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Parametric Optimization of Steel Shell Towers of High-Power Wind Turbines

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Abstract

This paper presents a parametric optimization task of steel conic shell towers of wind-powered generators. The minimum weight of the steel tower has been considered as purpose function when produced capacity of the wind-powered generators has been fixed at the target level. The tower’s height, diameters of the middle surface of the tower’s conic shell at the base level and at a height if the wind turbine installation, thicknesses of the tower’s conic shell, and also the diameter of the wind wheel were considered as design variables. The wind loads applied on steel shell tower have been presented in dependence of design variables and considered as state variables. The optimization problem formulated as non-linear programming task has been solved by improved gradient method. Optimum design decisions of the steel conic shell towers for the wind-powered generators with produced capacity of wide range have been defined. An optimum tower’s height and weight in dependence of produced capacity of wind-powered generators have been plotted.

Keywords: wind-powered generator; steel shell tower; parametric optimization; non-linear programming; gradient method; software.

1. Introduction

Tall towers of wind-powered generators are relatively new types of steel structures widely applied. At the present time several thousand steel towers for wind-powered generators are manufactured every year in the world. The increasing tendency for production volume of these structures is observed [1].

At the beginning of 70th years of the last century in many countries installation of wind-powered generators have been initiated when the first energy crisis erupt. The first researches devoted to the searching of the optimum wind turbine’s parameter have been performed. But most of these researches were concentrated mainly on cost study for wind turbine proper, optimization of turbine’s output capacity and cost minimization of electric energy [2], [3]. The problem of optimization of tall towers for wind-powered generators has been attended slightly. It is easy to explain for the period when low-powered wind turbines were used so long as the tower’s cost was not so big and was almost not perceptible in general construction budget [4].

But during last decade a typical capacity of the wind-powered generators increased single-order [5]; steel towers for wind turbine come up to the height 100 m and more, and tower’s cost reaches 15…20 % from the construction budget for whole wind-powered generator [6].
So, at present time the wind-powered generator Enercon E-126 (with height 141 m and produced capacity 7 MWt) located near the German city Emden (Fig. 1, b) is height and capacity record-holder. In 2011 year Norway has announced plan to construct the largest world’s wind turbine to date: the wind turbine height will be 162,45 m, diameter of the wind wheel will be 144,78 m; the cost of the pilot model will be 67,5 million USA dollars.

![Wind-powered generators with welded steel conic shell towers](image)

Fig. 1. Wind-powered generators with welded steel conic shell towers

Taken into account increasing the ratio between the cost of the steel tower and the construction budget for whole wind-powered generator, formulation an optimization problem for this kind of steel structure is considered as quite actual.

2. Parameters determined operation of the wind-powered generators

A kinetic energy $E$ of the incoming uniform wind flow with mass $m$ and with velocity $v_0$ at certain a distance in front of the wind-powered generator can be expressed, J:

$$E = 0.5mv_0^2.$$  \hspace{1cm} (1)

The value $m = \rho_a v_0 A_w$ in the cross-section in front of the wind turbine is mass rate of air flow with density $\rho_a$ ($\rho_a = 0,125 \times 10^{-3}$ t $\cdot$ s$^2$/m$^4$) passing through the blade-swept area $A_w$ per time unit. Herewith, the power of the air flow $N_w$ incoming on wind-powered generator equals to the complete kinetic energy:

$$N_w = 0.5mv_0^2 = 0,5\rho_a v_0^3 A_w.$$  \hspace{1cm} (2)

The flow energy causes the force $F_x$ and generating capacity of the rotor $W$. Wind-powered engine produces from the wind flow only a part of the wind flow energy, i. e. engine capacity:

$$W = 0.5C_p \rho_a v_0^3 A_w,$$  \hspace{1cm} (3)

where $C_p = W/N_w$ – utilization factor for wind energy.

