Measuring Pancharatnam’s relative phase for SO(3) evolutions using spin polarimetry

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In polarimetry, a superposition of internal quantal states is exposed to a single Hamiltonian and information about the evolution of the quantal states is inferred from projection measurements on the final superposition. In this framework, we here extend the polarimetric test of Pancharatnam’s relative phase for spin $-\frac{1}{2}$ proposed by Wagh and Rakhecha [Phys. Lett. A 197, 112 (1995)] to spin $j \geq 1$ undergoing noncyclic SO(3) evolution. We demonstrate that the output intensity for higher spin values is a polynomial function of the corresponding spin $-\frac{1}{2}$ intensity. We further propose a general method to extract the noncyclic SO(3) phase and visibility by rigid translation of two $\pi/2$ spin flippers. Polarimetry on higher spin states may in practice be done with spin polarized atomic beams.

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I. INTRODUCTION

In polarimetric experiments, a superposition of internal quantal states evolves in a single spatial beam under a single Hamiltonian. Information about the Pancharatnam relative phase $\Phi$ is then inferred from projection measurements on the final superposition. It is thus possible to measure the relative phase without using spatially separated beams, as in interferometry. This advantage with polarimetry has proved useful in measurements of phases of quantal states. Indeed, this technique was used in the first experiments that measured the cyclic adiabatic Berry phase $\Phi$ for two-level systems in terms of polarization of light [2] and neutron spin [3, 4]. Later, a test of Pancharatnam’s relative phase in noncyclic spin $-\frac{1}{2}$ polarimetry was put forward [6] and carried out [7]. In this paper, we demonstrate that the polarimetric scheme for spin $-\frac{1}{2}$ proposed in [6] may be extended to spin $j \geq 1$ states in noncyclic SO(3) evolution.

The polarimetric advantage translates into higher precision when working with matter waves. This comes from a better utilization of the particle source in the polarimetric setup, which allows more of the incoming particles to be used in the experiment [5]. The higher effective intensity improves the precision and enables experiments with low-flux particle sources [5]. Moreover, polarimetry is a more robust method, less sensitive to spatial, mechanical, and thermal disturbances than interferometry. However, the relative phases can only be measured indirectly in polarimetry, which complicates the theoretical analysis of the measured data. Here, we show how this complication can be overcome in SO(3) polarimetry and propose a general method to extract the relative phase and visibility in such experiments.

Polarimetric tests of noncyclic relative phases for higher spin states may in practice be done with polarized atomic beams. An interesting application for such systems could be to verify the noncyclic geometric phase [8] formula $-m\Omega$ for spin projections $-j \leq m \leq j$ subtending the geodesically closed solid angle $\Omega$ in the space of directions in ordinary three dimensional space. Such an experiment would extend on the atom interferometry test of the $m$ dependence of the cyclic Berry phase carried out in Ref. [11].

In the following section, Pancharatnam’s relative phase is analyzed for spin $-j$ in SO(3) evolutions. Sec. III describes the noncyclic relative phase in spin $-\frac{1}{2}$ polarimetry and in Sec. IV it is extended to $j \geq 1$. The paper ends with the conclusions.

II. PANCHARATNAM RELATIVE PHASE FOR SO(3) EVOLUTION

The Euler representation of SO(3) evolutions may be expressed in terms of the unitarity ($\hbar = 1$ from now on)

$$U (\delta, \xi, \zeta) = e^{i(\delta + \zeta)J_z} e^{-i2\xi J_y} e^{i(\delta - \zeta)J_z}. \quad (1)$$

Here, for notational convenience we have expressed $U$ in terms of the SU(2) parameters $\delta, \xi, \zeta$ that are related to the standard Euler angles $\alpha, \beta, \gamma$ as $\delta = -(\alpha + \gamma)/2, \xi = \beta/2, \zeta = -(\alpha - \gamma)/2$. Any $J_z$ eigenket $|jm\rangle$ undergoes the SO(3) evolution

$$|jm\rangle \rightarrow U (\delta, \xi, \zeta)|jm\rangle \quad (2)$$

yielding the Pancharatnam relative phase $\Phi_m^{(j)}$ between $|jm\rangle$ and $U (\delta, \xi, \zeta)|jm\rangle$ as

