EFFECT OF HALO BIAS AND LYMAN LIMIT SYSTEMS ON THE HISTORY OF COSMIC REIONIZATION

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ABSTRACT

We extend the existing analytical model of reionization by Furlanetto et al. to include the biasing of reionization sources and additional absorption by Lyman limit systems. Both effects enhance the original model in non-trivial ways, but do not change its qualitative features. Our model is, by construction, consistent with the observed evolution of the galaxy luminosity function at $z \lesssim 8$ and with the observed evolution of Lyα forest at $z \lesssim 6$. We find that the same model can match the Wilkinson Microwave Anisotropy Probe/Planck constraint on the Thompson optical depth and the South Pole Telescope and EDGES constraints on the duration of reionization for values of the relative escape fraction that are consistent with the observational measurements at lower redshifts. However, such a match is only possible if dwarf galaxies contribute substantially to the ionizing photon budget. The latter condition is inconsistent with simulations and observational upper limits on the escape fraction from dwarfs at $z \sim 3$. Whether such a disagreement is due to the different nature of $z > 6$ galaxies, the inadequacy of simulations and/or some of the observational constraints, or indicates an additional source of ionizing radiation at $z > 8$ remains to be seen.

Key words: cosmology: theory – intergalactic medium – methods: analytical

1. INTRODUCTION

After the recombination at redshift $z \sim 1090$ (Hinshaw et al. 2012), the hydrogen in the universe was mostly neutral. The epoch of reionization (EoR) started at $z \sim 20$ with the formation of the first star-forming galaxies and quasars. According to the current understanding of reionization, the ultraviolet radiation from these objects was the primary source of energy that stripped electrons off the hydrogen atoms (Gnedin 2008), leaving most intergalactic matter highly ionized (Fan et al. 2006).

Presently, there are three primary observational probes that provide information about the EoR. The first one is the Lyα forest. The first one is the Lyα forest, which can trace the distribution of neutral hydrogen in the universe and the small-scale structure of ionized bubbles, but only up to $z \sim 6$ (i.e., only late stages of the EoR can be probed with this method; Fan 2008; Fan et al. 2006; Gallerani et al. 2010). Another probe is the surveys of Lyα emitters (Malhotra & Rhoads 2004; Kashikawa et al. 2006; McQuinn et al. 2007; Ouchi et al. 2010; Bolton & Haehnelt 2013), which can statistically constrain the size distribution of ionized bubbles from the clustering strength of Lyα bright galaxies. Finally, Thomson scattering of cosmic microwave background (CMB) photons off the free electrons produced during the reionization, measured by the Planck (Planck Collaboration et al. 2013a, 2013b), the Wilkinson Microwave Anisotropy Probe (WMAP; Bennett et al. 2012; Hinshaw et al. 2012), and the South Pole Telescope (SPT; Schaffer et al. 2011), can be used to place global, integral constraints on the overall timing and duration of the reionization epoch.

The most promising type of observation, which should become possible in the near future, is the detection of the 21 cm transition of neutral hydrogen (see Morales & Wyithe 2010 for a recent comprehensive review of this large field). The 21 cm emission is a direct probe of the large-scale distribution of neutral hydrogen during the reionization era, and therefore, is a complementary probe to the Lyα forest at lower redshifts. Hence, theoretical studies of EoR are relevant at this time.

The EoR can be studied theoretically with numerical, seminumerical, and analytical models. Until recently, numerical simulations of the EoR have been limited to small volumes or low resolution (cf. Trac & Gnedin 2011 and references therein). These simulations are way too computationally expensive to be used in Markov Chain Monte Carlo analysis for obtaining constraints on cosmological parameters or for producing multiple mock data sets for future observations. To circumvent the above-mentioned problems, seminumerical and analytical models are used (see Zahn et al. 2011 for comparison of radiative transfer simulations and approximate, seminumeric models; Zhou et al. 2013 for comparison of seminumeric and semianalytic approaches; Battaglia et al. 2012 for a new method for modeling inhomogeneous cosmic reionization on large scales).

As the mass in the collapsed objects in the universe was growing, the total number of ionizing photons increased. At some point before $z \sim 6$, the number of photons became sufficient to ionize all of intergalactic hydrogen. Direct calculations of the number of ionizing photons and intergalactic baryons allows us to track the global ionized fraction (cf. Kuhlen & Faucher-Giguère 2012 and references therein). However, all the information about morphology of ionized regions is completely lost in such straightforward calculations. A more detailed treatment of this process can be achieved with the analytical model based on the Press–Schechter-like formalism, pioneered by Furlanetto et al. (2004, hereafter FZH04). This model allows us to track the bubble size distribution and the mean free path (MFP) of ionizing photons, as well as the power spectrum of ionized regions, which can be used for calculating secondary anisotropies of the CMB and for the expected 21 cm signal. In this paper, we present a logical extension of this analytical model that accounts for the biasing of the ionizing sources and additional absorptions by Lyman limit systems (LLSs).

