A NEW FUNDAMENTAL DUALITY IN NUCLEI
AND ITS IMPLICATIONS FOR
QUANTUM MECHANICS

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The Liquid Drop Models (LDM) and the Independent Particle Models (IPM) have been known to provide two conflicting pictures of the nucleus. The IPM being quantum mechanical, is believed to provide a fundamental picture of the nucleus and hence has been focus of the still elusive unified theory of the nucleus. It is believed that the LDM at best is an effective and limited model of the nucleus. Here, through a comprehensive study of one nucleon separation energy, we give convincing evidence that actually the LDM is as fundamental and as basic for the description of the nucleus as the IPM is. As such the LDM and the IPM provide simultaneously co-existing complementary duality of the nuclear phenomena. This fundamental duality also provides solution to the decades old Coester Band problem in the nucleus. Similarity and differences with respect to the well known wave-particle duality, as envisaged in Bohr’s Complementarity Principle, is pointed out. Thereafter implications of this new Duality in the nucleus for quantum mechanics is discussed.

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In the search for a "unified theory" of nuclear physics, in spite of intense efforts in the last 70 years or so, the dream remains unfulfilled. A large number of models have been proposed to study the nucleus in specific situations and in which cases these are quite successful. A single comprehensive theory of the nuclear phenomena however is still lacking.

Broadly speaking, all these large number of models can be classified as being of two kind: Liquid Drop Models (LDM) and Independent Particle Models (IPM). There is a basic and underlying conflict between these two: as the LDM requires a strongly interacting nucleus displaying a classical liquid drop character, while the IPM consists of weakly interacting and independent particles in a shell and which is quantum mechanical in character. Whether these diverse models are of the LDM kind or the IPM kind is determined by the underlying initial philosophy which goes into building the Hamiltonian of the model. Thereafter incorporated are various kind of corrections: primarily of the IPM compensating kind if the initial model was LDM, and primarily of the LDM compensation kind if the model was IPM initially. This is the broad picture of all the large number of models in nuclear physics [1]. Henceforth we shall use the acronym LDM and IPM to classify all these large number of nuclear models in use today. As IPM is basically quantum mechanical in character, it is felt that the putative "unified theory" would be intrinsically of IPM in character with suitable correction terms. This dream still remains unfulfilled in spite of intense efforts in this direction. The LDM is believed to provide at best a tentative and approximate picture of the nucleus. In fact many a "purist" would have it banished but for the fact that it offers a simple and straightforward description of several effects in nuclei. However, can we take the fact of the failure to obtain a viable "unified theory" of the nucleus as of now, as hinting at the possibility that perhaps the LDM intrinsically has "more" in it than what it has been credited for?

To understand the basis and the relationship between the LDM and IPM, we study the single neutron separation energies $S_N$ and the single proton separation energies $S_p$ in depth. In fact, as already well known, successful fitting of $S_N$ as a function of neutron numbers $N$ ( for various fixed proton numbers $Z$ ) and of fitting $S_p$ as a function of $Z$ ( for various fixed $N$ ), has been one of the initial checks on the validity of a particular model. As it turns out practically all the models come out successful in this regard. We have checked this within various models ( indicated below ) and confirm that by and large all these models are very successful in plotting separation energies as indicated above - that is $S_N$ vs $N$ (for
various fixed $Z$, both magic and non-magic) and $S_p$ vs $Z$ (for various fixed $N$, both magic and non-magic). The success of all these models as to the above separation energies indicates that if we were to plot separation energies slightly differently - that is: $S_N$ vs $Z$ (for various fixed $N$) and $S_p$ vs $N$ (for various fixed $Z$), then too we should get equally successful fits for all these models. We have checked that this too is always true (hundreds of cases - both magic and non-magic) \textbf{EXCEPT} for just three cases (to be discussed below). There is no a priori reason for the failure in just these three cases while the same models are so successful in fitting \textbf{ALL} the the separation energies plotted for any $N$ or any $Z$ and plotted in any consistent manner. This unexpected failure of separation energies in just these three cases is very puzzling.

