Probing hidden spin-2 mediator of dark matter with NA64\(e\), LDMX, NA64\(\mu\) and M\(^3\)

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The connection between Standard Model (SM) particles and dark matter (DM) can be introduced via hidden spin-2 massive mediator. In the present paper we consider the simplified benchmark link between charged lepton sector of SM and DM particles which are considered to be a hidden Dirac fermions from the dark sector. The regarding couplings are established through the dimension-5 operators involving spin-2 mediator field and the energy-momentum tensors of both SM and DM sectors. We study in detail the implication of this scenario for the lepton fixed-target facilities, such as NA64\(e\), LDMX, NA64\(\mu\) and M\(^3\). In particular, for the specific experiment we discuss in detail the missing-energy signatures of spin-2 boson production followed by its invisible decay into stable DM pairs. Moreover, we derive the expected reaches of these experiments for the projected statistics of the leptons accumulated on the target. We also discuss the implication of both nuclear and atomic form-factor parametrizations for the differential spectra of hidden spin-2 boson emission, the total cross-section of its production and the experimental reach of the fixed-fixed target facilities for probing hidden spin-2 DM mediator.

I. INTRODUCTION AND FRAMEWORK

The nature of the dark matter (DM) particles remains puzzling for decades. The indirect evidences for the DM are associated with galaxy rotation velocities, large scale structures, cosmic microwave background anisotropy, gravitational lensing, etc. Some extensions of the standard model (SM) imply connection between SM and DM via an idea of portals. For instance, the dark photon portal \([1–4]\) and Higgs boson portal \([5–8]\). Such portal scenarios suggest a systematic probing of the connection between SM and DM particles which are assumed to be independent.

In the present paper we show that corresponding setup (i) has a very broad phenomenological implication and can be examined through the missing energy signatures in the existent (NA64\(e\) \([25–37]\) and NA64\(\mu\) \([38, 39]\)) and the projected (LDMX \([40–43]\) and M\(^3\) \([44, 45]\)) lepton fixed-target experiments. These signatures can be obtained by bremsstrahlung-like reaction of the production of a spin-2 boson by the charged lepton \(l^-\) impinging on a nucleus \(N\). The corresponding diagrams are shown in Fig. 1. In the present paper we will focus on mainly invisible channel of dark spin-2 boson decay into DM particles, such that \(\text{Br}(G \rightarrow \chi \chi) \simeq 1\). Furthermore, for the \(G\)-boson production cross-section calculation we exploit the equivalent photons approximation, also known as the Weizsacker–Williams (WW) approach that is typically exploited for the hidden particle yield estimate at both beam dump and fixed-target experiments. In particular, this approach provides a fairly reasonable approximation (i.e., at the level of \(\lesssim 2\%\)) for the exact-tree-level production cross-sections of both hidden spin-0 and spin-1 bosons \([39, 46, 47]\).

In addition, we also discuss in detail the impact of both nuclear and atomic form-factor parametrizations on: (i) the differential spectra of \(G\) boson emission, (ii) the total cross-section of its production (iii) the experimental reach of the fixed-fixed target facilities for probing hidden spin-2 boson.

The paper is organized as follows. In Sec. II we derive...
explicitly the double differential cross-section of G-boson production in WW approach. In Sec. III we discuss the set of form-factors that are typically used for the calculation of regarding cross-sections. In this section we also study the impact of various form-factor parametrizations on the virtual photon flux distribution from the charged particles. In Sec. IV we provide a description of the missing energy signatures for the analysis of DM production at lepton fixed-target experiments, such as NA64e, LDMX, NA64μ and M3. In Sec. V we discuss the impact of form-factor parametrization on both differential and total cross-sections of hidden spin-2 boson production. In Sec. VI we obtain the constraints on the parameter space of spin-2 DM mediator from the NA64, LDMX, NA64μ and M3 facilities. In this section we also study implication of the form-factors for the expected reach of the regarding experiments.

II. THE PRODUCTION CROSS SECTION OF DM MEDIATOR IN WW APPROACH

Let us consider the kinematic variables of bremsstrahlung-like process 2 → 3 in the laboratory frame:

\[ l^{-}(p) + N(P_i) \rightarrow l^{-}(p') + N(P_f) + G(k), \]

where \( p = (E_l, p) \) is the momentum of incoming charged lepton, \( p' = (E_l', p') \) is the momentum of outgoing lepton, \( k = (E_G, k) \) is the momentum of spin-2 mediator, \( P_i = (M, 0) \) and \( P_f = (P_0^f, P_f) \) are the momenta of the initial and outgoing nucleus respectively. We define four-momentum transfer to nucleus as \( P_i - P_f = q = (q_0, q) \), such that the photon virtuality takes the form \( t = -q^2 > 0 \). In order to calculate the differential cross-section of G-boson production in nuclear interaction one can exploit the Weizsäcker-Williams approximation, implying that the energy of incoming lepton is much higher than \( m_l \) and \( m_G \). In this case, the incoming charged particle is replaced by its effective photon flux, such that the phase space of the process \( l^{-}(p)N(P_i) \rightarrow l^{-}(p')N(P_f)G(k) \) is reduced to the Compton-like process \( l^{-}(p)\gamma(q) \rightarrow l^{-}(p')G(k) \). In particular, one can obtain the following expression for the double differential cross-section [48–50]:

\[
\frac{d\sigma(p + P_i \rightarrow p' + P_f + k)}{d(pk)d(kP_i)}|_{WW} = \frac{\alpha\chi}{\pi(p'P_i)} \frac{d\sigma(p + q \rightarrow k + p')}{d(pk)}|_{t=t_{min}},
\]

where the flux of virtual photon \( \chi \) from nucleus is expressed through the elastic form-factor \( F(t) \) as follows:

\[
\chi = Z^2 \int_{t_{min}}^{t_{max}} \frac{t - t_{min}}{t^2} F^2(t) dt,
\]

where \( Z \) is the atomic number of the nucleus, the explicit expressions for the form-factors \( F(t) \) are discussed below in Section III. The photon flux for the inelastic form-factor is proportional to \( \alpha \), and thus can be ignored in the calculation for heavy nuclei \( Z \propto \mathcal{O}(100) \). The WW approach in Eqs. (3) and (4) implies that the virtuality \( t \) has its minimum \( t = t_{min} \) when \( q \) is collinear with \( k - p \). In particular, the expression for \( t_{min} \) is derived below (see, e. g. Eq. (11)). For ultra-relativistic incident lepton in laboratory frame we have:

\[
d(kp)d(kP_i) \sim |J(\cos(\theta_G), E_G)| d\sigma(\theta_G) dE_G \sim M|p||k|d\cos(\theta_G)dE_G,
\]

where \( \theta_G \) is the angle between the initial lepton direction and the momentum of the produced G-boson and \( J(\cos(\theta_G), E_G) \) is the Jacobian of the transformation from \( (kp) \) and \( (kP_i) \) variables to \( \cos \theta_G \) and \( E_G \). So that by substituting Eq. (5) into Eq. (3), we get the following expression for the double differential cross-section after some algebraic simplification:

