Non-Hermiticity and decoherence are two different effects of the open systems. Each of them has triggered many interesting phenomena. In this work, we rewrite the non-Hermitian Lindblad master equation into a linear one, under which the eigenvalues and eigenmatrices are easy to be obtained. Based on this, we consider an open system including both non-Hermitian and decoherence effects, which can be realized in the system of linearly coupled active and passive resonators. We find that there are new-style exceptional points and steady-states without PT-symmetry or pseudo-Hermiticity in this system. And we demonstrate that they in the presented system have different properties from the cases of the normal open quantum systems. Our work opens a new way to further explore the open systems.

I. INTRODUCTION

Non-Hermiticity and decoherence are the two different effects and produce different properties, even though both of them are used to describe the open systems where quantum systems of interest interact with their environments. The non-Hermitian Hamiltonians, whose the energy spectra are generally characterized as complex eigenvalues, refers to the open systems describing the progressive loss of populations. One of interesting properties of these non-Hermitian Hamiltonians is the presence of exceptional points (EPs) [1, 2], where two or more complex eigenvalues coincide and the corresponding eigenstates coalesce. The EPs have many intriguing properties [3–9] that have no counterparts in Hermitian systems. On the other hands, pseudo-Hermitian Hamiltonians with even parity-time (PT) symmetry have entirely real energy spectra, so that it enables one to realize the intriguing properties of non-Hermiticity in the steady systems [10–15]. Compared to the rigorous conditions of pseudo-Hermitian or PT-symmetry, a new-type non-Hermitan system with one eigenvalue being real and the other being complex has been proposed [16].

Different form the non-Hermiticity that is induced by the interaction of systems with their environments, another dynamics of an open quantum system is characterized by the presence of decoherence terms, describing the loss of energy, coherence, and information into the environments. As we all known, the time evolution of an open quantum system weakly interacting with a Markovian environment, under general hypotheses, can be described by the Lindblad master equation, composed of a Hermitian Hamiltonian part, and a Lindblad dissipative term [17–19]. Meanwhile, the numerical calculations of the Lindblad master equation have been broad applied in the open systems [20, 21]. To further explore this equation, a work [22–24] suggested to rewrite the Lindblad equation to a linear equation (a matrix multiplying a vector, if considering discrete systems), which is easy to be solved and can be easy applied to the vary systems. Up to now, most of works address the Lindblad master equation by means of the contact to the environment whose effect is describes by non-Hermitian parts of Hamiltonians [25–27]. This idea has also been applied to another open systems [28–30]. However, few works is taken into account both the non-hermiticity and the decoherence at the same time.

In this work, we focus on a system with the non-hermiticity and the decoherence. We address this system via the linearization method, whose Hamiltonian is only required time-independent. We rewrite the non-Hermitian Lindblad equation to a linear equation, whose the eigenvalues and eigenmatrices are easy to obtain. We theoretical demonstrate the non-Hermitian Liouvillian exceptional points (NLEPs) in this system which have different properties comparing to normal exceptional points (NLEPs) or Liouvillian exceptional points (LEPs). More importantly, we find that there is a new-style steady state without PT-symmetry or pseudo-Hermiticity in this system, where the imaginary part of one of its eigenvalues being zero and the other being negative. By numerically solving the Lindblad master equation, we verify the novel properties of this system.

The contents of this paper are as follows. In section II, we introduce the theoretical framework, mapping the non-Hermitian Lindblad master equation into a linear form, and point out the condition of the steady-state. In section III, we consider a two-level open system with and without coupling in order to illustrate the interaction between non-hermiticity and decoherence. Discussions and conclusions are given in section VI.
II. LINDBLAD EQUATION FOR A GENERAL OPEN SYSTEM

In this section, we focus on a general open quantum system including gain, losses and decoherence. This quantum system of interest coupling to an environment can be experimentally realized, (e.g., active and passive cavities or sink and source systems) by coupling to an environment. Under appropriate approximation, the time evolution of an open quantum system can be expressed using a non-Hermitian Lindblad master equation

\[
\frac{\partial \rho(t)}{\partial t} = -i[H\rho(t) - \rho(t)H^\dagger] + \sum_\mu D[\hat{\Gamma}_\mu]\rho(t),
\]

