RESUMMATION FOR HEAVY QUARK PRODUCTION NEAR PARTONIC THRESHOLD

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Abstract

We review some techniques of resummation applied to heavy quark production in hadronic scattering and electroproduction near partonic threshold, and discuss the reduction of factorization scale dependence in resummed cross sections.

1 Introduction

In this talk, we will discuss high-order corrections to heavy quark production in QCD. To get a better confidence in our ability to predict cross sections for various heavy quark reactions, including open flavor and quarkonia production, we need to gain control over a class of corrections associated with what is sometimes called partonic threshold. This talk will describe the nature of these corrections, and report on the progress of the past few years \cite{1, 2, 3, 4, 5, 6} in their resummation to all orders in perturbation theory.

Corrections due to resummation may turn out to be small, in which case our confidence in low-order perturbative cross sections should increase, or, they may turn out to be large, and may afford sensitivity to QCD dynamics in a regime where all orders of perturbation theory are relevant. It is possible that both scenarios apply in different cross sections, and/or in different kinematic ranges of the same cross section.

\footnote{Talk presented at Workshop on Physics With Electron Polarized Ion Collider, EPIC '99, April 8-11, 1999, Indiana University Cyclotron Facility.}
2 Threshold Resummation for Hard Inclusive Scattering

Inclusive heavy quark production is just one example of a QCD process at large momentum transfer. We may consider the class of cross sections in which we sum over all final states that include any heavy system \( F = Q\bar{Q}, \ldots \), which can only be produced by a short-distance process in partonic scattering.

Suppose for simplicity that the total mass \( Q \) of the system \( F \) is of order \( \sqrt{S} \), the total (hadronic) center of mass energy, and that the rapidity \( y \) of the produced system is not a large parameter. Such a cross section can be expressed to leading power by the factorized expression

\[
\frac{d\sigma_{AB\rightarrow F\times}}{dQ^2dy} = \sum_{ab} \int_{Q^2/S}^1 dz \int dx_a dx_b \phi_{a/A}(x_a, \mu^2)\phi_{b/B}(x_b, \mu^2) \\
\times \delta \left( z - \frac{Q^2}{x_a x_b S} \right) \hat{\sigma}_{ab\rightarrow F\times} \left( z, e^{-2y} \frac{x_a}{x_b}; \frac{Q}{\mu}, Q, \alpha_s(\mu^2) \right),
\]

which is illustrated in Fig. [1]. The \( \phi \)'s are parton distributions (in some factorization scheme, like DIS or \( \overline{\text{MS}} \)), while \( \hat{\sigma} \) is a partonic hard-scattering function, which at lowest order is the Born cross section for \( a + b \rightarrow F + X \),

\[
\hat{\sigma} = \sigma_{\text{Born}} + \frac{\alpha_s}{\pi} \hat{\sigma}^{(1)} + \ldots.
\]

These relations may be written down for both unpolarized and polarized cross sections. Corrections in either case are power-suppressed \( 2 \); beginning at \( \mathcal{O}(1/Q^2) \) for unpolarized or longitudinally polarized cross sections, and at \( \mathcal{O}(1/Q) \) for transversely polarized cross sections.

In Eq. (1), the limit \( Q^2/x_a x_b S \rightarrow 1 \) is associated with corrections that tend to enhance the cross section. These enhancements are the object of threshold resummation. We will discuss below the origin of this terminology, why such corrections can sometimes be large, and how they can be resummed to all orders. Much of the following discussion follows Ref. [8].

The kinematics of the partonic process require that \( x_a x_b S \geq Q^2 \), so that \( z \leq 1 \) in Eq. (1). At \( z = 1 \), the partons have just enough energy to produce

\[2\]The extension to single-particle inclusive cross sections has recently been carried out.
the observed final state, with no extra hadronic radiation. This is what we shall refer to as the elastic limit \[9\], or partonic threshold. It is important to distinguish partonic threshold from the usual concept of a threshold. In particular, in heavy quark production, we shall assume that the heavy quarks of mass \(M_Q\) are produced with nonzero velocity \(\beta\), and hence with a total invariant mass \(Q^2 > 4M_Q^2\). Thus, only for \(\beta = 0\) does partonic threshold coincide with true threshold. For the Drell-Yan production of highly relativistic lepton pairs with \(Q^2 \gg 4m_\ell^2\), partonic threshold still refers to \(z = 1\), and is a source of potentially large corrections.