N. E. Zhukovskyj and A. Betz showed that in ideal case the utilization factor $C_p$ can reach the value $16/27 = 0,593$ only. As newly-incoming air parcel has nowhere to go, so it’s not possible to stop wind flow completely in order to use wind energy with hundred-per-cent efficient. The wind flow can be decelerated only and the maximum possible rate of this deceleration is 59,3%.
If deceleration factor of the wind flow for the ideal turbine (Fig. 2) is defined as:

\[
\tau = \left(\frac{v - v_1}{v_0}\right),
\]

then according to (Betz, 1920) [7]:

\[
W = 0.5\rho_a v^3 A_{w_0} \times 4\pi (1 - \tau)^2 = 2\rho_a v^3 A_{w_0} \tau (1 - \tau)^2,
\]

\[
F_x = 0.5\rho_a v^2 A_{w_0} \times 4\pi (1 - \tau) = 2\rho_a v^2 A_{w_0} \tau (1 - \tau),
\]

then maximization \( W \) with variable \( \tau \) gives \( \tau = 1/3 \), whence follows \( C_p = 16/27 \).

From the (5) and (6) it easy to obtain an expression for pulling force transmitted on tower of the wind-powered generator:

\[
F_x = 1.5W/V.
\]

Modern wind-powered generators are provided by the control systems for the angle of attack of the rotor. This system ensures, so called, the energy characteristic, which define wind engine capacity in dependence of wind velocity \( v_0 \) for non-decelerated wind flow (Fig. 3). Wind-powered engine starts to operate by wind velocity \( v_{\text{min}} \). The nominal value of the produced capacity \( W_{\text{nom}} \) is reached when the velocity of the wind flow equals to the value \( v_{\text{stab}} \), keeping up at this level by the automatic control system up to the wind velocity \( v_{\text{max},1} \). Wind velocity \( v_{\text{max},1} \) is the maximum possible wind velocity, it allows of operating of the wind-powered engine without damage. When exceeding this velocity, wind-powered engine is disabled by means of rotation of blades in feathered position.

\[
\begin{array}{c}
\text{W} \\
\hline
\text{v}_{\text{stab}} & \text{v}_{\text{max},1} & \text{v}_{\text{max},2} \\
\text{W}_{\text{nom}} & & \\
\end{array}
\]

Fig. 3. Produced capacity description

3. Problem formulation

Optimization problem is formulated as searching of the optimum design decision of the tower for wind-powered generator, which is designed as a steel conic shell with diameter varied linearly along shell length. Nearly every modern wind-powered generator has the same structural design decision, see. Fig. 1.
Not only parameters of the tower’s structural decision (diameters and thicknesses of the steel conic shell tower, tower height), but also the wind wheel’s diameter $D_w$ are intended to subject to optimization. The latter and capacity of the wind-powered generator $W$ accordingly are restricted by tower’s height $H$.

Fig. 4. Design variables of the steel conic shell tower

The tower’s structure was divided into the some number $n_s$ parts along tower’s height with orientation on using finite element method for determination stress-and-strain state of the tower’s conic shell. For each tower’s part the constant thickness $t_i$ and constant diameter of the middle surface at lower level $D_i$ were considered as characteristic cross-sectional dimensions (Fig. 4).

3.1. Design variables

The main parameters of the steel conic shell tower were considered as design variables, namely: $H$ – the tower’s height; $D_{\text{min}}$ – minimum diameter of the middle surface of the tower’s conic shell at a height of $H$ (the height of wind wheel above earth’s surface level); $D_{\text{max}}$ – maximum diameter of the middle surface of the tower’s conic shell at the base level; $t_i$ – the thickness of $i^{th}$ part of tower’s conic shell, $i = 1 \ldots n_s$, where $n_s$ – quantity of the tower’s shell parts; $D_w$ – the diameter of the wind wheel.

