Tessellation in Architecture from Past to Present

Ecenur Kızılörenli, Feray Maden
Yaşar University, Department of Architecture, Üniversite Cad., No:37-39 Ağaçlı Yol
Bornova 35100 İzmir, Turkey
ecenur.kizilorenli@yasar.edu.tr

Abstract. Tessellation, which has examples of use in art and architecture, is the covering of a surface using one or more geometric shapes without overlapping or gaps. Based on Roman mosaics, the tessellation has an important place in architecture since the ancient times. Through the history, different patterns have been used by many cultures for various applications ranging from decorative covering elements to multi-functional latticework screens. The tessellation has still been used in contemporary architecture since it not only allows creating the geometrical surface in an order but also provides multi-functionality to the surface when applied as shading elements. The tessellation can be reviewed under three categories such as regular, semi-regular and demi-regular tessellations. Two- and three-dimensional examples of the tessellations can be seen in contemporary architecture either as façade elements or patterns used for structural elements. Because the tessellation plays a significant role in architecture in terms of geometrical or structural design, the interest on this topic has been increased in recent years. Due to their great potentials, more studies should be conducted on the tessellations. For this reason, within the scope of this paper, the applied examples of the tessellations in buildings from past to present are examined which include both static and kinetic ones. In this paper, the geometric design principles, combination methods and iteration processes of the examples are also presented. As well as providing a deeper understanding of such tessellation methods, this study will serve as a basis of reference for future studies in this field.

1. Introduction
The word “tessellation” derives from the Latin word “tessella,” which means square stone used in ancient Roman mosaics. Meaning as a cube-shaped piece of clay, stone or glass used in making mosaics, the word “tessella” is based on the Greek word “tessera” that means small square [1]. In fact, tessellation is a method mostly used in mathematics, art and architecture which can be defined as covering a surface by using one or more geometric shapes without overlapping or gap. In other words, patterns are created by repeating certain shapes [2].

The tessellation has a significant role in architecture since the ancient times [3]. Throughout the history, different patterns have been used by many cultures for structural elements, decorative surface coverings, non-structural elements and floor coverings. It is known that one of the first studies on this topic was conducted by Johannes Kepler who studied the polygonal tessellations systematically and defined the minimum set of tessellations as regular and semi-regular [4]. Another pioneering study on the tessellation types was conducted by Fedorov in 1891. In addition to these studies, Graünbaum and Shephard (1986) attempted to define and visualize the concept of tessellation [5]. The studies on tessellation has continued in the twenty-first century as well [6-8]. It is possible to see impressive...
examples of the tessellation in contemporary architecture not only as a static building element but also as kinetic ones.

2. Shapes That Can Tessellate
According to Seymour and Britton [1], the tessellation can be created with any type of triangle if the surface has a regular pattern. However, there is a rule to create a pattern with regular polygons. First, the side number of the polygon is divided to 360° to find the exterior angle. If the interior angle of the polygon is a factor of 360°, then the polygon can tessellate. For instance, the side number is equal to 3 in triangle and the interior angle is 60° which means it is a factor of 360°. In square, the interior angle equals to 90° that is a factor of 360°. Likewise, the interior angle is 120° in hexagon which is again a factor of 360°. A regular pattern can be created using triangle, square or hexagon. However, it is not possible to generate a regular tessellation with other polygons since the interior angles are not a factor of 360° (Figure 1).

3. Types of Tessellation
To create a tessellation, any point selected on the pattern must have the same configuration as any other point. A simple method is used to determine the type of tessellation. First, a vertex on the pattern is chosen and then the number of sides of the polygons positioned around that vertex is written side by side in a clockwise or counterclockwise direction. For instance, the tessellation composed of hexagons is called as 6.6.6 tessellation since there are 3 hexagons at each vertex. It should be noted that regular geometries can merge edge to edge. Grünbau and Shephard used a formula using regular \( n \)-gon (\( n \)) to calculate the probabilities of the combinations [5]:

\[
\frac{n_1-2}{n_1} + \cdots + \frac{n_r-2}{n_r} = 2
\]  
(Eq. 1)

According to this calculation, if the side number of a regular polygon (\( n_i \)) is subtracted at each step, the result is equal to 2 when \( n_r = 1 \). That means the polygon can cover the surface. Unlimited repetition of the tessellation is associated with the rotational axes being on the reflection axes. There are mainly three types of tessellation which are regular, semi-regular and semi-regular.

