The state $|IV\rangle$ (Babel of entanglement)

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Abstract

Maybe active discussions about entanglement in quantum information science demonstrate some immaturity of this rather young area. So recent tries to look for more accurate ways of classification devote rather encouragement than criticism.

Some tries of deeper analyze of conception of entanglement recently cased an active criticism [1]. Really, the examples with quantum optics\(^1\) and the vacuum entanglement maybe too complicated for such questions. Oversimplified treatment of a behaviour of a quantum systems may be relevant with even simpler examples.

A problem — are not very accurate constructions used in many works on quantum information science in relation with the definition of a compound quantum system. It may be discussed from both mathematical and physical point of view.

From mathematical point of view, in about 99% of quantum computing papers I saw, is given “simplified” definition of the tensor product\(^2\). An accurate definition of the tensor product uses a quotient of some infinite-dimension space \(^2\). Such a definition could be considered as too cumbersome for using in the quantum computing, but it defines tensor product in basis-independent way.

Such a basis independent view is important also in the physical applications. Let us consider two spin-half systems. There is a quite relevant note of Feynman [3, 12-1] in relation with hydrogen atom, i.e., the system with the proton and electron.

The first question we have to answer is: What are the base states for the system? Now the question has been put incorrectly. There is no such thing as “the” base states, because, of course, the set of base states you may choose is not unique. New sets can always be made out of linear combinations of the old. There are always many choices for the base states, and among them, any choice is equally legitimate. So the question is not what is the base set, but what could a base set be? We can choose any one we wish for our own convenience. It is usually best to start with a base set which is physically the clearest. It may not be the solution to

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\(^1\)Photon — is massless relativistic particle with spin 1.
\(^2\)The other fresh “invention” is tensor product of Lie groups, like “$SU(2) \otimes SU(2)$”!
any problem, or may not have any direct importance, but it will generally make it easier to understand what is going on.

The first set of basic states introduced there are

\[ |1\rangle = |++\rangle, \quad |2\rangle = |+--\rangle, \quad |3\rangle = |--\rangle, \quad |4\rangle = |--\rangle. \quad (1) \]

Feynman asked next

You may say, “But the particles interact, and maybe these aren’t right base states. It sounds as though you are considering the two particles independently.” Yes, indeed! The interaction raises the problem, that is the Hamiltonian for the system, but interaction is not involved in the question, of how to describe the system. What we choose for the base states has nothing to do with what happens next. It may be that the atom cannot ever stay in one of these base states, even if it is started that way. That’s another question. That’s the question: How do the amplitudes change with time in a particular (fixed) base? In choosing the base states, we are just choosing the “unit vectors” for our description.

So, also other set of base states is introduced next \[ \text{[3, 12-3]} \]

\[ |I\rangle = |++\rangle, \quad |II\rangle = |--\rangle, \quad |III\rangle = \frac{1}{\sqrt{2}}(|+-\rangle + |-+\rangle), \]
\[ |IV\rangle = \frac{1}{\sqrt{2}}(|+-\rangle - |-+\rangle). \quad (2) \]

It is the set of stationary states, convenient for description of the dynamics of the quantum system.

The citations above may be considered in relation with the question: Why such active discussions about the entanglement was revived in quantum computing, if the “regular” quantum mechanics during about a half century managed almost without even mentioning of the term?

Really, if choosing of the base states is matter of convenience, there is no big difference between \[ |III\rangle = (|01\rangle + |10\rangle)/\sqrt{2} \] and \[ |3\rangle = |01\rangle \] and so the term “entanglement” would be rather redundant. It should be mentioned, that possibility of using arbitrary basis for a single system seems quite clear, but here was emphasized an example with the basis ambiguity problem for the compound system and not with respect to only so-called local transformations, but for arbitrary change of basis in product space.

The theory of quantum algorithms may be considered as a modification of the theory of classical algorithms using specific artificial procedure of “quantization,” then any discrete structure like the bit or the array of bits corresponds to the elements of basis in some Hilbert space. Such a construction by definition introduces some preferred bases, but it is just a problem discussed above, and it may contradict natural properties of quantum systems.

Of course, the notion of entanglement\(^3\) is very natural and convenient in the mathematical theory of quantum computations, but the fact alone is not a guarantee of relevance to quantum mechanics.

\(^3\)In the theory of quantum communications most often used more difficult conception related with the mixed states entanglement, but it is not discussed in present note.
Seems the “naïve” definition of compound system and permanent discussion about entanglement as a mysterious and wonderful phenomenon in quantum information science silently supposes rather classical point of view, that if we have two systems in the state \(|0\rangle\), it is absolutely clear, that the state \(|0\rangle|0\rangle\) is a “natural” and well defined state for a compound system, but the states like \(|IV\rangle = (|01\rangle - |10\rangle)/\sqrt{2}\) suppose some weird manipulations with such classical states like \(|01\rangle\) and \(|10\rangle\).

It is not so, say for two spin-half systems the most natural state is just \(|IV\rangle\), because it is so-called singlet state and most often it is the ground state of the quantum system. So it is strange to consider entanglement as demonstration of some nonusual properties of the system — it is much simpler to find or prepare hydrogen atom in the ground state \(|IV\rangle\), than in any non-entangled one.

References

[1] S. J. van Enk, \(|0\rangle|1\rangle + |1\rangle|0\rangle\), quant-ph/0507189 [and references therein and references in references therein]

[2] S. Lang, Algebra, (Addison-Wesley, Reading, MA, 1965).

[3] R. P. Feynman, R. B. Leighton, and M. Sands, The Feynman lectures on physics III. Quantum mechanics, (Addison-Wesley, Reading, MA, 1965).