Transition prediction with hypersonic cross-flow model on HIFiRE-5

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Abstract. The work develops a localized hypersonic cross-flow transition criterion considering the influence of cross-flow intensity and surface roughness. A cross-flow extension of hypersonic modified \( \gamma - \text{Re}_\theta \) transition model based on Chant2.0 computing platform is implemented. The extended transition model is used to predict the cross-flow transition on the elliptic cone (HIFiRE-5) in multiple states, and the predicted results are in good accordance with the experimental results.

1. Introduction

The accurate prediction of hypersonic three-dimensional boundary layer transition induced by cross-flow instability is one of the main difficulties in hypersonic vehicle's aerodynamic design. In real flight, the cross-flow instability mechanism is widespread and becomes the dominant factor in transition of the three-dimensional boundary layer [1].

The transition model based on RANS equation is a powerful means to solve practical engineering problems. In respect of cross-flow transition models, researches domestic and abroad mainly focuses on transition criteria and transition models in low-speed field [1], and has achieved good prediction results. There are relatively few studies on hypersonic cross-flow transition models, which are mainly based on existing models [2][3][4]. There are also independent models with hypersonic cross-flow transition prediction capabilities [5].

This paper constructs a hypersonic cross-flow transition criterion based on DNS data extended by stability theory. The criterion considers the influence of cross-flow intensity and surface roughness. The hypersonic modified \( \gamma - \text{Re}_\theta \) transition model based on the Chant2.0 computing platform [1] has been locally extended, thereby constructing a C- \( \gamma - \text{Re}_\theta \) model which is suitable for hypersonic cross-flow transition prediction. The model is suitable for hypersonic cross-flow transition prediction. The numerical results is in good agreement with the quiet tunnel experiment and flight test in the prediction of Ma6 elliptic cone (HIFiRE-5).

2. Governing equations and basic turbulence/transition model

The finite volume method is applied to solve the Reynolds Average Navier-Stokes (RANS) equations. For the inviscid flux, it adopts the AUSM format and uses second-order precision MUSCL format to discretize. Moreover, the Van-Leer and Minmod hybrid limiter is used to capture the shock wave
discontinuity. The second-order central difference format discretizes the viscous flux. The time advancement utilizes the LU-SGS implicit method, and the MPI technology is applied for parallel calculation.

2.1. SST turbulence model
The widely-used and highly-reliable Shear-Stress Transport (SST) turbulence model proposed by Menter is applied. The mixing function \( F_i \) is designed to combine the \( k-\omega \) model with the \( k-\varepsilon \) model. Therefore, the SST model has both the \( k-\omega \) model's strong robustness in the boundary layer and the \( k-\varepsilon \) model's excellent performance in free flow. The transport equation form is as follows, the definition and specific parameter settings in the model can be found in the literature [6].

\[
\frac{\partial (\rho k)}{\partial t} + \frac{\partial (\rho U k)}{\partial x_i} = \bar{P}_k - \beta^* \rho \omega k + \frac{\partial}{\partial x_j} \left[ (\mu + \sigma_i \mu_j) \frac{\partial k}{\partial x_j} \right]
\]

\[
\frac{\partial (\rho \omega)}{\partial t} + \frac{\partial (\rho U \omega)}{\partial x_i} = \frac{\gamma}{\nu_f} \bar{P}_\omega - \beta \rho \omega^2 + \frac{\partial}{\partial x_j} \left[ (\mu + \sigma_i \mu_j) \frac{\partial \omega}{\partial x_j} \right] + 2\rho(1-F_1)\frac{\sigma_{\omega \sigma}}{\omega} \frac{\partial k}{\partial x_i} \frac{\partial \omega}{\partial x_i}
\]

2.2. \( \gamma-\text{Re}_\theta \) transition model
The \( \gamma-\text{Re}_\theta \) transition model contains two transport equations, the equation of \( \gamma \) and \( \text{Re}_\theta \). The intermittent factor \( \gamma \) characterizes the time ratio of the flow in turbulent and laminar flow. The local transition onset momentum thickness Reynolds number \( \text{Re}_\theta \) is used to capture the non-local influence of turbulence intensity and to avoid non-local calculations in the model. The dimensionless form of the transport equation is as follows, and the definitions and parameter settings are in reference [7].

