A lower bound on the right-handed neutrino mass from leptogenesis

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Abstract

In the seesaw model, the baryon asymmetry of the Universe can be generated by the decay of the lightest right-handed neutrino, $\nu_R$. For a hierarchical spectrum of right-handed neutrinos, we show that there is a model independent upper bound on the CP asymmetry produced in these decays:

$$\epsilon < \frac{3}{8}\frac{m_{\nu_3}}{M_{\nu_R}^2} \langle H_u \rangle^2.$$ 

This implies that $\epsilon$ and the mass $M_{\nu_R}$ of the lightest right-handed neutrino are not independent parameters, as is commonly assumed. If $m_{\nu_3} = \sqrt{\Delta m_{\text{atm}}^2}$ and the $\nu_R$ are produced thermally, then leptogenesis requires a reheating temperature of the Universe $T_{\text{reh}} > M_{\nu_R} > 10^8$ GeV. Reasonable estimates of $\nu_R$ production and the subsequent washout of the asymmetry, as made by Buchmüller and Plümer, imply $M_{\nu_R} > 10^9$ GeV, and $T_{\text{reh}} > 10^{10}$ GeV. Implications for the gravitino problem are also discussed.

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1 Introduction

The discovery of neutrino oscillations has been one of the most exciting experimental results in the last years. Although a deficit in the flux of solar neutrinos, observed for the first time in the late 60s [1], suggested that neutrinos oscillate, it was not until the Super-Kamiokande experiment [2] that the oscillation hypothesis acquired strength[1]. As is widely known, these results are nicely explained if neutrinos have a small mass. Explaining why the neutrino mass scale is so small is one of the current unsolved problems in particle physics. An elegant solution is the see-saw mechanism [4], which consists on adding a heavy Majorana fermion per generation to the Standard Model (SM) particle content, singlet with respect to the SM gauge group, and coupled to the Higgs doublet through a Yukawa coupling. Although very appealing, this model suffers from a serious hierarchy problem, since the right-handed neutrinos produce a (large) quadratically-divergent radiative correction to the Higgs mass. Hence, in this letter we will concentrate on the supersymmetric version of the see-saw mechanism, that stabilizes the Higgs mass against the dangerous quadratic divergences that otherwise appear.

The leptonic part of the corresponding superpotential reads

$$W_{\text{lep}} = e_R^T Y_e L \cdot H_d^T + \nu_R^T Y_\nu L \cdot H_u - \frac{1}{2} \nu_R^T M \nu_R,$$  

where $L_i$ and $e_{Ri}$ ($i = e, \mu, \tau$) are the left-handed lepton doublet and the right-handed charged-lepton singlet, respectively, and $H_d$ ($H_u$) is the hypercharge $-1/2$ ($+1/2$) Higgs doublet. $Y_e$ and $Y_\nu$ are the Yukawa couplings that give masses to the charged leptons and generate the neutrino Dirac mass, and $M$ is a $3 \times 3$ Majorana mass matrix that does not break the SM gauge symmetry.

[1] Other hints for neutrino oscillations have been reported in [1].
It is natural to assume that the overall scale of \( \mathcal{M} \), denoted by \( M \), is much larger than the electroweak scale or any soft mass. Therefore, at low energies the right-handed neutrinos are decoupled and the corresponding effective Lagrangian contains a Majorana mass term for the left-handed neutrinos:

\[
\delta \mathcal{L}_{lep} = e_R^T Y_e L \cdot H_d - \frac{1}{2} \nu^T \mathcal{M}_\nu \nu + h.c.,
\]

(2)

with

\[
\mathcal{M}_\nu = m_D^T \mathcal{M}^{-1} m_D = Y_\nu^T \mathcal{M}^{-1} \nu (H_u^0)^2,
\]

(3)
giving neutrino masses suppressed with respect to the typical fermion masses by the inverse power of the large scale \( M \).

