanism for heavy quarkonia with large transverse momentum $p_T$ is fragmentation \cite{3}, the production of a parton which subsequently decays into the quarkonium state and other partons. In the region, where fragmentation dominates, theoretical calculation becomes simplified and more reliable. Measurements of large-$p_T$ $S$-wave spin-triplet charmonia production cross section at the Fermilab Tevatron has led to a remarkable progress in heavy quarkonium physics based on nonrelativistic QCD (NRQCD) \cite{4}. In high-energy $p\bar{p}$ collisions, gluon production rate is dominant and inclusive production of a heavy quark pair $Q\overline{Q}$ via subsequent decay of this almost on-shell gluon is enhanced by the gluon propagator \cite{5}. Unexpectedly large measured production rate of direct $J/\psi$ and $\psi'$ at large $p_T$ at the Tevatron \cite{6} was explained by the gluon fragmentation into a color-octet $Q\overline{Q}$ pair followed by a nonperturbative NRQCD transition into the spin-triplet $S$-wave quarkonia \cite{7}. Importance of the gluon fragmentation mechanism has been tested also in inclusive $J/\psi$ production in $Z^0$ decay \cite{8}. There are still two open problems in the field. One is the polarization of prompt $J/\psi$ at the Tevatron \cite{9,10}. The other is the cross section for exclusive $e^+e^- \rightarrow J/\psi + \eta_c$ \cite{11,12} and $e^+e^- \rightarrow J/\psi + c\bar{c} + X$ at $B$-factories \cite{13,14}. However, the color-octet mechanism in NRQCD has been tested successfully in various ways. Comprehensive reviews on NRQCD phenomenology can be found in Ref. \cite{15}. In the rest of this paper, we shall restrict our discussion to the gluon fragmentation into a color-octet spin-triplet $Q\overline{Q}$ pair evolving into a $S$-wave heavy quarkonium.

In the NRQCD factorization formalism \cite{4}, the fragmentation function $D(z,\mu)$ for a parton splitting a heavy quarkonium is expressed as a linear combination of NRQCD matrix elements, which can be regarded as phenomenological parameters. Here, $z$ is the momentum fraction of the final hadron relative to the decaying parton and $\mu$ is the hard-scattering scale of the process. Corresponding short-distance factors depend on $z$ and are calculable in perturbation theory. Most of the phenomenologically relevant short-distance factors have been calculated to leading order in $\alpha_s$. They all begin at order $\alpha_s^2$ or higher, with the exception of
the color-octet $^3S_1$ term in the gluon fragmentation function, which begins at order $\alpha_s$. The color-singlet $^3S_1$ channel is suppressed because the short-distance factor begins at order $\alpha_s^3$. Since the color-octet $^3S_1$ term dominates, the high-$p_T$ gluon fragmentation phenomena in heavy quarkonium production, the next-to-leading order correction of order $\alpha_s^2$ to this term is particularly important. Unfortunately, two available results for the color-octet $^3S_1$ term disagree with each other [16, 17]. Therefore, it is worthwhile to calculate this important function in an independent way. Since both previous calculations employed the Feynman gauge, we shall present our results in the light-cone gauge.

The light-cone gauge is a physical gauge where the gluon field $A^\mu$ has vanishing light-cone projection $A \cdot n = 0$, where $n$ is an arbitrary light-like $(n^2 = 0)$ vector appearing in the gauge-fixing term in the QCD Lagrangian. Derivation of the Altarelli-Parisi evolution of parton densities [18] is one of the best examples of the use of the light-cone gauge. In a light-cone gauge, in which the fragmentation function was originally defined [19], there is a great simplification in QCD calculations. The eikonal line as well as the ghost decouples from the gluon, since the coupling, proportional to $n^\mu$, is orthogonal to the gluon propagator. However, one drawback in higher-order calculations is existence of the spurious pole $1/k \cdot n$ in the gluon propagator

$$\frac{i}{k^2 + i\epsilon} \left[-g^{\mu\nu} + \frac{k^\mu n^\nu + n^\mu k^\nu}{k \cdot n}\right], \epsilon \to 0^+,$$

