Configuration Entropy for Quarkonium in a Finite Density Plasma

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ABSTRACT: In the recent years many examples appeared in the literature where the configuration entropy (CE), introduced by Gleiser and Stamatopoulos, plays the role of an indicator of stability of physical systems. It was observed that, comparing states of the same system, the lower is the value of the CE, the more stable is the state. In this letter we investigate the behaviour of the CE in a new context. We consider quasi-states of quarkonium (a vector meson made of a heavy quark anti-quark pair) inside a plasma at finite density. It is known that the density increases the dissociation effect for quasi-particles inside a plasma. So, increasing the density of a thermal medium corresponds to reducing the stability of the quasi-particles. In order to investigate how this situation is translated in the Configuration Entropy context, we use a recently developed holographic AdS/QCD model for heavy vector mesons. The quasi-normal modes describing the quasi-states are obtained and the corresponding CE is calculated. We find, for bottomonium and charmonium 1S quasi-states, that the CE increases with the quark density, or quark chemical potential, of the medium. This result shows that the CE works again as an indicator of stability, represented in this case by the dissociation effect associated with the density.

KEYWORDS: Gauge-gravity correspondence, Phenomenological Models
1 Introduction

An interesting tool to investigate the stability of physical systems is the configuration entropy (CE), introduced by Gleiser and Stamatopoulos in refs. [1, 2] (see also [3]). An increase in the value of the CE is associated with a decrease in the stability. Such a behavior was observed in many different physical systems, such as: compact astrophysical objects [4] and holographic AdS/QCD models [5–10]. There are other many interesting applications of configuration entropy in the literature, as for example [11–30].

The purpose of the present letter is to investigate the application of the configuration entropy to a physical system of great interest currently: heavy mesons inside a quark gluon plasma with finite density. The interest in such a system comes from the quark gluon plasma (QGP). This very short-lived state of matter, where quarks and gluons are not confined, is produced in heavy ion collisions and consists of a strongly interacting thermal medium. It is a highly non-trivial task to build up a picture of the QGP from the particles that reach the detectors. For reviews about QGP, see for example [31–34]. One of the important available sources of information is the abundance of heavy vector mesons, made of $c\bar{c}$ or $b\bar{b}$ quarks in the final products of heavy ion collisions. These particles are partially dissociated in the plasma and their degree of dissociation depends on the temperature and density of the plasma. So, it is possible to relate their relative abundance with the properties of the pre-existing medium.

It is possible to describe the thermal behavior of heavy vector mesons inside a plasma using holographic models [35–39]. The dissociation of charmonium and bottomonium is represented in these references as the decrease in the peaks of the thermal spectral functions, that represent the amplitude of finding a particle with a given energy inside the medium. This
dissociation process can alternatively be analyzed through the quasinormal modes, that are normalizable solutions, with complex frequencies, for the fields that describe the mesons. The real parts of the frequencies are related to the masses and the imaginary parts to the widths of the quasi-states. In refs. [40–42], quasi-normal modes for heavy vector mesons were studied using the holographic model of [38, 39].

The configuration entropy[3] was motivated by the well known information entropy of Shannon [43] that is defined, for a discrete variable $x$ that may have the values $x_n$ with probabilities $p_n$ as

$$- \sum_n p_n \log p_n , \quad (1.1)$$

and measures the amount of information one gains in getting to know the value of $x$. Note that the probabilities satisfy the normalization condition: $\sum_n p_n = 1$.

The continuous version of eq. (1.1), in position space, reads

$$S = - \int d^d r \epsilon(\vec{r}) \log \epsilon(\vec{r}) , \quad (1.2)$$

where

$$\epsilon(\vec{r}) = \frac{\rho(\vec{r})^2}{\int d^d r |\rho(\vec{r})|^2} , \quad (1.3)$$

is a normalized function $\int d^d r \epsilon(\vec{r}) = 1$, called (spatial) modal fraction. In order to introduce the configuration entropy, one considers the momentum space version, by fourier transforming:

$$\tilde{\rho}(\vec{k}) = \frac{1}{(2\pi)^{d/2}} \int d^d r \rho(\vec{r}) \exp(-i\vec{k} \cdot \vec{r}) . \quad (1.4)$$

The CE is defined as

$$\tilde{S} = - \int d^d k \tilde{\epsilon}(\vec{k}) \log \tilde{\epsilon}(\vec{k}) , \quad (1.5)$$

where

$$\tilde{\epsilon}(\vec{k}) = \frac{|\tilde{\rho}(\vec{k})|^2}{\int d^d k |\tilde{\rho}(\vec{k})|^2} , \quad (1.6)$$

is the (momentum space) modal fraction, that is also normalized: $\int d^d k \tilde{\epsilon}(\vec{k}) = 1$.

