Topological Defects in a Deformed Gauge Theory

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Abstract

In this paper, we will analyse the topological defects in a deformation of a non-abelian gauge theory using the Polyakov variables. The gauge theory will be deformed by the existence of a minimum measurable length scale in the background spacetime. We will construct the Polyakov loops for this deformed non-abelian gauge theory, and use these deformed loop space variables for obtaining a deformed loop space curvature. It will be demonstrated that this curvature will vanish if the deformed Bianchi identities are satisfied. However, it is possible that the original Bianchi identities are satisfied, but the deformed Bianchi identities are violated at the leading order in the deformation parameter, due to some topological defects. Thus, topological defects could be produced purely from a deformation of the background geometry.

1 Introduction

Topological defects can be analysed using the Polyakov variables, and these Polyakov variables are defined using the the holonomies of the gauge fields \cite{1, 2, 3, 4}. In this paper, we will call these holonomies of the gauge fields as Polyakov loops, as they were introduced by Polyakov \cite{1}. So, these Polyakov loops would be constructed using the gauge fields as the holonomies of closed loops in spacetime. They are also called as the Dirac phase factors in the physics literature. They do not depend on the parameterization chosen, and they capture some interesting topological properties of the gauge theory. In fact, they resemble the Wilson’s loops, but unlike the Wilson loops, no trace
is taken over the gauge group for Polyakov loops. Thus, in this paper, there is a difference between these Polyakov loops, and the usual Wilson’s loops. This is because Wilson’s loops only represented by a number, but Polyakov loops are gauge group-valued functions of the infinite-dimensional loop space \([1]\). So, they can be used to analyze various interesting structures in the gauge theory, and this includes topological defects produced by the existence of non-abelian monopoles. It may be noted that recently Polyakov loops have been used for analyzing various interesting physical systems including fractional M2-branes \([5]\). They have also been used for analyzing three dimensional supersymmetric gauge theory \([6]\), and these theories are important to study systems like M2-branes and D2-branes. The four dimensional supersymmetric gauge theories have also been analyzed using this formalism \([7]\). In fact, this formalism was used to analyze the non-abelian monopoles in four dimensional supersymmetric gauge theories. Thus, it is possible to use this formalism to analyzing various interesting generalizations of the usual gauge theories. So, in this paper, we will analyze the effect of topological defects on a deformed non-abelian gauge theory. This gauge theory will be deformed by the existence of a minimum measurable length scale in the background geometry.

Such a deformation of the gauge theories by the existence of a minimum measurable length scale in the background geometry is in turn motivated from low energy effects of quantum gravity. This is because almost all the approaches to quantum gravity restrict the measurement of spacetime below the Planck scale. The string theory is one of the most important approaches for analyzing quantum gravity, and the fundamental string is the smallest probe available in perturbative string theory, and so it is not possible to probe spacetime below the string length scale in string theory \([8,9,10,11]\). Thus, the string length acts, which is given by \(l_s = \alpha'\) as a minimum measurable length in string theory. Furthermore, if non-perturbative effects are taken into consideration, then it is possible to have \(D0\)-brane which is a point like object. However, it has been argued that even in presence of such brane, there is an intrinsic minimal length of the order of \(l_{min} = l_s g_s^{1/3}\), where \(g_s\) is the string coupling constant \([13,14]\). The total energy of the quantized string depends on the excitation \(n\) and winding number \(w\), and under T-duality the \(n\) and \(w\) gets interchanged, as \(R \rightarrow l_s^2 / R\) and \(n \rightarrow w\). So, a description of string theory below \(l_s\) is the same as the description above it, and so it can be argued from T-duality that the string theory cannot be described below the string length scale \([13]\). The T-duality has been used to construct an effective path integral for the center of mass of the string, and analyze the corresponding Green’s function \([15,16]\). This has been done by analyzing strings propagating in spacetime with compactified additional dimensions. It has been demonstrated that this Green’s function also has a minimal length associated with it \([15,16]\). So, string theory due to T-duality has a minimal length associated with it. It may be noted that this minimal length can be different from Planck length \([13]\). This is because the the Planck length \(l_{PL}\) can be expressed as \(l_{PL} = g_s^{1/4} l_s\) \([13]\). It has also been argued that a minimal length may exist in models of quantum gravity, such as the loop quantum gravity \([17]\). The physics of black holes restricts the measurement to scales larger than the Planck scale. This is because the energy needed to probe a region of spacetime below Planck scale is greater than the energy needed to form a mini black hole in the region of spacetime \([18,19]\). So, if we try to probe
the spacetime at a scale smaller than the Planck scale, a mini black hole will form in that region of spacetime, and this will in turn restrict our ability to analyse that region of spacetime. It has also been argued that this length can be much larger than the Planck length, and its scale would be fixed by present experimental data [20, 21]. So, it may be possible to have such effects observed in future experiments, and thus it would be interesting to study different aspects of such effects.

However, the problem with the existence of such a minimum measurable length scale is that it is not consistent with the foundations principles of ordinary quantum mechanics. This is because the ordinary quantum mechanics is based on the the Heisenberg uncertainty principle, and according to this principle it is possible to detect the position of a particle with arbitrary accuracy, if the momentum is not measured. Thus, according to the Heisenberg uncertainty principle there is no bound on the accuracy to which the length can be measured as long as the momentum is not measured. Thus, in principle, according to the Heisenberg uncertainty principle, we can analyse the spacetime at a length scale smaller than the Planck scale, and so no minimum measurable length scale exists. However, the Heisenberg uncertainty principle can be modified to incorporate the existence of a minimum measurable length scale. This can be done by deforming the usual Heisenberg uncertainty principle $\Delta x \Delta p \geq \frac{1}{2}$, to $\Delta x \Delta p \geq \frac{1}{2} (1 + \beta (\Delta p)^2)$, where $\beta$ is a parameter in the theory. This modified Heisenberg uncertainty principle is called the generalized uncertainty principle (GUP).

As the Heisenberg uncertainty principle is closely related to the Heisenberg algebra, such a deformation of the Heisenberg uncertainty principle will generate a deformation of the Heisenberg algebra. In this GUP deformed Heisenberg algebra, the commutator of momentum and position operators is a function of momentum, $[x_i, p_j] = i(\delta_{ij} + \beta (p^2 \delta_{ij} + 2p_i p_j))$ [12, 22, 23]. This deformation of the Heisenberg algebra will also produce a deformation of the coordinate representation of the momentum operator. In fact, it is possible to write the deformed momentum operator, to the first order in $\beta$, as $p_i = -i \partial_i (1 - \beta \partial_j \partial^j)$. In this paper, we will analyse a relativistic version of this deformation, and the corresponding gauge theory using the Polyakov loop formalism.

