Patchy He II reionization and the physical state of the intergalactic medium

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ABSTRACT

We present a Monte Carlo model of He II reionization by quasi-stellar objects (QSOs, quasars) and its effect on the thermal state of the clumpy intergalactic medium (IGM). The model assumes that patchy reionization develops as a result of the discrete distribution of QSOs. It includes various recipes for the propagation of the ionizing photons, and treats photoheating self-consistently. The model predicts the fraction of He III, the mean temperature in the IGM, and the He II mean optical depth – all as a function of redshift. It also predicts the evolution of the local temperature versus density relation during reionization. Our findings are as follows. The fraction of He III increases gradually until it becomes close to unity at $z \sim 2.8–3.0$. The He II mean optical depth decreases from $\tau \sim 10$ at $z \gtrsim 3.5$ to $\tau \lesssim 0.5$ at $z \lesssim 2.5$. The mean temperature rises gradually between $z \sim 4$ and $z \sim 3$ and declines slowly at lower redshifts. The model predicts a flattening of the temperature–density relation, with significant increase in the scatter during reionization at $z \sim 3$. Towards the end of reionization, the scatter is reduced and a tight relation is re-established. This scatter should be incorporated in the analysis of the Ly$\alpha$ forest at $z \lesssim 3$. Comparison with observational results of the optical depth and the mean temperature at moderate redshifts constrains several key physical parameters.

Key words: intergalactic medium – quasars: general – cosmology: theory – dark matter – large-scale structure of Universe.

1 INTRODUCTION

The physical properties and the thermal history of the intergalactic medium (IGM) play an important role in shaping the observed structure in the Universe. Proper modelling of the evolution of the IGM is therefore essential for the interpretation of the rapidly accumulating data on the high-redshift Universe. These data include the secondary temperature and polarization fluctuations in high-resolution measurements of the cosmic microwave background (CMB), and absorption features in the spectra of quasi-stellar objects (QSOs, quasars), which probe the ionized and neutral gaseous components, respectively. Since the distribution of gas follows the underlying mass, these data can be used to constrain the linear mass power spectrum (Croft et al. 1998, 2002; McDonald et al. 2000, 2004; Viel, Haehnelt & Springel 2004).

The gaseous component of the IGM follows, by and large, the distribution of the underlying mass distribution of the gravitationally dominant dark matter (DM). The temperature, ionization state and other physical properties are, however, determined by energetic feedback incurred by QSO and galactic activities. This feedback is manifested in photoionization and heating of the IGM by radiation emitted by QSOs and galaxies, and in mechanical energy from supernovae explosions of massive stars producing galactic winds that can shock-heat the IGM and enrich it with metals (e.g. Cowie & Songaila 1998; Aracil et al. 2004).

Absorption features in QSO spectra indicate that mechanical energy in galactic winds stirs up the IGM only in the vicinity of galaxies. Observations (Adelberger et al. 2003) have shown that winds can evacuate the neutral hydrogen from within a distance of up to $1–2\, h^{-1}$ Mpc away from Lyman break galaxies (LBGs). Away from galaxies, analysis of the Ly$\alpha$ forest strongly suggests that the moderate-density IGM that makes up most of the volume in space is only affected by ionizing ultraviolet (UV) photons emitted by QSOs and galaxies.

The gaseous IGM is mainly made of a primordial composition of hydrogen and helium, with the former being the dominant element. At $z \lesssim 1000$ these elements recombined and remained neutral.

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until the appearance of stars (and possibly very high-redshift QSOs) able to produce sufficient photons to cause significant reionization. Measurements of the polarization anisotropies of the CMB by the Wilkinson Microwave Anisotropy Probe (WMAP) satellite call for an ionized IGM at redshifts as high as $z \sim 17 \pm 5$ (Kogut et al. 2003; Spergel et al. 2003). The Gunn–Peterson trough in QSO spectra seems to occur at $z \approx 6$ (Becker et al. 2001; Djorgovski et al. 2001) and might imply a complex reionization history of the IGM with more than a single phase of reionization (Cen 2003; Wyithe & Loeb 2003b). Ionizing radiation from galaxies is also sufficient to singly ionize helium (Benison et al. 2002) at $z > 6$. In any case, observations of the Lyα forest show that by $z \sim 6$ most of the hydrogen in the Universe was photoionized and that the IGM is in a quiet state governed by the gravitational drag of the dark matter and the photoheating of the ionizing radiation (see Rauch 1998, for a review). During this stage a tight relation is established between the temperature and density (e.g. Theuns et al. 1998). The enhancement of QSO activities peaking at $z \sim 2$ is expected to disturb this quiet state by the production of hard photons energetic enough to doubly ionize helium (e.g. Zheng et al. 2004). Modelling double helium reionization is the focus of the current paper. One of our basic assumptions is that QSOs are the main sources for Heii reionization at low redshifts. This assumption is sustained by the evidence from observations pointing to Heii reionization at redshifts concurrent with the escalation of QSO activities. Although star formation activity also peaks at similar redshifts, radiation from galaxies is too soft to contribute significantly to Heii reionization (Benson et al. 2002; Wyithe & Loeb 2003a), unless Population III stars are considered (e.g. Venkatesan, Tumlinson & Shull 2003).

In recent years, commendable efforts have been made to observe an Heii Gunn–Peterson trough, at the Lyα wavelength of 304 Å, in quasar absorption spectra. Estimation of the Heii optical depth from the spectrum of the quasars Q0302−003 ($z = 3.286$; Dobrzycki & Bechtold 1991; Heap et al. 2000) and of HE 2347−4342 ($z = 2.885$; Smette et al. 2002; Zheng et al. 2004) indicate a sharp decline in the optical depth at redshift $z \sim 2.8$. At $z \gtrsim 2.8$ the optical depth is $\tau \gtrsim 4$ and at $z \lesssim 2.7$ it is $\tau \lesssim 1$. This decline implies that Heii reionization occurred at $z \sim 3$. The optical depth of the quasar HS 1700+64 ($z = 2.743$; Davidsen, Kriss & Zheng 1996), $\tau = 1.00 \pm 0.07$ at redshift $z = 2.4$, agrees with the general picture described above. It is important to stress that there may be other explanations for the large fluctuations in the Heii optical depth. For example, in Smette et al. (2002), a model is described where optical depth fluctuations along the line of sight are a local effect due to nearby ‘soft’ and ‘hard’ sources, and are unrelated to the global reionization of the IGM.

We aim at a detailed modelling of the state of the IGM during Heii reionization. Two approaches can be adopted in achieving this goal. The first is to employ hydrodynamical simulations that include radiative transfer and a good recipe for modelling the distribution of QSOs. An effort towards this goal has been made by Sokasian, Abel & Hernquist (2001, 2002), who implemented their radiative transfer numerical code in a cosmological simulation of a box of $67 h^{-1}$ Mpc (run by V. Springel). The simulations offer a valuable insight into the evolution of ionized regions and their topology. The effect of Heii reionization on the thermal properties of the IGM have not been modelled in these simulations. This is because radiative transfer is not merged self-consistently with the gas dynamics in the simulations. This is a technical limitation that is expected to be overcome in the near future. However, the simulations are limited by the available CPU power, preventing a thorough exploration of the key physical parameters. The alternative approach is to develop fast semi-analytic models that capture the essential ingredients of the reionization process. This approach has been adopted to study a variety of other problems in structure formation (e.g. Bi & Davidsen 1997; Kauffmann, Nusser & Steinmetz 1997; Somerville & Primack 1999; Kauffmann et al. 1999; Cole et al. 2000; Benson et al. 2001). The basis of semi-analytic models is the synthesis of the knowledge acquired from diverse simulations and analytic reasoning. This synthesis allows us to study a given physical problem with greater detail than individual simulations offer directly. In this paper we propose a Monte Carlo model for studying Heii reionization assuming that QSOs are the sole sources of the ionizing radiation. For an assumed cosmology and a form for the QSO luminosity function, the model provides detailed information about the physical state of the IGM. The model is easy to run and allows a thorough exploration of the key physical processes. We confront the model with available observational data, and provide predictions for the equation of state of the IGM, i.e. the local temperature–density relation (hereafter $T$–$\rho$ relation). We compare the Heii mean optical depth, $\tau(z)$, to the optical depths calculated from the Heii Lyα forest of the three quasars mentioned above (Q0302−003, HE 2347−4342 and HS 1700+64) and use this to constrain our model. We also compare the mean temperature, $T(z)$, from our model to the temperatures at the mean density calculated from the Lyα spectra of nine quasars (Schaye et al. 2000) to place further constraints.

