Abstract

According to Mach’s principle inertia has its reason in the presence of all masses in the universe. Despite there is a lot of sympathy for this plausible idea, only a few quantitative frameworks have been proposed to test it. In this paper a tentative theory is given which is based on Mach’s criticism on Newton’s rotating bucket. Taking this criticism seriously, one is led to the hypothesis that the rotation of our galaxy is the reason for gravitation. Concretely, a functional dependence of the gravitational constant on the size, mass and angular momentum of the milky way is proposed that leads to a spatial, but not to a temporal variation of $G$. Since Newton’s inverse-square law is modified, flat rotation curves of galaxies can be explained that usually need the postulate of dark matter. While the consequences for stellar evolution are discussed briefly, a couple of further observational coincidences are noted and possible experimental tests are proposed.

1 Introduction

I start summarizing briefly some experimental and observational facts in gravitational physics which are not fully understood yet: dark matter, dark energy, the pioneer anomalous acceleration and the discrepant $G$ measurements. In section 2, Mach’s principle and in particular the rotating bucket filled with water, is discussed in detail. The reader will find there a step-by-step motivation using heuristic arguments and thought experiments that led to the central hypothesis which is summarized at the end of the section. In section 3, the current observational evidence and all further possible tests of the theory are outlined. A discussion of open theoretical problems and an outlook is found in section 4.

1.1 Dark matter

Doppler-shift measurements allow to determine the velocity of gas clouds orbiting around spiral galaxies. These velocities $v$ were expected to show a Keplerian behaviour $v^2 = GM/r$, $r$ being the distance to the galactic center, i.e. do decay with $r^{-1/2}$ outside the range of visible matter. Contrarily to that, a large number of galaxies show flat rotation curves (Mathewson et al. 1992; Persic and Salucci 1995), i.e. constant velocity $v$ far out the visible radius. This region being called halo, the standard explanation postulates a large amount of non visible (dark) matter inthere. However, there is not only disagreement whether dark matter is predominantly cold (baryonic) or hot, but no satisfactory observational evidence for the postulated objects or particles in laboratory physics. Even if such an evidence will occur some day, further surprising observational facts, for example the Tully-Fisher relation or the recently discovered $M(BH)$-$\sigma$ relation (Ferrarese and Merrit.
the Hubble constant was that the Hubble constant suffered from a huge uncertainty for a long time. While most of the methods yielded a value around $80 \text{kms}^{-1}/\text{Mpc}$, the analysis of high-redshift supernovae Ia suggested a value near $50 \text{kms}^{-1}/\text{Mpc}$. Since the increasing precision of the different methods left no doubt, the most reasonable conclusion from the datasets was that the Hubble constant increased with time (The actual precise value is $71.4 \text{kms}^{-1}/\text{Mpc}$, Bennet 2003), i.e. the expansion of the universe is accelerated (Perlmutter et al. 1997; Riess et al. 1998; Perlmutter et. al. 1997; 1998; 1999).

To explain this data, the standard cosmological model assumes a density with repulsive instead of attractive properties called dark energy (DE). According to the latest WAMP measurements (Bennet 2003) its relative amount is 0.73, yielding with dark and visible matter (0.23 and 0.04) altogether $\Omega \approx 1$.

While this argument is numerically correct, one must raise the question if the above concept of defining $\Omega$, according to which kinetic energy transforms into Newtonian gravitational energy, is tenable. Independently of $\Omega$ being smaller or greater than 1, one should expect a decrease of $H$ with time, which is excluded by observation. Given the fact that dark and visible matter are attractive and dark energy is repulsive, encompassing both fractions in $\Omega$ seems to be an ill-defined addition of different quantities (in agreement, Starobinsky 2003).

Instead of being happy with two forms of matter (DM, DE) to which no counterpart in the laboratories has been detected yet, cosmological data should raise doubts on a simple law on energy conservation that uses the Newtonian potential energy.

### 1.3 The Pioneer anomalous acceleration

The two spacecrafts Pioneer 10 and 11 that left the solar system\(^1\), show an unmodeled, approximately constant acceleration $a_p$ of about $8.7 \times 10^{-10} \text{ms}^{-2}$ towards the sun (Anderson et. al. 2001). Despite much effort, no systematic reason has been found yet. There is also a diurnal and annual residual acceleration not yet explained. While the authors still favour an undetected systematic error as an explanation for the anomalous acceleration, they admit that this would require some unlikely coincidences. As in the case of MOND, the numerical coincidence of $a_p$ with $c^2/R_U$ has launched a couple of speculations (Anderson et. al. 2001, p. 43 ff.). To avoid the enormous difficulties of filtering out $a_p$ from noisy data, further space missions are currently proposed (Nieto and Turyshve 2003).

### 1.4 Discrepant $G$ measurements

A couple of years ago, discrepant measurements of Newton’s constant $G$, which led to a raise in the CODATA uncertainty attracted attention (Uzan 2002).

