Fuzzy Analytical Hierarchy Processes for Damage State Assessment of Arch Masonry Bridge

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Abstract

The present work proposes a fuzzy analytical hierarchy approach for decision making in the maintenance programming of masonry arch bridges. As a practical case, we propose to classify the degradation state of the Mohammadia masonry bridge. A large number of criteria and sub-criteria are combined to classify this type of bridges through visual inspections. The main criteria (level 1) considered in this work are the history of the bridge, the environmental conditions, the structural capacity and the professional involvement of the bridge. In addition, these criteria are subdivided into several sub-criteria (level 2) which are, in turn, subdivided into sub-criteria (level 3). Considering these criteria and sub-criteria, weights Wi are calculated by fuzzy geometric mean method of Buckley. Subsequently, expert scores were assigned to calculate the overall score CS reflecting the degradation of the considered infrastructure. Thereafter, the masonry arch bridges are classified respecting the French IQOA scoring system using the overall scores value CS. The proposed classification method gave similar results provided by an expert’s study realized previously as part of a national patrimony preservation policy. The obtained results are in good agreement, which makes this method an effective scientific tool for decision-making in view of prioritization of the maintenance after systematic inspection of masonry bridges such as the bridge studied in this work.

Keywords: Masonry Bridges; Fuzzy Analytical Hierarchy Process; Degradation Degree Score; Classification of Bridge Degradation.

1. Introduction

Masonry bridges constitute a significant heritage within the road network. The average life of these structures exceeds, on average, a century of service. Consequently, the management of this heritage is essential through rigorous monitoring and regular maintenance. This inspection task proved to be very complex, given the large number of factors and the complex nature of the decision-making problem [1]. In this context, early in 1980’s, Saaty [2] proposed an Analytical Hierarchy Method (AHP) to solve the problem of decision-making complexity. This latter used the hierarchical structure that helps experts to make a simple classification. Since this date, several efforts were made by researchers to perform and improve this method [3-7]. Indeed, the AHP method can be generalized to determine the risks associated with bridges [8-10]. Thus, a scoring system is combined to help engineers establish a bridge reinforcement scheme using the AHP approach [11-13].
To overcome the problem of difficulty for experts to provide precise numerical values of scoring inherent to each criterion, scientists developed the Fuzzy Analytical Hierarchical Process (FAHP) [14-17]. This latter makes it possible for experts to give several indices for each factor. As an example, Sasmal and Ramanjaneyulu [15] proposed this methodology for grading the condition of reinforced concrete (RC) bridges and concluded that this helped engineers and decision-makers to overcome the problem of prioritizing bridge maintenance and rehabilitation. Moreover, the method was successfully applied for Risk prioritization in megaprojects [18]. In fact, these authors combined the fuzzy version with fuzzy TOPSIS to evaluate a risk ranking considering three criteria, namely, cost, time and quality of the project. Furthermore, the FAHP method has been applied in the decision-making process for the purchase of a property in a competitive context where several choices are available. As such, Shahidan et al. [19] applied the FAHP method to help purchaser selecting car and comparing the important criteria and sub-criteria needed to choose easily a car in Malaysia. The latter suggest the methodology as a guide to be implemented to other multiple criteria decision-making problems.

The objective of this work is to apply the FAHP method in the case of a masonry bridge. This approach consists of evaluating the current state of this type of structure in order to present a guide for the future maintenance and management. The main criteria used to assess the degree of degradation are the history, environmental conditions, structural capacity and professional involvement of the bridge. These criteria are divided into a set of sub-criteria, as the case requires. Thereafter, score is assigned by a group of experts to each criterion and each sub-criterion. The strong point of the method is to multiply these scores by weights calculated by the FAHP method. The sum of these products determines an overall CS score which defines the degree of degradation of the bridge. Using the French IQOA scoring system, the structure is finally classified according to the value obtained from CS. A lower CS score indicates a good condition of the bridge. On the other hand, a high CS score indicates a very pronounced degradation state. Therefore, bridges with a high CS are to be prioritized during maintenance campaigns.

