Composite p-branes on Product of Einstein Spaces

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Abstract

A multidimensional gravitational model with several scalar fields, fields of forms and cosmological constant is considered. When scalar fields are constant and composite p-brane monopole-like ansatz for the fields of forms is adopted, a wide class of solutions on product of \( n + 1 \) Einstein spaces is obtained. These solutions are composite p-brane generalizations of the Freund-Rubin solution. Some examples including the \( AdS_m \times S^k \times \ldots \) solutions are considered.
1 Introduction

Recently an interest to Freund-Rubin type solutions [2, 3, 4] in multidimensional models with \( p \)-branes “living” on product of Einstein spaces appeared (see, for example, [13, 14, 15] and references therein). This interest was inspired by papers devoted to duality between a certain limit of some superconformal theory in \( d \)-dimensional space and string or M-theory compactified on the space \( AdS_{d+1} \times W \), where \( AdS_{d+1} \) is \((d+1)\)-dimensional anti-de Sitter space and \( W \) is a compact manifold (e.g. sphere \( S^m \)) [6] (see also [7, 8, 9, 10, 11, 12] etc.)

In this paper we obtain a rather general class of solutions defined on product of \( n + 1 \) Einstein spaces for the multidimensional gravitational model with fields of forms and scalar fields. These solutions generalize the Freund-Rubin solutions [2] to the composite \( p \)-brane case with constant scalar fields. They follow just from the equations of motion when certain restriction on intersections of “extended” \( p \)-brane worldvolumes is imposed.

In the non-composite case the “cosmological derivation” of some special (static) solutions with \( p \)-branes was performed in [26].

We note that in the pure gravitational model with cosmological constant the solutions describing the product of Einstein spaces were considered in [16, 17]. These solutions were also generalized to some other matter fields, e.g. scalar one (see [18] and references therein).

It was shown in [5] that the solutions in \( D = 10, 11 \) supergravities representing \( D3, M2, M5 \) branes interpolate between flat-space vacuum and compactifications to AdS space. The AdS spaces appear in the “near-horizon” limit. The solutions obtained here are also related to the so-called Madjumdar-Papapetrou type solutions with intersecting composite \( p \)-branes (see [22, 23, 27] and references therein). This correspondence will be considered in detail in a separate publication [25].

We note that in [27] a large variety of so-called “block-orthogonal” Madjumdar-Papapetrou (MP) type \( p \)-brane solutions was obtained. These solutions may be related to Lie algebras: simple or hyperbolic [27, 29]. The non-extremal ”block-orthogonal" solutions were obtained earlier in [28]. The “block-orthogonal” MP solutions contain the “orthogonal” ones (see [31, 32, 33, 34, 35, 21, 20, 22, 23] and references therein) as a special case.

The Freund-Rubin solutions [2] in \( D = 11 \) supergravity [1]: \( AdS_4 \times S^7 \) and \( AdS_7 \times S^4 \), correspond to “electric” \( M2 \)-branes “living” in \( AdS_4 \) and \( S^4 \) respectively or, equivalently, to “magnetic” \( M5 \) branes “living” in \( S^7 \) and \( AdS_7 \). The “popular” \( AdS_5 \times S^5 \) solution in \( IIB \) \( (D = 10) \) supergravity model (see, for example [14, 15]) corresponds to a composite self-dual configuration with two \( D3 \) branes living in \( AdS_5 \) and \( S^5 \) respectively and corresponding to a 5-form.

In Sect. 2 we outline the general approach with arbitrary forms and dilaton fields on a product of \( (n + 1) \) manifolds. In Sect. 3 we give a general solution for static fields on a product of \( (n + 1) \) Einstein spaces. Several examples of solutions are presented. Among them the solution of \( D = 11 \) supergravity on the manifold \( AdS_2 \times S^2 \times M_2 \times M_3 \) is considered. \( AdS_2 \times S^4 \times T^m \) solutions of \( D = 11 \) supergravity and others were listed in [14, 15]). This solution corresponds to the “near horizon” limit of the Madjumdar-Papapetrou type solution of \( D = 11 \) supergravity describing a bound state of \( M2 \) and
$M5$ branes \[27\] with the intersection rule corresponding to the Lie algebra $A_2 = sl(3)$ \[38\].

