Suppression of the nuclear rainbow in the inelastic nucleus-nucleus scattering

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Abstract. The nuclear rainbow observed in the elastic α-nucleus and light heavy-ion scattering is proven to be due to the refraction of the scattering wave by a deep, attractive real optical potential. The nuclear rainbow pattern, established as a broad oscillation of the Airy minima in the elastic cross section, originates from an interference of the refracted far-side scattering amplitudes. It is natural to expect a similar rainbow pattern also in the inelastic scattering of a nucleus-nucleus system that exhibits a pronounced rainbow pattern in the elastic channel. Although some feature of the nuclear rainbow in the inelastic nucleus-nucleus scattering was observed in experiment, the measured inelastic cross sections exhibit much weaker rainbow pattern, where the Airy oscillation is suppressed and smeared out. To investigate this effect, a novel method of the near-far decomposition of the inelastic scattering amplitude is proposed to explicitly reveal the coupled partial-wave contributions to the inelastic cross section. Using the new decomposition method, our coupled channel analysis of the elastic and inelastic ¹²C+¹²C and ¹⁶O+¹²C scattering at the refractive energies shows unambiguously that the suppression of the nuclear rainbow pattern in the inelastic scattering cross section is caused by a destructive interference of the partial waves of different multipoles. However, the inelastic scattering remains strongly refractive in these cases, where the far-side scattering is dominant at medium and large angles like that observed in the elastic scattering.

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1 Introduction

The optical model (OM) studies of elastic heavy-ion (HI) scattering usually shows a strong absorption that suppresses the refractive (large-angle) scattering, and elastic HI scattering occurs mainly at the surface. However, with the nuclear rainbow pattern observed in the elastic scattering of some α-nucleus and light HI systems, the absorption turns out to be much weaker and the refractive, far-side scattering becomes dominant at medium and large angles [1,2,3]. Not only a fascinating semiclassical analog to the atmospheric rainbow, the nuclear rainbow also enables the determination of the real nucleus-nucleus optical potential (OP) with much less ambiguity, down to small internuclear distances [3]. The pattern of the nuclear rainbow is usually characterized by a broad oscillation of the Airy minima [1,2,3] in the elastic cross section. The observation of these minima, especially, the first Airy minimum A1 that is followed by a broad shoulder-like maximum, not only facilitates the determination of the real OP but also provides an useful probe of the cluster structure of light nuclei [3,5].

Similarly to the atmospheric rainbow, the nuclear rainbow can be interpreted as a pattern resulted from an interference of the two scattering amplitudes, as shown by the barrier-wave/internal-wave (BI) or near-side/far-side (NF) decomposition of the elastic scattering amplitude. The BI method proposed by Brink and Takigawa [6] describes elastic HI scattering in terms of the internal waves penetrating through the potential barrier into the nuclear interior, and barrier waves reflected from the barrier. On the other hand, the NF decomposition suggested by Fuller [7] splits the elastic scattering amplitude into the near-side (N) and far-side (F) components, corresponding to the waves deflected to the near side and far side of the scattering center, respectively. These two interpretations are complementary, and the broad Airy oscillation of the nuclear rainbow pattern is given by the interference of two far-side amplitudes [3,8,9,10]. These are either the barrier fBF and internal fIF far-side amplitudes [6], or fF< and fF> far-side amplitudes with the orbital momenta L smaller or larger than a critical value LR associated with the rainbow angle θR [7].
From such a pattern of the nuclear rainbow, a similar Airy structure is expected to be seen also in the inelastic scattering of a light HI or α-nucleus system that shows a strong rainbow pattern in the elastic scattering. In fact, some feature of the nuclear rainbow was observed in the inelastic light HI scattering [11,12,13,14,15,16,17,18,19,20], and some of these data were analyzed using the BI [13] and NF [17,18,19,20] decomposition methods. Although these studies have confirmed the dominance of the far-side scattering at large angles, the Airy oscillation pattern could not be clearly identified in the inelastic cross section. Such suppression of the nuclear rainbow was assumed by some feature of the nuclear rainbow was observed in the inelastic scattering [11,12,13,14,15,16,17,18,19,20]

\[ \frac{\hbar^2}{2\mu_{\beta}} \left[ \frac{d^2}{dR^2} + k^2 - \frac{L(L + 1)}{R^2} \right] - \langle \beta(LI') | V | \beta(LI) \rangle \times \chi_{\beta,I}(k, R) = \sum_{\beta'L'I'} \langle \beta(LI') | V | \beta'(L'I') \rangle \chi_{\beta',\beta}(k', R), \]

where \( \beta \) and \( \beta' \) denote the entrance and exit channels, respectively; \( \mu_{\beta} \) is the reduced mass, \( \hbar k = \sqrt{2\mu_{\beta}E_{\beta}} \) is the center-of-mass (c.m.) momentum, and \( \chi_{\beta,I}(k, R) \) is the scattering wave function at the internuclear radius.

