Fertilizer Transportation Problem Using Vogel Approximation Method

R H Kankarofi¹, U Ayakubu¹, I M Sulaiman², M Mamat², Sukono³, and M P A Saputra⁴

¹Department of Mathematics, Yusuf Maitama Sule University, Kano, Nigeria
²Faculty of Informatics and Computing, Universiti Sultan Zainal Abidin, Campus Kuala Nerus, 22200, Terengganu, Malaysia.
³Department of Mathematics, Faculty of Mathematics and Natural Science, Universitas Padjadjaran, Indonesia
⁴Master Program in Mathematics, Faculty of Mathematics and Natural Science, Universitas Padjadjaran, Indonesia

Corresponding author: usman.abbas84@yahoo.com

Abstract. This research work was designed to optimize the processes of transporting fertilizer from Kano State Agricultural Supply Company, KASCO, Maiduguri Road, Kano, also implement the distribution route that lead to the reduction of operating cost of transportation problems that usually occurs due to the nature of bad roads to some selected local government in Kano state Nigeria. To minimize the cost of shipping fertilizer from the company to the destinations a mathematical optimization model (Vogel Approximation method) has been used to ease the cost and the distribution of fertilizer to the various locations.

1. Introduction

The transportation problem was first formulated and developed by [6], and solved optimally for complex business problem. The transportation problem usually appears in a transportation tableau and often described by means of Linear Mathematical Programming model. By utilizing the classical methods such as Least-Cost method, Simplex method or North-West corner method, researchers are optimistic that the initial basic feasible solution of any transportation problem can be obtained [8]. Finally, in this research the optimality of the given transportation problem is verified by Vogel Approximation method.

A transportation problem includes $m$ sources, in which each of the source contain available $S_i$ where $i = 1, 2, 3, 4, \ldots, m$ unit of a homogeneous products with $n$ dimensions, each of which requires $d_j$ where $j = 1, 2, 3, 4, \ldots, n$ units for some positive integers $S_i$ and $d_j$. The cost $C_{ij}$ used to transport a unit of product to the $j^{th}$ destination from the source $i^{th}$ is given for every $i$ and $j$. The main aim is developing an integral transportation plan that would satisfy all demands at minimum total shipping cost which is considered from the current inventory. It is supposed that total demand and total supply are equal, i.e.

$$\sum_{j=1}^{n} d_j = \sum_{i=1}^{m} S_i$$  (1)

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Equation (1) is certain by forming either a false destination with a demand equivalent to the supply, if the overall demand is less than total supply, or a fictitious source with a supply equivalent to the shortage, or with the whole demand exceeding the total supply. The problem of transportation often arises when planning for the distribution of goods and services to different destinations which is originating from different supply locations.

2. Method for transportation problems

The mathematical models have been discovered to minimize the cost of transportation. This can be seen as a special case of the minimum cost flow problem. [2] said that the application of the transportation problem tends to involve a very large number of variable and constraints, thus, a straightforward simplex method computer application may need an excessive computation effort, for instance, George B. Dantzing (1951) solved the transportation model using the idea of Linear Programming [1,6]. Beamo [1] presented the simplex method standard for solving linear programming formulation of transportation problem. Since then the problem has the classical common literature in most text. This demonstration is regards as among the first significant contribution for solving the transportation problem. Hakim [9], studied independently, which was not related to Koopmans (1947), and referred to it as "the transportation system optimum utilization". These contributions aided in developing the methods of transportation methods involving a number of number destinations and a number of shipping source. Each unit transportation cost function is a linear in nature. Frank [12] constructed an algorithm and use it to reach the transportation problem optimal solution for the convex case. Fortunately, a key characteristic is most of the $X_{ij}$ coefficient in the constraints are zeros and relatively few none-zero coefficients give the idea in a distinctive pattern [2, 3, 4, 5, 7, 10, 11, 13].

