Experiment and theory: the case of the Doppler effect for photons

Giuseppe Giuliani

Dipartimento di Fisica, Università degli Studi di Pavia, via Bassi 6, I-27100 Pavia, Italy

E-mail: giuseppe.giuliani@unipv.it

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Abstract

In 1907, Einstein suggested an experiment with flying atoms for corroborating time dilation. In that paper, the flying atom was conceived as a flying clock: the reference to the Doppler effect was only indirect (the experiments by Stark to the first order of $v/c$). In 1922, Schrödinger showed that the emission of a light quantum by a (flying) atom is regulated by the conservation laws of energy and linear momentum. Therefore, the Doppler effect for photons is the consequence of the energy and momentum exchange between the atom and the photon: a central role is played by the quantum energy jump $\Delta E$ of the transition (a relativistic invariant). The first realization of the experiment devised by Einstein was by Ives and Stilwell (1938). Since then and until now, experiments of this kind have been repeated in search of better precision and/or a deviation from the predictions of special relativity. The striking feature is that all the papers dealing with these experiments completely neglect Schrödinger’s dynamical treatment. There are different origins of this omission: pragmatic (agreement between formulae, wherever coming from, and experiments), historical (deep rooting of the wave theory of light), and epistemological (neglect of basic epistemological rules).

1. Introduction

If light is treated as an electromagnetic wave in vacuum, it is straightforward to derive the formula for the Doppler effect by using, for instance, the transformation equations of the four—wavevector $(\omega/c, \hat{k})$:

$$\omega = \omega' \frac{\sqrt{1 - B^2}}{1 - B \cos \theta}; \quad B = \frac{v}{c}.$$  (1)

As usual, we are dealing with two inertial frames $(O, O')$ whose axis are parallel and oriented along the same directions: $O'$ is considered in motion with respect to $O$ with velocity $v$ along the positive direction of the common $x \equiv x'$ axis; $\theta$ is the angle that the wavevector $\hat{k}$ forms with the $x$ axis.
If we introduce the concept of a photon and write for its energy $E_p = \hbar \omega$, equation (1) becomes valid also for the photon energy:

$$E_p = E'_p \sqrt{1 - \frac{B^2}{1 - B \cos \theta}}.$$  \hspace{1cm} (2)

Alternatively, equation (2) can be derived by treating photons as relativistic particles whose energy—momentum four—vector is $(E/c, \vec{p})$, with $E = \hbar \omega$ and $p = \hbar \omega/c$.

The light source and its physical state do not enter equations (1), (2). Therefore, the interpretation of equation (1) reads: if $\omega'$ is the angular frequency of the light wave measured in the reference system $O'$, then the angular frequency of the same wave measured by $O$ is given by equation (1). A similar interpretation holds for equation (2). Notice that the source needs not to be at rest in $O'$.

Equations (1), (2) are also used for describing the Doppler effect when experiments deal with photons emitted or absorbed by atoms/nuclei. Moreover, since around 1905 till nowadays, the (transverse) Doppler effect for light has been interpreted as an experimental corroboration of time dilation both in research papers and in textbooks.

This paper discusses these issues in the frameworks of the wave and the corpuscular descriptions of light; it also discusses some historical passages that bring out the underlying epistemological aspects. Therefore, it may be of some interest for researchers and for teachers at university or high school levels.

2. The Doppler effect of photons

2.1. Stark and Einstein around 1905: atoms as clocks

Within the theoretical framework around 1900, the emission of light by matter was due to the harmonic vibrations of point electrical charges (electrons). On the basis of this model, Stark concluded that lines emitted by atoms in flight should be Doppler shifted. In 1905, he succeeded in detecting this shift (to the first order in $v/c$) for light emitted by hydrogen atoms [1]. This achievement, together with the later discovery of the Stark effect, earned him the Nobel prize in physics in 1919.

Einstein commented on the results obtained by Stark with these words.

In an important paper published last year, Mr J Stark (J Stark, *Ann. d. Phys.* 21 (1906), 401) demonstrated that the moving positive ions of canal rays emit line spectra by identifying the Doppler effect and following it quantitatively. He also undertook experiments with the intention of detecting and measuring an effect of the second order (proportional to $(v/c)^2$); however, the experimental arrangement, which was not set up specifically for this purpose, was not adequate for achieving reliable results.

