Dynamics of two ferromagnetic insulators coupled by superconducting spin current

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A conventional superconductor sandwiched between two ferromagnets can maintain coherent equilibrium spin current. This spin supercurrent results from the rotation of odd-frequency spin correlations induced in the superconductor by the magnetic proximity effect. In the absence of intrinsic magnetization, the superconductor cannot maintain multiple rotations of the triplet component but instead provides a Josephson type weak link for the spin supercurrent. We determine the analogue of the current-phase relation in various circumstances and show how it can be accessed in experiments on dynamic magnetization. In particular, concentrating on the magnetic hysteresis and the ferromagnetic resonance response, we show how the spin supercurrent affects the nonequilibrium dynamics of magnetization which depends on a competition between spin supercurrent mediated static exchange contribution and a dynamic spin pumping contribution. Depending on the outcome of this competition, a mode crossing in the system can either be an avoided crossing or mode locking.

Superconductivity is characterized by a $U(1)$ symmetry breaking complex order parameter and the dissipationless supercurrent proportional to its phase gradient. In the context of spin transport, analogous spin superfluidity has been discussed in various scenarios [1]. Recent work on coherent spin transport in multilayers containing ferromagnetic (F) and superconducting (SC) elements has opened the question of dissipationless spin transport in superconductors [2–4]. While there are notions of dissipationless or conserved spin currents in such systems [5, 6], it has remained unclear in which sense such spin currents can be observed in experiments on dynamical magnetization. Here we clarify the situation by showing how spin supercurrents (SS) naturally arise in ferromagnetic resonance (FMR) experiments involving two or more magnets, how it is linked to the gradient of the direction of the triplet order parameter, and how it mediates magnetic interactions.

What distinguishes the superconducting currents from normal persistent ones [7–9] is their robustness against disorder and interactions and the large length scales on which they occur. In F/SC systems SS depends on the magnetic proximity effect and its range is set by the coherence length of the SC.

Here we study SS in possibly the simplest superconducting system in which it can exist, namely a SC sandwiched between two ferromagnetic insulators (FI) with noncollinear magnetizations (Fig. 1). This system was considered already in the 1960s by de Gennes who showed that the SC mediates an antiferromagnetic interaction between the magnetic moments of the two FIs [10]. We demonstrate that this interaction can also be interpreted as an equilibrium spin current, and generalize it for a SC with a finite length and finite spin scattering. We consider the magnetization dynamics of two FIs coupled by spin pumping and SS, and show that spin supercurrents can lead to decreased or increased damping of the FMR modes of the trilayer, as compared to unhybridized modes in the normal (N) state.

We first illustrate the concept of SS with a scheme based on the linearized Usadel equation. Consider a SC of length $d$ coupled with two non-collinear FIs, with magnetizations on $y$-$z$ plane pointing in directions $m_1$ and $m_2$ forming an angle $\phi$. The effective exchange field at the SC/FI interface leads to a partial conversion of the conventional singlet superconducting condensate into a triplet component [11, 12]. Thus, near $T_c$, the Matsubara Green’s function (GF) for frequency $\omega_m > 0$ has the...
The triplet condensate is determined by the Usadel equation [24], using the spin-mixing BC [15, 25]

$$D \mathbf{n} \cdot \hat{\mathbf{g}} \hat{\mathbf{g}}_{\mathbf{x} = x_n} = i G_n [\mathbf{m}_n \cdot \sigma \tau_3 \hat{\mathbf{g}}],$$

(6)

where $\hat{\mathbf{g}}$ are the $8 \times 8$ quasiclassical Green’s functions in Keldysh-Nambu-Spin space and $\sigma$-products are time-convolutions. The above BC gives Eq. (3b) as a special case. In the BC we take into account only the effective exchange field, [15] and neglect terms associated with interfacial spin relaxation and decoherence. [26–28]

By taking the trace over spin of (6), and integrating over the energy, the spin current through the nth interface is [29]

$$j_{x,n}(t) = G_n \{S(t,x_n) \times \mathbf{m}_n(t) - N_0 \dot{\mathbf{m}}_n(t)\},$$

(7)

where $S$ contains both the equilibrium spin density $\chi(x, x')$. Substituting Eq. (3b) into Eq. (2), we find the SS

$$j^{eq}_{x x} = G_1 G_2 \chi(x_1, x_2) \sin \phi.$$

(5)

In analogy to SNS-junction, the FSF trilayer can be considered a spin Josephson junction [19, 20]. The middle layer is weak in the sense that it cannot support multiple phase windings, since there is no energy penalty for the vanishing condensate function. In order to have strong SS with a total phase winding larger than $\pi$ would require the condensate to be restricted to $U(1)$ symmetry, as happens e.g. in easy-plane ferromagnets [1, 21]. Figure 2b shows the current-angle relation, which deviates from sine function at high $T$ and at strong coupling.

As an equilibrium current, $j^{eq}_{x x}$ is conserved through the SC.[22] Figure 2a shows its magnitude as a function of $d$. In the thin-film limit $d \ll \xi_0$, where $\xi_0$ is the SC coherence length, each interface induces a homogeneous exchange field with amplitude $h_n = G_n/d$. [23] The total exchange field is $\mathbf{h} = h_1 \mathbf{m}_1 + h_2 \mathbf{m}_2$ is limited by the Chandrasekhar-Clogston critical value $h_c = \Delta_0/\sqrt{2}$. With fixed $h_n$ the SS is proportional to $d$. When $d \to \infty$, the supercurrent vanishes as exp($-d/\xi_0$). This exponential factor measures the overlap between the two proximity fields. Maximal SS is obtained when the maximum volume of SC is proximitized by both FIs at $d = \xi_0$.

