Radiation of a laser-channeled electron

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Abstract. The possibility of electrons trapping by the channels formed by electromagnetic waves may open new techniques for beam reflection and steering. This work describes relativistic electron dynamics in the field formed by crossed laser fields together with the emitted radiation spectrum at such propagation. The plane wave approximation for the laser field is used, and the results of both analytical and numerical analysis are provided.

1. Introduction

Nowadays channeling of both charged and neutral particles has been widely applied for beam shaping, deflection and transportation. And, moreover, the very term “channeling” that originates from specific propagation of charged particles in aligned crystals (crystal channeling) \cite{1,2} is getting wider today absorbing new phenomena that could be successfully described within channeling phenomenology, from X-ray quanta channeling in nanostructures \cite{3} and propagation of electrons in plasma media \cite{4,5,6} or nanotubes \cite{7,8,9} to controlled particles movement in the fields of different nature \cite{10,11,12,13} that form, let say, “potential channels”. Hence, suppose we have created a system to be characterized by a set of well oriented channels, in other words, by periodic potential wells with respect to the charged projectiles. Propagation of particles in such systems is to be considered as channeling, since they perform oscillations in periodic potential channels and since such oscillations are orientation-dependent with defined critical angle. Together with such similarities, the phenomena are united not only by the methods used for their description but also by the application fields. Nevertheless, some particular points about these channeling processes differ essentially.

In this work the motion of relativistic electron trapped by the potential well formed by standing electromagnetic waves is considered. The potential well (channel) could be generated either by crossing laser beams or by electromagnetic waves propagating in a waveguide. On the contrary to electron channeling in crystals, in this specific case no medium is present inside the channel, hence, no inelastic processes would take place at propagation in such a field. Besides, channeling phenomenon in crystals is known mostly for fast particles (to prevent particle diffraction in crystals), since, their large longitudinal momenta allow averaging the atomic raw/plane potential along the direction of the projectiles that form channels inside the crystal (the basics of channeling). Typically, for channeling experiments the beam alignment has to be done along the main crystallographic directions, passing nearby of which the beams...
of relativistic charged particles might be channeled. But in the case of, let us call it so, “laser-
channeling” an averaged potential could be calculated for nonrelativistic projectiles too that
was previously shown [11, 14]. The dynamics of relativistic beams, namely, relativistic laser-
channeled electrons is analyzed in this work. Apart of additional interest applying the proposed
 technique for hadron beams shaping, which is under investigation by our group and a subject of
separate publication, we have first presented below spectral characteristics of relativistic laser-
channeled electrons comparing this type of radiation with channeling radiation in crystals.

![Figure 1](image)

**Figure 1.** a: General system geometry. The region, where standing waves form the channels, is
separated in the center. The peaks of transverse electrical component of the field fast propagating
in the $k_s = k_1 + k_2$ direction shapes the potential barrier. b: Dependence of the barrier height
of a channel potential (normalized by the value $[e^2u_0^2k^2]/[8m\omega_0^2]$) from the particle velocity
normalized by the speed of light.

2. Basics of channeling in a laser field
Channeling phenomenon takes place during the propagation of relativistic particles through a
crystalline target under small angles to the main crystallographic planes (for planar channeling)
or axes (for axial channeling). Due to its velocity, the particle moving along either the plane
or the axis barely feels each separate atom rather than moves in an averaged field of the whole
atomic plane or string. Such a concept of continuum potential approximation for the description
of the inner structure potential for a monocrystalline target was introduced by Lindhard [2]. The
potential well forms longitudinally equipotential channel preferable for the particle propagation.
When the transverse energy of a particle is less than potential barrier height it becomes channeled
and performs transverse oscillations inside the potential well together with relativistic motion
along the channel. On the one hand, such a classical treatment is common for channeling of
charged particles in crystals, on the other hand, similar approach could be applied to a grasp
of other cases, i.e. to a particle propagating either in the field of crossed laser beams, or in
a waveguiding structure together with electromagnetic waves. Being mathematically similar,
these two latter situations will be described below, giving an example of electron channeled in
electromagnetic field.

