Cyclotron Resonance in Strongly Magnetized Plasmas and Gamma Ray Burst

S. Son

18 Caleb Lane, Princeton, NJ 08540

Sung Joon Moon

28 Benjamin Rush Lane, Princeton, NJ 08540

(Dated: November 6, 2012)

A plausible scenario for the gamma ray and the hard x-ray burst in a strongly magnetized plasma, based on the collective plasma maser instability, is proposed. The physical parameters with which this scenario becomes relevant are estimated. The attractive feature of this scenario over the conventional cyclotron radiation theory is discussed.

PACS numbers: 98.70.Rz, 98.70.Qy, 52.35.Hr

INTRODUCTION

According to a recent study, the thermal electron gyro-motion in a strongly magnetized plasma could lead to instabilities of electromagnetic (E&M) waves [1]. This instability, so-called the maser instability, has significant implications on strongly magnetized plasmas [2–7]. The goal of this paper is to estimate its relevance to the strongly magnetized astrophysical plasmas [5, 6]. The gamma ray burst, gamma ray or hard x-ray burst in the strongly magnetized plasma could lead to the maser instability, so-called the maser instability, has significant implications on strongly magnetized plasmas [1]. This instability regime is ubiquitous in strongly magnetized plasmas [1]. To summarize briefly, the kinetic motion of a relativistic electron is solved in the presence of an E&M mode to the second order, and the ensemble average of the electron kinetic loss (gain) is taken over the electron distribution. Equating this energy loss to the E&M wave growth, the E&M growth (decay) rate is obtained. The growth rate for a general propagation direction \( \theta \), which is defined to be the angle between the magnetic field and the photon wave-vector, is

\[
\Gamma = + \frac{1}{c^2} \left[ \frac{1}{2} \frac{\omega_{pe}^2}{\omega^2} \left( \frac{\Omega_1}{\gamma} - \frac{\beta}{\gamma} \right) \right]_{f=0} - \left[ \frac{1}{2} \frac{\omega_{pe}^2}{\omega^2} \left( \frac{\Omega_2}{\gamma} - \frac{\beta}{\gamma} \right) \right]_{f=0},
\]

where \( \omega(\omega) \) is the wave vector (frequency) of the E&M mode, \( \beta = (\beta_1, \beta_\parallel) = v/c, \beta_1^2 = \beta^2 + \beta^2_\perp, \omega_{pe} = \sqrt{4\pi ne^2/m_e} \) is the plasma frequency, \( \Omega(\beta) = (1 - \beta^2)^{-1/2} \), \( f \) is the electron distribution with the normalization of \( \int f d^3\beta = 1 \), \( \langle A \rangle_{S=0} = \int \delta(S)Ad^3\beta \) is the integration in the velocity space with the constraint \( S = 0 \), and \( \Omega_1 \) and \( \Omega_2 \) are obtained below. Defining \( \zeta \) to be the ratio of the wave energy density to the wave energy density in vacuum, \( \zeta = E_w/(E_\gamma^2/4\pi) \), it is assumed in our analysis that \( \zeta > 0 \), which is the case for most of E&M waves in dense plasmas. For a generic angle \( \theta \), there could be two independent modes: TE and TM modes. The wave vector is given as \( k = k \cos \theta \hat{\xi} + \sin \theta \hat{\gamma} \). Let us define the TE (TM) mode as \( E_1 = E_1 \cos(k \cdot r - \omega t) \hat{x} \) and \( B_1 = E_1(k \omega/c) \cos(k \omega/c) \cos(\theta \hat{y} - \sin \theta \hat{z}) \) (\( E_1 = E_1 \cos(k \cdot r - \omega t) \cos \theta \hat{x} - \sin \theta \hat{z} \) and \( B_1 = E_1(k \omega/c) \cos(k \cdot r - \omega t) \hat{y} \)). \( \Omega_1 \) and \( \Omega_2 \) for the TE mode are

