Relativistic Hartree-Fock theory. Part I: density-dependent effective Lagrangians

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Effective Lagrangians suitable for a relativistic Hartree-Fock description of nuclear systems are presented. They include the 4 effective mesons \( \sigma, \omega, \rho \) and \( \pi \) with density-dependent meson-nucleon couplings. The criteria for determining the model parameters are the reproduction of the binding energies in a number of selected nuclei, and the bulk properties of nuclear matter (saturation point, compression modulus, symmetry energy). An excellent description of nuclear binding energies and radii is achieved for a range of nuclei encompassing light and heavy systems. The predictions of the present approach compare favorably with those of existing relativistic mean field models, with the advantage of incorporating the effects of pion-nucleon coupling.

PACS numbers: 21.30.Fe, 21.60.Jz, 21.10.Dr, 24.10.Cn, 24.10.Jv

I. INTRODUCTION

The most actively investigated fields in present days nuclear structure and reaction studies are the production and exploration of isotopes with extreme neutron-to-proton number ratios, the so-called exotic nuclei \([1, 2, 3, 4, 5, 6, 7]\). In the last 20 years, technological breakthroughs in producing radioactive nuclear beams have opened up entirely new and exciting frontiers for nuclear physics \([8]\). With the development of unstable nuclear beams \([2, 5]\), a number of unexpected phenomena have been discovered like neutron and proton skins and halos \([3, 10, 11, 12]\), modifications of shell closures \([13]\), soft excitation modes \([14, 15]\), the enhancement of fusion cross sections induced by the extended matter distributions \([16, 17]\), etc. With further developments, many other new features are likely to be found.

In the universe, novae and supernova explosions leading to neutron stars, X-ray and gamma-ray bursts, all depend upon reactions involving nuclei that do not naturally occur on Earth. There still remain many mysteries in the origin of elements \([18]\), with the rapid neutron-capture process accounting for the formation of about half of \( A > 60 \) stable nuclei in nature. The \( r \)-process is the most
complex nucleosynthesis process from the astrophysics as well as nuclear physics point of view \[19, 20\]. Understanding the time scales and energies for such processes, and hence the stellar evolution itself, should be connected with the understanding of the exotic nuclear systems, either in astrophysical conditions or in the extreme nuclear environment. The exotic nuclei thus provide also a new testing standard for our understanding of phenomena of astrophysical interest.

Many of the above physics issues can be successfully studied in the framework of self-consistent mean field approaches. Non-relativistic Hartree-Fock approaches based on Skyrme type forces \[21, 22\], local density approximations of the Brueckner G-matrix \[23, 24\], and effective Gogny type forces \[25\] have been developed. A quantitative understanding of nuclear matter as well as finite nuclei was thus obtained. In parallel, the relativistic Brueckner-Hartree-Fock approach was applied to nuclear matter, at first without full self-consistency \[26\], and then fully self-consistent calculations were carried out by Brockmann and Machleidt \[27, 28\], and by Ter Haar and Malfliet \[29\]. Based on a one-boson-exchange interaction and combined with relativistic Brueckner-Hartree-Fock approach, the relativistic approach offers an appealing framework to investigate and explore macroscopic properties of hot and dense nuclear matter \[29\]. Using a density-dependent parametrization of the Dirac-Brueckner G-matrix in nuclear matter, an effective Dirac-Brueckner-Hartree-Fock model for finite nuclei was achieved by Boersma and Malfliet \[30, 31\], which also offers a wide range of applications in the nuclear physics and astrophysics domains.

During the past years, the relativistic mean field (RMF) theory \[32, 33\] has received wide attention due to its successful description of numerous nuclear phenomena \[34, 35, 36, 37, 38, 39\]. The RMF can be viewed as a relativistic Hartree approach with a no-sea approximation. In the framework of the RMF, the nucleons interact via the exchanges of effective mesons, and photons. For nuclear structure applications, the relevance of relativity is not the need for relativistic kinematics but that of a covariant formulation which maintains the distinction between scalar and vector fields (more precisely, the zeroth component of Lorentz four-vectors). The representations with large scalar and vector fields in nuclei, of the order of a few hundred MeV, provide simpler and more efficient descriptions than non-relativistic approaches \[40, 41, 42\]. The dominant evidence is the spin-orbit splitting. Other evidence includes the density dependence of the optical potential, the observation of approximate pseudo-spin symmetry, etc. With a very limited number of parameters adjusted to reproduce selected empirical observables, the RMF theory can describe very satisfactorily general properties such as the masses and radii of nuclei in the whole periodic table, or specific aspects like the observed kink in the isotopic shifts of the Pb-region \[43\]. It gives naturally the spin-orbit potential, the origin of the pseudo-spin symmetry \[44, 45\], as a relativistic symmetry \[46, 47, 48\], the spin symmetry in the anti-nucleon spectrum \[49\], etc. It also performs quite well for nuclei far away from the line of $\beta$-stability with proper treatment of the pairing correlation and continuum effects \[39, 50, 51, 52\].
In spite of the success, there are still some shortcomings in the RMF theory. A too small value of the symmetry energy would be obtained if the coupling constant $g_\rho$ takes on its experimental value. Actually, important contributions due to the exchange terms are missing in the model. Among the effective mesons which mediate the nucleon interaction, the pion meson might be the most important one. Because of the limit of the approximation itself, the one pion exchange cannot contribute within the framework of the RMF if parity is to be conserved. The RMF cannot take into account the effective spin-spin interaction coming from the nucleon density through the anti-symmetrization of the wave function. This effect is more important in spin-unsaturated systems.

Early attempts to investigate the structure of nuclear matter and finite nuclei in the relativistic Hartree-Fock (RHF) approximation have been made. The effective Lagrangians did not include density dependence or meson self-couplings. They gave a satisfactory description of the charge densities of magic nuclei but failed on the incompressibility. The nuclei were not bound enough, this defect being more important for light nuclei. By introducing additional density-dependent terms in the Lagrangian density, better agreement with the experiment and significant improvement on the incompressibility were obtained. The density dependence was simulated by the nonlinear self-couplings of the $\sigma$-field, but chiral symmetry is not conserved. To recover the chiral symmetry, the non-linear self-interaction (NLSI) of the scalar field with zero-range was introduced in Ref. With the zero-range limit, the nonlinear self-couplings are expressed in terms of the products of six and eight nucleon spinors, where the exchange contributions can be evaluated by the Fierz transformation. It is shown that, in the RHF formalism the explicitly isovector-independent NLSI generates strongly density-dependent scalar, vector and tensor nucleon self-energies. The description of nuclei thus obtained is in reasonable agreement with experimental data.

Although some progress have been obtained in the relativistic Hartree-Fock description of nuclear systems, it is still a long way compared to the RMF, either on the quantitative descriptions of nuclear systems or their extrapolation to the exotic region. Especially compared with the wide applications of RMF, the disparity seems more remarkable. Of course, the technical and numerical difficulties are increased strongly with the inclusion of the exchange terms. On the other hand, there is still no appropriate effective Lagrangian for the RHF approach, which does not break the chiral symmetry nor increase the complexity of the theory itself.

Based on the above considerations, we have constructed a density-dependent relativistic Hartree-Fock (DDRHF) theory by quantizing a Hamiltonian which contains density-dependent meson-nucleon couplings. For the first time, the RHF approach is systematically applied to effective Lagrangians with density-dependent meson-nucleon couplings. The main finding is that, with a number of adjustable parameters comparable to that of RMF Lagrangians, one can obtain a good quantitative description of nuclear systems without dropping the Fock terms. The pseudo-spin symmetry in finite
nuclei within our DDRHF model has been investigated in Ref. [59]. In this paper we present the method for obtaining the DDRHF effective Lagrangians and the results for the selected nuclei. Systematical applications of the DDRHF in nuclear matter and finite nuclei will be shown in a forthcoming paper [60].

This paper is organized as follows. The derivation of the DDRHF is briefly introduced in Section II. In Section III are given the parametrizations of DDRHF and some initial applications in nuclear matter and in the set of selected nuclei. A short summary is given in Section IV. The calculations of center-of-mass corrections for energies, radii and charge densities are explained in the Appendix.

II. THEORY FRAMEWORK

A. General Formalism

The general framework can be found in several papers published in the literature. We follow closely the formalism and notations of Refs. [54, 55]. The detailed expressions for nuclear matter and finite nuclei in the spherical symmetry approximation were already given in these two references, except that they were established for the case of density-independent coupling constants. Therefore, we recall here only the main steps and we briefly explain how to treat the additional rearrangement terms brought about by our density-dependent Lagrangian.

The starting point is an effective Lagrangian density $\mathcal{L}(x)$ constructed from the degrees of freedom associated with the nucleon ($\psi$), two isoscalar mesons ($\sigma$ and $\omega$), two isovector mesons ($\pi$ and $\rho$) and the photon ($A$) fields. The stationarity condition of the action integral $\int d^4x \mathcal{L}(x)$ variations of the physical fields $\phi$ ($\phi = \psi, \sigma, \omega, \rho, \pi$ and $A$) leads to the Euler-Lagrange equations, from which one can deduce the equations of motion for the meson, photon and nucleon fields [55]. The meson and photon fields obey inhomogeneous Klein-Gordon equations and a Proca equation with source terms, respectively, whereas the nucleon field obeys a Dirac equation.

