Turbulence suppression using phase conjugation for mode-division multiplexing over a 340-m free-space link

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The orbital angular momentum (OAM) of photons presents a degree of freedom for enhancing the secure key rate of free-space quantum key distribution (QKD) through mode-division multiplexing (MDM). However, atmospheric turbulence can lead to substantial modal crosstalk, which is a long-standing challenge to MDM for free-space QKD. We demonstrate that digital phase conjugation is an effective method for mitigating atmospheric turbulence. We experimentally characterize seven OAM modes after propagation through a 340-m outdoor free-space link and observe a suppression of average modal crosstalk from 37.0\% to 13.2\% by implementing real-time digital phase conjugation. The crosstalk can be further reduced to 3.4\% when adopting a mode spacing $\Delta \ell$ of 2. We implement a classical MDM system as a proof-of-principle demonstration, and the bit error rate is reduced from $3.6 \times 10^{-2}$ to be less than $1.3 \times 10^{-7}$ through the use of phase conjugation. We also propose a practical and scalable scheme for high-speed, mode-multiplexed QKD through a turbulent link. Our method can be useful to various free-space applications that require turbulence suppression.

In recent decades, QKD has attracted increasing interest because it can guarantee communication security based on fundamental laws of quantum mechanics\textsuperscript{1}. Free-space QKD\textsuperscript{2,6} can guarantee communication security between mobile nodes such as aircraft and satellites. In addition, free space presents lower loss than fibers and thus is favorable to loss-sensitive applications such as quantum teleportation\textsuperscript{2} and entanglement distribution\textsuperscript{3}. Due to the intrinsically low brightness of quantum light sources, the secure key rate of QKD is significantly lower than the data transfer rate of classical communication protocols. Thus, it remains highly desirable to enhance the secure key rate of QKD. The spatial degree of freedom is a promising candidate for boosting capacity of both quantum and classical communication through MDM\textsuperscript{9,10} or high-dimensional encoding\textsuperscript{11-18} and is compatible with polarization- and wavelength-division multiplexing. In particular, slowly diverging spatial modes such as OAM modes are commonly used as a basis set in free-space communication compared to alternative basis sets such as discrete spot arrays\textsuperscript{19} which are unsuitable for a long-distance link. However, atmospheric turbulence inevitably leads to strong modal crosstalk between spatial modes\textsuperscript{20,21}, which severely degrades the channel capacity of a free-space link. We next summarize several previous works to show the typical level of crosstalk between OAM modes in turbulent, outdoor free-space channel. In a 150-m link\textsuperscript{15}, the crosstalk fluctuates between 60\% and 80\% depending on time. In a 300-m intra-city link\textsuperscript{14}, the crosstalk is 11\% with a mode spacing $\Delta \ell$ of 4. In a 340-m cross-campus link\textsuperscript{7}, the crosstalk is measured to be in the range between 70\% and 80\%. In a 3-km link\textsuperscript{11}, a camera is used to measure the images of bright OAM superposition and an artificial neural network (ANN) is applied for image recognition, resulting in a bit error rate of $1.7 \times 10^{-2}$. Hence, the turbulence can be a serious concern for crosstalk-sensitive applications such as QKD. A more comprehensive summary is listed in Supplementary Section 1.

Adaptive optics is the most common method for turbulence correction and has been widely adopted for astronomical imaging\textsuperscript{22}. A conventional adaptive optics system consists of a wavefront sensor and a deformable mirror at the receiver. The wavefront sensor measures the aberrated phase of an incoming beacon beam (typically a Gaussian beam), and subsequently the deformable mirror corrects the phase aberration of the incoming beam based on the feedback from wavefront sensor as post-turbulence compensation\textsuperscript{23}. However, as shown in Fig. 1(a), different OAM modes (Laguerre-Gauss modes with radial index $p = 0$ and OAM index $\ell = -1, 0, 1$) exhibit mode-dependent amplitude and phase distortions after propagation through the same turbulent link\textsuperscript{17}. Therefore, a conventional post-turbulence single-plane phase-only adaptive optics system is unable to correct both amplitude and phase distortions for different OAM modes simultaneously, even in principle\textsuperscript{24,25}. Furthermore, the effectiveness of adaptive optics for OAM communication has mostly been tested in numerical simulations\textsuperscript{25-27} or in lab-scale links with emulated, slowly varying, fully controllable turbulence\textsuperscript{28-31}. To the best of our knowledge, there is only one experimental demonstration using adaptive optics for OAM communication through an outdoor link\textsuperscript{17}, and the crosstalk is reduced from 80\% to 77\% by using both adaptive optics and a fast steering mirror simultaneously. This level of crosstalk is too large to guarantee secure QKD\textsuperscript{32}. Therefore, adaptive optics...
has achieved very limited performance enhancement in outdoor free-space OAM communication links despite numerous simulations and lab-scale experiments. Other methods for turbulence suppression, such as a multiple-input multiple-output (MIMO) algorithm, and an ANN, cannot be applied to QKD because these algorithms require a large number of photons for digital signal processing and thus are inappropriate for quantum applications that operate at a single-photon level.

