Dark matter kinetic decoupling with a light particle

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Received March 15, 2017
Revised July 25, 2017
Accepted January 8, 2018
Published January 29, 2018

Abstract. We argue that the acoustic damping of the matter power spectrum is not a generic feature of the kinetic decoupling of dark matter, but even the enhancement (overshooting) can be realized depending on the nature of the kinetic decoupling when compared to that in the standard cold dark matter model. We consider a model that exhibits a sudden kinetic decoupling and investigate cosmological perturbations in the standard cosmological background numerically in the model. We also give an analytic discussion in a simplified setup. Our results indicate that the nature of the kinetic decoupling could have a great impact on small scale density perturbations.

Keywords: cosmological perturbation theory, cosmology of theories beyond the SM, dark matter theory

ArXiv ePrint: 1703.02338
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1 Introduction

The nature of dark matter (DM) has not been uncovered regardless of accumulated evidences of its existence from cosmic to galactic scale structures of the Universe (see ref. [1] for a recent review). The precise measurements of the cosmic microwave background anisotropies, for example, do not only support its existence but also precisely determine the DM present mass density [2]. There have been vigorous efforts to identify the nature of DM both in direct and indirect ways. One of such efforts is to seek imprints of DM interactions with other particles like baryons, photons, and neutrinos on the large scale structure of the Universe [3–20].

The imprints attract growing interests also in the context of the small scale problems in the standard ΛCDM model (see ref. [21] for a recent review)\(^1\) where the dark energy (Λ) and cold and only gravitationally interacting DM (CDM) are assumed. The DM interactions result in suppressions of small scale density perturbations, the cutoff scale of which is determined by the horizon scale at the time of the kinetic decoupling [25,26] (see [3,7,27–29] for early estimates of the cutoff scale). The reduced number of subgalactic halos is concordant with the observed number of the dwarf galaxies, which is overpredicted in the standard CDM model [30–42].

Although previous studies considered that the kinetic decoupling leads to a damped oscillation (acoustic damping) of the resultant matter power spectrum, the physics of the kinetic decoupling is not fully understood. For example, it is not clear if the acoustic oscillation is always damped in spite of the nature of the kinetic decoupling. In Paper I [43], we considered that charged massive particles with the lifetime longer than the cosmic time of the recombination account for the DM mass density. It was shown that the resultant matter power spectrum can be enhanced when compared to that in the standard ΛCDM model, depending on the suddenness of the kinetic decoupling. The Coulomb scattering between charged massive particles and baryons becomes inefficient when the electron and positron (\(e^−e^+\)) annihilate. At that time the momentum transfer rate per Hubble time decreases suddenly with the decreasing temperature by the Boltzmann factor of the \(e^±\) number density, which results in the enhancement (overshooting) of the DM density perturbations that enter

\(^1\)It has also been reported that cosmological hydrodynamic simulations incorporating baryonic processes in the standard CDM model reproduce the observed small scale structure [22–24].
the horizon around the $e^{-}e^{+}$ annihilation.\footnote{Note that an apparently similar enhancement was hinted in ref. \cite{26}. However, its mechanism is different from that discussed in paper I \cite{43}. In ref. \cite{26}, the kinetic decoupling of DM occurs before the $e^{-}e^{+}$ annihilation. On the other hand, in paper I \cite{43}, the kinetic decoupling of charged massive particles is triggered by the $e^{-}e^{+}$ annihilation. Therefore, in the former case, the $e^{-}e^{+}$ annihilation affects the evolution of the DM cosmological perturbations only through changes in the expansion rate and the gravitational potentials. In the latter case, on the other hand, the evolution of the DM bulk velocity potential is changed by a direct coupling to that of the thermal plasma (see eq. (3.1)).} We note that the overshooting of the resultant matter power spectrum at small scales can also be realized in models where an early matter domination is considered \cite{44, 45}. However, we do not consider such an early matter dominated era and just assume the standard thermal history. The overshooting mechanism due to the sudden kinetic decoupling, which will be discussed in this paper, is quite different from the one arising in models with an early matter dominated era.

In this paper, following paper I \cite{43}, we show that the overshooting of the resultant matter power spectrum is not limited in the case of charged massive particles, but just the suddenness of the kinetic decoupling is the essence of the overshooting and thus it could be applicable to a broad class of models. We take a phenomenological DM model with a pseudo scalar coupling of a DM particle to a light mediator that also couples to hidden neutrinos as an example \cite{33, 34, 42}. In a certain parameter region, the DM kinetic decoupling happens when the mediator becomes non-relativistic and its number density drops with the Boltzmann factor. It results in the virtually instantaneous kinetic decoupling, while the usual kinetic decoupling leaves an intermittent collisions. The resultant matter power spectrum shows oscillating features as usual, but the oscillation amplitude is not damped.

The paper is organized as follows. In section 2, first we describe how to quantify the suddenness of the kinetic decoupling by introducing the momentum transfer rate, which also plays an important role in the evolution of the DM cosmological perturbations. Then we describe the model setup and its thermal history considered in this paper. In section 3, first we present evolution equations of the DM cosmological perturbations. Then we give an analytic result in a simplified assumption to explain why we expect that the acoustic damping disappears when the kinetic decoupling occurs suddenly. Finally we numerically follow the co-evolution of the cosmological perturbations of DM, mediators, hidden neutrinos, and the standard model (SM) degrees of freedom to confirm the expectation. Section 4 is devoted to concluding remarks. Throughout this paper we use the \textit{Planck} 2013 cosmological parameters \cite{46} to be consistent with ref. \cite{42}: $\Omega_{b}h^{2} = 0.022068$, $\Omega_{c}h^{2} = 0.12029$, $H_{0} = 67.11$, $\ln(10^{10}A_{s}) = 3.098$, and $n_{s} = 0.9624$.

