The DARK MATTER problem

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1. Scope of the dark matter problem

1.1. Several dark matter problems?

There are several dark matter problems, each on a different scale. The standard way to prove the existence of dark matter is to compare dynamical estimates of the mass of an object (galaxy, cluster) with an evaluation of its population contents. Historically, we can distinguish four distinct problems:
1. the galactic force law in the solar neighbourhood
2. galactic halos
3. groups and clusters of galaxies
4. the Universe as a whole.

In each case there is more matter inferred dynamically than can be accounted for by known matter components. This mass discrepancy is usually attributed to additional (dark) matter, assuming that Newton’s laws are valid. Only in Modified Newtonian Dynamics (MOND, cf. Milgrom, 1983) is the discrepancy attributed to a modification of the force law at low densities. This theory has been worked out in detail only for the explanation of spiral galaxy rotation curves. The evidence for dark matter from extended rotation curves of spiral galaxies is considered the strongest, and this topic has thus spawned the most adhoc alternatives. Not only MOND, but also explanations based on magnetic forces have been considered. (cf. Battaner 1992, but see Cuddeford & Binney 1993). For a review of alternative theories of gravity, see Sanders (1990).

In these lecture notes, I will only discuss certain aspects of the whole dark matter problem, with a distinct emphasis on dark matter in spirals, including the Milky Way. The lectures were given to an audience of scientists working mainly in celestial mechanics, hence the inclusion of a lot of introductory material.

1.2. Dark matter in the solar neighbourhood

Oort (1932, 1960) was the first to perform an analysis of the vertical equilibrium of the stellar distribution in the solar neighbourhood. He found that there is more mass in the galactic disk than can be accounted for by star counts.
A reanalysis of this problem by Bahcall (1984abc) argued for the presence of a dark “disk” of a scaleheight of 700 pc. This was called into question by Bienaymé, Robin & Crézé (1987), and by Kuijken & Gilmore (1989ab, 1991). The newest result is based on a sample of stars with HIPPARCOS distances and Coravel radial velocities, within 125 pc of the Sun. Crézé et al. (1998) find that there is no evidence for dark matter in the disk: all the matter is accounted for by adding up the contributions of gas, young stars and old stars.

1.3. DARK MATTER IN GROUPS AND CLUSTERS

An analysis of 21 radial velocities of galaxies in the Coma cluster, assuming virial equilibrium, led Zwicky (1933) to conclude that the integrated mass-to-light ratio $\Upsilon$ of those galaxies must be of order 100 - 500. This result has been confirmed in more recent studies of clusters of galaxies (cf. sec. 4.2).

A classical paper on dark matter in the Local Group is Kahn & Woltjer (1959). Several notions still of interest are introduced in this paper. One is on the persistence of warps. A warp in the outer HI layer of the Milky Way had been discovered by the work of Burke (1957) and Kerr (1957). Kahn & Woltjer note that for any non-spherical distribution of mass a warp will suffer from differential precession. Thus a tidally pulled warp, with an integral sign warp shape, will wrap up and corrugate the disk in at most a few Gyr. It is clear that warps suffer from a similar persistence problem as do spiral arms.

The most interesting argument in Kahn & Woltjer (1959) is called the “timing argument”, and is based on the notion that M31 approaches our Galaxy with a relative speed of 119 km s$^{-1}$. Such a local deviation from the Hubble flow can only be due to the presence of mass in the Local Group. If M31 approaches us for the first time, a total mass for the Local Group of $\sim 3 \times 10^{12} M_\odot$ follows. In principle the mass could reside in each of the two principal galaxies, or else it could be much more spread out in the entire volume of the Local Group. In fact Kahn & Woltjer (1959) propose a much more spread out distribution in gaseous form, and study the potential effect of magnetic forces due to an intergalactic wind blowing on the outer gas layer of our Galaxy thereby causing the warped shape. (The notion that gravity is the dominant force in the dynamics of galaxies, including interactions between them, was firmly established only with the article of Toomre & Toomre (1972) on galactic bridges and tails).

1.4. DARK MATTER IN SPIRAL GALAXIES

1.4.1. Expectations

The notion of dark matter in and around galaxies was only gradually developed, and became accepted during the 1970’s. This slow development is due to the complexity of the problem: our knowledge of stellar populations in
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galaxies drastically improved due to new observational techniques and more systematic studies. Furthermore, the development of numerical simulations provided two very important concepts: the idea that dynamically hot matter is needed to stabilize disks (Ostriker & Peebles 1973), and the notion of merging of galaxies due to mutual interaction (Toomre & Toomre 1972, Toomre 1977).