\[
\Omega_1(\theta) = \frac{c^2k^2}{\omega} \cos \theta \left( \cos \theta - \frac{ck}{\omega} \beta_\parallel \right) - \frac{\omega_{ce}}{\gamma} \beta_\perp^2 \frac{\beta_\perp}{2}, \\
\Omega_2(\theta) = c k \left( 1 - \beta_\perp^2 - \frac{ck}{\omega} \beta_\parallel \cos \theta \right) \cdot \gamma.
\]
For the TM mode, they are given as
\[ \Omega_1(\theta) = \frac{c^2 k^2}{\omega} \cos \theta \left( 1 - \frac{ck}{\omega} \beta_\| \cos \theta \right) - \frac{\omega_{ce}}{\gamma} \cos \theta \]
\[ \Omega_2(\theta) = ck \left( 1 - \frac{\beta_\|^2}{2} \right) \cos \theta - \frac{ck}{\omega} \beta_\| . \quad (3) \]
The non-relativistic limit of Eq. (1) is
\[ \Gamma = \frac{\pi}{2} \frac{\omega_{pe}^2}{\zeta c^2 k^2} \left( \frac{\beta_\|^2}{2} \Omega_1(\theta) \frac{df}{dc} \right) - \frac{f(v)}{c} \Omega_2(\theta) , \quad (4) \]
where \( \{ \) is the ensemble average with \( v_z = v_r = (\omega - \omega_{ce})/k \) or \( \langle A \rangle = \int A \delta(v - v_r) d^3 v \). When \( \theta = 0 \), the Eqs. (1) and (4) are reduced to the previously obtained results [1].

For more rigorous treatment when \( \theta \neq 0 \), there are infinite expansions of the Bessel function \( J_n(k \cos \theta r_g) \), where \( r_g \) is the electron gyroradius. However, the unstable wave identified in our analysis has the condition \( k \cos \theta r_g < 1 \) so that \( J_1(k \cos \theta r_g) \approx 1 \). For this reason, Eq. (1) is derived assuming \( J_1 = 1 \).

**CASE WHERE \( \theta = 0 \) AND \( \omega \neq ck \)**

Consider the case where the E&M field propagates parallel with the magnetic field (\( \theta = 0 \)). There is no instability if \( \omega = ck \), because \( \Omega_1 = 0 \). For simplicity, we refer to the first (second) term on the right-hand side of Eqs. (1) and (4) as the gyro-lasing (gyro-damping) term. For \( \omega > ck \), the semi-classical instability growth rate is shown to be [1]
\[ \Gamma = \frac{\pi}{2} \frac{\omega_{pe}^2}{\zeta c^2 k^2} \left( \frac{v_r}{c} \Omega_1 - (1 - \beta_\|^2) - \frac{ck}{\omega} \beta_r \right) f_\| (v_r) , \quad (5) \]
where \( \beta_\| = v_\| / c, \beta_r = v_r / c = (\omega - \omega_{ce})/c, v_\| \) is the electron thermal velocity, and \( f_\| (v_r) = \int f \delta(v_z - v_r) d^3 v \). The instability criterion is \( \Gamma > 0 \). The maximum possible instability, ignoring the gyro-damping term, is estimated to be
\[ \Gamma_{max} \approx 0.19 \times \frac{\omega_{pe}^2}{(ck)^2} \Omega_1 , \quad (6) \]
where it is assumed that \( \zeta \approx 1 \).

