Schedule Risk Analysis by Different Phases of Construction Project Using CPM-PERT and Monte-Carlo Simulation

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Abstract. Construction project is an important activity along with the increasing need of infrastructure and residential building. Construction projects with different investment value and complexity level affect the completion duration of construction work. The higher the value or the more complex the building, the longer time required to accomplish the construction project. During the construction process, it is possible to find risks both from internal and external management which potentially cause project overdue that will lead to cost overruns. This study focused on assessing and managing schedule risk in three main phases of construction project, such as: feasibility study, design, and construction. A certain sample of residential building project was chosen as study case. In this study, mathematical approaches are used by performing CPM-PERT and Monte-Carlo simulation method. The results of this study can be used to help project managers to minimize potential risks by projecting simulation results and to assess if a project can be completed in timely manner according to given schedule.

Keywords: schedule risk, construction project, CPM, PERT, monte carlo simulation

1. Introduction

Timeliness is one indicator to determine success of a project, in addition to cost and performance [1]. Project lateness will affect the total cost, since the workers need to be on duty in longer duration than the original schedule. Therefore, to minimize the possibility of project lateness, it is required a proper project scheduling. According to [2], here are many significant issues in project scheduling such as: the estimation of activity duration, resources allocation, and an unstructured scheduling. The main problem of project scheduling is the uncertainty, especially in resource availability. Uncertainty is a situation that can not be specified, which has many possibility that indicated by the lack of information about the result of an event [3]. The uncertain situation faced by the project manager is also known as risk, which if it happens, has positive or negative effect to duration, cost, scope or quality of project [4]. On a construction project, many factors like material procurement delay, human resource problem, weather, and bureaucracy have potential to cause risks [5]. Therefore, risk analysis and risk control technique are required to obtain effective and efficient project completion process. Risk control includes the identification, assessment, and prioritization of risks in a project. It allows the management to control over project schedule, estimation, and quality. By implementing risk analysis and management, the right decision can be made as early as possible to avoid lateness in the project. This research aims to build model from a complex project management system by considering potential risk factors, so the possibility of project lateness can be minimized.
1.1. Project Risk Management

Time is one measurement to determine project completion success. The inability to manage time, inavailability of technical skills to deal with sudden changes, and difficulty to visualize process makes it difficult for management to monitor project progress status [6]. Practically, both projects and the environment will continue to change in a sustainable manner, focusing on the importance of recognizing potential risks and opportunities, therefore quick decision making are needed on many alternative strategies [3]. According to Weick [7], project manager ability to understand situation on alternative action will determine how well the strategic decisions are made that impacting the company performance. Consequently, an advanced project management skill is needed to control the risk of uncertain situation.

1.2. Potential Risks of Construction Project

Project risks, as specified by Zou et al [8], are categorized into four different phase according to project life-cycle such as feasibility study, design, construction, and operation. Risks on feasibility phase, mostly related to clients and governmental agencies. Client need, strategic brief, and practical project feasibility study which also consider the price fluctuation of construction material are the main focus of this phase. The next stage is design phase where designers play the most important role to understand and satisfy clients’ wants and need. Risks on this phase are related to government bureaucracy, design defect and change that come from both client and contractors as well. After the design is fixed, project get into the construction phase, the risks on this phase are likely to correspond with contractors and subcontractors, such as variation of project delay, unavailability of sufficient amount of skilled workers, lack of coordination, and occurrence of safety accident. The possible risks on final phase after construction work is completed may only came from the funding for the project operation and difficult government regulation related to facility management, environment sustainability. Overall, the majority of risks occurred in the pre-operation phase [8].

2. Methodology

There are several approaches that can be used in project management such as Critical Path Method (CPM) and Project Evaluation Technique (PERT). Those methods help management in generating deterministic decisions that is associated with project completion time [6]. By using CPM, project managers are enabled to understand the critical activities that determine overall duration during the completion of project. While PERT, emphasize on scheduling technique by estimating the time required for each task using three-point estimators. PERT assumed that the duration of each project activity is a random variable and follows Beta distribution [9]. Another approach for risk analysis is simulation technique. Monte Carlo is simulation method for risk analysis by using probabilistic approach from many uncertain activity combination to evaluate the uncertainty. According to Barraza [10], the advantage of Monte Carlo simulation is able to provide near-realistic estimation results by balancing the probability value of each activity as a critical value.

