Reemergence of missing Shapiro steps in the presence of in-plane magnetic field

Bassel Heiba Elfeky\textsuperscript{1}, Joseph J. Cuozzo\textsuperscript{2}, Neda Lotfizadeh\textsuperscript{1}, William F. Schiela\textsuperscript{1}, William M. Strickland\textsuperscript{1}, Dylan Langone\textsuperscript{1}, Enrico Rossi\textsuperscript{2}, and Javad Shabani\textsuperscript{1}

\textsuperscript{1}Center for Quantum Information Physics, Department of Physics, New York University, NY 10003, USA and \textsuperscript{2}Department of Physics, William & Mary, Williamsburg, VA, 23187, USA

(Dated: October 14, 2022)

In the presence of a 4\(\pi\)-periodic contribution to the current phase relation, for example in topological Josephson junctions, odd Shapiro steps are expected to be missing. While missing odd Shapiro steps have been observed in several material systems and interpreted in the context of topological superconductivity, they have also been observed in topologically trivial junctions. Here, we study the evolution of such trivial missing odd Shapiro steps in Al-InAs junctions in the presence of an in-plane magnetic field \(B^\theta\). We find that the odd steps reappear at a crossover \(B^\theta\) value, exhibiting an in-plane field angle anisotropy that depends on spin-orbit coupling effects. We interpret this behavior by theoretically analyzing the Andreev bound state spectrum and the transitions induced by the non-adiabatic dynamics of the junction. Our results highlight the complex phenomenology of missing Shapiro steps and the underlying current phase relations in planar Josephson junctions designed to realize Majorana states.

INTRODUCTION

Josephson junctions (JJs) fabricated on semiconductor structures with epitaxially grown superconductors have recently attracted attention due to their propitious characteristics\textsuperscript{1–9} and applications in quantum computing\textsuperscript{10–18}. In the presence of a Zeeman field\textsuperscript{19–21} or a phase bias\textsuperscript{22–24}, and a strong spin-orbit coupling (SOC) interaction, such high-quality JJs have shown signatures of topological superconductivity\textsuperscript{21–24}, which can host Majorana zero modes useful for fault-tolerant quantum computation\textsuperscript{25,26}. However, robust implementation and signatures of topological superconductivity remain ambiguous\textsuperscript{27–31}.

To harness the potential of topological superconductivity, it is essential to be able to identify unambiguously the topological character of the states in a JJ. Topological JJs exhibit a unique fractional Josephson effect which is inaccessible with DC measurements due to relaxation processes to the ground state. Consequently, detecting the fractional Josephson effect requires measurements on timescales shorter than the relaxation time\textsuperscript{32–37}; timescales that are accessible using microwave excitations\textsuperscript{38–42}.

When a microwave bias is applied to a JJ, the periodic modulation of the current bias becomes phase locked with the dynamics of the junction and results in constant voltage steps in the voltage-current characteristic known as Shapiro steps. The Andreev bound states (ABSs) of a conventional JJ in the short ballistic regime are 2\(\pi\)-periodic in phase \(\phi\), resulting in Shapiro steps at values of \(n\frac{2\pi}{f}\), where \(f\) is the frequency of the microwave drive, and \(n\) is an integer. When the current phase relation (CPR) is 4\(\pi\)-periodic, as expected for a topological JJs, the fractional Josephson effect results in Shapiro steps only at \(n\frac{4\pi}{f}\), resulting in missing odd Shapiro steps. Missing Shapiro steps have been observed in different material systems and are usually attributed to the presence of a topological state\textsuperscript{38–40,42–44}. In practice, even for a topological JJ, a 4\(\pi\)-periodic component CPR coexists with a 2\(\pi\)-periodic component in which case the absence of odd Shapiro steps depends on the details of the junction, and the frequency and power of the microwave radiation\textsuperscript{45–47}.

Recent work\textsuperscript{48} has experimentally shown that topologically trivial JJs can also exhibit missing odd Shapiro steps as predicted previously by other theoretical works\textsuperscript{14,45,49–51}. This can happen when ABSs have a large probability of undergoing a Landau-Zener transition (LZT) at \(\phi \sim \pi\), and a negligible probability of crossing into the continuum, are present. Other mechanisms responsible for missing Shapiro steps have also been proposed involving a bias-dependent junction resistance\textsuperscript{52}, or the presence of multiband superconducting states\textsuperscript{53}. Therefore, the observation of 4\(\pi\)-periodic supercurrent \(I_{4\pi}\) or missing Shapiro steps is a necessary signature of topological superconductivity but is not conclusive. Given that an in-plane magnetic field \(B^\theta\) is one of the ingredients required to drive a JJ to a topological transition, understanding how missing Shapiro steps
depend on $B^\theta$ is essential to distinguish a trivial JJ from its topological counterpart.

In this work, we present measurements on highly-transparent epitaxial Al-InAs JJs in the presence of an in-plane magnetic field $B^\theta$ and SOC effects, conditions associated with inducing topological superconductivity. For $B^\theta = 0 \text{ mT}$, we observe missing odd Shapiro steps with no applied field due to the presence of a topologically trivial $I_{4\pi}$ as observed previously. As $B^\theta$ is increased, these missing Shapiro steps eventually reappear and no topological signatures are observed up to the junction critical field $B^\theta_c$. The reappearance of the missing steps exhibits angle anisotropies that depend on the angle-dependent $B^\theta_c$ and carrier density associated with SOC interaction effects. Our results show the complex dependence of topologically trivial $I_{4\pi}$ on the applied in-plane field magnitude and direction, and SOC effects.

