Light vector hybrid states via QCD sum rules

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(Dated: March 26, 2022)

Abstract

Vector hybrid states with light quarks $u,d,s$ are investigated via QCD sum rules. The results show that the masses of the $q\bar{q}g$ ($q = u,d$), $q\bar{s}g$, and $s\bar{s}g$ states with $J^{PC} = 1^{--}$ are about 2.3-2.4, 2.3-2.5, and 2.5-2.6 GeV, respectively. It suggests that the recently discovered $Y(2175)$ could not be a pure $s\bar{s}g$ vector hybrid state.
It has been long expected that in QCD, besides the conventional $q\bar{q}$ mesons and $qqq$ baryons, exotic states such as the multiquark states and hybrid states, should exist as a consequence of the non-perturbative aspect of QCD \cite{1, 2}. A multiquark state is composed of more than three quarks and anti-quarks; a hybrid state contains valence gluon(s), besides valence quarks. There has been lots of work on the hybrid states with exotic quantum numbers, such as $J^{PC} = 0^{--}, 0^{+-}, 1^{+-}, 2^{+-}$ etc., which cannot be obtained by using only a quark and an anti-quark \cite{2}. In this paper, we will investigate the light flavor hybrid states with quantum numbers $J^{PC} = 1^{--}$.

In the present experimental spectrum of vector mesons, some states are argued to be vector hybrid candidates or contain hybrid components. For instance, the $\rho(1450)$ and $\omega(1420)$ were proposed to have significant hybrid components because their decay patterns are different from those expected for radially excited $q\bar{q}$ states \cite{3, 4, 5, 6}. The heavier state $\omega(1600)$ was suggested to be $2S$-hybrid mixtures \cite{6}. For the strange counterpart, $K^*(1410)$, both of its mass and decay pattern have not been understood yet \cite{7, 8}. The $K^*(1410)$ is too light to be a $2S$-state and its decay pattern is also against the $2S$ assignment \cite{8}. Its largest decay channel is $K^*\pi$, with a branching fraction larger than 40%; the branching fraction of the $K\pi$ channel is $6.6 \pm 1.3\%$, and the one of the $K\rho$ channel is less than 7\% \cite{9}. In Ref. \cite{8}, it was suggested that the low mass of the $K^*(1410)$ state might be due to the presence of additional hybrid mixing states.

The recent discovered charmonium-like vector state $Y(4260)$ \cite{10} was suspected to be a $c\bar{c}g$ hybrid state \cite{11}. Recently, a structure at about 2175 MeV was observed by the BABAR Collaboration, and it is consistent with a resonance with a mass of $M_X = 2175 \pm 10 \pm 15$ MeV and width of $\Gamma_X = 58 \pm 16 \pm 20$ MeV \cite{12}. It was observed in $e^+e^- \rightarrow \phi f_0(980)$ via initial state radiation, hence has quantum numbers of $J^{PC} = 1^{--}$. Immediately, a suggestion of an $ss\bar{s}\bar{s}$ tetraquark state appeared \cite{13}. Such an interpretation was criticized for its possible large width, and then an $s\bar{s}g$ hybrid interpretation was proposed \cite{14}. It was pointed out in Ref. \cite{15} that the hybrid suggestion would make sense if the $Y(4260)$ is a $c\bar{c}$ hybrid and $m_c - m_s \simeq (M_Y - M_X)/2 = 1.04$ GeV.

To understand all the states mentioned above, an important step is to calculate the masses of vector hybrid states. Isgur and Paton estimated the masses of the $q\bar{q}g$ ($q = u, d$), $n\bar{s}g$ and $s\bar{s}g$ states to be about 1.9, 2.0 and 2.1 GeV, respectively, by using the flux-tube model \cite{16}. However, they stated that the reliability was no better than $\pm 100$ MeV, and the spin-dependent perturbations were not considered. In this paper, we study the light flavor hybrid state with $J^{PC} = 1^{--}$ by using the method of QCD sum rules (QCDSR) which has been proved to be very successful in many hadronic problems \cite{17, 18}. The central idea of QCDSR is to calculate the correlation function of an interpolating current with definite $J^{PC}$ quantum numbers from both the phenomenological side.
\( q^2 > 0 \) and the QCD side \( q^2 \ll 0 \) through operator product expansion (OPE). The expression on the phenomenological side can be related to that on the QCD side via dispersion relation to get sum rules, and then hadronic parameters can be determined.

