Modification of Landau Levels in a 2D Ring Due to Rotation Effects and Edge States

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The properties of a 2D quantum ring under rotating and external magnetic field effects are investigated. The Landau levels and their inertial effects on them are initially analyzed. Among the results obtained, it is emphasized that the rotation lifted the degeneracy of Landau levels. The second part deals with the electronic confinement in a 2D ring modeled by a hard wall potential. The eigenstates are described by Landau states as long as they are not too close to the ring edges. On the other hand, near the ring edges, the energies increase monotonically. These states are known as edge states. Edge states have a significant role in the physical properties of the ring. Thus, the Fermi energy and magnetization are analyzed. In the specific case of magnetization, two approaches are considered. In the first approach, an analytical result for magnetization is obtained but without considering rotation. Numerical results show the de Haas-Van Alphen (dHvA) oscillations. In the second approach, rotating effects are considered. In addition to the dHvA oscillations, the Aharonov–Bohm-type (AB) oscillations are verified, which are associated with the presence of edge states. The effects of rotation on the results are discussed and it is found that rotation is responsible for inducing AB oscillations.

1. Introduction

A 2D electron gas (2DEG) confined in a mesoscopic ring and immersed in an external magnetic field exhibits a variety of phenomena.[1-11] Among them, we highlight that a quantum ring can carry a persistent current, which is periodic in the AB flux \( \Phi \) with a period \( \Phi_0 = h/e \), the flux quantum.[12,13] Persistent currents have been detected on metallic rings[14,15] and in experiments on a semiconductor ring in the GaAs/GaAlAs system.[16] Here, we are interested in magnetization,[17-23] which is proportional to the persistent current in a single-isolated ring.[14-17,24]

In the 2DEG bulk case Landau, and at zero temperature, the magnetization has sharp saw-tooth oscillations with a constant amplitude.[12-28] These oscillations are known as dHvA oscillations and were measured for the first in 1930 by de Haas, and van Alphen.[29] However, a more realistic model should consider the effect of edges on the electronic states of the sample. We can be modeled the edges of the sample by the hard walls. So, we consider a confinement potential, which is null inside the sample and that increases abruptly at the edges. This is the case of the infinite square well potential.[30,31] The energy eigenvalues are obtained numerically in this model.[32-35] By using this description for a case of a quantum dot, Sivan and Imry[36] showed that the discontinuous jumps in the oscillations are rounded because of the presence of edge states. Besides, AB oscillations are verified superimposed on the dHvA oscillations. The AB oscillations arise when the Fermi energy is within a bulk Landau level. So, when one edge state crosses the Fermi energy, an abrupt change in the magnetization occurs, which leads to AB oscillations. In the case of a quantum ring, we point out that these oscillations also occur when the Fermi energy moves into the gap of the Landau levels. In fact, the ring has two edges, and the crossing of the states of the outer edge with those of the inner edge at Fermi energy leads to AB oscillations.

It is interesting to note that rotating effects have been discussed in the literature.[37-41] In the context of mesoscopic rings, Merlin investigated rotation-induced effects.[42] He analyzed the electronic contribution to the moment of inertia in metal rings under rotating effects. In ref. [45], Merlin and Rojo discussed the persistent magnetic moment in rotating rings and cylinders in the mesoscopic regime. Vignale and Mashhoon also present another fascinating discussion on rotation-induced effects on mesoscopic rings in ref. [46] (see also ref. [13]). Namely, they showed that mesoscopic rings set in rotation with angular velocity \( \Omega \) carry a persistent current which is a periodic function of \( \Omega \), with period \( \Omega_0 = h/2\mu S \), where \( S \) is the area enclosed by the ring in a plane perpendicular to \( \Omega \). In other words, we can state that rotation plays the role of magnetic flux, that is, the problem of rotating rings is equivalent to the problem of non-rotating rings in the presence of magnetic flux. In refs. [47, 48], Aharonov and Carmi provided a significant analysis of equivalence between vector potentials and changes of the frames of references, in other words, of the relationship between electromagnetic fields and inertial forces. The idea is to show that rotation induces a phase shift in the interference pattern of the two-slit electron diffraction experiment, even

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in the absence of an external magnetic field. We refer the reader to ref. [49] for a more extensive review.

