Wet dark fluid in Brans-Dicke theory

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Abstract

Plane Symmetric LRSBI dark energy cosmological models are constructed in the frame of Brans-Dicke theory. Wet dark fluid is considered as the source of dark energy. In order to get viable models, the shear scalar is taken to be proportional to the expansion scalar which simulates an anisotropic relationship among the directional scale factors. The dynamics of the universe in presence of wet dark fluid have been discussed.

Keywords: Brans-Dicke Theory; Wet Dark Fluid; LRSBI model

1 Introduction

Recent observations from distant type Ia supernovae predict that currently the universe is undergoing a state of acceleration [1],[2]. This intriguing discovery has led to the idea of an exotic form of energy dubbed as dark energy which is believed to be responsible for the possible acceleration of the universe. The exact nature of dark energy is not yet known except the fact that dark energy violates the strong energy condition and clusters only at largest accessible scales. A simple candidate for dark energy can be a cosmological constant in the classical FRW model. However, the cosmological constant is entangled with many serious puzzles like the fine tuning problem and coincidence problem. Therefore a good number of alternative candidates have been proposed in recent times. Some alternative candidates for dark energy models are quintessence models [3], phantom models [4], ghost condensate...
or k-essence [6] and so on. The dark energy provides a negative pressure that generates an antigravity effect driving the acceleration. High resolution Cosmic Microwave Background Radiation (CMB) anisotropy data from Wilkinson Microwave Anisotropy Probe (WMAP) are in good agreement with the prediction of the Λ dominated cold dark matter model (ΛCDM) based upon the spatial isotropy and flatness of the universe [7], [8]. However, ΛCDM encounters some anomalous features at large scale. Eventhough the large scale anomalies in CMB anisotropy are still debatable, WMAP data suggest an asymmetric expansion with one direction expanding differently form the other two transverse directions at equatorial plane [9] and signal a non-trivial topology of the large scale geometry of the universe [10], [11].

The issue of global anisotropy of the universe can be simply dealt with a simple modification of the FRW model. Recently, some plane symmetric Bianchi-I models or Locally Rotationally Symmetric Bianchi-I (LRSBI) models have been proposed to address the issues related to the smallness in the angular power spectrum of the temperature anisotropy [12], [13], [14], [15]. For a planar symmetry, the universe looks the same from all the points but the points all have a preferred axis. However, it may be noted here that, there still persists uncertainty on these large angle anisotropies and they remain as open problems. LRSBI models are more general than the usual FRW models and are based on exact solutions to the Einstein Field equations with homogeneous but anisotropic flat spatial sections. LRSBI models have also been studied, in recent times, in different context [16], [17], [18], [19], [20].

Brans-Dicke (BD) theory is a natural alternative and a simple extension of Einstein general relativity. In BD theory, purely metric coupling of matter with gravity is preserved, thus the universality of free fall (equivalence principle) is ensured. It passes the experimental tests from solar system [21] and is able to provide an explanation of the accelerated expansion of the universe [22]. In Brans-Dicke theory, the gravitational constant is replaced with the inverse of a time-dependent scalar field, namely, \( \phi(t) = \frac{1}{8\pi G} \), and this scalar field couples to gravity with a coupling constant \( \omega \). Since the Brans-Dicke theory is an alternative to the general relativity and evokes wide interests in the modern cosmology, it is worthwhile to discuss dark energy models in this framework.

The value of \( \omega \) as obtained from solar-system experiments is \( \omega > 40000 \) [21]. However, when probing the larger scales, the limit obtained will be weaker than this result. In Ref. [23], the authors found that \( \omega \) is smaller than 40000 on a cosmological scale. Specifically, Wu and Chen [24] obtained the observational constraint on the Brans-Dicke model in a flat universe with cosmological constant and cold dark matter using the latest WMAP and SDSS data. They found that within 2\( \sigma \) range, the value of \( \omega \) satisfies
\( \omega < -120.0 \) or \( \omega > 97.8 \). They also obtained the constraint on the rate of change of \( G \) at present \( -1.75 \times 10^{-12} \, \text{yr}^{-1} < \frac{\dot{G}}{G} < 1.05 \times 10^{-12} \, \text{yr}^{-1} \) at 2\( \sigma \) confidence level.