\(^1\)The data available is in the range of 20-50 AU.
Still recently, the most precise values of Gundlach and Merkowitz (2000) and Quinn et al. (2001), which both used sophisticated versions of the torsion balance method, are discrepant. Some measurements are obtained by using superconducting gravimeters (Achilli et al. 1991). Interestingly, the accuracy of these instruments has been pushed to about $10^{-10} m/s^2$ - the same order of magnitude as $a_0$.

In general, some attention is given to the short-range of measurements of Newton’s law (Adelberger et al. 2001). On the other hand, few knowledge on $G$ can be obtained at the large scale. Even if a time variation of $G$ seems to be ruled out (see the excellent review on the change of physical constants, Uzan 2002, p. 25ff.), there is little observational evidence that constrains a pure spatial variation. While the atomic spectra of astrophysical data would immediately detect even slight variations of the ‘atomic’ constants like $c$, $\alpha$ and $\bar{h}$, it quite difficult to detect slight spatial variations of $G$ on a galactic scale.

I have listed - with a decreasing experimental evidence from section 1.1 to 1.4 - some observational problems which are quite well-known. Gravitational physics has however some unsatisfactory aspects from the theoretical point of view I shall address in the next section.

2 A new aspect of Mach’s principle: galaxies as rotating buckets?

2.1 Arbitrary elements in Newton’s theory

While Einstein claimed to be inspired by the idea of Mach, it has not been included into general relativity (Einstein 1917; Bondi 1952). From the Schwarzschild metric of a spherical mass distribution it is obvious that GR does not ‘care about’ distant masses, since it would take an identical form if the rest of the universe was empty.

Though not explicitly stated by him, Mach’s principle is is nowadays known as follows: The reason for inertia is that a mass is accelerated with respect to all other masses in the universe. The strength of gravitation can therefore be related to the mass distribution of the universe. A recent proposal using this paradigm is Barbour’s (2002) scale invariant theory, an overview on other Machian attempts can be found in Barbour and H. Pfister (eds.) (1995).

Mach criticized in particular Newton’s concept of absolute space. In his famous example of the rotating bucket filled with water\(^2\), Newton deduced the existence of an absolute, nonrotating space from the observation of the curved surface the water forms. Mach criticized this ‘non observable’ concept of absolute space as follows, suggesting that the water was rotating with respect to masses at large distance:

‘No one is competent to say how the experiment would turn out if the sides of the vessel increased in thickness and mass till they were ultimately several... [miles]... thick.’

Particularly clear is the formulation of the same problem given by Sciama (1953): ‘Using Mach’s principle we can predict that the angular velocity of the Earth, as deduced from a local dynamical experiment (such as the motion of a Foucault pendulum), will be the same as that deduced kinematically from the apparent motion of the fixed stars.’

Since in Newton’s theory there is no causal connection between these facts, Sciama considers this choice of inertial frames as one of two arbitrary elements in Newton’s theory, the other one being the value of the gravitational constant $G$.

In summary, one must conclude that either an important theoretical concept is still missing or the determination of inertial frames by the fixed stars and Foucault pendulum is just a meaningless coincidence.

In the next paragraph, I will outline how a particular proposal for a quantification of Mach’s principle can be related to the observation of dark matter. Then, I will discuss a new aspect of the rotating bucket.

2.2 A first approach to Mach’s Principle

When following the ideas of Mach, one raises the question how the masses of the universe could influence the gravitational interaction. The most general possibility is a functional dependence of Newton’s constant $G$:

\(^2\)for a very didactical presentation, see Will (1986).
\[ G = G(m_i, \vec{r}_i) \]  

(1)

Since the unit \( \frac{\text{m}^3}{\text{kg} \cdot \text{s}^2} \) seems to contain \( c^2 \), this leads us to consider terms of (1) with the unit \( \frac{\text{m}}{\text{s}^2} \), as the constant \( \tau = \frac{c^2}{8\pi G} \) used in general relativity. Since \( \sum_i \frac{m_i}{r_i} \) is obviously not reasonable, the next apparent guess is

\[ \frac{c^2}{G} = \sum_i \frac{m_i}{r_i}, \]  

(2)

Of course, other forms of (1) could still reflect Mach’s principle. One of the most interesting theories in this direction has been proposed by Barbour (2002, sec. 7). The proposal (2) has first been developed by Sciama (1953) in the context of a modified gravitational potential,

\[ \varphi = G \sum_i \frac{m_i}{r_i}, \]  

(3)

from which he deduced\(^3\) \( \varphi = -c^2 \). An application of this idea to the dynamics of topological defects will be discussed by Unzicker (2003). There is however another promising possibility, namely to insert (2) directly into Newton’s law. The potential instead can then be brought to the form

\[ \varphi = -c^2 \log \sum_i \frac{m_i}{r_i}, \]  

(4)

from which follows Newton’s law

\[-\nabla \varphi = -c^2 \sum_i \frac{m_i \vec{r}_i}{r_i^2} := -G(m_i, r_i) \sum_i \frac{m_i \vec{r}_i}{r_i^2}. \]  

(5)

Since the distant masses give the biggest contribution to the sum, \( G \) does practically not change on small scales and creates the illusion of an \( 1/r^2 \) law. The form (4) of the gravitational potential is however particularly interesting, since logarithmic potentials shows on the large scale a \( 1/r \)-decay of the gravitational field that is suitable for describing flat rotation curves of galaxies. Indeed, if we consider a spherical mass distribution with homogeneous density \( \rho \),

\[ \sum_i \frac{m_i}{r_i} = \int V \frac{\rho dV}{|\vec{x} - \vec{r}(V)|} = M \]  

(6)

holds for every point \( x \) outside the sphere, \( r \) being its distance to the center; this is well-known from potential theory. Thus outside the sphere (5) reduces to \( \frac{c^2 M}{r^2} = \frac{c^2}{r} \), yielding a \( 1/r \)-decay of the force.

