1. Introduction

A relatively small, non-premixed (buoyancy controlled) flame in the gravimetric field often exhibits a periodic motion known as “flame flickering” or “puffing (Fig. 1).” In general, flickering/puffing occurs in an axisymmetric fashion—the mushroom-like flame is formed in a relatively strong-acceleration flow field, and then the cap is pinched-off to divide the flame into two parts (cap and stem) (Kimura, 1965; Moses, 1993; Cetegen, 1998; J. Carpio, 2012; Abe et al., 2015). After the detachment of the cap, the remained stem temporarily maintains its length to keep the flame. However, a rolling-up flow generated by a vortex travels downstream to interact with the flame and alter its shape, producing the mushroom-like flame. The process is recycled at a certain duration (i.e., of a characteristic frequency), permitting the periodic mushroom flame deformation to continuously exhibit the flickering/puffing behavior. Because the observed gas dynamics resembles a longitudinal wave, the fluid periodic motion can be characterized in the axial direction. Thus, this type of dynamic behavior of the flame is often described as the “varicose mode” of the oscillation (Hamins et al., 1992; Cetegen and Ahmed, 1993; Meyer et al., 1999; Takahashi et al., 2007; Hu et al., 2015). Additionally, the flame occasionally exhibits another type of periodic motion of the sinuous meandering fashion as reported in previous studies (Cetegen and Dong, 2000; Manikantachari et al., 2015). Once this dynamics mode appears, the mushroom shape is no longer formed and the detachment of the cap becomes less pronounced. Moreover, the flame is kept connected, but exhibits a periodic-swinging, asymmetric, dynamic behavior resembling that of a transversal wave. Because the fluid periodic motion can be characterized by the perpendicular direction of the axis, this type of flame behavior is often referred to as the “sinuous mode” of the oscillation. Unlike the varicose mode, which has been substantially studied in the past (Hamins et al., 1992; Cetegen and Ahmed, 1993; Malalasekera et al., 1996), precise studies on the sinuous mode
are rather limited and many issues including its mechanism remain unknown, even though its existence has been clearly observed and pointed by early researchers (Cetegen and Dong, 2000).

![Diagram with varicose and sinuous modes](image)

**Fig. 1** Schematic depiction of the varicose and sinuous dynamic modes of a single-flame system (leftmost: Cetegen and Dong, 2000) and the in-phase and anti-phase instability modes of twin-flame system (rightmost: Toong et al., 1965) in buoyant diffusion flames.

According to the limited earlier research, the periodic frequency of the sinuous mode is a bit larger than that of the varicose mode. Interestingly, the appearance of the sinuous mode was found to be truly random and could be extremely difficult to control with precision. This “uncontrollable” feature could have been the primary reason for the less research attempts to reveal the character in detail, for instance, by a summary of its appearance and the critical condition surrounding it. In the best our knowledge, not a single study so far has successfully maintained the sinuous mode to diagnose its structure in detail. Worse, reproducing the sinuous mode numerically has proved to be a difficult task as well. From a scientific perspective, we perceive that a special technique should be developed to reveal the fundamental details of the oscillation of the sinuous mode, which is our motivation for the present work.

In terms of buoyancy-driven flame oscillation, we shall recognize the unique technique using the twin-flame system (Toong et al., 1965; Kitahata et al., 2009; Nagamine et al., 2017). When two identical flames interact with each other, the oscillation phase is not freely chosen but instead, is locked in two modes, namely, in-phase and anti-phase. With both flames being close enough, they tend to merge and flicker at the in-phase mode. As the distance between the flames increases, the flame motion temporary ceases before staggering to attain a stable asymmetric anti-phase mode appearance. Interestingly, the mode transition occurs at the critical distance, that is, the appearance of each mode is firmly controlled by the system-controllable parameter. Recently, Yang et al. (2019) and Bunkwang et al. (2020) conducted an extensive investigation on the phase-transition condition, which could be summarized well by a global parameter employing the length scale ratio and the Grashof number. It is worthy to note that the in-phase oscillation was quite similar to the flame motion observed at the varicose mode in the single-flame system. On the contrary, in the anti-phase mode of the twin-flame system, the “gap” between the flames exhibits a sinuous meandering fashion, which suggests that the anti-phase oscillation has similarity to the flame motion at the sinuous mode of the single-flame system. Furthermore, the oscillation frequency of the in-phase mode is slightly smaller than that of the anti-phase mode, which is similar to that observed at the varicose and sinuous modes of the single-flame system (Cetegen and Dong, 2000). Considering these characteristics, we deem it highly probable that the varicose/sinuous modes in the single-flame system exhibit a relative correspondence with the in-phase/anti-phase modes in the twin-flame system, which could be explained with a certain similarity rule. If this hypothesis is true, we can utilize the twin-flame system to understand the details of the sinuous mode in the single-flame system, in which the appearance and transition of each mode are well-controllable.

