Primordial Black Holes and Secondary Gravitational Waves from Higgs field

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Abstract

We devise a novel mechanism and for the first time illustrate that the Higgs model in particle physics can drive the inflation to satisfy CMB observations and simultaneously enhance the curvature perturbations at small scales to explain the abundance of dark matter in our universe created in the form of primordial black holes. The production of primordial black holes is accompanied by the second order gravitational waves induced by the first order Higgs fluctuations which is expected observable by space based gravitational wave detectors. We propose possible cosmological probes of Higgs field in the future observations for primordial black holes dark matter or stochastic gravitational waves.
INTRODUCTION

The detections of gravitational wave (GW) by the Laser Interferometer Gravitational Wave Observatory (LIGO) Scientific Collaboration and the Virgo Collaboration [1–7] started a new era of multimessenger astronomy. It was argued that GW observations can disclose the property of primordial black holes (PBHs) which could be explained as dark matter (DM) [8–10]. PBHs can be formed through gravitational collapses of highly overdense inhomogeneities with density contrast exceeding the threshold value at horizon reentry in the radiation era [11, 12]. Such large density contrast can arise from the primordial curvature perturbations in inflation. To produce abundant PBH DM, the power spectrum of the primordial curvature perturbations is required to reach the order $A_s \sim \mathcal{O}(0.01)$ [13–15]. However, the constrained amplitude of the scalar power spectrum from Planck 2018 measurements of the cosmic microwave background (CMB) anisotropy is $A_s = 2.1 \times 10^{-9}$ at the pivot scale $k_\ast = 0.05 \text{ Mpc}^{-1}$ [16]. In order to produce PBHs, we need mechanisms to enhance the curvature power spectrum at small scales by seven orders of magnitude of the CMB observed value. Such enhancement will induce second order GWs (SGWs) after the horizon reentry [17–30]. The observations of PBH DM and SGWs can provide novel probes of physics in the early universe.

Assuming the Higgs boson as inflaton and the inflationary Higgs potential in the form $\lambda \phi^4/4$, we find too small scalar spectral tilt $n_s$ and too big tensor scalar ratio $r$ to be allowed by CMB observations. To reduce $r$, a nonminimal coupling $\xi \phi^2 R$ between Higgs field and gravity was introduced in Higgs inflation [31, 32]. However this nonminimal coupling cannot provide strong enough curvature power spectrum at small scales. A new Higgs inflation model with non-minimally derivative coupling $G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi / M^2$ between the kinetic term of the Higgs field and Einstein tensor $G^{\mu\nu}$ was introduced to reconcile the observables $n_s$ and $r$ to satisfy CMB observations [33–35], unfortunately it cannot generate large enough power spectrum at small scales. There are other ways to reduce the tensor to scalar ratio $r$, for example in the Gauss-Bonnet inflation, but it requires a special relation between the inflationary potential and the coupling between the inflaton and the Gauss-Bonnet term [36]. In a single field inflation, it was claimed difficult to enhance the amplitude of the power spectrum to the order $\mathcal{O}(0.01)$ while keep the total e-folding number $N \simeq 50 - 60$ [39, 40]. Adopting the observed values of Higgs boson and top quark masses, the coupling
\( \lambda \) in the Higgs potential is allowed to become negative from the running of the Higgs self-coupling via the renormalization group equations. In critical Higgs inflation [37], near the critical point \( \lambda = \beta_\lambda = 0 \), the curvature power spectrum can be enhanced around the inflection point in the Higgs potential [38], however such an enhancement is again not big enough which is less than five orders of magnitude of the CMB measurement. Generalizing the coupling \( 1/M^2 \) to a special function \( g(\phi) = d/\sqrt{1 + (\phi - \phi_r)^2/c^2} \) in the non-minimally derivative coupling, an enhancement of the CMB power spectrum up to seven orders of magnitude at small scales was achieved, but the price to pay is to restrict the potential to be in the specific form \( \phi^{2/5} \) [41]. Further attempt by including the non-canonical kinetic term, similar to what done in k inflation [42, 43] and G inflation [44], was proposed to increase the curvature perturbation and achieve abundant production of PBH DM and SGWs [45]. In this mechanism the non-canonical kinetic term can succeed enhancing the perturbation power spectrum at small scales while keeping such effect negligible at large scales. However, such enhancement contributes up to 20 e-folds in inflation which effectively moves the field value \( \phi_* \) corresponding to the pivotal scale closer to the value at the end of inflation \( \phi_e \). This in turn leads the observed \( n_s \) and \( r \) inconsistent with CMB observations. It is fair to conclude that so far there is no available Higgs mechanism that can successfully satisfy observational requirements of inflation at large scales and simultaneously enhance the power spectrum at small scales.

