N-body Simulations of Star-Disc Captures in Globular Clusters

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ABSTRACT: The presence of protostellar disks can greatly increase the dissipation during close stellar encounters, leading to the formation of a significant population of binaries during the initial collapse and virialization of a cluster. We have used N-body simulations of collapsing globular clusters to find the major factors that determine the efficiency of binary formation through star-disk captures. This work serves the dual purpose of verifying the results of earlier analytic work as well as examining parameters not testable by that work. As in the earlier work, typical binary fractions of a few percent are found. For the parameters studied, the results are found to depend remarkably little upon disk evolution, the mass distribution of the stars, or their spatial distribution, though distributions in which the stars are highly clumped yield binary fractions larger by a factor of a few. The direct N-body integrations limit the models to relatively small values of N. Semiempirical relations are derived, however, which allow the results to be extrapolated to values of N appropriate to globular clusters.

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1 INTRODUCTION

A number of recent studies suggest that globular clusters contain a significant population of binaries (Pryor, Latham & Hazen 1988; Pryor et al 1991; Romani & Weinberg 1991). It is widely believed that the majority of these binaries must be primordial (i.e. formed at the epoch of cluster formation) since negligibly few would have formed subsequently through three-body capture processes (Hills 1976), and only the closest binaries can be ascribed to subsequent tidal capture events (Fabian, Pringle & Rees 1974). The presence of a number of primordial binaries in globular clusters has also long been suspected on theoretical grounds, since they play an important role in averting the onset of gravothermal catastrophe and cluster core collapse (Goodman 1989; Goodman & Hut 1989; McMillan, Hut & Makino 1990, 1991; Gao et al 1991).

In this paper, we consider in some detail a proposed mechanism for the formation of primordial binaries in globular clusters, namely that of captures due to dissipation by protostellar disks during close encounters in an initial phase of cluster violent relaxation. This possibility was examined by Murray, Clarke & Pringle (1992, henceforth MCP) using an idealised analytical model which predicted binary fractions of a few per cent—a number broadly compatible with the results of binary surveys in globular clusters. Such an analytical model is however only applicable to the case of the homologous collapse of an initially uniform density sphere, thus raising the suspicion that its results were heavily reliant on this (possibly unrealistic) initial condition. In order to remedy this uncertainty, therefore, we have undertaken a number of N-body simulations of the violent relaxation of stellar clusters, with the aim of discovering how the binary capture rate is affected by the initial global density profile, the degree of homogeneity and the spectrum of stellar masses involved.

Before setting out the star-disc binary formation mechanism in more detail, it is first necessary
to describe the context in which such a mechanism would operate in the early stages of globular cluster formation. We here follow a number of previous authors in assuming that stars form in globular clusters with a velocity dispersion that is substantially sub-virial, and that virialisation then proceeds on a dynamical time scale through the process of violent relaxation (e.g. Aarseth, Lin & Papaloizou 1988, henceforth ALP). Such a picture is clearly at odds with the way in which star clusters are currently observed to form in the Galactic disc, since here star formation appears to proceed in an ensemble of cloud clumps whose motions are already substantially virialised in the potential of the parent cloud. In globular clusters, however, a much more rapid (dynamical) process of fragmentation and star formation is indicated by a number of considerations. The narrow red giant branches of globular clusters are used to infer internal metallicity spreads $\Delta[\text{Fe/H}]$ ranging from 0.1 for metal poor clusters to 0.01 for metal rich clusters (Sandage & Katem 1977, 1982; Cohen 1979; Richer & Fahlman 1984; and Bolte 1987ab). Such small metallicity spreads imply that star formation must have occurred on a time scale less than a cluster dynamical time so as to avoid self-enrichment (Murray & Lin 1989). A similar limit may result from the narrow observed widths of the red giant branches of massive, young clusters in the Magellanic Clouds (Elson 1991). A dynamical constraint results from the fact that slower, less efficient star formation could lead to the clusters becoming unbound due to the loss of gas resulting from ionization-shock fronts caused by the first massive stars to form (Tenorio-Tagle et al. 1986; Lada, Margulis, & Dearborn 1984). If star formation indeed occurs on a dynamical time scale, this suggests that some external trigger (such as a cloud-cloud collision) must have brought about the sudden fragmentation of a cloud previously in approximate hydrostatic equilibrium (Murray & Lin 1989). In this case, the stars would fragment out of the background on essentially radially infalling orbits, causing the collapse of
the entire system. The formation of a singularity is however avoided due to the velocity dispersion generated by global gravitational instabilities during the collapse (ALP). Consequently, the system ‘bounces’ at a finite radius \(N^{-1/3}\) of the initial cluster radius for an initially homogeneous sphere containing \(N\) mass points) and rebounds into a state of approximate virial equilibrium.

It is in this context that we consider the role of star-disc interactions in the formation of binaries. Here, the idea is that two-body encounters between stars are dissipative if they involve a star’s passage close to (or through) the protostellar disc around another star (Pringle 1989; Larson 1990). Clarke & Pringle (1991a) (henceforth CP) however argued against such a process as an important source of binaries in the case of large \(N\) virialised clusters, owing to the prevalence of fast (non-capturing) encounters that would act so as to destroy the protostellar discs. The present situation is different, however, since the initial stellar velocity dispersion is very low: the binary formation rate changes during the collapse as a result of a trade-off between the competing effects of growing velocity dispersion and rising density. MCP quantified these effects using the analytical estimates of ALP for the growth of density and velocity dispersion during the collapse of a uniform sphere and applying a simple criterion for the dissipative energy loss during star-disc encounters. This exercise yielded a binary formation rate that rose during the collapse, peaking at the bounce, at which point the calculation was terminated due to the breakdown of the analytical estimates in the non-linear regime. MCP however anticipated that there would be little subsequent binary formation since the high velocity dispersion during the bounce would destroy most remaining discs.

