Pre-Equilibrium Stage and Phase Transition of Quark Matter Probed by Photon Interferometry

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U. Ornik¹†, M. Plümer²‡, A. Timmermann³§ and R.M. Weiner²¶

¹ GSI, Darmstadt, FRG
² Physics Department, Univ. of Marburg, Marburg, FRG
³ Max-Planck Institute of Meteorology, Hamburg, FRG

Abstract

We study single- and double-inclusive spectra of thermal photons, produced in heavy-ion collisions at $\sqrt{s} = 200$ AGeV within a realistic space-time framework which combines the Parton-Cascade-Model and 3-dimensional hydrodynamics (HYLANDER). This allows also for the first time to take into account pre-equilibrium effects for photon production. A rapid decrease in the width of the correlation function as the photon transverse momentum drops below $\sim 1.5$ GeV is a signature of the deconfinement phase transition.

¹† E.Mail: ORNIK@TPRI6O.GSI.DE
²‡ E.Mail: PLUEMER@MAILER.UNI-MARBURG.DE
³§ E. Mail: TIMMERMANN@DKRZ.D400.DE
²¶ E. Mail: WEINER@MAILER.UNI-MARBURG.DE
Under extreme conditions – expected to be realized, e.g., in high energy heavy ion collision experiments to be performed at the Relativistic Heavy-Ion Collider (RHIC) – the theory of strong interaction (quantum chromodynamics) predicts a phase transition from hadronic matter to a state of quasi-free quarks and gluons - the quark-gluon plasma (QGP). The detection of this transition is one of the most important and debated problems of high energy nuclear physics. The problem is the development of a suitable strategy for distinguishing between a system in which a QGP was formed at some early or intermediate stage of its evolution and a system which consisted of purely hadronic matter at all times. Two types of probes have been proposed for this strategy: hadronic probes and electromagnetic ones. Hadronic signals have two serious disadvantages:

(a) They appear only in the last stage of evolution of the system while the QGP, if it has been formed at all, exists only at the beginning of its evolution.

(b) There exists no satisfactory method yet to treat strong interactions in the soft (non-perturbative QCD) regime.

Electromagnetic probes on the other hand, such as direct photons and dileptons, due to their large mean free paths and small cross sections pass through the plasma with very little distortion from strong and electromagnetic interactions. Thus, photons and dileptons retain the memory of the early stages of the evolution \[1\].

Our aim is to study inclusive one- and two-photon spectra at the RHIC energy, \[\sqrt{s} = 200\] AGeV, in a realistic 3-dimensional framework. Particularly Bose-Einstein correlations (BEC) for thermal photons are very sensitive to the thermal history and the space-time development of the source. In a previous 1-dimensional calculation \[2\], it was pointed out that the transverse momentum dependence of the longitudinal correlation functions probes the deconfinement phase transition. A rapid change in the shape of the correlation function from a two component to a single component structure was observed. This change is directly related to the first order phase transition from the QGP to a hadronic phase. The sensitivity of BECs to a phase transition is
also considered in Ref. \[3\] where a small qualitative change of the transverse correlation functions was observed. However, the observation of this effect would require a very high experimental sensitivity of the correlation function of the order $10^{-3} - 10^{-4}$ which is quite unrealistic at present.

All calculations which have been performed so far \[2, 3, 4, 5\] are lacking a realistic space-time description of the source, ignore the pre-equilibrium stage and assume chemical equilibrium even in the early stages of the evolution. Moreover, previous studies \[2, 3, 4, 5\] of BEC’s of thermal photons used expressions for the rates of photons emitted from the hadronic system which did not take into account the contribution \[3\] from the $A_1$ meson and thus considerably underestimated the rates.

In order to improve this situation we include several new features for the description of the photon production at RHIC. For the space-time evolution we adopt a hybrid model, which implements an exact and fully 3-dimensional hydrodynamical simulation (HYLANDER \[3\]) and features of the Parton-Cascade-Model (PCM) \[8, 9, 10, 11\]. For the photon production rates in the quark-gluon phase, the mixed phase and the hadronic phase the same expressions were used as in Ref. \[12\].

