Active Optical Clock

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This letter presents the principles and techniques of active optical clock, a special laser combining the laser physics of one-atom laser, bad-cavity gas laser, super-cavity stabilized laser and optical atomic clock. As an example, a compact version of active optical clock based on thermal Strontium atomic beam shows a quantum-limited linewidth of 0.51 Hz, which is insensitive to laser cavity-length noise, and may surpass the recorded narrowest 6.7 Hz of Hg ion optical clock and 27Hz of very recent optical lattice clock. The estimated 0.1Hz one-second instability and 0.27Hz uncertainty are limited only by the relativistic Doppler effect may be improved to 10mHz by using cold atoms.

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FIG. 1: Birthday cake of Maser and Laser. Made by Townes, Basov, and Prokhorov, and won them the Nobel Prize in physics1964. The first quadrant is a “new continent” of lasers, good for active optical clocks and laser spectroscopy with supper-narrow linewidth, which is insensitive to cavity length variations.

In 1958, in a well-know paper, Schawlow and Townes proposed to extend the maser techniques to laser. Just two years later, the first laser was built by Maiman, also in this year, Goldenberg, Kleppner, and Ramsey invented Hydrogen maser, an active microwave atomic clock, for which scientists have enjoyed its excellent stability in a variety of applications since its invention. However, we never have an active optical clock so far. All the optical atomic clocks up to date, are not working in active mode. Here, 45 years after the invention of active microwave Hydrogen clock, this letter presents the principles and techniques of active optical clock, which is the optical frequency counterpart of active Hydrogen clock.

It’s a special laser combining the laser physics of one-atom laser[6,7], bad-cavity gas laser[8,9], super-cavity stabilized laser[10-12] and passive optical atomic clock[3-5,13,14]. A compact version based on thermal Strontium atomic beam shows a quantum-limited linewidth of 0.51 Hz, which will surpass the recorded narrowest 6.7 Hz of Hg ion clock[14] and 27Hz of very recent optical lattice clock[5]. The most interesting point is, the frequency of this active optical clock is insensitive to cavity-length noise, which is currently the limitation of available narrow linewidth laser light sources. The estimated 0.1Hz one-second instability and 0.25Hz uncertainty are limited by the relativistic Doppler effect may be improved by using cold atoms. The active optical clock provides a new way to optical atomic clock and precision laser spectroscopy, and it also opens a door to long-time coherence physics, say hundred-second even thousand-second coherence laser physics, the long-time counterpart of Atto-second physics.

Using the definition of \( a = \frac{\Gamma_{\text{cavity}}}{\Gamma_{\text{gain}}} \)[8], here \( \Gamma_{\text{cavity}} \) is the cavity loss rate, \( \Gamma_{\text{gain}} \) the frequency gain bandwidth of laser medium, when \( a << 1 \), a laser is working in the good-cavity limit, and in the bad-cavity regime while \( a \gg 1 \). Then let’s cut a cake into four quadrants as shown in Fig.1. Chronologically, the second quadrant was tasted by Gordon, Zeiger, and Townes in 1954 by building the first ammonia maser, then in 1960, Maiman tasted the forth one by building the famous first laser, Ruby laser. In the same year, Goldenberg, Kleppner, and Ramsey invented the best know Hydrogen maser, belongs to the second quadrant. The third quadrant was reached with the one-atom maser[15] in 1985 for cavity quantum electrodynamics. How about the first quadrant? Following a semiconductor device, a He-Ne 3.39\( \mu \)m gas laser[8] went into this bad-cavity regime of \( a = 1.4 \) in 1994, like the red cherry on the cake across the \( a = 1 \) line in Fig.1. All the conventional lasers are working in the good-cavity regime, the forth quadrant. In this letter, the laser will be pushed deep down into the bad-cavity regime \( a >> 1 \), the “new continent” of laser at first quadrant in the Fig.1.

