A Fast Scattering Model for C-Band Data Simulation in Forest Scenarios

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ABSTRACT
Synthetic aperture radar (SAR) data simulation of forest canopies is a common method used to study the interaction between radar and forests. The forest canopy contains a large number of small leaves, which require many computing resources to calculate their backscattering coefficients and affect the simulation efficiency of SAR data. To improve the simulation efficiency, an equivalent scattering matrix model is proposed to rapidly generate the backscattering coefficient of small leaves. The equivalent scattering matrix is generated by the coherence matrix. The tree crown is divided into several cuboids, and the backscattering coefficient matrices of leaves in each cuboid are replaced by an equivalent scattering matrix to reduce the calculation burden. To ensure the simulation accuracy, the relationship between the number of scatterers in each cuboid and the simulation error is analyzed. The experimental results show that the backscattering coefficients of small leaves generated by the equivalent scattering matrix model are consistent with the simulation results by the traditional Monte Carlo model. However, the computational efficiency of the equivalent scattering matrix model is higher than that of the Monte Carlo model, and the simulation efficiency is improved by 2 to 40 times.

INDEX TERMS
Synthetic aperture radar (SAR), forest canopy, backscattering coefficient, equivalent scattering matrix.

I. INTRODUCTION
Synthetic aperture radar (SAR) has the advantages of strong penetration and little influence by the weather and can work all day and in all weather. In recent years, SAR technology has been widely used in the field of forest remote sensing, such as tree height retrieval, forest biomass estimation, forest area estimation, and other image interpretation techniques [1], [2], [3], [4]. On the other hand, interpretation technology usually requires a large number of SAR images to verify the model, especially models related to deep learning technology. Although various SAR platforms have been developed successfully in recent years, it is still difficult to meet actual application requirements. For example, forest scene SAR images with known tree heights can be simulated for research on tree height inversion algorithms [4]. Because the tree height information is known, using human labor to obtain the data is not required, which is more convenient than using measured data. SAR simulation technology can obtain a large amount of available data with few resources and is a common method to obtain SAR data [5]. Therefore, the study of forest scene SAR data simulation technology has practical application value.

SAR data simulation technology for forest scenes has been studied for decades. Most of the early scattering models are based on radiative transfer theory, which only considers the radiation intensity and does not take into account the coherence of electromagnetic waves, so most of these models are incoherent scattering models [6], [7], [8], [9], [10]. The most representative scattering models are the water cloud model and the Michigan Microwave Canopy Scattering Model (MIMICS). The water cloud model regards vegetation...
The relationship between the backscattering coefficients of forests and meteorological factors was analyzed using this simulation model [29], [30].

The forest scene is a complex three-dimensional structure that contains a large number of scatterers, such as stems, branches, and leaves. To simulate SAR data more realistically, a large number of scatterers need to be simulated; therefore, the simulation efficiency is very low, and it is difficult to simulate large-scale forest scenes. Sun et al. proposed an equivalent scatterers method to improve the SAR raw data simulation efficiency of forest canopies [31]. In this model, the backscattering coefficients in a certain region are coherently superimposed to obtain an equivalent scatterer, which can replace the original scatterers, reduce the number of scatterers calculated in the SAR raw data simulation, and improve the simulation efficiency. Although this method can improve the simulation efficiency of SAR raw data, it still requires considerable time to calculate the backscattering coefficient of the forest canopy. To solve this problem, an equivalent scattering matrix model is proposed to further improve the simulation efficiency of SAR data in this paper.

II. SIMULATION MODEL

In this section, the SAR data simulation model is introduced, and then the scattering mechanism of the forest canopy is described.

