Moduli in Exceptional SUSY Gauge Theories

Peter Cho
Lyman Laboratory
Harvard University
Cambridge, MA 02138

Abstract

The low energy structures of $\mathcal{N} = 1$ supersymmetric models with $E_6$, $F_4$ and $E_7$ gauge groups and fundamental irrep matter contents are studied herein. We identify sets of gauge invariant composites which label all flat directions in the confining/Higgs phases of these theories. The impossibility of mapping several of these primary operators rules out previously conjectured exceptional self duals reported in the literature.
1. Introduction

Supersymmetric gauge theories possess a number of general characteristics which distinguish them from their non-supersymmetric counterparts. One such feature is the existence of flat directions along which the scalar potential vanishes. Up to local and global symmetry transformations, all points located on flat directions represent degenerate but physically inequivalent ground states. These different vacua can be described in terms of gauge dependent expectation values for matter fields within the microscopic theory. Alternatively, the potential’s flat directions may be labeled by gauge invariant combinations of the matter fields \([1, 2]\). In either case, a detailed analysis of the moduli space of vacua typically enters into the study of the infrared dynamics of any supersymmetric model.

Before one can begin to investigate how quantum effects may deform or even destroy the classical low energy picture of a particular theory, one must first identify an appropriate set of moduli space coordinates. It is relatively straightforward to perform this task in simple models such as \(\mathcal{N} = 1\) SUSY QCD with \(N_f \leq N_c + 1\) flavors. In this case, ground states are labeled by expectation values of composite meson and baryon operators built out of quark and antiquark superfields \([3]\). However, the problem of finding a suitable set of gauge invariant moduli grows harder for theories with more complicated color groups or matter contents. No prescription for how to solve this problem in an arbitrary model is known.

In this note, we identify sets of hadrons which act as moduli space coordinates for \(\mathcal{N} = 1\) theories based upon exceptional gauge groups containing \(N_f\) matter superfields in the fundamental irrep. Once these composite operator sets are established, one can straightforwardly analyze the vacuum structure of the models’ confining/Higgs phases. Identification of gauge invariant composites also represents a necessary first step in the search for dual descriptions of the exceptional theories’ nonabelian Coulomb phases. As the \(G_2\) model has already been thoroughly studied \([4, 5, 6]\) while the \(E_8\) model does not confine for any nonzero value of \(N_f\), we restrict our focus to the \(E_6\), \(F_4\) and \(E_7\) theories. As we shall see, the confining/Higgs phases of these theories terminate with quantum constraints. The physics of confinement within the exceptional gauge group models is consequently analogous to that for SUSY QCD with \(N_f \leq N_c\) quark flavors \([3]\). But their group theory structures are much more interesting and rich.

Our article is organized as follows. We first recall a few salient facts about exceptional group theory in section 2. We then construct the composite operators which label all
flat directions within the $E_6$, $F_4$ and $E_7$ models’ confining/Higgs phases in section 3. Several of these operators had not been known before, and we demonstrate in section 4 that their existence rules out previously conjectured self duals involving exceptional gauge groups. Finally, we close in section 5 with some thoughts on constructing genuine duals to exceptional SUSY gauge theories.

2. A little exceptional group theory

The group theory underlying the exceptional Lie algebra $E_6$ may seem much more alien than that for the more familiar classical groups. $E_6$ is defined in terms of $3 \times 3$ matrices with complex octonion elements whose multiplication rule is neither commutative nor associative [7,8]. Its 27-dimensional fundamental and 78-dimensional adjoint representations are large and unwieldy. The invariant tensors of $E_6$ include a symmetric $d_{\alpha\beta\gamma}$ symbol whose indices range over the fundamental rather than adjoint irrep. The generic symmetry breaking pattern

$$E_6 \rightarrow F_4 \rightarrow SO(8) \rightarrow SU(3) \rightarrow 1 \quad (2.1)$$

under which the rank-6 algebra decomposes is not intuitively obvious [9]. Given all these group theory difficulties, it is small wonder that $E_6$ model building is not commonplace.

However, a surprising amount of insight into $E_6$ can be gained by focusing upon its regular maximal subalgebras which involve only $SU(N)$ and $Sp(2N)$ factors. After constructing its extended Dynkin diagram and eliminating the central root, one readily finds that $E_6$ contains an amusing $SU(3) \times SU(3) \times SU(3) \times Z_3$ subgroup under which the fundamental decomposes as $27 \rightarrow (3,\overline{3},1)+(1,3,\overline{3})+(\overline{3},1,3)$. A field $Q$ that transforms according to the fundamental irrep may consequently be regarded as either a 27-dimensional vector or as a triplet $(L,M,N)$ of $3 \times 3$ matrices [10]. Recalling the embedding of the Standard Model inside $E_6$ Grand Unified Theories in which the first $SU(3)$ factor is identified with color while the remaining two are regarded as left and right handed factors [11,12], we name the individual fields within the 27 so as to indicate their color and electric charge assignments:

$$L = \begin{pmatrix} d_1 & u_1 & D_1 \\ d_2 & u_2 & D_2 \\ d_3 & u_3 & D_3 \end{pmatrix}, \quad M = \begin{pmatrix} N^0 & \overline{E} & \nu \\ E & \overline{N} & e \\ \overline{\nu} & \overline{e} & n^0 \end{pmatrix}, \quad N = \begin{pmatrix} \overline{d}_1 & \overline{d}_2 & \overline{d}_3 \\ \overline{\pi}_1 & \overline{\pi}_2 & \overline{\pi}_3 \\ \overline{D}_1 & \overline{D}_2 & \overline{D}_3 \end{pmatrix}. \quad (2.2)$$
As can be seen from these matrices, the 27 incorporates all the familiar members of a single Standard Model family along with some extra leptons and charge $\frac{1}{3}$ quarks.

$E_6$ also contains a regular $SU(2) \times SU(6)$ maximal subalgebra under which the fundamental breaks apart as $27 \rightarrow (1, 15) + (2, \bar{6})$. The 27 may therefore be written in terms of the antisymmetric $6 \times 6$ matrix

$$A = \begin{pmatrix}
0 & D_3 & -u_3 & -N^0 & -\overline{E} & -\nu \\
-D_3 & 0 & d_3 & -E & -\overline{N}^0 & -e \\
u & d_3 & 0 & -\overline{\nu} & -\overline{e} & -n^0 \\
N^0 & E & 0 & \overline{D}_3 & 0 & d_3 \\
\overline{E} & \overline{N}^0 & \overline{\nu} & -D_3 & 0 & \overline{d}_3 \\
e & n^0 & \overline{u}_3 & -\overline{d}_3 & 0 & 0
\end{pmatrix}$$ (2.3)

along with the $2 \times 6$ matrix

$$B = \begin{pmatrix}
d_1 & u_1 & D_1 & -\overline{d}_2 & -\overline{u}_2 & -\overline{D}_2 \\
d_2 & u_2 & D_2 & \overline{d}_1 & \overline{u}_1 & \overline{D}_1
\end{pmatrix}.$$ (2.4)

By examining the relationships between its two maximal subalgebras, one can figure out much about the group theory structure of $E_6$ itself.