2 Near-far decomposition of the inelastic scattering amplitude in the CC formalism

2.1 General formalism

We recall here briefly the coupled channel equations for the elastic and inelastic nucleus-nucleus scattering. The scattering wave function is obtained at each total angular momentum \( J^* \) of the nucleus-nucleus system from the solution of the following CC equations [25]

\[ J^* = \sqrt{2J_{\beta}E_{\beta}} \]

\[ \chi_{\beta,I}(k, R) \]

For the NF decomposition of the inelastic scattering amplitude, we apply the extended NF decomposition method by Fuller [7] to split the inelastic scattering amplitude of the coupled partial waves into the near-side and far-side components, so that the refraction in the inelastic scattering channel is studied on equal footing with that in the elastic channel, and the formation of the nuclear rainbow therein. Given the prominent nuclear rainbow pattern observed in the elastic \(^{12}\text{C}+^{12}\text{C}\) and \(^{16}\text{O}+^{12}\text{C}\) scattering at the refractive energies around 10 ~ 20 MeV/nucleon, we apply the extended NF decomposition method to the inelastic scattering to the \(^2\text{I}^+\) (4.44 MeV) state of \(^{12}\text{C}\) target in the coupled channel (CC) analysis of the inelastic scattering data measured for the \(^{12}\text{C}+^{12}\text{C}\) system at \( E_{\text{lab}} = 240 \) MeV [21,22], and the \(^{16}\text{O}+^{12}\text{C}\) system at \( E_{\text{lab}} = 200 \) and 260 MeV [10,23].
excitation of the target. Then, \(|L' - I'\) \leq L \leq L' + I',\) where \(I'\) is the spin of the excited target.

The diagonal matrix element \(V_{\alpha\beta}(R)\) of the projectile-target interaction in Eq. \(1\) is the nucleus-nucleus OP, and the nondiagonal matrix element \(V_{\alpha\beta}(R)\) is the transition potential, which is also dubbed as the inelastic scattering form factor (FF). The OP and inelastic scattering FF can be evaluated microscopically in the double-folding model (DFM) using the ground-state (g.s.) and transition nuclear densities, respectively, and an effective nucleon-nucleon (NN) interaction between the projectile- and target nucleons (see more details in Ref. \[25\]). From the solution of the CC equations \([1]\), we obtain the elastic scattering amplitude as

\[
f(\theta) = f_C(\theta) + \frac{1}{2ik} \sum_L (2L+1) \exp(2i\sigma_L)(S_L - 1)P_L(\cos \theta),
\]

where \(f_C(\theta)\) and \(\sigma_L\) are the Rutherford scattering amplitude and Coulomb phase shift, respectively; \(S_L\) is the diagonal element of the elastic scattering \(S\) matrix, and \(P_L(\cos \theta) \equiv P_{LM=0}(\cos \theta)\) is the Legendre function of the first kind. Within the CC formalism \([26\]), the amplitude of the inelastic scattering to an excited state of the target with spin \(I'\) and projection \(M'\) is written explicitly as

\[
f_{M',\theta}(\phi) = \frac{\sqrt{4\pi}}{2ik} \sum_{LL'} \sqrt{2L + 1} (L' - M'I'M'|L0) \times \exp[i(\sigma_L + \sigma_{L'})(S_L' - 1)P_L(\cos \theta)] (6)
\]

Here \(Y_{LM}(\theta, \phi)\) is the spherical harmonics, the Coulomb phase shift \(\sigma_{L'}\) is evaluated from the c.m. momentum \(k'\) in the exit channel, and \(S'_{L'}\) is the element of the inelastic scattering \(S\) matrix. The orbital angular momenta in the entrance and exit channels are linked with spin \(I'\) of the excited target by the triangular rule

\[
L' = L - I', L - I' + 2, \ldots, L + I' - 2, L + I',
\]

where the step of two angular-momentum units is implied by the parity conservation.

2.2 Multipole mixing of the partial waves

One can see from the expansion \([3\]) that the coupled partial waves of different multipoles can contribute coherently to the inelastic scattering amplitude at the same scattering angle \(\theta\) when \(I' \neq 0\). By expressing the selection rule \([4]\) as \(L' = L + K\), the inelastic scattering amplitude \([3]\) can be written in terms of the \(K\)-subamplitudes allowed by the selection rule \([4]\)

\[
f_{M',\theta}(\phi) = \sum_{K=-I'}^{I'} f^{(K)}_{M',\theta}(\phi).
\]

Like the elastic amplitude \([2]\), each \(K\)-subamplitude of \(f_{M',\theta}(\phi)\) can be expanded over the orbital momenta of the entrance channel \(L\) as

\[
f^{(K)}_{M',\theta}(\phi) = \sqrt{\frac{4\pi}{2ik}} \sum_L \sqrt{2L + 1} (L + K - M'I'M'|L0) \times \exp[i(\sigma_L + \sigma_{L+K})S_{L+K}Y_{L+K-M'I'}(\theta, \phi)]
\]

In terms of the inelastic scattering cross section, the contribution from each \(K\)-subamplitude is obtained at the given scattering angle as

\[
\frac{d\sigma_K}{d\Omega} = \sum_{M'} \left| f^{(K)}_{M',\theta}(\phi) \right|^2,
\]

so that the full cross section of the inelastic scattering to an excited state of the target with spin \(I'\) is

\[
\frac{d\sigma}{d\Omega} = \sum_{M'} \sum_{K=-I'}^{I'} \left| f^{(K)}_{M',\theta}(\phi) \right|^2.
\]

Thus, for an excited state with spin \(I' \neq 0\), the full inelastic scattering cross section is given by the interference of the \(K\)-subamplitudes with \(K = -I', I' + 2, \ldots, I' - 2, I'\).

