A Suboptimal PTS Algorithm Based on Particle Swarm Optimization Technique for PAPR Reduction in OFDM Systems

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A suboptimal partial transmit sequence (PTS) based on particle swarm optimization (PSO) algorithm is presented for the low computation complexity and the reduction of the peak-to-average power ratio (PAPR) of an orthogonal frequency division multiplexing (OFDM) system. In general, PTS technique can improve the PAPR statistics of an OFDM system. However, it will come with an exhaustive search over all combinations of allowed phase weighting factors and the search complexity increasing exponentially with the number of subblocks. In this paper, we workaround potentially computational intractability; the proposed PSO scheme exploits heuristics to search the optimal combination of phase factors with low complexity. Simulation results show that the new technique can effectively reduce the computation complexity and PAPR reduction.

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1. INTRODUCTION

Orthogonal frequency division multiplexing technique (OFDM) is a multicarrier modulation technology which can decrease the effect of the noise and interferences efficiently. Meanwhile, it has many advantages, such as: senior band efficiency and less impact of intersymbol interference. The high peak-to-average power ratio (PAPR) is the main drawback of the OFDM system, in which the OFDM transmitters require expensive linear amplifiers with wide dynamic range. Moreover, the amplifier nonlinearity will cause intermodulation products resulting in unwanted out-of-band power and increased interference.

OFDM is an attractive technique for achieving high bit rate wireless data transmission in frequency-selective fading channels [1]. Recently, many schemes of reduction in reductions PAPR have been proposed for OFDM system, as clipping [2] and peak windowing, block coding [3], nonlinear companding transform schemes [4, 5], active constellation extension [6], selective mapping [7, 8], and partial transmit sequences (PTs) [9–17], which are the most attractive ones due to good system performance and low complexity. Among these methods, PTS scheme is the most efficient approach and a distortionless scheme for PAPR reduction by optimally combining signal subblocks. In PTS technique, the input data block is broken up into disjoint subblocks. The subblocks are multiplied by phase weighting factors and then added together to produce alternative transmit containing the same information. The phase weighting factors, whose amplitude is usually set to 1, are selected such that the resulting PAPR is minimized. The number of allowed phase factors should not be excessively high, to keep the number of required side information bits and the search complexity within a reasonable limit. However, the exhaustive search complexity of the ordinary PTS technique increases exponentially with number of subblocks, so it is practically not realizable for a large number of subblocks. To find out a best weighting factor is a complex and difficult problem. In this paper, we present a novel approach to tackle the PAPR problem to reduce the complexity based on the relationship between the phase weighting factors and the subblock partition schemes.
The rest of this paper is organized as follow. In Section 2, definition of PAPR of OFDM system and the principles of PTS techniques are introduced. The particle swarm optimization (PSO) algorithm-based PTS OFDM system is examined in Section 3. The results of simulation are discussed in Section 4 and some conclusions for the proposed scheme are drawn in Section 5.

2. SYSTEM MODEL

2.1. OFDM systems and peak-to-average power ratio (PAPR)

In an OFDM system with \( N \) subcarriers, the complex baseband representation of an OFDM signal is expressed as

\[
x(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi (n/N)t}, \quad 0 \leq t \leq N - 1,
\]

(1)

where \( X = [X_0 \ X_1 \ \cdots \ X_{N-1}]^T \) is an input symbol sequence and \( t \) stands for a discrete time index. The PAPR of the OFDM signal sequence, defined as the ratio of the maximum power to the average power of the signal, can be expressed by

\[
PAPR(x) \triangleq \frac{\text{Max}_{0 \leq t \leq N-1} |x(t)|^2}{E[|x(t)|^2]},
\]

(2)

where \( E[\cdot] \) denotes the expected value [18].

2.2. Optimum partial transmit sequence-OFDM system model

The principle structure of PTS method is shown in Figure 1 as that in [15]. The known subblock partitioning can be classified into three categories. The first and the simplest category is called adjacent method which allocates \( N/M \) successive symbols to the same subblock. The second category is based on interleaving. In this method, the symbols with distance \( M \)
are allocated to the same subblock. The last one is called random partitioning method in which the input symbol sequence is partitioned randomly. The random partitioning is known as having the best performance in PAPR reduction [16]. It is well known that the PAPR performance will be improved as the number of subblocks $M$ is increased for OPTS technique, optimum PAPR can be found after searching $2^{M-1}$ computation if the number of subblock is $M$. A preset threshold can be used to reduce the computational complexity. We search the PAPR values through phase optimizer and the search is stopped once the PAPR drops below the preset threshold. By this way, the computational complexity can be significantly reduced.