The experimental data on binding energies and various separation energies is available in Ref. [2]. As a representation of the LDM we take the most comprehensive and the most respected compilation of their model calculations of the so called FRDM in Ref. [3]. Within the IPM the most respected and widely quoted is the comprehensive compilation of the calculation of mean field theory model of HFBCS-1 [4] (which we call HFB in short). These two are the most successful and widely accepted versions of the LDM and the IPM categories respectively. As one more representation of the IPM category we also use the published compilation of the so called INM [5].

We now have comprehensive plotting of $S_N$ vs $N$ (for various fixed $Z$ - both magic and non-magic); $S_N$ vs $Z$ (for various fixed $N$ - both magic and non-magic); $S_p$ vs $Z$ (for various fixed $N$ - both magic and non-magic); and $S_p$ vs $N$ (for various fixed $Z$ - both magic and non-magic). Plotted were the experimental numbers and compared with FRDM, HFB and INM model calculation results. As expected we find that all these models are by and large pretty much successful in fitting all these separation energies. This gives credence to the statement that all these are "successful" models of the nucleus. We find that ONLY for three cases there is a marked failure of the HFB and INM cases while FRDM is still successful in these cases also. These are: $S_N$ vs $Z$ for $N=126$ and $N=82$ and $S_p$ vs $N$ for $Z=82$. We plot these in Figs. 1, 2 and 3 respectively. The success of FRDM even in these three cases attests to the validity of basic underlying philosophy of liquid drop character of the nucleus with suitable IPM kind of corrections. Is the failure of the IPM models pointing towards the shortcoming of having ignored the liquid drop character of the nucleus?

For $S_N$ vs $Z$ for $N=N_{\text{magic}}$, the core consists of $N_{\text{magic}} + Z_{\text{magic}}$. Now as more protons are
added, the core would be affected say due to the mean field created by the extra protons each time. This is the well known effect in shell model; e.g. the electromagnetic properties of $O^{17}$ can only be explained by incorporating effective charges induced due to core polarization as a result of the extra neutron sitting outside the $O^{16}$ core. Now in the above separation energy we are picking up one neutron from this polarized core. This core remains the same as $Z$ goes from $Z_{\text{min}}$ to $Z_{\text{max}}$ (both of which are given by the empirical data).

Now any self consistent successful calculation in IPM (e.g. HFBCS-1) should have reproduced the $S_N$ vs $Z$ for $N=N_{\text{magic}}$ at $N=82$ and 126 respectively. It works for all the different neutron numbers and so it should have self consistently reproduced the above separation energy. So any such polarization should have been reproduced self-consistently by HFBCS-1. The same parameters of HFBCS-1 which successfully fit all the other separation energies (of the above kind as well), just cannot be changed arbitrarily for these two nuclei only. Hence clearly this failure should be taken as a fundamental conundrum. This suggests that, unknowingly or knowingly, we are missing some fundamental aspect of the nuclear reality, and which is manifesting itself only here in terms of the separation energies. What is it that this IPM kind of HFBCS-1 is missing? Quite clearly all possible conceivable corrections have, one way or the other, been included in these IPM kind of models.

We know that IPM definitely ignores the liquid drop character of the nucleus. It was never taken as reflecting any fundamental reality of the nucleus. But it may just be that this assumption was wrong and that the solution to this conundrum may lie in a proper incorporation of the LDM character in the IPM.

To understand the issue under discussion, we plot a schematic shell structure in Fig. 4. We assume that the neutron number is fixed at $N_{\text{magic}}$ so to say at $N=82$ and 126 respectively for Figs 1 and 2. For proton numbers, first one sees it as closed at $Z_{\text{magic}}$ so as to form a doubly magic closed core. This discussion would correspond to the plots in Figs. 1 and 2 of $S_N$ vs $Z$ for $N=82$ and $N=126$ respectively. That is one adds more protons on top of the magic number as the $Z$ number is changed. So for the $N=82$ and 126 cases, $Z_{\text{magic}}$ is 50 and 82 respectively (see figs. 1 and 2). We can treat the doubly magic core of all these nuclei for different $Z$ as forming the liquid drop kind of core. For different $Z$ numbers this core would be successively deformed or polarized more and more. In these separation energies we are pulling out the same single neutron for different $Z$ numbers. Now as stated above we expect all these polarization effects to be taken care of automatically in any IPM. As
expected IPM works well for all, except these three cases. Clearly it is not because of any fundamental shortcoming of these IPM, and hence, we are forced to conclude, that this may be arising from the ignored liquid drop character in these models.