In this work, we aim to establish the similarity rule, both experimentally and numerically, which is applicable to the appearance of the periodic behaviors in the single- and twin-flame systems. In order to examine the dynamic behaviors in a precise manner, high-speed imaging with Schlieren optical system is introduced for visualization of the direct flame, as well as the hot plume boundary (Bunkwang et al., 2020). Frequency analysis is performed by using the temporal signal of the thermocouples located near the burner exit. Moreover, three-dimensional numerical simulations are useful for
accurately studying the temporal variation of the thermal–flow structure during the flickering event. Furthermore, the similarity of the oscillation behaviors appearing in the single- and twin-flame systems is examined carefully, whereas the global parameter is summarized well to characterize the mode transition having the sinuous meandering behavior.

2 Methodology

2.1 Experimental approach

Figure 2 displays the schematic of a twin-flame system consisting of two identical burners aligned in the same horizontal plane. This configuration is basically the same as in our previous work (Bunkwang et al., 2020), thus, only a brief description was provided herein. The variable system parameters were fuel flow rate ($Q$), inner burner diameter ($D$), and the burner-separation distance ($L$). All experiments were performed at the open atmosphere and the experimental conditions are listed as shown in Table 1. Froude number is defined by $Fr = V/(gD)^{0.5}$, where $V$ [m/s] is characteristic velocity of fuel, $g$ [m/s$^2$] is gravitational acceleration and $D$ [m] is the characteristic length. Note that the Froude number being smaller than unity for all cases and the flame stays in laminar regime, not turbulent (see Table 1).

![Fig. 2 Experimental apparatus for the single-flame (right) and twin-flame (left) systems.](image)

| Burner diameter ($D$) [mm] | Flowrate ($Q$) [cc/min] | Jet Froude number $Fr = V/(gD)^{0.5}$ | Burner diameter ($D$) [mm] | Flowrate for two burners ($Q$) [cc/min] | Critical separation distance for the transition ($L_{cr}$) [mm] |
|---------------------------|-------------------------|--------------------------------------|---------------------------|--------------------------------------|----------------------------------|
| -                         | -                       | -                                    | 10                       | 500 (9.1)                            | 21                               |
| 18                        | 500 ~ 520 (3.2 ~ 3.33)   | 0.076 ~ 0.079                        | 18                       | 800 (5.1)                            | 17                               |
| 30                        | 700 ~ 900 (1.65 ~ 2.12)  | 0.030 ~ 0.039                        | 30                       | 1000 (2.4)                           | 14                               |
| 45                        | 1400 ~ 2300 (1.47 ~ 2.41)| 0.022 ~ 0.036                        | 45                       | 2000 (2.1)                           | 13                               |

The combustion field was surrounded with a mesh screen to prevent any external disturbances. Two K-type thermocouples (TCs) were placed near the burner exit to obtain the system dynamic frequency. For the single-flame system, the variable parameters were limited to only $Q$ and $D$. The random variation of the oscillation modes was...
monitored using the same procedure applied in the twin-flame system, although the TCs were installed in a different location (Fig. 2) for convenience. The research-grade of methane gas at the prescribed flow rate was issued from the circular-port burner(s) for each flame system. Moreover, the dynamic motion of the visible flame was recorded by a high-speed camera (CASIO EX-F1: 300 fps with 512 x 384 pixels). Schlieren imaging was also employed for visualization of the flame appearance, along with the hot plume, from the density variation shadow formation enveloped in the flame(s).

