Charm and bottom baryon masses in the $1/N_c$ expansion

E. Jenkins

\textsuperscript{a}Department of Physics, University of California, San Diego, 9500 Gilman Drive, La Jolla, California 92093, USA

The masses of heavy quark baryons are studied in an expansion in $1/N_c$, $SU(3)$ flavor symmetry breaking, and heavy-quark symmetry breaking. Very accurate model-independent mass relations are obtained for charm and bottom baryons.

1. INTRODUCTION

The $1/N_c$ expansion has proven useful for studying the spin-flavor properties of baryons containing light quarks. A spin-flavor symmetry for baryons is defined in large-$N_c$ QCD. Away from the $N_c \rightarrow \infty$ limit, $1/N_c$ corrections which break large-$N_c$ baryon spin-flavor symmetry can be classified in terms of the spin and flavor symmetries which remain for finite $N_c$. Since explicit $SU(3)$ flavor symmetry breaking $\epsilon \sim m_s/\Lambda_{QCD}$ is comparable to $1/N_c = 1/3$ for QCD baryons, a combined expansion in $1/N_c$ and $\epsilon$ is necessary to explain the symmetry-breaking pattern. The $1/N_c$ expansion has had considerable phenomenological success for $qqq$ baryons; $1/N_c$ suppression factors clearly are present in experimental data.

For baryons containing a single heavy quark $Q$ in HQET, there is spin-flavor symmetry in the large-$N_c$ limit as well as in the heavy quark limit $\overset{\rightarrow}{Q}$. The spin-flavor symmetry of heavy quark baryons with one heavy quark flavor contains a $SU(6)_\ell$ symmetry of the light degrees of freedom in the $N_c \rightarrow \infty$ limit, and a $SU(2)_Q$ spin symmetry of the heavy quark in both the $m_Q \rightarrow \infty$ and $N_c \rightarrow \infty$ limits. The spin-flavor symmetry of the light degrees of freedom is broken by corrections suppressed by factors of $1/N_c$ and $SU(3)$ flavor symmetry breaking, whereas the heavy-quark spin symmetry is broken by terms suppressed by factors of $1/N_c$ and heavy quark symmetry breaking $\delta_Q \sim \Lambda_{QCD}/m_Q$. Note that $SU(3)$ flavor symmetry and heavy quark spin symmetry are better symmetries for baryons than for mesons because violation of spin-flavor symmetry is suppressed by additional factors of $1/N_c$ for baryons.

Heavy quark symmetry for two heavy quark flavors $Q = c$ and $Q = b$ in HQET generalizes to heavy-quark spin-flavor symmetry $SU(4)_Q$, which relates the heavy quark spin-flavor properties of charm and bottom hadrons. Again, heavy quark spin-flavor symmetry is a better symmetry for baryons than for mesons because heavy quark spin-flavor symmetry violation is accompanied by factors of $1/N_c$.

The lowest-lying spin-flavor representation for $Qqq$ baryons consists of a completely symmetric spin-flavor representation of two light quarks combined with a single heavy quark with $J_Q = \frac{1}{2}$. Under light-quark spin and flavor, this representation decomposes into a $J_\ell = 0$ flavor $\overset{\rightarrow}{3}$, which consists of the isosinglet $\Lambda_Q(Qud)$ and the isodoublet $\Xi_Q(Qsq)$ with $J = J_\ell + J_Q = \frac{1}{2}$, and a $J_\ell = 1$ flavor $\overset{\rightarrow}{6}$, which consists of $\Sigma_Q, \Xi'_Q, \Omega_Q$ with $J = \frac{1}{2}$ and $\Sigma_Q^*, \Xi_Q^*, \Omega_Q^*$ with $J = \frac{3}{2}$.

The mass hierarchy of the lowest-lying charm and bottom baryon masses was predicted in a combined expansion in $1/N_c$, $\epsilon$ and $\delta_Q$ in Ref. \[1\]. Here, the theoretical hierarchy is compared with experiment for charm baryon masses, and the predicted pattern is seen. Bottom baryons are predicted to obey the same mass hierarchy with $1/m_c$ replaced by $1/m_b$.  

