ρ Meson Decays of Heavy Hybrid Mesons

Liang Zhang
Department of Physics, Beihang University, Beijing 100191, China

Peng-Zhi Huang
Department of Physics and State Key Laboratory of Nuclear Physics and Technology
Peking University, Beijing 100871, China and
Theoretical Physics Center for Science Facilities, CAS, Beijing 100049, China

We calculate the ρ meson couplings between heavy hybrid doublets $H^h/S^h/M^h/T^h$ and the conventional heavy meson doublets $H/S$ using the light-cone QCD sum rule method. The derived sum rules are stable with the variation of the Borel parameter within their corresponding working ranges. The extracted ρ meson couplings are rather small as a whole.

PACS numbers: 12.39.Mk, 12.38.Lg, 12.39.Hg  
Keywords: Heavy hybrid meson, QCD sum rule, Heavy quark effective theory

I. INTRODUCTION

Hadron states that do not fit into the constituent quark model have been studied widely over the past few decades. In recent years, the discovery of a number of unexpected exotic resonances such as the so called $XYZ$ mesons, etc., has revitalized the research of the existence of unconventional hadron states and their nature.

Theoretically, Quantum Chromodynamics (QCD), the fundamental theory of the strong interaction may allow a far richer spectrum than the conventional quark model. Unconventional hadrons include multi-quark states ($ggg\ldots$), glueballs ($qqg\ldots$), and hybrids ($q\bar{q}g\ldots$). Those with $J^{PC} = 0^{++}, 1^{+-}, 2^{++}, \ldots$ are called “exotic” states. They attracted much interest because they are not allowed by the constituent quark model and do not mix with the ordinary mesons.

Evidence of exotic mesons with $J^{PC} = 1^{--}$, e.g. $\pi(1400)$ [1], $\pi(1600)$ [2], have been reported in recent years. They are usually considered to be candidates of hybrid mesons and are studied extensively in various frameworks such as Lattice QCD, QCD sum rule, the flux tube model, AdS/QCD etc. The $1^{--}$ states have been studied in the framework of QCD sum rules in several works, including their masses [3] and decay properties [4].

If light hybrid mesons exist, there should also be hybrid mesons containing one heavy quark ($q\bar{Q}g$) and heavy quarkonium hybrids ($Q\bar{Q}g$), although the former could not be exotic. The hybrid mesons containing one heavy quark and heavy quarkonium hybrids have been studied in [3]. The masses of the heavy quarkonium hybrids were calculated in the heavy quark limit [6]. The masses and the pionic couplings to conventional heavy mesons of the hybrids containing one heavy quark were studied in [8]. In our previous work [13], we calculate the binding energy and the pionic couplings of hybrid mesons using Shifman-Vainshtein-Zakharov (SVZ) sum rules in the framework of heavy quark effective theory (HQET) [14], in which the expansion is performed in terms of $1/m_Q$, where $Q$ is the heavy quark involved. At the leading order of $1/m_Q$, the HQET Lagrangian respects the heavy quark flavor-spin symmetry, therefore heavy hadrons form a series of degenerate doublets. The two members in a doublet carry the same quantum number $j$, the angular momentum of the light components. The two $j_l = \frac{1}{2}$ $S$-wave conventional heavy mesons form a doublet $(0^-, 1^-)$ denoted as $H$ and the $j_l = \frac{1}{2} \frac{3}{2}$ $P$-wave doublets $(0^+, 1^+)/ (1^-, 2^+)$ are denoted as $S/T$. We denote the $j_l = \frac{3}{2} \frac{1}{2}$ $D$-wave doublets $(1^-, 2^-)/ (2^-, 3^-)$ as $M/N$. As far as the heavy hybrid containing one heavy quark are concerned, the two $j_l = \frac{1}{2}$ doublets with $P = +$ and $P = -$ are denoted as $S^h$ and $H^h$, respectively. Similarly, the two $j_l = \frac{3}{2}$ doublets with $P = +$ and $P = -$ are denoted as $T^h$ and $M^h$, respectively.

In our present work, we investigate the ρ meson couplings between heavy hybrid mesons and conventional heavy mesons using the light-cone QCD sum rules (LCQSR) [11], where the operator product expansion (OPE) of the $T$ product of two interpolating currents sandwiched between the vacuum and the ρ meson is performed near the light-cone. The QCD nonperturbative effects are included in the light-cone distribution amplitudes of the ρ meson.

The paper is organized as follows. We derive the sum rules for the ρ meson couplings between doublets $D^h$ and $D$ ($D = H/S/T/M$) in Sec. 11. The numerical analysis are given in Sec. 11. The last section is a short summary. The

*Electronic address: pzhuang@pku.edu.cn
details of the partial amplitudes of these \( \rho \) decay channels are presented in Appendix A. The light cone distribution amplitudes of the \( \rho \) meson employed in the present calculation are collected in Appendix B.

II. \( \rho \) MESON COUPLINGS

The interpolating currents for \( H^h \) and \( M^h \) used in our calculation read

\[
J_{H^h}^\dagger = \sqrt{\frac{1}{2}} \bar{h}_v g_s \gamma_5 \sigma_t \cdot Gq, \\
J_{H^h}^\alpha = \sqrt{\frac{1}{2}} \bar{h}_v g_s \gamma_\alpha \sigma_t \cdot Gq, \\
J_{M^h}^\alpha = \bar{h}_v g_s \left[ 3 G_t^\alpha \gamma_\beta + i \gamma_\beta \sigma_t \cdot G \right] q, \\
J_{M^h}^{\alpha_1 \alpha_2} = \sqrt{\frac{3}{2}} \bar{h}_v g_s \left[ G_t^{\alpha_1 \beta} \gamma_\beta \gamma_\alpha_2 + G_t^{\alpha_2 \beta} \gamma_\beta \gamma_\alpha_1 - \frac{2}{3} i g_t^{\alpha_1 \alpha_2} \sigma_t \cdot G \right] q, 
\]

where \( G_{\alpha \beta} = G^n \lambda / 2 \) and \( h_v(x) = e^{im_0 \cdot x} \frac{1+Q(x)}{2} Q(x) \). The subscript \( t \) denotes the transverseness of the corresponding Lorentz tensors to \( v \), the 4-velocity of the heavy quark. Specifically, we define

\[
g_t^{\alpha \beta} = g^{\alpha \beta} - v^{\alpha} v^{\beta}, \\
A_t^{\alpha_1 \alpha_2 \cdots \alpha_n} = A^{\alpha_1 \alpha_2 \cdots \alpha_n} - \sum_{i=1}^{n} (A^{\alpha_1 \cdots \alpha_i-1 \alpha_{i+1} \cdots \alpha_n} v_\alpha) v^{\alpha_i}. 
\]

where tensor \( A^{\alpha_1 \alpha_2 \cdots \alpha_n} \) is asymmetric to the interchange of its two arbitrary indexes. The overlapping amplitudes between the above currents and the corresponding hybrids are defined as

\[
\langle 0 | J_{H^h}^\dagger (v) | H_0^h (v) \rangle = f_{H^h}^n, \\
\langle 0 | J_{H^h}^\alpha (v) | H_0^h (v) \rangle = f_{H^h}^n \eta_{H^h}^\alpha (v, \lambda), \\
\langle 0 | J_{M^h}^\alpha (v) | M_0^h (v) \rangle = f_{M^h} \eta_{M^h}^\alpha (v, \lambda), \\
\langle 0 | J_{M^h}^{\alpha_1 \alpha_2} (v) | M_0^h (v) \rangle = f_{M^h} \eta_{M^h}^{\alpha_1 \alpha_2} (v, \lambda), 
\]

where \( \eta(v, \lambda) \) is the polarization tensor of the heavy hybrid. These polarization tensors are traceless, symmetric to their Lorentz index and \( \eta_{\alpha_1 \cdots \alpha_n} v^{\alpha} = 0 \), namely they are transversal to \( v \). Furthermore, we have the following projection operators:

\[
\sum_\lambda \eta^\alpha (v, \lambda) \eta^\beta (v, \lambda) = -g_t^{\alpha \beta}, \\
\sum_\lambda \eta^{\alpha_1 \alpha_2} (v, \lambda) \eta^{\beta_1 \beta_2} (v, \lambda) = \frac{1}{2} g_t^{\alpha_1 \beta_1} g_t^{\alpha_2 \beta_2} + \frac{1}{2} g_t^{\alpha_1 \beta_2} g_t^{\alpha_2 \beta_1} - \frac{1}{3} g_t^{\alpha_1 \alpha_2} g_t^{\beta_1 \beta_2}. 
\]