If all the mass of a galaxy is in the center, and the ionized gas can be considered as orbiting test particles, one expects a Keplerian rotation curve (rotation velocity $V_{\text{rot}}$ proportional to radius $R^{-1/2}$). If the mass distribution follows the light distribution, i.e. the mass-to-light ratio $\Upsilon$ is constant with radius, then, for an exponential light distribution (cf. Freeman 1970) one has a rotation curve which peaks at about 2.2 times the scalelength of the disk, and which thereafter slowly falls off towards the Keplerian behavior. Note that for a rotation curve that just peaks at the edge of the optical disk, there is only 64% of the mass enclosed inside the radius of the disk. Hence a substantial fraction of mass is already outside the optical radius in that case. If no turnover is reached, more mass must be outside. This is the justification for the statement in Freeman (1970) that for M33 and NGC 300 a lot of mass must be outside the last measured point on the rotation curve.

1.4.2. Early debates
Measurements with single dish telescopes with (for that time) very sensitive receivers provided the first hints that rotation curves stay flat at large radii. In particular, there was a debate about the rotation curve of M31 at large radii (cf. Roberts 1975, Emerson & Baldwin 1973, and Baldwin 1975), but also about possible sidelobe effects of Arecibo data (see discussion after Salpeter, 1978). Interferometer data free from this effect for 5 Scd galaxies (Rogstad & Shostak 1972) showed that rotation curves did not decline. Newer data for a number of galaxies observed with the Westerbork telescope settled these issues: a compilation of 25 rotation curves of spiral galaxies of various morphological types showed that all of them are roughly flat, or rising (cf. Bosma 1978, 1981a,b).

Numerical simulations of spiral galaxies also started in earnest in the early 1970s. Hohl (1971) found consistently that flat disks are very prone to the bar instability. A cure was devised by Ostriker & Peebles (1973), i.e. to embed a disk in a dynamically hot dark halo, which was presumed to be roughly spherical. The required halo masses interior to the disks are rather large, so that the total mass of a large galaxy at large radii can be easily $10^{12} \, M_\odot$. The absence of a decline in a rotation curve implies that mass increases linearly with radius. In this way a mass radius relationship can be established, e.g. for our Galaxy, using various tracers of the mass like outlying globular clusters, satellites, Local Group timing, etc. (Ostriker, Peebles & Yahil 1974).
The data in the late seventies from HI observations (Bosma 1978), show that extended flat rotation curves are ubiquitous for large spirals, and that only for small Sc galaxies the rotation curves are still rising. In a series of papers by Rubin et al. (1978, 1980, 1982, 1985) on Hα rotation curves a nice systematic behaviour as function of type and luminosity of the rotation curves for Sa, Sb and Sc spirals was established as well.

1.4.3. Mass modelling methods
Early ways of dealing with rotation curves were to take the observed form, and to "invert" it to derive a density law for the disk. Such a method is only valid if the mass resides indeed in the disk. If that is the case, for extended HI rotation curves, two major conclusions can be drawn: the mass-to-light ratio increases to very high numbers in the outer parts, and the ratio of HI gas mass to total mass stays roughly constant. (cf. Bosma 1978, Bosma & Van der Kruit 1979). In the early 70s it was not yet accepted that low mass stars (red dwarfs) could not account for the bulk of the mass in the disk; in the 90s the idea that dark matter is baryonic and residing in the disk resurfaced in the form of very cold gas, most of it undetectable (Pfenniger & Combes 1994). This idea was directly based on the constancy of the ratio of HI gas mass to total mass in the outer parts.

The inversion method leads to models which cannot be further analyzed. A more fruitful alternative is thus to consider a "realistic" mass distribution of disk and bulge, and to attribute the rotation curve discrepancy in the outer parts to an extended dark halo. Apart from specifying the bulge/disk decomposition, such a method requires a postulate concerning the mass-to-light ratio of the disk (and the bulge). An early application of this way of modelling was done by Kalnajs (1983), who demonstrated that no dark halo is necessary when the rotation curve does not extend far enough. As a rule, HI data extend at least twice as far as the "easy visible disk", and far enough to establish the discrepancy between the expected and the observed rotation curve, but optical (Hα) data usually do not extend far enough to reach this conclusion. This was further established by Kent (1986, 1987, 1988).

The question of disk stability in the presence of dark halos has been pursued vigourously. In particular, the competing influence of velocity dispersions in the disk on suppressing the bar instability has been addressed by Athanassoula & Sellwood (1986), who find that both a massive dark halo and high disk velocity dispersions slow down the development of a bar. Nevertheless, it is the initial velocity dispersion in the disk which determines the axial ratio of the bar (Athanassoula 1983). Toomre (1981) examined the question of spiral structure, and identified a mechanism called swing amplification, which could lead to disks with strong spiral structure. The presence of a dark matter halo is to lessen the dynamical influence of the disk, and for small disk/halo ratios the amplification may be suppressed altogether. Athanassoula, Bosma
& Papaioannou (1987) have used this theory to try to get limits on the possible mass-to-light ratios for the disk.

