Non-adiabatic geometric quantum computation with trapped ions

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We propose a non-adiabatic scheme for geometric quantum computation with trapped ions. By making use of the Aharonov-Anandan phase, the proposed scheme not only preserves the globally geometric nature in quantum computation, but also provides the advantage of non-adiabaticity that overcomes the problem of slow evolution in the existing adiabatic schemes. Moreover, the present scheme requires only two atomic levels in each ion, making it an appealing candidate for quantum computation.

Conventionally, the controllable operations in quantum computation (QC) are achieved on the basis of dynamical origins of quantum state evolutions \cite{1}. In recent years, for the purpose of being fault-tolerant to certain types of computational errors, there are considerable interest in exploiting the possibility of implementing quantum computation by geometrical means, which have been termed as holonomic quantum computation \cite{2,3,4}. Depending on the degenerate property of the eigenspace of the governing Hamiltonian, the holonomy can be either a simple Abelian Berry phase factor $\mathcal{F}$ or a general non-Abelian unitary transformation $\mathcal{U}$. It has been shown that the universal quantum computation can be implemented in principle by holonomies \cite{2,3,4}. Up to date, several experimental proposals have been suggested for geometric quantum computation, using such as the nuclear magnetic resonance \cite{5}, superconducting nanocircuits \cite{6,7}, trapped ions \cite{8,9}, and nonlinear optics \cite{10}.

The principle of the aforementioned geometric QC is rooted in the adiabatic evolution of quantum system, which may thus imply a slow computing speed. The adiabatically slow evolution may also challenge the sustainment of required coherence in QC. Therefore, geometric QC based on non-adiabatic evolution should be desirable. Very recently, Wang and Keiji suggested to exploit a non-adiabatic evolution to realize geometric QC in NMR system and superconductor nanocircuits \cite{11}. Indeed, geometric phase exists in non-adiabatic evolving quantum systems, which is in fact the Aharonov-Anandan (A-A) phase \cite{12}. Strictly speaking, the A-A phase depends on certain dynamical quantities such as the rotating angular speed of external (magnetic) field or state vector $i \hbar \partial_t \mathcal{|}\psi\rangle$. In this sense, the A-A phase differs from the adiabatic Berry phase. However, the dependence of the A-A phase on the angular speed is through the closed path loop depicted by the ending point of state vector, and hence is global in nature that largely retains the geometric sense of A-A phase. Accordingly, quantum computation based on the non-adiabatic A-A phase can be reasonably regarded as a kind of geometric QC.

In this work we propose a scheme for non-adiabatic geometric quantum computation with trapped ions. Besides removing the drawback of the slow adiabatic evolution, the proposed non-adiabatic scheme holds additional merits as follows. Firstly, there is no need to design the reverse evolving path to eliminate dynamical phases that occur in the Berry phase-based, adiabatic geometric QC operations in nondegenerated systems \cite{8,9}. Secondly, in comparison to the existing fully holonomic QC schemes \cite{4,10}, the present one does not involve the complicated construction of the degenerate eigenspace of driving Hamiltonian. The above two merits stem from the following observations. In a non-adiabatic quantum evolution, the geometric A-A phase is in general accompanied by a dynamical phase. However, if the evolving path is such designed that along it the state vector is always perpendicular to the driving (magnetic) field, the resulting phase factor after a non-trivial cyclic evolution will be purely geometric. This feature has been exploited by Suter et al in their seminal experiment for demonstrating the A-A phase \cite{13}. Finally, only two atomic levels of each ion are needed in our scheme. This merit alone is attractive, since the originally proposed ion-trap QC scheme required three levels \cite{10} and the recently proposed holonomic ion-trap QC required four levels \cite{16,17}. We notice that valuable efforts on improving ion-trap QC protocol by using only two levels have been carried out in dynamic schemes \cite{18,19,20}. In particular, the technique proposed in Ref. \cite{21} which effectively couples the electronic states of a pair of ions by virtually exchanging phonons, is similar to our present one. The major contribution of our work is to perform QC by geometric means. Viewing that the work of Ref. \cite{21} has in fact extended the ion-trap QC scheme to finite temperature, our geometric scheme may also hold to similar regime, although in the following we would restrict our description at zero temperature limit.

