Analysis of a selected sample of RR Lyrae stars in the LMC from OGLE-III *

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Abstract A systematic study of RR Lyrae stars is performed using a selected sample of 655 objects in the Large Magellanic Cloud (LMC) with long-term observations and numerous measurements from the Optical Gravitational Lensing Experiment III project. The phase dispersion method and linear superposition of the harmonic oscillations are used to derive the pulsation frequency and properties of light variation. It is found that a dichotomy exists in Oosterhoff Type I and Oosterhoff Type II for RR Lyrae stars in the LMC. Due to our strict criteria for identifying a frequency, a lower limit for the incidence rate of Blazhko modulation in the LMC is estimated in various subclasses of RR Lyrae stars. For fundamental-mode RR Lyrae stars, the rate of 7.5% is smaller than the previous result. In the case of the first-overtone RR Lyrae variables, the rate of 9.1% is relatively high. In addition to the Blazhko variables, 15 objects are identified to pulsate in the fundamental/first-overtone double mode. Furthermore, four objects show a period ratio around 0.6, which makes them very likely to be rare pulsators in the fundamental/second-overtone double mode.

Key words: stars: variables: other — galaxies: individual (LMC)

1 INTRODUCTION

RR Lyrae stars (RRLSs) are pulsating variables on the horizontal branch of the H-R diagram. They have short periods of 0.2 to 1 d and low metal abundances \( Z \) of 0.00001 to 0.01. Usually they can be easily identified by their light curves or color-color diagrams (Li et al. 2011). RRLSs are famous for the “Blazhko effect” (Blažko 1907), which is a periodic modulation of the amplitude and phase in the light curves. The theoretical underpinnings of the Blazhko effect are still a mystery today. The main photometric feature of the Blazhko effect is that the power spectra of the light curves are usually strongly dominated by a symmetric pattern around the main pulsation frequency \( f_0 \), i.e., \( kf_0 \) and \( kf_0 \pm f_{BL} \) where \( f_{BL} \) is the modulation frequency and \( k \) is the harmonic number (Kovács 2009). Moreover, the higher-order multiplets, such as quintuplets \( kf_0 \pm nf_{BL} \), are now also attributed to the Blazhko effect (Benkő et al. 2011). On the other hand, the amplitudes of modulation components are usually asymmetric so that one side could be under the detection limit in highly asymmetric cases.

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which may lead to the asymmetric appearance of the power spectra. Several models have been proposed to explain this effect, including the non-radial resonant rotator/pulsator (Goupil & Buchler 1994), the magnetic oblique rotator/pulsator (Shibahashi 2000), a 2:1 resonance model (Borkowski 1980), a resonance between radial and non-radial modes (Dziembowski & Mizerski 2004), and a 9:2 resonance model (Buchler & Kolláth 2011) and the convective cycle model (Stothers 2006). However, none of them can be used to interpret all the observational phenomena related to the Blazhko effect.

A systematic study of RRLs helps us to understand their nature, such as the rate of incidence for various pulsation modes, the distribution of modulation frequency and amplitude and the dependence of the light variation on their environment. This has been performed on the basis of some datasets with a large amount of data for variables. The early microlensing projects, including MACHO (Alcock et al. 2003) and the Optical Gravitational Lensing Experiment (OGLE) (Soszyński et al. 2008), mainly surveyed the Large Magellanic Cloud (LMC), the Small Magellanic Cloud (SMC) and the Galactic bulge. In addition, the All Sky Automated Survey (ASAS) (Szczygieł & Fabrycky 2007) contributed to the field of variable star research. Using the MACHO data, Alcock et al. (2000) and Nagy & Kovács (2006) analyzed the frequency of 1300 first overtone RRLs in the LMC and found an incidence rate for Blazhko variables of 7.5%. Meanwhile, Alcock et al. (2003) performed a frequency analysis of 6391 fundamental mode RRLs in the LMC that resulted in an incidence rate of 11.9% for Blazhko variables. With the OGLE-I data, Moskalik & Poretti (2003) searched for multi-periodic pulsators among 38 RRLs in the Galactic bulge. Mizerski (2003) completed a search for multi-period RRLs from the OGLE-II database. In addition, Collinge et al. (2006) compiled a catalog of 1888 fundamental mode RRLs in the Galactic bulge using the same database. Recently, Moskalik & Olech (2008) conducted a systematic search for multi-period RRLs in the globular cluster ω Centauri, and found the incidence rate of Blazhko modulation to be pretty high, about 24% and 38% for the fundamental and first-overtone RRLs, respectively. Jurcsik et al. (2009) derived an incidence rate of 47% from the dedicated Konkoly survey sample of 30 fundamental-mode RRLs in the Galactic field. Kolenberg et al. (2010) also claimed that at least 40% of a sample with 28 observations from the Kepler space mission were RRLs exhibiting the modulation phenomenon. The OGLE-III database released in 2009 (Soszyński et al. 2009) contained 24906 light curves that are preliminarily classified as RRLs in the LMC, making it the largest-ever sample of RRLs. This database covers a time span of about 10 years, allowing a large-scale analysis of the light variation in RRLs. Soszyński et al. (2009) analyzed the basic statistical features of RRLs in the LMC and divided them into four subtypes: RRab, RRc, RRd and RRε. With this sample of RRLs, the study of the structure of the LMC was undertaken. Pejcha & Stanek (2009) investigated the structure of the LMC stellar halo; Subramaniam & Subramanian (2009) found that RRLs in the inner LMC trace the disk and probably the inner halo; Feast et al. (2010) established a small but significant radial gradient in the mean periods of LMC RRLs. The data of RRLs in the SMC and the Galactic bulge were also released (Soszyński et al. 2010, 2011) and some work was also done with those data (Pietrukowicz et al. 2011). It is worth noting that these works mainly deal with the structure of the LMC rather than properties of the RRLs.

In this paper, we focus on the RRLs themselves based on the released OGLE-III database. With the phase dispersion method (PDM) and Fourier fitting methods, we make a precise systematic frequency analysis of 655 carefully selected RRLs in the LMC. We then classify the RRLs in the LMC and discuss the rate of incidence for the Blazhko modulation in various pulsation modes. The data and the sample are illustrated in Section 2, the method is introduced in Section 3, a detailed classification of RRLs in the LMC is described in Section 4 and a discussion is given in Section 5.

2 THE SAMPLE

Soszyński et al. (2009) presented a catalog of 24906 RRLs discovered in the LMC based on OGLE-III observations and classified them into 17693 fundamental-mode, 4958 first-overtone, 986 double-
Fig. 1 Histogram of the number of measurements in the $I$ band and the color-magnitude diagram of the OGLE-III RRLs. Upper panel: the blue dashed line represents the data originally from the catalog and the red solid line signifies those with photometric uncertainty smaller than 0.1 mag, which is the threshold when selecting our sample; the inset is the histogram of our sample in which the photometric uncertainty is less than 0.1 mag and the number of measurements is more than 1000. Lower panel: the gray dots and red dots correspond to all the RRLs from OGLE-III and those in our sample respectively; the dashed horizontal line at $\bar{I} = 18$ mag is our criterion for brightness. The arrow stands for the $A_V = 1$ vector.

mode and 1269 suspected second-overtone RRLs. Thanks to their generosity, all of the data have been released. This catalog has three columns recording the observational Julian date, magnitude in the $I$ or $V$ band and error associated with the magnitude. Since there are far fewer measurements in the $V$ band than in the $I$ band, our analysis mainly makes use of the $I$-band data. With over 20,000 objects, precise analysis of the frequency for all of them is an arduous task. Fortunately, the statistical properties can be ascertained by a much smaller sample. Thus, we concentrate our study on a sample of RRLs that was measured as many times as possible with high precision.

As the amplitude of some RRLs is rather small, about 0.1 mag, measurements with assigned photometric error bigger than 0.1 mag are dropped. In the released database, the number of measurements is usually not as numerous as those claimed in the OGLE-III web catalog. Most of them have fewer than 400 measurements, as can be seen in Figure 1, which displays the distribution of the number of measurements for all the RRLs. In our sample, only the objects with more than 1000 measurements are kept. To exclude the foreground stars, the criterion $\bar{I} \geq 18$ mag is added. In the $I/V − I$ diagram (Fig. 1), it can be seen that this brightness cutoff constrains the sources in the major RRL area and excludes some sparse sources that seem to be giant or foreground stars. With the limitation of brightness and number of measurements, our sample consists of 655 RRLs.

