We use an exact holon and spinon Landau-liquid functional which describes the holon - spinon interactions at all scattering orders, to study correlation functions of integrable multicomponent many-particle problems showing both linear and non-linear energy bands. Motivated by recent photoemission experiments, we consider specific cases when the dominant non-linear band terms are quadratic and apply our results to the evaluation of the half-filling 1D Hubbard model one-particle spectral functions beyond conformal-field theory.

PACS numbers: 05.30. ch, 64.60. Fr, 05.70.Jk, 75.40.-s
Recent experimental investigations on the one-particle spectral properties of quasi-one-dimensional (1D) insulators \cite{1} have confirmed that the low-energy physics of these materials is dominated by holons and spinons. The discovery of these elementary excitations followed from the diagonalization of the 1D Hubbard model \cite{2} by Bethe-ansatz (BA). Theoretical studies of the one-particle spectral properties of this model are mainly numerical \cite{1,3,4}. Analytical studies of the problem either focused on the limit of infinite on-site Coulomb interaction (when the charge - spin separation occurs at all energies) or to the metallic phase away from half filling, where conformal-field theory (CFT) has provided important information on correlation functions \cite{4–6}. Unfortunately, CFT does not allow the evaluation of one-particle correlation functions for the half-filling Mott insulator and the study of these functions at finite values of $U$ remains an open problem of great physical interest.

In this Letter we combine a Landau-liquid functional which describes exactly the holon and spinon interactions at all scattering orders to study the effects of the non-linearity of the holon and spinon bands in the asymptotics of the 1D Hubbard model one-particle correlation functions both at half filling and maximum magnetization. Our results apply directly to the study of the 1D insulators and allow for the description of the unusual spectral properties detected in the low-dimensional materials \cite{1}.

We consider the 1D Hubbard model in a magnetic field $H$ and chemical potential $\mu$ which describes $N = N^\uparrow + N^\downarrow$ interacting electrons and can be written as $\hat{H} = \hat{T} + U[\hat{D} - \frac{1}{2}\hat{N}] + \sum_{\alpha = c,s} \mu^h_\alpha 2\hat{S}_\alpha^z$. We have $\hat{T} = -t \sum_{j,\sigma} [c^\dagger_{j\sigma} c_{j+1\sigma} + h.c.]$ the “kinetic energy” ($t$ is the transfer integral), $\hat{D} = \sum_{j} \hat{n}_{j,\uparrow} \hat{n}_{j,\downarrow}$ measures the number of doubly occupied sites and $\mu^h_\uparrow = \mu$, $\mu^h_\downarrow = \mu_0 H$ ($\mu_0$ is the Bohr magneton) are the chemical potentials. Moreover, we define the charge and spin operators $\hat{S}^c_z = -\frac{1}{2}[N^\downarrow - \hat{N}]$, $\hat{S}^s_z = -\frac{1}{2}[\hat{N}^\uparrow - \hat{N}^\downarrow]$, $\hat{N} = \sum_{\sigma} \hat{N}_\sigma$, $\hat{N}_\sigma = \sum_{j} \hat{n}_{j,\sigma}$ where $\hat{n}_{j,\sigma} = c^\dagger_{j\sigma} c_{j\sigma}$ is the occupation number operator ($c^\dagger_{j\sigma}$ and $c_{j\sigma}$ are electron operators of spin projection $\sigma$ at site $j = 1, ..., N_a$). We define the charge density $n = n^\uparrow + n^\downarrow = N/N_a$ and a spin density $m = n^\uparrow - n^\downarrow$ in the intervals $0 \leq n \leq 1$ and $0 \leq m \leq n$, respectively, where $n^\sigma = N^\sigma/N_a$. The relevant Fermi momenta are $k_{F\sigma} = \pi n^\sigma$ and $k_F = \pi n/2$. We use units such that $a = \hbar = 1$, where $a$ is the lattice constant.
In the operational BA representation of Ref. [7] the holons and spinons are the holes of the c and s anticommuting pseudoparticles [3], respectively. In this basis the Slater-determinant levels of α pseudoparticles, with α = c, s, are exact energy eigenstates of the many-electron problem and the total momentum is additive in the pseudoparticle momenta q. The Hamiltonian contains pseudoparticle interaction terms, yet integrability implies that all the interactions are of zero-momentum forward-scattering type. The exact many-electron ground state (GS) is in this representation described by the α-pseudoparticle distribution, $N_\alpha^0(q) = \Theta(q_{F\alpha}^{(+1)} - q)$ and $N_\alpha^0(q) = \Theta(q - q_{F\alpha}^{(-1)})$ with $0 < q < q_\alpha^{(+1)}$ and $q_\alpha^{(-1)} < q < 0$, respectively, where $q_{F\alpha}^{(\pm1)} = \pm q_{F\alpha} + 0(1/N_\alpha)$ and $q_{F\alpha} = \pi N_\alpha/N_\alpha$. Here $N_c = N$ and $N_s = N_\perp$ and hence $q_{Fc} = 2k_F$ and $q_{Fs} = k_F\perp$. The pseudo-Brillouin zones are $q_\alpha^{(\pm1)} = \pm q_{\alpha} + O(1/N_\alpha)$ and $q_\alpha = \pi N_\alpha^*/N_\alpha$ where $N_c^* = N_s$ and $N_s^* = N_\perp$ and hence $q_c = \pi$ and $q_s = k_F\perp$. Note that at half filling, $N_c^h = N_c^* - N_c = 0$ (zero magnetization, $N_s^h = N_s^* - N_s = 0$) and the GS has no holons (spinons). Importantly, $N_\alpha$ and $N_\alpha^*$ are always conserving numbers. The energy eigenstates only involve the momentum distribution $N_\alpha(q)$ and for states differing from the GS by a small density of excited α pseudoparticles the excitation-energy spectrum can be expanded in the momentum deviations, $\Delta N_\alpha(q) = N_\alpha(q) - N_\alpha^0(q)$, what leads to a Landau functional [8], $\Delta E_L = \sum_{i=1}^{\infty} \Delta E_i$, which up to third order in $i$ reads

