Primordial black holes under the double inflationary power spectrum

Hee Il Kim
Basic Science Institute and Department of Physics, Sogang University, 121-742, Seoul, Korea

Recently, it has been shown that the primordial black holes (PBHs) produced by near critical collapse in the expanding universe have a scaling mass relation similar to that of black holes produced in asymptotically flat spacetime. Distinct from PBHs formed with mass about the horizon mass (Type I), the PBHs with the scaling relation (Type II) can be created with a range of masses at a given formation time. In general, only the case in which the PBH formation is concentrated at one epoch has been considered. However, it is expected that PBH formation is possible over a broad range of epochs if the density fluctuation has a rather large amplitude and smooth scale dependence.

In this paper, we study the PBH formation for both types assuming the power spectrum of double inflationary models in which the small scale fluctuations could have large amplitudes independent of the CMBR anisotropy. The mass spectrum of Type II PBHs is newly constructed without limiting the PBH formation period. The double inflationary power spectrum is assumed to be of double simple power-law which are smoothly connected. Under the assumed power spectrum, the accumulation of small PBHs formed at later times is important and the mass range is significantly broadened for both Types. The PBH mass spectra are far smoother than the observed MACHO spectrum due to our assumption of a smooth spectrum. In order to fit the observation, a more spiky spectrum is required.

97.60Lf, 98.80Cq

I. INTRODUCTION

The density fluctuations give an interesting way to form primordial black holes (PBHs) in the early universe. If the fluctuation amplitude of an overdense region is of order unity when the fluctuation enters into the cosmological horizon, the overdense region can evolve into a black hole. Early studies found that the PBHs so formed have mass about the horizon mass at the formation time (Type I) \([1,2]\). The PBHs can cover the complete mass range of black holes considered nowadays: from a mass of about the Planck mass \(\sim 2 \times 10^{-5} \text{g}\) to the mass of a black hole in the galactic bulge \(\sim 10^8 M_\odot\). The Hawking evaporation \([3]\) of PBHs with mass \(\lesssim 5 \times 10^{13} \text{g}\) can affect many early universe phenomena. In particular, PBHs with initial mass \(\approx 5 \times 10^{14} \text{g}\) would presently be at the final stage of their evaporation and could be observed through their energetic particle emission. Although PBHs have not yet been detected, the upper limits on the PBH number density have been extensively studied \([4,5]\). When the fluctuations are normalized to the CMBR anisotropy, PBH formation strongly constrains the spectral index of the density fluctuations, \(n \lesssim 1.25\) \([6,7]\). PBHs surviving today and the massive relics which may remain at the end of a PBH lifetime \([8]\) are natural candidates for dark matter. PBHs with mass \(\sim 0.5 M_\odot\) are also candidates for the observed massive compact halo objects (MACHOs) \([9,10]\). It has been proposed that PBHs with the MACHO mass can be produced in certain inflationary models \([11,12]\) or at the vanishing of the sound velocity at the cosmological quark-hadron phase transition \([13]\). If the MACHO PBHs form coalescing binaries, they could be detected by gravitational wave interferometers \([14]\).

On the other hand, critical phenomena have been exhibited in the gravitational collapse which produces a black hole. This was originally found by Choptuik in the numerical study of the gravitational collapse of massless scalar fields \([15]\). Especially, he found that for one parameter family of field configurations with controlling parameter \(p\), the black hole mass scales as \((p - p_c)^{\gamma_s}\) near and above the critical parameter \(p_c\) with a universal exponent \(\gamma_s \approx 0.37\). It has been shown subsequently that similar mass relations hold for other situations, such as gravitational waves \([16]\) and radiation fluid \([17]\). Interestingly, all the studies have found that \(\gamma_s \approx 0.37\). The analytic explanation has been given by perturbative analysis \([18]\).