The paper is organized as follows. A short theoretical background is presented in Section 2. In Section 3 we describe our model, which we use to investigate the process of patchy Heii reionization and its effect on the thermal evolution of the IGM. In Section 4 we try to fit observational measurements of the mean temperature and the mean optical depth of the IGM with our model for the power-law $\Lambda$CDM cosmological model implied by the WMAP data (Spergel et al. 2003). We also compute the local temperature–density relation for different redshifts. In Section 4.2 we repeat the same computation and comparison for a $\Lambda$CDM model with running spectral index (RSI) (Spergel et al. 2003) and for an open CDM (OCDM) cosmological model. We conclude with a summary and a discussion of the results in Section 5.

2 AN OVERVIEW OF HEII REIONIZATION

We assume that double helium reionization is mainly caused by ionizing photons emitted by QSOs, and that the contribution of galaxies is negligible. To justify this assumption, we have computed the emission rates of Heii ionizing photons from QSOs and galaxies and plotted them in Fig. 1. The calculation of the emission rate of QSOs is based on that of Madau, Haardt & Rees (1999, hereafter MHR), who performed a similar calculation for hydrogen (see Appendix A). The emission rate of galaxies is estimated using the GAlform semi-analytic model of galaxy formation. Specifically, we adopt the model and parameters of Benson et al. (2002), except for changes in the cosmological parameters required to match the cosmological model considered here. The reader is referred to that paper, and references therein, for a full description of the model. We choose to ignore the effects of H1 reionization on later star formation1 so as to obtain an upper limit on the number of ionizing photons produced at a given redshift. Note that this model tends to produce more photons than the more recent implementation of the Durham model presented in Baugh et al. (2005, see their fig. 1). As such, we overestimate

1 As shown by Benson et al. (2002), H1 reionization leads to a suppression of star formation at later times.
the IGM work to re-establish the tight T−ρ relation. Much of the thermal history of the IGM at 2 < z < 5 is governed by the way He II reionization proceeds.

3 THE MONTE CARLO MODEL

Given N random points in the IGM with specified gas density, temperature and ionization state at some high redshift, z_i, the model is designed to yield the corresponding quantities at any lower redshift. The model assumes that a uniform ionizing background for hydrogen and singly ionized helium has already been established prior to z_i. The model aims at following the physical state of the IGM as a result of He II reionization by the QSO population. At the early stages of He II reionization, the mean free path for the absorption of an He II ionizing photon, l_ph, is short, and the reionization proceeds in a patchy way as a result of the discrete distribution of QSOs. At these stages a QSO generates an almost fully ionized bubble as the bounding thin (thickness l_x) ionization front propagates outwards. The process continues until the QSO fades away. The mean lifetime of a QSO is ∼10^7 yr (e.g. Hosokawa 2002; Schirber, Miralda-Escudé & McDonald 2004; Porciani, Magliocchetti & Norberg 2004), which is much shorter than the recombination time-scale (order of 1 Gyr at z = 3) inside the bubbles. Therefore, we assume that the QSO radiation is emitted in bursts. At later stages when the filling factor of He III regions becomes significant, the mean free path, l_ph, becomes so large that a non-negligible fraction of the absorbed photons at a given time have actually been produced at significantly larger redshifts. We start the ionization of He II by QSOs at redshift z_i ≈ 6, with hydrogen and helium already singly ionized. We neglect the He II ionization by galaxies. Fig. 2 presents a schematic description of the Monte Carlo model.

3.1 Details of the model

3.1.1 Initialization

We work with N points representing random uncorrelated points in the IGM (we use N = 2 × 10^5). At each point we determine the He II abundance fraction, X, the temperature, T, and the density perturbation, δ. At the initial time, we set X = 0 for all the points and assign temperatures according to T = T_o(1 + δ)^{3/4}. At each point, the initial density contrast δ_i at some very high redshift, z_i = 6, is drawn randomly from a normal (Gaussian) probability distribution function with zero mean and with rms value σ_o.

3.1.2 The local density and its evolution

The ionized fraction and the adiabatic cooling/heating depend on the local gas density and its evolution in time. Given the initial mass power spectrum of the dark matter, we would like to have a recipe for deriving the gas density contrast as a function of time. Hydrodynamical simulations strongly support the picture in which the gas distribution in the IGM traces the dark matter density smoothed over the comoving Jeans length-scale, x_J, at which pressure forces roughly balance gravity (Hernquist et al. 1996; Theuns et al. 1998). We therefore estimate the gas density power spectrum by smoothing that of the dark matter on the Jeans length-scale (see Appendix C for details). We approximate the density contrast δ(z) at a point i as follows. The first step is to obtain the gas density variance (δ^2(z)) = σ^2(z) as a function of redshift. Given the linear power spectrum, we use the recipe described in Peacock (1999) to derive the corresponding non-linear power spectrum. This power spectrum...
3.1.3 The ionization probability

At each time-step the ionization state of helium is updated as follows. We assign a probability, $f_i$, for photoionizing He II at the point $i$, and draw a random number, $p_i$, between 0 to 1 from a uniform distribution. If $p_i < f_i$ then the point $i$ is ionized. We adopt the following form for the photoionization probability:

$$f_i = \frac{N_{\text{abs}}(\delta_i)}{N_{\text{He}} \sum_{j=1}^{N} (1 - X_j)(1 + \delta_j) P_{\text{bias}}(\delta_j)},$$

(4)

where $N_{\text{He}}$ is the mean comoving number density of all helium species, $X_j$ is the He II abundance fraction at point $j$, and $N_{\text{abs}}(z)$ is the total number of ionizing photons per unit comoving volume that are actually absorbed in the current redshift interval $[z(t), z(t + \Delta t)]$ (see below). At the beginning of each time-step the number of available ionizing photons is computed as $N_{\text{in}} + N_{Q} \Delta t$, where $N_{\text{in}}$ is the number of photons that have been transmitted from previous time-steps and $N_{Q} \Delta t$ is the number of photons that are produced by QSOs in the current time-step. The calculation of $N_{Q}$ is detailed in Appendix A. The number of available ionizing photons is tabulated in frequency bins, and $N_{\text{abs}}$ is computed in a self-consistent way as described in Appendix B.

The factor $P_{\text{bias}}$ takes account of the bias between the gas density distribution and the distribution of the He II ionizing photons. At the initial stages of He II reionization, the ionizing photons are preferentially absorbed near the sources. As He II reionization proceeds, the mean free path, $l_{ph}$, becomes large and the ionizing radiation eventually forms a uniform background. In this case the distribution of the ionizing photons is independent of $\delta$. Here we adopt the following form for $P_{\text{bias}}$:

$$P_{\text{bias}}(\delta) \propto (1 + \delta)^{b - 1},$$

(5)

where $b$ is the bias parameter between the distribution of the QSOs and the gas density field, and $s = s(l_{ph})$ is in general a function of the mean free path, $l_{ph}$, of an He II ionizing photon. We work with three choices for $s(l_{ph})$:

$$x_{\text{const}} = 1,$$

(6a)

$$x_{\text{linear}} = \begin{cases} \frac{x_j}{l_{ph}}, & \text{if } l_{ph} > x_j, \\ 1, & \text{otherwise,} \end{cases}$$

(6b)

and

$$x_{\text{exp}} = \begin{cases} e^{(1 - l_{ph}/x_j)}, & \text{if } l_{ph} > x_j, \\ 1, & \text{otherwise.} \end{cases}$$

(6c)

3.1.4 He II photoionization heating

Once a point is flagged for ionization, we update its ionized fraction and temperature according to

$$X_i \rightarrow 1$$

(7)

and

$$T_i \rightarrow T_i + \Delta T \left(1 - X_i^{\text{prev}} \right),$$

(8)

where $\Delta T$ is the increment in the temperature due to photoionization heating, and $X_i^{\text{prev}}$ is the ionization fraction before the current ionization. We also make an attempt at modelling the effect of a finite QSO lifetime, $t_{Q}$, on the reionization process. The model does not contain explicit information on the spatial distribution of the sources. The way we alleviate this problem is by assuming that each freshly ionized point remains ionized for a period of time equal to $t_Q$ if it has not

Figure 2. Flow chart of the Monte Carlo method.
been selected for ionization during this time. During this period of
time, for each time-step, some recombination and photoionization
heating occur, and can be treated as a local equilibrium.

The process of drawing random numbers for points continues
until all \( N_{\text{abs}} \) photons are absorbed.

