Anomalous Spin segregation in a weakly interacting two-component Fermi gas

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We explain the spin segregation seen at Duke in a two-component gas of $^6$Li as a mean-field effect describable via a collisionless Boltzmann equation. As seen in experiments, we find that slight differences in the trapping potentials in the two spin states drive small spin currents. Hartree-Fock type interactions convert these currents into a redistribution of populations in energy space, and consequently a long lived spin texture develops. We explore the interaction strength dependence of these dynamics, finding nontrivial dependence on system parameters and close agreement with experiment.

Spin waves in dilute cold gases such as spin polarized Hydrogen or $^8$Rb are driven by exchange effects and are manifestations of quantum coherence in a non-degenerate gas. Recent experiments at Duke on $^6$Li, with the interactions tuned to be very small, saw an unexpected spin segregation describable as a very long wavelength, low frequency, longitudinal spin wave. Their attempts to explain this behavior in terms of hydrodynamic spin-wave theory failed. Here we use a collisionless Boltzmann equation to explain this anomalous phenomenon. Our approach was motivated by discussions with John Thomas, who has recently explored a simplified version of this theory in a work with Du, Luo, and Clancy. A concurrent study by Piéchon, Fuchs, and Laloë reached similar conclusions. Spin waves in this collisionless Knudsen regime have been studied extensively, both theoretically and experimentally.

In the Duke experiments a cold gas ($T/T_F \sim 4$) of roughly $2 \times 10^5$ $^6$Li atoms, in their lowest hyperfine state (denoted $\sigma=\downarrow$), were prepared in an optical plus magnetic trap with a trapping potential of the form

$$U_\downarrow = \frac{1}{2}m\omega_R^2 r^2 + \frac{1}{2}m\omega_Z^2 Z^2,$$

with $\omega_R = 2 \pi \times 4360$Hz and $\omega_Z = 2 \pi \times 145$Hz. A radio pulse was used to coherently transfer atoms into a superposition of the $\downarrow$ and the next hyperfine level (denoted $\uparrow$). The subsequent dynamics were observed for several different bias magnetic fields, hence several different scattering lengths.

When the scattering length was small and negative they observed that after $\sim 100$ms of evolution, the two components of the gas segregate axially with the $\uparrow$ component moving inward, and the other moving outward. This spin texture persisted on timescales of a few seconds, much longer than the timescale for small oscillations. When the sign of the scattering length was changed, the $\uparrow$ moved outward and the $\downarrow$ moved inward. When the scattering length was tuned to zero, no dynamics was seen.

This behavior was surprising. According to hydrodynamic spin wave theory, the characteristic timescale for any oscillations should be given by the axial oscillator time, roughly two orders of magnitude faster than the observed dynamics. Moreover the only possible mechanism for driving this spin segregation is the very slight difference in the axial trap potential $\frac{d^2}{dt^2}(U_\uparrow - U_\downarrow) \sim 2\pi(4.4 \times 10^{-4})$Hz/µm². It is surprising that such a small difference in the trap leads to such dramatic density redistributions.

An intuitive picture of these dynamics is presented by Du et al. They note that since the time-scale of spin rearrangement is long compared to the oscillation period, local equilibrium is never attained. Instead, each atom’s spin dynamics is controlled by a mean field, averaged over its periodic trajectory. Low energy atoms that spend more time in high density regions experience a greater mean field. The $\downarrow$ atoms see a slightly weaker trapping potential and hence, for a given energy, have trajectories which extend over more space. This results in those atoms seeing a smaller mean field. The net result of the subsequent dynamics is a spin segregation in energy space.

Here we show how this behavior plays out in phase space. Following standard arguments, we derive an effective 1D collisionless Boltzmann equation. Solving this equation numerically, we reproduce the experimental observations.

At the temperatures of interest ($T \sim 27$µK) one only needs to consider s-wave scattering and the Hamiltonian reduces to

$$\hat{H}(t) = \sum_{\sigma=\uparrow,\downarrow} \int d\mathbf{r} \Psi_\sigma^\dagger(\mathbf{r}, t) \left( -\frac{1}{2m} \nabla_r^2 + U_\sigma(r) \right) \Psi_\sigma(\mathbf{r}, t) + g \int d\mathbf{r}_1 \Psi_\uparrow^\dagger(\mathbf{r}_1, t) \Psi_\downarrow(\mathbf{r}_1, t) \Psi_\downarrow(\mathbf{r}_1, t) \Psi_\uparrow(\mathbf{r}_1, t), \quad (1)$$

where the field operators obey the fermionic equal time anti-commutation relations \{\Psi_\sigma^\dagger(\mathbf{r}_1, t), \Psi_\sigma(\mathbf{r}_2, t)\} =
\[ \delta(r_1 - r_2)\delta_{\sigma,\sigma'} \text{, and } g = \frac{\delta_{\sigma,\sigma'}}{m} \text{ with s-wave scattering length } a. \]