In the last decades, there has been a growing study on the feasibility of rotating nanostructures.[30] For instance, Král and Sadeghpour showed that circularly polarized light generates a rotary motion of carbon nanotubes.[31] They also discuss possible applications of rotating microdevices. In ref. [52], the authors report on the construction and successful operation of a fully synthetic nanoscale electromechanical actuator incorporating a rotatable metal plate with a multi-walled carbon nanotube serving as the critical motion-enabling element. We must also emphasize the rapid progress in manufacturing, controlling, and understanding structures in the mesoscopic regime.[33] Thus, we have a favorable environment for the experimental investigation of the physical properties of mesoscopic systems in a rotating frame can be performed.

The rest of the paper is organized as follows. In Section 2, we analyze the behavior of electrons in the presence of external magnetic fields in a rotating frame. We obtain the energy spectrum and then discuss how rotation affects them. In Section 3, we consider electrons confined in a 2D quantum ring. To simulate the boundaries of the quantum ring, we introduce square well potential. In this case, the energy spectrum is obtained numerically, Halperin,[32] as well as MacDonald and Středa,[33] used this model to investigate the role played by edge currents in the quantization of the Hall conductivity. Bandos et al. study the excitonic Aharonov–Bohm effect in quantum rings described by hard wall potential.[34] Reimann et al. measured conductance oscillations in a 2DEG confined in a quantum dot described by hard wall potential.[35] In ref. [56], Lumb et al. investigated the generated charge currents and induced magnetization in a distorted GaAs/AlGaAs quantum disk. They explored the possibility of obtaining adiabatic magnetic pulses in the sub-picosecond range. In Section 4, we use the results from Section 3 to study the electronic states of the ring, Fermi energy, and magnetization. A detailed study of physical properties such as magnetization is challenging because of the complexity of the spectrum shape. However, by using some approximations, it was possible to obtain an expression for the magnetization, whose results are consistent with the literature when the ring is in an inertial frame of reference. When we added the rotation, we found that the analytical model was not viable. In this case, we consider another approach to obtain magnetization and the effects of rotation on it. Our results are summarized in Section 5.

2. Landau Levels

In this section, we present the physical environment under which we shall consider the motion of the electrons. We first write the Hamiltonian for a particle at rest concerning the rotating, followed by the insertion of the vector potential associated with a uniform magnetic field which allows us to obtain analytical solutions of the Schrödinger equation. According to ref. [57], the inclusion of rotating effects in nonrelativistic quantum mechanics can be accomplished through minimal substitution

\[ p' \to p' - \mu \mathbf{A}' \]  

(1)

where \( \mathbf{A}' \) is the gauge field for the rotating frame, with \( \tau = 0, 1, 2, 3 \) and \( \mu \) is the mass of the particle. The gauge field \( \mathbf{A}' \) is defined by

\[ \mathbf{A}' = (A_0, \mathbf{A}) = \left( -\frac{1}{2} \mathbf{V}^2, \mathbf{V} \right) \]  

(2)

where \( \mathbf{V} \) is the velocity. In the rotating frame, the velocity \( \mathbf{V} \) is given by \( \times \mathbf{r} \), where \( \omega \) is the angular velocity. The model includes the interaction of the electron with a uniform magnetic field. This interaction is included in the Hamiltonian of the system through the prescription

\[ p' \to p' - e \mathbf{A}' \]  

(3)

where \( e < 0 \) is the charge of the particle and \( \mathbf{A}' \) is the four-potential. Here, the four-potential of the electromagnetic field \( \mathbf{A}' \) has only the spatial component, \( \mathbf{A}' = (0, \mathbf{A}) \), where \( \mathbf{A} \) is the vector potential.

