Workspace Analysis and Dynamics Simulation of Manipulator based on MATLAB

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Abstract. Workspace analysis and joint moment calculation are the main contents of manipulator’s design and calculation. According to the joint layout characteristics of the light cooperative manipulator, the workspace of the manipulator is divided into two parts: the arm workspace and the wrist workspace. The geometric method and Monte Carlo method are used to calculate the workspace of the manipulator and to guide the determination of the structural parameters of the manipulator. Based on Simulink/Simmechanics library, the dynamics model of six-axis serial manipulator is built and the joint rotational moment is calculated. The simulation results are in good agreement with the results of Newton Euler method. The simulation model can be used to replace Newton Euler method in the design process of the manipulator to improve the design efficiency.

1. Introduction

The six-axis manipulator has high degree of freedom and versatility, and is suitable for various industrial occasions.¹ In the design of the manipulator, firstly, according to the application scenario of the manipulator, the workspace and load requirements of the manipulator should be determined. Then, the structural dimensions of the manipulator should be determined. Finally, the reducers and servo motors of each axis should be selected.

2. Workspace Analysis of Manipulator

Generally, the six-axis manipulator is divided into four parts: base, upper arm, lower arm and wrist. A common joint layout is that the base of the manipulator is connected with the upper arm through the first and second joints, the upper arm is connected with the lower arm through the third joint, and the end of the lower arm is the fourth, fifth and sixth joints connected sequentially. At present, most of the light cooperative manipulators in the market adopt this joint layout, such as UR series cooperative manipulators.

When designing a manipulator, the reachable range of the center point of the flange at the end of the manipulator is often defined as the workspace of the manipulator. However, when the center point of the flange of the manipulator is in the edge area of the workspace, the flexibility of the manipulator is limited, thus, not all the workspace is the effective workspace. Figure 1 shows the structure and workspace of UR5 manipulator.
The joint layout is very similar to that of the human arm: the length of upper arm is similar to that of lower arm, and the range of motion of the end of the lower arm is approximately spherical, pushing the wrist to the area where it needs to work; the fourth, fifth and sixth joints of the lower arm are similar to human wrist, with high flexibility and can rotate to different posture to complete the work.

In order to describe the reachable range and flexibility of the manipulator better, the workspace of the manipulator is divided into two parts: arm workspace and wrist workspace. The arm workspace is used to measure the effective operation range of the manipulator, and the wrist workspace is used to measure the flexibility of the wrist at the end of the manipulator. Figure 2 is a schematic illustration of workspace partitioning using the UR5 manipulator as an example.

The axes of the second, third and fourth joints of UR5 manipulator are parallel to each other, so the division of workspace is based on the intersection of the axes of the first joint and the axes of the fourth joint as show in figure 2. At this time, the shape of the arm workspace is a solid sphere.

The coordinate system is established with the intersection point of the first and second joint axes as the origin point. The range of motion of the arm workspace, i.e. the range of end point $P(x, y, z)$ of the lower arm, can be written out directly.
Monte Carlo method is used to calculate the wrist workspace because of that shape of the wrist workspace is irregular. The steps of using Monte Carlo method to calculate the workspace of manipulator are as follows: establishing coordinate system, selecting the position of each joint randomly, finding the position of the end point of manipulator relative to the base center in different positions and postures, drawing the point in the coordinate system, repeating this process, when the number of end points reaches sufficient quantity, the workspace of manipulator can be obtained.[2]

The coordinate system of the manipulator from the end of the lower arm to the center of the flange at the end of manipulator is established. As shown in Figure 3, the DH parameters of each coordinate system are as shown in Table 1.

