Determination of the strong coupling constant \( \alpha_s(M_Z) \) in next-to-next-to-leading order QCD using H1 jet cross section measurements

Daniel Britzger* for the collaboration†
Max-Planck-Institut für Physik (Werner Heisenberg Institute), München, Germany
E-mail: britzger@mpp.mpg.de

The strong coupling constant is one of the least known parameters of the Standard Model (SM) and a precise knowledge is of great importance for precision physics and searches for physics beyond the SM at the LHC. The value of the strong coupling constant \( \alpha_s(M_Z) \) is determined from inclusive jet and dijet cross sections in neutral-current deep-inelastic \( ep \) scattering (DIS) measured at HERA by the H1 collaboration using next-to-next-to-leading order (NNLO) QCD predictions. Using inclusive jet and dijet data together, the strong coupling constant is determined to be \( \alpha_s(M_Z) = 0.1157(20)_{\exp}(29)_{\mathrm{th}} \). Complementary, \( \alpha_s(M_Z) \) is determined together with parton distribution functions of the proton (PDFs) from jet and inclusive DIS data measured by the H1 experiment. The value \( \alpha_s(M_Z) = 0.1142(28)_{\mathrm{jet}} \) obtained is consistent with the determination from jet data alone. The impact of the jet data on the PDFs is studied. The running of the strong coupling is tested at different values of the renormalisation scale and the results are found to be in agreement with expectations.

*Speaker.
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1. Introduction

Jet production in neutral current deep-inelastic scattering (DIS) at HERA is directly sensitive to the strong coupling constant \( \alpha_s(M_Z) \) already in leading order in perturbative QCD (pQCD), since these cross section measurements are performed in the Breit frame of reference, where the virtual photon and the proton collide head on. In this work [1], the cross section predictions are performed in next-to-next-to-leading order (NNLO) accuracy, where the cross section predictions are obtained with the program NNLOJET [2, 3]. Using these new and improved predictions, the strong coupling constant \( \alpha_s(M_Z) \) is determined in two approaches, using data from the H1 experiment at HERA: First, a fit to all of the inclusive jet and dijet cross section data by H1 is performed, and in a second approach, a more global fit of parton distribution functions (PDFs) together with \( \alpha_s(M_Z) \) to normalised inclusive jet and dijet cross sections, as well as to inclusive neutral and charged current (CC) DIS cross sections by H1 is performed.

2. Determination of \( \alpha_s(M_Z) \) from H1 jet cross sections

The H1 experiment at HERA has measured cross sections for jet production in \( ep \) collisions in the Breit frame at different center-of-mass energies and for different kinematic regions. In this analysis, we consider data taken during different run periods in the years 1995 to 2007 [5, 6, 7, 8, 9]. Consistent to all these data sets, jets are defined using the \( k_t \) jet-algorithm with a parameter of \( R = 1 \), and jets are required to be contained in the pseudorapidity range \(-1 < \eta_{\text{jet}} < 2.5 \) in the laboratory frame. Data for inclusive jet, and inclusive dijet production, measured double-differentially as a function of the photon virtuality \( Q^2 \) and jet transverse momentum \( P_T \), or the average transverse momentum of the two hardest jets, \( \langle P_T \rangle \), respectively, is used. The selected data are summarised in table 1. The ratio of all H1 jet cross section measurements to the NNLO predictions is displayed in figure 1.

The value of the strong coupling constant is determined in a fit of NNLO calculations to the H1 jet data, where the \( \alpha_s \)-dependencies in the predictions, both in the partonic cross sections and in the PDF, are taken into account. The NNLO coefficients are calculated by the NNLOJET

| Data set [ref.] | \( \sqrt{s} \) [GeV] | \( Q^2 \) range | DIS kinematic range | Inclusive jets | Dijets | \( \eta_{\text{jets}} \geq 2 \) |
|-----------------|-----------------|-----------------|-------------------|--------------|-------|-------------------|
| 300GeV [5]      | 300             | 33              | 150 < \( Q^2 < 500 \) GeV \( ^2 \) | 7 < \( P_T \text{jet} < 50 \) GeV | \( P_T \text{jet} > 7 \) GeV | 8.5 < \( \langle P_T \rangle \) < 55 GeV |
| HERA-I [6]      | 319             | 43.5            | 5 < \( Q^2 < 100 \) GeV \( ^2 \) | 5 < \( P_T \text{jet} < 80 \) GeV | 5 < \( P_T \text{jet} < 50 \) GeV | \( P_T \text{jet} > 4 \) GeV |
| HERA-I [7]      | 319             | 65.4            | 150 < \( Q^2 < 1500 \) GeV \( ^2 \) | 5 < \( P_T \text{jet} < 50 \) GeV | \( P_T \text{jet} > 4 \) GeV | 5 < \( \langle P_T \rangle \) < 50 GeV |
| HERA-II [8, 9]  | 319             | 290             | 5.5 < \( Q^2 < 80 \) GeV \( ^2 \) | 4.5 < \( P_T \text{jet} < 50 \) GeV | \( P_T \text{jet} > 4 \) GeV | 5 < \( \langle P_T \rangle \) < 50 GeV |
|                 | 319             | 351             | 150 < \( Q^2 < 1500 \) GeV \( ^2 \) | 5 < \( P_T \text{jet} < 50 \) GeV | \( P_T \text{jet} > 12 \) GeV | 5 < \( \langle P_T \rangle \) < 50 GeV |