We assume that the magnetic field is normal to the plane of electronic motion. In polar coordinates, the vector potential that describes this configuration of the magnetic field is given by a symmetric gauge

\[ \mathbf{A} = \frac{B}{2} \phi \]  

(4)

In this way, the Hamiltonian for a particle at rest concerning the rotating frame is written as

\[ \mathcal{H} = \frac{1}{2\mu}(p - e\mathbf{A} - \mu \times \mathbf{r})^2 - \frac{1}{2} \mu(\times \mathbf{r})^2 \]  

(5)

We assume that the motion occurs in the xy plane and that in the rotating frame, the angular velocity has only the \( z \)-component, that is, \( \omega = \Omega z \), with \( \Omega \geq 0 \). In this case, the Schrödinger equation (in polar coordinates) takes the form

\[ \left[ -\frac{\hbar^2}{\mu r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) - \frac{\hbar^2}{\mu} \left( \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} - i \frac{\mu \omega^2}{\hbar^2} \frac{\partial}{\partial \phi} \right) \right] \psi_{n,m} = E_{n,m} \psi_{n,m} \]  

(6)

where

\[ \omega_1 = \sqrt{\omega_2^2 + 4\omega_2 \Omega}, \quad \omega_2 = \omega_c + 2\Omega \]  

(7)

are the effective frequencies, and \( \omega_c = eB/\mu \) is the cyclotron frequency.

By using wave functions in the form

\[ \psi_{n,m}(r, \phi) = e^{im\phi} f_{n,m}(r) \]  

(8)

with \( m = 0, \pm 1, \pm 2, \ldots \), and substituting in Equation (6), we obtain the radial equation

\[ \left[ -\frac{\hbar^2}{2\mu} \frac{d}{dr} \left( r \frac{d}{dr} \right) + V_{\text{eff}}(r) \right] f_{n,m} = E'_{n,m} f_{n,m} \]  

(9)
where
\[ V_{\text{eff}}(r) = \frac{\hbar^2}{2\mu} \left( \frac{m^2}{r^2} + \frac{\mu^2 \omega_1^2}{4\hbar^2} r^2 - \frac{|m| \mu \omega_1}{\hbar} \right) \]  \hspace{1cm} (10)

is the effective potential, and \( E_{n,m} = E_{\text{min}} - V_{\text{min}} \), where \( V_{\text{min}} \) is the minimal energy of the states given by
\[ V_{\text{min}} = \frac{1}{2} \hbar (|m| \omega_1 - \omega_2) \]  \hspace{1cm} (11)

The effective potential given by Equation (10) has a minimum at
\[ r_m = \lambda_1 \sqrt{2|m|} \]  \hspace{1cm} (12)

where \( \lambda_1 \) is the effective magnetic length, given by
\[ \lambda_1 = \sqrt{\frac{\hbar}{\mu \omega_1}} \]  \hspace{1cm} (13)

Equation (12) determines the radial position of the states, which is a measure of the relative position of the envelope function associated with a state of momentum angular \( m \). Furthermore, for \( r \to r_m \), the effective potential has the simple parabolic form
\[ V_{\text{par}}(r) = \frac{1}{2} \mu \omega_1^2 (r - r_m)^2 \]  \hspace{1cm} (14)

The solution of Equation (9) is well known and is given in terms of the confluent hypergeometric function of the first kind.\(^{59}\) The normalized wave eigenfunctions read
\[ \psi_{n,m}(r, \varphi) = \frac{1}{\lambda_1} \sqrt{\frac{\Gamma(|m| + n + 1)}{2^{|m|+1} n! \Gamma(|m| + 1)^2 \pi}} e^{i m \varphi} e^{-\frac{r}{\lambda_1}} \times \left( \frac{r}{\lambda_1} \right)^{|m|} \frac{\Gamma(|m| + n + 1)}{\Gamma(|m| + 1) 1_F_1\left(-n, |m| + 1, \frac{r^2}{2 \lambda_1^2}\right)} \]  \hspace{1cm} (15)

where \( n = 0, 1, 2, \ldots \), \( \Gamma(x) \) is the gamma function, and \( 1_F_1(a, b; x) \) is the confluent hypergeometric function of the first kind.\(^{59}\) The corresponding energy eigenvalues are
\[ E_{n,m} = (n + 1) \hbar \omega_1 + V_{\text{min}} \]  \hspace{1cm} (16)

where \( V_{\text{min}} \) is given by Equation (11).

