Upper critical field $H_{c2}$ in two-band superconductors

M.E. Palistrant

March 22, 2022

Institutul of Appleid Physics, Chishinau, Moldova

E-mail: statphys@asm.md

It was obtained the equation for the definition of upper critical field $H_{c2}$ in the system with two overlapping energetic bands on the Fermi surface, which is valid for the whole temperature interval $0 \leq T < T_c$. The analytic expressions for the value $H_{c2}$ at $T \sim T_c$ and $T \ll T_c$ cases were defined. The possibility of changing the curvature $H_{c2}(T)$ with changing of the electrons speed ratio $v_1/v_2$ on different cavities on the Fermi surface was revealed. It was obtained the qualitative accordance with experimental data for the intermetallic compound MgB$_2$.

Keywords: Superconductivity, two-band model, upper critical field, magnesium borid.

1 Introduction

The discovery of the high temperature of superconductive transition $T \sim 40K$ in the simple intermetallic compound MgB$_2$ [1] has simulated researches of this material properties both in the experimental and theoretical plans. An overview of the basic physical properties of MgB$_2$ can be seen, for example, in [2]. The significant result of these researches is the discovery of two energetic
gaps in the spectrum of the elementary excitations \[3\], \[4\] and the possibility of theoretical describing of this compound on the base of the two-band model \[5\] (see also \[6\]).

The theory of the thermodynamic and kinematic properties of superconductors (pure and doped) with overlapping energy bands on the Fermi surface was developed by Moskalenko and his collaborators (for the references to the articles see \[7\] - \[9\]). The typical feature of two-band model is the fact, that is gives not only the quantitative difference from the one energy band case, but leads to the qualitative new results \[7\]. Being built long before the discovery of the superconductivity in \(\text{MgB}_2\), this theory can generally describe the superconductive properties of this compound. Nowadays often happens rediscovering of already known results, which had been obtained in the previously mentioned articles.

Moskalenko two-band model \[5\] and its generalization for the anisotropic value of the energetic gaps \(\Delta_1\) and \(\Delta_2\) case \[10\], \[11\], confirm the experimental results for the thermal capacity \(C_S\), dependence on temperature, the penetration dept of the magnetic field \(\lambda(T)\) and other characteristics in the superconducting \(\text{MgB}_2\) compound.

At it is known, the superconductive metals undergo the transition from the superconductive phase into normal in the magnetic field at some its value. Researching the Ginzburg-Landau equation, Abrikosov has shown \[12\] that at some value of the exterior magnetic field \(H_{c1}\) (lower critical field) the mixed state realizes in the secondary type superconductors. This state is characterized by penetration of the magnetic field in the deep of superconductor as the exact lattice of the magnetic lines. The transitions into the normal state, which is relevant to the full penetration of the magnetic field into the superconductor, occurs in the moment when the field achieves the value of the upper critical field \(H_{c2}\). Definition of this value on the base of the Ginzburg-Landau theory is possible only in the critical temperature neighbourhood. The \(H_{c2}\) value is defined on the whole temperature interval in Goricov article \[13\] (see also Maki and Tsuzuki \[14\] ).

In the previously mentioned articles was considered an isotropic superconductor. It is interesting to research the magnetic properties of anisotropic superconductors because so are real superconductors. Overlapping of the energy bands \[5\] is one of the manifestation of anisotropy. There were obtained base equations of the electrodynamics of two-band superconductors in the work \[15\], which are valid both for pure and doped superconductors. Also
were researched the electromagnetic properties of two-band superconductors for temperatures close to the critical \((T_c - T \ll T_c)\).

The main purpose of the work is researching of pure two-band superconductor of the secondary type for arbitrary temperatures and external magnetic field close to the upper critical field and the definition of temperature dependence of the \(H_{c2}\) value.