The Maxwellian distribution for fully relativistic electrons is \( f(\beta) \approx \chi^3 \exp(\gamma \lambda) \), where \( \lambda = m_{e} c^2/T_e \). This is so-called the Maxwell-Juttner’s distribution [11]. When \( \lambda < 1 \), the distribution is peaked at \( \gamma = 3/\lambda \) with the width of \( \delta \beta \approx (\lambda/3)^2 \). We consider the case when \( \omega > ck > \omega_{ce} \). Assuming \( \beta \approx 1 \), the condition \( \Gamma > 0 \) can be re-casted as
\[ \int_{\delta \beta = 0} d^3 \beta \left( \frac{\nabla_\| S \cdot \nabla_\| f}{\nabla S(\beta)^2} \right) \left( \frac{\omega_{ce}}{\gamma ck} + 1 \right) \left( 1 - \frac{ck}{\omega} \right) \\
> \int_{\delta \beta = 0} d^3 \beta f(\beta_{res}) \left( 1 - \frac{ck}{\omega} \beta_{res} \right) \quad . \quad (7) \]
One necessary condition for the existence of the resonance is given as \( \omega/ck < \beta_\max + \omega_{ce}/\gamma_{\max} = \sqrt{1 + \omega_{ce}^2/c^2 k^2} \), where \( \gamma_{\max} = (1 - \beta_\max^2)^{-1/2} \) and \( \beta_\max = 1/\sqrt{1 + (\omega_{ce}/ck)^2} \). If the electron distribution has the relatively high slope near the resonance region (\( \gamma_{\max} \approx 3 \Delta r_e/m_r c^2 \)), Eq. (7) is possible since \( |\nabla S| \ll 1 \) near the resonance. The growth rate without the gyro-damping term can be estimated, using \( \nabla_\beta f \approx \gamma^2(3 - \lambda \gamma) f_\beta \) and \( \beta \approx 1 \), as
\[ \Gamma_{max} \approx \frac{\omega_{pe}^2}{(ck)^2} \left( \frac{d S/d \beta}{\nabla_\beta S|^2} \right) \int d^3 \beta \left[ \gamma_1 (3 - \lambda) f \frac{d S/d \beta}{\nabla_\beta S|^2} \right] \quad , \quad (8) \]
where \( d S/d \beta = (\beta \cdot \nabla S)/|\beta| \).

**CASE WHERE \( \theta \neq 0 \) AND \( \omega = ck \)**

The resonance condition for the TM mode is \( S = \beta_\| \cos \theta + \omega_{ce}/ck \gamma - 1 = 0 \). The gyro-damping term vanishes at the resonance because \( \Omega_1 = (c^2 k^2/\omega) S \cos \theta = 0 \) from Eq. (3). However, the gyro-damping term does not vanish (\( \Omega_2 = S \cos \theta (1 - \beta_\|^2/2 - \beta_r) \)). If \( \Omega_2 < 0 \) at \( S = 0 \), this term acts as an amplifying term instead of a damping term. Consider the semi-classical case. From the resonance condition \( \beta_r \cos \theta = 1 - \omega_{ce}/ck \), \( \Omega_2 \) is given by
\[ \Omega_2 = -ck \left[ \beta_r - \frac{1 - \beta_\|^2}{\beta_r} \right] . \quad (9) \]
The instability condition (\( \Omega_2 > 0 \)) is given as \( \beta_\|^2 > (1 - \omega_{ce}/ck)(1 - \beta_r^2/2) \). If \( \beta_r \approx 0 \), the condition is reduced to \( \beta_r > \sqrt{1 - \omega_{ce}/ck}, \) or \( \cos \theta = (1 - \omega_{ce}/ck)/\beta_r < \sqrt{1 - \omega_{ce}/ck} \). For a given Maxwellian distribution \( f_M \) with the normalization \( \int f d \beta_z = 1 \), the growth rate is
\[ \Gamma = \frac{\pi}{2} \frac{\omega_{pe}^2}{\zeta c^2 k^2} \left( \beta_r - \frac{1 - \beta_\|^2}{\beta_r} \right) f_M(\beta_r) . \quad (10) \]
The growth rate has the maximum similar to Eq. (9) when \( \beta_\|^2 / \beta_r < 2 \beta_\|^2 ) \). For given \( \omega_{ce} \) and \( ck \), the growth rate as a function of \( \theta \) has the maximum when \( \beta_\|^2 \cos \theta = \approx 1 - \omega_{ce}/ck \).

For the TE mode, let us consider semi-classical electrons first. With the resonance condition \( \beta_r \cos \theta = 1 - \omega_{ce}/ck \), we obtain \( \Omega_1 = ck (\cos^2 \theta - 1) \) and \( \Omega_2 = ck(1 - \beta_\|^2/2 - \beta_r \cos \theta) \) from Eq. (2). The instability criterion (\( \Gamma > 0 \)) is
\[ \beta_r > \frac{1 - \beta_\|^2}{1 - \cos^2 \theta + \cos \theta} . \quad (11) \]
The maximum growth rate, ignoring the gyro-damping term, is given similar to Eq. (6).