$$\Delta E_L = \sum_{\alpha,q} \Delta N_\alpha(q)\epsilon_\alpha(q) + \frac{1}{L} \sum_{\alpha,\alpha'} \sum_{q,q'} \Delta N_\alpha(q)\Delta N_{\alpha'}(q') \frac{1}{2} f_{\alpha,\alpha'}(q, q') +$$

$$+ \frac{1}{L^2} \sum_{\alpha,\alpha',\alpha''} \sum_{q,q',q''} \Delta N_\alpha(q)\Delta N_{\alpha'}(q')\Delta N_{\alpha''}(q'') \frac{1}{6} g_{\alpha,\alpha',\alpha''}(q, q', q'') + h.o.$$  

(1)

All the coefficients of the functional [8] can be exactly derived from the BA equations [8], yet for orders $i > 2$ the calculations are very lengthy. So far only the $i = 1$ band $\epsilon_\alpha(q)$ and $i = 2$ $f_{\alpha,\alpha'}$ function were derived [8]. From the band expressions the velocity $v_\alpha(q) = d\epsilon_\alpha(q)/dq$ and the function $a_\alpha(q) = dv_\alpha(q)/dq$ follow. The $f_{\alpha,\alpha'}$ function expression only involves the velocity and the two-pseudoparticle phase shift $\Phi_{\alpha,\alpha'}(q, q')$ [8]. This phase shift plays a central role in our critical theory – it is an interaction-dependent parameter associated with the zero-momentum forward-scattering collision of the α and $\alpha'$ pseudoparticles of momentum $q$ and $q'$, respectively. For instance, the phase-shift combinations $\xi_{\alpha\alpha'}^\perp = \delta_{\alpha,\alpha'} +$
\[ \sum_{j=\pm 1} (j)^i \Phi_{\alpha \alpha'}(q_{F\alpha}, jq_{F\alpha'}) \] with \( i = 0, 1 \) fully determined the \( \alpha \) anomalous dimensions of the generalized critical theory, as we discuss below.