The general model for the transportation problem from source to the destinations is to minimize the total cost, i.e. sources $i = 1,2,3,...,m$ has demanded for $d_j$ being received from the source $i$ to destination $j$ this process is directly proportional to the number distributed, with $C_{ij}$ denoting the per unit distribution cost, and letter $Z$ is the cost of total distribution and $X_{ij}$ for $i = 1,2,3,4,...,m$, and $j = 1,2,3,4,...,n$, represent the units number to be distributed to destination $j$ from source $i$, the problem of linear programming is as follows:

$$\text{minimize } \sum_{j=1}^{n} \sum_{i=1}^{m} C_{ij} X_{ij}$$

$$\text{subject to } \sum_{j=1}^{n} X_{ij} = S_i \quad \text{for } j = 1,2,3,...,m$$

$$\text{and } \sum_{i=1}^{m} X_{ij} = d_j \quad \text{for } i = 1,2,3,...,n$$

for all $X_{ij}$ non-negative, for all $i$ and $j$.

2.1. Northwest Corner Method

Commencing by the transportation tableau upper left corner and the set $X_{11}$ being as large as possible, then, obviously, $X_{11}$ can no more be larger than the smaller of $S_i$ and $d_j$.

- If $X_{11} = S_1$, then, crossing out the tableau first row is required. More, there is need for changing $d_1$ to $d_1 - S_1$.
- If $X_{11} = d_1$, then, crossing out the tableau first column is required. Also, there is need for changing $S_1$ to $S_1 - d_1$.
- If $X_{11} = S_1 = d_1$, then, crossing out either column 1 or row 1 but not both of them is needed.
  - By crossing out row, then, change $d_1$ to 0.
  - By crossing out the column, then, you change $S_1$ to 0.
This procedure is continuously applied to the most tableau Northwest cells not laying in a crossed-out column or row. Ultimately, a continuous process of this procedure will arrive at certain point with only one cell which can be assigned a value. Assigning the last cell, a value equivalent to its column or row demand, and crossing-out these cell’s column or row, produce a basic formulating solution.

2.2. Simplex Method

This method was summarized by the following steps:

- If the have an unbalance problem, then, balance the problem first.
- Employ any of the existing methods to find a basic formulating solution.
- For all basic variables, apply the fact that \( U_i = 0 \) and \( U_i + V_j = C_{ij} \) to obtain the \( U^S \) and \( V^S \) for current basic formulation solution.
- Suppose \( U_i + V_j - C_{ij} \leq 0 \) \( \forall \) non-basic variables, then the current basic formulation solution is optimal. If this is not the case, then enter the variable with the most positive \( U_i + V_j - C_{ij} \) into the basic using the pivoting procedure. This yield new basic formulating solution, then return to step three for a maximization problem. Continue as specified, but replacing the fourth step by the step that follows.
- If \( U_i + V_j - C_{ij} \geq 0, \forall \) nonbasic variables, then the present basic formulating solution is optimal, else, input the variable with negative \( U_i + V_j - C_{ij} \) into the base sign of the pivoting procedure. This yield a new basic formulating solution. Then return to step three.

2.3. Vogel Approximation Method

Calculating each column and row, a penalty equivalent to difference between the two least costs in column and row. Next is finding the column or row having the major penalty, select as the first basic variable, i.e. the variable in this column or row that has the least costs. For the method of Northwest corner, there is need to make this variable as large as possibly, and also crossing out column or row, change the demand or supply associated with the basic variable. Now re-compute new penalties and repeating the procedure continuously until only one uncrossed cell remains. Set this variable equal to the supply or do demand associates with the variable and cross out the variable's row and column [10].

3. Data Analysis

The data was collected from the marketing department of Kano State Agricultural and Supply Company (KASCO) Maiduguri road Kano. The request on the fertilizer to various destinations are always very high nowadays due to closure of Nigerian land border. This research will focus on six (6) local government area in Kano state which include Kano Municipal, Kura, Tudun Wada, Madobi, Gaya and Wudil). The information obtained are summarized and presented in table 2 and table 3 below. The general Transportation problem is detailed using the information that follows:

- A set of \( m \) supply points from which a good is shipped. Supply point \( i \) can supply at most \( S_i \) units.
- A set of \( n \) demand points to which the good is shipped. Demand point \( j \) must receive at least \( d_j \) units of the shipped good.
- Each unit product at supply point \( i \) and shipped to demand point \( j \) gives available cost of \( C_{ij} \).

The relevant data can be formulated in a transportation in Table 1.

