Multi-Target Adaptive Radar Waveform Design Based on SINR

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Abstract. Aiming at the radar waveform design problem of detecting multiple targets in the presence of clutter, a novel algorithm based on linear probability-weighted summation (PWS) of multi-target impulse response of signal to interference plus noise ratio (SINR) is proposed. In view of the traditional water-filling algorithm, the multi-target detection algorithm is further studied in a new way to improve the overall performance of the system. This method uses Jensen’s inequality to make a lot of derivations to determine the target function of the algorithm and the constraints. Simulation results show that the proposed algorithm has better detection performance and more accurate target information.

1. Introduction

Aiming at the problem of target detection and recognition in cognitive radar systems, Jiu Bo [1] proposes a method of maximizing target echo and expanding target mutual information to transform the traditional water-filling algorithm into general water-filling method in a clutter background in 2015. Fan Meimei [2] proposes that when multiple targets can be separated in the same beam, the radar transmit waveform is designed in the presence of only noise to achieve the role of radar search, detection and identification of targets in 2011. Wang Lulu [3, 4] solves a waveform optimization method for single extended target recognition. Each received echo is processed to obtain a priori information of the next transmission waveform of each target, and is used to guide each target to design the next transmit waveform In 2014. In 2017, Goodman [5-7] proposes the design of cognitive radar waveforms to complete the task of target recognition in the presence of noise and clutter. The paper considers random targets and determining targets, and uses detection probability as system performance.

2. Problem Formulation

2.1. Stochastic Multi-Target Signal Model

Supposing there are $i$ targets in the research process, $i = 1, 2, 3, ..., H$. Convolution operation is $\ast$. With the design of traditional multi-target water-filling algorithm, the model applied in this paper is shown in (1)

$$y(t) = w(t) \ast c(t) + w(t) \ast h_i(t) + n(t)$$  (1)

In the above formula (1), $w(t)$, $c(t)$, and $h_i(t)$ respectively represent the transmit waveform, environmental clutter, and multi-target impulse response in the model. Among them, multiple targets
obey Gaussian random distribution. The duration of the study is \( T_i \), and the noise is \( n(t) \). It can be known from the application of communication principle knowledge that the research process requires the energy spectral density (ESD) and power spectral density (PSD) of each waveform. Where, the noise power spectral density is \( P_n(f) \), and the clutter power spectral density is \( P_c(f) \). Multi-target duration time is \([0, T_i] \). Assuming \( g_i(t) = a(t)h_i(t) \), here \( a(t) \) is the identity matrix window function, which is a generalized complex Gaussian stationary process. Because of the existence of \( g_i(t) \), the ultimate goal is to make the multiple targets maintain a relatively stable state during the research process.

The research process is implemented in the frequency domain, so the paper use the Fourier transform, we can see that the multi-target frequency domain model is shown in (2)

\[
Y = WC + WG + N
\]  

(2)

2.2. SINR-Based Multi-target General Water-Filling Algorithm

According to the model established by equation (2) and the principle of information theory, we can know that the multi-target expression based on the SINR is shown in (3).

\[
\text{SINR} = \frac{\int_B \frac{|w(f)|^2 \sigma_{\mu}^2(f)}{P_n(f) + |w(f)|^2 P_c(f)} df}{T_i}
\]  

(3)

In the multi-target case, we can assume that the multi-target energy spectrum variance (ESV) is

\[
S_{\mu}(f) = \sum_{i=1}^{H} P_{i}(H_i)\sigma_{\mu}^2(f) - \left[ \sum_{i=1}^{H} P_{i}(H_i)\left(\sigma_{\mu}^2(f)\right)^{1/2} \right]^2
\]

Therefore, the new target ESV is

\[
S_{\mu}(f) = \sum_{i=1}^{H} P_{i}(H_i)\sigma_{\mu}^2(f) - \mu_{\mu}(f).
\]

3. SINR-Based Waveform Design for Stochastic Target in Signal-Dependent Interference

For the design of the multi-target radar transmit waveform based on the signal to interference plus noise ratio, the multi-target objective function after weighted summation is shown in equation (4)

\[
\text{SINRESV}_{\mu} = \frac{\int_B \frac{|w(f)|^2 S_{\mu}(f)}{T_i \left\{ P_n(f) + |w(f)|^2 P_c(f) \right\}} df}{T_i}
\]

\[
= \frac{\int_B \left[ \frac{|w(f)|^2 \left\{ \sum_{i=1}^{H} P_{i}(H_i)\sigma_{\mu}^2(f) - \mu_{\mu}(f) \right\}^2}{T_i \left\{ P_n(f) + |w(f)|^2 P_c(f) \right\}} \right] df}{T_i}
\]

\[
= \frac{\int_B \left[ \frac{|w(f)|^2 \left\{ \sum_{i=1}^{H} P_{i}(H_i)\left\{ \sigma_{\mu}^2(f) \right\} \right\}^2}{T_i \left\{ P_n(f) + |w(f)|^2 P_c(f) \right\}} \right] df}{T_i}
\]  

