An intuitive protocol for polarization-entanglement restoral of quantum dot photon sources with non-vanishing fine-structure splitting

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Generation of polarization-entangled photons from quantum dots via the biexciton-exciton recombination cascade is complicated by the presence of an energy splitting between the intermediate excitonic levels, which severely degrades the quality of the entangled photon source. In this paper we present a novel, conceptually simple and straightforward proposal for restoring the entanglement of said source by applying a cascade of time-dependent operations on the emitted photons. This is in striking contrast with the techniques usually employed, that act on the quantum emitter itself in order to remove the fine structure splitting at its root. The feasibility of the implementation with current technology is discussed, and the robustness of the proposed compensation scheme with respect to imperfections of the experimental apparatus is evaluated via a series of Monte Carlo simulations.

Quantum dots (QDs) have emerged as one of the most promising candidates for on-demand generation of non-classical light, due to their ability to generate single photons1–3 and even entangled ones4–7, especially via the biexciton-exciton recombination cascade mechanism8.

The presence of a fine structure splitting (FSS) between the intermediate excitonic levels9,10 has been for years the most severe obstacle in the generation of high quality polarization-entangled photon pairs: in a textbook example of Noether's theorem, the degeneracy of the levels is promptly lifted not only due to shape anisotropy but even because of disorder in the alloy of the dot, or strain and piezoelectrically induced electric fields which reduce the symmetry of the confinement potential11. This causes a residual FSS to appear even in highly symmetrical dots12, in spite of significant improvements in growth techniques that helped to reduce it significantly in recent years13–17.

Several approaches have been proposed to tackle this issue at its root18, i.e. relying on the application of external tuning mechanisms in the form of the electric19 and optical Stark effect20, and magnetic21 or strain22–24 fields in order to restore the symmetry and remove the source of the problem, or by annealing of the quantum dot to mitigate the severity of the effect25. While several attempts have been made to integrate the most successful of these solutions on semiconductor platforms in order to achieve compact and scalable entangled photon sources24, this usually implies a significant increase in fabrication complexity, a limited yield in terms of working dots and devices, or brings in new problems such as the quenching of the photoluminescence when electric field tuning is employed.

Most of these attempts undertaken to restore the quality of the emitted entangled photons, however, start from the assumption that the degradation of the entanglement properties of the source is an irreversible process, and thus focus on removing the FSS. A-posteriori solutions to mitigate FSS effects after the photons have been emitted are possible, but only a few proposal in this sense have emerged so far26–31 and, to our knowledge, none of them has been experimentally implemented at the time of this paper being written. Moreover, it looks like most of these proposals start by identifying the energy difference of the photons, inherited by the excitonic FSS of the quantum dot, as the issue to tackle: the underlying assumption is that such a difference makes the two possible decay channels of the biexciton-exciton cascade distinguishable6,28,32,33, thus breaking quantum superpositions...
Both phase terms are compensated by the operation of the excitonic part of the wavefunction, if the $t_1 = t_2 = t_{\text{prop}}$, as shown in Fig. 1. If the emitter is populated with a biexciton, it will always emit entangled photon pairs, the state each pair would be entangled to will be a (different) random time $t_{\text{prop}}$. The random nature of the interval $t_{\text{prop}}$ is known to remain coherent for several nanoseconds.
As discussed before, most approaches present in the literature have focused on shifting the energy of the two polarization states of the emitted photons so that they become degenerate after the emission. The approach we hereby propose however, builds directly on the theoretical framework developed by Stevenson and that we have recalled, and starts by considering the difference in phase and not in energy between the two excitonic photon states as the issue to tackle.

Results and discussion
Determinations of the conditions for entanglement restoral. Let’s assume that both the bieexciton and the exciton photons fly across a device which has the ability to impart a phase which is both different depending on the polarization and wavelength and changing linearly in time, so that we can write the phases introduced on either polarization of the photons as:

\[
\Phi_{VX}(t) = K_{VX} \ast t + \Phi_{VX}^0
\]

\[
\Phi_{HX}(t) = K_{HX} \ast t + \Phi_{HX}^0
\]

with the various K representing the slopes of the introduced phases for a given polarization and wavelength of a photon and the \(\Phi^0\) terms indicating the constant (that is, time independent) phases introduced by the compensation system.

