Chemical and Kinematic Properties of the Galactic Disk from the LAMOST and Gaia Sample Stars

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Abstract

We determined the chemical and kinematic properties of the Galactic thin and thick disk using a sample of 307,246 A/F/G/K-type giant stars from the LAMOST spectroscopic survey and Gaia DR2 survey. Our study found that the thick disk globally exhibits no metallicity radial gradient, but the inner disk (R ≤ 8 kpc) and the outer disk (R > 8 kpc) have different gradients when they are studied separately. The thin disk also shows two different metallicity radial gradients for the inner disk and the outer disk, and has a steep metallicity vertical gradient of d[Fe/H]/dz = -0.12 ± 0.0007 dex kpc⁻¹, which becomes flat when it is measured at increasing radial distance. The metallicity radial gradient on the other hand becomes weaker with increasing vertical distance. Adopting a Galaxy potential model, we derived the orbital eccentricity of sample stars and found a downtrend of average eccentricity with increasing metallicity for the thick disk. The variation of the rotation velocity with the metallicity shows a positive gradient for the thick disk stars and a negative one for the thin disk stars. Comparisons of our observed results with models of disk formation suggest that radial migration could have influenced the chemical evolution of the thin disk. The formation of the thick disk could be affected by more than one process: the accretion model could play an indispensable role, while other formation mechanisms, such as the radial-migration or disk-heating model could also have a contribution.

Key words: Galaxy: disk – Galaxy: evolution – Galaxy: formation – Galaxy: kinematics and dynamics – stars: abundances

1. Introduction

It has been suggested that the disk of the Milky Way can be divided into two components: the thin disk and the thick disk, as introduced by Gilmore & Reid (1983). The two components differ in their spatial distribution, metallicity, and kinematics. In the spatial distribution, the range of scale height and length for the thin disk are about 200–360 pc and 1.00–3.7 kpc, respectively, whereas the thick disk has a scale height of 600–1000 pc and a scale length of 2.0–5.5 kpc (Du et al. 2003, 2006; Bilir et al. 2006, 2008; Karanli et al. 2007; Jurić et al. 2008; Yaz & Karanli 2010; Chang et al. 2011; Jia et al. 2014; Chen et al. 2017; Wan et al. 2017), but these results are still contentious. The metallicity gradients of the two disks have been studied in previous works based on a variety of samples. For example, in the radial direction, Recio-Blanco et al. (2014) found no metallicity radial gradient for the thick disk while Coşkunoluğlu et al. (2012) and Li et al. (2018) derived a positive metallicity gradient. For the thin disk, most studies have found a negative metallicity radial gradient (e.g., Bilir et al. 2012; Boeche et al. 2013). In the vertical direction, Chen et al. (2011), Carrell et al. (2012), Li & Zhao (2017), and Tunçel Güçütkin et al. (2017) found a negative metallicity gradient of the thick disk, while Katz et al. (2011) and Li et al. (2018) derived a flatter gradient. For the thin disk, a negative metallicity gradient was also found in the vertical direction (Bilir et al. 2012; Duong et al. 2018), while Mikolaitis et al. (2014) derived a flatter gradient. In addition, some studies have also shown that there exists an obvious relationship between the rotation velocity and the metallicity for the disk stars. Lee et al. (2011) and Adibekyan et al. (2013) showed that this relationship is different for the thin and thick disk components.

Breifly, the thick-disk population has been characterized as containing stellar populations that are older (e.g., Wyse & Gilmore 1988; Bensby et al. 2014), enriched in [α/Fe] and metal poor (Prochaska et al. 2000; Lee et al. 2011; Fuhrmann et al. 2017), and showing higher velocity dispersions (Chiba & Beers 2000) compared with the typical thin-disk population. These different properties imply that the two populations may have different formation and evolutionary histories. Some models of formation have been proposed for the thick disk, such as the following scenarios: (a) accretion (Abadi et al. 2003), (b) heating (e.g., Quinn et al. 1993; Kazantzidis et al. 2008; Villalobos & Helmi 2008; Villalobos et al. 2010), (c) radial migration (e.g., Sellwood & Binney 2002; Schönrich & Binney 2009; Schönrich & McMillan 2017), and (d) a gas-rich merger (e.g., Brook et al. 2004, 2005). These models predict different trends between the kinematic properties and metallicity of disk stars, as well as between their kinematics and spatial distributions. For example, models of disk heating via satellite mergers or a growing thin disk can induce a notable increase in the mean rotation and velocity dispersions of the thick disk stars in comparison to models with an initial thick disk (Villalobos et al. 2010). Furthermore, models involving a gas-rich merger predict a rotational velocity gradient with galactocentric distance for disk stars near the solar radius (Brook et al. 2007). Sales et al. (2009) showed that the distribution of orbital eccentricities for nearby thick disk stars could provide constraints on these proposed formation models. Jing et al. (2016) determined a preferential scenario by comparing the orbital eccentricity distribution with these simulations, and their results agree with a gas-rich merger model of thick-disk formation. Therefore, the properties of...
metallicity and kinematics of the thick disk could provide strong constraints on formation.

To study the chemical and kinematic characteristics of the disks, it is necessary to separate the thick disk from thin disk. In general, there are four main ways to distinguish the local thick and thin disk stars at the solar annulus (Adibekyan et al. 2011): a pure kinematical approach (Bensby et al. 2003), a pure chemical approach (e.g., Lee et al. 2011; Duong et al. 2018), a combination of metallicity and spatial distribution (Jing et al. 2016), and a combination of kinematics, metallicity, and stellar age (Haywood 2008). Recently, researches found a gap in the \([\alpha/Fe]-[Fe/H]\) plane for star samples (e.g., Lee et al. 2011; Bovy et al. 2016; Duong et al. 2018; Liu et al. 2018), which has been widely used to separate thick and thin disk stars. The thin disk is often defined as the low-\([\alpha/Fe]\) population, with the thick disk population being described as high-\([\alpha/Fe]\). To understand the formation of the Galaxy components, we need more accurate chemical and kinematic information of a large number of stars. Large-scale spectroscopic surveys make this possible by providing ideal stellar atmospheric parameters such as metallicity and surface gravity. The ongoing Large Sky Area Multi-Object Fiber Spectroscopic Telescope survey (LAMOST, also called Guoshoujing Telescope; Zhao et al. 2012) has released more than five million stellar spectra with stellar parameters in the DR5 catalog. The kinematic studies require accurate proper motions and parallaxes with sufficiently small uncertainties. The second Gaia data release of Gaia survey (Gaia Collaboration et al. 2018a, 2018b) has provided an unprecedented sample of precisely and accurately measured sources. These data sets will provide a vast resource to study the details of the velocity distribution and give constraints on the formation of the galactic disk.

In the present study, we used data from the LAMOST spectroscopic survey and Gaia (DR2) survey to study the chemistry and kinematics of the Galactic disk of the Milky Way. The paper is structured as follows. Section 2 introduces observational data from LAMOST and Gaia, determines the distance and velocity of sample stars, and describes the sample selection. Section 3 presents the result of metallicity and \(\alpha\)-abundance variation with radial distance and vertical height. Section 4 investigates the distribution of orbital eccentricities and its trends with metallicity and vertical height, and presents the results of rotational velocity variation with radial distance, vertical height, and metallicity. Section 5 presents a discussion of the selection bias of our sample, and the formation and evolution of the thin disk and thick disk using our observed results. The summary and conclusions are given in Section 6.

2. Data

2.1. LAMOST and Gaia

The LAMOST, also called the GuoShouJing Telescope, is located at Xinglong Station of the National Astronomical Observatories at the Chinese Academy of Sciences (NAOC). It is a reflecting Schmidt telescope with an effective aperture of 3.6–4.9 m and 4000 fibers within a field of view of 5°. The LAMOST spectrograph has a resolution of \(R \sim 1800\) and an observed wavelength range that spans 3700 Å ∼ 9000 Å (Cui et al. 2012; Zhao et al. 2012). Its observable sky covers \(-10° \sim +90°\) DEC. and the survey reaches a limiting magnitude of \(r = 17.8\) (where \(r\) denotes magnitude in the SDSS \(r\)-band). In 2017 June, the LAMOST completed a five year survey operation, and in 2017 December the fifth LAMOST dataset was released, LAMOST DR5, containing more than 9 million spectra in total. Of these, \(\sim 5.34\) million are for A/F/G/K-type stars with estimated stellar atmospheric parameters as well as radial velocities, which provides a powerful basic dataset for astronomers to study the structure and formation of the Milky Way. For example, based on LAMOST DR3 data, Liu et al. (2017), Xu et al. (2018), and Wang et al. (2018) derived the stellar density profiles of the Milky Way and the spatial structure of the outer disk. Furthermore, Li et al. (2016, 2019) used LAMOST M giants to study the Sagittarius stream.

