Asymptotic solution for the electromagnetic scattering of a vertical dipole over plasmonic and non-plasmonic half-spaces

Mohsen Eslami Nazari | Weimin Huang

Electrical and Computer Engineering, Memorial University of Newfoundland, St.John’s, NL, Canada

Correspondence
Weimin Huang, Electrical and Computer Engineering, Memorial University of Newfoundland, St.John’s, NL, Canada.
Email: weimin@mun.ca

Funding information
Natural Sciences and Engineering Research Council of Canada, Grant/Award Number: RGPIN-2017-04508; Natural Sciences and Engineering Research Council of Canada, Grant/Award Number: RGPIIN-2017-507962

Abstract
A new asymptotic solution for the scattered electromagnetic fields of a vertical Hertzian dipole antenna in the presence of an imperfectly conducting half-plane for ordinary and plasmonic media is proposed. The scattered electric and magnetic field components are calculated from the intermediate Hertz potential expressed in terms of the Fourier-Bessel transforms associated with the Sommerfeld-type integral, which is difficult to evaluate due to the singularities of the integrand near the integration path and its oscillatory and slowly decaying integrand. Using the modified saddle point method, an approximate closed-form solution of the far-zone scattered electromagnetic fields including surface waves is presented. The new formulations are applied to calculate radiation patterns of different impedance half-planes for both ordinary and plasmonic media, that is, seawater, silty clay soil, silty loam soil and lake water as ordinary, and silver and gold as plasmonic media. Furthermore, a numerical evaluation of the proposed solution at various frequencies and comparisons with two alternative state of the art solutions shows that the proposed solution has higher accuracy in terms of the normalised root-mean-square error and the normalised maximum absolute error for plasmonic and non-plasmonic structures.

1 | INTRODUCTION

During the past decade, electromagnetic (EM) scattering of a vertical dipole (VED) radiating over a lossy half-space, widely known as the Sommerfeld half-space problem, has become an indispensable means in remote sensing [1, 2], ground penetrating radar (GPR) [3, 4], near field optics [5], terahertz (THz) devices [6], and plasmonic/nanophotonic applications [7, 8], where the EM waves propagating along the interface are of great attention.

Sommerfeld first presented a solution including an improper integral with a Bessel function kernel for the EM scattering of a vertical Hertzian dipole over an imperfect ground in [9]. This solution could not be evaluated in a closed form due to the oscillatory, slowly decaying integrand, and several singularities on the integration path. He deformed the integration path and acquired the spectral representation of the Hertz vector using hyperbolic branch cut integrals and the residue of a pole contributing the Zenneck [10] surface wave. He also presented an asymptotic closed-form expression for the term associated with the Zenneck surface wave in the Hertz vector using the error function of a complex argument.

Norton [14], who developed Van der pol's formulation for an arbitrary height of transmitter and receiver. Nonetheless, Norton's numerical distance parameter in his formulation was not quite accurate for field points far away from the interface caused by the truncation of a binomial expansion of a square root in his derivation. Subsequently, Bannister [15] extended...
Norton's far field dipole equations to the quasi near field region, while an inaccurate Norton's numerical distance parameter was adopted.

The saddle point method (SPM) of integration is another approach for approximating Sommerfeld integrals (SIs) in far field regions, in which the distance between the antenna and the observation point is considerable [16]. However, for a lossy half-space problem, the SPM method is not capable of approximating SIs since the Sommerfeld pole is close to the saddle point for this problem. Wait [17, 18] and Makarov et al. [19] used a multiplicative method for dealing with the pole near the saddle point. Van der Waerden [20] introduced the modified SPM which was applied to the Sommerfeld problem by Collin [21] as well as Bernard and Ishimaru [22]. Although the obtained solutions using SPMs are not similar to the Norton’s formulation [14], they are consistent with the Norton surface wave formulation for high contrast media and far field regions near the interface. Further developments on the scattering of EM waves over a lossy half-space have been conducted by numerous researchers, such as King [23–26], Wait [27, 28] and Green [29]. Nevertheless, it was revealed by Mahmoud et al. [30], Wait [31, 32] and Yokoyama [33] that the solutions for the scattered EM fields radiated by a VED over a lossy or dielectric coated half-space were not quite accurate due to the trapped surface wave. Consequently, this classical problem was reinvestigated by several researchers in the past few years [34–38]. Sarabandi et al. [39] proposed an analytical method based on the reflection coefficient approximation of integrands employing the Prony method to calculate SIs in near and far field regions. However, the accuracy of the proposed method is restricted when the antenna and observation point are located near the interface due to the number of image points. Esjabi Nazari and Huang [40] introduced a new method by decomposing the intermediate Hertz potential into three terms. The first and second term of the Hertz potential associated with the SIs are calculated employing hyperbolic functions, and the third term is approximated using SPM. However, this solution is only applicable for low frequencies, highly conductive surfaces and far field regions. It is worth mentioning that the accuracy and efficiency of all aforementioned methods are limited by the antenna and observation point locations and EM properties of each medium, that is, permittivity and conductivity of the regions.

Several numerical methods have been proposed in order to evaluate Sommerfeld-type integrals. Parhami et al. [41] introduced a method, which is valid for a VED, by deforming the path of integration to the steepest descent path. It should be noted that in this method, the integral is solved asymptotically for large distances between image and observation point, while it is solved numerically for small distances. Michalski [42] developed Parhami’s method by proposing a variation over the way of branch cut. Afterwards, Johnson and Dudley [43] proposed a numerical method, in which an analytical technique is utilised in order to reduce the oscillation of the Sommerfeld integrand. However, this method is only valid for small distances between image and observation point. Despite improvements in convergence properties of the SIs using the aforementioned numerical techniques, transformations are required to be applied to the Sommerfeld integrand, which increase the complexity of calculations and computational costs. Moreover, the obtained solutions are not valid for all source and observation point locations and EM properties of layers [39].

Here, the classical Sommerfeld half-space problem is reconsidered and a rigorous closed-form solution for the intermediate Hertz potential and the scattered electric and magnetic field components are presented using the modified SPM method and considering high-order surface wave. The theoretical development is validated by representative numerical results and compared with two alternative state of the art solutions referred to as the King [38] and the Norton-Bannister [15] solutions for ordinary and plasmonic media. The obtained results show that the proposed solution outperforms the conventional solutions at various frequencies and distances from the antenna and even for moderate media contrast.