Through the color-magnitude diagram of the 655 RRLs in comparison with the complete group of all the 24,906 RRL sources identified by Soszyński et al. (2009) in Figure 1 (top panel), we can see that the sample agrees with the majority of RRLs. The omission of the faint sources ($I > 19.5$) is mainly due to our requirement of high photometry quality. At the same time, some red RRLs with $V − I$ bigger than about 0.8 are also not included. The faint and red RRLs may be
caused by extinction as they coincide pretty well with the $A_V=1$ trend (the extinction law is taken from Mathis 1990) in Figure 1. Thus the missed faint and red stars should have intrinsically similar brightness and color like those of the majority, and their absence in our sample will not influence the statistical properties of light variation in RRLSs. This sample of 655 RRLSs has three advantages for performing frequency analysis. Firstly, more than six-hundred stars form a large enough sample to objectively obtain the statistical parameters and to understand the common properties exhibited by RRLSs in the LMC. Secondly, the sample size is reasonable so that we can both analyze the frequency accurately and check individual objects carefully to ensure that every result is as reliable as possible. Thirdly, our sample contains the highest-quality data for RRLSs in the OGLE-III project so that the derived properties of light variability should be highly reliable. As mentioned earlier, all stars in our sample have more than 1000 measurements. The time span is about 4000 d (i.e. close to 11 years), and the interval between two adjacent measurements is mostly shorter than 20 d. Based on the effect of random and uncorrected noise, using the least square fit of a sinusoidal signal, we can roughly estimate the frequency error to be $10^{-7}$, and the amplitude error to be $10^{-3}$ (Montgomery & Odonoghue 1999).

The objects in our sample are listed in Table 1, with the OGLE name, position, number of measurements both in the original OGLE catalog and in our calculation, the magnitude in the $I$ and $V$ bands that are the average over all the measurements, the subtype in terms of light variation, which will be discussed later, and the MACHO ID if available. It should be noticed that the $I$ magnitudes in the following tables are the average value of the Fourier fitting.

### 3 FREQUENCY ANALYSIS

Most studies of the RRLS frequencies (e.g. Kolenberg et al. 2010) use the Period04 software that analyzes the Fourier spectrum of the observed light curve. The advantage of Period04 is its high precision and intuitive power spectrum with both the frequency and amplitude shown, which makes it appropriate for analyzing multi-period light curves. However, the Fourier analysis fits a sinusoidal signal, but for RRLSs in the fundamental mode, the light curves are very asymmetric. Besides, RRLSs often have several harmonics, so that more than one period would be found for a single period RRLS. An alternative method for determining the frequency of light variation is the PDM (Stellingwerf 1978), which is independent of the shape of the light curve or an irregular distribution of measurements in the time domain. Although the PDM also brings about a strong signal for the harmonics, this can be overlooked in the folded phase curve, which needs a careful visual inspection. The manageable volume of our sample makes it possible to check the individual phase curves. Moreover, the result from the PDM provides a mutual check between the two common methods used in the study of variable stars.
The PDM looks for the best period of light variation in a range of trial periods by fixing the period of the minimum phase dispersion for the folded phase curve. The phase dispersion \( \Theta_{\text{PDM}} \) is defined as the ratio between the summed phase dispersion in all the phase bins to the phase dispersion of all the measurements. A perfect periodic light curve would produce \( \Theta_{\text{PDM}} = 0 \). The period of light variation of the RRLS sample is calculated in two steps. In the first step, the frequency range is set to the whole range for RRLS variation, i.e. from 1 c/d to 5 c/d, with a step size of \( 10^{-5} \) and 50 bins in the phase space \([0, 1.0]\). The PDM analysis yields the first estimation of frequency \( f_{\text{PDM}} \) at the minimum \( \Theta_{\text{PDM}} \). With this frequency, the folded phase light curve is plotted and visually inspected to exclude harmonics, often the double or triple, which result in a crude approximation of the main frequency, \( f_{\text{est}} = f_{\text{PDM}} \ast n \) (\( n = 1, 2, 3 \ldots \) according to the order of the harmonics in the phased light curves). In the second step, the frequency resolution is increased to \( 10^{-7} \) and the frequency range is shrunk to \([f_{\text{est}}-0.05, f_{\text{est}}+0.05]\]. Then an analysis using the PDM is performed once more to yield the minimum \( \Theta_{\text{PDM}} \) that calculates the frequency with higher accuracy. Thanks to the long time span and numerous measurements of RRLSs in the sample, such high precision in determining the frequency is achievable. This frequency is called the main pulsation frequency, \( f_0 \) or corresponding period \( P_0 \) in the following text, to distinguish among cases like the fundamental, first overtone and second overtone modes.

Once the main frequency is determined, the light curve is fitted by a linear superposition of its harmonic oscillations

\[
M(t) = M_0 + \sum_{k=1}^{n} (a_k \sin k\omega t + b_k \cos k\omega t),
\]

where \( n \) is the highest degree of the harmonics, \( M(t) \) the measured magnitude in the \( I \) or \( V \) band, \( M_0 \) the mean magnitude and \( \omega = 2\pi/P_0 \) the angular frequency. In fact, it is the phased light curve instead of the light curve itself that is fitted for the sake of higher significance

\[
M(t) = M_0 + \sum_{k=1}^{n} (a_k \sin 2\pi k\Phi t + b_k \cos 2\pi k\Phi t),
\]

where \( \Phi = (t - t_0)/P - |(t - t_0)/P| \), \( t \) is the time of observation and \( t_0 \) is the epoch of maximum brightness. Equation (2) can be re-written as

\[
M(t) = M_0 + \sum_{k=1}^{n} A_k \sin (2\pi k\Phi t + \phi_k),
\]

where \( A_k = \sqrt{a_k^2 + b_k^2} \) and \( \phi_k = \arctan(b_k/a_k) \). The parameters \( A_k \) and \( \phi_k \) can be transformed to the Fourier parameters \( R_{ij} = A_j/A_i \) and \( \phi_{ij} = j\phi_i - i\phi_j \) (\( i \) and \( j \) refer to different \( k \), Simon & Lee 1981), both of which are widely used in expressing the features of the light curves, and even to derive the physical parameters of the variables such as the metallicity (e.g. Jurcsik & Kovacs 1996).

In principle, the degree of the harmonics can be arbitrarily high, however, the highest degree in practice is set to 5 or 6, which is able to reflect the essential shape of the light curve. Among all the 655 stars, only 64 stars need the sixth harmonic and all the others have \( n \leq 5 \). Moreover, it avoids overfitting, even for very complex light curves such as that shown in Figure 2. The main pulsation frequency we derived by this method is almost the same as that of Soszyński et al. (2009), with a difference of \( \leq 0.0001 \). The upper four panels in Figure 2 show the process of determining the primary frequency, from the estimation of the frequency, through the accurate measurement of the frequency and the folded phase curve to the final fitting of the phase curve.

The secondary period is searched for in the residual after subtracting the variation in the main frequency with its harmonics. The method is the same as that for the principle period. The difference lies in the amplitude of the secondary oscillation being much smaller than the principle one. This
procedure is followed repeatedly until an assigned threshold is reached. This threshold is set to guarantee the significance of the derived period. The phase dispersion parameter Θ is related to the significance of the period; the smaller the better. Another related parameter is the probability $P_\Theta \left( F(P_\Theta/2, N - 1, \sum(n_i) - M) = 1/\Theta \right)$ (Stellingwerf 1978). However, Θ and $P_\Theta$ depends on the number and quality of measurements so they do not necessarily have the same cut-off value in different cases and become ambiguous in marginal cases. We developed an independent parameter, $S/N \equiv \frac{\Theta_{\text{min}}}{\sigma_\Theta}$, which reflects the S/N of the minimum Θ in the Θ distribution. Combining all the three parameters, the specific thresholds are set to $\Theta_{\text{PDM}} < 0.63 - 0.85$, $P_\Theta < 0.01 - 0.05$ and $S/N > 10 - 15$ for a reliable determination of period. Once the period is determined, a fitting is performed for the principle period, but the highest degree of the harmonics is taken to be three instead of six like in the first run. The four lower panels in Figure 2 show the procedure for determining a
secondary period, including the first approximation and the final determination of the frequency, as well as the fitting of the phased light curve.