The however big problem is that using (5), the flat rotation curves would appear outside a mass distribution of the size of the whole universe and not at the scale of a galaxy. Furthermore, due to the expansion of the universe, (2) would predict a relative increase of \( G \) which is of the order of \( dt/t_H \), the Hubble time, which is definitely ruled out by observations (Uzan 2002). Contrarily, the size of galaxies remains pretty constant at least for the present epoch. This leads to the hypothesis to apply the structure of (2) to a galactic scale. Indeed, the expression

\[ \frac{v^2}{G} = \sum_{\text{galaxy}} \frac{m_k}{r_i}, \]  

(7)

could yield the correct order of magnitude if \( v \) is a velocity characteristic for the galaxy. There are basically two options: \( v \) could be the Hubble velocity measured with the dipole anisotropy of the CMB, but rotation curves do apparently not depend on that.

The other option is that \( v \) is of the order of \( v_{\text{max}} \), the maximum rotational velocity of the galaxy. Even if (7) could in principle predict flat rotation curves, it cannot be the correct expression yet, since the varying distance earth-sun due to the elliptical orbit would cause a huge annual signal in \( G \).

In a certain sense the dependence on such a velocity seems even trivial, because \( v_{\text{max}} \) is the usual measure of the mass of the galaxy by equating the gravitational and centripetal force. This equality may be however of deep nature since it is a rotational velocity as \( v_{\text{max}} \) that acquires a surprising property in the rotating bucket thought experiment, once we try seriously to derive quantitative consequences from it. This will be investigated in the next section.

### 2.3 The bucket-galaxy analogy

In his famous criticism on Newton’s rotating bucket filled with water, Mach’s implicit conjecture was that the water surface even in a rotating bucket would remain flat, given that the ‘walls’ approach

\(^3\)Sciama 1953, eqn. 6, with a sign change.
the size of the universe or, if we refer to the preceding paragraph, to the size of a galaxy. If we suppose that the huge bucket can still be perceived as rotating from a test mass at the outside (distant galaxy or quasar), Mach’s statement has a dramatic implication.

Given that outside the bucket it appears as a rotating coordinate system, what force should act as centripetal force that keeps the water surface flat? If inertial and rotating frames are equivalent, by what transformation can we obtain equations of motion? One possible quantification of Mach’s principle is that the compensating centripetal force needed here is gravitation. In this case the need of considering rotating frames as inertial ones generates gravitation as a consequence of that transformation. I will try to develop this hypothesis with a slightly modified version of Newton’s bucket.

2.4 Universe consisting of two point masses.

Let’s analyze a gedanken (‘thought’) experiment with two masses \( m_1 \) and \( m_2 \) in a beyond empty universe, and let’s further assume that they rotate around their center of mass (COM).\(^4\) According to Newtonian mechanics, the centripetal force \( F_Z \) must be equal to the gravitational force \( F_G \):

\[
\frac{m_1 m_2}{m_1 + m_2} \frac{v^2}{r} = G \frac{m_1 m_2}{r^2},
\]

(8)

\( r \) being the distance between the two masses.\(^5\)

From a Machian point of view, it makes no sense to speak about a rotation, because there is no absolute space and the two masses define the inertial coordinate frame. Thus there are two possible solutions:\(^6\)

- a) since there is no measurable rotation, the two masses start moving towards each other due to gravitational attraction.
- b) since the situation is indistinguishable from the rotation, the outcome should be the same: no change in the relative distance \( r \). This means: when there is no rotation, there is no gravitation either.

Even if the two masses cannot measure it, a rotation could not be excluded in principle, a case in which a) offers no solution. No stable orbits in the two-body problem would be possible.

For that, solution b) seems to be much more logical, since even in a Machian context the classical mechanics of rotating systems is valid and need the concept of a centripetal force. b) suggests however that \( F_Z \) and \( F_G \) are identical. The rotation in case b) would just take the role of a (non observable) gauge field.

2.5 Non-measurable tangential velocities

The deeper reason for this somewhat surprising interpretation is that it is impossible to measure tangential velocities instantaneously. While radial velocities are usually measured by the redshift of atomic spectra, no similar objective method exists for tangential velocities. What can be measured is the change in position relative to other masses, but as long as the distance does not change, this is indistinguishable from a rotation of the coordinate system of the observer. Thus in the above case there is no way to measure tangential velocities, and no way to measure whatever rotation.