Putting aside the detailed dynamics and mechanism of inflation, when the Higgs field stays in the unstable phase of the Higgs potential during inflation, the quantum fluctuations of the Higgs field can produce abundant PBH DM [46]. In this mechanism, the Higgs field is not responsible for inflation. The reasonable question we intend to ask is whether the standard Higgs field model can drive inflation and produce abundant PBH DM without introducing other fields beyond the standard model. In this Letter, we devise a novel way in the framework of a single field inflation model with Higgs potential to enhance the primordial curvature perturbation at small scales while keep it negligible at large scales. We will show that this model is consistent with Planck 2018 data and can produce a significant abundance of PBH DM and SGWs to be detected by the future space-based GW detectors such as LISA [47, 48], TianQin [49], and TaiJi [50]. In our mechanism, the Higgs field not only drives inflation but also is responsible for the PBH DM content of our universe. It is interesting to note that our mechanism does not only work for the Higgs field, it is a general single field
inflationary model to explain the abundance of PBH DM and can be generalized to other inflationary field models, for example the T-model.

**THE ENHANCEMENT MECHANISM**

For a slow roll inflation with the non-canonical kinetic term \([1 + G(\phi)]\dot{\phi}^2/2\), the power spectrum of the primordial curvature perturbation is

\[
P_\zeta = \frac{H^4}{4\pi^2\dot{\phi}^2(1 + G)} \approx \frac{V^3}{12\pi^2 V^2_\phi}(1 + G), \tag{1}
\]

where \(V_\phi = dV/d\phi\) and the non-canonical kinetic term may arise from scalar tensor theory of gravity, G inflation \([44]\) or k inflation \([42, 43]\). If the function \(G(\phi)\) has a peak, then the power spectrum can be enhanced. Motivated by the \(\omega(\phi) = 1/\phi\) coupling in Brans-Dicke theory \([51]\), the function

\[
G(\phi) = G_a(\phi) = \frac{d}{1 + |\phi - \phi_p|/c} \tag{2}
\]

is used to enhance the power spectrum so as to produce abundant PBH DM and observable SGWs \([45]\), where \(d \sim \mathcal{O}(10^9)\) gives the amplitude of the peak, \(c \sim \mathcal{O}(10^{-10})\) controls the width of the peak and the number of e-folds before the end of inflation at the horizon exit for the pivotal scale, \(\phi_p\) determines the position of the peak which is related with the peak mass of PBH and the peak frequency of SGWs. Away from the peak, \(|\phi - \phi_p|/c \gg 1\), the function \(G_a(\phi)\) becomes negligible and the usual slow roll inflation resumes. At the horizon exit, the number of e-folds remaining in the inflation is

\[
N = \int_{\phi_e}^{\phi_*} (1 + G) \frac{V}{V_\phi} d\phi, \tag{3}
\]

where \(\phi_*\) is the field value at the horizon exit and \(\phi_e\) is the field value at the end of inflation. Due to the non-canonical term \(G\), the peak in \(G(\phi)\) contributes up to \(\sim 20\) e-folds, which effectively moves \(\phi_*\) closer to \(\phi_e\) in order to keep the total number of e-folds around 60. The effective e-folds contributed by the standard slow roll inflation then reduces to around 40, so that \(n_s\) and \(r\) in this mechanism become incompatible with CMB observations, if we choose allowed inflationary potentials in standard viable models, but this is the price to pay for the enhancement of the power spectrum at small scales due to the non-canonical coupling \(G(\phi)\). In particular, this mechanism does not work for the Higgs field.
We devise a new mechanism to enhance the primordial curvature perturbation at small scales while at the same time predict $n_s$ and $r$ from Higgs potential in consistent with Planck 2018 data. We invent a new coupling function $f(\phi)$ which has the chameleon effect to enhance the curvature perturbations at small scale, while at large scale it can adjust the predictions of $n_s$ and $r$ to meet CMB measurements. The non-canonical term, which might come from some kinds of scalar tensor theory of gravity, becomes

$$G = G_a + f(\phi). \quad (4)$$

In the end of the inflation, the scalar field rolls down to its minimum, the non-canonical term becomes negligible. In our new mechanism, the function $G_a(\phi)$ is general but not restricted to the form in Eq. (2). Introducing the function $f(\phi)$ we can modify the shape of the potential, so that when it is away from the peak $\phi_p$, the effect of $G_a(\phi)$ is negligible and the function $f(\phi)$ dominates. We can change the non-canonical field $\phi$ to the canonical field $\Phi$ by the transformation $d\Phi = \sqrt{f(\phi)}d\phi$. In terms of the canonical field, the potential changes to $U(\Phi) = V[\phi(\Phi)]$. To show how the mechanism works, without loss of generality, we take the potential $U(\Phi)$ in a power law form $U(\Phi) = U_0\Phi^n$. We have