Cosmological application of the critical phenomena was recently noticed by Niemeyer and Jedamzik in the study of the gravitational collapse of an overdense region in the expanding universe \([19,20]\). They found that if the horizon crossing amplitude \(\delta_H\) of the overdense region is near and above the critical amplitude \(\delta_H^c \approx 0.7\), then the black hole mass scales similarly, with \(\gamma_s \approx 0.36\) (Type II PBHs) \([21]\). The scaling relation can induce great changes in the mass spectrum because PBHs can form with a range of masses at a given time, while Type I PBHs have an initial mass of about the horizon mass at a given time. The mass spectrum of Type II PBHs has been studied for the case in which the PBHs form over a very short period \(\Delta H\) \([22,23]\), such as occurs for a blue-shifted density perturbation spectrum or the spiky spectrum associated with MACHO PBHs. Based on these mass spectra, the upper limits on the PBH number density and the spectral index have been revised \([24,25]\).

With normalization of the density fluctuation spectrum to the COBE CMBR anisotropy measurements, the PBHs can only form significantly under a simple power-law spectrum of the density fluctuations if the density fluctuation is largely blue-shifted with a fine-tuned spectral index \(\delta_\text{flat}^{1.7} < 0.2\). In this case, the PBH formation is
concentrated at the initial time $t_i$ when the fluctuation develops. Similarly, a narrow spiky fluctuation spectrum peaked at the MACHO scale is required for some MACHO PBH models [11,13,14]. Hence, it has been usually assumed that PBHs are formed at only one epoch or with only one fluctuation scale. For Type I PBHs this assumption leads to the $\delta$-function type mass spectrum [4]. For Type II PBHs, even if they are formed at one epoch, the PBHs can have a range of masses because of the scaling mass relation [21,23,24].

With rather large fluctuation amplitudes and a smooth power spectrum, the PBH formation is possible over a range of epochs (or fluctuation scales) and the accumulation of PBHs formed at different times should be considered. However, even in the models with large amplitude and smooth scale dependence [12], the accumulation effect has previously been neglected and the mass spectrum obtained by simply taking a short duration of PBH formation. With an assumption of large amplitude on small scales, we will investigate the accumulation effect in the mass spectrum of PBHs obtained based on the Press-Schechter method [26]. For this, we newly construct the mass spectrum which is more adequate to describe Type II PBHs formed over a range of epochs. Related work on the mass spectrum was done in Ref. [23] using the excursion set formalism, but that reference investigated the mass spectrum was done in Ref. [23] using the excursion set formalism, but that reference investigated the validity of the mass spectrum obtained by limiting the PBH formation epoch. The large amplitude power spectrum is natural in double inflationary models. In the double inflationary models, the small scale fluctuations could have large amplitude not constrained by the CMBR anisotropy, leading to enhanced formation of small PBHs. Though there are various double inflationary models, we will simply assume a double power-law spectrum. This kind of power spectrum has been studied in the double inflationary model of supergravity theory proposed to explain the MACHOs as PBHs [12]. From the mass spectrum of PBHs of both types, it will be shown that the PBH mass spectrum has difficulty explaining the observed MACHO spectrum with the assumed power spectrum of the density fluctuation.

The rest of the paper is organized as follows. In Sec. II, the PBHs of both types are reviewed. The mass spectra are formulated in Sec. III. PBH formation under the double-inflationary power spectrum is given in Sec. IV. The paper closes with some concluding remarks in Sec. V.

II. PBH FORMATION AND ITS MASS

We only consider the PBH formation in a universe with a hard equation of state for which the sound velocity $v_s = \sqrt{\gamma}$ with $0 < \gamma \lesssim 1$. Although we are interested in the radiation-dominated era (when $\gamma = 1/3$), we regard $\gamma$ as a parameter for convenience. For an overdense region in the fluctuated universe to collapse, the fluctuation amplitude of the region should be large enough to overcome the Jeans pressure of the region. Conversely, it should not be so large that the region is decoupled from the background universe. These conditions can be written for the horizon crossing amplitude $\delta_H$ as

$$\delta_H \leq \delta_H \leq \delta_H^c. \quad (1)$$

In addition, $\delta_H$ should be larger than the critical value needed for the trapped surface formation.

