Probing New Physics in the Neutrinoless Double Beta Decay Using Electron Angular Correlation

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Abstract

The angular correlation of the electrons emitted in the neutrinoless double beta decay (0ν2β) is presented using a general Lorentz invariant effective Lagrangian for the leptonic and hadronic charged weak currents. We show that the coefficient $K$ in the angular correlation $d\Gamma/d\cos\theta \propto (1 - K \cos\theta)$ is essentially independent of the nuclear matrix element models and present its numerical values for the five nuclei of interest ($^{76}$Ge, $^{82}$Se, $^{100}$Mo, $^{130}$Te, and $^{136}$Xe), assuming that the 0ν2β-decays in these nuclei are induced solely by a light Majorana neutrino, $\nu_M$. This coefficient varies between $K = 0.81$ (for the $^{76}$Ge nucleus) and $K = 0.88$ (for the $^{82}$Se and $^{100}$Mo nuclei), calculated taking into account the effects from the nucleon recoil, the $S$ and $P$-waves for the outgoing electrons and the electron mass. Deviation of $K$ from its values derived here would indicate the presence of New Physics (NP) in addition to a light Majorana neutrino, and we work out the angular coefficients in several $\nu_M$ + NP scenarios for the $^{76}$Ge nucleus. As an illustration of the correlations among the 0ν2β observables (half-life $T_{1/2}$, the coefficient $K$, and the effective Majorana neutrino mass $|\langle m \rangle|$) and the parameters of the underlying NP model, we analyze the left-right symmetric models, taking into account current phenomenological bounds on the right-handed $W_R$-boson mass and the left-right mixing parameter $\zeta$. 

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1 Introduction

It is now established beyond any doubt that the observed neutrinos have tiny but non-zero masses and they mix with each other, with both of these features following from the observation of the atmospheric and solar neutrino oscillations and from the long baseline neutrino oscillation experiments [1]. Theoretically, it is largely anticipated that the neutrinos are Majorana particles. Experimental evidence for the neutrinoless double beta decay (0ν2β) would deliver a conclusive confirmation of the Majorana nature of neutrinos, establishing the existence of physics beyond the standard model. This is the overriding interest in carrying out these experiments and in the related phenomenology [2].

We recall that 0ν2β-decays are forbidden in the standard model (SM) by lepton number (LN) conservation, which is a consequence of the renormalizability of the SM. However, being the low energy limit of a more general theory, an extended version of the SM could contain nonrenormalizable terms (tiny to be compatible with experiments), in particular, terms that violate LN and allow the 0ν2β decay. Probable mechanisms of LN violation may include exchanges by: Majorana neutrinos $\nu_{MS}$ [3, 4, 5] (the preferred mechanism after the observation of neutrino oscillations [1]), SUSY particles $\xi$ [6, 7, 8, 9, 10, 11], scalar bilinears (SBs) [12], e.g. doubly charged dileptons (the component $\xi^-$ of the $SU(2)_L$ triplet Higgs scalar etc.), leptoquarks (LQs) [13], right-handed $W_R$ bosons $\xi$ [14] etc. From these particles light $\nu$s are much lighter than the electron and others are much heavier than the proton. Therefore, there are two possible classes of mechanisms for the 0ν2β decay. With the light $\nu$s in the intermediate state the mechanism is called long range and otherwise it is referred to as the short range mechanism. For both these classes, the separation of the lepton physics from the hadron physics takes place [14], which simplifies calculations. According to the Schechter–Valle theorem [15], any mechanism inducing the 0ν2β decay produces an effective Majorana mass for the neutrino, which must therefore contribute to this decay. These various contributions will have to be disentangled to extract information from the 0ν2β decay on the characteristics of the sources of LN violation, in particular, on the neutrino masses and mixing. Measurements of the neutrinoless double beta decay in different nuclei will help in determining the underlying physics mechanism [17, 18].

Our aim in this paper is to examine the possibility to discriminate among the various possible mechanisms contributing to the 0ν2β-decays using the information on the angular correlation of the final electrons in the process $N_i(A, Z) \rightarrow N_f(A, Z + 2) + e^- + e^-$. A preliminary study along these lines was published by us in 2006 [19], with admittedly simplified treatment neglecting the nucleon recoil and the $P$-wave effects in the outgoing electron wave function. We rectify these shortcomings and provide in this paper a detailed account of the improved treatment. Restricting ourselves to the long-range mechanism, treating the electrons relativistically but with non-relativistic nucleons, we derive the angular correlation between the electrons using the general Lorentz invariant effective Lagrangian involving the leptonic and hadronic charged weak currents. Generally, this angular correlation can be expressed as $d\Gamma/d\cos \theta \sim 1 - K \cos \theta$, where $\theta$ is the angle between the electron momenta in the rest frame of the parent nucleus. Expressing $K = B/A$, with $-1 < K < 1$, we derive the analytic expressions for $A$ and $B$ for the effective Lagrangian characterized by the coefficients $e_{ai}^\beta$, encoding the standard, $(V - A) \otimes (V - A)$, and new physics contributions (see Eq. (1)). Essential steps of these derivations are presented in section 2. The analytic expressions derived here confirm the earlier detailed derivations by Doi et al. [5], and we specify where the treatment presented here transcends the earlier work. Specific cases are relegated to Appendix A (for the decays involving scalar nonstandard terms), Appendix B (for the vector nonstandard terms), and Appendix C (for the tensor nonstandard terms). We hope to return to the discussion of including the short-range mechanism, neglected in this paper, in future work.

Numerical analysis of the electron angular correlation is presented in section 3, and the coefficient $K$ for the various underlying mechanisms in 0ν2β-decays are worked out. In particular, numerical values of $K$ for the five nuclei of current experimental interest: $^{76}$Ge, $^{82}$Se, $^{100}$Mo, $^{130}$Te, and $^{136}$Xe are presented for the light Majorana neutrino $\nu_M$ case. Their values range from $K = 0.81$ (for the $^{76}$Ge nucleus) and $K = 0.88$ (for the $^{82}$Se and $^{100}$Mo nuclei). To study the uncertainty in the nuclear matrix elements, we have employed the so-called QRPA model with and without the p-n pairing for the $^{76}$Ge nucleus [20], and a more modern QRPA model, fixing the particle-particle pairing strength [21]. While the uncertainty due to the nuclear matrix element model is quite marked for $T_{1/2}$ in some cases, we show that it is rather modest for $K$, not exceeding 10% for the models discussed here. For the $\nu_M +$ NP scenarios, we remark that the nonstandard
coefficients $\epsilon_{V+}^{A}, \epsilon_{T_{R}}, \epsilon_{T_{L}}^{A}$ do not change the value of the angular coefficient $K$. The contribution of the scalar nonstandard term from the $\epsilon_{S}^{V+}F_{S}$ coefficients is found to be numerically small. So, what concerns the angular correlation, we have essentially three distinct scenarios: (i) Standard ($\nu_{M}$), (ii) R-parity violating SUSY ($\nu_{M} + \epsilon_{T_{R}}^{A}$), and (iii) left-right-symmetric models ($\nu_{M} + \epsilon_{V+}^{A}$). Numerical analysis of the coefficient $K$ in the extended $\nu_{M}$ + NP scenario is carried out for the decay of the $^{76}$Ge nucleus using the nuclear matrix element model already specified.

We take a closer look at the underlying physics behind the coefficients $\epsilon_{V+}^{A}$ in section 4. These coefficients appear in the context of the left-right symmetric models which are theoretically well motivated [22]. The form factors $g\kappa$ both the mixing of right-handed quarks and the mixing of left- and right-handed gauge bosons (see Ref. [5], Eq. (A.2.17)). Here and thereafter, a summation over the repeated indices is assumed; $\alpha, \beta = V \pm A, S \mp P, T_{L,R}$ ($O_{T_{R}} = 2\sigma^{\mu\nu}P_{\rho}, \sigma^{\mu\nu} = \frac{1}{2}[\gamma^{0}, \gamma^{\rho}]$), $P_{\rho} = (1 \mp \gamma_{5})/2$ is the projector, $\rho = L, R$; the prime indicates the summation over all the Lorentz invariant contributions, except for $\alpha = \beta = V - A, U_{ei}$ is the PMNS mixing matrix [25] and $V_{ud}$ is the CKM matrix element [1]. Note that in Eq. (1) the currents have been scaled relative to the strength of the usual $V - A$ interaction with $G_{F}$ being the Fermi coupling constant. The coefficients $\epsilon_{V+}^{A}$ encode new physics, parametrizing deviations of the Lagrangian from the standard $V - A$ current-current form and mixing of the non-SM neutrinos. In discussing the extension of the SM for the $0\nu2\beta$ decay, Ref. [5] considered explicitly only nonstandard terms with

$$
\epsilon_{V+}^{A} = \frac{\kappa g_{V}}{g_{V}^{u}}, \quad \epsilon_{V-}^{A} = \eta g_{V}^{u}, \quad \epsilon_{V+}^{A} = \lambda g_{V}^{u}.
$$

Implicitly, also the contributions encoded by the coefficients $\epsilon_{V-}^{A}$ are discussed arising from the non-SM contribution to $U_{ei}$ in $SU(2)_{L} \times SU(2)_{R} \times U(1)$ models with mirror leptons (see Ref. [5], Eq. (A.2.17)). Here $V, U'$ are the $3 \times 3$ blocks of mixing matrices for non-SM neutrinos, e.g., for the usual $SU(2)_{L} \times SU(2)_{R} \times U(1)$ model $V$ describes the lepton mixing for neutrinos from right-handed lepton doublets; for $SU(2)_{L} \times SU(2)_{R} \times U(1)$ model with mirror leptons [26] $U'$ ($V'$) describes the lepton mixing for mirror left(right)-handed neutrinos [5] etc. The form factors $g_{V}^{u}$ and $g_{V}^{d}$ are expressed through the mixing angles for left- and right-handed quarks. Thus, $g_{V}^{d} = \cos \theta_{C} = V_{ud}$ and $g_{V}^{u} = e^{i\delta} \cos \theta_{C}'$, with $\theta_{C}$ being the Cabibbo angle, $\theta_{C}'$ is its right-handed mixing analogue, and the CP violating phase $\delta$ arises in these models due to both the mixing of right-handed quarks and the mixing of left- and right-handed gauge bosons (see Ref. [5], Eq. (3.1.11)). The parameters $\kappa, \eta$, and $\lambda$ characterize the strength of nonstandard effects. Below, we give some illustrative examples relating the coefficients $\epsilon_{V-}^{A}$, $\epsilon_{V+}^{A}$, and the particle masses, couplings and the mixing parameters in the underlying theoretical models.

In the R-parity-violating (RPV) SUSY accompanying the neutrino exchange mechanism [6, 7, 8, 9, 10, 11], SUSY particles (sleptons, squarks) are present in one of the two effective 4-fermion vertices. (The other vertex
contains the usual $W_L$ boson.) The nonzero parameters are

$$
\epsilon_{V-A,i}^{S-P} = \frac{1}{2} \eta^{n_1}_{LR} U_{ni}, \quad \epsilon_{S-P,i} = 2 \eta^{n_1}_{LR} U_{ni},
$$

$$
\epsilon_{s + i}^{S-P} = -\frac{1}{4} \left( \eta^{n_1}_{LR} - 4 \eta^{n_1}_{LR} U_{ni} \right) U_{n_i}, \quad \epsilon_{T_R,i} = \frac{1}{16} \eta^{n_1}_{LR} U_{n_i}^{*},
$$

where the index $n$ runs over $e, \mu, \tau (1, 2, 3)$, and the RPV Minimal Supersymmetric Model (MSSM) parameters $\eta$ depend on the couplings of the RPV MSSM superpotential, the masses of the squarks and the sleptons, the mixings among the squarks and among the sleptons. Concentrating on the dominant contributions $\epsilon^{S-P}_{S+P,i}$ and $\epsilon_{T_R,i}$ (as the others are helicity-suppressed), one can express $\eta^{n_1}_{LR}$ and $\eta^{n_1}_{LR}$ as follows [10]

$$
\eta^{n_1}_{LR} = \sum_k \frac{\lambda_{11k} \lambda_{n1k}}{2 \sqrt{2} G_F} \sin 2\theta^{(k)} \left( \frac{1}{m_d^{(k)}} - \frac{1}{m_u^{(k)}} \right),
$$

$$
\eta^{n_1}_{LR} = \sum_k \frac{\lambda_{k11} \lambda_{n1k}}{2 \sqrt{2} G_F} \sin 2\theta^{(k)} \left( \frac{1}{m_d^{(k)}} - \frac{1}{m_u^{(k)}} \right),
$$

where $k$ is the generation index, $\theta^{(k)}$ are the squark and slepton mixing angles, respectively, $m_{d, u}$ are the sfermion mass eigenvalues, and $\lambda_{ijk}$ are the RPV-couplings in the superpotential.

For the mechanism with LQs in one of the effective vertices [13], the nonzero coefficients are

$$
\epsilon_{s + i}^{S-P} = \frac{\sqrt{2}}{4 G_M^2 M_S^2} \epsilon_V^{S-P}, \quad \epsilon_{S-P,i} = -\frac{\sqrt{2}}{4 G_M^2 M_S^2} \epsilon_S^{S-P},
$$

$$
\epsilon_{V-A,i}^{S-P} = -\frac{1}{2 G_F} \left( \alpha^{(L)}_S \frac{M_S^2}{M_V^2} + \alpha^{(L)}_V \frac{M_S^2}{M_V^2} \right), \quad \epsilon_{V-A,i}^{S-P} = -\frac{\sqrt{2}}{4 G_F} \left( \alpha^{(R)}_S \frac{M_S^2}{M_V^2} + \alpha^{(R)}_V \frac{M_S^2}{M_V^2} \right),
$$

where

$$
\epsilon^{\beta}_S = U e^{\beta} A_{ai},
$$

the parameters $\epsilon^{S(V)}_S$, $\alpha^{(L)}_{S(V)}$, and $\alpha^{(R)}_{S(V)}$ depend on the couplings of the renormalizable LQ-quark-lepton interactions consistent with the SM gauge symmetry, the mixing parameters and the common mass scale $M_{S(V)}$ of the scalar (vector) LQs [27].

The nonzero $\epsilon^{\beta}_S$ for the discussed models are collected in Table 1.

| Model           | Nonzero $\epsilon^{S}$                   |
|-----------------|------------------------------------------|
| with $W_{RS}$   | $\epsilon_{V-A,i}^{S-P}, \epsilon_{V+A,i}^{S-P}$ |
| RPV SUSY        | $\epsilon_{S-P,i}, \epsilon_{V-A,i}^{S-P}, \epsilon_{V+A,i}^{S-P}$ |
| with LQs        | $\epsilon_{S-P,i}, \epsilon_{V-A,i}^{S-P}, \epsilon_{V+A,i}^{S-P}$ |

The upper bounds on some of the $\epsilon^{\beta}_S$ parameters [6] from the Heidelberg-Moscow experiment were derived in Ref. [28] using the $S$-wave approximation for the electrons, considering nucleon recoil terms and only one nonzero parameter $\epsilon^{\beta}_{ai}$ in the Lagrangian [11] at a time.

The coefficients $\epsilon^{\beta}_{ai}$ entering the Lagrangian [11] can be expressed as

$$
\epsilon^{\beta}_{ai} = U_{ei}^{(\alpha, \beta)} U_{ci}^{(\alpha, \beta)},
$$

where $U_{ei}^{(\alpha, \beta)}$ are mixing parameters for non-SM neutrinos (see, e.g., Eq. [22]). As this Lagrangian describes also ordinary $\beta$-decays (without LN violation), the coefficients $\epsilon^{\beta}_{ai}$ are constrained by the existing data on precision measurements in allowed nuclear beta decays, including neutron decay [29]. For example, from these data we obtain the conservative bound

$$
|\epsilon_{V+A,i}^{S-P}| < 7 \times 10^{-2}.
$$
From Eqs. (5), (7), (8) and the bound $|\epsilon_{V+A}^p| < 7.9 \times 10^{-7}$ (see section 3.2) we can assume that the nonstandard mixing is small:

$$|U_{ei}V_{ei}| \lesssim 10^{-5}, \quad V_{ei} = U_{ei}^{(V+A,V+A)},$$  \hspace{1cm} (9)

### 2.2 Methods and approximations

We have calculated the leading order in the Fermi constant taking into account the leading contribution of the parameters $\epsilon_{n}^p$ to the decay matrix elements using the approximation of the relativistic electrons and non-relativistic nucleons. The wavefunction of an electron with the asymptotic momentum $p$ and the spin projection $s$ can be expanded in terms of spherical waves as [5] [30]

$$e_{p^s}(r) = e_{p^s_{1/2}}(r) + e_{p^s_{3/2}}(r) + \ldots$$  \hspace{1cm} (10)

We take into account the $S_{1/2}$ and the $P_{1/2}$ waves for the outgoing electrons:

$$e_{p^s_{1/2}}(r) = \left( \frac{\tilde{g}_{1/2}}{f_{1/2}} \right) \chi_s,$$  \hspace{1cm} (11)

$$e_{p^s_{3/2}}(r) = i \left( \frac{\tilde{g}_{3/2}}{f_{3/2}} \right) \chi_s,$$  \hspace{1cm} (12)

with $\tilde{r} = r/r$, $\tilde{p} = p/p$ and the two component spinor $\chi_s$. We use the approximate radial wave functions [5]

$$\left( \frac{\tilde{g}_{1/2}}{f_{1/2}} \right) = \tilde{A}_{\pm} \left[ 1 - \frac{1}{6}(\tilde{p}r)^2 \right],$$  \hspace{1cm} (13)

$$(\tilde{p}r)^2 = \left( \frac{3}{2} \alpha Z \right)^2 \left( \frac{r}{R} \right)^2 + 3\alpha Z \frac{r}{R} \varepsilon r + (pr)^2,$$  \hspace{1cm} (14)

$$\left( \frac{\tilde{g}_{3/2}}{f_{3/2}} \right) = \pm \tilde{A}_{\pm} \xi_{\pm} (\varepsilon r), \quad \xi_{\pm} = \frac{1}{2} \alpha Z + \frac{1}{3} (\varepsilon \pm \varepsilon_e) R,$$  \hspace{1cm} (15)

including the finite de Broglie wave length correction (FBWC) for the $S_{1/2}$ wave. Here $R$ is the nuclear radius, $\varepsilon$ is the electron energy and $\alpha$ is the fine structure constant. For the normalization constants $\tilde{A}_{\pm}$ we use the approximate Eq. (15) (see below).

