Anomaly Nucleation Constrains SU(2) Gauge Theories

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We argue for the existence of additional constraints on SU(2) gauge theories in four dimensions when realized in ultraviolet completions admitting an analog of D-brane nucleation. In type II string compactifications these constraints are necessary and sufficient for the absence of cubic non-abelian anomalies in certain nucleated SU(N > 2) theories. It is argued that they appear quite broadly in the string landscape. Implications for particle physics are discussed; most realizations of the standard model in this context are inconsistent, unless extra electroweak fermions are added.

I. Introduction

Despite its many successes, the standard model of particle physics is an incomplete description of Nature. Some of its shortcomings, such as the absence of a cold dark matter candidate, can be amended in quantum field theory; others, such as the absence of quantum gravity, require a more robust framework for ultraviolet completion. An important question is whether constraints on gauge theories in that framework differ from those of generic quantum field theories, and whether these differences have implications for low energy physics.

In this letter we focus our attention on SU(2) gauge theories, whose relevance for Nature cannot be overstated; they govern the weak interactions [1], the recently discovered Higgs boson [2], and perhaps dark matter. We demonstrate that there are additional constraints on these theories when realized in ultraviolet completions admitting certain dynamical processes. These include broad regions of the string landscape. Most realizations of the standard model in this context are inconsistent, unless extra electroweak fermions are added.

The physical argument relies critically on a beautiful property of string theory. There, gauge theories are often carried by charged objects, such as D-branes [3]. If these objects can pair produce and embed the SU(2) theory into an SU(N) theory, then the SU(2) theory must satisfy additional constraints necessary for consistency of the SU(N) theory, but not its own. D-brane nucleation is an example of such a process, and we will study the constraints in this light. A natural objection is that such a system is unstable. Indeed this is true, but the gauge theories arising in it must nevertheless be anomaly free.

String consistency conditions are stronger than those of quantum field theory. For example, it has been argued [4] that there are effective theories which do not admit string embeddings, and also that matter representations are constrained (see [5] for a recent discussion). Here, we obtain a different type of constraint, placed on one theory to ensure the consistency of another related by a dynamical process. Perhaps this basic mechanism could be applied to other transitions in the string landscape.

This letter is organized as follows. In section II we present the physical argument for new constraints. In section III we derive this idea in the string landscape. In section IV we discuss implications for particle physics.

II. The Physical Argument

It is already evident in quantum field theory that four-dimensional SU(2) gauge theories are special within the broader class of SU(N) theories. The latter receive SU(N)^3 anomaly contributions from Weyl fermions in complex representations of the gauge group, and consistency constrains the allowed representations. However, there is no such constraint on SU(2) theories, since SU(2) does not have complex representations. Furthermore, SU(2) theories with an odd number of Weyl fermion doublets are inconsistent [6]. This arises from the fact the π_d(SU(2)) = Z_2, but since π_d(SU(N > 2)) = 0, there is no corresponding constraint on those theories. The former constraints are stronger than the latter.

Our main point can be made in a simple example before turning to string theoretic realizations. Consider an SU(2) gauge theory in four dimensions with an even number of left-handed Weyl fermion doublets. This is a consistent quantum field theory.

Now suppose that this theory is UV-completed into a framework where gauge theories are carried by charged objects which have dynamics and can pair produce. If as a result of this process the SU(2) theory has embedded into a nucleated SU(N) theory, ensuring the absence of SU(N)^3 anomalies can place constraints on the chiral spectrum of the SU(2) theory. For example, suppose the latter is embedded via the unhiggsing of an adjoint scalar such that doublets embed either into the [χ](1) or [χ](3) of SU(N), henceforth [χ]_N or [χ]_V. The embedding defines a way to distinguish two types of doublets; for notational convenience, denote those of the first and second type as [χ]_2 and [χ]_l, respectively. Then SU(N)^3 anomaly cancellation requires that χ([χ]_N) ≡ #[χ]_V - #[χ]_N satisfies

0 = χ([χ]_N) = χ([χ]_2),

(1)

where the SU(2) constraint χ([χ]_l) = 0 exists due to the process and ensures the absence of nucleated anomalies.

