Aspects of heavy quark production in polarized proton-proton collisions

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Abstract

We examine the spin-dependence of $b$-quark production at large transverse momentum in polarized proton-proton collisions. The leading-order (LO) $2 \to 2$ subprocesses, $g+g \to Q+\overline{Q}$ and $q+\overline{q} \to Q+\overline{Q}$ have a large ‘analyzing power’, as measured by the average spin-spin asymmetry $<\hat{a}_{LL}>$, at large $p_T$, approaching $-100\%$. The contributions from next-to-leading order (NLO) $2 \to 3$ processes are also discussed and found to be dominated at large transverse momentum by subprocesses with a large and positive partonic level spin asymmetry leading to strong cancellations with the LO predictions. The results suggest that $b$-quark production might constitute an interesting test of the spin-dependence of NLO QCD but they also point up the importance of a full calculation of the complete $O(\alpha_S^3)$ spin-dependent corrections for a successful extraction of the polarized gluon distribution. Similarities to gluino pair production are also briefly discussed.
I Introduction

The first results on polarized deep-inelastic scattering from the EMC experiment \cite{1} were interpreted \cite{1, 2} as evidence that the total spin of the proton carried by quarks was rather small, in fact compatible with zero. This suggested that the polarization of sea quarks, including non-valence strange quarks, might be larger than naively expected from the quark model. An alternative explanation had also been suggested \cite{3, 4, 5}, based on the contribution from gluon polarization to the matrix element of the flavor-singlet axial current. There are strong arguments \cite{3, 7} that the glue polarization is unlikely to be as large as needed to explain the EMC data, but the ultimate resolution must come from experiment. The more recent, next generation SMC $e^-$– deuterium \cite{8} and SLAC $\mu$– $^3$He \cite{9} experiments have been recently analyzed \cite{10, 11, 12} and found to be compatible with each other and with the original EMC data. The possibility of a large gluon polarization therefore continues to be of interest.

The lack of the analog of polarized charged-current scattering implies that far less flavor separation is possible when examining spin-dependent parton distributions.\footnote{Polarized charged-current scattering could, in principle, be realized with a neutrino beam and a polarized nucleon target, but the very small cross-section is likely to make such an experiment impractical.}

The integral constraint on the polarized parton distributions arising from the spin-1/2 nature of the proton includes possible contributions from constituent parton orbital angular momentum and so is far less restrictive than the corresponding momentum sum rule. All of these observations have pointed to the desirability of more direct measurements of the polarized gluon distribution in polarized hadron collisions akin to the information available on the unpolarized gluon distribution from, say, direct photon production.
At the same time, the prospects [13, 14] for the availability of high luminosity polarized proton-proton collisions at the Relativistic Heavy Ion Collider (RHIC) at collider energies ($\sqrt{s} = 50 – 500$ GeV) have been extensively discussed. These facts have motivated an increasingly large literature discussing the spin-dependence and sensitivity to polarized parton distributions of many standard model processes. Familiar processes such as jet production [15, 16], direct photons [17], and weak boson production [18] have all been studied as have processes such as quarkonium production at both low $p_T$ [19]–[21] and high $p_T$ [22, 23] and double photon production [24]. Many of the results of these studies have found their way into the proposals [25] for spin-physics experiments using the proposed RHIC detectors. Using such facilities, a comprehensive program involving the extraction of the polarized parton densities and tests of the spin-structure of the standard model is envisaged.

The availability of high quality data on $b$-quark production from both UA1 [26] and CDF [27] combined with the availability of a full set of next-to-leading (NLO) order QCD calculations of heavy quark cross-sections [28, 29, 30] have proved to be a valuable new constraint on unpolarized gluon distributions [31, 32]. It is thus natural to explore the spin-dependence of $b$-quark production and its sensitivity to the polarized gluon density in polarized proton-proton collisions at RHIC energies. In general, one expects that theoretical estimates for $b$-quark production are more reliable than for charm production. In the following we will therefore concentrate on $b$-quarks.

Heavy quark production in polarized $pp$ collisions was first examined by Contogouris et al. [20] who considered only the spin-dependence of the lowest-order 2 → 2 processes $g+g, q+\bar{q} \rightarrow Q+\bar{Q}$ and their impact on the total $b$-quark production cross-section. (An earlier study by Hidaka [33] derives some of the same helicity-dependent cross-sections
as in Ref. [20] for use in the study of spin-dependence in charmonium production. Photoproduction of charm via the similar leading-order $\gamma + g \to Q + \overline{Q}$ process has recently been examined in Ref. [34].