For a homogeneously broadened single-mode laser, the quantum-limited linewidth of a bad-cavity laser de-
scribed by a modified Schawlow-Townes formula[8],

\[
\Delta \nu_{\text{laser}} = \frac{\Gamma_{\text{cavity}}}{4\pi \pi_{\text{cavity}}} N_{sp} \left( \frac{1}{1 + a} \right)^2 \left\{ 1 + \frac{4 \pi (\nu - \nu_0)^2}{\Gamma_{\text{gain}} + \Gamma_{\text{cavity}}} \right\},
\]

(1)

Here \( N_{sp} = N_p / (N_p - N_s) \) is the spontaneous-emission factor, \( N_s, N_p \) are the populations of the lower and upper levels, \( \nu - \nu_0 \) is the detuning of the mode frequency \( \nu \) from the center frequency \( \nu_0 \) of the gain profile, and \( \pi_{\text{cavity}} \) is the steady-state number of photons in laser mode. For an ideal four-level laser at zero detuning (\( \nu = \nu_0 \)), Eq.(1) reduced to the Schawlow-Townes formula in standard laser text books[16,17] \( \Delta \nu_{\text{laser}} = \Gamma_{\text{cavity}} / (4\pi \pi_{\text{cavity}}) \) when laser operating in the good cavity limit \( (a \ll 1) \). While entering the bad-cavity regime, the results from HeNe 3.39\mu m gas laser[8] agreed very well with theory as expressed Eq.(1). The physics behind the factor \((1 + a)^{-2}\) in Eq.(1) is the memory effect of atomic polarization[8,9,18].

Let’s construct a gas laser with thermal Strontium (\(^{88}\text{Sr}\)) atom beam by adjusting the one-atom \(^{138}\text{Ba}\) laser[6,7] technically. The lasing transition is at the 689nm line of \(5s5p\) \(^3\text{P}_1\)(\(m = 0\)) - \((5s)^2\) \(^1\text{S}_0\), in which the \(^3\text{P}_1\) state (lifetime \(\tau_{sp} = 21\mu s\)) has 7.6kHz decay rate to the \(^1\text{S}_0\) state [19]. The first adjustment is to increase the atomic beam flux \(R_p\), to make the average number of atoms in the laser mode to be \(N_{\text{transit}} \gg 1[20]\). The second adjustment is to increase the laser cavity length to 4cm, and increase the cavity mode waist to 800\mu m. Then the atom-cavity coupling constant \(g\) is decreased, which is[7], \(g = (\mu / h) \sqrt{2 \pi \hbar \omega_{\text{atom}} / V_{\text{mode}}} = 2\pi \times 1.0kHz\), where \(\mu\) the electric dipole moment, \(\omega_{\text{atom}}\) the transition frequency, and \(V_{\text{mode}}\) the mode volume. The most probable velocity of atoms in beam \(v_{\text{probable}}\) is 505m/s while the \(^{88}\text{Sr}\) oven is operating at the temperature of 630°C. The atomic transit-time through the cavity mode after averaging over transverse Gaussian profiles[7] is \(t_{\text{transit}} = \sqrt{\pi / \nu_0} / v = 2.8\mu s\). Then the transit-time broadening[7] is \(\Gamma_{\text{transit}} / 2\pi = 4 / 2\pi t_{\text{transit}} = 220kHz\). We have \(a = \Gamma_{\text{cavity}} / \Gamma_{\text{gain}} = 50\) while the cavity decay rate is \(\Gamma_{\text{cavity}} / 2\pi = 11MHz\). For \(g t_{\text{transit}} = 0.018 \ll \pi\), the atomic transition probability is \(\sin^2(\sqrt{n + 1} g t_{\text{transit}})\), then the photon emission rate, i.e., the laser emission coefficient [20] is \(K = \Gamma_{\text{transit}} \sin^2(\sqrt{n + 1} g t_{\text{transit}})\), where \(n\) is the number of photons in laser mode. The rates are shown in Fig.3, and all parameters of one-atom laser with \(^{138}\text{Ba}\) atom[6,7], active optical frequency standards with thermal \(^{88}\text{Sr}\) and cold \(^{40}\text{Ca}\) atoms are listed in Table 1. It will be showed at last \(n \gg 1\), thus the photon-number rate equation will be approximated by[7,20],

\[
\frac{dn}{dt} = R_p \sin^2 \left( \sqrt{n} g t_{\text{transit}} \right) - n \Gamma_{\text{cavity}}.
\]

(2)

The steady-state solution of Eq.(2) can be written in a dimensionless form[20],

\[
r_\eta = \frac{n_\nu}{\sin^2 \sqrt{n_\nu}},
\]

