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Epidemic model dynamics and fuzzy neural-network optimal control with impulsive traveling and migrating: case study of COVID-19 vaccination

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Abstract

To suppress the epidemics caused by a virus such as COVID-19, three effective strategies listing vaccination, quarantine and medical treatments, are employed under suitable policies. Quarantine motions may affect the economic systems and pharmaceutical medications may be recently in the developing phase. Thus, vaccination seems the best hope of the current situation to control COVID-19 epidemics. In this work, the dynamic model of COVID-19 epidemic is developed when the quarantine factor and the antiviral factor are established as free variables. Moreover, the impulsive populations are comprehended for traveling and migrating of individuals. The proposed dynamics with impulsive distractions are employed to generate the online data. Thereafter, the equivalent model is developed by using only the daily data of symptomatic infectious individuals and the optimal vaccination policy is derived by utilizing the closed-loop control topology. The theoretical framework of the proposed schemes validates the reduction of symptomatic infectious individuals by using fewer doses of vaccines comparing with the scheduling vaccination.

Key words: COVID-19, Modified SEIAR model, Impulsive migration, Optimal control, Discrete-time systems, Fuzzy rules emulated networks.

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1. Introduction

The COVID-19 epidemic has become a global pandemic rapidly more than other records in the history of RNA viruses outbreaks such as severe acute respiratory syndrome (SARS) in 2002 and middle east respiratory syndrome (MERS) in 2012. At the time this paper has been written, the peak of the third wave is currently observed that has provoked the big wave of economic, civil and health anxieties [1]. By utilizing suitable policies such as vaccination, quarantine and medical treatments, the authority may be able to control the epidemic. Nonetheless, pharmaceutical medications for this coronavirus are still in the developing phase and quarantine may cause the economic issue. In the meantime, the vaccination policy seems to be a worthy option. Unfortunately, the limitation in vaccine production causes an issue because of the high demand [2], especially for the developing countries. Currently, 23.7% of the world population has been fully vaccinated and only 1.3% of people in low-income countries have a chance to get at least one dose of vaccination [3]. Therefore, the expansion of sufficient vaccination policy with predicting the epidemic dynamics and minimizing the usage is essential [4, 5].

Investigating epidemic dynamics by numerical concept, mathematical models have been formulated by nonlinear ordinary differential equations (ODE) to predict the spread of infectious individuals or to evaluate the defense policies [6, 7]. Therefore, the model dynamics can contribute with sufficient information to design the adequate strategies [8, 9], especially for the case of COVID-19 when the abundant data can be simply acquired according to the big-data era [10, 11]. In general, dynamics of epidemics have been derived under the modifications of SEIR model [12, 13] when the individuals have been categorized as follows: Susceptible (S), Exposed (E), Infectious (I) and Recovered (R). It is obvious that the COVID-19 epidemic is an unstable open-loop system with an exponential increase of the infected individuals at the spreading period [14].

Moreover, by considering the epidemic dynamics as the controlled plants, the appropriate defenses i.e. the vaccination policies have been developed such that
the next-generation matrix [15], the compensator controller [16], the optimal controller based on fuzzy fractional derivatives [17], the vaccination based on threshold dynamics [18] and the modified sliding mode control [19]. However, those approaches are model based schemes and comprehensive state variables are generally required [20]. From the practical point of view, it’s very difficult to monitor and collect all states in real-time according to the continuous-time manner [21, 22]. Furthermore, only some states are available at a certain sampling interval [23–25] and the impulsive changes caused by immigration should be considered because the coronavirus can spread by the travel and immigrant of people [26, 27]. Resulting, the dynamics of epidemic models can be reconsidered as a class of impulsive control systems (ICS) such that [28–30].

ICS schemes have been proposed by some works for a class of discrete-time systems such that [31, 32] and linear systems [33]. For the optimal–control problems with ICS, they have a few schemes such as a linear-quadratic (LQ) controller [35], adaptive dynamic programming (ADP) [36] and Pontryagin’s maximum principle [26]. For the dynamics with partial knowledge of models, the neural optimal controller [37], the piecewise-constant optimal controller[38], models reducing order [39], neuro-fuzzy inference system [40] and the optimal control based on passivity [41] have been developed with the full state observer and measurement. Moreover, the model dynamics must be well-defined with the appropriate accuracy because of the impulsive data acquisition,[34].

In this work, the conventional SEIR is firstly augmented as the SqvEIAR epidemic model which includes the quarantined individuals and subgroups of ineffectively vaccinated and effectively vaccinated individuals. Thus, general dynamics of SqvEIAR are formulated as a class of discrete-time impulsive control systems when the vaccination policy is represented as the control effort. Secondly, the equivalent model is established by an adaptive network called fuzzy rules emulated network (FREN) on the impulsive axis. The learning law is utilized to improve the model’s performance with the convergent analysis of internal signals. Finally, the optimal vaccination policy is derived by using only the data of symptomatic infectious ($I$) on the impulsive axis when the impulsive
disturbances of susceptible (S), exposed (E), symptomatic infectious (I) and asymptomatic infectious (A) individuals are consolidated to mimic impulsive traveling and migrating. The analysis of closed-loop performance is conducted by the Lyapunov method with the extension of negative control–direction.

The main contributions of this paper are briefly expressed as the followings:

• Unlike SEIR and its modifications in [26, 28, 30], the vaccination is directly employed as the impulsive control input by the proposed SqvEIAR dynamics. Therefore, the equivalent model and the controller are derived according to the impulsive traveling and migrating of individuals.

• By utilizing only the daily data of symptomatic infectious individuals (I) on the impulsive axis, the equivalent model is established and the full state observer, which is generally required such as the works in [22, 24, 26], is completely omitted here.

• The controlled system of SqvEIAR dynamics on the impulsive axis is negative control direction by nature. The proposed controller is directly designed for this case when the closed-loop performance is guaranteed.

The rest of this paper is organized as follows. Section 2 introduces the SqvEIAR dynamics and problem formulation according to the controlled system with vaccination policy. In section 3, the equivalent model is established by utilizing an adaptive network FREN and the daily data of symptomatic infectious individuals. The optimal vaccination policy is derived in section 4 with the performance analysis. Numerical systems and comparison results are provided in section 5 to validate the proposed scheme. Finally, conclusions and future works are given in section 6.

2. Mathematical model and problem formulation

In this section, a mathematical model called SqvEIAR is attained by integrating state variables of quarantined people and vaccinated individuals with the
standard SEIAR. Thereafter, some problems with the impulsive iteration and sampling for the monitoring of symptomatic infectious individuals are clarified.