2 The system of equations for the order parameters \(\Delta_n\).

If the exterior magnetic field is great enough, the order parameters \(\Delta_m (m = 1, 2)\) of two-band superconductor is small enough, and we can use equations ref. [15] for pure two-band superconductor:

\[
\Delta_m^* (\vec{x}) = \frac{1}{\beta} \sum_{m'} \sum_{n} V_{nm} \int d\vec{y} g_{n'n}(\vec{y}, \vec{x}/\omega) \Delta_m^* (\vec{y}) g_{n'n}(\vec{y}, \vec{x}/ -\omega) . \quad (1)
\]

We restricted here by linear terms on \(\Delta_n\) quantities in comparison with given in [15] because in the \(H = H_{c2}\) point occurs solutions with the infinitely small values \(\Delta_m\). Green function defines by equation at presence of the magnetic field [16]:

\[
g_{nn'}(r, r'/\omega) = e^{i\varphi(r, r')/\omega} g_{nn'}^0 (r, r'/\omega) \quad (2)
\]

Were \(g_{nn'}^0\) - Green function of an electron in normal metal without magnetic field. The presence of the magnetic field is taken into account by the phase multiplier

\[
\varphi(r, r') = e \int_{r'}^{r} A(\vec{l}) d\vec{l} . \quad (3)
\]

We decompose in equation (1) the normal metal function \(g_{nn'}^0\) into the row by he Bloch functions \(\psi_{nk}(\vec{x}) = e^{i(\vec{k}\cdot\vec{x})} U_{nk}(\vec{x})/\sqrt{N} (U_{nk} - \text{the Bloch amplitude}):\)

\[
g_{nn'}^0(\vec{y}, \vec{x}/\omega) = \sum_{\vec{k}, \vec{k}'} g_{nn'}^0(\vec{k}, \vec{k}'/\omega) \psi_{nk}(\vec{y}) \psi_{nk}(\vec{x}) \quad (4)
\]
and use approaching of the diagonal Green functions, which were developed in works [17]. Besides choose the vector potential as

\[
A_x = A_z = 0; \quad A_y = H_0 x.
\]  

(5)

(magnetic field is guided along the z axis).

Equation (1) will accept the next view:

\[
\Delta^* (r) = \sum_n V_{nm} \frac{1}{\beta} \sum_\omega \int d^3r' \Delta^*_n (r') \int \frac{d^3kd^3q}{(2\pi)^6} g_n^0 (\vec{k}/\omega) \times \\
\times g_n^0 (\vec{q} - \vec{k}/\omega) \exp \left[ i \vec{q} \cdot (\vec{r} - \vec{r}') + i \epsilon (x + x') (y' - y) H_0 \right] U_n \vec{k} (\vec{r}) \times \\
\times U^*_n \vec{k} (\vec{r}) U_{n\vec{q}-\vec{k}} (\vec{r}') U_{n\vec{q}-\vec{k}} (\vec{r}).
\]  

(6)

\[
g_n^0 (\vec{k}/\omega) = \frac{1}{i\omega - \xi (k)}.
\]  

(7)

After integrating by impulse \( k \), we make the averaging by elementary cells, in order to exclude from consideration fast oscillations of function \( \Delta_n \) (which are stipulated by Bloch functions) and to save just dependence on these functions' coordinates (incipient due to the exterior magnetic field presence), because just last dependence is observed. As a result we obtain:

\[
\Delta^*_m (r) = \sum_n V_{nm} \frac{1}{\beta} \sum_\omega \int d\Omega \int \frac{d^3q}{(2\pi)^3} \frac{N_n \omega}{2|\omega|} \frac{i}{2i\omega + \vec{q} \cdot \vec{v}_n} \times \\
\times \int d^3r' \Delta^*_n (r') \exp \left[ i \epsilon (r - r') + i \epsilon H_0 (x + x') (y' - y) \right],
\]  

(8)

where \( v_n \) and \( N_n \) - are accordingly electron speed and electron density of state on n-th cavity of the Fermi surface. Applying to the calculation of the equation’s (8) left part Maki and Tsuzuki methodic [14], and supposing that \( \Delta_n \) depends just on \( x^1 \), we obtain on this way:

\[
\Delta^*_m (x) = \sum_n V_{nm} \frac{\pi N_n}{\beta v_n} \int_1^\infty \frac{du}{u} \int_{-\infty}^\infty dx' \Delta^*_n (x') \times \\
\times \frac{J_0 \left[ (x^2 - x'^2) e^{\epsilon H_0 \sqrt{u^2 - 1}} \right]}{sh \frac{2|x - x'| u}{\beta v_n}} - \theta (|x - x'| - \delta_n).
\]  