In this report, we extend this analysis and discuss two important aspects of the problem: $p_T$ dependence and NLO effects. In Sec. II we analyze the spin-dependence of the $p_T$-dependent cross-section using the $2 \to 2$ processes. This is motivated by the fact that the existing data extend over a wide range of transverse momentum. We find a dramatic dependence of the observable spin-spin asymmetries on $p_T$. In Sec. III we discuss the likely consequences of next-to-leading order (NLO) effects. These are both vertex and box-diagram corrections to the $2 \to 2$ processes, as well as the $2 \to 3$ contributions. We find that a large cancellation between the spin-dependence of the two types of processes is likely at large $p_T$. This may make $b$-quark production a less sensitive probe of the polarized gluon density than naively imagined on the basis of the leading-order predictions. It might provide, however, an important test of the spin-dependent matrix-elements of QCD beyond the leading order. We do not attempt in this study any systematic analysis of the detectability of heavy quark production in the proposed RHIC detectors nor make any quantitative estimates of the sensitivity of such experiments to the extraction of the polarized gluon distributions.

II 2 → 2 processes

We begin by discussing the spin-dependence of the leading-order $2 \to 2$ subprocesses, $q + \overline{q} \to Q + \overline{Q}$ and $g + g \to Q + \overline{Q}$. The cross-section for $q(q_1) + \overline{q}(q_2) \to Q(k_1) + \overline{Q}(k_2)$ can be written as

$$\frac{d\hat{\sigma}}{dt} = \frac{4\pi\alpha_s^2}{9s^2} \left[ \frac{\hat{t}^2 + \hat{u}^2 + 2M^2\hat{s}}{\hat{s}^2} \right]$$

(1)
where
\[ \tilde{t} \equiv M^2 - \hat{t}, \quad \tilde{u} \equiv M^2 - \hat{u}, \quad (2) \]
and \( M \) is the heavy quark mass. The partonic level longitudinal spin-spin asymmetry is defined by
\[ \hat{a}_{LL} \equiv \frac{d\hat{\sigma}(++) - d\hat{\sigma}(+-)}{d\hat{\sigma}(++) + d\hat{\sigma}(+-)} \quad (4) \]
where \( \pm \) refers to the helicity of the incoming partons. In this case, it is easily seen that
\[ \hat{a}_{LL}(q\bar{q} \rightarrow Q\bar{Q}) = -1 \quad (5) \]
due to helicity conservation and the assumed masslessness of the initial state quarks.

For the subprocess \( g(q_1) + g(q_2) \rightarrow Q(k_1) + \bar{Q}(k_2) \), the unpolarized cross-section, i.e. averaged (summed) over initial (final) spins can be written in the factorized form
\[ \frac{d\hat{\sigma}}{dt} = \frac{\pi \alpha_s^2}{8s^2} \left( \frac{4}{3\hat{u}t} - \frac{3}{s^2} \right) \left[ (\tilde{u}^2 + \tilde{t}^2) + \left( \frac{4M^2\hat{s}}{\hat{u}\hat{t}} \right) (\tilde{u}\tilde{t} - M^2\hat{s}) \right] \]
\[ = \frac{1}{2} \left( \frac{d\hat{\sigma}(++)}{dt} \right. + \left. \frac{d\hat{\sigma}(+-)}{dt} \right) \quad (6) \]
where \( \tilde{t}, \tilde{u} \) are defined above. A simple calculation using FORM \[34\] gives for the difference in helicity-dependent cross-sections
\[ \frac{d\hat{\sigma}(++)}{dt} - \frac{d\hat{\sigma}(+-)}{dt} = -\frac{\pi \alpha_s^2}{4s^2} \left( \frac{4}{3\hat{u}t} - \frac{3}{s^2} \right) \left[ (\tilde{u}^2 + \tilde{t}^2) - \left( \frac{2M^2\hat{s}}{\hat{u}\hat{t}} \right) (\tilde{u}^2 + \tilde{t}^2) \right] \quad (7) \]
which, after some manipulation, can be shown to be identical to the equivalent result of Ref. [20]. The corresponding partonic level spin asymmetry is then simply given by
\[ \hat{a}_{LL}(gg \rightarrow Q\bar{Q}) = -\frac{(\tilde{u}^2 + \tilde{t}^2) - 2M^2\hat{s}(\tilde{u}^2 + \tilde{t}^2)\hat{u}\hat{t}}{(\tilde{u}^2 + \tilde{t}^2) + 4M^2\hat{s}(\hat{u}\hat{t} - M^2\hat{s})/\hat{u}\hat{t}}. \quad (8) \]
We note that these expressions for the total and helicity-dependent cross-sections are quite similar to existing results for gluino pair-production [37] via $g + g \rightarrow \tilde{g} + \tilde{g}$. The only difference arises in the color-dependent pre-factor which in the case of color-octet gluino production is $(1/\tilde{t}\tilde{u} - 1/\tilde{s}^2)$. The resulting partonic level asymmetry is then identical to Eqn. 8. This similarity is not surprising as both involve the pair-production of colored, spin-1/2 fermions via gluon fusion.