Here we propose and demonstrate that digital phase conjugation can be used to effectively suppress atmospheric turbulence for OAM communication through a 340-m free-space link. Phase conjugation is also referred to as time reversal, and Fig. 1 illustrates how to transmit high-fidelity spatial modes from Alice to Bob by using phase conjugation. As shown in Fig 1(a), Bob first transmits standard OAM modes to Alice, and the modes received by Alice are distorted both in amplitude and phase. By contrast, perfect OAM modes can be transmitted to Bob if Alice uses a phase-conjugating mirror as shown in Fig 1(b). For an arbitrary incident spatial mode $A(x,y)e^{i\phi(x,y)}$, the mode reflected by a phase-conjugating mirror becomes $A(x,y)e^{-i\phi(x,y)}$. After propagation through the same turbulent link, the OAM modes received by Bob become the phase conjugate of the originally transmitted OAM modes and can in principle have a perfect spatial mode quality, assuming a link without beam clipping. The phase-conjugating mirror can be digitally implemented by a phase-only spatial light modulator (SLM), because spatial amplitude and phase modulation can be simultaneously realized by a diffractive hologram. In order to compensate for time-varying turbulence, the hologram needs to be updated dynamically in real time. From a technical point of view, the spatial modes transmitted from Bob to Alice can be regarded as probe beams that enable fast characterization of turbulence and thus allow Alice to perform pre-turbulence mode generation for each spatial mode. Although digital phase conjugation has been employed for aberration correction in biological tissues and multimode fibers, it has not been experimentally applied to free-space optical OAM communication.

Our phase conjugation experimental schematic is presented in Fig. 2. We first characterize the modal crosstalk matrix of the phase conjugation system, and then use the setup to realize a two-channel OAM communication system. A static diffractive hologram is displayed on SLM 1 at Bob’s side to generate two 780 nm OAM probe beams of horizontal and vertical polarization, respectively. These two OAM probe beams are combined by a polarizing beamsplitter (PBS) and then transmitted to Alice through a retroreflector. These two orthogonally polarized probe beams allow simultaneous digital phase conjugation for two different OAM modes, which facilitates the realization of two-channel OAM communication system discussed later. The retroreflector of 127 mm diameter is installed on a building rooftop that is 170 m away, resulting in a round trip distance of 340 m. At Alice’s side, a PBS is used to separate the two aberrated OAM modes, and a coherent reference plane wave is combined with two separated OAM probe beams by a beamsplitter (BS). A camera (Cam 1) is used to record the interference fringes, and a 45° polarizer is inserted before Cam 1 to enhance the interference pattern visibility. Through the standard off-axis holography analysis, the amplitude and phase of the received modes can be retrieved with a single-shot measurement from Cam 1. Hence, Cam 1 is used as a wavefront sensor in our setup. To provide the coherent reference plane wave for off-axis holography, a single-mode fiber is used to guide the continuous-wave 780 nm light source from Bob to Alice. We emphasize that this single-mode fiber can be avoided by using alternative wavefront sensors such as a Shack-Hartmann sensor or complex field direct measurement. Based on the measured amplitude and phase of the aberrated OAM modes, Alice computes the corresponding...
diffractive hologram and displays it on SLM 2 to generate the phase conjugates of the received OAM modes with desired amplitude and phase at the first diffraction order (see Supplementary Section 2). Alice uses a 785 nm laser diode as the light source and transmits the phase-conjugated modes to Bob. Due to the negligible dispersion of free space, the difference in wavefront distortion between 780 nm and 785 nm wavelength can be ignored\(^{17}\). In the experiment we use seven Laguerre-Gauss modes of \( \ell = -3, -2, \cdots, 3 \) to test the performance of the phase conjugation system. The aperture diameter of both telescopes is 5 cm, resulting in a Fresnel number product of \( N_f = D^2/\lambda z = 9.4 \), where \( D = 5 \text{ cm}, \lambda = 780 \text{ nm}, \) and \( z = 340 \text{ m} \). The beam waist radius of the OAM modes is \( w_0 = 10 \text{ mm} \) after beam expansion of the telescope. The turbulence structure constant\(^{45} \) \( C_n^2 \) is measured to be in the range of \( 2.2 \times 10^{-15} \text{ m}^{-2/3} \) to \( 8.6 \times 10^{-15} \text{ m}^{-2/3} \), the Fried parameter \( r_0 \) ranges from 0.16 m to 0.07 m, and thus \( D/r_0 \) is between 0.31 and 0.70. Additional experimental details can be found in Supplementary Section 2.

