YUKAWA COUPLING EVOLUTION IN SUSY GUTS

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ABSTRACT

The scaling behavior and fixed points in the evolution of fermion Yukawa couplings and mixing angles are discussed. The relevance of fixed points in determining the top quark mass is described.

The unification of gauge couplings is a well-known feature of unified theories. To understand the masses of the observed fermions requires ideas beyond the standard model. With certain assumptions about the interactions in the GUT theory, relationships arise between various fermion masses. A mild assumption of minimality yields the well-known relation between the bottom and tau Yukawa couplings at the GUT scale $\lambda_b = \lambda_\tau$. A more ambitious program is the attempt to formulate ansatze for the Yukawa coupling matrices, and obtain relationships between fermion masses and CKM relations as well.

Unlike the gauge couplings or the CKM matrix elements, infrared fixed point solutions exist for the Yukawa coupling. Since the top quark is the only known fermion with a mass of order of the electroweak scale, it is usually the only particle for which the fixed point solution is relevant. The bottom and tau Yukawa couplings can be large in a multi-Higgs doublet model, where the smallness of the masses can be attributed to the properties of the vacuum.

The fixed point of the top quark Yukawa coupling in the minimal supersymmetric standard model (MSSM) can be obtained approximately by setting

$$\frac{d\lambda_t}{dt} = \frac{\lambda_t}{16\pi^2} \left( -\sum c_i g_i^2 + 6\lambda_t^2 + \lambda_b^2 \right) = 0,$$

with $c_1 = 13/15$, $c_2 = 3$, $c_3 = 16/3$. This is only accurate to about 10% in practice because the gauge couplings are themselves evolving. An analysis of the two-loop RGEs in the MSSM using experimental input for the gauge couplings yields an effective fixed point of $\lambda_t^f \approx 1.1$ near the electroweak scale $\mu = M_Z$ as shown in Figure 1. Top quark Yukawa couplings exceeding the fixed point value at the GUT scale evolve rapidly to the fixed point, while the approach from below is more gradual.

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The prediction for the $m_b/m_\tau$ ratio provides motivation for the fixed point solution. If we take as inputs $m_\tau = 1.784$ GeV and the running mass $m_t(m_b) = 4.25$ GeV, a large top quark Yukawa coupling is needed to counteract the evolution due to the gauge couplings which alone yield too large a value for the $m_b/m_\tau$ ratio. In the standard model the effective fixed point solution implies that the top quark is heavy $m_t > 200$ GeV. In the MSSM the Yukawa coupling must be large ($\approx 1$) at the electroweak scale, implying a linear correlation between $m_t$ and $\sin \beta$ (neglecting contributions from $\lambda_b$ and $\lambda_\tau$ which have a significant effect only for very large $\tan \beta$),

$$m_t(m_t) = (192 \text{GeV}) \sin \beta,$$

where $\tan \beta$ is the ratio of the vevs of the two Higgs doublets in the MSSM. This solution is indicated as the bands shown in Figure 2. As $\alpha_3(\mu)$ is increased, $\lambda_t(\mu)$ must be correspondingly increased to preserve the $m_b/m_\tau$ prediction. Hence for larger input $\alpha_3(M_Z)$, the solutions tend to display more strongly the fixed point character as shown in Figure 2. The fixed point does not require that $\tan \beta$ be small, but allows for large $\tan \beta$ if $m_t$ is sufficiently large. However if $m_t^{\text{pole}}$ is below 160 GeV, the fixed point gives $\tan \beta < 2$ with interesting consequences for Higgs boson phenomenology.

Corrections to the Yukawa couplings at the GUT scale arise similarly to the much-discussed threshold corrections to the gauge couplings. These two types of threshold corrections are correlated since they arise from the spectrum of massive states in the GUT theory.

The fixed point solution is largely independent of the supersymmetric spectrum and therefore of the exact nature of supersymmetry breaking. It is known that the minimal supergravity models require that $\tan \beta > 1$ in order to achieve the required symmetry breaking. This is manifested in the (tree-level) relation

$$\frac{1}{2} M_Z^2 = \frac{m_{H_1}^2 - m_{H_2}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2.$$
Here $m_{H_1}$ and $m_{H_2}$ are the masses of the two Higgs doublets, and $\mu$ is the Higgs mass parameter appearing in the superpotential. For $\tan\beta$ near one, $\mu$ must be large to permit the substantial cancellation to achieve the correct $M_Z$. Various criteria can be chosen to determine where this cancellation becomes unnatural; this choice is largely a matter of taste. This unnaturalness is ameliorated somewhat with the inclusion of one-loop corrections. In any case, the large value of $\mu$ has interesting consequences for the supersymmetric spectrum. The $\tan\beta = 1$ direction in field space is a D-flat direction of the supersymmetric theory. Consequently an associated Higgs eigenstate is precisely massless at tree level for $\tan\beta = 1$.

