Depinning of a vortex chain in a disordered flow channel

R. Besseling\textsuperscript{1}, T. Dröse\textsuperscript{2(∗)}, V. M. Vinokur\textsuperscript{3} and P. H. Kes\textsuperscript{1}

\textsuperscript{1} Kamerlingh Onnes Laboratorium, Leiden University
P.O. Box 9504, 2300 RA Leiden, the Netherlands
\textsuperscript{2} I. Institut für Theoretische Physik, Universität Hamburg
Jungiusstrasse 9, D-20355 Hamburg, Germany
\textsuperscript{3} Materials Science Division, Argonne National Laboratory
Argonne, IL 60439, USA

(received 26 July 2002; accepted in final form 27 February 2003)

PACS. 74.60.Ge – Flux pinning, flux creep, and flux-line lattice dynamics.
PACS. 71.45.Lr – Charge-density-wave systems.
PACS. 83.50.Lh – Slip boundary effects (interfacial and free surface flows).

Abstract. – We study depinning of vortex chains in channels formed by static, disordered vortex arrays. Depinning is governed either by the barrier for defect nucleation or for defect motion, depending on whether the chain periodicity is commensurate or incommensurate with the surrounding arrays. We analyze the reduction of the gap between these barriers as a function of disorder. At large disorder, commensurability becomes irrelevant and the pinning force is reduced to a small fraction of the ideal shear strength of ordered channels. Implications for experiments on channel devices are discussed.

The depinning and dynamics of periodic elastic media in a random potential have received a great deal of recent attention [1]. It was shown, in particular for vortex lattices (VLs) in superconductors, that the depinning transition in most cases involves plastic deformations [2]. A system in which plastic depinning can be studied in a controlled way is that of narrow, weak pinning flow channels in a superconducting film [3]. In such a system, strongly pinned vortices in the channel edges (CEs) provide confinement as well as an effective pinning potential for chains of vortices inside the channel. By changing the magnetic field \( B \), one can vary the ratio between channel width and lattice spacing and thus induce incommensurability between the vortex spacing inside and outside the channel. This leads to topological defects in the channel which sensitively affect the threshold for plastic flow, as evidenced by oscillations of the critical current density \( J_c \) vs. field [3].

Simulations of channels with perfect hexagonal vortex arrays in the CEs showed sharp peaks in \( J_c \) for channel widths \( w \) equal to an integer number \( n \) of vortex row spacings, \( i.e. \ w = nb_0 \). At mismatch \( w \neq b_0 \) defects occurred in the channel and \( J_c \) vanished due to the small Peierls barrier for defect flow [4]. This, however, contrasts the smooth oscillations found experimentally. Moreover, in reality the CE arrays may contain quenched positional disorder.
due to random pinning. This should modify $J_c$ and its dependence on commensurability. In this letter we consider the simplest near-to-commensurability situation $w \sim b_0$, and investigate the effect of disorder on depinning of a single vortex chain. We find that commensurate chains, with a periodicity equal to that of the VL in the CEs, depin at a force $f_n$ below the ideal shear strength $f_c$ by nucleation of defect pairs. At incommensurability, $f_c$ is determined by the pinning force $f_d$ of existing defects. At weak disorder, a gap is found between $f_n$ and $f_d$, but for larger disorder commensurability becomes irrelevant and $f_c$ saturates at a small fraction of $f_0^c$. This has important consequences for the interpretation of the experiments [3]. Generally, our results are relevant to a wealth of 1D problems including models for interface growth [5], dynamics of Josephson junctions [6] and charge density waves (CDWs).

