THE COBE DIFFUSE INFRARED BACKGROUND EXPERIMENT SEARCH FOR THE COSMIC INFRARED BACKGROUND. II. MODEL OF THE INTERPLANETARY DUST CLOUD

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ABSTRACT

The COBE Diffuse Infrared Background Experiment (DIRBE) was designed to search for the cosmic infrared background (CIB) radiation. For an observer confined to the inner solar system, scattered light and thermal emission from the interplanetary dust (IPD) are major contributors to the diffuse sky brightness at most infrared wavelengths. Accurate removal of this zodiacal light foreground is a necessary step toward a direct measurement of the CIB. The zodiacal light foreground contribution in each of the 10 DIRBE wavelength bands ranging from 1.25 to 240 μm is distinguished by its apparent seasonal variation over the whole sky. This contribution has been extracted by fitting the brightness calculated from a parameterized physical model to the time variation of the all-sky DIRBE measurements over 10 months of liquid He cooled observations. The model brightness is evaluated as the integral along the line of sight of the product of a source function and a three-dimensional dust density distribution function. The dust density distribution is composed of multiple components: a smooth cloud, three asteroidal dust bands, and a circumsolar ring near 1 AU. By using a directly measurable quantity that relates only to the IPD cloud, we exclude other contributors to the sky brightness from the IPD model. High-quality maps of the infrared sky with the zodiacal foreground removed have been generated using the IPD model described here. Imperfections in the model reveal themselves as low-level systematic artifacts in the residual maps that correlate with components of the IPD. The most evident of these artifacts are located near the ecliptic plane in the mid-IR and are less than 2% of the zodiacal foreground brightness. Uncertainties associated with the model are discussed, including implications for the CIB search.

Subject headings: cosmology: observations — diffuse radiation — infrared: general — infrared: solar system — interplanetary medium

1. INTRODUCTION

The effort to understand the nature of the “zodiacal cloud” or interplanetary dust (IPD) cloud is of long standing. The first hypothesis for the cause of the zodiacal light was formulated over three centuries ago by Cassini (1685). He proposed on the basis of careful visual observations that the brightness pattern seen in the night sky could be caused by a lenticular cloud of dust centered on the Sun with its main axis lying in the ecliptic plane. This was an amazingly astute conclusion. Although much later in formulation, another insightful interpretation of the structure of the IPD cloud was forwarded by Fessenkov in the early 1940s (Struve 1943). Fessenkov considered the IPD distribution as a prolate spheroid surrounded by a dust torus formed from the fragmentation of the asteroids in the asteroid belt. Although even these early efforts came to reasonable conclusions as to the general shape of the IPD cloud, more recent data, such as those from the IRAS satellite and those from the COBE Diffuse Infrared Background Experiment (DIRBE), show the cloud structure to be rather more complex. The primary objective of the DIRBE, a search for the extragalactic infrared background, requires formulation of a detailed description of the IPD cloud so that its contribution to the sky brightness can be modeled and accurately removed from measurements of the diffuse sky brightness. The aim of this paper is to describe the model developed by the DIRBE team.

A wide variety of evidence indicates that within a few astronomical units (AU) of the Sun the solar system is filled with dust of cometary and asteroidal origin. The IPD reveals itself as a diffuse component of the sky brightness, attributed to the scattering of sunlight in the UV, optical, and near-IR, and the thermal reradiation of absorbed energy in the mid-IR and far-IR. At infrared wavelengths from approximately 1–100 μm, the signal from the IPD is a major contributor to the diffuse sky brightness and dominates the mid-IR (~10–60 μm) sky in nearly all directions except very low Galactic latitudes. With the aid of mid-IR photometric measurements, our picture of the zodiacal cloud has evolved over the last 15 years from a relatively simple smooth distribution of dust to one of increasing complexity. The terms “zodiacal cloud” and “IPD cloud” now encompass several distinct components, each of which may possess different grain properties and experience different orbital dynamics (Dermott et al. 1996; Leinert et al. 1998). In addition to a smooth background distribution arising from a mix of dust associated with asteroidal and cometary debris, there are also contributions from smaller scale structures. The IRAS data showed, and the DIRBE data confirm, that imposed on the brightness of the main cloud there are contributions from asteroidal dust bands (Low et al. 1984; Spiesman et al. 1995, and references therein). More recently, a circumsolar ring theoretically proposed to arise from dust spiraling in from the outer solar...
system and being resonantly trapped by the Earth in orbits near 1 AU (Gold 1975; Jackson & Zook 1989; Marzari & Vanzani 1994a, 1994b; Dermott et al. 1994) was compellingly confirmed by Rechat et al. (1995) through use of a simple IPD model and the DIRBE data. Finer scale features such as dust trails in cometary orbits have also been observed in IRAS data (Sykes et al. 1986).

The issue as to the ultimate source of the dust is still open, but satellite observations of the sky brightness and in situ measurements of dust particles are advancing our understanding of the relative contributions from comets, asteroids, and the interstellar medium. From a study of 200 captured dust particles, Schramm, Brownlee, & Wheelock (1989) find that it is possible to ascribe 45% of the particles to cometary origin and 37% to asteroidal origin. Using dynamical analysis to construct the configuration of the IPD cloud from comets and asteroids, Liou, Dermott, & Xu (1995) found a match to the IRAS brightness profiles using a mixture of 74% cometary and 26% asteroidal dust. More recently, using numerical modeling to determine the evolution of dust from asteroidal collisions, Durda & Dermott (1997) find that 34% of the IPD could arise from collisional destruction of asteroidal family members (10%) and main-belt members (24%). In addition to the dust formed from members of the solar system, Baguhl et al. (1995) find in the Ulysses satellite dust-detection data a distinct signature in the impact directions indicating that interstellar dust is flowing into the solar system. The contribution to the brightness of the IPD from the interstellar dust is small, as shown by Grogan, Dermott, & Gustafson (1996), and it is not considered in the modeling described in this paper. Although there have been advances in estimating the source of the dust, reality appears to be yet more complex. Divine (1993) has shown from an interpretation of ground-based and satellite measurements that the IPD is best represented as composed of five distinct dust-family components. On the basis of this work, Staubach, Divine, & Grün (1993) have estimated the sky brightness arising from the five-component cloud. Clearly in the future there will be a strong symbiotic relationship between the modeling of the brightness of the IPD and the growing intricate and precise theoretical and observational studies of its nature.

Data from IRAS (Neugebauer et al. 1984; Beichman 1987) have been instrumental in advancing studies of the infrared zodiacal emission. However, the IRAS database does have some limitations, including relatively uncertain calibration of the photometric zero-point, wavelength coverage only from 12 to 100 μm, and sparse solar elongation sampling of most celestial directions. Diffuse infrared sky brightness measurements have been greatly extended by the Diffuse Infrared Background Experiment (DIRBE), launched as part of the Cosmic Background Explorer (COBE) in 1989. The DIRBE is an absolutely calibrated photometer with an instrumentally established photometric zero point, designed to survey the sky in 10 broad photometric bands from 1.25–240 μm. As described in more detail in § 2, the unique DIRBE scanning strategy and dense, continuous sampling of the sky at solar elongations from 64°–124° over a 10 month interval have provided a previously unequaled infrared photometric database from which to study the IPD in both scattering and thermal wavelength regimes.

A major thrust in photometric studies of the IPD is the construction of models that can be used to reproduce the observed brightness distribution. The development of such models is not always motivated by a wish to understand the IPD for itself, but rather as a means for removing an obfuscating foreground that masks contributions from Galactic and extragalactic sources. The focus of this paper is to describe the model for the IPD that has been used to remove the zodiacal foreground from each of the 10 DIRBE bands to permit investigation of the residual sky brightness for evidence of the cosmic infrared background (CIB; defined here as all diffuse infrared radiation of extragalactic origin). Since it was anticipated that the CIB may be faint compared to the IPD and Galactic foregrounds, the aim has been to generate an IPD model that could reproduce the zodiacal brightness to within a few percent or better.

A number of models and modeling techniques have been developed to describe the infrared zodiacal light (e.g., Boulanger & Perault 1988; Jones & Rowan-Robinson 1993; Wheelock et al. 1994; Dermott et al. 1996). All models, including the one described here, have limitations brought about primarily because of the general lack of detailed information on IPD grain properties, populations, and spatial distribution. No previous modeling effort, however, has been designed to take advantage of the spectral, temporal, and angular coverage that DIRBE provides.

The remainder of this paper describes the development of a parameterized physical model of the IPD in which the parameters are determined by requiring the model brightness to match the measured time-dependence of the sky brightness in fixed celestial directions over the whole sky. Section 2 briefly describes the DIRBE instrument, scan strategy, and data reduction methods. In § 3 the motivation behind the chosen modeling approach is discussed. The details of the model itself are presented in § 4. Results of the modeling, including uncertainties, are presented in § 5. Section 6 contains a discussion of the implications of the model uncertainties for the CIB search, and conclusions are presented in § 7.

This paper is part of a series outlining the analysis of DIRBE data in the search for the cosmic infrared background. Paper I (Hauser et al. 1998) reports the observational results of the DIRBE search, compares them with previous work, and briefly discusses their implications. The foreground removal techniques used in the DIRBE search are described in two separate papers: this paper (Paper II) and Arendt et al. (1998, hereafter Paper III), which describes the Galactic foreground removal procedures and summarizes the systematic uncertainties arising from the total foreground discrimination process. A further discussion of the cosmological implications of the results described in Paper I is presented by Dwek et al. (1998, hereafter Paper IV).

2. DIRBE INSTRUMENT AND DATA OVERVIEW

The data used in this analysis are broadband photometric measurements obtained with the DIRBE instrument during its 10 months of cryogenic operation (1989 December 11 to 1990 September 21) aboard the Cosmic Background Explorer (COBE) satellite. A detailed description of the COBE mission has been given by Boggess et al. (1992), and the DIRBE instrument has been described by Silverberg et al. (1993). The DIRBE provided a simultaneous survey of the sky in 10 photometric bands at 1.25, 2.2, 3.5, 4.9, 12, 25, 60, 100, 140, and 240 μm using a 0'7 × 0'7 field of view. Linear polarization data were gathered in the
three shortest wavelength bands, but those data are not used in the model described here.

A stable DIRBE photometric system was established using a combination of internal and celestial sources (COBE DIRBE Explanatory Supplement 1997). The short-term instrumental gain stability, during the 10 months of cryogenic operation at 1.8 K, was maintained using observations of a stable internal stimulator ~6 times per orbit. The long-term gain stability was monitored using bright stable celestial sources observed during normal sky scans, which were placed on an instrument-unique relative photometric system. The combination of using both the internal stimulators and celestial sources achieved a relative photometric system that was stable to ~1% over the cryogenic mission at most wavelengths.

The data at 100 μm, however, had a special photometric problem. The Ge:Ga detector used at these wavelengths exhibited a gain instability referred to as photon induced responsivity enhancement (PIRE; COBE DIRBE Explanatory Supplement 1997). The PIRE effect is a form of hysteresis evidenced by an increase in responsivity when the DIRBE beam swept across a bright extended region, such as the inner Galaxy and the Cygnus spiral arm. The brightness recorded in pixels in the vicinity of these regions depended on the direction from which they were approached (i.e., on the brightness of the pixels previously scanned), with differences at 100 μm as large as 25%. A nonlinear correction was developed to mitigate the PIRE effect on the data. This correction significantly reduced the effect, leaving residual scan-dependent differences of the order of 5% or less, although in the brightest Galactic regions differences can still be as large as 10%. In contrast, the Ge:Ga detector used at 60 μm, for which a PIRE correction was not implemented, was better behaved, registering a difference of no more than 3%. Since the direction from which each pixel was approached in the survey varied in a systematic way with time, the PIRE effect may have created a spurious apparent temporal modulation of the brightness. Such modulation would not be coherent over a span of more than a few degrees and would occur only at low Galactic latitudes. Since our search for the CIB excluded regions at low Galactic latitude (Paper I), this does not pose a serious problem for the DIRBE investigation.