The stellar atmospheric parameters \([Fe/H], [\alpha/Fe],\) and surface gravity used in the present study are from LSS-GAC DR4 catalog. The LAMOST Spectroscopic Survey of the Galactic Anticentre (LSS-GAC) is a major component of the LAMOST Experiment for Galactic Understanding and Exploration. LSS-GAC Stellar Parameter Pipeline at Peking University [LSP3] (Xiang et al. 2015, 2017b) determines atmospheric parameters by template matching with the MILES spectral library (Sánchez-Blázquez et al. 2006). The MILES spectral library consists of 985 stars spanning a wide range of stellar atmospheric parameters. The wavelength coverage of the spectra is 3525–7410 Å, and the spectral resolution is about 2.5 Å. The latter is close to that of the LAMOST spectra. The current implementation of LSP3 has achieved an accuracy of 150 K, 0.25 dex, and 0.15 dex for the effective temperature, surface gravity, and metallicity, respectively, for the LSS-GAC spectra of F/G/K-type stars with a signal-to-noise ratio \((S/N)\) per pixel higher than 10 (Xiang et al. 2015). Values of \(\alpha\)-element (Mg, Si, Ca and Ti) to iron abundance ratio \([\alpha/Fe]\) have also been derived with LSP3, with precisions similar to those achieved by the APOGEE survey for the giant stars (Xiang et al. 2017a). In addition, the radial velocity is from LAMOST DR5 catalog. The determination of the radial velocity makes use of the ELODIE library (Wu et al. 2011).

The Gaia is a space-based mission mostly concentrating on astrometry that was launched by the European Space Agency (ESA) in December 2013 (Gaia Collaboration et al. 2016). The Gaia mission has already released its second set of data, the Gaia DR2, which provides accurate positions, parallaxes, and proper motions for 1.3 billion sources brighter than magnitude \(G \sim 21\) mag, and line of sight velocities for 7.2 million stars brighter than \(GRVS = 12\) mag (Gaia Collaboration et al. 2018a, 2018b). The median uncertainty for the bright sources \((G < 14\) mag) is 0.04 mas, 0.1 mas at \(G = 17\) mag, and 0.7 mas at \(G = 20\) mag for the parallax, and 0.05, 0.2, and 1.2 mas yr\(^{-1}\) for the proper motions, respectively. A more detailed description of the astrometric content of the Gaia DR2 can be found in Lindegren et al. (2018).

2.2. Distance and Velocity Determination

In this work, we use the Gaia DR2 proper motion and parallax data and restrict parallax uncertainties \((\varpi_{\text{error}}/(\varpi - \varpi_{\text{err}}))\) to smaller than 20%. The quantity \(\varpi\) denotes stellar parallax, and \(\varpi_{\text{err}}\) is the global parallax zero-point of Gaia observations. Butkevich et al. (2017) confirmed that various instrumental effects of the Gaia satellite, and particularly a certain kind of basic-angle variation, can bias the parallax zero-point of an astrometric solution derived from observations. This global parallax zero-point was determined in
Lindgren et al. (2018) based on observations of quasars: $\omega_{zp} = -0.029 \text{ mas}$. Therefore, it is necessary to subtract parallax zero-point ($\omega_{zp}$) when parallax is used to calculate astrophysical quantities (Li et al. 2019). We also restrict the error of proper motion in R.A. ($\mu_{\alpha, \ast}$) and in DEC. direction ($\mu_{\delta, \ast}$) to smaller than 0.2 mas yr$^{-1}$. We select giant stars by restricting $0 < \log(g) < 3.5$, and restrict these stars with $S/N > 20$ in the g-band, radial velocity uncertainties smaller than 10 km s$^{-1}$, and error of [Fe/H] smaller than 0.2 dex. In total, we obtain 307,246 giant stars.

The Gaia DR2 provides precise position and parallax for an unprecedented number of objects, and some astrophysical quantities such as distance and velocities can be inferred using those data. The above restrictions are important when deriving distance and velocities, especially when parallaxes are involved, because the effects of the observational errors on the parallaxes and the proper motions can lead to potentially strong biases (Luri et al. 2018). By comparison of the heliocentric distance computed by inverting the Gaia DR2 parallax with that derived by Bailer-Jones et al. (2018) using the geometrical distance prior for our sample (parallax uncertainties smaller than 20%), we found that the heliocentric distances of stars are precisely determined simply by inverting the parallax ($1/(\omega - \omega_{zp})$) for nearby ($1/(\omega - \omega_{zp}) < 2$ kpc) sample stars. Below we show how determine distances and velocities of stars for nearby ($1/(\omega - \omega_{zp}) < 2$ kpc) and distant ($1/(\omega - \omega_{zp}) > 2$ kpc) stars.

2.2.1. Nearby Sample Stars

For our sample, 143,303 out of 307,246 (~46.6%) are nearby ($1/(\omega - \omega_{zp}) < 2$ kpc) stars whose heliocentric distances are computed by inverting the parallax from Gaia DR2: $d = 1/(\omega - \omega_{zp})$. We transformed the Galactic coordinates ($l$, $b$) and heliocentric distances for the stars into a Cartesian galactocentric coordinate system ($x$, $y$, $z$), and derived the projected distance from the Galactic center using coordinate transformations (Bond et al. 2010):

$$
\begin{align*}
x &= R_\odot - d \cos(l) \cos(b) \\
y &= -d \sin(l) \cos(b) \\
z &= d \sin(b).
\end{align*}
$$

Here, we adopt the distance from the Sun to the Galactic center, $R_\odot = 8.2$ kpc (Bland-Hawthorn & Gerhard 2016). Furthermore, $d$ is the distance from the star to the Sun, and $l$ and $b$ are the Galactic longitude and latitude. The proper motions together with the radial velocity are used to derive Galactic velocity components ($U$, $V$, $W$), and galactocentric cylindrical components $V_c$ and their errors. Here we adopt a local standard of rest velocity $V_{LSR} = 220$ km s$^{-1}$, and the solar peculiar motion ($V_U^0$ = pec, $V_V^0$ = pec, $V_W^0$ = pec) = (10.0 km s$^{-1}$, 11.0 km s$^{-1}$, 7.0 km s$^{-1}$) (Tian et al. 2015; Bland-Hawthorn & Gerhard 2016).

2.2.2. Distant Sample Stars

For the 163,943 (~53.4%) distant ($1/(\omega - \omega_{zp}) > 2$ kpc) stars of our sample, we use the Bayesian approach following Bailer-Jones (2015), Astraatmadja & Bailer-Jones (2016a, 2016b), and Luri et al. (2018) to determine the stellar distance and velocity. According to Bayes formula, the posterior probability $P(\theta | x)$ of an observed star can be obtained, where $x$ and $\theta$ are the observed data vector and parameter vector that we expect to obtain, respectively. The posterior probability $P(\theta | x)$ denotes the probability distribution of the parameters under given observed data. The data vector are the parallax ($\omega$), proper motion in R.A. ($\mu_{\alpha, \ast}$) and DEC. ($\mu_{\delta, \ast}$), written as the column vector,

$$
x = (\omega - \omega_{zp}^0, \mu_{\alpha, \ast}^0, \mu_{\delta, \ast}^0)^T.
$$

