Can $X(3872)$ be a $J^P = 2^-$ tetraquark state?

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In this article, we test the nature of $X(3872)$, which is assumed to be a P-wave $[cq]$-scalar-diquark $[\bar{c}\bar{q}]$-axial-vector-antidiquark tetraquark state with $J^P = 2^-$. The interpolating current representing the $J^P = 2^-$ state is proposed. Technically, contributions of the operators up to dimension six are included in the operator product expansion (OPE). The mass obtained for such state is $m_{2^-} = (4.38 \pm 0.15)\text{ GeV}$. We conclude that it is impossible to describe the $X(3872)$ structure as $J^P = 2^-$ tetraquark state.

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The state $X(3872)$ was first discovered by Belle \cite{1} in the $\pi^+\pi^-J/\psi$ mode and then confirmed by the CDF \cite{2}, D0 \cite{3}, and BABAR \cite{4} Collaborations in the same decay channel. The most recent measure of its mass is \cite{5}

$$m_X = (3871.85 \pm 0.27\text{(stat)} \pm 0.19\text{(syst)})\text{ MeV},$$

with a width of $\Gamma_X < 1.2\text{ MeV}$. Belle \cite{1} and CDF \cite{6} propose that it proceeds through the $X \to J/\psi\rho \to J/\psi\pi^+\pi^-$ decay. Since a charmonium state has isospin zero, it cannot decay into $X \to J/\psi\omega$, so the $X(3872)$ is identified as an “exotic” state. According to the CDF analysis of the decay angular distribution \cite{6} and the invariant $\pi^+\pi^-$ mass distribution \cite{7} of the $J/\psi\pi^+\pi^-$ decay mode, only $1^+$ and $2^-$ assignments are possible. The close proximity of $X(3872)$ mass to the $D\bar{D}^*$ threshold indicates that $X(3872)$ might be a loosely bound $D\bar{D}^*$ molecular state, whose quantum number is $J^P = 1^+$. Also, an angular analysis applied to the $2\pi$ mass distribution in $J/\psi\rho$ favors the quantum number $J^P = 1^+$ \cite{8}. In compliance with these quantum numbers, many literatures have appeared in the past years. Its possible interpretations include the molecular state, tetraquark state and hybrid charmonium (see reviews \cite{9}-\cite{15} and references therein). Using QCD sum rules (QCDSR) \cite{16}, Nielsen et al. discuss the possibility that it is possible to describe the $X(3872)$ structure as a mixed molecule-charmonium state and study its strong decay and radiative decay \cite{17,18}.

Very recently, the BABAR collaboration has performed angular distribution analysis of the decay $B \to J/\psi\omega K$, indicating that P-wave between $J/\psi$ and $\omega$ is favored, so that quantum numbers $J^P = 2^-$ is preferred \cite{19}. In this case, the most conventional explanation is the $1^1D_2$ charmonium state $\eta_{c2}(1D)$. In Ref \cite{20}, the radiative transition processes $\eta_{c2}(1D) \to J/\psi(\psi') + \gamma$ is investigated within several phenomenological potential models with the assumption that $X(3872)$ is a $\eta_{c2}(1D)$ charmonium, which are in contradiction
with the existing BABAR measurements \cite{21}. The data on its $D^0\bar{D}^0\pi^0$ decay mode \cite{22} also contradict the $1^1D_2$ charmonium interpretation of the $X(3872)$ \cite{23}. The decay of $B \to \eta_{c2}X$ is studied in NRQCD factorization framework, which indicates that $X(3872)$ is unlikely to be a $1^1D_2$ charmonium state \cite{24}. Thus, we have to resort to exotic explanations for the $J^P = 2^-$ quantum numbers. In Ref. \cite{25}, it's shown that the molecular interpretation appears to be untenable, but the tetraquark interpretation may be a viable candidates to be $X(3872)$ with $J^P = 2^-$. Follow their opinion, we study the mass of $X(3872)$ as a P-wave $[cq]$-scalar-diquark $[\bar{c}\bar{q}]$-axial-vector-antidiquark tetraquark state with $J^P = 2^-$ using the QCDSR.

The interpolating current representing a $J^P = 2^-$ P-wave tetraquark state with $[cq]$-scalar-diquark and $[\bar{c}\bar{q}]$-axial-vector-antidiquark fields is adopted as

$$j_{\mu\nu} = \frac{\varepsilon_{abc}\varepsilon_{dec}}{\sqrt{2}}[(q_d^T C\gamma_5 c_b)D_\mu (\bar{q}_d\gamma_\nu C\bar{c}^T e_e)] . \quad (2)$$

Herein the index $T$ represents matrix transposition, $C$ means the charge conjugation matrix, $D^\mu$ denotes the covariant derivative, while $a$, $b$, $c$, $d$, and $e$ are color indices.