\[
\frac{d\sigma(p + P_i \rightarrow p' + P_f + k)}{dxd\cos(\theta_G)}|_{WW} = \frac{\alpha\chi E_p^2 x\beta_G}{\pi (1 - x)} \frac{d\sigma(p + q \rightarrow k + p')}{d(pk)}|_{t=t_{min}},
\]

where \( \alpha = e^2/(4\pi) \approx 1/137 \) is the fine structure constant, \( x = E_G/E_l \) is the energy fraction of spin-2 mediator that it carries away and \( \beta_G = \sqrt{1 - m_G^2/(xE_l)^2} \) is the typical velocity of G-boson. By solving the mass-shell equations for both outgoing electron and nucleus:

\[
p^2 = (q + p - k)^2 = m_l^2, \quad P_f^2 = (P_i - q)^2 = M^2,
\]

we get the following auxiliary expressions: \( q_0 = -t/(2M) \) and \( |q|^2 = t^2/(4M^2) + t \). Furthermore, by taking into account the small typical energy transferred to the nucleus \( |q|/M \ll 1 \), we get both approximate expressions \( t \approx |q|^2 \) and \( q_0 \approx -|q|^2/(2M) \) for the photon virtuality and nucleus energy transfer respectively. The value of \( q_0 \) is fairly small and thus can be neglected in the calculation, since the typical momentum is \( |q| \lesssim \mathcal{O}(100) \) MeV and the mass of nucleus is of the order of \( M \approx \mathcal{O}(100) \) GeV. On the other hand, the photon flux \( \chi \) is sensitive to the photon virtuality \( t \approx |q|^2 \), as long as the screening effects due to the atomic electrons should be taken into account (see e. g. Sec. III for detail).

Next, let us introduce the Mandelstam variables in the following form:

\[
s_2 = (p + q)^2, \quad u_2 = (p - k)^2, \quad t_2 = (p - p')^2.
\]

It is worth mentioning that Eq. (7) implies, \( s_0^2 - |q|^2 + u_2 + 2q_0V_0 - 2(q, V) = m_l^2 \), where we define the three-vector as \( V = p - k \). Then for the small energy transferred to the nucleus \( |q|^2 \ll s_2, t_2, u_2 \), one obtains:

\[
t = \frac{(u_2 - m_l^2)^2}{4|V|^2 \cos^2(\theta_{qV})},
\]
where $\theta_{qV}$ is the angle between vector $q$ and $V$. Therefore keeping only leading terms in $m_{G}^{2}/E_{l}^{2}$, $m_{\ell}^{2}/E_{l}^{2}$, $m_{l}^{2}/E_{l}^{2}$ and $\theta_{G}^{\ell}$, we obtain the approximate expression for the absolute value of the three vector, $|V| \simeq E_{l}(1-x)$. Let us introduce the following auxiliary functions:

$$U \equiv m_{l}^{2} - u_{2} \simeq E_{l}^{2}\theta_{G}^{\ell}x + m_{G}^{2}(1-x)/x + m_{l}^{2}x > 0, \quad (10)$$

thus by exploiting Eqs. (9) and (10) one can obtain the expressions for minimum of the virtuality $t = t_{min}$, implying that $\theta_{qV} \simeq \pi$, such that the three vector $q$ is almost collinear with $k - p$:

$$t_{min} \simeq |q|^{2} \simeq U^{2}/(4E_{l}^{2}(1-x)^{2}). \quad (11)$$

It is worth noting that the WW approximation for the $2 \rightarrow 3$ cross-section implies that $t_{min}$ is the function of both $x$ and $\theta_{G}$ variables, so that it is fairly accurate approach for the exact tree level cross-section (for detail, see e. g. Ref. [39] and references therein). Moreover, we note that $\theta_{qV} \simeq \pi$ as long as $\theta_{G} \ll 1$ and $q$ is collinear with $k - p$, therefore this yields, $(q, k) \simeq |q||k| \simeq Ux/(2(1-x))$, and we finally get the following expressions for Mandelstam terms expressed through the both $x$ and $\theta_{G}$ variables:

$$u_{2} = m_{l}^{2} - U \lesssim 0, \quad (12)$$

$$t_{2} = -2(q, k) + t + m_{G}^{2} \simeq -Ux/(1-x) + m_{G}^{2} \lesssim 0, \quad (13)$$

$$s_{2} = 2m_{l}^{2} + m_{G}^{2} - t - t_{2} - u_{2} \simeq U/(1-x) + m_{l}^{2} \simeq 0. \quad (14)$$

Note that both the energy conservation law, $q_{0} + E_{l} = E_{l}^{'} + E_{G}$, and the condition, $q_{0}/E_{l} \ll 1$, imply that $E_{l} \simeq E_{G} + E_{l}^{'}$, thus for the energy fraction $x = E_{G}/E_{l}$ we get respectively its minimum and maximum values in the following form $\hat{x}_{\min} \simeq m_{G}/E_{l}$ and $\hat{x}_{\max} \simeq 1 - m_{l}/E_{l}$.

By exploiting FeynCalc package [51, 52] for the Wolfram Mathematica routine [53], we get matrix element for process $l^{-} \gamma \rightarrow l^{-}G$ (see e. g. Appendix (A)):

$$|\mathcal{M}_{l^{-} \gamma \rightarrow l^{-}G}|^{2} = \frac{c_{l}^{2}e^{2}u_{2}s_{2} - 2m_{l}^{2}[((t_{2} + u_{2})^{2} + (u_{2} - m_{G}^{2})^{2})[4u_{2}(2m_{l}^{2} - s_{2}) - m_{G}^{2}t_{2}]}{4t_{2}(u_{2} - m_{l}^{2})^{2}(s_{2} - m_{l}^{2})^{2}} - \frac{c_{l}^{2}e^{2}}{A^{2}} \frac{m_{G}^{2}R(m_{l}, m_{G}, t_{2}, u_{2})}{12t_{2}(u_{2} - m_{l}^{2})^{2}(s_{2} - m_{l}^{2})^{2}}, \quad (15)$$
where $R(m_1, m_G, t_2, u_2)$ is regular expression for $m_l$ and $m_G$ that is given in the Appendix A. Note that in Eq. (15) we set universal coupling of $G$ boson to both charged lepton and photon, such that $c_1 = c_e = c_μ = c_γ$. It implies that the unitarity of the scenario is not violated at low energies as soon as $m_G \to 0$ (see, e.g. Ref. [10] and references therein for detail). The differential cross section for process $2 \to 2$ is
\[
\frac{dσ_{2→2}}{d(pk)} = 2 \frac{dσ_{2→2}}{du_2} = \frac{1}{8π(s_2 - m_1^2)} |M_{l→G}|^2 , \tag{16}
\]
where $|M_{l→G}|^2$ is defined by (15). As a result, the double-differential cross section of the bremsstrahlung-like process $IN \to lNG(\to \gamma \chi)$ takes the following form:
\[
\frac{dσ(p + P_l \to p' + P_{l'} + k)}{dxdcos(θ_G)} |_{WW} = \frac{αχ E^2 xβ_G}{π(1 - x) 8π(s_2 - m_1^2)} |M_{l→G}|^2 . \tag{17}
\]
In order to verify our calculation for the matrix element squared of spin-2 particle production $e^+(p)γ(q) \to G(k)e^−(p')$ one can exploit the crossing symmetry for the well known process of electron positron pair annihilation $e^+(p_ε+)e^−(p_ε−) \to G(p_G)γ(p′_G)$, for which the amplitude squared can be found in Ref. [10]. In particular, for the massless fermion the regarding transition element reads as follows [10]:
\[
|M_{e^+e^-→G}|^2 = \frac{c^2 e^2 (s^2_2 + 2t_2(s_2 + t_2) - 2m^2_2t_2 + m^4_G)}{A^2} \times (4t_2(s_2 + t_2) - m^2_G(s_2 + 4t_2)) , \tag{18}
\]
where the Mandelstam variables are defined by:
\[
\hat{s}_2 \equiv (p_ε+ + p_ε−)^2 , \quad \hat{t}_2 \equiv (p_ε− - p_G^−)^2 , \quad \hat{u}_2 \equiv (p_ε− - p_G^−)^2 . \tag{19}
\]
The crossing-symmetry implies the momentum replacement for the initial process $e^+e^- \to γG$ in the following form: $p_ε+ \to -p_ε−$ and $p_G^− \to -p_γ$. The regarding Mandelstam variables transform as follows: $s_2 \to t_2$, $t_2 \to u_2$ and $u_2 \to s_2$, where the notations for $s_2$, $u_2$, and $t_2$ are introduced (see e.g. Eq. (8)) in order to match with conventional labels of the authors of Ref. [50]. Finally this yields the following expression for the matrix element squared:
\[
|M_{e^+γ→Ge^-}|^2 = \frac{c^2 e^2 (t^2_2 + 2u_2(u_2 + t_2) - 2m^2_2u_2 + m^4_G)}{4u_2t_2(u_2 + t_2 - m^2_G)} \times (4u_2(u_2 + t_2) - m^2_G(t_2 + 4u_2)) \tag{20}
\]
It is worth noting that Eq. (15) tends to Eq. (20) in the massless lepton limit as soon as $m_l \to 0$.