2.6 The influence of distant masses

We continue now the gedanken experiment by placing a third mass \( M \) at a constant distance \( R \) from the COM of \( m_1 \) and \( m_2 \) (see sketch fig. 1). To maintain the analogy with the universe (\( M \) as distant galaxy or quasar), we shall assume that \( R \gg r \) and \( M \gg m \), but in a way that both \( M/R \gg m/r \) and \( M/R^2 \ll m/r^2 \) holds,\(^7\) even if only the very first

\(^4\)Let’s assume that time and length measurements are possible. There is an interesting idea of of Barbour (2000) that relates the perception of time to the change configurations. According to that idea, time would not even pass in this thought experiment.

\(^5\)The reduced mass appears at the l.h.s.

\(^6\)A similar situation has been analyzed by Chubykalo (1995).

\(^7\)This implicates also negligible tidal forces.
condition is essential. The appearance on $M$ has a couple of consequences how $m_1$ and $m_2$ ‘perceive the universe’:

It makes sense now to speak about a rotation of $m_1$ and $m_2$, when their sight lines crosses the ‘quasar’ $M$.

While $m_1$ and $m_2$ alone had a ‘gauge freedom’ choosing any rotating coordinate system up to a angular velocity corresponding to $\omega_{\text{max}} = c/r$, the constraint is now $c/R$, which limits their possibility to consider themselves as nonrotating. That means, if $v$ exceeds $cR$, they cannot ‘gauge away’ their rotation any more by assuming $M$ rotating.

When rotating, $m_1$ and $m_2$ may determine their mass ratio by the excentricity of their COM, i.e. when the minimum (maximum) distance from $m_1$ to $M$ is different from that one of $m_2$. Without $M$, this was impossible.

Taking the option b) of the previous section, $m_1$ and $m_2$ need a centripetal force if they want to keep $r$ constant, a necessity that is induced by $M$.

The central hypothesis of this proposal is that gravitation is generated by this necessity and perceived as a $1/r^2$ attraction law that depends on the product of the attracting masses.

Gravitation is however not an illusion (when talking about two masses we still could see it like that), because three masses prohibit the description in a global inertial system.

Since it is not illusory, gravitation acts with radial symmetry also in the third dimension, while it was ‘primarily invented’ to act towards a rotation axis. I will come back to this point in section 3.8.

If we equate the ‘induced’ centripetal force with the Newtonian\textsuperscript{8} force, the same condition as (8) holds, but the gravitational force cannot be ‘gauged away’ any more:

$$G(m_1 + m_2) = v^2r,$$

which is apparently similar to eqn. (7).

### 2.7 Generalizing the three-masses-thought experiment

We are seeking now a general formula that contains (9) as a special case and yields the functional dependence of $G$. There are three problems to be addressed: how to generalize, $m$, $r$, and $v$.

![Figure 2: If we place additional masses $m_3$, $m_4$ which rotate around their COM, all masses $m_1-m_4$ contribute to the gravitational interaction.](image)

**Masses.** We observe that with Newtonian interaction, the motion of $m_1$ and $m_2$ is not quite affected by the presence of $m_3$ and $m_4$, while $m_3$ and $m_4$ feel well the presence of $m_1$ and $m_2$. Therefore, in a rotating system, the sum should be taken over the masses inside a given radius only. This limiting radius must enter the seeked formula, thus one cannot uphold eqn. (7).

**Velocity.** Given that the inner and the outer pair of masses need not to orbit with the same velocity, a reasonable ‘medium rotating velocity’ has to be determined. One must also consider that there should be a cancelation of clockwise and counterclockwise rotating masses; otherwise $G$ would immediately depend of the temperature of a star. This excludes the possibility of introducing a term like $\sum_i v_i^2$. The natural measure for the rotation of a system of mass points is the angular momentum $\vec{L}$. Thus, a reasonable way to define a medium tangential velocity $\bar{v}$
is
\[ \vec{v}^2 = \frac{\vec{L}^2}{IM} = \frac{(\sum_{i} m_i \vec{r}_i \times \vec{v}_i)^2}{\sum_{i} m_i r_i^2 \sum_{i} m_i}, \tag{10} \]
when \( I \) is the moment of inertia, \( r_i \) is the distance vector from the rotation axis, and the sum is taken over all the masses of the rotating system for which \( r_i < r \) holds. Thus, \( r \) being the distance from the rotation axis,
\[ G(r, r_i, m_i, v_i) = \frac{r \vec{L}^2}{IM^2} \tag{11} \]
is the proposal I favour and shall discuss in the next section.

I should address here however some unsatisfactory theoretical aspects of (11). Technically, one may replace \( \sum_i m_i r_i^2 \sum_i m_i \) in the denominator of (10) by \( (\sum_i m_i r_i)^2 \), which yields the correct units as well.