For Type I PBHs, it has been found from the study of the evolution of a spherical overdense region in the expanding universe that the bounds for the PBH formation, $\delta_H^c$ and $\delta_H$, are scale independent and should be of the order of $\gamma$. It has been usually taken that $\delta_H^c = \gamma$ and $\delta_H = 1$. Also, it has been assumed that the trapped surface forms with these bounds at nearly the horizon time. Then the PBH mass can be approximated, neglecting the slight dependence on the fluctuation amplitude, as [36]

$$M_{BH}(t) = \gamma^{3/2} M_H = \gamma^{3/2} M_H \left( \frac{t}{t_i} \right)$$

$$= \gamma^{3/2} / (1+3 \gamma) (1+3 \gamma) M_H^{(1+3 \gamma) M_i^{(1+3 \gamma)}} \quad (2)$$

where $M_H$ is the horizon mass, the subscript ‘$i$’ represents the quantity at $t_i$, the time when the density fluctuation develops, and $M \propto M_H^{3/2}$ is the mass contained in the overdense region with comoving wavenumber $k$ at $t_i$. Note that the mass of PBHs formed at a given time is fixed as given in Eq. (2). So, all the quantities, $M_{BH}, M_H, M_i$, and $k$ have one-to-one correspondences. Larger PBHs can form only if the larger fluctuations cross the horizon at later times.

Distinct from Type I PBHs, Niemeyer and Jedamzik investigated the PBH formation by numerically simulating the evolution of an overdense region in the expanding universe [2,24]. Applying the three different configurations of the overdense region, they found that the critical phenomena also exist in the PBH formation and the resulting PBH mass scales as

$$M_{BH} = kM_H (\delta_H - \delta_H^c)^{\gamma_s} \quad (3)$$

where $k \sim 3, \delta_H^c \approx 0.7$, and the critical exponent, $\gamma_s \approx 0.36$, is similar to that obtained in the asymptotically flat cases [26]. In this case, the PBH formation mass is not unique at a given time any more but covers a range of masses, in principle, from zero to infinity. However, we introduce the Planck mass $M_{Pl}$ as a minimum PBH mass. Though $\delta_H^c$ has not yet been determined for Type II PBHs, we take $\delta_H^c = 1$ as the upper limit for PBH formation. Actually, $\delta_H^c$ is not crucial in this work. Then the possible mass range for a given time $t$ is from $M_{Pl}$ to $kM_H(t)(\delta_H - \delta_H^c)^{\gamma_s} \sim 2M_H(t)$. Because the critical amplitude $\delta_H^c = 0.7$ is larger than that taken for Type I PBHs $\delta_H^c = 1/3$, one may suspect that Type I PBHs always form more abundantly than Type II PBHs. However, this is not so because the mass shift to smaller PBHs should be considered for Type II PBHs. The details are given in Sec. IV.
III. MASS SPECTRUM OF PBHS

The mass spectrum of PBHs can be constructed based on the Press–Shechter method [23]. To do this we start from the description of an overdense region at $t_i$. We only consider Gaussian fluctuations. For an overdense region with mass scale $M$ and size $R$, the smoothed density field $\delta_M$ is defined by

$$\delta_M(x) = \int d^3y \delta(x + y)W_M(y)$$

(4)

where $\delta(x) \equiv (\rho(x) - \bar{\rho})/\bar{\rho}$, $\bar{\rho}$ is the background energy density of the universe, and $W_M(x)$ is the smoothing window function of scale $M$. The dispersion $\sigma_M$, the standard deviation of the density contrast of the regions with $M$, is given by

$$\sigma_M^2 = \frac{1}{V_W} \langle \delta_M^2(x) \rangle = \frac{1}{V_W} \int \frac{d^3k}{(2\pi)^3} |\delta_k|^2 W_k^2(M)$$

(5)

where $V_W \sim R^3$ denotes the effective volume filtered by $W_M$, and $\delta_k$ and $W_k$ are the Fourier transforms of $\delta(x)$ and $W_R(x)$, respectively.

