About the construction of p-adic QFT limit of TGD

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16. October 1995
Abstract

The p-adic description of Higgs mechanism provides excellent predictions for elementary particle and hadron masses. In this work the construction of p-adic field theory limit of Quantum TGD is considered. Topological condensate defined as a manifold obtained by gluing together p-adic spacetime regions with different values of \( p \) is identified as 'quantum average' space time defined as absolute minimum of effective action \( S_e \) associated with Kähler Dirac action defining configuration space geometry. p-Adic spacetime topology is ultrametric in accordance with the analogy with spin glass phase implied by the vacuum degenerarity of Kähler action. Symmetry arguments suggest that \( S_e \) in presence of fermion fields is just the super symmetrized Kähler Dirac action. Super symmetrization motivated by \( N = 1 \) super symmetry generated by right handed neutrino replaces H-coordinates with their super counterparts and requires color singletness of physical states appears as consistency condition. p-Adic \( Diff(M^4) \) and canonical transformations of \( CP_2 \) localized with respect to \( M^4 \) are approximate symmetries of the effective action and broken only by the nonflatness of the induced metric. Super gauge invariances in interior allow only massless many boson states with vanishing electroweak and color quantum numbers including graviton in accordance with the hypothesis that elementary particle quantum numbers reside on boundaries. Boundary components are idealized with world lines and Kac Moody Dirac spinors used in mass calculations describe the conformal degrees of freedom of the boundary component. Perturbation theory in powers of \( p \) converges extremely rapidly: UV divergences are absent but natural infrared cutoff is present.
1 Introduction

The p-adic description of Higgs mechanism provides excellent predictions for elementary particle and hadron masses (hep-th@xxx.lanl.gov 9410058-62). The description was based on rather general assumptions motivated by Quantum TGD. Canonical correspondence between p-adic and real numbers is an essential element of the model. p-Adic thermodynamics gives mass squared as a thermal expectation value and massivation results from the thermal excitation of Planck mass states. p-Adic length scale hypothesis states that fundamental basic length scales are given by $L_p = \sqrt{p}L_0$, $L_0 \simeq 1.824 \cdot 10^4 \sqrt{G}$ and that physically most interesting primes correspond to primes near prime powers of two. p-Adic super conformal invariance at boundary components together with the concept of Kac-Moody spinor are also cornerstone of the approach. The four-dimensionality of the algebraic extension allowing the existence of p-adic square root in the vicinity of p-adic real axis is essential element and this suggests that 4-dimensional p-adic conformal invariance might have key role in the the construction of p-adic field theory limit of TGD but it turns out that the actual approximate gauge symmetry in the interior of spacetime contains $Diff^4(M^4)$ and is therefore much larger than the group of p-adic conformal transformations ($Conf^4(M^4)$). What remained lacking from the spectrum was graviton. This is understandable since only the boundary degrees of freedom of 3-surface were taken into account in the calculation and it is interior degrees of freedom, which correspond to gravitational interaction. The topological description of graviton emission should be in terms of the decay of 3-surface via topological sum vertex rather than the decay of boundary component as in case of gauge interactions.

The study of the low energy field theory limit [Pitkanen] defined as a second quantized field theory inside p-adic convergence cube with side $L_p$ provides the practical manner to calculate at energies below Planck mass. This field theory limit is essentially the p-adic version of the standard model understood in perturbative sense since gauge group and the spectrum of light states apart from some exotic states are identical. The construction of this theory demonstrates how p-adicity solves the divergence problems of ordinary QFT.

i) The construction of the momentum eigenstates involves elegant number
theory. The discretization of momenta caused by the finite size of the convergence cube implies automatically the absence of infrared divergences.

ii) p-Adicity implies automatically the absence of ultraviolet divergences. Perturbation theory fails for certain critical size \( L_p \) of the p-adic convergence cube if self energy loops contain massless particles. p-Adic planewaves actually make possible to construct p-adic theory of integration and p-adic Fourier analysis, which is a central technical tool in the construction of the theory.

The second step in the program is the formulation of p-adic field theory limit of TGD inside convergence cube of size \( L_p \) taking into account only boundary degrees of freedom and treating boundary components as point like objects but taking into account the conformal degrees of freedom of boundary component somehow. The basic idea is to generalize the p-adic counterpart of the standard model to a second quantized field theory using as basic fields Kac-Moody spinors rather than ordinary finite component fields and to deduce basic vertices from the requirements of gauge and conformal invariance.

The third step would be the formulation of field theory description in length scales \( L > L_p \). This theory is completely analogous to nonperturbative QCD describing hadrons as basic objects. An attractive possibility is that this theory can be constructed as second quantized theory in the discrete lattice formed by p-adic convergence cubes (basic dynamical objects) and that vertices are essentially the n-point functions of the field theory inside single converge cube.

The fourth step would be the formulation of quantum field description for gravitational interaction in terms of the interior degrees of freedom. One should also construct a description for the topology changing reactions p-adic cubes describing for instance hadron reactions.

During the work it has become clear that time is perhaps ripe for an attempt to formulate the general p-adic field theory including both boundary and interior degrees of freedom. The construction could be based on the following basic ingredients.

a) Field theory limit is defined as a purely fermionic QFT on quantum averaged spacetime defined as an absolute minimum of effective action: minimization involves also a variation in the degrees of freedom associated with
3-surface whereas in the definition of Kähler function the absolute minimum is defined for the variations keeping the 3-surface fixed. Average spacetime depends on quantum state. The enormous vacuum degeneracy of Kähler action (any surface, which is submanifold of \( M^4 \times Y^2 \), where \( Y^2 \) is Lagrange submanifold of \( CP_2 \) having vanishing Kähler form) implies analogy with spin glass phase and suggests ultrametricity of the effective spacetime topology. A very general realization of ultrametricity is provided by the concept of topological condensate defined as a manifold obtained by gluing together spacetime regions with different values of p-adic prime \( p \). The hypothesis about RG invariance of Kähler action and symmetry arguments suggest that effective action is just the supersymmetrized p-adic counterpart of Kähler Dirac action defining configuration space geometry.

b) Criticality suggests that effective action allows p-adic conformal transformations of \( M^4 \) as symmetries. Actually Kähler action allows all diffeomorphisms of \( M^4 \) as symmetries in the excellent approximation that induced metric is flat. Also the canonical transformations of \( CP_2 \) coordinates localized with respect to \( M^4 \) coordinates are symmetries in same approximation in the interior of \( X^4 \). Both symmetries become exact if one requires that the infinitesimal generators of these transformations annihilate physical states.

c) \( N = 1 \) supersymmetry generated by right handed neutrino is realized if one assumes that quantized \( H \) coordinates contains super part. In \( CP_2 \) degrees of freedom super symmetrization is defined in preferred \( CP_2 \) coordinates, for which the action of \( su(2) \times u(1) \) is linear. These coordinates are determined up to a color rotation and internal consistency requires that physical states are color singlets (each p-adic region of spacetime is color singlet). This implies that \( H \) coordinate becomes dynamical quantum field and gauge bosons and gravitons correspond to 2- and 4-fermion bound states respectively. \( N = 1 \) supersymmetry and closely related Kappa symmetry make it possible to extend conformal symmetry to super conformal symmetry on boundaries. \( Diff(M^4) \) and canonical transformations have similar extension in the interior of \( X^4 \).

d) In the interior of \( X^4 \) the quantization in fermionic degrees of freedom relies on the properties of p-adic planewaves. \( Diff(M^4) \) and local canonical invariance pose additional constraints on the states. \( Diff(M^4) \) invariance allows massless boson states annihilated by all \( Diff(M^4) \) generators except translations. Graviton is contained in the spectrum but many fermion states are not possible in accordance with the basic assumption that partons cor-
respond to boundary components. p-Adic perturbation theory is free from
UV divergences but infrared cutoff is necessarily present.

e) The practical treatment of boundary components involves idealization to
world line and the use of Kac Moody spinors to describe the vibrational
degrees of freedom of boundary component. The assumption that Super
Virasoro generators are sums of Super Virasoro generators associated with
boundary component time coordinate and with 2-dimensional boundary com-
ponent explains Kac Moody Dirac equation used in the description of Higgs
mechanism. Conformal $so(3, 1)$, $so(4)$ and $su(3)$ are approximate symmetries
in boundary degrees of freedom. Second quantization must be performed for
Kac Moody spinors.

Although this rough general formulation need not yet offer practical cal-
culation receipes and need not even be correct, it makes possible to under-
stand p-adic topology as a property of quantum averaged spacetime and
demonstrates that p-adic super conformal invariance on boundaries, crucial
for p-adic description of Higgs mechanism, is not in conflict with the basic
principles of TGD. In principle the formulation offers a possibility to derive
vertices for simpler p-adic conformal field theory limit formulated in terms of
Kac Moody spinors only and treating boundary components as point like ob-
jects. If the general formulation is correct then the main unsolved technical
problem is the construction of the interaction vertices for N-S and Ramond
typer Kac Moody spinors.

The program of the work is following:
a) The general ideas of Quantum TGD are described.
b) p-Adic building blocks of the theory are described.
c) The concept of effective action is defined and the p-adization of spacetime
in terms of topological condensate concept is described.
d) The symmetries of the theory are discussed.
e) A formulation of field theory limit in terms of p-adicized Kähler Dirac
action in the interior of $X^4$ is discussed in detail.
f) Appendix contains general formulas for currents associated with symme-
tries.
2  Quantum TGD and p-adic field theory limit

In the following some basic facts about TGD are considered and shown to lead to quite unique general receipe for the construction of Quantum TGD.

2.1  Construction of configuration space metric and spinor structure

The construction of Quantum TGD reduces to the construction of geometry for configuration space consisting of all possible 3-surfaces in space $H = M_4 \times CP_2$. What is needed is metric and spinor structure. The construction is based on the assumption that the geometry in question is Kähler geometry. This means that configuration space allows complex structure and metric can be expressed in terms of Kähler function. A further requirement is $Diff(X^4)$ invariance and this requires that the definition of Kähler function associates to a 3-surface 4-surface at which $Diff^4$ can act.

The value of Kähler function for a given 3-surface $X^3$ is defined as absolute minimum of Kähler action in the set of 4-surfaces having $X^3$ as submanifold is defined as Maxwell action for Maxwell field defined by induced Kähler form of $CP_2$. Action contains also Chern-Simons term associated with boundaries of 4-surface. Definition associates to a given 3-surface the absolute minimum spacetime surface identifiable as 'classical history'. One interesting feature of the construction is that Kähler function defines generalized catastrophe theory in configuration space (studied in the third part of the book). Kähler action is analogous to thermodynamic free energy and can have several extrema and absolute minimum requirement selects one of the extrema just as minimization of free energy selects the thermodynamic phase. The properties of absolute minimum 4-surface can change discontinuously along 'Maxwell line' just as thermodynamic phase transition occurs along Maxwell line.

The definition of spinor structure necessitates the definition of anticommuting gamma matrices. These are defined for a given $X^3$ as linear superpositions of fermionic oscillator operators associated with second quantized Dirac action for induced spinors in the classical space time surface $X^4(X^3)$. The action is just free Dirac action on background defined by induced spinor connection in $X^4(X^3)$. 

The proposed more explicit construction of configuration space metric is carried on the set of 3-surfaces belonging to ‘lightcone boundary’ \( \delta M_4^4 \times CP_2 \). The motivation is that \( Diff(X^4) \) invariance dictates the metric from its initial values on light cone boundary. The generalized conformal structure of the light cone boundary having metric dimension 2 plus the assumption that metric allows infinite-dimensional Kac Moody symmetry (Lie algebra corresponds to the group of maps from \( \delta M_4^4 \) to \( su(3) \)) plays essential role in the construction. The conformal structure of light cone boundary and Kac Moody algebra structure should have some counterparts in the construction of conformal field field limit. It is to be emphasized that the construction of configuration space geometry and spinor structure is by no means a closed chapter of TGD and the support for the essential correctness of the picture comes from various applications rather than from rigorous mathematical formulation.

### 2.2 Some basic properties of Kähler action

#### 2.2.1 Criticality of the Kähler action

Quantum TGD should define a theory of everything. For instance, gauge couplings should be predictions of the theory. Kähler action contains Kähler coupling strength \( \alpha_K \) as free parameter and one should have some principle to determine the value of this parameter.

The key observation is that the vacuum functional of the theory is uniquely defined as exponential of Kähler function from the requirement that two ill defined determinants, namely the determinant of the configuration space metric and the Gaussian determinant associated with perturbative functional integration around minimum of Kähler function cancel each other. The vacuum functional is analogous to thermodynamic partition function and Kähler coupling strength is analogous to temperature. Therefore a natural requirement is that the value of Kähler coupling strength \( \alpha_K \) is such that Quantum TGD describes critical system.

The duality of \( CP_2 \) Kähler form supports the criticality hypothesis: in standard complex coordinates of \( CP_2 \) the geometric realization of duality transformation is as an orientation changing isometry \( \xi_1 \leftrightarrow \xi_2 \). In certain \( N = 1 \) supersymmetric theories duality transformation relates strong cou-
pling phase for particles with weak coupling phase for magnetic monopoles.
In self dual case coupling constant and action must be critical and particles and magnetic monopoles must be equivalent. The so called $CP_2$ type extremals [Pitkanen] identified as elementary particles can indeed be regarded as magnetic monopoles in homological sense. What is remarkable is that elementary particles create no long range $1/r^2$ type magnetic field since magnetic flux flows in $CP_2$ degrees of freedom: this is possible by to the nontrivial homology of $CP_2$. Also magnetic monopoles in ordinary sense are in principle possible but are not favoured by the minimization of Kähler action: magnetic $1/r^2$ field gives positive contribution to the action whereas electric $1/r^2$ field gives negative contribution and is therefore favoured.

2.2.2 Coupling constant evolution hypothesis

Coupling constant evolution hypothesis is based on the assumption that Kähler coupling strength is a fixed point of coupling constant evolution and on the observation that Kähler action can be interpreted as EYM action for induced gauge fields (electroweak gauge fields and gluons) and induced metric. Besides this one must include the field defined by induced gamma matrices $\Gamma_\alpha = \Gamma_k \partial_\alpha$, which was identified as a field describing the presence of fermions in [Pitkanen]. It seems that this interpretation is not correct but that new boson field is in question. The hypothesis gives nontrivial constraints between various coupling strengths in accordance with general features of the coupling constant evolution and p-adic field theory limits should provide quantum level realization for the coupling constant evolution hypothesis. The criticality is expected to fix the value of Kähler coupling strength as a quantity analogous to critical temperature so that in principle all coupling constants come out as predictions.

2.2.3 Vacuum degeneracy of Kähler action and canonical invariance

Kähler action allows enormous vacuum degeneracy: any 4-surface with $CP_2$ projection, which is Lagrange manifold is vacuum extremal. Lagrange manifold is defined as a submanifold of $CP_2$ with vanishing induced Kähler form with dimension $d \leq 2$. Canonical transformations leave Kähler form invariant and are obviously exact dynamical symmetries in vacuum sector.
The vacuum degeneracy implies that the second variation of Kähler action, whose inverse would define propagator in the usual quantization, is highly degenerate. In fact, for the standard imbedding of $M^4$ the lowest order term in action is fourth order in the deviation of $CP_2$ coordinates from constant values so that canonical quantization around flat solution is not possible. In Quantum TGD this kind of quantization is not needed. If field theory limit is defined as a field theory defined on quantum averaged spacetime defined by effective action this problem is not encountered. Since $CP_2$ coordinates are treated classically they define external fields for the quantum theory formulated in terms of fermionic fields only. This is in accordance with the result of Quantum TGD that boson quanta correspond fermion-antifermion bound states on boundaries. The existence of long range classical gauge boson fields, in particular $Z^0$ fields, have many applications in TGD so that the appearance of these fields in the formulation of quantum theory is a rather satisfactory feature.

The vacuum degeneracy makes TGD:eish Universe analogous to spin glass phase. Fermions correspond to spins, induced gauge potentials and induced metric correspond to couplings between spins and the presence of vacuum degeneracy implies that one must take average over a large number of vacuum configurations with different gauge fields. Induced Kähler form is analogous to the frustration field. Ultrametricity is the basic property of the configuration space for the minima of effective action in spin glass phase. This suggests that the effective spacetime topology associated with effective action is ultrametric. P-Adic ultrametricity is especially attractive since it makes possible p-adic conformal invariance. The most general form of p-adic topology is obtained by gluing together p-adic regions of spacetime with different p-adic $p$ along their common boundaries in a manner to be described later. Topological condensate is p-adic manifold $.. < p_1 < p_2...$ consisting of p-adic surfaces glued together along common boundaries with the property that the sizes of the surfaces form increasing hierarchy.

3 p-Adic building blocks

The construction of p-adic field theory limit necessitates the generalization of the basic tools of standard physics such as differential and integral calculus, the concept of Hilbert space, Riemannian geometry, group theory, action
principles, probability and unitary concepts to p-adic context. The construction of all these tools relies heavily on the canonical identification of p-adic and real numbers.

3.1 Canonical identification

The canonical identification $x = \sum m x_m p^n \in R_p \rightarrow \sum x_n p^{-n} \equiv x_R \in R$ of p-adic and real numbers is the cornerstone of p-adic TGD. The success of the elementary particle mass calculations and the possibility to understand connection between p-adic and real probabilities suggests that canonical identification is related to some deeper mathematical structure. The following inequalities hold true:

\[
(x + y)_R \leq x_R + y_R \\
|x|_p \leq (xy)_R \leq x_R y_R
\]  

(1)

where $|x|_p$ denotes p-adic norm. These inequalities can be generalized to case of $(R_p)^n$ (linear space over p-adic numbers).

\[
(x + y)_R \leq x_R + y_R \\
|\lambda vert_p \leq (\lambda y)_R \leq \lambda_R y_R
\]  

(2)

where the norm of the vector $x \in T^n_p$ is defined in some manner. The case of Euclidian space suggests the definition

\[
(x_R)^2 = (\sum_n x_n^2)_R
\]  

(3)

These inequalities resemble those satisfied by vector norm. The only difference is the failure of the linearity in the sense that the norm of scaled vector is not obtained by scaling the norm of original vector. Ordinary situation prevails only if scaling corresponds to power of $p$.

These observations suggests that the concept of normed space or Banach space might have generalization and physically the generalization might apply to the description of highly nonlinear systems such as TGD. The nonlinearity would be concentrated in the nonlinear behaviour of the norm under scaling.
Amusingly, the p-adic counterpart of Minkowskian norm

$$ (x_R)^2 = (\sum_k x_k^2 - \sum_l x_l^2)_R $$

produces nonnegative norm. Clearly the p-adic space with this norm is analogous to future light cone.

