Heavy Quarks and QCD Matter

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I present recent results on the theory of QCD matter production in high energy heavy ion collisions and on the interactions of heavy quarks in such environment. The centrality and rapidity dependence of hadron production is evaluated in semi-classical approach. The energy loss of heavy quarks in matter is computed. The heavy–to–light meson ratio (e.g., $D/\pi$) at moderate transverse momenta is demonstrated to be both sensitive to the density of color charges in the medium and infrared stable.

1. Foreword

This conference celebrates the contributions made by Helmut Satz to the theory of super–dense matter. Helmut was among the very first who started to think about the subject at the time of its infancy. His work on QCD matter over the years brought the field of statistical QCD to maturity. His strong support was vital in establishing experimental heavy ion programs at CERN and BNL. All of this has been already described at the conference by people who have a first–hand knowledge of the history of the field (in which the previous Bielefeld 1980 meeting was a major milestone). What I would like to add to these accounts is Helmut’s influence on young physicists: he is, and has always been, an inspiration for people entering the field. Many of us, like myself, were brought into the field and encouraged by him. Working with Helmut is both enriching and enjoyable. I wish new generations of physicists will discover this for themselves.

2. Statistical QCD: from small $x$ to high $T$

2.1. Quantum statistics at small $x$

Small $x$ physics is not usually considered to belong to the realm of statistical QCD. Nevertheless, especially at this conference, it is worthwhile to emphasize that the concepts of statistical approach provide a very useful perspective in small $x$ physics as well.

Let us begin by noting that a parton fluctuation with a given Bjorken $x$ and transverse momentum $k_\perp$ inside a hadron with a momentum $P$ has, in the Lab frame, a lifetime given by the uncertainty relation:

$$t \sim \frac{xP}{k_\perp^2}. \quad (1)$$

\textsuperscript{*Invited talk at the International Conference on “Statistical QCD”, Bielefeld, August 26-30, 2001.}
This shows that the partons at larger \( x \), and smaller \( k_\perp \), live much longer than partons at small \( x \) and large \( k_\perp \).

Let us denote by \( \varphi = \{ \psi, \bar{\psi}, A \} \) the set of parton fields with \( x < x_0 \) (with \( x_0 \) setting some arbitrary boundary between “fast” and “slow” fields), and by \( \phi \) the set of the same fields, but with \( x > x_0 \). Suppose that we want to compute an expectation value of some observable \( \mathcal{O} \). In doing so, we have to take account of the fact that partons at larger \( x > x_0 \) are effectively “frozen”; this is done by employing the form familiar from the treatment of statistical systems with random, frozen impurities:

\[
\langle \mathcal{O} \rangle = \int D\phi \rho(\phi) \frac{\int D\varphi \mathcal{O}(\varphi, \phi) \exp(iS(\varphi, \phi)/\hbar)}{\int D\varphi \exp(iS(\varphi, \phi)/\hbar)}.
\]  

(2)

where \( S \) is the action, \( \rho(\phi) \) describes the distribution of large \( x \) partons, and we have explicitly written down the Planck constant \( \hbar \). The meaning of (2) is simple – the “frozen” fields \( \varphi \) are not a dynamical part of the system of the “fast” fields \( \phi \); rather, they act as impurities, or sources. This formulation is the basis for McLerran–Venugopalan model of hadron structure at small \( x \); since glasses are among the physical systems with large relaxation time, one may also call such system a “color glass condensate” (see [1,2] and references therein). Renormalization group equations with respect to the changing scale \( x_0 \) allow then to reconstruct QCD evolution: partons radiated by sources at larger \( x \) themselves become sources for radiation at even smaller \( x \).

At sufficiently small \( x \) and/or large atomic number of the nucleus, the density of partons will become very large and the system will thus cease to be dilute. What will it look like? What kind of dynamics will govern its properties? To address these questions, let us first note that for a system with large number of gluons the action is large, \( S \gg \hbar \). Such systems are appropriately described by using the semi–classical approximation. To go further, we need to establish the dependence of the action on the coupling constant. To do this, let us re-scale the gluon fields in the QCD Lagrangian as follows: \( A_a^\mu \rightarrow \tilde{A}_a^\mu = gA_a^\mu \).