The paper is organized as follows. In Section 2, the brief outline of the FZH04 model is presented. In Section 3, we describe the key steps of our method, with the details of our implementation to follow. In Section 4, the results are presented and we discuss the benefits and possible applications of our model. In Section 5 we conclude.
2. THE FZH04 MODEL

The FZH04 model is based on the idea of calculating the total number of emitted photons in a large enough region of space with known overdensity and comparing this number with the number of baryons in that region. If \( m_{\text{coll}} \) is the mass of the collapsed objects and \( m \) is the mass of some region of space, the condition for the region to be ionized is formulated as

\[
m = \zeta m_{\text{coll}}.
\]

We can interpret \( \zeta \) as ratio of the total number of hydrogen ionizing photons produced by one collapsed hydrogen atom \( N_{\gamma/c} \) to the number of photons required to ionize one hydrogen atom \( N_{i/H} \) (which would be 1 if there are no recombinations),

\[
\zeta = \frac{N_{\gamma/c}}{N_{i/H}}.
\]

In the presence of recombinations,

\[
N_{i/H}(t) = 1 + \int_0^t \frac{dt}{t_{\text{rec}}},
\]

where \( t_{\text{rec}} \) is the average recombination time that depends, among other factors, on the clumping factor of the gas. Both \( N_{\gamma/c} \) and \( N_{i/H} \) can, in principle, vary in space. Note that the gas clumping is only important in the regime when recombinations dominate over the first ionization. In this paper, we follow FZH04 and neglect recombinations.

Now let us consider a region in space, and neglect recombinations for now. It is ionized if the number of hydrogen ionizing photons in it is greater than the number of hydrogen atoms times \( N_{\gamma/c} \). In order to compare these numbers, we first have to find \( m_{\text{coll}} \), which is the function of local overdensity. It can be done using, for example, the Press–Schechter model (Press & Schechter 1974; Bond et al. 1991; Lacey & Cole 1993),

\[
f_{\text{coll}} = \text{erfc} \left( \frac{\delta_c(z) - \delta_m}{\sqrt{2}\sigma^2(M_{\text{min}}) - \delta^2(m)} \right),
\]

where \( \sigma^2(m) \) is the variance of density fluctuations on the scale \( m \) and \( M_{\text{min}} \) is the minimum mass of an ionizing source (we discuss this mass later in Section 3.2). Hereafter, we choose convention (adopted by FZH04) in which \( \sigma(m) \) is evaluated at the reference redshift \( z_\ast = 0 \) and \( \delta_c(z) \) is a function of redshift. Such a convention is convenient because \( \delta_c(z) \) becomes the only quantity that explicitly depends on time.

Hence, the condition that the bubble of mass \( m \) at redshift \( z \) with overdensity \( \delta_m \) is ionized becomes

\[
\frac{1}{\zeta} \leq f_{\text{coll}}.
\]

Equation (3) can be rewritten as a constraint on local density:

\[
\delta_m \geq \delta_c(z) \equiv \delta_c(z) - \sqrt{2K(\zeta)}[\sigma^2(M_{\text{min}}) - \delta^2(m)]^{1/2},
\]

where \( K(\zeta) = \text{erf}^{-1}(1 - \zeta^{-1}) \). Using the excursion set formalism and the function \( \delta_c(m, z) \) as a barrier, one can find the distribution of the masses (and scales) of ionized regions.

A more detailed description of this method can be found in FZH04. In the remainder of this paper, we assume that the reader is familiar with the excursion set formalism, the concept of barrier crossing, and Monte Carlo sampling of possible random-walk trajectories as a method to compute the first barrier crossing.

3. OUR APPROACH

The method described in the original work by Furlanetto et al. (2004) assumes the constant photon per collapsed baryon ratio. Later in Furlanetto et al. (2006), it was extended by making \( \zeta \) a function of halo mass and local overdensity assuming power-law relationship between galaxy mass and \( \zeta \). In our approach, we take into account the efficiency of photon production (not only the total number of emitted photons) of each galaxy individually as a function of all three parameters: redshift, local overdensity, and the bubble size; also we track the process of merging of ionized bubbles. Below we give a general overview of our approach step by step; it is followed by detailed explanation of each component.

