ENHANCED GAUGE SYMMETRIES IN SUPERSTRING THEORIES

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ABSTRACT

Certain four-dimensional $N = 4$ supersymmetric theories have special vacua in which massive charged vector supermultiplets become massless, resulting in an enhanced non-abelian gauge symmetry. We show here that any two $N = 4$ theories having the same Bogomolnyi spectrum at corresponding points of their moduli spaces have the same enhanced symmetry groups. In particular, the $K_3 \times T^2$ compactified type II string is argued to have the same enhanced symmetry groups as the $T^6$-compactified heterotic string, giving further evidence for our conjecture that these two string theories are equivalent. A feature of the enhanced symmetry phase is that for every electrically charged state whose mass tends to zero as an enhanced symmetry point is approached, there are magnetically charged and dyonic states whose masses also tend to zero, a result that applies equally to N=4 super Yang-Mills theory. These extra non-perturbative massless states in the $K_3$ compactification result from $p$-branes wrapping around collapsed homology two-cycles of $K_3$. Finally, we show how membrane ‘wrapping modes’ lead to symmetry enhancement in D=11 supergravity, providing further evidence that the $K_3$-compactified D=11 supergravity is the effective field theory of the strong coupling limit of the $T^3$-compactified heterotic string.
1. Introduction

Non-abelian gauge symmetry is the crucial ingredient of the standard model of particle physics. It is hoped that the gauge symmetries of the standard model will eventually be understood in terms of some more fundamental theory that unifies the electro-weak and strong forces with gravity, such as superstring theory. Various ways in which non-abelian gauge symmetry can emerge from such unified theories have been proposed in the past, many of them involving extra compact dimensions. The most obvious way that this can happen is if the non-abelian gauge symmetry is already an ingredient of the candidate fundamental theory. An example is the $E_8 \times E_6$ gauge symmetry of the heterotic string theory compactified on a Calabi-Yau manifold, which has its origin in the $E_8 \times E_8$ gauge symmetry of the ten-dimensional theory. Another possibility is the Kaluza-Klein (KK) mechanism in which the gauge symmetry arises from the isometry group of an internal compact space. A third way is the Halpern-Frenkel-Kač (HFK) mechanism for symmetry enhancement in toroidally-compactified string theories at weak string coupling (see, for example, [1]). The simplest example of the HFK mechanism occurs for the bosonic string compactified on a circle of radius $R$; for generic values of $R$, there is only a $U(1)$ gauge symmetry of Kaluza-Klein origin but at the self-dual radius this is enhanced to $SU(2)$. Another example is the toroidal compactification of the heterotic string to four dimensions for which the gauge group is $U(1)^6 \times G$. The group $G$ is $U(1)^{22}$ for generic compactifications, but is a non-abelian group of rank 22 [2] at special points in the moduli space of the compactification. A form of the HFK mechanism is also applicable to the type II string in the covariant lattice approach [3], in which the fermionic constructions of [4] are recovered as special cases. However, in the type II string this mechanism does not lead to realistic theories [5].

In these string theory examples, the HFK symmetry enhancement is due to massive spin one states in the spectrum becoming massless at special points of the moduli space. Some of these massive states can be interpreted as due to string
winding modes, and the possibility of symmetry enhancement by this mechanism has therefore been seen as an intrinisically ‘stringy’ one. However, we will show that symmetry enhancement in fact occurs for any theory which has an \( N = 4 \) supersymmetric low-energy effective action and which has the requisite massive states; these might arise either as perturbative states (e.g. Kaluza-Klein modes) or as solitons. The symmetry enhancement is understood in terms of the effective field theory and is independent of the nature of the compactifying space, e.g. whether or not it has isometries.

Mass eigenstates of any four-dimensional theory with \( N \geq 2 \) supersymmetry can be chosen to be simultaneous eigenstates of the central charges appearing in the supersymmetry algebra, and the mass of any central charge eigenstate is bounded by the values of its central charges [6]; this is generally called the ‘Bogomolnyi bound’. In the supergravity context, i.e. for theories with local supersymmetry, the \( N(N-1) \) central charges are the electric and magnetic charges associated with the \( N(N-1)/2 \) abelian gauge fields in the graviton supermultiplet. In this case, the Bogomolnyi bound [7] is a lower bound on the ADM energy of an asymptotically-flat spacetime given in terms of the total electric and magnetic charges, and the asymptotic values of the scalar fields. These scalar field values parameterise the possible classical vacua, and can be interpreted as coupling constants of the theory. Classically, the electric and magnetic charges can be chosen arbitrarily but in the quantum theory the Dirac-Schwinger-Zwanziger (DSZ) quantization condition restricts their values to a lattice, given the existence of electrically and magnetically charged states of each type. The mass of any ‘Bogomolnyi state’ saturating the Bogomolnyi bound is then determined by its location on the charge lattice and the choice of vacuum, i.e. the scalar field expectation values.

For pure \( N \geq 2 \) supergravity theories without matter coupling, the only zero mass states saturating the bound are those with zero electric and magnetic charges. A new feature emerges for \( N \geq 2 \) Maxwell/Einstein supergravity theories, i.e. \( N \geq 2 \) supergravity coupled to abelian vector supermultiplets, as a result of the additional electric and magnetic charges. As shown in an explicit \( N = 2 \) example
[8], there can be charged states that saturate the bound but whose masses vanish at special points of the scalar field target space, i.e. for special choices of the vacuum. Some of these massless states have spin one and this results in an enhancement of the abelian symmetry to a non-abelian group. However, these $N = 2$ results only hold in a semi-classical approximation and the masses will in general receive quantum corrections [8], even when the $N = 2$ theory is the low-energy effective action for a superstring theory, such as the type II string compactified on a Calabi-Yau manifold or the heterotic string compactified on $K_3 \times T^2$. Here we shall concentrate on theories with at least $N = 4$ supersymmetry as in such cases the masses and charges of Bogomolnyi states are believed to receive no quantum corrections, thus justifying semi-classical reasoning based on a non-renormalizable effective theory. We shall show that for any theory for which the effective four-dimensional field theory is an $N = 4$ Maxwell/Einstein supergravity, a non-abelian gauge symmetry will appear at special points of the sigma-model target space, provided only that there exist Bogomolnyi states in the spectrum carrying the appropriate charges.

For the toroidally compactified heterotic string the effective four dimensional field theory is, for generic compactifications, N=4 supergravity coupled to 22 abelian vector multiplets. The required electrically charged Bogomolnyi states arise in string perturbation theory, so that there exist points in moduli space of enhanced gauge symmetry, in agreement with the results of the HFK mechanism. More significantly, the same supergravity theory is also the effective four dimensional field theory of either the type IIA or the type IIB superstring compactified on $K_3 \times T^2$. This fact had been noted on various occasions in the past but was considered to be merely a coincidence. We argued in [9] that the massive spectrum could also be the same once account was taken of non-perturbative states resulting from wrapping $p$-brane solitons of the ten-dimensional string round homology cycles of $K_3 \times T^2$, and a study of these ‘wrapping modes’ led us to conjecture that the toroidally compactified heterotic and $K_3 \times T^2$ compactified type II superstrings are actually equivalent. We pointed out that this conjecture implies the existence of special points of enhanced symmetry in the moduli space of $K_3 \times T^2$ compacti-
fications of type II superstrings and we suggested that this might occur as a result of some non-perturbative mechanism involving \(p\)-brane solitons.