Information entropies like $S$ and $\tilde{S}$, based in conjugate variables in the sense of eq. (1.4), satisfy the so called entropic uncertainty relation [44, 45] that, for this d-dimensional case takes the form:

$$S + \tilde{S} \geq d(1 + \log(\pi)) . \quad (1.7)$$

So, one could guess that a variation of the configuration entropy, defined in momentum space, $\tilde{S}$ could be associated with a particular variation of the conjugate quantity, $S$, defined
in position space. Such a conjecture was investigated recently in Ref. [45], where it was found that, for the case of an anti-de Sitter black hole, when the temperature varies, both $\tilde{S}$ and $S$ vary but their sum remains constant. Here we will investigate this behavior for the case of heavy vector mesons inside a plasma when the density varies.

This letter is organized in the following way: in section 2 we review the holographic model for heavy vector mesons in a plasma. In section 3 we develop the calculation of the configuration entropy for charmonium and bottomonium 1S states. Then in section 4 we present and analyses the results obtained and finally in section 5 we make some final remarks.

2 Holographic heavy vector mesons at finite density

Heavy vector mesons are described holographically [38, 39] by a vector field $V_m = (V_\mu, V_z)(\mu = 0, 1, 2, 3)$, living in a five dimensional curved space, that is assumed to be dual to the four dimensional gauge theory current $J_\mu = \bar{\psi} \gamma^\mu \psi$. The curved five dimensional space is just an anti-de Sitter space for the case when the mesons are in the vacuum (vanishing temperature and density). Additionally, there is a scalar background. The action reads

$$\mathcal{I} = \int d^4x dz \sqrt{-g} e^{-\phi(z)} \left\{ -\frac{1}{4 g_5^2} F_{mn} F^{mn} \right\}, \quad (2.1)$$

where $F_{mn} = \partial_m V_n - \partial_n V_m$. The background scalar field $\phi(z)$ has the form:

$$\phi(z) = \kappa^2 z^2 + M z + \tanh \left( \frac{1}{M z} - \frac{\kappa}{\sqrt{\Gamma}} \right). \quad (2.2)$$

The parameters $\Gamma, \kappa$ and $M$ represent, in an effective way, respectively: the string tension, the mass of the quarks and a large mass scale associated with the decay of the heavy meson into a non hadronic state. The values found in Ref. [39] for charmonium and bottomonium are respectively

$$\kappa_c = 1.2, \sqrt{\Gamma_c} = 0.55, M_c = 2.2; \ \kappa_b = 2.45, \sqrt{\Gamma_b} = 1.55, M_b = 6.2; \quad (2.3)$$

where all quantities are expressed in GeV. The geometry dual to a finite temperature medium is in general a black hole one. For the case when the medium additionally has a finite chemical potential $\mu$ the black hole has charge[46–48]. In particular, it is a 5-d anti-de Sitter charged black hole space with metric

$$ds^2 = \frac{R^2}{z^2} \left( -f(z) dt^2 + d\vec{x} \cdot d\vec{x} + \frac{dz^2}{f(z)} \right), \quad (2.4)$$

where

$$f(z) = 1 - \frac{z^4}{z_h^4} - q^2 z_h^2 z^4 + q^2 z^6, \quad (2.5)$$
and \( f(z_h) = 0 \). The relation between the horizon position \( z_h \) and the temperature \( T \) of the black hole, is obtained requiring that there is no conical singularity at the horizon:

\[
T = \frac{|f'(z)|_{z=z_h}}{4\pi} = \frac{1}{\pi z_h} - \frac{q^2 z_h^5}{2\pi}. \tag{2.6}
\]

The parameter \( q \), proportional to the black hole charge, is related to the density of the medium, or quark chemical potential, \( \mu \) of the gauge theory. The quantity \( \mu \) works as the source of correlators of the quark density operator \( \bar{\psi}\gamma^0\psi \). So it should appear in the Lagrangian multiplying the quark density. In the holographic description, the time component \( V_0 \) of the vector field plays this role. So, one considers a particular solution for the vector field \( V_m \) with only one non-vanishing component:

\[
V_0 = A_0(z) (V_z = 0, V_i = 0). \tag{2.7}
\]

So, specifying both \( z_h \) and \( q \), the values of the temperature and the chemical potential are fixed and contained into the metric \( (2.4) \).