Using the kinematic relations

\[
\tilde{t} = \frac{\tilde{s}}{2}(1 - \beta y), \quad \tilde{u} = \frac{\tilde{s}}{2}(1 + \beta y)
\]  

(9)

where $y \equiv \cos(\theta^*)$ gives the center-of-mass scattering angle and $\beta \equiv \sqrt{1 - 4M^2/\tilde{s}}$ is the heavy quark speed, we plot $\hat{a}_{LL}$ versus $y$ for several values of $\sqrt{\tilde{s}}/2M$ in Fig. 1(a). We note that very near threshold, i.e. $\sqrt{\tilde{s}} \gtrsim 2M$ the asymmetry is $+1$, independent of $y$, while for large energies it reduces to the massless quark limit $\hat{a}_{LL} = -1$, first found by Babcock, Monsay, and Sivers [38]. For comparison, we plot in Fig. 1(b) the corresponding $\hat{a}_{LL}$ for the $g + g \rightarrow g + g$ and $q + g \rightarrow q + g(\gamma)$ subprocesses which give important contributions to jet and direct photon production.

For the case of massive final state quarks, the individual helicity-dependent differential cross-sections can be integrated, so we can define a partonic level spin-spin asymmetry for the total cross-section via

\[
\hat{A}_{LL} \equiv \frac{\hat{\sigma}(++) - \hat{\sigma}(+-)}{\hat{\sigma}(++) + \hat{\sigma}(+-)}.
\]

Integrating Eqns. 6 and 7 we find that

\[
\hat{A}_{LL}(\tilde{s}) = \frac{2(9\beta^2 - 17) \log \left(\frac{1+\beta}{1-\beta}\right) + 15\beta(5 - \beta^2)}{(33 - 18\beta^2 + \beta^4) \log \left(\frac{1+\beta}{1-\beta}\right) + \beta(31\beta^2 - 59)}.
\]

(11)
We plot this asymmetry in Fig. 2 as a function of $\sqrt{\hat{s}}/2M$ and note again the (slow) change from $+1$ at threshold to its asymptotic value of $-1$.

The observable longitudinal spin-spin asymmetry is given by

$$A_{LL} \equiv \frac{\sum_{i,j} \int dx_a \int dx_b [\Delta f_i(x_a, Q^2)\Delta f_j(x_b, Q^2)] \hat{a}_{ij}^{LL} d\hat{\sigma}_{ij}}{\sum_{i,j} \int dx_a \int dx_b [f_i(x_a, Q^2)f_j(x_b, Q^2)] d\hat{\sigma}_{ij}}$$

where the $\hat{a}_{ij}^{LL}$ and $d\hat{\sigma}_{ij}$ are the relevant partonic level asymmetries and hard-scattering cross-sections respectively, summed over all possible initial parton states, $(i,j)$. The information on the spin-dependent parton distributions is contained in the $\Delta f(x, Q^2)$, defined via $\Delta f(x, Q^2) \equiv f_+(x, Q^2) - f_-(x, Q^2)$ where $f_+(f_-)$ denotes the parton distribution in a polarized nucleon with helicity parallel (antiparallel) to the parent nucleon helicity.

A useful measure of the intrinsic spin-dependence in an observable process is the average value of the partonic level spin-spin asymmetry, defined via

$$\langle \hat{a}_{LL} \rangle \equiv \frac{\sum_{i,j} \int dx_a \int dx_b [f_i(x_a, Q^2)f_j(x_b, Q^2)] \hat{a}_{ij}^{LL} d\hat{\sigma}_{ij}}{\sum_{i,j} \int dx_a \int dx_b [f_i(x_a, Q^2)f_j(x_b, Q^2)] d\hat{\sigma}_{ij}} = \sum_{ij} \langle \hat{a}_{LL}^{ij} \rangle$$

which can be equated with an ‘average analyzing power’ for the reaction in question. Each of the individual ‘average asymmetries’

$$\langle \hat{a}_{LL}^{ij} \rangle \equiv \frac{\int dx_a \int dx_b [f_i(x_a, Q^2)f_j(x_b, Q^2)] \hat{a}_{LL}^{ij} d\hat{\sigma}_{ij}}{\sigma_{tot}}$$