## 2 Sudden kinetic decoupling with a light mediator

The following momentum transfer rate determines not only the kinetic decoupling temperature \cite{47, 48} but also the density perturbation evolution around the kinetic decoupling \cite{42}:

$$
\gamma = \frac{1}{6m_{\chi}T} \sum_{s_{TP}} \int d^{3}P_{TP} \left( \frac{2}{(2\pi)^{3}} \right) \frac{f_{eq}}{f_{eq}^{TP}} (1 \mp f_{eq}) \int_{-4p_{TP}^{2}}^{0} dt(-t) \frac{d\sigma}{dt} v, \tag{2.1}
$$

where the subscript (TP) denotes a particle in a thermal bath with which a DM particle elastically interacts, $T$ is the temperature of the thermal bath, $t$ is the Mandelstam variable that represents the momentum transfer squared, and $f_{eq} = \exp\{(-p \cdot u - \mu)/T\} \mp 1\}^{-1}$ with $\mu$ and $u$ being respectively the chemical potential and the bulk velocity. The $\mp$ in $\gamma$ ($\pm$ in

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\footnote{Note that an apparently similar enhancement was hinted in ref. \cite{26}. However, its mechanism is different from that discussed in paper I \cite{43}. In ref. \cite{26}, the kinetic decoupling of DM occurs before the $e^{-}e^{+}$ annihilation. On the other hand, in paper I \cite{43}, the kinetic decoupling of charged massive particles is triggered by the $e^{-}e^{+}$ annihilation. Therefore, in the former case, the $e^{-}e^{+}$ annihilation affects the evolution of the DM cosmological perturbations only through changes in the expansion rate and the gravitational potentials. In the latter case, on the other hand, the evolution of the DM bulk velocity potential is changed by a direct coupling to that of the thermal plasma (see eq. (3.1)).}
is for fermions and bosons, respectively. The above formula is valid as long as the DM mass is much larger than the temperature and the mass of the particle in the thermal bath.

The DM kinetic decoupling occurs when the momentum transfer per Hubble time drops below unity: \( \gamma(T_d) = H(T_d) \), where \( H^2 = 4\pi^2 g_* T_\gamma^4/(45 m_{Pl}^2) \) in the radiation dominated era with \( T_\gamma \) being the photon temperature, \( g_* \) being the effective number of massless degrees of freedom for the energy density, and \( m_{Pl} \) being the Planck mass. The suddenness of the kinetic decoupling is parametrized by \( n \) being an index when we approximate the momentum transfer rate per Hubble time around the kinetic decoupling by a simple scaling law as

\[
\frac{\gamma}{H} \approx \left( \frac{T}{T_\gamma} \right)^{n+4}.
\]  

For example, \( n = 0 \) when a DM fermion elastically interacts with a relativistic fermion through the Fermi interaction [26]. The sudden kinetic decoupling corresponds to much larger \( n \), meaning

\[
\frac{1}{H^2} \frac{d\gamma}{dt} \bigg|_{\gamma = H} \gg 1,
\]  

which leaves an interesting imprint on the resultant matter power spectrum as we will see in the next section. Large \( n(= O(10)) \) is achieved when a DM particle elastically interacts with a massive particle and its momentum transfer rate drops suddenly due to the Boltzmann suppression when the massive particle becomes non-relativistic. In paper I [43], the \( e^-e^+ \) annihilation triggers the kinetic decoupling of charged massive particles. Below we demonstrate how such a sudden kinetic decoupling can be realized in a phenomenological model with a light mediator.

In ref. [42] a general aspect of the DM kinetic decoupling in phenomenological models was studied. There we extend the SM with a DM sector which contains a Dirac DM particle \( (\chi) \), a bosonic mediator \( (\phi) \), and \( N_\nu/2 \) copies of hidden Dirac neutrinos \( (\nu) \) [33, 34]. The interactions are given by

\[
L_S \supset g_\chi \bar{\chi} \phi \chi + g_\nu \bar{\nu} \phi \nu ,
\]  

\[
L_V \supset g_\chi \phi \gamma^\mu \chi \phi \mu + g_\nu \phi \gamma^\mu \nu \phi \mu ,
\]  

\[
L_{PS} \supset ig_\chi \phi \gamma^5 \chi + ig_\nu \phi \gamma^5 \nu ,
\]  

\[
L_{PV} \supset g_\chi \phi \gamma^\mu \gamma^5 \chi \phi \mu + g_\nu \phi \gamma^\mu \gamma^5 \nu \phi \mu ,
\]

for the scalar, vector, pseudo scalar, and pseudo vector mediators, respectively.

We assume that the hidden neutrinos are different from the SM neutrinos, and take their masses such that their energy density at present is negligible. Since light mediators and hidden neutrinos act as dark radiation, the deviation of the effective number of neutrino species from the standard model value \( (\Delta N_{\text{eff}}) \) can be sizable, which could constrain the model. We assume that the DM sector decouples from the SM sector in the early Universe, and take the ratio of the hidden neutrino temperature to the photon temperature, \( r (= T_\nu/T_\gamma) \), such that \( \Delta N_{\text{eff}} \) is concordant with the observations. More specifically, the following inequality should be satisfied:

\[
\frac{r}{r_0} < \left( \frac{\Delta N_{\text{eff}}^{\text{obs}}}{N_\nu} \right)^{1/4},
\]  

where \( \Delta N_{\text{eff}}^{\text{obs}} \) denotes the upper bound on \( \Delta N_{\text{eff}} \) from observations. The ratio of the SM neutrino temperature to the photon temperature is denoted by \( r_0 \), and takes a value of
(4/11)^{1/3}$ after the $e^-e^+$ annihilation ($T_\gamma \sim 100$ keV). The combined constraint from “Planck TT+lowP+lensing+BAO” [2] gives $\Delta N_{\text{eff}}^{\text{obs}} \simeq 0.65$ (95% confidence level), which implies that $r/r_0 < 0.57$ for $N_\nu = 6$.

In the above formula, we evaluate $\Delta N_{\text{eff}}$ long after the freeze-out of $\chi$’s and $\phi$’s, since the constraint comes from the observation of cosmic microwave background anisotropies that are sensitive to the Universe at the recombination ($T_\gamma \sim 1$ eV). If (pseudo) scalar $\phi$’s also contribute to $\Delta N_{\text{eff}}$, which may be the case when we consider the constraint from the big bang nucleosynthesis ($T_\gamma \sim 100$ keV), $N_\nu$ should be replaced by $N_\nu + 4/7$. Note that one needs to take account of the evolution of $r/r_0$ through the annihilation of $\chi$’s and the decay of $\phi$’s, when one fixes its value at some time. For example, $r/r_0$ increases by a factor of $(N_\nu + 2 + 4/7)^{1/3}/(N_\nu + 4/7)^{1/3}$ after the DM freeze-out when compared to that before. Such effects may be important when we consider the ultraviolet completion of the present model, although we simply parametrize the temperature ratio between the DM and SM sectors by $r/r_0$ here. The model we consider here can be regarded as an effective theory that describes lower energy physics in the DM sector than the DM mass. Our aim is to study the kinetic decoupling of DM with a light mediator, which occurs long after the decoupling of heavy particles. Thus the phenomenological model is sufficient for our purpose.