### 3.1.5 He II recombination

Recombination reduces the He III abundance fraction by

\[
\frac{dX}{dt} = -\alpha_{\text{HeIII}}^{(2)}(1+z)^3n_eX(1+\delta)^2,
\]

where \( \alpha_{\text{HeIII}}^{(2)} \) is the He II case B recombination coefficient (e.g. Verner & Ferland 1996) and \( n_e \) in the mean comoving number density of electrons.

At every time-step we approximate the reduction in the He III abundance fraction at every point \( i \) due to recombination:

\[
X_i \rightarrow X_i \exp \left[ -\alpha_{\text{HeIII}}^{(2)}(T_i)(1+z)^3(1+\delta_i)n_e\Delta t \right].
\]

### 3.1.6 Adiabatic cooling/heating

The main cooling process in the moderate-density IGM at redshifts
\( z \lesssim 6 \) is adiabatic cooling. The adiabatic cooling/heating is calculated from the thermodynamic dependence between the temperature and density of a monatomic ideal gas,

\[
\frac{T}{T_0} = \left( \frac{\rho}{\rho_0} \right)^{2/3},
\]

where \( T_0 \) and \( \rho_0 \) are the temperature and the mean density at some initial redshift, \( z_0 \). The mean density is \( \rho \sim (1+z)^3 \), so that

\[
T = T_0 \left( \frac{1+z}{1+z_0} \right)^2 \left( \frac{1+\delta}{1+\delta_0} \right)^{2/3}.
\]

Therefore, from equation (12), the adiabatic cooling/heating at every
time-step is

\[
T_i(z) = T_i(z + \Delta z)
\times \left( \frac{1+z}{1+z + \Delta z} \right)^2 \left( \frac{1+\delta_i(z)}{1+\delta_i(z + \Delta z)} \right)^{2/3}.
\]

### 3.1.7 Photoionization heating by H I and He I

Our basic assumption is that at redshifts \( z \lesssim 6 \) hydrogen is fully
ionized and helium is singly ionized. We also assume a local thermal equilibrium between the background radiation and recombination at every time-step. For simplicity we treat He I as extra H I particles, and use the Miralda-Escudé & Rees (1994) analysis for the IGM heating by the He I photoionization background radiation to calculate the temperature increment at every point \( i \):

\[
T_i \rightarrow T_i + \frac{2}{3k} E \alpha_{\text{HeI}}^{(2)}(T_i)(1+z)^3(1+\delta_i)n_e\Delta t,
\]

where \( E \) is the average energy of the absorbed photons minus the
average energy lost per recombination, and \( \alpha_{\text{HeI}}^{(2)} \) is the H I case B
recombination coefficient (e.g. Verner & Ferland 1996).

### 3.1.8 Compton cooling

Compton cooling resulting from free electrons scattering off the CMB photons is non-negligible at redshifts \( 3 < z < 6 \). This cooling time-scale is given by (Peebles 1968)

\[
\frac{1}{\tau_{\text{Compton}}} = \frac{1161.3\left(1+X_e^{-1}\right)}{(1+z)^2 \left[ 1 - T_{\text{CMB}}^4(1+z)/T_e \right]} \text{Gyr,}
\]

where \( X_e \) is the fraction of free electrons, \( T_{\text{CMB}}^4 \) is the temperature of the CMB at the present day, and \( T_e \) is the temperature of the free electrons. At every time-step the temperature decrease at every point \( i \) due to Compton cooling is

\[
T_i \rightarrow T_i - \frac{T_i\Delta t}{\tau_{\text{Compton}}}. \tag{16}
\]

### 3.1.9 Other cooling processes

We have assessed the importance of the following cooling processes:
collisional ionization, recombination, dielectronic recombination, collisional excitation and bremsstrahlung (e.g. Theuns et al. 1998). We have found that all of these processes are negligible for \( \delta \lesssim 100 \) in the temperature range of interest to us, \( T \lesssim 10^5 \text{K} \). One should also keep in mind that we assume an instantaneous photoionization and photoheating (equation 7). Rapid energy loss in line excitations and collisional ionization immediately follows and brings the temperature to \( T \lesssim 10^4 \text{K} \). This rapid cooling is implicitly included in the temperature increment \( \Delta T \) in equation (8). The parameter \( \Delta T \) is treated as a free parameter in our model.

### 3.2 The model input

A basic input of the model are the cosmological parameters. We will present results for three variants of the cold dark matter (CDM) cosmological model. The first is the \( \Lambda \)CDM cosmological model of a flat universe with a cosmological constant and a power-law power spectrum of density fluctuations. The second is also a \( \Lambda \)CDM model with similar cosmological parameters but with a running spectral index (RSI) for the power spectrum. The third is an open CDM (OCMD) model, similar to our \( \Lambda \)CDM model but with no cosmological constant term. The main parameters defining these cosmologies are listed in Table 1. The parameters of the \( \Lambda \)CDM and RSI cosmologies are taken from the first-year WMAP data analysis (Spergel et al. 2003). The OCDM model is not currently viable in view of the WMAP measurements and the observations of Type Ia supernovae.

| \( \Omega_0 \) | \( \Omega_{\Lambda} \) | \( \Omega_b \) | \( h \) | \( n \) | \( \sigma_8 \) |
|---|---|---|---|---|---|
| \( \Lambda \)CDM | 0.270 | 0.730 | 0.0463 | 0.72 | 0.99 | 0.90 |
| RSI | 0.268 | 0.732 | 0.0444 | 0.71 | var | 0.84 |
| OCDM | 0.300 | 0.000 | 0.0463 | 0.70 | 1.00 | 0.90 |

Table 1. Parameters of the three cosmological models considered here. The parameters of the power-law \( \Lambda \)CDM and the running spectral index \( \Lambda \)CDM models are taken from Spergel et al. (2003). The open CDM model has zero dark energy component. The parameters \( \Omega_0 \), \( \Omega_{\Lambda} \) and \( \Omega_b \) are, respectively, the mass density (in units of the critical value) of the dark matter, the dark energy (cosmological constant) and the baryon component. The Hubble constant, \( h \), is given in units of 100 \( \text{km s}^{-1} \text{Mpc}^{-1} \), \( n \) is the power spectrum power law, and \( \sigma_8 \) is the rms value of density fluctuations in spheres of radius 8 \( h^{-1} \text{Mpc} \). The spectral index of the RSI model is a function of wavenumber, \( k \), and is given by \( n(k) = n(k_0) + (dn/d\ln k) \ln(k/k_0) \) with \( k_0 = 0.05 \text{ Mpc}^{-1} \) and \( d\ln k = -0.031 \).
supernovae (e.g. Knop et al. 2003; Riess et al. 2004). Nevertheless, we run our Monte Carlo model with this cosmological model for the sake of comparison.

The Monte Carlo model has also to be fed with the input parameters pertaining to the physical processes discussed previously. These parameters are summarized in Table 2. The first parameter in this table, $f_{N_{Q}}$, is the QSO emission factor. In Appendix A we describe our calculation of the QSO emission rate of ionizing photons per unit comoving volume, $N_{Q}(z)$. The calculation is based on the MHR luminosity function. Although we work with the basic shape of the ionized will stay ionized at least for the QSO lifetime. Therefore, the QSO life-time, $t_{Q}$, is the time for which the QSO is active.

$T_{0}(z = 6)$ is the mean temperature at redshift $z = 6$, $\Delta T$ is the temperature increment induced by photoionization of He II, $E$ is the cross-section weighted excess photon energy for hydrogen ionization, $x_{f}(z = 3)$ is the comoving Jeans length at redshift $z = 3$. $b$ is the QSO distribution bias parameter, and $s$ is the scaling function between the QSOs and the ionizing emission distributions.

**Table 2.** The input parameters of the Monte Carlo model: $f_{N_{Q}}$ is the QSO emission rate factor, $t_{Q}$ is the mean QSO lifetime, $T_{0}(z = 6)$ is the initial mean temperature at redshift $z = 6$, $\Delta T$ is the temperature increment induced by photoionization of He II, $E$ is the cross-section weighted excess photon energy for hydrogen ionization, $x_{f}(z = 3)$ is the comoving Jeans length at redshift $z = 3$, $b$ is the QSO distribution bias parameter, and $s$ is the scaling function between the QSOs and the ionizing emission distributions.

| Parameter | Values | Units |
|-----------|--------|-------|
| $f_{N_{Q}}$ | 0.2, 0.3, 0.4, 0.5, 1.0 | |
| $t_{Q}$ | 0, 5, 10, 20, 40, 60 | Myr |
| $T_{0}(z = 6)$ | 1.0, 2.0, 3.0, 4.0, 5.0, 6.0 | $10^{4}$ K |
| $\Delta T$ | 1.0, 1.5, 1.84 | $10^{4}$ K |
| $E$ | 1.27, 2.54, 3.81 | eV |
| $x_{f}(z = 3)$ | 0.05, 0.1, 0.2, 0.3, 0.4 | $h^{-1}$ Mpc |
| $b$ | 1.0, 2.0, 3.0, 5.0, 8.0 | |
| $s$ | const, linear, exp | |

where $X = n_{He}/(n_{H} + n_{He} + n_{e}) \approx 0.07$ is the fraction of particles that participate in the photoionization process from the total number of particles in the gas, $(\epsilon) \approx 1.96 \times 10^{-10}$ erg is the mean energy per photon of photons with energies greater than 4 Ry that are emitted by QSOs, and $\epsilon_{0} = 4$ Ry $\approx 7.82 \times 10^{-11}$ erg is the energy spent per He II ionization.