2. Methodology for Degradation Assessment Procedure

This study aims to develop a numerical model based on the FAHP method to assess the condition of masonry arch bridges. Figure 1 shows the flowchart giving the five steps of the model. As indicated by the flowchart, the first step is to identify the degradation factors having a direct influence on the condition state of the bridge. Secondly, the hierarchical structure of the model is built. Once done, the FAHP method can be applied in order to calculate the overall weights of the factors and the sub-factors as will be detailed later. In the next step, an overall CS score index is calculated and used to classify the degree of degradation of the masonry bridge in the last step.

2.1. Identification of Degradation Factors

Factors that might have an impact on the degradation of the structure were selected after a process of a census and collection. This selection was based on expert judgment after inspections and visual surveys without recourse to in-situ measurements and tests [20]. According to the literature and the assessments of experts, thirty-five factors and sub-factors were retained in this study. Among other criteria, the history plays a major role in the condition of the bridge. Combined with the environmental conditions, the age of the masonry bridge has a negative impact on its state of health. Added to this, the structural capacity of the bridge such as the apparent disorder of the foundations, deflection and deformation of the superstructure and thus the condition of the equipment are also retained. The last criterion retained is the professional involvement which contains design involvement and supervision of the structure.
2.2. Development of the Hierarchical Structure

The constructed hierarchical structure contains four levels as shown in Figure 2. Level 0 consists in defining the problem. This consists in evaluating the degree of degradation of the masonry bridge. Level 1 includes the four main criteria indicating the state of degradation of the studied structure. Level 2 identifies the different sub-criteria included in each criterion. The last level groups the sub-criteria that result from the last five sub-criteria of level 2.

![Diagram of the hierarchical structure]

Figure 2. The hierarchy structure of adopted assessment criteria

2.3. Calculation of Weights by the Fuzzy Analytic Process (FAHP) Method

It is difficult for experts to provide precise numerical values from comparison ratios for several reasons. Indeed, the ambiguity and the complexity of information emitted by human during decision making often arises. Appropriately, the Fuzzy Analytical Hierarchy Process (FAHP) approach is used in this study to deal with uncertainty in such situations.
2.3.1. The Triangular Fuzzy Numbers (TFN)

Among the various forms of fuzzy number, the Triangular Fuzzy Number (TFN) is the most widely used in the literature. This triangular fuzzy number denoted \( \tilde{A} \) is a function of a triplet \((l, m, u)\), where \( l \), \( m \) and \( u \) are the lower, middle and upper bounds of TFN \([21, 22]\). TFN is intuitive, easy to use, computationally simple and useful for processing calculation in a fuzzy environment. Figure 3 presents the membership function of TFN \([23]\) that is expressed as follows:

\[
\mu_{\tilde{A}}(x) = \begin{cases} 
\frac{x-l}{m-l}, & l \leq x \leq m \\
\frac{u-x}{u-m}, & m \leq x \leq u \\
0, & \text{otherwise}
\end{cases}
\]  

(1)

Figure 3. The curve of triangular fuzzy number \([23]\)

Given any two TFNs \( \tilde{A}_1 = (l_1, m_1, u_1) \) and \( \tilde{A}_2 = (l_2, m_2, u_2) \), the main operation of triangular fuzzy numbers are \([24]\):

\[
\tilde{A}_1 \oplus \tilde{A}_2 = (l_1, m_1, u_1) \oplus (l_2, m_2, u_2) = (l_1 + l_2, m_1 + m_2, u_1 + u_2)
\]  

(2)

\[
\tilde{A}_1 \otimes \tilde{A}_2 = (l_1, m_1, u_1) \otimes (l_2, m_2, u_2) = (l_1 \times l_2, m_1 \times m_2, u_1 \times u_2)
\]  

(3)

\[
\tilde{A}_1 (/) \tilde{A}_2 = (l_1, m_1, u_1) (/) (l_2, m_2, u_2) = (l_1 / l_2, m_1 / m_2, u_1 / u_2)
\]  

(4)

\[
\tilde{A}_1^{-1} = (l_1, m_1, u_1)^{-1} = (1 / l_1, 1 / m_1, 1 / u_1)
\]  

(5)

2.3.2. The fundamental scale used to compare two criteria

To choose an appropriate TFN, Experts are invited to make a comparison of the relative importance of two criteria at the same time. Figure 4 illustrates the scaling scheme of the appreciation procedure.

