Distributed generation hosting capacity in electric distribution network in the presence of correlated uncertainties

Sajjad Solat1 | Farrokh Aminifar2 | Heidarali Shayanfar1

1 Center of Excellence for Power Systems Automation and Operation, School of Electrical Engineering, Iran University of Science and Technology, Tehran, Iran
2 School of Electrical and Computer Engineering, College of Engineering, University of Tehran, Tehran, Iran

Correspondence
Heidarali Shayanfar, Center of Excellence for Power Systems Automation and Operation, School of Electrical Engineering, Iran University of Science and Technology, Tehran 13114–16846, Iran.
Email: hashayanfar@iust.ac.ir

Abstract
High penetration of renewable distributed generations is one of the most current challenges of electric distribution networks due to resources uncertainties, and hosting capacity limitation of the networks. The challenges may be more critical when uncertainties are highly correlated. The effect of correlated uncertainties of wind speed and load on the distributed generation hosting capacity of distribution networks is evaluated. A combination of point estimation method, and inverse Nataf transformation is proposed for correlated uncertainties modelling. The efficiency of some ANM schemes on the hosting capacity improvement is also studied in the presence of correlated uncertainties. To do so, an optimisation framework with the objective of maximising the installed capacity of distributed generation subject to network operational constraints such as voltage deviation is proposed. The proposed optimisation problem is formulated in a mixed-integer quadratically constrained program form, and is solved via CPLEX solver. The proposed method is implemented on the 33-bus standard test system. The results demonstrate the significant effect of the correlation between uncertainties on the distributed generation hosting capacity of network.

1 | INTRODUCTION

The development of electric distribution networks (EDNs) is focused on customers in recent years, so that the behaviour of the customers is shifting from passive (only consumer) to active (producer and consumer or prosumer) [1]. In traditional EDNs, operators saw only the incremental load in front of themselves and their overall strategy was to manage production to meet the demand. Over time, with utilisation of the load management plans by EDN operators, attention was drawn to the potential value of network subscribers. Nowadays, by increasing environmental concerns, technology advances and rising the fossil fuel prices, use of generative (e.g. distributed generation [DG]) and non-generative (e.g. energy storage) resources has been expanded by EDN customers (prosumers) [1].

One of the latest changes in EDN is the presence of active subscribers, grid connected DGs at the top of them. Along with the valuable advantages of DGs for the operation of EDN, such as power loss minimisation [2], voltage profile improvement [2], reliability enhancement [3], etc., there are some challenges as well, for example, overvoltage [4] and feeder congestion [5]. So, the maximum allowed installation capacity of DGs in EDN without the network augmentation requirement, referred to as DG hosting capacity (DGHC), is limited [6].

DGHC indicates the potential of EDN for the gradual transformation to a more active EDN [7]. Along side operational aspects of EDN, the uncertainties affecting DGs output power should be taken into consideration for accurate DGHC evaluation. DGHC evaluation methods can be categorised into two groups: (i) deterministic [8, 9], and (ii) stochastic [10–12]. The basic idea of the deterministic methods is evaluation of DGHC either under the normal condition or the worst scenario; the results are hence unrealistic or conservative, respectively. These methods are however simple and easy to implement. The stochastic techniques, on the other hand, consider the uncertainties in DGHC evaluation. These techniques bring more realistic results but with higher computational burden. In this paper we focus on wind speed and load uncertainties and their correlation effect on DGHC; hence the stochastic approach is...
necessarily adopted. The most crucial point in stochastic approach is how the uncertainties are modelled.

Different techniques have been proposed for dealing with uncertainties. Three common techniques, as presented in [13], include possibilistic, probabilistic, and hybrid possibilistic and probabilistic. The backbone of possibilistic approaches is allocating appropriate fuzzy membership functions to input uncertain variables. Then, the membership functions of desired output variables are found using α-cut method. Finally a defuzzification strategy, usually centroid method, is used to defuzzify the output and obtain a crisp output value [13]. The idea of probabilistic approach is the allocation of appropriate probability distribution functions (PDFs) to input uncertain variables and then analysing the output using Monte Carlo simulation [14], scenario-based analysis [15], or point estimation methods (PEMs) [16]. When some input variables are modelled with known PDF while the others are modelled with fuzzy membership function, the hybrid approach is used. In this paper, the probabilistic approach is adopted and the uncertain variables include wind speed and load with Weibull and normal PDFs, respectively [13]. For the sake of analysis, PEM is preferred thanks to its low computation burden and high accuracy in the low number of uncertain variables.

Besides dealing with uncertainties, modelling the correlation between uncertain variables is of significant interest for more accurate analysis [17]. Reference [18] has developed a multi-objective optimal power flow model for DG planning considering the correlated uncertainties between wind speed, light intensity, and load demand using Spearman rank correlation coefficient. Reference [19] proposed a probabilistic small signal stability-constrained EDN reconfiguration model considering the correlated uncertainties in loads and DGs. The proposed method combines the Cholesky decomposition and PEM for correlation modelling. Reference [20] has investigated the optimal operation of distribution feeder reconfiguration strategy in the smart grids with high penetration of plug-in electric vehicles and correlated wind power generation. This reference uses unscented transformation for correlation modelling. A probabilistic computational model for correlated wind farms was developed using Copula theory in [21]. Due to inability of PEM to model the correlated uncertainties, some modification is essential to make it capable to capture them. In [19] and [22], the Cholesky decomposition was combined with PEM for modelling the correlated uncertainties.

