Bound on Noncommutative Standard Model with Hybrid Gauge Transformation via Lepton Flavor Conserving $Z$ Decay

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Abstract

The $Z \to e^+e^-$ decay is studied basing on the noncommutative standard model (NCSM) with the hybrid gauge transformation. It is shown that if the latter is not included, the noncommutative correction to the amplitude of the $Z \to e^+e^-$ appears only as a phase factor, so that there is no new physical effect on the decay width. However, when the hybrid gauge transformation is included, the noncommutative effect appears in the two-body decay process. The discrepancy between the experimental branch ratio and the standard model prediction allows us to set the bound on the noncommutative parameters.

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The concept of noncommutative (NC) space-time was firstly introduced by Snyder in 1947\cite{1}. Interest on NC space-time was revived since it appeared in the string theory and other quantum gravity models as effective theories in low energy limit\cite{2,3,4,5}. In a popular NC model the NC space-time is characterized by a coordinate operator satisfying

\[ [\hat{x}_\mu, \hat{x}_\nu] = i\theta_{\mu\nu} = \frac{ic_{\mu\nu}}{\Lambda_{NC}^2}, \]

where \( \theta_{\mu\nu} \) is a constant antisymmetric matrix. Its elements have a dimension of \((mass)^{-2}\). Here \( c_{\mu\nu} \) is a real antisymmetric matrix, whose dimensionless elements are assumed to be of order unity, and the NC scale \( \Lambda_{NC} \) characterizes the threshold where the NC effect becomes relevant and its role can be compared to that of \( \hbar \) in quantum mechanics. The existence of a finite \( \Lambda_{NC} \) implies the existence of a fundamental space-time distance below which the space-time coordinates become fuzzy. By Weyl-Moyal correspondence, the quantum field theory in NC space-time is equivalent to that in ordinary space-time with the normal product of the field variables replaced by the star product, defined by \cite{6}

\[ \phi_1(x) \ast \phi_2(y) = \exp \left( \frac{i}{2} \theta_{\mu\nu} \partial_\mu \partial_\nu \right) \phi_1(x)\phi_2(y) |_{y \to x}. \]

Using this method, a noncommutative extension of the standard model (NCSM) has been proposed\cite{7}, where the SU(N) Lie algebra is generalized to the enveloping algebra via the Seiberg-Witten map (SWM)\cite{3}. The SWM is a map between the noncommutative field and its counterpart in ordinary space-time as a power series of the NC parameter \( \theta_{\mu\nu} \). The NCSM predicts the NC-corrected particle vertex and many new interactions beyond the standard model, for instance, the \( Z - \gamma - \gamma \) and \( Z - g - g \) vertices. The rich phenomenological implication has been intensively examined in high energy processes for possible experimental signal or give a bound on the noncommutative scale \( \Lambda_{NC} \)\cite{8,9}.

On the other hand, the neutrino oscillation experiments have shown convincing evidence of massive neutrinos and leptonic favor mixing\cite{10,11}, so that in constructing NCSM the neutrino mass should be included. It is found that the hybrid gauge transformation and hybrid SWMs are needed to accommodate the seesaw mechanism\cite{11}. The most popular mechanism for generating neutrino mass, and the gauge invariance of NC gauge theory. The hybrid gauge transformation and hybrid SWM have been adopted in the Higgs sector of NCSM to ensure covariant Yukawa terms\cite{7}. In this scenario, the Higgs fields feel a “left” charge and a “right” charge in the NC gauge theory and transforms from left side and right
side correspondingly. Although it is only applied to the Higgs sector in Ref. [7], in Ref. [11] it was shown that this method can in principle be extended to consider fermion fields. A new physics predicted by the hybrid gauge transformation is the tree-level coupling between the neutrino and the photon

$$i\kappa e(\hat{A}_\mu \nu - \nu \hat{A}_\mu),$$

(3)

where \(\hat{A}\) and \(\nu\) are the photon and neutrino fields, respectively. In NCQED, to maintain the gauge invariance the charge is quantized to -1, 0, and 1, corresponding to the interaction terms \(e\hat{A}_\mu \psi\), \(e(\hat{A}_\mu \psi - \psi \hat{A}_\mu)\), and \(e\psi \hat{A}_\mu\). However, in the NCSM based on the enveloping algebra, the Seiberg-Witten map can overcome the constraint of charge quantization and guarantee the gauge invariance at the same time. So one can loosen the constraint on charge quantization and arbitrarily set the electric charge in (3) as \(\kappa e\). The photon-neutrino interaction can lead to interesting phenomena, and has been discussed by many authors (see Ref. [12] and the references therein).