Another attractive feature of the see-saw mechanism is that it provides a natural mechanism to generate the observed Baryon Asymmetry of the Universe (BAU), which we parametrize as

\[
D \quad \text{satisfies} \quad \eta_B = \frac{n_B - n_{\bar{B}}}{s},
\]

with

\[
\eta_B \approx \left( \frac{\delta}{\text{B-L violating processes}} \right)
\]

This corresponds approximately to \( \Gamma_{\nu R} \) being out of equilibrium when the right-handed neutrinos decay.

Let us briefly review the mechanism of generation of the BAU through leptogenesis. At the end of inflation, a certain number density of right-handed neutrinos, \( n_{\nu_R} \), is produced, that depends on the cosmological scenario. These right-handed neutrinos decay, with a decay rate that reads, at tree level,

\[
\Gamma_D = \Gamma(\nu_R \to \ell_i H_u) + \Gamma(\nu_R \to \bar{\nu}_L H_u) = \frac{1}{8\pi} (Y_\nu Y_{\nu^\dagger})_{i,i} M_i.
\]

(4)
The out of equilibrium decay of the lightest right-handed neutrino \( \nu_{R1} \), creates a lepton asymmetry given by

\[
\eta_L = \frac{n_{\nu_L} - n_{\bar{\nu}_L}}{s} = \frac{n_{\nu_{R1}} + n_{\bar{\nu}_{R1}}}{s} \epsilon \delta.
\]

(5)
The value of \( (n_{\nu_{R1}} + n_{\bar{\nu}_{R1}})/s \) depends on the particular mechanism to generate the right-handed (s)neutrinos. On the other hand, the CP-violating parameter

\[
\epsilon_i = \frac{\Gamma_{D_i} - \bar{\Gamma}_{D_i}}{\Gamma_{D_i} + \bar{\Gamma}_{D_i}},
\]

(6)
where \( \bar{\Gamma}_{D_i} \) is the CP conjugated version of \( \Gamma_{D_i} \), is determined by the particle physics model that gives the masses and couplings of the \( \nu_R \). Finally, \( \delta \) is the fraction of the produced asymmetry that survives washout by lepton number violating interactions after \( \nu_R \) decay. To ensure \( \delta \approx 1 \), lepton number violating interactions (decays, inverse decays and scatterings) must be out of equilibrium when the right-handed neutrinos decay. This corresponds approximately to \( \Gamma_{D_i} < H |_{T=M_i} \), where \( H \) is the Hubble parameter at the temperature \( T \), and can be expressed in terms of an effective light neutrino mass \( \overline{\nu}_{L1} \), \( \overline{\nu}_{L1} \), as

\[
\overline{\nu}_{L1} = \frac{8\pi (H_u^0)^2}{M_1^2} \Gamma_{D_1} = (Y_\nu Y_{\nu^\dagger})_{11} \frac{(H_u^0)^2}{M_1} \lesssim 5 \times 10^{-3} \text{eV}.
\]

(7)

This requirement has been carefully studied \( \overline{\nu}_{L1} \); the precise numerical bound on \( \overline{\nu}_{L1} \) depends on \( M_1 \), and can be found in \( \overline{\nu}_{L1} \).

The last step is the transformation of the lepton asymmetry into a baryon asymmetry by non-perturbative B+L violating (sphaleron) processes \( \overline{\nu}_{L1} \), giving

\[
\eta_B = \frac{C}{C-1} \eta_L,
\]

(8)
where \( C \) is a number \( O(1) \), that in the Minimal Supersymmetric Standard Model takes the value \( C = 8/23 \).
Although the supersymmetric leptogenesis scenario is a very attractive framework to generate the BAU, it is not free of problems. Namely, there is a potential conflict \[14\] between the gravitino bound \[15, 16\] on the reheat temperature, and the thermal creation of right-handed neutrinos. In a plasma at high temperature, gravitinos are abundantly produced, and their late decay could modify the abundances of light nuclei, contrary to observation. This sets an upper bound on the reheat temperature that will have an important role in our discussion.