where $k$ is the momentum of the gluon. One should be very careful in dealing with the pole in evaluating loop integrals. As conventional methods, Cauchy-Principal-Value (CPV) or Mandelstam-Leibbrandt (ML) prescriptions [20, 21] have been used for a long time. The two prescriptions follow from different canonical quantizations, light-front (LF) and equal-time (ET), respectively [22]. It turns out that LF quantization (CPV) is untenable beyond the tree level as it conflicts with causality: the resulting theory cannot be renormalized. The ML prescription is causal; renormalization has been proved not only for the Green functions at any perturbative order [23] but also for composite operators [24]. All previous calculations beyond-leading order had to employ some prescription for the spurious pole. The CPV prescription is used in Refs. [19, 23, 26]. The ML prescription is employed in the leading order [27] and the next-to-leading order [28, 29]. Comprehensive reviews on these prescriptions can be found in Refs. [30, 31, 32].

In our light-cone gauge calculation, we introduce a new method in determining the spu-
rious pole appearing in our calculation for the fragmentation function. We first employ a light-cone gauge to express the fragmentation function in terms of one-loop scalar integrals using the gauge-invariant definition of Collins and Soper [33]. However, we do not impose the sign of $i\epsilon$ in the spurious poles and keep the scalar integrals from evaluation unlike the conventional ways. Our light-cone-gauge result for the fragmentation function is same as that for the Feynman gauge, which is guaranteed by the gauge-invariant Collins–Soper definition. Therefore, the scalar integrals involving spurious poles can be determined by comparing with the Feynman-gauge result. The spurious poles are finally identified with poles coming from the eikonal-line contribution in the Feynman gauge. Because all the poles are well-defined in the Feynman gauge, the spurious-pole contributions in the light-cone gauge are completely determined based on gauge invariance. Our result agrees with a previous calculation of Braaten and Lee [17] in the Feynman gauge, but disagrees with the other previous calculation [16].

II. COLLINS-SOPER DEFINITION AND LIGHT-CONE GAUGE

The fragmentation function $D_{g\rightarrow H}(z, \mu)$ gives the probability that a gluon produced in a hard-scattering process involving momentum transfer of order $\mu$ decays into a hadron $H$ carrying a fraction $z$ of the gluon’s longitudinal momentum. This function can be defined in terms of the matrix element of a bilocal operator involving two gluon field strengths in a light-cone gauge [19]. In Ref. [33], Collins and Soper introduced a gauge-invariant definition of the gluon fragmentation function that involves the matrix element of a nonlocal operator consisting of two gluon field strengths and eikonal operators. One advantage of this definition is that it avoids subtleties associated with products of singular distributions. The gauge-invariant definition is also advantageous for explicit perturbative calculations, because it allows the calculation of radiative corrections to be simplified by using the Feynman gauge.

The gauge-invariant definition of Collins and Soper for the gluon fragmentation function for splitting into a hadron $H$ is [33]

$$D_{g\rightarrow H}(z, \mu) = \frac{(-g_{\mu\nu})}{16\pi(N - 1)k^+} \int_{-\infty}^{+\infty} dx^- e^{-ik^+x^-} \times \langle 0 | G_{\mu}^{+\nu}(0) \mathcal{E}(0^-)_{cb} \mathcal{P}_{H(zk+, \alpha_\perp)} \mathcal{E}(x^-)_{ba} G_{\nu}^{+\mu}(0^+, x^-, 0_\perp)|0 \rangle. \quad (2)$$

The operator $\mathcal{E}(x^-)$ in (2) is an eikonal operator that involves a path-ordered exponential
of gluon field operators along a light-like path:

\[ \mathcal{E}(x^-)_{ba} = P \exp \left[ +ig \int_{z^-}^\infty dz^- A^+(0^+, z^-, 0) \right]_{ba}, \]  