In the QCDSR approach, the mass of the particle can be determined by considering the two-point correlation function

$$\Pi_{\mu\nu,\alpha\beta}(q^2) = i \int d^4xe^{iq.x} < 0|T[j_{\mu\nu}(x)j_{\alpha\beta}^+(0)]|0 > . \quad (3)$$

The QCDSR attempts to link the hadron phenomenology with the interactions of quarks and gluons, which is obtained by evaluating the correlation function in two ways: an approximate description of the correlation function in terms of intermediate states through the dispersion relation, a description of the same correlation function in terms of QCD degrees of freedom via OPE.

In the phenomenological side, the correlation function is calculated by inserting a complete set of intermediate states with the same quantum numbers as the tetraquark state. Parametrizing the coupling of the $J^P = 2^-$ tensor state to the current $j_{\mu\nu}$ in term of the parameter $f_X$ as

$$\langle 0|j_{\mu\nu}(0)|X \rangle = f_X \varepsilon_{\mu\nu}, \quad (4)$$

where $\varepsilon_{\mu\nu}$ is the relevant polarization tensor. Using Eq. (4) in the phenomenological side of Eq. (3), we obtain

$$\Pi_{\mu\nu,\alpha\beta} = \frac{f_X^2}{m_X^2 - q^2} \left\{ \frac{1}{2}(g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha}) \right\} + \text{other structures} + ... , \quad (5)$$

where the only structure which contains the contribution of the tensor meson has been kept. In calculations, we have performed summation over the polarization tensor using

$$\varepsilon_{\mu\nu}\varepsilon_{\alpha\beta}^* = \frac{1}{2} T_{\mu\alpha}T_{\nu\beta} + \frac{1}{2} T_{\mu\beta}T_{\nu\alpha} - \frac{1}{3} T_{\mu\nu}T_{\alpha\beta} , \quad (6)$$
with

\[ T_{\mu\nu} = -g_{\mu\nu} + \frac{q_{\mu}q_{\nu}}{m_X^2}, \quad (7) \]

In the OPE side, we single out the structure \( \frac{1}{2}(g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha}) \) whose coefficient is denoted as

\[ \Pi^{(1)}(q^2) = \int_{4m_c^2}^{\infty} ds \frac{\rho^{\text{OPE}}(s)}{s - q^2}, \quad (8) \]

where the spectral density is \( \rho^{\text{OPE}}(s) = \frac{1}{\pi} \text{Im}\Pi^{(1)}(s) \). After equating the two sides, assuming quark-hadron duality, and making a Borel transformation, the sum rule can be written as

\[ f_X^2 e^{-m_X^2/M^2} = \int_{4m_c^2}^{s_0} ds \rho^{\text{OPE}}(s)e^{-s/M^2}, \quad (9) \]

with \( M^2 \) the Borel parameter.

In calculations, we work at the leading order in \( \alpha_s \) and consider vacuum condensates up to dimension six, with the similar techniques in Refs. \cite{26, 27}. After tedious calculation, the concrete forms of spectral densities read:

\[ \rho^{\text{OPE}}(s) = \rho^{\text{pert}}(s) + \rho^{(\bar{q}q)}(s) + \rho^{(g^2G^2)}(s) + \rho^{(g\bar{q}q\sigma Gq)}(s) + \rho^{(\bar{q}q)^2}(s), \quad (10) \]

with

\[ \rho^{\text{pert}}(s) = \frac{1}{5 \times 2^{13} \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^4} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^4} (\alpha^4 + \alpha^2 \beta - 2\alpha^2 + \alpha \beta^2 - 3\alpha \beta + 2\alpha + \beta^3 - \beta^2 + \beta - 1)r(m_c, s)^5, \]

\[ \rho^{(\bar{q}q)}(s) = \frac{\langle \bar{q}q \rangle}{3 \times 2^{13} \pi^6} m_c \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^4} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^2} (2\beta^2 + 2\alpha \beta - 2\beta - 1)r(m_c, s)^3, \]

\[ \rho^{(g^2G^2)}(s) = \frac{\langle g^2G^2 \rangle}{3 \times 2^{13} \pi^6} m_c \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^4} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta} (\alpha + \beta - 1)^2(2\alpha^2 + \alpha - 2\beta - \beta)r(m_c, s)^2, \]

\[ \rho^{(g\bar{q}q\sigma Gq)}(s) = \frac{\langle g\bar{q}q \cdot Gq \rangle}{2^{10} \pi^4} m_c \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^4} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^2} (4\alpha^2 \beta + 6\alpha \beta^2 - 2\alpha \beta - \alpha^2 - \beta)r(m_c, s)^2, \]

\[ \rho^{(\bar{q}q)^2}(s) = \frac{\langle \bar{q}q \rangle^2}{3 \times 2^{13} \pi^6} m_c^2 \left( \frac{2m_c^2}{3} - \frac{s}{6} \right) \sqrt{1 - 4m_c^2/s}, \quad (11) \]

with \( r(m_c, s) = (\alpha + \beta)m_c^2 - \alpha \beta s \). The integration limits are given by \( \alpha_{\min} = \left( 1 - \sqrt{1 - 4m_c^2/s} \right)/2, \alpha_{\max} = \left( 1 + \sqrt{1 - 4m_c^2/s} \right)/2, \) and \( \beta_{\min} = \alpha m_c^2/(s \alpha - m_c^2) \).
TABLE I: Upper limits in the Borel window obtained from the sum rule for different values of $\sqrt{s_0}$.