is both a measure of the spin-dependence (via the $\hat{a}_{LL}$) of a specific contributing initial parton configuration and of its importance in the cross-section. This quantity is a useful figure of merit for the examination of the spin-dependence in any specific reaction. The observable asymmetries, $A_{LL}$, which require information about the (presently unknown) $\Delta f_i(x, Q^2)$, will of course be smaller than this quantity, barring accidental
cancellations, since $|\Delta f(x, Q^2)/f(x, Q^2)| < 1$. Given a set of putative polarized distributions, the observable $A_{LL}$ can be easily calculated and estimates of the measurability of any particular asymmetry can be made by using the total cross-section and luminosity (and appropriate cuts for the realistic detection of the events) to estimate the total event rates. In this report, we will focus on calculations of $<\hat{a}_{LL}>$ as it provides a reasonable glimpse into the relative importance of the various contributions to the observable spin-dependence.

For $b$-quark production in $pp$ collisions in the range of energies accessible to RHIC ($\sqrt{s} = 50 - 500$ GeV), we find that when calculating the total cross-section the average partonic center of mass energy, $\sqrt{<s>/2M}$, is in the range $1.2 - 1.9$. This fact and the results shown in Fig. 1(a) and 2 then imply that the observable $A_{LL}$, when typical polarized gluon distributions (such as those of Ref. [39]) are included, does indeed yield a not unreasonably small and positive asymmetry, just as found by Contogouris et al. in Ref. [20]. The total cross-section is, of course, dominated by events at low transverse momentum, i.e. for values of $\sqrt{s}/2M \lesssim 2$ and for large values of $|y|$, i.e., $|y| \approx 1$. From Figs. 1(a) and 2 we can see that as the center-of-mass energy is increased and as $\theta^* \rightarrow \pi/2$, i.e., when the transverse momentum is increased, the partonic level spin-spin asymmetries rapidly approach $-1$ for the dominant gluon fusion process, changing dramatically the expected magnitude and even sign of the observable asymmetry.

Motivated by the excellent UA1 [26] and CDF [27] data on $b$-quark production as a function of transverse momentum, we calculate the average $<\hat{a}_{LL}>$ mentioned above for $b$-quark production in $pp$ collisions at the highest RHIC energy, $\sqrt{s} = 500$ GeV, for the integrated production cross-section, namely $\sigma(p_T > p_T(min))$ versus $p_T(min)$. We use the $2 \rightarrow 2$ subprocesses above, a (relatively) recent set of leading-order (LO)
unpolarized parton distributions \(^{10}\) (as that is all that is required in Eqn. 13) corresponding to \(\Lambda^{1-\text{loop}} = 177\text{ MeV}.\) In addition, we assume \(M = m_b = 5\text{ GeV}\) and use a momentum scale \(Q^2 = (M^2 + p_T^2)/4\) as suggested by Ref. \(^{32}\). These assumptions reasonably reproduce the UA1 integrated cross-section (when rapidity cuts are included) in proton-antiproton collisions, provided one uses an overall multiplicative \(K\)-factor of roughly \(K = 2.5\), a fact which is consistent with many other analyses \(^{29}\).

The resulting average asymmetry in the integrated cross-section in proton-proton collisions is shown in Fig. 3(a) as a function of \(p_T(\text{min})\). As expected, the value at \(p_T(\text{min}) = 0\), corresponding to the total cross-section, gives a small positive average asymmetry consistent with our observations above. For values of \(p_T(\text{min}) \gtrsim 2M\), the spin asymmetry in the dominant gluon fusion contribution is already approaching its asymptotic value of \(-1\) so that the ‘analyzing power’ of this reaction, based on the \(2 \rightarrow 2\) processes, seems optimally large. This reaction is unique in that all contributing lowest order processes are characterized by the same maximal partonic level asymmetry, at least at large \(p_T\).

For comparison, we plot in Fig. 3(b) the corresponding average \(\langle \hat{a}_{LL} \rangle\) for the \(2 \rightarrow 2\) processes contributing to direct photon production, namely \(q + g \rightarrow q + \gamma\) and \(q + \bar{q} \rightarrow g + \gamma\). In this case also, increasing \(p_T\) leads to \(|y| \rightarrow 0\) where \(\hat{a}_{LL}\) is reduced compared to the larger values in the forward and backward direction found at lower \(p_T\). This helps to explain the (slow) decrease in \(\langle \hat{a}_{LL} \rangle\) from to the \(qg/gq\) contributions. The partial cancellation between the \(qg\) and \(q\bar{q}\) contributions is also evident.