Figure 4: The fundamental scale used to compare a pair of criteria

2.3.3. Construction of Comparison Matrices

The pair-wise comparison matrix \([\tilde{A}]\) is constructed by collection of pair-by-pair scores as follows \([25, 26]\):

\[ [A] = \{(\bar{a}_{ij})\}_{n \times n} = \begin{bmatrix}
(1,1,1) & (l_{12}, m_{12}, u_{12}) & \cdots & (l_{1n}, m_{1n}, u_{1n}) \\
(l_{21}, m_{21}, u_{21}) & (1,1,1) & \cdots & (l_{2n}, m_{2n}, u_{2n}) \\
\vdots & \vdots & \ddots & \vdots \\
(l_{n1}, m_{n1}, u_{n1}) & (l_{n2}, m_{n2}, u_{n2}) & \cdots & (1,1,1)
\end{bmatrix}
\]

Where, \(i\) and \(j\) vary from 1 to \(n\) (number of parameters) and \((\bar{a}_{ij})\) indicates the expert's preference of \(i^{th}\) criterion over \(j^{th}\) criterion. The lower triangular matrix \([A]\) is computed by the Equation 7:

\[
(\bar{a}_{ij}) = (\bar{a}_{ij})^{-1} = \left( \frac{1}{u_{ij}}, \frac{1}{m_{ij}}, \frac{1}{l_{ij}} \right), \text{ for } i < j
\]

All the comparison matrices between the criteria and sub-criteria of the assumed hierarchical structure will be presented in the results section.

### 2.3.4. Fuzzy Geometric Mean Method

According to Buckley [27], the matrix \([A]\) is aggregated by fuzzy geometric mean \(\bar{r}_i\) using the expression:

\[
\bar{r}_i = (\prod_{j=1}^{n} \bar{a}_{ij})^{\frac{1}{n}}, i = 1, 2, \ldots, n
\]

### 2.3.5. Fuzzy Weights

Fuzzy weights \(\bar{w}_i\) are computed by multiplying each fuzzy geometric mean \(\bar{r}_i\) by a vector summation as follows [25]:

\[
\bar{w}_i = (l w_i, m w_i, u w_i) = \bar{r}_i \odot (\bar{r}_1 \oplus \bar{r}_2 \oplus \cdots \oplus \bar{r}_n)^{-1}, i = 1, 2, \ldots, n
\]

After that, the fuzzy weights \(\bar{w}_i\) must be defuzzified by the method known as center of area (COA) as follows [28]:

\[
w_i = \frac{lw_i + mw_i + uw_i}{3}, i = 1, 2, \ldots, n
\]

This step is followed by the normalization of the defuzzified weights following the expression:

\[
nw_i = \frac{w_i}{\sum_{i=1}^{n} w_i}, i = 1, 2, \ldots, n
\]

### 2.3.6. Consistency Test

In order to assure the consistency of the judgment matrix, a defuzzification process was performed using method proposed by Kwong and Bai [28]. Each TFN in the pair-wise matrix is converted to crisp number \(c_{ij}\) as follows:

\[
c_{ij} = \frac{l_{ij} + 4m_{ij} + u_{ij}}{6}, i, j = 1, 2, \ldots, n
\]

Then, the crisp pair-wise comparison matrix \([C]\) is constructed as follows [29, 30]:

\[
[C] = (c_{ij})_{n \times n} = \begin{bmatrix}
1 & c_{12} & \cdots & c_{1n} \\
c_{21} & 1 & \cdots & c_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
c_{n1} & c_{n2} & \cdots & 1
\end{bmatrix}
\]

Next, the consistency index, \(CI\) [30, 31], is calculated as:

\[
CI = (\lambda_{\text{max}} - n) / (n - 1)
\]

Where \(\lambda_{\text{max}}\) is the maximum eigenvalue of the matrix\([C]\). The consistency ratio \(CR\) is given by the expression:

\[
CR = \frac{CI}{RI}
\]

Where, \(CI\) is the consistency index and \(RI\) is the Random Index given in Table 1. According to Saaty [31], matrices with \(CR\) values \(\leq 10\%\) are accepted, otherwise \(CR\) values greater than 10\% are rejected.