1. In the first step, we find the UV luminosity-to-mass ratio of galaxies (i.e., ionizing sources) as a function of redshift. This can be done by using abundance matching between the halo mass function (Tinker et al. 2008) and the observed luminosity functions of high-redshift galaxies (Bouwens et al. 2007, 2010). The main uncertainty in this step is the extrapolation of the abundance-matched luminosity-to-mass ratio to higher redshifts, since observations only cover the redshift range \( z \lesssim 8 \), while reionization starts at earlier times. In Section 3.1, we describe in detail the abundance matching process and our method of extrapolation to higher redshifts that is based only on the theoretically computable evolution of the halo mass function, without any additional parameters.

2. The observations provide us with information about the galaxy UV luminosity at the wavelength \( \sim 1600 \, \text{Å} \). However, the ionizing radiation has wavelengths below 912 Å. To convert one into another we need to know the typical spectrum of the galaxy and the relative escape fraction of the photons at these two wavelengths. For the typical spectrum, we used Starburst99 (Leitherer et al. 1999, 2010; Vázquez & Leitherer 2005) and found that ratio of fluxes at \( \lambda < 912 \, \text{Å} \) and \( \lambda = 1600 \, \text{Å} \) is \( r_{\text{int}} = 0.241 \) (see Appendix A for details). For the escape fraction, we use a simple, two-parameter model which is discussed in Section 3.2. Combining these two factors, we can rewrite luminosity as

\[
L_{\sim 912 \, \text{Å}}(M, z) = r_{\text{int}} f_{\text{esc}, \text{rel}}(M) L_{1600 \, \text{Å}}(M, z).
\]

3. In this step, we calculate the number of ionizing photons. In FZH04, the authors used the parameter \( \zeta \), which measures the ratio of ionizing photons to collapsed hydrogen atoms. We find it more convenient to use another parameter, \( N_{\gamma/H} \), the ratio of the number of hydrogen ionizing photons to the total number of hydrogen atoms in the region (these two parameterization are related to each other by the collapsed fraction). The luminosity-to-mass ratio for ionizing sources,
calculated in the first step, allows us to find \( \dot{N}_{\gamma}/H \) at the mean cosmic density as a function of redshift,

\[
\dot{N}_{\gamma}/H(\delta) = \int \frac{L_{\lambda < 912\,\text{Å}}(M, \delta, z) \, dn/dM(z)}{n_{H,0}(1 + \delta)} \, dM,
\]

where \( dn/dM \) is the halo mass function and \( n_{H,0} \) is the mean number density of hydrogen atoms,

\[
n_{H} = \frac{\Omega_{b}}{\Omega_{m}(1 - Y_{p})} \rho_{\text{crit}} / m_{p}.
\]

At this point we need to account for the halo bias. In overdense as well as in underdense regions, the halo mass function is different, which makes \( \dot{N}_{\gamma}/H \) a function of local overdensity \( \delta \). Details of how the bias is implemented in our calculations are provided in Section 3.3. Another important factor is the absorption of ionizing radiation by LLSs, which are not modeled in the FZH04 formalism and need to be included as a separate component. This absorption depends on the MFP of an ionizing photon at a given redshift and the size distribution of ionized bubbles. In Section 3.4, we describe in detail what model for LLSs we use and how we calculate the fraction \( f_{\text{LLS}}(R, \delta) \) of ionizing photons that can reach the boundary of an ionized bubble of size \( R \) (in comoving units) and hit a neutral hydrogen atom. Taking the bias and LLS effects into account, Equation (5) becomes

\[
\dot{N}_{\gamma}/H(z, \delta, R) = f_{\text{LLS}}(z, R) \times \int \frac{L_{\lambda < 912\,\text{Å}}(M, \delta, R) \, dn/dM(z, \delta, R)}{n_{H,0}(1 + \delta)} \, dM,
\]

where \( (dn/dM)(z, \delta, R) \) is a halo mass function at overdensity \( \delta \) on scale \( R \).

4. In the most general case, \( \dot{N}_{\gamma}/H \) is the function of redshift, local overdensity, and scale (bubble size). The bubble of size \( R \) at a redshift \( z \) with an overdensity \( \delta \) is ionized when \( \dot{N}_{\gamma}/H(z, \delta, R) \) reaches the fraction of non-collapsed hydrogen atoms,

\[
N_{\gamma}/H(z, \delta, R) = 1 - f_{\text{coll}}.
\]

where \( f_{\text{coll}} \) is taken from Equation (2) with \( M_{\text{min}} \) being the minimum mass of a halo that is capable of retaining its gas after (local) reionization (Hoeft et al. 2006; Okamoto et al. 2008). In this paper, we take that mass to be \( 10^{8} M_{\odot} \), since the minimum mass of a halo capable of retaining its gas has not yet been calibrated as a function of local conditions; however, in the future the model can be improved by adopting it as function of redshift and other parameters. This correction is not particularly important, though, since \( f_{\text{coll}} \) is expected to be small.

