Charmonium and Bottomonium from Classical $SU(3)$ Gauge Configurations

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Abstract

The charmonium and bottomonium spectra computed from a potential defined from a single gauge configuration, obtained from solving the classical field equations, is discussed. The theoretical spectra shows good agreement with the measured states.

A discussion of possible interpretations, within the same non-relativistic potential model, for the new charmonia states $X(3872)$, $\chi_{c1}(2P)$ and $Y(4260)$ is performed. In particular, we give predictions for electromagnetic E1 transitions for various scenarios.

1 Introduction and Motivation

In [1] it was proposed a generalized Cho-Faddeev-Niemi ansatz for the SU(3) gauge fields. For the simplest form of the ansatz, the classical field equations were solved and a potential for heavy quarkonia motivated. The single configuration potential is coulombic for short interquark distances and grows exponentially for large interquark distances. In this way, the potential provides quark confinement. Then, assuming that the quark interaction is a pure vectorial interaction, the spectra of charmonium was investigated.

As described in [1], the single configuration potential is able to describe the charmonium spectra with an error of less than 3% ($\sim 100$ MeV). However, the prediction for the $1S$ and $2S$ hyperfine splitting is about half of the experimental value. This result is probably due to the definition of the potential from a single gauge configuration. In the same work, the theoretical predictions for the leptonic widths of $1^{-+}$ states where computed and the results are in-line with the experimental values.

In [2] the investigation was extended to include the E1 electromagnetic transitions together with an analysis of the bottomonium system. For $B\bar{B}$, the spectra is reproduced with an error of less than 1% ($\sim 100$ MeV) and the theoretical
|        | Charmonium    | Bottomonium  |
|--------|---------------|--------------|
| $1^3S_1$ | 3097          | $1^3S_1$     |
|         | 0 $J/\psi(1S)$ | 9460         |
| $2^3S_1$ | 3659          | $2^3S_1$     |
|         | -27 $\psi(2S)$ | 10023        |
| $1^3D_1$ | 3688          | $1^3D_1$     |
|         | -83 $\psi(3770)$ | 10159       |
| $3^3S_1$ | 4164          | $3^3S_1$     |
|         | -95 $Y(4260)$ | 10385        |
| $2^3D_1$ | 4155          | $2^3D_1$     |
|         | 2 $\psi(4160)$ | 10476        |
| $4^3S_1$ | 4669          | $4^3S_1$     |
|         | -              | 10727        |
| $3^3D_1$ | 4636          | $3^3D_1$     |
|         | -              | 10796        |
| $5^3S_1$ | 11065         | $5^3S_1$     |
|         | 46            | $\Upsilon(10860)$ |

Table 1: $J^{PC} = 1^{--}$ charmonium and bottomonium spectra. The table shows the theoretical state, the mass prediction, the difference between the experimental measured mass and the theoretical prediction. All numbers are in MeV.

prediction for the $b\bar{b}$ leptonic widths reproduces the same level of accuracy as found in charmonium.

In what concerns the electromagnetic transitions, the single configuration potential is able to reproduce well the quoted particle data book numbers \cite{pdg}.

### 2 Charmonium and Bottomonium Spectra

In table \cite{1} we show the theoretical $J^{PC} = 1^{--}$ spectra; see \cite{1, 2} for details. The third column is the deviation of the theoretical mass to the experimental measured mass, in MeV. The overall agreement between theoretical and particle spectra is good. The only particles which don’t fit well in the theoretical spectra are $\psi(4040)$ and $\psi(4415)$. In what concerns these two states, the experimental information is scarce and the particle data book comments that the “interpretation of these states as a single resonance is unclear because of the expectation of substantial threshold effects in this energy region”. Curiously, both particle masses are essentially the sum of $J/\psi$ with light $J^{PC} = 0^{++}$ mesons.

