The cosmic coincidences of primordial-black-hole dark matter

Yi-Peng Wu\textsuperscript{a}, Elena Pinetti\textsuperscript{abf}, and Joseph Silk\textsuperscript{acde}

\textsuperscript{a}Laboratoire de Physique Théorique et Hautes Energies (LPTHE),
UMR 7589 CNRS and Sorbonne Université, 4 Place Jussieu, F-75252, Paris, France
\textsuperscript{b}Dipartimento di Fisica, Università di Torino and INFN,
Sezione di Torino, via P. Giuria 1, I-10125 Torino, Italy
\textsuperscript{c}Institut d’Astrophysique de Paris, UMR 7095 CNRS and Sorbonne Université, 98 bis boulevard Arago, F-75014 Paris, France
\textsuperscript{d}Department of Physics and Astronomy, The Johns Hopkins University,
3400 North Charles Street, Baltimore, Maryland 21218, USA
\textsuperscript{e}Beecroft Institute for Particle Astrophysics and Cosmology,
University of Oxford, Keble Road, Oxford OX1 3RH, United Kingdom and
\textsuperscript{f}Theoretical Astrophysics Department, Fermi National Accelerator Laboratory, Batavia, Illinois, 60510, USA

(Dated: January 28, 2022)

If primordial black holes (PBHs) contribute more than 10% of the dark matter (DM) density, their energy density today is of the same order as that of the baryons. Such a cosmic coincidence might hint at a mutual origin for the formation scenario of PBHs and the baryon asymmetry of the Universe. Baryogenesis can be triggered by a sharp transition of the rolling rate of inflaton from slow-roll to (nearly) ultraslow-roll phases that produce large curvature perturbations for PBH formation in single-field inflationary models. We show that the baryogenesis requirement drives the PBH contribution to DM, along with the inferred PBH mass range, the resulting stochastic gravitational wave background frequency window, and the associated cosmic microwave background tensor-to-scalar ratio amplitude, into potentially observable regimes.

I. INTRODUCTION

Primordial black holes (PBHs) are one of the most interesting dark matter (DM) candidates that have been severely constrained by joint astrophysical and cosmological observations\textsuperscript{11,12}. The asteroid-mass window $M_{\text{PBH}}/M_\odot \sim 10^{-16} - 10^{-12}$ for PBHs to be all DM\textsuperscript{15,18} will be tested in the near future by femtolensing of gamma ray bursts\textsuperscript{19,21}, microlensing of x-ray pulsars\textsuperscript{22}, primary photons measured by next-generation MeV detectors\textsuperscript{7,23}, neutron star disruption\textsuperscript{24}, white dwarf explosions\textsuperscript{25}, MeV photons from the Galactic Center\textsuperscript{26} and radio emission measurable with the next-generation radio telescopes\textsuperscript{27}. Even if PBHs only occupy a small fraction of the DM density today, their existence could be (or could have been\textsuperscript{28}) probed by gravitational wave observations through binary mergers, either mutual or with neutron stars (for recent reviews, see\textsuperscript{29,30}).

If PBHs really constitute more than 10% of the DM density today, the PBH density up to matter-radiation equality is of the same order as that of the baryons, namely $\Omega_{\text{PBH}}/\Omega_{\text{eq}} \sim O(1)$. Such a cosmic coincidence would hint at a mutual origin for PBHs and the baryon asymmetry of the Universe. In this Letter, we argue that the PBH-baryon density coincidence could be a natural consequence due to baryogenesis triggered by inflation models for PBH formation. We show that the abundance of PBHs and baryons are indirectly correlated to each other via the dynamics of inflation, and, thus, the scenario is very different from previous suggestions that existing PBHs create baryon asymmetry\textsuperscript{31,32} or account for cosmic coincidence\textsuperscript{11,43}. (See, also,\textsuperscript{44} for cosmic coincidence from asymmetric DM\textsuperscript{45,47} collapses into PBHs.)