The Hamiltonian operator, i.e., the (00) component of the energy-momentum tensor $T^{\mu\nu} \equiv \frac{\partial \mathcal{L}}{\partial (\partial_{\mu} \phi_i)} \partial^{\nu} \phi_i - g^{\mu\nu} \mathcal{L}$, can be formally obtained through the general Legendre transformation

$$\mathcal{H} = T^{00} = \frac{\partial \mathcal{L}}{\partial \dot{\phi_i}} \dot{\phi_i} - \mathcal{L}. \quad (1)$$

The next step is to use the Klein-Gordon and Proca equations to formally express the meson and photon fields in terms of nucleon fields $\psi$ only. Thus, one would be able to obtain the Hamiltonian in nucleon space in a form suitable for studies of nuclear systems. We make the simplifying assumption of neglecting the time component of the four-momenta carried by the mesons, i.e., the meson fields are assumed to be time independent. This amounts to neglect the retardation effects. The energies involved are small compared to the masses of the exchanged mesons, so that this approximation should
be valid for the \( \sigma \), \( \omega \) and \( \rho \)-induced interactions, and also, to a lesser extent for the pion. Then, the meson propagators have the usual Yukawa form,

\[
D_\sigma(x_1, x_2) = \frac{1}{4\pi} e^{-m_\sigma|x_1 - x_2|},
\]

(2)

and the nucleonic Hamiltonian can be written as

\[
H = \int d^3x \left[ \bar{\psi} \left[-i\gamma \cdot \nabla + M \right] \psi \right] + \frac{1}{2} \int d^3x_1 d^3x_2 \sum_{i=\sigma,\omega,\rho,\pi} \bar{\psi}(x_1) \psi(x_2) \Gamma_i(1, 2) D_i(x_1, x_2) \psi(x_2) \psi(x_1),
\]

(3)

where the interaction vertices are generalizations of those of Ref. [55]:

\[
\Gamma_\sigma(1, 2) = -g_\sigma(1) g_\sigma(2),
\]

(4a)

\[
\Gamma_\omega(1, 2) = +g_\omega(1) \gamma^\mu(1) g_\omega(2) \gamma^\mu(2),
\]

(4b)

\[
\Gamma_\rho(1, 2) = +g_\rho(1) \gamma^\mu(1) \bar{\tau}(1) \cdot g_\rho(2) \gamma^\mu(2) \tau(2),
\]

(4c)

\[
\Gamma_\pi(1, 2) = -\left[ \frac{\hbar}{m_\pi} \bar{\tau}_5 \gamma^\mu \partial^\nu \right]_1 \cdot \left[ \frac{\hbar}{m_\pi} \bar{\tau}_5 \gamma^5 \gamma^\mu \partial^\nu \right]_2,
\]

(4d)

\[
\Gamma_A(1, 2) = +\frac{e^2}{4} \left[ \gamma^\mu(1 - \tau_3) \right]_1 \left[ \gamma^\mu(1 - \tau_3) \right]_2.
\]

(4e)

The nucleon field operators \( \psi \) and \( \psi^\dagger \) are expanded as [55]:

\[
\psi(x) = \sum_i \left[ f_i(x) e^{-ic_i^\dagger x} c_i + g_i(x) e^{ic_i^\dagger x} d_i^\dagger \right],
\]

(5a)

\[
\psi^\dagger(x) = \sum_i \left[ f_i^\dagger(x) e^{ic_i^\dagger x} c_i + g_i^\dagger(x) e^{-ic_i^\dagger x} d_i \right],
\]

(5b)

where \( f_i(x) \) and \( g_i(x) \) are complete sets of Dirac spinors, \( c_i \) and \( c_i^\dagger \) represent annihilation and creation operators for nucleons in a state \( i \), while \( d_i \) and \( d_i^\dagger \) are the corresponding ones for antinucleons. Since we will restrict ourselves to the Hartree-Fock approximation level and study the exchange corrections to the mean field approach, we keep the same level of approximation, the so-called no-sea approximation, i.e., the \( d \) and \( d^\dagger \) terms are omitted in the expansions. Their inclusion leads to divergences and requires a cumbersome renormalization procedure [34]. The \( \{ f_i \} \) states will be determined by the self-consistent HF scheme.

The trial ground state \( |\Phi_0\rangle \) is the HF Slater determinant built on the lowest \( \{ f_i \} \) states. The energy functional, i.e., the expectation value of the Hamiltonian can be obtained as

\[
E \equiv \langle \Phi_0 | H | \Phi_0 \rangle = \langle \Phi_0 | T | \Phi_0 \rangle + \sum_i \langle \Phi_0 | V_i | \Phi_0 \rangle.
\]

(6)

For the two-body interaction parts \( \langle \Phi_0 | V_i | \Phi_0 \rangle \), one obtains two types of contributions, the direct (Hartree) and exchange (Fock) terms. For example, the \( \sigma \)-meson contributions are:

\[
E^D_\sigma = -\frac{1}{2} \int dx \ g_\sigma \langle \Phi_0 | \rho_\sigma | \Phi_0 \rangle \int dx' g_\sigma \langle \Phi_0 | \rho_\sigma | \Phi_0 \rangle D_\sigma(x, x')
\]

(7a)

\[
E^E_\sigma = +\frac{1}{2} \int dx \int dx' \sum_{\alpha,\beta} [g_\sigma f_\alpha f_\beta]_x D_\sigma(x, x') [g_\sigma f_\beta f_\alpha]_x,
\]

(7b)
where \( \rho_s \) is the scalar density. The states \( \{ f_\alpha \} \) are solutions of a Dirac equation containing a self-energy term \( \Sigma \). The stationarity of the energy \( \Sigma \) with respect to variations of the \( \{ f_\alpha \} \) determines self-consistently \( \Sigma \).

To be complete, we would like to mention about the contributions from the one-pion exchange. At the Hartree-Fock level, it is well known from general arguments (chiral symmetry) and from the construction of the NN potential (with pair suppression mechanism) that one should adopt the pseudo-vector coupling to obtain reasonable results in the one-pion exchange approximation. Here, we use the pseudo-vector coupling for the pion, as it is done in Ref. [55]. At the Hartree (RMF) level, the contributions from the one-pion exchange are reduced to zero if the variational space respects parity conservation while in RHF these contributions appear naturally.

In the recent years, the RMF with density-dependent meson-nucleon couplings (DDRMF) has been actively developed [61, 62, 63, 64, 65, 66]. In this work, the density-dependent relativistic Hartree-Fock (DDRHF) theory is derived by jointly using the RHF approach with an effective Lagrangian whose meson-nucleon couplings are density-dependent. Similarly to the DDRMF, the coupling constants \( g_\sigma \), \( g_\omega \), \( g_\rho \) and \( f_\pi \) are taken as functions of the baryonic density \( \rho_b \), the zeroth component of the nucleon current \( j^\mu = \bar{\psi} \gamma^\mu \psi \). For the \( \sigma \)- and \( \omega \)-mesons, the density-dependent behaviors of \( g_\sigma \) and \( g_\omega \) are chosen as

\[
g_i(\rho_b) = g_i(\rho_0) f_i(\xi) \quad \text{for } i = \sigma, \omega \tag{8}
\]

where

\[
f_i(\xi) = a_i \frac{1 + b_i(\xi + d_i)^2}{1 + c_i(\xi + d_i)^2}, \tag{9}
\]

is a function of \( \xi = \rho_b / \rho_0 \), and \( \rho_0 \) denotes the saturation density of symmetric nuclear matter. For the functions \( f_i(x) \), five constraint conditions \( f_i(1) = 1, f_i''(1) = f_i''(1) \) and \( f_i''(0) = 0 \) are introduced to reduce the number of free parameters.

For \( g_\rho \) and \( f_\pi \), an exponential density-dependence is adopted

\[
g_\rho(\rho_b) = g_\rho(0) e^{-a_\rho \xi}, \tag{10a}
\]

\[
f_\pi(\rho_b) = f_\pi(0) e^{-a_\pi \xi}. \tag{10b}
\]

Because of the energy-momentum conservation condition, the density-dependence in meson-nucleon couplings will lead to a new term in the HF self-energy, the rearrangement term \( \Sigma_R^\mu \). Thus, the total HF self-energy is a sum of a Hartree (direct) term \( \Sigma_D^\mu \), Fock (exchange) term \( \Sigma_F^\mu \) and rearrangement term \( \Sigma_R^\mu \).
B. Nuclear Matter

In nuclear matter, the self-energy $\Sigma$ can be written quite generally as

$$\Sigma(p, p_F) = \Sigma_S(p, p_F) + \gamma_0 \Sigma_0(p, p_F) + \gamma \cdot \hat{p} \Sigma_V(p, p_F),$$

(11)

where $\hat{p}$ is the unit vector along $p$ and $p_F$ is the Fermi momentum. The tensor piece $\gamma_0 \gamma \cdot \hat{p} \Sigma_T(p)$ does not appear in the HF approximation for nuclear matter. The scalar component $\Sigma_S$, time component $\Sigma_0$ and space component $\Sigma_V$ of the vector potential are functions of $p = (E(p), p)$.