### 2.2 Numerical approach

Figure 3 shows the numerical model we adopted herein to examine the precise thermal–flow structure during the flickering phenomenon. We utilized Fire Dynamics Simulator (FDS, version 6.5.3) developed by the National Institute of Standards and Technology (McGrattan et al., 2017) to simulate time-dependent, 3-D heat, and mass transport processes under finite-rate reactions using an Arrhenius model, as in our previous study (Mochizuki et al., 2017). The simplified chemical model was acceptable for the present work since ignition and extinction were not the governing processes on the flickering flame dynamics. As assured from our preliminary test, the periodic flame motions reproduced by the present model are insensitive to the imposed initial conditions but are characterized by the imposed boundary conditions. All surfaces of the domain boundary were set to “open boundary (i.e., free flow incoming and outgoing),” except at the bottom surface (non-slip floor), as indicated in Fig. 3 (left). To ensure a non-reactive, smooth inert wall with the temperature fixed at 25°C, non-slip conditions were applied to the burner surface. A fully developed parabolic flow profile was employed at the inlet of the burner. Also, because of the orthogonal structured grid system employed in the FDS, the shape of the circulated port was not very smooth. However, this did not cause any severe problems as long as the ejecting fuel flow rate was properly imposed, mainly because the major dynamics motion of interest is driven by buoyancy and gas dynamics above the jet (Clement, 2000). Pure methane was issued through the burners toward the ambient air at the standard state. The temperature at the top surface of the burners was maintained at 1000°C to promote ignition and anchoring.

![Fig. 3 Numerical model applied to the twin-flame system](image)

The adopted numerical grid is shown in Fig. 3 (right). A uniform grid was applied around the burners, stretched toward the outer boundaries to reduce the computational cost. The minimum grid size was 1 mm, which is satisfactory to obtain the buoyancy-driven dynamics. It makes sense that the adopted grid may not precisely resolve the reaction structure, so that the maximum temperature might not be correct, thus, accordingly, we cannot expect a quantitative agreement between the simulation and experiment results. Because the purpose of the simulation is to elucidate further the thermal–flow structure at various modes, we believe that the present strategy is satisfactory. More than 6 million cells were adopted for all the simulations performed in this study. The impact of mesh size on the simulation results was carefully checked prior to the production runs to confirm that the variation in the obtained characteristic oscillation frequency was less than 2% against all the pre-running cases. Performed conditions are mainly near the transition point of twin-flame systems under the range of burner diameter and fuel flowrate is from 10 to 18 mm and 500 to 1000 cc/min, respectively.
3. Experimental and numerical results
3.1 Periodic axisymmetric/asymmetric behaviors in the single- and twin-flame systems

Figures 4 and 5 respectively illustrate typical time-sequential images of the periodic axisymmetric and asymmetric dynamics found in the single- and twin-flame systems. For reference purposes, we have added flame images from the work of Cetegen and Dong (2000) in Fig. 4(a). Only time-sequential flame images were shown, and no other information (e.g., flow pattern, plume shape) was provided. Note that the flame axis was slightly tilted, which could probably be from the environmental disturbance, which suggests the quite complex task of obtaining the “ideal” axisymmetric behavior. Nevertheless, the flame clearly displayed the typical varicose pattern described by the top-cap detachment as a result of the neck getting squeezed by the buoyancy-induced flow to assume a periodic motion.