The nucleon matrix elements of the color singlet quark currents are [8] [31] [32] [33]

$$\langle P(k'|\bar{u}(1 \mp \gamma_5) d|N(k)) \rangle = \bar{\psi}(k') \left[ F_{S}^{(3)}(q^2) \mp F_{P}^{(3)}(q^2) \gamma_5 \right] \tau_+ \psi(k),$$  \hspace{1cm} (16)

$$\langle P(k')|\bar{u}\gamma^\mu(1 \mp \gamma_5) d|N(k)\rangle = \bar{\psi}(k') \left[ g_{V}(q^2)\gamma^\mu \mp g_{A}(q^2)\gamma_5\gamma^\mu \gamma_5 - ig_{\omega}(q^2) \frac{\sigma\omega q}{2m_p} \mp g_{\rho}(q^2)\gamma_5 q^\mu \right] \tau_+ \psi(k),$$  \hspace{1cm} (17)

$$\langle P(k')|\bar{u}\sigma^{\mu\nu}(1 \mp \gamma_5) d|N(k)\rangle = \bar{\psi}(k') \left[ J^{\mu\nu} \mp \frac{2}{3} \tau^\mu \tau^\nu \right] \tau_+ \psi(k),$$  \hspace{1cm} (18)

$$J^{\mu\nu} = T_{1}^{(3)}(q^2)\sigma^{\mu\nu} + \frac{T_{2}^{(3)}}{m_p}(\gamma^\mu q^\nu - \gamma^\nu q^\mu) + \frac{T_{3}^{(3)}}{m_p^2}(\sigma^{\mu\nu} q_\rho q^\rho - \sigma^{\rho\nu} q_\rho q^\mu),$$  \hspace{1cm} (19)

where

$$\psi = \left( \begin{array}{c} P \vspace{0.2cm} \\ N \end{array} \right)$$  \hspace{1cm} (20)

is a nucleon isodoublet.

The non-relativistic structure of the nucleon currents in the impulse approximation is derived using Refs [32] [33], see Appendices A, B, and C. We have calculated the nucleon recoil terms including the recoil terms due to the pseudoscalar form factor.
Table 2: Expressions for $\mathcal{A}$ in Eqs. (21) and (22) for the stated choice of $c_\beta^\alpha$.

| $\epsilon$   | $\mathcal{A}$                                                                 |
|--------------|-------------------------------------------------------------------------------|
| $\epsilon_{V'-A}$ | $\mathcal{A}_0 + 4C_1 |\mu||\mu_{V'-A}|c_{02} + 4C_1 |\mu_{V'-A}|^2$             |
| $\epsilon_{V'^+A}$ | $\mathcal{A}_0 + 4C_0 |\mu||\mu_{V'^+A}|c_{03} + 4C_1 |\mu_{V'^+A}|^2$             |
| $\epsilon_{S'}^{-+P}$ | $\mathcal{A}_0 + C_3 |\epsilon_{S'^+A}|c_2 + C_5 |\epsilon_{S'^+A}|^2$         |
| $\epsilon_{S'^+A}$ | $\mathcal{A}_0 + C_2 |\epsilon_{S'^+A}|c_1 + C_4 |\epsilon_{S'^+A}|^2$         |
| $\epsilon_{S'^++P}$ | $\mathcal{A}_0 + 4C_0 |\mu||\mu_{S'^+P}|c_{04} + 4C_1 |\mu_{S'^+P}|^2$             |
| $\epsilon_{S'^+P}$ | $\mathcal{A}_0 + 4C_0 |\mu||\mu_{S'^+P}|c_{03} + 4C_1 |\mu_{S'^+P}|^2$             |
| $\epsilon_{S'^+P}$ | $\mathcal{A}_0 + 4C_0 |\mu||\epsilon_{S'^+P}|c_4 + 4C_1 |\epsilon_{S'^+P}|^2$          |
| $\epsilon_{S'^++P}$ | $\mathcal{A}_0 + 4C_1 |\mu||\epsilon_{S'^++P}|c_3 + 4C_1 |\epsilon_{S'^++P}|^2$        |
| $\epsilon_{T'^+P}$ | $\mathcal{A}_0 + 4C_0 |\mu||\epsilon_{T'^+P}|c_06 + 4C_1 |\epsilon_{T'^+P}|^2$       |

| $\epsilon_{T_h}'$, $\epsilon_{T_h}$ | $\mathcal{A}_0 + C_2 |\mu||\epsilon_{T_h}'|c_5 + C_3 |\epsilon_{T_h}'|^2$ |

Table 3: Expressions for $\mathcal{B}$ in Eq. (22) for the stated choice of $c_\beta^\alpha$.

| $\epsilon$   | $\mathcal{B}$                                                                 |
|--------------|-------------------------------------------------------------------------------|
| $\epsilon_{V'-A}$ | $\mathcal{B}_0 + 4D_1 |\mu||\mu_{V'-A}|c_{02} + 4D_1 |\mu_{V'-A}|^2$             |
| $\epsilon_{V'^+A}$ | $\mathcal{B}_0 + 4D_0 |\mu||\mu_{V'^+A}|c_{01} + 4D_1 |\mu_{V'^+A}|^2$             |
| $\epsilon_{S'}^{-+P}$ | $\mathcal{B}_0 + |\mu||\epsilon_{S'^+A}|(D_2c_2 + D_3s_2) + D_3 |\epsilon_{S'^+A}|^2$    |
| $\epsilon_{S'^+A}$ | $\mathcal{B}_0 + |\mu||\epsilon_{S'^+A}|(D_2c_1 + D_3s_1) + D_4 |\epsilon_{S'^+A}|^2$    |
| $\epsilon_{S'^+P}$ | $\mathcal{B}_0 + 4D_0 |\mu||\mu_{S'^+P}|s_{04} + 4D_1 |\mu_{S'^+P}|^2$             |
| $\epsilon_{S'^++P}$ | $\mathcal{B}_0 + 4D_0 |\mu||\epsilon_{S'^++P}|s_{03} + 4D_1 |\epsilon_{S'^++P}|^2$        |
| $\epsilon_{S'^++P}$ | $\mathcal{B}_0 + |\mu||\epsilon_{S'^++P}|(D_3c_4 + D_3s_4) + 4D_3 |\epsilon_{S'^++P}|^2$      |
| $\epsilon_{T'^+P}$ | $\mathcal{B}_0 + 4D_0 |\mu||\epsilon_{T'^+P}|s_{06} + 4D_1 |\epsilon_{T'^+P}|^2$          |

| $\epsilon_{T_h}'$, $\epsilon_{T_h}$ | $\mathcal{B}_0 + C_2 |\mu||\epsilon_{T_h}'|c_5 + C_3 |\epsilon_{T_h}'|^2$ |

2.3 Electron angular correlation

Taking into account the dominant terms introduced in the Appendices A, B, and C in the closure approximation [5], we obtain the differential width in $\cos \theta$ for the $0^+(A, Z) \rightarrow 0^+(A, Z + 2)e^-e^-$ transitions:

$$\frac{d\Gamma}{d\cos \theta} = \frac{\ln 2}{2} |M_{\text{GT}}|^2 A (1 - K \cos \theta),$$

where $\theta$ is the angle between the electron momenta in the rest frame of the parent nucleus and the angular correlation coefficient is

$$K = \frac{\mathcal{B}}{\mathcal{A}}, \quad -1 < K < 1.$$  \hspace{1cm} (22)

The Gamow–Teller nuclear matrix element $M_{\text{GT}}$ is defined in Eq. (51) below.

The expressions for $\mathcal{A}$ and $\mathcal{B}$ for different choices of $c_\alpha^\beta$, with only one coefficient considered at a time, are shown in Tables 2 and 3.

In these tables

$$c_i = \cos \psi_i, \quad s_i = \sin \psi_i$$  \hspace{1cm} (23)

and

$$\mu = \langle m \rangle / m_e, \quad \mu_\alpha^\beta = m_\alpha^\beta / m_e,$$  \hspace{1cm} (24)
with the standard effective Majorana mass \( m = \sum_i U_{ei}^2 m_i \) and the nonstandard ones:

\[
m_{S-P} = \sum_i U_{ei}^2 S_{SP,i} m_i, \quad m_{V-A} = \sum_i U_{ei}^2 V_{VA,i} m_i, \quad m_{T_L} = \sum_i U_{ei}^2 T_{LR,i} m_i.
\]

The quantities \( A \) and \( B \) for all zero \( \epsilon_{i}^\beta \) are

\[
A_0 = C_1 |\mu|^2, \quad B_0 = D_1 |\mu|^2
\]

and the relative phases are

\[
\psi_0 = \arg(\mu_{V-A}^*), \quad \psi_1 = \arg(\mu_{V+A}^*), \\
\psi_2 = \arg(\mu_{V-A}^*), \quad \psi_3 = \arg(\mu_{S-P}^*), \\
\psi_4 = \arg(\mu_{S-P}^*), \quad \psi_5 = \arg(\mu_{S+P}^*).
\]

The coefficients \( C_i \) and \( C_i^{(SP,T)} \) in Table 2 are

\[
C_0 = (\chi_F^2 - 1) A_{01}, \\
C_1 = (\chi_F - 1)^2 A_{01}, \quad C_1 = (\chi_F + 1)^2 A_{01}, \\
C_2 = (\chi_F - 1)(\chi_2 - A_{03} - \chi_1 A_{01}), \\
C_3 = - (\chi_F - 1)(\chi_2 + A_{03} - \chi_1 A_{04} - \chi_1 A_{05} + \chi_R A_{06}), \\
C_4 = \chi_2^2 A_{02} - \frac{2}{9} \chi_1 \chi_2 - A_{03} + \frac{1}{9} \chi_1^2 A_{04}, \\
C_5 = \chi_2^2 A_{02} - \frac{2}{9} \chi_1 \chi_2 - A_{03} + \frac{1}{9} \chi_1^2 A_{04} + \chi_2^2 A_{05} - \chi_2 A_{06} + \chi_2 A_{06};
\]

\[
C_0^{SP} = - (\chi_F - 1) \chi_F^{SP} A_{00}, \\
C_1^{SP} = (\chi_F^{SP})^2 A_{01}, \\
C_2^{SP} = - (\chi_F - 1) (\chi_B^{SP} + \chi_D^{SP}) A_{05} + (\chi_F - 1) \chi_P^{SP} A_{06}, \\
C_3^{SP} = (\chi_F^{SP} - \chi_D^{SP})^2 A_{05} + (\chi_F - 1) \chi_P^{SP} A_{06}, \\
C_4^{SP} = (\chi_B^{SP} + \chi_D^{SP})^2 A_{05} - (\chi_B^{SP} - \chi_D^{SP}) \chi_P^{SP} A_{05} + \chi_P^{SP} A_{04}, \\
C_5^{SP} = (\chi_B^{SP} - \chi_D^{SP})^2 A_{05} + (\chi_B^{SP} - \chi_D^{SP}) \chi_P^{SP} A_{05} + \chi_P^{SP} A_{04};
\]

\[
C_0^T = \frac{T_1^{(3)}}{g_A} (\chi_F - 1) A_0^T, \\
C_1^T = \left( \frac{T_1^{(3)}}{g_A} \right)^2 A_{01}, \\
C_2^T = - (\chi_F - 1) \left[ (T_1^{(3)} + \frac{T_1^{(3)}}{g_A} + \frac{T_1^{(3)}}{g_A} - \frac{T_1^{(3)}}{g_A}) A_{01} + \left( \frac{1}{3} T_1^{(3)} + \frac{T_1^{(3)}}{g_A} - \frac{T_1^{(3)}}{g_A} \right)^2 A_{02} \right], \\
C_3^T = (\chi_B^{3} + \chi_D^{3}) \frac{T_1^{(3)}}{g_A} + \chi_R^{3} - \chi_R^{3} \chi_R^{3} \chi_R^{3} A_{09} + \left( \frac{1}{3} T_1^{(3)} + \frac{T_1^{(3)}}{g_A} - \frac{T_1^{(3)}}{g_A} \right)^2 A_{03}.
\]

The coefficients \( D_i \) and \( D_i^{(SP,T)} \) entering in Table 3 are:

\[
D_0 = (\chi_F^2 - 1) B_{01},
\]
\[ D_1 = (\chi_F - 1)^2 B_{01}, \quad D_{1+} = (\chi_F + 1)^2 B_{01}, \]
\[ D_{2-} = (\chi_F - 1)\chi_{2-} B_{03-}, \quad D_2 = -(\chi_F - 1)\chi_{1+} B_{04}, \]
\[ D_3 = (\chi_F - 1)(\chi_2 + B_03 - \chi_2 B_05), \]
\[ D_{3-} = -((\chi_F - 1))(\chi_{1-} B_{04} - \chi_2 B_{05} + \chi_4 B_{06}), \]
\[ D_4 = -\chi_2^2 B_{02} + \frac{1}{9} \chi_1^2 B_{04}, \]
\[ D_5 = \chi_2^2 B_{02} - \frac{1}{9} \chi_1^2 B_{04} - \chi_2^2 B_{08} + \chi_4 B_{07} B_{07} - \chi_2^2 B_{09}; \quad (31) \]

\[
\begin{align*}
D_{0-}^{SP} &= (\chi_F - 1)\chi_F^{SP} B_{00-}, \\
D_1^{SP} &= -((\chi_F)^2 B_{01}^{SP}, \\
D_2^{SP} &= -(\chi_F - 1)(\chi_B^{SP} + \chi_D^{SP}) B_{05}^{SP} + (\chi_F - 1)\chi_2^{SP} B_{06}^{SP}, \\
D_2^{SP} &= (\chi_F - 1)\chi_P^{SP} B_{02}^{SP}, \\
D_2^{SP} &= (\chi_F - 1)(\chi_B^{SP} - \chi_D^{SP}) B_{05}^{SP} + (\chi_F - 1)\chi_2^{SP} B_{06}^{SP}, \\
D_3^{SP} &= (\chi_B^{SP} + \chi_D^{SP})^2 B_{02}^{SP} + (\chi_B^{SP} + \chi_D^{SP}) \chi_P^{SP} B_{03}^{SP} + (\chi_P)^2 B_{04}^{SP}, \\
D_3^{SP} &= (\chi_B^{SP} - \chi_D^{SP})^2 B_{02}^{SP} + (\chi_B^{SP} - \chi_D^{SP}) \chi_P^{SP} B_{03}^{SP} + (\chi_P)^2 B_{04}^{SP}; \quad (32) \end{align*}
\]

\[
\begin{align*}
D_0^T &= \frac{T_0^{(3)}}{g_A} (\chi_F - 1) B_0^T, \\
D_1^T &= -\left(\frac{T_1^{(3)}}{g_A}\right)^2 B_1^T, \\
D_2^T &= -(\chi_F - 1) \left[ (\chi_R^{T_2} + \chi_R^{T_1} + \chi_R^{T_2} - \chi_R^{T_1}) B_{01} + \left(\frac{1}{3} \chi_R^{T_2} - 2 \chi_R^{T_1}\right) B_{02}^T \right], \\
D_3^T &= (\chi_R^{T_2} + \chi_R^{T_1} + \chi_R^{T_2} - \chi_R^{T_1} B_{02} + \left(\frac{1}{3} \chi_R^{T_2} - 2 \chi_R^{T_1}\right)^2 B_{03}^T, \quad (33) \end{align*}
\]

where the integrated phase space factors are
\[
\begin{pmatrix}
A_0, & A_0^{(SP,T)} \\
B_0, & B_0^{(SP,T)}
\end{pmatrix} = \frac{1}{\ln 2 (m_e R)^2} \int \left( a_0, \ a_0^{(SP,T)} \right) d\Omega_{0w},
\]

with the phase space element \(d\Omega_{0w}\) defined as follows:
\[
d\Omega_{0w} = m_e^{-5} |p_1| |p_2| |e_1| |e_2| (\delta(e_1 + e_2 + E_f - E_t) d\varepsilon_1 d\varepsilon_2 d(\hat{p}_1 \cdot \hat{p}_2)). \quad (35)
\]

The constant \(a_{0w}\) and the kinematic factors \(a_{0w}, a_{0w}^{(SP,T)}\), \(b_{0w}\) and \(b_{0w}^{(SP,T)}\) entering above are defined as follows:
\[
a_{0w} = (G_F g_A)^4 |V_{ud}|^4 m_e^9 / (64\pi^5), \quad (36)
\]

\[
\begin{align*}
a_{01} &= \alpha_+ + \beta_+, & a_{02} &= \left(\frac{\varepsilon_1}{m_e}\right)^2 \beta_+, & a_{03} &= 2\frac{\varepsilon_1}{m_e} \beta_+, & a_{04} &= \frac{4}{9} \beta_+, \\
a_{05} &= \frac{4}{3} \left(\frac{\zeta}{m_e R} - 2 \alpha_+\right), & a_{06} &= \frac{8}{m_e R} \alpha_-, & a_{07} &= \frac{4}{3} \left(\frac{m_e R}{m_e R}\right)^2 \left(\zeta \alpha_+ - 2 m_e R \alpha_+\right), \\
a_{08} &= \left(\frac{2}{3 m_e R}\right)^2 [\zeta^2 \alpha_+ + 4 m_e R (m_e R \alpha_+ - \zeta_\alpha)], & a_{09} &= \left(\frac{4}{m_e R}\right)^2 \alpha_+; \quad (37)
\end{align*}
\]

8
\[ a_{00}^{SP} = \alpha_-, \quad a_{01}^{SP} = \alpha_+, \quad a_{02}^{SP} = \frac{1}{(m_e R)^2} (\alpha_+ + \beta_+), \]
\[ a_{03}^{SP} = \frac{1}{3m_e R} \left[ \frac{\zeta}{m_e R} (\alpha_+ + \beta_+) - 2\alpha_- \right], \]
\[ a_{04}^{SP} = \frac{1}{9} \left\{ \left( \frac{\zeta}{2m_e R} \right)^2 + 1 \right\} \alpha_+ + \left( \frac{\zeta}{2m_e R} \right)^2 \beta_+ - \frac{\zeta}{m_e R} \alpha_- \right), \]
\[ a_{05}^{SP} = \frac{1}{m_e R} (\alpha_+ + \beta_+), \]
\[ a_{06}^{SP} = \frac{1}{6} \left[ \frac{\zeta}{m_e R} (\alpha_+ + \beta_+) - 2\alpha_- \right] = \frac{m_e R}{2} a_{03}^{SP}; \quad (38) \]
\[ a_{00}^T = 2\beta_-, \quad a_{01}^T = 16\alpha_+ = 16a_{01}^{SP}, \quad a_{02}^T = \frac{8\zeta \beta_+}{m_e R}, \]
\[ a_{03}^T = \left( \frac{8\zeta}{m_e R} \right)^2 \beta_+; \quad (39) \]
\[ b_{01} = \gamma_+ + \delta_+, \quad b_{02} = \left( \frac{\varepsilon_{21}}{m_e} \right)^2 \delta_+, \]
\[ b_{03} = \frac{2\varepsilon_{21}}{m_e} \delta_+, \quad b_{03-} = \frac{2\varepsilon_{21}}{m_e} \delta_- \]
\[ b_{04} = \frac{4}{9} \delta_+, \quad b_{04-} = \frac{4}{9} \delta_- \]
\[ b_{05} = \frac{8}{3} \gamma_+, \quad b_{05-} = \frac{4 \zeta \gamma_-}{3m_e R} \]
\[ b_{06-} = \frac{8 \zeta \gamma_-}{m_e R}, \quad b_{07} = \frac{16}{3} \left( \frac{\zeta \gamma_+}{m_e R} \right)^2 \]
\[ b_{08} = \frac{16}{9} \left[ \left( \frac{\zeta}{2m_e R} \right)^2 - 1 \right] \gamma_+, \quad b_{09} = \left( \frac{4}{m_e R} \right)^2 \gamma_+; \quad (40) \]
\[ b_{00}^{SP} = \gamma_-, \quad b_{01}^{SP} = \gamma_+ = \frac{3}{8} b_{05}, \]
\[ b_{02}^{SP} = \frac{1}{(2m_e R)^2} (\gamma_+ + \delta_+), \quad b_{02-}^{SP} = \frac{1}{3} \gamma_- = \frac{1}{3} b_{06-}, \]
\[ b_{03}^{SP} = \frac{\zeta}{6m_e R^2} (\gamma_+ + \delta_+), \]
\[ b_{04}^{SP} = \frac{1}{9} \left\{ \left( \frac{\zeta}{2m_e R} \right)^2 - 1 \right\} \gamma_+ + \left( \frac{\zeta}{2m_e R} \right)^2 \delta_+ \}
\[ b_{05}^{SP} = \frac{1}{m_e R} (\gamma_+ + \delta_+), \quad b_{06}^{SP} = \left( \frac{\zeta}{m_e R} \right)^2 (\gamma_+ + \delta_+); \quad (41) \]
\[ b_{00-}^T = 4\gamma_- = 4b_{00}^{SP}, \quad b_{01}^T = 16\gamma_+ = 6b_{05}, \]
\[ b_{02}^T = \frac{8 \zeta \delta_+}{m_e R}, \quad b_{03}^T = \left( \frac{8 \zeta}{m_e R} \right)^2 \delta_+; \quad (42) \]

where \( \varepsilon_{21} = \varepsilon_2 - \varepsilon_1 \) is the difference in the electron energy. The characteristic features of the \( P_{1/2} \)-wave are expressed as
\[ \zeta = 3\alpha Z + (\varepsilon_1 + \varepsilon_2) R \]
and the Coulomb corrections appear as the following combinations

\[
\alpha_\pm = \frac{|\alpha_{-1-1}|^2 \pm |\alpha_{11}|^2}{2}, \quad \beta_\pm = \frac{|\alpha_{11}|^2 \pm |\alpha_{-11}|^2}{2}, \\
\gamma_+ = 2\text{Re}(\alpha_{11}^* \alpha_{-1-1}), \quad \gamma_- = 2\text{Im}(\alpha_{11}^* \alpha_{-1-1}), \\
\delta_+ = 2\text{Re}(\alpha_{-11}^* \alpha_{1-1}), \quad \delta_- = 2\text{Im}(\alpha_{-11}^* \alpha_{1-1}),
\]

(44)

with \(\alpha_{ij} = \tilde{A}_i(\varepsilon_2)\tilde{A}_j(\varepsilon_1)\).