We consider a channel at $T = 0$ with boundaries (CEs) formed by two static, disordered arrays with vortex positions $R_{n,m} = r_{n,m} + d_{n,m}$, where $r_{n,m}$ denote the hexagonal lattice with spacing $a_0 = 2b_0/\sqrt{3}$ (see the inset to fig. 1), the channel width is defined by the spacing between rows $m = \pm 1$ and $d_{n,m}$ are random shifts. We restrict ourselves to longitudinal shifts [7] and choose $d_{n,m} = d_n \vec{e}_x$ such that the strain $(d_{n+1} - d_n)/a_0$ is uniformly distributed in the interval $[-\Delta, \Delta]$ with $\Delta$ the disorder parameter. The field $B$ sets both the vortex density in the CEs ($\rho_c = (a_0b_0)^{-1} = B/\Phi_0$ with $\Phi_0$ the flux quantum) and the density $\rho_c = (aw)^{-1}$ inside the channel. Hence, vortices in the channel have an average spacing $a = a_0b_0/w$ which can be commensurate with the CE arrays ($w = b_0$, $a = a_0$) or incommensurate ($a \neq a_0$). The equation of motion for vortex $i$ in the channel is

$$\gamma \partial_t r_i = f - \sum_{j \neq i} \nabla V(r_i - r_j) - \sum_{n,m} \nabla V(r_i - R_{n,m}).$$

(1)

$V(r)$ is the interaction potential, $j$ labels other vortices in the channel, $\gamma = B\Phi_0/\rho_l$ with $\rho_l$ the flux flow resistivity and $f = J\Phi_0$ is the drive along $x$ due to a uniform current density $J$ applied perpendicular to the channel.

For $\Delta = 0$ the vortex chain can be described by a Frenkel-Kontorova model for interacting particles in a periodic potential [4]. The ratio between the chain stiffness and the height of the periodic potential is given by $g \propto \lambda/a_0 \gg 1$ with $\lambda$ the penetration depth. A commensurate
chain depins uniformly at a critical force $f_c = \mu = 2a_0c_{66}/\pi \sqrt{3}$ ($c_{66}$ is the shear modulus) with a velocity $v = \sqrt{f^2 - \mu^2}/\gamma$. At incommensurability, defects of size $l_d = 2\pi a_0/\sqrt{g} \gg a_0$ are generated. Their pinning barrier and the critical force are essentially vanishing. For $f < \mu$ and small defect density, given by $c_d = |a_0^{-1} - a^{-1}| \lesssim l_d^{-1}$, the motion of defects leads to a low mobility regime, where $v = c_d v_d^{0} a_0$ with $v_d^{0} = (\pi^2 \sqrt{g}/2\gamma)f$ the pure defect velocity [8].

We start with a numerical study of the behavior in the presence of disorder. Equation (1) was solved using a modified London form for $V(r)$ which yields the correct shear modulus [4]. We used cyclic boundary conditions, channels of length $L \geq 1000a_0$ and we recorded the velocity $v(f) = \langle \dot{x}_i \rangle_{i,t}$ and vortex positions $x_i(t)$.

The data points in fig. 1 show $f$-$v$ curves of a commensurate and an incommensurate chain for $\Delta = 0.025$. We first focus on the commensurate case. Compared to the result for ordered CEs (dashed line), $f_c$ in the presence of disorder is clearly reduced. The origin of this reduction is that the random strains lower the energy barrier for formation of discommensuration pairs in the chain (interstitial/vacancy pairs in the 2D crystal formed by the chain and the CEs).

We show the depinning process in detail in fig. 2a by plotting the time evolution of the displacements $u_i = x_i - ia_0$ [9]. At $t = t_{11}$, the force is increased to a value $f > f_c$. The motion starts at an unstable site in the chain by nucleation of a vacancy/interstitial pair shown as steps of $\pm a_0$ in $u$. We denote the force at which this local nucleation occurs by $f_n$. The process at this site repeats periodically with rate $R_n \propto (f - f_n)^\beta$ and a depinning exponent $\beta = 0.46 \pm 0.04$, as previously reported for 1D periodic media [10]. Due to the nucleation center, a domain forms with defect density $c_d = R_n/\langle v_d \rangle$ and a net velocity $v = c_d\langle v_d \rangle a_0 = R_n a_0$ with $\langle v_d \rangle$ the average defect velocity $\simeq v_d^{0}$, see below. In a larger system (fig. 2b), a distribution of unstable sites $p$ with local rate $R_{n,p}$ initially leads to the growth of several domains. However, when two domains with rates $R_{n,1} > R_{n,2}$ meet, their interstitials and vacancies annihilate and domain 1, with the larger nucleation rate, then expands at the cost of domain 2 with a rate $\sim (R_{n,1} - R_{n,2}^2)$. The stationary state, covering the entire system, is thus governed by the nucleation center with the smallest local threshold $f_n^{\text{min}}$.

