PRICING AND COORDINATION OF COMPETITIVE RECYCLING AND REMANUFACTURING SUPPLY CHAIN CONSIDERING THE QUALITY OF RECYCLED PRODUCTS

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Abstract. Considering the quality of recycled products, we develop a game model of a multi-level competitive recycling and remanufacturing supply chain with two manufacturers and multiple recyclers. Being focus on two mainstream game models, namely the manufacturer-recycler cooperation game model and the manufacturer-led Stackelberg game model, we explore the connection between optimal pricing decisions and performance levels of the supply chain members. Although researches indicate that the quality of recycled products will not affect the pricing decisions in the forward supply chain, it is positively related to the recycling price, the repurchase price, and the overall profit in the reverse supply chain, and the intensity of competition among manufacturers or recycled products will affect the pricing decisions and the performance levels of the two models. In the manufacturer-led Stackelberg game model, the supply chain does not reach the Pareto optimum, which uses the recycling cost sharing contract to achieve the coordination. Afterwards, the profits of the two manufacturers and multiple recyclers in the supply chain are increased, and the overall profit of the supply chain system is higher than that of the manufacturer-led Stackelberg game model. Finally, numerical analysis is conducted to verify the proposed coordination mechanism and its effectiveness.

1. Introduction. With the focus on the increasingly depleted resources, energy shortage and deteriorating ecological environment, countries around the world is exploring the circular economy and the green sustainable development [8, 1, 30]. The recycling and remanufacturing activities of used products can save the energy consumption, and reduce the consumption of raw materials and environmental pollution of waste disposal [22]. On the another hand, it can bring competitiveness to a manufacturing enterprises such as less costs and more profits, which is helpful to establish a green manufacturing image. Compared with manufacturing new products, remanufactured products have lower cost and lower environmental influence.

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Therefore, the recycling and remanufacturing industry has become an important way to realize a circular economy and a green sustainable development. The reasonable pricing and coordination strategies for the used products in recycling and remanufacturing supply chain can improve the efficiency and the benefit of the supply chain management, as well the scale and the efficiency of the entire manufacturing and remanufacturing industry. The open issue of pricing and coordination in the recycling and remanufacturing supply chain has attracted attention from academia, enterprises, and governments.

Many scholars develop pricing models from different perspectives, such as corporate social responsibility, consumer environmental preference, emission tax, government subsidies, service level, and horizontal fairness concern behavior. However, these research works are mainly based on the relatively simple market structure with only one of the manufacturers, the retailers, the recyclers, and the product. They rarely consider the complex market structure with the multiple manufacturers and the multiple recyclers. Along with the manufacturing and remanufacturing industry tends to maturate, the products in the market become homogeneous, and the substitutability of products leads to more fierce competition of the intra industry such as Midea and Gree. Therefore, a competitive decision-making problem exists among members of the closed-loop supply chain when it comes to multiple manufacturers and multiple recyclers. Due to the degree of product substitution and the intensity of competition, it has different effects on the pricing decision and the performance level in the recycling and remanufacturing supply chain. Establishing and optimizing the supply chain competition strategy is helpful to improve the competition of the supply chain, it is also an important support for achieving the goals of the market development.

The quality factors of the recycled products will directly affect the recycling and repurchase prices, remanufacturing costs, closed-loop supply chain node corporate profits, and the recycling rates, and the remanufacturing rate is an example in the recycling and remanufacturing process. Recycling used products with low quality may cost companies a lot. Some existing research results focus on the pricing of the reverse supply chains, but the differences in the quality of recycling of used products are ignored. Most of them assumed that all the used products have the same of the quality status, as well the cost and the treatment methods for recycling used products. In fact, consumers have obvious differences in the time length, the frequency, the environment, the habits, and the methods of using products, the quality of the recycled used products varies greatly. Different treatment methods should be choosen when it comes to remanufacturing or disposal.

Therefore, we conducted the research on the pricing and coordination of the supply chain considering the competition and the quality of recycled products. The related literatures are divided into three parts: (1) competitive supply chain pricing; (2) supply chain pricing considering the quality of recycled products; and (3) closed-loop supply chain coordination.

1.1. Competitive supply chain pricing. On the issue of supply chain competition, Wei et al. surveyed the profit problem of the closed-loop supply chain affected by both the recycling channels and the retailer competition, and pointed out that the profit of the closed-loop supply chain will be affected by the recycling scale during the recycling effort and the degree of competition among retailers. Han and Xue focused on a closed-loop supply chain system with two competing manufacturers and a single retailer. Han and Xue conducted a research on the
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evolution process of recycling channels. Gu and Gao [9] found that if the manufacturer is willing to transfer the cost savings to the retailer in remanufacturing activities, the retailer’s recycling is the best channel. Jena and Sarmah [16] developed a decision model in three scenarios. Wu and Han [35] surveyed the optimal production decision. [32, 39] focused on a closed-loop supply chain system with two competing manufacturers, a single retailer, and a single recycler. Wang et al. [32] analyzed the influence of the award-punishment mechanism on the optimal decision-making in closed-loop supply chain. Yao and Teng [39] found that the growing competition among manufacturers is good for increasing both the sales of new products and the recycling of used products. Ma et al. [21] analyzed the remanufacturing strategy of duopoly enterprises, and the influence factors on the equilibrium strategy are discussed including the competition, the substitution, the production cost, and the remanufacturing cost. In [33], the competition between a supplier and a recycler has more pricing power than a manufacturer with limited production capacity. Zhao et al. [45] found that fierce competition in dual recycling channel will affect new products both the best wholesale prices and the best retail prices. Under the influence of the assimilation and the contrast, Wu et al. [36] researched how to develop their remanufacturing and pricing strategies between two competing original equipment manufacturers. These researches mainly concern the competition influence between two manufacturers on the supply chain, and most of them assume that the manufacturer or the retailer is responsible for recycling. Few articles can consider competition between manufacturers and competition between recycled products when multiple recyclers are responsible for recycling.