However, when the ionization fraction is not very high, the electrons produced by photoionization may have a large probability of interacting with He I and lose energy in line excitations and collisional ionization. Furthermore, when secondary electrons are produced, the initial energy must be shared among several electrons, and so the final energy per electron is reduced. Therefore, we also explore lower values of $\Delta T$.

The cross-section weighted photon energy, $E$, determines the photoheating due to the H I and He I ionizing backgrounds, assuming local equilibrium between the background radiation and the recombination processes (equation 14). Following Miralda-Escudé & Rees (1994) we neglect the temperature dependence of $E$ (which is valid for low temperatures, since the energy lost in recombination is then negligible). In the case of pure hydrogen and for QSO spectral energy distribution $L(\nu) \propto \nu^{-1.5}$, Miralda-Escudé & Rees (1994) found $E = 0.28$ Ry $\approx 3.81$ eV. Since $E$ should also include the He II ionizing photon background and since we use $L(\nu) \propto \nu^{-1.8}$ (see Appendix A2), we explore lower values of $E$ as well.

The next parameter, $x_{f}(z = 3)$, is the Jeans length (comoving) at redshift $z = 3$. The Jeans length, $x_{J}$, is defined as the scale over which the dark matter density should be smoothed to yield an estimate of the gas density (see Appendix C). Large values of $x_{J}$ yield smaller fluctuations, which in turn decreases the recombination rate, and also affects the adiabatic cooling. We assume a simple evolution of $x_{J}$ that depends only on redshift:

$$x_{J}(z) = x_{J}(z = 3) \left( \frac{4}{1 + z} \right)^{1/2}. \quad (18)$$

The QSO bias parameter, $b$, determines the QSO distribution from the gas density distribution, and the scaling function, $s$, scales the ionizing emission distribution from the QSO distribution. We used three different scaling functions: constant, linear and exponential (see equations 6a, 6b and 6c).

**4 RESULTS**

We run our Monte Carlo model for a wide range of the input parameters listed in Table 2. For every set of parameters, the Monte Carlo model provides:

(a) the He II actual volume filling factor and the volume filling factor of regions with He II fraction above 0.9 as a function of redshift;

(b) the mean clumping factor for all regions and for regions with He II fraction above 0.9 as a function of redshift;

(c) the mean temperature as a function of redshift for the mean density regions (regions with $|\delta| < 0.2$) and for all regions;

(d) the He II mean optical depth as a function of redshift; and

(e) the IGM equation of state, i.e. the $T - \rho$ relation.

2 Although the Jeans length is in principle specified by the temperature of the IGM, we choose to treat it here as a free parameter of the model. The shape of the Jeans filtering window is quite uncertain – even in the linear regime, it depends non-trivially on the reionization history (e.g. Hui & Gnedin 1997; Nusser 2000). The situation becomes more complicated when non-linear effects become important.
We compare our results with available observational data. These data include (i) the mean temperature in regions of densities around the mean $|\delta| < 0.2$ estimated from nine quasar Lyα spectra (Schaye et al. 2000), and (ii) the He ii mean optical depth, $\tau_{\text{He ii}}(z)$, measured from the spectrum of the quasars HS 1700+64 (Davidsen et al. 1996), Q0302−003 (Dobrzycki & Bechtold 1991; Heap et al. 2000), and HE 2347−4342 (Smette et al. 2002; Zheng et al. 2004).

Unfortunately, the available observational data are very limited. The uncertainties in the mean temperature data are large, while the uncertainties in the mean optical depth data are fairly small. There is a huge scatter in the optical depth measurements around redshift $z = 2.8$ (i.e. a scatter larger than expected on the basis of the reported error bars), and it is therefore not meaningful to use these measurements to compute a mean optical depth. Nevertheless the data still provide useful constraints on the model input parameters.

As a measure of the ‘goodness of fit’ of a given choice of parameters we use a $\chi^2$ statistic. For the temperature data we define $\chi^2_1$ as

$$\chi^2_1 = \sum \frac{(T_{\text{data}} - T_{\text{model}})^2}{\sigma^2_1},$$

where $T_{\text{data}}$ are the measured temperatures in the data and $T_{\text{model}}$ are the model predicted temperatures at the same redshifts as the data. The estimate of the errors in the data, $\sigma_1$ (Schaye et al. 2000), should serve as a general guide only. We also define a $\chi^2$ statistic for the optical depth measurement. We interpret these data as measurements of the local optical depth rather than the mean optical depth. Therefore, we write $\chi^2_1$ as

$$\chi^2_1 = \sum \frac{(\tau_{\text{data}} - \tau_{\text{model}})^2}{(\sigma^2_{\tau_{\text{data}}} + \sigma^2_{\tau_{\text{model}}})},$$

where the $\tau_{\text{data}}$ are the measured optical depths in the data and $\tau_{\text{model}}$ are the model predicted mean optical depths at the same redshifts as the data. The observational errors in the data are $\sigma_{\tau_{\text{data}}}$, and the scatter about the mean optical depth as estimated in the model is $\sigma_{\tau_{\text{model}}} = \langle (\tau - \langle \tau \rangle)^2 \rangle^{1/2}$ (see Fig. 4).

4.1 Results with a power-law $\Lambda$CDM cosmology

We first present results for the $\Lambda$CDM cosmological model (see Table 1). We discuss other cosmologies in Section 4.2.

In general, the best fits of the model to the observations, with $\chi^2_1 \leq 1$ and $\chi^2_1 \leq 1$, are obtained for $f_{\text{fr}} \approx 0.3$, Jeans scale of $x_J(z = 3) \approx 0.2−0.4$ $h^{-1}$ Mpc, and QSO lifetime $t_0 \lesssim 10$ Myr. We found no constraints on the bias parameter $b$, and the scaling function $s$.

The initial mean temperature is $T_{\text{ini}}(z = 6) \lesssim 40000$ K. The temperature increment is $\Delta T \approx 10000−15000$ K, which, as expected, is smaller than the estimate of Miralda-Escudé & Rees (1994), who find $\Delta T \approx 18400$ K for a fully ionized region. This is due to energy losses by line excitation and collisional ionization in partially ionized regions. We found a correlation between the temperature increment, $\Delta T$, and the hydrogen photoionization heating parameter, $E$. For lower temperature increment, $\Delta T \approx 10000$ K, the heating parameter is $1.27 \leq E \leq 3.81$ eV, but for higher temperature increment, $\Delta T \approx 15000$ K, the heating parameter is only $E \approx 1.27$ eV.

Fig. 3 summarizes some of our main results for the $\Lambda$CDM cosmology with parameters (listed in the top left panel) for which the Monte Carlo model yields one of the best fits to the observational data, $\chi^2_1 = 0.76$ and $\chi^2_1 = 0.75$.
Figure 3. Results for the power-law $\Lambda$CDM cosmological model for the choice of the set of parameters as indicated in the figure. This set of parameters gives $\chi^2_T = 0.76$ and $\chi^2_\tau = 0.75$. (a) The He III fraction (see text) as a function of redshift. The solid line is the mean fraction of He III. The dashed line is the volume filling factor of regions with He III fraction above 0.9. (b) The mean clumping factor, $\langle 1 + \sigma^2 \rangle$, as a function of redshift. The solid line is the mean clumping factor. The dashed line is the clumping factor for regions with He III fraction above 0.9. (c) The He II mean optical depth as a function of redshift. The solid line is the mean optical depth calculated from the model. The symbols are measurements of the He II optical depth from the spectrum of the He II Gunn–Peterson effect towards the quasars HS 1700+64 (triangles; Davidsen et al. 1996), Q0302−003 (squares; Dobrzycki & Bechtold 1991; Heap et al. 2000), and HE 2347−4342 (circles; Smette et al. 2002; Zheng et al. 2004). The noise in the line is due to the limited number of points used in the Monte Carlo model. (d) The mean temperature as a function of redshift. The solid line is the mean temperature of the mean density regions, calculated from regions with $|\delta| < 0.2$. The dashed line is the overall mean temperature, calculated from all regions. The circles with error bars are the observed mean temperatures calculated from nine quasar Ly$\alpha$ spectra (Schaye et al. 2000).

is in reasonable agreement with the data. Comparing with panel (a), we see that the temperature curves peak at the redshift for which full reionization is reached. At higher redshifts each He III particle can recombine more than once, resulting in an efficient photoheating by the He II ionizing photons. At lower redshifts, the recombination is greatly reduced and, subsequently, the photoheating rate decreases and adiabatic cooling dominates. At these low redshifts, the decline of the solid lines does not seem to be fast enough to match the observational data. The dashed curves, however, fall nicely among the data points. This difference between the dashed and solid curves is due to the slower adiabatic cooling of regions with $|\delta| < 0.2$, relative to the overall volume-weighted cooling, which is dominated by the expanding low-density regions.