When both the densities of \( \alpha \) pseudoparticles, \( n_{\alpha} \), and of \( \alpha \) pseudoholes, \( n_{h\alpha} \), are finite the dominant \( \epsilon_{\alpha}(q) \) band terms are linear in the momentum for low-energy excitations. We emphasize that in this case only the terms of order \( i = 1 \) and \( i = 2 \) of the functional (1) are relevant for the low-energy physics. Upon linearization of the \( \epsilon_{\alpha}(q) \) bands and replacement of \( f_{\alpha,\alpha'}(q, q') \) by \( f_{\alpha,\alpha'}(q_{F\alpha}, \pm jq_{F\alpha'}) \), the spectrum (1) leads to the CFT spectrum of Ref. [5]. This reveals that our basis is the natural representation for the critical-point operator algebra. For these densities CFT provides asymptotic correlation functions expressions [5].

The functional (1) is universal for a wide class of integrable multicomponent models. As we illustrate below for the 1D Hubbard model, occurs often in these systems that as the density \( n_{\alpha} \) or \( n_{h\alpha} \) becomes small the dominant low-energy \( \alpha \) band term is non linear and of the general form \( \epsilon_{\alpha}(q) \approx \pm |q| - Q_{\alpha} j^j / j! \mathcal{M}_{\alpha} \), where \( Q_{\alpha} = 0 \) when \( n_{\alpha} \to 0 \) and \( Q_{\alpha} = q_{\alpha} \) when \( n_{h\alpha} \to 0 \), \( \mathcal{M}_{\alpha} \) is a positive generalized mass, and \( j \) is a positive integer, \( j = 1, 2, 3, 4, ..., \infty \). Thus, \( j \) characterizes the curvature of the band at the pseudo-Fermi points, that is, \( j = 1 \) is linear (the usual case described by CFT), \( j = 2 \) is quadratic and so on (\( j = \infty \) describes the dispersionless band). We use the notation \( \{\alpha, j\} \) for a \( \alpha \) band of type \( j \). The \( c \) and \( s \) bands of the present model are represented in Figs. 7 and 8, respectively, of Ref. [9]. For \( U > 0 \) the \( c \) band is quadratic around \( q = \pm \pi \) and \( q = 0 \) and at \( m = 0 \) and \( U > 0 \) the \( s \) band is only quadratic around \( q = 0 \). In the limit of \( U \to \infty \) its width vanishes and it becomes flat, whereas the width of the \( c \) band is always \( 4t \). The \( c \) and \( s \) pseudo-Fermi surfaces are at \( q = \pm 2k_F \) and \( q = \pm k_{F\downarrow} \), respectively, and at low energy the bands can be linearized around them outside the quadratic or flat regions. Therefore, there occur band changes \( \{c, 1\} \to \{c, 2\} \) as \( n_c \to 0 \) and \( n_{c}^h \to 0 \) when \( 2k_F \to 0 \) and \( 2k_F \to \pi \), respectively, and \( \{s, 1\} \to \{s, 2\} \) as \( n_s \to 0 \) when \( k_{F\downarrow} \to 0 \), and \( \{s, 1\} \to \{s, \infty\} \) as \( U \to \infty \) at \( m = 0 \) and the \( s \) band becomes flat \( (\epsilon_{s}(q) = 0 \implies \{s, \infty\}) \). For each \( \{\alpha, j\} \) branch the low-energy critical spectrum has contributions from terms up to order \( i = j + 1 \) of the functional (1).