2.1. Model of SqvEIAR epidemic

A flow diagram of SqvEIAR model is depicted in Fig. 1. In this model, the total population is divided into nine groups, i.e. susceptible $S(t)$, exposed $E(t)$, symptomatic infectious $I(t)$, asymptomatic infectious $A(t)$, recovered $R(t)$, ineffectively vaccinated $V^i(t)$, effectively vaccinated but still unprotected $V(t)$, protected $P(t)$ and quarantined $Q(t)$ individuals. Thus, the dynamic system of

$$\Lambda = \epsilon_L E + (1 - q_L)I + d_L A$$

Figure 1: Flow diagram of the proposed SqvEIAR epidemic model.
SqvEIAR is derived as follows:

\[ \dot{S}(t) = -\beta \Lambda(t) S(t) - [\lambda + \upsilon(t)] S(t), \]
\[ \dot{Q}(t) = \lambda S(t), \]
\[ \dot{V}^i(t) = (1 - e) \upsilon(t) S(t) - \beta \Lambda(t) V^i(t), \]
\[ \dot{V}(t) = e \upsilon(t) S(t) - \beta \Lambda(t) V(t) - \omega V(t), \]
\[ \dot{E}(t) = \beta \Lambda(t) [S(t) + V(t) + V^i(t)] - \tau E(t), \]
\[ \dot{I}(t) = p \tau E(t) - [\alpha + \varphi] I(t) + (1 - z) n A(t), \]
\[ \dot{A}(t) = (1 - p) \tau E(t) - n A(t), \]
\[ \dot{R}(t) = z n A(t) + [\alpha + \varphi] I(t), \]
\[ \dot{P}(t) = \omega V(t), \]  

(1)

where

\[ \Lambda(t) = \epsilon_L E(t) + [1 - q_L] I(t) + d_L A(t), \]  

(2)

and all parameters are described by Table 1.

From the control engineering point of view, in this work, the control input is assigned as the vaccination policy \( \upsilon(t) \) and the output is the number of symptomatic infectious individuals \( I(t) \). Then, the controlled system under our investigation is conceptually rewritten as

\[ \dot{I}(t) = F_c(\upsilon(t), I(t), \Xi(t)), \]  

(3)

where \( F_c(\cdot) \) is an unknown function and \( \Xi(t) \) denotes other states such that \( S(t), Q(t), V^i(t), V(t), E(t), A(t) \) and \( R(t) \). Theoretically, the system in (3) is discretized by Euler approximation with the sampling time \( T_s \) as

\[ I(k + 1) = T_s F_c(\upsilon(k), I(k), \Xi(k)) + I(k), \]
\[ = F_d(\upsilon(k), I(k), \Xi(k)), \]  

(4)

where \( k \) is the sampling–index and \( F_d(\cdot) \) is the unknown nonlinear function. It’s worth to emphasize that \( F_d(\cdot) \) in (4) is not precisely required to design a
| Parameter | Description | Remark |
|-----------|-------------|--------|
| $\beta$   | Transmission rate | According to the initial population |
| $e$       | Vaccine efficacy  | Average efficacy |
| $\omega$  | Progressive rate  | Protected group |
| $\xi$     | Progressive rate  | Infected group |
| $p$       | Fraction         | Exposed group |
| $r$       | Recovery rate    | Infected group |
| $a$       | Antiviral factor | Antiviral therapies |
| $f$       | Fraction         | Infected group |
| $n$       | Recovery rate    | Asymptomatic group |
| $z$       | Fraction         | Asymptomatic group |
| $\lambda$ | Quarantine rate  | Approximated policy $[0,1)$ |
| $\epsilon_L$ | Infectivity reduction | $\Lambda(t)$:Exposed group |
| $q_L$     | Infectivity factor | $\Lambda(t)$:Infected group |
| $d_L$     | Infectivity reduction | $\Lambda(t)$:Asymptomatic group |

Table 1: Descriptions Model’s parameters.

Figure 2: SqvEIAR on Impulsive-axis
model-free controller. Furthermore, in this work, only the relation between the input $v(k)$ and the output $I(k+1)$ is obliged by the format of IF-THEN rules.

Fig. 2 illustrates the concept of SqvEIAR dynamics as the controlled plant mentioned above. By utilizing impulsive sampling interval $\kappa$ and the first sampling–index $\kappa_j$ of the $j$th day as the block diagram in Fig. 2, the vaccination policy $v(k)$ is attended in the impulsive axis as

$$v(k) = \begin{cases} 0 & \text{if } k \neq \kappa_j, \\ v(\kappa_j) & \text{otherwise}. \end{cases}$$ (5)

Therefore, it’s clear that the output $I(\kappa_j)$ can be rewritten as the function of $v(\kappa_j)$ such that

$$I(k) = \begin{cases} F_d(I(k-1), \Xi(k-1)) & \text{if } k \neq \kappa_j, \\ F_d(I_{k-1}) + G_u(v(\kappa_j)) & \text{otherwise}. \end{cases}$$ (6)

where $G_u(-)$ denotes the unknown function which makes a change of $I(k)$ caused by the impulsive input or the vaccination policy $v(\kappa_j)$.

2.2. A class of impulsive controlled plants

By considering SqvEIAR dynamics as the controlled plant in Fig. 2 at the impulsive index $k = \kappa_j$, the input $u(\kappa_j)$ is the vaccination policy $v(\kappa_j)$ and the output state $x(\kappa_j)$ is the number of infected individuals $I(\kappa_j)$. Thus, the system dynamics (6) can be represented as

$$x(k) = F_d \circ F_d \circ \cdots F_d(x(\kappa_j-1) + 1),$$

$$= F_d^n(x(\kappa_j-1) + 1),$$ (7)

where $n_j$ is the number of sampling intervals during $\kappa_{j-1}$ to $\kappa_j$. By using one step-back of (7), it leads to

$$x(k - 1) = F_d^{n_j}(x(\kappa_{j-1})).$$ (8)

Recalling (6), when $k = \kappa_j$, the dynamics (7) can be obtained as

$$x(\kappa_j) = F_d(x_{k-1}) + G_u(u(\kappa_j)).$$ (9)
By substitute $x_{k-1}$ from (8) into (9), it yields

$$x(\kappa_j) = F_d(F_d^n(x(\kappa_{j-1}))) + G_u(u(\kappa_j)),$$

$$= F_T(x(\kappa_{j-1}), u(\kappa_j)),$$

where $F_T(-)$ is the unknown function that represents the dynamics of the controlled plant on the impulsive axis.

Without loss of generality, our problem formulation can be concluded that founding the control policy $u(\kappa_j)$ when the function $F_T(-)$ is unknown and $x(\kappa_j)$, $j = 1, 2, \ldots$ is only the measurable state.