Mathematically, the DGHC evaluation is an optimisation problem. A rich body of research has done on modelling of this problem. In [23], a multi-objective optimal power flow was used for DGHC evaluation from the DG investor and network operator points of view and the technique was applied for problem solving. A mixed-integer non-linear multi-period optimal power flow model was developed in [24] for DGHC evaluation and it was solved with simple branch and bound solver. Reference [25] proposed a mixed-integer second-order cone programming model for evaluation of DGHC and solved the model by MOSEK solver. A mixed-integer linear programming model was proposed in [26] to evaluate the DGHC and was solved with a novel heuristic method. Also many research have proposed the non-linear optimal power flow model for modelling the DGHC evaluation problem and solved it by meta-heuristic approaches such as genetic [27] and particle swarm optimisation [28] algorithms.

In addition to above mentioned perspectives, different approaches have been proposed for DGHC enhancement. Conductor reinforcement [29], using passive harmonic filter [30], FACTs devices installation [31], and energy storage integration [32] are among the traditional approaches for DGHC improvement. The implementation of traditional approaches are costly and controversial; especially when the main question comes up: who is responsible to pay the network reinforcement expenses for DGHC improvement? [10]. Static network reconfiguration (SNR) [24], power factor control (PFC) of DGs [24], dynamic network reconfiguration (DNR) [25], reverse power flow permission (RPFP) [33], and demand side management [23] are among the active network management (ANM) schemes for DGHC improvement. These methods are more worthwhile and cost-effective than traditional ones.

In spite of vast research activities to assess DGHC, to the best knowledge of the authors, the correlated uncertainties effect on DGHC has not been investigated so far. The main purpose of this paper is to fill this gap. To do so, a novel effective technique is proposed here by the help of which DGHC evaluation in the presence of uncertainties is conducted. Also some of the ANM schemes for DGHC improvement including SNR, DNR, PFC, and RPFP, are selected to investigate their efficiency in the presence of correlated uncertainties.

The major contributions are as follows:

- A mixed-integer quadratically constrained program (MIQCP) model is developed for DGHC evaluation of active EDN. The privilege of proposed model is its capability in consideration of different ANM schemes for DGHC evaluation.
- The effect of correlated uncertainties on DGHC in the presence of different ANM schemes including PFC, RPFP, SNR, and DNR is investigated.

The structure is as follows: the outline of the proposed method is expressed in Section 2. Section 3 describes the correlated uncertainties modelling technique based on the combination of PEM and inverse Nataf transformation. Mathematical formulation for DGHC evaluation is presented in Section 4. The implementation results of the proposed method are analysed in Section 5. Finally the paper is concluded in Section 6.

## 2 | PROPOSED METHOD OUTLINE

The outline of the proposed method is depicted in Figure 1. As can be seen, the proposed method has two phases. At the first, the correlated uncertainties including load and wind speed are modelled. The advantage of this research in this step is that it proposes a precise, fast and low computational approach for correlated uncertainties modelling. The output of first step is used as the input of the second phase. In the second phase, the DGHC evaluation model is formulated as a MIQCP model and solved using CPLEX solver. The superiority of this research in this step is that it proposes a simple model that captures
3 | CORRELATED UNCERTAINTIES MODELLING

Two types of correlation between uncertainties are investigated in this paper which include: (1) the correlation of wind speed profiles of two separated groups of wind farms located within EDN and (2) the correlation between each of these two wind speed profiles with load.

3.1 | Point estimation method

The PEM technique is a simple method with low computational burden for uncertainty modelling. PEM represents each uncertain variable with \( K \) points named concentrations [16]. For an input uncertain variable \( y_i \), the \( k \)th concentration \( \zeta_{i,k} \) is a pair of a location \( \mu_i \) which is described below and a weighting factor \( \omega_{i,k} \) which denotes the importance of \( y_i \) in output variable evaluation.

\[
y_{i,k} = \mu_i + \zeta_{i,k} \sigma_i, \quad i = 1, 2, \ldots, m; \quad k = 1, 2, \ldots, K.
\]  

(1)

Different versions of PEM have been developed. In this paper the 2\( m + 1 \) PEM [16] is used. Standard locations and weights in this version are calculated as follows:

\[
\zeta_{i,k} = \frac{\lambda_{i,3}}{2} + \frac{(-1)^{3-k}}{\lambda_{i,4} - \frac{3}{4} \lambda_{i,3}^2} \quad k = 1, 2 \& \zeta_{i,k} = 0,
\]  

(2)

\[
\omega_{i,k} = \frac{(-1)^{3-k}}{\zeta_{i,k}(\zeta_{i,1} - \zeta_{i,2})} \quad k = 1, 2,
\]  

(3)

\[
\omega_{i,3} = \frac{1}{m} - \frac{1}{\lambda_{i,4} - \lambda_{i,3}^2}.
\]  

(4)

According to (1)–(4), it is clear that the 2\( m + 1 \) PEM method estimates each uncertain variable \( y_j \) by three points, \( K = 3 \), that one of them is always equal to the mean value of variable \( \mu_i \). For statistical evaluation of variable \( \zeta_\rho \) \( \zeta_\rho = F(y) \) in the presence of \( m \) input uncertain variables \( y_j \) using 2\( m + 1 \) PEM, the below deterministic input vectors \( (Y_\rho) \) are used. So the function \( F \) has to be evaluated in 2\( m + 1 \) iterations.