For the normalization constants in the approximation including terms up to \((\alpha Z)^2\) [5]

\[
\tilde{A}_{\pm 1} = \sqrt{\frac{\varepsilon + m_e}{2\varepsilon}} F_0(Z, \varepsilon), \\
F_0 = 4\frac{1}{\Gamma^2(2\gamma_1 + 1)} (2pR)^2(\gamma_1 - 1)|\Gamma(\gamma_1 + iy)|^2 e^{\pi y}, \\
\gamma_1 = \sqrt{1 - (\alpha Z)^2}, \quad y = \alpha Z / p,
\]

we have

\[
\left( \begin{array}{c}
\alpha_+ \\
\beta_+
\end{array} \right) = \frac{1}{2}(\varepsilon_1 \varepsilon_2 \pm m_e^2)C_{00}, \quad \left( \begin{array}{c}
\alpha_- \\
\beta_-
\end{array} \right) = \frac{1}{2}(\varepsilon_2 \pm \varepsilon_1)m_e C_{00},
\]

(46)

\[
\gamma_+ = \delta_+ = \frac{1}{2} |P_1| |P_2| C_{00}, \quad \gamma_- = \delta_- = 0,
\]

(47)

where

\[
C_{00} = \frac{F_0(Z, \varepsilon_2)F_0(Z, \varepsilon_1)}{\varepsilon_2 \varepsilon_1}.
\]

(48)

Note that using Eq. (47) the expressions for \(B\) from Table 3 are reduced to the form shown in Table 4.

### Table 4: Expressions for \(B\) in Eq. (22) for the stated choice of \(\varepsilon^\gamma_0\) for the \(\tilde{A}_{\pm 1}\) from Eq. (45).

| \(\varepsilon\) | \(B\) |
|----------------|------------------|
| \(\varepsilon_{V-A}^V\) | \(B_0 + 4D_1 |\mu_0|^2 + 4D_1 |\mu_0|^2\) |
| \(\varepsilon_{V-A}^V\) | \(B_0 + 4D_1 |\mu_0|^2 + 4D_1 |\mu_0|^2\) |
| \(\varepsilon_{V-A}^V\) | \(B_0 + 4D_1 |\mu_0|^2 + 4D_1 |\mu_0|^2\) |
| \(\varepsilon_{S-P}^S\) | \(B_0 + 4D_2 |\mu_0|^2 + 4D_2 |\mu_0|^2\) |
| \(\varepsilon_{S-P}^S\) | \(B_0 + 4D_2 |\mu_0|^2 + 4D_2 |\mu_0|^2\) |
| \(\varepsilon_{T-Q}^T\) | \(B_0 + 4D_3 |\mu_0|^2 + 4D_3 |\mu_0|^2\) |
| \(\varepsilon_{T-Q}^T\) | \(B_0 + 4D_3 |\mu_0|^2 + 4D_3 |\mu_0|^2\) |

In the definitions of \(C_i\) and \(D_i\) we use some combinations of nuclear parameters (which are assumed to be real) similar to the ones in Ref. [5]. Thus,

\[
\chi_{2\pm} = \chi_{GT\omega} \pm \chi_{F\omega} - \frac{1}{9} \chi_{1\mp}; \quad \chi_{1\pm} = (\chi_{GT} - 6\chi_{F}) \pm 3\chi_{F};
\]

\[
\chi_F = \left( \frac{g_V}{g_A} \right)^2 \frac{M_F}{M_{GT}}; \quad \chi_k = \frac{g_V}{g_A} \frac{M_k}{M_{GT}}, \quad k = P, R, RT;
\]

\[
\chi_k = \frac{M_k}{M_{GT}}, \quad k = T, GT, RC_\sigma, RT_\sigma;
\]
\[
\chi^{SP} = \frac{F^{(3)}_g}{gV} \chi_F; \quad \chi^{SP} = \frac{F^{(3)}_g}{gV} \left( \frac{\gamma V}{gA} \right)^2 \frac{M_F}{M_{GT}};
\]

\[
\chi^{SB} = \frac{F^{(3)}_g}{gA} \frac{M_B}{M_{GT}}; \quad \chi^{SP} = \frac{F^{(3)}_g}{gV} \frac{M_D}{M_{GT}};
\]

\[
\chi^T_k = \frac{T^{(3)}_k}{gA} \chi_k, \quad k = R, RT, RC_\sigma, RT_\sigma; \quad \chi^T_k = \frac{T^{(3)}_k}{gA} \frac{M_T}{M_{GT}}, \quad k = GT, T,
\]

where the index \( F \) refers to Fermi, \( GT \) to Gamow–Teller, \( T \) to tensor, \( P \) to the \( P \)-wave effect and \( R \) to the recoil effect. If \( \chi \) has prime or the index \( \omega \) than the same has the according matrix element in the numerator. The nuclear matrix elements defined below contain the operator \( \tau^a_{+_k} = (\tau_1 + i\tau_2)^a/2 \) converting the \( a \)-th neutron into the \( a \)-th proton, and the initial (final) nuclear state are denoted by \(|0^+_a\rangle \) (\(|0^+_f\rangle\))

\[
M_F = \sum_N \langle 0^+_f | \sum_{a \neq b} h_+^{(r_{ab}, E_N)} \tau^a_{+_i} \tau^b_{+_f} |0^+_i\rangle, \quad (50)
\]

\[
M_{GT} = \sum_N \langle 0^+_f | \sum_{a \neq b} h_+^{(r_{ab}, E_N)} \sigma_a \cdot \sigma_b \tau^a_{+_i} \tau^b_{+_f} |0^+_i\rangle, \quad (51)
\]

\[
M_T = \sum_N \langle 0^+_f | \sum_{a \neq b} h_+^{(r_{ab}, E_N)} \left[ \left( \sigma_a \cdot \hat{r}_{ab} \right) \left( \sigma_b \cdot \hat{r}_{ab} \right) - \frac{1}{3} \sigma_a \cdot \sigma_b \right] \tau^a_{+_i} \tau^b_{+_f} |0^+_i\rangle, \quad (52)
\]

\[
M_{GT} = \sum_N \langle 0^+_f | \sum_{a \neq b} h_+^{(r_{ab}, E_N)} \sigma_a \cdot \sigma_b \tau^a_{+_i} \tau^b_{+_f} |0^+_i\rangle, \quad (53)
\]

\[
M_P = \sum_N \langle 0^+_f | \sum_{a \neq b} h_+^{(r_{ab}, E_N)} \frac{i\tau^{+ab}}{2r_{ab}} \left( \left( \sigma_a \cdot \sigma_b \right) \cdot \left[ \hat{r}_{ab} \times \hat{r}_{+ab} \right] \right) \tau^a_{+_i} \tau^b_{+_f} |0^+_i\rangle, \quad (54)
\]

\[
M_R = \sum_N \langle 0^+_f | \sum_{a \neq b} h_+^{(r_{ab}, E_N)} \frac{R}{2r_{ab}} \left( \left( \sigma_a \times D_b + D_a \times \sigma_b \right) \right) \tau^a_{+_i} \tau^b_{+_f} |0^+_i\rangle, \quad (55)
\]

\[
M_{GT,\omega} = \sum_N \langle 0^+_f | \sum_{a \neq b} h_{0\omega}^{(r_{ab}, E_N)} \sigma_a \cdot \sigma_b \tau^a_{+_i} \tau^b_{+_f} |0^+_i\rangle, \quad (56)
\]

\[
M_{F,\omega} = \sum_N \langle 0^+_f | \sum_{a \neq b} h_{0\omega}^{(r_{ab}, E_N)} \tau^a_{+_i} \tau^b_{+_f} |0^+_i\rangle, \quad (57)
\]

\[
M_B = \sum_N \langle 0^+_f | \sum_{a \neq b} \frac{iR}{2r} h_+^{(r_{ab}, E_N)} \hat{r}_{ab} \left( \sigma_a B_b - B_a \sigma_b \right) \tau^a_{+_i} \tau^b_{+_f} |0^+_i\rangle, \quad (58)
\]

\[
M_D = \sum_N \langle 0^+_f | \sum_{a \neq b} \frac{iR}{2r} h_+^{(r_{ab}, E_N)} \left( D_a - D_b \right) \tau^a_{+_i} \tau^b_{+_f} |0^+_i\rangle, \quad (59)
\]

\[
M_{GT}^{R} = \sum_N \langle 0^+_f | \sum_{a \neq b} h_+^{(r_{ab}, E_N)} \frac{iR}{r} \sigma_a \cdot \sigma_b \tau^a_{+_i} \tau^b_{+_f} |0^+_i\rangle, \quad (60)
\]

\[
M_D^{R} = \sum_N \langle 0^+_f | \sum_{a \neq b} h_+^{(r_{ab}, E_N)} \frac{iR}{2r} \left( \hat{r}_{ab} \cdot \left( T_a - T_b \right) \right) \tau^a_{+_i} \tau^b_{+_f} |0^+_i\rangle, \quad (61)
\]

\[
M_{RC, \omega} = \sum_N \langle 0^+_f | \sum_{a \neq b} h_{0\omega}^{(r_{ab}, E_N)} \frac{iR}{2r} \left( \hat{r}_{ab} \cdot \sigma_a C_b - C_a \hat{r}_{ab} \cdot \sigma_b \right) \tau^a_{+_i} \tau^b_{+_f} |0^+_i\rangle, \quad (62)
\]
In the above expressions, the neutrino potentials \( h_i(r_{ab}, \langle E_N \rangle) \) are defined as follows:

\[
\begin{align*}
    h_+(r_{ab}, \langle E_N \rangle) &= \frac{R}{4\pi^2} \int \frac{d\mathbf{k}}{\omega} \left( \frac{1}{\omega + A_1} + \frac{1}{\omega + A_2} \right) e^{i\mathbf{k} \cdot \mathbf{r}} \simeq R H(r, \bar{A}), \\
    h_0(r_{ab}, \langle E_N \rangle) &= \frac{1}{2\pi^2 \varepsilon_{12}} \int \frac{d\mathbf{k}}{\omega} \left( \frac{1}{\omega + A_1} - \frac{1}{\omega + A_2} \right) e^{i\mathbf{k} \cdot \mathbf{r}} \\
    &\simeq 2H(r, \bar{A}) + r \frac{\partial}{\partial r} H(r, \bar{A}), \\
    h_{0\omega}(r_{ab}, \langle E_N \rangle) &= h_+ - \bar{A}R h_0, \\
    h_R(r_{ab}, \langle E_N \rangle) &= -\frac{\bar{A}}{m_p} \left[ \frac{2}{\pi} \left( \frac{R}{r} \right)^2 - \bar{A} R h_+ \right],
\end{align*}
\]

with

\[
H(r, \bar{A}) = \frac{1}{2\pi^2} \int \frac{d\mathbf{k}}{\omega} \frac{e^{i\mathbf{k} \cdot \mathbf{r}}}{\omega + \bar{A}}.
\]

where \( r_{ab} \) is the distance between the nucleons \( a \) and \( b \), and \( \langle E_N \rangle \) is the average energy of the intermediate nucleus \( N \).

To derive the expressions for \( A \) and \( B \) shown in Tables 2 and 3 we have used the formulas:

\[
\begin{align*}
    C_1^A (\epsilon_{S^+P}^P) &= 2 M_{GT} \frac{m_{S^+P}^P}{m_e} \chi_{\epsilon_{S^+P}^P}, \\
    C_2^A (\epsilon_{S^+P}^P) &= 0, \\
    C_3^A (\epsilon_{S^+P}^P) &= 2 M_{GT} \epsilon_{S^+P}^P \chi_{\epsilon_{S^+P}^P}, \\
    C_4^A (\epsilon_{S^+P}^P) &= 0, \\
    C_5^A (\epsilon_{S^+P}^P) &= 2 M_{GT} \epsilon_{S^+P}^P (\bar{\chi}_{B}^P - \chi_{\epsilon_{S^+P}^P}).
\end{align*}
\]

\[
\begin{align*}
    Z_1^X (\epsilon_{V^-A}^-) &= M_{GT} (\mu + 2\mu_{V^-A}^-)(\chi_{\epsilon_{V^-A}^-} - 1), \\
    Z_1^X (\epsilon_{V^+A}^+), Z_2^X (\epsilon_{V^+A}^+), Z_3^X (\epsilon_{V^+A}^+), Z_4^X (\epsilon_{V^+A}^+), Z_5^X (\epsilon_{V^+A}^+) &= \pm M_{GT} \epsilon_{V^+A}^+ (\chi_{V^+A}^+ + \chi_{\epsilon_{V^+A}^+}), \\
    Z_6^X (\epsilon_{V^-A}^-) &= \frac{1}{3} M_{GT} \epsilon_{V^-A}^- \chi_{\epsilon_{V^-A}^-}, \\
    Z_7^X (\epsilon_{V^-A}^-) &= M_{GT} \epsilon_{V^-A}^- \chi_{\epsilon_{V^-A}^-}.
\end{align*}
\]

\[
\begin{align*}
    W_1^U (\epsilon_{T^a_3 T^b_3}^a) &= -4M_{GT} T^3_l T^3_l \frac{T^3_l}{g_A}, \\
    W_4^U (\epsilon_{T^a_3 T^b_3}^a) &= -2M_{GT} T^3_l T^3_l (\chi_{RC^a} + \chi_{RC^b} + \chi_{RT^a} + \chi_{RT^b} - 2 T^3_l), \\
    W_2^U (\epsilon_{T^a_3 T^b_3}^a) &= 2iM_{GT} T^3_l T^3_l \frac{T^3_l}{g_A} \chi_{GT}, \\
    W_7^U (\epsilon_{T^a_3 T^b_3}^a) &= -4iM_{GT} T^3_l T^3_l \frac{T^3_l}{g_A} \left( \frac{1}{3} \chi_{GT} + 2 \chi_{T} \right).
\end{align*}
\]
For all other arguments $\epsilon^B_\alpha$ these nucleon matrix elements have zero values, except for

$$Z_1^X (\epsilon^V_{\mp A} = 0) = M_{GT}\mu (\chi_F - 1).$$

(76)

We have calculated the numerical values of the integrated kinematic factors $A_{0i}$, $A^{(SP,T)}_{0i}$, $B_{0i}$, and $B^{(SP,T)}_{0i}$ for all the five nuclei of current experimental interest. We shall use them in the results shown below in Table 6 for the angular coefficient $K$. However, as we will focus in this paper mainly on the $0\nu 2\beta$ decay of the $^{76}\text{Ge}$ nucleus, we give the values of these factors for this nucleus in Table 5, where we have used

$$Q = E_i - E_f - 2m_e = 2.039 \text{ MeV},$$

(77)

taken from Ref. [35], the scaling factor for the neutrino potentials

$$R = r_0 A^{1/3}, \quad r_0 = 1.1 \text{ fm},$$

(78)

and the values of $g_A = 1.254$ and $|V_{ud}| = 0.97377$ [1]. The values of $A_{03}$ and $B_{03}$ are of the order of $10^{-44}$ yr$^{-1}$. Hence these values are not given in Table 5 and the terms with $A^T_{03}$ and $B_{03}$ can be safely neglected.

Table 5: The integrated kinematic $A$- and $B$-factors [in $10^{-15}$yr$^{-1}$] for the $0^+ \rightarrow 0^+$ transition of the $0\nu 2\beta$ decay of $^{76}\text{Ge}$.

| $A_{0i}$ | $B_{0i}$ |
|----------|----------|
| $A_{01}$ | $B_{01}$ |
| $A_{02}$ | $B_{02}$ |
| $A_{03}$ | $B_{03}$ |
| $A_{04}$ | $B_{04}$ |
| $A_{05}$ | $B_{05}$ |
| $A_{06}$ | $B_{06}$ |
| $A_{07}$ | $B_{07}$ |
| $A_{08}$ | $B_{08}$ |
| $A_{09}$ | $B_{09}$ |
| $A^{SP}_{09}$ | $B^{SP}_{09}$ |
| $A^{SP}_{01}$ | $B^{SP}_{01}$ |
| $A^{SP}_{02}$ | $B^{SP}_{02}$ |
| $A^{SP}_{03}$ | $B^{SP}_{03}$ |
| $A^{SP}_{04}$ | $B^{SP}_{04}$ |
| $A^{SP}_{05}$ | $B^{SP}_{05}$ |
| $A^{SP}_{06}$ | $B^{SP}_{06}$ |
| $A^{T}_{01}$ | $B^{T}_{01}$ |
| $A^{T}_{02}$ | $B^{T}_{02}$ |
| $A^{T}_{03}$ | $B^{T}_{03}$ |

We recall that the analytic expressions associated with the coefficients $\epsilon^V_{\mp A}$ given in this section and the values of $A_{0i}$ from Table 5 confirm the results of Ref. [5]. The analytic expressions associated with the coefficients $\epsilon^V_{\mp A}$, $\epsilon^{SP}_{\mp P}$, $\epsilon_{TL,R}^{SP}$, and the values of $A^{(SP,T)}_{0i}$, $B_{0i}$, $B^{(SP,T)}_{0i}$ from Table 5 transcend the earlier work.