For simplicity let us consider laser beams wave surface intensity distribution to be uniform
and the problem to be two-dimensional (shown in figure 1,a), thereby two plane electromagnetic
waves of the same frequency $\omega_0$ and amplitude $u_0$ are aligned with some nonzero angle to $Oz$-
axis so that the sum of their wave vectors $k_s = k_1 + k_2$ is co-directional with $Oz$ (longitudinal).
The magnetic components of both electromagnetic waves are aligned normal to $xOz$, so all
the forces will be situated in the $xOz$-plane. Having fixed $z$-coordinate and measured either
3. Field potential and particle dynamics

Let us consider relativistic electron in a field characterized by the following potential

$$\varphi = 0; \quad A = -u_0 \frac{k_x}{k_0} \cos (k_x x) \cos (\omega_0 t - k_0 z) e_x + u_0 \sin (k_x x) \sin (\omega_0 t - k_0 z) e_z,$$

(1)

where $k_{x,z}$ are the absolute values of transverse and longitudinal components of $k_{1,2}$, $u_0$ is the vector potential amplitude, $\omega_0$ is the characteristic field frequency. Since the terms of the right side of the expression for a vector-potential can posses comparable values for relativistic particle, we have to analyze this expression keeping all terms without any approximation at the beginning. Such a field configuration could be the result of either crossing two laser beams or propagation of electromagnetic wave inside an infinite planar waveguide parallel to $Oz$-axis. As above mentioned, the particle trajectory can be presented as a sum of averaged smooth component $\bar{x}_i$ and rapid oscillations $\chi_i$, $x_i = \bar{x}_i + \chi_i$. The latter, on the contrary to the former term $\bar{x}_i$, is characterized by both small amplitude and high frequency. Solving the equation of motion for $\chi_i$, we can reduce high oscillation part of the trajectory

$$[\chi_x(t), \chi_z(t)] = \frac{e u_0 k}{m \omega_0^2 (1 - \beta_0 \sin \alpha)^2} \left[ \frac{\sin \alpha - \beta_0}{\gamma}, \frac{\cos \alpha}{\gamma^3} \right] \cos \Phi_1 \sin \Phi_2,$$

(2)

where $\Phi_1 = k \bar{x} \cos \alpha$, $\Phi_2 = \omega_0 (1 - \beta_0 \sin \alpha) t - k_0 \bar{z}_0$, $e$ and $m$ are the electron charge and rest mass, $\chi_{x,z}$ are the small additions to the particle averaged trajectory, $\gamma = \left\{ 1 - \left[ (\bar{d}\bar{x}/dt)^2 + (\bar{d}\bar{z}/dt)^2 \right] / c^2 \right\}^{-1/2}$ is the Lorentz factor, $\beta_0 (\approx \beta_z)$ is the initial relativistic electron velocity and $k = \omega_0 / c$. The averaged trajectory $\bar{x}_i$ is defined by the motion equations

$$m \frac{d}{dt} \left( \gamma \frac{\bar{d}\bar{x}}{dt} \right) + A(\omega_0, \alpha, \beta_0) \sin (2k \bar{x} \cos \alpha) = 0 \quad \text{and} \quad m \frac{d}{dt} \left( \gamma \frac{\bar{d}\bar{z}}{dt} \right) = 0,$$

(3)

where $\alpha$ is the angle of the wave vector $k$ with respect to the $Ox$-axis. Neglecting small rapid oscillations and, in the first approximation, assuming $\gamma$ to be defined only by $z$-velocity, $\gamma \approx 1 / (1 - \beta_0^2)^{-1/2}$, we can write down the averaged potential

$$U_{eff} = U_0 - \frac{e^2 u_0^2 k^2 (- \cos (2\alpha) - 2 \beta_0 \sin \alpha + (1 + \cos^2 \alpha) \beta_0^2)}{8 \gamma m \omega_0^2 (1 - \beta_0 \sin \alpha)^2 \cos^2 \alpha} \cos (2k x \cos \alpha)$$