2 Loop Space

In this section, we will construct the Polyakov loops for a deformed gauge theory, which will be deformed by the deformation of the Heisenberg algebra by GUP. It is also possible to define a relativistic version of the GUP deformed Heisenberg algebra, and study the quantum field theory corresponding to such a deformed algebra [27, 28, 29, 30, 31, 32]. Thus, the full covariant algebra can be written as, $[\hat{x}^\mu, \hat{p}_\nu] = i\partial_\nu [1 + \beta \hat{p}^\rho \hat{p}_\rho] + 2i\beta \hat{p}^\rho \hat{p}_\rho$. The generalized uncertainty for this deformed algebra can be expressed as $\Delta x'^\mu \Delta p_\mu \geq 1/2 (1 + 3\beta \Delta p^\rho \Delta p_\rho + 3\beta \langle p^\rho \rangle \langle p_\rho \rangle)$, and this generalized uncertainty can be used to obtain the following bound $\Delta x'^\mu_{\min} = \sqrt{3\beta} \sqrt{1 + 3\beta \langle p^\rho \rangle \langle p_\rho \rangle}$ [27]. So, there exists a minimum length $l_s$ and a minimum time $t_s$ in this algebra, such that $l_s = \sqrt{3\beta}$ and $t_s = \sqrt{3\beta}$. To the first order in $\beta$, we can write the deformed momentum operator as $p_\mu = -i \partial_\mu (1 - \beta \partial^\rho \partial_\rho) + O (\beta^2)$. It is possible to define a gauge covariant derivative which is consistent with the existence of a minimum
length scale as
\[ D_\mu = (1 - \beta D^\rho D_\rho) D_\mu, \]
where \( D_\mu = \partial_\mu + i A_\mu T_A. \) Here \( T_A \) are the generators of the Lie algebra \([T_A, T_B] = i f^C_{AB} T_C. \) Now as the covariant derivative transform \( D_\mu \rightarrow U D_\mu U^{-1}, \) so the deformed covariant derivative transform as [28]
\[ D_\mu \rightarrow -i (1 - \beta U D^\rho U^{-1} U D_\rho U^{-1}) U D_\mu U^{-1} = -i U (1 - \beta D^\rho D_\rho) D_\mu U^{-1} = U D_\mu U^{-1}. \] (2)
So, the GUP deformed covariant derivative still transforms like a regular covariant derivative. It is possible to show that the Bianchi identity will hold, but the algebraic manipulations are long (we used the package Quantum Mathematica to prove this result),
\[ [D_\lambda, [D_\mu, D_\nu]] + [D_\mu, [D_\nu, D_\lambda]] + [D_\nu, [D_\lambda, D_\mu]] = [(1 - \beta D^\rho D_\rho) D_\lambda, [(1 - \beta D^\nu D_\nu) D_\mu, (1 - \beta D^\sigma D_\sigma) D_\nu]] + [(1 - \beta D^\nu D_\nu) D_\lambda, [(1 - \beta D^\sigma D_\sigma) D_\mu, (1 - \beta D^\rho D_\rho) D_\nu]] + [(1 - \beta D^\sigma D_\sigma) D_\lambda, [(1 - \beta D^\rho D_\rho) D_\mu, (1 - \beta D^\nu D_\nu) D_\nu]] = 0. \] (3)
Motivated from the definition of the usual field tensor \( F_{\mu\nu} = -i [D_\mu, D_\nu], \) the deformed field tensor is defined as,
\[ F_{\mu\nu} = -i [D_\mu, D_\nu]\]
\[ = -i [(1 - \beta D^\rho D_\rho) D_\mu, (1 - \beta D^\rho D_\rho) D_\nu] = (1 - \beta D^\rho D_\rho) [(1 - \beta D^\rho D_\rho) F_{\mu\nu} - \beta (D^\rho F_{\nu\rho} D_\mu - D^\rho F_{\mu\rho} D_\nu)] - \beta (F_{\mu\rho} D^\rho D_\nu - F_{\nu\rho} D^\rho D_\mu), \]
\[ F_{\mu\nu} = 2 \beta D^\rho D_\rho F_{\mu\nu} - \beta (D^\rho F_{\nu\rho} D_\mu - D^\rho F_{\mu\rho} D_\nu) - \beta (F_{\mu\rho} D^\rho D_\nu - F_{\nu\rho} D^\rho D_\mu) \]
\[ = F_{\mu\nu} + \beta \tilde{F}_{\mu\nu}, \]
(4)
where \( F_{\mu\nu} = -i [D_\mu, D_\nu] \) is the un-deformed field tensor, and
\[ \tilde{F}_{\mu\nu} = -2 D^\rho D_\rho F_{\mu\nu} - (D^\rho F_{\nu\rho} D_\mu - D^\rho F_{\mu\rho} D_\nu) - (F_{\mu\rho} D^\rho D_\nu - F_{\nu\rho} D^\rho D_\mu) \]
(5)
Writing out equation (4) in terms of the undeformed potential \( A_\mu, \) we have
\[ D_\mu = [1 - \beta (\partial^\rho + i A_\rho)(\partial_\rho + i A_\rho)](\partial_\mu + i A_\mu). \] (6)
It is now clear that we can consider a modified potential \( A_\mu, \) differing from the undeformed \( A_\mu, \) by a term proportional to \( \beta: \)
\[ A_\mu = A_\mu + \beta \tilde{A}_\mu \] (7)
with the extra term obtained from equation (4)
\[ \tilde{A}_\mu = (\partial^\rho + i A_\rho)(\partial_\rho + i A_\rho)(\partial_\mu + i A_\mu). \] (8)
We now wish to define loop space variables based on this deformed field tensor $F_{\mu\nu}$ to study topological obstructions in this GUP spacetime.