At arbitrary temperatures and with finite spin-orbital/spin-flip scattering times $\tau_{so/ff}$, we determine the SC spin accumulation in the dirty limit from the full Usadel equation [24], using the spin-mixing BC [15, 25]

$$D \mathbf{n} \cdot \hat{\mathbf{g}} \hat{\mathbf{g}}_{\mathbf{x} = x_n} = i G_n [\mathbf{m}_n \cdot \sigma \tau_3 \hat{\mathbf{g}}],$$

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We introduce a Stoner-Wohlfarth type free energy (per interface area) to describe the effect of SS on the magnetic configuration of the FI/SC/FI trilayer

\[ F(m_1, m_2) = \sum_{n=1,2} \left[ -\mu_0 M_n H - K_A^n (m_n \cdot z)^2 + K_B^n (m_n \cdot x)^2 \right] + F_{sc}(m_1 \cdot m_2, \Delta), \quad (8) \]

where \(|M_n| = M_n\) is the FI magnetic moment per interface area. The free energy includes the Zeeman energy from external magnetic field \(H\), the SC free energy \(F_{sc}\), and the in-plane easy axis/out-of-plane anisotropy energies \(K_A^n / K_B\). In the thin-film limit, at \(T = 0\) and without any spin scattering \(F_{sc}(h) = dN_0(|h|^2 - \Delta^2)/2\). In the general case we use the SC energy functional of Ref. 32.

In a static setting, the coupling between the magnets can be described as an effective magnetic field \(\mu_0 H_{eff,1} = -J_\sigma m_2 / M_1\), which can be identified with SS via the spin-transfer torque

\[ j_{\tau,1}^{eq} = -M_1 \times \mu_0 H_{eff,1}^{sc} = J_\sigma m_1 \times m_2, \quad (9) \]

with exchange constant \(J_\sigma = dF_{sc}/d\cos \phi \geq 0\). At weak coupling Eq. (9) coincides with Eq. (5).

When the SC exchange energy is small compared to the anisotropy energies, SS modifies the coercive fields [33] (Fig. 3a). In the SC state, the coercive fields for AP-to-P switching increase by \(J_\sigma / M_n\), and the coercive fields for P-to-AP switching decrease by \(J_0 / M_n\) relative to the N state. With a strong SS (Fig. 3b) the anisotropies cannot force the magnets into a binary parallel/antiparallel configuration space and the exchange interaction may induce a spin-flop transition, in which the two magnets collectively rotate from AP to P configuration [34].

At low temperatures the SC transition is of the first order as a function of \(\phi\) and exhibits hysteresis (Fig. 2b).[35–38] If there is a strong uniaxial anisotropy in the FIs, it is possible to study the N-to-SC hysteresis[37, 38], as opposed to the ordinary magnetic hysteresis resulting from anisotropies. By applying the external magnetic field perpendicular to the easy axis direction, the exchange field changes continuously. If the SC transition is continuous, there is also no magnetic hysteresis. If the transition is of the first order, the N-to-SC and magnetic hysteresis become entangled. We show in the supplementary how the two can be disentangled in the magnetization curve.[24]

We now consider the dynamical effects due to SS. In contrast to most studies of F/SC hybrid structures [39, 40], we take both magnets as dynamical. We study the small-angle dynamics around the equilibrium configuration with the ansatz

\[ m_n(t) = m_n(0) + Re(m_n(\omega) \exp(-i\omega t)), \quad (10) \]

where \(m_n(\omega)\) is a small perturbation perpendicular to \(m_n(0)\). The dynamical magnetization induces a time-dependent exchange field at the SC interfaces, which in turn induces a time-dependent spin density in the SC.

The dynamics of the FI magnetizations are described by the classical Landau-Lifshitz-Gilbert (LLG) equation, supplemented by a spin current term [41]

\[ \dot{M}_n = -\gamma M_n \times \mu_0 H_{eff,n}^{(0)} + \alpha_n (M_n \times \dot{M}_n) + \gamma j_{\tau,n}, \quad (11) \]

where \(H_{eff,n}^{(0)}\) is the effective magnetic field in the N state including the external magnetic field and anisotropy fields, \(\gamma\) is the gyromagnetic ratio, and \(\alpha_n\) is the Gilbert damping coefficient, which can be controlled with an additional heavy metal layer next to the FI (Fig. 1b). The spin current is given by Eq. (7).