We consider the pseudo scalar mediator in this paper since it has a different property from the others. The spin-averaged invariant amplitude squared for $\nu \rightarrow \chi \nu$ scales as $|\mathcal{M}|^2 \propto (-t)^2$ in the cases of the pseudo scalar mediator, while $|\mathcal{M}|^2 \propto (-t)m^2_\chi, E_\gamma^2 m^2_\chi, E_\gamma^2 m^2_\nu$ in the case of the scalar, vector, and pseudo vector mediators, respectively. Here we assume that the DM ($m_\chi$) and mediator ($m_\phi$) masses are much larger than the hidden neutrino energy ($E_\nu$). The different scaling originates from the fact that in the small momentum transfer limit, where the DM particle is regarded as at rest before and after the collision, the expectation value of $\chi \gamma^5 \chi$ vanishes. It follows that $\nu \rightarrow \chi \nu$ through the pseudo scalar mediator decouples much earlier than those through the others with the model parameters being fixed. Thus in such a case, the DM kinetic decoupling is determined by $\chi \phi \rightarrow \nu \nu$, the momentum transfer rate of which drops suddenly when $\phi$ becomes non-relativistic: $T_\nu = m_\phi$. Figure 1 sketches the thermal history of the DM sector in this model. Below we see what a parameter choice realizes such a thermal history.

The momentum transfer rate per Hubble time for $\nu \rightarrow \chi \nu$ through the pseudo scalar mediator is given by

$$\frac{\gamma}{H} \simeq 2 \left( \frac{r}{r_0} \right)^2 \left( \frac{N_\nu}{6} \right) \left( \frac{\alpha_\chi}{4.7 \times 10^{-2}} \right) \left( \frac{\alpha_\nu}{10^{-12}} \right) \left( \frac{1 \text{ GeV}}{m_\chi} \right)^3 \left( \frac{10 \text{ keV}}{m_\phi} \right)^4 \left( \frac{T_\nu}{10 \text{ keV}} \right)^6,$$

(2.9)

where $\alpha_\chi = g_\chi^2/(4\pi)$, $\alpha_\nu = g_\nu^2/(4\pi)$. As we see from this expression, for minuscule $\alpha_\nu$, the decoupling of $\chi \nu \rightarrow \chi \nu$ occurs before $\phi$ becomes non-relativistic, i.e., the mediator mass becomes larger than the hidden neutrino energy.

Note that even when $\alpha_\nu$ is minuscule and thus $\chi \bar{\chi} \rightarrow \nu \nu$ annihilation is inefficient, the correct DM relic abundance is obtained through $\chi \bar{\chi} \rightarrow \phi \phi$ annihilation, which is independent of $\alpha_\nu$:

$$\Omega_\chi h^2 \simeq \frac{0.12}{2} \left( \frac{r}{r_0} \right) \left( \frac{4.7 \times 10^{-2}}{\alpha_\chi} \right)^2 \left( \frac{m_\chi}{1 \text{ GeV}} \right)^2 \left( \frac{m_\chi/T_\gamma r_0}{13.4 \frac{1}{r} \frac{10^2}} \right)^2.$$

(2.10)

\footnote{In this case eq. (2.9) is not valid any longer since there we assume that the mediator mass is much larger than the hidden neutrino energy. Essentially $m_\phi^2$ in the denominator is replaced by $T_\nu^2$ in addition to a overall multiplication by a hidden Coulomb logarithm $\sim \ln(T_\nu^{2}/\max|m_\phi^2, \alpha_\nu T_\gamma^2|)$.}
Here the factor $(1/10^2)$ in the last parentheses can be achieved when $\chi$’s are copiously produced at $T_\nu = 3m_\chi/13.4\left(r_0/r\right)(1/10^2)$, for example, by decay of some long-lived particles, although we do not specify the production mechanism. In the following we focus on such a case.

Even after the $\chi\nu \to \chi\nu$ decoupling, $\phi \leftrightarrow \nu\bar{\nu}$ and $\chi\phi \to \chi\phi$ reactions are still efficient as long as $\phi$ is relativistic. $\phi$’s are kept in thermal equilibrium with $\nu$’s through the decay and inverse decay, which are as efficient as

$$
\Gamma_H \simeq 2 \times 10^{11} \left(\frac{r}{r_0}\right)^2 \left(\frac{N_\nu}{6}\right) \left(\frac{\alpha_\nu}{10^{-12}}\right) \left(\frac{m_\phi}{10 \text{ keV}}\right)^2 \left(\frac{10 \text{ keV}}{T_\nu}\right)^3,
$$

where $\zeta(x)$ is the Riemann zeta function and the decay rate at rest is evaluated as

$$
\Gamma = \frac{\alpha_\nu}{4} N_\nu m_\phi.
$$

The prefactor $(\pi^2 m_\phi/12\zeta(3)T_\nu)$ represents the boost factor for relativistic $\phi$ so that the above estimate gives the lower bound.

Let us discuss $\nu$’s after the $\chi\nu \to \chi\nu$ decoupling. It should be noted that $\phi \leftrightarrow \nu\bar{\nu}$ reaction is efficient for $\phi$’s, but not for $\nu$’s. This is because the inverse decay is suppressed by the Boltzmann factor but the number density of $\phi$ ($n_\phi$) is also small due to the same factor, while the number density of $\nu$ ($n_\nu$) is huge and only a tiny fraction of them experiences the inverse decay. If $\nu$ does not have any other interaction, it starts to freely stream just after the inverse decay becomes inefficient. This impacts the evolution of the cosmological perturbations and the resultant matter power spectrum as we see in appendix B. One can control the free-streaming of $\nu$’s without changing the dynamics of $\chi$’s by introducing the
Figure 2. Momentum transfer rate per Hubble time as a function of the scale factor (normalized such that \( a = 1 \) at present) with \( m_\phi = 6 \text{ keV}, \alpha_\chi = 4.7 \times 10^{-2}, m_\chi = 1 \text{ GeV}, \) and \( r = r_0 \). The rate starts to deviate from that with a massless mediator \( (m_\phi = 0) \) around \( T_\nu = m_\phi \) (vertical dashed line) and crosses the Gamow criterion \( (\gamma/H = 1, \text{ horizontal line}) \) steeply.

scalar mediator given in eq. (2.4) as well as the pseudo scalar mediator and taking tiny \( g_{\chi,s} \) and sizable \( g_{\nu,s} \). If \( m_{\phi,s} \) is larger than \( T_\nu \) of interest, the effective interaction reads as

\[
\mathcal{L} \supset (g_{\nu,s}/m_{\phi,s})^2 (\bar{\nu}\nu)^2/2 \quad \text{and the reaction rate per Hubble time is evaluated as}
\]

\[
\langle \sigma v \rangle_{\nu} = 0.3 \left( \frac{r}{r_0} \right)^2 \left( \frac{m_{\phi,s}}{m_{\chi,s}} \right)^4 \left( \frac{1 \text{ keV}}{T_\nu} \right)^3,
\]

where the total cross section before thermally averaged is given by

\[
\langle \sigma v \rangle = \frac{s^2(N_{\nu} + 7/4)}{96\pi E_{\nu 1} E_{\nu 2}} \left( \frac{g_{\nu,s}^2}{m_{\phi,s}^2} \right),
\]

with \( s \) being the Mandelstam variable that represents the center-of-mass energy squared.