II. PBHS FROM INFLATION

Let us focus on single-field inflation for PBH formation\textsuperscript{18,65,74,77}. The generic assumption is that the inflaton $\phi$ receives a sudden deceleration on comoving scales $k_0$ through the entropy mode in different slow-roll phases of inflation. In terms of the first slow-roll parameter $\epsilon_H \equiv -\dot{H}/H^2$ and, thus, largely enhances the power spectrum of the curvature perturbation $P_\zeta$\textsuperscript{66}. Such an enhancement is due to the temporal dominance of the entropy mode in the curvature perturbation\textsuperscript{63,67,68,79}. Dilatation symmetry of the de Sitter background requires the momentum scaling at each phase of inflation with $k 

The analytic structure of the USR power spectrum at the end of inflation ($N = N_{\text{end}}$) has been intensively investigated\textsuperscript{74,79}. Dilatation symmetry of the de Sitter background requires the momentum scaling at each phase
with different values of $\delta$ to satisfy \cite{74,79}

$$P_\zeta = \begin{cases} 
\frac{A_{\text{CMB}}}{A_{\text{PBH}}(k/k_0)^{1/4}}, & k < k_{\text{min}}, \\
A_{\text{PBH}}(k/k_0)^{6+2\delta}, & k_{\text{min}} < k < k_0, \\
A_{\text{PBH}}(k/k_0)^{6+2\delta}, & k_0 < k < k_{\text{end}},
\end{cases}$$

where $A_{\text{CMB}} \approx 2.2 \times 10^{-9}$ measured on comis microwave background (CMB) scales has negligible contribution to PBH formation. $k_0$ is the pivot scale for the enhancement and shall be fixed by the desired peak scale $M_{\text{PBH}}$ in the PBH mass function. $k_{\text{min}} \approx k_0(A_{\text{CMB}}/A_{\text{PBH}})^{1/4}$ is the beginning scale of the $k^4$ growth driven by the Leach-Sasaki-Wands-Liddle mechanism \cite{65} (sometimes also called the steepest growth \cite{72,80}). The amplitude $A_{\text{PBH}}$ is determined by USR parameters as

$$A_{\text{PBH}} \approx A_{\text{CMB}} \left( \frac{k_0}{k_{\zeta}} \right)^{6+4\delta} = A_{\text{CMB}} e^{-N_s(6+4\delta)}. \quad \text{(3)}$$

Note that in the template \cite{2} one should use the value of $\delta < -3$ found in the deceleration phase $N_0 < N < N_*$ since \cite{1} must become positive for $N > N_*$ to increase $\epsilon_H$ and terminate inflation. The positive rolling rate in the final acceleration phase ($N > N_*$) is constrained by $\delta$ in the deceleration phase (with respect to the conformal symmetry due to nonviolation of the adiabatic condition \cite{79}) so that the scaling of $P_\zeta(k)$ for $k_0 < k < k_{\text{end}}$ is the same as $k_0 < k < k_*$. 

In the case of exact USR ($\delta \rightarrow -3$), the inflaton potential $V(\phi)$ is completely flat so that $\phi$ is exactly massless, where quantum diffusion led by short wavelength modes well inside the horizon may have an important impact on the classical trajectory of $\phi$ \cite{54,57,110}. A non-Gaussian tail in the high-sigma limit of the probability distribution of $\zeta$ can significantly raise the resulting PBH abundance from USR inflation \cite{57,105,111}, indicating the real amplitude $A_{\text{PBH}}$ estimated by the Gaussian spectrum \cite{2} (based on the linear relation $\zeta = -H/\dot{\phi} \phi$) should be smaller than expected. To suppress the effect of quantum diffusion, we adopt an upper bound $\delta < -3.1$, which corresponds to an effective mass $m_\phi \equiv (V_\phi)^{1/2} > H_*/2$ for the inflaton fluctuation $\delta \phi$ \cite{92}.

### III. BARYOGENESIS VIA INFLATION

Now, we show that baryogenesis can be triggered by USR inflation. Scalar fields naturally develop large vacuum expectation values (VEVs) during inflation due to the high energy background expansion at the scale of $H_*$ (possibly as high as $10^{13-14}$ GeV \cite{81,82}). These large VEVs provide suitable initial conditions for baryogenesis driven by the Affleck-Dine (AD) mechanism \cite{83,87}: (1) The stochastic nature of the inflationary fluctuations always allows CP-violating VEVs arising from theories with CP invariant Lagrangian \cite{86,88}, and (2) the post-inflationary relaxation of a $B$ or $B-L$ violating scalar condensate is an out-of-equilibrium process.