Figure 4(b) shows an image of the flickering cycle for a varicose mode captured by a Schlieren optical system for visualization of the variation of hot plume shape, as well as the direct flame shape, with time. Necking (local squeezing of the hot plume) occurred upstream to allow the mushroom-shaped formation of the flame and traveled downstream over time. Eventually, the top part of the flame was detached to generate the fragment. It is understood that the hot plume and the flame were similarly deformed at an earlier stage, whereas their inconsistency was pronounced when the detaching process was near. This suggests that air entrainment induced by buoyancy may not reduce the temperature much but only sufficiently initiate a local extinction to separate the flame. Once the local extinction is experienced, the top part of the remained stem stays there, even when the necking generated at the upstream travels downstream. This triggers the formed mushroom flame to display the periodic “varicose” behavior. Nevertheless, the buoyancy flow shall be responsible for the whole dynamics within the gas-phase and control the periodic frequency of the system. Figure 4(c)
illustrates a cycle of the in-phase oscillation presented at a shorter burner-separation distance (15 mm in this case) for a twin-flame system. Although the flame shape differed to that observed in Fig. 4(b), the essential physics is basically identical. Moreover, we could see that the inner layers of the hot plume interacted and merged relatively upstream, whereas the outer layers were strongly deformed to cause the necking. Also, the hot plume over the flames merged sufficiently to form a (unified) cluster of the hot product gases. Although not very clear, the hot plume zone between the flames merged relatively; thus, although visually two separate flames were formed, thermally only a single, merged plume, which is responsible for the whole dynamics behavior, was present [Fig. 4(d)].

![Time-sequential images of pulsating fire(s) with sinuous (a)(b) and anti-phase (c)(d) modes, respectively.](image)

Fig. 5 shows the detailed time-response during the asymmetric flame behaviors (sinuous mode in the single-flame system, anti-phase mode in the twin-flame system). Similar to Fig. 4(a), the time-sequential images of the sinuous mode as reported by Cetegen and Dong (2000) in Fig. 5(a) were added as a reference. The flame displayed a winding shape although not very clear, which could be suspected as a random perturbation in the flame behavior. Figure 5(b) displays the Schlieren-captured evolution process by which sinuous meandering was achieved. The hot plume that formed over the flame showed an asymmetry even though the flame did not show such behavior. Triggered by the asymmetric perturbation over the flame (traveling upstream), the flame tip started to wind, after which the whole flame achieved the sinuous meandering mode. A cycle of the anti-phase oscillation at a (relatively) larger burner-separation distance (21 mm) for the twin-flame system is depicted in Fig. 5(c). Two flames flickered asymmetrically with alternating detachment of the cap, which was not observed in the single-flame system [Fig. 5(b)]. Accordingly, the hot plume over the flames fluctuated radically, although the “cooled” ambient gas in the gap between the flames penetrated the hot gas zone [Fig. 5(d)] with its shape exhibiting sinuous meandering similar to the flame shape in Fig. 5(b). It is worth noting that the
sinuous meandering cannot be stably reproduced with the present simulation, which entails the difficulty of studying the sinuous mode of the oscillation.

### 3.2 Transition in single- and twin-flame systems

![Image](https://via.placeholder.com/150)

Fig. 6 (a) Typical optical image-sequence of coexisting varicose and sinuous modes for a single-flame system ($D = 30\, \text{mm}, \, Q = 760\, \text{cc/min}, \, \Delta t = 0.1\, \text{s}$)

Fig. 6 (b) Typical optical image-sequence of the coexisting flame transitions from the in-phase- to the anti-phase mode for a twin-flame system ($L =18\, \text{mm}, \, D = 18\, \text{mm}, \, Q = 900\, \text{cc/min}, \, \Delta t = 0.1\, \text{s}$).