Model Description. For quantum logic with trapped ions, we assume that each ion has two relevant internal states $|0\rangle$ and $|1\rangle$ with energy separation $\omega_0$, and, as usual \cite{19}, can be selectively addressed by lasers. Consider, for instance, the $j$th ion being exposed to a traveling-wave laser field $\mathbf{E}(\mathbf{z}) = \mathbf{E}_0 \cos(\mathbf{k} \cdot \mathbf{z} - \omega_L t + \phi)$ with frequency $\omega_L$, wave vector $\mathbf{k}$, and phase $\phi$. Here $\mathbf{z} = z_0 \mathbf{z}(a + a^\dagger)$ is
by the geometric evolution of the eigenstates of can be performed by geometric means, we first consider the ion chain, and Hamiltonian reads
\[ H \equiv \sigma^z_0 \equiv \{ \omega_0 - \omega_L \}/2 \]
Eq. (1) \[ \tilde{\omega} \]
In the rotating frame (with angular velocity \( \omega_L \)), the state \(|+\rangle \) rotates back to \(|+\rangle \) around \( \Omega_2 = \{ -\tilde{\omega}_j, 0, (\omega_0 - \omega_L)/2 \} \) along the curve BDA on the Bloch sphere. According to the A-A phase theory, after the above cyclic evolution, the state \(|+\rangle \) will acquire a geometric phase \( e^{i\gamma} \), with \( \gamma = 4 \arctan[2\tilde{\omega}_j/(\omega_0 - \omega_L)] \). Note that during the above operation, the state vector keeps always perpendicular to the effective magnetic field, thus no dynamical phase is accumulated in the evolution. Similarly, the state \(|-\rangle \) will acquire A-A phase \( e^{-i\gamma} \) at the same time.
Now consider the evolution of logic states \( |0\rangle = \frac{1}{\sqrt{2}}(|+\rangle - |-\rangle) \), and \( |1\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle) \). After the above operations, they evolve to states
\[ |0\rangle \rightarrow \cos \gamma |0\rangle + \sin \gamma |1\rangle, \]
\[ |1\rangle \rightarrow \cos \gamma |1\rangle - \sin \gamma |0\rangle. \]
We see here the geometric A-A phase plays a role of rotating the logic states. Particularly, complete state flipping between \(|0\rangle \) and \(|1\rangle \) can take place at \( \gamma = \pi/2 \). Note also that the possible value of \( \gamma \) ranges from 0 to 2\( \pi \), implying the ability of arbitrary rotation between \(|0\rangle \) and \(|1\rangle \). The state evolution of the performed qubit described by Eq. (3) is expressed in the rotating frame with frequency \( \omega_L \), in which other free (not performed) qubits would have relative phase accumulations in the non-resonant case of \( \omega_L \neq \omega_0 \). Conventionally, a more convenient choice is to express states in the interaction picture with respect to \( \mathcal{H}_0 = \frac{\omega_0}{2} \sum \sigma_j^z \) (equivalently, a rotating frame with frequency \( \omega_0 \) around \( z \)-axis). Accordingly, Eq. (3) can be recast in the interaction picture as
\[ |0\rangle \rightarrow e^{-i\omega_D \tau/2} \cos \gamma |0\rangle + e^{i\omega_D \tau/2} \sin \gamma |1\rangle, \]
\[ |1\rangle \rightarrow e^{i\omega_D \tau/2} \cos \gamma |1\rangle - e^{-i\omega_D \tau/2} \sin \gamma |0\rangle, \]
where \( \omega_D = \omega_0 - \omega_L \) and \( \tau \) is the total operation time on the performed qubit.
Interestingly, the above state rotation (flipping) is performed by non-resonant pulses via geometrical means. Now we show that by resonant pulses, a pure phase shift gate of single qubit can be geometrically realized. In resonant case, the laser-frequency-associated rotating frame coincides with the interaction picture defined by \( \mathcal{H}_0 \), in which the (jth) qubit Hamiltonian reads \( \mathcal{H}^{(j)} = \tilde{\omega}_j \left[ \sigma_j^+ e^{i\phi} + \text{H.c.} \right] = \Omega_j \cdot \sigma_j \), with \( \Omega_j = \{ \tilde{\omega}_j \cos \phi, \tilde{\omega}_j \sin \phi, (\omega_0 - \omega_L)/2 \} \). We see that in the rotating frame the effective magnetic field constantly has zero z-axis component for arbitrary laser phase \( \phi \), i.e., it lies in the \( x \)-\( y \) plane. To realize the single bit phase gate, we first switch on a \( \pi \)-pulse with laser phase at certain value, say \( -\phi_0 \). The logic state \(|0\rangle \) and \(|1\rangle \) would rotate around the effective magnetic field \( \{ \tilde{\omega}_j \cos \phi_0, -\tilde{\omega}_j \sin \phi_0, 0 \} \) to \(|1\rangle \) and \(|0\rangle \), respectively.
recast to a similar form as the single qubit [23] operation, one can focus on state evolution in the sub-
demonstrated in Ref. 22 by numerical simulation starting
tive two state dynamics (e.g. Rabi oscillations) has been
and \( \Sigma \)
where \( \Phi = 4\phi_0 \). With the help of this phase shift operation, the additional phase factor in Eq. 11 can be can-
celled out by properly choosing \( \phi_0 \). More importantly, together with this phase shift gate, the qubit state rotation
Eq. 11 constitutes a complete logic set for arbitrary single qubit operation.