The symbol “T” stands for matrix transpose. The data vector ($x$) is in units of mas, mas yr$^{-1}$, and mas yr$^{-1}$, respectively, and has the following covariance matrix:

$$
\Sigma = \begin{pmatrix}
\sigma_\omega^2 & \sigma_\omega \sigma_{\mu_{\alpha, \ast}} \rho(\omega, \mu_{\alpha, \ast}) & \sigma_\omega \sigma_{\mu_{\delta, \ast}} \rho(\omega, \mu_{\delta, \ast}) \\
\sigma_{\mu_{\alpha, \ast}} \rho(\omega, \mu_{\alpha, \ast}) & \sigma_{\mu_{\alpha, \ast}}^2 & \sigma_{\mu_{\alpha, \ast}} \sigma_{\mu_{\delta, \ast}} \rho(\mu_{\alpha, \ast}, \mu_{\delta, \ast}) \\
\sigma_{\mu_{\delta, \ast}} \rho(\omega, \mu_{\delta, \ast}) & \sigma_{\mu_{\alpha, \ast}} \sigma_{\mu_{\delta, \ast}} \rho(\mu_{\alpha, \ast}, \mu_{\delta, \ast}) & \sigma_{\mu_{\delta, \ast}}^2
\end{pmatrix},
$$

where $\rho(i, j)$ denotes the correlation coefficient between the astrometric parameters $i$ and $j$, and $\sigma_k$ is the error of astrometric parameters $k$. The parameter vectors are heliocentric distance ($d$), tangential speed ($v$), and travel direction ($\phi$, increasing anti-clockwise from north), written as

$$
\theta = (d, v, \phi)^T,
$$

in units of pc, km s$^{-1}$, and radians, respectively. If there is no error in data vector, components of the parameter vectors $\theta$ are given by the simple geometrical transformation

$$
\begin{align*}
d &= \frac{10^3}{\omega - \omega_{zp}} \\
v &= 4.74 \sqrt{\frac{\mu_{\alpha, \ast}^2 + \mu_{\delta, \ast}^2}{\text{mas yr}^{-1} (\omega - \omega_{zp})}} \text{ km s}^{-1} \\
\phi &= \arctan \left( \frac{\mu_{\delta, \ast}}{\mu_{\alpha, \ast}} \right).
\end{align*}
$$

The 3D posterior over the parameters $\theta$ is calculated according to the Bayes formula

$$
P(\theta | x) \propto P(x | \theta) P(\theta),
$$

where $P(x | \theta)$ is the likelihood probability. The likelihood probability usually represents an adopted model. The symbol $\theta$ represents the parameters of this model. The likelihood probability $P(x | \theta)$ is a multidimensional Gaussian distribution centered on $m$,

$$
m = \begin{pmatrix}
\frac{10^3}{d} & c_2 \frac{10^3 \sin \phi}{d} & c_2 \frac{10^3 \cos \phi}{d}
\end{pmatrix}^T,
$$

where $c_2 = (pc \text{ mas yr}^{-1})/(4.74 \text{ km s}^{-1})$, which is a result of the conversion of units of astrophysical quantities, and $m$ represents a set of theoretical values predicted by this model. The likelihood probability can be written as

$$
P(x | \theta) \propto \exp \left[ -\frac{1}{2} (x - m(\theta))^T \Sigma^{-1} (x - m(\theta)) \right].
$$

Here a separable prior distribution is used (Luri et al. 2018):

$$
P(\theta) = P(d) P(v) P(\phi)
$$

[Where $P(d)$ represents the distribution of $d$, and $P(v)$ and $P(\phi)$ are the distributions of $v$ and $\phi$ respectively].
with
\[
P(d) \propto \begin{cases} 
  d^2e^{-d/L(a,b)} & d > 0 \\
  0 & d \leq 0
\end{cases}
\]  \tag{10}
\[
P(v) \propto \begin{cases} 
  \left(\frac{v}{v_{\text{max}}}\right)^{\alpha - 1}(1 - \frac{v}{v_{\text{max}}})^{\beta - 1} & 0 \leq v \leq v_{\text{max}} \\
  0 & \text{otherwise}
\end{cases}
\]  \tag{11}
\[
P(\phi) \propto \frac{1}{2\pi}.
\]  \tag{12}

The prior distribution of distance is the exponentially decreasing space density introduced in Bailer-Jones (2015). Here, we adopt the length scale of Galactic longitude- and latitude-dependent (Bailer-Jones et al. 2018) \(L(a, b)\), which is obtained by fitting a spherical harmonic model. The prior over speed is a beta distribution, and we adopt \(\alpha = 2, \beta = 3\) and \(v_{\text{max}} = 750\ \text{km s}^{-1}\). The prior over the angle \(\phi\) is uniform.

Although the 3D posterior probability distribution \(P(\theta|\chi)\) can be obtained using Equations (6), (8) and (9), the posterior probability does not have a simple form. Therefore, we characterize the posterior probability using the Markov chain Monte Carlo (MCMC) sampler EMCEE (Goodman & Weare 2010; Foreman-Mackey et al. 2013). We run each chain using 100 walkers and 100 steps, for a total of 10,000 random samples drawn from the posterior distribution \(P(\theta|\chi)\).

We also sample 10,000 random samples for radial velocity and assume uniform priors on radial velocity. The radial velocity and its error are provided by LAMOST catalog and do not depend on parallax and proper motion. For each star, we obtain 10,000 posterior samples including its heliocentric distance (\(d\)), tangential speed (\(v\)), direction of travel (\(\phi\)) and radial velocity (\(r_v\)). We then directly use these random samples to derive Cartesian galactocentric coordinates (\(x, y, z\)), the projected distance from the Galactic center, Galactic velocity components (\(U, V, W\)), and the galactocentric cylindrical component \(V_\varphi\) as described in Section 2.2.1. Here we assume the same parameters for \(R_\odot, V_{\text{LSR}}, \text{and solar peculiar motion as presented in Section 2.2.1.}\) We choose the median as an estimator of these astrophysical quantities, and the standard deviation of the quantities defines the uncertainty in the estimate value.

### 2.3. Sample Selection

The top panel of Figure 1 shows the range of galactocentric radius covered by our sample of 307,246 giant stars within \(4 < R < 15\ \text{kpc}\), extending up to 6 kpc in height from the Galactic plane. In this work, we combine the kinematics with the chemical abundances ([\(\alpha/Fe\]) and [\(Fe/H\)]) to distinguish the local thick-disk and thin-disk stars. The thick-disk stars are believed to have higher [\(\alpha/Fe\)] ratios and lower [\(Fe/H\)] than the thin-disk stars based on several previous element abundance analyses (Bensby et al. 2003, 2005). Therefore, the dividing line in [\(\alpha/Fe\)] versus [\(Fe/H\)] is widely adopted to define high-[\(\alpha/Fe\)] and low-[\(\alpha/Fe\)] populations. However, the adopted dividing line between the high- and low-[\(\alpha/Fe\)] populations is different in some studies. The [\(\alpha/Fe\)]-[\(Fe/H\)] distribution is shown in the bottom panel of Figure 1. As shown in this panel, there is a gap that divides sample stars into low-[\(\alpha/Fe\)] and high-[\(\alpha/Fe\)] populations. The determination of our dividing line between the high- and low-[\(\alpha/Fe\)] populations follows Adibekyan et al. (2011) and Recio-Blanco et al. (2014).
Figure 3. Top panel: [Fe/H] distribution for the high-[α/Fe] and low-[α/Fe] populations. Bottom panel: [Fe/H] distribution for the thin-disk and thick-disk components selected according to a gap in the [α/Fe] versus [Fe/H] plane and kinematics. They can be described by a two-Gaussian model with peaks at [Fe/H] ∼ −0.21 with σ_{Fe/H} ∼ 0.20 for the thin disk, and [Fe/H] ∼ −0.52 with σ_{Fe/H} ∼ 0.23 for the thick disk with [Fe/H] > −1.2 dex. The metallicity functions of the global thick disk are not Gaussian however, and it has an extended metallicity tail.

As shown in Figure 2, we divide the sample into eight metallicity bins from [Fe/H] = −2.4 to 0.5 and identify the minima in the [α/Fe] histograms for each bin. We determine eight separation points in the [α/Fe]-versus-[Fe/H] plane according to minima in the [α/Fe] histograms for each bin: (−2.4, 0.16), (−0.7, 0.16), (−0.5, 0.16), (−0.35, 0.15), (−0.25, 0.14), (−0.15, 0.13), (−0.05, 0.12), and (0.5, 0.12). The separation curve in the bottom panel of Figure 1 is the simple connection of these separation points. The high-[α/Fe] population is defined as stars above the separation curve (red line in the bottom panel of Figure 1), while the low-[α/Fe] population is defined as the stars below it. The high-[α/Fe] population extends from [Fe/H] ≈ −2.4 to 0.1 and the low-[α/Fe] from [Fe/H] ≈ −0.9 to 0.4, as shown in the top panel of Figure 3.