$$\Phi_m^{(j)} = \arg jm |U (\delta, \xi, \zeta)|jm\rangle = 2m\delta + \arg d_m^{(j)}(\xi), \quad (3)$$

where $d_m^{(j)}(\xi) \equiv \langle jm|e^{-i2\xi J_y}|jm\rangle$ is real valued (see, e.g., Eq. (6.2.16) of Ref. [12]). This latter property implies that $\arg d_m^{(j)}(\xi)$ only takes the values 0 or $\pi$. Thus, for $m = 0$ spin projections these are the only possible values of Pancharatnam’s relative phase in SO(3) evolution.

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The amount of interference is measured by the visibility $V$, which in the SO(3) case reads

$$V^{(j)}_m = |\langle jm|U(\delta, \xi, \zeta)|jm\rangle| = |d^{(j)}_{m,n}(\xi)|. \quad (4)$$

For cyclic evolution where $\xi = 0$ (modulo $\pi$), we have maximal interference contrast $V^{(j)}_m = 1$. In the case where $m \neq 0$, the angle $\xi = \pi/2$ corresponds to the spin flip $m \rightarrow -m$ for which $V^{(j)}_m = 0$. Depending upon the explicit functional form of $d^{(j)}_{m,n}(\xi)$, there may exist further $\xi$ values for which the visibility vanishes.

We notice that if $U(\delta, \xi, \zeta)$ is parallel transporting, the Pancharatnam relative phase can be identified with the noncyclic geometric phase. In such a case $\Phi^{(j)}_m = -m\Omega$, $\Omega$ being the solid angle enclosed by the path and its shortest geodesic closure in the space of directions in ordinary three dimensional space. For example, such a parallel transporting unitarity could be realized by a sequence of SO(3) rotations along great circles in this space.

III. WAGH-RAKHECHA SETUP

Consider the Wagh-Rakhecha setup \[ Fig. 1 \] sketched in Fig. 1. A single beam of spin polarized particles with $j = m = \frac{1}{2}$ and magnetic moment $\mu$ is sent through a series of devices. A superposition of the two orthogonal states $|\frac{1}{2}, \pm \frac{1}{2}\rangle$ is created by rotating $\pi/2$ around an axis perpendicular to the quantization axis of the initial state. Under the influence of the unitarity $U$ the components of the superposition acquire opposite Pancharatnam relative phases. Another $-\pi/2$ rotation is applied and the output intensity is subsequently measured along the initial quantization axis.

![Fig. 1: Conceptual view of the Wagh-Rakhecha setup for measuring noncyclic relative phases in polarimetry. Particles spin polarized in the $z$ direction and carrying a magnetic moment $\mu$ are sent through an SO(3) unitarity, surrounded by two $\pi/2$ spin flippers. By rigid translation of the spin flippers at relative distance $L_0 = n\pi v/|\mu B|$, $n$ integer and $v$ the particle speed, the noncyclic relative phase is extracted from the output intensities registered at the analyzer.](image)

For a cyclic evolution, the output state differs from the initial state by a rotation of $\Phi^{(j)}_{\frac{1}{2}}$ about the initial quantization axis. For a noncyclic spinor evolution, however, an extra phase shift $\pm \frac{1}{2}\phi$ must be applied to the spin eigenkets $|u_{\pm}\rangle \equiv |\frac{1}{2}, \pm \frac{1}{2}\rangle$ and the relative phase can thereafter be inferred from the oscillations of the intensity as measured along the initial quantization axis, when $\phi$ varies. The extra phase shift $\phi$ is implemented by a guiding magnetic field $B\hat{z}$ put over the entire setup and the variation of $\phi$ is achieved by translating the pair of flippers, keeping their relative distance $L_0$ fixed. By choosing $L_0 = n\pi v/|\mu B|$, $n$ integer and $v$ the particle speed, one obtains the output intensity \[ Eq. (4) \]

$$I^{(j)}_{\frac{1}{2}} = \cos^2 \frac{\xi}{2} \cos^2 \delta + \sin^2 \frac{\xi}{2} \sin^2 (\frac{\pi}{2} - \phi). \quad (5)$$