III. THE VIRTUAL PHOTON FLUX FUNCTION

In this section we discuss the impact of different atomic and nuclear form-factor parametrization $F(t)$ for the photon flux $χ$ which is given by Eq. (4). The latter affects the differential and total cross sections of hidden spin-2 boson production, that will be discussed in Section V.

The nuclear form-factor in the laboratory frame is associated with charge density of nucleus through the Fourier transformation for both spin-0 and spin-1/2 (see e.g. Refs. [39, 46, 47, 49, 50, 54–56] and references therein for detail). The atomic form-factor can be represented as the nuclear form-factor that takes into account the screening of the nucleus by Coulomb field due to the atomic electrons. Indeed, in the limit $t \to 0$ nuclear form-factor tends to $F_{nuc}(t) \to 1$ in opposite to the atomic form-factor, which tends to $F_{atom}(t) \to 0$ as $t \to 0$. In addition, screening charge density of the atomic form-factor can be represented as a convolution of the nuclear charge density with the specific screening density. In particular, following L. Schiff [57] to obtain the atomic form-factor one should multiply the nuclear form-factor by the screening term $t/(t_a + t)$.

First, let us consider the elastic atomic form-factor that was studied by Y. Tsai [49] and L. Schiff [58] in the following form [50]:
\[
FRS(t) = \frac{t}{(t_a + t)} \frac{1}{(1 + t/t_a)} , \tag{21}
\]
where $\sqrt{t_a} = 1/R_a$ is a momentum transfer associated with nucleus Coulomb field screening due to the atomic electrons, with $R_a$ being a typical magnitude of the atomic radius $R_a = 111Z^{-1/3}/m_e$, $\sqrt{t}_d = 1/R_n$ is the typical magnitude associated with nuclear radius $R_n$, such that $R_n \approx 1/\sqrt{d}$ and $d = 0.164A^{-2/3} GeV^2$. Since the integration with respect to $t$ in Eq. (4) is dominated by $t \gtrsim t_{min}$, therefore the magnitude of $t_{min}$ (see e.g. Eq. (11)) defines the typical form-factor approach to be considered. In particular, if $t_{min}/t_a \ll 1$, then the complete screening regime takes place, which implies that nuclei transfer momentum is small and the typical atomic elastic form-factor is much less then unity, $FRS(t) \approx t/t_a \ll 1$. On the other hand, as soon as $t_{min}/t_a \gg 1$, then no screening regime occurs. In this case the atomic elastic form-factor is scaled as $FRS(t) \approx 1/(1 + t/t_a)$ and the nuclear size effects dominate, it implies also that the nuclear elastic form-factor is highly lost. It is worth noting, the parameter space of interest $1 MeV \leq m_G \leq 1 GeV$ and $10 GeV \leq E_0 \leq 100 GeV$ both screening and nuclear size parameters can contribute to the virtual photon flux and thus to the total yield of the $G$-boson production.

Next, let us consider the nuclear Helm’s form-factor $F_{Helm}(t)$ that corresponds to the inverse Fourier transformation of the nucleus charge density $\rho(r)$. The latter can be represented as the convolution of the spherically
uniform charge inside the nucleus and the Gaussian profile implying better accounting of the the edge of nucleus [59]. Both the nuclear Helm’s form-factor $F_{H_{nuc}}(t)$ and atomic Helm’s form-factor $F_H(t)$ read as follows respectively [60, 61]:

$$F_{H_{nuc}}(t) = \frac{3j_1(\sqrt{t}R_H)}{\sqrt{t}R_H} \exp{-s_H^2 t/2},$$

$$F_H(t) = t/(t_a + t) F_{H_{nuc}}(t),$$

where $j_1(x)$ - first spherical Bessel function of the first kind, the effective nuclear radius $R_H$ can be parameterized as

$$R_H = \sqrt{c_H^2 + 7/3\pi^2 a_H^2 - 5s_H^2},$$

where $s_H = 0.9$ fm is the nuclear shell thickness, $a_H = 0.52$ fm and $c_H = (1.23A^{1/3} - 0.6)$ fm. It is worth noting that we also set $F_{H_{nuc}}$ to zero for $t \gtrsim (4.49/R_H)^2$, it implies that we consider only non-negative values of the Helm’s form-factor, since it vanishes at $t \approx (4.49/R_H)^2$ and the dominant contribution for the photon flux is due to the typical range $t \lesssim (4.49/R_H)^2$ (for details see e. g. Ref. [61]).

Finlay, let us consider now the exponential atomic form-factor corresponding Gaussian charge distribution that reads as follows [62]:

$$F_E(t) = \frac{t/(t_a + t)}{\exp\left(-tR_{exp}^2/6\right)},$$

where the mean radius of nucleus is defined as $R_{exp} = (0.91A^{1/3} + 0.3)$ fm. As was discussed above, the screening term $t/(t_a + t)$ in Eq. (24) is introduced to take into account the shielding of the nucleus Coulomb field due to the atomic electrons.