To estimate the effects of including polarized gluon distributions, we evaluate the observable asymmetry, \(A_{LL}\), using a simplified set of assumptions. We assume that the

\(^2\)The qualitative results are likely to be insensitive to the details of the unpolarized parton distributions, since the estimates in Eq. 13 should be understood as an indication of what can be expected from polarized distributions.
quark sea is unpolarized so that there is no contribution from the $q\bar{q}$ diagrams. For the polarized gluon distribution, we use the ansatz, $\Delta G(x, Q^2) = x^\alpha G(x, Q^2)$ which yields a value of the integrated contribution to the proton spin (at $Q_0^2 = 10$ GeV$^2$) of

$$\Delta G \approx 0.5, 2.0, 4.5 \quad \text{for} \quad \alpha = 1.0, 0.5, 0.25 \quad ,$$  \hspace{1cm} (15)

where

$$\Delta G = \int_0^1 dx \Delta G(x, Q_0^2) .$$

This range of values spans many of the models which have been discussed in the literature. The resulting asymmetries are shown in Fig. 4 and demonstrate the large variation in $A_{LL}$ which is possible.

This analysis based on the $2 \to 2$ subprocesses suggests that not only could $b$-quark production at large $p_T$ be an excellent process for the extraction of the polarized gluon distributions (based on the large value of the average asymmetry) but that the dramatic change in the sign of $A_{LL}$ implied by Fig. 3(a) would constitute an interesting test of the spin-structure of the QCD matrix elements. All of this interesting spin structure is in a region where perturbative QCD is applicable and at energies where it has been thoroughly tested. In addition, the energy is not so high as to require the use of resummation techniques [11] which are necessary when working at very low $x$. The large $K$ factor necessary to reproduce the data, however, is reminder that radiative corrections play an important role in the detailed structure of the observable cross-section and we examine the possible effects of such next-to-leading order effects in the next section.
III NLO corrections and $2 ightarrow 3$ processes

The ability to confront or even to extract unpolarized gluon distributions from $b$-quark production data relies on the existence of a full set of next-to-leading order corrections to heavy quark production \cite{28, 29, 30} which are by now standard. Clearly any attempt to probe the polarized gluon distributions will also eventually require a complete analysis of the spin-dependent $O(\alpha_s^3)$ corrections as well. In this section we make a few comments on the likely effect that such corrections might play in modifying the results found in Sec. II. We do not attempt a complete analysis of the spin-dependent radiative corrections.

The finite $O(\alpha_s^3)$ corrections remaining after the extraction of divergent pieces to be associated with parton evolution will come in two varieties: those involving vertex and box-diagram corrections to the $2 \rightarrow 2$ processes and those arising from $2 \rightarrow 3$ subprocesses such as $g + g \rightarrow Q + \overline{Q} + g$. The first class of corrections, retaining as it does the same basic helicity structure of the final state heavy quarks, will still yield a partonic level spin-spin asymmetry approaching $-100\%$ at large transverse momentum, i.e., $\hat{a}_{LL} \rightarrow -1$, when $\sqrt{s}$ becomes large compared to the final state quark masses. The approach to this value could well be different but at large enough values of $p_T$, say $p_T \gtrsim 4M$, we expect the same general features as shown in Fig. 3(a) to appear. We cannot, for example, argue on general grounds that the value of $\hat{a}_{LL}$ near threshold will remain maximally large at $+1$.

Long before the complete NLO calculations were available, it was argued \cite{42} that gluon fragmentation to heavy quark pairs, via $2 \rightarrow 3$ subprocesses such as

\[ g + g \rightarrow g + g \rightarrow Q + \overline{Q} + g \]
\[ g + q \rightarrow g + q \rightarrow Q + \overline{Q} + q \],
is likely to be a large, even the dominant source of heavy quarks at large transverse momentum, i.e., for $p_T > M$, yielding much, if not all, of the needed $K$ factor. The standard argument put forth is that at $90^\circ$ in the parton-parton center-of-mass frame, gluon fusion is much more likely to produce a gluon pair rather than quarks, i.e.,

$$\frac{\sum |M(gg \to gg)|^2}{\sum |M(gg \to q\bar{q})|^2} \approx 200.$$  \hspace{1cm} (16)