In this paper, we address the same problem using an N-body code. This has a number of advantages over the method described above, being more flexible in terms of the initial conditions that it can handle and allowing the calculation to be pursued into the non-linear regime–i.e. through
and beyond the bounce phase. We therefore consider both uniform and isothermal initial density profiles, these corresponding respectively to the limits in which the proto-globular cloud was predominantly confined by an external medium and by self-gravity (we do not, however, consider the deviations from spherical symmetry induced by the external trigger for fragmentation and star formation: see Boily, Clarke & Murray 1992). We also consider the case of an initially clumpy mass distribution (motivated by the appearance of young globular clusters in the Magellanic clouds (Els"{o}n 1991)) and investigate the effects both of including a spectrum of stellar masses and of varying the initial stellar velocity dispersion. An obvious disadvantage of N-body codes, compared with the analytical method of MCP, is however the steeply rising computational cost with increasing values of N. Since it is clearly unfeasible to run simulations with N of order the number of stars in a globular cluster, and since a variety of effects mean that the scaling of binary formation rate with N is not clear from first principles, we have been forced to experiment with different values of N. When (in the uniform density case) a value of N is attained (> $N_{\text{min}}$) for which the results agree with the analytical estimates for that N, the analytical results can then be used to extrapolate into the high N regime appropriate to globular clusters. The comparison of differing initial conditions is then made for models with $N = N_{\text{min}}$. In practice, we find that this implies simulations involving 2000 particles.

The structure of the paper is as follows. In § 2 we set out the numerical method and the range of models that we explore. In § 3 we briefly describe the results of these models. Section 4 contains a detailed discussion of the results; where possible we have contrasted the results with the analytical model of MCP, and present semi-empirical expressions for the scaling of binary yields with disc and cluster properties. Section 5 briefly presents the conclusions.
2. THE MODELS

2.1. Numerical Method

The most straightforward means by which the formation of binaries can be studied is by the use of direct N-body integration of the orbits and interactions of the stars of a collapsing cluster of protostars. For long term integrations, the method is limited to \( N \lesssim 10^3 \) (Aarseth 1985). In principle, larger \( N \), up to \( \sim 10^4 \), could be used in the present study, for which the clusters need only be followed for a few dynamical times. So as to make the most efficient use of computation time, we have in practice limited most of our models to 2000 stars, so as to be able to investigate the effects of changing several parameters. To ensure that the models do adequately represent the evolution of larger \( N \) systems, we have run some models with as many as \( 2 \times 10^4 \) stars (see below).

The program used is described in Aarseth (1985). It employs a direct integration of the motion of the stars, with separate integration time steps for each star. To further improve computational efficiency, the contribution to the force on each star is divided into two components, one from close neighbors, and the other from more distant particles, with the latter being updated at longer intervals. The effects of close encounters between stars are weakened by the use of a softened potential, in which the interparticle force varies as

\[
F_i \propto \frac{x_i}{(r^2 + \epsilon^2)^{3/2}},
\]

(2.1.1)

where the index \( i \) indicates the component, \( r \) is the interparticle separation, and \( \epsilon \) is the constant softening parameter. To ensure that softening does not affect the capture rates, we set \( \epsilon \ll R_d \), where \( R_d \) is the disk radius, in all results below.
The energy loss due to encounters with stellar disks is treated impulsively. During the integration of a star’s motion, the nearest neighbour distance is calculated. If this distance is less than the sum of the disk radii, then, when the stars reach periastron, their kinetic energy is decreased by an amount equal to the total kinetic energy of the stellar disks at periastron (CP), or

\[ \Delta E = \frac{1}{2} \left( M_{d1} V_1^2 + M_{d2} V_2^2 \right), \]  

where \( M_{di} \) and \( V_i \) are the disk masses and relative speeds in the center of mass frame of particles 1 and 2, respectively. We note that the method of treating encounters does not account for the relative orientations of the disks, does not allow for (rare) three-body interactions, and assumes that the disks are entirely destroyed during the encounter, thus ignoring the effects of further interactions. The assumption that the disks are completely disrupted should give an upper limit to the energy loss per encounter (CP), but also implies that each disk can be involved in only one encounter, and is in this latter sense a pessimistic assumption. We do not feel that further experimentation with the interaction prescription is warranted at this stage, pending further detailed work on the nature of star-disc interactions (Clarke & Pringle in preparation).

The N-body integration method described above has been tested against the numerical method used by MCP, for systems with \( 10^4 \) stars which were initially distributed with uniform density, and whose disk radii and masses were held constant with time. The number of binaries, \( N_{\text{bin}} \), found by the two methods agreed to within 10% at the time that the analytical calculation had to be abandoned due to the perturbations becoming non-linear. Such excellent agreement helps to confirm both the simplified analysis of the previous work, as well as the lack of any affect of \( \epsilon \) upon the results.
2.2. Models

The advantage of the current method is that it allows us to examine a more realistic range of parameters than could be studied with the previous analytic method. Most important are the effects of varying the number of stars, their initial kinetic energy, the spatial distribution of the stars, and the effects of a distribution of stellar (and disk) masses and radii. The last three could not be examined using the method employed by MCP.

The parameters of the models used are summarized in Table 1. Listed are: the model number; the number of stars, \( N \); the stellar density distribution, \( \rho_*(R) \) where \( R \) is the radius within the cluster (see below); the initial ratio of kinetic energy to potential energy of the cluster, \( Q \), (0.5 for virial equilibrium); whether or not disk evolution is included; the initial disk radii; and the initial mass function of the stars. Also listed in each case are the number of binaries formed \( (N_{\text{bin}}) \) and the time \( (t/\tau_c) \) (where \( \tau_c \) is the cluster crossing time) at which the number of binaries is evaluated. For small \( N \) models (\( \approx 2000 \)) the binary yield is normally evaluated after two crossing times, whereas at large \( N \) we have been forced to terminate our calculations earlier, either due to the computational expense of running for longer times or because of the accumulation of energy errors in the N-body integration to an unacceptable degree. In all cases we estimate that the incompleteness of our binary yields is \( \approx 25\% \).

Models 1-16 examine the role of cluster properties. Each of these models uses equal mass stars, with \( M_* = 1 \, M_\odot \) assumed. The disk radii and masses, \( R_d = 10^{-3} \, \text{pc} \), and \( M_d = M_*/2 \), respectively, are held constant in time: we discuss what values of \( R_d \) might be expected in practice in § 4.4. The half-mass radius of each model is \( r_h \approx 2 \, \text{pc} \), with some variation due to the random placement of the stars. The number of stars used in these models varies from 50 to \( 10^4 \).
Given the uncertainties in the form of $\rho_\ast(R)$ to be expected following star formation in clusters, three extremes have been tested. If star formation occurred within initially pressure-bound clouds, then the resulting stellar distribution might be expected to be fairly uniform, reflecting the gas density (models 1-7). Alternatively, in initially self-gravitating clouds, more centrally condensed density distributions are expected. A singular isothermal sphere represents one extreme distribution for a hydrostatic, self-gravitating cloud, and so we adopt $\rho_\ast \propto 1/R^2$ in models 8 to 13.