Note that the general virtual photon flux (4) depends on both fraction of energy $x$ and emission angle of spin-2 boson $\theta_G$ through the function of the lower limit $t_{min}(x, \theta_G)$ (see e. g. Eq. (11) for detail). So that it is instructive to study the the dependence of $\chi$ upon $t_{min}$ for various form-factor parametrizations. Intriguingly, that the integration over $dt$ in the virtual photon flux (4) of the Tsaí-Schiff’s elastic form-factor (21) can be performed in the analytical way through the elementary functions. The latter simplification can be exploited for the reducing of the computational time of $\chi_{TS}$ integration. As the result, the virtual photon flux in analytic form for
Tsai-Schiff’s form-factor reads as follows:
\[
\chi_{TS} = \frac{Z^2 q^2}{(t_a - t_d)^3} \left( |C_1^\chi + C_2^\chi t_{\min}| + \right.
\]
\[
\left. + |C_3^\chi + C_4^\chi t_{\min}| \ln \left( \frac{t_{\min} + t_d}{t_{\min} + t_a} \right) \right) \tag{25}
\]

where the functions \( C_1^\chi, C_2^\chi, C_3^\chi \) and \( C_4^\chi \) are defined by the following expressions respectively:
\[
C_1^\chi = \left( \frac{t_d(t_a - t_d)}{t_d + t_{\max}} + \frac{t_a(t_a - t_d)}{t_a + t_{\max}} \right),
\]
\[
-2(t_a - t_d) + (t_a + t_d) \ln \left( \frac{t_d + t_{\max}}{t_a + t_{\max}} \right),
\]
\[
C_2^\chi = \left( \frac{t_a - t_d}{t_d + t_{\max}} + \frac{t_a - t_d}{t_a + t_{\max}} + 2 \ln \left( \frac{t_d + t_{\max}}{t_a + t_{\max}} \right) \right),
\]
\[
C_3^\chi = -(t_a + t_d), \quad C_4^\chi = -2. \tag{29}
\]

The regarding flux and exponential form-factor can be expressed through the special function as follows
\[
\chi_E = Z^2 \left( \frac{t_a + t_{\min}}{t_a + t_d} e^{-R_{exp} t/3} + \frac{3 + R_{exp}^2 (t_a + t_{\min})}{3} e^{R_{exp} t/3} \right) \tag{27}
\]
\[
\times \left. \frac{R_{exp}(t_a + t_{\min})}{3} \right|_{t_{\min}} \tag{28}
\]
where \( E(x) = \int_{-\infty}^{x} e^t / t \, dt \) is the exponential integral function. Contrary, photon flux with Helm’s form-factor cannot be integrated over \( t \) in analytical way, in what forms we perform the numerical integration for both \( \chi_H \) and \( \chi_{H_{\text{act}}} \). In addition, the numerical calculations reveal that general photon flux (4) depends weakly on \( t_{\max} \) as long as \( t_{\max} \gg t_{\min}, t_a, t_d \). On the other hand, one can show that the typical values of \( t_{\min}, t_a \) and \( t_d \) don’t exceed the magnitude of the order of \( \mathcal{O}(1) \, \text{GeV}^2 \). In what follows we set \( t_{\max} \approx 10 \, \text{GeV}^2 \) for the numerical estimations.

In the left panel of Fig. 2 we show the virtual photon fluxes \( \chi \) as a function of \( t_{\min} \) for the lead target material of the NA64e experiment with atomic number of \( Z = 82 \) and particle number of \( A = 207 \) that parameterize both typical screening and nuclear radius. In the right panel of Fig. 2 we choose \( \chi_{TS} \) as a benchmark photon flux and show that the relative differences between the typical momentum squared \( 10^{-10} \, \text{GeV}^2 \) \( \lesssim t_{\min} \lesssim 10^{-3} \, \text{GeV}^2 \) all the atomic photon fluxes match with a reasonable accuracy at the level of \( \lesssim 5\% \). In order to illustrate the impact of the screening effect we show both the nuclear and atomic Helm’s photon fluxes in Fig. 2. In particular, for the screening region \( t_{\min} \lesssim t_a \approx 10^{-9} \, \text{GeV}^2 \) the nucleus photon flux \( \chi_{H_{\text{act}}} \) exceeds the atomic one \( \chi_H \) by 10% approximately. The latter one is associated also with the smaller slope of \( \chi_H \) in the screening range \( t_{\min} \lesssim t_a \).

It is worth noting, that there are also form-factor models that consider the charge density as the sum of Gaussian, Fourier-Bessel functions and Klein-Nystrand’s charge density [63-67]. In addition, the Fermi distribution for the nucleus charge is also discussed in the literature [60, 66]. We note that these nucleus form-factor parametrizations are beyond the scope of the present paper, even though we would expect that they provide a similar results for the flux shown in Fig. 2.

\[ \chi_{TS} = \frac{Z^2 q^2}{(t_a - t_d)^3} \left( |C_1^\chi + C_2^\chi t_{\min}| + \right. \]
\[ \left. + |C_3^\chi + C_4^\chi t_{\min}| \ln \left( \frac{t_{\min} + t_d}{t_{\min} + t_a} \right) \right) \tag{25} \]

IV. MISSING ENERGY SIGNAL

In this section we discuss the setups for the fixed target experiments such as NA64e (SPS, CERN) [25–37, 68], LDMX (Fermilab) [40–43], NA64e (SPS, CERN) [38, 39] and M³ (Fermilab) [44, 45], that can potentially probe invisible decay of \( G \rightarrow \chi \bar{\chi} \) in the associated charged lepton missing energy process \( IN \rightarrow ING \rightarrow \chi \bar{\chi} \), where the label \( l = (e, \mu) \) denotes either electron or muon beam and \( N \) denotes the nucleus of the target. Given the benchmark coupling Eq. (1), the spin-2 Dark Matter mediator \( G \) can decay through the different channels. In particular, as soon as \( m_G \gtrsim 2 m_l \) the invisible decay \( G \rightarrow l^+ l^- \) is allowed with a specific decay width [9]

\[
\Gamma(G \rightarrow l^+ l^-) = \frac{c_G^2 m_G}{160 \pi} \left( \frac{m_G}{\Lambda} \right)^2 \times \]
\[
(1 - 4 m_l^2/m_G^2)^{3/2} (1 + 8 m_l^2/(3 m_G^2)) \tag{31},
\]

where \( m_l \) is the mass of the charged lepton. The Lagrangian (1) also implies that for \( m_G \gtrsim 2 m_N \) the invisible decay into fermion DM pair \( G \rightarrow \chi \bar{\chi} \) is kinetically allowed with a decay width

\[
\Gamma(G \rightarrow \chi \bar{\chi}) = \frac{c_G^2 m_G}{160 \pi} \left( \frac{m_G}{\Lambda} \right)^2 \times \]
\[
(1 - 4 m_N^2/m_G^2)^{3/2} (1 + 8 m_N^2/(3 m_G^2)) \tag{27},
\]

where \( m_N \) is the mass of the hidden Dirac DM fermion. In this paper we focus on the processes of the invisible channel of \( G \)-boson decay into pair of hidden dark fermions, \( G \rightarrow \chi \bar{\chi} \) with Br\((G \rightarrow \chi \bar{\chi}) \simeq 1 \) for the sufficiently light DM particles, \( m_G \gtrsim 2 m_N \). It implies that decay widths obey the condition \( \Gamma_{G \rightarrow \chi \bar{\chi}} \gg \Gamma_{G \rightarrow \ell^+ \ell^-} \) and therefore the coupling constants are chosen to be \( c_G \gg c_{\ell} \) for the parameter space of interest \( 1 \, \text{MeV} \lesssim m_G \lesssim 1 \, \text{GeV} \). As the result, this benchmark conditions impose the rapid decay of spin-2 DM mediator to \( \chi \bar{\chi} \) pair after its production in the process \( IN \rightarrow ING \).

