The Family Collider

W-Y. Pauchy Hwang

Asia Pacific Organization for Cosmology and Particle Astrophysics, Institute of Astrophysics, Center for Theoretical Sciences, and Department of Physics, National Taiwan University, Taipei 106, Taiwan

(September 22, 2014)

Abstract

Granting that the $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ Standard Model is valid (or, partially valid), for the real world, we propose the $\mu^+e^-$ collider in the $10^2 GeV$ range as the family collider. This family collider may work efficiently in producing the family Higgs particles and detecting the effects of family gauge bosons, with the range of sub-sub-fermi’s (a few $10^{-2}$ fermi’s).

PACS Indices: 12.60.-i (Models beyond the standard model); 98.80.Bp (Origin and formation of the Universe); 12.10.-g (Unified field theories and models).

1 Introduction

This is the experimental collider project dreamed by a theoretical physicist.

Imagine that the lepton world is also governed by the family forces and family particles, but the quark world does not feel it. Family forces are assumed in the sub-sub-fermi range ($10^{-2}$ fermi), which is so much shorter than the mutual distances between two electrons in the same atom. Family Higgs particles may be around $(80-120 GeV)$, and which could convert an electron into a muon.

Noting that $SU_c(3)$ couples alone to the quark world for no reasons, $SU_f(3)$ is assumed to couple alone to the lepton world - making the asymptotical freedom everywhere and escaping the QED Landau’s ghost for all particles.

The dreamed collider is based upon the $(\mu^+e^-)$ collisions at the center-of-mass energies of $(80 - 120) GeV$ - a collider based on two well-controlled beams (so far).

2 The Basic Physics

There were many speculations about the origin of mass; among them, the one in the (old) Standard Model [1], though "ugly", might be the closest one in the thinking. Along this line, the author worked out the origin of mass [2] upon introducing the massive family gauge bosons.

In [2], it is demonstrated that, before spontaneous symmetry breaking (SSB) in the generalized family Higgs mechanism, the system does not have mass terms; upon SSB, every mass term appears as a result. This works for masses of the various Higgs, of the leptons, of the quarks, and of the various gauge bosons.

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1Correspondence Author; Email: wyhwang@phys.ntu.edu.tw
Early on, we propose\(^3\) \(\left( (\nu_\tau, \tau)_L, (\nu_\mu, \mu)_L, (\nu_e, e)_L \right) \) \((\equiv \Psi(3,2))\) as the \(SU_f(3)\) triplet and \(SU_L(2)\) doublet. In fact, this is a natural proposal so long as the idea of the family gauge theory can be adopted. Basically, we introduce another \(SU(3)\) that covers the lepton world and protects it from the QED Landau ghost (and makes it asymptotically free). Note that in notation we put \(\left( (\nu_\tau, \tau)_L \right)\) as the first member of \(SU_f(3)\) to emphasize the family effects.

In writing out the \(SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)\) Standard Model in detail \(^4\), the neutrino mass term assumes a unique form:

\[
\frac{i h}{2} \bar{\Psi}_L(3,2) \times \Psi_R(3,1) \cdot \Phi(3,2) + h.c., \quad (1)
\]

where \(\Psi(3,i)\) is the neutrino triplet just mentioned above (with the first label for \(SU_f(3)\) and the second for \(SU_L(2)\)). The cross-dot (curl-dot) product is somewhat new, referring to the singlet combination of three triplets in \(SU(3)\). The Higgs field \(\Phi(3,2)\) is new in this effort \(^3\), because it carries some nontrivial \(SU_L(2)\) charge.

Indeed, this off-diagonal neutrino mass term offers us a natural way to describe neutrino oscillations, since the neutral part of \(\Phi(3,2)\) could each receive vacuum expectation values.

