Communication and Consensus Co-Design for Low-Latency and Reliable Industrial IoT Systems

Hyowoon Seo, Student Member, IEEE, Jihong Park, Member, IEEE, Mehdi Bennis, Senior Member, IEEE and Wan Choi, Senior Member, IEEE

Abstract

Designing fast and reliable distributed consensus protocols is a key to enabling mission-critical and real-time controls of industrial Internet of Things (IIoT) nodes communicating over wireless links. However, chasing both low-latency and reliability of a consensus protocol at once is a challenging task. The problem is even aggravated under wireless connectivity that is slower and less reliable, compared to wired connections presumed in traditional consensus protocols. To tackle this issue, we investigate fundamental relationships between consensus latency and reliability under wireless connectivity, and thereby co-design communication and consensus protocols for low-latency and reliable IIoT systems. Specifically, we propose a novel communication-efficient distributed consensus protocol, termed Random Representative Consensus (R2C), and show its effectiveness under gossip and broadcast communication protocols. To this end, we derive closed-form end-to-end (E2E) latency expression of R2C that guarantees a target reliability, and compare this with a baseline consensus protocol, referred to as Referendum Consensus (RC).

Index Terms

Distributed consensus, distributed ledger technology (DLT), Byzantine Fault Tolerance (BFT), gossip protocol, broadcast protocol, industrial Internet of Things (IIoT), Industry 4.0.

I. INTRODUCTION

We are currently witnessing Industry 4.0 causing a paradigm shift from heavyweight and rigid architectures towards lightweight and flexible systems [1], [2]. One key is cyber-physical systems (CPSs) that wirelessly interconnect a variety of nodes ranging from mobile devices to sensors and actuators in smart factories. This enables mission-critical controls in real time by sharing the sheer amount of indispensable information across nodes [3], while enhancing human and machine safety in many tasks including manufacturing and inventory tracking, as well as remotely and autonomously controlled vehicles [4].

H. Seo and W. Choi are with the School of Electrical Engineering, Korea Advanced Institute of Science and Technology (KAIST), Daejeon 34141, Korea (e-mail: hyowoonseo@kaist.ac.kr, wchoi@kaist.edu). J. Park and M. Bennis are with the Centre for Wireless Communications, University of Oulu, Oulu 90014, Finland (email: {jihong.park, mehdi.bennis}@oulu.fi) (Corresponding author: Wan Choi)
To this end, CPS operations should enable multiple nodes to carry out valid control actions in proper order, in the face of interactions, malfunctions, and adversarial attacks. We address this problem by leveraging distributed ledger technology (DLT), in which every node stores a ledger containing a consensual sequence of valid control actions. For candidate actions proposed by multiple nodes, each action in the ledger of a single node is validated by itself and all other validating nodes, e.g., via majority rule. Then, the order of valid actions is determined by a pre-defined consensual ordering policy, e.g., based on each action’s average validated time.

Designing a distributed consensus protocol performing these action validating and ordering operations is of prime interest in this paper. Towards supporting mission-critical and real-time CPS tasks over wireless links, the consensus protocol needs to account for wireless system characteristics and thereby optimize its operations under the trade-off between consensus latency and reliability. This raises the following two fundamental questions.

Q1. How does communication affect the trade-off consensus latency and reliability?

With a fixed amount of faulty nodes carrying out adversarial and malicious actions, a consensus becomes more reliable when more participating nodes validate actions in the consensus. However, too many validators incur their huge communication overhead. This increases consensus latency, resulting in the trade-off between consensus latency and reliability.

Q2. How to enable fast and reliable consensus under wireless connectivity in a distributed way?

To minimize consensus latency, on the one hand, its consensus protocol can be optimized by adjusting the number of validators as many as required for guaranteeing a target reliability. On the other hand, the communication protocol of validators can also be optimized. These consensus and communication protocols should be co-designed, under the aforementioned trade-off between consensus latency and reliability.

We answer these questions by proposing a novel distributed consensus protocol, termed Random Representative Consensus (R2C). As illustrated in Fig. 1 in R2C, only randomly selected representative nodes validate the consensus process. Furthermore, we seek for its communication-efficient design, by investigating the R2C implementations with two different communication protocols: 1) gossip based R2C wherein a single message is disseminated through multi-hop communications, and 2) broadcast based R2C in which every message can be disseminated by
Fig. 1. An illustration of our proposed (1) random representative consensus (R2C) protocol compared to (2) a baseline referendum consensus (RC) protocol, under (a) gossip and (b) broadcast communication protocols.

To show its effectiveness and feasibility, we examine the end-to-end (E2E) latency and reliability of R2C by deriving their closed-form expressions. Here, E2E latency measures the delay an action experiences from the request for validation to the completion and update of the validation. Reliability is studied in terms of resiliency against faulty nodes and robustness against missing validators.

Compared to a baseline scheme, referred to as Referendum Consensus (RC) where all nodes are validators, we show that R2C can reach its consensus faster while achieving a target reliability requirement. Moreover, we compare gossip and broadcast protocols under RC and R2C, among which broadcast based R2C achieves the fastest consensus, for a sufficiently small amount of faulty nodes. Although broadcasting consumes larger single-hop transmission power than gossiping, the total energy consumption of each consensus is minimized under broadcast based R2C, thanks to its lowest consensus latency.

A. Related Work

The problem of reliability and fault tolerance of consensus protocols has long been studied, mostly under peer-to-peer network architectures with a (relatively) small number of nodes [5]–[8]. However, recent interest in value transfer applications, such as crypto-currencies and smart
contracts, has triggered a rapid development of distributed consensus protocols for large-scale systems with low-latency.

In terms of the scalability, Blockchain is one of the most popular and reliable consensus protocols, utilized in many applications ranging from crypto-currency [9] to distributed machine learning [10] and drone-aided mobile edge computing [11]. However, since Blockchain allows permission-less participation of any nodes [9], it may suffer from large consensus delays (e.g., several minutes-hours) that are ill-suited for mission-critical and real-time IIoT control applications.

From the low-latency perspective, permissioned consensus protocols are now emerging as alternatives. These methods are built on Byzantine fault tolerant (BFT) algorithms [5], [8] that require exchanging voting information prior to the consensus process, hindering their scalability increase. In view of this, Hashgraph is one compelling algorithm, in which the consensus is locally carried out at each node without exchanging voting information [12], thereby achieving its scalability with low consensus latency.

Nonetheless, most of the aforementioned algorithms postulate that nodes are communicating over fast and reliable wired links. To support large-scale systems, wireless connectivity is mandatory in consensus operations, and its impact on consensus reliability and latency should be carefully examined. On this account, wireless distributed consensus protocols have recently been studied in several works [10], [11], [13]–[18]. For instance, a Hashgraph-motivated wireless distributed consensus protocol has been introduced in [13], in the context of distributed wireless spectrum access applications. For power grid applications, an Ethereum-based smart contract structure and its operation protocol has been studied in [19].

Still, most of the preceding works on wireless distributed consensus protocols are application-oriented, and void of clarifying the relationship between wireless communication and consensus protocol operations. To the best of our knowledge, this work is the first of its kind that investigates not only IIoT application-specific consensus protocols, but also the fundamentals of fast and reliable consensus over wirelessly connected nodes by co-designing communication and consensus protocols.
B. Contributions and Organization

The contributions of this paper are summarized as follows.

- We propose a novel communication-efficient distributed consensus scheme, R2C protocol (Sec. IV), and compare its effectiveness with a baseline method, RC protocol (Sec. III).

- We derive the minimum required number of R2C validators under gossip and broadcast communication protocols, guaranteeing a target resiliency probability against faulty nodes and a target robustness probability against missing validators (Propositions 3-4 in Sec. IV-B and C).

- We derive the E2E latency expressions of RC and R2C under gossip and broadcast protocols, guaranteeing a target resiliency and robustness requirement (Propositions 1-2 in Sec. III-B and Propositions 5-6 in Sec. IV-D).

- We thereby provide a distributed consensus and wireless communication co-design guideline, claiming that R2C with wireless broadcast is a suitable fast and reliable consensus solution for IIoT systems. Its effectiveness and feasibility are underpinned by both analysis and numerical evaluations (Sec. V).

The remainder of this paper is organized as follows. In Sec. II, we explain the system architecture including network model and the communication protocols in detail. In Sec. III, we study the baseline RC protocol, and in Sec. IV we propose the R2C protocol. In Sec. V, we numerically evaluate the effectiveness of RC and R2C under gossip and broadcast communication protocols, followed by the conclusion in Sec. VI.

II. System Model

In this section, we describe the network model and communication protocols under study. The communication protocol incorporates two types of message disseminating protocols: gossiping and broadcasting.

A. Network and Channel Model

The network under study consists of a set $\mathcal{N}$ of $N + 1$ static nodes that are placed in a square grid. The network is assumed to be permissioned [5], [20], in which each node knows the identities of the other nodes. The nodes can be interpreted as any network edges ranging...
from mobile devices to sensors, actuators, and controllers in Industry 4.0. The coordinates of the \(i\)-th node for \(i \in \mathcal{N}\) are denoted as \((x_i, y_i)\). The distance between the nodes \(i\) and \(k\) is thereby given as \(R_{ik} = R_{ki} = \sqrt{(x_i - x_k)^2 + (y_i - y_k)^2}\). For the sake of convenience, we hereafter assume \(\sqrt{N+1}\) is a positive integer.