For Gaussian fluctuations, the probability that the region of size $R$ has density contrast in the range of $(\delta_i, \delta_i + d\delta_i)$ is

$$P(M, \delta_i)d\delta = \frac{1}{\sqrt{2\pi}\sigma_M} \exp\left(-\frac{\delta_i^2}{2\sigma_M^2}\right) d\delta_i$$

(6)

The fluctuation amplitudes at $t_i$ are related to the amplitudes at the horizon crossing time as follows,

$$\delta_i = \left(\frac{M}{M_{Hi}}\right)^{-\frac{3}{2}} \delta_H, \quad \sigma_M = \left(\frac{M}{M_{Hi}}\right)^{-\frac{3}{2}} \sigma_H$$

(7)

Since the smoothed density field under the window with size $R$ sees only the structures larger than $R$,

$$F(M, \delta_i) = \int_{\delta_c}^{\infty} P(M, \delta_i)d\delta_i$$

(8)

can be interpreted as the fraction of the overdense regions larger than $R$ and with $\delta_i > \delta_c$. And the fraction of the regions with $\delta_i > \delta_c$ in the range of $(M, M + dM)$ is given by

$$f(M, \delta_c)dM = -\frac{\partial F}{\partial M}dM$$

(9)

For Type I PBHs, from Eq. (1) and (7), the condition for PBH formation can be written for $\delta_i$ as [14]

$$\alpha \equiv \gamma \left(\frac{M_i}{M_{Hi}}\right)^{-\frac{3}{2}} \leq \delta_i \leq \left(\frac{M_i}{M_{Hi}}\right)^{-\frac{3}{2}} \equiv \beta$$

(10)

From the PBH formation condition, the fraction of overdense regions which will evolve to PBHs is $f_{PBH}(dM = f(M, \alpha)dM - f(M, \beta)dM) \simeq f(M, \alpha)dM$. Multiplying by $\bar{\rho}_i/M$, this fraction can be converted to the number density. After a change of variables, the initial mass spectrum of PBHs in the range of $(M_{BH}, M_{BH} + dM_{BH})$ is given by [27]

$$n_{BH}(M_{BH})dM_{BH} = \frac{2}{\sqrt{\pi}} \left(\frac{\bar{\rho}_i}{M_{Hi}}\right) \left(\frac{M_{BH}}{M_{Hi}}\right)^{-\frac{3}{2}} \times$$

$$\left[\frac{M_{BH}}{\sigma_H} \frac{\partial \sigma_H}{\partial M_{BH}} - \frac{1}{\sigma_H} \right] \exp\left(-\frac{\gamma^2}{2\sigma_H^2}\right) \frac{dM_{BH}}{M_{BH}}$$

(11)

Here a factor of 2 is included in accordance with the Press–Shechter prescription for normalization [23]. The number density of all the PBHs at time $t$ is then given by

$$\beta_BH(t) = \left(\frac{a(t)}{a(t_i)}\right)^{-3} \int_{M_{BH1}}^{M_{BH2}} n_{BH}(M_{BH})dM_{BH}$$

(12)

where $a(t)$ is the cosmic scale factor, $M_{BH2} = \gamma^{3/2} M_{BH1}(t)$, and $M_{BH1} = M_{Hi}$ if $M_*(t) \approx 10^9(t/\text{sec})^{1/3} \text{g} < M_{Hi}$ where $M_*(t)$ is the PBH mass whose lifetime is $t$, else $M_{BH1} = M_*(t)$. If PBHs leave stable massive relics at the end of their evaporation lifetimes, then $M_{BH1} = M_{Hi}$.