On the other hand, for charged leptons, the Standard-Model choice is \(\Psi^\dagger(\bar{3},2)\Psi^{\dagger C}(3,1)\Phi(1,2) + c.c.,\) which gives three leptons an equal mass; but, in view of that if \((\phi_1, \phi_2)\) is an \(SU(2)\) doublet then \((\phi_2^\dagger, -\phi_1^\dagger)\) is another doublet, we could form \(\tilde{\Phi}^\dagger(3,2)\) from the doublet-triplet \(\Phi(3,2)\).

\[
\frac{i h_C}{2} \bar{\Psi}_L(3,2) \times \Psi_R^{\dagger C}(3,1) \cdot \tilde{\Phi}^{\dagger}(3,2) + h.c. \quad (2)
\]

Here vacuum expectation values of \(\Phi^0(3,2)\) give rise to the imaginary off-diagonal (hermitian) elements in the \(3 \times 3\) mass matrix, so removing the equal masses of the charged leptons.

Here the couplings \(h_C\) and \(h\) are closely related to the coupling strength \(\kappa\) for the family gauge bosons \(^5\).

We wish to note that the last entity is the main basis of the proposed family collider. Here the coupling \(\eta^\prime_1 \mu^+ e^-\) (see below for explanation) is defined (for the direct production of the mixed family Higgs \(\eta^\prime_1\)).

As said in \(^2\), we suppose that, before the spontaneous symmetry breaking (SSB), the Standard Model does not contain any parameter that is pertaining to "mass", but, after the SSB, all particles in the Standard Model acquire the mass terms as it should - we call it "the origin of mass".

In our Standard Model \(^4\), we begin with the Standard-Model Higgs \(\Phi(1,2)\), the purely family Higgs \(\Phi(3,1)\), and the mixed family Higgs \(\Phi(3,2)\), with the first label for \(SU_f(3)\) and the second for \(SU_L(2)\). We need another triplet \(\Phi(3,1)\) since all eight family gauge bosons are massive \(^5\).

The three Higgs fields sound too many but we are forced to have them. In fact, the quartic interactions for complex scalar fields can easily explain the repulsive nature between them, and the attractive forces can be built up among the related three complex fields \(^6\). They give rise to that in \(^2\). The "ignition" turns out to come from the channel in the purely family Higgs \(\Phi(3,1)\).
The potential among the three Higgs is given as follows:

\[ V_{\text{Higgs}} = \mu^2 \Phi^\dagger(3,1)\Phi(3,1) + \lambda(\Phi^\dagger(1,2)\Phi(1,2) + \cos\theta_P\Phi^\dagger(3,2)\Phi(3,2))^2 \]

\[ + \lambda(-4\cos\theta_P)(\Phi^\dagger(3,2)\Phi(1,2))(\Phi^\dagger(1,2)\Phi(3,2)) \]

\[ \lambda(\Phi^\dagger(3,1)\Phi(3,1) + \sin\theta_P\Phi^\dagger(3,2)\Phi(3,2))^2 + \lambda(-4\sin\theta_P)(\Phi^\dagger(3,2)\Phi(3,1))(\Phi^\dagger(3,1)\Phi(3,2)) \]

\[ + \lambda^2\Phi^\dagger(3,1)\Phi(3,1)(\Phi^\dagger(1,2)\Phi(1,2)) + \text{(terms in is' and in decay).} \] (3)

Here the interaction between \( \Phi(3,1) \) and \( \Phi(1,2) \), as characterized by the \( \lambda' \) term, is rather small or vanishes identically.

Exercises in the U-gauge give us what is going on among physical particles. In the U-gauge, we have

\[ \Phi(1,2) = (0, \frac{1}{\sqrt{2}}(v+\eta)), \quad \Phi^0(3,2) = \frac{1}{\sqrt{2}}(u_1+\eta'_1, u_2+\eta'_2, u_3+\eta'_3), \quad \Phi(3,1) = \frac{1}{\sqrt{2}}(w+\eta', 0, 0), \] (4)

all in columns. The five components of the complex triplet \( \Phi(3,1) \) get absorbed by the \( SU_f(3) \) family gauge bosons and the neutral part of \( \Phi(3,2) \) has three real parts left - together making all eight family gauge bosons massive.