First we consider the space of loops in undeformed spacetime, with a fixed base point. A loop is parameterized by the coordinates $\xi^\mu(s)$,

$$C : \{\xi^\mu(s) : s = 0 \to 2\pi, \xi^\mu(0) = \xi^\mu(2\pi)\}, \quad (9)$$

where $\xi^\mu(0) = \xi^\mu(2\pi)$ is the chosen (but arbitrary) base point. Next we define the loop space variable

$$\Phi[\xi] = P_s \exp i \int_0^{2\pi} A^\mu(\xi(s)) \frac{d\xi_\mu}{ds}. \quad (10)$$

where $P_s$ denotes ordering in $s$ increasing from right to left. From this we can define its logarithmic derivative as a kind of loop space connection

$$F^\mu[\xi] = i \Phi^{-1}[\xi] \frac{\delta}{\delta \xi^\mu(s)} \Phi[\xi]. \quad (11)$$

The derivative in $s$ is taken from below. It may be noted as the loop variable $\Phi[\xi]$ only depends on $C$ and not the manner in which $C$ is parameterized, so labeling it with a fixed point is over complete. In fact, any other parameterization of $C$ will only change the variable in the integration and not the loop space variable $\Phi[\xi]$.

We can obtain a formula relating $F^\mu[\xi]s$ to the spacetime curvature $F_{\mu\nu}$ by first defining a parallel transport from a point $\xi(s_1)$ to a point $\xi(s_2)$ as

$$\Phi[\xi : s_1, s_2] = P_s \exp i \int_{s_1}^{s_2} A^\mu(\xi(s)) \frac{d\xi_\mu}{ds}. \quad (12)$$

Thus

$$F^\mu[\xi]s = \Phi^{-1}[\xi : s, 0] F^{\mu\nu} \Phi[\xi : s, 0] \frac{d\xi_\nu(s)}{ds}. \quad (13)$$

This formula can be understood as follows. We parallel transport from a fixed point along a fixed path to another fixed point. After reaching that point, we will take a detour then turn back along the same path till we reach the original point. Thus, the phase factor generated by going along the path from the original point to final point will be canceled by the phase factor generated by going from the final point back to the original point. However, there will be a contribution generated by the transport along the infinitesimal circuit along the final point, which is proportional to the spacetime curvature at that point.

We can repeat the same construction using our deformed variables. So, we can define a deformed loop variable with a deformed connection. As this deformed connection, is a connection in the deformed theory, we can write

$$\Phi[\xi] = P_s \exp i \int_0^{2\pi} A^\mu(\xi(s)) \frac{d\xi_\mu}{ds}. \quad (14)$$

Here again $P_s$ denotes ordering in $s$ increasing from right to left. Now we can define the logarithmic derivative of this deformed variable as a deformed loop space connection

$$F^\mu[\xi]s = i \Phi^{-1}[\xi] \frac{\delta}{\delta \xi^\mu(s)} \Phi[\xi]. \quad (15)$$
We can also deformed a parallel transport from a point $\xi(s_1)$ to a point $\xi(s_2)$ as
\[
\Phi[\xi : s_1, s_2] = P_s \exp i \int_{s_1}^{s_2} A^\mu(\xi(s)) \frac{d\xi_\mu(s)}{ds}.
\] (16)

Now we can use this deformed parallel transport to go from a fixed point to another fixed point, along a fixed path. We can also take a detour from that final fixed point and go back to the initial fixed point along the same path. So, the phase generated by going to the final fixed point will exactly cancel the phase generated by going back to the initial fixed point. However, to take a detour, we will have to produce an infinitesimal circuit along the final point. This infinitesimal circuit will produce a contribution, and as we are using the deformed parallel transport, we can write this contribution as
\[
F^\mu[\xi(s)] = \Phi^{-1}[\xi : s, 0] F^{\mu\nu}(\xi(s)) \Phi[\xi : s, 0] \frac{d\xi_\nu(s)}{ds}.
\] (17)

Now since the GUP deformation in the covariant derivative is first order in $\beta$, in this expression we can actually replace the deformed $\Phi$ by the undeformed $\Phi$
\[
F^\mu[\xi(s)] = \Phi^{-1}[\xi : s, 0] F^{\mu\nu}(\xi(s)) \Phi[\xi : s, 0] \frac{d\xi_\nu(s)}{ds} = \Phi^{-1}[\xi : s, 0] [F^{\mu\nu} + \beta \bar{F}^{\mu\nu}] (\xi(s)) \Phi[\xi : s, 0] \frac{d\xi_\nu(s)}{ds} = F^\mu[\xi(s)] + \beta \bar{F}^\mu[\xi(s)].
\] (18)

Now this is important to note that if the original $F^{\mu\nu} = 0$, then $F^\mu[\xi(s)] = 0$. However, it is possible that even if $F^{\mu\nu} = 0$, we can have $\bar{F}^{\mu\nu} \neq 0$, and so $F^{\mu}[\xi(s)] \neq 0$. This would mean that even if $F^\mu[\xi(s)] = 0$, we can still have $\bar{F}^{\mu}[\xi(s)] \neq 0$, and so $F^{\mu}[\xi(s)] \neq 0$. Thus, there could be a contribution to the Polyakov loop produced solely from the deformation of the background geometry.

### 3 Topological Defects

We can regard $F^\mu[\xi(s)]$ as the connection in the loop space as it represents the change in phase of $\Phi[\xi]$ as one moves in the loop space. It is interesting to note that the connection is loop space $F^\mu[\xi(s)]$ is proportional to the field strength in spacetime $F^\mu[\xi(s)]$. As $F^\mu[\xi(s)]$ acts as a connection in the loop space, we can define covariant derivative in loop space $\Delta_\mu(s) = \delta / \delta \xi^\mu(s) + i F^\mu[\xi(s)]$. This covariant derivative can be used to define the curvature of the loop space $-i G^\mu[\xi(s_1), s_2] = G^\mu[\xi(s_1, s_2)]$ as the commutator of these covariant derivatives $[\Delta_\mu[\xi(s_1)], \Delta_\nu[\xi(s_2)]]$. So, we obtain the following expression for the curvature of the deformed loop space
\[
G^\mu[\xi(s_1, s_2)] = \frac{\delta}{\delta \xi^\mu(s_2)} F^\nu[\xi(s_1)] - \frac{\delta}{\delta \xi^\nu(s_1)} F^\mu[\xi(s_2)] + i [F^\mu[\xi(s_1), F^\nu[\xi(s_2)]] + \beta G^\mu[\xi(s_1, s_2)].
\] (19)