4 CLASSIFICATION

RRLs are basically considered as pure radial pulsators. According to the pulsation modes, they are classically divided into four types: RRd that pulsates in the fundamental (FU) mode, RRc in the first overtone (FO) mode, RRd in the second overtone mode (SO) and RRd in the double (FU and FO) modes. Alcock et al. (2000) introduced a new system of notation that uses digits to mark the primary pulsation mode to replace the letters, i.e. RR0, RR1, RR2 and RR01 instead of RRab, RRc, RRd and RRd, which is more intuitive with regard to mnemonics. When the modulation of period and amplitude is considered, Nagy & Kovács (2006) adopted additional letters to classify RRLs, where PC stands for period change, BL for the Blazhko effect and MC for closely spaced multiple frequency components. To more unambiguously express the variation type, we combine both notations into a more detailed designation to denote the phenomenological classification of RRLs by following Nagy & Kovács (2006).

The identification of the main pulsation mode is clear for separating the RR0 and RR1 classes, as demonstrated in many previous studies. The period is longer and the amplitude is mostly larger in RR0 RRLs than in RR1, and the shape of the light curves of RR0-type RRLs is more asymmetric than that of the RR1-type, as shown in the period-amplitude and period-skewness diagrams in Figure 3, where the skewness is calculated from the phased light curve. The definition of skewness is $E[(x - \mu)^3]/\sigma^3$, where $x$ is the observed magnitude, $\mu$ the mean of $x$, $\sigma$ the standard deviation of $x$, and $E(t)$ represents the expected value of the quantity $t$. The gap between RR0 and RR1 is also clearly shown in the plot of period versus Fourier coefficients in Figure 4. Examples of phased light curves in our sample are shown in Figure 5.

The puzzle arises from the identification of RR2-type RRLs – those pulsating in the second-overtone mode. The RR2 stars are believed to have an even shorter period, a slightly smaller amplitude, and a more symmetric light curve. In the period-amplitude diagram, they should be located to the left of RR1 stars, e.g. the magenta points in figure 2 of Soszyński et al. (2009). In the period-amplitude diagram of our sample (Fig. 3, top), no clear gap is found in the group of stars with a shorter period. There is also no apparent peak in the period distribution of all the single-mode RRLs shown in Figure 6, which is very similar to that of all the RRLs in the LMC (Soszyński et al. 2009). Concerning the shape of the light curves, their skewness is calculated. As is discernable in the bottom panel of Figure 3, the separation between RR0 and RR1 is again apparent with RR1 being systematically more symmetric, but no further subgroups can be distinguished in the relatively symmetric cloud of points. Thus, if an RR2 group is to be assigned, the borderline would be very arbitrary both in the period-amplitude and period-skewness diagrams. Because there are no systematic features, we focus on identifying a group of RR2 stars. In fact, it is also possible that the stars with a shorter period and smaller amplitude in their sinusoidal light curve may be metal-rich RR1 cases (Bono et al. 1997a).

4.1 Single Period RRLs

The notation “RR-SG” refers to the single period RRL, i.e. no additional frequency is found in the residual after removing the primary frequency and its harmonics. In our sample, 556 stars were found to be RR-SG stars – 84.9% of the sample of all 655 RRLs. Out of these RR-SG stars, 424 (76%) are RR0-SG in the FU mode and 132 (24%) are RR1-SG RRLs in the FO mode. There are three times as many RR0-SG RRLs as RR1-SG RRLs.

In Table A.1 we list their period of light variation, minimum phase dispersion $\Theta$, amplitude and mean magnitude in the $I$ band, as well as subtype. The histogram of the RRL periods (Fig. 6)
Fig. 3 The period-amplitude and period-skewness diagrams for all RRLSs in our sample. The definition of skewness is given in the text. The meanings of the symbols are shown in the legend where $P_{1st}$ and $P_{2nd}$ mean the primary and secondary period respectively in double-mode RRLSs, and the meaning of specific classifications is described in Sect. 4.

Fig. 4 Period versus Fourier coefficients. The symbols obey the same legend as in Fig. 3. The Fourier coefficients are defined in Eq. (3) with $\phi$ divided by $2\pi$. 
displays two classical prominent peaks at 0.58 d and 0.34 d for RR0 and RR1 stars, respectively. The average period of our sample for RR0 stars $\bar{P}_{RR0-SG}$ is 0.587 d and that for RR1 stars $\bar{P}_{RR1-SG}$ is 0.328 d.

We compare our division of the subtype into RR0 and RR1 with that of Soszyński et al. (2009), without considering RR2 stars. One star (ID: 303, OGLE ID: OGLE-LMC-RRLYR-14697) is found to be discrepant, which is classified as RR1 in our work and RR0 in their work.

Figure 3 is the main criterion for classifying this star, which is specially denoted by a pentagon. In both the period-amplitude and the period-skewness diagrams, this star is located in the central area of RR1 stars. When compared with the RR0 and RR1 stars in the phased light curve (Fig. 5), its shape is apparently asymmetric, but not in the same way as RR0 stars (its slope is not as steep as that for RR0 stars). On the other hand, its small amplitude and short period cause it to be regarded as a member of the RR1 group. This example also indicates that the skewness of the light curve of one class covers a wide range, and makes the separation of RR2 stars difficult.

The distribution of RR-SG in Figure 3 is composed of two typical parts: one that forms a sequence of RR0-SG and the other that has a bell shape for RR1-SG. The distribution of RR0-SG also appears to be composed of two parts: one which is densely clumped on the left to form a linear shape,
Fig. 6 Period distribution of all the single-mode RR-SG stars, with a blue dashed line for RR0-SG, a green dashed line for RR1-SG and a gray solid line for the sum of RR0-SG and RR1-SG RRLSs.

Fig. 7 Period-amplitude (I-band and V-band) diagram of RR0-SG stars. Contours show the distribution of the bulge RR0-SG stars from [a]: Collinge et al. (2006) and [b]: Szczygieł et al. (2009) as well as [c] those from the LMC RR0-SG stars in this work. The two solid lines in the V-band diagram are from Szczygieł et al. (2009), showing the fitting of the Bailey diagram for the Oo I and II RRLSs respectively, while the dashed line is the border separating the Oo I type and Oo II type RRLSs in the Galactic RR0 stars.

and the other which is loosely distributed on the right. This distributional shape is reminiscent of the dichotomy into Oosterhoff (Oo) I and II classes of Galactic RRLSs due to different evolving phases. Such dichotomy was found in the Galactic fundamental-mode RRLs (Szczygieł et al. 2009).

Figure 7 compares the Bailey diagram of RR0-SG stars in the LMC with that in the Galactic bulge (Collinge et al. 2006) and the Galactic field (Szczygieł et al. 2009), where the contours represent the density of RR0-SG stars, the solid and dashed lines are from equations (2), (3), (4) and
(5) of Szczygiel et al. (2009) for their fitting to the Oo I (left), Oo II (right) groups and their border-
lines, respectively, in the Galactic field RRLSs. It can be seen that the RR0 stars in the LMC can be
divided into the Oo I and Oo II groups as well as into the Galactic field and bulge. Although the two
groups are not fitted, the solid and dashed lines from Szczygiel et al. (2009) generally agree with the
distribution. Moreover, the Oo I group’s RR0 stars are clearly dominant over the Oo II stars. This is
consistent with the fact that the average period \( \bar{P}_{\text{RR0-SG}} = 0.587 \text{ d} \) is more appropriate for 0.549 d
being the average of Oo I clusters than for 0.647 d being the average of Oo II clusters (Clement &
Rowe 2000; van Agt & Oosterhoff 1959). According to the contour diagram, the Oo I group of RR0
in LMC stars has a slightly longer period (\( \Delta \log P \sim 0.02 \)) than those in the bulge. If the appearance
of the bulge and field in the contour diagram is taken into account, the Oo I distribution forms a
series from the bulge to the field and then to the LMC from left to right, i.e. the period from short
to long. This sequence coincides with that of the average metallicity in these three environments.
Since metallicity influences the opacity and the mechanism for the light variation of an RRLS is the
\( \kappa \) mechanism, the shift of the Oo I group may be caused by the difference in metallicity. Indeed,
metal abundance also influences the distribution of Oo I and Oo II RR0 stars in the Galactic field
(Szczygiel et al. 2009).