(9)
The introduction of $\delta_n$ corresponds to cutting of the inter-electron interaction in the impulse space in vicinity of n-th cavity of the Fermi surface. It is easy to notice, that at $T = T_c$, $H_{c2} = 0$ equation (9) turns to the equation for the critical temperature definition. On the base of the last equation the $\delta_n$ value defines:

$$
\delta_n = \frac{v_n}{2\gamma e_0\omega_D},
$$

$e_0$ - is the base of natural logarithm, $\omega_D$ - Debay frequeny. The solution of the equation (9) we search in the view [17]:

$$
\Delta_m(x) = \Delta_m e^{-eH_0x^2}. \tag{10}
$$

Using formula (10), we can obtain equation (9) in the next view:

$$
\Delta_m^* = \sum_n \sqrt{\frac{2}{\pi}} \frac{V_{nm}N_n}{\beta v_n} \Delta_n^* \int_1^\infty \frac{du}{u} \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dx' \theta(|x - x'| - \delta_n) \times
$$

$$
\times \exp \left[-eH_0(x^2 + x'^2)\right] \frac{I_0 \left(\frac{\zeta^2(u^2 - 1)}{4(u^2 + 1)}\right)}{\text{sh} \left(\frac{\zeta u}{\beta v_n}\right)} =
$$

$$
= \sum_n V_{nm}N_n \rho_n^{-1/2} \Delta_n^* \int_1^\infty \frac{du}{u} \int_0^\infty d\zeta \exp \left[-\frac{\zeta^2}{4}(u^2 + 1)\right] \frac{I_0 \left(\frac{\zeta^2(u^2 - 1)}{4(u^2 + 1)}\right)}{\text{sh} \left(\frac{\zeta u}{\beta v_n}\right)} \tag{11},
$$

were

$$
\delta'_n = (eH_0)^{1/2}\delta_n; \quad \rho_n = \frac{v_n^2 eH_0}{(2\pi T)^2}. \tag{12}
$$

3 Upper critical field $H_{c2}$ definition.

Basing on (11) it is easy to obtain the equation for the upper critical field definition in the next view:

$$
-\Delta_m^* + \sum_n V_{nm}N_n \rho_n^{-1/2} \Delta_n^* \ln \frac{2\gamma \omega_D^2}{\pi T_c} + \sum_n V_{nm}N_n \Delta_n^* \ln \frac{T_c}{T} - f(\rho_n) = 0, \tag{13}
$$

where

$$
f(\rho_n) = \rho_n^{-1/2} \int_0^\infty d\zeta \int_1^\infty \frac{du}{u \text{sh} \left[\frac{u \zeta}{\rho_n^{-1/2}}\right]} \times
$$

5
\[
\times \left\{ 1 - \exp \left[ -\frac{\zeta^2}{4}(u^2 + 1) \right] I_0 \left[ \frac{\zeta^2}{4}(u^2 - 1) \right] \right\}. \quad (14)
\]

Here was used the next correlation:
\[
\rho_{n}^{1/2} \int_{1}^{\infty} \frac{du}{u} \int_{\delta_n}^{\infty} \frac{d\zeta}{\sinh[u\zeta\rho_{n}^{1/2}]} = \ln \frac{2\rho_{n}^{1/2}}{\delta_n e_0}, \quad (15)
\]

Equality to zero of the system (13) determinant corresponds to the presence of non-zero solutions, that is the connected pairs forming. The field, in the presence of which such solutions can appear, is the upper critical field \(H_{c_2}\). So the \(H_{c_2}\) value defines from the condition of the system (13) solvability:
\[
a f(\rho_1) f(\rho_2) + B_1 f(\rho_1) + B_2 f(\rho_2) + C = 0, \quad (16)
\]
where
\[
B_n = N_n V_{nn} - a \xi_{c}^{(n)}; \quad (n = 1, 2)
\]
\[
C = 1 - N_1 V_{11} \xi_{T}^{(1)} - N_2 V_{22} \xi_{T}^{(2)} + a \xi_{T}^{(1)} \xi_{T}^{(2)}; \quad a = N_1 N_2 (V_{11} V_{22} - V_{12} V_{21});
\]

\[
\xi_{T}^{(n)} = \ln \frac{2\gamma \omega_{D}^{(n)}}{\pi T}; \quad \xi_{c}^{(n)} = \ln \frac{2\gamma \omega_{D}^{(n)}}{\pi T_{c}}
\]