3 Analysis of the electron angular correlation

3.1 Qualitative analysis

If the effects of all the interactions beyond the SM extended by the $\nu_{MS}$, which we call the “nonstandard” effects, are zero (i.e., all $\epsilon_\alpha^B = 0$), then $K = B_{01}/A_{01}$. Its values are given in Table 6 for various decaying
nuclei. We will concentrate on the case of $^{76}$Ge nucleus in the following. In this case the correlation \[21\] is proportional to $1 - 0.81 \cos \theta$. (Note that in the limit of $m_s/(E_i - E_f) \to 0$ we have $\alpha_+ + \beta_+ = \gamma_+ + \delta_+$ and $K = 1$.) Tables 2 and 4 show that the presence of the “nonstandard” parameters $\epsilon_{T_L}$, $\epsilon_{T_R}$ or $\epsilon_{T_{\perp}}$ does not change the value of $K$ and therefore the form of the angular correlation. The presence of any other parameter $\epsilon^\beta_\alpha$ does change this correlation. Due to the large values of $A^{SP}_{0i}$, $B^{SP}_{0i}$, $A^T_{0i}$ and $B^T_{0i}$ for $i \geq 2$ there are three additional nonstandard parameters that can change the form of the angular correlation, namely, $\epsilon_{S+P}$, $\epsilon_{T_L}$ and $\epsilon_{T_R}$.

Table 6: The values of angular correlation coefficient $K$ for various decaying nuclei for the SM extended by the $\nu_{M_S}$.

| Nucleus | $K$ |
|---------|-----|
| $^{76}$Ge | 0.81 |
| $^{82}$Se | 0.88 |
| $^{100}$Mo | 0.88 |
| $^{136}$Te | 0.85 |
| $^{136}$Xe | 0.84 |

Using Table 1 and taking into account the fact that $|\mu_\alpha^\beta|$ are suppressed in comparison with $|\epsilon_\alpha^\beta|$ by the factor $m_s/m_e$ (the chiral suppression), we find the coefficient $K$ and the set $\{\epsilon\}$ of nonzero $\epsilon_\alpha^\beta$'s that change the $1 - 0.81 \cos \theta$ form of the correlation for the SM plus $\nu_{M_S}$, see Table 7 (the lower two entries). They correspond to the following extensions of the SM: $\nu_{M_S}$ plus RPV SUSY \[10\], $\nu_{M_S}$ plus right-handed currents (RC) (connected with right-handed $W$ bosons \[5\] or vector LQs \[13\]), and $\nu_{M_S}$ plus scalar LQs (SLQ) \[13\]. Hence, the angular coefficient $K$ can signal the presence of these NP interactions.

Table 7: The angular correlation coefficient $K$ for various SM extensions for decays of $^{76}$Ge.

| SM extension | $\{\epsilon\}$ | $K$ |
|--------------|-----------------|-----|
| $\nu_{M_S}$  | $-$             | 0.81|
| $\nu_{M_S}$ + RPV SUSY | $\epsilon_{S+P}, \epsilon_{T_R}$ | $-1 < K < 1$ |
| $\nu_{M_S}$ + RC | $\epsilon_{V+A}$ | $-1 < K < 1$ |
| $\nu_{M_S}$ + SLQ | $\epsilon_{S+P}$ | $-1 < K < 1$ |

We remark here that in our earlier analysis \[19\] we had neglected the $P$-wave and recoil effects, which is not a good assumption. Our current study shows that these effects give significant contribution to the terms with $\epsilon_{V+A}, \epsilon_{S+P}$ and $\epsilon_{T_R}$. Hence, they have to be included in any realistic analysis of the data, as and when it becomes available. Including them, not only the model called $\nu_{M_S}$ + RC but also the models $\nu_{M_S}$ + RPV and $\nu_{M_S}$ + SLQ can essentially change the angular coefficient $K$ from being 0.81 in the decay of the $^{76}$Ge nucleus. Left-right symmetric models belong to the class $\nu_{M_S}$ + RC and we have studied these models in detail in section 4, where the correlations among the parameters $K$, $T_{1/2}$ and either $m_{W_R}$ or $\zeta$ are worked out for the case $|\langle m \rangle| \neq 0$, $\cos \psi_i = 0$ considered in section 3.2.

Note that the decay half-life and angular correlation do not give any bounds on the parameters $\epsilon_{T_R}$ and $\epsilon_{T_L}$ because the according expressions for $A$ and $B$ do not depend on them.

### 3.2 Quantitative analysis

Let us now consider some particular cases for the parameter space. We will analyze only the terms with $\epsilon_{V+A}$ as the corresponding nuclear matrix elements have been worked out in the literature. We use various types of QRPA model for the $^{76}$Ge nucleus \[20\] [21] as a test case.

Using the case of $|\langle m \rangle| = 0$, which gives conservative upper bounds on $|\mu_\alpha^\beta|$ and $|\epsilon_\alpha^\beta|$, the decay half-life is expressed from Eq. \[21\] as

$$T_{1/2} = \ln 2/\Gamma = (|M_{GT}|^2 A)^{-1}.$$ \[79\]

From Eq. \[79\], using Tables 2, 5 and the values of the nuclear matrix elements reported in Refs. \[20\] [21], we have the following expressions for the half-life [in yr] for various choices of the parameters $|\mu_{V+A}|$ and...
that the dispersion in the half-lives is less marked for the coefficient $T_{1/2}$ provides a useful indication of the strong role in the numerical analysis. However, as we show below, the nuclear-model dependence of the angular coefficient $K$ is rather modest.

The fact that the dependence of $K$ on the nuclear matrix elements is much weaker than the uncertainty in $T_{1/2}$ from this source is illustrated in Table 8 for QRPA models [20, 21] for the assumed values of the parameters: $|\mu_{1}^{V-A}| = |\epsilon_{1}^{V+A}| = 5 \times 10^{-7}$, $|\epsilon_{2}^{V+A}| = 5 \times 10^{-9}$. It is clear from Table 8 that measuring $K$ with 10% accuracy (or better) produces useful experimental data that could be sensitive to the new physics. We note that for the parameters $\mu_{1}^{V-A}$ the angular coefficient does not depend actually on the nuclear matrix elements as it is seen from Tables 2, 3 (for $|\mu| = 0$) and Eqs. (28), (31): $K = (\chi_{F} + 1)^{2} B_{01}/[(\chi_{F} + 1)^{2} A_{01}] = B_{01}/A_{01} \approx 0.81$.

Table 8: $T_{1/2}$ and $K$ for the fixed values of the parameters $|\mu_{1}^{V-A}|, |\epsilon_{1}^{V+A}|$ for decay of $^{76}$Ge for the case of $|(m)| = 0$ in QRPA without (with) p-n pairing [20] [pnQRPA with $g_{pp} = 1.02(1.06)$ [21].

| $T_{1/2}/(10^{25}\,\text{yr})$ | $|\mu_{1}^{V-A}| = 5 \times 10^{-7}$ | $|\mu_{2}^{V-A}| = 5 \times 10^{-7}$ | $|\epsilon_{1}^{V+A}| = 5 \times 10^{-9}$ | $|\epsilon_{2}^{V+A}| = 5 \times 10^{-9}$ |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| $K$             | 0.81(0.81)       | 0.81(0.81)       | 0.81(0.81)       | 0.81(0.81)       |

Using the numerical results given above, the current lower bound $T_{1/2} > 1.6 \times 10^{25}$ yr for the $^{76}$Ge nucleus [37] yields the upper bounds on the parameters $|\mu_{1}^{V-A}|$ and $|\epsilon_{2}^{V+A}|$ shown in Table 8. The bound on $|\epsilon_{1}^{V-A}|$ is stronger than the others shown in this table due to the relatively large values of the recoil and $P$-wave matrix elements in this case. The bounds on $|\epsilon_{1}^{V+A}|$ given in Table 9 are comparable with the bounds $|\epsilon_{1}^{V-A}| < 4 \times 10^{-9}$, $|\epsilon_{2}^{V+A}| < 6 \times 10^{-7}$ given in Ref. [28].

Table 9: Upper bounds on $|\mu_{1}^{V-A}|, |\epsilon_{2}^{V+A}|$ for decays of $^{76}$Ge for the case of $|(m)| = 0$ in QRPA.

| Nuclear model | $|\mu_{1}^{V-A}|$ | $|\mu_{2}^{V-A}|$ | $|\epsilon_{1}^{V+A}|$ | $|\epsilon_{2}^{V+A}|$ |
|---------------|-----------------|-----------------|-----------------|-----------------|
| pnQRPA with $g_{pp} = 1.02(1.06)$ [21] | 2.6(2.9) $\times 10^{-9}$ | 4.5(5.0) $\times 10^{-9}$ | | |
| QRPA without (with) p-n pairing [20] | 5.0(11) $\times 10^{-9}$ | 5.4(6.5) $\times 10^{-9}$ | 4.8(13) $\times 10^{-9}$ | 7.9(25) $\times 10^{-9}$ |
Hence, using Eq. (79) we obtain

\[ |\alpha_I^{(L)}| \leq 1.1 \times 10^{-9} \left( \frac{M_I}{100 \text{ GeV}} \right)^2, \quad |\alpha_I^{(R)}| \leq 2.6 \times 10^{-7} \left( \frac{M_I}{100 \text{ GeV}} \right)^2, \quad I = S, V. \]  

(83)

• Consider a more general case of \(|\langle m \rangle| \neq 0\), \(\cos \psi_i = 0\), where the index \(i\) depends on \(\alpha, \beta\) (as above, we take only one nonzero \(\epsilon_\alpha^\beta\) at a time). Using Tables 2 and 4 we have

\[ A = C_1 |\mu|^2 + 4C_i |\mu_\alpha^\beta|^2, \]

\[ K_A = D_1 |\mu|^2 + 4D_i |\mu_\alpha^\beta|^2, \]

(84)

and

\[ A = C_1 |\mu|^2 + C_i |\epsilon_\alpha^\beta|^2, \]

\[ K_A = D_1 |\mu|^2 + D_i |\epsilon_\alpha^\beta|^2. \]

(85)

Hence, using Eq. (79) we obtain

\[ |\mu|^2 = (\lambda_1 - \lambda_2 K)/T_{1/2}, \]

\[ |\epsilon_\alpha^\beta|^2 = (-\lambda_3 + \lambda_4 K)/T_{1/2} = 4|\mu_\alpha^\beta|^2, \]

(86)

with the coefficients

\[ \lambda_1 = \frac{D_i}{|M_{GT}|^2 \Delta_i}, \quad \lambda_2 = \frac{C_i}{|M_{GT}|^2 \Delta_i}, \]

\[ \lambda_3 = \frac{D_1}{|M_{GT}|^2 \Delta_i}, \quad \lambda_4 = \frac{C_1}{|M_{GT}|^2 \Delta_i}. \]

(87)

where \(\Delta_i = C_1 D_i - D_1 C_i\).

Using Eqs (80)–(87) we have for \(\epsilon_{V+A}^{V+A} \neq 0\)

\[ |\mu|^2 = (7.9 + 10 K) \times 10^{12}/T_{1/2}, \quad |\epsilon_{V+A}^{V+A}|^2 = (5.1 - 6.3 K) \times 10^{12}/T_{1/2} \]

(88)

and for \(\epsilon_{V-A}^{V-A} \neq 0\)

\[ |\mu|^2 = (7.7 + 10 K) \times 10^{12}/T_{1/2}, \quad |\epsilon_{V-A}^{V-A}|^2 = (1.9 - 2.4 K) \times 10^{8}/T_{1/2}, \]

(89)

with \(T_{1/2}\) in years. Fig. 1 shows the correlation among \(|\langle m \rangle|,\ T_{1/2},\ K\ (left)\) and the correlation among \(|\epsilon_{V+A}^{V+A}|,\ T_{1/2},\ K\ (right)\) for the choice of a nonzero \(\epsilon_{V+A}^{V+A}\). Fig. 2 shows the same for the parameter \(\epsilon_{V-A}^{V-A}\). It is clear from Figs 1 and 2 that the closer is \(K\) to 1 for the fixed value of \(T_{1/2}\), the weaker is bounded \(|\langle m \rangle|\) and stronger is bounded \(|\epsilon_{V-A}^{V-A}|\). The correlations among \(|\epsilon_{V-A}^{V-A}|,\ T_{1/2},\ K\) will be used in the next section in the analysis of left-right symmetric models.

Note that if several \(\epsilon_\alpha^\beta\) are nonzero in the considered model than the respective interference terms should be taken into account.

• To extract \(|\mu|,\ |\mu_\alpha^\beta|,\ |\epsilon_\alpha^\beta|,\ c_i\) in the general case of \(|\langle m \rangle| \neq 0,\ c_i \neq 0\) we need to analyze the data on at least two decaying nuclei. This analysis will be presented for the five nuclei already discussed in a forthcoming paper [38].
4 Electron angular correlation in left-right symmetric models

The experimental bounds on the $\epsilon^R_\beta$ are connected with the masses of new particles, their mixing angles, and other parameters specific to particular extensions of the SM \[5,4,8,10,12,13\]. To illustrate the kind of correlations that the measurements of $T_{1/2}$ and the angular correlation coefficient $K$ in the $0\nu2\beta$ decay would imply, we work out the case of the left-right symmetric models \[22\]. In the model $SU(2)_L \times SU(2)_R \times U(1)$ the parameters $\eta$ and $\lambda$ (see Eq. \[2\]) are expressed through the masses $m_{W_L}$ and $m_{W_R}$ of the left- and right-handed $W$ bosons and their mixing angle $\zeta$ \[5\]:

$$\eta = -\tan \zeta, \quad \lambda = (m_{W_L}/m_{W_R})^2,$$

under the condition

$$m_{W_L} \ll m_{W_R}.$$ \[91\]

Eqs. \[2\] and \[6\] and the relation \[5\]

$$V_{ei} = V'_{ei}$$ \[92\]

of the $SU(2)_L \times SU(2)_R \times U(1)$ model yield

$$\epsilon^{V+A}_{V^+} = \lambda g_V' U_{ei} V_{ei}, \quad \epsilon^{V-A}_{V^+} = \eta U_{ei} V_{ei}. \quad \epsilon'$$ \[93\]

To reduce the number of free parameters, we assume the equality of the form factors of the left- and right-handed hadronic currents:

$$g_V = g'_V.$$ \[94\]

The small masses of the observable $\nu$s are likely described by the seesaw formula that in the simplest case gives

$$m_{i} \sim m_D^2/M_R, \quad M_R \gg m_D,$$ \[95\]

with the Dirac mass scale $m_D$ (for the charged leptons and the light quarks $m_D \sim 1$ MeV) and the mass scale $M_R$ of right $\nu_{MS}$ (in the majority of theories $M_R > 1$ TeV). In the left-right symmetric models these scales arise usually from the two scales of the vacuum expectation values of Higgs multiplets \[22\]. In the seesaw mechanism, the values of the mixing parameters $V_{ei}$ (for $i$ numbering light mass states) have the same order of magnitude as $m_D/M_R$. In our discussion we use two rather conservative values (compare with Eq. \[93\])

$$\epsilon = 10^{-6}, \ 5 \times 10^{-7}$$ \[96\]

for the mixing parameter

$$\epsilon = |U_{ei} V_{ei}|.$$ \[97\]

We recall that here the summation index $i$ runs only over the light neutrino mass eigenstates (the summation over the total mass spectrum including also heavy states gives strictly zero due to the orthogonality condition \[5\]).

From Eqs. \[90\], \[93\], \[94\], and \[97\] we have

$$m_{W_R} = m_{W_L} \left(\epsilon/|\epsilon^{V^+}_{V^+}|\right)^{1/2}, \quad \zeta = -\arctan \left(|\epsilon^{V^+}_{V^+}| / \epsilon\right).$$ \[98\]

Using Eq. \[91\] we note the approximate equality of $m_{W_L}$ and the mass of the observed charged gauge boson $W_1$ ($m_{W_1}$=80.4 GeV \[1\]).

The correlation among $m_{W_R}$ ($\zeta$), $K$, and $T_{1/2}$ for the case of $|\langle m\rangle| \neq 0, \cos \psi_i = 0$ (see section 3.2) is shown in Fig. 3 (4) for the two chosen values of $\epsilon$. The numerical results for these figures have been obtained using Eqs. \[88\] and \[89\]. It is clear from Fig. 3 (4) that the closer is $K$ to 1 for the fixed value of $T_{1/2}$ the stronger is the lower bound on $m_{W_R}$ (the upper bound on $\zeta$). However this bound is weaker than the one $m_{W_R} > 715$ GeV, obtained from the electroweak fits \[1\]. There is still a more stringent bound $m_{W_R} > 1.2$ TeV, obtained in Ref. \[39\] for the $0\nu2\beta$ decay mediated by heavy Majorana neutrinos using arguments based on the vacuum stability \[6\] and additional theory input. We assume $m_{W_R} \geq 1$ TeV in the next figure.
While experiments in the 0ν2β decay would measure the product of the quantities called λ and the neutrino mixing matrix elements U_{e
u} V_{e
nu} in Eq. (93), collider experiments at the Tevatron and the LHC can, in principle, measure λ by determining m_{W_R}. Assuming these logically independent possibilities, we plot the differential width (21) vs. \cos \theta in Fig. 5 for a set of values of \langle |m| \rangle and m_{W_R}, taking \nu^{V+A} at a time and assuming \epsilon = 10^{-6}. In this figure, we consider the values of \langle |m| \rangle, starting from \langle |m| \rangle \leq 0.03 eV up to \langle |m| \rangle = 5 meV, covering two of three scenarios of neutrino mass hierarchies and mixing angles: normal and inverted mass hierarchies (see Ref. [40] for a recent discussion and update). It is seen that the sensitivity of the electron angular correlation to the right-handed W-boson mass m_{W_R} increases with decreasing values of the effective Majorana neutrino mass \langle |m| \rangle, as can be seen from Fig. 5 (right), where this correlation is shown for \langle |m| \rangle=5 meV, 10 meV.