Another essential feature of the calibration was the monitoring of the shape and centroids of the beams for each detector. This was done by assembling the transit data from selected bright celestial sources. The location of any transit was determined to the order of 1.5, which is a very small fraction of the 42° DIRBE beam. We found no solid angle variations in any band during the 10 months of cryogenic operation. Any variation must be smaller than the accuracy with which we were able to measure the solid angle, which was better than 1% from 1.25 to 25 μm, and better than 5% from 60 to 240 μm. These numbers include the uncertainty due to the possible contribution of stray light. The month-to-month beam centroid peak-to-peak variations during the cold mission were at most 1'. Errors in the centroid correction for wavelengths from 1.25 to 4.9 μm are certainly much less than this because the reference beams for these bands were formed during intervals in which the variation about the mean was much less than 1'. Beam centroid errors for the longer wavelength bands, for which a single reference beam was used for the entire cold mission, are estimated to be less than 0.5.

The absolute gain calibration at each wavelength was based upon observations of a single well-calibrated celestial object. The selected calibration objects were Sirius (1.25–12 μm), NGC 7027 (25 μm), Uranus (60 and 100 μm), and Jupiter (140 and 240 μm). The uncertainty in the absolute gain calibration is ~3%–5% at 1.25–12 μm and ~10%–15% at 25–240 μm (Paper I). The DIRBE intensity measurements are reported in units of MJy sr⁻¹, quoted at the 10 nominal wavelengths with effective bandwidths calculated assuming the observed source energy distribution is νIν = constant. This necessitates the use of a color correction when computing the zodiacal light contribution (§4).

A critically important part of the DIRBE calibration, and one that distinguishes the DIRBE from most other infrared instruments, was establishing a true zero point for the photometric system. This was accomplished instrumentally. The DIRBE optical system was a totally off-axis design with multiple field and pupil stops that limited external stray light to levels well below the sensitivity limit for the measurements at all wavelengths, νIν < 1 nW m⁻² sr⁻¹. The DIRBE instrument was equipped with a cold chopper, which modulated the beam at 32 Hz, and a cold shutter located at a field stop that could completely shut out sky light from the detectors. When the shutter was open, each detector measured the difference between the sky brightness and that of an internal beam stop maintained at a temperature below 3 K so as to emit essentially no flux at the DIRBE wavelengths. The shutter was closed about 6 times per orbit and the instrumental offset signal at 32 Hz was measured. This offset was stable throughout the entire 10 month cryogenic mission and was subtracted from the data obtained while scanning the sky. A real radiative instrumental offset was present, largely due to emission from the junction field effect transistors (JFETs) used in the signal amplifiers for the detectors that were mounted in the detector assemblies. This offset, which was significant only at wavelengths of 60 μm and longer, was confirmed by turning off the JFETs sequentially and measuring the change in offset in the other detectors. The offset uncertainty was carefully assessed using special on-orbit tests, which showed that the size of the offset did not depend on whether the shutter was open or closed. The zero-point uncertainty, which is substantially below the sky brightness at all wavelengths, does not affect the IPD model, but does affect the systematic uncertainty in the CIB at long wavelengths (Paper I).

The observing strategy for DIRBE was designed to monitor the change in the sky brightness contribution from the IPD toward each celestial line of sight as a function of time, thus providing a richer infrared database for studying the IPD cloud than previously available. Like IRAS, the COBE spacecraft was placed in a Sun-synchronous polar orbit at 900 km altitude, but, unlike IRAS, the spacecraft also rotated (rate ~0.8 rpm). The COBE spin axis was nominally held fixed at an angle of 94° from the Sun, with the DIRBE instrument line of sight angled 30° from the spin axis. The 30° cant caused the DIRBE to execute a cycloidal track on the sky as the COBE both rotated and orbited the Earth. With a half-angle of 30°, the viewing swath enclosing this track covered half of the sky after one orbit of the spacecraft (Fig. 1, top left). Because the field of view of the DIRBE was 0.7 × 0.7 and the COBE orbital velocity was about 3.5 minute⁻¹ the one-orbit scan swath had gaps in coverage. Full sampling of half of the sky was achieved after
Fig. 1—DIRBE 12 μm maps of the sky for various integration periods. Top left: one orbit. Top right: 1 day. Bottom left: 1 week. Bottom right: 10 months. Maps are Mollweide projections in ecliptic coordinates.
about 1 day of observations (Fig. 1, top right); the highest
density of observations occurred along the edges of the
viewing swath, and the lowest density occurred at the center
of the swath. It also follows from the scanning geometry
that the solar elongation angle (angle between the Sun and
the line of sight) at which an observation was made is
dependent on the location within the viewing swath: those
positions along the swath edges closest to the Sun were
viewed at elongations $e \sim 64^\circ$, and those along the edges
farthest from the Sun were viewed at $e \sim 124^\circ$, with
a smooth continuum between. 

The COBE orbit precessed 1° per day, so the DIRBE
viewing pattern on the sky shifted by this amount along
the ecliptic plane each day. Complete sky coverage was thus
achieved within 4 months, with more uniform sampling
in 6 months. Figure 1 (bottom left and right) shows the
average of all observations of the sky at 12 µm for periods
of 1 week and the full mission, respectively. Because of the 1°
day $^{-1}$ orbital precession, a fixed celestial location near the
ecliptic plane was observed at solar elongations from 64° to
124° over the course of 60 days each half-year. Above ecliptic
latitudes of $\sim 60^\circ$ the amplitude of the full range in
elongation decreases about 2° per degree of latitude and is
zero directly at the ecliptic poles. In this way, all accessible
solar elongation angles in the range 64° to 124° were
sampled for each location on the sky.

The photometric data from the DIRBE time-ordered
scans described above were converted into maps of the sky
by associating each observation with a pixel in the quadri-
lateralized spherical cube projection (O’Neill & Laubscher
1976; COBE DIRBE Explanatory Supplement 1997) in
geocentric ecliptic coordinates, epoch 2000. The map
resolution was chosen such that pixels are roughly 0:32 on
a side, yielding 393,216 approximately equal-area pixels. A
set of forty-one weekly sky maps was created for the period
of cryogenic operation, in which all individual observations
of pixels sampled during 1 week were robustly averaged to
produce a single average intensity per pixel at each wave-
length. Pixels within $\sim 60^\circ$ of the ecliptic plane typically
have $\sim 10$–15 samples per week over which to average; the
most densely sampled pixels near ecliptic latitudes $|\beta| = 60^\circ$
have $\sim 30$ samples per week. The averaging interval of 1
week was chosen so as not to overly smear apparent tempo-
ral variations of the zodiacal light (compare the bottom left
and top right panels of Fig. 1) and yet provide a reasonably
compact database. This set of weekly sky maps is the basis
for this study; specifically the pass 3b version of these pro-
ducts released through the National Space Science Data
Center (NSSDC) in 1997.

All analysis has been performed using data from the origi-
nal sky-cube coordinate system. For the purpose of illus-
tration, maps shown in this paper are reprojected into an
ecliptic coordinate Mollweide projection. The Mollweide
projection is an equal-area projection with the convenient
properties that longitudes are equally spaced at each lati-
tude and all lines of constant latitude are straight horizontal
lines (although unequally spaced).

3. MOTIVATION AND MODELING TECHNIQUE

The development of the IPD cloud model was driven by
requirements imposed by the ultimate goal of determining
the contribution of the isotropic CIB signal in the infrared.
Thus, the method described here was designed to provide a
means for accurately subtracting the zodiacal signal from
the DIRBE observations at 1.25–240 µm while preserving
any isotropic component of the sky brightness unrelated to
the IPD; i.e., our method includes no arbitrary zero-point
constants. The scheme follows what is now a canonical
approach adopted in early models (Haug 1958; Leinert et
al. 1976) of representing the IPD cloud geometric and radi-
ative characteristics in a parametric form. This procedure
has been used in a number of recent investigations (e.g.,
Murdock & Price 1985; Deul & Wolstencroft 1988;
Rowan-Robinson et al. 1990; Jones & Rowan-Robinson
1993). The physics of the IPD cloud is embedded in the
representations of the scattering and emissivity functions
and the form factors for the various components (i.e., main
cloud, asteroidal bands, and circumsolar ring).

Figures 2a and 8a illustrate the nature of the difficulty in
generating such a model. The observed infrared sky bright-
ness is a complex mix of foregrounds due to interplanetary
dust, starlight, and interstellar dust, as well as an extra-
galactic background that is presumed to be isotropic. The
relative mixture of foregrounds is wavelength dependent;
only in the mid-IR does emission from the IPD contribute
90% or more of the total sky brightness. Although the
shape of the underlying zodiacal “lower envelope” is
clearly visible in the figures from 1.25–100 µm, spatially
disentangling the Galaxy from the IPD signal with high
precision is challenging (Hauser 1988). The problem is addi-
tionally complicated by the presence of low-contrast,
smaller scale structures within the IPD cloud such as the
dust bands and circumsolar ring (§§ 4.2.2 and 4.2.3).

In order to determine the contribution from interplan-
etary dust without modifying any of the other contribu-
tions, the one unique signature of the zodiacal light is used:
it is the only component of the sky brightness that is not
fixed on the celestial sphere. For an Earth-based observer,
the IPD brightness observed in a given celestial direction on
a given day depends on the observer’s viewing aspect.
Changes in viewing aspect resulting from the DIRBE
cycloidal scan strategy and motion through the IPD cloud
cause different path lengths through the dust to be sampled
(and thus different portions of the dust density and tem-
perature distributions), which gives rise to apparent tempo-
ral variations in the brightness observed toward a fixed
celestial direction. Figure 3 is a schematic representation of
how the zodiacal cloud signal is modulated in two direc-
tions. Toward the ecliptic pole (Fig. 3a), the main causes for
variation of the brightness are the motion of the Earth with
respect to the inclined midplane of the dust distribution, as
well as the motion of the Earth radially due to its orbital
eccentricity. At lower latitudes (Fig. 3b), the apparent tem-
poral variation is primarily due to the changing solar elon-
gation of the line of sight as the Earth orbits about the Sun.