$$n_s = 1 - \frac{n + 2}{2N}, \quad (5)$$
$$r = \frac{4n}{N}. \quad (6)$$

Without the enhancement in small scale curvature perturbation, $N \sim 60$, it is easy to see that no chaotic inflation is consistent with observational constraints. However when there is the enhancement, the effective number of e-folds $N$ for the canonical field is around 40, so that taking $n = 1/3$, we get $n_s = 0.971$ and $r = 0.033$. If we take $n = 2/3$, we get $n_s = 0.967$ and $r = 0.067$. Therefore, depending on the function $G_a(\phi)$ and the model parameters, it is possible that the predictions of these models are consistent with CMB constraints $n_s = 0.9649 \pm 0.0042$ (68% CL) and $r_{0.05} < 0.06$ (95% CL) [52]. Given the power law form for $U(\Phi)$ and $V(\phi)$, we can get the function $f(\phi)$,

$$f(\phi) = \frac{1}{n^2} \left( \frac{1}{U_0} \right)^{\frac{2}{n}} V^{\frac{2}{n} - 2} V_\phi^2. \quad (7)$$

From the above argument, we see that our mechanism does not restrict to a specific potential form. Now we show how the Higgs potential can be used to drive inflation successfully in
consistent with CMB observations and generate large peak in the curvature perturbations at small scales.

For the Higgs potential \( V = \lambda \phi^4 / 4 \), we first take the model \( U(\Phi) = U_0 \Phi^n \) with \( n = 1/3 \) as an example and label it as H1. In this case, the function \( f(\phi) = f_0 \phi^{22} \) with \( f_0 = 9(\lambda / U_0)^6 / 256 \). In the low energy regime after inflation, the Higgs field runs away from the peak and the function \( f(\phi) \) becomes negligible leading to the negligible non-canonical term. Choosing the parameters \( c, d, \phi_*, \phi_p, \lambda \) and \( f_0 \) as shown in Table I, and solving the equations for the background and the perturbations numerically, we get \( n_s = 0.9686 \), \( r = 0.0374 \) and \( N = 55.3 \). The chosen parameter set and the results are shown in Tables I and II. The power spectrum of the primordial curvature perturbations is shown in Fig. 1.

When the overdense region generated by the primordial curvature perturbations reenters the horizon in radiation era, it can be a seed to cause gravitational collapse to form PBHs. The current fractional energy density of PBHs with mass \( M \) to DM is \([13, 53]\)

\[
Y_{\text{PBH}}(M) = \frac{\beta(M)}{3.94 \times 10^{-9} \left( \frac{\gamma}{0.2} \right)^{1/2} \left( \frac{g_*}{10.75} \right)^{-1/4}}
\times \left( \frac{0.12}{\Omega_{\text{DM}} h^2} \right) \left( \frac{M}{M_\odot} \right)^{-1/2},
\]

where \( M_\odot \) is the solar mass, \( \gamma = 0.2 \) \([54]\), \( g_* \) is the effective degrees of freedom at the formation time, \( \Omega_{\text{DM}} \) is the current energy density parameter of DM, the fractional energy density of PBHs at the formation is related to the power spectrum of the primordial curvature perturbations as \([55–57]\)

\[
\beta(M) \approx \sqrt{\frac{2}{\pi}} \frac{\sqrt{P_\zeta}}{\mu_c} \exp \left( -\frac{\mu_c^2}{2P_\zeta} \right),
\]

where \( \mu_c = 9\delta_c / 4 \) and \( \delta_c \) is the critical density perturbation for the PBH formation. We take \( \Omega_{\text{DM}} h^2 = 0.12 \) \([58]\) and \( \delta_c = 0.4 \) \([57, 59–62]\). Substituting the power spectrum into Eq. (8), we get the abundance of PBH DM and the result is shown in Fig. 2. We also show the peak mass and the peak abundance of PBH DM in Table II.

