Unifying dark components and crossing the phantom divide
with a classical Dirac field

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In this paper we consider a spatially flat Friedmann-Robertson-Walker (FRW) cosmological model
with cosmological constant, containing a stiff fluid and a classical Dirac field. The proposed cos-
mological scenario describes the evolution of effective dark matter and dark energy components
reproducing, with the help of that effective multifluid configuration, the quintessential behavior.
We find the value of the scale factor where the effective dark energy component crosses the phantom
divide. The model we introduce, which can be considered as a modified ΛCDM one, is characterized
by a set of parameters which may be constrained by the astrophysical observations available up to
date.

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I. INTRODUCTION

According to the standard cosmology the total en-
ergy density of the Universe is dominated today by both
dark matter and dark energy densities. The dark mat-
ter, which includes all components with nearly vanishing
pressure, has an attractive gravitational effect like usual
pressureless matter and neither absorbs nor emit radia-
tion. The dark energy component in general is consid-
ered as a kind of vacuum energy with negative pressure
and is homogeneously distributed and, unlike dark mat-
ter, is not concentrated in the galactic halos and in the
clusters of galaxies. The observational data provide comp-
pelling evidence for the existence of dark energy which
dominate the present day Universe and accelerates its
expansion.

In principle, any matter content which violates the
strong energy condition and possesses a positive energy
density and negative pressure, may cause the dark en-
ergy effect of repulsive gravitation. So the main problem
of the modern cosmology is to identify this form of dark
energy that dominates the universe today.

In the literature the most popular candidates are cos-
mological constant Λ, quintessence and phantom mat-
ter. Their equation of state is given by \( w = p/\rho \), where
\( w = -1 \), \( w > -1 \) and \( w < -1 \) respectively. Dark en-
ergy composed of just a cosmological term \( \Lambda \) is fully
consistent with existing observational data. However,
these data do not exclude the possibility of explaining
the observed acceleration with the help of phantom mat-
ter. The cosmological constant can be associated with a
time independent dark energy density, the energy density
of quintessence scales down with the cosmic expansion,
and the energy density of phantom matter increases with
the expansion of the universe.

Mostly, the attention has been paid to dark energy as
high energy scalar fields, characterized by a time varying
equation of state, for which the potential of the scalar
field plays an important role. Among scalar field models
we can enumerate quintessence models [1], Chameleon
fields [2], K-essence [3], Chaplygin gases [4], tachyons [5],
phantom dark energy [6], etc.

In general the crossing of the phantom divide cannot
be achieved with a unique scalar field [7] this fact has
motivated a lot of activity oriented toward different ways
to realize such crossing [8]. For instance, in Ref. [9] it
was explored the so called kinetic k-essence models [10],
i.e. cosmological models with several k-fields in which
the Lagrangian does not depend on the fields themselves
but only on their derivatives. It was shown that the dark
energy equation of state transits from a conventional to
a phantom type matter. Note that formally, one can get
the phantom matter with the help of a scalar field by
switching the sign of kinetic energy of the standard scalar
field Lagrangian [6]. So that the energy density \( \rho_{ph} = -(1/2)\dot{\Phi}^2 + V(\Phi) \) and the pressure \( p_{ph} = -(1/2)\dot{\Phi}^2 - V(\Phi) \) of the phantom field leads to \( \rho_{ph} + p_{ph} = -\dot{\Phi}^2 < 0 \),
violating the weak energy condition.

In the Universe nearly 70% of the energy is in the form
of dark energy. Baryonic matter amounts to only 3 – 4%,
while the rest of the matter (27 %) is believed to be in
the form of a non-luminous component of non-baryonic
nature with a dust like equation of state \( (w = 0) \) known

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as cold dark matter (CDM). In this case, if the dark energy is composed just by a cosmological constant, then this scenario is called ΛCDM model.