More unpleasantly, the ‘sum over all masses of the rotating system’ is not really a well-defined quantity. It is the angular momentum as ingredient of the sum that should tell if a system is rotating, not an a-priori selection of masses. The reason for this poor logic is that in the above thought experiment the distant mass \( M \) (‘Quasar’) plays a decisive role without entering eqn. (8), respectively (9). Paradoxically, the physics of the three-masses situation is neither changed by the amount of \( M \) nor by its precise distance \( R \), as long as \( R \gg r \) holds. How should \( M \) and \( R \) enter any formula if there values are not important? \( M \) just ‘communicates’ the frame of reference without playing any further role. It seems however that one inevitably runs into this paradox once we take Mach’s criticism of Newton’s bucket seriously.\(^9\) To remedy this, eqn. (11) took \( M \) out ‘by hand’, and should be for that regarded only as a first tentative to arrive at quantitative predictions.

### 2.8 Rotating subsystems

While fig. 2 is a primitive model of the universe if we consider \( m_1 - m_4 \) as our galaxy and \( M \) as the rest, our galaxy contains subsystems like the solar system that rotate with respect to the (already rotating) galaxy at a distance \( r \) from the rotation axis.

Following the arguments given above, it is impossible for the solar system to consider itself as non-rotating, not even with respect to the milky way. Consequently, this additional rotation with respect to the galaxy should create an extra force. While its structure must be again Newtonian, one may express this new effect by a correcting term that analogous to (11) but containing the angular momentum, the size and the mass of the solar system. Similar arguments hold for subsequent systems like planet-moon etc.

The definite quantitative form of the correction has to be still derived. The inner systems are compelled to consider themselves rotating by the outer ones. If we return to the argument in section 2.4, there is however a ‘gauge freedom’ left that allows us to assume the boundary of the universe to move with a tangential velocity \( c \). One may also suggest that there is a connection of the corresponding centripetal acceleration \( c^2/R_U \) that seems to be detected at the boundaries of the respective subsystems (\( a_0 \) at galaxies, \( a_p \) at the solar system).

### 2.9 Conclusion

I summarize here the assumptions and collect the formulas for quantitative predictions.

The milky way and galaxies in general are regarded as ‘Newton’s buckets’ in which the Machian hypothesis holds. Since there must be a centripetal force, gravitation is generated by this necessity. Thus one may construct a functional dependence of \( G \) in a way that \( F_Z = F_G \) have the same value for an orbital motion in the milky way. \( G \) on a given distance \( R \) from the axis of rotation depends therefore on the angular momentum and mass distribution of the galaxy:

\[ G(r, r_i, m_i, v_i) = \frac{r \vec{L}^2}{IM^2} = \frac{\vec{v}^2 r}{M} = \frac{r (\sum_i m_i \vec{r}_i \times \vec{v}_i)^2}{\sum_i m_i r_i^2 (\sum_i m_i)^2}, \tag{12} \]
while the sums are taken over the masses \( m_i \) with velocities \( v_i \) and distances from the rotation axis \( r_i \), whereby \( r_i < r \) must hold. \( r \) is again the distance from the rotation axis of the position where \( G \) is measured. \( \vec{v} \) represents a characteristic velocity of the galaxy.

\(^9\)However, even summation or integration over space coordinates we are used to, could turn out to be ill-defined, since space without matter is meaningless in a Machian sense.
3 Observational and experimental tests

3.1 Reproduction of Newton’s law

Eqn. (12) contains $\vec{L}$, $M$, and $r$ as variables. If we assume in a very crude approximation the milky way as a rigidly rotating flat disk with constant density, $M \sim \vec{v}^2$ holds, thus $G$ increases linearly with the distance from the center. If $r_0$ is the distance of the solar system from the center of the milky way, the variation is however too small to be perceived at the scales of celestial mechanics, and even smaller for $1/r^2$-gravity tests on earth.

3.2 Rotation curves and dark matter

Since for gas clouds in the halo of a galaxy very large distances $r$ occur in Newton’s law $a = G\frac{\vec{M} \vec{R}}{r^2}$, where $M$ is the total mass of the galaxy, the functional dependence (12) causes the acceleration $a$ following the law

$$a = \frac{\vec{v}^2 r M}{M r^2} = \frac{\vec{v}^2}{r},$$

where $\vec{v}$ is fixed by (10) and should have the same order of magnitude as $v_{\text{max}}$. The $1/r$ decay of the acceleration obviously predicts a rotation curve which is flat outside the (visible) mass distribution. So far, no nonvisible mass distributions have to be postulated. The evidence for a relatively higher amount of dark matter in low-luminosity-galaxies is addressed below (3.3). Since this proposal does not arbitrarily modify Newton’s potential just in the galactic plane, it seems not to be in conflict with the further observational constraints listed by Aguirre et al. (2001).

3.3 Variation of $G$ and its consequences for stellar evolution

When talking about a variation of $G$, almost all proposals in the literature deal with a time evolution of $G$. Against this popular idea originated by Dirac (1938) an overwhelming observational evidence has been collected (Uzan 2002). In agreement with this facts, this proposal predicts no time evolution, since the quantities $\vec{L}$, $M$ and $R$ of galaxies are constants at least in the present period.

However, it predicts both a huge spatial variation of $G$ inside a galaxy and different values of $G$ for each galaxy, e.g. for each rotating mass distribution. This raises the question how the apparent uniformity of star populations matches this prediction. In particular, one has to investigate how the mass-luminosity-relation and correlated quantities like stellar lifetime is affected by a different $G$\(^{10}\).