The interpolating currents for the doublets \( S^h \) and \( T^h \) can be obtained by simply adding \( \gamma_5 \) to the currents in Eq. (1):

\[
J_{S_1}^\dagger = \sqrt{\frac{1}{2}} \bar{h}_v g_s \gamma_5 \sigma_t \cdot Gq, \\
J_{S_1}^\alpha = \sqrt{\frac{1}{2}} \bar{h}_v g_s \gamma_\alpha \sigma_t \cdot Gq, \\
J_{T_1}^\alpha = \bar{h}_v g_s \gamma_5 \left[ 3 G_t^\alpha \gamma_\beta + i \gamma_\beta \sigma_t \cdot G \right] q, \\
J_{T_2}^{\alpha_1 \alpha_2} = \sqrt{\frac{3}{2}} \bar{h}_v g_s \left[ G_t^{\alpha_1 \beta} \gamma_\beta \gamma_\alpha_2 + G_t^{\alpha_2 \beta} \gamma_\beta \gamma_\alpha_1 - \frac{2}{3} i g_t^{\alpha_1 \alpha_2} \sigma_t \cdot G \right] q. 
\]
The corresponding overlapping amplitudes and projection operators can be defined similarly to Eq. (3) and (4), respectively.

To derive the sum rules for the ρ meson couplings of these heavy hybrids to conventional heavy mesons, we need the following interpolating currents of conventional heavy meson doublets $H$ and $S$:

$$J_{H_0}^\dagger = \sqrt{\frac{T}{2}} h_\nu \gamma_5 q,$$

$$J_{H_1}^{\dagger \alpha} = \sqrt{\frac{T}{2}} h_\nu \gamma_1^\alpha q,$$

$$J_{S_0}^\dagger = \sqrt{\frac{T}{2}} h_\nu q,$$

$$J_{S_1}^{\dagger \alpha} = \sqrt{\frac{T}{2}} h_\nu \gamma_1^\alpha \gamma_5 q.$$

The overlapping amplitudes between the above currents and the corresponding heavy mesons are

$$\langle 0 | J_{H_0}(0) | H_0(v) \rangle = f_{H_0},$$

$$\langle 0 | J_{H_1}^{\alpha}(0) | H_1(v, \lambda) \rangle = f_{H_1} e_\alpha^H (v, \lambda),$$

$$\langle 0 | J_{S_0}(0) | S_0(v) \rangle = f_{S_0},$$

$$\langle 0 | J_{S_1}^{\alpha}(0) | S_1(v, \lambda) \rangle = f_{S_1} e_\alpha^S (v, \lambda).$$

Here we present the derivation of the sum rule for the coupling constant $g_{H_1 \rho}^{H_1 \rho}$ and $g_{H_1 \rho}^{H_1 \rho}$, where $p$ denotes the orbital momentum of the final $\rho$ meson, superscript ‘0’ and ‘1’ denote the total momentum of the final $\rho$ meson. $g_{H_1 \rho}^{H_1 \rho}$ and $g_{H_1 \rho}^{H_1 \rho}$ are defined through the decay amplitude for the channel $H_1^h \rightarrow H_1 + \rho$:

$$\mathcal{M}(H_1^h \rightarrow H_1 + \rho) = I \left[ (e^+ \cdot q) (e^+ \cdot q_t) - (e^+ \cdot q) (e^+ \cdot q_t) \right] g_{H_1 \rho}^{H_1 \rho} + I (e^+ \cdot q_t) (e^+ \cdot q) g_{H_1 \rho}^{H_1 \rho},$$

where $q$ is the momentum of the final vector meson, $\eta$, $e^+$ and $e^+$ are the polarization vector of the initial hybrid meson $H_1^h$, final $H_1$ heavy meson and $\rho$ meson, respectively, the isospin factor $I = 1, 1/\sqrt{2}$ for the charged and neutral $\rho$ meson, respectively.

We consider the following correlation function:

$$i \int dx e^{-ik \cdot x} \langle \rho(q) | J_{H_1}^{\beta}(0) J_{H_1}^{\alpha*(x)}(0) \rangle = I \left[ e_\beta^q q_t^\alpha - q_t^\beta e_\alpha^q \right] G_{H_1 \rho, H_1 \rho}^{H_1 \rho}(\omega, \omega') + I g_{H_1 \rho}^{H_1 \rho} (e \cdot q_t) G_{H_1 \rho, H_1 \rho}^{H_1 \rho}(\omega, \omega'),$$

where $\omega = 2k \cdot v$ and $\omega' = 2(k - q) \cdot v$. Here $G_{H_1 \rho, H_1 \rho}^{H_1 \rho}(\omega, \omega')$ can be related to $g_{H_1 \rho}^{H_1 \rho}$ by the dispersion relation

$$G_{H_1 \rho, H_1 \rho}^{H_1 \rho}(\omega, \omega') = \int_0^{\infty} ds_1 \int_0^{\infty} ds_2 \frac{\rho_{H_1 \rho}^{H_1 \rho}(s_1, s_2)}{(s_1 - \omega - i\epsilon)(s_2 - \omega' - i\epsilon)} + \int_0^{\infty} ds_1 \frac{\rho_{H_1 \rho}^{H_1 \rho}(s_1)}{s_1 - \omega - i\epsilon} + \int_0^{\infty} ds_2 \frac{\rho_{H_1 \rho}^{H_1 \rho}(s_2)}{s_2 - \omega' - i\epsilon} + \cdots, \quad (10)$$

with

$$\rho_{H_1 \rho}^{H_1 \rho}(s_1, s_2) = f_{H_1} f_H g_{H_1 \rho}^{H_1 \rho} \delta(s_1 - 2\Lambda_{H_1}) \delta(s_2 - 2\Lambda_H) + \cdots. \quad (11)$$

The case of $G_{H_1 \rho, H_1 \rho}^{H_1 \rho}(\omega, \omega')$ is similar. On the other hand, $G_{H_1 \rho, H_1 \rho}^{H_1 \rho}(\omega, \omega')$ can be calculated explicitly at the quark level when $\omega, \omega' \ll 0$, and be expressed by the $\rho$ meson light-cone distribution amplitudes

$$G_{H_1 \rho, H_1 \rho}^{H_1 \rho}(\omega, \omega') = -\frac{1}{4} \int_0^{\infty} dt \int D\alpha e^{it(\frac{2}{3} \omega + \frac{2}{3} \omega')} \frac{m^2}{(q \cdot v)^3} \left[ -2f_{\rho}m^3 \bar{\Psi}(\alpha) - 2f_T^\rho m^2 T(\alpha)(q \cdot v) + f_{\rho}m[6\bar{\Psi}(\alpha) + 2\bar{\Psi}(\alpha)](q \cdot v)^2 + 2f_T^\rho T(\alpha) + 2T_1(\alpha) + 2T_2(\alpha)(q \cdot v)^3 \right],$$

$$C_{H_1 \rho, H_1 \rho}^{H_1 \rho}(\omega, \omega') = \frac{1}{4} \int_0^{\infty} dt \int D\alpha e^{it(\frac{2}{3} \omega + \frac{2}{3} \omega')} \frac{m}{q \cdot v}$$
\[
\left[ f_s m^2 [\mathcal{V}(\alpha) + A(\alpha)] - 2 f_{\rho}^T m [\mathcal{T}_1(\alpha) - \mathcal{T}_2(\alpha) + S(\alpha)(q \cdot v) - 2 f_{\rho} [\mathcal{V}(\alpha) + A(\alpha)](q \cdot v)^2 \right].
\]

where \( u \equiv \alpha_2 + \alpha_3 \) and \( \bar{u} \equiv 1 - u \).

