Nuclear dependence in transverse momentum distribution for Drell-Yan pair

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In terms of multiple scattering picture, I compute the nuclear dependence in Drell-Yan transverse momentum distribution, \( \frac{d\sigma}{dQ^2 dq_T^2} \), in hadron-nucleus collisions. I present the results for large \( q_T \) region and discuss the possible suppression in small \( q_T \) region.

1. INTRODUCTION

Recently, it was found experimentally that the transverse momentum distribution of Drell-Yan pair in hadron-nucleus collisions shows nuclear dependence not linearly proportional to \( A \), the atomic number of the target [1]. In addition, the average of squared transverse momentum, \( \langle q_T^2 \rangle \), grows approximately as \( A^{1/3} \) [1,2]. This observed non-linear nuclear dependence indicates that multiple-scattering is important in hadron-nucleus collisions. In this talk, I present a calculation of double-scattering in QCD perturbation theory for nuclear dependence of Drell-Yan \( q_T^2 \) spectrum, \( \frac{d\sigma}{dQ^2 dq_T^2} \).

Although double-scattering is in principle a power correction (or known as a high twist effect) to the leading single scattering, it is not necessary small because of \( A^{1/3} \) enhancement from large nuclear size. In terms of QCD factorization generalized to higher twist [3], Luo, Qiu and Sterman (LQS) developed a consistent perturbative treatment of multiple scattering at parton level [4]. Since Drell-Yan pair does not interact strongly once produced, the observed non-linear nuclear dependence is a result of multiple scattering between the incoming beam parton and nuclear matter before the pair was produced. Drell-Yan \( \frac{d\sigma}{dQ^2 dq_T^2} \) has two observed physical scales: \( Q^2 \) and \( q_T^2 \). When \( Q^2 \) and \( q_T^2 \) are both large, \( \frac{d\sigma}{dQ^2 dq_T^2} \) is effectively an one-scale process. Method developed by LQS [4] for double scattering can be naturally applied. However, when \( q_T^2 \ll Q^2 \), a resummation of large \( \ell n(Q^2/q_T^2) \) is necessary for a reliable prediction of Drell-Yan \( q_T^2 \) spectrum [3].

2. PARTON LEVEL DOUBLE SCATTERING

Consider the Drell-Yan process in hadron-nucleus collisions, \( h(p') + A(p_A) \rightarrow l\bar{l}(q) + X \), where the lepton pair has invariant mass \( (Q) \) and transverse momentum \( (q_T) \). As an example, the scattering amplitude for the lowest order double-scattering, as shown in Fig. 1, has the following general form:

\[
M \sim \int dx_1 \frac{1}{x_1 - x_{1A} + i\epsilon} \frac{1}{x_1 - x_{1B} + i\epsilon} F(x_1, x),
\]

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where \( x_1 \) and \( x \) are parton momentum fractions, as labeled in Fig. 1, and \( x_1 \) needs to be integrated. The sum of \( x_1 \) and \( x \) is fixed by the kinematics, such as \( x'p' \), \( p_3 \) and \( q \).

In Eq. (2), the function \( F(x_1, x) \) is a non-vanishing and smooth function when \( x_1 = x_{1A} \) and/or \( x_1 = x_{1B} \), and it is proportional to the parton fields of momentum \( x_1 p \) and \( x p \) with \( p \equiv p_A / A \). Taking the pole at \( x_1 = x_{1A} \) (or at \( x_1 = x_{1B} \)) corresponds to putting the propagator labeled by “A” (or “B”) in Fig. 1 on its mass shell. In the region of interests \((x's - 2p_3 \cdot p > 0)\), both potential poles in Eq. (2) are in the same half of the complex plane. When \( q_T \neq 0, x_{1A} \neq x_{1B} \). Contour integration of \( dx_1 \) yields

\[
M \sim \frac{F(x_{1A}, x_{tot} - x_{1A})}{x_{1A} - x_{1B}} - \frac{F(x_{1B}, x_{tot} - x_{1B})}{x_{1A} - x_{1B}} \sim M_{\text{soft-hard}} - M_{\text{double-hard}},
\]

where \( x_{tot} \) is the sum of the total momentum fraction from the target, and is a function of \( x' \), \( p_3 \) and \( q \). Note that two amplitudes in Eq. (2) have the opposite sign.