RESULTS

Fig. 1a presents the junction heterostructure studied. An InAs near-surface quantum well is grown between two layers of In$_{0.81}$Ga$_{0.19}$As which is then capped with a thin layer of epitaxial Al grown in situ. Two JJs, JJ1 and JJ2, are fabricated on two different wafers grown under slightly different growth conditions (see Supplementary Information). The junctions are defined using a selective wet etch of the Al and are $w = 4 \mu\text{m}$ wide and $l \sim 100 \text{ nm}$ long. Given $l$ of the junctions, the calculated mean free path to be $l_{\text{mfp}} \approx 150 - 250 \text{ nm}$, and the superconducting coherence length $\xi \approx 530 - 630 \text{ nm}$, the junctions are expected to be in the short ($l < \xi$) ballistic ($l < l_{\text{mfp}}$) regime.

To get insight into the dynamics of such highly transparent junctions, we first perform tight binding simulations of an Al-InAs junction using realistic parameters and calculate the energy spectrum of the ABSs shown in Fig. 1b (simulation details are provided in Supplementary Information). The calculations of these wide junctions present a com-
plex ABS spectrum with hundreds of modes. For a junction with width larger than the coherence length \((w > \xi)\), modes with momentum primarily along the transverse direction behave effectively as “long junction” modes. Consequently, these modes develop a detachment gap \(\delta\) from the continuum when the phase difference across the junction \(\phi = 0\), as indicated in Fig. 1b. The number of long junction modes and their \(\delta\) is sensitive to several factors (density \(n\), \(w\),...). When the junction is highly transparent, the gap at \(\phi = \pi\) is sufficiently small to allow Landau-Zener transitions (LZTs) when the system is diabatically driven\(^{45,48,49}\). The combination of a large detachment gap and a small gap at \(\phi = \pi\) for these long junction modes gives rise to a \(4\pi\)-periodic contribution to the CPR, causing a topologically trivial junction to have both \(2\pi\)- and \(4\pi\)-periodic supercurrent channels\(^{46,47}\). In the presence of a magnetic field in the plane of the junction, the Zeeman effect splits the ABSs and eventually leads to the closing of the detachment gap of the long junction modes, as seen in Fig. 1c. The \(4\pi\)-periodic trajectory of long junction modes is then suppressed due to transitions to the continuum. Additionally, LZTs may occur between long junction modes and other modes with negligible detachments gaps, leading to transitions to the continuum mediated by conventional ABSs and suppressing \(I_{4\pi}\).

To experimentally investigate such trivial \(4\pi\)-modes, we examine the microwave response of JJ1 in a DC current-biased setup. The measurements are carried out at \(T = 30\) mK where the junction exhibits no hysteresis, as seen in Supplementary Fig. S2. In Fig. 1d, we present \(V/dI\) as a function of the DC current bias and RF power at \(f = 12\) GHz in addition to a histogram of the voltage distribution. For this value of \(f\), we can identify all the integer Shapiro steps along with subharmonic Shapiro steps. Subharmonic Shapiro steps are expected at high frequencies due to the anharmonicity associated with the forward skewness of the CPR in highly transparent junctions\(^{39,54–59}\). The presence of a \(4\pi\)-periodic supercurrent channel, with critical current \(I_{4\pi}\), is expected to result in missing odd Shapiro steps\(^{44,45,48–51}\) when the energy of the photon irradiating the JJ, \(hf\), is less than \(hf_{4\pi} \approx 2eI_{4\pi}R_n\)\(^{46,47}\). Fig. 1e shows a similar Shapiro map for \(f = 7\) GHz where we see that the first odd Shapiro step is missing indicating the presence of a finite \(I_{4\pi}\) even though the JJ is in a topologically trivial regime. For JJ1, at \(B^\theta = 0\) mT, we find \(f_{4\pi} \approx 8.2\) GHz corresponding to \(I_{4\pi} = 52.1\) nA. Considering the Josephson frequency, \(f_J \equiv \frac{2eI_c}{h}\), for JJ1, we get \(f_{4\pi}/f_J \approx I_{4\pi}/I_c\) corresponding to 6.5% of the supercurrent being carried by a \(4\pi\)-periodic supercurrent channel.

We next consider the dependence of the critical current \(I_c\) in JJ1 on a magnetic field, without a microwave bias, as seen in the differential resistance map in Fig. 1f where the in-plane magnetic field is applied along the junction, \(B^\theta\). The critical field, \(B_{c^\theta}\), is seen to be \(\sim 620\) mT. Similar measurements performed at different \(\theta\) values are presented in Supplementary Fig. S3. The field dependence data show no topological signatures such as a minimum in \(I_c\)\(^{21}\), indicating that the junctions are topologically trivial for all the values of \(B^\theta\) up to the critical field \(B_{c^\theta}\).
in Fig. 2 imply that \( f_{4\pi} \) does not scale proportionally with \( f_1 \). In fact, the ratio \( f_{4\pi}/f_1 \) generally increases as a function of in-plane field strength. This indicates that the suppression of \( I_{4\pi} \) is not simply proportional to the critical current \( I_c \), implying that the response of diabatically driven long junction modes to an in-plane field is distinct from conventional “short junction” modes that make up the rest of the spectrum in 2DEG JJs and the entire spectrum in narrow junctions e.g., nanowire junctions.