In the present work, we have constructed some cosmological models for LRSBI universe in the framework of BD theory with a self interacting potential. Wet dark fluid is considered as the source of dark energy. The paper is organised as follows: In section 2, the basic equations for LRSBI universe are derived. Wet dark fluid is discussed in Section 3. In Section 4, we have constructed a cosmological model considering an exponential expansion of the volumetric expansion. In Section 5, cosmological model for power law expansion is discussed. The dynamics of universe in presence of wet dark fluid are investigated for respective models. Finally, we summarize our results in the Section 6.

2 Basic Equations

We consider here the generalized Brans-Dicke (GBD) theory with a self interacting potential. In this GBD theory, the BD parameter is considered as a function of the scalar field \( \phi \). The action for GBD in Jordan frame is given by [25, 26]

\[
S = \int d^4x \sqrt{-g} \left[ \phi R - \frac{\omega(\phi)}{\phi} \phi^\alpha \phi_{,\alpha} - V(\phi) + L_m \right],
\]

where, \( \omega(\phi) \) is the modified BD parameter, \( V(\phi) \) is the self-interacting potential, \( R \) is the scalar curvature and \( L_m \) is the matter Lagrangian. The unit system we choose here is \( 8\pi G_0 = c = 1 \). Variation of the action in (1) with respect to the metric tensor \( g_{ij} \) and the scalar field \( \phi \), the field equations are obtained as,

\[
G_{ij} = \frac{\omega(\phi)}{\phi^2} [\phi_i \phi_j - \frac{1}{2} g_{ij} \phi^\alpha \phi_{,\alpha}] + \frac{1}{2} [\phi,_{ij} - g_{ij} \Box \phi]
\]

\[
\Box \phi = \frac{T}{2\omega(\phi) + 3} - \frac{2V(\phi) - \phi \frac{\partial V(\phi)}{\partial \phi}}{2\omega(\phi) + 3} - \frac{\partial \omega(\phi)}{\partial \phi} \phi^i \phi_{,i}
\]