A possible realization for the USR inflation to affect the dynamics of a charged scalar $\sigma$ is given by

$$L = \mathcal{L}_{\phi} + |\partial \sigma|^2 + m_\sigma^2 |\sigma|^2 + \frac{c_1}{\Lambda} |\sigma|^2 \phi$$

$$+ \frac{c_2}{\Lambda} \partial \mu \phi [\sigma \partial \mu \sigma - \sigma^* \partial \mu \sigma^*] + \mathcal{O}(\Lambda^{-2}) \cdots ,$$

where $c_1$ and $c_2$ are real constants of $\mathcal{O}(1)$. CP invariance is imposed on the Lagrangian for demonstrative purposes, yet it is not a necessary condition for AD baryogenesis. Similar couplings for enhanced charged scalar production from rolling inflaton as a chemical potential can be found in \cite{89,91}. We ask $H_* \ll \Lambda < M_{\text{P}}$ for the cutoff $\Lambda$. Note that the $c_2$ term violates the conserved current $j^\mu = i(\sigma^* \partial^\mu \sigma - \sigma \partial^\mu \sigma^*)$, which is identified as a baryon number for convenience.

In terms of the mass eigenstates $\sigma_{\pm}$, where $\sigma \equiv (\sigma_- + i\sigma_+)/\sqrt{2}$, the charged scalar is decomposed into a pair of decoupled canonical real scalars, $L_{\sigma_{\pm}} = \frac{1}{2} (\partial \sigma_{\pm})^2 + \frac{1}{\pi} m_{\sigma_{\pm}}^2 \phi^2$, with asymmetric (nondegenerate) masses as

$$m_{\pm}^2 = m_\sigma^2 + \frac{c_1 + c_2}{\Lambda} \phi.$$ 

One can see that the phase transition of $\delta$ for PBH formation also changes the effective masses as $\phi$ becomes $-3H \phi \approx -(3 + \delta)\sqrt{2\pi M_{\text{P}}^2 H_*^2}$. For $m_{\sigma} \sim H_*$ and $H_* \gg 0.1M_{\text{P}}$, the sharp decrease of $\epsilon_H$ for the $P_\zeta$ enhancement usually leads to $m_{\pm} \approx m_\sigma$ in the $\phi$-deceleration (USR) phase.

The sudden transition of $m_{\pm}$ from the primary slow-roll phase (with $\delta \rightarrow 0$) to the deceleration phase with $\delta < -3$ drives the original VEVs of $\sigma_{\pm}$ out of equilibrium in their potential, triggering the coherent motion of these scalar condensates. The analytic solutions for the coherent motion of $\sigma_{\pm}$ are given in \cite{92}. At the end of inflation, the VEVs of the mass eigenstates are led by

$$\sigma_{\pm} \sim e^{-\Delta_{\pm} N_{\text{end}}}, \quad \bar{\sigma}_{\pm} \sim -\Delta_{\pm} e^{-\Delta_{\pm} N_{\text{end}}}, \quad \text{(6)}$$

where $\Delta_{\pm} = 3/2 - \sqrt{9/4 - m_{\pm}^2/H_*^2}$ is nothing but the negative branch of the conformal weight for a massive scalar in de Sitter \cite{93}. The late-time approximation used in \cite{0} applies when $m_{\pm}/H_* < 3/2$.

Assuming the standard reheating process driven by the coherent oscillation of $\phi$, one can numerically solve the relaxation of $\sigma_{\pm}$ from the end of inflation to reheating completion (or radiation domination) \cite{88,92}. Here, we consider the decay of $\phi$ into radiation via a perturbative channel with a decay width $\Gamma_\phi$. Thus, the approximated time scale at the beginning of radiation domination is $t_r \sim 1/\Gamma_\phi$. The final baryon asymmetry in radiation domination reads

$$Y_B = \frac{n_B(t_r)}{s(t_r)} = \frac{\sigma_+(t_r)\bar{\sigma}_-(t_r) - \sigma_-(t_r)\bar{\sigma}_+(t_r)}{s(t_r)}, \quad \text{(7)}$$

where $s(t) \approx 2\pi^2 g_*(T^3(t))/45$ is the entropy production and $T = (90/\pi^2 g_*(M_{\text{P}}^2 H_*^2))^{1/4}$ is the temperature. Note that
Y_B = Y_B(\delta, N_e, N_{end}) as those parameters of USR inflation enter through the initial conditions \([6]\).

We highlight the generic property of the final baryon asymmetry with examples given in Fig. 1. In general, \(Y_B\) is sensitive to \(N_{end}\) since initial conditions \([6]\) are exponentially diluted by the e-fold numbers, but it approaches a constant value when \(N_e \sim \mathcal{O}(1)\) depending on the value of \(\delta\). For \(\delta = -3.15\), examples in Fig. 1 indicate that \(Y_B \approx\) const. when \(N_e \gtrsim 2\), which corresponds to \(A_{PBH} \gtrsim 10^{-3}\). This asymptotic constant behavior of \(Y_B\) is the most important property for resolving the coincidence problem for PBH DM.