This suggestion, and the conjecture, became much more plausible after Witten pointed out that for every point in the heterotic string moduli space at which the symmetry was enhanced, the corresponding point in the \(K_3\) moduli space was associated with a limit in which certain homology two-cycles of \(K_3\) \cite{10} collapse to zero area. To see the relevance of this observation, we recall the identification in \cite{9} of massive modes carrying 22 of the 28 electric charges with the wrapping modes of the 2-brane soliton of the type IIA superstring around the 22 homology cycles of \(K_3\). Since the mass of these states is proportional to the area of the two-cycle the mass should vanish when the two-cycle collapses to zero area. Moreover, since these states are BPS saturated they belong to ultrashort multiplets which can turn into massless vector supermultiplets as the mass goes to zero, as required for symmetry enhancement.

One potential problem with this mechanism of symmetry enhancement, which was discussed in the context of \(N=2\) superstring compactifications in \cite{11}, is that the concept of a \(p\)-brane soliton is intrinsically semi-classical; the description of states as \(p\)-brane wrapping modes makes sense only when the \(p\)-brane tension is large and the \(p\)-cycles around which they are wrapped are large relative to the scale set by the \(p\)-brane core. It is clear that one needs additional input from supersymmetry to justify the extension of \(p\)-brane wrapping modes to the limit in which the \(p\)-cycle collapses. In the case of \(N=2\) superstring compactifications, the effective supergravity theory is not uniquely determined by supersymmetry so that considerations of renormalization must play a role. The picture now emerging \cite{12,13,14} is of a stringy generalization of the work of Seiberg and Witten \cite{15} and Ceresole \textit{et al.} \cite{9}. Here we shall be examining this mechanism for symmetry enhancement in the context of \(N=4\) superstring compactifications. In this case, the effective supergravity theory \textit{is} determined uniquely by supersymmetry (once the spectrum of massless states and the Yang-Mills gauge group are known).
As we shall see, for theories with $N = 4$ supersymmetry the masses of Bogomolnyi states are also determined entirely by the low-energy effective field theory. Thus, once the existence of a massive Bogomolnyi state has been established at some particular point in the moduli space of vacua, by whatever means, its mass at other points in the moduli space is determined by the effective low-energy supergravity theory. In particular, the mass of certain Bogomolnyi states must vanish at special points in this moduli space purely as a consequence of N=4 supersymmetry. Since these states must fill out vector supermultiplets the proof of symmetry enhancement in N=4 supersymmetric theories rests on the existence of the relevant massive states at generic points in moduli space. Although we provide a characterisation of the relevant states in various superstring compactifications, we rely on semi-classical methods to establish their existence in the spectrum when these states are not in the perturbative string spectrum. As in our previous work [9] on the non-perturbative Bogomolnyi spectrum, which we amplify here, we identify the required states as $p$-brane wrapping modes.

The results so obtained provide further evidence for the conjectured equivalence of the $K_3 \times T^2$ compactified type II string to the $T^6$ compactified heterotic string. However, it is possible that these two theories have identical Bogomolnyi spectra but differ in their non-Bogomolnyi spectra. Indeed, since supersymmetry provides no clues to the spectrum of states in full, unshortened, supermultiplets it might seem unlikely that any two $N = 4$ theories would be the same even if their spectra of shortened supermultiplets were to coincide in every detail. Of course, it may be that there are specifically ‘stringy’ reasons, not directly related to supersymmetry, why the non-Bogomolnyi spectra must also coincide, but it is also possible that there is a sense in which there are no non-Bogomolnyi states in the full theory, despite their occurrence in perturbation theory. States occurring in perturbation theory that do not saturate a Bogomolnyi bound (strong or weak) can be expected to correspond to particles that become unstable either at higher orders of perturbation theory or non-perturbatively because there would appear to be nothing to prevent their decay into Bogomolnyi states. In this case they could not appear in an exact
S-matrix.

Another theory for which the effective four-dimensional field theory is an N=4 supergravity is D=11 supergravity compactified on $K_3 \times T^3$ [16]. In fact, this effective field theory is identical to that of the $K_3 \times T^2$ compactified type II string. Until recently this was thought to be just another coincidence, but in [9] it was pointed out that, when account is taken of the p-brane solitons in the higher dimension, the soliton spectrum in four dimensions is the same for D=11 supergravity compactified on $B \times T^3$ as for a type II superstring compactified on $B \times T^2$, where $B$ is either $T^4$ or $K_3$. This result hinted at an 11-dimensional interpretation of the D=10 type IIA superstring theory but it was not clear then how such an interpretation could be consistent with the fact that the critical dimension of perturbative string theory is D=10. It was pointed out in [17] that this difficulty is resolved non-perturbatively by the existence in type IIA string theory of the BPS saturated extreme black holes, found earlier by Horowitz and Strominger [18], since the corresponding quantum states can be interpreted as towers of massive Kaluza-Klein states. It was subsequently argued in [10], for similar reasons, that D=11 supergravity theory is the effective field theory for the strong coupling limit of the type IIA superstring. Specifically, the D=11 interpretation of the type IIA superstring implies an identification of the string coupling constant with $R^{2/3}$ where $R$ is the radius of the extra dimension. Thus, string perturbation theory is a perturbation expansion about $R = 0$, which explains why perturbative superstring theory seems to require a spacetime of dimension D=10.

It was further argued in [10] that the seven-dimensional field theory obtained by $K_3$ compactification of D=11 supergravity is the effective action for the strong coupling limit of the $T^3$ compactified heterotic string. Further evidence for this proposition is the fact that the $T^3$ compactified heterotic string can be viewed [19] as a double-dimensional reduction on $K_3$ of the D=11 fivebrane soliton of [20]. If this proposition is indeed true then it follows that the $K_3 \times T^3$ compactified D=11 supergravity is the effective four-dimensional field theory for a limit of the $T^6$ compactified heterotic string in which one of the coupling constants (moduli)
becomes large. An obvious question that arises in this case is whether the symmetry enhancement known to occur for the heterotic string can be seen from the standpoint of D=11 supergravity. Since the latter is merely an effective field theory it is not obvious that this should be possible. Indeed, it is not possible within the confines of Kaluza-Klein theory but we shall show that symmetry enhancement can be understood once $p$-brane solitons of D=11 supergravity are included. If there were some consistent quantum theory in 11-dimensions underlying D=11 supergravity one might consider this result to be evidence of its equivalence, after compactification, to a superstring theory. There are reasons to believe [17,19] that this 11-dimensional theory is the D=11 supermembrane of [21], although we are a long way from understanding its quantization.

A caution is in order before we continue with the derivation of the results described above. In the toroidally compactified bosonic string the HFK mechanism leads to (perturbative) symmetry enhancement in the effective four-dimensional field theory but, since the effective non-abelian gauge theory is asymptotically free, the conventional wisdom is that all particles carrying non-abelian charge (which includes the massless non-abelian gauge bosons) are confined. Thus, the extra massless states found in perturbation theory via the HFK mechanism are not likely to be present in the full theory. If it were possible to bypass perturbation theory and deal directly with the full quantum string theory*, one might expect to find a transition to a confining phase at special points of the moduli space, rather than the occurrence of additional massless states at these points.

The status of the HFK mechanism in the toroidally compactified heterotic string is quite different because the effective non-abelian gauge theory at special points of moduli space is not asymptotically free; in fact, the beta function vanishes. Since confinement no longer operates to remove massless particles with non-abelian charge from the spectrum it might be thought that here, in contrast to the bosonic string, the extra massless particles found in perturbation theory (either by the

* Because of the tachyon it is not clear that this exists.
HFK mechanism or, as explained here, as a consequence of $N = 4$ supersymmetry) indicate the existence of extra massless states in the full theory. However, the infrared divergences due to unconfined non-abelian gauge fields do not allow a standard interpretation of the Hilbert space in terms of particles with definite charge quantum numbers. Instead, the existence of vector fields whose masses tend to zero as one approaches special points in moduli space signals a transition to a non-abelian Coulomb phase at these points in which there is a non-abelian gauge symmetry associated with long-range forces; other theories with such phases have been investigated in [22].