Unsurprisingly we found a tight relation between the shape of the mean temperature curve and the combination of the heating sources $\Delta T$ and $E$. In the case we have investigated so far, with low temperature increment $\Delta T = 10000$ K and moderate hydrogen photoionization heating parameter $E = 2.54$ eV, the temperature rises 5000–6000 K during $3 \lesssim z \lesssim 4$, and never rises above 18 200 K. We also present two other typical cases in Fig. 5. In panel (a) of Fig. 5 we present a case of low $\Delta T = 10000$ K and high $E = 3.81$ eV. In this case the rise in the temperature during reionization is around 7500 K. The high hydrogen photoionization heating parameter does not allow the IGM to cool at higher redshifts, $4 \gtrsim z \gtrsim 6$, and the temperature reaches a maximum around 19 000 K. In panel (b) of Fig. 5 we present a case of high $\Delta T = 15000$ K and low $E = 1.27$ eV. In this case the rise in the temperature during reionization exceeds 8000 K and can reach 11 000 K. The temperature can reach a maximum of 20 000 K.

It is instructive to examine the temperature fluctuations in space. This can be quantified in our Monte Carlo model by computing the rms of the temperature fluctuations, $T_{\text{rms}} = \langle (T - \langle T \rangle)^2 \rangle^{1/2}$. In panel (b) of Fig. 4 we show the temperature rms as a function of redshift. The temperature rms of regions with mean density (the solid lines) is affected mainly by helium photoheating. When the IGM reaches full ionization, the photoheating is not effective any
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Figure 4. The rms values of optical depth and the temperature fluctuations in the power-law ΛCDM cosmological model for the set of parameters indicated in the figure. (a) The He II optical depth rms as a function of redshift. The noise in the curve is due to the limited number of points used in the Monte Carlo model. (b) The rms of temperature fluctuations as a function of redshift. The solid line is the temperature rms in regions with $|\delta| < 0.2$. The dashed line is the overall temperature rms, calculated from all regions. The rms value of the fluctuations in a variable $Z$ is $Z_{\text{rms}} = \langle (Z - \langle Z \rangle)^2 \rangle^{1/2}$.

Figure 5. The mean temperature as a function of redshift in the power-law ΛCDM cosmological model for two sets of parameters as indicated in the panels. The rest of the model parameters are the same for both cases: $f_{N_0} = 0.3$, $t_Q = 0$ Myr, $t_Q = 0$ Myr, $x_J(z = 3) = 0.3$, $b = 5$ and $s = \text{linear}$. The set of parameters of panel (a) gives $\chi^2_T = 0.93$ and $\chi^2_\tau = 0.77$. The set of parameters of panel (b) gives $\chi^2_T = 1.0$ and $\chi^2_\tau = 0.94$. The solid line is the mean temperature of the mean density regions, calculated from regions with $|\delta| < 0.2$. The dashed line is the overall mean temperature, calculated from all regions. The circles with error bars are the observed mean temperature calculated from nine quasar Lyα spectra (Schaye et al. 2000).

The mean and the temperature rms decreases to approximately the same value before ionization.

Finally, we consider the effect of He II ionization on the $T-\rho$ relation. In the absence of He II reionization, we expect a tight $T-\rho$ relation resulting from adiabatic cooling and photoheating (e.g. Hui & Gnedin 1997). We assume in the Monte Carlo model that at $z = 6$ the $T-\rho$ relation is such that $T \propto (1 + \delta)^{2/3}$. To emphasize the effect of patchy helium reionization, we do not include any scatter in the $T-\rho$ relation at $z = 6$.

Fig. 6 presents the results for the same set of parameters as the above. The six panels in the figure correspond to the $T-\rho$ relation at different redshifts. At the bottom of each panel, an effective equation of state of the form $T_{\text{eff}} \propto (1 + \delta)^{\gamma - 1}$ is given. This effective equation of state is derived by the linear least-squares fit between $\log(T)$ and $\log(1 + \delta)$.

In Fig. 6 we see that, during the early stages of reionization, when mostly high-density regions are being ionized, the slope of the $T-\rho$ relation is steepened and the scatter increases significantly. At $z \sim 3-4$ a non-negligible scatter of about 50 per cent is introduced. Around $z = 3$, where the reionization of low-density regions is much more significant, the slope of the temperature–density relation flattens. When the helium in the IGM is fully ionized, the photoionization heating becomes inefficient and the main process affecting the gas temperature is adiabatic cooling. The cooling reduces the scatter, and by $z = 1$ a tight $T-\rho$ relation is restored.

In Fig. 7 we present the $T-\rho$ relation as derived from two sets of observed quasar absorption-line systems from Bryan & Machacek (2000): APM 08279+5255 at $\langle z \rangle = 3.4$ (left-hand panel) and HS 1946+7658 at $\langle z \rangle = 2.7$ (right-hand panel), and compare with the $T-\rho$ relation from the Monte Carlo model at the same redshifts.

At $z = 2.7$ the slope of the effective equation of state from the model, $\gamma_{\text{model}} = 1.392 \pm 0.002$, fits the slope of the data from the HS 1946+7658 absorption lines, $\gamma_{\text{data}} = 1.31 \pm 0.15$, within 0.6σ. At $z = 3.4$ the slope of the effective equation of state from the model, $\gamma_{\text{model}} = 1.748 \pm 0.008$, fits the slope of the data from the APM...
Figure 6. The $T$–$\rho$ relation for the power-law ΛCDM cosmological model. The values of the key parameters are indicated in the figure. The six panels show the temperature as a function of the density contrast $\delta$ at different redshifts. The dots are the local temperature values. $T_{\text{eff}}$ is the effective equation of state assuming $T_{\text{eff}} \propto (1 + \delta)^{\gamma - 1}$ for all points ionized and non-ionized. The effective temperature is calculated from the least-squares linear fit between $\log(T)$ and $\log(1 + \delta)$.

Figure 7. The $T$–$\rho$ relation as derived from two sets of observed absorption-line systems (Bryan & Machacek 2000) and from the Monte Carlo model with the same parameter set as in Fig. 6. The effective equation of state, $T_{\text{eff}} \propto (1 + \delta)^{\gamma - 1}$, is calculated from the least-squares linear fit between $\log(T)$ and $\log(1 + \delta)$. Right panel: HS 1946+7658 at $\langle z \rangle = 2.7$ (large circles); data from the Monte Carlo model at the same redshift (small dots). Left panel: APM 08279+5255 at $\langle z \rangle = 3.4$ (large circles); data from the Monte Carlo model at the same redshift (small dots).

08279+5255 absorption lines, $\gamma_{\text{data}} = 1.55 \pm 0.14$, within 1.4$\sigma$. The coefficient of the effective equation of state from the model fits the data from the observations within 1.2$\sigma$ at $z = 2.7$ and within 0.3$\sigma$ at $z = 3.4$. In Fig. 8 we present the slopes, $\gamma$, of the effective equation of state at the mean density of the nine quasar Ly$\alpha$ spectra studied by Schaye et al. (2000, open circles) and the slopes calculated from the Monte Carlo model from regions with $|\delta| < 0.2$ at the same redshift.
redshifts (filled circles). As expected, the reheating of the gas at the time of reionization, \(z \sim 3-4\), causes a decrease in \(\gamma\). After the reionization, the IGM gradually evolves again towards an ionization equilibrium and \(\gamma\) increases. To compare the slopes from the observational data with those from the model, we calculated \(\eta\), the distance in \(\sigma\) between the two \(\gamma\):

\[
\eta = \frac{|\gamma_{\text{data}} - \gamma_{\text{model}}|}{\sqrt{\sigma_{\text{data}}^2 + \sigma_{\text{model}}^2}},
\]

where \(\gamma_{\text{data}}\) and \(\sigma_{\text{data}}\) are the observational data slopes and their 1\(\sigma\) errors, and \(\gamma_{\text{model}}\) and \(\sigma_{\text{model}}\) are the Monte Carlo model slopes and their 1\(\sigma\) errors (see Table 3). From Fig. 8 one can see that in general \(\gamma\) from the Monte Carlo model have higher values than \(\gamma\) from the observational data, although for most redshifts \(\eta < 3\) (see Table 3).