| Table 1. Random Consistency Index (RI) |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| n   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  |
| RI  | 0.525 | 0.882 | 1.109 | 1.248 | 1.342 | 1.406 | 1.450 | 1.485 |
2.4. Evaluation of the Degradation Degree Score

In this part of the research, once the overall weights are determined by the FAHP method, each factor has an expert score denoted "ESC", ranging from 0 to 10. The relationship between ESC and the scale is shown in Table 2.

| ESC  | State condition of scaling          |
|-----|------------------------------------|
| [0, 2] | no risk grade noticed               |
| [2, 4] | low-grade risk                      |
| [4, 6] | moderate risk                       |
| [6, 8] | high risk                           |
| [8, 10] | super high risk                     |

Next, the average score $ACS$ is calculated by the Equation 16:

$$ACS = \frac{1}{n} \sum_{i=1}^{n} ESC_i$$  \hspace{1cm} (16)

Where $n$ is the number of experts. Finally, the comprehensive score CS (commonly called the overall score) for degradation of masonry bridges [32] is defined by Equation 17:

$$CS = \sum_{i=1}^{26} W_i \times ACS_i$$  \hspace{1cm} (17)

2.5. Classification of Masonry Bridge Degradation According to IQOA

The French scoring system IQOA [33] (quality assessment of engineering structures) is used in this research to classify the degree of degradation of masonry bridge. Table 3 illustrates the principle of evaluating and classifying structures using the overall value of the CS scores. Class 1 corresponds to a CS score of 0 to 2 and designed a good apparent condition. While Class 3U (CS score from 8 to 10) represents extremely critical condition with serious structural failure.

| Class | CS   | Apparent Condition                                           |
|-------|------|--------------------------------------------------------------|
| 1     | [0, 2] | Good overall state                                           |
| 2     | [2, 4] | Equipment failures or minor structure damage. Non urgent maintenance needed |
| 2E    | [4, 6] | Equipment failures or minor structure damage. Urgent maintenance needed |
| 3     | [6, 8] | Structure deterioration. Non urgent maintenance needed        |
| 3U    | [8, 10] | Serious structure deterioration. Urgent maintenance needed    |

3. Case Study: Masonry Bridge of Mohammadia

3.1. Localization and Description of the Bridge

A masonry bridge located at Mohammadia city was selected for our case study. The bridge in question is located in the north-west of Algeria (Figure 5). This bridge measures 110 meters long and 7.50 meters wide including ten stone arch spans, as shown in Figure 6.

![Figure 5. Location of the studied bridge, a) Global location into Algeria map, b) Zoom to Mascara province and c) Capture image near the bridge of Mohammadia city](image-url)
It should be noted that the bridge has been closed to traffic since the commissioning of the new bridge as a result of the deviation of the roadway. Despite this, a rehabilitation operation is desired and remains to be included in the framework of the preservation of the national heritage. The photos in Figures 7.a to 7.d highlight the types of damage, stated during our on-site inspection of the structure.

Figure 6. Overview of the studied masonry bridge: a) Upstream side and B) Downstream side

Figure 7. Photos showing the damage state revealed on the bridge a) Roadway, b) Abutment on the upstream side, c) Arc number one, upstream side and d) Arcs number four and five on the downstream side

The construction of the bridge dates back to the colonial period so its age will exceed a century of service, according to preliminary information. This criterion works against the state health of the bridge. In addition, as shown in Figures 7(b-c), there are open transverse cracks in the abutment wall. The same figures show a massive degradation of the joints between the stones over the entire surface of the return wall, as well as the presence of vegetation. Additionally, Figure 7(d), shows advanced pavement settlement caused by very heavy scour under the foundation. In addition, very pronounced breaks in the body of the vault were noted with detachment of the stones (Figure 7.d). As a first judgment, this structure will be classified as highly deteriorated and therefore requiring major rehabilitation work. In the following sections, we will analyze the resulted weights values performed using the fuzzy analytical hierarchy approaches. This approach will allow us to determine the overall score which determines the classification of the structure and to make a decision concerning the urgency to proceed to maintenance and rehabilitation of the bridge.

