Assessing the ultra-central flow puzzle in the Bayesian era

A.V. Giannini, M. N. Ferreira, M. Hippert, D.D. Chinellato, G. S. Denicol, M. Luzum, J. Noronha, T. Nunes da Silva and J. Takahashi
(The ExTrEMe Collaboration)

1 Instituto de Física Gleb Wataghin, Universidade Estadual de Campinas, Rua Sérgio Buarcque de Holanda 777, 13083-859 São Paulo, Brazil
2 Illinois Center for Advanced Studies of the Universe
Department of Physics, University of Illinois at Urbana-Champaign, 1110 W. Green St., Urbana IL 61801-3080, USA
3 Instituto de Física, Universidade Federal Fluminense, Av. Milton Tavares de Souza, Niterói, Brazil, Zip Code: 24210-346,
4 Instituto de Física, Universidade de São Paulo, Rua do Matão, 1371, Butantã, 05508-090, São Paulo, Brazil
5 Departamento de Física - Centro de Ciências Físicas e Matemáticas, Universidade Federal de Santa Catarina, Campus Universitário Reitor João David Ferreira Lima, Florianópolis 88040-900, Brazil

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An outstanding problem in the field of relativistic heavy-ion collisions is the inability for any simulation model to accurately describe experimental flow data in extremely central collisions — in particular, models always predict either an elliptic flow that is too large or triangular flow that is too small (or both). We reassess the status of this puzzle in light of recent progress in Bayesian parameter estimation, in which a large model parameter space can be efficiently explored to determine what parameters are necessary for a good fit to experimental results, and how well state-of-the-art models are able to describe data. We explore predictions for flow in ultra-central collisions from multiple recent Bayesian models that were tuned to various observables in different collision systems at typical centralities. We find that, while ultra-central data can now be described with better accuracy than in previous calculations, tension with experimental observation remains, and progressively gets worse as one goes to more central collisions. Thus, the physics of ultra-central collisions is still not fully understood. A resolution to this puzzle will be an important step in solidifying our understanding of the physics of the strong interactions in these extreme conditions, as well as increasing our confidence in the results of precision analyses.

I. INTRODUCTION

High energy heavy-ion collisions are currently understood via complex, multistage hybrid hydrodynamic simulations usually composed of i) an initial condition model; ii) a pre-equilibrium phase; iii) a hydrodynamical expansion; iv) the conversion from fluid to particles (“particization”); v) final state dynamics. Each one of these ingredients requires its own model that contains various inputs. Recently, the (potentially large) parameter space associated to these simulations started to be systematically constrained by means of Bayesian parameter estimation. Of particular interest are the constraints put on the temperature dependence of transport coefficients such as shear and bulk viscosities that characterize important properties of the quark-gluon plasma — the strongly interacting and deconfined medium produced in heavy-ion collisions — but are notoriously difficult to calculate from first principles, either numerically [1,2] or via perturbative techniques [3,4].

There are several recent comprehensive Bayesian analyses focused on heavy-ion (as well as light-heavy ion) collisions [5,13]. While each analysis focuses on a particular set of collision systems and selected observables, all data considered in these studies come from measurements performed at typical centralities (covering at minimum 5% of the total cross section). Therefore, any constraints produced by these studies do not rely on the regime of ultra-central collisions, even though there exist data, for example, on flow harmonics at the energies considered [14,15]. This opens up the opportunity to test whether (and how) the constraints coming from non-ultra-central collisions affects the description of observables measured at ultra-central collisions.

The physics of ultra-central collisions is one where the impact parameter nearly vanishes, thus fixing (on average) the collision geometry (rotationally invariant for non-deformed nuclei). Therefore, any quantity must be driven by event-by-event fluctuations of the nuclear ge-
ometry. This is qualitatively different from non-central collisions where anisotropic flow develops due to stronger pressure gradients along the shorter direction of the “almond” shaped system formed by the collision participants \[16\]. In the latter case, the magnitude of elliptic flow depends crucially on the collision dynamics. Different models for particle production in the early-time system (for example a Glauber-type model versus a Color-Glass-Condensate picture \[17\]) predict different spatial eccentricities, which then determine final elliptic flow. In ultra-central collisions, on the other hand, all flow harmonics are generated only by fluctuations, and specifically are thought to be dominated by initial-state fluctuations driven by quantum fluctuations in nucleon positions within the nucleus before collision. Further, central collisions achieve the highest temperatures and have the longest lifetime. Because of this, modern hydrodynamic models should be most reliable in central collisions, and one would expect the best agreement with measured data.