4.2 Results with the RSI and OCDM cosmologies

We have also run our Monte Carlo model for the RSI and the OCDM cosmological models (see Table 1) and have tried to fit the model parameters to the observations of the mean optical depth and mean temperature. We found no major differences among the results in the three cosmological models we consider.

As in the ΛCDM cosmology, by fitting the observations with \(x_1^2 \leq 1\) and \(x_2^2 \leq 1\), we get \(f_{\theta_0} \approx 0.3\) for both the RSI and the OCDM models. The QSO lifetime in the RSI and the OCDM models is \(t_0 \leq 5\) Myr. A Jeans scalelength of \(x_1(z = 3) \approx 0.1-0.4 h^{-1} \text{Mpc}\) fits the RSI model, while \(x_1(z = 3) \approx 0.2-0.4 h^{-1} \text{Mpc}\) fits the ΛCDM model, and only \(x_1(z = 3) \approx 0.4 h^{-1} \text{Mpc}\) fits the OCDM model. While in the ΛCDM the bias parameter is not constrained, we find \(b \approx 2-8\) and \(b \approx 2-3\) for the RSI and OCDM models, respectively. As in the ΛCDM cosmology, we found no constraints on the scaling function (equation 6). In the RSI and OCDM cosmological models, the initial mean temperature is \(T_{\text{TH}}(z = 6) \leq 30,000\) K, and temperature increment of \(\Delta T \approx 10,000-15,000\) K. For \(\Delta T \approx 10,000\) K, the hydrogen photoionization heating parameter varies between 2.54 \(\lesssim E \lesssim 3.81\) eV, while in the ΛCDM model \(1.27 \lesssim E \lesssim 3.81\) eV. In all cosmological models, the heating parameter is 1.27 eV for \(\Delta T \approx 15,000\) K.

5 DISCUSSION

We have presented a model for following the thermal properties of the IGM during patchy He II reionization. The model assumes that radiation from QSOs is the main cause of He II reionization and neglects the contribution from galaxies. This is consistent with our finding that the model can account for He II reionization in a way consistent with the available observations of the mean temperature and He II optical depth. Further, according to the GALFORM semi-analytic model for galaxy formation, the contribution of galaxies to the He II ionizing radiation is negligible compared to QSOs even at redshifts as high as \(z \sim 6\). We have also neglected contributions from thermal emission in shock-heated gas (Miniati et al. 2004) and also from redshifted soft X-rays produced at \(z \sim 20\) (Ricotti & Ostriker 2004). These contributions could be important for He II, especially at \(z > 3\), and will be studied in detail in a future paper.

We have considered three different cosmological models — a standard power-law ΛCDM, a ΛCDM with a running spectral index (RSI), and an open CDM — and found no significant differences between them. Table 4 lists the input parameters for which the Monte Carlo model matches best the observational data.
Table 4. Summary of the best-fitting parameters derived from comparison with the observed optical depth and temperature evolution for $\chi^2_T \leq 1$ and $\chi^2_{\tau} \leq 1$. 'NC' implies no constraints on the parameter in the relevant range.

| Parameter | $\Lambda$CDM | RSI | OCDM | Units |
|-----------|--------------|-----|------|-------|
| $f_{NQ}$  | 0.3          | 0.3 | 0.3  |       |
| $t_Q$     | $\leq 10$    | $\leq 5$ | $\leq 5$ | Myr |
| $T_0(z = 6)$ | $\leq 4.0$ | $\leq 3.0$ | $\leq 3.0$ | $10^4$ K |
| $\Delta T$ | 1.0–1.5     | 1.0–1.5 | 1.0–1.5 | $10^5$ K |
| $E$       | 1.27–3.81    | 1.27–3.81 | 1.27–3.81 | eV |
| $x_f(z = 3)$ | 0.2–0.4   | 0.1–0.4 | 0.4 | $h^{-1}$ Mpc |
| $b$       | NC           | NC  | NC   |       |
| $s$       | NC           | NC  | NC   |       |
| $z_{ion}$ | 2.8–2.9      | 2.8–2.9 | 2.8–2.9 |       |

A key parameter is the emission factor $f_{NQ}$, which multiplies the overall emission rate derived from the MHR luminosity function. In all cosmologies, an emission factor of $f_{NQ} = 1$ yields results that are inconsistent with the relevant observational data (see panels on the right of Fig. 9). For $f_{NQ} = 1$, reionization occurs too early at $z = 4.1$, and shifts the typical behaviour of the mean temperature and optical depth to earlier times. Too low a value for $f_{NQ}$ does not match the data either. For $f_{NQ} = 0.2$ the reionization occurs too late ($z \approx 2.5$), and the mean optical depth does not reach high enough values around $z = 3$ (see panels on the left of Fig. 9). To match the data we need $f_{NQ} \approx 0.3$. This result agrees with several observations (Giroux & Shull 1997; Savaglio et al. 1997; Songaila 1998; Kepner et al. 1999; Smette et al. 2002) and simulations (Theuns et al. 1998; Efstathiou et al. 2000), which suggest a softer UV radiation field than the standard UV background due to QSO emission.

The He III fraction rises modestly at the initial stages of reionization. The rise becomes more rapid as the He III fraction gets close to unity (full reionization), at redshift $z_{ion}$. In the simulations of Sokasian et al. (2002) the full reionization occurs at a redshift of $z \approx 3.8$ (see their fig. 3, models 1–3), while we found that full reionization is never achieved before $z_{ion} = 3$. The reason for this difference is that we tune our model to match the temperature evolution. In our model the temperature increases until full reionization is reached, and declines monotonically afterwards.

The optical depth, for He II Ly$\alpha$ absorption, is $\tau_{He II} \sim 10$ at $z \gtrsim 3.5$ with a slow decline. Around $z_{ion}$ the decline is much more rapid.

![Figure 9](https://example.com/figure9.png)

Figure 9. The filling factor, temperature and optical depth evolution as a function of redshift in a $\Lambda$CDM cosmology for $f_{NQ} = 1.0$ (right panels) and $f_{NQ} = 0.2$ (left panels). For $f_{NQ} = 1.0$ the set of parameters gives $\chi^2_T = 2.3$ and $\chi^2_{\tau} = 7.9$, and for $f_{NQ} = 0.2$ gives $\chi^2_T = 3.2$ and $\chi^2_{\tau} = 0.1$. © 2005 RAS, MNRAS 361, 1399–1414
from $\tau_{\text{He II}} \sim 10$ at $z \sim 3.5$ to $\tau_{\text{He II}} \sim 0.5$ at $z \sim 2.5$. At $z \lesssim 2.5$ the decline is gradual again and reached $\tau_{\text{He II}} \sim 0.3$ at $z = 1.0$. We need more observational data to have better constraints on Monte Carlo model parameters. Sokasian et al. (2002) also compute the mean optical depth in their simulations; however, they show results up to $z = 2.8$. Some of their models also match the observations up to that redshift, but we cannot make a more direct comparison of their models with ours at lower redshifts.

An important application of the model is to study the temperature–density relation in the IGM, Patchy He II reionization is expected to produce a 50 per cent scatter in this relation at $3 < z < 4$ (see also Hui & Haiman 2003). At lower redshifts, when the helium in the IGM becomes fully ionized, photoheating is no longer effective and the main process governing the gas temperature is adiabatic cooling. The scatter is then reduced and a tight $T - \rho$ relation is rapidly established. The increase in the scatter at $3 < z < 4$ should be taken into account in the analysis of the Ly$\alpha$ forest data and can be detected in high-resolution spectra. The temperature–density relation from our Monte Carlo model fits the data from Bryan & Machacek (2000) around $z \sim 3$ reasonably well, but more observational data are needed, especially in low-density regions, to be able to put better constraints on the model.