In terms of new fields, \( \tilde{G}_a^\mu_{\nu} = gG_a^\mu_{\nu} = \partial_\mu \tilde{A}_a^\nu - \partial_\nu \tilde{A}_a^\mu + f^{abc} \tilde{A}_a^b \tilde{A}_a^c \), and the dependence of the action on the coupling constant is given by

\[
S \sim \int \frac{1}{g^2} \tilde{G}_a^\mu_{\nu} \tilde{G}_a^\mu_{\nu} \, d^4x.
\]  

(3)

Let us now consider a classical configuration of gluon fields; by definition, \( \tilde{G}_a^\mu_{\nu} \) in such a configuration does not depend on the coupling, and the action is large, \( S \gg \hbar \). The number of quanta in such a configuration is then

\[
N_g \sim \frac{S}{\hbar} \sim \frac{1}{\alpha_s} \rho_4 V_4,
\]  

(4)

where we re-wrote (3) as a product of four–dimensional action density \( \rho_4 \) and the four–dimensional volume \( V_4 \).

The effects of non–linear interactions among the gluons become important when \( \partial_\mu \tilde{A}_a^\mu \sim \tilde{A}_a^2 \) (this condition can be made explicitly gauge invariant if we derive it from the expansion of a correlation function of gauge-invariant gluon operators, e.g., \( \tilde{G}^2 \)). In momentum space, this equality corresponds to

\[
Q_s^2 \sim \tilde{A}_a^2 \sim (\tilde{G}^2)^{1/2} = \sqrt{\rho_4};
\]  

(5)
$Q_s$ is the typical value of the gluon momentum below which the interactions become essentially non-linear.

Consider now a nucleus $A$ boosted to a high momentum. By uncertainty principle, the gluons with transverse momentum $Q_s$ are extended in the longitudinal and proper time directions by $\sim 1/Q_s$; since the transverse area is $\pi R_A^2$, the four-volume is $V_4 \sim \pi R_A^2/Q_s^2$. The resulting four-density from (4) is then
\[ \rho_4 \sim \alpha_s N_g \frac{Q_s^2}{\pi R_A^2} \sim Q_s^4, \] where at the last stage we have used the non-linearity condition (5), $\rho_4 \sim Q_s^4$.

Identifying the number of gluons in the infinite momentum frame with the gluon structure function $N_g \sim xG(x, Q_s^2)$, we arrive at the condition
\[ Q_s^2 \sim \alpha_s \frac{xG_A(x, Q_s^2)}{\pi R_A^2}, \] originally derived in [8–10] as the criterion for “parton saturation” (for a discussion of saturation in terms of the partons in the final state, see [3,4]). This simple derivation [5] illustrates that the physics in the high-density regime can potentially be understood in terms of classical gluon fields.

2.2. Classical QCD and particle production in heavy ion collisions

The energy dependence of saturation scale $Q_s$ is determined by the $x-$ dependence of the gluon structure function (see (7)). In spite of significant uncertainties in determination of the gluon structure functions, the following observation [9] is very important: the HERA data exhibit scaling when plotted as a function of variable
\[ \tau = \frac{Q^2}{Q_s^2} \left( \frac{x}{x_0} \right)^\lambda, \] where $\lambda \approx 0.25 \div 0.3$. In saturation scenario, this scaling translates in the following $x$ dependence of dimensionful scale $Q_s$:
\[ Q_s^2(x) = Q_0^2 \left( \frac{x_0}{x} \right)^\lambda. \] Since the rapidity $y$ and Bjorken variable are related by $\ln 1/x = y$, (9) leads to the dependence of the saturation scale $Q_s^2$ on rapidity:
\[ Q_s^2(s; \pm y) = Q_s^2(s; y = 0) \exp(\pm \lambda y). \] Let us now evaluate the rapidity and centrality dependences of hadron production in heavy ion collisions basing on this picture [12,13]. We need to evaluate the leading tree diagram describing emission of gluons on the classical level, see Fig. 1.