In general, if we assume the zero of our time scale to correspond with the initialization of the biexciton, the state resulting from such an operation can be written as a function of the time intervals \(t_1\) and \(t_2\) (with \(t_1\) and \(t_2\) being the random times before the emission of the biexciton and of the exciton photons, respectively) as:

\[
\begin{align*}
&\left| H_{XX} > e^{i\Phi_{VX}(t_1 + \frac{t_2}{2})} \otimes H_{X} > e^{i\Phi_{HX}(t_1 + t_2 + \frac{t_2}{2})} + |V_{XX} > e^{i\Phi_{VX}(t_1 + t_2 + \frac{t_2}{2})} \otimes V_{X} > e^{i\Phi_{HX}(t_1 + t_2 + \frac{t_2}{2})} \right| \right| e^{iFSS/2}/\hbar
\end{align*}
\]

where \(t_{prop}^{XX}\) (prop being short for propagation) is the time of flight of the biexciton (exciton) photon from the quantum dot to the entrance of the compensation system, and \(t_{Start}^{XX}\) is the time at which the ramping of the different phases begins. With our choice of the zero of the time scale, we will thus have that if \(t_{prop}^{XX} = t_{Start}^{XX}\), the phase ramp for the biexciton photon will start when an XX photon emitted at \(t = 0\) (immediately after the dot has been excited) will reach the compensation system.

Substituting relations (3) and collecting all terms we can get an overall state of the form:

\[
\begin{align*}
&\left| H_{XX} > e^{i\Phi(t)} \otimes H_{X} > e^{i\Phi(t_1 + t_2 + \frac{t_2}{2})} + |V_{XX} > e^{i\Phi(t_1 + t_2 + \frac{t_2}{2})} \right| \end{align*}
\]

If \(t_{Start}^{XX} = t_{prop}^{XX}\), the phase term no longer depends on random variables \(t_1\) and \(t_2\) can be straightforwardly determined:

\[
\begin{align*}
K_{VX} - K_{Hx} &= -\frac{FSS}{\hbar} \\
(K_{VX} - K_{Hx}) &= -(K_{VX} - K_{Hx})
\end{align*}
\]

If these conditions are met, the final 2 photon state resulting from the quantum dot emission after the compensation system will always be the same for each biexciton-exciton photon pair, as all of the other terms depend only on experimental parameters, such as the time of flight, that are assumed to remain constant over time: this implies that this residual constant phase term does not infringe on the resulting degree of entanglement, but merely changes the pure state all the photons pair will be entangled to, and can be easily compensated for using an additional static (i.e. non-time-dependent) quantum gate.

Figure 1 provides a graphical and intuitive representation of the workings of the scheme we are proposing and of the conditions we have derived, in an ideal case where the other phase terms are simply assumed to be zero and the ramps are perfectly synchronized (these hypothesis are not necessary for the scheme to work, but merely simplify our analysis): condition (6) implies that the phase differences imparted between the two \(|H_{XX} H_{X}>\) and \(|V_{XX} V_{X}'>\) by the operation on the biexciton line and that on the exciton line are equal but opposite. Provided the differential phase ramps are synchronized, the differential phase introduced on the biexciton photon emitted at \(t_1\) will be perfectly compensated when the exciton photon, emitted at \(t_1 + t_2\), crosses the compensation system. Likewise, condition (5) will ensure that the phase introduced by the evolution of the state due to the FSS is compensated as well when the exciton photon crosses the compensation system we have described. This removes the randomness of the final state.

Notably, one important advantage of this differential compensation approach is its insensitivity to any jitter present in the system, including those related to the excitation scheme employed, and to the starting point of the ramps which, as discussed before, would only determine the final entangled state, but not the degree of entanglement (concurrence) of the source. This implies that sophisticated initialization schemes such as two-photon resonant excitation of the biexciton are not necessary for the protocol to work, and even the much deprecated non-resonant excitation would suffice, the jitter introduced by the feeding of the quantum dot with charges from its surrounding having no impact on the final outcome.

Moreover, since all photon pairs will be transformed to the target state and none discarded, the technique we are proposing is in principle a lossless one: a major improvement with respect to time-gating techniques.