For example, it is interesting to inquire how the $E_6$ invariant $Q^3 \equiv d_{\alpha\beta\gamma} Q^\alpha Q^\beta Q^\gamma$ decomposes under the $SU(3)^3$ and $SU(2) \times SU(6)$ subgroups. In the former case, $Q^3$ must break apart into a linear combination of $\det L + \det M + \det N$ and $\text{Tr}LMN$, for only these terms are cubic in $Q$ and invariant under the cyclic $Z_3$ permutation symmetry. Similarly, $Q^3$ must be expressible as a linear combination of $\text{Pf} A$ and $\epsilon_{ij} B_i^a A^{ab} B_j^b$. After equating these two linear combinations and solving for their unknown coefficients, we find

$$d_{\alpha\beta\gamma} Q^\alpha Q^\beta Q^\gamma = \det L + \det M + \det N - \text{Tr}LMN = \text{Pf} A + \frac{1}{2} \epsilon_{ij} B_i^a A^{ab} B_j^b.$$ (2.5)

Numerical values for $d_{\alpha\beta\gamma}$ may readily be computed from this result. We tabulate the independent, nonvanishing components of this symmetric symbol in Appendix A.

Maximal regular subalgebra considerations also shed light upon $E_6$ symmetry breaking. As (2.4) indicates, $E_6$ breaks down to $F_4$ when a single field $Q \sim 27$ develops a vacuum expectation value. Working with the basis in (2.2), we can rotate this vev into the form $\langle Q \rangle = (\langle L \rangle, \langle M \rangle, \langle N \rangle) \propto (0, 1_{3 \times 3}, 0)$ which remains invariant under only an $SU(3)_C \times SU(3)_L+R$ subgroup of $SU(3)_C \times SU(3)_L \times SU(3)_R$. The nonvanishing value for $\langle M \rangle$ also implies that the antisymmetric matrix $A$ in (2.3) develops a vev proportional to $\sigma_2 \otimes 1_{3 \times 3}$. The $SU(2) \times SU(6)$ subgroup of $E_6$ consequently reduces to $SU(2) \times Sp(6)$.
As a check, one can verify that $F_4$ contains both $SU(3) \times SU(3)$ and $SU(2) \times Sp(6)$ as maximal regular subalgebras under which its 26-dimensional irrep respectively decomposes as $(\bar{3}, \bar{3}) + (\bar{3}, 3) + (1, 8)$ and $(2, 6) + (1, 14)$ \cite{13}.

If a second $Q \sim 27$ field acquires a nonvanishing expectation value proportional to $(0, \lambda_3, 0)$ where $\lambda_3$ denotes the third $SU(3)$ Gell-Mann matrix, $E_6$ breaks to $SO(8)$. The fundamental irrep then decomposes as $27 \rightarrow 8_v + 8_s + 8_c + 3(1)$. As we outline in Appendix B, the low energy structure of $\mathcal{N} = 1$ supersymmetric $SO(8)$ theories with $N_f$ vectors, $N_f$ spinors and $N_f$ conjugate spinors can be understood for any number of flavors. It is consequently instructive to determine how $SO(8)$ is embedded within $E_6$ in order to gain insight into models based upon the exceptional gauge group. We utilize the Dynkin basis matrices $P(E_6 \rightarrow F_4)$, $P(F_4 \rightarrow SO(9))$ and $P(SO(9) \rightarrow SO(8))$ listed in ref. \cite{13} to project all 27 weights in the fundamental of $E_6$ down to $SO(8)$. After matching the results with the weights for the 8-dimensional vector, spinor and conjugate spinor irreps, we identify how each element within $Q = (L, M, N) \sim 27$ transforms under the $SO(8)$ subgroup:

$$Q = \begin{pmatrix} s_2 & c_6 & v_2 \\ s_3 & c_7 & v_3 \\ s_5 & c_8 & v_5 \end{pmatrix}, \begin{pmatrix} \alpha_1 & v_8 & c_4 \\ v_1 & \alpha_2 & s_1 \\ c_5 & s_8 & \alpha_3 \end{pmatrix}, \begin{pmatrix} s_4 & s_6 & s_7 \\ c_1 & c_2 & c_3 \\ v_4 & v_6 & v_7 \end{pmatrix} \quad (2.6)$$

where $v_i \in 8_v$, $s_i \in 8_s$, $c_i \in 8_c$ and $\alpha_1, \alpha_2, \alpha_3 \sim 1$. Comparing this basis for $Q$ with the colored and electrically charged one in (2.2), we observe that the down quarks and electrons together with their antiparticles form complete $SO(8)$ irreps as do the up quarks and neutrinos. Clearly, the $SO(8)$ subgroup contains and generalizes Pati-Salam $SU(4)$ which treats leptons as fourth colored quarks \cite{14}.

As we shall see in the following sections, SUSY gauge theories based upon $E_6$, $F_4$ and $SO(8)$ are closely linked. We will therefore find it useful to keep in mind the relations summarized in Table 1 between these exceptional groups and their maximal subalgebras.

\begin{table}[h]
\begin{tabular}{c c c c c c c c}
$E_6$ & \xrightarrow{(27)} & $F_4$ & \xrightarrow{(27)} & $SO(8)$
\hline
SU(3)$_C \times SU(3)_L \times SU(3)_R$ & $\rightarrow$ & SU(3)$_C \times SU(3)_{L+R}$ & $\rightarrow$ & SU(3)$_C \times U(1)_{3V} \times U(1)_{8V}$

SU(2) $\times$ SU(6) & $\rightarrow$ & SU(2) $\times$ Sp(6) & $\rightarrow$ & SU(2)$^4$
\end{tabular}
\caption{Symmetry breaking patterns among exceptional groups and their maximal regular subalgebras}
\end{table}
3. Confining exceptional theories