### 3.2 Existence of square roots of p-adically real numbers and 4- and 8-dimensional extensions of p-adic numbers

The existence of square root plays key role in various p-adic considerations.

a) The p-adic generalization of the representation theory of the ordinary groups and Super Kac Moody and Super Virasoro algebras exists provided an extension of p-adic numbers allowing square roots of p-adically real numbers is used. The reason is that the matrix elements of the raising and lowering operators as well as oscillator operators typically involve square roots. Also the definition of anticommuting Gamma matrix fields in the representations of Super Virasoro algebra involves square root function $\sqrt{Z}$ of algebraically extended p-adic number.

b) The existence of square root is a necessary ingredient in the definition of p-adic unitarity and quantum probability concepts.

c) p-adic Riemannian geometry necessitates the existence of square root since the definition of the infinitesimal length $ds$ involves square root.

For $p > 2$ the minimal dimension for the algebraic extension allowing square roots is $D = 4$. For $p = 2$ the dimension of the minimal extension is $D = 8$. This implies that the dimension of the p-adic Riemannian geometries considered in the first part of the paper is multiple of 4 or 8 depending on whether $p > 2$ or $p = 2$ so that p-adic Riemannian geometry gives very deep explanation for spacetime and imbedding space dimensions.

In $p > 2$ case the general form of extension is

$$ Z = (x + \theta y) + \sqrt{p}(u + \theta v) $$

(5)
where the condition $\theta^2 = x$ for some p-adic number $x$ not allowing square root as a p-adic number. For $p \mod 4 = 3$ $\theta$ can be taken to be imaginary unit. This extension is natural for p-adication of spacetime surface so that spacetime can be regarded as a number field locally. Imbedding space can be regarded as a cartesian product of two 4-dimensional extensions locally.

In $p = 2$ case 8-dimensional extension is needed to define square roots. The extension is defined by adding $\theta_1 = \sqrt{-1} \equiv i$, $\theta_2 = \sqrt{2}$, $\theta_3 = \sqrt{3}$ and the products of these so that the extension can be written in the form

$$Z = x_0 + \sum_k x_k \theta_k + \sum_{k<l} x_{kl} \theta_{kl} + x_{123} \theta_1 \theta_2 \theta_3$$

Clearly, $p = 2$ case is exceptional as far as the construction of the conformal field theory limit is considered since the structure of the representations of Virasoro algebra and groups in general changes drastically in $p = 2$ case. The result suggest that in $p = 2$ limit spacetime surface and $H$ are in same relation as real numbers and complex numbers: spacetime surfaces defined as the absolute minima of 2-adiced Kähler action are perhaps identifiable as surfaces for which the imaginary part of 2-adically analytic function in $H$ vanishes.

The physically interesting feature of p-adic group representations is that if one doesn’t use $\sqrt{p}$ in the extension the number of allowed spins for representations of $SU(2)$ is finite: only spins $j < p$ are allowed. In $p = 3$ case just the spins $j \leq 2$ are possible. If 4-dimensional extension is used for $p = 2$ rather than 8-dimensional then one gets the same restriction for allowed spins.

3.3 p-Adic spacetime and topological condensate as generalized manifold

p-Adic topology and differentiability rather than ordinary topology and differentiability are useful concepts above some length scale $L_0$ of the order of $10^4$ Planck lengths as suggested by the mass scale argument. The p-adic realization of the Slaving Principle suggests that various levels of the topological condensate correspond to different values of p-adic prime $p$: the larger the
length scale the larger the value of $p$ and the course the induced real topology. If the most interesting values of $p$ indeed correspond Mersenne primes the number of most interesting levels is finite: at most 12 levels below electron length scale: actually also primes near prime powers of two seem to be physically important. According to the experiences achieved from the study low energy field theory limit and contrary to the original expectations, $L_p = \sqrt{p}L_0$ serves as infrared cutoff rather than ultraviolet cutoff for the p-adic field theory. If length scale resolution is larger than $L_p$ ordinary real physics should be an excellent approximation.

The existence of p-adic definite integral (relying on the canonical identification between p-adic and real numbers) and of p-adic valued inner product makes possible to define the concept of Riemannian metric and related structures on spacetime surface and the concept of submanifold with induced metric and spinor structure makes sense. Basic symmetry groups (in particular, Poincare group) acting as isometries of the imbedding space have p-adic counter parts. It is possible define variational principles for the fields defined in p-adic spacetime using p-adization of ordinary real action principle and the minimization of the Kähler action leads to the p-adic counterparts of field equations having same form as in real case.

The p-adic realization of the topological condensate concept requires rather general manifold concept. This manifold has locally p-adic topology but decomposes into regions with different values of p-adic prime $p$. These different regions are glued together along their boundaries. The mathematical description of the gluing operation for two p-adic topologies with different values of $p$ is based on the use of canonical identification map. What is involved is the identification map identifying the boundaries of regions with topologies $p_1$ and $p_2$. A possible manner to perform this identification is via two steps:

a) The points $x_{p_1}$ of $R^n_{p_1}$ (region of boundary is subset of $R^n_{p-1}$) are mapped via canonical identification to the points of $R^n$: $x_{p_1} \rightarrow (x_{p_1})_R$.

b) These points of $R^n$ are expressed using $p_2$-adic pinary expansion and mapped via the inverse of the canonical identification map to $R^n_{p_2}$, which is two valued for real numbers with finite number of $p_2$-nary digits.

A more general but essentially equivalent manner to describe gluing operation is to map the common boundaries of $p_1$- and $p_2$-adic regions to regions of
and to identify these two regions of $\mathbb{R}^n$ with differentiable invertible map.

The possible difficulties in the construction are related to the nonuniqueness of the inverse of the canonical identification map implying that real numbers with finite number of pinary digits have two inverse images.

The construction generalizes to the gluing together p-adic submanifolds of $H$. For $p_1$ and $p_2$ imbedding space is regarded as $p_1$ and $p_2$-adic manifold respectively. What is required is that the real counterparts for the imbeddings space points associated with identified boundary points are identical. This means that the condition

$$ (h^k(x_{p_1}))_R = (h^k(x_{p_2}))_R \text{ for } (x_{p_1})_R = (x_{p_2})_R $$

holds true. For large values of $p$ already the lowest pinary digit gives an excellent discretization of the p-adic convergence cube and in this approximation the conditions defining gluing are rather simple numerically. Analytically the gluing conditions do not look tractable.

### 3.4 p-Adic unitarity and probability concepts

p-Adic unitarity and probability concepts lead to highly nontrivial conclusions concerning the general structure of the S-matrix. S-matrix can be expressed as

$$ S = 1 + i\sqrt{p}T $$

$$ T = O(p^0) $$

for $p \mod 4 = 3$ allowing imaginary unit in its four-dimensional algebraic extension. Using the form $S = 1 + iT$, $T = O(p^0)$ one would obtain in general transition rates of order inverse of Planck mass and theory would have nothing to do with reality. Unitarity requirement implies iterative expansion of $T$ in powers of $p$ (Pitkänen, the few lowest powers of $p$ give extremely good approximation for the physically most interesting values of $p$.

The relationship between p-adic and real probabilities involves the hypothesis (Pitkänen) that transition probabilities depend on the experimental resolution. Experimental resolution is defined by the decomposition of the
state space \( H \) into direct sum \( H = \oplus H_i \) so that experimental situation cannot differentiate between different states inside \( H_i \). Each resolution defines different real transition probabilities unlike in ordinary quantum mechanics. Physically this means that the experimental arrangements, where one monitors each state in \( H_i \) separately differ from the situation, when one only looks whether the state belongs to \( H_i \). One application is related to the momentum space resolution dependence of the transition probabilities. More exotic application \cite{Pitkanen} is related to \( Z^0 \) decay widths: the total annihilation rate to exotic lepton pairs (unmonitored) is essentially zero: if one would (could) monitor each exotic lepton pair one would obtain simply sum of the rates to each pair.

3.5 p-Adic thermodynamics and transition probabilities

The p-adic field theory limit as such is not expected to give a realistic theory at elementary particle physics level. The point is that particles are expected to be either massless or possess Planck masses. The p-adic description of Higgs mechanism documented shows that p-adic thermodynamics provides the proper formulation of the problem. What is thermalized is Virasoro generator \( L_0 \) (mass squared contribution is not included to \( L_0 \) so that states do not have fixed conformal weight). Temperature is quantized purely number theoretically in low temperature limit \((\exp(H/kT) \to p^{L_0/T}, T = 1/n)\): in fact, partition function does not even exist in high temperature phase. The extremely small mixing of massless states with Planck mass states implies massivation and predictions for elementary particle masses are in excellent agreement with experimental masses. Thermodynamic approach also explains the emergence of the length scale \( L_p \) for a given p-adic condensation level and one can develop arguments explaining why primes near prime powers of two are favoured.

In the calculation of S-matrix thermodynamics approach should work also. What is is needed is thermodynamical expectation value for transition amplitudes squared over incoming and outgoing states. In this expectation value 3-momenta are fixed and only mass squared varies.
3.6 p-Adic cancellation of UV divergences

p-Adic cancellation of the ultraviolet divergences is based on very simple observations. The necessary infrared cutoff $L_p$ implies infrared cutoff and discretization for momentum: what one has is just quantum field theory in finite 4-dimensional box. Loop integrals over the loop momenta are replaced by summation over momenta and for a given value of the propagator expression one always gets integer valued degeneracy factor $N$ giving the number of momentum combinations giving rise to this particular value of the propagator expression. Since integer has always p-adic norm not larger than one this means that the summations over the loop momenta converge trivially provided the integers $N$ are finite.

For standard $M^4$ metric it was found that for spacelike propagator momentum $P_m$ the degeneracy factor $N = N(P_m, k)$ is for certain values of propagator momentum $k^2$ infinite as real number and although the p-adic norm of $N(P_m, k)$ is not larger than one it is ill defined p-adically. It was found that the gravitational warping of the standard imbedding of $M^4$ ($CP_2$ coordinates cease to be constant and $CP_2$ projection of spacetime becomes geodesic circle) to flat nonstandard imbedding transforms the induced metric $m_{\alpha\beta}$ to $m_{\alpha\beta} - p^2KR^2P_\alpha P_\beta$, where $P$ is the total momentum associated with the p-adic convergence cube. Replacing the standard inner product for momenta with the inner product defined by the warped metric the degeneracies associated with the p-adic propagator expression become finite.

The necessity to take into account the gravitational warping of the flat spacetime means that one cannot totally neglect the action of the fermionic matter on the background spacetime geometry. The back-reactions related to other conserved charges are also expected to be present but one might hope that these effects are small. In principle one can take into account back reactions exactly by interpreting field equations as expectation values between various states. In particular, $H$-coordinates can depend on various conserved charges besides four momentum, boundary components serve as sources of classical gauge fields, etc..
3.7 p-Adic Fourier analysis

The definition of p-Adic integration is based on two ideas. The ordering for the limits of integration is defined using canonical correspondence. \( x < y \) holds true if \( x_R < y_R \) holds true. The integral functions can be defined for Taylor series expansion by defining indefinite integral as the inverse of the differentiation. As far as physical applications is considered this definition does not seem to be the correct one since p-adic plane waves are not analytic functions. p-Adic planewaves are certainly needed since ordinary exponential functions \( \exp(ikx) \) have unphysical properties and orthonormalization does not have seem any obvious physical meaning. For p-adic planewaves orthogonality is natural in light of their periodicity properties. Therefore in the sequel all functions are assumed to be expressible as p-adic Fourier series using p-adic plane wave basis whose detailed definition is given in [Pitkänen].

In the region \( 0 \leq x_R \leq (p-1) \) the basis functions are given by

\[
f^k(x) = a^{kx}, \quad k = 0, \ldots, p-1
\]

\[
a^{p^{-1}} = 1
\]

In smaller regions \( 0 \leq (x - x_n)_R \leq (p-1)/p \) the same functional form applies expect that p-adic \( k \) is scaled to \( k/p \). One can associate with the cm coordinate \( x_n = n \) of this region p-adic planewave with momentum so that one obtains planewaves with momenta \( m/p + k, \quad m, k = 0, \ldots, p-1 \). This process of going to smaller and smaller length scales can be continued indefinitely and one obtains p-adic planewaves with infrared cutoff in momentum. Momentum eigenstate basis has as its real analog so called wavelet basis [Combes et al.]: this basis has been found to be especially effective in the numerical treatment of wave equations.

p-Adic planewaves satisfy periodic boundary conditions \( f_k(0) = f_k(p-1) = 1 \). One can define also p-adic planewaves satisfying antiperiodic boundary conditions as square roots of p-adic planewaves for odd values of momentum \( k \). The square root indeed exists for the extension allowing square root of real p-adic number. These two kinds of planewave basis are essential for the realization of Ramond and N-S type representations of Super Virasoro.

The orthonormalization of p-adic planewaves gives definition of p-adic integration in the range \( (0, p-1) \). All other p-adic planewaves except constant
\( k = 0 \) planewave have vanishing integral. Constant planewave gives integral, which can be chosen to be some constant. This definition of integration is of central importance in the formulation of variational principles based on Lagrangian density.

Although p-adic planewaves are real they allow complex conjugation type operation defined by \( k \to -k \). p-Adic planewaves possess infrared cutoff and this means that p-adic momentum \( k \) can have only a finite number of non-negative pinary digits whereas \(-k = k(p-1)(1 + p + p^2 + \ldots)\) has infinite number of pinary digits. Therefore only positive values of \( k \) appear in the Fourier expansion of any function \( f \) and the conjugate of \( f \) appearing in the definition of the inner product do not belong to the Hilbert space in question so that Hilbert space and its dual are not identical for p-adic planewaves. For higher dimensional p-adic planewaves all momentum components have finite number of pinary digits and this implies some new features in quantization. For instance, for fermion field positive energy modes creating fermions have p-adic momenta have \( k_i > 0 \) for each \( i \) whereas negative energy modes destructing antifermions have p-adic momenta \( k_i < 0 \) for each \( i \). The generalization of positive/negative energy concept to all momentum components makes possible to realize approximate \( Diff(\mathbb{M}^4) \) invariance in a nontrivial manner.

There is problem related to the definition of derivative for p-adic planewave. p-adic planewave is not differentiable in ordinary p-adic sense. The analogy with real planewaves

\[
a^{kx} = \exp \left( \frac{i 2\pi k x}{p-1} \right) = \ln(a) k
\]

suggests

\[
da^{kx}/dx = \ln(a) k a^{kx} = \frac{i 2\pi k}{(p-1)} a^{kx}
\]

which however does not exist p-adically. There are several manners to avoid the problem. Consider first the tricky manners:

i) Define \( \pi \) as the p-adic counterpart of real \(\pi\) obtained via the canonical
identification. The presence of the imaginary unit is troublesome since p-adic planewaves are real in the ordinary sense.

ii) Define $\ln(a)$ as the p-adic counterpart of $\ln(a_R)$ defined by the canonical identification. In this case one avoids imaginary units.

Neither of these tricks is needed in the formulation of the p-adic field theory limit of TGD since the action is well defined apart from a proportionality factor $\ln(a)^{-D}$, $D = 4$ in interior and $D = 3$ on boundaries and by defining the interaction measure $dx$ to be proportional to $1/\ln(a)$ for each coordinate $x^k$ the action becomes independent of $\ln(a)$. This is seen by following arguments:

a) For surfaces representable as maps from $M^4$ to $CP_2$ (the restriction is not essential) $CP_2$ coordinates must allow p-adic Fourier expansion and therefore $CP_2$ part of the induced metric is necessarily proportional to $\ln(a)^2$. The same must be true for $M^4$ part of the induced metric and the imbedding is of the following general form

\[ m^k = \ln(a)x^k + \text{Fourier expansion} \]
\[ s^k = \text{Fourier expansion} \tag{12} \]

The form of $M^4$ coordinate does not lead to difficulties since metric of $H$ does not depend on $M^4$ coordinates and $m^k$ appears only in p-adic planewaves $exp(p_km^k) = a^{kp}x^k$. In $M^4$ coordinates p-adic planewaves are formally similar to ordinary planewaves.

b) Also the super symmetrization of Kähler action to obtain effective action is possible by the replacement $h^k \rightarrow h^k_S \equiv h^k + \bar{\Psi}\gamma^k\Psi$. The supersymmetrized planewaves $exp(m_S^k p_k)$ are well defined

\[ exp(p_km^k_S) = a^{p_k}x^k exp(p_k\bar{\Psi}\gamma^k\Psi) \tag{13} \]

b) Induced spinor connection is proportional to $\ln(a)$ and in Dirac operator $\ln(a)$ dependences coming from induced gamma-matrices and covariant derivative cancel each other. Kähler Dirac action density is proportional to $\ln(a)^D$, $D = 4(3)$ in interior (boundary) and by defining coordinate differential $dx^k$ so that it contains $\ln(a)$ factor one obtains completely well defined action density.
4 Length scale hypothesis

p-Adic length scale hypothesis states that the length scales $L_p = \sqrt{p}L_0$ associated with primes near prime powers of 2, especially Mersenne primes are physically especially interesting. The results of the mass calculations give convincing support for the hypothesis and explain the emergence of $L_p$ for a given $p$ in terms of p-adic thermodynamics for Super Virasoro representations. There are also many arguments giving partial explanation for Mersenne primes and primes near prime powers of two. In the following one of the arguments for believing that primes near prime powers of two are physically most interesting is produced and the physical interpretation of $L_p$ and $L_0$ is considered.

4.1 Length scales defined by prime powers of two and Finite Fields

Mersenne primes ought to be physically especially interesting primes by arguments relying on 2-adic fractality. Above $M_{127}$ there is extremely large gap for Mersenne primes and this suggests that there must be also other physically important primes. Certainly all primes near powers of 2 define physically interesting length scales by 2-adic fractality but there are too many of them. The first thing, which comes into mind is to consider the set of primes near prime powers of two containing as special case Mersenne primes. One can indeed find a good argument in favour of these length scales, which play central role in the p-adic description of Higgs mechanism.