Figure 6 shows a comparison of the sequential images during the mode transition in the single- and twin-flame systems. As mentioned earlier, the appearance of the sinuous mode is not well-controllable and can be found by chance. Therefore, the images presented here were extracted from a specific period of the phenomenon of interest. Nevertheless, important behaviors could be observed at the onset of the transition. In Fig. 6(a), we could see ($0 < t < 1.4\, \text{s}$) that the flame initially showed a typical laminar flow and the hot plume had few wrinkles. As the flame height increased, the plume exhibited a “puffing” motion to achieve the varicose mode. Although such a motion continued for a while, the flame tended to “cease” ($5.1 < t < 5.9\, \text{s}$). During such a temporal “freezing” behavior, the post-flame zone (hot plume) continuously displayed the “puffing” motion. Moreover, the plume showed an axisymmetric (varicose mode) motion that converted gradually into an asymmetric (sinuous mode) one, especially at the downstream (away from the flame). Simultaneously, the remained stem part of the flame gently elongated with time, while the flame top interacted with the asymmetrical dynamics motion downstream and restarted to flicker at an asymmetric mode, revealing the sinuous meandering. Once the flame exhibited the sinuous motion, the mode was sustained for a while ($8.8 < t < 10.2\, \text{s}$) by varying the flame height over time. Although not shown here, after a certain time, the mode transition recurred to the varicose mode with some time delay. From this observation, we could say that the mode transition is completed within a definite time “reset” (i.e., time delay) and does not just instantaneously happen. Therefore, it would be difficult to point out which mode is more stable. On a further note, we must admit that it is difficult, if not almost impossible, to sustain the sinuous mode for a long period of time.
Figure 6(b) displays the mode transition in the twin-flame system. The transition of one mode to the other is faster than in the single-flame system [Fig. 6(a); Note that the time interval of the images is one order of magnitude smaller than in Fig. 6(b)], which could be explained by the rather easier asymmetric plume formation over the flame in the twin-flame system. As discussed above, a bit larger burner-separation distance would enable the flames to exhibit the anti-phase mode with no in-phase mode. Alternatively, a bit smaller distance would allow the flames to exhibit the in-phase mode but not the anti-phase mode. This suggests that there is better control of the mode transition in the twin-flame system, compared to the single-flame system. Prior to the transition, although with the nearly identical flames (at in-phase), the hot plume over the flame appeared slightly different from each other. Eventually, this asymmetry feature became more pronounced once the transition (to anti-phase) was completed.

As suggested by the above, a fixed burner-separation distance yields a quicker transition with no “cease” time. Nevertheless, as also described earlier, the in-phase-to-anti-phase mode transition can be done manually by varying the burner-separation distance, which results in the temporary “ceasing” of the flame motion with the variation of the distance across the critical value. Such a no-motion feature for the transition onset is qualitatively similar to that we observed in the single-flame transition.

### 3.3 Structure during transition in the twin-flame(s) system

Although our numerical model may not easily reproduce the mode transition for a single-flame system (even with the intentional addition of a weak disturbance), the case is different for the twin-flame system. Figure 7 shows a typical example. For convenience, we intentionally constructed the temperature contour plot, where the temperature of the blue zone is less than 25°C (ambient gas temperature). In the early stage, we could see that the in-phase motion was fairly reproduced. As depicted by the Schlieren images in Fig. 6(b), the hot plumes above the flames were initially nearly identical, however, their shapes gradually deformed and eventually reformed asymmetrically. Importantly, our prediction clearly showed that the low-temperature zones between the flames (gap zone) are “winding” and their heights are increasing until the transition is complete, which was difficult to ascertain through the optical measurement. After the transition was succeeded by the full anti-phase motion of the flames, the gap zone displayed a behavior quite similar to the sinuous meandering flame for the single-flame system. Both flames in the anti-phase motion were well-surrounded by fresh air, promoting combustion between the burners.

![Fig. 7 Demonstration of the predicted flame transition from the in-phase mode to the anti-phase mode for the twin-flame system ($L = 21$ mm, $D = 10$ mm, $Q = 500$ cc/min, $\Delta t = 0.2$ s).](image)

### 3.4 Characteristic frequency of two modes of oscillation

Figure 8 shows the measured temperature fluctuations recorded by the TCs and the analyzed data, for the single-flame system. A typical time-variation of the TC signal placed at the height of 2D above the burner exit is depicted in Fig. 8(a), which reflects a clear periodic motion. We analyzed the measured signal by using fast-Fourier transform (FFT) to obtain the system frequency ($f$). As shown in Fig. 8(b), the obtained characteristic frequencies were 10.7, 12.7, and 21.4 Hz, where the peak of 10.7 Hz corresponded to the varicose mode oscillation while its secondary harmonics can be found around 21.4 Hz. Interestingly, the additional peak could be found at 12.7 Hz, which is slightly higher than that of the varicose mode oscillation, and which corresponds to the sinuous mode oscillation. The harmonized frequency of sinuous meandering was always higher than that of the varicose mode, which is similar to that observed in Fig. 8(c) in the case of the twin-flame system. As we pointed out previously (Mochizuki et al., 2017; Bunkwang et al., 2020), there is a critical burner-separation distance ($L_{cr}$) for the mode change (in-phase and anti-phase of the flame dynamics mode),
in which a sudden discontinuous jump of the frequency can be found. Moreover, the anti-phase oscillation frequency is expected to be always higher than that of the in-phase.