2.3 Near-far decomposition of the scattering amplitude

As mentioned above, the NF decomposition method by Fuller \([7]\) is a very helpful tool to analyze the interference of the near-side and far-side scattering amplitudes in the elastic scattering \([23\]). Namely, the elastic scattering amplitude is decomposed into the near-side (\(f_N\)) and far-side (\(f_F\)) components as

\[
f(\theta) = f_N(\theta) + f_F(\theta) = f_{C}^{(N)}(\theta) + f_{C}^{(F)}(\theta)
\]

\[
+ \frac{1}{2ik} \sum_L (2L+1) \exp(2i\sigma_L)(S_L - 1) \times \left[ Q_{L}^{(-)}(\cos \theta) + Q_{L}^{(+)}(\cos \theta) \right],
\]

where \(f_{C}^{(NF)}(\theta)\) is the near-side (far-side) component of the Rutherford scattering amplitude \([7]\), the relative strength of the near-side and far-side nuclear scattering is given by \(Q_{L}^{(-)}(\cos \theta)\) and \(Q_{L}^{(+)}(\cos \theta)\), respectively,

\[
Q_{L}^{(\pm)}(\cos \theta) = \frac{1}{2} \left[ P_{L}(\cos \theta) \pm \frac{2i}{\pi} Q_{L}(\cos \theta) \right],
\]

where \(Q_{L}(\cos \theta) \equiv Q_{LM=0}(\cos \theta)\) is the Legendre function of the second kind. It is well established \([23,27\]) that the nuclear rainbow pattern observed in the elastic \(\alpha\)-nucleus and light HI scattering is determined entirely by the far-side component of the elastic amplitude \([8]\). The nuclear rainbow pattern is a broad oscillation of the Airy minima at medium and large scattering angles that results from an interference between \(f_{F,\alpha}(\theta)\) and \(f_{F,\alpha}(\theta)\) subamplitudes of the far-side component in \([8]\), with \(L\) being smaller or larger than a critical partial wave \(L_{R}\) associated with the rainbow angle \(\theta_{R}\) \([27]\).
It is natural to expect a similar rainbow pattern also in the inelastic scattering cross section of a nucleus-nucleus system that exhibits a pronounced nuclear rainbow in the elastic scattering channel. For this purpose, a NF decomposition of the inelastic scattering amplitude should be done for each projection $M_{I'}$ of the target spin $I'$, with the contributions of all allowed $K$-subamplitudes treated explicitly. However, such a detailed decomposition method is so far unavailable, and only some general discussion on possible Airy structure of the inelastic scattering cross section was made based on the Airy pattern established in the elastic scattering cross section. We note here an early attempt to extend the NF decomposition method to the inelastic HI scattering by Dean and Rowley, where the near-side and far-side scattering amplitudes obtained for each $M_{I'}$ magnetic substate of the target excitation with $I' \neq 0$ were shown to be not in phase. However, the refractive Airy pattern of the nuclear rainbow was not discussed at all in Ref. [28]. To close this gap of the scattering theory, we suggest in the present work a method of the NF decomposition of the inelastic scattering amplitude to investigate explicitly the Airy oscillation pattern in the inelastic scattering cross section. Thus, the NF decomposition is generalized to decompose the inelastic scattering amplitude using the associated Legendre functions as

$$f_{M_{I'}}(\theta, \phi) = f_{N_{M_{I'}}}^{(K)}(\theta, \phi) + f_{F_{M_{I'}}}^{(K)}(\theta, \phi) = \frac{\sqrt{4\pi}}{2iK} \sum_{L,K} \sqrt{2L + 1} \langle L' - M_{I'}, I'M_{I'} | L \rangle \times A_{L',M_{I'}} \exp[i(\sigma_L + \sigma_{L'}^\prime) \exp(-i\pi M_{I'} \phi) \times S_{L',L}^{(\pm)} \left[ Q_L^{(\pm)}(\cos \theta) + Q_{L'-M_{I'}}^{(\pm)}(\cos \theta) \right].$$

(11)

where $Q_L^{(\pm)}(\cos \theta) = \frac{1}{2} \left[ P_{LM}(\cos \theta) \pm \frac{2i}{\pi} Q_{LM}(\cos \theta) \right]$, and

$$A_{L,M} = \sqrt{\frac{2L + 1}{4\pi} \frac{(L + M)!}{(L - M)!}}.$$

Here $P_{LM}(\cos \theta)$ and $Q_{LM}(\cos \theta)$ are the associated Legendre functions of the first- and second kind, respectively. We note that the inelastic scattering FF includes both the Coulomb and nuclear contributions, and the inelastic Coulomb scattering amplitude is not treated separately as in the elastic scattering channel.