In this Letter we study the effects of non-linearity on the asymptotics of correlation
functions. In particular, following recent photoemission data \[1\], we consider the one \(\sigma\) electron (or hole) spectral function \(A_\sigma(k, \omega)\), momentum distribution \(N_\sigma(k) = \sum_\omega A_\sigma(k, \omega)\), and density of states \(D_\sigma(\omega) = \sum_k A_\sigma(k, \omega)\). There are no previous analytical results for these functions at finite values of \(U\) and (I) for the \(n = 1\) Mott-Hubbard insulator and (II) the fully polarized ferromagnetic \((m = n)\) initial GS’s. At \(n = 1\) and \(m = 0\) there are neither holons nor spinons in the initial \(N\)-GS and the final \(N - 1\) states have one holon and one spinon. Importantly, the numerical energy dispersions of Ref. \[1\], which agree with the photoemission data, can be described by our holon and spinon bands \(\epsilon_\alpha(q)\). At large \(U\) the right (or left) holon line, which we denote by \(\omega^R(k)\) [or \(\omega^L(k)\)], of Fig. 3 of the first paper of Ref. \[1\] is in the thermodynamic limit slightly \(\omega\) shifted and refers to \(\omega > 0\) (or \(\omega < 0\)) and reads \(\omega^R(k) = 2t + \epsilon_c(\pi/2 + k)\) [or \(\omega^L(k) = 2t + \epsilon_c(\pi/2 - k)\)]. It corresponds in our representation to the creation of the spinon at \(q = -\pi/2\) (or \(q = \pi/2\)) and the holon in the domain \(|q| \in [\pi/2, \pi]\) (or \(|q| \in [0, \pi/2]\)). We emphasize that the final \((N - 1)\)-GS corresponds to \(k = \pi/2 = k_F\) and (due to the slight \(\omega\) shift) to \(\omega = 2t\). The right holon line is quadratic in \(\omega\) around the GS energy because of the \(j = 2\) character of the \(c\) band at \(q \approx \pm \pi\). To illustrate the physical importance of our non-linear critical theory, we evaluate below the \(\omega\) dependence of the \(\omega^R(k)\) peak which in the \(N_a \to \infty\) limit exists at \(k = k_F = \pi/2\), between the \(k = 3\pi/7\) and \(k = 4\pi/7\) peaks of the figure. [Below we measure \(\omega\) from the GS, i.e. \(\omega^R(k) = \epsilon_c(\pi/2 + k)\).] We also find that the \(n = 1\) non-linear effects lead to a non-Luttinger-liquid divergent behavior for \(D_\sigma(\omega)\).

The low-energy critical theory with both \(\{\alpha, 1\}\) and \(j > 1 \{\alpha, j\}\) bands (for our model the total number of bands is two) requires the use of terms up to the order \(i = j + 1\) in the functional \([\|\|]\). For \(i = 3\) the calculations are lengthy and are technically similar to the the ones presented in the Appendix of Ref. \[8\] for order \(i = 2\). Here they involve substitution of the third-order deviation expansions in the BA equations of Ref. \[8\]. This leads to \(g_{\alpha,\alpha',\alpha''}(q, q', q'') = \bar{g}_{\alpha,\alpha',\alpha''}(q, q', q'') + \bar{g}_{\alpha,\alpha',\alpha''}(q', q'', q) + \bar{g}_{\alpha',\alpha,\alpha''}(q'', q, q')\) where