Accompanied by the production of PBHs, the scalar perturbations can induce SGWs during radiation. The equation for the Fourier components of the second order tensor perturbations \( h_k \) is \([19, 20]\)

\[
h_k'' + 2H h_k' + k^2 h_k = 4S_k,
\]
where $h'_k = dh_k/d\eta$, the scalar source

$$S_k = \int \frac{d^3\tilde{k}}{(2\pi)^{3/2}} e_{ij}(k)|\tilde{k}^i\tilde{k}^j| \left[ 2\Phi_k\Phi_{k-\tilde{k}} + \frac{1}{\mathcal{H}^2} \right]$$

$$\times \left( \Phi'_k + \mathcal{H}\Phi_k \right) \left( \Phi'_{k-\tilde{k}} + \mathcal{H}\Phi_{k-\tilde{k}} \right),$$

(10)

$\mathcal{H} = 1/\eta$, $e_{ij}(k)$ is the polarization tensor, the Bardeen potential $\Phi_k = \Psi(k\eta)\phi_k$, the transfer function $\Psi$ in the radiation era is

$$\Psi(x) = \frac{9}{x^2} \left( \frac{\sin(x/\sqrt{3})}{x/\sqrt{3}} - \cos(x/\sqrt{3}) \right),$$

(11)

and $\phi_k$ is related with $P_\zeta$ as

$$\langle \phi_k\phi_{\tilde{k}} \rangle = \delta^{(3)}(k + \tilde{k}) \frac{2\pi^2}{k^3} \left( \frac{2}{3} \right)^2 P_\zeta(k).$$

(12)

The power spectrum of the SGWs is defined as

$$\langle h_k(\eta)h_{\tilde{k}}(\eta) \rangle = \frac{2\pi^2}{k^3} \delta^{(3)}(k + \tilde{k}) P_h(k,\eta),$$

(13)

and the fractional energy density is

$$\Omega_{GW}(k,\eta) = \frac{1}{24} \left( \frac{k}{aH} \right)^2 \frac{P_h(k,\eta)}{P_\zeta(k,\eta)}.$$

(14)

Combining Eqs. (9)-(14) and the primordial power spectrum $P_\zeta$, we obtain $\Omega_{GW}$ and the result is shown in Fig. 3.

To show that the mechanism can give different $n_s$ and $r$, we take the Higg potential with the power law $U(\Phi) = U_0\Phi^{2/3}$ as an example and label it as H2. Taking the parameter set in Table I, we get $n_s = 0.9698$, $r = 0.0632$ and $N = 61.4$. The power spectrum of the primordial curvature perturbations, the PBH abundance and the energy density of SGWs are listed in Figs. 1, 2 and 3 respectively.

In order to show that our treatment is not specific to potential form, we generalize our discussion to the T-model inflation below. For the T-model inflation [63–65]

$$V = V_0 \tanh^{2m} \left( \frac{\phi}{\sqrt{6\alpha}} \right),$$

(15)

we can derive the attractor $n_s = 1 - 2/N$ and $r = 12/N^2$ which are consistent with Planck 2018 data for $N = 50 – 60$. The T-model with $m = 1/6$ and $\alpha = 1$ combined with the power law $U(\Phi) = U_0\Phi^{1/3}$ is labelled as T1 and the T-model with $m = 1/3$ and $\alpha = 1$ combined
with the power law $U(\Phi) = U_0 \Phi^{2/3}$ is labelled as T2. The model parameters and the results are shown in Tables I and II and Figs. 1, 2 and 3.

From these results, we see that our mechanism is general and appropriate to both the Higgs field and the T-model. We have shown that both models are consistent with Planck 2018 data. A significant abundance of PBH DM and SGWs can be produced in our mechanism applied and not restricted to the two models discussed, which is expected detectable by LISA/Tianqin/Taiji detectors.

| Model | $d$ | $c$ | $\phi_p$ | $\phi_s$ | $\lambda/V_0$ | $f_0$ | $N$ | $n_s$ | $r$ | $k_{\text{peak}}$/Mpc$^{-1}$ |
|-------|-----|-----|---------|---------|---------------|------|-----|-------|-----|---------------------|
| H1    | $1.05 \times 10^{10}$ | $2.04 \times 10^{-10}$ | 1.344 | 1.40 | $1.24 \times 10^{-9}$ | 1 | 62.3 | 0.9681 | 0.0383 | $4.66 \times 10^{12}$ |
| T1    | $4.72 \times 10^9$ | $8.89 \times 10^{-11}$ | 0.451 | 0.81 | $1.68 \times 10^{-9}$ | 36 | 55.6 | 0.9686 | 0.0369 | $2.29 \times 10^{12}$ |
| H2    | $7.13 \times 10^9$ | $1.94 \times 10^{-10}$ | 1.750 | 1.88 | $6.40 \times 10^{-10}$ | 1 | 64.2 | 0.9694 | 0.0641 | $3.67 \times 10^{12}$ |
| T2    | $8.90 \times 10^9$ | $4.75 \times 10^{-11}$ | 0.835 | 1.35 | $2.95 \times 10^{-9}$ | 36 | 63.4 | 0.9704 | 0.0597 | $5.24 \times 10^{12}$ |