To do this, we introduce the unintegrated gluon distribution $\varphi_A(x, k_t^2)$ which describes the probability to find a gluon with a given $x$ and transverse momentum $k_t$ inside the nucleus $A$. As follows from this definition, the unintegrated distribution is related to the gluon structure function by
\[ xG_A(x, p_t^2) = \int_{k_t^2}^{p_t^2} dk_t^2 \varphi_A(x, k_t^2); \]
when $p_t^2 > Q_s^2$, the unintegrated distribution corresponding to the bremsstrahlung radiation spectrum is

$$\varphi_A(x, k_t^2) \sim \frac{\alpha_s}{\pi} \frac{1}{k_t^2}. \quad (12)$$

In the saturation region, the unintegrated gluon distribution has only logarithmic dependence on the transverse momentum:

$$\varphi_A(x, k_t^2) \sim \frac{S_A}{\alpha_s}; \quad k_t^2 \le Q_s^2, \quad (13)$$

where $S_A$ is the nuclear overlap area, determined by the atomic numbers of the colliding nuclei and by centrality of the collision.

The differential cross section of gluon production in a $AA$ collision can now be written down as [8,14]

$$E \frac{d\sigma}{d^2p} = \frac{4\pi N_c}{N^2_c - 1} \frac{1}{p_t^2} \int dk_t^2 \alpha_s \varphi_A(x_1, k_t^2) \varphi_A(x_2, (p - k)^2_t), \quad (14)$$

where $x_{1,2} = (p_t/\sqrt{s}) \exp(\pm \eta)$, with $\eta$ the (pseudo)rapidity of the produced gluon; the running coupling $\alpha_s$ has to be evaluated at the scale $Q^2 = \max\{k_t^2, (p - k)^2_t\}$. The rapidity density is then evaluated from (14) according to

$$\frac{dN}{d\eta} = \frac{1}{\sigma_{AA}} \int d^2p_t \left( E \frac{d\sigma}{d^2p} \right), \quad (15)$$

where $\sigma_{AA}$ is the inelastic cross section of nucleus–nucleus interaction. Evaluation of

Figure 1. Mueller diagram of classical gluon emission.
Eqs. (14) and (15) leads to the following simple analytical formula [13], which exhibits the scaling properties of hadron multiplicity in nucleus–nucleus collisions:

\[
\frac{dN}{dy} = c \ N_{\text{part}} \left( \frac{s}{s_0} \right)^{\lambda/2} e^{-\lambda|y|} \left[ \ln \left( \frac{Q_s^2(s)}{\Lambda_{QCD}^2} \right) - \lambda|y| \right] \left[ 1 + \lambda|y| \left( 1 - \frac{Q_s}{\sqrt{s}} e^{(1+\lambda/2)|y|} \right)^4 \right],
\]

with \( Q_s^2(s) = Q_s^2(s_0) \ (s/s_0)^{\lambda/2} \). This formula expresses the predictions of high density QCD for the energy, centrality, rapidity, and atomic number dependences of hadron multiplicities in nuclear collisions in terms of a single scaling function. Once the energy–independent constant \( c \sim 1 \) and \( Q_s^2(s_0) \) are determined at some energy \( s_0 \), Eq. (16) contains no free parameters.

The results for the Au–Au collisions at \( \sqrt{s} = 130 \text{ GeV} \) based on Eq. (16) are presented in Figs 2 and 3. One can see that the agreement with the data is quite good. If it persists at higher energies, one may conclude that parton saturation indeed adequately describes the initial conditions created in relativistic heavy ion collisions. Since saturation provides good conditions for parton thermalization [7], we may expect that the final goal of producing the equilibrated QCD matter in the laboratory may be within reach. We thus have to look for the probes which can be used for its diagnostics. One, by now famous, probe of QCD matter is the heavy quarkonium [16]. Another probe is provided by high \( p_t \) jets (see [17]). I am now going to discuss a recent proposal, involving heavy quarks at high \( p_t \) [18].
3. Heavy quark energy loss in QCD matter

Let us begin by recalling the basic features of gluon radiation caused by propagation of a fast parton (quark) through QCD medium. As was pointed out in [19], the accompanying radiation is determined by multiple rescattering of the radiated gluon in the medium. The gluon, during its formation time given again by (1)

\[ t_{\text{form}} \simeq \frac{\omega}{k_\perp^2}, \quad (17) \]

accumulates a typical transverse momentum

\[ k_\perp^2 \simeq \mu^2 \frac{t_{\text{form}}}{\lambda}, \quad (18) \]

with \( \lambda \) the mean free path and \( \mu^2 \) the characteristic momentum transfer squared in a single scattering. This is the random walk pattern with an average number of scatterings given by the ratio \( t_{\text{form}}/\lambda \).