![Figure 3. Coordinate system of the wrist workspace.](image)

### Table 1. DH parameter of wrist workspace coordinates.

| Ordinate order | Connecting rod offset $d_i$ (mm) | Twist angle $\alpha_{i,1}$ ($^\circ$) | Joint angle $\theta_i$ ($^\circ$) |
|----------------|---------------------------------|-----------------------------------|-------------------------------|
| 1              | 0                               | 90                                | 0+θ1                          |
| 2              | 110                             | -90                               | 0+θ2                          |
| 3              | 94.5                            | 90                                | 0+θ3                          |
| 4              | 82.5                            | 0                                 | 0                             |

Coordinate system of the wrist workspace is non-standard DH coordinate system, adjacent coordinate system transformation according to the following formula:

$$T_i = Trans(z, d_i) Rotate(x, \alpha_{i,1}) Rotate(z, \theta_i)$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \end{bmatrix} \begin{bmatrix} \cos \alpha_{i,1} & \sin \alpha_{i,1} & 0 & -d_i \\ -\sin \alpha_{i,1} & \cos \alpha_{i,1} & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \cos \theta_i & -\sin \theta_i & 0 & 0 \\ \sin \theta_i & \cos \theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

(2)

Among them, $R$ is 3x3 matrix, $P$ is 3x1 matrix and $\mathbf{P}$ is the origin of coordinate system 4 expressed in the global coordinate system. Given the rotation angles of the 4th, 5th and 6th joints with random
functions, the origin of coordinate system 4 (the center point of the end flange) relative to the global coordinate system is obtained by the above coordinate transformation, and the points are repeatedly taken out in the coordinate system. Finally, the UR5 wrist workspace point map is as Figure 4:

\[
\begin{align*}
-125.5 \leq x & \leq 125.5 \\
-125.5 \leq y & \leq 125.5 \\
-192.5 \leq z & \leq -27.5
\end{align*}
\] (4)

When the manipulator moves, the center of the wrist workspace moves within the scope of the arm workspace. All space swept by the wrist workspace are all positions that can be reached by the center point of the end flange. This method can accurately measure the effective operating space and the flexibility of the end joints of the manipulator. It has certain significance to guide the structural design of the manipulator.

3. Dynamics Simulation of Manipulator

After the workspace design of the manipulator is completed, it is necessary to calculate the required moment of each joint in the course of movement, and select the servo motors and reducers. The joint moment of universal six-axis manipulator can be calculated by Newton-Euler method. Craig gives the formula of Newton-Euler method for serial manipulator. The Newton-Euler method is simple and
reliable, but the calculation is slightly cumbersome, and the formulas need to be deduced again when the structure of the manipulator changes.

Simmechanics, a multi-body dynamics simulation toolbox, is provided in MATLAB. The dynamics model of the manipulator based on Simmechanics is established as follows.[3] The coordinate system of the UR5 manipulator is established by DH parameter method,[4] as shown in Fig. 5. The coordinate transformation parameters are shown in Table 2.

![Coordinate System of the UR5 Manipulator](image)

**Figure 5.** Coordinate system of the UR5 manipulator.

**Table 2.** DH parameters of wrist coordinates for UR5 manipulator.

| Ordinate order | Joint angle $\theta_i$ (°) | Connecting rod length $a_{i-1}$ (mm) | Twist angle $\alpha_{i-1}$ (°) | Connecting rod offset $d_i$ (mm) |
|----------------|-----------------------------|--------------------------------------|-----------------------------|----------------------------------|
| 1              | $0+\theta_1$                | 0                                    | 0                           | 80                               |
| 2              | $90+\theta_2$               | 0                                    | 90                          | 110                              |
| 3              | $0+\theta_3$                | 425                                  | 0                           | 0                                |
| 4              | $-90+\theta_4$              | 392                                  | 0                           | 0                                |
| 5              | $0+\theta_5$                | 0                                    | $-90$                       | 94.5                             |
| 6              | $0+\theta_6$                | 0                                    | 90                          | 0                                |

The coordinate transformation matrix of adjacent connecting rods is:

$$
T_i = \begin{bmatrix}
1 & 0 & 0 & a_{i-1} \\
0 & \cos \alpha_{i-1} & -\sin \alpha_{i-1} & 0 \\
0 & \sin \alpha_{i-1} & \cos \alpha_{i-1} & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
\cos \theta_i & -\sin \theta_i & 0 & 0 \\
\sin \theta_i & \cos \theta_i & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

The three-dimensional model of the manipulator is established by using CAD software, and the DH coordinate system is established in the CAD software. The inertia tensor and centre of mass of the
links relative to the coordinate system is calculated. The physical model of the manipulator is established in Simulink according to the connection relationship of the links of the manipulator, and the inertia attributes of the links are set up. Finally, the manipulator’s dynamics model is as figure 6.