Next, we turn to the $f$-$v$ curves in fig. 1 at incommensurability ($w/b_0 = 0.98$, small defect
disorder strengths $\Delta$. We define the pinning force for a single defect as a significant threshold force due to pinning of the existing defects by random strains in the data one retains a low-mobility regime with $d$ density becomes $\gtrsim d$. The typical disorder strengths $\Delta_c$ and $\Delta_s$ are indicated. The insets show $f$-$v$ curves for $\Delta > \Delta_c$.

density $c_d \sim 1/2l_d$). In contrast to the curve for $\Delta = 0$ (full line), the data for $\Delta = 0.025$ show a significant threshold force due to pinning of the existing defects by random strains in the CEs. We define the pinning force for a single defect as $f_d(x)$ with a distribution $\{f_d\}$ along the channel and a maximum value $f_d^{\text{max}}$. The shape of the distribution is roughly Gaussian and has a width $\sim 0.1 \mu$. The threshold force for a single defect will be $f_c = f_d^{\text{max}}$. If the defect density is low, the defects occupy only the highest values of $\{f_d\}$ and $f_c \lesssim f_d^{\text{max}}$ which in fig. 1 is $f_c \sim 0.2 \mu$. Once defects are depinned, their velocity $v_d$ fluctuates spatially and $(v_d)$ is smaller than $v_d^0$. However, these effects strongly decrease with velocity and for $f \gtrsim 2f_d^{\text{max}}$ one retains a low-mobility regime with $dv/df \simeq c_d v_d^0 a_0$, as seen in fig. 1. Considering the regime of larger drive, the data exhibits a velocity upturn for $f \sim f_{n}^{\text{min}}$. At this driving force new defect pairs start to nucleate at a strong disorder fluctuation with a rate that exceeds the one at which existing defects travel through the system.

Let us now discuss the disorder dependence of the threshold forces. Shown in fig. 3 are $f_c$ data vs. $\Delta$ for commensurate and incommensurate chains. For $w = b_0$ the minimum from many disorder realizations is plotted, i.e. $f_c = f_{n}^{\text{min}}$. With growing disorder $f_{n}^{\text{min}}$ decreases sharply while for the incommensurate chain with $w/b_0 = 0.98$, $f_c \sim f_d^{\text{max}}$ grows linearly. This behavior changes at $\Delta \simeq \Delta_c$, where $\Delta_c$ is defined as the disorder strength where for $w = b_0$ defects first appear spontaneously. Above $\Delta_c$ both curves (in)decrease more slowly with disorder and eventually merge. This change in behavior is due to the fact that favorable nucleation sites in the commensurate case act as strong pinning site for defects in the incommensurate case. The simulations show that the presence of pinned defects strongly affects the formation of a new nucleus. As a result, $f_{n}^{\text{min}}$ decreases more slowly at larger disorder, i.e. at a higher density of pinned defects. The curve for $w/b_0 = 0.95$, for which the density of static, mismatch induced defects is $c_d \simeq l_d^{-1}$, shows a reduced threshold force and merges with the other curves at a disorder strength $\Delta = \Delta_s$. For $\Delta > \Delta_s$, disorder-induced defects start to overlap (i.e. their density becomes $\gtrsim l_d^{-1}$) and $f_c$ decreases further. The large disorder also has a pronounced effect on the shape of the $f$-$v$ curves. As shown in the insets to fig. 3, the typical low mobility regime at weak mismatch has vanished and all curves exhibit linear behavior, except in a small regime for $f$ just above $f_c$.