3.2. Results Analysis

The calculations at the base of the algorithm explained in section 2 were carried out using the MS Excel environment. Indeed, MS Excel remains the most widespread and the most accessible to the greatest number of designers and
researchers. Table 4 summarizes the triangular fuzzy numbers pair-by-pair of all criteria and sub-criteria. According to expert’s assessments, it is assessed, for example that criterion C1 is of equal importance with itself. This is interpreted by the fuzzy triangular number TFN (1, 1, 1) (see Figure 4). Similarly, criterion C1 is classified as low intermediate to C2, which is interpreted by a TFN (1/3, 1/2, 1). In the same time, this same criterion C1 is classified very weak intermediate to C4. This is interpreted by a TFN (1, 2, 3). Thus, all the comparison matrices between the criteria and sub-criteria of the hierarchical structure are constructed as presented in Table 4.

Table 4. Pair-wise comparison matrices according to the assumed hierarchical structure

| Factors | C1      | C2      | C3      | C4      |
|---------|---------|---------|---------|---------|
| C1      | (1,1,1) | (1/3,1/2,1) | (1/7,1/6,1/5) | (1,2,3) |
| C2      | (1,2,3) | (1,1,1)  | (1/4,1/3,1/2) | (2,3,4) |
| C3      | (5,6,7) | (2,3,4)  | (1,1,1)  | (4,5,6) |
| C4      | (1/3,1/2,1) | (1/4,1/3,1/2) | (1/6,1/5,1/4) | (1,1,1) |
| C1.1    | (1,1,1) | (1/3,1/2,1) | (1/3,1/2,1) | (1,1,1) |
| C1.2    | (1,2,3) | (1,1,1)  | (1/3,1/2,1) | (1,2,3) |
| C1.3    | (1,2,3) | (1,1,1)  | (1,1,1)  | (1,1,1) |
| C1.4    | (1,1,1) | (1/2,3)  | (1,1,1)  | (1,1,1) |
| C1.5    | (1,1,1) | (1/3,1/2,1) | (1,1,1)  | (1,1,1) |
| C2.1    | (1,1,1) | (1,1,1)  | (1,1,1)  | (1,2,3) |
| C2.2    | (1,1,1) | (1,1,1)  | (1/2,3)  | (3,4,5) |
| C2.3    | (1,1,1) | (1/3,1/2,1) | (1,1,1)  | (3,4,5) |
| C2.4    | (1/3,1/2,1) | (1/5,1/4,1/3) | (1/5,1/4,1/3) | (1,1,1) |
| C2.5    | (1/3,1/2,1) | (1/4,1/3,1/2) | (1/5,1/4,1/3) | (1/3,1/2,1) | (1,1,1) |
| C3.1    | (1,1,1) | (1,1,1)  | (1,1,1)  | (4,5,6) |
| C3.2    | (1,1,1) | (1,1,1)  | (1/2,3)  | (5,6,7) |
| C3.3    | (1/6,1/5,1/4) | (1/7,1/6,1/5) | (1,1,1)  | |
| C3.1.1  | (1,1,1) | (1/3,1/2,1) | (1/3,1/2,1) | (1/7,1/6,1/5) | (1,1,1) |
| C3.1.2  | (1,2,3) | (1,1,1)  | (1,1,1)  | (1,1,1)  | (2,3,4) |
| C3.1.3  | (1,2,3) | (1,1,1)  | (1,1,1)  | (1,1,1)  | (3,4,5) |
| C3.1.4  | (5,6,7) | (1,1,1)  | (1,1,1)  | (1,1,1)  | (2,3,4) |
| C3.1.5  | (1/6,1/5,1/4) | (1/4,1/3,1/2) | (1/5,1/4,1/3) | (1/4,1/3,1/2) | (1,1,1) |
| C3.2.1  | (1,1,1) | (1/2,3)  | (3,4,5)  | |
| C3.2.2  | (1/3,1/2,1) | (1,1,1)  | (1,1,1)  | |
| C3.2.3  | (1/5,1/4,1/3) | (1,1,1)  | (1,1,1)  | |
| C3.3.1  | (1,1,1) | (1/3,1/2,1) | (1,1,1)  | (1,1,1) |
| C3.3.2  | (1,2,3) | (1,1,1)  | (1,2,3)  | (2,3,4) |
| C3.3.3  | (1,1,1) | (1/3,1/2,1) | (1,1,1)  | (1,1,1) |
| C3.3.4  | (1,1,1) | (1/4,1/3,1/2) | (1,1,1)  | (1,1,1) |
| C3.4    | (1,1,1) | (1,1,1)  | (1,1,1)  | |
| C4.1    | (1,1,1) | (1,1,1)  | |
| C4.2    | (1/3,1/2,1) | (1,1,1)  | |
| C4.1.1  | (1,1,1) | (1,1,1)  | |
| C4.1.2  | (1,1,1) | (1,1,1)  | |
| C4.2.1  | (1,1,1) | (1,1,1)  | |
| C4.2.2  | (1,1,1) | (1,1,1)  | |
Table 5 summaries local weights $w_i$ obtained after realizing the consistency tests. These results obtained by the FAHP are projected on the hierarchical structure as illustrated by Figure 8.