In the above equations, \( T = g^{ij} T_{ij} \) is the trace of the energy momentum tensor \( T_{ij} \), \( \Box \) is the de Alembert’s operator. The BD theory reduces to Einstein’s general relativity in the limit of a constant scalar field and an infinitely large BD parameter \( \omega \). However, this consideration may not hold always good as pointed out in Refs. [20, 27, 28].
A plane symmetric LRSBI model is considered through the metric
\[
ds^2 = -dt^2 + A^2 dx^2 + B^2 (dy^2 + dz^2),
\]
where \(A\) and \(B\) are the directional scale factors and are considered as functions of cosmic time only. The metric corresponds to considering \(yz\)-plane as the symmetry plane and \(x\) as the axis of symmetry. The eccentricity of such a universe is given by \(e = \sqrt{1 - A^2/B^2}\). The expansion scalar \(\theta\) for this metric can be expressed as
\[
\theta = \frac{\dot{A}}{A} + 2\frac{\dot{B}}{B},
\]
where, an overhead dot represents ordinary time derivative. Defining the directional Hubble parameters along the the axis of symmetry and symmetry plane as \(H_1 = \frac{\dot{A}}{A}\) and \(H_2 = \frac{\dot{B}}{B}\), the mean Hubble parameter can be written as \(H = \frac{1}{3}(H_1 + 2H_2)\) and \(\theta = 3H\). The shear scalar for the plane symmetric defined in (4) is expressed as
\[
\sigma^2 = \frac{1}{2}[\Sigma_i H_i^2 - \frac{1}{3}\theta^2] = \frac{1}{3}(H_1 - H_2)^2
\]
where, \(\Sigma_i\) is the sum over the directional Hubble parameters. The shear scalar may be taken to be proportional to the expansion scalar which envisages a linear relationship between the directional Hubble parameters \(H_1\) and \(H_2\),
\[
H_1 = kH_2
\]
which leads to an anisotropic relation between the directional scale factors \(A\) and \(B\) as \(A = B^k\). Here, \(k\) is an arbitrary positive constant that takes care of the anisotropic nature of the model. If \(k = 1\), the model reduces to be isotropic and otherwise the model is anisotropic. The mean Hubble parameter can now be expressed as \(H = \frac{1}{3}(k + 2)H_2\). The field equations, for a perfect fluid distribution with energy momentum tensor \(T_{ij} = (\rho + p)u_iu_j + pg_{ij}\), now assume the explicit forms
\[
9(2k + 1)H^2 = (k + 2)^2 \left[ \frac{\rho}{\phi} + \frac{\omega(\phi)}{2} \left( \frac{\dot{\phi}}{\phi} \right)^2 - 3H \left( \frac{\dot{\phi}}{\phi} \right) + \frac{V(\phi)}{2\phi} \right],
\]
\[
6(k+2)\dot{H} + 27H^2 = (k+2)^2 \left[ -\frac{p}{\phi} - \frac{\omega(\phi)}{2} \left( \frac{\dot{\phi}}{\phi} \right)^2 - \frac{6H}{(k + 2)} \left( \frac{\dot{\phi}}{\phi} \right) - \frac{\ddot{\phi} + V(\phi)}{2\phi} \right],
\]
\[
3(k^2 + 3k + 2)\dot{H} + 9(k^2 + k + 1)H^2 = (k+2)^2 \left[ -\frac{p}{\phi} - \frac{\omega(\phi)}{2} \left( \frac{\dot{\phi}}{\phi} \right)^2 - \frac{3(k+1)H}{(k + 2)} \left( \frac{\dot{\phi}}{\phi} \right) - \frac{\ddot{\phi} + V(\phi)}{2\phi} \right],
\]
and the Klein-Gordon wave equation for the scalar field,
\[
\frac{\ddot{\phi}}{\phi} + 3H \frac{\dot{\phi}}{\phi} = \frac{\rho - 3p}{2\omega(\phi) + 3} - \frac{\partial \omega(\phi)}{\partial \phi} \left( \frac{\phi}{\dot{\phi}} \right)^2 - \frac{2V(\phi)}{2\omega(\phi) + 3}
\] (11)
where, \( \rho \) is the rest energy density and \( p \) is the proper pressure.

Subtracting eqn(10) from eqn(9), we can obtain the evolution equation for the BD scalar field,
\[
- \frac{\dot{H}}{H} - 3H = \frac{\dot{\phi}}{\phi},
\] (12)
which can also be expressed as,
\[
(q - 2)H = \frac{\dot{\phi}}{\phi},
\] (13)
where, \( q = -1 - \frac{\dot{H}}{H^2} \) is the deceleration parameter. It should be mentioned here that a positive deceleration parameter describes a decelerating universe whereas a negative \( q \) implies an accelerating one. Eqn (13) implies that, for a non-static universe \( (H \neq 0) \), a constant scalar field will give us a decelerating universe with \( q = 2 \).

The general expressions for the BD parameter and the self interacting potential can be obtained from the field eqns (8)-(10) as,
\[
\omega(\phi) = \left( \frac{\dot{\phi}}{\phi} \right)^2 \left[ -\frac{\rho + p}{\phi} - \frac{\dot{\phi}}{\phi} + \frac{3kH \dot{\phi}}{k + 2} - \frac{6H}{k + 2} - \frac{18(1 - k)}{(k + 2)^2} H^2 + 18(1 - k) \right],
\] (14)
\[
V(\phi) = 2\phi \left[ \frac{9(2k + 1)H^2}{(k + 2)^2} - \frac{\rho}{\phi} - \frac{\omega(\phi)}{2} \left( \frac{\dot{\phi}}{\phi} \right)^2 + 3H \frac{\dot{\phi}}{\phi} \right].
\] (15)

3 Wet Dark fluid

Wet dark fluid model is constructed in the spirit of generalized Chaplygin Gas model. In wet dark fluid model, a physically motivated equation of state derived from an equation of state meant to treat water and aqueous solution is suggested with properties relevant to dark energy.