![Graph](image)

**Fig. 8** Time-sequence of the TC signal (a) and FFT data corresponding to (b) the single-flame case of \( D = 30 \) mm and \( Q = 850 \) cc/min. (c) Flickering characteristic frequency versus the separate on distance for the twin-flame case of \( D = 10 \) mm, \( Q = 500 \) cc/min.

Figure 9 provides a summary of the frequencies for the varicose and sinuous mode oscillations. To ensure reproducibility, we repeated the frequency analyses several times. The standard deviation of the typical run was about 0.2 for the varicose mode and 0.4 for the sinuous mode, which ensured the quality of the data worthy of discussion. Figures 9(a) to 9(c) describes the sensitivity of oscillation on the imposed fuel flow rate for each burner size. Because the current dynamic oscillatory motion was purely buoyancy controlled, we expected that the characteristics of the oscillation dynamics would be insensitive to the momentum of the jet (i.e., supplied fuel flow rate). The results confirmed the hypothesis, suggesting that the characteristic frequencies for the varicose and sinuous modes can be determined by the imposed scale \( (D) \).

![Graph](image)

**Fig. 9** Analysis of the flickering characteristic frequency of various fuel flow rates and burner diameters for the single-flame system (a–c). (d) Frequency difference between the single-flame and twin-flame systems.

Figure 9(d) displays a plot of the frequencies against the imposed burner scale (of diameter \( D \)) to capture the behavior of the characteristic frequency for each oscillation mode. Although the data is limited, it is evident that the characteristic frequencies for both the varicose and sinuous modes decreased, along with their difference \( (DF) \), as the burner scale.
increased. Assuming that the decrease follows a linear trend as shown in the dashed line, both mode frequencies merged at $D = 60$ mm, where the flame dynamic motion became highly fluctuated and two modes oscillation (varicose and sinuous) could be hardly identified. Similarly, we could identify the in-phase and anti-phase oscillation frequencies for a given separation distance for the twin-flame system. At the transition condition, the TC signals included a mixture of the minimum in-phase oscillation frequency and the maximum of anti-phase oscillation frequency, and the difference of the frequencies for both dynamic modes were obtained. We will discuss the similarity in the difference of the two-mode frequencies ($\Delta f$) for a given scale of interest in the following section.

4. Discussion

4.1 Frequency difference ($\Delta f$) versus a given burner scale

As implied above, sinuous meandering for the single-flame system is similar to the anti-phase oscillation for the twin-flame system. When such asymmetric oscillation mode appears, a frequency “jump” identified with $\Delta f$ is experienced. A luminous flame for the single-flame system indicates the “winding” feature of a sinuous mode, whereas the “winded” dark zone formed by two “winding” luminous flames over twin jets is characteristic of a twin-flame system. In other words, the winded dark zone is formed over the gap zone between twin burners defined as the separation distance $L$. Comparatively speaking, the characteristic length of the sinuous meandering associated to the single-flame system is the burner scale, whereas that for the twin-flame system is the burner-separation distance, and not the burner size itself. Recalling that the current flame behavior is controlled by buoyancy (either in varicose mode or sinuous mode for the single-flame system, or either in the in-phase or anti-phase modes for the twin-flame system), any feature of oscillation, including $\Delta f$, can be summarized using the characteristic length scale.

We attempted to summarize the $\Delta f$, as the most featured dynamics value in the present study, obtained with the single- and twin-flame systems, as shown in Table 2.

### Table 2 Experimental conditions for two systems of flickering flame

| Burner diameter | Flowrate | Frequency difference | Burner diameter | Flowrate | Critical burner-separation distance | Effective diameter | Frequency difference |
|-----------------|----------|----------------------|-----------------|----------|-------------------------------------|---------------------|----------------------|
| $D$ [mm]        | $Q$ [cc/min] | $\Delta f = f_{\text{sinuous}} - f_{\text{varicose}}$ [Hz] | $D$ [mm] | $Q$ [cc/min] | $L_{\text{crit}}$ [mm] | $D_{\text{eff}}$ [mm] | $\Delta f = f_{\text{in-phase}} - f_{\text{anti-phase}}$ [Hz] |
| 18              | 500 ~ 520 | 2.88                 | 18              | 800      | 17                                  | 22.7                | 1.70                 |
| 30              | 700 ~ 900 | 2.50                 | 30              | 1000     | 14                                  | 29.6                | 1.40                 |
| 45              | 1400 ~ 2300 | 0.80                | 45              | 2000     | 13                                  | 39.0                | 0.75                 |