In a FMR experiment, the external field has a static component and a small transverse dynamic part \(H_{rf}\). The linear response to \(H_{rf}\) given by LLG Eq. (11) is

\[ \dot{\chi}_n^{-1}(\omega) M_n(\omega) = \mu_0 H_{rf}(\omega) - m_n(0) \times j_{\tau,n}(\omega) / M_n, \quad (12) \]

where \(\chi_n^{-1}\) are the \(2 \times 2\) magnetic susceptibilities of the uncoupled magnets. [24] Since the magnets are insulating, there are no eddy currents inside the FIs and we can neglect the direct coupling between the SC and the rf field.[42, 43]

In the parallel configuration, the coupled dynamics of the two magnets is described by a matrix susceptibility

\[ \dot{\chi}_tot^{-1}(\omega) = \begin{pmatrix} \dot{\chi}_1^{-1}(\omega) + J_0 - J_{11}(\omega) M_1 M_2 \hfill & -J_0 - J_{12}(\omega) M_1 M_2 \\ J_0 - J_{11}(\omega) M_1 M_2 \\ -J_0 + J_{12}(\omega) M_1 M_2 \end{pmatrix}, \quad (13) \]

where \(J_0 = -G_1 G_2 \chi(x_1, x_2)\) is the static exchange constant, and

\[ \delta \dot{J}_{ij}(\omega) = -G_1 G_2 \left[ \chi(\omega, x_1, x_2) - \chi(x_1, x_2) \right] \]

\[ -N_0 G_1 \omega \delta \dot{J}_{ij} \delta \bar{\gamma}, \quad (14) \]
are the dynamic corrections related to spin pumping and other finite-frequency processes. Here, \( \chi \) is the dynamic spin susceptibility, [24, 44] related to the static spin susceptibility by \( \chi(0, x_i, x_j) = \chi(x_i, x_j)\).

To illustrate the effect of SS on the FMR properties, we first consider a fully symmetric trilayer. In the P configuration, the eigenmodes are the acoustic and optical modes for which \( m_1(t) = \pm m_2(t) \), respectively. [34] In the acoustic mode, the magnetizations are always parallel to each other and there is no SS [34]. The magnets are only coupled by the residual part of the susceptibility, \( \chi(\omega) = \chi(0) \approx \omega \chi'(0) \). The imaginary part \( \text{Im} \chi'(0) \) contributes directly to dissipation and can be included in the Gilbert damping coefficient. It describes the relaxation of quasiparticles, and vanishes at low \( T \) where quasiparticles cannot be excited due to SC gap [45]. This leads to the usual decrease of the FMR linewidth in the SC state (Fig. 4a). The real part \( \text{Re} \chi'(0) \) shifts the resonance frequency.

In the optical mode the magnetizations precess out-of-phase and are strongly coupled by the SS. In this case, the effective magnetic field is shifted by \( 2 J_0/M_n \). The resonance field difference between acoustic and optical modes at fixed frequency gives a direct measure of SS. However, measuring the optical mode can be difficult as a symmetrically applied field excites only the acoustic mode. Optical mode can be excited by longitudinal FMR pumping [46, 47], or by breaking the symmetry. For the optical mode the non-equilibrium spin currents pumped by the two magnets partially cancel in the SC spacer. In the thin-film limit this cancellation is exact and the dissipation in the spacer does not affect the linewidth.

In an asymmetric trilayer, SS can have a drastic effect on the linewidths of the FMR modes. For illustration, let us neglect the spin pumping and consider only the effect of SS together with the intrinsic damping of the magnets. In the N state the magnets are uncoupled, and the eigenmodes are the Kittel modes of the individual magnets with linewidths \( \Delta H_n \) proportional to Gilbert damping constants \( \alpha_n \). In the SC state, the SS hybridizes the modes so that their linewidths become weighted averages \( \Delta H_n = (\Delta H_n^0 + p_n \Delta H_n^0)/(1 + p_n) \), where \( p_1 = 0 \) and \( p_2 = \infty \) in the uncoupled system, and \( p_1 \to M_1/M_2 \) and \( p_2 \to M_2/M_1 \) in the strongly coupled system. [48] The top panel of Fig. 4b shows a numerical evaluation for the linewidth of such a system, including the spin pumping contribution. In particular, if one magnet has a lower intrinsic damping than the other magnet, the linewidth of the related mode increases below the SC transition.

The SS-mediated exchange interaction and spin pumping depend on temperature in opposite ways; spin pumping vanishes at zero temperature, whereas SS vanishes in the normal state. The competition between these two processes can be studied at a mode crossing between two FMR modes (Fig. 5), which can be engineered e.g. by having FI films with different thicknesses. In general, thinner films will have stronger anisotropy fields. A mode crossing may then be seen by rotating the applied in-plane magnetic field relative to the anisotropy axis. [49]

In the N state (Fig. 5a) the magnets are only coupled by spin pumping. The dissipative component of spin pumping generated by spin relaxation in the SC layer gives rise to mode attraction. Its signature is the sudden change of the mode linewidths at the mode crossing (inset of Fig 5a) [30, 50]. In the SC state (Fig. 5b), the SS mediated exchange coupling dominates over spin pumping, creating a regular avoided crossing.

So far the dynamical properties of SS have been experimentally studied only in systems with ferromagnetic metals (FM) [2–4], and an increase in FMR linewidth below \( T_c \) has been observed in systems which include multiple FM or heavy metal layers with strong spin-orbit coupling. In some of these experiments [2], there is nominally only a single magnet, but the heavy metal layers close to the ferromagnetic instability can be magnetized by the SS-mediated magnetic proximity effect. [51]

With minor changes, our theory can describe SS-mediated coupling in FM/SC/FM trilayer. The form of the magnetic susceptibility (13) is otherwise unchanged from the insulating case, but the susceptibilities for the spin density at the interfaces are replaced by the susceptibilities for the total spin density of the FM conduction electrons, and the spin mixing conductance is replaced by the s-d coupling inside the FM. In contrast to an FI, in FM the spin current is not absorbed in a layer of atomic thickness, but penetrates into the FM at the range of \( \xi_F = \sqrt{D_F/2T_c} \), where \( D_F \) is the diffusion coefficient of the FM. [12] The long-range triplet component [12] penetrating deep into the FM is exactly the component non-collinear to its exchange field, and the one related to SS. Because FMs support eddy currents, there is also an electromagnetic coupling between the layers. [42, 43]. Finding the magnitude of spin currents in a metallic system will be left for further work.