Star formation may not be expected to follow a smooth density law, but may occur in clumps. This is observed in molecular clouds today (Shu, Adams, & Lizano 1985), and is also predicted in globular clusters if star formation occurs as the result of thermal instability (Murray & Lin 1989), in which case stars form in regions where the gas was initially overdense relative to the background. To test this, the stars in model 14 are initially distributed randomly within ten subclumps of half-mass radius 0.4 pc, which themselves are distributed randomly throughout the cluster radius.

The final cluster parameter of interest is the kinetic energy of the stars. To test its effect, the initial value of $Q$ is varied between 0.01 and 0.05.

Models 17-19 examine the dependence on disc parameters. In model 17 the disc radii are increased by a factor five compared with previous models (note that this is identical to decreasing the cluster radius by the same factor). In models 18 and 19 we examine the effect of viscous evolution of the discs for two (uniform and isothermal) models with $N = 2000$, and $Q = 0.01$. The disk evolution is approximated as in Lin & Pringle (1990), such that the disk radii and masses vary as

$$\frac{R_d}{R_{d0}} = \left(\frac{M_{d0}}{M_d}\right)^2,$$

(2.2.1)
and

\[ M_d = M_{d0} \left[ 1 + 5 \left( \frac{t}{\tau_{\nu0}} \right) \right]^{-1/5}, \quad (2.2.2) \]

where

\[ \tau_{\nu} = \left( \frac{R_d^3}{GM_*} \right)^{1/2} \left( \frac{M_*}{M_d} \right)^{2/5} \eta^{-1} \quad (2.2.3) \]

is the viscous time scale, \( \eta \) is an adjustable parameter, and \( M_* \) is the total mass of the disk and central star. The models discussed below all assume \( \eta = 0.001 \): for the disk and cluster parameters given above, this gives \( \tau_{\nu0} = 1.9 \) Myr, comparable to the initial free-fall time \( \tau_{ff} = 1.5 \) Myr for the uniform density clusters with \( N = 2000 \).

The potential role of variations in stellar and disc properties is examined in models 20-22. In each, the stellar distribution is assumed to follow a Salpeter mass function, with stellar masses in the range 0.5-2.5 \( M_\odot \), giving a mean stellar mass of \( \langle M \rangle = 1 M_\odot \). The variation of the disk masses and radii with stellar mass are uncertain, and will depend upon the early evolution of protostellar fragments prior to star formation (see discussion in § 4.5). For simplicity, we assume a constant disc to star mass ratio and take \( M_{d0} = 0.5M_* \) for all stars. We also assume \( R_{d0} \propto M_* \) (with \( R_{d0} = 10^{-3} \) pc for \( M_* = \langle M \rangle \)) but discuss this assumption critically in § 4.5 below.

3. RESULTS

3.1. Variation with N (Uniform Models)

Figure 1 shows \( N_{bin} \) vs. time, and the resulting distribution of semimajor axes, \( a \), for models 1-5. The time at which the system reaches maximum compression before re-expanding and virializing is approximately one initial free-fall time of the clusters, or approximately \( 3 N_{500}^{-1/2} \) Myr, where \( N_{500} = N/500 \). Also shown are the results of the analytical calculation (MCP) at the time that the
perturbations become non-linear. Figure 2 illustrates the onset of destructive star-disc collisions during the bounce for $N = 10^4$. It also demonstrates that the rate of binary formation has, indeed, slowed down greatly by the end of the simulation, which is not apparent from Figure 1a.

The distribution of $a$ is similar in each of the models, so that only the two extreme cases are shown in Figure 1b. From the figure, it can be seen that the distribution is similar to that found by Clarke & Pringle (1991b) for small $N$ systems, and predicted by MCP for large $N$ systems. The distribution in log $a$ has a peak near $a = R_d$, and for $a < R_d$, the number of binaries with semimajor axes less than a given $a$ varies approximately as $N_{bin}(< a) \propto a$.

### 3.2. Variation with Density Law and Initial Kinetic Energy

The solid and dotted lines in Figure 3 show the evolution of $N_{bin}$ with time for Models 3 and 6, two uniform clusters containing 2000 stars with different values of the initial kinetic energy ($Q = 0.05$ and 0.01 respectively). Again the crosses mark the corresponding analytical results when the perturbations go non-linear. At this stage, it is evident that the binary yield is rather sensitive to $Q$ (differing by approximately 50% after 2.5 Myr); the similar post-bounce yields however diminish the contrast in the total number of binaries. For higher $N$ values, the shutting off of binary production after the bounce leads to the marked $Q$-dependence being preserved in the total yield of binaries (model 4 cf model 7, Table 1).

Also shown in Figure 3 (short-dashed and long-dashed lines) are the corresponding plots for Models 10 and 12, $N = 2000$ models in which the initial density profile is that of an isothermal sphere. It is immediately apparent that whereas the time-dependence of the binary capture rate is quite different as compared with the uniform case (note the absence of a well defined ‘bounce’
phase of peak binary production in the isothermal case) the over-all binary yield is changed by less than a factor two. The isothermal case is considerably less sensitive to $Q$, at this $N$, implying that the collapse process is more efficient in erasing any trace of the initial conditions than in the uniform case. At larger $N$, however, this $Q$ dependence of binary production is better preserved in the isothermal case as well (Figure 4).

### 3.3 Clumpy Initial Conditions

Figure 5 shows the evolution of Models 6 and 14, both of which have $N = 2000$ and $Q = 0.01$. In Model 6, the stars are distributed uniformly, whereas in model 14 they follow a clumpy distribution as described in § 2.2. As expected, the clumps collapse ahead of the overall infall of the cluster, and the binary yield is increased by a factor of about two relative to the uniform case. We have also run a model (17) corresponding to a single clump in model 14, the similar binary fraction obtained in this case confirming that model 14 evolves like an ensemble of independent clumps over several cluster crossing times. We have also included in Table I a selection of low $N$ models (1, 15-17) that can be used in order to estimate binary fractions from clumpy initial conditions.

### 3.4. The Role of Disk Evolution

Figure 6a compares the evolution of models 6 and 18, both of which are uniform density clusters with $N = 2000$, and $Q = 0.01$. In model 6, the disk properties do not evolve with time, while in model 18 $R_d$ and $M_d$ evolve as described in § 2.2 above. At early times ($\lesssim 1.4$ Myr), the increase in $R_d$ due to viscous evolution increases the capture rate relative to the unevolving system. At later times, however, the capture rates are approximately equal in the two clusters, an effect that can be traced to the steep increase in $N_{\text{loss}}$ at late times in the viscously evolving case (dotted and long
dashed curves in Figure 6a). (By the last times shown, $N_{\text{loss}} = 297$ in model 18, as compared to 111 in model 6.) The primary effect of disk evolution is thus to increase the overall encounter rate significantly, while increasing the rate of captures only slightly.