The graphical structure of the $2 \to 3$ diagrams (dominated by $t$ - and $u$-channel gluon exchange) is very different from that of the $2 \to 2$ diagrams (coming from $s$-channel annihilation and heavy quark exchange) leading to different kinematics, and presumably, a different spin structure. In fact, the spin-spin asymmetries for the underlying $2 \to 2$ processes shown in Fig. 1(b), namely $g + g \to g + g$ and $q + g \to q + g$, are all large and positive which leads to the disappointing possibility that there might be substantial cancellations with the spin-effects present in the $g + g, q + \bar{q} \to Q + \bar{Q}$ processes. Using the asymmetries in Fig. 1(b) (in the large $p_T$ limit we use the values at $y = \cos(\theta^*) \to 0$ of roughly $0.6 - 0.8$) and the observed necessary $K$ factor of $K \approx 2.5$, we might estimate very crudely that the average asymmetry could be reduced to something of the order

$$<\hat{a}_{LL}> \approx \frac{1 \cdot (-1) + (K - 1) \cdot (0.6 - 0.8)}{1 + (K - 1)} \approx 0.08 - (-0.04) \approx 0$$ \hspace{1cm} (17)

instead of $<\hat{a}_{LL}> = -1$.

To examine this possibility more quantitatively, we choose to model the effects of the $2 \to 3$ subprocesses by following one rather typical pre-radiative correction analysis of such effects by Glover, Hagiwara, and Martin [43]. They 'regularize' the $2 \to 3$ contributions by requiring that

$$\frac{p_T(Q\bar{Q})}{m(Q\bar{Q})} > \epsilon, \quad \theta_{Q\bar{Q}}, \theta_{Q\bar{Q}_c} > \delta$$ \hspace{1cm} (18)
where the notation $a + b \rightarrow c + Q + \overline{Q}$ collectively denotes the subprocesses $g + g \rightarrow g + Q + \overline{Q}$, $q + g \rightarrow q + Q + \overline{Q}$, and $q + \overline{q} \rightarrow g + Q + \overline{Q}$. In Eqn. 18, $\epsilon$ and $\delta$ are cutoffs which we take to be $\epsilon = 0.2$ and $\delta = 20^\circ$ respectively. These conditions ensure that the final light parton, $(c)$, is hard and acollinear to the initial parton directions while the angular cut excludes collinear gluon emission from the final heavy quarks. The remaining phase space configurations are further restricted to three regions, described by the following cuts:

‘A’: three-jet configuration

$$d(i,j) > 1 \quad \text{for } (i,j) = (Q, \overline{Q}), (c, Q), (c, \overline{Q})$$

$$p_T(i) > 10 \text{ GeV} \quad \text{for } i = Q, \overline{Q}, c \quad (19)$$

‘B’: heavy quark excitation

$$\text{Min}(p_T(Q), p_T(\overline{Q})) < 5 \text{ GeV} \quad (20)$$

‘C’: collinear $Q\overline{Q}$ production

$$d(Q, \overline{Q}) < 1. \quad (21)$$

In the equations above, $d(i,j)$ is the separation in the pseudorapidity-azimuthal angle plane, namely,

$$d(i,j) = \left[ (\Delta \eta_{ij})^2 + (\Delta \phi_{ij})^2 \right]^{1/2}. \quad (22)$$

The cut-off dependence of these prescriptions has been extensively discussed in Ref. 43 where it is also found that the massless limit of the exact $2 \rightarrow 3$ matrix elements[35] is an excellent approximation at all but the very smallest values of $p_T(\text{min})$. Making use
of this simplified model of the important contributions arising from $t$- and $u$-channel gluon exchange processes, we plot in Fig. 5 the resulting integrated cross-sections using both the $2 \rightarrow 2$ and these regularized $2 \rightarrow 3$ subprocesses, here collected by initial parton combination ($gg$, $qg$, and $q\bar{q}$.) As expected, the latter diagrams dominate the cross-section at large transverse momentum, in this case even roughly reproducing the required $K \approx 2.5$ factor (which is somewhat fortuitous). These results are consistent with earlier calculations which make similar assumptions \cite{44} but they do seem to overestimate somewhat the contribution from the $qq$ initial states compared to the exact NLO calculation \cite{29}.