**IV. PBH DARK MATTER**

As the inflaton decays during reheating, the enhanced curvature perturbation on scales \(k > k_0\) is inherited by the density perturbation of the radiation. Soon after reentry into the horizon in radiation domination, PBHs are formed at high-sigma peaks of the density contrast \(\Delta\) smoothed over a given comoving scale \(R = 1/(aH) = 1/k\). The comoving scale \(R\) can be expressed in terms of the horizon mass parameter \(M_H = \frac{2\pi}{H^3} \rho_R\), with \(\rho_R\) the energy density of the radiation dominated Universe, as

\[
R(M_H) = \frac{1}{k_{eq}} \left(\frac{M_H}{M_{eq}}\right)^{1/2} \left(\frac{g_*}{g_{eq}}\right)^{1/6},
\]

where \(M_{eq} = 2.9 \times 10^{17} M_\odot\) and \(g_{eq} \approx 3\) are the horizon mass and the number of relativistic degrees of freedom at matter-radiation equality. We use \(k_{eq} = 0.01\text{Mpc}^{-1}\) and \(g_* = 106.75\) for \(M_H < 1.5 \times 10^{-7} M_\odot\) where the temperature of the Universe is higher than 300 GeV.

The mass fraction \(\beta(M_{PBH}, M_H)\) of a flat Universe that collapses into PBHs with mass \(M_{PBH}\) at a given horizon mass \(M_H\) can be obtained from the density parameter \(\Omega_{PBH}(R)\) of PBHs at the corresponding scale \(R(M_H)\) as

\[
\beta(M_{PBH}, M_H) = d\Omega_{PBH}/d\ln M_{PBH},
\]

where \(\Omega_{PBH}\) is usually estimated via threshold statistics:

\[
\Omega_{PBH}(R) = \int \cdots \int_{\Delta} \frac{M_{PBH}}{M_H} f_c(y_i)P(\Delta, y_1, \sigma_i)dy_1 \cdots dy_i.
\]

Here \(P(\Delta, y_i, \sigma_i)\) is the joint probability distribution of \(\Delta\) and \(y_i\) are components of its first and second order spatial derivatives. \(f_c(y_i)\) describes spatial constraints to ensure the selected peaks are local maxima in space \([94–98]\). All \(y_i\) are Gaussian random fields if \(\Delta\) is Gaussian. \(\sigma_i\) stands for the \(i\)th spectral moment of the Gaussian field \(\Delta\) smoothed by the window function \(W(kR)\). The form of \(\sigma_i\) is defined as

\[
\sigma_i^2(R) = \int_0^{\infty} k^{2i} W^2(kR) P_{\Delta}(k)d\ln k,
\]

where \(P_\Delta\) is the dimensionless power spectrum. \(\Delta_{c}\) is the threshold value above which the density contrast will collapse to form a PBH. Therefore, the mass fraction \(\beta = \beta(M_H, \Delta_{c}, \sigma_i)\) is a general function of the smoothed scale \(R\) (or, namely, \(M_H\)), the threshold \(\Delta_{c}\), and the spectral moment \(\sigma_i\).

Even for inflation close to the USR limit \((\delta \to -3)\), we find that \(M_{PBH} \approx M_H\) can be a good approximation for resolving the PBH mass function \(f(M_{PBH})\) \([92]\). Such a monochromatic relation leads to a simple expression of the PBH density at matter-radiation equality

\[
\Omega_{PBH,eq} = \int \beta(M_H) \left(\frac{M_{eq}}{M_H}\right)^{1/2} d\ln M_H,
\]

where \((M_{eq}/M_H)^{1/2} \sim a_{eq}/a\) accounts for the relative growth of PBH density during radiation domination. The PBH mass function defined from the PBH-to-DM ratio,

\[
f_{PBH} \equiv \frac{\Omega_{PBH,eq}}{\Omega_{DMeq}} = \frac{1}{\Omega_{DMeq}} \int f(M_H)d\ln M_H,
\]

implies \(f(M_H) = \beta(M_H)(M_{eq}/M_H)^{1/2}/\Omega_{DMeq}\).