3.2. State variables

The parameters of the tower’s design decision depended on design variables were considered as state variables, namely:

- $D_i$ – diameter of the middle surface at the lower level of $i^{th}$ part of tower’s conic shell, $i = 1 \ldots n_s$, $D_i$ varies linearly along shell length:
  \[ D_i = D_{\text{max}} - (i-1)(D_{\text{max}} - D_{\text{min}})/n_s. \]  \tag{8}

- $h_s$ – the length of the $i^{th}$ part of tower’s conic shell along the shell axis:
  \[ h_s = H/n_s; \]  \tag{9}

- $z_i$ – the height of the centre of $i^{th}$ part of tower’s conic shell above earth’s surface level, $i = 1 \ldots n_s$:
  \[ z_i = (i - 0.5)h_s. \]  \tag{10}
Besides, a wind velocity \( v_H \) at a height of \( H \) (the height of wind wheel above earth’s surface level) was also considered as state variable. Based on \( v_H \) value a produced capacity of wind turbine \( W \) was determined. At the same time, wind velocity \( v_H \) was calculated in dependence of average annual wind velocity \( v_0 \) at a height of ten meters above earth’s surface level (knowing value for terrain where wind-powered generator will be installed) by the following formula (\( H \) in m):

\[
v_H (H) = v_0 \left( \frac{H}{10} \right)^{0.14} = v_0 e^{0.14 (\ln H - \ln 10)}.
\] (11)

3.3. Wind loads on tower as a state variables

Wind loading on steel conic shell towers of wind-powered generators was determined by the methodology proposed in [8]. This method is accepted in the second edition of the Ukrainian design codes ДБН В.1.2-2:2013 which is under development at the present time [9].

The load-bearing capacity of the steel tower’s shell structure should be verified for two wind load cases. The first wind load case is determined by the maximum possible wind velocity \( v_{H,\text{max},1} \) at a height of \( H \) above earth’s surface level (at the height of installation of the wind wheel) by which wind wheel’s blades of the wind-powered generator are under operate conditions. The corresponding wind velocity \( v_{0,\text{max},1} \) at a height ten meters above earth’s surface level is calculated as (\( H \) in m):

\[
v_{0,\text{max},1} (H) = \frac{v_{H,\text{max},1} (H/10)^{0.14}}{v_{H,\text{max},1} e^{0.14 (\ln H - \ln 10)}}.
\] (12)

The second wind load case is determined by the maximum possible wind velocity \( v_{0,\text{max},2} \) at a height ten meters above earth’s surface level. This wind velocity characterizes the considered terrain where wind-powered generator will be installed.

The load-bearing capacity of the steel tower’s shell structure should be verified for two wind load cases determined by the wind velocities \( v_{0,\text{max},1} \) and \( v_{0,\text{max},2} \) mentioned above so long as it is not known beforehand which of them more critical. The fact of the matter is that under the lower wind velocity \( v_{0,\text{max},1} (v_{0,\text{max},1} < v_{0,\text{max},2}) \) together with concentrated pulling load of the wind turbine \( F_x(v_{0,\text{max},1}) \) apply on steel tower’s structure. At the same time under the higher wind velocity \( v_{0,\text{max},2} \) the distributed wind load \( q_{i,2}(H, z_i, D_i) \) on tower’s structure is greater, but the pulling force of the wind turbine is not applied to the tower’s top due to rotation of the blades in feathered position.

3.3.1. Wind load case caused by the maximum wind velocity \( v_{0,\text{max},1} \)

Distributed wind load on conic shell tower of wind-powered generator is calculated by the following formula:

\[
q(z_i) = \frac{1}{2} \rho_a v_{0,\text{max},1}^2 C_h(z_i) C_{\text{aer}} C_d D_i,
\] (13)

where the wind velocity \( v_{0,\text{max},1} \) in m/s, mass air density \( \rho_a = 0.125 \times 10^{-3} \) t·s\(^2\)/m\(^4\), and aerodynamic factor \( C_{\text{aer}} = 0.8 \). The factor \( C_{\text{in}} \) takes into account the wind pressure increment along tower’s height, \( C_h \) is calculated as:

\[
C_h(z_i) = k_r^2 \ln^2 \left( z_i / z_0 \right),
\] (14)

where \( z_0 \) and \( k_r \) – roughness parameter of earth’s surface and terrain type factor respectively. The dynamic factor \( C_d \) takes into account a fluctuating part of the wind flow, \( C_d \) is determined by the following formula:

\[
C_d(H) = 1 + \xi_d(H) \zeta_d(H),
\] (15)

where \( \xi_d(H) \) – the factor of turbulence intensity at a height \( H \):

\[
\zeta_d(H) = \frac{7}{\ln(H/z_0)}.
\] (16)
\[ \zeta_d(H) \] – the factor of dynamic sensitivity determined by the following formula:

\[ \zeta_d(H) = \sqrt{1 + 33k_s(H)} = \sqrt{1 + 33e^{-1.5(1+\sqrt{H/H_0})}}; \]  