**Table 5. Local weights of criteria and sub-criteria**

| $w_i$ | Consistency test |
|-------|------------------|
| 0.125 | $\lambda_{\text{max}}=4.106$, CI=0.035, RI=0.882, CR=3.92% < 10% |
| 0.225 | $\lambda_{\text{max}}=5.190$, CI=0.047, RI=1.109, CR=4.24% < 10% |
| 0.565 | $\lambda_{\text{max}}=5.273$, CI=0.068, RI=1.109, CR=6.09% < 10% |
| 0.085 | $\lambda_{\text{max}}=0.002$, CI=0.580, RI=0.525, CR=0.41% < 10% |
| 0.157 | $\lambda_{\text{max}}=5.197$, CI=0.049, RI=1.109, CR=4.41% < 10% |
| 0.232 | $\lambda_{\text{max}}=4.033$, CI=0.011, RI=0.882, CR=1.21% < 10% |
| 0.219 | $\lambda_{\text{max}}=4.033$, CI=0.011, RI=0.882, CR=1.21% < 10% |
| 0.219 | $\lambda_{\text{max}}=4.033$, CI=0.011, RI=0.882, CR=1.21% < 10% |
| 0.173 | CR not verified for comparison between two criteria |
| 0.221 | CR not verified for comparison between two criteria |
| 0.320 | CR not verified for comparison between two criteria |
| 0.266 | CR not verified for comparison between two criteria |
| 0.104 | CR not verified for comparison between two criteria |
| 0.088 | CR not verified for comparison between two criteria |
| 0.443 | CR not verified for comparison between two criteria |
| 0.471 | CR not verified for comparison between two criteria |
| 0.085 | CR not verified for comparison between two criteria |
| 0.099 | CR not verified for comparison between two criteria |
| 0.260 | CR not verified for comparison between two criteria |
| 0.307 | CR not verified for comparison between two criteria |
| 0.088 | CR not verified for comparison between two criteria |
| 0.577 | CR not verified for comparison between two criteria |
| 0.241 | CR not verified for comparison between two criteria |
| 0.182 | CR not verified for comparison between two criteria |

In fact, "structural capacity of the bridge" received a maximum importance (56.5%) followed by "environmental conditions" (22.5%), and in last position, "bridge history state" (12.5%) and "professional involvement conditions" (8.5%) received a relatively minimal effect importance in the degradation process of the masonry bridge. According to the hierarchical model, the overall weights $W_i$ of sub-factors are equal to the product of local weights of their father factors (of lower level). Table 6 summarizes $W_i$ results with the source local weights by levels. Visibly, analyzing Figure 9, one can easily note that there are considerable differences in the overall weights relating to the retained sub-factors. These differences relate their importance and influences on the state of degradation masonry bridge.
Table 6. Overall weights of sub-factor