The technique used here exploits these effects to separate
the light scattered and emitted by the IPD from all other
brightness contributions by imposing an analytical form for
the interplanetary dust density distribution, thermal emis-
sion characteristics, and scattering phase function. This
parameterized model is then optimized to match the
observed temporal variations in selected directions. The for-
mulation of the parameterized model is described in § 4.
Once the best-fit model parameters are determined, the
brightness of the zodiacal light is then evaluated for all
DIRBE weekly observations via a line-of-sight integral of
the three-dimensional model.
Fig. 2a — DIRBE mission-averaged sky brightness in 10 wave bands, both before and after removal of the IPD signature. Top: As observed sky, logarithmic scale. Center: Sky after removal of zodiacal light (ZL), scaled identically with top. Bottom: Same as center, but on a linear scale expanded to show defects in the residuals. Units are in MJy sr$^{-1}$. Sixteen color contour levels are used. The minimum and maximum scaling values used for each band are listed as follows, where the first set of numbers are the logarithmic scaling limits, and the second set are the linear scale: 1.25 μm: [0.063, 31.6], [0.05, 0.3]; 2.2 μm: [0.04, 31.6], [0, 0.2]; 3.5 μm: [0.032, 31.6], [−0.01, 0.2]; 4.9 μm: [0.1, 15.8], [0, 0.2]; 12 μm: [1.58, 79.4], [0, 2]; 25 μm: [3.98, 79.4], [0.5, 3]; 60 μm: [1, 79.4], [0, 15]; 100 μm: [1, 79.4], [0, 15]; 140 and 240 μm: [1, 794.3], [0, 20]. All maps are Mollweide projections in ecliptic coordinates.
4. PARAMETRIC IPD MODEL

The DIRBE IPD model is a parameterized physical model whose formulation is similar, but not identical, to that used in creating the IRAS Sky Survey Atlas (Wheelock et al. 1994). The DIRBE model consists of the integral along the line of sight of the product of a source function and a three-dimensional density distribution function. The emissivity function includes both thermal emission and scattering terms. The three-dimensional dust density distribution is composed of multiple components—a smooth cloud, three dust bands, and a circumsolar dust ring just beyond 1 AU. Earlier versions of the model have been described by Reach et al. and Franz et al. (1996).

4.1. The Brightness Integral

The model for the IPD foreground computes the brightness of the zodiacal light observed at wavelength $\lambda$ for each pixel $p$ at time $t$ as the integral along the line of sight of scattered and thermal emission contributions, summed over each density component $c$:

$$ Z_\lambda(p, t) = \sum_c \int n_c(X, Y, Z)[A_{c,\lambda}F_\phi^\lambda \Phi_\lambda(\Theta) + (1 - A_{c,\lambda})E_{c,\lambda}B_\lambda(T)K_\lambda(T)]ds, \quad (1) $$

where $n_c(X, Y, Z)$ is the three-dimensional density for each of the components, $A_{c,\lambda}$ is the albedo for component $c$ at wavelength $\lambda$, $F_\phi^\lambda$ is the solar flux, $\Phi_\lambda(\Theta)$ is the phase function at scattering angle $\Theta$, $E_{c,\lambda}$ is an emissivity modification factor that measures deviations from the blackbody thermal radiance function $B_\lambda(T)$, and $K_\lambda(T)$ is the DIRBE color-correction factor appropriate for $B_\lambda(T)$. The dust grain temperature $T$ is assumed to vary with distance from the Sun as $T(R) = T_0 R^{-\delta}$. The derived model value of $\delta = 0.467$ is very close to the theoretical value of 0.5 for large gray grains in radiative equilibrium.

As in the case of IRAS photometry, the DIRBE broadband photometric measurements $I_\lambda$ are quoted at fixed nominal wavelengths with bandwidths determined assuming the source energy distribution is constant. Since the model parameters are optimized by fitting to the DIRBE data, a color correction $K_\lambda(T)$ must be applied to each evaluation of the blackbody source term in order to compute the zodiacal brightness in a fashion consistent with the DIRBE database; this is equivalent to convolving the source function with the DIRBE spectral response. A color correction of 1.0 is used for the scattering source function. The color corrections are taken from those tabulated in the COBE DIREBE Explanatory Supplement (1997).

The treatment of the albedos $A_{c,\lambda}$ and emissivity modification factors $E_{c,\lambda}$ is important for this work. The aim is to achieve high accuracy in modeling the zodiacal light from 1.25–240 $\mu$m, but the accuracy of the absolute calibration from wavelength to wavelength in the mid-IR is relatively poor ($\delta$). Furthermore, it is unlikely that the assumption of a single grain temperature at each distance $R$ is entirely accurate. Models for the emission from interplanetary dust...
predict that a small range of temperatures contributes to the mid-IR emission, expanding to a wider range at shorter wavelengths (Reach 1988). The wavelength-dependence of the albedo is essentially unknown, and a constant albedo is unlikely. In order to allow for the imprecision of the band-to-band calibration, as well as the restrictive nature of the spectral model, the factors $A_{c,\lambda}$ and $E_{c,\lambda}$ are allowed to be free parameters. Albedos are allowed to be nonzero only at the shorter wavelengths (1.25, 2.2, and 3.5 $\mu$m), and $E_{c,\lambda}$ is set to 1.0 at 1.25 and 2.2 $\mu$m. The $E_{c,\lambda}$ is normalized to unity at 25 $\mu$m. As coded, each density component could have its own particle properties, as delineated by $A_{c,\lambda}$ and $E_{c,\lambda}$. In practice, however, we allowed for three groups of $A_{c,\lambda}$ and $E_{c,\lambda}$: one for the smooth cloud, one for all of the dust bands, and one for the circumsolar ring. In addition, at 1.25–3.5 $\mu$m, where the dust bands and circumsolar ring have a small contribution to the cloud brightness, the ring and dust band values for $A_{c,\lambda}$ and $E_{c,\lambda}$ were not optimized but rather assumed to be identical to the values found for the smooth cloud at those wavelengths.

There is no definitive work that establishes a phase function in the infrared. Initial modeling attempts used the Henyey-Greenstein formulation determined by Hong (1985) for visible data, which allowed for no dependence of the phase function on wavelength. The final model incorporated a wavelength-dependent three-parameter functional form that was capable of reproducing the shape of Hong’s function, but could also be optimized for the infrared. The functional form of the phase function $\Phi(\Omega)$ is

$$\Phi(\Omega) = N[C_{0,\lambda} + C_{1,\lambda}\Theta + e^{i2.1\Theta}],$$

where $\Theta$ is the scattering angle in radians. The three free parameters for each wavelength are $C_{0,\lambda}$, $C_{1,\lambda}$, and $C_2$. The factor $N$ is not free; it is a function of the three $C$ parameters and serving to normalize the phase function such that the integral over $4\pi$ steradians is 1.

4.2. Model Components and Geometry

The model calculations are performed in heliocentric ecliptic coordinates $(X, Y, Z)$. The coordinate transformation for a grid point at a distance $s$ from the Earth along a line of sight at geocentric ecliptic coordinates $(\lambda, \beta)$ is

$$X = R_\oplus \cos \lambda_\oplus + s \cos \beta \cos \lambda,$$
$$Y = R_\oplus \sin \lambda_\oplus + s \cos \beta \sin \lambda,$$
$$Z = s \sin \beta,$$
$$R = \sqrt{X^2 + Y^2 + Z^2},$$

where $R_\oplus$ is the Earth-Sun distance and $\lambda_\oplus$ is the heliocentric longitude of the Earth ($\lambda_\oplus = \pi - \lambda_\oplus$) on the date of observation. The eccentricity of the Earth’s orbit was included in the model.

The integration along the line of sight was performed by Gauss-Laguerre quadrature with 50 points. For ecliptic latitudes $|\beta| \leq 20^\circ$, Simpson’s rule with $\sim 200$ steps was used in order to more accurately evaluate density gradients in small-scale structures closer to the ecliptic plane. Integration along the line of sight is performed from the Earth to an outer radial cutoff of 5.2 AU from the Sun, which roughly corresponds to the orbit of Jupiter.

All dust density components were assumed to be intrinsically time independent and to have a plane of symmetry (although not necessarily the same plane in all cases). The only exception to this is the trailing blob in the circumsolar ring, which follows the Earth in its orbit. Isodensity contours of the total model density and its components are shown in Figure 4. The parameterized form used for each of these components is described in detail below.

4.2.1. Smooth Cloud

The center of the smooth dust cloud was allowed to be offset from the Sun by $(X_0, Y_0, Z_0)$, so that the cloud coordinates were translated as follows:

$$X' = X - X_0,$$
$$Y' = Y - Y_0,$$
$$Z' = Z - Z_0,$$

$$R_\xi = \sqrt{X'^2 + Y'^2 + Z'^2}. \tag{4}$$

The symmetry plane of the smooth cloud was also allowed to be tilted with respect to the ecliptic plane, so that the vertical structure is dictated by the height above the tilted midplane:

$$Z_\xi = X' \sin \Omega \sin i - Y' \cos \Omega \sin i + Z' \cos i, \tag{5}$$

where $i$ and $\Omega$ are the inclination and ascending node of the midplane, respectively.

The density of the smooth cloud was presumed to be of a form that is separable into radial and vertical terms:

$$n_\xi(X, Y, Z) = n_0 R_\xi^{-a} f(\xi), \tag{6}$$

where $\xi \equiv |Z_\xi/R_\xi|$. The separation into radial and vertical terms is typical of several models for the cloud density in the literature (e.g., as summarized by Giese, Kneissl, & Rittich 1986). However, it should be noted that there are some theoretical reservations as to its applicability to modeling the IPD cloud (Banaskiewicz, Fahr, & Scherer 1994; Fahr, Scherer, & Banaskiewicz 1995). The radial power-law index is motivated by the radial distribution expected for particles under the influence of Poynting-Robertson drag, which causes their orbital semimajor axes to decay such that their equilibrium distribution is $1/R$. The assumption that the vertical distribution depends only on $\xi$ is motivated by the fact that Poynting-Robertson drag does not affect the orbital inclination of particles as they spiral into the Sun. The radial power-law index, $\alpha$, is a free parameter.

The vertical distribution $f(\xi)$ was written in a form representing a widened, modified fan model:

$$f(\xi) = e^{-\beta \varphi}, \tag{7}$$

where

$$\varphi = \begin{cases} \xi^2/2\mu & \text{for } \xi < \mu, \\ \xi - \mu/2 & \text{for } \xi \geq \mu, \end{cases}$$

and $\beta$, $\gamma$, and $\mu$ are free parameters. (Note that $\beta$ is not to be confused with the geocentric ecliptic latitude.) The vertical distribution is critically important in connecting the local density of the dust, which is very well determined by the DIRBE data, with the integrated column density. This model is somewhat different from the one fitted to the IRAS data in production of the IRAS Sky Survey Atlas (Wheelock et al. 1994) because we used a spherical radius rather than a cylindrical one. Furthermore, the function $g$, which replaces $\xi$ in the exponential of the IRAS model, rolls off at small values and avoids the cusp in the midplane that
is present in traditional fan models. Isodensity contours of
the smooth cloud model are shown in Figure 4b.

4.2.2. Dust Bands

The dust bands were discovered in the IRAS data (Low et al. 1984) and are believed to be asteroidal collisional debris (Dermott et al. 1984; Sykes et al. 1989). The dust bands have been studied using the DIRBE data (Spiesman et al. 1995), confirming the observational results from IRAS data and extending them to the near-IR. In particular, the parallactic and spectroscopic distances to the bands are less than the distance to the asteroid belt, so that the material producing them is likely to be debris spiraling into the Sun under Poynting-Robertson drag. Three band pairs that appear at ecliptic latitudes around ±1.4°, ±15°, and ±15° in the sky maps are included. These have been attributed (Sykes et al. 1989; Reach et al. 1997) to a blend of the Themis and Koronis families (+1°), the Eos asteroid family (+10°), and the Maria/Io family (+15°). All band pairs were centered on the Sun, but were allowed to be inclined with respect to the ecliptic plane. Each band pair had its own inclination and ascending node transformation.