Below, we analyze a FRW universe having cosmological constant and filled with a stiff fluid and a classical Dirac field (CDF). With this matter configuration, we will see that the FRW universe evolves from a non-accelerated stage at early times to an accelerated scenario at late times recovering the standard ΛCDM cosmology. The CDF may be justified by an important property: in a spatially flat homogeneous and isotropic FRW spacetime it behaves as a "perfect fluid" with a energy density, not necessarily, positive definite. This "perfect fluid" can be seen as a kind of "dust". In particular motivated by the fact that the dark matter is generally modelled as a system of collisionless particles [11, 12], we have the possibility of giving to cold dark matter content an origin based on the nature of the CDF. On the other hand, the stiff fluid is an important component because at early times, it could describe the shear dominated phase of a possible initial anisotropic scenario, dominating upon the remaining components of the model.

The organization of the paper is as follows: In Sec. II we present the dynamical field equations for a FRW cosmological model with a matter source composed by a stiff fluid and a CDF. In Sec. III the behavior of the dark energy component is studied. In Sec. IV we conclude with some remarks.

II. DYNAMICAL FIELD EQUATIONS

We shall adopt a spatially flat, homogeneous and isotropic spacetime described by the FRW metric

\[ ds^2 = dt^2 - a^2(t) \left( dx^2 + dy^2 + dz^2 \right), \]

where \( a(t) \) is the scale factor. The spacetime contains a cosmic fluid composed by (i) a stiff fluid \( \rho_s = \rho_{s0}/a^6 \) and (ii) a homogeneous classical Dirac field \( \psi \). The Einstein-Dirac equations are

\[ 3H^2 - \Lambda = \frac{\rho_{s0}}{a^6} + \rho_D, \] \hspace{1cm} \text{(2)}

\[ (\Gamma^i \nabla_i - \alpha) \psi = 0, \] \hspace{1cm} \text{(3)}

where \( H = \dot{a}/a \) is the Hubble expansion rate, \( \alpha \) is a constant and the dot denote differentiation with respect to the cosmological time. Here \( \rho_D \) represents the energy density of the CDF.

The dynamical equation for the CDF in curved spacetime can be obtained using the vierbein formalism. So, \( \Gamma^i \) are the generalized Dirac matrices, which satisfy the anticommutation relations

\[ \{ \Gamma^i, \Gamma^k \} = -2g^{ik} I, \] \hspace{1cm} \text{(4)}

with the metrics tensor \( g^{ik} \) and \( I \) the identity \( 4 \times 4 \) matrix. They can be defined in terms of the usual representation of the flat space-time constant Dirac matrices \( \gamma^i \) as

\[ \Gamma^0 = \gamma_0, \quad \Gamma^\beta = \gamma^\beta \frac{1}{a}, \] \hspace{1cm} \text{(5)}

where the Dirac matrices \( \gamma^i \) can be written with the Pauli matrices \( \sigma^\beta \) as

\[ \gamma^0 = i \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \gamma^\beta = i \begin{pmatrix} 0 & -\sigma^\beta \\ \sigma^\beta & 0 \end{pmatrix}, \] \hspace{1cm} \text{(6)}

and

\[ \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \] \hspace{1cm} \text{(7)}

with \( I \) the identity \( 2 \times 2 \) matrix. The symbol \( \nabla_i = \partial_i + \Sigma_i \) denotes the spinorial covariant derivatives, being the spinorial connection \( \Sigma_i \) defined by \( \nabla_i \Gamma_k = 0 \). Then it leads to

\[ \Sigma_0 = 0, \quad \Sigma_\beta = \frac{1}{2} \rho H \Gamma^0 \Gamma_\beta. \] \hspace{1cm} \text{(8)}

The constant \( \alpha \) will be associated, later on, with the total observable matter.

Coming back to the Dirac equation (3), it takes the form

\[ \Gamma^0 \left( \partial_i + \frac{3}{2} H + \alpha \right) \psi(t) = 0. \] \hspace{1cm} \text{(9)}