(17)

where \( k_s(H) = e^{-1.5(1+\sqrt{H/H_0})} \) – the factor of range of pulsations, and \( \omega_1 \) – the first natural frequency. At a first approximation, the frequency \( \omega_1 \) can be estimated by the following formula \( \omega_1 = 46/H \).

Finally, the distributed wind load on each \( i^{th} \) part of conic shell tower is defined as:

\[ q_{i,1}(H, z_i, D_i) = \gamma_{fm,w} \frac{D_i^2}{2} v_{0,\text{max},1} C_{aer} D_i k_s^2 \times \ln^2 \left( \frac{z_i}{z_0} \right) \left( 1 + \frac{7\sqrt{1 + 33k_s(H)}}{\ln(H/z_0)} \right), \]

(18)

where \( \gamma_{fm,w} \) – safety factor for wind loading.

In this load case the pulling force of the wind turbine defined based on ideal turbine model proposed by N. E. Zhukovskyj and A. Betz, is calculated by the formula presented below:

\[ F_x(D_w) = \gamma_{fm,w} \frac{\pi D_w^2}{9} \rho_w v_{H,\text{max},1}^3. \]

(19)

3.3.2. Wind load case caused by the maximum wind velocity \( v_{0,\text{max},2} \)

The distributed wind load on each \( i^{th} \) part of conic shell tower is defined as:

\[ q_{i,2}(H, z_i, D_i) = \gamma_{fm,w} \frac{D_i^2}{2} v_{0,\text{max},2} C_{aer} D_i k_s^2 \times \ln^2 \left( \frac{z_i}{z_0} \right) \left( 1 + \frac{7\sqrt{1 + 33k_s(H)}}{\ln(H/z_0)} \right). \]

(20)

3.4. Purpose function

The tower of the wind-powered generator is an auxiliary structure intended to ensure an efficient operability of the wind turbine. If the optimization problem of tower for wind turbine with target capacity \( W^* \) is considered, then this purpose can be achieved by choosing the corresponded diameter \( D_w \) of the wind wheel and corresponded wind velocity \( v_{0}(H) \), which varies in dependence of tower’s height \( H \). It follows from the expression for produced capacity \( W \) of the wind-powered generator, which is estimated based on ideal turbine model proposed by N. E. Zhukovskyj and A. Betz’s taken into account average annual wind velocity \( v_0 \) as:

\[ W\left( v_H(H), D_w \right) = \frac{16}{27} \frac{\rho_a v_H^3}{2} \frac{\pi D_w^2}{4}. \]

(21)

Naturally, that at the same time it should be aspire to the weight minimization of steel tower. The theoretical weight of the tower’s conic shell can be calculated as:

\[ G(D_i, t_i) = \pi \rho \sum_{i=1}^{n} t_i D_i, \]

(22)

Where \( \rho = 7.8 \text{ t/m}^3 \) – the steel density.

Under such problem formulation an equality of the produced capacity \( W \) of the wind-powered generator to the value of the target capacity \( W^* \) was considered as additional constraint expressed in the form of inequalities:

\[ W^* - \varepsilon_W \leq W\left( v_H(H), D_w \right) \leq W^* + \varepsilon_W, \]

(23)

here \( \varepsilon_W \) – small positive number.
3.5. System of constraints

The load-bearing capacity of the steel shell tower should be verified for two wind load cases mentioned above.