The momentum transfer rate per Hubble time for \( \chi\phi \rightarrow \chi\phi \) is given by

\[
\frac{\gamma}{H} = \frac{4 (p_{\phi}) \sigma n_{\phi} m_{\phi} \to 10^6 \left( \frac{r}{r_0} \right)^2 \left( \frac{\alpha_\chi}{4.7 \times 10^{-2}} \right)^2 \left( \frac{1 \text{ GeV}}{m_{\chi}} \right)^3 \left( \frac{T_\nu}{10 \text{ keV}} \right)^2,}
\]

where \( (p_{\phi}) \) is the thermal average of the absolute value of the three-momentum of \( \phi \). The cross section is constant;

\[
\sigma = \frac{2\pi\alpha_\chi^2}{m_{\chi}^2}.
\]

In the final expression we take the relativistic limit of \( \phi \). Once we take finite \( m_{\phi} \) into account, the momentum transfer rate per Hubble time starts to drop around \( T_\nu = m_{\phi} \) as shown in figure 2. More importantly, the kinetic decoupling happens more suddenly than that in the massless mediator case: \( n = 10–20 \) (see eq. (2.2)).
3 Evolution of the cosmological perturbations

Let us describe the evolution of the cosmological perturbations in the perfect fluid limit: the equation of continuity and Euler equation of the DM fluid density perturbation ($\delta_\chi$) and the bulk velocity potential ($\theta_\chi$) are given by

$$\dot{\delta}_\chi = -\theta_\chi - \frac{1}{2}\ddot{h}, \quad \dot{\theta}_\chi = -\frac{\dot{a}}{a}\theta_\chi + k^2c_\chi^2\delta_\chi + \gamma_0a(\theta_\nu - \theta_\chi),$$

with dots denoting the derivatives with respect to the conformal time ($\tau$), the subscript of 0 denoting the quantity at the leading order (i.e., homogeneous and isotropic part), and $k$ being the absolute value of the wavenumber. The sound speed is defined as

$$c_\chi^2 = \frac{T_{\chi 0}}{m_\chi} \left(1 - \frac{1}{3}\frac{d\ln T_{\chi 0}}{d\ln a}\right),$$

with the temperature evolving according to

$$\frac{d\ln(a^2T_{\chi 0})}{d\tau} = 2\gamma_0a \left(\frac{T_{\chi 0}}{T_{\chi 0}} - 1\right).$$

Note that the bulk velocity potential of the $\phi$ fluid is kept the same as that of the $\nu$ fluid since $\phi$’s are kept in thermal equilibrium with $\nu$’s (see the discussion around eq. (2.11)). This is why $\theta_\nu$ appears in the Euler equation although the direct coupling between DM and $\nu$ is not efficient due to the suppression of $a_\nu$ (see the discussion around eq. (2.9)). Hereafter we take the synchronous gauge:

$$ds^2 = a^2(\tau)\left(-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right), \quad h_{ij} = \frac{\partial_i \partial_j}{\partial^2} h + \left(\frac{\partial_i \partial_j}{\partial^2} - \frac{\delta_{ij}}{3}\right)6\eta,$$

where $x^i (\partial_i)$ denotes (the derivative with respect to) the comoving coordinate, $G = 1/m_{Pl}^2$ is the Newton constant, $\rho$ and $P$ respectively denote energy density and pressure, and the summation runs over all the degrees of freedom.

Before following numerically the co-evolution of the cosmological perturbations of DM, mediators, (both SM and hidden) neutrinos, baryons, photons, and the gravitational potentials, let us derive an analytic result in a simplified case, where we ignore the sound speed of DM, i.e., take a heavy DM limit while keeping $\gamma_0$ being intact. In fact, the heavy DM limit is also important when we validate that DM can be described as a perfect fluid since the effects of the imperfectness are suppressed by $k\sqrt{T_{\nu 0}/m_{\chi}/(aH)}$ [42]. This point will be examined closely in appendix A. Furthermore we assume that the radiation component is dominated by photons and hidden neutrinos, i.e., we ignore the contribution from SM neutrinos, which freely stream and cannot be approximated by a perfect fluid. On the other hand, photons can be regarded as a perfect fluid due to the efficient Compton scattering with baryons. In addition we introduce the self-interaction between hidden neutrinos to prevent them from freely streaming (see the discussion around eq. (2.13) and also appendix B), which simplifies the equations.

Under these assumptions we obtain a closed set of the equations:

$$\dot{\delta}_{\gamma(\nu)} = -\frac{4}{3}\theta_{\gamma(\nu)} - \frac{2}{3}\ddot{h}, \quad \dot{\theta}_{\gamma(\nu)} = \frac{1}{4}k^2\delta_{\gamma(\nu)}, \quad \ddot{h} + \frac{1}{\tau}\dot{h} = -\frac{6}{\tau^2}(1 - f_\nu)\delta_\gamma + f_\nu\delta_\nu,$$
where \( f_\nu = \rho_\nu / (\rho_\gamma + \rho_\nu) \) is the ratio of the hidden neutrino energy density to the whole radiation (photon + hidden neutrino) energy density. The solutions are given by \([50, 51]\)

\[
\dot{h} = -\frac{24C}{\tau} \left( \frac{2}{x} \sin x + \frac{2}{x^2} \cos x - \frac{2}{x^2} - 1 \right),
\]

(3.7)

\[
\delta_\gamma = \delta_\nu = -8C \left( \frac{2}{x} \sin x - \cos x + \frac{2}{x^2} \cos x - \frac{2}{x^2} \right),
\]

(3.8)

\[
\theta_\gamma = \theta_\nu = -\frac{6C}{\tau} (-x \sin x - 2 \cos x + 2),
\]

(3.9)

where \( x = k\tau/\sqrt{3} \). Here we normalize the cosmological perturbations such that

\[
\dot{h} = 2C(k^2\tau), \quad \delta_\gamma = \delta_\nu = -\frac{2}{3} C(k\tau)^2, \quad \theta_\gamma = \theta_\nu = -\frac{1}{18} C(k^4\tau^3),
\]

(3.10)

in the small \( x \) limit, which coincide with those for the adiabatic mode given in eq. (96) of ref. [49]. Note that \( \delta_\nu = \delta_\gamma \) and \( \theta_\nu = \theta_\gamma \) for the adiabatic mode as long as both \( \nu \)'s and \( \gamma \)'s can be regarded as perfect fluids, i.e., their free-streaming can be neglected.