4.2 Multiple Period RRLSs

There are 99 (15.1\%) RRLSs with variation detected in the residual of the light curve after remov-
ing the principle frequency and its harmonics. They are further classified into several subclasses
according to the number of additional frequencies and their locations relative to the main frequency,
including RR01, RR-BL, RR-MC, RR-PC and miscellaneous subtypes.

4.2.1 RR01 stars

RR01 refers to the RRLSs pulsating in double radial modes; one is the FU mode and the other
is the FO mode, which are classical RRd stars. In our sample, 15 (2.3\% of the sample) stars are
classified as RR01 stars. Seven of them have two detected frequencies. The other eight stars have
three detected frequencies, but the third frequency is either the sum or the difference of the first and
second frequency and is thus dependent.

All these 15 stars are listed in Table A.2, with the period and amplitude in the FU mode denoted
by \( P_0 \) and \( A_0 \) respectively, the minimum phase dispersion \( \Theta_0 \) used in deriving the FU mode, the
mean magnitude in the \( I \) band, the period and amplitude ratios between FO and FU modes and the
minimum phase dispersion \( \Theta_1 \) used in deriving the FO frequency.

In the period-magnitude diagram (Fig. 3, top), both the FU and FO periods of these double mode
RRLSs are shown by black open triangles. The periods are either located in the region representing
the long side of the single FO mode or that of the short side of the FU RRLSs. Their amplitudes
are small, all smaller than 0.3 mag, which is comparable to that of the RR1 stars. The dominance of
the FO mode in RR01 can explain this small amplitude. It can be seen that the amplitude of the FU
mode of RR01 stars is much smaller than those single-mode FU stars at corresponding short periods,
but comparable to those single-mode FO stars. In fact, RR01 stars distinguish themselves from the
other single-mode FU stars by their small amplitude in the period-amplitude diagram. Meanwhile,
their light curves appear different because of a positive skewness from the FO RRLSs, most of which
have a negative skewness although both are more symmetric than the FU RRLSs. The lower panel
of Figure 3 shows such a difference.

The period ratio \( P_{1}/P_{0} \) ranges from 0.7422 to 0.7465, with an average of 0.7436. The amplitude
ratio \( A_{1}/A_{0} \) is significantly larger than one in 13 stars, but two stars have ratio values smaller than
one (but larger than 0.8), with an average of 1.533. It can be concluded that the RR01 stars mainly
pulsate in the FO mode. In the Petersen diagram for RRLSs (Fig. 8), the period and amplitude ratios
Fig. 8 Petersen’s diagram for the RR01 stars (upper) and the amplitude ratio (lower) versus \( \lg P \) in the LMC. The solid symbols denote our results while the hollow ones represent the LMC data from the MACHO project (Alcock et al. 2000).

are plotted versus the FU period, and compared with the results of Alcock et al. (2000) from the MACHO data. The increase of both ratios with the period is clear and consistent with Alcock et al. (2000). The distribution of the period ratio overlaps completely with that of Alcock et al. (2000). The ratio of the amplitude does not rise as high as the Alcock et al. (2000) result, although the rising trend with period is the same. Moreover, the RR01 stars whose dominant mode is fundamental (i.e. \( A_1/A_0 < 1 \)) have smaller \( P_1/P_0 \) than the average, specifically, their \( P_1/P_0 \) values are all smaller than 0.743, even when including those from Alcock et al. (2000).

4.2.2 RR-BL stars

RR-BL stars refer to the Blazhko stars. As mentioned in the Introduction, the early identification of RR-BL stars was the symmetric appearance of the power spectrum. With developments in the study of the Balzhko effect, some asymmetric patterns are considered to be variations in it. Thus, in the frequency pattern, they may appear as: (1) two frequencies with one closely spaced frequency around the principle frequency (RR-BL1), (2) three frequencies with two side frequencies closely and symmetrically distributed around the main frequency (RR-BL2), and (3) more than three frequencies with multiple components at closely spaced frequencies (RR-MC). All of them are a consequence of the modulation of the amplitude and/or phase, which can be explained by the modulation of a single (sinusoidal or non-sinusoidal) oscillation (Szeidl & Jurcsik 2009; Benkő et al. 2011).
RR-BL1 stars

RR-BL1 stars have one frequency close to the main frequency, which can be bigger or smaller. Alcock et al. (2000) marked them as $\nu 1$, and here we use the definition of Nagy & Kovács (2006) to mark them as BL1. There are 41 (6.3% of all the 655 stars in the sample) such RR-BL1 stars, forming a much larger group than other multi-period RRLs. Taking into account the main pulsation mode, they are classified into 32 RR0-BL1 stars in the FU mode and 9 RR1-BL1 stars in the FO mode. The main pulsation period of RR0-BL1 stars ranges from 0.38 d to 0.76 d and that of RR1-BL1 from 0.26 d to 0.37 d. The main pulsation amplitude of RR0-BL1 stars ranges from 0.106 mag to 0.747 mag and that of RR1-BL1 from 0.073 mag to 0.352 mag. In Figures 3 and 4, these RR-BL1 stars are homogeneously mixed with other single-mode RRLs, which means they have a normal main period and amplitude of light variation like those single-mode stars.

The sum of all the differences between every side frequency and main frequency is defined as $\delta f = \sum_i \Delta f_i$, where $\Delta f_i = f_i - f_0$ and $i = 1, 2...$ represents the frequency at the first overtone, second overtone and so on. This expression is usually used to characterize the asymmetry of the frequency distribution. For RR-BL1 stars, there is only one side frequency, resulting in $\delta f = f_1 - f_0$. About 75% (24 out of 32) of these RR0-BL1 stars and 67% (6 out of 9) of these RR1-BL1 stars have a positive $\delta f$. This is very different from the fraction of 37% for RR1-BL1 stars derived from the MACHO data by Alcock et al. (2000). However, it agrees well with the percentage of 80% for RR0-BL1 stars from the study of Blazhko variables by Kovács (2002).

For RR0-BL1 stars, the Blazhko periods vary from about 23 d to over 1500 d with an average of about 180 d and rms of 348 d. For RR1-BL1 stars, the Blazhko periods vary from 6.4 d to about 3000 d with an average of 745 d and rms of 1404 d. The shortest modulation period (6.4 d) is comparable to that found for RR1-BL in LMC by Nagy & Kovács (2006) and is also consistent with those in the Galactic field for RR0-BL stars (Jurcsik et al. 2005b), i.e. around 6 d, which is about 20 times the main pulsation period. At the other end of the scale, the longest modulation period of $\sim$ 3000 d is comparable to the time span of the data that are available; it is at least partly limited by the observational time coverage. With the continuation of the OGLE project, a longer modulation period can be expected. The distribution of the Blazhko periods of RR-BL1 stars is shown in Figure 9. The RR0-BL1 stars exhibit a normal distribution with the peak around 60–70 d. The RR1-BL1 stars show a rather scattered distribution over a much wider range; although the group of only nine RR1-BL1 objects makes this statistical significance less convincing. It seems that there is a preferred range of Blazhko periods for RR0-BL1 from 32 d to 200 d ($\lg P$ from 1.5 to 2.3), but there is no such preferred range for RR1-BL1, which agrees well with the result of Nagy & Kovács (2006).