**Two-Bit Gate via Non-adiabatic A-A Phase.** To com-
plete the quantum logic gate for quantum computation, non-
trivial two-bit gate such as the controlled NOT, or equival-
ently, the conditional phase shift (CPS) gate, would be
required. Below we show how the CPS gate can be imple-
mented via geometric means. Consider two qubits (e.g. the
the jth and kth ones) irradiated by two lasers with fre-
quencies \( \omega_{L,1} \) and \( \omega_{L,2} \), and phases \( \phi_1 \) and \( \phi_2 \). By setting
\( \omega_{L,1} > \omega_0 \) and \( \omega_{L,2} < \omega_0 \), and correspondingly denoting the detunings by \( \delta_1 = \omega_{L,1} - \omega_0 \) and \( \delta_2 = \omega_{L,2} - \omega_0 \), the effective coupling between the two-bit states \( |00\rangle \) and \( |11\rangle \) can be established via virtually exchanging phonons, and the resulting two-bit effective Hamiltonian reads [22]

\[
\hat{H}^{(j,k)} = 4 \sum_{m=1}^{4} E_m |m\rangle \langle m| + g_{jk} \left[ e^{-i(\omega_{L,1} + \omega_{L,2}) t} \right. \\
\left. \times e^{i(\phi_1 + \phi_2)} \sigma_j^+ \sigma_k^+ + \text{H.c.} \right] .
\]

Here notations \( \{ |\bar{1}\rangle = |11\rangle, |\bar{2}\rangle = |00\rangle, |\bar{3}\rangle = |10\rangle, |\bar{4}\rangle = |01\rangle \} \) are introduced for the two-bit computational basis states. Up to the first-order expansion of the Lamb-Dicke parameter \( \eta \) in Eq. 11, the effective coupling strength can be obtained via second-order perturbation theory as

\[g_{jk} = g_j g_k \left( \frac{1}{3} \eta^2 - \frac{2}{3} \eta^3 \right),\]

where \( g_{jk} = \tilde{\omega}_{j,k} \eta \) is the one-phonon frequency of single ion transition. The four basis-state energies \( E_m \) \( (m = 1, \ldots, 4) \) contain also the ac Stark shifts. The effective interaction couples only between \( |\bar{1}\rangle \) and \( |\bar{2}\rangle \), but leaves \( |\bar{3}\rangle \) and \( |\bar{4}\rangle \) inactive with respect to the laser operation in study. This effec-
tive two state dynamics (e.g. Rabi oscillations) has been demonstrated in Ref. 22 by numerical simulation starting
from the original Hamiltonian. As a result, for two-bit operation, one can focus on state evolution in the sub-
space \( \{ |\bar{1}\rangle, |\bar{2}\rangle \} \) in which the two-bit Hamiltonian (6) is recast to a similar form as the single qubit [23]