**Table I.** The chosen parameter sets and the results. H represents the Higgs potential and T represents the T-model, 1 represents the case $n = 1/3$ and 2 represents the case $n = 2/3$. H1 means the model with Higgs potential and the power law potential $U(\Phi) = U_0 \Phi^{n}$ with $n = 1/3$.

| Model | $P_{\zeta(\text{peak})}$ | $M_{\text{peak}}/M_\odot$ | $Y_{\text{FBH}}^{\text{peak}}$ | $f_c$/Hz |
|-------|-----------------|---------------------|----------------|-----------|
| H1    | $1.16 \times 10^{-2}$ | $1.70 \times 10^{-13}$ | $3.57 \times 10^{-2}$ | $8.11 \times 10^{-3}$ |
| T1    | $1.21 \times 10^{-2}$ | $7.05 \times 10^{-13}$ | $7.64 \times 10^{-2}$ | $3.54 \times 10^{-3}$ |
| H2    | $1.15 \times 10^{-2}$ | $2.73 \times 10^{-13}$ | $2.64 \times 10^{-2}$ | $6.40 \times 10^{-3}$ |
| T2    | $1.10 \times 10^{-2}$ | $1.34 \times 10^{-13}$ | $7.12 \times 10^{-3}$ | $9.13 \times 10^{-3}$ |

**Table II.** The results for the primordial power spectrum, the peak mass and abundance of PBH and the peak frequency of SGWs with the chosen parameter sets shown in Table I. H represents the Higgs potential and T represents the T-model, 1 represents the case $n = 1/3$ and 2 represents the case $n = 2/3$. H1 means the model with Higgs potential and the power law potential $U(\Phi) = U_0 \Phi^{n}$ with $n = 1/3$.  

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FIG. 1. The results for the power spectrum of the primordial curvature perturbations. H represents the Higgs potential and T represents the T-model, 1 represents the case $n = 1/3$ and 2 represents the case $n = 2/3$. H1 means the model with Higgs potential and the power law potential $U(\Phi) = U_0 \Phi^n$ with $n = 1/3$.

FIG. 2. The results for the PBH abundance. The shaded regions show the observational constraints on the PBH abundance. The models are the same as those in Fig. 1.

FIG. 3. The results for SGWs along with the sensitivity curves for different GW detectors. These SGWs can be detected in LISA/TaiJi/TianQin because their amplitudes are above the sensitivity limits. The models are the same as those in Fig. 1.
CONCLUSION

In this work we proposed a novel mechanism to resolve the contradiction in the original non-canonical kinetic mechanism of simultaneously requiring the enhancement of the curvature perturbations at small scales and keeping the model predictions in consistent with CMB observations at large scales. We found that with this mechanism Higgs field inflationary model becomes viable. In our method, the function $G_a(\phi)$ peaks near the end of inflation thereby the enhancement of the power spectrum happens at small scales only. Such peak contributes about 20 e-folds during the enhancement. To keep the number of e-folds to be $50 - 60$, the field value $\phi_*$ at the horizon exit moves closer to the field value at the end of inflation and the slow-roll contributions to $n_s$ and $r$ are changed. Away from the peak, the function $G_a(\phi)$ is negligible and the usual slow roll inflation applies, the non-canonical term with the function $f(\phi)$ ensures the power spectrum at large scales to be consistent with CMB observations. In our mechanism, the observables $n_s$ and $r$ are not sensitive to the inflaton potential, where both Higgs potential and the general T-models can be employed to describe the Planck 2018 observations. The mechanism does not restrict the functions $G_a(\phi)$ and $f(\phi)$ to particular forms used in this Letter, other forms are permitted.

The Higgs boson of the standard model of particle physics is responsible not only for the masses of elementary particles, but can act as an inflaton to drive inflation to meet CMB measurements. Furthermore, we have shown that it can explain the DM content of our universe in the form of PBHs. The SGWs induced by the large first order Higgs fluctuations at small scales can be observed by the space based GW observatories, such as LISA, Taiji and Tianqin. Future GW observations can grasp more signatures of Higgs field through PBHs DM and SGWs.

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[1] B. P. Abbott et al. (LIGO Scientific and Virgo Collaborations), Observation of Gravitational Waves from a Binary Black Hole Merger, Phys. Rev. Lett. 116, 061102 (2016), arXiv:1602.03837.

