The role of isospin filtering reactions in the $S = -1$ sector

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Abstract. The present study reveals interesting constraining effects of isospin filtering reactions on the low energy constants present in meson-baryon chiral effective Lagrangian, particularly, on the next-to-leading order constants. Our model has been developed within the framework of Unitarized Chiral Perturbation Theory and has been fitted to two-body scattering data in the sector of $S = -1$. In addition, the model was further elaborated by means of the inclusion of high-spin hyperonic resonances.

1 Introduction and formalism

The plausible explanation of the $\Lambda(1405)$ resonance as a molecular state arising from coupled channel meson-baryon re-scattering in the strangeness $S = -1$ sector employing the lowest order chiral Lagrangian is one of the most important successes of Unitarized Chiral Perturbation Theory (UChPT). This scheme consists of a non-perturbative sum of properly arranged topologies derived from an effective chiral Lagrangian. Such a Lagrangian is basically a momentum expansion whose building blocks preserve the same symmetries as Quantum Chromodynamics (QCD). Being more precise, the unitarization in coupled channels is implemented by solving the Bethe-Salpeter equation (BSE) which, although it is defined as a complex system of integral equations, can be converted into a system of algebraic equations by means of the on shell factorization [1] giving the final expression: $T_{ij} = (1 - V_i G_l)^{-1} V_{lj}$. Here, $T_{ij}$ stands for the scattering amplitude which connects the incoming $i$-channel with the outgoing $j$-channel, and $G_l$ is the loop function solved employing dimensional regularization.

Finally, $V_{ij}$, the interaction kernel, is derived from the chiral Lagrangian whose leading order contribution gives rise to the Weinberg-Tomozawa (WT) term and to the meson-baryon vertices needed to build the Born diagrams.

$$\mathcal{L}^{(1)}_{\phi B} = i\langle \bar{B} \gamma_\mu [D^\mu, B] \rangle - M_0 \langle \bar{B} B \rangle - \frac{1}{2} D \langle \bar{B} \gamma_\mu \gamma_5 [d^\mu, B] \rangle - \frac{1}{2} F \langle \bar{B} \gamma_\mu [u^\mu, B] \rangle , \quad (1)$$

where $M_0$ is the common baryon octet mass in the chiral limit. The constants $D, F$ denote the axial vector couplings of the baryons to the mesons. $B$ is the baryon octet field and the symbol $\langle \ldots \rangle$ stands for the trace in flavour space, while the pseudoscalar meson octet field $\phi$ enters as: $u_\mu = iu^\dagger \partial_\mu U u^\dagger$, where $U(\phi) = u^2(\phi) = \exp \left( \sqrt{2}i \phi / f \right)$. 

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Despite this theoretical breakthrough, the aim for more precise calculations led the community to extend this approach with the inclusion of higher order terms and to explore higher energies. In particular, to compute the next-to-leading order (NLO) contributions of the Lagrangian in S wave one can use the prescription:

\[ \mathcal{L}_{\phi B}^{(2)} = b_D \langle B|\chi_+|B\rangle + b_F \langle B|\chi_+|B\rangle + b_0 \langle BB|\chi_+|BB\rangle + d_1 \langle B|u\mu|[u^\mu|B]\rangle + d_2 \langle B|u\mu|[u^\mu|B]\rangle + d_3 \langle B|\rho|u\mu|\rho B\rangle + d_4 \langle BB|[u^\mu|u\mu|BB]\rangle, \tag{2} \]

where \( \chi_+ = 2B_0 (u^\dagger Mu^\dagger + uMu) \) breaks chiral symmetry explicitly via the quark mass matrix \( M = \text{diag}(m_u,m_d,m_s) \) and \( B_0 = -\langle 0|\bar{q}q|0\rangle/f^2 \) relates to the order parameter of spontaneously broken chiral symmetry, with \( f \) being the pseudoscalar decay constant in the chiral limit. The coefficients \( b_D, b_F, b_0 \) and \( d_i \) make up the low energy constants (LEC) set at this order. As these parameters are not fixed by the symmetries of the underlying QCD theory, the fitting procedures to the experimental data play a key role.