Monte Carlo simulation has been widely used for the purpose of risk analysis in many project management studies. Research from Tysiak and Sereseanu [11] implemented CPM and Monte Carlo simulation method for risk analysis in IT project. The estimation of timeliness possibility for construction schedule became the main focus of research from Ganame and Chaudhari [12]. While a research from Yuan et al [13] applied Monte Carlo simulation to develop an early warning system prototype for construction schedule.

This study is distinct from other approaches [11] [12] [13], which emphasize on the potential risk analysis by different phase of construction project schedule using CPM-PERT and Monte Carlo simulation. Since the majority risks of construction project occured in the pre-operation stage [8], it became the main focus of this study. By implementing this approach, project managers’ assessment and decision can be more accurate to potential risks characteristics on pre-operation stage such as feasibility, design, and construction phase.
2.1. PERT and CPM
PERT assumed that the activity duration follows Beta distribution [14], so that three point estimator such as optimistic duration (a), pessimistic duration (b), and most likely duration (m) are used to calculate the mean (μ) and the variance (σ²) for each activity duration from state i to j with the following formula:

\[
\mu_{(i,j)} = \frac{a_{(i,j)} + 4m_{(i,j)} + b_{(i,j)}}{6}, \quad \sigma^2_{(i,j)} = \frac{(b_{(i,j)} - a_{(i,j)})^2}{36}
\]

(1)

Critical path which represent the longest duration from the first state (start) to the latest state (finish) of the project is the key to find the critical activities in PERT network by arranging project schedule that consist of earliest start time (ES), earliest finish time (EF), latest start time (LS), latest finish time (LF), and slack time (SL).

Earliest start time (ES) and earliest finish time (EF) are calculated forward from the first activity using the following formula:

\[
EF_{(i,j)} = ES_{(i,j)} + \mu_{(i,j)}, \quad ES_{(i,j)} = \max(EF_{(i,j')})
\]

(2)

Latest finish (LF) and latest start (LS) are calculated backward from the last activity with the formula below:

\[
LF_{(last)} = EF_{(last)}, \quad LS_{(i,j)} = LF_{(i,j)} - \mu_{(i,j)}, \quad LF_{(i,j)} = \min(LS_{(i,j')})
\]

(3)

Slack time (SL) or the measurement for identifying critical activities is determined using the following formula:

\[
SL_{(i,j)} = LS_{(i,j)} - ES_{(i,j)}
\]

(4)

If there are project activities that have SL value equal to 0, then the activities are identified as critical that form critical path on the PERT network. Project completion time is calculated by adding the duration of all critical activities.

2.2. Monte Carlo Simulation
Monte Carlo simulation is a method that depends on the generation of random or pseudo-random number from given distribution [15]. As stated by Hong et al [16], Beta-PERT distribution is more suitable to used in Monte Carlo simulation for estimating construction activity duration. Beta-PERT is the improvement of Beta distribution which can better fit to uniform, normal, and lognormal distribution [17]. It relies on α and β values (α > 0; β > 0) which determine the distribution shape that calculated using formula below [17]:

\[
\alpha = \frac{(\mu - x_{min})(2x_{prob} - x_{min} - x_{max})}{(x_{prob} - \mu)(x_{max} - x_{min})}, \quad \beta = \frac{x_{max} - \mu}{\mu - x_{min}}
\]

(5)

The \(x_{min}, x_{prob}, x_{max}\) and \(\mu\) in formula (6) respectively represent the minimum, most probable, maximum and mean of activity duration estimation value.