\[
\{Y_\rho(i, k) = [\mu_1, \ldots, \mu_{i-1}, y_{i,k}, \ldots, \mu_m] \quad k = 1, 2; \quad b = 1, \ldots, 2m,
\]  

(5)

\[
\{Y_\rho(i, k) = [\mu_1, \mu_{i+1}, \ldots, \mu_m] \quad k = 3; \quad b = 2m + 1.
\]

The occurrence coefficients for \( b = 1 \) to \( 2m \) are calculated in (3). The equivalent occurrence coefficient for \( b = 2m + 1 \) is calculated as follows [16]:

\[
\omega_0 = \sum_{i=1}^{m} \omega_{i,3} = 1 - \sum_{i=1}^{m} \sum_{k=1}^{K} \omega_{i,k}(\zeta_{i,k})^2.
\]  

(6)

After calculation of all \( \zeta_{i,k} \), different statistical raw moments can be estimated as follows:

\[
E[\xi] \cong \sum_{i=1}^{m} \sum_{k=1}^{K} \omega_{i,k}(\zeta_{i,k})' - \omega_0 (\zeta_{i,1})'.
\]  

(7)

The major advantage of the PEM is its simplicity and low computation burden. But it has some limitations, one of them is that the output representative locations for any random variable approximately follow the normal PDF [34]; while, the input random variables may not necessarily follow this type of PDF. Therefore, it is required to map the representative locations from normal PDF to a desired space, for example Weibull space for wind speed. Another limitation is that, in the presence of more than one input random variable, the PEM estimates the locations for each random variable independent of the other. So this technique alone is not appropriate for modelling of correlated uncertainties. For this purpose in this paper, the inverse Nataf transformation is merged with PEM to make an effective tool for correlated uncertainties modelling.

3.2 | Nataf transformation

Nataf transformation is used when the input data are correlated and follow a specific (non-normal) marginal PDF and must be analysed in an uncorrelated normal space [35]. Suppose a set of \( m \) randomly distributed input variables \( Y = \{y_1, y_2, \ldots, y_i, \ldots, y_m\} \) with separate cumulative distribution function (CDF) for each variable, \( \Theta_j(y_j) \). The correlation matrix of input variables, \( C_j \), is assumed as follows:

\[
C_j = \begin{pmatrix}
\rho_{001} & \cdots & \rho_{01m} \\
\vdots & \ddots & \vdots \\
\rho_{0m1} & \cdots & \rho_{0mm}
\end{pmatrix},
\]  

(8)

where, \( \rho_{0ij} \) is the correlation coefficient between \( y_i \) and \( y_j \) variables. According to (9), the input matrix \( Y \) can be mapped
from random distribution space to matrix $W$ in standard normal distribution space with correlation matrix $C_w$.

$$
\begin{align}
\Gamma(w_i) &= \Theta_i(y_i), \\
w_i &= \Gamma^{-1}(\Theta_i(y_i)), \\
C_w &= \begin{pmatrix}
\rho_{11} & \cdots & \rho_{1m} \\
\vdots & \ddots & \vdots \\
\rho_{m1} & \cdots & \rho_{mm}
\end{pmatrix},
\end{align}
$$

(9)

(10)

$\rho_{ij}$ in matrix $C_w$ is calculated as follows:

$$
\rho_{ij} = \begin{cases} 
T(y_i, y_j, \rho_{0ij}) \rho_{0ij} & i \neq j \\
1 & i = j
\end{cases}
$$

(11)

$$
\gamma_i = \frac{\sigma_{y_i}}{\mu_{y_i}}.
$$

(12)

The function $T$ is an empirical relation and depends on primary PDF. So, according to the PDF used for modelling the uncertain input variables, the function $T$ varies. For example, if primary PDF is normal, $T$ is equal to 1 and if it is Weibull, $T$ is formulated as below [36]:

$$
T(y_i, y_j, \rho_{0ij}) = 1.063 - 0.004\rho_{0ij} - 0.2(y_i + y_j)
- 0.001\rho_{0ij}^2 + 0.337(y_i^2 + y_j^2) + 0.007\rho_{0ij}(y_i + y_j)
- 0.007y_j.
$$

(13)

Then, matrix $C_w$ is decomposed to a lower triangular matrix $L_0$ using Cholesky decomposition [37] as follows:

$$
C_w = L_0 L_0^T.
$$

(14)

Using $L_0$, the correlated standard normal distributed variables $W = \{w_1, ..., w_i, ..., w_m\}$ converts to an uncorrelated ones, $U$, preserving the previous PDF as follows:

$$
U = L_0^{-1} \times W.
$$

(15)

As discussed in the previous section, the representative locations obtained from the PEM are uncorrelated normal distributed data. So, the inverse Nataf transformation can be used to turn the PEM resultant representative locations as uncorrelated standard normal distributed variables $U$ to the correlated randomly distributed variables $Y$. As seen, the combination of PEM with inverse Nataf transformation can model the correlated uncertainties using the marginal PDF of each uncertain variable without the need for their joint PDF. It is worth noting that, each row of obtained matrix $Y_i$ is a vector that was discussed previously in Equation (5). So, using $m$ input uncertain variables, the number of rows of $Y$ (the number of scenarios that must be studied) is equal to $2^m + 1$. Also, the occurrence coefficients of each of these scenarios are the $\omega_{ik}$ calculated in (3) and (6).