We list the main frequency and other variation parameters of RR-BL1 stars in Table A.3. The amplitude ratio ($A_1/A_0$) of RR0-BL1 varies from 0.119 to 0.382 with an average of 0.237 and that of RR1-BL1 varies from 0.324 to 1.081 with an average of 0.574. RR1-BL1 have a larger amplitude ratio than RR0-BL1, with one star (ID: 126) being even larger than one but its main amplitude is small, only 0.087 mag.

The asymmetric frequency can be regarded as the extreme case of the amplitude asymmetry in the Blazhko effect when the invisible symmetric component is completely submersed in noise. In Figure 10 there is an example of such a situation (star ID: 78, OGLE ID: OGLE-LMC-RRLYR-09295). The two upper figures are the frequency-ThetaPDM diagram and the phased light curves of the first-loop period while searching for the main frequency of 1.7927. The lower left figure shows the results of searching for the secondary frequency at 1.8108, and the lower right figure shows that no more reliable frequency values can be derived since all three parameters at $f=1.7746$ ($\Theta_{PDM}=0.86$, $P_0=0.015$ and S/N=8.7) are below our threshold. On the other hand, it may be expected that this frequency would be detected given a higher sensitivity of observation or a lower threshold. This example further supports that the absence of another frequency component in RR1 stars is caused by the asymmetry of the modulated amplitude.
Fig. 9 One example (Star 78) of the BL1 star shows asymmetric frequency which is the result of extreme amplitude asymmetry in the Blazhko effect when the invisible component is completely submerged in the noise, where the legend is the same as in Fig. 2. Notice that the Θ-Frequency diagram is different from the power spectrum, and the “triplets” shown in the first figure are only the main frequency and two aliases.

RR-BL2 Stars

For RR-BL2 stars, the secondary frequencies indicate the modulation of the amplitude and phase. There are 11 (1.7%) RR-BL2 stars and they can be divided into two subtypes, four RR0-BL2 and seven RR1-BL2 based on the main pulsation mode. This percentage (1.7%) is much smaller than that of the RR-BL1 stars (6.3%). It is also low in comparison with the results of previous studies, which will be discussed later. We think such low percentage is mainly due to the very strict criteria we used for identifying a frequency, which meant that some third frequencies were dropped, like the case of Star 78 shown in Figure 9. This percentage of 1.7% should be taken as the lower limit of the percentage of RR-BL2 stars.

The differences between the side and main frequencies are shown in Table A.4, i.e. $\Delta f_+$ and $\Delta f_-$. They are both smaller than 0.1. In addition, the differences between $\Delta f_+$ and $\Delta f_-$ are all smaller than 0.003. It is these two features that bring them into the RR-BL2 class. Regarding the ratio of the two amplitudes, $A_+$ and $A_-$ ($A_+/A_-$), it changes from 0.76 to 1.60. This range of ratio means the two components have pulsation amplitudes at the same order, or we are only sensitive to such a range of values. This is understandable since a large ratio of the two amplitudes would surely make the weak component invisible and move the star into the RR-BL1 group. Such a bias can only be alleviated by a highly sensitive observation. This fact can also account for the low percentage of BL2 stars.
As shown in Figure 3 by the asterisks, the RR-BL2 stars have ordinary periods and amplitudes in the principle pulsation mode. The amplitude ranges from 0.283 mag to 0.567 mag and the period from 0.465 d to 0.647 d, with an average of $A_0 = 0.44$ mag and $P_0 = 0.58$ d for RR0-BL2. For RR1-BL2 stars, the amplitude ranges from 0.158 mag to 0.265 mag and the period from 0.270 d to 0.489 d, with an average $A_0 = 0.21$ mag and $P_0 = 0.33$ d.

According to $\Delta f_+$ and $\Delta f_-$, the modulation period varies from about 43 to over 2700 d with an average of 1349 d for RR0-BL2; and from 12 to 2902 d with an average of 1288.5 d for RR1-BL2. The distribution of the modulation frequencies is shown in Figure 10. Because the volume of the RR-BL2 stars is small, the distribution does not exhibit any outstanding features. However, the situation becomes clearer when the RR-BL1 stars are included, which is reasonable since BL1 stars can be regarded as the extreme case of RR2 and both are Blazhko variables. Consequently, our sample of 655 RRLs contains 52 Blazhko stars.

In Figure 10, the period distribution of all the RR0-BL and RR1-BL stars is shown. Because of the dominance of RR-BL1 stars, the distribution of RR-BL stars is similar to that of RR-BL1 stars, i.e. with a preferred range of period from a few tens to a couple of hundred days. In regards to the modulation amplitude, a correlation is found with the main pulsation amplitude. As shown in Figure 10 (bottom), a linear fitting results in $A_i = 0.106 \times A_0 + 0.057$ and the correlation coefficient is 0.605, which means there is a significant correlation. The error we adopted here is the maximum of the photometric error assigned in the catalog which is apparently bigger than the error in the fitting. The order of the error is mostly around 0.1 mag. This correlation was neither found before nor predicted in any models for the Blazhko effect. It indicates that the Blazhko modulation is related to the main pulsation mode and it should be taken into account in models. On the contrary, Jurcsik et al. (2005a) found that the largest possible value of the modulation amplitude, defined as the sum of the Fourier amplitudes for the first four modulation frequency components, increases towards shorter period variables.
RR-MC stars

We have four RR-MC stars in our sample. All of them have three side frequency components. One RR-MC star is in the FU mode and the other three are in the FO mode. According to the structure of the side frequencies, they show three patterns, similar to the RR-MC stars described in Nagy & Kovács (2006): (a) two of the three frequencies are symmetric with respect to \( f_0 \); (b) none is symmetric with respect to the others, but three frequencies are on both sides of \( f_0 \); (c) three side frequencies are all on one side of \( f_0 \). In Table A.5, the four RR-MC stars are shown with their patterns in the column “Notes.” These stars with multiplets may also be Blazhko stars (Benkő et al. 2011).

4.2.3 RR-PC stars

RR-PC refers to period-changing stars. It is difficult to distinguish PC stars from MC stars or BL stars since they all have closely spaced side frequencies. Nagy & Kovács (2006) defined the PC stars as those that have close components which cannot be eliminated within three prewhitening cycles or their separation from the main pulsation component is \( \leq \sim 1/T \), where \( T \) is total time span. This definition is followed in our work. In our sample, we find that no star has any frequency detected after four prewhitening loops, i.e. the number of frequencies is not larger than four. The two stars which have a fourth frequency component both have at least one frequency with separation from the main pulsation component being \( \leq \sim 1/T \). Following the definition, they are classified as RR-PC stars. In addition, there are some RRRLs that have fewer than four frequency components but with some frequency whose separation from the main component is \( \leq \sim 1/T \); the number of such stars is 18. Altogether, there are 20 RR-PC stars, which is about 3% of the sample. Their light variation properties are listed in Table A.6. Our attempt to analyze the variation in period is hampered by the large interval between adjacent measurements which ranges from 0.003 d to 300 d with a mean value of about 3.6 d, that is to say, the interval is several periods long.

4.2.4 Other RRRLs

Eight (1.2%) multi-period RRRLs in our sample cannot be classified into the above subtypes. They have more than one pulsation frequency, but their other frequencies are not close to the main frequency. Table 2 shows their frequencies and other variation parameters.