In this study, we use 10000 replications for simulating various project completion time. Random numbers are generated from Beta inverse using the parameter \(\alpha\) and \(\beta\) to create all replications. The generated random numbers or pseudonumbers are used as the simulated activity duration (x) which expressed as the following formula:

\[
x = f^{-1}(p | \alpha, \beta) = \{x : f(x|\alpha, \beta) = p\}
\]

(6)

The p value represents the probability of occurrence that is generated randomly from uniform distribution with the interval 0 to 1 as shown as the formula below:

\[
p = f(x|\alpha, \beta) = \frac{1}{B(\alpha, \beta)}x^{\alpha-1}(1-x)^{\beta-1}I_{[0,1]}(x)
\]

(7)
Project completion time \((T)\) is calculated by the critical path which consists of critical activities that already simulated for 10000 different project scenarios, therefore the \(T\) value also generated from \(T_1\) to \(T_{10000}\). Since the project completion time \((T)\) is assumed to follow normal distribution \([6]\), standard score of normal distribution \(N(\mu, \sigma^2)\) is calculated as follow:

\[
Z = \frac{T_i - \mu}{\sigma} \quad (8)
\]

The mean \((\mu)\) and standard deviation \((\sigma)\) are calculated using formula (1) for all simulated project completion time \((T_1, T_2, ..., T_{10000})\). The standard score of normal distribution is used to determine the timeliness possibility of a given project completion time as noted as \(p(T)\) in cumulative distribution formula below:

\[
p(T) = \Phi(Z) = \frac{1}{\sigma \sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{(t-\mu)^2}{2\sigma^2}} dt \quad (9)
\]

### 3. Results and Discussions

The activity list of a certain residential building project and the estimated duration of each activity such as optimistic duration \((a)\), pessimistic duration \((b)\), and most likely duration \((m)\) are given in Table 1.

| Activity Code | State \((i, j)\) | Activity Name                          | Immediate Predecessor | Estimated Duration (days) | Phase        |
|---------------|-----------------|----------------------------------------|-----------------------|--------------------------|--------------|
| A             | \((1, 2)\)      | Strategic Brief & Survey               | -                     | 9 10 15                  | Feasibility  |
| B             | \((2, 3)\)      | Budgeting & Feasibility Studies        | A                     | 10 11 13                 |              |
| C             | \((3, 4)\)      | Concept Design                         | B                     | 10 14 16                 | Design       |
| D             | \((4, 5)\)      | Develop Design & Regulation Compliance | C                     | 13 15 21                 |              |
| E             | \((5, 6)\)      | Technical Design                       | D                     | 9 12 14                 |              |
| F             | \((6, 7)\)      | Enabling Works                         | E                     | 8 10 11                 | Construction |
| G             | \((7, 8)\)      | Sub Structure Work                     | F                     | 23 25 28                |              |
| H             | \((8, 9)\)      | Column & Curtain Wall                  | G                     | 18 21 22                |              |
| I             | \((9, 10)\)     | Plumbing                               | H                     | 5 8 9                   |              |
| J             | \((9, 12)\)     | Ceiling                                | H                     | 4 9 10                  |              |
| K             | \((9, 11)\)     | Roof Installation                      | H                     | 10 16 20                |              |
| L             | \((10, 12)\)    | Electrical Work                        | I                     | 6 9 10                  |              |
| M             | \((11, 12)\)    | Roof Isolation                         | K                     | 4 5 7                   |              |
| N             | \((12, 13)\)    | Plastering & Mist Coat                 | L, M                  | 12 14 18                |              |
| O             | \((13, 15)\)    | Windows, Frames, and Doors Installation| N                     | 7 11 19                 |              |
| P             | \((15, 17)\)    | Painting                               | P                     | 15 18 22                |              |
| Q             | \((12, 14)\)    | Tiling & Flooring                      | J, L                  | 12 15 15                |              |
| R             | \((14, 16)\)    | Decoration                             | O                     | 8 12 13                 |              |
| S             | \((16, 17)\)    | Landscaping                            | R                     | 6 7 10                  |              |
| T             | \((17, 18)\)    | Cleaning & Final Control               | P, S                  | 2 5 6                   |              |
| U             | \((18, 19)\)    | Project Closure & Post Review          | T                     | 6 7 10                  |              |

Based on the Table 1, the project activities then arranged to a network called PERT chart by considering its predecessor and successor, as shown in Figure 1.
Figure 1. PERT Chart of Project

PERT chart only gives information about the sequence of project activities, but the critical path which has important role to determine the project duration need to be identified with CPM method. The critical path analysis is shown in Table 2.