\[
\frac{\partial (\rho \gamma)}{\partial t} + \frac{\partial (\rho U \gamma)}{\partial x_j} = P_\gamma + E_\gamma + \frac{1}{\text{Re}_\theta} \frac{\partial}{\partial x_j} \left[ (\mu^* \frac{\rho}{\sigma_f}) \frac{\partial \gamma}{\partial x_j} \right]
\]

\[
\frac{\partial (\rho \text{Re}_\theta)}{\partial t} + \frac{\partial (\rho U \text{Re}_\theta)}{\partial x_j} = P_\text{Re}_\theta + \frac{1}{\text{Re}_\theta} \frac{\partial}{\partial x_j} \left[ (\sigma_{\mu \mu} \mu^* \frac{\rho}{\sigma_f}) \frac{\partial \text{Re}_\theta}{\partial x_j} \right]
\]

The transition model is embedded in \( k-\omega \) SST turbulence model. The transition model is accomplished by the effect of the effective intermittent factor \( \gamma_{\text{eff}} \) on the generation term \( P_k \) and the dissipation term \( D_k \) of the turbulent kinetic energy equation:

\[
\bar{P}_k = \gamma_{\text{eff}} P_k, \quad \bar{D}_k = \min(\max(\gamma_{\text{eff}}, 0.1), 1.0)D_k
\]

3. Hypersonic cross-flow transition criterion and cross-flow extension

3.1. Hypersonic cross-flow transition criterion
The core idea of the \( \gamma-\text{Re}_\theta \) transition model is to no longer integrate the Momentum Thickness Reynolds number \( \text{Re}_\theta \), but to use the transport equation to solve \( \text{Re}_\theta \). The original model cannot predict the cross-flow transition. Following, Langtry [8] expanded it with low-speed cross-flow transition prediction capabilities. This paper refers to the construction form of Langtry's low-speed cross-flow transition criterion [8], and constructs a prediction criterion suitable for the hypersonic cross-flow transition based on the stability extension of DNS test data [9].

As shown in Figure 1, the expanded " \( \text{Re}_{\text{EF,crit}} - h \) " data is separated into nine groups and each with 3 data points, a total of 27 data points. Factors include angle of attack and surface roughness. Based on the above data, referring to the form of low-speed cross-flow transition criterion [8], a localized hypersonic cross-flow transition criterion considering surface roughness is constructed [10]:
\[ Re_{CF} = C_1 \cdot \ln \left( \frac{h}{\mu} \right) + C_2 + f(H_{crossflow}) \]  

(6)

Figure 1. Stability extension of transition data for elliptic cone.

Among them, \( H_{crossflow} \) is the non-dimensionlized streamwise vorticity [8]. Roughness height "\( h \)" is obtained by surface roughness measurement, \( l \mu = 1 \mu m \). \( C_1 \), \( C_2 \) is obtained by the least square method, \( C_1 = -9.618 \), \( C_2 = 128.33 \), and the lifting function is defined based on the 0° ellipsoid data:

\[ f(H_{crossflow}) = 6000 \left[ 0.1066 - \Delta H_{crossflow} \right] + 50000 \left( 0.1066 - \Delta H_{crossflow} \right)^2 \]  

(7)

\[ \Delta H_{crossflow} = H_{crossflow} \left[ 1.0 + \min \left( 1.0, 0.4 \right) \right] \]  

(8)

3.2. Cross-flow extension

The extension method of the cross-flow effect in the transition model is similar to that in the reference[8] which is embedded in the momentum thickness Reynolds number \( Re_{\theta} \)- transport equation as a source term \( D_{CF}, F_{\theta} \), comes from the \( \gamma - Re_{\theta} \) transition model to ensure that the source term is turned on in the boundary layer and closed outside the boundary layer:

\[ \frac{\partial \left( \rho \bar{e}_{\theta} \right)}{\partial t} + \frac{\partial \left( \rho U_j \bar{e}_{\theta} \right)}{\partial x_j} = P_{\theta} + D_{CF} + \frac{1}{Re_{\theta}} \frac{\partial}{\partial x_j} \left[ \sigma_{\theta_i} \left( \mu + \mu_i \right) \frac{\partial \bar{e}_{\theta_i}}{\partial x_j} \right] \]  