Constraints of the ultimate limit state were formulated as verifications that the actual stresses arisen in design cross-section of \(i\)th part of tower’s conic shell, here \(i = 1 \ldots n_s\), caused by \(k\)th load case combination, should not exceed the critical stresses of compression (by the local stability):

\[
\frac{N_i}{A_i} + \frac{M_{i,k}}{W_i} \leq \sigma_{cr,i},
\]

where \(\sigma_{cr,i}\) – the critical stress for \(i\)th part of the tower’s conic shell; \(N_i\) – the design axial force in \(i\)th part of the tower’s conic shell which is calculated as:

\[
N_i = \gamma_{fm,S}G_w + \gamma_{fm,SW}\pi\rho_s h_s \sum_{j=i}^{n_s} t_j D_j;
\]

\(M_{i,k}\) – the design bending moment for \(i\)th part of the tower’s conic shell subjected to \(k\)th load case, which is determined by the formulas presented below:

- For the first wind load case \((k = 1)\):

\[
\begin{align*}
M_{n_s,1} &= F_x h_s + 0.5q_{n_s,1}h_s^2; \\
M_{i-1,1} &= M_{i,1} + h_s \left( F_x + \sum_{j=n_s}^{i} q_{j,1}h_s \right) + 0.5q_{1,1}h_s^2; \\
&\vdots \\
M_{1,1} &= M_{2,1} + h_s \left( F_x + \sum_{j=n_s}^{1} q_{j,1}h_s \right) + 0.5q_{1,1}h_s^2;
\end{align*}
\]

- For the second wind load case \((k = 2)\):

\[
\begin{align*}
M_{n_s,2} &= 0.5q_{n_s,2}h_s^2; \\
M_{i-1,2} &= M_{i,2} + h_s \sum_{j=n_s}^{i} q_{j,2}h_s + 0.5q_{1,2}h_s^2; \\
&\vdots \\
M_{1,2} &= M_{2,2} + h_s \sum_{j=n_s}^{1} q_{j,2}h_s + 0.5q_{1,2}h_s^2;
\end{align*}
\]

here \(G_w\) – concentrated axial load applied at a height \(H\) of tower caused by self weight of the rotor, the value of \(G_w\) is estimated using empirical dependence \((G_w\) in t, \(D_w\) in m):

\[
G_w = 0.013D_w^2;
\]

\(\gamma_{fm,r}, \gamma_{fm,sw}\) – safety factors for self weight loads caused by the rotor and steel tower conic shell respectively; \(A_i\) and \(W_i\) – area and second moment of inertia of the cross-section of \(i\)th part of the tower’s conic shell correspondingly:

\[
\begin{align*}
A_i &= \pi D_i t_i; \\
W_i &= 0.25 \pi D_i^2 t_i.
\end{align*}
\]
For steel conic shell with cone angle $\beta \leq 60^\circ$ the critical stress $\sigma_{cr,i}$ for $i$th part of the conic shell is determined in dependence of it reduced radius $r_{m,i}$. The latter is calculated according to the requirements of ДБН В.2.6-163:2010 [9] by the following formula:

$$r_{m,i} = \left(0.5D_i - 0.1h_i \tan \beta\right) / \cos \beta,$$

or in dependence of design variables:

$$r_{m,i} = \left(\frac{D_{\text{max}} - D_{\text{min}}}{20H}\right) \frac{h_i}{1 + \left(\frac{D_{\text{max}} - D_{\text{min}}}{2H}\right)^2}.$$

The critical stress $\sigma_{cr,i}$ for $i$th part of the tower’s conic shell is calculated as lower value:

$$\sigma_{cr,i} = \min \{\sigma_{cr,1,i}, \sigma_{cr,2,i}\},$$

where

$$\sigma_{cr,1,i} = \psi_i R_y = \left(0.97 - \left(0.00025 + 0.95 \frac{R_y}{E}\right) \frac{r_{m,i}}{t_i}\right) R_y;$$

$$\sigma_{cr,2,i} = \frac{E t_i}{r_{m,i}} = 1.819 \left(\frac{r_{m,i}}{t_i}\right)^{-0.44}.$$

Here $R_y$ – the design steel resistance; $E$ – modulus of elasticity.