## 2 Upper bound on the CP asymmetry

As was explained in the introduction, the out of equilibrium decay of $\nu_{RS}$ generates a lepton asymmetry that is proportional to the CP asymmetry, $\epsilon$. To compute the CP asymmetry, it is convenient to work in the flavour basis in which the charged-lepton Yukawa matrix, $Y_e$, and the gauge interactions are flavour-diagonal (therefore, all the lepton flavour mixing is in $Y_e$). In this basis, the neutrino mass matrix can be diagonalized by the MNS \[17\] matrix $U$ according to

$$U^T M_\nu U = \text{diag}(m_1, m_2, m_3) \equiv D_m,$$

where $U$ is a unitary matrix that relates flavour to mass eigenstates

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}.$$  

(10)

On the other hand, one can always choose to work in a basis of right-handed neutrinos where $\mathcal{M}$ is diagonal

$$\mathcal{M} = \text{diag}(M_1, M_2, M_3) \equiv D_M,$$

with $M_i \geq 0$. In this basis, the CP asymmetry can be readily computed, yielding the result

$$\epsilon_i \simeq -\frac{1}{8\pi} \frac{1}{|Y_e Y_\nu^\dagger|_{ii}} \sum_j \text{Im} \left\{ \left[ Y_\nu Y_\nu^\dagger \right]_{ij} \right\} f \left( \frac{M_j^2}{M_i^2} \right),$$

(12)

where

$$f(x) = \sqrt{x} \left( \frac{2}{x-1} + \ln \left[ \frac{1+x}{x} \right] \right).$$

(13)

Here, we will assume that the masses of the right-handed neutrinos are hierarchical. In this case, the lepton asymmetry is essentially generated in the decay of the lightest right-handed neutrino, so is proportional to

$$\epsilon_1 \simeq -\frac{3}{8\pi} \frac{1}{|Y_e Y_\nu^\dagger|_{ii}} \sum_j \text{Im} \left\{ \left[ Y_\nu Y_\nu^\dagger \right]_{ij} \right\} \left( \frac{M_j}{M_1} \right) = -\frac{3}{8\pi} \frac{M_1}{(H_u^0)^2} \frac{1}{|Y_e Y_\nu^\dagger|_{ii}} \text{Im} \left\{ \left[ Y_\nu M_\nu^\dagger Y_\nu^T \right]_{11} \right\}. $$

(14)

The value of the CP asymmetry depends on the details of the model, however, we will show that there exists a model independent upper bound on the CP asymmetry with several interesting physical consequences.

To derive the upper bound on $|\epsilon_1|$, we will use the parametrization of the Yukawa couplings introduced in \[19\]. There, it was proved that the most general Yukawa coupling that satisfies eq.(3) is given by

$$Y_\nu = \frac{1}{(H_u^0)^2} D \sqrt{A} R D \sqrt{m} U^+,$$

(15)

where, in an obvious notation, $D \sqrt{A} \equiv +\sqrt{DA}$, and $R$ is a (complex) orthogonal matrix. Substituting in eq.(14), one gets

$$\epsilon_1 \simeq -\frac{3}{8\pi} \frac{M_1}{(H_u^0)^2} \sum_j m_j^2 \text{Im}(R_{1j}^2) \frac{\sum_j m_j R_{1j}^2}{\sum_j m_j^2}.$$

(16)

\[2\]See \[18\] for a discussion on leptogenesis with degenerate right-handed neutrinos.
Then, using the orthogonality condition $\sum_j R_{1j}^2 = 1$, it is straight-forward to show that

$$|\epsilon_1| \lesssim \frac{3}{8\pi} \frac{M_1}{\langle H_u^0 \rangle^2} (m_3 - m_1).$$

(Notice that the upper bound, like the CP asymmetry, goes to zero when the light neutrinos are degenerate in mass.) A hierarchical spectrum of right-handed neutrinos strongly suggests a spectrum of left-handed neutrinos also hierarchical, otherwise a big conspiracy would be needed between $Y_\nu$ and $M$ to produce a non-hierarchical spectrum. Therefore, we can assume $m_3 \gg m_1$, hence

$$|\epsilon_1| \lesssim \frac{3}{8\pi} \frac{M_1 m_3}{\langle H_u^0 \rangle^2}.$$  \hspace{1cm} (18)

Equations (18) and (17) are the main results of this letter. They are valid in the seesaw model with hierarchical right-handed neutrino masses, and arbitrary Yukawa matrices. A formula similar to eq.(18) can be found in [12], who estimate $\epsilon$ to be our bound, but say that unspecified cancellations can allow it to be larger. The limit eq.(18) is present in [21], who use it to argue in favour of non-thermal right-handed sneutrino production. Numerically similar bounds have been found in specific models [14, 22].