Every node communicates with the other nodes over wireless channels. The wireless channels are assumed to follow the standard path loss model and Rayleigh fading model \([21]\). Specifically, the path loss between nodes \(i\) and \(k\) is given as

\[
PL_{dB}(R_{ik}) = PL_{dB}(R_0) + 10\eta \log_{10} \left( \frac{R_{ik}}{R_0} \right),
\]

where \(PL_{dB}(R_0) = 20 \log_{10} \frac{\lambda}{4\pi R_0}\) denotes the path loss at the reference distance \(R_0\), and \(\lambda\) is the wavelength, e.g., \(\lambda = 0.125\) meters at 2.4 GHz industrial, scientific and medical (ISM) radio bands. The term \(\eta \geq 2\) indicates the path loss exponent, which ranges between 2.7−3.5 for urban outdoor scenarios and between 1.6−3.3 for indoor scenarios \([21]\). Furthermore, the multi-path effect on the channel from node \(i\) to node \(k\) is characterized by the Rayleigh fading model. The fading gain \(H_{ik} \sim \text{Exp}(1)\) identically and independently for all \(i, k \in \mathcal{N}\) with \(i \neq k\). Consequently, when a signal is transmitted from node \(i\) to \(k\) with transmit power \(P_t\), the signal-to-noise ratio (SNR) is represented as

\[
\text{SNR}_{ik} = \frac{H_{ik} R_{ik}^{-\eta} P_t}{(4\pi)^2 P_{\text{noise}}},
\]

where \(P_{\text{noise}}\) is the additive white Gaussian noise (AWGN) power and the reference distance of the path loss model in \([1]\) is set to \(R_0 = 1\) meter. The absolute time is globally synchronized periodically with GPS \([22], [23]\) and split into time slots of fixed time interval

\[
\tau = \frac{M}{B \log(1 + \rho)} \text{ seconds},
\]

where \(M\) is the maximum size of message sent by nodes during the consensus protocol in bits, \(\rho\) is the target signal-to-noise (SNR) of each transmission, and \(B\) in Hz denotes bandwidth utilized for the transmission. An SNR outage occurs if SNR is below \(\rho\). Since \(H_{ik}\) is exponentially distributed, the SNR outage probability is given as

\[
\epsilon_{ik} = 1 - \exp \left( -\rho R_{ik}^{-\eta} \frac{(4\pi)^2 P_{\text{noise}}}{\lambda^2 P_t} \right).
\]

For each outage event, we consider the type-I hybrid automatic repeat request (HARQ), where the message transmissions are repeated until the first success.
B. Communication Protocol

During consensus operations, every node can become either a message source or its destination. Each message is then disseminated from a single \textit{source} to multiple \textit{destinations}, according to either gossip or broadcast communication protocol, as detailed next.

1) \textit{Gossip Protocol}: The nodes make use of multi-hop communication for disseminating messages in the gossip protocol. Unlike a typical gossip protocol in peer-to-peer wired network, the spread of information is constrained by communication coverage in wireless environment. Thus, we consider a neighbor gossip protocol, where direct communications are only available to the nodes located within the coverage of a transmitter. Specifically, in the network model under study, neighbors are located $R$ meters apart from each other. Assume that all transmit nodes including the source and relays utilizes the same transmit power $P_t = P_{t,g}$, which can be fairly small since we assume that the direct communication is only done between the neighbors.

2) \textit{Broadcast Protocol}: The nodes communicates with other nodes within a single hop in the broadcast protocol, which means that any two nodes can be directly linked through a wireless channel. Compared to the gossip protocol, each transmit node in broadcast protocol must cover larger areas, and thus, we assume that the transmission power $P_t = P_{t,b}$ used by each node in broadcast protocol is larger than that of gossip protocol, i.e., $P_{t,b} \geq P_{t,g}$.

In the mean time, for the efficient use of radio resources, we preallocate \textit{dissemination time duration} $w_i \tau$ seconds for message dissemination by node $i \in \mathcal{N}$ as a source. Let $W_i$ be a random variable denoting the number of time slots required for message dissemination to all destinations.
from source $i$. In both protocols, $w_i$ is determined to achieve

$$\Pr[W_i \leq w_i] \geq \zeta,$$

for some target dissemination success probability $0 \leq \zeta < 1$.

Moreover, the location of the source node can affect the performance of the message dissemination. Thus, we consider two topologies (a) the source located at the corner point of the network and (b) the source node located at the center point of the network, as illustrated in Fig. [2] when analyzing the performance of the protocols in the later sections.

C. Distributed Ledger Architecture

Every node possesses a distributed ledger which stores a chain of valid actions. Throughout the paper, we suppose that the distributed ledger is a kind of replicated state machine [7] that takes validity of the proposed actions and their timestamps recorded during the validation processes as an input, and outputs a series of valid actions. Note that the distributed ledgers are updated and synchronized by the consensus protocol. Moreover, we use cryptographic techniques to prevent fabrication of the messages and detect corrupted messages. We assume that the messages are contain public-key signatures [24], message authentication codes [25], and message digests produced by collision-resistant hash functions [26]. Throughout the paper, a message $M$ signed by node $i$ is denoted by $[M]_i$. Note that a method of signing a digest of a message and appending it to the raw message is widely used rather than signing the full message in practice. Thus, it can be understood that the raw message $M$ and the encrypted digest of $M$ are included in the signed message $[M]_i$.

III. BASELINE: REFERENDUM CONSENSUS (RC)

In this section, we introduce our baseline scheme, referendum consensus (RC) protocol, where all nodes become validators and participate in the consensus process. The basics of RC follows the Practical Byzantine Fault Tolerance (PBFT) protocol that aims to reach consensus on the validity of proposed actions [8]. In addition, as a light-weight permissioned DLT for supporting IIoT systems, RC seeks to reach a consensus on the order of valid actions [12], as detailed next.

In RC, any new action proposed by a node is validated and then attached to a chain of valid actions stored at each node’s ledger. To this end, nodes play either one of the following roles.
A proposer proposes a new action to the validators. Alice in Fig. 3 is a validator.

A validator validates the proposed actions and shares the validated result with other validators to determine whether to accept the proposed action or not. Bob, Carol and David are validators.

We assume that the proposer does not validate the proposed action itself and all nodes except for the proposer are validators in the RC protocol. It is also assume that the system is aware of the number of faulty node $F \leq N$, which can be learned from the previous consensus rounds for other proposed actions. A consensus process for a single proposed action is done over wireless channels of a single frequency band of bandwidth $B$ and if there are multiple proposed actions at the same time, the consensus processes can be done in parallel by leveraging orthogonal frequency bands of bandwidth $B$ with typical Listen-Before-Talk approach or distributed spectrum access via Consensus-Before-Talk [13].

A. Operational Structure of RC

The RC protocol (refer to Fig. 3) is composed of four phases as follows:

**Phase 1 (Action Proposal)** Suppose node $p \in \mathcal{N}$ is a proposer and denote the action proposed by the proposer $p$ as $A_p$. In addition, define the set of validators as $\mathcal{V}_p = \mathcal{N}\{p\}$. In this phase, the proposer becomes a source and the validators become destinations in the communication perspective. The proposer first selects a vacant frequency band of bandwidth $B$ and initiates the RC protocol by disseminating a signed action proposal message.
where the message is a tuple of the proposed action $A_p$, the absolute time when the action is proposed $T(A_p)_p$ and the randomly chosen sequence of validator indices $S(V_p)$, e.g., 1 - 2 - · · · - $N + 1$, which inform the validators of the committing order of the validated results in Phase 3. Note that the dissemination time duration given to proposer $p$ is $w_p$ time slots.

**Phase 2 (Local Validation)** After receiving the proposed action, each validator goes through a local validation of the proposed action based on information given by the distributed ledger. Here, the term local validation implicitly means that the acceptance of the proposed action is yet to be decided. A validator determines that the proposed action is locally valid if the action does not contradict with other valid actions already stored in the ledger. A double spend problem in a crypto-currency system can be a good example for the contradiction between the newly proposed action and the existing actions. The local validity of the proposed action $A_p$ determined by validator node $v \in V_p$ is denoted by $V(A_p)_v$, which is the binary information that takes 1 if locally valid, and 0 if locally invalid. The validator also records a timestamp when it finishes the local validation of the proposed action and the timestamp recorded at node $v$ is denoted by $T(A_p)_v$. Timestamps of the same proposed action may differ in distinct validator nodes depending on the message dissemination method, channel condition and local computing time.

**Phase 3 (Commit)** After the local validation, the validators disseminate commit messages by taking turns with a Round Robin time-division approach. The committing order is informed by the proposer and is specified in $S(V_p)$ of the action proposal message. Note that in this phase, the committing validator node becomes a source and the rest of the nodes, i.e., the other validators and the proposer, are destinations. When it is node $v$’s turn to commit, the node disseminates a signed commit message

$$[M_{\text{commit},v}(A_p)]_v = [V(A_p)_v, T(A_p)_v].$$

(7)

The dissemination time duration of the commit message given to validator $v$ is $w_v$ time slots for all $v \in V_p$. We assume that the dissemination of the commit messages are done over the same
frequency band of which is used for the action proposal.