1.5. **Importance of the dark matter problem today**

The dark matter problem is nowadays a complex of problems, which is pervasive in every aspect of extragalactic astronomy and cosmology. This can be gleaned from previous reviews and symposia, e.g. Trimble 1987, Kormendy & Knapp 1987, Holt & Bennett 1995, Zaritsky 1998). Another way of seeing this is to look at the way the apparently simple problem of the origin of the Hubble sequence has evolved. The classic view: ellipticals and bulges form quickly, and spirals disks are built up gradually, still has its supporters (e.g. Sandage 1986). However, new ideas about the importance of secular evolution modifying the Hubble type of spirals, and producing ellipticals from merging disk galaxies (Toomre 1977), are now used together with numerical simulations of dark matter mixed in with gas and star formation recipes destined to model the formation of galaxies in a cosmological framework (e.g. Navarro, Frenk & White 1996, 1997). Although there are still a plethora of assumptions necessary to proceed from numerical simulations to observations which can test them, such an integrated approach seems to hold promising keys for future developments.

2. **Dark matter in the Galaxy**

New data at virtually all wavelength bands are now available for our Galaxy. In terms of the mass distribution, the most valuable contributions come from a near IR map obtained with the COBE satellite (Dwek et al. 1995), and the ongoing microlensing experiments (MACHO, EROS, OGLE, etc.). For the solar neighbourhood dynamics, the new data from the HIPPARCOS satellite will provide fresh insights into old problems. For the determination of the Galactic rotation curve beyond the solar radius, see the reviews by Fich & Tremaine 1991, Merrifield 1993, and Olling & Merrifield 1998. In general, it is assumed that the Galactic rotation curve remains more or less flat at large radii, consistent with data on satellites such as outlying globular clusters and dwarf spheroidals. Thus the rotation curve of our Galaxy is similar to that of any other larger spiral.

2.1. **Escape speed for a simple spherical model**

We can tie in local observations of high velocity stars with the notion of the escape speed (cf. Binney & Tremaine 1987). For a simple spherical model we have:
The corresponding escape speed is

\[ V_e^2 = 2V_c^2 [1 + \ln \frac{r_\ast}{r}] \quad \text{at} \quad r < r_\ast \]

\[ V_e^2 = 2V_c^2 \frac{r_\ast}{r} \quad \text{at} \quad r \geq r_\ast \]  

Substituting \( V_c = 220 \text{ km s}^{-1} \), \( r = R_\odot = 8.5 \text{ kpc} \) and an estimate for the escape speed of 500 km s\(^{-1}\), we find \( r_\ast = 4.9R_\odot \simeq 41 \text{ kpc} \), and \( M(r=r_\ast) = 4.6 \times 10^{11} \text{ M}_\odot \). Note that Olling and Merrifield (1998) advocate lower values for the distance of the Sun to the Galactic Center and for the rotation velocity at the distance of the Sun. Recent estimates of the escape speed from HIPPARCOS data are discussed in Meillon et al. (1997), and fall within a range of 400 to 550 km s\(^{-1}\).

## 2.2. Disk Heating and Massive Black Holes

The phenomenon of disk heating, i.e. the increase in velocity dispersion of stellar populations as function of age is well known for the solar neighborhood. Stars are thought to be born with a velocity dispersion similar to the gas, i.e. 10 km s\(^{-1}\). Due to scattering of the stars in the fluctuations of the galactic potential, the peculiar velocities of the star with respect to the circular velocity increase, and an ensemble of stars of a given age has thus a higher velocity dispersion with increasing age.

Several mechanisms have been proposed for the disk heating: 1) massive black holes as being the constituents of dark matter halos 2) giant molecular clouds, and 3) shearing bits and pieces of spiral arms. Predictions for the shape of the velocity ellipsoid (the normalized length of the velocity vectors in the three principal directions) have been made by Lacey (1984, 1991) for each of these three scenario’s. The new HIPPARCOS results will certainly lead to a more profound examination of this problem for the solar neighbourhood (e.g. Gomez et al. 1997, Dehnen & Binney 1998).

Lacey & Ostriker (1985) analyse the effect of massive black holes, assuming they are the only constituent of the dark halo, on the dynamical heating of the disk. They derive an upper limit of about \( 10^6 \text{ M}_\odot \) for our Galaxy. Rix & Lake (1993) point out that for small Sc galaxies this upper limit will have to be lowered to \( 10^4 \text{ M}_\odot \), since their potential well is much shallower. Very heavy black holes could even wreck the fragile disk of those galaxies altogether. In view of this effect, most people exclude massive black holes as an important constituent of dark matter.
2.3. Improved Local Disk Model

Crézé et al. (1998) analyzed the new HIPPARCOS data and the associated Coravel radial velocity databases, and created a proper sample of stars for which they could reanalyze the galactic force law in the z-direction. Their best solution is \( \rho_0 = 0.076 \pm 0.015 \, M_\odot \, \text{pc}^{-3} \), which does not leave any room for disk dark matter.