Expressing $L' = L + K$ in the generalized NF decomposition, we obtain explicitly the near-side and far-side components of each $K$-subamplitude of the inelastic scattering amplitude as

$$f_{L,M_{I'}}^{(K)}(\theta, \phi) = f_{N,M_{I'}}^{(K)}(\theta, \phi) + f_{F,M_{I'}}^{(K)}(\theta, \phi) = \sqrt{4\pi} \frac{1}{2iK} \sum_L \sqrt{2L + 1} \langle L + K - M_{I'}, I'M_{I'} | L \rangle \times A_{L,M_{I'}} \exp[i(\sigma_L + \sigma_{L+K}^\prime - M_{I'} \phi)] S_{L,M_{I'}}(\cos \theta) \times Q_{L'-M_{I'}}^{(\pm)}(\cos \theta) + Q_{L'-M_{I'}}^{(\pm)}(\cos \theta).$$

(12)

Thus, the generalized NF decomposition allows us to determine the near-side and far-side contributions from each $K$-subamplitude to the partial and full inelastic cross section, and to study the formation of the nuclear rainbow in the inelastic nucleus-nucleus scattering in the same manner as done for the elastic scattering using the Fuller method.

### 3 Elastic and inelastic $^{12}$C+$^{12}$C and $^{16}$O+$^{12}$C scattering at the refractive energies

The diagonal matrix element $V_{\beta\beta}(R)$ of the projectile-target interaction in the CC equations is determined by the total optical potential $U_0(R)$. The new version of the density dependent CDM3Y3 interaction with the rearrangement term included is used in the double-folding calculation of the real optical potential $V_0(R)$. Because the nuclear rainbow is a subtle effect that can be observed only when the absorption of the dinuclear system is weak, the imaginary OP in the flexible Woods-Saxon (WS) form is usually used for a proper identification of the rainbow pattern. Thus, we have

$$U_0(R) = N_R V_0(R) + i W_0(R) + V_C(R),$$

where

$$W_0(R) = -\frac{W_C}{1 + \exp[(R - R_V)/a_V]].$$

The Coulomb potential $V_C(R)$ is obtained by folding the two uniform charge distributions with their mean-squared radii chosen to be close to the measured charge radii of the two nuclei. The nuclear g.s. densities used in the DFM calculation are taken as the Fermi distributions with parameters chosen to reproduce the empirical matter radii of the considered nuclei. The renormalization $N_R$ of the real folded potential and the WS parameters are adjusted in each case to the best CC description of the elastic scattering data, and a small deviation of $N_R$ from unity validates the use of the folding model. The best-fit OP parameters used in the present CC study of the elastic and inelastic $^{12}$C+$^{12}$C and $^{16}$O+$^{12}$C scattering are given in Table 1.

The nondiagonal matrix element $V_{\beta\beta'}(R)$ in the CC equations is given by the inelastic form factor $U_{I'}(R)$ that accounts for inelastic scattering to the target excited state with spin $I'$ (see details in Ref. [29]).

$$U_{I'}(R) = N_R V_{I'}(R) - i \delta_{I'I} \frac{\partial W_0(R)}{\partial R} + V_C(R),$$

(14)

where the real nuclear $V_{I'}(R)$ and Coulomb $V_C(R)$ inelastic form factors are calculated in the DFM using the nuclear transition densities of the excited states of $^{12}$C obtained in the resonating group method (RGM). The nuclear deformation lengths $\delta_{I'}$ are determined by the collective-model prescription using the measured $B(EI')$ transition rates of the considered excited states of $^{12}$C. All the CC calculations have been done using the code ECIS97 written by Jacques Raynal that provides the detailed output of the elastic and inelastic scattering.
of the corresponding scattering amplitudes. It should be recalled that the sequential iteration method implemented in the ECIS code was developed by Raynal to tackle the inelastic HI scattering with a strong Coulomb contribution, focusing in particular on the scattering experiments being carried out at GANIL at that time [24]. By using the recurrence relations for the Coulomb excitation integrals in the CC calculations [35], the ECIS integration of the coupled equations is highly stable and accurate up to very large radii with sufficiently high number of partial waves. For the 12C+12C and 16O+12C systems under the present study, the ECIS integration up to \( R_{\text{max}} \approx 25 \) fm in steps of \( dr = 0.05 \) fm is needed to ensure the convergence of the calculated cross section, taking into account up to 180 partial waves. At the considered energies, the CC results obtained using the nonrelativistic and relativistic kinematics are about the same.

The elastic and inelastic 12C+12C scattering has been widely studied at energies ranging from the Coulomb barrier [10] up to about 200 MeV/nucleon [36]. While the elastic 12C+12C scattering at the barrier energies was shown to be of interest for nuclear astrophysics [37], the scattering data measured for this system at the refractive energies around 20 MeV/nucleon [21,22] exhibit a nuclear rainbow pattern that enabled an unambiguous determination of the real OP down to small distances [38,24]. In particular, the 12C+12C scattering data measured accurately at \( E_{\text{lab}} = 240 \) MeV [21,22] are very important for our study because this energy was found optimal for the observation of the first Airy minimum A1 of the nuclear rainbow scattering [24]. As shown above in Fig. 1, the data of the inelastic 12C+12C scattering to the 2+ (4.44 MeV) state of 12C measured at this same energy [21,22] do not have any minimum that can be interpreted as the remnant of A1, at angles near the location of A1 established in the elastic cross section. Another light HI system that exhibits a prominent rainbow pattern in the elastic scattering is 16O+12C [23]. Unlike 12C+12C, the 16O+12C system does not have the boson symmetry, and the angular evolution of the Airy pattern could be observed with the increasing energy. The strongest rainbow pattern, the deep A1 minimum followed by an exponential fall-off of the rainbow shoulder, is well confirmed in the elastic 16O+12C scattering data measured at \( E_{\text{lab}} = 200 \) MeV [23]. The question why the inelastic 16O+12C scattering data measured at this same energy [16] does not show a similar Airy pattern (see lower panel of Fig. 1) is so far unanswered. In the present work, we try to explain the suppression of the first Airy minimum in the inelastic 12C+12C and 16O+12C scattering cross sections at the energies where A1 was clearly identified in the elastic cross sections measured for these systems.