Let us study Fig 1 and 2 carefully. First the HFB plot with respect to the experimental plot in Fig. 1. We notice that there is a clear pattern in the manner in which HFB is missing the experimental points. First for Z=81 the HFB point is opposite in direction to the experimental point. Next right from Z=82 upwards the HFB plot runs parallel to the experimental plot almost at a constant distance from it. This constant difference between the two lines is significant and shows that it is independent of the Z number. On the basis of figure 4, it suggests that this constant difference must be a function of the liquid drop core, a fixed number for all Z numbers, that is of

\[ A_{\text{magic}} = N_{\text{magic}} + Z_{\text{magic}} \]

which is 126+82=208 for this case. We know that in liquid drop models, in the binding energy the most significant term is a volume term ( contributing the dominant \( a_vA \) term to the binding energy with \( a_v = 16 \text{ MeV} \) ). So here too we notice that the almost constant difference of about 0.5 MeV between the theory and the experiment can be understood to be arising from a volume term \( b_vA_{\text{magic}} \) which is due to the liquid drop like core for all these nuclei. For the case under study with \( A_{\text{magic}} = 208 \) we get \( b_v = 0.0024 \text{ MeV} \). Note that this LDM kind of correction is, clearly beyond the periphery of all the corrections incorporated in HFBCS-1 ( or any other IPM for that matter ).

Next the situation for N=82 case ( Fig.2 ) is the same with about a similar value for \( b_v \). For the other IPM model considered here, that is the INM, it also similarly misses the experimental points completely and systematically ( with no structure whatsoever ), in the same manner as the HFB model does ( in fact more clearly and cleanly! ). This too supports the discussion above indicating here also the necessity of and LDM kind of correction.

Now let us study \( S_p \) vs N for Z=82 (Fig. 3). Here too FRDM fits the data quite well. This supports, as discussed earlier, the basic underlying LDM philosophy of FRDM. To understand the manner in which the IPM are failing here, we need to have a figure like Fig. 4 but now exchanging symbols N and Z. Note that HFB has even opposite behaviour at N=106, 121 and 125. Most interestingly it fits the data nicely for N=126 and above. So given magic Z=82 fixed here, the relevant neutron magic number is N=126. Hence the ”core” is doubly magic A=82+126=206. Thus the lower N numbers in the figure may be
considered as "holes" in this liquid drop like "core". Hence, as discussed above the almost constant difference between the HFB data and experiment below N=126 is due to the LDM effect with about the same $b_v$ value. The INM is also missing the data in about the same manner as discussed above. Therefore the same conclusion as to the role of LDM is relevant here too. Hence we conclude that to explain the three anomalies, it is necessary to include both IPM and LDM simultaneously.

If this be so, then this co-existence of the LDM and IPM characteristic should obviously be present in all the nuclei, light and heavy. However we find that for smaller magic numbers like N/Z=20, 50 etc there are no prominent effects like in the above three heavy nuclei cases. We plot these in Figs. 5 and 6. At best one may note some pre-cursor effects, like for example for the INM cases in Figs. 5 and 6.

This indicates that though, the above IPM+LDM effect must be present in all nuclei, it does not manifest itself that clearly and prominently for lighter nuclei. For lighter nuclei apparently the sensitivity of the liquid drop rigidity is not so prominent so as to stand out above what may be parametrized in any good IPM. What we notice is that this liquid drop becomes rigid enough to stand out above whatever the best IPM models can parametrize, only for heavy nuclei. The sensitivity and freedom of good parametrization (as apparently done in HFB etc) is good enough to mask and liquid drop effects that should be present in light nuclei as well. However, for heavy nuclei, even the best parametrization fails to mask the same, wherein there appears as a much stronger liquid drop effect. The separation energy, plotted in the special manner, as done here, has allowed us a peep through a narrow window at this unique phenomenon.