A. SAR DATA SIMULATION MODEL

The geometric model of the SAR simulated system is shown in Fig. 1. The radar movement direction is the positive direction of the y-axis, the vertical ground plane upward is the positive direction of the z-axis, and the direction of the x-axis is determined according to the y-axis and z-axis. The transmitted signal of the SAR system usually has a large time-bandwidth product; therefore, linear frequency modulation (LFM) is used. The simulation process can be regarded as a stop-go model[32]. At one location, the radar emits electromagnetic waves and then receives the reflected echoes from...
all targets in the scene as Equation (1),
\[
S^{pq}(t, \tau) = \sum_{i=1}^{N} \sigma^p_{i} W_{a}(\theta) \text{rect}(\frac{t-2r_i(t)/c}{T_p}) \\
\times \exp(-\frac{4\pi r_i(\tau)}{\lambda}) \exp(j\pi k_{r}(\tau - \frac{2r_i(t)/c}{c})^2)
\]
where \( S \) represents the SAR echoes reflected from the forest canopies, \( p \) and \( q \) represent the polarization channel of the transmitting antenna and receiving antenna, respectively; \( \sigma^p_{i} \) denotes the coefficient of the \( k_{th} \) scatterer, \( T_p \) represents the pulse width, \( \lambda \) represents the wavelength, and \( c \) represents the speed of light. \( t \) and \( \tau \) represent the range time and the azimuth time, respectively; \( r_i(t) \) represents the distance between the radar and scatterer at time \( t \); \( k_{r} \) represents the chirp slope.

The receiving and transmitting process is repeated at each location, and the SAR raw data of the forest scene are obtained. Then, the SAR raw data are converted into SAR images using imaging processing techniques.

**B. FOREST CANOPY SCATTERING MODEL**

The backscattering coefficient of the forest canopy can be approximated as direction scattering and double scattering, as shown in Fig. 2. The backscattering coefficient consists of four parts as (2),
\[
\sigma_{all} = \sigma_{g} + \sigma_{t} + \sigma_{gl} + \sigma_{tg}
\]
where \( \sigma_{g} \) represents the direct backscattering from the ground surface after the attenuation of the canopy, \( \sigma_{t} \) represents the direction backscattering from the tree, \( \sigma_{gl} \) represents the double-bounce backscattering between the ground surface and the tree, and \( \sigma_{tg} \) represents the double-bounce backscattering between the tree and the ground surface.

Trees can be divided into trunks, branches, twigs, and leaves, so the backscattering coefficient of \( \sigma_{t} \) and \( \sigma_{tg} \) can be written as (3) and (4),
\[
\sigma_{t} = \sigma_{trunk} + \sigma_{branch} + \sigma_{twig} + \sigma_{leaf}
\]
\[
\sigma_{tg} = \sigma_{trunk-g} + \sigma_{branch-g} + \sigma_{twig-g} + \sigma_{leaf-g}
\]
where \( \sigma_{trunk}, \sigma_{branch}, \sigma_{twig}, \) and \( \sigma_{leaf} \) are the direction backscattering of trunks, branches, twigs, and leaves, respectively. \( \sigma_{trunk-g}, \sigma_{branch-g}, \sigma_{twig-g}, \) and \( \sigma_{leaf-g} \) are the double-bounce backscattering between trunks, branches, twigs, and leaves, and the ground, respectively.

When calculating the double backscattering \( \sigma_{gl} \), the tree is regarded as a whole, so \( \sigma_{gl} \) is not decomposed into different backscattering coefficients in this paper.

**III. EQUIVALENT SCATTERING MATRIX MODEL**

Forest canopies contain a large number of small leaves, and this requires many computing resources to calculate their backscattering coefficients and reduces the simulation efficiency of SAR data. To improve the simulation efficiency of SAR data, an equivalent scattering matrix model is proposed in this paper. This model generates the scattering matrix to replace the backscattering coefficient matrix of leaves and improves the simulation efficiency.

It is assumed that the leaves are equal in size and randomly distributed in the forest canopy. When the radar works in the C-band, after coherent superposition, the backscattering coefficient of leaves in the canopy can be assumed to meet the Gaussian distribution condition \([33],[34],[35]\),

1. A large number of scatterers in a resolution cell of a homogeneous medium.
2. The range distance is much larger than many radar wavelengths.
3. The surface is much rougher compared with the scale of the radar wave.