This is the phenomenon we wish to investigate broadly in the landscape. In certain corners such constraints have already been derived and been noted to be stronger than anomaly cancellation; it is in these corners that we derive the relationship to anomalies in nucleated theories. In other corners, we utilize nucleation processes and dualities to argue for their existence. Perhaps explicit derivations are possible there, as well.
III. Traversing the Landscape

We will begin in type IIA, since there the relationship between the chiral spectrum of four-dimensional theories and topological consistency conditions is simple.

Large Volume Type IIA Compactifications

Consider a large volume type IIA compactification (See \[\text{[7]}\] for reviews) on a compact Calabi-Yau threefold \(X\) with an antiholomorphic orientifold involution with fixed point locus a three-cycle \(\pi_0 \in H_3(X, \mathbb{Z})\) wrapped by a spacetime filling O6-plane. Stacks of \(N_a\) spacetime-filling D6-branes which wrap a generic three-cycle \(\pi_a\) and its orientifold image \(\pi'_a\) give rise to \(U(N_a)\) gauge theories in four dimensions; the \((1)\subset U(N_a)\) is often massive, giving \(SU(N_a)\) in the infrared. Chiral matter is localized at points of D6-brane intersection in \(X\); the possible representations are bifundamentals of two unitary groups or two-index tensor representations of one.

We would like to study the chiral spectrum of a distinguished D6-brane stack on \(\pi_N\) and its image with \(U(N)\) gauge symmetry. Since the D6-branes and O6-plane carry Ramond-Ramond charge, Gauss’ law requires

\[
N(\pi_N + \pi'_N) + \sum_{a \neq N} N_a (\pi_a + \pi'_a) - 4\pi_{O6} = 0.
\]

(2)

This is the D6-brane tadpole cancellation condition \[\text{[3]}\]. The topological intersection numbers of the branes compute the chiral spectrum as

\[
\chi(\pi_a, \pi_b) = \pi_a \cdot \pi_b, \quad \chi(\pi_a) = \frac{1}{2}(\pi_a \cdot \pi'_a + \pi_a \cdot \pi_{O6})
\]

\[
\chi(\pi_a, \pi'_b) = \pi_a \cdot \pi'_b, \quad \chi(\pi'_a) = \frac{1}{2}(\pi_a \cdot \pi'_a - \pi_a \cdot \pi_{O6}).
\]

Using these and intersecting \[\text{[2]}\] with \(\pi_N\) gives

\[
T_N \equiv \chi(\pi_N) + (N-4)\chi(\pi'_N) + (N+4)\chi(\pi'_N) = 0,
\]

(3)

a constraint on the chiral spectrum necessary for D6-brane tadpole cancellation, and thus global consistency.

This interplay between D6-brane tadpole cancellation and constraints on the chiral spectrum has been discussed extensively in the type IIA literature; see \[\text{[8]}\] for critical early works. In particular, \(T_N = 0\) for \(N > 2\) is the \(SU(N)^3\) anomaly cancellation condition; such anomalies do not exist for \(N = 1, 2\). In addition, certain \(U(1)\) anomalies are cancelled by a combination of the condition \(T_N = 0\) and axionic couplings via the Green-Schwarz mechanism; these include, for example, \(U(1)^3\) anomalies for the particular \(U(1) \subset U(N)\) \[\text{[9]}\]. This gives a low energy interpretation of the constraints \(T_2 = 0\) and \(T_1 = 0\); they play a partial role in \(U(1)\) anomaly cancellation.

We would like to present a different physical understanding of the \(T_2\) and \(T_1\) constraints. Though D6-brane charge cancellation in \(X\) is required for consistency, stability is not. To the system we have discussed, add a single D6 on \(\pi_N\) and a \(\overline{D6}\) on a distant but homologous cycle \(\pi_N\) (as well as their orientifold images). Such a configuration can be reached (with energy cost) by nucleating a \(D6-\overline{D6}\) pair on \(\pi_N\) and its image, and then unhiggsing the adjoint scalar associated to the combined D6-brane system.

Though there is a force between the brane anti-brane pair and they will annihilate via open string tachyon condensation \[\text{[10]}\], the worldvolume gauge theories of the D-branes must nevertheless be anomaly free prior to annihilation; in particular, the \(U(N+1)\) theory on \(\pi_N\) must not have \(SU(N+1)^3\) anomalies. Similar ideas regarding anomaly cancellation after brane nucleation have been studied in ten- and six-dimensional theories \[\text{[11]}\].