**Remark 1.** It worth to note that the unknown function $F_T(-)$ is only dependent on the state at $\kappa_{j-1}$ and the control effort at $\kappa_j$. Unlike the works on impulsive systems such as [32, 35, 37], the equivalent model developed in this work is required only the state at the previous impulsive axis and the current control effort. That will reduce the number or sampling data.

3. Equivalent model with impulsive sampling

3.1. Equivalent model based on FREN

By employing the dynamic-linearization [44, 45] with the unknown function $F_T(-)$ in (10), there exist functions $f_T(x(\kappa_{j-1}))$ and $g_T(x(\kappa_{j-1}))$ for the affine dynamics such that

$$x(\kappa_j) = f_T(x(\kappa_{j-1})) + g_T(x(\kappa_{j-1}))u(\kappa_j).$$

Furthermore, by utilizing the results in [46] with dynamics (1)-(4), we have

$$|f_T(x(\kappa_{j-1}))| \leq \alpha_f |x(\kappa_{j-1})|,$$

and

$$|g_T(x(\kappa_{j-1}))| \leq \alpha_g |x(\kappa_{j-1})|,$$

where $\alpha_f$ and $\alpha_g$ are positive constants. It’s worth clarifying that the parameter $\alpha_f$ characterizes virus states such that $\alpha_f < 1$ and $\alpha_f > 1$ according to the virus
at decreasing and increasing phases, respectively. The parameter \( \alpha_g \) represents the control gain \( g_T(\cdot) \) in (11). Furthermore, both parameters \( \alpha_f \) and \( \alpha_g \) are unknown and only their existence is required for further analysis.

For the unknown affine dynamics (11), the equivalent model based on FREN can be established as

\[
\hat{x}(\kappa_j) = f_m(x(\kappa_{j-1})) + g_m(x(\kappa_{j-1}))u(\kappa_j),
\] (14)

where \( \hat{x}(\kappa_j) \) is the estimated state. Functions \( f_m(x(\kappa_{j-1})) \) and \( g_m(x(\kappa_{j-1})) \) are derived by FREN’s computation as

\[
f_m(x(\kappa_{j-1})) = \beta_f^T(\kappa_{j-1})\phi(\kappa_{j-1}),
\] (15)

and

\[
g_m(x(\kappa_{j-1})) = \beta_g^T(\kappa_{j-1})\phi(\kappa_{j-1}),
\] (16)

respectively, where \( \phi(\kappa_{j-1}) \) is the membership vector of \( x(\kappa_{j-1}) \) and \( \beta_f(\kappa_{j-1}) \) and \( \beta_g(\kappa_{j-1}) \) are adjustable parameters.

By utilizing (15) and (16), the network architecture is established in Fig. 3 where \( N \) denotes as the number of IF-THEN rules such that:

IF \( x(\kappa_{j-1}) \) is Small (\( \mu_1 \)) THEN \( f_m(x(\kappa_{j-1})) \) should be Small and \( g_m(x(\kappa_{j-1})) \) should be Small.

Thereafter, the learning laws are derived to tune all adjustable parameters \( \beta_{f,g}(\kappa_j) \) with the estimation error \( \hat{e}(\kappa_{j}) \) defined by

\[
\hat{e}(\kappa_{j}) = x(\kappa_{j}) - \hat{x}(\kappa_{j}).
\] (17)

It’s worth remarking that \( x(\kappa_{j}) \) is stationary during \( \kappa_j \) to \( \kappa_{j+1} \). Resulting, the estimation error in (17) is rewritten according to the \( i^{th} \) inner iteration as

\[
\hat{e}(\kappa_{j}, i + 1) = x(\kappa_{j}) - \hat{x}(\kappa_{j}, i + 1),
\] (18)

where

\[
\hat{x}(\kappa_{j}, i + 1) = \beta_f^T(\kappa_{j-1}, i)\phi(\kappa_{j-1}, i) + \beta_g^T(\kappa_{j-1}, i)\phi(\kappa_{j-1}, i)u(\kappa_{j}),
\] (19)
∀i = 0, 1, 2, · · · , i_{max} and \( \hat{x}(\kappa_j, 1) \) is the first estimated state at \( k = \kappa_j \) before initiating the inner iterative learning law.

Let’s define the cost function \( \hat{E}(\kappa_j, i + 1) \) over the \( i^{th} \)-iteration as

\[
\hat{E}(\kappa_j, i + 1) = \frac{1}{2} \hat{e}^2(\kappa_j, i + 1).
\] (20)

Employing the gradient search, the learning law of \( \beta_f(\kappa_j - 1, i) \) is obtained as

\[
\beta_f(\kappa_j - 1, i + 1) = \beta_f(\kappa_j - 1, i) - \eta_f \frac{\partial \hat{E}(\kappa_j, i + 1)}{\partial \beta_f(\kappa_j - 1, i)},
\] (21)

where a positive constant \( \eta_f \) is the learning rate. By utilizing the chain rule according to (18) and (20), we have

\[
\frac{\partial \hat{E}(\kappa_j, i + 1)}{\partial \beta_f(\kappa_j - 1, i)} = \frac{\partial \hat{E}(\kappa_j, i + 1)}{\partial \hat{e}(\kappa_j, i + 1)} \frac{\partial \hat{e}(\kappa_j, i + 1)}{\partial \beta_f(\kappa_j - 1, i)} - \hat{e}(\kappa_j, i + 1) \phi(\kappa_j - 1, i).
\] (22)

Substitution (22) into (21), the learning law for \( \beta_f(\kappa_j - 1, i) \) is obtained as

\[
\beta_f(\kappa_j - 1, i + 1) = \beta_f(\kappa_j - 1, i) + \eta_f \hat{e}(\kappa_j, i + 1) \phi(\kappa_j - 1, i).
\] (23)
By repeating the similar procedure with (21) to (23) with $\beta_g(-)$, we have

$$\beta_g(\kappa_{j-1}, i + 1) = \beta_g(\kappa_{j-1}, i) - \eta_g \frac{\partial \hat{E}(\kappa_j, i + 1)}{\partial \beta_g(\kappa_{j-1}, i)},$$

(24)

where $\eta_g$ is the learning rate. By utilizing the chain rule again, it yields to

$$\frac{\partial \hat{E}(\kappa_j, i + 1)}{\partial \beta_g(\kappa_{j-1}, i)} = \frac{\partial \hat{E}(\kappa_j, i + 1)}{\partial \hat{e}(\kappa_j, i + 1)} \frac{\partial \hat{e}(\kappa_j, i + 1)}{\partial \beta_g(\kappa_{j-1}, i)} = -\hat{e}(\kappa_j, i + 1) \phi(\kappa_{j-1}, i) u(\kappa_j),$$

(25)

Thus, the learning law for $\beta_g(\kappa_{j-1}, i)$ is derived as

$$\beta_g(\kappa_{j-1}, i + 1) = \beta_g(\kappa_{j-1}, i) + \eta_g \hat{e}(\kappa_j, i + 1) \phi(\kappa_{j-1}, i).$$

(26)

To terminate the inner iteration, two conditions are employed as: i: the maximum iteration number $i \geq i_{\text{max}}$ and ii: the limited estimation error $|\hat{e}(\kappa_j, i + 1)| \leq \varepsilon_o$, where $\varepsilon_o$ is a designed parameter.