The work reported in \cite{1, 2} and summarized in table \cite{1} is based on the non-relativistic analysis of a confining potential derived from a single configuration. Our previous work suggests that either one should include the contribution from other configurations and/or one should perform a coupled-channel analysis. We are currently engaged in extending our previous studies in both ways. Anyway, in what concerns the spectra, the single channel analysis shows that the potential is able to explain the observed states if one allows for an error of $\sim 100$ MeV.

In the following we report on the predictions of the single channel analysis for the potential obtained in \cite{1} for the new charmonium states $X(3872)$, $\chi_{c2}(2P)$ and $Y(4260)$ (we follow the particle data book notation). In order to be able to distinguish the possible quantum number assignments, when possible, we also report on our predictions for electromagnetic E1 transitions. We call the reader attention to the good agreement between theoretical predictions
and experimental measures of charmonium and bottomonium electromagnetic widths - see tables 4, 5,6 of [2].

3 \( X(3872) \)

In what concerns the quantum numbers of \( X(3872) \), experimentaly only the parity, \( C = +, \) is known. Belle Collaboration [4] has performed an analysis of possible quantum numbers and conclude in favor of \( J^{PC} = 1^{++}, 2^{++} \). In table 2, we report the charmonium states compatible with these quantum numbers and whose mass differs from the \( X(3872) \) by 100 MeV, including the E1 electromagnetic widths for the assignement favoured by Belle data.

4 \( \chi_{c2}(2P) \)

According to the particle data book, this is a \( J^{PC} = 2^{++} \) with a mass of 3929 ± 5 MeV. In our model, the \( J = 2 \) states around this mass value are \( 2^{3}P_{2} \), with \( J^{PC} = 2^{++} \) and mass 4048 MeV; the \( 1^{3}F_{2} \), with \( J^{PC} = 2^{++} \) and mass 3932 MeV.

If this state is a \( 2^{3}P_{2} \) state, it has a large E1 width of 140 KeV for the transition to \( \psi(2S) \) and a E1 width of 59 KeV to \( J/\psi(1S) \).

5 \( Y(4260) \)

This \( \sigma \) state is a \( J^{PC} = 1^{--} \) with a mass of 4259±10 MeV. According to the particle data book, the “interpretation as due to two interfering resonances is not excluded”. In our study, possible candidates with a mass between ~ 4160 MeV and ~ 4360 MeV are: \( 3^{3}S_{1} \), 4155 MeV; \( 2^{3}D_{2} \), 4327 MeV; \( 2^{3}D_{2} \), 4230 MeV; \( 2^{3}D_{2} \), 4230 MeV; \( 1^{3}G_{4} \), 4260; \( 1^{3}G_{3} \), 4161 MeV; \( 3^{3}P_{0} \), 4300 MeV. Only the state \( 3^{3}S_{1} \) has \( J^{PC} = 1^{--} \), as it should be for a state produced via initial state radiation.

If \( Y(4260) \) is a \( 3^{3}S_{1} \) state, its larger E1 electromagnetic widths are transitions to \( \chi(2P) \) states, namely: \( 2^{3}P_{0} \), \( \Gamma = 358 \) KeV; \( 2^{3}P_{1} \), \( \Gamma = 475 \) KeV; \( 2^{3}P_{2} \), \( \Gamma = 233 \) KeV. The corresponding widths for \( \chi(1P) \) states being: \( 1^{3}P_{0} \), \( \Gamma = 4 \) KeV; \( 1^{3}P_{1} \), \( \Gamma = 9 \) KeV; \( 1^{3}P_{2} \), \( \Gamma = 13 \) KeV, making them hard to measure. Given the values

| Mass  | \( J^{PC} \) | E1 Electromagnetic Transition  |
|-------|-------------|--------------------------------|
| 3938  | \( 1^{++} \) | \( \psi(2S) + \gamma \) 63     |
|       |             | \( J/\psi(1S) + \gamma \) 48   |
| 3932  | \( 2^{++} \) |                                |

Table 2: \( X(3872) \) possible interpretations and E1 electromagnetic transitions.
for the various widths, it seems that a combine investigation of $Y(4260)$ and $\chi_c(2P)$ could be helpfull in understanding the nature of this particle.

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