Next, we consider the \( I_{4\pi} \) dependence on the applied in-plane field direction, \( \theta \). A topologically non-trivial \( I_{4\pi} \) is expected to be sensitive\(^{60} \) to \( \theta \); on the other hand, the angle dependence of a trivial \( I_{4\pi} \) resulting from LZT is ambiguous and can depend on several contributing effects from Zeeman, orbital and SOC interactions. Fig. 3a and b show Shapiro maps with \( f = 3.5 \) GHz at \( B^0 = 200 \) mT for \( \theta = 30^\circ \) and \( \theta = 90^\circ \). Unlike the \( \theta = 0^\circ \) case presented in Fig. 2b, the first step appears to partially reemerge for \( \theta = 30^\circ \) and completely reemerges for \( \theta = 90^\circ \), which indicates an angle anisotropy of \( I_{4\pi} \). To determine more precisely the threshold value of \( B^0 \) above which the first step reappears, we calculate \( Q_{12} \) as a function of \( B^0 \) where the ratio \( Q_{12} = \frac{Q^{(y)}}{Q^{(x)}} \) represents the strength of the first step with respect to the second found by binning the voltage distribution and calculating the max step size/bin count of the first (second) step, \( s_1 \) (\( s_2 \)). More details about the extraction of \( Q_{12} \) from the data are provided in the Supplementary Information. We then identify the crossover field \( B^0_{co} \) for the in-plane angle \( \theta \) as the value of \( B^0 \) for which \( Q_{12} \approx 1 \). Fig. 3c shows the evolution of \( Q_{12} \) with \( B^0 \) for \( \theta = 0^\circ \) and \( \theta = 90^\circ \). In both cases, the first step is suppressed up to the crossover value \( B^0_{co} \), and is fully present for values \( B^0 > B^0_{co} \). The scaling of \( Q_{12} \) is seen to exhibit clear anisotropy with respect to \( B^0 \): \( \theta = 0^\circ \) shows a \( B^0_{co} \approx 400 \) mT, whereas \( \theta = 90^\circ \) shows a \( B^0_{co} \approx 200 \) mT.

In Fig. 3e, we present a polar plot of \( B^0_{co} \) (at \( f = 3.5 \) GHz) as a function \( \theta \). A large variation in crossover field is observed; however, we note that the critical field \( B^0_{c} \) for \( \theta = 0^\circ \) and \( 90^\circ \) are significantly different \((B^0_c = 620 \text{ mT} \text{ and } B^0_{90} = 320 \text{ mT})\) as seen in Fig. 3d, similar to other Al-InAs junctions\(^{61} \). When normalized by their respective critical fields to account for the angle-dependence of \( B^0_c \), the crossover fields become quantitatively similar and in fact match a fit of \( B^0_{co}/B^0_{c} = 67\% \) as seen in Fig. 3f. This suggests that the anisotropy observed in \( B^0_{co} \) is likely due to the variation in critical field and implies that JJ1 has weak SOC effects.

---

**Figure 3. Angle dependence of reemergence of missing Shapiro step.** Shapiro maps at \( B^0 = 200 \) mT for a \( \theta = 30^\circ \) and b \( \theta = 90^\circ \). c Calculated \( Q_{12} \) and d \( I_c/R_n \) as a function of in-plane magnetic field \( B^0 \) for in-plane field angles \( \theta = 0^\circ \) and \( 90^\circ \). e, f The crossover field \( B^0_{co} \), field value at which missing Shapiro step first fully reemerges, presented in e units of Tesla and f normalized by the corresponding critical field \( B^0_{c} \), as a function of \( \theta \).

**Missing Shapiro steps in an in-plane magnetic field.** In Fig. 2, we present Shapiro maps for various magnetic field strengths applied along the junction for \( f = 3.5 \) GHz and \( f = 6.4 \) GHz. At \( B^{0\circ} = 0 \) mT, the first Shapiro step is seen to be missing for both frequencies since \( f < f_{4\pi} \). At \( B^{0\circ} = 200 \) mT, the first step almost completely emerges for \( f = 6.4 \) GHz while still being missing for \( f = 3.5 \) GHz. At \( B^{0\circ} \approx 300 \) mT, the first step starts emerging for \( f = 3.5 \) GHz, eventually completely appearing at \( B^{0\circ} = 400 \) mT. This behavior implies a decrease of \( I_{4\pi} \) as a function of in-plane field strength, consistent with the mechanisms described in Fig. 1c. We note that the data presented
Reemergence of missing Shapiro steps at finite gate voltage. One of the unique advantages of using a semiconductor-based system is the ability to have electrostatic tunability of the carrier density and SOC interaction using a gate. To study the trivial $I_4\pi$ dependence on such properties, we focus on JJ2 fabricated on the same heterostructure presented in Fig. 1 but equipped with a top gate. JJ2 is expected to have a stronger SOC interaction than JJ1 even at zero gate voltage ($V_g = 0$ V) due to the presence of a gate dielectric Al$_2$O$_3$ layer (see Supplementary Information) that tends to increase the carrier density and consequently SOC interaction. JJ2 is markedly hysteretic at 30 mK due to thermal effects$^{62}$ and so it is studied at 800 mK where it shows no hysteresis. At $B^\theta = 0$ mT and $V_g = 0$ V, JJ2 exhibits a missing first Shapiro step as seen in Supplementary Fig. S8 even though at $T = 800$ mK the overall transparency is expected to be reduced. Further, Supplementary Fig. S4 shows that JJ2 exhibits a similar $B^\theta_c$ anisotropy to that of JJ1. However, we note that for JJ2, $B^\theta_c$ also depends on $V_g$.