The CC results for the elastic and inelastic 12C+12C scattering describe well the data as shown in Fig. 2. The dominance of the far-side cross sections at medium and large angles indicates that both the elastic- and inelastic 12C+12C at the considered energies are strongly refractive. The nuclear rainbow pattern is well established in the elastic 12C+12C scattering, with the first Airy minimum A1 unambiguously identified [21] at the scattering angle \( \theta_{\text{c.m.}} \approx 41^\circ \) based on the NF decomposition [9] of

| \( ^{12}\text{C}+^{12}\text{C} \) | \( E_{\text{lab}} \) (MeV) | \( N_R \) | \( J_R \) (MeV fm \(^3\)) | \( W_V \) (MeV) | \( R_V \) (fm) | \( a_V \) (MeV fm \(^3\)) | \( J_V \) (MeV fm \(^3\)) | Data |
|---|---|---|---|---|---|---|---|---|
| 12C+12C | 240 | 1.067 | 336.0 | 19.29 | 5.743 | 0.595 | 117.5 | [21,22] |
| 16O+12C | 200 | 0.936 | 300.4 | 13.32 | 6.150 | 0.502 | 72.06 | [16,23] |
| 16O+12C | 260 | 0.930 | 291.8 | 18.50 | 5.756 | 0.550 | 83.92 | [16,23] |
the elastic scattering amplitude. The data of the inelastic $^{12}\text{C}+^{12}\text{C}$ scattering to the $2_1^+$ state of $^{12}\text{C}$ are reproduced reasonably by the CC calculation [1], but the Airy structure seen in the elastic cross section is smeared out in the inelastic cross section. Given the dominance of the far-side scattering at medium and large angles, the suppression of $\text{A1}$ in the inelastic cross section is definitely not caused by the near-side/far-side interference, but more likely by a destructive interference of the far-side subamplitudes.

![Fig. 3](image1.png)

**Fig. 3.** The same as Fig. [2](#) but for the elastic and inelastic $^{16}\text{O}+^{12}\text{C}$ scattering at $E_{lab} = 200\text{ MeV}$ [10,23].

One can see in the partial-wave expansion of the inelastic scattering amplitude [3] that the inelastic scattering cross section at the given scattering angle contains the contributions from the subamplitudes of different partial waves ($L' \neq L$) when the spin of the excited state is nonzero ($I' \neq 0$). For the inelastic $^{12}\text{C}+^{12}\text{C}$ scattering to the $2_1^+$ state of $^{12}\text{C}$ shown in Fig. [2](#), each $L$-component of the inelastic scattering amplitude is resulting from an interference of the three $K$-subamplitudes with $K = L' - L = 2, 0, 2$. The partial inelastic scattering cross sections [7] given by the three $K$-subamplitudes (summed over all partial waves $L$) are shown separately in the lower panel of Fig. [2](#). By tracing the angular evolution of the corresponding far-side cross sections, we have identified the first Airy minimum $\text{A1}$ in the partial inelastic $^{12}\text{C}+^{12}\text{C}$ cross section with $K = 0$ at $\theta_{c.m.} \approx 40^\circ$ which is close to the locations of $\text{A1}$ in the elastic $^{12}\text{C}+^{12}\text{C}$ cross section. While a slight remnant of $\text{A1}$ with $K = 0$ can still be seen in the calculated inelastic scattering cross section [8], it is not observed in the measured data. Rather weak rainbow patterns of the two partial inelastic $^{12}\text{C}+^{12}\text{C}$ cross sections with $K \neq 0$ were found which are shifted in angles, with $\text{A1}$ located at $\theta_{c.m.} \approx 33^\circ$ and $52^\circ$ in the partial inelastic cross section with $K = 2$ and $K = -2$, respectively. One can see in the lower panel of Fig. [2](#) that the partial inelastic cross sections with $K = \pm 2$ are much weaker than that with $K = 0$ at medium and large angles, and no remnants of $\text{A1}$ with $K = \pm 2$ can be seen in the total inelastic cross section.

![Fig. 4](image2.png)

**Fig. 4.** The same as Fig. [2](#) but for the elastic and inelastic $^{16}\text{O}+^{12}\text{C}$ scattering at $E_{lab} = 260\text{ MeV}$ [10,23].