Evolution of the disk radii also affects the distribution of $a$, as shown in Figure 6b. Because more captures occur when $R_d$ has a greater value, the peak in the distribution is shifted to a larger $a$ by about a factor of two, corresponding to the value of $R_d$ near the time of maximum stellar density, with a corresponding increase in the width of the distribution.

The change in $N_{\text{bin}}$ is even less striking for the case of centrally condensed systems, as is apparent in Figure 7a, which shows the evolution of models 12 and 19. The capture rate in these systems decreases slowly with time, in contrast to the uniform density models which are dominated by captures during the bounce phase. As a result, in the viscous case, many encounters occur before the disks evolve significantly. The increase in the encounter rate at late times relative to the case in which the disks do not evolve occurs at the expense of an increase in $N_{\text{loss}}$. Rather than resulting in a shift in the peak of the distribution in $a$, the effect is to broaden the distribution, which is now almost uniform in the range $1.8 \lesssim \log a \lesssim 2.8$, as shown in Figure 7b.

3.5. The Role of the IMF

Models 20-22 are equivalent to models 18, 6 and 19, except that the stellar mass distribution follows a Salpeter function over a factor five in mass: in each case, the resultant binary yield differs from the equal mass case by $\lesssim 50\%$.

Figure 8 demonstrates the mass dependence of the binary formation process: in Figure 8a the fraction of stars of a given mass that are contained in binaries is plotted as a function of stellar mass.
(models 20 and 22), illustrating the preferential incorporation of higher mass stars into binaries. This same effect is also illustrated in Figures 8b and 8c which plot the mass ratio $M_2/M_1$ as a function of $M_2$ for binaries in which $M_1$ lies in the range $0.7 - 1.3 M_{\odot}$ (in the event that both binary members lie in this range, $M_1$ is taken to be the star with mass closest to $1 M_{\odot}$). In each case the distribution resulting from random pairing from the mass function is also illustrated: again, the preference for high mass companions is demonstrated, particularly for the isothermal case.

4. DISCUSSION

We now discuss the behaviour of the models described above and attempt to understand these results by comparing them with the scalings suggested by analytical estimates. In this way, we are able, in certain regimes, to propose semi-empirical expressions for the resultant binary fraction as a function of model parameters. Before considering various regimes in detail, however, we first lay out the analytical dependences that govern the rate of binary formation by star-disc capture.

For the star-disc capture prescription used here (i.e. in which the relative orbital energy of two stars is reduced by an amount proportional to the disc’s orbital kinetic energy, if periastron is less than $R_d$, and is unchanged if periastron is greater than $R_d$, independent of orbital inclination) the appropriate capture rates as a function of disc and local cluster variables are given by equations (2.2.8) and (2.2.9) in CP (note that, strictly speaking, such a formulation is only appropriate to the case of a Gaussian local velocity dispersion, a condition that is not necessarily exactly satisfied during violent relaxation). The scalings that can be extracted from these equations may be written as

$$\Gamma_{\text{cap}} \propto n_o R_d f_r/v_*$$  \hspace{1cm} (4.1)
where $\Gamma_{\text{cap}}$ is the capture rate per star, $n_o$ and $v_*$ are respectively the local stellar number density, and velocity dispersion and $f_r$ is a reduction factor for the case in which star-disc encounters are predominantly destructive (i.e. non-capturing):

$$f_r = 1 \ (v_* \ll V_c) \quad (4.2)$$

and

$$f_r \propto \left(\frac{V_c}{V_*}\right)^2 \ (v_* \gtrsim V_c) \quad (4.3)$$

where

$$V_c = \left(\frac{4GM_d}{R_d}\right)^{1/2} \quad (4.4)$$

### 4.1 Uniform Density Case

The results of the uniform density N-body calculations are of particular interest since they admit detailed comparison with the analytical calculation of MCP. As is evident from Figure 1a, the N-body results produce consistently more binaries than the MCP calculation. This effect can be readily understood by noting that the MCP calculations are terminated when the velocity perturbations become non-linear (i.e. during the ‘bounce’ phase), whereas the N-body results can be pursued over a number of crossing times. At low N, a number of binaries are formed post-bounce. As N is increased (for constant $R_d$ and cluster half mass radius, $R_h$) the importance of post-bounce encounters decreases considerably and this progressively improves the agreement between the two
models. This effect is illustrated in Figure 9 by the × (denoted 1-7 for progressively higher \(N\)) whereas the open circle contains the predictions of the MCP method for these models. Quantitatively, one may understand the steep decline in binary formation efficiency for \(N \gtrsim 2000\) by noting that star-disc encounters start to become predominantly destructive (as opposed to capturing) once the velocity dispersion exceeds \(V_c\) (equation (4.4)). For the values of \(R_h, M_d\) and \(R_d\) of our models, the post-bounce velocity dispersion exceeds \(V_c\) for \(N \gtrsim 2000\) and thus one would anticipate the shutting off of post-bounce captures in clusters larger than this.

We deduce from this that the MCP results give a reasonable prediction of the binary fraction for uniform clusters of \(\sim 10^4\) stars, providing estimates that are less than a factor two below the ‘true’ N-body results in this regime. This success then leads us to enquire more deeply as to the way the MCP results scale with model parameters, an investigation that is computationally prohibitive with the N-body code at such high \(N\).