Amplitude \( M_{\text{soft-hard}} \) corresponds to the residue at the pole “A”, at which \( x_1 \approx 0 \) and \( x \approx x_{tot} \). We call this type of double-scattering a soft-hard scattering. In this case, the first scattering is effectively soft and not localized. It represents a long range correlation of the color field inside the nucleus. The second scattering, on the other hand, is localized in a distance space \( \sim 1/x_{tot} p \sim 1/Q \). Because of the long range correlation of color fields, the soft-hard double-scattering does not have the classical double-scattering picture. Its contribution to the cross section can not be expressed in terms of a product of two localized partonic cross sections.

Amplitude \( M_{\text{double-hard}} \) in Eq. (2) corresponds to the residue at the pole “B”. When \( q_T \neq 0 \), both \( x_1 = x_{1B} \) and \( x = x_{tot} - x_{1B} \) are finite. We call this type of double-scattering a double-hard scattering. In this case, both partonic scatterings are hard, and they are localized at a distance \( \sim 1/x_{1B} p \) and \( \sim 1/(x_{tot} - x_{1B}) p \), respectively. Such a double-hard subprocess resembles the classical double-scattering picture. Its contribution to the cross section can be expressed in terms of a product of two localized partonic cross sections.

From Eq. (2), we see that the double-scattering contribution to the cross section has three types of contributions: the soft-hard \( \sim M_{\text{soft-hard}}^2 \), the double-hard \( \sim M_{\text{double-hard}}^2 \),
and the contribution from the interference terms. The interference contribution has an opposite sign in comparison with the other two contributions. Consequently, the interference terms give nuclear suppression, while both the soft-hard and double-hard terms give the nuclear enhancement. From Eq. (2), the interference contribution is proportional to \( F^*(0, x_{tot})F(x_{1B}, x_{tot} - x_{1B}) \) plus its complex conjugate. When \( q_T \) is large, \( x_{1B} \neq 0 \), overlap in phase space for \( F^* \) and \( F \) is clearly small because of the difference in parton momenta. However, when \( q_T \rightarrow 0 \), \( x_{1B} \sim O(q_T^2/Q^2) \rightarrow 0 \). The overlap becomes large and the interference term becomes more important.

3. ENHANCEMENT IN LARGE \( q_T \) REGION

In large \( q_T \) region, the interference between the soft-hard and the double-hard subprocesses is not important, and consequently, the double-scattering contribution gives nuclear enhancement to the Drell-Yan transverse momentum distribution. The contribution from a double-hard subprocess can be expressed as [3]:

\[
\frac{d\sigma^{DH}}{dQ^2dq_T^2dy} = \sum_{a,b,c=q,g,u} \int dx' f_{c/h}(x') T_{ab}^{DH}(x_a, x_b, A) \cdot \frac{1}{2x'x_b} \cdot H_{bc}^{DH}(x_b, x') \times \frac{12\pi \alpha_s^2}{x's + u - Q^2 - u} \cdot \left( \frac{4\pi \alpha_{em}^2}{9Q^2} e_a^2 \right) .
\]

(3)

In Eq. (3), \( f_{c/h}(x') \) is the parton distribution of parton “c” from the beam hadron \( h \), and \( x_a, x_b \) are the momentum fractions of the partons “a” and “b” from the target respectively. The \( s, t, u \) are Mandelstam variables defined as

\[
s = (p + p')^2 = 2p \cdot p' ; \quad t = (p' - q)^2 = -2p' \cdot q + Q^2 ; \quad u = (p - q)^2 = -2p \cdot q + Q^2 .
\]

(4)

In Eq. (3), \( H_{bc}^{DH}(x_b, x') \) represents the calculated partonic part. Function \( T_{ab}^{DH}(x_a, x_b, A) \) is a four parton matrix element (or correlation function) [3]. For a cold nucleus, we use the following model [3]:

\[
T_{ab}^{DH}(x_a, x_b, A) = CA^{1/3} f_a/N(x_a)f_b/N(x_b),
\]

(5)

where \( C = 0.35/(4r_0^2) \text{GeV}^2 \), with \( r_0 = 1.1 - 1.25 \). At finite temperature, \( T_{ab}^{DH} \) depends on the density matrix at the given temperature.