3.2. Model performance analysis

In general, the performance of learning laws developed under the gradient search is obviously related to the setting of learning rates. The following theorem is employed to select the learning rates $\eta_f$ and $\eta_g$ with the convergence of the estimation error.

Theorem 3.1. For a class of impulsive system dynamics (14), the estimation error (17) of the equivalent model based on FREN (14) is a convergent sequence along with the iteration axis when the learning rates $\eta_f$ and $\eta_g$ are selected by the following conditions:

$$0 \leq \eta_f < \frac{\gamma_f}{\phi_M},$$

(27)

and

$$0 \leq \eta_g < \frac{\gamma_g}{u_M^2 \phi_M},$$

(28)

where $0 < \gamma_f < 1$, $0 < \gamma_g < 1$, $u_M = \max |u(\kappa_j)|$, $\forall j$ and $\phi_M = \sup_{x \in \mathbb{X}} \{(||\phi(x)||^2)\}.$

Proof. By recalling the universal function approximation of FREN in [42], it exists the ideal parameters $\beta_f^*(\kappa_{j-1})$ and $\beta_g^*(\kappa_{j-1})$ such that

$$x(\kappa_j) = \beta_f^T(\kappa_{j-1}) \phi(\kappa_{j-1}, i) + \beta_g^T(\kappa_{j-1}) \phi(\kappa_{j-1}, i) u(\kappa_j) + \varepsilon_s(k),$$

(29)
where \( \varepsilon_s(k) \) is a bounded residue error. During the inner iteration \( i^{th} \), \( \phi(\kappa_{j-1}, i) \) is fixed, thus, the relation in (29) can be rewritten as

\[
x(\kappa_j) = \beta_f^T(\kappa_{j-1})\phi(\kappa_{j-1}) + \beta_g^T(\kappa_{j-1})\phi(\kappa_{j-1})u(\kappa_j) + \varepsilon_s(k),
\]

(30)

By using (29) with (14-16), we have

\[
\hat{e}(\kappa_j, i + 1) = x(\kappa_j) - \hat{z}(\kappa_j, i + 1),
\]

\[
= \beta_f^T(\kappa_{j-1})\phi(\kappa_{j-1}) + \beta_g^T(\kappa_{j-1})\phi(\kappa_{j-1})u(\kappa_j) + \varepsilon_s(k)
\]

\[
= \beta_f^T(\kappa_{j-1}, i)\phi(\kappa_{j-1}) + \beta_g^T(\kappa_{j-1}, i)\phi(\kappa_{j-1})u(\kappa_j) + \varepsilon_s(k),
\]

(31)

where

\[
\tilde{\beta}_f^T(\kappa_{j-1}, i) = \beta_f^T(\kappa_{j-1}) - \beta_f(\kappa_{j-1}, i),
\]

(32)

and

\[
\tilde{\beta}_g^T(\kappa_{j-1}, i) = \beta_g^T(\kappa_{j-1}) - \beta_g(\kappa_{j-1}, i).
\]

(33)

With one step back on \( i^{th} \) iteration, the learning laws in (23) and (26) can be rearranged as

\[
\beta_f(\kappa_{j-1}, i) = \beta_f(\kappa_{j-1}, i - 1) + \eta_f \hat{e}(\kappa_j, i)\phi(\kappa_{j-1}),
\]

(34)

and

\[
\beta_g(\kappa_{j-1}, i) = \beta_g(\kappa_{j-1}, i - 1) + \eta_g \hat{e}(\kappa_j, i)u(\kappa_j)\phi(\kappa_{j-1}),
\]

(35)

respectively. By applying (34) and (35) with (32) and (33), respectively, we obtain

\[
\tilde{\beta}_f^T(\kappa_{j-1}, i) = \beta_f(\kappa_{j-1}) - \beta_f(\kappa_{j-1}, i - 1) - \eta_f \hat{e}(\kappa_j, i)\phi(\kappa_{j-1}),
\]

\[
= \beta_f^T(\kappa_{j-1}, i - 1) - \eta_f \hat{e}(\kappa_j, i)\phi(\kappa_{j-1}),
\]

(36)

and

\[
\tilde{\beta}_g^T(\kappa_{j-1}, i) = \beta_g(\kappa_{j-1}) - \beta_g(\kappa_{j-1}, i - 1) - \eta_g \hat{e}(\kappa_j, i)u(\kappa_j)\phi(\kappa_{j-1}),
\]

\[
= \beta_g^T(\kappa_{j-1}, i - 1) - \eta_g \hat{e}(\kappa_j, i)u(\kappa_j)\phi(\kappa_{j-1}).
\]

(37)
By substituting (36) and (37) into (31), it leads to

\[
\dot{e}(\kappa_j, i+1) = \left[ \tilde{\beta}_f (\kappa_{j-1}, i) - \eta_f \hat{e}(\kappa_j, i) \phi(\kappa_{j-1}) \right]^T \phi(\kappa_{j-1}) + \left[ \tilde{\beta}_g (\kappa_{j-1}, i) - \eta_g \hat{e}(\kappa_j, i) u(\kappa_j) \phi(\kappa_{j-1}) \right]^T \phi(\kappa_{j-1}) u(\kappa_j) + \varepsilon_s(k),
\]

where

\[
\Phi(\kappa_{j-1}) = \dot{\beta}_f (\kappa_{j-1}, i) - \eta_f \hat{e}(\kappa_j, i) \phi(\kappa_{j-1}) + \dot{\beta}_g (\kappa_{j-1}, i) - \eta_g \hat{e}(\kappa_j, i) u(\kappa_j) \phi(\kappa_{j-1}) + \varepsilon_s(k),
\]

Recalling the conditions in (27) and (28), it’s clear that

\[
\Phi(\kappa_{j-1}) \leq \left[ 1 - \eta_f \phi(\kappa_{j-1}) \right] \phi(\kappa_{j-1}) + \frac{\gamma_f}{\phi_M} \phi(\kappa_{j-1}) \phi(\kappa_{j-1}) \leq \phi(\kappa_{j-1}) < 1.
\]