The kinematic approach to defining the thick and thin disk stars proposed by Bensby et al. (2003) was adopted by many studies (e.g., Reddy et al. 2006; Coşkunoğlu et al. 2012; Bensby et al. 2014; Li & Zhao 2017). This method assumes that the Galactic velocities (U_{LSR}, V_{LSR}, W_{LSR}) have Gaussian distribution given by the equation (Bensby et al. 2003)

\[ f(U, V, W) = k \cdot \exp \left( \frac{U_{LSR}^2}{2\sigma_U^2} - \frac{(V_{LSR} - V_{sym})^2}{2\sigma_V^2} - \frac{W_{LSR}^2}{2\sigma_W^2} \right) \]

where

\[ k = \frac{1}{(2\pi)^{3/2}\sigma_U\sigma_V\sigma_W}. \]

Here, \( \sigma_U, \sigma_V, \) and \( \sigma_W \) are the characteristic velocity dispersions, and \( V_{sym} \) is the asymmetric drift; their values are listed in Table 1. \( U_{LSR}, V_{LSR}, \) and \( W_{LSR} \) are stellar velocity relative to the Local Standard of Rest. By dividing the thick-disk probability (TD) by the thin-disk (D) probability, we obtain the relative probabilities for thick-disk-to-thin-disk (TD/D) as follows:

\[ \frac{TD}{D} = \frac{X_{TD} \cdot f_{TD}}{X_D \cdot f_D}. \]

| X     | \( \sigma_U \) | \( \sigma_V \) | \( \sigma_W \) | \( V_{sym} \) |
|-------|----------------|----------------|----------------|----------------|
| Thin disk (D) | 0.94 | 35 | 20 | 16 | −15 |
| Thick disk (TD) | 0.06 | 67 | 38 | 35 | −46 |
| Halo (H) | 0.0015 | 160 | 90 | 90 | −220 |

Here, \( X \) is the observed fraction of stars for the populations in the solar neighborhood, and \( X_{TD} \) and \( X_D \) represent the fraction for the thick disk and the thin disk, respectively. Their values are listed in Table 1. The factors \( f_{TD} \) and \( f_D \) represent the Gaussian distribution of Galactic velocities for the thick and thin disk, and can be calculated with Equations (13) and (14) for a given star with Galactic velocities \( (U_{LSR}, V_{LSR}, W_{LSR}) \). The factors TD and D are the probabilities that the given stars belong to the thick disk and the thin disk, respectively. We selected TD/D > 5 (implying those stars are five times more likely to be thick disk stars than thin disk stars) from the high-[α/Fe] population as the thick disk stars, and those TD/D < 0.2 from low-[α/Fe] population as the thin disk stars. The Toomre diagram is also an effective way of displaying the sample, and has been widely used to define halo stars (e.g., Venn et al. 2004; Nissen & Schuster 2010; Bonaca et al. 2017).

In this work, we use the Toomre diagram to exclude halo stars. Figure 4 presents the Toomre diagram of the sample stars, and the dashed line shows the values of the total spatial velocity \( v_{tot} = \sqrt{U_{LSR}^2 + V_{LSR}^2 + W_{LSR}^2} = 180 \text{ km s}^{-1} \). The thick-disk, thin-disk, and halo stars are represented by blue, red, and black dots, respectively. Also, we limited the disk sample stars within \( v_{tot} < 180 \text{ km s}^{-1} \) (Nissen & Schuster 2010; Bensby et al. 2014; Bonaca et al. 2017).

We find that our sample of the thick disk contains few metal-poor stars (645 thick disk stars with \(-1.8 < [Fe/H] < -1.2\) dex and 75 stars with \(-2.4 < [Fe/H] < -1.8\) dex), and they account for about 2.4% of the thick disk sample. It is well

| observed fraction of stars for the populations in the solar neighborhood (X), Asymmetric Drift (V_{sym}), and Characteristic Velocity Dispersions (\( \sigma_U, \sigma_V, \) and \( \sigma_W \)) | X     | \( \sigma_U \) | \( \sigma_V \) | \( \sigma_W \) | \( V_{sym} \) |
|--------------------------------------------------|-------|----------------|----------------|----------------|----------------|
| Thin disk (D)                                    | 0.94  | 35             | 20             | 16             | −15            |
| Thick disk (TD)                                  | 0.06  | 67             | 38             | 35             | −46            |
| Halo (H)                                         | 0.0015| 160            | 90             | 90             | −220           |
known that the halo stars are more metal-poor (e.g., Carollo et al. 2007, 2010; Du et al. 2018; Li et al. 2019) and the existence of the metal-weak thick disk (MWTD) has been confirmed by many studies (e.g., Morrison et al. 1990; Beers & Sommerlarsen 1995; Chiba & Yoshii 1998; Martin & Morrison 1998; Chiba & Beers 2000; Beers et al. 2002, 2014; Tian et al. 2019). These metal-poor stars may therefore be contaminated by halo stars or, at least partially, are MWTD stars. However, we find that these metal-poor stars of the thick disk ([Fe/H] < −1.2 dex) have a higher mean rotational velocity, \( V_\odot \sim 114 \text{ km s}^{-1} \), than halo stars of the slight spin (Carollo et al. 2010; Deason et al. 2017), and this value is consistent with the result of Carollo et al. (2010) where the mean rotational velocity of the MWTD ([Fe/H] < −0.8 dex) was found to be 125 ± 4 km s\(^{-1}\). Therefore, the assumption that our metal-poor stars may be contaminated by halo stars can be excluded. Alternatively, they probably belong to the MWTD. Carollo et al. (2010) reported that the metallicity random stars that are likely members of the MWTD is −1.8 ≤ [Fe/H] ≤ −0.8 dex, and therefore we exclude 75 stars with [Fe/H] < −1.8 dex for the thick disk. In summary, our sample of the thick disk contains both the canonical thick disk and the MWTD.

Here we briefly summarize our selection criteria of the thin disk and thick disk. We obtained 29,966 thick disk stars with high-[\( \alpha/\text{Fe} \)], [Fe/H] > −1.8 dex, TD/D > 5, \( V_\odot < 180 \text{ km s}^{-1} \), and 179,092 thin disk stars with low-[\( \alpha/\text{Fe} \)] and TD/D < 0.2. The bottom panel of Figure 3 indicates that the metallicity distribution of the thick- and thin-disk populations can be described by a two-Gaussian model with peaks at [Fe/H] ~ −0.21 and standard deviation of σ[Fe/H] ~ 0.20 for the thin disk, and [Fe/H] ~ −0.52 with σ[Fe/H] ~ 0.23 for the thick disk stars with [Fe/H] > −1.2 dex. The metallicity function of the global thick disk has an extended metallicity tail, which indicates the existence of the MWTD.

3. Metallicity and \( \alpha \)-abundance Gradient of the Galactic Disk

In this section, we use our sample stars to examine the observed gradients of metallicity with \( R \) and \( z \), as well as the gradient of \( \alpha \)-abundance with \( |z| \) for the thin disk and the thick disk populations. First, we introduce the estimation of gradient and its error.

3.1. Gradient and its Error Estimation

As an example, we only introduce the estimation of metallicity radial gradient and its error. For a given sample, the total number of stars is \( N \) and each star is arranged in order, numbered \( i \) \((i = 1, 2, 3, ..., N) \). Also, we consider [Fe/H], as a function of radial distance, \( R_0 \), and [Fe/H], has a noise \( \sigma_r \). The maximum likelihood estimation is used to estimate gradient, and the likelihood function is as follows:

\[
P([\text{Fe/H}] \mid R, \sigma, k, b) = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{[\text{Fe/H}] - kR_i - b}{{2\sigma^2}}},
\]

where \( k \), \( b \), and \( \sigma \) denote fitting slope, intercept, and [Fe/H] error, respectively. The likelihood function is a Gaussian function. The slope and intercept can be evaluated by maximizing the likelihood function. A more detailed discussion on fitting the slope can be found in Hogg & Bovy (2010).

We adopt the Bayesian approach to estimate the error of the metallicity radial gradient. The data vectors include metallicity, [Fe/H] error, and radial distance. The parameter vectors are \( k \) and \( b \). The posterior over the parameters is

\[
P(k, b \mid [\text{Fe/H}], \sigma) \propto P([\text{Fe/H}] \mid R, \sigma, k, b)P(k, b),
\]

where the marginalization is used as a likelihood function, and \( P([\text{Fe/H}] \mid R, \sigma, k, b) \) is given in Equation (16). We use uniform priors

\[
P(k, b) = P(k)P(b)
\]

with

\[
P(k) \propto \begin{cases} 
1 & -1 < k < -1 \\
0 & \text{otherwise} \end{cases}
\]

\[
P(b) \propto \begin{cases} 
1 & -5 < b < 5 \\
0 & \text{otherwise} \end{cases}
\]

The posterior distribution can be obtained, and we characterize it by using Markov chain Monte Carlo (MCMC) sampler EMCEE. We run 300 burn-in steps to let the walkers explore the parameter space and then run each chain using 100 walkers and 100 steps to get a total of 10,000 random gradients from the posterior distribution. The standard deviation of 10,000 random gradients is used to define the uncertainty of the gradient in the estimation.