The article is organised as follows. In Section 2, the scattered electric and magnetic field components using the intermediate Hertz potential are derived in terms of two-dimensional Fourier transforms. A rigorous closed-form solution for the intermediate Hertz potential and scattered electric and magnetic field components in the far field region are proposed in Section 3. Finally, in Section 4, numerical evaluation of the proposed solution at various frequencies and comparisons with the conventional methods, that is, King and Norton-Bannister solutions, for both ordinary and plasmonic media are presented in terms of the normalised root-mean-square error (NRMSE) and the normalised maximum absolute error (NMAE). Finally, the conclusion is presented in Section 5.

2 | FORMULATION OF THE PROBLEM

The scattered electric ($\vec{E}$) and magnetic ($\vec{H}$) field components radiated by a VED located on the z-axis of the cylindrical coordinate system at height $\hat{b}$ above a lossy half-space, as shown in Figure 1, can be expressed as [28]:

$$\vec{E} = \frac{I\Delta l}{j\omega\epsilon_0} \left[ \frac{\partial^2 \Pi_\omega}{\partial y^2} y + \frac{\partial^2 \Pi_\omega}{\partial x^2} x + \frac{\partial^2 \Pi_\omega}{\partial z^2} z + \left( \frac{\partial^2}{\partial z^2} + k_0^2 \right) \Pi_\omega z \right]$$

$$\vec{H} = I\Delta l \left[ \frac{\partial \Pi_\omega}{\partial y} x - \frac{\partial \Pi_\omega}{\partial x} y \right]$$

in which $I$ denotes the current source, $\Delta l$ is the antenna length, $\omega$ is the angular frequency of the source, $\epsilon_0$ denotes the permittivity of free space and $k_0$ is the wavenumber in free space. The special Fourier transform of the intermediate Hertz potential $\Pi_\omega$ can be written as:

$$\Pi_\omega = \left[ e^{-|x-b|\gamma_0} + e^{-(x+b)\gamma_0} \frac{\gamma_0 - \frac{1}{2}\gamma_1}{2\gamma_0} \right] + \left[ e^{-|x-b|\gamma_1} + e^{-(x+b)\gamma_1} \frac{\gamma_0 + \frac{1}{2}\gamma_1}{2\gamma_0} \right]$$.
where in \( \gamma_m \) \((m = 0, 1)\) and complex refractive index \(n_{om}\) can be expressed as:

\[
\gamma_m = \sqrt{k^2 - k_0^2 n_{om}^2}, \quad n_{om} = \sqrt{\frac{\epsilon_m(f)}{\epsilon_0}}
\]

where in \( k^2 = k_x^2 + k_y^2 \), \( \epsilon_0 \) is the permittivity of free space and \( \epsilon_m \) denotes the complex permittivity of medium \( m \) depending on the frequency. It is worth mentioning that the complex refractive index presented in (4) is valid for both plasmonic and non-plasmonic media at different frequencies. To determine the scattered electric field components (I), the inverse spatial Fourier transform is applied to (3). Therefore, we have

\[
\Pi_z = \frac{1}{4\pi^2} \int_{-\infty}^{+\infty} \left( \frac{e^{-|z-h|\gamma_0}}{2\gamma_0} + \frac{e^{-(z-h)\gamma_0}}{2\gamma_0} \right) e^{i(k_x x + k_y y)} dk_x dk_y - P
\]

and

\[
P = \frac{1}{4\pi^2} \int_{-\infty}^{+\infty} \frac{1}{\gamma_0 + 1/m_1} \frac{e^{-(z-h)\gamma_0}}{\gamma_0} e^{i(k_x x + k_y y)} dk_x dk_y.
\]

By changing the Cartesian integration variables in (5) and (6) into the polar form, the double integrals are converted to single integrals as follows:

\[
\Pi_z = \frac{1}{4\pi} \left[ \int_0^\infty k \left( \frac{e^{-|z-h|}}{\gamma_0} + \frac{e^{-(z+h)\gamma_0}}{\gamma_0} \right) J_0(kp) dk - 2P \right]
\]

wherein

\[
P = \int_0^\infty k \frac{1}{\gamma_0 + 1/m_1} \frac{e^{-(z+h)\gamma_0}}{\gamma_0} J_0(kp) dk
\]

In (8), \( J_0(kp) \) indicates the Bessel function of the first kind of order zero and \( \rho \) denotes the horizontal distance between the source and the observation point in the cylindrical coordinate as shown in Figure 1. By using the Sommerfeld identity [44], (7) can be simplified as:

\[
\Pi_z = \frac{1}{4\pi} \left( \frac{e^{-ik_0 R_0}}{R_0} + \frac{e^{-ik_0 R_1}}{R_1} - 2P \right).
\]

According to the geometry of the problem shown in Figure 1, \( R_0 \) and \( R_1 \) can be calculated as:

\[
R_0 = (\rho^2 + (z_0 - h)^2)^{1/2}, \quad R_1 = (\rho^2 + (z_0 + h)^2)^{1/2}
\]

where \( z_0 \) denotes the \( z \) coordinate of the observation point. To present an approximate closed-form solution for the scattered electric and magnetic field components, \( P \), which is a Sommerfeld-type integral, should be evaluated.

### 3 | EVALUATION OF THE INTEGRAL

Various analytical solutions have been proposed for seeking an approximate closed-form solution for the scattered electric and magnetic field components. However, proposing a general closed-form solution for different antenna and observation point locations with an arbitrary value of the complex refractive index is the main difficulty in evaluating them.

