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The signed loop approach to the Ising model: foundations and critical point.  (English) [Zbl 1276.82009]
J. Stat. Phys. 152, No. 2, 353-387 (2013).

Summary: The signed loop approach is a beautiful way to rigorously study the two-dimensional Ising model with no external field. In this paper, we explore the foundations of the method, including details that have so far been neglected or overlooked in the literature. We demonstrate how the method can be applied to the Ising model on the square lattice to derive explicit formal expressions for the free energy density and two-point functions in terms of sums over loops, valid all the way up to the self-dual point. As a corollary, it follows that the self-dual point is critical both for the behaviour of the free energy density, and for the decay of the two-point functions.

MSC: 82B20 Lattice systems (Ising, dimer, Potts, etc.) and systems on graphs arising in equilibrium statistical mechanics
82B27 Critical phenomena in equilibrium statistical mechanics
82D40 Statistical mechanics of magnetic materials

Keywords: Ising model; signed loops; critical point; free energy density; two-point functions

Full Text: DOI arXiv

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