We begin our study of exceptional SUSY gauge theories by considering a model with the symmetry group

$$G = E_6 \times \left[ SU(N_f) \times U(1)_R \times Z_{6N_f} \right]_{\text{global}}$$  \hspace{1cm} (3.1a)

and matter content

$$Q \sim (27; \Box; R = 1 - 4/N_f, 1).$$  \hspace{1cm} (3.1b)

Several points about this model should be noted. Firstly, it is reminiscent of supersymmetric quantum chromodynamics inasmuch as it contains only fundamental irrep matter. But since we do not choose to incorporate antiquarks transforming as $\overline{27}$ along with the quarks in (3.1b), this $E_6$ model is chiral. Secondly, the classical field theory remains invariant under phase rotations which count quark number. In the quantum theory, only a discrete $Z_{6N_f}$ subgroup of $U(1)_Q$ survives as a nonanomalous symmetry \cite{13–19}. Finally, the $E_6$ model is asymptotically free so long as its one-loop Wilsonian beta function coefficient

$$b_0 = \frac{1}{2} \left[ 3K(\text{Adj}) - \sum_{\text{matter reps } \rho} K(\rho) \right] = 3(12 - N_f)$$  \hspace{1cm} (3.2)

remains positive. Its infrared dynamics are consequently nontrivial provided $N_f < 12$.

Quark field expectation values break the $E_6$ gauge symmetry at generic points in moduli space according to the pattern displayed in (2.1). We can use this symmetry breaking information to count the gauge invariant operators which act as coordinates on the moduli space of degenerate vacua for small values of $N_f$. In Table 2, we list the initial parton matter degrees of freedom, the unbroken color subgroup and the number of matter fields eaten by the superHiggs mechanism as a function of $N_f$. The remaining uneaten parton fields correspond to independent hadrons in the low energy effective theory which represent either Goldstone bosons resulting from global $SU(N_f)$ symmetry breaking or massless moduli labeling D-flat directions of the $E_6$ scalar potential. The counting results

---

1 We adopt the $E_6$ index values $K(27) = 6$ and $K(78) = 24$. For later use, we also record the indices $K(26) = 6$ [$K(56) = 12$] and $K(52) = 18$ [$K(133) = 36$] of the fundamental and adjoint irreps of $F_4$ [$E_7$].
Table 2: Number of independent massless hadrons in the $E_6$ model

| $N_f$ | Parton DOF | Unbroken Subgroup | Eaten DOF | Hadrons |
|-------|------------|-------------------|----------|---------|
| 1     | 27         | $F_4$             | 78 − 52 = 26 | 1       |
| 2     | 54         | $SO(8)$           | 78 − 28 = 50 | 4       |
| 3     | 81         | $SU(3)$           | 78 − 8 = 70 | 11      |
| 4     | 108        | 1                 | 78        | 30      |

displayed in the last column of Table 2 essentially fix the forms of the composites within the $N_f \leq 4$ $E_6$ models:

$$B = Q^3 \sim (1; 3R, 3)$$

$$C = Q^6 \sim (1; 6R, 6)$$

$$D = Q^{12} \sim (1; 12R, 12).$$

(3.3)

This proposed hadron set satisfies several consistency checks. We first recall that $E_6$ irreps belong to one of three different “triality” equivalence classes [13]. The 27-dimensional fundamental irrep has triality 1, whereas the singlet has triality 0. Since all gauge invariant composites are $E_6$ singlets, they must contain a multiple of three quark constituents. The hadrons in (3.3) clearly satisfy this necessary condition.

We next tally the number of composite degrees of freedom as a function of $N_f$ in Table 3 and compare with the results of Table 2. For $N_f \leq 3$, $B$ and $C$ precisely account for the required number of massless fields. On the other hand, the hadron count exceeds the needed number of composites by one when $N_f = 4$. A single relation must exist among $B$, $C$ and $D$ in this case. The quantum constraint is restricted by dimensional analysis, $SU(4)$ invariance and discrete symmetry considerations. It schematically appears in superpotential form as

$$W_{N_f=4} = X \left[ D^2 + C^4 + C^3B^2 + C^2B^4 + CB^6 + B^8 - \Lambda_4^{24} \right]$$

(3.4)

where $X \sim (1; 1; 2, 0)$ represents a Lagrange multiplier field. The undetermined numerical coefficients multiplying each term in this expression can in principle be determined by Higgsing $E_6$ down to $SO(8)$. Their values are then fixed by the requirement that the exact
quantum constraint in (B.4) be recovered after the $E_6$ hadrons in (3.3) are decomposed in terms of the $SO(8)$ composite operators listed in (B.3).

Although we have not attempted to implement this tedious procedure to deduce the precise form of $W_{N_f=4}$, we at least know that the coefficient of its first term must not vanish if the first term in (B.4) is to be recovered along the $SO(8)$ flat direction. As a result, the point $B = C = 0$, $D \propto \Lambda_1^{12}$ lies within the deformed quantum moduli space. Since an $SU(4) \times U(1)_R \times Z_{12}$ subgroup of the full global symmetry in (3.1d) remains unbroken at this point, it is instructive to compare ’t Hooft anomalies calculated in the microscopic $E_6$ theory and the low energy sigma model. We could compute the hadronic global anomalies in terms of the independent fluctuations about vevs which satisfy the constraint in (3.4). But as the quantum numbers of $X$ are precisely opposite to those of the fluctuation which is removed by the constraint, it is easier to instead retain all components of $B$, $C$ and $D$ and include anomaly contributions from the Lagrange multiplier field $X$ as well. We then find that the parton and hadron level $SU(4)^3$, $SU(4)^2U(1)_R$, and $U(1)^{1,3}_R$ global anomalies precisely match. Moreover, the $SU(4)^2Z_{12}$, $Z_{12}^{1,3}$, $U(1)^2Z_{12}$ and $U(1)_RZ_{12}^2$ anomalies match modulo 12 [19]. This anomaly agreement indicates the hadrons in (3.3) form a complete set of moduli which label flat directions in the $N_f = 4$ $E_6$ theory. On the other hand, the parton and hadron level global anomalies do not match for $N_f > 4$, and the disagreement cannot be eliminated by including additional color-singlet fields into the low energy spectrum without disrupting the $N_f = 4$ results. So we conclude the $E_6$ model’s confining phase terminates at this stage.