The zero-order Bessel function in (8) can be written as the sum of two Hankel functions of the first and second kinds with the same argument as

\[
J_0(kp) = \frac{1}{2} \left[ H_0^\gamma(kp) + H_0^{\gamma_0}(kp) \right]
\]

(11)

By substituting (11) into (8), \( P \) can be written as:

\[
P = \frac{1}{2} \int_{-\infty}^{+\infty} k \frac{1}{\gamma_0 + 1/m_1} \frac{e^{-(z+h)\gamma_0}}{\gamma_0} H_0^\gamma(kp) dk.
\]

(12)

Moreover, for moderate and high media contrast, (8) can be further simplified as:

\[
P = \frac{1}{2} \int_{-\infty}^{+\infty} \frac{ik\beta}{\gamma_0 + 1/m_1} \frac{e^{-(z+h)\gamma_0}}{\gamma_0} H_0^\gamma(kp) dk
\]

(13)

where \( \beta = \gamma_1/\gamma_0 n_0^2 \). By changing the integral variable \( k \) to \( \zeta = \arccos(k/k_0) \), \( P \) becomes
\[
\frac{j k_0 \beta}{2} \int_{-j \infty}^{j \infty} \frac{H_0^1(k_0 \rho \cos \zeta)}{\sin \frac{\pi \rho}{2} \cos \frac{\pi z}{2}} \ e^{-iv \kappa_0 (z + b) \sin \zeta} \, d\zeta
\]  

(14)

in which \( \alpha_0 = \sin^{-1}(\beta) \). By using the first term of the asymptotic expansion of the Hankel function of the first kind in the far field region, (14) can be expressed as:

\[
e^{2 \beta \rho} \int_{-j \infty}^{j \infty} \frac{\sqrt{\cos \zeta}}{\sin \frac{\pi \rho}{2} \cos \frac{\pi z}{2}} \ e^{-iv \kappa_0 (z + b) \sin \zeta} \, d\zeta
\]  

(15)

where \( \theta_2 \) is defined as:

\[
\theta_2 = -\tan^{-1} \left( \frac{z + b}{\rho} \right) = \frac{\pi}{2} + \theta_1.
\]  

(16)

Although extra terms for the asymptotic expansion of the Hankel function in (15) may increase the accuracy of the integral evaluation, it is quite accurate in the far-field region since the NRMSE values for the real and imaginary parts of the proposed approximation of the Hankel function are 0.004 and 0.0042, respectively.

By deforming the integration path via the substitution \( \cos \theta = -i t^2 \) and using the modified SPM, \( P \) can be expressed as:

\[
P \approx j \sqrt{\pi} P e^{-W_\beta} \ \text{erfc} \left( j \sqrt{W_\beta} \right) e^{-j k_0 R_1} \frac{R_1}{\beta R_1} \]  

(17)

in which \( \text{erfc} \) is the Gauss error function, and \( W_\beta \) and \( P_\beta \) may be expressed as:

\[
P_\beta = \frac{-j k_0 R_1}{2} \beta^2, \quad W_\beta \approx P_\beta \left( 1 + \frac{b + z}{\beta R_1} \right)^2.
\]  

(18)

### 3.1 Scattered E-field components

In order to calculate the scattered electric field components over the lossy half-space, the intermediate Hertz potential (9) along with (17) are substituted into (1). The \( x \) component of the scattered electric field may be expressed as:

\[
\overrightarrow{E}_x = \frac{i \Delta \rho}{4 \pi \omega \epsilon_0} \left[ x(b - z) \left[ R_0^2 k_0^2 - 3(1 + j k_0 R_0) \right] e^{-j k_0 R_1} \frac{R_1}{R_0^3} \right.
\]

\[
- x(b + z) \left[ R_1^2 k_0^2 - 3(1 + j k_0 R_1) \right] e^{-j k_0 R_1} \frac{R_1}{R_0^3} + \sqrt{2 \pi \beta k_0} \ e^{-j \beta x} \left[ T_1 + T_2 + T_3 \ \text{erfc} \left( j \sqrt{W_\beta} \right) - j T_4 \right]
\]  

(19)

wherein \( T_1 \) to \( T_4 \) with their sub variables can be acquired from the following equations.

\[
T_1 = \frac{k_0 \beta^2 x e^{W_\beta}}{2 R_1 \sqrt{\pi W_\beta}} \left[ 1 - \cos^2 \theta_1 \right] \frac{1}{\beta^2}
\]

\[
T_2 = -e^{W_\beta} \frac{e^{-jk_0 R_1}}{R_1^2} \left[ (1 - j k_0 R_1) (b + z) + C_1 R_1 \right]
\]

\[
T_3 = -2(1 + j k_0 R_1) C_2 - j k_0 x e^{-W_\beta} e^{-jk_0 R_1} \frac{b + z}{R_1^2} + C_2
\]

\[
T_4 = C_3 e^{-jk_0 R_1} - \frac{j k_0 x C_1 e^{-jk_0 R_1}}{\sqrt{\pi W_\beta} \frac{1}{R_1^{1.5}}}
\]

\[
C_1 = \frac{-j k_0 \beta^2}{2} \left( 1 + \frac{\cos \theta_1}{\beta} \right) \frac{\cos \theta_1 \left( 1 + \frac{\cos \theta_1}{\beta} \right) + 2 R_1}{\beta R_1}
\]

\[
C_2 = \frac{(b + z) e^{-W_\beta} e^{-jk_0 R_1}}{2 R_1^2} \left( C_1^2 R_1^2 + j k_0 x R_1 + 2.5 \right)
\]

\[
C_3 = -\frac{j k_0 x}{2 R_1^2} \left( -\sin(2\theta_1)\sin \theta_1 - \cos \theta_1 (\beta^2 - \cos^2 \theta_1) \right)
\]

\[
C_5 = \frac{-j k_0 \beta}{2} \left[ (1 + \cos \theta_1)(1 + \sin \theta_1 + \beta \cos \theta_1) \right]
\]

\[
C_6 = \frac{e^{-W_\beta} e^{-jk_0 R_1}}{R_1^2} \left( -R_0^2 C_3 + \frac{x}{2} - x(1 + j k_0 R_1) \right)
\]

\[
C_7 = \frac{1}{2 R_1^2 W_\beta} \left[ 2 C_3 R_1^2 W_\beta - C_1 (x W_\beta + R_1^2 C_3) \right]
\]  

(20)

The \( y \)-component of the scattered electric field can be expressed as:

\[
\overrightarrow{E}_y = \frac{-i \Delta \rho}{4 \pi \omega \epsilon_0} \left[ y(b - z) \left[ R_0^2 k_0^2 - 3(1 + j k_0 R_0) \right] e^{-j k_0 R_1} \frac{R_1}{R_0^3} \right]
\]

\[
- y(b + z) \left[ R_1^2 k_0^2 - 3(1 + j k_0 R_1) \right] e^{-j k_0 R_1} \frac{R_1}{R_0^3} + \sqrt{2 \pi \beta k_0} \ e^{-j \beta x} \left[ T'_1 + T_2 + T'_3 \ \text{erfc} \left( j \sqrt{W_\beta} \right) - j T'_4 \right]
\]  