An immediate consequence of eq.(18) is that the CP asymmetry, $\epsilon$, and the lightest right-handed neutrino mass, $M_1$, are not completely independent parameters, contrary to what is often assumed in the literature. $\epsilon$ clearly depends on $M_1$, but also on the unknown couplings of the $\nu_R$, so it would seem reasonable to parametrize the predictions of leptogenesis with $\epsilon$, $M_1$ and $m_1$, regarding them as independent parameters. However, the upper bound eq.(18) sets a constraint on this parametrization: for instance, if $\epsilon > 10^{-6}$, the reference value chosen by Buchmuller and Plimacher(BP), then $M_1 > 4 \times 10^9$ GeV.

This bound is not significantly weakened as the right-handed neutrinos become less hierarchical. If $M_3 > 2M_2$ and $M_2 > 2M_1$, then

$$|\epsilon_1| \lesssim \frac{1}{2\pi} \frac{M_1}{\langle H_u^0 \rangle^2} \frac{1}{\langle Y_\nu Y_\nu^T \rangle_{11}} \text{Im}\left\{ Y_\nu^\dagger M^\dagger Y_\nu^T \right\} \lesssim \frac{1}{2\pi} \frac{M_1}{\langle H_u^0 \rangle^2} (m_3 - m_1),$$

i.e., a factor 4/3 larger than the bound eq.(17). As before, for hierarchical left-handed neutrinos, $m_3 - m_1$ can be safely approximated by $m_3$. Nonetheless, this mild hierarchy of right-handed masses could be compatible with degenerate left-handed neutrinos, with a certain amount of fine-tuning. In that case, $m_3 - m_1 \sim \Delta m_{\text{atm}}^2/2m_3$, hence, the maximum CP asymmetry decreases as the overall scale of neutrino masses increases.

So far, we have not implemented in our bound the out-of-equilibrium condition,

$$\tilde{m}_1 = \sum_j m_j |R_{1j}|^2 < 5 \times 10^{-3} \text{eV}. \hspace{1cm} (20)$$

When one does this, the bound on $\epsilon$ only becomes slightly stronger, and the improvement is numerically irrelevant. Therefore, and for the sake of clarity, we will use eq.(18) in the forthcoming discussion.

Notice that from eq.(20) we can obtain an upper bound on the lightest left-handed neutrino mass, $m_1$. There is a well-known bound on $\tilde{m}_1$, eq.(6), from requiring that lepton number violating $\nu_R$ decays be out of equilibrium at $T \sim M_1$ (to avoid washing-out any lepton asymmetry present at that time). The bound $\tilde{m}_1 < 5 \times 10^{-3}$ eV is usually applied to $m_1$, assuming that $\tilde{m}_1 \simeq m_1$. This is not immediately obvious; $m_1$ is the $\nu_{L_1}$ mass, and $\tilde{m}_1$ the rescaled $\nu_{R_1}$ decay rate. However, using the orthogonality of $R$, it is clear that $m_1 \leq \tilde{m}_1$, so this assumption is justified. This upper limit implies that, in the minimal seesaw model considered here, leptogenesis cannot generate the baryon asymmetry if the $\nu_L$ are degenerate[14, 22]. More complicated models are required [23].

## 3 Lower bound on the lightest right-handed neutrino mass

In this section we will derive a bound on $M_1$ using the lower bound on the CP asymmetry, and the information available on neutrino masses and cosmology, for the case of hierarchical right-handed and left-handed neutrinos.