5. The last step in our model is computing \( \dot{N}_{\gamma}/H \). In the original FZH04 model, \( \zeta \) was constant and \( N_{\gamma}/H \) was a function of \( z \) only, hence Equation (7) could be integrated directly. In the case when \( \dot{N}_{\gamma}/H \) depends on \( R \) (or, equivalently, on \( m \)), \( \delta \), and \( z \), it is no longer valid to directly integrate Equation (7) because both \( \delta \) and \( R/m \) can be functions of time. In addition, ionized bubbles can merge, thus combining two separate bubbles with \( m_{1} \) and \( m_{2} \) into a single bubble of mass \( m = m_{1} + m_{2} \).
Such an approach has been used, for example, by Gnedin (2008) and Volonteri & Gnedin (2009), and we show an updated version of abundance matching between the galaxy UV luminosity and the halo virial mass in Figure 1 for four values of cosmological redshift.

However, as is apparent from Figure 1, the luminosity–mass relations obtained by abundance matching are redshift-dependent. Hence, any extension to higher redshifts will require extrapolation and will again be subject to unknown biases.

Trenti et al. (2010) circumvented that limitation by matching luminosity functions not with the actual halo mass functions, but with the difference between the two halo mass functions at two redshifts separated by a fixed time interval of 200 Myr. With such matching, the luminosity–mass relations become redshift-independent.

In this paper, we propose another method of extrapolation. Our main motivation is to find a physically plausible relation between the galaxy luminosity and some property of a dark matter halo, and then match that property to the halo mass.

Given the relation between \( L \) and \( M_\Delta \), we can derive \( L(M_\text{vir}, z) \) at any redshift by abundance matching of two theoretical mass functions, \( n(>M_\Delta) \) and \( n(>M_{\text{vir}}) \), which are known at any redshift, i.e.,

\[
L(M_{\text{vir}}, z) \equiv L(M_\Delta(M_{\text{vir}}, z)).
\]

This procedure does not involve any extrapolation in time, and, hence, all time evolution comes through the physical evolution of the halo mass function.

### 3.2. Escape Fraction

The most sensitive parameter for any reionization model is the escape fraction of ionizing photons from galaxies. Different definitions of \( f_{\text{esc}} \) exist in the literature. When the actual ionizing emissivity of all stars in a galaxy is known, \( f_{\text{esc}} \) stands for a fraction of the radiation that leaves the galaxy—it is called the absolute escape fraction then. In reality, the total ionizing flux from stars in high-redshift galaxies is unknown. The observed quantity is the UV luminosity (at wavelength \( \lambda \approx 1600 \) Å) of a galaxy, and, hence, we are interested not in the absolute, but in the relative escape fraction, \( f_{\text{esc,rel}} = f_{\text{esc}}(<912 \) Å)/\( f_{\text{esc}}(1600 \) Å) that enters Equation (8).

A large body of work exists on modeling the escape fractions from high-redshift galaxies (see, e.g., Razoumov & Sommer-Larsen 2010; Sribinovsky & Wyithe 2010; Wyithe et al. 2010; Fernandez & Shull 2011; Mitra et al. 2013 for the latest models). In this paper, we adopt the second-to-the-simplest model with only two parameters—the minimum mass \( M_{\text{crit}} \) and the amplitude \( f_{\text{esc,rel,0}} \):

\[
f_{\text{esc,rel}}(M) = \begin{cases} f_{\text{esc,rel,0}}, & M > M_{\text{crit}}, \\ 0, & M \leq M_{\text{crit}}. \end{cases}
\]

The mass \( M_{\text{crit}} \) in Equation (9) may play multiple roles: it can be the minimal mass of FZH04 that corresponds to the virial mass of a halo with temperature \( 10^4 \) K (the critical temperature for production of ionizing photons; Choudhury et al. 2008); it can be a minimum mass of a halo capable of retaining photoionized gas (Okamoto et al. 2008), or, alternatively, it can be the critical mass from Gnedin et al. (2008) who found that dwarf galaxies below that mass have very low escape fractions.