Despite this, for ultra-central events involving 1% or less of the total number of collisions, for nearly a decade there is a universal inability for hydrodynamic models to simultaneously describe measured flow harmonics \(v_2\). Most notably, either the calculated elliptic flow \(v_2\) is larger than measurement, or the predicted triangular flow is too small (or both) \[18\] \[19\]. While the size of the discrepancy varies considerably (even when the problem was first noticed, some calculations approached 10% agreement at 0–1% centrality \[18\] Fig. 2b), it is striking how universal it is even at a qualitative level, in a regime that naively should be well understood.

Attempts to better understand this puzzle have been made: while the \textit{magma} model \[20\] is consistent with flow measurements in the 0–1% centrality bin \[14\] in an initial-state based description, this is not true anymore when using it as initial condition to a full hydrodynamic simulation \[21\]; Ref. \[21\] also explored alternate initial conditions, such as a modified version of the \textit{magma} model, as well as IP-Jazma initial conditions \[22\] (without sub-nucleonic degrees of freedom), which reproduces several features present in the IP-Glasma model without using lumpy initial conditions. No simultaneous description of \(v_3\) \[2\] has been achieved. The role of sub-nucleonic fluctuations was explored in \[23\] and did not help solve the issue. Reference \[24\] studied the effect of initial-state nucleon-nucleon correlations, and Ref. \[25\] studied the effect of bulk viscosity, both in the context of the IP-Glasma + hydrodynamics model. Furthermore, it has been pointed out in \[26\] that including an octupole deformation on the lead nucleus can somewhat improve the description of \(v_2\) – \(v_3\), but only at the expense of worsening the ratio of triangular flow cumulants, \(v_3\) – \(v_3\). Despite all of these efforts, the accurate and simultaneous description of flow harmonics in ultra-central collisions remains an open problem.

In this work, we reassess the status of the ultracentral flow puzzle in light of state-of-the-art hybrid hydrodynamic models that have been tuned with extensive model-to-data comparison \[6\] \[9\] \[12\] \[27\]. In particular, these models show excellent agreement with data in a range of typical centralities, and represent our best understanding of the collision system to date. It is therefore interesting to investigate to what extent these models can describe ultra-central data, and what we can learn from the centrality dependence of these models. This work follows similar study using one of these models \[12\], which we discuss and compare to in the following.

This paper is organized as follows: a summary of the simulation chains employed in Refs. \[6\] \[9\] \[12\] \[27\] is presented in section \[11\] in section \[11\] we make use of these simulation chains to study ultra-central Pb-Pb collisions at the TeV regime, comparing our results with experimental data when possible. Lastly, conclusions from this comparison are drawn in section \[11\].

II. MULTISTAGE HYBRID HYDRODYNAMIC SIMULATIONS

While modern Bayesian analyses share the basic structure described above, each analysis is unique due to \(i\) the specific details of how each stage is implemented and \(ii\) the collision system(s) (as well as the center of mass energy) and observable considered in each case. Here we provide a brief summary of the ingredients used in Refs. \[6\] \[9\] \[12\] \[27\]; we refer the reader to these references for a complete description of each simulation.