3This statement supposes that the Universe is radiation dominated when the $\nu_R$ decay. If it is matter dominated (e.g. by a scalar condensate), leptogenesis might be possible.
From eqs. (8,15), one obtains

$$M_1 \approx \eta_B \frac{1 - C}{C} \left[ \frac{n_{\nu R} + n_{\nu L}}{s} \frac{3}{8 \pi (H_0^2 s)} m_3 \delta \right]^{-1}. \quad (21)$$

In this inequality, $\eta_B$ is constrained by Big Bang Nucleosynthesis to lie in the range $(0.3 - 0.9) \times 10^{-10}$, and $m_3 \approx \sqrt{\Delta m^2_{\text{atm}}}$ by atmospheric neutrino data to be within $0.04 - 0.08$ eV. The washout parameter, $\delta \lesssim 1$, should be calculated case by case by integrating the full Boltzmann equations (11). On the other hand, the remaining quantity, the right-handed neutrino density over the entropy density, depends crucially on the mechanism of generation of right-handed neutrinos: thermal production, or non-thermal production (for instance during preheating) lead to different bounds on $M_1$. Let us discuss each case separately.

**Thermal production**

In this case, the right-handed neutrinos are generated by scatterings in the thermal bath. When the number density of right-handed neutrinos is thermal at $T > M_1$, the prediction for the ratio of $n_{\nu R}$ to the entropy density, $s$, is $n_{\nu R} / s \approx 0.4 / g_s$, where $g_s \approx 230$ is the number of propagating states in the supersymmetric plasma. Assuming that the asymmetry was not washed-out after being generated ($\delta \sim 1$) gives a conservative lower bound of $M_1 \gtrsim 10^8$ GeV. However, the numerical results of BP suggest that this is improbable: if the Yukawa couplings are large enough to produce a thermal density $n_{\nu R}$, then the asymmetry will be partially washed-out by lepton number violating interactions after the decay of the right-handed neutrinos. Therefore, scaling our bound by $n_{\nu R} / s \delta \sim 0.04 / g_s$, we obtain

$$M_1 \gtrsim 10^9 \left( \frac{\eta_B}{5 \times 10^{-11}} \right) \left( \frac{0.06 eV}{m_3} \right) \left( \frac{2 \times 10^{-4}}{n_{\nu R} / s \delta} \right) \text{GeV. } \quad (22)$$

The thermal production of right-handed neutrinos requires a reheat temperature larger than $M_1$. A typical value might be $T_{rch} \sim 10 M_1$, so we get

$$T_{rch} \gtrsim 10^{10} \left( \frac{\eta_B}{5 \times 10^{-11}} \right) \left( \frac{0.06 eV}{m_3} \right) \left( \frac{2 \times 10^{-4}}{n_{\nu R} / s \delta} \right) \left( \frac{T_{rch}}{10 M_1} \right) \text{GeV, } \quad (23)$$

or, using $T_{rch} > (1 - 10)M_1$,

$$T_{rch} \gtrsim 10^8 - 10^{10} \text{ GeV } \quad (24)$$

being more probable the large values in this range, since they take into account the unavoidable washout effects. The bound (24) applies to hierarchical right-handed neutrinos, irrespective of the form of the Yukawa matrix. It is the same as the estimate made in (14) for specific texture models.

It is enlightening to compare this lower bound on the reheat temperature with the upper bound obtained from gravitino overproduction (13). (16). Gravitinos are produced by scattering in the thermal bath at a rate $\sim T^3 / m^2_{3/2}$, and then decay with a lifetime $\sim m^2_{3/2} / m^2_{\text{G}}$. Their decay products can disassociate light elements, jeopardizing the successful predictions of Big Bang Nucleosynthesis (25). To prevent this, gravitinos should not be abundantly produced, and this in turn imposes an upper bound on the reheat temperature (26):

$$T_{rch} \lesssim 10^9 - 10^{12} \text{ GeV } \quad (25)$$

The first value, $T_{rch} \sim 10^9$ GeV, is the most commonly used in the literature, although the weaker bound $T_{rch} \sim 10^{12}$ GeV, corresponding to $3 \text{ TeV} < m_{3/2} < 10 \text{ TeV}$, cannot be precluded. Gravitinos can also be produced efficiently in the oscillations of the inflaton (25), if the inflatino mixes significantly with the gravitino (26). This leads to a stronger, although model-dependent, bound on $T_{rch}$, which we do not consider.