Here the original curvature in loop space is given by
\[
G^\mu[\xi(s_1, s_2)] = \frac{\delta}{\delta \xi^\mu(s_2)} F^\nu[\xi(s_1)] - \frac{\delta}{\delta \xi^\nu(s_1)} F^\mu[\xi(s_2)] + i [F^\mu[\xi(s_1), F^\nu[\xi(s_2)]]],
\] (20)
and \( \tilde{G}_{\mu\nu}[\xi(s_1, s_2)] \) is the correction to the original loop space curvature. It may be noted that the deformed loop connection, \( \tilde{F}_{\mu\nu}[\xi(s)] \) represents a change in phase \( \Phi \) as one moves in the deformed loop space. So, for a deformed gauge connection \( A_{\mu} \), it is possible to construct the holonomy using the deformed field tensor \( F^{\mu\nu} \). However, now \( F^{\mu\nu}[\xi(s)] \) is also a connection in the deformed loop space, and so we can construct the corresponding holonomy. Thus, we can go from a fixed point in the deformed loop space to another fixed point, and then take a detour back to the initial point. We will go back along the same path we initially took, and so the contribution of going to the final point will exactly cancel the contribution of going back to the initial point. However, to take a detour, we will have to make an infinitesimal circuit, and this will have a contribution. As we are moving in the deformed loop space, this contribution would be equal to \( \tilde{G}_{\mu
u}[\xi(s_1, s_2)] \). It may be noted that in spacetime, this would appear as sweeping out an infinitesimal two dimensional surface enveloping a three dimensional volume. Now the value of this deformed loop space curvature will depend on what is inside this volume. This loop space curvature can be used to analyse the presence of a topological defect in the original theory. This is because if a monopole is not present in the spacetime, then this deformed loop space curvature term vanishes. This can be seen by showing that this deformed loop space curvature is proportional to left-hand side of the Bianchi identity.

In fact, following closely similar arguments for the usual gauge theories [2], we consider variations of the curve in two orthogonal directions \( \lambda \) and \( \kappa \). Now first we define three displaced curves,

\[
\begin{align*}
(\xi^\mu(s))_\lambda &= (\xi^\mu(s))_\lambda + \Delta^\mu_\lambda \delta(s - s_1) \\
(\xi^\mu(s))_\kappa &= (\xi^\mu(s))_\kappa + \Delta^\mu_\kappa \delta(s - s_2) \\
(\xi^\mu(s))_s &= (\xi^\mu(s))_s + \Delta^\mu_s \delta(s - s_2),
\end{align*}
\]

(21)

where the Kronecker delta \( \delta^\mu_\lambda \) means that the variation is zero if \( \mu \neq \lambda \), and similarly for \( \delta^\mu_\kappa \). Then by definition

\[
\frac{\delta}{\delta \xi^\kappa(s_2)} F_{\alpha}[\xi(s_1)] = \lim_{\Delta \to 0} \lim_{\Delta' \to 0} \frac{1}{\Delta \Delta'} \{ \Phi^{-1}[\xi_1] \Phi[\xi_3] - \Phi^{-1}[\xi] \Phi[\xi] \}. \tag{22}
\]

It may be noted that the right-hand side usually has the implicit indices \( \lambda \) and \( \kappa \) as noted above.

Next we calculate the value of \( \Phi^{-1}[\xi_2] \Phi[\xi_3] - \Phi^{-1}[\xi] \Phi[\xi] \). Using parallel transport along these paths, we obtain

\[
\Phi[\xi_i] = \Phi[\xi] - i \int ds \Phi[\xi : 2\pi, s] F(\xi(s)) \Phi(\xi : s, 0), \tag{23}
\]

where

\[
F(\xi(s)) = F^{\mu\nu}(\xi(s)) \frac{\partial \xi^\nu(s)}{\partial s} \Delta \delta^\lambda_\mu \delta(s - s_1). \tag{24}
\]

Furthermore, we also obtain,

\[
\Phi[\xi_2] = \Phi[\xi] - i \int ds \Phi[\xi : 2\pi, s] F(\xi(s)) \Phi[\xi : s, 0], \tag{25}
\]
where

\[ F(\xi(s)) = F^{\mu\nu}(\xi(s)) \frac{d\xi_\nu(s)}{ds} \Delta^\prime s_\mu^\nu \delta(s - s_2). \quad (26) \]

Finally, we obtain

\[ \Phi[\xi_3] = \Phi[\xi_1] - i \int ds \Phi[\xi_1 : 2\pi, s] F(\xi_1(s)) \Phi[\xi_1 : s, 0], \quad (27) \]

where

\[ F(\xi_1(s)) = F^{\mu\nu}(\xi_1(s)) \frac{d\xi_{1\nu}(s)}{ds} \Delta^\prime s_\mu^\nu \delta(s - s_2). \quad (28) \]

We can also write similar expressions for \( \Phi[\xi : 2\pi, s] \) and \( \Phi[\xi_1 : s, 0] \). Now collecting all these these, we obtain the following expression,

\[
\frac{\delta}{\delta \xi_\mu(s_2)} F_\nu[\xi|s_1] = \Phi^{-1}[\xi : s_1, 0] D^\nu F^{\mu\nu}(\xi(s_2)) \times \frac{d\xi_\nu(s_1)}{ds_1} \Phi[\xi : s_1, 0] \delta(s_2 - s_1) \\
+ \Phi^{-1}[\xi : s_2, 0] F_{\mu\nu}(\xi(s_2)) \Phi[\xi : s_2, 0] \times \frac{d}{ds_1} \delta(s_2 - s_1) \\
+ i[F_{\mu}[\xi|s_2], F_\nu[\xi|s_1]] \theta(s_1 - s_2). 
\]

\[
\frac{\delta}{\delta \xi_\nu(s_1)} F_\mu[\xi|s_2] = \Phi^{-1}[\xi : s_2, 0] D^\mu F^{\nu\tau}(\xi(s_1)) \times \frac{d\xi_\tau(s_2)}{ds_2} \Phi[\xi : s_2, 0] \delta(s_1 - s_2) \\
+ \Phi^{-1}[\xi : s_1, 0] F_{\rho\mu}(\xi(s_1)) \Phi[\xi : s_1, 0] \times \frac{d}{ds_2} \delta(s_1 - s_2) \\
+ i[F_{\tau}[\xi|s_1], F_\mu[\xi|s_2]] \theta(s_2 - s_1). \quad (29) \]

So, the loop space curvature can be written as,

\[
\mathcal{G}_{\mu\nu}[\xi(s_1, s_2)] = \frac{\delta}{\delta \xi_\mu(s_2)} F_\nu[\xi|s_1] - \frac{\delta}{\delta \xi_\nu(s_1)} F_\mu[\xi|s_2] \\
+ i[F_{\mu}[\xi|s_1], F_\nu[\xi|s_2]] \\
= \Phi^{-1}[\xi : s_1, 0] \left[ [D_\mu, F_{\nu\tau}] + [D_\nu, F_{\tau\mu}] + [D_\tau, F_{\mu\nu}] \right] \\
\times \frac{d\xi_\tau(s_1)}{ds} \delta(s_1 - s_2). \quad (30) 
\]