As long as we are away from special points in moduli space there is no problem in providing a standard particle interpretation for the spectrum. At first sight there would appear to be no difficulty in extrapolating this spectrum to those special points at which some massive Bogomolnyi states become massless. Symmetry enhancement in the toroidally compactified heterotic string has so far only been analysed in string perturbation theory, with the result that electrically charged Bogomolnyi perturbative string states become massless at special points in moduli space. However, the heterotic string has non-perturbative magnetically charged Bogomolnyi states arising from BPS monopole and dyonic solutions of the low-energy theory [23,24,25]. We shall show that for every electrically charged state whose mass tends to zero as one approaches a special point in moduli space, there is also a magnetically charged state and (assuming S-duality [26]) an infinite set of dyonic ones whose masses also approach zero. The interpretation of this is not clear, but possibly signals a non-abelian Coulomb phase of a type rather different from those discussed in [22]. However, it does show how symmetry enhancement might be consistent with the conjectured S-duality of the heterotic string: whenever the mass of a perturbative string state tends to zero, so do the masses of its magnetically charged $SL(2, \mathbb{Z})$ partners. Thus, even for the toroidally compactified heterotic string we learn something more about the symmetry enhancement mechanism from the analysis based on $N = 4$ supersymmetry than we learn from the HFK mechanism. A similar picture emerges for the type II string on $K_3 \times T^2$. 

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In fact, since this feature of the enhanced symmetry phase depends only on $N = 4$ supersymmetry it applies equally to $N = 4$ super Yang-Mills theory, i.e. as the Higgs expectation value tends to zero, all charged massive states, including the magnetic monopoles and dyons, become massless together. Whatever the correct interpretation of this may be, we wish to stress here that it is a general feature. In particular, if, as argued in [9] and again here, the Bogomolnyi spectrum of the $K_3 \times T^2$ compactified type II string is the same as that of the toroidally compactified heterotic string at some generic points of their respective moduli spaces, then whatever happens to the heterotic string at special points also happens to the type II superstring.

The thrust of this paper is that one can establish symmetry enhancement in superstring compactifications to four dimensions preserving $N = 4$ supersymmetry merely by an analysis of the low-energy effective field theory. We do this in two steps. First, as shown in the following section, the mass of a Bogomolnyi state with a given charge vector is determined entirely by this effective field theory, so that symmetry enhancement is the consequence of the mere existence in the spectrum of certain states. We examine some consequences of this result in the context of specific $N=4$ superstring compactifications in section 3; in particular we extend the results of the perturbative HFK mechanism for the toroidally compactified heterotic string to the full non-perturbative string theory. Second, as we explain in section 4, the existence of the states relevant to symmetry enhancement for $K_3$ compactifications can be deduced by consideration of the $p$-brane soliton solutions of the effective supergravity theory [27-32] in a limit in which semi-classical methods are reliable, because $N = 4$ supersymmetry tells us what happens to these states in all other regions of parameter space. In section 5 we explain how these ideas can be used in the context of compactifications of D=11 supergravity.

The results of sections 2 and 3 were announced at the Strings '95 conference [33]. At the same meeting, the results of [10] were also announced, including some discussion of symmetry enhancement in compactified type II theories.
2. Symmetry Enhancement in $N = 4$ Supergravity

We now wish to see what can be learned directly from an analysis of the effective four-dimensional theory for any theory which has at least $N = 4$ local supersymmetry in $d = 4$. We shall assume the existence of certain Bogomolnyi states and postpone most of our discussion of their origin until the following sections. The bosonic massless fields of an $N \geq 4$ supergravity theory are the four-dimensional space-time metric $g_{\mu\nu}$, scalars $\phi^i$ taking values in a sigma-model target space $M$ with metric $g_{ij}$ and vector fields $A^I_\mu$ with field strengths $F^I_{\mu\nu}$. The gauge group has rank $k$ and is abelian for generic points in the moduli-space, in which case $I = 1, \ldots, k$. The lagrangian, omitting fermions, is

$$L = \sqrt{-g} \left( \frac{1}{4} R - \frac{1}{2} g_{ij}(\phi) \partial_\mu \phi^i \partial^\mu \phi^j - \frac{1}{4} m_{IJ}(\phi) F^{I\mu\nu} F^J_{\mu\nu} - \frac{1}{8} \varepsilon^{\mu\nu\rho\sigma} a_{IJ}(\phi) F^{I}_{\mu\nu} F^J_{\rho\sigma} \right)$$

(2.1)

for some matrix functions $m, a$, with $m$ being positive definite. These massless fields may couple to massive fields, which can be viewed as sources for $(g, A, \phi)$. It is a feature of such theories that the mass of any field configuration satisfies a classical bound of the form [7,9,25,34]

$$M^2 \geq Z^A R_{AB}(\tilde{\phi}) Z^B$$

(2.2)

where

$$Z = \begin{pmatrix} p^I \\ q_I \end{pmatrix}$$

(2.3)

and $p$ and $q$ are the magnetic and Noether electric charges defined by integrals over the two-sphere at spatial infinity, as in [9]. The matrix $R$ is a function of the asymptotic values $\tilde{\phi}^i$ of the scalar fields. In the quantum theory, a similar bound applies to all quantum states, with the numbers $\tilde{\phi}^i$ now to be interpreted as the expectation values of the scalar fields $\phi^i$, parameterising the possible vacua.
The charge vector $Z$ satisfies the DSZ quantization condition, which implies that $q$ takes values in some lattice $\Gamma$ and $p$ takes values in the dual lattice $\tilde{\Gamma}$. The matrix $R$ is a continuous function of $\bar{\phi}$, so that the masses of Bogomolnyi states are also continuous functions of $\bar{\phi}$. Under certain circumstances, to be elaborated shortly, the matrix $R_{AB}(\bar{\phi})$ has a (fixed) number of zero eigenvalues, for all values of $\bar{\phi}$. This might make it appear that there should be extra massless particles for all values of the moduli, but this is not the case for two reasons. First, at any given point on moduli space there may be no points in the charge lattice that lie in the Kernel of $R_{AB}(\bar{\phi})$. Second, as we shall see in more detail later, not all points in the lattice of charges allowed by the DSZ quantization condition actually occur in a given theory. In particular, in string theory only those points in the electric charge lattice that are consistent with the physical state conditions of perturbative string theory can correspond to states in the string spectrum, and there are no such points whose charges are in the kernel of $R_{AB}$ for generic points in moduli space. For special values of $\bar{\phi}$, however, a finite number of string states have charge vectors in the kernel, so that they become massless. Conversely, the mass of a Bogomolnyi state with given charge vector can vanish only for certain values of $\bar{\phi}$. We shall return shortly to consider the circumstances under which all conditions for massive Bogomolnyi states to become massless at special points in moduli space can be satisfied in a string theory, but we shall first examine some general consequences of $N = 4$ or $N = 8$ supersymmetry in the event that this phenomenon occurs.