\[
\bar{g}_{\alpha,\alpha',\alpha''}(q, q', q'') = 4(\pi)^2\{a_\alpha(q)\Phi_{\alpha,\alpha'}(q, q')\Phi_{\alpha,\alpha''}(q, q'')
\]
The associate low-energy processes can be classified into three types, (i) GS - GS transitions associated with variations \( \Delta N_\alpha = N_\alpha - N_\alpha^0 \) which only change \( q_{F\alpha} \), (ii) finite momentum \( K = \sum_\alpha \mathcal{D}_\alpha 2q_{F\alpha} \) processes associated with variations \( \mathcal{D}_\alpha = J_\alpha - J_\alpha^0 \), and (iii) a number \( N_{ph}^\alpha = 0, 1, 2, \ldots \) of elementary particle - hole processes around the Fermi point \( q_{F\alpha}^{(i)} \). In the absence of transitions (i) we define \( q_{F\alpha} \) relatively to the initial GS and otherwise relatively to the final GS. Independently of the form of the band \( \epsilon_\alpha(q) \), at low energy the above processes lead to a momentum of the form \( K = k_0 + \sum_\alpha \Delta P_\alpha \) where \( \Delta P_\alpha = \frac{2\pi}{N_\alpha} \sum \Delta \int N_{\alpha l} \mathcal{D}_\alpha + \sum \mathcal{M}_\alpha \) and in the present case, \( k_0 = \sum_\alpha \mathcal{D}_\alpha 2q_{F\alpha} \). Let us denote by \( \sum_{\alpha^0}, \sum_{\alpha^1}, \) and \( \sum_{\alpha^2} \) the summations over linear branches, quadratic (or other \( j > 2 \) non-linear) branches, and all types of branches, respectively. The \( \{\alpha, 2\} \) band reads \( \epsilon_\alpha(q) = \pm |q| - Q_\alpha |q|^2 / 2m_\alpha^*, \) with \( \mathcal{M}_\alpha \equiv m_\alpha^* \) and \( m_\alpha^* = 1/a_\alpha(0) \) and
$m^*_\alpha = 1/a_\alpha(q_\alpha)$ for pseudoparticles and pseudoholes, respectively. These masses play the same role as the effective mass of Ref. [9]. For the Hubbard model $m^*_s$ and $m^*_c$ are given by Eqs. (47) and (48), respectively, of Ref. [9]. Introducing in Eqs. (1)-(3) the band expressions (with $q$ parameters. Note that $\Delta E_n$ and $\xi_n$, $\xi_m$ are the same roles as the effective mass of Ref. [10]. For the Hubbard model $\xi^0 = 0$ (or $\xi^0 = \alpha c$, $\xi^0 = 0$) for the initial GS.

When all $\alpha$ bands are linear, the critical-theory expressions involve the anomalous dimensions, $2\Delta^c_\alpha = \sum_{i=0,1}(\mathcal{N}^i_n)^2 + N^p_{\alpha,0} = \sum_{i=0,1} \xi^0_{\alpha\alpha'} + \frac{\Delta N_{\alpha'}}{2} = \sum_{i=0,1} \xi^0_{\alpha\alpha'}\mathcal{D}_{\alpha'}$, $\bar{\xi}_\alpha = v_\alpha\xi_\alpha^0$ with $v_\alpha = q_{\alpha \alpha}/m_\alpha$ and $v_\alpha = q_{\alpha \alpha}/m_\alpha^*$ (or $\bar{v}_\alpha = [q_\alpha - q_{\alpha \alpha}]/m_\alpha$ and $v_\alpha = [q_\alpha - q_{\alpha \alpha}]/m_\alpha^*$), $\xi^0_{\alpha\alpha'} = \bar{m}_\alpha = \bar{v}_\alpha = \frac{2\Delta^c_\alpha}{\Delta N_{\alpha'}}$ and $\xi_\alpha$ is independent of $\bar{v}_\alpha$ and contains no pseudoparticle interaction parameters. Note that $q_{\alpha \alpha} = \pi \Delta N_{\alpha}/N_{\alpha}$ (or $q_\alpha - q_{\alpha \alpha} = -\pi \Delta N_{\alpha}/N_{\alpha}$) in the limit when $n_\alpha = 0$ (or $n^h_{\alpha} = 0$) for the initial GS.

When all $\alpha$ bands are linear, the critical-theory expressions involve the anomalous dimensions, $2\Delta^c_\alpha = \sum_{i=0,1}(\mathcal{N}^i_n)^2 + \xi N^p_{\alpha,0} = \sum_{i=0,1} \xi^0_{\alpha\alpha'} + \frac{\Delta N_{\alpha'}}{2} = \sum_{i=0,1} \xi^0_{\alpha\alpha'}\mathcal{D}_{\alpha'}$, $\bar{\xi}_\alpha = v_\alpha\xi_\alpha^0$ with $v_\alpha = q_{\alpha \alpha}/m_\alpha$ and $v_\alpha = q_{\alpha \alpha}/m_\alpha^*$ (or $\bar{v}_\alpha = [q_\alpha - q_{\alpha \alpha}]/m_\alpha$ and $v_\alpha = [q_\alpha - q_{\alpha \alpha}]/m_\alpha^*$), $\xi^0_{\alpha\alpha'} = \bar{m}_\alpha = \bar{v}_\alpha = \frac{2\Delta^c_\alpha}{\Delta N_{\alpha'}}$ and $\xi_\alpha$ is independent of $\bar{v}_\alpha$ and contains no pseudoparticle interaction parameters. Note that $q_{\alpha \alpha} = \pi \Delta N_{\alpha}/N_{\alpha}$ (or $q_\alpha - q_{\alpha \alpha} = -\pi \Delta N_{\alpha}/N_{\alpha}$) in the limit when $n_\alpha = 0$ (or $n^h_{\alpha} = 0$) for the initial GS.
These symmetries imply that the terms of the critical spectrum (4) which contain no linear velocity terms are of non-interacting pseudoparticle character. Following this remarkable property we find the following expressions for the asymptotics of correlation functions