In conclusion, we have presented a detailed study of the electron angular correlation for the long range decays in a general theoretical context. This information, together with the ability of observing these decays in several nuclei, would help greatly in identifying the dominant mechanism underlying these decays. At present, no experiment is geared to measuring the angular correlation in 0ν2β decays, as the main experimental thrust is on establishing a non-zero signal unambiguously in the first place. We note that the running experiment NEMO3 has already measured the electron angular distribution for the two decays. At present, no experiment is geared to measuring these decays in several nuclei, would help greatly in identifying the dominant mechanism underlying these decays. At present, no experiment is geared to measuring the angular correlation in 0ν2β decays, as the main experimental thrust is on establishing a non-zero signal unambiguously in the first place. We note that the running experiment NEMO3 has already measured the electron angular distribution for the two decays. At present, no experiment is geared to measuring these decays in several nuclei, would help greatly in identifying the dominant mechanism underlying these decays. At present, no experiment is geared to measuring the angular correlation in 0ν2β decays, as the main experimental thrust is on establishing a non-zero signal unambiguously in the first place. We note that the running experiment NEMO3 has already measured the electron angular distribution for the two decays. At present, no experiment is geared to measuring these decays in several nuclei, would help greatly in identifying the dominant mechanism underlying these decays. At present, no experiment is geared to measuring the angular correlation in 0ν2β decays, as the main experimental thrust is on establishing a non-zero signal unambiguously in the first place. We note that the running experiment NEMO3 has already measured the electron angular distribution for the two decays. At present, no experiment is geared to measuring these decays in several nuclei, would help greatly in identifying the dominant mechanism underlying these decays. At present, no experiment is geared to measuring the angular correlation in 0ν2β decays, as the main experimental thrust is on establishing a non-zero signal unambiguously in the first place. We note that the running experiment NEMO3 has already measured the electron angular distribution for the two decays. At present, no experiment is geared to measuring these decays in several nuclei, would help greatly in identifying the dominant mechanism underlying these decays.
and the combinations
\[
\ell_{\mu}^{L,R} = \frac{s_{\mu}^{L,R}(2y, 1x)}{\omega + A_1} - \frac{s_{\mu}^{L,R}(1y, 2x)}{\omega + A_2},
\]
\[\ell_{\nu}^L = \frac{s_{\nu}^L(2y, 1x)}{\omega + A_1} - \frac{s_{\nu}^L(1y, 2x)}{\omega + A_2},
\]
(105)
(106)
of electron currents
\[s_{\mu}^{L,R}(2y, 1x) = \bar{e}_2(y)\gamma_\mu(1 \mp \gamma_5)e_i(x), \quad s_{\nu}^L(2y, 1x) = \bar{e}_2(y)\gamma_\lambda(1 - \gamma_5)e_i(x),
\]
(107)
e_i(x) \equiv e_{p,s_i}(x),
the matrix element is expressed as
\[
R_{0\nu}^{SP} = \frac{1}{\sqrt{2}} \left( \frac{G_F|V_{ud}|}{\sqrt{2}} \right)^2 2 \sum_i \int dx dy \frac{dk}{(2\pi)^3 2\omega} 
\times \sum_N \left[ m_i \left( J_{\mu L}^{R} \ell_{\mu L}^{R} - J_{\mu L}^{L} \ell_{\mu L}^{L} \right) + k^\lambda \left( J_{\mu L}^{R} \ell_{\mu R}^{L} - J_{\mu L}^{L} \ell_{\mu L}^{L} \right) \right],
\]
(108)
where \( r = y - x \). By using the identities
\[s_{\mu}^{L,R}(1y, 2x) = s_{\mu}^{R,L}(2x, 1y), \quad s_{\nu}^L(1y, 2x) = -s_{\nu}^L(2x, 1y),
\]
(109)
the algebraic formula
\[2(am \pm bn) = (a + b)(m \pm n) + (a - b)(m \mp n),
\]
(110)
the constant
\[C_{0\nu} = \frac{G_F^2 |V_{ud}|^2 2m_e}{8\sqrt{2}\pi R},
\]
(111)
and the neutrino potentials
\[(H_j, H_{\omega j}, H_{\kappa j}') = 4\pi \int \frac{dk}{(2\pi)^3} \frac{e^{ikx}}{(\omega + A_j)^2},
\]
(112)
the matrix element \((108)\) is expressed as
\[R_{0\nu}^{SP} = -C_{0\nu} \sum_i \sum_N \left( \frac{m_i}{m_e} M_{SP}^m + M_{SP}^k \right),
\]
(113)
Each part of this matrix element is expressed as a sum of nonvanishing (indexed by \( n \)) and vanishing (indexed by \( c \)) terms, in the closure approximation:
\[M_{SP}^{m,k} = \{M_{SP}^{m,k}\}_n + \{M_{SP}^{m,k}\}_c,
\]
(114)
\[\{M_{SP}^{m}\}_n = \frac{R}{2} \int dx dy T_N(H_1 + H_2)
\times \left[ (A_1 + A_{1R})F_5^0 + (A_3 + \tilde{A}_{3R})F_5^+ + B_{1R}F_5^0 + (B_3 + \tilde{B}_{3R})F_5^+ \right],
\]
(115)
\[\{M_{SP}^{m}\}_c = \frac{R}{2} \int dx dy T_N(H_1 - H_2)
\times \left[ (A_1 + A_{1R})F_5^0 + (A_3 + \tilde{A}_{3R})F_5^- + B_{1R}F_5^0 + (B_3 + \tilde{B}_{3R})F_5^- \right],
\]
(116)
\{ M^k_{SP} \}_n = \frac{R}{2m_e} \int dx dy T_N \{ (H_{\omega 1} - H_{\omega 2}) \left[ -(A^i_1 + \tilde{A}^i_{4R})E^i_+ + B_{2R}E_- \right] \\
+ (H^1_{k1} + H^2_{k2}) \left[ -(A^i_2 + A^i_{2R})E^i_+ + (A^i_{5k} + \tilde{A}^i_{5R})E^i_+ + (B^i_1 + B^i_{1R})E_+ \right] \}, \tag{117}

\{ M^k_{SP} \}_c = \frac{R}{2m_e} \int dx dy T_N \{ (H_{\omega 1} - H_{\omega 2}) \left[ -(A^i_1 + \tilde{A}^i_{4R})E^i_- + B_{2R}E_+ \right] \\
+ (H^1_{k1} - H^2_{k2}) \left[ -(A^i_2 + A^i_{2R})E^i_- + (A^i_{5k} + \tilde{A}^i_{5R})E^i_- + (B^i_1 + B^i_{1R})E_- \right] \}, \tag{118}

with

\begin{equation}
T_N = g_A^2 (F \sum_a r_a^a |N \rangle \langle N | \sum_b r_b^b |I \rangle \delta(x - r_a) \delta(y - r_b)) . \tag{119}
\end{equation}

The electron currents are defined as:

\begin{align*}
F_+ &= \frac{1}{2} [u(yx) \mp u(xy)], & F_{5\pm} &= \frac{1}{2} [u_5(yx) \pm u_5(xy)], \\
F^\mu_+ &= \frac{1}{2} [u^\mu(yx) \pm u^\mu(xy)], & F^5_{\pm\mp} &= \frac{1}{2} [u^5_\mp(yx) \mp u^5_\mp(xy)], \\
F^{\mu\nu}_+ &= \frac{1}{2} [u^{\mu\nu}(yx) \pm u^{\mu\nu}(xy)], & F^{5\mu}_{\pm\mp} &= \frac{1}{2} [u^{5\mu}_\mp(yx) \mp u^{5\mu}_\mp(xy)],
\end{align*}

\begin{equation}
E_\pm = F_\pm + F_{5\pm}, & E^5_\pm = F^{05}_{\mp} + F^{50}_{\pm}. \tag{120}
\end{equation}

with

\begin{align*}
u(yx) &= \bar{e}_2(y)\gamma_5 e^c_1(x), & u_5(yx) &= \bar{e}_2(y)\gamma_5 e_1^c(x), \\
u^\mu(yx) &= \bar{e}_2(y)\gamma^\mu e^c_1(x), & u_5^\mu(yx) &= \bar{e}_2(y)\gamma_5 \gamma^\mu e_1^c(x), \\
u^{\mu\nu}(yx) &= -i\bar{e}_2(y)\sigma^{\mu\nu} e^c_1(x), & u_5^{\mu\nu}(yx) &= -i\bar{e}_2(y)\gamma_5 \sigma^{\mu\nu} e_1^c(x). \tag{121}
\end{align*}

The nucleon operator matrix elements are defined as follows:

\begin{align*}
\tilde{A} = A + A^P, & \quad \tilde{B} = B + B^P, \tag{122}
\end{align*}

\begin{align*}
A_1 &= 2g^0_0 \epsilon_0 \sigma_0, & A_{1R} &= -g^0_0 \epsilon_0 \sigma_0 \epsilon_0 B_+, \\
A_2 &= 2g^0_0 \epsilon_0 \sigma^1, & A_{2R} &= -g^0_0 \epsilon_0 \sigma_0 \epsilon_0 B_+, \\
A^1_3 &= g^0_0 \epsilon_0 \sigma^1_+, & A^1_{3R} &= -g^0_0 \epsilon_0 \sigma^1_+ \epsilon_0 B^1_+, & A^1_{3R} &= -g^0_0 \epsilon_0 \sigma^1_+ \epsilon_0 B^1_+, \\
A_4 &= g^0_0 \epsilon_0 \sigma^1_-, & A^1_{4R} &= g^0_0 \epsilon_0 \sigma^1_- \epsilon_0 B^1_+, & A^1_{4R} &= g^0_0 \epsilon_0 \sigma^1_- \epsilon_0 B^1_+, \\
A^1_5 &= i \epsilon_{ijk} A^1_4, & A^1_{5R} &= i \epsilon_{ijk} \tilde{A}^1_{4R}, \tag{123}
\end{align*}

\begin{align*}
B_{1R} &= -g^0_0 \epsilon_0 \sigma_0 \epsilon_0 \epsilon_0 B_+, & B_{2R} &= -g^0_0 \epsilon_0 \sigma_0 \epsilon_0 \epsilon_0 B_+, \\
B^1_3 &= g^0_0 \epsilon_0 \epsilon_0 \sigma^1_+, & B^1_{3R} &= -g^0_0 \epsilon_0 \epsilon_0 \sigma^1_+ \epsilon_0 B^1_+, & B^1_{3R} &= -g^0_0 \epsilon_0 \epsilon_0 \sigma^1_+ \epsilon_0 B^1_+, \\
B_{4R} &= g^0_0 \epsilon_0 \epsilon_0 \sigma^1_-, & B^1_{4R} &= g^0_0 \epsilon_0 \epsilon_0 \sigma^1_- \epsilon_0 B^1_+, & B^1_{4R} &= g^0_0 \epsilon_0 \epsilon_0 \sigma^1_- \epsilon_0 B^1_+, \tag{124}
\end{align*}

with

\begin{align*}
\sigma^1_+ &= \sigma^1_+ \epsilon_0 \pm \sigma^0_a \epsilon_0 \epsilon_0 \sigma^1_-, & B_{\pm} &= B_\pm \epsilon_0 I_0 \pm I_a B_b, & B^1_{\pm} &= \sigma^0_b B_a \pm B_a \sigma^0_a, \\
C_{\pm} &= C_a I_b \pm I_a C_b, & D^1_{\pm} &= D^1_{\pm} \epsilon_0 I_0 \pm I_a D^1_b, & P^1_{\pm} &= P^1_{a} I_b \pm I_a P^1_b. \tag{125}
\end{align*}

Under the exchange of running indices \(a\) and \(b\) (i.e. \(x \leftrightarrow y\)), nuclear operators \(A\), electron currents \(E_+\) and \(F_+\) and neutrino potentials \(H_i\) and \(H_{\omega i}\) are even, while \(B\), \(E_-\), \(F_-\), and \(H_{ki}\) are odd.
The constants are defined as:

\[ G_V = \frac{9V}{g_A} \left[ (U_{ei} + \epsilon V^{-A}_{-A,i}) + \epsilon V^{-A}_{+A,i} \right], \quad G_A = \left( U_{ei} + \epsilon V^{-A}_{-A,i} \right) - \epsilon V^{-A}_{+A,i}, \]

\[ G^0 = G(\epsilon = 0), \quad G_V^0 = \frac{9V}{g_A} U_{ei}, \quad G_A^0 = U_{ei}, \]  

(126)

\[ \varepsilon_S = \frac{F_S^{(3)}}{g_A} \left( \varepsilon_{S-P, i} + \varepsilon_{S-P, i}^{+} \right), \quad \varepsilon_P = \frac{F_P^{(3)}}{g_A} \left( \varepsilon_{S-P, i} - \varepsilon_{S-P, i}^{+} \right), \]

\[ \varepsilon'_S = \frac{F_S^{(3)}}{g_A} \left( \varepsilon_{S+P, i} + \varepsilon_{S+P, i}^{+} \right), \quad \varepsilon'_P = \frac{F_P^{(3)}}{g_A} \left( \varepsilon_{S+P, i} - \varepsilon_{S+P, i}^{+} \right). \]  

(127)

Note that in the notations of Ref. [5]:

\[ t = u + u_5, \quad t^l = u^0 + u_5^{0l}. \]  

(128)

Since the nucleon recoil term \( P_a \) behaves as an even parity operator while the neutrino momentum \( k \) and the recoil terms \( B_a, C_a, D_a \) as odd ones, each of the \( A_j, k \cdot A_j, B_j, k \cdot B_j \) has a definite parity. The operators

\[ A_1, A_3^i, A_4^i, A_{3R}^i, A_{4R}^i; \quad B_3^i, B_{3R}^i; \quad r \cdot B_A^i, \ r^l A_{2R}^i, \ r^l A_{5R}^i, \]  

(129)

have even parity and the operators

\[ A_{1R}^i, A_{3R}^i, A_{4R}^i; \quad B_{1R}^i, B_{2R}^i, B_{3R}^i; \quad r \cdot B_A^i, \ r^l A_{2R}^i, \ r^l A_{5R}^i, \]  

(130)

have odd parity. The odd-parity operators do not contribute to the \( 0^+ \to J^+ \) transition in the case where both the electrons are in the S-wave state (the \( S - S \) case) with no de Broglie wave length correction (no FBWC).

Using the definitions of neutrino potentials

\[ h_+ = \frac{R}{2} (H_1 + H_2), \quad h_0 = \frac{1}{\varepsilon_{21}} (H_1 - H_2), \]

\[ h_{\omega +} = \frac{R^2}{2} (H_{\omega 1} + H_{\omega 2}), \quad h_{\omega 0} = \frac{R}{\varepsilon_{21}} (H_{\omega 1} - H_{\omega 2}), \]

\[ h_{\omega +}^l r^l = -i \frac{r R}{2} (H_{\omega 1}^l + H_{\omega 2}^l), \quad h_{\omega 0}^l r^l = -i \frac{r}{\varepsilon_{21}} (H_{\omega 1}^l - H_{\omega 2}^l), \]  

(131)

in the \( S - S \) case with no FWBC, Eqs. (115), (117) are reduced to

\[ \{ M_{SP}^{(n)} \}_{n, S - S} = \int dx dy T_N h_+ \left[ A_1 F_{5+}^0 + (A_4^i + A_{4R}^i) F_{5+}^i \right], \]  

(132)

\[ \{ M_{SP}^{(n)} \}_{c, S - S} = \frac{\varepsilon_{21} R}{2} \int dx dy T_N h_0 (B_3^i + B_{3R}^i) F_{s+}^i, \]  

(133)

\[ \{ M_{SP}^{(n)} \}_{n, S - S} = \frac{\varepsilon_{21}}{2 m_e} \int dx dy T_N h_{\omega 0} (A_1^i + A_{1R}^i) E_i^+ \]

\[ + \frac{2}{m_e R} \int dx dy T_N \frac{i R}{2r} h_0^l r^l \cdot B_{AR} E_+^l, \]  

(134)

\[ \{ M_{SP}^{(n)} \}_{c, S - S} = \frac{\varepsilon_{21}}{2 m_e} \int dx dy T_N \frac{i R}{r} h_0^l r^l (-A_{2R}^l E_+^l + A_{5R}^l E_+^l), \]  

(135)
where $E$, $F$ are taken for $x=0$, $y=0$.

For the $0^+ \to 0^+$ transition we have

\begin{align}
\sum_i \frac{m_i}{m_e} \sum_N \{M_{SP}^m\}_{S-S} &= g_A^2 C_1 A_{SP}^{0+}, \quad (136) \\
\sum_i \sum_N \{M_{SP}^k\}_{S-S} &= g_A^2 \frac{2}{m_e R} C_{4R}^{B} E_+, \quad (137)
\end{align}

with

\begin{align}
C_1^A &= \left\langle \frac{m_i}{m_e} h_+ A_1 \right\rangle, \quad C_{4R}^B = \left\langle \frac{i R}{2 r} h_+^{l} \hat{r} \cdot B_{4R} \right\rangle, \quad (138)
\end{align}

where $\hat{r} = r/r$ and $\langle X \rangle = \sum_i \sum_N \langle 0^+ | X | 0^+ \rangle$, with $h = h(r, E_N)$.

In the $S - P_{1/2}$ case with no FBWC for the $0^+ \to 0^+$ transition we have

\begin{align}
\{M_{SP}^m\}_{n,S-P_{1/2}} &= \int dx dy T_N h_+ \left( A_{3R}^{i} F_{5+}^{i} + B_{3R}^{i} F_{4+}^{i} \right), \quad (139) \\
\{M_{SP}^m\}_{c,S-P_{1/2}} &= \frac{\varepsilon_{21} R}{2} \int dx dy T_N h_0 \left( A_{3R}^{i} F_{5-}^{i} + B_{3R}^{i} F_{4+}^{i} \right), \quad (140)
\end{align}

\begin{align}
\{M_{SP}^k\}_{n,S-P_{1/2}} &= -\frac{1}{2} \frac{\varepsilon_{21}}{m_e R} \int dx dy T_N h_{\omega} A_{iR}^{i} E_{-}^{i} \\
+ \frac{2}{m_e R} \int dx dy T_N \frac{i R}{2 r} h_+^{i} \hat{r} \left[ -A_2 E_+^{i} + (A_5^{i} + A_{5R}^{i}) E_+^{i} \right], \quad (141) \\
\{M_{SP}^k\}_{c,S-P_{1/2}} &= -\frac{1}{2} \frac{\varepsilon_{21}}{m_e R} \int dx dy T_N h_{\omega} A_{iR}^{i} E_{-}^{i} \\
+ \frac{1}{2} \frac{\varepsilon_{21}}{m_e R} \int dx dy T_N \frac{i R}{r} h_0^{i} \hat{r} \left[ -A_2 E_+^{i} + (A_5^{i} + A_{5R}^{i}) E_+^{i} \right]. \quad (142)
\end{align}