Interestingly enough, there is a potential conflict between equations (24) and (25). If the right-handed neutrinos are produced thermally, the preferred gravitino bound $T_{rch} \lesssim 10^9$ GeV is below the preferred leptogenesis bound $T_{rch} \gtrsim 10^{10}$ GeV. There are various solutions to this possible problem. If the gravitino is heavy, $T_{rch} > 10^{10}$ GeV is consistent with BBN. It is also possible to construct models where the gravitino is the LSP, that allow reheat temperatures $< 10^{11}$ GeV (16, 25). Alternatively, the number of right-handed neutrinos produced thermally may be insufficient to generate the baryon asymmetry. In this case, other (non-thermal) production mechanisms should come into play (24).
Non-thermal production

Right-handed neutrinos could be coupled to the inflaton, and hence produced perturbatively [31, 32] in inflaton decay. A right-handed sneutrino condensate could also be generated by the Affleck-Dine mechanism [21]. In these cases, $M_1 < T_{reh}$ is not required. Then, it is clear that the gravitino problem will be easily avoided, since it is possible to create rather heavy particles with relatively low reheat temperatures, that do not endanger the predictions of Big Bang Nucleosynthesis.

There is nonetheless a bound on $M_1$ in this scenario, and on the temperature of the Universe when the $\nu_R$ decay, $T_\Gamma < M_1$. This has been considered in [21], for the decay of a $\tilde{\nu}_R$ condensate. We briefly review their discussion here, applied to the general case of non-thermally produced $\nu_R$ and $\tilde{\nu}_R$. We assume a distribution of right-handed neutrinos that decay instantaneously, at a time equal to the right-handed neutrino lifetime, to radiation that dominates the Universe. The decay products are thermalized because they have gauge interactions. Probably, the right-handed neutrinos only contribute a fraction of the energy density of the Universe, $\rho_U$, when they decay; however, the baryon to entropy ratio will be maximum if the right-handed neutrinos dominate the Universe. Hence, if we assume $\rho_U \approx \rho_{\nu_R}$, we will obtain a lower bound on $\epsilon_1$ and therefore on $M_1$. When the right-handed neutrinos decay

$$M_1 \, n_{\nu_R} \simeq \frac{g_\phi \pi^2}{30} T^4_\Gamma \sim s \, T_\Gamma,$$

which combined with eq.(21) gives [21]

$$M_1 > T_\Gamma \gtrsim 5 \times 10^5 \left( \frac{\eta_B}{5 \times 10^{-11}} \right) \left( \frac{0.06 \text{GeV}}{m_3} \right) \text{ GeV.}$$

There is a lower bound on $T_\Gamma$, because if $M_1$ is large (implying large $\epsilon_1$) but $T_\Gamma$ is small, then the entropy produced per $\nu_R$ decay is large, and dilutes the asymmetry. The bound on $T_\Gamma$ sets the minimum temperature possible in the Universe at the time that the lepton asymmetry is generated. Consequently, it also represents a bound on the temperature at which all the unwanted relics (gravitinos, moduli,...) cannot be overproduced, since any relics produced after this moment cannot be diluted by entropy production (otherwise, the baryon asymmetry would also be diluted).

A particular example of non-thermal $\nu_R$ production is to create the population of right-handed neutrinos during preheating. The previous lower limit on $M_1$, eq.(27), applies to this scenario. In addition, the lower bound on $\epsilon_1$ from eq.(18) implies that neutrinos must be relatively strongly coupled to the inflaton in the model of [31], as we will now see.