We can also observe that from a quantum circuit perspective, the scheme hitherto proposed is equivalent to a cascade of two temporally-dependent phase gates, one operating on the biexciton and one operating on the exciton photon: in this model, the degradation of the entanglement due to the FSS is equivalent to the action of a phase gate \(R_{FSS}(t_2)\), which turns what would otherwise be a perfect Bell state into a random one, \(|\Phi(t_2)\rangle)\) If
the conditions we have derived are fulfilled, the sequential action of the two phase gates $R_{XX}(t_1)$ and $R_{X}(t_1 + t_2)$ would compensate the effect of the first phase gate, and result in the recovery of the original Bell state $|\Phi^+\rangle$.

It should be noted that the scheme (and subsequent proposed implementation) we are proposing are somewhat similar to the one suggested by Wang et al. in their paper\textsuperscript{30}. Their formalism, however, adopted a frequency domain approach, focused on the removal of the energy difference between the two excitonic photon, and neglected the effect of other, time-independent phase terms on the outcome state. We believe our approach to provide a more natural, elegant, physically complete, and simple to grasp picture. Notably, in view of its circuit model interpretation, our model could be easily applied to photonic quantum circuit design. Obviously, however, if one considers only frequency translation\textsuperscript{39}, the two can be seen as rather complementary: in fact, our solution will restore the energy degeneracy as well, and the two formalisms can be easily reconciled in the framework of signal theory using the concept of instantaneous angular frequency\textsuperscript{40}.

If we define the energy scale according to Fig. 2, we will have that:

$$\omega^V_X = \frac{E_X}{\hbar} + \frac{FSS}{2\hbar}$$
$$\omega^H_X = \frac{E_X}{\hbar} - \frac{FSS}{2\hbar}$$

(7)

For an angle modulated wave $s(t) = A \cos(\omega_c t + \theta(t)) = A \cos(\Phi(t))$, phase modulation will introduce an instantaneous angular frequency defined as $\omega_i(t) = \frac{d\Phi(t)}{dt}$, so that in our case, for a signal affected by the ramp, we will have:

$$\omega^V_X(t) = \omega^V_X + KV$$
$$\omega^H_X(t) = \omega^H_X + KH$$

(8)

And substituting Eq. 5 and 7 we will have that $\omega^V_X(t) - \omega^H_X(t) = 0$, which implies that the resulting photons will also be degenerate, and that the two approaches are consistent with each other. Indeed, the resulting state will be entangled in both polarization and energy, but we stress that while the two effects (of phase correction and frequency translation) cannot be decoupled, what is needed to restore polarization entanglement is exclusively the phase correction, and the frequency translation should be regarded in principle as a side effect. Our discussion in the final part of our manuscript (see Fig. 4) will help clarify that a simple frequency correction (ref 30) will not in general deliver a state whose fidelity to the ideal Bell state is unitary.

**Proposed implementation and feasibility analysis**

In deriving the conditions that allow to restore the entanglement of the source, we have not made any assumption on the nature of the system that is supposed to implement them. It is quite obvious however that a natural realization of such a scheme could be based on the Pockels effect, as was also proposed in Ref.\textsuperscript{30}; a birefringent crystal would introduce a phase difference between polarizations aligned along the two optical axes, and the application of a transversal external electric field would allow to tune such a phase difference and change it in time by modifying the material's index ellipsoid: electro-optical phase modulators are indeed devices designed for this specific purpose, and that could be employed to implement our proposal.

For instance, in the most widely used material, LiNbO\textsubscript{3}, the relationship between the applied field and the variation of the refractive index is described by the tensor\textsuperscript{41}:

![Graphical representation of the excitonic levels and fine-structure splitting. The zero in the energy scale is assumed to be the ground state.](image-url)
So that variations of the refractive index with the application of an external electric field can be written, in first approximation, as:

\[
\text{Application of an external field in the } z \text{ direction will affect the ordinary and extraordinary refractive indexes in a different way, owing to the different terms involved in the tensor product, and thus allow for an externally controlled (and potentially temporally-varying) phase gate.}
\]

Once the FSS, which is a property of the dot and can be measured experimentally is known, the relationship between the slope of the voltage ramp and the FSS can be easily determined in order to fulfill condition (5), while condition (6) can in turn be easily fulfilled by employing two phase modulators operated with opposite voltage ramps, one affecting the biexciton photon while the other affects the exciton, as shown in Fig. 3.