\[\text{Table 3: Hadron degree of freedom count}\]

| $N_f$ | Hadrons | $B$ | $C$ | $D$ | constraint |
|-------|---------|-----|-----|-----|------------|
| 1     | 1       | 1   |     |     | -          |
| 2     | 4       | 4   |     |     | -          |
| 3     | 11      | 10  | 1   |     | -          |
| 4     | 30      | 20  | 10  | 1   | -1         |

\[2\text{ As we discuss in Appendix B, the } SO(8) \text{ theory which results from Higgsing the } N_f > 4 \text{ } E_6 \text{ model exists in either a nonabelian Coulomb or free electric phase. Since the } N_f > 2 \text{ } SO(8) \text{ theory does not confine at the origin of moduli space, neither does its } N_f > 4 \text{ } E_6 \text{ progenitor.}\]
Given the hadron set in (3.3), we can investigate the low energy structure of the $E_6$ models with $N_f < 4$ quark flavors. Consistency with known results along the $SO(8)$ flat direction requires that dynamical superpotentials be generated in these theories. Since the $E_6$ model in (3.1) is chiral, we cannot simply add a mass term to $W_{N_f=4}$ and integrate out heavy flavors. However, the nonperturbative superpotentials’ forms are fixed up to numerical factors by dimensional analysis, holomorphy and symmetry considerations:

\[
W_{N_f=1} = \left[ \frac{\Lambda_{33}^3}{B^2} \right]^{1/9},
\]

\[
W_{N_f=2} = \left[ \frac{\Lambda_{2}^{30}}{B^4} \right]^{1/6},
\]

\[
W_{N_f=3} = \left[ \frac{\Lambda_{3}^{27}}{C^3 + CB^4 + B^6} \right]^{1/3}.
\]

As in SUSY QCD with $N_f < N_c$ flavors [20], these quantum terms destabilize the $N_f < 4$ $E_6$ models’ ground states.

Once we know the low energy $E_6$ spectrum, we can readily deduce the confining phase particle content of the $F_4$ model which results from Higgsing (3.1):

\[
G = E_6 \times \left[ SU(N_f) \times U(1)_R \times Z_{6N_f} \right] \overset{\langle Q_{N_f} \rangle}{\longrightarrow} H = F_4 \times \left[ SU(N_f - 1) \times U(1)_{R'} \times Z_{6(N_f-1)} \right]
\]

\[
Q \sim (27; \Box; R = 1 - 4/N_f, 1) \quad \quad Q' \sim (26; \Box; R' = 1 - 3/(N_f - 1), 1)
\]

\[
\Phi' \sim (1; \Box; R' = 1 - 3/(N_f - 1), 1)
\]

\[
\downarrow \mu < \Lambda_{E_6} \quad \quad \downarrow \mu < \Lambda_{F_4}
\]

\[
B = \quad \rightarrow \quad + \quad + \quad +
\]

\[
C = \quad \rightarrow \quad + \quad + \quad + \quad +
\]

\[
D = \quad \rightarrow \quad + \quad + \quad + \quad +
\]

\[
\text{(3.6)}
\]

It is important to note that this diagram is commutative. The same $F_4$ hadronic spectrum is found whether one breaks $E_6$ at high energies and then allows the unbroken $F_4$ gauge group to confine at $\mu < \Lambda_{F_4}$ or if one instead performs a flavor decomposition of the $E_6$ hadrons at $\mu < \Lambda_{E_6}$. The two components of the $B$ baryon which transform according

\[3\] The $E_6$ and $F_4$ models’ scales are related by the matching condition $\Lambda_{E_6}^{N_f - 4} = \langle Q_{N_f} \rangle^3 \Lambda_{F_4}^{9 - N_f} \propto \langle B^{N_f, N_f, N_f} \rangle^{9 - N_f}$. 

8
to the fundamental and singlet irreps of $SU(N_f - 1)$ are respectively identified with the color-singlet $\Phi'$ field and quark vev $\langle Q^{N_f} \rangle$ in the microscopic $F_4$ theory. All the remaining hadrons on the LHS of (3.4) represent composites of the 26-dimensional $Q'$ quarks.

As a check, one can verify that global anomaly matching demonstrates the $N_f - 1 = 3$ $F_4$ spectrum is saturated by the gauge invariant moduli

$$
M' = Q'^2 \sim (1; \boxed{2R'}/2, 2) \quad O' = Q'^5 \sim (1; \boxed{5R'}/5)
$$

$$
B' = Q'^3 \sim (1; \boxed{3R'}/3, 3) \quad C' = Q'^6 \sim (1; \boxed{6R'}/6)
$$

$$
N' = Q'^4 \sim (1; \boxed{4R'}/4, 4) \quad P' = Q'^9 \sim (1; \boxed{9R'}/9)
$$

along with a field $X' \sim (1; 1; 2, 0)$. This last object acts as a Lagrange multiplier whose equation of motion yields the quantum constraint

$$
P'^2 + O'^2 N'^2 + O'^2 M'^4 + O'^2 N'M'^2 + O'M'^5 B' + O'M'^3 N'B' + O'M'^2 N'^2 B' + N'^3 C' + N'^2 B'^2 + N'^2 M'^2 C' + N'^2 M'^2 B' + N'M'^4 C' + N'M'^4 B'^2 + M'^6 C' + M'^6 B'^2 = \Lambda'_3^{18}
$$

(3.8)

that comes from Higgsing the $E_6$ relation in (3.4). Since $F_4$ has only real representations, there is no group theory obstruction to integrating out matter fields from this constraint and deriving the dynamical superpotentials in the $F_4$ models with one and two quark flavors.

We conclude our survey of exceptional SUSY theories by briefly sketching the outlines of an $E_7$ model with nonanomalous symmetry group

$$
G = E_7 \times [SU(N_f) \times U(1)_R \times Z_{12N_f}]_{\text{global}},
$$

(3.9a)

matter content

$$
Q \sim (56; \boxed{R}; R = 1 - 3/N_f, 1)
$$

(3.9b)

and Wilsonian beta function coefficient $b_0 = 6(9 - N_f)$. At generic points in moduli space, the gauge group breaks according to the pattern

$$
E_7 \rightarrow E_6 \rightarrow SO(8) \rightarrow 1,
$$

(3.10)

and the pseudoreal fundamental irrep of $E_7$ decomposes as $56 \rightarrow 27 + \overline{27} + 2(1) \rightarrow 2(8_v + 8_s + 8_c) + 8(1)$. Using this group theory information to count massless degrees of freedom
in exactly the same fashion as for the $E_6$ and $F_4$ theories, we find that the $E_7$ model’s confining/Higgs phase spectrum looks like

\[
\begin{align*}
M &= Q^2 \sim (1; \begin{array}{c}2R, 2 \end{array}) \\
B &= Q^4 \sim (1; \begin{array}{c}4R, 4 \end{array}) \\
C &= Q^6 \sim (1; \begin{array}{c}6R, 6 \end{array}) \\
D &= Q^8 \sim (1; \begin{array}{c}8R, 8 \end{array}) \\
E &= Q^{12} \sim (1; \begin{array}{c}12R, 12 \end{array}) \\
F &= Q^{18} \sim (1; \begin{array}{c}18R, 18 \end{array}).
\end{align*}
\]  

