Spin-dependent transmission through a chain of rings: influence of a periodically modulated spin-orbit interaction strength or ring radius

B. Molnár, P. Vasilopoulos, and F. M. Peeters

Departement Natuurkunde, Universiteit Antwerpen (Campus Drie Eiken), B-2610 Antwerpen, Belgium

We study ballistic electron transport through a finite chain of quantum circular rings in the presence of spin-orbit interaction of strength α. For a single ring the transmission and reflection coefficients are obtained analytically and from them the conductance for a chain of rings as a function of α and of the wave vector k of the incident electron. We show that due to destructive spin interferences the chain can be totally opaque for certain ranges of k the width of which depends on the value of α. A periodic modulation of the strength α or of the ring radius widens up the gaps considerably and produces a nearly binary conductance output.

In recent years the study of spintronics devices, which utilize the spin rather than the charge of an electron, has been intensified mainly because they are expected to operate at much higher speeds than the conventional ones and have potential applications in quantum computing. One such device is a single ring in the presence of the Rashba coupling or spin-orbit interaction (SOI) which results from asymmetric confinement in semiconductor nanostructures. It is important in materials with a small band gap such as InGaAs. An important feature of electron transport through a ring is that, even in the absence of an external magnetic field, the difference in electron transport through a chain of identical rings in the presence of SOI of strength α/ℏ cannot be neglected. In the presence of spin-orbit interaction of strength α/ℏ, the spin interferences the chain can be totally opaque for certain ranges of α/ℏ in region I and II, respectively, and k denotes the incident wave vector. The coefficients f(μ)/(|g(μ)|) are the amplitudes of the spin state μ = 1, 2 for electrons incident from the left (right) lead and r(μ)/(|t(μ)|) those of the spin state which are reflected to the left (entering to the right) of the ring. One can show that the spinor χ(μ)(0) = (−1)μ+1Uχ(μ)(π), with the unitary operator U having the form

$$ U = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} $$

A similar expansion can be made for the wave functions in the upper and lower arms of the ring. The result is

$$ \Psi_{up}(\varphi) = \sum_{\mu,j=1}^{2} a_{j}^{(\mu)} e^{in_{\mu}^{\varphi} \chi(\mu)(\varphi)}, $$

$$ \Psi_{down}(\varphi) = \sum_{\mu,j=1}^{2} b_{j}^{(\mu)} e^{-in_{\mu}^{\varphi} \chi(\mu)(-\varphi)}, $$

with n_{μ}^{\varphi} = (−1)^μ jka + Φ(μ)/2π the solutions of the equation $k^2 a^2 = E_{n,\varphi}/\hbar \Omega$ that ensure energy conservation.

Reflection and transmission coefficients. Applying the boundary conditions one can verify that the amplitudes g(μ) and t(μ) are connected to r(μ) and f(μ) by a transfer matrix in the form of a unitary operator.
matrix $L$ independent of the spin index $\mu$ as
\[ L[r^{(\mu)}, f^{(\mu)}]^T = [g^{(\mu)}, t^{(\mu)}]^T. \]  
(8)

The matrix $L$ above can be written in an analytic form
\[ L = \frac{1}{T} \begin{bmatrix} 1 & -R \sin(ka \pi) \\ R & (T)^2 - (R)^2 \end{bmatrix}, \]  
(9)

where $T$ and $R$ are the functions of $ka$ and $\Delta_{AC} = (\Phi_{AC}^{(1)} - \Phi_{AC}^{(2)})/2$ (hence of SOI strength $\alpha$)

\[ T = \frac{i \sin(\Delta_{AC}/2) \sin(ka \pi)}{\sin^2(\Delta_{AC}/2) - \cos(ka \pi) - i \sin(ka \pi)/2}, \]  
(10)

\[ R = \frac{1 + 3 \cos(2ka \pi) + 4 \cos \Delta_{AC}/8}{\sin^2(\Delta_{AC}/2) - \cos(ka \pi) - i \sin(ka \pi)/2}. \]  
(11)

Let us assume that an electron enters the ring from the left with an arbitrary spin orientation ($f^{(\mu)}$ are arbitrary complex numbers) but $g^{(1)} = g^{(2)} = 0$, i.e., that there is no incident electron current from the right. Then the electron is reflected without changing its original spin-orientation with the probability amplitude $R = -L_{12}/L_{11}$; on the other hand, it is transmitted with the transmission coefficient $T = -L_{12}L_{21}/L_{11} + L_{22}$ but its spin is unitarily rotated by $U$. One can show that the standard relation $|T|^2 + |R|^2 = 1$ is held.