[2] B. P. Abbott et al. (Virgo, LIGO Scientific), GW151226: Observation of Gravitational Waves from a 22-Solar-Mass Binary Black Hole Coalescence, Phys. Rev. Lett. 116, 241103 (2016), arXiv:1606.04855.

[3] B. P. Abbott et al. (VIRGO, LIGO Scientific), GW170104: Observation of a 50-Solar-Mass Binary Black Hole Coalescence at Redshift 0.2, Phys. Rev. Lett. 118, 221101 (2017), arXiv:1706.01812.

[4] B. P. Abbott et al. (Virgo, LIGO Scientific), GW170814: A Three-Detector Observation of Gravitational Waves from a Binary Black Hole Coalescence, Phys. Rev. Lett. 119, 141101 (2017), arXiv:1709.09660.

[5] B. P. Abbott et al. (Virgo, LIGO Scientific), GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral, Phys. Rev. Lett. 119, 161101 (2017), arXiv:1710.05832.

[6] B. P. Abbott et al. (Virgo, LIGO Scientific), GW170608: Observation of a 19-solar-mass Binary Black Hole Coalescence, Astrophys. J. 851, L35 (2017), arXiv:1711.05578.

[7] B. Abbott et al. (LIGO Scientific, Virgo), GWTC-1: A Gravitational-Wave Transient Catalog of Compact Binary Mergers Observed by LIGO and Virgo during the First and Second Observing Runs, Phys. Rev. X 9, 031040 (2019), arXiv:1811.12907.

[8] P. Ivanov, P. Naselsky, and I. Novikov, Inflation and primordial black holes as dark matter, Phys. Rev. D 50, 7173 (1994).

[9] S. Bird, I. Cholis, J. B. Muñoz, Y. Ali-Haimoud, M. Kamionkowski, E. D. Kovetz, A. Raccanelli, and A. G. Riess, Did LIGO detect dark matter?, Phys. Rev. Lett. 116, 201301 (2016), arXiv:1603.00464.

[10] M. Sasaki, T. Suyama, T. Tanaka, and S. Yokoyama, Primordial Black Hole Scenario for the
Gravitational-Wave Event GW150914, Phys. Rev. Lett. 117, 061101 (2016), [erratum: Phys. Rev. Lett. 121, 059901 (2018)], arXiv:1603.08338.

[11] B. J. Carr and S. W. Hawking, Black holes in the early Universe, Mon. Not. Roy. Astron. Soc. 168, 399 (1974).

[12] S. Hawking, Gravitationally collapsed objects of very low mass, Mon. Not. Roy. Astron. Soc. 152, 75 (1971).

[13] H. Di and Y. Gong, Primordial black holes and second order gravitational waves from ultra-slow-roll inflation, J. Cosmol. Astropart. Phys. 07 (2018) 007, arXiv:1707.09578.

[14] Y. Lu, Y. Gong, Z. Yi, and F. Zhang, Constraints on primordial curvature perturbations from primordial black hole dark matter and secondary gravitational waves, J. Cosmol. Astropart. Phys. 12 (2019) 031, arXiv:1907.11896.

[15] G. Sato-Polito, E. D. Kovetz, and M. Kamionkowski, Constraints on the primordial curvature power spectrum from primordial black holes, Phys. Rev. D 100, 063521 (2019), arXiv:1904.10971.

[16] Y. Akrami et al. (Planck), Planck 2018 results. X. Constraints on inflation, arXiv:1807.06211.

[17] S. Matarrese, S. Mollerach, and M. Bruni, Second order perturbations of the Einstein-de Sitter universe, Phys. Rev. D 58, 043504 (1998), arXiv:astro-ph/9707278.

[18] S. Mollerach, D. Harari, and S. Matarrese, CMB polarization from secondary vector and tensor modes, Phys. Rev. D 69, 063002 (2004), arXiv:astro-ph/0310711.

[19] K. N. Ananda, C. Clarkson, and D. Wands, The Cosmological gravitational wave background from primordial density perturbations, Phys. Rev. D 75, 123518 (2007), arXiv:gr-qc/0612013.

[20] D. Baumann, P. J. Steinhardt, K. Takahashi, and K. Ichiki, Gravitational Wave Spectrum Induced by Primordial Scalar Perturbations, Phys. Rev. D 76, 084019 (2007), arXiv:hep-th/0703290.