Thus, the deformed loop space curvature is proportional to the deformed Bianchi identity in the spacetime. It is known that the Bianchi identity are satisfied in absence of a topological defect in spacetime, \([D_\mu, F_{\nu\tau}] + [D_\nu, F_{\tau\mu}] + [D_\tau, F_{\mu\nu}] = 0\), and so the loop space curvature vanishes in absence of a topological defect in spacetime, \(\mathcal{G}_{\mu\nu}[\xi(s_1, s_2)] = 0\). However, if a monopole exists in spacetime, then Bianchi identity are not satisfied \([D_\mu, F_{\nu\tau}] + [D_\nu, F_{\tau\mu}] + [D_\tau, F_{\mu\nu}] \neq 0\).
\[ \mathcal{D}_\tau, \mathcal{F}_{\mu \nu} \neq 0. \] Now if the world-line of a monopole goes through the point represented by \( s_1 \), then the loop space curvature does not vanish \( \mathcal{G}_{\mu \nu}[\xi(s_1, s_2)] \neq 0. \) However, if the topological defect only contributes at the order \( \beta \), then the original Bianchi identity will be satisfied, and the deformed Bianchi identity will be violated at the order \( \beta \). Thus, the loop space curvature is proportional will also have a contribution at the order \( \beta \) and it will not vanish. So, it is possible to produce topological defects in the gauge theory from the deformation of the background geometry by minimum measurable length scale. It would be interesting to analyse the consequences of such a deformation further.

4 Monopole Charge

Now we will finally obtain an expression for the non-abelian monopole charge in such deformed field theories. It is possible to obtain the non-abelian monopole charge for the usual gauge theories using the concept of loop of loops [2]. In this section, we will generalize this construction to deformed gauge theories, and thus obtain a generalized monopole charge for deformed gauge theories. It is also possible to construct a loop in the loop space by using the connection in the loop space, \( \mathcal{F}_{\mu \nu}[\xi] \). In order to do that, we define \( \Sigma \) as

\[ \Sigma : \{ \xi^\mu(s) : s = 0 \rightarrow 2\pi, \ t = 0 \rightarrow 2\pi \}, \] (31)

where

\[ \xi^\mu(t : 0) = \xi^\mu(t : 2\pi), \quad t = 0 \rightarrow 2\pi, \]
\[ \xi^\mu(0 : s) = \xi(2\pi : s), \quad s = 0 \rightarrow 2\pi. \] (32)

So, for each \( t \), we have \( \xi^\mu(t : s) \) and this represents a closed loop \( C(t) \ s = 0 \rightarrow 2\pi, \)

\[ C(t) : \{ \xi^\mu(t : s), s = 0 \rightarrow 2\pi \}. \] (33)

Here \( C(t) \) traces out a closed loop as \( t \) varies, and it shrinks to a point for \( t = 0 \) and \( t = 2\pi \). Now using \( \Sigma \), we can construct a loop in the loop space. Thus, for the usual un-deformed gauge theories, this will be given by

\[ \Theta(\Sigma) = P_t \exp i \int_0^{2\pi} dt \int_0^{2\pi} \mathcal{F}_{\mu \nu}[\xi][t, s] \frac{\partial \xi^\mu[\xi][t, s]}{\partial t}. \] (34)

This loop in the loop space is a parameterized surface in spacetime. Thus, this loop in the loop space encloses a volume. So, it can be used to measure the monopole inside such a volume. We will now generalize this construction to deformed gauge theories, and then apply that deformed formalism to analyze the monopole charge for the deformed gauge theory.

However, as the deformation by the generalized uncertainty principle, deforms \( \mathcal{F}_{\mu \nu}[\xi][t, s] \) to \( \mathcal{F}_{\mu \nu}[\xi][t, s] \), we can construct a loop in the loop space of deformed theories using

\[
\Theta(\Sigma) = P_t \exp i \int_0^{2\pi} dt \int_0^{2\pi} \mathcal{F}_{\mu \nu}[\xi][t, s] \frac{\partial \xi^\mu[\xi][t, s]}{\partial t} = P_t \exp i \int_0^{2\pi} dt \int_0^{2\pi} [\mathcal{F}_{\mu \nu}[\xi][t, s] + \beta \tilde{\mathcal{F}}_{\mu \nu}[\xi][t, s]] \frac{\partial \xi^\mu[\xi][t, s]}{\partial t} = \Theta(\Sigma) + \beta \tilde{\Theta}(\Sigma). \] (35)
This Θ measures the charge of a non-abelian monopole for a deformed gauge theory, since Θ(Σ) = ζ, where ζ is the generalized monopole charge enclosed by the surface Σ. Note that Θ(Σ) = I the group identity represents the vacuum, i.e., no topological charge is enclosed by the surface. As an example, let us consider a gauge theory with SO(3) as its gauge group. In this case, monopole charges are +1 for no monopole, and −1 for a monopole. If a monopole is not present, then Θ(Σ) will wind fully around the gauge group and will equal to the identity. However, in presence of a monopole, Θ(Σ) cannot wind fully around the gauge group and will equal the identity. It may be noted that the expression Θ(Σ) = ζ is interesting as it can be used to evaluate the monopole charge. It is possible to demonstrate that this result holds for all non-abelian Yang-Mills theories with gauge group is SU(N)/ZN. The monopole charge for such a gauge group is given by ζ = exp i2πr/N, where r = 0, 1, 2, · · · , (N − 1). Thus, with this modification this result can be applied to Yang-Mills theory with any gauge group. It is interesting to note that even the change has a β contribution coming from deformation. This occurs because the topological defects can occur at the order β, even if they do not occur in the original theory. Thus, even in deformed gauge theories, the Polyakov loops space formalism can be used to analyse the topological defects.