The squared modulus of the matrix element \(113\), summed over the polarizations $s_j$ of the electrons and multiplied by the phase space element \(155\), yields the differential decay rate for the $0^+ \to 0^+$ transition

\begin{align}
d\Gamma = \sum_{s_1, s_2} |R_{00}^{SP}|^2 \frac{m_e^5}{2 \pi^3} d\Omega_{0\nu} = \frac{a_{0\nu}}{(m_e R)^4} \left[ A_0^{SP} - \hat{p}_1 \cdot \hat{p}_2 B_0^{SP} \right] d\Omega_{0\nu}, \quad (143)
\end{align}

with $a_{0\nu}$ being defined in Eq. \(36\). Here the coefficients are

\begin{align}
A_0^{SP} &= \sum_{i=1}^{4} |M_i|^2, \quad (144) \\
B_0^{SP} &= \text{Re}(M_1 M_1^* + M_2 M_2^* + M_3 M_3^* + M_4 M_4^*) \quad (145)
\end{align}

with

\begin{align}
M_1 &= \alpha_1^{2-1} \left\{ -C_1^A + \frac{2}{m_e R} C_{4R}^{B} \right\} + \left\{ \frac{m_e R}{3} \left( \frac{\zeta}{m_e R} - 2 \right) C_{3R}^{A} + \frac{\varepsilon_{21} R}{3} \{C_{3R}\}_{c} \right\} \frac{r}{2R} \\
+ \frac{\varepsilon_{21} R}{6 m_e} \left\{ \{C_2^A\}_{c} - \{C_5^A\}_{c} - \{C_{5R}\}_{c} - \{C_{4R}\}_{c} \right\} \frac{r}{2R} + \frac{1}{6} \left( \frac{\zeta}{m_e R} - 2 \right) \left( C_2^A - C_5^A - C_{5R}^A - \{C_{4R}\}_{c} \right) \\
+ \left\{ \frac{\alpha Z}{2 m_e R} \left\{ \{C_2^A\}_{c} + \{C_{4R}\}_{c} - 3 C_{4R}^{B} \right\} \right\}, \quad (146)
\end{align}

\begin{align}
M_2 &= \alpha_1^{2} \left\{ -C_1^A + \frac{2}{m_e R} C_{4R}^{B} \right\} + \left\{ \frac{m_e R}{3} \left( \frac{\zeta}{m_e R} + 2 \right) C_{3R}^{A} - \frac{\varepsilon_{21} R}{3} \{C_{3R}\}_{c} \right\} \frac{r}{2R} \\
+ \frac{\varepsilon_{21} R}{6 m_e} \left\{ \{C_2^A\}_{c} - \{C_5^A\}_{c} - \{C_{5R}\}_{c} - \{C_{4R}\}_{c} \right\} \frac{r}{2R} + \frac{1}{6} \left( \frac{\zeta}{m_e R} + 2 \right) \left( C_2^A - C_5^A - C_{5R}^A - \{C_{4R}\}_{c} \right) \\
+ \left\{ \frac{\alpha Z}{2 m_e R} \left\{ \{C_2^A\}_{c} + \{C_{4R}\}_{c} - 3 C_{4R}^{B} \right\} \right\}, \quad (146)
\end{align}

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\[ + \frac{\varepsilon_{21}}{m_e} \left( \left\{ C_2^A \right\}_e - \left\{ C_5^A \right\}_e - \left\{ C_5^R \right\}_e - C_4^A \left( \frac{r}{2R} + \frac{\zeta}{m_e R} + 2 \right) \left( C_2^A - C_5^A - C_5^R - \left\{ C_4^R \right\}_e \right) \right] \\
+ \left[ \frac{\left( \alpha Z \right)^2}{2m_e R} \left( \left\{ C_4^A \right\}_e + \left\{ C_4^R \right\}_e - 3C_{44RF}^B \right) \right] \right], \tag{147}
\]

\[ M_3 = \alpha_{11}^* \left[ \left\{ 2 \frac{m_e}{m_e R} C_{44RF}^B \right\} + \left( \varepsilon_{21} \frac{R}{6m_e} + \frac{\varepsilon_{21}}{m_e} + 2 \right) \left( \left\{ C_2^A \right\}_e - \left\{ C_5^A \right\}_e - \left\{ C_5^R \right\}_e - C_4^A \right) \frac{r}{2R} \\
+ \left[ \frac{\left( \alpha Z \right)^2}{2m_e R} \left( \left\{ C_4^A \right\}_e + \left\{ C_4^R \right\}_e - 3C_{44RF}^B \right) \right] \right], \tag{148}
\]

\[ M_4 = \alpha_{11}^* \left[ \left\{ 2 \frac{m_e}{m_e R} C_{44RF}^B \right\} + \left( \varepsilon_{21} \frac{R}{6m_e} \right) \left( \frac{\varepsilon_{21}}{m_e} - 2 \right) \left( \left\{ C_2^A \right\}_e - \left\{ C_5^A \right\}_e - \left\{ C_5^R \right\}_e - C_4^A \right) \frac{r}{2R} \\
+ \left[ \frac{\left( \alpha Z \right)^2}{2m_e R} \left( \left\{ C_4^A \right\}_e + \left\{ C_4^R \right\}_e - 3C_{44RF}^B \right) \right] \right], \tag{149}
\]

where \( \alpha_{ij} = \tilde{A}_i (\varepsilon_2) \tilde{A}_j (\varepsilon_1) \) and the nucleon matrix elements are

\[
C_{33R}^B = \left\langle \frac{m_i}{m_e} i h_0 \hat{r} \cdot B_{33R} \right\rangle, \quad \left\{ C_{33R}^B \right\}_e = \left\langle \frac{m_i}{m_e} i h_0 \hat{r} \cdot B_{33R} \right\rangle, \\
C_{33R}^A = \left\langle \frac{m_i}{m_e} i h_0 \hat{r} \cdot A_{33R} \right\rangle, \quad \left\{ C_{33R}^A \right\}_e = \left\langle \frac{m_i}{m_e} i h_0 \hat{r} \cdot A_{33R} \right\rangle, \\
C_{44R}^A = \left\langle i h_0 \hat{r} \cdot A_{44R} \right\rangle, \quad \left\{ C_{44R}^A \right\}_e = \left\langle \frac{r}{R} h_0 \hat{r} \cdot A_{44R} \right\rangle, \\
\{ C_2^A \}_e = \left\langle \frac{r}{R} h_0 \hat{r} \cdot \hat{r}_+ A_2 \right\rangle, \quad C_2^A = \left\langle h_+ A_2 \right\rangle, \\
\{ C_5^R \}_e = \left\langle \frac{R}{r} h_0 \hat{r} \cdot \hat{r}_+ A_{55}^{ij} \right\rangle, \quad C_5^R = \left\langle h_+ \hat{r} \cdot A_{55}^{ij} \right\rangle, \\
C_{44RF}^B = \left\langle i R \frac{r_+^2 + r_+ \hat{r} \cdot B_{44R}}{2r} \right\rangle, \tag{150}
\]

with \( r_+ = y + x = r_+ \hat{r}_+ \).

The terms in the first brackets in Eqs. (140)–(149) come from the \( S - S \) case, the terms in the second brackets come from the \( S - P_{1/2} \) case and in the third brackets there are the most important terms due to the \( P_{1/2} - P_{1/2} \) case and FBWC.

Assuming now \( \langle m \rangle \neq 0 \) for the dominant terms we have

\[
M_1 = \alpha_{11}^* \left[ \left\{ Z_1^X - C_1^A + \frac{2}{m_e R} \right\} C_{44R}^B \right] \\
+ \left[ \frac{\varepsilon_{21}^2 R}{6m_e} \left( \left\{ C_2^A \right\}_e - \left\{ C_5^A \right\}_e \right) \frac{r}{2R} + \frac{\zeta}{m_e R} \frac{r}{2R} \left( C_2^A - C_5^A \right) \right], \tag{151}
\]

\[
M_2 = \alpha_{11}^* \left[ \left\{ Z_1^X + C_1^A + \frac{2}{m_e R} \right\} C_{44R}^B \right] \\
+ \left[ \frac{\varepsilon_{21}^2 R}{6m_e} \left( \left\{ C_2^A \right\}_e - \left\{ C_5^A \right\}_e \right) \frac{r}{2R} + \frac{\zeta}{m_e R} \frac{r}{2R} \left( C_2^A - C_5^A \right) \right], \tag{152}
\]

\[
M_3 = \alpha_{11}^* \left[ \left\{ Z_1^X + \frac{2}{m_e R} \right\} C_{44R}^B \right] \\
+ \left[ \frac{\varepsilon_{21}^2 R}{6m_e} \left( \frac{\varepsilon_{21}}{m_e} + 2 \right) \left( \left\{ C_2^A \right\}_e - \left\{ C_5^A \right\}_e \right) \frac{r}{2R} + \frac{\zeta}{m_e R} \frac{r}{2R} \left( C_2^A - C_5^A \right) \right], \tag{153}
\]

23
\[ M_4 = \alpha_{-11} \left\{ Z^X_1 + \frac{2}{m_e R} C^B_4 R \right\} + \left( \frac{\varepsilon_0 R_1}{6} \left( \frac{\varepsilon_0 R_2}{2} - 2 \right) \left( \{C^A_2\}_c - \{C^A_5\}_c \right) \frac{r}{2 R} + \frac{1}{6 m_e R} (C^A_2 - C^A_5) \right\}. \] (154)

In the expressions for \( M_1, \ldots, M_4 \), the terms with \( \zeta \) are due to the inclusion of the \( P \)-wave in the electron wave function and those with \( C^B_4 R \) are from the inclusion of the nucleon recoil effect. Note that some of the subdominant terms should be taken into account in case of large cancellation among the dominant terms.

**B 0ν2β decay rate for vector nonstandard terms**

In this appendix we in general follow the derivation of Ref. [5]. However in addition to Ref. [5] we keep in more details the terms associated with the parameters \( \epsilon_{Y^\pm A} \) and the pseudoscalar form factor.

The nucleon currents in the impulse approximation up to order \( p/m_p \) in the nonrelativistic expansion are [32, 34]:

\[ J_{V\mp A}^\mu(x) = \sum_a \tau_a \delta(x - r_a) \left[ g^{a0}(g_V I_a + g_A C_a) + g^{am}(\pm g_A \sigma_{am} - g_V D^m_a \mp g_A P^m_a) \right], \] (155)

with \( C_a, D^m_a, P^m_a \) given in Eq. (101).

In terms of \( S_{L\mu\nu}, V_{a\mu\nu}, J_{\alpha\beta}^\mu (\alpha, \beta = L, R) \) [5] the matrix element

\[ R_{0V}^A = C_{0V} \sum_i \sum_N \frac{R}{2m_e} \int dx dy 4\pi \frac{dk}{(2\pi)^3} \omega \left( m_i \epsilon_{L\mu\nu}^a S_{L\mu\nu} + J_{L\mu\nu}^a \epsilon_{R\mu\nu} + J_{RL\mu\nu} \right), \] (156)

may be expressed as

\[ R_{0V}^A = C_{0V} \sum_i \sum_N \left( \frac{m_i}{m_e} M_{V^\pm A}^m + M_{V^0 A}^m \right), M_{V^0 A}^m = \{ M_{V^0 A}^m \}_n + \{ M_{V^0 A}^m \}_c. \] (157)

The analogues of the Eqs. (C.2.11), (C.2.23), and (C.2.24) from Ref. [5] are as follows:

\[ \{ M_{V^0 A}^m \}_n \equiv \{ M_{m_e}^m \}_n = \frac{R}{2} \int dx dy T_N (H_1 + H_2) \left[ (X_1 + \tilde{X}_{1R})E_+ + (Y_1^i + \tilde{Y}_{1R}^i)E_+^i \right], \] (158)

\[ \{ M_{V^0 A}^m \}_c \equiv \{ M_{m_e}^m \}_c = \frac{R}{2} \int dx dy T_N (H_1 - H_2) \left[ (X_1 + \tilde{X}_{1R})E_- + (Y_1^i + \tilde{Y}_{1R}^i)E_-^i \right], \] (159)

\[ \{ M_{V^+ A}^m \}_n \equiv \{ M_{V^+ A(a)}^m \}_n = \frac{R}{m_e} \int dx dy T_N \left\{ (H_{\omega_1} - H_{\omega_2}) \right\}, \]

\[ \times \left[ (X_3 + \tilde{X}_{5R})F_0^+ + Y_3R F_5^0 - (X_4^i + \tilde{X}_{4R}^i)F_0^i + (Y_4^i + \tilde{Y}_{4R}^i)F_5^i \right] + (H_{k_1} - H_{k_2}), \]

\[ \times \left[ (X_5^i + \tilde{X}_{5R}^i)F_0^0 + (Y_5^i + \tilde{Y}_{5R}^i)F_0^0 - (X_4^i + \tilde{X}_{4R}^i)F_0^0 - (Y_4^i + \tilde{Y}_{4R}^i)F_0^0 \right], \] (160)

\[ \{ M_{V^+ A}^m \}_c \equiv \{ M_{V^+ A(a)}^m \}_c = \frac{R}{m_e} \int dx dy T_N \left\{ (H_{\omega_1} + H_{\omega_2}) \right\}, \]

\[ \times \left[ (X_3 + \tilde{X}_{5R})F_0^0 + Y_3R F_5^0 - (X_4^i + \tilde{X}_{4R}^i)F_0^i + (Y_4^i + \tilde{Y}_{4R}^i)F_5^i \right] + (H_{k_1} - H_{k_2}), \]

\[ \times \left[ (X_5^i + \tilde{X}_{5R}^i)F_0^0 + (Y_5^i + \tilde{Y}_{5R}^i)F_0^0 - (X_4^i + \tilde{X}_{4R}^i)F_0^0 - (Y_4^i + \tilde{Y}_{4R}^i)F_0^0 \right], \] (161)

with \( \tilde{X} = X + X^P, \tilde{Y} = Y + Y^P \). The operators \( X \) and \( Y \) are defined in [5], except for the operator \( Y_{6R}^i = -Y_{5R}^i \) which is defined to remove the minus sign from the Eqs. (160) and (161); \( X_1 = X_{1S}, Y_1 = Y_{1S}. \)

The additional operators are

\[ X_1^{P_{4\sigma\tau}} = G^2 A P^i_{\alpha\sigma\tau} \] \[ X_3^{P_{4\sigma\tau}} = X_2^{P_{4\sigma\tau}} = X_4^{P_{4\sigma\tau}} = \] \[ X_5^{P_{4\sigma\tau}} = G_A \varepsilon_{A\sigma\tau} P^i_{\alpha\sigma\tau}, \]

\[ X_6^{P_{4\sigma\tau}} = -G_A \varepsilon_{A\sigma\tau} \left[ \delta_{\sigma\tau} P^i_{\sigma\tau} - (P^i_{\sigma\tau} + P^i_{\sigma\tau}) + \right] + iG_A \varepsilon_{ijkl} P^i_{\sigma\tau}, \]

\[ X_1^{P_{4\sigma\tau}} = G^2 A P^i_{\alpha\sigma\tau} \] \[ X_2^{P_{4\sigma\tau}} = X_3^{P_{4\sigma\tau}} = X_4^{P_{4\sigma\tau}} = \] \[ X_5^{P_{4\sigma\tau}} = G_A \varepsilon_{A\sigma\tau} P^i_{\alpha\sigma\tau}, \]

\[ X_6^{P_{4\sigma\tau}} = -G_A \varepsilon_{A\sigma\tau} \left[ \delta_{\sigma\tau} P^i_{\sigma\tau} - (P^i_{\sigma\tau} + P^i_{\sigma\tau}) + \right] + iG_A \varepsilon_{ijkl} P^i_{\sigma\tau}, \] (162)
with

\[ P_{\sigma^+} = \sigma^+_a P_{b^+} + \sigma^+_b P_{a^+}. \]  

(163)

Under the exchange of running indices \(a\) and \(b\), nuclear operators \(X\), electron currents \(E_+\) and \(F_+\) and neutrino potentials \(H_i\) and \(H_{\omega i}\) are even, while \(Y, E_-, F_-, \) and \(H_{ki}\) are odd.

New constants are defined as:

\[ \varepsilon_V = \frac{g_Y}{g_A} \left( \epsilon_{V+A}^{+} + \epsilon_{V+A}^{-} \right), \quad \varepsilon_A = \epsilon_{V+A}^{+} - \epsilon_{V-A}^{+}, \]  

(164)

The operators

\[ X_1, \; X_{1R}^1; \; Y_i^1, \; Y_{1R}^i; \]  

\[ X_3, \; X_{3R}^i, \; X_{4R}^{Pli}, \; r \cdot X_{5R}^i, \; r'X_{6R}^{ik}; \]  

\[ Y_4, \; Y_{6R}^i, \; r \cdot Y_{5R}^i, \; r'Y_{4R}^{ik}, \]  

(165)

have even parity and the operators

\[ X_{1R}; \; Y_{1R}^i, \; X_{5R}^i, \; X_{4R}^{Pli}, \; r \cdot X_{5R}^i, \; r'X_{5R}^{Pli}, \]  

\[ Y_{3R}, \; Y_{6R}^i, \; r \cdot Y_{5R}^i, \; r'Y_{6R}^{ik}, \; r'Y_{4R}^{ik}; \]  

(166)

have odd parity.

Using the definitions of the neutrino potentials from Eq. (131) and

\[ h_\omega = \frac{R^2}{2}(H_{\omega 1} + H_{\omega 2}) \]  

(167)

in the \( S - S \) case with no FBWC we have

\[ \{M_{V_A}^m\}_{n,S-S} = \int dx dy T_N h_+(X_1 + X_{1R}^P)E_+, \]  

(168)

\[ \{M_{V_A}^m\}_{c,S-S} = \frac{\varepsilon_{21} R}{2} \int dx dy T_N h_0(Y_{1R}^i + Y_{1R}^{Pli})E_+, \]  

(169)

\[ \{M_{V_A}^k\}_{n,S-S} = \frac{\varepsilon_{21}}{m_e} \int dx dy T_N h_0 \left[(X_3 + X_{5R}^{Pli})F_0^0 + (X_5^i + X_{4R}^{Pli})F_0^i\right] \]  

\[ + \frac{4}{m_e R} \int dx dy T_N \frac{i R}{2r} h_0' Y_{4R}^{ik} \left[ Y_{5R}^{Pli} F_{5+} + Y_{6R}^{Pli} F_{5+}\right], \]  

(170)

\[ \{M_{V_A}^k\}_{c,S-S} = \frac{2}{m_e R} \int dx dy T_N h_0(Y_{1R}^i + Y_{5R}^{Pli})F_0^i \]  

\[ + \frac{\varepsilon_{21}}{m_e} \int dx dy T_N \frac{i R}{r} h_0' Y_{4R}^{ik} \left( X_{3R}^{Pli} F_0^0 + X_{5R}^{Pli} F_0^i\right), \]  

(171)

where \(E\) and \(F\) are taken for \(x = y = 0\).