I will show here briefly that the principle of relativity in conjunction with the principle of the constancy of the velocity of light makes it possible to predict the above effect. As I showed in an earlier paper, it follows from these principles that a uniformly moving clock runs at a slower rate as judged from a ‘stationary’ system than as judged by a co—moving observer. If $\nu$ denotes the number of the clock’s strokes per unit time for the observer at rest, and $\nu_0$ the corresponding number for the co—moving observer, then

$$\frac{\nu}{\nu_0} = \sqrt{1 - \left(\frac{v}{c}\right)^2}.$$  \hspace{1cm} (3)

\[1\] Stark’s discovery of the effect bearing his name has been the result of a research program. As discussed in [2], the contemporaneous and independent discovery of the same effect by Lo Surdo was an accidental one.
or to the first approximation

\[ \frac{\nu - \nu_0}{\nu_0} = -\frac{1}{2} \left( \frac{\nu}{c} \right)^2. \] (4)

The atom ion of the canal rays that emits and absorbs radiation of certain frequencies is thus to be conceived as a fast—moving clock, and the relation just indicated can therefore be applied to it [3, p 232].

Both Stark and Einstein assumed that the atom is a clock. Clearly, this assumption is justified only if the atom is the seat of some periodic motion. This motion could be, for instance, the supposed harmonic motion of bound electrons or the supposed elliptical motion of the electron in Bohr’s model of a hydrogen atom. However, in the latter case, the frequency of the light emitted by the hydrogen atom differs from the frequency of the electronic motion.

Anyway, the advent of quantum mechanics forbids any description of atoms as seats of periodic motion of electrons: atoms are not clocks\(^2\).

Nevertheless, let us come back to Einstein. After having assumed that atoms are clocks, he applied the time dilation formula to them. He thus obtained equation (3) that yields the ratio between the ‘proper’ frequency \(\nu_0\) of an atom—clock and the frequency \(\nu\) of the same atom—clock for an observer that sees the atom—clock in uniform motion with velocity \(v\).

Einstein did not refer to the formula for the Doppler effect (1) derived two years before in his theory of special relativity by treating light as an electromagnetic wave [4, p 161]. In this paper, Einstein also proved that the energy of a ‘light complex’, i.e. the light energy contained in a sphere moving at the speed of light with respect to an inertial frame, in passing from this frame to another, changes in exactly the same way as the light frequency. Of course, this means that the light energy and its frequency are connected by a relativistic invariant: \(U = (Nh)\nu\). However, Einstein did not take this step: as stressed and commented by Pais, Einstein kept special relativity well separated from the light quanta hypothesis [5, p 909].

2.2. Schrödinger, 1922: atoms emit light quanta endowed with energy and momentum

In 1922 Schrödinger dealt with the radiation emitted by atoms in motion in terms of light quanta [6]. Schrödinger makes it clear from the beginning that, once we accept Einstein’s idea that the quantum \(h\nu\) ‘always carries’ a linear momentum \(h\nu/c\), we have to recognize that the emission of a quantum \(h\nu\) by an atom produces a ‘jump’ in its velocity and that this jump is responsible for the Doppler shift.

The problem posed by Schrödinger is illustrated in figure 1 (not used by Schrödinger). In the reference frame of the measuring apparatus, it is solved by writing down the conservation equations for energy:

\[ E_p = \gamma_1 E_1 - \gamma_2 E_2 \] (5)

and momentum:

\[ \gamma_1 \frac{E_1}{c^2} v_1 \cos \theta_1 = \gamma_2 \frac{E_2}{c^2} v_2 \cos \theta_2 + \frac{E_p}{c} \] (6)

\[ \gamma_1 \frac{E_1}{c^2} v_1 \sin \theta_1 = \gamma_2 \frac{E_2}{c^2} v_2 \sin \theta_2. \] (7)

\(^2\) The fact that we now use atomic clocks should not confuse us: in these clocks, a quantum transition between two atomic levels is used as a locking parameter of the resonant frequency of a quartz oscillator.
Figure 1. Emission of a light quantum by the atom A in motion. The light quantum is emitted along the direction \( A \rightarrow O \). O is the entrance slit of the spectrograph. The subscript 1 denotes the quantities before the emission; subscript 2 denotes the quantities after the emission.