Therefore, a set of random numbers with a Gaussian distribution satisfying certain conditions can be generated to approximate the backscattering coefficient of leaves without calculating the backscattering coefficient of each leaf according to radiative transfer theory to improve the simulation efficiency of the SAR data. The principle of the simulation model is shown in Fig. 3. Fig. 3(a) is the traditional Monte Carlo model, which simulates a large number of needles and discs randomly distributed in the crown to approximate the leaves \([17]\). Fig. 3(b) is the equivalent scattering matrix model proposed in this paper. First, the tree crown is divided into several cuboids, and the length of the cuboid is adjusted so that the projection area of the leaves in the same cuboid does not exceed 1 pixel in the SAR image. Then, an equivalent scatterer is used to replace all the leaves in the cuboid. Finally, the scattering matrix of the equivalent scatterer is generated as the scattering matrix of the leaves in the cuboid.

The equivalent scattering matrix model needs to solve two problems: 1) how to divide the tree crown and 2) how to generate an equivalent scattering matrix to replace the scattering coefficient of the leaves in the cuboid. These two problems will be discussed below.

**A. TREE CROWN SEGMENTATION**

The tree crown is divided into several cuboids, each of which has an equivalent scatterer in the center to replace the leaves...
in the cuboid. To improve the simulation efficiency, it is necessary to reduce the number of equivalent scatterers as much as possible. However, too few equivalent scatterers will increase the simulation error. In [31], a method to divide tree crowns is proposed, and the phase error of the backscattering coefficient after coherent superposition is analyzed. In this paper, the tree crown division method is used, as shown in Fig. 4. The bottom of the tree is the coordinate origin $O$, and the direction of the radar beam is the positive direction of the $x$-axis. In the plane determined by the radar movement direction and the $x$-axis, the direction perpendicular to the $x$-axis is the $y$-axis direction and the side of the radar movement direction is the positive $y$-axis direction. The direction of the $z$-axis is determined according to the right-hand rule. The tree crown is meshed along the three coordinate axes, and the lengths of the cuboid are $D_x$, $D_y$, and $D_z$. The position of the equivalent scatterer is in the center of each cuboid.

The geometric relationship between the radar and the cuboid is shown in Fig. 5. $M$ is the position of the radar and $A$, $B$, $C$, $D$, $E$, $F$, $G$, and $H$ are eight vertices of the cuboid. $D_x$, $D_y$, and $D_z$ are the lengths of the cuboid. $R_1$, $R_2$, and $R_3$ are the distances between the radar $M$ and vertices $A$, $C$, and $E$, respectively. $P_{rx}$ denotes the distance difference between radar $M$ and points $A$ and $C$, and $P_{rz}$ denotes the distance difference between radar $M$ and points $A$ and $E$ and can be expressed as (5) and (6):

$$P_{rx} = R_2 - R_1 < AC = \sqrt{D_x^2 + D_y^2}$$  \hspace{1cm} (5)  

$$P_{rz} = R_3 - R_1 = \sqrt{R_1^2 + D_z^2} - R_1$$
$$= R_1(1 + \frac{D_z^2}{2R_1^2} + o(\frac{D_z^2}{R_1^2})) - R_1 \approx \frac{D_z^2}{2R_1^2}$$  \hspace{1cm} (6)  

The radar is usually far away from the scatterers. For the convenience of analysis, $R_1$ is approximated by $R$, which denotes the distance from the radar to the center of the simulation scene.

$$R_{rz} \approx \frac{D_z^2}{2R}$$  \hspace{1cm} (7)
The projection length of the cuboid in the range direction of the SAR image \( P_r \) can be expressed as

\[
P_r \leq \sqrt{P_{rx}^2 + P_{rz}^2} < P_{rx} + P_{rz} = \sqrt{D_r^2 + D_z^2 + \frac{D^2_r}{2r}}
\]  

(8)

It is easy to obtain the projection length of the cuboid in the azimuth direction \( P_a \), and this can be expressed as

\[
P_a = D_y
\]  

(9)

Let \( D_r \) and \( D_a \) represent the range and azimuth pixel space of the SAR image, respectively. There is only one equivalent scatterer in each cuboid. To ensure that there is no blank space in the SAR images, the projected area of cuboids in SAR images should not exceed one pixel, and we can obtain Equation (10).

\[
\begin{align*}
D_r & \geq \sqrt{D_r^2 + D_z^2 + \frac{D^2_r}{2r}} \\
D_a & \geq D_y
\end{align*}
\]  

(10)