If total supply equals to the total demand, then the problem is said to be a balanced transportation problem. Let \( X_{ij} \) be a number of units shipped from supply point \( i \) to demand point \( j \) then the general programming representation of a transportation problem is
\[
\text{minimize } = \sum_{j=1}^{n} \sum_{i=1}^{m} c_{ij} x_{ij}
\]

such that \( \sum_{j} x_{ij} \leq S_i \) (\( i = 1, 2, 3, ... , m \)) \( \rightarrow \) supply constraints

and \( \sum_{i} x_{ij} \geq d_j \) (\( j = 1, 2, 3, ... , n \)) \( \rightarrow \) demand constraints

where \( x_{ij} \geq 0 \)

### Table 1: Transportation Tableau

| Demand | Demand 2 | ... | Demand n |
|--------|----------|-----|----------|
| S_1    | C_{11}   | ... | C_{1n}   |
| S_2    | C_{21}   | ... | C_{2n}   |
| S_3    | ...      | ... | ...      |
| S_m    | C_{m1}   | ... | C_{mn}   |

### 3.1. Data presentation

Total bags of fertilizer supplied from the KASCO company and total bags demanded for the respective local government.

### Table 2 Fertilizer demand by local governments

| Unit | Municipal | Kura | Wudil | Madobi | Tudun Wada | Gaya | Total Supply |
|------|-----------|------|-------|--------|------------|------|--------------|
| 1    | 20,100    | 16,700 | 15,000 | 11,700 | 14,800     | 11,000 | 89,300      |
| 2    | 19,300    | 16,000 | 14,000 | 11,200 | 13,800     | 10,100 | 84,400      |
| 3    | 19,300    | 16,100 | 13,600 | 11,400 | 13,700     | 10,300 | 84,400      |
| 4    | 19,000    | 16,100 | 14,100 | 11,400 | 13,900     | 10,400 | 85,400      |
| 5    | 19,700    | 16,000 | 14,600 | 11,000 | 14,600     | 10,500 | 86,400      |
| **Total Demand** | **97,400** | **80,900** | **71,300** | **57,200** | **70,800** | **52,300** |               |

### Table 3. Cost of transportation per bag in naira (₦)

| Unit | Municipal | Kura | Wudil | Madobi | Tudun Wada | Gaya |
|------|-----------|------|-------|--------|------------|------|
| 1    | 20        | 30   | 39    | 31     | 50         | 51   |
| 2    | 25        | 36   | 45    | 35     | 55         | 56   |
| 3    | 26        | 35   | 47    | 36     | 54         | 54   |
| 4    | 21        | 28   | 39    | 29     | 51         | 50   |
| 5    | 18        | 31   | 40    | 29     | 50         | 49   |
The Method of generating optimal solution considered in this research work is Vogel approximation method. Considering Table 4

### Table 4. Destination Table 1

|   | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  |
|---|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1 | $C_{11}$ | $C_{12}$ | $C_{13}$ | $C_{14}$ | ... | $C_{1n}$ | $S_1$ | $U_1$ |
| 2 | $X_{12}$ | $X_{13}$ | $X_{14}$ | $X_{15}$ | ... | $X_{1n}$ |       |     |
| 3 | $C_{21}$ | $C_{22}$ | $C_{23}$ | $C_{24}$ | ... | $C_{2n}$ | $S_2$ | $U_2$ |
| 4 | $X_{21}$ | $X_{22}$ | $X_{23}$ | $X_{24}$ | ... | $X_{2n}$ |       |     |
| 5 | $C_{31}$ | $C_{32}$ | $C_{33}$ | $C_{34}$ | ... | $C_{3n}$ | $S_3$ | $U_3$ |
| 6 | $X_{31}$ | $X_{32}$ | $X_{33}$ | $X_{34}$ | ... | $X_{3n}$ |       |     |
| 7 | $C_{m1}$ | $C_{m2}$ | $C_{m3}$ | $C_{m4}$ | ... | $C_{mn}$ | $S_m$ | $U_m$ |
| 8 | $X_{m1}$ | $X_{m2}$ | $X_{m3}$ | $X_{m4}$ | ... | $X_{mn}$ |       |     |
| 9 | $V_1$  | $V_2$  | $V_3$  | $V_4$  | ... | $V_n$   |       |     |

where,
- $C_{ij}$ is the cost of transporting a unit of the products from $i^{th}$ origin to $j^{th}$ destination.
- $X_{ij}$ represents the unknown number of units to be shipped from source $i$ to destination $j$.
- $U_i$ is the quantity of the commodity available at source $i$.
- $V_j$ is the quantity of the commodity needed at destination $j$.
- Basic variables can be found by using $C_{ij} = U_i + V_j$ and non-basic variable by $X_{ij} = C_{ij} - (U_i + V_j)$