Despite these differences, our framework suggests that the experimentally observed enhancement below \( T_c \) is
likely to be a result of SS-mediated hybridization between the FMR modes. In interpreting the FMR data in systems with superconducting interlayers and multiple magnets, one should not rely on the spin pumping picture with a single dynamical magnet, but instead model the magnetization dynamics of the whole structure.

Conclusions. We have studied the properties of SS in FI/SC/FI systems and shown how they can mediate damping between ferromagnetic insulators even though they, as equilibrium currents, are themselves nondissipative. In an analogy to Josephson junctions, the SS can be characterized via the spin current - magnetization angle relation. This can be accessed by studying temperature dependent modifications to the FMR frequencies in FI/SC/FI setups.

The phenomena described here can be studied in various different FI/SC combinations, provided they can be suitably stacked or placed next to each other. The FI can be for example GdN, EuS/EuO, or ferrimagnetic YIG, which have been recently studied in combination with superconductors such as Nb, NbN or Al. [33, 52–57] Our results can be used to design magnetic resonator structures where the SS mediates a tunable coupling between the resonators.

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FIG. 5. Absorbed rf power as a function of external field angle $\theta$ and amplitude $\mu_0 H$ (in units of $K_B^1/M_1$). Dashed lines indicate the resonance fields and the insets show the linewidths. a) Mode locking in the N state is the most evident from the linewidth broadening/narrowing near the crossing. b) Avoided crossing in the SC state. The linewidths do not depend strongly on the proximity to the crossing. Parameters are as in Fig. 4a except for $\omega M_1/\gamma = 1.75$, $K_A^1 = 0.1$, $K_B^2 = 1.1$, $\Delta_0 M_n/\gamma = 10$, given in units of $K_B^1$.

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Supplementary material

The FI/SC/FI trilayer is described by the Hamiltonian

$$H = H_{SC} + H_{imp} + \sum_{n=1,2} \left( H_{FI}^{(n)} + H_{ex}^{(n)} \right),$$  \hspace{1cm} (S1)

where $H_{SC}$ is the BCS Hamiltonian for an $s$-wave superconductor, assumed infinite in $y$ and $z$ directions, but with a finite thickness $d$ in the $x$ direction. $H_{imp}$ is the random impurity potential in the SC, containing both non-magnetic and magnetic impurities, which gives the spin-orbit and spin-flip self-energies, respectively. We assume that the elastic mean-free path is short so that the spin-orbit and spin-flip self-energies, respectively. We assume that the elastic mean-free path is short so that the SC can be described with the quasiclassical Usadel equation.

The Hamiltonians describing the FIs and the SC/FI interfaces are \cite{23}

$$H_{FI}^{(n)} = -\sum_{ij} J_{ij} S_i \cdot S_j + H_{an},$$ \hspace{1cm} (S2)

$$H_{ex}^{(n)} = -J_{ex,n} \sum_{i \in I_n} \Psi^\dagger(r_i) S_i \cdot \sigma \Psi(r_i),$$ \hspace{1cm} (S3)

where $J_{ij}$ is the exchange coupling between the localized spins $S_i$ at FI lattice sites $r_i$. $H_{an}$ includes the possible anisotropy fields. $I_n$ is the set of spins at the interface between $n$th FI and the SC, and the spin-mixing conductances are related to the interfacial exchange coupling $J_{ex,n}$ by $G_n = J_{ex,n} S_n a$, where $S_n$ is the local spin at the interface and $a$ is the FI lattice constant. \cite{23} The Nambu field operator

$$\Psi^\dagger(r) = \left( \psi_{\uparrow}^\dagger(r) \ \psi_{\downarrow}^\dagger(r) \ - \psi_{\uparrow}(r) \ \psi_{\downarrow}(r) \right)$$ \hspace{1cm} (S4)

is chosen so that the $s$-wave SC order parameter is proportional to unit matrix in spin space.

Quasiclassical theory

When the elastic mean-free path is short, the SC specified by the above Hamiltonian can be described at the quasiclassical limit by using the Keldysh-Usadel equation \cite{23, 29, 59}

$$-\{\tau_3 \partial_{t'} \hat{g} \} + \partial_{t} (D \hat{g} \circ \partial_{t} \hat{g}) = [\Delta \tau_1 + \hat{\Sigma}, \hat{g}]$$ \hspace{1cm} (S5)

with the boundary conditions given by Eq. (6) of the main text. Above, $\hat{g}$ is the quasiclassical Green’s function (GF) in $8 \times 8$ space consisting of Keldysh, Nambu and spin indices. It obeys normalization condition $\hat{g} \circ \hat{g} = \delta(t, t')$. The elastic spin relaxation is determined by spin-orbit and spin-flip scattering self-energies $\hat{\Sigma} = \hat{\sigma} \cdot \hat{g} \hat{\sigma} / 6 \tau_{so} + \tau_3 \hat{\sigma} \cdot \hat{g} \tau_3 / 6 \tau_{sf}$, where $\tau_{so/sf}$ are the scattering rates. The order parameter $\Delta$ is solved from the self-consistency equation