Four of them have one period around 0.3 d and the other around 0.5 d, yielding a period ratio around 0.6 which is the canonical period ratio between the SO and FU modes (Bono et al. 1997b). So we suspect that although we cannot find single SO mode pulsating RRRLs, they can co-exist with the FU mode. The four RR02 candidates are shown by red triangles in Figure 3 where \( P_{\text{1st}} \) and \( P_{\text{2nd}} \) refer to the primary and secondary periods, respectively. Like the RR01 stars, the interaction between the two modes has led to their amplitude and period being on the short/long end of the FU/SO mode, which brings them together in the period-amplitude diagram (Fig. 3). Moreover, Star

| ID  | \( P_0 \)  | \( P_1 \)  | \( P_2 \)  | \( A_0 \)  | \( i \)  | \( P_{\text{short}}/P_{\text{long}} \) | \( A_{\text{short}}P/A_{\text{long}}P \) | Type   |
|-----|------------|------------|------------|------------|-------|-----------------|-----------------|-------|
| 21  | 0.601141   | 0.997709   | 0.466      | 19.032     | 0.6025 | 3.399            |                  | RR0-D1 |
| 34  | 0.379702   | 0.613512   | 0.109      | 18.667     | 0.6189 | 1.946            |                  | RR02  |
| 159 | 0.576922   | 0.332432   | 0.536      | 18.733     | 0.5762 | 0.455            |                  | RR02  |
| 181 | 0.546876   | 0.815054   | 0.409      | 18.191     | 0.6931 | 4.964            |                  | RR02  |
| 228 | 0.304926   | 0.499391   | 0.49175    | 0.231      | 18.599 | 0.6106           | 1.712            | RR02  |
| 248 | 0.599241   | 0.464902   | 0.413      | 18.806     | 0.7758 | 0.147            |                  |       |
| 452 | 0.529150   | 0.997639   | 0.445      | 18.483     | 0.5304 | 2.092            |                  | RR0-D1 |
| 535 | 0.298121   | 0.499435   | 0.248      | 18.655     | 0.5969 | 2.879            |                  | RR02  |
Fig. 11 The RR02 candidate stars in the Petersen diagram. The black solid lines delineate the rough range of RR02 stars by models from Poretti et al. (2010), and the red dashed line that from Nemec et al. (2011). The blue, green, black and red dots show the candidate RR02 stars from the MACHO, CoRoT and Kepler datasets as well as our results from the OGLE-III datasets respectively, with the name or ID number labeled (see Section 6.4 for details).

228 seems to have a long modulation period for its FU mode, from the fact that a third frequency is found to be close to its FU frequency.

Only four stars have been clearly claimed to be RR02 stars in the past two years. All of them were discovered in a space mission. They are V350 Lyr (Benkő et al. 2010) and KIC 7021124 (Nemec et al. 2011) from the Kepler mission, and CoRoT 101128793 (Poretti et al. 2010) and V1127 Aql (Chadid et al. 2010) from the CoRoT mission. We found that star MACHO 18.2717.787, unidentified by Nagy & Kovács (2006) from the MACHO dataset, could also be such a double-mode RR02 star with the period ratio of 0.5810. Poretti et al. (2010) computed a grid of linear RRLS models in a large stellar parameter space which delineated a rough range of the RR02 stars in the Petersen diagram. A similar work was presented by Nemec et al. (2011) who used the Warsaw pulsation hydrocode including turbulent convection.

In Figure 11, the range of RR02 in the Petersen diagram defined by the two models is delimited by solid and dashed lines, respectively. It can be seen that the two models generally agree with each other but also disagree at the short fundamental periods in particular. In this diagram, Star MACHO 18.2717.787, denoted by a blue dot, is definitely inside the model range. The four RR02 candidates from our sample are shown by red dots with the other five such stars denoted by dots in other colors. Stars 159 and 535 are undoubtedly inside the range defined by both models. Star 228 is just outside
the upper border of the models, but cannot be excluded since the models have some uncertainty. The only star which apparently deviates from the models is Star 34 although it is not too far. Puzzlingly, all nine stars follow a trend that the period ratio increases with the FU period, which is opposite to what is expected from the models. Interestingly, such a discrepancy between observation and model occurs in exactly the same way in RR01, the other double-mode star (see fig. 5 of Alcock et al. 2000).

Two stars have a secondary period of around 1 d. This frequency is not taken as the alias because the phased light curves at this frequency show reliable periodicity. We hereby denote them as RR0-D1, following Alcock et al. (2000). The other three stars cannot be classified into any of the classes described above. We just mark them with “?” in Table 2.

5 DISCUSSION AND SUMMARY

The incidence rates of each subclass are shown in Table 3. The majority, 85%, represents single-mode pulsators. The RRLs exhibiting the Blazhko effect (sum of RR-BL1 and RR-BL2) are the second most numerous group. With 52 RR-BL stars, they are 7.9% of the sample. The incidence rates of Blazhko stars are compared with previous results in Table 4. As our identification of a frequency is quite strict, the percentages in our sample should be taken as the lower limit of the Blazhko incidence rate.

For RRLs in the LMC, the Blazhko variables (sum of RR-BL2 and RR-BL1 stars) occur less frequently in RR0 (7.5%) than in RR1 stars (9.1%). For RR1 stars, we can see an increasing trend of the incidence rates from 2.0% and 7.5% in previous work to 9.1% in the present work. This can be explained by the longer time span and more precise data. But this trend does not appear in RR0 stars although the data have been improved for RR1 stars. To further analyze the reason, it is found that the incidence rate of RR-BL1 is comparable to previous work: the rate is 6.7% for RR0-BL1 and 5.1% for RR1-BL1, compared to 6.5% (Alcock et al. 2003) and 3.5% (Nagy & Kovács 2006) respectively. However, regarding the RR-BL2 stars, the incidence rate of RR1-BL2 is comparable to previous work, with 5.1% to 3.5%. Meanwhile, the incidence rate of RR0-BL2 is abnormally low, with only 0.8% compared to 5.4% in Alcock et al. (2003). As mentioned in the previous section, the reason may lie in our very strict criteria for accepting a frequency, which can change the classification of a BL2 star into a BL1 star in the case of high asymmetry of the amplitude for the two side frequencies. This explanation finds support in the fact that the RR0-BL2 stars have the amplitude ratio \( A_+ / A_- \) not far from unity. Another possible reason is that, among RR0-BL1 stars, 75% (24 out of 32) have \( f_+ \), and all the four RR0-BL2 stars have \( A_+ > A_- \). In the work of Kovács (2002), 80% of RR0-BL1 stars were found to have an \( A_+ \) component. We suspect that for RR0-BL stars, there may be some unknown effect that makes \( A_+ \) much larger than \( A_- \), causing a lot of \( A_- \) components to be missed. This effect does not appear in RR1 stars, for example, Alcock et al. (2000) found 37% of RR1-BL1 stars have \( A_- \) components, and in our sample only 43% of RR1-BL2 stars have \( A_+ > A_- \). Based on equation (45) from Benkő et al. (2011), most of the RR0 stars have \( \pi < \phi_m < 2\pi \); but for those RR1 stars, \( \phi_m \) is evenly distributed between zero and \( 2\pi \).

People used to believe that the incident rates of the Blazhko variables are lower in the LMC than in the Galaxy. However, this is only true for RR0 stars and may not be true for RR1, as the work of Nagy & Kovács (2006) has already suggested. From our work, with the long time span of observation, the Blazhko incidence rate for RR1 stars is larger in the LMC than in the Galactic bulge, as the rates are 5.1% and 4.0% for RR1-BL1 and RR1-BL2 respectively in our LMC sample in comparison with 3.1% and 1.5% from Moskalik & Poretti (2003) or 2.9% and 3.9% from Mizerski (2003) for the bulge RRLS sample. On the other hand, both the improvement of the observational precision and the extension of observational span increase the possibility of detecting the Blazhko effect. Based on the data from the Kepler mission, Kolenberg et al. (2010) and the Konkoly Blazhko Survey (Jurcsik et al. 2009), the incidence rate can exceed 40%, but they are small samples with no more than 30 ob-
Table 3  Statistical Results of the RRLS Classification

| Type    | Short description | Number | Percent | Subtype     | Number |
|---------|-------------------|--------|---------|-------------|--------|
| RR-SG   | Single-period     | 556    | 84.9%   | RR0-SG      | 424    |
|         |                   |        |         | RR1-SG      | 132    |
| RR01    | FU/FO double mode | 15     | 2.3%    |             |        |
| RR02    | FU/SO double mode | 4      | 0.6%    |             |        |
| RR-BL   | One close component | 52 | 7.9%    | RR0-BL1     | 32     |
|         |                   |        |         | RR1-BL1     | 9      |
|         | Two symmetric components | |        | RR0-BL2    | 4      |
|         |                   |        |         | RR1-BL2     | 7      |
| RR-MC   | Multiple close components | 4 | 0.6%    | RR0-MC      | 1      |
|         |                   |        |         | RR1-MC      | 3      |
| RR-PC   | Period changing   | 20     | 3.1%    | RR0-PC      | 11     |
|         |                   |        |         | RR1-PC      | 9      |
| RR-D1   | Second frequency at unity | 2 | 0.3%    | RR0-D1      | 2      |
| RR-?    | Mysterious double mode | 2 | 0.3%    | RR0-others  | 2      |