Fig. 10 Schematic illustration of the equivalent diameter technique (a) and the experimental correlation of the effective length scale for both different systems (b).
In order to calculate the “equivalent gap scale” for the twin-flame system, we introduced the effective diameter ($D_{ef}$) at the gap zone comprising two particular areas (indicated in red and blue), thus, $D_{ef}/D = [4/\pi \times (1 - \pi/4 + L/D)]^{1/2}$. Here, the first term on the right-hand side represents the area of flow distribution [red area in Fig. 10(a)], which was approximately 17.4% of the cross-sectional flow area for the twin circular burners. The second term, the blue area, was modified through the length scale ratio ($L/D$), and its effective diameter was calculated to include the equivalent cross-sectional area of the opening gap zone between the twin-flame [Fig. 10(a): top view]. A summary of the $\Delta f$ for both systems in terms of $D_{ef}$ or $D$ is shown in Fig. 10(b), which suggests that the relationship between $\Delta f$ (the frequency difference between the varicose (in-phase) and the sinuous (anti-phase) modes of the dynamic flames) and $D$ or $D_{ef}$ follows a linear model, and that their correlation is inversely linear. This fact quite resembles the well-known $St-Fr^{-1}$ scaling of the flickering flames, where the dynamic frequency ($f$) is inversely correlated with the imposed fire scale ($D$).

The sinuous meandering behavior of the sinuous mode for the single-flame system and the anti-phase mode for the twin-flame system suggests the potential insight of an approach to determine the sinuous mode, which in general, randomly appears and is hardly controlled. Because the meandering motion occurs when a hot flame is surrounded by cold air (in the single-flame system), and when cold air is surrounded by hot flames (in a twin-flame system), we can assert that sinuous meandering would occur when there are two thermal boundary layers existing at a certain length scale. Due to the large temperature variation in the thermal layers, a strong shear layer is introduced by buoyancy. Shear layers at the edge of the scale (gap) generate vortices, get detached, and then flow downstream. In this regard, we can attribute the cause of the sinuous meandering to the detachment of the vortices from the two edges of the scale of interest, which is a quite similar scenario as that of the so-called Karman vortex formed downstream of an obstacle. Accordingly, the Karman vortex exhibits asymmetric in a crossed flow, which shows the sinuous meandering fashion. Also, although it is known that there is in-line oscillation mode at double frequency (the so-called in-line mode), we did not observe its occurrence herein. Unlike the Karman vortex that forms stably in a uniform flow, the flames we studied were formed under an accelerating flow field. This might be the reason why the clear sinuous motion appeared at the limited range of the imposed conditions. In the twin-flame system, because the evolved vortices weakened in the (surrounded) hot zone and rather confined in the cold zone, the clear winding motion can be sustained in a solid manner. By contrast, in the single-flame system, the vortices are released to the cold ambient with no confined mechanism, which makes it difficult to keep a stable sinuous mode. This hypothesis is supported by the trends we observed in the experiments.

In summary, it is important to note that the sinuous oscillation behavior might be essentially the same for the single-flame system and the twin-flame system, or further, to that of the Karman Vortex structure in fluid motion. To confirm this hypothesis along with other perspectives, we will summarize the motion behavior based on the Reynolds number.

### 4.2 Interpretation of mode transition

As suggested above, generation of Karman vortex would be the key to show asymmetric flame dynamics. To support this, let us look for Reynolds number at the transition from axisymmetric to asymmetric flame dynamics. Figure 11 shows a plot depicting the relationship between $St$ and $Re$, where the blue and red symbols represent the varicose and sinuous modes of the single-flame system, the star and plus signs denote, respectively, the initial condition to start a pulsation for the single-flame system and the transition from the in-phase to the anti-phase mode of the twin-flame system, and the vertical dashed line represents the region of the characteristic wake flow reported by Taneda (1956), Betchov and Szewczyk (1963), and Williamson (1996). Here, Strouhal number is defined as: $St = fD/V$, where $V$ [m/s] is characteristic velocity of fuel, $f$ [Hz] is characteristic frequency of flame oscillation and $D$ [m] is the characteristic length.