**V. THE COSMIC COINCIDENCE**

In the standard Λ-cold dark matter (CDM) Universe \([99]\), the cold dark matter density today \(\Omega_{CDM} = 0.265\) and the redshift \(z_{eq} = 3402\) gives \(\Omega_{CDM,eq} = 0.42\) and \(\Omega_{Beq} = n_B n_{Beq} = 0.08\). This shows that \(\Omega_{PBH,eq}/\Omega_{Beq} \approx 0.5 - 5\) for \(f_{PBH} = 0.1 - 1\). Here \(m_B = 0.938\) GeV is the averaged nucleon mass and \(n_{Beq} = |Y_B|/t_{eq}\) is the baryon number density at matter-radiation equality. Using \(H_{eq} = H_0 \sqrt{\Omega_{Λ0} + 2\Omega_m(a_0/aeq)^3}\) with \(\Omega_{Λ0} = 1 - \Omega_{m0} = 0.6847\) and \(H_0 = 67.36\) km s\(^{-1}\) Mpc\(^{-1}\), we find an expectation value \(|Y_B| = 6.25 \times 10^{-11}\) at \(t_{eq}\).

An example of a parameter scan for the \(|Y_B|\) at the beginning of radiation domination is given in Fig. 2 with \(\delta = -3.15\), \(m_\sigma/H_* = 0.5\) and \(\Lambda/M_P = 0.3\).
FIG. 2. (Upper panel-) Contours of the final baryon asymmetry $|Y_B|$ in radiation domination with $\delta = -3.15$, $m_\nu = H_* / 2$, $\Lambda = 0.3 M_p$, where $H_* = 2.37 \times 10^{13} \text{ GeV}$ and $\Gamma_\phi = 10^{13} \text{ GeV}$ are used. The region $2.506 < N_* < 2.515$ corresponds to the PBH-to-DM ratio $0.1 < f_{\text{PBH}} < 1$ for $\delta = -3.15$ at the pivot scale $k_0 = 9.46 \times 10^{13} \text{ Mpc}^{-1}$. (Lower panel-) $f_{\text{PBH}}$ and $|Y_B|$ as functions of $N_*$ at $N_{\text{end}} = 20$ and $\delta = -3.15$ from $f_{\text{PBH}} = 1$ at $N_* = 2.515$ to $f_{\text{PBH}} = 10^{-10}$ at $N_* = 2.3$.

$Y_B$ is assumed to be a conserved quantity until matter-radiation equality. $Y_B \gtrsim 10^{-10}$ can be reached with $N_* > 1.5$ (for $N_{\text{end}} < 18$), which translates to $A_{\text{PBH}} > 4.4 \times 10^{-5}$. Changing $A_{\text{PBH}}$ by 1 order of magnitude roughly corresponds to a 0.35 variation in $N_*$. The PBH abundance is exponentially sensitive to the peak value $\nu \equiv \Delta / \sigma_0$ in all statistical methods, which means that a tiny change in $A_{\text{PBH}}$ will result in a large difference to $\Omega_{\text{PBH}}$ or $f_{\text{PBH}}$. As a result, the condition $f_{\text{PBH}} > 0.1$ for PBH to be an important DM contributor, indeed, specifies a very precise parameter space for the USR inflation.

To explore the fiducial parameter space for PBH DM, we adopt the standard Press-Schechter (PS) method [90] based on the linear density relation $P_\Delta = 16/81(kR)^3 P_\zeta$ to obtain the mass fraction $\beta_{\text{PS}}(M_H) = \text{erfc}(\nu \zeta / \sqrt{2})$ in terms of inflation parameters $\{\delta, N_*, N_{\text{end}}\}$ with a detailed analytic expression given in [92]. The mass function $f(M_H)$ based on $\beta_{\text{PS}}(M_H)$ with various choices of $\delta$ is displayed in Fig. 3 (blue shadowed regions).

We compare the PBH abundance with the existing observational constraints in the literature. The Galactic constraints are displayed in green. They include the bounds from: the local flux of $e^\pm$ measured by Voyager1 [4], the MeV diffuse flux observed by the INTEGRAL/SPI detector [10], the 511 keV line in our Galaxy [8,9], the primary photons detected by the Comptel experiment [23]. The red curves refer to the extragalactic constraints, comprising the diffuse neutrino background measured by Super-Kamiokande [11] and the extragalactic background radiation [5]. The black lines denote the constraints from the energy injection on the cosmic microwave background at recombination [10] as well as the evaporating constraints from the 21 cm signal observed by EDGES [103,105]. The figure does not include the bounds from the heating of the interstellar medium in dwarf galaxies [3], recently found in the analysis of [106]. The dynamical constraints based on the destruction of white dwarfs and neutrons stars by PBHs are displayed in purple. They are shown with dash-dotted lines, since they are controversial [101,106]. The interested reader can find a comprehensive discussion on these constraints and future prospects in [12,30,107].