It is interesting to see if the hybrid gauge transformation will lead to other phenomenological effects. In Ref. [13], the anomalous \(Z - \nu - \nu\) interaction is derived and the invisible \(Z\) decay process \(Z \rightarrow \nu \bar{\nu}\) is studied. It is shown that for \(\kappa = 1\), the current experimental result \(\Gamma_{\text{invisible}} = (499.0 \pm 1.5)\) MeV [14] allows us to set the bound \(\Lambda_{NC} \geq 140\) GeV on the noncommutative scale.

Besides the invisible decay, it is also of interest to investigate the \(Z \rightarrow l^+ l^-\) channel. In the standard model, the \(Z\) boson decays into lepton pairs through the lepton flavor conserving (LFC) interaction at the tree level. Up to now, the current experimental data produces \(\text{Br}(Z \rightarrow e^+ e^-) = 3.363 \pm 0.004\%\), \(\text{Br}(Z \rightarrow \mu^+ \mu^-) = 3.366 \pm 0.007\%\) and \(\text{Br}(Z \rightarrow \tau^+ \tau^-) = 3.370 \pm 0.0023\%\) [14]. On the other hand, the theoretical prediction from SM, including the loop correction, is \(\text{Br}(Z \rightarrow e^+ e^-) = \text{Br}(Z \rightarrow \mu^+ \mu^-) = 3.3346\%\) and \(\text{Br}(Z \rightarrow \tau^+ \tau^-) = 3.3338\%\) [15]. In this paper, we focus on the \(Z \rightarrow e^+ e^-\) and in the following calculation the zero lepton mass approximation is adopted. The gap between the experimental results and the theoretical prediction is of order 0.03\% and exhibits possible existence of new physics beyond the standard model. Motivated by this, various models beyond the SM have been discussed [15-17]. In Ref. [17], the same issue has been discussed in the NCSM framework without the hybrid gauge transformation. However, our detailed analysis [9] showed that the NC effect only appears in the \(Z - l - l\) vertex as a phase factor, so that no physical deviation appears. Here we study \(Z \rightarrow e^+ e^-\) in the framework of NCSM with hybrid gauge
transformations. From the viewpoint of gauge invariance, the hybrid feature also effects the charged lepton interaction. To see this, we briefly review our earlier results\cite{9}. The action of lepton in NCSM can be written as

$$\hat{S}_{\text{lepton}} = i \int d^4x [\bar{\hat{\Psi}} L \gamma \mu D_{\mu} \hat{\Psi} L + \bar{\hat{l}} R \gamma \mu D_{\mu} \hat{l} R]$$

(4)

with \(\bar{\hat{\Psi}} L\) and \(\bar{\hat{l}} R\) denoting the doublet lepton and the right-handed singlet lepton, respectively. Under the hybrid gauge transformation, the \(\hat{\Psi} L\) and \(\hat{l} R\) transform as

$$\delta_\Lambda \left( \begin{array}{c} \hat{\nu} L \\ \hat{l} L \end{array} \right) = ig Y \left( -\frac{1}{2} + \kappa \right) \Lambda \ast \left( \begin{array}{c} \hat{\nu} L \\ \hat{l} L \end{array} \right) - \kappa \left( \begin{array}{c} \hat{\nu} L \\ \hat{l} L \end{array} \right) \ast \Lambda,$$

(5)

$$\delta_\Lambda \hat{l} R = ig Y \left[ (1 + \kappa) \Lambda \ast \hat{l} R - \kappa \hat{l} R \ast \Lambda \right],$$

(6)

where \(\Lambda\) is the gauge parameter. Under the gauge transformation above, the covariant derivatives in Eq. (4) is

$$D_{\mu} \hat{\Psi} L = \partial_{\mu} \hat{\Psi} L - ig L \hat{A}_a^{a} T^a \ast \hat{\Psi} L - \left( \frac{1}{2} + \kappa \right) g Y \hat{B}_\mu \ast \hat{\Psi} L + i\kappa g Y \hat{\Psi} l \ast \hat{B}_\mu,$$

(7)

$$D_{\mu} \hat{l} R = \partial_{\mu} \hat{l} R - i\kappa g Y B_\mu \ast \hat{l} R + i\kappa g Y \hat{l} R \ast B_\mu,$$

(8)

where \(\hat{A}_\mu^a\) and \(g_L\) are the \(SU(2)_L\) gauge fields and a coupling constant. To get the appropriate particle vertex, we should replace the fermion and gauge fields in Eqs. (4), (7), and (8) by their classical counterparts via appropriate Seiberg-Witten maps. The detailed formation of Seiberg-Witten map is given in Ref. \cite{11}, where the so-called \(\theta\)– exact formation is adopted to include the contribution of all \(\theta\) orders. From the deformed Lagrangian\cite{9} one can then obtain the Feynman rule of the \(Z - l - l\) interaction

$$\frac{i e}{\sin 2\theta_W} \gamma^\mu (C_V - C_A^5) e^{\frac{p_1 \cdot p_2}{2\cos \theta_W}} + \frac{2\kappa e \sin \theta_W}{\cos \theta_W} \gamma^\mu \sin \left( \frac{1}{2} p_1 \theta p_2 \right),$$