After invoking the double Borel transformation \( B_{\omega}^{T_1} B_{\omega}^{T_2} \), we extract the double dispersion relation part of Eq. (10):

\[
f_{H^1} f_{H} g_{H}^{\rho} e^{-2\omega \lambda_{H}/T - 2u_{0}\lambda_{H}/T}
\]

\[
= m_{\rho}^2 \left\{ \frac{1}{2} f_{\rho} m_{\rho}^2 [\mathcal{T}^{-1}(u_0) - \mathcal{T}^{-2}(u_0)] - f_{\rho} m_{\rho} [\tilde{\Phi}^{-1}(u_0) + 2 \tilde{\Psi}^{-1}(u_0) + A^{-1}(u_0)]
\]

\[
+ f_{\rho}^T [\mathcal{T}^{[0]}(u_0) + 2\mathcal{T}_1^{[0]}(u_0) + 2\mathcal{T}_2^{[0]}(u_0)] T f_{0}(\frac{\omega'}{T}) \right\},
\]

\[
f_{H^1} f_{H} g_{H}^{\rho} e^{-2\omega \lambda_{H}/T - 2u_{0}\lambda_{H}/T}
\]

\[
= m_{\rho} \left\{ f_{\rho} m_{\rho}^2 [\mathcal{V}^{-1}(u_0) + A^{-1}(u_0)] + f_{\rho} m_{\rho} [\mathcal{T}_1^{[0]}(u_0) - \mathcal{T}_2^{[0]}(u_0) + S^{[0]}(u_0)] T f_{0}(\frac{\omega'}{T})
\]

\[
- \frac{1}{2} f_{\rho} [\mathcal{V}^{[1]}(u_0) + A^{[1]}(u_0)] T^2 f_1(\frac{\omega'}{T}) \right\},
\]

where

\[
u_0 = \frac{T_1}{T_1 + T_2}, \quad T = \frac{T_1 T_2}{T_1 + T_2}.
\]

The function \( f_n(x) \) which is introduced while subtracting the contribution of continuum is defined as

\[
f_n(x) = 1 - e^{-x} \sum_{i=0}^{\infty} \frac{x^i}{i!}.
\]

The definitions of \( F^{[\alpha]} \) are

\[
F^{[0]}(u_0) \equiv \int_0^{u_0} \mathcal{F}(u_0, \alpha_2, u_0 - \alpha_2) d\alpha_2,
\]

\[
F^{[1]}(u_0) \equiv \mathcal{F}(u_0, u_0, 0) - \int_0^{u_0} d\alpha_2 \frac{\partial \mathcal{F}(1 - \alpha_2 - \alpha_3, \alpha_2, \alpha_3)}{\partial \alpha_3} \bigg|_{\alpha_3=u_0-\alpha_2},
\]

\[
F^{[2]}(u_0) \equiv \frac{\partial \mathcal{F}(1 - \alpha_2 - \alpha_3, \alpha_2, \alpha_3)}{\partial \alpha_2} \bigg|_{\alpha_2=u_0-\alpha_2} + \frac{\partial \mathcal{F}(u_0 - \alpha_3, u_0, \alpha_3)}{\partial \alpha_3} \bigg|_{\alpha_3=0} - \int_0^{u_0} d\alpha_2 \frac{\partial^2 \mathcal{F}(1 - \alpha_2 - \alpha_3, \alpha_2, \alpha_3)}{\partial \alpha^2} \bigg|_{\alpha_3=u_0-\alpha_2},
\]

\[
F^{[-1]}(u_0) \equiv \int_0^{1} \int_0^{1-\alpha_2} \mathcal{F}(1 - \alpha_2 - \alpha_3, \alpha_2, \alpha_3) d\alpha_3 d\alpha_2 - \int_0^{u_0} \int_0^{u_0-\alpha_2} \mathcal{F}(1 - \alpha_2 - \alpha_3, \alpha_2, \alpha_3) d\alpha_3 d\alpha_2,
\]

\[
F^{[-2]}(u_0) \equiv \int_0^{1} \int_0^{1-\alpha_2} \int_0^{\alpha_3} \mathcal{F}(1 - \alpha_2 - \alpha_3, \alpha_2, \alpha_3) dx d\alpha_3 d\alpha_2 - \int_0^{u_0} \int_0^{u_0-\alpha_2} \int_0^{\alpha_3} \mathcal{F}(1 - \alpha_2 - \alpha_3, \alpha_2, \alpha_3) dx d\alpha_3 d\alpha_2
\]

\[
- \int_0^{1} \int_0^{1-\alpha_2} \mathcal{F}(1 - \alpha_2 - \alpha_3, \alpha_2, \alpha_3) dx d\alpha_3 d\alpha_2.
\]

Using the above mentioned method, we obtain the sum rules of other \( \rho \) meson coupling constants as follows. Their definitions are presented in Appendix A.

\[
f_{H^1} f_{H} g_{H}^{\rho} e^{-2\omega \lambda_{H}/T - 2u_{0}\lambda_{H}/T}
\]

\[
= \frac{1}{6} m_{\rho} \left\{ \frac{1}{2} f_{\rho} m_{\rho}^4 [\tilde{\Phi} - \tilde{\Psi} - A]^{-2}
\]

\[
+ 4 f_{\rho}^T m_{\rho}^3 [\mathcal{T}^{-1}(u_0) - 2 \mathcal{T}_1^{-1}(u_0) + 2 \mathcal{T}_2^{-1}(u_0) - 2 \mathcal{T}_4^{-1}(u_0) + S^{-1}(u_0)]
\]

\[
+ f_{\rho} m_{\rho} [-4 \mathcal{V}(u_0) - 4 \tilde{\Phi}(u_0) + 2 \tilde{\Psi}(u_0) + 2 \tilde{\Psi}(u_0) - 3 \mathcal{A}(u_0)] T f_{0}(\frac{\omega'}{T})
\]

\[
- f_{\rho}^T m_{\rho} [\mathcal{T}_1^{-1}(u_0) + 2 \mathcal{T}_2^{-1}(u_0) - 2 \mathcal{T}_4^{-1}(u_0) - 2 S^{-1}(u_0)] T^2 f_1(\frac{\omega'}{T}) - f_{\rho} [\mathcal{V}^{[2]}(u_0) + \mathcal{A}^{[2]}(u_0)] T^3 f_2(\frac{\omega'}{T}) \right\},
\]
\[f_{\mathcal{H}^1}f_{\mathcal{S}_{1,S_1}}e^{-2\tilde{u}_0\Lambda_{\mathcal{H}}/T - 2\tilde{u}_0\Lambda_{S}/T} = \frac{1}{4} m^2 \rho \left\{ f^\rho m^2_\rho [4\Phi - 2\tilde{\Phi} - 2\tilde{\Psi} - \mathcal{A}]^{(-2)} + 8 f^\rho m^2_\rho [T^{[-1]}(u_0) + T^{[-1]}_1(u_0) + 2 T^{[-1]}_2(u_0) + 2 T^{[-1]}_4(u_0) + 2 S^{[-1]}(u_0)] + 4 f^\rho \mathcal{Y}^{[0]}(u_0) + 4 f^\rho \mathcal{A}^{[0]}(u_0) T f_0(\frac{\omega'}{T}) \right\}, \]

\[f_{\mathcal{H}^2}f_{\mathcal{S}_{2,S_2}}e^{-2\tilde{u}_0\Lambda_{\mathcal{H}}/T - 2\tilde{u}_0\Lambda_{S}/T} = \frac{1}{4\sqrt{2}} m^2 \rho \left\{ 2 f^\rho m^2_\rho [A^{[-1]}(u_0) - 2\mathcal{A}^{[-1]}(u_0)] - 2 f^\rho m^2_\rho [T^{[0]}(u_0) - T^{[0]}_1(u_0) + 2 \mathcal{Y}^{[0]}(u_0) T f_0(\frac{\omega'}{T}) - f^\rho \mathcal{A}^{[1]}(u_0) - 2 \mathcal{Y}^{[1]}(u_0) T^2 f_1(\frac{\omega'}{T}) \right\}, \]