We shall consider here only those Bogomolnyi states which preserve half the supersymmetry of the vacuum. States which preserve some, but less than half, the supersymmetry are also sometimes called ‘Bogomolnyi states’ but they saturate a stronger bound and will not be relevant to the phenomena to be discussed here, for reasons to be explained below. Bogomolnyi states that preserve half the supersymmetry fit into ultra-short $N = 4$ or $N = 8$ massive supermultiplets with highest spin $h$; these have the same spectrum of helicity states as the corresponding massless supermultiplets with highest spin $h$ (apart from the obvious charge
doubling). In a given vacuum, the ultra-short supermultiplets are precisely those that saturate the bound (2.2). However, as the moduli are varied, it is possible that states that fit into 16 ultra-short multiplets saturating (2.2) for a given point in moduli space may combine into a long multiplet that does not saturate (2.2) for other points. The number of ultra-short Bogomolnyi supermultiplets is therefore conserved modulo 16 (but note that not every combination of 16 ultra-short supermultiplets can be combined into an unshortened supermultiplet because the latter has highest spin $h \geq 2$ whereas an ultra-short supermultiplet has highest spin $h \geq 1$).

Thus, as $\bar{\phi}$ is continued to $\bar{\phi}_0$, the ultrashort Bogomolnyi supermultiplets with charge vector $Z_0$ must continue (at least modulo 16) to massless supermultiplets with the same highest spin. We do not expect the new massless supermultiplets to have highest spin $h \geq 2$, as these would lead to well-known inconsistencies*. Since all $N = 8$ supermultiplets have highest spin of at least two we should not expect any massive supermultiplets to become massless at special points in the moduli space of compactifications that preserve $N = 8$ supersymmetry, e.g. the $T^6$ compactification of the type II superstring. We shall verify this prediction below. For $N = 4$ there remain two possibilities: $h = 1$ and $h = 3/2$.

Consider first the $h = 3/2$ case†. The existence of additional massless spin-$3/2$ states implies an enhanced $N > 4$ local supersymmetry, but this is possible only if all massless states belong to the graviton supermultiplet, since there are no massless matter supermultiplets (with $h \leq 1$) for $N > 4$. Moreover, the total number of massless vectors would increase since the $N = 4$ supermultiplet with $h = 3/2$ contains vector fields. In the cases of most interest to us here, the toroidally compactified heterotic string or type II on $K_3 \times T^2$, the number of massless vector fields at a generic point in the moduli space is already 28, so that we would need

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* These inconsistencies might be avoided if an infinite number of supermultiplets become massless. This occurs in a ‘decompactification limit’ in which some compact dimensions become non-compact, or in a null string limit. Such phenomena, which we will not consider here, are associated with points on the boundaries of moduli space.

† This has been recently considered in [35].
an effective $N > 4$ supergravity with more than 28 vector fields. There is no such theory (the $N = 8$ theory has exactly 28). Moreover, the gauge group of the massless vector fields would have to be non-abelian, for reasons explained below, and it is difficult to reconcile this with a vanishing cosmological constant in a pure supergravity theory. For these reasons, we exclude the possibility of additional $h = 3/2$ massless supermultiplets. Since partially shortened supermultiplets saturating a stronger bound must have highest spin $h \geq 3/2$, this exclusion explains why we may restrict our attention to ultrashort multiplets.

This leaves the possibility that massive $N = 4$ vector multiplets become massless at special points in the moduli space of a compactification preserving $N = 4$ supersymmetry. Ultra-short massive vector multiplets come in central charge doublets which couple to the vector field $A^0$ of the corresponding central charge. Consider the case in which only one such charge doublet with vector fields $A^+, A^-$ (and their superpartners) becomes massless. Since the effective massless theory now contains three vector fields with a trilinear $A^0 A^+ A^-$ coupling, consistency implies that the original $U(1)^k$ gauge symmetry is enhanced to the non-abelian group $U(1)^{k-1} \times SU(2)$. Note that there are also additional massless scalars, but that these have quartic interactions (as required by $N = 4$ supersymmetry) so that their expectation values do not constitute new moduli. More generally, several charged doublets may become massless simultaneously, leading to an enhanced symmetry group of higher dimension. The rank, however, must remain equal to $k$ (and the maximal rank of the maximal simple subgroup equal to $k - 6$). This is because each of the additional massless vector multiplets is charged with respect to one of the $k$ original $U(1)$'s.

We now investigate the conditions under which the matrix $\mathcal{R}$ has zero eigenvalues. Consider first the case of $N = 8$ supergravity, which can be interpreted as the effective theory for the toroidally compactified type II superstring since the moduli space of this compactification is $E_7/(SU(8)/\mathbb{Z}_2)$ and it preserves $N = 8$ supersymmetry. The scalar fields of $N = 8$ supergravity take values in the coset space $E_7/(SU(8)/\mathbb{Z}_2)$ and can be represented by a $56 \times 56$ matrix function $\mathcal{V}(x)$ taking
values in $E_7$. This transforms under rigid $E_7$ transformations with parameter $\Lambda$ and local $SU(8)$ transformations with parameter $h(x)$ as

$$V(x) \to h(x)V(x)\Lambda^{-1}. \quad (2.4)$$

The $56 \times 56$ $R$-matrix for this theory is given by

$$R = V^tV, \quad (2.5)$$

where $V^t$ is the transpose of $V$. For every point in the coset space $E_7/[SU(8)/Z_2]$, the matrix $V$ is non-degenerate and so $R$ must have non-zero determinant. This means that, as predicted, there can be no points at which $R$ has zero eigenvalues.

We now turn to $N = 4$ supergravity coupled to $m$ vector multiplets. The number of vector fields is $k = 6 + m$, since the supergravity multiplet contains six vector fields. The scalars take values in the coset space $G/H$ where $G = SL(2;\mathbb{R}) \times O(6,m)$ and $H = U(1) \times O(6) \times O(m)$. The scalars can be represented by a $2k \times 2k$ matrix function $V(x)$ taking values in $G$. In formulating the theory [36,37,9], it is also useful to introduce a scalar-dependent $6 \times k$ matrix $t$, of rank 6, which converts $SO(6,m)$ indices into $SO(6)$ indices. Introducing the $12 \times 2k$ matrix $K = \mathbb{1} \otimes t$ where $\mathbb{1}$ is the $2 \times 2$ identity matrix, the $R$-matrix takes the form

$$R = V^tK^tKV. \quad (2.6)$$

As in the $N = 8$ case, the determinant of $V$ is non-zero but, by construction, the rank of $K^tK$ is precisely 12, so $R$ has precisely $2m$ eigenvectors with zero eigenvalue (when $m = 0$, $K^tK$ is the identity matrix).

To investigate the nature of these zero-eigenvalue eigenvectors it is convenient to split the modulus $\tilde{\phi} = (\varphi, \lambda) \in G/H$ into a complex coordinate $\lambda \in SL(2,\mathbb{R})/U(1)$ and coordinates $\varphi \in O(6,m)/O(6) \times O(m)$ and rewrite the
$N=4$ Bogomolnyi mass formula in the form [38,23]

$$M^2 = \frac{1}{16} (p \ q) [S \otimes (M + L)] \begin{pmatrix} p \\ q \end{pmatrix}$$  \hspace{1cm} (2.7)$$

where

$$S = \frac{1}{\lambda_2} \begin{pmatrix} |\lambda|^2 & \lambda_1 \\ \lambda_1 & 1 \end{pmatrix}$$  \hspace{1cm} (2.8)$$

is an $SL(2,\mathbb{R})$ matrix depending on $\lambda = a_4 + ie^{\Phi_4} = \lambda_1 + i\lambda_2$, where $a_4$ and $\Phi_4$ are scalars that we shall refer to as the four-dimensional axion and dilaton respectively. For the toroidally compactified heterotic string, $\Phi_4$ comes from the ten-dimensional dilaton $\Phi_{10}$, while for the type II string compactified on $K_3 \times T^2$, $\Phi_4$ comes not from the ten-dimensional dilaton but from one of the $T^2$ moduli. Here, $L$ is the invariant metric of $O(6,m)$ (with six eigenvalues of +1 and $m$ of −1) and $M(\varphi)$ is an $O(6,m)$ matrix whose explicit form is given, for $m = 22$, in [38,23].