Besides, the system of constraints should also include additional conditions which take into account peculiarities of the considered kind of structure, namely:

- constraint on tower’s height $H$ of the wind-powered generator, by which an operation condition for the wheel with diameter $D_w$ is satisfied:

$$H \geq H_{\text{min}} = D_w + 10\text{ m};$$

- constraint on wind wheel diameter $D_w$ taken into account manufacturing capability of the wind wheel’s blade of maximum possible length $0.5D_{w,\text{max}}$:

$$D_w \leq D_{w,\text{max}}.$$

Additional constraints on cross-sectional sizes $D_i$ for the $i$th part of the tower’s conic shell can be also included to the system of constraints as required considering transportability of the tower’s parts with maximum possible diameters $D_{i,\text{max, gab}}$:

$$D_i \leq D_{i,\text{max, gab}}.$$

4. Parametric optimization methodology

The parametric optimization problem of the tower’s conic shell formulated above has been solved by improved method of projection purpose function’s negative gradient on the surface of the active constraints using software OptCAD [10].

As you know, gradient methods are based on construction of the sequence of the design decisions which ensure the convergence to the optimum point with the minimum value of the purpose function [11]. Proposed improvement of the gradient method [12] allows to perform at the stage of formation the gradient matrix the selection of linear-independent
constraints under the condition of constant lengths of gradient vectors of active constraints as well as vector of purpose function’s gradient.

Solution algorithm of parametric optimization problem includes the following steps.

Step 1. Definition of the start design decision and description of the initial data for optimization calculation. The start values for design variables are specified, namely: tower’s height $H$; minimum $D_{\text{min}}$ and maximum $D_{\text{max}}$ diameters of the middle surface of the tower’s conic shell at a height $H$ and at foundation level respectively; thicknesses $t_i$ for each part of the conic shell; diameter of the wind wheel $D_{w}$.

Step 2. Calculation the node coordinates for finite-element model of the tower’s structure and state variables (9), (10) in dependence of design variables. At the next steps the values of the node coordinates and values of the state variables are re-calculated considering the current value of the tower’s height $H$.

Step 3. Calculation geometrical cross-sectional properties for each part of tower’s conic shell (29)–(30) and state variables (8) in dependence of design variables. At the next iterations the values of geometrical cross-sectional properties and state variables are re-calculated considering current values of the minimum $D_{\text{min}}$ and maximum $D_{\text{max}}$ diameters of the middle surface of the tower’s conic shell, as well as thicknesses $t_i$ of shell’s parts.

Step 4. Calculation the values of the wind loads, applied on the tower’s structure (18)–(20) in dependence of the design variables. At the next steps the values of the wind loads $q_i$ and $F_i$ are re-calculated taken into account current values of the tower’s height $H$ and diameters $D_i$ of the middle surfaces of the conic shell’s parts.

Step 5. Linear static analysis. Using finite element method for each design load case combination the linear displacements for all nodes of the finite element model are determined, as well as stresses values for all design section of the tower’s structure.

Step 6. Verification of all constraints of mathematical model (24), (36)–(38) for all load case combinations in all design sections of the structure. Then active (breached) constraints of the mathematical model should be identified.

Step 7. Calculation the current value of the purpose value as well as purpose function’s gradient; determination of the desired increment of the purpose function.

Step 8. Forming the vector of residuals and the matrix of gradients of active linear-independent constraints with triangular structure.

Step 9. Calculation the increment of the design variables and improved approximation of the optimum design decision.

Step 10. Checking the stop conditions of the iterative searching. If the current design decision fulfils all constraints of the mathematical model with assumed precision, then it should go to the step 11. In other case it should turn to the step 2.

Step 11. The optimum design decision of the tower structure is defined.

5. Results of optimization

A number of optimization problems has been formulated with different values of target capacity $W^*$ of the wind-powered generator and solved using methodology presented above. Initial data are as follow: average annual wind velocity $v_0 = 7$ m/s; maximum possible wind velocity $v_{l,\text{max},1} = 20$ m/s for wind wheel’s blades; maximum possible wind velocity $v_{l,\text{max},2} = 30$ m/s for considered terrain; roughness parameter of earth’s surface $z_0 = 0.05$ and terrain type factor $k_r = 0.19$ as for rural area with houses and trees of low altitude; the design steel resistance $R_s = 3,823$ t/sm²; safety factor for wind loading $\gamma_{\text{fm},w} = 1.2$, for loading of the rotor self weight $\gamma_{\text{fm},r} = 1.2$; for loading of the tower’s conic shell self weight $\gamma_{\text{fm},w} = 1.05$; maximum diameter of the wind wheel $D_{w,\text{max}} = 150$ m. Besides, constraints (38) on maximum cross-sectional dimensions for each part of the tower’s conic shell were not considered.