In previous studies, the relevance of the NLO terms [2] as well as the Born terms [3] of the chiral \( SU(3) \) Lagrangian for the \( K\Xi \) channels was proved establishing the experimental \( K^- p \to K\Xi \) cross-section data as a very important ingredient to obtain more reliable values of the NLO parameters. The main novelty in [3] with respect to [2] was the systematic addition of the \( u \)- and \( s \)-channel diagrams to a kernel consisting of WT term and the NLO one. As an obvious consequence, very different parameterizations for the NLO LEC were obtained. But, paradoxically, both reached very similar goodness in order to describe the experimental data. The analysis of the isospin components of the \( K^- p \to K\Xi \) cross-section for these LEC sets in [3] showed very dissimilar isospin-distribution patterns, a fact that clearly points out the need to explore reactions acting as isospin selectors by means of these LEC sets in [3] showed very dissimilar isospin-distribution patterns, a fact that clearly points out the need to explore reactions acting as isospin selectors by means of which more realistic values of NLO LEC are obtained. Therefore, motivated by the previous findings, we have performed a new fit that includes additional experimental data from the reactions \( K^- p \to \eta\Lambda, \eta\Sigma^0 \) which proceed via single \( I = 0 \) and \( I = 1 \) component respectively. Likewise, the secondary \( K_L^0 \) beam, at K-Long Facility for JLab, on liquid hydrogen might induce pure isovector reactions. Among them, the \( K_L^0 p \to K^+\Xi^0 \) process is specially interesting given its sensitivity to the NLO terms. Thus, the measurement of such a cross section could provide very valuable information. To check the predictive capacity of the model, we include a prediction for the corresponding total cross section.

Like in [2], the inclusion of additional high-spin and high-mass resonances plays a double role: it improves the description of the experimental cross sections and, additionally, it allows us to study the stability of the NLO coefficients. In this study, \( \Lambda(1890), \Sigma(2030) \) and \( \Sigma(2250) \) (with \( J^P = 7/2^+, 5/2^- \) and \( 3/2^+ \), respectively) have been incorporated into the channels estimated to be sensitive to the NLO terms, namely the \( \bar{K}N \to Y \to K\Xi(\eta\Lambda) \) processes. These transition amplitudes have been implemented adopting the Rarita-Schwinger method that permits building resonant amplitudes from effective lagrangians, which allow one to couple 1/2 spins with the higher ones by means of tensorial structures [4]. Eventually, the scattering amplitudes are then rewritten according to the following prescription:

\[ T_{ij} = T_{ij}^{RS} + \sum_{J^P} T_{ij}^{J^P}, \tag{3} \]

where \( T_{ij}^{RS} \) is the scattering amplitude obtained from the chiral lagrangian and unitarized by means of the BS equation, while \( T_{ij}^{J^P} \) accounts for the corresponding resonant term with \( J^P \) quantum numbers, which take the values \( J^P = 3/2^+, 5/2^-, 7/2^+ \) for the \( K^- p \to K^0\Xi^0, K^+\Xi^- \) reactions.
processes while for the $K^- p \rightarrow \eta \Lambda$ one, only the $\Lambda(1890)$ resonance ($J^\pi = 3/2^+$) is implemented.

2 Results and discussion

| $b_0$ (GeV$^{-1}$) | $b_1$ (GeV$^{-1}$) | $b_2$ (GeV$^{-1}$) | $d_1$ (GeV$^{-1}$) | $d_2$ (GeV$^{-1}$) | $d_3$ (GeV$^{-1}$) | $d_4$ (GeV$^{-1}$) | $\chi^2_{d.o.f}$ |
|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|------------------|
| WT+Born+NLO       | $0.13 \pm 0.04$   | $0.12 \pm 0.01$   | $0.21 \pm 0.02$   | $0.15 \pm 0.03$   | $0.13 \pm 0.03$   | $0.30 \pm 0.02$   | $0.25 \pm 0.03$   | 1.14             |
| WT+Born+NLO+RES   | $-0.07 \pm 0.01$  | $0.13 \pm 0.01$   | $0.27 \pm 0.02$   | $0.14 \pm 0.03$   | $0.13 \pm 0.01$   | $0.40 \pm 0.02$   | $0.02 \pm 0.02$   | 0.96             |

Table 1. Values of the NLO parameters and the corresponding $\chi^2_{d.o.f}$, defined in [2], for both fits.