It may be noted as Gµν[ξ(s1, s2)] is analogous to Fµν in deformed loop space, it can be constructed using a logarithmic derivative of Θ(Σ). So, basically, we can argue that the logarithmic derivative of Θ(Σ) would produce a connection in this loop of loop space. In fact, this has been done for ordinary loop space [2], and the same argument can be used for deformed loop space by using deformed quantities. Now for s1 ̸= s2, Gµν[ξ(s1, s2)] does not enclose any volume, and so for this Θ(Σ) = I, which is the group identity, and its logarithmic derivative vanishes. This also occurs for s1 = s2, of ξ(s) does not intersect with a monopole worldline, which we can represent by Y(τ). So, in that case again θ(Σ) = I. However, when s1 = s2, and x(τ) intersects a monopole worldline Y(τ), Gµν[ξ(s1, s2)] corresponds to Σ enclosing a monopole. Now for original undeformed loop space variable we have [2],

\[
G_{µν}[ξ(s1, s2)] = − \frac{π}{g} \int dτκ[ξ(τ)] \frac{dξ_σ(s_1)}{ds} \frac{dY_ρ(τ)}{dτ} δ(ξ(s) − Y(τ))δ(s_1 − s_2), \tag{36}
\]
where exp iπκ = ζ. However, in deformed loop space, we can write Gµν[ξ(s1, s2)] = Gµν[ξ(s1, s2)] + βGµν[ξ(s1, s2)], so we obtain

\[
G_{µν}[ξ(s1, s2)] = − \frac{π}{g} \int dτκ[ξ(τ)] \frac{dξ_σ(s)}{ds} \frac{dY_ρ(τ)}{dτ} δ(ξ(s) − Y(τ))δ(s_1 − s_2)
− βGµν[ξ(s1, s2)]. \tag{37}
\]
So, even if Gµν[ξ(s1, s2)] = 0, due to the original Bianchi identity being satisfied, we still have

\[
− \frac{π}{g} \int dτκ[ξ(τ)] \frac{dξ_σ(s)}{ds} \frac{dY_ρ(τ)}{dτ} δ(ξ(s) − Y(τ))δ(s_1 − s_2) ≠ 0, \tag{38}
\]
and so the deformation of the loop space produces a topological defect in space-time. Thus, we will have demonstrated that a monopole contribution can generated from deformation of loop space variables. It may be noted that monopoles
in general have been analyzed in loop space using a duality which reduces to electromagnetic Hodge duality for abelian theories [35, 36, 37, 38]. However, as far as we know, all such constructions use the loop space formalism, and we are not aware of any proof of this duality using space-time variables alone. Therefore we are restricted at present to such a discussion in loop space only.

We would like to point out that solitonic solutions of the 't Hooft-Polyakov type are sometimes called non-abelian monopoles, but the magnetic charge carried by them is usually an abelian magnetic charge (with symmetry breaking into a $U(1)$ subgroup). As far as we know, no solutions of the pure Yang-Mills equation i.e., (without the introduction of symmetry breaking), with a non-abelian monopole charge has been constructed, either using spacetime variables or loop variables. Furthermore, the monopoles (with symmetry breaking) are solutions only outside of a sphere of a finite radius, usually interpreted as the size of the monopole. Inside of this sphere, not much is known, since the interactions due to the original non-abelian forces become non-negligible. Here we are interested in studying the genuinely non-abelian magnetic charge, without involving the Higgs fields. Their existence in ordinary spacetime is governed by topology. In spacetimes with a minimum length scale, as we study here, the obstruction to the vanishing of the relevant loop space curvature indicates also the topological nature of this obstruction, which by analogy we think of as generalized monopoles. As no non-abelian monopole solutions in ordinary spacetime are known, to construct one for GUP spacetime would really be interesting, but perhaps not feasible at present. In this paper we have demonstrated that minimal length in spacetime can give rise to a certain topological charge. However, we would like to point out that it is possible that an object would not exist even if such a charge is allowed to exist [39]. So, we only demonstrate that such an object can exist due to the existence of a topological charge produced by minimal length.

5 Validity of the Approximation

In this section, we will argue that the higher order contributions cannot cancel the topological defects produced at a certain order in $\beta$. Thus, we will be able to demonstrate that the results obtain in this paper are not a consequence of the approximation that we have used. This is because if we had considered the deformation to the next order, then we would get higher order contribution to the field strength, which would produce higher order contributions to the loop space variables. Thus, if we analyzed the theory to the order $\beta^2$, then the corrected field strength would be given by

$$ F_{\mu\nu} = F_{\mu\nu} + \beta \tilde{F}_{\mu\nu} + \beta^2 \tilde{F}_{\mu\nu}. \quad (39) $$

This would occur because $D_\mu$ will also have a $\beta^2$ contribution to it. This will in turn produce a $\beta^2$ contribution to the connection,

$$ A_\mu = A_\mu + \beta \tilde{A}_\mu + \beta^2 \tilde{A}_\mu. \quad (40) $$

Now by using this new connection in the loop space formalism, we can obtain the $\beta^2$ contribution to all the loop space variables. Thus, by using the expression
of the connection to the order $\beta^2$, we obtain

$$
\Phi[\xi] = P_s \exp i \int_0^{2\pi} A^\mu(\xi(s)) \frac{d\xi_\mu}{ds} \\
= P_s \exp i \int_0^{2\pi} [A^\mu + \beta \tilde{A}^\mu + \beta^2 \tilde{\tilde{A}}^\mu](\xi(s)) \frac{d\xi_\mu}{ds},
$$

(41)

We can also write, to the $\beta^2$ order,

$$
\Phi[\xi : s_1, s_2] = P_s \exp i \int_{s_1}^{s_2} A^\mu(\xi(s)) \frac{d\xi_\mu}{ds} \\
= P_s \exp i \int_{s_1}^{s_2} [A^\mu + \beta \tilde{A}^\mu + \beta^2 \tilde{\tilde{A}}^\mu](\xi(s)) \frac{d\xi_\mu}{ds}.
$$

(42)

Finally, we can also obtain $F^\mu[\xi(s)]$ to the order $\beta^2$ as

$$
F^\mu[\xi(s)] = \Phi^{-1}[\xi : s, 0]F^\mu(\xi(s))\Phi[\xi : s, 0] \frac{d\xi_\mu}{ds}.
$$

(43)

Thus, we can demonstrate that to the order $\beta^2$,

$$
F^\mu[\xi(s)] = \Phi^{-1}[\xi : s, 0]\left[\left[F^\mu + \beta \tilde{F}^\mu + \beta^2 \tilde{\tilde{F}}^\mu\right](\xi(s))
\times \Phi[\xi : s, 0] \frac{d\xi_\mu}{ds}\right].
$$

(44)