Full local thermalization is not reached instantaneously in ultrarelativistic heavy-ion collisions. Non-equilibrium models \[8, 13\] which take into account parton-parton interactions in the early stages of the collision suggest that local thermal equilibrium is reached after some time on the order of $\sim 0.1 - 2$ fm/c. For high energies, as will be reached, e.g., in heavy-ion collisions at RHIC, this type of scenario is generally accepted as the most realistic description presently available. Thus it becomes necessary to consider photon emission in the absence of chemical equilibrium. This was done in Ref. \[14\] and we shall apply this procedure below. The fugacities $\lambda_q, \lambda_\bar{q}, \lambda_g$ for quarks and gluons are introduced via the statistical distributions of the particles:

$$f_{q,\bar{q}} = \frac{\lambda_{q,\bar{q}}}{\lambda_{q,\bar{q}} + e^{p_0/T}}$$  \hspace{1cm} (1)

and for gluons analogously, replacing the $+$ by $-$. Here $p_0$ denotes the energy in the rest frame
of the particle. The expressions for the photon rates of the Compton and annihilation process in a non-equilibrium plasma are given as \[14\]

**Compton:**

\[
E_x \frac{dN}{d^3k d^4x} = \frac{5}{9} \frac{2\alpha_s}{\pi^4} \lambda_q \lambda_g T^2 e^{-E/T} \sum_{n=0}^{\infty} \frac{(-\lambda_q)^n}{(n+1)^2} \left[ \ln \left( \frac{12E}{g^2\kappa^2 T(n+1)} \right) + \frac{1}{2} - C \right]
\]

(2)

and

**Annihilation:**

\[
E_x \frac{dN}{d^3k d^4x} = \frac{5}{9} \frac{2\alpha_s}{\pi^4} \lambda_g \lambda_q T^2 e^{-E/T} \sum_{n=0}^{\infty} \frac{\lambda_g^n}{(n+1)^2} \left[ \ln \left( \frac{12E}{g^2\kappa^2 T(n+1)} \right) - 1 - C \right],
\]

where \( g = \sqrt{4\pi\alpha_s} \) and \( C = 0.57721... \). In Ref. \[14\] \( \kappa \) is considered to be a function of \( \lambda_q \) and \( \lambda_g \). We shall put \( \kappa \) to 1, which corresponds to a simplified thermal quark mass of \( m_q^2 = \frac{1}{6} g^2 T^2 \).

In our model the space-time evolution of \( Au + Au \) collisions at \( \sqrt{s} = 200 \) AGeV can be divided into two stages.

1. The first stage is associated with the pre-equilibrium dynamics, which is governed by hard and semihard QCD processes. The development of the initial nuclear parton distributions is calculated within the PCM \[8, 9, 10, 11\]. It is found \[8\] that for \( Au + Au \) collisions at RHIC kinetical equilibrium is established for quarks and for gluons at the latest after 2 fm/c. Gluons reach both chemical and thermal (local) equilibrium. Chemical equilibration of quarks, on the other hand, takes much longer as the chemical equilibration time for light quarks is too large to compete with the dilution of the expanding system. The fugacity of light quarks e.g. reaches only a value of \( \sim 0.6 \) instead of 1 as would correspond to full equilibrium \[11\]. Actually, the equilibrium QGP at the RHIC experiment is dominated by gluons.

2. As soon as the hot and dense system has reached local thermal equilibrium it is more realistic to describe the further evolution of the fireball within a hydrodynamical model.

The equations of relativistic hydrodynamics are solved exactly and fully 3-dimensionally \[15, 7\] with the numerical code HYLANDER. In our calculation we adopt an equation of state

\[\text{Note, that the rates in (2) diverge, if } \lambda_q > 1, \text{ which corresponds to a gluonic condensate.}\]
(EOS) which was obtained by lattice-gauge simulations. The principal ingredients of hydrodynamics are the initial conditions which are obtained directly from the PCM. At the “initial time” \( t_i = 2.4 \text{ fm/c} \) we extract the velocity field, the baryon-density distributions and the energy-density profile from the PCM and use it in the hydrodynamical calculation.

In order to obtain the single-inclusive spectra for thermal photons, it is necessary to integrate the photon rates over the space-time region defined by the space-time evolution and the thermal and chemical history of the hot and dense matter. The fugacities, the temperature field and the four velocity field are obtained in the pre-equilibrium stage from the PCM and afterwards from hydrodynamics.