TGD Universe is critical at quantum level and criticality is related closely to scaling invariance. This suggests that unitary irreducible representations of p-adic scalings $x \rightarrow p^m x$, $m \in \mathbb{Z}$ should play central role in quantum theory. Unitarity requires that scalings are represented by a multiplication with phase factor and the reduction to a representation of finite cyclic group $Z_m$ requires that scalings $x \rightarrow p^m x$, $m$ some integer, act trivially. In ordinary complex case the representations in question correspond to phase factors

$$\Psi_k(x) = |x|^{ik2\pi/(\ln(p))} = \exp(i\ln(|x|)k2\pi/\ln(p)), \quad k \in \mathbb{Z}$$

and the reduction to a representation of $Z_m$ is also possible but there is no good reason for restricting the consideration to discrete scalings.

a) The Schrödinger amplitudes in question are p-adic counterparts of
ordinary complex functions $\Psi_k(x) = \exp(i\ln(|x|)k \frac{2\pi}{\ln(p)})$, $k \in \mathbb{Z}$. They have unit p-adic norm, they are analogous to planewaves, they depend on p-adic norm only and satisfy the scaling invariance condition

\[
\Psi_k(p^m x | p \to p_1) = \Psi_k(x | p \to p_1) \\
\Psi_k(x | p \to p_1) = \Psi_k(|x| | p \to p_1) \\
|\Psi_k(x | p \to p_1)|_p = 1 \tag{14}
\]

which guarantees that these functions are effectively functions on p-adic numbers with cutoff performed in $m$:th power.

b) The solution to the conditions is suggested by the analogy with the real case:

\[
\Psi_k(x | p \to p_1) = \exp(i kn(x) \frac{2\pi}{m}) \\
n(x) = \ln_p(N(x)) \in \mathbb{N} \tag{15}
\]

where $n(x)$ is integer (the exponent of the lowest power of p-adic number) and $k = 0, 1, ..., m - 1$ is integer. The existence of the functions is however not obvious. It will be shortly found that the functions in question exist in $p > 2$-adic for all $m$ relatively prime with respect to $p$ but exist for all odd $m$ and $m = 2$ in 2-adic case.

c) If $m$ is prime (!) the functions $K = \Psi_k$ form a finite field $G(m, 1) = \mathbb{Z}_m$ with respect to p-adic sum defined as the p-adic product of the Schrödinger amplitudes

\[
K + L = \Psi_{k+l} = \Psi_k \Psi_l \tag{16}
\]

and multiplication defined as

\[
KL = \Psi_{kl} \tag{17}
\]

So, if the proposed Schödinger amplitudes possessing definite scaling invariance properties are physically important then the length scales defined by
the prime powers of two must be physically special since Schrödinger amplitudes or equivalently, the p-adic scaling momenta \( k \) labeling them, have natural finite field structure. By Slaving Hierarchy Hypothesis also the p-adic length scales near prime powers of two (and perhaps of prime \( p > 2 \), too) are therefore physically interesting.

As shown in [Pitkänen], for \( p > 2 \) only those finite fields \( G(p_1, 1) \) for which \( p_1 \) is factor of \( p - 1 \) are realizable as representation of phasefactors whereas for \( p = 2 \) all \( G(p_1, 1) \) allow this kind of representation. Therefore \( p = 2 \)-adic numbers are clearly exceptional. In p-adic case the functions \( \Psi_p(x, |p \rightarrow p_1) \) give irreducible representations for the group of p-adic scalings \( x \rightarrow p^m x, m \in \mathbb{Z} \) and the integers \( k \) can be regarded as scaling momenta. This suggests that these functions should play the role of the ordinary momentum eigenstates in the quantum theory of fractal structures. The result motivates the hypothesis that prime powers of 2 and also of \( p \) define physically especially interesting p-adic length scales: this hypothesis will be of utmost importance in future applications of TGD.

The ordinary p-adic planewaves associated with translations can be constructed as functions \( f_k(x) = a^{kx}, k = 0, \ldots, n, a^n = 1 \). For \( p > 2 \) these planewaves are periodic with period \( n \), which is factor of \( p - 1 \) so that wavelengths correspond to factors of \( p - 1 \) and generate finite number of physically favoured length scales. These planewaves form finite fields for prime factors \( n \) of \( p - 1 \).

There are also additional arguments supporting the special role of primes near prime powers of two.

\( a) \) If 2-adicity is fundamental then 2-adic thermodynamics implies temperature quantization in the form \( \exp(1/T) \rightarrow 2^n \). An interesting possibility is that the allowed 2-adic temperatures are not only integers but also primes: \( n = p_1 \). If for p-adic primes \( p \) Boltzmann factor is as near as possible to an allowed 2-adic Boltzmann factor: \( \exp(T) \rightarrow p^n \sim 2^{p_1} \), one obtains \( p \sim 2^{p_1} \).

\( b) \) Fast Fourier transform is fastest, when the number of wavevectors is power of two so that spacetime volume has size \( 2^m L_0 \), where \( L_0 \) is discretization interval.
4.2 Physical interpretation of $L_p$

The tentative interpretation of $L_p$ proposed in Pitkänen was as a lower cutoff length scale for p-adic field theory. The study of the low energy field theory counterpart of p-adic TGD [Pitkänen] however demonstrated that $L_p$ is in fact infrared cutoff and UV divergences cancel without any momentum cutoff. This results give justification for the basic TGD:eish picture of the topological condensate. p-Adic convergence cubes of size $L_p$ are the p-adic version of topological field quanta introduced in Pitkänen, and in certain sense largests 3-surfaces possible at a given p-adic condensation level. The topological condensation at level $p_1 > p$ is possible and at this level these structures become particles perhaps describable using field theory approach below length scales $L_{p_1}$. Therefore the original assumption is correct only in the sense that $L_p$ is a natural lower cutoff length scale at level $L_{p_1}$, when extended objects are treated as point particles. One must also consider the possibility that the formation of larger structures by gluing p-adic convergence cubes together by tubes connecting together neighbouring boundaries. This mechanism is in fact in key role in the suggested hadron physics, nuclear physics and condensed matter applications of p-adic TGD (see the third part of the book). For instance, the color flux tubes connecting valence quarks can be identified as join along boundaries bonds.

There is also a second, purely kinematical, length scale generation mechanism: the construction of p-adic planewaves [Pitkänen] demonstrated that there is a preferred set of p-adic wavelengths, which are proportional to the factors of $p - 1$, whose number is relatively small for the physically most interesting primes $p$.

Length scale hypothesis leads to a picture of the topological condensate as more or less hierarchical structure consisting of almost flat 3-surfaces $... < p_1 < p_2 ...$ obeying effective p-adic topology so that the size of the surface increases with $p$ and has rough order of magnitude given by $L_p$. This concept of 3-space replaces ordinary 3-space with many sheeted structure. For instance, ordinary macroscopic bodies can be regarded as 3-surfaces, whose outer boundaries correspond to the surface of the body and boundaries are possible even in the astrophysical length scales.

An obvious question is what happens above the length scale $L_p$. a) p-Adic field theory description certainly fails. The application of the length
scale hypothesis in condensed matter length scales suggests that upper bound for the size of the 3-surface at condensate level \( p \) is typically of order \( 10^2 L_p \). For nuclei and hadrons the length scale is of order few \( L_p \).

b) The basic condensation level for atoms is \( p \simeq 2^k \), \( k = 131 \) and \( L_{131} \) is about one half of Bohr radius of hydrogen atom. The p-adic counterpart of the hydrogen atom seems to differ in many respects from real hydrogen atom. Coulomb potential contains factor \( 1/2\pi \) coming from the Fourier transform of photon propagator and is not p-adically well defined since \( e^2 \) is certainly rational so that one must work in momentum space and use discrete p-adic planewaves. Because of \( \pi \) the standard expression for the energy levels of hydrogen atom does not make sense p-adically. The p-adic versions of the spherical harmonics do exist (as homogenous polynomials of coordinate variables) and states with definite \( J \) provide a good ansatz for Schrödinger equation. There are number theoretic problems with the definition of energy \( E = M - \Delta E \). The point is that rest mass defined as the square root of \( M^2 \) thermal expectation value for electron at \( k = 137 \) level need not exist: the reduction of p-adic Schrödinger or Dirac equation to the standard form however requires this. It is unclear whether the energies are sufficiently close to the energies of the real hydrogen atom. All this suggests that real Schrödinger equation provides the correct description of hydrogen atom.

d) Quite generally the success of the real physics in long length scales suggests that p-adicity is not important in length scales above \( L_p \). The generation of lattice structures in condensed matter physics might be regarded as a direct manifestation of underlying p-adicity (basic structural unit being p-adic converge cube of size \( L_p \)) although real physics gives a good description of the situations.

All this suggests that although the p-adic condensation levels obey p-adic topology also in length scales \( L > L_p \) real physics must provide an excellent description of physics in length scales \( L > L_p \) and that p-adicity becomes important only below \( L < L_p \). The only manner to avoid contradiction is that ordinary real physics works, when the length scale resolution is \( L_p \), which in numerical practice means discretized physics using rational numbers. In the more formal level this means that the effective action associated with action obtained by functionally integrating in Kähler action degrees of freedom with momenta larger than \( 1/L_p \) defines in a good approximation spacetime with real topology. This hypothesis is in accordance with the fact that real conti-
nuity implies p-adic continuity. The conclusion is in conflict with the original 
idea that length scale cutoff leads to effective p-adic topology and more in 
concordance with the more standard assumption of p-adic physics that p-
adicity becomes important below some cutoff length scale \( l \) usually assumed 
to be equal to Planck length scale. Another manner to say the same thing 
is that quantum fluctuations in spacetime geometry are negligible in length 
scales larger than \( L_p \) so that effective action obtained by averaging Kähler 
Dirac action and integrating over small length scale degrees of freedom is real 
rather than p-adic.

An attractive possibility is that p-adic field theory description at length 
scales \( L > L_p \) can be based on discrete QFT in the lattice formed by p-adic 
convergence cubes and that the vertices are given by n-point functions of the 
convergence cube.

### 4.3 Physical interpretation of the length scale \( L_0 \)

The fundamental p-adic mass scale \( L_0 \) can be deduced from the experimental 
value of electron mass in p-adic description of Higgs mechanism.

\[
L_0 = \frac{\pi}{m_0} \\
m_e = \sqrt{\frac{(5 + \frac{2}{3} + \Delta)m_0}{\sqrt{M_{127}}}} \\
\Delta \approx .0156 \\
\]

\( \Delta \) corresponds to a small correction to the electron mass coming from Coulomb 
energy and topological mixing of lepton families. This gives

\[
L_0 = \frac{\pi \sqrt{(5 + \frac{2}{3} + \Delta)}}{m_e \sqrt{M_{127}}} \\
\]

Using \( Gm_e^2 \approx 17.5145 \cdot 10^{-46} \) this gives

\[
L_0(\Delta = 2^{-6}) \approx 1.824 \cdot 10^4 \sqrt{G} \\
\]
The $L_0$ coefficient is by a factor 1.096 larger than its value $L_0 = 2^{10}k\sqrt{G}$, $k \simeq 16.25$ deduced in [Pitkänen] from the first attempts to understand hadronic mass spectrum using p-adic super conformal invariance. Convenient reference scales are $L_{127} \simeq 2.89E-12\ m$ and $L_{107} \simeq 2.82E-15\ m$. The definition of the length scales should not be taken too literally since the definition necessarily contains some unknown numerical factor. Rather remarkably, the length scale $L_0$ is of same order of magnitude as the length scale at which the renormalized standard model coupling constants become equal and some kind of New Physics probably emerges.

The original hypothesis was that the length scale at which p-adicity sets on is of order $L_0 \sim 10^4\sqrt{G}$. Above this length scale spacetime should begin to look approximately flat piece of Minkowski space and the field theoretic description should make sense. This hypothesis is probably not correct as the following arguments demonstrate.

The basic problem is to understand the appearance of the square root of $p$ instead of the naively expected $p$ in the expression of length scale. The first idea to come into mind is that $M^4$ coordinate $m^k$ is essentially the square root of p-adic spacetime coordinate $x^k$. The argument was based on analogy with Brownian motion. $M^4$ coordinates as functions of p-adic spacetime coordinates are analogous to the spatial coordinate $x$ of particle in Brownian motion and therefore one has

$$\langle x^2(t) \rangle = Dt$$

It has however turned difficult to realize this idea in Lorentz invariant manner.

An elegant Lorentz invariant manner to understand the appearance of square root is to assume that the p-adic version of $M^4$ metric contains $p$ as a scaling factor.

$$ds^2 = p m_{kl} L_0^2 dm^k dm^l$$

$$L_0 = 1$$

(22)

where $m_{kl}$ is the standard $M^4$ metric $(1, -1, -1, -1)$ and $L_0 = 1$ holds true in units used and actually corresponds to the distance $L_0 \sim 10^4\sqrt{G}$. The
p-adic distance along k:th coordinate axis from origin to the point \( m^k = (p-1)(1+p+p^2+...) = -1 \) on the boundary of the fundamental convergence cube of the square root function, is

\[
\sqrt{p}((p - 1)(1 + p + ...)) L_0 \to \frac{(p - 1)(1 + p^{-1} + p^{-2} + ...)}{\sqrt{p}} L_0 = \sqrt{p}L_0 = L_p
\]  

(23)

What is remarkable that the shortest distance in the range \( m^k = 1, \ldots m - 1 \) is actually \( L_0/\sqrt{p} \) rather than \( L_0 \) so that p-adic numbers in range span the entire \( R_+ \) at the limit \( p \to \infty \). What is remarkable is p-adic topology indeed approaches real topology in the limit \( p \to \infty \) since the length of the discretization step approaches to zero. This interpretation is in accordance with the interpretation of p-adic space time as effective spacetime surfaces spanned by most probable 3-surfaces and with the intuitive idea that in very long length scales quantum effects become vanishingly small so that effective spacetime surface becomes continuous in real sense.

In TGD the 'radius' \( R \) of \( CP_2 \) is fundamental length scale and is of same order as Planck length [Pitkanen]. Concerning the formulation of conformal field theory limit the basic problem is

i) whether one should can take \( R \) as basic length scale and predict \( L_0 \) or

ii) whether the value of \( L_0 \) is a prediction of Quantum TGD and must be taken as given in the formulation of field theory limit.

The general physical picture is in accordance with the latter alternative. This means that in the expression of the induced metric the parameter \( R \) must be expressed in terms of \( L_0 \) and one has following kind of expression

\[
R^2 = kpL_0^2
\]

\[
kp \to x \sim 10^{-6}
\]

(24)

\( k \) is very large number, of order \( 10^{-6}p \). It is illustrative to look how large the contribution of \( CP_2 \) metric to induced metric is.

i) If the gradients of \( CP_2 \) coordinates are of order \( O(\sqrt{p}) \), the contribution of \( CP_2 \) metric to the components of the induced metric is of order \( kp^2 \), where as the contribution of \( M^4 \) coordinates is of order \( p \). This implies that the
contribution if $CP_2$ metric to the distance is roughly a fraction $x \sim 10^{-6}$ from the contribution of $M^4$ coordinates so that gravitational effects should be important immediately below elementary particle length scales $L_p$. This is not in accordance with the expectation that gravitation becomes important in Planck length scales, only.

ii) If the gradients of $CP_2$ coordinates are of order $O(p)$ then the contribution of $CP_2$ metric to the induced metric is of order $O(p)$ p-adically and in accordance with the weakness and even the size of the coupling strength of gravitational interaction for physically interesting primes $p$.

5 Topological condensate as minimum of effective action

The construction of p-adic TGD necessitates a satisfactory answer to the question 'Why the effective spacetime topology is p-adic?'. One can consider two different answers but only one of them seems to be entirely satisfactory and is based on the observation that observed spacetime is actually some kind of quantum average defined as minimum of the effective action.

5.1 p-Adicity from length scale cutoff?

Consider first the wrong answer. The original explanation for the p-adicity of effective spacetime topology was in terms of length scale cutoff. The concept of the length scale cutoff plays fundamental role in the description of the critical phenomena. The TGD: eish counterpart of this procedure is the smoothing out of the topological details of size smaller than some given length scale $L$. This procedure should give the 3-space of General Relativity: the presence of details is described by energy momentum tensor and various particle densities. The difficulty is that the usual momentum cutoff of quantum field theories doesn’t provide a solution to this problem. The geometric description of this procedure provided by various lattice models seems to be more in the spirit of TGD:eish description of the cutoff procedure.

Smoothing out procedure (or coarse graining) need not respect differentiability for the simple reason that differentiability in Planck length scales does not correlate the values of functions at distances much larger than Planck
length. This suggests that smoothed out spacetime surfaces obey p-adic topology instead of real topology. A possible definition for the smoothing out procedure is as p-adic cutoff: replace $x_p$ appearing as the argument of $h^k$ with its first $N$ pinary digits. This means that all details of the 3-surface related to the higher pinary digits of the expansion are smoothed out: by p-adic continuity these details are the smaller the higher the corresponding pinary digit is. The presence of the details must be described somehow and the use of p-adically differentiable densities is a natural manner to do this.

This interpretation need not be the correct one.

a) The appearance of the favoured p-adic length scales corresponding to powers of $p$ and also the appearance of the favoured values of $p$ is not in accordance with the arbitrariness of p-adic cutoff length scale.

b) p-Adic QFT turns out to be UV finite without any p-adic length scale cutoff.

c) Despite several attempts it has not possible to define precisely what the concept RG transformation and RG invariance of Kähler action means in this interpretation.

In fact, in light of the interpretation to be suggested below the smoothing out procedure should lead from p-adic topology to real topology, when the cutoff length scale is larger than $L_p$ rather than from real to p-adic!

5.2 p-Adic spacetime as absolute minimum of effective action

In TGD the geometry of 3-surface and the corresponding unique classical spacetime surface are subject to quantum fluctuations and the criticality of the TGD:ish Universe suggests that these fluctuations are important in all length scales. Therefore the observed 3-space and spacetime should correspond to nontrivial quantum averages. These averages obviously depend on quantum state. p-Adic topology could be one of the basic properties of this average spacetime.

The problem is to formulate the concept of quantum average for 3-space and spacetime. In ordinary QFT the concept of effective action is in decisive role. Effective action is typically obtained by coupling some fields appearing in the action exponential to external currents and by functionally integrating
over some fields appearing in the action exponential to get effective action in terms of new field $\Phi$.

$$\Phi_{\text{new}} = \frac{1}{\Gamma} \frac{\delta \Gamma}{\delta J(x)}$$

$$\Gamma = \int \exp(S(\Phi) + J\Phi) D\Phi \equiv \exp(S_{\text{eff}}(\Phi_{\text{new}})) \quad (25)$$

The absolute minima of the effective action define the vacuum average of the field $\Phi(x)$ in presence of external current $J$.

In TGD the corresponding construction could proceed as follows. Kähler Dirac action defines configuration space geometry. What one is interested in is the integration over configuration space degrees of freedom to get effective action, whose absolute minima would define p-adic space time concept. The effective spacetime is characterized by $H$ coordinates $h^k_{\text{new}}(x)$ and the simplest coupling of these coordinates to ordinary $H$-coordinates is obtained by coupling Kähler potential to the external Kähler $j^\alpha_{\text{new}}$ current defined by $h^k_{\text{new}}$

$$S_K(h) + S_D \rightarrow S_K(h) + S_D + \int j^\alpha_{\text{new}}(x) A^\alpha(x) \sqrt{g} d^4x$$

$$j^\alpha_{\text{new}} = (D_\beta j^{\alpha\beta})_{\text{new}}$$

$$j^{\alpha\beta}_{\text{new}} = g^{\alpha\mu}_{\text{new}} g^{\beta\nu}_{\text{new}} j^{\mu\nu}_{\text{new}} \quad (26)$$

Here the subscript 'new' means that the induced Kähler field and induced metric are determined entirely by the new coordinates $h^k_{\text{new}}$. The matrix element of the exponential of the effective action in a given quantum state is defined as the average obtained by integrating over configurations space degrees of freedom that is over all possible 3-surfaces. It must be emphasized that the resulting effective action depends on spinor fields since one doesn’t integrate over them. Effective spacetime is determined as the absolute minimum of effective action and need not be unique in a given quantum state: in thermodynamics the nonuniqueness means possibility of several phases and criticality suggest the existence of a large degeneracy.