The solid dots in Figure 9 show the variation of binary fraction, \(f_{\text{bin}}\), (by the MCP method) as a function of the initial kinetic energy parameter, \(Q\), for a variety of values of \(N\) in the range \(10^4\) to \(10^6\), whilst the solid line is a fit to these points of the form \(f_{\text{bin}} \propto Q^{-1/2}\). Such a scaling may be understood by considering the expressions for the capture rate per star as a function of local variables (equations (4.1) to (4.4) above). One crude estimate of the resultant binary fraction may be obtained by multiplying the initial \(\Gamma_{\text{cap}}\) by the cluster free-fall time, so that, noting that initially \(v_* \propto (QN/R_h)^{1/2}\) we obtain \(f_{\text{bin}} \propto (R_d/R_h)Q^{-1/2}\) (if the initial velocity dispersion is less than \(V_c\): \(Q \lesssim Q_c\)) and \(f_{\text{bin}} \propto M_d/(NQ^{3/2})\) for \(Q \gtrsim Q_c\). The former scaling, with binary fraction independent of \(N\), is reproduced by the solid points in Figure 9; for higher \(N\), \(Q\) can become greater than \(Q_c\) and the diamonds in Figure 9 (\(N = 10^6\)) indicates the \(Q^{-3/2}\) scaling (dashed line) in this case.
At the low \( Q \) end, as well, the binary fraction deviates from the \( Q^{-1/2} \) law, since the velocity dispersion cannot be held close to its initial value for a cluster collapse time if this initial value is much smaller than the initial two-body free-fall velocity. If such is the case, then two-body effects drive up the velocity dispersion on less than a cluster collapse time, and the resultant binary fraction is accordingly lower than an extrapolation of the \( Q^{-1/2} \) scaling; the limiting \( Q \) value in this case is \( Q_{\text{min}} \sim N^{-2/3} \). This effect is illustrated by the + in Figure 9 for the case \( N = 10^4 \). However, in the case of a stellar cluster in which the initial stellar separation is of order a Jeans length, this limiting initial velocity dispersion is of order the sound speed: in reality, pressure differentials during fragmentation will always impart a velocity dispersion of at least this order, so that the \( Q \lessgtr Q_{\text{min}} \) case would not be encountered in practice.

To summarise the results above, the resultant binary fraction in the case of a uniform high \( N \) cluster can be expressed by the following semi-empirical formulae:

\[
\begin{align*}
    f_{\text{bin}} &= 3\% (R_d/10^{-3} R_h)(Q/10^{-2})^{-1/2} \quad (Q_{\text{min}} \lessgtr Q \lessgtr Q_c) \\
    f_{\text{bin}} &= 1\% (M_d/M_\ast)(Q/10^{-2})^{-3/2}(N/10^6)^{-1} \quad (Q > Q_c)
\end{align*}
\] (4.1.1)

where

\[
    Q_{\text{min}} \sim N^{-2/3}
\] (4.1.3)

and

\[
    Q_c \sim 6 \times 10^{-3} (M_d/M_\ast)(10^{-3} R_h/R_d)(N/10^6)^{-1}
\] (4.1.4).

In each case the scalings have been derived using the arguments above, whilst the coefficients have been fit to the results of the MCP calculations in the range \( N = 10^4 - 10^6 \) and \( Q = 10^{-4} - 0.1 \).
We note, in passing, that it is remarkable how well these scalings work (based as they are on initial capture rates) when one considers that the bulk of binary captures occur well into the ‘bounce’ phase. The dependence of the binary fractions on $Q$ indicates that the system in some sense ‘remembers’ its initial conditions during the onset of the bounce.

The above expressions refer to the total yield of binaries generated by this process, whose semi-major axes are initially distributed as $N_{\text{bin}}(<a) \propto a$ for $a \lesssim R_d$. It is likely, however, that the ultimate distribution of separations is mainly governed by subsequent orbital evolution of the protobinary due to gravitational interaction with remnant disc material. For example, if as little as 10% of the binary mass remains in circumbinary orbit following capture, then orbital energy transfer from binary to disc can cause the binary orbit to shrink over a time scale of a few thousand orbital periods (Artymowicz et al 1991). For binaries with separations $\sim 100$ A.U., substantial orbital shrinkage could then occur before the dispersal of the disc on a time scale $10^6 - 10^7$ years (Skrutskie et al 1991). If, however, such spiralling in does not occur during the disc lifetime, many of the binaries produced above would be destroyed, being wider than the hard-soft borderline $a_{hs} \sim 0.4 R_{pc}(10^6/N)$ A.U., where $R_{pc}$ is the cluster half mass radius in parsecs. The relevant destruction time scale is $\sim (R_h/a)$ cluster crossing times (Binney & Tremaine 1987) and is therefore considerably longer than the lifetime of protostellar discs. Thus we conclude that either substantial orbital evolution occurs during the pre-main sequence stage (e.g. efficient binary-disc coupling), in which case $f_{\text{bin}}$ above represents the ultimate binary fraction, or else only a fraction $a_{hs}/R_d$ of these binaries survive. In the latter case the ultimate binary fraction (for $Q \leq Q_c$) would be reduced to $0.6\%(Q/10^{-2})^{-1/2}(10^4/N)$.

4.2 Isothermal Case.
In the case of clusters with an initially isothermal density profile, we have no model for the growth of density and velocity perturbations, and thus cannot undertake the type of comparison described above. We can however gain some insights into the binary formation process by consideration of Figures 3 and 4. First, it is clear from comparison of these and Figure 1a) that the binary formation history of isothermal systems is not characterised by a well-defined peak, since the spread of arrival times at the origin in inhomogeneous systems does not result in a single ‘bounce’. Instead, the innermost regions of the cluster collapse first, and outer shells, collapsing later, interact during their infall with initially inward lying material now in the process of re-expansion from the origin. As a result, the outer parts of the cluster acquire a velocity dispersion that is a substantial fraction of virial during their infall phase. This effect is apparent in both Figures 3 and 4, from which we can distinguish two phases of binary formation. For the first ∼ 0.2 of a cluster free fall time scale (‘early’ phase) the binaries originate from the innermost regions which are collapsing in a manner similar to a homogeneous system (ALP): during this time, therefore, the parts of the cluster that form most of the binaries ‘remember’ the initial conditions, an effect that is apparent from the ∼ $Q^{-1/2}$ dependence of the binary yield during this phase (note also that the ‘early’ binary fraction is insensitive to $N$: cf equation (4.1.1) above). Subsequently, however, (‘late’ phase) the number of binaries formed is more or less independent of $Q$, an effect that reflects the erasing of initial conditions through shell crossing in the outer parts of the cluster. As $N$ is increased beyond $\approx 2000$, where the virial velocity becomes comparable with $V_c$, the relative importance of ‘late’, as compared with ‘early’, captures decreases, due to the increasing predominance of destructive encounters in the ‘late’ regime, and thus the $Q$ dependence is better preserved in the total binary yields at higher $N$ (models 10 and 11 cf 12 and 13).
Despite the very different histories of binary formation in uniform and isothermal models, it is notable how insensitive is the total binary yield to gross changes in the global density profile. Comparison of models 1-4, 6 and 7 with models 8-13 indicates that at comparable times the binary yields in the two cases differ by less than a factor two, the tendency being for isothermal models to produce somewhat fewer binaries than corresponding uniform models.