**Multi-ring conductance.** If we have $N$ rings, the single-ring result can be easily generalized for a chain if the rings only touch each other, cf. inset of Fig. 3(d1). First, one has to calculate the joint transfer matrix $\tilde{L}$

\[ \begin{bmatrix} g_N^{(\mu)} \\ f_N^{(\mu)} \end{bmatrix} = \tilde{L} \begin{bmatrix} r_1^{(\mu)} \\ f_1^{(\mu)} \end{bmatrix} = L_N ... L_1 \begin{bmatrix} r_1^{(\mu)} \\ f_1^{(\mu)} \end{bmatrix}, \]  
(12)

then apply the boundary condition $g_N^{(\mu)} = 0$, that is, after the last ring there is only outgoing wave function. Then the reflection ($\tilde{R}$) and transmission ($\tilde{T}$) coefficients are written in terms of $L$ and the conductance $G$ reads

\[ G = 2(e^2/h) \left| \tilde{T} \right|^2 = 2(e^2/h) \left| L_{12}L_{21}/L_{11} - L_{22} \right|^2. \]  
(13)

We note that the spin of the electrons exiting the chain is rotated by $U = U(N)...U(1)$ in respect to that of the incident electrons.

**Numerical results.** In Fig. 1 the conductance $G(ka)$, through a chain of $N = 101$ rings, is shown as a function of the incident wave vector $ka$ for various values of $\alpha$. Because $G(ka)$ is an even and periodic function of $ka$ with period 1 we show it only within one period of $ka$, for $5 \leq ka \leq 6$. In the absence of SOI the conductance $G$ oscillates with high values but it never drops to zero. In other words, a chain without SOI is never totally reflective. But for finite non zero values of $\alpha$ it takes a "square-wave" form and has zero value in a finite range of $k$, the width of which strongly depends on $\alpha$. Outside these ranges $G$ always oscillates with high values.

The occurrence of the gaps in $G(ka)$ is attributed to the Aharonov-Casher effect due to the spin precession induced by the SOI and the destructive interference between the electron spins travelling in the clockwise and counter-clockwise directions. In the most extreme cases, if the SOI strength has certain well-defined values $\alpha = (h^2/2m^*a) \sqrt{4(n+1)^2 - 1} \equiv \alpha_n$, for $n$ an arbitrary integer, the difference between $\Phi_{AC}^{(1)} = (2n + 1)\pi$ and $\Phi_{AC}^{(2)} = -2\pi - \Phi_{AC}^{(1)} = -(2n + 3)\pi$ renders the spin interference destructive and leads to the widest gap because each ring is non-transparent for any value of $ka$. With the effective mass of InAs $m^* = 0.023m_0$ and a ring radius $a = 0.25\mu$m the smallest value ($n = 0$) of $\alpha$ which can produce total reflection in the chain (and used for the results shown) is $\alpha_0 = 1.147 \times 10^{-11}$eV/m.