A comment on the treatment of boundary components in quantum averaging is in order. The success of the ordinary QFT suggests that boundary
components can be replaced with quantum averages in vibrational degrees of freedom but averaging does not make sense in cm degrees of freedom. Quantum averaging should give rise to some boundary 3-surface or a family of 3-surfaces (degeneracy is possible) not depending much on the details of the surrounding spacetime. Degeneracy is indeed expected and corresponds to conformal and modular degeneracy. The idealization of the boundary component with a point like particle looks rather natural so that effective spacetime contains boundaries as worldlines. Kac Moody spinors provide economical manner to describe conformal degrees of freedom. The coupling of the wordline dynamics to interior dynamics is in principle determined by the boundary conditions on the boundaries and the conservation of various charges.

What can one conclude about the general form of effective action?

a) RG invariance of the effective action has been one of the basic hypothesis of Quantum TGD [Pitkanen]. The problem is to define this concept precisely. For simplest RG invariant actions correspond to free field theories and for these theories effective action is identical with the original action. This observation suggests a precise definition for RG invariance: the effective action associated with Kähler action in the absence of spinor fields is indeed formally identical with the Kähler action but the effective spacetime topology need not be the real one. The same conclusion is achieved if one assumes that effective action possesses same approximate symmetries and the characteristic vacuum degeneracy selecting Kähler action uniquely.

b) An attractive possibility is that the effective spacetime topology is that of a hierarchical topological condensate \(< p_1 < p_2 ...\) consisting of pieces possessing p-adic topology with various values of \(p\). The gluing of two topologically different pieces \(p_1\) and \(p_2\) should be performed along their boundaries. This gluing operation can be performed using canonical correspondence.

c) The enormous vacuum degeneracy of the Kähler action and associated exact \(Diff(M^4)\) and canonical invariances suggests analogy with spin glass phase in the sense that there exists enormously large number of vacuum spacetimes serving as background for fermionic dynamics. The averaging over these vacuum spacetimes is completely analogous to the averaging over the bonds strengths in spin glass phase. The basic feature of the configuration space of minima in spin glass phase is ultrametricity. Ultrametricity is the basic feature of p-adic topology and therefore the topological condensate
as a hierarchy of p-adic topologies $p_1 < p_2 < ...$ is perhaps the most general realization of the ultrametricity.

d) What about the form of effective action, when spinor fields are non-vanishing? RG invariance suggests that the general form remains invariant but something happens for $H$-coordinates appearing as arguments of the action. TGD allows $N = 1$ supersymmetry generated by covariantly constant right handed neutrino and this suggests the solution to the problem. What happens is that $H$-coordinates are replaced by super coordinates $h^k \rightarrow h^k + K \bar{\Psi} \gamma^k \Psi$. The replacement is coordinate invariant only provided one can select preferred coordinates for $H$. For $M^4$ the standard Minkowski coordinates are obviously the preferred coordinates. For $CP_2$ the complex coordinates for which the action of $su(2) \times u(1)$ is linear are preferred coordinates. These coordinates are defined only modulo color rotation and consistency requires that physical states are color singlets. This implies that each p-adic region each color singlet so that color confinement in strong sense results.

The interpretation of the p-adic spacetime as effective quantum spacetime is in principle possible in arbitrarily short length scales and for all spacetime topologies and the approximate symmetries are not essential for this in this respect. Later it will be found that p-adic conformal and canonical invariances seem to be completely general approximate symmetries of the zero action extremals of the effective action broken by gravitational effects only. Therefore p-adic spacetime concept is not just a convenient approximation working at the flat spacetime limit as originally believed but an essential part of Quantum TGD. The only approximation made in the p-adic field theory limit is the freezing of the spacetime degrees of freedom by using quantum averaged spacetime determined by the effective action and it might be even possible to develop a formalism to calculate the corrections to this approximation.

5.3 The value of the Kähler coupling strength?

The requirement that the effective action is identical with the super symmetrized Kähler Dirac action should fix the value of Kähler coupling strength $g_K^2$ and thus make theory unique. There are two independent heuristic arguments leading to same estimate for the Kähler coupling constant.
a) Coupling constant evolution argument \cite{Pitkanen} led to the estimate

\[
\alpha_K = \frac{28}{27} \alpha_{em}(low)
\]

\[
\alpha_{em}(low) \simeq \frac{1}{137}
\]

(27)

b) The experience with the ordinary field theories suggests that elementary particle mass scale is related to the exponent of the inverse of \(\alpha_K\): \(L_{elem} = \exp(c/\alpha_K)/\sqrt{G}\), where \(c\) is some constant. A generalization of this formula is suggested by two observations:

i) Mersenne prime \(M_{127}\) defines the largest elementary particle length scale and is also a member in the Combinatorial Hierarchy \(M(n) = M_{M(n-1)}\) of the Mersenne numbers (possibly all primes) given by 3, 7, 127, \(M_{127}\), ...

ii) The exponent of the Kähler action for \(CP_2\) type extremals describing elementary particles defines natural large dimensionless parameter of order \(10^{19}\) and in \cite{Pitkanen} an explanation of the mass elementary particle mass scale in terms of this number was suggested.

These observations suggest the following formula for the fundamental length scale \(L(127)\) as a prediction of TGD:

\[
L(127) = \exp(S_K(CP_2))\sqrt{G}
\]

\[
L(127) = \sqrt{M_{127}L_0}
\]

\[
S_K(CP(2)) = \frac{\pi}{8\alpha_K}
\]

\[
L_0 = k10^4\sqrt{G}
\]

\[
k \simeq 1.824
\]

(28)

These formulas give

\[
\alpha_K = .9641 \frac{137.03604}{\alpha_{em}(low)} \cdot \frac{28}{27} \alpha_{em}(low)
\]

(29)

The two estimates give identical results if one has \(\alpha_{em}(low) = .9641/137.03604\): this assumption is consistent with the electromagnetic coupling constant evolution and gives \(\alpha_K = 137.06\). \(\frac{1}{\alpha_K} = 137\) would give \(k = 1.77 < 1.824\), which
is however too small value. In any case, the two heuristic arguments are consistent which each other and internal consistency predicts the sale $L_0$ correctly. $\alpha_K$ has a natural upper bound corresponding to $L_0 = \sqrt{G}$ and given by $\alpha_K \geq .0089$. The great task is to understand the generation of the length scale $L_0$ or equivalently the low energy value of the fine structure constant.

It is quite possible that conformal invariance gives conditions on the value of $g_K$. The construction of unitary representations of conformal symmetry should lead to a definite value for Kähler coupling $g_K$ (conformal central charge $c$ vanishes). The conformal charges contain parts coming from Dirac action and from Kähler action and the part coming from Kähler action depends is proportional to $1/g_K^2$. Therefore the requirement that conformal current commutators contain no anomalies and that vacuum expectation of Virasoro generator $L_0$ should have quantized value fixed by unitarity, could pose condition on the value of Kähler coupling strength.

5.4 Approximate symmetries of the p-adized Kähler action

The criticality together with two-dimensional analogy suggests that 4-dimensional conformal transformations act as approximate symmetries of p-adic Kähler action. Also the existence of p-adically analytic extrema of p-adic Kähler action suggests the same. Actually the symmetry group is much larger. When gravitational effects can be neglected $Diff(M^4)$ acting on $M^4$ coordinates is approximate symmetry. Besides this canonical transformations of $CP_2$ localized with respect to $M^4$ coordinates act as dynamical symmetries in the interior of the spacetime surface. The emergence of $Diff(M^4)(M^4)$ suggests that the theory does not differ much from what might be called topological field theory in the interior of spacetime surface. It however turns out that p-adic quantization of Dirac action allows massless states, presumably including also graviton. Actually these states correspond to the representations of certain Super Virasoro algebra with the property that all generators except $L_0$ annihilate vacuum whereas $L_0$ gives the energy of the (massless) state.

p-Adic diffeomorphisms acting on $M^4$ via the formula $M \rightarrow W(M)$ and canonical transformations of $CP_2$ local with respect to $M^4$ act as exact sym-
metries of p-adic Kähler action, when gravitational effects are neglected. This motivates the realization of these symmetries as exact symmetries by requiring that conformal and canonical charges annihilate physical states.

The change of the action in general transformation can be decomposed into two parts: \( \delta S = \delta S_1 + \delta S_2 \) corresponding to the change induced by the change of the induced Kähler form and of the metric.

a) For the extremals of Kähler action any variation of the action reduces to total divergence, which is the divergence of the current associated with infinitesimal transformation and the vanishing of this divergence means just charge conservation. In present case the total divergence contains only the contribution \( \delta S_1 = T^{\alpha\beta} \delta g_{\alpha\beta} \) coming from the change of the induced metric since the induced Kähler form remains invariant for both diffeomorphisms and canonical transformations. The contribution coming from the metric is in general very small small by the weakness of the gravitational interaction and vanishes in the approximation that \( CP_2 \) contribution to the induced metric is zero.

b) The idea is to pose the vanishing of the total divergence as a quantum condition on the physical states: this condition states that \( Diff(M^4) \) (or perhaps only conformal) and canonical (local with respect to \( M^4 \)) charges are conserved. A more stronger condition that these charges annihilate physical states is indeed the basic constraint on physical states in field theories. The condition makes sense as a condition for the effective action since the minima of the effective action can be regarded as effective spacetimes associated with quantum states. Also the breaking of conformal symmetry in the sense that \( L_0 \) is conserved but does not annihilate physical states is possible.

The symmetries defined in question are extremely general symmetries as the following examples demonstrate.

a) \( Diff(M^4) \) and local canonical symmetries are exact symmetries in the usual sense for the vacuum extremals: any 4-surface with \( CP_2 \) projection, which is Lagrange manifold (having vanishing induced Kähler form). Lagrange manifolds are in general 2-dimensional, which means that a very rich variety of topologies becomes possible for the vacuums. This vacuum degeneracy implies the analogy between TGD and spin glass. Diffeomorphisms of \( M^4 \) are symmetries in the vacuum sector.

b) Spacetimes representable as maps \( M^4 \rightarrow CP_2 \) and carrying nonvanishing Kähler field and satisfying Maxwell equations \( j^\alpha = 0 \) are examples of
nonvacuum extremals. An interesting possibility is that also the p-adically analytic maps $M^4 \to CP^2$ to be described in next section define actually exact solutions of the effective action. One example of zero action minima of Kähler action are so called massless solutions discussed in [Pitkänen,]. These are defined as maps $M^4 \to CP^2$

$$s^k = f^k(p \cdot m, \epsilon \cdot m)$$

(30)

where $p$ is massless four-momentum vector $p \cdot p = 0$ and $\epsilon$ is polarization vector orthogonal to $p$: $\epsilon \cdot p = 0$ and the functions $f^k$ are arbitrary. In this case canonical transformations are exact symmetries.

c) String like objects of type $X^2 \times Y^2 \subset M^4 \times CP^2$ are vacuum solutions and suitable deformations of these objects with vanishing action and nontrivial Kähler form probably define nonvacuum extremals with vanishing action. The identification of hadronic strings as objects of this type was suggested in [Pitkänen,]. The hadronic string tension would be generated dynamically and be due to the presence of fermions and nonvanishing Kähler fields. In fact, classical gluon field is proportional to Kähler field: $G^A \propto H^A J_{\alpha\beta}$, where $H^A$ is the Hamiltonian of color transformation. It should be noticed that color symmetries are canonical transformation so that physical states must be color singlets also by the canonical invariance.

d) The so called $CP^2$ type extremals obtained by deforming the standard imbedding of $CP^2$ so that the $M^4$ projection of the surface is light like curve

$$m^k = f^k(s)$$

$$m_{kl} \frac{df^k}{ds} \frac{df^l}{ds} = 0$$

(31)

These surfaces are metrically equivalent with $CP^2$ and are vacuum solutions. Canonical and conformal symmetries are exact symmetries for these surfaces. In [Pitkänen,] it was suggested that elementary particles correspond to $CP^2$ extremals and that Feynmann diagrams correspond to surfaces obtained by gluing these extremals together by connected sum operation. The action of these solutions is nonvanishing and these surfaces as such are not absolute minima of effective action. One can however consider more general solutions obtained as a topological sum of $CP^2$ type extremal and background space-time surface carrying nontrivial Kähler Coulomb field, which gives a negative
contribution to action so that total action and its real counterpart vanishes and one obtains absolute minimum. The exponent of Kähler action defines in natural manner elementary particle length scale as suggested already in [Pitkänen].

These examples encourage the conjecture that the field theory limit of TGD defined as a fermionic field theory in the average spacetime defined by the physical state allows local canonical transformations and $Diff(M^4)$ as gauge symmetries under rather general conditions. The approximate invariances are not needed in the definition of the super symmetrized Kähler action. The surprising success of the mass calculations based on super conformal invariance also supports the realization as gauge symmetry. It might well be that these symmetries are spoiled 'in large' by gravitational effects. A purely p-adic feature of these symmetries is however that one can restrict the symmetries to be sufficiently small to force the change in the induced metric small enough. This is achieved if the parameter $t$ in the exponential $exp(tJ)$ of Lie-algebra generator has p-adic norm smaller than $p^{n/2}$: these transformations indeed form a group for any integer $n$. In this manner the configuration space decomposes into disjoint blocks and one obtains different realization of symmetries inside each block. This is all what is need to deduce the most important consequences of conformal invariance (such as universality of the mass spectrum). These disjoint blocks are clearly analogous to the degenerate vacua of the spin glass phase.

### 5.5 Gravitational warping

The absolute minimum of effective action depends on the quantum numbers of the state. In particular, there is dependence on four-momenta and various charges of the quantum state. Gravitational warping represents perhaps the simplest dependence on the quantum numbers of the state. Gravitational warping is completely analogous with the existence of infinitely many imbeddings of flat 2-dimensional space $E^2$ to $E^3$ obtained by deforming any plane of $E^3$ without stretching it. The physical reason for the warping is the gravitational mass of the p-adic convergence cube. The mass of the p-adic convergence cube deforms spacetime surface and average effect is momentum dependent scaling of some metric components. In GRT gravitational warping clearly does not make any sense. The simplest physical consequence of
the deformation is that photons propagating in condensate spend in general longer time than vapour phase photons to travel a given distance [Pitkänen].

A rather general nonstandard imbedding of $M^4$ representing gravitational warping is obtained by assuming that $CP^2$ projection of spacetime surface is one-dimensional curve:

\[
s = K p P_k m^k \\
g_{\alpha\beta} = m_{\alpha\beta} - K^2 p^2 P_\alpha P_\beta \\
m_{\alpha\beta} = p n_{\alpha\beta}
\]

(32)

Here $s$ is the curve length coordinate, $P_k$ is total four-momentum and one has $|K| = 1$.

The scaling of the metric components has two nice consequences as far as the convergence of the perturbation theory is considered:

a) Assuming that the spectrum of off-mass shell four-momenta for spinors is not changed the propagator expression has no poles since one has always $k^2 = g^{\alpha\beta} k_\alpha k_\beta \neq M^2$ in warped metric.

b) For $m_{00} = 1 \to 1 - \delta$ the expression $k_+^2 = 0$ becomes $k_-^2 = -\delta k_0^2$ and this number becomes p-adically very large, when p-adic norm of $k_0$ increases. This guarantees the convergence of the propagator expression as well as finite degeneracies for all values of $P$.

Gravitational warping cannot make spacetime metric Euclidian if boundary contribution to the mass squared is given by the standard stringy mass formula. In the rest frame the deformation of the metric is just $g_{00} = p(1 - K^2 p M^2)$. Stringy mass spectrum is of form $M^2 = kn$, $|k|_p = 1$ so that the p-adic norm of the mass squared is not larger than one and the contribution to the metric is of order $O(p^2)$ p-adically. It is worth noticing that stringy mass formula gives upper bound for the mass contained in a region of size $L_p$: $M \leq M_{\text{max}} \sim \sqrt{p} 10^{-3} M_{Pl}$. An interesting possibility is that stringy mass formula might hold true even in astrophysical length scales. For Sun $L_p$ is of order of the size of solar system and upper bound for mass is much larger than solar mass. For neutron star of size of order $10^4$ meters the mass is not far from its maximum value for $L_p \sim 10^4 m$. 

39
5.6 Idealization of boundary components with point like objects

In TGD boundary components of 3-surface are carriers of elementary particle quantum numbers. In particular, the state of elementary particle depends on modular degrees of freedom of the boundary component. In [Pitkänen], the so called elementary particle vacuum functionals were constructed and the p-adic description of Higgs mechanism relies on the p-adized version of these functionals. A purely p-adic feature of the construction is the discretization of p-adic moduli space. The route from ordinary modular degrees of freedom of Quantum TGD to discrete p-adic modular degrees of freedom of p-adic TGD should take place via the construction of 'effective' elementary particle functionals by coupling modular degrees of freedom to the p-adic modular degrees of freedom somehow. How this process is carried out in detail is not considered here.

As already found one cannot perform quantum averaging for the cm degrees of freedom of boundary component and p-adic spacetime surface is the average spacetime obtained by keeping the center of mass positions of boundaries at some reference time fixed. The practical treatment of boundary degrees of freedom in the low energy field theory limit suggests the idealization of boundary component with point like object. This necessitates quantum mechanical averaging over the modular degrees of freedom. This quantum mechanical averaging gives to the mass squared the dominating modular contribution. The modular contribution to mass squared is indeed quantum mechanical rather than thermal average: this issue was left open in the p-adic description of Higgs mechanism.

5.6.1 Induced Dirac spinors on the world line

The simplest description of the point like limit is obtained by neglecting totally the vibrational degrees of freedom. The 3-dimensional orbit of the boundary component is replaced with the world line of a point like particle and the Dirac action for the induced spinors on the boundary component is replaced with the Dirac action on with world line. The induced spinor connection on the world line reduces to a pure gauge with a suitable choice of $CP_3$ vielbein basis and a suitable gauge for the Kähler potential on world line and one can wonder whether the couplings of the fermions to the classical
gauge fields are actually trivial (note that the emission of a gauge boson corresponds to the splitting of the boundary component world line). This is not probably not the case: the point is that boundary component cm degrees of freedom are not frozen and one must treat these degrees of freedom quantum mechanically so that there is no general gauge choice transforming the gauge potentials away. If the couplings of the fermions to the classical background fields were trivial the problems associated with the classical long range \( Z^0 \) fields (the possibility large parity breaking effects) considered in the third part of the book would have an easy solution. Physical intuition and the previous work with TGD however suggest that classical background fields are important.