Note that we found that though present in any nuclei, light or heavy, it was actually getting manifested clearly in separation energy studies, only for heavier nuclei. Hence one expects that for Infinite Nuclear Matter the presence of LDM character should be even more manifest in any IPM study. Hence it should have something to say about a resolution of the Coester Band problem.

Indeed, this Duality also provides an answer to the well known several decades old puzzle, the Coester Band problem [6]. It is known that the best realistic interactions (like the Reid Interaction), using the best techniques, cannot reproduce the Nuclear Matter properties. They miss the binding energy and the saturation density of Nuclear Matter in a systematic manner, along the so called Coester Band. It is found [6] that when the saturation density
is fitted, the binding energy comes out too low (as compared to the 16 MeV value). Clearly these results fall within the ambit of our definition of the IPM. Hence on the basis of our discussion here, all these IPM are ignoring the LDM effects at a fundamental level. The corrections in the Coester Band are therefore clearly arising due to the non-inclusion of the LDM part.

The above discussion makes it clear that the resolution of the separation energy anomalies in the three (clear cut) cases is the unambiguous inclusion of the LDM character in the IPM approximation. This liquid drop property is beyond any IPM approaches with a correction of any other kind or an approximation of any other kind. It is an additional effect which shows itself when all that conceivably should and could have been done, has been successfully executed. Nuclear physics is teaching us that the IPM approach has an unanticipated limitation. One can only get a complete description of the nuclear phenomenon within IPM by including the orthogonal liquid drop characteristics as well. IPM is quantum mechanical while the LDM is classical in character. The fact that these two - classical and quantum, are appearing side by side with equal fundamental relevance is a surprise. But it is clear that the two co-exist simultaneously and in a complementary manner to provide a complete description of the nucleus.

Hence this is pointing to a fundamental "Duality" in nuclear physics. The two apparently conflicting points of view - the LDM (classical in character) and IPM (quantum mechanical in character) are providing dual description of the nucleus. They actually co-exist and are required simultaneously in a complementary manner to provide a complete and consistent description of the nucleus. Hence this is a fundamental duality in physics.

We have found here that the LDM is not an approximation of an IPM of any kind. And also that neither is it some kind of an effective model of the nucleus. It is as fundamental in describing nucleus as an IPM is. In fact, both of these are simultaneously needed to describe the nucleus. As such these are complementary in nature to provide a new Duality in nuclear physics.

Hence we suggest that the nuclear phenomena should schematically be represented as

\[ \langle \text{nucleus} \rangle = \langle \text{IPM} \rangle + \langle \text{LDM} \rangle \]

This new Duality between the classical Liquid Drop Model and the quantum Independent Particle Model of the nucleus is very reminiscent of the wave-particle duality of photon and matter particles in quantum mechanics. Though reminiscent, it is quite independent of the
same.

The wave-particle duality has been a source of much confusion; while de Broglie thought that actually it is "wave AND particle" while Bohr argued for "wave OR particle" [7]. Today as part of the Copenhagen interpretation, Bohr’s view is more dominant. Bohr’s point of view is dubbed as the Complementarity Principle [7]. The wave and particle aspects of a quantum entity is exclusive in nature, as per Bohr.

This exclusiveness is an essential aspect of the Complementarity Principle. Bohr believed that there was a single quantum reality of say a photon or an electron, and due to classical measurement, it reduces to two distinct classical realities - that of a wave nature or a particle nature. Now as logically it would be inconsistent for a single quantum reality to manifest itself simultaneously as two conflicting classical properties of wave and particle nature. Hence if this were the whole truth then there is nothing wrong with Bohr’s logic - that the wave and the particle aspects be exclusive of each other. Experimentally there has been support of Bohr’s Complementarity Principle of exclusive wave-particle duality and this is well documented [7]. However, ever since the de Broglie suggestion and later due to Bohm’s work, efforts have been around to find photon and matter particles like electron manifesting both wave and particle characters simultaneously in a single experiment [7]. One should mention the suggestion of a suitable such experiment by Ghose, Home and Agarwal [8] and its claimed experimental confirmation by Mizobushi and Ohtoke [9]. They show that actually both the wave and particle aspects of photon and matter particles may co-exist simultaneously. One should also mention the recent claims as to same effect by Afshar [10].