(21)

The only difference between the prime parameters, \( T'_1, T'_3 \) and \( T'_4 \), and original parameters is the \( x \) parameter. In other words, by changing \( x \) to \( y \) in \( T_1, T_3, T_4 \) and their sub variables,
prime parameters can be obtained. By calculating the x- and y-components of the scattered electric field over the lossy half-space, cross polarised components are obtained since the polarisation of the antenna is vertical. By using (1), the z-component of the scattered electric field can be calculated. Thus, we have

\[
\frac{\mathbf{E}_z}{\mathbf{j} k_0^2} = \frac{1}{4\pi} \int \left[ e^{-jk_0R_0} \left( \frac{e^{-jk_0R_1}}{R_0} + \frac{e^{-jk_0R_1}}{R_1} \right) \right. \\
+ \cos^2\theta_0 (1 + jk_0R_0) e^{-jk_0R_0} \left( -\sec\theta_0 + jk_0R_0 + 3 \right) \\
- \frac{jk_0R_0}{1 + jk_0R_0} + \cos^2\theta_1 (1 + jk_1R_1) e^{-jk_1R_1} \left( -\sec\theta_1 \right) \\
+ jk_0R_1 + 3 + \frac{jk_0R_1}{1 + jk_0R_0} - 2k_0^2P - 2T_5 
\]

(22)

in which \( T_5 \) to \( T_8 \) and their sub variables are obtained from the following equations.

\[
T_5 = -\frac{\pi k_0^2}{2} e^{-jk_0R_0} e^{j\pi x} \\
\beta \left[ -j e^{W e} C_1 + T_7 \text{erfc}(j\sqrt{W e}) - j T_8 \right] \\
T_6 = -\cos^2\theta_1 e^{-W e} e^{-jk_1R_1} \left( \frac{1}{2} + jk_0R_1 + \frac{R_1C_1}{\cos\theta_1} \right) \\
T_7 = -(1 + 2jk_0R_1) C_8 + \frac{jk_0R_0}{R_1^{1.5}} e^{-W e} e^{-jk_1R_1}\cos^2\theta_1 + C_0 \\
T_8 = C_4 e^{-jk_1R_1-W e} \left[ C_1 - jk_0\cos\theta_1 - \frac{W e \cos^2\theta_1 + R_1C_1}{2R_1^{1.5}W e} \right] \\
C_8 = -\frac{e^{-jk_1R_1-W e}}{2R_1^{1.5}} \left[ -1 + (h+z) C_4 + \cos^2\theta_1 \left( jk_0R_1 + \frac{5}{2} \right) \right] \\
C_9 = \frac{e^{-jk_0R_1-W e}}{2R_1^{1.5}} \left[ -2R_1 C_{10} + 2R_1 C_1 \right] \\
+C_1(1 + 2jk_0R_1) \cos\theta_1] \\
C_{10} = \frac{jk_0R_1}{2R_1} \left[ (1 + \beta) \sin^2\theta_1 \cos\theta_1 + 2\sin^2\theta_1 + (\beta + \cos\theta_1) \right] \\
\sin^2\theta_1 - \frac{2 \cos^2\theta_1 \beta \omega^2}{\beta R_1^2} \right] 
\]

(23)

### 3.2 | Scattered H-field components

The scattered magnetic field components can also be calculated using the intermediate Hertz potential (9). In other words, by substituting (9) along with (17) into (2), different components of the scattered magnetic field can be obtained. The x-component of the scattered magnetic field may be expressed as

\[
\mathbf{H}_x = -\frac{1}{4\pi} \left[ e^{-jk_0R_0} \left( 1 + jk_0R_0 \right) + e^{-jk_0R_1} \left( 1 + jk_0R_1 \right) \right] \\
- e^{jk_0R_1} \beta \sqrt{2\pi k_0} \left( \text{erfc}(j\sqrt{W e}) e^{-W e} \right) T_9 \\
+ \frac{jC_{11}}{y\sqrt{\pi W e} R_1} \right] 
\]

(24)

wherein \( T_9 \) and \( C_{11} \) can be expressed as follows:

\[
T_9 = \frac{C_{11}}{y\sqrt{R_1}} + \frac{1 + 2jk_0R_1}{2R_1^{1.5}} \\
C_{11} = -\frac{jk_0\beta^2}{2R_1} \left( 1 - \cos^2\theta_1 \right).
\]

The y-component of the scattered magnetic field over the lossy half-space can also be written as:

\[
\mathbf{H}_y = -\frac{1}{4\pi} \left[ e^{-jk_0R_0} \left( 1 + jk_0R_0 \right) + e^{-jk_0R_1} \left( 1 + jk_0R_1 \right) \right] \\
- e^{jk_0R_1} \beta \sqrt{2\pi k_0} \left( \text{erfc}(j\sqrt{W e}) e^{-W e} \right) T_9 \\
+ \frac{jC_3}{x\sqrt{\pi W e} R_1} \right] 
\]

(26)

in which \( T_9' \) can be obtained by changing \( C_{11} \) and \( y \) to \( C_3 \) and \( x \) in (25), respectively.

### 4 | RESULTS

To evaluate the accuracy and efficiency of the recently developed method for the calculation of the intermediate Hertz potential and the scattered electric and magnetic field components over the lossy half-space, the NRMSE and the NMAE, in where the standard deviation is used for normalisation, are utilised for each frequency. It should be noted that the NRMSE is calculated using the reference and calculated values expressed as
Table 1: Media parameters

| Parameter | Seawater | Silty loam soil | Silty clay soil | Lake water | Silver | Gold |
|-----------|----------|----------------|----------------|------------|--------|------|
| $F$       | 300 MHz  | 1.8 GHz        | 108 MHz        | 9.4 GHz    | 351.87 THz | 420.52 THz |
| $k_0$ (m$^{-1}$) | 6.28     | 37.7           | 2.26           | 196.87     | $7.36 \times 10^6$ | $8.80 \times 10^6$ |
| $H$       | 0.5 m    | 2.5 m          | 1.5 m          | 0.5 m      | 48 nm   | 220 nm |
| $R$       | $8/k_0$  | $10/k_0$       | $10/k_0$       | $10/k_0$  | $10/k_0$ | $20/k_0$ |
| $\sigma_0$ (S/m) | 4        | 0.04           | 0.012          | 17.51      | $6.30 \times 10^6$ | $7.20 \times 10^6$ |
| $\epsilon_r$ | 80       | 4.64           | 23.06          | 60.98      | $-36.93$ | $-17.2$ |
| $|n_{\infty}|$ | 252.77   | 4.66           | 23.15          | 69.58      | 36.93   | 17.24 |
| $\tan \delta$ | 2.99     | 0.09           | 0.086          | 0.55       | 0.012   | 0.075 |