Table 1: Summary of the kinematic ranges of the inclusive jet and dijet data taken by the H1 experiment.


As a consequence of that approach, the PDFs, which are taken as external input to the fit, are only also considered in the fit. The value chosen for values of \( \alpha_s \) is assigned, which is closely related to \( \mu_0 \) in case of inclusive jet cross sections, and \( \mu_0 \) of 20 GeV is a typical scale of the jet data studied. As a consequence of that approach, the PDFs, which are taken as external input to the fit, are only evaluated at \( \mu_0 \), where it is found that state-of-the-art PDF sets by different groups agree in general very well [1]. For the central result, the NNPDF3.1 PDF set is used. The renormalisation \( \mu_R \) and factorisation \( \mu_F \) scales in this analysis are chosen to be \( \mu_R^2 = \mu_F^2 = Q^2 + P_T^2 \), where \( P_T \) denotes \( P_T^{\text{jet}} \) in case of inclusive jet cross sections, and \( \langle P_T \rangle \) for dijets. To each data point, a representative scale value \( \mu \) is assigned, which is closely related to \( \mu_R \) and \( \mu_F \). This value is used to group the data for tests of the assumption of the running of \( \alpha_s \), and for additional cuts, as discussed below. The goodness-of-fit quantity, \( \chi^2 \), which is subject to the minimisation algorithm, accounts for experimental, hadronisation and PDF uncertainties. Correlations of the uncertainties among the different data sets and running periods, and also statistical correlations of the data, are taken into account.

The \( \alpha_s(M_Z) \) values obtained in fits to the individual data sets are all found to be consistent [1]. Also, very reasonable values of \( \chi^2/\text{n}_{\text{dof}} \) are found in these studies, such indicating a good agreement of data and NNLO predictions. A significantly reduced experimental uncertainty is obtained in a fit to all inclusive jet and dijet cross sections. Such a fit is denoted as ‘H1 jets’, and a value of \( \alpha_s(M_Z) = 0.1143 (9)_{\text{exp}} (43)_\text{th} \) is determined.

The \( \alpha_s(M_Z) \) value obtained from H1 jet data restricted to \( \mu > 28 \text{ GeV} \) is

\[
\alpha_s(M_Z) = 0.1157 (20)_{\text{exp}} (6)_{\text{had}} (3)_{\text{PDF}} (2)_{\text{PDF}} \alpha_s (3)_{\text{PDFset}} (27)_{\text{scale}}
\]

(2.1)

with \( \chi^2 = 63.2 \) for 91 data points. The cut value for this main result of \( \mu > 28 \text{ GeV} \) was chosen such, that the scale and experimental uncertainty are balance: a lower cut results in significantly
smaller experimental uncertainties, but larger scale uncertainties, and vice-versa for a larger cut on $\bar{\mu}$. As indicated, uncertainties due to experimental and theoretical sources are estimated separately. Three PDF related uncertainties are assigned to the fitted $\alpha_s(M_Z)$ results: The ‘PDF’ uncertainty originates from the data used for the NNPDF3.1 PDF determination \cite{12}; The ‘PDFset’ uncertainty is defined as half of the maximum difference of the results from fits when using different PDF sets, i.e. ABMP16, CT14, MMHT or HERAPDF2.0; The ‘PDF$\alpha_s$’ uncertainty is defined as the difference of results from repeated fits using PDFs of the NNPDF3.1 series determined with $\alpha_{\mathrm{PDF}}(m_Z)$ values differing by 0.002. The hadronisation and scale uncertainties are estimated by propagating the respective uncertainties of the NNLO cross section predictions to the $\alpha_s(M_Z)$ fit result.

The running of the strong coupling constant as a function of the renormalisation scale, $\alpha_s(\mu_R)$, is studied by repeating the ‘H1 jets’ fit for groups of data points with similar values of $\bar{\mu}$. The resulting values of $\alpha_s(\mu_R)$, displayed at a representative value $\bar{\mu}$, and the respective value of $\alpha_s(M_Z)$, are displayed in figure 2. The values are in agreement with the expectation from the QCD renormalisation group equation and with $\alpha_s$-determinations at NNLO in other reactions and at similar scales.