Figure 1: Hard-scattering cross section in cut (unitarity) diagram notation.

Typical hard-scattering cross sections \(\hat{\sigma}(z\ldots)\) in Eq. (1) are distributions in the variable \(z\) rather than simply functions of \(z\), because they include contributions from virtual as well as real gluons. We are interested in a class of potentially large, positive corrections due to such distributions, which occur in all \(\sigma^{(\alpha)}\). We next explain in what sense they are “large”, and why they are positive to all orders.

At order \(\alpha_s^n\), the leading logarithmic distributions in Eq. (1) are of the form (suppressing color factors, which depend on parton type)

\[-\frac{\alpha_s^n}{n!} \left[ \frac{\ln^{2n-1}((1-z)^{-1})}{1-z} \right]_+, \tag{3}\]

whose integral with a smooth function \(\mathcal{F}(z)\) (such as the convolution of
parton distributions in Eq. (1) is defined as

\[- \frac{\alpha_s^n}{n!} \int_0^1 dz \frac{\mathcal{F}(z) - \mathcal{F}(1)}{1 - z} \ln^{2n-1} \left( (1 - z)^{-1} \right) \]

\[= \frac{\alpha_s^n}{n!} \int_0^1 dz \mathcal{F}'(1) \ln^{2n-1} \left( (1 - z)^{-1} \right) + \ldots \sim \frac{\alpha_s^n}{n!} \mathcal{F}'(1)(2n - 1)! + \ldots (4) \]

where we have kept only the first term in the expansion of \( \mathcal{F}(z) \) about \( z = 1 \). It is evident that such terms give, at least formally, contributions that grow even faster than \( n! \) at \( n \)th order. If they had alternating signs, these contributions might add up to a finite number somehow, but they are all of the same sign.

The signs in Eq. (4) come directly from the manner in which hard-scattering cross sections are computed. The fully inclusive Drell-Yan cross section \( d\sigma/dQ^2 \) illustrates the situation. The convolution in Eq. (1) factors into simple products of functions for each pair of initial-state partons \( a \) and \( b \), under moments with respect to \( \tau = Q^2/S \);

\[ \sigma_{AB}^{\text{DY}}(N, Q) = \int_0^{1} d\tau \tau^{N-1} \frac{d\sigma_{AB}^{\text{DY}}(\tau, Q)}{dQ^2} \]

\[= \sum_{ab} \phi_{a/A}(N, \mu) \hat{\sigma}_{ab}^{\text{DY}}(N, Q, \mu) \phi_{b/B}(N, \mu). \]

(5)

Identifying \( \sigma_{AB}^{\text{DY}} \) as the contribution to the cross section from the parton combination \( a + b \), we find

\[ \hat{\sigma}_{ab}^{\text{DY}}(N, Q, \mu) = \prod_{i=a,b} \frac{1}{\int_0^1 dx x^N \phi_i(x, \mu)} \int_0^{1} d\tau \tau^{N-1}\frac{d\sigma_{AB}^{ab}(\tau)}{dQ^2}. \]

(6)

Neglecting parton labels, this is the ratio of the moment of a physical cross section to a product of moments of parton distributions. Because \( \hat{\sigma} \) is, by construction, dependent only on short-distance behavior, the ratio may be computed in perturbation theory, as illustrated schematically in Fig. 2 (for the DIS scheme). The numerator is a moment of the perturbative partonic Drell-Yan cross section, while the denominator is the product of moments of two perturbative parton distributions. For quark-antiquark processes, the parton distributions are the same, so the denominator is the square of squared partonic amplitudes, summed over final states.
Figure 2: Schematic representation of moments of the Drell-Yan partonic hard-scattering function.