Restricting ourselves to the metric (1), the general solution of the latter equation consistent with Eq. (2) is given by

\[ \psi(t) = \frac{1}{a^{3/2}} \begin{pmatrix} b_1 e^{-i\alpha t} \\ b_2 e^{-i\alpha t} \\ d_1^* e^{i\alpha t} \\ d_2^* e^{i\alpha t} \end{pmatrix}, \] \hspace{1cm} \text{(10)}

with arbitrary complex coefficients \( b_1, b_2, d_1 \) and \( d_2 \). The only nonvanishing component of the energy-momentum tensor for the CDF is

\[ T^D_{00} = \frac{\alpha}{a^3} \left( |b_1|^2 + |b_2|^2 - |d_1|^2 - |d_2|^2 \right) \equiv \frac{\rho_D}{a^3}, \] \hspace{1cm} \text{(11)}

where \( \rho_D = \alpha(b^2 - d^2) \), \( b^2 = |b_1|^2 + |b_2|^2 \) and \( d^2 = |d_1|^2 + |d_2|^2 \). For positive values of \( \rho_D \) this source formally behaves as a perfect fluid representing a classical dust. However, the CDF will allow us to extend the analysis for negative values of the energy density. Here, we restrict us to the physical sector \( d^2 < b^2 \) and without loss of generality we choose \( b^2 = 1 \); this means that \( \rho_D \geq 0 \).

From Eq. (11) we see that the energy density of the CDF is given by \( \rho_D = \rho_D/a^3 \). Thus the Eq. (2) can be rewritten in the following form

\[ 3H^2 = \rho_m + \rho_x, \] \hspace{1cm} \text{(12)}
From this it is clear that this cosmological scenario exhibits an accelerated expansion since there is a stage where $\ddot{a} > 0$.

$$\rho_m = \frac{\alpha}{a^3}, \quad (13)$$

$$\rho_x = \frac{\rho_{s0}}{a^6} - \frac{\alpha d^2}{a^3} + \Lambda. \quad (14)$$

We shall associate the positive part of the Dirac energy–momentum tensor with the pressureless matter giving rise to the total (“true”) observable matter $\rho_m$, while one may assume that its negative part, along with the stiff fluid and the cosmological constant constitute the effective dark energy component $\rho_x$.

The general solution of the Einstein equation (12) with sources (13) and (14) takes the form

$$a^3(t) = \frac{\alpha(1 - d^2)}{2\Lambda} \left[ -1 + \cosh \sqrt{3\Lambda}t \right]$$

$$+ \sqrt{\frac{\rho_{s0}}{\Lambda}} \sinh \sqrt{3\Lambda}t, \quad (15)$$

where we have setting the initial singularity at $t = 0$. In the Fig. 1 it is showed the behavior of the scale factor and its derivatives.

III. DARK ENERGY EVOLUTION

In order to have a positive $\rho_x$, we choose the parameters of the models according to the following restriction

$$\alpha^2 d^4 < 4\Lambda \rho_{s0}. \quad (16)$$

Since $a(t)$ is an increasing function of time, this effective dark energy component decreases up to it reaches a minimum value $\rho_{xc} = \Lambda - \alpha^2 d^4/4\rho_{s0}$ at $a_c = (2\rho_{s0}/\alpha d^2)^{1/3}$ where the dark component crosses the phantom divide and begins to increase with time (see Fig. 2). Fundamentally, the effective dark energy component crosses the phantom divide due to presence of the $d$–parameter. In fact, the cosmological evolution of dark energy depends on the negative term $-\alpha d^2/a^3$, see Eq. (14), because it produces a minimum at $\dot{\rho}_x(a_c) = 0$, showing the importance of considering the CDF as a source of the Einstein equation.

Assuming that total matter and dark energy are coupled only gravitationally, then they are conserved separately, so we have that

$$\dot{\rho}_m + 3H \rho_m = 0, \quad (17)$$

$$\dot{\rho}_x + 3H (1 + w_x) \rho_x = 0, \quad (18)$$

where we have assumed the equation of state $p_x = w_x \rho_x$ for the dark energy component. Taking into account that the dark energy component has a variable state parameter $w_x = w_x(a)$, we define as $a_m = (\rho_{s0}/\Lambda)^{1/6}$ the value of the scale factor where this equation of state coincides with the matter one, that is $w_x = 0$. So that, the above restriction $\alpha^2 d^4 < 4\Lambda \rho_{s0}$ now becomes $a_m < a_c$. In terms

FIG. 1: We show the behavior of the scale factor (straight line) and its derivatives $\dot{a}$ (dotted line) and $\ddot{a}$ (dashed line). From this it is clear that this cosmological scenario exhibits an accelerated expansion since there is a stage where $\ddot{a} > 0$.

