Ultracold neutron accumulation in a superfluid-helium converter with magnetic multipole reflector

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Abstract

We analyze accumulation of ultracold neutrons (UCN) in a superfluid-helium converter vessel surrounded by a magnetic multipole reflector. Employing formulas valid for trapped UCN in mechanical equilibrium we solved the rate equation and obtained results for the saturation density of low field seeking UCN for any field strength of the multipolar field. The addition of magnetic storage can increase the density and energy of the UCN produced and serves to mitigate the effects of wall losses on the source performance. This work was performed in preparation of the UCN source project SuperSUN at the ILL.

Keywords: ultracold neutrons, UCN source, neutron EDM, neutron decay

1 Introduction

Mirror reflection of neutrons is an effect of the neutron optical potential which is mainly due to coherent s-wave scattering of neutrons by atomic nuclei in condensed matter [1]. Neutrons with energy sufficiently low to become totally reflected under any angle of incidence are called ultracold neutrons (UCN). This peculiar property enables experimentalists to store them in "neutron bottles" made of suitable materials with small cross sections for neutron absorption and providing well depths up to about 300 neV [2, 3]. Storage time constants of many hundreds of seconds and the possibility to employ also magnetic fields and gravity for trapping and manipulation have made UCN a versatile tool to investigate various phenomena of fundamental physics complementary to experiments at high-energy particle accelerators [4, 5, 6].

Among recent experiments with UCN feature a first demonstration of gravity resonance spectroscopy with the goal to search for deviations from Newton’s gravity law at the micrometer length scale [7], searches for “mirror dark matter” [8, 9], a test of Lorentz invariance [10], searches for axion-like particles [11, 12, 13], a demonstration of the effect of accelerated matter on the neutron wave [14] and of the stability of the Berry geometrical phase for spin ½ particles under the influence of noise fluctuations [15]. Earlier work with UCN on the geometrical phase was published in [17, 18], while its first demonstration with cold neutrons can be found in [16].

Long standing are efforts to improve the accuracy of the weak axial-vector and vector coupling constants of the nucleon derived from precise values of the neutron lifetime [19, 20, 21, 22, 23] and the beta-asymmetry [24, 25, 26, 27]. Among other applications these values are crucial input for calculations of weak reaction rates in big-bang nucleo-synthesis and stellar fusion [28, 29], and of the efficiency of neutrino detectors [30]. Also long standing is the search for a non-vanishing neutron electric dipole moment (EDM), which would violate the symmetries of time reversal (T)
and thus via the CPT theorem also the combined symmetries of charge conjugation and parity (CP). This search was proposed already in 1950 by Purcell and Ramsey [31] and has become a prominent route to investigate new mechanisms of CP violation beyond the standard model complex phase of the weak quark mixing CKM matrix, and the matter-antimatter asymmetry in the universe [32]. At the present best level of sensitivity, still no EDM was observed [33]. Several projects are in preparation or underway with the goal to gain at least an order of magnitude in sensitivity [34, 35, 36, 37, 38, 39, 40, 41].

Most studies with UCN are counting statistics limited and will strongly profit from new UCN sources which are currently being developed in various laboratories around the world [42, 43, 44, 45, 46, 47, 48, 49]. They are all based on the "superthermal" UCN production scheme proposed in 1975 by one of the authors together with Mike Pendlebury [50], using either superfluid $^4$He or solid deuterium as a medium for neutron conversion. Here we focus on UCN production in a converter of superfluid $^4$He installed at the end of a neutron guide, wherein neutrons with energy 1 meV, respectively, wavelength 0.89 nm may loose nearly their entire energy in single scattering events. At low temperature only few excitations are present in the helium that are able to scatter UCN back to higher energies. With the vanishing absorption cross section of $^4$He it becomes possible to accumulate UCN within a converter with reflective boundaries before releasing them to an experiment at room temperature. While an earlier attempt to bring this technique to life was hampered by extraction losses (nonetheless producing record UCN densities for its time) [51], a more efficient method was developed recently by one of the authors together with his co-workers [49, 52, 53]. UCN are extracted through a cold UCN valve and a short vertical UCN guide section, superseding lossy separation window, screens and gaps for thermal insulation between the converter and the UCN guide of the earlier scheme. Work is in progress to bring the technique to maturity for a new user facility at the ILL, and in particular to perform a neutron lifetime experiment using magnetic trapping [54, 55, 56]. Also other groups have recognized the potential advantages of a superfluid helium converter feeding UCN to experiments at room temperature [43, 47], and in some experiments this type of converter is employed in situ [34, 36, 57].