It is interesting to mention that there are many interesting previous studies using holography to describe thermal effects and heavy flavors like for example [49–64].

3 Configuration entropy of the heavy mesons

3.1 Energy Density

The quantity that is relevant for the determination of the configuration entropy of heavy mesons is the energy density, that is the \( T_{00} \) component of the energy momentum tensor. We assume that in this phenomenological model \( T_{mn} \) is obtained from the action in the same way as in general relativity. That means, writing the action as

\[
\int d^4x dz \sqrt{-g} L \]

the energy momentum tensor has the form:

\[
T_{mn}(z) = \frac{2}{\sqrt{-g}} \left[ \frac{\partial(\sqrt{-g} L)}{\partial g^{mn}} - \frac{\partial}{\partial x^p} \frac{\partial(\sqrt{-g} L)}{\partial (\partial g^{mn}/\partial x^p)} \right]. \tag{3.1}
\]

So, for the action \( (2.1) \) the energy density for the vector field is

\[
\rho(z) = \frac{e^{-\phi(z)}}{g_5^2} \left[ g_{00} \left( \frac{1}{4} g^{mp} g^{nq} F_{mn} F_{pq} \right) - g^{mn} F_{0n} F_{0m} \right]. \tag{3.2}
\]

Considering the metric \( (2.4) \) and a plane wave solution in the \( x^\mu \) directions, in the meson rest frame \( V_\mu = \eta_\mu v(p, z) e^{-i\omega t} \), with \( \eta_\mu = (0, 1, 0, 0) \), the energy density takes the form

\[
\rho(z) = \frac{z^2 e^{-\phi(z)}}{2 R^2 g_5^2} \left[ |\omega|^2 |v|^2 + f^2 |\partial_z v|^2 \right]. \tag{3.3}
\]
In order to obtain the energy density for a meson inside the plasma, one has to find the solution for the field $v$ representing the corresponding quasistate and plug it into eq. (3.3). At zero temperature, states are represented holographically by normalizable solutions of the gravity field equations. This type of solutions are called normal modes and satisfy trivial boundary conditions. On the other hand, at finite temperature, the solutions that represent the quasistates are the so called quasinormal modes, that are also normalizable solutions of the field $v$ but satisfy non trivial boundary conditions. At finite $T$ there is an event horizon at $z = z_h$ where one has to impose infalling boundary conditions. Additionally, the normalizability condition requires that the fields vanish at the boundary $z = 0$. Satisfying both conditions requires, in general, solutions corresponding to complex frequencies $\omega$. The real part, Re($\omega$), is related to the thermal mass and the imaginary part, Im($\omega$), is related to the thermal width. We will see in the next section how to obtain these solutions.

### 3.2 Quasinormal modes

As in the previous section, we consider $V_z = 0$ and $V_\mu = \eta_\mu v(p, z)e^{-i\omega t}$, with $\eta_\mu = (0, 1, 0, 0)$. Introducing the electric field component $E = \omega V_1$, the equations of motion coming from action (2.1) with the metric (2.4) take the form:

$$E'' + \left(\frac{f'}{f} - \frac{1}{z} - \phi'\right)E' + \frac{\omega^2}{f^2} E = 0,$$

where $('') represents derivative with respect to the radial $z$ coordinate.

One has to impose the normalizability condition at $z = 0$ and the infalling condition at $z = z_h$. It is convenient, in order to impose the boundary conditions at the horizon, to re-write the field equations in such a way that they separate into a combination of infalling and outgoing waves. One introduces the coordinate $r_*$, implicitly defined by the relation $\partial_\star = -f(z)\partial_2$ with $r_*(0) = 0$, for $z$ in the $0 \leq z \leq z_h$. In addition let us introduce the field

$$\psi = e^{-\frac{B(z)}{2}} E,$$

with $B(z) = \log(z/R) + \phi$. Then, Eq. (3.4) reduce to the form:

$$\partial_{\star}^2 \psi + \omega^2 \psi = U \psi.$$ 

The potential $U(z)$, obtained this way, diverge at $z = 0$ so one must impose $\psi(z = 0) = 0$. At the horizon $U(z = z_h) = 0$ so one expects to find *infalling* $\psi = e^{-i\omega r_*}$ and *outgoing* $\psi = e^{+i\omega r_*}$ wave solutions for equation (3.6). Only the first kind of solutions are physically allowed. The Schrödinger like equation can be expanded near the horizon leading to the following expansion the field solution:

$$\psi = e^{-i\omega r_*(z)} \left[1 + a^{(1)}(z - z_h) + a^{(2)}(z - z_h)^2 + \ldots\right].$$
One can solve recursively for $a^{(n)}$. The first coefficient obtained is:

$$a^{(1)} = \frac{(2 - q^2 z_h^0)}{2(q^2 z_h^0 + i\omega z_h - 2)} \left( z_h \left( \frac{k^2}{2 - q^2 z_h^0} + 2\kappa^2 \right) - \frac{\text{sech}^2 \left( \frac{\sqrt{2} \kappa}{M z_h} \right)}{M z_h^2} + \frac{1}{z_h} + M \right).$$