Fig. 1 shows the separate contributions of (i) the pre-equilibrium stage, (ii) the equilibrated QGP phase, (iii) the mixed phase and (iv) the hadronic phase to the single-inclusive spectra of thermal photons for the reaction \( Au + Au \) at \( \sqrt{s} = 200 \text{ AGeV} \). In the transverse momentum region \( k_\perp > 1.5 \text{ GeV} \) the pre-equilibrium outshines all other phases. This is due to the high initial effective temperatures of \( T_i \sim 950 \text{ MeV} \) in the PCM. Only at \( k_\perp \sim 1 \text{ GeV} \), the contribution from the purely hadronic phase becomes comparable to that of the pre-equilibrium, while the rate of photons emitted from the equilibrium QGP and from the mixed cannot compete with the pre-equilibrium yield in the transverse momentum range considered here (1 GeV \( \leq k_\perp \leq 3 \text{ GeV} \)). Thus, only for photons within the transverse momentum window \( k_\perp \sim 1 - 2 \text{ GeV} \) there is a chance that the correlations are sensitive to the presence of two separate phases: the pre-equilibrium stage, and the purely hadronic phase.

In ultrarelativistic nuclear and particle collision experiments BEC have been measured almost exclusively for pions and kaons so far. These particles decouple from the hydrodynamical system on the freeze-out hypersurface and thus carry only information on the late stage of the evolution. BEC for thermal photons on the other hand are sensitive to the entire space-time evolution of the system. They carry information on the pre-equilibrium QGP, the fully equilibrated QGP,

\[ \text{(2),(3).} \]
the phase transition and the hadronic matter.

We consider the second order correlation function $C_2(\vec{k}_1, \vec{k}_2)$ for identical particles:

$$C_2(\vec{k}_1, \vec{k}_2) = \frac{P_2(\vec{k}_1, \vec{k}_2)}{P_1(\vec{k}_1)P_1(\vec{k}_2)},$$

(4)

where $\vec{k}_i$ ($i = 1, 2$) are the three momenta of the particles and $P_1(\vec{k})$ and $P_2(\vec{k}_1, \vec{k}_2)$ are the one- and two-particle inclusive distributions.

For the general case of a Gaussian density matrix and a totally chaotic source the thermal Wick theorem holds and the two-particle inclusive distribution can be written in the form

$$P_2(\vec{k}_1, \vec{k}_2) = P_1(\vec{k}_1)P_1(\vec{k}_2) + \left| \int d^4x \, w \left( x, \frac{k_1 + k_2}{2} \right) e^{i(k_1 - k_2)x} \right|^2,$$

(5)

where $w(x, k)$ represents the source function which describes the mean number of particles of four-momentum $k$ emitted from a source element centered at the space-time point $x$. In order to examine the correlation function in longitudinal and transverse direction it is convenient to introduce the quantities $k = (k_\perp \cosh y, k_\perp \cos \psi, k_\perp \sin \psi, k_\perp \sinh y)$, $K = \frac{1}{2}(k_1 + k_2) = (K_0, K_t, \vec{K}_\perp)$ and $q = k_1 - k_2 = (q_0, q_t, \vec{q}_\perp)$ and the projection $q_{\text{out}}$, which is the component of $\vec{q}_\perp$ parallel to $\vec{K}_\perp$. The analysis of the BECs in the direction of $q_t, q_{\text{out}}$ reflects properties of the longitudinal and transverse space-time expansion of the system.

Fig. 2a shows the Bose-Einstein correlation function in longitudinal direction for thermal photons produced in $Au + Au$ collisions at $\sqrt{s} = 200$ AGeV. It is calculated for the momentum configuration $k_{1\perp} = k_{2\perp}$, $\psi_1 = \psi_2 = 0$ and $y_1 = 0$ as a function of the rapidity difference of the two photons $\Delta y = y_2 - y_1$. In Fig. 2b the correlation function is plotted as a function of $q_{\text{out}} = q_\perp \frac{\vec{k}_1 \cdot \vec{q}_\perp}{k_1 q_\perp}$. We consider the case $y_1 = y_2 = 0, \psi_1 = \psi_2 = 0$. Figs. 2a and 2b reveal that the shapes and the widths of the correlation functions change drastically as $k_\perp$ increases from 1 to 2 GeV. At 1 GeV the BEC functions have an exponential shape and a small width whereas the shape turns into a gaussian and the width increases strongly as $k_\perp$ reaches values of 2 GeV and higher.

This characteristic behaviour is due to the fact that low transverse momentum ($k_\perp \sim 1$ GeV)
photons are produced both in the pre-equilibrium and the hadronic phase (see Fig. 1). Photons with \( k_\bot > 1.5 \text{ GeV} \), on the other hand, are almost exclusively emitted from the pre-equilibrium stage. This is reflected in the widths of the photon correlation functions (Fig. 2a and 2b), which are inversely proportional to the size of the emission region. The broadest correlation function \( (k_\bot = 5 \text{ GeV}) \) is related to the hot pre-equilibrium stage which has minimum spatial extension whereas the correlation function for pairs of lower momenta is dominated by the late hadronic stage which has a large spatial extension.