This result has consequences for an idea of Pfenniger & Combes (1994), i.e. that the dark matter is in the form of cold gas, which is almost undetectable due to its fractal structure. This idea is partly designed to explain the evolutionary sequence from Sd (gas rich, dark matter important) to Sa galaxies (gas poor, and apparently less dark matter within the optical radius) (cf. Pfenniger, Combes & Martinet 1994). It also explains the coincidence noted by Bosma (1978) that the ratio of total mass to gas mass surface density becomes constant in the outer parts. Carignan et al. (1990) restated that result by noting the similarity in shape between the computed rotation curve for the HI component with that of the dark halo component.

For the solar neighbourhood, the rotation curve for the disk component has to rise to at least about 180 km s\(^{-1}\) to make a roughly flat total rotation curve with bulge and disk alone. For an exponential disk which places the Sun roughly at the position of turnover of the rotation curve of the disk component (\( \sim 2.2 \) times the scalelength), this corresponds to a surface density of about 100 \( M_\odot \, \text{pc}^{-2} \). Gould, Bahcall & Flynn (1996) evaluate the “visible” components as follows: 13 \( M_\odot \, \text{pc}^{-2} \) due to the (ordinary) gas, 14 \( M_\odot \, \text{pc}^{-2} \) due to young stars, and 12 \( M_\odot \, \text{pc}^{-2} \) due to dwarfs. Thus if all matter is in a thin disk, about 60 \( M_\odot \, \text{pc}^{-2} \) is “missing”. If this matter is distributed as cold gas in a disk, it is hard to see why such a gas rich disk does not go unstable and forms stars. Gerhard & Silk (1996) propose instead that such matter is distributed in a flattened halo.

2.4. Microlensing Results

The recent results from the micro-lensing surveys have led to the construction of new Galactic mass models, with emphasis on their capability to predict the microlensing rate. The results from several years of the MACHO survey (Alcock et al. 1997), combined with the EROS results (Renault et al. 1998), leave little room for compact objects in the galactic halo with masses in the range of \( 10^{-7} \) to \( 10^{-3} \, M_\odot \) (cf. Alcock et al. 1998). The most likely mass of the handful events detected in the direction of the Large Magellanic Cloud is about 0.5 \( M_\odot \), which leads to a debate about the real location of the objects giving rise to the lensing phenomenon: they could belong to an outlying tidal streamer of the LMC (cf. Zhao 1998). Analysis of mass models taking into account a number of dynamical contraints leads to the result that only
a fraction of the mass in the dark halo of the Galaxy could be made up by MACHO’s (cf. Alcock et al. 1998).

2.5. Satellites of the Milky Way

These galaxies are excellent tools for studying the dark matter problem. Not only do their movement around the Galaxy lead to estimates of the total mass of the Galaxy or the Local Group, their internal dynamics show that they themselves are probably dominated by the dark matter.

Lin & Lynden-Bell (1982) studied the dynamics of the Magellanic Stream, and find that an extended dark halo is needed to model this complicated system. Lynden-Bell (1994) emphasizes the coincidence between the location of the Draco and Ursa Minor dwarf spheroidals and the Magellanic Stream along a great circle, as well as the possible existence of an older Fornax-Leo-Sculptor stream which could betray the orbital path of larger satellites. The recent HI map of Putnam et al. (1998) of the Magellanic system, with the detection of a possible leading stream, calls for a new study of the interaction between the Magellanic Clouds and our Galaxy, in order to get more insight in the extended dark halo around our Galaxy.

2.6. Dwarf galaxy halos

The phase space evolution of dark matter can put a constraint on the nature of it, as follows (cf. Tremaine & Gunn 1979): it can be shown that there is a minimum mass for neutrino’s which they should have if they are to constitute the dark matter in dwarf galaxies. This has spurred the quest for a thorough study of dwarf galaxy dynamics. Not only dwarf spheroidals can give an answer, also gas rich dwarfs can help here. The ability of dark matter candidates to cluster or not on small scales has led to the important distinction of cold dark matter (CDM), which can cluster on small scales, and hot dark matter (HDM), which cannot, but might be important e.g. on the scales of clusters and superclusters. (cf. Blumenthal et al. 1984).

For dwarf spheroidals, kinematics of individual bright stars allows estimates of the mass-to-light ratios. A recent update (Mateo 1994) shows that for the Draco and Ursa Minor dwarf spheroidals, the mass-to-light ratio is about 100. Better surface photometry is now available from the work of Irwin & Hadzidimitriou (1995), and more velocities are being collected for individual stars in all dwarf spheroidals orbiting our Galaxy, with ever greater accuracy (cf. Olszewski 1998). The conclusion that some of these systems have a high central density of dark matter (as high as 1 M_☉ pc^{-3} for the Ursa Minor dwarf) seems fairly secure, although tidal effects from the Galaxy remain an important source of uncertainty.