We start from the two-point correlation function of the interpolating vector current \( J_\mu(x) \)

\[
\Pi_{\mu\nu}(q^2) = i \int d^4 x e^{iq \cdot x} \langle 0| T\{J_\mu(x)J^\dagger_\nu(0)\}|0\rangle = (-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2})\Pi_1(q^2) + \frac{q_\mu q_\nu}{q^2}\Pi_0(q^2).
\]

The interpolating current for a vector hybrid state with quantum numbers \( J^{PC} = 1^{--} \) is taken as

\[
J_\mu(x) = g_s \bar{\psi}_A(x)\gamma^\nu\gamma^5\lambda^a_{\mu\nu} G^{\mu\nu}_n(x)\psi^b_B(x),
\]

where \( g_s \) is the strong coupling constant, \( A, B = u, d, s \) and \( a, b = 1, 2, ..., 8 \) are flavor and color indices, respectively. \( G^{\mu\nu}_n(x) = \varepsilon^{\mu\nu\alpha\beta} G^{n,\alpha\beta}_n(x)/2 \) is the dual field strength of \( G^{\mu\nu}_n(x) \). The overlapping amplitude of this current with the vector hybrid state \( X \) is defined as

\[
\langle 0|J_\mu(0)|X\rangle = f_X m_X^3 \epsilon_\mu,
\]

where \( f_X, m_X \) and \( \epsilon_\mu \) are the decay constant, mass and polarization vector of the hybrid state, respectively.

For the phenomenological side, we can obtain the imaginary part of the correlation function as

\[
\frac{\text{Im}\Pi(q^2)}{\pi} = f_X^2 m_X^6 \delta(q^2 - m_X^2) + \rho^h(q^2)\theta(q^2 - s_0^h)
\]

From the dispersion relation without subtractions (because subtractions will be removed by the Borel transform, we can neglect them here), we have

\[
\Pi(q^2) = \frac{f_X^2 m_X^6}{m_X^2 - q^2 - i\epsilon} + \int_{s_0}^\infty \frac{\rho^h(s)}{s - q^2 - i\epsilon} ds,
\]

where \( \rho^h(s) \) represents the spectral function of the continuum states, and \( s_0 \) is the threshold parameter.

The correlation function can be treated in the framework of the operator product expansion (OPE), then the short and long distance interactions are separated. The short distance interactions are encoded in the Wilson coefficients which can be calculated by using perturbative QCD at large \( Q^2 = -q^2 \), and the long distance effects are parameterized in a sets of universal quark and gluon condensates [17]. When performing operator product expansion (OPE), we consider operators up to eight dimension. The relevant Feynman diagrams for deriving the QCD sum rules for light vector hybrid states are shown in Fig. 1.
After tedious calculations, the correlation function of the $s\bar{s}g$ vector hybrid is obtained as

$$
\Pi_{s\bar{s}g}(p^2) = \left( -\frac{\alpha_s}{240\pi^3} p^6 + \frac{5\alpha_s}{48\pi^3} m_s^2 p^4 - \frac{4\alpha_s}{9\pi} m_s \langle \bar{s}s\rangle p^2 + \frac{1}{144\pi^2} \langle g_s^2 G^2 \rangle p^2 - \frac{1}{16\pi^2} m_s^2 \langle g_s^2 G^2 \rangle \right)
$$

$$
- \frac{49\alpha_s}{144\pi} m_s \langle g_s \bar{s}G_s \rangle - \frac{1}{64\pi^2} \langle g_s^3 G^3 \rangle \ln (-p^2)
$$

$$
+ \frac{m_s^2}{32\pi^2 p^2} \langle g_s^3 G^3 \rangle + \frac{m_s}{12\pi^2} \langle \bar{s}G_s \rangle \langle g_s^2 G^2 \rangle - \frac{\pi\alpha_s}{36\pi^2} \langle \bar{s}s \rangle \langle g_s \bar{s}G_s \rangle,
$$

where $\langle g_s \bar{s}G_s \rangle = \langle g_s \bar{s}\sigma^{\mu\nu} t^n G_{\mu\nu}^m \rangle$ with $t^n = \lambda^n/2$ being SU(3) generators, $\langle g_s^3 G^3 \rangle = \langle g_s^3 f^{lmn} G_{\gamma\delta}^l G_{\gamma}^m G_{\gamma}^n \rangle$. Similarly, the correlation functions of the $q\bar{q}g$, and $q\bar{q}g$ ($q = u, d$) vector hybrid states are

$$
\Pi_{q\bar{q}g}(p^2) = \left( -\frac{\alpha_s}{240\pi^3} p^6 + \frac{3\alpha_s}{64\pi^3} m_s^2 p^4 - \frac{\alpha_s}{18\pi} m_s \langle q\bar{q}G_q \rangle p^2 - \frac{\alpha_s}{6\pi} m_s \langle \bar{s}s \rangle p^2 + \frac{1}{144\pi^2} \langle g_s^2 G^2 \rangle p^2 
$$