where \( A^\mu(x) \) is the matrix-valued gluon field in the adjoint representation: \( [A^\mu(x)]_{ac} = if^{abc} A^\mu_b(x) \). The operator \( \mathcal{P}_{H(p^+, p)} \) in Eq. (2) is a projection onto states that, in the asymptotic future, contain a hadron \( H \) with momentum \( p = (p^+, p^- = (m_H^2 + p_{\perp}^2)/p^+ + p_{\perp}) \), where \( m_H \) is the mass of the hadron. The hard-scattering scale \( \mu \) in Eq. (2) can be identified with the renormalization scale of the nonlocal operator. The prefactor in the definition (2) has, therefore, been expressed as a function of the number of spatial dimensions \( N = 3 - 2\epsilon \). This definition is particularly useful when we use dimensional regularization to regularize ultraviolet divergences. If the production process of the hadron \( H \) can be described by perturbation theory, one can use the definition (2) to calculate the fragmentation function \( D_{g\rightarrow H}(z, \mu) \) as a power series in \( \alpha_s \). In Ref. [33], complete sets of Feynman rules for this perturbative expansion for quark and gluon fragmentation functions are given. Since Ma first used the definition [34], the method became popular in quarkonium physics [17, 35, 36] because of various reasons listed above. By inserting the eikonal operator (3), the operator consisting of two gluon fields with different locations becomes gauge invariant. At higher order in \( \alpha_s \), there are numerous diagrams which have gluons coupled to the eikonal lines. In the light-cone gauge, the contribution of the eikonal operator disappears since the gluon decouples from the eikonal line. Therefore, there is a great reduction in the number of Feynman diagrams. On the other hand, the spurious pole contribution of the gluon propagator appears in the light-cone gauge. However, the gauge invariance of this definition (2) provides the gauge transformation of the eikonal line contribution in the Feynman gauge into the spurious pole contribution in the light-cone gauge. By comparing the final results for the gauge-invariant quantity \( D_{g\rightarrow H}(z, \mu) \) from the two gauges, the spurious pole coming from the gluon propagator in the light-cone gauge can be fixed unambiguously.

III. PERTURBATIVE CALCULATION

In this section we perform the next-to-leading-order calculation for the fragmentation function for a gluon splitting into a color-octet spin-triplet \( \Omega_c \) pair of order \( \alpha_s^2 \) in a light-cone gauge. Although we carry out the calculation using a different gauge, we use the same
FIG. 1: Leading order Feynman diagram for $g \rightarrow QQ$.

conventions as those presented in Ref. [17]. We do not reproduce the description of the theoretical background of the fragmentation function for heavy quarkonium production in NRQCD factorization formalism which is well explained in Ref. [4]. Based on the NRQCD factorization formalism [4], the fragmentation function is written in a factorized form [17]:

$$D_{g\rightarrow H}(z) = [(N-1)d_T(z) + d_L(z)] \langle O_8^H (3S_1) \rangle,$$

where $d_T$ and $d_L$ are the short-distance coefficients for the transverse and longitudinal contributions and $\langle O_8^H (3S_1) \rangle$ is the color-octet $3S_1$ matrix element defined in Ref. [4].

There is only one lowest-order diagram in both Feynman and light-cone gauge, which is shown in Fig. 1. The circles connected by the double pair of lines represent the nonlocal operator consisting of the gluon field strengths and the eikonal operators in the definition [2]. The momentum $k = (k^+, k^-, \mathbf{k}_\perp)$ flows into the circle on the left and out of the circle on the right. The cutting line represents the projection onto states which, in the asymptotic future, include a $QQ$ pair with total momentum $p = (zk^+, p^2/(zk^+), 0_\perp)$. The appearance of the diagrams for both gauges is the same in this order, since the circle should emit a gluon. With the Feynman rules of Ref. [3] and following the method of extracting the short-distance coefficients of the fragmentation function in Ref. [17], we can read off the order-$\alpha_s$ terms in the short-distance functions $d_T(z)$ and $d_L(z)$ as

$$d_T^{(LO)}(z) = \frac{\pi\alpha_s \mu^2}{8N(N-1)m_Q^3} \delta(1-z),$$

$$d_L^{(LO)}(z) = 0.$$

We have neglected the relative momentum of the heavy quark in the $Q\overline{Q}$ rest frame so that the invariant mass of the pair is $p^2 = 4m_Q^2$. The LO results (5) and (6) agree with previous
FIG. 2: The Feynman diagrams of order $\alpha_s^2$ for $g \to Q\overline{Q}$ with $Q\overline{Q}$ final states. There are additional contributions from the complex-conjugate diagrams.

calculations in the Feynman gauge \[17, 37\]. Note that there is no reason for worrying about the spurious pole in this leading-order calculation.