We can obtain the DM cosmological perturbations by solving

\[
\dot{\delta}_\chi = -\theta_\chi - \frac{1}{2} \dot{h}, \quad \dot{\theta}_\chi = -\frac{\dot{a}}{a} \theta_\chi + \gamma_0 a (\theta_\nu - \theta_\chi),
\]

(3.11)

where eqs. (3.7) and (3.9) are substituted. We can make an ansatz for the solutions as \([26]\)

\[
\delta_\chi = -12C \left( -\text{Ci}(x) + \frac{1}{x} \sin x + \frac{1}{x^2} \cos x - \frac{1}{x^2} + f_1 \ln x + f_2 \right),
\]

(3.12)

\[
\theta_\chi = \frac{12C}{\tau} (f_1 - 1),
\]

(3.13)

where the cosine integral follows

\[
\text{Ci}(x) = \gamma_E + \ln x + \int_0^x \cos t - \frac{1}{t} dt, \quad \frac{d}{dx} \text{Ci}(x) = \frac{\cos x}{x},
\]

(3.14)

with the Euler’s constant being \( \gamma_E \simeq 0.577 \). We end up with a simple set of equations:

\[
f_1 + \gamma_0 a f_1 = \gamma_0 a \left( \cos x + \frac{1}{2} x \sin x \right), \quad \dot{f}_2 + \dot{f}_1 \ln x = 0.
\]

(3.15)

In the pure CDM limit, where \( \gamma_0 = 0 \), we can set \( f_1 = 1 \) and \( f_2 = \gamma_E - 1/2 \), which satisfies eq. (3.15) and the initial condition of \( \delta_\chi = -1/2 C(k\tau)^2 \) for \( x \to 0 \) and \( \theta_\chi = 0 \), coinciding with that for the adiabatic mode given in eq. (96) of ref. [49]. Note that in this limit the above solution reproduces the well-known logarithmic growth of the CDM density perturbations, \( \delta_{\text{CDM}} \propto \ln a \), inside the horizon.

Supposing that \( \gamma_0 a \tau \to \infty \) for \( \tau \to 0 \), we obtain the formal solutions of eq. (3.15):

\[
f_1 = \int_0^\tau \frac{d\tau'}{\tau'} \exp \left( -\int_{\tau'}^\tau \frac{d\tau''}{\tau''} \gamma_0 a'' \tau'' \right) \gamma_0 a' \tau' \left( \cos x' + \frac{1}{2} x' \sin x' \right),
\]

(3.16)

\[
= \cos x + \frac{1}{2} x \sin x - \int_0^\tau \frac{d\tau'}{\tau'} \exp \left( -\int_{\tau'}^\tau \frac{d\tau''}{\tau''} \gamma_0 a'' \tau'' \right) \left( -\frac{1}{2} x' \sin x' + \frac{1}{2} x'^2 \cos x' \right),
\]

\[
f_2 = \int_0^\tau \frac{d\tau'}{\tau'} (f_1' - 1) + (1 - f_1) \ln x + \gamma_E - \frac{1}{2},
\]

(3.17)
where (double) primes denote quantities evaluated at \( \tau' \) (\( \tau'' \)). The initial condition of the formal solutions takes a form of

\[
\begin{align*}
    f_1 &= \cos x + \frac{1}{2} x \sin x, \\
    f_2 &= \text{Ci}(x) - f_1 \ln x - \frac{1}{2} \cos x,
\end{align*}
\]

for \( \tau \to 0 \), which results in

\[
\delta_{\chi} = \frac{3}{4} \delta_{\nu} = \frac{3}{4} \delta_{\gamma}, \quad \theta_{\chi} = \theta_{\nu} = \theta_{\gamma}.
\]

This initial condition for interacting DM coincides with that discussed in paper I \cite{43}, where the residual gauge degrees of freedom in the synchronous gauge are also examined. Again note that we consider the adiabatic mode and the second equalities are held in the perfect fluid limit of \( \nu ' \)s and \( \gamma ' \)s.

We are interested in the resultant late time density perturbations and thus let us take \( \tau \to \infty \) in eqs. (3.16) and (3.17). Then \( f_1 \) and \( f_2 \) become functions of \( k \). In general we need to rely on numerical methods to evaluate the integrals in eqs. (3.16) and (3.17). On the other hand, if the momentum transfer rate per Hubble time follows a simple scaling law (corresponding to eq. (2.2)),

\[
\gamma_0 a \tau = \left( \frac{d}{\tau} \right)^{n-4},
\]

we can analytically evaluate the integrals with the help of the steepest descent method as done in ref. \cite{26}. There \( n \) is set to be 0, but we can generalize the result to an arbitrary value of \( n \) as

\[
f_1 \approx \left( \frac{4\pi}{5+n} q \right)^{1/2} \exp (-R) \left[ \cos \left( I - \frac{4+n \pi}{5+n} \right) + q \cos \left( I - \frac{4+n 3\pi}{5+n} \right) \right],
\]

where \( R \) and \( I \) are respectively defined as the real and imaginary parts of

\[
R + iI = \frac{5+n}{4+n} q \exp \left( \frac{4+n \pi}{5+n} \right),
\]

with

\[
q = \frac{1}{2} \left( \frac{k \tau_d}{\sqrt{3}} \right)^{(4+n)/(5+n)}.
\]

The factor of \( \exp (-R) \) represents the damping of the acoustic oscillations for the density perturbations that are subhorizon around the kinetic decoupling (\( k > 1/\tau_d \)). This damping is often called the acoustic damping in the literature and originates from intermittent collisions (collision interval longer than the oscillation period) around the kinetic decoupling. Note that the acoustic damping is different from the diffusion (Silk) damping and from the collisionless (free-streaming) damping \cite{25}. In the large \( n \) limit (effectively \( n = 10–20 \) in figure 2), which corresponds to the sudden kinetic decoupling,

\[
R \approx \frac{\pi}{4+n} \frac{k \tau_d}{2\sqrt{3}}, \quad I \approx \frac{5+n k \tau_d}{4+n \sqrt{3}}.
\]