## 2 The model

We consider the model governed by the action

\[
S = \int_M d^Dz \sqrt{|g|} \{ R[g] - 2\Lambda - h_{\alpha\beta} g^{MN} \partial_M \varphi^\alpha \partial_N \varphi^\beta \\
- \sum_{a \in \Delta} \frac{\theta_a}{n_a!} \exp[2\lambda_a(\varphi)] (F^a)^2 \},
\]

where $g = g_{MN} dz^M \otimes dz^N$ is the metric, $\varphi = (\varphi^\alpha) \in \mathbb{R}^l$ is a vector from dilatonic scalar fields, $(h_{\alpha\beta})$ is a non-degenerate symmetric $l \times l$ matrix $(l \in \mathbb{N})$, $\theta_a \neq 0$,

\[
F^a = dA^a = \frac{1}{n_a!} F^a_{M_1...M_n} dz^{M_1} \wedge ... \wedge dz^{M_n}
\]

is a $n_a$-form ($n_a \geq 2$) on a $D$-dimensional manifold $M$, $\Lambda$ is a cosmological constant and $\lambda_a$ is a 1-form on $\mathbb{R}^l$ : $\lambda_a(\varphi) = \lambda_{a\alpha} \varphi^\alpha$, $a \in \Delta; \alpha = 1, ..., l$. In (2.1) we denote $|g| = |\det(g_{MN})|$, \[2.3\]

\[
(F^a)^2 = F^a_{M_1...M_n} F^a_{N_1...N_n} g_{M_1N_1} ... g_{M_nN_n},
\]

where $\lambda_a(\varphi) = h^{\alpha\beta} \lambda_{a\beta}$, where $(h^{\alpha\beta})$ is a matrix inverse to $(h_{\alpha\beta})$. In (2.4)

\[
Z_{MN} = Z_{MN}[\varphi] + \sum_{a \in \Delta} \theta_a e^{2\lambda_a(\varphi)} Z_{MN}[F^a, g],
\]

where

\[
Z_{MN}[F^a, g] = \frac{1}{n_a!} \left[ \frac{n_a - 1}{2 - D} g_{MN}(F^a)^2 + n_a F^a_{M_1...M_n} F^a_{M_2...M_n} \right].
\]

In (2.5) and (2.6) $\Delta[g]$ and $\nabla[g]$ are Laplace-Beltrami and covariant derivative operators respectively corresponding to $g$.

**Multi-index notations.** Let us consider the manifold

\[
M = M_0 \times M_1 \times ... \times M_n.
\]
We denote $d_i = \dim M_i \geq 1$; $i = 0, \ldots, n$. $D = \sum_{i=0}^{n} d_i$. Let $g^i = g^i_{m_m} (y_i) dy^m \otimes dy^n$ be a metric on the manifold $M_i$, $i = 0, \ldots, n$. Here we use the notations of our previous papers [19, 20, 24, 23]. Let any manifold $M_\nu$ be oriented and connected. Then the volume $d_i$-form

$$\tau_i \equiv \sqrt{|g^i(y_i)|} \, dy^1 \wedge \ldots \wedge dy^{d_i}, \tag{2.11}$$

and the signature parameter

$$\varepsilon_i \equiv \text{sign}(\det(g^i_{m_m})) = \pm 1 \tag{2.12}$$

are correctly defined for all $i = 0, \ldots, n$.

Let $\Omega = \Omega(n + 1)$ be a set of all non-empty subsets of $\{0, \ldots, n\}$. The number of elements in $\Omega$ is $|\Omega| = 2^{n+1} - 1$. For any $I = \{i_1, \ldots, i_k\} \in \Omega$, $i_1 < \ldots < i_k$, we denote

$$\tau(I) \equiv \hat{\tau}_{i_1} \wedge \ldots \wedge \hat{\tau}_{i_k}, \tag{2.13}$$

$$\varepsilon(I) \equiv \varepsilon_{i_1} \times \ldots \times \varepsilon_{i_k}, \tag{2.14}$$

$$M_I \equiv M_{i_1} \times \ldots \times M_{i_k}, \tag{2.15}$$

$$d(I) \equiv \sum_{i \in I} d_i, \tag{2.16}$$

where $d_i$ is both, the dimension of the oriented manifold $M_i$ and the rank of the volume form $\tau_i$ and $\hat{\tau}_i$ is the pullback of $\tau_i$ to the manifold $M$: $\hat{\tau}_i = p_i^* \tau_i$, where $p_i : M \to M_i$, is the canonical projection, $i = 0, \ldots, n$.