For this work, a dust band density based on the migrating model (Reach 1992) was used, but with a simpler analytic formulation that is easier to evaluate and optimize. A modification was added in the form of a multiplicative factor that allowed for only “partial” migration, i.e., a cutoff at a minimum radius:

\[
n_{B_i}(X, Y, Z) = \frac{3n_{3B_i}}{R} \exp \left[ -\left( \frac{\zeta_{B_i}}{\delta_{(B_i)}} \right)^6 v_{B_i} + \left( \frac{\zeta_{B_i}}{\delta_{(B_i)}} \right)^{p_{B_i}} \right] \times \left( 1 - \exp \left[ -\left( \frac{R}{\delta_{R_{B_i}}} \right)^{20} \right] \right),
\]

where \(n_{3B_i}\) is the density at 3 AU of band \(i\), \(\zeta_{B_i} = |z_{B_i}/R_{i}|\), and \(\delta_{(B_i)}, v_{B_i}\), and \(p_{B_i}\) are adjustable shape parameters. The parameter \(\delta_{R_{B_i}}\) determines the distance to which band \(i\) migrates in toward the Sun.

4.2.3. Circumsolar Ring

The Earth temporarily traps migrating dust particles into resonant orbits near 1 AU if they are in low-eccentricity orbits, as is expected for asteroidal debris (Jackson & Zook 1989; Marzari & Vanzani 1994a, 1994b; Dermott et al. 1994, 1996). We have confirmed the existence of the dust ring near 1 AU by subtracting a smooth cloud model from two weekly sky maps, revealing the signature of the ring in remarkable agreement with the predictions (Reach et al. 1995). For the DIRBE IPD model, an empirical ring density function was developed to emulate the numerical simulations of Dermott et al. (1994). It consists of a circular toroid with an enhancement in a three-dimensional blob trailing the Earth. This representation ignores the fact that the trailing blob follows the Earth in an equally eccentric orbit. Neglect of this effect is expected to introduce only a small error, although it will affect a large range in ecliptic latitude.
This effect may be worth consideration in future models. As with the smooth cloud and dust bands, the symmetry plane of the ring complex (ring + trailing blob) was allowed to be inclined with respect to the ecliptic plane; the vertical distance above the ring midplane is denoted by $Z_R$ and is computed using equation (5), with the optimized values of $\Omega_{RB}$ and $I_{RB}$. The three-dimensional ring dust density distribution is modeled as

$$n_R(X, Y, Z) = n_{SR} \exp \left[ -\frac{(R - R_{SR})^2}{2\sigma_{SR}^2} \right] - \frac{|Z_R|}{\sigma_{SR}} - \frac{|Z_R|}{\sigma_{TB}}.$$

where the subscript SR stand for the circumsolar ring and TB stands for the trailing blob. The $\sigma$ values are free parameters for scale lengths in the $R, Z, \theta$ coordinates. Also free are the radial locations of the peak density of the ring $(R_{SR})$ and blob $(R_{TB})$ and the peak densities $n_{SR}$ and $n_{TB}$.

The angle $\theta$ is the heliocentric ecliptic longitude relative to the mean longitude of the Earth. We have presumed that the ring structure is fixed with respect to the mean longitude of the Earth and not its true longitude. In these coordinates, the Earth moves in an epicycle about its mean longitude with an amplitude of $2\degree$. The location of the trailing enhancement is $\theta_{TB} \sim 10\degree$, so the epicyclic motion of the Earth changes its distance from the enhancement by $\sim 10\%$–$20\%$ over the year. Isodensity contours of the dust ring model are shown in Figure 4d. The map in Figure 4d cuts through the ring at 1 AU in two places, yielding the two cross-sectional slices. Note that the orientation of the map is rotated about the $Z$ axis by $\theta_{TB}$ so that it cuts through the trailing blob.

### 4.3. Fitting Technique

The parameters of the IPD model are determined using the Levenberg-Marquardt nonlinear least-squares optimization algorithm (Bevington 1969). The fit is constrained using only the observed time variation along independent lines of sight, while ignoring their underlying photometric baselines. This is achieved by forcing the mean of the model over all time samples to match the mean of the data for each individual line of sight. In this way, the optimization procedure only has enough information to match the amplitude and the temporal variation of each line of sight, with no assumption about the morphology or spectrum of nonvarying components. The goodness of fit is defined so that the model is optimized to match the modulation as follows. Let $t$ be the observation time (an index over the weekly sky maps), $p$ be the celestial position (an index over selected pixels), and $\lambda$ be the wavelength (an index over spectral bands). The observed brightness is $I_j(p, t)$, and the model evaluated for the same conditions is $\bar{I}_j(p, t)$. Then the goodness of fit is defined

$$\chi^2 = \sum_{j, \lambda, p, t} \frac{\sigma^2_j(p, t)}{\sigma^2_j(p, t)} \times \left( [I_j(p, t) - \langle I_j(p, t) \rangle] - [\bar{Z}_j(p, t) - \langle \bar{Z}_j(p, t) \rangle] \right)^2.$$

In this equation, $\sigma^2_j(p, t)$ is the estimated uncertainty on $I_j(p, t)$, as derived from the quadrature sum of random measurement errors and the uncertainty in the temporal stability of the gain calibration. A typical value for $\sigma_j$ in the 1.25–4.9 $\mu$m bands is 0.005 MJy sr$^{-1}$. The 12–100 $\mu$m standard deviation values are dominated by the temporal gain calibration uncertainty: typical values are best expressed in terms of $\sigma_j/I_j$, and are $\sim 0.0076, 0.0076, 0.015$, and 0.022, respectively. The 140 and 240 $\mu$m bands, which are dominated by instrumental noise, have typical standard deviations of 7 and 4 MJy sr$^{-1}$.

The parameters $T_0$, $A_{\lambda,j}$, and $E_{\lambda,j}$ are highly correlated with each other and cannot be determined independently. The value of $T_0$ was determined by running a preliminary fit that assumed that the smooth cloud is the dominant component and that the spectrum in the mid-IR is that of a pure blackbody; i.e., the albedos and emissivities at 4.9, 12, and 25 $\mu$m were set to $A_j = 0$ and $E_j = 1$, and the model solved for $T_0$. This value of $T_0$ was then fixed and subsequently used in all further fits for geometry and source terms.

Initially, the complete set of model parameters (geometry and source terms) were determined simultaneously using data from all 10 DIRBE wavelength bands. In order to avoid excessive computational requirements, the fitting data set was chosen from a subset of the 41 weekly averaged DIRBE maps. A spatial grid was established for each wavelength that sampled a sky pixel every $\sim 5\degree$ for ecliptic latitudes $\leq 30\degree$, and every $\sim 10\degree$ above that. In addition to excluding observations within $10\degree$ of the Galactic plane, only "quiet" pixels that were not overtly situated on a strong photometric gradient were chosen for each wavelength. All available high-quality weekly averages were used for each chosen line of sight; this translates to a maximum of 40 time samples (for pixels near the ecliptic poles) and an average of about 20 time samples for pixels near the ecliptic plane. No chosen line of sight had fewer than eight time samples. Each wavelength used $\sim 800$ lines of sight after exclusion of the Galactic plane. The "quiet" pixel criterion did not constrain the $\sim 800$ lines of sight to be precisely the same at each wavelength, since local photometric gradients are also wavelength dependent.

Further analysis showed that the relatively high detector noise and small contribution of the IPD signal at 140 and 240 $\mu$m caused these bands to possess very little influence on the derived model geometry. In addition, the number of longer wavelength data points used in the fitting data set was insufficient to overcome inherent random errors in these data. Ultimately, these data constraints forced adoption of a two-step fitting procedure. In the first step, the complete set of model parameters (geometry and source terms) were found simultaneously using data from the 1.25–100 $\mu$m wavelength bands. In the second step, the model geometry found for 1.25–100 $\mu$m was assumed to be applicable to the 140 and 240 $\mu$m bands. In this case, the nonlinear problem reduces to a linear least-squares solution for the 140 and 240 $\mu$m model source terms. Rather than sampling the sky every few degrees, the complete DIRBE weekly data set at these two wavelengths is used in a separate fit to derive the 140 and 240 $\mu$m source terms, once again relying only upon temporal variations.

### 5. RESULTS

The result of the modeling process described above is a formulation that can be used to describe the modulations in the IPD foreground observed by DIRBE to an accuracy of 2% or better. This section describes the results of the fitting
process, starting with the small subset of data used to produce the model, and working up to the production of maps of the full sky in which the zodiacal foreground has been removed based on our model. Consistency of the model with the observed modulation and observed structure in the IPD is also discussed in this section, whereas the question of the uniqueness of the model is addressed in § 6.

5.1. Optimized Model

There are nearly 90 potential free parameters in this model, including both density terms and particle thermal emissivity and scattering properties. In the final optimization run, fewer than 50 of the parameters were actually determined from the fitting procedure. Treatment of the albedos and emissivities has been discussed in § 4; in addition, some of the dust band and circumsolar ring density shape parameters were held fixed. This was necessitated in part by limitations in the fitting data set and in part because of deficiencies in the analytical model that would cause one density component to attempt to compensate for inadequacies in another.

The idea of using only the apparent time variation to determine the zodiacal light was tested on simulated DIRBE observations (including noise) of a model IPD cloud. The fitting procedure was able to recover the properties of the smooth cloud and circumsolar ring well. However, some angular variations of the low-contrast dust bands were difficult to recover, partly because of the large grid spacing in the fitting data set. To remedy this, some of these parameters were set to values derived by Reach et al. (1997) using angular filtering techniques to isolate the dust band structure. In addition, tests on real data showed that the circumsolar ring component would sometimes be fit in a way that attempted to flatten it to compensate for the dust bands. For this reason, the parameters $\theta_{rb}$ and $\sigma_{sr}$ were fixed to values derived from visual examination of DIRBE data.

The final 1.25–100 $\mu$m fitting data set consisted of ~800 lines of sight per wavelength and a total number of 87,035 observations (including all wavelengths and weekly samples). This constitutes only ~0.13% of the weekly averaged data set. There are 45 model parameters, leaving 80,425 degrees of freedom for the optimization. Final parameter values are summarized in Tables 1 and 2, together with a short description of each parameter. Also listed is the 68% joint confidence uncertainty for each fitted parameter, which is derived from the maximum projection of the principal axis of the 45 parameter 68% confidence ellipsoid onto the axis for that individual parameter.

The final total $\chi^2$ per degree of freedom for the optimized fit to the 1.25–100 $\mu$m data was 2.1. The $\chi^2$ per degree of freedom achieved for the individual bands from 1.25 to 100 $\mu$m was 2.2, 1.9, 2.0, 3.6, 2.3, 1.5, 1.4, and 1.7, respectively. These reduced $\chi^2$'s are higher than one would expect a priori. Some part of this excess appears to be due to the fact that the brightness of the IPD is intrinsically time dependent (see § 5.6), but this variation was not recognized until after the present modeling effort was completed. As discussed in § 5.6, the intrinsic time variation is small and ignoring it in the model is not believed to have caused significant error in the residual sky brightness or the conclusions of our CIB search.

Although the formulation of the IPD modeling problem has been approached differently by various researchers, it is interesting to compare the findings for some of the major parameters. This is done in Table 3, where comparisons are made with five recent investigations.

5.2. Residuals within the Fitting Data Set

Figure 5 shows the IPD model brightness overlaid on the data for three pixels selected from the fitting data set used to optimize the model parameters. The zodiacal model intensity has been offset to match the mean observed pixel brightness, just as is done in order to fit the data. Within the error bars, the fit looks good on a pixel-by-pixel basis. Section 3 and Figure 3 provide an explanation of the shape of the intensity variation.