### 3.3. Halo Bias

The original FZH04 takes into account halo bias by making \( f_{\text{con}} \) to be the function of scale and overdensity. However, it is not the only manifestation of bias. As pointed out in Furlanetto et al. (2006), high dense regions tend to have higher abundance of massive halos, i.e., the bias factor modifies the halo mass function in the regions with non-zero cosmic overdensity \( \delta \), and, therefore, forces the photon production rate to become a function of local overdensity. In the mentioned paper, the authors assume power-law relationship between galaxy mass and \( \zeta \). We consider a more general case, which allows to use the actual observed luminosity functions. To calculate the total luminosity in a given region, we need to integrate the luminosity–mass
relation \( L_{\text{912}}(M, z) \) from Section 3.2 over the halo mass function to find the average UV luminosity produced by a collapsed baryon at redshift \( z \),

\[
L_{\text{tot}}(z, \delta, \sigma^2) = \int L_{\text{912}}(M, z) \times \frac{dn}{dM}(z, \delta, \sigma^2) dM, \quad (10)
\]

where we use the bubble mass \( m \) instead of its \( R \) as a variable, as a more convenient variable (in this formulation there is no need to assume a particular geometric shape for a bubble). We adopt the mass function from Tinker et al. (2008) with the halo bias model from Tinker et al. (2010), modified in the following way to ensure the exact conservation of ionizing photons:

\[
\frac{dn}{dM} = f_{\text{coll}}^{\text{PS}} / f_{\text{coll}}^{\text{Tinker}} \times \frac{dn_{\text{Tinker}}}{dM}. \quad (11)
\]

The factor \( f_{\text{coll}}^{\text{PS}} / f_{\text{coll}}^{\text{Tinker}} \) is required, since the original Press–Schechter form for the halo mass function allows us to maintain the exact photon conservation independently of the smoothing scale by virtue of the property

\[
\left\langle f_{\text{coll}}^{\text{PS}}(\delta, \sigma^2) \right\rangle = f_{\text{coll}}^{\text{PS}}(\delta = 0, \sigma^2 = 0),
\]

while fits provided in Tinker et al. (2010) do not guarantee such normalization. We further discuss the conservation of photons in the FZH04 model in Section 3.6.

The Lagrangian bias that is used in Tinker et al. (2010) works only on large scales (small \( \sigma^2 \)) and small overdensities. In an ideal case, we should know local nonlinear Eulerian bias. Such information might be extracted from large-scale simulations, as was done in Roth & Porciani (2011). Unfortunately, at the moment these models cannot cover all the redshifts and scales that are considered in our reionization model.

3.4. Lyman Limit Systems

In the original FZH04 model, the universe remains 100% ionized after the end of reionization. The MFP of an ionized photon in such a universe would be limited by the cosmic horizon only. In reality, however, the MFP for ionizing radiation is much shorter than the cosmic horizon even at lower redshifts (Songaila & Cowie 2010); it is limited by absorption in the LLSs. Hence, the LLSs are not modeled by the original FZH04 model and need to be accounted for separately.

If the MFP is much larger than the size of an ionized bubble, the majority of photons produced inside a bubble will be able to reach the bubble edge and ionize fresh neutral hydrogen outside the bubble. In the opposite case, when the MFP is small compared to the bubble size, the probability of absorption by LLSs is much higher and only galaxies near the edge of the bubble contribute to ionizing fresh neutral hydrogen beyond the edge. In Appendix B, we calculate the fraction \( f_{\text{LLS}}(z, R) \) of ionizing photons emitted inside the bubble that are available for ionizing fresh neutral hydrogen (and, hence, contributing to \( N_{\gamma / \text{HI}}(z, \delta, R) \)). For such a calculation, distributions of galaxies and LLSs inside every bubble is needed; such distributions are not part of the Press–Schechter formalism and, hence, have to be added as external assumptions in the model. In this paper, we consider two limiting cases: a homogeneous distribution (galaxies are distributed uniformly inside a bubble) and a highly clumped distribution (all galaxies are located at the bubble center). LLSs in both cases are assumed to be uniformly distributed inside the bubble.

The average abundance of LLSs is not a free parameter, however; it has been constrained observationally up to \( z \approx 6 \) (Songaila & Cowie 2010). For our purpose, it is more convenient to use the MFP due to LLS, \( l_{\text{LLS}} \), directly, hence we adopt Equation (8) from Songaila & Cowie (2010):

\[
l_{\text{LLS}}(z) = 50 \left[ \frac{1 + z}{4.5} \right]^{-4.44} \text{Mpc}.
\]

This fit is made in the redshift interval 0–6. In our model, we extrapolate this fit to higher redshifts, where its accuracy cannot be verified, but, at present, there is no alternative.