The contribution of the modular degrees of freedom to mass squared should be taken into account somehow. The only manner seems to be as a constraint on the effective mass squared of spinors. The Dirac equation for induced spinors contains a term proportional to the trace of the second fundamental form of the worldline

\[
\Gamma^\mu D_\mu \Psi = \frac{1}{2} \Gamma_k H^k
\]

\[
H^k = g^{\mu \nu} D_\mu D_\nu h^k
\]

(33)

\( H^k \) decomposes into \( M^4 \) and \( CP_2 \) contributions and the description of Higgs mechanism proposed in [Pitkänen] was based on the identification of \( CP_2 \) part of \( H^k \) as Higgs field. If \( H^k \) is covariantly constant quantity along the world line. It seems natural to require that the square of \( CP_2 \) part \( S^k \) of \( H^k \) is equal to the modular contribution to the mass squared on the world line

\[
M^2 (mod) = S^k S_k
\]

(34)

In general \( S^k \) is not covariantly constant but one could perhaps add the covariant constancy of \( S_k \) along world line as an additional condition:

\[
D_t S^k = 0
\]

(35)

The use of the constraints is in accordance with the concept of the effective action. What constraints express is the state dependence of the average
spacetime surface on quantum numbers. An additional constraint comes from the requirement that electromagnetic gauge group, which is subgroup of local $so(4) \times u(1)$ is exact gauge group. This implies that induced $W^\pm$ fields mixing different charge states of fermion vanish on world line. Altogether this means that the $CP_2$ projection of the world line is highly determined: a detailed consideration can be found in context of the symmetries. The motion in $M^4$ degrees of freedom is determined by the boundary conditions stating the conservation of four-momenta and other charges.

5.6.2 Generalization of Kac Moody Dirac operator to worldline

The simplest point like limit implies that spinors have only finite number of degrees of freedom and therefore a loss of conformal degrees of freedom necessary to obtain Planck mass excitations. According to the mass calculations elementary particle masses contain besides the dominating modular contribution a contribution from spin, electroweak and color degrees of freedom, which can be calculated using p-adic thermodynamics with energy replaced with the Virasoro generator $L_0$. Therefore it seems to be an overidealization to eliminate totally the conformal degrees of freedom. Of course, one could use the total mass squared including the thermal contribution in the Dirac equation for ordinary induced spinors. Probably this is not wise. The mass calculations provide a better solution of the problem: replace $H$-spinors with the Kac-Moody spinors constructed as representations of Super Virasoro algebra [Pitkänen]. One can define the coupling of the world line gauge fields to Kac Moody spinors and also the coupling of $H^k\Gamma_k$ is well defined and due to covariant constancy is combination of the flat space gamma matrices. The possibility to eliminate neutral gauge potentials by gauge transformations suggests that in $CP_3$ degrees of freedom actually no dependence on $CP_2$ coordinates remains and KM spinors in $CP_2$ degrees of freedom behave as flat space spinors in cm degrees of freedom.

p-Adic description of Higgs mechanism led to the concept of Kac Moody Dirac operator acting on Ramond or N-S type representation. For Ramond type representation the operator is sum of three terms.

$$
D = p^k \gamma_k (M^4) + M_{op} + G_0 \\
M_{op} = b_k \gamma^k (CP_2)
$$
\[ M_{\text{op}}^2 = M^2 + M_{\text{mod}}^2 \] (36)

The first term is \( G_0 \) acting on Super Virasoro, presumably acting on conformal degrees of freedom of two-dimensional boundary component defined as \( m^0 = \text{constant} \) slice of 3-dimensional boundary. The second term is Dirac operator in \( M^4 \) cm degrees of freedom. The third term is mass operator, which is linear combination of \( CP_2 \) type gamma matrices anticommuting with \( G_0 \) and is required to give correct conformal weight for states, which should be massless in absence of the modular contribution \( M_{\text{mod}}^2 \) to mass squared (second paper of [Pitkanen]). The presence of \( M_{\text{op}} \) can be interpreted as a breaking of the super conformal symmetry.

What is nice is that the tachyonic vacuum weight of \( L_0 \), which remained mysterious in the mass calculations, can be understood in terms of the induced spinor concept. A second great mystery of the mass calculations was the origin of the additional cm part in \( G_0(\text{tot}) \), in particular mass operator. It would be very nice if \( G_0(\text{tot}) \) would actually correspond to Super Virasoro generator. This is indeed the case if Super Virasoro generators for the boundary component idealized to point are defined as sums of two terms. The first term corresponds to the Super conformal transformations of proper time coordinate for which induced metric \( g_{tt} \) is constant. Second term corresponds to the Super Virasoro generator acting on the two-dimensional boundary component in standard manner. If proper time is used the super generators \( G_n \ n > 0 \) annihilate the state automatically and \( G_n(\text{tot}) \) reduces to \( G_n \). \( G(\text{tot}, t) \) defined as

\[
G(\text{tot}, t) = \sum_n G_n(\text{tot})a^{nt}
\]

\[
G_n(\text{tot}, t) = G_n(\text{worldline}) + G_n(\text{bound})
\]

\[
G(\text{worldline}, t) = \frac{1}{2}(\gamma^t D_t^+ - D_t^- \gamma^t)
\] (37)

however reduces to \( G(\text{bound}, s) \) in proper time frame \( t = s \) apart from the cm part

\[
G_0(\text{worldline}) = p_s \gamma^s = \gamma_k(M^4) \frac{dm_k}{ds} + \gamma_k(CP_2) \frac{ds_k}{ds} + ... \\
\equiv p_k \gamma^k(M^4) + p_k(CP_2) \gamma^k(CP_2)
\] (38)
If $t$ is proper time the contribution is indeed constant.

The proper definition of the fermionic KM Dirac action density on the world line is therefore as

$$\bar{\Psi} G(tot, t) \Psi = \bar{\Psi} (G(\text{worldline}, t) + G(\text{bound}, t)) \Psi$$

$$= \bar{\Psi} (\gamma^s \frac{d}{ds} + G(\text{bound}, s)) \Psi$$

(39)

where the last expression holds true in proper time coordinate $s$. KM Dirac equation states that

$$G_0(tot) \Psi = -M_{op} \Psi$$

$$M_{op} = \frac{H^k \gamma_k}{2} \gamma_k \Psi$$

(40)

The mass term $M_{op}$ comes from the fact that induced gamma matrices are not covariantly constant: $D_\alpha \Gamma^\alpha = H^k \gamma_k$, where $H^k$ are the components of the second fundamental form of the world line. The breaking of the conformal symmetry via the generation of a nonvanishing negative (tachyonic) vacuum conformal weight $h(vac)$ requires that $M_{op}$ is nonvanishing. The value of $h(vac)$ is $-2$ ($-1$) for isospin 1/2 ($-1/2$) fermions and $-5/2$ for bosons. The presence of this term implies that world line cannot be a geodesic line $CP_2$ degrees of freedom. In $M^4$ degrees of freedom accelerated motion causes additional breaking of conformal symmetry and small change in mass squared. If second fundamental form is covariantly constant then only the mass of the particle changes and higher Virasoro conditions remain true. Although electroweak doublet Higgs particle is definitely absent from the mass spectrum of the theory, second fundamental form however appears in the KM Dirac equation in the role of classical Higgs field so that the original identification of the Higgs field as the trace of the second fundamental form is not totally wrong.

The treatment of N-S case proceeds in similar manner. The new feature is that KM Dirac operator has conformal weight 1/2 and corresponds to $G^{1/2}(tot)$ for world line containing ordinary conformal part and the part acting on the world line:
\[ G(\text{tot}, t) = G(\text{worldline}, t) + G(\text{bound}, t) \]
\[ G(\text{tot}, t) = \sum_n G(\text{tot}, n + 1/2) a^{(n+1/2)t} \]
\[ G(\text{wordline}) = \gamma^t D_t^\gamma - D_t^{-\gamma^t} \quad (41) \]

In bosonic case one expands \( G(\text{tot}) \) with respect to half odd integer p-adic planewaves on world line. Bosonic KM action density on world line is of form

\[ L_B = \bar{\Psi} G^2(\text{tot}, t) \Psi \quad (42) \]

If propertime coordinate is used and second fundamental form is covariantly constant equations of motion reduce to Virasoro conditions for \( L_n, \ n > 0 \) and \( n = 0 \) term gives breaking of conformal symmetry.

Virasoro conditions are automatically true if \( G_n, n > 1/2 \) annihilates the states automatically and this must be assumed in order to achieve Super Virasoro invariance. Again the equation of motion implies additional mass squared term with geometric interpretation

\[ G^{1/2}(\text{tot}) \Psi = -M_{\text{op}}^{1/2} \Psi \]
\[ M_{\text{op}}^{1/2} = -\frac{\gamma^{1/2}_k H^k}{2} \quad (43) \]

The operator \( M_{\text{op}}^{1/2} \) is expressible as linear combination of \( M^4 \) gamma matrices and \( CP_2 \) type gamma matrices \( \gamma^{1/2} k, \ k = 0, 3 \) and the square of \( M_{\text{op}}^{2} \) is such that operators with conformal weight \( h = 5/2 \) create light states (mass comes only from modular contribution to the mass of higher genus bosons).

### 5.7 Second quantization of the field theory limit and p-adic thermodynamics

p-Adic field theory limit is not all what is needed to construct a realistic theory. One must also perform second quantization both in interior and boundary degrees of freedom. Although not very much will be done in this direction in this paper it is useful to have rough idea about what is involved.
Consider first second quantization in boundary degrees of freedom.

a) The basic point is that field theory inside p-adic convergence cube allows only very few light states for both the interior and boundary components of single convergence cube and in practice the light excitations of boundary components must correspond to elementary particles.

b) In practice single boundary component can in a good approximation be idealized with a point like object described by Kac-Moody spinor in order to take into account the conformal degrees of freedom. The cm coordinates of the boundary components are kept fixed in the definition of the quantum averaged spacetime and these degrees of freedom must be quantized. The experience with QFT suggests that one can second quantize boundary component Super Virasoro representations regarded as quantum fields in the interior of the convergence cube: second quantized N-particle state corresponds to state of N topologically condensed small 3-surfaces with boundary.

c) The action should be just the generalization of free KM action obtained by minimal substitution to realize gauge invariances. Gauge field is defined by N-S representation. Since N-S representation contains bosonic KM Dirac operator and its conjugate one obtains polynomial action fourth order in operators creating N-S states. The difficult technical problem is the construction of vertices that is the action of N-S representation as a gauge potential on Ramond representations and N-S representations.

d) Classical background fields defined associated with effective spacetime surface are also present and couple to the Kac Moody spinors.

e) The experience with the p-adic counterpart of standard model suggests that in boundary degrees of freedom single p-adic convergence cube with size \( L_p \) is the natural quantization volume. The propagators of the theory are apart from additive constant equal to Super Virasoro generators \( p^k \gamma_k - G_0 + H^k \Gamma_k \) (fermions) and \( p^2 - L_0 + h \) (bosons).

One cannot exclude the possibility is that the second quantization of Kac-Moody spinors is directly related to the p-adic nondeterminism for the dynamics of boundary components. The world lines of boundary components are determined by second order differential equations, which allow integration constants depending on finite number of positive pinary digits. For instance, p-adic geodesics of \( M^4 \) have \( m^k \) and \( dm^k/ds \) depending on finite number of positive pinary digits. This nondeterminism implies degeneracy of the absolute minima of effective action in accordance with spin glass analogy.
possible manner to treat the situation is to use vacuum functional depending on world line. This functional is analogous to the action exponential for particle and should be essentially equal to the action exponential associated with Kac-Moody spinor and therefore generalized Dirac action. The calculation of S-matrix elements necessitates averaging over the different vacuum configurations and therefore one must perform summation over all possible different world line configurations. Since world lines can split these configurations in fact are counterparts of Feynmann graphs. It would not be surprising if this summation, which resembles functional integration would not give second quantized field theory for Kac Moody spinors. It seems however that this line of argument is not correct although the value of the effective action for space time is not expected to depend much on the positions of particle world lines.

Boundary degrees of freedom do not give rise to graviton in the scenario explaining Higgs mechanism and this suggests that graviton must correspond to interior degrees of freedom. The p-adic realization of $Diff(M^4)$ invariance indeed allows massless states for the p-adic quantization volume and graviton and its possible companions correspond to entire p-adic convergence cube or small cube inside it (recall that localized p-adic plane waves are possible). Graviton emission corresponds to the emission of small 3-surface via topological sum vertex and the exchange of graviton can occur between different p-adic convergence cubes interpreted as particles such as hadron and leptons. The general rules look again quite obvious. Propagators are same as in previous case. Vertices correspond to operators creating a state with the quantum numbers of the emitted particle, say graviton. N-particle vertices are essentially time ordered n-point functions integrated over the interior of the p-adic convergence cube and slashed between initial and final states. The vertices describing the emission of several particles via topological sum vertex are possible but probably the rates processes involving several exchanges are extremely low since gravitational coupling characterizes the strength of these interactions.

The description of hadron reactions necessarily involves topology changing processes for p-adic 3-surfaces and p-adic field theory limit in cube $L_p$ is not all what is needed to describe these processes just as perturbative QCD does not provide full understanding of hadronic reactions. An open problem is how to construct the formalism describing topological particle reactions in
length scales $L > L_p$. Real physics probably gives a satisfactory description for these processes but in a more ambitious approach one should somehow glue the $p$-adic QFT:s associated with neighbouring $p$-adic convergence cubes together. One possibility is to construct $p$-adic manifolds from $p$-adic convergence cubes as Feynmann diagram type objects. The lines of these diagrams would correspond to the sequences of convergence cubes glued together along their sides. The sum over the Feynmann diagrams would be reproducible from a second quantization of some field theory in the discrete lattice formed by the convergence cubes the vertices of the theory being defined in terms of $n$-point functions associated with single convergence cube. In [Pitkänen], the concepts of $p$-adic light cone and $p$-adic manifold (constructed by gluing together $p$-adic convergence cubes) were introduced: in [Pitkänen] it was suggested $p$-adic manifolds might be the counterparts of Feynmann diagrams and a geometric second quantization for the $p$-adic conformal field theory was suggested. In this formalism single line of Feynmann diagram was supposed to be a part of so called $p$-adic light cone. As already noticed it seems that only the sequences of $p$-adic convergence cubes of size $L_p$ are needed and it is possible to perform second quantization in the discrete lattice formed by these cubes.

$p$-Adic thermodynamics for boundary degrees of freedom seems to be necessary in order to get predictions comparable with experiments. $p$-Adic thermodynamics provides an elegant description of Higgs mechanism and consistency requires the thermal averaging of reaction rates, too. It is not clear whether the $p$-adic version of the standard model, which should provide the practical calculational tool, allows to use thermal masses as such. The technical problem is that $p$-adic square roots for thermal $M^2$ appearing in Dirac equation need not be $p$-adically real number. This problem is avoided if one uses Kac-Moody spinors truncated to spinors of $H$ and introduces mass matrix $M^k \Gamma_k$ in $CP_2$ degrees of freedom with the property $M_k M^k = M^2$ with respect to Euclidian metric. This mass matrix is just the counterpart of $CP_2$ contribution to the trace of the second fundamental form.

### 6 Symmetries of the theory

In interior of $X^2$ the basic symmetries of the theory broken include $Diff(M^4)$, canonical transformations localizable with respect to $M^4$ coordinates and
containing local color transformations as a subgroup. These symmetries are broken by gravitational effects only. Point particle limit for boundary components allows besides conformal symmetries and local canonical symmetries local \( so(4) \times u(1) \) broken down to electromagnetic gauge group and local \( so(3,1) \) broken down to local version of the isotropy group of 4-momentum of world line. All these symmetries posses super extension: fermionic generators are obtained by commuting the bosonic generators with local \( N = 1 \) super symmetries and kappa symmetries defined by covariantly constant right handed neutrino. Localization of symmetries is possible using arbitrary Fourier expandable functions of \( M^4 \) coordinates and \( Diff(M^4) \) algebra.

6.1 \( N = 1 \) supersymmetry and kappa symmetry

In quantum TGD right handed neutrino generates \( N = 1 \) supersymmetry. \( N = 1 \) supersymmetry and the related Kappa symmetry provide the means for extending p-adic conformal symmetry to super conformal symmetry: fermionic Super Virasoro generators are obtained by localizing of \( N = 1 \) super (Ramond case) symmetry or Kappa symmetry (N-S case). The supersymmetrization of local color and local canonical transformations, Lorentz transformations and electroweak gauge transformations is achieved by taking anticommutators with Super Virasoro algebra. This is what is needed since in the papers [Pitkanen] devoted to p-adic Higgs mechanism p-adic super conformal symmetry together with super versions of color and other symmetries were found to lead to excellent predictions for particle mass spectrum.

6.1.1 Definitions of \( N = 1 \) super symmetry and kappa symmetry

\( N = 1 \) super symmetry generated by covariantly constant right handed neutrino spinor has following form

\[
\begin{align*}
\Psi & \rightarrow \Psi + \epsilon u_R \\
h^k & \rightarrow h^k + \epsilon \bar{u}_R \gamma^k \Psi + \bar{c} \bar{\Psi} \gamma^k u_R
\end{align*}
\]

(44)

Here \( \epsilon \) refers to the oscillator operator creating right handed neutrino spinor with constant spatial dependence and \( \bar{c} \) is the hermitian conjugate of this
operator: the anticommutator of these operators is proportional to unit operator.

The is also a second supersymmetry generated by right handed neutrino analogous to the kappa symmetry of super string models [Green et al]. This symmetry is needed on boundary components only to construct N-S type representations and is defined as

\[
\Psi \rightarrow \Psi + Ku_R
\]
\[
h^k \rightarrow h^k + \epsilon \bar{u}_R \gamma^k K \Psi + \bar{e} \Psi K \gamma^k u_R
\]
\[
K = n_k\gamma^k
\]
\[
n^k \equiv \frac{D_0 \Gamma^\alpha}{\sqrt{H^k H_k}} = \frac{\gamma^k H_k}{\sqrt{H^k H_k}}
\]

(45)

This supersymmetry doesn’t annihilate on mass shell states as in string model. The action of Dirac operator gives the trace of the second fundamental form and this implies analogy with kappa symmetry of string model apart from the presence of normalization factor. Normalization factor and therefore the direction of unit vector \(n^k\) is well defined unless the boundary component in question is minimal surface. For boundary components idealized to world lines the value of \(H^k \gamma_k\) is covariant constant defined by the nonvanishing vacuum weight of Kac Moody spinor so that there are no problems. Kappa symmetry has the properties needed to generate N-S type generators if one assumes that leptons move in integer modes and quarks move in half odd integer modes.