The efficiency of a UCN accumulator at an external neutron beam relies on loss rates being as low as possible. Most critical are those losses which occur when UCN hit the walls of the converter vessel. They are proportional to the frequency of collisions and thus depend on the size and shape of the converter vessel. From transmission measurements through superfluid $^4$He at 1.25 K a mean free path of 17 m was derived for the 0.89 nm neutrons most effective for UCN production [55]. Hence, the vessel can be made several meters long without significant reduction in UCN density. On the other hand, the lateral dimensions of the converter vessel should match the size of the available neutron beam and guide it to avoid dilution of the incident flux. The mean free path of UCN in a vessel with such geometry is therefore only in the order of 5 – 10 cm, leading to high typical frequencies of UCN wall collisions of 50 – 100 per second. It thus becomes challenging to obtain long UCN storage time constants which however are a prerequisite for accumulation of a high saturated UCN density. Values measured for narrow vessels are normally well below 200 s. For instance, in a recent experiment on UCN production, a rather short storage time constant of 67 s was obtained for a vessel held at 0.7 K, which consisted of a 1 m long 7 × 7 cm$^2$ tube of BeO with Be windows on each end and included a short pipe from stainless steel. That, nonetheless, a record UCN density was obtained demonstrates the potential of the method [49]. To our knowledge the Cryo-EDM collaboration achieved with $\tau = 160$ s the so far highest value for a helium converter enclosed within matter boundaries, using a 3 m long tubular vessel with diameter 63 mm, made of Be coated copper and closed off by Be windows [34].

Magnetic trapping of UCN offers a viable way for a drastic improvement of the storage properties of the converter vessel, ultimately limited only by the neutron lifetime $\tau_\beta \approx 880$ seconds.
It relies on the potential energy $\pm \mu B$ of the neutron magnetic moment $\mu$ of 60 neV/T in a magnetic field $B$. Suitable configurations of magnetic field gradients keep low field seeking UCN away from walls where otherwise the collisional losses occur. A group at NIST has demonstrated storage time constants consistent with the neutron lifetime within a helium converter equipped with a superconducting magnetic quadrupole UCN reflector [57, 59]. The apparatus was designed to perform neutron lifetime measurements, for which a complete suppression of UCN wall contacts is mandatory. On the other hand, for the sake of enhancing the output of a UCN source, combined magnetic and material trapping turns out to be particularly beneficial. In addition, the phase space for UCN accumulation can be much increased using a higher multipole order.

In this paper we provide an analytic treatment of the rate equation for UCN production and storage in a superfluid-helium converter confined by material walls and surrounded by a magnetic mirror. We show that, combining a converter vessel possessing good (but not exceptional) storage properties with a magnetic mirror of high multipole order, one may achieve a saturated UCN density close to the theoretical limit defined by an ideal experimental bottle, i.e. a square well potential without imaginary part. That the magnet needs to generate only part of the trapping potential is of great practical value for constructing a device using standard superconducting wire technology.

## 2 Rate equation and system definition

The temporal evolution of the spectral UCN density in a closed converter irradiated with the cold beam is governed by a simple rate equation. UCN production is characterized by a spectral rate density $p$ that depends on the spectral flux of the incident beam, and a loss term is due to finite lifetime $\tau$ of UCN in the converter,

$$\frac{dn(\epsilon_0, t)}{dt} = p(\epsilon_0) - \frac{n(\epsilon_0, t)}{\tau(\epsilon_0)}.$$  (1)

Here we label stored neutrons by their total energy $\epsilon_0$, defined as their kinetic energy at the point of lowest potential energy in the trap. The quantities $n(\epsilon_0, t)$ and $p(\epsilon_0)$ denote the real space density, respectively, the spatial UCN production rate density, each per energy interval of a group of UCN with total energy in the range $(\epsilon_0, \epsilon_0 + d\epsilon_0)$. The saturated spectral UCN density obtains when UCN losses balance UCN production for $t \gg \tau$ after having switched on the beam. It is given by

$$n_\infty(\epsilon_0) = p(\epsilon_0) \tau(\epsilon_0).$$  (2)

If one wants to fill a trap with UCN up to a cutoff energy set by the trapping potential $V_{\text{trap}}$, what matters is the saturated total UCN density which is obtained from

$$n_\infty = \int_0^{V_{\text{trap}}} n_\infty(\epsilon_0) \, d\epsilon_0.$$  (3)

Many applications of UCN sources involve filling external traps with as many UCN as possible, followed by a long time for holding or manipulation, during which the density in the source can be refreshed. Therefore $n_\infty$ is a useful parameter of the converter to be optimized.

We consider a system as schematically shown in fig. 1. A cylindrical converter vessel with diameter $2R$ is situated within a multipole magnet and illuminated homogeneously by a cold neutron beam passing along the $r = 0$ axis. UCN accumulation takes place over a length $L \gg R$ between a beam window and a UCN valve fully immersed in the helium as in the apparatus

\footnote{For experiments involving external traps with poor storage properties it will be better to drain UCN frequently from a partially charged source. However, also in this case a long UCN storage time constant is an asset as it will raise the time-averaged UCN content of the converter.}
Figure 1: Schematic of the UCN accumulator comprised of a superfluid $^4$He converter with multipole magnet and system for UCN extraction. On the left a cut view is shown for $n = 12$; filled (open) squares indicate electric current flowing into (out of) the plane. The neutron optical potentials are: $V$ for the cylindrical part over the length $L$, $\tilde{V}$ at the beam window and the UCN valve, and $\geq \tilde{V}$ for the UCN extraction system.