![Diagram](image)

Fig. 2. The effect of threshold corrections on the Yukawa coupling unification condition $\lambda_b(M_G) = \lambda_\tau$ with $m_b(m_b) = 4.25$ GeV for $\alpha_3(M_Z) = 0.11$ and 0.12.

When there is a hierarchy of masses in the Yukawa matrices, the evolution of the quark masses and CKM mixing angles is given as a simple scaling. If only one generation is heavy (with Yukawa couplings $\simeq 1$) and the mixing angles of this generation with the light generations is small, the evolution is given by scaling equations. The CKM mixing angles evolve as

$$\frac{dW_1}{dt} = -\frac{W_1}{8\pi^2} \left( \lambda_1^2 + \lambda_2^2 \right),$$

(4)

in the MSSM where $W_1 = |V_{cb}|^2, |V_{ub}|^2, |V_{ts}|^2, |V_{td}|^2$, the CP-violation parameter $J$ and

$$\frac{dW_2}{dt} = 0,$$

(5)

where $W_2 = |V_{us}|^2, |V_{cd}|^2, |V_{tb}|^2, |V_{cs}|^2, |V_{ud}|^2$. The solution of Eq. (4) is given by the scaling equation

$$W_1(M_G) = W_1(\mu) \exp \left\{-\frac{1}{8\pi^2} \int_{\mu}^{M_G} \left( \lambda_1^2 + \lambda_2^2 \right) d\ln \mu' \right\}.$$

(6)
The lightest two generations do not affect the evolution, and one does not need the mixing between the first two generations to be small for the universal scaling described above to occur. This makes the scaling universality an especially good approximation since the Cabbibo angle is the largest of the quark mixings. The scaling behavior can be demonstrated to all orders in perturbation theory.

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References

1. M. Chanowitz, J. Ellis and M. Gaillard, *Nucl. Phys.* B128 (1977) 506.
2. B. Pendleton and G. G. Ross, *Phys. Lett.* B98 (1981) 291; C. T. Hill, *Phys. Rev.* D42 (1981) 691.
3. C. D. Froggatt, I. G. Knowles and R. G. Moorhouse, *Phys. Lett.* B249 (1990) 273; *Phys. Lett.* B298 (1993) 356.
4. V. Barger, M. S. Berger, and P. Ohmann, *Phys. Rev.* D47 (1993) 1093; V. Barger, M. S. Berger, T. Han and M. Zralek, *Phys. Rev. Lett.* 68 (1992) 3394.
5. M. Carena, S. Pokorski, and C. E. M. Wagner, *Nucl. Phys.* B406 (1993) 59; W. Bardeen, M. Carena, S. Pokorski, and C. E. M. Wagner, Munich preprint MPI-Ph/93-58.
6. V. Barger, M. S. Berger, P. Ohmann, and R. J. N. Phillips, *Phys. Lett.* B314 (1993) 351.
7. P. Langacker and N. Polonsky, University of Pennsylvania preprint UPR-0556-T (1993).
8. S. Kelley, J. L. Lopez, D. V. Nanopoulos, H. Pois, and K. Yuan, *Nucl. Phys.* B398 (1993) 3.
9. M. Olechowski and S. Pokorski, *Nucl. Phys.* B404 (1993) 590.
10. B. de Carlos and J. A. Casas, *Phys. Lett.* B309 (1993) 320.
11. V. Barger, M. S. Berger, and P. Ohmann, in preparation.
12. E. Ma and S. Pakvasa, *Phys. Lett.* B86 (1979) 43, *Phys. Rev.* D20 (1979) 2899; K. Sasaki, Z. Phys. C32 (1986) 149; K. S. Babu, Z. Phys. C35 (1987) 69; M. Olechowski and S. Pokorski, *Phys. Lett.* B257 (1991) 388; V. Barger, M. S. Berger, and P. Ohmann, *Phys. Rev.* D47 (1993) 2038; K. S. Babu and Q. Shafi, *Phys. Rev.* D47 (1993) 5004.