Wet fluid dark energy model is modeled through the equation of state (eos)
\[
p = \gamma(\rho - \rho^*),
\] (16)
where, $\gamma$ and $\rho^*$ are positive constants. The motivation is that the internal attraction between molecules can generate negative pressure. It should be noted here that the wet dark fluid eos contains two parts, one behaves as the usual barotropic cosmic fluid and the other behaves as a cosmological constant and hence unifies the dark energy and dark matter components. The velocity of sound for this eos is $C_s^2 = \gamma$. For stability of a model the velocity of sound should be $C_s^2 \geq 0$ and for causality, $C_s^2 \leq 1$. Hence, $\gamma$ should lie in the range of $0 \leq \gamma \leq 1$. $\gamma = 0$, refers to the case of dust matter and $\gamma = 1$ refers to a stiff fluid with a bulk viscosity or a cosmological constant. However, there are no such constraints for $\rho^*$ and it can be treated as a free parameter.

The energy conservation equation for matter field is given by

$$\dot{\rho} + 3H(\rho + p) = 0.$$  \hspace{1cm} (17)

For wet dark fluid eos, (17) can be integrated to get

$$\rho = \frac{\gamma \rho^*}{1 + \gamma} + \left( \rho_0 - \frac{\gamma \rho^*}{1 + \gamma} \right) \left( \frac{a_0}{a} \right)^{3(1+\gamma)}$$  \hspace{1cm} (18)

where $a = (AB^2)^{\frac{1}{3}}$ is the average radius scale factor of the universe. $a_0$ and $\rho_0$ are respectively the scale factor and the rest energy density at the present epoch. Since the field equations (8)-(11) are highly nonlinear in nature, in order to get viable cosmological models, we require some additional physical conditions. Keeping an eye on the accelerated expansion of the universe, here, we will consider the exponential and power law of expansion of the scale factor corresponding to a constant and variable (decaying) mean Hubble rate i.e

$$H = H_0,$$  \hspace{1cm} (19)

and

$$H = \frac{m}{t},$$  \hspace{1cm} (20)

where $H_0$ and $m$ are positive constants.

4 Model with exponential law of expansion

If the Hubble rate is a constant quantity i.e. $H = H_0=$constant, then the scale factor is given by $a = e^{H_0(t-t_0)}$ corresponds to an exponential law of expansion and describes a de Sitter type universe. The average scale factor in the present time $a_0$ is taken here to be 1 and $t_0$ is the cosmic time in the present epoch. The directional scale factors along the transverse and
longitudinal directions are \( A = A_0 e^{-\frac{3kH_0(t-t_0)}{(k+2)}} \) and \( B = B_0 e^{-\frac{3kH_0(t-t_0)}{(k+2)}} \), where \( A_0 \) and \( B_0 \) represent the values at the present time. The deceleration parameter and jerk parameter for this choice of the Hubble rate, are \( q = -1 \) and \( j = \frac{\ddot{a}}{aH^2} = 1 \). The observational constraints as set upon these parameters in the present epoch from type Ia supernova and X-ray cluster gas mass fraction measurements are \( q_0 = -0.81 \pm 0.14 \) and \( j_0 = 2.16 \pm 0.81 \). Experimentally it is challenging to measure the deceleration parameter and jerk parameter and one needs to observe objects of redshift \( z \geq 1 \). Therefore, current observational results for these quantities may not be reliable.