![Shapes that can tessellate](image1.png)
3.1. Regular Tessellation
In regular tessellation, the same number of specific $n$-gon are found at each selected vertex. A regular polygon having equal sides and interior angles must be used while creating this type of tessellation [9]. There are only three examples of this defined category which are 3.3.3.3.3.3 tessellation consisting of triangles, 4.4.4.4 tessellation consisting of squares and 6.6.6 tessellation consisting of hexagons (Figure 2).

![Figure 2. Regular Tessellation](image)

3.2. Semi-Regular Tessellation
It consists of two or more regular polygons arranged in the same at each vertex. The side lengths of the polygons in the semi-regular tessellation must be the same length for the pattern to form and iterate. There are only eight types of semi-regular tessellations (Figure 3).

![Figure 3. Semi-regular tessellation](image)

3.3. Demi-Regular Tessellation
The semi-regular tessellation has been defined by some researchers as the combination of regular and semi-regular tessellations. There are 20 examples of this category (Figure 4).
4. Tessellation in Ancient Times
The use of tessellation in architecture dates back to ancient times. It is possible to find different tessellated patterns in buildings not only as surface covering but also as decoration element. One of the earliest examples was found on wall decorations in 4000 BC which was used by the Sumerians [10]. It is known that the tessellation was also used by other civilizations such as Egyptians, Persians, Romans, Greeks, Arabs, Japanese and Chinese, but the used patterns varied from culture to culture. For instance, mosaics made of square blocks were used in antiquity while more complex geometric patterns were used in Arab culture [11-13]. Some tessellation examples from those periods are shown in Figure 5-9.
5. Tessellation in Islamic Architecture

Islamic religion, culture, art and architecture spread across the continents. As a result of this, distinct characteristics of Islamic architecture were used by the masters in many buildings constructed in different regions. Since the human and natural figures are not used in Islamic architecture, geometric figures and patterns became more dominant representing the unity and the universe. Even though very complex geometries are seen in Islamic architecture, they are derived from regular polygons and simple grids. Various geometric patterns have been used in Islamic architecture such as radial and periodic patterns. The radial patterns revolve around the centre point whereas the periodic patterns are created by translation and symmetry in two independent directions [14]. Girih and star patterns are two types of the periodic pattern which are based on equilateral triangles, squares and hexagons forming the regular tessellation. In most patterns, squares are used since they form a lattice system. However, hexagonal and triangular shapes also provide advantages of ease of fabrication and iteration as they are dual of each other. There is a study showing the compass-straight edged structures of the patterns step by step.
composed of square and hexagon [15]. When the patterns used in Islamic architecture are examined, it is seen that most of them consist of the intersection of regular pentagons [16].

5.1. Girih Patterns
Girih pattern can be defined as an ornamentation style used in Islamic architecture which is based on a set of five striped tiles. This set consists of a regular decagon, a rhombus, a regular pentagon and a bowtie-like shape (Figure 10). The sides of each polygon are equal, and the lines emerging from the midpoints of both sides intersect with each other at 72 and 108 degrees [7].

![Figure 10. Girih Pattern](image)

The date when Girih pattern was first used in architecture is still unknown [7], but it is known that this pattern was used in the buildings of Seljuk and Ilkhanians. It was also used in decorative arts in the 14th century. One of the well-known examples of this pattern can be found in *Ottoman Green Mosque* in Bursa, Turkey (Figure 11). Another example is *Abbasid Al-Mustansiriya Madrasah* (Figure 12) that is one of the standing buildings in Baghdad demonstrating the development of geometric ornaments [17]. The main entrance of the building consists of three conical arches, star and polygon figures. Other entrances are decorated with zigzag geometric patterns and square patterns.

![Figure 11. Ottoman Green Mosque](image) ![Figure 12. Abbasid Al-Mustansiriya Madrasah](image)

5.2. Star Patterns
Star patterns are one of the complex patterns used in Islamic architecture, and little is known about how they originated and designed. In Islamic architecture, the use of circle is a way of representing the unity, the God and the qibla that is the direction towards the Kaaba [18, 19].