described in [60]. Shown is a butterfly valve but also different types may be envisaged, such as an iris diaphragm valve. For experiments at room temperature UCN are released into a window-less extraction system with a short vertical guide section as described in [53][2]. In the section for UCN accumulation the cylindrical wall possesses a neutron optical potential $V + iW$ [1][2][3], with

$$V = \frac{2\pi \hbar^2}{m_n} \sum_l N_l b_l, \quad W = \frac{\hbar}{2} \sum_l N_l v \sigma_l(v),$$  \hspace{1cm} (4)

where $m_n$ is the neutron mass, $N_l$ is the atomic number density of the nuclear species $l$ with coherent bound neutron scattering length $b_l$, and $\sigma_l(v)$ is the sum of cross sections for neutron capture and scattering at (thermal) neutron velocity $v$, which is usually proportional to $1/v$. The beam window and the UCN valve are made of (or coated with) a material with neutron optical potential $\tilde{V} + i\tilde{W}$.

A radial $n$-polar magnetic field with modulus

$$B(r) = B_R \left( \frac{r}{R} \right)^{\frac{n}{2} - 1}$$  \hspace{1cm} (5)

can be generated as shown with a regular arrangement of an even number of $n$ straight current bars on the outer cylinder surface, with opposite current in adjacent bars (in practice one employs long racetrack coils). A neutron moving in such a field has a magnetic potential energy of

$$V_m(r) = \pm V_{mR} \left( \frac{r}{R} \right)^{\frac{n}{2} - 1},$$  \hspace{1cm} (6)

where $V_{mR}$ is defined to be positive so that the positive (negative) sign in eq. 6 stands for neutrons in the low (high) field seeking spin state. The trapping potential of the converter vessel filled with superfluid helium ($V_{\text{He}} = 18.5 \text{ neV}$) is given by

$$V_{\text{trap}} = \min \left( V + V_{mR}, \tilde{V} \right) - V_{\text{He}}.$$  \hspace{1cm} (7)

\footnote{It is also conceivable to place the UCN valve closer to (or within) the extraction guide. This would however considerably increase the surface of wall material exposed to the UCN during accumulation. Here we analyse the system as shown in fig. 1.}
3 UCN losses from the converter

The inverse of the time constant appearing in the loss term in eq. 11 i.e. the rate of UCN loss normalized to the number of UCN in the converter, is comprised of several contributions,

$$\tau^{-1}(\epsilon_0, T) = \tau^{-1}_{\text{wall}}(\epsilon_0) + \tau^{-1}_{\text{slit}}(\epsilon_0) + \tau^{-1}_{\text{up}}(T) + \tau^{-1}_{\text{abs}} + \tau^{-1}_{\text{depol}}(\epsilon_0) + \tau^{-1}_\beta. \quad (8)$$

From left to right, the partial rates are due to UCN loss at collisions with the walls of the converter vessel, escape of UCN through apertures caused by manufacturing imperfections of the vessel assembly and through slits of an imperfect UCN valve, upscattering by thermal excitations in the helium, absorption by $^3$He impurities, UCN depolarization at wall collisions, and neutron beta decay. Note that the rate $\tau^{-1}_{\text{depol}}$ may become relevant only if trapping is at least partly magnetic. In an experimental study a depolarization probability per wall collision of $7 \times 10^{-6}$ was measured for a bottle made of Be [61]. This corresponds to a rate smaller than $\tau^{-1}_\beta$ even for the fastest neutrons in the narrow trap geometry discussed here. Like the first two rates in eq. 8 it will be further suppressed due to the multipole magnet. We therefore neglect $\tau^{-1}_{\text{depol}}$. For temperatures below 1 K [60] [62],

$$\tau^{-1}_{\text{up}}(T) \approx \frac{(T [K])^7}{100 \text{s}}, \quad (9)$$

so that for $T < 0.5$ K the rate $\tau^{-1}_{\text{up}}$ contributes less than 10% of $\tau^{-1}_\beta$. The rate $\tau^{-1}_{\text{abs}}$ can be suppressed below any relevant level by purification of the helium from $^3$He using a superleak [53] [63] [64] or the heat flush technique [65]. As a result, we are left with $\tau^{-1}_{\text{wall}}$ and $\tau^{-1}_{\text{slit}}$ as dominating contributions, and the rate due to neutron decay sets an ultimate lower limit for a perfect converter.

For the losses due to wall collisions we want to apply an analytic description. If we assume the trapped UCN in mechanical equilibrium we can use formulas derived in the book [2] where the authors analyzed the effect of the earth’s gravitational field on neutrons moving in a bottle. We adapt the notation to our case and replace the height parameter $h$ by the radial coordinate $r$ characterizing the multipolar magnetic field strength. At any point in the trap accessible for low field seeking neutrons with total energy $\epsilon_0$ the kinetic energy is given by $E(r) = \epsilon_0 - |V_m(r)|$. Note that we neglect the gravitational field, which is a good approximation for a horizontal source with less than 10 cm diameter. The angular averaged probability $\overline{\pi}$ for UCN loss during a wall collision is a function of the kinetic energy $E$ with which neutrons hit the wall. For a boundary between superfluid helium and the neutron optical potential $V + iW$ with small losses, i.e. $f = W/V \ll 1$, it is given by

$$\overline{\pi}(E) = 2f \operatorname{Re} \left( \frac{V'}{E} \arcsin \sqrt{\frac{E}{V'}} - \sqrt{\frac{V'}{E} - 1} \right), \quad V' = V - V_{\text{He}}. \quad (10)$$