We also denote by

$$\delta_i^I = \sum_{j \in I} \delta_i^j \tag{2.17}$$

the indicator of $i$ belonging to $I$: $\delta_i^I = 1$ for $i \in I$ and $\delta_i^I = 0$ otherwise.

### 3 The solution

The solution reads as following. The metric is defined on the manifold (2.10) and has the following form

$$g = \hat{g}^0 + \hat{g}^1 + \ldots + \hat{g}^n, \tag{3.1}$$

where $g^i$ is a metric on $M_i$ satisfying the equation

$$\text{Ric}[g^i] = \xi_i g^i, \tag{3.2}$$

$\xi_i = \text{const}, i = 0, \ldots, n$. Here $\text{Ric}[g^i]$ is Ricci-tensor corresponding to $g^i$ and $\hat{g}^i = p_i^* g^i$ is the pullback of the metric $g^i$ to the manifold $M$ by the canonical projection: $p_i : M \to M_i$, $i = 0, \ldots, n$. Thus, all $(M_i, g^i)$ are Einstein spaces.

The fields of forms and scalar fields are the following

$$F^a = \sum_{I \in \Omega_a} Q_{aI} \tau(I), \tag{3.3}$$

$$\varphi^a = \text{const}. \tag{3.4}$$
where $Q_{aI}$ are constants, $\Omega_a \subset \Omega$ are subsets, satisfying the relations
\begin{equation}
I \in \Omega_a, \ a \in \Delta, \mathrm{and \ the \ Restriction \ presented \ below. \ \ The \ parameters \ of \ the \ solution \ obey \ the \ relations} 
\end{equation}
\begin{equation}
d(I) = n_a, \quad (3.5)
\end{equation}
\begin{equation}
I \in \Omega_a, \ a \in \Delta, \mathrm{and \ the \ Restriction \ presented \ below. \ \ The \ parameters \ of \ the \ solution \ obey \ the \ relations} 
\end{equation}
\begin{equation}
\sum_{a \in \Delta} \theta_a e^{2\lambda_a(\varphi)} \sum_{I \in \Omega_a} (Q_{aI})^2 \varepsilon(I) = 0, \quad (3.6)
\end{equation}
\begin{equation}
\xi_i = \frac{2\Lambda}{D-2} + \sum_{a \in \Delta} \theta_a e^{2\lambda_a(\varphi)} \sum_{I \in \Omega_a} (Q_{aI})^2 \varepsilon(I) \left[ \delta^n_a - \frac{n_a-1}{D-2} \right], \quad (3.7)
\end{equation}
i = 0, \ldots, n.

The solution is valid if the following restriction on the sets $\Omega_a, \ a \in \Delta$, similar to that from [23] (see also [22]) is satisfied.

**Restriction.** For any $a \in \Delta$ and $I, J \in \Omega_a, \ I \neq J$, we put
\begin{equation}
d(I \cap J) \leq n_a - 2. \quad (3.8)
\end{equation}

This restriction guarantees the block-diagonal structure of the $Z_{MN}$-tensor in (2.7) (see relation (5.6) from the Appendix).

The solution mentioned above may be verified by a straightforward substitution of the fields from (3.1)-(3.4) into equations of motion (2.4)-(2.9) while formulas from the Appendix are keeping in mind. We note that due to the relations $dF_a = 0$ the potential form $A^n_a$ satisfying $F^a = dA^n_a$ exists at least locally, $a \in \Delta$.

We note that the **Restriction** is satisfied if the number of 1-dimensional manifolds among $M_i$ is no more than 1.