(3)

with

\[
\begin{align*}
n_\nu &= n (g t_{\text{transit}})^2, \\
r_\eta &= \frac{N_{\text{transit}}}{N_{\text{threshold}}}, \\
N_{\text{threshold}} &= \frac{\Gamma_{\text{cavity}}}{g^2 t_{\text{transit}}}, \\
N_{\text{transit}} &= R_p t_{\text{transit}},
\end{align*}
\]

(4)

Where \(N_{\text{transit}}\) is the number of atoms in the cavity mode, and \(N_{\text{threshold}}\) is the threshold atom number for lasing, thus \(r_\eta\) has the meaning of the pumping parameter of conventional laser. In order to decrease the quantum-limit laser linewidth, set \(r_\eta = 2\). Around \(r_\eta = 2\), the solution of Eq.(3) is shown in Fig.4.
From Eq.(4), an atomic flux of $R_a = 4.3 \times 10^{11}$ atoms/s is needed to satisfy $r_\eta = 2$. With the solution of $n_\nu \approx 2$ at $r_\eta = 2$ showed in Fig.4, from Eq.(4), the steady-state number of photons in laser cavity is $\pi_{\text{cavity}} = n_\nu / (g \Gamma_{\text{transit}})^2 = 6.2 \times 10^3$, hence the output power of laser $P = \pi_{\text{cavity}} \nu \Gamma_{\text{cavity}}$ is $0.12 \mu W$.

The laser linewidth of Eq.(1), at zero detuning will be reduced to,

$$\Delta \nu_{\text{laser}} = \frac{\Gamma_{\text{cavity}}}{4 \pi \pi_{\text{cavity}}} \frac{(1 + r_\eta)}{2} \left( \frac{1}{1 + a} \right)^2.$$ \hspace{1cm} (5)

Putting all the $^{88}\text{Sr}$ numerical parameters into Eq.(5), we have $\Delta \nu_{\text{laser}} = 0.51 Hz$.

As a laser light source, this 0.51Hz linewidth almost reaches the best-known 0.16Hz linewidth of a cavity stabilized laser[11], which is achieved within a 9mm$^3$ wooden enclosure lined internally with lead foam[21]. As an optical frequency standard, this 0.51Hz linewidth surpasses the recorded narrowest 6.7 Hz linewidth of Hg ion clock[14] and measured 27Hz linewidth of most recent optical lattice clock[5].

When $N_{sp}$ is expressed in $r_\eta$, the Eq.(5) agrees with the result $\Delta \nu_{\text{laser}} = \Gamma_{\text{cavity}}(1 + \theta^2)/(8\pi \pi_{\text{cavity}})$ with $\theta^2 = r_\eta$ from quantum theory of micromaser[22] where the bad-cavity effect is not included. The text-book Schawlow-Townes formula[16,17] is valid only within the good-cavity regime, which gives a standard text-book example: a gas laser with milliHertz linewidth. Unfortunately, due to the vibrations of cavity length, this textbook example of milliHertz linewidth[16,17] has never been achieved. At the bad-cavity regime, as the first and second quadrants shown in Fig.1, the linewidth of a laser or maser will be further modified to be much smaller than the good-cavity Schawlow-Townes limit with a factor of $(1 + a)^{-2}$ as shown in Eq.(1), and results in the original Schawlow-Townes formula[1,8]. The elegant experiments of this bad-cavity effect on laser linewidth have been performed thoroughly in small gas laser in Woerdman group recently, with clear theoretical explanation[8].

The center frequency of a good-cavity gas laser or a super-cavity stabilized laser follows the cavity length variation almost perfectly to the level of mHz, and even more tightly is possible[10-12]. Thus the final technical limitations on the available laser linewidth are from the variations of the cavity length[10-12] caused by the environmental vibrations, thermal expansion, body-force which produces distortion, long-term creep, and thermal Brownian motion noise[23]. This formidable hurdle is cleared here by the bad-cavity effect. In the bad-cavity regime, the laser center frequency doesn’t follow the cavity length variation exactly[8], but in a form of “cavity pulling” shift, which is a well-known shift in Hydrogen maser$[2,24]$, $\Delta \nu_{\text{cavity-pulling}} = (\Gamma_{\text{gain}} / \Gamma_{\text{cavity}}) (\nu - \nu_0)$. It is $\Delta \nu_{\text{cavity-pulling}} = 2 \times 10^{-3} (\nu - \nu_0)$ with the $^{88}\text{Sr}$ atomic beam value of $a = 50$. When the cavity spacer consists of Zerodur or ULE(Ultra-Low Expansion) and optically contacting to cavity mirrors, $\pm 0.2Hz/s$ cavity mode drift[3,10-12] only causes a $\pm 4mHz/s$ shift of the laser center-frequency.