(1)

where $\hat{H}$ is a non-Hermitian Hamiltonian of the system with gain and losses. $\rho(t)$ is the density matrix of the system and the dissipator $D[\hat{\Gamma}_\mu]\rho(t)$ is a linear operator and quadratic in the Lindblad operators $\hat{\Gamma}_\mu$ describing directly the dissipative effects of the environment. Without loss of generality, the most general quantum dynamical semigroup form of the dissipator can be written as

\[
D[\hat{\Gamma}_\mu](t) = \hat{\Gamma}_\mu(t)\hat{\Gamma}_\mu^\dagger + \frac{1}{2}\{\hat{\Gamma}_\mu^\dagger,\rho(t)\} + \frac{1}{2}\{\rho(t),\hat{\Gamma}_\mu^\dagger\},
\]

(2)

The effect of $D[\hat{\Gamma}_\mu]$ on the density matrix $\rho(t)$ can be divided into two parts: The first one represents the quantum jump terms $\hat{\Gamma}_\mu(\rho(t)\hat{\Gamma}_\mu^\dagger)$, and the second one is the continuous non-unitary decoherence terms $\frac{1}{2}[\hat{\Gamma}_\mu^\dagger,\rho(t)] + \frac{1}{2}[\rho(t),\hat{\Gamma}_\mu^\dagger]$, which describes the continuous loss of energy, information, and coherence of the system into the environment.

Given a non-Hermitian Lindblad master equation as in Eq. (1), one can associate a so-called Liouvillian superoperator $\hat{L}$ to it. We rewrite a $N \times N$ density matrix $\rho(t)$ into a column vector, e.g., $\hat{\rho}(t) = \begin{bmatrix} \rho(1,1) & \cdots & \rho(1,N) \\ \vdots & \ddots & \vdots \\ \rho(N,1) & \cdots & \rho(N,N) \end{bmatrix}^T$, and by using the Hilbert-Schmidt expression of the density matrix. After a straightforward algebra, we can recast the non-Hermitian Hamiltonian Lindblad equation (1) into a linear form

\[
\frac{\partial \hat{\rho}(t)}{\partial t} = (\hat{H}' + \hat{G}') \hat{\rho}(t),
\]

(3)

where $\hat{H}' = -i(\hat{H} \otimes I - I \otimes \hat{H}^\dagger)$, and $\hat{G}' = \sum_\mu \hat{\Gamma}_\mu \otimes \hat{\Gamma}_\mu^\dagger - \frac{\hat{\Gamma}_\mu^\dagger}{2} \otimes I - I \otimes \frac{\hat{\Gamma}_\mu^\dagger}{2}$, while $\hat{L}'$ which is a linear superoperator operating in Liouville space, is represented by a $N^2 \times N^2$ matrix, acting on a basis of vector in this reduced space. It’s worth noting that $\hat{L}'$ is included in two different effects of the open system, one is the non-Hermiticity and the other is the decoherence. The non-Hermiticity is attributed to the non-Hermitian systems rather than the environmental decoherence-induced non-Hermitian parts of Hamiltonians. Eq.(3) indicates that given a non-Hermitian Hamiltonian, together with the decoherence terms, one can fully describe the quantum dynamics of a dissipative system within the Lindblad formalism. This framework has an obvious advantage: the eigenvalues and eigenmatrices of the open system that are defined via the relation

\[
L\rho^i = \lambda^i \rho^i,
\]

(4)

thus can be obtained exactly. And in non-Hermitian systems, the condition of the steady state is that eigenvalues satisfy

\[
\text{Im} \lambda_i = 0 \text{ and } \text{Im} \lambda_j \neq i \leq 0.
\]

(5)

The first equation ensures that at least one eigenstate is steady, and the second equation avoids the fact that the other eigenstates exponentially grow. In the following section, taking into consideration of the non-Hermiticity and decoherence, we will predict that there are new-style degenerate points and new-style stable states in this type of open system.