Therefore, the optimal solution $Z$ can be calculated using

$$
\text{minimize } Z = \sum_{j=1}^{n} \sum_{i=1}^{m} C_{ij} X_{ij}
$$

where, $X_{11} = 11,000$, $X_{13} = 71,300$, $X_{15} = 7,000$, $X_{24} = 48,200$, $X_{25} = 36,200$, $X_{35} = 27,600$, $X_{36} = 52,300$, $X_{42} = 80,900$, $X_{44} = 4,500$, $X_{51} = 86,400$

$$
Z = C_{11}X_{11} + C_{13}X_{13} + C_{15}X_{15} + C_{24}X_{24} + C_{25}X_{25} + C_{35}X_{35} + C_{36}X_{36} + C_{42}X_{42} + C_{44}X_{44} + C_{51}X_{51}
$$

$$
= 20(11,000) + 39(71,300) + 50(7,000) + 35(52,700) + 55(31,700) + 54(32,100) + 54(52,300) + 28(80,900) + 29(4,500) + 18(86,400)
$$

$$
= 220,000 + 2,780,700 + 350,000 + 1,844,500 + 1,743,500 + 1,733,400 + 2,824,200 + 2,265,200 + 2,265,200 + 130,500
$$

$$
Z = 15,447,200
$$

(3) updating the result in naira we have ₦15,447,200.
Table 5. Destination Table 2

|       | 1  | 2  | 3  | 4  | 5  | 6  | Supply |
|-------|----|----|----|----|----|----|--------|
| 1     | 20 | 30 | 39 | 31 | 50 | 51 | 89,300 |
| 2     | 25 | 36 | 45 | 35 | 55 | 56 | 84,400 |
| 3     | 26 | 35 | 47 | 36 | 54 | 54 | 84,400 |
| 4     | 21 | 28 | 39 | 29 | 51 | 50 | 85,400 |
| 5     | 18 | 31 | 40 | 29 | 50 | 49 | 86,400 |
| Demand| 97,400 | 80,900 | 71,300 | 57,200 | 70,800 | 52,300 | D=S $429,900 |
| $V_j$ | $V_1$ | $V_2$ | $V_3$ | $V_4$ | $V_5$ | $V_6$ |

where $1 \leq i \leq 5$ is the number of row and $1 \leq j \leq 6$ the number of column. Therefore, the total demand is equal to the total supply i.e. $D = S = 429,900$.

Table 6. Row Penalty

| ROW   | PEN 1 | PEN 2 | PEN 3 | PEN 4 | PEN 5 | PEN 6 | PEN 7 | PEN 8 | PEN 9 | PEN 10 |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|--------|
| ROW 1 | 10    | 1     | 8     | 1     | -     | -     | -     | -     | -     | -      |
| ROW 2 | 10    | 10    | 1     | 10    | 1     | 1     | 1     | 1     | 55    |        |
| ROW 3 | 9     | 9     | 1     | 1     | 7     | 0     | 0     | 0     | -     | -      |
| ROW 4 | 7     | 7     | 1     | 11    | 1     | 1     | -     | -     | -     | -      |
| ROW 5 | 1     | -     | -     | -     | -     | -     | -     | -     | -     | -      |