Condition (16) considers in the two limit cases:

a. \(\rho_n \ll 1(T_c - T \ll T_c)\); b. \(\rho_n \gg 1(T \ll T_c)\),

for which functions \(f(\rho_n)\) are defined in works [14]:
\[
f(\rho_n) = \frac{7}{6} \zeta(3)\rho_n - \frac{31}{10} \zeta(5)\rho_n^2 + \frac{381}{28} \zeta(7)\rho_n^3, \quad \rho_n \ll 1. \quad (17)
\]
\[
f(\rho_n) = \ln \frac{2(2\gamma\rho_n)^{1/2}}{e_0} - \frac{1}{\pi^2 \rho_n} \left[ \zeta'(2) + \frac{\zeta(2)}{2} \ln \frac{2}{\pi^2 \gamma \rho_n} \right], \quad \rho_n \gg 1. \quad (18)
\]

Case a.\(\rho_n \ll 1\) \((T_c - T \ll T_c)\) Subject to the low electronic-phonon binding, expression (16) accepts the next view in this case:
\[
B_1 f(\rho_1) + B_2 f(\rho_2) + C_a = 0,
\]
were
\[ C_a = \left[ -(N_1 V_{11} + N_2 V_{22}) + 2a\xi \right] \ln \frac{T_c}{T}. \]

(19)

(It is supposed as usually \( \xi^{(1)}_c = \xi^{(2)}_c = \xi_c = \ln \frac{2\omega_D}{\pi T_c} \)).

During obtaining of (19) had been used equality to zero determinant of the next system of equations:
\[ \Delta^{\ast}_{mc} = \sum_n V_{nm} N_n \Delta^{(v)}_{ne} \xi_c, \]

(20)

This system defines the critical temperature \( T_c \) of superconductor. Basing on formulas (19) and (20), (17) and (12) we obtain expression for the \( H_{c2} \) value (see also [18], [19]):
\[ H_{c2}(T) = \frac{4\pi^2 T_c^2}{\epsilon} \left[ v_1^2 \eta_1 + v_2^2 \eta_2 \right]^{-1} \frac{6}{7\xi(3)} \theta. \]

\[ 1 + \theta \left( \frac{v_1^2 \eta_1 + v_2^2 \eta_2}{v_1 \eta_1 + v_2 \eta_2} \right) \frac{31}{10} \xi(5) \left( \frac{6}{7\xi(3)} \right)^2 - \frac{3}{2}, \]

(21)

\[ \theta = 1 - \frac{T}{T_c}, \quad \eta_1 = \frac{1}{2}(1 + \eta) \quad \eta_2 = \frac{1}{2}(1 - \eta); \]

\[ \eta = \frac{N_1 V_{11} - N_2 V_{22}}{\sqrt{(N_1 V_{11} - N_2 V_{22})^2 + 4N_1 N_2 V_{12} V_{21}}}. \]

Coefficients \( \eta_1, \eta_2, \eta \) satisfy the expressions:
\[ 0 \leq \eta_1 \leq 1, \quad 0 \leq \eta_2 \leq 1, \quad \eta_1 + \eta_2 = 1, \quad 0 \leq |\eta| \leq 1. \]

Case b. \( \rho_n \gg 1 \quad (T \ll T_c) \)

Basing on equation (16) and using (12) and (18) we obtain for the \( H_{c2}(T) \) value the next expression in this case:
\[ H_{c2}(T) = \frac{2\gamma \omega_D^2}{ev_1 v_2} \exp(2 - \Sigma), \]

(22)

were
\[ \Sigma = \eta^+ \pm \sqrt{(\ln \lambda - \eta^{-})^2 + \frac{4N_1 N_2 V_{12} V_{21}}{a^2} - 4F(\rho_0)}, \]