Reference [31] assumes a model of chaotic inflation, where the right-handed neutrino interacts with the inflaton, $\phi$, via $g_\phi \nu_R \nu_R$. The effective mass of the $\nu_R$ is $M + g_\phi(t)$, so, as $\phi$ oscillates, it goes through zero for sufficiently large $\phi$ oscillations. Significant numbers of $\nu_R$ can be produced while they are effectively massless. The energy density in right-handed neutrinos, divided by the energy density of the inflaton, is of order [31]

$$\frac{\rho_{\nu_R}}{\rho_\phi} \simeq \frac{4}{3 \pi^2} \frac{m_\phi^2}{\phi_0^2} q = \frac{g^2}{3 \pi^2},$$

where $m_\phi \sim 10^{-6} m_{pl}$ is the oscillation frequency of the inflaton, $\phi_0 \simeq m_{pl}/3$ is its initial value at the start of oscillations, and $q \equiv g^2 \phi_0^2 / m_\phi^2$. The final asymmetry can be estimated as [31]

$$\frac{n_{B-L}}{s} \simeq 9 \times 10^{-8} \, \epsilon_1 \, \frac{T_{reh}}{10^9 \text{GeV}} \, \frac{10^{15} \text{GeV}}{M_1} \, \frac{q}{10^{10}}.$$  

Implementing our bound in this estimate, we find

$$\frac{n_{B-L}}{s} \lesssim 2 \times 10^{-8} \, \frac{T_{reh}}{10^9 \text{GeV}} \, \frac{q}{10^{10}},$$

so $q \gtrsim 10^8$ is required. This corresponds to $g \gtrsim 0.06$.

Loopholes

There are various ways of evading our lower bound on $M_1$ and $T_{reh}$.
The right-handed neutrinos are produced by a non-thermal process, no lower bound on $M_\text{reh}$ must be at least as large as $\eta$. If right-handed neutrinos are thermally produced, then the lower bound on $T_{\text{reh}}$ from leptogenesis is not disturbing. However, the seesaw without SUSY suffers from a hierarchy problem.

4 Summary

In leptogenesis scenarios, the baryon asymmetry is generated in the out-of-equilibrium decay of right-handed neutrinos and sneutrinos in the early Universe. The asymmetry is proportional to $\epsilon_1$, which parametrizes CP violation in these decays.

We have shown that there exists a model independent upper bound on $\epsilon_1$ as a function of $M_1$:

$$|\epsilon_1| \lesssim \frac{3}{8\pi} \frac{M_1}{\langle H_u^0 \rangle^2} (m_3 - m_1).$$

(31)

where $m_3$ is the heaviest light neutrino mass (we take $(m_3 - m_1) \simeq \sqrt{\Delta m^2_{\text{atm}}}$), and $M_1$ is the lightest right-handed neutrino mass. This bound assumes hierarchical right-handed neutrino masses.

The observed baryon asymmetry $\eta_{B-L} \sim 10^{-10}$ sets a lower limit on $\epsilon_1$, and therefore on $M_1$. If the right-handed neutrinos are thermally produced, then $\eta_{B-L} \lesssim 10^{-2-3} \epsilon_1$ and $M_1 > 10^{8-9}$ GeV. The reheat temperature must be at least as large as $M_1$, so this implies $T_{\text{reh}} > 10^8 - 10^{10}$ GeV. Numerical and analytic results suggest $\eta_{B-L} \lesssim 10^{-3} \epsilon_1$ for $T_{\text{reh}} = 10 M_1$. These parameters correspond to the lower bound $T_{\text{reh}} > 10^{10}$ GeV, that in some scenarios conflicts with the upper bound from gravitino overproduction.

We briefly discussed various loopholes in the limit on $\epsilon_1$, and in the bound derived from it on $T_{\text{reh}}$. If the right-handed neutrinos are produced by a non-thermal process, $M_1$ can be much larger than $T_{\text{reh}}$, so there is no lower bound on $T_{\text{reh}}$. There are analytic approximations for the lepton asymmetry due to right-handed neutrinos generated during preheating: implementing the bound in these formulae, we find that very efficient $\nu_R$ production ($q > 10^8$) is required to get a large enough asymmetry.

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