(4)

The channel potential depth (or height) is a function of the system geometry and particle characteristics: $U_{am} = U_{am}(\alpha, u_0, \omega_0, \beta_0, m)$. And it defines the effective potential $U_{eff} =$
$U_0 - U_{am} \cos(2kx \cos \alpha)$. Having fixed all described parameters except $\beta_0$, we can get the dependence of a channel potential depth on the particle velocity (shown in figure 1,b).

In dependence with the particle velocity $\beta_0$, from the analysis we can reveal the change in a channel spatial position. Herein it is important to remind that the electron $z$-velocity might be either co-directed or opposite directed with respect to high-frequency sum laser field propagation; that is the reason of a $\beta_0$ sign change. As seen from the dependence in figure 1,b, the potential depth curve suffers a qualitative change, from slowly decreased part at positive values to sharply felt down to the minimum at negative values. This feature corresponds to the potential change from attracting one to scattering one. For attracting potential the $\beta_0$-parameter is in the region of positive $U_{am}$, and the potential channels’ axes lie at $x_n = (0.5 + n)\pi/k_z$ while the peaks of the field are at $x_n = n\pi/k_z$. On the contrary, for scattering potential (for $\beta_0$ near the minimum of potential amplitude $U_{am}$) the potential channels’ axes lie at $x_n = \pi n/k_z$ and the field peaks at $x_n = (0.5 + n)\pi/k_z$.

In the applicability range of analytical solutions for the system shown above the computer simulations are in rather good agreement with analytically derived results, allowing us to go beyond the range of small transverse energies. Supposing the particle oscillations in the field of this potential near the channel bottom, the averaged component of its trajectory is expressed by harmonic function of a fixed frequency $\omega_1$. The total energy of channeled electron in such a case is much less than the height of the channel potential barrier. But higher total transverse energy of a particle results in less linear process of these oscillations and the trajectory becomes such as shown in figure 3,a with larger averaged oscillation period and amplitude.

Summing up, a charged particle moving in the described configuration of electromagnetic field could be trapped and successfully channeled. The dynamics of such a particle could be treated with the described methodology, approved by the numerical experiments conducted. Of course, the radiation of such a channeled electron is of great interest either for diagnostic purposes, or for future applications. First results of its analysis is presented below.

![Figure 2](image.png)

Figure 2. a: Dimensionless spectral power distribution $f(\xi) = \left(\frac{\omega^2}{e^2 c^3} a_z^2 \gamma^2\right) dW/d\omega$ versus dimensionless frequency $\xi(\omega) = \omega/2\gamma^2\omega_2$ for a large number of small oscillations. b: Matching analytical and numerical calculations for the photon yield from 20 rapid oscillations of one relativistic electron initially positioned near channel bottom with $\beta_0 = 0.9997$, zero initial transverse velocity channeled in a field of laser beams of frequency $\omega_0 = 7.5 \cdot 10^{15}$ Hz and intensity 300 MV/m crossed at the angle of $\eta = 30^\circ$. 
4. Radiation of a relativistic laser-channeled particle