Table 4  The Incidence Rates of the Blazhko Effect in the Sample Compared to Previous Results

| Ref. | Bulge (a) | Bulge (c) | Bulge (d) | LMC (b) | ω Cen (e) | LMC (h) |
|------|-----------|-----------|-----------|---------|-----------|---------|
| RR0  | 150       | 1942      | 1888      | 6391    | 70        | 478     |
| RR0-BL1 | 16.7%     | 12.5%     | 8.8%      | 6.5%    | 4.3%      | 6.7%    |
| RR0-BL2 | 6.0%      | 7.4%      | 14.9%     | 5.4%    | 18.6%     | 0.8%    |
| RR0-MC | 4.4%      | 3.5%      | 0.3%      | 0.2%    |           | 0.2%    |
| RR0-PC | 6.3%      | 2.9%      |           |         |           | 2.3%    |

| Ref. | Bulge (a) | Bulge (c) | LMC (f) | LMC (g) | ω Cen (e) | LMC (h) |
|------|-----------|-----------|---------|---------|-----------|---------|
| RR1  | 65        | 771       | 1327    | 1332    | 81        | 177     |
| RR1-BL1 | 3.1%      | 2.9%      | 1.8%    | 3.5%    | 21.0%     | 5.1%    |
| RR1-BL2 | 1.5%      | 3.9%      | 0.2%    | 4.0%    | 4.9%      | 4.0%    |
| RR1-MC | 5.3%      | 0.4%      | 1.0%    |         |           | 1.7%    |
| RR1-PC | 10.6%     | 14.0%     |         |         |           | 4.5%    |

Notes: The references in the table are: (a): Moskalik & Poretti (2003), (b): Alcock et al. (2003), (c): Mizerski (2003), (d): Collinge et al. (2006), (e): Moskalik & Olech (2008), (f): Alcock et al. (2000), (g): Nagy & Kovács (2006), (h): this work.

jects. Thus such a comparison may not be conclusive as the observation and the analysis techniques are not uniform, and the samples are very different. According to these observations of the LMC, the SMC, the Galactic bulge and ω Cen, there is no clear relation between the incidence rate and metallicity. What causes the difference in the Blazhko incidence rate in different environments is unclear, which could be part of the difficulty in understanding the mechanism for the Blazhko effect.

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### Table A.1 Parameters of Light Variation for the RR-SG Stars

| ID | Period  | $\Theta_{PDM}$ | $A_0$ | $\bar{I}$ | Subtype   |
|----|---------|----------------|-------|-----------|-----------|
| 1  | 0.611821| 0.1361         | 0.559 | 18.845    | RR0-SG    |
| 2  | 0.408541| 0.1926         | 0.261 | 18.551    | RR1-SG    |
| 4  | 0.555120| 0.0749         | 0.806 | 18.720    | RR0-SG    |
| 5  | 0.497251| 0.0814         | 0.755 | 18.889    | RR0-SG    |
| 6  | 0.511593| 0.1681         | 0.514 | 18.733    | RR0-SG    |
| 8  | 0.551528| 0.0953         | 0.638 | 18.807    | RR0-SG    |
| 9  | 0.515681| 0.1468         | 0.412 | 18.464    | RR0-SG    |
| 10 | 0.545564| 0.1076         | 0.669 | 18.807    | RR0-SG    |

### Table A.2 Parameters of Light Variation for the RR01 Stars

| ID | Type | $P_0$  | $\Theta_0$ | $A_0$  | $\bar{I}$ | $P_1/P_0$ | $\Theta_1$ | $A_1/A_0$ |
|----|------|--------|------------|-------|-----------|-----------|------------|-----------|
| 3  | RR01 | 0.464248| 0.61       | 0.169 | 18.816    | 0.7434    | 0.59       | 1.279     |
| 53 | RR01 | 0.463814| 0.55       | 0.158 | 18.768    | 0.7435    | 0.50       | 1.492     |
| 157| RR01 | 0.465272| 0.57       | 0.169 | 18.890    | 0.7431    | 0.48       | 1.628     |
| 174| RR01 | 0.493743| 0.57       | 0.178 | 18.782    | 0.7446    | 0.45       | 1.525     |
| 249| RR01 | 0.511593| 0.62       | 0.121 | 18.664    | 0.7456    | 0.39       | 2.026     |
| 264| RR01 | 0.461786| 0.51       | 0.235 | 18.886    | 0.7429    | 0.57       | 1.306     |
| 330| RR01 | 0.460572| 0.47       | 0.247 | 18.850    | 0.7427    | 0.59       | 1.073     |
| 332| RR01 | 0.468276| 0.73       | 0.106 | 18.845    | 0.7431    | 0.37       | 2.613     |
| 382| RR01 | 0.481604| 0.84       | 0.128 | 18.692    | 0.7441    | 0.57       | 1.938     |
| 483| RR01 | 0.534026| 0.65       | 0.098 | 18.482    | 0.7465    | 0.26       | 2.608     |
| 528| RR01 | 0.459523| 0.60       | 0.234 | 18.611    | 0.7422    | 0.50       | 0.810     |
| 531| RR01 | 0.465609| 0.65       | 0.146 | 18.669    | 0.7430    | 0.47       | 1.055     |
| 558| RR01 | 0.5817725| 0.52    | 0.164 | 18.773    | 0.7440    | 0.49       | 1.452     |
| 622| RR01 | 0.462159| 0.47       | 0.213 | 18.874    | 0.7431    | 0.55       | 1.363     |
| 642| RR01 | 0.468788| 0.57       | 0.295 | 18.842    | 0.7427    | 0.42       | 0.839     |

### Table A.3 Parameters of Light Variation for the RR-BL1 Stars

| ID | Type | $f_0$  | $\Theta_0$ | $A_0$  | $\bar{I}$ | $\Delta f$ | $A_1/A_0$ |
|----|------|--------|------------|-------|-----------|-----------|-----------|
| 7  | RR0  | 1.545554| 0.50       | 0.106 | 18.056    | 0.010905  | 0.382     |
| 20 | RR0  | 1.592310| 0.15       | 0.747 | 18.597    | 0.020002  | 0.186     |
| 78 | RR0  | 1.792749| 0.27       | 0.515 | 18.975    | 0.018073  | 0.212     |
| 103| RR0  | 1.791820| 0.42       | 0.419 | 18.770    | –0.001242 | 0.381     |
| 125| RR0  | 1.816095| 0.37       | 0.423 | 18.809    | 0.009155  | 0.308     |
| 191| RR0  | 1.639542| 0.27       | 0.443 | 18.598    | –0.000641 | 0.232     |
| 193| RR0  | 2.148450| 0.25       | 0.563 | 18.751    | 0.005300  | 0.204     |
| 217| RR0  | 1.941551| 0.33       | 0.477 | 18.910    | 0.025364  | 0.283     |
| 230| RR0  | 2.056198| 0.24       | 0.594 | 18.768    | 0.022642  | 0.188     |
| 239| RR0  | 1.596092| 0.45       | 0.265 | 18.696    | 0.017372  | 0.313     |
| 253| RR0  | 1.814577| 0.13       | 0.583 | 18.586    | –0.004017 | 0.119     |
| 261| RR0  | 1.971605| 0.24       | 0.530 | 18.886    | 0.007993  | 0.213     |
| 275| RR0  | 1.883136| 0.17       | 0.485 | 18.627    | 0.035635  | 0.138     |
| 280| RR0  | 2.079711| 0.23       | 0.664 | 19.010    | 0.043438  | 0.203     |
| 283| RR0  | 2.627460| 0.31       | 0.491 | 19.285    | 0.016204  | 0.279     |
| 311| RR0  | 1.875675| 0.16       | 0.645 | 18.733    | 0.030124  | 0.217     |
| 313| RR0  | 1.722907| 0.37       | 0.393 | 18.765    | 0.010309  | 0.338     |
| 325| RR0  | 1.849624| 0.27       | 0.543 | 18.882    | 0.010813  | 0.256     |
| 347| RR0  | 1.830031| 0.12       | 0.641 | 18.692    | –0.003638 | 0.119     |
| 377| RR0  | 1.587357| 0.31       | 0.293 | 18.736    | 0.030942  | 0.222     |
| ID  | Type | \(f_0\) | \(\Theta_0\) | \(A_0\) | \(I\) | \(\Delta f_1\) | \(\Delta f_2\) | \(\Delta f_3\) | \(A_1/A_0\) | \(A_2/A_0\) | \(A_3/A_0\) |
|-----|------|-------|--------|-------|----|------|------|------|-------|-------|-------|
| 427 | RR0  | 1.777755 0.25 0.502 18.721 | 0.022702 | 0.284 |
| 444 | RR0  | 1.969073 0.18 0.667 18.862 | 0.002295 | 0.206 |
| 464 | RR0  | 1.717309 0.28 0.377 18.766 | −0.009179 | 0.327 |
| 469 | RR0  | 1.829113 0.25 0.592 18.883 | 0.016978 | 0.272 |
| 517 | RR0  | 1.662463 0.42 0.227 18.537 | 0.026333 | 0.345 |
| 579 | RR0  | 1.560733 0.21 0.493 18.602 | −0.023590 | 0.244 |
| 597 | RR0  | 1.323251 0.37 0.197 18.507 | 0.011120 | 0.254 |
| 624 | RR0  | 1.671056 0.27 0.360 18.391 | −0.007175 | 0.286 |
| 639 | RR0  | 2.189412 0.15 0.619 18.762 | 0.014528 | 0.173 |
| 645 | RR0  | 1.720961 0.25 0.354 18.651 | 0.019355 | 0.200 |
| 652 | RR0  | 1.560733 0.21 0.493 18.602 | −0.023590 | 0.244 |
| 668 | RR0  | 1.323251 0.37 0.197 18.507 | 0.011120 | 0.254 |
| 675 | RR0  | 1.671056 0.27 0.360 18.391 | −0.007175 | 0.286 |
| 689 | RR0  | 2.189412 0.15 0.619 18.762 | 0.014528 | 0.173 |