Thus, the estimation error along the \(i\)-iteration can be simplified as

\[
\dot{e}(\kappa_j, i+1) = \Phi(\kappa_{j}) \dot{e}(\kappa_j, i).
\]

To simplify, let’s define

\[
\Phi(\kappa_j) = 1 - \eta_f \phi(\kappa_{j-1}) - \eta_g u^2(\kappa_j) \phi(\kappa_{j-1})^2.
\]

Thus, the estimation error along the \(i\)-iteration axis with the proposed learning algorithm. The proof is completed.

\[
\Phi(\kappa_j) \leq \left[ 1 - \frac{\gamma_f}{\phi_M} \phi(\kappa_{j-1}) \right] \phi(\kappa_{j-1}) + \frac{\gamma_g}{u^2_M \phi_M} u^2(\kappa_j) \phi(\kappa_{j-1}) \phi(\kappa_{j-1}) \leq \phi(\kappa_{j-1}) < 1.
\]
Remark 2. For the controllable systems, it’s required that $|g_m(x_{\kappa_{j-1}})| \neq 0$. This is one of our advantages of the proposed equivalent model when the condition in (43) is always satisfied. Furthermore, the evidence will be demonstrated in the section on numerical results.

4. Impulsive optimal controller for vaccination policy

In this section, the optimal vaccination policy for SqvEIAR dynamics is derived on the impulsive axis. To simplify, the impulsive index $\kappa_j$ in (10) is solely noted as $j$. Thus, the controlled plant (10) can be simply rewritten as

$$x(j) = F_T(x(j-1), u(j)),$$

and the equivalent model in (14) can be also simplified as

$$\dot{x}(j) = f_m(x(j-1)) + g_m(x(j-1))u(j).$$

Let’s define the unity function $r(j)$ as

$$r(j) = \gamma_x x^2(j-1) + \gamma_u u^2(j),$$

where $\gamma_x$ and $\gamma_u$ are positive constants. Thereafter, the long term cost function $J(j)$ is given as

$$J(j) = r(j) + \gamma r(j+1) + \gamma^2 r(j+2) + \gamma^3 r(j+3) + \cdots,$$

$$= r(j) + \gamma J(j+1),$$

where $0 < \gamma < 1$ is a discount factor. The optimal solution can be obtained when $\frac{\partial J(j)}{\partial u(j)} = 0$. By using (46) and (47), it yields

$$\frac{\partial J(j)}{\partial u(j)} = \frac{\partial r(j)}{\partial u(j)} + \gamma \frac{\partial J(j+1)}{\partial u(j)},$$

$$= 2\gamma_u u(j) + \gamma \left[ \frac{\partial J(j+1)}{\partial r(j+1)} \frac{\partial r(j+1)}{\partial x(j)} \frac{\partial x(j)}{\partial u(j)} \right],$$

$$= 2\gamma_u u(j) + \gamma \left[ 2\gamma_x x(j) \frac{\partial x(j)}{\partial u(j)} \right].$$

(48)
By setting $\frac{\partial J(j)}{\partial u(j)} = 0$, the ideal-optimal control law $u^*(j)$ is derived as

$$u^*(j) = -\gamma \frac{\gamma_x}{\gamma_u} x(j) \frac{\partial x(j)}{\partial u(j)},$$

(49)

When the function $F_T(-)$ in (44) is unknown, it’s obvious that $\frac{\partial x(j)}{\partial u(j)}$ cannot be determined directly because of the unknown function $F_T(-)$ in (44). Furthermore, the relation of $x(j)$ with respect to $u(j)$ also leads to the causality problem according to the diagram in Fig. 2.

To utilize the control law based on (48) and (49), the equivalent model (45) is employed as $x(j) \rightarrow \hat{x}(j)$. By recalling (45), it leads to

$$\frac{\partial x(j)}{\partial u(j)} \approx \frac{\partial \hat{x}(j)}{\partial u(j)} = g_m(x(j - 1)).$$

(50)

Thus, the relation in (48) can be rearranged as

$$\frac{\partial J(j)}{\partial u(j)} = 2\gamma_u u(j) + \gamma \left[2\gamma_x f_m(x(j - 1)) + g_m(x(j - 1))u(j)\right] g_m(x(j - 1)),$$

$$= 2 \left\{ \gamma_u + \gamma \gamma_x g_m^2(x(j - 1))u(j) + \gamma \gamma_x g_m(x(j - 1))f_m(x(j - 1)) \right\}.$$  (51)

By setting $\frac{\partial J(j)}{\partial u(j)} = 0$, thus, the control law is obtained as

$$u(j) = -\frac{\gamma \gamma_x g_m(x(j - 1))}{\gamma_u + \gamma \gamma_x g_m^2(x(j - 1))} f_m(x(j - 1)).$$

(52)

It’s clear that the proposed control law in (52) is a model free controller utilizing under the estimated functions $f_m(-)$ and $g_m(-)$ from FREN. Furthermore, only the data of infected individuals on the impulsive axis $x(j) = I(\kappa_j)$ is required. Unlike the works in [26, 41], the full-state observers are strictly demanded at all sampling intervals $k$ as real-time monitoring.

Therefore, the closed-loop performance is guaranteed by employing the Lyapunov method and and algebraic inequality procedures as the following theorem.

**Theorem 4.1.** By utilizing the vaccination policy $\nu(\kappa_j)$ on the impulsive axis via the control law (52) for the SqvEIAR dynamics in (1), the closed-loop performance is guaranteed under the number of infected individuals when the designed parameters are given as the following conditions:
i: \( \gamma_u \) in (46) is defined as a time varying parameter \( \gamma_u(j - 1) \) such that
\[
\gamma_u(j - 1) = \alpha_u g_m(x_{j-1}),
\]
where \( \alpha_u \) is a positive constant and

ii: \( \gamma_x \) and \( \gamma \) in (46) and (47), respectively, are satisfied
\[
\frac{\gamma \gamma_x}{\alpha_u + \gamma \gamma_x} \leq \frac{1 - \alpha_f g_m^{\text{min}}}{\alpha_g f_m^{\text{max}}}. \tag{54}
\]

Proof. By employing the control law (52) with the dynamics (11), it yields
\[
x(j) = f_T(x_{j-1}) - \frac{\gamma \gamma_x g_m(x_{j-1}) g_T(x_{j-1})}{\gamma_u + \gamma \gamma_x g_m^2(x_{j-1})} f_m(x_{j-1}). \tag{55}
\]

Let’s define the Lyapunov candidate function \( L(j) \) as
\[
L(j) = |x(j)|. \tag{56}
\]