6.1.2 Supersymmetrization of Kähler Dirac action

\(N = 1\) supersymmetry suggests also a proper manner to generalize the Kähler Dirac action to p-adic context to obtain effective action. Dirac action as such transforms by total divergence in supersymmetry, which does not act on \(H\)-coordinates. One can however extend the super supersymmetry by replacing \(H\)-coordinates with super coordinates:

\[
h^k \rightarrow h^k + J^k
\]
\[ J^k = K \Psi \Gamma^k \Psi \]
\[ K = K_0 p^n, \quad n > 0 \quad (46) \]
in Kähler Dirac action. This replacement breaks general coordinate invariance unless one fixes some physically preferred coordinates for \( H \). In \( M^4 \) degrees of freedom the preferred coordinates are just the standard Minkowski coordinates. The simplest candidate are complex coordinates with the property that the action of \( su(2) \times u(1) \) is linear. These coordinates are unique only apart from color rotation and internal consistency requires that physical states are color singlets: note that each p-adic regions is separately color singlets. The nice feature of this choice is that color symmetry is exact and therefore super symmetrization works without any assumptions about the symmetries p-adic spacetime surface. That color confinement is related to super symmetry is in accordance with the recent results of Seiberg and Witten [Seiberg and Witten].

The parameter \( K \) appearing in the defining formula for the super coordinate must be chosen to be proportional to some positive power of \( p \) in order to obtain converging perturbation theory. The convergence of the perturbation theory is extremely rapid for physically interesting values of \( p \). The deviation of the induced metric from flat metric must of order \( K^2 \). For \( K \propto p \) this means that the \( CP_2 \) part of induced metric is of order \( O(p^2) \) as it should be. Same applies to the gradients of spinor fields.

### 6.1.3 Why supersymmetry is so nice?

Supersymmetrization is physically well motivated as the following considerations show.

a) \( Diff(M^4) \) symmetry implies that the oscillator operators associated with the quantized fermion fields create only electroweakly neutral massless states. Graviton is contained in the spectrum of the massless states. Each boundary component however carries its own quantized spinor fields and the total fermion number of 3-surface can at most be the number of its boundary components. This means that elementary particle quantum numbers reside on boundary components in topological condensate. This implies explanation of family replication phenomenon in terms of boundary topology excellent predictions for lepton masses.

b) Supersymmetrization gives color confinement as a consistency condition:
each p-adic region with fixed value of $p$ is color singlet.
c) Elementary bosons other than graviton correspond to boundary components. Boundary component spinor fields are indeed independent degrees of freedom. If $CP_2$ coordinates are supersymmetrized the induced gauge potentials on the boundaries have fermionic quantum parts and in the lowest order gauge boson quanta can be regarded as fermion antifermion pairs: this is just what physical consideration have forced to conclude years ago. The couplings of the bosonic quanta are automatically of correct form and the interpretation of Kähler action as EYM action generalizes to the quantum context. It is easy to verify that the quantum contribution to the induced metric is automatically of order $p$ so that gravitational interaction strength has correct order of magnitude automatically.
c) In TGD quark color triplets correspond to color partial waves, which are expressions in $CP_2$ coordinates fixed uniquely by group theoretical considerations [Pitkanen]. There is however a problem in flat $M^4$ background since $CP_2$ coordinates are constant. One possibility is to allow color partial waves in $CP_2$ cm degrees of freedom of quark. Second possibility is classical description by fixing the classical color charges associated with the world line of point like quark. Supersymmetrization suggests a third manner to overcome the difficulty. The partial waves can be constructed using purely quantal $CP_2$ coordinates $K\bar{\Psi}\Gamma^k\Psi$ and are clearly a purely nonperturbative phenomenon!

6.1.4 Can one reproduce the Super Virasoro representations associated with elementary particles?

A basic test for the tentative scenario is whether it can reproduce the general structure making possible the succesfull mass calculations performed in [Pitkanen]. The previous considerations suggest that Kac Moody spinors correspond to representations of Super Virasoro transformations, which act on world line coordinate $t$ and on the complex coordinate of the boundary component so that generators are simply sums of the corresponding generators and one can understand string mass formula.

The rules for constructing Kac Moody spinors are following.
a) Single two-dimensional spinor corresponds to a p-adic counterpart of a unitary discrete series $so(4)$ representation $(c,h) = (0,0)$ representations
Kac Moody counter part of $H$-spinor corresponds to 4-fold tensor product of these representations, two representations in $M^4$ and two representation in $CP^2$ degrees of freedom. These representations give rise to Kac Moody version of Lorentz group $so(3, 1)$ and $so(4)$. To get Kac Moody spinor fields one adds two additional tensor factors. The group $U(1)$ associated with Kähler potential of $CP^2$ gives rise to additional tensor factor and color $su(3)$ gives further tensor factor. It turned out that $su(3)$ is unique among Lie groups in that it allows physically acceptable $(c, h) = (0, 0)$ Super Virasoro representation.

b) Gauge bosons correspond to N-S type representations in each tensor factor. Quarks correspond to Ramond representations in each tensor factor. Leptons are exceptional in the sense that they correspond to N-S representations in $su(3)$ factor rather than being Ramond type in every tensor factor.

c) The super partners of various particles have quantum numbers of lepton-quark in N-S representations so that N-S type super generators transform quarks into leptons and vice versa. In Ramond representations super generators conserve fermion number.

Is it possible to reproduce these structures in the proposed scenario? One can restrict the consideration to boundaries and each boundary component carries its own spinor fields satisfying its own Super Virasoro conditions.

a) Right handed neutrino generates $N = 1$ super symmetry and respects fermion number conservation whereas kappa symmetry transforms leptons into quarks. Localizing $N = 1$ supersymmetry generators using half odd integer or integer powers of $M$ one obtains anticommuting Super Virasoro generator in Ramond and N-S type representations respectively. Kappa symmetry has the properties needed to generate N-S type generators if one assumes that leptons move in integer modes and quarks move in half odd integer modes obtained as square roots of planewaves associated with leptons.

b) Although the action of Kappa symmetry transforms quarks into right handed neutrino and vice versa, the states of N-S representations are eigenstates of baryon and lepton number: in particular, bosons possess vanishing $B$ and $L$. The point is that the state, which is created by applying several operators $F_{n+1/2}$ vanishes unless the oscillator operators associated with right handed neutrino and its antiparticle anticommute pairwise to identity: this in turn implies that the quark number of the state vanishes. G-parity rule stating that only even or odd number of $F_{n+1/2}$'s appear in the operator cre-
ating the state implies that all bosons have $B = L = 0$.
c) One can construct local canonical transformations and as a special case
color transformations by multiplying the Hamiltonians with an analytic function of $M^4$ coordinates. Super symmetrization of the algebra is achieved by calculating the commutators with fermionic super Virasoro generators.
d) Local $so(4) \times u(1)$ is obtained makes sense as broken symmetry, when boundary components are idealized with world lines and localization is achieved by multiplying the generators with some function basis $f_n(t) = t^n$ world line coordinate. Again one must assume that leptons and quark type KM spinors on world line correspond to real p-adic plane waves and their square roots respectively.

One must keep in mind that the previous arguments say nothing about the realization of super conformal or $Diff(M^4)$ invariance in the interior. In the interior $Diff(M^4)$ seems to be a more appropriate symmetry and super version of this symmetry for fermions probably involves only Ramond representations and $N = 1$ symmetry since the kappa symmetry is need not be well defined in interior (the trace of the second fundamental form is not covariantly constant and can vanish).

6.2 The action of super $Diff(M^4)$ (conformal transformations)

$Diff(M^4)$ (conformal) transformation can be regarded as active transformations in $M^4$ (boundary components). One must replace the ordinary coordinate $h^k$ with $h^k_S$ in the expression for the infinitesimal generators of the $Diff(M^4)$ (conformal) transformation

$$ j^k(h) \frac{\partial}{\partial h^k} \rightarrow j^k(h_S) \partial_k \\
\partial_k \equiv \frac{\partial}{\partial h^k_S} \quad (47) $$

to guarantee that $j^k(h_S)$ is invariant under super $N = 1$ supersymmetry and Kappa symmetry. The action of the infinitesimal $Diff(M^4)$ (conformal) transformation on $H$-spinors is spin rotation
\[ \delta \Psi = \epsilon \partial_k j_l \Sigma^{kl} \Psi \]  

so that fermions possess conformal spin \( J = 1/2 \).

To derive the explicit form of Super Diff\((M^4)\) (Virasoro) generators one needs the explicit form of the infinitesimal generators of Diff\((M^4)\) (conformal) transformation, super symmetry and kappa symmetry as vector fields in the cartesian product of \( H \) and spinor space

\[
J_C = j^k \partial_k + \delta \Psi \frac{\partial}{\partial \Psi} + \delta \bar{\Psi} \frac{\partial}{\partial \bar{\Psi}} \\
J_F = j_F^k \frac{\partial}{\partial h^k} + \epsilon O(F) u_R \frac{\partial}{\partial \Psi} + \bar{\epsilon} u_R O(F) \frac{\partial}{\partial \Psi} \\
\bar{j}_F^k = \bar{\epsilon} u_R \gamma^k O(F) \bar{\Psi} + \epsilon \bar{\Psi} O(F) \gamma^k u_R \\
O(F = super) = 1 \\
O(F = kappa) = n^k \gamma_k \\
n^k \gamma_k \equiv \frac{H^k \gamma_k}{\sqrt{H^k H^k}} \\
Anti(\epsilon, \bar{\epsilon}) = 1
\]

(49)

Here subscript \( F = super/kappa \) refers to super symmetry or kappa symmetry and \( C \) to Diff\((M^4)\) (conformal) symmetry.

It is not at all clear whether kappa symmetry makes sense in interior whereas on the boundaries kappa symmetry is necessary in order to reproduce the physical picture suggested by the mass calculations. The generator of the might-be Kappa symmetry is obtained by multiplying \( u_R \) in the definition of super symmetry by \( K = n_k \gamma^k \), where \( n_k \) is the unit vector field associated with the trace of the super symmetrized second fundamental form. For minimal surfaces \( n_k \) is ill defined. The trace vanishes for the standard imbedding of \( M^4 \) and also for the gravitationally warped surfaces if the \( CP_2 \) projection of the surface is geodesic circle. On the boundaries situation is different. When boundary components are idealized with world lines with the property that \( S^k S_k = M^2 \) (\( S_k \) is the bosonic part of the ordinary second
fundamental form) hold true, the unit vector exists. Note that the existence of $1/\sqrt{H_k^k}$ does not produce problems if the bosonic part of $H^k$ is nonvanishing. It will be found that the value of $H^kH_k$ on boundaries is completely fixed.

The simplest guess is that super $Diff(M^4)$ (Virasoro) generators are obtained by suitably localizing $N = 1$ super symmetry and kappa invariance. The explicit form of the generators can be deduced using following arguments. For simplicity the consideration is restricted to conformal generators on boundary components. The generalization to $Diff(M^4)$ generators acting on interior is obvious: the only difference is that both quarks and leptons correspond to Ramond type representations in interior and only $N = 1$ supersymmetry is needed.

a) The square of the super generator is lightlike translation $u_R\gamma^k u_R$ and therefore proportional to the generator $L_{-1}$ multiplied by suitable p-adic number in 4-dimensional extension. Same applies to kappa symmetry. Hence the conformal weight of super and kappa symmetry generators is $-1/2$. Super generator and kappa generator behave essentially as $G_{-1/2}$ and it turns out that the anticommutators of the local super/kappa symmetry generators generate the bosonic parts of Virasoro generators. Virasoro generators however contain spin rotation term and therefore also local super generators must contain analogous term, which must vanish for super generator and kappa generator. A little consideration shows that this term must act as vector field in $M^4$ degrees of freedom.

b) The simplest guess for Ramond generators is following

\begin{align}
G_k &= G_k^0 + G_k^1 \\
G_k^0 &= M_{S}^{k+1/2} J_{Super} \\
G_k^1 &= k_R(n) M_{S}^{1/2} (\bar{\epsilon} u_R L_n \gamma^k \Psi + \epsilon \bar{\Psi} \gamma^k L_n u_R) \partial_k
\end{align}

For N-S generators one has

\begin{align}
G_{n-1/2}^0 &= M_{S}^{n} J_{Kappa} \\
G_{n-1/2}^1 &= k_{N-S}(n) (\bar{\epsilon} u_R K L_n \gamma^k \Psi + \epsilon \bar{\Psi} \gamma^k L_n K u_R) \partial_k \\
K &= \gamma^k n_k
\end{align}
where $M_S$ is complex $M^4$ super coordinate. The action of $L_n$ on $u_R$ is spin rotation. The dependence of the parameter $k_R(n)$ ($k_{N-S}(n)$) on $n$ is fixed by the requirement that anticommutators of the super generators are correct.

To see that the ansatz works for a suitable choice of the parameters $k_n$ some calculation is needed. It is enough to study the anticommutator of Ramond generators since the calculation for N-S case is essentially identical.

a) Super coordinate is annihilated by the action of local supersymmetries $G^0_n$ so that only the derivative with respect to $\Psi$ or $\bar{\Psi}$ appearing in the fermionic part appears in the anticommutator of local supersymmetries $G^0_n$ and one has

\[
\text{Anti}(G^0_m, G^0_n) = 2M_S^{n+m}\text{Anti}(\bar{\epsilon}, \epsilon)\bar{u}_R\gamma^k u_R \partial_k = 2M_S^{n+m}\bar{u}_R\gamma^k u_R \partial_k \propto L_{n+m}
\]

(52)

The action of the light like translation on analytic functions is equivalent with the action of $d/dM_S$ and one obtains apart from multiplicative constant the bosonic part of the Virasoro generator. Same conclusion holds true in N-S case.

b) The action of the fermionic part of the super symmetry $G^0_n$ on $G^1_m$ takes place via fermionic derivative and gives

\[
G^0_m G^1_n + (n \leftrightarrow m) = k_R(n)M_S^{m+1/2}\text{Anti}(\epsilon\bar{\epsilon})(M_S^{1/2}\bar{u}_R\gamma^k L_n u_R \partial_k + (n \leftrightarrow m)\propto 2k_R m S^{n+m}\bar{u}_R\gamma^k u_R \partial_k \propto L_{n+m}
\]

(53)

If $k_R(n)$ is of form $k_R(n) = k_R/(n+1)$, this term is also proportional to $L_{n+m}$ and combines with the commutator of the super generators to give $2L_{n+m}$ with a proper choice of the constant $k_R$. Same conclusion holds true in N-S case.

c) Spin rotation part of the Virasoro generator comes from the action of the spin rotation term $G^1_n$ on the fermionic part of local super-symmetry $G^0_m$ (the action on bosonic part vanishes) and gives a term

\[
G^1_n G^0_m + (m \leftrightarrow n)
\]

57
\[
=k_R(n)M_S^{1/2}(\bar{e}u_R L_n \gamma^k \Psi + \epsilon \bar{\Psi} \gamma^k L_n u_R)\partial_k Z^{m+1/2}(\bar{e}u_R \partial \Psi + \epsilon u_R \partial \Psi) \\
\propto k_R(n)(m + 1/2)M_S^m(\bar{e}u_R L_n \gamma^k \Psi + \epsilon \bar{\Psi} \gamma^k L_n u_R)(\bar{e}u_R \partial \Psi + \epsilon u_R \partial \Psi) + (n \leftrightarrow m) \tag{54}
\]

For the anticommutator to be proportional to the spin rotation term associated with \( L_{n+m} \), it must be proportional to \( n + m + 1 \). This is the case if one has (in accordance with the conclusion of b)

\[
\begin{align*}
  k_R(n) &= \frac{k_R}{n+1} \quad n \neq 0 \\
  k_R(-1) &= 0 
\end{align*} \tag{55}
\]

For this choice the anticommutator of the oscillator operators associated with covariantly constant right handed neutrino gives c-number term and the action of derivative operator reduces to the action of the light like translation generator and therefore the anticommutator is proportional to the spin rotation term of \( L_{m+n} \). With a proper choice of \( k_R \) and normalization of Super Virasoro generators it is possible to get correct anticommutation relations. In N-S case one obtains similar result. Note that \( n = -1 \) does not produce trouble since spin rotation term vanishes in this case and only the second term in anticommutator contributes.

The general expressions for \( Diff(M^4) \) currents can be deduced from the general formulas of appendix. The contribution of fermions to the conformal current has besides the term proportional to energy momentum tensor also a spin term proportional to \( \bar{\Psi} \Gamma^\alpha \partial_{jk} \Sigma^{kl} \Psi \). The spin rotation term does not vanish.

### 6.3 Local color symmetry and local canonical transformations in interior

Local canonical transformations are generated by Hamiltonians, which have arbitrary dependence on \( M^4 \) coordinates

\[
H_{loc} = f(m^k)H \tag{56}
\]
The change in the induced Kähler form is vanishing so that the change in action comes from the change of induced metric and is very small. Conformal canonical transformations are obtained by restricting the functions $f$ to functions $M^n$. The algebra contains local color transformations as a subalgebra.

The action of the infinitesimal canonical transformation on the induced spinor fields has following form

$$ J = j^k A_k + \partial_k j^i \Sigma^{kl} + k H $$

In this expression $h^k$ is replaced with $k^k$ everywhere. Here the covariant derivative contains also the $U(1)$ part necessitated by the acceptable spinor structure of $H$ and $k$ is integer, whose value is determined by the coupling to Kähler potential and proportional to fermion number, which is one third for quarks. $H$ denotes the Hamiltonian of the canonical transformation. The action of the canonical transformation on spinor connection implies additional terms in conserved current, which cancels the noncovariant piece proportional to $A_k$ in the conserved current and only the spin rotation term and Hamiltonian term are left. Hamiltonian term. The variation of $\Gamma_\alpha$ gives also a contribution to the current.

Canonical transformations generalize to supercanonical transformations just as conformal transformations do and one obtains Ramond and N-S type representations of super canonical symmetries. The currents associated with super canonical transformations are obtained by applying the general receipes of Appendix.

6.4 Realization of $Diff(M^4)$ invariance allows graviton.

$Diff(M^4)$ tranformations and their super counterparts act as approximate symmetries of the theory in the interior of the spacetime surface. This means that the approximate symmetry group is even larger than the group of p-adic conformal symmetries and that the theory is analogous to topological field theory. This is rather satisfactory feature since it gives final justification for the hypothesis that partons correspond to boundary components. The result however raises the question whether the spectrum in the interior contains any $Diff(M^4)$ invariant states besides vacuum state. Physical intuition
suggests that graviton and possibly some massless partners of the graviton should be contained in the spectrum (after all, the starting point of TGD was to construct Poincare invariant theory of gravitation!) The problem is to understand how graviton can circumvent the extremely tight constraint of $Diff(M^4)$ invariance. Standard anticommutation relations for fermions seem to imply essentially topological field theory. It turns out that p-adic massless of fermions allow to circumvent the difficulty.