Table 7. Column Penalty

|       | COL 1 | COL 2 | COL 3 | COL 4 | COL 5 | COL 6 |
|-------|-------|-------|-------|-------|-------|-------|
| PEN 1 | 2     | 2     | 0     | 0     | 1     | 1     |
| PEN 2 | 1     | 2     | 0     | 2     | 1     | 1     |
| PEN 3 | -     | (2)   | 0     | 2     | 1     | 1     |
| PEN 4 | -     | -     | 0     | 2     | 1     | 1     |
| PEN 5 | -     | -     | 0     | -     | 1     | 1     |
| PEN 6 | -     | -     | -     | -     | 1     | 1     |
| PEN 7 | -     | -     | -     | -     | 3     | (4)   |
| PEN 8 | -     | -     | -     | -     | 1     | (2)   |
| PEN 9 | -     | -     | -     | -     | 55    | 66    |
| PEN10 | -     | -     | -     | -     | 55    | -     |

where, PEN=penalty and COL=column. From table 5 above we have
• Row 5 has the highest penalty and 18 is the smallest cost, therefore we assign 86,400 to it and cancel row 5.
• Row 1 has the highest penalty also 20 is the smallest cost, so, we assign 11,000 and cancel column 1, since it is balanced.
• Column 2 has the highest penalty and 28 is the smallest cost, therefore, we assign 80,900 and cancel column 2.
• Row 3 has the highest penalty and 36 is the smallest cost, therefore, we assign 57,200 and cancel column 4.
• Row 1 has the highest penalty and 39 is the smallest cost, therefore, we assign 71,300 and cancel column 3.
• Row 1 has the highest penalty and 50 is the smallest cost, again we assign 7,000 and cancel row 1.
• Column 6 has the highest penalty and 50 is the smallest cost, now we have to assign 4,500 and cancel row 4.
• Column 6 has the highest penalty and 54 is the smallest cost. Therefore, put 27,200 and cancel row 3.
• Column 6 has the highest penalty and assigns 20,600 to 56 cells since it is the only remaining cell in that column, then cancel column 6.
• The only remaining cell is 55, so, we assign 63,800 and cancel both row 2 and column 5.

4. Result and Discussion

In this section, we clearly discuss the cost of transportation of fertilizer from the KASCO company to the required destinations. Considering the table 4 Destination table 1 after assigned the values, it indicated that the fertilizer distribution profile is as follows:

• 11,000 bags of fertilizer from unit (1) should be sent to Municipal, 71,300 bags should be sent to Wudil and 7,000 bags should be sent to Tudun Wada while no any bag of fertilizer should be sent to Kura, Madobi, or Gaya local government.
• From unit (2) 52,700 bags should be sent to Madobi, 31,700 bags to Tudun Wada, and no any bag should be sent to Municipal, Kura, Wudil and Gaya local government.
• In the unit (3) 32,100 bags of fertilizer should be sent to Tudun Wada, 52,300 to Gaya, but no any bag to Municipal, Kura, Wudil and Madobi local government.
• From unit (4) 80,900 bags should be sent to Kura, 4,500 bags should be sent to Madobi, and no any bag to Municipal, Gaya, Tudun Wada and Wudil local government.
• Unit (5) should send 86,400 bags of fertilizer to Municipal but no any bag to the remaining local governments.

Therefore, from table 2 and table 3. Initially, the cost of transportation was as follows:

\[
\text{Initial Cost} = (20,100 \times 20) + (16,700 \times 30) + (15,000 \times 39) + (11,700 \times 31) + (14,800 \times 50) + (11,000 \times 51) + (19,300 \times 25) + (16,000 \times 36) + (14,000 \times 45) + (11,200 \times 35) + (13,800 \times 55) + (10,100 \times 56) + (19,300 \times 26) + (16,100 \times 35) + (13,600 \times 47) + (11,400 \times 36) + (13,700 \times 54) + (10,300 \times 54) + (19,000 \times 21) + (16,100 \times 28) + (14,100 \times 39) + (11,900 \times 29) + (13,900 \times 51) + (10,400 \times 50) + (19,700 \times 18) + (16,000 \times 31) + (14,600 \times 40) + (11,000 \times 29) + (14,600 \times 50) + (10,500 \times 49)
\]
\[
= 15,939,500
\]

The cost of transportation was amount to ₦15,939,500. Nevertheless, the Vogel approximation method generally gives a better initial solution. Now, by comparing the total cost of transportation from the optimal solution equation (3) and the initial cost of transportation equation (4) we observe that there is discount of ₦492,300. Table 8 shows the number of bags of fertilizer demanded from
selected local government in Kano state and the number of bags that should be sent from the KASCO Company after using Vogel approximation method of transportation.