Due to its flexibility, each analysis employed the (boost invariant) TRENTo parametrization \[25\] to model the initial condition of their simulation; the notable difference being that Refs. \[7\] \[8\] \[12\] \[27\] allow for fluctuations of nucleon shape via sub-nucleonic degrees of freedom (“hot spots”) while \[6\] \[9\] considers a symmetric nucleon with only a varying radius. Once the initial energy density is obtained, the system is evolved according to a free-streaming prescription until a given proper-time \(\tau_{fs}\). References \[7\] \[8\] \[12\] \[27\] rely on a centrality independent value for \(\tau_{fs}\). The JETSCAPE collaboration \[9\], on the other hand, allowed \(\tau_{fs}\) to fluctuate event by event with some power of the mean energy density deposited at the transverse plane at the initial time with respect to some (arbitrarily defined) scale. The normalization and the power to which \(\tau_{fs}\) depends on the mean energy density were constrained by their Bayesian analysis. Further, while Refs. \[6\] \[7\] \[9\] assume a scale-invariant system of massless particles moving at the speed of light, Refs. \[8\] \[12\] \[27\] allow for a varying (though still uniform within a system) speed.

At the end of the free-streaming phase, it is assumed that the medium can be described by the relativistic viscous hydrodynamics equations of motion. All simulations solve second order Israel-Stewart-like hydrodynamic equations \[29\] \[32\]. An equation of state is needed in order to close the system of equations; all simulations use the the HotQCD \[33\] equation of state at high tem-
temperatures while the low temperature regime is described by a hadron resonance gas with different particle content, matched to the hadronic afterburner used in each calculation.

Regarding the temperature dependence of the first-order transport coefficients, each calculation uses a similar parametrization for the bulk viscosity: an unnormalized (possibly skewed) Cauchy distribution. The functional form assumed for the temperature dependence of the shear viscosity are also similar — two regions separated by a sudden change in slope. References \[6, 9\] assume two linear sections surrounding a variable transition temperature, while Refs. \[7, 8, 12, 27\] include possible curvature in the high-temperature region.

The switch from a fluid description to a particle description is made at an isotherm surface of temperature \( T_{\text{sw}} \). Fluid cells are then converted into particle distributions which are independently sampled from the switching hypersurface.

Due to the presence of dissipative corrections in the hydrodynamic phase, the distribution function to be sampled deviates from the equilibrium one. There are different ways to estimate the out-of-equilibrium correction. One option is to add a \( \delta f \) contribution, which is linear in the dissipative currents, to the equilibrium distribution, so that the total distribution is \( f = f_{\text{eq}} + \delta f \). Such Ansatz is, however, valid as long as \( \delta f \ll f_{\text{eq}} \), otherwise it may lead to negative distribution functions at higher momenta \[34\]. In order to avoid any possibility of negative probability densities, another option is to make use of “modified” equilibrium distribution, where viscous corrections are taken into account by re-scaling the temperature and momenta of the equilibrium distribution by factors that depend on out-of-equilibrium quantities \[35\]. The JETSCAPE collaboration has explored four different models for correcting the equilibrium distribution of hadrons; in this work we follow the setup that makes use of a the \( \delta f \) correction from the 14-moment approximation, identified as “Grad” in \[6, 9\], which gave a better fit to the experimental data studied in that work. The other analyses, on the other hand, make use of the Pratt-Torrieri-Bernhard modified equilibrium distribution \[7, 30\].

The hadrons sampled from the freeze-out hypersurface are then evolved according to transport equations. References \[7, 12\] use the UrQMD afterburner \[37, 38\], while Refs. \[6, 8, 9, 27\] use SMASH in their main analyses. The JETSCAPE collaboration presented a systematic comparison of results with SMASH \[29, 40\] and UrQMD. There were no discernible differences in results for the observables studied, as long as one matches the low-temperature regime of the equation of state to the particle content present in each model.

From a systematic data-to-model comparison, one can extract a posterior distribution, which represents the probability density for the set of model parameters to have a certain value, given the value of measured data. The maximum of the posterior in this multidimensional parameter space is known as the Maximum a Posteriori (MAP) point, and represents the most probable set of model parameters. With these parameters, the models have an excellent agreement with the experimental data used in these analyses. The Duke analysis \[7\] tuned their model to p-Pb and Pb-Pb data at 5.02 TeV, while JETSCAPE \[6, 9\] performed a simultaneous analysis of 2.76 TeV Pb-Pb collisions and 0.2 TeV Au-Au collisions. The first Trajectum analysis \[8, 27\] made a simultaneous analysis of 2.76 and 5.02 TeV Pb-Pb collisions and 5.02 TeV p-Pb collisions, while the second Trajectum analysis \[12\] used 2.76 and 5.02 TeV Pb-Pb data.