3. PARTICLE SWARM OPTIMIZATION-BASED PTS

Basically, the PSO [19–27] technique-based PTS technique described below can be implemented by appropriately changing the optimization for block $W$ in Figure 1. In this context, the population is called a swarm and the individuals are called particles. Resembling the social behavior of a swarm of bees to search the location with the most flowers in a field, the optimization procedure of PSO is based on a population of particles which fly in the solution space with velocity dynamically adjusted according to its own flying experience and the flying experience of the best among the swarm.

Figure 2 shows the flow chart of a PSO algorithm. During the PSO process, each potential solution is represented as a particle with a position vector $x$, referred to as phase weighting factor and a moving velocity represented as $W$ and $v$, respectively. Thus for a $K$-dimensional optimization, the position and velocity of the $i$th particle can be represented as $W_i = (W_{i,1}, W_{i,2}, ..., W_{i,K})$ and $V_i = (v_{i,1}, v_{i,2}, ..., v_{i,K})$, respectively. Each particle has its own best position $W_i^P = (W_{i,1}, W_{i,2}, ..., W_{i,K})$ corresponding to the individual best objective value obtained so far at time $t$, referred to as $pbest$. The global best ($gbest$) particle is denoted by $W_G^G = (W_{g,1}, W_{g,2}, ..., W_{g,K})$, which represents the best particle found so far at time $t$ in the entire swarm. The new velocity $v_i(t + 1)$ for particle $i$ is updated by

$$v_i(t + 1) = wv_i(t) + c_1r_1(W_i^P(t) - W_i(t)) + c_2r_2(W_G^G(t) - W_i(t)),$$

(7)

where $v_i(t)$ is the old velocity of the particle $i$ at time $t$. Apparent from this equation, the new velocity is related to the old velocity weighted by weight $w$ and also associated to the position of the particle itself and that of the global best one by acceleration factors $c_1$ and $c_2$. The $c_1$ and $c_2$ are therefore referred to as the cognitive and social rates, respectively, because they represent the weighting of the acceleration terms that pull the individual particle toward the personal best and global best positions. The inertia weight $w$ in (7) is employed to manipulate the impact of the previous history of velocities on the current velocity. Generally, in population-based optimization
methods, it is desirable to encourage the individuals to wander through the entire search space, without clustering around the local optima, during the early stage of the optimization.

A suitable value for \( w(t) \) provides the desired balance between the global and local exploration ability of the swarm and, consequently, improves the effectiveness of the algorithm. Experimental results suggest that it is preferable to initialize the inertia weight to a large value, giving priority to global exploration of the search space, linear decreasing \( w(t) \) so as to obtain refined solutions [20–22]. For the purpose of intending to simulate the slight unpredictable component of natural swarm behavior, two random functions \( r_1 \) and \( r_2 \) are applied to independently provide uniform distributed numbers in the range \([0, 1]\) to stochastically vary the relative pull of the personal and global best particles. Based on the updated velocities, new position for particle \( i \) is computed according to the following equation:

\[
W_i(t + 1) = W_i(t) + v_i(t + 1) \tag{8}
\]

The populations of particles are then moved according to the new velocities and locations calculated by (7) and (8), and tend to cluster together from different directions. Thus, the evaluation of each associate fitness of the new population of particles begins again. The algorithm runs through these processes iteratively until it stops. In this paper, the current position can be modified by [24]

\[
w(t) = w_{\text{max}} - \left( \frac{w_{\text{max}} - w_{\text{min}}}{\text{iter}_{\text{max}}} \right) \times \text{iter}, \tag{9}
\]

where \( w_{\text{max}} \) is the initial weight, \( w_{\text{min}} \) is the final weight, \( \text{iter}_{\text{max}} \) is maximum number of iterations, and iter is the current iteration number. The procedures of standard PSO can be summarized as follows.

**Step 1.** Initialize a population of particles with random positions and velocities, where each particle contains \( K \) variable.

**Step 2.** Evaluate the fitness values of all particles, let \( \text{pbest} \) of each particle and its objective value equal to its current position and objective value, and let \( \text{gbest} \) and its objective value equal to the position and objective value of the best initial particle.