With the general form of the self-energy, the Dirac equation in infinite nuclear medium can be written as [54]

$$[\gamma \cdot p^* + M^*_S] u(p, s) = \gamma_0 E^* u(p, s).$$

(12)

The starred quantities are defined by

$$p^* = p + \hat{p} \Sigma_V(p),$$

$$M^*_S = M + \Sigma_S(p),$$

$$E^* = E(p) - \Sigma_0(p),$$

(13)

where $M^*_S$ is the scalar nucleon mass, and $E$ is the single-particle energy in the medium. The calculation of the self-consistent self-energies follows the method indicated in Refs. [54, 55]. In DDRHF, the density-dependence of the coupling constants gives additional contributions to the vector self-energy $\Sigma_0$, namely the direct ($\Sigma^D_R$) and exchange ($\Sigma^E_R$) rearrangement terms $\Sigma_R$:

$$\Sigma_R = \Sigma^D_R + \Sigma^E_R + \Sigma^D_R + \Sigma^E_R + \Sigma^E_R + \Sigma^E_R + \Sigma^E_R,$$

(14)

where

$$\Sigma^D_R = \frac{1}{\rho_0} \left[ - \frac{g_\sigma}{m_\sigma^2} \frac{\partial g_\sigma}{\partial x} + \frac{g_\omega}{m_\omega^2} \rho_0 \frac{\partial g_\omega}{\partial x} + \frac{g_\rho}{m_\rho^2} \rho_0 \frac{\partial g_\rho}{\partial x} \right],$$

(15a)

$$\Sigma^E_R = \frac{1}{\rho_0} \left[ \frac{\delta \Sigma^E_\sigma}{\partial g_\sigma} \frac{\partial g_\sigma}{\partial x} + \frac{\delta \Sigma^E_\omega}{\partial g_\omega} \frac{\partial g_\omega}{\partial x} + \frac{\delta \Sigma^E_\rho}{\partial g_\rho} \frac{\partial g_\rho}{\partial x} + \frac{\delta \Sigma^E_\pi}{\partial f_\pi} \frac{\partial f_\pi}{\partial x} \right],$$

(15b)

with $x = \rho_b/\rho_0$. The rearrangement term $\Sigma_R$ can also be explicitly written in terms of self-energies. For example, the contribution from the $\sigma$-meson reads as

$$\Sigma_{R,\sigma} = \frac{\partial g_\sigma}{\partial \rho_b} \frac{1}{g_\sigma} \sum_\tau \frac{1}{\pi^2} \int \left[ \tilde{M}(p) \Sigma^\tau_\sigma(p) + \Sigma^\tau_{\sigma,0}(p) + \hat{P}(p) \Sigma^\tau_{\sigma,\tau}(p) \right] p^2 dp.$$
From the self-energies, the starred functions (13) can be determined. Finally, a self-consistent procedure is achieved for a given baryonic density \( \rho_b \) and relative neutron excess \( \beta \equiv (N - Z)/A \) as,

\[
0\Sigma_S, \ 0\Sigma_V \rightarrow \begin{cases} 
\hat{M}(p) &\rightarrow \begin{cases} 
\rho_s \\
\Sigma E_S 
\end{cases} \\
\hat{P}(p) &\rightarrow \Sigma V(p)
\end{cases} \rightarrow 1\Sigma_S \rightarrow \text{New iteration.}
\tag{17}
\]

C. Spherical Nuclei

In spherical nuclei, a single-particle state with energy \( E_a \) is specified by the set of quantum numbers \( \alpha = (a, m_a) \) and \( a = (\tau_a, n_a, l_a, j_a) \), where \( \tau_a = 1 \) for neutrons and \(-1\) for protons. The Dirac spinor \( f_\alpha(r) \) in the expansion (5) can be written as,

\[
f_\alpha(r) = \frac{1}{r} \left( \begin{array}{c} iG_a(r) \\ F_a(r)\hat{\sigma} \cdot \hat{r}
\end{array} \right) \hat{\gamma}_a(r),
\tag{18}
\]

where \( \chi_2^\pm(\tau_a) \) is the isospinor, \( G_a \) and \( F_a \) correspond to the radial parts of upper and lower components, respectively, \( \hat{\gamma}_a \) is the spinor spherical harmonics. The Dirac spinor (18) is normalized according to

\[
\int dr f_\alpha^\dagger(r)f_\alpha(r) = \int dr \left[ G_a^2(r) + F_a^2(r) \right] = 1.
\tag{19}
\]

With the help of the Dirac spinors (18) the expression for the total energy (6) can be obtained \[55\].

Then, the Hartree-Fock equations can be derived by requiring the total binding energy \( E_B \)

\[
E_B = \langle \phi_0 | H | \phi_0 \rangle - AM = E - AM
\tag{20}
\]

to be stationary with respect to norm-conserving variations of the spinors \( f_a \) (or, of \( G_a \) and \( F_a \)):

\[
\delta \left[ E_B - \sum_a E_a \int f_a^\dagger f_a dr \right] = 0.
\tag{21}
\]

The Hartree-Fock equations in coordinate space take the form:

\[
E_a G_a(r) = - \left[ \frac{d}{dr} - \frac{\kappa_a}{r} \right] F_a(r) + [M + \Sigma_S(r) + \Sigma_0(r)] G_a(r) + Y_a(r),
\tag{22a}
\]

\[
E_a F_a(r) = + \left[ \frac{d}{dr} + \frac{\kappa_a}{r} \right] G_a(r) - [M + \Sigma_S(r) - \Sigma_0(r)] F_a(r) + X_a(r),
\tag{22b}
\]

where \( \Sigma_S \) and \( \Sigma_0 \) contain the contributions of all mesons to the direct (Hartree) potential and the rearrangement potential \( \Sigma_R \), \( X_a \) and \( Y_a \) are the convolution integrals of the non-local exchange (Fock) potentials with \( F_a \) and \( G_a \), respectively. The detailed expressions of the Hartree and Fock potentials can be found in Ref. [55]. Similarly to nuclear matter, the rearrangement potential \( \Sigma_R \) due to the
density-dependence of the meson-nucleon couplings can be obtained from the variation of the energy functional with respect to the baryonic density $\rho_b$. As an example, the rearrangement potential due to the $\sigma$-N coupling can be written as

$$
\Sigma^{(\sigma)}_R (r) = \frac{\partial g_\sigma}{\partial \rho_b} \left[ \rho_\sigma(r) \sigma(r) + \frac{1}{g_\sigma} \frac{1}{r^2} \sum_\alpha j_\alpha^2 \left( G_\alpha(r) Y_\alpha^{(\sigma)}(r) + F_\alpha(r) X_\alpha^{(\sigma)}(r) \right) \right]. \tag{23}
$$

Notice that the rearrangement potential is local.

The system of coupled integro-differential equations [22] is rather cumbersome to solve. It is possible to transform it into an equivalent system of coupled differential equations, which can be solved iteratively by the standard techniques used in the RMF approach [51]. To this end, one can follow the method of Ref. [55] and rewrite the functions $X_\alpha(r)$ and $Y_\alpha(r)$ in the form:

$$
X_\alpha(r) = \frac{G_\alpha(r) X_\alpha(r)}{G_\alpha^2(r) + F_\alpha^2(r)} G_\alpha(r) + \frac{F_\alpha(r) Y_\alpha(r)}{G_\alpha^2(r) + F_\alpha^2(r)} F_\alpha(r) \equiv X_{\alpha,G_\alpha}(r) G_\alpha(r) + X_{\alpha,F_\alpha}(r) F_\alpha(r), \tag{24a}
$$

$$
Y_\alpha(r) = \frac{G_\alpha(r) Y_\alpha(r)}{G_\alpha^2(r) + F_\alpha^2(r)} G_\alpha(r) + \frac{F_\alpha(r) Y_\alpha(r)}{G_\alpha^2(r) + F_\alpha^2(r)} F_\alpha(r) \equiv Y_{\alpha,G_\alpha}(r) G_\alpha(r) + Y_{\alpha,F_\alpha}(r) F_\alpha(r). \tag{24b}
$$

Then, the integro-differential equations [22] are formally transformed into a homogeneous differential system,

$$
E_a G_\alpha(r) = - \left[ d \frac{d}{dr} - \frac{\kappa_\alpha}{r} - Y_{\alpha,F_\alpha}(r) \right] F_\alpha(r) + [ M + \Sigma S(r) + \Sigma_0(r) + Y_{\alpha,G_\alpha}(r) ] G_\alpha(r), \tag{25a}
$$

$$
E_a F_\alpha(r) = + \left[ d \frac{d}{dr} + \frac{\kappa_\alpha}{r} + X_{\alpha,G_\alpha}(r) \right] G_\alpha(r) - [ M + \Sigma S(r) - \Sigma_0(r) - X_{\alpha,F_\alpha}(r) ] F_\alpha(r), \tag{25b}
$$

which has the same structure as the RMF equations. This can be solved iteratively as in the RMF approach.

It is very important to treat the $T=1$ pairing correlations if one aims at a quantitative description of finite nuclei. From a mean field point of view, one should ultimately adopt a Hartree-Fock-Bogoliubov approach. At this stage we only use the BCS approximation [67, 68] to include pairing effects, as it is widely done in non-relativistic and relativistic mean field approaches. The pairing matrix elements are calculated with a zero-range, density-dependent interaction of the type [69]

$$
V(r_1, r_2) = V_0 \delta(r_1 - r_2) \left[ 1 - \frac{\rho_b(r)}{\rho_0} \right], \tag{26}
$$

with $V_0 = -850$ MeV.fm$^3$ and $\rho_0$ denotes the saturation density of nuclear matter. The active pairing space is limited to the single-particle states below $+5$ MeV. The unbound states are calculated by imposing a box boundary condition, and one checks that the overall results are not affected by the choice of the box size. The Hartree-Fock-BCS equations are solved self-consistently by iteration just like in RMF-BCS calculations [35]. This is a standard procedure where the presence of the Fock terms does not bring any additional difficulty.
The correction of the center-of-mass motion is treated in a microscopic way as in Ref. [66]. The detailed expressions are given in Appendix A. For the radii, both direct and exchange corrections (see (A19)) are taken into account to be consistent with the theory itself. To calculate the charge densities and radii we take into account both the proton form factor and the center-of-mass corrections (see Appendix A).

III. EFFECTIVE INTERACTIONS

The long standing problem in the RHF approach is to provide an appropriate quantitative description of finite nuclei and nuclear matter. The present work aims at determining such effective interactions for the DDRHF. As already done with the RMF theory [66], the multi-parameter fitting is performed with the Levenberg-Marquardt method [70]. In the DDRHF, 8 parameters for the density-dependence of $g_\sigma$ and $g_\omega$ are reduced to 3 with the five constraint conditions mentioned in Subsec. II.A. Among the effective mesons which mediate the interaction, the pion meson is the most important one. Within the RMF model the $\sigma$-meson takes into account only the two-pion exchange in a phenomenological way. In the Hartree approximation the one-pion exchange does not contribute because of parity conservation. This approach cannot take into account the effective spin-spin interaction coming from the nucleon density through the anti-symmetrization of the wave function. Because of its relatively small mass the pion cannot be approximated by a zero range force with a rather trivial exchange term. In nuclear matter and in spin-saturated systems the one-pion exchange plays only a minor role. However, it has an important influence on single-particle properties in spin-unsaturated systems [53]. In the DDRHF, the contributions of one-pion exchange can be included naturally.