The rapid oscillations and the square profile of $G(k)$ stem from the fact the chain contains many identical rings. In general, the low transmission values for a single ring become almost zero for a chain with many rings while those values near the maximum 1 remain nearly unchanged. However, this is not exactly true if $ka$ is a half integer. For such a $ka$ we have $\tilde{L}L = -\mathcal{I}$, where $\mathcal{I}$ is the identity matrix; consequently, the conductance of a finite chain depends on the parity of $N$. More precisely, if $N$ is odd $G$ equals the single ring conductance, while
A more regular behavior can be obtained if only $\alpha$ is modulated and the radius is kept fixed. For a chain of $N = 101$ rings having the same radius $a$ but with SOI $\alpha$ changing from ring to ring as $\alpha_i = \alpha$, if $i$ is odd, and $\alpha_i = 0$, if $i$ is even, the resulting $G(k, \alpha)$ is shown in Fig. 3(c). Such a profile for $\alpha$ could be created by applying a gate to only to the $i$-th ring, $i$ for odd. The contour plot is more symmetric and has definite dark gaps as in the case of a single waveguide. Relative to Fig. 3(a) we see more gaps for constant $\alpha$ and variable $ka$. Another way of appreciating the results of Fig. 3(c) is shown in Fig. 4 in which $G(k, \alpha)$ is plotted along the lines of Fig. 3(c) at $\alpha/\alpha_0 = 0, 0.4, 0.6, 0.8$ and 1. Notice that in each panel $\alpha$ changes from ring to ring as shown in Fig. 3(d2). As for the widening of the gaps conductance gaps in Figs. 3(c) and Fig. 4 relative to those in Fig. 3(a), it can be understood qualitatively with the help of Eq. (2): upon periodically varying the ring radius or strength $\alpha$, $\omega_{so}$ and the energy levels change from ring to ring thus creating the usual superlattice barriers or wells. Depending then on the incident electron's energy one has the usual gaps or bands in the transmission.

The results presented here are valid for chains of strictly one-dimensional rings. They can be extended to rings of finite width $w$ provided the inequality $w \ll a$ holds and, e.g., an infinite well confinement is assumed along the radial direction. In this case the radial and angular motion are decoupled and the energy levels, given by Eq. (2), are shifted by $\hbar^2 l^2 / 2m^* w^2$, where $l$ is an integer. The results presented above correspond then to the lowest $l = 1$ mode.

In summary, we studied ballistic electron transport through chains of rings in the presence of SOI, of strength $\alpha$, and showed that gaps in the conductance, as a function of $\alpha$ and/or the electron's wave vector $k$, occur due to destructive interferences between electron spins travelling in the clockwise and counter-clockwise directions. In particular, we showed that periodic modulations of $\alpha$ or of the ring radius widen these gaps and produce a nearly square-wave conductance. The full gaps in the conductance plotted in Figs. 2-4 occur whether the incident electrons are spin polarized or not. Accordingly, the results are pertinent to the development of the spin transistor where a spin-dependent and binary conductance output is necessary with as a good control as possible.

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* Department of Physics, Concordia University, 1455 de Maisonneuve Quest, Montréal, Quebec, Canada H3G 1M8
3 E. I. Rashba, Sov. Phys. Solid State 2, 1109 (1960).
Y. Aharonov and A. Casher, Phys. Rev. Lett. 53, 319 (1984); A. G. Aronov and Y. B. Lyanda-Geller, ibid 70, 343 (1993); S. L. Zhu and Z. D. Wang, ibid 85, 1076 (2000); D. Frustaglia, M. Hentschel, and K. Richter, ibid 87, 256602 (2001); J. B. Yau, E. P. De Poortere, and M. Shayegan, ibid 88, 146801 (2003).

S. Datta and B. Das, Appl. Phys. Lett. 56, 665 (1990).

F. E. Meijer, A. F. Morpurgo, and T. M. Klapwijk, Phys. Rev. B 66, 033107 (2002).

D. Grundler, Phys. Rev. Lett. 84, 6074 (2000).

B. Molnar, F. M. Peeters, and P. Vasilopoulos, Phys. Rev. B 69, 155335 (2004). A constant term $\hbar \omega_0^2/4\Omega$ is neglected in the expression for the eigenvalues and in Eq. (1). The neglect is acceptable for $ka \gg \omega_0/2\Omega$.

J. B. Xia, Phys. Rev. B 45, 3593 (1992); T. Choi, S. Y. Cho, C. M. Ryu, and C. K. Kim, ibid 56, 4825 (1997).

X. F. Wang and P. Vasilopoulos, Appl. Phys. Lett. 82, 940 (2003).