In Fig. 4, we present measurements performed on JJ2 at $f = 3.4$ GHz for $V_g = -5$ V and $+10$ V at different $B^\theta_0$ and $B^{90\theta}_0$ values. For $\theta = 0^\circ$, $V_g = -5$ V shows $B^\theta_0 = 125$ mT while $V_g = +10$ V shows $B^{90\theta}_0 = 225$ mT. The difference between $V_g = -5$ V and $+10$ V is reconciled when considering $B^{\theta_0}/B^{90\theta_0}(V_g)$, as seen in Fig. 4i, where both $V_g$ values exhibit a $B^{\theta_0}(V_g)/B^{90\theta_0}(V_g)$ of $\sim$ 40%. For $\theta = 90^\circ$, the data presented in Fig. 4j show a $B^{\theta_0}/B^{90\theta_0}$ ratio of $\sim$ 57% and $\sim$ 65% for $V_g = -5$ V and $+10$ V, respectively. While the $\theta = 90^\circ$ case exhibits similar $B^{\theta_0}/B^{90\theta_0}$ values to that reported for JJ1, the $\theta = 0^\circ$ case shows a significant discrepancy for both $V_g$ values. It is evident here that for JJ2, the angle anisotropy is not simply accounted for by considering $B^{\theta_0}_c$ and that other effects play a role in the suppression of $I_4\pi$, consistent with the expectation of JJ2 having stronger SOC effects in comparison to JJ1. In the following, we discuss the origin of such suppression of $I_4\pi$ and the observed angle anisotropy by considering the ABS spectrum.

Theoretical analysis. Following the picture presented in Fig. 1c, we first consider the suppression of $I_4\pi$ in terms of transitions between the long junction modes to the continuum, related mainly to the
detachment gap $\delta$. Using tight-binding simulations, we calculate the energy spectrum of the ABS spectrum in an InAs-Al junction. Fig. 5a shows a linear decrease in $\delta$ as a function of the Zeeman field $\Delta_Z^\theta$. The decrease in $\delta$ results in a higher probability of undergoing LZTs to the continuum, suppressing the $4\pi$-component of the CPR. In the absence of SOC effects ($\lambda_{SOC} = 0$), corresponding to the black line in Fig. 5a, the suppression of $\delta$ as a function of $B^\theta$ shows no $\theta$-dependence.

In the presence of strong SOC effects, the Fermi surface of the quantum well has an anisotropic response to an in-plane Zeeman field, creating an anisotropic suppression of $\delta$ in the ABS spectrum. For $\lambda_{SOC} = 7.5$ meV $\cdot$ nm, Fig. 5a illustrates that a larger $\Delta_Z^\theta$ in the $\theta = 0^\circ$ (green line) direction is needed than in the $\theta = 90^\circ$ (orange line) direction to suppress $\delta$ by the same amount. However, Fig. 4i and j shows $B^\theta_{090}/B^\theta_{000} < B^\theta_{090}/B^\theta_{000}$. This indicates that the presence of strong SOC (as expected for JJ2) enhances the lack of correlation between the suppression of $I_{4\pi}$ and of $\delta$.

We thus consider the suppression of $I_{4\pi}$ in terms of mode-to-mode coupling. Due to the large number of ABS modes in our junctions, a result of the large width $w$, we have a very dense ABS spectrum. Consequently, we have several quasi-avoided crossings between ABSs and between ABSs and the continuum. In the presence of a Zeeman field, the ABS spectrum becomes even more complex, with more quasi-avoided crossings and new protected crossings. A fully microscopic description of the JJ would require the determination of the dynamics of a multi-level Landau-Zener problem. This is a problem that is computationally prohibitive to solve. However, to gain a qualitative understanding, we can estimate the relevant multi-mode couplings by calculating the wave function overlap between a long junction mode at $\theta = \phi_i$ and all positive energy Andreev mid-gap states at $\theta = \phi_f$ far from the avoided crossing at $\theta = \pi$, as shown schematically in Fig. 5b. This allows to estimate the probability that an occupied ABS, when $\phi \approx \pi$, can either transition to an ABS with a large detachment from the continuum and therefore contribute to $I_{4\pi}$, or transition to an ABS with a small $\delta$ and therefore contribute solely to $I_{2\pi}$. We provide a detailed discussion of the calculations in the Supplementary Information. In Fig. 5c, we present a histogram of the wave function overlaps $|\langle \psi_m(\phi_i)|\psi_m(\phi_f) \rangle|^2$ between a long junction mode $|\psi_m\rangle$ and modes $|\psi_m\rangle$ for $\phi_i = 0.6\pi$ and $\phi_f = 1.41\pi$. At $\Delta_Z = 0$, we observe a distribution localized at zero except for a single outlier shown in the inset. This outlier corresponds to an overlap with another long junction mode. At finite $\Delta_Z^\theta$ and $\lambda_{SOC} = 7.5$ meV $\cdot$ nm, more states develop a non-zero overlap with the long junction mode evident from the histogram distribution. The histogram distribution also shows that the system is more sensitive to $\Delta_Z^\theta$ in the $\theta = 0^\circ$ direction than the $\theta = 90^\circ$ direction with the $\theta = 0^\circ$ case exhibiting a broader distribution. These results suggest that the distribution of the overlaps between ABS states across $\theta = \pi$, through their effect on Landau-Zener transitions, may play an important role in the anisotropy observed in Fig. 4i and j for JJ2, where a strong SOC interaction is present.