Let us estimate \( N_G \) the number of \( G \) produced by the lepton beam at fixed target as follows

\[
N_G \simeq \text{LOT} \cdot \frac{\rho N_A}{A} \frac{L_T}{x_{\text{min}}} \int_{x_{\text{min}}}^{x_{\text{max}}} dx \frac{d\sigma_{2-3}(E_l)}{dx} \text{Br}(G \rightarrow \chi \bar{\chi}), \tag{33}
\]

where LOT is number of leptons accumulated on target, \( \rho \) is target density, \( N_A \) is Avogadro’s number, \( A \)
is atomic weight number. \( L_T \) is the effective interaction length of the lepton in the target, \( \frac{d\sigma_{WW}}{dx}, \text{GeV}^2 \) is the differential cross-section of the lepton missing energy process \( LN \rightarrow lNG(\rightarrow \chi \bar{\chi}) \), \( E_l \) is the initial energy of lepton beam, \( x_{\text{min}} \) and \( x_{\text{max}} \) are the minimal and maximal fraction of missing energy respectively for the regarding experimental setup, \( x \equiv E_{\text{miss}}/E_l \), where \( E_{\text{miss}} \equiv E_G \). The \( x_{\text{min}} \lesssim x \lesssim x_{\text{max}} \) cuts are determined by specific fixed–target facility. In what follows, we describe below the input benchmark parameters of the lepton fixed-target experiments.

### A. NA64e

The spin-2 mediator of DM can be produced in the reaction of ultra-relativistic electrons of \( E_e \approx 100 \text{ GeV} \) scattering off the nuclei of an active target \( eN \rightarrow eNG \) followed by rapid \( G \rightarrow \chi \bar{\chi} \) decay into DM particles. The fraction of the primary electron energy \( E_{\text{miss}} = xE_e \) can be carried away by \( \chi \bar{\chi} \) pair, that passes the NA64e detector without energy deposition. The remaining part of the beam energy fraction, \( E_{e,\text{GCC}} \approx (1 - x)E_e \), can be deposited in the electromagnetic calorimeter (ECAL) of NA64e by the recoil electrons. So that, the production of the hidden spin-2 boson can be observed as an excess of events with a single electromagnetic shower of energy \( E_{e,\text{GCC}} \) above the predicted background [26]. In this paper we carry out an estimate that implies the localization of that electromagnetic shower in the first radiation length of the lead target. As a result one can set \( L_T \approx X_0 \) in Eq. (33), where \( X_0 \approx 0.56 \text{ cm} \) is the typical radiation length of the electron in the lead target. In addition, the candidate event is required to have a missing energy in the range \( E_{e,\text{GCC}} \lesssim 0.5E_e \approx 50 \text{ GeV} \), that leads to the specific energy fraction cut \( x_{\text{min}} \approx 0.5 \) for the NA64e facility in Eq. (33).

Moreover we note that the electromagnetic calorimeter of the NA64e serves as the active target for the incident electron beam and contains \( 6 \times 6 \) Shahalyk-type modules which are made of the both plastic scintillator (Sc) and lead (Pb, \( \rho \approx 11.34 g \text{ cm}^{-3}, A = 207 \text{ g mole}^{-1}, Z = 82 \)) plates. It is worth mentioning that the production of \( G \)-boson inside the scintillator plate is subdominant due to its smaller density, \( \rho(\text{Sc}) \ll \rho(\text{Pb}) \), and larger radiation length, \( X_0(\text{Sc}) \gg X_0(\text{Pb}) \). In what follows in the numerical calculations we neglect the contribution of Sc to the production rate of the spin-2 mediator.

To conclude this subsection we note that NA64e uses the electron beam from the \( H4 \) beam line at the CERN SPS. The regarding beam intensity is estimated to be of the order of \( \approx 10^7 \) electrons per spill of \( \approx 4.6 \text{ seconds} \), however the number of good spills per day is expected to be \( \approx 4000 \). As a result, about \( \approx 120 \) days are needed to collect EOT \( \approx 5 \times 10^{12} \) at the \( H4 \) beam line for the projected statistics of the NA64e. In this work we also perform the analysis of the sensitivity of NA64e to probe DM for the EOT \( \approx 3.22 \times 10^{11} \) (see e. g. Ref. [32] for detail). The regarding statistics has been already collected by NA64e during previous experimental runs in 2016-2021.
The Light Dark Matter Experiment (LDMX) is the projected electron fixed-target facility at Fermilab, that aims probing the relic DM particles in the mass range between 1 MeV and 1 GeV. It can be considered as a facility that is complimentary to NA64\(e\) experiment due to its unique electron missing momentum technique [69]. It is remarkable that the missing-energy cuts and active veto system of both NA64\(e\) and LDMX experiments provide a significant background suppression at the level of \(\lesssim 10^{-12}\).

The projected LDMX experiment would employ the target, the silicon tracker and both the hadron and electromagnetic calorimeter which are located downstream. The missing energy cut of the recoil electron is set to be \(E_{\text{rec}}^{\text{cut}} \lesssim 0.3 E_e\), that implies \(x_{\text{min}} = 0.7\) in Eq. (33). We perform the analysis of the LDMX sensitivity for aluminium target (Al) \((\rho = 2.7 \text{ g cm}^{-3}, A = 27 \text{ g mole}^{-1}, Z = 13)\) with a thickness of \(L_T \simeq 0.4 X_0 \simeq 3.56 \text{ cm}\), where \(X_0 = 8.9 \text{ cm}\) is a radiation length of the electron in the aluminum. The energy of the beam is chosen to be \(E_e \simeq 16 \text{ GeV}\) and the projected statistics corresponds to EOT \(\simeq 10^{16}\) for the final phase of experimental running after 2027 (see e. g. Ref. [43] and references therein for detail).

**B. LDMX**

**C. NA64\(\mu\)**

The NA64\(\mu\) experiment is the fixed-target facility at the CERN SPS that searches for the dark sector particles in the muon beam missing momentum mode \(\mu N \rightarrow \mu NG\). It can be considered as a complementary experiment to NA64\(e\). For the sensitivity estimation of NA64\(\mu\) we set the energy of the muon beam to be \(E_{\mu} \simeq 160 \text{ GeV}\) and choose MOT \(\simeq 5 \times 10^{13}\) as the projected statistics for the muons accumulated on target. The NA64\(\mu\) experiment employs the lead Shashlyk-type electromagnetic calorimeter that serves as a target with a thickness of \(L_T \simeq 40 X_0 \simeq 22.5 \text{ cm}\). We note that one can neglect the muon stopping loss in the lead target of \(\simeq 22.5 \text{ cm}\) for the ultra-relativistic muons of \(E_{\mu} \simeq 160 \text{ GeV}\) due to small energy attenuation in the lead \(\langle dE_{\mu}/dx \rangle \simeq 12.7 \times 10^{-3} \text{ GeV/cm}\) (see e. g. Ref. [70] for detail).