Integration of (12), along with the condition of (19) yields,
\[
\phi = \phi_0 e^{-3H_0(t-t_0)},
\]
where, \( \phi_0 \) is the value of the scalar field in the present epoch. The scalar field decreases exponentially with the growth of cosmic time and vanishes at a later epoch. In terms of the scale factor,
\[
\phi = \phi_0 a^{-3}.
\]
Consequently, the rest energy density and pressure can be expressed as,
\[
\rho = \frac{\gamma \rho^*}{1 + \gamma} + \left( \rho_0 - \frac{\gamma \rho^*}{1 + \gamma} \right) \left( \frac{\phi}{\phi_0} \right)^{1+\gamma},
\]
and
\[
p = -\frac{\gamma \rho^*}{1 + \gamma} + \gamma \left( \rho_0 - \frac{\gamma \rho^*}{1 + \gamma} \right) \left( \frac{\phi}{\phi_0} \right)^{1+\gamma}.
\]
The rest energy density and pressure in the model evolve with the scalar field. They decrease from higher values in the past to low values in a later period. Using the fact that \( \frac{\dot{\phi}}{\phi} = -3H_0 \) and \( \frac{\ddot{\phi}}{\phi} = 9H_0^2 \) we get the BD parameter from eqns (14), (19), (23) and (24) as
\[
\omega(\phi) = \omega_0 + \omega_1 \phi^\gamma,
\]
where, \( \omega_0 = -2 \left[ \frac{\gamma^3 k^2 (k+2)(k+2) + 3k^2 + 2k(2k+1) + 3}{(k+2)^2} \right] \) and \( \omega_1 = - \left[ \frac{(\gamma+1)\rho_0 - \gamma \rho^*}{9H_0^2} \right] \phi_0^{-1+\gamma} \). The variable BD parameter becomes a constant for the lower limit of \( \gamma \), whereas it varies linearly with the scalar field for its upper limit. The allowed range of the BD parameter is \( \omega_0 + \omega_1 \leq \omega(\phi) \leq \omega_0 + \omega_1 \phi \). The role played by the parameter \( \rho^* \) is quite interesting. In the absence of this parameter, the cosmic fluid behaves as a perfect barotropic fluid with the usual relation \( p = \gamma \rho \) and \( \omega_1 \) turns out
to be negative. Consequently, the BD parameter assumes a much higher negative value in the early phase of cosmic evolution. However, in presence of this parameter, the value of $\omega(\phi)$ is bit lifted up because of the positive contribution from $\rho^*$. For the particular choice of $\rho^* = \left(1 + \frac{1}{\gamma}\right) \rho_0$ when $\omega_1$ vanishes, $\omega(\phi)$ behaves as constant $\omega_0$. From a dimensional consistency as demanded by the Klein-Gordon wave equation (11), for $\gamma \neq 0$, the value of $\omega_0$ should be $-1.5$ which favours the anisotropic parameter $k$ to be $4$. However, for $\gamma = 0$, the value of $\omega$ is decided by the parameters $\rho^*, \rho_0, \phi_0$ and $H_0$. The scalar field decreases with time and therefore, for any value of $\gamma$ else than zero, the BD parameter evolves to a constant $\omega_0$ at late time of evolution.

The self interacting potential can be expressed as

$$V(\phi) = V_0 + V_1 \phi^{1+\gamma},$$

(26)

where, $V_0 = -\frac{2\gamma \rho^*}{1+\gamma}$ and $V_1 = -2 \left(\rho_0 - \frac{2\rho^*}{1+\gamma}\right) \phi_0^{-(1+\gamma)}$. It interesting to note that, the self interacting potential does not depend upon the anisotropic parameter $k$, rather it depends upon the parameters of the wet dark fluid.

For a lower limit of $\gamma$, the self interacting potential varies linearly with the scalar field and for the upper limit it varies in a quadratic manner. For a particular choice of the parameter $\rho^* = \left(1 + \frac{1}{\gamma}\right) \rho_0$, when the BD parameter behaves like a constant with values $\omega_0 = -1.5$, the self interacting potential behaves as constant with the value of $V(\phi) = V_0 = -2\rho_0$. With the evolution of the scalar field with time, the self interacting potential evolves to a constant value of $-\frac{2\gamma \rho^*}{1+\gamma}$ at a later epoch with vanishing scalar field. However, in the absence of the parameter $\rho^*$ in the wet dark energy eos, the potential vanishes. In other words, the presence of the parameter $\rho^*$ induces a self interacting potential even in the absence of a scalar field.