$$\Delta(\omega) = \frac{\lambda}{16i} \int_{-\Omega_D}^{\Omega_D} d\epsilon \text{Tr}[\hat{\tau}_1 \hat{g}^K(\epsilon, \epsilon - \omega)],$$ \hspace{1cm} (S6)

where the coupling constant $\lambda$ and the cutoff $\Omega_D$ can be eliminated in favor of SC order parameter $\Delta_0$ at $T = 0$ in the absence of pair-breaking effects. \cite{23}.

The spin current density in $x$-direction and the spin density are defined as

$$j_x(\omega, x) = \frac{N_0}{16} \int_{-\infty}^{\infty} d\omega \text{Tr}[\sigma D(\hat{g} \circ \partial_{x} \hat{g})(\epsilon, \epsilon - \omega)] K,$$ \hspace{1cm} (S7)

$$S(\omega, x) = -\frac{N_0}{8} \int_{-\infty}^{\infty} d\omega \text{Tr}[\sigma \tau_3 \hat{g}^K(\epsilon, \epsilon - \omega); x].$$ \hspace{1cm} (S8)

For the SC we use the free energy functional \cite{32, 58}

$$F_{sc}[\hat{g}, \Delta] = N_0 \int dx \left( \frac{\Delta^2}{\lambda} \right)$$ \hspace{1cm} (S9)

$$- \frac{\pi T}{2} \sum_\omega \text{Tr}\{ (\omega_n + i\hbar \cdot \sigma) \tau_3 \hat{g} + \Delta \tau_1 \hat{g} - \frac{D}{4} (\nabla \hat{g})^2$$

$$+ \frac{1}{12 \tau_{so}} (\sigma \hat{g}) \cdot (\sigma \hat{g}) + \frac{1}{12 \tau_{sf}} (\sigma \tau_3 \hat{g}) \cdot (\sigma \tau_3 \hat{g}) \} \right),$$

which is regularized by subtracting the normal state energy with $\hat{g} = \text{sgn}(\omega_n) \tau_3$ and $\Delta = 0$.

Spin susceptibility at the thin-film limit

In equilibrium, spatially averaging over Eq. (S5) and using the BC of Eq. (6), we find an effective position-independent Usadel equation

$$[i(\epsilon - \hbar \cdot \sigma) \tau_3 + \Delta \tau_1 + \hat{\Sigma}, \hat{g}] = 0.$$ \hspace{1cm} (S10)

This equation is valid at the thin-film limit $d \ll \xi_0, l_{sd}$, where $l_{sd} = [D/(\tau_{so}^{-1} + \tau_{sf}^{-1})]^{1/2}$ is the normal state spin diffusion length. Equations (S6) and (S10) together with the normalization condition constitute a nonlinear group of equations which we solve numerically. In the thin-film limit we drop the spatial indices and define the static susceptibility as $S = \chi(0) \hbar$. It is related to the nonlocal susceptibility by $\chi(0) = \lim_{d \to 0} \chi(0, x_1, x_j) d$.

Let us denote by $\chi_{\text{homog}}(\omega)$ the usual spin susceptibility to an external in-plane magnetic field \cite{61}. It is related to the nonlocal thin-film susceptibility by a simple shift: $\chi(\omega) = \chi_{\text{homog}}(\omega) - N_0$. These susceptibilities vanish at different limits; $\chi_{\text{homog}}(0) = 0$ at $T = 0$ in the absence of spin scattering \cite{62}, whereas the nonlocal susceptibility vanishes in the normal state. Figure S1a shows the static spin susceptibility for $\tau_{so}^{-1} = \tau_{sf}^{-1} = 0$.

The dynamic spin susceptibility $\chi(\omega)$ is defined as a spin response to a dynamic perturbation of the exchange field

$$S(\omega, x) = \sum_{j=1,2} \chi(\omega, x_1, x_j) G_j m_j(\omega).$$ \hspace{1cm} (S11)
Generally, $\tilde{\chi}$ is a $3 \times 3$ matrix. In the thin-film limit, it is diagonal in the standard circular basis
\begin{equation}
 e_\pm = \mp(x \pm iy)/\sqrt{2}, \quad e_z = z,
\end{equation}
where the $z$-axis is chosen along the static component $h(0)$. The components of $\tilde{\chi}$ are extracted as $\chi^{\mu\nu} = e^\dagger_\mu \tilde{\chi} e_\nu$. In the parallel case, or in the thin-film limit, the susceptibilities $\chi(\omega, x, y)$ can be simultaneously diagonalized, and we drop the second index: $\chi^+ = \chi^{++}$. We call $\chi^z$ the longitudinal susceptibility and $\chi^{+(--)}$ the left(right)-handed transverse susceptibility. The longitudinal component is only induced if $\mathbf{m}_1(0)$ and $\mathbf{m}_2(0)$ are non-collinear, since only then $\mathbf{m}_1(\omega) \cdot \mathbf{m}_2(0) \neq 0$.