(9)

\[ D_{CF} = c_{CF} \cdot \rho \min(Re_{CF} - Re_{\theta}, 0) \cdot F_{\theta} \]  

(10)

4. Numerical results

4.1. HIFiRE-5b flight test

The flight test model is an elliptic cone with a 7-degree half-angle on the minor axis. And the elliptic cone section of the payload was 861.05mm in length. It holds a 2:1 aspect ratio and a 2.5-mm-radius nose tip. The maximum Mach number reached 7.9, and the maximum unit Reynolds number was 5.0 \times 10^7/m. The test adopts the surface distributed thermocouple to infer the transition front. The transition model's calculation state is at ballistic time \( t = 515.1s \), the freestream unit Reynolds number is \( 1 \times 10^7 \) and the freestream Mach number is about 7.82. As shown in fig 2, from left to right are the flight test results [11], the calculation results of the C-\( \gamma \)-Re\( \theta \) model and the original \( \gamma \)-Re\( \theta \) model. From the figure, we find that the modified transition model predicts the dominating hypersonic cross-flow transition on the vehicle surface well. Furthermore, the transition location and the transition front of the three-
dimensional boundary layer fit the flight test well. The original $\gamma$-Re$_0$ model is not capable of predicting hypersonic cross-flow transition because its transition criterion is not suitable for hypersonic speed, and almost no transition occurs on the surface of the vehicle.

![Figure 2. Cross-flow transition on HIFiRE-5b flight vehicle.](image)

### 4.2. HIFiRE-5 wind tunnel experiment

In wind-tunnel experiment, the model is 38.1% scaled of the flight test[12], the images of experimental cross-flow transition(fig3(a)) is obtained by TSP(Temperature Sensitive Paint) technology. The numerical calculation is consistent with the wind tunnel experiment. The elliptic cone has a 2:1 aspect ratio and a 7-degree half-angle on the minor axis. It holds a 1-mm-radius nose tip, and the length of the elliptic cone section was 328.1mm. The Mach number Ma=6, the Reynolds number range is $10.3 \sim 13.0 \times 10^{6}$/m, the inflow turbulence is set to Tu=0.1% consulting the silent wind tunnel level, and the angle of attack is zero. The grid amount for the numerical calculation of the half-mode grid amount is about 15 million, and the normal grid spacing of the first layer on the object surface is $y^+=1$. This experiment is one of the few hypersonic cross-flow transition wind tunnel tests with surface roughness information. The surface roughness is $0.17 \sim 0.42 \mu m$[12], 0.3$\mu m$ is used in this article.

In figure 3, we make a comparison between the heat flow distribution of quiet tunnel (fig3(a)) under different freestream Reynolds number conditions[12] and the heat flow distribution predicted by the C-$\gamma$-Re$_0$ transition model(fig3(b)). The freestream unit Reynolds numbers from top to bottom are $10.3 \times 10^6$/m, $11.3 \times 10^6$/m and $13.0 \times 10^6$/m. The transition model can simulate the transition front on the elliptical cone surface dominated by the steady cross-flow vortex in the quiet tunnel. The shape is symmetrical 'two lung lobes', and the starting position of transition predicted by the model is also relatively accurate. Due to the mesh resolution and modeling, the transition model can only describe the overall transition area and cannot describe the details of the transition flow field (such as details of the 'stripe' high heat flow area caused by the cross-flow standing vortex).

![Figure 3. Cross-flow transition on HIFiRE-5 of (a)wind tunnel[12] & (b)transition model.](image)

Under flight conditions or quiet tunnel conditions, the cross-flow transition is usually dominated by stationary cross-flow vortex, which is affected by surface roughness[12]. The C-$\gamma$-Re$_0$ model can
reflect the effect of surface roughness on stationary cross-flow. In figure 4(a), the wind tunnel experiment[12] the surface roughness varies with other flow parameters precisely the same. The increase in roughness causes transition to move forward. By setting different roughness parameters (0.3 μm & 10 μm) in the model, the effect of surface roughness on the stationary cross-flow transition is reflected (figure 4(b)).

Figure 4. The influence of surface roughness on stationary cross-flow transition.
(a)wind tunnel[12]  (b)transition model

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