3.2. Radial Gradients of the Galactic Disk

The metallicity gradient with radial distance for the thick-disk stars is given in the top panel of Figure 5. The metallicity radial gradient of the thick disk is \( d[\text{Fe/H}] / dR = +0.0006 \pm 0.0005 \text{ dex kpc}^{-1} \). Our result therefore indicates that the thick disk has a mainly flat radial distribution, which is consistent with the results of Recio-
Blanco et al. (2014), Mikolaitis et al. (2014), and Peng et al. (2018) as shown in Table 2. However, our result is different from what was obtained by Coşkunolu et al. (2012), who used about 17,000 F-type and G-type dwarfs from RAdial Velocity Experiment (RAVE) Data Release 3 (DR3). Also, Li & Zhao (2017) and Li et al. (2018) used 2035 thick-disk giant stars from LAMOST DR3 and disk stars from Apache Point Observatory Galactic Evolutionary Experiment data release 13 (DR13 hereafter) combined Tycho-Gaia data to derive a flat gradient for the thick disk. The bottom panel of Figure 5 shows several interesting features. When we study the variation of metallicity radial gradient with radial distance for the thick disk, we find two different metallicity gradients in the inner (R \leq 8 \text{kpc}) and the outer disk (R > 8 \text{kpc}). The inner (R \leq 8 \text{kpc}) disk of the thick disk has a positive gradient \(d[\text{Fe/H}]/dR = +0.022 \pm 0.001 \text{dex kpc}^{-1}\) while the outer disk (R > 8 \text{kpc}) has a negative gradient \(d[\text{Fe/H}]/dR = -0.016 \pm 0.0009 \text{dex kpc}^{-1}\).

The top panel of Figure 6 shows the metallicity gradient with radial distance for the thin-disk stars. Our results show that the thin disk has a negative metallicity gradient along the radial direction, \(d[\text{Fe/H}]/dR = -0.05 \pm 0.0002 \text{dex kpc}^{-1}\). This result is consistent with the results summarized in Table 2 where it can be seen that the thin disk has a negative metallicity gradient (\(-0.058 \leq d[\text{Fe/H}]/dR \leq -0.027 \text{dex kpc}^{-1}\)). The metallicity radial gradient of the thin disk is in good agreement with the results from previous works. It is interesting that two different radial metallicity gradients are also found for the inner disk (R \leq 8 \text{kpc}) and the outer disk (R > 8 \text{kpc}) of the thin disk. The inner (R \leq 8 \text{kpc}) disk of the thin disk has a positive gradient \(d[\text{Fe/H}]/dR = +0.03 \pm 0.002 \text{dex kpc}^{-1}\) while the outer disk (R > 8 \text{kpc}) has a negative gradient \(d[\text{Fe/H}]/dR = -0.054 \pm 0.0002 \text{dex kpc}^{-1}\).

Curir et al. (2012) reported that a positive radial metallicity slope for the inner early Galactic disk (R \lesssim 10 \text{kpc}), combined with the usual decreasing slope in the outer disk (R \gtrsim 10 \text{kpc}), in their N-body simulations (including radial migration), plays a critical role in producing a positive rotation–metallicity correlation in the thick disk. As shown in Table 2, a positive rotational velocity gradient with metallicity for the thick disk has been confirmed by many studies, which provides important evidence for this hypothetical relationship between metallicity and radial distance for the early Galactic disk. Furthermore, Curir et al. (2014) showed that the initial metallicity imprint could not be “washed out” by secular dynamical processes. This variation trend of metallicity distribution with radial distance for the early Galactic disk is consistent with our results concerning the thick and thin disk radial metallicity, and only the turning point of the radial metallicity slope is different. Our results show the variation trend that the metallicity radial gradient is positive in the inner disk and negative in the outer disk for the thick disk and thin disk, which may be this initial metallicity imprint.

3.3. Vertical Chemical Abundance Gradients of the Galactic Disk

We also consider the variations of metallicity [Fe/H] and \(\alpha\)-abundance distributions versus vertical distance \(z\) for the disk stars and study their vertical gradient. The metallicity vertical gradient of the thick disk is \(d[\text{Fe/H}]/dz = -0.074 \pm 0.0009 \text{dex kpc}^{-1}\) as shown in the top panel of Figure 7. This result is roughly consistent with the results from some previous studies, such as Katz et al. (2011), Mikolaitis et al. (2014), and Duong et al. (2018) in Table 2. Chen et al. (2011) on the other hand used a sample of 1728 red horizontal-branch (RHB) stars with 0.5 \(< z < 3 \text{kpc}\) from SDSS DR8 and Li & Zhao (2017) used 2035 thick-disk giant stars from LAMOST DR3 to derive a gradient for the thick disk that was steeper than our result.

The bottom panel of Figure 7 shows the \(\alpha\)-abundance gradient with \(z\) for the thick disk, which is used to derive a very flat gradient of \(d[\alpha/\text{Fe}]/dz = +0.008 \pm 0.0002 \text{dex kpc}^{-1}\). Duong et al. (2018) and Li et al. (2018) also found an almost flat gradient for the thick disk, in good agreement with our result. Mikolaitis et al. (2014), using about 2000 F/G/K-type dwarfs and giants from the GES DR1, measured a steeper gradient for the thick disk: \(d[\alpha/\text{Fe}]/dz = +0.033 \pm 0.002 \text{dex kpc}^{-1}\) for the main sample. When they use a clean sample (S/N > 40), the result is \(d[\alpha/\text{Fe}]/dz = +0.011 \pm 0.005 \text{dex kpc}^{-1}\) which is consistent with our result.

Figure 8 also shows the trends of vertical metallicity and \(\alpha\)-abundance for the thin disk. The top panel of this figure indicates that the thin disk has an obvious metallicity vertical gradient \(d[\text{Fe/H}]/dz = -0.12 \pm 0.0007 \text{dex kpc}^{-1}\), which is in good agreement with results from previous works. For example, Bilir et al. (2012), Duong et al. (2018), and Li et al. (2018) reported \(d[\text{Fe/H}]/dz = -0.109 \pm 0.008 \text{dex kpc}^{-1}\), \(d[\text{Fe/H}]/dz = -0.18 \pm 0.01 \text{dex kpc}^{-1}\), and \(d[\text{Fe/H}]/dz = -0.09 \pm 0.001 \text{dex kpc}^{-1}\), respectively.

The [\alpha/\text{Fe}] gradient with vertical height for the thin-disk stars is given in the bottom panel of Figure 8. The [\alpha/\text{Fe}] vertical gradient of the thin disk is \(d[\alpha/\text{Fe}]/dz = +0.05 \pm 0.0002 \text{dex kpc}^{-1}\). This result agrees with the result of Mikolaitis et al. (2014), \(d[\alpha/\text{Fe}]/dz = +0.041 \pm 0.004 \text{dex kpc}^{-1}\), and is slightly steeper than result given by Li et al. (2018): \(d[\alpha/\text{Fe}]/dz = +0.022 \pm 0.0001 \text{dex kpc}^{-1}\). However, Duong et al. (2018) derived a very flat gradient \(d[\alpha/\text{Fe}]/dz = +0.008 \pm 0.002 \text{dex kpc}^{-1}\) using data from the GALAH survey internal data release.
Table 2
Metalslicity [α/Fe] and Rotational Velocity Gradients of the Thick Disk and Thin Disk in the Literature

| Author          | d\([\text{Fe/H}]\)/dR   | d\([\text{Fe/H}]\)/dz   | d\([\alpha/Fe]\)/dz | \(dV_r/d[\text{Fe/H}]\) | Notes                          |
|-----------------|-------------------------|-------------------------|------------------------|--------------------------|--------------------------------|
| Chen et al. (2011) | ...                      | ...                      | ...                    | ...                      | RHB stars, 0.5 < |z| < 3 kpc               |
| Katz et al. (2011)  | ...                      | −0.12 ± 0.01             | ...                    | ...                      | RHB stars, 1 < |z| < 3 kpc               |
| Lee et al. (2011)   | ...                      | −0.068 ± 0.009           | ...                    | ...                      | ...                            |
| Bilir et al. (2012) | +0.017 ± 0.008           | −0.034 ± 0.003           | ...                    | ...                      | ...                            |
| Coşkunoğlu et al. (2012) | +0.016 ± 0.011       | ...                      | ...                    | ...                      | ...                            |
| Adıbekyan et al. (2013) | ...                      | +0.008 ± 0.007           | −0.072 ± 0.006         | +0.033 ± 0.002           | ...                            |
| Recio-Blanco et al. (2014) | ...                      | −0.021 ± 0.029           | −0.037 ± 0.016         | +0.011 ± 0.005           | ...                            |
| Li & Zhao (2017)    | ...                      | −0.035 ± 0.01            | −0.164 ± 0.010         | ...                      | ...                            |
| Duong et al. (2018) | ...                      | −0.058 ± 0.003           | −0.007 ± 0.002         | ...                      | ...                            |
| Li et al. (2018)    | +0.031 ± 0.001           | ...                      | −0.086 ± 0.001         | −0.001 ± 0.001           | ...                            |
| Peng et al. (2018)  | ...                      | −0.001 ± 0.020           | ...                    | ...                      | ...                            |
| Tunçel Güçtekin et al. (2019) | ...                      | −0.164 ± 0.014           | ...                    | ...                      | ...                            |
| This work          | −0.0006 ± 0.0005         | −0.074 ± 0.0009          | +0.008±0.0002          | +30.87 ± 0.001           | ...                            |