### 3.1 “Electro-magnetic” form of solution

Due to relation
\begin{equation}
* \tau(I) = \varepsilon(I) \delta(I, I) \tau(I), \quad (3.9)
\end{equation}
where $* = *[g]$ is the Hodge operator on $(M, g)$,
\begin{equation}
\bar{I} = \{0, \ldots, n\} \setminus I
\end{equation}
is “dual” set and $\delta(I, I) = \pm 1$ is defined by the following relation
\begin{equation}
\tau(\bar{I}) \wedge \tau(I) = \delta(I, I) \tau(\{0, \ldots, n\}), \quad (3.11)
\end{equation}
the electric “brane” living in $M_\bar{I}$ (see [21]) may be interpreted also as a magnetic “brane” living in $M_I$. The relation (3.3) may be rewritten in the “electro-magnetic” form as following
\begin{equation}
F^a = \sum_{I \in \Omega_a} Q_{aIe} \tau(I) + \sum_{J \in \Omega_{am}} Q_{aJm} * \tau(J), \quad (3.12)
\end{equation}
where $\Omega_a = \Omega_{ae} \cup \bar{\Omega}_{am}, \Omega_{ae} \cap \bar{\Omega}_{am} = \emptyset, \bar{\Omega}_{am} \equiv \{ J | J = \bar{I}, I \in \Omega_{am} \}$, and $Q_{aIe} = Q_{aI}$ for $I \in \Omega_{ae}$ and $Q_{aJm} = Q_{aJ} \varepsilon(J) \delta(J, J)$ for $J \in \Omega_{am}$.
4 Some examples

Here we consider some examples of the obtained solutions when $\varepsilon_0 = -1$ and all $\varepsilon_i = 1$, $i = 1, \ldots, n$, i.e. “our space” $(M_0, g^0)$ is pseudo-Euclidean space and the “internal spaces” $(M_i, g^i)$ are Euclidean ones. We also put $\theta_a = 1$ and $n_a < D - 1$ for all $a \in \Delta$.

4.1 Solution with one $p$-brane

Let $\Omega_a = \{I\}$, $\lambda_a = 0$ for some $a \in \Delta$ and $\Omega_b$ are empty for all $b \neq a, b \in \Delta$. Equations (3.6) are satisfied identically in this case and (3.7) read

$$\xi_i = \frac{2\Lambda}{D - 2} + \varepsilon(I)Q^2[\delta^i_0 - \frac{n_a - 1}{D - 2}],$$

$$i = 0, \ldots, n,$$ where $Q = Q_{al}$.

4.1.1 $p$-brane does not “live” in $M_0$

For $I = \{1, \ldots, k\}, 1 \leq k \leq n$, we get $\varepsilon(I) = 1$ and

$$\xi_0 = \xi_{k+1} = \ldots = \xi_n = \frac{2\Lambda}{D - 2} - Q^2\frac{n_a - 1}{D - 2},$$

$$\xi_1 = \ldots = \xi_k = \frac{2\Lambda}{D - 2} + Q^2[1 - \frac{n_a - 1}{D - 2}].$$

For $\Lambda = 0, Q \neq 0$ we get $\xi_0 = \xi_{k+1} = \ldots = \xi_n < 0$ and $\xi_1 = \ldots = \xi_k > 0$. These solutions contain the solutions with the manifold

$$M = AdS_{d_0} \times S^{d_1} \times \ldots \times S^{d_k} \times H^{d_{k+1}} \times \ldots \times M_n.$$ (4.4)

Here $H^d$ is $d$-dimensional Lobachevsky space; $M_n = H^{d_n}$ for $k < n$ and $M_n = S^{d_n}$ for $k = n$.

For $2\Lambda = Q^2(n_a - 1)$ we get a solution with a flat our space: $M = \mathbb{R}^{d_0} \times S^{d_1} \times \ldots \times S^{d_k} \times \mathbb{R}^{d_{k+1}} \times \ldots$. We may consider the fine-tuning of the cosmological constant, when $\Lambda$ and $Q^2$ are of the Planck order but $\xi_0$ is small enough in agreement with observational data.