**DISCUSSION**

By studying the microwave response of an epitaxial Al-InAs JJ, we observe signatures of a $4\pi$-periodic contribution to the CPR attributed to topologically-trivial LZT between long junction modes. With the application of an external magnetic field, the $I_{4\pi}$ is observed to be suppressed differently to $I_{2\pi}$ and eventually disappears at a crossover field. In a device with weak SOC (JJ1), we observe an isotropic suppression of $I_{4\pi}$ with an applied magnetic field when the device’s angle anisotropy in $B^\theta$ is taken into account. In the gate tunable device (JJ2) with a significantly larger SOC, an anisotropic suppression of $I_{4\pi}$ is observed, which cannot be accounted for by the device’s $B^\theta$ angle anisotropy. We attribute the anisotropy to SOC effects which introduce a non-trivial angle $\theta$ dependence in the coupling of long junction modes to other Andreev mid-gap states lacking a detachment gap, suggesting multi-level LZTs. Our results indicate that such anisotropy in in-plane magnetic field and dependence on SOC effects need to be considered when differentiating between topologically trivial and non-trivial $I_{4\pi}$ and requires other correlated signatures to make claims about topological superconductivity.

**METHODS**

**Fabrication details.** The devices were fabricated by electron beam lithography using spin-coated PMMA resist. To define the mesa features of the junction, Al is removed with Transene Al etchant type-D followed by a wet etch down to the buffer layer using a III-V etchant consisting of phosphoric acid ($H_3PO_4$, 85%), hydrogen peroxide ($H_2O_2$, 30%) and deionized water in a volumetric ratio of 1:1:40. The junction gap and contacts
Figure 5. Theoretical ABS spectrum analysis in the presence of a Zeeman field. a Calculation of the detachment gap $\delta$ for a junction with $w = 500$ nm wide and $l = 100$ nm as a function of the Zeeman energy $\Delta Z$ for $\lambda_{SOC} = 0$ and for $\theta = 0^\circ$ and $\theta = 90^\circ$ with $\lambda_{SOC} = 7.5$ meV·nm. b Schematic of the Andreev bound state spectrum illustrating which states are used to calculate wavefunction overlaps. c Distribution of wavefunction overlaps with $\lambda_{SOC} = 7.5$ meV·nm between $\phi_i = 0.6\pi$ and $\phi_f = 1.41\pi$ for $\Delta Z = 0$ and $0.3\Delta$ for $\theta = 0^\circ$ and $90^\circ$. Inset: outlier overlaps where $|\psi_m\rangle$ is a long junction mode.

were subsequently defined by a wet etch using Transene Al etchant type D. For JJ2, 60 nm of aluminum oxide dielectric was then deposited by atomic layer deposition to electrically isolate the gate electrodes. Next, a top gate, leads, and bonding pads consisting of 5 nm Cr and 60 nm Au were deposited via electron beam evaporation for JJ2. JJ1 does not have a dielectric layer or gates.

Measurements details. Our measurements are performed in an Oxford Triton dilution refrigerator fitted with a vector magnet. For JJ1, all measurements were performed at 30 mK where there is no hysteresis observed as seen in Fig. S1b. For JJ2, all measurements were performed at 800 mK to avoid the effects of hysteresis since the junction is hysteretic at 30 mK but not 800 mK as seen in Fig S5. Standard dc and lock-in techniques are used at low frequencies (17 Hz) with current excitation of $I_{ac} = 10$ nA in a four-point geometry using a current-biased configuration by sweeping $I_{dc}$ and the differential resistance $dV/dI$ using an SRS860 lock-in amplifier as well as the voltage drop $V$ across the junction using a Keithley DMM6500. We measure the switching or critical current at which the junction switches from the superconducting to the normal resistive state. All current bias sweeps are done from negative to positive unless specified otherwise.

REFERENCES

[1] P. Krogstrup, N. L. B. Ziino, W. Chang, S. M. Albrecht, M. H. M. Madsen, E. Johnson, J. Nygård, C. Marcus, and T. S. Jespersen, Epitaxy of semiconductor–superconductor nanowires, Nature Materials 14, 400 (2015).

[2] J. Shabani, M. Kjaergaard, H. J. Suominen, Y. Kim, F. Nichele, K. Pakrouski, T. Stankovic, R. M. Lutchyn, P. Krogstrup, R. Feidenhans’l, S. Kraemer, C. Nayak, M. Troyer, C. M. Marcus, and C. J. Palmstrøm, Two-dimensional epitaxial superconductor-semiconductor heterostructures: A platform for topological superconducting networks, Physical Review B 93, 155402 (2016).

[3] M. Kjaergaard, H. Suominen, M. Nowak, A. Akhmerov, J. Shabani, C. Palmstrøm, F. Nichele, and C. Marcus, Transparent Semiconductor-Superconductor Interface and Induced Gap in an Epitaxial Heterostructure Josephson Junction, Physical Review Applied 7, 034029 (2017).

[4] C. G. L. Bøttcher, F. Nichele, M. Kjaergaard, H. J. Suominen, J. Shabani, C. J. Palmstrøm, and C. M. Marcus, Superconducting, insulating and anomalous metallic regimes in a gated two-dimensional semiconductor–superconductor array, Nature Physics 14, 1138 (2018).

[5] W. Mayer, J. Yuan, K. S. Wickramasinghe, T. Nguyen, M. C. Dartiailh, and J. Shabani, Superconducting proximity effect in epitaxial al-InAs heterostructures, Applied Physics Letters 114, 103104 (2019).

[6] J. S. Lee, B. Shojaei, M. Pendharkar, A. P. McFadden, Y. Kim, H. J. Suominen, M. Kjaergaard, F. Nichele, H. Zhang, C. M. Marcus, and C. J. Palmstrøm, Transport Studies of Epi-Al/InAs Two-Dimensional Electron Gas Systems for Required Building-Blocks in Topological Superconductor Networks, Nano Letters 19, 3083 (2019).