Technically, we have set $r_\eta = 2$ before. By Eq.(3) and Fig.4, one can get a larger photon number $\pi_{\text{cavity}}$ by increasing the pumping parameter $r_\eta$, but the disadvantage is spontaneous-emission factor will go up too. It is a technical trade-off between $\pi_{\text{cavity}}$ and $N_{sp}$ to minimize the laser linewidth in a practical set-up. Since an $^{88}\text{Sr}$ atomic beam of $R_a = 2 \times 10^{12}$ atoms/s flux has been achieved[25] at the oven temperature of 630°C, it is possible to reach much higher flux while 2cm nozzle array to satisfy the $r_\eta = 2$ requirement on high atomic flux. When the inhomogeneous broadening is close to the homogeneous broadening[26], the laser linewidth of Eq.(1) will increase by a factor of 3. The relative motion between atoms and cavity in the direction of cavity mode axis due to vibrations can be neglected. The transverse velocity distribution of thermal atomic beam will cause the inhomogeneous broadening of gain profile. It is predicted two-photon Doppler cooling of $^{88}\text{Sr}$ via $(5s)^2 1S_0 - 5s6s 1S_0$ transition can achieve a Doppler limit of $57\mu K$[27], this means a transverse velocity of $v_{\text{transverse}} = 0.075m/s$, and a narrowed inhomogeneous broadening of $2\pi \times 108kHz$, which is less than the transit time broadening $\Gamma_{\text{transit}} = 2\pi \times 200kHz$ of laser cavity. Given there is 0.2 micro-radian angle deviation between the 1D transverse cooling beam and the laser cavity axis, for atomic beam with velocity of 505m/s, will result in a line broadening of $\Delta \nu_{\text{laser-Doppler}} = 146kHz$.

An accuracy of this thermal $^{88}\text{Sr}$ active optical clock is estimated as follow, and the estimated major corrections and uncertainties are listed in Table II. The atom density in laser cavity is $6 \times 10^7$ atoms/cm$^3$ in the thermal $^{88}\text{Sr}$ beam case, gives 0.08Hz density-dependent frequency shift with the measured cold $^{88}\text{Sr}$ atoms result[19]. The recoil-induced shift of stimulated emission $-4.7kHz$ can be corrected with an uncertainty less than $1mHz$[19]. For the standing wave in the laser cavity, the residual
TABLE I: Parameters of one-atom laser and active optical frequency standards.

| Laser                  | One-atom $^{138}$Ba | Atomic-beam $^{88}$Sr | Cold-atom $^{40}$Ca |
|------------------------|----------------------|------------------------|---------------------|
| Gain medium            | $^{138}$Ba           | $^{88}$Sr              | $^{40}$Ca           |
| $\Gamma_{p-}\gamma/2\pi(kHz)$ | 50                   | 7.6                    | 0.32                |
| $g/2\pi(kHz)$          | 300                  | 1.0                    | 0.205               |
| $v_{\text{probable}}(m/s)$ | 120                 | 505                    | 10                  |
| $\Gamma_{\text{transit}}(\mu s)$ | 0.2           | 2.8                    | 143                 |
| $\Gamma_{\text{cavity}}/2\pi(kHz)$ | 3.100     | 220                    | 4.3                 |
| $\Gamma_{\text{cavity}}/2\pi(MHz)}$ | 0.15          | 11                     | 0.22                |
| $\alpha$ $\Gamma_{\text{cavity}}/\Gamma_{\text{gain}}$ | 0.05          | 50                     | 50                  |
| $K = g^2\Gamma_{\text{transit}}(s^{-1})$ | $3.2 \times 10^5$ | 110                    | 237                 |
| $R_g(\text{atom/s})$   | $2.3 \times 10^7$   | $4.3 \times 10^{11}$  | $8.2 \times 10^7$   |
| 2$^{nd}$ Doppler(Hz)   | 270                  | 615                    | 0.25                |
| Photons in cavity      | 15                   | 6200                   | 60                  |
| $P_{\text{output-power}}(nW)$ | 0.004         | 120                    | 0.023               |
| $\Delta r_{\text{laser}}(r_0 = 2)(Hz)$ | 7,500         | 0.51                   | 1.1                 |