Combining (18) and (17) we obtain

\[ N_{\text{coh}} = \frac{t_{\text{form}}}{\lambda} = \sqrt{\frac{\omega}{\mu^2 \lambda}} \quad (19) \]

describing the number of scattering centers which participate, coherently, in the emission of the gluon with a given energy \( \omega \). For sufficiently large gluon energies, \( \omega > \mu^2 \lambda \), when the coherent length exceeds the mean free path, \( N_{\text{coh}} > 1 \). In this situation the standard Bethe-Heitler energy spectrum per unit length describing independent emission of gluons at each center gets suppressed:

\[ \frac{dW}{d\omega dz} = \left( \frac{dW}{d\omega dz} \right)_\text{BH} \cdot \frac{1}{N_{\text{coh}}} = \frac{\alpha_s C_R}{\pi \omega \lambda} \cdot \sqrt{\frac{\mu^2 \lambda}{\omega}} \cdot \frac{\alpha_s C_R}{\pi \omega} \sqrt{\frac{\hat{q}}{\omega}}. \quad (20) \]

Here \( C_R \) is the “color charge” of the parton projectile (\( C_R = C_F = N_c^2 - 1 \) for the quark case we are interested in).

In (20) we have substituted the characteristic ratio \( \mu^2/\lambda \) by the so-called gluon transport coefficient \[ \hat{q} \equiv \rho \int \frac{d\sigma}{dq^2} q^2 dq^2, \quad (21) \]

which is proportional to the density \( \rho \) of the scattering centers in the medium and describes the typical momentum transfer in the gluon scattering off these centers.

The transport coefficient for cold nuclear matter was expressed in [20] as

\[ \hat{q} \simeq \frac{4\pi^2 \alpha_s N_c}{N_c^2 - 1} \rho \left[ xG(x, Q^2) \right], \quad (22) \]

with \( \rho \simeq 0.16 \text{ fm}^{-3} \) the average nuclear density and \( [xG(x)] \) the gluon density in a nucleon. Taking \( \alpha_s \simeq 0.5 \) and \( [xG(x)] \simeq 1 \) (at \( x < 0.1 \)), yields

\[ \hat{q}_{\text{cold}} \simeq 0.01 \text{ GeV}^3 \simeq 8 \rho. \quad (23) \]
This estimate is an agreement with the result of the analysis of the gluon $p_{\perp}$ broadening from the experimental data on $J/\psi$ transverse momentum distributions \cite{21}, which in the present notation yielded
\[ \hat{q} = (9.4 \pm 0.7) \rho. \] (24)

An estimate \cite{20} for a hot medium based on perturbative treatment of gluon scattering in quark–gluon plasma with $T \sim 250$ MeV resulted in the value of the gluon transport coefficient of about factor twenty larger than (23):
\[ \hat{q}_{\text{hot}} \simeq 0.2 \text{ GeV}^3 \simeq 20 \hat{q}_{\text{cold}}. \] (25)

Multiplying (20) by the length $L$ of the medium traversed\footnote{For the sake of simplicity we assume here that the medium is static and uniform.} we arrive at the following expression for the inclusive energy distribution of gluons radiated by a quark:
\[ \frac{dW}{d\omega} \simeq \frac{\alpha_s C_F}{\pi} \sqrt{\frac{\omega_1}{\omega}}, \quad \omega < \omega_1 \equiv \hat{q} L^2. \] (26)

The fact that the medium induced radiation vanishes for $\omega > \omega_1$ has a simple physical explanation, as according to (19) the formation time of such gluons starts to exceed the length of the medium:
\[ t_{\text{form}} = \lambda \cdot \sqrt{\frac{\omega}{\mu^2 \lambda}} = \sqrt{\frac{\omega}{\hat{q}}} = L \cdot \sqrt{\frac{\omega}{\omega_1}} > L. \]

Another important feature of medium induced radiation is the relation between the transverse momentum and the energy of the emitted gluon. Indeed, from (17) and (18) (see also (21)) we derive
\[ k_{\perp}^2 \simeq \sqrt{\hat{q}} \omega. \] (27)

This means that the angular distribution of gluons with a given energy $\omega$ is concentrated at a characteristic energy- (and medium-) dependent emission angle
\[ \theta \simeq \frac{k_{\perp}}{\omega} \sim \left( \frac{\hat{q}}{\omega^3} \right)^{1/4}. \] (28)