The parameter space for $0.1 < f_{\text{PBH}} < 1$ based on $\beta_{\text{PS}}$ with $\delta = -3.15$ is given in Fig. 1. $f_{\text{PBH}}$ is invariant with respect to $N_{\text{end}}$. The difference in $N_*$ for $f_{\text{PBH}} = 1$ and $f_{\text{PBH}} = 0.1$ is $O(10^{-2})$. The mass function computed by the peak statistics [94] with additional spatial constraints [98,98] shows a $10^{-2}$ difference in $N_*$. The uncertainty of $N_*$ due to nonlinear effect between $\Delta$ and $\zeta$ could be as large as $O(10^{-1})$ [92]. $N_{\text{end}} < 16.5$ is excluded by the minimal $e$-fold number for the pivot scale $k_0$ to pass the spatial curvature constraint and the finite deviation of scale-invariant $P_\zeta$ in single-field inflation [81].

Nonperturbative contributions to the curvature perturbation $\zeta$ due to quantum diffusion of the inflaton dynamics can play an important role in the resulting PBH abundance [57,108,111]. In general, the non-Gaussian tail of $\zeta$ in the limit of USR inflation ($\delta \to -3$) can raise the PBH abundance from the standard Gaussian prediction $\beta_{\text{PS}}$ by some 10 orders of magnitude so that the real $N_*$ for $0.1 < f_{\text{PBH}} < 1$ might be shifted toward a smaller value. However, for the given example in Fig. 2 with $\delta = -3.15$ and $k_0 = 9.46 \times 10^{13} \text{ Mpc}^{-1}$,
we find $\beta_{PS}(N_e = 2.5)/\beta_{PS}(N_e = 2.2) \gg 10^{100}$ for the viable range of $N_{end}$. This implies that the effect of quantum diffusion seems unlikely to shift the parameter space for $0.1 < f_{PBH} < 1$ to $N_e < 2$, leaving the $O(1)$ ratio $\Omega_{PBHeq}/\Omega_B$ nearly unchanged in the scenario.

VI. SUMMARY AND DISCUSSION

PBHs fostered by the USR transition during inflation can contribute as a significant DM component. We have shown that such an USR transition of the inflationary DM can contribute as a significant DM component. We have shown that such an USR transition of the inflationary mechanism. The resulting baryon asymmetry is nearly unchanged in the scenario.

ACKNOWLEDGMENTS

We are grateful to Kalliopi Petraki for helpful discussions and the full support on this project. E. P. is supported by the Fermi Research Alliance, LLC under Contract No. DE-AC02-07CH11359 with the U.S. Department of Energy, Office of High Energy Physics; Department of Excellent grant 2018-2022, awarded by the Italian Ministry of Education, University and Research (MIUR); Research grant of the Italo-French University, under Bando Vinci 2020. Y.-P. W. was supported by the Agence Nationale de la Recherche (ANR) Accueil de Chercheurs de Haut Niveau (ACHN) 2015 grant (“TheIntricateDark” project). The project has received funding from the European Union’s Horizon 2020 research and innovation programme under Grant Agreement No. 101002846 (ERC CoG “CosmoChart”).