For a soft-hard subprocess, the contribution has the following general form [3]:

\[
\frac{d\sigma^{SH}}{dQ^2dq_T^2dy} = \sum_{a,b,c} \int dx' f_{c/h}(x') \Phi_{ac}^{SH}(x_a, x', A) \cdot \frac{1}{2x's} \cdot \frac{12\pi \alpha_s^2}{x's + u - Q^2} \cdot \left( \frac{4\pi \alpha_{em}^2}{9Q^2} e_a^2 \right) .
\]

(6)

with

\[
\Phi_{ac}^{SH}(x_a, x', A) = \left[ \frac{\partial^2}{\partial x_a^2} \left( \frac{1}{x_a} T_a(x_a, A) H_{ac}^{SH}(x_a, x') \right) \right] \cdot \frac{2q_T^2}{(x's + u - Q^2)^2} + \left[ \frac{\partial}{\partial x_a} \left( \frac{1}{x_a} T_a(x_a, A) H_{ac}^{SH}(x_a, x') \right) \right] \cdot \frac{2(Q^2 - u)}{x's(x's + u - Q^2)} .
\]

(7)
In Eq. (7), $H^{SH}_{ac}$ is the calculated partonic part, and $T_a(x_a, A)$ is the soft-hard matrix element. For a cold nucleus, LQS proposed the following model [4]:

$$T_a(x_a, A) = \lambda^2 A^{4/3} f_{a/N}(x_a),$$

(8)

where $\lambda^2 \sim 0.05 - 0.1 \text{ GeV}^2$, and $\lambda$ is estimated from the data on di-jet momentum imbalance [8].

Combining contributions from double-hard and soft-hard subprocesses, and using the models for $T_{DH}^{ab}$ and $T_a$, large nuclear enhancement for Drell-Yan production in hadron-nucleus collisions was obtained [3]. A typical enhancement in large $q_T$ region is shown in Fig. 2.

4. SUPPRESSION IN SMALL $q_T$ REGION

When $q_T \to 0$, $x_{1A} \sim x_{1B} \sim 0$, both soft-hard and double-hard contributions become divergent. But these divergences are canceled by the interference terms from Eq. (2), because

$$M \rightarrow \left( \frac{F(x_{1A}, x_{tot} - x_{1A})}{x_{1A} - x_{1B}} \right)_{\text{soft-hard}} - \left( \frac{F(x_{1B}, x_{tot} - x_{1B})}{x_{1A} - x_{1B}} \right)_{\text{double-hard}} \rightarrow \text{finite},$$

(9)

as $q_T \to 0$. In addition, similar to the single-scattering, double-scattering subprocesses develop the collinear and soft divergences when $q_T \to 0$. Although all divergences are canceled after including necessary virtual diagrams and proper collinear subtractions, there are large $\ln(Q^2/q_T^2)$ for every power of $\alpha_s$. A systematic resummation at the presence of double-scattering is necessary for a reliable prediction in small $q_T$ region [3].

Because of very small $A$-dependence in Drell-Yan $d\sigma/dQ^2$, and large nuclear enhancement for large $q_T$ region, one expects nuclear suppression in small $q_T$ region. This is consistent with above discussion on the role of quantum interference between the soft-hard and double-hard subprocesses. It is known that nuclear shadowing in the parton distribution of momentum fraction $x$ is due to quantum interference of multi-parton recombination in the longitudinal direction [4]. If the same thought is applied to the transverse direction, a nuclear suppression in small $q_T$ region is expected from quantum interference between soft-hard and double-hard subprocesses. The expected ratio of double-scattering to single-scattering contribution in small $q_T$ region is sketched in Fig. 2 [4].

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