FIG. 2: We show the behavior of $\rho_x$ as a function of time. The dashed line represents the limit case $a_c = a_m$, i.e. $\alpha^2 d^4 = 4\Lambda \rho_{s0}$, while straight and dotted lines represent a typical case satisfying the condition $a_c > a_m$ since $\alpha^2 d^4 < 4\Lambda \rho_{s0}$. 

where
FIG. 3: We show the behavior of the dark energy state parameter \( w_x \) (dashed line) and the state parameter for the full source content \( w_T \) (straight line). It is clear that the dark energy violates the dominant energy condition while the full source content does not violate it. Note also that the dark energy component remains in the phantom region as it enters to it after crossing the phantom divide.

of the above parameters, \( a_c \) and \( a_m \), the dark energy state parameter \( w_x \) can be written as

\[
w_x = \frac{1 - (a/a_m)^6}{1 - 2(a/a_c)^3 + (a/a_m)^6}.
\] (19)

On the other hand we can consider the state equation for the full source content. For this 2–component system we define the total pressure \( p_T = w_T \rho_T \), where \( p_T = p_x \) and the total energy density \( \rho_T = \rho_m + \rho_x \). This implies that

\[
w_T = \frac{w_x}{1 + r},
\] (20)

where \( r = \rho_m/\rho_x \). In the Fig. 4 we compare the behaviors of the state parameters of the dark energy and the full source content.

Let us consider more in detail the ratio of energy densities of matter and dark energy. The defined above ratio takes the form

\[
r = \frac{\alpha a^3}{\Lambda a^6 - \alpha d^2 a^3 + \rho s_0}.
\] (21)

It is easy to show that this ratio has a maximum at \( a = a_c = (\rho s_0/\Lambda)^{1/6} \) (at this point the \( d^2r/da^2 < 0 \)). This maximum value is given by

\[
r_{\max} = \alpha \sqrt{\frac{\rho s_0}{\Lambda}} \frac{2\rho s_0 - \alpha d^2 \sqrt{\rho s_0/\Lambda}}{2\rho s_0 - \alpha d^2 \sqrt{\rho s_0/\Lambda}}.
\] (22)

It is clear that for cosmological scenarios where \( r_{\max} < 1 \) the dark energy component always dominates over the dark matter during all cosmological evolution. Thus in order to have stages where dark matter dominates over dark energy we have to require that \( r_{\max} > 1 \). So in this case we would have two values for the scale factor where the energy density of dark matter equals the energy density of the dark energy:

\[
a_{\pm} = \left[ \frac{\alpha (1 + d^2) \pm \sqrt{\alpha^2 (1 + d^2)^2 - 4\Lambda \rho s_0}}{2\Lambda} \right]^{1/3}.
\] (23)

Thus for cosmological scenarios where \( r_{\max} > 1 \) at the beginning the dark energy dominates over the dark matter until the scale factor reaches the value \( a = a_- \) where the dark matter energy density equals the energy density of dark energy and it begins to dominate. This stage of domination of the dark matter is prolonged to the moment where the scale factor reaches the value \( a = a_+ \) and the dark energy starts to dominate again over the dark matter (see Fig. 4).
A. Constraints on cosmological parameters

The proposed scenario is characterized by four parameters which may be constrained by the astrophysical observations available up to date. Since we have considered flat FRW cosmological scenarios the dimensionless density parameters are constrained today as

$$\Omega_{m,0} + \Omega_{x,0} = 1. \quad (24)$$

From Eqs. (12)–(14) we have that

$$3H^2 = \frac{\alpha}{a^3} + \frac{\rho_{s0}}{a^6} - \frac{\alpha d^2}{a^3} + \Lambda. \quad (25)$$