\[f_{\mathcal{H}^3}f_{\mathcal{S}_{3,S_3}}e^{-2\tilde{u}_0\Lambda_{\mathcal{H}}/T - 2\tilde{u}_0\Lambda_{S}/T} = \frac{3}{40\sqrt{2}} m^2 \rho \left\{ f^\rho m^2_\rho [4\Phi - 2\tilde{\Phi} + 2\tilde{\Psi} + \mathcal{A}]^{(-3)} - 2 f^\rho m^2_\rho [A^{[-1]}(u_0) - 4\mathcal{A}^{[-1]}(u_0)] + 4 f^\rho m^2_\rho [T^{[0]}(u_0) + 5 T^{[0]}_1(u_0) + 5 T^{[0]}_2(u_0) - T^{[1]}_1(u_0) - 4 \mathcal{Y}^{[1]}(u_0) T^2 f_1(\frac{\omega'}{T}) + 6 f^\rho \mathcal{A}^{[1]}(u_0) T^2 f_1(\frac{\omega'}{T}) \right\}, \]

\[f_{\mathcal{H}^4}f_{\mathcal{S}_{4,S_4}}e^{-2\tilde{u}_0\Lambda_{\mathcal{H}}/T - 2\tilde{u}_0\Lambda_{S}/T} = \frac{1}{12\sqrt{2}} m^2 \rho \left\{ f^\rho m^2_\rho [2\Phi - 2\tilde{\Phi} + 2\tilde{\Psi} + \mathcal{A}]^{(-2)} + 4 f^\rho m^2_\rho [T^{[-1]}(u_0) + 2 T^{[-1]}_1(u_0) + 2 T^{[-1]}_2(u_0) + 2 T^{[-1]}_4(u_0) - 2 S^{[-1]}(u_0)] - 2 f^\rho m^2_\rho [\mathcal{Y}^{[0]}(u_0) + 2 \mathcal{Y}^{[0]}(u_0) + 2 \tilde{\Phi}^{[0]}(u_0) + 2 \tilde{\Psi}^{[0]}(u_0) - 3 \mathcal{A}^{[0]}(u_0) T f_0(\frac{\omega'}{T}) - f^\rho \mathcal{Y}^{[2]}(u_0) - 2 \mathcal{A}^{[2]}(u_0) T^3 f_2(\frac{\omega'}{T}) \right\}, \]

\[f_{\mathcal{S}^1}f_{\mathcal{H}_{\mathcal{S}_{S_1}}}e^{-2\tilde{u}_0\Lambda_{\mathcal{H}}/T - 2\tilde{u}_0\Lambda_{S}/T} = \frac{3}{2\sqrt{2}} m^2 \rho \left\{ 2 f^\rho m^2_\rho [T^{[-1]}_1(u_0) + T^{[-1]}_4(u_0)] + f^\rho \mathcal{Y}^{[0]}(u_0) T f_0(\frac{\omega'}{T}) \right\}, \]

\[f_{\mathcal{S}^2}f_{\mathcal{H}_{\mathcal{S}_{S_2}}}e^{-2\tilde{u}_0\Lambda_{\mathcal{H}}/T - 2\tilde{u}_0\Lambda_{S}/T} = \frac{1}{6} m^2 \rho \left\{ \frac{1}{2} f^\rho m^2_\rho [4\Phi - 2\tilde{\Phi} - 2\tilde{\Psi} - \mathcal{A}]^{(-2)} - 4 f^\rho m^2_\rho [T^{[-1]}(u_0) - 2 T^{[-1]}_1(u_0) + 2 T^{[-1]}_2(u_0) - 2 T^{[-1]}_4(u_0) + 2 S^{[-1]}(u_0)] + f^\rho m^2_\rho [-4 \mathcal{Y}^{[0]}(u_0) - 4 \mathcal{Y}^{[0]}(u_0) + 2 \tilde{\Phi}^{[0]}(u_0) + 2 \tilde{\Psi}^{[0]}(u_0) - 3 \mathcal{A}^{[0]}(u_0) T f_0(\frac{\omega'}{T}) + f^\rho \mathcal{Y}^{[2]}(u_0) - 2 \mathcal{A}^{[2]}(u_0) T^3 f_2(\frac{\omega'}{T}) \right\}, \]

\[f_{\mathcal{S}^3}f_{\mathcal{H}_{\mathcal{S}_{S_3}}}e^{-2\tilde{u}_0\Lambda_{\mathcal{H}}/T - 2\tilde{u}_0\Lambda_{S}/T} = -\frac{1}{4} m^2 \rho \left\{ f^\rho m^2_\rho [4\Phi - 2\tilde{\Phi} - 2\tilde{\Psi} - \mathcal{A}]^{(-2)} \right\} \]
\[-8f_\rho^T m_\rho [T^{[-1]}(u_0) + T_1^{[-1]}(u_0) + 2T_2^{[-1]}(u_0) + T_4^{[-1]}(u_0) + S^{[-1]}(u_0)] + 4f_\rho \mathcal{V}^{[0]}(u_0) + A^{[0]}(u_0) T f_0(\omega'_T)\),
\]
\[
f_\rho f_\rho g^{\alpha_0}_{0} S_{\rho} e^{-2\theta_{\alpha_0} \Lambda_{\alpha_0} / T} - \frac{1}{2} f_\rho m_\rho^3 T^{[-2]} + f_\rho m_\rho [6\Phi^{[-1]}(u_0) + 2\tilde{\Phi}^{[-1]}(u_0) + A^{[-1]}(u_0)]
\]
\[
+ f_\rho^T [T^{[0]}(u_0) + 2T_1^{[0]}(u_0) + 2T_2^{[0]}(u_0)] T f_0(\omega'_T)\),
\]
\[
f_\rho f_\rho g^{\alpha_1}_{1} S_{\rho} e^{-2\theta_{\alpha_1} \Lambda_{\alpha_1} / T} - \frac{1}{2} f_\rho m_\rho^3 [\mathcal{V}^{[-1]}(u_0) + A^{[-1]}(u_0)] - f_\rho^T m_\rho [T_1^{[0]}(u_0) - T_2^{[0]}(u_0) + S^{[0]}(u_0)] T f_0(\omega'_T)
\]
\[
- \frac{1}{2} f_\rho [\mathcal{V}^{[1]}(u_0) + A^{[1]}(u_0)] T^2 f_1(\omega'_T)\),
\]
\[
f_\rho f_\rho g^{d_1}_{1} S_{\rho} e^{-2\theta_{d_1} \Lambda_{d_1} / T} - \frac{1}{2} f_\rho m_\rho^3 [2\Phi + 2\tilde{\Phi} + 2\tilde{\Psi} + A]^{[-2]}
\]
\[
- 4f_\rho^T m_\rho [T^{[-1]}(u_0) - 2T_1^{[-1]}(u_0) + 2T_2^{[-1]}(u_0) + 2T_4^{[-1]}(u_0) - 2S^{[-1]}(u_0)]
\]
\[
- 2f_\rho m_\rho [2\mathcal{V}^{[0]}(u_0) + 2\Phi^{[0]}(u_0) + 2\tilde{\Phi}^{[0]}(u_0) + 2\tilde{\Psi}^{[0]}(u_0) - 3A^{[0]}(u_0)] T f_0(\omega'_T)
\]
\[
+ f_\rho^T m_\rho [T^{[1]}(u_0) + 4T_1^{[1]}(u_0) + 2T_2^{[1]}(u_0) - 2T_4^{[1]}(u_0) + 4S^{[1]}(u_0)] T^2 f_1(\omega'_T) - f_\rho [\mathcal{V}^{[2]}(u_0) - 2A^{[2]}(u_0)] T^3 f_2(\omega'_T)\),
\]
\[
f_\rho f_\rho g^{d_2}_{1} S_{\rho} e^{-2\theta_{d_2} \Lambda_{d_2} / T} - \frac{1}{2} f_\rho m_\rho^3 [2\Phi + 2\tilde{\Phi} + 2\tilde{\Psi} + A]^{[-2]}
\]
\[
- 4f_\rho^T m_\rho [T^{[-1]}(u_0) - 2T_1^{[-1]}(u_0) + 2T_2^{[-1]}(u_0) + 2T_4^{[-1]}(u_0) - 2S^{[-1]}(u_0)] + 2f_\rho [\mathcal{V}^{[0]}(u_0) - 2A^{[0]}(u_0)] T f_0(\omega'_T)\),
\]
\[
f_\rho f_\rho g^{p_1}_{1} S_{\rho} e^{-2\theta_{p_1} \Lambda_{p_1} / T} - \frac{1}{2} f_\rho m_\rho^3 [A^{[-1]}(u_0) - 2\mathcal{V}^{[-1]}(u_0)]
\]
\[
+ 2f_\rho^T m_\rho [T_1^{[0]}(u_0) - T_2^{[0]}(u_0) + 2S^{[0]}(u_0)] T f_0(\omega'_T) - f_\rho [A^{[1]}(u_0) - 2\mathcal{V}^{[1]}(u_0)] T^2 f_1(\omega'_T)\),
\]
\[
f_\rho f_\rho g^{p_2}_{1} S_{\rho} e^{-2\theta_{p_2} \Lambda_{p_2} / T}
\]
\[
= \frac{3}{40\sqrt{2}} m_\rho \left\{ 4f_\rho m_\rho^4 \tilde{\Psi}^{[-3]} + 2f_\rho^T m_\rho^3 T^{[-2]} - 4f_\rho m_\rho^3 [A^{[-1]}(u_0) - 4\Phi^{[-1]}(u_0)]
\]
\[
- 4f_\rho^T m_\rho [T^{[0]}(u_0) + 5T_1^{[0]}(u_0) + 5T_2^{[0]}(u_0)] T f_0(\omega'_T) + 6f_\rho [A^{[1]}(u_0)] T^2 f_1(\omega'_T)\right\},
\]
\[
f_\rho f_\rho g^{f_2}_{1} S_{\rho} e^{-2\theta_{f_2} \Lambda_{f_2} / T} - \frac{1}{2} f_\rho m_\rho^3 \tilde{\Psi}^{[-3]} - f_\rho^T m_\rho T^{[-2]} + 8f_\rho A^{[-1]}(u_0)\right\}.
\]
\[
(17)
\]
III. NUMERICAL ANALYSIS