### 3.3 Load and wind speed uncertainty modelling

As a common practice, active and reactive load demands uncertainty is modelled with normal PDF [38]:

$$
\begin{align}
\tilde{f}_{pd}(P_d) &= \frac{1}{\sigma_d \sqrt{2\pi}} \exp\left(\frac{-\left(P_d - \mu_d\right)^2}{2\sigma_d^2}\right).
\end{align}
$$

(16)

So, $\Gamma$ in (9) is replaced with normal CDF with known mean and standard deviation values. According to [39], the wind speed data can accurately be modelled with two parameters Weibull PDF as follows:

$$
\begin{align}
\tilde{f}_{wind}(ws) &= \frac{k}{\xi} \left(\frac{ws}{\xi}\right)^{k-1} \exp\left[-\left(\frac{ws}{\xi}\right)^k\right].
\end{align}
$$

(17)

So, $\Gamma$ in (9) is replaced with Weibull CDF with known scale and shape factors. After wind speed modelling, the wind speed energy converts to the electrical energy using wind turbine characteristic with equation (18) [40].

$$
\begin{align}
P(ws) &= \begin{cases} 
0 & \text{if } ws \leq ws_{lo}, \\
\frac{P_{rated} \times (ws-ws_{lo})}{(ws_{hi}-ws_{lo})} & \text{if } ws_{lo} \leq ws \leq ws_{hi}, \\
P_{rated} & \text{if } ws_{hi} \leq ws \leq ws_{hi}. 
\end{cases}
\end{align}
$$

(18)

### 4 PROBLEM FORMULATION FOR DGHC EVALUATION

The criterion of this paper in DGHC determination is the maximum installed capacity of DGs in EDN in such a way that the expected value of network active power loss in the presence of DGs does not exceed that of the basic network excluding DGs. The EDN technical constraints such as bus voltage and feeder congestion limitations are included as well. The proposed model for DGHC evaluation which considers four ANM schemes simultaneously including SNR, DNR, PFC and RPFP is formulated as an MIQCP model in the following.

#### 4.1 Objective

The objective function of DGHC evaluation problem is to maximise the summation of installed capacity of DGs at candidate buses $B_d$ for DG installation.

$$
\text{Maximise } \sum_{d \in B_d} PDG_d.
$$

(19)

#### 4.2 Basic constraints

The basic constraints include the power flow equations. In this paper, the conic form [41] of power flow equations due to its
convexity and accuracy is used. The active and reactive power balances at each bus of the network are emphasised in (20) and (21).

\[
PG_{a,b} + \sum_{(c,d) \in F^b} (P_{c,d} - r_{cd}u_{c,d}) = \sum_{(c,d) \in F^b} P_{c,d} + PD_{a,b} + g_a u_{a,b} \quad \forall (a,b) \in B \ & \ & \forall b \in H, \tag{20}
\]

\[
QG_{a,b} + \sum_{(c,d) \in F^b} (Q_{c,d} - x_{cd}u_{c,d}) = \sum_{(c,d) \in F^b} Q_{c,d} + QD_{a,b} + b_a u_{a,b} \quad \forall (a,b) \in B \ & \ & \forall b \in H. \tag{21}
\]

The expected value of the total active power loss considering all the possible scenarios, \( H \), with occurrence coefficient of \( \omega_j \), is calculated as:

\[
TL = \sum_{\forall b \in H} \sum_{\forall (a,b) \in F^b} r_{a,b} \times \Psi_{a,b} \times \omega_j \tag{29}
\]

It is worth noting that, inequality (27) is the relaxed form of non-linear equality \( \Psi_{a,b} = \frac{P_{a,b}^2 + Q_{a,b}^2}{v_{a,b}^2} \). One requirement of convex relaxation in (27) to obtain a convex MIQCP model, is that for the obtained solution of the relaxed problem, (27) should be binding (i.e. it should be equality in the optimal solution), otherwise, the obtained solution is not a feasible solution of the original non-convex model. Constraint (28) limits the capacity of branch \((a, b)\) to its thermal capacity \( \Psi_{a,b} \) if it is in service in scenario \( b \), \( \alpha_{a,b} - \beta_{a,b} = 1 \).

\[
\Psi_{a,b} \leq (\alpha_{a,b} - \beta_{a,b}) \times \bar{\Psi}_{a,b} \quad \forall (a,b) \in F^b \ & \ & \forall b \in H. \tag{28}
\]

The active (in service) branches of the network change in each scenario \( b \) due to network reconfiguration. According to (38) and (39), if there is a branch between two buses \( a \) and \( b \) in scenario \( b \), \((a,b) \in F^b \) and \( \alpha_{a,b} - \beta_{a,b} = 1 \), the voltage of these two buses is limited to Ohm’s law; otherwise, the constraint is relaxed.