This yields the extreme values

$$I^{(j)}_{\frac{1}{2}} = \cos^2 \delta \quad \text{max}$$

$$I^{(j)}_{\frac{1}{2}} = \cos^2 \delta \quad \text{min}$$

at $\phi = \zeta$ and $\phi = \zeta + \pi/2$, respectively, upon translation of the flippers. Now, up to a sign, the Pancharatnam relative phase modulo $\pi$ may be obtained as

$$\cos^2 \Phi^{(j)}_{\frac{1}{2}} = \cos^2 \delta + \arg d^{(j)}_{\frac{1}{2}, \frac{1}{2}}(\xi)$$

$$= \cos^2 \delta = \frac{I^{(j)}_{\frac{1}{2}}}{1 - I^{(j)}_{\frac{1}{2}} + I^{(j)}_{\frac{1}{2}}}. \quad (7)$$

where we have used that $\arg d^{(j)}_{\frac{1}{2}, \frac{1}{2}}(\xi)$ is an integer multiple of $\pi$. Similarly, we obtain the visibility as

$$V^{(j)}_{\frac{1}{2}} = |\cos \xi| = \sqrt{1 - I^{(j)}_{\frac{1}{2}} + I^{(j)}_{\frac{1}{2}}}. \quad (8)$$

Thus, a cyclic evolution is characterized by $I^{(j)}_{\frac{1}{2}} = I^{(j)}_{\frac{1}{2}} = \frac{1}{2}$ and spin flip corresponds to the case where $I^{(j)}_{\frac{1}{2}} = 1$ and $I^{(j)}_{\frac{1}{2}} = 0$.

The practical advantage of polarimetry may be limited by the modulo $\pi$ property of the phase measurement, as is clear from the appearance of $\cos^2 \Phi^{(j)}_{\frac{1}{2}}$ in Eq. \[ Eq. 7 \]. Physically, this arises from the final $z$ projection in polarimetry: states with opposite phases give the same $|z|$-intensity. Interferometric experiments, on the other hand, measure modulo $2\pi$, which in particular allows verification of the Pauli anticommutation, \[ Eqs. 14, 15 \] as well as the $\pi$ phase shift associated with the sign of $d^{(j)}_{\frac{1}{2}, \frac{1}{2}}(\xi)$.

IV. MEASURING HIGHER SPIN PHASES

Consider the Wagh-Rakhecha setup \[ Fig. 1 \] shown in Fig. 1, now for spin $j \geq 1$ associated with a magnetic moment $\mu$ and undergoing an arbitrary SO(3) evolution. Prepare a $J_z$ eigenket $|jm\rangle$ as input and apply a $\pi/2$ flip around the $y$ axis. Each component of the resulting superposition of $|jm\rangle$ states acquires an extra variable phase shift $\nu \phi$ implemented by the Zeeman split due to a guiding
magnetic field $B \mathbf{z}$ put over the entire setup. The SO(3) evolution is followed by a $-\frac{\pi}{2}$ flip around the $y$ axis, and the output intensity in the $|jm\rangle$ channel is detected.

This prescription corresponds to the output intensity

$$I_{j'm',m}^{(j)} = |\langle jm| \tilde{U}(\delta, \xi, \zeta, \phi) |jm\rangle|^2$$

with the unitarity

$$\tilde{U}(\delta, \xi, \zeta, \phi) = e^{\frac{i}{2} J_y} e^{\phi J_z} U(\delta, \xi, \zeta) e^{-\frac{i}{2} \phi J_y}.$$  

(10)

$I_{j'm',m}^{(j)}$ may be evaluated by introducing a decomposition of the $|jm\rangle$ state into spin-$\frac{1}{2}$ states $u_+$ and $u_-$ according to (see, e.g., Ref. [12])

$$|jm\rangle = \frac{C_{jm}}{(2j)!} \sum_P |u_+^{(1)}\rangle \otimes \ldots \otimes |u_+^{(j+m)}\rangle$$

$$\otimes |u_-^{(j+m+1)}\rangle \otimes \ldots \otimes |u_+^{(2j)}\rangle$$

(11)

with the normalization constant

$$C_{jm} \equiv \sqrt{\frac{(2j)!}{(j+m)!(j-m)!}}.$$  

(12)

and by treating the operator $\tilde{U}$ as a product of $\tilde{U}^{(i)}$ operators, each rotating the spin-$\frac{1}{2}$ subspace $i$ separately, as

$$\tilde{U} = \tilde{U}^{(1)} \otimes \ldots \otimes \tilde{U}^{(2j)}.$$  

(13)

The summation sign $P$ in Eq. (11) refers to a sum of all permutations of the labels of the $u_+$ states.