The parameters in the distribution amplitudes of $\rho$ meson take the values from [12]. We use the values at the scale $\mu = 1 \text{ GeV}$ in our calculation under the consideration that the heavy quark behaves almost as a spectator of the decay processes in our discussion in the leading order of HQET:

\[
\begin{array}{cccccccccc}
\mu^2 [\text{MeV}] & \mu^2 [\text{MeV}] & a_2 & a_2 & \zeta_{3p} & \omega_{3p} & \omega_{3p} & \omega_{3p} & \zeta_4 & \omega_4 & \zeta_4 & \tilde{\zeta}_4 \\
216(3) & 165(9) & 0.15(7) & 0.14(6) & 0.030(10) & -0.09(3) & 0.15(5) & 0.55(25) & 0.07(3) & -0.03(1) & -0.03(5) & -0.08(5)
\end{array}
\]

The working interval of the Borel parameter $T$ of the mass sum rules for $H$ and $S$ is about $0.8 < T < 1.1 \text{ GeV}$ [13], which is very close to that of the mass sum rules for $D^h$ $(D = H/S/M/T)$ [8]. So we set $u_0 = 1/2$ in our calculation. This choice of $u_0$ will enable us to subtract the continuum contribution cleanly, while the asymmetric choice will lead to the very difficult continuum subtraction [14].

The binding energy and the overlapping amplitudes of doublets $H/S$ [13] and $H^h/M^h, S^h/T^h$ [8] used in our numerical analysis of the sum rules for the above $\rho$ meson couplings are as follows.

| $H$ | $S$ | $H^h/M^h$ | $S^h/T^h$ |
|-----|-----|-----------|-----------|
| $\Lambda \ [\text{GeV}]$ | 0.50 | 1.15 | 2.0 | 2.5 |
| $f \ [\text{GeV}^{3/2}]$ | 0.25 | 0.40 | 1.1 GeV$^{7/2}$ | 1.6 GeV$^{7/2}$ |

The working interval of Borel parameter $T$ is obtained by requiring the stability of the coupling constant to the variation of $T$ and that the pole contribution is larger than 40%. The resulting sum rule is plotted with $\omega_c = 2.8, 3.0, 3.2 \text{ GeV}$ in Fig. 1.

FIG. 1: The sum rules for (a) $g_{H^0_H, H^0_S}^\rho$ and (b) $g_{H^1_H, H^1_S}^\rho$ with continuum threshold $\omega_c = 2.8, 3.0, 3.2 \text{ GeV}$.

We notice that

\[
\begin{align*}
g_{H^0_H, H^0_H}^\rho & = g_{H^0_H, H^0_H}^\rho, \\
g_{H^1_H, H^0_H}^\rho & = g_{H^1_H, H^0_S}^\rho = g_{H^1_H, H^0_S}^\rho = g_{H^1_H, H^0_H}^\rho, \\
g_{S^1_H, S^1_H}^\rho & = g_{S^1_H, S^1_H}^\rho = g_{S^1_H, S^1_H}^\rho = g_{S^1_H, S^1_H}^\rho, \\
g_{H^1_H, S^1_H}^\rho & = g_{H^1_H, S^1_H}^\rho = g_{H^1_H, S^1_H}^\rho = g_{H^1_H, S^1_H}^\rho, \\
g_{M^1_H, H^0_H} & = g_{M^1_H, H^0_H} = -\frac{1}{2} g_{M^1_H, H^0_H} = -\frac{\sqrt{6}}{3} g_{M^1_H, H^0_H}, \\
g_{M^2_H, H^0_H} & = g_{M^2_H, H^0_H} = -\frac{\sqrt{6}}{2} g_{M^2_H, H^0_H} = -\sqrt{6} g_{M^2_H, H^0_H}, \\
g_{M^1_H, H^1_H} & = g_{M^1_H, H^1_H} = -\frac{\sqrt{6}}{2} g_{M^2_H, H^0_H} = -\sqrt{6} g_{M^2_H, H^0_H}, \\
g_{M^2_H, H^1_H} & = g_{M^2_H, H^1_H} = -\frac{\sqrt{6}}{2} g_{M^2_H, H^0_H} = -\sqrt{6} g_{M^2_H, H^0_H},
\end{align*}
\]
the light-cone distribution amplitudes of $T$ amplitudes, the uncertainty in significant, while such a correction is under control in the case of the bottom quark.

The extracted coupling constants are collected in Table I. These numerical values are rather small as a whole. The meson couplings between heavy hybrid meson and conventional heavy mesons at the

$$g_{M^1 H_{1 \rho}}^{S0} = g_{M^1 H_{2 \rho}}^{S1} = g_{M^1 S_{1 \rho}}^{S1} = - \frac{1}{2} g_{H_{1 \rho}}^{S0} = - \frac{\sqrt{6}}{3} g_{M^1 S_{1 \rho}}^{S1},$$

$$g_{M^1 H_{1 \rho}}^{d1} = g_{M^1 H_{2 \rho}}^{d2} = g_{M^1 S_{1 \rho}}^{d2} = \frac{1}{2} g_{H_{1 \rho}}^{d1} = - \frac{\sqrt{6}}{3} g_{M^1 S_{1 \rho}}^{d2},$$

$$g_{M^1 H_{1 \rho}}^{d1} = g_{M^1 H_{2 \rho}}^{d2} = g_{M^1 S_{1 \rho}}^{d2} = \frac{1}{2} g_{H_{1 \rho}}^{d1} = - \frac{\sqrt{6}}{3} g_{M^1 S_{1 \rho}}^{d2},$$

$$g_{T^1 H_{1 \rho}}^{S0} = g_{T^1 S_{1 \rho}}^{S1} = g_{T^1 S_{1 \rho}}^{S1} = - \frac{1}{2} g_{H_{1 \rho}}^{S0} = - \frac{\sqrt{6}}{3} g_{T^1 S_{1 \rho}}^{S1},$$

$$g_{T^1 H_{2 \rho}}^{d1} = g_{T^1 H_{2 \rho}}^{d2} = g_{T^1 S_{1 \rho}}^{d2} = - \frac{1}{2} g_{H_{1 \rho}}^{d1} = - \frac{\sqrt{6}}{3} g_{T^1 S_{1 \rho}}^{d2},$$

$$g_{T^1 H_{2 \rho}}^{p1} = g_{T^1 S_{1 \rho}}^{p1} = - \frac{1}{2} g_{H_{1 \rho}}^{p1} = - \frac{\sqrt{6}}{3} g_{T^1 S_{1 \rho}}^{p1},$$

$$g_{T^1 S_{1 \rho}}^{p1} = g_{T^1 S_{1 \rho}}^{p2} = - \frac{1}{2} g_{H_{1 \rho}}^{p1} = - \frac{\sqrt{6}}{3} g_{T^1 S_{1 \rho}}^{p2},$$

$$g_{T^1 S_{1 \rho}}^{p2} = g_{T^1 S_{1 \rho}}^{f2} = - \frac{1}{2} g_{H_{1 \rho}}^{p2} = - \frac{\sqrt{6}}{3} g_{T^1 S_{1 \rho}}^{f2}.$$