Thus by repeating this argument we have used for in this paper, to the order $\beta^2$, we can demonstrate that to the order $\beta^2$,

$$
G_{\mu\nu}[\xi(s_1, s_2)] = \frac{\delta}{\delta \xi_{\mu}(s_2)}F_{\mu}[\xi[s_1]] - \frac{\delta}{\delta \xi_{\mu}(s_1)}F_{\mu}[\xi[s_2]] \\
+ i[F_{\mu}[\xi[s_1]], F_{\mu}[\xi[s_2]]] \\
= \Phi^{-1}[\xi : s_1, 0]\left[\left[D_{\mu}, F_{\sigma\tau}\right] + \left[D_{\nu}, F_{\tau\mu}\right] + \left[D_{\tau}, F_{\mu\nu}\right]\right] \\
\times \Phi[\xi : s_1, 0] \frac{d\xi_{\tau}(s_1)}{ds} \delta(s_1 - s_2).
$$

(45)

where we have considered all the covariant derivatives and the field strengths deformed to the order $\beta^2$. This is because the $F^\mu[\xi(s)]$ also contain the $\beta^2$ terms. It could be demonstrated by repeating the calculations we did to the order $\beta$, that the Bianchi identity also holds iteratively for higher order $\beta$ deformations, and thus it would hold for $\beta^2$ deformation. Thus, we can argue that it would be possible for the $G_{\mu\nu}[\xi(s_1, s_2)]$ not to be zero at the order $\beta^2$, even if it is zero at the order $\beta$. Thus, topological defects can occur at higher order, even if they do not occur at lower order. However, if $G_{\mu\nu}[\xi(s_1, s_2)] \neq 0$ at the order $\beta$, then it cannot vanish at any higher order. This is because at higher order say $\beta^2$, the contribution to $G_{\mu\nu}[\xi(s_1, s_2)]$ will come from $\tilde{F}_{\mu\nu}$ which is of order $\beta^2$, and no additional contribution will come at the order $\beta$. Now as $\beta < 1$, the $\beta^2$ contribution cannot cancel the $\beta$ contributions to the loop space variables. Thus, the topological defect which is present at the order $\beta$ cannot be eliminated by considering by considering higher order corrections to the loop space. It may be noted that at $\beta^2$ order $\Theta(\Sigma) = \Theta + \tilde{\Theta} + \beta^2\tilde{\tilde{\Theta}}$. 


In fact, this argument can be made iteratively for the loop space variables at any order. Thus, if a topological defect exists at the order $\beta^n$, it cannot be eliminated at the order $\beta^{n+m}$, when $m \geq 1$. This is because the $D_\mu$ will have an contribution to the order $\beta^n$ at the order $n$ and $\beta^{n+m}$ at the order $n+m$. So, the field strength $F_{\mu\nu}$ at the order $n$ will also contain terms proportional to $1 \cdots \beta^n$, and the field strength $F_{\mu\nu}$ at the order $n+m$ will contain terms proportional to $1 \cdots \beta^{n+m}$. So, the connection $A_\mu$ will also contain terms proportional to $\beta^n$ and $\beta^{n+m}$ at the orders $\beta^{n}$ and $\beta^{n+m}$, respectively. Now repeating the argument used in this section, we can define the loop space variables for each the deformation at any order, and $G_{\mu\nu}[\xi(s_1, s_2)]$ would also be given in terms of the Bianchi identity at the corresponding order of the deformation parameter. This implies that $G_{\mu\nu}[\xi(s_1, s_2)]$ will contain terms proportional to $\beta^n$ at the order $n$ and $\beta^{n+m}$ at the order $n+m$. Now if $G_{\mu\nu}[\xi(s_1, s_2)] \neq 0$ at $\beta^n$, then this contribution cannot be canceled at the order $\beta^{n+m}$, because $\beta < 1$. Thus, the topological defects produced at any order cannot be eliminated by considering higher order contributions in the deformation parameter. We would also like to point out that the $\Theta(\Sigma)$ will also contributions proportional to $1 \cdots \beta^n$ at $n$ order, and $1 \cdots \beta^{n+m}$ at $n+m$ order.

6 Conclusion

In this paper, we were able to analyse the deformation of a gauge theory by the existence of a minimum measurable length scale. This was done using the loop space formalism. We explicitly constructed the loop space variable for this deformed theory. This loop space variable was then used for constructing the loop space curvature. This curvature did not vanish in presence of a non-abelian monopole. Hence, we were able to demonstrate that the non-vanishing of the loop space curvature indicates the existence of a topological obstruction even for deformed gauge theories. We have also constructed an explicit expression for the charge of a non-abelian monopole using the loop in the loop space. However, it was possible to consider configurations, for which the original field strength vanished, but the deformation did not vanish. Using these field configurations, it was possible to demonstrate that the Polyakov connection can get contributions purely from the deformation, and the loop space curvature can also get $\beta$ order contributions, even if originally it vanished. Thus, it is possible the deformation of gauge theories by the deformation of the background geometry can give rise to topological defects. We have also demonstrated that higher order corrections cannot cancel the topological defects produced at a certain order in the loop space formalism. So, the presence of a minimum length actually may create topological obstructions like monopoles. It thus does not seem to be an artifact of any approximation, but what may happen if quantum effects are taken into account, in the way we propose. It may be noted that the production of magnetic monopoles and even electric charge by quantum gravitational effects is not a new idea, and such charges have been constructed using Wheeler-DeWitt approach [33, 34]. However, all such work was done only for abelian gauge theories, and this is the first time it has been proposed that quantum gravity may produce monopoles in non-abelian gauge theories. We would like to point out that we have only used the deformed gauge theories to obtain such results, however, such a deformation of gauge theories occurs due to quantum gravity.
This is because a deformation of quantum mechanics can occur due to a low energy effects from quantum gravity [20, 21], and the corresponding deformation of quantum field theories (including gauge theories) can also occur from such quantum gravitational effects [27, 28, 29, 30, 31, 32]. This deformation of gauge theory, from quantum gravitational effects, is the deformation we have used to obtain the results of this paper. Thus, it is possible that the topological defects produced from the deformation studied in this paper, could occurs due to quantum gravitational effects because of the existence of minimal length in spacetime.

