Neutrinos Confronting Large Extra Dimensions

J. Maalampi\textsuperscript{1,2*}, V. Sipiläinen\textsuperscript{3†} and I. Vilja\textsuperscript{4‡}

\textsuperscript{1}Department of Physics, University of Jyväskylä, Finland
\textsuperscript{2}Helsinki Institute of Physics, FIN-00014 University of Helsinki, Finland
\textsuperscript{3}Department of Physics, FIN-00014 University of Helsinki, Finland
\textsuperscript{4}Department of Physics, FIN-20014 University of Turku, Finland

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Abstract

We study neutrino physics in a model with one large extra dimension. We assume the existence of two four-dimensional branes in the five-dimensional space-time, one for the ordinary particles and the other one for mirror particles, and we investigate neutrino masses and mixings in this scheme. Comparison of experimental neutrino data with the predictions of the model leads to various restrictions on the parameters of the model. For instance, the size of the extra dimension, $R$, turns out to be bounded from below. Cosmological considerations seem to favor a large $R$. The usual mixing schemes proposed as solutions to the solar and atmospheric neutrino anomalies are compatible with our model.
1 Introduction

The possibility that the physical space has more than three dimensions [1] has attracted a great deal of attention recently. Refs. [2]–[4] pioneered the idea that the presence of extra compact spatial dimensions, some of which may be as large as fractions of millimeter, could lower the "fundamental" Planck scale $M_*$, i.e. the Planck scale in the higher-dimensional space-time, down to the TeV energy range, alleviating thereby the hierarchy problem. If the number of new dimensions is $n$ and we assume that all of them have the same radii of the size $R$, the four-dimensional Planck scale $M_{Pl}$ is related to the fundamental Planck scale through $M_{Pl}^2 \sim M_*^{n+2} R^n$. Given the experimental fact that no deviations from the Newton law has shown up down to $R \simeq 1$ mm [5], it is immediately clear from this relation that one extra dimension would not be enough for the solution of the hierarchy problem. On the other hand, for $n = 2$ one gets $R \sim 0.1$ mm, which is in an interesting range in view of the gravitational force experiments discussed in [5, 6].

The basic assumption of the extra dimension scenario is that all the fields charged under the Standard Model (SM) gauge group are localized on a brane, the familiar 3+1-dimensional space-time, embedded in the 4+n-dimensional space, called the bulk [2]. Gravitons and other particles with no SM interactions are not confined to the brane but are free to propagate in the bulk as well. This brane-bulk structure is suggested by some string theoretical observations.

The large extra dimension scenario is particularly intriguing from the neutrino physics point of view. The right-handed chiral components of neutral leptons ($\nu_R$), if they exist, are the only SM particles that can live in the bulk [7, 8]. The Yukawa coupling between $\nu_L$, residing on the brane, and $\nu_R$ is suppressed by a factor $M_* / M_{Pl}$, providing a new and elegant explanation for the lightness of neutrinos. Perhaps the most natural conventional explanation is due to the seesaw mechanism, which requires the existence of a new mass scale around $10^{12}$ GeV or higher. In the extra dimension scheme such a high mass-scale is neither needed nor naturally appears, but small neutrino masses follow from the suppression of the Yukawa couplings due to the large bulk volume.

The existence of the bulk neutrinos has another interesting consequence. The bulk
neutrinos $\nu_{lR}$ ($l = e, \mu, \tau$) appear in four dimensions as Kaluza-Klein excitations having Dirac mass terms with the left-handed bulk components $\nu_{lL}$ that originate in the quantized internal momenta in compact extra dimensions. The existence of these new degrees of freedom enrich the neutrino spectrum and the neutrino oscillation patterns. Theoretical and phenomenological aspects of this scenario have been addressed in several recent papers (see e.g. [9]–[25]).

The localization of the SM fields on the brane may be explained in terms of non-perturbative effects in string theory [3]. An alternative and perhaps more intuitive approach is achieved in the context of an effective field theory. In this method one introduces a five-dimensional scalar field (for $n = 1$) with an effective potential that has a domain wall type profile in the extra dimension [26]–[28]. The origin of the scalar field is usually left unspecified. For example the so-called radion may be a natural, though not the only, possibility for such a scalar field. The radion field is associated to the extra dimension components of the metric tensor and therefore is always present in the model.