[1] V. Poulin, J. Lesgourgues and P. D. Serpico, JCAP 03, 043 (2017) [arXiv:1610.10051 [astro-ph.CO]].
[2] S. Clark, B. Dutta, Y. Gao, L. E. Strigari and S. Watson, Phys. Rev. D 95, no.8, 083006 (2017) [arXiv:1612.07738 [astro-ph.CO]].
[3] M. Boudaud and M. Cirelli, Phys. Rev. Lett. 122, no.4, 041104 (2019) [arXiv:1807.03075 [astro-ph.HE]].
[4] J. Cang, Y. Gao and Y. Z. Ma, [arXiv:2108.13256 [astro-ph.CO]].
[5] H. Kim, [arXiv:2007.07739 [hep-ph]].
[6] H. Niikura, M. Takada, N. Yasuda, R. H. Lupton, T. Sumi, S. More, T. Kurita, S. Sugiyama, A. More and M. Oguri, et al. Nature Astron. 3, no.6, 524-534 (2019) [arXiv:1701.02151 [astro-ph.CO]].
[7] H. Niikura, M. Takada, S. Yokoyama, T. Sumi and S. Masaki, Phys. Rev. D 99, no.8, 083503 (2019) [arXiv:1901.07120 [astro-ph.CO]].
[8] P. Montero-Camacho, X. Fang, G. Vasquez, M. Silva and C. M. Hirata, JCAP 08, 031 (2019) [arXiv:1906.05950 [astro-ph.CO]].
[9] N. Smyth, S. Profumo, S. English, T. Jeltema, K. McKinnon and P. Guhathakurta, Phys. Rev. D 101, no.6, 063005 (2020) [arXiv:1910.01285 [astro-ph.CO]].
[10] R. J. Nemiroff and A. Gould, Astrophys. J. Lett. 452, L111 (1995) [astro-ph/9505019].
[11] R. Laha, J. B. Muñoz and T. R. Slatyer, Phys. Rev. D 101, no.12, 123514 (2020) [arXiv:2004.06267 [astro-ph.CO]].
[12] B. Dassgupta, R. Laha and A. Ray, Phys. Rev. Lett. 125, no.10, 101101 (2020) [arXiv:1912.01014 [hep-ph]].
[13] B. Carr, K. Kohri, Y. Sendouda and J. Yokoyama, arXiv:2002.12778 [astro-ph.CO].
[14] J. Cang, Y. Gao and Y. Ma, JCAP 05, 051 (2021) [arXiv:2011.12244 [astro-ph.CO]].
[15] H. Niikura, M. Takada, N. Yasuda, R. H. Lupton, T. Sumi, S. More, T. Kurita, S. Sugiyama, A. More and M. Oguri, et al. Nature Astron. 3, no.6, 524-534 (2019) [arXiv:1701.02151 [astro-ph.CO]].
[16] H. Niikura, M. Takada, S. Yokoyama, T. Sumi and S. Masaki, Phys. Rev. D 99, no.8, 083503 (2019) [arXiv:1901.07120 [astro-ph.CO]].
[17] P. Montero-Camacho, X. Fang, G. Vasquez, M. Silva and C. M. Hirata, JCAP 08, 031 (2019) [arXiv:1906.05950 [astro-ph.CO]].
[18] N. Smyth, S. Profumo, S. English, T. Jeltema, K. McKinnon and P. Guhathakurta, Phys. Rev. D 101, no.6, 063005 (2020) [arXiv:1910.01285 [astro-ph.CO]].
[19] R. J. Nemiroff and A. Gould, Astrophys. J. Lett. 452, L111 (1995) [astro-ph/9505019].
[20] S. Jung and T. Kim, Phys. Rev. Res. 2, no.1, 013113 (2020) [arXiv:1908.00078 [astro-ph.CO]].
[21] A. Katz, J. Kopp, S. Sibiryakov and W. Xue, JCAP 12, 005 (2018) [arXiv:1807.11495 [astro-ph.CO]].
[22] Y. Bai and N. Orlowski, Phys. Rev. D 99, no.12, 123019 (2019) [arXiv:1812.01427 [astro-ph.HE]].
M. Tristram, A. J. Banday, K. M. Górski, R. Keski-talo, C. R. Lawrence, K. J. Andersen, R. B. Barreiro, J. Borrill, H. K. Eriksen and R. Fernandez-Cobos, et al. [arXiv:2010.01139 [astro-ph.CO]].