The character of the mean-adjusted residuals $((I_j - \langle I_j \rangle) - (Z_j - \langle Z_j \rangle))$ for the complete data set used to fit the time variation is illustrated in Figure 6. The histograms in Figures 6a–6b show that the overall means of the residuals from the fit are very close to zero within a roughly Gaussian distribution function of narrow width. A search for systematic trends in the residuals versus ecliptic latitude and solar elongation is presented for 1.25, 3.5, 25, and 100 $\mu$m in Figure 6. The 25 $\mu$m trend with solar elongation shows some systematic structure, which is also seen at 12 $\mu$m.

5.3. Residuals for a Weekly Map

The optimized DIRBE IPD model brightness was evaluated for each of the 41 weekly sky maps described in § 2 using the model parameters (Tables 1 and 2) and equation (1). Figure 7 illustrates the IPD modeling results for one of these weekly maps (DIRBE mission week 22, which consists of averaged observations over the interval 1990 April 16–22, inclusive). For each pixel, the model was evaluated for the same time as the average time of observation of that pixel during the week. The results for the 25 $\mu$m data are shown in order to illustrate most clearly deficiencies of the model. The observed sky brightness is presented in the top left panel of Figure 7, and the corresponding IPD model intensity for this week is shown in the center left panel on the same scale. The residual map that results from subtracting the IPD model from the data is provided in the bottom left panel of Figure 7 on a linear scale whose maximum is ~20 times fainter than that used to display the observed sky brightness. The brightest residual features are emission from the Galactic plane. There are, however, lower level systematic residuals that arise from imperfections in the zodiacal model. The most noticeable of these residual imperfections lie near the ecliptic plane and solar elongation extrema. An excess residual in the 12 and 25 $\mu$m maps at solar elongation extrema was also indicated in the results for the fitting data set described in § 5.2.

Individual contributions to the IPD model brightness from the smooth cloud, dust bands, and circumsolar ring components are given in the top, center, and bottom right panels of Figure 7, respectively. The intensity stretch for the center and bottom right panel is expanded ~20 times compared to the top right panel in order to show detail. The intensity from the circumsolar ring is asymmetric with respect to the Earth-leading and trailing halves of the sky; the brighter half corresponds to that containing the trailing “blob” (§ 4.2.3). Comparison of the residuals in Figure 7 (bottom left) with the individual component contributions in Figures 7 (top right) and 7 (bottom right) shows that some of the lower level structure in the residuals correlates with the dust bands and circumsolar ring intensity contours. There is
TABLE 1
DENSITY AND GEOMETRICAL PARAMETERS OF THE IPD MODEL

| Parameter          | Description          | Final Value   | Uncertainty   |
|--------------------|----------------------|---------------|---------------|
| Smooth Cloud (Widened Modified Fan) |                       |               |               |
| $n_0$ (AU$^{-1}$)  | Density at 1 AU      | $1.13 \times 10^{-7}$ | $6.4 \times 10^{-10}$ |
| $z$                | Radial power-law exponent | 1.34         | 0.022         |
| $\beta$            | Vertical shape parameter | 4.14         | 0.067         |
| $\gamma$           | Vertical power-law exponent | 0.942       | 0.025         |
| $\mu$              | Widening parameter   | 0.189        | 0.014         |
| $i$ (deg)          | Inclination          | 2.03         | 0.017         |
| $\Omega$ (deg)     | Ascending node       | 77.7         | 0.6           |
| $X_0$ (AU)         | x offset from Sun    | 0.0119       | 0.0011        |
| $Y_0$ (AU)         | y offset from Sun    | 0.00548      | 0.00077       |
| $Z_0$ (AU)         | z offset from Sun    | $-0.00215$   | 0.00043       |
| Dust Band 1        |                      |              |               |
| $n_{B1}$ (AU$^{-1}$) | Density at 3 AU      | $5.59 \times 10^{-10}$ | $7.20 \times 10^{-11}$ |
| $\delta_{B1}$ (deg) | Shape parameter      | 8.78         | Fixed         |
| $v_{B1}$           | Shape parameter      | 0.10         | Fixed         |
| $p_{B1}$           | Shape parameter      | 4            | Fixed         |
| $l_{B1}$ (deg)     | Inclination          | 0.56         | Fixed         |
| $\Omega_{B1}$ (deg)| Ascending node       | 80           | Fixed         |
| $\delta_{RB1}$ (AU)| Inner radial cutoff  | 1.5          | Fixed         |
| Dust Band 2        |                      |              |               |
| $n_{B2}$ (AU$^{-1}$) | Density at 3 AU      | $1.99 \times 10^{-9}$ | $1.28 \times 10^{-10}$ |
| $\delta_{B2}$ (deg) | Shape parameter      | 1.99         | Fixed         |
| $v_{B2}$           | Shape parameter      | 0.90         | Fixed         |
| $p_{B2}$           | Shape parameter      | 4            | Fixed         |
| $l_{B2}$ (deg)     | Inclination          | 1.2          | Fixed         |
| $\Omega_{B2}$ (deg)| Ascending node       | 30.3         | Fixed         |
| $\delta_{RB2}$ (AU)| Inner radial cutoff  | 0.94         | 0.025         |
| Dust Band 3        |                      |              |               |
| $n_{B3}$ (AU$^{-1}$) | Density at 3 AU      | $1.44 \times 10^{-10}$ | $2.34 \times 10^{-11}$ |
| $\delta_{B3}$ (deg) | Shape parameter      | 15.0         | Fixed         |
| $v_{B3}$           | Shape parameter      | 0.05         | Fixed         |
| $p_{B3}$           | Shape parameter      | 4            | Fixed         |
| $l_{B3}$ (deg)     | Inclination          | 0.8          | Fixed         |
| $\Omega_{B3}$ (deg)| Ascending node       | 80.0         | Fixed         |
| $\delta_{RB3}$ (AU)| Inner radial cutoff  | 1.5          | Fixed         |
| Solar Ring         |                      |              |               |
| $n_{SR}$ (AU$^{-1}$) | Density at 1 AU      | $1.83 \times 10^{-8}$ | $1.27 \times 10^{-9}$ |
| $R_{SR}$ (AU)      | Radius of peak density| 1.03        | 0.00016       |
| $\sigma_{SR}$ (AU) | Radial dispersion    | 0.025        | Fixed         |
| $\sigma_{zSR}$ (AU)| Vertical dispersion  | 0.054        | 0.0066        |
| $l_{SR}$ (deg)     | Inclination          | 0.49         | 0.063         |
| $\Omega_{SR}$ (deg)| Ascending node       | 22.3         | 0.0014        |
| Trailing Blob      |                      |              |               |
| $n_{TB}$ (AU$^{-1}$) | Density at 1 AU      | $1.9 \times 10^{-8}$ | $1.42 \times 10^{-9}$ |
| $R_{TB}$ (AU)      | Radius of peak density| 1.06        | 0.011         |
| $\sigma_{rTB}$ (AU)| Radial dispersion    | 0.10         | 0.0097        |
| $\sigma_{zTB}$ (AU)| Vertical dispersion  | 0.091        | 0.013         |
| $\theta_{TB}$ (deg) | Longitude with respect to Earth | $-10$ | Fixed |
| $\sigma_{\theta TB}$ (deg) | Longitude dispersion | 12.1 | 3.4 |

an ecliptic plane excess in the leading half of the week 22 residual map near elongation 80°. The similarity of this excess to the brightness profile of the ring indicates that there is an insufficient model contribution from the circum-solar ring.

5.4. Average Residuals

Mission-averaged maps of the sky with zodiacal light removed were formed by averaging together the available weekly residual maps, of which Figure 7 (bottom left) is one example. Because of the imperfections in the IPD model at extreme solar elongation angles described previously, a solar elongation constraint was implemented by which weekly averaged observations for which $e < 68^\circ$ and $e > 120^\circ$ were excluded from the average. In addition, data from three of the 41 weeks were excluded from the average for the 1.25–100 $\mu$m data because of low numbers of normal survey observations in those weeks (COBE DIRBE Explanatory Supplement 1997).

Figure 2 shows the observed mission-averaged sky
portions of the residual maps (Fig. 2 chosen to emphasize any visible defects in low-intensity Galaxy and any extragalactic light. Although the removal averaged residual maps would appear as clean images of the success of zodiacal light removal compared to the original data, in order to illustrate the brightness at all 10 DIRBE wavelengths, in addition to the mission-averaged maps after zodiacal light removal. Mission-averaged maps that have had the IPD signal removed are shown on two scales—the first on the same logarithmic scale as the measured data, in order to illustrate the removal of the IPD signal were perfect, the mission-averaged maps after zodiacal light removal. In general, these features are relatively faint. For example, the 12 μm excess near the ecliptic plane is of order 2% of the zodiacal signal in the near- and far-IR looks relatively free of defects even on the expanded linear scale of Figure 2c, there are clearly systematic defects present in the 4.9–60 μm residual maps. These wavelengths present the most rigorous test of the IPD model, since it is at these wavelengths that the zodiacal foreground is strongest. In general, there are systematic residual features parallel to the ecliptic plane and within about 15° of it at these wavelengths. Such features presumably arise from deficiencies in the model. In general, these features are relatively faint. For example, the 12 μm excess near the ecliptic plane is of order 2% of the zodiacal signal.

### TABLE 2

**SOURCE FUNCTION PARAMETERS OF THE IPD MODEL**

| Parameter                  | Description                       | Final Value | 68% Joint Confidence Uncertainty |
|----------------------------|-----------------------------------|-------------|---------------------------------|
| $T_0$ (K)                  | Temperature at 1 AU               | 286         | Fixed                           |
| $\delta$                   | Temperature power-law exponent    | 0.467       | 0.0041                          |
| $C_{0,1}$ (sr$^{-1}$)      | 1.25 μm phase function parameter | $-0.942$    | Fixed                           |
| $C_{1,1}$ (rad$^{-1}$ sr$^{-1}$) | 1.25 μm phase function parameter | 0.121       | Fixed                           |
| $C_{2,1}$ (rad$^{-1}$)     | 1.25 μm phase function parameter | $-0.165$    | Fixed                           |
| $C_{0,2}$ (sr$^{-1}$)      | 2.2 μm phase function parameter  | $-0.527$    | Fixed                           |
| $C_{1,2}$ (rad$^{-1}$ sr$^{-1}$) | 2.2 μm phase function parameter | 0.187       | Fixed                           |
| $C_{2,2}$ (rad$^{-1}$)     | 2.2 μm phase function parameter  | $-0.598$    | Fixed                           |
| $C_{0,3}$ (sr$^{-1}$)      | 3.5 μm phase function parameter  | $-0.431$    | Fixed                           |
| $C_{1,3}$ (rad$^{-1}$ sr$^{-1}$) | 3.5 μm phase function parameter | 0.172       | Fixed                           |
| $C_{2,3}$ (rad$^{-1}$)     | 3.5 μm phase function parameter  | $-0.633$    | Fixed                           |