\( E_p \) is the energy of the light quantum; \( E_1 \) and \( E_2 \) are the rest energies of the atom before and after the emission, respectively; \( \gamma_1 \) and \( \gamma_2 \) are the relativistic factors before and after the emission.

After a somewhat lengthy analytical manipulation we get:

\[
E_p = E_p^0 \sqrt{\frac{1 - v_1^2/c^2}{1 - (v_1/c) \cos \theta_1}} \tag{8}
\]

with

\[
E_p^0 = \Delta E \left( 1 - \frac{\Delta E}{2E_1} \right), \quad (v_1 = 0), \tag{9}
\]

where \( \Delta E = (E_1 - E_2) \); then, \( \Delta E \) is the energy difference between the two states of the atomic transition. \( E_p^0 \) is the measured light quantum energy when the atom is at rest before the emission. Both \( \Delta E \) and \( E_p^0 \) are relativistic invariants since they depend only on rest energies. The term \( \Delta E/2E_1 \) is in general negligible, unless we are dealing with \( \gamma \) photons emitted by free nuclei.

Schrödinger did not write equation (8). He stopped at an intermediate formula containing both atoms’ velocities, before and after the emission [6, p 303]:

\[
\nu = \nu^* \frac{1}{\sqrt{\gamma_1 [1 - (v_1/c) \cos \theta_1] \times \gamma_2 [1 - (v_2/c) \cos \theta_2]}}, \tag{10}
\]

where

\[
\nu^* = \frac{E_1^2 - E_2^2}{2\hbar \sqrt{E_1E_2}}. \tag{11}
\]

A simple calculation suggested by the fact that the atom’s velocity after the emission is determined by its velocity before the emission and by the energy—momentum conservation leads to the more significant equations (8) and (9) [7, p 197–203]. Obviously, Schrödinger was well aware of the relation between the two velocities; this renders the fact that he stopped at equation (10) more intriguing.

Schrödinger’s treatment can be applied also to the case of a photon absorbed by an atom in flight with respect to the laboratory reference frame [7, p 201–202]. The energy \( E_p \) that a photon must have for being absorbed by the atom is again given by equation (8), where, in this case:

\[
E_p^0 = \Delta E \left( 1 + \frac{\Delta E}{2E_1} \right) \tag{12}
\]
is the energy of the photon absorbed by an atom at rest before the absorption and, of course, now \( \Delta E = E_2 - E_1 \).

Equation (8) is formally identical to equation (2) or, since \( E_p = \hbar \omega \), to equation (1): however, the meanings of these formulae are quite different. While equations (1), (2) ignore the emitting/absorbing particle and its physical state, equation (8), through equation (9) for emission or equation (12) for absorption, focuses on the energy difference \( \Delta E \) between the two quantum levels of the emitting/absorbing particle and the energy \( E_p^0 \) emitted/absorbed when the particle is at rest before emission/absorption (equations (9), (12)). Furthermore: while equations (1), (2) connect quantities measured in two reference frames, Schrödinger’s treatment uses only the reference frame of the experimental apparatus owing to the use of the relativistic invariants \( \Delta E \) and \( E_p^0 \).

In order to further clarify the physical meaning of Schrödinger’s treatment, let us illustrate the features of the emission/absorption process as described by equations (8) and (9), (12). In the case of photons emitted/absorbed, the increase/decrease of their energy with respect to the quantity \( \Delta E \) is due to an energy—momentum exchange with the emitting/absorbing particle. For instance, let us consider an atom that emits a photon. If the photon is emitted in the forward direction, its energy is increased, with respect to \( \Delta E \), by exactly the same amount by which the kinetic energy of the atom is decreased; if the photon is emitted in the backward direction, its energy is decreased, with respect to \( \Delta E \), by exactly the same amount by which the kinetic energy of the atom is increased. In the case of absorption, the photon energy required for exciting the atom, when the photon flights in the opposite direction to that of the atom, is decreased, with respect to \( \Delta E \), by exactly the same amount by which the kinetic energy of the atom is decreased; when the photon is chasing the atom, the photon energy required for exciting the atom, is increased, with respect to \( \Delta E \), by exactly the same amount by which the kinetic energy of the atom is increased.