To accurately calculate the effective fraction of photons due to LLSs at some redshift, we should know not only the bubble sizes but also the merging statistics of these bubbles. For instance, if we have a large number of small bubbles that merge into larger ones only during the late stages of reionization, the effective fraction will always be close to unity. In the opposite case, if all galaxies are highly clustered at the centers of a few large bubbles, the bubble size will be limited by the MFP and a large fraction of ionizing photons will be lost in LLSs. Luckily, the FZH04 model contains the information about bubble merging.

3.5. Merging of Bubbles

The procedure of searching the progenitors using excursion set formalism was first described in Lacey & Cole (1993) for the Press–Schechter model. Here, we follow the same idea. It was already applied to the FZH04 model in Furlanetto & Oh (2005), where the distribution of bubble progenitors was found. However, the authors did not use this information for correcting the number of ionizing photons.

Let us imagine that at some time \( t \) we have a distribution of ionized bubbles in space and assume that there is a barrier that can describe this distribution. During some time \( \Delta t \) each of the bubbles was growing according to the photon production rate from Equation (7), as well as merging with other bubbles. In order to calculate \( N_{\gamma / \text{HI}} \) for a bubble of size \( m \) at \( t + \Delta t \) we need to know its progenitors, which is illustrated in Figure 2. A solid line corresponds to the barrier at time \( t \). If we start a random walk at some point below the barrier (a black square) assuming that this point belongs to the new barrier at \( t + \Delta t \), the trajectories that cross the barrier at time \( t + \Delta t \) will correspond to the progenitors of the current bubble. Two random walks are shown in the figure: the dashed one crosses the barrier and therefore corresponds to the ionized bubble at time \( t \); the dotted random walk does not cross the barrier, thus it corresponds to the region that was neutral at time \( t \) and got ionized during the time interval between \( t \) and \( t + \Delta t \).

If we know the distribution \( f(\sigma^2(m)) \) of the barrier crossings at time \( t \), we can write \( N_{\gamma / \text{HI}} \) as

\[
N_{\gamma / \text{HI}}(t + \Delta t, \delta, m') \times m'(1 + \delta')
\]

\[
= \int_{\sigma^2(m')} f(\sigma^2(m)) \times m(1 + \delta)
\]

\[
\times (N_{\gamma / \text{HI}}(t, \delta, m) + \dot{N}_{\gamma / \text{HI}}(t, \delta, m) \Delta t) \, d\sigma^2(m), \quad (12)
\]
where we took into account the initial numbers of photons in each bubble and their growth during $\Delta t$. The function $f(\sigma^2(m))$ in Equation (12) depends on the barrier at time $t$. Solving Equation (12) for every $m'$ gives us the barrier at time $t + \Delta t$.

For numerical integration it is very important to choose right time step, $\Delta t$. We found good convergence for time steps less than 10 Myr. For the beginning of integration we chose $z = 14.5$, because insignificant number of photons were produced in earlier epochs (with our luminosity function model).

The described approach makes the model more computationally expensive than the original FZH04 approach, but still much faster than a full numerical simulations.

### 3.6. Conservation of Photons

The conservation of photons in the models based on the Press–Schechter formalism might be tricky, because there is no implicit mechanism that would force the conservation. The ionized fraction of the universe calculated with the excursion set approach is systemically below the ionized fraction calculated directly by counting all emitted photons (i.e., $\zeta f_{\text{coll}}(\sigma^2 = 0)$—see Figure 3). In the original FZH04 paper, the authors propose to rescale the volumes of bubbles in order to match the number of emitted photons (i.e., ionized baryons).

However, the photon number non-conservation in the FZH04 model is due to trajectories that never cross the barrier. These trajectories correspond to spatial locations that are never fully ionized internally, but some ionizing photons are still emitted inside them (there are just not enough of them). The contribution of these photons to the total ionized fraction is included only after the region is merged with another, fully ionized region, i.e., all ionizing photons are eventually accounted for, but some of the emitted photons in the FZH04 model are treated as if they ionize a hydrogen atom not at the moment of their emission but at some later moment. As a result, the end of reionization coincides for all models from Figure 3 that neglect the explicit photon absorption in LLS, but the actual reionization histories for different models differ among themselves. Since in the model that accounts for the halo bias, more photons are deposited into larger bubbles, it matches the direct calculation of photons slightly better.

### 4. RESULTS

The significance of LLSs is demonstrated in Figure 3, where we show four reionization histories with LLS effects turned on and off. The corresponding barriers at $z = 6.7$ are shown in the top panel of Figure 4. In the bottom panel, the first crossing distributions for the barriers are presented. Additional absorption by LLSs leads to the upturn of the barrier on large scales (or small $\sigma^2(m)$), because in large ionized bubbles only galaxies near the edge contribute to the further expansion of a
bubble. LLSs affect the reionization history at later stages, when the average bubble size is large, significantly slowing down the reionization process and making the end of reionization more gradual.