The analytical spectral intensity distribution was found for electron moving close to the bottom of the potential channel. For the large smooth oscillations the spectral intensity distribution was calculated numerically. Let consider the case of small-amplitude averaged oscillations and particle velocity, close to the speed of light, directed along the channel. Then we can rewrite the trajectory equations in the assumption of absence of longitudinal particle oscillations, since they are $\gamma^2$-times less than transverse ones for the ultra-relativistic case

$$z(t) \approx v_0 t \quad \text{and} \quad x(t) \approx a_1 \sin(\omega_1 t) + a_2 \sin(\omega_2 t), \quad (5)$$

where $\omega_1^2 = (2\epsilon_0 k^2 \cos \alpha)^2 (\gamma m \omega_2)^{-2} [\cos(2\alpha) - 2\beta_0 \sin \alpha + (1 + \cos^2 \alpha) \beta_0^2]$ and $\omega_2 = \omega_0 (1 - \beta_0 \sin \alpha)$. Thus, the radiation by the electron at large distances is defined by the expressions

$$[A_x, A_z, \varphi] = -\left[\frac{1 - \beta_0 \cos \theta}{\sin \theta \cos \phi}, \beta_0, 1\right] \frac{e}{r(1 - \beta_0 \cos \theta)} \sum_{n,m} e^{-i\omega_{n,m}(t-r/c)} J_n(\nu a_1) J_m(\nu a_2), \quad (6)$$

where $\omega_{n,m} = (n \omega_1 + m \omega_2) / (1 - \beta_0 \cos \theta)$ and $\nu = (\omega(n, m) \sin \theta \cos \phi) / c$. Assuming the frequency of averaged oscillations is a multiple of the frequency of rapid oscillations $\omega_2 = N \omega_1$ (which is surely possible for some strict system and particle parameters) and using expressions (6) it becomes possible to express the expansion of the radiation field potentials in spherical harmonics, and, hence, to write down the spectral intensity distribution $1^{[15, 16]}

$$\frac{d^2W}{d\omega d\Omega} = \frac{\omega_1}{2\pi c N_0} \sum_{n=1}^{\infty} \frac{\sin^2 \left(\frac{\pi N_0}{\omega_1} \left(\frac{\omega(1 - \beta_0 \cos \theta)}{\omega_1} - n\right)\right)}{\pi^2 \left(\frac{\omega(1 - \beta_0 \cos \theta)}{\omega_1} - n\right)^2} \left(|A_n|^2 - |\varphi_n|^2\right), \quad (7)$$

Here $N_0$ is the number of full averaged oscillations. For small values $\theta$ we deal with small-amplitude averaged oscillations with respect to the width of potential well and the field, where $\omega_2 / \omega_1 = N \gg 1$. Hence, the radiation spectrum peaks are located at frequencies $\Omega_1 = \omega_1 / (1 - \beta_0 \cos \theta)$, $\Omega_2 = \omega_2 / (1 - \beta_0 \cos \theta)$. And taking into account that the radiation by ultra relativistic particle fans out in a narrow forward direction cone and performing integration by the angle, we can write down the expression of radiation spectral power distribution for relativistic laser-channeled electron

$$\frac{dW}{d\omega} = \sum_{i=1}^{2} \frac{e^2 \omega_i^3 a_i^2 \gamma_i^2}{c^3} \zeta_i (1 - 2\zeta_i + 2\zeta_i^2) \Theta(\pi N_i(1 - \zeta_i)), \quad (8)$$

where $N_i$ is the numbers of $\omega_i$-frequency oscillations, $\zeta_i = \omega / (2\gamma^2 \omega_i)$, $\Theta(x) = 0.5 + \sin(2x) / \pi - \sin^2(x) / \pi x$. The expression (8) shows that for small-amplitude averaged oscillations the full radiation spectrum includes two curves of a similar form (see figure 2,a) and different absolute amplitudes as well as maximum frequencies.

The numerical calculation of spectral radiation distribution for relativistic particle can be performed by integration of well-known expressions $[16]$. The simulation results match analytically calculated curves for the case of small averaged oscillations radiation (see figure 2,b). Taking into account rather good correspondence of analytical and numerical results, shown in figure 2,b, the code has been used to calculate the radiation by relativistic electron in the case of large-amplitude averaged oscillations shown in figure 3,a, which goes beyond analytical treatment. The radiation spectrum for this trajectory is shown in figure 3,a. Obviously, several peaks in figure 3,b characterize spectral structure of nonlinear $\bar{x}(t)$ function harmonic expansion. The less is averaged oscillations amplitude, the more sinusoidal the electron trajectory $\bar{x}(t)$ becomes and the closer the results of figure 3,b gets to the ones of figure 2,a.