Let us briefly explain how this graph is made. First, select the diameter of the burner and starting with small fuel flow rate to form the perfect steady laminar diffusion flame. Then increase the fuel flowrate and the initial varicose oscillation is identified. This is the point shown with asterisk in Fig. 11. Since then “varicose” mode always appears as increase of the fuel flowrate, hence, blue symbols are plotted over the Fig. 11. Selecting smaller burner ($D = 10$ mm) or taking lower fuel flowrate, sinuous oscillation mode is hardly observed, resulting that no red plot is made in the figure. When a larger burner is employed, a chance to appear the sinuous mode is pronounced. Importantly, we successfully plot the first chance to identify the sinuous oscillation mode, which corresponds to the very left red symbol (noted with red circle in the figure) for all diameters. The condition where red and blue points coexist means that both varicose and sinuous modes have been identified randomly (namely, the mode change frequently occurs).
Fig. 11 A plot depicting the relationship between $St$ and $Re$. The blue and red markers respectively represent the varicose and sinuous modes of the single-flame. The star indicates the onset of flame flickering in the single-flame system. The plus sign indicates the anti-phase mode at the critical condition (transition) of the twin-flame system. The vertical dashed line represents the region of the characteristic wake flow reported by past studies.

The results clearly show that three representative cases for single flame system and four representative cases for twin-flame system at various fire scales consistently summarized by Reynolds number defined by $D$ (for single-flame system) and $D_{eff}$ (for twin-flame system). Perceivably, appearance of the flame pulsation (axisymmetric motion) appeared in the region 1 for single flame system, where the vortex ring starts to form behind a bluff body in fluid trajectories for characteristic wake flow. As Reynolds number increases, sinuous meandering shall appear (noted with red circle as mentioned) around $Re \approx 49$, where the boundary of regions 1 and 2. Accordingly, co-exist of varicose and sinuous modes (actually switching two modes randomly) are indicated mostly in region 2. Interestingly, the transition conditions extracted from twin-flame system satisfactory fall in region 2. It implies that Reynolds number based on the effective diameter ($D$ for single-flame system and $D_{eff}$ for twin-flame system) could be key parameter to determine the transition. Importantly, as denoted in the figure, region 2 is identical to have wake shedding in the fluid flow to appear well-known Karman-Vortex in the fluid. Obviously such vortex street is not-symmetric (rather asymmetric) fashion, which is resembling sinuous mode for the single flame system and anti-phase oscillation for twin-flame system.

Remember that all flames in this work is in the region of buoyancy-controlled condition ($Fr < 1$) so that the Reynolds number is not controlling the whole flame dynamics. What we suggest in this study is that the critical Reynolds number showing the wake shedding condition may play as “trigger” of the mode transition for buoyancy-controlled flame motion. Although this is just a hypothesis predicted by the present work under the limited range of the parameter, further study is needed to confirm.

5. Concluding remarks

In order to reveal the similarity of the appearance of the sinuous meandering motion, we have experimentally and numerically compared two different flame systems (single-flame and twin-flame). We have successfully demonstrated the physical similarity of the flame dynamics and the frequency jump when the mode changes from the axisymmetric
behavior to the asymmetric one. There is a close resemblance between the sinuous mode (asymmetric flowing characteristics) of the single-flame system and the anti-phase mode of the twin-flame system. Moreover, the alternately staggered flowing pattern of the anti-phase mode exhibiting the winding-plume was characterized as the sinuous meandering. If a proper length scale is introduced, the frequency jump can be summarized accordingly by the characteristic length scale following an inverse relationship. Furthermore, the asymmetric dynamics quite resembles the Karman vortex feature. We have also attempted to prove the similarity between the two by examining the critical Reynolds number that is associated with the asymmetric dynamic behavior.

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