A travelling wave, velocity matched phase modulator is conveniently characterized by its half-wave voltage \( V_{\pi} \) that is, the value of the applied voltage that induces a phase shift of \( \pi \) on a wave travelling through it. As the electro-optic coefficients are different for the two axes, we can denote with \( V_{V,}\pi \) and \( V_{H,}\pi \) the values of \( V_{\pi} \) for \( V- \) and \( H \)-polarized wave respectively. The induced phase shift of a photon wave packet entering the modulator at time \( t \) for either polarization can then be easily written as

\[
\phi_{V,H}(t) = V_{V,H}(t) \cdot \frac{\pi}{V_{V,H}}.
\]

If the voltage is varying linearly in time, we can calculate the slope of induced phase by trivially taking the time derivative of this expression, and easily rewrite Eq. 5 as:

\[
\frac{dV}{dt} \left( \frac{1}{V_{V,\pi}} - \frac{1}{V_{H,\pi}} \right) \cdot \pi = -\frac{\text{FSS}}{\hbar}
\]

to determine the slope of the voltage ramp.

If for instance we consider again a \( z \)-cut \( \text{LiNbO}_3 \) integrated EOMs, and assume a \( V_{\pi} \) of 3 Volts for the TM mode and a 3 times larger \( V_{\pi} \) for the TE modes, we will get a slope of the voltage ramp \( \frac{dV}{dt} \) of around 2 V/(ns*\( \mu \)eV), which appears to be within the capabilities of modern high frequency EOMs if only a FSS of a few \( \mu \)eV has to be corrected, as it customarily found in dots grown on high symmetry substrates\(^{13,14}\). Our protocol would allow to employ these as entangled photon sources for quantum communication without requiring active techniques for the zeroing of the FSS, such as the application of strain fields, which are usually demanding in terms of fabrication complexity.

Moreover, while the estimation we have provided implies that the proposed scheme can be implemented with currently available technology, advances in electro-optic material science\(^{42}\) and integration\(^{43}\) can pave the way for the development of compact quantum photonic circuits that perform a similar task and push the performance much further.

**Sensitivity to imperfections of the compensation scheme.** In order to estimate the robustness of our approach to imperfections of the setup, a simple Monte Carlo simulation was performed, generating density matrices for different values of the mismatches.

![Figure 3. Proposed implementation of the compensation scheme using two electro-optic modulators operated in a parallel configuration, as also envisioned by Wang et al.](image-url)
This was done by first of all assuming that $t_{XX}^{\text{prop}} = t_{XX}^{\text{start}}$ and $t_{X}^{\text{prop}} = t_{X}^{\text{start}}$, that is, a perfect syncing of the start of the ramp with the laser pulse, and by ignoring every other constant phase term. By creating random values for variables $t_1$ and $t_2$, a large number of density matrixes was generated and averaged until a suitable convergence was reached. In all of these simulations we have assumed a lifetime of the exciton state $X$ of 1 ns, and half of that for the biexciton state, as it is often the case for III-V semiconductor quantum dots, and the FSS was chosen to be 3 µeV (corresponding to a spin precession of around 4.6 Rad/ns).

Figure 4 shows the behavior of the concurrence of the resulting density matrixes and their fidelity to the Bell state $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|HH\rangle + |VV\rangle)$ as a function of the errors $\Delta \omega_1$ and $\Delta \omega_2$: it can be observed that $\Delta \omega_2$ has a less severe impact due to the term being weighted by the random variable $t_1$, which is related to the shorter lifetime of the biexciton. For comparison, without the compensation scheme applied the same quantum dot would exhibit a fidelity and concurrence of 0.52 and 0.21 respectively. It can be also noticed that while $t_1$, the time that passes between the excitation of the quantum dot and the emission of the biexciton photon, plays no role in the level of the entanglement without our compensation scheme, it becomes important when the latter is applied: this is due to the differential phase gates being dependent on $t_1$ and $t_1 + t_2$.

In order to also understand the impact of the constant phase term, a new series of simulations was performed. In this case, conditions to perfectly restore the entanglement of the state have been assumed to be fulfilled ($\Delta \omega_1 = \Delta \omega_2 = 0$), and concurrence and fidelity to the states $|\Phi^+\rangle$ and $|\Phi^-\rangle$ were calculated as a function of the time delay $\delta t = (t_{XX}^{\text{prop}} - t_{XX}^{\text{start}}) - (t_{X}^{\text{prop}} - t_{X}^{\text{start}})$, again for a quantum dot having a 3 µeV FSS.