Gluon bremsstrahlung off a heavy quark differs from the case of a massless parton (produced in a process with the same hardness scale) in one respect: gluon radiation is suppressed at angles smaller than the ratio of the quark mass $M$ to its energy $E$. Indeed, the distribution of soft gluons radiated by a heavy quark is given by
\[ dP = \frac{\alpha_s C_F}{\pi} \frac{d\omega}{\omega} \frac{k_{\perp}^2}{(k_{\perp}^2 + \omega^2 \theta_0^2)^2}, \quad \theta_0 \equiv \frac{M}{E}, \] (29)

where the strong coupling constant $\alpha_s$ should be evaluated at the scale determined by the denominator of (29). Equating, in the small-angle approximation, $k_{\perp}$ with $\omega \theta$ we conclude that the formula (29) differs from the standard bremsstrahlung spectrum
\[ dP_0 \simeq \frac{\alpha_s C_F}{\pi} \frac{d\omega}{\omega} \frac{dk_{\perp}^2}{k_{\perp}^2} = \frac{\alpha_s C_F}{\pi} \frac{d\omega}{\omega} \frac{d\theta^2}{\theta^2}. \] (30)
by the factor

\[ dP_{HQ} = dP_0 \cdot \left(1 + \frac{\theta^2_0}{\theta^2} \right)^{-2} \]  

(31)

This effect is known as the “dead cone” phenomenon. Suppression of small-angle radiation has a number of interesting implications, such as perturbative calculability of (and non-perturbative \( \Lambda/M \) corrections to) heavy quark fragmentation functions \([22,23]\), multiplicity and energy spectra of light particles accompanying hard production of a heavy quark \([24]\).

In the present context we should compare the angular distribution of gluons induced by the quark propagation in the medium with the size of the dead cone. To this end, for the sake of a semi-quantitative estimate, we substitute the characteristic angle (28) into the dead cone suppression factor (31) and combine it with the radiation spectrum (20) to arrive at

\[ I(\omega) = \omega \frac{dW}{d\omega} = \frac{\alpha_s C_F}{\pi} \sqrt{\omega_1 \omega} \left(1 + (\ell \omega)^{3/2}\right)^2, \]  

(32)

where

\[ \ell \equiv \hat{q}^{-1/3} \left(\frac{M}{E} \right)^{4/3}. \]  

(33)

To see whether the finite quark mass essentially affects the medium induced gluon yield, we need to estimate the product \( \ell \omega \) for the maximal gluon energy \( \omega \simeq \omega_1 \) to which the original distribution (20) extends:

\[ \ell \omega_1 = \hat{q}^{-1/3} \left(\frac{M}{E} \right)^{4/3} \cdot qL^2 = \left(\frac{E_{HQ}}{E} \right)^{4/3}, \quad E_{HQ} \equiv M\sqrt{\hat{q}L^3}. \]  

(34)

This shows that the quark mass becomes irrelevant when the quark energy exceeds the characteristic value \( E_{HQ} \) which depends on the size of the medium and on its “scattering power” embodied into the value of the transport coefficient.

Which regime is realized in the experiments on heavy quark production in nuclear collisions? Taking \( M = 1.5 \) GeV for charm quarks and using the values (23) and (25) we estimate

\[ E_{HQ} = \sqrt{\hat{q}_{\text{cold}}} L^{3/2} M \simeq 20 \ \text{GeV} \left(\frac{L}{5 \text{ fm}}\right)^{3/2}, \]  

(35)

\[ E_{HQ} = \sqrt{\hat{q}_{\text{hot}}} L^{3/2} M \simeq 92 \ \text{GeV} \left(\frac{L}{5 \text{ fm}}\right)^{3/2}, \]  

(36)

for the cold and hot matter, respectively. We observe that for the transverse momentum (energy) distributions of heavy mesons the inequality \( E \ll E_{HQ} \) always holds in practice, especially for the hot medium. We thus conclude that the pattern of medium induced gluon radiation appears to be qualitatively different for heavy and light quarks in the kinematical region of practical interest.