Let us try to fix the notations further; see Ch. 13, Ref. [1]. For \( \eta' \) going through SSB, we have the following terms, neglecting the small \( \lambda' \) term,

\[ \frac{\mu^2}{2}(\eta' + w)^2 + \left( \frac{\epsilon_2}{4}u_iu_i + \frac{\eta_2}{4}u_i^2 \right)(\eta' + w)^2 + \frac{\lambda^2}{4}(\eta' + w)^4. \] (5)

SSB means that all the linear terms add up to zero, resulting the change in sign of the mass term, \( \frac{1}{4}(2\lambda^2w^2(\eta')^2) \). (Here, for notations, \( \epsilon_2, \eta_2, \text{etc.}, \text{see} [2]. \) Note that, for real fields, a factor of \( \frac{1}{2} \) should be factored out.

The same applies to other SSB fields, even though the original minus signs are generated by other fields. For example, this applies for the SM Higgs \( \eta \).

In other word, the three "related" scalar (Higgs) fields \( \Phi(1,2), \Phi(3,2), \text{and} \Phi(3,1) \) should be "equivalent" among themselves. The spontaneous symmetry breaking (SSB) is happening for all of them, actively or passively. It is easy to see that only one SSB-driving term is enough for all the three Higgs fields. As for the SSB-driving term (ignition), we use [2] the purely family term, \( \mu^2\Phi^\dagger(3,1)\Phi(3,1) \).

From the expressions of \( u_iu_i \) and \( v^2 \), we obtain [2]:

\[ v^2(3\cos^2\theta_P - 1) = \sin\theta_P\cos\theta_Pw^2. \] (6)

And the SSB-driven \( \eta' \) yields

\[ w^2(1 - 2\sin^2\theta_P) = -\frac{\mu^2}{\lambda} + (\sin2\theta_P - \tan\theta_P)v^2. \] (7)

These two equations show that it is necessary to have the driving term, since \( \mu^2 = 0 \) implies that everything is zero. Also, \( \theta = 45^\circ \) is the (lower) limit.

What follows below about the masses of \( \eta, \eta', \text{and} \eta'_{1,2,3} \) from [2]:
The mass squared of the SM Higgs $\eta$ is $2\lambda \cos^2 \theta_P u_i u_i$, as known to be (125 GeV)$^2$. The famous $v^2$ is the number divided by $2\lambda$, or $(125 \text{ GeV})^2/(2\lambda)$. Using PDG’s for $e$, $\sin^2 \theta_W$, and the $W$-mass [7], we find $v^2 = 255 \text{ GeV}$. So, $\lambda = \frac{5}{3}$, a simple model indeed.

The mass squared of $\eta'$ is $-2(\mu^2 - \sin \theta_P u_i^2 + \sin \theta_P u_i^2)$). The other condensates are $u_i^2 = \cos \theta_P v^2 + \sin \theta_P w^2$ and $u_{i,3}^2 = \cos \theta_P v^2 - \sin \theta_P w^2$ while the mass squared of $\eta_1'$ is $2\lambda u_i^2$, those of $\eta_{2,3}'$ be $2\lambda u_{i,3}^2$. The mixings among $\eta_i'$ themselves are neglected in this paper.

There is no SSB for the charged Higgs $\Phi^\pm$ (3, 2). The mass squared of $\phi_1$ is $\lambda(\cos \theta_P v^2 - \sin \theta_P w^2) + \frac{1}{2} m_i u_i$ while $\phi_{2,3}$ be $\lambda(\cos \theta_P v^2 + \sin \theta_P w^2) + \frac{1}{2} m_i u_i$. (Note that a factor of $1/2$ appears in the kinetic and mass terms when we simplify from the complex case to that of the real field; see Ch. 13 of [1].)