The point is that p-adic planewaves with positive p-adic momentum $k$ are only needed and $k \to -k$ defines complex conjugate and functions expressible as Fourier series of negative momenta belong to the dual of the function space. This implies that fermionic creation operators correspond to momenta $k$ with each $k_i > 0$ and possessing finite number of positive pinary digits. The following modified definition of fermion vacuum so that time and spatial directions are treated in same manner

$$a_n|vac\rangle = 0, n_i < 0 \text{ for some } i \tag{58}$$

The formula states that oscillator operator acts as creation operator only provided all components of the four momentum are positive. The anticommutators of the fermionic oscillator operators

$$Anti(A_m, A_{-n}) = \delta(m - n) \tag{59}$$

are of the standard form.

The commutation relations of $Diff(M^4)$ generators in p-adic planewave basis $J(\epsilon, k)$, where $\epsilon$ is polarization vector characterizing the direction of flow induced by generator and $k$ with $k_i > 0$ for each $i$, is momentum of the p-adic planewave read as

$$Comm(J(\epsilon_p, p_1), J(\epsilon_2, p_2)) = \epsilon_1 \cdot p_2 J(\epsilon_2, p_1 + p_2) - \epsilon_2 \cdot p_1 J(\epsilon_1, p_1 + p_2) \tag{60}$$

On any light like geodesic $\epsilon \cdot \epsilon = 0$, with $p$ and $\epsilon$ parallel, these commutation relations become trivial. The central extension parameter $c$ for $Diff(M^4)$
are expected to be vanishing since the only source of c-number terms is normal ordering of the fermion generators in the expressions for translation generators $J(\epsilon, 0)$. In the lowest order the only possible source of normal ordering terms is spin rotation term, which however vanishes identically for translations. One can again define $\text{Diff}(M^4)$ generators associated with negative p-adic momenta as Hermitian conjugates of the positive momentum generators and they act in the dual of the function space.

$\text{Diff}(M^4)$ generators $J(\epsilon, -n)$ and other gauge group generators must annihilate physical states if $n$ is nonzero and $n_i$ are nonnegative:

$$J(\epsilon, -n)|\text{phys}\rangle = 0 \quad n_i \geq 0, \quad n \neq 0 \quad (61)$$

and it turns out that these conditions allow massless states. Translations correspond to generators $J(\epsilon, 0)$ and these generators cannot annihilate non-vacuum physical states. That complete $\text{Diff}(M^4)$ invariance is not required is in accordance with the fact that $\text{Diff}(M^4)$ invariance broken by gravitation.

Consider now the action of $\text{Diff}(M^4)$ generators $J(\epsilon, -n)$ on a state created by a fermionic creation operator with light like momentum $p$ with positive components. The action of $\text{Diff}(M^4)$ generator on the oscillator operator is induced from its action on planewave with momentum $p$ and the action must be of the form

$$\text{Comm}(J(\epsilon, -n), a_p) \propto \epsilon \cdot p a_{p-n} \quad (62)$$

The action vanishes if $\epsilon$ and $p$ are parallel or $\epsilon$ lies in transversal polarization space to $p$. The action gives annihilation type operator, when $n$ has momentum component $n_{\text{perp}}$ in transversal polarization space of $p$ since $n_{\text{perp}}$ has positive components so that states created using only oscillator operators with momenta in direction of $p$ are automatically annihilated by these $\text{Diff}(M^4)$ generators. When $\epsilon$ is in light like direction conjugate to $p$ and $n$ is in two-dimensional Minkowskian place spanned by $p$ and its light like conjugate the action is nonvanishing. The resulting oscillator operator is creation type operator for $p_3 - n_3 \geq 0$ so that one obtains finite number of conditions. Actually the conditions are equivalent with the ordinary Virasoro conditions.
The representation in question is reducible since there are several types of fermionic oscillator operators labeled by their electroweak quantum numbers. This means that nontrivial solutions of the Virasoro conditions are obtained and the number of the solutions is actually infinite. Therefore there are good hopes of obtaining graviton and its companions as $\text{Diff}(M^4)$ invariant states. Additional constraints come from Super Diff generators.

There are also additional conditions coming from the canonical invariance of the states. Canonical transformations induce $CP_2$ spin rotation and $U(1)$ transformation on the states and this means that canonical invariants must be constructed as Fourier components with momentum $p$ of spin vector currents $\bar{\Psi}\gamma^k\Psi$ invariant under $CP_2$ spin rotations and $U(1)$ rotations. $\Psi$ can have either lepton or quark type chirality.

\[
\begin{align*}
B(L, p, \epsilon) &= \bar{L}EL(p) \\
B(q, p, \epsilon) &= \bar{q}Eq(p) \\
E &= \epsilon_k\gamma^k \\
E \cdot p &= 0
\end{align*}
\]

(63)

Here $\gamma^k$ is $M^4$ gamma matrix and creation operators corresponds to either quark or lepton chirality: spin indices are written explicitly. Canonical invariance drops from consideration massless one-fermion states and only integer spin states with vanishing $B$ and $L$ remain under consideration. The quantities $B(p, \epsilon)$ can consists of quark or lepton type creation operators.

The constraints coming from Super symmetry are satisfied if one drops from $\bar{\Psi}\gamma^k\Psi$ away the part quadratic in oscillator operators associated with right handed neutrino and replaces $M^4$ coordinate with its super counterpart in p-adic planewes so that p-adic planewaves become supersymmetric. In a concise form one can express the invariants $B$ as

\[
\begin{align*}
B(L, p, \epsilon) &= \bar{L}EL(p) - \bar{\nu}_RE\nu_R \\
B(q, p, \epsilon) &= \bar{q}Eq(p) \\
p \cdot \epsilon &= 0
\end{align*}
\]

(64)

Only two transversal polarization vectors are needed. The operators $B(F, p, \epsilon)$ commute with each other and the commutators of $\text{Diff}(M^4)$ generators with
operators $B$ are proportional to operators $B$ themselves and Diff invariance conditions for state with momentum $p = n$ reduce to Virasoro conditions for $L_k,k = 1,\ldots,n$. This implies that Diff($M^4$) invariant states form algebra and the problem is to identify the generators of this algebra as monomials of operators $B$. The generators are labeled by the numbers $N(L)$ and $N(B)$ of $B(L,\ldots)$ and $B(q,\ldots)$ in the monomial, by polarization tensor $\epsilon_{kl\ldots n}$ in transversal degrees of freedom (spin) and by the total momentum $p$. A possible interpretation of the generators is as elementary bosons and the products of generators as many boson states with collinear momenta. If this interpretation is adopted one obtains dilaton, antisymmetric tensor ($J = 0$) and graviton as elementary bosons. Higher spin states can perhaps be regarded as many boson states.

a) $N = 1$ states are just zero momentum spin one states created by $B(F,\epsilon,0), F = L, q$.

b) $N = 2$ states are of type $B(L)B(L), B(q)B(q)$ and $B(L)B(q)$. For $p > 2$ states Virasoro conditions allow no solutions and only possible states have $p = 0$ or $p = 1$. $N = 2$ states with zero momentum are obtained as products of spin 1 generators: also spin two is possible. $p = 1$ states are possible, when zero modes are allowed. The states are of the form

\[ \sum_{\epsilon_1,\epsilon_2} c_{\epsilon_1,\epsilon_2}(B(F_1,\epsilon_1, p = 1)B(F_2,\epsilon_2, 0) - B(F_2,\epsilon_1, 1)B(F_1,\epsilon_2, 0)) \]  

$L_1$ Virasoro condition is satisfied by antisymmetry under exchange $F_1 \leftrightarrow F_2$. $B(L)B(q)$ states allow dilaton, antisymmetric tensor ($J = 0$) and graviton. For $B(F)B(F)$ only antisymmetric tensor state is possible.

c) For $N = 3$ one obtains $p = 0$ and $p = 1$ states constructed as products of $N = 1$ and $N = 2$ states: the largest spin of this kind of state is $J = 3$. It seems that no $N = 3, p = 1$ generating states exist. Spin one state with nonvanishing momentum is possible and this is due to the fact that only two polarization vectors are at use.

d) For $N = 4$ one obtains products of $p = 0,1$ generating states. Spin $J = 4$ is possible for two graviton state. Also $p = 2$ graviton and scalar state containing $B(L,\epsilon,p = 2)$ is possible in leptonic sector and is constructed using the operator $B(L,\epsilon,p = 2)B^2(L,\epsilon_+,0)B(L,\epsilon_-,0)$. This state is obtained as a special case of a more general construction procedure.
The states constructed do not include any higher spin exotic states. States with arbitrarily large values of $N$ and $p$ are possible and state space has natural shell structure as the following arguments show.

i) To construct a state containing $B(L, \epsilon, p = 2)$ operators construct first a zero momentum vacuum using the operator $B(L, \epsilon_+, 0)^2 B(B(L, \epsilon_-, 0)$. This state has spin $J = 1$ since the lacking right handed neutrino pair acts as a spin 1 hole. This state is annihilated by leptonic creation operators with $p = 0$. For quarks corresponding state is created by $B^2(q, \epsilon_+, 0)B^2(q, \epsilon_-, 0)$ and has vanishing spin. This state satisfies gauge conditions and is analogous to full electron shell in atomic physics. Apply to this state operators $B(F, \epsilon, 2)$. The resulting states satisfy gauge conditions automatically by fermion statistics.

ii) The procedure can be continued. Fill first $p = 1$ shell using using 3 (4) leptonic (quark like) operators $B(F, \epsilon, p = 2 = n)$. By commutativity of $B$ operators only the $p = 1$ fermionic creation operators are effective in this state. The new vacuum has spin $J = 2(J = 0)$ for $F = L (F = q)$ and applying $B(F, \epsilon, p = 4 = 2n)$ operators one obtains a new shell containing $p \leq 2$ fermionic creation operators.

iii) It is obvious that the procedure can be continued indefinitely and one obtains infinite number of states. The resulting states can be regarded as many particle states if full shell is regarded as a new vacuum state. The vacuum states in turn can be regarded as many boson states constructed from previous vacuum state and states are just products of leptonic and quark type states unlike the states obtained directly from the vacuum. This suggests that only the $N = 2, p = 1$ generators should be regarded as operators creating single particle states. The peculiar consequence of the supersymmetry is the possibility of states possessing arbitrarily large spin consisting of $J = 1$ holes made from pairs of right handed neutrinos.

6.5 Local $so(4) \times u(1)$ is broken in standard manner

Kac-Moody spinors used in the mass calculations are representations of the local $so(4)$ and $u(1)$ Kac Moody algebras and Quantum TGD as well as the success of the mass calculations suggests that local $so(4) \times u(1)$ do not act as gauge symmetry of the Kähler Dirac action.

The natural identification of $so(4) \times u(1)$ is as the gauge group of $CP_2$ spinor connection. Global $U(1)$ is exact symmetry. $so(4)$ rotations are exact
symmetries only for the surfaces possessing one-dimensional $CP_2$ projection implying vanishing of the induced gauge fields. This condition is in general not true. Condition however holds true, when the orbits of the boundary components are idealized with world lines so that global $so(4)$ is good approximate symmetry at parton level, whereas it is broken in the interior of the spacetime surface.

The localized $so(4) \times u(1)$ transformations are not exact symmetries in the point particle limit. The change of Dirac action density in a local gauge transformation reduces to a total divergence by the conservation of the charges associated with the global transformations if the the trace of the second fundamental form vanishes: this is not however the case.

\[
\delta L_D = D_t(\bar{\Psi} \text{Anti}(\Gamma^t, Q)\Psi f_n(t)) - X
\]
\[
X = D_t(\bar{\Psi} \text{Comm}(\Gamma^t, Q)D_t \Psi) f_n(t)
\]
\[
f_n(t) = t^n
\]

Here $Q$ is the generator of the local gauge transformation. Nonconservation term in general doesn’t vanish: it differs from conserved charge in that anti-commutator of $A$ and $\Gamma^t$ is replaced with commutator and the commutator does not in general vanish.

The term $X$ characterizing local charge nonconservation leads to the standard form of electroweak symmetry breaking. For left handed charges $Q$ ($I^t_L$ and $I^t_R$ associated $W^\pm$ bosons) it is impossible to find $\Gamma_t$ commuting with $Q$. For either purely vectorial or axial generator it is possible to satisfy the commutativity condition. Electromagnetic charge is the only purely vectorial electroweak charge and the vectorial generator $\Sigma_{12}$ appearing in electromagnetic charge commutes with $\Gamma^t$ when the condition

\[
\Gamma^t = a(t)\gamma^0 + b(t)\gamma^3
\]

is satisfied: here the standard definition of $CP_2$ vielbein is used [Pitkanen].

The commutativity of $\Gamma^t$ with $Q_{em}$ can be taken as additional condition on the time development for the world line of the boundary component in $CP_2$ degrees of freedom: this condition is equivalent with the conditions stating that the projection of the worldline is orthogonal to $e^A$, $A = 1, 2$:
Here $e^A_k$ are the components of the vielbein vectors of $CP_2$ and implies that induced $W^\pm$ fields vanish on the world line. This guarantees that different charged states for spinor field are not mixed and electromagnetic charge conservation results. The conditions imply that in standard coordinates $(r, \Theta, \Phi, \Psi)$ (Pitkanen) the coordinates $\Theta$ and $\Phi$ are constant on the world line of the particle. One must however notice that p-adic differentiability allows $\Theta$ and $\Phi$ to depend on finite number of positiveinary digits of time variable so that these variables are constant only below some time scale.

It is interesting to examine this condition, when $M^4$ projection of world line is geodesic line. Combining the condition with the requirement that $S^k$ is covariantly constant along world line (this guarantees the condition $S_kS_k = M^2$) one obtains the conditions

$$e^A_k \frac{ds^k}{dt} = 0, \ A = 1, 2$$

(68)

where $A$ and $B$ are p-adic constants depending on finite number of pinary digits of the time coordinate. The resulting conditions give two second order differential equations with 4 p-adic integration constants corresponding to fixing arbitrarily finite number of pinary digits for $(r, \Psi)$ and $(dr/dt, d\Psi/dt)$ as a function of $t$. One must notice that conservation laws give constraints, which reduce p-adic nondeterminism. For instance, p-adic geodesic lines of $M^4$ should look like ordinary geodesic lines. Furthermore, if p-adic derivative is algebraic operation as it is for p-adic planewaves p-adic nondeterminism is lost.

6.6 Lorentz invariance and local Lorentz transformations

The first problem related to the realization of p-adic Lorentz invariance is whether the p-adic convergence cube for square root function is invariant under Lorentz transformations or its subgroup. Real word geometric intuition
suggests that this is cannot be the case. p-Adic Lorentz group is however quite different from its real counterpart. The existence of the exponent for Lie-algebra element requires that the element is proportional to $\sqrt{p}$ at least. This means that the real counterparts of Lorentz transformations related continuously to identify are small and these transformations form a group. If the convergence cube has size $L_p$ then Lorentz transformation with generators are proportional to $\sqrt{p}$ at least leave the convergence cube invariant.

Momentum spectrum is not invariant under small Lorentz transformations. p-Adic convergence cube implies infrared cutoff in momentum: choosing scale suitably suitable the momentum components have the expansion

$$P = \sum_{k>1} P_k p^{-k}$$

(70)

The cutoff is obviously particle in the box effect. Small Lorentz transformations do not leave this expansion invariant since they induce non allowed p-adic digits to momentum components: the contributions of these digits to the real counterparts of momenta are extremely small. Perhaps one must just accept the noninvariance of the momentum spectrum under Lorentz transformations as a physical fact. The lowest order contribution to the energy is given by rest mass and is of order $O(p^{1/2})$ and Lorentz invariant since small Lorentz transformations induce only $O(p^{3/2})$ correction to energy.

One must emphasize that there are also 'large' Lorentz transformations for which nondiagonal matrix elements are of order $O(p^0)$. The orthogonality conditions for the matrix elements of these transformations give strong number theoretic constraints on the matrix elements of these transformations. An especially interesting subgroup of Lorentz group is rational Lorentz-transformations. It is impossible to generate them using exponential map since exponent function does not converge for Lie-algebra element having p-adic norm one. It would be interesting to understand the detailed physical consequences deriving from the factorization of the p-adic Lorentz transformations into a product of large and small transformations. Perhaps the so called nonrelativistic regime corresponds to small Lorentz transformations leaving p-adic convergence cube invariant.

Local $so(3,1)$ appears also in the context of Kac-Moody spinors and one can wonder whether these transformations are local symmetries for boundary-
components in the point like limit. Exactly the same arguments as before can be carried out as for local $\text{so}(4)$ to find that those local Lorentz transformations for which the charge generator $Q = q_{kl} \Sigma^{kl}$ commutes with $\Gamma^t$ define local worldline symmetries broken by gravitation only. Not surprisingly, this group is the isotropy group of four-momentum and for p-adic geodesic lines the group in question is independent of $t$ in sufficiently short time scales.

7 Kähler Dirac action and general field equations

In the sequel Kähler action and field equations will be discussed in a more detailed form.

7.1 Kähler Dirac action

Kähler action density in the interior is given by

$$L_K = \frac{1}{4g_k^2} J^\mu \nu J^\mu \nu \sqrt{g}$$  \hspace{2cm} (71)

g_k^2 is p-adic version of Kähler coupling strength. The experience with field theory limit of TGD suggests that p-adic Kähler coupling strength is related to the real coupling strength via the relationship

$$g_k^2 = g_K^2 p$$  \hspace{2cm} (72)

In this manner the real counterpart of the p-adic Kähler coupling strength does not deviate too much from its real value. Kähler action can contain also a boundary term, which is Chern Simons type term, which we do not bother to write explicitely here.

Dirac action density is given by

$$L_D = \left( \bar{\Psi} \Gamma^\alpha D^\alpha_+ \Psi - \bar{\Psi} D^\alpha_- \Gamma^\alpha \Psi \right) \sqrt{g}$$  \hspace{2cm} (73)

$$\Gamma^\alpha = \Gamma_k \partial^k$$
and differs from the ordinary Dirac action in that gamma matrices are dynamical variables obtained by inducing the gamma matrices of $H$ to 4-surface. In [Pitkänen], it was suggested that induced gamma matrices describe the presence of fermions but it seems that the proper interpretation is as a new bosonic field. Dirac action contains also parts associated with the boundaries of $X^4$ and each boundary component carries its own spinor field $\Xi$ so that boundaries become carriers of fermion numbers: this is the only manner to achieve conservation of fermion number currents in Quantum TGD. The Dirac action for boundary component has exactly the same form as the interior action and we do not bother to write it explicitly.