Nowadays, this energy–momentum exchange is basically exploited by those who laser–cool atoms or use saturation spectroscopy.

Schrödinger’s approach can be generalized by taking into account the dependence of the energy of the photon emitted/absorbed on the gravitational potential. As shown, for instance by Møller [10, pp 401–407], it is sufficient to rewrite, for the emission case, equation (9) as follows:

\[
E_p^0 \approx \Delta E \left( 1 - \frac{\Delta E}{2E_1} \right) \left( 1 + \frac{\phi}{c^2} \right),
\]  

(13)

where \( \phi \) is the gravitational potential and the \( \approx \) sign is due to the approximation for small gravitational potential. An analogous correction must be made for equation (12) (absorption case).

Schrödinger’s paper has been rapidly forgotten; its derivation has been rediscovered by Davisson [8]; more recent ones can be found, for instance, in the books by French [9, pp 197–199] and Møller [10, pp 401–407]. No one quotes Schrödinger’s paper. The idea of describing the interaction of photons with particles by writing down the conservation equations for energy and linear momentum was used some years later by Compton [11] and Debye [12], without quoting Schrödinger, for explaining the Compton effect. The reasons why Schrödinger’s paper has been forgotten are not clear: we can only guess some plausible ones. First of all, around 1920, the concept of light quanta had not yet been accepted by the scientific community; secondly, the fact that Schrödinger stopped at equations (10) and (11) might have obscured the relevance of the paper.
2.3. Ives and Stilwell, 1938: atoms as clocks, again

In the late 1930s, Ives and Stilwell set up an apparatus for the realization of the experiment devised by Einstein in 1907 [13]. The experiment was fully financed by the Bell Telephone Laboratories, where Ives was then working. It was a weird twist of fate that a test of relativity suggested by its founder had been taken up, performed and interpreted by an anti—relativist.

Ives (1882–1953) has been a staunch opponent of relativity. He built his views, characterized by an ether—based theory, through a series of papers written over about fifteen years: they have been collected in a volume [14]. Unfortunately, the grossly anti—relativist preface by one of the editors throws an unfavourable shadow on Ives’ papers. A review of this book by Arthur Miller can be found in [15]. Ives’ theory is a very intricate one and relies on a procedure for clock synchronization based on the use of two pairs of rods and clocks, one pair being, by assumption, not affected by their motion in the ether. By assumption, the velocity of light is isotropic and equal to c only in a reference frame at rest in the ether. Ives’ coordinate transformations converge to Lorentz’s as the velocities of rods and clocks used for synchronization go to zero. Thus, in principle, the two coordinate transformations could never coincide. According to Miller, ‘Ives’s Larmor—Lorentz theory was never developed to the point where it could be seriously considered as an alternative to the special relativity theory’.

The article’s title is unequivocal: ‘An experimental study of the rate of a moving atomic clock’. As in Einstein’s paper [3], the atom is considered as a clock. Ives and Stilwell observe that, as far as the transverse Doppler effect is concerned

...it would be extremely difficult to be sure that observation was made exactly at right angles to the direction of the rays, and very small deviations from this direction would introduce shifts of the order of magnitude of the expected effect [13, p 215].

However, this difficulty

...can be avoided by observing not at right angles, but in two directions, with and against the motion of the particles; the observation being made simultaneously by the use of a mirror in the tube. Under these conditions the displaced Doppler lines are observed corresponding to motion towards and away from the observer, and the effect to be observed is a shift of the centre of gravity of the displaced lines with respect to the undisplaced line. As shown in an earlier paper of this series, this shift of centre of gravity is expressed by the equation

$$\lambda = \lambda_0 \left(1 - v^2/c^2\right)^{1/2}$$

where v is the observed or measured velocity of the positive particles [13, p 216].

Since the experimental setup used by Ives and Stilwell has inspired many experimenters up to the present day, it is worth discussing it in some detail: see the appendix. The experiment allows the measurement of the transverse Doppler effect in the approximation of small velocities: of course, in the conclusions, Ives and Stilwell interpret their results as a confirmation of Ives’s ether based theory.