A further look of these equations tells that $3 \cos^2 \theta_P - 1 > 0$ and $2 \sin^2 \theta_P - 1 > 0$. A narrow range of $\theta_P$ is allowed (greater than 45° while less than 57.4°, which is determined by the group structure). For illustration, let us choose $\cos \theta_0 = 0.6$ and work out the numbers as follows: (Note that $\lambda = \frac{5}{8}$ is used.)

$$6w^2 = v^2, \quad -\mu^2 / \lambda = 0.32v^2;$$
$$\eta: \quad m(\eta) = 125 \text{ GeV}, \quad v^2 = (250 \text{ GeV})^2;$$
$$\eta': \quad m(\eta') = 51.03 \text{ GeV}, \quad w^2 = v^2/6;$$
$$\eta_1': \quad m(\eta_1') = 107 \text{ GeV}, \quad u_1^2 = 0.7333v^2;$$
$$\eta_{2,3}: \quad m(\eta_{2,3}) = 85.4 \text{ GeV}, \quad u_{2,3} = 0.4667v^2;$$
$$\phi_1: \quad \text{mass} = 100.8 \text{ GeV}; \quad \phi_{2,3}: \text{mass} = 110.6 \text{ GeV}. \quad (8)$$

All numbers appear to be reasonable. Since the new objects need to be accessed in the lepton world, it would be a challenge for our experimental colleagues. Note that they are smooth functions of $\theta$ as long as it is between 45° and 57.4°.

As for the range of validity, $\frac{1}{3} \leq \cos^2 \theta_P \leq \frac{1}{2}$. The first limit refers to $w^2 = 0$ while the second for $\mu^2 = 0$.

We may fix up the various couplings, using our common senses. The cross-dot products would be similar to $\kappa$, the basic coupling of the family gauge bosons. The electroweak coupling $g$ is 0.6300 while the strong QCD coupling $g_s = 3.545$; my first guess for $\kappa$ would be about 0.1. The masses of the family gauge bosons would be estimated by using $\frac{1}{2} \kappa \cdot w$, so slightly less than 10 GeV. (In the numerical example with $\cos \theta_P = 0.6$, we have $6w^2 = v^2$ or $w = 102 \text{ GeV}$. This gives $m = 5 \text{ GeV}$ as the estimate.) So, the range of the family forces, existing in the lepton world, would be 0.04 fermi.

For the quark world, or the lepton world, which the $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ Minkowski space-time supports [6], the story is also fixed if the so-called "gauge-invariant derivative", i.e. $D_\mu$ in the kinetic-energy term $-\bar{\Psi}\gamma_\mu D_\mu \Psi$, is given for a given basic unit [1].

Thus, we have, for the up-type right-handed quarks $u_R$, $c_R$, and $t_R$,

$$D_\mu = \partial_\mu - igc \frac{\lambda^a}{2} G^a_\mu - i\frac{2}{3} g' B^a_\mu, \quad (9)$$

and, for the rotated down-type right-handed quarks $d'_R$, $s'_R$, and $b'_R$,

$$D_\mu = \partial_\mu - igc \frac{\lambda^a}{2} G^a_\mu - i(-\frac{1}{3})g' B^a_\mu. \quad (10)$$
On the other hand, we have, for the $SU_L(2)$ quark doublets,

$$D_\mu = \partial_\mu - ig_\mu \frac{\lambda^a}{2} C^a_\mu - i g \frac{\bar{\tau}}{2} \cdot \vec{A}_\mu - i \frac{1}{6} g' B_\mu.$$  \hspace{1cm} (11)$$