For convenience in later calculations we have included the projection on the real part of the expression to ensure that $\overline{\pi}(E < 0) = 0$ without need to specify the range of $E$ in advance as positive. The function $\overline{\pi}(E)$ rises monotonously with $E$ from $\overline{\pi} = 0$ for $E = 0$ to $\overline{\pi} = \pi f$ for $E = V'$ (for $E > V'$ we may set $\overline{\pi} = 1$ since we are not interested here in calculating the dynamics of marginally trapped neutrons). For $E = V'/2\pi \overline{\pi} \approx 1.14f$. Since we consider a long trap for which $2\pi RL \gg \pi R^2$, we can neglect losses due to $\overline{W}$ at the end disks, and the argument of $\overline{\pi}(E)$ becomes

$$E = \epsilon_0 - V_{mR}. \quad (11)$$

The magnetic multipole suppresses wall losses because first, only a fraction of neutrons has sufficient energy to hit the lossy wall and second, because those with $\epsilon_0 > V_{mR}$ hit the wall due
to magnetic deceleration only with kinetic energy in the range $0 < \epsilon_0 - V_{mR} < V_{trap} - V_{mR}$, and $\mathcal{P}(\epsilon_0 - V_{mR}) < \mathcal{P}(\epsilon_0)$. A further suppression is due to a reduced rate of UCN wall collisions, which we calculate next. As a result of phase space transformation under the influence of a conservative potential, in the spatial region accessible for the low field seekers, the spectral real space UCN density is given by

$$n(\epsilon_0, t, r) = \text{Re} \sqrt{\frac{\epsilon_0 - |V_m(r)|}{\epsilon_0}} n(\epsilon_0, t, 0),$$

(12)

where projection on the real part ensures $n(\epsilon_0, t, r) = 0$ for $r > R^*$ defined by $\epsilon_0 = |V_m(R^*)|$.

We define an effective real space volume of the source for neutrons with total energy $\epsilon_0$ by

$$\gamma(\epsilon_0) = \frac{2}{\pi} L \text{Re} \int_0^R \sqrt{\frac{\epsilon_0 - |V_m(r)|}{\epsilon_0}} r dr.$$

(13)

The spectral UCN density averaged over the entire volume of the converter is then given by

$$n(\epsilon_0, t) = \gamma'(\epsilon_0) n(\epsilon_0, t, 0),$$

(14)

and the reduced quantity

$$\gamma'(\epsilon_0) = \frac{\gamma(\epsilon_0)}{\pi R^2 L} = 2 \text{Re} \int_0^1 \sqrt{1 - \frac{V_{mR}}{\epsilon_0} \frac{r^2}{\epsilon_0} - 1} r dr.$$

(15)

was derived using eq. 6. The values for $\gamma'$ listed in table 1 show that the reduction of real space density with respect to a potential well of same depth becomes less significant for higher multipolarity.

The spectral flux density of neutrons at any point in the vessel, per unit area and per energy interval about $\epsilon_0$, is given by the gas kinetic relation

$$J(\epsilon_0, t, r) = \frac{1}{4} n(\epsilon_0, t, r) v(\epsilon_0, r).$$

(16)

$2\pi RLJ(\epsilon_0, t, R)$ is the spectral rate of UCN collisions with the cylindrical wall of the helium container. The speed $v(\epsilon_0, R)$ of the neutrons as they hit the wall is related to the speed at $r = 0$ through

$$v(\epsilon_0, R) = \text{Re} \sqrt{\frac{\epsilon_0 - V_{mR}}{\epsilon_0}} v(\epsilon_0, 0).$$

(17)

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3We do not consider the high field seekers further. They hit the wall with higher energies and will leave the trap much faster than the low field seekers. For a calculation of the polarization of the neutrons they should however be properly taken into account.
With eq. 12 we thus obtain

\[ J(\epsilon_0, t, R) = \frac{\epsilon_0 - V_{mR}}{\epsilon_0} J(\epsilon_0, t, 0) \Theta (\epsilon_0 - V_{mR}), \tag{18} \]

where the step function \( \Theta (\epsilon_0 - V_{mR}) = 1 \) for \( \epsilon_0 > V_{mR} \), and 0 otherwise.