| $\sqrt{s_0}$ (GeV) | $M_{\text{max}}^2 (\text{GeV}^2)$ |
|-------------------|-----------------------------|
| 4.6               | 2.3                         |
| 4.7               | 2.4                         |
| 4.8               | 2.6                         |
| 4.9               | 2.8                         |
| 5.0               | 2.9                         |

To extract the mass $m_X$, we take the derivative of Eq. (9) with respect to $M^2$ and then divide the result by itself

$$m_X^2 = \int_{4m_c^2}^{s_0} ds \rho^{\text{OPE}}(s) s e^{-s/M^2} / \int_{4m_c^2}^{s_0} ds \rho^{\text{OPE}}(s) e^{-s/M^2}. \quad (12)$$

Before the numerical analysis of Eq. (12), we first specify the input parameters. The quark mass is taken as $m_c = 1.23 \text{ GeV}$ [28]. The condensates are $\langle \bar{q}q \rangle = -0.23 \text{ GeV}^3$, $\langle g\bar{q}\sigma \cdot Gq \rangle = m_0^2 \langle \bar{q}q \rangle$, $m_0^2 = 0.8 \text{ GeV}^2$, and $\langle g^2 G^2 \rangle = 0.88 \text{ GeV}^4$ [16]. Complying with the standard procedure of the QCDSR, the threshold $s_0$ and Borel parameter $M^2$ are varied to find the optimal stability window. There are two criteria (pole dominance and convergence of the OPE) for choosing the Borel parameter $M^2$ and threshold $s_0$. In general, the continuum threshold $s_0$ is a parameter of the calculation which is connected to the mass of the studied state, by the relation $\sqrt{s_0} \approx (m_X + 0.5 \text{ GeV})$.

Concretely, the contributions from the high dimension vacuum condensates in the OPE are shown in Fig. 1. We have used $\sqrt{s_0} \geq 4.6 \text{ GeV}$. From this figure it can be seen that for $M^2 \geq 2.0 \text{ GeV}^2$, the contribution of the dimension-6 condensate is less than 16% of the total contribution and the contribution of the dimension-5 condensate is less than 20% of the total contribution, which indicate a good Borel convergence. Therefore, we fix the uniform lower value of $M^2$ in the sum rule window as $M_{\text{min}}^2 = 2.0 \text{ GeV}^2$. The upper limit of $M^2$ is determined by imposing that the pole contribution should be larger than continuum contribution. Fig. 2 demonstrates the contributions from the pole terms with variation of the Borel parameter $M^2$. We show in Table II the values of $M_{\text{max}}^2$ for several values of $\sqrt{s_0}$. In Fig 3 we plot the tetraquark state mass in the relevant sum rule window, for different values of $\sqrt{s_0}$. It can be seen that the mass is very stable in the Borel window with the corresponding threshold $\sqrt{s_0}$. The final estimate of the $J^P = 2^-$ tetraquark state is obtained as

$$m_X = (4.38 \pm 0.15) \text{ GeV}. \quad (13)$$
FIG. 1: The OPE convergence for the $J^P = 2^-$ tetraquark state. The I and II correspond to the contributions from the $D = 6$ term and the $D = 5$ term, respectively. Notations $\alpha$, $\beta$, $\gamma$, $\lambda$ and $\rho$ correspond to threshold parameters $\sqrt{s_0} = 4.6\text{ GeV}, 4.7\text{ GeV}, 4.8\text{ GeV}, 4.9\text{ GeV}$ and $5.0\text{ GeV}$, respectively.

FIG. 2: Contributions from pole terms with variation of the Borel parameter $M^2$ in the case of $J^P = 2^-$ tetraquark state. Notations $\alpha$, $\beta$, $\gamma$, $\lambda$ and $\rho$ correspond to threshold parameters $\sqrt{s_0} = 4.6\text{ GeV}, 4.7\text{ GeV}, 4.8\text{ GeV}, 4.9\text{ GeV}$ and $5.0\text{ GeV}$, respectively.

In summary, by assuming $X(3872)$ as a $[cq][\bar{c}q]$ tetraquark state with quantum numbers $J^P = 2^-$, the QCDSR approach has been applied to calculate the mass of the resonance. Our numerical results are $m_X = (4.38 \pm 0.15)\text{ GeV}$, which indicates that $X(3872)$ is unlikely to be a $J^P = 2^-$ tetraquark state. Thus, $J^P = 1^+$ assignment for the quantum numbers of the $X(3872)$ is favored.
FIG. 3: The mass of the $J^P = 2^-$ tetraquark state as a function of $M^2$. Notations $\alpha$, $\beta$, $\gamma$, $\lambda$ and $\rho$ correspond to threshold parameters $\sqrt{s_0} = 4.6 \text{ GeV}$, $4.7 \text{ GeV}$, $4.8 \text{ GeV}$, $4.9 \text{ GeV}$ and $5.0 \text{ GeV}$, respectively.

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