The model predicts a gradual rise in the mean temperature between $z \sim 4$ and $z \sim 3$. Our results match the observed mean temperature as a function of redshift (Schaye et al. 2000). A possible discrepancy is that adiabatic cooling is not efficient enough to cause a rapid decline in the temperature at $z \lesssim 2$, as the observations indicate. However, the observational situation is still inconclusive. Although the temperature increase inferred from the observations is probably robust (Theuns et al. 2002), the quantitative behaviour is less certain.

There is a weak degeneracy between the temperature increment, $\Delta T$, and the energy, $E$, corresponding to photoheating by the diffuse hydrogen ionizing background. An energy of $E \sim 1.27$ eV works well with the full range $10 000 \lesssim \Delta T \lesssim 15 000$ K. On the other hand, the higher energies $E \sim 2.54–3.81$ eV works only with $\Delta T$ close to $10 000$ K. In any case, the inferred values for $\Delta T$ are consistent with the assumption that, as a result of energy losses in partially ionized regions, the temperature increment must be smaller than the estimates of Miralda-Escudé & Rees (1994) for fully ionized regions.

We have also found that only short QSO lifetimes fit the data: $t_{Q} \lesssim 10$ Myr for the $\Lambda$CDM model and $t_{Q} \lesssim 5$ Myr for the RSI and OCDM models. The initial mean temperature has an upper limit of $T_{0} (z = 6) \sim 40 000$ K for the $\Lambda$CDM model and an upper limit of $T_{0} (z = 6) \approx 30 000$ K for the RSI and OCDM models. The Jeans scalelength of $x_{J} (z = 3) \approx 0.1–0.4 h^{-1}$ Mpc fits the RSI model, while $x_{J} (z = 3) \approx 0.2–0.4 h^{-1}$ Mpc fits the $\Lambda$CDM model, and only $x_{J} (z = 3) \approx 0.4 h^{-1}$ Mpc fits the OCDM model. While in $\Lambda$CDM the bias parameter is not constrained, in RSI $b \approx 2$–8 and in OCDM $b \approx 2$–3. In all cosmologies we found no constraints on the scaling function.

In numerical hydrodynamical simulations of the Ly$\alpha$ forest, He II reionization is often modelled as a result of a uniform ionizing background of photons. This recipe for He II reionization leads to a sudden jump in the mean IGM temperature at about $z \sim 3$. Our findings imply that this recipe is oversimplified and may lead to incorrect conclusions. An incorporation of models similar to ours in this type of simulation is needed.

The distribution of the column densities ratio $\eta = N(\text{He II})/N(\text{H I})$ contains important information on the properties of the ionizing sources (e.g. FUSE observations; Zheng et al. 2004). Therefore, it will be very interesting to look at the distribution of $\eta$ as a diagnostic of the reionization epoch. To do so, one must include H I reionization, and therefore take into account the ionizing radiation from galaxies. It is not trivial to add it to our Monte Carlo model since the model is based on the assumption that the ionizing sources are short-lived, which is incorrect for galaxies. Therefore, adding the ionizing radiation from galaxies requires a different approach, which will be presented in a future paper.

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APPENDIX A: THE QSO IONIZING PHOTON EMISSION RATE

Our calculation of the QSO emission rate of He ii ionizing photons per unit comoving volume is based on Madau et al. (1999, MHR) for hydrogen. First we calculate the QSO emission rate in a flat universe without a cosmological constant ($\Omega_0 = 1$, $\Omega_\Lambda = 0$). Later in Appendix A6, we extend this calculation to fit all cosmologies with any desirable $\Omega_0$ and $\Omega_\Lambda$.

Table A1. Parameters of the double power-law luminosity function (Pei 1995; MHR).

| Parameter | Value |
|-----------|-------|
| $\beta_1$ | 1.64  |
| $\beta_2$ | 3.52  |
| $z_*$      | 1.9   |
| $\zeta$    | 2.58  |
| $\xi$      | 3.16  |
| $M_*(0)$   | $-22.35$ |
| $\log [\phi_s(h_0)/\text{Gpc}^{-3}]$ | 2.95 |

A1 QSO luminosity function

Following MHR, we represent the quasar blue luminosity function ($\lambda_B = 4400 \text{ Å}$) as a double power law:

$$\phi(L, z) = \frac{\phi_s/L_*(z)}{[L_B/L_*(z)]^{\beta_1} + [L_B/L_*(z)]^{\beta_2}},$$  \hspace{1cm} (A1)

where $\beta_1$ and $\beta_2$ are the power-law indices for faint and bright quasars, respectively. The position of the break $L_*(z)$ is

$$L_*(z) = L_*(0)(1+z)^{\alpha - 1} \frac{e^{\zeta z}(1+e^{\zeta z})}{e^{\zeta z} + e^{\xi z}}.$$  \hspace{1cm} (A2)

where

$$L_*(0) = L_0 \times 10^{-0.4(M_0(0) - M_B)} = 4.67 \times 10^{29} \text{ erg Hz}^{-1} \text{ s}^{-1},$$  \hspace{1cm} (A3)

$M_0 = 5.48$ and $L_0 = 3.44 \times 10^{18} \text{ erg Hz}^{-1} \text{ s}^{-1}$ are the Solar magnitude and luminosity, respectively, and the rest of the luminosity parameters can be found in Table A1.

In Table A1, $\phi_s$ is given for a Hubble constant $h = 0.5$. From this value we calculate $\phi_s(h) = \phi_s(0) \times (h/0.5)^3 \times 10^{-7} \text{ Mpc}^{-3}$.  \hspace{1cm} (A4)

A2 The QSO spectral energy distribution

The luminosity spectrum of a ‘typical’ QSO is assumed to have a power-law spectral energy distribution (SED), $L(v) \propto v^{-\alpha}$, with different slopes in different wavelength ranges (MHR):

$$L(v) \propto \begin{cases} v^{-0.3}, & \text{if } 2500 \text{ Å} < \lambda < 4400 \text{ Å}, \\ v^{-0.8}, & \text{if } 1050 \text{ Å} < \lambda < 2500 \text{ Å}, \\ v^{-1.8}, & \text{if } \lambda < 1050 \text{ Å}. \end{cases}$$  \hspace{1cm} (A5)

The QSO ionizing flux, $S$, is the number of ionizing photons emitted by a QSO per second,

$$S = \int_0^\nu L(v) \frac{dv}{h v},$$  \hspace{1cm} (A6)

where $h$ is the Planck constant. For convenience we represent $S$ as a function of wavelength, $\lambda$, and since the ionization wavelength of helium ($\lambda = 228$ Å) is less than 1050 Å,

$$S = \frac{L_B}{1.8 h} \left( \frac{4400}{2500} \right)^{-0.3} \left( \frac{2500}{1050} \right)^{-0.8} \left( \frac{1050}{\lambda} \right)^{-1.8} \left|_{228} \right. \text{ Å}^{-1},$$  \hspace{1cm} (A7)

where $L_B$ is the luminosity in the blue band (in units of erg Hz$^{-1}$ s$^{-1}$), and neglecting ionization of heavier elements.
A3 QSO ionizing flux function

Equations (A1) and (A7) allow us to write the number of emitted ionizing photons per unit time per unit (comoving) volume as

$$\Phi(S, z) = \frac{\phi_s/S(z)}{[S/S(z)]^{1/5} + [S/S(z)]^{1/6}},$$

where $S_3(z)$ for helium ionization is

$$S_3(z) = \frac{L_\nu(z)}{1.8h} \left( \frac{4400}{2500} \right)^{0.3} \left( \frac{2500}{1050} \right)^{0.8} \frac{(1050)}{\lambda} \left| \frac{228 \text{ Å}}{0.8} \right| = 1.057 \times 10^{42} (1 + z)^{1.8} \text{ photon s}^{-1}. \quad (A9)$$

A4 $k$-correction

Taking into account the wavelength dependence of the power $\alpha_\nu$ in the QSO spectrum, one should replace the $k$-correction $(1+z)^{\alpha_\nu}$ in equation (A9) with

$$K_k(z) = \begin{cases} (1+z)^{0.3-1}, & \text{if } 2500 \text{ Å} < 4400 \frac{\text{Å}}{1+z} < 4400 \text{ Å}, \\ K_1(1+z)^{0.8-1}, & \text{if } 1050 \text{ Å} < 4400 \frac{\text{Å}}{1+z} < 2500 \text{ Å}, \\ K_1 K_2(1+z)^{0.4-1.8}, & \text{if } 4400 \frac{\text{Å}}{1+z} < 1050 \text{ Å}, \end{cases} \quad (A10)$$

where

$$K_1 = \left( \frac{4400}{2500} \right)^{0.3-0.8} \text{ and } K_2 = \left( \frac{4400}{1050} \right)^{0.4-1.8}. \quad (A10)$$