**Step 3.** Update the velocity and position of each particle according to (7) and (8).

**Step 4.** Evaluate the objective values of all particles.

**Step 5.** For each particle, compare its current objective value with the object value of its \( \text{pbest} \). If current value is better, then update \( \text{pbest} \) and its object value with the current position and objective value. Furthermore, determine the best particle of current warm with the best objective values. If the objective value is better than the object value of \( \text{gbest} \), then update \( \text{gbest} \) and its objective value with the position and objective value of the current best particle.

**Step 6.** Termination criteria: if a predefined stopping criterion is met, then output \( \text{gbest} \) and its objective value; otherwise go back to Step 3.

### 4. SIMULATION RESULTS AND DISCUSSIONS

To evaluate and to compare the performance of the suboptimal PTS, numerous computer simulations have been conducted to determine the PAPR improvements. QPSK modulation is employed with \( N = 256 \) subcarriers. The phase weighting factors \( W = [0, 2\pi] \) have been used. In order to generate the complementary cumulative distribution function (CCDF) [18] of the PAPR, 10000 random OFDM frames have been generated. The sampling rates for an accurate PAPR need to be increased by 4 times. The complementary distribution function (CDF) of the PAPR is one of the most frequently used performance measures for PAPR reduction techniques. The CDF of the amplitude of a signal sample is given by CDF = 1 – exp(PAPR). In the performance comparison, the parameter of CCDF is defined as

\[
\text{CCDF} = P_r(\text{PAPR} > \text{PAPR}_0) = 1 - P_r(\text{PAPR} \leq \text{PAPR}_0) \tag{10} = 1 - (1 - \exp(-\text{PAPR}_0))^N.
\]

In Figure 3, some results of the CCDF of the PAPR are simulated for the OFDM system with 256 subcarriers, in which \( M = 16 \) subblock employing random partition and the phase weight factor \( W = \{\pm 1\}^M \) uniformly distributed random variables are used for PTS. As we can see that the CCDF of the PAPR is gradually promoted upon increasing the numbers of generations due to the limited phase weighting factor. As the numbers of generation are increased, the CCDF of the PAPR has been improved. For
a generation \( G_n = 40 \), we can see that the PSO-based PTS technique is capable of attaining a near OPTS technique performance, when \( P_e (\text{PAPR} > \text{PAPR}_0) = 10^{-3} \).

In Figure 3, we compare the PAPR performance of different numbers of particle generations \( G_n \), for \( c_1 = c_2 = 2 \). Basically, the PAPR performance is improved with \( G_n \) increasing. However, the degree of improvement is limited when \( G_n \) is above 40. On the other hand, the computational complexity is increased with \( G_n \). Only a slight improvement is attained for increasing \( G_n = 20 \) to 40. The computational complexity of \( G_n = 40 \) is double of that of \( G_n = 20 \). Hence, based on the trade-off between the PAPR reduction and computational complexity, \( G_n = 20 \) is a suitable choice for our proposed PSO-based PTS technique.

Figure 4 shows the simulated results of the PSO-assisted PTS technique, in comparison against normal OFDM for number of subblocks \( M \). \( M \) is one of value in the set \{2, 4, 8, 16, 32\}. In particular, the PAPR of an OFDM signal exceeds 12 dB for \( 10^{-3} \) of the possible transmitted OFDM blocks. However, by introducing PTS approach with \( M = 16 \) clusters partition with phase factors limited to \( W = 4 \), the \( 10^{-3} \) PAPR reduces to 7.5 dB. In short, new approach can achieve a reduction of PAPR by approximately 3.5 dB at the \( 10^{-3} \) PAPR. Thus, the performance of the techniques is better for larger \( M \) since larger numbers of vectors are searched for larger \( M \) in every update of the phase weighting factors. Moreover, it can be observed that probability of very high peak power has been increased significantly if PTS techniques are not used. As the number of subblocks and the set of phase weighting factor are increased, the performance of the PAPR reduction becomes better. However, the processing time gets longer because of much iteration. From Figure 4, as expected, the improvement increases as number of clusters increases. Thus, using the PSO technique, we can obtain better results than presented previously.

The subblock partition for proposed suboptimal method involves dividing the subblocks into multiple disjoint subblocks. Therefore, determining which subblock partition method produces the best performance is important. Figure 5 shows that the subblock partition for proposed suboptimal method involves three dividing subblocks: adjacent method, interleaving method, and pseudorandom method. In the viewpoint of PAPR reduction, pseudorandom subblock partitioning has better performance than others.