To uncover the underlying physics of the phenomena described above, we now propose an
analytical description of our system. The energy of a vortex in the channel at \( r_0 = (x, 0) \) due to interaction with shifted vortices in the CEs is

\[
V(r_0) = (2\pi)^{-2} \int dk V(k) \rho_e(k) e^{i k \cdot r_0}.
\]  

\( V(k) = 2\pi U_0 / (k^2 + \lambda^{-2}) \) with \( U_0 = \Phi_0^2 / 2\pi \mu_0 \lambda^2 \), and \( \rho_e(k) \) are the Fourier transforms of the London potential and the vortex density in the edge, respectively. For weak disorder \( (\nabla \cdot d \ll 1) \), \( \rho_e \) can be decomposed \[11]\: \rho_e(r_e, d) \simeq (B/\Phi_0)(1 - \nabla \cdot d + \delta \rho_e), \] where \( \delta \rho_e = \sum_i \cos[K_i(r_e - d(r_e))] \) and \( K_i \) spans the reciprocal lattice. Substitution in eq. (2) yields two terms: a quasi-periodic potential \( V^p \simeq (\mu/k_0) \sin[k_0(x - d)] \), with \( k_0 = 2\pi/a_0 \) \[12\] due to \( \delta \rho_e \) of the vortex rows nearest to the CEs, and a random, nonlocal contribution coming from the density fluctuations: \( V^r(r_0) = -(B/\Phi_0) \int dr_e V(r_0 - r_e) \nabla \cdot d(r_e) \) with correlator

\[
\langle V^r(0)V^r(x) \rangle \simeq 4\Delta^2 U_0^2(\lambda/a_0)^{1+\alpha} e^{-(x/\lambda)^2}.
\]

Here \( \alpha \) depends on the disorder correlations between the rows \( m \) in the CEs. We choose \( \partial_x d(x) \) to be independent of row number, which yields \( \alpha = 2 \).

To obtain a continuum description of the chain in terms of the displacement field \( u(x) \), the vortex density \( \rho_e \) in the channel is decomposed, as was done for \( \rho_e \). The resulting interaction of the chain with the CEs is \( H_p = a_0^{-1} \int (V^p + V^r) \delta \rho_e(x, u) - \partial_x u \) dx. For \( \lambda > a_0 \), both \( V^r \) and \( \partial_x u \) vary slowly while \( V^p \) and \( \delta \rho_e \) oscillate rapidly and only two terms in \( H_p \) remain \[13\]. The force \(-\delta H_p/\delta u \) on the chain thus contains a commensurate term and a random compression term \(-\partial_x V^r(x) \), which is independent of \( u \). We obtain the equation of motion for \( u \) by adding the \textit{intra-chain} force \( \kappa \partial_x^2 u \), with stiffness \( \kappa \simeq U_0 \pi(\lambda/a_0) \) \[14\], resulting in

\[
\gamma \partial_t u = f + \kappa \partial_x^2 u - \mu \cos(k_0 u) - \partial_x V^r(x).
\]

The reduced stiffness is now \( g = \kappa/(k_0 \mu a_0^2) = 3\pi(\lambda/a_0) \). Equation (4) describes the transverse displacements of an elastic string in a tilted washboard potential with random phase \( \phi(x) = \ldots \)
− \int_{-\infty}^{\infty} dx' V^r(x')/\kappa$, see fig. 4. It corresponds to a commensurate CDW with random forward scattering [15] rather than the usual model for a CDW in which the commensurability potential is ignored either due to strong direct random coupling to \( u \) or due to large mismatch [16].

Next, the dependence of \( f_{\text{min}}^c \) and \( f_{\text{max}}^c \) on the strain \( \Delta \) can be addressed by considering a deformation in the CEs of wavelength \( l_{\text{dis}} \): \( \partial_x d(r) = \Delta \sin(2\pi x/l_{\text{dis}}) \). When \( l_{\text{dis}} \gg \lambda \), the last term in eq. (4) simplifies to \(-\partial_x V^r \simeq g^2 \Delta U_0/(2\sqrt{3} l_{\text{dis}}) \cos(2\pi x/l_{\text{dis}})\). To describe nucleation (for \( f \geq \mu/2 \)), we need the string displacements \( \delta(x) = u - u_0 \) relative to the minimum of the tilted washboard potential given by \( u_0 = -\kappa_0^{-1} [2(1 - f/\mu)]^{1/2} \). The restoring force for such displacements is \( \mu k_0^2 u_0 \delta(x) \), while the elastic force is \( \kappa \partial_x^2 \delta \). Balancing these with \( \delta V^r \) yields \( \delta(x) = \delta_0 \cos(2\pi x/l_{\text{dis}}) \). The amplitude of the displacement \( \delta_0 \sim l_{\text{dis}}/(l_0^2 + k_0 |u_0| l_{\text{dis}}^2) \) is largest for a wavelength \( l_{\text{dis}} = l_0/\sqrt{k_0 |u_0|} \) resulting in \( \delta_0^{\text{max}} \simeq 3\kappa_0 g^{3/2} \Delta/(8\pi \sqrt{k_0 |u_0|}) \). Since the critical displacement at which the string depins is \( \simeq |u_0| \) [8], the minimum nucleation force follows from \( |u_0| = \delta_0^{\text{max}} \):