For gas rich dwarfs, 21-cm HI line studies using the WSRT or the VLA, have yielded results for e.g. DDO 154 (Carignan & Freeman 1985), DDO 170
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Lake, Schommer & Van Gorkom 1990), NGC 3109 (Jobin & Carignan 1990) and DDO 105 (Broeils 1992). All these galaxies are dark matter dominated, and their dark halos, when modeled as isothermal spheres, or with Hernquist profiles, appear to be concentrated enough to exclude massive neutrinos as their constituents. Recent work by De Blok and his collaborators (e.g. De Blok & McGaugh 1997) on low surface brightness late type disk galaxies leads to a similar conclusion.

However, a new problem arises related to numerical simulations of cosmological models. Moore (1994) and also Navarro, Frenk & White (1996) find core radii for dwarf galaxies from their numerical simulations, which are even smaller than those observed in the above mentioned four systems, although Kravtsov et al. (1998) claim to have solved this problem. In any case, the rotation curve data for dwarf galaxies can be used to constrain cosmological models via such simulations, and it may well be possible that a standard CDM $\Omega = 1$ model is ruled out by them (e.g. Navarro 1998).

3. Dark matter in spiral galaxies

We saw already in section 1 how the study of extended galaxy rotation curves showed that dark matter is needed to explain their shape in the outer parts of spiral galaxies. Here we will concentrate on modern work, which still has not succeeded in determining whether the inner parts of galaxies are dominated by dark matter or not.

3.1. Maximum disks?

It is customary to construct composite disk/halo mass models of spirals assuming a “maximum disk” solution, or to adopt a “best fit”. In such models, the data from surface photometry are used, assuming a constant mass-to-light ratio, to calculate the expected rotation curve for the visible components, bulge and disk. From the observed HI gas density, and a suitable factor to include helium, a rotation curve is calculated also, and quadratically added to the first one. The resulting curve is then compared with the observed rotation curve, and an additional dark halo component is introduced when necessary. For extended HI rotation curves, such an analysis has been done by several authors, e.g. Begeman, Broeils & Sanders (1991). The constancy of the mass-to-light with radius is usually justified by the absence of colour gradients, indeed, data of De Jong (1996) shows that colour gradients are small, and if present, in the sense that disks become bluer outwards. In the latter case the use of near infrared data is preferred, since it accounts better for the contribution to the mass of the old stellar disk.

Athanassoula et al. (1987) introduce criteria from spiral structure theory, and in particular those for swing amplification (Toomre 1981), in order to
get limits on the dynamical importance of the disk. This leads to a range of values possible for \((M/L)_{\text{disk}}\), in case one is asking for the possibility to have \(m = 2\) structures and for the suppression of \(m = 1\) structures. They find that the requirement to have halos with non hollow cores usually is consistent with the absence of \(m = 1\) structures, and that such models are preferred when considerations of stellar populations and the buildup of Sc disks at a constant rate of star formation over a Hubble time are taken into account.

Bottema (1993), from an analysis of velocity dispersions, claims that the maximum velocity of the disk component is 63% of the maximum observed velocity. The path to this result is strewn with assumptions, the most important of which are that disks are exponential with a velocity ellipsoid close to that in the solar neighbourhood, that Freeman’s (1970) law holds, and that \((B-V)_{\text{old disk}} = 0.7\) for all disks. For NGC 3198 his result corresponds closely to the “no \(m = 2\)” solution proposed by Athanassoula et al. (1987).

Recent work by Navarro (1998), based on fitting rotation curves with a dark halo profile which fits well the cosmological simulations of Navarro, Frenk & White (1996), show that in his mass models the dark matter also dominates in the inner parts of spiral galaxies. Indeed, his decompositions for NGC 3198 are so dark matter dominated that \(m = 2\) structures will not be swing amplified at any radius.

However, Debattista & Sellwood (1998) produce a clear argument in favour of maximum disks: the dynamical friction of a bar against a dark halo slows it down, and only in a maximum disk situation does the corotation radius at the end of the simulation extend to roughly 20% further than the end of the bar. If the halo is more concentrated, as in Navarro’s models, the bar slow down is so strong that corotation is at several times the bar length, completely inconsistent with current notions about bar pattern speeds. From realistic hydrodynamical simulations of the gas flow in barred spirals, which well mimic the observed dust lanes as regions of strong shocks, Athanassoula (1992) places corotation at about \(1.2 \pm 0.2\) times the bar length. Other determinations of bar pattern speeds, based e.g. on the location of rings, which are presumed to be linked to resonances, concur with this (e.g. Elmegreen 1996 for a review).