| Author          | d\([\text{Fe/H}]\)/dR   | d\([\text{Fe/H}]\)/dz   | d\([\alpha/Fe]\)/dz | \(dV_r/d[\text{Fe/H}]\) | Notes                          |
|-----------------|-------------------------|-------------------------|------------------------|--------------------------|--------------------------------|
| Lee et al. (2011) | ...                      | ...                      | ...                    | ...                      | G-type dwarfs                 |
| Bilir et al. (2012) | ...                      | −0.041 ± 0.003           | −0.109 ± 0.008         | ...                      | ...                            |
| Coşkunoğlu et al. (2012) | ...                      | −0.043 ± 0.005           | ...                    | ...                      | ...                            |
| Adıbekyan et al. (2013) | ...                      | −0.033 ± 0.007           | ...                    | ...                      | ...                            |
| Mikolaitis et al. (2014) | ...                      | −0.044 ± 0.009           | −0.107 ± 0.009         | +0.041 ± 0.004           | ...                            |
| Recio-Blanco et al. (2014) | ...                      | −0.028 ± 0.018           | −0.057 ± 0.016         | +0.036 ± 0.006           | ...                            |
| Guiglion et al. (2015) | ...                      | +0.058 ± 0.008           | ...                    | ...                      | ...                            |
| Prieto et al. (2016) | ...                      | ...                      | ...                    | ...                      | ...                            |
| Duong et al. (2018) | ...                      | −0.18 ± 0.01             | +0.008 ± 0.002         | ...                      | ...                            |
| Li et al. (2018)    | −0.044 ± 0.001           | −0.091 ± 0.001           | +0.022 ± 0.001         | ...                      | ...                            |
| Peng et al. (2018)  | −0.027 ± 0.031           | ...                      | ...                    | ...                      | ...                            |
| Tunçel Güçtekin et al. (2019) | ...                      | −0.308 ± 0.018           | ...                    | ...                      | ...                            |
| This work          | −0.05 ± 0.0002           | −0.12 ± 0.0007           | +0.05 ± 0.0002         | −17.03 ± 0.001           | AFGK-type giants              |

Figure 6. Top panel: Metallicity radial gradient for the thin disk stars. Bottom panel: Metallicity radial gradient for the inner disk (\(R < 8 \text{kpc}\)) and outer disk (\(R > 8 \text{kpc}\)) of the thin disk.

Figure 7. Variation of metallicity and α-abundance with vertical height for the thick-disk stars. Top panel: α-abundance gradient with |z| for the thick-disk stars. Bottom panel: Metallicity gradient with |z| for the thick-disk stars.
3.4. Variation of Metallicity Gradients with Radial Distance and Vertical Height

We now study the variation of metallicity radial gradients versus vertical height and metallicity vertical gradients versus radial distance for the thin- and thick-disk stars. Several subsamples are separated in galactocentric radial distance and vertical height. For each subsample, the gradient and its error are obtained by the method introduced in Section 3.1. Results of gradients and the number of stars in each subsample are given in Tables 3 and 4. The curves in Figure 9 are formed from the simple connection of data points from Tables 3 or 4.

In Figure 9, the top panel gives the variation of metallicity radial gradient versus vertical distance, showing that it flattens with increasing vertical height for the thin disk, and the thick disk (blue solid line) also flattens slightly with increasing vertical height. The bottom panel gives the variation of metallicity vertical gradient versus radial distance, which also shows an almost constant metallicity vertical gradient in the inner region and a flattening with increasing radial distance in the outer region for the thin disk, while the thick disk (blue dashed line) is always slightly flat on average.

4. Kinematical Properties of the Galactic Disk

4.1. Distribution of Stellar Orbital Eccentricities

We study the orbital properties of the thin and thick disk stars by adopting a Galaxy potential model. We use a recent Galactic potential model provided by Schönrich & McMillan (2017). This model includes components that represent the contribution of the cold gas disks near the Galactic plane, as well as the thin and thick stellar disk, a bulge component, and a dark-matter halo. Based on this Galactic potential model, we compute the orbital eccentricities of the thin- and thick-disk stars, $e$, defined as $e = (r_{apo} - r_{peri})/(r_{apo} + r_{peri})$, where $r_{peri}$ and $r_{apo}$ denote the closest approach of an orbit to the Galactic center and the farthest extent of an orbit from the Galactic center.

Figure 10 shows the normalized distributions of orbital eccentricities for the thick- and thin-disk stars. It can be seen that the distribution of orbital eccentricities for the thick- and thin-disk stars is obviously different. The eccentricity distribution of the thin-disk sample stars peaks at $e \sim 0.12$, with narrow widths, and includes very few high-eccentricity stars ($e > 0.3$). In contrast, the distribution for the thick-disk stars shows higher eccentricities and peaks at $e \sim 0.42$, with large widths, and extends to higher eccentricities, up to $e \sim 0.8$.

Sales et al. (2009) demonstrated that the orbital eccentricity distribution could help to probe the formation mechanisms of the thick disk. Four simulation models for the thick disk formation are provided: a radial-migration model (e.g., Sellwood & Binney 2002; Schönrich & Binney 2009), a gas-rich merger model (e.g., Brook et al. 2004, 2005), an accretion
model (Abadi et al. 2003), and a heating scenario (e.g., Quinn et al. 1993; Kazantzidis et al. 2008). Generally, the distribution of stellar orbital eccentricities generated by violent models such as disk heating and accretion include higher-eccentricity stars \((e > 0.6)\). A discussion of the formation mechanisms of the thick disk is given in Section 5.

Figure 9. Top panel: Variation of metallicity radial gradients vs. vertical distance for the thin disk (black dotted line) and thick disk (blue solid line). Bottom panel: Variation of metallicity vertical gradient vs. radial distance for the thin disk (black dotted line) and thick disk (blue solid line). Data points are from Table 3 for the top panel and Table 4 for the bottom panel.

Figure 10. Normalized distributions of orbital eccentricities for the thick disk (black dotted line) and thin disk (red solid line).

### 4.2. Distribution of Stellar Orbital Eccentricities with Metallicity and Vertical Height

We study the variation of orbit eccentricities with metallicity and vertical height for the thin and thick disk. Several subsamples are separated in metallicity and vertical height. More detailed information about subsamples is listed in Tables 3 and 5. The trend of orbital eccentricities with metallicity and vertical height is shown in Figure 11, and the curves are formed from the simple connection of data points from Tables 3 or 5.

The top panel of Figure 11 indicates a downward trend in average orbital eccentricity with increasing metallicity, which is almost constant however in the range of \(-1.0 < [\text{Fe/H}] < -0.6\) dex for the thick disk. In general, the stars with poor metallicity form at an earlier time, which means that the thick-disk stars which have lower orbit eccentricity are the youngest ones. We could think that it is possible that these young stars in the thick disk could originate in the thin disk, reaching the thick disk through radial migration or other mechanisms. However, the orbital eccentricity has a slight uptrend with increasing metallicity for the thin disk. This comparison also indicates that stars with the highest orbital eccentricities are probably the youngest since they are the most metal-rich. These two trends of orbital eccentricity are consistent with the results of Lee et al. (2011) and Peng et al. (2018). The bottom panel of Figure 11 indicates that there is no obvious correlation between orbital eccentricity and vertical height for the thick disk and the thin disk.