I. Affleck and M. Dine, Nucl. Phys. B 249, 361 (1985).

A. D. Linde, Phys. Lett. 160B, 243 (1985).

A. D. Dolgov, Phys. Rept. 222, 309 (1992).

M. Dine, L. Randall and S. D. Thomas, Nucl. Phys. B 458, 291-326 (1996) [arXiv:hep-ph/9507453 [hep-ph]].

M. Dine and A. Kusenko, Rev. Mod. Phys. 76, 1 (2003) [arXiv:hep-ph/0303065 [hep-ph]].

Y. P. Wu and K. Petraki, JCAP 01, 022 (2021) [arXiv:2008.08549 [hep-ph]].

L. T. Wang and Z. Z. Xianyu, JHEP 02, 044 (2020) [arXiv:1910.12876 [hep-ph]].

A. Bodas, S. Kumar and R. Sundrum, JHEP 02, 079 (2021) [arXiv:2010.04727 [hep-ph]].

Y. P. Wu, E. Pinetti, K. Petraki and J. Silk, [arXiv:2105.00118 [hep-ph]].

I. Antoniadis, P. O. Mazur and E. Mottola, JCAP 09, 024 (2012) [arXiv:1103.4164 [gr-qc]].

J. M. Bardeen, J. R. Bond, N. Kaiser and A. S. Szalay, Astrophys. J. 304, 15 (1986).

A. M. Green, A. R. Liddle, K. A. Malik and M. Sasaki, Phys. Rev. D 70, 041502 (2004) [astro-ph/0403181].

S. Young, C. T. Byrnes and M. Sasaki, JCAP 07, 045 (2014) [arXiv:1405.7023 [gr-qc]].

T. Suyama and S. Yokoyama, [arXiv:1912.04687 [astro-ph.CO]].

Y. P. Wu, Phys. Dark Univ. 30, 100654 (2020) [arXiv:2005.08441 [astro-ph.CO]].

N. Aghanim et al. [Planck], Astron. Astrophys. 641, A6 (2020) [arXiv:1807.06209 [astro-ph.CO]].

B. J. Carr, Astrophys. J. 201, 1 (1975).

F. Capela, M. Pshirkov and P. Tinyakov, [arXiv:1402.4671 [astro-ph.CO]].

R. Laha, P. Lu and V. Takhistov, [arXiv:2009.11837 [astro-ph.CO]].

S. Clark, B. Dutta, Y. Gao, Y. Z. Ma and L. E. Strigari, Phys. Rev. D 98, no.4, 043006 (2018) [arXiv:1803.09390 [astro-ph.HE]].

A. Hektor, G. Hütsi, L. Marzola, M. Raidal, V. Vaskonen and H. Veermäe, Phys. Rev. D 98, no.2, 023503 (2018) [arXiv:1803.09697 [astro-ph.CO]].

S. Mittal, A. Ray, G. Kulkarni and B. Dasgupta, [arXiv:2107.02190 [astro-ph.CO]].

G. Defilipp, E. Granet, P. Tinyakov and M. H. G. Tytgat, Phys. Rev. D 90, no.10, 103522 (2014) [arXiv:1409.0469 [gr-qc]].

A. Kashlinsky, Y. Ali-Haimoud, S. Cl esse, J. Garcia-Bellido, L. Wyrzykowski, A. Achucarro, L. Amendola, J. Annis, A. Ar bey and R. G. Aren dt, et al. [arXiv:1903.04424 [astro-ph.CO]].

J. M. Ezquiaga, J. García-Bellido and V. Vennin, JCAP 03, 029 (2020) doi:10.1088/1475-7516/2020/03/029 [arXiv:1912.03599 [astro-ph.CO]].

D. G. Figueroa, S. Raatikainen, S. Rasanen and E. Tomberg, [arXiv:2012.06551 [astro-ph.CO]].

C. Pattison, V. Vennin, D. Wands and H. Assadullahi, [arXiv:2101.05741 [astro-ph.CO]].

M. Biagetti, V. De Luca, G. Franciolini, A. Kehagias and A. Riotto, [arXiv:2105.07810 [astro-ph.CO]].

R. Saito and J. Yokoyama, Phys. Rev. Lett. 102, 161101 (2009) [erratum: Phys. Rev. Lett. 107, 069901 (2011)] [arXiv:0812.4339 [astro-ph]].

R. Saito and J. Yokoyama, Prog. Theor. Phys. 123, 867-886 (2010) [erratum: Prog. Theor. Phys. 126, 351-352 (2011)] [arXiv:0912.5317 [astro-ph.CO]].

R. Saito and J. Yokoyama, Phys. Rev. Lett. 122, no.20, 211101 (2019) [arXiv:1810.11000 [astro-ph.CO]].

N. Bartolo, V. De Luca, G. Franciolini, A. Lewis, M. Peloso and A. Riotto, Phys. Rev. Lett. 122, no.21, 211301 (2019) [arXiv:1810.12118 [astro-ph.CO]].

S. Cl esse, J. García-Bellido and S. Orani, [arXiv:1812.11011 [astro-ph.CO]].

G. Domènech, [arXiv:2109.01398 [gr-qc]].

A. Kusenko and M. E. Shaposhnikov, Phys. Lett. B 418, 46-54 (1998) [arXiv:hep-ph/9709492 [hep-ph]].

G. White, L. Pearce, D. Vagie and A. Kusenko, [arXiv:2105.11655 [hep-ph]].