### 3.2. Declining rotation curves

Some galaxies have rotation curves which decline just beyond the optical image, and stay more or less flat thereafter. Early examples are NGC 5033 and NGC 5055 in Bosma (1978, 1981a), and also NGC 5908 (Van Moorsel 1982). Two more examples, NGC 2683 and NGC 3521, were given by Casertano & Van Gorkom (1991), who speculated that declining curves are linked with disks having short scalelengths. However, Broeils (1992) finds cases of declining curves for galaxies with large disk scalelengths.
Declining rotation curves, because of the additional identifiable feature in the rotation curve, might hold out a promise to enable us to discriminate between the various mass models. Since one expects them to be found amongst galaxies with high rotational velocities, I made a small survey with the VLA of a number of galaxies with $W_R > 400$ km/s in collaboration with Van Gorkom, Gunn, Knapp and Athanassoula. Several new cases of galaxies with declining rotation curves were found. In Bosma (1998) a preliminary account is given for the most spectacular case, NGC 4414, for which also radial velocities and velocity dispersion information was obtained.

Unfortunately, the range in disk mass-to-light ratios for that galaxy cannot be constrained very easily, in spite of the feature. However, the velocity dispersion data allow the evaluation of the Toomre $Q$-parameter, which is found to be about 1.1 for a maximum disk model, but 2.3 for a “no $m = 2$” model. The latter value is definitely too high to allow spiral structure from swing amplification. A weak global spiral pattern is present in the old disk (Thornley 1996). Therefore, it seems unlikely that the inner parts of bright disk galaxies are dark matter dominated.

3.3. Warps

As already noted by Kahn & Woltjer (1959), any non-spherical mass distribution will cause a differential precession of a warp shape, and thus leads to an increasing corrugation of the outer disk. The observed statistics of warps are such that at least 50% of all spirals are thought to be warped (cf. Bosma 1991). To explain the frequency of warped HI disks, it is thus necessary to have recourse to a mechanism which can keep them going. Several proposals have been made, none of them entirely satisfactory (cf. Binney 1992).

Bosma (1991) also shows that the frequency of warps depends on the ratio of halo core radius to optical radius of the galaxy: galaxies for which this ratio is small do not have warped HI disks. This is usually attributed to dynamical friction between a misaligned disk and a dark halo. If the dark halo is strongly concentrated, such misalignments are short lived, as is shown also by numerical simulations (Dubinski & Kuijken 1995).

3.4. Shape of dark matter halos

3.4.1. Polar ring galaxies

A special class of galaxies are the polar ring galaxies. These are small S0 galaxies seen edge-on, and have an additional ring (annulus) of matter orbiting over the poles. A first study of one of them, A0136 - 0801 (Schweizer, Whitmore & Rubin 1983), indicated that the material over the pole had roughly the same rotation speed as the stellar disk. From this, it was concluded that the dark halo must be nearly spherical. However, later studies using more accurate data, and including proper modelling of the self gravity of the polar ring,
changed this conclusion. Sackett et al. (1994) show that for NGC 4650A the dark matter halo is quite flattened, but new data for this galaxy by Arnaboldi et al. (1997) and modelling by Combes & Arnaboldi (1996) shows that the situation is even more complicated yet.

3.4.2. **Axisymmetry of the disks**
Since the natural shape of dark matter is triaxial (Binney 1978), as is confirmed as well from the cosmological N-body simulations, it is surprising at first sight that disk galaxies are roughly axisymmetric in the outer parts. Yet this is borne out even for very high quality HI data. Schoenmakers (1998) analyzed data for several well studied spiral galaxies, and find a very high degree of axisymmetry. The same conclusion can be drawn from the work of Rix & Zaritsky (1995) on face-on spirals. This means that the process of disk galaxy formation is such that the original triaxial dark matter halo shape is modified by the formation of the disk, e.g. due to dissipation (cf. Dubinski 1994). In any case the shape of the dark matter halo close to the disk should be either oblate or prolate.

3.4.3. **Flaring gas layers**
A direct way to study the vertical shape of the dark halo is using the variation of the thickness of the gas layer with radius in edge-on galaxies. Predictions for this can be modelled quite straightforwardly from multicomponent disk/halo mass models (e.g. Athanassoula & Bosma 1988, Bosma 1994), but in practice the effects of angular resolution and sensitivity, small warps, lopsidedness, residual inclination effects, etc. may make the observational determination difficult. Moreover, the problem is directly dependent on the assumed value of the velocity dispersion of the gas. Even so, Olling (1996ab) determined for the edge-on galaxy NGC 4244 that its dark matter halo must be quite flattened, like an E5 - E9 shape. This result contrasts with the results of Hofner & Sparke (1994), who found much rounder halos, based on their interpretation of the warping behavior of the HI disks of several inclined galaxies.

3.5. **Tidal tail extent**
In their survey of galactic bridges and tails, Toomre & Toomre (1972) succeeded in producing numerical models of tidal encounters between galaxies which, for a given moment in the evolution sequence, and using the information on the spatial orientation, resembles closely the observed system such as the M51 system, the Antennae (NGC 4038/39), etc. The basic postulate in such models is that gravity is the dominant force (it was frequently thought before that that bridges and tails needed to be explained by invoking magnetic forces).