(3.11)

These composites are restricted by a single quantum constraint when $N_f = 3$. After Lagrange multiplier contributions are taken into account, it is again straightforward to verify that the $SU(N_f)^3$, $SU(N_f)^2U(1)_R$, $SU(N_f)^2Z_{12N_f}$, $U(1)_R^{1,3}$, $Z_{12N_f}^{1,3}$, $U(1)_R^2Z_{12N_f}$ and $U(1)_R^2Z_{12N_f}^2$ ‘t Hooft anomalies calculated at the partonic and hadronic levels match for $N_f = 3$ but disagree for $N_f = 4$. The $E_7$ model consequently ceases to confine at this juncture.

4. Exceptional self duals

Much of the progress made during the past few years in understanding nonperturbative aspects of $\mathcal{N} = 1$ supersymmetric gauge theories has stemmed from Seiberg’s key insight that vacuum structures of strongly interacting models can sometimes be described in terms of weakly coupled duals [21]. Although a number of such strong-weak pairs have been discovered, no systematic method for determining dual theories has so far been developed. Even finding new examples of duality remains a highly nontrivial problem. In particular, constructing general duals to models involving exceptional gauge groups other than $G_2$ represents an outstanding challenge.

Within the past year, a few investigators have claimed to find examples of $E_6$, $F_4$ and $E_7$ self duals involving special numbers of fundamental irrep matter fields [22,23,24]. The symmetry groups for both members of the alleged dual pairs are identical, and their matter contents are nearly the same. As a result, ‘t Hooft anomalies in the electric and magnetic theories’ continuous global symmetry groups match in a rather trivial fashion. Moreover, a few gauge invariant composites within the electric theory appear to map onto
magnetic counterparts. Based upon this circumstantial evidence, these self dual models have been conjectured to represent the first examples of $E_6$, $F_4$ and $E_7$ duality.

Csàki and Murayama have recently called into question the exceptional self duals’ validity [19]. They noticed that certain discrete anomalies in the electric and magnetic theories do not agree. As these authors pointed out, this defect might conceivably be remedied through nonperturbative generation of accidental symmetries. The basic veracity of the exceptional duals has therefore remained in doubt. Fortunately or unfortunately, we can now use our knowledge of the low energy $E_6$, $F_4$ and $E_7$ spectrum to rule out these self duality proposals.

We first examine the $N_f = 6$ $E_6$ theory of Ramond [22]. The basic structure of his dual pair is outlined below:

$$ G = E_6 \times [SU(6) \times U(1)_R] \quad \iff \quad \tilde{G} = E_6 \times [SU(6) \times U(1)_R] $$

$$ Q^i \sim (27; \begin{array}{c} \square \end{array}; \frac{1}{3}) $$

$$ q_i \sim (27; \begin{array}{c} \square \end{array}; \frac{1}{3}) $$

$$ b^{ijk} \sim (1; \begin{array}{c} \square \end{array}; 1) $$

$$ \tilde{W}_{\text{tree}} = b^{ijk} q_i q_j q_k. $$

Because the electric and magnetic quarks share the same $R$-charge assignments, $R$ symmetry matching dictates that every electric composite $Q^n$ which is not introduced into the dual as an elementary field must be identified with either some $q^n$ magnetic composite or else a hadron involving dual field strength tensors. Since $Q$ and $q$ transform according to conjugate representations of the nonabelian flavor group, the $Q^n \leftrightarrow q^n$ identification can be consistent only if the composites transform according to real irreps of $SU(6)$. As Ramond observed, this condition is satisfied for the $C = Q^6$ baryon in (3.3):

$$ C = Q^6 \sim \left(1; \begin{array}{c} \square \end{array}; 2\right) \quad \iff \quad c = q^6 \sim \left(1; \begin{array}{c} \square \end{array}; 2\right). $$

However, it is impossible to similarly map the $D = Q^{12}$ baryon onto $q^{12}$. Instead, $D$ can only be paired with a hadron containing 6 dual quarks and 2 dual field strength tensors:

$$ D = Q^{12} \sim \left(1; \begin{array}{c} \square \end{array}; 4\right) \quad \iff \quad d = q^6 \tilde{W}^2 \sim \left(1; \begin{array}{c} \square \end{array}; 4\right). $$

Although it is not at all obvious that one can in fact construct such a $d$ chiral operator which is primary, we will assume this mapping is possible. We are then similarly forced to make the converse identification

$$ D' = Q^6 W^2 \sim \left(1; \begin{array}{c} \square \end{array}; 4\right) \quad \iff \quad d' = q^{12} \sim \left(1; \begin{array}{c} \square \end{array}; 4\right). $$
Up to this point, Ramond’s dual pair appears to pass all anomaly matching and operator mapping tests. But the $E_6$ self dual must also withstand careful scrutiny along the $F_4$ and $SO(8)$ flat directions. In the former case, the electric and magnetic theories in (4.1) are deformed as follows:

$$H = F_4 \times [SU(5) \times U(1)_R] \quad \iff \quad \tilde{H} = E_6 \times [SU(5) \times U(1)_R]$$

\[ Q^i \sim (26; \square; \frac{2}{5}) \]
\[ \Phi^i \sim (1; \square; \frac{2}{5}) \]
\[ W_{\text{tree}} = 0 \]

\[ \Phi \sim (1; \square; 1; 0, 4, \frac{1}{2}) \]
\[ \Phi' \sim (1; 1, \square; 1, 0, 4, \frac{1}{2}) \]

while the magnetic theory reduces to

\[ W'_{\text{tree}} = b_{ijk} q_i q_j q_k + m_{ij} q_i q_j q' + \phi^i q_i q' q' + q' q' q'. \]
\[ \tilde{I} = E_6 \times [SU(4)_{v+s+c} \times (Z_3)_B \times U(1)_R] \]

\[ q_i \sim (27; \square; 0, \frac{1}{6}) \quad \phi^i \sim (1; \square; 0, \frac{1}{2}) \quad b^{ijk} \sim (1; \square\square\square; 0, \frac{3}{2}) \]

\[ q' \sim (27; 1; 1, \frac{2}{3}) \quad \phi'^i \sim (1; \square; 2, \frac{1}{2}) \quad m'^{ij} \sim (1; \square; 2, 1) \quad (4.6) \]

\[ q'' \sim (27; 1; -1, \frac{2}{3}) \quad \phi''^i \sim (1; \square; -2, \frac{1}{2}) \quad m''^{ij} \sim (1; \square; -2, 1) \]

\[ \tilde{W}_{\text{tree}} = b^{ijk} q_i q_j q_k + m'^{ij} q_i q_j q'^i + m''^{ij} q_i q_j q''^i + \phi^i q_i q'^i q''^i + \phi'^i q_i q'' q'^i + \phi''^i q_i q'' q'^i + q' q' q' + q'' q'' q'' . \]

Only the diagonal subgroup of the \( SO(8) \) model’s nonabelian flavor symmetry and a discrete \( Z_3 \) subgroup of its abelian baryon number remain intact within the microscopic \( E_6 \) theory. In principle, the full global symmetry group should be restored at the infrared fixed point \([23,25]\).