In Figure 6, for a fixed number of clusters, the phase weighting factor can be chosen from a larger set of \{2, 4, 8, 16\}. It is shown that the added degree of freedom in choosing the combining phase weighting factors provides an additional reduction. When the number of phase weighting factor \( W = 2 \) and number of subblocks \( M = 4 \), PAPR can be reduced about 2.78 dB at \( 10^{-4} \) from 12 dB to 9.22 dB. When \( W = 4 \) and \( M = 4 \), at \( 10^{-3} \) PAPR can be reduced about 4.2 dB from 12 dB to 7.8 dB. As the number of subblocks and the set of phase weighting factor are increased, the performance of the PAPR reduction becomes better. However, the processing time gets longer because of much iteration.

In this section, a threshold is also applied to reduce calculation complexity and is calculated from the CCDF equation, which has given optimal threshold for the number of subblocks as follows. When \( N \) subcarriers and \( M \) subblocks are assumed, the probability that the PAPR will exceed certain of \( \text{PAPR}_0 \) is represented [18] as

\[
P_e (\text{PAPR} > \text{PAPR}_0) = (1 - (1 - \exp (\text{PAPR}_0^{N/M}))^M).
\]  (11)

From (11), \( \text{PAPR}_0 \) of threshold \( \zeta \), which is satisfied with given probability CCDF, can be represented as

\[
\zeta = 10 \log (- \ln (1 - 10^{p \log_{10} M / N}))
\]  (12)

which \( p \) is \( \Pr \{ \}. \)
The acceleration factors $c_1$ and $c_2$ when $N = 128$, $M = 4$, and $W = 2$. It can be seen that when the acceleration factors increase, the level of CCDF being 0.1%, the acceleration factors resulted in the PAPR depression increasing. For example, at $c_1 = 2$ and $c_2 = 2$, the PAPR is 6.8 dB. By these two examples of the acceleration factors $c_1$ and $c_2$, the improvement in PAPR reduction is about 1.5 dB. Furthermore, we see that the PAPR reduction of $c_1 = 2$ and $c_2 = 2$; and $c_1 = 2.5$ and $c_2 = 2$ have similar performance. Hence, after taking the effect of the

The CCDF, $P_r(PAPR > PAPR_0)$ for $M = 8$, is shown in Figure 7. The $10^{-4}$ PAPR of an original OFDM frame was 12 dB. OPTS and the PSO-PTS improved on this by 7.6 dB with nearly the same performance for $M = 8$, while the performance loss with the iteration PTS is 8.4 dB. The iteration number of proposed technique is shown in Table 1. For $M = 8$, the OPTS technique requires 128 iterations per OFDM frame, while iteration PTS technique requires 16 iterations and the PSO-PTS technique without a threshold requires 88 iterations per OFDM frame. The complexity of iteration PTS is only 12.5% (16/128) of that of the PTS technique. The PSO-PTS technique with a threshold value is exhibited a lower complexity that only requires 23 iterations per OFDM frame. Thus, compared to the OPTS technique, the complexity of the PSO-PTS with threshold is only 18% (23/128 = 0.18).

Table 1: The computational complexity of the OPTS, IPTS and PSO-PTS techniques with phase weighting factor $W = 2$.

| Method               | Computation complexity | $P_r(PAPR > PAPR_0) = 0.0001$ |
|----------------------|------------------------|-------------------------------|
| OPTS                 | $W^{M-1} = 2^{M-1} = 128$ | 7.6 dB                        |
| IPTS                 | $M \times M = 2 \times 8 = 16$ | 8.4 dB                        |
| PSO-PTS              | $V \times O(W^3) = (1 + G_0) \times O(W^3) = (1 + 10) \times (2^3) = 11 \times 8 = 88$ | 8.0 dB                        |
| PSO-PTS with threshold | 23                     | 7.6 dB                        |

Figure 6: Comparisons of PSO-PTS technique under different phase weight factors and number of subblocks.

Figure 7: Comparison of the PSO-PTS technique with threshold PAPR, iterative PTS, PSO-PTS, and OPTS methods when $M = 8$.