We discuss the impact of potentially mixing the thin disk and thick disk in \(-0.6 < [\text{Fe/H}] < -0.2\) dex on trends of the top panel of Figure 11. If our thick-disk sample with high eccentricity were to have a strong effect on the thin disk in the top panel of Figure 11, average orbital eccentricity of the thin-disk in \(-0.6 < [\text{Fe/H}] < -0.2\) dex would be higher than that of thin-disk stars with \([\text{Fe/H}] > -0.2\). However, the orbital

| [Fe/H] (dex) | \(N_{\text{stars}}\) | Mean \(e\) | \(\sigma_e\) |
|-------------|-----------------|-----------|-----------|
| \(-1.8 \leq [\text{Fe/H}] < -1.4\) | 266 | \ldots | \ldots |
| \(-1.4 \leq [\text{Fe/H}] < -0.8\) | 3775 | 0.4571 | 0.165 |
| \(-0.8 \leq [\text{Fe/H}] < -0.7\) | 2793 | 0.4366 | 0.154 |
| \(-0.7 \leq [\text{Fe/H}] < -0.6\) | 4722 | 0.4330 | 0.152 |
| \(-0.6 \leq [\text{Fe/H}] < -0.5\) | 5221 | 0.4259 | 0.145 |
| \(-0.5 \leq [\text{Fe/H}] < -0.4\) | 4723 | 0.4070 | 0.145 |
| \(-0.4 \leq [\text{Fe/H}] < -0.3\) | 3909 | 0.3908 | 0.141 |
| \(-0.3 \leq [\text{Fe/H}] < 0\) | 4823 | 0.3660 | 0.129 |
| \(0 \leq [\text{Fe/H}] < 0.1\) | 15646 | 0.1306 | 0.066 |
| \(0.1 \leq [\text{Fe/H}] < 0.2\) | 19450 | 0.1377 | 0.068 |
| \(0.2 \leq [\text{Fe/H}] < 0.3\) | 26090 | 0.1413 | 0.068 |
| \(0.3 \leq [\text{Fe/H}] < 0.4\) | 31150 | 0.1467 | 0.069 |
| \(0.4 \leq [\text{Fe/H}] < 0.5\) | 32297 | 0.1522 | 0.072 |
| \(0.5 \leq [\text{Fe/H}] < 0.6\) | 28962 | 0.1563 | 0.069 |
| \(0.6 \leq [\text{Fe/H}] < 0.7\) | 17201 | 0.1607 | 0.069 |
| \(0.7 \leq [\text{Fe/H}] < 0.8\) | 8222 | 0.1686 | 0.070 |

Table 5

Number and Mean Orbital Eccentricities (and its Standard Deviation) of the Stars at Different [Fe/H] Bins for the Thick Disk and Thin Disk

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eccentricity of the thin disk has an uptrend with increasing metallicity. Furthermore, the number of thin-disk stars is six times that of thick-disk stars in our samples. Thus, the thick disk has little effect on the thin disk. The top panel of Figure 11 indicates that the average orbital eccentricity of the thick disk has a downtrend with increasing metallicity within $-0.6 < [\text{Fe/H}] < -0.2$ dex. In order to confirm that this downtrend is not affected by the thin-disk stars with low eccentricity, we strengthen the conditions for the selection of thick-disk stars within $-0.6 < [\text{Fe/H}] < -0.2$, and we restrict $\text{TD}/D > 20$ (TD/D is explained in Section 2.3) and $e > 0.2$ for the thick-disk stars in $-0.6 < [\text{Fe/H}] < -0.2$. We find that the orbital eccentricity trend for restricted thick-disk stars is similar to the top panel of Figure 11, indicating that the thin disk has little effect on the thick disk in Figure 11. This also indicates that potential mixing between the thin disk and thick disk has little effect on Figure 10.

4.3. Rotational Velocity Gradients with Radial Distance, Vertical Height, and Metallicity

Gradients of rotational velocity are important properties for the Galactic disk and can provide useful clues as to its formation and evolution. For example, gradients of rotation velocity have been predicted by some models of disk formation, and have been observed by some studies. Thus, observational clues are provided for model prediction. Also, rotational velocity gradients can be used to restrict model parameters; for example, Curir et al. (2012) reported that the gradient of $V_\phi$ with $[\text{Fe/H}]$ of the thick disk plays a critical role in determination of the variation trend of the metallicity of the Galactic disk with $R$ at the time of Galactic disk formation. Here, we examine the observed gradients of $V_\phi$ with $R$, $|z|$ and $[\text{Fe/H}]$ using our sample stars.

Figure 12 displays the distribution of rotational velocity with radial distance (left panel) and vertical height (right panel) for the thin disk (bottom panel) and thick disk (top panel). In the radial direction, very small rotational velocity gradients of $-1.827 \pm 0.0009 \text{ km s}^{-1} \text{ kpc}^{-1}$ and $-1.40 \pm 0.0001 \text{ km s}^{-1} \text{ kpc}^{-1}$ are derived for the thick disk and the thin disk, respectively. In the vertical direction, there also exist two small rotational velocity gradients of $-9.27 \pm 0.001 \text{ km s}^{-1} \text{ kpc}^{-1}$ and $-9.4 \pm 0.001 \text{ km s}^{-1} \text{ kpc}^{-1}$ for the thick disk and the thin disk, respectively. These results are in general agreement with those of Lee et al. (2011). Lee et al. (2011) also observed flat gradients: $dV_\phi/dR = -5.6 \pm 1.1 \text{ km s}^{-1} \text{ kpc}^{-1}$ and $dV_\phi/|z| = -9.4 \pm 1.3 \text{ km s}^{-1} \text{ kpc}^{-1}$ for the thick disk, and $dV_\phi/dR = -0.1 \pm 0.6 \text{ km s}^{-1} \text{ kpc}^{-1}$ and $dV_\phi/|z| = -9.2 \pm 1.2 \text{ km s}^{-1} \text{ kpc}^{-1}$ for the thin disk.
5. Discussion of Selection Bias and the Formation and Evolution of the Galactic Disk

One may be concerned about biases in our sample that might arise from the selection of A/F/G/K-type giant stars, which may lead to the loss of more massive and younger stars in our sample. Our sample has no massive stars, but this is not a serious problem, because there are considerable low-mass young giant stars according to the result of Wu et al. (2019), who measured the age and mass for a sample of 640, 986 red giant branch (RGB) stars of the Galactic disk from the LAMOST DR4. Furthermore, the selection of giant stars may also lead to bias against old stars and neglect some young stars in our initial sample. Some previous studies suggest that ages are about 9–10 Gyr for the high-[α/Fe] stars in the Galactic disk (e.g., Bensby et al. 2014; Bergemann et al. 2014; Wu et al. 2019). Therefore, the thick disk may contain few young stars, and this selective bias can have little effect on the thick disk. If selecting giant sample stars strongly favors old stars over young stars in the thin disk, the metallicity distribution of the thin disk can be shifted lower. Thus, this bias might produce misleading correlations between [Fe/H] and the spatial (R, |z|) and kinematic parameters ($V_\phi$). However, as discussed above, our results are highly consistent with other studies for the thin disk. Therefore, the impact of this bias on the thin disk may also be negligible.

Now, we discuss the implication of our results for the formation and evolution of the Galactic disk. Because a model or simulation is affected by many factors, such as unavoidable numerical effects, and some assumptions that are required in their construction, models and simulations of formation and evolution of the Galactic disk may not accurately represent physical reality. Therefore, it is difficult to compare our observational results with expectations from the results of models and simulations quantitatively. Therefore, we only compare qualitatively our observational results with expectations from the published radial-migration, gas-rich, merger, accretion, and disk-heating models. More detailed quantitative comparisons require the construction of more physically realistic models and simulations.

First of all, we discuss the formation and evolution of the thin disk. According to the radial-migration model (e.g., Sellwood & Binney 2002; Schönrich & Binney 2009; Minchev & Famaey 2010; McMillan 2017), in the inner region of the Galactic disk, gas density is higher and chemical abundance is richer than in the outer region. Therefore, most stars formed in the inner disk should be metal rich, whereas those born in the outer disk should be metal poor. This indicates that the thin disk may have a negative metallicity gradient with radial distance at earlier times. These stars could be affected by radial migration: stars of the thin disk that are born in the outer disk move inward to the solar neighborhood, while metal-rich stars formed in the inner disk migrate outward into the solar neighborhood. Therefore, radial migration can flatten the metallicity radial gradient. Loebman et al. (2011) confirmed, using N-body simulations of radial migration, a weak metallicity radial gradient for the thin disk and showed that the thin disk in the solar neighborhood ($R = 7–11$ kpc and $|z| = 0.3–2.0$ kpc) has a gradient of $d[Fe/H]/dR = -0.02$ dex kpc$^{-1}$. This gradient is slightly flatter than what is shown by our results: $d[Fe/H]/dR = -0.05 \pm 0.0002$ dex kpc$^{-1}$.