Thus, the change of (56) can be obtained as
\[
\Delta L(j) = L(j) - L(j - 1),
\]
\[
= |x(j)| - |x(j - 1)|,
\]
\[
= |f_T(x_{j-1}) - \frac{\gamma \gamma_x g_m(x_{j-1}) g_T(x_{j-1})}{\gamma_u + \gamma \gamma_x g_m^2(x_{j-1})} f_m(x_{j-1})| - |x(j - 1)|,
\]
\[
= |f_T(x_{j-1}) - \alpha_o(x_{j-1}) f_m(x_{j-1}) g_T(x_{j-1})| - |x(j - 1)|,
\]
\[
\leq |f_T(x_{j-1})| + |\alpha_o(x_{j-1}) f_m(x_{j-1}) g_T(x_{j-1})| - |x(j - 1)|, \tag{57}
\]
where
\[
\alpha_o(x_{j-1}) = \frac{\gamma \gamma_x g_m(x_{j-1})}{\gamma_u + \gamma \gamma_x g_m^2(x_{j-1})}. \tag{58}
\]

Recalling (12) and (13), we have
\[
\Delta L(j) \leq \alpha_f |x(j - 1)| + |\alpha_o(x_{j-1}) f_m(x_{j-1})| - |x(j - 1)|,
\]
\[
\leq \left[ \alpha_f + \alpha_g |\alpha_o(x_{j-1}) f_m(x_{j-1})| - 1 \right] |x(j - 1)|. \tag{59}
\]

In order to obtain \( \Delta L(j) \leq 0 \), it’s required that
\[
\alpha_f + \alpha_g |\alpha_o(x_{j-1}) f_m(x_{j-1})| \leq 1, \tag{60}
\]

or
\[ |\gamma_o(x_{j-1})f_m(x_{j-1})| \leq \frac{1 - \alpha_f}{\alpha_g}. \]  
(61)

By using \( \gamma_o(x_{j-1}) \) (58) and \( \gamma_u \) in (53), it yields
\[ \left| \frac{\gamma \gamma x g_m(x_{j-1})}{\gamma_u + \gamma \gamma x g_m(x_{j-1})} f_m(x_{j-1}) \right| \leq \frac{1 - \alpha_f}{\alpha_g}, \]
\[ \left| \frac{\gamma \gamma x g_m(x_{j-1})}{\alpha_u g_m^2(x_{j-1}) + \gamma \gamma x g_m^2(x_{j-1})} f_m(x_{j-1}) \right| \leq \frac{1 - \alpha_f}{\alpha_g}, \]
\[ \left| \frac{\gamma \gamma x}{\alpha_u + \gamma \gamma x g_m(x_{j-1})} f_m(x_{j-1}) \right| \leq \frac{1 - \alpha_f}{\alpha_g}. \]  
(62)

The designed parameters \( \gamma, \gamma_x \) and \( \alpha_u \) are positive constants, thus, the relation in (62) can be rearranged as
\[ \frac{\gamma \gamma x}{\alpha_u + \gamma \gamma x} < \frac{1 - \alpha_f}{\alpha_g} \left| \frac{g_m(x_{j-1})}{f_m(x_{j-1})} \right|. \]  
(63)

or
\[ \frac{\gamma \gamma x}{\alpha_u + \gamma \gamma x} \leq \frac{1 - \alpha_f}{\alpha_g} \frac{g_{\text{min}}}{f_{\text{max}}} \leq \frac{1 - \alpha_f}{\alpha_g} \left| \frac{g_m(x_{j-1})}{f_m(x_{j-1})} \right|. \]  
(64)

That fulfills the condition in (54). Thus, the proof is completed. \( \square \)

**Remark 3.** To acquire the parameters such as \( \alpha_f, \alpha_g, f_{\text{max}} \) and \( g_{\text{min}} \), the design engineers can determine it by their experience according to the controlled plant or monitoring the response which will be demonstrated by the section of numerical results.

By considering SqvEIAR dynamics with the vaccination policy, it’s obvious that the infected individual should be decreased by increasing the vaccination. Thus, in this case, the controlled plant has a negative control direction. Therefore, the analysis of Theorem 4.1 is extended by the following Lemma.

**Lemma 4.1.** For the case of negative control direction of SqvEIAR dynamics (1) according to the impulsive vaccination \( \nu(k_j) \) (52), the closed-loop performance under Theorem 4.1 is still valid via the following condition:
\[ 0 < \frac{\alpha_f - 1}{\alpha_g} \frac{g_{\text{min}}}{f_{\text{max}}} \leq \frac{\gamma \gamma x}{\alpha_u + \gamma \gamma x} \leq \frac{\alpha_f + 1}{\alpha_g} \frac{g_{\text{min}}}{f_{\text{max}}}. \]  
(65)
Proof. Let’s define the Lyapunov function $\mathcal{L}(j)$ as

$$
\mathcal{L}(j) = x^2(j).
$$

Recalling dynamics in (55) and (58), the change of Lyapunov function $\Delta \mathcal{L}(j)$ is derived as

$$
\Delta \mathcal{L}(j) = x^2(j) - x^2(j - 1),
$$

$$
= [f_R(x_{j-1}) - \gamma_\alpha(x_{j-1})f_m(x_{j-1})g_R(x_{j-1})]^2 - [x(j-1)]^2,
$$

$$
= -2\gamma_\alpha(x_{j-1})f_m(x_{j-1})f_R(x_{j-1}) + f_R^2(x_{j-1})
$$

$$
+ \gamma_\alpha^2(x_{j-1})f_m^2(x_{j-1})g_R^2(x_{j-1}) - x^2(j - 1).
$$

According to the SqvEIAR dynamics in (1) and (11) and the equivalent model in (14), it’s clear that $I(k)$ is always positive or $x(\kappa_j) \geq 0$ leading to $f_m(x_{j-1}) \geq 0$ and $f_R(x_{j-1}) \geq 0$. Thereafter, by using (12) and (13), we obtain

$$
\Delta \mathcal{L}(j) \leq 2\gamma_\alpha(x_{j-1})f_m(x_{j-1})\alpha_f\alpha_gx^2(j - 1) + \alpha_f^2x^2(j - 1)
$$

$$
+ \alpha_g^2\gamma_\alpha^2(x_{j-1})f_m^2(x_{j-1})x^2(j - 1) - x^2(j - 1),
$$

$$
\leq \left[\alpha_f^2 + 2\gamma_\alpha(x_{j-1})f_m(x_{j-1})\alpha_f\alpha_g + \alpha_g^2\gamma_\alpha^2(x_{j-1})f_m^2(x_{j-1}) - 1\right]
$$

$$
x^2(j - 1),
$$

$$
\leq \left[\alpha_f + \alpha_g\gamma_\alpha(x_{j-1})f_m(x_{j-1})\right]^2 - 1]x^2(j - 1).
$$