Due to atmospheric turbulence and aberration of the telescope system, the OAM probe beams received by Cam 1 at Alice’s side exhibit clear distortions as shown in the top row of Fig. 3(a). Alice generates the phase conjugates of these aberrated modes and transmits them to Bob. The phase-conjugated modes received by Cam 2 at Bob’s side are shown in the bottom row of Fig. 3(a), which exhibit improved mode fidelity compared to those received by Alice. To quantify the modal crosstalk of the phase-conjugated modes, we display a densely encoded diffractive hologram on SLM 3 as a mode demultiplexer\(^{12} \), and a camera (Cam 3) is placed at the Fourier plane of SLM 3 to measure the crosstalk matrix\(^{15} \) (see Supplementary Section 2). When Alice transmits standard OAM modes, the crosstalk matrix of the modes received by Bob is shown in Fig. 3(b). The average value of the diagonal elements is 63.0%, and thus the average crosstalk is 37.0%. Therefore, this link cannot support secure QKD with OAM encoding in the absence of phase conjugation since the crosstalk is higher than the security error threshold of 23.7% for a seven-dimensional system\(^{12} \). By contrast, when Alice transmits phase-conjugated OAM modes to Bob, the average crosstalk is reduced to 13.2% as shown in Fig. 3(c), which allows for secure QKD operating at a single-photon level. In addition, it is well known that the modal crosstalk can be reduced by increasing the mode spacing \( \Delta \ell \). We calculate the crosstalk matrix with a mode spacing \( \Delta \ell = 2 \) by post-selecting the data of \( \ell = -3, -1, 1, 3 \), and the average crosstalk can be further reduced from 10.0% to 3.4% by using phase conjugation as shown in Fig. 3(d,e). These results are the lowest crosstalk ever achieved in an outdoor free-space link to the best of our knowledge (see Supplementary Section 1 for a comprehensive comparison to other previous works). Although we use classical light to characterize the crosstalk matrix, we emphasize that our method can be readily applied to quantum applications that operate at a single-photon level.

Ideally, a realization of phase conjugation can completely eliminate the aberration and achieve zero crosstalk. Here we attribute the nonzero crosstalk observed in our experiment to the following reasons. First, the operational bandwidth of our digital phase conjugation system is limited. The image transfer time from Cam 1 to computer memory is 4 ms, the computation time for diffractive hologram generation is 1 ms, and the refresh time of SLM 2 is 5 ms, resulting in a total response time of \( \approx 10 \text{ ms} \) and hence an operational bandwidth of 100 Hz. By contrast, the characteristic frequency of turbulence can be tens of to hundreds of hertz\(^{22} \). We believe that the operational bandwidth can be improved to exceed 1 kHz with faster devices such as a 100 kHz wavefront sensor\(^{43} \) and 22 kHz SLM\(^{46} \). Second, the mode fidelity of our phase conju-
gate generation is not perfect. The phase conjugate generation fidelity can be further improved by calibrating and correcting the residual aberration of the SLM. Third, beam clipping should be avoided in a free-space link in order to enable a perfect realization of phase conjugation. In our experiment, beam clipping can occur at the telescope aperture as well as at the retroreflector, and the link transmittance for $\ell = \pm 3$ is $\approx 10\%$ lower than that of $\ell = 0$ (see Supplementary Section 3). It should be noted that the retroreflector is unnecessary and can be removed in a realistic free-space link. In addition, low-cost and large-diameter Fresnel lenses (1 in diameter lens is commercially available)\(^4\) can be used in a long-distance link to avoid beam clipping. The static aberration of the Fresnel lens should not be a concern because it can be corrected by phase conjugation as part of the overall channel aberration.

It should be noted that the data transfer rate of OAM communication is not limited by the SLM refresh rate but is decided by the modulation speed of the transmitter\(^3\). To clarify this point, we demonstrate a proof-of-principle classical communication system with two-channel OAM multiplexing. We choose $\ell = 2$ for channel 1 and $\ell = 3$ for channel 2. Bob generates horizontally polarized $\ell = 3$ mode and vertically polarized $\ell = 2$ mode and transmits them to Alice. It should be noted that the polarization has negligible effect on beam propagation due to the small birefringence of turbulent free space. Hence, Alice can measure two aberrated OAM modes simultaneously with a single-shot measurement. We emphasize even though we are using orthogonally polarized OAM modes to facilitate turbulence characterization, this is unnecessary and can be avoided by transmitting different OAM modes in different time slots to Alice using a spatial mode switch\(^4\).