Table 2. Critical Path Analysis

| Activity Code | State (i, j) | Estimated Duration (days) | Critical Path Analysis from PERT Chart (days) | Critical Activity Identification |
|---------------|-------------|---------------------------|---------------------------------------------|--------------------------------|
| A             | (1, 2)      | 9 10 15 11 11 0 11 11 0 11 0 11 0 | Critical                                  |
| B             | (2, 3)      | 10 11 13 11 11 22 22 11 22 11 22 0 | Critical                                  |
| C             | (3, 4)      | 10 14 16 14 11 11 11 22 22 11 22 0 | Critical                                  |
| D             | (4, 5)      | 15 13 21 16 16 36 36 52 52 36 52 0 | Critical                                  |
| E             | (5, 6)      | 9 12 14 12 52 52 52 64 64 64 64 0 | Critical                                  |
| F             | (6, 7)      | 10 11 11 11 11 11 11 64 64 64 64 0 | Critical                                  |
| G             | (7, 8)      | 23 25 28 25 25 74 74 99 99 99 99 0 | Critical                                  |
| H             | (8, 9)      | 19 21 22 21 99 99 99 120 120 120 120 0 | Critical                                  |
| I             | (9, 10)     | 9 8 9 8 120 120 120 124 124 124 124 4 | Non-Critical                             |
| J             | (9, 12)     | 4 9 10 8 120 120 120 133 133 133 133 13 | Non-Critical                             |
| K             | (9, 11)     | 10 16 20 16 120 120 120 136 136 136 136 0 | Critical                                  |
| L             | (10, 12)    | 6 9 10 9 128 128 128 137 137 137 137 14 | Non-Critical                             |
| M             | (11, 12)    | 4 5 7 5 136 136 136 141 141 141 141 0 | Critical                                  |
| N             | (12, 13)    | 12 14 18 14 141 141 141 155 155 155 155 0 | Critical                                  |
| O             | (13, 15)    | 7 11 19 12 155 155 155 167 167 167 167 0 | Critical                                  |
| P             | (15, 17)    | 15 18 22 18 167 167 167 185 185 185 185 0 | Critical                                  |
| Q             | (12, 14)    | 12 15 15 15 137 137 137 152 152 152 152 14 | Non-Critical                             |
| R             | (14, 16)    | 8 12 13 12 152 152 152 164 164 164 164 14 | Non-Critical                             |
| S             | (16, 17)    | 6 7 10 7 164 164 164 171 171 171 171 14 | Non-Critical                             |
| T             | (17, 18)    | 2 5 6 5 185 185 185 190 190 190 190 0 | Critical                                  |
| U             | (18, 19)    | 6 7 10 7 190 190 190 197 197 197 197 0 | Critical                                  |

Length of Critical Path (Project Duration) = \( EF_{(last)} = LF_{(last)} = 197 \) days

The critical activities can be determined based on the activity with zero slack time (SL). According to Table 2, the critical activities are: A-B-C-D-E-F-G-H-K-M-N-O-P-T-U. As a complement to default PERT chart, the critical path by project phase is shown in Figure 2.
According to critical path analysis using CPM-PERT method, the critical project duration is 197 days which consists of feasibility study phase (22 days), design phase (42 days), and construction phase (133 days). The duration represents the shortest time possible to complete the project, that means there are always uncertain conditions that make project completion duration become exceeded.

In this situation, simulation technique become useful to tackle the problem. By using simulation technique, the uncertain conditions are then simulated for a number of replications, so the management is able to understand any situation that possible to happen in the future. Since the critical path is already identified using CPM-PERT method, Monte Carlo is performed for 10000 replications to simulate the duration of critical activities which generated using Beta inverse function as expresed in formula (7). The simulation result is shown in Table 3 below.