With similar considerations as for the case of $\rho$-N coupling in DDRMF, an exponential density-dependent behavior for the $\pi$-N coupling is adopted here. As further constraints, we impose that the coupling constants $g_\rho(0)$ and $f_\pi(0)$ reach their experimental values at $\rho=0$. Thus, there are in total 8 free parameters in the DDRHF: the mass $m_\sigma$ of the $\sigma$-meson, the couplings constants $g_\sigma(\rho_0)$, $g_\omega(\rho_0)$ and the 3 parameters describing their density dependence, and the $a_\rho$ and $a_\pi$ parameters for the density dependence of the $\rho$-N and $\pi$-N couplings. This gives the new effective interaction PKO1 shown in Table I where the masses of the nucleon, $\omega$, $\rho$- and $\pi$-mesons are respectively taken as $M = 938.9\text{MeV}$, $m_\omega = 783.0\text{MeV}$, $m_\rho = 769.0\text{MeV}$ and $m_\pi = 138.0\text{MeV}$, and the experimental values for the coupling constants $g_\rho$ and $f_\pi$ are respectively 2.629 and 1.0. In order to investigate the role of one-pion exchange, two other parameterizations (see Table I) are also developed as PKO2 without $\pi$-N coupling, where $g_\rho(0)$ is free to be adjusted, and PKO3 with free $g_\rho(0)$, where $a_\pi$ is adjusted by hand. In these parameterizations, the masses of the nuclei $^{16}\text{O}$, $^{40}\text{Ca}$, $^{48}\text{Ca}$, $^{56}\text{Ni}$, $^{68}\text{Ni}$, $^{90}\text{Zr}$, $^{116}\text{Sn}$, $^{132}\text{Sn}$,
TABLE I: The effective interactions of the DDRHF: PKO1, PKO2 and PKO3. The masses of nucleon, $\omega$, $\rho$- and $\pi$-mesons are respectively taken (in MeV) as $M = 938.9$, $m_\omega = 783.0$, $m_\rho = 769.0$ and $m_\pi = 138.0$.

| PKO1   | 525.769084 | 8.833239 | 10.729933 | 2.629000 | 1.000000 | 0.076760 | 1.231976 | 0.151989 |
| PKO2   | 534.461766 | 8.920597 | 10.550553 | 4.068299 | 0.631605 | 0.151021 |
| PKO3   | 525.667686 | 8.895635 | 10.802690 | 3.832480 | 0.635336 | 0.934122 |

| PKO1   | 1.384494  | 1.513190 | 2.296615 | 0.380974 | 1.403347 | 2.008719 | 3.0466  | 0.330770 |
| PKO2   | 1.375772  | 2.064391 | 3.052417 | 0.330459 | 1.451420 | 3.574373 | 5.4783  | 0.246668 |
| PKO3   | 1.244635  | 1.566659 | 2.074581 | 0.400843 | 1.245714 | 1.645754 | 2.1770  | 0.391293 |

$^{182}$Pb, $^{194}$Pb, $^{208}$Pb and $^{214}$Pb are fitted. The bulk properties of symmetric nuclear matter, i.e., the saturation density $\rho_0$, the compression modulus $K$ and the symmetry energy $J$ at saturation point are also included as constraints to improve the parameterizations. These inputs have been used to minimize the least square error,

$$
\chi^2(a) = \sum_{i=1}^{N} \left[ \frac{y_i^{\exp} - y(x_i; a)}{\sigma_i} \right]^2,
$$

where $y_i$ and $\sigma_i$ are the observables and corresponding weights. The vector $a$ is the ensemble of the free parameters. Notice that the radii are not included as observables in the fitting procedure. Only the masses of the above nuclei and the bulk properties of nuclear matter are taken into account as observables. The reason is that the radii are much less sensitive to the changes in the parameters. For the parameter search, the best choice is to find a set of observables which have more or less the same sensitivity to the parameters. There would be no special difficulty to include the radii in the list of fitted data, but we have not made this choice. It is thus more remarkable that the radii calculated with our models have a good agreement with the data, as it will be seen in Table V.

The properties of nuclear matter such as the saturation point, the compression modulus $K$ and the symmetry energy $J$ are just empirically determined quantities which are known with some errors and therefore, they can be slightly varied to improve the description of finite nuclei. In fact, it is possible to obtain fairly good description of radii by properly choosing the values of bulk properties of nuclear matter. The isospin related properties in nuclei can also be well described with an appropriate choice of the symmetry energy $J$. In Table II are shown the compression modulus $K$, the saturation baryonic density $\rho_b$, the symmetry energy $J$, the scalar (Dirac) mass $M_\pi^s$, the non-relativistic and relativistic effective masses $M_{NR}^*$ and $M_R^*$ at the saturation point of nuclear matter. The results are calculated by the DDRHF with PKO1, PKO2 and PKO3, the RMF with PK1, PK1R, PKDD, NL3 and DD-ME1, and the RHF approach with set e in Ref. 55, HFSI 56, ZRL1 57.
It is found that the values of the compression modulus $K$, saturation density $\rho_0$, symmetry energy $J$ and the binding energy per particle $E/A$ provided by the DDRHF approach are all in the currently acceptable regions [72], namely $K \sim [250\text{MeV}, 270\text{MeV}]$, $J \sim [32\text{MeV}, 36\text{MeV}]$ and $E/A \sim 16\text{MeV}$. Compared with earlier RHF applications [55], the improvement on the compression modulus is quite significant. It is worthwhile to mention that the values of effective mass $M^*_\text{NR}$ predicted by the DDRHF with PKO series are quite close to the empirical value $M^* = 0.75$ [73].

### TABLE II: The bulk properties of symmetric nuclear matter calculated in DDRHF with PKO1, PKO2 and PKO3. The results predicted by the RMF theory with PK1, PKDD, NL3 and DD-ME1, and the RHF approach with set e in Ref. [55], HFSI [56], ZRL1 [57] are also given for comparison.

|       | $K$(MeV) | $\rho_0$(fm$^{-3}$) | $J$(MeV) | $E/A$(MeV) | $M^*_S(p_f)/M$ | $M^*_\text{NR}(p_f)/M$ | $M'_R/M$ |
|-------|---------|---------------------|---------|-----------|---------------|----------------------|--------|
| PKO1  | 250.24  | 0.1520              | 34.371  | -15.996   | 0.5900        | 0.7459               | 0.7272 |
| PKO2  | 249.60  | 0.1510              | 32.492  | -16.027   | 0.6025        | 0.7636               | 0.7447 |
| PKO3  | 262.47  | 0.1530              | 32.987  | -16.041   | 0.5862        | 0.7416               | 0.7229 |
| PK1   | 282.69  | 0.1482              | 37.641  | -16.268   | 0.6055        | 0.6811               | 0.6642 |
| PK1R  | 283.67  | 0.1482              | 37.831  | -16.274   | 0.6052        | 0.6812               | 0.6639 |
| TM1   | 281.16  | 0.1452              | 36.892  | -16.263   | 0.6344        | 0.7074               | 0.6900 |
| NL3   | 271.73  | 0.1483              | 37.416  | -16.250   | 0.5950        | 0.6720               | 0.6547 |
| PKDD  | 262.18  | 0.1500              | 36.790  | -16.267   | 0.5712        | 0.6507               | 0.6334 |
| TW99  | 240.27  | 0.1530              | 32.767  | -16.247   | 0.5549        | 0.6371               | 0.6198 |
| DD-ME1| 244.76  | 0.1520              | 33.069  | -16.202   | 0.5779        | 0.6574               | 0.6403 |
| Ref.  | 465.00  | 0.1484              | 28.000  | -15.750   | –             | –                    | –       |
| HFSI  | 250.00  | 0.1400              | 35.000  | -15.750   | 0.6100        | –                    | –       |
| ZRL1  | 250.00  | 0.1550              | 35.000  | -16.390   | 0.5800        | –                    | –       |

For the description of finite nuclei, the comparisons between the DDRHF and the previous RHF approaches [55, 56, 57] are shown in Table III. It is found that the DDRHF provides successful quantitative descriptions of binding energies, spin-orbit splittings and charge radii for the nuclei $^{16}$O, $^{40}$Ca, $^{48}$Ca, $^{90}$Zr and $^{208}$Pb, where the deviations from the data on the binding energies are less than 1.5 MeV. Compared with the previous RHF results, the present DDRHF brings substantial improvements on these observables.