The radius of a state given by Equation (12) depends on both the angular number \( m \) and magnetic field strength as well as angular velocity. States with the quantum numbers \( m \) and \(-m\) have the same radius. The radius \( r_m \) shrinks as the strength of the magnetic field, or the angular velocity is increased. These results are displayed in Figure 1. The flux enclosed between consecutive radii is given by
\[ \pi B (r_{m+1}^2 - r_m^2) = \phi_0 \sqrt{\frac{\omega_1}{\omega_2 + 4 \Omega}} \]  \hspace{1cm} (17)

It is interesting to investigate the rotating effects in the energy eigenvalues, given by Equation (16). When \( \Omega = 0 \), Equation (11) shows that states with \( m \geq 0 \) have zero minimum potential energy. In that case, as shown in Equation (16), the energy does not depend on the angular number \( m \). It follows from this result that the states with \( m \geq 0 \) with the same quantum number \( n \) have the same energy. Then, there is the formation of infinitely degenerate energy levels, called Landau levels. The energy gap between the consecutive Landau levels corresponds to \( \hbar \omega_1 \). States with \( m < 0 \) belong to Landau levels of quantum number \( n + |m| \). The states with angular number \( m < 0 \) have a minimum potential energy \( \hbar \omega_1 \). On the other hand, when \( \Omega > 0 \) all states have a minimum potential energy and are proportional to a combination of both the frequencies \( \omega_1 \) and \( \omega_2 \). As the first term of Equation (16) is proportional to \( \omega_1 \), then the Landau degeneracy is lifted. In addition, the spacing between consecutive energy levels is given by \( \hbar \omega_1 \). Therefore, the energy separation between the adjacent levels increases as the strength of the magnetic field or the angular velocity increases.

In Figure 2, we show the first four Landau levels for the case \( B = 0.1 \) Tesla. The dots and continuous lines correspond to the states and levels, respectively. We can see the effect of the minimum potential on the states: if \( \Omega = 0 \), then the states with \( m \geq 0 \) degenerate, while the states with \( m < 0 \) belong to higher Landau levels. On the other hand, the angular velocity increases the energy separation between the adjacent levels. The slope of the Landau levels for states with \( m > 0 \) is given by \( \hbar (\omega_1 - \omega_2)/2 \), that
If the states occupy the region next middle of the ring and away from the edges, the confining potential \( V(r) \) does not influence the behavior of the states. Thus, it is evident that the equation for the harmonic oscillator given by Equation (9) describes the radial motion of the electron. Consequently, wavefunctions and eigenvalues are given by Equations (15) and (16), respectively.

For states localized near the edges of the quantum ring, however, we must take into account the effects of the confining potential \( V(r) \). In fact, the edges of the sample are modeled by hard walls as shown in Equation (18). We can analyze the electronic profile at the inner edge, and ignore the effect of the outer edge. Subsequently, we must seek solutions that vanish as \( r \to r_m \to \infty \). The wave functions have the form

\[
\chi_{\nu,m}(r, \varphi) = e^{\imath \varphi} f_{\nu,m}(r)
\]

where \( f_{\nu,m}(r) \) satisfies the radial differential equation\(^{[32,33]}\)

\[
\frac{\hbar^2}{2\mu} \frac{d^2}{dr^2} + V_{\text{par}}(r) + V(r) f_{\nu,m}(r) = E_{\nu,m} f_{\nu,m}(r)
\]

Here \( E_{\nu,m} = E_{\nu,m} - V_{\text{min}} \), where \( V_{\text{min}} \) is given by Equation (11), \( V_{\text{par}} \) is the parabolic potential given in Equation (14), and \( V(r) \) is the confining potential, defined by Equation (18).

The solution of Equation (20) satisfying the condition \( f_{\nu,m}(r) \to 0 \) as \( r \to r_m \to \infty \) is given by\(^{[33,68]}\)

\[
f_{\nu,m}(r) = e^{-\frac{r_m}{\lambda_1}} \left[ c_1 M \left( -\frac{\nu}{2} - \frac{1}{2}, \frac{r - r_m}{\lambda_1} \right) + c_2 \left( \frac{r - r_m}{\lambda_1} \right)^\nu M \left( -\frac{\nu}{2} - \frac{3}{2}, \frac{r - r_m}{\lambda_1} \right) \right]
\]

where \( c_1 \) and \( c_2 \) are constant, and \( \nu \) correspond to the zeros of \( f_{\nu,m}(r) \). Here, \( \nu \) is not only restricted to taking integer values, but \( \nu \) can take any positive real values.