A5 The QSO emission rate of ionizing photons per unit comoving volume

The QSO emission rate of ionizing photons per unit comoving volume can be written as

$$N_{Q}(z) = \int_{\Delta \nu} \Phi(S, z) S \, dS,$$

where $S_{min}$ = 0.018 $S_3$(0) corresponds to $M_B$ $\approx$ $-18$, and therefore $L_{min} \approx 0.018 L_{4}(0)$ (Cheng et al. 1985). Using equation (A8) we write $N_{Q}(z)$ as

$$N_{Q}(z) = hS/S(z) \int_{\Delta \nu} x \, dx \int_{0.018} \frac{x^{1/3} + x^{2/3}}{x^{1/3} + x^{2/3}}. \quad (A12)$$

where $x \equiv S/S(z)$. Numerical integration gives

$$\int_{0.018} \frac{x^{1/3} + x^{2/3}}{x^{1/3} + x^{2/3}} = 2.298. \quad (A13)$$

Equations (A12), (A4) and (A9) yield the QSO emission rate of helium-ionizing photons:

$$N_{Q}(z) = 2.16 \times 10^{49} K_k(z) \frac{e^{\nu/1050}}{e^{\nu/1050} + e^{\nu/2500}} \left( \frac{h}{0.5} \right)^3 \text{ Mpc}^{-3}. \quad (A14)$$

A6 The cosmological factor

Until now we have assumed a flat universe cosmology with $\Omega_0 = 1$ and $\Omega_\Lambda = 0$. To obtain $N_{Q}(z)$ in different cosmologies, we multiply the above result by a cosmological factor $f_{\cosmo}(\Omega_0, \Omega_\Lambda)$ so that

$$N_{Q}(z) \mapsto f_{\cosmo}(\Omega_0, \Omega_\Lambda)N_{Q}(z). \quad (A15)$$

where

$$f_{\cosmo}(\Omega_0, \Omega_\Lambda) = \frac{\text{dr}(1.0)/dz}{\text{dr}(\Omega_0, \Omega_\Lambda)/dz} = \sqrt{(1 - a)(1 - a + a^3 - a \Omega_\Lambda + a)}, \quad (A16)$$

and $\text{dr}/dz$ is the rate of change of comoving distance with redshift.

APPENDIX B: THE NUMBER OF ABSORBED PHOTONS PER REDSHIFT INTERVAL BIN

We write the total number of ionizing photons that were absorbed at time interval $[t, t + \Delta t]$ as

$$N_{abs}(z) = [N_0(z) \Delta t + N_{in}] \left[ 1 - e^{-\Delta t/\tau_{\cosmo}(z)} \right]. \quad (B1)$$

where $N_0(z)$ is the QSO ionizing photon emission rate per unit comoving volume, $\Delta t$ is the time-step, $N_{in}$ is the number of photons received from previous time intervals, and the mean free path of the He II ionizing photons is

$$l_{\text{ion}}(z) = \left\{ \frac{n_{\text{He II}}[1 + \delta(z)](1 + z)^3 \sigma_{\text{ion}}}{\text{He II}} \right\}^{-1}, \quad (B2)$$

where $n_{\text{He II}}$ is the mean comoving number density of He II, $\delta(z)$ is the local density perturbation, and $\sigma_{\text{ion}}$ is the mean ion–photon cross-section for He II.

The ion–photon cross-section at frequency $\nu$ is

$$\sigma_{\text{ion}}(\nu) = 8.61 \times 10^{-18} \text{ cm}^2 \left( \frac{\nu}{10^{19} \text{ Hz}} \right)^{-4} \frac{e^{-4 \arctan \nu/10^{19} \text{ Hz}}}{1 - e^{-4 \arctan \nu/10^{19} \text{ Hz}}}, \quad (B3)$$

where $\epsilon = (\nu/\nu_{\text{He II}} - 1)^{-1/2}$, and $\nu_{\text{He II}} = 1.3 \times 10^{16}$ Hz is the frequency at the He II ionization energy, $E = 54.42$ eV (Cen 1992).

Following Theuns et al. (1998) we approximate the Cen (1992) ion–photon cross-section as having simple power-law dependence on frequency:

$$\sigma_{\text{ion}}(\nu) = 1.9 \times 10^{-18} \text{ cm}^2 \left( \frac{\nu}{10^{19} \text{ Hz}} \right)^{-3}. \quad (B4)$$

B1 Frequency binning

Because the frequency dependence of the cross-section, we divide the frequency interval of ionizing photons into 10 bins, from $\nu_{\text{He II}}$ to infinity, and calculate the total number of absorbed photons in each bin. We define the luminosity as the photon energy rate per frequency (in units of erg Hz$^{-1}$ s$^{-1}$). The luminosity of a ‘typical’ QSO at wavelengths $\lambda < 1050$ Å is assumed to behave as $L(\nu) \propto \nu^{-1.8}$ (equation A5). To have equal luminosity in each bin, we choose the frequency at the edge of every bin $j$ to be

$$\nu_{j+1}^{1.8} = \nu_{1050} \left( \frac{\nu_{j}^{1050}}{\nu_{1050}} \right)^{-1.8} - f_B \left( \frac{\nu_{\text{He II}}}{\nu_{1050}} \right)^{-1.8}, \quad (B5)$$

where $\nu_{1050} = 2.85 \times 10^{15}$ Hz is the frequency at wavelength $\lambda = 1050$ Å, and $f_B = 0.1$ is the reciprocal of the number of bins. For each bin we calculated the mean ion–photon cross-section

$$\sigma_{\text{ion}}^j = \sigma_{\text{ion}} \frac{1}{\nu_{j+1}^{1050}} \left( \frac{\nu}{\nu_{1050}} \right)^{-1.8} \frac{d\nu}{\nu_{j}^{1050}} \left( \frac{\nu}{\nu_{1050}} \right)^{-1.8} \text{ d}v, \quad (B6)$$

and the QSO ionizing photon emission rate

$$N_j(z) = f_B N_0(z). \quad (B7)$$
B2 The number of absorbed photons

To get more accurate results for the radiative transfer, we first calculate the number of redshifted photons that were transferred to a lower frequency bin. Next, since we choose 10 frequency bins, we solve a set of 10 differential equations for the photon transmission,

\[
\frac{dN_{\text{trs}}^i}{dt} = \left[ N_0(z) - n_{\text{HeII}} - \sum_{j=1}^{10} N_{\text{trs}}^j(z) \right] c \sigma_{\text{ion}} N_{\text{trs}}^i(z),
\]

(B8)

where \(N_0(z) = \sum_{j=1}^{10} N_{\text{trs}}^j(z) \Delta t + N_{\text{trs}}^i(z)\) is the total number of photons that are available for ionization and \(N_{\text{trs}}^i\) is the previous number of transmitted photons in bin \(j\). The actual number of the absorbed ionizing photons is

\[
N_{\text{abs}}(z) = N_0(z) - \sum_{j=1}^{10} N_{\text{trs}}^j(z),
\]

(B9)

where here \(N_{\text{trs}}^j\) is the current number of transmitted photons in bin \(j\).

APPENDIX C: THE GAS DENSITY FIELD

We assume that the gas density traces the dark matter density smoothed over the Jeans length-scale, \(x_J\), determined by the balance of pressure and gravitational forces. Under this assumption we compute the variance of the gas density field, \(\sigma^2\), from the dark matter non-linear (dimensionless) power spectrum, \(\Delta^2_{\text{NL}}(k)\) and a smoothing window function, \(f_k\).

\[
\sigma^2 = \int \Delta^2_{\text{NL}}(k) |f_k|^2 \frac{dk}{k}.
\]

(C1)

We use the recipe outlined in Peacock (1999) to derive the non-linear power spectrum from the linear one. For the smoothing window we choose a Gaussian function \(f_k = e^{-k_2 x_J^2/2}\) (Hui & Gnedin 1997; Nusser 2000), where \(x_J\) is the Jeans length-scale.

The clumping factor can be defined as

\[
\frac{f_{\text{clump}}}{(\rho)} = \frac{\langle \rho^2 \rangle}{\langle \rho \rangle^2} = (1 + \delta^2) = 1 + \langle \delta^2 \rangle.
\]

(C2)

where \(\rho = \langle \rho \rangle(1 + \delta)\) is the gas density at any point in the Universe, \(\langle \rho \rangle\) is the mean gas density, and \(\delta\) is the density fluctuation.

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