For the lepton side, we introduce the family triplet, $(\nu^R_\tau, \nu^R_\mu, \nu^R_e)$ (column), under $SU_f(3)$. Since the minimal Standard Model does not see the right-handed neutrinos, it would be a natural way to make an extension of the minimal Standard Model. Or, we have, for $(\nu^R_\tau, \nu^R_\mu, \nu^R_e)$,

$$D_\mu = \partial_\mu - i \kappa \frac{\bar{\lambda}}{2} F^a_\mu.$$  \hspace{1cm} (12)$$

and, for the left-handed $SU_f(3)$-triplet and $SU_L(2)$-doublet ($(\nu^L_\tau, \tau^L), (\nu^L_\mu, \mu^L), (\nu^L_e, e^L)$) (all columns),

$$D_\mu = \partial_\mu - i \kappa \frac{\bar{\lambda}}{2} F^a_\mu - i g \frac{\bar{\tau}}{2} \cdot \vec{A}_\mu + i \frac{1}{2} g' B_\mu.$$  \hspace{1cm} (13)$$

The right-handed charged leptons form the triplet $\Psi^C_R(3,1)$ under $SU_f(3)$, since it were singlets their common factor $\Psi^L_R(3,2)\Psi^R_R(1,1)\Phi(3,2)$ for the mass terms would involve the cross terms such as $\mu \rightarrow e$.

In other words, the quarks don’t see the family, i.e. $SU_f(3)$, but leptons see fully the family.

These allow us to write down how the quarks and the leptons enter in the standard manner.

In our example, the masses for family Higgs $\eta'$ and $\eta'_{1,2,3}$ are $(50 - 110) \text{ GeV}$ - accessible only through the lepton world. The implication of the family gauge theory is in fact a multi-GeV or sub-sub-fermi gauge theory - the leptons are shielded from this $SU_f(3)$ theory against the QED Landau’s ghost.

The masses of quarks are diagonal, or the singlets in the $SU_f(3)$ space (in the old-fashion way), those of the three charged leptons are $m_0 + a \lambda_2 + b \lambda_5 + c \lambda_7$ (before diagonalization, with real $a$, $b$, and $c$) and the masses of neutrinos are purely off-diagonal, i.e. $a' \lambda_2 + b' \lambda_5 + c' \lambda_7$. This result is very interesting and very intriguing.

Neutrinos oscillate among themselves, giving rise to a lepton-flavor-violating interaction (LFV). As argued in the other context, there are other oscillation stories, such as the oscillation in the $K^0 - \bar{K}^0$ system, but there is a fundamental ”intrinsic” difference here - the $K^0 - \bar{K}^0$ system is composite while neutrinos are ”point-like” Dirac particles. We have standard Feynmann diagrams for the kaon oscillations but similar diagrams do not exist for point-like neutrino oscillations - our proposal solves the problem, maybe in a unique way.

Thinking it through, it is true that neutrino masses and neutrino oscillations may be regarded as one of the most important experimental facts over the last thirty years [7].

In fact, certain LFV processes such as $\mu \rightarrow e + \gamma$ [7], $\mu + A \rightarrow A^* + e$, $e^+ + e^- \rightarrow \mu^+ + e^-$, etc., are closely related to the most cited picture of neutrino oscillations [7]. In recent publications [8], it was pointed out that the cross-generation or off-diagonal neutrino-Higgs interaction may serve as the detailed mechanism of neutrino oscillations, with some vacuum expectation value of the family Higgs, $\Phi(3,1)$ and $\Phi^0(3,2)$. So, even though we haven’t seen, directly, the family gauge bosons and family Higgs particles, we already see the manifestations of their vacuum expectation values.
We suspect that neutrino oscillations and lepton-flavor violating reactions (or decays) would, eventually, help us to decide, in the great details, the whereabouts of the Standard Model.

Besides the three Higgs, the primary prediction of our Standard Model is the existence of the force of a new kind - i.e., the family force mediated by the family gauge bosons. As said above, we could use $\frac{1}{2}\kappa w$ as an estimate of the mass(es) of the family gauge bosons. My first guess is for some feeble force - $\kappa = 0.1$. Our numerical example corresponds to $w = 102 \text{ GeV}$, so as to the family gauge boson mass of $5 \text{ GeV}$.