The halo bias modifies the halo mass function, resulting in higher abundance of massive halos in overdense regions, and, consequently, a higher photon production rate. That lowers the barrier on all scales. However, the effect caused by bias is minor compared to the one due to LLSs.

The computational efficiency of this method allows us to explore the parameter space of our model for the escape fraction (Equation (9)). The results are presented in Figure 5. The solid contour lines show the end of reionization for the model with LLSs in assumption of uniform distribution of galaxies and LLS. The dashed contour lines show the corresponding duration of reionization, the redshift interval between 20% and 99% levels of ionization (following the definition from Zahn et al. 2012).

The duration of reionization ($\Delta z$) varies from 1.6 to 2.4 across the parameter space presented in the figures. Current constraints on $\Delta z$ from CMB are too broad and cannot rule out any of our model parameters. The lower limit on $\Delta z$ made by EDGES (Bowman & Rogers 2012) is $\Delta z > 0.06$ with 95% confidence level, which is much smaller than any of our predictions.

Other observational constraints in our parameter space ($M_{\text{crit}}$ and $f_{\text{esc,rel},0}$) are shown in Figure 6. For all our reionization models consistent with the Planck value for the Thompson optical depth ($\tau = 0.092 \pm 0.013$; Planck Collaboration et al. 2013a), the measurement of the secondary anisotropies by SPT (Zahn et al. 2012) does not yet provide any significant constraint, because reionization proceeds faster than the SPT upper limit on the duration of reionization. We also show in Figure 6, mostly for the sake of illustration, the observational constraint on $f_{\text{esc,rel},0}$ at $z \sim 3$ from Nestor et al. (2013). We emphasize that the latter constraint is obtained well after reionization, and, therefore, may not apply to galaxies at $z > 6$.

The constraints shown in Figure 6 argue in favor of models with low $M_{\text{crit}}$ (assuming reasonable value of $f_{\text{esc,rel},0}$) and therefore only models with substantial contribution of ionizing photons from dwarf galaxies are allowed.

**Figure 6.** Observational constraints in the parameter space ($M_{\text{crit}}, f_{\text{esc,rel},0}$) of our reionization model (which, by construction, matches the observed evolution of the galaxy UV luminosity function for $z \lesssim 8$ and the observed opacity of the Ly$\alpha$ forest at $z \lesssim 6$). The solid and dot-dashed lines show the Planck $2\sigma$ constraint ($\tau > 0.092 - 2 \times 0.013$) on the Thompson optical depth (Planck Collaboration et al. 2013a) for a model with direct counting of ionizing photons (solid lines) and for a model with bias and LLS (dot-dashed lines). The shaded region is the SPT $1\sigma$ constraint on the duration of reionization from Zahn et al. (2012). For reference, we also show with the dotted vertical lines the observed constraint on the relative escape fraction at $z \sim 3$ from Nestor et al. (2013, see the text for detail). Constraints on the duration of reionization from EDGES (Bowman & Rogers 2012) are presently too weak to be able to exclude any portion of the displayed parameter space.

5. CONCLUSIONS

In this paper, we extend the analytical model for cosmic reionization from FZH04 to account for the absorption of ionizing radiation by LLSs, for the halo bias effect, and for merging of ionized bubbles.

The halo bias does not affect the end of reionization since it does not change the total number of galaxies (and, hence, the total number of ionizing photons). However, the halo bias causes a redistribution of ionizing photons between overdense and underdense regions, and therefore modifies the size distribution of ionized bubbles by increasing the abundance of large bubbles and suppressing small ones.

Additional absorption of ionizing radiation by LLSs is significant only during the late stages of reionization, when sufficiently large bubbles (with sizes exceeding the photon MFP due to LLSs) are abundant.

Our model is, by construction, consistent with the observed evolution of the galaxy luminosity function at $z \lesssim 8$ and with the observed evolution of Ly$\alpha$ forest at $z \lesssim 6$. We also find that, in agreement with other studies (Robertson et al. 2013 and references therein), a so-constrained model is able to match the WMAP/Planck constraint on the Thompson optical depth and the SPT and EDGES constraints on the duration of reionization for reasonable values of the relative escape fraction (i.e., those that are consistent with the observational measurements at lower redshifts), but only if dwarf galaxies ($M_h \lesssim 10^9 M_\odot$) contribute substantially or even dominate the ionizing photon budget. This conclusion highlights the importance of extrapolation of the luminosity function to low-mass, dim galaxies, since they are not observed in high-redshift surveys. This uncertainty may be as important as the uncertainty of extrapolation to high redshifts.
The latter condition is inconsistent with cosmological simulations that (marginally) resolve the escape of ionizing photons from galaxies (Gnedin et al. 2008) or with observational upper limits on the escape fraction at $z \sim 3$ (Fernández-Soto et al. 2003). Whether such a disagreement is due to different nature of $z > 6$ galaxies, inadequacy of simulations and/or some of the observational constraints, or indicates an additional source of ionizing radiation at $z > 8$ remains to be seen.