Teller (1948), in criticizing Dirac’s idea, derived a $G^7$-dependence of the luminosity $L$. While this is approximately correct for a star with given mass under temporal variations of $G$, the argument does not apply to the question what kind of stars would be formed under conditions with a spatially different $G$\(^{11}\).

For main sequence stars I consider here for simplicity, $R \sim M$ (Shu 1982)\(^{12}\) is the condition for stars to exist. This reflects the fact that the inner temperature $T_i$ is determined by parameters of the nuclear reactions and $T_i \sim G\frac{M}{R}$ due to the virial theorem\(^{13}\).

Thus $G\frac{M}{R} = \text{const.}$ should approximately hold for any main sequence. Consequently, stars that we observe in the inner parts of the milky way, where according to (12) $G$ is lower, should have either a smaller radius or a greater mass than in standard models. Since size rather than mass should determine the instability region in the HR-diagram, one would expect the latter possibility and a mass-to-light-ratio $\Gamma$ proportional to $G^{-1}$. Even if there is still a huge uncertainty regarding $\Gamma$, this prediction seems to hit the right direction.

As a consequence, large high-luminosity galaxies with a lower medium value of $G$ would show a higher mass-to-light ratio. This is in agreement with the observational fact that low-luminosity galaxies seem to be relatively more dominated by dark matter.

3.4 The Tully-Fisher-relation

Both conventional dark-matter models and alternative post-newtonian potentials do not explain the luminosity dependence $L \sim v^4$ discovered by Tully and Fisher.

\(^{10}\)There could be even an influence on cepheid-based distances measurements.

\(^{11}\)A similar error would be to to confuse the issues of the mass-luminosity-relation for a single star with the mass-to-light ratio of an ensemble of stars.

\(^{12}\)On the upper main sequence, $R \sim M^{0.6}$.

\(^{13}\)To be precise the preconditions for the virial theorem have to be reanalyzed for the present proposal, see Landau and Lifshitz 1973.
Since the dependence $G \sim R$ should increase $G$ with $v$, (12) influences again the mass-to-light ratio. Therefore, an increase of $L$ with a higher power of $v$ is predicted as in conventional models. (12) is therefore approximately in agreement with observation.

### 3.5 Globular clusters

Due to the difficulties outlined in section 2.7, unfortunately it is not clear how to apply eqn. (12) to globular clusters which are located in the halo. Given that a spatial variation of $G$ is expected for the disc, it is at least possible that $G$ may have different values at the cluster positions. Since clusters are nonrotating objects, there should be a discrepancy towards lower $G$ if any. Though a detailed theoretical study is needed, a considerable change of the HR-diagram would be the consequence, as indeed it is observed. Since a lower $G$ raises the Chandrasekhar limit

$$M_C \approx (\hbar c)^2 G^{-2} m_p^2,$$

(14)
($m_p$ being the proton mass and $\hbar$ Plack’s constant), much less supernovae can occur, which is in agreement with the low metallicity of population II stars.

The apparent stability and absence of gravitational contraction is currently explained with gravothermal oscillations, which seems to be a complicated and non obvious mechanism.

### 3.6 The Pioneer anomaly

I outlined in section 2.8 the possible influence of rotating subsystems in a galaxy, like the solar system. Qualitatively, eqn. (12), calculated with the parameters of the solar system, predicts a slight increase of $G$ with distance from the center of rotation (the sun) which results as an extra acceleration towards the sun. The large contributions of Jupiter and Saturn to the angular momentum of the solar system suggest that an extra acceleration due to an increased $G$ could be ‘switched on’ at that distance, as indeed the form of the diagram in Anderson et. al. (2001), p. 19, shows. A preliminary quantitative analysis however, predicts an effect which is too big by several orders of magnitude. This could indicate that (12) has to be replaced by a similar formula.

### 3.7 $G$ and absolute $g$ measurements

The next level of a rotating subsystem to which a possible correction has to be applied is the earth-moon-system; in a more general sense, even free fall experiments on earth are subject to these possible corrections. Given the considerable discrepancies in the $G$ measurements it would be desirable if parameters like exact time and position on earth would be recorded.

Given that the proposed model predicts a spatial variation of $G$ that depends on the distance from the galactic center, an annual signal of $G$ due to the earth orbit is expected. Since the relative amplitude is $1AU/R_\odot \approx 6 \cdot 10^{-10}$, an amplitude of 6 nGal in $g$ with maximum in Summer should be observed. Absolute $g$-measurements of the earth’s gravity field can be done by superconducting gravimeters (SG) that reach a precision of about 0.1 $nm/s^2$. Unfortunately the earth has lots of noisy signals in its field, starting from the difficulties in modeling nutation effects up to atmospheric and hydrological disturbances. Currently, there is an unmod- eled signal/uncertainty of about 20 nGal (Kroner 2003). However, the worldwide linking of SG data in combination with the excellent data of the beginning GRACE mission should make such a signal detectable in the future.