[7] W. Mayer, M. C. Dartiailh, J. Yuan, K. S. Wickramasinghe, E. Rossi, and J. Shabani, Gate controlled anomalous phase shift in al/inas josephson junctions, Nature Communications 11, 212 (2020).
[8] B. H. Elfeky, N. Lotfizadeh, W. F. Schiela, W. M. Strickland, M. Dartiailh, K. Sardashti, M. Hatetipour, P. Yu, N. Pankratova, H. Lee, V. E. Manucharyan, and J. Shabani, Local Control of Supercurrent Density in Epitaxial Planar Josephson Junctions, Nano Letters 21, 8274 (2021).

[9] W. M. Strickland, M. Hatetipour, D. Langone, S. M. Farzaneh, and J. Shabani, Controlling Fermi level pinning in near-surface InAs quantum wells (2022), arXiv:2206.01057 [cond-mat].

[10] T. W. Larsen, K. D. Petersson, F. Kuemmeth, T. S. Jespersen, P. Kroghstrup, J. Nygård, and C. M. Marcus, Semiconductor-nanowire-based superconducting qubit, Phys. Rev. Lett. 115, 127001 (2015).

[11] F. Luthi, T. Stavenga, O. W. Enzing, A. Bruno, C. Dickel, N. K. Langford, M. A. Rol, T. S. Jespersen, J. Nygård, P. Kroghstrup, and L. DiCarlo, Evolution of nanowire transmon qubits and their coherence in a magnetic field, Phys. Rev. Lett. 120, 100502 (2018).

[12] A. Kringhøj, L. Casparis, M. Hell, T. W. Larsen, F. Kuemmeth, M. Leijnse, K. Flensberg, P. Kroghstrup, J. Nygård, K. D. Petersson, and C. M. Marcus, Anharmonicity of a superconducting qubit with a few-mode josephson junction, Phys. Rev. B 97, 060508 (2018).

[13] L. Casparis, M. R. Connolly, M. Kjaergaard, N. J. Pearson, A. Kringhøj, T. W. Larsen, F. Kuemmeth, T. Wang, C. Thomas, S. Gronin, G. C. Gardner, M. J. Manfra, C. M. Marcus, and K. D. Petersson, Superconducting gateon qubit based on a proximitized two-dimensional electron gas, Nature Nanotechnology 13, 915 (2018).

[14] L. Casparis, T. W. Larsen, M. S. Olsen, F. Kuemmeth, P. Kroghstrup, J. Nygård, K. D. Petersson, and C. M. Marcus, Gateon benchmarking and two-qubit operations, Phys. Rev. Lett. 116, 150505 (2016).

[15] J. O’Connell Yuan, K. S. Wickramasinghe, W. M. Strickland, M. C. Dartiailh, K. Sardashti, M. Hatetipour, and J. Shabani, Epitaxial superconductor-semiconductor two-dimensional systems for superconducting quantum circuits, Journal of Vacuum Science & Technology A 39, 033407 (2021).

[16] A. Danilenko, D. Sabonis, G. W. Winkler, O. Erdlandsson, P. Kroghstrup, and C. M. Marcus, Few-mode to mesoscopic junctions in gateon qubits (2022).

[17] A. Hertel, M. Eichinger, L. O. Andersen, D. M. T. van Zanten, S. Kallatt, P. Scarlino, A. Kringhøj, J. M. Chavez-Garcia, G. C. Gardner, S. Gronin, M. J. Manfra, A. Gyenis, M. Kjaergaard, C. M. Marcus, and K. D. Petersson, Gate-tunable transmon using selective-area-grown superconductor-semiconductor hybrid structures on silicon (2022).

[18] W. M. Strickland, B. H. Elfeky, J. O. Yuan, W. F. Schiela, P. Yu, D. Langone, M. G. Vavilov, V. E. Manucharyan, and J. Shabani, Superconducting resonators with voltage-controlled frequency and nonlinearity (2022), arXiv:2210.02491 [cond-mat, physics:quant-ph].

[19] M. Hell, M. Leijnse, and K. Flensberg, Two-Dimensional Platform for Networks of Majorana Bound States, Physical Review Letters 118, 107701 (2017).

[20] F. Pientka, A. Keselman, E. Berg, A. Yacoby, A. Stern, and B. I. Halperin, Topological Superconductivity in a Planar Josephson Junction, Physical Review X 7, 021032 (2017).

[21] M. C. Dartiailh, W. Mayer, J. Yuan, K. S. Wickramasinghe, A. Matos-Abiague, I. Žutić, and J. Shabani, Phase Signature of Topological Transition in Josephson Junctions, Physical Review Letters 126, 036802 (2021).

[22] H. Ren, F. Pientka, S. Hart, A. T. Pierce, M. Kosowsky, L. Lunczer, R. Schlereth, B. Scharf, E. M. Hankiewicz, L. W. Molenkamp, B. I. Halperin, and A. Yacoby, Topological superconductivity in a phase-controlled Josephson junction, Nature 569, 93 (2019).

[23] A. Fornieri, A. M. Whiticar, F. Setiawan, E. Portolés, A. C. C. Drachmann, A. Keselman, S. Gronin, C. Thomas, T. Wang, R. Kallaher, G. C. Gardner, E. Berg, M. J. Manfra, A. Stern, C. M. Marcus, and F. Nichele, Evidence of topological superconductivity in planar Josephson junctions, Nature 569, 89 (2019).

[24] A. Banerjee, O. Lesser, M. A. Rahman, H. R. Wang, M. R. Li, A. Kringhøj, A. M. Whiticar, A. C. C. Drachmann, C. Thomas, T. Wang, M. J. Manfra, E. Berg, Y. Øreg, A. Stern, and C. M. Marcus, Signatures of a topological phase transition in a planar Josephson junction, arxiv 10.48550/ARXIV.2201.03453 (2022), publisher: arXiv Version Number: 1.