Furthermore, a rotation velocity gradient of $dV_\phi/d[Fe/H] = -19$ km s$^{-1}$ dex$^{-1}$ for the thin disk is also found by Loebman et al. (2011) for younger stars (identified with the thin-disk component with low [α/Fe]) in the solar neighborhood ($R = 7–9$ kpc and $|z| = 0.1–1$ kpc), which is consistent with our result of $dV_\phi/d[Fe/H] = -17.03 \pm 0.001$ km s$^{-1}$ dex$^{-1}$. Therefore, we conclude that the metallicity radial gradient and rotation velocity gradient with [Fe/H] for the thin disk can be explained by the radial-migration model.

Brook et al. (2007) predicted a correlation between rotation velocity and radial distance for the Galactic disk stars using N-body simulations of the gas-rich merger model. These latter authors suggested that there exists a rotation velocity gradient with radial distance for the thick disk (that they refer to as disk stars) in the solar neighborhood ($6 < R < 10$) and that there is no [α/Fe] gradient with vertical height for the thin disk. These two properties differ from our results of $dV_\phi/dR = -1.404 \pm 0.0001$ km s$^{-1}$ kpc$^{-1}$ and $d[α/Fe]/dz = +0.05 \pm 0.0002$ for the thin disk. Therefore, we conclude that the gas-rich merger model may not explain the lack of rotation velocity gradient with radial distance or the existence of a [α/Fe] gradient in the vertical direction for the thin disk.

Concerning the formation and evolution of the thin disk, Loebman et al. (2011) reported a metallicity radial gradient of $d[Fe/H]/dR = 0.00$ dex kpc$^{-1}$ for the thick disk from their N-body simulations of radial migration, and Loebman et al. (2016) also showed a metallicity vertical gradient of $d[Fe/H]/dz \approx -0.03$ dex kpc$^{-1}$. These two results are consistent with our results of $d[Fe/H]/dR = -0.0006 \pm 0.0005$ dex kpc$^{-1}$ and $d[Fe/H]/dz = -0.074 \pm 0.0009$ dex kpc$^{-1}$, respectively. However, Loebman et al. (2011) reported a rotation velocity gradient of $dV_\phi/d[Fe/H] = +8$ km s$^{-1}$ dex$^{-1}$ for older stars ($8 \leq$ Age $\leq 10$ Gyr, identified with the thick disk component with high [α/Fe]) in the solar neighborhood ($R = 7–9$ kpc and $|z| = 0.1–1$ kpc), which differs from our result of $dV_\phi/d[Fe/H] = +30.87 \pm 0.001$ km s$^{-1}$ dex$^{-1}$. Figure 14 gives comparisons of observed orbital eccentricities with the accretion, heating, radial-migration, and merger models depicting the formation scenarios of the thick disk from Sales et al. (2009). Our result shows that the radial-migration model lacks high-orbital-eccentricity stars. In summary, the radial-migration model can explain metallicity radial and vertical gradients but not rotation velocity gradients and distribution of orbital eccentricities for the thick disk. This may indicate that radial migration did not play a major role in the formation and evolution of the thick disk, but was involved to a lesser extent.

Using N-body simulations of the dynamical heating of a pre-existing thin disk, Villalobos et al. (2010) reported that the $V_\phi$ gradients with $R$ and $|z|$ depend sensitively on the orbital inclination of the infalling satellite that produced the initial thick disk. These latter authors showed that the thick disk exhibits a very weak trend of $V_\phi$ with $R$ for the low orbital
inclination, and that the correlation between the two quantities becomes stronger with increasing incidence angle. However, the correlation between \( V_c \) and \( z_0 \) is very weak for high-orbital inclinations and this correlation becomes stronger with decreasing incidence angle. Our gradient of \( V_c \) with \( R \) for the thick disk is in good agreement with their low-orbital-inclination stars, while our correlation between \( V_c \) and \( z_0 \) is consistent with high-orbital-inclination stars. This indicates that the dynamical heating model may not explain the shortage of the rotation velocity gradient with radial distance and vertical height. In addition, Figure 14 shows that the absence of the peak at high \( e \approx 0.8 \) in our orbital eccentricity distribution excludes the dynamical heating model. We conclude that the dynamical heating model could not have played a significant role in the process of formation and evolution of the thick disk, but may have been involved to a lesser extent. In addition, the relative shortage of low-eccentricity stars in our observation excludes the gas-rich merger model, and our eccentricities distribution is more consistent with the accretion model. However, our observation shows more stars at \( 0.3 < e < 0.6 \) than the accretion model would predict. There could be several reasons for the accretion model not fitting the data. First, the thick disk may be formed by combined processes, and other formation mechanisms such as those of the radial-migration or disk-heating models could also contribute to the distribution of orbital eccentricities. Second, due to selective criteria, the sample is not complete enough to derive the distribution of orbit eccentricities for the thick-disk stars. Finally, it is possible that the simulation cannot precisely duplicate the formation of the Milky Way.

In summary, comparisons of our observed results with the predictions of the radial migration, gas-rich merger, accretion, and disk-heating models suggest that radial migration may have influenced the structure and chemical evolution of the thin disk, but could not have played a significant role in the formation of the thick disk. The formation of the thick disk could have been affected by more than one process. The accretion model could have played an indispensable role in the formation of the thick disk, and other formation mechanisms, such as those of the radial-migration or disk-heating models, could also have contributed to its formation.

6. Summary and Conclusions

Based on a sample of 307,246 giant stars from the LAMOST spectroscopic survey and Gaia DR2 survey located at \( 4 \lesssim R \lesssim 15 \) kpc and extending up to 6 kpc in height from the Galactic plane, we investigated kinematics and metallicity distribution of the Galactic disk. First of all, the sample stars were divided into thin-disk and thick-disk components according to the chemical abundances \((\text{Fe/H})\) and \([\alpha/\text{Fe}]\) and kinematics. In total, we obtained 179,092 thin-disk stars and 29,966 thick-disk stars, and the metallicity distribution of the thin disk can be described by a Gaussian model with a peak at \([\text{Fe/H}] \sim -0.21\) and a standard deviation of \(\sigma_{\text{Fe/H}} \sim 0.20\). The metallicity distribution of the thick disk has an extended metallicity tail, but it can be described by a Gaussian model with a peak at \([\text{Fe/H}] \sim -0.52\), \(\sigma_{\text{Fe/H}} \sim 0.23\) in the range of \([\text{Fe/H}] > -1.2\) dex. Our large sample with a wide spatial range is ideal for investigating chemical and kinematical properties of the thin and thick disks.

For the thin disk, we summarize our results as follows:

1. The thin disk has a negative metallicity gradient with radial distance, which becomes flatter with increasing vertical height. However, the inner disk \((R < 8 \) kpc) of the thin disk has a positive metallicity gradient and the outer disk \((R > 8 \) kpc) has a negative metallicity gradient with radial distance.

2. The thin disk has a negative metallicity gradient with vertical height, but this gradient is invariable in the inner region and then becomes flat with increasing radial distance. Also, the thin disk has a positive \([\alpha/\text{Fe}]\) gradient with vertical height.

3. The thin-disk stars have low orbital eccentricities (peak at \( \sim 0.12\)). The orbital eccentricities show a slight uptrend with increasing metallicity, and have no obvious relationship with vertical height. The thin disk has a flat rotational velocity gradient with radial distance and vertical height, and has a negative rotational velocity gradient with metallicity.

For the thick disk, there exist more controversies on chemical and kinematic properties. Our results are summarized as follows:

1. The thick disk has no metallicity gradient with radial distance, while the inner disk \((R < 8 \) kpc) has a positive metallicity gradient and the outer disk \((R > 8 \) kpc) has a negative metallicity gradient.

2. The thick disk has a negative metallicity gradient with vertical height, but this gradient becomes flatter on average with increasing radial distance. The thick disk has a positive \([\alpha/\text{Fe}]\) gradient with vertical height.

3. The orbital eccentricities of the thick disk are higher, and orbital eccentricity peaks at \(\sim 0.42\), with larger widths, and extends up to \( e \sim 0.8\). Furthermore, a downtrend exists with increasing metallicity, and no obvious relationship is seen with vertical height. The thick disk has a flat rotational velocity gradient with radial distance and vertical height, but has a positive rotational velocity gradient with metallicity.

Our results are in agreement with most previous studies of metallicity, \([\alpha/\text{Fe}]\), and rotational velocity. More properties are also found for the thin disk and thick disk, such as two different metallicity radial gradients for the inner and outer disks.
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