In the thin-film limit finite frequency transverse spin susceptibility can be obtained by solving the perturbed Usadel equation
\begin{equation}
 [\hat{\Delta}(0) + \hat{\Sigma}(1)]\hat{g}(12) - \hat{g}(12)[\hat{\Delta}(2) + \hat{\Sigma}(2)] \\
 + \hat{\Sigma}(12)\hat{g}_0(2) - \hat{g}_0(1)\hat{\Sigma}(12) \\
 = i[\hat{g}_0(1)(h(\omega) \cdot \sigma)\tau_3 - (h(\omega) \cdot \sigma)\tau_3\hat{g}_0(2)],
\end{equation}
where $\hat{g}_0(0)$ is the static GF at Matsubara frequency $\epsilon_1 = i\omega_n$ and $\hat{g}(12)$ is the perturbation induced by the left polarized driving field $h(\omega) = h^e(\omega)e_z$, and
\begin{align}
 \hat{\Delta}(\epsilon) &= -i(\epsilon - h\sigma_3)\tau_3 + \Delta \tau_1, \\
 \hat{s}(\epsilon) &= \sqrt{[\hat{\Delta}(\epsilon) + \hat{\Sigma}(\epsilon)]^2}, \\
 \hat{g}_0(\epsilon) &= \hat{s}(\epsilon)^{-1}[\hat{\Delta}(\epsilon) + \hat{\Sigma}(\epsilon)].
\end{align}
Using the normalization condition
\begin{equation}
 \hat{g}_0(0)^2 = 1, \quad \hat{g}_0(1)\hat{g}(12) + \hat{g}(12)\hat{g}_0(2) = 0,
\end{equation}
and the relation
\begin{equation}
 \tau_3\hat{g}_0(1)\hat{g}(12)\tau_3 + \hat{g}_0(1)\tau_3\hat{g}(12)\tau_3\hat{g}_0(2) \\
 = -2[\tau_3(1)\tau_3(1) - \tau_3(1)\tau_3(2)]\hat{g}(12),
\end{equation}
the solution for the Matsubara GF is found as
\begin{equation}
 \hat{g}(12) = \frac{i[\tau_3 - \tau_3(1)\tau_3(2)]h(\omega) \cdot \sigma}{s_1(1) + s_2(1) + \frac{1}{3\tau_0} + \frac{g_{11}(1)g_{12}(2) - g_{12}(1)g_{11}(2)}{3\tau_0}}.
\end{equation}
The energy-resolved spin susceptibility is obtained by taking the trace
\begin{equation}
 \chi^+(\omega, x, y) = -\frac{N_0}{8} \text{Tr} \left( (e^+ \cdot \sigma)^T \tau_3 \hat{g}(12) / h_\omega \right)
\end{equation}
The solution for real frequencies is obtained with analytical continuation
\begin{equation}
 \chi^{+}(\omega) = \int_{-\infty}^{\infty} d\epsilon \left[ \chi(1, R, 2R) f_2 - f_1 \chi(1, A, 2A) \\ + (f_1 - f_2) \chi(1, R, 2A) \right],
\end{equation}
where $1^{R/A}$ and $2^{R/A}$ stand for the replacements $i\omega_m \rightarrow \epsilon \pm i\Gamma$, $i\omega_m \rightarrow \epsilon - \omega \pm i\Gamma$, $f_1 = \text{tanh}(\epsilon/(2T))$, $f_2 = \text{tanh}((\epsilon - \omega)/(2T))$, and $\Gamma$ is an infinitesimally small quantity denoting the correct solution branch. In the numerical solution, we use a finite $\Gamma < 10^{-2}\Delta_0$ to broaden the BCS divergence in the density of states, which makes the numerical solution converge faster.

The solution Eq. (S19) depends on the numerically determined equilibrium GFs and the integral (S21) is evaluated numerically. The low-frequency corrections to the static spin susceptibility are shown in Fig. S1b. In the normal state, there are no spectral changes, and only the last term in Eq. (S21) contributes:
\begin{equation}
 \chi^{\pm}_{\Delta}(\omega) = \frac{2N_0\omega}{\omega - 2h + i\tau_{sn}^{-1}},
\end{equation}
with $\tau_{sn}^{-1} = \frac{4(\tau_{so}^{-1} + \tau_{sf}^{-1})}{3} + 2\Gamma$. We see that $\Gamma$ acts as an additional mechanism for spin relaxation. In general,
For longitudinal susceptibility, there is no simple analytical solution, but the Usadel equation can be written as a 4-component matrix equation, which is solved numerically [29]. For small frequencies, the longitudinal and transverse susceptibility differ near $T_c$, where a small change in the modulus of $h$ can have a large effect on $\Delta$.

Since both $h(t)$ and $S(t)$ are real, the left- and right-handed susceptibilities are related by $\chi^-(\omega) = \chi^+(\omega^*).$ Also, the transverse spin susceptibility at zero frequency and the static nonlinear spin accumulation can be related to each other by considering an adiabatic rotation of the total exchange field, so that when $\chi(0, x_i, x_j)$ for $i, j \in \{1, 2\}$ can be simultaneously diagonalized, we have

$$S^{eq}(0, x_i) = \sum_{j=1,2} \chi^\pm(0, x_i, x_j)G_jm_j(0).$$

This relation is needed to show the exact cancellation of spin supercurrents for the acoustic mode in P configuration.