[25] C. Nayak, S. H. Simon, A. Stern, M. Freedman, and S. Das Sarma, Non-Abelian anyons and topological quantum computation, Reviews of Modern Physics 80, 1083 (2008).

[26] D. Aasen, M. Hell, R. V. Mishmash, A. Higginbotham, J. Danon, M. Leijnse, T. S. Jespersen, J. A. Folk, C. M. Marcus, K. Flensberg, and J. Alicea, Milestones Toward Majorana-Based Quantum Computing, Physical Review X 6, 031016 (2016).

[27] S. Mohapatra, S. Mathimalar, S. Chaudhary, and K. V. Raman, Observation of zero-bias conductance peak in topologically-trivial hybrid superconducting interfaces, Journal of Physics Communications 3, 045005 (2019).

[28] H. Pan, W. S. Cole, J. D. Sau, and S. Das Sarma, Generic quantized zero-bias conductance peaks in superconductor-semiconductor hybrid structures, Physical Review B 101, 024506 (2020).

[29] J. Cayao and P. Burset, Confinement-induced zero-bias peaks in conventional superconductor hybrids, Physical Review B 104, 134507 (2021).

[30] P. Yu, J. Chen, M. Gomanko, G. Badawy, E. P. A. M. Bakkers, K. Zuo, V. Mourik, and S. M. Frolov, Non-Majorana states yield nearly quantized conductance in proximatized nanowires, Nature Physics 17, 482 (2021).

[31] H. Pan and S. Das Sarma, On-demand large con-
ductance in trivial zero-bias tunneling peaks in Majorana nanowires, Physical Review B 105, 115432 (2022).

[32] H.-J. Kwon, K. Sengupta, and V. M. Yakovenko, Fractional ac Josephson effect in p- and d-wave superconductors, The European Physical Journal B - Condensed Matter 37, 349 (2003).

[33] M. D. Shaw, R. M. Lutchyn, P. Delsing, and P. M. Echternach, Kinetics of nonequilibrium quasiparticle tunneling in superconducting charge qubits, Physical Review B 78, 024503 (2008).

[34] J. M. Martinis, M. Ansmann, and J. Aumentado, Energy Decay in Superconducting Josephson-Junction Qubits from Nonequilibrium Quasiparticle Excitations, Physical Review Letters 103, 097002 (2009).

[35] D. I. Pikulin and Y. V. Nazarov, Phenomenology and dynamics of a Majorana Josephson junction, Physical Review B 86, 140504 (2012).

[36] P. M. Badiane, L. I. Glazman, M. Houzet, and J. S. Meyer, Ac Josephson effect in topological Josephson junctions, Comptes Rendus Physique 14, 840 (2013).

[37] D. J. van Weerom, A. Geresdi, and L. P. Kouwenhoven, One minute parity lifetime of a NiTiN Cooper-pair transistor, Nature Physics 11, 547 (2015).

[38] L. P. Rokhinson, X. Liu, and J. K. Furdyna, The fractional a.c. Josephson effect in a semiconductor—superconductor nanowire as a signature of Majorana particles, Nature Physics 8, 795 (2012).

[39] J. Wiedemann, E. Bocquillon, R. S. Deacon, S. Hartinger, O. Herrmann, T. M. Klapwijk, L. Maier, C. Ames, C. Brüne, C. Gould, A. Oiwa, K. Ishibashi, S. Tarucha, H. Buhmann, and L. W. Molenkamp, 4\pi-periodic Josephson supercurrent in HgTe-based topological Josephson junctions, Nature Communications 7, 10303 (2016).

[40] E. Bocquillon, R. S. Deacon, J. Wiedemann, P. Leubner, T. M. Klapwijk, C. Brüne, K. Ishibashi, H. Buhmann, and L. W. Molenkamp, Gapless Andreev bound states in the quantum spin Hall insulator HgTe, Nature Nanotechnology 12, 137 (2017).

[41] R. Deacon, J. Wiedemann, E. Bocquillon, F. Domínguez, T. Klapwijk, P. Leubner, C. Brüne, E. Hankiewicz, S. Tarucha, K. Ishibashi, H. Buhmann, and L. Molenkamp, Josephson Radiation from Gapless Andreev Bound States in HgTe-Based Topological Junctions, Physical Review X 7, 021011 (2017).

[42] D. Laroch, D. Bouman, D. J. van Weerom, A. Proutska, C. Murthy, D. I. Pikulin, C. Nayak, R. J. J. van Gulik, J. Nygård, P. Krogstrup, L. P. Kouwenhoven, and A. Geresdi, Observation of the 4\pi-periodic Josephson effect in indium arsenide nanowires, Nature Communications 10, 245 (2019).

[43] C. Li, J. C. de Boer, B. de Ronde, S. V. Ramankutty, E. van Heumen, Y. Huang, A. de Visser, A. A. Golubov, M. S. Golden, and A. Brinkman, 4\pi-periodic Andreev bound states in a Dirac semimetal, Nature Materials 17, 875 (2018).

[44] R. Fischer, J. Picó-Cortés, W. Himmel, G. Platero, M. Grifoni, D. A. Kozlov, N. N. Mikhailov, S. A. Dvoretsky, C. Strunk, and D. Weiss, 4\pi-periodic supercurrent tuned by an axial magnetic flux in topological insulator nanowires, Physical Review Research 4, 013087 (2022).