We now focus on the comparison of results between the DDRHF with PKO1, PKO2 and PKO3 and the RMF with the non-linear PK1 [66], NL3 [71] and the density-dependent PKDD [66], DD-ME1 [65]. The reference nuclei are those selected to determine these seven parameterizations, i.e., $^{16}$O, $^{40}$Ca, $^{48}$Ca, $^{56}$Ni, $^{58}$Ni, $^{90}$Zr, $^{112}$Sn, $^{116}$Sn, $^{124}$Sn, $^{132}$Sn, $^{182}$Pb, $^{194}$Pb, $^{204}$Pb, $^{208}$Pb, $^{214}$Pb, and $^{210}$Po. Table IV shows the masses (binding energies) of these nuclei, where the numbers in boldface denote the selected observable used in the corresponding parameterization. The last row of Table IV...
TABLE III: The properties of the nuclei $^{16}$O, $^{40}$Ca, $^{48}$Ca, $^{90}$Zr and $^{208}$Pb predicted by the DDRHF with PKO1, PKO2, and PKO3, and the RHF approaches with set e in Ref. [55], HFSI [56], ZRL1 [57]. The total binding energies $E$, the proton spin-orbit splitting $\Delta_{LS}$ for the 1p shell of $^{16}$O and 1d shell of $^{40}$Ca and $^{48}$Ca nuclei are given in MeV. The r.m.s. charge radii $r_c$ are given in fm. The experimental data are taken from Ref. [74] for the binding energies, and from Refs. [75, 76, 77] for charge radii, and from Ref. [78] for spin-orbit splitting.

| Set        | $^{16}$O | $^{40}$Ca | $^{48}$Ca | $^{90}$Zr | $^{208}$Pb |
|------------|----------|-----------|-----------|-----------|-----------|
|            | $E$      | $r_c$     | $\Delta_{LS}$ | $E$      | $r_c$     | $\Delta_{LS}$ | $E$      | $r_c$     | $E$      | $r_c$     |
| EXP        | -127.63  | 2.693     | 6.32      | -342.05  | 3.478     | 5.94      | -415.99  | 3.479     | 5.01     | -783.89  | 4.270     | -1636.45  | 5.504     |
| PKO1       | -128.33  | 2.676     | 6.36      | -343.28  | 3.443     | 6.63      | -417.37  | 3.451     | 5.59     | -784.61  | 4.251     | -1636.92  | 5.505     |
| PKO2       | -126.94  | 2.659     | 6.25      | -340.93  | 3.427     | 6.47      | -415.55  | 3.450     | 6.15     | -782.56  | 4.247     | -1636.63  | 5.509     |
| PKO3       | -128.34  | 2.687     | 6.20      | -343.37  | 3.448     | 6.63      | -416.80  | 3.459     | 5.27     | -784.89  | 4.254     | -1637.42  | 5.505     |
| Ref. [55]  | -99.52   | 2.73      | 7.3       | -280.8   | 3.47      | 8.0       | -349.44  | 3.47      | 4.1      | -673.2   | 4.26      | -1406.08  | 5.47      |
| HFSI       | -128.64  | 2.73      | 6.4       | -341.2   | 3.48      | 7.05      | -414.24  | 3.48      | 3.27     | -779.4   | 4.26      | -1622.4   | 5.52      |
| ZRL1       | -127.68  | 2.71      | 6.3       | -341.2   | 3.44      | 7.1       | -417.12  | 3.49      | 2.6      | -787.5   | 4.25      | -1636.96  | 5.49      |

presents the r.m.s. deviations from the data, defined as

$$\Delta \equiv \left[ \frac{1}{N} \sum_{i} \left( \frac{E_{i}^{\text{Exp}} - E_{i}^{\text{Cal}}}{N} \right)^{2} \right]^{\frac{1}{2}}. \quad (28)$$

It is seen that the DDRHF reproduces well the binding energies of the selected nuclei as compared with the data, where few (only 4 cases for PKO1) deviations are larger than 1.5 MeV. From the r.m.s. deviations in Table IV it is also demonstrated that the DDRHF achieves successful quantitative descriptions comparable to the RMF, even slightly better for the selected nuclei. Since the selected nuclei cover from the light to heavy regions, one may conclude that the DDRHF with PKO series can provide appropriate quantitative descriptions of the masses of finite nuclei throughout the periodic table.

In the DDRHF parameterizations, the radii are not included as fitted observables. The charge radii of the selected nuclei calculated by the DDRHF and RMF are shown in Table V. As one can see from the r.m.s. deviations, the DDRHF gives satisfactory quantitative descriptions of the charge radii, less good but still comparable to the RMF predictions. It is also found that large deviations only exist in the light region. For heavy nuclei, the DDRHF has the same quality of fit as the RMF. It should also be kept in mind that other corrections beyond the mean field can affect the radii. For instance, the corrections from the RPA correlations may increase the charge radii for $^{40}$Ca.
TABLE IV: The binding energies of the selected nuclei used in the various models. The results are calculated by the DDRHF with PKO1, PKO2, and PKO3, and the RMF with PK1, PKDD, NL3 and DD-ME1. The experimental data are taken from Ref. [74]. The boldface values correspond to nuclei used in the various fits.

|      | EXP | PKO1  | PKO2  | PKO3  | PK1   | NL3   | PKDD  | DD-ME1 |
|------|-----|-------|-------|-------|-------|-------|-------|--------|
| O    | -127.619 | -128.325 | -126.942 | -128.094 | -127.127 | -127.808 | -127.926 |
| Ca   | -342.052 | -343.275 | -340.930 | -343.374 | -341.709 | -342.579 | -343.653 |
| Ca   | -415.991 | -417.372 | -415.554 | -416.799 | -415.077 | -415.944 | -415.012 |
| Ni   | -483.998 | -483.061 | -484.548 | -480.807 | -483.956 | -483.599 | -484.479 |
| Ni   | -506.454 | -502.679 | -503.739 | -500.912 | -504.033 | -503.395 | -504.013 |
| Ni   | -590.430 | -591.370 | -588.581 | -590.693 | -591.685 | -591.456 | -591.241 |
| Zr   | -783.893 | -784.605 | -782.444 | -784.891 | -783.859 | -784.879 | -784.206 |
| Sn   | -953.529 | -951.651 | -950.488 | -951.472 | -952.562 | -953.370 | -952.468 |
| Sn   | -988.681 | -987.276 | -985.808 | -987.119 | -988.491 | -987.699 | -988.066 |
| Sn   | -1049.963 | -1049.619 | -1047.818 | -1049.918 | -1049.162 | -1049.884 | -1048.113 |
| Sn   | -1102.920 | -1103.585 | -1103.171 | -1102.106 | -1103.503 | -1105.459 | -1102.648 |
| Pb   | -1411.650 | -1412.159 | -1411.832 | -1413.027 | -1415.184 | -1416.309 | -1415.216 |
| Pb   | -1525.930 | -1523.023 | -1524.502 | -1523.546 | -1525.536 | -1525.733 | -1525.474 |
| Pb   | -1607.520 | -1606.269 | -1607.230 | -1606.737 | -1607.851 | -1609.906 | -1607.770 |
| Pb   | -1636.446 | -1636.913 | -1636.625 | -1637.435 | -1637.443 | -1640.584 | -1637.387 |
| Pb   | -1663.298 | -1660.797 | -1658.841 | -1661.099 | -1659.382 | -1662.551 | -1656.084 |
| Pb   | -1645.228 | -1646.350 | -1645.877 | -1647.242 | -1648.443 | -1650.755 | -1648.039 |
| △   | 1.6177  | 1.8745  | 2.0489  | 1.8825  | 2.2506  | 2.3620  | 2.7561  |
TABLE V: Same as Table [IV] for charge radii. The experimental data are taken from Refs. [75, 76, 77].

|     | EXP | PKO1 | PKO2 | PKO3 | PK1 | NL3 | PKDD | DD-ME1 |
|-----|-----|------|------|------|-----|-----|------|--------|
| ^16O | 2.693 | 2.6763 | 2.6593 | 2.6871 | 2.6957 | 2.7251 | 2.6988 | 2.7268 |
| ^40Ca | 3.478 | 3.4428 | 3.4266 | 3.4479 | 3.4433 | 3.4679 | 3.4418 | 3.4622 |
| ^48Ca | 3.479 | 3.4506 | 3.4500 | 3.4594 | 3.4675 | 3.4846 | 3.4716 | 3.4946 |
| ^56Ni | 3.6903 | 3.6943 | 3.6944 | 3.7085 | 3.7122 | 3.7162 | 3.7315 |
| ^58Ni | 3.776 | 3.7199 | 3.7213 | 3.7264 | 3.7383 | 3.7435 | 3.7442 | 3.7613 |
| ^68Ni | 3.8501 | 3.8518 | 3.8613 | 3.8621 | 3.8773 | 3.8681 | 3.8926 |
| ^90Zr | 4.270 | 4.2505 | 4.2466 | 4.2537 | 4.2522 | 4.2689 | 4.2534 | 4.2725 |
| ^112Sn | 4.593 | 4.5635 | 4.5640 | 4.5672 | 4.5704 | 4.5861 | 4.5722 | 4.5901 |
| ^116Sn | 4.625 | 4.5930 | 4.5944 | 4.5981 | 4.5984 | 4.6149 | 4.6004 | 4.6212 |
| ^124Sn | 4.677 | 4.6481 | 4.6523 | 4.6545 | 4.6536 | 4.6685 | 4.6567 | 4.6781 |
| ^132Sn | 4.6967 | 4.7053 | 4.7011 | 4.7064 | 4.7183 | 4.7102 | 4.7270 |
| ^182Pb | 5.3710 | 5.3687 | 5.3683 | 5.3708 | 5.3900 | 5.3709 | 5.3896 |
| ^194Pb | 5.442 | 5.4333 | 5.4311 | 5.4320 | 5.4327 | 5.4506 | 5.4329 | 5.4539 |
| ^204Pb | 5.482 | 5.4856 | 5.4883 | 5.4839 | 5.4869 | 5.5027 | 5.4877 | 5.5038 |
| ^208Pb | 5.504 | 5.5054 | 5.5093 | 5.5046 | 5.5048 | 5.5204 | 5.5053 | 5.5224 |
| ^214Pb | 5.559 | 5.5633 | 5.5644 | 5.5629 | 5.5658 | 5.5820 | 5.5635 | 5.5779 |
| ^210Po | 5.5394 | 5.5425 | 5.5383 | 5.5370 | 5.5539 | 5.5371 | 5.5544 |
| Δ | 0.0269 | 0.0299 | 0.0225 | 0.0204 | 0.0177 | 0.0188 | 0.0163 |
IV. CONCLUSIONS

In this work, we have introduced an effective Lagrangian suitable for a description of nuclear systems in a relativistic Hartree-Fock framework. This Lagrangian contains the 4 effective mesons $\sigma, \omega, \rho$ and $\pi$ with density-dependent meson-nucleon couplings. The number of adjustable free parameters is 8, i.e., comparable with the currently existing Lagrangians where the Fock terms are not treated. We recall that the pion can contribute only through its Fock term if parity is to be conserved. The criteria for determining the model parameters are the reproduction of the binding energies in a number of selected nuclei, and the bulk properties of nuclear matter (saturation point, compression modulus, symmetry energy).