3. Simultaneous $\alpha_s$ and PDF determination

In a complementary approach, which is then denoted as ‘PDF+$\alpha_s$’ fit, the value of $\alpha_s(M_Z)$ is determined together with the non-perturbative PDFs. This fit considers the H1 normalised inclusive jet and dijet cross sections, as well as H1 polarised and unpolarised inclusive NC and CC DIS cross section data. The latter data samples are equivalent to the one used in the H1PDF2012 PDF fit \cite{11}. The correlation of the uncertainties between the jet data and the inclusive DIS data are account for by using normalised jet cross sections \cite{7,8,9}.

The ansatz for the fit follows closely the methodology of previous studies, such as the one of HERAPDF2.0 \cite{10} or H1PDF2012. In brief, the PDFs are parameterised at a low starting scale with 12 fit parameters, and are evolved using the DGLAP formalism to higher scales. In this analysis, all predictions are performed in NNLO accuracy, and the DIS cross sections are performed in the zero-mass variable flavour number scheme. The DIS data are restricted to $Q^2 > 10\text{GeV}^2$, and the jet data to $\bar{\mu} > 2m_b$. The scales $\mu_R$ and $\mu_F$ are set to $Q^2$ for inclusive DIS, and for jet predictions to $Q^2 + P_T^2$.

The value of $\alpha_s(M_Z)$ is determined in the PDF+$\alpha_s$ fit to

$$\alpha_s(M_Z) = 0.1142(11)_{\text{exp, had, PDF}}(2)_{\text{mod}}(2)_{\text{par}}(26)_{\text{scale}},$$

where ‘mod’ and ‘par’ denote the model and parameterisation uncertainties, similar to the HERAPDF2.0 approach. The scale uncertainty is estimated by repeating the fit with scale factors 0.5 and 2 applied to $\mu_R$ and $\mu_F$ simultaneously to all calculations involved. This $\alpha_s(M_Z)$-value is consistent with the result from the $\alpha_s$-fit presented above, and with other determinations, as displayed in figure 3.

This fit yields $\chi^2/n_{\text{dof}} = 1539.7/(1529 - 13)$, which indicates a good agreement between predictions and data. The resulting PDFs, denoted as H1PDF2017 [\text{NNLO}]$^1$, are thus able to describe

$^1$The PDF H1PDF2017 [\text{NNLO}] is available in the LHAPDF format, with experimental, hadronisation and $\alpha_s(M_Z)$ uncertainties included.
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Figure 2: Values of $\alpha_s(M_Z)$ obtained from fits to ‘H1 jets’ data points with similar values of $\mu_R$ (full circles) in comparison to values from other experiments and processes, where all values are obtained at least in NNLO accuracy. The fitted values of $\alpha_s(M_Z)$ are translated to $\alpha_s(\mu_R)$ using the solution of the QCD renormalisation group equation as they also enter the calculations. The inner error bars display the experimental uncertainties and the outer error bars indicate the total uncertainties.

Figure 3: Summary of $\alpha_s(M_Z)$ values obtained in the $\alpha_s$-fits to multiple H1 jet data sets with different cuts on $\tilde{\mu}$ (denoted as H1 Jets), and in the H1PDF2017 [NNLO] PDF+$\alpha_s$-fit. The inner error bars indicate the experimental uncertainty and the outer error bars the total uncertainty. The results are compared to the world average and to other NNLO $\alpha_s(M_Z)$ determinations from DIS data.

the 141 jet data points and the inclusive DIS data simultaneously. When comparing H1PDF2017 [NNLO] to NNPDF3.1 at a scale $\mu_F = 20\text{GeV}$, the gluon and the singlet momentum distributions are in reasonable agreement (not shown). Remarkably, H1PDF2017 [NNLO] tends to have a higher gluon distribution at lower values of $x$, which is found to be rather similar to NNPDF3.1sx [13]. Although only H1 data is included in the PDF determination, and additionally $\alpha_s(M_Z)$ is a free parameter in that case, the uncertainties on the gluon distribution are fairly competitive between H1PDF2017 and NNPDF3.1. This is due to the H1 jet data, which provides stringent constraints on the gluon distributions.

4. Summary

The new next-to-next-to-leading order pQCD calculations (NNLO) for jet production cross sections in neutral-current DIS are exploited for a determination of the strong coupling constant $\alpha_s(M_Z)$ using inclusive jet and dijet cross section measurements published by the H1 collaboration. Two methods are explored to determine the value of $\alpha_s(M_Z)$, and precise values for $\alpha_s(M_Z)$ are determined in NNLO accuracy. The values are found to be consistent with each other, with other determinations, and with the world average value. This is the first precision extraction of $\alpha_s(M_Z)$ from jet data at NNLO involving a hadron in the initial state.
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