We need to calculate the effective equation of state parameter defined through the pressure to energy density ratio, $\omega_{\text{eff}} = \frac{p}{\rho}$ to know the evolution of the universe through its expansion history. The effective equation of state parameter can be calculated from (23) and (24) as,

$$\omega_{\text{eff}} = \gamma - \frac{1 + \gamma}{1 + \left[\frac{\rho_0(1+\gamma)}{\gamma \rho^*} - 1\right] \left(\frac{\phi}{\phi_0}\right)^{1+\gamma}}.$$  

(27)

The effective eos, for $\gamma > 0$, decreases from positive values at the initial epoch to behave as a cosmological constant with $\omega_{\text{eff}} = -1$, at a later epoch when the scalar field vanishes. At a given time, the effective eos is decided by the parameters $\gamma$ and $\rho^*$. One should note the role played by the parameter
\( \rho^* \). In the absence of this parameter the effective eos is simply given by 
\( \omega_{\text{eff}} = \gamma \), which can take only positive values as decided from the constraints on the velocity of sound. But the inclusion of \( \rho^* \) into the eos modifies the relation and make the effective eos an evolving one. In other words, \( \rho^* \) incorporates some negative pressure simulating the dark energy necessary for the accelerated expansion.

## 5 Model with power law expansion

If we use the power law expansion with the Hubble parameter behaving as \( H = \frac{m}{t} \), with \( m \) is a positive constant, the average scale factor behaves as \( a = \left( \frac{t}{t_0} \right)^m \) and the scale factors along the transverse and longitudinal directions read as \( A = \left( \frac{t}{t_0} \right)^{\frac{m}{1+m}} \) and \( B = \left( \frac{t}{t_0} \right)^{\frac{m}{1+m}} \). The deceleration parameter for this model is \( q = m - 1 \), which turns out to be a constant quantity. In order to be in the safe zone for accelerated expansion model, the predicted deceleration parameter should be negative and that can be achieved only if \( m > 1 \). In terms of the deceleration parameter we can express the parameter \( m \) as \( m = \frac{1}{1+q} \). Considering the observational constraints from Ref. [29], we put the constraints on \( m \) to be \( 3 < m < 20 \). The jerk parameter can be calculated to be \( j = \frac{(m-1)}{m} \) and corresponding to the constraints of deceleration parameter, we can have \( 2/3 < j < 17 \).

The scalar field for this model becomes
\[
\phi = \phi_0 \left( \frac{t}{t_0} \right)^{1-3m} \tag{28}
\]
and in terms of the scale factor
\[
\phi = \phi_0 (a)^{1-3m} \tag{29}
\]

It is obvious from (28) and (29) that, the scalar field decreases with expansion of the universe and vanishes at large cosmic time. The behavior of the scalar field is only decided by \( m \) or more specifically the constant negative deceleration parameter.

The energy density and pressure for this model with power law expansion read as
\[
\rho = \frac{\gamma \rho^*}{1+\gamma} + \left( \rho_0 - \frac{\gamma \rho^*}{1+\gamma} \right) \left( \frac{\phi}{\phi_0} \right)^\frac{3m(1+\gamma)}{3m-1}, \tag{30}
\]
and

\[ p = -\frac{\gamma \rho^*}{1 + \gamma} + \gamma \left( \rho_0 - \frac{\gamma \rho^*}{1 + \gamma} \right) \left( \frac{\phi}{\phi_0} \right)^{\frac{3m(1+\gamma)}{3m-1}}. \]  

(31)

Just like the previous model, the energy density and pressure evolve with the scalar field from large values at the initial epoch to vanish at large cosmic time.