For $\Delta \mathcal{L}(j) \leq 0$, it’s required that

$$
-1 \leq \alpha_f + \alpha_g\gamma_\alpha(x_{j-1})f_m(x_{j-1}) \leq 1,
$$

or

$$
-1 \leq \alpha_f + \alpha_g\gamma_\alpha\gamma_g\gamma_{\gamma_g}\gamma_{g_m}(x_{j-1})f_m(x_{j-1}) - \frac{\gamma_{\gamma_g}\gamma_{g_m}(x_{j-1})f_m(x_{j-1})}{\gamma_{\gamma_g}\gamma_{g_m}(x_{j-1})f_m(x_{j-1})} \leq 1.
$$

By using (53), thus, the relation in (70) is rearranged as

$$
-1 \leq \alpha_f + \alpha_g\gamma_\alpha(\gamma_g\gamma_{\gamma_g}\gamma_{g_m}(x_{j-1})f_m(x_{j-1}) \leq 1,
$$

or

$$
-\frac{1 + \alpha_f}{\alpha_g} \leq \frac{\gamma_\alpha\gamma_g\gamma_{\gamma_g}\gamma_{g_m}(x_{j-1})f_m(x_{j-1})}{(\alpha_f + \gamma_{\gamma_g}\gamma_{g_m}(x_{j-1})} \leq \frac{1 - \alpha_f}{\alpha_g}.
$$
For the case of negative control direction, we have

\[ g_m(x(\kappa_{j-1})) < 0, \quad (73) \]

or

\[ g_m(x_{j-1}) = -|g_m(x_{j-1})|. \quad (74) \]

By using (74), the relation in (72) can be rearranged as

\[ \frac{1 + \alpha_f}{\alpha_g} \frac{|g_m(x_{j-1})|}{f_m(x_{j-1})} \geq \frac{\gamma \gamma_x}{\alpha_u + \gamma_x} \geq -\frac{1 - \alpha_f}{\alpha_g} \frac{|g_m(x_{j-1})|}{f_m(x_{j-1})}, \quad (75) \]

or

\[ \frac{\alpha_f - 1}{\alpha_g} \frac{|g_m(x_{j-1})|}{f_m(x_{j-1})} \leq \frac{\gamma \gamma_x}{\alpha_u + \gamma_x} \leq \frac{\alpha_f + 1}{\alpha_g} \frac{|g_m(x_{j-1})|}{f_m(x_{j-1})}. \quad (76) \]

Thus, the relation in (76) is conclusively conducted as

\[ \frac{\alpha_f - 1}{\alpha_g} \frac{g_{m_{\text{min}}}}{f_{m_{\text{max}}}} \leq \frac{\gamma \gamma_x}{\alpha_u + \gamma_x} \leq \frac{\alpha_f + 1}{\alpha_g} \frac{g_{m_{\text{min}}}}{f_{m_{\text{max}}}}. \quad (77) \]

The proof is completed. \( \square \)

**Remark 4.** The negative control direction of SqvEIAR dynamics under the proposed vaccination policy can be validated by the plot of \( g_m(\kappa_j) \) in Fig. 7. The negative value of \( g_m(\kappa_j) \) is obviously observed along with the operation.

For the conclusion, the Algorithm 1 is given to represent the learning laws for the equivalent model and the determination of the proposed vaccination policy according to the condition (65) of Lemma 4.1.

5. Controller setting and numerical results

In this section, the design of the proposed scheme is demonstrated and the validation results are given by the numerical system of SqvEIAR dynamics (1) altogether with impulsive traveling of individuals.
Algorithm 1: Daily vaccination policy and inner-loop of equivalent model.

**Input:** $I(\kappa_j)$ or $x(\kappa_j)$: Daily sampling

**Output:** $u(\kappa_j)$ or $v(\kappa_j)$: Daily vaccination policy

**Data:** Membership function $\mu_l$ for $l = 1, 2, 3$, Model’s parameters, Controller’s parameters, $\beta_f(\kappa_{j-1}, 0)$, $\beta_g(\kappa_{j-1}, 0)$, $u(\kappa_{j-1})$ and $x(\kappa_{j-1})$.

1. Determine $\phi(\kappa_{j-1}, i)$ in (19) and $\hat{e}(\kappa_j, 1)$ by setting $i = 0$.
2. Keep $\phi(\kappa_{j-1}, 0)$ constant as $\phi(\kappa_{j-1})$.
3. for $i \geq i_{\text{max}}$ or $|\hat{e}(\kappa_j, i + 1)| \leq \varepsilon_o$ do
   4. Determine $\hat{x}(\kappa_j, i + 1)$ by (19).
   5. Determine $\hat{e}(\kappa_j, i + 1)$ by (18).
   6. Update $\beta_f(\kappa_{j-1}, i + 1)$ and $\beta_g(\kappa_{j-1}, i + 1)$ by (23) and (26), respectively.
   7. Set $i := i + 1$.
8. Set $f_m(x(j - 1)) = \beta_f^T(\kappa_{j-1}, i_{\text{final}})\phi(\kappa_{j-1})$ and $g_m(x(j - 1)) = \beta_g^T(\kappa_{j-1}, i_{\text{final}})\phi(\kappa_{j-1})$.
9. if Condition (65) = True then
   10. Determine $u(\kappa_j)$ by (52).
11. else if $j \geq 2$ then
   12. Recall $f_m(x(j - 2))$ and $g_m(x(j - 2))$ and determine $u(\kappa_j)$.
13. else
   14. Recall $f_m(x(j - 1))$ and $g_m(x(j - 1))$ from the previous iteration and determine $u(\kappa_j)$.
| Parameters | Values | Parameters | Values |
|------------|--------|------------|--------|
| $S(1)$     | 8,000  | $e$        | 0.7    |
| $Q(1)$     | 0      | $\omega$  | 0.1    |
| $V(1)$     | 0      | $p$        | 0.1    |
| $E(1)$     | 1,000  | $r$        | 0.3    |
| $I(1)$     | 500    | $a$        | 0.3    |
| $A(1)$     | 500    | $\bar{q}$ | 0.965  |
| $N(1)$     | 10,000 | $n$        | 0.3    |
| $z$        | 0.02   | $\lambda$ | 0.1    |
| $\lambda$ | 0.1    | $\epsilon_L$ | 0       |
| $q_L$      | 0.5    | $d_L$      | 1      |

Table 2: Initial and parameter values.