Intractable problems with these candidate \( E_6 \) duals become apparent when one tries to trace the mappings of the \( D = Q^{12} \) and \( d' = q^{12} \) baryons. Global quantum number matching dictates the generalizations

\[ Q^{12-n} \quad \Leftrightarrow \quad q^{3+n} q'^{3-n} \tilde{W}^2 \]

\[ Q^{3+n} W^2 \quad \Leftrightarrow \quad q^{12-n} q''^n \quad (4.7) \]

of the operator identifications \((4.3a,b)\) in the dual pair \((4.4)\). These relations are not obviously incorrect. However, the only electric partner allowed by color, flavor, baryon number and R-charge conservation to the \( d' = q^{12} \) baryon in the magnetic \( E_6 \) theory of \((4.6)\) is the \( SO(8) \) glueball \( W^2 \). This relation looks implausible given that the \( E_6 \) glueball \( \tilde{W}^2 \) also maps to \( W^2 \). Global quantum number matching similarly requires the pairing

\[ D'' = SW^2 \sim (8_s; 1, \square, 1, \frac{5}{2}) \quad \Leftrightarrow \quad d'' = q^{11} q' \sim (1; \square\square\square\square; 1, \frac{5}{2}), \quad (4.8) \]

for no other combination of \( SO(8) \) vectors, spinors, conjugate spinors and field strength tensors has the same global charges as \( d'' \). But as \( SW^2 \) is not gauge invariant, this last mapping is clearly impossible.

Mapping other composites beside the \( q^{12} \) type baryons in the candidate \( SO(8)-E_6 \) dual pair is also problematic. For instance, the \( SO(8) \) invariant \( SVC \) has the same \( R \) charge
and number of degrees of freedom as the $E_6$ hadrons $b$, $q^5q'$, $q^5q''$ and $qq'q''$ [23]. However, this mapping is faulty from a $(Z_3)_B$ standpoint. Since all nonredundant, primary elements of the $E_6$ chiral ring cannot be consistently identified along the $SO(8)$ flat direction, Ramond’s self dual pair must be rejected.

Similar difficulties plague the $N_f = 4$ $E_7$ dual pair of Distler and Karch [23]:

$$G = E_7 \times [SU(4) \times U(1)_R] \quad \iff \quad \tilde{G} = E_7 \times [SU(4) \times U(1)_R]$$

$$Q^i \sim (56; \Box; \frac{1}{4})$$

$$W_{\text{tree}} = 0$$

Among the $E_7$ hadrons listed in (3.11), $M = Q^2 \sim \Box$, $C = Q^6 \sim \Box$ and $D = Q^8 \sim \Box$ transform according to real representations of the $SU(4)$ flavor group. They may consequently be identified with $m = q^2$, $c = q^6$ and $d = q^8$ in the magnetic theory. On the other hand, $E = Q^{12} \sim \Box$ and $F = Q^{18} \sim \Box$ transform according to complex $SU(4)$ irreps. $E$ has the same quantum numbers as $e = q^4 \tilde{W}^{2}$. But this operator identification suffers from the same sorts of problems along the $SO(8)$ flat direction as we have seen for the $Q^{12}$ baryon in Ramond’s $E_6$ dual pair. Furthermore, the only possible dual counterpart to $F$ allowed by $SU(4) \times U(1)_R$ is $f = q^6 \tilde{W}^{3}$. However, the electric superfield is bosonic whereas its magnetic partner is fermionic. As there is no consistent way to map the $F$ baryon within the dual $E_7$ theory, the duality conjecture in (4.9) is fatally flawed.

All similar exceptional self dual pairs which have been reported in the literature can be ruled out in an analogous fashion. These invalid self dual examples serve as useful reminders that ‘t Hooft anomaly matching represents a necessary but insufficient condition for establishing duality.

5. Conclusion

With the demise of the self dual proposals, virtually nothing is known about the existence of duals to $\mathcal{N} = 1$ supersymmetric theories based upon exceptional groups other than $G_2$. No viable magnetic counterparts to $E_6$, $F_4$ or $E_7$ models with vanishing tree level superpotentials have so far been uncovered. However, careful study of the $SO(8)$ flat directions that exist in all these theories may yield some valuable clues. In particular,
looking for generalizations of the magnetic $SU(3N_f-5) \times Sp(2N_f-2)$ counterpart to the $SO(8)$ model with $N_f$ vectors, $N_f$ spinors and $N_f$ conjugate spinors outlined in Appendix B represents an obvious starting point in the search for exceptional duals. It is important to note that only an $SU(N_f) \times SU(2)$ subgroup of the $SO(8)$ theory’s $SU(N_f)^3$ flavor symmetry is realized at short distances in its magnetic partner. This suggests that the full nonabelian flavor groups in the $E_6$, $F_4$ and $E_7$ models which we have studied might also not exist at all energy scales within their duals. Instead, we believe it is more likely that only an $SU(2)$ subgroup embedded inside $SU(N_f)$ such that $N_f \rightarrow N_f$ is realized in the ultraviolet. If this conjecture is incorrect, matching global anomalies and operator composites between the microscopic exceptional gauge theories and their mystery dual partners looks formidably difficult.