These relations result from the heavy quark flavor-spin symmetry.

| $g_{M^1 H_{1 \rho}}^{p0}$ | $g_{M^1 H_{1 \rho}}^{p1}$ | $g_{M^1 S_{1 \rho}}^{S0}$ | $g_{M^1 S_{1 \rho}}^{S1}$ | $g_{M^1 H_{1 \rho}}^{d1}$ | $g_{M^1 H_{1 \rho}}^{d2}$ | $g_{M^1 H_{1 \rho}}^{p1}$ | $g_{M^1 H_{1 \rho}}^{p2}$ | $g_{M^1 S_{1 \rho}}^{f0}$ | $g_{M^1 S_{1 \rho}}^{f1}$ | $g_{M^1 S_{1 \rho}}^{f2}$ |
|------------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| 0.2                    | 0.5                   | -1.0                  | 0.09                  | 0.12                  | 0.21                  | -0.12                 | -0.01                 | 0.02                  | 0.07                  |
| $g_{S^0 H_{1 \rho}}^{S0}$ | $g_{S^0 H_{1 \rho}}^{S1}$ | $g_{S^0 S_{1 \rho}}^{S1}$ | $g_{S^0 S_{1 \rho}}^{S1}$ | $g_{S^0 H_{1 \rho}}^{d1}$ | $g_{S^0 H_{1 \rho}}^{d2}$ | $g_{S^0 S_{1 \rho}}^{d2}$ | $g_{S^0 S_{1 \rho}}^{d2}$ | $g_{S^0 S_{1 \rho}}^{p1}$ | $g_{S^0 S_{1 \rho}}^{p2}$ | $g_{S^0 S_{1 \rho}}^{f2}$ |
| 1.4                    | -0.4                  | 0.14                  | -0.22                 | -0.18                 | 0.09                  | -0.09                 | -0.02                 | -0.15                 | 0.06                  |

TABLE I: The absolute values of the coupling constants. The units of the $P$-, $D$-wave and $F$-wave coupling constants are GeV$^{-1}$, GeV$^{-2}$ and GeV$^{-3}$ respectively.

The extracted coupling constants are collected in Table I. These numerical values are rather small as a whole. The annihilation of the gluon degree of freedom in the decay processes may be responsible for these weak couplings.

IV. CONCLUSION

Using appropriately constructed interpolating currents for the heavy hybrid mesons containing one heavy quark ($qQg$), we calculated the $\rho$ meson couplings between heavy hybrid meson and conventional heavy mesons at the leading order of HQET within the framework of LCQSR. The obtained sum rules for the $\rho$ meson couplings are stable with the variations of the Borel parameter and the continuum threshold. The extracted couplings are rather small as a whole.

The errors in our calculation lie in the inherent inaccuracy of LCQSR: the omission of the higher twist terms in the OPE near the light-cone, the variation of the binding energy and the coupling constant with the Borel parameter $T$ in the working interval and the continuum threshold $\omega_c$, the omission of the higher conformal partial waves in the light-cone distribution amplitudes of $\rho$ meson, the uncertainty of the parameters in these light-cone distribution amplitudes, the uncertainty in $f$’s and $\bar{A}$’s. As far as the charm quark is concerned, the $1/m_Q$ correction may be significant, while such a correction is under control in the case of the bottom quark.
We hope that our calculation may be helpful to the experimental search of these heavy hybrid mesons and the understanding of their strong interaction with conventional heavy mesons. Moreover, the extracted coupling constants in our work might shed further light on the nature of the XYZ mesons.

Acknowledgments

This work is supported by the National Natural Science Foundation of China under Grants No. 11105007.

Appendix A: The $\rho$ decay amplitudes of heavy hybrid mesons

The $\rho$ decay amplitudes considered in the text are as follows.

\begin{align}
\mathcal{M}(H_0^0 \rightarrow H_0 + \rho) &= I(e^+ \cdot q_1)(e^- \cdot q_2) g_{H_0^{0}H_0\rho}^{\rho}, \\
\mathcal{M}(H_+^0 \rightarrow H_1 + \rho) &= I(\epsilon^{+} \cdot e^{-} \cdot q^{+} - \epsilon^{-} \cdot e^{+} \cdot q^{-}) g_{H_+^{0}H_1\rho}^{\rho}, \\
\mathcal{M}(H_1^+ \rightarrow H_0 + \rho) &= I(\epsilon^{+} \cdot e^{-} \cdot q^{+} - \epsilon^{-} \cdot e^{+} \cdot q^{-}) g_{H_1^{+}H_0\rho}^{\rho}, \\
\mathcal{M}(H_1^+ \rightarrow H_1 + \rho) &= I(e^+ \cdot q_1)(e^+ \cdot \eta_1) g_{H_1^{+}H_1\rho}^{\rho} + I \left[ (e^+ \cdot \eta_1)(e^+ \cdot q_1) - (e^+ \cdot \epsilon_1^+)(\eta_1 \cdot q_1) \right] g_{H_1^{+}H_1\rho}^{\rho}, \\
\mathcal{M}(H_1^0 \rightarrow S_1 + \rho) &= I(e^+ \cdot \epsilon_1^+ q^+_1 e^{-} \cdot q^-_1) g_{H_1^{0}S_1\rho}^{\rho} + I \left[ (e^+ \cdot q_1)(e^+ \cdot q_2) - \frac{1}{3} (e^+ \cdot \epsilon_1^+ q_2^2) \right] g_{H_1^{0}S_1\rho}^{\rho},
\end{align}
\[ \mathcal{M}(H^b_0 \to S_0 + \rho) = I(e^* \cdot \eta_h)g^{S_0}_{H^b_0, S_0 \rho} + I\left[(\eta \cdot q_t)(e^* \cdot q_t) - \frac{1}{3}(e^* \cdot \eta_h)q_t^2\right]g^{d_1}_{H^b_0, S_0 \rho}, \]  
(A6)

\[ \mathcal{M}(H^b_0 \to S_1 + \rho) = I(e^* \cdot \eta_h)g^{S_1}_{H^b_0, S_1 \rho} + I\left[(\eta \cdot q_t)(e^* \cdot q_t) - \frac{1}{3}(e^* \cdot \eta_h)q_t^2\right]g^{d_1}_{H^b_0, S_1 \rho}, \]  
(A7)

\[ \mathcal{M}(M^b_1 \to H_0 + \rho) = I(e^* \cdot \eta_h)g^{M^b_1}_{H_0, H_0 \rho}, \]  
(A8)

\[ \mathcal{M}(M^b_1 \to H_1 + \rho) = I\left[(\eta \cdot \eta^*_t)(e^* \cdot q_t) - (\eta \cdot q_t)(e^* \cdot \epsilon^*_t)\right]g^{p_1}_{M^b_1, H_1 \rho}, \]  
(A9)

\[ \mathcal{M}(M^b_2 \to H_0 + \rho) = 2Ii\eta_{\alpha\alpha_2} \left[ e^{\alpha_1 \alpha_2} q_t^2 - \frac{1}{3} g^{\alpha_1 \alpha_2} (e^* \cdot q_t) \right]g^{d_2}_{M^b_2, H_0 \rho}, \]  
\[ + 2Ii\eta_{\alpha\alpha_2} \left[ e^{\alpha_1 \alpha_2} q_t^2 (e^* \cdot q_t) - \frac{1}{3} g^{\alpha_1 \alpha_2} (e^* \cdot q_t) \right]g^{d_2}_{M^b_2, H_0 \rho}, \]  
(A10)