| Level 1 | Level 2 | Level 3 |
|---------|---------|---------|
| Factors wi | Sub-Factors wi | Sub-Factors wi | W_i |
| C1 0.125 | C1.1 0.157 | - | - | 0.020 |
| | C1.2 0.232 | - | - | 0.029 |
| | C1.3 0.219 | - | - | 0.027 |
| | C1.4 0.219 | - | - | 0.027 |
| | C1.5 0.173 | - | - | 0.022 |
| C2 0.225 | C2.1 0.221 | - | - | 0.050 |
| | C2.2 0.320 | - | - | 0.077 |
| | C2.3 0.266 | - | - | 0.060 |
| | C2.4 0.104 | - | - | 0.024 |
| | C2.5 0.088 | - | - | 0.020 |
| C3 0.565 | C3.1 0.443 | C3.1.1 0.099 | 0.025 |
| | C3.1.2 0.246 | 0.062 |
| | C3.1.3 0.260 | 0.065 |
| | C3.1.4 0.307 | 0.077 |
| | C3.1.5 0.088 | 0.022 |
| | C3.2 0.471 | C3.2.1 0.577 | 0.154 |
| | C3.2.2 0.241 | 0.064 |
| | C3.2.3 0.182 | 0.049 |
| C3.3 0.085 | C3.3.1 0.198 | 0.010 |
| | C3.3.2 0.430 | 0.021 |
| | C3.3.3 0.198 | 0.010 |
| | C3.3.4 0.174 | 0.008 |
| C4 0.085 | C4.1 0.644 | C4.1.1 0.500 | 0.027 |
| | C4.1.2 0.500 | 0.027 |
| | C4.2 0.356 | C4.2.1 0.500 | 0.015 |
| | C4.2.2 0.500 | 0.015 |

Figure 9. Overall weights of sub-factor

For a validation purpose, the obtained results are compared to the results published by Bakhtavar et al. [34]. In the latter, authors presented an example of weights calculation by the fuzzy geometric mean method. Observing Figure 10, one can easily note that the current results are in good agreement with those given by Bakhtavar et al.

Figure 10. Results comparison of the calculated fuzzy weights to those obtained by [34]

At this stage, three experts are invited to give notes (ESC1, ESC2 and ESC3) to assess the degradation state of the bridge. Indeed, for each factor, three scores were assigned as given in Table 7. The average of the scores to each sub-criterion is multiplied by the overall relating weight. The last calculation to be made in this case study is to calculate the overall score as expressed by Equation 17. The resulted overall score is given at the end of Table 7. Accordingly, the structure is finally classified according to the IQOA grading system as 3U, which is equivalent to a severely deteriorated state of the bridge. This means that immediate maintenance is necessary and imperative.
Table 7. The final results and the classification of the studied masonry bridge

| Number | Factor | ESC 1 | ESC2 | ESC3 | Average score (ASC) | overall weight (W) | ASC × W |
|--------|--------|-------|------|------|--------------------|-------------------|---------|
| 1      | C1.1   | 8.000 | 9.000| 9.000| 8.667              | 0.020             | 0.170   |
| 2      | C1.2   | 8.000 | 9.000| 9.000| 8.667              | 0.029             | 0.250   |
| 3      | C1.3   | 8.000 | 7.000| 6.000| 7.000              | 0.027             | 0.191   |
| 4      | C1.4   | 8.000 | 9.000| 10.000| 9.000              | 0.027             | 0.246   |
| 5      | C1.5   | 8.000 | 10.000| 10.000| 9.333              | 0.022             | 0.201   |
| 6      | C2.1   | 8.000 | 7.000| 8.000| 7.667              | 0.050             | 0.382   |
| 7      | C2.2   | 8.000 | 9.000| 10.000| 9.000              | 0.072             | 0.648   |
| 8      | C2.3   | 8.000 | 8.000| 9.000| 8.333              | 0.060             | 0.500   |
| 9      | C2.4   | 8.000 | 6.000| 4.000| 6.000              | 0.024             | 0.141   |
| 10     | C2.5   | 8.000 | 10.000| 9.000| 9.000              | 0.020             | 0.179   |
| 11     | C3.1.1 | 8.000 | 5.000| 6.000| 6.333              | 0.025             | 0.158   |
| 12     | C3.1.2 | 8.000 | 10.000| 9.000| 9.000              | 0.062             | 0.554   |
| 13     | C3.1.3 | 8.000 | 9.000| 8.000| 8.333              | 0.065             | 0.543   |
| 14     | C3.1.4 | 8.000 | 9.000| 10.000| 9.000              | 0.077             | 0.692   |
| 15     | C3.1.5 | 8.000 | 10.000| 10.000| 9.333              | 0.022             | 0.206   |
| 16     | C3.2.1 | 8.000 | 8.000| 10.000| 8.667              | 0.154             | 1.333   |
| 17     | C3.2.2 | 8.000 | 10.000| 8.000| 8.667              | 0.064             | 0.556   |
| 18     | C3.2.3 | 8.000 | 10.000| 10.000| 9.333              | 0.049             | 0.453   |
| 19     | C3.3.1 | 8.000 | 10.000| 9.000| 9.000              | 0.010             | 0.086   |
| 20     | C3.3.2 | 8.000 | 9.000| 10.000| 9.000              | 0.021             | 0.186   |
| 21     | C3.3.3 | 8.000 | 9.000| 9.000| 8.667              | 0.010             | 0.082   |
| 22     | C3.3.4 | 8.000 | 9.000| 9.000| 8.667              | 0.008             | 0.073   |
| 23     | C4.1.1 | 8.000 | 8.000| 8.000| 8.000              | 0.027             | 0.219   |
| 24     | C4.1.2 | 8.000 | 9.000| 10.000| 9.000              | 0.027             | 0.247   |
| 25     | C4.2.1 | 8.000 | 9.000| 9.000| 8.667              | 0.015             | 0.132   |
| 26     | C4.2.2 | 8.000 | 10.000| 9.000| 9.000              | 0.015             | 0.137   |