### Magnetic susceptibility of FI/SC/FI trilayer

In this section, we derive the non-collinear matrix susceptibility for a coupled magnetic system. To find the susceptibility, one first solves the equilibrium problem by finding a local minimum of the free energy, Eq. (8). This determines the static magnetization directions

$$m_{1/2}(0) \parallel H^{[0]}_{eff,i/2} - \frac{J_g m_{2/1}(0)}{M_{1/2}}.$$  

In LLG equation the modulus of the magnetization is fixed and there are only two degrees of freedom for each magnet. To remove the third dimension, we rotate Eq. (11) to each magnet’s individual eigenbasis in which $m_n(0)$ points in the z-direction and project on the $x$-$y$ plane.

Identifying in Eq. (12) the parts which are proportional to $M_1(\omega)$ and $M_2(\omega)$, and doing the thin-film approximation $\hat{\chi}(\omega, x_i, x_j)G_j \approx \hat{\chi}(\omega)h_j,$ we find the magnetic susceptibility for a non-collinear static configuration

$$\hat{\chi}^{-1}_{tot}(\omega) = \left(\hat{\chi}^{-1}_1(\omega) + \frac{L_{11}(\omega)}{M_1^2} - \frac{L_{21}(\omega)}{M_1M_2} \right) + \hat{\chi}^{-1}_2(\omega) + \frac{L_{22}(\omega)}{M_2^2},$$

$$\hat{\chi}^{-1}_1(\omega) = -K_{B} / M_1^2,$n

$$\hat{\chi}^{-1}_2(\omega) = -K_{B} / M_2^2.$$ 

$$\mu_0 H^{[0]}_{eff,i} = \mu_0 H\cos(\theta - \phi_i) + [2K_A \cos(\phi_i)^2 - K_B] / M_i$$

where $\phi_i(\theta)$ is the angle between the anisotropy axis $z$ and $m_i(0)$. We assume that the static magnetization is always in-plane. The spin currents not included

![FIG. S2. FMR dispersion (bottom) and magnetic hysteresis (top) in a symmetric bilayer a) in the N state and b) in the SC state at $T = 0.01T_c$. In the N state the magnets are only weakly coupled by pumped quasiparticle spin currents. In the SC state the magnets are also coupled by SS. SC is evaluated in the thin-film limit. Parameters: $dN_0 \Delta^2 = 0.5$, $K^2 = 0.01$, $\Delta_0 M_1 / \gamma = 10^3$, in units of $K^2_B$; $\hbar_n = 0.3 \Delta_0$, $\alpha_n = 0.01$, $\tau_{so/af} = 0$. External field is at $1^\circ$ angle to the anisotropy axis.](image)
into the effective magnetic field are

\[ \dot{L}_{ij}(\omega) = -dh_ih_j \left[ \tilde{D}_i \tilde{\chi}(\omega) \tilde{D}_j - \chi(0) \delta_{ij} \right] - dN_0 h_i \omega \delta_{ij} \delta_3, \]

(S25e)

where \( \tilde{\chi} = \text{diag}(\chi^+, \chi^x, \chi^+ \chi^x) \) is the spin susceptibility in the SC eigenbasis, and the \( 2 \times 3 \) matrices \( \tilde{D}_n = PD(0, \phi_n - \phi_{sc}, 0) \) are a product of a projection \( P \) to the \( x \)-plane, and of spin-1 Wigner matrices \( \tilde{D}^1 \). \( \phi_{sc} \) is the direction of \( h(0) \).

The eigenmodes are found by diagonalizing \( \tilde{\chi}^{-1}_{\text{tot}}(\omega) \) and finding the values of \( \omega \) for which the real part of an eigenvalue vanishes. A typical dispersion for a symmetric system is shown in Fig. S2.

In parallel configuration the magnetic susceptibility can be written as Eqs. (13-14) with

\[ \chi^{-1}_{\text{tot}}(\omega)^{++} = \frac{\omega - \gamma \mu_0 H_{\text{eff}, i}^{(0)} \pm i \alpha_i \omega}{\gamma M_i}, \]

(S26a)

\[ \chi^{-1}_{\text{tot}}(\omega)^{+-} = -K_B^i / M_i^2 \]

(S26b)

\[ \mu_0 H_{\text{eff}, i}^{(0)} = \mu_0 H + 2K_A^i / M_i - K_B^i / M_i, \]

(S26c)

and with spin susceptibility \( \tilde{\chi} = \text{diag}(\chi^{++}, \chi^{--}) \) which includes only the transverse components. Here we have separated the N and SC state contributions to the effective fields into \( H_{\text{eff}, i} \) and \( -J / M_i \), respectively, and did not assume the thin-film limit.

Since \( M_n(t) \) and \( H_n(t) \) are real, the \( -- \) and \(--\) components can be obtained by \( \chi^{-1}(-\omega)^{\mu\nu} = [\chi^{-1}(\omega)^{\mu\nu}]^* \). The spin current \( j_{\mu i}^{\pi}(\omega) = \xi_{ij}(\omega) m_{\mu j}(\omega) \), where \( \xi \) is a response function, also obeys the same reality condition \( \xi_{ij}^+(\omega) = [\xi_{ij}^-(\omega)]^* \).