For the loss term in eq. 1 due to collisions with the cylindrical wall we can write

\[ \frac{n(\epsilon_0, t)}{\tau_{\text{wall}}(\epsilon_0)} = \frac{2}{R^2} \pi (\epsilon_0 - V_{mR}) J(\epsilon_0, t, R). \tag{19} \]

Further evaluation using eq. 18, eq. 16 for \( r = 0 \), eq. 14, and inserting \( \pi \) from eq. 10 leads us to

\[ \tau_{\text{wall}}^{-1}(\epsilon_0) = \frac{v(\epsilon_0, 0)}{R \gamma'} \left( \frac{\epsilon_0 - V_{mR}}{V'} - \sqrt{\epsilon_0 - V_{mR}} \left( 1 - \frac{\epsilon_0 - V_{mR}}{V'} \right) \right). \tag{20} \]

For the calculation of the corresponding expression for losses through slits it is reasonable to assume them to be situated at \( r = R \), e.g. at the seam of the tube or at its connections to the circular windows for the cold beam. Assuming that any UCN hitting a slit will be lost and denoting the total surface of all slits by \( A \) their contribution to the loss term in eq. 1 is given by

\[ \frac{n(\epsilon_0, t)}{\tau_{\text{slit}}(\epsilon_0)} = \frac{A}{2 R \pi RL} J(\epsilon_0, t, R), \tag{21} \]

neglecting the small surface of the disks at the ends. Hence,

\[ \tau_{\text{slit}}^{-1}(\epsilon_0) = \frac{Av(\epsilon_0, 0)}{4 \gamma(\epsilon_0)} \frac{\epsilon_0 - V_{mR}}{\epsilon_0} \Theta (\epsilon_0 - V_{mR}), \tag{22} \]

and we see that the ordinary gas kinetic expression represented in the first fraction on the right hand side becomes reduced for \( V_{mR} > 0 \) for the same reason as the wall losses discussed before.

### 4 UCN production

In absence of the magnetic multipole field and for homogeneous irradiation of the converter with the cold neutron beam, the UCN production rate density is position independent,

\[ p_0 = p_0^\uparrow + p_0^\downarrow = \int_0^{V_{\text{trap}}} p(\epsilon_0) \, d\epsilon_0 = K V_{\text{trap}}^{3/2}. \tag{23} \]

For an unpolarized beam, as always assumed hereafter, the production rates for the two components with spin parallel (\( \uparrow \)) and anti-parallel (\( \downarrow \)) to the magnetic field are equal, i.e. \( p_0^\uparrow = p_0^\downarrow = p_0/2 \). The \( V_{\text{trap}}^{3/2} \) dependence follows for a homogeneous population of states within a sphere in momentum space. The factor \( K \) due to single phonon emission has been calculated on the basis of neutron scattering data and confirmed in several experiments [52, 66, 67, 68], albeit with modest experimental accuracy limited by detection efficiency and other corrections. For unpolarized neutrons and UCN with maximum energy determined by \( V_{\text{trap}} = V - V_{He} = 233 \) neV for Be or Ni with natural isotopic composition, it is given by

\[ K \approx 5 s^{-1} \, \text{cm}^{-3} \Phi_{0.89 \mu \text{m}} \left[ 10^9 \text{cm}^{-2} \text{s}^{-1} \text{nm}^{-1} \right] / (233 \text{neV})^{3/2}, \tag{24} \]

where \( \Phi_{0.89 \mu \text{m}} \) is the differential unpolarized neutron flux density at a neutron wave length of 0.89 nm [69]. The flux unit is chosen numerically close to values available at existing facilities.
Table 2: Values for $\kappa$ as defined in eq. (28) for various values of $n$ and $V_m R/V_{\text{trap}}$.

e.g. the monochromatic beam H172a at the ILL [70]. An additional, usually smaller contribution to UCN production is due to multi-phonon processes [71, 72].

In the converter with the magnetic multipole switched on, the UCN production rate density becomes position dependent. For the low field seekers,

$$p^\uparrow (r) = \frac{K}{2} \text{Re} (V_{\text{trap}} - |V_m (r)|)^{3/2}.$$  \hfill (25)

$\text{Re} (...)$ accounts for the fact that in regions where $|V_m (r)| > V_{\text{trap}}$ no neutrons with total energy smaller than the trapping depth can be produced. After averaging over the volume of the converter vessel and using eq. (6) we obtain

$$p^\uparrow = KV_{\text{trap}}^{3/2} \int_0^1 \text{Re} \left( 1 - \frac{V_m r}{V_{\text{trap}}} \right)^{3/2} r dr.$$  \hfill (26)

Note that, with the definition of $\gamma' (\epsilon_0)$ given in eq. (15) we can write the spectral mean UCN production rate density as

$$p^\uparrow (\epsilon_0) = \frac{3}{4} K \gamma' (\epsilon_0) \sqrt{\epsilon_0},$$  \hfill (27)

leading to eq. (28) after integration over $\epsilon_0$.

The ratio of total production rates for low field seeking UCN with the magnetic multipole switched on and switched off,

$$\kappa = \frac{p^\uparrow}{p^\uparrow_0},$$  \hfill (28)

is smaller than unity due to the phase space reduction by the magnetic multipole. The values quoted in table 2 demonstrate the positive effect of high multipole order $n$ on $\kappa$ and hence on the saturated UCN density calculated in the next section. There are however practical limits. First, thermal insulation between the magnet and the much colder helium container necessitates an annular gap over which the field would drop too strongly if $n$ is chosen too large. Second, the maximum field strength achievable with a given maximum current density in the current bars around the converter of given diameter decreases with $n$. For $R = 5$ cm, and taking into account the results given in the next section, $n \approx 12$ seems a reasonable choice, which also is still feasible using standard NbTi superconducting wire technology.