The loop space formalism has been used to construct loop space duality for ordinary Yang-Mills theories [35, 36, 37, 38]. This duality reduces to the usual electromagnetic Hodge duality for abelian gauge theories. So, even though the Hodge duality cannot be generalized to non-abelian gauge theories, this loop space duality can be used to construct a dual potential even in case of non-abelian gauge theories. This dual potential has also been used for constructing a Dualized Standard Model [40, 41, 42, 43, 44], and which has in turn been used for explaining the difference of masses between different generations of fermions [45, 46]. This model has also been used for analysing the Neutrino oscillations [47], Lepton transmutations [48], and off-diagonal elements of the CKM matrix [49]. The dual potential used for obtaining the Dualized Standard Model has also been used for constructing the ’t Hooft’s order-disorder parameters [50, 51, 52]. It would be interesting to repeat this analysis for a gauge theory deformed by a minimum measurable length. Thus, we can use the results of this paper to construct a dual potential for gauge theories deformed by generalized uncertainty principle. This dual potential can in turn be used for constructing a deformed version of Dualized Standard Model. This deformed Dualized Standard Model can be used for analyzing the effect on generalized uncertainty principle on the off-diagonal elements of the CKM matrix, difference of masses between different generations of fermions, Neutrino oscillations and Lepton transmutations. It would also be interesting to analyse the ’t Hooft’s order-disorder parameters for gauge theories deformed by generalized uncertainty principle.

Acknowledgments

We would like to thank Mohammed Khalil for proving the Bianchi identity for deformed gauge theories.

References

[1] A. M. Polyakov, Nucl. Phys. B164, 171 (1980)
[2] H. M. Chan and S. T. Tsou, Some Elementary Gauge Theory Concepts, World Scientific (1993)
[3] H. M. Chan, P. Scharbach and S. T. Tsou, Ann. Phys. 167 454 (1986)
[4] H. M. Chan and S. T. Tsou, Act. Phys. Pol. B17, 259 (1986)
[5] M. Faizal and S. T. Tsou, Int. J. Theor. Phys. 54, 896 (2015)
[6] M. Faizal, Europhys. Lett. 103, 21003 (2013)
[7] M. Faizal and S. T. Tsou, Europhys. Lett. 107, 20008 (2014)
[8] D. Amati, M. Ciafaloni and G. Veneziano, Phys. Lett. B216, 41 (1989)
[9] A. Kempf, G. Mangano, and R. B. Mann, Phys. Rev. D52, 1108 (1995)
[10] L. N. Chang, D. Minic, N. Okamura, and T. Takeuchi, Phys.Rev. D65, 125027 (2002)
[11] L. N. Chang, D. Minic, N. Okamura, and T. Takeuchi, Phys. Rev. D65, 125028 (2002)
[12] S. Benczik, L. N. Chang, D. Minic, N. Okamura, S. Rayyan, and T. Takeuchi, Phys. Rev. D66, 026003 (2002)
[13] S. Hossenfelder, Living Rev. Rel. 16, 2 (2013)
[14] M. R. Douglas, D. N. Kabat, P. Pouliot and S. H. Shenker, Nucl. Phys. B 485, 85 (1997)
[15] A. Smailagic, E. Spallucci and T. Padmanabhan, [hep-th/0308122]
[16] M. Fontanini, E. Spallucci and T. Padmanabhan, Phys. Lett. B 633, 627 (2006)
[17] P. Dzierzak, J. Jezierski, P. Malkiewicz, and W. Piechocki, Acta Phys. Polon. B 41, 717 (2010)
[18] M. Maggiore, Phys. Lett. B304, 65 (1993)
[19] M. I. Park, Phys. Lett. B659, 698 (2008)
[20] S. Das and E. C. Vagenas, Phys. Rev. Lett. 101, 221301 (2008)
[21] I. Pikovski, M. R. Vanner, M. Aspelmeyer, M. Kim and C. Brukner, Nature Phys. 8, 393 (2012)
[22] L. J. Garay, Int. J. Mod. Phys. A 10, 145 (1995)
[23] C. Bambi, F. R. Urban, Class. Quantum Grav. 25, 095006 (2008)
[24] K. Nozari, Phys. Lett. B 629, 41 (2005)
[25] A. Kempf, J. Phys. A 30, 2093 (1997)
[26] A. F. Ali, S. Das, and E. C. Vagenas, Phys. Rev. D84, 44013 (2011)
[27] M. Kober, Phys. Rev. D82, 085017 (2010)
[28] M. Kober, Int. J. Mod. Phys. A26, 4251 (2011)
[29] M. Faizal, Int. J. Geom. Meth. Mod. Phys. 12, 1550022 (2015)
[30] M. Faizal and S. I. Kruglov, Int. J. Mod. Phys. D 25, 1650013 (2016)
[31] M. Faizal and B. Majumder, Ann. Phys. 357, 49 (2015)
[32] V. Husain, D. Kothawala and S. S. Seahra, Phys. Rev. D 87, 025014 (2013)
[33] R. Garattini and B. Majumder, Nucl. Phys. B 883, 598 (2014)
[34] R. Garattini, Phys. Lett. B 666, 189 (2008)
[35] H. M. Chan, J. Faridani and S. T. Tsou, Phys. Rev. D 52, 6134 (1995)
[36] M. Faizal and S. T. Tsou, Found. Phys. 45, 1421 (2015)
[37] M. Faizal and S. T. Tsou, Eur. Phys. J. C 75, 316 (2015)
[38] H. M. Chan, J. Faridani and S. T. Tsou, Phys. Rev. D 53, 7293 (1996)
[39] O. Aharony, N. Seiberg and Y. Tachikawa, JHEP 1308, 115 (2013)
[40] H. M. Chan, J. Bordes and S. T. Tsou, Int. J. Mod. Phys. A 14, 2173 (1999)
[41] H. M. Chan and S. T. Tsou, Acta Phys. Polon. B 28, 3027 (1997)
[42] H. M. Chan and S. T. Tsou, Acta Phys. Polon. B 33, 4041 (2002)
[43] H. M. Chan, Int. J. Mod. Phys. A 16, 163 (2001)
[44] H. M. Chan and S. T. Tsou, Acta Phys. Polon. B 28, 3041 (1997)
[45] J. Bordes, H. M. Chan, J. Faridani, J. Pfaudler and S. T. Tsou, Phys. Rev. D 58, 013004 (1998)
[46] J. Bordes, H. M. Chan, J. Faridani, J. Pfaudler and S. T. Tsou, Phys. Rev. D 60, 013005 (1999)
[47] J. Bordes, H. M. Chan, J. Pfaudler and S. T. Tsou, Phys. Rev. D 58, 053003 (1998)
[48] J. Bordes, H. M. Chan and S. T. Tsou, Phys. Rev. D 65, 093006 (2002)
[49] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652-657 (1973)
[50] G. ’t Hooft, Nucl. Phys. B 138, 1 (1978)
[51] H. M. Chan and S. T. Tsou, Phys. Rev. D 57, 2507 (1998)
[52] H. M. Chan and S. T. Tsou, Phys. Rev. D 56, 3646 (1997)