**All Cloud Components**

| Parameter                  | Description                       | Final Value | 68% Joint Confidence Uncertainty |
|----------------------------|-----------------------------------|-------------|---------------------------------|
| $A_1$                      | Albedo at 1.25 μm                 | 0.204       | 0.0013                          |
| $A_2$                      | Albedo at 2.2 μm                  | 0.255       | 0.0017                          |
| $A_3$                      | Albedo at 3.5 μm                  | 0.210       | 0.019                           |
| $E_{10}$                   | Emissivity modification factor at 3.5 μm | 1.66       | 0.088                           |
| $E_{20}$                   | Emissivity modification factor at 4.9 μm | 0.997      | 0.0036                          |
| $E_{30}$                   | Emissivity modification factor at 10 μm | 0.958      | 0.0026                          |
| $E_{40}$                   | Emissivity modification factor at 25 μm | 1.00       | Fixed                           |
| $E_{50}$                   | Emissivity modification factor at 60 μm | 0.733      | 0.0055                          |
| $E_{60}$                   | Emissivity modification factor at 100 μm | 0.647     | 0.012                           |
| $E_{70}$                   | Emissivity modification factor at 140 μm | 0.677     | ...                             |
| $E_{10}$                   | Emissivity modification factor at 240 μm | 0.519  | ...                             |

**Smooth Cloud**

| Parameter                  | Description                       | Final Value | 68% Joint Confidence Uncertainty |
|----------------------------|-----------------------------------|-------------|---------------------------------|
| $A_1$                      | Albedo at 1.25 μm                 | 0.204       | Fixed to smooth cloud           |
| $A_2$                      | Albedo at 2.2 μm                  | 0.255       | Fixed to smooth cloud           |
| $A_3$                      | Albedo at 3.5 μm                  | 0.210       | Fixed to smooth cloud           |
| $E_{10}$                   | Emissivity modification factor at 3.5 μm | 1.66       | Fixed to smooth cloud           |
| $E_{20}$                   | Emissivity modification factor at 4.9 μm | 0.359     | 0.054                           |
| $E_{30}$                   | Emissivity modification factor at 12 μm | 1.01       | 0.15                            |
| $E_{40}$                   | Emissivity modification factor at 25 μm | 1.00       | Fixed                           |
| $E_{50}$                   | Emissivity modification factor at 60 μm | 1.25       | 0.30                            |
| $E_{60}$                   | Emissivity modification factor at 100 μm | 1.52      | 0.65                            |
| $E_{70}$                   | Emissivity modification factor at 140 μm | 1.13      | ...                             |
| $E_{10}$                   | Emissivity modification factor at 240 μm | 1.40      | ...                             |

**Dust Bands**

| Parameter                  | Description                       | Final Value | 68% Joint Confidence Uncertainty |
|----------------------------|-----------------------------------|-------------|---------------------------------|
| $A_1$                      | Albedo at 1.25 μm                 | 0.204       | Fixed to smooth cloud           |
| $A_2$                      | Albedo at 2.2 μm                  | 0.255       | Fixed to smooth cloud           |
| $A_3$                      | Albedo at 3.5 μm                  | 0.210       | Fixed to smooth cloud           |
| $E_{10}$                   | Emissivity modification factor at 3.5 μm | 1.66       | Fixed to smooth cloud           |
| $E_{20}$                   | Emissivity modification factor at 4.9 μm | 1.06       | 0.0089                          |
| $E_{30}$                   | Emissivity modification factor at 12 μm | 1.06       | 0.00078                         |
| $E_{40}$                   | Emissivity modification factor at 25 μm | 1.00       | Fixed                           |
| $E_{50}$                   | Emissivity modification factor at 60 μm | 0.873     | 0.0042                          |
| $E_{60}$                   | Emissivity modification factor at 100 μm | 1.10      | $7.5 \times 10^{-6}$           |
| $E_{70}$                   | Emissivity modification factor at 140 μm | 1.15      | ...                             |
| $E_{10}$                   | Emissivity modification factor at 240 μm | 0.858  | ...                             |

**Smooth Cloud**

| Parameter                  | Description                       | Final Value | 68% Joint Confidence Uncertainty |
|----------------------------|-----------------------------------|-------------|---------------------------------|
| $A_1$                      | Albedo at 1.25 μm                 | 0.204       | Fixed to smooth cloud           |
| $A_2$                      | Albedo at 2.2 μm                  | 0.255       | Fixed to smooth cloud           |
| $A_3$                      | Albedo at 3.5 μm                  | 0.210       | Fixed to smooth cloud           |
| $E_{10}$                   | Emissivity modification factor at 3.5 μm | 1.66       | Fixed to smooth cloud           |
| $E_{20}$                   | Emissivity modification factor at 4.9 μm | 1.06       | 0.0089                          |
| $E_{30}$                   | Emissivity modification factor at 12 μm | 1.06       | 0.00078                         |
| $E_{40}$                   | Emissivity modification factor at 25 μm | 1.00       | Fixed                           |
| $E_{50}$                   | Emissivity modification factor at 60 μm | 0.873     | 0.0042                          |
| $E_{60}$                   | Emissivity modification factor at 100 μm | 1.10      | $7.5 \times 10^{-6}$           |
| $E_{70}$                   | Emissivity modification factor at 140 μm | 1.15      | ...                             |
| $E_{10}$                   | Emissivity modification factor at 240 μm | 0.858  | ...                             |
TABLE 3

COMPARISON OF RECENT IPD MODELING PRIME PARAMETERS

| Quantity                              | Murdock & Price (1985) | Deul & Wolstencroft (1988) | Rowan-Price et al. (1990) | Wheelock et al. (1994) | Jones & Rowan-Robinson (1993) | This Investigation (1998) |
|---------------------------------------|------------------------|----------------------------|---------------------------|------------------------|-------------------------------|--------------------------|
| Database                              | ZIP                    | IRAS                       | IRAS                      | IRAS                   | IRAS                          | DIRBE                    |
| Main cloud line of nodes (deg)        | 77–110°                | 70                         | 79                        | 69                     | 76                            | 77.7                      |
| Main cloud inclination (deg)          |                        |                            |                           |                        |                               |                          |
| Radial density exponent               | 1.6–3.0°               | 2.3                        | 1.1                       | 1.73                   | 1.35                          | 2.03                      |
| Number of bands                       | 0                      |                            | 1                         |                        |                               |                          |
| Circumsolar ring?                     | N                      |                            | N                         | N                      | Y                             |                          |
| $T$ in Kelvin at 1 AU                 | 280°                   | 238                        | 255                       | 266                    | 255                           | 286                      |
| Temperature law exponent              |                        |                            |                           |                        |                               |                          |

*a* Results insensitive to the value within the range shown.

*b* Included a "small grain" component that can be considered compensating for the contribution from the asteroidal bands and/or main-belt dust.

*c* Included a "broad band" component that could be compensating for the ring and/or main-belt asteroidal dust.

*d* The effective color temperature.

5.5. **Contribution of IPD Signal to Total Sky Brightness**

The IPD model provides quantitative estimates of the contribution of the zodiacal foreground to the observed sky brightness at each of the 10 DIRBE wavelengths. Figures 8 and 9 illustrate the observed sky intensity and corresponding IPD model intensity for representative directions in the sky. For regions not dominated by strong Galactic sources, the estimated fraction of the total sky brightness due to zodiacal light varies from about $\frac{1}{2}$ in the near-IR (1.25–3.5 μm) to more than 90% in the mid-IR (4.9–60 μm). Even in the far-IR, the zodiacal contribution is not completely negligible at high Galactic latitudes: ~20% of the 240 μm emission in the Lockman Hole (direction of minimum H I column density, Lockman, Jahoda, & McCammon 1986) can be attributed to zodiacal emission, as shown in Figure 9 (*bottom*). The values used to make the plots in Figure 9 are also presented in Table 4, with further details about the locations in the figure legend.

![Figure 5](image-url)  

*Fig. 5.*—Sky brightness vs. time as observed by DIRBE for four different wave bands and three different sky locations. *Left:* The north ecliptic pole. *Center:* The north Galactic pole. *Right:* The ecliptic plane (near ecliptic longitude 180°). Error bars include random and systematic uncertainties. Smooth curve through the data is that generated from the IPD model, with an offset added to raise the zodiacal contribution to the mean of the observed pixel photometry.
Fig. 6.—Overall character of residuals in the fitting data set for four representative wavelengths: (a) Left: 1.25 μm, right: 3.5 μm. (b) Left: 25 μm, right: 100 μm. For each wavelength band, the histogram of residuals, residuals as function of ecliptic latitude, and residuals as function of solar elongation are plotted from bottom to top.
Fig. 7.—Data, IPD model and residual maps for week 22 (90106-90112) at 25 μm. Top left: The as observed sky. The scale is linear: [0, 100] MJy sr$^{-1}$. Center left: Sky brightness for this week as predicted by the IPD model, on the same scale as the top left panel. The IPD model intensities shown here are broken down into the brightness due to the smooth cloud, bands, and ring in the top, center, and bottom right panels, respectively. Bottom left: The residual map (observed − model), on a linear scale [0, 5] MJy sr$^{-1}$. Top right: Brightness for the smooth cloud model component, on the same scale as the top left panel. Center right: Brightness for the combined bands components, scale = [0, 5.5]. Bottom right: Brightness for the ring model component, scale = [0, 5.5]. All maps are Mollweide projections in ecliptic coordinates.
**TABLE 4**

IPD Model Component Intensities $I_r$ (MJy sr$^{-1}$) at Two Selected Locations and Times (see Fig. 9)

| Position ($\lambda, \beta$) | 1990 Day | IPD Component | Wavelength |
|-----------------------------|----------|---------------|------------|
|                             |          |               | 1.25 $\mu$m | 2.2 $\mu$m | 3.5 $\mu$m | 4.9 $\mu$m | 12 $\mu$m | 25 $\mu$m | 60 $\mu$m | 100 $\mu$m | 140 $\mu$m | 240 $\mu$m |
| (122°, 0°)$^a$ .......... | 109      | Smooth Cloud  | 0.307      | 0.196      | 0.133      | 0.679      | 28.476   | 58.063    | 19.155    | 6.909      | 3.346      | 1.066      |
|                             |          | Dust Bands    | 0.0210     | 0.0132     | 0.00837    | 0.0141     | 1.938    | 3.992     | 2.440     | 1.232      | 0.430      | 0.222      |
|                             |          | Ring + Blob   | 0.0284     | 0.0189     | 0.0164     | 0.109      | 3.324    | 5.120     | 1.618     | 0.779      | 0.368      | 0.122      |
|                             |          | Total ZL      | 0.354      | 0.229      | 0.156      | 0.808      | 33.875   | 67.232    | 23.214    | 8.993      | 4.152      | 1.404      |
| (137°, 46°)$^b$ .......... | 129      | Smooth Cloud  | 0.144      | 0.0951     | 0.0762     | 0.449      | 14.669   | 26.365    | 7.461     | 2.572      | 1.222      | 0.384      |
|                             |          | Dust Bands    | 0.000854   | 0.000574   | 0.000515   | 0.00114    | 0.0924   | 0.146     | 0.0655    | 0.0302     | 0.0101     | 0.00511    |
|                             |          | Ring + Blob   | 0.00637    | 0.00426    | 0.00379    | 0.0251     | 0.735    | 1.118     | 0.350     | 0.168      | 0.0795     | 0.0241     |
|                             |          | Total ZL      | 0.151      | 0.0998     | 0.0807     | 0.476      | 15.483   | 27.619    | 7.877     | 2.770      | 1.312      | 0.413      |

$^a$ Corresponds to DIRBE pixel 162811.

$^b$ Corresponds to DIRBE pixel 64552.