As can be seen from Fig. 5, this has no effect on the level of concurrence, and thus on the degree of entanglement, but it can be exploited to finely tune the final state which is generated by the application of the compensation scheme.

In conclusion, we proposed a conceptually simple and intuitive way to compensate the effects of FSS on entanglement quality.

Conclusions
The scheme we have discussed for restoring the entanglement of photons emitted from quantum dots using the biexciton-exciton recombination cascade appears to be quite robust, flexible and powerful, and it appears to be possible to implement it with currently available technology. As the implementation we have suggested is based on the repurposing of devices already widely employed in the (classical) field of information technology and telecommunications, it represents an elegant solution that could significantly lower infrastructural costs associated with a "quantum internet". This is especially true since in principle no photon losses are introduced by our approach, which would result result in a higher bitrate. Moreover, by introducing the paradigm shift of a-posteriori compensation of the effect of the FSS, constraints on the sources that could be employed for such a purpose are significantly relaxed, and fabrication of devices integrating semiconductor quantum dots as entangled photon sources for quantum information processing are simplified.

Methods
Monte Carlo simulations were performed using a custom-written Python code. Large numbers of density matrixes were generated and averaged until a suitable convergence was reached, defined as a less than $10^{-6}$ relative change in the matrix elements from the previous iteration.

Fidelity of the converged density matrixes to any Bell state was calculated as:

$$\Delta \omega_1 = K_{V_X} - K_{H_X} - \frac{\text{FSS}}{\hbar}$$
$$\Delta \omega_2 = K_{V_{XX}} - K_{H_{XX}} + K_{V_X} - K_{H_X}$$

(11)

Figure 4. Fidelity (left) and Concurrence (right) of the state obtained after the compensation scheme for a dot with an FSS of 3 µeV, as a function of the errors $\Delta \omega_1$ and $\Delta \omega_2$. All of the other time-independent phase terms are assumed to be 1 for simplicity.
where we have used $\rho_B$ and $\rho$ to indicate the density matrix of the target Bell state and the one resulting from our simulation, respectively.

Concurrence was calculated using the standard formula:

$$C(\rho) = \max(0, \frac{1}{4} - \frac{\sqrt{\lambda_1}}{4} - \frac{\sqrt{\lambda_2}}{4} - \frac{\sqrt{\lambda_3}}{4} - \frac{\sqrt{\lambda_4}}{4})$$

With $\lambda_n$ being the square roots of the eigenvalues, in decreasing order, of the operator $R = \rho \Sigma \rho^\dagger \Sigma$, and

$$\Sigma = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

**Data availability**

Our simulations have been performed using a custom-written code in the 3.7 version of the Python language. The code uses the Scipy and Numpy Python libraries, and was run via the Anaconda Python distribution, with typical computation times of less than 2 h on a 3.4 GHz quad-core Intel i3 machine with 16 Gbyte of RAM. All code is available free of charge upon request.