In order to measure the momentum of incident and outgoing muon, the NA64\(\mu\) facility employs two magnet spectrometers. We choose the typical cut for the outgoing muon as \(E_{\text{cut}}^{\mu} \lesssim 0.5E_{\mu} \simeq 80 \text{ GeV}\), that corresponds to \(x_{\text{min}} = 0.5\) in Eq. (33). We note that for NA64\(\mu\) facility approximately \(\simeq 120\) days are needed to accumulate statistics of MOT \(\simeq 5 \times 10^{13}\) relative to EOT \(\simeq 5 \times 10^{12}\) for NA64\(e\) facility. This can be explained by the increased intensity of muon beam line at M3 that is higher by factor of \(\simeq 10\) than the intensity of electron beam at H4.
The muon missing momentum experiment (M³) at Fermilab is the projected fixed target facility that aims probing dark sector particles by employing the muon-specific missing energy signature $\mu N \rightarrow \mu NG (\rightarrow \chi \bar{\chi})$. It utilizes the muon beam of $E_\mu \simeq 15$ GeV impinging on the tungsten target (W) ($\rho = 19.3 \text{ g cm}^{-3}$, $A = 184 \text{ g mole}^{-1}$, $Z = 74$) of the typical thickness of $L_T \simeq 50X_0 \simeq 17.5 \text{ cm}$, where $X_0 \simeq 0.35 \text{ cm}$ is the radiation length of electron in the tungsten. The regarding facility also exploits a downstream detector to veto the Standard model background. It aims to collect MOT $\simeq 10^3$ during $\approx 3$ months of the experimental running. The cut on the missing momentum of muon is chosen to be $E_{\mu \text{rec}}^{\text{min}} \lesssim 9$ GeV, this yields the lower limit on the energy fraction $x_{\text{min}} \simeq 0.4$ in Eq. (33).

To conclude this subsection we note that for muons of $E_\mu \simeq 530 \text{ MeV}$ through the target medium of 50X₀. This allows one to neglect $\langle dE_\mu / dx \rangle$ in the numerical calculation of the yield of G-boson at M³. As a result that justifies the exploiting of Eq. (33) for the estimate of $N_G$ with $L_T \simeq 50X_0$ at M³ facility.
V. THE DIFFERENTIAL AND TOTAL CROSS SECTIONS

In this section we study both the single-differential and total cross sections in the WW approximation for the set of the form-factors discussed in Section III. The results are presented for the benchmark parameters of the lepton fixed target experiments which can potentially probe the invisible signatures associated with a lepton missing energy in bremsstrahlung-like process \( lN \rightarrow lNX(\rightarrow \chi \bar{\chi}) \). The total \( \sigma_{tot} \) cross-sections are obtained by the numerical integration of the double-differential cross section (6) for the specific experimental cuts. In the left panels of Figs. 3, 4, 5, and 6 we show the differential cross-sections of \( G \)-boson production as function of energy fraction \( x = E_G/E_l \) for the NA64\( e \), LDMX, NA64\( \mu \) and \( M^3 \) experiments respectively and for the set of form-factors discussed in Sec. III. These cross-sections are also presented for the specific set of \( G \)-boson masses, \( m_G = 5 \) MeV, \( m_G = 250 \) MeV and \( m_G = 1 \) GeV.

Let us describe now the typical properties and the general kinematics of \( G \)-boson production by the charged lepton beams impinging on the solid target. It is worth mentioning that both NA64\( e \) and LDMX electron cross-sections have a peak in the region \( x \lesssim 1 \) for the relatively heavy masses \( m_G \gtrsim 250 \) MeV, it means that the signal of \( G \)-boson production is strongly forward peak, such that the dominant part of the initial beam energy transfers to the hidden spin-2 boson. On the other hand, for the relatively light masses \( m_G \gtrsim 5 \) MeV the sharp forward peak is mitigated and the regarding differential cross-section is in the soft bremsstrahlung-like regime. It implies that the cross-section peaks in the infra-red (IR) region \( x = E_G/E_e \ll 1 \). However, for the higher masses \( m_G \gtrsim 250 \) MeV the IR peak is smeared as long as the energy fraction is small.

Remarkably that both NA64\( \mu \) and \( M^3 \) muon cross-sections are in the soft bremsstrahlung-like regime \( d\sigma_{\mu}/dx \propto \mathcal{O}(1/x) \) as long as \( x \ll 1 \) for all masses in the range \( 1 \) MeV \( \lesssim m_G \lesssim 1 \) GeV, since the mass of spin-2 boson is comparable to the muon mass in that region, \( m_G \propto \mathcal{O}(m_\mu) \).

Let us describe now the impact of various form-factor parametrizations on the shape of the differential cross-sections. In order to compare these cross-sections we choose Tsai-Shiff’s form-factor as a benchmark one, since this latter is exploited widely in the calculations of WW experiments (see e. g. Refs. [39, 45, 50, 70] and references therein for detail). In what follows in the right panels of Figs. 3, 4, 5, and 6 we show the relative differences between \( (d\sigma/dx)_G \) and the set of \( (d\sigma/dx)_{H_{el}} \), \( (d\sigma/dx)_H \) and \( (d\sigma/dx)_E \) respectively for the specific experiments and the specific masses of \( G \)-boson. We recall again for clarity, that the subscriptions of \( F_{TS}(t) \), \( F_H(t) \) and \( F_E(t) \) are related to the atomic elastic form-factors with screening term, while the label of \( F_{H_{el}}(t) \) is associated with the nuclear elastic form-factor that doesn’t take into account the screening effect.

It is worth mentioning that given the mass \( m_G \) all the atomic form-factor cross-sections for the NA64\( e \) experiment match with a reasonable accuracy at the level of \( 2\% - 10\% \) as soon as \( 0.2 \lesssim x \lesssim 1 \). However, the sizable deviation (i. e. at the level of \( \mathcal{O}(2) \)) between the nuclear and atomic cross-sections appears only for the small mass region long as \( m_G \lesssim 5 \) MeV. These effects are shown in the right panel of Fig. 3. The regarding impact of the form-factor parametrization on the differential cross-section shape is shown in the right panel of Fig. 4 for the LDMX electron fixed-target experiment. In addition we note that the shapes of the cross-sections can be varied for \( x \ll 1 \) and \( m_G \gtrsim 1 \) GeV, however this energy fraction region doesn’t provide a sizeable contribution to the total cross-section of heavy masses.

For the muon beam cross-sections of the NA64\( \mu \) experiment the impact of the different form-factor parametrization can be estimated at the level of \( \lesssim 2\% - 8\% \) for energy fraction in the range \( 0.2 \lesssim x \lesssim 1 \). The latter is shown in the right panel of Fig. 5. One can see also from Fig. 6 that the shape of the differential cross-section for the \( M^3 \) experiment can be varied significantly due to the form-factors as long as \( x \ll 1 \) and \( m_G \lesssim 1 \) GeV.