Because the massless matrix elements are known to be a good approximation in this case, we can also make use of the corresponding partonic level asymmetries for the massless $2 \rightarrow 3$ subprocesses which were first evaluated in Ref. \cite{45}. We use these to evaluate the resulting average asymmetry, $\langle \hat{a}_{LL} \rangle$, now incorporating all the $2 \rightarrow 2$ and $2 \rightarrow 3$ subprocesses. The resulting average asymmetry in the integrated cross-section is shown in Fig. 6 as a function of $p_T(min)$. In each of the three kinematic regions, the $q\bar{q}$ contributions are negligible and in the ‘$A$’ configuration the individual contributions to the average asymmetry from the $gg$ and $qq$ configurations are consistent with existing results \cite{15} on the spin asymmetries in three-jet events, namely small and positive for the $qq$ configurations and small and negative for the $gg$ type. In both the ‘$B$’ and ‘$C$’ classes, the spin-spin asymmetries arising from the $gg$ and $qg$ contributions are relatively large and positive, reflecting their connection to the underlying $gg \rightarrow gg$ and $qg \rightarrow qg$ origin as seen in Fig. 1(b). The intuitive and qualitative estimates quoted above seem to be borne out in this approximate calculation showing that there may well be a large cancellation in the spin-dependence.
We also note that the $2 \rightarrow 3$ configurations make a small negative contribution to the asymmetry in the integrated cross-section at very small values of $p_T(\text{min})$, somewhat reducing the asymmetry in the total cross-section. This is, however, in the region where the massless approximation for the matrix-elements and partonic asymmetries is worst and, in fact, may likely give the wrong sign for $\hat{a}_{LL}$. (Compare the purely massless $2 \rightarrow 2$ matrix elements and asymmetries to the exact ones which would give a constant asymmetry of $-1$ at threshold instead of $+1$.) Combined with the fact that the NLO corrections to the $2 \rightarrow 2$ diagrams will likely give similar positive asymmetries near threshold, it seems that the asymmetry in the total cross-section, or at least for $p_T(\text{min}) < M$, could have a form quite similar to that obtained with the LO $2 \rightarrow 2$ diagrams alone. Once again, however, a complete NLO spin-dependent calculation will be required to verify these conjectures. One possible experimental difficulty with attempts at extracting spin information from such low $p_T$ data is that the UA1 and CDF data both have lower bounds on $p_T(\text{min})$ of roughly 5 and 10 GeV respectively. The total $b$-production cross-section that UA1 quotes is then extracted from extrapolations of the $p_T$-dependent data to $p_T(\text{min}) \rightarrow 0$ and to the full-rapidity interval by using the QCD predictions for the rapidity variation. It is in just this region that one would like real data to probe the spin-dependence.

We note that a very similar analysis could be applied to gluino pair production using the $2 \rightarrow 2$ cross-sections and asymmetries of Ref. [37], provided one could generalize the results for $g + g \rightarrow \tilde{g} + \tilde{g} + g$ of Ref. [46] to include spin-dependence. In this case, however, the use of any massless approximation for the matrix elements would likely be unjustified due to the presumably large gluino mass. Any realistic appraisal of the spin-dependence of gluino production at supercollider energies will seemingly require a much more detailed analysis than has been done to date.
Finally, it has been pointed out [47] that the *transverse* spin-dependent quark and antiquark distributions (often called ‘transversity’) can be probed using the $q + \bar{q} \rightarrow Q + \bar{Q}$ contribution to heavy quark production. While this annihilation process does indeed have a large intrinsic transverse polarization asymmetry, $\hat{a}_{TT}$, the overall contribution of this subprocess to the total cross-section is never large in $pp$ collisions at the energies we discuss, so that the corresponding average asymmetry $<\hat{a}_{TT}>$, is never more than $-5\%$.

**IV Conclusions**

While these results do not constitute a complete analysis of the spin-dependence of $b$ quark production, they are suggestive of some general trends which we believe will be a robust feature of the full calculation. Inclusion of the polarized parton distributions for a realistic evaluation of the observable asymmetries, $A_{LL}$, will no doubt change the cancellations found above to some extent but we certainly expect that the sizable cancellation found above between LO and NLO effects will persist in any complete analysis. This might well imply that $b$-quark production may not be the most sensitive process from which to attempt to constrain or extract information on the polarized gluon distribution, despite the highly suggestive LO results.