4.3 Clumpy clusters

The existence of hierarchical clustering in the initial conditions changes both the character of the collapse and the resultant binary yield as compared to the case of smooth initial conditions. In a system consisting of \( N_c \) clumps, with initial filling factor \( f_v \), the clumps collapse on themselves on a time scale equal to \( f_v^{1/2} \) times the cluster free fall time scale. If these time scales are separated by a factor of more than a few, the clumps will have undergone violent relaxation (and re-expanded to a dimension \( \sim \) half their initial sizes) by the time the whole cluster reaches maximum compression. Since the maximum collapse factor of the whole system is \( \sim N_c^{1/3} \), the clumps will not merge at the bounce in any cluster for which \( f_v \ll 8/N_c \). Consequently, such systems (such as model 14) evolve as an ensemble of \( N_c \) independent clumps, an expectation that we have confirmed by comparing the binary yield in this case with that from a single such clump (model 17). The binary fraction may therefore be increased in clumpy models due to two effects: a) the reduced system size and consequently increased density and b) if \( N/R \) is smaller for each clump than for the cluster as a whole, the lowered internal velocity dispersion reduces the incidence of fast, non-capturing encounters. From models 15, 16 and 1-5 in Table I (identical models apart from progressively greater \( N \) in the range 50 to \( 10^4 \)) we deduce that the binary yield increases with increasing \( N \) (because of increased density) until such point that the virial velocity becomes comparable with
for the disc to cluster radius of these models, the optimum binary yield is obtained for clusters numbering one to two thousand (Figure 9).

4.4 Dependence on Disc Parameters

We now consider the dependence of binary yield on disc parameters implied by equations (4.1)-(4.4). In the case where the velocity dispersion is below $\sim V_c$ the determining factor is $R_d$ alone, since most encounters that intersect the disc result in capture. Conversely, where the velocity dispersion exceeds $V_c$, only a fraction of star-disc encounters result in capture: mainly those that hit the disc within a radius well inside $R_d$, this critical radius depending on the strength of star-disc interaction, i.e. $M_d$.

For the case of clusters that remain in the former regime for much of a free-fall time, the chief disc quantity that determines the binary yield is $R_d$, the initial value of which being fixed by the angular momentum retained, or acquired, during the fragmentation process. One scenario, assumed by MCP, comes about from an analogy with the angular gained by protogalactic clouds. During their fragmentation, gravitational torques from neighboring fragments give the condensations an angular velocity such that their energy of rotation is a constant fraction ($\lambda$) of their gravitational binding energy. Taking the value $\lambda \approx 0.07$ found in cosmological simulations of fragmentation (Layzer 1963; Barnes & Efstathiou 1987) and assuming that the initial fragment dimensions are $\sim$ the initial interstellar separation, we thus obtain the scaling

$$R_d/R_h \sim 10^{-3}(N/10^6)^{-1/3}(\lambda/0.07). \quad 4.4.1$$

It may plausibly be argued, however, that such a picture is only appropriate in the case that fragments have had sufficient time to redistribute angular momentum in this way and that this may
not be the case in the ‘prompt initial fragmentation’ scenario envisaged here. If this is so, then the angular momentum per fragment is either gained as a result of the star formation process of cloud-cloud collision followed by thermal and gravitational instability, or that which it inherits from the rotation of the gaseous protocluster. The angular momentum expected to result in the former case is uncertain, but will almost certainly not be less than that expected in the latter case, which can therefore be treated as a lower limit. Provided that the initial cluster rotation is fixed by $\lambda$, due to encounters during the fragmentation of the protocluster clouds, then the resultant mean disc radius is again similar to that derived above (the reason for this may be understood by noting that as the fragments condense out of the background, their densities are comparable with the mean cluster density and thus the break-up angular velocity of cluster and of individual fragments are similar).

While these different processes lead to similar disk radii, they do have different consequences for the way that disc radii scale with stellar mass (see § 4.5 below) and also because the latter scenario ties the disc radii more directly to the rotation of the parent cluster. Variations in binary fractions would, in this case, be related to the angular momentum imparted to the cluster during its initial fragmentation. Such a scenario could, in principle, be tested observationally, though it would be extremely difficult, given the small rotational energies expected (and observed) in clusters.

In the numerical calculations we have taken $R_d = 5 \times 10^{-4} R_h$ in most of our models. Such a value is comparable to that implied by equation (4.4.1) for higher $N$ models. It is an order of magnitude greater than that employed by MCP, due to the higher densities for the star forming gas assumed in the previous work. Such differences should be taken as reflecting uncertainties in disk radii expected to result from the star formation process. We adopt the larger values throughout this study in order to reduce the computational inaccuracies involved in calculating very close
encounters.

We stress that though any value of $R_d$ may be linked, by the above arguments, to a cluster rotation rate, all our simulations have been undertaken with zero angular momentum clusters. We note that since a rotating cluster collapses by a factor $\lambda^2$ before being held up by centrifugal forces, and since a uniform cluster collapses by a factor $N^{1/3}$ before entering the ‘bounce’, rotation is dynamically unimportant for all clusters for which $N \lesssim 10^7(0.07/\lambda)^6$. A link between disk radii and angular momentum of the parent cluster does not, therefore, conflict with the observed lack of rotation in globular clusters.

We now turn to the case in which $M_d$ and $R_d$ undergo viscous evolution during the cluster collapse. Here, our N-body results show a remarkable insensitivity to such evolution on a time scale comparable with the cluster free-fall time scale. This result can be traced to the fact that angular momentum conservation requires $M_d$ evolves as $R_d^{-1/2}$ (for any viscosity prescription) so that the resultant capture rates scale as $\Gamma_{\text{cap}} \propto R_d$ and $\Gamma_{\text{cap}} \propto R_d^{-1/2}$ in the low velocity and high velocity regimes respectively. Thus we find that although viscous evolution boosts the capture rate somewhat at early times (slightly larger discs) this is offset by the reduced disc mass at late times (smaller fraction of capturing encounters). This insensitivity to viscous evolution results from the relatively large values of the initial disc radii used in our N-body calculations, which mean that the crossover from $v_s < V_c$ (equation 4.4) to $v_s > V_c$ occurs relatively early in the collapse. For smaller disc initial radii (as employed by MCP), viscous evolution can boost the binary yield by a factor of a few.

4.5 Dependence on Stellar Mass Function.
It is well known that in the process of violent relaxation, stars acquire a specific energy that is independent of their mass, thus behaving, in this respect, like test particles in the rapidly varying cluster potential. It is therefore no surprise that the global properties of the collapse are little affected by the introduction of a spread of stellar masses in models 20-22.