**Phase 4 (Global Validation and Action Ordering)** When each validator collects more than $N - F$ local validity on the proposed action received from distinct validators, it determines the global validity of the proposed action. Specifically, RC protocol determines the global validity based on the majority rule. Namely, if there are more collected ‘locally valid’ than ‘locally invalid’, then the validator determines the proposed action to be globally valid and vice versa. On one hand, if every node is non-faulty and the distributed ledgers are perfectly synchronized, then the collected votes on the local validity will be either all ‘locally valid’ or all ‘locally invalid’. On the other hand, if there are $F$ faulty nodes that can harm the global validity of the proposed action, it is known that the as long as the condition $N > 3F$ holds, the system is resilient against the faulty nodes [8].

The globally valid actions are the candidates of the actions that will be recorded on the distributed ledgers. Though the majority of validators vote for ‘locally valid’ for the proposed action, however, the validated time of the proposed action may differ at distinct validator nodes. This in turn may cause asynchrony on the order of valid actions between the distinct distributed ledgers, if there are multiple actions that are undergoing consensus processes at similar time frame. Accordingly, the validators should also reach consensus on the order of the globally valid actions, based on the collected timestamps. Particularly in RC protocol, each validator take the average of the collected timestamps and if all the timestamps are correctly received, the validators will get the *consensual timestamp* of the proposed action, which is denoted by

$$
C(A_p, V_p) = \sum_{v \in V_p} T(A_p)_v.
$$

The order of the valid actions are organized based on the consensual timestamp. Due to the reorganization of the valid actions, there might be some contradictory actions that are violating the causal relation of the actions and in such cases. The inaccepted actions are announced to be retried later on or discarded.

**B. E2E Latency of RC**

As defined earlier, E2E latency is the time interval between the action proposal and global validation and action ordering. In order to focus on the impact of the wireless communication,
we assume that the local computation load is relatively small compared to the local computation capability and thus the local computing time is negligibly small. Then the E2E latency of RC protocol is obtained as

\[ L_{RC} = \tau \sum_{i=1}^{N+1} w_i, \]  

(9)

with communication success probability larger than or equal to \( \zeta^{N+1} \), for some \( 0 \leq \zeta < 1 \). We get (9), since a single round of RC is comprised of \( N + 1 \) turns of independent message dissemination opportunities in total and the success probability comes from (5).

1) Gossip-based RC: In the gossip-based RC protocol, sources disseminate messages to destinations via the gossip protocol described in Sec. II-B. Consider the following example which illustrates the gossip-based RC protocol (see Fig. 3a).

**Example 1. Operation of Gossip-based RC Protocol.**

Consider a network, which is composed of Alice, Bob, Carol, and David, and Alice is proposing an action \( A_a \). Here, we suppose Alice-Bob, Bob-Carol, and Carol-David are geographically neighbors, while other pairs are located distant from each other. Assume each node can only communicate with neighbors.

**Phase 1** Alice initiates the RC protocol by proposing the action. In addition, Alice decides the committing order, say Bob-Carol-David, and propose it together with the action. Since Bob is the only node inside the communication coverage of Alice, only Bob hears the proposal. Thus, Bob relays the action proposal message to Carol and Carol relays it to David.

**Phase 2** Bob, Carol and David locally decide whether to accept \( A_a \) or not, and record the timestamp when \( A_a \) is validated.

**Phase 3** As Alice informed, Bob first commits its locally validated results to the others. Only Alice and Carol hear the message since only those two are inside the coverage of Bob. Thus, Carol relays the commit message to David. Carol and David commits in order with similar way.

**Phase 4** Once all of the locally validated results are collected at each node, he/she decides the global validity of \( A_a \) and compute the consensual timestamp which is needed for valid action ordering.

In the model described in Section II-A, where \( N + 1 \) nodes are composing a square network, E2E latency of gossip-based RC protocol is lower bounded as follows.

**Proposition 1.** The E2E latency of gossip-based RC protocol can be lower bounded as

\[ L_{RC,g} \geq \begin{cases} 
\frac{(3\sqrt{N+1}-2)(N+1) - \sqrt{N+1}}{2} \tau, & \text{for odd } \sqrt{N+1}, \\
\frac{(3\sqrt{N+1}-2)(N+1)}{2} \tau, & \text{for even } \sqrt{N+1}.
\end{cases} \]  

(10)
Proof: The proof is provided in Appendix A.

Note that the bound (10) is tight with guaranteeing communication success probability approximately equal to 1, if SNR outage probability of the communication between the neighbors (46) is sufficiently small.

2) Broadcast-based RC: In the broadcast-based RC protocol, sources disseminate messages to destinations via broadcast protocol described in Sec. [II-B]. Following example illustrates the broadcast-based RC protocol (see Fig. 3b).

Example 2. Operation of Broadcast-based RC Protocol.

Now suppose all nodes can communicate with other nodes directly, without any constraints on geographical locations.

**Phase 1** Alice initiates the RC protocol by proposing the action and the committing order as in Example 1. Since everyone is inside the coverage of Alice, they can directly get the action proposal message from Alice.

**Phase 2** Bob, Carol and David locally decide whether to accept $A_a$ or not, and record timestamp of the local decision.

**Phase 3** Bob first commits its locally validated results directly to the other and both Carol and David commit in the same way.

**Phase 4** Once all of the locally validated results are collected at each node, he/she decides the global validity of the proposed action and the order of valid proposed actions by computing the consensual timestamp.

Again, in general scenario with $N + 1$ nodes, we have the following E2E latency of the broadcast-based RC.

**Proposition 2.** The E2E latency of broadcast-based RC is

$$L_{RC,b} = \sum_{i=1}^{N+1} \left[ \log \left( \frac{1 - \zeta \frac{1}{N} \left( \frac{1}{\epsilon_{i,\text{max}}} \right) \tau}{\log \epsilon_{i,\text{max}}} \right) \right] \tau,$$

with communication success probability larger than $\zeta^{N+1}$, for $0 \leq \zeta < 1$, where $\epsilon_{i,\text{max}}$ denotes the maximum among all SNR outage probabilities between node $i$ and all the other nodes.

**Proof:** The proof is provided in Appendix B.

Similar to the gossip protocol, for sufficiently small $\epsilon_{b,i,\text{max}}$, the dissemination outage probability becomes close to zero.

IV. PROPOSED: RANDOM REPRESENTATIVE CONSENSUS (R2C)

In this section, we propose the random representative consensus (R2C) protocol which reduces the consensus latency while guaranteeing a target reliability of the consensus process. As
discussed in Q1, too many validators incur long consensus latency, and by contrast too small validators threaten consensus reliability. Balancing between latency and reliability, R2C seeks the minimum number of validators to achieve a target reliability, thereby reducing the consensus latency, as elaborated in the following subsections.

A. Operational Structure of R2C

For a given proposer node $p$, we suppose $\tilde{N}$ representative nodes out of the entire $N$ nodes can play the role of a validator, while the other $N - \tilde{N}$ nodes become acceptors, who do not validate the proposed actions, but only aggregate the validated results and determine whether to accept or reject the proposed action.

The R2C operations follow the same procedures of RC in Sec. III except for the following changes at each phase.

Phase 1 (Action Proposal) At first, the proposer $p$ uniformly randomly selects $\tilde{N}$ representative validators from the set $V_p$. We denote the chosen representative subset as $\tilde{V}_p$. Then the proposer initiates the consensus protocol by disseminating the action proposal message $[M_{\text{proposal}}(A_p, \tilde{V}_p)]_p = [A_p, T(A_p)_p, S(\tilde{V}_p)]_p$ to the network.

Phase 2 (Local Validation) Unlike the RC protocol, the local validation is only done by the members of $\tilde{V}_p$ in the R2C protocol.

Phase 3 (Commit) The members in $\tilde{V}_p$ take turn to commit the validated results based on the commit order $S(\tilde{V}_p)$ informed by the proposer.

Phase 4 (Global Validation and Action Ordering) All members including the proposer, validators and acceptors go through a global validation process same as in the RC protocol. The consensual timestamp of R2C protocol is

$$C(A_p, \tilde{V}_p) = \sum_{v \in \tilde{V}_p} T(A_p)_v. \quad (12)$$

Due to its missing validators compared to RC, the reliability of R2C should be more carefully examined. For this reason, we study the resilience of R2C against faulty nodes and its robustness against missing validators in the following subsections.
B. Resiliency

Against \( F \) faulty nodes, the baseline RC protocol becomes resilient if the number \( N \) of validators satisfies \( N > 3F \) \[8\]. Likewise, R2C protocol becomes resilient if the number \( \tilde{N} \) of representative validators satisfies \( \tilde{N} > 3\tilde{F} \). Due to the randomly selected representatives, the resilience of R2C is guaranteed stochastically. For a target \( \alpha \) the resilience outage probability \( \alpha \), the resilience definition of R2C is described as below.

**Definition 1.** For fixed number of representatives \( \tilde{N} \) and random number of faulty representative nodes \( \tilde{F} \), the R2C protocol is \( \alpha \)-resilient if

\[
\Pr[\tilde{N} > 3\tilde{F}] \geq \alpha, \tag{13}
\]

for some target resiliency probability \( 0 < \alpha \leq 1 \).