Evaluating it today (where we set \( a = 1 \)) we have that

$$\rho_{crit} = 3H^2_0 = \frac{\rho_{s0}}{a^6} - \frac{\alpha \rho_{s0} - \alpha d^2 + \Lambda}, \quad (26)$$

so the two dimensionless density parameters are given by

$$\Omega_{m,0} = \frac{\rho_{m}(a = 1)}{\rho_{crit}} = \frac{\alpha}{\alpha + \rho_{s0} - \alpha d^2 + \Lambda}, \quad (27)$$

$$\Omega_{x,0} = \frac{\rho_{x}(a = 1)}{\rho_{crit}} = \frac{\rho_{s0} - \alpha d^2 + \Lambda}{\alpha + \rho_{s0} - \alpha d^2 + \Lambda}. \quad (28)$$

Now it may be shown that in general, for a flat FRW cosmology deceleration parameter \( q \) is given by

$$q = -\frac{\ddot{a}}{a} = \frac{1}{2} + \frac{3 p_x}{2 \rho_x}. \quad (29)$$

Taking into account that we have \( p_x = \omega_x \rho_x \) and \( \rho_x = \rho_m + \rho_x \) the deceleration parameter is given by

$$q = \frac{1}{2} + \frac{3 \omega_x (1 - \Omega_m)}, \quad (30)$$

and evaluating it today (i.e. \( a = 1 \)) and using (19) we obtain

$$\alpha (1 - d^2)(1 - 2q_0) + 2\rho_{s0}(2 - 2q_0) = 2\Lambda (1 + q_0). \quad (31)$$

Thus, for accelerated scenarios, \( q < 0 \), we require a positive cosmological constant. Other constraint may be introduced by taking into account the moment when the universe has started to accelerate again. In other words this is related to the moment when the Universe starts violating the strong energy condition, i.e. \( \rho + p \geq 0 \) and \( \rho + 3p \geq 0 \). So we must require the inequality \( \rho + 3p < 0 \). Now from condition \( \ddot{a} = 0 \) and the equivalent Friedmann equation

$$\frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 3p), \quad (32)$$

we conclude that \( \rho + 3p = 0 \), which implies that \( \rho_m + 3\omega_x \rho_x = 0 \), obtaining the condition

$$\alpha (1 - d^2)(1 + z_{acc})^3 + 4\rho_{s0}(1 + z_{acc})^6 = 2\Lambda. \quad (33)$$

where the equation \( 1/a = (1 + z) \) was used. Here \( z_{acc} \) is the value of the redshift where the universe starts to accelerate again.

In conclusion we have the four conditions (27), (28), (31) and (33) for the four parameters \( \alpha, \rho_{s0}, d \) and \( \Lambda \) of our model.

Note that from Eqs. (31) and (33) we have that

$$\rho_{s0} = \frac{\alpha(1 - d^2)}{K \rho_{s0}}, \quad (34)$$

where

$$K = \frac{4(1 + q_0) z^2 - 2(2 - q_0)}{(1 - 2q_0) - (1 + q_0) z}, \quad \bar{z} = (1 + z_{acc})^3. \quad (35)$$

Since \( \rho_{s0} \) is related to the energy density of the CDF we must require that \( K > 0 \). So from Eqs. (27), (28), (31) and (33) we have that

$$\alpha = 3H^2_0 \Omega_{m,0}, \quad (36)$$

$$\rho_{s0} = \frac{6H^2_0}{K(2 + \bar{z}) + 2(1 + 2\bar{z}^2)}, \quad (37)$$

$$d^2 = 1 - \frac{2K}{\Omega_{m,0}[K(2 + \bar{z}) + 2(1 + 2\bar{z}^2)]}, \quad (38)$$

$$\Lambda = \frac{3H^2_0 \bar{z}(K + 4\bar{z})}{K(2 + \bar{z}) + 2(1 + 2\bar{z}^2)}, \quad (39)$$

where we have used the Eqs. (21) and (54).

Now the four model parameters need to be constrained. We made this by using the Eqs. (36)–(39) and by considering the increasing bulk of observational data that have been accumulated in the past decade. The present expansion rate of the universe is measured by the Hubble constant. From the final results of the Hubble Space Telescope Key Project to measure the Hubble constant we know that its present value is constrained to be \( H_0 = 72 \pm 8 \text{ km s}^{-1} \text{ Mpc}^{-1} \), or equivalently \( H_0^{-1} = 9.776 h^{-1} \text{ Gyr} \), where \( h \) is a dimensionless quantity and \( 0.64 < h < 0.8 \) [12].