Table 1 presents the results of optimization calculations. Optimum tower’s height and optimum tower’s weight in dependence of produced capacity of wind-powered generator are given by Fig. 5 and Fig. 6 respectively.

| $W^*$, MWt | $H$, m | $G$, t | $D_{\text{max,m}}$, m | $D_{\text{max,m}}$, m | $D_{\text{w,m}}$, m |
|------------|--------|--------|------------------------|------------------------|------------------------|
| 1,0        | 75,65  | 39,76  | 1,881                  | 4,769                  | 65,65                  |
| 2,0        | 98,13  | 90,74  | 2,520                  | 6,335                  | 88,13                  |
| 3,0        | 114,57 | 147,71 | 2,975                  | 7,482                  | 104,57                 |
| 4,0        | 128,02 | 209,00 | 3,347                  | 8,422                  | 118,02                 |
| 5,0        | 139,60 | 273,80 | 3,667                  | 9,233                  | 129,60                 |
| 6,0        | 149,89 | 341,55 | 3,951                  | 9,955                  | 139,89                 |
| 7,0        | 159,22 | 411,90 | 4,209                  | 10,609                 | 149,22                 |
Additionally, a number of optimization problems has been solved taken into account the constraints (38) on maximum cross-sectional dimensions $D_i,_{\text{max, gab}} = 4,5$ m for each part of the tower’s conic shell. Table 2 presents the results of these optimization calculations.

Table 2. Results of optimization taken into account the constraints on maximum cross-sectional dimensions of the tower’s conic shell

| $W^*$, MWt | $H$, m  | $G$, t   | $D_{\text{min, m}}$ | $D_{\text{max, m}}$ | $D_{w, m}$ |
|------------|---------|----------|---------------------|---------------------|-----------|
| 1,0        | 75,65   | 39,95    | 1,93                | 4,50                | 65,65     |
| 2,0        | 98,13   | 90,74    | 3,42                | 4,50                | 88,13     |
| 3,0        | 114,57  | 165,36   | 4,50                | 4,50                | 104,57    |
| 4,0        | 128,02  | 246,51   | 4,50                | 4,50                | 118,02    |
| 5,0        | 139,60  | 340,13   | 4,50                | 4,50                | 129,60    |
| 6,0        | 149,89  | 445,22   | 4,50                | 4,50                | 139,89    |

As you see from Table 2, the optimum values of the tower’s height $H$ and wind wheel $D_w$ are independent from cross-sectional sizes of the tower. For this case optimum tower’s weight in dependence of produced capacity of wind-powered generator are given by Fig. 7.
6. Conclusion

This paper presents a parametric optimization task of steel conic shell towers of wind-powered generators. The minimum weight of the steel tower has been considered as purpose function when produced capacity of the wind-powered generators has been fixed at the target level. The tower’s height, diameters of the middle surface of the tower’s conic shell at the base level and at a height if the wind turbine installation, thicknesses of the tower’s conic shell, and also the diameter of the wind wheel were considered as design variables. The wind loads applied on steel shell tower have been presented in dependence of design variables and considered as state variables. The optimization problem formulated as non-linear programming task has been solved by improved gradient method. Optimum design decisions of the steel conic shell towers for the wind-powered generators with produced capacity of wide range have been defined. An optimum tower’s height and weight in dependence of produced capacity of wind-powered generators have been plotted.

It should be noted that the results of the optimization problems presented by the paper are in a good accordance with [13] where tall towers for large wind turbines have been studied. Namely, main data for 5MW turbine proposed by Engström et al. [13] includes turbine diameter 126 m and tower weight 350 t; main data for 3MW turbine includes tower diameter 100 m and tower weight 176,5 t (to detail comparing, see Table 2). That is why there is a good justification of general judgment and recommendation.

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