If the power spectrum can be approximately treated as a $\delta$-function type as in the blue-shift perturbation with the CMBR normalization or the spiky spectrum in some inflationary models, it is usual to assume that the PBH formation occurs at only one epoch. In this case, the PBHs form with only one mass for Type I PBHs, and the initial number density is approximately expressed by [14]

$$\beta_BH(t) = \sigma_H \exp\left(-\frac{\gamma^2}{2\sigma_H^2}\right)$$

(13)

For Type II PBHs, even if the power spectrum resembles a $\delta$-function, the PBHs can form with a range of masses from about $M_{Hi}$ to $2M_{Hi}(t)$. Their mass spectrum can be obtained from the probability distribution of $\delta_i$ (or $\delta_H$), Eq. (6), as follows,

$$n_{BH}(M_{BH})dM_{BH} = \frac{1}{\sqrt{2\pi}\sigma_H^3} \bar{\rho}_i \left(\frac{M_{BH}}{M_{Hi}}\right)^{-3/2} \times$$

$$\left(\frac{M_{BH}}{kM_H}\right)^{1/\gamma_s} \exp\left(-\frac{\delta_H^2}{2\sigma_H^2}\right) \frac{dM_{BH}}{M_{BH}}$$

(14)

where, from Eq.(3),

$$\delta_H = \delta_H^* + \left(\frac{M_{BH}}{kM_H}\right)^{1/\gamma_s}$$

(15)
The excursion set formalism of [23] validated Eq. (14) for the blue-shifted fluctuation spectrum with small CMBR anisotropy and the spiky spectrum for MACHO PBHs [24]. However, the above expressions Eq. (13) and (14) are inappropriate to describe the accumulation of PBHs formed at later times. In particular, for Type II PBHs, the accumulation of newly formed PBHs contributes to all masses below $\sim 2M_H(t)$, so that the shape of the mass spectrum is time dependent while that of Type I PBHs, Eq. (11), does not depend on time. For Type I PBHs, the newly formed PBHs just add to the total number density at masses higher than the older PBHs.

One of the main purpose of this work is to formulate the mass spectrum for Type II PBHs which describes the accumulation effect more properly. For this we start from the interpretation of the probability distribution, Eq. (6). As mentioned before, the smoothed field can see the structures larger than the window size. So, Eq. (6) can be interpreted as the fraction of the overdense regions which are larger than the scale $M$ and have amplitude in the range of $(\delta_i, \delta_i + d\delta_i)$. The contribution from larger scales can be eliminated by differentiation. Then the fraction of the regions between $(M, M + dM)$ and $(\delta_i, \delta_i + d\delta_i)$ is given by

$$f(M, \delta_i)\delta_i dM = -\frac{\partial \tilde{P}(M, \delta_i)}{\partial M}d\delta_i M .$$

(16)

Applying a change of variables $(M \rightarrow M_H, \delta_i \rightarrow M_{BH})$ and the Press-Schechter prescription, we can find the mass spectrum for the PBHs. The number density of PBHs in the ranges of $(M_{BH}, M_{BH} + dM_{BH})$ and $(M_H, M_H + dM_H)$ can be derived to be

$$\bar{n}_{BH}dM_H dM_{BH} = -\sqrt{\frac{2}{\pi}} \frac{\tilde{\rho}_H}{M_{BH}^2} \left( \frac{M_H}{M_{BH}} \right)^{-3/2} \times \left( \frac{M_{BH}}{kM_H} \right)^{1/\gamma_s} \left( \frac{\delta_i^2}{\sigma_H^2} - 1 \right) \frac{M_H \partial \sigma_H}{\sigma_H^2} \frac{\partial M_H}{M_H} dM_{BH} .$$

(17)

The mass spectrum at time $t$ is then given by

$$n_{BH}(M_{BH}, t)M_{BH} = \left( \int_{M_{BH}}^{M_{BH}^2} \bar{n}_{BH}dM_{BH} \right) dM_{BH}$$