Now assuming a flat universe, i.e. Eq. (24) is valid, Perlmutter et al. [14] found that the dimensionless density parameter \( \Omega_{m,0} \) may be constrained to be \( \sim 0.3 \), implying that \( \Omega_{x,0} \sim 0.7 \) [12, 13], and the present day deceleration parameter \( q_0 \) may be constrained to be \( -1 < q_0 < -0.64 \) [12, 13, 16].

For consistency we need also to compare the age of the Universe determined from our model with the age of the oldest stellar populations, requiring that the Universe is older than these stellar populations. Specifically, the age of the universe \( t_0 \) is constrained to be \( t_0 > 11-12 \text{ Gyr} \) [12, 17].

So let us calculate the age of the universe from Friedmann equation (25). This equation may be written as

$$H^2 = \frac{H_0^2}{a^6} \left( \rho_{s0} + \frac{\alpha a^3(1 - d^2)}{\alpha + \rho_{s0} - \alpha d^2 + \Lambda} \right), \quad (40)$$
obtaining Eq. (21) when the expression (40) is evaluated today. Then the age of the universe may be written as

\[ t_0 = \int_0^{\tau_0} \frac{dz}{H(z)} = \frac{1}{H_0} \int_0^{\tau_0} \sqrt{\frac{\alpha (1-d^2) + \rho_m + \Lambda}{(1+z)^3}} \, dz. \] (41)

In the Table I we include some values obtained from Eqs. (30)–(39) for the parameters \( \alpha, d, \rho_m, \Lambda \) and \( K \) corresponding to some given values of the parameters \( H_0, q_0 \) and \( z_{acc} \). For the Hubble parameter \( H_0 \) is considered both possible values \( H_0- \) for \( h = 0.64 \) and \( H_0+ \) for \( h = 0.8 \), and for the matter dimensionless density parameter we have taken \( \Omega_{m,0} = 0.3 \). The last two columns represent the age of the universe determined from the model parameters for \( h = 0.8 \) and \( h = 0.64 \) respectively. Clearly in the proposed model there are configurations which are not allowed from oldest stellar age since there exist combinations of the parameters \( \alpha, d, \rho_m, \Lambda \) which give \( t_0 > 11-12 \) Gyr satisfying the stellar population constraints.

| \( H_0 \) | \( q_0 \) | \( Z_{acc} \) | \( \alpha \) (in units of \( 3H_0^2 \)) | \( d^2 \) | \( \rho_m \) (in units of \( 3H_0^2 \)) | \( \Lambda \) (in units of \( 3H_0^2 \)) | \( K \) | \( H_{0+,t_0} \) (Gyr) | \( H_{0-,t_0} \) (Gyr) |
|--------|--------|--------|------------------|--------|------------------|------------------|--------|------------------|------------------|
| \( H_0+ \) -0.68 | 0.587 | 3.86 \times 10^{-2} | 0.38 | 1.7 \times 10^{-4} | 10^{-2} | 14 | 10 | 12 |
| \( H_0- \) -0.68 | 0.587 | 2.85 \times 10^{-3} | 0.38 | 1.3 \times 10^{-4} | 7.6 \times 10^{-3} | 14 | 10 | 12 |
| \( H_0+ \) -0.68 | 0.94 | 3.86 \times 10^{-3} | 0.29 | 1.05 \times 10^{-6} | 10^{-2} | 2619 | 12 | 15.7 |
| \( H_0- \) -0.68 | 0.94 | 2.85 \times 10^{-3} | 0.29 | 7.73 \times 10^{-7} | 7.5 \times 10^{-3} | 2619 | 12.5 | 15.7 |
| \( H_0+ \) -0.9 | 0.6 | 3.9 \times 10^{-3} | 0.964 | 3.5 \times 10^{-4} | 1.2 \times 10^{-2} | 0.38 | 10 | 12.7 |
| \( H_0- \) -0.9 | 0.6 | 2.8 \times 10^{-3} | 0.964 | 2.6 \times 10^{-4} | 9.1 \times 10^{-3} | 0.38 | 10 | 12.7 |
| \( H_0+ \) -0.9 | 1 | 3.9 \times 10^{-3} | 0.815 | 7.2 \times 10^{-5} | 1.2 \times 10^{-2} | 9.9 | 12.5 | 15.6 |
| \( H_0- \) -0.9 | 1 | 2.8 \times 10^{-3} | 0.815 | 5.3 \times 10^{-5} | 8.9 \times 10^{-3} | 9.9 | 12.5 | 15.6 |