Interestingly, the damping scale, where \( R \) becomes of order of unity, is proportional to \( n \) and thus shifted to a smaller length scale with increasing \( n \). From this observation we expect that
the instantaneous kinetic decoupling leaves the resultant matter power spectrum oscillating but not damped.\footnote{The overshooting of the matter power spectrum due to the sudden kinetic decoupling was first pointed out in paper I \cite{43}. This phenomena has also been observed in ref. \cite{52} where density perturbations in a model with long-lived charged massive particles have been investigated. In the paper, the coupling between charged massive particles and baryons has been instantaneously switched off at the lifetime of the charged massive particle, which corresponds to the instantaneous kinetic decoupling discussed here. We thank the authors of ref. \cite{52} for the clarification of their calculations \cite{53}.}

It should be noted that $R$ roughly corresponds to the ratio of the time scale of the sudden kinetic decoupling to that of the acoustic oscillation. When the kinetic decoupling proceeds more rapidly than the oscillation, i.e.,

$$\frac{1}{\gamma_0/H} \frac{d(\gamma_0/H)}{dt} \bigg|_{\gamma_0=H} > \frac{k c_s}{a},$$

(3.25)

with the sound speed of the DM-neutrino fluid being $c_s$, the acoustic damping does not occur. For larger $k$, the density perturbations damp through intermittent collisions around the kinetic decoupling. In fact $R$ can be represented as the ratio of the left hand side to the right hand side of eq. (3.25):

$$R \sim \left( \frac{k c_s}{a} \right) / \left( \frac{1}{\gamma_0/H} \frac{d(\gamma_0/H)}{dt} \bigg|_{\gamma_0=H} \right).$$

(3.26)

By using eq. (3.20) one can find that this expression roughly reproduces eq. (3.24).

Let us confirm the above expectation by numerically solving the co-evolution of the cosmological perturbations of DM, mediators, (both SM and hidden) neutrinos, baryons, photons, and the gravitational potentials. To this end we modify the public code \textsc{Camb} suitably \cite{54}. Here we take account of the finite sound speed of the DM fluid and assume that the scattering between $\nu$'s is efficient (see the discussion around eq. (2.13) and also appendix B). The resultant linear matter power spectrum is shown in figure 3. The dark acoustic oscillation is seen at $k > 50 \, h/\text{Mpc}$ and its amplitude damps at $k > 400 \, h/\text{Mpc}$. The powers at intermediate $k$, on the other hand, are not only undamped as expected but in fact are enhanced when compared to those in the standard CDM model.

Figure 4 is helpful in understanding the origin of the overshooting. Here we show the evolution of the cosmological perturbations (including the standard CDM density perturbation for reference) with $k = 80$ and $480 \, h/\text{Mpc}$. Initially $\delta_\chi$ and $\delta_{\text{CDM}}$ coincide with each other, but for $k = 80 \, h/\text{Mpc}$ they end up with the opposite sign and the absolute value of $\delta_\chi$ is larger than $\delta_{\text{CDM}}$. The bulk velocity potential of the $\chi$ fluid follows that of the $\phi$ fluid (which is equal to that of the $\nu$ fluid) due to the $\chi\phi \to \chi\phi$ scattering until the kinetic decoupling. This drives the oscillation of $\delta_\chi$ as well as $\theta_\chi$. If $\chi$'s kinetically decouple from $\nu$'s suddenly when $\delta_\chi = (3/4) \delta_\nu \simeq 0$ and $\theta_\chi = \theta_\nu$ is around the oscillation peak, which is in fact the case for $k = 80 \, h/\text{Mpc}$ as can be seen in the top panel of figure 4, $\theta_\chi$ starts to decay as $\theta_\chi \propto 1/a$ but drives $\delta_\chi$ to the opposite sign before it completely decays. Intuitively this can be understood by the analogy to the harmonic oscillator. In the tight coupling limit, i.e., before the kinetic decoupling, the evolution of $\delta_\chi$ as well as $\theta_\chi$. If $\chi$'s kinetically decouple from $\nu$’s suddenly when $\delta_\chi = (3/4) \delta_\nu \simeq 0$ and $\theta_\chi = \theta_\nu$ is around the oscillation peak, which is in fact the case for $k = 80 \, h/\text{Mpc}$ as can be seen in the top panel of figure 4, $\theta_\chi$ starts to decay as $\theta_\chi \propto 1/a$ but drives $\delta_\chi$ to the opposite sign before it completely decays. Intuitively this can be understood by the analogy to the harmonic oscillator. In the tight coupling limit, i.e., before the kinetic decoupling, the evolution of $\delta_\chi = (3/4) \delta_\nu$ and $\theta_\chi = \theta_\nu$ can be regarded as a particle oscillating in the quadratic potential. The DM density perturbation and the pressure from $\nu$'s correspond to a position of the particle and the quadratic potential for it, respectively. Suppose that the potential disappears instantaneously at some time, which is an analogy to the instantaneous kinetic decoupling. The behavior of the particle after that depends on its position at the time
Figure 3. Linear matter power spectra extrapolated to the present ($z=0$). We compare those in the standard CDM model and in the interacting DM model with the same parameter choice as in figure 2. The latter clearly shows the oscillation featuring the acoustic oscillation. Interestingly the powers at the oscillation peaks are larger when compared to those in the standard CDM model.

of the potential disappearance. If it is at the oscillation peak, it does not move after that; meanwhile if it is at the bottom of the potential, it starts to stream freely (although in reality the bulk velocity potential redshifts), which results in the overshooting of the particle. The cosmological perturbations with $k = 80 \, h/\text{Mpc}$ correspond to the case where the potential disappears when the particle comes to the bottom of the potential. This is why $\delta_\chi$ is overshot.

In contrast, for the cosmological perturbations with $k = 480 \, h/\text{Mpc}$ depicted in the bottom panel of figure 4, the kinetic decoupling does not occur suddenly when compared to the oscillation time scale (wavenumber times the sound speed). $\theta_\chi$ starts to deviate from $\theta_\nu$ when $\delta_\chi$ takes a value of zero, i.e., when the particle comes to the bottom of the potential in the analogy discussed above. However, intermittent collisions after that prevent $\theta_\chi$ with $k = 480 \, h/\text{Mpc}$ from freely decaying ($\theta_\chi \propto 1/a$), unlike that with $k = 80 \, h/\text{Mpc}$. Therefore, in this case, no overshooting occurs and the density perturbation damps as in the usual case.