(18)

where $M_{H1} = M_{BH}/[k(\delta_{BH}^2 - \delta_{BH1}^2)^{\gamma_s}]$ and $M_{H2} = M_H(t)$. The number density of PBHs at $t$ is

$$\beta_{BH}(t) = \left( \frac{a(t)}{a(t_i)} \right)^{-3} \int_{M_{BH1}}^{M_{BH2}} n_{BH}(M_{BH})dM_{BH}$$

(19)

where $M_{BH2} = kM_H(t)(\delta_{BH}^2 - \delta_{BH1}^2)^{\gamma_s}$ and $M_{BH1} = M_{BH1}(t)$, $M_{BH1}(t) < M_{BH1}$ if $M_H(t) < M_{BH1}$, otherwise $M_{BH1} = M(t)$. If PBHs leave stable relics, then we can take $M_{BH1} = M_{clic}$.

IV. PBHS UNDER THE DOUBLE INFLATIONARY POWER SPECTRUM

With the COBE normalization on the CMBR anisotropy and a simple power-law spectrum, the PBHs can form significantly only if the spectrum is strongly blue-shifted. The PBH formation is then usually assumed to occur only at one epoch and the accumulation of black holes formed at later times is neglected. However, if the power spectrum can have larger amplitude, it is expected that the PBH formation is possible over a rather broad period of times. For practical purposes, one would require the large amplitude to occur on scales sufficiently small as to be irrelevant to the CMBR anisotropy while the fluctuation amplitudes on large scales are constrained by the CMBR anisotropy $\delta_H \sim 10^{-5}$. This kind of power spectrum can be generated by double inflationary models [12, 14, 31]. In particular, the double inflation model with one scalar field has been studied in Refs. 13, 14. In the double inflationary models, the first inflation gives the density fluctuation which is responsible for the CMBR anisotropy on large scales. The second inflation generates the smaller scale density fluctuations which are irrelevant to the CMBR anisotropy. If the small scale fluctuations have a rather large amplitude, then the PBH formation can be enhanced. Phenomenologically, the large amplitude on small scales in the double inflation model could resolve the problems of the standard cold dark matter scenario [13]. It has been also claimed that MACHO PBHs could be produced in models with large amplitude at the MACHO scale $\sim 1M_{\odot}$, such as the double inflation from supergravity theory [12] and chaotic new inflationary potential [13, 14]. The double inflation models with two scalar fields might suffer from the initial condition problem if our universe originated from an inflationary domain of Planckian density [33]. However, the study on this problem is not the subject of this work and we assume that the double inflation is realized in a proper way.

We consider only the fluctuation scales larger than $M_{H1}$, the horizon scale at the reheating time of the second inflation $(= t_i)$. The PBHs from the fluctuations smaller than $M_{H1}$, if any, will be diluted away by the second inflation. Also, for simplicity, we assume that the power spectrum larger than $M_{H1}$ is a combination of two simple power-law spectra which are given by

$$\sigma_{H}(M_{BH}) = \sigma_{H1} \left( \frac{M_H}{M_{BH1}} \right)^{(1-n)/4} \text{ for } M_{H} < M_{BH1}$$

and

$$\sigma_{H0} \left( \frac{M_H}{M_{BH0}} \right)^{(1-n_{obs})/4} \text{ for } M_{H} \gg M_{BH1}$$

(20)

where the present values $M_{BH0} \approx 10^{56}g$, $\sigma_{H0} \approx 10^{-4}$ [3] and $n_{obs} = 1.2 \pm 0.3$ from the COBE data [34], and the breaking scale $M_{BH1}$ is well below $10^{15}M_{\odot}$ which is the horizon mass at the matter-radiation equal time $t_i$. The two simple power-law spectra are assumed to
be connected in such a way that \( \sigma_H \) decreases monotonically to the second spectrum for \( M_H > M_H^i \). Also the change of \( \sigma_H \) is assumed to be smooth satisfying \((M_H/\sigma_H)(d\sigma_H/dM_H) \ll 1\). If the spectra were connected sharply or discontinuously at \( M_H^i \), PBHs or other structures with the mass of about \( M_H^i \) might be produced significantly. If \( M_H^i \sim 10^{58}\text{g} \), such PBHs could be related to the MACHOs. However, with the above assumptions, we will not consider such PBH formation in this work. Also, by assuming \( n_{\text{obs}} \simeq 1 \), the PBHs from the fluctuation larger than \( M_H^i \) are neglected.