Next, we seek for the minimum number of representative validators for achieving \( \alpha \)-resiliency. Since the representatives are chosen uniformly by the proposer, \( \tilde{N} \) can be seen as a random variable which follows the hypergeometric distribution with probability mass function

\[
\Pr[\tilde{F} = f] = \frac{\binom{F}{f}\binom{N-F}{\tilde{N}-f}}{\binom{N}{\tilde{N}}}, \forall f \in \{0, \ldots, F\}. \tag{14}
\]

Accordingly, the resiliency outage probability can be equivalently expressed as

\[
\Pr\left[\tilde{F} < \frac{\tilde{N}}{3}\right] = 1 - \frac{\binom{\tilde{N}}{\frac{\tilde{N}}{3}}\binom{N-\tilde{N}}{F-\frac{\tilde{N}}{3}}}{\binom{N}{\tilde{N}}} \ {}_3\text{F}_2\left[1, \left\lceil \frac{\tilde{N}}{3}\right\rceil - F, \left\lceil \frac{\tilde{N}}{3}\right\rceil - \tilde{N} \middle| \frac{\tilde{N}}{3} + 1, N + \left\lceil \frac{\tilde{N}}{3}\right\rceil + 1 - F - \tilde{N} ; 1\right], \tag{15}
\]

where \( {}_3\text{F}_2[\cdot] \) is the generalized hypergeometric function. A straightforward way of obtaining the condition on \( \tilde{N} \) for \( \alpha \)-resiliency is to compute the inverse function of (15), however, since it includes a hypergeometric function, it is less tractable. Alternatively, we can take advantage of the fact that the hypergeometric distribution can be approximated to the normal distribution when \( \tilde{N} \) is sufficiently large, \( N \) and \( F \) are large compared to \( \tilde{N} \), and \( \frac{F}{\tilde{N}} \) is not close to 0 or 1. Thus from the normal approximation, (15) yields to

\[
\Pr\left[\tilde{F} < \frac{\tilde{N}}{3}\right] \approx \frac{1}{2} \left[1 + \text{erf}\left(\frac{\frac{\tilde{N}}{3} - \mu_{\tilde{F}} - \phi}{\sigma_{\tilde{F}} \sqrt{2}}\right)\right], \tag{16}
\]

where \( \text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_{0}^{x} e^{-t^2} \, dt \) is the error function, \( \mu_{\tilde{F}} = \frac{\tilde{F}}{\tilde{N}} \) and \( \sigma_{\tilde{F}} = \sqrt{\frac{\tilde{F}N \tilde{N} - F N - \tilde{N}}{N \tilde{N} - 1}} \) are the mean and standard deviation of the hypergeometric random variable \( \tilde{F} \), respectively, and \( 0 < \phi < 1 \) is the correction factor that comes from the approximation of the probability function.
Fig. 4. Illustration of resiliency probability versus the number of representative validators obtained from hypergeometric and approximated normal approach, when $N = 80$ and $F = 5, 15, 25$.

of a discrete random variable to a continuous random variable. In [27], [28], an approximation for the error function $\text{erf}(x)$, of which the derivation is based on the method that is a generalization of Hermite-Pade approximation, was proposed in a simple form as

$$
\text{erf}(x) \approx g(x) = \left[ 1 - e^{-x^2 \frac{4 + ax^2}{1 + ax^2}} \right]^{\frac{1}{2}}, \ x \geq 0,
$$

(17)

where the constant $a \approx 0.14$ is chosen to achieve a relative precision better than 0.004 uniformly for all real $x \geq 0$. For $x < 0$, the identity $\text{erf}(-x) = -\text{erf}(x)$ can be used. Note that the approximated error function $g(x)$ in (17) can be easily inverted analytically as

$$
g^{-1}(x) = \left[ -\frac{2}{\pi a} - \frac{\log(1 - x^2)}{2} + \sqrt{\left( \frac{2}{\pi a} + \frac{\log(1 - x^2)}{2} \right)^2 - \frac{1}{a} \log(1 - x^2)} \right]^{\frac{1}{2}}, \ 0 \leq x < 1.
$$

(18)

Thus, we approximate the inverse of the error function as $\text{erf}^{-1}(x) \approx g^{-1}(x)$. Note that for the range of $-1 < x \leq 0$, the identity $\text{erf}^{-1}(x) = -\text{erf}^{-1}(x)$ can also be used. In Fig. 4, we compare the resiliency probabilities obtained from two different approaches, i.e., the original hypergeometric distribution and approximant with approximated error function. As shown in the figure, the approximation is tight for different number of faulty nodes $F$ and the number of random representative validators $\tilde{N}$. In result, we have the condition of $\tilde{N}$ that approximately achieves $\alpha$-resiliency of the R2C protocol as follows.

**Proposition 3.** $\alpha$-resiliency of the R2C protocol can be approximately achieved if the number of representatives $\tilde{N}$ satisfies the following condition

$$
\tilde{N} > N_\alpha,
$$

(19)
where \( N_\alpha = \frac{\phi A + B N + \sqrt{2\phi ABN - 2\phi^2 B + B^2 N^2}}{A^2 + 2B} \), \( A = \frac{1}{3} - \frac{F}{N} \) and \( B = \frac{F(N-F)}{(N-1)N^2} (g^{-1}(2\alpha - 1))^2 \).

**Proof:** From (16) and (17), we have the condition

\[
\frac{1}{2} \left( 1 + g \left( \frac{\tilde{N}/3 - \mu_F - \phi}{\sigma_F \sqrt{2}} \right) \right) \geq \alpha,
\]

achieving \( \alpha \)-resiliency. By rearranging this to satisfy the condition for \( \tilde{N} \), we have (19). 

Note that \( N_\alpha \) depends on \( N, F, \) and \( \alpha \). Thus, the number of random representative validators can be easily chosen to achieve \( \alpha \)-resilience if we have the knowledge of \( N \) and \( F \).

**C. Robustness**

We also seek for the condition that ensures the robustness against missing validators, especially by showing that the consensual timestamp of the valid actions are . We first define a consensus distortion function \cite{29}

\[
|D(A_p, \mathcal{V}_p, \tilde{\mathcal{V}}_p)| = |C(A_p, \mathcal{V}_p) - C(A_p, \tilde{\mathcal{V}}_p)|,
\]

which measures the absolute difference between the RC consensual timestamp \( C(A_p, \mathcal{V}_p) \) in (8) and the R2C consensual timestamp \( C(A_i, \tilde{\mathcal{V}}_p) \) in (12). Both \( C(A_p, \mathcal{V}_p) \) and \( C(A_p, \tilde{\mathcal{V}}_p) \) are random values, where the randomness of \( C(A_p, \mathcal{V}_p) \) comes from the channel uncertainty and that of \( C(A_p, \tilde{\mathcal{V}}_p) \) comes from the randomly chosen representative validator set as well as the channel uncertainty. Intuitively, if the distortion is small, it means that the representatives are well representing the consensual timestamp of the entire network, so that the valid action ordering by the random representative validators will be the same as the ordering done by the all the participants in the network. On the other hand, if the distortion is large, the ordering of the valid actions by the representatives might be different to that done by all nodes. In this context, we define \((\beta, \gamma)\)-robustness of the R2C protocol as follows.

**Definition 2.** The R2C is \((\beta, \gamma)\)-robust if

\[
\Pr[|D(A_p, \mathcal{V}_p, \tilde{\mathcal{V}}_p)| \leq \beta] \geq \gamma,
\]

for some acceptable consensus distortion \( \beta \geq 0 \) and target robustness probability \( 0 \leq \gamma \leq 1 \).

Note that the number of validators should be sufficiently large in order to make the distortion smaller than \( \beta \). Again, we seek for the condition of \( \tilde{N} \) that guarantee \((\beta, \gamma)\)-robustness of the R2C protocol. A straight forward way of obtaining the condition is to derive the exact distribution
of $D(A_p, V_p, \tilde{V}_p)$. However, as mentioned before, $D(A_p, V_p, \tilde{V}_p)$ is a jointly distributed random variable, where the randomness comes from the random selection of the representative set $\tilde{V}_p$ and the number of transmissions required for the successful information delivery, so the distribution is complicated and hard to express in a tractable form.

Alternatively, we approximate $D(A_p, V_p, \tilde{V}_p)$ to follow a normal distribution with zero mean and variance $\sigma_D^2$. A justification of the normal approximation is as follows. First, assume that all nodes determines that the proposed action is valid, that is $V(A_p)_v = 1$ for all $v \in V_p$. Then, we can write

$$D(A_p, V_p, \tilde{V}_p) = \frac{1}{N} \sum_{i \in V_p} T(A_p)_i - \frac{1}{N} \tilde{N} \sum_{j \in \tilde{V}_p} T(A_p)_j. \quad (23)$$

Then for a given some realization $T(A_p)_v = t_v$ for all $v \in V_i$, the consensual timestamp of the RC protocol $C(A_p, V_p) \mid \mathcal{T} = \frac{1}{N} \sum_{v \in V_p} t_v$ can be seen as a population mean over a set $\mathcal{T} = \{t_v \mid v \in V_p\}$, while the consensual timestamp of the R2C protocol $C(A_p, \tilde{V}_p) \mid \mathcal{T} = \frac{1}{\tilde{N}} \sum_{v \in \tilde{V}_p} t_v$ is a sample mean where the samples are chosen over the set $\mathcal{T}$ without replacement. It is known that from the Central Limit Theorem (CLT), $C(A_p, V_p) \mid \mathcal{T} - C(A_p, \tilde{V}_p) \mid \mathcal{T}$ follows normal distribution if the population of the $\mathcal{T}$ is infinite. Though we assume finite $N$, Fig. 5 shows that the approximation is quite tight so long as $N$ is sufficiently large. Thus, we have

$$\Pr[|D(A_i, V_i, \tilde{V}_i)| \leq \beta] \approx \text{erf} \left( \frac{\beta}{\sigma_D \sqrt{2}} \right). \quad (24)$$

Approximating the inverse error function via (18), we provide the following proposition.