A year later, Robert Clark Jones, he too at Bell Laboratories, interpreted Ives and Stilwell’s results in a special relativity approach, assuming, of course, that atoms can be treated as clocks. Jones did not attack Ives’ standpoint; he wrote instead:

The conceptual background of these theories (Larmor’s and Lorentz’s) is not the one which is most popular with physicists today, however, and for this reason it seemed worthwhile to obtain the theoretical predictions from the point of view of the special
theory of relativity, particularly since the relativistic point of view yields the results in so simple a manner. The theoretical predictions we shall obtain here are identical with those obtained by Ives from electron theory [16, p 337] (my italics).

Some years later, a similar experiment with canal rays was carried out by Gerhard Otting [17]. The contrast with Ives and Stilwell’s paper is striking. Otting does not comment on either the formulae or their interpretation: he is interested only in the correspondence between the formulae and the experimental data.

The experiment was repeated again in the sixties by Mandelberg and Witten [18]. The authors recall that

An analysis of the experiments of Ives and Stilwell and of Otting indicates that although their reported experimental points seem to fit the curve with an accuracy of about 2 to 3%, the experimental uncertainty is more nearly 10–15% [18, p 529].

Hence the necessity of repeating the experiment. The basic experimental setup was the same as that of the older experiment: only the precision was improved. According to Mandelberg and Witten ‘The experimental result is that the exponent in the quadratic expression for the Doppler shift, \((1 - \beta^2)^{1/2}\), is found to be 0.498 \(\pm\) 0.025 [18, p 529].’ ‘This implies an over-all precision in this experiment of 5% the limit on the accuracy being imposed by the width of the beam lines [18, p 536].’

3. The Doppler effect for photons as a direct consequence of Lorentz transformations

Starting about 1970, the Doppler shift of radiation emitted/absorbed by atoms/nuclei in flight has been viewed as a direct consequence of Lorentz transformations: the idea that the atoms are clocks has gone, at least as an explicit statement.

3.1. Experiments with \(\gamma\) photons

Olin et al used an experimental set up similar to that of Ives and Stilwell [19]: the Doppler shift of 8.64 MeV \(\gamma\) photons emitted by \(^{20}\)Ne nuclei was studied. The velocity of the emitting nuclei was 0.012 \(c\) or 0.049 \(c\). The detector was an annular \(Ge(Li)\) junction that measures the energy of \(\gamma\) photons. In order to test special relativity through the transverse Doppler effect, the authors discuss the formula:

\[
E(\theta) = E_0 \frac{F(\beta)}{1 - \beta \cos \theta},
\]

where the significant quantity is the photon energy and the function \(F(\beta)\) is equal to \((1 - \beta^2)^{1/2}\) if special relativity is correct (\(\beta = v/c\)). They write:

This phenomenon (Doppler shift) is a geometrical property of space—time, and is intimately connected with the problem of synchronization of clocks in different frames of reference [19, p 1633].

3.2. Direct observation of the transverse Doppler shift in hydrogen

The transverse Doppler shift of the \(H_\alpha\) hydrogen line has been observed directly (i.e. perpendicularly with respect the direction of the atoms’ motion) by Hasselkamp et al [20]. The detector was a photomultiplier used in the single photon counting mode. According to the authors:
Equation (1) is a consequence of the Lorentz transformation of time. The experimental confirmation of the validity of (1) is therefore a verification of time dilation [20, p 152].

3.3. Lasers enter the scene

While, from Ives and Stilwell’s experiment, the Doppler shift has been studied by measuring the energy of the emitted photons, a major change occurred with the appearance of lasers: the measured quantity became the energy of the photons absorbed by the atoms in flight. The measuring techniques varied from the simple use of lasers for exciting a quantum transition of the atoms in flight [21], to the utilization of two photon absorption [22] or saturation spectroscopy [23]; collinear (probing laser beam parallel/antiparallel to the atoms’ motion), orthogonal (probing laser beam perpendicular to the atoms’ motion) and variable (different angles between the directions of the probing laser beam and the atoms’ motion) geometries have been used. For a rather recent review, see [24].

These papers consider the Doppler shift as a direct consequence of Lorentz transformations and compare the predictions of special relativity with those of the Mansouri—Sexl kinematic test theory of special relativity [25]. This theory is based on the assumption that the speed of light is isotropic only in a hypothetical preferred reference frame and use generalized coordinates transformations that take into account also the possibility of different clock synchronization procedures.