The distance from the radar to the center of scene \( R \) is much larger than the pixel space of the SAR image, so the length of \( D_c \) can be very large. For example, letting \( D_r/2 = D_z \) and \( D_a/2 = D_y \) from Equation (10), it can be obtained that

\[
D_c \leq \sqrt{(D_r - \sqrt{D_r^2 + D_z^2/4}) \times 2R}
\]  

(11)

In general, the two-dimensional pixel space of the SAR image is approximately equal. Let \( D_r = D_a = D \), then

\[
D_c \leq \sqrt{(2 - \sqrt{2}) \times D \times R}
\]  

(12)

According to the above analysis, the relationship between the maximum length of the cuboid \( D_c \) and the pixel spacing of the SAR image can be obtained. Assuming that the distance between the radar and the center of the simulation scene is 5000 m and the two-dimensional pixel space of the SAR image is equal, the relationship between the pixel space of the SAR image and the length of the cuboid \( D_c \) is shown in Fig. 6. When the pixel space is 1 m, the maximum length of \( D_c \) is approximately 50 m, which is much larger than the pixel space of the SAR image.

**B. EQUIVALENT SCATTERING MATRIX**

Generating the equivalent scattering matrix is another key step. Lee et al. proposed a method to simulate polarimetric scattering data by a coherence matrix or covariance matrix [33]. The simulation process is as follows. For the coherence matrix or covariance matrix \( C_m \), compute \( C_m^{1/2} \), where

\[
C_m^{1/2}(C_m^{1/2\dagger}) = C_m
\]  

(13)

Simulate a complex random vector \( v \) that has the complex normal distribution \( C_N(0, I) \), where \( I \) denotes the identity matrix. This can be easily accomplished by independently generating the real and imaginary components of each complex element of \( v \) with a normal distribution having mean of zero and variance of 1/2.

Compute \( u = C_m^{1/2}v \). The complex vector \( u \) has a \( C_N(0, I) \) distribution, and \( u \) is a single-look SAR pixel. This procedure can be easily verified, since

\[
u = C_m^{1/2} \quad E[uv^\dagger] = C_m^{1/2}E[vv^\dagger](C_m^{1/2\dagger}) = C_m
\]  

(15)

According to the above method, if the coherent matrix \( C_m \) that satisfies the polarization characteristics of leaves is found, the equivalent scattering matrix of leaves can be simulated by the method above. In this paper, the coherent matrix is generated as follows.

1. \( M \times N \) small leaves with a random direction are generated, and these leaves are approximated by needles or disks. The forward scattering matrix \( S_f \) and backscattering matrix \( S_b \) are calculated using Foldy’s approximation [17].

2. The leaves are divided into \( M \) groups, and each group has \( N \) leaves. The scattering matrices of each group are coherently superimposed to obtain forward scattering matrices \( \overline{S}_f \) and backscattering matrices \( \overline{S}_b \). Therefore, \( M \) forward scattering matrices and backscattering matrices can be obtained.

\[
\overline{S}_f = \sum_{i=1}^{N} S_{fi}
\]  

(16)

\[
\overline{S}_b = \sum_{i=1}^{N} S_{bi}
\]  

(17)

where \( S_{fi} \) and \( S_{bi} \) denote the forward scattering matrix and backscattering matrix of the \( i \)th scatterer, respectively.

3. The forward scattering coherence matrix \( C_f \) and the backscattering coherence matrix \( C_f \) are obtained by the statistical average of the \( M \) scattering matrices. According to the coherence matrix, it is easy to obtain the forward scattering matrix \( \overline{S}_f \) and backscattering matrix \( \overline{S}_b \).
Due to the different numbers of leaves in each cuboid, the intensity of the coherence matrix needs to be adjusted. Under the condition of Gaussian distribution [33], [34], [35], assuming that there are $N$ leaves in a cuboid, the backscattering coefficient of the leaves satisfies the amplitude of $\rho$, and the phase is uniformly distributed between $(-\pi, \pi)$. After coherent superposition, the variance can be expressed as (18)

$$\sigma^2 = \frac{N}{2}E(A^2)$$

The variance of $K$ leaves can be obtained by multiplying by the coefficient $a$

$$a = \frac{K}{N}$$

(19)

It is easy to prove that the coherence matrix $C_m$ and variance have the same proportional relationship; therefore, the coherence matrix of different leaves can be obtained by multiplying the existing coherence matrix by coefficient $a$.