\[
f_{\text{min}}^c/\mu \simeq 1 - \left[ 4\Delta g^{3/2}/(5\sqrt{3}) \right]^{4/3}.
\] (5)

For an existing defect at zero drive, the pinning energy is \( E_{\text{d,p}}(x) = a_0^{-1} \int dx' \partial_x u_d(x' - x)V^r(x') \) with \( u_d \) the familiar shape of a sine-Gordon kink [8] centered at \( x \) and of size \( l_d \). Optimal pinning occurs for deformations in the CEs with \( l_{\text{dis}} = l_d \), similar to the “optimal” size of a nucleation center for \( f \simeq \mu/2 \). The maximum defect pinning force is then given by

\[
f_{\text{max}}^c/\mu \simeq \Delta g^{3/2}/\sqrt{3}.
\] (6)

The results (5) and (6) are plotted in fig. 3, using \( g = 9 \) as in the simulations. Equation (5) agrees with the numerical data, while eq. (6) can be considered as upper bound. The curves merge at \( \Delta_c \sim \sqrt{3} g^{-3/2}/2 \), where pinned defects can appear spontaneously in the channel [17].

The disorder strength \( \Delta_c \), where the density of disorder induced defects becomes \( \sim l_d^{-1} \), can be estimated by equating the typical (rms) pin energy of a defect with its bare energy \( \sim \mu a_0 \sqrt{g} \). The former is estimated using the previous form for \( E_{\text{d,p}} \) and eq. (3): \( \langle (E_{\text{d,p}})^2 \rangle^{1/2} \simeq 2U_0 (g/3\pi)^{(2+\alpha)/2} g^{-1/4} \Delta \) leading to

\[
\Delta_c \simeq (3\pi/4) g^{-5/4},
\] (7)
also in reasonable agreement with the data in fig. 3. These results show that the effect of disorder rapidly grows on increasing the interaction range \( \lambda/a_0 \).

Finally, we shortly discuss how these results carry over to channels with multiple chains near matching (\( w/b_0 \simeq n \), with \( n \) an integer \( \geq 2 \)). Without disorder \( f_c \) has sharp peaks at matching of height \( f_c^0 = \mu b_0/w = \mu/n \) [4]. With disorder, the behavior of \( f_c \) is similar to that in fig. 3 [18]. In fig. 4(c) we show the depinning process for \( w/b_0 = 3, \Delta = 0.02 \) and \( f = 0.7 f_c^0 \). As observed, the defects involved with depinning consist of stacks of discommensurations, coupled between the chains. Each stack is terminated by a pair of dislocations with opposite Burgers vector along the CEs. This quasi-1D behavior at weak disorder can also be described by eq. (4) by substituting \( \mu/n \) for \( \mu \). For larger disorder the threshold near matching reduces to \( \sim 30\% \) of \( f_c^0 \). The merging of the commensurate and weakly incommensurate threshold forces now occurs for \( \Delta_* = \Delta_c/n^{1/4} \). For a realistic value of \( \lambda/a_0 \sim 4 \), we estimate \( \Delta_* \sim 0.03 \) meaning that rms strains in the CEs \( \lesssim 2\% \) already cause strong disorder. At strong disorder and large mismatch the quasi-2D nature of the system causes new phenomena: for \( w/b_0 \simeq n \pm 1/2 \) transitions from \( n \to n \pm 1 \) rows occur, involving pinning of dislocations with misaligned Burgers vector [19]. The threshold force then exceeds \( f_c \) near matching [19].
In summary, depinning of a vortex chain in a channel formed by disordered vortex arrays with (nearly) the same periodicity occurs by nucleation of defect pairs or motion of existing defects. The gap between the barriers for these two processes and the sharp peak in $f_c$ at commensurability vanish with increasing disorder. For large disorder in the channel edges $f_c$ saturates at a small fraction of the ideal lattice strength.