For the \(0^+ \rightarrow 0^+\) transition we have

\[ \sum_i \frac{m_i}{m_e} \sum_N \{M_{V_A}^m\}_{S-S} = g_A^2 \left\{ Z_1^X + Z_{1R}^{XP} \right\} E_+, \]  

(172)

\[ \sum_i \sum_N \{M_{V_A}^k\}_{S-S} = g_A^2 \left[ \frac{\varepsilon_{21}}{m_e} \left( Z_3^X + Z_{5R}^{XP} + \{Z_{3R}\}_{c} F_0^0 + \frac{4}{m_e R} Z_{4R}^{P0} F_{5+}\right) \right], \]  

(173)

with

\[ Z_1^X = \left( \frac{m_i}{m_e} h_+ X_1 \right), \quad Z_{1R}^{XP} = \left( \frac{m_i}{m_e} h_+ X_{1R}^P \right), \quad Z_3^X = \langle h_+ X_3 \rangle, \]  

\[ Z_{4R}^{Y} = \left( \frac{i R}{2r} h_0' \cdot Y_{5R} \right), \quad Z_{5R}^{XP} = \langle h_0 X_{5R}^P \rangle, \quad \{Z_{3R}\}_{c} = \left( \frac{i R}{r} h_0' \cdot X_{3R} \right). \]  

(174)
In the $S - P_{1/2}$ case with no FBWC for the $0^+ \to 0^+$ transition we have

$$\{M_{VA}^m\}_{n,S-P_{1/2}} = \int dx dy T_N h_+ Y_{1R}^i E^i, \quad (175)$$

$$\{M_{VA}^m\}_{c,S-P_{1/2}} = \frac{\varepsilon_{21} R}{2} \int dx dy T_N h_0 Y_{1R}^i E^i, \quad (176)$$

$$\{M_{VA}^k\}_{n,S-P_{1/2}} = \frac{\varepsilon_{21} m_e}{2} \int dx dy T_N h_{0\omega} (X_{4R}^i F^i_+ + Y_{6R}^i F^i_{-}) \quad + \frac{4}{m_e R} \int dx dy T_N \frac{i R}{2r} h_{0\omega} ^{(3)} \left[ (X_{4R}^{ik} + X_{4R}^{plk}) F^k_+ + (Y_{6R}^{ik} + Y_{6R}^{plk}) F^k_{-} \right], \quad (177)$$

$$\{M_{VA}^k\}_{c,S-P_{1/2}} = \frac{\varepsilon_{21} m_e}{2} \int dx dy T_N h_{0\omega} (X_{4R}^i F^i_+ + Y_{6R}^i F^i_{-}) \quad + \frac{\varepsilon_{21} m_e}{2} \int dx dy T_N \frac{i R}{r} h_{0\omega} ^{(3)} \left[ (X_{4R}^{ik} + X_{4R}^{plk}) F^k_+ + (Y_{6R}^{ik} + Y_{6R}^{plk}) F^k_{-} \right]. \quad (178)$$

The decay rate for the $0^+ \to 0^+$ transition takes the form

$$d\Gamma = \sum_{s_1,s_2} |R_{0\omega}|^2 \frac{m_0^5}{4\pi^2} d\Omega_{0\omega} = \frac{a_{0\omega}}{(m_e R)^2} \left[ A_{0VA}^V - \hat{p}_1 \cdot \hat{p}_2 B_{0VA}^V \right] d\Omega_{0\omega}, \quad (179)$$

where the coefficients are

$$A_{0VA}^V = \sum_{i=1}^4 |N_i|^2, \quad (180)$$

$$B_{0VA}^V = \text{Re}(N_1 N_2^* + N_3 N_3^* + N_4 N_4^*), \quad (181)$$

with

$$N_1 = \alpha_{-1} \left\{ \left[ Z_1^X + Z_{1R}^{XP} - \frac{4}{m_e R} Z_{1R}^Y \right] + \frac{m_e r}{6} \left( \left( \frac{\zeta}{m_e R} - 2 \right) Z_{1R}^Y + \frac{\varepsilon_{21} R}{2m_e} \left( Z_{1R}^Y \right) \right) + \frac{2}{3} \left( \frac{\zeta}{m_e R} - 2 \right) \left( Z_6^Y + Z_{4R}^{XP} + Z_{6R}^Y \right) \right\}, \quad (182)$$

$$N_2 = \alpha_{11} \left\{ \left[ Z_1^X + Z_{1R}^{XP} + \frac{4}{m_e R} Z_{1R}^Y \right] + \frac{m_e r}{6} \left( \left( \frac{\zeta}{m_e R} + 2 \right) Z_{1R}^Y + \frac{\varepsilon_{21} R}{2m_e} \left( Z_{1R}^Y \right) \right) + \frac{2}{3} \left( \frac{\zeta}{m_e R} + 2 \right) \left( Z_6^Y + Z_{4R}^{XP} + Z_{6R}^Y \right) \right\}, \quad (183)$$

$$N_3 = \alpha_{-1} \left\{ \left[ Z_1^X + Z_{1R}^{XP} - \frac{\varepsilon_{21}}{m_e} \left( Z_3^X + Z_{5R}^{XP} + Z_{3R}^Y \right) \right] + \frac{r}{6 R} \left( \zeta Z_{1R}^Y + \frac{1}{2} \varepsilon_{21} (\varepsilon_{21} + 2 m_e) R^2 Z_2^Y \right) + \frac{1}{3} \varepsilon_{21} \zeta \left( \frac{2}{3} \left( \{ Z_4^X \} + \{ Z_{6R}^X \} \right) \right) \right\}, \quad (184)$$

$$N_4 = \alpha_{-1} \left\{ \left[ Z_1^X + Z_{1R}^{XP} + \frac{\varepsilon_{21}}{m_e} \left( Z_3^X + Z_{5R}^{XP} + Z_{3R}^Y \right) \right] + \frac{r}{6 R} \left( \zeta Z_{1R}^Y + \frac{1}{2} \varepsilon_{21} (\varepsilon_{21} - 2 m_e) R^2 Z_2^Y \right) - \frac{1}{3} \varepsilon_{21} \zeta \left( \frac{2}{3} \left( \{ Z_4^X \} + \{ Z_{6R}^X \} \right) \right) \right\}, \quad (185)$$

where the terms in the first brackets in Eqs. (182)–(185) come from the $S - S$ case and the terms in the second ones come from the $S - P_{1/2}$ case. The terms in the third brackets in Eqs. (184)–(185) are the most
important terms of those that come from the $p/m$ factor
and the fact that $Z$ agrees with the Eq. (C.3.7) of Ref. \[5\] taking into account the correspondence with their notations:

$$m_i \frac{i}{m_c} 2R h_0 r \cdot Y_{1R}, \quad \{Z^Y_{1R}\}_c = (\frac{m_i}{m_c} i \frac{i}{m_c} h_0 r \cdot Y_{1R}),$$

$$Z_6^Y = (\frac{1}{2r} h^t_{0*}(2i)_{4}^p Y_{6}^q), \quad Z_4^{YP} = (\frac{1}{2r} h^t_{0*}(2i)_{4}^p Y_{4}^p), \quad \{Z^Y_{6R}\}_c = (\frac{1}{2r} h^t_{0*} r \cdot Y_{6R}),$$

$$Z_6^X = (\frac{i}{2R} h_0 r \cdot Y_{6R}), \quad \{Z^X_6\}_c = (\frac{1}{2r} h^t_{0*}(2i)_{4}^p Y_{6}^q), \quad Z_4^X = (\frac{i}{2R} h_0 r \cdot X_{4R}), \quad \{Z^X_4\}_c = (\frac{1}{2r} h^t_{0*}(2i)_{4}^p X_{4}^p),$$

$$Z_4^X = (\frac{i}{2R} h_0 r \cdot X_{4R}), \quad \{Z^X_4\}_c = (\frac{1}{2r} h^t_{0*}(2i)_{4}^p X_{4}^p), \quad Z_5^X = (\frac{i}{2R} h_0 r \cdot X_{5R}), \quad Z_{5R}^X = (\frac{i}{2R} h_0 r \cdot X_{5R}).$$  \[(186)\]

The dominant terms give

$$N_1 = \alpha_{1-1}^* \left\{ \left[ Z_1^X - \frac{4}{m_c R} Z_4^X \right] + \left[ \frac{2}{3} \left( \frac{\zeta}{m_c R} - 2 \right) Z_6^X \right] \right\}$$

$$N_2 = \alpha_{11}^* \left\{ \left[ Z_1^X - \frac{4}{m_c R} Z_4^X \right] + \left[ \frac{2}{3} \left( \frac{\zeta}{m_c R} + 2 \right) Z_6^X \right] \right\}$$

$$N_3 = \alpha_{1-1}^* \left\{ \left[ Z_1^X - \frac{2z}{m_c} Z_3^X \right] + \left[ \frac{1}{3} \left( \frac{z}{m_c} + 2 \right) Z_4^X \right] \right\}$$

$$N_4 = \alpha_{-11}^* \left\{ \left[ Z_1^X - \frac{2z}{m_c} Z_3^X \right] + \left[ \frac{1}{3} \left( \frac{z}{m_c} - 2 \right) Z_4^X \right] \right\}$$

that agrees with the Eq. (C.3.7) of Ref. \[5\] taking into account the correspondence with their notations:

$$Z_1^X = Z_1, \quad Z_3^X = Z_3, \quad Z_6^Y = Z_6,$$

$$Z_4^X = Z_4^R, \quad Z_4^X = Z_5^R, \quad Z_4^X = Z_5^R,$$  \[(191)\]

and the fact that $Z_2$ is absent, as we have calculated only the leading contribution of the parameters $c_0^4$. Recall that in Ref. \[5\] the pseudoscalar form factor is not taken into account. However the terms associated with this form factor do not contribute to the dominant terms \[(187)\] - \[(190)\]. Note that in the expressions for $N_1$ and $N_2$ given above, the terms with $\zeta$ are due to the inclusion of the $P$-wave in the electron wave function and the ones with $Z_1^Y$ are due to the nucleon recoil effect. We remark that some of the subdominant terms, like those with $Z_4^{YP}$, $Z_4^X$, $Z_6^X$, $Z_5^X$ and $Z_5^X$, should be taken into account in the case of large cancellation amongst the dominant terms. The same is valid for the contribution due to the pseudoscalar form factor $g_A T_0^4$, which yields corrections at about 10% to the dominant terms.

### C $0\nu2\beta$ decay rate for tensor nonstandard terms

The nucleon currents in the impulse approximation up to order $p/m_p$ in the nonrelativistic expansion are used \[(32)\] \[(34)\], $J_{\mu}^{p+}$, from Eq. \[(100)\] and

$$J_{TM}^{\mu+}(x) = T_{1}^{(3)} \sum_a \sigma^{\mu} \delta(x - r_a) \left\{ (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\sigma} g^{\nu\rho}) T_a^{k} + g^{\mu\nu} g^{\rho\sigma} \varepsilon_{kmn} \sigma_{a}^{k} \right\}$$

$$+ \frac{i}{2} \varepsilon^{\mu
u\rho\sigma} \left\{ (g_{\rho\sigma} g_{\mu\nu} - g_{\mu\sigma} g_{\nu\rho}) T_a^{k} + g_{\mu\nu} g_{\sigma\rho} \varepsilon_{kmn} \sigma_{a}^{k} \right\}$$

$$T_{a}^{k} = \left( i \left( T_{a}^{(3)} \right)^{(3)} - 2 T_{2}^{(3)} \right) q^{k} I_{a} + T_{1}^{(3)} [q_a \times Q_{a}] / (2 T_{1}^{(3)} m_p),$$  \[(192)\] \[(193)\]

where, as before, $q^{\mu} = p^{\mu} - p'^{\mu}$ is the 4-momentum transferred from hadrons to leptons, $Q^{\mu} = p^{\mu} + p'^{\mu}$, $p^{\mu}$ and $p'^{\mu}$ are the initial and final 4-momenta of a nucleon. We neglect the dipole dependence of the form factors $T_{a}^{(3)}$ and $T_{2}^{(3)}$ on the momentum transfer and omit the zero argument of the form factors.
Consider the pure $T_{L,R}$ case assuming $\langle m \rangle = 0$. In terms of the hadronic currents

$$J_{L,R}^{\mu\nu} = (F|j_{L,R}^{\mu\nu}\rangle \langle N|j_{L,R}^{\mu\nu}|I\rangle, \quad J_{L,R}^{\mu\nu L} = (F|j_{L,R}^{\mu\nu}|N\rangle\langle N|j_{L,R}^{\mu\nu}|I\rangle),$$

(194)

$$\hat{j}_{L,R}^{\mu\nu} = c_{L,R}^{\nu} j_{L,R}^{\mu\nu} + c_{L,R}^{\mu} j_{L,R}^{\mu\nu}, \quad \hat{j}_{L,R}^{\mu\nu L} = c_{L,R}^{\nu} j_{L,R}^{\mu\nu L} + c_{L,R}^{\mu} j_{L,R}^{\mu\nu L},$$

(195)

$$\tilde{j}_{L}^{\mu\nu} = U_{e\nu} j_{L}^{\mu\nu},$$

(196)

and the leptonic tensors

$$t_{1}^{\alpha\mu\nu}(2y, 1x) = \frac{1}{\omega + A_{1}} \frac{t_{1}^{\alpha\mu\nu}(1y, 2x)}{\omega + A_{2}},$$

(197)

$$t_{2}^{\alpha\mu\nu}(2y, 1x) = \frac{1}{\omega + A_{1}} \frac{t_{2}^{\alpha\mu\nu}(1y, 2x)}{\omega + A_{2}},$$

(198)

$$t_{3}^{\alpha\mu\nu}(2y, 1x) = \frac{1}{\omega + A_{1}} \frac{t_{3}^{\alpha\mu\nu}(1y, 2x)}{\omega + A_{2}},$$

(199)

$$t_{4}^{\alpha\mu\nu}(2y, 1x) = \frac{1}{\omega + A_{1}} \frac{t_{4}^{\alpha\mu\nu}(1y, 2x)}{\omega + A_{2}},$$

(200)

with the electron currents defined as

$$t_{1}^{\alpha\mu\nu}(2y, 1x) = \bar{\epsilon}_{2}(y) \gamma_{\alpha}(1 - \gamma_{5}) \sigma_{\mu\nu} \epsilon_{5}\gamma_{1}(x),$$

$$t_{2}^{\alpha\mu\nu}(2y, 1x) = \bar{\epsilon}_{2}(y) \gamma_{\alpha}(1 - \gamma_{5}) \gamma_{\lambda} \sigma_{\mu\nu} \epsilon_{1}\gamma_{1}(x),$$

$$t_{3}^{\alpha\mu\nu}(2y, 1x) = \bar{\epsilon}_{2}(y) \gamma_{\alpha}(1 - \gamma_{5}) \gamma_{\lambda} \sigma_{\mu\nu} \epsilon_{1}\gamma_{1}(x),$$

$$t_{4}^{\alpha\mu\nu}(2y, 1x) = \bar{\epsilon}_{2}(y) \gamma_{\alpha}(1 - \gamma_{5}) \gamma_{\lambda} \sigma_{\mu\nu} \epsilon_{1}\gamma_{1}(x),$$

(201)

the matrix element is expressed as

$$R_{0\nu}^{T} = \frac{1}{\sqrt{2}} \left( G_{F} |V_{ud}| \right) \times \int d\mathbf{x} dy \frac{d\mathbf{k}}{(2\pi)^{3}} \frac{e^{i\mathbf{k} \cdot \mathbf{r}}}{2\omega} \sum_{\alpha,\nu} \left[ m_{i} \left( J_{L,R}^{\mu\nu} t_{1}^{\alpha\mu\nu} + J_{L,R}^{\mu\nu L} t_{2}^{\alpha\mu\nu} \right) + k^{4} \left( J_{L,R}^{\mu\nu} t_{3}^{\alpha\mu\nu} + J_{L,R}^{\mu\nu L} t_{4}^{\alpha\mu\nu} \right) \right].$$

(202)

For the electron currents we have the identities

$$t_{1}^{\alpha\mu\nu}(1y, 2x) = -t_{2}^{\mu\nu\alpha}(2y, 1x),$$

$$t_{1}^{\alpha\mu\nu}(1y, 2x) = t_{2}^{\mu\nu\alpha}(2y, 1x).$$

(203)

Using Eqs (110), (111), and (112), the matrix element (202) is expressed as

$$R_{0\nu}^{T} = C_{0\nu} \sum_{i} \sum_{N} \left( \frac{m_{i}}{m_{e}} M_{T}^{m} + M_{T}^{c} \right),$$

(204)

$$M_{T}^{m,k} = \{ M_{T}^{m,k} \}_{n} + \{ M_{T}^{m,k} \}_{c},$$

(205)

with nonvanishing ($n$) and vanishing ($c$) in the closure approximation parts:

$$\{ M_{T}^{m} \}_{n} = R \int dx dy T_{N}(H_{1} + H_{2}) \times \left[ (U_{1} + \bar{U}_{1} R) F_{5}^{0} \right] + \left[ (U_{3} + \bar{U}_{3} R) F_{5}^{0} \right] + \left[ (V_{3} + \bar{V}_{3} R) F_{5}^{0} \right],$$

(206)

$$\{ M_{T}^{m} \}_{c} = R \int dx dy T_{N}(H_{1} - H_{2}) \times \left[ (U_{1} + \bar{U}_{1} R) F_{5}^{0} \right] + \left[ (U_{3} + \bar{U}_{3} R) F_{5}^{0} \right] + \left[ (V_{3} + \bar{V}_{3} R) F_{5}^{0} \right],$$

(207)
\begin{align}
\{M^k_L\}_n &= \frac{R}{m_e} \int dxdyT_N(H_{\omega 1} - H_{\omega 2}) \left[ \tilde{V}_{2RE_+} + (U_4^i + \tilde{U}_{4R}^i)F^{0i} + (U_6^i + \tilde{U}_{6R}^i)F^{ij} \right] \\
&+ (H_{i1}^i + H_{i2}^i) \left[ (V_4^i + \bar{V}_{4R}^i)E_+ + (U_2^i + \tilde{U}_{2R}^i)F^{0i} + (U_5^i + \tilde{U}_{5R}^i)F^{ij} + (U_7^i + \tilde{U}_{7R}^i)F^{0j} \right] \\
&+ (U_8^{ij} + \tilde{U}_{8R}^{ij})F^{-j} \right], \\
\{M^k_L\}_c &= \frac{R}{m_e} \int dxdyT_N(H_{\omega 1} + H_{\omega 2}) \left[ \tilde{V}_{2RE_+} + (U_4^i + \tilde{U}_{4R}^i)F^{0i} + (U_6^i + \tilde{U}_{6R}^i)F^{ij} \right] \\
&+ (H_{i1}^i - H_{i2}^i) \left[ (V_4^i + \bar{V}_{4R}^i)E_+ + (U_2^i + \tilde{U}_{2R}^i)F^{0i} + (U_5^i + \tilde{U}_{5R}^i)F^{ij} + (U_7^i + \tilde{U}_{7R}^i)F^{0j} \right] \\
&+ (U_8^{ij} + \tilde{U}_{8R}^{ij})F^{-j} \right],
\end{align}

(208)

where the nucleon operators are

\begin{align}
\tilde{U} &= U + U^P, \quad \tilde{V} = V + V^P,
\end{align}

(210)

\begin{align}
U_1 &= -2G_A^0(\varepsilon_{T1} + \varepsilon_{T2})\sigma_{a\sigma b}, \quad U_{1R}^P = G_A^0(\varepsilon_{T1} + \varepsilon_{T2})P^{a\sigma b}, \\
U_2 &= 2iG_A^0(\varepsilon_{T1}' + \varepsilon_{T2}')\sigma_{a\sigma b}, \quad U_{2R}^P = -iG_A^0(\varepsilon_{T1}' + \varepsilon_{T2}')P^{a\sigma b}, \\
U_3 &= -G_A^0(\varepsilon_{T1} + \varepsilon_{T2})\sigma_5^+, \quad U_{3R}^P = -iG_A^0(\varepsilon_{T1} + \varepsilon_{T2})\varepsilon_{ijk}P^{ijk}, \\
U_5 &= -iG_A^0(\varepsilon_{T1} + \varepsilon_{T2})C_{a\sigma b}^+ - iG_A^0(\varepsilon_{T1} + \varepsilon_{T2})\varepsilon_{ijk}D^{ijk}, \\
U_7 &= -iG_A^0(\varepsilon_{T1}' + \varepsilon_{T2}')\sigma_5^+, \quad U_{7R}^P = -iG_A^0(\varepsilon_{T1}' + \varepsilon_{T2}')\varepsilon_{ijk}P^{ijk}, \\
U_6 &= -iG_A^0(\varepsilon_{T1}' + \varepsilon_{T2}')C_{a\sigma b}^+ + G_A^0(\varepsilon_{T1}' + \varepsilon_{T2}')\varepsilon_{ijk}D^{ijk}, \\
U_8 &= -iG_A^0(\varepsilon_{T1}' + \varepsilon_{T2}')C_{a\sigma b}^+ - G_A^0(\varepsilon_{T1}' + \varepsilon_{T2}')\varepsilon_{ijk}D^{ijk},
\end{align}

(211)
The new constants are defined as:

\[ a_{\alpha} = \left( T_{\alpha}^{i} \right)^{T}, \quad V_{\alpha} = \left( T_{\alpha}^{i} \right)^{T}, \quad V = \left( T_{\alpha}^{i} \right)^{T}, \quad \frac{1}{2} \mathbf{V}_{\alpha} = \left( T_{\alpha}^{i} \right)^{T}, \quad \mathbf{F} = \mathbf{V}_{\alpha} \]

with

\[ T_{\alpha}^{i} = T_{\alpha}^{i} \pm i \epsilon_{\alpha} T_{\alpha}^{i}, \quad T_{\alpha}^{i \pm} = \frac{1}{2} \left( T_{\alpha}^{i} \pm i \epsilon_{\alpha} T_{\alpha}^{i} \right), \]

\[ \frac{1}{2} \mathbf{V}_{\alpha} = \mathbf{V}_{\alpha} \pm i \epsilon_{\alpha} \mathbf{V}_{\alpha}, \quad \frac{1}{2} \mathbf{F} = \mathbf{F} \pm i \epsilon_{\alpha} \mathbf{F}, \]

Under the exchange of indices \( a \) and \( b \), nuclear operators \( U \), electron currents \( F_{+} \) and neutrino potentials \( H_{i} \) and \( H_{\beta} \) are even, while \( V \), \( F_{-} \), and \( H_{\beta} \) are odd.