We close by noting an intriguing possibility regarding the phase structure of the $E_6$ model in (3.1). Since the nonanomalous R-charge assignment for the matter fields is unique, the scaling dimensions of all composite operators are fixed by the relation $D = 3R/2$. This simple formula implies $D(B = Q^3) = 9/10$ in the $N_f = 5$ model. General lore holds that a magnetic dual becomes free at long distances when the mass dimensions of all electric composites included into the dual as fundamental fields reduce to less than unity [21]. So if $B$ is the only elementary composite within the dual, the $E_6$ theory with $N_f = 5$ flavors may exist in a free magnetic phase. Of course, if any parts of the $C = Q^6$ or $D = Q^{12}$ hadrons enter into the dual as well, then the $N_f = 5$ model cannot be free since these fields’ dimensions are greater than unity. We note that certain portions of $C$ possess exactly the right symmetry properties to be identified with bilinear meson fields along the $SO(8)$ flat direction, and such mesons appear in the magnetic superpotential of the $SU(3N_f-5) \times Sp(2N_f-2)$ theory. So the existence or absence of a free magnetic phase for the $E_6$ model will only be known for certain after a genuine dual is found.

Acknowledgments

I thank Per Kraus for stimulating my interest in exceptional SUSY models and Howard Georgi for many enlightening group theory discussions. This work was supported by the National Science Foundation under Grant #PHY-9218167.
Appendix A. Component values of the $E_6$ $d_{\alpha\beta\gamma}$ symbol

We tabulate here the independent, nonvanishing components of the totally symmetric $E_6$ $d_{\alpha\beta\gamma}$ symbol in the basis where the 27 decomposes under $SU(3)^3$ as $Q = (L, M, N)$ with

$$L = \begin{pmatrix} Q^1 & Q^2 & Q^3 \\ Q^4 & Q^5 & Q^6 \\ Q^7 & Q^8 & Q^9 \end{pmatrix} \quad M = \begin{pmatrix} Q^{10} & Q^{11} & Q^{12} \\ Q^{13} & Q^{14} & Q^{15} \\ Q^{16} & Q^{17} & Q^{18} \end{pmatrix} \quad N = \begin{pmatrix} Q^{19} & Q^{20} & Q^{21} \\ Q^{22} & Q^{23} & Q^{24} \\ Q^{25} & Q^{26} & Q^{27} \end{pmatrix}.$$  
(A.1)

After inserting these forms for $L$, $M$ and $N$ into (2.5), we can simply read off the values for $d_{\alpha\beta\gamma}$. The nonzero components all equal either 1 or $-1$ as indicated in the tables below:

| $d_{\alpha\beta\gamma} = 1$ |
|---------------------------|
| 1,5,9 | 10,14,18 | 19,23,27 |
| 2,6,7 | 11,15,16 | 20,24,25 |
| 3,4,8 | 12,13,17 | 21,22,26 |

| $d_{\alpha\beta\gamma} = -1$ |
|---------------------------|
| 1,6,8 | 2,4,9 | 3,5,7 | 4,10,20 | 5,13,20 | 6,16,20 | 7,11,24 | 8,14,24 | 9,17,24 |
| 1,10,19 | 2,13,19 | 3,16,19 | 4,11,23 | 5,14,23 | 6,17,23 | 7,12,27 | 8,15,27 | 9,18,27 |
| 1,11,22 | 2,14,22 | 3,17,22 | 4,12,26 | 5,26,15 | 6,18,26 | 10,15,27 | 11,13,18 | 12,14,16 |
| 1,12,25 | 2,15,25 | 3,18,25 | 7,10,21 | 8,13,21 | 9,16,21 | 19,24,26 | 20,22,27 | 21,23,25 |

It is amusing to note that only $45 \times 3! = 270$ components of $d_{\alpha\beta\gamma}$ out of the possible $27^3 = 19,683$ are nonvanishing in the basis (A.1).

Appendix B. $SO(8)$ theory with equal numbers of vector, spinor and conjugate spinor fields

The $E_6$ model in (3.1) with $N_f + 2$ fundamental matter fields reduces to an $SO(8)$ theory with $N_f$ vectors, $N_f$ spinors and $N_f$ conjugate spinors at points in moduli space.
where two of the 27’s develop nonvanishing expectation values. The same $SO(8)$ theory can be reached by starting from an $SO(10)$ model with $N_f + 2$ 10-dimensional vectors and $N_f$ 16-dimensional spinors. Since duals to $SO(10)$ SUSY gauge theories containing arbitrary numbers of vectors and spinors have recently been uncovered [26], the vacuum structure of the $SO(8)$ model can be studied for all values of $N_f$. We sketch its principle features in this appendix.

The microscopic $SO(8)$ model has the full symmetry group

$$G = SO(8) \times [SU(N_f)_V \times SU(N_f)_S \times SU(N_f)_C \times U(1)_B \times U(1)_V \times U(1)_R]_{\text{global}} \ (B.1)$$

and matter content

$$V^i \sim (8_V; \Box, 1, 1; 0, -2, R)$$
$$Q^i_s \sim (8_S; 1, \Box, 1; 1, 1, R) \ (B.2)$$
$$Q^i_c \sim (8_C; 1, 1, \Box; -1, 1, R)$$

where $R = 1 - 2/N_f$. This theory confines everywhere throughout moduli space for $N_f \leq 2$. The gauge invariant composites

$$M^{ij} = V^i V^j \quad O_{sv}^{[ij]} = \frac{1}{2!} Q_s[i] \Gamma^{[ij]} C Q_s[j]$$
$$L_s^{[ij]} = Q_s^i C Q_s^j \quad O_{cv}^{[ij]} = \frac{1}{2!} Q_c[i] \Gamma^{[ij]} C Q_c[j]$$
$$L_c^{[ij]} = Q_c^i C Q_c^j \quad O_{sc}^{[ij]} = \frac{1}{2!} Q_s[i] \Gamma^{[ij]} C Q_c[j]$$
$$N^{iij} = Q_s^i \Gamma^{ij} C Q_c^j \quad P_{ij}^{[ij]} = \frac{1}{2!} Q_s[i] \Gamma^{[ij]} C Q_c[j]$$

act as confining phase moduli space coordinates. In the $N_f = 2$ model, these hadrons are not all independent. Instead, they are related by the single quantum constraint

$$P^2 - 4O_{sc}O_{sv}O_{cv} + 8N^4 + 2[M^2 O_{sc}^2 + L_s^2 O_{cv}^2 + L_c^2 O_{sv}^2]$$
$$- 8[N^2 MO_{sc} + N^2 L_s O_{cv} + N^2 L_c O_{sv}] + 16N^2 ML_s L_c - 2M^2 L_s^2 L_c^2 = \Lambda_2^{12}. \ (B.4)$$

One can check that this exact relation reduces to the correct constraint for $N_f = N_c = 4$ SUSY QCD along the flat direction where vevs for the two vectors break $SO(8)$ down to $SO(6) \simeq SU(4)$.