As $S$ is unimodular, any zero mass states in the spectrum correspond to the zero eigenvalues of the $(6+m) \times (6+m)$ matrix $M(\varphi) + L$. There is a special point in $\varphi$-space at which $M = \mathbf{1}$ (the identity matrix). A general point $\varphi$ in moduli space is obtained by acting with an $O(6,m)$ transformation represented by a matrix $\Omega(\varphi)$, in which case $M(\varphi) = \Omega(\varphi)^t \Omega(\varphi)$. Thus $M + L = \Omega^t(\mathbf{1} + L)\Omega$ has $m$ zero eigenvalues and so the matrix $\mathcal{R} = S \otimes (M + L)$ has $2m$ zero eigenvalues, for all values of $\bar{\phi} = (\lambda, \varphi)$, in agreement with our analysis above. Furthermore, a given $(6+m)$-vector $V$ will be in the kernel of $M(\varphi) + L$ if and only if $\Omega(\varphi)V$ is in the kernel of $\mathbf{1} + L$. In particular, this requires the $O(6,m)$ norm, $V^2 \equiv V^I L_{IJ} V^J$, of $V$ to be strictly negative. Thus, an electrically charged Bogomolnyi state with $q = V$ and $p = 0$ can never become massless if $V^2 \geq 0$, whereas if $V^2 < 0$ it will be massless for precisely those values of $\varphi$ for which $V = \Omega(\varphi) V$ is in the kernel of $\mathbf{1} + L$. Moreover, for any $V$ with $V^2 < 0$, there will be values of $\varphi$ for which $V$ represents a massless state, since there will be $O(6,m)$ transformations that rotate $V$ into the kernel of $\mathbf{1} + L$. 

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Instead of dealing with electric charge vectors $q \in \Gamma$, it is sometimes useful to consider the charge vectors $\hat{q} \in \hat{\Gamma}$ where $\hat{q} = \Omega(\varphi)q$ and $\hat{\Gamma}$ is the lattice obtained from $\Gamma$ by acting with the $O(6, m)$ transformation $\Omega(\varphi)$. For electrically charged states, the mass formula (2.7) is

$$M^2 = \frac{1}{16\lambda_2} \hat{q}^t (1 + L) \hat{q}$$

so that instead of considering fixed charges and varying the matrix $M$, we can vary the lattice $\hat{\Gamma}$ and keep the lattice metric fixed. We are interested in those points in the lattice $\hat{\Gamma}$ that also lie in the kernel of $1 + L$. It can happen that $\hat{\Gamma}$ has no basis vectors that are annihilated by $1 + L$, in which case no massless states arise for that value of $\varphi$. If $\hat{\Gamma} \cap \ker(1 + L)$ is non-trivial, the next question to be addressed is whether the potential massless states arise in a given theory, since not all points in $\hat{\Gamma}$ need correspond to physical states. The lattice $\hat{\Gamma}$ of allowed charges can be decomposed into $O(6, m; \mathbb{Z})$ orbits, each of which has a particular value of the $O(6, m)$ norm $q^2 = \hat{q}^2$. As we have just seen, only those orbits with $q^2 < 0$ can give rise to massless states. In string theory, most states have $q^2 \geq 0$ so that only a small part of the spectrum can give rise to extra massless states in this way.

From the mass formula (2.7), it is clear that if an electric Bogomolnyi state with charge $(p, q) = (0, V)$ becomes massless at some special modulus $\varphi_0$, then any state with charge vector $(p, q) = (mV, nV)$ with $m, n$ integers will have a mass that also tends to zero as $\varphi \to \varphi_0$. This fact has some interesting implications for string theory compactifications, which we now examine in further detail.
3. Application to Superstring Compactifications

We shall now show how the results just obtained apply to specific superstring compactifications. For both the generic $T^6$ compactification of the heterotic string and the $K_3 \times T^2$ compactified type II superstrings the effective four-dimensional field theory is $N=4$ supergravity coupled to 22 abelian vector supermultiplets, so the discussion of section 2 applies with $m = 22$. In each case the scalar field sigma-model of the effective four-dimensional field theory has a target space given by the product of $O(22,6)/[O(22) \times O(6)]$, parameterised by $\varphi$, and $SL(2,\mathbb{R})/U(1)$, parameterised by $\lambda$. The equations of motion of the supergravity theory are invariant under the continuous symmetry group

$$SL(2;\mathbb{R}) \times SO(6,22)$$ \hspace{1cm} (3.1)

Quantum effects break this to the discrete group

$$SL(2;\mathbb{Z}) \times SO(6,22;\mathbb{Z}) .$$ \hspace{1cm} (3.2)

For both the $T^6$ compactified heterotic string and the $K_3 \times T^2$ compactified type II superstring this discrete group is conjectured to be a symmetry of the spectrum. In the case of the heterotic string, $SO(6,22;\mathbb{Z})$ is the T-duality group and $SL(2;\mathbb{Z})$ is the S-duality group. As we shall see later, for the type II string the situation is less straightforward: the T-duality group of perturbative symmetries contains $SL(2;\mathbb{Z})$ and an $SL(2;\mathbb{Z}) \times O(4,20;\mathbb{Z})$ subgroup of $O(6,22;\mathbb{Z})$, while the $SL(2;\mathbb{Z})$ S-duality group acting on the dilaton is also a subgroup of $O(6,22;\mathbb{Z})$. The full symmetry (3.2) was conjectured in [9] to be a symmetry of the non-perturbative spectrum and has recently been investigated further in [39]. Note that although T-duality is known to be a symmetry of the perturbative spectrum its status as a symmetry group of the full non-perturbative spectrum remains conjectural.

Consider the $T^6$ compactified heterotic string. In this case the parameters $\varphi$ are the $T^6$ moduli and the real and imaginary parts of the complex variable $\lambda$
are the constant values of the axion and dilaton fields [38]. For weak coupling, i.e. \( g_4 << 1 \), where \( g_4 \equiv \langle e^{\Phi_4} \rangle \) and is the same as the ten-dimensional string coupling \( \langle e^{\Phi_{10}} \rangle \) for the heterotic string on \( T^6 \), the electrically charged Bogomolnyi states arise as modes of the fundamental string and have charges \( q \) for which \( q^2 \) is even and satisfies \( q^2 \geq -2 \). We have shown that states with \( q^2 \geq 0 \) do not become massless, which is just as well since that would have meant higher-spin supermultiplets becoming massless. Also, the \( q^2 = -2 \) states indeed fit into vector supermultiplets, in agreement with our earlier analysis. Thus, only Bogomolnyi states with \( q^2 = -2 \) can become massless and for a given value of \( \varphi \), the extra massless states are the ones satisfying \( q^2 = -2 \) and such that \( \hat{q} \in \ker(1 + L) \). For generic values of \( \varphi \), there will be no such massless states, but for special values there will a finite number of vectors in the charge lattice satisfying these conditions, and these can be identified with the root vectors of the enhanced gauge algebra [2]. Conversely, the general analysis of section 2 shows that for a given charge vector satisfying these conditions there is always a vacuum for which this happens.