5 Saturated UCN density

The saturated spectral UCN density follows from eq. (2) with eq. (27) which already includes the necessary spatial average over the converter volume,

$$n^\uparrow_\infty (\epsilon_0) = \frac{3}{4} K \gamma' (\epsilon_0) \sqrt{\epsilon_0} \tau (\epsilon_0, T).$$  \hfill (29)
Figure 2: Saturated density \( n_\infty \) of low field seeking UCN in a converter vessel with diameter 10 cm, held at 0.5 K and surrounded by a 12-pole, respectively, 8-pole magnet. Calculations employ eq. [30] for \( V_{\text{trap}} = V - V_{\text{He}} = 233 \text{ neV} \) (i.e. for a converter vessel made entirely of Be), and for various values of \( f = W/V \). The solid lines show results for the best value of \( f \) previously achieved for Be, while the upmost curves show a hypothetical situation for illustration. Values are given for an unpolarized differential neutron flux density of \( \Phi_{0.89 \text{nm}} = 10^9 \text{cm}^{-2}\text{s}^{-1}\text{nm}^{-1} \) at \( \lambda = 0.89 \text{ nm} \), as available at the monochromatic beam position H172a at the ILL. A characteristic time constant \( \tau_0 \) is calculated for neutrons with velocity \( v = \frac{3}{4} v_{\max} \), using eq. [20] for \( V_{mR} = 0 \) and \( \epsilon_0 = \frac{9}{16} \sqrt{\gamma} \).

Hence, using eq. [3] and writing out all relevant arguments characterizing the multipolar magnetic field and the converter, the saturated total mean UCN density in the converter is given by

\[
n_\infty (V_{mR}, n, V, f, \bar{V}, T) = \frac{3}{4} K \int_0^{V_{\text{trap}}} \frac{\gamma' (\epsilon_0, V_{mR}, n) \sqrt{\epsilon_0}}{\tau^{-1} (\epsilon_0, V_{mR}, n, V, f, T)} d\epsilon_0.
\]  

The dependence on \( \bar{V} \) is contained in the upper limit of integration, see eq. [4]. From the various contributions to the rate constant \( \tau^{-1} \) (see eq. [3], we retain the terms due to wall collisions, upscattering and neutron beta decay, assuming that wall losses can entirely be described by eq. [20] and that there is no \(^3\text{He}\) in the converter and there are no slits in the vessel.

Figure 2 shows results for a vessel made entirely (i.e. \( \bar{V} = V \)) of Be and exemplary for multipole magnets with multipole order \( n = 8 \) and \( n = 12 \), as a function of the magnetic potential at the cylindrical wall of the vessel, normalized to the trapping depth. The UCN density increases with \( n \) as expected due to the increase in trapping phase space. Beryllium has become a standard material for UCN trapping, with a best reported experimental value of \( f = 3 \times 10^{-5} \) in the low temperature limit [73, 74], despite much smaller theoretical values (the
Figure 3: Saturated density of low field seeking UCN in converter vessels with diameter 10 cm, $V = 252$ neV, $f = 3 \times 10^{-5}$, $T = 0.5$ K, and beam window and UCN valve (see fig. 1) coated with three different materials with values of $\tilde{V}$ as indicated in the legend. The upper (lower) curve of each pair is for multipolarity $n = 12$ ($n = 8$), and all values are given for $\Phi_{0,89nm} = 10^9 \text{cm}^{-2} \text{s}^{-1} \text{nm}^{-1}$ (unpolarized). The kinks visible for the two upper pairs of curves appear at fields for which $V_{mR} = \tilde{V} - V$.

Finding that these were never reached was termed "anomalous losses" and has triggered many experimental investigations and speculations. However, it might be more realistic to consider even larger values for $f$, assuming that efficient cleaning procedures cannot be applied in situ (e.g., baking is excluded in presence of indium seals). As obvious from the curves, the poorer the neutron optical UCN storage performance, the larger will be the improvement of UCN density due to the multipole magnet. For the situations reported in the introduction, with experimental loss rate ratios as high as $\tau_1^{-1}/\tau_1^{-1} \simeq 5.5$ [34], respectively 13 [49], a multipole magnet would indeed be very useful. Some of the curves exhibit a maximum of $n^\infty$ for values $V_{mR}/V_{trap} < 1$. This can be understood as resulting from the competition of storage time constants $\tau$ and the effective trap volume $\gamma'$ entering eq. [30]. For bad values of $f$ the optimum obtains for $V_{mR}/V_{trap}$ close to 1, while for a trap with excellent storage properties the multipolar field reduces the UCN density even at low field values because the factor $\gamma' < 1$ then dominates over a marginal gain in $\tau$. For illustration of this behaviour we also added a curve for an unrealistic converter vessel with hypothetical $f = 3 \times 10^{-6}$.