Very recently, Chou et al have studied the transverse Doppler effect and the gravitational red shift using an optical clock [26]. The unique feature of optical clocks consists of the use of a single ion at rest in an electromagnetic trap. As far as the transverse Doppler shift is concerned, Chou et al have compared the frequency of the probing laser when the ion is at rest (with respect to the laboratory) with that of the same probing laser when the ion is set in harmonic motion along a direction approximately perpendicular to the direction of the laser beam: as in the other experiments with lasers, the experiment checks the absorption of photons by the ion in motion. Chou et al interpret their results on the basis of the equation:

$$\frac{\delta f}{f_0} = \frac{1}{\gamma(1 - v_l/c)} - 1 \approx -\frac{1}{2} \frac{\langle v^2 \rangle}{c^2}$$

(15)

where $v_l$ denotes the component of the ion speed along the direction of the probing laser beam; equation (15) is, of course, another way of writing equation (1) by averaging over time. $\langle v_l \rangle = 0$ because the ion’s motion is harmonic; $1/\gamma \approx -1/2(\langle v^2 \rangle / c^2)$ because $v \approx 10 \text{ ms}^{-1} \ll c$. Since the experimental data fit equation (15), Chou et al conclude that they have experimentally tested time dilation.

4. Experiments, formulae and theories: what are we measuring?

Sections 2, 3 show that all the papers written after Schrödinger’s seminal article completely ignore its treatment; instead, they rely on equations (1), (2) that compare the angular frequencies of a light wave or the photon energies in two distinct reference frames. Let us begin with equation (1). It is well known that we can derive a relativistic Doppler formula valid for both acoustic or light signals [27, 28]:

$$\frac{\omega_a}{\omega_e} = \frac{1 - (v_a/V) \cos(V, \vec{v}_a)}{1 - (v_e/V) \cos(V, \vec{v}_e)} \sqrt{1 - \frac{v_e^2}{c^2}} \sqrt{1 - \frac{v_a^2}{c^2}}.$$
The reference system is the one in which the medium is at rest; \( V \) is the signal velocity; \( v_e \) and \( v_a \) the emitter and the absorber velocity. For light in vacuum, this formula reduces to formula (1). Equation (16) is obtained by assuming that either the source emits signals of ideally null duration at a specified time interval or a periodic wave. In the first case, the phenomenon’s period is the time interval between two consecutive signals; in the latter, it is the wave period.

However, atoms and nuclei do not emit waves or null duration signals, but photons endowed with energy and linear momentum: therefore, equation (1) cannot be applied to atoms or nuclei. Nevertheless, equation (1) describes the experimental data: this is due to the fact that equation (8) reduces to (1) by assuming \( \omega' = E^0_p/\hbar \), where \( E^0_p \) is given by equation (9) for emission or by equation (12) for absorption and that the velocity entering equation (1) is the atom/nucleus velocity before emission/absorption. Then, equation (1) can be used only through a series of conceptual shifts in passing from a treatment of the emission/absorption process in term of photons, based on relativistic invariants \((\Delta E, E^0_p)\) and one reference system (that of the experimental apparatus), to a formula belonging to the wave theory of light and connecting two quantities (the angular frequencies) in two distinct reference frames. The use of equation (2) does not involve the wave theory of light: however, also in this case, the various conceptual and approximation steps should be made explicit.

The experiments discussed in sections 2, 3 are easily explained using Schrödinger’s approach within the reference frame of the experimental apparatus. Let us first consider the experiments in which the flying particle emits photons. In the laboratory reference frame, given an atom/nucleus and the two quantum levels of the transition (i.e. \( \Delta E \)), equation (9) yields \( E^0_p \); alternatively, \( E^0_p \) can be measured directly when the particle, before emission, is at rest. Then equation (8) predicts the energy of the photon emitted by the flying particle in terms of the measured velocity \( v_1 \) and the angle \( \theta_1 \). If the experimental test is positive, it corroborates a prediction of the joint use of relativistic dynamics and quantum mechanics.