**Proposition 4.** For given location of the proposer and the message dissemination method, the variance of $D(A_p, V_p, \tilde{V}_p)$ is

$$\sigma_D^2 = \frac{\tau^2(N - \tilde{N})}{N N^2} \psi, \quad (25)$$

and the necessary number of representatives for $(\beta, \gamma)$-robustness is

$$\tilde{N} > N_{(\beta, \gamma)}, \quad (26)$$

where $\psi = \sum_{v \in V_p} \left( E[Z_{pv}^2] + \frac{1}{N-1} \sum_{j \in V_p, j \neq v} E[Z_{pv}] E[Z_{pj}] \right)$, $N_{(\beta, \gamma)} = \left[ \frac{1}{N} + \frac{\beta^2 N}{2\tau^2(g^{-1}(\gamma))^2\psi} \right]^{-1}$, and $Z_{pv}$ is the number of transmissions required for successful message delivery from node $p$ to $v$.

**Proof:** The proof is provided in Appendix C.

Unlike the condition for the $\alpha$-resiliency, $N_{(\beta, \gamma)}$ is dependent on $N$, $F$, $\beta$ and $\gamma$, but also on the the location of the proposer $p$. Also note that the message disseminating method affects
the term $\psi$, thus we write $N_{(\beta,\gamma),g}$ and $N_{(\beta,\gamma),b}$ for the gossip and broadcast protocols in R2C, respectively.

D. E2E Latency of $\alpha$-Resilient and $(\beta, \gamma)$-Robust R2C

From Propositions 3 and 4 we approximately achieve $\alpha$-resiliency and $(\beta, \gamma)$-robustness by the R2C protocol when the number of random representative validators satisfy

$$\tilde{N} > \max \left( N_\alpha, N_{(\beta,\gamma)} \right), \quad (27)$$

for the given proposer $p$. Meanwhile, since the validators are chosen randomly by the proposer, the consensus latency will vary depending on the chosen validator set. Thus, for the given proposer $p$, the expected consensus latency of the R2C protocol is obtained as

$$L_{R2C} = \left[ w_{p,\zeta} + \frac{\tilde{N}}{N} \sum_{v \in \tilde{V}_p} w_{v,\zeta} \right] \tau, \quad (28)$$

with communication success probability larger than $\zeta^{\tilde{N}+1}$, where $w_p$ and $w_v$ are the dissemination time duration as discussed in Sec. II-B and $\tilde{N}$ is from the marginalization taken over all possible representatives subset $\tilde{V}_p \subset V_p$ and $|\tilde{V}_p| = \tilde{N}$.

1) Gossip-based R2C: Similar to Example 1, the gossip protocol is used for message dissemination in the gossip-based R2C. However, the gossip-based R2C exploits only a smaller set of validators for consensus. In order to obtain E2E latency of the $\alpha$-resilient and $(\beta, \gamma)$-robust gossip-based R2C, we must first find out the necessary number of representatives $\tilde{N}$ for given proposer $p$. 

![Fig. 5. Bound on the consensus failure probability due to time stamp distortion vs. the number of random representative validators $\tilde{N}$, obtained from empirical experiment, normal and approximated normal approaches, when $N = 80$ and $\beta = 0.5, 1, 2$.](image-url)
Suppose the proposer $p$ proposes an action $A_p$ at time $T(A_p)_p$, and since we are assuming that the local computation time at each node is negligible, the timestamp of the proposed action at a validator node $v \in V_p$ can be expressed as

$$T(A_p)_g,v = T(A_p)_p + \tau Z_{g,pv}, \quad (29)$$

where $Z_{g,pv}$ is the random variable that denotes the number of time slots that is required to deliver a message from node $p$ to node $v$. From (8), the consensual timestamp of $A_p$ in the gossip-based RC protocol can be expressed as

$$C(A_p, V_p)_g \approx T(A_p)_p + \tau \sum_{v \in V_p} Z_{g,pv}, \quad (30)$$

and the consensual timestamp in the gossip-based R2C protocol can be expressed as

$$C(A_p, \tilde{V}_p)_g \approx T(A_p)_p + \tau \sum_{v \in \tilde{V}_p} e_{pv}, \quad (31)$$

where $\tilde{V}_p$ is the random representative validators set, $\tilde{N}_{g,p} = |\tilde{V}_p|$ and $e_{pv}$ is the number of edges of the shortest paths from node $p$ to $v$ as defined in Appendix [A].

To achieve $(\beta,\gamma)$-robustness, we derive the bounds on the number of representatives by leveraging Proposition 4. For different proposer locations, the lower bound numbers of representatives under gossip-based R2C are described in the following case study.

**Case Study 1. $N_{(\beta,\gamma),g}$ in Gossip-based R2C.**

For $(\beta,\gamma)$-robustness, the number of representatives in gossip-based R2C must be no smaller than

$$N_{(\beta,\gamma),g} = \left[ \frac{1}{N} + \frac{\beta N}{2\tau^2(g^{-1}(\gamma))^2\psi_g^g} \right]^{-1}, \quad (33)$$

where $\psi_g$ varies depending on the location of the proposer node $p$.

**Topology (a) (see Fig. 2)** - For any non-negative integer $\sqrt{N + 1} > 1$, we have

$$\psi_g^{(a)} = \frac{(N + 1)((13N - 24\sqrt{N + 1} + 16)N + 12(\sqrt{N + 1} - 1))}{6(N - 1)}, \quad (34)$$

**Topology (b)** - For odd non-negative integer $\sqrt{N + 1} > 1$, we have

$$\psi_g^{(b)} = \frac{(13N^2 - 4N - 8)N}{24(N - 1)}. \quad (35)$$

For example, when $N = 80$, $\beta = 1$ and $\gamma = 0.1$, we have $\lceil N_{(\beta,\gamma),g}^{(a)} \rceil = 67$ and $\lceil N_{(\beta,\gamma),g}^{(b)} \rceil = 48$ for the topologies (a) and (b), respectively.
Consequently, the number of representatives for $\alpha$-resiliency and $(\beta, \gamma)$-robustness in the gossip-based R2C must satisfy

$$N \geq \max(N_{\alpha}, N_{(\beta, \gamma)}),$$

where $N_{\alpha}$ is fixed number for given $N$, $F$ and $\alpha$ as mentioned in Sec. IV-B. Accordingly we can derive the E2E latency of the gossip-based R2C as follows.

**Proposition 5.** The E2E latency of the gossip-based R2C is lower bounded as

$$\tilde{L}_{g}^{(a)} \geq \begin{cases} \frac{3\sqrt{N+1}}{2} - \frac{\sqrt{N+1} - 1}{N} \tilde{N}_{g}^{(a)} + 2(\sqrt{N+1} - 1), \quad \text{for odd } \sqrt{N+1}, \\ \frac{3\sqrt{N+1}}{2} - \frac{\sqrt{N+1} - 1 - 2}{2N} \tilde{N}_{g}^{(a)} + 2(\sqrt{N+1} - 1), \quad \text{for even } \sqrt{N+1}, \end{cases} \quad \tau,$$

if the proposer is located at the corner point of the network (i.e., topology (a) in Fig. 4), and

$$L_{R2C,g}^{(b)} \geq \left( \frac{3}{2} \sqrt{N+1} - 1 \right) \tilde{N}_{g}^{(b)} + \sqrt{N+1} - 1 \quad \tau,$$

if the proposer is located at the center point of the network, where $\tilde{N}_{g}^{(a)}$ and $\tilde{N}_{g}^{(b)}$ are the number of randomly chosen representatives in the cases (a) and (b), respectively.

**Proof:** The results follow from the proof of Proposition 1 in Appendix A with (28).

Similar to the bound (10), the bounds (37) and (38) are tight if SNR outage probability of a communication between two neighbors is sufficiently small.