At each order, both the numerator and denominator in Fig. 2 have double-logarithmic terms like Eq. (3). Before moments, the perturbative Drell-Yan cross sections include distributions in $1 - z$, like Eq. (3) above, while in the deeply inelastic scattering cross section the same sort of terms depend on Bjorken $x$ through $1 - x$. After moments, both give double-logarithmic $\alpha_n^a \ln^{2n} N$ at $n$th order, with $N$ the moment variable. These leading logarithms are the finite remainders of corrections from $n$ pairs of real and virtual gluons that attach to the scattered quarks in DIS and the annihilating pair in DY. In the denominator, each DIS parton distribution, which is itself of the form of a cross section, has both incoming and outgoing quarks, while in the numerator, DY involves incoming quarks only.

On physical grounds, the numerator must vanish as $z \to 1$ and the denominator must vanish as $x \to 1$. The reason for this may be seen by recalling the relations of $z$ and $x$ to the invariant mass $W$ of hadrons in the final state for the two cases:

\[
\begin{align*}
\text{DY} & : \quad W^2 \leq \frac{1}{4} Q^2 (1 - z)^2, \\
\text{DIS} & : \quad W^2 \sim Q^2 (1 - x)/x.
\end{align*}
\] (7)

The limits $z \to 1$ and $x \to 1$ thus both correspond to nearly elastic scattering: for Drell-Yan, the annihilation of a quark pair into an electroweak vector boson, for DIS, the scattering of a quark into a nearly massless jet of particles. It is not difficult to verify that in these limits, the partonic cross section is suppressed. Indeed, in these limits, we expect the coherent scattering
of hadronic bound states, whose contributions are normally suppressed by a
power of \( Q \) compared to incoherent partonic scattering, to dominate [11]. The
outgoing quarks give an extra suppression in DIS when the hard-scattering
function \( \hat{\sigma} \), Fig. 2, is computed in perturbation theory. That is, the product
of DIS denominators is suppressed even more than the single DY numerator.
Then the ratio grows with moment \( N \), from the elastic limit in \( z \) space. This
is the source of the terms shown in Eq. (3) in \( \hat{\sigma} \), and is the reason why they
all have the same sign.

The Drell-Yan cross section is the benchmark for the resummation of
singular distributions. As above, singular distributions at \( z = 1 \) translate into
logarithms of the moment variable \( N \). Logarithms of \( N \) (to all logarithmic
order, not just leading or next-to-leading logarithm) in the moments of the
inclusive Drell-Yan cross section exponentiate [8, 9],
\[ \hat{\sigma}^{\text{DY}}_{q\bar{q}}(N, Q, \mu) = \sigma^{\text{Born}}(Q) e^{C(\alpha_s) + E(N, Q, \mu, \alpha_s)}, \]
(8)
where \( \alpha_s \) stands for \( \alpha_s(\mu^2) \). In the exponent, the function \( C \) is known to two
loops, while the function \( E \), which organizes all logs of \( N \), has the following
form in the \( \overline{\text{MS}} \) scheme,
\[ E(N, Q, \mu, \alpha_s) = - \int_0^1 dx \frac{x^{N-1} - 1}{1 - x} \left[ \int \mu^2 \frac{d\mu^2}{\mu^2} g_1 (\alpha_s[\mu^2]) + g_2 (\alpha_s [Q^2]) \right]. \]
(9)
The functions \( g_1 \) and \( g_2 \) are finite series in \( \alpha_s \) [9], given for \( \overline{\text{MS}} \) by
\[ g_1(\alpha_s) = 2C_F \left( \frac{\alpha_s}{\pi} + \frac{1}{2} K \left( \frac{\alpha_s}{\pi} \right)^2 \right) + \ldots, \quad g_2(\alpha_s) = \mathcal{O}(\alpha_s^2), \]
(10)
where
\[ K = C_A \left( \frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{5}{9} n_f. \]
(11)
Eqs. (8) and (9) resum all logarithms of \( N \) in the sense of an order-by-order
expansion, by reexpanding the running couplings in terms of \( \alpha_s \). The re-
summed integrals, however, are ill-defined for \( x \to 1 \), no matter how large
\( Q^2 \) is, since the perturbative running coupling diverges at \( \mu^2 = \Lambda^2 \). Such a
divergence is sometimes called an infrared renormalon, and may give hints
on the structure of power corrections to Eq. (1). Most importantly, it is possible to regulate the singularity in the running coupling without affecting the leading power behavior of the resummed cross section [13]. Different approaches to this problem can give somewhat different predictions [14, 15, 16], but in any reasonable prescription the threshold corrections remain relatively modest, although nonnegligible in at least some physically relevant regions.