According to the generated forward scattering matrix $S_f$ and backscattering matrix $S_b$, the direction scattering matrix and double scattering matrices of the forest canopy can be simulated. The direction equivalent scattering matrix $S_{1e}$ of the leaves can be calculated as (20)

$$S_{1e} = T_c(L)S_fT_s(L)$$

(20)

where $T_c$ denotes the transfer matrix of the forest canopy, $L$ denotes the length of the electromagnetic wave through the forest canopy, and $M_{hh}$ and $M_{vv}$ denote the extinction matrix of horizontal polarization and vertical polarization, respectively. The transfer matrix and extinction matrix are calculated in [15]. The extinction matrix reflects the attenuation ratio of electromagnetic waves of different frequencies passing through the tree canopy. For the high frequency band electromagnetic waves, most of their energy is attenuated by the tree canopy and hardly reaches the surface.

The double equivalent scattering matrix $S_{2e}$ can be calculated as (22)

$$S_{2e} = T_g(L_g)R_gT_r(L_{rg})S_fT_e(L_e)$$

(22)

where $T_c(L_e)$ is the transfer matrix from radar to forest canopy, $T_g(L_{rg})$ is the transfer matrix from the forest canopy to the ground surface, $T_g(L_g)$ is the transmission matrix from the ground surface to radar, and $R_g$ is the ground surface Fresnel reflection matrix. $L_e$, $L_{rg}$ and $L_g$ are the transmission paths shown in Fig. 7.

**C. ERROR ANALYSIS OF THE COHERENCE MATRIX**

In the proposed model, the coherence matrices of the leaves are obtained by statistical averaging and have some errors compared with the real coherence matrix. In this section, the error of the coherence matrix obtained is analyzed.

First, a $3 \times 3$ matrix $C_{3v}$ is used as the coherence matrix and can be obtained from the measured SAR image of the canopy region. In this experiment, let $C_{3v}$ as (23)

$$C_{3v} = \begin{bmatrix} 3 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 3 \end{bmatrix}$$

(23)

which is the expression of the volume scattering component in the Freeman–Durden decomposition[35], and $P_v$ denotes the contribution of volume scattering; since no other scattering components are involved here, the value here is 1.

Second, a large number of scatterers are simulated, and their scattering matrices are calculated according to the method proposed in (13)-(15). Finally, the coherence matrix $C_{3v}$ of the simulated data is calculated. The error of two coherence matrices $e_{3v}$ is defined as (24)

$$e_{3v} = \frac{\text{sum}(\text{abs}(C_{3v} - C_{3v}^\text{abs})))}{\text{sum}(\text{abs}(C_{3v}^\text{abs})))} \times 100\%$$

(24)

The different numbers of scatterers are simulated, and the errors of the coherence matrix are calculated. Each experiment was repeated 10 times, and the results were averaged. The relationship between the number of scatterers and the coherence matrix error $e_{3v}$ is shown in Fig. 8. It can be seen...
that with the increase in the number of simulated scatterers, the coherence matrix error \( e_3 \) decreases obviously. When the number of scatterers is approximately 10000, the error is approximately 2%. With the increase in the number of scatterers, the reduction rate of the coherence error slows down. When the number of scatterers is 40000, the error is reduced to approximately 1%, which is only 1% less than when the number of scatterers is 10000.

**D. ERROR ANALYSIS OF THE GAUSSIAN DISTRIBUTION**

There are a large number of leaves in the forest canopy, and they are small in size and random in direction. When the number of leaves is large enough, the real and imaginary backscattering coefficients can be considered to satisfy a Gaussian distribution in the SAR images [34]. Therefore, it is necessary to ensure that each cuboid is large enough to accommodate enough scatterers and meet the Gaussian distribution condition. In this section, first, the Gaussian distribution error of the measured data is calculated, and then the relationship between the number of scatterers and Gaussian error is analyzed by simulation.