$$
- \frac{1}{64\pi^2} m_s^2 \langle g_s^2 G^2 \rangle + \frac{3\alpha_s}{64\pi} m_s \langle g_s \bar{q}G_q \rangle + \frac{\alpha_s}{192\pi} m_s \langle g_s \bar{s}G_s \rangle - \frac{1}{64\pi^2} \langle g_s^3 G^3 \rangle \ln (-p^2)
$$

$$
+ \frac{5m_s^2}{384\pi^2 p^2} \langle g_s^3 G^3 \rangle + \frac{m_s}{24\pi^2} \langle \bar{q}G_q \rangle \langle g_s^2 G^2 \rangle - \frac{\pi\alpha_s}{72\pi^2} \langle \bar{q}q \rangle \langle g_s \bar{s}G_s \rangle - \frac{\pi\alpha_s}{72\pi^2} \langle \bar{s}s \rangle \langle g_s \bar{s}qG_q \rangle,
$$

and

$$
\Pi_{q\bar{q}g}(p^2) = \left( -\frac{\alpha_s}{240\pi^3} p^6 + \frac{1}{144\pi^2} \langle g_s^2 G^2 \rangle p^2 - \frac{1}{64\pi^2} \langle g_s^3 G^3 \rangle \ln (-p^2) - \frac{\pi\alpha_s}{36\pi^2} \langle \bar{q}q \rangle \langle g_s \bar{s}qG_q \rangle,
$$

respectively.
The Borel transform are defined as

\[
\mathcal{B}_{M_B^2}f(p^2) = \lim_{-p^2/n \to \infty} \frac{(-p^2)^{n+1}}{n!} \left( \frac{d}{dp^2} \right)^n f(p^2).
\]  

(9)

Performing the Borel transform to both the phenomenological side and the QCD side, and using the quark-hadron duality to approximate the continuum contribution, we obtain the following sum rules

\[
\begin{align*}
 f^2_{s\bar{s}g} m_{s\bar{s}g}^6 e^{-m^2_{s\bar{s}g}/M_B^2} &= \int_0^{s_0} dp^2 \left( \frac{\alpha_s}{240\pi^3} p^6 - \frac{5\alpha_s}{48\pi^3} m_s^2 p^4 + \frac{4\alpha_s}{9\pi} m_s \langle s\bar{s} \rangle p^2 - \frac{1}{144\pi^2} \langle g_s^2 G^2 \rangle p^2 \\
 &+ \frac{1}{16\pi^2} m_s^2 \langle g_s^2 G^2 \rangle + \frac{49\alpha_s}{144\pi} m_s \langle g_s \bar{s} G s \rangle + \frac{1}{64\pi^2} \langle g_s^3 G^3 \rangle \right) e^{-p^2/M_B^2} \\
 &- \frac{m_s^2}{32\pi^2} \langle g_s^3 G^3 \rangle - \frac{m_s}{12} \langle \bar{s}s \rangle \langle g_s^2 G^2 \rangle + \frac{\pi \alpha_s}{36} \langle \bar{s}s \rangle \langle g_s \bar{s} G s \rangle, \\
 f^2_{q\bar{q}g} m_{q\bar{q}g}^6 e^{-m^2_{q\bar{q}g}/M_B^2} &= \int_0^{s_0} dp^2 \left( \frac{\alpha_s}{240\pi^3} p^6 - \frac{3\alpha_s}{64\pi^3} m_s^2 p^4 + \frac{\alpha_s}{18\pi} m_s \langle q\bar{q} \rangle p^2 + \frac{\alpha_s}{6\pi} m_s \langle \bar{s}s \rangle p^2 - \frac{1}{144\pi^2} \langle g_s^2 G^2 \rangle p^2 \\
 &+ \frac{1}{64\pi^2} m_s^2 \langle g_s^2 G^2 \rangle - \frac{3\alpha_s}{64\pi} m_s \langle g_s \bar{q} G q \rangle - \frac{\alpha_s}{192\pi} m_s \langle g_s \bar{s} G s \rangle + \frac{1}{64\pi^2} \langle g_s^3 G^3 \rangle \right) e^{-p^2/M_B^2} \\
 &- \frac{5m_s^2}{384\pi^2} \langle g_s^3 G^3 \rangle - \frac{m_s}{24} \langle q\bar{q} \rangle \langle g_s^2 G^2 \rangle + \frac{\pi \alpha_s}{72} \langle q\bar{q} \rangle \langle g_s \bar{s} G s \rangle + \frac{\pi \alpha_s}{72} \langle \bar{s}s \rangle \langle g_s \bar{q} G q \rangle, \\
 f^2_{q\bar{q}} m_{q\bar{q}}^6 e^{-m^2_{q\bar{q}}/M_B^2} &= \int_0^{s_0} dp^2 \left( \frac{\alpha_s}{240\pi^3} p^6 - \frac{1}{144\pi^2} \langle g_s^2 G^2 \rangle p^2 + \frac{1}{64\pi^2} \langle g_s^3 G^3 \rangle \right) e^{-p^2/M_B^2} \\
 &+ \frac{\pi \alpha_s}{36} \langle q\bar{q} \rangle \langle g_s \bar{q} G q \rangle,
\end{align*}
\]  