Most Islamic geometric patterns (IGPs) are based on regular polygons such as hexagons and octagons. Stars, the basic geometry of these IGPs, are formed by the combination of the vertices of these regular polygons. There is a group used to create star geometry ranging from hexagons, which are considered the simplest of all polygons, to much more complex polygons. Rosettes are formed from these stars (Figure 13).
An example of the use of star pattern can be seen in the *Abbasid Palace* in Baghdad which strongly represents the architectural features of the late Abbasid and early Seljuk periods [20]. The building has geometric patterns made of carved brick and terracotta, and some of the earliest examples of the rosettes attached to 8- and 12-point star patterns were used (Figure 14). Another example is the *Sultan Hassan Complex* located in Cairo (Figure 15). Although there are many different patterns in the structure, the most striking ones are the 6-, 8-, 10- and 12-star patterns and rosettes. The 16-star patterns found on the panels of the wooden pulpit are surprisingly intricate [20].

![Figure 13. Evolution of the Star Pattern](image)

6. **Tessellation in Contemporary Architecture**

It is possible to see the examples of simple or complex tessellations in contemporary architecture. One of the two-dimensional examples of tessellation applied on a flat surface is seen on the ceiling of the *Yale Art Gallery* building designed by Louis Khan in 1953 (Figure 16). The waffle slab was created using regular tessellation made of equilateral triangles. Unlike previous example, a structural system called diagrid is used in the *London Swiss-Re* building designed by Norman Foster in 2004, which is an example of structural application of tessellation (Figure 17). A regular triangular tessellation pattern is used to cover the building facade.

![Figure 14. Abbasid Palace](image)  ![Figure 15. Sultan Hassan Complex](image)
Another two-dimensional application of the tessellation is seen in the façade of the Melbourne Federation Square building designed by Lab Architecture (Figure 18). Contrary to previous examples, the façade does not consist of regular tessellation. Rather, it is composed of nonidentical triangular panels that do not repeat on the surface. On the other hand, the two-dimensional tessellation inspired by versatile floral patterns is used in the Ravensbourne College designed by Foreign Office Architects (Figure 19). The facade features a pattern of two irregular pentagons and rhombuses. Due to the rotational symmetry of the tessellation, different sizes of openings can be created in different locations on the facade.

One of the most important examples of three-dimensional tessellation can be seen in the roof of the British Museum in London designed by Foster and Partners (Figure 20). Formed of triangular tessellation, the roof geometry was generated using a mathematical model in which the side lengths of the triangles were optimized based on an algorithm [21]. The tessellated patterns can be generated using both regular or irregular geometries. For instance, the façade of the Storey Hall in Melbourne has a three-dimensional irregular tessellation (Figure 21). The most striking feature of this building is that the rhombus pattern continues from exterior surface to the interior walls and ceiling.

Apart from the static examples of the tessellation, it is also possible to see the impressive examples designed kinetically. For instance, the façade of the Latvia Pavilion has a regular square tessellation.
which consists of 100,000 coloured and transparent plastic elements positioned to represent nature [22] (Figure 22). The façade elements are free to move independently in reaction to wind movement as creating an oscillating motion. On the other hand, in the Dancing Pavilion built in Brazil, there are 345 round mirror elements that make full rotation in the horizontal direction (Figure 23). Inside the building, there are sensors that capture the music and movement of the people dancing. The data collected by the sensors activate the motors and the mirrors on the façade accordingly. Since this building was in temporary use, the façade does not have a functional purpose such as temperature control or daylight control.

![Figure 22. Latvia Pavilion](image22)
![Figure 23. Dancing Pavilion](image23)

More complex façade systems have been proposed and implemented in the last decades. One of the well-known façades is the Al-Bahr Towers that has 1049 umbrella-like components organized into hexagonal units (Figure 24). Based on regular triangular tessellation, the responsive façade is controlled by a central building management system (BMS) [23]. Performing a folding movement, the system works in real time and provides mainly daylight and thermal control. There is also a façade system with hybrid movements such as rotation and translation in a much more complex way than the previous example (Figure 25). Inspired by Mashrabiya, which is based on the Islamic star and rosette patterns, the south façade of the Institute Du Monde Arabe has a regular square tessellation consisting of camera-like diaphragms. At low altitude levels, the diaphragms open by means of a photoelectric cell and close in the opposite direction when the sun is brighter. The expansion and contraction mechanism of these lens-like apertures is regulated by sliders to automatically control the amount of light entering the building [24].