The discussion in this paper is based on the observation that the scalar field responsible for the localization of the SM fermions must form two kinks, i.e. a kink and an anti-kink (or any number of such kink/anti-kink pairs), in order to be continuous in a compactified extra dimension. That kind of profile leads to interesting consequences since in addition to the “usual” brane, where the SM world resides, there necessarily exists at some distinct point of the extra dimension another brane, a mirror brane. In the mirror brane there live particles whose gauge interactions are identical to those of the SM particles but with reversed chiralities, that is, they are mirror particles [29]. Besides gravity, the ordinary brane and the mirror brane can communicate with each other only via possible inert neutrinos, that is, the right-handed ordinary neutrinos $\nu_{lR}$ and the left-handed mirror neutrinos $N_{lL}$ that live in the bulk.

We will study in this paper the neutrino sector of the brane-mirror brane scenario. We neglect for simplicity any flavour mixing and consider neutrino masses and mixing within a single family of neutrinos and mirror neutrinos. The neutrino sector under study thus consists of the fields $\nu_L$, $\nu_R$, $N_L$ and $N_R$. In Sec. 2 a concrete realization of our scheme is constructed. In its framework we derive an effective Lagrangian from which the neutrino
mass matrix is obtained. Unlike in many previous works, our mass matrix is finite since we integrate out the massive Kaluza-Klein modes. In Sec. 3 we study numerically the generic constraints neutrino data set upon the various fundamental parameters of the model, such as the size of the extra dimension, the mass of the sterile neutrino the model predicts and the fundamental mass scale (the vev of the Higgs doublet) of the mirror brane. Sec. 4 is devoted to discussion of the obtained results. Concluding remarks are presented in Sec. 5.

2 Theory

Let us now construct our model for a two-brane world, where one of the branes traps the SM fermions and the other one the mirror fermions. The gauge inert states, the right-handed neutrino $\nu_R$ and the left-handed mirror neutrino $N_L$, are assumed to propagate freely in the whole higher-dimensional bulk. For the localization mechanism of the gauge-active fermions, based on effective field theory approach, we follow Refs. [27, 28]. For simplicity and clarity we restrict ourselves in the following to the case where there is only one extra dimension, but the extension of the analysis to the cases of several extra dimensions is quite straightforward.

We assume the extra dimension to be compactified on a circle of radius $R$. The coordinate system reads $z = (x^\mu, y)$, where $\mu = 0, 1, 2, 3$ and $y \sim y + 2\pi R$. The scalar field responsible for the localization of fermions on the 4-dimensional membranes is denoted by $\Phi$. It is assumed to have two kinks, one at $y = 0$ where its value grows from $-f$ to $+f$ and another one at $y = y_*$ where its value drops back to $-f$. Between the kinks $\Phi$ is supposed to remain constant. Fermions are localized on a finite-width wall around the points in the fifth dimension where $\Phi = 0$ as, heuristically speaking, their position-dependent masses are there the smallest. Outside these positions field fluctuations are strongly suppressed due to higher masses [28].

The five-dimensional spinors are decomposed as

$$L = \begin{pmatrix} N_R' \\ \nu_L' \end{pmatrix}, \quad \Psi = \begin{pmatrix} \nu_R'' \\ N_L'' \end{pmatrix}, \quad (1)$$

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where $\nu'_L$ and $\nu'_R$ are related to the ordinary neutrinos and $N'_L$ and $N'_R$ to mirror neutrinos. These fields are functions of the five-dimensional space-time. The appropriateness of our notation will become clear towards the end of this section.

We assume the relevant part of the brane-world action to be of the form

$$S = \int d^4xdy[\bar{L}(i\Gamma^A\partial_A - m + \Phi)L + i\bar{\Psi}\Gamma^A\partial_A\Psi + (\kappa H^*\bar{\Psi}L + h.c.)],$$

where $A = 0, \ldots, 4$, $m$ is a mass parameter, $\kappa$ is a dimensionful Yukawa coupling, and $H$ is the neutral component of the standard SU(2)-doublet Higgs field. The gamma matrices are given by

$$\Gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}, \quad \Gamma^4 = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix},$$

and all the fields are taken to be periodically continuous, e.g. $L(x, 0) = L(x, 2\pi R)$. The term $\bar{L}\Phi L$ takes care of the localization of the active neutrinos on the branes. In general the action could also include Dirac and Majorana mass terms for bulk neutrinos \[24\], but we neglect them in the present discussion. It should also be remarked that the presence of the inert neutrinos in the bulk is an assumption, because we have omitted in the action the term $\bar{\Psi}\Phi\Psi$, which would have localized the inert neutrinos on the branes.