TABLE I: In this table we show some values of the model parameters obtained for given \( H_0, q_0 \) and \( z_{acc} \) (\( \Omega_{m,0} = 0.3 \)).

### B. The effect of the d–parameter

Now it is interesting to get some insights concerning the nature of the \( d \)-parameter for studying the effect on the cosmological evolution of the negative term \( -\alpha d^2/\alpha^3 \) in the Eq. (14) for the dark energy component \( \rho_x \). It can be shown that in the proposed cosmological scenario the dominant energy condition (DEC) is violated thanks to the presence of this parameter. Effectively, if \( d = 0 \) the dark energy state parameter \( \omega_x \) and the state parameter of the full source content \( \omega_x \) are given by

\[ \omega_x = \frac{\rho_\text{m}}{\rho_\text{m} + \Lambda 6^2} \quad \omega_x = \frac{\rho_\text{m} - \Lambda 6}{\rho_\text{m} + \Lambda 6 + \alpha 3}, \] (42)

respectively. From these expressions we see that always \(-1 < \omega_x < 1 \) which implies that now the dark energy component satisfies DEC, as well as \( \omega_x \). Their general behavior is showed in the Fig. 5 (compare with Fig. 3).

However, the fulfilment of the DEC does not imply that the universe has a decelerated expansion. From Eq. (32) we may write that

\[ \frac{6\ddot{a}}{a} = -(\rho_x + 3p_x) = -\left( \frac{4\rho_m + \alpha (1 - d^2)}{a^3} - 2\Lambda \right), \] (43)

and putting \( d = 0 \) we see that the accelerated expansion is realized if \( a > a_{acc} = (\alpha + \sqrt{\alpha^2 + 32\Lambda \rho_m})/4\Lambda \). Other property of the \( d \)-parameter to be considered is its effect on the deceleration parameter \( q_0 \). From Eq. (38), which is independent of the Hubble parameter \( H_0 \), and using Eqs. (35) we can express the deceleration parameter \( q_0 \) as a function of the \( z_{acc} \) obtaining

\[ q_0(z_{acc}) = \frac{1 - \frac{\bar{z}}{2} \left( \frac{6d^2 \Omega_{m,0} \bar{z}}{2(2 \bar{z}^2 + 1)} + 4 \bar{z} + 3d^2 \Omega_{m,0} \bar{z} - 3 \Omega_{m,0} + 4 \right) \bar{z}}{6 - 6 \Omega_{m,0} \bar{z} + 4 \bar{z} + 3d^2 \Omega_{m,0} - 3 \Omega_{m,0} + 4}, \] (44)

where as before \( \bar{z} = (1 + z_{acc})^3. \) We can see that in this case \( q_0(z_{acc}) \) rapidly tends to the value

\[ q_0(\infty) = -1 + \frac{3}{2}(1 - d^2)\Omega_{m,0}, \] (45)
obtaining for $d = 0$ and $\Omega_{m,0} = 0.3$ the value $q_{0,\infty} = -0.55$, and from this value the deceleration parameter reaches the value $q_{0,\infty} = -1$ for $d \approx 1$ (see Fig. 4). So the parameter $d$ affects directly the range of validity of the deceleration parameter which is constrained to be in the range $-1 < q_0 < 0$. Note that the value $q_{0,\infty} = -1$ for $d = 1$ is independent of the value of the dimensionless density parameter $\Omega_{m,0}$.