The energy spectrum of the system is given by

\[
E_{\nu,m} = \left( \nu + \frac{1}{2} \right) \hbar \omega_1 + V_{\text{min}}
\]

where \( \omega_1 \) and \( V_{\text{min}} \) are given by Equations (7) and (11), respectively. The requirement that the wave function \( f_{\nu,m}(r) \to 0 \) as \( r \to \infty \) leads to the following result\(^{[33]}\)

\[
c_2 = 2 \frac{\Gamma(1 + \nu)}{\Gamma(2 + \nu)} \tan(\nu \pi / 2)
\]

Furthermore, we shall impose that the wave function vanishes at \( r = r_a \). Combining this result with the above equation, we numerically compute the values of \( n \).

For states located far from the edges, that is, those with large values of \( r_a - r_m \) compared to the magnetic length \( \lambda_1 \), Equation (23) provides

\[
\sin \pi \nu = 0
\]

which means that \( \nu = n = 0, 1, 2, \ldots \). In this way, we recover the result obtained at the beginning of this section, namely that states
Figure 4. Energy levels of a 2D quantum ring (Equation (22)) as a function of the radial \( r_m \) for \( B = 1.0 \) T and some values of \( \Omega \). We consider the four lowest subbands. b–d) Dashed black lines in the figure correspond to the case \( \Omega = 0 \). When \( \Omega > 0 \), a slope appears in the subbands.

4. Numerical Results

In this section, we perform a numerical approach to the results found in Section 3. For our purposes, we shall investigate the energy spectrum, the Fermi energy, and the magnetization of a 2D quantum ring. As noted by Avishai et al.,\textsuperscript{[15]} if the ring width is much greater than the magnetic length, given by Equation (13), then the ring presents the conditions for edge states as well as Landau states to occur. With that in mind, we consider a quantum ring with inner and outer radii given by \( r_a = 600 \) nm and \( r_b = 900 \) nm, respectively. The heterostructure is made of GaAs, and the effective mass of the electron is \( \mu = 0.067 \mu_e \), where \( \mu_e \) is the electron mass. In the numerical analysis of both Fermi energy and the magnetization as a function of the magnetic field, we consider a sample with \( N = 1100 \) spinless electrons.

4.1. Energy

In Figure 4, we plot \( E_{\nu,m} \) as a function of \( r_m \), given by Equations (22) and (12), respectively. We use \( B = 1.0 \) Tesla and different angular velocities \( \Omega \). The continuous lines correspond to the four lowest subbands. The dashed black lines in Figure 4b–d corresponds to the case \( \Omega = 0 \). In Figure 4a, we can observe that the behavior of the electronic states in subbands changes according to the position they occupy inside the ring. The states do not see the confining potential near the middle of the ring. For this case, the values of \( \nu \) are computed from Equation (24), and consequently, the energies are given by Equation (16). We have already discussed the results of this configuration in Section 2. Such states are known as bulk states. In contrast, electronic states centered near the ring edges strongly depend on quantum number \( m \). As we see in Figure 4a, the energy \( E_{\nu,m} \) increases monotonically as \( r_m \) approaches one of the edges. These states are commonly referred to as edge states. It is evident the slope of subbands when \( \Omega = 50 \) GHz (solid red line), \( \Omega = 75 \) GHz (solid green line), and \( \Omega = 100 \) GHz (solid blue line), as we can observe in Figures 4b–d, respectively. In effect, the minimum potential energy (11) is negative for states with \( m > 0 \) (see Section 2).

In Figure 5, we show the evolution of the electronic states as a function of the magnetic field for the two lowest subbands. States belonging to the outer edge reduce their energy as \( B \) increases, while those states belonging to the inner edge run upward in energy. Figure 5a shows that the states of the 2D quantum ring converge into degenerate Landau levels at the bottom of the subbands. On the other hand, Figure 5b–d reveals that there is no convergence; instead, there is a high concentration of states in the region near the bottom of the subbands. The reason for this behavior is that the degeneracy at the bottom of the subbands (Landau levels) is removed due to rotation effects. These results can be seen in Figure 6, where we evaluate the energy as a function of \( \Omega \) for \( B = 1.0 \) T. We can verify the formation of Landau
energy indicated by the black dashed line. As can be seen from Figure 7a,b, the Fermi energy decreases with increasing angular velocity. To explain these observations, we recall that one of the physical implications due to the rotation of the electronic states is to decrease the energy of states with the angular number \( m > 0 \). Figure 7b shows that the rotation shifts all the oscillation peaks to the left.

