NEUTRINO INTERACTIONS IN MATTER*

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If a fermion is travelling through a medium, it can have matter-induced magnetic and electric dipole moments. These contributions conserve chirality, and can be non-vanishing even for a Majorana neutrino. Several implications for neutrino physics are discussed.

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1 Neutrino propagation in matter

This subject became popular when Wolfenstein calculated neutrino refractive index in matter, and subsequently Mikheyev and Smirnov recognized resonant nature of flavor oscillations triggered by matter effects. Application to the solar neutrino problem is now standard [1].

To set up the stage, we outline a covariant calculation leading to Wolfenstein’s result. One evaluates the self-energy function to determine the dispersion relation, which can give the refractive index [2, 3, 4]. Consider a massless neutrino for simplicity. Chirality dictates that its self-energy is of the form

\[ \Sigma = RSL, \]

where \( R \) and \( L \) are chirality projection operators. In vacuum, the most general form of \( S \) is

\[ S = a(k^2)k. \]

Thus, the pole of the propagator is at \( k^2 = 0 \), i.e., the particle is massless to all orders.

In a medium characterized by the center-of-mass velocity \( v^\mu \),

\[ S = a\not{k} + b\not{v} \]

in general. Thus, the dispersion relation changes, which is responsible for a refractive index different from unity.

2 Electromagnetic vertex

![Figure 1: The effective vertex with photon.](image)

In general, the vertex in Fig. 1 can be written as

\[ \bar{u}(k')\Gamma_\lambda(k, k', \nu)u(k)A^\lambda, \]

where the vector \( v^\mu \) has been defined earlier. Conservation of charge implies the condition:

\[ q^\lambda \Gamma_\lambda = 0. \]

In the vacuum where \( v^\mu \) does not exist, the neutrino has no charge, so that

\[ \Gamma_\lambda(k, k, 0) = 0. \]

These imply the following most general form for \( \Gamma_\lambda \) in the vacuum [4, 5, 6, 7]:

\[ \Gamma_\lambda = (q^2\gamma_\lambda - q_\lambda\not{q})(R + r\gamma_5) + i\sigma_{\lambda\rho}q^\rho(D_M + D_E\gamma_5). \]
In matter, additional terms are possible because of the 4-vector $v$ [8, 9]:

$$\Gamma'_{\lambda} = iD'_E(\gamma_{\lambda}v_{\rho} - \gamma_{\rho}v_{\lambda})q^{\rho}\gamma_{5} + iD'_M\epsilon_{\lambda\rho\alpha\beta}\gamma^{\rho}\gamma_{5}q^{\alpha}v^{\beta}.$$

(8)

It is easy to understand these terms in co-ordinate space if all form factors are assumed to be momentum independent. The vacuum part can then be written as

$$\bar{\psi}\gamma_{\lambda}\gamma_{5}\psi\partial_{\rho}F^{\lambda\rho}(R + r\gamma_{5}) + \bar{\psi}\sigma_{\lambda\rho}\psi F^{\lambda\rho}(D_{M} + D_{E}\gamma_{5}),$$

(9)

whereas the extra terms in a medium are

$$D'_E\bar{\psi}\gamma_{\lambda}\gamma_{5}\psi v_{\rho}F^{\lambda\rho} + D'_M\bar{\psi}\gamma_{\lambda}\gamma_{5}\psi v_{\rho}\tilde{F}^{\lambda\rho}.$$  

(10)

In the non-relativistic limit, since $\bar{\psi}\gamma_{0}\gamma_{5}\psi \to 0$, $\bar{\psi}\vec{\gamma}_{5}\gamma_{5}\psi \to \vec{\sigma}$, in a frame where $v^{\rho} = (1, \vec{0})$, we can write (10) as

$$D'_E\vec{\sigma} \cdot \vec{E} + D'_M\vec{\sigma} \cdot \vec{B}.$$  

(11)

Hence, these are new contributions to dipole moments [8, 9]. Notice that these terms are chirality conserving, and can be non-zero even for a Majorana neutrino [8]. Both these properties are different from the vacuum dipole moment terms.