\[
\begin{align*}
v_{a,b} & \leq O \times (1 - (\alpha_{a,b} - \beta_{a,b})) \\
\ & \ + r_a P_{a,b} + s_a Q_{a,b} \quad \forall (a,b) \in F^b \ & \ & \forall b \in H, \quad \tag{24}
\end{align*}
\]

\[
\begin{align*}
\ & \ + (r_a^2 + s_a^2) \Psi_{a,b} \quad \forall (a,b) \in F^b \ & \ & \forall b \in H,
\end{align*}
\]

\[
\begin{align*}
u_{a,b} & \geq O \times (1 - (\alpha_{a,b} - \beta_{a,b})) \\
\ & \ + r_a P_{a,b} + s_a Q_{a,b} \quad \forall (a,b) \in F^b \ & \ & \forall b \in H, \quad \tag{25}
\end{align*}
\]

\[
\begin{align*}
\ & \ + (r_a^2 + s_a^2) \Psi_{a,b} \quad \forall (a,b) \in F^b \ & \ & \forall b \in H.
\end{align*}
\]

Apart from (24) and (25), voltage of each bus is inherently limited to specified upper and lower bounds.

\[
u \leq u_{a,b} \leq \bar{u} \quad \forall a \in B \ & \ & \forall b \in H. \quad \tag{26}
\]

The squared current magnitude of any network branch in terms of branch power and squared voltage magnitude of bus is limited as:

\[
\Psi_{a,b} \times u_{a,b} \geq (P_{a,b}^2 + Q_{a,b}^2), \quad \forall (a,b) \in F^b \ & \ & \forall b \in H. \quad \tag{27}
\]

The DNR scheme changes the network topology in real-time using remotely controlled switches according to the operation condition. Constraints (31) to (36) control the buses connectivity and assures the radiality of network.

\[
\sum_{(a,b) \in F^b} (\alpha_{a,b} - \beta_{a,b}) = NB - 1, \quad \forall b \in H, \quad \tag{31}
\]

\[
(\beta_{a,b} + \beta_{h,a,b}) = \alpha_{a,b}, \quad \forall b \in H \ & \ & \forall (a,b) \in F^b, \quad \tag{32}
\]

\[
\sum_{i \in B \setminus (a)} \beta_{a,b} = 1, \quad \forall b \in H \ & \ & \forall a \in B \setminus \{B_{\text{grid}}\}, \quad \tag{33}
\]

\[
\sum_{i \in B} \beta_{a,b} = 0, \quad \forall b \in H \ & \ & \forall a \in B_{\text{grid}}, \quad \tag{34}
\]

\[
\alpha_{a,b} = \alpha_{h,a,b} \quad \forall i \in H, \quad \tag{35}
\]

\[
\alpha_{a,b} = \beta_{h,a,b} = \beta_{a,b} = 0, \quad \forall b \in H \ & \ & \forall (a,b) \in F^b. \quad \tag{36}
\]

In these constraints, \( \alpha_{a,b} - \beta_{a,b} \) reveals the status (in/out of service) of branch \((a, b)\) and \( \beta_{ab} \) determines the candidate bus \( b \) for connecting to bus \( a \). The branch \((a, b)\) is closed if
\[ \alpha_{ab,h} - \beta_{ab,h} = 1 \] and bus \( b \) is the candidate to connect to bus \( a \) if \( \beta_{ab} = 1 \). It should be noted that each bus has only one candidate to connect except the substation bus which has no candidate to connect.

### 4.4 SNR constraints

The SNR scheme provides an optimal configuration at planning stage for EDN to accommodate maximum DGs capacity and the network topology is fixed in operation stage. For SNR implementation, it is sufficient to force the variables \( \alpha_{ab,h} \) as well as \( \beta_{ab,h} \) in (31)–(36) to be equal in all states. Mathematically, it can be done by replacing \( \alpha_{ab,h} \) and \( \beta_{ab,h} \) by \( \alpha_{ab} \) and \( \beta_{ab} \), respectively, that are independent of \( h \) index.

### 4.5 PFC constraint

One of the ANM schemes for DGHC improvement is PFC of DGs. It is supposed that the wind DGs are with reactive power support capability. So, the PFC scheme can be applied for DGHC improvement by controlling the \( p_f_{dga} \) in constraint (37).

\[
-P_{G_{a,b}} \times \tan^{-1}(p_f_{dga}) \leq Q D_{G_{a,b}} \leq P_{G_{a,b}} \times \tan^{-1}(p_f_{dga}),
\]

\( \forall b \in H \) and \( \forall a \in B_{d_g} \). 

According to (37), the wind DGs can be operated between \( p_f_{dga} \) leading power factor and \( p_f_{dga} \) lagging power factor. So they can produce or consume reactive power depending on operational conditions of system and objective function optimality.

### 4.6 RPFP constraint

Apart from technical limitation of EDN for embedding more DG such as bus voltage, one of the key limitations is the reverse power flow prohibition of substation feeder. The amount of active power imported from substation is bounded according to (38). For the RPFP scheme that permits the reverse power flow of substation feeder, \( PR \) should be replaced by a positive value.

\[
-PR \times \sum_{a \in B_{d_k}} P D_{G_a} \leq P_{G_{a,b}} \quad \forall b \in H \text{ and } \forall a \in B_{grid}.
\]

The flowchart shown in Figure 2 depicts the proposed approach of this paper in DGHC evaluation, considering the correlated uncertainties of input variables including wind speed profiles and load.