### 4.3. Magnetization

Now, we analyze the magnetization profile as a function of the magnetic field in a 2D quantum ring. At zero temperature, the magnetization is derived from the relation

\[
\mathcal{M} = -\frac{dU}{dB}
\]

where \( U \) is the internal energy, which is expressed in terms of energy eigenvalues as

\[
U = \sum_{\nu,m} E_{\nu,m}
\]

The magnetic moment carried for each electronic state is given by

\[
\mathcal{M}_{\nu,m} = -\frac{dE_{\nu,m}}{dB}
\]

and consequently, the magnetization may be rewritten as

\[
\mathcal{M} = \sum_{\nu,m} \mathcal{M}_{\nu,m}
\]

Since the spectrum of the system is obtained numerically, we cannot use Equation (28) to obtain the magnetic moment carried by a state. A way to overcome this limitation is to consider some approximations. We assume that the angular velocity is zero. The number of states in a subband can be computed from

\[
N_N = \frac{\Phi}{\Phi_0}
\]

where \( \Phi = \pi(r_0^2 - r_i^2)B \) and \( \Phi_0 = h/e \) are the magnetic flux and magnetic flux quantum, respectively. We can estimate the number of edge states and energy of an edge state, respectively, as

\[
N_E = n_{\nu} \frac{r_i}{\lambda}
\]

and

\[
E_{\nu,m} = \left(\nu + \frac{1}{2}\right) \frac{\hbar \omega_0}{\lambda} + i \Delta E
\]

where \( \Delta E \approx \hbar \omega_0/N_s, \; \nu = n = 0, 1, 2, \ldots \) and \( 1 \leq i \leq N_s \). The Prefactor \( n_{\nu} \) in Equation (31) counts the number of the edges. For a ring with two edges, \( n_{\nu} = 2 \). On the other hand, for a quantum dot, \( n_{\nu} = 1 \). The corresponding energy of a bulk state is obtained by making \( i = 0 \) in Equation (32).

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**Figure 7.** Fermi energy of a 2D ring as a function of a) the number of electrons and b) the magnetic field. In (a) and (b), the black dashed line corresponds to the usual Fermi energy of a quantum ring without edge states (2DEG bulk).
Equation (34), the magnetization reads given by Equation (34), as function of the magnetic field. The red line describes the usual bulk oscillations of the magnetization.

Using the results expressed by Equations (30) and (32), we can show that the internal energy can be written as

$$U = \hbar \omega_c \left\{ \left[ \frac{p^2}{2} + \left( p + \frac{1}{2} \right) q \right] N_i + \frac{p}{2} (N_E + 1) \right. \left. + \frac{N_E + N_i (q - 1)}{2N_E} \right\}$$

(33)

where $p = N/N_i$ and $q = N/N_i - p$. Consequently, using the Equation (34), the magnetization reads

$$\mathcal{M} = -\mathcal{M}_0 \left\{ (2p + 1) N - 2p(p + 1) N_i \right. \left. + p \left[ \frac{3}{2} N_E + 1 \right] + \left[ 1 + (q - 1) \frac{N_i}{N_E} \right] \right. \left. \times \left[ \frac{3}{2} (N_E + 1) + \frac{5}{2} (q - 1) N_i - 2N \right] \right. \left. - \frac{N}{N_E} - \frac{1}{2} \right\}$$

(34)

where $\mathcal{M}_0 = \hbar e/2\mu$.

The magnetization in a 2D quantum ring as a function of the magnetic field given by Equation (34) is shown in Figure 8. The solid red line describes the usual bulk oscillations of the magnetization in a 2D quantum ring. These oscillations are called dHvA oscillations and result from the depopulation of a subband in the Fermi energy. Each abrupt change in the magnetization corresponds to a jump of the Fermi energy from one level to the next lower level. The solid blue line shows the effect of the edge states in the dHvA oscillations. As we can observe, the edge states remove the discontinuous jumps in the magnetization. Now the density of states is not composed of $\delta$ functions. Besides, the amplitude of the oscillations decreases. When the magnetic field is strong enough to make only one occupied subband, the magnetization tends to the value $-N\mathcal{M}_0$.