### 3.8 The accelerated universe and dark energy

In the vicinity of rotating galaxy, a $1/r$-decay of the gravitational field is predicted, which means a stronger field up to a scale of galaxy halos. However, since the largest rotating structures are galaxies, the angular momenta of different galaxies will cancel out on larger scales. Therefore, this proposal predicts no gravitational interaction at all for intergalactic, cosmological scales. Distant galaxies and quasars are only ‘used’ as frame of reference (as the distant mass $M$ in the thought experiment) that however influences the appearance of gravity in the described Machian picture.

Given that a couple of observations may have to be reinterpreted following this Machian idea, there is at least no obvious contradiction with the observed high-redshift supernovae. Contrarily, a constant non decelerated Hubble expansion could be understood in this picture without postulating a
new form of matter called dark energy.

Rather than postulating a repulsive gravity that has not shown up elsewhere in physics, one may interpret the high redshift supernova data in terms of a violation of energy conservation on cosmological scales. It is well-known that energy conservation can be deduced from a radially symmetric force field. Inverting that theorem, one deduces that if energy conservation is violated, the force field cannot obey radial symmetry any more. This absence of radial symmetry on intergalactic scales is actually what appears in a Machian interpretation of the rotating bucket and what is included in eqn. (12).

Regarding the observed self-similar fractal structure of galaxy distributions one should keep in mind that phase transitions in thermodynamics that show similar structures are satisfiable modeled with a next-neighbour interaction and therefore do not require a long-range interaction; in this aspect, the present proposal is in agreement with observation.

3.9 Galaxy evolution and radial symmetry

At the heart of this proposal stands the relation between gravitational and centripetal forces. While gravitation obeys radial symmetry, rotational forces are directed towards an axis and act, so to speak, in two dimensions. If rotation indeed generates three-dimensional gravity, a homogeneous distribution of matter in the early universe, for which we have evidence from CMB, should be contracted along one dimension, because centrifugal forces may compensate only two.

Observing the shape of disc galaxies one must raise the question why most the universe is dominated by structures that violate radial symmetry so obviously, if Newton’s radial law with constant \( G \) is really the only interaction that matters. Despite much progress in detail, we do not really understand why galaxies have the shape and size they have. Is it a coincidence that the surface of all galaxies matches approximately \( 14 \) the surface of a sphere with radius \( R_u \) of the universe?

It is well-known that galaxies, clusters and superclusters show a hierarchical, selfsimilar structure (Mandelbrot 1983). Recently, the fractal dimension of that structure has been measured to \( 2 \pm 0.2 \) (Roscoe 2002), promoting the conjecture that the universe has a fractal dimension of two. Why should a radially symmetric interaction produce such a particular form? Once we have evidence that energy conservation is violated on large scales, there is no need to keep radial symmetry as a dogma.

4 Theoretical Problems

I collect briefly some unconventional implications of this proposal that have to be investigated with more stringent methods than the heuristic arguments given in section 2.

The modification of the gravitational constant \( G \) leads to a force that depends on the positions \( \vec{r} \) of other masses. Moreover, the spatial dependence does refer to the distance from the center of rotation, not the distance to a given point \( \vec{R} \).

Energy conservation. The spatially dependent \( G \) in (12) obviously disagrees with Newton’s 3rd law on large scales. As a consequence, energy conservation in its conventional form is violated on large scales (see section 3.8). In view of the high-redshift supernovae, this theoretical challenge that has to be taken however anyway.

Since the Newtonian potential energy seems to do not a good job on intergalactic scales, a different Lagrangian has to be derived that allows to deduce equations of motion from a variational principle. Eqn. (12) suggests that velocities and vector valued quantities may enter that Lagrangian.

Relation to the universe. Since this proposal discarded all versions of Mach’s principle that included all masses in the universe\(^{15}\) and focussed on the effects on a galactic level, one may raise the question what meaning remains for the ‘fundamental acceleration’ \( c^2/R_u \) that actually appears. This has to be understood investigating galaxy-formation processes, as well as the coincidence \( \frac{c^2 R_u}{M_u} \approx \frac{c^2 R_G}{M_G} \).

\(^{14}\)Assuming milky way as one of \( 10^{11} \) standard galaxies with \( r = 10 \text{kpc} \) and \( R = 1.3 \cdot 10^{26} \text{m} \) matches up to a factor 2.

\(^{15}\)The rest of the universe had only the meaning of a distant frame of reference like the mass \( M \) in fig. 1.
The compatibility with general relativity remains entirely to be clarified. Only one point is satisfying yet: The Machian interpretation of the Newton’s bucket seems to be in agreement with the equivalence principle, since masses do react on inertia and gravitation in the same way, and the two interactions are in a sense equated by definition.

Outlook. It is hard to be happy with the paradox situation that distant masses create the necessity of gravity without their parameters $M$ and $R$ showing up in the formalism. Given this logical difficulties, It may be justified to discard Mach’s comment on the rotating bucket. If one concludes to do that, he should believe however in absolute space.