In this Section we consider the mass dependence of the binary formation process: specifically, we derive scalings for the distribution of companion masses, $M_2$, to stars of (fixed) mass $M_1$. To clarify the argument we consider here only the case $M_2 \ll M_1$, since in this case the degree of gravitational focusing of an encounter (for given boundary conditions at infinity) is $\sim$ independent of $M_2$. We note, from the property of violent relaxation alluded to above, that the velocity dispersion is independent of mass at all times, and that (owing to the lack of mass segregation) the mass function also remains spatially uniform at all times. It follows, therefore, that any deviation of the binary pairing process from random selection from the mass function must result from the nature of the star-disc interaction process: from the dissipative prescription employed and from the dependence of disc mass and radii on stellar mass.

In the numerical simulations, and in the scalings below, we employ the interaction prescription equation (2.1.2) and also the scalings $M_d \propto M_*$ and $R_d \propto M_*$. The disc mass scaling results from the (not unreasonable) assumption that the initial division of fragment mass into star and disc is scale free. The disc radius scaling results from assuming a scenario in which condensations are spun up to a constant fraction of break up during fragmentation (see discussion in § 4.4 above). If such is the case, fragments collapse by a constant factor from dimensions $\sim$ a Jeans length, $R_J$. Since, for fluctuations of various densities in an isothermal medium, $R_J \propto M_J$ (the Jeans mass) it follows that in this case $R_d \propto M_*$. We note, however, that if we adopt the alternative view (i.e. that the
angular momentum of condensations is inherited from the rotation of the parent cluster: see § 4.4) the resultant scaling would be $R_d \propto M_*^3$. We mention this point in order that the uncertainties entering these scalings can be appreciated.

The prescription for energy loss on star-disc interaction (equation 2.1.2) implies

$$\Delta E \propto M_{d1} M_2^2 + M_{d2} M_1^2$$

(4.5.1)

whereas the initial energy of the relative orbit $E_{orb} \propto M_2$. If $M_d \propto M_*$ (see above) then it follows that $\Delta E/E_{orb} \propto M_2$ if only $M_1$ has a disc whereas $\Delta E/E_{orb}$ is independent of $M_2$ if both stars have a disc or if $M_2$ only has a disc. If $M_1$ has a disc (whether or not $M_2$ has a disc) it follows that it is the dimension of $M_1$'s disc that mainly fixes whether an encounter takes place (since $R_d \propto M_*$), whereas if only $M_2$ has a disc it is clearly the size of its disc that is the determinant.

Putting these effects together we conclude that if both stars have discs the process is $\sim$ independent of $M_2$ (i.e. random picking from the mass function) whereas if only one star has a disc the capture process is biased against companions of low $M_2$. We can therefore understand the downturn in the companion mass function in Figure 8b and 8c as resulting both from the form of the energy loss prescription used and from the fact that lighter stars have smaller discs, and are thus less likely to be involved in star-disc encounters.

5. CONCLUSIONS

We have used N-body simulations of globular cluster collapse in order to clarify what are the major factors that determine the efficiency of binary formation through star-disc captures.

We find that in the case of large $N$ clusters with a smooth density profile, the main determinants
are the ratio of disc to cluster radii \( R_d/R \) and the initial stellar velocity dispersion (equation 4.1.1). Both these quantities are determined by the details of the fragmentation process, and the former also by the rotation of the protoglobular cloud. The maximum binary yield is obtained when the initial stellar velocity dispersion is as low as possible, that is of order the sound speed in the star forming gas. Using disc radii implied by the rotation rates estimated for protoglobulars from cosmological simulations, we obtain a maximum binary yield of 3\% in such systems, independent of \( N \) (equations 4.1.1, 4.1.3 and 4.4.1). It is notable that this maximum yield scales as \( \lambda \) where \( \lambda \) is the ratio of rotational to gravitational energy of the protostellar condensation (assumed to be 0.07 in the estimate above). If, instead of resulting from the process of star formation, the angular momentum of the protostars is inherited from the parent cluster, then we expect that more rapidly rotating clusters should yield a higher binary fraction.

The effect of viscous disc evolution (on a time scale comparable with the cluster collapse time scale) is to cause discs simultaneously to grow in size (binary promoting) and to shrink in mass (binary inhibiting). Our numerical simulations use rather large initial disc radii, for reasons of computational economy, and in this case further viscous growth hardly affects the binary yield: the effects of the above two processes roughly cancel. In previous (analytical) work using smaller initial disc sizes we showed that viscous evolution can boost the binary yield calculated above by a factor of a few (MCP).

Our N-body simulations (with \( N \) as high as 5000) show that the binary yield is remarkably insensitive both to changes in the initial stellar density profile and to the inclusion of a range of stellar masses. Changing the initial density distribution from uniform to that of an isothermal sphere changes the nature of the collapse, since, in the latter case, the inner regions collapse first
and, in re-expanding, interact with material infalling from larger radii. As a result, the history of binary formation is no longer marked by a well defined peak in production at maximum compression (the ‘bounce’). Remarkably, however, the binary yields are rather similar in the two cases, with comparable uniform models exceeding the isothermal models by less than a factor two.

The insensitivity of over-all binary yields to the inclusion of a spectrum of stellar masses may be understood by noting that, in the process of violent relaxation, the specific energy attained by each particle is independent of mass. The mass dependence of the binary pairing process is therefore governed only by the way that the star-disc energy loss prescription and the masses and radii of discs scale with stellar mass. Such details, which are not well understood theoretically at present, will determine the precise form of the binary statistics as a function of mass. We note, however, that since it is a reasonable expectation that more massive stars possess larger discs, it is likely that the binary pairing process is more biased toward massive stars than would result from random pairing from the mass function (Figure 8).

We stress that all the binary yields quoted here are total binary yields, without reference to whether they are hard or soft and are thus likely to survive in the cluster environment. In fact, since most star-disc capture binaries are formed with orbital velocities comparable with the local velocity dispersion at the time of formation, and since, in high $N$ clusters, most binaries are formed when the velocity dispersion is sub-virial, it follows that most of the binaries formed in this way are soft. If one assumes that all initially soft binaries are destined to be dissolved by encounters with field stars, then the resultant yields of surviving binaries are reduced considerably, to $\lesssim 1\%$. Calculations of the interaction between a pre-main sequence binary and material in circumbinary orbit however indicate that binary orbits can be efficiently shrunk during the pre-main sequence
stage, even where the mass of gas left in circumbinary orbit is a rather modest fraction of the binary mass (\(\sim 10\%\); Artymowicz et al 1991). If such is the case, then the bulk of binaries formed through star-disc captures could survive, although naturally, in this case, their period distributions would then reflect this subsequent orbital evolution, rather than the initial capture parameters.