The Feynman diagrams for the fragmentation function for $g \to Q\overline{Q}$ at order $\alpha_s^2$ consist of virtual corrections, for which the final state is $Q\overline{Q}$, and real-gluon corrections, for which the final state is $Q\overline{Q}g$. The diagrams with virtual-gluon corrections to the left of the cutting line are shown in Fig. 2. The black blob in Fig. 2(a) includes the vertex corrections and propagator corrections shown in Fig. 3. In the Feynman gauge, only the diagram in Fig. 2(b) vanishes, because the gluon attached to the eikonal line gives a factor of $n^\mu$. On the other hand, all the diagrams except for Fig. 2(a) vanish in the light-cone gauge. If we use the threshold-expansion method of Braaten and Chen \[38\], we can simplify the structure of the expression without employing the projection method. With the threshold expansion, we can keep the full structure of color and spin. Here we utilize the dimensionally regularized
FIG. 3: One loop correction diagrams for $g^* \rightarrow Q \bar{Q}$.

threshold expansion method of Braaten and Chen [37, 39]. With the Dirac equation and the usual methods for reducing tensor integrals into scalar integrals, we factorize each virtual correction diagram into the leading order diagram in Fig. 1 times a multiplicative factor. In the light-cone gauge, the ghost decouples since its coupling to the gluon is orthogonal to the gluon propagator (1), so the gluon propagator correction factor shown in Fig. 3(d) does not have ghost contribution.

The virtual corrections contribute only to the transverse short-distance function $d_T(z)$ defined in Ref. [17]:

$$d_T^{(virtual)}(z) = d_T^{(LO)}(z) \times 2 \text{Re} \left[ \Lambda + \Pi + \delta Z_Q + \Delta \right].$$

(7)
where \( \Lambda \) is the vertex correction factor. The expressions \( \delta Z_Q \) and \( \Pi \) are given by

\[
\delta Z_Q^{\text{LC}} = i \frac{16 \pi \alpha_s \mu^{2 \epsilon}}{3} \left[ (2 - N) I_{AD} + p^2 I_{ADD} + 2 p \cdot n I_{BCD} \right],
\]

\[
\delta Z_Q^{\text{F}} = i \frac{16 \pi \alpha_s \mu^{2 \epsilon}}{3} \left[ (4 - N) I_{AD} + p^2 I_{ADD} \right],
\]

\[
\Pi^{\text{LC}} = -i 6 \pi \alpha_s \mu^{2 \epsilon} \left\{ \left[ 7 + \frac{1}{N} - \frac{2 n_f}{3} \left( 1 - \frac{1}{N} \right) \right] I_{AB} - 8 p \cdot n I_{ABC} \right\},
\]

\[
\Pi^{\text{F}} = -i 6 \pi \alpha_s \mu^{2 \epsilon} \left[ 3 + \frac{1}{N} - \frac{2 n_f}{3} \left( 1 - \frac{1}{N} \right) \right] I_{AB},
\]

where the superscripts \( \text{F} \) and \( \text{LC} \) stand for Feynman gauge and light-cone gauge, respectively.

The scalar integrals appearing in Eqs. (8)-\( (11) \) are of the form

\[
I_{AB} = \int \frac{d^{N+1}l}{(2\pi)^{N+1}} \frac{1}{AB \cdots},
\]

where the denominator \( AB \cdots \) can be a product of 1, 2, 3, or 4 of the following factors:

\[
A = l^2 + i\epsilon,
\]

\[
B = (l - p)^2 + i\epsilon = l^2 - 2l \cdot p + 4 m_Q^2 + i\epsilon,
\]

\[
C = (p - l) \cdot n + i\epsilon,
\]

\[
D = (l - p/2)^2 - m_Q^2 + i\epsilon = l^2 - l \cdot p + i\epsilon.
\]

The values for the integrals are in the Appendix of Ref. [17] except for

\[
I_{AAD} = \frac{-i}{(4\pi)^2(2m_Q^2)} \left( \frac{4\pi}{m_Q^2} \right)^\epsilon \frac{\Gamma(1+\epsilon)}{\epsilon_{\text{IR}}(1+2\epsilon)},
\]

\[
I_{ADD} = \frac{i}{(4\pi)^2(2m_Q^2)} \left( \frac{4\pi}{m_Q^2} \right)^\epsilon \frac{\Gamma(1+\epsilon)}{\epsilon_{\text{IR}}}.
\]