Figure 4: Membership functions of $I(\kappa_{j-1})$ or $x(\kappa_{j-1})$. 
5.1. Parameters design and setting

Table 2 represents all parameter values of SqvEIAR dynamics in (1) and initial values. Fig. 4 illustrates the membership functions $\mu$ of FREN in Fig. 3 when $N = 3$ and $I(\kappa_{j-1})$ or $x(\kappa_{j-1}) \in [0, 1000]$. It’s worth to denote that, in this work, the setting of the range of $x(\kappa_{j-1})$ is double of $I(1)$ in Table 2.

The vaccination policy is in the range of $[0, 1]$. Thus, the parameter $u_M$ is given as $u_M = \max(v) = 1$. With 3 membership functions, we have $\phi_M = 3$. By recalling the results from Theorem 3.1 where $\gamma_f = 0.7$ and $\gamma_g = 0.7$, thus, the learning rates for $\beta_f$ and $\beta_g$ can be determined as

$$0 \leq \eta_f < \frac{\gamma_f}{\phi_M} = \frac{0.7}{3} = 0.2333,$$  
(78)

and

$$0 \leq \eta_g < \frac{\gamma_g}{u_M^2 \phi_M} = \frac{0.7}{1 \times 3} = 0.2333,$$  
(79)

respectively. In this work, we select $\eta_f = 0.2$ and $\eta_g = 0.2$.

Thereafter, the parameters of the controller are designed according to Theorem 4.1 and Lemma 4.1. The sampling time $k$ is given as $T_s = 0.001$ [day] in this simulation. Let’s simply select $g_m^{\text{min}} = 200$, $f_m^{\text{max}} = 700$, $\alpha_f = 0.7$ and $\alpha_g = 0.7$. It yields $\gamma = 0.7$, $\gamma_x = 0.5$ and $\alpha_u = 0.25$ such that

$$\frac{\gamma \gamma_x}{\alpha_u + \gamma \gamma_x} = 0.5833 \leq \frac{\alpha_f + 1}{\alpha_g} \frac{g_m^{\text{min}}}{f_m^{\text{max}}} = 0.6939.$$  
(80)

Thus, the condition in Lemma 4.1 has been fulfilled.

5.2. Validation results

By testing SqvEIAR dynamics (1) without applying the controller ($v(t) = 0$) and quarantine ($\lambda = 0$), the population of symptomatic infectious individuals $I(\kappa_j)$ is depicted by the A-plot in Fig. 5. Next, the parameter $\lambda$ is give as $\lambda = 0.5$ for quarantining. Thus, the population of $I(\kappa_j)$ is shown by the B-plot in Fig. 5. Thereafter, the C-plot in Fig. 5 represents $I(\kappa_j)$ of SqvEIAR dynamics when applying only the vaccination policy as $v(t) = u(\kappa_j)$ in (52) and $\lambda = 0$. It’s obvious that the peak of symptomatic infectious individuals is definitely reduced.
Figure 5: Plots of symptomatic infectious $I(\kappa_j)$ populations.

Figure 6: Estimated function $f_m(\kappa_j)$. 
Figure 7: Estimated function $g_m(\kappa_j)$.

Figure 8: Vaccination policy $u(\kappa_j)$. 
Fig. 6 displays the plot of $f_m(\kappa_j)$. It’s clear that the setting as $f_m^{\text{max}} = 700$ in Section 5.1 has been validated. The plot of $g_m(\kappa_j)$ is illustrated in Fig. 7. It proves the case of negative control direction regarding to the establishment of Lemma 4.1. Furthermore, it can be observed $|g_m(\kappa_j)|_{\text{min}} = 400$. Thus, the setting as $g_m^{\text{min}} = 200$ in Section 5.1 has been also validated. The vaccination policy is represented by the plot in Fig. 8 and the details are fully illustrated by Fig. 9.

5.3. Results with impulsive traveling and migrating

The effects of impulsive immigrating and traveling are considered in this test. Fig. 10 presents the impulsive moving pattern which contains four groups such that $S(\kappa_j)$, $E(\kappa_j)$, $I(\kappa_j)$ and $A(\kappa_j)$. It’s worth emphasizing that those varying individuals are assumed to be unknown. Thus, the controller generates the vaccination policy $v(t)$ by using the equivalent model only.

To demonstrate the performance of the proposed vaccination policy, the scheduling vaccination policy [18] is firstly utilized with $v(t) = 0.7$ along 4
Figure 10: Impulsive immigrating pattern.

Figure 11: Symptomatic infectious $I(\kappa_j)$ with Impulsive immigrating.
weeks (28 days). Thereafter, the proposed scheme is employed by using the same setting in Section 5.1. Fig. 11 illustrates the plots of $I(\kappa_j)$ with different vaccination policies. The result of no vaccination is firstly considered with the higher number of symptomatic infectious individuals. Afterward, the scheduling vaccination is employed. It’s clear that the number of symptomatic infectious individuals is significantly decreased and 7,389 doses of vaccine are used. Finally, the proposed vaccination policy is utilized. Only 6,478 doses of vaccine are used but the number of symptomatic infectious individuals is obviously reduced according to the result from the scheduling vaccination. Furthermore, the proposed vaccination policy is represented by the plot in Fig. 12. After day 23rd, the vaccination policy is reached zero because the epidemic is under control. On day 25th, it has large immigrating amount of $I(\kappa_j = 25)$. Thus, the vaccination policy is spontaneously increased. That will suggest the authorities gaining control of the epidemic.

Figure 12: Vaccination policy $u(\kappa_j)$ with Impulsive immigrating.
6. Conclusions

In this paper, the optimal vaccinated strategy has been derived by using only the daily data of symptomatic infectious individuals and considering the impulsive immigrants of susceptible, exposed, symptomatic infectious and asymptomatic infectious individuals. SqvEIAR dynamics have been developed to accomplish the vaccination’s effectiveness, antiviral factor and quarantine. By utilizing only the daily data of symptomatic infectious individuals the equivalent model has been established with the impulsive axis. Therefore, the negative control direction of the controlled SqvEIAR dynamics with the vaccination policy has been validated by the negative value of the estimated function $g_m(\kappa_j)$. By employing the proposed vaccination policy, the number of symptomatic infectious individuals has been significantly reduced with the fewer usage vaccines. Furthermore, the adaptive algorithm has validated the fast response according to the impulsive traveling and migrating of individuals.

The optimal vaccination proposed in this work may provide a feasible non-pharmaceutical policy for the authority to control the COVID-19 epidemic. Currently, the vaccines are all in the intensively developing phase and the new variants of the coronavirus are persistently discovered thus the new or updated data will be included to enchant the performance as our upcoming future work.

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