Figure 6. Illustration of the manipulator’s dynamics model.

During the working process of the manipulator, the second joint receives the greatest load. Assuming that the other joints are fixed in accordance with the position shown in Figure 3, the second joint moves from -90 to 90 degrees, which takes 50 seconds. The position, velocity and acceleration of the second joint are generated by the trajectory planning function "jtraj". The motion data generated by trajectory planning function are input into the second joint actuator, and the reaction moment curve of the second joint varying with time is obtained, as shown in Figure 7.

Figure 7. The reaction moment curve of the second joint varying with time.

The Newton Euler method uses the knowledge of rigid body mechanics to derive the recursive formula of kinematics, and then derives the mathematical model of robot dynamics. The Newton Euler method calculates the torque of each joint as follows.

\[
\begin{align*}
[i+1] \omega_{i+1} &= [i+1] R \omega_i + [i+1] \dot{\theta}_{i+1} \times [i+1] Z_{i+1} \\
[i+1] \dot{\omega}_{i+1} &= [i+1] R \dot{\omega}_i + [i+1] \dot{\theta}_{i+1} \times [i+1] Z_{i+1} + [i+1] R \omega_i \times [i+1] \dot{\theta}_{i+1} \times [i+1] Z_{i+1} \\
[i+1] \dot{v}_{i+1} &= [i+1] R \left( \dot{v}_i + \dot{\omega}_i \times P_{i+1} \right) \\
[i+1] \ddot{v}_{i+1} &= [i+1] R \left( \ddot{v}_i + \dot{\omega}_i \times P_{i+1} + \dot{\omega}_i \times \left( \omega_i \times P_{i+1} \right) \right) \\
\dot{v} &= \dot{v}_i + \dot{\omega}_i \times P_{i+1} + \dot{\omega}_i \times \left( \omega_i \times P_{i+1} \right)
\end{align*}
\]
According to the Euler equation of motion, the resultant force and resultant moment of each connecting rod are obtained:

\[ F_i = m_i \dot{v}_i \]
\[ N_i = I_{ci} \dot{\omega}_i + v_c \times I_{ci} \omega_c \]  
(7)

\[ f_i \] is used to express the force for connecting rod applied by previous rod, and \( n_i \) is used to express the moment connecting rod receives.

\[ f_i = F_i + R_{i.e} f_{i.e} \]
\[ n_i = N_i + R_{i.e} n_{i.e} + P_{c_i} \times F_i + P_{c_i} \times R_{i.e} f_{i.e} \]  
(8)

In Simulink, Newton Euler method is used to calculate the moment change process of the second joint in the process, and the results of Newton Euler method are subtracted from the simulation results. The difference changes with time as shown in Figure 8.

![Figure 8. Deviation between simulation model and Newton Euler method.](image)

As can be seen from the figure, the simulation results are in good agreement with the results of Newton Euler method, and the simulation model can be used instead of Newton Euler method.

4. Conclusion

In order to describe the reachable range and flexibility of the manipulator, the workspace of the UR5 manipulator is divided and calculated. The Monte Carlo method is a random sampling method used in the calculation of the wrist workspace. Compared with the actual workspace, the workspace obtained by Monte Carlo method still has some errors, but by increasing the number of sampling points, we can get a higher accuracy, so as to approximate the actual workspace until the demand is met. Because the shape of the arm workspace is regular, the analytic formula of the arm workspace is written directly, while the shape of the wrist space is irregular, the Monte Carlo method is used for analysis. When the joint layout of manipulator adopts other forms, the division and analysis method of workspace should be selected according to the situation.

The dynamics model of the manipulator is established based on Simmechanics library, and the joint moment is calculated. The results are compared with those of Newton Euler method. The simulation model is in good agreement with those of Newton Euler method. The simulation model is more intuitive and visual, and the model can be established or changed more quickly, which is more suitable for the rapid development environment. Because the working environment of the second joint is the
worst, this paper only simulates the motion of the second joint alone. If necessary, it can simulate the force moment of all joints when joints move together.

**Reference**

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