(23)
\[ \eta^\pm = \frac{1}{\alpha} (N_1 V_{11} \pm N_2 V_{22}), \quad \lambda = \frac{v_1}{v_2}, \]

\[ F(\rho_0) = \frac{1}{\rho_0} \left[ P(\ln \rho_0)^2 + Q(\ln \rho_0 + R) \right], \quad (24) \]

\[ \rho_0 = \frac{e v_1 v_2}{4 \pi^2 \gamma^2} H_{c_2}(0). \quad (25) \]

\[ H_{c_2}(0) = \frac{2 \gamma \omega_0^2}{e v_1 v_2} \exp(2 - \alpha), \quad (26) \]

\[ \alpha = \eta^\pm + \sqrt{(\ln \lambda - \eta(-))^2 + \frac{4N_1 N_2 V_1 V_2}{\alpha^2}}, \quad (27) \]

\[ P = \frac{\zeta(2)}{4 \pi^2} \left( \lambda + \frac{1}{\lambda} \right), \quad (28) \]

\[ Q = \frac{1}{2 \pi^2} \left( \lambda + \frac{1}{\lambda} \right) \left[ \frac{\zeta(2)}{2} \ln(x_1 x_2) - \zeta'(2) \right] + \]

\[ + \frac{\xi(2)}{2 \pi^2} \left[ \varepsilon^\pm - (\lambda + \frac{1}{\lambda}) \xi_T \right], \quad x_1 = \frac{8 \gamma}{\epsilon_0}, x_2 = \frac{\pi^2 \gamma}{2}; \quad (29) \]

\[ R = \frac{\zeta(2)}{4 \pi^2} \left\{ \left( \lambda + \frac{1}{\lambda} \right) \ln x_1 \ln x_2 - (\ln \lambda)^2 \right\} - \]

\[ - \frac{\xi'(2)}{2 \pi^2} \left[ \left( \lambda + \frac{1}{\lambda} \right) \ln x_1 + \left( \lambda - \frac{1}{\lambda} \right) \ln \lambda \right] + \frac{1}{\pi^2} \left[ \frac{\xi(2)}{2} \ln x_2 - \zeta'(2) \right] \times \]

\[ \times \left[ \varepsilon^\pm - (\lambda + \frac{1}{\lambda}) \xi_T \right] - \frac{\zeta(2)}{2 \pi^2} \ln \lambda \left[ \varepsilon^\pm - (\lambda - \frac{1}{\lambda}) \xi_T \right], \quad (30) \]

In our case (T close to zero) expression can be adduced to the view:

\[ H_{c_2}(T) = H_{c_2}(0) \left[ 1 + \frac{2}{sv(\lambda)} F(\rho_0) \right] \quad (31) \]
were

\[ \nu(\lambda) = \sqrt{(\ln \lambda - \eta(-))^2 + \frac{4N_1N_2V_{12}V_{21}}{a^2}}, \quad s = \pm 1 \]  

(32)

or

\[
\frac{H_{c2}(T)}{H_{c2}(0)} = 1 + \frac{16\gamma}{e_0^2\pi^2} \left( \frac{T}{T_c} \right)^2 e^{\nu(\lambda) - s\nu(1)} \left\{ \left( \lambda \gamma^+ + \frac{1}{\lambda} \gamma^- \right) + \right.
\]

\[ \left[ \zeta'(2) + \zeta(2)\ln \frac{4T}{e_0\pi T_c} + \frac{s}{2} \nu(\lambda) - \frac{s}{2} \nu(1) \right] + \]

\[ \left. + \frac{\zeta(2)}{2} \left( \lambda \gamma^+ - \frac{1}{\lambda} \gamma^- \right) \ln \lambda \right\}, \]  

(33)

\[
\gamma^\pm = \frac{1}{2} \left[ 1 \pm \eta(-) - \ln \nu(\lambda) \right], \quad H_{c2}(0) = \frac{\pi^2 T_c^2 e_0^2}{2\gamma e v_1 v_2} \exp \left[ \nu(1) - \nu(\lambda) \right]. \]  

(34)