As a corollary to this work, it was found that for slow encounters between spirals the objects merge to form an elliptical like object, and Toomre (1977)
produced a sequence of peculiar galaxies which he considered in various stages of a merging process. This merger hypothesis proves very pervasive, and now forms an integral part of the “bottom-up” scenario’s of galaxy and structure formation. A full observational study of the sequence of galaxies discussed by Toomre (1977) can be found in Hibbard & Van Gorkom (1996).

Since the study of Toomre & Toomre (1972) was done without dark halos, it was interesting to see whether the addition of halos to N-body simulations would change the story. Barnes (1988) did this for the Antennae (NGC 4038/39), with success. The growing capabilities of supercomputers to address the gravitational N-body problem led Dubinski, Mihos & Hernquist (1996) to reexamine the problem, with the specific intent to delimit the extent of the dark halo. They thus considered models with different halo extent, and found that for large halo to disk mass tail forming is inhibited. Dubinski, Hernquist & Mihos (1998) extended these calculations, in order to establish a clear criterion to delimit halo extent. However, Barnes (1998) shows that their conclusions are not true for haloes with very shallow density profiles, and thus tidal tail extent may not be a helpful indicator which can be used to rule out very large halos (see also Springel & White 1998).

3.6. Extended dark halos

The evidence for very extended dark halos around spiral galaxies does not come from a single tracer such as HI. Other tracers are brought to bear, such as satellites, binary galaxy statistics, etc. The use of these tracers is much less straightforward, and needs the development of mass estimators, which take into account the statistical effects of the orbits of the tracer galaxy around the parent galaxy.

An example of this is the work by Zaritsky & White (1994) and Zaritsky et al. (1997) on a sample of spiral galaxies with dwarf satellites. These authors have been slowly collecting data on dwarfs around large, inclined spirals, in order to put together a well defined sample in which the satellite distribution around the primaries can be treated statistically. They conclude that large massive halos, with radii as large as 200 kpc, do indeed exist around large spirals. However, the statistics of the satellite distribution show some interesting details, which are not fully understood in the framework of current ideas on galaxy formation and evolution.

A completely different way to probe the extent of dark halos around individual galaxies comes from the analysis of weak but measurable changes to the shapes of distant galaxies due to the gravitational lensing by foreground galaxies. Brainerd et al. (1996) report on one such analysis, and find that large halos, of \( \sim 100 \, h^{-1} \) kpc do indeed exist, in agreement with the conclusions from Zaritsky & White (1994).
4. Dark Matter in other systems, and at larger scales

I will treat these subjects only briefly, to show mainly that the previous chapters are only the tip of the iceberg as far as the dark matter problem is concerned. For more information one can consult general cosmology textbooks, and Binney & Tremaine (1987). This subject is developing very rapidly, so I have selected some highlights only.

4.1. Dark Matter in Ellipticals

For ellipticals, it has been much more difficult to establish that they too are imbedded in a dark matter halo. This is due to the fact that their luminosity distribution drops off very steeply with radius, and that they, ordinarily, do not contain neutral hydrogen. Moreover, the interpretation of the stellar velocity data in the outer parts depends also on the anisotropy of the velocity dispersion tensor. For a few systems which do have neutral hydrogen, the data show generally a flat rotation curve, hence a dark halo is inferred. The best studied system this way is IC 2006, which has an outer ring of HI. Indications are that the halo around this system is close to axisymmetric (cf. Franx, Van Gorkom & De Zeeuw 1994).

Recent efforts in detailed modelling of high quality stellar radial velocity and velocity dispersion data have resulted in the demonstration that dark matter is a necessary ingredient to get good fits of the models to the data. This was done for NGC 2434 (Rix et al. 1997) and NGC 6703 (Gerhard et al. 1998).

Other tracers can be used. A fruitful one comes from the kinematics of planetary nebulae. The detection of such objects around individual galaxies have now become routine and for a few ellipticals the data indicate the presence of dark mass at large radii. This has been shown in particular for NGC 3384 (Tremblay et al. 1995) and NGC 5128, also known as Centaurus A (Hui et al. 1995). For this last system, the dynamics of the outer shells detected in neutral hydrogen also indicate the presence of a dark halo (Schiminovich et al. 1994).

The detection of X-ray gas in and around elliptical galaxies in clusters has lead early on to the detection of large dark matter halos around ellipticals like M87 and NGC 1399, which are in the center of clusters. For normal ellipticals, only recent data confirm the presence of extended dark matter halos around them. In particular, Buote & Canizares (1998) use a geometrical test to show the presence of dark matter around three field ellipticals.
4.2. Groups and clusters

The traditional way to demonstrate the presence of dark matter has been to collect radial velocity data from the individual members of a group and cluster, from optical and/or radio data, and to apply some form of the virial theorem. This has been pioneered by Zwicky (1933), and became popular in the 70s and 80s. Data for many groups and clusters indicate high mass-to-light ratios, of order 100 - 500, which is much higher than expected for the mass-to-light ratios of individual galaxies which are of order 5 - 10 if they do not contain much dark matter.