\[ CS = \sum_{i=1}^{26} W_i \times ASC_i = 8.519 \]  
Class 3U, Serious structure deterioration. Urgent maintenance needed

4. Conclusions

The method used for carrying out the expertise of this study is commonly known as the Fuzzy Hierarchical Analytical Process (FAHP). This approach allowed us to assess the current state of the masonry bridge (subject of this study). Conclusions from this work are presented as follows:

- It is difficult for experts to provide accurate numeric values from comparison reports. In addition, the decision-making process when appraising the health state of a structure such as masonry bridges is complex and uncertain. Among the factors contributing to this complexity is the ambiguity of information during a visual inspection. Adding to this, the large number of criteria affecting the priority of judgment. This certainly leads experts to make different judgments.

- The fuzzy hierarchical analytical process is needed as a reliable and efficient method to overcome the problem of uncertainty in expert judgments.

- Four-level hierarchical structure was sufficiently constructed, integrating 26 sub-criteria. Score results showed that the structural capacity gained the greatest impact on bridge deterioration (weight equal to 56.5%) followed by environmental conditions (weight equal to 22.5%). Factors of bridge history and professional involvement had the lowest impact (weight equal to 12.5%, and 8.5%).
The use of the IQOA grading system has been successfully combined. Indeed, using this classification system, it was possible to calculate a CS index which characterizes the degree of degradation of the structure. The obtained CS (8.519) located in the interval [8, 10] classifies the structure as highly degraded (Class 3U according to IQOA).

Bridge of masonry situated at Mohammadia was successfully expertized using FAHP approach, as has been proposed in this article. The bridge is classified in the category U3. Consequently, the structure presents a high structural risk and requires very urgent maintenance.

The FAHP method has demonstrated its effectiveness in eliminating the uncertainties and ambiguities present during an appraisal of masonry bridges. For this reason, this methodology is recommended for use in future survey applications to assess the condition of existing masonry structures in general.

5. Declarations

5.1. Author Contributions

Conceptualization, M.L. and A.M.; methodology, M.L., AM. and A.D.; software, M.L. and A.D. validation, M.L. and A.D.; formal analysis, M.L. and A.D.; investigation, M.L.; resources, M.L.; data curation, M.L.; writing—original draft preparation, M.L., AM. and A.D.; writing—review and editing, M.L. and A.D.; visualization, M.L. and A.D.; supervision, M.L. and A.D. All authors have read and agreed to the published version of the manuscript.

5.2. Data Availability Statement

The data presented in this study are available in article.

5.3. Funding

The authors received no financial support for the research, authorship, and/or publication of this article.

5.4. Conflicts of Interest

The authors declare no conflict of interest.

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