(3.8)

This expansion leads to the following form for the infalling boundary conditions for the field and its derivative at the horizon:

$$E(z_h) = e^{-i\omega r^*_v(z_h) + \frac{B(z_h)}{2}},$$

(3.9)

$$E'(z_h) = \left(-i\omega r^*_v(z_h) + \frac{B'(z_h)}{2} + a^{(1)}_j\right) E(z_h).$$

(3.10)

Then one solves eq. (3.4) numerically integrating from the horizon, using a method that consists of imposing these infalling boundary conditions and search for complex frequencies that provide solutions vanishing on the boundary: $E(z = 0) = 0$. The results are the quasinormal frequencies and the corresponding solutions are the quasinormal modes, that represent the heavy meson quasi-states in the thermal medium.

### 3.3 Entropy

The solutions for the gravity fields that holographically describe the heavy vector mesons are complex. So, the actual form of the Lagrangian density is $F^*_mn F^mn$. The Configuration entropy is calculated form the solutions $v(p, z)$ corresponding to the the quasinormal modes $v_n(z)$, described in the previous section. One considers the Fourier transform of the energy density $\rho(z)$ in coordinate $z$: $\tilde{\rho}(k)$. It is convenient, for the computation of the CE, to split $\tilde{\rho}(k) = (C(k) + iS(k)) / \sqrt{2}$, where

$$C(k) = \int_0^{z_h} \rho(z) \cos(kz) dz,$$

$$S(k) = \int_0^{z_h} \rho(z) \sin(kz) dz.$$

(3.11)

(3.12)

In terms of these components, the modal fraction reads:

$$\tilde{\epsilon}(k) = \frac{S^2(k) + C^2(k)}{\int_{-\infty}^{\infty} [S^2(k') + C^2(k')] dk'}.$$

(3.13)

For this one dimensional case, the CE (1.5) reads

$$\tilde{S} = -\int_{-\infty}^{\infty} \tilde{\epsilon}(k) \log \tilde{\epsilon}(k) dk.$$

(3.14)

### 4 Results

The idea is to investigate the dependence of the configuration entropy $\tilde{S}$ and of the associated conjugate quantity $S$ on the density of the plasma. The relevant quantity is not the absolute
value of the entropy but rather the variation with the density. So, it is convenient to introduce:

\[
\Delta \tilde{S}(\mu) = \tilde{S}(\mu) - \tilde{S}(\mu = 0)
\]
\[
\Delta S(\mu) = S(\mu) - S(\mu = 0).
\]  

(4.1)

(4.2)

We calculated these variations, for bottomonium and charmonium, considering four representative temperatures. In Figure 1 we show plots for the variations \(\Delta \tilde{S}\), \(\Delta S\) and also their sum, for the bottomonium 1S state. One notes that the first quantity, that corresponds to the variation of the CE, increases with the density, while the second decreases. The sum, \(\Delta \tilde{S} + \Delta S\) shows a very small variation.

In figure 2 we present similar plots for the charmonium 1S state. In this case we choose a lower temperature interval because \(c\bar{c}\) states dissociate at lower temperatures. As in the bottomonium case, one finds that the CE increases with the density while the conjugate quantity \(\Delta S\) decreases and their sum exhibits a much smaller (positive) variation.

It is known that, as the density of the plasma increases, the dissociation degree of heavy mesons increases. So, they become more unstable in the sense of their tendency to “melt” in the plasma. So, the results shown in figures 1 and 2 for the CE are consistent with the interpretation that instability corresponds in general to an increase in the value of this quantity.