Next we consider an interesting further opportunity which takes advantage of the fact that the multipole magnet increases not only storage time constants but also the potential energy of the neutron at the cylindrical wall. Hence, the trapping depth of the converter vessel becomes larger if the disks providing axial confinement are made of a material with larger neutron optical...
potential $\tilde{V} > V$ (remember eq. [7] and see fig. 1). Since the surface of the disks is small, one may employ materials which would be unsuited for the entire vessel, for an unfavorably large value of $\tilde{f} = \tilde{W}/\tilde{V}$ or because they might be difficult to produce or deposit in high quality on large surfaces. While diamondlike carbon has already been studied in some detail [75, 76, 77, 78], further candidate materials able to extend the spectrum for UCN trapping beyond the Be cutoff have been the scope of recent investigations [79]. Particularly promising is boron nitride in the cubic phase (cBN). Its neutron optical potential of 324 neV is even larger than that of diamond (304 neV) but due to the large absorption cross section of the isotope $^{10}$B, cBN also has an excessively large value of $\tilde{f} = 1.5 \times 10^{-2}$. Enrichment of the weakly absorbing $^{11}$B however may reduce $\tilde{f}$ down to $3.3 \times 10^{-5}$, along with a further increase of $\tilde{V}$ to a theoretic value of up to 350 neV, depending also on the high-density cubic phase content of the specimen. Using experiments on transmission with time-of-flight analysis and cold neutron reflectometry, the authors of [79] have demonstrated that their 2 $\mu$m thick $^{11}$BN deposit on a circular silicon wafer possesses a value of $315 \pm 10$ neV. The deviation from the ideal value is mainly due to a cubic phase content of 90%, which was measured independently by IR spectroscopy.

Figure 3 shows saturated UCN densities calculated for a Be trap with axial UCN confinement provided by Be, $^{11}$BN as reported, or cubic C$_3$N$_4$, the latter possessing an extraordinarily large theoretical value of $\tilde{V} = 391$ neV. Calculations were this time performed with a fixed value of $f = 3 \times 10^{-5}$. As a function of magnetic field at the cylindrical wall, the curves for $\tilde{V} > V$ start

Figure 4: Saturated density of low field seeking UCN in converter vessels with diameter 10 cm, $V = 252$ neV, $T = 0.5$ K, $f = 3 \times 10^{-5}$ (solid curves) or $f = 10^{-4}$ (dashed). Beam window and UCN valve (see fig. 1) are coated with $^{11}$BN ($\tilde{V} = 315$ neV). The vessels are surrounded by magnets with multipole order varied between $n = 6$ and 24. Values are given for $\Phi_{0.89nm} = 10^{9}$cm$^{-2}$s$^{-1}$nm$^{-1}$ (unpolarized).
Figure 5: Saturated density of low field seeking UCN in converter vessels with diameter 10 cm, $V = 252$ neV, $T = 0.5$ K and surrounded by a 12-pole magnet. Solid curves are for $f = 3 \times 10^{-5}$, dashed ones for $f = 10^{-4}$. $\tilde{V} - V_{\text{He}}$ is varied between 294 neV and 60 neV. Values are given for $\Phi_{0.89 \text{nm}} = 10^9 \text{cm}^{-2} \text{s}^{-1} \text{nm}^{-1}$ (unpolarized).

with a slope larger than in the case $\tilde{V} = V$ shown in fig. 2. This is due to the increase of $V_{\text{trap}}$ and therewith of the integration range in eq. (30) for $V_{mR} < \tilde{V} - V$. Kinks in the curves appear at $V_{mR} = \tilde{V} - V$ where the full trapping depth is reached. While for a high quality Be trap with $\tilde{V} = V$ the gain in UCN density is not too impressive (lowest two curves), for a magnetic 12-pole and $\tilde{V} = 315$ neV, a technically feasible field of $B_R = 2.5$ T enhances the saturated low field seeker UCN density by a factor 2.2 from 820 cm$^{-3}$ to 1820 cm$^{-3}$. The magnetic multipole is thus an attractive device even for a vessel with good storage properties.

Figure 4 shows the dependence of saturated UCN density on the multipole order, for traps made of Be with $f = 3 \times 10^{-5}$, respectively $f = 10^{-4}$, each with end windows made of $c^{11}\text{BN}$ with $\tilde{V} = 315$ neV. For the worse trap a field of $B_R = 2.5$ T enhances the saturated low field seeker UCN density by a higher factor 3.6 from 390 cm$^{-3}$ to 1400 cm$^{-3}$. Hence, while without multipole the worse $f$ suppresses the UCN density by more than a factor two, with field it is only 22% lower than for the better trap. The multipole field thus stabilizes the output of the source; it becomes much less sensitive to the loss coefficient of the Be surface or if its quality degrades with time.

Figure 5 shows, again for traps with the cylindrical section made of Be with $f = 3 \times 10^{-5}$, respectively $f = 10^{-4}$, the dependence of saturated UCN density on the upper bound of the trapped UCN energy spectrum. This is easily calculated using eq. (30) by setting the potential $\tilde{V} - V_{\text{He}}$ to appropriate values. We see that, with decreasing $\tilde{V}$, the magnetic field needed to optimize the UCN density decreases as well. The reason is that a converter with lower trapping
potential accumulates UCN with smaller energies for which wall collisional losses are smaller due to the energy dependence of $\mu(E)$ defined in eq. 10 and due to wall collisions occurring at a smaller rate. In addition, the effective volume $\gamma'$ decreases with $V_mR$ faster than for neutrons with higher energies, as obvious from table 1. If one wants to extract (or the experiment connected to the source is able to use) only a spectrum with low-energy UCN of say, up to 60 neV as provided for instance by a magnetic bottle with a trapping field of 1 T, the multipole magnet around the source will be useless for $f = 3 \times 10^{-5}$. For the worse trap with $f = 10^{-4}$ it will however still lead to a gain factor attaining a maximum of 1.4 at $B_R \approx 0.75$ T.