Expression (33) can be also obtained from formula (33) of work [18]. Let’s consider simplified case \( V_{22} = 0 \). At that on the base of definitions (24) and (32) we obtain:

\[
\eta^\pm = -\frac{\lambda_{11}}{\lambda_{12}\lambda_{21}}, \quad \lambda_{11} = N_1V_{11} \quad \lambda_{12} = N_2V_{12}, \quad \lambda_{21} = N_1V_{21}
\]

\[
\nu(\lambda) = \sqrt{\left( \ln \lambda + \frac{\lambda_{11}}{\lambda_{12}\lambda_{21}} \right)^2 + \frac{4}{\lambda_{12}\lambda_{21}}},
\]

\[
\gamma^\pm = \frac{1}{2} \left[ 1 \mp \frac{\lambda_{11}}{\lambda_{12}\lambda_{21}} + \ln \nu(\lambda) \right].
\]  

(35)

For \( s = 1 \) and \( V_{22} = 0 \) the expression for \( H_{c2}(T) \) (33) turns into corresponding expression of work [8]. For the temperature hear to the value \( T_c \) in the limit case \( V_{22} = 0 \) we obtain:

\[
\frac{H_{c2}(T \sim T_c)}{H_{c2}(0)} = \frac{8\gamma v_1 v_2}{e_0^2 [v_1^2 \eta_1 + v_2^2 \eta_2]} \exp(\nu(\lambda) - \nu(1)) \frac{6}{7\zeta(3)} \times \]

\[ \times \left( 1 - \frac{T}{T_c} \right) \left\{ 1 + \left( 1 - \frac{T}{T_c} \right) \left[ \frac{v_1^2}{v_2^2} \eta_1 + \frac{v_2^2}{v_1^2} \eta_2 \right] \frac{31}{10} \zeta(5) \left( \frac{6}{7\zeta(3)} \right)^2 - 3 \left( \frac{6}{7\zeta(3)} \right)^2 \right\} \}, \]

(36)
were
\[ \eta_{1,2} = (1 \pm \eta)/2, \quad \eta = \frac{\lambda_{11}}{\lambda_{12}\lambda_{21}\nu(1)} \quad (37) \]

Assuming in formulas (33)-(36) \( v_1 = v_2 \) we obtain corresponding ratios for usual one-band superconductor. These ratios have the view:
\[ \frac{H_{c2}(T \to 0)}{H_{c2}(0)} = 1 + \frac{16\gamma}{\pi^2e_0^2} \left( \frac{T}{T_c} \right)^2 \left\{ \zeta(2)\ln \frac{T}{T_c} + \zeta'(2) + \zeta(2)\ln \frac{4}{\pi e_0} \right\}, \quad (38) \]
\[ \frac{H_{c2}(T \to T_c)}{H_{c2}(0)} = \frac{8\gamma}{e_0^2} \frac{6}{7\zeta(3)} \left( 1 - \frac{T}{T_c} \right) \left\{ 1 + \left( 1 - \frac{T}{T_c} \right) \left[ \frac{31}{10}\zeta(5) \left( \frac{6}{7\zeta(3)} \right)^2 - \frac{3}{2} \right] \right\} \quad (39) \]
and coincide with the results of works [13], [14].

4 The discussion of the results.

We obtained equation (16), on the base of which the value of the upper critical field in the two-band system can be calculated on the whole temperature interval \( 0 \leq T < T_c \). The analytic solutions of this equation were obtained for \( T \to T_c \) (21) and \( T \to 0 \) (33). It is easy to notice that \( H_{c2} \) depends on the correlations of the speeds \( v_1 \) and \( v_2 \) of the electrons on the Fermi surface, and on the constants of the electronic-phonon interaction \( \lambda_{nm} \).

If \( H_{c2}^0 \) and \( T_{c0} \) are introduced (upper critical field and critical temperature of the one-band low-temperature superconductor), on the base of (34) we obtain:
\[ \frac{H_{c2}(0)/H_{c2}^0(0)}{(T_c/T_{c0})^{v_1/v_2}} \text{exp}(v(1) - v(\lambda)) \quad (40) \]

The numerical estimations (40) let us do the conclusion, that the upper critical field of two-band superconductors for \( T = 0 \) can exceed the value of \( H_{c2}^0(0) \) for usual superconductors by two-three orders. These big values \( H_{c2}(0) \) are provided by high \( T_c \) and by ratio \( v_1/v_2 > 1 \) or \( \gg 1 \).