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2. $1/N_c$ Expansion

In the $1/m_Q$ expansion of HQET, the mass of a hadron containing a single heavy quark is given by

$$M(H_Q) = m_Q + \bar{\Lambda} - \frac{\lambda_1}{2m_Q} - d_H \frac{\lambda_2}{2m_Q} + \cdots,$$

(1)

where

$$\lambda_1 = \langle H_Q(v)\bar{Q}v(i\mathcal{D})^2 Qv|H_Q(v)\rangle,$$

(2)

denotes the matrix elements of the $1/m_Q$-suppressed operators in the heavy hadron, $d_H = -4(J_t \cdot J_Q)$, and corrections of order $1/m_Q^2$ have been neglected. Eq. (1) can be applied to mesons and baryons, but the values of $\bar{\Lambda}$, $\lambda_1$ and $\lambda_2$ will be different in the two cases.

The spin-$\frac{1}{2}$ $\Lambda$, and spin-$\frac{3}{2}$ and $\frac{5}{2}$ baryons have masses given by the $1/m_Q$ expansions

$$T_Q = m_Q + \bar{\Lambda}_T - \frac{\lambda_1 \ell}{2m_Q} + \cdots,$$

$$S_Q = m_Q + \bar{\Lambda}_S - \frac{\lambda_1 S}{2m_Q} - 2\frac{\lambda_2 S}{m_Q} + \cdots,$$

(3)

$$S_Q^* = m_Q + \bar{\Lambda}_S - \frac{\lambda_1 S}{2m_Q} + \frac{\lambda_2 S}{m_Q} + \cdots,$$

respectively. Large-$N_c$ spin-flavor symmetry implies that the hadronic matrix elements of these baryons are equal in the $N_c \to \infty$ limit. For finite $N_c$, the matrix elements have expansions in terms of $1/N_c$ operators given by

$$\bar{\Lambda} = N_c \mathbf{1} + \frac{J_Q^2}{N_c},$$

$$\bar{\Lambda}_T = \left( \frac{1}{m_Q} + \frac{1}{N_c^2} \right) N_Q J_T^2,$$

(4)

$$\bar{\Lambda}_S = \left( \frac{1}{m_Q} - \frac{1}{N_c} \right) \frac{1}{m_Q} (J_t \cdot J_Q),$$

where $N_Q$ is the heavy quark number operator which is equal to 1 for baryons containing a single heavy quark, $\mathbf{1}$ is the unit operator for baryons, and $J_T^2$ and $J_Q^2$ are the spins of the light degrees of freedom and the heavy quark, respectively. In Eq. (4), it is to be understood that each $1/N_c$ operator is accompanied by an unknown, dimensionful, $\mathcal{O}(1)$ coefficient, which has been suppressed for simplicity. Eq. (4) makes a number of interesting predictions. For instance, $\bar{\Lambda}_T$ and $\bar{\Lambda}_S$ are equal at leading order $N_c$ in the $1/N_c$ expansion. However, at order $1/N_c$, the two matrix elements are not equal, but are split by a contribution which is order $1/N_c^2$ relative to the leading $\mathcal{O}(N_c)$ term. Similar remarks apply for the $\lambda_1$ matrix elements.

The generalization of Eq. (4) to include $SU(3)$ flavor symmetry and its breaking is provided in Ref. [1]. At the time of this work, the $\Xi'_c$ mass had not been measured, and the $1/N_c$ analysis was used to successfully predict $\Xi'_c = 2580.8 \pm 2.1$ MeV [2], to be compared with the subsequent experimental value $\Xi'_c = 2576.5 \pm 2.3$ MeV. This theoretical prediction and its precision required the $1/N_c$ expansion.