The equation of motion for $L$ is

$$i\Gamma^A\partial_A L = (m - \Phi)L,$$

where a term $\kappa^*H\Psi$ has been neglected as in classical approximation $\Psi = 0$. Supposing that $\Phi = \Phi(y)$, and writing $N'_L = N_R(x)g_R(y)$ and $\nu'_L = \nu_L(x)g_L(y)$, one ends up with

$$ig_L(y)\sigma^\mu\partial_\mu\nu_L = N_R(\partial_y + m - \Phi(y))g_R(y),$$
$$ig_R(y)\bar{\sigma}^\mu\partial_\mu N_R = \nu_L(-\partial_y + m - \Phi(y))g_L(y).$$

The left-hand side of Eqs. [4] being classically zero, it is trivial to see that $g_{L,R}(y)$ have solutions behaving so that $g_R(y)$ tends to confine near $y = y_*$ and $g_L(y)$ near $y = 0$. In other words, since $\nu_L$ and $N_R$ really get localized on different branes, the chosen action reproduces exactly the features that we have been looking for. The consistency of the theory requires that $\int_0^{2\pi R} dy(\Phi(y) - m) = 0$, which for thin branes reduces to
\[ \pi - \Delta \theta = \pi m / f , \] where \( \Delta \theta = 2\pi - y_* / R \). Slightly more contrived considerations, which we shall not present here, reveal that the Higgs field has a twin-peaked profile with maxima at \( y = 0 \) and \( y = y_* \). One may thus approximate (somewhat symbolically, cf. the appendix of Ref. \[7\])

\[ \nu'_L = \sqrt{\delta(y)} \nu_L(x) , \quad N'_R = \sqrt{\delta(y - y_*)} N_R(x) , \quad H = \sqrt{\delta(y)} h_-(x) + \sqrt{\delta(y - y_*)} h_+(x) , \] (5)

where \( h_-(h_+) \) can be viewed as a vev of the Higgs field in “our” brane (the mirror brane). By substituting these, together with the familiar Kaluza-Klein expansion

\[ \begin{align*}
\left( \begin{array}{c}
\nu'_R(x,y) \\
N'_L(x,y)
\end{array} \right) &= \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{\infty} \left( \begin{array}{c}
\nu_{Rn}(x) \\
N_{Ln}(x)
\end{array} \right) e^{iny/R},
\end{align*} \] (6)

to the original action, one has finally

\[ S = \int d^4 x \left\{ iN^\dagger_R \bar{\sigma}^\mu \partial_\mu N_R + i\nu^\dagger_L \sigma^\mu \partial_\mu \nu_L + \sum_{n=-\infty}^\infty \left[ i\nu^\dagger_{Rn} \bar{\sigma}^\mu \partial_\mu \nu_{Rn} + iN^\dagger_{Ln} \sigma^\mu \partial_\mu N_{Ln} \\
+ \frac{in}{R} (\nu^\dagger_{Rn} N_{Ln} - N^\dagger_{Ln} \nu_{Rn}) + u (h^*_+ \nu^\dagger_{Rn} \nu_L + h^*_+ N^\dagger_{Ln} N_R e^{in\Delta \theta} + h.c.) \right] \right\} , \] (7)

where

\[ u = \frac{\kappa}{\sqrt{2\pi R}} , \] (8)

and \( \kappa \) is taken to be real.

The first six \( n \)-dependent terms of the action \(\[7\] \) disappear due to the equations of motion of the Kaluza-Klein excitations \( \nu_{Rn} \) and \( N_{Ln} \) (with \( n \neq 0 \)),

\[ \begin{align*}
N_{Ln} &= \frac{iR}{n} (i\bar{\sigma}^\mu \partial_\mu \nu_{Rn} + uh^*_- \nu_L) , \\
\nu_{Rn} &= -\frac{iR}{n} (i\sigma^\mu \partial_\mu N_{Ln} + uh^*_+ N^\dagger_{Ln} e^{in\Delta \theta} ) .
\end{align*} \] (9)