Let us study now the specific shape of the total cross-sections of \( G \)-boson production. In the left panel of Fig. 7 we show the regarding cross-sections for the NA64\( e \), LDMX, NA64\( \mu \) and \( M^3 \) facilities, which are calculated for the benchmark Tsai-Shiff’s atomic elastic form-factor. One can see from Fig. 7 that both NA64\( e \) and NA64\( \mu \) cross-sections are calculated for the lead (Pb) target \((Z \approx 82)\), nevertheless the the total electron beam cross-section \( \sigma_{el}^{tot} \) is generally larger by factor of \( \approx 5 \) than the muon beam cross-section \( \sigma_{\mu}^{tot} \), even though they have a comparable energies of the impinging beams \( E_e \approx E_\mu \approx \mathcal{O}(100) \) GeV. That implies the advantage of using the electron beam instead of muon beam at the CERN SPS facility in the mass range \( 1 \) MeV \( \lesssim m_G \lesssim 1 \) GeV. However, if one compares the LDMX electron cross-section for the aluminium (Al) target \((Z \approx 13)\) with the \( M^3 \) cross-section for the muon beam impinging on the tungsten (W) target \((Z \approx 74)\), one can conclude that the regarding advantage of using the electron beam is compensated by the nucleus charge suppression \( \lesssim 13/74 \), even though both LDMX and \( M^3 \) experiments have a comparable beam energies, \( E_\mu \approx E_e \approx 15 \) GeV. In addition we note from the NA64\( \mu \) and \( M^3 \) cross-sections that the greater energy of the muon beam implies greater rate of the \( G \)-boson production, \( \sigma_{\mu}^{tot}(160 \text{ GeV}) \gtrsim \sigma_{\mu}^{tot}(16 \text{ GeV}) \). To conclude this section, we specify briefly the impact of the form-factor parametrization on the total cross-section. In the right panel of Fig. 7 the relative differences of the total cross-sections are shown for the set of atomic form-factors. In the small mass region \( m_G \lesssim 100 \) MeV these differences are relatively small, i. e. at the level of \( \lesssim 2\% \). However, the regarding discrepancies can be as large as \( \gtrsim 8\% \) for the heavy mass region \( m_G \lesssim 1 \) GeV.
VI. THE EXPERIMENTAL LIMITS

In this section we study current and expected reach of the lepton fixed-target experiments. The limit on the coupling $c_l/\Lambda$ can be obtained as follows. By exploiting both Eq. (33) for the number of produced $G$-bosons and the result on its production cross-section with specific experimental cuts, one can require that the number of signal events is $N_G \gtrsim 2.3$. As a result, this yields 90\% C.L., exclusion limit on the coupling constant $c_l/\Lambda$ for the background free case and null result of the fixed-target facilities. In the left panel of Fig. 8 we show by the dashed lines the projected sensitivities of NA64, LDMX, NA64$\mu$ and $M^3$ fixed target facilities.

It is worth mentioning that the expected reach of the LDMX is rather strong, i.e. at the level of $c_l/\Lambda \gtrsim 10^{-5}$ GeV$^{-1}$, even though the typical cross-section of the $G$-boson production is fairly small (see e.g. the blue line in the left panel of Fig. 7). The enhanced expected reach of the LDMX is associated with fairly large number of the electrons that will be accumulated on target. In particular, by the final phase of data taking LDMX plans to accumulate EOT $\simeq 10^{16}$, as we discussed it above in Sec. IV B.

In addition, from the projected sensitivities of both NA64$\mu$ and $M^3$ muon experiments shown in Fig. 8, one can conclude that compared to the $M^3$ option with 16 GeV beam muons and MOT $\simeq 10^{13}$, the higher energy muons of NA64$\mu$ (e.g. $E_{\mu} \simeq 160$ GeV) with MOT $\simeq 5 \times 10^{15}$ allow examining a wider region in the parameter space of the spin-2 boson scenario. Nevertheless, the ultimate projected bounds of the NA64$\mu$ experiment for EOT $\simeq 5 \times 10^{12}$ (see e.g. the green dashed line in the left panel of Fig. 8 at the level of $c_l/\Lambda \gtrsim 10^{-4}$ GeV$^{-1}$), can be ruled out even by the $M^3$ facility at the level of $c_l/\Lambda \gtrsim 7 \times 10^{-5}$ GeV$^{-1}$ (see e.g. the red dashed line in the left panel of Fig. 8). In the left panel of Fig. 8, we show by the solid green line the excluded limit of the NA64$\mu$ experiment (at the level of $c_l/\Lambda \gtrsim 7 \times 10^{-4}$ GeV$^{-1}$) for the current accumulated statistics of EOT $\simeq 3.22 \times 10^{11}$. In particular, from the analysis of the NA64$\mu$ data collected during 2016-2021 runs no signal events were found for the background free case (see e.g. Ref. [32]), as we discussed it in Sec. IV A.

It is worth mentioning that authors of Ref. [10] have been already considered the bounds on the light spin-2 mediator from $e^+e^- \rightarrow \gamma + E_{\text{miss}}$ at BaBar experiment [71]. The analysis was carried out for mainly invisible decay mode $\text{Br}(G \rightarrow \chi\bar{\chi}) \simeq 1$. The regarding exclusion limit yields the magnitude of the coupling constant at the level of $c_l/\Lambda \gtrsim 2 \times 10^{-4}$ GeV$^{-1}$ for the masses in the range $1\text{MeV} \lesssim m_G \lesssim 1\text{GeV}$. These bounds are shown by the solid black line in the left panel of Fig. 8. One can conclude that it rules out the current experimental constraints of the NA64$\mu$ facility for EOT $\simeq 3.22 \times 10^{11}$.

To conclude this section, we discuss briefly the impact of the form-factor parametrization on the experimental limits $c_l/\Lambda$ for the fixed target facilities. In the right panel of Fig. 8 the relative differences of the coupling constants are shown for the set of atomic form-factors. In the small mass region $m_G \lesssim 100\text{MeV}$ these differences are relatively small, i.e. at the level of $\lesssim 2\%$. However, the regarding discrepancies can be as large as $\gtrsim 8\%$ for the heavy mass region $m_G \lesssim 1\text{GeV}$.

VII. CONCLUSION

In the present paper we have discussed in detail the probing of the massive spin-2 boson $G$ through its invisible decay into pair of DM particles $\chi$ with $\text{Br}(G \rightarrow \chi\bar{\chi}) \simeq 1$. The regarding scenario implies that the $G$-boson serves as a mediator between charged leptons of SM and DM sector. The benchmark simplified coupling of this model involves the dimension-5 operators of massive $G_{\mu\nu}$ field and the energy momentum tensors of both SM and DM particles. We have studied explicitly the missing energy signatures for the projected and existing lepton fixed-target experiments, such as NA64$\mu$, LDMX, NA64$\mu$ and $M^3$. Namely, by exploiting the WW approach, we have calculated $G$-boson production cross-section in the process $lN \rightarrow lNG$ followed by its invisible decay into DM particles $G \rightarrow \chi\bar{\chi}$ for the specific fixed-target experiments. We have calculated the expected reach for the regarding experiments, implying the background free case and null result for DM detection. Moreover, we have also discussed in detail the impact of both nuclear and atomic form-factor parametrizations on (i) the differential spectra of $G$ boson emission, (ii) the total cross-section of its production (iii) the experimental reach of the fixed-target facilities for probing hidden spin-2 boson. It was found that the total yield of spin-2 boson production and the regarding experimental reach can be affected by the form-factor at the level of $\lesssim 8\% - 10\%$ for the large mass region $m \lesssim 1\text{GeV}$. For the light masses of $G$-boson, $m_G \lesssim 100\text{MeV}$, the implication of the form-factors for the yield and experimental reach is estimated at the level of $\lesssim 2\%$.