Figure 8: CCDFs comparison of the PSO-based PTS scheme with different combinations of acceleration constants when $N = 128$, $M = 4$, and $W = 2$. 
reduction and the computational complexity into account, $c_1 = 2$ and $c_2 = 2$ is a suitable choice for our proposed PSO-based PTS technique. The values of $c_1$ and $c_2$ affect the behavior of the swarm in different ways: a bigger $c_1$ can increase the attraction of $W^G_i$ for every particle and prevent the particle converging to $W^G_i$ quickly, while a bigger $c_2$ can decrease the attraction of $W^G_i$ and prompt the swarm converging to the same $W^G_i$.

5. CONCLUSION

In this paper, we analyze the PAPR reduction performance which is derived by using adjacent, interleaved, and random subblock partitioning methods. Random subblock partitioning method has derived the most effective performance, and interleaved subblock partition method has derived the worst. As the number of subblocks is increased, PAPR can be further reduced. Moreover, we formulate the phase weighting factors searching of PTS as a particular combination optimization problem and we apply the PSO technique to search the optimal combination of phase weighting factors for PTS to obtain almost the same PAPR reduction as that of optimal PTS while keeping low complexity. Simulations results show that PSO-based PTS method is an effective method to compromise a better tradeoff between PAPR reduction and computation complexity. By appropriate selection of phase weighting factors according to the required performance and tolerable complexity, the proposed partition scheme can be adaptive to QOS requirement. We illustrated that with this method we can develop algorithms which can achieve better performance-complexity tradeoff than the existing approaches. Additionally, the performance of the proposed method was slightly degraded compared to that of optimum method, PTS. However, the complexity of the proposed method was remarkably lower than that of optimum method.