(9)

where \(p_1\) (\(p_2\)) is the ingoing (outgoing) lepton momentum, \(p_1 \theta p_2 \equiv p_1^\mu p_1^\nu p_2^\nu, C_V = -\frac{1}{2} + 2 \sin^2 \theta_W, C_A = -\frac{1}{2}\), and \(\theta_W\) denotes the Weinberg angle. We have applied the equation of motion to the electron external line and omitted the vanishing terms due to the on-shell condition.
Using the Feynman rule in Eq. (9), the derivative decay width of $Z \rightarrow e^+ e^-$ can be easily obtained in the $Z$ boson rest frame
\[
\frac{d\Gamma}{d \cos \theta d \phi} = \frac{M_Z}{48\pi^2} \left[ \frac{e^2}{\sin^2 \theta_W} \left( C_V^2 + C_A^2 \right) + \left( 4\kappa^2 - 2\kappa C_V \frac{e^2}{\cos^2 \theta_W} \sin^2 \left( \frac{1}{2} p_1 \theta p_2 \right) \right) \right]. \tag{10}
\]

In the calculation, we omit the lepton mass. As mentioned, the NC parameter $\theta_{\mu\nu}$ is a fundamental constant that breaks the Lorentz symmetry. Following the method adopted in Ref. [18], one can decompose $\theta_{\mu\nu}$ into two types: the electric-like components $\theta_E = (\theta_{01}, \theta_{02}, \theta_{03})$ and the magnetic-like components $\theta_B = (\theta_{23}, \theta_{31}, \theta_{12})$. Both of them are assumed to be directionally fixed in a primary, unrotated reference. That is, when discussing phenomena in the laboratory frame, the Earth’s rotation should be included. Defining $(\hat{X}, \hat{Y}, \hat{Z})$ to be the orthonormal basis of this primary frame, $\theta_E$ and $\theta_B$ are
\[
\begin{align*}
\theta_E &= \frac{1}{\Lambda_E} (\sin \eta \cos \xi \hat{X} + \sin \eta \sin \xi \hat{Y} + \cos \eta \hat{Z}), \tag{11} \\
\theta_B &= \frac{1}{\Lambda_B} (\sin \eta \cos \xi \hat{X} + \sin \eta \sin \xi \hat{Y} + \cos \eta \hat{Z}), \tag{12}
\end{align*}
\]
where $\eta$ and $\xi$ denote the polar angular and azimuth angular of NC parameter with $0 \leq \eta \leq \pi$ and $0 \leq \xi \leq 2\pi$, respectively. Since we are in the $(\hat{x}, \hat{y}, \hat{z})$ frame on Earth, it is necessary to find an appropriate transformation matrix correlating the primary and laboratory reference frames. Following Ref. [19], we have
\[
\begin{pmatrix}
\hat{X} \\
\hat{Y} \\
\hat{Z}
\end{pmatrix} = \begin{pmatrix}
cos \delta \cos \zeta + s \delta c \zeta & c \delta c \zeta & s \delta c \zeta - c \delta s c \zeta \\
-s \delta c \zeta + c \delta s \zeta & -s \delta c \zeta - c \delta s c \zeta & s \delta c \zeta \\
-c \delta s \zeta & s \delta & c \delta c \zeta
\end{pmatrix} \begin{pmatrix}
x \\
y \\
z
\end{pmatrix}, \tag{13}
\]
where the abbreviation $c_\alpha = \cos \alpha$ and $s_\alpha = \sin \alpha$, with $\alpha = a$, $\delta$ and $\zeta$ respectively, are used. Here, $\delta$ and $a$ define the location and orientation of the experiment site, with $-\frac{\pi}{2} \leq \delta \leq \frac{\pi}{2}$ and $0 \leq a \leq 2\pi$, $\zeta = \omega t$ is the rotation angle, and $\omega = 2\pi/23h56m4.09s$ is the earth’s angular velocity. Ignoring the Earth’s revolution, the collider machine returns to its original position after one day. Using Eqs. (11), (12), and (13), we get
\[
p_2 \theta p_1 = -\frac{s}{2\Lambda_{NC}^2} (\sin \theta \cos \phi \Theta^x_E + \sin \theta \sin \phi \Theta^y_E + \cos \theta \Theta^z_E) \tag{14}
\]
with
\[
\begin{align*}
\Theta^x_E &= s_\eta c_\zeta (c_\alpha s_\zeta + s_\delta s_\alpha c_\zeta) + s_\eta s_\zeta (-c_\alpha s_\zeta + c_\delta s_\alpha s_\zeta) - c_\eta c_\delta c_\alpha, \\
\Theta^y_E &= s_\eta c_\zeta c_\delta c_\alpha + s_\eta s_\zeta c_\delta c_\zeta + c_\eta c_\delta, \\
\Theta^z_E &= s_\eta c_\zeta (s_\alpha s_\zeta - s_\delta c_\alpha) + s_\eta s_\zeta (-s_\eta c_\zeta - s_\delta c_\alpha s_\zeta) + c_\eta c_\delta c_\alpha. \tag{15}
\end{align*}
\]
Substituting Eq. (14) into Eq. (10), we obtain the decay width of $Z \to e^+e^-$ in the laboratory frame.