\[ \mathcal{M}(M^b_2 \to H_1 + \rho) = 2Ii\eta_{\alpha\alpha_2} \left[ e^{\alpha_1 \alpha_2} q_t^2 (e^* \cdot q_t) - \frac{1}{3} g^{\alpha_1 \alpha_2} (e^* \cdot q_t) \right]g^{d_2}_{M^b_2, H_1 \rho}, \]  
\[ + 2Ii\eta_{\alpha\alpha_2} \left[ e^{\alpha_1 \alpha_2} q_t^2 (e^* \cdot q_t) - \frac{1}{3} g^{\alpha_1 \alpha_2} (e^* \cdot q_t) \right]g^{d_2}_{M^b_2, H_1 \rho}, \]  
(A11)

\[ \mathcal{M}(M^b_0 \to S_0 + \rho) = I(\eta \cdot q_t)g^{S_0}_{M^b_0, S_0 \rho} + I\left[(\eta \cdot q_t)(e^* \cdot q_t) - \frac{1}{3}(\eta \cdot \eta^*_t)q_t^2\right]g^{d_1}_{M^b_0, S_0 \rho}, \]  
(A12)

\[ \mathcal{M}(M^b_0 \to S_1 + \rho) = I(e^* \cdot \eta_h)g^{S_1}_{M^b_0, S_1 \rho} + I\left[(\eta \cdot q_t)(e^* \cdot q_t) - \frac{1}{3}(\eta \cdot \eta^*_t)q_t^2\right]g^{d_1}_{M^b_0, S_1 \rho}, \]  
(A13)

\[ \mathcal{M}(M^b_0 \to S_0 + \rho) = 2Ii\eta_{\alpha\alpha_2} e^{\alpha_1 \alpha_2} q_t^2 g^{d_2}_{M^b_2, S_0 \rho}, \]  
(A14)

\[ \mathcal{M}(M^b_2 \to S_1 + \rho) = 2Ii\eta_{\alpha\alpha_2} \left[ e^{\alpha_1 \alpha_2} q_t^2 - \frac{1}{3} g^{\alpha_1 \alpha_2} (e^* \cdot \epsilon^*_t) \right]g^{d_1}_{M^b_2, S_1 \rho}, \]  
\[ + 2Ii\eta_{\alpha\alpha_2} \left[ e^{\alpha_1 \alpha_2} q_t^2 (e^* \cdot q_t) - \frac{1}{3} g^{\alpha_1 \alpha_2} (e^* \cdot q_t) \right]g^{d_1}_{M^b_2, S_1 \rho}, \]  
(A15)

\[ \mathcal{M}(T^b_1 \to H_0 + \rho) = I(\eta \cdot \eta^*_t)g^{T^b_1}_{H_0, H_0 \rho} + I\left[(\eta \cdot q_t)(e^* \cdot q_t) - \frac{1}{3}(\eta \cdot \eta^*_t)q_t^2\right]g^{d_1}_{T^b_1, H_0 \rho}, \]  
(A20)

\[ \mathcal{M}(T^b_1 \to H_1 + \rho) = I(\eta \cdot \eta^*_t)g^{T^b_1}_{H_1, H_1 \rho} + I\left[(\eta \cdot q_t)(e^* \cdot q_t) - \frac{1}{3}(\eta \cdot \eta^*_t)q_t^2\right]g^{d_1}_{T^b_1, H_1 \rho}, \]  
(A21)

\[ \mathcal{M}(T^b_0 \to H_0 + \rho) = I(e^* \cdot \eta_h)g^{T^b_0}_{H_0, H_0 \rho} + I\left[(\eta \cdot q_t)(e^* \cdot q_t) - \frac{1}{3}(\eta \cdot \eta^*_t)q_t^2\right]g^{d_1}_{T^b_0, H_0 \rho}, \]  
(A22)

\[ \mathcal{M}(T^b_0 \to H_1 + \rho) = I(\eta \cdot \eta_h)g^{T^b_0}_{H_1, H_1 \rho} + I\left[(\eta \cdot q_t)(e^* \cdot q_t) - \frac{1}{3}(\eta \cdot \eta^*_t)q_t^2\right]g^{d_1}_{T^b_0, H_1 \rho}, \]  
(A23)
The definitions of the distribution amplitudes of the $\rho$ meson light-cone distribution amplitudes.

\[ M(T^h_1 \to H_1 + \rho) = I e^{\eta \cdot e^\nu} g_{T^h_1 H_1 \rho} + I [\eta \cdot T^h_1 (e^* \cdot q) - \frac{1}{3} \eta \cdot e^\nu (\eta \cdot q)] g_{T^h_1 H_1 \rho} \]
\[ + I [\eta \cdot e^\nu (q_\perp \cdot e^\nu) + e^\nu \cdot e^\nu (q_\perp \cdot \eta)] g_{T^h_1 H_1 \rho} \]
\[ M(T^h_1 \to S_0 + \rho) = I e^{\eta \cdot q} g_{T^h_1 S_0 \rho} \]
\[ M(T^h_1 \to S_1 + \rho) = I [\eta \cdot T^h_1 (e^* \cdot q) - \frac{1}{3} \eta \cdot q \cdot e^\nu] g_{T^h_1 S_1 \rho} \]
\[ + I \left[ \left( \eta \cdot (q_\perp \cdot e^\nu) + \frac{1}{2} (q_\perp \cdot \eta) (e^* \cdot e^\nu) - \frac{2}{3} (e^* \cdot \eta) (e^* \cdot q) \right) g_{T^h_1 S_1 \rho} \right] \]
\[ M(T^h_2 \to H_0 + \rho) = I \eta_{\alpha_1 \alpha_2} (e^{\alpha_1 e^\nu} q_\perp \alpha_2 + e^{\alpha_2 e^\nu} q_\perp \alpha_1) g_{T^h_2 H_0 \rho} \]
\[ M(T^h_2 \to H_1 + \rho) = I \eta_{\alpha_1 \alpha_2} \left[ \eta \cdot e^\nu (\eta \cdot q) - \frac{1}{3} g_{T^h_2 H_1 \rho} \right] \]
\[ + I \eta_{\alpha_1 \alpha_2} \left[ \left[ \eta \cdot e^\nu (\eta \cdot q) - \frac{1}{3} g_{T^h_2 H_1 \rho} \right] \right] \]
\[ M(T^h_2 \to S_0 + \rho) = I \eta_{\alpha_1 \alpha_2} \left[ \eta \cdot e^\nu (\eta \cdot q) - \frac{1}{3} g_{T^h_2 S_0 \rho} \right] \]
\[ + I \eta_{\alpha_1 \alpha_2} \left[ \left[ \eta \cdot e^\nu (\eta \cdot q) - \frac{1}{3} g_{T^h_2 S_0 \rho} \right] \right] \]
\[ M(T^h_2 \to S_1 + \rho) = I \eta_{\alpha_1 \alpha_2} \left[ \eta \cdot e^\nu (\eta \cdot q) - \frac{1}{3} g_{T^h_2 S_1 \rho} \right] \]
\[ + I \eta_{\alpha_1 \alpha_2} \left[ \left[ \eta \cdot e^\nu (\eta \cdot q) - \frac{1}{3} g_{T^h_2 S_1 \rho} \right] \right] \]

Appendix B: The definitions of the $\rho$ meson light-cone distribution amplitudes.