\[
NRMSE = \left( \frac{\sum_{i=1}^{N} (\chi_i - \hat{\chi}_i)^2}{N} \right)^{1/2} \sigma_\chi 
\]

wherein $\chi$ represents the reference values, which are the numerical solutions obtained by the high-order global adaptive quadrature method [45], $\hat{\chi}$ denotes the calculated values, which are the proposed analytical solutions, $N$ is the number of reference or calculated values and $\sigma_\chi$ is the standard deviation of the reference values. In order to calculate the NRMSE for each frequency, the amplitude of the scattered electric field (i.e. $\sqrt{|E_x|^2 + |E_y|^2 + |E_z|^2}$) is considered as $\chi$ while $\theta$ (elevation angle in Figure 1) is changed between 0 to $\pi/2$ with the resolution of $\pi/200$ ($N = 100$). The numerical solution for the amplitude of the scattered electric field is also considered as $\chi$, while $\theta$ is changed between 0 to $\pi/2$ with the resolution of $\pi/200$. The next accuracy metric is the NMAE defined as the maximum deference between the proposed analytical and numerical solutions for the scattered electric field, which is normalised by the standard deviation of the numerical results. Therefore, the NMAE can be expressed as:

\[
NMAE = \frac{\max_{i=1,2,\ldots,N} |\chi_i - \hat{\chi}_i|}{\sigma_\chi}.
\]

Here, the proposed solution of the scattered E and H fields are compared with the King and Norton-Bannister solutions, while the rigorous numerical computation of the SIs using the high-order global adaptive quadrature method [45] is considered as reference. Moreover, the elevation pattern of the scattered electric field is compared with the numerical solution. Six various problems have been selected for analysis, with parameters listed in Table 1. Silver and gold illustrate plasmonic media, which have been considered in optical frequency range, and others represent ordinary media.

4.1 Non-plasmonic media

The proposed formulations are applied to six different ordinary media listed in Table 1. In this table, the soil composition is characterised by the percentage of soil constituents, which are sand, clay, silt and water. Silty loam soil consists of roughly equal amounts of silt and sand and a little less clay. On the other hand, silty clay soil has more clay than silt. The complex relative permittivity of seawater as well as lake water are calculated using the Meissner and Wentz model [46], which is based on the double Debye model and is quite accurate at higher frequencies. On the other hand, for silty loam and silty clay soil, the complex relative permittivity is calculated by the developed Dobson model [47, 48].

The frequency of the antenna over each medium in Table 1 is related to the application of wave scattering over that region. For seawater, the very high frequency (VHF) band has been selected since the pulsed radars operating at this frequency band are employed for remote sensing of ocean surface in order to extract the speed and direction of ocean surface currents in real time [2, 49]. For the silty loam soil scattering problem, the global system for mobile communications (GSM) frequency band has been considered since finding the pattern of the scattered EM fields, coverage area and blind zones over earth surface are of interest [1, 50]. Therefore, GSM-1800 (1.8 GHz) has been selected for this scattering problem. On the other hand, the frequency modulation (FM) broadcast band has been selected for the silty clay soil scattering problem due to its application in finding the coverage area of the passive radars and FM broadcast radio systems [51]. For radio oceanography applications, X-band marine radar is commonly used to scan the water surface with high temporal and spatial resolutions [52], and this frequency range has been selected for the lake water scattering problem.

The elevation pattern of the scattered electric field, $|E| = (|E_x|^2 + |E_y|^2 + |E_z|^2)^{1/2}$, over the selected ordinary media listed in Table 1 is calculated using the proposed method and compared with the numerical and conventional (King-Bannister and King solutions) methods. Figure 2 depicts the elevation pattern of the scattered electric field for a VED over selected non-plasmonic media in different frequencies and distances from the antenna. The horizontal and vertical axes, respectively, correspond to $\theta = 90^\circ$ and $\theta = 0^\circ$, as shown in Figure 1. As can be seen in Figure 2, the King and Norton-Bannister results are close to each other and become indistinguishable when the media contrast is sufficiently high [53]. In Figure 2(a), which has
been obtained for seawater at VHF band, the proposed solution has a good agreement with the numerical method not only for low angles, but also near the interface, in which groundwave contribution is high. In order to evaluate the accuracy of the proposed solution when compared with the conventional solutions, the NRMSE and NMAE are calculated shown in Table 2. As can be seen in this table, both NRMSE and NMAE of the proposed solution are lower than the conventional solutions for both scattered E and H fields. The elevation pattern of the scattered electric field for a VED and its comparison with the proposed, Norton-Bannister and King solutions over (a) seawater, (b) silty loam soil, (c) silty clay soil and (d) lake water as non-plasmonic media.

![Graph](image.png)

**FIGURE 2** The elevation pattern of the scattered electric field for a VED and its comparison with the proposed, Norton-Bannister and King solutions over (a) seawater, (b) silty loam soil, (c) silty clay soil and (d) lake water as non-plasmonic media.