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**Fig. 8a**

IPD contribution to the observed sky brightness. The lower curve in each plot is the ZL brightness for the time and locations computed using the DIRBE IPD model. (a) DIRBE observations of the infrared sky brightness on 1990 January 19 at solar elongation 90°, ecliptic longitude 179°. The 140 and 240 $\mu$m data have been averaged and smoothed. (b) Mission-averaged intensity profile in the Galactic plane.
Quantitative uncertainties associated with the IPD model are discussed in the following sections. These include both the errors associated with fitting the observed time dependence with the chosen model, as well as uncertainty in the true IPD brightness resulting from the fact that other model formulations might fit the time dependence of the observations equally well. This issue of model "nonuniqueness" is of critical importance for cosmological studies such as the search for the CIB. As a simple illustration, the zodiacal brightness at high ecliptic latitudes is influenced at the few percent level by the modeled contribution of the near-Earth IPD structures. Changing the formulation for the 10° asteroidal bands so that they migrate in completely toward the Sun, rather than having an inner cutoff radius as in our model, increases the contribution for the dust bands shown in the bottom panel of Figure 9 by a factor of 5 at high ecliptic latitude. A change in the band inner cutoff has little effect on the model at low ecliptic latitudes. Changes in the residual sky brightness resulting from different inner radial cutoffs for the bands are within the uncertainties of the isotropic components of the IPD model discussed in § 6.

5.6. Consistency of Model as a Function of Time

It is difficult to specify precisely the errors inherent in our modeling technique of fitting the time variations of the sky brightness. This is particularly true because the level and nature of the errors depend both on epoch and spatial location. To give some insight on this point, Figure 10 presents maps of the 12 μm sky brightness in ecliptic coordinates for four weeks during the mission. These weeks have been selected so that two of the maps were obtained approximately half a year after one of the earlier maps. Data at 12 μm were used because the IPD signal is the predominant signal in the sky and because all features in the model are evident in the data. These specific images are "constant-time maps" created by interpolating linearly between contiguous weekly averaged maps (in which the mean time of observation of each pixel is not a constant, but depends upon the times in which it was scanned during that week). In "constant-time" maps the brightness at each pixel rep-
resresents a view of the IPD cloud at a fixed time, thus eliminating the small brightness fluctuations in our weekly sky maps arising from different mean observation times per pixel.

These constant-time maps have been used to create Figure 11, which shows the variation of the brightness of the sky, the IPD model, and the residuals (difference between measured brightness and model) as a function of ecliptic longitude for two representative latitudes. This figure also shows the average of the weekly residual results. Both the sky maps and line plots disclose much about the nature of the modeling errors.

Comparing the maps separated by half a year shows that the residuals in the ecliptic polar regions are quite comparable, but there are clear indications of variable errors in the vicinity of the ecliptic plane. Note in particular the large-scale inversions of the strength of the residuals between maps separated by half a year in the center portion of the maps. This results mainly from an inadequate representation of the circumsolar ring and the Earth-trailing blob. Errors resulting from deficiencies in the model of the dust bands are also visible. Although the errors can be easily seen in these maps, it is essential to realize that these obvious artifacts result from modeling errors that are only a few percent of the intensity of the zodiacal emissions. But the nature of the errors is complex; some features of this complexity are illuminated by the latitude-cut line plots of Figure 11.

In the plots for $\beta = 80^\circ$, it is seen that the longitudinal dependence of the zodiacal emission is seemingly very well tracked by the model. But there is a noticeable phase shift between the observations and the model. This phase difference is vividly translated into a wide separation of residuals over a half-year interval. Although not shown graphically, for weeks 6 and 32, this separation reaches a maximum around longitude 180°, whereas for weeks 12 and 38 the peak separation appears around longitude 240°. The cause of this temporal variation in the phase shift between the observations and the model is not known.

The model does reproduce well the striking differences in the amplitude of the time variation of the IPD signal in different directions. For example, in the data of weeks 6 and 32, the variation in the sky at a latitude of 80° for longitude of 100° is $\sim 0.1$ MJy sr$^{-1}$. This is in contrast to the observations at 270° where the variation is $\sim 3.7$ MJy sr$^{-1}$. The range of temporal variation at high latitudes is strongly dependent on longitude. That the model captures this distinctive feature of the temporal variation indicates that the basic geometry of the IPD cloud is represented reasonably well in the model. A comparison of the weekly results with the mission-averaged values indicates that the error in the modeled zodiacal contribution ranges from 0% to 4% of the IPD modeled brightness, implying an error in the average residual ranging from about 0% to 42%.

The plots for $\beta = 0^\circ$ demonstrate what can be seen in the corresponding full-sky maps—there are strong anti-correlated variations in the residuals separated by half a year. All elements of the model, i.e., main cloud, circumsolar ring, and bands, combine to produce these effects. Although the residuals behave quite differently in the two halves of the year, it should be noted that the differences in the residuals only represent a modest lack of fidelity in the model, ranging from about 0% to 3.8% of the IPD brightness in a pixel.

The characteristics of the DIRBE data set that make it well suited for modeling of the IPD signal based upon the observed time variation of the sky brightness (highly repetitive sampling, stable photometry) also permit some unique analyses of the global deficiencies in the modeling. One of the best methods for discerning the general nature of the model defects over the whole sky comes from an analysis of the weekly residuals maps. As noted above, a grand residual sky map was produced at each wavelength by subtracting the weekly map calculated from the IPD model from the map of weekly averaged observations and then averaging these weekly residual maps over all the weeks of the mission as described in § 5.4. Using the average residual map as a fiducial measure, a distribution function representing the quality of each weekly residual map was constructed by forming the difference between the weekly residual map and the mission-averaged residual map, expressed as a fraction of the IPD model brightness for that week:

$$\langle [I_j(w, p) - Z_j(w, p)] - \langle I_j(w, p) \rangle - Z_j(w, p) \rangle_w / Z_j(w, p), \quad (11)$$
Fig. 10.—DIRBE 12 μm (data model) residual maps for mission week pairs spaced 6 months apart: Weeks 6 (top left) and 32 (bottom left) correspond to midweek observations times of 1989 December 28 and 1990 June 28, respectively, and are displayed on a linear stretch of [0.456, 3.511] MJy sr$^{-1}$. Similarly, weeks 12 (top right) and 38 (bottom right) correspond to 1990 February 8 and 1990 August 9, respectively, and the scale = [0.439, 3.987] MJy sr$^{-1}$. Maps are Mollweide projections in ecliptic coordinates.
Fig. 11a

Fig. 11b

Fig. 11.—The 12 μm intensity profiles as a function of ecliptic longitude for ecliptic latitudes of 80° (a) and 0° (b). For each latitude the top two panels show the sky data for two weeks separated by 26 weeks, and the next two panels show the corresponding data from the IPD model. The bottom panel shows an overlay of the differences of the sky and IPD model for each of the two weeks, as well as the run of the mission-averaged residual.
where this expression is evaluated for each visible pixel $p$ in the weekly map. If the modeling were perfect, the distribution functions of these weekly pixel differences would be Gaussians with mean zero.

Using again the $12 \mu m$ data for illustration, Figure 12 shows two examples of such distribution functions for mission weeks 12 and 38. These distribution functions are created after removing the bottom and top 0.5% of the data to remove the effects of discrepant outliers. The number of pixels contributing to such distribution functions is a bit more than those in half the sky, i.e., about 205,000 per week. As can be seen in Figure 12, the distribution for week 12 is not Gaussian, whereas the distribution for week 38 is nearly Gaussian. These two examples also illustrate that the means of the distribution functions are not stable with time. This instability in the means is shown in Figure 13, where the time variation of the means of the weekly distribution functions are shown for the 1.25, 3.5, 12, and $240 \mu m$ data. A five-term harmonic fit to the means with a 1 yr period is also shown. The use of a 1 yr fitting period is based on the conclusion that one problem with the model arises from assuming the circumsolar ring to be circular, whereas the orbit of the Earth about the Sun is eccentric. It is interesting to note that the phase of the fit for the $12 \mu m$ data is almost $180^\circ$ out of phase with those for the other wavelengths. It is believed that this could be caused by the strong dependence of the $12 \mu m$ result on the assumed model structure for the circumsolar ring and trailing blob.

The plots in Figure 13 also show a high-frequency variation of the means, with a period of the order of 27 days and an amplitude of 1%–2%. Such variations are not seen in the photometry of the discrete standard objects used to stabilize our photometric system on long timescales, so they are not artifacts arising from the calibration process. Such variations are also seen in the average behavior of the deviations of subsets of isolated pixels relative to harmonic fits of the observations, i.e., it is not a phenomenon arising only in some special direction(s) in the sky or from the modeling of the IPD cloud. It is most strongly seen in the $4.9 \mu m$ data. The approximate 27 day period suggests a cause related to the Moon or to solar rotation. Since the Moon crosses the DIRBE scan swath twice per lunar month, any lunar effect would be expected to have a 14 day period. Furthermore, the DIRBE off-axis response was measured in flight and found to be orders of magnitude less at all angular distances from the Moon retained in the DIRBE data analysis than the residual effects seen in Figure 13. A more likely cause is therefore solar modulation of the zodiacal light brightness due to solar rotation and variation of the UV flux associated with sunspots. This is supported by the fact that the variations are correlated with the Mg $\Pi$ absorption strength. A similar UV-flux variation periodicity is seen in

![Figure 12](image_url)

**Fig. 12.**—Representative distribution functions of the ratio $(I_j(w, p) - Z_j(w, p)) / Z_j(w, p)$ where $\langle I_j(w, p) - Z_j(w, p) \rangle_w$ is the mission-averaged value. **Left:** Result for mission week 12. **Right:** The same result for week 38; i.e., one displaced by half a year where the expectation would be for similar results, which clearly are not obtained.
Fig. 13.—Samples of the run of the full-sky mean for the \([I_j(w, p) - Z_j(w, p)] - <I_j(w, p) - Z_j(w, p)>_w/Z_j(w, p)\) ratio as a function of time for the (a) 1.25, (b) 3.5, (c) 12, and (d) 240 \(\mu m\) bands.

the sky Ly\(\alpha\) background (Quémerais, Sandel, & de Toma 1996). A more thorough discussion of this discovery will be presented in a separate paper (Kelsall et al. 1998). Although low- and high-frequency variations are evident in the residuals, these effects are small, and the modeling results at all wavelengths reproduce the mean behavior of the observed sky to within a few percent of the level of the IPD contribution. Table 5 summarizes the deviations of the.