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**References**

1. Arakawa, Y. & Holmes, M. J. Progress in quantum-dot single photon sources for quantum information technologies: A broad spectrum overview. *Appl. Phys. Rev.* 7, 21309 (2020).
2. Somaschi, N. et al. Near-optimal single-photon sources in the solid state. *Nat. Photonics* 10, 340–345 (2016).
3. Senellart, P., Solomon, G. & White, A. High-performance semiconductor quantum-dot single-photon sources. *Nat. Nanotechnol.* 12, 2 (2017).
4. Shields, A. J. Semiconductor quantum light sources. *Nat. Photonics* 1, 215–223 (2007).
5. Orieux, A., Versteegh, M. A. M., Jons, K. D. & Ducci, S. Semiconductor devices for entangled photon pair generation: A review. *Rep. Progr. Phys.* 80, 076001 (2017).
6. C. Arakawa, Y. & Holmes, M. J. Progress in quantum-dot single photon sources for quantum information technologies: A broad spectrum overview. *Appl. Phys. Rev.* 7, 21309 (2020).
7. Somaschi, N. et al. Near-optimal single-photon sources in the solid state. *Nat. Photonics* 10, 340–345 (2016).
8. Senellart, P., Solomon, G. & White, A. High-performance semiconductor quantum-dot single-photon sources. *Nat. Nanotechnol.* 12, 2 (2017).
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10. Orieux, A., Versteegh, M. A. M., Jons, K. D. & Ducci, S. Semiconductor devices for entangled photon pair generation: A review. *Rep. Progr. Phys.* 80, 076001 (2017).
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2. Somaschi, N. et al. Near-optimal single-photon sources in the solid state. *Nat. Photonics* 10, 340–345 (2016).
3. Senellart, P., Solomon, G. & White, A. High-performance semiconductor quantum-dot single-photon sources. *Nat. Nanotechnol.* 12, 2 (2017).
4. Shields, A. J. Semiconductor quantum light sources. *Nat. Photonics* 1, 215–223 (2007).
5. Orieux, A., Versteegh, M. A. M., Jons, K. D. & Ducci, S. Semiconductor devices for entangled photon pair generation: A review. *Rep. Progr. Phys.* 80, 076001 (2017).
6. Alipourian, N. et al. Entangled photon pairs from semiconductor quantum dots. *Phys. Rev. Lett.* 96, 130501 (2006).
7. Schwartz, I. et al. Deterministic generation of a cluster state of entangled photons. *Science* 354, 434–437 (2016).
8. Yamamoto, Y., Santori, C. & Pelton, M. Regulated and entangled photons from a single quantum dot. *Phys. Rev. Lett.* 84, 2513–2516 (2000).
9. Gong, M., Zhang, W., Guo, G. C. & He, L. Exciton polarization, Fine-structure splitting, and the asymmetry of quantum dots under uniaxial stress. *Phys. Rev. Lett.* 106, 227401 (2011).
10. Alipourian, N. et al. Entangled photon pairs from semiconductor quantum dots. *Phys. Rev. Lett.* 96, 130501 (2006).
11. Schwartz, I. et al. Deterministic generation of a cluster state of entangled photons. *Science* 354, 434–437 (2016).
12. Yamamoto, Y., Santori, C. & Pelton, M. Regulated and entangled photons from a single quantum dot. *Phys. Rev. Lett.* 84, 2513–2516 (2000).
13. Gong, M., Zhang, W., Guo, G. C. & He, L. Exciton polarization, Fine-structure splitting, and the asymmetry of quantum dots under uniaxial stress. *Phys. Rev. Lett.* 106, 227401 (2011).
14. Alipourian, N. et al. Entangled photon pairs from semiconductor quantum dots. *Phys. Rev. Lett.* 96, 130501 (2006).
15. Schwartz, I. et al. Deterministic generation of a cluster state of entangled photons. *Science* 354, 434–437 (2016).
16. Yamamoto, Y., Santori, C. & Pelton, M. Regulated and entangled photons from a single quantum dot. *Phys. Rev. Lett.* 84, 2513–2516 (2000).
17. Wang, Z., Rastelli, A., Kuroda, T. & Sanguinetti, S. Droplet epitaxy of semiconductor nanostructures for quantum photonic devices. *Nat. Mater.* 18, 799–810 (2019).
18. Basso Basset, F. et al. High-yield fabrication of entangled photon emitters for hybrid quantum networking using high-temperature droplet epitaxy. *Nano Lett.* 18, 505–512 (2018).
19. Chung, T. H. et al. Selective carrier injection into patterned arrays of pyramidal quantum dots for entangled photon light-emitting diodes. *Nat. Photonics* 10, 782–787 (2016).
17. Skiba-Szymanska, I. et al. Universal growth scheme for quantum dots with low fine-structure splitting at various emission wavelengths. Phys. Rev. B 80, 041313 (2011).
18. Plumhof, J. D., Trotta, R., Rastelli, A. & Schmidt, O. G. Experimental methods of post-growth-tuning of the excitonic fine structure splitting in semiconductor quantum dots. Nanoscale Res. Lett. 7, 336 (2012).