2) **Broadcast-based R2C:** Broadcast-based R2C is similar to Example 2, but exploits a smaller set of validators for consensus. Suppose the proposer $p$ starts disseminating the action proposal message at time $T(A_p)_p$. Then the timestamp of the proposed action at validator $v \in \tilde{V}_p$ is

$$T(A_p)_b,v = T(A_p)_p + \tau Z_{b,pv}. \quad (39)$$

Assuming that $V(A_p)_v = 1$, $\forall v \in V_p$, the consensual timestamp of the proposed action $A_p$ in the RC protocol with wireless broadcasting is

$$C(A_p, V_p)_b = T(A_p)_p + \frac{\tau}{N} \sum_{v \in V_p} Z_{b,pv}. \quad (40)$$

Similarly, the consensual timestamp in the R2C protocol with the broadcast protocol can be expressed as

$$C(A_p, \tilde{V}_p)_b = T(A_p)_p + \frac{\tau}{\tilde{N}_b} \sum_{v \in \tilde{V}_p} Z_{b,pv}, \quad (41)$$

where $\tilde{N}_b = |\tilde{V}_p|$. Following case study shows the lower bound on the number of representatives for $(\beta, \gamma)$-robust broadcast-based R2C protocol.
Case Study 2. \( N_{(\beta, \gamma)} \) in Broadcast-based R2C.

For \((\beta, \gamma)\)-robustness, the number of representatives in broadcast-based R2C must be no smaller than

\[
N_{(\beta, \gamma), b} = \left[ \frac{1}{N} + \frac{\beta^2 N}{2\tau^2 (g^{-1}(\gamma))^2 \psi_b} \right]^{-1},
\]

where \( \psi_b = \sum_{v \in V} \left( \frac{1}{1 - \epsilon_{pv}} \right)^{\frac{\beta}{\gamma}} + \frac{1}{(1 - \epsilon_{pv})(1 - \epsilon_{pj})} \). For example, when \( N = 80, \beta = 1 \) and \( \gamma = 0.1 \), we have \( \lceil N_{(\beta, \gamma), b} \rceil = 6 \) for the topologies (a) and (b), respectively.

The number of representatives for the \( \alpha \)-resiliency and \((\beta, \gamma)\)-robustness in broadcast-based R2C must satisfy

\[
\tilde{N} \geq \max(N_{\alpha}, N_{(\beta, \gamma), b}),
\]

where \( N_{\text{res}}(\alpha) \) is fixed number for given \( N, \gamma \) and \( \alpha \) as discussed in Sec. IV-B. Accordingly we can derive the E2E latency of the broadcast-based R2C as follows.

**Proposition 6.** The E2E latency of broadcast-based R2C is

\[
L_{RC, b} = \frac{\tilde{N}_b}{N} \sum_{i \in V_p} \left[ \log \left( \frac{1 - \zeta^{-1}}{\log \epsilon_{i, \text{max}}} \right) + \log \left( \frac{1 - \zeta^{-1}}{\log \epsilon_{p, \text{max}}} \right) \right] \tau, \tag{44}
\]

with communication success probability greater than or equal to \( \zeta^{\tilde{N}_b + 1} \).

**Proof:** The results follow from the proof of Proposition 2 in Appendix B with (9).

V. Simulation Results

In this section, we numerically evaluate the performance of the RC and R2C protocols and validate the analytic results obtained in the previous sections. We fix the transmission powers used by each nodes for a message transmission in gossip and wireless broadcast protocols as \( P_{g,t} = 2.5 \) mW and \( P_{b,t} = 100 \) mW, respectively, and the noise power as \( P_{\text{noise}} = 10^{-10} \) mW. In addition, we let the distance between the two neighboring nodes as \( R = 10 \) meters and fix the path loss exponent as \( \eta = 3 \) and the target dissemination success probability is fixed to \( \zeta = 0.9999 \). We assume that the system operates in the ISM band with carrier frequency of 2.4 GHz.

Fig. 6 illustrates the E2E latency of RC and R2C with gossip (denoted by (G)) and broadcast (denoted by (B)) protocols with respect to the target resiliency probability \( \alpha \). From the figure, we can easily find out the trade-off between the latency and the resiliency of R2C protocol. For achieving \( \alpha \) close to 1, the number of representative validators \( \tilde{N} \) in R2C must be as large as
Fig. 6. E2E latency of RC and R2C with gossip (G) and broadcast (B) protocols versus target resiliency probability $\alpha$, when $F = 5, 25$.

Fig. 7. E2E latency of R2C with gossip and broadcast protocols versus (a) the target robustness probability ($\gamma$) with fixed acceptable consensus distortion ($\beta = 1$), and (b) the target robustness probability ($\gamma$) with fixed acceptable consensus distortion ($\beta = 1$).

Another interesting feature found in the result is that when utilizing broadcast protocol, the increment of E2E latency is smaller than that of gossip protocol. This makes the system achieve low latency consensus while guaranteeing a small loss of resiliency against the faulty nodes.

Fig. 7a and 7b illustrate the E2E latency of RC and R2C with gossip and broadcast protocols with respect to the acceptable consensus distortion and target robustness probability, respectively, in two topologies mentioned in Fig. 2 when $N + 1 = 81$ and the target SNR $\rho = 10$ dB. In Fig. 7a we can see that as the acceptable consensus distortion gets smaller, R2C incur larger delay since it requires a larger number of representative validators. R2C jointly designed with broadcast protocol outperforms other designs in terms of achieving low E2E latency with small acceptable distortion. Similarly, in Fig. 7b co-design of R2C with broadcast protocol can achieve lowest E2E latency, while guaranteeing robustness more than any other approaches.

Fig. 8a and 8b illustrate the E2E latency and corresponding normalized energy consumption
versus the number of faulty nodes $F$, respectively, of RC and R2C with gossip and broadcast protocols in topology (a) and (b) described in Fig. 2. The reliability factors are fixed to $\alpha = 0.01$, $\beta = 1$ and $\gamma = 0.1$. As shown in Fig. 8a, the E2E latency of R2C is relatively lower than that of RC, while R2C with broadcast-based message dissemination can further reduce latency better than using the gossip approach. Note that for small number of faulty nodes the dominant factor that determines the E2E latency of R2C comes from guaranteeing $(\beta, \gamma)$-robustness, while for large number of faulty nodes, guaranteeing $\alpha$-resiliency is the dominant factor. In the mean time, the total energy consumption is much larger when using broadcast approach in RC, however, if there are small number of faulty nodes in the network, R2C with broadcast approach can dramatically reduce energy consumption.

VI. Conclusion

Towards supporting mission-critical and real-time controls in IIoT systems, we proposed a novel communication-efficient distributed consensus protocol, i.e., Random Representative Consensus (R2C). For both gossip and broadcast communication protocols, we derived the closed-form expressions of the E2E latency and reliability of R2C. These expressions clarify fundamental relationships between consensus latency and reliability under wireless connectivity, thereby providing a guideline on co-designing distributed consensus and wireless communication protocols. The effectiveness of R2C was validated by both analysis and numerical evaluations, for two different network topologies while considering uniformly distributed faulty nodes. To validate its feasibility in more realistic scenarios, investigating more general network topologies and spatial node distributions is an interesting topic for future research.
Consider a message delivery from an arbitrary source node $i$ to destination node $k$, where $i, k \in \mathcal{N}$, $i \neq k$, and $k$ is not necessarily be a neighbor of node $i$. In graph theory, a walk is a finite or infinite sequence of edges which join a sequence of nodes and a path is a walk in which all nodes and vertices are distinct. In this perspective, a route of the message delivery from node $i$ to $k$ can be seen as a path between the two nodes. A single message may flow over various paths, while we focus on the one with the shortest amount of time upon all successful message delivery. Let $Z_{g,ik}$ be a random variable which denotes the least number of time slots required to deliver a message from source $i$ to destination $k$ via the gossip protocol. Among all the possible paths for the message flow, we define the shortest paths as the paths which are comprised of the minimum number of edges, and define the number of edges comprising the shortest paths between the node $i$ and $k$ as $e_{ik}$ and the number of shortest paths between the two nodes as $s_{ik}$. Intuitively, the shortest paths are the dominant factors that determines $Z_{g,ik}$, if there are sufficiently large number of independent shortest paths from node $i$ to node $k$. Furthermore, a convergence of $Z_{g,ik}$ to $e_{ik}$ if $s_{ik} \to \infty$ is intuitive trivial, since at least one shortest path without any outage will exist among $s_{ik}$ shortest paths.

**Lemma 1.** Given a source-destination node pair $(i, k)$, the random variable $Z_{g,ik}$ converges in distribution to a constant random variable as

$$Pr[Z_{g,ik} = z] = \begin{cases} 1, & z = e_{ik} \\ 0, & \text{elsewhere.} \end{cases}$$

if the shortest paths do not have any internal edge in common, i.e., edge-independent, and $s_{ik} \to \infty$.