Given any Bogomolnyi vector supermultiplet with \((p, q) = (0, V)\) such that \( V^2 = -2 \), there are also supermultiplets with \((p, q) = (V, nV)\) and \((p, q) = (2V, nV)\) which are represented for weak coupling and at generic points in the moduli space by BPS monopoles and BPS dyons [23,24,25]. It has been shown [38,23] that the conjectured \( SL(2, \mathbb{Z}) \) symmetry of the heterotic string spectrum implies Bogomolnyi states with \((p, q) = (mV, nV)\) for all co-prime integers \( m, n \). We saw in section 2 that all these states must become massless together. Thus, S-duality and N=4 supersymmetry imply that as an enhanced symmetry point of the heterotic string is approached there is an infinite set of dyon states whose masses tend to zero, in addition to the purely electric and purely magnetic states. The interpretation of the magnetic and dyon states as due to quantization of solitons presumably fails at points of enhanced symmetry because the sizes of the monopole and dyons approach infinity as their mass approaches zero\(^*\). This phenomenon is presumably

\(^*\) It is possible that they may be considered as quanta of wave solutions obtained by boosting a soliton solution to the speed of light while keeping the mass fixed.
another indication of a phase transition to a special type of non-abelian Coulomb phase.

It is instructive to explore further the consistency of the conjectured S-duality [26,38,23] of the four-dimensional heterotic string with the phenomenon of symmetry enhancement. At strong coupling, $g_4 >> 1$, the four-dimensional heterotic string is conjectured to be related to the weakly coupled theory by S-duality, with the roles of electric and magnetic charges interchanged. If so, then at generic points in moduli space, electric states with $(p, q) = (0, V)$ are perturbative string states for weak coupling and represented at strong coupling by solitons of the dual theory, while magnetic states with $(p, q) = (V, 0)$ are solitons of the weakly coupled theory but are perturbative states of the dual theory. It is expected that the theory can be smoothly continued in $g_4$ without encountering any phase transition, in which case S-duality implies that the electric and magnetic charges should be on exactly the same footing. This would mean, in particular, that magnetically charged vector states become massless at strong coupling through the HFK mechanism in the dual theory. S-duality implies that magnetically charged states occur as perturbative states of the dual strong-coupling theory, and if, as we are assuming, these can be continued in $g_4$ back to magnetically charged states at weak coupling, it is clear that magnetic states with masses tending to zero at the special points must be present in the weakly-coupled theory, even though their perturbative description as solitons in the weak coupling theory breaks down.

The string theory coupling constant is $g_{10} = \langle e^{\Phi_{10}} \rangle$, but the dilaton $\Phi_{10}$ appears differently in the two theories, as we shall now argue. In particular, $\Phi_{10}$ occurs in the $SL(2; \mathbb{R})/U(1)$ coset space for the heterotic string (and so can identified with $\Phi_4$) while the type II dilaton lies in the $O(6, 22)/O(6) \times O(22)$ coset space after compactification on $K_3 \times T^2$. This means that perturbative effects in the heterotic string can be non-perturbative in the type II string, and vice versa. To see this, we shall focus on the subgroup $O(4, 20) \times SL(2; \mathbb{R})_S \times SL(2; \mathbb{R})_T \times SL(2; \mathbb{R})_U$ of (3.1). For both superstring compactifications, $SO(2, 2) \sim SL(2; \mathbb{R})_T \times SL(2; \mathbb{R})_U$ acts on the moduli space of $T^2$ and $SL(2; \mathbb{R})_S$ acts on the dilaton and axion fields.
arising in the usual way from the $D = 10$ dilaton $\Phi_{10}$ and the antisymmetric tensor gauge field. The space $O(4, 20)/[O(4) \times O(20)]$, modulo the discrete group (3.2), is the moduli space for $K_3$ and for the Narain construction of heterotic strings in six dimensions. For the heterotic string, $SL(2, \mathbb{R})_T \times SL(2, \mathbb{R})_U \subset O(6, 22)$. For the type II string, however, the RR charges are inert under S-duality [9] and do not couple to the ten-dimensional dilaton $\Phi_{10}$ [40,10] (see also [41]), so that $\Phi_{10}$ cannot be identified with $\Phi_4$, which does couple to all charges. Thus it must be the case that $SL(2, \mathbb{R})_S \subset O(6, 22)$, and in fact $SL(2, \mathbb{R})_S \times SL(2, \mathbb{R})_U \subset O(6, 22)$, so that the $SL(2, \mathbb{R})_S$ and $SL(2, \mathbb{R})_T$ factors are interchanged compared with the heterotic string, as in [42]. In particular, for the heterotic string, $\Phi_4$ coincides with the $D = 10$ dilaton $\Phi_{10}$ while for the type II string they are distinct: $\Phi_{10}$ lies in $O(6, 22)/[O(6) \times O(22)]$ while $\Phi_4$ is one of the $T^2$ moduli. It is interesting to note that the equivalence of the heterotic and type II superstring compactifications together with T-duality in each theory implies invariance of the spectrum under the full group $O(4, 20) \times SL(2; \mathbb{R})_S \times SL(2; \mathbb{R})_T \times SL(2; \mathbb{R})_U$ since what is non-perturbative in one is perturbative in the other.

The main implication of section two for the $K_3 \times T^2$ compactification of the type II superstring is that symmetry enhancement will occur if the Bogomolnyi spectrum includes electrically charged states with $q^2 < 0$. These states do not occur in perturbation theory but they may appear in the non-perturbative spectrum as Ramond-Ramond (RR) solitons [9]; we shall address this question in the following section. Note, however, that if the conjectured equivalence to the heterotic string is correct the type II string studied using perturbation theory in the $T^2$ modulus $g_4 \equiv \langle e^{\Phi_4} \rangle$ should give the same results as the heterotic string expanded in the usual way order by order in the string coupling $g$. For the type II string on $K_3 \times T^2$, the usual string perturbation theory in $g$ is not useful as the RR soliton effects are non-perturbative in $g$, whereas an expansion in $g_4$ should yield results equivalent to those of the usual perturbative heterotic string.
4. Bogomolnyi states

We have seen in the preceeding sections that symmetry enhancement in superstring theories need not be associated with toroidal compactifications. The fact that such an association has been made in the past is simply a reflection of the fact that this is the only example that can be discovered by conventional perturbative string theory methods. Fortunately, N=4 supersymmetry is such a strong requirement that in this case it is not necessary to solve the strongly coupled theory to obtain the information needed to determine whether a given N=4 supersymmetric theory will exhibit non-perturbative symmetry enhancement. If the existence of a Bogomolnyi state with a given charge vector can be established for some value of the coupling constants, then there will be a state with that charge vector for all values of the coupling constants, with mass given by the Bogomolnyi mass formula. In this section we will discuss the Bogomolnyi spectrum.