Note that for the light-cone gauge integrals \( I_{BCD} \) in Eq. (8) and \( I_{ABC} \) in Eq. (10) have spurious poles \( 1/(p - l) \cdot n \) that require prescriptions. Those poles for the Feynman gauge are unambiguously defined as in Eq. (15). Therefore, we postpone evaluating the scalar integrals for the time being. If we use the ML prescription, each light-cone-dependent integral has ultraviolet (UV) and IR structures which are different from the values shown in the Appendix of Ref. [17]. Effectively, the ML prescription transforms a double pole into an IR pole and makes the integral satisfy naive power counting rules. Note also that \( Z_Q \) and \( \Pi \) are gauge dependent. The values in the Feynman gauge agree with well-known
ones that can be found, for example, in Ref. \[17\]. The result using the ML prescription is known only for the UV poles. We have full agreement in the UV poles if we use the ML prescription: the gluon propagator correction term is proportional to the QCD beta function as \[\Pi = \frac{33 - 2n_f}{12\pi\epsilon_{\text{UV}}}\] \[21, 40\] and \[\delta Z_Q^{\text{LC}} = \frac{\alpha_s}{3\pi\epsilon_{\text{UV}}}\] \[41\]. All of them are listed, for example, in Ref. \[42\].

The contribution from the remaining diagrams shown in Fig. 2 (b)-(e), which have gluon couplings to the eikonal lines, is expressed as \(\Delta\). Their values are expressed in terms of one-loop scalar integrals:

\[
\Lambda^{\text{LC}} = i \frac{2\pi\alpha_s \mu^2}{3} \left[ 9 \left( \frac{7}{N} + 1 \right) I_{AB} + \left( N + \frac{18}{N} - 67 \right) I_{AD} - p^2 I_{AAD} \right. \\
+ 2 \, p \cdot n \, (9I_{ACD} + I_{BCD} - 36I_{ABC}) \right],
\]

\[
\Lambda^{\text{F}} = i \frac{2\pi\alpha_s \mu^2}{3} \left[ 9 \left( \frac{1}{N} + 1 \right) I_{AB} + \left( N + \frac{18}{N} - 47 \right) I_{AD} - p^2 I_{AAD} \right],
\]

\[
\Delta^{\text{LC}} = 0,
\]

\[
\Delta^{\text{F}} = i \frac{12}{3} \pi\alpha_s \mu^2 \left[ I_{AB} - 2I_{AD} + p \cdot n \left( I_{ACD} + I_{BCD} \right) \right].
\]

The explicit value of the vertex correction factor \(\Lambda^{\text{F}}\) in the Feynman gauge shown in Eq. \[20\] agrees with the result in Ref. \[17\]. The UV dependence of the vertex correction factor \(\Lambda^{\text{LC}}\) in the light-cone gauge shown in Eq. \[19\] agrees with the result using the ML prescription in Refs. \[41, 42\] where only the UV contribution is given: \(\Lambda^{\text{LC}}_Q = -\delta Z_Q^{\text{LC}} = -\alpha_s/(3\pi\epsilon_{\text{UV}})\).

The integral \(I_{AAD}\) has a Coulomb singularity as well as a logarithmic IR divergence due to the exchange of a gluon between the on-shell heavy quark and anti-quark. Dimensional regularization puts power infrared divergence like the Coulomb singularity to zero, so only the logarithmic IR divergence remains in the integral \(I_{AAD}\). Then the integral is effectively expressed by \(I_{ADD}\) via the equation \(I_{ADD} = (N - 4)I_{AAD}\). It is important to notice that various correction factors in the Feynman and the light-cone gauge involve different combinations of the same scalar integrals. Straight-forward sums for both gauges produce a common result

\[
d_T^{(\text{virtual})}(z) = d_T^{(\text{LO})}(z) \frac{4\pi\alpha_s}{3} \Re \left\{ i \left[ - \left( 7N - \frac{18}{N} + 51 \right) I_{AD} + 6n_f \left( 1 - \frac{1}{N} \right) I_{AB} \right. \\
+ 18 \, p \cdot n \left( I_{ACD} + I_{BCD} \right) + p^2 (8I_{ADD} - I_{AAD}) \right] \right\}. \quad (23)
\]

Thus the non-vanishing contributions from the gluon coupling to the eikonal line in the
Feynman gauge, $\Delta^F$, is simply distributed to other correction factors in the light-cone gauge via additional gluon propagator terms.