Thus, we see that while each analysis is substantially similar, there are various differences in model as well as the choice of collision systems and observables. In principle, these differences can significantly impact the final posterior distribution and the predicted values for other observables. For example, the maximum bulk viscosity \( \zeta/s(T) \) from the MAP of Refs. \[6, 9\] is more than a factor 5 higher than any of the other analyses.

In the next section, we extend the application of these simulation chains to flow in ultra-central Pb+Pb collisions, using knowledge obtained from the Bayesian parameter estimation. We assess to what extent the Bayesian constraints help us to understand the properties of the system formed in these extreme conditions.

### III. RESULTS

Here we apply the simulation chains described in the previous section to calculate two-particle flow harmonics in ultra-central Pb+Pb collisions. We use 4 different models that we will refer to as Duke \[7\], JETSCAPE \[6, 9\], Trajectum 1 \[8, 27\], and Trajectum 2 \[12\]. We performed simulations using the maximum a posteriori (MAP) parameter values from the respective Bayesian analyses of Duke \[7\], JETSCAPE’s “Grad” setup \[9\], and Trajectum 1 \[8, 27\].

For Trajectum 2, we follow Ref. \[12\] and use a sampling of the posterior (using 20 random samples), with the average and standard deviation of the results representing the midpoint and uncertainty of the model prediction. This latter calculation uses more of the information from the Bayesian analysis in order to quantify systematic uncertainty. Note that there may be nontrivial correlations between parameters, or redundancies. Different sets of parameters may result in a roughly equivalent description of the data that was used in the Bayesian analysis. If one uses only the MAP point to predict new observables (in this case, flow observables in different centrality bins), one cannot rule out the possibility that a different combination of parameters could give an almost equal description of the original data, but a quite different prediction for the new observables. By using the full posterior to predict ultra-central flow, we have a better idea of whether it would be possible to re-tune parameters to fit all centralities at the same time.


FIG. 1. Anisotropic flow \(v_n\) in 1% centrality bins, with \(n = 2, 3, 4\) for \(\text{Pb+Pb}\) collisions at 5.02 TeV predicted by the MAP parameters from the Duke [7] and \text{Trajectum 1} [8, 27], Bayesian analyses compared to measurements from the ALICE collaboration [11]. Also included is the result from \text{Trajectum 2} from Ref. [12] for comparison. Duke and \text{Trajectum 1} results have been slightly offset horizontally for better visibility.

Here we study only \(\text{Pb-Pb}\) collisions and limit the application of each model to the center-of-mass energy where the respective Bayesian analysis was carried out. In order to maximize statistics and allow for analysis of extremely central collisions, we select centrality via the total initial energy at \(\tau \to 0\) (i.e., the output of \text{TRENTo}). We note that in experiment, as well as the Bayesian analyses cited, centrality is selected via final multiplicity. While results can generally depend on how centrality is selected [14, 42], here we study the basic flow observable \(v_n\), where differences due to centrality determination are generally much smaller than the discrepancy between theory and experiment [12] — in the case of \text{Trajectum 1}, we verified this explicitly by comparing to results with centrality selected via final multiplicity.

In Figure 1, we show the results of \(v_2\), \(v_3\), and \(v_4\), predicted by the Duke and \text{Trajectum 1} setups, compared to measurements from the ALICE Collaboration [11]. We note excellent agreement in both cases at centralities above \(\sim 2\%\), both models having been trained from experimental data with the same range in transverse momentum, with the smallest centrality bin representing a combined \(0-5\%\). However, the centrality dependence (particularly for \(v_2\)) appears to deviate from the experimental trend starting at the 1–2% centrality bin. We can compare this to the results of \text{Trajectum 2} in Ref. [12] Fig. 14) that we have added to Fig. 1 and which shows the same trend.