For $3 \leq N_f \leq 5$, the $SO(8)$ theory no longer confines at the origin of moduli space. Instead, it exists in a nonabelian Coulomb phase which can be described in terms of a dual model with the symmetry group

$$\tilde{G} = [SU(\tilde{N}_c) \times Sp(2\tilde{N}_c')]_{\text{local}} \times [SU(N_f) \times SU(2) \times U(1)_B \times U(1)_V \times U(1)_R]_{\text{global}} \ (B.5)$$
where $\tilde{N}_c = 3N_f - 5$ and $\tilde{N}_c' = N_f - 1$. The dual has matter content

\[
\begin{align*}
q &\sim (\square, 1; \square, 1; 0, Y_q, R_q) \\
q' &\sim (\square, \square; 1, 1; 0, Y_{q'}, R_{q'}) \\
q'' &\sim (\square, 1; 1, 1; 0, Y_{q''}, R_{q''}) \\
q''' &\sim (\square, 1; 1, 1; 0, Y_{q'''}, R_{q'''}) \\
\bar{q} &\sim (\square, 1; 1, 2N_f - 1; 0, Y_{\bar{q}}, R_{\bar{q}}) \\
s &\sim (\square, 1; 1, 1; 0, Y_s, R_s) \\
t &\sim (1, \square; 1, 2N_f - 2; 0, Y_t, R_t) \\
m &\sim (1, 1; \square, 1; 0, Y_m, R_m) \\
\ell_s &\sim (1, 1; 1, 2N_f - 1; 2, Y_{\ell_s}, R_{\ell_s}) \\
\ell_C &\sim (1, 1; 1, 2N_f - 1; -2, Y_{\ell_C}, R_{\ell_C}) \\
n &\sim (1, 1; \square, 2N_f - 1; 0, Y_n, R_n)
\end{align*}
\]

where the magnetic fields' hypercharge and R-charge assignments are given by

\[
\begin{align*}
Y_q &= \frac{4N_f(N_f - 2)}{\tilde{N}_c} \quad R_q = -\frac{2N_f - 7}{\tilde{N}_c} R \\
Y_{q'} = Y_{q''} &= -\frac{2N_f(N_f - 1)}{\tilde{N}_c} \quad R_{q'} = R_{q''} = \frac{N_f + 2}{\tilde{N}_c} R \\
Y_{\bar{q}} &= -\frac{4N_f(N_f - 2)}{N_c} \quad R_{\bar{q}} = -\frac{N_f^2 - 12N_f + 16}{N_f \tilde{N}_c} \\
Y_s &= \frac{4N_f(N_f - 1)}{N_c} \quad R_s = \frac{4N_f^2 - 10N_f + 8}{N_f \tilde{N}_c} \\
Y_t = Y_{\ell_s} = Y_{\ell_C} &= 2N_f \\
Y_m &= -4N_f \\
Y_n &= 0 \\
R_t = R_{\ell_s} = R_{\ell_C} &= 2R \\
R_m &= 2R \\
R_n &= 3R.
\end{align*}
\]

(B.7)

The magnetic theory also has a tree level superpotential which schematically looks like

\[
\tilde{W} = mqsq + q'' sq''' + nq\bar{q} + \ell_s q'' \bar{q} + \ell_C q''' \bar{q} + q' sq' + q' \bar{q} t.
\]  

(B.8)

The electric $SO(8)$ and magnetic $SU(\tilde{N}_c) \times Sp(2\tilde{N}_c')$ pair satisfy all the standard anomaly matching, operating mapping and confinement recovery tests of duality [21].

Finally for $N_f \geq 6$, the $SO(8)$ model loses asymptotic freedom and becomes a free field theory in the far infrared.
References

[1] D. Mumford and J. Fogarty, Geometrical Invariant Theory (Springer, 1982).
[2] M.A. Luty and W. Taylor, Phys. Rev. D53 (1996) 3399.
[3] N. Seiberg, Phys. Rev. D49 (1994) 6857.
[4] I. Pesando, Mod. Phys. Lett A10 1995, 1871.
[5] S.B. Giddings and J.M. Pierre, Phys. Rev. D52 (1995) 6065.
[6] P. Pouliot, Phys. Lett. B359 (1995) 108.
[7] F. Gürsey, in Second Workshop on Current Problems in High Energy Particle Theory, edited by G. Domokos and S. Köresi-Domokos, (Johns Hopkins Press, Baltimore, 1977), p. 3.
[8] Y.I. Kogan, A.Y. Morozov, M.A. Olshanetskii and M.A. Shifman, Sov. J. Nucl. Phys. 43 (1986) 1022.
[9] A.G. Elashvili, Funk. Anal. Pril. 6 (1972) 51.
[10] T.W. Kephart and M.T. Vaughn, Ann. Phys. 145 (1983) 162.
[11] F. Gursey, P. Ramond and P. Sikivie, Phys. Lett. B177 1976.
[12] A. DeRujula, H. Georgi and S.L. Glashow, in Fifth Workshop on Grand Unification, edited by K. Kung, H. Fried and P. Frampton (World Scientific, Singapore, 1984) p. 88.
[13] R. Slansky, Phys. Rept. 79 (1981) 1.
[14] J.C. Pati and A. Salam, Phys. Rev. D10 (1974) 275.
[15] G. ’t Hooft, Phys. Rev. Lett. 37 (1976) 8; Phys. Rev. D14 (1976) 3432.
[16] J. Preskill, S. Trivedi, F. Wilczek and M. Wise, Nucl. Phys. B363 (1991) 207.
[17] L. Ibáñez and G. Ross, Phys. Lett. 260B (1991) 291; Nucl. Phys. B368 (1992) 3; L. Ibáñez, Nucl. Phys. B398 (1993) 301.
[18] T. Banks and M. Dine, Phys. Rev. D45 (1992) 1424.
[19] C. Csàki and H. Murayama, hep-th 9710103.
[20] I. Affleck, M. Dine and N. Seiberg, Nucl. Phys. B241 (1984) 493.
[21] N. Seiberg, Nucl. Phys. B435 (1995) 129.
[22] P. Ramond, Phys. Lett. B390 (1997) 179.
[23] J. Distler and A. Karch, Fortsch. Phys. 45 (1997) 517.
[24] A. Karch, Phys. Lett. B405 (1997) 280.
[25] R.G. Leigh and M.J. Strassler, Nucl. Phys. B496 (1997) 132.
[26] M. Berkooz, P. Cho, P. Kraus and M. Strassler, Phys. Rev. D56 (1997) 7166.