In comparison with other RMF models, the present density-dependent RHF approach is at the same level of quantitative description, or slightly better as far as the set of selected nuclei are concerned. This applies not only for binding energies but also for observables not considered in the fitting procedure such as charge radii. Because the nuclei selected for the parameter fitting cover a wide range of masses, it can be expected that the present density-dependent RHF approach can perform quite well throughout the periodic table and out to the borderlines of the region of bound nuclei.

One may wonder if one has gained any benefit over the RMF approach by including the Fock terms. The answer is twofold. Firstly, there is less arbitrariness in RHF since one does not drop the exchange terms while they are known to be important. Even though their effects can be partly simulated by renormalizing the coupling constants, this cannot be completely correct: for instance, the pion will never appear explicitly in RMF. Secondly, there are other physical properties not discussed in this paper, where the Fock terms and particularly the pion contributions are quite important for explaining the nuclear data. This is in particular the case of the isospin dependence of the nucleon effective mass [58], or the evolution of the single-particle energies along isotopic and isotonic chains [79].

Several aspects remain to be investigated. The next step is to improve the treatment of pairing correlations within the RHF by setting up a computational scheme for a relativistic Hartree-Fock-Bogoliubov approach. Another extension is to include the tensor coupling of the $\rho$-meson and to redetermine the optimal effective Lagrangians for describing finite nuclei.

Acknowledgments

This work is partly supported by the National Natural Science Foundation of China under Grant No. 10435010, and 10221003, and the European Community project Asia-Europe Link in Nuclear Physics and Astrophysics CN/Asia-Link 008(94791).
1. Corrections to the total energy

The corrections from the center-of-mass motion to the total energy have been systematically discussed in Ref. [80]. Here for completeness, we just recall the corresponding formula in this subsection. The correction of the center-of-mass motion on the binding energy is taken into account in a non-relativistic form as,

$$ E_{\text{c.m.}} = -\frac{1}{2MA} \langle P_{\text{c.m.}}^2 \rangle, \quad (A1) $$

where the center-of-mass momentum $P_{\text{c.m.}} = \sum_i p_i$ and the expectation value of its square is

$$ \langle P_{\text{c.m.}}^2 \rangle = \sum_a v_a^2 p_{aa}^2 - \sum_{a,b} v_a^2 v_b^2 p_{ab} \cdot p_{ab}^* + \sum_{a,b} v_a u_a v_b u_b p_{ab} \cdot p_{ab}^*. \quad (A2) $$

To calculate the above expression, we express $P_{\text{c.m.}}^2$ in second quantized form,

$$ P_{\text{c.m.}}^2 = \sum_{ab} p_a \cdot p_b = \sum_{\alpha\alpha'} p_{\alpha\alpha'}^2 + \sum_{\alpha\beta\alpha'} p_{\alpha\alpha'} \cdot p_{\beta\beta'}^c c_{\alpha}^c c_{\alpha'}^c. \quad (A3) $$

In the spherical case, we obtain

$$ \langle \hat{P}_{\text{c.m.}}^2 \rangle = -\sum_{\alpha\kappa} v_{\alpha\kappa}^2 \sum_{m=-j}^j \langle \alpha\kappa m | \Delta | \alpha\kappa m \rangle \quad (A4) $$

$$ - \sum_{\alpha,\beta,\kappa,K} v_{\alpha\kappa} v_{\beta\kappa} (v_{\alpha\kappa} v_{\beta\kappa} + u_{\alpha\kappa} u_{\beta\kappa}) \sum_{m=-j}^j \sum_{M=-J}^J \langle \alpha\kappa m | \nabla | \beta\kappa m \rangle \langle \beta\kappa M | \nabla | \beta\kappa M \rangle, $$

where

$$ \sum_{m=-j}^j \langle \alpha\kappa m | \Delta | \alpha\kappa m \rangle = j^2 \sum_{\eta=\pm} \int dr r^2 \psi_{\alpha\kappa}^{(\eta)} \Delta \psi_{\alpha\kappa}^{(\eta)}, \quad (A5) $$

and

$$ \sum_{m=-j}^j \sum_{M=-J}^J \langle \alpha\kappa m | \nabla | \beta\kappa M \rangle \langle \beta\kappa M | \nabla | \beta\kappa M \rangle = j^2 j^2 \sum_{\eta=\pm} \left\{ \begin{array}{c} J \cr l \end{array} \right\}^2 \left\{ \begin{array}{c} j \cr 1 \end{array} \right\}^2 \left\{ \begin{array}{c} J \cr L \end{array} \right\}^{1/2} \left\{ \begin{array}{c} l \cr \frac{1}{2} \end{array} \right\} \right\} \times \left[ \begin{array}{c} L \delta_{l_{(\eta)}, L_{(\eta)} - 1} A^{(\eta)}_{\alpha\beta} B^{(\eta)}_{\beta\alpha} + l \delta_{l_{(\eta)}, l_{(\eta)} - 1} A^{(\eta)}_{\beta\alpha} B^{(\eta)}_{\alpha\beta} \right]. \quad (A6) $$

The corresponding formulas for $A, B$ read as,

$$ A^{(\eta)}_{\alpha\beta} = \int dr r^2 \psi_{\alpha\kappa} \left( \frac{d}{dr} + \frac{L^{(\eta)} + 1}{r} \right) \psi_{\beta\kappa} \quad (A7a) $$

$$ B^{(\eta)}_{\alpha\beta} = \int dr r^2 \psi_{\alpha\kappa} \left( \frac{d}{dr} - \frac{L^{(\eta)}}{r} \right) \psi_{\beta\kappa} \quad (A7b) $$

In the above expressions, $\eta$ denotes the corresponding quantities of the upper ($\eta = +$) or lower ($\eta = -$) components of the Dirac spinor [18].
2. Corrections to radii

The definition of the center-of-mass coordinate is

\[ r_{\text{c.m.}} = \frac{1}{A} \sum_{i=1}^{A} r_i . \]  

(2.8)

The nucleon r.m.s. radius is defined as (taking protons as representative),

\[ r_p^2 = \frac{1}{Z} \langle \phi | \sum_{i=1}^{Z} (r_i - r_{\text{c.m.}})^2 | \phi \rangle . \]  

(2.9)

With the center-of-mass correction, it then becomes

\[ r_{\text{c.m.},p}^2 = \frac{1}{Z} \langle \phi | \sum_{i=1}^{Z} (r_i - r_{\text{c.m.}})^2 | \phi \rangle = r_p^2 + \delta r_p^2, \]  

(2.10)

where the correction term \( \delta r_p^2 \) reads as

\[ \delta r_p^2 = -\frac{2}{Z} \sum_{i=1}^{Z} \langle \phi | r_i \cdot r_{\text{c.m.}} | \phi \rangle + \langle \phi | r_{\text{c.m.}}^2 | \phi \rangle . \]  

(2.11)

In right hand of above equation, there exist the following type operators

\[ \hat{r}(i, j) = \sum_{i,j} r_i \cdot r_j = \sum_i r_i^2 + \sum_{i \neq j} r_i \cdot r_j. \]  

(2.12)

Similar as the quantization of the center-of-mass momentum, it can be quantized as

\[ \hat{r}(i, j) = \sum_{\alpha \alpha'} (r^2)_{\alpha \alpha'} c^\dagger_{\alpha} c_{\alpha'} + \sum_{\alpha \alpha' \beta \beta'} r_{\alpha \alpha'} \cdot r_{\beta \beta'} c^\dagger_{\alpha} c_{\beta} c_{\alpha'} c_{\beta'}, \]  

(2.13)

With the BCS state,

\[ |\phi\rangle = |\text{BCS}\rangle = \prod_k \left( u_k + v_k c^\dagger_k c^\dagger_k \right) |0\rangle, \]  

(2.14)

the expectation can be obtained as

\[ \langle \phi | \hat{r}(i, j) | \phi \rangle = \sum_{\alpha} v^2_{\alpha} (r^2)_{\alpha \alpha} - \sum_{\alpha \beta} v^2_{\alpha} v^2_{\beta} r_{\alpha \beta} \cdot r_{\beta \alpha} + \sum_{\alpha \beta} v_{\alpha} v_{\beta} u_{\alpha} u_{\beta} r_{\alpha \beta} \cdot r_{\beta \alpha}. \]  

(2.15)