*The parameters of one-atom $^{138}$Ba laser are averaged over the standing wave transverse Gaussian profile. But for the parameters of $^{88}$Sr and $^{40}$Ca active optical frequency standards only the transit times and the transit-time broadening are averaged over the transverse Gaussian profiles.

In the recent $^{138}$Ba experiment[28], the number of photons in the cavity has reached more than 2500, it means the laser quantum-limit linewidth is 17Hz at good cavity regime.

TABLE II: Estimated major corrections and uncertainties of the thermal $^{88}$Sr beam active optical frequency standard. All values are in Hz.

| Effect                     | Correction | Uncertainty |
|----------------------------|------------|-------------|
| 2$^{nd}$-order Doppler     | 615        | 0.25        |
| Light shift                | 8          | 0.08        |
| Recoil shift               | 4.737      | $1 \times 10^{-3}$ |
| 1$^{st}$-order Zeeman      | 0          | 0.02        |
| 2$^{nd}$-order Zeeman      | 4          | 0.04        |
| Blackbody shift            | 1          | 0.01        |
| Collision shift            | 0.08       | 0.01        |
| cavity pulling             | 0.1        | 0.01        |
| Recoil asymmetry            | 0.02       | $1 \times 10^{-3}$ |
| Total uncertainty          |            | 0.27        |

first-order Doppler effect only broaden the line symmetrically, it does not cause shift of laser frequency[28]. The second-order Doppler broadening of 1.6kHz can be neglected, but its asymmetry contributes to frequency uncertainty of clock. Assuming there is 0.1°C uncertainty of oven temperature, the uncertainty of induced second-order Doppler broadening 0.25Hz is set as frequency uncertainty. The light shift induced by stray light from pumping laser is estimated to be 8Hz providing one tenth of the pumping laser intensity ($\pi$-pulse for moving atoms) goes into the cavity and the effective frequency detuning of the stray light is 1Hz supposing the pumping laser is locked to the output laser of active optical clock finally. Its uncertainty is set to be one percent of this light shift. The recoil effect will cause the asymmetry of lineshape on the broad gain background. Considering the $\pm 4.7kHz$ recoil frequency positions and the 220kHz gain bandwidth, we set a frequency correction of 0.02Hz. When the cavity mode detuning is 5Hz, the cavity pulling is 0.1Hz, and the cavity pulling may be calibrated to an uncertainty less than 0.01Hz. For 0.1°C oven temperature fluctuation during one second sampling-time, the corresponding instability of clock is 100mHz from the main source due to the Second-Doppler effect, which gives a limited Allan variance of $\sigma(\tau) = 2 \times 10^{-16}/\sqrt{\tau}$.

The parameters of cold $^{40}$Ca atom listed in Table I show the second-order Doppler shift decreases dramatically, and the 0.4ms long lifetime of $^3P_1$ state of $^{40}$Ca atom allowing the pumping laser to be put far away from the cavity to minimize the light shift induced by the stray light of pumping laser beam. Thus one can expect an absolute frequency uncertainty will be less than 10mHz with cold atoms. Particularly, once atoms trapped in Lamb-Dick regime of optical lattice with “magic wavelength” trapping laser[5], the limits from Doppler effect on instability and uncertainty may be almost eliminated, in this case, the active optical frequency standard based on the lattice atoms can be called optical lattice laser[29].

Another conceivable scheme is the “two-photon active optical clock”, the most attractive one will be “Hydrogen 1S-2S two-photon active optical clock” combining the Hydrogen 1S-2S two-photon spectroscopy[30] with the two-photon laser[31] under the principles and techniques presented in this letter.

This active optical clock can use any “free” medium: neutral atoms, ions and molecules. It’s expected the extension of the principles and techniques of active optical clock presented in this letter will have a great effect on fundamental physics such as Lorentz invariance test and gravitational wave detection, not only limited to precision laser spectroscopy and optical atomic clocks.

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