For $|jm\rangle$ states, symmetrization brings $(2j)!$ terms in the sum in Eq. (11). However, it is not necessary to work with a fully symmetrized state since the symmetrization deals only with the labeling of the $u_\pm$ states, while the number of $u_+$ and $u_-$ states remains the same. The transformation properties are thus unaffected by the symmetrization. We only need to consider simplified states of the form

$$|jm\rangle \equiv C_{jm} u_+^{j+m} u_-^{j-m}.$$  

(14)

From the spin-$\frac{1}{2}$ case we already know that $\tilde{U}^{(i)}$ acts upon $|u_+^{(i)}\rangle$ and $|u_-^{(i)}\rangle$ as

$$\tilde{U}^{(i)} |u_+^{(i)}\rangle = a |u_+^{(i)}\rangle + b |u_-^{(i)}\rangle,$$

$$\tilde{U}^{(i)} |u_-^{(i)}\rangle = a^* |u_+^{(i)}\rangle + b^* |u_-^{(i)}\rangle,$$

(15)

where

$$a = \cos \xi \cos \delta - i \sin \xi \sin (\zeta - \phi),$$

$$b = i \cos \xi \sin \delta + \sin \xi \cos (\zeta - \phi).$$

(16)

Now, $\tilde{U}$ applied to the state in Eq. (14) yields

$$\tilde{U}|jm\rangle = \sum_{m''} |jm''\rangle \frac{C_{jm}}{C_{jm''}} \sum_{\nu} \left( j + m''' \nu \right)$$

$$\times \left( j - m''' \nu \right) a^{\nu} b^{j+m'''-\nu} (-b^*)^{j+m'''-\nu}$$

$$\times (a^*)^{\nu-m'''-m''} = \sum_{m'} |jm''\rangle \tilde{U}_{j'm',m}^{(j)}.$$  

(17)

where we have used binomial expansion and the summation range of the integer $\nu$ is chosen so that the arguments of all factorials are positive. Thus, the intensity $I_{j'm',m}^{(j)}$ for an incident $|jm\rangle$ state analyzed in the $|jm\rangle$ channel reads

$$I_{j'm',m}^{(j)} = |\tilde{U}_{j'm',m}^{(j)}|^2 = \left( \frac{C_{jm}}{C_{jm''}} \right)^2$$

$$\times \left( \sum_{\nu} (-1)^{\nu} \left( j + m''' \nu \right) \left( j - m''' \nu \right) \right)$$

$$\times \left| a^{\nu} b^{j+m'''-\nu} \right|^2$$

$$= \frac{(j+m)! (j-m')!}{(j+m)! (j-m)!}$$

$$\times \sum_{\nu} (-1)^{\nu} \left( j + m''' \nu \right) \left( j - m''' \nu \right)$$

$$\times \left( I^{(j)} \right)^{\nu-m'''-m''} \left( 1 - I^{(j)} \right)^{j+m'''-\nu},$$

(18)

where we have used the identities $|a|^2 = I^{(j)}$ and $|b|^2 = 1 - I^{(j)}$. Notice that, changing the sign of one of $m, m'$ is equivalent to the change of variables $I^{(j)} \rightarrow 1 - I^{(j)}$. Changing the sign of both $m$ and $m'$ yields the same intensity due to the rotational symmetry of the setup.

We proceed by looking for extreme points by solving

$$\frac{\partial I_{j'm',m}^{(j)}}{\partial \phi} = \frac{dI_{j'm',m}^{(j)}}{dI^{(j)}} \frac{\partial I^{(j)}}{\partial \phi} = 0.$$  

(19)

From this it is evident that all $I_{j'm',m}^{(j)}$ have extreme points at $\phi = \zeta$ or $\phi = \zeta + \pi/2$ corresponding to those in the spin-$\frac{1}{2}$ case. Thus, the problem of finding $\cos^2 \delta$ and $|\cos \xi|$ is reduced to using Eqs. (7) and (8) after having determined $I^{(j)}_{\text{min}}$ and $I^{(j)}_{\text{max}}$ from the measured intensities.