![Figure 24. Al-Bahr Towers](image24)
![Figure 25. Institute Du Monde Arabe](image25)

7. Conclusion
The tessellation has been used in architecture for centuries not only to cover the building surfaces but also to create the structural forms or systems. In this paper, the examples of the tessellation have been presented starting from past to present in order to reveal its potentials. First of all, the tessellation and its basic rules have been introduced. Then, the examples varying in different cultures in ancient times have been examined in which the tessellation was used for covering surfaces or decorating non-structural elements. When the tessellation examples used in Islamic architecture are investigated, it has been seen that more complex geometries can also be generated using regular shapes and tessellation types. The
use of tessellations in architecture continued and played a significant role in the twentieth century. Although this usage was first seen on two-dimensional flat surfaces, it was later used three dimensionally to create spatial structures or surfaces. With the technological development, the tessellation has started using in kinetic architecture, especially as passive or active facade systems. These examples showed us that innovative solutions can be developed by integrating the tessellation with the concept of motion. As a result, it can be claimed that the tessellation method provides great advantages in architecture since it facilitates constructing the geometries by simply repeating regular or non-regular shapes with a set of rules and allows generating various geometric patterns two or three dimensionally. Moreover, it gives designer the freedom to create even more complex forms using a simple design approach and to explore a higher-dimension using computational tools.

References
[1] D. Seymour and J. Britton, Introduction to Tessellations. ERIC, 1989.
[2] E. Gjerde, Origami tessellations: awe-inspiring geometric designs. AK Peters/CRC Press, 2008.
[3] W. Chang, “Application of tessellation in architectural geometry design,” in E3S Web of Conferences, vol. 38, p. 3015, 2018.
[4] J. Kepler, The harmony of the world, vol. 209. American Philosophical Society, 1997.
[5] B. Grünbaum and G. C. Shephard, Tilings and patterns. Courier Dover Publications, 1987.
[6] C. Greco, “Modular tessellation and architecture,” 2014.
[7] P. J. Lu and P. J. Steinhardt, “Decagonal and quasi-crystalline tilings in medieval Islamic architecture,” Science (80-. ), vol. 315, no. 5815, pp. 1106–1110, 2007.
[8] C. Goodman-Strauss, “Tessellations,” arXiv Prepr. arXiv1606.04459, 2016.
[9] L. C. Kinsey and T. E. Moore, Symmetry, Shape and Space: An Introduction to Mathematics Through Geometry. Springer Science & Business Media, 2006.
[10] C. A. Pickover, The math book: from Pythagoras to the 57th dimension, 250 milestones in the history of mathematics. Sterling Publishing Company, Inc., 2009.
[11] K. Dunbabin, Mosaics of the Greek and Roman world. Cambridge University Press, 1999.
[12] R. Field, Geometric patterns from Roman mosaics. Tarquin, 2017.
[13] “Hull Museum Collections.” http://museumcollections.hullcc.gov.uk/ (accessed Mar. 16, 2021).
[14] R. F. Tennant, “Islamic tilings of the Alhambra Palace: teaching the beauty of mathematics,” 2009.
[15] E. Broug, Islamic geometric patterns. Thames & Hudson London, 2008.
[16] R. Sarhangi, “Interlocking star polygons in Persian architecture: the special case of the decagram in mosaic designs,” Nexus Netw. J., vol. 14, no. 2, pp. 345–372, 2012.
[17] Y. Tabbaa, The transformation of Islamic art during the Sunni revival. University of Washington Press, 2001.
[18] K. Critchlow, Islamic patterns. Thames and Hudson London, 1976.
[19] S. Akkach, Cosmology and architecture in premodern Islam: An architectural reading of mystical ideas. SUNY Press, 2012.
[20] Y. Abdullahi and M. R. Bin Embi, “Evolution of Islamic geometric patterns,” Front. Archit. Res., vol. 2, no. 2, pp. 243–251, 2013.
[21] B. Kolarevic, Architecture in the digital age: design and manufacturing. taylor & Francis, 2004.
[22] S. Jordana, “Latvia Pavilion for Shanghai World Expo 2010 | ArchDaily,” [Online] 2010. https://www.archdaily.com/48851/latvia-pavilion-for-shanghai-world-expo-2010 (accessed Aug. 26, 2020).
[23] K. Cilento, “Al Bahar Towers Responsive Facade / Aedas | ArchDaily,” [Online] 2012. https://www.archdaily.com/270592/al-bahar-towers-responsive-facade-aedas (accessed Jul. 14, 2020).
[24] M. Schumacher, M.-M. Vogt, and L. A. Cordón Krumme, “New MOVE,” New MOVE, 2019, doi: 10.1515/9783035613629.