Since gauge invariance holds for both the virtual and the real-gluon corrections separately, the equality of the virtual corrections in the Feynman and the light-cone gauge is a consequence of gauge invariance. As we commented previously, the light-cone dependent integrals in the Feynman gauge result have no ambiguities form spurious poles. On the other hand, we have not fixed the sign of the $i\epsilon$ in the spurious pole of the integrals which are obtained in the light-cone gauge. Since we have found exact agreement between the two results in the two gauges, we may simply use the values obtained from the Feynman gauge calculation. Note that the integral $I_{ABC}$ disappears in Eq. (23), so the only light-cone dependent integrals that survive are $I_{ACD}$ and $I_{BCD}$. The values for these integrals are independent of the sign of the $i\epsilon$ in the definition of $C$ in Eq. (15). The expansion of Eq. (23) in $\epsilon$ reproduces the result of Braaten and Lee [17]:

$$d_T^{(virtual)}(z) = d_T^{(LO)}(z) \frac{\alpha_s}{\pi} \left( \frac{\pi \mu^2}{m_Q^2} \right)^\epsilon$$

$$\times \left[ 3(1 - \epsilon) \Gamma(1 + \epsilon) \frac{1}{2 \epsilon} + \beta_0 \frac{1}{\epsilon} + \frac{177 - 10n_f}{18} - \frac{\pi^2}{2} + 8 \ln 2 + 6 \ln^2 2 \right], \quad (24)$$

where $\beta_0 = (33 - 2n_f)/6$.

The Feynman diagrams for the real-gluon corrections to the fragmentation function for $g \to Q\bar{Q}$ can also be calculated in both gauges. We draw the 5 left-half diagrams only, which must be multiplied by their complex conjugates to give a total of 25 diagrams. The real-gluon correction is a tree-level calculation. Therefore, there is no spurious pole problem. In the Feynman gauge, all 25 diagrams contribute, while only 9 diagrams in the light-cone gauge. In the latter gauge, diagrams 4(a) and 4(b) vanish. The real-gluon correction contribution is also gauge invariant. Employing either gauge, we reproduce the real-correction contribution given in Ref. [17] before the phase-space integral is performed:

$$d_T^{(real)}(z) = \frac{\pi \alpha_s \mu^{2\epsilon}}{8N(N-1)m_Q^3} \times \frac{3\alpha_s}{\pi \Gamma(1 - \epsilon)} \left( \frac{\pi \mu^2}{m_Q^2} \right)^\epsilon \left( \frac{1}{z} - \frac{1}{z(1-z)} \right)^2 \int_{(1-z)/z}^\infty dx \frac{t^{1-\epsilon}}{x^2}, \quad (25)$$

$$d_L^{(real)}(z) = \frac{\pi \alpha_s \mu^{2\epsilon}}{8N m_Q^3} \times \frac{3\alpha_s}{\pi \Gamma(1 - \epsilon)} \left( \frac{\pi \mu^2}{m_Q^2} \right)^\epsilon \left( \frac{1-z}{z} \right)^2 \int_{(1-z)/z}^\infty dx \frac{t^{-\epsilon}}{x^2}, \quad (26)$$

where $t = (1 - z)(zx + z - 1)$, $x = 2q \cdot p/p^2$, $q$ is the final-state gluon momentum, and $p$ is the $Q\bar{Q}$ momentum. The final results for the real-gluon correction contribution of Braaten
FIG. 4: The Feynman diagrams of order $\alpha_s^2$ for $g \to Q\bar{Q}$ with $Q\bar{Q}g$ final states. There are a total of 25 diagrams, but only the left halves of the diagrams are shown.