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Figure 8. Left panel: 90\% C.L. limits on $c_l/\Lambda$ coupling constant for the fixed-target and BaBar experiment \cite{10} as a function of $G$-boson mass $m_G$. For all expected reaches, we imply that $\text{Br}(G \to \chi \bar{\chi}) \simeq 1$ and $m_G \gtrsim 2m_\chi$. In addition, in the left panel all curves for the fixed-target facilities imply the Tsai-Shiff’s form-factor in the cross-section. The green dashed line is the projected sensitivity of the NA64 experiment for $EOT \simeq 5 \times 10^{12}$, the red dashed line is the expected reach of the $M^3$ facility for MOT $\simeq 10^{14}$, the violet dashed line is the projected sensitivity of the NA64 experiment for MOT $\simeq 5 \times 10^{13}$ and the blue dashed line is the expected reach of the LDMX facility for $EOT \simeq 10^{16}$. The green solid line represents the excluded at 90\% C.L. bound of the NA64 experiment for $EOT \simeq 3.22 \times 10^{11}$. Right panel: the relative difference is expressed as $(\mathcal{O}_H - \mathcal{O}_{TS})/\mathcal{O}_{TS}$ (right upper) and $(\mathcal{O}_E - \mathcal{O}_{TS})/\mathcal{O}_{TS}$ (right bottom) for atomic Helm’s and the exponential form-factors respectively, where $\mathcal{O} = c_l/\Lambda$.

Appendix A: Matrix element for process $l^-\gamma \to G^l^-$

In this section, we collect an expressions for the Feynman diagrams \cite{9,72} associated with spin-2 boson production and the matrix element squared for process $l^-\gamma \to G^l^-$. In particular, we use the expression for polarization sum of spin-2 mediator:

$$\sum_s e_{\mu\nu}(k,s)e_{\alpha\beta}(k,s) = 1/2(P_{\alpha\mu}P_{\beta\nu} + P_{\beta\mu}P_{\alpha\nu} - 2/3P_{\mu\nu}P_{\alpha\beta}),$$

(A1)

where $P_{\mu\nu} = g_{\mu\nu} - k_\mu k_\nu/m_G^2$. The vertex for the outgoing $G$-boson in case of incoming $l(p_1)$ and outgoing $l(p_2)$ leptons is

$$G^{\mu\nu} = -\frac{ie_l}{4\Lambda} \left\{ (p_2 + p_1)^{\mu\nu} + (p_2 + p_1)^{\nu\mu} - 2g^{\mu\nu}(p_2 + p_1 - 2m_l) \right\}.$$  

(A2)

For the incoming $V^\alpha(k_1)$ and outgoing $V^\beta(k_2)$ massless vector bosons one has

$$G^{\mu\nu} = -\frac{ie_l}{\Lambda} \left\{ \eta^{\alpha\beta} k_1^\mu k_2^\nu + \eta^{\nu\alpha}((k_1,k_2)\eta^{\alpha\beta} - k_1^\beta k_2^\alpha) - \eta^{\mu\beta} k_1^\alpha k_2^\nu + 1/2 \eta^{\nu\beta}(k_1^\alpha k_2^\mu - (k_1,k_2)\eta^{\beta}) + (\mu \leftrightarrow \nu) \right\}.$$  

(A3)
Finally the vertex for 4-point interaction is

\[
\begin{align*}
|M_{l^- \gamma ightarrow Gl^-}|^2 &= \frac{\alpha^2 e^2}{\Lambda^2} u_2 s_2 - 2 m_t^2 [(t_2 + u_2)^2 + (u_2 - m_G^2)^2] [4 u_2 (2 m_t^2 - s_2) - m_G^2 t_2] \\
&\quad \frac{4 t_2 (u_2 - m_t^2)^2 (s_2 - m_t^2)^2}{s_2 - m_t^2} \left( \frac{\alpha^2 e^2}{\Lambda^2} m_t^2 R(m_t, m_G, t_2, u_2) \right) - \frac{\alpha^2 e^2}{\Lambda^2} m_t^2 R(m_t, m_G, t_2, u_2)
\end{align*}
\]

As a result, by using FeynCalc package [51, 52] for the Wolfram Mathematica [53], we get matrix element squared for process \( l^- \gamma \rightarrow l^- G \):

\[
|M_{l^- \gamma ightarrow Gl^-}|^2 = \frac{\alpha^2 e^2}{\Lambda^2} u_2 s_2 - 2 m_t^2 [(t_2 + u_2)^2 + (u_2 - m_G^2)^2] [4 u_2 (2 m_t^2 - s_2) - m_G^2 t_2] \\
\frac{4 t_2 (u_2 - m_t^2)^2 (s_2 - m_t^2)^2}{s_2 - m_t^2} \left( \frac{\alpha^2 e^2}{\Lambda^2} m_t^2 R(m_t, m_G, t_2, u_2) \right) - \frac{\alpha^2 e^2}{\Lambda^2} m_t^2 R(m_t, m_G, t_2, u_2)
\] 

where \( R(m_t, m_G, t, u) \) is regular expression for \( m_t \) and \( m_G \):

\[
R(m_t, m_G, t, u) = 24 m_t^4 t + 24 m_t^8 [m_G^2 + 5 m_t^2 t - 6 u] + \\
+ 2 m_t^6 [24 m_t^6 + 6 m_t^4 (15 t - 8 u) - 3 m_t^2 t (33 t + 92 u) + 2 t (13 t^2 + 18 t u + 90 u^2)] + \\
+ 2 m_t^4 [12 m_t^4 + 12 m_t^2 (5 t - 6 u) + m_t^2 (72 u^2 - 113 t^2 - 288 t u)] + \\
+ 4 m t (t + 3 u) (13 t + 42 u) - (7 t^3 + 64 t^2 u + 144 t^3 u + 240 u^3)] + \\
+ m_t^2 [12 m_t^2 (3 t - 4 u) - 3 m_t^2 (29 t^2 + 80 t u - 48 u^2)] + m_t^2 (70 t^3 + 544 t^2 u + 696 t u^2 - 96 u^3) - \\
- m_t^2 t (27 t^3 + 364 t^2 u + 852 t u^2 + 912 u^3) + 2 t (t^4 + 46 t^3 u + 92 t^2 u^2 + 216 t u^3 + 180 u^4)] + \\
+ [-3 m_t^2 (3 t^2 + 16 t u - 8 u^2) + m_t^2 (15 t^3 + 128 t^2 u + 168 t u^2 - 48 u^3)] - \\
- m_t^2 (9 t^4 + 122 t^3 u + 362 t^2 u^2 + 384 t u^3 - 24 u^4) + \\
+ m_t^2 t (3 t^4 + 66 t^3 u + 240 t^2 u^2 + 552 t u^3 + 408 u^4) - \\
- 2 t (t + u) (7 t^3 + 24 t^2 u + 72 t u^2 + 72 u^3)],
\]
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