### 5 IMPLEMENTATION AND RESULTS

The modified 33-bus standard test system [41] shown in Figure 3 is used for implementation of the proposed method. This radial network has 33 buses, 35 lines, 32 (normally closed) sectionalising switches and 5 (normally open) tie switches located on branches 8–21, 9–15, 12–22, 18–33 and 25–29. These tie switches are used for network reconfiguration. The nominal voltage of network is 12.66 kV and the peak load is 3.715 MW and 2.3 MVAR. The on-load tap changer transformer
at substation (bus 1) adjusts the voltage of bus 1 in the range of 1.02 to 1.05 p.u. to control the voltage of all the buses of the network in the range of 0.95 to 1.05 p.u. The thermal limits of all the feeders are adjusted to be 6.6 MVA. The other network characteristics can be found in [41].

The active and reactive loads of each bus are modelled using normal PDF with mean value of nominal load of the bus and standard deviation equals to 10% of mean value. For installation of wind DGs, eight wind sites in two groups A and B are considered. The wind sites of the group A locate at buses 12, 18, 22 and 25 and the sites of group B locate at buses 6, 7, 28 and 33. The wind turbine characteristic is given in Table 1. The maximum capacity of each turbine is considered to be 4 MW. All of the wind sites are exposed to wind speed approximated with Weibull PDF with scale and shape factor equal to 7.28 and 2.01, respectively. The correlation coefficients between the uncertain input variables is considered as matrix below:

\[
C_R = \begin{pmatrix}
1 & \alpha & \beta \\
\alpha & 1 & \theta \\
\beta & \theta & 1
\end{pmatrix}
\] (39)

The information of this matrix can be summarised with \(\text{corr} = [\alpha, \beta, \theta]\). The parameters \(\alpha, \beta\) and \(\theta\) are the correlation coefficients between wind speed of group A and B sites, wind speed of group A sites with load and wind speed of group B sites with load, respectively. Using the proposed method, totally \(2 \times 3 + 1 = 7\) states are sufficient for simultaneous modelling of considered uncertainties. For example, Table 2 provides the representative locations for wind speeds and load as output states for \(\text{corr} = [0.9, 0, 0]\). Applying the turbine characteristic of Table 1 to the wind speeds of each state, the output electric power of wind turbines is obtained. Also, the load of each bus is updated using representative locations of load for each state. The wind speed is usually the same all over the EDN area; so the wind speed profiles of sites A and B are correlated. On the other hand, the wind speed and load can be correlated or not. For analysing the effect of correlation between uncertainties on DGHC, four cases below are considered:

1. Case A: only wind speeds are correlated \((\alpha = 0.9, \beta = \theta = 0)\).
2. Case B: all the uncertainties are positively correlated \((\alpha = \beta = \theta = 0.9)\).
3. Case C: none of the uncertainties are correlated \((\alpha = \beta = \theta = 0)\).
4. Case D: wind speeds are positively correlated while are negatively correlated with load \((\alpha = 0.9, \beta = \theta = -0.9)\).

The MIQCP optimisation model has been developed in GAMS software and is solved using the CPLEX solver. All tests have been performed on a PC of 2.8-GHz and 4-Gb RAM.

### 5.1 DGHC of the basic network

For benchmarking purposes, the DGHC of the basic EDN for the four cases of the correlation is evaluated without applying of the ANM schemes. The results are shown in Table 3.
### Table 4: DGHC Evaluation Using the PFC Scheme

| Bus | Case A | Case B | Case C | Case D |
|-----|--------|--------|--------|--------|
| 6   | 0      | 0      | 0      | 0      |
| 7   | 0.679  | 0.92   | 0.271  | 0.36   |
| 28  | 0.961  | 0.997  | 0.954  | 0.934  |
| 33  | 1.379  | 1.386  | 1.676  | 1.363  |
| DGHC of site B (MW) | 3.019 | 3.303 | 2.901 | 2.657 |
| Gain in site B (%) | 14.8 | 11.5 | 38.8 | 20.5 |
| 12  | 2.016  | 2.058  | 2.566  | 1.923  |
| 18  | 1.178  | 1.19   | 1.281  | 1      |
| 22  | 0      | 0.176  | 0.037  | 0      |
| 25  | 1.672  | 1.971  | 2.017  | 1.458  |
| DGHC of site A (MW) | 4.866 | 5.395 | 5.901 | 4.381 |
| Gain in site A (%) | 9.4 | 10.4 | -3.5 | 7.3 |
| Total DGHC (MW) | 7.885 | 8.698 | 8.802 | 7.038 |
| Total gain (%) | 11.4 | 10.9 | 7.2 | 11.9 |

A is studied as a usual practice (standard case) and the results of three other cases are compared with it to demonstrate the effect of different correlation coefficients between uncertainties on DGHC. The results of cases B and D in the comparison with A show that the positive and negative correlations between load and wind speed increase and decrease the DGHC, respectively, without changing the optimal location. The results of case C indicates the significance of correlation consideration on accurately evaluation of DGHC. As can be seen, DGHC evaluation using cases C and D leads to an optimistic and pessimistic assessment of DGHC, respectively; while the results of case A is more realistic. As bus voltage and feeder thermal capacity are the limiting constraints in all of the cases, so the farther the DG location is to the substation, the larger the DG nominal capacity. As a result, the DGHC in wind sites of group A with the buses farther from substation is greater than group B.

### 5.2 DGHC Evaluation Using the PFC Scheme

This section aims to investigate the effect of PFC scheme on DGHC for the predefined cases of correlation. The PFC scheme tries to improve the DGHC by optimally control of $p_{f_{dc}}$ and facilitating the voltage control. The $p_{f_{dc}}$ is controlled in the range of 0.9 lag to 0.9 lead and the results are gathered in Table 4. Note that, compared with the basic network (see Table 3), the DGHC of sites group B improves significantly in cases C (38.8%) and D (20.5%); but the DGHC of sites group A decreases little in case C (−3.5%). This suggests that the DGHC improvement using the PFC scheme depends on DG siting and the correlation between uncertainties; so must be evaluated on a case-by-case basis.