IV. CONCLUDING REMARKS

Since today the observations constraint the value of $\omega$ to be close to $\omega = -1$, we have considered broader cosmological scenarios in which the equation of state of dark energy changes with time. The two principal ingredients of the model are a stiff fluid which dominates at early time and a CDF. The positive part of the latter was associated with a dark matter component while its negative part was considered as a part of the dark energy component and was the responsible that the effective dark energy density crossed the phantom divide (see and compare the Figs. 3 and 5). At the end, this cosmological model becomes accelerated recovering the standard $\Lambda$CDM cosmology.

In general, the model may be seen as a continuation of the inflation era. In the inflationary paradigm the scalar field $\Phi$, driven by the potential $V(\Phi)$, generates the inflationary stage. In the slow roll limit $\dot{\Phi}^2 \ll V(\Phi)$, with $\omega_{\Phi} \approx -1$, we have a superluminal expansion while in the kinetic–energy dominated limit $\dot{\Phi}^2 \gg V(\Phi)$, with $\omega_{\Phi} \approx 1$, we have a stiff matter scenario characterized by a subluminal expansion. Taking into account that our model has a variable equation of state, we can think it as a transient model which interpolates smoothly between different barotropic eras, as for instance, radiation dominated era, matter dominated era and so on. In other words, from Eqs. (19)–(21) we see that $w_x \rightarrow 1$ (and $w_T \rightarrow 1$) for $a \rightarrow 0$ implying that the energy density $\rho_x$ behaves like $1/a^6$ and matching, after inflation, with the kinetic–energy mode of the scalar field $\rho_{\Phi} \propto 1/a^6$. Now from Eq. (19) we see that $\rho_x$ passes through a radiation dominated stage (i.e. $\omega_x = 1/3$) for

$$a_{rad} = \frac{a_m}{2^{2/3}} \left[ \left( \frac{a_m}{a_c} \right)^3 + \sqrt{8 + \left( \frac{a_m}{a_c} \right)^6} \right], \tag{46}$$

behaving like $\rho_x \approx 1/a_{rad}^4$ and dominating over $\rho_m$. After that, at $a = a_m$, we have $w_x = 0$ and the effective dark component behaves as a pressureless source obtaining a matter–dominated stage. Finally the model evolves from this state to a vacuum-energy dominated scenario. It is interesting to note that in the a matter–dominated stage if the condition (11) is fulfilled then the total energy density is given by $\rho_x = \alpha(1 - d^2)/a_m^3 + 2\Lambda$, and if $a_m = a_c$ then $\rho_x = \alpha/a_m^3$, being $\rho_x = 0$, implying that we have

FIG. 5: We show the behavior of $w_x$ and $w_T$ as a function of the scale factor. The dashed line represents the behavior of the dark energy state parameter, while straight line represents the behavior of the state parameter of the full source content. Both satisfy the DEC since in this case $d = 0$.

FIG. 6: We show the behavior of the deceleration parameter $q_0$ as a function of $z_{\text{acc}}$ for three values of the parameter $d$ (0, 1/2, 1) with $\Omega_{m,0} = 0.3$. We can see that the deceleration parameter rapidly tends to the values $(-0.55, -0.6625, -1)$ respectively.
at this stage only the dark matter component.

The above results indicate that a cosmological sce-

nario based on a CDF component and the effective multi-
fluid configuration \( P_c \) can, in certain cases, reproduce the
quintessential behavior (see Figs. [3] and [5]). In fact, the
state parameter of the total matter content \(-1 < w_r < 1\)
is constrained the same as the state parameter of the
scalar field in quintessence models. In this manner we
avoid the use of scalar fields and particular classes of
potentials for describing the dark energy component.

Finally, all the parameters of the model has been ex-
pressed in terms of the observable quantities which may
be constrained by the astrophysical observational data.
In effect, in the Table 2 some values of the model param-
eters \( \alpha, d, \rho_{s0} \) and \( \Lambda \) were included, which correspond
to some given values of the parameters \( H_0, q_0, z_{acc} \) and
\( \Omega_{m,0} \) constrained by astrophysical observations.

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