The remaining two terms of the action then yield

\[ \mathcal{L}_{n\neq0} = \sum_{n \neq 0} \left[ \left( iu \frac{R}{n} h^*_+ h^*_- N^\dagger_{Rn} \nu_L e^{-in\Delta \theta} + h.c. \right) + u \frac{R}{n} h^*_- \nu^\dagger_L \sigma^\mu \partial_\mu N_{Ln} \\
- u \frac{R}{n} h^*_+ N^\dagger_{Rn} \bar{\sigma}^\mu \partial_\mu \nu_{Rn} e^{-in\Delta \theta} \right] , \] (10)
or, by applying Eqs. (9) again,

\[ \mathcal{L}_{n \neq 0} = u^2 s(\Delta \theta) R(h_+ h^*_- N^\dagger_L \nu_L + h.c.) + \sum_{n \neq 0} i u^2 R^2 \frac{R^2}{n^2} h_- \nu^\dagger_L \sigma^\mu \partial_\mu (h^*_- \nu_L) \\
+ \sum_{n \neq 0} i u^2 R^2 \frac{R^2}{n^2} h_+ N^\dagger_R \sigma^\mu \partial_\mu (h^*_+ N_R), \]

(11)

where terms including higher derivatives of \( \nu_{Rn} \) and \( N_{Ln} \) have been neglected, and

\[ s(\Delta \theta) = i \sum_{n \neq 0} e^{-in\Delta \theta} n = \pi - \Delta \theta \quad (\Delta \theta \neq 0). \]

(12)

This on-shell Lagrangian could be equivalently obtained by integrating out the massive Kaluza-Klein excitations.

Note that since \( h^*_\pm \) are constants in the leading order, they can well be taken out of the derivatives in Eq. (11). Examinations show that the values of \( h_\pm \) are essentially dependent on the widths of the ordinary and mirror branes. The quantity \( h_+ \) can be considered here as a free parameter while \( h_- \) is bound by the usual Higgs scalar expectation value \( |h_-| = 174 \text{ GeV} \).

Combining finally Eq. (11) and Eq. (7) (for \( n = 0 \)) with suitable rescalings of the fields so that the kinetic terms take the canonical form, one ends up with a Lagrangian

\[ \mathcal{L}_{\text{eff}} = \frac{1}{\sqrt{1 + \frac{\pi^2}{3} u^2 R^2 |h_\pm|^2}}, \]

and the identity \( \sum_{n \neq 0} n^{-2} = \pi^2/3 \) has been used. In this Lagrangian, where all the massive Kaluza-Klein modes are integrated out, only the lowest order effects of the extra dimension (i.e. field rescalings and mass terms) are kept.
3 Neutrino phenomenology

According to the Lagrangian $\mathcal{L}_{\text{eff}}$ the neutrino mass term of the model is given by

$$\mathcal{L}_M = -(\nu_L \nu_R \mathcal{N}_R \mathcal{N}_L) \begin{pmatrix} 0 & a & b & 0 \\ a & 0 & 0 & 0 \\ b & 0 & 0 & c \\ 0 & 0 & c & 0 \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R \\ \mathcal{N}_R \\ \mathcal{N}_L \end{pmatrix},$$

(14)

where

$$a = u n_- |h_-|, \quad b = u^2 s(\Delta \theta) R n_+ n_- |h_+ h_-|, \quad c = u n_+ |h_+|,$$

(15)

with appropriate redefinitions of the fields, and we have associated $\nu_R$ and $\mathcal{N}_L$ with the zero modes of the respective Kaluza-Klein states. More familiarly the mass Lagrangian can be presented in the form

$$\mathcal{L}_M = -\frac{1}{2} \mathcal{N}_R^T M \mathcal{N}_L + \text{h.c.},$$

(16)

where $\mathcal{N}_L^T = (\nu_L \nu_L^c \mathcal{N}_L^c \mathcal{N}_L)$, and $M$ is the matrix appearing in Eq. (14).

As one can see from the results above, the dependence of the neutrino mass Lagrangian on the fundamental parameters of the theory, such as the radius $R$, the values of the scalar, $h_\pm$, the coupling $\kappa$, and the relative positions of the brane and mirror brane (i.e. $\Delta \theta$), is quite non-trivial and difficult to analyse analytically. We will therefore study numerically the generic constraints the present neutrino data sets on the model by varying unknown parameters within conceivable range of values and plotting the ensuing predictions for various measurable quantities.