***

This project is supported by the Nederlandse Stichting voor Fundamenteel Onderzoek der Materie (FOM) and also by U.S. DOE, Office of Science under contract #W-31-109-ENG-38 during RB’s stay at Argonne. VMV is supported by U.S. DOE, Office of Science under contract #W-31-109-ENG-38. TD acknowledges financial support from the Graduiertenkolleg “Physik nanostrukturierter Festkörper”, University of Hamburg.

REFERENCES

[1] Koshelev A. E. and Vinokur V. M., Phys. Rev. Lett., 73 (1994) 3580; Giamarchi T. and Le Doussal P., Phys. Rev. Lett., 76 (1996) 3408; Balents L. and Fisher M. P. A., Phys. Rev. Lett., 75 (1995) 4270; Blatter G. et al., Rev. Mod. Phys., 66 (1994) 1125.
[2] Jensen H. J. et al., Phys. Rev. Lett., 60 (1988) 1676.
[3] Pruymboom A. et al., Phys. Rev. Lett., 60 (1988) 1430; Theunissen M. H. et al., Phys. Rev. Lett., 77 (1996) 159.
[4] Besselings R. et al., Phys. Rev. Lett., 82 (1999) 3144.
[5] Krug J., Phys. Rev. Lett., 75 (1995) 1795.
[6] Malomed B. A., Phys. Rev. B, 39 (1989) 8018.
[7] In the experiment, $d'$ is assumed to be suppressed due to screening currents along the channel edges. See also Plourde B. L. T. et al., Phys. Rev. B, 66 (2002) 054529.
[8] Böttiker M. and Landauer R., Phys. Rev. A, 23 (1981) 1397.
[9] Representative movies are available at http://lions1.leidenuniv.nl/wwwhome/rbessel.
[10] Myers C. R. and Sethna J. P., Phys. Rev. B, 47 (1993) 11171; Cule D. and Hwa T., Phys. Rev. Lett., 77 (1996) 278.
[11] Giamarchi T. and Le Doussal P., Phys. Rev. B, 52 (1995) 1242.
[12] We obtain $\mu = U_0/6\pi a_0 \equiv 2a_0 \epsilon_0$ with $\epsilon_0 = a_0 c_{\text{eff}} / (2\pi b_0)$ as expected for a harmonic potential between rows spaced by $b_0$ with intra-row spacing $a_0$, Frenkel J., Z. Phys., 376 (1926) 572.
[13] The product $V^p \rho$ in fact yields $(\mu/k_0) \sin[k_0(u-d)]$. The resulting extra random phase $-k_0 d$ in the cos term in (4) is ignored since it is small compared to the phase $\phi$ arising from $V^r$.
[14] The full, dispersive expression for the stiffness is $\kappa(k) = U_0 \pi (\lambda/a_0) / \sqrt{T+\lambda^2 k^2}$. Thus eq. (4) applies for $k < \lambda^{-1}$.
[15] Feigelman M. V. and Vinokur V. M., Solid State Commun., 45 (1983) 595; Vinokur V. M. and Mineev M. B., Zh. Eksp. Teor. Fiz., 88 (1985) 1809.
[16] Fukuyama H. and Lee P. A., Phys. Rev. B, 17 (1978) 535; Littlewood P. B., Phys. Rev. B, 33 (1986) 6694.
[17] Since, for depinning, the “optimal” wavelengths are at least $l_d$, the results are valid when $l_d > 2\pi \lambda$. Using $l_d = 2\pi a_0 / \sqrt{f}$, this corresponds to the range $\lambda/a_0 \lesssim 10$.
[18] Besselings R., PhD Thesis, Leiden University (2001); Besselings R. et al., to be published.
[19] Kokubo N. et al., Phys. Rev. Lett., 88 (2002) 247004.