For assessment of the effect of $p_{f_{dc}}$ control range on DGHC, this range is changed from 0.4 to 1 using 0.05 steps and the DGHC is evaluated for all desired cases. The results are presented in Figure 4. For example, if $p_{f_{dc}}$ can be controlled in the range of 0.8 lag to 0.8 lead, the DGHC in case A is equal to 8.21 MW. As can be seen, in all of the cases, the greater the range of $p_{f_{dc}}$ control, the greater the DGHC. However, the efficiency of the PFC scheme on DGHC improvement differs from case to case. Obviously, the bus voltage constraint in case D is more challenging than the other cases due to negative correlation of the DGs output power and the load; therefore the PFC scheme plays a more effective role in this case and increases the DGHC up to 30%; while without consideration of correlation (case C), DGHC improvement using the PFC scheme limits to 13%. Also, as expected, the bus voltage constraint in case A is more limiting than case B, due to lack of correlation between output power of DGs and load. As a result, the PFC scheme efficiency on DGHC is up to 25% for case A versus 20% for case B.

### 5.3 DGHC Evaluation Using the RPFP Scheme

For execution of RPFP, the $Pr$ value in (38) is adjusted to be 10%. It means that, some surplus active power generation can be injected from EDN into the upstream network. The results of this analysis is reported in Table 5. As can be seen, the total DGHC for all the cases is improved more than 10%. For investigation of the potential of RPFP scheme on DGHC improvement, the $Pr$ parameter is increased from 0% to 50% by 5% steps and the DGHC is calculated and reported in Figure 5. As can be seen, this scheme has a significant impact on DGHC improvement. Compared with the PFC scheme, the RPFP scheme more effectively improves the DGHC, because it makes the EDN more flexible and makes the constraints more relaxed.

Also it is shown that the maximum DGHC of cases A, B and D using the RPFP scheme is approximately the same; while, if the correlation of uncertainties is neglected, the DGHC will
TABLE 5  DGHC evaluation using the RPFP scheme

| Bus | Case A | Case B | Case C | Case D |
|-----|--------|--------|--------|--------|
| 6   | 0      | 0.106  | 0      | 0.667  |
| 7   | 0.916  | 1.092  | 0.47   | 0.667  |
| 28  | 0.836  | 0.866  | 0.853  | 0.788  |
| 33  | 1.145  | 1.148  | 1.441  | 1.133  |
| DGHC of site B (MW) | 2.897 | 3.212 | 2.764 | 2.588 |
| Gain in site B (%) | 10.2 | 8.5 | 32.2 | 17.4 |
| 12  | 1.954  | 2.028  | 2.637  | 1.874  |
| 18  | 1.028  | 0.989  | 1.192  | 0.91   |
| 22  | 0.159  | 0.438  | 0.634  | 0      |
| 25  | 1.982  | 2.203  | 2.603  | 1.768  |
| DGHC of site A (MW) | 5.123 | 5.658 | 7.066 | 4.552 |
| Gain in site A (%) | 15.2 | 15.8 | 15.5 | 11.5 |
| Total DGHC (MW) | 8.02 | 8.87 | 9.83 | 7.14 |
| Total gain (%) | 13.3 | 13.05 | 19.7 | 13.5 |

FIGURE 5  DGHC for different PR in the RPFP scheme

be significantly greater. The main reason is that the bus voltage and feeder capacity constraints in the presence of correlation between uncertainties reach their threshold earlier. Its worth noting that although the RPFP scheme significantly increases the DGHC, but the regulations must also be provided to take advantage of this scheme for absorbing more DGs in EDN.

For DGHC evaluation using the PFC and RPFP schemes simultaneously, the $p_{f_{d_{g}}}$ and PR are changed from 0.5 to 1 and 0% to 50%, respectively, and DGHC is calculated. The results are depicted in Figure 6. Its obvious that merging the PFC scheme with the RPFP independent of $p_{f_{d_{g}}}$ improves the DGHC significantly. As can be seen, the higher the PR, the less the effect of the $p_{f_{d_{g}}}$ on the DGHC. This is because the big value of PR sufficiently relaxes the EDN constraints to accommodate more DGs. For investigation of the importance of correlation consideration in DGHC evaluation, the same study is done on case C and the results are reported in Figure 7. Compared with Figure 6, it can be concluded that in the presence of correlated uncertainties in case A, the network constraints are further stimulated and as a result the PFC scheme becomes more effective on DGHC improvement.