(10, 11, 12)

The running coupling constant can be taken as \(\alpha_s = 4\pi / \left[ (11 - 2/3n_f) \ln (M_B^2/\Lambda_{QCD}^2) \right]\) with three active flavors. For numerical analysis, we use the following inputs: 

\[\Lambda_{QCD} = 220 \text{ MeV},\]
\[m_s(2 \text{ GeV}) = 94 \text{ MeV},\]
\[\langle q\bar{q} \rangle = -(0.024 \text{ GeV})^3 ,\]
\[\langle \bar{s}s \rangle = 0.8 \times \langle q\bar{q} \rangle,\]
\[\langle g_s^2 G^2 \rangle = 0.48 \text{ GeV}^4,\]
\[\langle g_s q G q \rangle = m_0^2 \langle q\bar{q} \rangle \text{ with } m_0^2 = 0.8 \text{ GeV}^2,\]
\[\langle g_s^3 G^3 \rangle = 0.045 \text{ GeV}^6.\]

According to Ref. [22], the physical information of \(m_X\) and \(f_X\) can be extracted by fitting the right hand side and the left hand side with \(m_X^2\) and \(f_X^2\) as free parameters. The \(\chi^2\) fit is done in a reasonable interval of values of \(M_B^2\) to guarantee that the contributions of the operators from dimension 8 are less than 10%, and those of the continuum are less than 50%. The results are
TABLE I: The values of $f_X$, $m_X$, and $\sqrt{s_0}$ from fitting the left hand side to the right hand side.

|       | $m_X$ (GeV) | $f_X$ (MeV) | $\sqrt{s_0}$ (GeV) |
|-------|-------------|-------------|-------------------|
| $q\bar{q}g$ | 2.33        | 10.8        | 2.6               |
|        | 2.34        | 12.3        | 2.8               |
|        | 2.43        | 13.9        | 3.0               |
| $q\bar{s}g$ | 2.34        | 11.0        | 2.7               |
|        | 2.41        | 12.6        | 2.9               |
|        | 2.50        | 14.2        | 3.1               |
| $s\bar{s}g$ | 2.54        | 11.3        | 2.8               |
|        | 2.54        | 12.8        | 3.0               |
|        | 2.62        | 14.3        | 3.2               |

given in Table I. The results show that the masses of the $1^{--}$ hybrid states are well above 2 GeV. Compared with the results from the flux tube model [16], the values obtained here are 400-500 MeV higher.

In this paper, we calculate the masses and decay constants of the light flavor $1^{--}$ hybrid states. The masses of all the $q\bar{q}g$, $q\bar{s}g$ and $s\bar{s}g$ vector states are above 2 GeV, being about 2.3-2.4, 2.3-2.5, and 2.5-2.6 GeV, respectively. From these results, the mesons $\rho(1450)$, $\omega(1420)$, $\omega(1600)$, $K^*(1410)$, and etc. would not be pure hybrid states, but they could contain hybrid mixtures. Compared with these states, the recently discovered vector state $Y(2175)$ is stated closer to the position of the predicted hybrid state, $s\bar{s}g$. However, its mass is still lower than the one obtained here, which imply that the $Y(2175)$ state could not be a pure hybrid state. This state could be an excited $s\bar{s}$ state [23] or its mixing with a hybrid state. Both effort from the experimental side and the theoretical side are necessary to identify the nature of the $Y(2175)$ state. Experimentally, other decay channels of the state should be hunted out; theoretically, decay properties of the vector $s\bar{s}g$ hybrid state and the $2^3D_1$ state with mass of 2175 MeV have been investigated by using the flux tube model and the $3P_0$ model [14, 24]. By using the information on the mass of vector hybrid states, possible mixing between the $s\bar{s}g$ and the $2^3D_1$ state could be studied.

**Acknowledgments**

We sincerely acknowledge D.V. Bugg for useful discussion on the $Y(2175)$ state. This work is partially supported by the NSFC grant Nos. 90103020, 10475089, 10435080, 10447130, CAS Knowledge Innovation Key-Project grant No. KJCX2SWN02 and Key Knowledge Innovation
Project of IHEP, CAS (U529). One of the authors (WZG) would like to thank National Natural Science Foundation, Grant Number 10405009, for financial support.

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