Near the magnetic fields of interest \( a(B) = -3.5(B - B_0)a_B/G \). We have set \( h = 1 \) throughout, and we work in the Larmor frame rotating with a frequency equal to that of the \( |\downarrow\rightarrow\uparrow| \) transition for a uniform gas.

Given the small scattering lengths, and low densities in this experiment (\( a \sim 4a_B, n \sim 10^{12} \text{cm}^{-3} \)), the mean collision time \( \tau = 1/(n\sigma v) \approx 10 \text{ s} \) is much longer than the timescale of the experiment, and interactions only enter at the mean field level. From experimental studies of relaxation in a single component gas \([1]\), it appears that the time between background collisions \( \tau_n \), due to an imperfect vacuum, is also on the order of several seconds. For times short compared to \( \tau \) and \( \tau_n \), one can describe the system in terms of a collisionless Boltzmann equation, where the system is described by a Hartree-Fock approximation. The long timescales involved in collisional relaxation also explain why a simple hydrodynamic theory does not capture the physics of the phenomenon.

Following \([16]\) we use the Heisenberg equations for \( \Psi_\sigma(r, t) \) to derive the equations of motion for the spin-dependent Wigner function

\[
\overline{F} = \left( \begin{array}{cc} f_{\uparrow\uparrow}(p, R, t) & f_{\uparrow\downarrow}(p, R, t) \\ f_{\downarrow\uparrow}(p, R, t) & f_{\downarrow\downarrow}(p, R, t) \end{array} \right)
\]

\[ f_{\sigma\sigma'}(p, R, t) = \int dr e^{-ip\cdot r} \langle \Psi_\sigma^\dagger(R - \frac{r}{2}, t) \Psi_{\sigma'}(R + \frac{r}{2}, t) \rangle, \]

which is the quantum analogue of the classical distribution function. Here \( p \) represents the momentum, \( r = r_1 - r_2 \) is the relative coordinate and \( R = r_1 + r_2 \) is the center of mass coordinate. Treating interactions in the Hartree-Fock approximation \([16]\), the resulting equations involve densities and currents such as \( <s_{\sigma\sigma'}(R, t)> = \langle \Psi_\sigma^\dagger(R, t) \Psi_{\sigma'}(R, t) \rangle \) and \( <j_{\sigma\sigma'}(R, t)> = \int \frac{dp}{(2\pi)^3} p f_{\sigma\sigma'}(p, R, t) \).

We define \( s_\sigma = s_\sigma, s_\uparrow = s_\uparrow + s_\downarrow \) and analogously for the spin currents. Here \( s_\pm \) refer the spin raising and lowering operators that are related to the transverse components of the spin \( s_x \) and \( s_y \) in the usual way \( s_+ = s_x + is_y \). Throughout this paper we use upper-case
potential matrix include the direct contribution to for-
square brackets and braces. The diagonal terms of the
tators and anti-commutators are respectively given by
and the effective potentials are

given by

\begin{align*}
\delta \frac{\partial}{\partial t} F + \frac{1}{m} \nabla_R F &= i(\nabla R \overline{\nabla}, F) + \frac{1}{2} \{\nabla_R \overline{\nabla}, \nabla_p F\} \\
\overline{\nabla} &= \begin{pmatrix} U_1^{\text{eff}} & -g s_+ \\ -g s_- & U_1^{\text{eff}} \end{pmatrix} \\
\text{and the effective potentials are } U_1^{\text{eff}}(R, t) &= U_1(R) + g s_1(R, t) \text{ and } U_1^{\text{eff}}(R) = U_1(R) + g s_1(R, t). \text{ Commutators and anti-commutators are respectively given by square brackets and braces. The diagonal terms of the potential matrix include the direct contribution to forward scattering, while the off diagonal components represent the exchange contribution.}
\end{align*}
The amplitude of these oscillations depends on the difference in the trap frequencies seen by the ↑ and ↓ atoms. These oscillations are not captured in the analysis presented in [8].