Then the mass spectrum of Type I PBHs with the assumed power spectrum is given by

\[
n_{BH}(M_{BH}) dM_{BH} = \frac{n + 3}{4} \sqrt{\frac{2}{\pi}} \gamma_{\text{obs}} \left( \frac{M_{BH}}{M_{Hi}} \right)^{-\frac{3}{2}} \frac{1}{\sigma_{H}} \exp \left( -\frac{\gamma^2}{2\sigma_{H}^2} \left( \frac{M_{BH}}{M_{BH}} \right) \right) dM_{BH}.
\]

(21)

Fig. 1 shows the mass spectrums for various parameters. The PBH mass starts from \( M_{BH} = \gamma^{3/2}M_{H}(t_i) \). For an \( n = 1 \) scale invariant spectrum, the mass spectrum has no cutoffs and scales as \( n_{BH} \propto M_{BH}^{-5/2} \). For \( n < 1 \) \((n > 1)\), there are exponential cutoffs at small (large) scales. However, as the fluctuation amplitude grows or as \( n \) approaches 1, the cutoffs develop more slowly and the mass range is significantly broadened.

\[
\tilde{n}_{BH} dM_H dM_{BH} = \frac{n + 3}{4} \sqrt{\frac{2}{\pi}} \gamma_{\text{obs}} \left( \frac{M_{H}}{M_{Hi}} \right)^{-\frac{3}{2}} \frac{1}{\sigma_{H}} \exp \left( -\frac{\gamma^2}{2\sigma_{H}^2} \right) dM_{BH} dM_{BH}.
\]

(22)

For Type II PBHs, the mass spectrum is more complicated. With the assumed double power spectrum, Eq. (17) becomes

FIG. 2. The mass spectrum of Type II PBHs today with the same parameter set of Fig. 1.

Fig. 2 shows the mass spectrum \( n_{BH}(M_{BH}, t_0) \). The PBH mass spectrum extends down to \( M_{Pi} \). There is an extra condition for Type II PBHs: requiring positive number density requires \( \delta_{H}^2 \geq \sigma_{H}^2 \). This condition causes a rapid drop in the mass spectrum at small scales for \( n \geq 1 \) or at large scales for \( n < 1 \). The drop can be seen in III of Fig. 2a. The mass range is also significantly broadened when the fluctuation amplitude is large or \( n \) approaches 1. Since \( \delta_{H} \) is a function of \( M_{BH} \), the role of the exponential term \( \exp(-\frac{\delta_{H}^2}{2\sigma_{H}^2}) \) is very different to that of \( \exp(-\frac{\delta_{H}^2}{2\sigma_{H}^2}) \) in the Type I mass spectrum which leads to cutoffs in the mass spectrum. Interestingly, the mass spectrum for Type II PBHs scales in the same manner as for Type I PBHs, \( n_{BH} \propto M_{BH}^{-5/2} \), in spite of the enhanced formation of small PBHs. This is mainly because \( \exp(-\frac{\delta_{H}^2}{2\sigma_{H}^2}) \) is a function of \( (M_{BH}/M_{H}) \) for \( n = 1 \) and the main contributio
from $M_{BH} \approx M_H$. For $n > 1$, the exponential cut-off develops similarly at large PBH mass and there is an $M_{BH}^{-5/2}$ decrease at small PBH mass. However for $n < 1$ there is no exponential cutoff at small mass because of the accumulation of small PBHs formed at later times. Instead, the mass spectrum at small mass scales as $n_{BH} \propto M_{BH}^{(1-\gamma)/\gamma}$.