When $\cos^2 \delta$ has been found we may determine the desired value of $\cos^2 \Phi_{j'}$ in terms of Chebyshev polynomials as

$$\cos^2 \Phi_{j'} = \cos^2 \left( 2m\delta + \arg d_{m',m}(\xi) \right)$$

$$= \cos^2 \left( 2m\delta \right) = \left| T_{2m}(\cos \delta) \right|^2,$$

(20)

where we have used that $\arg d_{m',m}(\xi)$ is an integer multiple of $\pi$. The first few cases are

$$\cos^2 \Phi_{j=\frac{1}{2}} = \cos^2 \delta,$$

$$\cos^2 \Phi_{j=1} = \left( -1 + 2 \cos^2 \delta \right),$$

$$\cos^2 \Phi_{j=\frac{3}{2}} = \left( -3 \cos \delta + 4 \cos^3 \delta \right)^2.$$  

(21)

Notice here that for all $m = 0$, the noncyclic relative phase is trivially $0$ or $\pi$ independent of $\delta$. Thus, although the intensity and $\cos^2 \delta$ can be calculated for the $m = 0$
cases, they are unrelated to the Pancharatnam relative phase because \( \delta \) does not appear in the phase expression.

We may also obtain the visibility as a function of \( |\cos \xi| \) from the standard expression of the matrix elements \( d_{m,m}(\xi) \). It yields

\[
\nu_m^{(j)} = \nu_{-m}^{(j)} = \left| \sum_{\nu} (-1)^{\nu+j+m} \binom{j+m}{\nu} \binom{j-m}{j+m-\nu} \frac{1}{\cos^2 \xi} \nu^{-m} \left( 1 - \cos^2 \xi \right)^{j+m-\nu} \right|. \tag{22}
\]

The first few cases are

\[
\begin{align*}
\nu_{\frac{1}{2}}^{(j)} &= |\cos \xi|, \\
\nu_{0}^{(j)} &= |2 \cos^2 \xi - 1|, \\
\nu_{1}^{(j)} &= \cos^2 \xi, \\
\nu_{\frac{1}{2}}^{(j)} &= |3 \cos^3 \xi - 2 \cos \xi|, \\
\nu_{\frac{3}{2}}^{(j)} &= |\cos^3 \xi|.
\end{align*}
\tag{23}
\]

Notice here that the visibilities for \( m = 0 \) are well-defined in terms of \( |\cos \xi| \).

In general, it is impossible to find closed expressions for \( \cos^2 \delta \) in terms of the measured intensities. An interesting exception, however, is the case of spin-coherent states \( |m=0; \xi\rangle \), characterized by \( m=j \). For such states, the intensity reads

\[
I_{j,j}^{(j)} = \left( I_{j,j}^{(\frac{1}{2})} \right)^{2j}, \tag{24}
\]

which allows a direct evaluation of \( \cos^2 \delta \) and \( |\cos \xi| \) in terms of the extreme values of \( I_{j,j}^{(j)} \) according to

\[
\begin{align*}
\cos^2 \delta &= 1 - 2 I_{j,j;\min}^{(j)} \left( 1 - 2 I_{j,j;\max}^{(j)} \right) \\
|\cos \xi| &= \sqrt{1 - 2 I_{j,j;\max}^{(j)} + 2 I_{j,j;\min}^{(j)}}.
\end{align*}
\tag{25}
\]

The reason for this is that when \( m = j \) the state in Eq. \( 14 \) consists of a term with all spin components in the same direction, i.e.,

\[
|jj\rangle = |u_+^{(1)}\rangle \otimes \ldots \otimes |u_+^{(2j)}\rangle.
\tag{26}
\]

This implies that the intensity for the \( |jj\rangle \) state is nothing but the product of \( 2j \) spin-\( \frac{1}{2} \) intensities.