Among the right hand of above equation, the terms \( r_{\alpha \beta} \cdot r_{\beta \alpha} \) and \( r_{\alpha \beta} \cdot r_{\alpha \beta} \) can be obtained as

\[ r_{\alpha \beta} \cdot r_{\beta \alpha} = \int drr'dr'' \sum_{\alpha \beta} \left[ \mathcal{A}_{\alpha \beta} G_{\alpha} G_{\beta} + \mathcal{B}_{\alpha \beta} F_{\alpha} F_{\beta} \right]_r \left[ \mathcal{A}_{\alpha \beta} G_{\alpha} G_{\beta} + \mathcal{B}_{\alpha \beta} F_{\alpha} F_{\beta} \right]_{r'}, \]  

\[ r_{\alpha \beta} \cdot r_{\alpha \beta} = \int drr'dr'' \sum_{\alpha \beta} \left[ \mathcal{A}_{\alpha \beta} G_{\alpha} G_{\beta} + \mathcal{B}_{\alpha \beta} F_{\alpha} F_{\beta} \right]_r \left[ \tilde{\mathcal{A}}_{\alpha \beta} G_{\alpha} G_{\beta} + \tilde{\mathcal{B}}_{\alpha \beta} F_{\alpha} F_{\beta} \right]_{r'}, \]  

where

\[ \mathcal{A}_{\alpha \beta} = (-1)^{l_a} l_b \begin{pmatrix} j_b & j_a & 1 \\ l_a & l_b & \frac{1}{2} \end{pmatrix} C^0_{l_b,010} \quad \mathcal{B}_{\alpha \beta} = (-1)^{l_a} l_b \begin{pmatrix} j_b & j_a & 1 \\ l_a' & l_b' & \frac{1}{2} \end{pmatrix} C^0_{l_b',010}, \]  

\[ \tilde{\mathcal{A}}_{\alpha \beta} = (-1)^{l_b} l_a \begin{pmatrix} j_b & j_a & 1 \\ l_a & l_b & \frac{1}{2} \end{pmatrix} C^0_{l_a,010} \quad \tilde{\mathcal{B}}_{\alpha \beta} = (-1)^{l_b} l_a \begin{pmatrix} j_b & j_a & 1 \\ l_a' & l_b' & \frac{1}{2} \end{pmatrix} C^0_{l_a',010}. \]  

(2.16a, b)

(2.17a, b)
In the end, the correction terms \( A^{11} \) for the proton and neutron radii can be expressed as,

\[
\delta r^2_p = -\frac{1}{A} \left[ (2\bar{r}^2_p - \bar{r}^2_M) + (2\bar{r}^2_{\bar{p}} - \bar{r}^2_{\bar{M}}) \right], \quad \delta r^2_n = -\frac{1}{A} \left[ (2\bar{r}^2_n - \bar{r}^2_M) + (2\bar{r}^2_{\bar{n}} - \bar{r}^2_{\bar{M}}) \right],
\]

(A18a)

where

\[
\bar{r}^2_p = \frac{1}{Z} \int r^4 \rho_b^{(p)} dr, \quad \bar{r}^2_{\bar{p}} = -\frac{1}{Z} \sum_{\alpha\beta} v_\alpha v_\beta \left( v_\alpha v_\beta \mathbf{r}_{\alpha\beta} \cdot \mathbf{r}_{\bar{\alpha}\bar{\beta}} - u_\alpha u_\beta \mathbf{r}_{\alpha\beta} \cdot \mathbf{r}_{\bar{\alpha}\bar{\beta}} \right),
\]

(A19a)

\[
\bar{r}^2_n = \frac{1}{N} \int r^4 \rho_b^{(n)} dr, \quad \bar{r}^2_{\bar{n}} = -\frac{1}{N} \sum_{\alpha\beta} v_\alpha v_\beta \left( v_\alpha v_\beta \mathbf{r}_{\alpha\beta} \cdot \mathbf{r}_{\bar{\alpha}\bar{\beta}} - u_\alpha u_\beta \mathbf{r}_{\alpha\beta} \cdot \mathbf{r}_{\bar{\alpha}\bar{\beta}} \right),
\]

(A19b)

\[
\bar{r}^2_M = \frac{1}{Z} \left( Z \bar{r}^2_p + N \bar{r}^2_n \right), \quad \bar{r}^2_{\bar{M}} = \frac{1}{A} \left( Z \bar{r}^2_{\bar{p}} + N \bar{r}^2_{\bar{n}} \right).
\]

(A19c)

3. Corrections to the charge density

In general, the charge density is obtained from the calculated proton distribution corrected by the center-of-mass motion and finite proton size \([23, 24]\). The first correction is done by using the proton density \( \rho_{\text{c.m.}} \) in the center-of-mass system, which is related to the HF density through

\[
\rho_{\text{HF}} = \frac{4}{B^3 \pi^2} \int e^{-r^2/B^2} \rho_{\text{c.m.}}(\mathbf{r} - \mathbf{r'}) dr',
\]

(A20)

with \( B = [\hbar/M\omega A]^{1/2} \). This formula is valid for a system with the harmonic-oscillator center-of-mass correction, which gives the center-of-mass correction on the energy as

\[
E_{\text{c.m.}} = -\frac{3}{4} \hbar \omega = -\frac{1}{2MA} \langle P^2_{\text{c.m.}} \rangle.
\]

(A21)

In term of the center-of-mass momentum, the factor \( B \) can be written as,

\[
B^{-2} = \hbar^{-1} M A \omega = \hbar^{-2} M A h \omega = \frac{2}{3} \hbar^{-2} \langle P^2_{\text{c.m.}} \rangle.
\]

(A22)

The HF density \( \rho_{\text{HF}} \) is then expressed as

\[
\rho_{\text{HF}} = \frac{8}{3} \sqrt{\frac{2}{3\pi}} \int \exp \left[ -\frac{2}{3} \langle P^2_{\text{c.m.}} \rangle r'^2 \right] \rho_{\text{c.m.}}(|\mathbf{r} - \mathbf{r'}|) dr'.
\]

(A23)

Such that, the contributions from the center-of-mass motion to the density can be treated in microscopic way by introducing the expectation of the center-of-mass momentum \( \langle \rangle_{\text{c.m.}} \).

The finite proton is also taken into account by convoluting \( \rho_{\text{c.m.}} \) with a Gaussian representing the proton form factor,

\[
\rho_{\text{ch}}(r) = \frac{1}{2\pi^2 r} \int_0^\infty k \sin(kr) \rho_{\text{HF}}(k) \exp \left[ -\frac{1}{4} k^2 (B^2 - a^2) \right] dk,
\]

(A24)
where the Gaussian is \((a\sqrt{\pi})^{-3}\exp(-r^2/a^2)\), with \(a = \sqrt{2/3} \langle r_p \rangle_{\text{r.m.s.}}, \langle r_p \rangle_{\text{r.m.s.}} = 0.8 \text{ fm} \) and \(\rho_{\text{HF}}(k)\) is the Fourier transform of the HF proton density. When \(a > B\), this expression can be reduced into the following form,

\[
\rho_{\text{ch}}(r) = \int dr_2 (a\sqrt{\pi})^{-3} \exp \left[ -(r - r_2)^2/a^2 \right] \rho_{\text{c.m.}}(r)
\]

\[
= \int dr_1 dr_2 (a\sqrt{\pi})^{-3} (B\sqrt{\pi})^{-3} \exp \left[ -(r - r_2)^2/a^2 + (r_2 - r_1)^2/B^2 \right] \rho_{\text{HF}}(r_1) \tag{A25}
\]

\[
= \int dr_1 \left[ (a^2 - B^2) \pi \right]^{-3/2} \exp \left[ -(r - r_1)^2/(a^2 - B^2) \right] \rho_{\text{HF}}(r_1) .
\]

Denoting by \(\lambda^2 = 1/(a^2 - B^2)\), the integral can be expressed as,

\[
\rho_{\text{ch}}(r) = \frac{\lambda^3}{\pi^{3/2}} \int \exp \left[ -\lambda^2(r - r')^2 \right] \rho_{\text{HF}}(r') dr'. \tag{A26}
\]

In the spherical case, the integral reads as

\[
\rho_{\text{ch}}(r) = \frac{\lambda^3}{\pi^{3/2}} \int \exp \left[ -\lambda^2(r - r')^2 \right] \rho_{\text{HF}}(r') dr'
\]

\[
= \frac{\lambda^3}{\pi^{3/2}} \int r'^2 dr' \rho_{\text{HF}}(r') \int_0^\pi \sin \vartheta d\vartheta \exp \left[ -\lambda^2(r - r')^2 \right] \int_0^{2\pi} d\varphi
\]

\[
= \frac{\lambda^3}{\pi^{3/2}} \int r'^2 dr' \rho_{\text{HF}}(r') \exp \left[ -\lambda^2(r^2 + r'^2) \right] \int_0^\pi \sin \vartheta d\vartheta \exp(2\lambda^2 rr' \cos \vartheta) \times 2\pi \tag{A27}
\]

\[
= \frac{\lambda^3}{\pi^{3/2}} \int r'^2 dr' \rho_{\text{HF}}(r') \exp \left[ -\lambda^2(r^2 + r'^2) \right] \frac{e^{2\lambda^2 rr'} - e^{-2\lambda^2 rr'}}{2\lambda^2 rr'} \times 2\pi .
\]

Finally, one can write the charge density distribution combined with the center-of-mass motion and proton size corrections as,

\[
\rho_{\text{ch}}(r) = \frac{\lambda}{\sqrt{\pi} r^2} \int r' dr' \rho_{\text{HF}}(r') \left[ e^{-\lambda^2(r-r')^2} - e^{-\lambda^2(r+r')^2} \right] . \tag{A28}
\]