**TABLE 2** Performance comparison between the proposed and conventional solutions for the ordinary media

| Medium         | E-field NRMSE | E-field NMAE | H-field NRMSE | H-field NMAE | E-field NRMSE | E-field NMAE | H-field NRMSE | H-field NMAE |
|----------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| Seawater       | 0.0355       | 0.0571       | 0.0654       | 0.1317       | 0.0649       | 0.1313       | 0.0112       | 0.0238       |
| Silty loam soil| 0.1182       | 0.2216       | 0.1459       | 0.2585       | 0.1458       | 0.2584       | 0.0170       | 0.0452       |
| Silty clay soil| 0.1098       | 0.2028       | 0.1555       | 0.2794       | 0.1528       | 0.2780       | 0.0320       | 0.0610       |
| Lake water     | 0.044        | 0.1042       | 0.0601       | 0.1218       | 0.0601       | 0.1218       | 0.0522       | 0.1096       |
field over silty loam soil is shown in Figure 2(b). The proposed solution follows the numerical solution, particularly on the pattern nulls and also on the interface, which corresponds to the groundwave contribution. The NRMSE and NMAE evaluation of the scattered E and H fields reveal that the proposed solution outperforms the conventional solutions even for moderate media contrast. In Figure 2(c), the elevation pattern of the scattered electric field over silty clay soil has been obtained. It can be observed from the figure that the proposed solution agrees well with the numerical solution, particularly in high angles and near the interface, and further, the NRMSE and NMAE values for silty clay soil problem listed in Table 2 are lower than the conventional solutions. Figure 2(d) illustrates the elevation pattern of the scattered electric field for lake water, in which discrepancies between results are not noticeable. However, the NRMSE and NMAE values listed in Table 2 substantiate that the proposed solution is more accurate than the King and Norton-Bannister solutions. To evaluate the accuracy of the proposed solution for the scattered magnetic field, the NRMSE and NMAE values for the H-field over non-plasmonic media listed in Table 1 are calculated, and are shown in Table 2. As evident, all the NRMSE and NMAE values for the H-fields are smaller than the conventional solutions for the ordinary media.

To assess the robustness of the proposed solution in terms of NRMSE at various frequencies, the NRMSE value is calculated for the scattered electric field components for the whole frequency range, which is between 100 MHz and 100 GHz for non-plasmonic media. In other words, the NRMSE value is calculated for each frequency while the observation point angle is changed between $\theta = 0^\circ$ and $\theta = 90^\circ$. It should be noted that the real and imaginary parts of the dielectric constant for the ordinary media listed in Table 1 depend on frequency. Therefore, the relative permittivity should be calculated for each frequency for the NRMSE calculation. For seawater, the relative permittivity depends on temperature and also salinity and varies with frequency [54]. In the NRMSE calculation of seawater, the salinity of seawater and lake water

---

**Figure 3**  The NRMSE of the scattered electric field for a VED at various frequencies and its comparison with the Norton-Bannister and King solutions over (a) seawater, (b) silty loam soil, (c) silty clay soil and (d) lake water as non-plasmonic media.
has been assumed as 35 and 0, respectively, while the temperature is 17°C. For the silty clay and loam soil, the relative permittivity depends on frequency, temperature and also the texture of the soil [55]. For this scattering problem, the temperature has been assumed as 23°C.

Figure 3 illustrates the NRMSE comparison of the proposed and conventional solutions over ordinary media in a wide variety of frequency ranges, while the antenna height ($\hat{b}$) and distance of the observation point from the origin of the coordinate system ($R$) are assumed to be $\lambda/10$ and $k_0/10$, respectively. As is obvious from this figure, the NRMSE value of the scattered electric field components is better than the conventional solutions (i.e. King and Norton-Bannister solutions) in all frequencies shown in Figure 3(a–d). In order to evaluate the accuracy of each scattered electric field component, that is, $E_\rho$ and $E_z$, the NRMSE has been calculated over the presented ordinary media. Figure 4 shows the NRMSE comparison of the scattered electric field for each component, that is, $E_\rho$ and $E_z$, over non-plasmonic media at various frequencies. As can be seen in this figure, the $E_\rho$ component obtained by the proposed method has better accuracy in comparison with the $E_z$ component. In other words, the $E_\rho$ component has more impact on the NRMSE value of the scattered electric field over non-plasmonic media since its NRMSE value is greater than the other component. Also, the NMAE comparison between the proposed and the conventional methods shown in Figure 5 substantiates that the proposed solution outperforms the conventional solution over non-plasmonic media since the value of NMAE is better than the King and the Norton-Bannister solutions in all frequency range.

4.2 | Plasmonic media

In order to evaluate the accuracy of the proposed solution for plasmonic media, silver and gold have been considered and the elevation patterns of the scattered electric field have been
compared with the numerical solutions at 351.87 and 420.52 THz for silver and gold, respectively. It should be noted that for light-matter interaction in THz frequencies, the real part of the permittivity attains negative value and varies with frequency [56]. Figure 6(a) and (b) depict the elevation patterns of the scattered electric field over the silver and gold, respectively, as plasmonic media. As is obvious by these two figures, the proposed solution agrees well with the numerical method, especially in high angles and near the interface where the surface wave contribution is high. Moreover, the NRMSE and NMAE values mentioned in Table 3 reveal that the proposed solution outperforms the conventional solution for the plasmonic media. In order to evaluate the accuracy of the proposed solution for magnetic field, the NRMSE and NMAE values of the H-field for silver and gold have been shown in Table 3. As evident, all the NRMSE and NMAE values for the H-field are smaller than the conventional solutions.

Similar to non-plasmonic media, the NRMSE evaluation of the scattered electric field for silver and gold are accomplished, while the frequency is changed between 300 and 900 THz. Figure 7 demonstrates the NRMSE value of the scattered electric field at various frequencies for plasmonic media. As can be seen in this figure, the NRMSE value of the scattered electric field is less than the King and Norton-Bannister solutions in the optical frequency range. The NRMSE value of the silver in Figure 7(a) is relatively similar to gold in Figure 7(b) since the real and imaginary parts of the permittivity for both of them are quite similar in the optical frequency range. To evaluate the accuracy of each scattered electric field component over plasmonic media, the NRMSE has been calculated for \( E_\rho \) and \( E_z \) components over silver and gold and compared with the conventional solutions, as shown in Figure 8. As observed in this figure, the \( E_\rho \) and \( E_z \) components obtained by the proposed method have better
accuracy in comparison with the conventional solutions over plasmonic media. In addition, the accuracy of the proposed solution is evaluated in terms of NMAE for plasmonic media. Figure 9 depicts the NMAE of the proposed solution for the scattered electric field intensity over plasmonic media. As is obvious, the proposed solution outperforms the conventional solutions at various frequencies. It is worth mentioning that the complex relative permittivity $\varepsilon'$ for the plasmonic media varies with frequency. Here, the Drude model [57] is employed for the calculation of the complex
FIGURE 8 The NRMSE comparison of each scattered E-field component for a VED at various frequencies over (a) silver and (b) gold.