The experiments with lasers deserve a separate discussion because they deal with absorption of photons by atoms/nuclei. In the laboratory reference frame, given an atom/nucleus and the quantum levels of the transition (i.e. \( \Delta E \)), equation (12) yields \( E^0_p \); alternatively, \( E^0_p \) can be measured directly when the particle, before absorption, is at rest. Then, given \( E^0_p \), the particle velocity \( v_1 \) and the angle \( \theta_1 \), equation (8) yields the energy that the laser photon must have in order to be absorbed by the flying particle. If the experimental test is positive, i.e. if the flying particle absorbs the laser photon, then it corroborates a prediction obtained by the joint use of relativistic dynamics and quantum physics.

The measurement of the gravitational red shift by Chou et al [26, p 1632], calls for a final remark. The generalization of Schrödinger’s treatment that takes into account the gravitational potential yields:

\[
E_p \approx \Delta E \left( 1 + \frac{\Delta E}{2E_1} \right) \left( 1 + \frac{\phi}{c^2} \right) \frac{1 - v_1^2/c^2}{1 - (v_1/c) \cos \theta_1},
\]

where, as already pointed out in section 2.2, the \( \approx \) sign comes in for small gravitational potentials. Equation (17) describes both effects contemporaneously: the Doppler effect due to the velocity of the absorbing particle and the gravitational one. Instead, in the quoted paper, the two effects seem to derive from two distinct theoretical backgrounds.

Why does Schrödinger’s treatment keep being neglected? The first and basic answer is a pragmatic one: as pointed out, formulae (1), (2) describe the experimental data. However,
physics is not a game between formulae and experiments: formulae belong to theories with a well defined application domain. Therefore, formulae cannot be extrapolated from a theory and applied to phenomena belonging to other theoretical frameworks. As Heinrich Hertz put it some time ago.

The very fact that different modes of representation contain what is substantially the same thing, renders the proper understanding of any one of them all the more difficult. Ideas and conceptions which are akin and yet different may be symbolized in the same way in the different modes of representation. Hence for a proper comprehension of any one of these, the first essential is that we should endeavour to understand each representation by itself without introducing into it the ideas which belong to another [29, p 21] (my italics).

Applied to our case, this means that we should not mix ideas and formulae coming from two different 'representations' as the undulatory and the corpuscular theory of light. We should instead define their domains of applications and find out when, how and why their predictions coincide.

But there is another reason: the 19th century has deeply embedded the undulatory description of light in the background knowledge of physicists. Within this knowledge, the Doppler effect has been constantly viewed as a wave phenomenon: the emergence of the light quantum did not change this rooted habit. The prevailing influence of the undulatory description emerges also in the language: for instance, the locution ‘photon frequency’ is often used instead of ‘photon energy’ and, in general, when dealing with a photon the equations are written in terms of frequencies instead of energies.

There are two questions left: (a) the Doppler effect as a direct consequence of Lorentz transformations; (b) the statement according to which the experimental corroboration of the Doppler effect for photons is a corroboration of time dilation.

About (a). As we have seen, the Doppler effect for photons is a consequence of the relativistic conservation laws for energy and momentum and the concept of a photon endowed with energy and linear momentum: the term ‘direct’ is clearly inappropriate since we must add to Lorentz transformations the laws of relativistic dynamics and the quantum concept of atoms/nuclei energy levels.

Instead, (b) is a sound statement. With a specification: it is an indirect corroboration. The relativistic factor \( \sqrt{1 - v^2/c^2} \), basically a time dilation factor, enters equation (8) through the relativistic conservation equations; therefore, an experimental corroboration of equation (8) is, primarily, a corroboration of relativistic dynamics and some quantum hypothesis. That the factor \( \sqrt{1 - v^2/c^2} \) is a time dilation factor, can be easily shown by ideal experiments with light signals of null duration. This kind of approach was firstly outlined by Bondi [30], who, however, uses also geometric considerations. Derivations of time dilation, length contraction and Lorentz transformations based only on ideal experiments with light signals of null duration can be found in, for instance, [31] and [7, pp 29–42].

Then, every corroboration of a formula containing the relativistic factor \( \sqrt{1 - v^2/c^2} \) can be considered as an indirect corroboration of time dilation. For example, a typical formula of this kind is the one giving the radius \( R \) of the circular trajectory of a point electrical charge that enters a uniform magnetic field perpendicularly to it. With obvious notations, we have (neglecting irradiation):

\[
R = \frac{1}{\sqrt{1 - v^2/c^2}} \frac{mv}{qB}.
\] (18)
Since equation (18) is derived by the joint use of the relativistic force law and the expression of Lorentz force, its experimental verification constitutes a corroboration of both laws and, indirectly, of time dilation.