Let us start the phenomenological analysis by diagonalizing the neutrino mass matrix obtained above. Since $M$ is real and symmetric, the diagonalization can be performed by an orthogonal matrix $O$. (We consider here just one neutrino family. The different families may have different mixing angles.)

If we define

$$O = \frac{1}{\sqrt{2}} \begin{pmatrix} \sin \alpha & \sin \alpha & \cos \alpha & -\cos \alpha \\ -\sin \beta & \sin \beta & -\cos \beta & -\cos \beta \\ -\cos \beta & \cos \beta & \sin \beta & \sin \beta \\ \cos \alpha & \cos \alpha & -\sin \alpha & \sin \alpha \end{pmatrix},$$

(17)
\[ \sin^2 \alpha = \frac{1}{2} + \frac{a^2 + b^2 - c^2}{2\sqrt{(a^2 + b^2 + c^2)^2 - 4a^2c^2}}, \quad \sin^2 \beta = \frac{1}{2} + \frac{a^2 - b^2 - c^2}{2\sqrt{(a^2 + b^2 + c^2)^2 - 4a^2c^2}}, \tag{18} \]

we obtain

\[ O^T M O = \begin{pmatrix} -m_+ & 0 & 0 & 0 \\ 0 & m_+ & 0 & 0 \\ 0 & 0 & -m_- & 0 \\ 0 & 0 & 0 & m_- \end{pmatrix} \equiv \begin{pmatrix} \sigma_1 m_+ & 0 & 0 & 0 \\ 0 & \sigma_2 m_+ & 0 & 0 \\ 0 & 0 & \sigma_3 m_- & 0 \\ 0 & 0 & 0 & \sigma_4 m_- \end{pmatrix}, \tag{19} \]

where the eigenvalues are

\[ m_{\pm} = \sqrt{\frac{1}{2} \left( a^2 + b^2 + c^2 \pm \sqrt{(a^2 + b^2 + c^2)^2 - 4a^2c^2} \right)}, \tag{20} \]

and \( \sigma_i \)'s are sign factors. Note that if \( b = 0 \), matrix \( O \) can be presented in a much simpler form. This case, however, is realized only in the special case of \( \Delta \theta = \pi \), i.e. when the two branes are in opposite locations in the fifth dimension.

The mass eigenstates are given by

\[ \chi_i = \sum_j \left( O_{ij}^T \mathcal{N}_L \right)_j + \sigma_i O_{ij}^T \mathcal{N}_R ^j \), \tag{21} \]

in terms of which the mass Lagrangian reads

\[ L_M = -\frac{1}{2} m_+ (\bar{\chi}_1 \chi_1 + \bar{\chi}_2 \chi_2) - \frac{1}{2} m_- (\bar{\chi}_3 \chi_3 + \bar{\chi}_4 \chi_4). \tag{22} \]

By defining

\[ \psi = \frac{1}{\sqrt{2}} (\chi_1 + \chi_2), \quad \phi = \frac{1}{\sqrt{2}} (\chi_3 - \chi_4), \tag{23} \]

one has finally

\[ L_M = -m_+ \bar{\psi} \psi - m_- \bar{\phi} \phi. \tag{24} \]

The neutrino sector thus consists of two Dirac neutrinos, as was expected as the theory conserves, by construction, lepton number.

From Eqs. (17), (21) and (23) it follows that the ordinary left-handed neutrino \( \nu_L \) is the following superposition of the mass eigenstate neutrinos:

\[ \nu_L = \cos \alpha \bar{\phi} \chi_L + \sin \alpha \bar{\psi} \chi_L. \]
The superposition orthogonal to this combination is the inert field $N_L$. As $\nu_L$ is the only active neutrino living in our brane, only the mixing angle $\alpha$ can be experimentally probed. The other angle $\beta$ parametrizes the mixing between the right-handed fields $\nu_R$ and $N_R$, and is therefore a measurable quantity only in the mirror world.