It has been noted in several phenomenological applications that threshold resummation, and even fixed-order expansions based upon it, significantly reduces sensitivity to the factorization scale [2, 4, 16, 17, 18, 19]. The reason for this reduction is easy to see in the dependence on the factorization scale, $\mu$ in Eq. (9). Indeed, for the Drell-Yan, and all related cross sections, the dependence on $\mu$ in the hard-scattering function matches that part of the scale dependence of the $\overline{\text{MS}}$ distributions due to the singular part of the splitting functions $P_{ii}(z)$. That is, $g_1$ in Eq. (11) is exactly the coefficient of $1/(1-z)_+$ in $P_{qq}(z)$. Because resummation matches phase space to all orders at partonic threshold [11], all associated factorization scale dependence is also matched. To be specific, we may write the moments of the Drell-Yan cross section in a form that resums the singular threshold distributions,

$$
\sigma^{DY}_{AB}(N, Q) = \sum_q \phi_{q/A}(N, \mu) \hat{\sigma}^{DY}_{qq}(N, Q, \mu) \phi_{\bar{q}/B}(N, \mu)
$$

$$
= \sum_q \phi_{q/A}(N, \mu) M_q(\mu, Q) \hat{\sigma}^{DY}_{qq}(N, Q, Q) \phi_{\bar{q}/B}(N, \mu) M_q(\mu, Q)
$$

$$
+ \mathcal{O}(1/N),
$$

where in the second line the factors $M_q(\mu, Q)$ absorb the $\mu$-dependence in the exponentials of Eq. (11). These factors compensate for the singular part of the evolution of the parton distributions, and the $\mu$-dependence of the resummed expression is suppressed by a power of the moment variable,

$$
\mu \frac{d}{d\mu} \left[ \phi_{q/A}(N, \mu) M_q(\mu, Q) \right] = \mathcal{O}(1/N).
$$

Of course, the importance of the remaining sensitivity to $\mu$ depends on the kinematics.
3 Resummation with Color Exchange

Beyond leading logarithms, there are important differences between the electro-
weak-induced Drell Yan cross sections and the QCD-induced top or jet cross
sections. These are due primarily to the presence of final-state radiation from
scattered quarks in the latter case, which is absent in the former, and to the
interplay of color exchange in the hard scattering with the soft radiation. As
in the case of Eq. (8), resummation is based first of all on the factorization
properties of the cross section in the neighborhood of the elastic limit [10].
The situation is illustrated in Fig. 3. Near the elastic limit, all gluons emitt-
ed into the final state have energies limited by $(1 - z)Q \ll Q$. Correspondingly,
gluons with energies of order $Q$ can appear only in virtual diagrams. Stan-
dard factorization methods may then be used to separate the relatively soft
but still perturbative gluons from the underlying hard scattering. This may
be done order-by-order in perturbation theory, keeping both the hard and
soft factors of $\hat{\sigma}$ free of soft and collinear divergences. The process of factor-
ization may be thought of as the construction of an effective field theory for
soft gluons in the presence of the hard scattering [1, 3].

In the relevant effective field theory, the incoming partons that annihilate
into the heavy quarks and the outgoing heavy quarks themselves are rep-
resented by ordered exponentials (Wilson lines). The Wilson lines are tied
gether in the amplitude and its complex conjugate at local vertices, $T_I$ and $T_J$ in Fig. 3, which describe the flow of color between the initial and final
states. Indices $I$ and $J$ label matrices in color space. We will consider here
the annihilation of light quarks (color indices $a_1$ and $a_2$) into heavy quarks
(indices $a_3$ and $a_4$),

$$ q_{a_1}(p_a) + \bar{q}_{a_2}(p_b) \rightarrow Q_{a_4}(p_1) + \bar{Q}_{a_3}(p_2), \quad (14) $$

with kinematic invariants,

$$ t_1 = (p_a - p_2)^2 - m^2, \quad u_1 = (p_b - p_2)^2 - m^2, \quad s = (p_a + p_b)^2. \quad (15) $$