For both Types, the dependence on the reheating time only appears in the mass spectrum for $n \geq 1$. The mass spectrum shifts to small mass as the reheating time decreases. Unfortunately, we now have an extra parameter $M_H^*$. The breaking scale $M_H^*$ must be determined, in addition to the shape of the power spectrum, to estimate the number density $\beta_{BH}$ or the density fraction $\Omega_{BH}$ of PBHs. This could be done for more concrete models. Although $\beta_{BH}$ and $\Omega_{BH}$ are not determined here, it is interesting to compare the mass spectrum of PBHs with that of MACHOs. From the data fit of 8 MACHO events, the fraction of MACHOs between $M_{MACHO}$ and $M_{MACHO} + dM_{MACHO}$ is found to be

$$f_{MACHO}dM_{MACHO} \propto M_{MACHO}^{-3.9}dM_{MACHO}$$

(23)

for $M_{MACHO} \geq 0.3M_{\odot}$. This fraction corresponds to a PBH mass spectrum which is far steeper than the spectrum for $n < 1$ shown in Fig. 1b and Fig. 2b. In Ref. [22], it was claimed that the second inflation with an $n < 1$ spectrum could produce PBHs peaked at the MACHO mass. However, they did not consider the accumulation effect. It seems difficult to generate the current mass spectrum with the assumed two simple power-law spectra unless the spectra are connected in a proper way near $M_{MACHO}$. The MACHO spectrum can arise for some $n > 1$ spectra, but the universe would be overclosed by the PBHs.

V. CONCLUDING REMARKS

In this paper, we study the formation of PBHs of two different types, the PBHs with horizon mass (Type I) and the PBHs with scaling mass relation (Type II), under the double inflationary power spectrum. The assumed power spectrum is a double simple power-law spectrum such that the small scale power spectrum gives PBH formation and large scale power spectrum generates the CMBR anisotropy. With a rather large fluctuation amplitude, the PBH formation occurs over a range of epochs and it is important to consider the accumulation of smaller mass PBHs formed at later times. For both types, the PBH mass ranges are significantly broadened. As expected, there are cutoffs which develop slowly in the mass spectrum for Type I PBHs. From the newly constructed mass spectrum for Type II PBHs, it is found that the mass spectrum scales in the same manner as Type I PBHs for $n = 1$. Due to the enhanced formation of small PBHs for Type II PBHs, the mass spectrum for $n < 1$ does not develop an exponential cutoff at small mass but scale as $M_{BH}^{(1-\gamma)/\gamma}$. This may give an unique feature of PBHs in the history of the universe.

With the assumed power spectrum, the PBH mass spectrum has difficulty explaining the current MACHO spectrum. In the present work, however, we did not consider the spiky spectrum [11] which has more rapid scale dependence. Since the resulting mass spectrum will also have a rapid scale dependence, one could explain the MACHO spectrum by controlling the model parameters. However, the Press-Schechter method could not be applicable if the $\sigma_H$ grows rapidly as can be seen for a simple power-law with $n < -3$. So, our mass spectrum could not be applicable to the spiky spectrum. This problematic issue will be studied in detail later.

The gamma-rays from evaporating PBHs can generate a diffuse $\gamma$-ray background (DGB) in the universe [49, 50]. However, it is difficult to generate the complete observed DGB in the $\gamma$-ray energy range $0.8\text{MeV} \leq E \leq 100\text{GeV}$ [38, 39] with the PBHs from a simple power-law spectrum [26, 30] (or $E^{-3}$ if the PBH emission can form a photosphere around the PBH [41]). On the other hand, the $\gamma$-ray spectrum for $E \lesssim 100\text{MeV}$ depends on the mass spectrum of PBHs in the range of $2 \times 10^{13} \lesssim M_{BH} \lesssim 5 \times 10^{14} \text{g}$ [7]. The double inflation model could be tuned to form sufficient PBHs to explain the observed $E \lesssim 100\text{MeV}$ DGB spectrum.

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