5.4  DGHC evaluation using the SNR and DNR schemes

Tables 6 and 7 present the results of DGHC evaluation using the SNR and DNR schemes, respectively. As can be seen, applying the SNR or DNR schemes alone does not significantly improve the DGHC. Although the DNR scheme is more flexible, here it has no particular advantages over the SNR, because among the seven probabilistic scenarios of wind speeds and network load in all the four cases, one scenario is more decisive than the others and further stimulates the network constraints.
Also the effect of simultaneously use of the SNR and DNR with the PFC ($\delta_{PF} > 0.9$) and RPFP ($PR = 10\%$) schemes is investigated on cases A and C to demonstrate the importance of correlation consideration in DGHC evaluation. The results are shown in Figure 8. It is obvious that, merging the PFC and RPFP with the SNR and DNR schemes, appreciably increases their effectiveness. It is also observed that in merging with the SNR and DNR schemes, the efficiency of the RPFP scheme is more improved than PFC. Finally, Table 8 shows the results of DGHC evaluation and its gain for all the four cases using all the desired ANM schemes. It can be seen that in each case of correlation, the EDN operator can increase the DGHC of EDN more than 30\% using different ANM schemes.
6 CONCLUSION

Here, a model for investigating the effect of correlated uncertainties of wind speeds and load on DGHC of EDN is proposed. The efficiency of some ANM schemes on DGHC improvement in the presence of correlated uncertainties was investigated as well. The combination of PEM with inverse Nataf transformation was proposed for correlated uncertainties modelling. The advantages of this model include its simplicity and low computational burden.

The DGHC evaluation problem was modelled as a MIQCP model and was solved using CPLEX solver. The efficiency of the proposed model was shown studying the 33-bus standard test system considering the four cases of correlation between uncertainties. The results show the importance of correlation consideration on DGHC evaluation of EDN. In all the cases of correlation, the EDN operator can improve the DGHC more than 30% using different ANM schemes. However, the fundamental question is what is the incentive for the EDN operator with for-profit and monopolistic structure to take advantage of these costly schemes?

NOMENCLATURE

A. Correlated uncertainties model

- \( P_d \): Load demand [MW]
- \( f_{pd}(P_d) \): PDF of load demand
- \( \mu_d \): Average of load demand [MW]
- \( \sigma_d \): Standard deviation of load demand [MW]
- \( ws \): Wind speed m/s
- \( c/k \): Scale/shape factors of Weibull PDF
- \( P(ws) \): Electrical output power of wind turbine [MW]
- \( \mu_{j, i} / \sigma_{j, i} \): Average/standard deviation of input uncertain variable \( y_j \)
- \( \lambda_{3, i} \): Skewness of \( y_j \)
- \( \lambda_{4, i} \): Kurtosis of \( y_j \)
- \( \gamma_i \): Coefficient of variation of input variable \( y_i \)
- \( \mu_{yi} / \sigma_{yi} \): Average/standard deviation of \( y_i \)
- \( \theta_{yi} \): CDF of \( y_i \)
- \( \Gamma \): CDF of standard normal distribution
- \( L_0 \): Lower triangular matrix
- \( B \): Set of network buses
- \( B_{grid} \): Set of substation bus
- \( B_{dc} \): Set of candidate buses for DG installation
- \( B_a \): Set of buses that can connect to bus \( a \)
- \( F_b \): Set of all network branches (from bus, to bus)
- \( H \): Set of all probable states
- \( a, b, c \): Indices for buses
- \( h \): Index for state
- \( NB \): Number of network buses
- \( O \): Very big number (\( \approx 20 \))
- \( z_{ab} \): Impedance of branch [p.u.]
- \( y_{ab} \): Shunt admittance of bus [p.u.]
- \( PD_{a,h} \): Active power demand [MW]
- \( QD_{a,h} \): Reactive power demand [MVAr]
- \( g_a \): Conductance of bus [p.u.]
- \( b_a \): Susceptance of bus [p.u.]
- \( r_{ca} \): Resistance of branch [p.u.]
- \( x_{ca} \): Reactance of branch [p.u.]
- \( scale_{dc} \): Ratio of output power to capacity of DG
- \( \theta \): Squared maximum voltage magnitude [p.u.]
- \( \varphi \): Squared minimum voltage magnitude [p.u.]
- \( \omega_i \): Occurrence coefficient
- \( TL \): Maximum expected active power loss of network [MW]
- \( PR \): Permitted reverse power flow [MW]
- \( \bar{P}_{ab} \): Squared maximum current magnitude of branch [p.u.]
- \( P_{d, a} \): DG minimum power factor
- \( S_{ab} \): Apparent power injected to bus [MVA]
- \( S_{cab} \): Apparent power flow of branch [MVA]
- \( PG_{a,b} \): Active power injected to bus [MW]
- \( Q_{a,b} \): Active power flow of branch [MW]
- \( QC_{a,b} \): Reactive power injected to bus [MVAr]
- \( Q_{ca,b} \): Reactive power flow of branch [MVAr]
- \( \varphi_{ab} \): Squared voltage magnitude of bus [p.u.]
- \( \bar{\varphi}_{ab} \): Squared current magnitude of branch [p.u.]
- \( PDG_a \): DG nominal capacity installed at bus [MW]
\[ P_{\text{grid},a,h} \] Active power imported from substation [MW]

\[ Q_{\text{grid},a,h} \] Reactive power imported from substation [MVAr]

\[ QD_{\text{DG},a,b} \] Reactive power consumed/produced by DG [MVAr]

\[ \alpha_{a,b} \] Binary variable: 1 if one of buses \( a \) or \( b \) is candidate to connect to another one, 0 otherwise

\[ QD_{\text{DG},a,b} \] Reactive power consumed/produced by DG [MVAr]

\[ \beta_{a,b} \] Binary variable: 1 if bus \( b \) is candidate to connect to bus \( a \), 0 otherwise

\[ TL \] Expected value of total active power loss of network [MW]

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