Table 8. Demand before and after

| Local Government | Number of Bags Demanded                                      | Number of Bags Supply                                      | Total   |
|------------------|-------------------------------------------------------------|------------------------------------------------------------|---------|
| Municipal        | 20,100 from unit (1), 19,300 from unit (2), 19,300 from unit (3), 19,000 from unit (4) and 19,700 from unit (5). | 11,000 from unit (1) and 86,400 from unit (5)               | 97,400  |
| Kura             | 16,700 from unit (1), 16,000 from unit (2), 16,100 from unit (3), 16,100 from unit (4) and 16,000 from unit (5).    | 80,900 from unit (4)                                       | 80,900  |
| Wudil            | 15,000 from unit (1), 14,000 from unit (2), 13,600 from unit (3), 14,100 from unit (4) and 14,600 from unit (5).       | 71,300 from unit (1)                                       | 71,300  |
| Madobi           | 11,700 from unit (1), 11,200 from unit (2), 11,400 from unit (3), 11,900 from unit (4) and 11,000 from unit (5).       | 52,700 from unit (2) and 4,500 from unit (4)               | 57,200  |
| Tudun Wada       | 14,800 from unit (1), 13,800 from unit (2), 13,700 from unit (3), 13,900 from unit (4) and 614,600 from unit (5).        | 7,000 from unit (1), 31,700 from unit (2) and 32,100 from unit (3). | 70,800  |
| Gaya             | 11,000 from unit (1), 10,100 from unit (2), 10,300 from unit (3), 10,400 from unit (4) and 10,500 from unit (5).         | 52,300 from unit (3)                                       | 52,300  |

Nonetheless each and every local government get the same number of bags demanded from the company, but only units have been changed for some local government in order to reduce the cost of transportation. Municipal demand 97,400 bags of fertilizer which will cost the company ₦2,139,900 but after using this method their demand change to 11,000 bags from unit (1) and 86,400 from unit (5) which cost ₦1,775,200. Kura demand 80,900 bags which will cost ₦2,587,300 if their demand has been change to 80,900 bags from unit (4) then the price will have reduced to ₦2,265,200. Wudil demand 71,300 which will cost ₦2,988,100, now their demand changes to 71,300 from unit (1) which cost ₦2,780,700. Madobi demand 57,200 bags which will cost ₦1,829,200 but after using Vogel approximation method their demand changed to 52,700 bags from unit (2) and 4,500 bags from unit (4) which cost ₦1,975,000. Tudun Wada demanded 70,800 bags which will cost ₦3,677,700 but if their demand has been change to 7,000 bags from unit (1), 31,700 bags from unit (2) and 2,100 bags from unit (3) the transportation cost was increased to ₦3,826,900.

Lastly, Gaya local government demanded 52,300 bags which will cost ₦2,717,600 but if their demand change to 52,300 bags from unit 3 then the cost of transportation becomes ₦2,824,200.

The Table 9 shows the cost of transportation of fertilizer demanded before and after using Vogel’s method.

Now we observed that in order to minimize the cost of transportation and distribution of fertilizer by using Vogel approximation method, we have to change some request and increase others without reducing the demand of each and every local government.
Table 9: Cost before and after

| Local Government | Transportation Cost Before | Transportation Cost After |
|------------------|-----------------------------|---------------------------|
| Municipal        | ₦2,139,900                 | ₦1,775,200               |
| Kura             | ₦2,587,300                 | ₦2,265,200               |
| Wudil            | ₦2,988,100                 | ₦2,780,700               |
| Madobi           | ₦1,829,200                 | ₦1,975,000               |
| Tudun Wada       | ₦3,677,700                 | ₦3,826,900               |
| Gaya             | ₦2,717,300                 | ₦2,824,200               |
| **Total**        | **₦15,939,500**            | **₦15,447,200**          |

5. Conclusion

This research was carried out in order to obtain the minimum cost of transporting fertilizer from source to the destinations. The KASCO Company was able to know how much they have to spend to supply their customers based on the cost of transportation to minimize the total cost of transport. The Vogel approximation method was utilized and found to be efficient and it is determined towards profitability of the company.

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