### Table 3. Monte Carlo Simulation Result

| Critical Activity Code | Estimated Duration (days) | Simulated Duration of Critical Activities (days) | Average and Std Dev of Simulated Duration by Phase (days) | Histogram of Simulated Duration by Phase (days rounded) |
|------------------------|---------------------------|--------------------------------------------------|---------------------------------------------------------|-------------------------------------------------------|
| A                      | 9 10 15                   | 9.0068 10.674 14.4704                             | Avg=21.8296  StdDev=1.1580                              | 19 21 26                                              |
| B                      | 10 11 13                  | 10.0216 11.1622 12.8689                            |                                                         |                                                       |
| C                      | 10 14 16                  | 10.1936 13.6733 15.9775                            | Avg=41.1602  StdDev=2.0271                              | 33 41 50                                              |
| D                      | 13 15 21                  | 13.0307 15.6557 20.5089                            |                                                         |                                                       |
| E                      | 9 12 14                   | 9.1523 11.8312 13.9382                             |                                                         |                                                       |
| F                      | 8 10 11                   | 8.1396 9.8340 10.9698                              | Avg=132.5823  StdDev=3.8088                            | 120 133 145                                           |
| G                      | 23 25 28                  | 23.0273 25.1678 27.9006                            |                                                         |                                                       |
| H                      | 18 21 22                  | 18.2968 20.6685 21.9922                            |                                                         |                                                       |
| K                      | 10 16 20                  | 10.2076 15.6471 19.9261                            |                                                         |                                                       |
| M                      | 4 5 7                    | 4.0241 5.1704 6.8536                                |                                                         |                                                       |
| N                      | 12 14 18                  | 12.0123 14.3275 17.6958                            |                                                         |                                                       |
| O                      | 7 11 19                   | 7.0460 11.6152 18.4261                              |                                                         |                                                       |
| P                      | 15 18 22                  | 15.0912 18.1584 21.8300                            |                                                         |                                                       |
| T                      | 2 5 6                    | 2.3330 4.6650 5.9904                                |                                                         |                                                       |
| U                      | 6 7 10                   | 6.0115 7.3282 9.9174                                |                                                         |                                                       |

**Simulated Project Duration (μ=195.5720 days ; σ=4.4982)**

From the simulation, 10000 various possible combinations of project duration are generated that plotted into histogram that shown in Figure 3 below.
Figure 3. Histogram of Simulated Project Duration

Histogram that is shown on figure 3 looks rather similar to normal distribution pattern, which are symetrical and centered about its mean. It is supported by assumption [6] that project completion duration follows normal distribution. Through on the assumption, various schedule timeliness possibilities are generated by following cumulative normal distribution function with parameter \( \mu \) dan \( \sigma \) which obtained from the simulation into formula (9) and (10). The graph of schedule timeliness possibilities both for each phase and overall project (with additional 5% and 95% possibility indicators) are shown in figure 4 below.

Figure 4. Schedule Timeliness Possibilities

The graphs shows that the shorter project deadline the less likely the project can be finished on time. The longer its deadline the more likely the project can be completed in timely manner. If project schedule given to management is based on the CPM-PERT result, which is 197 days, the timeliness possibility of the project is \( p(197) = \Phi(0.3071) \approx 62.06\% \). In other words, the schedule or project deadline have potential risk of lateness 37.94\%. According to the simulation result, timeliness possibilities for each phases duration can be identified as well, the critical duration for feasibility study is 22 days then the timeliness possibility for feasibility phase is \( p(22) = \Phi(0.1472) \approx 55.85\% \), for 42 days duration of design phase has timeliness possibility \( p(42) = \Phi(0.4143) \approx 66.07\% \), while the construction phase has \( p(133) = \Phi(0.1097) \approx 54.37\% \) timeliness possibility.
4. Conclusion
The ability of a company to complete projects in timely manner become business competitive advantage, especially in the increasingly fierce business competition environment. CPM-PERT is a method that allows the management to estimate the duration of a construction project, so that the financial budget and resources can be planned properly. In this study, a simulation technique is used as a complementary to CPM-PERT method to give project duration estimation and the potential risk value of lateness from a given project schedule. Monte Carlo method is used to simulate each activity duration to analyze the completion timeliness possibility both for each phase and overall project. According to the simulation, the management has possibility to finish a construction project in 197 days (as it estimated by CPM-PERT) is 62.04%, in 203.1 ≈ 204 days is 95%. While 100% timeliness possibility can be obtained in minimum 208.6 ≈ 209 days. Those possibility range will help the management to determine schedule for their project, both for each project phase and or overall project. Potential risk analysis which explain the category and its severity will be the main focus for further research.

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