The dependence \( H_{c2}(T)/H_{c2}(0) \), which was obtained on the base of formulas (33) and (34) of the value \( H_{c2} \) for \( T \sim 0 \) and \( T \sim T_c \) correspondingly and their extrapolations, are given in the fig.1.
Fig. 1. The temperature dependence $\frac{H_{c2}(T)}{H_{c2}(0)}$ at $\lambda_{11} = 0, 2, \lambda_{12} = 0, 3$ and values $v_1/v_2 = 1, 2, 3$ and 4 (curves 1-4 correspondingly).

It is easy to see, that with growth of $v_1/v_2$, the curvature in this dependence changes. Curves 3 and 4 give the curvature, which was observed during the experiment in a row of oxidation ceramics. So if there are heavy carriers in the second band (low speeds on the Fermi surface), the two-band model qualitatively describes the behavior of $H_{c2}$ as a function of temperature in these materials.

Obtained above results let us do the conclusion about qualitative describing of the experimental data by the behavior of the ratio $\frac{H_{c2}(T)}{H_{c2}(0)}$ as a function of temperature [2] in the intermetallic compound $MgB_2$. This theory give also the big values of $H_{c2}(0)$ in two-band system in comparison with one-band case. For receiving of the quantitative accordance with the experimental data it is necessary to know the theory parameters conformably to $MgB_2$, which should be evaluated from the existing experimental data on
the base of two-band model.

**Acknowledgment.**

I is very grateful to Prof. T.Mishonov for his interest to our works, describes theory of two-band superconductors and for given references about investigation property of MgB2.

**References**

[1] J. Nagamatsu, N.Nakagowa at al.,*Nature* **410** 63 (2001).

[2] P.C. Canfield, S.L. Bud’ko, D.K. Finemore, *Physica.C*, **385**, 1(2001).

[3] X.K. Chen, M.J. Konstantinovich et al., *Phys.Rev.Lett.* **87**, 157002 - 1.

[4] S.Tsuda, T. Yokoya at al., *Phys.Rev.Lett.* **87**, 177006-1 (2001).

[5] V.A.Moskalenko , *Fiz.Met.Metalloved* 503(1959);*Phys.Met. and Metallog.* **8**, 25 (1959).

[6] H.Suhle, B.T. Matthies and L.R.Walker, *Phys.Rev. Lett.*, **3**, 552 (1959).

[7] M.E.Palistrant, arXiv: *conden.matter / 0305496* (2003).

[8] V.A. Moskalenko, M.E. Palistrant, V.M. Vackalyuk, *Usp. Fiz. Nauk* **161**, 155 (1991); *Sov. Phys.Usp.* **34**, 717 (1991); arXiv:cond-mat/0309671.

[9] L.Z. Kon, arXiv: *cond-mat/0309707*(2003).

[10] T.Mishonov, E. Penev, J.O. Indekeu and V.I. Pokrovsky, arXiv: *cond-mat/0209342*; *Phys.Rev.B*, **68**, 104517 (2003).

[11] T.Mishonov, S.Drechsler and E.Penev, *Modern Physics Lett. B*, **17**, 755 (2003).

[12] A.A. Abrikosov, *Zh. Eksp. Teor.Fiz.* **32**, 1442, (1957).

[13] L.P. Gor’kov, *Zh. Eksp. Teor.Fiz.* **37**, 833, (1959).
[14] K. Maki and T. Tsuzuki, *Phys. Rev. A*, **139**, 868, (1965).

[15] V.A. Moskalenko, *Zh. Eksp. Teor. Fiz.* **51**, 1163, (1966).

[16] L.P. Gor’kov, *Zh. Eksp. Teor. Fiz.* **36**, 1918, (1959).

[17] V.A. Moskalenko, M.E. Palistrant, *Dokl. Akad. Nauk SSSR* **162**, 532, (1965); *Zh. Eksp. Teor. Fiz.* **49**, 770 (1965); *Sov. Phys. JETP* **22**, 536 (1966).

[18] M.E. Palistrant and Dedju, *Phys. Lett. A* **24**, 537, (1967); *Investigation on the Quantum Theory of Many-Particle Systems (Kishinev, Stiinta, 1969)*, p.55.

[19] V.A. Moskalenko and M.E. Palistrant, *Statistical Physics and Quantum Field Theory M., Nauka*, 262 (1973) [in Russian].