**Proof:** The outage probability of a single hop communication between two neighbors can be written as

$$\epsilon_g = 1 - \exp\left(-\rho R^g \frac{(4\pi)^2 P_{\text{noise}}}{\lambda^2 P_{g,t}} \right),$$

from [4]. Suppose there exist $s_{ik}$ shortest paths between node $i$ and $k$ and let $T_1, \ldots, T_{s_{ik}}$ be the random variable that denotes the number of transmission required for a successful message delivery over those shortest paths. Then, we can model $T_s$ as a random variable which follows the negative binomial distribution as

$$Pr[T_s = t] = \begin{cases} (t-1) \epsilon_{ik}^t (1 - \epsilon_g)^{e_{ik}}, & t \geq e_{ik}, \\ 0, & t < e_{ik}. \end{cases}$$

for $s \in \{1, \ldots, s_{ik}\}$, where $t$ is a non-negative integer.
Let $U_{ik}$ be a random variable which denotes the minimum number of transmissions for successful message delivery from node $i$ to node $k$, over all the paths from node $i$ to node $k$ excluding the shortest paths. We assume that $U_{ik}$ is following some p.m.f. $Pr[U_{ik} = u]$, which is obviously $Pr[U_{ik} = u] = 0$ for $u \leq e_{ik}$, since no paths without the shortest paths can deliver a message with less than or equal to $e_{ik}$ transmissions. Then we can write

$$Z_{g,ik} = \min\{T_1, \ldots, T_{s_{ik}}, U_{ik}\}, \quad (48)$$

where the random variables $T_1, T_2, \ldots, T_{s_{ik}}$ are identically and independently distributed following p.m.f. of the negative binomial distribution (47) with parameter $e_{ik}$. Then the cumulative distribution function (c.d.f.) of $Z_{g,ik}$ can be derived as

$$Pr[Z_{g,ik} \leq z] = 1 - Pr[Z_{g,ik} > z] = 1 - Pr[\min\{T_1, \ldots, T_{s_{ik}}, U_{ik}\} > z] = 1 - Pr[T_1 > z, \ldots, T_{s_{ik}} > z, U_{ik} > z] = 1 - (1 - Pr[U_{ik} \leq z]) (1 - Pr[T_s \leq z])^{s_{ik}}, \quad (52)$$

where (52) follows from the assumption that all the paths are independent. If there exist a sufficiently large number of shortest paths, i.e., $s_{ik} \to \infty$, we have

$$\lim_{s_{ik} \to \infty} Pr[Z_{g,ik} \leq z] = \begin{cases} 
0, & z < e_{ik}, \\
1, & z \geq e_{ik}, 
\end{cases} \quad (53)$$

since $Pr[U_{ik} \leq z] = 0$ and $Pr[T_s \leq z] = 0$ for $z < e_{ik}$, and $0 < Pr[T_s \leq z] < 1$ for $z \leq e_{ik}$. From (52), we also have

$$Pr[Z_{g,ik} = z] = (Pr[Z_{g,ik} \leq z] - Pr[Z_{g,ik} \leq z - 1]). \quad (54)$$

Thus, from (53) we have (45) and this completes the proof of Lemma 1.

Put differently, if there are infinitely many edge-independent shortest paths from the node $i$ to $k$, there may exist at least one path that can guarantee a successful message delivery from the node $i$ to $k$ without occurring any outages during the delivery. In fact, the total number of shortest paths $s_{ik}$ is finite and the number of shortest paths that are edge-independent is much smaller than $s_{ik}$. Therefore, the random variable $Z_{g,ik}$ can be lower bounded as

$$Z_{g,ik} \geq e_{ik}, \quad (55)$$

in the considered network model considered. From (53), we can also conclude that

$$w_i \geq \max_k e_{ik}, \quad (56)$$

for all $i \in \mathcal{N}$. In the gossip protocol, the SNR outage probability between any two neighboring nodes are fixed to

$$\epsilon_g = 1 - \exp\left(-\rho R_g \frac{(4\pi)^2 P_{\text{noise}}}{\lambda^2 P_{g,t}}\right), \quad (57)$$
for given transmit power $P_{g,t}$ and target SNR $\rho$, from (4). For small $\epsilon_g$, the bound (56) tight.
For sufficiently small $\epsilon_g$, we approximately get
\[
\Pr \left[ \max_k Z_{g,ik} > \max_k \epsilon_{ik} \right] \approx 0. \tag{58}
\]

In the mean time, in the square network considering in this paper, the number of edges that comprises shortest paths is $e_{ik} = \tilde{x}_{ik} + \tilde{y}_{ik}$, where $\tilde{x}_{ik} = |x_k - x_i|/R$ and $\tilde{y}_{ik} = |x_k - y_i|/R$, and from simple combinatorics the number of shortest paths between the two nodes can be readily derived as $s_{ik} = \frac{(\tilde{x}_{ik} + \tilde{y}_{ik})!}{\tilde{x}_{ik}!\tilde{y}_{ik}!}$. Since we assume square network composed of $N + 1$ nodes, and from (56) and (9), we have (10).

\textbf{APPENDIX B}

\textbf{PROOF OF PROPOSITION 2}

Let $Z_{b,ik}$ be a random variable which denotes the number of time slots required to deliver a message from source node $i$ to destination node $k$ via the broadcast protocol. In this case, messages are delivered in a single hop without any help from relays. Since the messages are sent repeatedly until successful delivery, the random variable $Z_{b,ik}$ follows the geometric distribution with pdf
\[
\Pr[Z_{b,ik} = z] = \begin{cases} 
\epsilon_{ik}^{z-1} (1 - \epsilon_{ik}), & \forall z \geq 1 \\
0, & \text{elsewhere},
\end{cases} \tag{59}
\]
where $\epsilon_{ik} = \Pr[\text{SNR}_{ik} < \rho]$ is the SNR outage probability (4) with transmit power $P_t = P_{t,b}$. Then the dissemination outage probability can be upper bounded as
\[
\Pr \left[ \max_k Z_{b,ik} > w_{i,\zeta} \right] = 1 - \Pr \left[ \max_k Z_{b,ik} \leq w_i \right] \tag{60}
= 1 - \prod_{k \in N, k \neq i} \Pr [Z_{b,ik} \leq w_i] \tag{61}
= 1 - \prod_{k \in N, k \neq i} (1 - \epsilon_{ik}^{w_i}) \tag{62}
\leq 1 - \left( 1 - \epsilon_{i,\max}^{w_i} \right)^N, \tag{63}
\]
where $\epsilon_{i,\max} = 1 - \exp \left( -\rho \max_k P_{g,ik}^{n} \frac{P_{t,\text{max}}}{P_t} \right)$ is the SNR outage probability between node $i$ and the node located maximum distance apart from node $i$. The equality (61) holds from the independency of the channels between the source and the destinations, and the inequality (63) holds from the fact that the SNR outage probability is the largest when the node $k$ is located farthest apart from the node $i$ among all the destinations from (4). Since the dissemination time duration achieve dissemination outage proabability smaller than $\zeta$, we have a bound as follows.
Remark 1. In broadcast protocol, the dissemination time duration is bounded as

\[ w_i \geq \left\lceil \frac{\log \left( 1 - \zeta \frac{\tilde{N}}{N} \right)}{\log \epsilon_{i,\text{max}}} \right\rceil, \]  

since (63) must be smaller than or equal to \( \zeta \) from (5).

From Remark 1 and (9), we get (2).

APPENDIX C

PROOF OF PROPOSITION 4

Throughout the proof, we fix a proposer as node \( p \) without loss of generality and use \( D \) and \( T_i \) instead of \( D(A_p, \mathcal{V}_p, \tilde{\mathcal{V}}_p) \) and \( T_i(A_p) \) from (23), respectively, for simple notation.

We first show that \( \mathbb{E}[D] = 0 \). Let \( \tilde{\mathcal{V}}_p \in \left\{ \tilde{\mathcal{V}}_{p,1}, \ldots, \tilde{\mathcal{V}}_{p,\left( \frac{N}{\tilde{N}} \right)} \right\} \) be the randomly chosen validator set by the proposer, where \( \tilde{\mathcal{V}}_{p,1}, \ldots, \tilde{\mathcal{V}}_{p,\left( \frac{N}{\tilde{N}} \right)} \) be all possible sets of cardinality \( \tilde{N} \) that can be chosen from \( \mathcal{V}_p \) with equal probability \( \frac{1}{\left( \frac{N}{\tilde{N}} \right)} \). For given \( \tilde{\mathcal{V}}_p = \tilde{\mathcal{V}}_{p,l} \), we have

\[ \mathbb{E}[D] = \mathbb{E} \left[ \frac{1}{N} \sum_{i \in \mathcal{V}_p} T_i - \frac{1}{N} \sum_{j \in \tilde{\mathcal{V}}_{p,l}} T_j \right] \]

\[ = \mathbb{E} \left[ \frac{1}{N} \sum_{i \in \tilde{\mathcal{V}}_{p,l}} T_i - \frac{N - \tilde{N}}{NN} \sum_{j \in \tilde{\mathcal{V}}_{p,l}} T_j \right] \]

\[ = \frac{1}{N} \sum_{i \in \tilde{\mathcal{V}}_{p,l}} \mathbb{E}[T_i] - \frac{N - \tilde{N}}{NN} \sum_{j \in \tilde{\mathcal{V}}_{p,l}} \mathbb{E}[T_j], \]

where \( \tilde{\mathcal{V}}_{p,l} = \mathcal{V}_p \setminus \tilde{\mathcal{V}}_{p,l} \) is the complement of \( \tilde{\mathcal{V}}_{p,l} \). By marginalizing (67), we have

\[ \mathbb{E}[D] = \frac{1}{\left( \frac{N}{\tilde{N}} \right)} \sum_{l=1}^{\left( \frac{N}{\tilde{N}} \right)} \mathbb{E} \left[ D \mid \tilde{\mathcal{V}}_{p,l} \right] \]

\[ = \frac{1}{\left( \frac{N}{\tilde{N}} \right)} \sum_{l=1}^{\left( \frac{N}{\tilde{N}} \right)} \left( \frac{1}{N} \sum_{i \in \tilde{\mathcal{V}}_{p,l}} \mathbb{E}[T_i] - \frac{N - \tilde{N}}{NN} \sum_{j \in \tilde{\mathcal{V}}_{p,l}} \mathbb{E}[T_j] \right) \]

\[ = \frac{N - \tilde{N}}{N} \left( \frac{1}{N} \sum_{i \in \mathcal{V}_p} \mathbb{E}[T_i] - \frac{1}{NN} \sum_{j \in \mathcal{V}_p} \tilde{N} \mathbb{E}[T_j] \right) \]

\[ = 0. \]