This cancellation is in itself, however, a very interesting prediction which depends sensitively on the spin structure of NLO QCD and could possibly provide a useful test of the matrix element structure of the strong interactions beyond the leading order. If the polarized gluon density were measured in other reactions and found large, then the smallness of the observable $A_{LL}$ in $b$-quark production might then be attributable to this cancellation and not to the inherent smallness of $\Delta G(x, Q^2)$.
Next-to-leading corrections to jet or direct photon production, for example, for which the $O(\alpha_3^2)$ diagrams contain the same kinematical structure would not be expected to yield such a dramatic change in spin-dependence from the leading order result. This is already apparent from the NLO corrections for direct photon production which have recently appeared. The same conclusion is also suggested by the fact that jet (direct photon) production is dominated at leading order by spin-one gluon (spin-1/2 quark) exchange and this feature is retained by NLO calculations and is confirmed by data on jet-jet and jet-$\gamma$ angular distributions. The very different kinematical structures found in the 2 $\rightarrow$ 2 processes ($t$- and $u$-channel $Q$ exchange and $s$-channel production) and in the 2 $\rightarrow$ 3 processes ($t$- and $u$-channel gluon exchange) could presumably be probed by similar $b$-quark pair angular distributions but this is experimentally difficult. The spin-dependence of this process might well provide a unique glimpse into the interplay between LO and NLO corrections in QCD. The only similar case evident so far in the study of standard QCD processes at polarized colliders is the case of double photon production which gets dominant contributions from both $q + \bar{q} \rightarrow \gamma + \gamma$ via Born diagrams and from $g + g \rightarrow \gamma + \gamma$ via box-diagrams and exhibits something of the same partial cancellation of spin-dependence.

Finally, and perhaps most importantly, these results reinforce the notion that the reliable extraction of spin-dependent parton distributions from data obtained in polarized proton-proton collisions may necessarily be much more dependent on the availability of radiative corrections than has been the case for unpolarized distributions in the past.
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Figure Captions

Fig. 1. The partonic level spin-spin asymmetry, \( \hat{a}_{LL} \), versus the center-of-mass scattering angle, \( y = \cos(\theta^*) \) for various partonic processes.

(a) The asymmetry for \( g + g \rightarrow Q + \bar{Q} \) with the solid (dashed, dotdash, dotted) lines corresponding to \( \sqrt{s}/2M = 1.1 (1.5, 2, 5) \).

(b) The asymmetry \( \hat{a}_{LL} \) for \( g + g \rightarrow g + g \) (solid), \( q + g \rightarrow q + g(\gamma) \) (dotdashed) and \( g + q \rightarrow q + g(\gamma) \) (dashed).

Fig. 2. The spin-spin asymmetry, \( \hat{A}_{LL} = \Delta \hat{\sigma}(\hat{s})/\hat{\sigma}(\hat{s}) \) for the integrated total cross-section for \( g + g \rightarrow Q + \bar{Q} \) versus \( \sqrt{s}/2M \).

Fig. 3. (a) The average asymmetry, \(<\hat{a}_{LL}>\), for \( b \)-quark production in \( pp \) collisions at \( \sqrt{s} = 500 \text{ GeV} \) using only \( 2 \rightarrow 2 \) processes. The total \((gg, q\bar{q})\) contributions correspond to the solid (dashed, dotted) line. (b) The same average asymmetry for direct photon production using \( 2 \rightarrow 2 \) sub-processes. The total \((qg, q\bar{q})\) contributions correspond to the solid (dashed, dotted) line.

Fig. 4. The observable asymmetry \( A_{LL} \) in the integrated \( b \)-quark cross-section \((\sigma(p_T > p_T(\text{min}))))\), versus \( p_T(\text{min})(\text{GeV}) \). A polarized gluon distribution of the form \( \Delta G(x, Q^2) = x^\alpha G(x, Q^2) \) is used with \( \alpha = 1.0, (0.5, 0.25) \) for the dotted (dashed, solid) curves.

Fig. 5. Integrated cross-section for \( b \)-quark production, \( \sigma(p_T > p_T(\text{min}))(nb) \), versus \( p_T(\text{min})(\text{GeV}) \), in proton-proton collisions at \( \sqrt{s} = 500 \text{ GeV} \). The upper (lower) dotted lines correspond to \( 2 \rightarrow 2 \) \((2 \rightarrow 3) \) \( gg \) processes. The upper (lower) dotdash lines correspond to \( 2 \rightarrow 2 \) \((2 \rightarrow 3) \) \( q\bar{q} \) processes. The dashed line corresponds to \( 2 \rightarrow 3 \) \( qg \) processes and the solid line corresponds to the total.
Fig. 6. The average asymmetry $<\hat{a}_{LL}>$ in the integrated $b$-quark production cross-section versus $p_T(min)$(GeV). The solid line is the result of including both the $2 \rightarrow 2$ and the ‘regularized’ $2 \rightarrow 3$. The dotted line is the total average asymmetry when one considers $2 \rightarrow 2$ subprocesses only, i.e. the result shown in Fig. 3(a).
Fig. 1(b)
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http://arxiv.org/ps/hep-ph/9310346v1
Fig. 2

$\Delta \sigma(\hat{s}) / \sigma(\hat{s})$
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http://arxiv.org/ps/hep-ph/9310346v1
Fig. 3(b)
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\sigma(p_T>p_T^{\text{min}}) \text{ (nb)}
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