III. EFFECT OF THE INTERACTION BETWEEN NON-HERMITICITY AND DECOHERENCE

A. Two-level open systems without coupling $\Omega$

In this section, we first study a two-level system only coupled to an environment. The two-level system consisting of lower level $|0\rangle$ and higher level $|1\rangle$ coupled to an environment with loss $\gamma_1$ and gain $\gamma_2$. On the other hands, there is the spontaneous emission $\Gamma$, which populations decay from $|1\rangle$ to $|0\rangle$. When ignore the coupling (e.g., $\Omega = 0$) between two levels in Fig.1, the Hamiltonian is simplified to

\[
\hat{H} = \begin{pmatrix} -i\gamma_1 & 0 \\ 0 & i\gamma_2 \end{pmatrix},
\]

(6)
matrices are \( \rho_i \) and \( \lambda_i \), whose eigenvalues are properties. We first analyze the energy spectrum of the Liouvil-12ian, whose the eigenvalues are \( \gamma \). Therefore, the non-Hermitian Liouvillian master equation becomes

\[
\dot{\rho}(t) = -i[H\rho(t) - \rho(t)\hat{H}^\dagger] + D[\hat{\Gamma}_-]\rho(t)
\]

(8)

Based on the framework in the last section, we can obtain the linear form superoperator of the Eq.(8)

\[
\mathcal{L}' = i \begin{pmatrix} -\gamma_1 & 0 & 0 & \Gamma \\ 0 & \frac{-\gamma_2 - \gamma_1 - \Gamma}{2} & \frac{-\gamma_2 - \gamma_1 - \Gamma}{2} & 0 \\ 0 & \frac{-\gamma_2 - \gamma_1 - \Gamma}{2} & \frac{-\gamma_2 - \gamma_1 - \Gamma}{2} & 0 \\ 0 & 0 & 0 & \frac{-\gamma_2 - \gamma_1 - \Gamma}{2} \end{pmatrix}
\]

(9)

The Liouvillian can present several interesting properties. We first analyze the energy spectrum of the Liouvillian, whose the eigenvalues are \( \lambda_1 = -i\gamma_1 \), \( \lambda_2 = i(\gamma_2 - \Gamma) \), \( \lambda_3 = \lambda_4 = i(\gamma_2 - \gamma_1 - \Gamma)/2 \) and the eigemtrices are \( \rho^1 = (1, 0, 0, 0)^T \), \( \rho^2 = (\frac{1}{\gamma_2 + \gamma_1}, 0, 0, 1)^T \), \( \rho^3 = (0, 1, 0, 0)^T \), \( \rho^4 = (0, 0, 1, 0)^T \). Obviously, the real part of the eigenvalues are zero, so we plot the imaginary part of these eigenvalues as function of \( \gamma_2/\gamma_1 \) in Fig.2(a).

We note that, there is an exceptional point for \( \gamma_1 + \gamma_2 = \Gamma \), \( \lambda_1 \) and \( \lambda_2 \) coalesce to \( -i\gamma_1 \), \( \rho^1 \) and \( \rho^2 \) coalesce to \( (1, 0, 0, 0)^T \). This result means that this is a specific degenerate point that there is no coherence between |0\rangle and |1\rangle because of no coupling. Unlike Lindblad exceptional point only induced by the decay channel, this degenerate point is jointly induced by non-Hermitian effect and decoherence effect, so this point is referred to as non-Hermitian Liouvillian exceptional points (NLEPs).