The GF-3 satellite works in the C-band, which has high frequency and weak penetration. In the forest scene, the scattering mainly comes from the leaves in the canopy. In this paper, the Gaussian distribution fitting error of forest canopy data is analyzed by using the measured GF-3 satellite fully polarized SAR images. The fitting error is calculated by the Kolmogorov–Smirnov (K–S) method. Ten GF-3 fully polarized SAR images covered by dense forests were selected. These data were obtained between July and August 2018. It is located between 42°–48°N latitude and 119°–125°E longitude zone. The area is located in the northeast of China and grows a large amount of deciduous forest. Forest areas with different sizes are between 0.006 and 0.012, which are close to the distribution condition. When the number of scatterers in each pixel is too small to satisfy the Gaussian distribution condition, the number of scatterers in each pixel is set to 4, 8, 16, and 32. The backscattering coefficients of all scatterers were coherently superimposed as the final scattering coefficients of each pixel, and the K–S fitting errors of channel I and channel Q were calculated. Each experiment was repeated 50 times to obtain the average value. The simulation results are shown in Table 2. It can be seen that when the number of scatterers in each pixel is the same, the larger the simulated image is, the smaller the K–S fitting error is, which is the same as the GF-3 data. On the other hand, keeping the image size unchanged, the more scatterers there are in each pixel, the smaller the K–S fitting error is. Compared with the GF-3 data, when the scatterers in each pixel are 4, the K–S fitting error is significantly larger than that of GF-3 data, indicating that the number of scatterers in each pixel is too small to satisfy the Gaussian distribution condition. When the number of scatterers in each pixel is 16, the K–S fitting errors of images with different sizes are between 0.006 and 0.012, which are close to the measured data. Therefore, to reduce the simulation error, it is necessary to ensure that cuboid volumes are large enough to accommodate enough scatterers.

According to the calculation results of the GF-3 satellite data, images of the same size are simulated. Some random numbers with uniform distribution are generated in each pixel. After coherent superposition, the image has the characteristic of Gaussian distribution, which is consistent with the distribution of leaves. A uniformly distributed random function is used to generate two numbers in the range of −0.5 to 0.5, which are the real and imaginary parts of the scattering coefficient, respectively. The number of scatterers in each pixel was set to 4, 8, 16, and 32. The backscattering coefficients of all scatterers were coherently superimposed as the final scattering coefficients of each pixel, and the K–S fitting errors of channel I and channel Q were calculated. Each experiment was repeated 50 times to obtain the average value. The simulation results are shown in Table 2. It can be seen that when the number of scatterers in each pixel is the same, the larger the simulated image is, the smaller the K–S fitting error is, which is the same as the GF-3 data. On the other hand, keeping the image size unchanged, the more scatterers there are in each pixel, the smaller the K–S fitting error is. Compared with the GF-3 data, when the scatterers in each pixel are 4, the K–S fitting error is significantly larger than that of GF-3 data, indicating that the number of scatterers in each pixel is too small to satisfy the Gaussian distribution condition. When the number of scatterers in each pixel is 16, the K–S fitting errors of images with different sizes are between 0.006 and 0.012, which are close to the measured data. Therefore, to reduce the simulation error, it is necessary to ensure that cuboid volumes are large enough to accommodate enough scatterers.

### TABLE 1. The K–S error of GF-3 satellite data.

| Size   | Channel | HH   | HV   | VH   | VV   |
|--------|---------|------|------|------|------|
| 50×50  | I       | 0.0135 | 0.0127 | 0.0123 | 0.0139 |
|        | Q       | 0.0135 | 0.0122 | 0.0125 | 0.0138 |
| 80×80  | I       | 0.0089 | 0.0104 | 0.0101 | 0.0099 |
|        | Q       | 0.0083 | 0.0122 | 0.0113 | 0.0102 |
| 100×100| I       | 0.0080 | 0.0085 | 0.0085 | 0.0085 |
|        | Q       | 0.0078 | 0.0084 | 0.0084 | 0.0081 |
| 200×200| I       | 0.0075 | 0.0075 | 0.0075 | 0.0075 |
|        | Q       | 0.0073 | 0.0077 | 0.0076 | 0.0076 |
| 300×300| I       | 0.0066 | 0.0065 | 0.0066 | 0.0066 |
|        | Q       | 0.0066 | 0.0067 | 0.0065 | 0.0067 |