Since the discrepancy between model and data does not exceed 10% here, one might wonder whether there still remains a puzzle — even in modern Bayesian analyses, it is difficult to fit all measurements at all centralities at better than \(\sim 10\%\) accuracy. Further, these models were not specifically tuned to reproduce these data, so it might be possible to obtain a better fit (while still retaining a good fit at other centralities) by re-tuning parameters.

To answer this, we first note that the discrepancy with measurement does not appear to be random — while having a slightly smaller magnitude than in past calculations, it has exactly the same qualitative features (the predicted ratio \(v_2/v_3\) being too large). Second, we note that there exist experimental data for more central bins (though only at lower collision energies), and it would be useful to know whether agreement continues to worsen. Finally, we can partially answer the question of the possibility of retuning parameters to fit ultra-central data by including the systematic uncertainty from the full Bayesian posterior.

We address all of these in Fig. 2 where we show \(v_2\), \(v_3\), and \(v_4\) measured by the CMS Collaboration [15] compared to predictions from the JETSCAPE Bayesian analysis [6, 9], as well as \text{Trajectum 1} [8, 27] and \text{Trajectum 2} [12]. In the latter case, we include a sampling of the full posterior to estimate the Bayesian systematic uncertainty, as described at the beginning of this section. To better see the individual bins, including ultra-central bins down to 0–0.02% centrality, we plot centrality on a log scale.

Here we see again that there is good agreement at moderate centralities [7] but a discrepancy appears and indeed

\[\text{ Each of the original Bayesian analyses were performed using} \]
continues to get worse as one goes to more and more central collisions. In particular, the experimental elliptic flow continues to decrease with centrality for all measured bins, while the model predictions become almost constant. Even with the included estimate of systematic uncertainty in the Trajectum 2 prediction, the agreement becomes poor.

In Fig. 3, we show the ratio \( v_3 / v_2 \), a simple illustration of how the most difficult aspect for simulation models to fit — relative values of elliptic and triangular flows. Experimental covariances are not available, so here we estimate the uncertainty on the ratio by assuming uncorrelated uncertainties between \( v_2 \) and \( v_3 \). This is almost certainly an overestimate, but even so, the discrepancy is clear — the ratio of triangular to elliptic flow in each model simulation reaches an almost constant value, while the experimental data continues to increase with a reduction in centrality, all the way down to 0–0.02% centrality (and possibly beyond). In this kinematic range, experimental \( v_3 \) actually reaches parity with \( v_2 \), a feature that is not seen in any simulation.

This indicates that there is still a problem, common to all these models, that is unlikely to be fixed by a further tuning of parameters.

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**IV. CONCLUSIONS**

We find that simulation models that have been tuned with the most recent Bayesian analyses all fail to describe flow in ultra-central collisions. While flow in moderately-central collisions is described almost perfectly in some cases (the combined 0–5% bin being the most central used to train each model), discrepancies appear at centralities near the percent level or lower, and worsen as centrality is decreased (i.e., more central). These discrepancies follow the universal behavior seen in the past — either elliptic flow is predicted to be too large, or triangular flow too small. In particular, the main problem appears to be in the centrality dependence of \( v_2 \), which has a qualitatively different trend in models compared to experimental data. We note that, while none of the models included ultra-central data in their respective Bayesian analyses, when we include information from the full Bayesian posterior, this discrepancy remains. This indicates that it may not be possible to adjust parameters to fit ultra-central data, while maintaining an equivalent fit to the rest of the data.

Therefore, we conclude that the ultra-central flow puzzle persists even today. There appears to be a universal feature in modern heavy-ion collision hydrodynamic models that prevents a correct description of flow in ultra-central collisions, despite naively being precisely the type of collisions that should be best described by hydrodynamics (being the largest and longest-lived systems). It will be of interest to identify this feature, and consistently repair these models. This will not only give potential physics insight, but will also allow for more confidence in simulation results, including the precise determinations of system properties in future Bayesian analyses.

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