Thus, the rapid change of the shape of the correlation functions in the region \( k_\bot = 1 - 2 \) GeV is a direct consequence of the fact that at \( k_\bot \sim 1 \text{ GeV} \) the contribution of the hadronic phase to the photon yield becomes comparable to the contribution of the pre-equilibrium phase (which dominates above \( k_\bot \sim 1.5 \text{ GeV} \)), i.e., that for this momentum range there are significant contributions from two separate sources.

One may ask how it is possible for the hadronic phase to emit 1 GeV photons at a rate comparable to the pre-equilibrium contribution while the contributions from the equilibrium QGP remain smaller by about an order magnitude. The reason is that at a given temperature the emission rates from a hadronic resonance gas are considerably larger than the rates from a system of quasi-free quarks and gluons. The larger rates from the hadron gas are mainly due to the contribution the \( A_1 \) meson [6] which were not taken into account in previous studies of BEC’s of thermal photons. Thus the change in the shape of the correlation function is indeed due to a change in the production mechanism of photons between quark-matter and the hadronic resonance gas. This implies that the transition from a hadron gas to a deconfined quark-gluon stage can be seen by photon-interferometry.

In Fig. 3 we show how the correlation function would behave in the absence of this change of production mechanism associated with the phase transition. To be specific, we have taken the equation of state for a resonance gas and connected it to the PCM solution at \( t=2.4 \text{ fm/c} \). For the entire “thermal” history (including the pre-equilibrium with its effective temperatures) we
used the photon rates obtained for the hadron gas in Ref. [6]. As far as the early pre-equilibrium stages are concerned, this is of course not a realistic scenario. The only reason we discuss it here is to demonstrate explicitly that the change in shape of the correlation function observed in Figs. 2a and 2b is in fact a consequence of the different production mechanisms in the hadronic and the quark-gluon phase. Indeed, as can be seen in Fig. 3 for the “purely hadronic” scenario the correlation function does not exhibit a pronounced $k_\perp$ dependence anymore. Moreover, the BEC function does not have the two-component structure characteristic for the contribution from two separate phases. Rather, the curves all have an approximate Gaussian shape.

From our hybrid model, which connects the PCM with relativistic 3d-hydrodynamics and includes photon production in the early pre-equilibrium stage, we conclude that photon interferometry in the transverse momentum region between 1 and 3 GeV can serve as a probe of the change in the production mechanism of thermal photons associated with the transition from a system of quasi-free quarks and gluons to a hadronic resonance gas. Whereas the inclusive one-photon spectrum is dominated by the extremely "hot" pre-equilibrium phase it turns out that the two-photon correlation function exhibits a strong transverse momentum dependence which is a signature of the deconfinement phase transition.

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Figure Captions

Fig. 1 Single-inclusive spectrum of pre-equilibrium and thermal photons for $Au + Au$ collisions at $E_{lab} = 200$ AGeV as a function of the transverse momentum at rapidity $y=0$. The contributions of the different phases are displayed separately.

Fig. 2a Bose-Einstein correlations of photons as a function of the rapidity difference of the photons calculated for different values of the transverse momentum $k_\perp$.

Fig. 2b Bose-Einstein correlations of photons as a function of the variable $q_{out}$ calculated for different transverse momenta $k_\perp$ of one photon.

Fig. 3 Bose-Einstein correlations of photons as a function of the rapidity-difference of the photons calculated for different transverse momenta $k_\perp$ of one photon. For the calculation we took a purely hadronic scenario.
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Fig. 1

Single-inclusive spectrum of thermal $\gamma$

$dN/dk_{\perp} dy$

$10^{-4}$

$10^{-3}$

$10^{-2}$

$10^{-1}$

$10^0$

$10^1$

$10^2$

$10^3$

$10^4$

$k_{\perp}$ (GeV)

Preequilibrium

Hadronic phase

Mixed phase

QGP phase
Fig. 2a

Fig. 2b

longitudinal BEC

transversal BEC

$\Delta y$

$C_2$

$q_{\text{out}}$

$C_2$

$1 \text{ GeV}$

$2 \text{ GeV}$

$3 \text{ GeV}$

$5 \text{ GeV}$
Fig. 3

longitudinal BEC (hadronic scenario)