A similar picture can be seen in the CC results for the elastic and inelastic $^{16}\text{O}+^{12}\text{C}$ scattering at $E_{lab} = 200\text{ MeV}$ plotted in Fig. [3](#). While the prominent $\text{A1}$ minimum is located at $\theta_{c.m.} \approx 65^\circ$ in the elastic cross section, it seems to disappear in the inelastic $^{16}\text{O}+^{12}\text{C}$ scattering cross section. Such an effect was found also in the results of the earlier CC analysis of the inelastic $^{16}\text{O}+^{12}\text{C}$ scattering [5] as well as those of a cluster folded model study [9]. As discussed for the $^{12}\text{C}+^{12}\text{C}$ system, three different Airy oscillation patterns can be seen in the partial inelastic $^{16}\text{O}+^{12}\text{C}$ cross sections given by the subamplitudes with $K = 2, 0, -2$, with $\text{A1}$ located at $\theta_{c.m.} \approx 49^\circ, 62^\circ$, and $84^\circ$, respectively. Again, the location of $\text{A1}$ with $K = 0$ is quite close to the locations of $\text{A1}$ in the elastic $^{16}\text{O}+^{12}\text{C}$ cross section. The full inelastic scattering cross section [5] includes the contributions from all allowed $K$-subamplitudes, and their out-of-phase interference smears out the individual $\text{A1}$ minima seen in the partial inelastic cross sections [7]. While a slight remnant of $\text{A1}$ with $K = 0$ is seen in the calculated inelastic cross section (solid line in the lower panel of Fig. [3](#)), it cannot be clearly resolved in the measured data. The same CC results for the elastic and inelastic $^{16}\text{O}+^{12}\text{C}$ scat-
tering at \(E_{\text{lab}} = 260\) MeV are compared with the data [10][23] in Fig. 4. We found that the absorption becomes slightly stronger (see Table 1) with the increasing energy, and the Airy oscillation pattern is weakened and shifted to smaller angles (with A1 in the elastic cross section located at \(\theta_{c.m.} \approx 50^\circ\)). The weaker Airy oscillation pattern of each partial inelastic cross section can still be seen but the remnant of A1 with \(K = 0\) disappears in both the calculated inelastic scattering cross section and measured data (lower panel of Fig. 3). In conclusion, the results of our CC analysis shown in Figs. 2-4 explain naturally why the Airy oscillation pattern of the nuclear rainbow is strongly suppressed in the measured data of the inelastic \(^{12}\text{C} + ^{12}\text{C}\) and \(^{16}\text{O} + ^{12}\text{C}\) scattering to the \(2_1^+\) state of \(^{12}\text{C}\) at the rainbow energies.

![Fig. 5. The same CC results as those in Fig. 3 for the elastic and inelastic \(^{16}\text{O} + ^{12}\text{C}\) scattering at \(E_{\text{lab}} = 200\) MeV, obtained with a less absorptive OP (13) with \(W_V \rightarrow W_V/3\).](image)

It is well-known that the nuclear rainbow is formed by the interference of the far-side scattering waves, refracted by the attractive real OP [21][3]. That’s the reason why the nuclear rainbow could be observed only when the absorption of the scattering system is weak enough for the far-side trajectories to survive at the medium and large scattering angles. In practice, the absorptive strength of the OP is often reduced to artificially enhance the far-side scattering amplitude for a proper identification of the Airy oscillation pattern [24]. The results of the CC calculation of the elastic and inelastic \(^{16}\text{O} + ^{12}\text{C}\) scattering at 200 MeV given by a less absorptive OP (with \(W_V \rightarrow W_V/3\)) are plotted in Fig. 5 and one can trace in the elastic cross section the whole pattern of the nuclear rainbow including the first (A1), second (A2), and third (A3) Airy minima. The same Airy oscillation pattern can be seen also in the three partial inelastic scattering cross sections [7] given by the three K-subamplitudes [8], but the locations of the Airy minima are shifted to the smaller angles when \(K = 2\), and to the larger angles when \(K = -2\). It is very essential to emphasize again that the Airy oscillation pattern in the partial inelastic cross section with \(K = 0\) remains about the same as that observed in the elastic scattering thanks to an in-phase interference of the partial waves with \(L' = L\). When \(K \neq 0\), the out-of-phase interference of the partial waves with \(L' \neq L\) smears out the different Airy oscillation patterns in the full inelastic \(2_1^+\) scattering cross section. Because the partial inelastic cross section with \(K = 0\) is substantially larger that those with \(K = \pm 2\) at medium and large angles, the remnant of the first Airy minimum A1 with \(K = 0\) can be very well seen in the full inelastic scattering cross section (solid line in the lower panel of Fig. 3) when a reduced absorption \(W_V\) was used in the CC calculation. In fact, the broad rainbow shoulder following A1 with \(K = 0\) is still visible in the data measured at \(E_{\text{lab}} = 200\) MeV for the inelastic \(^{16}\text{O} + ^{12}\text{C}\) scattering to the \(2_1^+\) state of \(^{12}\text{C}\) (see lower panel of Fig. 3).

It becomes clear now that there is no unique Airy pattern of the nuclear rainbow in the full (far-side) cross section of the inelastic nucleus-nucleus scattering to an excited nuclear state with nonzero spin. In such a case, only the Airy oscillation pattern of the partial inelastic cross section [7] given separately by each K-subamplitude [8] can be determined in the same manner as done in the case of elastic scattering. Thus, the detailed locations of the Airy minima A1, A2, and A3 in the inelastic \(^{16}\text{O} + ^{12}\text{C}\) scattering cross section deduced visually by Ohkubo et al. from the calculated angular distribution [34] are not properly founded.