Since the overshooting is realized when the DM cosmological perturbations do not experience intermittent collisions around the kinetic decoupling, the condition of the overshooting is given by eq. (3.25). Therefore the density perturbations with a smaller wavenumber exhibit the overshooting in figure 4, but those with a larger wavenumber do not. Eq. (2.3) is roughly the condition of the overshooting for the first oscillation peak of the resultant matter power
spectrum since the corresponding cosmological perturbations enter the horizon around the kinetic decoupling and thus $k c_s / a \simeq H$ when $\gamma = H$ as long as the entropy density of the fluid is dominated by the radiation degrees of freedom (neutrino in the present case).

We remark that the above argument on how and why the overshooting occurs is concordant with the result and conjecture of paper I [43]. Before closing this section, we comment on the effect of the gravitational potential. Precisely speaking, the gravitational potential ($\dot{h}$) also impacts the evolution of $\delta_\chi$ (see eq. (3.1)), but it is subdominant. This is because both $\dot{h}$ and $\theta_\chi$ decay inversely proportionally to $a$ when subhorizon and after the kinetic decoupling, but the latter ($\theta_\nu = \theta_\gamma \propto x_{kd} \tau_{kd}$) dominates the former ($\dot{h} \propto 1/\tau_{kd}$) for the wavenumber with which $\theta_\chi$ takes a peak value at the time of the kinetic decoupling ($\delta_\nu (x_{kd}) = 0$ and $x_{kd} \simeq 4$ for the first oscillation peak of the matter power spectrum) as we can see from eqs. (3.7)–(3.9).

4 Conclusion

We studied the impact of the suddenness of the DM kinetic decoupling on the evolution of the DM cosmological perturbations and the resultant matter power spectrum. We took a phenomenological model with the pseudo scalar mediator coupling to DM and hidden neutrinos as an example. If the mediator-neutrino coupling is suppressed, the DM-neutrino scattering becomes inefficient in the early Universe, but they couple with each other indirectly through the DM-mediator scattering and the mediator decay and inverse decay into neutrinos. Then the DM kinetic decoupling proceeds when the mediator becomes non-relativistic. The momentum transfer rate per Hubble time suddenly drops as the number density of the mediator is reduced by the Boltzmann factor.

We numerically followed the co-evolution of cosmological perturbations of DM, mediators, neutrinos, and the SM degrees of freedom. Our calculation relies on the formulation developed in ref. [42]. The resultant matter power spectrum is featured by the dark acoustic oscillation and the powers at the first oscillation peaks are not damped but enhanced when compared to those in the standard CDM model, while eventually damped at smaller length scales. This supports the conjecture in paper I [43] that the sudden kinetic decoupling leads to an overshooting of the matter power spectrum and this phenomena can happen in a broad class of models where the suddenness of the kinetic decoupling is satisfied.

We also analytically studied the evolution of the DM cosmological perturbations in a simplified setup to provide a further support to the conjecture. The analytic result hints the criterion for determining whether the density perturbation with a given wavenumber is overshot or damped. We note that the overshooting is different from the enhancement that can occur in a non-standard background evolution like an early matter dominated era, in which new features of the resultant matter power spectrum have been reported [44, 45] and expected [55] recently. Our result shows that the physics of the kinetic decoupling still needs to be more clarified even if we assume the standard background evolution of the Universe.

In this paper we focused on the case where the dark acoustic oscillation can be seen at subgalactic scales, which could attract interest in the context of the issues reported in the structure formation in the standard CDM model. Even if the kinetic decoupling occurs in the earlier stage of the Universe and thus the resultant dark acoustic oscillation is found at much smaller scales, the kinetic decoupling may impact the minimal halo mass of the Universe to leave the observational effect, for example, through the indirect detection signal of DM. Our analysis is restricted within the linear level of the cosmological perturbations, while it
Figure 4. Time evolution of the cosmological perturbations with \( k = 80 \, h/\text{Mpc} \) (top) and \( 480 \, h/\text{Mpc} \) (bottom). We multiply the bulk velocity potentials of the DM fluid (\( \theta_\chi \)) and the neutrino fluid (\( \theta_\nu \)) by a factor of 10 just for presentation. We set the parameters of the interacting DM model to be the same as in figure 2.

will be interesting to study the non-linear matter distribution of the Universe in this type of model. The vigorous efforts into the detailed understanding of the physics of the DM kinetic decoupling are essential for the identification of the nature of DM.

Acknowledgments

We thank Toyokazu Sekiguchi for carefully reading the manuscript and giving a fruitful feedback. This work was partially supported by IBS under the project code IBS-R018-
A Effects of the imperfectness of the DM fluid

As stressed in section 3, the acoustic damping discussed in this paper has a different physical origin from the diffusion damping and from the collisionless damping. The perfect fluid approximation, where the collisionless damping is neglected, is taken in section 3 since it provides an ideal setup for studying the acoustic damping. One may, on the other hand, would like to see what happens in a realistic case, where all such mechanisms may play equally important roles. Actually, in the phenomenological model taken in this paper (see section 2), the DM mass cannot be arbitrarily large; otherwise the $\chi\phi \rightarrow \chi\phi$ scattering decouples too early to leave the dark acoustic oscillation on the matter power spectrum above subgalactic scales (see eq. (2.15)). In this section we examine to what extent the free-streaming of DM, i.e., imperfectness of the DM fluid impacts the resultant matter power spectrum.

To this end, in principle, we need to follow the full Boltzmann hierarchy somehow \[26, 42\], while it is beyond the scope of this paper. Instead we rely on the imperfect fluid approximation, where the next leading order contributions in terms of $k\sqrt{T_{\nu0}/m_{\chi}}/(aH)$ are incorporated \[42\]. There we introduce the anisotropic inertia ($\sigma_{\chi}$) and the entropy perturbation ($\pi_{\chi}$) of the DM fluid in addition to the density perturbation and the bulk velocity potential. They evolve with the conformal time as follows:

\begin{align*}
\dot{\delta}_{\chi} &= -\theta_{\chi} - \frac{1}{2} \dot{h}, \\
\dot{\theta}_{\chi} &= -\frac{\dot{a}}{a} \theta_{\chi} - k^2 \sigma_{\chi} + k^2(c_\chi^2 \delta_{\chi} + \pi_{\chi}) + \gamma_0 a (\theta_{\nu} - \theta_{\chi}), \quad (A.1) \\
\dot{\sigma}_{\chi} &= -2\frac{\dot{a}}{a} \sigma_{\chi} + \frac{4}{3} \frac{T_{\nu0}}{m_{\chi}} \theta_{\chi} + \frac{2}{3} \frac{T_{\nu0}}{m_{\chi}} (h + 6\eta) - 2\gamma_0 a \sigma_{\chi}, \quad (A.2) \\
\dot{\pi}_{\chi} &= -2\frac{\dot{a}}{a} \pi_{\chi} - \frac{1}{a^2} \frac{d(a^2 c_\chi^2)}{d\tau} \delta_{\chi} - \left(\frac{5}{3} \frac{T_{\nu0}}{m_{\chi}} - c_\chi^2 \right) \theta_{\chi} - \frac{1}{2} \left(\frac{5}{3} \frac{T_{\nu0}}{m_{\chi}} - c_\chi^2 \right) \dot{h} \\
&- 2\gamma_0 a \left[ \pi_{\chi} - \frac{\delta T_{\nu}}{m_{\chi}} - \left(\frac{T_0}{m_{\chi}} - c_\chi^2 \right) \delta_{\chi} \right] + 2\gamma_0 a \left(\frac{T_{\nu0}}{T_{\nu0}} - 1 \right) \frac{T_{\nu0}}{m_{\chi}} \frac{\delta \gamma}{\gamma_0}. \quad (A.3)
\end{align*}

In the present model, the momentum transfer rate is a function solely of the neutrino temperature since $\phi$’s are in thermal equilibrium with $\nu$’s. Thus we can relate their perturbations as $\delta \gamma/\gamma_0 = d \ln \gamma_0 / d \ln T_{\nu0} T_{\nu0}$. The perturbation of the neutrino temperature is determined by the density perturbation of the neutrino as $\delta T_{\nu}/T_{\nu0} = \delta_\nu/4$.

We solve the above equations numerically by suitably modifying \textsc{camb} \[54\]. Figure 5 compares the linear matter power spectrum in the imperfect fluid approximation with that in the perfect fluid approximation shown in figure 3. The power at the first oscillation peak is changed by 10%, while those at higher oscillation peaks changed more drastically. We need to take into account the full Boltzmann hierarchy, i.e., the effects of the free-streaming carefully at such smaller length scales. The power spectra in the two approximations coincide with each other at large length scales ($k \lesssim 100 h/\text{Mpc}$), which seems concordant with the rough estimate.
Figure 5. Linear matter power spectrum in the imperfect fluid (dashed) is overplotted in figure 3. They differ from each other above $k \sim 100 \ h/\text{Mpc}$. The power at the first oscillation peak ($k \sim 80 \ h/\text{Mpc}$) is not impacted significantly (10% at most). We also show the power spectrum with the DM mass being larger ($m_\chi = 1 \rightarrow 100 \ GeV$) but $\gamma_0$ being fixed (dotted).

given in ref. [42]: the effects of the higher order terms of the Boltzmann hierarchy on the density perturbations are suppressed by $k\sqrt{T_{\nu,0}/m_\chi/(aH)}$, which takes a maximum value at the matter radiation equality. This ratio is larger than unity for $k > 430 \ h/\text{Mpc}$ ($m_\chi/1 \ GeV)^{1/2}$. To make this point clear we also show the linear matter power spectrum for a larger DM mass. Here we replace $m_\chi = 1 \ GeV \rightarrow 100 \ GeV$ in eqs. (A.1)–(A.4) while keeping $\gamma$ being intact. The power spectra in the perfect and imperfect fluid approximations are concordant with each other in the plotted region ($k < 600 \ h/\text{Mpc}$). This supports the above estimate of the effects of the higher order terms of the Boltzmann hierarchy.

B Effects of the free-streaming of hidden neutrinos

As discussed in section 2, in the phenomenological model considered in this paper, the self-interaction between hidden neutrinos is controlled by the additional interaction such as the scalar mediator, while the pseudo scalar mediator is kept dominant for the DM phenomenology. In section 3, we assume that the scattering between $\nu$’s is efficient and thus $\nu$’s do not freely stream around the DM kinetic decoupling. One may wonder how the free-streaming of the hidden neutrinos impact the evolution of the DM cosmological perturbations and the resultant matter power spectrum. To address this point, we consider the extreme case, i.e., the free-streaming neutrino limit, although in reality at least before the mediator becomes non-relativistic, the inverse decay of $\phi$ is efficient to prevent $\nu$’s from streaming freely.

In this limit, the evolution of the hidden neutrino cosmological perturbations is identical to those of the SM neutrinos and thus not repeated here (see eq. (49) of ref. [49]). We solve the co-evolution of the cosmological perturbations of DM, mediators, hidden neutrinos, and the SM degrees of freedom numerically by suitably modifying CAMB [54]. The resultant matter power spectrum is shown in figure 6. It still shows the dark acoustic oscillation, but the
Figure 6. Linear matter power spectrum in the free-streaming limit of \( \nu \)'s is overplotted in figure 3. The amplitudes of the oscillations are lower than those in the perfect fluid limit of \( \nu \)'s due to the collisionless damping of the \( \nu \) cosmological perturbations. We also plot the linear matter power spectrum in the case of the gradual kinetic decoupling (scalar mediator case, \( \gamma/H \propto T^4 \)) that is shown in figure 2 in ref. [42]. The amplitudes of the oscillations in this case are reduced when compared to those in the case of the sudden kinetic decoupling. Note that \( \nu \)'s are assumed to freely stream in the case of the gradual kinetic decoupling as well.

Powers at the oscillation peaks are suppressed when compared to those in the case of the perfect fluid shown in figure 3. This is because the free-streaming of \( \nu \)'s transfers the initial powers of the density perturbation (monopole) and the bulk velocity potential (dipole) of \( \nu \) to higher multipoles of the Boltzmann hierarchy (see, e.g., [49, 56]). As we discussed in section 3, the dark acoustic oscillation is driven by the bulk velocity potential of \( \nu \). It follows that the DM density perturbation and bulk velocity potential also lose their initial powers. In the plot we also show the linear matter power spectrum in the case of the scalar mediator taken from figure 2 in ref. [42] for reference. In this case \( \nu \)'s are assumed to freely stream (i.e., free-streaming limit), while \( \gamma/H \propto T^4 \) and thus the kinetic decoupling is not sudden, but gradual. This is why the ratio of the power at the second oscillation peak to that at the first one in this case is larger when compared to that in the case of the sudden kinetic decoupling.

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