I consider it as a nice feature of the proposal outlined here that it does not introduce further arbitrary parameters into Newton’s theory but tries to get rid of the arbitrary parameter $G$. Deviating from the law of energy conservation - even at large scales - seems however a too expensive toll. Thus much more theoretical understanding has to be achieved before one can claim the validity of such an approach. Given the enormous riddles astronomers are observing with increasing precision, this proposal may however be worth to be tested in detail.

Acknowledgement. Hannes Hoff prevented me from giving up this idea at an early stage, and contributed important arguments. This work wouldn’t have been possible either without the uncountable inspiring discussions with Karl Fabian. Comments of Ettore Minguzzi are acknowledged.

References

Achilli, V., P. Baldi, S. Focardi, P. Gaspevini, F. Palmonavi, and R. Sabadini (1991). The brasimone experiment: a measurement of the gravitational constant $G$ in the 10-100 m range of distance. *Chaiers du CEGS*, 241-246.

Adelberger et.al., E. G. (2001). Sub-millimeter tests of the gravitational inverse square law. *arXiv: astro-ph/9810302*.

Aguirre, A., C. P. Burges, A. Friedland, and D. Nolte (2001). Astrophysical constraints on modifying gravity at large distances. *arXiv: hep-ph/0105083*.

Anderson et. al. (2001). Study of the anomalous acceleration of Pioneer 10 and 11. *arXiv: gr-qc/0104064*.

Barbour, J. (2000). *The End of Time*. Oxford University Press.

Barbour, J. (2002). Scale-invariant gravity: particle dynamics. *arXiv: gr-qc/0211021*.

Barbour, J. and H. Pfister (eds.) (1995). *Mach’s Principle*. Boston: Birkhäuser.

Bennet, C. (2003). http://www.gsfc.nasa.gov/scienceques2002/20030307.htm.

Bondi, H. (1952). *Cosmology*. Cambridge: University Press.

Bosma, A. (1998). Dark matter in disc galaxies. *arXiv: astro-ph/9812013*.

Chubykalo, A. E. (1995). Principle of Mach, equivalence principle and concepts of inertial mass. *arXiv: gr-qc/9510055*.

Dirac, P. A. M. (1938). A new basis for cosmology. *Proc. Roy. Soc. London A* 165, 199–208.

Einstein, A. (1917). *Sitzungsberichte der Preussischen Akademie der Wissenschaften, phys.- math. Klasse*, 142.

Ferrarese, L. and D. Merrit (2000). A fundamental relation between supermassive black holes and their host galaxies. *arXiv: astro-ph/0005053*.

Gundlach, J. and M. Merkowitz (2000). Measurement of Newton’s constant using a torsion balance with angular acceleration feedback. *arXiv: gr-qc/0006043*.

Kroner, C. (2003). personal communication, Geophysical Observatory Jena.

Landau, L. D. and E. M. Lifshitz (1973). *Theoretical Physics - Classical Field Theory*, Volume I. Moscow: Nauka.

Mandelbrot, B. (1983). *The fractal geometry of nature* (3rd ed.). New York: Freeman.

Mathewson, D. S., V. L. Ford, and M. Buchhorn (1992). *ApJS 81*, 413.

Milgrom, M. (1998). The modified dynamics - a status review. *arXiv: astro-ph/9810302*. 
Nieto, M. M. and S. G. Turyshev (2003). Finding the origin of the Pioneer anomaly. *arXiv: gr-qc/030817.*

Perlmutter, S., G. Aldering, M. Della Valle, and others (1997). Discovery of a supernova at half the age of the universe and its cosmological implications. *arXiv: astro-ph/9712212.*

Perlmutter et. al. (1997). *Astrophysical Journal* 483, 565.

Perlmutter et. al. (1998). *Nature* 391, 51.

Perlmutter et. al. (1999). *Astrophysical Journal* 517, 565.

Persic, M. and P. Salucci (1995). Rotation curves of 967 spiral galaxies. *arXiv: astro-ph/9502091.*

Quinn, T., C. C. Speake, S. Richman, R. S. Davis, and A. Picard (2001). A new determination of G using two methods. *Physical Review Letters* 87(11), 111101.

Riess, A. G., A. V. Filippenko, P. Callis, and others (1998). Observational evidence from supernovae for an accelerating universe and a cosmological constant. *arXiv: astro-ph/9805201.*

Roscoe, D. F. (2002). *General Relativity and Gravitation* 34, 577–602.

Sciama, D. W. (1953). On the origin of inertia. *Monthly Notices of the Royal Astronomical Society* 113, 34–42.

Scott, e. a. (2001). Cosmological difficulties with modified Newtonian dynamics (or: La fin du MOND ?). *arXiv: astro-ph/0104435.*

Shu, F. H. (1982). *The physical universe.* Mill Valley, California: University Science Books.

Starobinsky, A. (2003). personal communication, MarcelGrossmann Meeting X.

Teller, E. (1948). On the variation of physical constants. *Physical Review* 73(801).

Unzicker, A. (2003). *arXiv: to appear.*

Uzan, J.-P. (2002). The fundamental constants and their variaion: Observational status and theoretical motivations. *arXiv: hep-ph/0205340.*

Will, C. (1986). *Was Einstein right ?* New York: Basic Books.