Designing problem-solving questions to measure mathematical thinking type modeling

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Abstract. This research aims to describe the design results of problem-solving which can be used to measure mathematical thinking type modeling. This is design research consisting: Preliminary Design, Focus Group Discussion (FGD), Trial, Observation & Interview, and Retrospective Analysis. The conclusion is the questions are problem-solving which can be used to measure mathematical thinking type modeling. FGD and test trial analysis show that the problem can be used to measure mathematical thinking type modeling. From the results of the design and FGD, the questions which have been designed have met the modeling indicators which are identifying problems, making assumptions, and mathematical models, analyzing solutions, iterating, implementing models. From test trial analysis, modeling indicators can be seen from the answer of the subjects. For example from the question number 2, subjects identify the problem by making a bonus arrangement obtained by an online motorcycle driver, and they assume the bonus in the first month in the form of variables. After that, through the iteration process and analyzing each iteration the subjects make a mathematical model, bonus patterns for online motorcycle driver in the n-th month, then subjects implement the model created so that the last month bonus obtained.

1. Introduction
Mathematics learning at schools must include the balance between mathematics with numbers and without numbers such as Figures, tables, patterns and so forth so it is very important to direct students' minds [1]. In learning mathematics at schools, there are two activities that should be done by the teacher, namely math counting activities or can be called empirical thinking and mathematical thinking activities [2]. This shows that in school learning there must be a balance between math and mathematical thinking. However, in Indonesia learning at school has more mathematical calculations or empirical thinking than mathematical thinking.

Empirical thinking or mathematical numeracy is more related to the use of facts, concepts and processes of mathematics. Whereas mathematical thinking is not always bound to mathematical procedures where in the mathematical thinking process there is a strategy as a frame of mind for students to arrive at the final goal. Mathematical thinking means the activity of developing a mathematical perspective, assessing the process and applying it [1], where the mathematical thinking process has various types, namely abstraction, reasoning, proof, representation, symbolization, modeling and mathematical [3].

The process of thinking mathematically is very important to be owned by students in order to develop students’ knowledge because the process is what students use to find solutions to problem solving
problems. This is in line with Raven [4], Lesh and Harel [5], Katagiri [6], Breen and O’shea [7], Wahyuningrum and Suryadi [8] which states that mathematical thinking is very important to be developed in students because it is very useful for solving problem solving.

In solving problems, students are faced with a problem and asked to find a solution to the problem. This is in line with Teare, Butterworth, Thwaites, and Day et al, which states that in solving problems one is asked to find a solution that might be a calculated value or a way to do something where the problem might have more than one solution [9,10,11].

The average PISA 2012 Mathematics score of Indonesian students is 375, this score is very far with the average international score of 494 and in PISA 2015 the average score is 386, this score is still far below the international average of 490 [12,13]. This is still far below expectations and relatively low.

This fact shows that education in Indonesia is still experiencing major problems that have not been resolved by the government. The learning process in school is one of the factors of this problem. Therefore, the government is investigating the causes of these problems. Is this due to the lack of mathematical thinking of students in solving a problem. This is what needs further investigation. One type of mathematical thinking that is currently becoming a trend is modeling. Modeling is the process of understanding mathematics through real situations [14], by representing the problem in mathematical terms [15,16]. There are several terms in modeling, namely concretization, objectification, mathematical sense and estimation of result [3]. The modeling process consists of several stages, namely identifying problems, making assumptions, making mathematical models, analyzing solutions, iterating and implementing models [17]. Modeling is very important because it has a role in measuring student success in mathematics [18]. So this article entitled "Designing Problem-Solving Questions to Measure Mathematical Thinking Type Modeling".

2. Research methodology
This is design research because it aims to describe the design results of problem solving that can be used to measure mathematical thinking type modeling. The subject of this study consisted of 3 people (initials) with the criteria that the subject has studied the material related to the problem designed by the researcher and the willingness of the subject. The implementation of this research consists of several stages: Preliminary design, Focus Group Discussion (FGD), Trial, Observation and Interview and Restropective Analysis.

3. Results and discussion
3.1. Preliminary design
After conducting a literature review regarding problem solving. Researchers looked for some examples of problem solving problems that can measure mathematical thinking type modeling as inspiration for researchers to design questions. In problem number 1, the researcher is inspired by the problem with the card game context which can be solved by splitv or by working backwards [19]. So the idea emerged from the researchers to use the context of the game race "Race in Melody" with the game conditions and rules that had been designed by researchers.

In question number 2, the researcher was inspired by the problem that served $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{49} = 0.5$ [19]. From these problems the researcher changes the direction of the problem and use the real context in the problem.

3.2. Focus Group Discussion (FGD)
On 25 August 2018, the FGD was conducted together with an unscheduled mathematics lecturer named Weni Dwi Pratiwi, S.Pd. M.Sc. according to him the two questions included problem solving that could measure mathematical thinking type modeling. In terms of language, he corrects words that are typographical, and he also advises that language use is simpler. On August 29, 2018 the FGD was conducted together with an UNSRI mathematics lecturer, Scristia, S. Pd, M. Pd. according to him the
problem has been categorized as problem solving which can measure mathematical thinking type modeling. However, the question must be corrected in language and he explained the location of the improvement. In question number 1, he said that the context could not be understood so that the storyline must be corrected. Whereas in matter number 2, he suggested replacing the numbers because it was too small so it was not realistic to use bonus contests for online motorcycle taxi drivers. On September 1, 2018, the FGD was conducted with one of the teachers at the Kusuma Bangsa Palembang High School, namely Feriyanto, S.Pd. According to him, problem number 1 is too difficult so the storyline must be corrected and clarified to make it easier to understand and he agrees to problem number 2 because it also involves factorial or SPLTV in the alternative solution. Then, on September 15, 2018, the researchers conducted another FGD with Weni Dwi Pratiwi, S.Pd. M.Sc and Feriyanto, S, Pd at different times to show improvements to the problem and they stated that the matter could be used. So, the problem changes as in the following table:

| Table 1. Problem-Solving Design to measure mathematical thinking type modeling after FGD. |
| No. | Questions |
| --- | --- |
| 1. | Amel, Bastian and Fay play in games "Berpacu dalam Melodi". They just finished the guessing song and have got their scores. Next, they play the round of lyrics. The lyric content round consists of 3 rounds, participants who fail to answer will give the score to the participants who can answer as many as the scores of participants who can answer. Pay attention to the information at the end of the contents of the following lyrics! |
| | Round | Who can answer |
| | 1 | Fay |
| | 2 | Bastian |
| | 3 | Amel |
| | Thus, the final score obtained by each participant is Amel 60, Bastian 40 and Fay 140. |
| | a. Make a mathematical model of the initial score (the score in the song round) of each participant! |
| | b. Make a mathematical model of the scores of each participant in the round of lyrics in rounds I, II and III! |
| | c. Determine the initial score (the score in the round song) of each participant using the model you created! |
| 2. | An online motorcycle taxi application will give a bonus every month for the first 2 years joining as an online motorcycle taxi driver, provided that the driver is able to penetrate the target set every month. Here are the bonus provisions that will be given by the application! |
| | Month | Bonus earned (in rupiah) |
| | 1 | certain amount |
| | 2 | More than 60,000 from the first month |
| | 3 | More than 80,000 from the second month |
| | 4 | More than 100,000 from the third month |
| | etc |
| Andy has worked 2 years as an online motorcycle taxi in the application and has always managed to reach the target. On the 12th month he received a bonus of Rp1,820,000. |
| | a. Determine the bonus Andy got in the nth month! |
| | b. What is the last bonus Andy got? |
3.3. Trial, observation and interview
The questions of the design were tested on research subjects with financial HD, IF and MA. The results of observations regarding the answers of research subjects are associated with the results of interviews obtained.

3.4. Retrospective analysis
Figure 1 show in problem number 1, the subject of HD research has been able to identify problems. This can be seen from what he wrote about the information contained in the problem. From the results of the interview, he was also able to explain his understanding of the question in his own language. This is what is called concretization [20].

Figure 1. Result of answer number 1 subject HD.

Figure 1, he made an assumption by calculating the initial score of each participant in the variables namely x, y and z. This is what is called objectification where from the context of the gameshow comes the idea of someone to make an assumption. After making assumptions, the student models the scores of each participant in each round through his understanding of the problem situation. This subject is also said by the subject through the interview. At this stage, the idea of spontaneity appears to model real problems into mathematical situations or can be called mathematical sense [21].

Figure 2, the subject analyzed the model he had made. Linking with information that is known in the problem and he determines the next step he will take to solve the problem. In this case, the subject uses the SPLTV concept to determine the initial score of each participant. However, inaccuracy makes the results obtained by students less precise. And from the results of the interview, the subject said that he did not interpretate (iterating) the results of the answers, this is the cause of the subject did not improve the location of the inaccuracies he made.
Figure 2. Result of answer number 1 subject HD.

Figure 3 show in problem number 2, the IF subject also identifies the problem first. In the interview, he was also able to explain his understanding of the problems presented using his own language. In modeling this is called concretization [20] where one is able to understand the problem and concretize the abstract problem in its own way.

Figure 3 show in making assumptions, IF subjects have different understandings with HD subjects. IF subjects prefer to first calculate each bonus increase until the 12th month. Then, he made a bonus
arrangement obtained each month by calculating the difference between the 12th month bonus and the bonus increase for the month so that a bonus was obtained in the 11th month. And so on so that he gets a bonus arrangement from the first month to the 12th month. This is called objectification [22] where from the context of the bonus of an online motorcycle taxi driver, the subject thinks of an idea as the first step for him to solve the problem.

Figure 3, the IF subject begins to observe and analyze the bonus structure he made to make the bonus model obtained in the nth month. He first simplifies number by making it multiplied by $10^4$. Then after that, he thought how many times did he make a bonus in the first, second, third and so on.

So that in figure 4, the subject IF gets a bonus model in the nth month. This process can occur because of the mathematical sense [21] possessed by the IF subject, that is, he has the idea of spontaneously forming a bonus pattern in the nth month. In the interview, the subject of IF poured out what he thought, namely as follows:

Researcher: "What do you think about so you get a pattern like the one you wrote down?"

IF: "Confused as well as how to deliver it, the book was searched first. The first tribe, the second tribe and so on, see what it might be like, after that it was changed to $n$ because it was the people who wanted to find the nth tribe."

Figure 4, the IF subject implements the model to find out the last bonus obtained by the online motorcycle taxi driver.

![Bonus Pattern](image)

**Figure 4.** Result of next answer of number 2 subject IF.

4. Conclusion

The conclusion is the problem can be used to measure mathematical thinking type modeling. From the results of the design and FGD, the questions which have been designed have met the modeling indicators which are identifying problems, making assumptions, and mathematical models, analyzing solutions, iterating, implementing models. From test trial analysis, modeling indicators can be seen from the answer of the subjects. For example from the question number 2, subjects identify the problem by making a bonus arrangement obtained by an online motorcycle driver, and they assume the bonus in the first month in the form of variables. After that, through the iteration process and analyzing each iteration the subjects make a mathematical model, bonus patterns for online motorcycle driver in the $n$-th month, then subjects implement the model created so that the last month bonus obtained.

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