4.1.2 $p$-brane “lives” in $M_0$

For $I = \{0, \ldots, k\}, 0 \leq k \leq n$, we get $\varepsilon(I) = -1$ and

$$\xi_{k+1} = \ldots = \xi_n = \frac{2\Lambda}{D - 2} + Q^2\frac{n_a - 1}{D - 2},$$

$$\xi_0 = \ldots = \xi_k = \frac{2\Lambda}{D - 2} - Q^2[1 - \frac{n_a - 1}{D - 2}].$$

For $\Lambda = 0, Q \neq 0$, we get $\xi_{k+1} = \ldots = \xi_n > 0$ and $\xi_0 = \ldots = \xi_k < 0$. The solutions contain the solutions with the manifold

$$M = AdS_{d_0} \times H^{d_1} \times \ldots \times H^{d_k} \times S^{d_{k+1}} \times \ldots \times M_n.$$ (4.7)
Here \( M_n = S^{d_a} \) for \( k < n \) and \( M_n = H^{d_a} \) for \( k = n \).

For \( 2\Lambda = Q^2(D - n_a - 1) \) we get a solution with a flat our space: \( M = \mathbb{R}^{d_0} \times S^{d_1} \times \ldots \times S^{d_k} \times \mathbb{R}^{d_{k+1}} \). We may also consider the fine-tuning mechanism here.

### 4.2 Solution with two \( p \)-branes

#### 4.2.1 Composite solution on \( M_0 \times M_1 \)

Let \( n = 1, d_0 = d_1 = n_a = d, \Omega_a = \{I_0 = \{0\}, I_1 = \{1\}\} \), for some \( a \) and other \( \Omega_b \) are empty. Denote \( Q_0 = Q_{a I_0} \) and \( Q_1 = Q_{a I_1} \). For the field of form we get from (3.3)

\[
F^a = Q_0 \hat{\tau}_0 + Q_1 \hat{\tau}_1. \tag{4.8}
\]

When \( \lambda_a \neq 0 \) the equations (3.6) are satisfied if and only if \( Q_0^2 = Q_1^2 = Q^2 \). Relations (3.7) read

\[
\xi_0 = \frac{2\Lambda}{D - 2} - Q^2 e^{2\lambda_a(\phi)}, \tag{4.9}
\]

\[
\xi_1 = \frac{2\Lambda}{D - 2} + Q^2 e^{2\lambda_a(\phi)}. \tag{4.10}
\]

For \( \Lambda = 0 \) and \( Q \neq 0 \) we get the solution defined on the manifold \( M = AdS_d \times S^d \). For odd \( d \) the form (4.8) is self-dual (see subsection 3.1). The solution describes a composite \( p \)-brane configuration containing \( AdS_5 \times S^5 \) solution in \( IIB \) supergravity as a special case.

#### 4.2.2 Near-horizon limit for \( A_2 \)-dyon in \( D = 11 \) supergravity

Here we consider the extension of the Madjumdar-Papapetrou solution [30] to \( D = 11 \) supergravity, describing a bound state of two \( p \)-branes: one electric (\( M_2 \)) and one magnetic (\( M_5 \)) [27]. This solution has an unusual intersection rule corresponding to the Lie algebra \( A_2 = sl(3) \). The solution is defined on the manifold (2.10) with \( n = 3, D = 11 \) and has the following form

\[
g = H^2 g^0 - H^{-2} dt \otimes dt + \hat{g}^2 + \hat{g}^3, \tag{4.11}
\]

\[
F^a = \nu_1 dH^{-1} \wedge dt \wedge \hat{\tau}_2 + \nu_2 (\ast_0 dH) \wedge \hat{\tau}_2, \tag{4.12}
\]

where \( H \) is the harmonic function on \((M_0, g^0)\); metrics \( g^0, g^2, g^3 \) are Ricci-flat, \( \varepsilon_1 = +1, \nu_1^2 = \nu_2^2 = 1 \), \( \text{rank} F^a = 4 \) and \( d_0 = 3, d_2 = 2, d_3 = 5 \).

Let \( g^0 = dR \otimes dR + R^2 \hat{g}[S^2], H = C + \frac{M}{R} \), where \( C \) and \( M \) are constants and \( g[S^2] \) is the metric on \( S^2 \). For \( C = 1 \) the 4-dimensional section of the metric describes extremally charged Reissner-Nordström black hole of mass \( M \) in the region out of the horizon: \( R > 0 \).