This relationship between the \(T_2\) (or \(T_1\)) constraint and nucleated anomalies can be derived. After adding this pair, there is a \(U(N+1)\) theory on \(\pi_N\) and its image, a \(U(1)\) theory on the \(\overline{D6}\) on \(\pi_N\) and its image, and the \(U(N_a)\) theories are left untouched. Quantitatively, we have added zero (in homology) to \(\text{[2]}\), which now reads

\[
(N + 1)(\pi_N + \pi'_N) + \sum_{a \neq N} N_a (\pi_a + \pi'_a) - 4\pi_{O6} = 0.
\]

\[
-\pi_N - \pi'_N = 0.
\]

(4)

Intersecting \(\pi_N\) with this equation, the terms in the first line give the contribution to \(SU(N+1)^3\) anomalies from chiral fermions localized at \(D6-\overline{D6}\) intersections, whereas \(-\pi_N \cdot \pi_N = 0\), but \(-\pi_N \cdot \pi'_N\) can be non-zero. The latter counts chiral fermions localized at these \(D6-\overline{D6}\) intersections, and the relative sign is important since the GSO projection in this sector projects out the opposite chirality fermion. In all, this calculation gives \(T_{N+1} = 0\), with the subtlety that \(\chi(\pi + \pi')\) receives contributions from both \(D6-\overline{D6}\) and \(D6-\overline{D6}\) intersections.

In summary, the computation before and after nucleation give \(T_N = 0\) and \(T_{N+1} = 0\), respectively. This can be iterated \(M\) times, giving the relationship between the constraint in the setups without and with \(M\ \overline{D6}\)-branes,

\[
T_N = 0 \leftrightarrow T_{N+M} = 0.
\]

(5)

This immediately gives a simple understanding of the \(T_2\) and \(T_1\) conditions: they are necessary and sufficient for the absence of \(SU(N + M)^3\) anomalies in this nucleated \(U(N+M)\) theory with \(N + M > 2\). This should be contrasted with their role in \(U(1)\) anomaly cancellation; since there axionic terms are also required, they are necessary but not sufficient for \(U(1)\) anomaly cancellation.

More Derivations in the Landscape

We would like to discuss two more derivations of the constraints in the landscape: one in type IIB, and the other in type I and their heterotic \(SO(32)\) duals.

Consider large volume type IIB flux compactifications with intersecting stacks of D7-branes carrying abelian worldvolume fluxes \(F_2\). The brane stacks wrap divisors \(D_a\) of a Calabi-Yau threefold \(X\) and carry \(U(N_a)\) gauge symmetry. For brevity, consider the case without orientifolds and a distinguished \(U(N)\) theory, this time with a \(D7\)-brane stack on \(D_N\) with flux \(F_N\). The \(D7\) and \(D5\)
tadpole cancellation conditions read
\[ N D_N + \sum_{a \neq N} N_a D_a = 0 \]
\[ N D_N \wedge F_N + \sum_{a \neq N} N_a D_a \wedge F_a = 0, \]
respectively, with Poincaré duality implied. Wedging the D7 tadpole with \( F_N \wedge D_N \) and the D5 tadpole with \( D_N \) gives \( \sum N_a D_a \wedge D_N \wedge (F_N - F_A) = 0 \). Rewriting in terms of the spectrum, this gives \( \sum_a N_a \chi(D_N, \pi_a) = \chi(D_N) = 0 \), a constraint necessary for D7 and D5 tadpole cancellation. Now consider a brane nucleated system with a D7 on \( D_N \) and a \( D7^\perp \) on a distant but homologous divisor \( D_N \) with worldvolume fluxes \( F_N \) and \( F_N \) in the same cohomology class. This adds zero in cohomology to the D7 and D5 tadpole condition but gives the constraint \( \chi(D_N+1) = 0 \). Taking \( N = 2 \), the constraint on the \( U(2) \) theory is necessary and sufficient for the absence of cubic non-abelian anomalies in the nucleated theory.

We see that \( SU(N > 2)^3 \) anomaly cancellation and the additional \( T_2 = 0 \) and \( T_1 = 0 \) constraints are a consequence of D7 and D5 tadpole cancellation. Since background three-form fluxes contribute to the D3 tadpole, they can be added without spoiling these chiral spectrum constraints. These are the fluxes critical in the popular moduli stabilization scenarios \([12]\) which give most of the known landscape of string vacua. Thus, the constraints appear broadly in the known landscape.