### TABLE 2. The K–S error of simulation data.

| Size   | Channel | 4   | 8   | 16  | 32  |
|--------|---------|-----|-----|-----|-----|
| 50×50  | I       | 0.0150 | 0.0135 | 0.0128 | 0.0116 |
|        | Q       | 0.0152 | 0.0132 | 0.0127 | 0.0119 |
| 80×80  | I       | 0.0121 | 0.0089 | 0.0076 | 0.0075 |
|        | Q       | 0.0120 | 0.0092 | 0.0081 | 0.0074 |
| 100×100| I       | 0.0107 | 0.0082 | 0.0074 | 0.0070 |
|        | Q       | 0.0109 | 0.0088 | 0.0075 | 0.0071 |
| 200×200| I       | 0.0090 | 0.0073 | 0.0067 | 0.0063 |
|        | Q       | 0.0091 | 0.0073 | 0.0067 | 0.0063 |
| 300×300| I       | 0.0085 | 0.0065 | 0.0059 | 0.0051 |
|        | Q       | 0.0085 | 0.0066 | 0.0058 | 0.0052 |

### E. LEAF DENSITY AND PIXEL SPACING

According to the above analysis, to ensure that the projected area of the cuboid in the SAR image does not exceed one pixel, the volume of the cuboid should not be too large. At the same time, to satisfy the Gaussian distribution conditions, the cuboid must be large enough to accommodate a sufficient number of leaves. Therefore, it is necessary to consider the pixel spacing of the SAR image and the density of leaves to judge whether the constraint conditions can be satisfied.

Suppose \( V_e \) denotes the volume of the cuboid, \( m_{\text{leaf}} \) denotes the density of leaves, and \( N_{\text{min}} \) denotes the minimum number of scatterers in the cuboid. To satisfy the constraint
condition, Equation (25) should be satisfied.

\[ V_e \geq \frac{N_{\text{min}}}{m_{\text{leaf}}} \]  \hspace{1cm} (25)

According to Equation (10), let \( D_x = D_y = D_z = D/2 \) When \( D_z \) takes the maximum value, Equation (25) can be expressed as

\[ V_e = \frac{D^2}{8} \sqrt{(2 - \sqrt{2}) \times D \times R} \]  \hspace{1cm} (26)

\[ m_{\text{leaf}} \geq \frac{8N_{\text{min}}}{\sqrt{(2 - \sqrt{2}) \times D \times R}} \]  \hspace{1cm} (27)

Assume that the distance from the radar to the center of the simulation scene \( R = 5000 \text{ m} \). The relationship between leaf density \( m_{\text{leaf}} \) and pixel spacing \( D \) is shown in Fig. 9. The pixel spacing is inversely proportional to the leaf density. When the pixel spacing of the SAR image is 1 m, the required leaf density is approximately 15/m³. When the pixel spacing of the SAR image is 0.3 m, the required leaf density is approximately 50/m³. With the decrease in pixel spacing, the requirement of leaf density increases sharply. Therefore, when the pixel spacing of the simulated SAR image is small, this proposed method can only simulate the forest canopies with high leaf density.

The equivalent scattering matrix model is only used to approximate the backscattering coefficients of leaves in the canopy, so only the backscattering coefficients of leaves are generated here and compared. The simulation parameters are shown in Table 3.

### TABLE 3. The simulation parameters.

| Parameters                  | Value       |
|-----------------------------|-------------|
| frequency                   | 5.4 GHz     |
| incidence angle             | 45°         |
| number of trees             | 16          |
| tree high                   | 10 m        |
| leaf density                | 250/m³      |
| species                     | broadleaf tree |
| volumetric water content of leaf | 35%          |
| region                      | 100 m x 100 m |

The backscattering coefficients of leaves generated by the two methods were projected onto a 200 x 200 image with a pixel spacing of 0.5 m x 0.5 m, the density of leaves was 830 m⁻³, and the backscattering coefficients within each pixel cell were coherently superimposed. Fig. 10 are the simulation results of the Monte Carlo model, and Fig. 11 shows the simulation results of the equivalent scattering matrix method (\( D_x = D_y = 0.3 \text{ m}, D_z = 20 \text{ m} \)). It can be seen that the simulation results of the two simulation models are very similar.