The issue of in-medium quenching of inclusive particle spectra was recently addressed in \([23]\). The \( p_{\perp} \) spectrum is given by the convolution of the transverse momentum distribution in an elementary hadron–hadron collision, evaluated at a shifted value \( p_{\perp} + \epsilon, \)
with the distribution $D(e)$ in the energy $e$ lost by the quark to the medium-induced gluon radiation:

$$\frac{d\sigma^{\text{med}}}{dp_\perp^2} = \int de \ D(e) \ \frac{d\sigma^{\text{vac}}}{dp_\perp^2}(p_\perp + e) \equiv \frac{d\sigma^{\text{vac}}}{dp_\perp^2}(p_\perp) \cdot Q(p_\perp),$$

(37)

with $Q(p_\perp)$ the medium dependent quenching factor. The two facts, namely that in the essential region $e \ll p_\perp$ and that the vacuum cross section is a steeply falling function, allow one to simplify the calculation of the quenching factor $Q$ by adopting the exponential approximation for the $e$-integral in (37):

$$Q(p_\perp) \simeq \int de \ D(e) \ \exp \left\{ \frac{e}{p_\perp} \cdot \mathcal{L} \right\} \equiv \frac{d}{d \ln p_\perp} \ln \left[ \frac{d\sigma^{\text{vac}}}{dp_\perp^2}(p_\perp) \right].$$

(38)

This integral results in the Mellin moment of the quark distribution,

$$Q(p_\perp) = \tilde{D}(\nu) = \exp \left[ -\nu \int_0^\infty d\omega \ N(\omega) \ e^{-\nu \omega} \right], \quad \nu = \frac{\mathcal{L}}{p_\perp},$$

(39)

where $N(\omega)$ is the integrated gluon multiplicity defined according to (see [25] for details)

$$N(\omega) \equiv \int_\omega^\infty d\omega' \ \frac{dW(\omega')}{d\omega'}.$$

(40)

The use of (39) furnishes our final result:

$$Q_H(p_\perp) \simeq \exp \left[ -\frac{2\alpha_s C_F}{\sqrt{\pi}} \ \hat{q} / p_\perp \ L \ \sqrt{\frac{\mathcal{L}_H}{p_\perp}} + \frac{16\alpha_s C_F}{9\sqrt{3}} \ L \ \left( \frac{\hat{q} \ M^2}{M^2 + p_\perp^2} \right)^{1/3} \right].$$

(41)

The first term in the exponent in (41) represents the quenching of the transverse momentum spectrum which is universal for the light and heavy quarks, (modulo the difference of the $\mathcal{L}$ parameters determined by the $p_\perp$ distributions in the vacuum). The second term is specific for heavy quarks. It has a positive sign, which means that the suppression of the heavy hadron $p_\perp$ distributions is always smaller than that for the light hadrons. This is a straightforward consequence of the fact that the heavy quark mass suppresses gluon radiation. At very high transverse momenta, both terms vanish – this is in accord with the QCD factorization theorem, stating that the effects of the medium should disappear as $p_\perp \to \infty$. How fast this regime is approached depends, however, on the properties of the medium encoded in the value of the transport coefficient $\hat{q}$ and in the medium size $L$.

Constructing the ratio of the quenching functions, we estimate the heavy-to-light enhancement factor as

$$\frac{Q_H(p_\perp)}{Q_L(p_\perp)} \simeq \exp \left[ \frac{16\alpha_s C_F}{9\sqrt{3}} \ L \ \left( \frac{\hat{q} \ M^2}{M^2 + p_\perp^2} \right)^{1/3} \right].$$

(42)

Basing on (42) we find [18] that hot QCD matter leads to a strong, factor of $2 \div 3$, medium–dependent enhancement of the heavy quark yields with respect to the yield of light quarks at moderately large $p_t > M$. Experimentally, this effect should manifest itself as an enhancement of the heavy–to–light ratios such as $D/\pi$. It will also be of interest to study the $B/D$ ratio.
I wish to thank Helmut Satz for inspiring my interest in the topics discussed in this talk. I am grateful to my collaborators Yuri Dokshitzer, Eugene Levin and Marzia Nardi, with whom the results presented above were obtained. I am indebted to Rolf Baier, Jean-Paul Blaizot, Kari Eskola, Miklos Gyulassy, Keijo Kajantie, Larry McLerran, Al Mueller, Berndt Müller, Vesa Ruuskanen, Xin-Nian Wang, and Raju Venugopalan for very useful discussions. This work was supported by the U.S. Department of Energy under Contract No. DE-AC02-98CH10886.

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