As a final example, consider the type I or \( SO(32) \) heterotic string compactified to four dimensions on a Calabi-Yau threefold \( X \) endowed with a holomorphic vector bundle \( V = \bigoplus_{m=K+1}^{K+L} V_m \oplus \bigoplus_{i=1}^K L_i \), where the structure group of \( V_m \) and \( L_i \) are \( U(N_m) \) and \( U(1) \), respectively. This is more generic than the common ansatz of \( V \) with \( SU(N) \) structure group. The four-dimensional gauge algebra has an \( SU(N_i)^3 \) factor, and in \([13]\) it was shown that the \( SU(N_i)^3 \) anomaly is \( A_{SU(N_i)^3} \sim 2 \int c_1(L_i) \times \text{Tad} \), where \( T \) is a four-form expression which must be cohomologically trivial for consistency via D5 tadpole cancellation and the B-field Bianchi identity in the type I and heterotic string, respectively.

Again, \( SU(N_i)^3 \) anomaly cancellation is ensured by a topological consistency condition and there is also a constraint for \( N_i = 2 \). It is interesting that such a constraint exists in the heterotic string, since D-brane nucleation has played major a role thus far and does not exist in the heterotic string. Via nucleation of magnetized D9-branes in the type I case the \( SU(2) \) constraints can likely be related to nucleated \( SU(N_i)^3 \) anomaly cancellation. Doing so in a way consistent with D7-brane tadpole cancellation likely requires introducing an instability in the gauge bundle, pointing to a heterotic interpretation.

**Existence Arguments**

Though (to our knowledge) similar constraints have not been explicitly derived in four-dimensional compactifications of M-theory, F-theory, and the heterotic \( E_8 \times E_8 \) superstring, we would like to present arguments in favor of their existence, utilizing the existence of nucleation-type processes and string dualities.

We began by discussing type IIA intersecting D6-brane compactifications. If supersymmetric, these lift to compactifications of M-theory on seven manifolds with \( G_2 \) holonomy, in which case vector (chiral) multiplet data is captured by codimension four (seven) singularities in the geometry. An important question is whether the \( D6 \)-\( D6 \) annihilation process critical to the physics of the constraints in IIA has a known M-theory lift, even locally. Such gravitational solutions have in fact been constructed \([14]\); prior to annihilation the system is described by a bolt singularity, and afterwards a Taub-NUT. If such a process relates an \( SU(2) \) and \( SU(N) \) theory it is plausible that \( SU(N)^3 \) anomaly cancellation can be used to constrain the \( SU(2) \) theory obtained after annihilation.

The T-Dual type IIB picture with D7-branes lifts to F-theory. In its weak coupling limit, F-theory is simply a geometrization of type IIB. \( D7-\overline{D7} \) nucleation modifies the IIB axiodilaton profile, and to the author’s knowledge the geometric F-theory lift is not known; it certainly must modify the geometry significantly, as the presence of the new seven-branes introduces new moduli. Perhaps the appropriate modification of the geometry can be extended outside of the weakly coupled regime; if so, consistency requires the absence of nucleated anomalies. As further evidence, it is known \([15]\) that \( SU(N)^3 \) anomalies are automatically cancelled in \( d = 4 \) F-theory compactifications with appropriately specified “\( T_4 \)-flux,” and the mechanism is equivalent to D7 and D5 tadpole cancellation in the weak coupling limit, which were critical above.

Finally, consider the \( E_8 \times E_8 \) heterotic string on a Calabi-Yau threefold. Suppose the compactification has an F-theory dual; then vector bundle moduli which Higgs \( E_8 \times E_8 \) to the four-dimensional gauge group map to complex structure moduli in F-theory, which determine the Higgsing of two \( E_8 \) seven-branes via unfolding. The additional seven-brane moduli associated to the nucleation process suggested in the last paragraph would require passing to a different heterotic vector bundle with more moduli: since supersymmetry would be broken via the nucleation process, the new bundle must be unstable. This may provide an avenue for an explicit derivation.

**Symplectic Realizations of \( SU(2) \)**

We would like to note another possibility, where an \( SU(2) \) gauge theory is realized as \( Sp(1) \). In known cases, there are no constraints analogous to \( T_2 = 0 \); e.g. in type IIA there are no constraints on their chiral spectra necessary for D6 tadpole cancellation. This matches nicely with the fact that such theories would nucleate \( Sp(N) \) rather than \( U(N) \) theories, which do not have cubic non-abelian anomalies. See \([16]\) for a IIA discussion.