For the TE mode of the fully relativistic electrons, we consider only when \( \theta = \pi/2 \). The resonance surface is
given as \( S = 1 - \omega_c/\gamma \omega \) so that \( \omega = \omega_c/\gamma \) and \( |\nabla_\beta S| = (\omega_c/\gamma)\gamma/\beta = \gamma^2/\beta \). Note also that \( \Omega_1 = -\omega \) and \( \Omega_2 = (1 - \beta^2/2)(ck) \) from Eq. (2). A similar analysis to Eq. (7) can be used, and the instability criterion \( (\Gamma > 1) \) becomes

\[
\int_{S=0} d^3 \beta \left[ \Omega_1 \nabla_\beta S \cdot \nabla_\beta = \frac{1}{|\nabla S(\beta)|^2} \right] > \int_{S=0} d^3 \beta \left[ \frac{(1 - \beta^2/2)\omega}{M} \right].
\]

(12)

Note that \( \nabla_\beta S \cdot \nabla_\beta < 0 \) for a positive gyro-lasing term since \( \Omega_1 < 0 \). Assuming \( \beta \approx 1 \), Eq. (12) and is simplified to

\[
\frac{|\nabla_\beta S|}{f} > \gamma^2.
\]

(13)

In the case of the Maxwellian plasma, \( |\nabla f_M|/f_M = (3\gamma^2 - \lambda \gamma^3)\beta \), where \( \lambda = m_e c^2/T_e \). If \( \lambda \gamma < 2 \), Eq. (13) and \( \nabla_\beta S \cdot \nabla_\beta < 0 \) are satisfied. Assuming \( \beta \approx 1 \), it is estimated that

\[
\Gamma \approx \frac{\pi}{2} \omega_{pe}^2 f \frac{1}{c^2 \epsilon^2} \int_{S=0} \frac{d\beta}{\gamma} d^3 x.
\]

(14)

Note that \( df/d\beta \) can be as large as \( \gamma^4 \), and the maximum growth rate is given as

\[
\Gamma_{\max} \approx \frac{\pi}{2} \frac{1}{\zeta} \omega_{pe}^2 \omega \int_{S=0} \gamma d^3 \beta.
\]

(15)

The comparison between the case when \( \theta = 0 \) and when \( \theta \neq 0 \) is in order. For the case when \( \theta = 0 \), \( \Omega_2 \) vanishes if \( \omega = ck \), thus the instability can occur only when \( \omega \neq ck \); this requires a very high electron density for gamma rays or hard x-rays. On the other hands, when \( \theta \neq 0 \), an explosive instability is possible even when \( \omega = ck \gg \omega_{pe} \). Therefore, the case when \( \theta \neq 0 \) is more probable than the case when \( \theta = 0 \).

**THE INSTABILITY THEORY VERSUS THE INCOHERENT CYCLOTRON RADIATION THEORY**

The theoretical consideration in our study has the following distinctive features, compared to the conventional cyclotron radiation theory:

(i) The fastest growing E&M mode would be the one parallel (or perpendicular) to the magnetic field. The intensity of a collective E&M mode is proportional to \( \exp(\gamma t \cos \theta) \) when \( \theta = 0 \) and to \( \exp(\gamma t \cos (\theta - \pi/2)) \) when \( \theta = \pi/2 \), so that the peak angle is narrow when \( \gamma t \gg 1 \). Therefore, in a certain direction, more intense photons would be observed than predicted from the conventional incoherent cyclotron radiation, whose intensity is proportional to \( \cos^2 \theta \). The actual energy power requirement for the gamma ray burst might be less than that of the conventional theory [4].

(ii) As the electrons emit the photons via the instability, the temperature becomes further anisotropic, rendering the E&M wave perpendicular to the magnetic field unstable to the Weibel instability. In turn, the Weibel instability mitigates the temperature anisotropy, emitting low-frequency photons in the perpendicular direction.
(iii) Our analysis suggests that the gamma ray burst could be originated from a more compact and dense object than conventionally believed. In a highly dense plasma considered here, an incoherent photon has a short mean free path due to the gyro-damping as well as the Thomson scattering, while the collective photons could overcome these degradations through the gyro-lasing. In other words, the compact and dense plasma could be optically thin for the coherent photons, while optically thick for the incoherent photons.