Table A.4 Parameters of Light Variation for the RR-BL2 Stars

| ID  | Type | \(f_0\) | \(\Theta_0\) | \(A_0\) | \(I\) | \(\Delta f_1\) | \(\Delta f_2\) | \(\Delta f_3\) | \(A_1/A_0\) | \(A_2/A_0\) | \(A_3/A_0\) |
|-----|------|-------|--------|-------|----|------|------|------|-------|-------|-------|
| 165 | RR0  | 2.150372 0.27 0.567 18.753 | 0.022871 | 0.022862 | 0.290 | 0.182 | 1.593 |
| 237 | RR0  | 2.044692 0.15 0.619 18.762 | 0.014528 | 0.173 |
| 441 | RR0  | 1.545543 0.32 0.283 18.699 | 0.016791 | 0.260 | 0.214 | 1.215 |
| 460 | RR0  | 1.594151 0.13 0.559 18.434 | 0.000376 | 0.477 | 0.759 |
| 594 | RR0  | 1.741086 0.42 0.367 18.641 | 0.000422 | 0.373 | 0.233 | 1.601 |
| 92  | RR1  | 2.988331 0.45 0.189 18.534 | 0.000337 | 0.477 | 0.759 |
| 227 | RR1  | 3.646038 0.38 0.265 19.032 | 0.000708 | 0.269 | 0.272 | 0.989 |
| 357 | RR1  | 3.560084 0.38 0.265 19.032 | 0.000708 | 0.269 | 0.272 | 0.989 |
| 357 | RR1  | 3.699046 0.48 0.176 18.933 | 0.006638 | 0.292 | 0.256 | 1.141 |
| 529 | RR1  | 3.031503 0.64 0.185 18.845 | 0.000433 | 0.477 | 0.759 |
| 380 | RR1  | 3.14803 0.41 0.224 18.773 | 0.006638 | 0.292 | 0.256 | 1.141 |

Table A.5 Parameters of Light Variation for the RR-MC Stars

| ID  | Type | \(f_0\) | \(\Theta_0\) | \(A_0\) | \(I\) | \(\Delta f_1\) | \(\Delta f_2\) | \(\Delta f_3\) | \(A_1/A_0\) | \(A_2/A_0\) | \(A_3/A_0\) |
|-----|------|-------|--------|-------|----|------|------|------|-------|-------|-------|
| 219 | RR0  | 0.50 0.319 18.675 | 1.903236 | −0.001661 | −0.000565 | 0.000533 | 0.568 | 0.341 | 0.303 | a |
| 156 | RR1  | 0.53 0.224 18.810 | 2.645540 | 0.000053 | −0.001795 | −0.000964 | 0.428 | 0.335 | 0.330 | b |
| 161 | RR1  | 0.75 0.176 18.933 | 2.817171 | 0.000403 | −0.000312 | −0.000804 | 0.772 | 0.468 | 0.480 | c |

Table A.6 Parameters of Light Variation for the RR-PC Stars

| ID  | Type | \(f_0\) | \(\Theta_0\) | \(A_0\) | \(I\) | \(\Delta f_1\) | \(\Delta f_2\) | \(\Delta f_3\) | \(A_1/A_0\) | \(A_2/A_0\) | \(A_3/A_0\) |
|-----|------|-------|--------|-------|----|------|------|------|-------|-------|-------|
| 413 | RR0  | 1.717188 0.51 0.298 18.576 | −0.000309 | 0.590 |
| 442 | RR0  | 1.791474 0.39 0.310 18.775 | 0.000204 | 0.278 |
| 550 | RR0  | 1.864087 0.32 0.460 18.719 | −0.000301 | 0.272 |
| 496 | RR1  | 3.656314 0.71 0.147 18.955 | −0.000279 | 0.503 |
| 651 | RR1  | 3.309307 0.47 0.268 18.693 | 0.000244 | 0.378 |
Table A.6 —Continued.

| ID | Type | \(f_0\) | \(\Theta_0\) | \(A_0\) | \(\bar{I}\) | \(\Delta f_i\) | \(A_1/A_0\) |
|----|------|---------|------------|--------|-------|-------------|------------|
| 247 | RR0 | 2.093590 | 0.39 | 0.483 | 18.999 | 0.000253 | 0.246 |
| 473 | RR1 | 2.447215 | 0.58 | 0.221 | 18.514 | 0.001115 | 0.459 |
| 581 | RR0 | 1.674840 | 0.38 | 0.388 | 18.730 | 0.000257 | 0.226 |
| 28 | RR1 | 2.981115 | 0.74 | 0.152 | 19.091 | -0.000213 | 0.686 |
| 67 | RR1 | 2.451110 | 0.45 | 0.253 | 18.554 | 0.000265 | 0.388 |
| 297 | RR1 | 3.127205 | 0.67 | 0.229 | 18.713 | 0.000274 | 0.516 |
| 586 | RR1 | 2.726628 | 0.47 | 0.210 | 18.815 | 0.000278 | 0.327 |
| 407 | RR0 | 2.141042 | 0.27 | 0.471 | 19.002 | 0.000609 | 0.276 |
| 312 | RR0 | 2.117445 | 0.56 | 0.384 | 18.902 | -0.000226 | 0.684 |
| 495 | RR0 | 2.075435 | 0.23 | 0.410 | 18.597 | 0.000258 | 0.173 |
| 501 | RR0 | 1.716627 | 0.42 | 0.355 | 18.582 | 0.000232 | 0.362 |
| 176 | RR1 | 2.743459 | 0.71 | 0.169 | 18.627 | -0.000926 | 0.459 |
| 519 | RR1 | 2.662524 | 0.40 | 0.249 | 18.678 | -0.000535 | 0.311 |
| 435 | RR0 | 1.639894 | 0.61 | 0.283 | 18.684 | -0.000840 | 0.262 |
| 596 | RR0 | 1.369461 | 0.56 | 0.279 | 18.713 | 0.000316 | 0.637 |

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