\[
\tilde{H}_R^{(j,k)} = \frac{\tilde{\omega}_D}{2} \Sigma^z + g_{jk} \left( e^{i\Phi} \Sigma^+ + e^{-i\Phi} \Sigma^- \right),
\]

where \( \Phi = \phi_1 + \phi_2 \), and the two-bit Pauli matrices are introduced as \( \Sigma^+|\bar{1}\rangle \langle 2| = \pm |\bar{2}\rangle \), \( \Sigma^+|\bar{2}\rangle = |\bar{1}\rangle \), and \( \Sigma^-|\bar{1}\rangle = |\bar{2}\rangle \). Note also that the Hamiltonian 11 has been expressed in the rotating frame with respect
to \( \Sigma^z \) with the rotation frequency \( \omega_{L,1} + \omega_{L,2} \); thus
\( \tilde{\omega}_D = (\Delta_1 - \Delta_2) - (\omega_{L,1} + \omega_{L,2}) \). Simple comparison of Eq. (7) with Eq. (2) indicates that an arbitrary rotation be-
tween \( |\bar{1}\rangle \) and \( |\bar{2}\rangle \) can be performed geometrically as that in the single bit case.

In the spirit of pulse-sequence operations in dynamic scheme based on an XY spin model [24, 25, 26], we shall in the following show that the CPS gate can be implemented geometrically as

\[
U_{\text{CPS}} = e^{-i\pi/4} e^{i\eta \sigma_j^z} e^{-i\eta \sigma_k^z} e^{-i\eta \sigma_j^z} e^{-i\eta \sigma_k^z}/2 \\
\times U_{jk}(\pi/4) e^{-i\eta \sigma_j^z}/2 U_{jk}(\pi/4) e^{-i\eta \sigma_j^z}/2. \quad (8)
\]

Here the vector Pauli operator \( \sigma = (\sigma_x, \sigma_y, \sigma_z) \), unit vector \( n_j = (1, 1, -1)/\sqrt{3} \), and \( n_k = (1, -1, 1)/\sqrt{3} \). In the interaction picture with respect to the original free ion Hamiltonian, the two-bit rotation operator \( U_{jk}(\alpha) \) is defined via \( U_{jk}(\alpha)|00\rangle = |01\rangle, U_{jk}(\alpha)|10\rangle = |10\rangle \), and

\[
U_{jk}(\alpha)|00\rangle = \cos \left( \frac{\alpha}{2} \right) |00\rangle + i \sin \left( \frac{\alpha}{2} \right) |11\rangle,
\]

\[
U_{jk}(\alpha)|11\rangle = \cos \left( \frac{\alpha}{2} \right) |11\rangle + i \sin \left( \frac{\alpha}{2} \right) |00\rangle. \quad (9)
\]

Since the arbitrary rotation and phase shift gate of single qubit have been built up, the single bit operations in Eq. (8) can be readily implemented via geometric means by properly combining the single bit logic ele-
ments. Viewing the similarity between the two-bit re-
duced Hamiltonian 11 in the subspace \( \{ |00\rangle, |11\rangle \} \) and
the single bit Hamiltonian 23, the two-bit rotation of
Eq. (9) can be straightforwardly implemented by the fol-
lowing two-step procedures:

(i) In the two-bit rotating frame with frequency \( \omega_{L,1} + \omega_{L,2} \) around \( \Sigma^z \), performing cyclic evolution for the eigen-
states of \( \Sigma^y \) by controlling the laser phases \( \Phi \) similarly
as in the single bit case, one can geometrically rotate the states \( |00\rangle \) and \( |11\rangle \). Expressed in the interaction picture which also corresponds to \( \tilde{H}_0 \equiv \tilde{E}_1|\bar{1}\rangle \langle |11\rangle + \tilde{E}_2|00\rangle \langle 00| \), this operation can realize the following state transformation

\[
|00\rangle \rightarrow e^{-i \tilde{\omega}_D \tau/2} \cos \Gamma |00\rangle + e^{i \tilde{\omega}_D \tau/2} \sin \Gamma |11\rangle,
\]

\[
|11\rangle \rightarrow e^{i \tilde{\omega}_D \tau/2} \cos \Gamma |11\rangle - e^{-i \tilde{\omega}_D \tau/2} \sin \Gamma |00\rangle. \quad (10)
\]

Here \( \Gamma \) is the geometric A-A phase determined by the
evolution contour of the two bit state vector, and \( \tilde{\omega}_D \tau \) is the
detuning-induced phase accumulation.