Although the inelastic \(^{16}\text{O} + ^{12}\text{C}\) scattering to the \(0_2^+\) (Hoyle) and \(3_1^-\) excited states (at \(E_x \approx 7.65\) and 9.64 MeV, respectively) were not measured at \(E_{\text{lab}} = 200\) MeV, the CC prediction of the inelastic cross sections for these states should be of interest for the revelation of the nuclear rainbow pattern therein. The CC results for the inelastic \(^{16}\text{O} + ^{12}\text{C}\) scattering at 200 MeV obtained with the nuclear transition densities of the \(0_2^+\) and \(3_1^-\) states of \(^{12}\text{C}\) given by the RGM [30] and the same OP as given in Table 1 are shown in Fig. 4. One can see that the refractive (far-side) scattering is also dominant at medium and large angles in the inelastic scattering to the Hoyle (02+) state, with \(K = 0\) (or \(L' = L\)) and \(d\sigma/K/d\Omega = d\sigma/d\Omega\). In this case, spin of the excited state is zero and there is no interference of the scattering subamplitudes with different \(K\). As a result, the Airy pattern in the angular distribution of the inelastic scattering to the Hoyle state is determined with a single (\(K = 0\)) inelastic scattering amplitude, in the same manner as done for the elastic \(^{16}\text{O} + ^{12}\text{C}\) scattering. Thus, the deep minimum of the inelastic \(0_2^+\) cross section can be confirmed as the first Airy minimum A1 which is located at about the same angle as A1 of the elastic cross section (see upper panels of Figs. 3 and 5).
the inelastic of suppression of the Airy pattern as discussed above for elastic scattering cross section (8). This is the same effect lower panel of Fig. 6 that the out-of-phase interference of Airy minimum $A_1$ of the partial inelastic cross section $16$ Fig. 6. The same CC results as those in Fig. 2 but for the inelastic scattering amplitude for all partial wave contributions, which does not mix different multipoles in the inelastic scattering amplitude, and the Airy pattern in the inelastic cross section can be determined consistently in the same manner as done for the elastic scattering. In light of this result, an accurate measurement of the inelastic $\alpha$ or light ion scattering to the $0^+_1$ excitation of the $^{12}$C target should be of interest for the future studies of the nuclear rainbow scattering as well as the $\alpha$-cluster structure of the Hoyle state $^{16}$.[10,11].

Last but not least, we gratefully notice that over the years our nuclear scattering study has been relied on several versions of the coupled channel code ECIS written by Jacques Raynal. This state-of-the-art computer code of nuclear scattering is still being actively used in the community, and Jacques’ important contribution to the development of the nuclear physics research is strongly appreciated by many of us.

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References

1. M.E. Brandan, M.S. Hussein, K.W. McVoy, and G.R. Satchler, Comments on nuclear and particle physics, Vol. 22 (Gordon and Breach, New York, 1996), p. 77.
2. M.E. Brandan and G.R. Satchler, Phys. Rep. 285, 143 (1997).
3. D.T. Khoa, W. von Oertzen, H.G. Bohlen, and S. Ohkubo, J. Phys. G 34, R111 (2007).
4. F. Michel, G. Reidemeister, and S. Ohkubo, Phys. Rev. C 61, 041601 (2000).
5. S. Ohkubo, Phys. Rev. C 93, 041303 (2016).
6. D.M. Brink and N. Takigawa, Nucl. Phys. A 279, 159 (1977).
7. R.C. Fuller, Phys. Rev. C 12, 1561 (1975).
8. F. Michel, F. Braun, G. Reidemeister, and S. Ohkubo, Phys. Rev. Lett. 85, 1823 (2000).
9. R. Anni, Phys. Rev. C 63, 031601 (2001).
10. N. Rowley, H. Doubre, and C. Marty, Phys. Lett. B 69, 147 (1977).

4 Summary

The present work explains why the Airy pattern of nuclear rainbow is suppressed in the inelastic $^{12}$C+$^{12}$C and $^{16}$O+$^{12}$C scattering to the $0^+_1$ state of $^{12}$C at the refractive energies, where a strong rainbow pattern has been observed in the elastic scattering. For this purpose, the near-far decomposition method by Fuller is generalized to determine the near-side and far-side components of the inelastic scattering amplitude for all partial wave contributions. Using the generalized NF decomposition method, our coupled channel analysis of the elastic and inelastic $^{12}$C+$^{12}$C and $^{16}$O+$^{12}$C scattering at the energies under study shows unambiguously that the destructive interference of the inelastic partial waves of different multipoles suppresses the Airy oscillation pattern in the inelastic scattering cross section. Nevertheless, the inelastic scattering remains strongly refractive in these cases, with the dominant far-side scattering at medium and large scattering angles.