3 Calculations at the leading order

To the leading order in the Fermi constant [10, 11],

$$\Gamma_{\lambda} = \mathcal{T}_{\lambda}^{\rho}\gamma^{\rho}L,$$

$$\mathcal{T}_{\lambda}^{\rho} = \mathcal{T}_{T}R_{\lambda}^{\rho} + \mathcal{T}_{L}Q_{\lambda}^{\rho} + \mathcal{T}_{P}P_{\lambda}^{\rho},$$

(12)

(13)

where, with $\tilde{g}_{\lambda\rho} = g_{\lambda\rho} - q_{\lambda}q_{\rho}/q^{2}$ and $\tilde{v}_{\lambda} = \tilde{g}_{\lambda\rho}v^{\rho}$,

$$R_{\lambda}^{\rho} = \tilde{g}_{\lambda\rho} - Q_{\lambda}^{\rho},$$

$$Q_{\lambda}^{\rho} = \tilde{v}_{\lambda}\tilde{v}^{\rho}/\tilde{v}^{2},$$

$$P_{\lambda}^{\rho} = i\epsilon_{\lambda\rho\alpha\beta}q^{\alpha}v^{\beta}/\sqrt{(q \cdot v)^{2} - q^{2}}.$$  

(14)

(15)

(16)

The form factors $\mathcal{T}_{T}$, $\mathcal{T}_{L}$, $\mathcal{T}_{P}$ have been calculated in the leading order in $G_{F}$, in the general case when the incoming and the outgoing neutrinos in Fig. 3 may or may not be the same. From this, one can obtain various physical effects, as described below.

Radiative decay: In the vacuum, this is suppressed due to leptonic GIM. A medium full of electrons and not muons or taons is not flavor symmetric. Thus, the GIM mechanism is no more operative. The rates are enhanced tremendously [12, 13].

Modification of forward scattering amplitude: This occurs through the electromagnetic vertex, if the electron scatters from the photon. This was originally supposed to be large, as large as the Wolfenstein term [14]. Later, it was pointed out that protons can also scatter from the photon, and this cancels exactly the electron-scattering contribution in the forward scattering amplitude for neutrinos [15, 16].
**Induced electric charge**: Neutrinos in medium have a small charge induced by matter effects \[\varepsilon_{\text{ind}} = -\left( e G_F / \sqrt{2} \right) (1 + 4 \sin^2 \theta_W) (e^2 r_D^2)^{-1} \] where the Debye screening length is given by \[r_D^2 = T/n_e e^2\] for a non-relativistic plasma at temperature \(T\). This gives \(\varepsilon_{\text{ind}} \approx -2 \times 10^{-32} (1 \text{ cm}/r_D)^2\). This is not measurable even for the densest known plasma for which \(r_D \approx 10^{-4} \text{ cm}\).

**Plasmons**: Plasmons can decay into \(\nu \bar{\nu}\). The rate has been calculated \[\textnormal{[11]}\]. Neutrinos can produce plasmons in a medium \[\textnormal{[11, 18]}\].

### 4 Outlook

The effects may be large compared to the vacuum effects, but still not hopeful for being observable. However, the subject is interesting, and I am sure that the resonant neutrino oscillation is not the only interesting physical effect. Maybe some other process, like the Majoronic decay of neutrinos \[\textnormal{[19]}\], or other systems like the early universe \[\textnormal{[20, 21]}\], will yield some non-trivial effects. Or maybe in more non-trivial matter distribution as in a crystal, some of the electromagnetic effects will be enhanced enough to be observable.

I end by thanking my friend José F Nieves, with whom I have done most of the work on this subject. I also thank R Cowsik for some discussions after my talk at the conference.

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