There exist several empirical constraints on the masses and mixing angles of neutrinos, coming from laboratory experiments, astrophysical observations and cosmological considerations. In our case, where flavour mixings are neglected, mixings occur between an active and a sterile neutrino of each family. It turns out that the existing laboratory bounds on the mixing angle $\alpha$ between the electron neutrino and a heavier neutrino in various mass ranges of the heavier neutrino mass are in general ineffective in our model. This is because the mass of the extra neutrino turns out to be quite low for a plausible choice of the model parameters. The only relevant laboratory constraints are due to the electron neutrino mass measurement in the tritium beta decay experiments [30] and the neutrino oscillation experiments. The upper limit of the electron neutrino mass from the beta decay experiment is [30]

$$m_{\nu_e} < 2.3 \text{ eV}. \quad (25)$$

As far as the electron neutrino is concerned the most stringent oscillation limit comes from the Bugey disappearance experiment [31]. This limit has been approximated as follows in our numerical calculation:

$$\sin^2 2\alpha < 0.1 \quad \text{for } 100 \text{ eV}^2 > |\delta m^2| > 2 \text{ eV}^2,$$
$$\sin^2 2\alpha < 0.02 \quad \text{for } 2 \text{ eV}^2 > |\delta m^2| > 0.04 \text{ eV}^2,$$
$$\sin^2 2\alpha < 0.1 \quad \text{for } 0.04 \text{ eV}^2 > |\delta m^2| > 0.01 \text{ eV}^2. \quad (26)$$

The active-sterile mixing can also be constrained by cosmological arguments. If the mixing is too large, neutrino oscillations, acting as an effective interaction, would bring the sterile neutrino in equilibrium before neutrino decoupling, and the resulting excess in energy density would endanger the standard scheme for the nucleosynthesis of light elements (BBN) [32]. This leads to the following bound for $\nu_e \leftrightarrow \nu_s$ mixing [33]:

$$|\delta m^2| \sin^2 2\alpha < 5 \times 10^{-8} \text{ eV}^2 \quad \text{for } |\delta m^2| < 4 \text{ eV}^2,$$
$$\sin^2 2\alpha < 10^{-8} \quad \text{for } |\delta m^2| > 4 \text{ eV}^2. \quad (27)$$

This bound is, however, avoided if there was a suitable net lepton number in the early universe [34].
With applying these constraints, we search the allowed regions of the parameter space numerically using a Monte Carlo analysis where we have varied three unknown parameters: the radius of the extra dimension $R$, the mirror brane Higgs value $h_+$ and the mirror brane position parameter $\Delta \theta$. The extra dimension radius is varied from the Planck scale to a millimeter scale and the mirror brane Higgs value between $10^{-5}$ MeV and $10^{10}$ MeV. We consider these parameter ranges wide enough for a representative analysis. Both $R$ and $h_+$ are randomized so that their logarithms are evenly distributed. For the position parameter $\Delta \theta$ an even distribution between 0 and $\pi$ has been taken.

In addition to these parameters the action depends on the dimensionful coupling $\kappa$. The natural scale for it could possibly be set by the higher-dimensional Planck scale $M_*$, and therefore we write (with $M_{Pl}^2 = M_*^2 R$)

$$\kappa = \frac{\kappa'}{\sqrt{M_*}} = \kappa' \left( \frac{R}{M_{Pl}^2} \right)^{1/6},$$

(28)

where $\kappa'$ is a dimensionless coupling constant. Its value is not really known, but a plausible choice would be a number relatively close to unity. We have performed our analysis for the values $\kappa' = 1$ and $\kappa' = 0.01$.

A central parameter in the scheme is obviously the size $R$ of the extra dimension. From the experiments that probe the effects of gravity in short distances one knows that

$$R \ll 1 \text{ mm.}$$

(29)

From the dependence of the elements of the mass matrix $M$ on $R$ follows an upper bound on the larger mass eigenvalue $m_+$ as a function of $R$, as presented in Fig. 1. When the limit (29) is saturated, the largest allowed value is $m_+ \sim 10$ eV, but larger values are allowed for a smaller $R$.