This work was done with significant usage of CosmoloPy Python package.7

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APPENDIX A

GALAXY SPECTRUM

The observations of galaxies (Bouwens et al. 2007, 2010) provide us with information about the UV luminosity at wavelength $\lambda = 1600 \text{ Å}$. To model the reionization process, we need to know the ionizing luminosity (wavelengths $<912 \text{ Å}$). In this Appendix, we compute the ratio of the intrinsic ionizing luminosity to the UV luminosity from galaxies. We use a simple fit to the stellar spectrum from Ricotti et al. (2002):

$$x \equiv \frac{h \nu}{1 \text{Ry}}, \quad (A1)$$

$$s_{\nu}(v) = \begin{cases} 5.5, & x < 1 \\ x^{-1.8}, & 1 < x < 2.5 \\ 0.4x^{-1.8}, & 2.5 < x < 4 \\ 2 \times 10^{-3} x^3/\left(\exp(x/1.4) - 1\right), & 4 < x. \end{cases} \quad (A2)$$

The intrinsic ionizing luminosity can be written as

$$L_{\nu}^{\text{int}}(< 912 \text{ Å}) = r_{\text{int}} v L_{\nu}^{\text{obs}}(1600 \text{ Å}), \quad (A3)$$

where $r_{\text{int}}$ is a constant that converts the observed value $v L_{\nu}^{\text{obs}}(1600 \text{ Å})$ into ionizing luminosity $L_{\nu}^{\text{int}}(< 912 \text{ Å})$. The value of $r_{\text{int}}$ can be found from the stellar spectrum ($L_{\nu} \sim s_{\nu}$):

$$r_{\text{int}} = \int_{1}^{\infty} s_{\nu} dx, \quad (A4)$$

Since we are interested in photon flux density rather than energy flux, we calculate the average energy of an ionizing photon:

$$\langle E \rangle = \frac{\int_{1}^{\infty} h \nu \times s_{\nu} dx}{\int_{1}^{\infty} s_{\nu} dx} = 1.93 \text{ Ry}. \quad (A5)$$

Finally, the flux of ionizing photons can be expressed as

$$\hat{N}_{\nu} = r_{\text{int}} v L_{\nu}^{\text{obs}}(1600 \text{ Å})/\langle E \rangle. \quad (A6)$$

APPENDIX B

FRACTION OF ESCAPED PHOTONS

Here, we consider a stand-alone ionized bubble and calculate the fraction of photons that end up absorbed by LLSs.

We consider two limit cases. In the first case, we assume galaxies and LLSs are distributed uniformly inside the bubble. If the MFP is larger than the bubble size, the number of photons that reach the boundary of the bubble and ionize a neutral atom is proportional to the bubble volume. Another regime is when MFP is much smaller that the bubble size, in this case the number of photons that can ionize bubble surroundings is proportional to its surface area.

Let the MFP at that epoch be $l_{\text{LLS}}$, and we consider a bubble of size $R$ and a galaxy located at distance $h$ from the center. The fraction of photons emitted by this galaxy that will reach the boundary of the bubble is

$$x = \frac{\sqrt{h^2 + R^2} - 2 Rh \cos(\theta)}{R}.$$

$$f_{\text{eff}}(h) = \frac{1}{4\pi R^2} \int_{0}^{\pi} d\theta 2\pi \sin(\theta)x^2 \exp(-x/l_{\text{LLS}}).$$

Now we have to find the average over the volume (since galaxies are assumed to be distributed uniformly inside the bubble):

$$f_{\text{LLS}}(R/\lambda) = \frac{1}{\frac{4}{3} \pi R^3} \int_{0}^{R} 4\pi r^2 f_{\text{eff}}(h) dh.$$

In result, we have the $f_{\text{LLS}}$ as a function of $R/l_{\text{LLS}}$. Note that $l_{\text{LLS}}$ is the function of redshift.

In the second case, galaxies are clumped in the centers of bubbles, but LLSs are still distributed uniformly. In this case, the fraction of not-absorbed photons is simply

$$f_{\text{LLS}}(R/l_{\text{LLS}}) = 1 - \exp(-R/l_{\text{LLS}}).$$

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