To be more specific, let us consider the Thomson scattering. The decay rate is given as \( \Gamma_T = n_e \sigma_{TC} \equiv n_e r_e^2 c \), where \( r_e = e^2/m_e c^2 \) is the classical electron radius. From Eq. (10), the ratio \( \Gamma_{\max}/\Gamma_T \) is given as \( \Gamma_{\max}/\Gamma_T \equiv (1/k r_e) \). Then, the Thomson scattering is weaker than the gyro-lasing if the photon wavelength is longer than the classical electron radius, whose photon energy roughly corresponds to 100 MeV, which is the case in our scenario. This suggests that the photon can get amplified, overcoming the Thomson scattering when the gyro-lasing from the instability is strong. However, as the instability gets weaker, the Thomson scattering becomes eventually the dominant decay mechanism of the photons, where the photon mean free path is \( l \equiv (1/n_e r_e^2) \). For example, for \( n_e = 10^{18}/cc, l \equiv 10^6 \) cm. For a given object, the collective photons could be excited inside the object and sustained via the gyro-lasing. But, if there is no instability on the surface of the object, the photons will decay due to the Thomson scattering within rather short mean free path; in order for the photons to escape from the object, it is necessary that the gyro-lasing from the instability exists on the surface of the object. The delicate interplay between the Thomson scattering and the gyro-lasing in the process of the collective photons escaping from an object is beyond the scope of this paper.

(iv) The growth rate of the instability is sensitive to the slope of the distribution function at the resonance. Consider a shock region where two plasmas of different drifts violently encounter. It is plausible that the parallel and perpendicular electron temperatures are comparable but the distribution could have two sharp humps in the parallel direction, at which case the instability could be strong.

(v) Finally, our theory provides an escape scenario of photons from dense plasmas in a changing magnetic field. A typical example would be the decreasing magnetic field (e.g., the foot point of the solar corona). Consider the TE mode when a photon propagates with \( \theta = \pi/2 \) from \( r = 0 \) and the magnetic field decreasing with the increasing \( r \) (or \( B = B(r) \hat{z} \) with \( B(r_1) < B(r_2) \) if \( r_1 > r_2 \)). From the resonance condition \( \omega = \omega_{ce}/\gamma \), the relativistic factor \( \gamma \) of a resonant electron should decrease with the decreasing \( \omega_{ce} \). Assuming the electron temperature remains the same along the photon propagation, the gyro-lasing term stays positive if \( \lambda \gamma < 2 \) at \( r = 0 \) (as analyzed in Sec. 4). Then, collective photons are radiated primarily in the direction of (perpendicular to) the magnetic field as photons from the incoherent cyclotron radiation suffer a severe damping and photons not parallel with (or perpendicular to) the magnetic field experience a weaker gyro-lasing term than the photons parallel (or perpendicular) to the magnetic field.

**SUMMARY**

A scenario of the gamma ray burst, based on the recent radiation theory [1], is proposed and examined. The previous analysis [1] is generalized to an arbitrary angle. The estimation shows that the coherent burst of 10 keV to 1 MeV photons is plausible in a relativistic plasma when \( \gamma_0 = 10 \sim 1000 \) and the electron density is higher than some critical value, \( n_e > 10^{18} \sim 10^{26} \text{ cm}^{-3} \). It is shown that a rather compact dense object with less available energy could cause the short gamma ray burst and that the observed gamma rays would be coherent rather than incoherent. In addition, the coherent photons of lower frequency comparable to the plasma frequency might be observed due to the Weibel instability. This is particularly relevant to the case when \( \theta = \pi/2 \) because the low frequency coherent photons (high frequency coherent photons) from the Weibel instability (from the instability studied here) can be observed simultaneously. The above conjecture might be useful in verifying whether the scenario proposed here would account for some of the short gamma ray burst events observed in the satellites [6].

While our estimation is rather focused on the gamma and hard x-rays, a similar mechanism would manifest in generating soft x-rays in the inertial confinement fusion plasma [14]. The electron beam of \( \gamma > 10 \sim 100 \) and the magnetic field of \( 10^8 \text{ gauss} \) can be readily generated in laboratories. Even a magnetic field of \( 10^9 \text{ gauss} \) might be possible [15]. Then, the photon generated from the instability may have energy between 10 eV and 1 keV. Complications would be the electron quantum diffraction effect and the degeneracy [13, 17]. The plausibility study is in progress.

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