The variable BD parameter can be expressed as

\[ \omega \phi = \omega_0 \phi + \omega_1 \phi^{\frac{3m-1}{3m-1}}, \]  

(32)

where, \( \omega_0 = \frac{3m((k+2)(k-3mk+2)-6m(1-k))}{(1-3m)^2(k+2)^2} \) and \( \omega_1 = -\frac{(\gamma+1)\rho_0 - \rho^*}{(1-3m)^2} \phi_0^{\frac{3m-2}{3m-1}} \). We have used the fact \( \dot{\phi} = \frac{1-3m}{t} \) and \( \ddot{\phi} = \frac{3m(3m-1)}{t^2} \) to derive above relation (32) from (14). It is interesting to note that, the BD parameter is a function of the scalar field even in the lower limit of \( \gamma \), in which it decreases with the scalar field. In other words, the BD parameter assumes lower values in the past and larger values in the late time of cosmic evolution. If we consider the upper bound of \( \gamma \), the BD parameter evolves linearly with the scalar field. Just like the previous model, \( \rho^* \) has a significant role upon the behavior of the BD parameter. For the particular choice \( \rho^* = \left( 1 + \frac{1}{\gamma} \right) \rho_0 \), it behaves as a pure constant which can be equated to \(-1.5\), from dimensional consistency of the Klein-Gordon wave equation.

The self interacting potential for this model is given by

\[ V(\phi) = V_0 + V_1 \phi^{\frac{3m(1+\gamma)}{3m-1}}, \]  

(33)

where,

\[ V_1 = (\gamma - 1) \left( \rho_0 - \frac{\gamma \rho^*}{1 + \gamma} \right) \phi_0^{\frac{3m(1+\gamma)}{1-3m}}. \]  

(34)

Since, \( m > 1 \), the self interacting potential increases with the increase in the scalar field. Like the previous model, the scalar field does not depend on the anisotropic exponent \( k \) and depend on the parameters of the wet dark fluid. For a choice of \( \rho^* = \left( 1 + \frac{1}{\gamma} \right) \rho_0 \) or \( \gamma = 1 \), the self interacting potential becomes independent of the scalar field and equal to \(-\frac{2\rho^*}{14\gamma}\). This is the same value the potential assumes at a later epoch. In other words, there is an induced self interacting potential in the absence of the scalar field, because of the parameter \( \rho^* \).
The effective equation of state $\omega_{\text{eff}}$ can be calculated from (30) and (31) as

$$\omega_{\text{eff}} = \gamma - \frac{1 + \gamma}{1 + \left[\frac{(1+\gamma)\rho_0}{\gamma\rho^*} - 1\right] \left(\frac{\phi}{\phi_0}\right)^{3m(1+\gamma)}}. \quad (35)$$

The effective eos decreases from $\gamma$ in the beginning to behave like a cosmological constant with $\omega_{\text{eff}} = -1$ at a late epoch of cosmic evolution. As it is clear from (16) that, in the absence of the parameter $\rho^*$, the effective eos is a constant quantity i.e. $\gamma$. The presence of this parameter makes the effective eos an evolving one.

6 Conclusion

In the present work, we have investigated the possibility of cosmic acceleration in the frame work of generalized Brans-Dicke scalar tensor theory of gravitation for a plane symmetric universe. The cosmic fluid is considered to be a barotropic fluid satisfying the equation of state for wet dark fluid. The motivation behind choosing such an equation of state is much clear. The interaction among the molecules in aqueous solution provides a negative pressure which may simulate, in the present case, a cosmological constant. We have considered two kinds of volume expansion namely, the power law expansion and the exponential law of expansion to study the effect. The presence of the extra term in the barotropic fluid eos, makes the effective eos an evolving one which evolves from a constant quantity in the beginning to a cosmological constant at a later epoch of cosmic evolution. The scalar field is found to decrease with the increase in the cosmic time. The self interacting potential increases with the increase in scalar field. In an initial epoch, the self interacting potential is having a large value and decreases with time to have a constant value decided by the equation of state parameter at a later epoch.

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