Equation (34) shows that the edge states affect the behavior of the dHvA oscillations without considering rotating effects. As mentioned above, the energies are determined numerically using the transcendental equation (23), and it isn’t easy to get accurate physics information from the model. An alternative method to overcome this is calculating the magnetization with small steps $\Delta B$. The approximate expression is

$$\mathcal{M} \approx \frac{-\Delta U}{\Delta B}$$

(35)

Figure 9 illustrates a density plot as a function of the magnetic field and angular velocity of a 2D quantum ring by using the approximate formula (35). In Figure 9a, we can observe an oscillatory pattern along a line with constant angular velocity. These oscillations correspond to the dHvA effect already observed when we discuss the magnetization in Figure 8. As we can see, the amplitude of these oscillations increases with an increase in the magnetic field. Such an effect is associated with the presence of edge states.

Rotating effects are evident in Figure 9a. As we can see, the rotation shifts the dHvA oscillations to lower magnetic fields. In addition to this, rotation decreases the amplitude of the dHvA oscillations. In the absence of rotating effects ($\Omega = 0$), we can observe the position of the valleys and peaks of the dHvA oscillations in Figure 9a agrees with those observed in Figure 8. For strong magnetic fields, magnetization tends much faster to the value $-N\mathcal{M}_0$ when $\Omega \to 0$.

Besides the dHvA oscillations, AB-type oscillations are observed along a line with constant angular velocity. These oscillations are more evident when considering a smaller magnetic field range, as shown in Figure 9b. The origin of these oscillations is related to the crossing of the edge states at the Fermi energy, as we can infer from Figure 5. Figure 9b also shows that AB-type oscillations are also present along a line with a constant magnetic field. In other words, the rotation induces AB-type oscillations, which is an expected result. Indeed, as shown in Figure 6, the rotation induces energy level crossings. We emphasize that the nearly periodic behavior of AB-type oscillations is related to a small number of occupied subbands.

5. Conclusions

In summary, we have studied the physical implications of rotating effects on the electronic structure of a 2D quantum ring immersed in a uniform magnetic field considering edge effects. Initially, we addressed the problem of an electron in a region without
confinement potential. This initial approach illustrated many aspects of the impact of rotation on Landau levels. The energy eigenvalues are written in terms of minimum potential energy. The minimum potential provides essential information about the energy the electron acquires in a given state of the system since it depends on the quantum number \( m \) and the cyclotron and rotation frequencies. Among the various results investigated, we point out that the rotation lifts the Landau-levels degeneracy.

Subsequently, the confinement of electrons in a quantum ring was considered. We believe in the hard wall potential model to describe the quantum ring. Due to the shape of the confinement potential, states inside the ring and far away from the edges do not feel the potential. The situation is then similar to that studied in Section 2. On the other hand, close to the edges, the walls significantly influence electronic states. These states are known as edge states. We verified that they play an essential role in Fermi energy and magnetization. The importance of edge states in physical phenomena has been reported in other works, for example, in refs. [32, 35, 36].

The effects of rotation on Fermi energy and magnetization are also evident in our analyses. One of the most important results of our investigation has revealed that The rotation induces AB-type oscillations at magnetization. At this point, we must remember that Vignale and Mashhoon[46] had already verified that rotation induces another equilibrium property, specifically, persistent current. The results we have reported here emphasize the importance of investigations of physical phenomena in mesoscopic systems set in rotation. Finally, we considered high rotational frequencies of \( 10^9 \) to observe rotation effects. This detail was mentioned in refs. [45, 46] in which the authors show that much higher rotational frequencies are required to produce appreciable quantum rotational effects.

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Conflict of Interest

The authors declare no conflict of interest.

Data Availability Statement

Research data are not shared.

Keywords

Aharonov–Bohm oscillations, bound states, edge states, Haas-van Alphen oscillations, Landau levels
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