Another way to study dark matter comes from X-ray data, assuming hydrostatic equilibrium. The enclosed mass within a given radius depends on the temperature of the hot gas giving rise to the X-ray emission, its radial gradient, and the gradient of the gas density. Mapping the latter using X-ray imaging is rather straightforward, but the determination of the temperature gradient is rather more difficult. In the 80s, data of the Einstein satellite were used to determine estimates of the dark matter content of several clusters of galaxies. Further improvement of the data came from the Rosat and ASCA satellites, and soon high quality data will come from the AXAF and XMM facilities.

A third way to determine masses of clusters is using gravitational lensing, using arcs and arclets. This has grown from the first demonstration of that arcs are due to lensing (Soucail et al. 1988) to an impressive field in its own right, in particular with imaging data from the Hubble Space Telescope, and the development of reliable estimates of the mass of the lensing object from the distorted shapes of more distant galaxies.

An interesting study comparing all three methods for a number of distant clusters is the one by Smail et al. (1996). They find reasonable agreement between the results from the X-ray and lensing data, but the optical data do not correspond that well, presumably due to the influence of both substructure and interlopers in the samples of galaxies used to determine the cluster velocity dispersion. In any case, they confirm the high mass-to-light ratios found previously, and give an upper bound to $\Omega$ of $\sim 0.4$.

Compact groups of galaxies are a somewhat special case of ordinary groups, but their properties are quite interesting (see Hickson 1997 for a review). If there is little dark matter in a common halo around such groups, numerical simulations show that the galaxies should merge quite fast into one object. However, inclusion of a large extended common halo, such a the one found for HCG 62 from X-ray data, will retard the merging timescale to longer than a Hubble time (cf. Athanassoula, Makino & Bosma 1997). X-ray observations of sparser groups also indicate high mass-to-light ratios, and thus the presence of dark matter in them.
4.3. **Dark matter in the Universe**

4.3.1. *Large scale structure*

Work on the distribution of galaxies has shown the presence of large scale structure in the Universe. Large scale flows exist between these structures, and indicate the presence of dark matter on ever larger scales. A review of this field has been made by Dekel (1994). From a variety of methods, one can infer that in general $\Omega$ stays less than 1, but at least as large as 0.2 - 0.3.

4.3.2. *Big Bang nucleosynthesis and the amount of baryons*

One of the colloraries of the standard Big Bang model is the calculation of the nucleosynthesis of the primordial elements. A first calculation was done in the seventies, and compared with the observational data on $D^2$, $^3He$, $^4He$ and $^7Li$. These calculations and comparison data are now more and more refined, but recent work still has it that the upper limit to baryonic dark matter is $\Omega_B \leq 0.02 h^{-2}$ with $h \equiv H_0/(100 \text{ km s}^{-1} \text{ Mpc}^{-1})$. (e.g. Copi, Schramm & Turner 1995). The difficulties surrounding the determination of the Hubble Constant need not to be emphasized, but a lower limit for $h$ is 0.4, and more likely $h \simeq 0.75$ (Madore et al. 1998). Thus $\Omega_B$ is at most 0.13, and more likely about 0.04, which is lower than the values observed in groups and clusters if those were typical for the Universe as a whole. It is interesting to note that for some clusters, like the Coma cluster, the gas fraction as detected by X-rays alone might exceed the upper bound derived from Big Bang nucleosynthesis if the Universe is at closure density (cf. White et al. 1993).

4.3.3. *$\Omega = 1$ ?*

In the early 80’s, the most popular scenario for cosmology is the inflation scenario, which could account for the fact that the observed matter density in the universe is close to the critical density. In fact, it was postulated that the matter density, expressed in terms of the critical density, $\Omega$, is exactly 1. Thus, compared to the results from Big Bang nucleosynthesis, there is a lot of dark matter not in baryonic form, but most likely in the form of a Weakly Interacting Massive Particle (WIMP). This hypothesis still persists until today, since it is theoretically very attractive. However, the actual measurements of the matter density, though difficult, come out to be $\Omega \sim 0.2$ (cf. Bahcall, Lubin & Dorman 1995), which lead some people to think that all of the dark matter could still be baryonic. Only further work will tell what the real answer is.

5. **Concluding remarks**

From this brief review, one can see that the study of the local Universe - our Galaxy, its near neighbours, and nearby galaxies in general - provides many
constraints on the dark matter problem, including on what it actually may consist of. The study of the more distant Universe is complementary to this, and though the next generation of large telescopes should provide a lot more information on it, only a combined approach will solve the entire dark matter problem.

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