4 Results and discussion

Let us now proceed to our numerical results. In Fig. 2 we present a scatter plot in the $(R, \sin \alpha)$-plane, obtained by allowing the model parameters vary as indicated above, with and without taking the cosmological constraint, Eq. (27), into account (marked in
the plot by crosses and dots, respectively). Fig. 2a corresponds to the case $\kappa' = 0.01$. Let us now look at the general features displayed by this figure. Comparison of Eqs. (8), (15) and (28) implies that $a, c \gg b$ for small values of $R$. From Eq. (18) it can then be inferred that, depending on the relative sizes of $h_+$ and $h_-$, either $\sin^2 \alpha \simeq 1$ ($h_+ < h_-$) or $\sin^2 \alpha \simeq 0$ ($h_+ > h_-$). In other words, for small values of the size $R$ of the extra dimension, the mixing angle $\alpha$ has values either close to 0 or close to $\pi/2$, the other values being forbidden. The small values of $\alpha$ correspond to the case where $a < c$, that is, where the ordinary active neutrino $\nu_L$ is predominantly the lighter mass eigenstate $\phi_L$ and the sterile mirror neutrino $N_L$ is predominantly the heavier state $\psi_L$. The values of $\alpha$ close to $\pi/2$ in turn correspond to the situation where $a > c$, and $\nu_L$ is predominantly the heavier state $\psi_L$ and the sterile mirror neutrino $N_L$ is predominantly the lighter state $\phi_L$, the both neutrino masses now being below the experimental limit $m_\pm \lesssim 2.3$ eV.

It is important to note that the size of the extra dimension, $R$, is bounded from below in our scenario. This is an obvious consequence of the fact that the quantities $a$ and $c$ increase with decreasing $R$, and the mass of the predominantly active neutrino state $\simeq \nu_L$,
whose mass experiments test, is determined mainly by these quantities, since for small $R$ $a, c \gg b$. In the case of $\kappa' = 0.01(1.0)$ the lower limit is $R \gtrsim 4 \times 10^{-7} (5 \times 10^{-5})$ mm. More generally the lower limit of $R$ is obtained from

$$\frac{\kappa' h_-}{\sqrt{2\pi}} < (RM_{Pl})^{1/3} \times 2.3 \text{ eV}, \quad (30)$$

as long as $\kappa' \lesssim 0.1$ (cf. the solid line in Fig. 1).

When the cosmological constraint, Eq. (27), is taken into account (crosses in Figs. 2a and 2b), large values of $R$ are favoured ($R \gtrsim 3 \times 10^{-2}$ mm), unless the mixing angle $\alpha$ is very close to 0 or $\pi/2$. This is because for smaller $R$ the squared mass difference $m^2_+ - m^2_-$

**Figure 2:** The neutrino mixing angle $\sin(\alpha)$ as function of extra dimension radius $R$ for $\kappa' = 0.01$ (a) and $\kappa' = 1.0$ (b). Crosses indicate points allowed both oscillations and cosmology, whereas dot are points allowed by oscillations only.
tends to increase, jeopardizing the fulfillment of the cosmological condition. Let us note that in the parameter space covered by our Monte Carlo analysis, the cases allowed by the cosmological condition are concentrated to the region of large mixing angle $\alpha$, \textit{i.e.} to the case where the active neutrino is heavier than the sterile one. Nevertheless, also the small values of the mixing angle are allowed, \textit{i.e.} the case where the sterile neutrino can be relatively heavy, albeit with quite a small part of the parameter space.

The step-like behavior of the scatter plot in the region $10^{-5} \text{mm} \lesssim R \lesssim 10^{-3} \text{mm}$ is due to the oscillation constraint, Eq. (26). As remarked before, this bound together with the upper bound of 2.3 eV of the electron neutrino mass, is the only laboratory constraint that is effective for our model. The reason is the relatively small values of the inert neutrino mass the theory allows, as displayed by Fig. 1.

Fig. 2b is the same scatter plot as in Fig. 2a, except that now $\kappa' = 1$. A comparison with the previous case reveals the interesting fact, namely that for a large values of the coupling $\kappa'$ only the large mixing angles $\alpha$ are allowed. This can be understood as follows. The ratio of the mass matrix elements $a$ and $c$,

$$
\frac{a}{c} = \frac{n_-|h_-|}{n_+|h_+|} = \frac{|h_-|}{|h_+|} \sqrt{\frac{1 + \pi^2 u^2 R^2 |h_+|^2/3}{1 + \pi^2 u^2 R^2 |h_-|^2/3}} \approx 1,
$$