From Chebyshev’s inequality, we have

\[ \Pr \left[ |D| \geq \sqrt{\beta} \right] \leq \frac{\text{Var}(D)}{\beta}. \]
where from the law of total variance

$$\text{Var}(D) = \mathbb{E} \left[ \text{Var} \left( D \mid \tilde{V}_p \right) \right] + \text{Var} \left( \mathbb{E} \left[ D \mid \tilde{V}_p \right] \right).$$  \hspace{1cm} (73)

The first term of the right-hand side in (73) can be rewritten as

$$\mathbb{E} \left[ \text{Var} \left( D \mid \tilde{V}_p \right) \right] = \frac{1}{N^2} \sum_{l=1}^{N} \text{Var} \left( \frac{1}{N} \sum_{i \in \tilde{V}_p,l} T_i - \frac{N - \tilde{N}}{\tilde{N}N} \sum_{j \in \tilde{V}_p,l} T_j \right),$$ \hspace{1cm} (74)

$$= \frac{1}{N^2} \sum_{l=1}^{N} \left( \sum_{i \in \tilde{V}_p,l} \text{Var}(T_i) + \frac{(N - \tilde{N})^2}{N^2} \sum_{j \in \tilde{V}_p,l} \text{Var}(T_j) \right),$$ \hspace{1cm} (75)

$$= \frac{1}{N^2} \sum_{i \in \tilde{V}_p} \left( \frac{(N - \tilde{N})}{N} \text{Var}(T_i) + \frac{\tilde{N}(N - \tilde{N})^2}{N^2N^3} \text{Var}(T_i) \right),$$ \hspace{1cm} (76)

$$= \frac{(N - \tilde{N})}{N^2N^2} \sum_{i \in \tilde{V}_p} \text{Var}(T_i),$$ \hspace{1cm} (77)

and the second term as

$$\text{Var} \left( \mathbb{E} \left[ D \mid \tilde{V}_p \right] \right) = \frac{1}{N^2} \sum_{l=1}^{N} \left( \frac{1}{N} \sum_{i \in \tilde{V}_p,l} \mathbb{E}[T_i] - \frac{N - \tilde{N}}{\tilde{N}N} \sum_{j \in \tilde{V}_p,l} \mathbb{E}[T_j] \right)^2,$$ \hspace{1cm} (78)

$$= \frac{(N - \tilde{N})}{N^2N^2} \sum_{i \in \tilde{V}_p} \left( \mathbb{E}[T_i]^2 - \frac{1}{(N-1)} \sum_{j \in \tilde{V}_p,l \neq i} \mathbb{E}[T_i] \mathbb{E}[T_j] \right).$$ \hspace{1cm} (79)

From (73), (77) and (79), and since \(\text{Var}(T_i) = \mathbb{E}[T_i^2] - \mathbb{E}[T_i]^2\), we have

$$\text{Var}(D) = \frac{\tau^2(N - \tilde{N})}{N^2N^2} \sum_{i \in \tilde{V}_p} \left( \mathbb{E}[T_i^2] - \frac{1}{N-1} \sum_{j \in \tilde{V}_p,l \neq i} \mathbb{E}[Z_i] \mathbb{E}[Z_j] \right).$$ \hspace{1cm} (80)

By substituting (80) into (72), we have (25). This finishes the proof.

REFERENCES

[1] K. Schwab, The fourth industrial revolution. Currency, 2017.
[2] M. Bernard, “Why everyone must get ready for the 4th industrial revolution,” Forbes (Blog), 2016.
[3] G. Brown, “Ultra-reliable low-latency 5G for industrial automation,” tech. rep., Qualcomm, 2017.
[4] M. Hermann, T. Pentek, and B. Otto, “Design principles for industri 4.0 Scenarios,” in Proc. 49th Hawaii Int. Conf. System Sciences (HICSS), pp. 3928–3937, Jan. 2016.
[5] L. Lamport, R. Shostak, and M. Pease, “The byzantine generals problem,” 1982.
[6] M. J. Fischer, N. A. Lynch, and M. S. Paterson, “Impossibility of distributed consensus with one faulty process,” J. ACM, vol. 32, pp. 374–382, Apr. 1985.
[7] F. B. Schneider, “Implementing fault-tolerant services using the state machine approach: A tutorial,” ACM Computing Surveys (CSUR), vol. 22, no. 4, pp. 299–319, 1990.
[8] M. Castro, B. Liskov, et al., “Practical byzantine fault tolerance,” in OSDI, vol. 99, pp. 173–186, 1999.
[9] S. Nakamoto, “Bitcoin: A peer-to-peer electronic cash system, http://bitcoin.org/bitcoin.pdf.”
[10] H. Kim, J. Park, M. Bennis, and S. Kim, “Blockchained on-device federated learning,” IEEE Communications Letters, p. 1, 2019.
[11] G. Lee, J. Park, W. Saad, and M. Bennis, “Performance analysis of blockchain systems with wireless mobile miners,”
[12] L. Baird, “The swirls hashgraph consensus algorithm: Fair, fast, byzantine fault tolerance,” Swirls Tech Reports SWIRLDS-TR-2016-01, Tech. Rep., 2016.
[13] H. Seo, J. Park, M. Bennis, and W. Choi, “Consensus-before-talk: Distributed dynamic spectrum access via distributed spectrum ledger technology,” in Proc. IEEE Int. Symp. Dynamic Spectrum Access Networks (DySPAN), pp. 1–7, Oct. 2018.
[14] P. Danzi, A. E. Kalr, . Stefanovi, and P. Popovski, “Delay and communication tradeoffs for blockchain systems with lightweight iot clients,” IEEE Internet of Things Journal, vol. 6, pp. 2354–2365, Apr. 2019.
[15] Y. Liu, K. Wang, Y. Lin, and W. Xu, “LightChain: A lightweight blockchain system for industrial internet of things,” IEEE Transactions on Industrial Informatics, vol. 15, pp. 3571–3581, June 2019.
[16] J. Al-Jaroodi and N. Mohamed, “Blockchain in industries: A survey,” IEEE Access, vol. 7, pp. 36500–36515, 2019.
[17] C. T. Nguyen, D. T. Hoang, D. N. Nguyen, D. Niyato, H. T. Nguyen, and E. Dutkiewicz, “Proof-of-stake consensus mechanisms for future blockchain networks: Fundamentals, applications and opportunities,” IEEE Access, vol. 7, pp. 85727–85745, 2019.
[18] Y. Zhao, J. Zhao, L. Jiang, R. Tan, and D. Niyato, “Mobile edge computing, blockchain and reputation-based crowdsourcing iot federated learning: A secure, decentralized and privacy-preserving system,”
[19] P. Danzi, S. Hambridge, . Stefanovi, and P. Popovski, “Blockchain-based and multi-layered electricity imbalance settlement architecture,” in Proc. and Computing Technologies for Smart Grids (SmartGridComm) 2018 IEEE Int. Conf. Communications, Control, pp. 1–7, Oct. 2018.
[20] E. Androulaki, A. Barger, V. Bortnikov, C. Cachin, K. Christidis, A. D. Caro, D. Enyeart, C. Ferris, G. Laventman, Y. Manevich, S. Muralidharan, C. Murthy, B. Nguyen, M. Sethi, G. Singh, K. Smith, A. Sorniotti, C. Stathakopoulou, M. Vukoli, S. W. Cocco, and J. Yellick, “Hyperledger fabric: A distributed operating system for permissioned blockchains,”
[21] A. Goldsmith, “Wireless communications,” 2005.
[22] B. R. Calder and A. McLeod, “Ultraprecise absolute time synchronization for distributed acquisition systems,” IEEE Journal of Oceanic Engineering, vol. 32, pp. 772–785, Oct. 2007.
[23] A. Mahmood, M. I. Ashraf, M. Gidlund, and J. Torsner, “Over-the-air time synchronization for urllc: Requirements, challenges and possible enablers,”
[24] R. L. Rivest, A. Shamir, and L. Adleman, “A method for obtaining digital signatures and public-key cryptosystems,” Communications of the ACM, vol. 21, no. 2, pp. 120–126, 1978.
[25] G. Tsudik, “Message authentication with one-way hash functions,” ACM SIGCOMM Computer Communication Review, vol. 22, no. 5, pp. 29–38, 1992.
[26] R. Rivest, “The md5 message-digest algorithm,” tech. rep., 1992.
[27] S. Winitzki, “Uniform approximations for transcendental functions,” in International Conference on Computational Science and Its Applications, pp. 780–789, Springer, 2003.
[28] S. Winitzki, “A handier approximation for the error function and its inverse,” A lecture note obtained through private communication, 2008.
[29] T. M. Cover and J. A. Thomas, Elements of information theory. John Wiley & Sons, 2012.