### FMR linewidth

Here we give the definition of the FMR linewidth used in the main text. Typically in FMR experiment, rf frequency is held fixed while sweeping the external field strength \( H \). The power of the rf drive is given by \[ P/A = \sum_{n=1,2} \langle H_n(t) \cdot \partial_t M_n(t) \rangle = \sum_{\lambda \in \text{eigs}} \omega \text{Im} \chi_\lambda(\omega) |H_\lambda|^2, \]

(S27)

where \( \chi_\lambda \) are the eigenvalues of the matrix susceptibility \( \tilde{\chi} \), and \( H_\lambda \) are the projections of the rf field along the corresponding eigenvector.

The linewidth is defined as the difference \( \Delta H = (2/\sqrt{3})(H_{\text{min}} - H_{\text{max}}) \) between the minimum and the maximum of the field derivative \( dP/dH \). Near resonance, \( dP/dH \) is dominated by the resonant eigenmode \( \lambda \) and other modes can be neglected. To determine the linewidth, we linearize the susceptibility of the resonant eigenmode, expressing it in terms of resonance field \( \mu_0 H_{\text{res}} \), weight factor \( W \) and linewidth \( \mu_0 \Delta H \),

\[ \chi_\lambda(H)^{-1} \approx \mu_0 (H - H_{\text{res}}) - i \mu_0 \Delta H \frac{\omega}{W}. \]

(S28)

The above parameters are defined by the equations

\[ 0 = \text{Re}[\chi_\lambda(H_{\text{res}})^{-1}], \]

(S29a)

\[ \frac{1}{W} = \left. \frac{d \text{Re}[\chi_\lambda(\omega)^{-1}]}{d \mu_0 H} \right|_{H=H_{\text{res}}}, \]

(S29b)

\[ \mu_0 \Delta H = -W \text{Im}[\chi_\lambda(H_{\text{res}})^{-1}]. \]

(S29c)

In bulk ferromagnets, the field linewidth can be identified with Gilbert damping \( \alpha = \gamma \mu_0 \Delta H / \omega \).

### Effect of the first order SC transition on the magnetic hysteresis

At low temperatures, the transition between SC and the N state as a function of the exchange field is of the first order.\[35, 36\] In the phase diagram, the first order transition is accompanied by a region in which both phases are possible as metastable states, albeit only one of them is thermodynamically stable away from the phase boundary (Fig. S3a). Experimentally, the metastability shows as hysteresis or state memory when the induced exchange field is swept over the phase boundary.\[37, 38\]

The FI/SC/FI trilayer provides a way of probing the transition and the SC-N hysteresis via magnetization measurements. The requirement for such measurement is that it must be possible to vary the total exchange field induced into the SC continuously as a function of the external field. This requirement is met if the FI s have an in-plane easy-axis and the magnetic field is applied perpendicular to it.

The FI s individually induce the exchange fields \( h_1, h_2 > 0 \) into the SC. In the AP and P configurations, the total fields are \( |h_1 - h_2| \) and \( h_1 + h_2 \), respectively. By changing the magnetization configuration, we thus sweep a horizontal segment of the phase diagram as shown in Fig. S3 for three different temperatures (blue, orange and red lines). If \( h_1 \) and \( h_2 \) are chosen properly, we cross the first order phase boundary at a low temperature (blue). Let us denote the thermodynamic transition point by \( h_{\text{th}} \). If there is a measurable SC-N hysteresis, the N-to-SC and SC-to-N transitions occur at fields \( h_{c1} < h_{\text{th}} \) and \( h_{c2} > h_{\text{th}} \), respectively.

Let us then consider the magnetization in a symmetric trilayer as the external field is varied. In the normal state the magnets are uncoupled. At zero field the magnetizations lie along the anisotropy axis. Increasing the field twists the magnetizations until they point towards the external field (red dashed line in Fig. S3). The P and
FIG. S3. Effect of the first order SC transition on the magnetic hysteresis. a) Phase diagram as a function of temperature and exchange field. Red, orange and blue lines show the range of total exchange fields, when the individual magnets induce a field of strength $h_1$ and $h_2$. Blue shading indicated the region of metastability where both SC and N state are possible. Dashed black line indicates the thermodynamic phase transition. b) Magnetization along the external field ($M_x$) as a function of the field strength $H$. Red dashed line and the solid orange and blue lines correspond to the temperatures indicated in panel (a). Red dashed line is the high temperature (normal state) behavior. Orange line is the behavior at intermediate temperatures when the SC transition is continuous. Blue line shows the behavior at low temperatures in which the SC transition is of the first order.

AP configurations are degenerate and when the field is reduced to zero, the system may end up in either one.

The SC state on the other hand favors the AP configuration, and the slope of the magnetization curve (orange and blue lines) is lower. The first order transition (blue line) shows in the magnetization curve as hysteresis. However, the magnetic hysteresis itself is not necessarily evidence of SC-N hysteresis, but only indicates a first-order transition between SC and N states. The shaded region in Fig. S3 is the magnetic hysteresis obtained by assuming that the SC transition always occurs when $h$ crosses the thermodynamic field $h_{th}$, with no hysteresis. With this assumption, the P-to-AP and AP-to-P transitions occur in Fig. S3b at the same total magnetization, but the field strength depends on the direction of transition.

The evidence for SC-N hysteresis is the difference in the magnetization $M$ at which the transitions occur, since there is a one-to-one correspondence between $M$ and $h$. 