In this case a convenient basis for the $T'$s is one that represents color singlet
and octet exchange in the $s$-channel,

$$ (T_1)_{a_1} = \delta_{a_1a_2}\delta_{a_3a_4}, \quad (T_2)_{a_1} = \sum_c (T_c^{(F)})_{a_2a_1} (T_c^{(F)})_{a_4a_3}. \quad (16) $$
Other bases, particularly singlet exchange in the $s$- and $u$-channels, are also interesting [1, 3]. As in Fig. 3, each choice of effective vertices leads to a separate soft function $S_{IJ}$, which depends on $(1 - z)Q$ only, rather than $Q$ itself.

The (virtual) hard-scattering functions, $h_I(Q)$ and $h_J^*(Q)$, are labelled by the same color exchange indices. As in most factorizations and constructions of effective field theories, the new vertices require renormalization. Thus we renormalize the soft functions [3, 20],

$$S_{IJ}^{(un)}((1 - z)Q) = Z_{JJ} S_{IJ}^{(ren)}((1 - z)Q) Z_{IJ},$$  \hspace{1cm} (17)

and the hard functions

$$h_I^{(un)}(Q) = Z_{IJ}^{-1} h_J^{(ren)}(Q),$$  \hspace{1cm} (18)

where the $Z_{KL}$ are cutoff-dependent renormalization constants.

Solutions to the renormalization group equation for $S_{IJ}$ that follows from (17) are generally ordered exponentials [3, 20], but at leading logarithm in $S_{IJ}$, which is next-to-leading logarithm in the overall cross section, we can diagonalize the anomalous dimension, and separate the evolution of particular linear combinations of composite color vertices [6].

We can apply these results to the production of a pair of heavy quarks with total invariant mass $Q \geq 2M_Q \equiv 2m$ at fixed rapidity $y$. The cross section is, as usual, a convolution of hard-scattering functions $\hat{\sigma}_{ab}$ with parton distributions $\phi_q/A$ and $\phi_{\bar{q}}/B$, as in Eq. (1), with $F = Q\bar{Q}$. Corresponding to the Drell-Yan result, (8), we now have, to next-to-leading logarithmic accuracy, choosing $\mu = Q$,

$$\hat{\sigma}_{q\bar{q} \rightarrow Q\bar{Q}}(N) = \sum_{IJ} S_{IJ}^{(0)} h_I(Q) h_J^*(Q) e^{C_{IJ}(\alpha_s) + E_{IJ}(N, \alpha_s)},$$  \hspace{1cm} (19)

where again $\alpha_s$ stands for $\alpha_s(Q^2)$. The function $C'$ is known to one loop only at this time. To next-to-leading log, we need only the lowest-order soft functions, $S_{IJ}^{(0)} \sim \delta_{IJ}$. The function $E_{IJ}$, which contains the logs of the moment variable $N$ has a form very similar to the Drell-Yan case, but now with a dependence on the effective color vertices, through a third function, $g_3$,

$$E_{IJ}^{(ab)}(N, \alpha_s) = - \int_0^1 dx \frac{x^{N-1} - 1}{1 - x} \left[ \int_{(1-x)^2 Q^2}^{Q^2} \frac{d\mu'^2}{\mu'^2} \ g_1 \left( \alpha_s [\mu'^2] \right) \right].$$
Figure 3: Representation of the factorization of the hard scattering function 
\( \hat{\sigma} \) near the elastic limit. The second part shows the soft-gluon matrix \( S_{IJ} \) as a cut diagram for the scattering of incoming ordered exponentials (double lines - the incoming partons in the eikonal approximation) to give outgoing ordered exponentials (bold lines - the outgoing heavy quarks in the eikonal approximation). For simplicity, only a few of the possible gluon interactions with the ordered exponentials are shown.

\[
\hat{\sigma} = h_i(Q) h^*_j(Q) S_{IJ}
\]

\[
S_{IJ} = \begin{array}{c}
\quad \quad \\
T_i \\
\quad \quad \\
T_j
\end{array}
\]