is approximately equal to 1 when the product $u^2 R^2$ is large enough. From Eq. (18) one can deduce that in this case $\sin^2 \alpha \gtrsim 1/2$. In the case $\kappa' = 1$ one is practically always in this regime, taken into account the lower limit for $R$ from the electron neutrino mass measurement. In the previous case of $\kappa' = 0.01$ this regime is reached, because of the smaller value of $u$, only at the upper end of the allowed $R$ range, as can be seen in Fig. 2a. The lower bound for $R$, still given by the experimental upper limit for the electron mass, is now larger than in the case of $\kappa' = 0.01$ since the neutrino mass (for small $b$) is proportional to $\kappa'$, as seen from Eqs. (15), (20) and (28). When $R$ is increased, all possibilities from $\nu_L \simeq \psi_L$ to the maximal mixing $\nu_L = (\phi_L + \psi_L)/\sqrt{2}$ become open. At low $R$ cosmology forces $\nu_L$ to be very close to the heavier mass state $\psi_L$, but when $R \gtrsim 3 \times 10^{-2} \text{mm}$, $\nu_L$ may develop a large $\phi$ component as well.

Fig. 3 presents the region in the parameter space $(\sin^2 2\alpha, \delta m^2)$ (with $\delta m^2 = m_+^2 - m_-^2$) reached by our model in the case $\kappa' = 1$. The only constraint taken into account here is
the upper bound 2.3 eV for electron neutrino mass. We have checked that the inclusion of other laboratory bounds do alter Fig. 3, but only cuts $\sim 1\%$ more points from the region which is anyway excluded by oscillation experiments. The line between the cosmologically allowed (lower part) and excluded (upper part) regions has been drawn to Fig. 3, too. For $\kappa' = 0.01$ the allowed region would extend to larger mass differences, with virtually no change in cosmologically allowed region. As one can see, our model allows for all the active-sterile mixing schemes discussed as possible solutions to the solar neutrino and atmospheric neutrino anomalies. In particular the so-called SMA solution ($|\delta m^2| \approx (3 - 10) \times 10^{-6}$ eV$^2$, $\sin^2 2\alpha \approx (0.2 - 1.3) \times 10^{-2}$) of the solar neutrino anomaly can be realized in this model. Note, however, that the solar LMA solution is excluded by cosmology bound.
5 Conclusions

We have investigated a five-dimensional model with two four-dimensional branes, a brane for ordinary SM particles and another brane for mirror particles. By looking at neutrino masses and mixings, we have shown that confrontation of the predictions of the model with data constrains the extra dimension physics remarkably. The experimental upper bounds on the masses of the known neutrinos directly restrict the extra dimension size, independently of how neutrinos actually mix. The value of brane-bulk neutrino coupling $\kappa'$ determines the minimal value of extra dimension size $R$, a larger coupling corresponding to a larger $R_{\text{min}}$. The nature of neutrino mixing depends crucially on the coupling $\kappa'$. For $\kappa' \sim 1$ the predominantly active neutrino, i.e. the ordinary neutrino, is never lighter than the predominantly sterile neutrino. For smaller values, $\kappa' \sim 0.01$, also the opposite situation is allowed. In both cases active and sterile neutrino can mix maximally when $R$ is large enough.

As may be seen from Fig. 1, also the largest possible mass $m_{\nu}^{\text{max}}$ of the heavier neutrino (of each flavour) depends essentially on $\kappa'$, a smaller $\kappa'$ (i.e. a smaller $R_{\text{min}}$) enabling $m_{\nu}^{\text{max}}$ to be higher. For $\kappa' = 0.01$ the maximal neutrino mass is $\sim 1$ keV. However, already for $\kappa' \sim 10^{-3}$ the neutrino mass may be of the order of $1$ MeV with $R_{\text{min}} \sim 10^{-10}$ mm. Therefore the issue of the value of $\kappa'$ is emphasized.

If also cosmological constraints are taken into account, the room for allowed neutrino mixings is largely suppressed. With cosmological bound the model clearly favor large radius $R \gtrsim 10^{-2}$ mm. Only mixing angles very close to $\pi/2$ (and equally to 0 for some values of $\kappa'$) make exception, all the radii down to the minimal one being then allowed. In this case no direct method exists for observing mirror brane neutrinos.

The model we have studied is the simplest possible in the sense that only one extra dimension is introduced. This kind of approach is not fully realistic since it is known that the hierarchy problem cannot be accounted for unless the number of extra dimensions is at least two. It is clear that with more than one extra dimension the brane-bulk structure becomes much more complicated than in the five-dimensional case we have studied in this paper, increasing the flexibility in the values of the model parameters. Nevertheless, we
expect that our model exhibits the generic features that also appear in higher-dimensional treatments.

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