In previous experiments on bosons [7], the spin dynamics were much faster than such collective modes. This difference can be attributed to the ratios of the mean field interaction energy to the trap frequency \( \lambda = g^{1D} / \omega_z \). In the current experiment \( \lambda \sim 0.2 \) while in [7], \( \lambda \sim 10 \).

Fig. 2(b) shows that both the magnitude and timescale for spin segregation seen in [1] is strongly dependent on the difference in trapping frequencies \( (\delta \omega = \omega_{\uparrow} - \omega_{\downarrow}) \). Had this frequency difference in [1] been an order of magnitude larger (blue/thick curve in Fig. 2(b)), the dynamics would have been much more complicated and much less dramatic.

As previously discussed, an important observation in [7] was that the spin segregation in [1] can be viewed as a segregation in energy space. We illustrate this effect in Fig. 3 by plotting the phase space distributions for \( a = -4.5a_B \) for \( t = 0, 100 \) and \( 200 \) ms respectively. One sees that the phase space distributions are not separately functions of \( Z \) and \( p_Z \), but instead depend on \( \omega_z^2 Z^2 + p_Z^2 / m \).

Finally we note that this spin segregation is very robust. We can illustrate this by exciting a large amplitude spin dipole mode at \( t = 0 \). We find that spin segregation occurs even as the ↑ and ↓ atoms slosh around in the trap, out of phase with one another. As may be expected for a gas in the Knudsen regime, oscillations on timescales much shorter than the interactions do not change the long term dynamics. Nonetheless, it would be interesting to observe this stability experimentally.

**Summary and Conclusions:** Using standard techniques [16], we have derived a collisionless Boltzmann equation which reproduces the anomalous spin waves seen in [1]. This is an exciting regime for spin waves, as the system is far from local equilibrium. Remarkably we find an ergodicity where the phase space distribution function is only a function of energy – but is not a simple exponential.

Our numerical simulations indicate that this spin segregation depends strongly on the difference in the trapping frequencies seen by the two spin species. Moreover, despite being in a nondegenerate regime, substantial quantum coherences are found in this system. We believe that much can be learned from studying how these collisionless dynamics evolve into hydrodynamics as the scattering length is made larger.

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[1] X.Du, L.Luo, B.Clancy, and J.E. Thomas, Phys. Rev. Lett. 101 150401 (2008).
[2] B.R. Johnson, J.S. Denker, N.Bigelow, L.P.Levy, J.H. Freed, and D.M. Lee, Phys. Rev. Lett. 52 1508 (1984).
[3] L.P. Levy and A.E. Ruckenstein, Phys. Rev. Lett. 52, 1512 (1984).
[4] M.Ö.Oktel and L.S. Levitov, Phys. Rev. Lett. 88, 230403 (2002).
[5] J.E.Williams, T. Nikuni, and Charles W. Clark, Phys. Rev. Lett. 88, 230405 (2002).
[6] J.N.Fuchs, D.M.Gangardt, F.Laloë, Phys. Rev. Lett. 88, 230405 (2002).
[7] H.J. Lewandowski, D.M. Harber, D.L. Whitaker, and E.A. Cornell, Phys. Rev. Lett. 88 070403 (2002).
[8] X.Du, Y.Zhang, J.Petricka, and J.E.Thomas, www.arxiv.org/cond-mat/0901.3702v1.
[9] F.Pieżchon, J.N.Fuchs and F.Laloë, www.arxiv.org/cond-mat/0901.4008v1.
[10] N.P. Bigelow, J.H. Freed and D.M. Lee, Phys Rev. Lett., 63 1609-1612 (1989).
[11] V.P. Silin, Sov. Phys. JETP 6, 945 (1958).
[12] A.J. Legett, J.Phys C 4 448 (1970).
[13] C. Lhuillier and F. Laloë, J.Phys (Paris) 43 197 (1982).
[14] C. Lhuillier and F. Laloë, J.Phys (Paris) 43 225 (1982).
[15] J. W. Jeon and W. J. Mullin, J. Phys. France 49 (1988).
[16] Quantum Statistical Mechanics, L. P. Kadanoff, and G. Baym, W. A. Benjamin, Inc. 1962.
[17] Numerical Recipes Third Edition: The Art of Scientific Computing W.H. Press, S.A. Teukolsky, W.T. Vetterling, and B.P.Flannery, Cambridge University Press, 2007.
[18] Bose-Einstein Condensation in Dilute Gases C.J.Pethick and H.Smith, Cambridge University Press, 2002.