Realizing \( Sp(N) \) theories can require additional geometric constraints; e.g. in IIA, D6-brane three-cycles must be orientifold invariant, and in F-theory codimension two singular loci must induce an automorphism of codimension one fibers. Additionally, conventional grand unification is difficult when \( SU(2)_L \) is realized as \( Sp(1) \).
IV. Implications for Particle Physics

Given the necessity of ultraviolet completion, it is important to study the potential implications of these constraints for physics beyond the standard model.

Model-building from the bottom-up in this context \([17]\), the standard model (or MSSM) itself is incomplete: one must specify more input data, labeling \(SU(2)\), doublets as \(\mathbb{C}^{1}\) or \(\mathbb{C}^{2}\), according to their embedding into the nucleated theory. Our scheme for counting is as follows: for each set of \(F\) fermion doublets with the same standard model quantum numbers, consider all possible tuples (\(#\mathbb{C}^{1}\), \(#\mathbb{C}^{2}\)) such that the sum is \(F\). Then the three families of quark and lepton doublets split as \((3, 0)\), \((2, 1)\), \((1, 2)\) or \((0, 3)\). These contribute to \(T_2\) as

\[
T_2 \equiv \{\pm 1, \pm 3\} \quad \text{and} \quad T_2^Q = \{\pm 3, \pm 9\},
\]

where there is a factor of 3 for color in \(T_2^Q\). In all, this gives 16 possibilities for \(T_2^SM \equiv T_2 + T_2^Q\), only two of which satisfy \(T_2^SM = 0\). In the MSSM, the Higgsinos split as \((1, 0)\) or \((0, 1)\), contributing

\[
T_2^{\tilde{h}_u} \in \{\pm 1\} \quad \text{and} \quad T_2^{\tilde{h}_d} \in \{\pm 1\}
\]

with only 6 of the 64 possibilities being consistent.

Most standard model and MSSM configurations do not satisfy the additional constraint on the \(SU(2)\) spectrum. Without probabilistic information about the likelihood of realizing one configuration over another in the landscape, it is difficult to draw definitive conclusions.

Conservatively, though, there are only two possibilities that should be considered.

1) If \(T_2^SM = 0\), then the quark doublet sector exhibits a \((2+1)\) family non-universality \([18]\).

2) If \(T_2^SM \neq 0\), then new electroweak fermions are required for the absence of nucleated anomalies.

Identical statements hold when considering \(T_2^{MSSM}\). The quark doublet non-universality could have further implications; for example, in type II realizations with \(U(2)\) gauge symmetry the diagonal \(U(1)\) will forbid some quark Yukawa couplings in string perturbation theory.

The second possibility is striking: it provides a new theoretical motivation for exotic electroweak fermions. In many cases (see \([19]\) for systematic studies in type II) these new states are vector-like with respect to the standard model but chiral under another symmetry, in which case they give smaller corrections to precision electroweak observables than do chiral exotics, but nevertheless have protected mass. If protected from decay, the neutral components of the exotics are excellent WIMP dark matter candidates; see \([20]\) for a broad discussion in type II. These exotic particles could be discovered at LHC or in direct detection experiments in the near future.

Note that since the \(T_3 = 0\) condition is just the \(SU(3)^3\) anomaly cancellation condition, there is not a similar motivation for new colored fermions. Thus, the additional constraints motivate exotics which are not in complete GUT multiplets. One might naively think that such exotics would ruin gauge coupling unification (GCU), and in fact they will if added as chiral supermultiplets to the MSSM. However, in theories without weak scale supersymmetry grand unification could be a virtue of these exotic \(SU(2)_L\) states; see \([21]\) for a broad treatment. For example, pairs of exotic \((1, 2)_{\pm 1/2}\) fermions have the quantum numbers of Higgsinos and can improve GCU; this is the minimal extension \([22]\) of the standard model giving dark matter and grand unification.

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[17] That is, when such constraints hold; when SU(2) is realized as Sp(1), for example, this is not the case.

[18] For brevity, we have not considered the possibility that $e_\gamma$ or $\nu_\tilde{\gamma}$ is realized as an antisymmetric tensor; in that analysis the family non-universality can be relaxed. Adjoint breaking of SU(5) GUTs yields such configurations.

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