Bearing in mind our previous findings, we have carried out two different fits. On the one hand, the WT+Born+NLO fit corresponds to a unitarized calculation employing an interaction kernel built from Eqs. (1) and (2). This is to say, a model that depends on 16 fitting parameters: the pseudoscalar decay constant, the 2 axial vector couplings, the 6 subtraction constants showing up in the dimensional regularization, and the 7 NLO low energy constants. On the other hand, the inclusion of the resonant contributions described by Eq. (3) results in the WT+Born+NLO+RES fit. These phenomenological terms involve 13 new parameters on which, for brevity, we omit thorough information. The same large amount of $K^- p$ elastic and inelastic cross section data has been taken into account for both procedures, as well as, branching ratios at threshold and the precise SIDDHARTA value of the energy shift and width of kaonic hydrogen. For more specific details, see [5].

![Figure 1](https://example.com/figure1.png)

Figure 1. $K^- p \rightarrow \eta \Lambda, \eta \Sigma^0, K^+ \Xi^-, K^0 \Xi^0$ total cross sections for the WT+Born+NLO fit, with its error bands (gray area) estimated as specified in [5], and for the WT+Born+NLO+RES fit. Experimental data has been taken from [6–16].
The NLO coefficients corresponding to the WT+Born+NLO fit are displayed in Table 2, from which we can appreciate a notable homogeneity in their values, especially, if we compare them with those of [2, 3]. This fact underpins the idea that the use of experimental data coming from isospin filtering processes promotes the reliability of LEC. From the comparison between rows of Table 2, one can note an overall stability. The constants with small variations are those that are present in the NLO couplings of the processes being analyzed (Table VIII in [2]), while $b_0$ and $d_4$ only show up in elastic processes which are not directly taken into account in the driving terms of the amplitude needed to fit the present data.

Another eye-catching feature in Table 2 is the remarkable 16% improvement in the $\chi^2_{d.o.f.}$ when the resonant amplitudes are incorporated. This fact is clearly reflected in Fig. 1, where only the total cross sections sensitive to NLO are represented. The top panels, corresponding to the cross sections of the $\eta$ channels, show the good agreement with data reached by both models which are able to reproduce, for instance, the resonant structure of the $\Lambda(1670)$ seen in $K^-p \rightarrow \eta\Lambda$. It should be mentioned that WT+Born+NLO+RES (dashed line) does a better job in describing the experimental $K^-p \rightarrow \eta\Lambda$ cross section in the energies ranging from 1850 to 2200 MeV because of the explicit inclusion of the $\Lambda(1890)$ resonance. This resonance is also relevant to improve the reproduction of the $K^-p \rightarrow K^0\Xi^0$ cross section (left bottom panel) above threshold, where WT+Born+NLO (solid line) overshoots the scattering data slightly. The combined effect of the $\Sigma(2030)$ and $\Sigma(2250)$ resonances provides a clear bump structure reaching its maximum at around 2100 MeV. The WT+Born+NLO model presents a good agreement with data within the whole energy range for the $K^-p \rightarrow K^+\Xi^-$ cross section. There, the resonant terms interfere destructively, resulting in a slight reduction of strength around the maximum. Finally, in Fig. 2, we present a prediction for the $K_L^0p \rightarrow K^0\Xi^0$ cross section whose experimental measurement could contribute to a better knowledge of the NLO LEC.

![Figure 2](https://example.com/figure2.png)

**Figure 2.** Total cross sections of the $K^0_Lp \rightarrow K^+\Xi^0$ reactions for both models. The experimental points of the $I = 1 \ K^-n \rightarrow K^0\Xi^-$ reaction, taken from [17, 18] and divided by two.
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