6 Conclusions

As our analysis shows, a multipole magnet can lead to a large gain in the saturated density of low field seeking UCN because the presence of the field reduces the number of neutrons hitting the material walls and reduces the energy and wall collision rate of those that do. A 12-pole magnet with field $B_R = 2.5$ T on a radius of $R = 5$ cm seems technically feasible using existing NbTi superconducting wire technology, as investigated in an independent study using a finite element code. Based on results of experimental work done by other groups, a promising candidate vessel consists of a Be trap closed off by disks coated with $c^{11}$BN. Alternative materials are diamondlike carbon with $V$ close to 300 neV depending on the abundance of sp$_3$ chemical bonds, or enriched $^{58}$Ni with $\overline{V} = 346$ neV and a theoretical $f = 8.6 \times 10^{-5}$, which however is magnetic and UCN depolarization might be an issue that needs experimental study. We note that, in order to extract the full benefits, the incoming cold beam will need to be transported by a supermirror, with a top layer deposit of the UCN reflecting material with neutron optical potential $V$. An experimental investigation of this concept is currently underway at the ILL.

Our benchmark converter is able to provide a saturated low field seeker UCN density almost as high as an unrealistic, perfect trap with $f = 0$ and Be cutoff, for which one calculates $n_\infty^\uparrow = 2060$ cm$^{-3}$. In addition, the densities quoted in table 3 for various values of the parameter $f$ highlight a valuable practical advantage offered by a magnetic multipole reflector. Even for $f$ by a factor 13 worse than the best value of $3 \times 10^{-5}$ previously achieved for Be, $n_\infty^\uparrow$ would still exceed 1000 cm$^{-3}$ and only be a factor 1.8 smaller than in the best case. On the other hand, for a pure Be trap without magnet the same lack of performance in $f$ would reduce the UCN density by a factor 6.3. Obviously, a converter vessel equipped with multipole magnet is then also much less sensitive to degradation of its surface quality with time.

Up to now, we have considered only the low field seekers. Without the magnetic field the high field seekers are however equally well trapped, so that in this case (and still assuming the usual situation of an unpolarized cold neutron beam for UCN production) the total UCN density is a factor 2 higher. For experiments requiring polarized UCN such as a neutron lifetime experiment using magnetic trapping or the neutron EDM experiment, this factor is of no use and the benefit of the magnetic multipole mirror is obvious.

As an important detail not affecting our conclusions we note that, in addition to the multipole

| $f$                  | $3 \times 10^{-5}$ | $1 \times 10^{-4}$ | $2 \times 10^{-4}$ | $4 \times 10^{-4}$ |
|----------------------|---------------------|---------------------|---------------------|---------------------|
| $n_\infty^\uparrow$ (cm$^{-3}$, for $B_R = 2.5$ T) | 1820                | 1400                | 1200                | 1040                |
| $n_\infty^\uparrow$ (cm$^{-3}$, without magnet)   | 820                 | 390                 | 230                 | 130                 |

Table 3: Values for the saturated density of low field seeking UCN as defined in eq. 30 for a converter vessel with $R = 5$ cm, $V = 252$ neV, $\overline{V} = 315$ neV, $T = 0.5$ K, surrounded by a 12-pole magnet, and for several values of $f$. Values are given for an unpolarized differential flux density of $\Phi_{0.89\mu m} = 10^9$ cm$^{-2}$s$^{-1}$nm$^{-1}$. 

| $f$ | $3 \times 10^{-5}$ | $1 \times 10^{-4}$ | $2 \times 10^{-4}$ | $4 \times 10^{-4}$ |
|-----|---------------------|---------------------|---------------------|---------------------|
| $n_\infty^\uparrow$ (cm$^{-3}$, for $B_R = 2.5$ T) | 1820                | 1400                | 1200                | 1040                |
| $n_\infty^\uparrow$ (cm$^{-3}$, without magnet) | 820                 | 390                 | 230                 | 130                 |
field it will be necessary to apply a weaker bias field in the order of some 10 mT along the converter axis to avoid depolarization in the region around \( r = 0 \), where the multipole field is zero. In addition, we can consider using axial magnetic pinch fields to increase \( V_{\text{trap}} \) and thereby the density of the UCN, however for an extended energy spectrum. For extraction the field on one end needs to be ramped down, so that an iris type UCN valve might be more appropriate for this case. An extended UCN spectrum would be interesting if UCN of any velocity are beneficial for the experiment, such as UCN transmission experiments, or in combination with a phase space transformation by letting the UCN rise against the gravitational field. Note however that extraction of such a UCN spectrum will be a challenge. On the other hand, some studies might be performed in situ using static pinch fields, such as experiments on UCN upscattering in superfluid \(^4\text{He}\), for which any increase in UCN density will be very welcome.

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