and Lee [17] are straight-forwardly reproduced:

$$d_T^{(\text{real})}(z) = \frac{\pi \alpha_s \mu^{2\epsilon}}{8N(N-1)m_Q^3} \times \frac{\alpha_s}{\pi} \left( \frac{\pi \mu^2}{m_Q^2} \right)^\epsilon \Gamma(1+\epsilon) \times \left[ -\frac{3(1-\epsilon)}{2\epsilon_{\text{UV}}\epsilon_{\text{IR}}} \delta(1-z) + \frac{3(1-\epsilon)}{\epsilon_{\text{UV}}} \left( \frac{z}{1-z} + \frac{1-z}{z} + z(1-z) \right) \right.$$  

$$-\frac{6}{z} \left( \ln(1-z) \right)_+ + 6(2-z+z^2) \ln(1-z) \right], \quad (27)$$

$$d_L^{(\text{real})}(z) = \frac{\pi \alpha_s}{8Nm_Q^3} \times \frac{3\alpha_s}{\pi} \frac{1-z}{z}. \quad (28)$$

The infrared divergence cancels after summing the real and virtual correction contributions shown in Eqs. (24) and (27). Employing the $\overline{\text{MS}}$ scheme, $\alpha_s$ and the operator are renormalized as in Ref. [17]. After renormalization, the final answers for $d_T(z)$ and $d_L(z)$ of Braaten and Lee [17] are reproduced:

$$d_T(z, \mu) = \frac{\pi \alpha_s(\mu)}{48m_Q^3} \left\{ \delta(1-z) + \frac{\alpha_s(\mu)}{\pi} \left[ A(\mu) \delta(1-z) + \left( \ln \frac{\mu}{2m_Q} - \frac{1}{2} \right) P_{gg}(z) \right. \right.$$  

$$+6(2-z+z^2) \ln(1-z) - \frac{6}{z} \left( \ln \frac{1-z}{1-z} \right)_+ \right\}, \quad (29)$$

$$d_L(z) = \frac{3\alpha_s}{\pi} \frac{1-z}{z}. \quad (28)$$
where the coefficient $A(\mu)$ is
\[
A(\mu) = \beta_0 \left( \ln \frac{\mu}{2m_Q} + \frac{13}{6} \right) + \frac{2}{3} - \frac{\pi^2}{2} + 8 \ln 2 + 6 \ln^2 2 ,
\] (30)
and $P_{gg}(y)$ is the gluon splitting function:
\[
P_{gg}(z) = 6 \left[ \frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z) + \frac{\beta_0}{6} \delta(1-z) \right].
\] (31)

The transverse term $d_T(z)$ in Eq. (29) still disagrees with that of Ref. [16]. Our final answer for the longitudinal fragmentation function is obtained by setting $\epsilon \to 0$ in Eq. (28):
\[
d_L(z, \mu) = \frac{\alpha_s^2(\mu)}{5m_Q^2} \frac{1-z}{z}.
\] (32)

The longitudinal term, $d_L(z)$ agrees with that of Braaten and Lee [17] as well as that of Beneke and Rothstein [43]. The dependence on the spectroscopic state of the produced quarkonium of this fragmentation function can be found in Ref. [17].

IV. DISCUSSION

We have calculated the next-to-leading order correction to the color-octet $^3S_1$ term in the gluon fragmentation function in a light-cone gauge without using conventional prescriptions for the spurious pole. The gauge-invariant definition of the fragmentation function of Collins and Soper allows us to fix the ambiguities from spurious poles in the light-cone gauge by comparing with the result obtained in the Feynman gauge. Our result agrees with that of Braaten and Lee [17] which disagrees with that of Ref. [16]. The light-cone gauge considerably simplifies the calculation procedure for both the real and the virtual corrections. At least at one-loop level, the spurious pole problem can be resolved. This problem does not appear in the real corrections because they come from tree-level diagrams, but it does appear in the virtual corrections. We reduced the virtual correction in the color-octet $^3S_1$ fragmentation function in the light-cone gauge to a linear combination of scalar integrals. After naive cancellations among the scalar integrals, ignoring the ambiguity from spurious poles, the correction reduces to scalar integrals that are independent of the sign of $i\epsilon$ in the denominator $k \cdot n + i\epsilon$. As a byproduct, the renormalization constants in the light-cone gauge were obtained at one-loop level. Their UV dependencies agree with the previous calculations within the ML prescription. They might be useful for other calculations, such as the next-to-leading order corrections to other fragmentation functions.
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