FIGURE 9 The NMAE of the scattered electric field for a VED at various frequencies and its comparison with the Norton-Bannister and King solutions over (a) silver and (b) gold.

relative permittivity at each frequency, which can be expressed as:

\[ \varepsilon_r(\omega) = 1 + \frac{j\sigma_0}{\omega\varepsilon_0(1 - j\omega\tau)} \]  \hspace{1cm} (29)

wherein \( \tau \) represents the average time between collisions experienced by an electron and can be written as

\[ \tau = \frac{\sigma_0 m}{n_0 e^2} \]  \hspace{1cm} (30)

in which \( m \) denotes the electron mass, \( n_0 \) is the electron density of the metal and \( e \) represents the elementary charge. Subsequently, (29) is utilised for each frequency so as to calculate the real and imaginary parts of the complex relative permittivity for the NRMSE and the NMAE calculations at various frequencies.

5 | CONCLUSION

The authors have developed a new asymptotic solution for the far-zone EM fields of a VED radiating over an imperfectly conducting half-plane. The solution has been evaluated for the both ordinary and plasmonic media. Accuracy comparison indicates that the proposed solution outperforms the conventional solutions like King and Norton-Bannister solutions, in terms of NRMSE and NMAE at various frequencies and distances from the antenna in the far field region. Possible future enhancements to the method...
involve developing the solution for slightly rough surfaces in the presence of a VED in the near and far field regions.

ACKNOWLEDGMENTS

The work was supported in part by Natural Sciences and Engineering Research Council of Canada (NSERC) Discovery Grants to W. Huang (RGPIN-2017-04508 and RGPAS-2017-507962). Also, the authors would like to thank Professor K. A. Michalski from Texas A&M University for his technical assistance.