| Region                  | Band (\(\mu m\)) | \(\langle\text{Mean}\rangle\) (%) | \(\langle\text{Mean}\rangle\) (%) | \(\langle\text{Median}\rangle\) (%) |
|-------------------------|------------------|---------------------------------|---------------------------------|---------------------------------|
| Full sky…………………   | 1.25             | 0.07 ± 1.76                    | 1.49 ± 1.77                     | 0.11 ± 1.27                     |
|                         | 2.2              | 0.04 ± 0.92                    | 0.73 ± 0.92                     | 0.09 ± 0.60                     |
|                         | 3.5              | -0.11 ± 1.59                   | 1.26 ± 1.59                     | -0.04 ± 1.09                    |
|                         | 4.9              | 0.35 ± 1.07                    | 0.88 ± 1.12                     | 0.17 ± 0.92                     |
|                         | 12               | 0.30 ± 0.68                    | 0.57 ± 0.74                     | 0.17 ± 0.71                     |
|                         | 25               | 0.20 ± 0.30                    | 0.28 ± 0.36                     | 0.11 ± 0.30                     |
|                         | 60               | 0.10 ± 1.25                    | 0.99 ± 1.26                     | 0.07 ± 1.05                     |
|                         | 100              | 0.52 ± 4.10                    | 2.74 ± 4.14                     | 0.03 ± 2.45                     |
|                         | 140              | 0.82 ± 12.85                   | 9.00 ± 12.88                    | 0.24 ± 10.11                    |
|                         | 240              | 0.77 ± 21.22                   | 16.6 ± 21.23                    | 1.23 ± 16.38                    |
| High-quality region B… | 1.25             | 1.34 ± 2.63                    | 1.41 ± 2.07                     | 0.99 ± 2.54                     |
|                         | 2.2              | 1.04 ± 1.44                    | 0.53 ± 0.91                     | 0.70 ± 1.47                     |
|                         | 3.5              | 1.64 ± 1.96                    | 0.88 ± 1.28                     | 0.46 ± 2.39                     |
|                         | 4.9              | 1.60 ± 1.70                    | 0.83 ± 1.37                     | 0.75 ± 2.21                     |
|                         | 12               | 0.84 ± 1.07                    | 0.37 ± 0.59                     | 0.68 ± 1.21                     |
|                         | 25               | 0.65 ± 0.82                    | 0.42 ± 0.55                     | 0.53 ± 0.94                     |
|                         | 60               | 0.75 ± 1.54                    | 0.75 ± 0.98                     | 0.38 ± 1.68                     |
|                         | 100              | 1.51 ± 2.87                    | 1.07 ± 1.68                     | 0.49 ± 3.16                     |
|                         | 140              | 4.90 ± 13.51                   | 6.29 ± 8.16                     | -0.54 ± 18.51                   |
|                         | 240              | 13.02 ± 31.31                  | 11.38 ± 15.70                   | -0.22 ± 40.55                   |
weekly residual sky maps from the mission-averaged residual sky maps, as illustrated in Figure 12. Table 5 also shows similar statistics for the high-quality region B used in our search for the CIB (Paper III).

6. UNCERTAINTIES IN COSMOLOGICAL RESULTS

The uncertainties and errors in the model of the IPD thermal emission and scattered light can be characterized in many different ways; e.g., measurements of residual variations as a function of time, or amount of residual emission that is correlated with components of the IPD model. The errors evident as temporal variations of the residual emission are reduced by averaging the data over time for the length of the DIRBE cold-era mission. Model errors that are apparent as residual structure in the maps can often be minimized by selection of "good" regions of the maps (typically at high latitude) for use in further analyses. However, since we ultimately seek to determine the level of an isotropic CIB, the most significant form of uncertainty is that which affects the amount of emission from any potential isotropic or nearly isotropic IPD component. This is an issue of model uniqueness: Are there any other models that can give an equally good fit to the apparent time variations of the IPD emission and scattered light and yet lead to substantially different isotropic residuals?

To evaluate the uncertainty of a CIB measurement due to the uncertainty in the isotropic or nearly isotropic components of the IPD model, we attempted to fit the IPD cloud using several different functional forms (kernels) for the density distribution of the main IPD cloud. The intensity differences between models that fit the time variations equally well are taken as a measure of the uncertainty in the IPD model.

For this analysis, the shape and position of the small-scale components of the IPD were fixed, except for the inclination and line of nodes of the 10° dust bands. Functional forms for the main cloud that were tested included the IRAS model (Wheelock et al. 1994), a modified fan model, an ellipsoid model, a sombrero model (Giese 1986), and a widened modified fan model. The IRAS and sombrero models fared distinctly worse at χ² = 2.562 and 3.187, respectively. The model intensities were evaluated at five high-latitude regions of interest for measurement of the CIB (Paper I): the north and south ecliptic poles, the north and south Galactic poles, and the Lockman Hole region, which contains the location of lowest Galactic H I column density. Among the three models with the lowest χ², the largest intensity differences were found at the NGP region. At wavelengths from 1.25–100 μm the ellipsoid model was brighter than the widened modified fan model, which was brighter than the modified fan model. Thus, we have taken the differences between the ellipsoid and the modified fan models at the NGP as an indication of the uncertainty in the IPD models. These uncertainties are listed in Table 7. The uncertainties at 140 and 240 μm were obtained by scaling the 100 μm uncertainty by the mean color of the IPD emission at 100 μm relative to the emission at 140 and 240 μm, respectively.

By choosing the largest variations among good IPD models at high latitudes, we may be overestimating the uncertainty of the IPD model at a typical high-latitude location. However, we cannot be certain that there are not other models of the IPD cloud that can fit the time variation of the data, as well as the models we have investigated, and yet lead to larger differences in the predicted model intensity. Most important, the existence of a real isotropic component in the IPD contribution to the sky brightness that is not represented in any of these semiempirical models cannot be discerned directly by fitting the time variation in the DIRBE data. Independent arguments limiting the likely intensity of such a component are made in Paper IV and summarized in Paper I.

An additional systematic error that needs to be recognized arises from the fact that, as noted in § 5.6, the weekly mean deviations of the residual sky maps (after the modeled IPD contribution is removed) from the mission-averaged residual map show a strong harmonic feature with a period of 1 yr. Since the DIRBE observations did not cover a full year, averaging this harmonic variation over an incomplete cycle introduces a small systematic "truncation" error. These errors have been computed for both the whole sky and a smaller region used for the analysis in Paper I. Since the main cosmological results of Paper I are based upon

### Table 6

| Wavelength (μm) | Present Model | Modified Fan Model | Widened Modified Fan Model | Ellipsoidal Model |
|-----------------|--------------|-------------------|---------------------------|------------------|
| 1.25            | 1.00         | 1.05              | 1.06                      | 1.08             |
| 2.2             | 1.00         | 1.11              | 1.12                      | 1.14             |
| 3.5             | 1.00         | 1.00              | 1.00                      | 1.03             |
| 4.9             | 1.00         | 1.01              | 1.01                      | 1.03             |
| 12              | 1.00         | 0.98              | 0.98                      | 1.00             |
| 25              | 1.00         | 0.97              | 0.98                      | 1.01             |
| 60              | 1.00         | 0.94              | 0.95                      | 0.99             |
| 100             | 1.00         | 0.93              | 0.95                      | 1.00             |

### Table 7

| Wavelength (μm) | Uncertainty (nW m⁻² sr⁻¹) |
|-----------------|---------------------------|
| 1.25            | 15                        |
| 2.2             | 6                         |
| 3.5             | 2.1                       |
| 4.9             | 5.9                       |
| 12              | 138                       |
| 25              | 156                       |
| 60              | 26.7                      |
| 100             | 6.3                       |
| 140             | 2.3                       |
| 240             | 0.5                       |
small selected areas of the sky, it is important to check whether the magnitude of the error seen for the whole sky applies to these selected areas. A study of the distribution functions of the deviations of the weekly residuals was performed for the high-quality region B (HQB), which contains two patches at high northern and southern Galactic and ecliptic latitudes. The variation of the means of these distribution functions is similar to that for the whole sky. The grand average of these weekly means has a truncation error of about 1% or less of the IPD contribution, except at 240 μm, as shown in Table 8. The truncation error has a generally small impact on the overall uncertainties of the IPD model at every DIRBE wavelength.

7. CONCLUSIONS

This paper describes in some detail the construction of a physically motivated, parametric model of the IPD cloud contribution to the infrared sky brightness using the most distinctive and only unique feature of that contribution: its apparent temporal variation in fixed celestial directions induced by the orbiting of the Earth about the Sun. The effort to arrive at a model satisfactory for our search for the cosmic infrared background (CIB) turned out to be more arduous than expected, since the data revealed the complex cloud structures not known when the DIRBE investigation was planned. The model presented here is the survivor from a large number of parameterizations and fitting procedures that we have investigated. This extensive effort, although time consuming, was essential since it was driven by the desire to achieve an ~1% precision in the identification of the IPD signal. It is critical to note that “precision” is the correct word, for although the model well represents the time variations, it is not a unique model. It is, in fact, impossible to determine a unique model from any set of data taken from within the IPD cloud. Only a mission flying well beyond the orbit of Jupiter could gather data permitting a unique solution.

Although not unique, the model described here is of rather high fidelity, in that it reproduces the apparent time variations over the sky and leaves quite stable residual maps at all wavelengths. The range of functional forms investigated allows an estimate of the uncertainties due to the lack of uniqueness of the final model. The modest size of these uncertainties has allowed successful conduct of our search for the CIB (Paper I). The resulting residual maps with the IPD contribution removed are the best all-sky absolutely calibrated photometric maps now available for study of large-scale Galactic phenomena from 1.25 to 240 μm, although evident artifacts do remain, especially at low ecliptic latitude.

The virtues of our approach for finding the contribution of the IPD cloud to the infrared sky brightness include the following:

1. Modeling the time variations only is a unique method that does not require extraneous zero-point constants and automatically excludes contributions from Galactic or extragalactic sources.
2. Modeling that uses the data from many wavelengths simultaneously provides an IPD structure that is consistent with the data and the same for all wavelengths.
3. The IPD cloud density function kernel found by modeling the time variations produces a level for the IPD contributions over 2 orders of magnitude in wavelength that is compatible with expectations, i.e., that a positive sky residual remains after removing the modeled IPD contribution.
4. The isotropic part of the model is only modestly sensitive to the functional forms chosen to parameterize the model.
5. The method readily permits incorporation of all known elements of the IPD cloud: a main cloud, the asteroidal bands, and a circumsolar ring containing dust particles in Earth-resonant orbits near 1 AU.
6. The model yields a global precision of the order of a few percent of the IPD contribution.

Though our approach produced a successful model for the IPD cloud, it is also clear that it is not a perfectly consistent model for the whole sky. Testing the time stability of the weekly residual sky maps after subtracting the modeled IPD contribution is a powerful tool for assessing the quality of the model. The result of that assessment is that the model did achieve the goal of matching the time variations. There are, however, apparent spatial artifacts in the results that are correlated with the various components of the IPD cloud and quite clearly reflect imperfections in the model. On the basis of this work, we feel that further improvements could be made in constructing a quasiphysical model that, without recourse to elaborate representations of the dust, should be able to achieve further improvements in the results.

Some possible changes in the modeling technique, in rough order of perceived priority, are as follows:

1. Force the trailing blob in the circumsolar ring to trail the Earth in its eccentric orbit.
2. Include a separate contributing form factor for the dust coming from the main-belt asteroids.
3. Incorporate more realistic representations for the asteroidal bands and the circumsolar ring, where now, for example, the representations use simple Gaussian distributions.
4. Include a source function scattering term at 4.9 and 12 μm since a significant fraction of the IPD particles are large enough to scatter these wavelengths efficiently.
5. Recognize a possible warpage of the symmetry plane as a function of distance from the Sun due to the influences of Jupiter, Mars, Earth, and Venus.
6. Include an intrinsic time variation of the IPD bright-
ness with a period equal to that seen in the data.

7. Permit variation of the albedo for the shorter wavelength bands to accommodate the clues in the observations that point to a variation with latitude, which may well result from the differences in the dust contributed by comets as compared to that coming from asteroids.

8. Add a contribution from a hot dust component.

9. Incorporate the deviations from the grand average residuals directly into an iterative modeling loop so as to limit the available solution space and push the results toward distribution functions of residuals that are Gaussian.

The data described in this paper are available to the public from the NSSDC through the COBE web site at http://www.gsfc.nasa.gov/astro/cobe/. The DIRBE weekly maps of the sky and the corresponding IPD model brightness are available as the “DIRBE Sky and Zodi Atlas (DSZA)” product; the mission-averaged, zodi-subtracted residual skymaps are contained in the “Zodi-Subtracted Mission Average (ZSMA)” maps.

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