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REFERENCES

[1] S. H. Han and J. H. Lee, “PAPR reduction of OFDM signals using a reduced complexity PTS technique,” IEEE Signal Processing Letters, vol. 11, no. 11, pp. 887–890, 2004.
[2] X. Li and L. J. Cimini Jr., “Effects of clipping and filtering on the performance of OFDM,” IEEE Communications Letters, vol. 2, no. 5, pp. 131–133, 1998.
[3] T. Jiang and G. Zhu, “Complement block coding for reduction in peak-to-average power ratio of OFDM signals,” IEEE Communications Magazine, vol. 43, no. 9, pp. S17–S22, 2005.
[4] T. Jiang and G. Zhu, “Nonlinear companding transform for reducing peak-to-average power ratio of OFDM signals,” IEEE Transactions on Broadcasting, vol. 50, no. 3, pp. 342–346, 2004.
[5] T. Jiang, W. Yao, P. Guo, Y. Song, and D. Qu, “Two novel nonlinear companding schemes with iterative receiver to reduce PAPR in multi-carrier modulation systems,” IEEE Transactions on Broadcasting, vol. 52, no. 2, pp. 268–273, 2006.
[6] Z. Yang, H. Fang, and C. Pan, “ACE with frame interleaving scheme to reduce peak-to-average power ratio in OFDM systems,” IEEE Transactions on Broadcasting, vol. 51, no. 4, pp. 571–575, 2005.
[7] C.-L. Wang and Y. Ouyang, “Low-complexity selected mapping schemes for peak-to-average power ratio reduction in OFDM systems,” IEEE Transactions on Signal Processing, vol. 53, no. 12, pp. 4652–4660, 2005.
[8] [9] T. Jiang, W. Yao, P. Guo, Y. Song, and D. Qu, “Two novel nonlinear companding schemes with iterative receiver to reduce PAPR in multi-carrier modulation systems,” IEEE Transactions on Broadcasting, vol. 51, no. 4, pp. 571–575, 2005.
[10] G. Zhu, H. Fang, and C. Pan, “ACE with frame interleaving scheme to reduce peak-to-average power ratio in OFDM systems,” IEEE Transactions on Broadcasting, vol. 51, no. 4, pp. 571–575, 2005.
[11] C.-L. Wang and Y. Ouyang, “Low-complexity selected mapping schemes for peak-to-average power ratio reduction in OFDM systems,” IEEE Transactions on Signal Processing, vol. 53, no. 12, pp. 4652–4660, 2005.
[12] H. Breiling, S. H. Muller, and J. B. Huber, “SLM peak power reduction without explicit side information,” IEEE Communication Letters, vol. 5, no. 6, pp. 239–241, 2001.
[13] S. H. Muller and J. B. Huber, “OFDM with reduced peak-to-average power ratio by optimum combination of partial transmit sequences,” Electronics Letters, vol. 33, no. 5, pp. 368–369, 1997.
[14] S. G. Kang, J. G. Kim, and E. K. Joo, “A novel subblock partition scheme for partial transmit sequence OFDM,” IEEE Transactions on Broadcasting, vol. 45, no. 3, pp. 333–338, 1999.
[15] W. S. Ho, A. S. Madhukumar, and E. Chin, “Peak-to-average power reduction using partial transmit sequences: a suboptimal approach based on dual layered phase sequencing,” IEEE Transactions on Broadcasting, vol. 49, no. 2, pp. 225–231, 2003.
[16] D.-W. Lim, S.-J. Heo, J.-S. No, and H. Chung, “A new PTS OFDM scheme with low complexity for PAPR reduction,” IEEE Transactions on Broadcasting, vol. 52, no. 1, pp. 77–82, 2006.
[17] L. Yang, R. S. Chen, Y. M. Siu, and K. K. Soo, “PAPR reduction of an OFDM signal by use of PTS with low computational complexity,” IEEE Transactions on Broadcasting, vol. 52, no. 1, pp. 83–86, 2006.
[18] A. D. S. Jayalath and C. Tellambura, “Adaptive PTS approach for reduce of peak-to-average power ratio of OFDM signal,” Electronics Letters, vol. 36, no. 14, pp. 1226–1228, 2000.
[19] L. J. Cimini Jr. and N. R. Sollenberger, “Peak-to-average power ratio reduction of an OFDM signal using partial transmit sequences,” IEEE Communication Letters, vol. 4, no. 3, pp. 86–88, 2000.
[20] S. G. Kang, J. G. Kim, and E. K. Joo, “A novel subblock partition scheme for partial transmit sequence OFDM,” IEEE Transactions on Broadcasting, vol. 45, no. 3, pp. 333–338, 1999.
[21] O.-I. Kwon and Y.-H. Ha, “Multi-carrier PAP reduction method using sub-optimal PTS with threshold,” IEEE Transactions on Broadcasting, vol. 49, no. 2, pp. 232–236, 2003.
[22] S. H. Han and J. H. Lee, “An overview of peak-to-average power ratio reduction techniques for multicarrier transmission,” IEEE Wireless Communications, vol. 12, no. 2, pp. 56–65, 2005.
[23] A. Ratnaweera, S. K. Halgamuge, and H. C. Watson, “Self-organizing hierarchical particle swarm optimizer with time-varying acceleration coefficients,” IEEE Transactions on Evolutionary Computation, vol. 8, no. 3, pp. 240–255, 2004.
[24] M. Clerc and J. Kennedy, “The particle swarm—stability, convergence and potential benefits of a complex space,” IEEE Transactions on Evolutionary Computation, vol. 6, no. 1, pp. 58–73, 2002.
[25] Y. Shi and R. Eberhart, “A modified particle swarm optimizer,” in Proceedings of the IEEE International Conference on Evolutionary Computation (ICEC ’98), pp. 69–73, Anchorage, Alaska, USA, May 1998.
[26] D. J. Krusienski and W. K. Jenkins, “Design and performance of adaptive systems based on structured stochastic optimization strategies,” IEEE Circuits and Systems Magazine, vol. 5, no. 1, pp. 8–20, 2005.
[23] W.-C. Liu, “Design of a multiband CPW-fed monopole antenna using a particle swarm optimization approach,” *IEEE Transactions on Antennas and Propagation*, vol. 53, no. 10, pp. 1–7, 2005.

[24] J. Robinson and Y. Rahmat-Samii, “Particle swarm optimization in electromagnetics,” *IEEE Transactions on Antennas and Propagation*, vol. 52, no. 2, pp. 397–407, 2004.

[25] A. Torn and A. Zilinkas, *Global Optimization*, vol. 350 of Lecture Notes in Computer Science, Springer, New York, NY, USA, 1989.

[26] J. Kennedy, R. C. Eberhart, and Y. Shi, *Swarm Intelligence*, Morgan Kaufmann, San Mateo, Calif, USA, 2001.

[27] H.-L. Hung, Y.-F. Huang, and J.-H. Wen, “A particle swarm optimization based multiuser detector for DS-CDMA communication systems,” in *Proceedings of the IEEE International Conference on Systems, Man and Cybernetics (SMC ’06)*, vol. 3, pp. 1956–1961, Taipei, Taiwan, October 2006.