$^1$ This condition is not necessary for $N \gg 1$ case, while for other values of $N$ it is important.
Figure 3. a: Numerical simulation for the large amplitude averaged oscillations of relativistic channeled electron with total transverse energy close to the height of the potential barrier. b: Numerical results for the photon yield from 20 large-amplitude averaged oscillations of the same electron initially positioned near channel border in the same fields system.

Conclusions

The dynamics of relativistic electron in electromagnetic field inherent for the system of two crossed laser beams or for the infinite bilplanar waveguiding structure carrying the electromagnetic wave was described. The possibility and characteristics of relativistic electron channeling regime in such a configuration was shown. Analytical expressions describing the electron dynamics in the system were supported by fully coincident numerical experiment results. The averaged potential function (4) presents the way for manipulating channel width and height by varying the parameters of laser beams or electromagnetic wave traveling the waveguide. This channeling regime of a charged particle in electromagnetic field could be interesting for future applications on accelerators for beams steering and shaping; especially, with curved axial waveguiding structures implementation. The second interesting point described is relativistic electron radiation in the considered system, which looks similar to channeling radiation: it is shifted to the high energy region due to the Doppler effect and has a tunable spectral peak frequency depending on both system field and particle parameters. Deeper studies are needed to answer the question on a new radiation source based on electron channeling in a crossed-laser field. Comparatively small photon yield obtained could be, for instance, compensated by the number of electrons, since the channel width for an IR laser is $\sim 10^{-5}$ cm.

This work was supported by the Ministry of Education and Science of the Russian Federation in the frames of Competitiveness Growth Program of National Research Nuclear University MEPhI, Agreement 02.A03.21.0005.

References

[1] Gemmel D S 1974 Rev Mod Phys 46 (1) 129
[2] Lindhard J 1965 K. Dan. Vidensk. Selsk. Mat.-Fys. Medd. 34 (14)
[3] Dabagov S B 2003 Phys. Usp. 46 1053
[4] Mangles S P D, Walton B R, Tzoufras M et al 2005 Phys. Rev. Lett. 94 (24) 24500
[5] Pukhov A, and Ter Vehn J M 2002 Appl. Phys. B 74 4
[6] Dik A V, Ligidov A Z, and Dabagov S B 2013 Nucl. Instr. Meth. B 309C 210
[7] Artru X, Fomin S P, Shul’ga N F, Ispirian K A, and Zhevago N K 2005 Phys. Rep. 412 89
[8] Dabagov S B, and Zhevago N K 2008 Riv. Nuovo Cimento 31 491
[9] Karabarbounis A, Sarros S, and Trikalinos Ch 2013 Nucl. Instr. Meth. Phys. Res. B 316 160
[10] Kapitza P L, and Dirak P A M 1933 Math. Proc. Cambridge 29 (02) 297
[11] Andreev A V, and Akhmanov S A 1990 Zh. Eksp. Teor. Fiz. 99 1668
[12] Balykin V I, Subbotin M V, and Letokhov V S 1996 Opt. Commun. 129 177
[13] Artru X, Ispirian K A, and Ispiryan M K 2007 ArXiv e-prints 0707.0148
[14] Frolov E N, Dik A V, and Dabagov S B 2013 Nucl. Instr. Meth. B 309C 157
[15] Landau L D, and Lifshitz E M 1988 Field Theory Moscow, Nauka
[16] Bagrov V G, Bisnovatyi-Kogan G S, Bordovitsyn V A et al 2002 Radiation theory of relativistic particles Moscow, Fizmatlit