As before, the \( g_i, i = 1, 2, 3 \) are finite functions of their arguments. In the \( \overline{\text{MS}} \) scheme, \( g_1 \) and \( g_2 \) are given for incoming light quarks by (10) above. Dependence on color exchange in the hard scattering is contained entirely in the new functions \( g_3^{(I)} \). To determine \( g_3^{(I)} \), we go to a color basis that diagonalizes the renormalization matrix \( Z_{IJ} \) in eqs. (18) and (17). In this basis,

\[
g_3^{(I)}[\alpha_s] = -\lambda_I[\alpha_s],
\]

where the eigenvalues \( \lambda_I \) of the corresponding anomalous dimension matrix

\[
+g_2 \left( \alpha_s \left[ Q^2 \right] \right) + g_3^{(I)} \left( \alpha_s \left[ (1-x)^2 Q^2 \right] \right) + g_3^{(J)*} \left( \alpha_s \left[ (1-x)^2 Q^2 \right] \right).
\]

(20)
are complex in general, and may depend on the directions of the incoming and outgoing partons.

The anomalous dimension matrix of the effective vertices $T_I$ in Fig. 3 for light to heavy quark annihilation in the singlet-octet basis of Eq. (16) [1] is:

$$\Gamma_{S'} = \frac{\alpha_s}{\pi} \left( \begin{array}{c}
-C_F (L_\beta + 1 + \pi i) \\
2 \ln \left( \frac{u_1}{t_1} \right) + \frac{C_F}{C_A} \ln \left( \frac{u_1}{t_1} \right) \\
C_F \left[ 4 \ln \left( \frac{u_1}{t_1} \right) - L_\beta - 1 - \pi i \right] \\
+ \frac{C_A}{2} \left[ -3 \ln \left( \frac{u_1}{t_1} \right) - \ln \left( \frac{m^2 s}{t_1 u_1} \right) + L_\beta + \pi i \right] \end{array} \right).$$

(22)

Here $L_\beta$ is the vertex function in the eikonal approximation for the production of a pair of heavy quarks with center of mass velocity $\beta$,

$$L_\beta = \frac{1 - 2m^2/s}{\beta} \left( \ln \frac{1 - \beta}{1 + \beta} + i\pi \right), \quad \beta = \sqrt{1 - 4m^2/s}. \quad (23)$$

Solving for the eigenvalues, substituting them in Eq. (19), and expanding the result to first order in $\alpha_s$, we can derive an explicit one-loop expression for the cross section for heavy quark production through light quark annihilation [1, 2, 4]. This result is consistent [5] with the explicit one-loop formulas given in [21] for the single-particle inclusive cross section.

Much the same considerations apply to electroproduction of heavy quarks. It is interesting to note that for the process [18]

$$\gamma^*(p_a) + g_i (p_b) \rightarrow Q_{a_4}(p_1) + \bar{Q}_{a_3}(p_2), \quad (24)$$

as for direct photon production [3, 5, 17] there is only a single color tensor, coupling the produced pair to the gluon in an octet state. As a result, there is only one eigenvalue, which in this case is given (in the conventions of Ref. [3]) by

$$\lambda_8 = \frac{\alpha_s}{\pi} \left( \frac{C_A}{2} \left[ \ln \frac{t_1 u_1}{m^2 s} - 1 + i\pi \right] + \left( \frac{1}{2} C_A - C_F \right) [L_\beta + 1] \right). \quad (25)$$

Resolved processes in photoproduction, however, involve nontrivial color mixing [22]. In the limit $\beta \rightarrow 0$, we can make contact with quarkonia production in the nonrelativistic QCD (NRQCD) formalism [23].
4 Conclusion

The use of resummation as a tool for heavy-quark and other hard-scattering cross sections may help to put the successes and shortcomings of fixed-order perturbative QCD into a larger context. The reduction of factorization scale dependence is an encouraging observation in this direction. It will be interesting to study as well renormalization scale independence [24]. Other directions for investigation include the inversion of the moments and the closely-related issue of power-suppressed corrections. The development of resummed polarized cross sections, for which next-to-leading order calculations are becoming available [25], is sure to be a further source of insight.

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