In the \( 0 < m \neq j \) case, the measured intensity could be a nonmonotonous polynomial function of the spin-\( \frac{1}{2} \) intensity. Thus, there is in general many possible \( I_{\frac{1}{2}}^{(j)} \) values for a given measured intensity. To remove this ambiguity we may use several intensity profiles. To illustrate this point let us consider the \( j = \frac{3}{2} \) case, where we have

\[
\begin{align*}
I_{\frac{1}{2}}^{(j)} &= 3 \left( I_{\frac{1}{2}}^{(\frac{1}{2})} \right)^2 \left( 1 - I_{\frac{1}{2}}^{(\frac{1}{2})} \right), \\
I_{\frac{3}{2}}^{(j)} &= I_{\frac{1}{2}}^{(\frac{1}{2})} \left( 3 I_{\frac{1}{2}}^{(\frac{1}{2})} - 2 \right)^2. \tag{27}
\end{align*}
\]

Both these intensities have extreme values at \( \phi = \zeta \) and \( \phi = \zeta + \pi/2 \), as shown in Fig. 2 for \( \delta = \zeta = \xi = \pi/5 \).
These extreme values correspond to the horizontal lines in Fig. 3 whose intersections with the theoretical curves in Eq. 29 can be matched to give the solutions $I_{\min}^{(\frac{1}{2})} \approx 0.43$ and $I_{\max}^{(\frac{1}{2})} \approx 0.77$ from which we obtain $\cos^2 \delta = \cos^2 \Phi^{(\frac{1}{2})} \approx 0.65$ and $|\cos \xi| \approx 0.81$ by using Eqs. 14 and 15. The latter value may be used to compute the visibility as $V^{(\frac{1}{2})} = |3\cos^3 \xi - 2\cos \xi| \approx 0.03$.

We finally notice that there might be cases where the problem in assigning unique extreme spin-$\frac{1}{2}$ values cannot be resolved, because of low visibility and the finite precision in the experimental data. For example, due to the almost vanishing visibility in the $(j, m) = (\frac{3}{2}, \frac{3}{2})$ case discussed above, there are crossings near $I^{(\frac{1}{2})} \approx 0.53$ in Fig. 3 that in any real experiment would potentially be difficult to tell that they in fact correspond to a spurious solution. This problem may be overcome either by increasing the resolution of the experiment or by looking at the intensity in more than two output channels. It is likely that in most cases, any ambiguity of this kind can be resolved in this way.

V. CONCLUSIONS

We have extended the polarimetric setup proposed for spin-$\frac{1}{2}$ in Ref. 4 and implemented in Ref. 5 for the same case, to spin $j \geq 1$ in noncyclic SO(3) evolution. The key feature that makes it possible to extend the noncyclic relative phase and visibility in such experiments is that the output intensity for any spin value is a polynomial function of the corresponding spin-$\frac{1}{2}$ intensity. This entails that the existence of phase shifts at distance $\pi/2$ corresponding to extreme intensities is general, and these extreme values can in turn be used to extract the desired quantities. This procedure becomes particularly simple in the case of spin-coherent states, where the noncyclic relative phase and visibility can be expressed directly in terms of the measured intensities. However, in the general case, such closed expressions do not exist and measurements in several output channels is needed.

An apparent extension of the present work is to consider polarimetric tests of the noncyclic relative phase for spin $j \geq 1$ in SU(2$j + 1$) evolution. Previously, the phase in the SU(3) case has been analyzed in Ref. 17, 18 and a three-channel optical interferometry experiment of the cyclic SU(3) geometric phase has been proposed in Ref. 19. From the perspective of the special unitary group, it is indeed expected that the Wagh-Rakhecha setup 6 should be possible to extend to higher spin in SO(3) evolution since the SU(2) group is locally isomorphic to the group of rotations in three dimensional space. However, no such isomorphism exists for higher $j$ and thus there is no simple way to extend the SU(2) method to higher SU(2$j + 1$) evolutions using the Wagh-Rakhecha setup, because an SU(2$j + 1 \geq 3$) operator does not work only as a rotation operator.

We hope that the present work will lead to further considerations of polarimetric phase measurements, in particular in connection to special unitary transformations on finite dimensional Hilbert spaces, as well as to experiments that tests the Pancharatnam relative phase using, e.g., polarized atomic beams.

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