[45] F. Domínguez, F. Hassler, and G. Platero, Dynamical detection of Majorana fermions in current-biased nanowires, Physical Review B 86, 140503 (2012).

[46] F. Domínguez, O. Kashuba, E. Bocquillon, J. Wiedemann, R. S. Deacon, T. M. Klapwijk, G. Platero, L. W. Molenkamp, B. Trauzettel, and E. M. Hankiewicz, Josephson junction dynamics in the presence of 2\pi - and 4\pi-periodic supercurrents, Physical Review B 95, 195430 (2017).

[47] J. Picó-Cortés, F. Domínguez, and G. Platero, Signatures of a 4\pi-periodic supercurrent in the voltage response of capacitively shunted topological Josephson junctions, Physical Review B 96, 125438 (2017).

[48] M. C. Duriaux, J. J. Cuozzo, B. H. Efeyk, W. Mayer, J. Yuan, K. S. Wickramasinghe, E. Rossi, and J. Shabani, Missing Shapiro steps in topologically trivial josephson junction on InAs quantum well, Nature Communications 12, 78 (2021).

[49] P.-M. Billangeon, F. Pierre, H. Bouchiat, and R. Deblock, ac Josephson Effect and Resonant Cooper Pair Tunneling Emission of a Single Cooper Pair Transistor, Physical Review Letters 98, 216802 (2007).

[50] J. D. Sau and F. Setiawan, Detecting topological superconductivity using low-frequency doubled Shapiro steps, Physical Review B 95, 060501 (2017).

[51] A. V. Galaktionov and A. D. Zaikin, Fractional Shapiro steps without fractional Josephson effect, Physical Review B 104, 054521 (2021).

[52] S. R. Mudi and S. M. Frolov, Model for missing Shapiro steps due to bias-dependent resistance, arXiv:2106.00495 [cond-mat] (2021), arXiv:2106.00495.

[53] J. J. Cuozzo, W. Yu, P. Davids, T. M. Menoff, D. B. Soh, W. Pan, and E. Rossi, Leggett modes in dirac semimetals (2022), arXiv:2205.15995 [cond-mat.supr-con].

[54] G.-H. Lee, S. Kim, S.-H. Jhi, and H.-J. Lee, Ultimately short ballistic vertical graphene Josephson junctions, Nature Communications 6, 6181 (2015).

[55] I. N. Askerzade, Effects of anharmonicity of current-phase relation in Josephson junctions (Review Article), Low Temperature Physics 41, 241 (2015).

[56] R. Snyder, C. Trimble, C. Rong, P. Folkes, P. Taylor, and J. Williams, Weak-link Josephson Junctions Made from Topological Crystalline Insulators, Physical Review Letters 121, 097701 (2018).

[57] A. Kringlehøj, L. Casparis, M. Hell, T. W. Larsen, F. Kuemmeth, M. Leijnse, K. Flensberg, P. Krogstrup, J. Nygård, K. D. Petersson, and C. M. Marcus, Anharmonicity of a superconducting qubit with a few-mode Josephson junction, Physical Review B 97, 060508 (2018).

[58] R. Panghotra, B. Raes, C. C. de Souza Silva, I. Cools, W. Keijers, J. E. Scheerder, V. V.
Moshchalkov, and J. Van de Vondel, Giant fractional Shapiro steps in anisotropic Josephson junction arrays, Communications Physics 3, 53 (2020).

[59] J. O’Connell Yuan, K. S. Wickramasinghe, W. M. Strickland, M. C. Dartailh, K. Sardashti, M. Hatefipour, and J. Shabani, Epitaxial superconductor-semiconductor two-dimensional systems for superconducting quantum circuits, Journal of Vacuum Science & Technology A 39, 033407 (2021).

[60] B. Scharf, F. Pientka, H. Ren, A. Yacoby, and E. M. Hankiewicz, Tuning topological superconductivity in phase-controlled Josephson junctions with Rashba and Dresselhaus spin-orbit coupling, Physical Review B 99, 214503 (2019).

[61] H. J. Suominen, J. Danon, M. Kjaergaard, K. Flensberg, J. Shabani, C. J. Palmstrøm, F. Nichele, and C. M. Marcus, Anomalous Fraunhofer interference in epitaxial superconductor-semiconductor Josephson junctions, Physical Review B 95, 035307 (2017).

[62] H. Courtois, M. Meschke, J. T. Peltonen, and J. P. Pekola, Origin of Hysteresis in a Proximity Josephson Junction, Physical Review Letters 101, 067002 (2008).

**Author contributions** B.H.E and W.F.S fabricated the devices on epitaxial Al-InAs heterostructures grown by W.M.S and D.L.. The measurements were performed by B.H.E and N.L. with J.S. providing input. J.S. conceived the experiment. B.H.E performed the data analysis. J.J.C. and E.R. developed the theoretical model and J.J.C. carried out the simulations. The manuscript was written by B.H.E., J.J.C., E.R., and J.S. with suggestions from all other authors.

**Acknowledgements** We thank Matthieu C. Dartailh for fruitful discussions. The NYU team acknowledges support by DARPA TEE award no. DP18AP900007. We acknowledge funding from DOE award no. DE-SC0022245. J.J.C also acknowledges support from the Graduate Research Fellowship awarded by the Virginia Space Grant Consortium (VSGC). J.J.C. and E.R. acknowledge William & Mary Research Computing for providing computational resources and/or technical support that have contributed to the results reported within this paper. URL: https://www.wm.edu/it/rc. W.F.S. acknowledges funding from an ND-SEG Fellowship. W.M.S. acknowledges funding from the ARO/LPS QuaCR Graduate Fellowship.