ORCID

Mohsen Eslami Nazari https://orcid.org/0000-0001-9960-5963

Weimin Huang https://orcid.org/0000-0001-9622-5041

REFERENCES

1. Nazari, M.E., Ghorbani, A.: Predicting a three-dimensional radar coverage area: Introducing a new method based on propagation of radio waves. IEEE Antennas Propag. Mag. 58(1), 28–34 (2016)
2. Eslami Nazari, M., Huang, W., Zhao, C.: Radio frequency interference suppression for HF surface wave radar using CEMD and temporal windowing methods. IEEE Geosci. Remote Sens. Lett. 17(2), 212–216 (2020)
3. Cross, J.D, Atkins, P.R.: Electromagnetic propagation in four-layered media due to a vertical electric dipole: A clarification. IEEE Trans. Antennas Propag. 63(2), 866–870 (2015)
4. Liu, H., Takahashi, K., Sato, M.: Measurement of dielectric permittivity and thickness of snow and ice on a brash ice lagoon using GPR. IEEE J. Sel. Topics Appl. Earth Observ. 7(3), 820–827 (2014)
5. Novotny, L.: Allowed and forbidden light in near-field optics. i. a single dipolar light source. J. Opt. Soc. Am. A. 14(1), 91–104 (1997)
6. Jeon, T.-I., Grischkowsky, D.: THz zenneck surface wave (THz surface plasmon) propagation on a metal sheet. Appl. Phys. Lett. 88(6), 061131–1–60113-3 (2006)
7. Ung, B., Sheng, Y.: Optical surface waves over metallo-dielectric nanostructures: Sommerfeld integrals revisited. Opt. Exp. 16(12), 9073–9086 (2008)
8. Nikitin, A.Y., et al.: In the diffraction shadow: Norton waves versus surface plasmon polaritons in the optical region. New J. Phys. 11(12), 123020 (2009)
9. Sommerfeld, A.: Propagation of waves in wireless telegraphy. Ann. Phys. 28, 665–736 (1909)
10. Ishimaru, A., Rockway, J.D., Lee, S.W.: Sommerfeld and zenneck wave propagation for a finitely conducting one-dimensional rough surface. IEEE Trans. Antennas Propag. 48(9), 1475–1484 (2000)
11. Wise, W.H.: Asymptotic dipole radiation formulas. Bell Syst. Tech. J. 8(4), 662–671 (1929)
12. Wise, W.H.: Sommerfeld and Zenneck wave propagation for a finitely conducting one-dimensional rough surface. Bell Syst. Tech. J. 8(4), 662–671 (1929)
13. Pol, B.V.D.: Theory of the reflection of the light from a point source by a finitely conducting flat mirror, with an application to radiotelegraphy. Physica. 2(1), 843–853 (1935)
14. Norton, K.A.: The propagation of radio waves over the surface of the earth and in the upper atmosphere. Proc. IRE. 25(5), 1203–1236 (1937)
15. Bannister, P.R.: New formulas that extend Norton’s far-field elementary dipole equations to the quasi-nearfield range (Tech. Rep. 6883). Naval Underwater Syst. Center, New London (1984)
16. Felsen, L.B., Marcuvitz, N.: Radiation and Scattering of Waves. Wiley-IEEE Press, New York (1994)
17. Wait, J.R.: Radiation from a vertical electric dipole located over laterally anisotropic groundplane. Elect. Lett. 26(1), 74–76 (1990)
18. Wait, J.R.: Wave Propagation Theory. Pergamon, New York (1981)
19. Makarov, G.I., Novikov, V.V., Rybachek, S.T.: Electromagnetic Waves Propagation Over the Earth’s Surface. Nauka, Moscow (1991)
20. Van der Waerden, B.L.: On the method of saddle points. Appl. Sci. Res. B. 2(1), 33–45 (1952)
21. Collin, R.E.: Hertzian dipole radiating over a lossy earth or sea: Some early and late 20th-century controversies. IEEE Antennas Propag. Mag. 46(2), 64–79 (2004)
22. Bernard, G., Ishimaru, A.: On complex waves. Proc. Inst. Elect. Eng. 114, 43–49 (1967)
23. King, R.W.P.: The electromagnetic field of a horizontal electric dipole in the presence of a three-layered region. J. Appl. Phys. 69(12), 7987–7995 (1991)
24. King, R.W.P.: The electromagnetic field of a horizontal electric dipole in the presence of a three-layered region: Supplement. J. Appl. Phys. 74(8), 4845–4848 (1993)
25. King, R.W.P., Sandler, S.S.: The electromagnetic field of a vertical electric dipole over the earth or sea. IEEE Trans. Antennas Propag. 42(3), 382–389 (1994)
26. King, R.W.P.: The electromagnetic field of a vertical electric dipole in the presence of a three-layered region. Radio Sci. 29(1), 97–113 (1994)
27. Wait, J.R.: Electromagnetic fields of a vertical electric dipole over a laterally anisotropic surface. Elect. Lett. 38(10), 1719–1723 (1990)
28. Wait, J.R.: The ancient and modern history of EM ground-wave propagation. IEEE Trans. Antennas Propag. 40(5), 7–24 (1998)
29. Green, H.E.: Derivation of the norton surface wave using the compensation theorem. IEEE Trans. Antennas Propag. 49(6), 47–57 (2007)
30. Mahmoud, S.F., King, R.W.P., Sandler, S.S.: Remarks on “the electromagnetic field of a vertical electric dipole over the earth or sea” and reply. IEEE Antennas Propag. 47(11), 1745–1747 (1999)
31. Yoshizawa, N., Iwai, T.: The electromagnetic field of a vertical electric dipole over the earth or sea”. IEEE Trans. Antennas Propag. 44(2), 271–272 (1996)
32. Wait, J.R.: Comment on “the electromagnetic field of a vertical electric dipole in the presence of a three-layered region” by Ronald W. P. King and Sheldon S. Sandler”. Radio Sci. 33(2), 251–253 (1998)
33. Yokoyama, A.: Comments on “the electromagnetic field of a vertical electric dipole over the earth and sea”. IEEE Trans. Antennas Propag. 43, 541–542 (1995)
34. Zhang, H.Q., Pan, W.Y.: Electromagnetic field of a vertical electric dipole on a perfect conductor coated with a dielectric layer. Radio Sci. 37(4), 13–1–13–7 (2002)
35. Zhang, H.Q., Li, K., Pan, W.Y.: The electromagnetic field of a vertical dipole on the dielectric-coated imperfect conductor. J. Electromag. Waves Appl. 18(10), 1305–1320 (2004)
36. Li, K., Lu, Y.: Electromagnetic field generated by a horizontal electric dipole near the surface of a planar perfect conductor coated with a uniaxial layer. IEEE Trans. Antennas Propag. 53(10), 3191–3200 (2005)
37. Michalski, K.A., Mosig, J.R.: On the surface fields excited by a Hertzian dipole over a layered halfspace: From radio to optical wavelengths. IEEE Trans. Antennas Propag. 63(12), 5741–5752 (2015)
38. King, R.W.P.: Electromagnetic field of a vertical dipole over an imperfectly conducting half-space. Radio Sci. 25, 149–160 (1990)
39. Sarabandi, K., Casciato, M.D., Koh, I.S.: Efficient calculation of the fields of a dipole radiating above an impedance surface. IEEE Trans. Antennas Propag. 50(9), 1222–1235 (2002)
40. Eslami Nazari, M., Huang, W.: An analytical solution of electromagnetic radiation of a vertical dipole over a layered half-space. IEEE Trans. Antennas Propag. 68(2), 1181–1185 (2020)
41. Parhami, P., Rahmat-Samii, Y., Mittra, R.: An efficient approach for evaluating Sommerfeld integrals encountered in the problem of a current element radiating over lossy ground. IEEE Trans. Antennas Propag. 28(1), 100–104 (1980)
42. Michalski, K.A.: On the efficient calculation of integral arising in the Sommerfeld halfspace problem. IEEE Proc. Part H-Microw. Antenna Propagat. 132, 312–318 (1985)
43. Johnson, W.A., Dudley, D.G.: Real axis integration of Sommerfeld integrals: Source and observation points in air. Radio Sci. 18(02), 175–186 (1983)
44. Golubovic, R., Polimeridis, A.G., Mosig, J.R.: Efficient algorithms for computing Sommerfeld integral tails. IEEE Trans. Antennas Propag. 60(5), 2409–2417 (2012)
45. Press, W.H., et al.: Numerical Recipes. In: The Art of Scientific Computing, 3rd ed. Cambridge University Press (2007)
46. Meissner, T., Wentz, F.J.: The complex dielectric constant of pure and sea water from microwave satellite observations. IEEE Trans. Geosci. Remote Sens. 42(9), 1836–1849 (2004)
47. Van Dam, R.: Calibration functions for estimating soil moisture from GPR dielectric constant measurements. Commun. Soil Sci. Plant Anal. 45, 01 (2014)
48. Dobson, M.C., et al.: Microwave dielectric behavior of wet soil-part ii: Dielectric mixing models. IEEE Trans. Geosci. Remote Sens. GE-23(1), 35–46 (1985)
49. Barnick, D.E.: Theory of HF and VHF propagation across the rough sea, 2, application to HF and VHF propagation above the sea. Radio Sci. 6(5), 527–533 (1971)
50. Kurner, T., Meier, A.: Prediction of outdoor and outdoor-to-indoor coverage in urban areas at 1.8 GHz. IEEE J. Sel. Areas Commun. 20(3), 496–506 (2002)
51. Barott, W.C., Himed, B.: Simulation model for wide-area multi-service passive radar coverage predictions. In: 2013 IEEE Radar conference, Ottawa, pp. 1–4 (2013)
52. Chen, X., et al.: Rain detection from X-band marine radar images: a support vector machine-based approach. IEEE Trans. Geosci. Remote Sens. 58(3), 2115–2123 (2020)
53. Michalski, K.A., Jackson, D.: Equivalence of the King and Norton-Bannister theories of dipole radiation over ground with extensions to plasmonics. IEEE Trans. Antennas Propag. 64(12), 5251–5261 (2016)
54. Somaraju, R., Trumpf, J.: Frequency, temperature and salinity variation of the permittivity of seawater. IEEE Trans. Antennas Propag. 54(11), 3441–3448 (2006)
55. Wobschall, D.: A theory of the complex dielectric permittivity of soil containing water: The semidisperse model. IEEE Trans. Geosci. Electron. 15(1), 49–58 (1977)
56. Geddes, C.: Reviews in Plasmonics 2016, vol. 2016. Springer, 01 (2017)
57. Nickelson, L.: Electromagnetic Theory and Plasmonics for Engineers. Springer, Singapore (2019).

How to cite this article: Nazari ME, Huang W. Asymptotic solution for the electromagnetic scattering of a vertical dipole over plasmonic and non-plasmonic half-spaces. IET Microw. Antennas Propag. 2021;15:704–717. https://doi.org/10.1049/mia2.12069