Now we put \( C = 0 \) and \( M = 1 \) (i.e. the so-called “near-horizon” limit is considered). We get the solution

\[
g = \hat{g}[AdS_2] + \hat{g}[S^2] + \hat{g}^2 + \hat{g}^3, \tag{4.13}
\]

\[
F^a = \nu_1 \hat{\tau}[AdS_2] \wedge \hat{\tau}_2 + \nu_2 \hat{\tau}[S^2] \wedge \hat{\tau}_2 \tag{4.14}
\]
defined on the manifold

\[ M = AdS_2 \times S^2 \times M_2 \times M_3. \] (4.15)

Here \( g[AdS^2] = R^{-2}[dR \otimes dR - dt \otimes dt] \) is the metric on \( AdS_2 \), \((M_i, g'_i)\) are Ricci-flat, \( i = 2, 3; \varepsilon_2 = +1, d_2 = 2, d_3 = 5 \) and \( \nu_2^2 = \nu_3^2 = 1 \).

**Remark 1.** The solutions (4.11)-(4.15) may be generalized to the case of so-called \( BD \)-models in dimension \( D \geq 12 \) [38]. In this case rank \( F^a \in \{4, \ldots, D-7\} \), \( d_2 = a - 2, d_3 = D - 2 - a \), and all scalar fields are zero. In this case the solution (4.11)-(4.12) describes \( A_2 \)-dyon with electric \( d_1 \)-brane and magnetic \( d_2 \)-brane, corresponding to \( F^a \)-form and intersecting in 1-dimensional time manifold. We note that \( B_{12} \)-model corresponds to the low-energy limit of the \( F \)-theory [37].

**Remark 2.** For \( M_2 = \mathbb{R}^2 \) and \( M_3 = \mathbb{R}^2 \times M_4 \), the metrics (4.11) and (4.13) may be obtained also for the solution with two \( M_2 \) branes and two \( M_5 \) branes [14, 15].

### 5 Appendix

Let \( F_1 \) and \( F_2 \) be forms of rank \( r \) on \((M, g)\) (\( M \) is a manifold and \( g \) is a metric on it). We define

\[
(F_1 \cdot F_2)_{MN} \equiv (F_1)_{MM_2\ldots M_r} (F_2)^{M_2\ldots M_r}_N
\]

\[ F_1 F_2 \equiv (F_1 \cdot F_2)^M_M = (F_1)_{M_1M_2\ldots M_r} (F_2)^{M_1M_2\ldots M_r}. \] (5.1)

For the volume forms (2.13) we get

\[
\frac{1}{d(I)!} (\tau(I) \tau(I)) = \varepsilon(I),
\]

\[
\frac{1}{(d(I) - 1)!} (\tau(I) \cdot \tau(I))_{m_i n_i} = \varepsilon(I) \delta^i_j g_{m_i n_i},
\]

where the indices \( m_i, n_i \) correspond to the manifold \( M_i, i = 0, \ldots, n \). The symbols \( \varepsilon(I) \) and \( \delta^i_j \) are defined in (2.14) and (2.17) respectively.

Let \( I, J \in \Omega, I \neq J \) and \( d(I) = d(J) \). Then

\[
\tau(I) \tau(J) = 0,
\]

and due to **Restriction**

\[
(\tau(I) \cdot \tau(J))_{MN} = 0.
\] (5.6)

### 6 Conclusions

In this paper we obtained exact solutions describing the product of \((n + 1)\) Einstein spaces for the gravitational model with fields of forms and (dilatonic) scalar fields. The solutions are given by the relations (3.1)-(3.7) and may be considered as a composite \( p \)-brane generalization of the Freund-Rubin solutions.

These solutions may be used in multidimensional cosmology as they provide a mechanism of compensation or reduction of the cosmological constant in our space by the use of fine-tuning of “big” \( \Lambda \) (of the Planck’s order) with the “charges” of \( p \)-branes.
Another interesting aspect is connected with a singling out of a special subclass of $AdS_m \times S^k \times \ldots$ solutions originating from the Madjumdar-Papapetrou type solutions: “orthogonal” and “block-orthogonal” \([27]\) in the “near-horizon” limit. Such compactifications are of interest for the string- or M-theories itself and for the studying of the quantum phenomena near the horizon of the black hole. Here the compactifications inheriting the “non-orthogonal” intersection rules, e.g. corresponding to different Lie algebras (simple or hyperbolic) \([27, 29]\) should be also considered.

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