We conclude, therefore, that it is not possible to identify uniquely the Airy pattern of the nuclear rainbow in the angular distribution of the inelastic nucleus-nucleus scattering to an excited state with nonzero spin. Semi-classically, such a refractive mixing of the partial waves of different multipoles is analogous to an optical prism refracting ray of light of different wave lengths. The only exception is the inelastic scattering to a monopole excitation which does not mix different multipoles in the inelastic scattering amplitude, and the Airy pattern in the inelastic cross section can be determined consistently in the same manner as done for the elastic scattering. In light of this result, an accurate measurement of the inelastic $\alpha$ or light ion scattering to the $0^+_1$ excitation of the $^{12}$C target should be of interest for the future studies of the nuclear rainbow scattering as well as the $\alpha$-cluster structure of the Hoyle state $^{16}$.[10,11].
11. H.G. Bohlen, M.R. Clover, G. Ingold, H. Lettau, and W. von Oertzen, Z. Phys. A 308, 121 (1982).
12. D.T. Khoa and O.M. Knyazkov, Z. Phys. A 328, 67 (1987).
13. D.T. Khoa, H. Bohlen, W. von Oertzen, G. Bartnitzky, A. Blazevic, F. Nuoffer, B. Gebauer, W. Mittig, and P. Roussel-Chomaz, Nucl. Phys. A 759, 3 (2005).
14. F. Michel and S. Ohkubo, Phys. Rev. C 70, 044609 (2004).
15. S. Hamada, Y. Hirabayashi, N. Burtebayev, and S. Ohkubo, Phys. Rev. C 87, 024311 (2013).
16. S. Ohkubo, Y. Hirabayashi, A.A. Ogloblin, Y.A. Gloukhov, A.S. Dem’yanova, and W.H. Trzaska, Phys. Rev. C 90, 064617 (2014).
17. A. Dem’yanova, V. Bragin, A. Ogloblin, A. Lebedev, J. Bang, S. Goncharov, S. Ershov, F. Gareev, and P. Korovin, Physics Letters B 184, 129 (1987).
18. A. Dem’yanova, E. Svinareva, S. Goncharov, S. Ershov, F. Gareev, G. Kazacha, A. Ogloblin, and J. Vaagen, Nucl. Phys. A 542, 208 (1992).
19. A. D’Arrigo, G. Fazio, G. Giardina, O. Goryunov, A. Ilyin, M. Sacchi, A. Shvedov, A. Taccone, I. Vishnevsky, and I. Zaiats, Il Nuovo Cimento A 107, 1353 (1994).
20. P. D’Agostino, G. Fazio, G. Giardina, O. Goryunov, M. Sacchi, A. Shvedov, I. Vishnevsky, and I. Zaiats, Nucl. Phys. A 583, 437 (1995).
21. H. Bohlen, X. Chen, J. Cranner, P. Frobrich, B. Gebauer, H. Lettau, A. Miczaika, W. von Oertzen, R. Ulrich, and T. Wilpert, Z. Phys. A 322, 241 (1985).
22. A. Dem’yanova, H. Bohlen, A. Danilov, S. Goncharov, S. Khelebnikov, V. Maslov, Y. Penionzkevich, Y. Sobolev, W. Trzaska, G. Tuyrin, and A. Ogloblin, Nucl. Phys. A 834, 473c (2010).
23. A.A. Ogloblin, Y.A. Glukhov, W.H. Trzaska, A.S. Dem’yanova, S.A. Goncharov, R. Julin, S.V. Khelebnikov, M. Mutterer, M.V. Rozhkov, V.P. Rudakov, G.P. Tiorin, D.T. Khoa, and G.R. Satchler, Phys. Rev. C 62, 044601 (2000).
24. D.T. Khoa, N.H. Phuc, D.T. Loan, and B.M. Loc, Phys. Rev. C 94, 034612 (2016).
25. D.T. Khoa and G. Satchler, Nucl. Phys. A 668, 3 (2000).
26. G. Satchler, Direct Nuclear Reactions (Clarendon, Oxford, 1983).
27. K.W. McVoy and M.E. Brandan, Nucl. Phys. A 542, 295 (1992).
28. D.R. Dean and N. Rowley, J. Phys. G 10, 493 (1984).
29. D.T. Khoa, Phys. Rev. C 63, 034007 (2001).
30. M. Kamimura, Nuclear Physics A 351, 456 (1981).
31. S. Raman, C. Malarkey, W. Milner, C. Nestor, and P. Stelson, Atomic Data and Nuclear Data Tables 36, 1 (1987).
32. T. Kibédi and R. Spear, Atomic Data and Nuclear Data Tables 80, 35 (2002).
33. J. Raynal, Computing as a Language of Physics (IAEA, Vienna, 1972) p. 75; J. Raynal, coupled-channel code ECIS97 (unpublished).
34. N. Alamanos, Eur. Phys. J. A 56, 212 (2020).
35. J. Raynal, Phys. Rev. C 23, 2571 (1981).
36. J. Hostachy, M. Buenerd, J. Chauvin, D. Lebrun, P. Martin, J. Lugol, L. Papineau, P. Roussel, N. Alamanos, J. Avrieux, and C. Cerruti, Nucl. Phys. A 490, 441 (1988).
37. L.H. Chien, D.T. Khoa, D.C. Cuong, and N.H. Phuc, Phys. Rev. C 98, 064604 (2018).
38. S. Ohkubo, Y. Hirabayashi, and A.A. Ogloblin, Phys. Rev. C 96, 024607 (2017).
$^{12}\text{C} + ^{12}\text{C}, E_{\text{lab}} = 240$ MeV

$^{16}\text{O} + ^{12}\text{C}, E_{\text{lab}} = 200$ MeV