Now as we have found a new Duality in nuclei here in this paper, wherein both the LDM and IPM co-exist. This is the only duality known outside the wave-particle duality of quantum mechanics. There would be a conflict between these two dualities, as per the Complementarity Principle of Bohr the duality is exclusive, while in the nuclear Duality, the two co-exist simultaneously. However as discussed above, there are convincing experimental evidences that Bohr’s exclusive Complementarity Principle may be wrong. In that case, there is no conflict between the wave-particle duality and the nuclear IPM-LDM duality.

Should quantum mechanics manifest itself at macroscopic distances? First obvious answer would be "no"! However there are several well known phenomena like superconductivity and superfluidity where clearly quantum phenomena is distinctly exhibited at macroscopic scales. So as to explain the Meissner effect it is necessary to assume that the macroscopic wave
function is unperturbed by the magnetic field. So to say, it becomes "rigid"! In recent years the so called "macroscopic quantum mechanics" has become popular due to the ongoing studies on Josephson junction. Macroscopic tunneling through the Josephson junction is routinely observed [11]. Even well above the classical-quantum crossover temperature, the macroscopic quantum effects in Josephson junction has been observed [12]. McDonald [13] has studied the sharpness of the quantized energy levels in two Josephson junctions and he states, "The fact that the myriad of interactions of $10^{12}$ electrons in a macroscopic body, a Josephson junction, can produce sharply defined energy levels, suggests a dynamical state effectively divorced from the complexity of its environment. The existence of this state, the macroscopic quantum state of superconductor, is well established."

This just consolidates that actually as shown in this paper, the basic physical reality may be

<reality> = ⟨micro⟩ + ⟨macro/classical⟩

It may happen that in a particular situation only micro quantum aspect dominates (as in say atomic physics) and in another where only the macro quantum reality dominates (as say in superconductivity). However, in some cases it may happen that the macroscopic part above may appear as entirely to be, as what we may be willing to call as "classical". For example, as we have observed in nuclear physics here, where the micro part is the IPM and the macro/classical part is the LDM. It seems that this may happen when both the micro and macro are co-existing in a physical phenomena - just as we have found here in the nucleus. This just blurs the distinction of what is macro (meaning classical really) and what is micro.

Deep reflections on the measurement dilemma has been forcing physicists to think beyond the deep divide between the micro and the macro as envisaged in Bohr’s Complementarity Principle. The speculation that there may be some unknown but intrinsic connection between the macro and the micro has actually been around for some time. For example as per Bell [14, p. 171], "Now in my opinion the founding fathers were in fact wrong on this point. The quantum phenomena do not exclude a uniform description of micro and macro world .. systems and apparatus. It is not essential to introduce a vague division of the world of this kind. This was indicated already by de Broglie in 1926, when he answered the question: "wave or particle?" by "wave and particle" ."

What has only been speculated above, as shown in this paper, has actually found concrete
expression in terms of a new Duality between the LDM and the IPM degrees of freedom in
nuclei. Hence there is no division between the micro and the macro/classical. In fact they
are part of the unity of Nature. Both of these co-existing simultaneously in a fundamental
manner, is what the nucleus is teaching us.

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FIG. 1: $S_N$ vs $Z$ for fixed $N=126$
FIG. 2: $S_N$ vs Z for fixed N=82
FIG. 3: $S_p$ vs N for fixed Z=82
FIG. 4: Schematic shell model for N and Z. This is for understanding $S_N$ vs $Z$ for $N=82$ and 126. As $Z$ is changed, the core for any $Z$ is $Z_{\text{magic}}$ ($Z$ including $Z_{\text{max}}$ is given by the empirically existing data).
FIG. 5: $S_N$ vs $Z$ for fixed $N=20$ and 50
FIG. 6: $S_p$ vs $N$ for fixed $Z=20$ and 50
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