19. Kowalik, K. et al. Influence of an in-plane electric field on exciton fine structure in InAs-GaAs self-assembled quantum dots. Appl. Phys. Lett. 86, 041907 (2005).
20. Muller, A., Fang, W., Laval, J. & Solomon, G. S. Creating polarization-entangled photon pairs from a semiconductor quantum dot using the optical Stark effect. Phys. Rev. Lett. 103, 217402 (2009).
21. Stevenson, R. M. et al. A semiconductor source of triggered entangled photon pairs. Nature 439, 178–182 (2006).
22. Trotta, R. et al. Universal recovery of the energy-level degeneracy of bright excitons in ingaas quantum dots without a structure symmetry. Phys. Rev. Lett. 109, 1–5 (2012).
23. Martin-Sánchez, J. et al. Strain-tuning of the optical properties of semiconductor nanomaterials by integration onto piezoelectric actuators. Semicond. Sci. Technol. 33, 39 (2018).
24. Zhang, Y. et al. Monolithically integrated microelectromechanical systems for on-chip strain engineering of quantum dots. Nano Lett. 16, 5785–5791 (2016).
25. Ellis, D. J. P. et al. Control of fine-structure splitting of individual InAs quantum dots by rapid thermal annealing. Appl. Phys. Lett. 90, 011907 (2007).
26. Stace, T. M., Milburn, G. J., Milburn, G. I. & Barnes, C. H. W. Entangled two-photon source using biexciton emission of an asymmetric quantum dot in a cavity. Phys. Rev. Condens. Matter Mater. Phys. 67, 085317 (2003).
27. Jones, N. S. & Stace, T. M. Photon frequency-mode matching using acousto-optic frequency beam splitters. Phys. Rev. A At. Mol. Opt. Phys. 73, 033813 (2006).
28. Coub, W. A. & Gambetta, J. M. Entangled photons on demand: Erasing which-path information with sidebands. Phys. Rev. B Condens. Matter Mater. Phys. 80, 241303 (2009).
29. Zhou, Z. Q. et al. Phase compensation enhancement of photon pair entanglement generated from biexciton decay in quantum dots. Phys. Rev. A. At. Mol. Opt. Phys. 81, 063802 (2010).
30. Wang, X. B., Yang, C. X. & Liu, Y. B. On-demand entanglement source with polarization-dependent frequency shift. Appl. Phys. Lett. 96, 201103 (2010).
31. Fognini, A., Ahmadi, A., Daley, S. J., Reimer, M. E. & Zwiller, V. Universal finestructure eraser for quantum dots. Opt. Express 26, 24487–24496 (2018).
32. Meiron, E. A. et al. Distilling entanglement from random cascades with partial 'which path' ambiguity. Phys. Rev. A At. Mol. Opt. Phys. 77, 062310 (2008).
33. Planer, G., Seliger, M. & Hohenester, U. Entangled photon sources based on semiconductor quantum dots: The role of pure dephasing. Phys. Rev. B Condens. Matter Mater. Phys. 78, 195410 (2008).
34. Stevenson, R. M. et al. Evolution of entanglement between distinguishable light states. Phys. Rev. Lett. 101, 170501 (2008).
35. Juska, G. et al. Conditions for entangled photon emission from (111)B site-controlled pyramidal quantum dots. J. Appl. Phys. 117, 134302 (2015).
36. Coish, W. A. & Gambetta, J. M. Entangled photons on demand: Erasing which-path information with sidebands. Phys. Rev. B Condens. Matter Mater. Phys. 80, 241303 (2009).
37. Hudson, A. J. et al. Coherence of an entangled exciton-photon state. Phys. Rev. Lett. 99, 266802 (2007).
38. Ward, M. B. et al. Coherent dynamics of a telecom-wavelength entangled photon source. Nat. Commun. 5, 1–6 (2014).
39. Wu, Z. & Grier, A. Serrodyne frequency translation using time-modulated metasurfaces. IEEE Trans. Antennas Propag. 68, 1599–1606 (2020).
40. Alencar, M. S. & da Rocha, V. C. Angle modulation. In Communication Systems 197–224 (Springer, 2005). https://doi.org/10.1007/1-84628-7097-3_6.
41. Saleh, B. E. A. & Teich, M. C. Fundamentals of Photonics (John Wiley & Sons Inc, 1991).
42. Liu, J. et al. Recent advances in polymer electro-optic modulators. RSC Adv. 5, 15784–15794 (2015).
43. Luo, K. H. et al. Nonlinear integrated quantum electro-optic circuits. Sci. Adv. 5, 1415 (2019).

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Author contributions
S.V. conceived the idea and developed the code used to simulate the density matrices with assistance from G.J., E.P. supervised the work and assisted in writing the manuscript. All authors reviewed the manuscript.

Competing interests
The authors declare no competing interests.

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