(ii) Tuning the laser frequencies in resonance with the
two ions [27], i.e., \( \tilde{\omega}_D = 0 \), a phase-shift gate associat-
ing with \( |00\rangle \) and \( |11\rangle \) can be implemented to cancel out the phase factors in Eq. (10). In the resonance case the
\( \omega_{L,1} + \omega_{L,2} \) rotating frame coincides with the interaction picture of \( \tilde{H}_0 \). As the one-bit resonant case, the effective
magnetic field corresponding to Eq. 11 now lies in the x-
y plane since \( \tilde{\omega}_D = 0 \). By successively choosing two
different values of the laser phase \( \Phi \), one can perform two \( \pi \) rotations on the states \( |11\rangle \) and \( |00\rangle \) around the effective
magnetic fields, and readily generate the A-A geometric phases, $e^{-i\vec{F}}$ and $e^{i\vec{F}}$, for states $|11\rangle$ and $|00\rangle$, respectively. Now, after a phase-shift operation with $\tilde{\Gamma} = \tilde{\omega}_D \tau / 2 - \pi / 4$, Eq. (10) becomes

$$
|00\rangle \rightarrow e^{-i\pi/4} \left[ \cos \Gamma |00\rangle + i \sin \Gamma |11\rangle \right] \\
|11\rangle \rightarrow e^{i\pi/4} \left[ \cos \Gamma |11\rangle + i \sin \Gamma |00\rangle \right].
$$

(11)

This is identical to Eq. (10), except for the additional global phases. Obviously, these global phases will not appear if we first generate a phase shift of $e^{i\pi/4}$ on $|00\rangle$ and $e^{-i\pi/4}$ on $|11\rangle$ at the same time, by the phase-shift gate just described, prior to the operation of Eq. (10).

We have thus realized the two-bit gate $U_{jk}$ as defined in Eq. (10). Together with the arbitrary one-bit operations (rotation and phase shift) described earlier, we can now readily implement the important CPS gate Eq. (5), whose role is to transform $|11\rangle \rightarrow e^{i\pi/4} |11\rangle$, while to keep other computational two-bit basis states unchanged.

**Conclusion and Discussion.** The proposed non-adiabatic geometric QC scheme based on the A-A phases is expected to overcome several drawbacks of the adiabatic schemes, namely, the slow evolution, need of refocusing to eliminate the dynamical phases, and continuous control over many fields to construct non-trivial loops in the parameter space. Viewing that the trapped-ion is one of the best exploited systems for quantum computation, and that the proposed scheme requires a relatively simple atomic level configuration, we suggest, as a first step, to exploit it as an interferometer for principle-proof of the non-adiabatic geometric A-A phase discussed in this work. We believe that the interference associated with the non-adiabatic A-A phase can be readily demonstrated by experiment in ion-trap systems.

As a possible QC architecture, the elementary operation steps in the proposed non-adiabatic geometric scheme are comparable to its dynamic counterparts. Specifically, the time scales for both the one-bit and two-bit geometric operations are about the same as those in the dynamic operations. Concerning the possible fault-tolerance, in the adiabatic case, quantum logic is tolerant to certain types of errors such as the field fluctuations that preserve the loop area in parameter space, i.e., the Berry phase. Similarly, in the non-adiabatic case, the A-A phase is of error tolerance to any fluctuations around the state-evolving path that preserve the path loop area. In principle, there exist many possible driving field deviations that can preserve the state path loop area. However, the most natural and possible errors appear to be random (but small) fluctuations of the laser phase, frequency, and coupling strength to the atomic levels, which equivalently result in fluctuations of the effective magnetic field. The global A-A phase is expected to be largely immune from this type of errors, and, at the same time, the net dynamic phase accumulation is approximately zero due to the cancellation of the positive and negative contributions.

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the detunings of the two lasers, the phase accumulations will be cancelled. Notice that the exchange of the laser detunings does not affect the Hamiltonian (7) in the subspace \(\{|1\rangle, |2\rangle\}\). We thus do not explicitly mention this point further in the two-bit operations.

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