**Figure 1.** Variations of configuration entropy \(\tilde{S}\), its position space dual \(S\) and their sum with respect to chemical potential for \(\Upsilon\) at temperatures \(T = 300, 400, 500, 600\) MeV
Figure 2. Variations of configuration entropy $\tilde{S}$, its position space dual $S$ and the their sum with respect to chemical potential for $J/\psi$ at temperatures $T = 200, 300, 400, 500$ MeV

Regarding the quantity $S(\mu)$, that is the position space conjugate of the CE, we found a result that is similar to the one obtained in [45]. The corresponding variation $\Delta S(\mu)$ has the opposite behavior, decreasing with $\mu$. The sum $\Delta S(\mu) + \Delta \tilde{S}(\mu)$ satisfies the inequality (1.7) in a somehow trivial way, in the sense that this quantity has a very small variation, compared to the individual variations of $S$ and $\tilde{S}$.

It is interesting to investigate if the dependence of the CE on the density $\mu$ can be expressed in the form of a scaling law. As an illustration, we plot in figure 3 the logarithm of the CE as a function of the density for $J/\psi$ at $T = 200$ MeV and for $\Upsilon$ at $T = 300$ MeV. From the analysis of this kind of plot, for different temperatures, one finds that there is an approximate scaling law of the form

$$
\log(\tilde{S}) = c_0 + c_1\mu + c_2\mu^2,
$$

where the coefficients $c_0, c_1, c_2$ depend on the temperature. We show on tables 1 and 2 the values obtained for these parameters at different temperatures and the error in the polynomial approximation for $J/\psi$ and $\Upsilon$, respectively.
Figure 3. The left panel shows the logarithm of configuration entropy as a function of $\mu$ for $J/\psi$ at temperature 200 MeV. The right panel shows the logarithm of configuration entropy as a function of $\mu$ for $\Upsilon$ at temperature 300 MeV. The dots are the values for $\log(S)$ calculated and the continuous lines are the second order polynomial adjusts.

| $T$ (GeV) | $c_0$    | $c_1(GeV)^{-1}$ | $c_2(GeV)^{-2}$ | percentual error (%) |
|-----------|-----------|-----------------|-----------------|----------------------|
| 0.2       | 1.263     | 0.248439        | 0.861552        | 0.112                |
| 0.25      | 1.44593   | 0.107797        | 0.670848        | 0.061                |
| 0.3       | 1.56016   | 0.0539618       | 0.525653        | 0.034                |
| 0.35      | 1.64409   | 0.0310821       | 0.421229        | 0.021                |
| 0.4       | 1.71412   | 0.0198792       | 0.345314        | 0.015                |
| 0.45      | 1.77766   | 0.0112524       | 0.295084        | 0.009                |
| 0.5       | 1.83739   | 0.00845286      | 0.249219        | 0.009                |

Table 1. Coefficients $c_0$, $c_1$ and $c_2$ of eq. 4.3 for for $J/\psi$ meson at different temperatures.

| $T$ (GeV) | $c_0$    | $c_1(GeV)^{-1}$ | $c_2(GeV)^{-2}$ | percentual error (%) |
|-----------|-----------|-----------------|-----------------|----------------------|
| 0.3       | 3.23953   | 0.302745        | 2.2972          | 0.046                |
| 0.35      | 3.88824   | 0.130368        | 1.9909          | 0.014                |
| 0.4       | 4.39979   | 0.0673866       | 1.71382         | 0.006                |
| 0.45      | 4.82115   | 0.0423925       | 1.46801         | 0.003                |
| 0.5       | 5.17289   | 0.0294709       | 1.2607          | 0.003                |
| 0.55      | 5.46933   | 0.0201726       | 1.09106         | 0.002                |
| 0.6       | 5.723     | 0.0153527       | 0.948492        | 0.001                |

Table 2. Coefficients $c_0$, $c_1$ and $c_2$ of eq. 4.3 for $\Upsilon$ meson at different temperatures.
5 Conclusions

We studied here the variation of the configuration entropy for the 1S states of charmonium (the $J/\psi$) and bottomonium (the $\Upsilon$) as a function of the quark density of the medium. The results show that the entropy increases with the density. This is consistent with the expectation that an increase in the instability of a physical system should correspond to an increase in the CE.

For the dual spatial entropy $S$ we found a behavior similar to the one found in the AdS black hole case[45]. The two quantities $S$ and $\tilde{S}$ vary in opposite ways while their sum has a very small variation. As discussed in [45] if $S$ and $\tilde{S}$ are interpreted in terms of information content stored in the energy distribution in position and momentum space, respectively, the increase in instability could be associated with a change in the way the system stores information. It must be remarked however that information, as defined in the Shannon entropy of eq. (1.1), is related to a probability distribution. So, it is only in the case that the modal fractions $\epsilon$ and $\tilde{\epsilon}$ can be interpreted as probability densities, that the one could associate the change in stability with a change in information content.

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