![FIGURE 9. The relationship between leaf density and pixel spacing.](image)

**FIGURE 10.** Simulation results of the Monte Carlo coherent scattering model.

**FIGURE 11.** Simulation results of the equivalent scattering matrix model.

Due to the speckle noise of SAR images, it is not meaningful to directly compare the value of a single pixel. Therefore, the average value of multiple pixels is taken for comparison.

The mean value of 9 pixels in the center of the canopy is used to calculate the backscattering coefficients. The backscattering coefficients of 16 trees are shown in Fig. 12 and Fig. 13. The horizontal coordinate axis represents the number of trees. The backscattering coefficients...
of the HH and VV polarization channels are approximately $-20\sim-19$ dB, and the backscattering coefficients of the HV polarization channels are approximately $-25\sim-24$ dB. The backscattering coefficients obtained by the two models are in the same range.

To further verify the effectiveness of the simulation model, different parameters are used to compare the model proposed in this paper and the Monte Carlo coherent scattering model. According to the parameters listed in Table 3, some parameters are modified, including tree height (H), leaf density (D), and leaf volume water content (LVWC). As in previous experiments, 16 trees were simulated in these experiments, and 9 points were averaged for each tree, and the results of 16 trees were averaged. The input parameters of the two models are the same. To reduce the interference of other factors, only the backscattering coefficient of leaves in the crown is calculated. The simulation results of the Monte Carlo coherent scattering model are shown in Table 4, and the simulation results of the equivalent scattering matrix model are shown in Table 5. The comparison results of the two models are shown in Fig. 14. The horizontal coordinate axis represents experiments 1 to 8, and the vertical coordinate axis is the difference between the two simulation models. It can be seen that the difference between the two simulation models is only about 0.1 dB.

According to the above experiments, the simulation results of the equivalent scattering matrix model proposed in this paper are very similar to those of the Monte Carlo coherent scattering model. Considering the randomness of the two models, it can be considered that the simulation results of the two models are consistent.

### B. SIMULATION EFFICIENCY

In this section, the simulation efficiency of the proposed model is verified. The equivalent scattering matrix model and Monte Carlo model are used to simulate the backscattering coefficients of the same parameters and forest scenes, and the simulation efficiency of the two methods is compared. The simulation parameters are shown in Table 6. The area of
the simulation scene is 100 m × 100 m. The tree heights are 10 m and 20 m. The leaf density is 250/m³ and 830/m³, the $D_x$ and $D_y$ of the cuboid are equal to the pixel spacing, and the $D_z$ of the cuboid is equal to 20 m. The computer configuration is shown in Table 7.

V. CONCLUSION

The simulation results are shown in Table 8. Time(M) is the simulation time required by the Monte Carlo model, and Time(E) is the simulation time required by the equivalent scattering matrix model. $N$ is the ratio of the time required for the two models. The simulation efficiency of the equivalent scattering matrix model is significantly higher than that of the Monte Carlo model. According to the difference in pixel spacing and tree geometry, the simulation efficiency is improved by 2-40 times.

The improvement in the simulation efficiency is related to the number of leaves in the cuboid. Since the backscattering matrix of leaves in each cuboid is replaced by only an equivalent scattering matrix, the more leaves there are in each cuboid, the greater the simulation efficiency will be improved. Under the constraint conditions, the maximum length of the cuboid along the z-axis is tens of meters, which is usually much higher than the height of the tree. When the height of the tree changes from 10 meters to 20 meters, the length of $D_z$ increases, so the volume of the cuboid becomes larger. Similarly, as the pixel spacing increases, the lengths of $D_x$ and $D_y$ also increase. On the other hand, the higher the leaf density is, the greater the number of leaves per unit volume, and the greater the simulation efficiency.

The equivalent scattering matrix model proposed in this paper is based on the Gaussian distribution, and the generated SAR images conform to the Rayleigh distribution characteristics. The model applies to the simulation of single-look amplitude SAR images in the homogenous region. With the improvement in image resolution and the difference in simulation scenes, SAR images show different distribution characteristics. The next step is to study the equivalent scattering matrix model that conforms to different distribution characteristics.

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