$$\chi_\vartheta(x, t) \propto \prod_{\check{\alpha}, \check{\alpha}', \check{\alpha}''} e^{-i k_0 x} \left[ \frac{1}{\left( x - t' v_\alpha t' \right)^{2 \Delta_\omega'}} \left( x \sum_{2 \Delta_\omega} \right) \right], \quad x \gg 2t/m_\alpha^*, \quad \alpha \in \check{\alpha}$$

$$\propto \prod_{\check{\alpha}, \check{\alpha}', \check{\alpha}''} 1/\left[ \left( -t' v_\alpha t' \right)^{2 \Delta_\omega'} \left( 2t/m_\alpha^* \right)^{2 \Delta_\omega} \right], \quad x = 0. \quad (5)$$

These $x$ and $t$ dependences can be understood in the following way. In the limit of low energy each $\alpha$ excitation branch corresponds to an independent momentum-energy tensor component and to one independent Minkowski space with light velocity $v_\alpha$. For bands with both finite $n_\alpha$ and $n^h_\alpha$ densities the velocity $v_\alpha$ is also finite and the metric is Lorentzian. However, for vanishing small values of $n_\alpha$ or $n^h_\alpha$ the metric becomes Galilean. In the Minkowschian case the asymptotic of correlation functions involves the variables $(x \pm v_\alpha t)$ associated with Lorentz transformations. On the other hand, in the case of Galilean symmetry space $x$ and time $t$ are transformed independently. This is consistent with the two asymptotic-expression regimes involving either $x$ or $t$. Moreover, the only combination of the time $t$ and mass $m_\alpha^*$ with dimensions of $x$ is $\text{const} \times \sqrt{t/m_\alpha^*}$. Note that the changes in the asymptotics only concern the metric whereas the $\alpha$ anomalous dimensions, whose values depend on $U/t, n,$ and $m$, remain the same. Both the CFT asymptotic correlation function expressions and expressions (4) are limiting cases valid for finite and vanishing, respectively, values of $n_\alpha$ or $n^h_\alpha$. As $n_\alpha$ or $n^h_\alpha$ is gently increased, we come into a small-density $\{\alpha, 2\} \rightarrow \{\alpha, 1\}$ band transition regime which is not described by these asymptotic expressions.

Our theory does not describe the case when all bands are of $j > 1$ type. When some of the bands are of $j > 2$ type we find expression (4) for $x \gg (t/M_\alpha)^{1/j}$, whereas the $x = 0$ expression becomes

$$\chi_\vartheta(x, t) \propto \prod_{\check{\alpha}, \check{\alpha}', \check{\alpha}''} 1/[\left( -t' v_\alpha t' \right)^{2 \Delta_\omega'} \left( t/M_\alpha \right)^{2 \Delta_\omega}]. \quad \text{Here } j \text{ is meant to be a function of } \alpha, \ i.e. \text{ different } \alpha \text{ bands may have different } j > 1 \text{ values.}$$

Fourier transforms of the above asymptotic expansions provide correlation-function expressions for values of momentum $k$ close to $k_0$ and low values of energy $\omega$ measured from the initial GS energy. For $m = 0$ and both $n = 1$ and small finite densities of holes $\delta = (1 - n)$
and low negative (positive) values of $\omega$ and/or values of $k$ close to $k_F$ our generalized theory leads to $A_\sigma(k_F, \omega) \propto |\omega|^{-7/8}$ and $N_\sigma(k) \propto |k-k_F|^{1/8}$ for the particle (hole) spectral function and momentum distribution. (At $n = 1$ we define the ground-state energy at the bottom and the top of the Mott-Hubbard gap [2] for particles and holes, respectively.) We emphasize that $A_\sigma(k_F, \omega) \propto |\omega|^{-7/8}$ is (for particles) the above $n = 1$ peak of the figure.