The new constants are defined as:

\[ \varepsilon_{T_{1}} = \frac{T_{1}^{(3)}}{g_{A}} \left( \epsilon_{T_{L_{1}}} + \epsilon_{T_{R_{1}}} \right), \quad \varepsilon_{T_{2}} = \frac{T_{1}^{(3)}}{g_{A}} \left( \epsilon_{T_{L_{2}}} - \epsilon_{T_{R_{2}}} \right), \]

\[ \varepsilon_{T_{1}} = \frac{T_{1}^{(3)}}{g_{A}} \left( \epsilon_{T_{L_{1}}} + \epsilon_{T_{R_{1}}} \right), \quad \varepsilon_{T_{2}} = \frac{T_{1}^{(3)}}{g_{A}} \left( \epsilon_{T_{L_{2}}} - \epsilon_{T_{R_{2}}} \right). \]

The even parity operators are

\[ U_{1}, U_{1}^{P_{1}}, k^{i}U_{2R}, U_{3}^{i}, U_{3}^{P_{1}}, U_{4}^{i}, U_{4}^{P_{1}}, U_{4}^{P_{1}}; \]

and the odd parity operators are

\[ k^{i}U_{2}^{P_{1}}, k^{i}U_{2R}^{P_{1}}, U_{3}^{i}, U_{3}^{P_{1}}, U_{4}^{i}, U_{4}^{P_{1}}, U_{4}^{P_{1}}; \]

Using the definitions of the neutrino potentials from Eqs. (210) and (211), in the \( S - S \) case with no FBWC we have

\[ \{ M_{T_{1}} \}_{n,S-S} = 2 \int dx dy T_{N} h_{+} \left[ \left( U_{1} + U_{1}^{P_{1}} \right) F_{5}^{0} + \left( U_{3} + U_{3}^{P_{1}} \right) F_{5}^{0} \right], \]

\[ \{ M_{T_{1}} \}_{e,S-S} = \varepsilon_{21} R \int dx dy T_{N} h_{0} \left[ V_{1}^{P_{1}} F_{5}^{0} + \left( V_{3} + V_{3}^{P_{1}} \right) F_{5}^{0} \right], \]

\[ \{ M_{T_{1}} \}_{n,S-S} = \frac{\varepsilon_{21}}{m_{e}} \int dx dy T_{N} h_{0} \left[ \left( U_{1} + U_{1}^{P_{1}} \right) F_{5}^{0} + \left( U_{3} + U_{3}^{P_{1}} \right) F_{5}^{0} \right], \]

where \( E \) and \( F \) are taken for \( x = y = 0 \).
For the $0^+ \to 0^+$ transition we have

$$
\sum_{i} \frac{m_i}{m_e} \sum_{N} \{ M_{T}^{i} \}_{S-S} = \frac{g_{A}^{2}}{m_{e}^{2}} \left[ 2(W_{1}^{U} + W_{1R}^{U})F_{5}^{0} + \varepsilon_{21} R (W_{1R}^{V} \cdot F_{4}^{0}) \right],
$$

(221)

$$
\sum_{i} \sum_{N} \{ M_{T}^{i} \}_{S-S} = \frac{g_{A}^{2}}{m_{e} R} (2W_{4R}^{V} + \{ W_{2R}^{V} \}_{c}) E_{+},
$$

(222)

with

$$
W_{1}^{U} = \left( \frac{m_{i}}{m_{e}} h_{+} U_{1} \right), \quad W_{1R}^{U} = \left( \frac{m_{i}}{m_{e}} b_{0} U_{1R} \right), \quad \{ W_{1R}^{V} \}_{c} = \left( \frac{m_{i}}{m_{e}} b_{0} V_{1R} \right),
$$

$$
W_{4R}^{V} = (\frac{R}{2r}) h_{+} \cdot V_{4R}, \quad \{ W_{2R}^{V} \}_{c} = (h_{+} V_{2R}).
$$

(223)

In the $S-P_{1/2}$ case with no FBWC for the $0^+ \to 0^+$ transition we have

$$
\{ M_{T}^{i} \}_{n, S-P_{1/2}} = 2 \int dxdy T_{N} h_{+} (U_{3R}^{i} F_{5}^{i} + V_{3R}^{i} F_{-}^{i}),
$$

(244)

$$
\{ M_{T}^{i} \}_{c, S-P_{1/2}} = \frac{\varepsilon_{21} R}{m_{e}} \int dxdy T_{N} h_{0} (U_{3R}^{i} F_{5}^{i} + V_{3R}^{i} F_{-}^{i}),
$$

(225)

$$
\{ M_{T}^{i} \}_{n, S-P_{1/2}} = \frac{\varepsilon_{21}}{m_{e}} \int dxdy T_{N} h_{0} U_{4R}^{i} F_{5}^{i} + \frac{4}{m_{e} R} \int dxdy T_{N} h_{+} U_{4R}^{i} F_{5}^{i} \left[ (U_{2} + U_{2R}) F_{0}^{i} + (U_{7} + U_{7R}) F_{0}^{i} \right],
$$

(226)

$$
\{ M_{T}^{i} \}_{c, S-P_{1/2}} = \frac{\varepsilon_{21}}{m_{e}} \int dxdy T_{N} h_{+} U_{4R}^{i} F_{5}^{i} + \frac{4}{m_{e} R} \int dxdy T_{N} h_{+} U_{4R}^{i} F_{5}^{i} \left[ (U_{2} + U_{2R}) F_{0}^{i} + (U_{7} + U_{7R}) F_{0}^{i} \right].
$$

(227)

The decay rate for the $0^+ \to 0^+$ transition takes the form

$$
d\Gamma = \sum_{s_{1}, s_{2}} |R_{0v}|^{2} \frac{m_{c}^{5}}{4 \pi^{3}} d\Omega_{0v} = \frac{a_{0v}}{(m_{c} R)^{2}} \left[ A_{0}^{T} \cdot \hat{p}_{1} \cdot \hat{p}_{2} B_{0}^{T} \right] d\Omega_{0v},
$$

(228)

where the coefficients are

$$
A_{0}^{T} = \sum_{i=1}^{4} |O_{i}|^{2},
$$

(229)

$$
B_{0}^{T} = \text{Re}(O_{1} O_{2}^{*} + O_{1}^{*} O_{2} + O_{3} O_{4}^{*} + O_{3}^{*} O_{4}),
$$

(230)

with

$$
O_{1} = \alpha_{1-1}^{*} \left\{ \left[ -2(W_{1}^{U} + W_{1R}^{U}) + 2 \frac{m_{c} R}{m_{e}} (W_{1R}^{V} + \{ W_{2R}^{V} \}_{c}) \right] + \left[ \frac{m_{c} R}{3} \left( \frac{\zeta}{m_{c} R} - 2 \right) W_{3R}^{U} + \frac{2}{m_{e} R} \left( 3(\alpha Z)^{2} \right) \left( W_{4R}^{V} + \frac{1}{2} \{ W_{2R}^{V} \}_{c} \right) \right] \right\},
$$

(231)

$$
O_{2} = \alpha_{11}^{*} \left\{ \left[ 2(W_{1}^{U} + W_{1R}^{U}) + \frac{2}{m_{e} R} (W_{4R}^{V} + \{ W_{2R}^{V} \}_{c}) \right] + \left[ -\frac{m_{c} R}{3} \left( \frac{\zeta}{m_{c} R} - 2 \right) W_{3R}^{U} + \frac{2}{m_{e} R} \left( 3(\alpha Z)^{2} \right) \left( W_{4R}^{V} + \frac{1}{2} \{ W_{2R}^{V} \}_{c} \right) \right] \right\},
$$

(232)

$$
O_{3} = \alpha_{1-1}^{*} \left\{ \left[ -\varepsilon_{21} R (W_{1R}^{V})_{c} + \frac{2}{m_{e} R} (W_{3R}^{V} + \{ W_{4R}^{V} \}_{c}) \right] + \left[ \frac{m_{c} R}{3} \left( \frac{\varepsilon_{21} R}{m_{c} R} + 2 \right) W_{3R}^{V} + \frac{2}{m_{e} R} \left( 3(\alpha Z)^{2} \right) \left( W_{4R}^{V} + \frac{1}{2} \{ W_{2R}^{V} \}_{c} \right) \right] \right\},
$$

(233)
\[
+ \frac{\varepsilon_{21} R}{3} \left( \frac{\varepsilon_{21}}{m_e} + 2 \right) \left( W_{4R}^U - \{ W_{2R}^U \}_{c} - \{ W_{7}^U \}_{c} - \{ W_{2R}^P \}_{c} - \{ W_{7R}^P \}_{c} \right) \\
- \frac{4}{3} \frac{\zeta}{m_e R} \left( W_{2}^U + W_{7}^U + W_{2R}^P + W_{7R}^P - \frac{1}{2} \{ W_{4R}^U \}_{c} \right) + \left[ - \frac{3(\alpha Z)^2}{m_e R} \left( W_{4R}^V + \frac{1}{2} \{ W_{2R}^P \}_{c} \right) \right] \right), \tag{233}
\]
\[
O_4 = \alpha_{-11}^{*} \left\{ \left[ \varepsilon_{21} R \{ W_{1R}^V \}_{c} + \frac{2}{m_e R} \left( W_{4R}^V + \{ W_{2R}^P \}_{c} \right) \right] \\
+ \left[ - \frac{m_e R}{3} \left( \frac{\varepsilon_{21}}{m_e} - 2 \right) W_{3R}^V - \zeta \frac{\varepsilon_{21} R}{6} \{ W_{3R}^V \}_{c} \right. \right. \\
+ \left[ \frac{\varepsilon_{21} R}{3} \left( \frac{\varepsilon_{21}}{m_e} - 2 \right) \left( W_{4R}^U - \{ W_{2}^U \}_{c} - \{ W_{7}^U \}_{c} - \{ W_{2R}^P \}_{c} - \{ W_{7R}^P \}_{c} \right) \\
- \frac{4}{3} \frac{\zeta}{m_e R} \left( W_{2}^U + W_{7}^U + W_{2R}^P + W_{7R}^P - \frac{1}{2} \{ W_{4R}^U \}_{c} \right) + \left[ - \frac{3(\alpha Z)^2}{m_e R} \left( W_{4R}^V + \frac{1}{2} \{ W_{2R}^P \}_{c} \right) \right] \right\}, \tag{234}
\]
where the terms in the first brackets in Eqs. \ref{231}–\ref{233} come from the \( S - S \) case and the terms in the second ones come from the \( S - P_{1/2} \) case. The terms in the third brackets in Eqs. \ref{231}–\ref{232} are the most important terms of those that come from the \( S - S \) case due to FBWC. Note that in the \( S - S \) case there is the contribution to Eqs. \ref{231} and \ref{232} from the \( (H_{\alpha 1} + H_{\alpha 2}) \) combination in Eq. \ref{209}. Therefore the contribution from the \( P_{1/2} - P_{1/2} \) case should not be taken into account.

The nuclear matrix elements are
\[
W_{3R}^U = \left\langle \frac{m_i}{m_e} i h + \hat{r} \cdot \hat{U}_{3R} \right\rangle, \quad \{ W_{3R}^U \}_{c} = \left\langle \frac{m_i}{m_e} i h_0 \hat{r} \cdot \hat{U}_{3R} \right\rangle, \\
W_{3R}^V = \left\langle \frac{m_i}{m_e} i h + \hat{r} \cdot \hat{V}_{3R} \right\rangle, \quad \{ W_{3R}^V \}_{c} = \left\langle \frac{m_i}{m_e} i h_0 \hat{r} \cdot \hat{V}_{3R} \right\rangle, \\
W_{4R}^U = \langle i h_0 \hat{r} \cdot \hat{U}_{4R} \rangle, \\
\{ W_{2}^U \}_{c} = \left\langle \frac{R}{r} h_0 \hat{r} \cdot \hat{U}_{2} \right\rangle, \quad \{ W_{2R}^P \}_{c} = \left\langle \frac{R}{r} h_0 \hat{r} \cdot \hat{U}_{2R} \right\rangle, \quad \{ W_{2R}^P \}_{c} = \left\langle \frac{R}{r} h_0 \hat{r} \cdot \hat{U}_{2R} \right\rangle, \\
\{ W_{7}^U \}_{c} = \left\langle \frac{R}{r} h_0 \hat{r} \cdot \hat{U}_{7} \right\rangle, \quad \{ W_{7R}^P \}_{c} = \left\langle \frac{R}{r} h_0 \hat{r} \cdot \hat{U}_{7R} \right\rangle. \tag{235}
\]
Assuming now \( \langle m \rangle \neq 0 \) for the dominant terms we have
\[
O_1 = \alpha_{-11}^{*} \left\{ \left[ Z_1^X - 2 \left( \frac{m_i}{m_e} \right) i h + \hat{r} \cdot \hat{U}_{4R} \right] \left\{ W_{4R}^U + \{ W_{2R}^P \}_{c} \right\} \right\}, \tag{236}
\]
\[
O_2 = \alpha_{11}^{*} \left\{ \left[ Z_1^X + 2 \left( \frac{m_i}{m_e} \right) i h + \hat{r} \cdot \hat{V}_{4R} \right] \left\{ W_{4R}^U + \{ W_{2R}^P \}_{c} \right\} \right\}, \tag{237}
\]
\[
O_3 = \alpha_{-11}^{*} \left\{ \left[ Z_1^X + 2 \left( \frac{m_i}{m_e} \right) i h + \hat{r} \cdot \hat{U}_{4R} \right] \left\{ W_{4R}^U + \{ W_{2R}^P \}_{c} \right\} \right\} + \left[ \frac{\varepsilon_{21} R}{3} \left( \frac{\varepsilon_{21}}{m_e} + 2 \right) \left( W_{4R}^U - \{ W_{2}^U \}_{c} - \{ W_{7}^U \}_{c} - \{ W_{2R}^P \}_{c} - \{ W_{7R}^P \}_{c} \right) \\
- \frac{4}{3} \frac{\zeta}{m_e R} \left( W_{4R}^U - \{ W_{2}^U \}_{c} - \{ W_{7}^U \}_{c} - \{ W_{2R}^P \}_{c} - \{ W_{7R}^P \}_{c} \right) \right\}, \tag{238}
\]
\[
O_4 = \alpha_{-11}^{*} \left\{ \left[ Z_1^X + 2 \left( \frac{m_i}{m_e} \right) i h + \hat{r} \cdot \hat{U}_{4R} \right] \left\{ W_{4R}^U + \{ W_{2R}^P \}_{c} \right\} \right\} + \left[ \frac{\varepsilon_{21} R}{3} \left( \frac{\varepsilon_{21}}{m_e} - 2 \right) \left( W_{4R}^U - \{ W_{2}^U \}_{c} - \{ W_{7}^U \}_{c} - \{ W_{2R}^P \}_{c} - \{ W_{7R}^P \}_{c} \right) \\
- \frac{4}{3} \frac{\zeta}{m_e R} \left( W_{4R}^U - \{ W_{2}^U \}_{c} - \{ W_{7}^U \}_{c} - \{ W_{2R}^P \}_{c} - \{ W_{7R}^P \}_{c} \right) \right\}. \tag{239}
\]
Again, in the above expressions, the terms with \( \zeta \) are due to the inclusion of the \( P \)-wave in the electron wave function and the ones with \( W_{2R}^P \) and \( W_{4R}^P \) \( (X = U, V) \) are due to the nucleon recoil effect.
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Figure 1: Correlation between the neutrino effective mass $|\langle m \rangle|$ (left) $| |\epsilon^{V+\bar{A}}| \rangle$ (right), the angular correlation coefficient $K$, and the half-life $T_{1/2}$ for the $0\nu2\beta$ decay of $^{76}$Ge for the case $\cos \psi_1 = 0$.

Figure 2: Correlation between the neutrino effective mass $|\langle m \rangle|$ (left) $| |\epsilon^{V-A}_{+}| \rangle$ (right), the angular correlation coefficient $K$, and the half-life $T_{1/2}$ for the $0\nu2\beta$ decay of $^{76}$Ge for the case $\cos \psi_1 = 0$. 
Figure 3: Correlation between the right-handed $W$-boson mass $m_{W_R}$, the angular correlation coefficient $K$, and the half-life $T_{1/2}$ for the $0\nu2\beta$ decay of $^{76}$Ge for the case $\cos \psi_1 = 0$ and $\epsilon = 10^{-6}$ (left) and for $\epsilon = 5 \times 10^{-7}$ (right).

Figure 4: Correlation between the mixing parameter $\zeta$, the angular correlation coefficient $K$, and the half-life $T_{1/2}$ for the $0\nu2\beta$ decay of $^{76}$Ge for the case $\cos \psi_1 = 0$ and $\epsilon = 10^{-6}$ (left) and for $\epsilon = 5 \times 10^{-7}$ (right).
Figure 5: Left: Differential width in $\cos \theta$ for the $0\nu\beta\beta$ decay of $^{76}$Ge for a fixed value of $\epsilon = 10^{-6}$ and $|\langle m \rangle| = 20, 30$ meV. The straight and dotted lines correspond to $m_{W_R} = 1$ TeV, $\infty$, respectively (the latter is the conventional case of the light Majorana neutrino exchange mechanism). Right: The same as the left figure but for smaller values of $|\langle m \rangle| = 5, 10$ meV. In addition, the dashed lines correspond to $m_{W_R} = 1.5$ TeV.