Due to the Earth’s rotation, any observable calculated in the NC space-time frame should depend on time. On the other hand, it is difficult to follow the experiments in time. It is therefore reasonable to average the cross section or decay width over a full day. For our problem, the time-averaged decay width is

$$
\langle \Gamma \rangle_T = \frac{1}{T_{\text{day}}} \int_0^{T_{\text{day}}} dt \int_{-1}^{1} d(cos \theta) \int_0^{2\pi} d\phi \frac{d\Gamma}{d\cos \theta d\phi}.
$$

In particular, we are interested in the NC correction of the branch ratio

$$
\Delta BR = \frac{\delta \Gamma}{\Gamma_0} \equiv \frac{\Gamma - \Gamma_{\text{SM}}}{\Gamma_{\text{SM}}}.
$$

The behavior of $\Delta BR$ for different $\kappa$, as well as the current experimental uncertainty, is shown in Fig. 1 as a function of the NC scale $\Lambda_{NC}$. In the numerical analysis, we use the input parameters of Ref. [15]. The location and orientation of laboratory frame are set to be $(\delta, a) = \left(\frac{\pi}{4}, \frac{\pi}{4}\right)$, where the LEP experiment measures the decay width of $Z \to e^+e^-$. We can see from Fig. 1 that $\Delta BR$ is sensitive to both $\kappa$ and $\Lambda_{NC}$. Clearly, the NC correction is significantly enhanced as $\Lambda_{NC}$ or $\kappa$ decreases. Compared with the experimental branch ratio $\text{Br}(Z \to e^+e^-) = 3.363 \pm 0.004\%$, for the choice of $\kappa = 1$, a bound on the noncommutative scale $\Lambda_{NC} \geq 150$ GeV is obtained by imposing the constraint $\Delta BR \leq 3 \times 10^{-4}$. As seen from Eq. (8), the NC correction of decay width also depends on the orientation of $\theta_E$ i.e., the parameter $\eta$. In Fig. 1 we have set $\eta = \frac{\pi}{2}$. It is thus necessary to investigate the sensitivity of $\Delta BR$ on $\eta$. The NC correction of the time-averaged decay width is presented as the function of parameter $\eta$ in Fig. 2. One can see from Fig. 2 that the NC effect produces a positive deviation from the SM branch ratio for the whole range of $\eta$. Despite the fact that a slightly peaked distribution appears, the curve is not sensitive to $\eta$. In this sense, the bound obtained from Fig. 1 should be credible.

In Fig. 3 we show the allowed region of $\Delta BR$ in the $(\kappa, \Lambda_{NC})$ plane for $-1 < \kappa < 1$. We see that as $\kappa$ increases, a higher bound on the NC scale appears. Furthermore, a lower limit $\kappa \geq 0.04$ is found for the forbidden region when we set $\kappa$ to zero. This means that in Eq. (9), for $\kappa = 0$ the NC correction to the magnitude of $Z \to e^+e^-$ only appears as a phase factor, indicating that there is no NC deviation to the decay width. Thus if we assume that the discrepancy between the experimental and the SM results is fully induced by the
noncommutative effect, the value of $\kappa$ can not be arbitrarily small. In the hybrid feature, additional sin-type deformation shows up in the Feynman rule of NC $Z - e - e$ interaction and leads to NC correction which is potentially detectable or allows us to set bound on the NC parameters in high-accuracy measurements of $Z$ decay width.

In conclusion, the $Z \to e^+ e^-$ channel provides an ideal process to understand not only the space-time noncommutativity, but also the mathematical structure of the corresponding gauge theory. We showed that the decay width is sensitive to both $\Lambda_{NC}$ and the parameter $\kappa$ for the freedom of the hybrid gauge transformation. In terms of the NC effect, the discrepancy between the experimental and SM results allows us to set a bound on the noncommutative parameters. Although the current experimental uncertainty is still a little
large, the next generation $Z$ factory with the Giga-$Z$ option of the International Linear Collider can generate $2 \times 10^9$ $Z$ events at resonance energy $[20, 21]$. We therefore expect that the high-luminosity $Z$ factory can significantly enhance the sensitivity to probe the noncommutative model via $Z$ boson decays.

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