Next, we discuss the steady state about this system. When \( \Gamma = \gamma_2 \), we can find \( \lambda_2 = 0 \) and the other eigenvalues are negative imaginary. This means that there is a steady state while the other states will exponentially decay except for \( \lambda_2 \). On the other hands, all populations \( \langle \rho_{ij} \rangle \) will exponentially decay if all eigenvalues are negative imaginary or exponentially grow when there is at least one eigenvalue that is positive imaginary. In order to better understand the properties of steady states, we plot the populations of the system as a function of \( \gamma_1 t \) for the different parameters (e.g., \( \gamma_2 = \gamma_1, 2\gamma_1, 3\gamma_1 \)) in Figs.2(b)-2(d). Obviously, one can see clearly there are three different evolution trends.

It’s worth noting that the steady state \( \rho^2 \) becomes \( (\gamma_2/\gamma_1, 0, 0, 1)^T \) for \( \Gamma = \gamma_2 \). In other words, the populations of |0\rangle and |1\rangle, which can be not equal, are entirely controlled by parameters. This property is quite different from that of general PT-symmetry non-hermitian systems. Further more, we numerically prove it by plotting the populations of |0\rangle and |1\rangle as a function of \( \gamma_1 t \) in the different conditions, as shown in Fig.3.

**B. two-level open systems with coupling \( \Omega \)**

When \( \Omega \neq 0 \), the non-heritian Hamiltonian becomes as

\[
\hat{H} = \begin{pmatrix} -i\gamma_1 & \Omega \\ \Omega & i\gamma_2 \end{pmatrix}
\]

(10)

combining with the decay channel in Eq.(7), the linear form superoperator of the non-Hermitian Liouvillian
master equation becomes

\[ \mathcal{L}' = i \begin{pmatrix} -\gamma_1 & \frac{\Omega}{2} & -\frac{\Omega}{2} & -\frac{\Gamma}{2} \\ -\frac{\Omega}{2} & 0 & -\gamma_2 - \gamma_1 - \Gamma & 0 \\ 0 & -\gamma_2 - \gamma_1 - \Gamma & -\frac{\Omega}{2} & -\frac{\Omega}{2} \\ \frac{\gamma_2 - \gamma_1 - \Gamma}{2} & 0 & -\frac{\gamma_2 - \gamma_1 - \Gamma}{2} & \gamma_2 - \Gamma \end{pmatrix} . \] (11)

Although the eigenvalues and the eigenstates of Eq. (11) have analytic expressions in principle, here we do not show them for reducing wordiness and directly plot them in the following figures.

When \( \Gamma = 0 \), this system is a normal non-hermitian and the energy spectra as the functions of \( \gamma_2/\gamma_1 \) are plotted in Figs. 4(a) and 4(b). We can clearly see that there are two EPs, at which four eigenvalues coalesce into only one value. We also find the PT-symmetry in the condition of \( \gamma_2 = \gamma_1 \). In this case, all the imaginary parts of the eigenvalues are 0, corresponding to all the eigenstates are steady states. Interestingly, if there is a decay channel (e.g. \( \Gamma \neq 0 \)) in the system, as shown in Figs. 4(c) and 4(d), some imaginary parts of the eigenvalues will split. So the normal EPs disappear and the NLEPs arise, which clearly indicates the origin of NLEPs. In this case, \( \rho^3 \) and \( \rho^4 \) coalesce to

\[ \rho_{NLEP} = (1 + \frac{\Xi + \xi}{2\Omega}, -\frac{\Xi + \xi}{2\Omega}, i\frac{\Xi + \xi}{2\Omega}, i\frac{\Xi + \xi}{2\Omega})^T . \] (12)

where \( \xi = \gamma_2 - \gamma_1 - \Gamma \) and \( \Xi = \frac{1}{3}[54\Gamma \Omega^2 + \sqrt{2916\Gamma^2 \Omega^2 - 27(\xi^2 - 4\Omega^2)^3}]^{1/3} \). Form this result above obtained, it indicates that there are two NLEPs in the system and there are coherence between \( |0\rangle \) and \( |1\rangle \), which is quite different from the case of no coupling. And the phase between \( |0\rangle \) and \( |1\rangle \) is \( \pi/2 \), which is the same as the case of the normal EP.