[1] A. Mueller and B. Sherrill, Ann. Rev. Nucl. Part. Sci. 43, 529 (1993).
[2] I. Tanihata, Prog. Part. Nucl. Phys. 35, 505 (1995).
[3] P. G. Hansen, A. S. Jensen, and B. Jonson, Ann. Rev. Nucl. Part. Sci. 45, 591 (1995).
[4] R. F. Casten and B. M. Sherrill, Prog. Part. Nucl. Phys. 45, S171 (2000).
[5] A. Mueller, Prog. Part. Nucl. Phys. 46, 359 (2001).
[6] B. Jonson, Phys. Rep. 389, 1 (2004).
[7] A. Jensen, K. Riisager, D. Fedorov, and E. Garrido, Rev. Mod. Phys. 76, 215 (2004).
[8] J. P. Schiffer, S. M. Austin, G. A. Baym, T. W. Donnelly, B. Filippone, S. Freedman, W. C. Haxton, W. F. Henning, N. Isgur, B. Jacak, et al., Nuclear Physics The Core of Matter, The Fuel of Stars (National Academic Press, Washington D. C., 1999).
[9] I. Tanihata, H. Hamagaki, O. Hashimoto, Y. Shida, N. Yoshikawa, K. Sugimoto, O. Yamakawa, T. Kobayashi, and N. Takahashi, Phys. Rev. Lett. 55, 2676 (1985).
[10] I. Tanihata, T. Kobayashi, O. Yamakawa, S. Shimoura, K. Ekuni, K. Sugimoto, N. Takahashi, T. Shimoda, and H. Sato, Phys. Lett. B 206, 592 (1988).
[11] R. E. Warner, J. H. Kelley, P. Zecher, F. D. Becchetti, J. A. Brown, C. L. Carpenter, A. Galonsky, J. Kruse, A. Muthukrishnan, A. Nadasen, et al., Phys. Rev. C 52, R1166 (1995).
[12] L. Chulkov, G. Kraus, O. Bochkarev, P. Egelhof, H. Geissel, M. Golovkov, H. Irnich, and Z. Janas, Nucl. Phys. A 603, 219 (1996).
[13] A. Ozawa, T. Kobayashi, T. Suzuki, K. Yoshida, and I. Tanihata, Phys. Rev. Lett. 84, 5493 (2000).
[14] T. Kobayashi, S. Shimoura, I. Tanihata, K. Katori, K. Matsuta, T. Minamisono, K. Sugimoto, W. Müller, D. L. Olson, T. J. M. Symons, et al., Phys. Lett. B 232, 51 (1989).
[15] T. Kobayashi, Nucl. Phys. A 538, 343c (1992).
[16] T. Alm, G. öpke, W. Bauer, F. Daffin, and M. Schmidt, Nucl. Phys. A 587, 815 (1995).
[17] A. Yoshida, N. Aoi, T. Fukuda, M. Hirai, M. Ishihara, H. Kobinata, Y. Mizoi, L. Mueller, Y. Nagashima, J. Nakano, et al., Nucl. Phys. A 588, 109c (1995).
[18] M. Arnould, S. Goriely, and M. Rayet (2001), arXiv: astro-ph/0101383.
[19] S. Goriely, P. Demetriou, H.-T. Janka, J. M. Pearson, and M. Samyn (2004), arXiv: astro-ph/0410429.
[20] S. Wanajo, S. Goriely, M. Samyn, and N. Itoh, Astrophys. J. 606, 1057 (2004), arXiv: astro-ph/0401412.
[21] D. Vautherin and D. M. Brink, Phys. Rev. C 5, 626 (1972).
[22] M. Beiner, H. Flocard, N. Van Giai, and P. Quentin, Nucl. Phys. A 238, 29 (1975).
[23] J. W. Negele, Phys. Rev. C 1, 1260 (1970).
[24] X. Campi and D. W. Sprung, Nucl. Phys. A 194, 401 (1972).
[25] J. Dechargé and D. Gogny, Phys. Rev. C 21, 1568 (1980).
[26] M. R. Anastasio, L. S. Celenza, W. S. Pong, and C. M. Shakin, Phys. Rep. 100, 327 (1983).
[27] R. Brockmann and R. Machleidt, Phys. Lett. B 149, 283 (1984).
[28] R. Brockmann and R. Machleidt, Phys. Rev. C 42, 1965 (1990).
[29] B. ter Haar and R. Malfliet, Phys. Rep. 149, 207 (1987).
[30] H. F. Boersma and R. Malfliet, Phys. Rev. C 49, 233 (1994).
[31] H. F. Boersma and R. Malfliet, Phys. Rev. C 49, 1495 (1994).
[32] L. D. Miller and A. E. S. Green, Phys. Rev. C 5, 241 (1972).
[33] J. Walecka, Ann. Phys. (N.Y.) 83, 491 (1974).
[34] B. Serot and J. D. Walecka, Adv. Nucl. Phys. 16, 1 (1986).
[35] P.-G. Reinhard, Reports on Progress in Physics 52, 439 (1989).
[36] P. Ring, Prog. Part. Nucl. Phys. 37, 193 (1996).
[37] B. D. Serot and J. D. Walecka, Adv. Nucl. Phys. 4, 1 (1964).
[38] P.-G. Reinhard, Revs. Mod. Phys. 75, 121 (2003).
[39] J. Meng, H. Toki, S. G. Zhou, S. Q. Zhang, W. H. Long, and L. S. Geng, Prog. Part. Nucl. Phys. 57, 470 (2006).
[40] R. J. Furnstahl and B. D. Serot, 2, A23 (2000).
[41] E. Epelbaum, W. Glöke, and U.-G. Meiβer, Nucl. Phys. A671, 295 (2000).
[42] R. J. Furnstahl, Lect. Notes Phys. 641, 1 (2004), and references therein, arXiv: nucl-th/0307111.
[43] M. M. Sharma, G. A. Lalazissis, and P. Ring, Phys. Lett. B 317, 9 (1993).
[44] A. Arima, M. Harvery, and K. Shimizu, Phys. Lett. B 30, 517 (1969).
[45] K. Hecht and A. Adler, Nucl. Phys. A 137, 129 (1969).
[46] J. N. Ginocchio, Phys. Rev. Lett. 78, 436 (1997).
[47] J. Meng, K. Sugawara-Tanabe, S. Yamaji, P. Ring, and A. Arima, Phys. Rev. C58, R628 (1998).
[48] J. Meng, K. Sugawara-Tanabe, S. Yamaji, and A. Arima, Phys. Rev. C59, 154 (1999).
[49] S.-G. Zhou, J. Meng, and P. Ring, Phys. Rev. Lett. 91, 262501 (2003).
[50] J. Meng and P. Ring, Phys. Rev. Lett. 77, 3963 (1996).
[51] J. Meng, Nucl. Phys. A 635, 3 (1998).
[52] J. Meng and P. Ring, Phys. Rev. Lett. 80, 460 (1998).
[53] S. Marcos, R. Niembro, M. L. Quelle, and J. Navarro, Phys. Lett. B 271, 277 (1991).
[54] A. Bouyssy, S. Marcos, J. F. Mathiot, and N. Van Giai, Phys. Rev. Lett. 55, 1731 (1985).
[55] A. Bouyssy, J. F. Mathiot, N. Van Giai, and S. Marcos, Phys. Rev. C 36, 380 (1987).
[56] P. Bernardos, V. N. Fomenko, N. V. Giai, M. L. Quelle, S. Marcos, R. Niembro, and L. N. Savushkin, Phys. Rev. C 48, 2665 (1993).
[57] S. Marcos, L. N. Savushkin, V. N. Fomenko, M. López-Quelle, and R. Niembro, J. Phys. G: Nucl. Part. Phys. 30, 703 (2004).
[58] W. H. Long, N. Van Giai, and J. Meng, Phys. Lett. B 640, 150 (2006).
[59] W. H. Long, H. Sagawa, J. Meng, and N. Van Giai, Phys. Lett. B 639, 242 (2006).
[60] W. H. Long, Ph. D. Thesis, Université Paris-Sud, 2005 (unpublished); W. H. Long, N. Van Giai, and J. Meng, in preparation (2006).
[61] R. Brockmann and H. Toki, Phys. Rev. Lett. 68, 3408 (1992).
[62] H. Lenske and C. Fuchs, Phys. Lett. B 345, 355 (1995).
[63] C. Fuchs, H. Lenske, and H. H. Wolter, Phys. Rev. C 52, 3043 (1995).
[64] S. Typel and H. H. Wolter, Nucl. Phys. A 656, 331 (1999).
[65] T. Nikšić, D. Vretenar, P. Finelli, and P. Ring, Phys. Rev. C 66, 024306 (2002).
[66] W. Long, J. Meng, N. Van Giai, and S.-G. Zhou, Phys. Rev. C69, 034319 (2004).
[67] A. M. Lane, Nuclear Theory (Benjamin, 1964).
[68] P. Ring and P. Shuck, The Nuclear Many-body Problem (Springer, 1980).
[69] J. Dobaczewski, W. Nazarewicz, T. R. Werner, J. F. Berger, C. R. Chinn, and J. Dechargé, Phys. Rev. C 53, 2809 (1996).
[70] W. H. Press, S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, Numerical Recipes in Fortran 77 (Press Syndicate of the University of Cambridge, London, 1992).
[71] G. A. Lalazissis, J. König, and P. Ring, Phys. Rev. C 55, 540 (1997).
[72] D. Vretenar, T. Nikšić, and P. Ring, Phys. Rev. C 68, 024310 (2003).
[73] M. Jaminon and C. Mahaux, Phys. Rev. C 40, 354 (1989).
[74] G. Audi and A. Wapstra, Nucl. Phys. A 595, 409 (1995).
[75] H. de Vries, C. de Jager, and C. de Vries, At. Data Nucl. Data Tables 36, 495 (1987).

[76] S. Dutta, R. Kirchner, O. Klepper, T. Kühl, D. Marx, G. Sprouse, R. Menges, U. Dinger, G. Huber, and S. Schroder, Z. Phys. A 341, 39 (1991).

[77] G. Fricke, C. Bernhardt, K. Heilig, L. Schaller, S. Schellenger, E. Shera, and C. W. Dejager, At. Data Nucl. Data Tables 60, 177 (1995).

[78] A.-M. Oros, Ph. D. thesis, University of Köln (1996).

[79] W. H. Long, H. Sagawa, J. Meng, and N. Van Giai, arXiv: nucl-th/0609076 (2006).

[80] M. Bender, K. Rutz, P.-G. Reinhard, and J. A. Maruhn, Eur. Phys. J. A 7, 467 (2000).