5. Conclusions

A survey of the experiments on the Doppler effect for photons (through about a century) shows that the explanation of these experiments has been initially given by considering the atoms as clocks; then by using the Doppler formula of the wave theory of light (or its purely formal reformulation in terms of photon energies). The corpuscular treatment of the Doppler effect put forward by Schrödinger in 1922 has been completely ignored in spite of the fact that, in other physical contexts, it is commonplace that during the emission/absorption of a photon by an atom/nucleus the emitting/absorbing particle exchanges energy and momentum with the photon: for instance, in the emission/absorption of photons by free nuclei, in the laser cooling of atoms and in saturation spectroscopy.

The origins of this omission are of different kinds: pragmatic (agreement between formulae, wherever coming from, and experiments), historical (deep rooting of the wave theory of light) and epistemological (neglect of basic epistemological rules).

Physics is not simply a game between formulae, from wherever they are, and experiments: one should not use formulae from one theory (the wave theory of light) to describe phenomena concerning photons which belong to another theory (the corpuscular theory of light). This epistemological criterium, put forward in its general form more than a century ago by Heinrich Hertz, should not be forgotten, particularly in teaching; however, also in research, the neglect of basic epistemological criteria may, in the long run, be leading us astray.

Appendix. Ives and Stilwell experiment

The formulae appearing below from (i) to (iv) are relativistic formulae, not Ives’.

(i) The wavelength $\lambda_B$ of the light emitted by the incoming atoms at a small angle $\theta$ to the beam direction is given by:

$$\lambda_B = \lambda_0 \frac{1 - B \cos \theta}{(1 - B^2)^{1/2}} \approx \lambda_0 \left(1 - B \cos \theta + \frac{1}{2} B^2\right),$$

where $\lambda_0$ is the natural wavelength, $B = v/c \ll 1$ and $v$ is the velocity of the atoms.

(ii) The wavelength $\lambda_R$ of the light emitted by the receding atoms at the angle $(\pi - \theta)$ to the beam direction is instead:

$$\lambda_R = \lambda_0 \frac{1 + B \cos \theta}{(1 - B^2)^{1/2}} \approx \lambda_0 \left(1 + B \cos \theta + \frac{1}{2} B^2\right).$$

(iii) The average of the two wavelengths is:

$$\lambda_Q = \frac{1}{2} (\lambda_B + \lambda_R) = \lambda_0 \left(1 + \frac{1}{2} B^2\right),$$

and equals the wavelength which would be observed at right angles to the beam.

(iv) The difference between the two wavelengths is:

$$2\lambda_D = \lambda_R - \lambda_B = 2\lambda_0 B \cos \theta,$$

where $\lambda_D$ represents the first order Doppler effect.
The emitting particles were hydrogen atoms. The observed line, on a photographic plate, was the \( H_\beta \) line. Ives and Stilwell:

(a) derived the experimental value of the atoms velocity \( v \) from (A.4), using for \( \lambda_0 \) the wavelength of the ‘undisplaced line’, i.e. the central line appearing on the plate together with the ‘displaced lines’ \( \lambda_B \) and \( \lambda_R \);

(b) predicted the value of the transverse Doppler shift by using this value of the atom’s velocity;

(c) compared the calculated value of the transverse Doppler shift with that measured according to (A.3);

(d) concluded by stating that the theoretical predictions agree with experimental results within measurement precision\(^4\).

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\(^4\) As a matter of fact, Ives and Stilwell used also an (a’) step (instead of (a)) in which the velocity of the emitting atom was assumed to be the velocity of the incoming \( H_2^+ \) or \( H_1^+ \) ion calculated through the relation \( eV = (1/2)Mv^2 \), where \( e \) is the charge of the ion, \( M \) its mass, \( v \) its velocity and \( V \) the accelerating potential. Ives and Stilwell found that both procedures for calculating the velocity of the emitting atoms resulted in a predicted transverse Doppler shift (point (b) above) in agreement with the measured one (point (c) above).
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