The family gauge bosons would then be in the vicinity of $5 \text{ GeV}$ or nearby, or the range of $0.04 \text{ fermi}$. Or, $0.04 \times 10^{-13} \text{ cm}$ in the effective range, between leptons (such as two electrons or an electron-positron pair) is too short to be detected for the entire atomic physics or the entire chemistry.

The precision experiments such as $g-2$ would eventually detect the residual family effects, since the existing $g-2$ calculations [9] is so far the QED calculation and should be completed by inclusion of other effects with the emphasis on family gauge bosons. We are looking forward to prospects in this directions.

Of course, we need to examine the precision part of atomic physics when the story becomes clear; even though the effects are tiny, the evolutions usually come from the tiny effects to begin with.

3 The $\mu^+e^-$ Collider

The idea to have the $\mu^+e^-$ collider at $(80-120) \text{ GeV}$ CM energies is to produce the family Higgs $\eta_{1,2,3}'$ directly. The $\epsilon_{abc}$ coupling in the lepton world means that so far the $\mu^+e^-$ collider would be only the available collider.

We have plenty of experience in constructing high-quality electron beams but may be lack of obtaining a $\mu^+$ beam in view of the short muon lifetime. There were the proposals of the $\mu=\mu^-$ collider. So, the proposal of the $\mu^+e^-$ collider might not be outrageous.

The cost for the $\mu^+e^-$ collider at, e.g., $120 \text{ GeV}$ should be in the ball part of LEP of LEPII previously at CERN.

The (old) standard wisdom is that, besides the QED background, you would see nothing in the absence of the $SU_f(3)$; but, in view of $SU_f(3)$, you should see $\eta_{1,2,3}'$ and many other family things. Do you want to gamble on this? I think that, until you check on this, we don’t know where the three generations (or the family) come from.

If the (new) Standard Model is substantiated, there are so many things both for theorists and for experimentalists.

4 Why is the hundred GeV $\mu^+e^-$ collider the only family collider?

The mixed family Higgs $\eta'_1$ can only be produced directly by the $\mu^+e^-\eta_1'$ coupling (in Eq. (2)), thus by the proposed family collider. The hadron collider, such as LHC at Geneva, has nothing with it; even the $e^+e^-$ collider could not do it directly. This is why the phenomena
of the three generations are clearly there but the objects such as $\eta'_1$ have been rather elusive, experimentally.

Of course, if we are satisfied with some indirect evidences, we could do it in many other ways. For example, we eventually should see the effects of the family gauge bosons, in messy environments (since these family bosons couple to the $e^+e^-$ pair), or through a clear deviation of $g-\Delta$ from the huge QED-Weak "background". Here so far and what follows, we assume that the ordering of the couplings, $g_W/g_c \sim \kappa/g_W$, is approximately valid. To disentangle the minute effects in the atomic physics environments would eventually becomes our goal - it is as simple as the QED-Weak situation.

One would argue against constructing a one-purpose collider, but the same argument held also for the $e^+e^-$ collider or for LHC. After all, it would be a nontrivial task to build the $\mu$ beam that could be used in the game, since the muon lifetime (at rest) is so short.

5 Are they too narrow to escape the detection?

Our experience in the detection of weak bosons $W^\pm$ and $Z^0$ with GeV widths is to use the scanning technique. Remembering that we assume the coupling strength $\kappa$ to be about one order smaller than the weak coupling $g_W$, we thus have to be able to fight to see the ten's $MeV$ in the one-hundred GeV environments - a resolution of $10^{-5}$ (in energy) is needed if the same scanning technique could be used.

This also implies that we may have missed important things if the coupling turns out to be too small. In that case, we have to invent new detection techniques in order to unravel more in the Nature.

Of course, there are plenty of techniques available to us; some of them with miracle invention may work; etc. The field of knowledge, in theory and in experiment, seems to be unlimited.

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