[21] J. Garcia-Bellido, M. Peloso, and C. Unal, Gravitational Wave signatures of inflationary models from Primordial Black Hole Dark Matter, J. Cosmol. Astropart. Phys. 09 (2017) 013, arXiv:1707.02441.

[22] R. Saito and J. Yokoyama, Gravitational wave background as a probe of the primordial black hole abundance, Phys. Rev. Lett. 102, 161101 (2009), [Erratum: Phys. Rev. Lett. 107, 069901 (2011)], arXiv:0812.4339.

[23] R. Saito and J. Yokoyama, Gravitational-Wave Constraints on the Abundance of Primordial
Black Holes, Prog. Theor. Phys. 123, 867 (2010), [Erratum: Prog. Theor. Phys.126,351(2011)], arXiv:0912.5317.

[24] E. Bugaev and P. Klimai, Induced gravitational wave background and primordial black holes, Phys. Rev. D 81, 023517 (2010), arXiv:0908.0664.

[25] E. Bugaev and P. Klimai, Constraints on the induced gravitational wave background from primordial black holes, Phys. Rev. D 83, 083521 (2011), arXiv:1012.4697.

[26] L. Alabidi, K. Kohri, M. Sasaki, and Y. Sendouda, Observable Spectra of Induced Gravitational Waves from Inflation, J. Cosmol. Astropart. Phys. 1209 (2012) 017, arXiv:1203.4663.

[27] N. Orlofsky, A. Pierce, and J. D. Wells, Inflationary theory and pulsar timing investigations of primordial black holes and gravitational waves, Phys. Rev. D 95, 063518 (2017), arXiv:1612.05279.

[28] T. Nakama, J. Silk, and M. Kamionkowski, Stochastic gravitational waves associated with the formation of primordial black holes, Phys. Rev. D 95, 043511 (2017), arXiv:1612.06264.

[29] K. Inomata, M. Kawasaki, K. Mukaida, Y. Tada, and T. T. Yanagida, Inflationary primordial black holes for the LIGO gravitational wave events and pulsar timing array experiments, Phys. Rev. D 95, 123510 (2017), arXiv:1611.06130.

[30] S.-L. Cheng, W. Lee, and K.-W. Ng, Primordial black holes and associated gravitational waves in axion monodromy inflation, J. Cosmol. Astropart. Phys. 07 (2018) 001, arXiv:1801.09050.

[31] D. I. Kaiser, Primordial spectral indices from generalized Einstein theories, Phys. Rev. D 52, 4295 (1995), arXiv:astro-ph/9408044.

[32] F. L. Bezrukov and M. Shaposhnikov, The Standard Model Higgs boson as the inflaton, Phys. Lett. B 659, 703 (2008), arXiv:0710.3755.

[33] C. Germani and A. Kehagias, New Model of Inflation with Non-minimal Derivative Coupling of Standard Model Higgs Boson to Gravity, Phys. Rev. Lett. 105, 011302 (2010), arXiv:1003.2635.

[34] C. Germani, Y. Watanabe, and N. Wintergerst, Self-unitarization of New Higgs Inflation and compatibility with Planck and BICEP2 data, J. Cosmol. Astropart. Phys. 1412 (2014) 009, arXiv:1403.5766.

[35] N. Yang, Q. Fei, Q. Gao, and Y. Gong, Inflationary models with non-minimally derivative coupling, Class. Quant. Grav. 33, 205001 (2016), arXiv:1504.05839.

[36] Z. Yi, Y. Gong, and M. Sabir, Inflation with Gauss-Bonnet coupling, Phys. Rev. D 98, 083521
[37] F. Bezrukov and M. Shaposhnikov, Higgs inflation at the critical point, Phys. Lett. B 734, 249 (2014), arXiv:1403.6078.

[38] J. M. Ezquiaga, J. Garcia-Bellido, and E. Ruiz Morales, Primordial Black Hole production in Critical Higgs Inflation, Phys. Lett. B 776, 345 (2018), arXiv:1705.04861.

[39] M. Sasaki, T. Suyama, T. Tanaka, and S. Yokoyama, Primordial black holes—perspectives in gravitational wave astronomy, Class. Quant. Grav. 35, 063001 (2018), arXiv:1801.05235.

[40] S. Passaglia, W. Hu, and H. Motohashi, Primordial black holes and local non-Gaussianity in canonical inflation, Phys. Rev. D 99, 043536 (2019), arXiv:1812.08243.

[41] C. Fu, P. Wu, and H. Yu, Primordial Black Holes from Inflation with Nonminimal Derivative Coupling, Phys. Rev. D 100, 063532 (2019), arXiv:1907.05042.