Today the masses of all singly charmed baryons are measured except for the spin-$\frac{5}{2}$ $\Omega_c^*$. The most suppressed mass combination

$$\frac{1}{4} \left[ (\Sigma_c^* - \Sigma_c) - 2(\Xi_c' - \Xi_c') + (\Omega_c^* - \Omega_c) \right],$$

(5)

which is suppressed by $\delta \xi / N_c^3$ relative to the $\mathcal{O}(N_c)$ baryon mass, can be used to extract $\Omega_c^* = 2770.7 \pm 5.9$ MeV. Thus, it is possible to evaluate the charm baryon mass hierarchy and compare with theory. For bottom baryons, only the $\Lambda_b^0$ mass is measured. Using heavy quark spin-flavor symmetry, it is possible to predict all of the other bottom baryon masses in terms of the charm baryon masses [3].

Figure 1 plots seven of the eight independent mass combinations of the lowest-lying charm baryon spin-flavor multiplet. [The mass splitting Eq. (4) is not plotted since it was used to determine the $\Omega_c^*$ mass.] The first mass combination is the leading order $N_c \Lambda + m_Q$ baryon mass where $Q = c$. The remaining mass splittings are order $N_c$ times $\frac{1}{N_c}$, $\frac{\epsilon}{N_c}$, $\frac{\epsilon^2}{N_c}$, $\frac{\delta}{N_c}$, and $\frac{\delta Q}{N_c^2}$, respectively. Fig. 1 shows that the $1/N_c$ splitting is comparable to the $\epsilon$ splitting, and that the $\delta Q/N_c$ splitting is a bit larger than the $\epsilon/N_c$ splitting for $Q = c$. The most suppressed splittings are consistent with the predicted $\frac{\epsilon^2}{N_c}$ and $\frac{\delta Q}{N_c^2}$ hierarchy as well.

It is possible to determine the $1/m_Q$-dependent contributions to the charm baryon mass splittings which do not violate heavy quark spin symme-
try by comparison with the analogous mass splittings for baryons containing no heavy quark. The $1/N_c$ expansions of $qqq$ and $Qqq$ baryon masses are given in the flavor symmetry limit by

$$M(qqq) = N_c 1 + \frac{1}{N_c} J^2,$$

$$M(Qqq) = N_c 1 + N_Q m_Q + \frac{1}{N_c} J^2 + \frac{1}{N_c^2} m_Q J^2,$$

which shows that the $N_Q$ and the $N_Q J^2$ splittings can be extracted by making this comparison.

Figure 2 plots the $1/m_Q$-dependent portion of the first five mass splittings of Fig. 1 together with the two heavy quark spin-violating mass splittings (points 6 and 7) of Fig. 1. The first mass (point 1) yields $m_Q$ or the charm quark mass at leading order in $1/m_Q$, whereas the six other mass splittings are order $\Lambda$ times the the dimensionless suppression factors $\delta_Q N_c^2$, $\epsilon Q Q N_c$, $\epsilon Q N_c J^2$, $\epsilon Q Q J^2$, $\delta_Q N_c$, and $\delta Q N_c$, respectively. It is interesting to note, for example, that the two $\epsilon Q Q$ splittings (points 4 and 7) are in good agreement, showing that the same heavy quark symmetry-violating parameter $\delta Q$ is governing heavy quark spin-conserving and spin-violating mass splittings.

3. CONCLUSIONS

The $1/N_c$ hierarchy of the $1/N_c$ expansion is evident in the masses of $Qqq$ baryons as well as $qqq$ baryons. The $1/N_c$ expansion, together with $SU(3)$ flavor violation and heavy-quark symmetry violation, gives a quantitative understanding of spin-flavor symmetry breaking for heavy quark baryons. An intricate pattern of spin-flavor symmetry breaking is predicted since $1/N_c$, $\epsilon$, and $\delta_Q$ for $Q = c$ are comparable in magnitude. The same $1/N_c$ hierarchy is expected to appear for bottom baryon masses, and it is possible to predict the bottom baryon mass splittings in terms of charm baryon mass splittings. Heavy quark spin-flavor symmetry is a better symmetry for $Qqq$ baryons than for heavy quark mesons because violation of the spin-flavor symmetry is suppressed by factors of $1/N_c$ as well as $1/m_Q$.

REFERENCES

1. E. Jenkins, Phys. Rev. D 54 (1996) 4515.
2. E. Jenkins, Phys. Rev. D 55 (1997) R10.