Finally we note that binary yields considerably higher than those quoted above for smooth initial conditions can be obtained if the initial stellar distribution is clumpy. In this case, overdense regions, collapsing ahead of the general cluster, may cause the system to behave like an ensemble of independent small \(N\) systems over a number of dynamical time scales. This may considerably increase the yield, both because the virial velocity of each clump may be less than that of the cluster as a whole, implying a preponderance of capturing (as opposed to disc destructive encounters) over many dynamical times, and also because of the enhanced density within each clump. For example, we find that by dividing a cluster into an ensemble of stellar ‘nests’, each containing several hundred stars and several tenths of a parsec in radius, we obtain binary fractions of \(\geq 15\%\). Such lumpiness in initial conditions would persist over several cluster crossing times, and would be compatible with the structure seen in the Magellanic globulars of about that age.

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Table 1. The Models.

| Model | $N_*$ | $\rho_\star(R)$ | Q | Evolution | $R_d$ (pc) | IMF | $N_{bin}$ | $\tau_{bin}^d$ |
|-------|-------|-----------------|---|-----------|-----------|-----|----------|--------------|
| 1     | 500   | u               | 0.05 | no        | $10^{-3}$ | U   | 26       | 2.0          |
| 2     | 1000  | u               | 0.05 | no        | $10^{-3}$ | U   | 70       | 2.0          |
| 3     | 2000  | u               | 0.05 | no        | $10^{-3}$ | U   | 122      | 2.0          |
| 4     | 5000  | u               | 0.05 | no        | $10^{-3}$ | U   | 112      | 2.0          |
| 5     | 10000 | u               | 0.05 | no        | $10^{-3}$ | U   | 141      | 2.0          |
| 6     | 2000  | u               | 0.01 | no        | $10^{-3}$ | U   | 134      | 1.3          |
| 7     | 5000  | u               | 0.01 | no        | $10^{-3}$ | U   | 203      | 1.0          |
| 8     | 500   | i               | 0.05 | no        | $10^{-3}$ | U   | 25       | 1.0          |
| 9     | 1000  | i               | 0.05 | no        | $10^{-3}$ | U   | 44       | 1.0          |
| 10    | 2000  | i               | 0.05 | no        | $10^{-3}$ | U   | 66       | 1.5          |
| 11    | 5000  | i               | 0.05 | no        | $10^{-3}$ | U   | 72       | 1.3          |
| 12    | 2000  | i               | 0.01 | no        | $10^{-3}$ | U   | 87       | 2.0          |
| 13    | 5000  | i               | 0.01 | no        | $10^{-3}$ | U   | 121      | 2.0          |
| 14    | 2000  | c               | 0.01 | no        | $10^{-3}$ | U   | 317      | 2.0          |
| 15    | 100   | u               | 0.01 | no        | $10^{-3}$ | U   | 5        | 2.0          |
| 16    | 50    | u               | 0.01 | no        | $10^{-3}$ | U   | 1        | 2.0          |
| 17    | 200   | u               | 0.01 | no        | $5\times10^{-3}$ | U | 39     | 10.0         |
| 18    | 2000  | u               | 0.01 | yes       | $10^{-3}$ | U   | 138      | 1.3          |
| 19    | 2000  | i               | 0.01 | yes       | $10^{-3}$ | U   | 91       | 2.0          |
| 20    | 2000  | u               | 0.01 | yes       | $10^{-3}$ | S   | 134      | 2.0          |
| 21    | 2000  | u               | 0.01 | no        | $10^{-3}$ | S   | 134      | 2.0          |
| 22    | 2000  | i               | 0.01 | yes       | $10^{-3}$ | S   | 68       | 2.0          |

$^a$u = uniform distribution, i = $\rho_\star \propto R^{-2}$, c = clumpy distribution (see text).

$^b$no = no disk evolution, yes = disks evolve as described in text.

$^c$U = single stellar mass, S = Salpeter IMF.

$^d$Time of evaluation of $N_{bin}$ in units of cluster crossing time.
Figure Captions

**Figure 1.** (a) $N_{bin}$ vs. time for models 1-5 (solid, dotted, short dashed, long dashed, dot-dashed curves respectively). The crosses indicate the values of $N_{bin}$ and time when the models of MCP became nonlinear. (b) Histograms of the resulting semimajor axis distributions for models 1 and 5 (solid and short dashed). The vertical tick marks indicate, from left to right, the softening lengths used in the models 5 and 1 and the disk radii.

**Figure 2.** The number of binaries (solid curve) and encounters which do not lead to capture (short dashed curve) for model 5.

**Figure 3.** $N_{bin}$ vs time for $N = 2000$ systems with unevolving disks: models 3 (solid), 6 (dotted), 10 (short dashed) and 12 (long dashed).

**Figure 4.** $N_{bin}$ vs time for $N = 5000$ isothermal systems: models 11 (solid) and 13 (long dashed).

**Figure 5.** $N_{bin}$ vs time for uniform and clumpy systems: models 12 (short dashed) and 15 (clumpy).

**Figure 6.** (a) $N_{bin}$ vs time for models 6 and 18 (short dashed and solid) and $N_{loss}$ vs time in each case (long dashed and dotted). (b) Histograms of resulting semi-major axis distribution (short dashed and solid respectively).

**Figure 7.** (a) As Figure 6a for models 12 (short dashed and long dashed) and 14 (solid and dotted). (b) Histograms of resulting semi-major axis distribution (short dashed and solid respectively).

**Figure 8.** (a) Fraction of stars in binaries as a function of stellar mass for models 20 (solid) and 22 (short dashed). (b) Mass ratio distribution for binaries containing one member in range $0.7 - 1.3M_\odot$ for model 20 (solid) compared with distribution expected in the case of random picking from the mass function (dotted). (c) as (b) for model 22.

**Figure 9.** Binary fraction as a function of $Q$ for clusters with $R_d/R = 5 \times 10^{-4}$. Crosses (1-7) are N-body results for successively higher $N$ in the range $50$ to $10^4$. Other points are calculated by MCP method: solid points corresponding to equation (4.1.1) (solid line), diamonds (for $N = 10^6$) to equation (4.1.2) (dotted line) and crosses to the regime $Q < Q_{min}$ (equation 4.1.3) for $N = 10^4$. 

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