(4)

As \( \sum_{i=1}^{H} P_{i}(H_i) = 1 \), applying Jensen’s inequality, PWS–SINR algorithm target function becomes

\[
\text{PWS–SINR} = \frac{\int_B \left[ \sum_{i=1}^{H} P_{i}(H_i) \left\{ \frac{|w(f)|^2 \left\{ \sigma_{\mu}^2(f) - \mu_{\mu}(f) \right\}^2}{T_i \left\{ P_n(f) + |w(f)|^2 P_c(f) \right\}} \right] df}{T_i}
\]  

(5)
The PWS–SINR algorithm under energy constraints target function proposed in this paper becomes

$$\max \sum_{i=1}^{n} P_i(H_i) \left\{ T_s \int_{\mathcal{R}} \left[ \frac{|w(f)|^2 \left\{ \sigma_n^2(f) - \mu_n(f) \right\}}{T_s \left[ P_n(f) + |w(f)|^2 P_i(f) \right]} df \right\} \right.$$ \hspace{1cm} (6)

subject to $$\int_{\mathcal{R}} |w(f)|^2 df \leq E_w$$

where $$\hat{\sigma}_n^2(f) = \sigma_n^2(f) - \mu_n(f)$$.

The final target model based on the PWS–SINR algorithm under energy constraints as (7)

$$\max \sum_{i=1}^{n} P_i(H_i) \left\{ T_s \int_{\mathcal{R}} \left[ \frac{|w(f)|^2 \hat{\sigma}_n^2(f)}{T_s \left[ P_n(f) + |w(f)|^2 P_i(f) \right]} df \right\} \right.$$ \hspace{1cm} (7)

subject to $$\int_{\mathcal{R}} |w(f)|^2 df \leq E_w$$

The multi-target function based on the SINRESV algorithm under energy constraints is

$$\max T_s \int_{\mathcal{R}} \left[ \frac{|w(f)|^2 \sum_{i=1}^{n} P_i(H_i) \hat{\sigma}_n^2(f)}{T_s \left[ P_n(f) + |w(f)|^2 P_i(f) \right]} df \right.$$ \hspace{1cm} (8)

subject to $$\int_{\mathcal{R}} |w(f)|^2 df \leq E_w$$

Finally, because the optimal transmit waveform energy spectrum is always positive, the expression $$|w(f)|^2$$ can be found according to the root formula

$$|w(f)|^2 = \max\left( \frac{P_n(f) \cdot \sum_{i=1}^{n} P_i(H_i) \hat{\sigma}_n^2(f)}{\lambda} - P_n(f), 0 \right)$$ \hspace{1cm} (9)

4. Simulation analysis

Supposing there are four targets in the whole process and all obey the distribution of trigonometric functions. Figure 2 shows the distribution of the total target spectrum and clutter spectrum at $$I = [0, 1]$$.

The distribution of the four target spectra and clutter spectra is shown in Figure 1. It can be seen from the simulation diagram in Figure 1 that the four targets are distributed independently.

![Multi-target spectrum](image)

**Figure 1.** Distribution of four targets and clutter on I.

4.1. SINR-Based Waveform Design for Stochastic Target in Signal-Dependent Interference
Figure 2. Distribution of total target spectrum and clutter spectrum on I.
Figure 2 shows the distribution of the total target spectrum and clutter spectrum at \( I = [0,1] \), and Figure 3 shows the distribution of the optimal transmit waveform spectrum under energy constraints. It can be seen from Figure 3 that when the energy is different, the distribution of the optimal transmission waveform also changes.

Figure 3. Optimal transmit waveform spectrum under different energies.

Figure 4. Relationship between energy spectrum and detection probability based on the SINR. Figure 4 illustrates the advantages of the SINR-based multi-target weighted probability (PWS − SINR) algorithm. It can be seen from Figure 4 that the basic trends of the detection probability of these three waveforms are basically the same, but the PWS − SINR algorithm has better detection performance.

5. Conclusions
The paper mainly proposes multi-target optimized radar transmission waveforms based on signal to interference plus noise ratio under bandwidth constraints and energy constraints. Simulation results show that the PWS − SINR method proposed in this paper is better than LFM signal and SINRESV algorithms in terms of target detection performance. The performance of multi-target radar optimal waveform detection and recognition has been proved through an example.

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