In the context of the purely massless effective four-dimensional $N \geq 4$ supergravity theory the Bogomolnyi states breaking half the supersymmetry can only arise from quantization of extreme black holes [43,44,45,46] for special values of the ‘dilaton coupling constant’, $a$. Specifically, they arise from $a = \sqrt{3}$ extreme black holes of the $N = 8$ supergravity [9] and from $a = \sqrt{3}$ and $a = 1$ extreme black holes of $N = 4$ theories [47]. One obstacle to the interpretation of quantized extreme black holes as Bogomolnyi states is the fact that only the $a = 1$ magnetic ones are completely non-singular. The way around this obstacle in the context of string theory is that the four-dimensional supergravity is interpreted as the effective theory of a compactified D=10 theory. In this case one can interpret the extreme black hole solutions of the four-dimensional theory as either (i) Kaluza-Klein modes of the massless D=10 fields, (ii) their magnetic duals, the KK monopoles [48], or (iii) as ‘wrapping modes’ of the $p$-brane solitons [9]. We shall argue that all the Bogomolnyi states needed for symmetry enhancement arise from non-singular, or otherwise physically acceptable, solutions of the D=10 theory describing weak coupling or of the dual effective theory describing strong coupling.
Let us first consider how higher dimensions help for the heterotic string. The $a = \sqrt{3}$ magnetic extreme black holes can now be interpreted either as Kaluza-Klein monopoles (for those whose magnetic field is of KK origin) or as compactified five-branes of the 10-dimensional theory (for those whose magnetic field is of antisymmetric tensor origin). Some of the latter are interpreted as BPS monopoles in four dimensions [25,49]. Thus, all of the magnetic extreme black holes breaking half the $N = 4$ supersymmetry are non-singular in the ten-dimensional context and it should therefore be possible to determine the corresponding quantum states by semi-classical methods at weak string coupling. The resulting non-perturbative magnetic states are therefore in the string spectrum for weak string coupling and because they belong to ultra-short supermultiplets they must remain in the spectrum (at least modulo 16) at strong coupling. The singularities of the electric extreme black holes cannot be completely removed in this way, but this difficulty can be resolved by identifying them with the perturbative Bogomolnyi states of the heterotic string. The particles in the perturbative string spectrum have electric and gravitational fields and might therefore be expected to appear as extreme black holes in the effective field theory. Indeed, the electric extreme black holes carry the same quantum numbers as the Bogomolnyi states in the perturbative heterotic string spectrum. Finally, such an identification is expected from considerations of S-duality [47,9], and corresponds to identifying the solitonic string or one-brane solution in ten dimensions with the fundamental string [9].

According to this picture, extreme black holes that break half the supersymmetry correspond either to fundamental string states or are the projection to four dimensions of KK monopoles or $p$-brane solitons that are non-singular in a higher dimension. This was implicit in our previous work [9] on U-duality in type II string theory, but since there we passed over issues related to the singularities of $p$-brane solitons of type II superstrings we now take another look.

The electrically and magnetically charged extreme black holes of the type II string compactified on either $T^6$ or $K_3 \times T^2$ can be interpreted as KK monopoles, or as compactified $p$-branes of either the type IIA or the type IIB ten-dimensional su-
pergravity [9]. Recall that these consist of an electric one-brane and magnetic five-brane in the Neveu-Schwarz/Neveu-Schwarz (NS-NS) sector and, in the Ramond-Ramond (RR) sector, $p$-branes with $p = 0, 2, 4, 6$ for type IIA and $p = 1, 3, 5$ for type IIB. Of these, only the NS-NS fivebrane and the type IIB self-dual threebrane is completely non-singular in string sigma-model variables. The NS-NS one-brane is to be identified with the fundamental string, as for the heterotic string. The type IIA magnetic $p$-brane solitons are all completely non-singular as solutions of 11-dimensional supergravity [50,17], which can be interpreted as the effective action of the type IIA superstring at strong coupling [10]. This justifies consideration of all magnetic extreme black holes in the effective four-dimensional theory, but this is not sufficient by itself because the non-perturbative string states expected on the basis of extreme black hole solutions include some electrically charged ones from the Ramond-Ramond (RR) sector [9]. The inclusion of these states (required by U-duality [9]) can also be justified by considerations of eleven dimensional supergravity to the extent that the 0-brane RR solitons can be identified with KK modes of D=11 supergravity compactified on a circle [17,10], and the RR 2-brane soliton has its origin in the 11-dimensional membrane soliton. In fact, apart from the KK modes and the KK monopoles, all $a = \sqrt{3}$ extreme black holes solutions of $N = 8$ supergravity in four dimensions can be regarded [9] as ‘compactifications’ of either the 2-brane [51] or the 5-brane [20] soliton of 11-dimensional supergravity, both of which are needed for U-duality of the $T^6$-compactified type II superstrings. The problem that remains to be confronted is that whereas the 11-dimensional 5-brane soliton is completely non-singular, the 2-brane soliton has a time-like singularity hidden by an event horizon [52] (as for the four-dimensional extreme Reissner-Nordstrom solution). This suggests either that we consider this type of singularity to be consistent with the apellation ‘soliton’, or that we identify the 11-dimensional 2-brane soliton with a fundamental 11-dimensional supermembrane.

We now turn to the type IIB superstring. The NS-NS sector string and five-brane are the same as those already discussed for the type IIA string while the RR sector solitons that break half the supersymmetry consist of a string, a five-brane,
and a self-dual three-brane. The self-dual three-brane solution is geodesically complete [50], but the RR string and five-brane solitons are not (the solutions may be found in [17,53]). The weakly coupled theory justifies inclusion of the charges corresponding to the NS-NS string (which is identified with the fundamental string) and the NS-NS five-brane and RR three-brane (which are non-singular solitons). As for the type IIA string, the inclusion of the other charges is justified by the fact that they are carried by acceptable solutions of the low-energy effective theory at strong coupling. In this case, the conjectured strong coupling limit is a dual type IIB string theory [10] with NS-NS and RR charges interchanged. As will be argued below, the RR five-brane is a non-singular soliton of the dual theory while the RR string should be identified with the dual fundamental string. Thus the inclusion of all types of charge is justified in the type IIB theory.

It was conjectured in [9] that this theory has an $SL(2;\mathbb{Z})$ duality symmetry. This includes a $\mathbb{Z}_2$ subgroup which interchanges weak and strong coupling regimes and it was this that led to the conjecture that the type IIB theory, like the four-dimensional heterotic string, is self-dual. This $\mathbb{Z}_2$ duality corresponds to a symmetry of the equations of motion of the IIB supergravity theory that interchanges the NS-NS with the RR solutions, so the RR string and fivebrane of the weakly coupled theory become the NS-NS string and fivebrane of the effective strongly coupled theory, as required.

Semi-classical quantization of the solitons in D=4 that can be interpreted as wrapping modes of the $p$-brane solitons in D=10 or D=11 will lead to supermultiplets of massive states. Since these $p$-brane solitons preserve half the supersymmetry, the ground state soliton supermultiplet will preserve half the supersymmetry of the vacuum state, which implies that these supermultiplets are ultra-short ones, as required for our arguments of section 2. This remains true as the coupling constants and the moduli of the compactification are varied, so that the multiplets associated with the $p$-brane solitons remain ultra-short and do not combine into longer multiplets. This in turn implies that the resulting four-dimensional states indeed saturate the bound (2.2). Certain of these will then become massless at the
special points in moduli space, as we shall argue below.

What the above arguments show is that each abelian gauge field in the effective four-dimensional $N = 4$ supergravity couples to a tower of massive charged states, irrespective of which superstring theory this is the effective action. This result can be considered to be a generalization, first, of KK theory (for which one finds only the KK states) and, second, of perturbative string theory (for which one finds only the KK states and the winding states). According to either KK theory or perturbative string theory, certain gauge fields (those coming from the metric or the two-form potential) are singled out for special treatment. From the new perspective, in which one also includes non-perturbative $p$-brane ‘wrapping’ modes, all abelian gauge fields are on the same footing. This is in accord with the conjectured S-duality [38] and U-duality [9] of the heterotic and type II strings.