Super symmetrization of Kähler Dirac action is achieved by adding Super symmetric part to the bosonic coordinates. This is in accordance with general coordinate invariance if physically preferred coordinates are chosen for $H$. The coordinates in question are standard Minkowski coordinates for $M^4$ and complex coordinates of $CP_2$ transforming linearly under $su(2) \times u(1)$: as a by product color singletness of physical states emerges as consistency requirement.

7.2 Field equations in the interior

Field equations are interpreted as field equations for the effective action defining fermionic field theory in average space time associated with a given quantum state. This implies the needed back-reaction from matter to geometry. In particular, for diagonal matrix elements of bosonic field equations one obtains different fermionic source term for each state and background space-time depends on the quantum numbers of the state. It is convenient to allow bosonic solution to depend on maximal number of commuting operators defining the quantum numbers of the state. In particular, background spacetime depends on various YM charges and on the four-momentum of the state. If boundary components are treated as point like objects, the square for the trace of the $CP_2$ part the second fundamental form gives modular contribution to the mass squared of fermion.

It seems natural to solve the field equations by starting from the standard imbedding of $M^4$, to solve fermionic Dirac equation and then calculate the change of the space time metric (or deviation of $CP_2$ coordinates from constancy) by requiring that bosonic field equation remains true in given state.
with momentum $P^k$. Gravitational warping is in some sense the lowest order backreaction effect and depends on the four-momentum of the state, only. This effect is necessary in order to guarantee well definedness of perturbation series.

The general field equations of the theory are obtained by varying the action with respect to $h^k$ and $\Psi$. It is convenient to use $h^k_S$, $h^k$ and $\Psi$ as basic variables and introduces the definition of super coordinate as a constraint to the action density:

$$L = L_K + L_D + L_{\text{const}} = L + \Lambda_k (h^k_S - h^k - K \bar{\Psi} \Gamma^k (h^m_S) \Psi) \quad (74)$$

The field equations are obtained by varying with respect to $\Lambda$, $h^k_S$, $h^k$ and $\Psi$.

Consider first the variation with respect to $h^k_S$ in the interior: the field equations are obtained from the field equations of nonsupersymmetrized action by replacing $h^k$ with $h^k_S$ and by adding constraining term $\Lambda_k$ to the right hand side.

$$\frac{\delta S}{\delta h^k_S(x)} = \partial_\alpha (\partial(L_K + L_D)) - \partial(L_K + L_D) \frac{\partial h^k_S}{\partial h^k_S} - F_k$$

$$F_k = \Lambda_k - K \Lambda_l \bar{\Psi} \text{Comm}(V_k, \Gamma^l) \Psi$$

$$\text{Comm}(A, B) \equiv AB - BA \quad (75)$$

The functional derivative of Kähler action with respect to $h^k(x)$ is

$$\frac{\delta S_K}{\delta h^k(x)} = D_\alpha (T^{\alpha \beta} \partial_\beta h^l h_{kl}) - j^\alpha J_{kl} \partial_\alpha h^l$$

$$T^{\alpha \beta} = \frac{1}{4g^2_K} (J^{\mu \alpha} J^{\beta \nu} - g^{\alpha \beta} J^{\mu \nu} J_{\mu \nu}/4) \sqrt{g}$$

$$j^\alpha \equiv D_\beta J^{\alpha \beta} \quad (76)$$

The covariant derivatives appering in the various expressions contain both the ordinary Riemannian connection acting on $X^4$ indices and the Riemannian connection of $H$ acting on $H$ indices.
The functional derivatives of Dirac action with respect to H-coordinates are

\[
\frac{\delta S_D}{\delta h^k(x)} = D_a(T_\alpha^\beta \partial_\beta h^k + \bar{\Psi} \Gamma_k D^\alpha \Psi) + \bar{\Psi} \Gamma_k F^k_l \partial_\alpha h^l \Psi
\]

\[
T_\alpha^\beta = \bar{\Psi} (\Gamma_\alpha D^\beta + \Gamma_\beta D^\alpha) \Psi \sqrt{g}
\]

Divergence of the energy momentum tensor comes from the variation of metric and term proportional to \( \Gamma^k \) comes from the variation of induced gamma matrix. The term proportional to spinor curvature \( F_{kl} \) comes from the variation of spinor connection and divergence of energy momentum current comes from the variation of metric.

The resulting interior field equations have the character of hydrodynamical conservation equations. Only four of them are independent by general coordinate invariance and for surfaces representable as maps \( M^4 \rightarrow \mathbb{C}P^2 \) the equations can be replaced with equations stating conservation of four-momentum density obtained by contracting the canonical energy momentum tensor \( T_\alpha^\beta \) of Kähler Dirac action with the gradients \( \partial_\beta m^k \). Probably the situation is same now since \( \Lambda^k \) vanishes for \( M^4 \) indices.

The variation with respect to \( h^k \) affects only the constraint term and gives

\[
\Lambda_k = 0 \quad (78)
\]

so that the constraint force \( \Lambda^k \) vanishes in both \( M^4 \) and \( \mathbb{C}P^2 \) degrees of freedom and field equations associated with \( H \) coordinates are just standard equations with \( h^k \) replaced with \( h^k_S \).

The variation with respect to \( \Psi \) gives the Dirac equation for the induced spinors in interior with \( h^k \rightarrow h^k_S \) replacement and additional source term coming from the variation of the constraint equation

\[
(\Gamma_\alpha D_\alpha + \frac{1}{2} \Gamma_k H^k) \Psi = K \Lambda_k \Gamma^k \Psi = 0 \quad (79)
\]
Here $H^k$ is the super symmetric generalization of the second fundamental form

$$H^k_S = g^{\alpha\beta} D_\beta \partial_\beta h^k_S$$  \hspace{1cm} (80)

The presence of second fundamental form implies mixing of $M^4$ chiralities. The constraint force term vanishes by previous results so that field equations are of same form as in standard theory.

The vanishing of $\Lambda^1_{k}$ implies complete decoupling of Kähler and Dirac actions as also happens in Quantum TGD. One just takes the absolute minimum of ordinary p-adic Kähler action and replaces the $H$ coordinates with their supersymmetrized versions with $\Psi$ solved iteratively from Dirac equation.

The equations for $H$-coordinates must be posed as conditions on physical states analogous to and probably equivalent with Virasoro conditions. This means that Super Virasoro condition could well be an approximation to the actual field equations true only in the limit that $CP^2$ metric gives no contributions to the induced metric. Same conclusion applies to canonical invariance and its super counterpart.

### 7.3 Field equations on boundaries

The field equations on boundaries can be derived straightforward manner.  

a) Dirac equation on boundary reads as

$$\left(\Gamma^\alpha D_\alpha + \frac{1}{2} \Gamma_k H^k\right) \xi = \Gamma^n \Psi$$  \hspace{1cm} (81)

Interior spinors act as a source term for boundary spinors.  

b) Field equations for $h^k(x)$ state that the functional derivatives of total action with respect to $h^k(x)$ vanish on boundaries. The functional derivative of total Kähler action including Chern Simons term on boundary is given by

$$\frac{\partial S_K}{\partial h^k(x)} = T^{\alpha\beta} \partial_\beta h^k - (J^{n\beta} - J^{n\beta}_*) J^k I \partial_\beta h^I$$  \hspace{1cm} (82)
\( J_\ast \) denotes the dual of induced \( CP_2 \) Kähler form. It is assumed that the ratio of the coefficients \( k_1 \) and \( k_2 \) multiplying Maxwell action and Chern Simons term respectively in the definition of Kähler function equals to \( k_1/k_2 = 1 \) (for various definitions see [Pitkänen]).

c) The functional derivative of the Dirac action with respect to \( h^k(x) \) on boundary is given by

\[
\frac{\partial S_D}{\partial h^k(x)} = D_\alpha (T^{\alpha\beta}_{DB} \partial_\beta h^k) + \bar{\xi} \Gamma_k D^\alpha \xi + \bar{\xi} \Gamma^\alpha F^k_l \partial_\alpha h^l + T^{\alpha\beta}_{DB} \partial_\beta h^k
\]

\( T^{\alpha\beta}_{DB} = \bar{\xi} (\Gamma^\alpha D^\beta + \Gamma^\beta D^\alpha) \xi \sqrt{g_3} \)

(83)

The index \( B \) refers to boundary in the above expressions. \( F_{kl} \) denotes the components of spinor curvature.

These equations have counterparts for Kac Moody spinors and should take care of conservation laws besides giving the KM Dirac equation.

8 General solution of field equations

8.1 Perturbative solution

The general solution of the field equations is obtained by expanding \( \Psi \) in powers of \( K \propto p \)

\[
\Psi = \sum_n K^n \Psi_n
\]

(84)

and solve \( \Psi_n \) iteratively power by power. Since the super part of \( H \) coordinate is proportional to \( K \propto p \), one indeed obtains rapidly converging power series in \( K \).

a) The lowest order \( (K^0) \) field equation is in flat space case just the ordinary massless Dirac equation for the induced spinors and classical field equations for the extremals of Kähler action. \( \Lambda^k \) vanishes in this order.

b) Substituting the solutions of \( K^0 \) equations to field equations and writing
them in first order in $K$ one obtains Dirac equation for $\Psi^1$ with source term coming from the currents $J^k$ defined by $\Psi^0$. $\Psi_1$ can be expressed as a convolution of propagator with the source term just as in ordinary quantum field theory.

c) One can continue the iteration in similar manner but in practice few lowest orders should give the essentially exact solution for physically interesting values of $p$ of order $10^{38}$. The condition for convergence is that gradients of $CP_2$ coordinates are of order $K$ at most.

8.2 Perturbative construction for the solution of field equations for nonwarped $M^4$

Flat space background theory should be all what is needed for the application of the theory in particle physics context. Although the gravitational warping of $M^4$ must be taken into account in realistic theory it is useful to look the construction theory first for the standard imbedding of $M^4$.

In the lowest order one obtains just the massless Dirac equation in the background determined by the classical induced spinor connection of $CP_2$. In the standard $M^4$ background one has

$$\gamma^k \partial_k \Psi = 0$$

(85)

The most natural basis for the spinors is the $p$-adic planewave basis. The use of planewaves is natural since $p$-adic convergence cube is the analog of the finite string world sheet obtained by the logarithmic mapping $z \to w = \ln(z)$ from infinite complex plane and $M^4$ coordinates are just the counterpart of complex world sheet coordinate $w = \tau + i\sigma$.

The next order in $p$ introduces interactions and the Dirac equation contains gauge potential term given by the super part of the $CP_2$ coordinates defined by $\Psi_0$. Also the induced metric contains term, which however gives only a total divergence to the action in accordance with the idea that gravitational interaction is weak and therefore $O(p^2)$ effect. One obtains formally massless Dirac equation for $\Psi_1$ so that $\Psi_0$ defines a source term. One can solve $\Psi_1$ as a convolution of the free field propagator defined by quantized $\Psi_0$ with source term. One must however take this argument very cautiously.
The normal ordering of the oscillator operators in the source term gives via anticommutators a term linear in oscillator operators and that mass term might be generated.

The procedure can be continued. In fourth order in \( p \) (order is simply the maximal number of fermion currents in interaction term) Kähler action density gives nontrivial contribution to the interaction Lagrangian. This contribution can be interpreted also as EYM action for the induced gauge field and metric as proposed in [Pitkanen]: the difference with respect to the standard theory is that quantum gauge fields are fermion antifermion composites.

There are good reasons to expect that perturbation expansion leads to the replacement of the planewaves \( f_P \) with their quantized counterpart obtained by replacing \( M^4 \) coordinate with its super symmetric version. In fact, if one takes \( M^4 \) super coordinates as coordinates instead of \( M^4 \) coordinates one obtains automatically super symmetrized planewaves. The exponential is just the TGD:eish counterpart for the standard vertex operators appearing in string model and n-point functions for various fields automatically contain this kind of vertex operator.

### 8.3 The effect of gravitational warping

Flat space approximation works for the vacuum expectation value of the bosonic field equations only. The general matrix element of the bosonic field equation implies the dependence of \( H \) coordinates on various conserved quantum numbers. The most important of these quantum numbers is 4-momentum. The previous considerations suggest that standard imbedding of \( M^4 \) is simply replaced with a warped version of \( M^4 \).

Gravitational warping is indeed necessary to make the loop summations appearing in perturbation theory well defined and to remove propagator poles. This is achieved assuming that

i) propagator is defined in terms of warped metric

ii) the plane wave state basis is defined as state basis associated with ordinary massless Dirac operator, to which Dirac operator reduces in lowest order approximation.

The use of standard planewave basis is in principle possible. A possible good
reason for using this state basis is that the planewave basis associated associated warped metric does not satisfy the required orthogonality relations unless one scales the sides of p-adic convergence cube in appropriate manner.

For the most general gravitational warping Dirac operator contains mass term proportional to second fundamental form $H^k$:

$$p^\alpha \gamma_\alpha \rightarrow m^{\alpha\beta} p_\beta + \frac{H^k \Gamma^k}{2} H^k \quad = \quad D^\alpha \partial_\alpha h^k$$

If this term is covariant constant mass term can be included in propagator. Covariant constancy means that the $CP^2$ projection of warped $M^4$ can be regarded as a path of particle in covariantly constant force field in $CP^2$. A little calculation shows that second fundamental form is proportional to $P^2$ for gravitational warping and therefore vanishes for massless states.

9 Some open questions

It is good to list some of the open questions not explicitly mentioned in the text.

9.1 Description of hadrons as p-adic string like objects?

The basic assumption of p-adic approach is super conformal invariance for boundary components. Hadron mass calculations of fourth paper of [Pitkanen] are based on general features of Super conformal invariance and doesn’t exclude either the description of hadron as 'bag' or as a string. Therefore one must consider also the possibility of more general spacetimes such as string like objects suggested in basic TGD as description of hadrons [Pitkanen]. The formulation of the p-adic field theory is formally possible arbitrary spacetime topology and approximate symmetries are same as in $M^4 \rightarrow CP^2$ provided $CP^2$ contribution to metric is small. A particularly interesting possibility is the description of hadrons as string like objects. In this case the basic vacuum topology would be $X^2 \times Y^2 \in M^4 \times CP^2$, $Y^2$ being Lagrange manifold and $X^2$ two-dimensional string like object in $M^4$. These surfaces are vacuum extremals and canonical transformations produce new vacua of
this type. More general vacua are obtained as maps $X^2 \times Y^2$ to $M^4$ and physical hadrons should correspond to ‘thickened’ strings with transversal dimension of order hadronic length. Quarks and gluons would correspond to boundary components for this surface. One obtains string model with dynamically generated string tension coming from the energy associated with the super part of $H$-coordinates. What is however important that the general features of hadronic mass spectrum are determined to high degree from the p-adic thermodynamics of boundary components and universality for critical systems suggests that the predictions of string model description do not differ much from the predictions of ‘bag’ model description.

9.2 Kac Moody spinor concept

There are good reasons to hope that the treatment of the boundary components by idealizing them with point like objects and using second quantized Kac Moody spinors provides good practical description of elementary particle physics. The justification of the Kac Moody spinor concept from the properties of p-adicized Kähler Dirac action remains however to be found.

The main technical problem related to the second quantization of Kac Moody spinors is the construction of the action of bosonic KM spinors on the fermionic and bosonic KM spinors to make possible the concrete realization of minimal coupling gauge action.

9.3 The values of length scale $L_0$ and Kähler coupling strength

In this work a consistency argument leading to correct values for the length scale $L_0$ and Kähler coupling strength was represented but the actual deduction of the value of $L_0$ and Kähler coupling strength from Quantum TGD remains lacking.

10 Appendix: Conserved currents

The currents associated with conformal and canonical transformation can be deduced using standard formulas. The general form of transformation is given by
\[ h^k \rightarrow h^k + \delta h^k \]
\[ \Psi \rightarrow \Psi + \delta \Psi \] (87)

The change of spinor field come from gauge rotation associated with the transformation. In \( C P_2 \) degrees of freedom there is also a term proportional to the Hamiltonian of the canonical transformation acting as \( U(1) \) gauge trasformation. The details are given in main text.

The change of super coordinate can be derived using the constraint equation
\[ h^k_S = h^k + K \bar{\Psi} \Gamma^k (h^k_S) \Psi \]. A little calculation shows that super part of coordinate suffers a mere spin rotation so that one has

\[ \delta h^k_S = \delta h^k + K \bar{\Psi} \text{Comm}(\partial h^{m}_{S} \Sigma_{mn}, \Gamma^k) \Psi \] (88)

From this consistency condition \( \delta h^k_S \) can be solved by writing \( \delta h^k_S \) in series with respect to \( K \)

\[ \delta h^k_S = \sum K^n \delta h^k_{S,n} \]
\[ \delta h^k_{S,n} = \bar{\Psi} \text{Comm}(\partial \delta h^m_{S,n-1} \Sigma_{mn}, \Gamma^k) \Psi \]
\[ \delta h^k_{S,0} = \delta h^k \] (89)

The bosonic contribution to the current is given by

\[ J^\alpha_K = J^\alpha_k \delta h^k_S \]
\[ J^\alpha_k = \frac{\partial L_K}{(\partial_{\alpha} h^k_S(x))} \]
\[ = (T^{\alpha \beta} \partial \delta h^l h_{kl}) - F^{\alpha \beta} J_{kl} \partial \delta h^l ) \delta h^k_S \]
\[ T^{\alpha \beta} = \frac{1}{4g^2} (J^\alpha J^\beta + g^{\alpha \beta} J^\mu J^\nu / 4) \sqrt{g} \] (90)

Fermionic contribution to the current is given by
\[ J_D^\alpha = J_1^\alpha + J_2^\alpha \]
\[ J_1^\alpha = \frac{\partial L_D}{\partial (\partial_\alpha \Psi(x))} \delta \Psi = \bar{\Psi} \Gamma^\alpha \delta \Psi \]
\[ J_2^\alpha = \frac{\partial L_D}{\partial (\partial_\alpha h^k(x))} \delta h^k_S \]
\[ = (T_D^{\alpha \beta} \partial_\beta h^k + \bar{\Psi} \Gamma_k D^\alpha \Psi) \delta h^k_S \]
\[ T_D^{\alpha \beta} = \bar{\Psi} (\Gamma^\alpha D^\beta + \Gamma_\beta D^\alpha) \Psi \sqrt{g} \]
\[ \delta \Psi = \partial_k j_k \Sigma^{kl} + kH \]  

(91)

In this formula \( \delta \Psi \) contains only spin rotation term and Hamiltonian term since the term proportional to spinor connection drops from the transformation formula. Similar formulas can be derived for boundary contributions. The spin rotation term gives fermion conformal spin \( 1/2 \). Fermions possess also canonical spin coming from spin rotation in \( CP_2 \) degrees of freedom and canonical \( u(1) \) charge, which is essentially fermion number (1/3 for quarks).
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