Moreover, \( \rho_1 \) becomes a steady state since \( Im[\lambda_1] = 0 \), and \( Im[\lambda_{2,3,4}] < 0 \) in this case. We plot the details of this state as the function of time in Fig. 5. In this example, like the example without coupling, the populations of this steady state do not need to be equal. On the other hands, unlike the example without coupling, there are coherence between \( |0\rangle \) and \( |1\rangle \) in this steady state. Specifically, although the steady state is a mixed state, the phase between \( |0\rangle \) and \( |1\rangle \) is still \( \pi/2 \), which is the same as the case of \( \rho_{NLEP} \).

IV. DISCUSSION

In order to further classify and understand the phenomenon of the phase between \( |0\rangle \) and \( |1\rangle \), we give the following discussion. No matter what \( \Omega \) is, the phase between \( |0\rangle \) and \( |1\rangle \) of NLEPs must be \( \pi/2 \). On the other hand, the phase of the steady state in above section is still \( \pi/2 \). This fact can be interpreted as: in the first row of the linear form superoperator \( \mathcal{L}' \), the first and the fourth elements are the real numbers, the second and the third elements are the imaginary numbers. It means that, for any eigenstates, there are \( i \) in \( \rho_{00,11} \) or \( \rho_{01,10} \). Note

FIG. 4. (Color online) The imaginary part (a) and real part (b) of the eigenvalues \( \lambda_1 \) (magenta solid line), \( \lambda_2 \) (green dashed line), \( \lambda_3 \) (red dash-dot line) and \( \lambda_4 \) (blue dotted line) in the system with \( \Omega = 2 \gamma_1 \), and \( \Gamma = 0 \). The imaginary part (c) and real part (d) of the eigenvalues in the system with \( \Gamma = \gamma_1 \), another parameters are the same as (a) and (b).

FIG. 5. (Color online) Each elements of \( \rho \) are 1/2 initially. (a) The probabilities \( \rho_{00} \) (blue dotted line), \( \rho_{11} \) (red dash-dot line), (b) the imaginary parts and (c) the real parts of \( \rho_{12} \) (green dotted line), \( \rho_{21} \) (magenta dash-dot line) as the function of time \( \gamma_1 t \). Other parameters are the same as those in Fig. 4.
that \(\rho_{ij}\) are the elements of the eigenmatrix \(\rho\). It is well known that \(\rho_{00,11}\) must be the real numbers, representing the population of \(|0\rangle\) and \(|1\rangle\), so \(\rho_{01,10}\) must be the imaginary numbers. In other words, for all eigenstates of \(\mathcal{L}\), the phase of between \(|0\rangle\) and \(|1\rangle\) must be \(\pi/2\), as long as \(\gamma_1, \Gamma \neq 0\). So this result also applies to the normal non-hermitian or decoherence systems. For example, this phase can be controled in the \(\mathcal{P}\)-\(\mathcal{T}\)-symmetry system, which \(\Gamma \neq 0\) and \(\gamma_1 = \gamma_2\), or there is not coherence between \(|0\rangle\) and \(|1\rangle\) in the normal decoherence system if \(\gamma_1 = 0\).

V. CONCLUSION

In summary, we have explored the systems with non-Hermitian and decoherence effect via rewriting the non-Hermitian Lindblad equation to a linear equation. As an example, the properties of the NLEPs and the steady state of the two-level system have been discussed. We find that NLEPs in this system have different properties from the normal EPs. Besides, we predict a new-type steady state and discuss the populations and the phase of the steady state. Finally, we demonstrate that there are a special phase of \(\pi/2\) in this system. Our system can be realized in the current experimental conditions. In short, we open a new way to further explore the open systems with non-Hermiticity and decoherence.

ACKNOWLEDGMENTS

This work is supported by the National Natural Science Foundation of China (Grants No. 11747107), the Natural Science Foundation of Guangdong province (Grants No. 2017A030307023), the Scientific Research Project of Hunan Province Department of Education (Grant Nos.19B060). Y N Guo is supported by the Program of Changsha Excellent Young Talents (kq1905005).

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