Having established the existence of certain Bogomolnyi states, we now turn to the question of which of these can become massless and so lead to symmetry enhancement. Consider first the compactification of the type IIA string on $K_3$. The resulting six dimensional string theory will have Bogomolnyi states, identifiable in the semi-classical regime as wrapping modes of the type IIA 2-brane around the homology 2-cycles of $K_3$ [9]. There are special points in the moduli space of $K_3$ at which the $K_3$ surface becomes singular due to the collapse of certain homology 2-cycles: the area of the 2-cycles tends to zero as the special point is approached. The mass of a state obtained by wrapping a 2-brane round one of these cycles is proportional to the area of the cycle, so that it is a natural candidate for a state that becomes massless at a special point in the full moduli space of metrics and antisymmetric tensors on $K_3^*$. In [10], it was shown that there is a one-to-one correspondence between the points in the Narain moduli space of toroidally compactified heterotic strings in six dimensions at which a non-abelian symmetry group

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* As pointed out by Aspinwall [hep-th/9507012], the extra antisymmetric tensor moduli must be adjusted in order to ensure that a point at which a two-cycle collapses corresponds to a point of enhanced symmetry. In the following, we shall assume that this adjustment has been made.
$G$ emerges and the points in the $K_3$ moduli space at which there is a singularity of type $G$. At such a singularity, $r$ two-spheres collapse to a point, where $r$ is the rank of the group. Each two-sphere is associated with a simple positive root of $G$, so that exactly the right number of states are becoming massless. The vector fields taking values in the Cartan sub-algebra of $G$ are perturbative states of the theory, so the situation is reminiscent of the HFK mechanism: there, the Cartan sub-algebra is associated with perturbative states, while the extra generators of enhanced gauge symmetry are non-perturbative. The arguments of [10] then imply that the Narain lattice of the heterotic string can be identified with the lattice of integral points in the real cohomology of $K_3$. Furthermore, the intersection form is given by the the Dynkin diagram of $G$, and this seems to be the source of the non-abelian interactions.

The states becoming massless can be reliably interpreted as wrapping modes for large mass when the semi-classical approximation is valid, so that in this regime the area of the two-cycle is given by the Bogomolnyi mass formula. As the mass tends to zero, it is not clear whether or not the semi-classical picture of the state as a wrapped brane remains tenable. Nonetheless, the Bogomolnyi mass formula remains reliable, and can be expected to still give the area of the cycle. Thus the state will become massless as the two-cycle collapses, irrespective of whether the semi-classical approximation is valid near the point at which the cycle collapses. At these special points in moduli space, there is symmetry enhancement of the six dimensional string theory, with the product of $U(1)^4$ and a rank 20 non-abelian group as the gauge group. This feature can be preserved by a subsequent compactification to four dimensions on $T^2$, which will allow extension of the gauge group to $U(1)^6$ times a rank 22 non-abelian group. Some of the extra symmetries come from the perturbative HFK mechanism. The enhanced symmetry groups of the toroidally compactified heterotic string and the type II string compactified on $K_3 \times T^2$ are the same at corresponding points of the two moduli spaces, and this implies that at least those parts of the Bogomolnyi spectra of the two theories that become massless somewhere in moduli space must agree.
We can now provide an explanation, in the context of type II compactifications, of why each electrically charged Bogomolnyi state that becomes massless in special vacua is accompanied by a magnetically charged state. Consider again the compactification of the type IIA superstring theory to D=6; as explained in [9], in addition to the 22 electrically charged states resulting from wrapped two-branes, the type IIA four-brane yields 22 ‘elementary’ two-branes in D=6 because of the 22 independent homology two-cycles of $K_3$ around which it can wrap. On subsequent reduction on $T^2$ each of these D=6 two-branes can wrap around the two-torus to produce a magnetically charged state of the four dimensional theory. There is one such magnetic four-brane wrapping mode for each electric membrane wrapping mode, as one would expect from the fact that the electric dual of a membrane in D=10 is a fourbrane. Moreover, it is clear from the above description of the D=10 origin of these states that each homology two-cycle of $K_3$ is associated with electric-magnetic pairs of Bogomolnyi states which must become simultaneously massless as the area of the two cycle vanishes. This result is in perfect accord with our earlier conclusion to this effect based solely on N=4 supersymmetry. Precisely for this reason, the result is true also for the toroidally compactified heterotic string, as mentioned earlier, although the mechanism for this remains obscure. Curiously, although the electric part of the symmetry enhancement mechanism is better understood for the heterotic string, it seems easier to understand the magnetic part of this mechanism for the $K_3 \times T^2$ compactified type II string. It would be interesting to understand the extra massless dyon states from this point of view as well.
5. Symmetry enhancement in D=11 supergravity compactifications

It has been conjectured [10] that the effective action for the strongly coupled D=7 heterotic string, at generic points in its moduli space, is the D=7 field theory found by $K_3$ compactification of D=11 supergravity. Here we shall provide further evidence for this conjecture by showing how the symmetry enhancement known to occur for the heterotic string at special points of its moduli space also occurs for D=11 supergravity.

The effective D=7 field theory obtained by $K_3$ compactification of D=11 supergravity is an N=2 (minimal) supergravity theory coupled to 19 vector supermultiplets. Since there are three vector gauge potentials in the supergravity multiplet there are 22 vectors in all, and the gauge group of the effective D=7 field theory is $U(1)^{22}$ (it is an abelian group of rank 22 but for reasons explained in [9] the charges are quantized so the group is compact). There is no need to go into the details of this compactification because the D=7 Maxwell/Einstein supergravity is completely determined by supersymmetry once one specifies the number of vector supermultiplets [54]; a summary of the features relevant to the current discussion can be found in [19]. From the standpoint of Kaluza-Klein theory these gauge potentials have no massive modes to which they might couple but they do couple to the wrapping modes of the two-brane soliton of D=11 supergravity around the 22 homology two-cycles of $K_3$. These massive modes are charged with respect to the gauge fields which are obtained from the three-form potential of D=11 supergravity. They are massive Bogomolnyi states carrying D=7 central charges, the D=11 dimensional origin of which is the two-form charge appearing in the D=11 superalgebra [55]. At points in the $K_3$ moduli space at which a two-cycle collapses, the associated Bogomolnyi state will become massless for reasons explained in the previous section *. Thus, the symmetry enhancement mechanism for compactification of D=11 supergravity on $K_3$ to D=7 is an intrinsically ‘membrany’ mechanism,

* Note that in this case the full moduli space is that of $K_3$ metrics, so that no adjustment of additional variables is necessary
analogous to the intrinsically ‘stringy’ mechanism for $T^3$ compactification of the heterotic string. This is in accord with the D=7 string/membrane duality picture elaborated in [19].

The D=11 fivebrane soliton yields 22 three-brane solitons in D=7 after wrapping around the homology two-cycles of $K_3$. These are the duals of the membrane wrapping modes just discussed. On further reduction to four-dimensions on $T^3$ each of these produces 22 magnetically charged Bogomolnyi states, so that we again find magnetic partners to each electrically charged state whose mass vanishes as a homology two-cycle of $K_3$ vanishes. As discussed above, the phenomenon of magnetic charges becoming massless simultaneously with electric ones is presumably indicative of some special non-abelian Coulomb phase. One might have supposed that such a phenomenon would be restricted to four dimensions since it is presumably related to infrared divergences of unconfined massless non-abelian gauge fields. However, we see from this example that symmetry enhancement in higher dimensions is related to an even stranger phenomenon. In D=7, for example, each electric state that becomes massless is partnered by a three-brane whose tension, and mass per unit volume, goes to zero, i.e. we have a null three-brane. This is an example of a phenomenon that happens much more generally: in many string theories and supergravity theories, there are points in moduli space at which $p$-brane solitons become null, generalising the phenomenon of black hole solitons becoming massless discussed here. Indeed, the higher dimensional origin of black hole solitons becoming massless is often in $p$-brane solitons that become null.

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