[42] C. Armendariz-Picon, T. Damour, and V. F. Mukhanov, $k$-inflation, Phys. Lett. B 458, 209 (1999), arXiv:hep-th/9904075.

[43] J. Garriga and V. F. Mukhanov, Perturbations in $k$-inflation, Phys. Lett. B 458, 219 (1999), arXiv:hep-th/9904176.

[44] T. Kobayashi, M. Yamaguchi, and J. Yokoyama, G-inflation: Inflation driven by the Galileon field, Phys. Rev. Lett. 105, 231302 (2010), arXiv:1008.0603.

[45] J. Lin, Q. Gao, Y. Gong, Y. Lu, C. Zhang, and F. Zhang, Primordial black holes and secondary gravitational waves from $k$ and $G$ inflation, Phys. Rev. D 101, 103515 (2020), arXiv:2001.05909.

[46] J. R. Espinosa, D. Racco, and A. Riotto, Cosmological Signature of the Standard Model Higgs Vacuum Instability: Primordial Black Holes as Dark Matter, Phys. Rev. Lett. 120, 121301 (2018), arXiv:1710.11196.

[47] K. Danzmann, LISA: An ESA cornerstone mission for a gravitational wave observatory, Class. Quant. Grav. 14, 1399 (1997).

[48] H. Audley et al., Laser Interferometer Space Antenna, arXiv:1702.00786.

[49] J. Luo et al. (TianQin), TianQin: a space-borne gravitational wave detector, Class. Quant. Grav. 33, 035010 (2016), arXiv:1512.02076.

[50] W.-R. Hu and Y.-L. Wu, The Taiji Program in Space for gravitational wave physics and the nature of gravity, Natl. Sci. Rev. 4, 685 (2017).

[51] C. Brans and R. Dicke, Mach’s principle and a relativistic theory of gravitation, Phys. Rev.
[52] P. A. R. Ade et al. (BICEP2, Keck Array), BICEP2 / Keck Array x: Constraints on Primordial Gravitational Waves using Planck, WMAP, and New BICEP2/Keck Observations through the 2015 Season, Phys. Rev. Lett. **121**, 221301 (2018), arXiv:1810.05216.

[53] B. Carr, F. Kuhnel, and M. Sandstad, Primordial Black Holes as Dark Matter, Phys. Rev. D **94**, 083504 (2016), arXiv:1607.06077.

[54] B. J. Carr, The Primordial black hole mass spectrum, Astrophys. J. **201**, 1 (1975).

[55] S. Young, C. T. Byrnes, and M. Sasaki, Calculating the mass fraction of primordial black holes, J. Cosmol. Astropart. Phys. 07 (2014) 045, arXiv:1405.7023.

[56] O. Özsoy, S. Parameswaran, G. Tasinato, and I. Zavala, Mechanisms for Primordial Black Hole Production in String Theory, J. Cosmol. Astropart. Phys. 07 (2018) 005, arXiv:1803.07626.

[57] Y. Tada and S. Yokoyama, Primordial black hole tower: Dark matter, earth-mass, and LIGO black holes, Phys. Rev. D **100**, 023537 (2019), arXiv:1904.10298.

[58] N. Aghanim et al. (Planck), Planck 2018 results. VI. Cosmological parameters, arXiv:1807.06209.

[59] I. Musco and J. C. Miller, Primordial black hole formation in the early universe: critical behaviour and self-similarity, Class. Quant. Grav. **30**, 145009 (2013), arXiv:1201.2379.

[60] T. Harada, C.-M. Yoo, and K. Kohri, Threshold of primordial black hole formation, Phys. Rev. D **88**, 084051 (2013), [Erratum: Phys. Rev.D 89,029903(2014)], arXiv:1309.4201.

[61] A. Escrivà, C. Germani, and R. K. Sheth, Universal threshold for primordial black hole formation, Phys. Rev. D **101**, 044022 (2020), arXiv:1907.13311.

[62] C.-M. Yoo, T. Harada, and H. Okawa, Threshold of Primordial Black Hole Formation in Nonspherical Collapse, arXiv:2004.01042.

[63] R. Kallosh and A. Linde, Non-minimal Inflationary Attractors, J. Cosmol. Astropart. Phys. 1310 (2013) 033, arXiv:1307.7938.

[64] R. Kallosh and A. Linde, Universality Class in Conformal Inflation, J. Cosmol. Astropart. Phys. 1307 (2013) 002, arXiv:1306.5220.

[65] Z. Yi and Y. Gong, Nonminimal coupling and inflationary attractors, Phys. Rev. D **94**, 103527 (2016), arXiv:1608.05922.