The definitions of the distribution amplitudes of the $\rho$ meson used in the text read as

\[ \langle 0 | \bar{u}(z) \gamma_\mu d(-z) | \rho^-(P, \lambda) \rangle = \int \rho \cdot m_\rho \left[ \rho \cdot \left( \gamma_\mu \cdot z \right) \right] \int_0^1 d u e^{i \xi p \cdot x} \varphi_\parallel(u, \mu^2) + e^{i \lambda \mu} \int_0^1 d u e^{i \xi p \cdot x} g_\perp(u, \mu^2) \]

\[ - \frac{1}{2} \rho \cdot \left( \gamma_\mu \cdot z \right) \int_0^1 d u e^{i \xi p \cdot x} g_3(u, \mu^2) \]

\[ \langle 0 | \bar{u}(z) \gamma_5 d(-z) | \rho^-(P, \lambda) \rangle = \frac{1}{2} \int \rho \cdot m_\rho \int_0^1 d u e^{i \xi p \cdot x} g_\parallel(u, \mu^2) \]

\[ \langle 0 | \bar{u}(z) \sigma_\mu d(-z) | \rho^- (P, \lambda) \rangle = i f_\rho \int \left[ e^{i \lambda \mu} p_\nu - e^{i \lambda \nu} p_\mu \right] \int_0^1 d u e^{i \xi p \cdot x} \varphi_\perp(u, \mu^2) \]

\[ + (p_\mu z_\nu - p_\nu z_\mu) \gamma_\nu \int_0^1 d u e^{i \xi p \cdot x} h_\parallel(u, \mu^2) \]

\[ \langle 0 | \bar{u}(z) \gamma_\mu d(-z) | \rho^-(P, \lambda) \rangle = \int \rho \cdot m_\rho \int_0^1 d u e^{i \xi p \cdot x} \varphi_\parallel(u, \mu^2) \]

\[ - \frac{1}{2} \rho \cdot \left( \gamma_\mu \cdot z \right) \int_0^1 d u e^{i \xi p \cdot x} g_\perp(u, \mu^2) \]

\[ - \frac{1}{2} \rho \cdot \left( \gamma_\mu \cdot z \right) \int_0^1 d u e^{i \xi p \cdot x} g_3(u, \mu^2) \]

\[ \langle 0 | \bar{u}(z) \gamma_5 d(-z) | \rho^-(P, \lambda) \rangle = \frac{1}{2} \int \rho \cdot m_\rho \int_0^1 d u e^{i \xi p \cdot x} g_\parallel(u, \mu^2) \]

\[ \langle 0 | \bar{u}(z) \sigma_\mu d(-z) | \rho^- (P, \lambda) \rangle = i f_\rho \int \left[ e^{i \lambda \mu} p_\nu - e^{i \lambda \nu} p_\mu \right] \int_0^1 d u e^{i \xi p \cdot x} \varphi_\perp(u, \mu^2) \]

\[ + (p_\mu z_\nu - p_\nu z_\mu) \gamma_\nu \int_0^1 d u e^{i \xi p \cdot x} h_\parallel(u, \mu^2) \]
\[
\phi \equiv \frac{1}{2} (e^{(\lambda)}_{\perp \nu} - e^{(\lambda)}_{\perp \mu}) \frac{m_{\rho}^2}{p \cdot z} \int_0^1 du e^{i\xi p \cdot z} h_3(u, \mu^2),
\]

\[
(0|\bar{u}(z)d(-z)|\rho^- (P, \lambda)) = -if_{\rho}^T (e^{(\lambda)}_{\perp} z) m_{\rho}^2 \int_0^1 du e^{i\xi p \cdot z} h_3^{(t)}(u, \mu^2).
\]  

(B1)

The distribution amplitudes \( \varphi_\parallel \) and \( \varphi_\perp \) are of twist-2, \( g_\perp^{(v)} \), \( g_\parallel^{(a)} \), \( h_\parallel^{(x)} \) and \( h_\parallel^{(t)} \) are twist-3 and \( g_3 \), \( h_3 \) are twist-4. All functions \( \phi = \{ \varphi_\parallel, \varphi_\perp, g_\perp^{(v)}, g_\parallel^{(a)}, h_\parallel^{(x)}, h_\parallel^{(t)}, g_3, h_3 \} \) are normalized to satisfy \( \int_0^1 du \phi(u) = 1 \).

The 3-particle distribution amplitudes of the \( \rho \) meson are defined as \([12, 13]\):

\[
(0|\bar{u}(z)gG_{\mu\nu}\gamma_\alpha\gamma_5 d(-z)|\rho^- (P, \lambda)) = f_\rho m_\rho p_\alpha [p_\mu e^{(\lambda)}_{\perp \mu} - p_\mu e^{(\lambda)}_{\perp \nu}] A(v, p z)
\]

\[
+ f_\rho m_\rho^2 e^{(\lambda)}_{\perp \nu} \frac{z}{p z} [p_\mu g_{\nu\alpha} - p_\nu g_{\alpha\mu}] \Phi(v, p z)
\]

\[
+ f_\rho m_\rho^2 e^{(\lambda)}_{\perp \nu} \frac{z}{(p z)^2} p_\alpha [p_\mu z_{\nu} - p_\nu z_{\mu}] \Psi(v, p z),
\]

\[
(0|\bar{u}(z)gG_{\mu\nu}i\gamma_\alpha d(-z)|\rho^- (P)) = f_\rho m_\rho p_\alpha [p_\mu e^{(\lambda)}_{\perp \mu} - p_\mu e^{(\lambda)}_{\perp \nu}] V(v, p z)
\]

\[
+ f_\rho m_\rho^2 e^{(\lambda)}_{\perp \nu} \frac{z}{p z} [p_\mu g_{\nu\alpha} - p_\nu g_{\alpha\mu}] \Phi(v, p z)
\]

\[
+ f_\rho m_\rho^2 e^{(\lambda)}_{\perp \nu} \frac{z}{(p z)^2} p_\alpha [p_\mu z_{\nu} - p_\nu z_{\mu}] \Psi(v, p z),
\]

\[
(0|\bar{u}(z)\sigma_{\alpha\beta}gG_{\mu\nu}(v z)d(-z)|\rho^- (P, \lambda)) = f_\rho^T m_\rho^3 e^{(\lambda)}_{\perp \nu} \frac{z}{2(p z)} [p_\alpha p_\mu g^{(\lambda)}_{\alpha\nu} - p_\beta p_\mu g^{(\lambda)}_{\beta\nu} - p_\alpha p_\nu g^{(\lambda)}_{\beta\mu} + p_\beta p_\nu g^{(\lambda)}_{\alpha\mu}] T(v, p z)
\]

\[
+ f_\rho^T m_\rho^2 [p_\mu e^{(\lambda)}_{\perp \mu} g^{(\lambda)}_{\nu\alpha} - p_\beta e^{(\lambda)}_{\perp \mu} g^{(\lambda)}_{\beta\alpha} + p_\alpha e^{(\lambda)}_{\perp \nu} g^{(\lambda)}_{\beta\alpha} + p_\beta e^{(\lambda)}_{\perp \nu} g^{(\lambda)}_{\alpha\mu}] T_1(v, p z)
\]

\[
+ f_\rho^T m_\rho^2 [p_\mu e^{(\lambda)}_{\perp \mu} g^{(\lambda)}_{\nu\alpha} - p_\alpha e^{(\lambda)}_{\perp \nu} g^{(\lambda)}_{\beta\alpha} + p_\beta e^{(\lambda)}_{\perp \nu} g^{(\lambda)}_{\alpha\mu}] T_2(v, p z)
\]

\[
+ \frac{f_\rho^T m_\rho^2}{p z} [p_\alpha p_\mu e^{(\lambda)}_{\alpha\beta} z_{\nu} - p_\beta p_\mu e^{(\lambda)}_{\beta\alpha} z_{\nu} - p_\alpha p_\nu e^{(\lambda)}_{\alpha\beta} z_{\mu} + p_\beta p_\nu e^{(\lambda)}_{\beta\alpha} z_{\mu}] T_3(v, p z)
\]

\[
+ \frac{f_\rho^T m_\rho^2}{p z} [p_\alpha p_\mu e^{(\lambda)}_{\alpha\beta} z_{\nu} - p_\beta p_\mu e^{(\lambda)}_{\beta\alpha} z_{\nu} - p_\alpha p_\nu e^{(\lambda)}_{\alpha\beta} z_{\mu} + p_\beta p_\nu e^{(\lambda)}_{\beta\alpha} z_{\mu}] T_4(v, p z),
\]

\[
(0|\bar{u}(z)gG_{\mu\nu}(v z)d(-z)|\rho^- (P, \lambda)) = if_\rho^T m_\rho^2 [e^{(\lambda)}_{\perp \mu} p_\nu - e^{(\lambda)}_{\perp \nu} p_\mu] S(v, p z),
\]

\[
(0|\bar{u}(z)i gG_{\mu\nu}(v z)\gamma_5 d(-z)|\rho^- (P, \lambda)) = if_\rho^T m_\rho^2 [e^{(\lambda)}_{\perp \mu} p_\nu - e^{(\lambda)}_{\perp \nu} p_\mu] S(v, p z).
\]  

(B2)

The distribution amplitudes \( A, V \) and \( T \) are of twist-3 and the other 3-particle distribution amplitudes are of twist-4.