The intensity of the 785 nm laser diode at Alice’s side is modulated at the rate of 20 Mbps with on-off keying format. The modulation rate is solely limited by our modulator bandwidth and can be readily improved to Gbps level by using commercially available high-speed modulators. A fiber coupler is used to split the beam, and a 10 m fiber delay line is used to de-correlate the signal streams. The two beams illuminate separate areas of SLM 2 to generate the corresponding phase conjugate of the two OAM modes transmitted by Bob. The two phase-conjugated OAM modes are combined by BS 1 and transmitted to Bob. It should be noted that both phase-conjugated modes are vertically polarized, and the horizontal polarization is an unused degree of freedom that can be further adopted for polarization encoding or multiplexing if needed. In the schematic shown in Fig. 2, we use BS 4 at the receiver to split the beam and subsequently use SLM 3 to perform projection onto two OAM modes. In the experiment we omit BS 4 and perform different OAM projections by switching the hologram for simplicity. The projection measurement realized by SLM 3 can be replaced by a low-loss OAM mode sorter\(^4\) for loss-sensitive applications such as QKD. Within every 4 s we collect $8 \times 10^4$ bits from the oscilloscope, and we collect a total of $8 \times 10^6$ bits over 400 s. The eye diagrams at different times for $\ell = 2$ is shown in Fig. 4(a). It can be seen that the data transfer rate is determined by the intensity modulator we use and is not limited by the refresh rate of SLM. Slight power fluctuation can be observed in the eye diagrams between dif-
Ren et al. use an SLM to apply a phase-only compensation to all different OAM modes simultaneously. Due to these limitations, the pre-turbulence compensation demonstrated in ref. 29 is essentially equivalent to the conventional post-turbulence compensation with back-propagating beams and thus does not exhibit a better performance than the conventional adaptive optics.

Based on the low crosstalk and high communication bandwidth of the system, here we propose a practical, scalable free-space QKD system with N-channel OAM multiplexing using phase conjugation for turbulence suppression. The schematic of the proposed QKD system is shown in Fig. 4(b). At Bob’s side, an SLM is used to generate and switch sequentially among N OAM modes. High-speed OAM mode switching can be readily achieved by using a digital micromirror device at 22 kHz 46 or an acousto-optic modulator at 500 kHz 48. Alice uses a wavefront sensor to measure the amplitude and phase of each OAM mode in real time. A densely encoded diffractive hologram 12 can be computed and displayed on an SLM to generate and multiplex N phase-conjugated modes simultaneously. The hologram needs to be updated dynamically at a speed faster than the turbulence characteristic frequency as discussed earlier. Although the densely encoded hologram typically has a low diffraction efficiency, this is not a problem for coherent-state-based QKD protocols, because strong loss is inherently needed to attenuate a classical, high-brightness laser to a single-photon level. The standard polarization-encoded decoy-state QKD protocol 6 can be used to enable secure communication, and the secure key rate of each channel is not limited by the SLM refresh rate but determined by the polarization modulation rate which can readily reach GHz level 50. In fact, by adding a high-speed polarization switch and attenuating the laser diode to a single-photon level, our classical MDM system can be immediately turned to a polarization-encoded OAM-multiplexed QKD system. Alternative coherent-state-based protocols such as time-bin encoding 51 and continuous-variable encoding 52 are also applicable to our scheme. Furthermore, wavelength-division multiplexing is compatible with our scheme because of the non-dispersive, broadband spectral response of free space. The major advantage of this phase conjugation QKD protocol is the low crosstalk as demonstrated in our experiment, which has not been achieved by any adaptive optics system in an outdoor turbulent link and there is still room for improvement. Moreover, this protocol cannot be replaced by classical turbulence suppression methods such as MIMO and ANN for quantum applications operating at a single-photon level as discussed earlier.

In conclusion, we experimentally demonstrate turbulence suppression in a 340-m free-space OAM communication link through the use of digital phase conjugation. The crosstalk induced by turbulence can be reduced from 37.0% to 13.2% with phase conjugation, and further down to 3.4% by using a mode spacing Δℓ of 2. We believe that lower crosstalk can be reasonably achieved by using faster equipment in a straightfor-
ward manner. A proof-of-principle classical communication system is realized to show the feasibility of high-speed communication with OAM multiplexing. In addition, a practical and scalable scheme for free-space QKD with OAM multiplexing is also proposed and analyzed. Based upon the scalability of the experimental implementation and low crosstalk of the data, we anticipate that digital phase conjugation can be useful to numerous free-space quantum and classical applications that require turbulence suppression.

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Competing interests
The authors declare no competing interests.

Author contributions
Y.Z. conceived and performed the experiment with assistance from J.Z., B.B., K.P., R.Z., and R.W.B. All authors contributed to the discussion of the results and the writing of the manuscript. A.E.W., Z.S., and R.W.B. supervised the project.

Data and materials availability
The data supporting this study are available in the manuscript and Supplementary Information. If needed, other relevant data may be available upon request.

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