On the other hand, the particle (hole) density of states is given by $D_\sigma(\omega) \propto |\omega|^{-3/16}$ and $D_\sigma(\omega) \propto |\omega|^{1/8}$ for $n = 1$ and small finite $\delta$ values, respectively. However, the latter expression (also predicted by CFT) is restricted to frequencies $|\omega| < E_c = \delta(\frac{2\pi}{L})^2 \frac{5}{32m^* c}$. In the limit of $n \to 1$ this domain shrinks to a single point and the spectral function diverges as $D(\omega) \propto |\omega|^{-3/16}$. In the $m = n$ case we consider creation of one $\downarrow$ electron and find for both $m = n$ and for small finite densities $n_\downarrow$, $A_\downarrow(k_{F\downarrow}, \omega) \propto |\omega|^{-1+\frac{1}{4}[1-n_\downarrow]}$ and $N_\downarrow(k) \propto |k-k_F\downarrow|^{\frac{1}{2}[1-n_\downarrow]}$, where $\eta_0 = (2/\pi)\tan^{-1}([4t\sin(\pi n)]/U)$ and $n < 1$, whereas the density of states is given by $D_\downarrow(\omega) \propto |\omega|^{-\frac{1}{2}+\frac{1}{4}[1-n_\downarrow]}$ and $D_\downarrow(\omega) \propto |\omega|^{\frac{1}{2}[1-n_\downarrow]}$ for $m = n$ and small finite values of $n_\downarrow$, respectively. The latter expression is restricted to energies $|\omega| < E_s = \delta(\frac{2\pi}{L})^2 \frac{1}{m^*}$. Again, in the limit of $n_\downarrow \to 0$ this domain shrinks to a point and the spectral function behaves as $D_\downarrow(\omega) \propto |\omega|^{-\frac{1}{2}+\frac{1}{2}[1-n_\downarrow]}$. These $D_\downarrow(\omega)$ expressions are not valid for $U \to \infty$ because then the bands are of $\{c,1\}$ and $\{s,\infty\}$ type and instead $D_\downarrow(\omega) \propto |\omega|^{-\frac{1}{2}}$. Finally, concerning the comparison of our results with previously obtained $m = 0$ and $U \to \infty$ expressions, while our theory does not apply to the $n = 1$ case of bands $\{c,2\}$ and $\{s,\infty\}$, it provides expressions for the $\{c,1\}$ and $\{s,\infty\}$ case which corresponds to finite values of $n$ and $\delta$. Importantly, our general expressions lead to the same results as Ref. [4], with the density of states given by $D_\sigma(\omega) \propto |\omega|^{-3/8}$ and $D_\sigma(\omega) \propto |\omega|^{1/8}$ for $U \to \infty$ and small finite values of $4t/U$, respectively [12]. Our non-linear critical theory results are expected to shed new light on the unusual properties of quasi-1D materials.

We thank D. K. Campbell, A. Luther, L. M. Martelo, and A. W. Sandvik for illuminating discussions and the support of PRAXIS under Grants No. 2/2.1/FIS/302/94 and BCC/16441/98. A. H. C. N. acknowledges support from the Alfred P. Sloan Foundation and the partial support provided by an US Department of Energy CULAR research grant.
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11. In analogy with the adiabatic vanishing of the Fermi-liquid quasiparticle interactions, which here is achieved by adiabatically changing $n_\alpha$ or $n_\alpha^h$.

12. By providing the suitable form for the $G(x, t)$ function of page 897 of Ref. [4], our theory explains how the $1/x$ divergence of the $u_s = 0$ integral $\int dt \, G(x = 0, t)e^{i\omega t}$ is removed.