1.2. Supply chain pricing considering the quality of recycled products. The cost of recycling and remanufacturing in the recycling and remanufacturing supply chain depends on the quality or the grade of the recycled products [3]. The research of Deng et al. [5] showed that the recycling rate and the profit of the manufacturer are affected by the quality of recycling. There exists an optimal quality coefficient of recycling that maximizes the total profit and the recycling rate of the manufacturer. Heydari et al. [14] found that refurbishment capability and recycling quality will affect the coordination of the supply chain. Considering the difference among the prices of recycled quality, the new products and the remanufactured products, Liu et al. [20] explored the pricing decision-making issue of remanufacturing closed-loop supply chains under different rights structures. For a two-stage closed-loop supply chain with a single manufacturer and a single retailer, Giri et al. [7] analyzed the influence of product quality and pricing strategy on the optimal decision-making. According to the above research work, developing a theoretical model including multi-products, multi-manufacturers, and multi-recyclers based on the quality of recycled products can better adapt to actual conditions.

1.3. Closed-loop supply chain coordination. The research on the contract coordination of the closed-loop supply chain also has an important influence on the development of the supply chain. Yi and Liang [41] developed a game model for the closed-loop supply chain consisting of a single manufacturer, a single retailer and a third-party collector, and the coordination strategy based on premium and penalty mechanism for the remanufacturing closed-loop supply was explored. Xu and Tang [37] developed a dual-channel closed-loop supply chain model for third-party recycling, and designed a profit sharing contract to achieve the coordination for supply chain. Chen et al. [4] combined the profit sharing mechanism with the
fee system to achieve the coordination of a dual-channel closed-loop supply chain. He et al. [13] constructed a closed-loop supply chain model with competitive recovery. Comparing the recovery rates in centralized or decentralized decision-making, they proposed two coordination mechanisms, namely contract mechanisms and authorization mechanisms, and the recovery efficiency is optimal under the contract mechanism. Panda et al. [25] considered two aspects of corporate profit and used product recycling. They analyzed the coordination of corporate social responsibility and the pricing in closed-loop supply chain. Giri et al. [6] conducted a research on the pricing decision for a closed-loop supply chain, and explored the coordination issues considering the chain with two dual channel in product sales and used product recycling. Arshad et al. [2] found that the manufacturer as a channel manager encourages retail competition to reduce prices, then uses the three-party coordination of the revenue sharing contracts to achieve the Pareto optimality. Zhang et al. [43] designed a coordination model integrated sharing contracts for retailers and manufacturers for a supply chain, which is good for sustainable development of the environmental and the economic. Based on the above, we can state that the pricing theory of the supply chain should be closely integrated with the coordination theory. The supply chain maximizes the total benefits through coordinating and distributing the benefits of each member. That motivates more members to participate in the used product recycling and remanufacturing activities.

This paper works on the pricing and coordination of the closed-chain supply chain within different market forces. A multi-level closed-loop supply chain system consisting of two competing manufacturers and multiple recyclers is constructed. The manufacturer is engaged in manufacturing and remanufacturing of products, meanwhile responsible for the product sales. Considering the quality of used products, multiple recyclers are responsible for the competitive recycling of used products. The optimal pricing and the profit strategy of the companies in supply chain nodes in the two models are explored, and their efficiency in closed-loop supply chain is compared between different market forces. In the manufacturer-led Stackelberg game model, the recycling and remanufacturing supply chain does not reach the Pareto optimum, but uses the recycling cost sharing contract to achieve the coordination.

The remainder of this paper is organized as follows. Section 2 analyzes the model and puts forward problem descriptions and hypotheses. We design the solution algorithm, get the proposition, and analyze the practical application of the proposition in Section 3. In Section 4, an improved coordination mechanism is proposed. Section 5 conducts the numerical case and discussed the influence of the main parameters on the results. The final section concludes this paper and presents the future directions of this work.

2. Model analysis.

2.1. Problem description. This paper is based on a multi-level closed-loop supply chain of recycling and remanufacturing. The system architecture is shown in Figure 1. The supply chain system is consisted of two competing manufacturers and multiple recyclers. There is a premise that the two competing manufacturers manufacture and remanufacture products can replace each other, which are responsible for selling the new and remanufactured products in market; Multiple recyclers form an recycler alliance to conduct competitive recycling of used products in market. Recyclers reprice the recycled products according to the designated recycling price,
meanwhile the pricing process considers the factors of the economy scale of their own market. In addition, these factors will all affect the decisions of manufacturers and recyclers, such as the intensity of competition between the two manufacturers, government subsidies for recycling used products, the various costs of recycling used products, the intensity of recycling competition between used products, and the quality of used products.

![System architecture of the recycling and remanufacturing supply chain](image)

**Figure 1.** System architecture of the recycling and remanufacturing supply chain

2.2. **Parameter definition.** The following tables give the parameter definition and their descriptions in this paper.

| Table 1. Indices |
|------------------|
| $m$ | Manufacturer ($m=1,2$) |
| $r$ | Recycler ($r=1,2,3,...R$) |

| Table 2. Decision variables |
|-----------------------------|
| $p_m$ | The unit sale price for new and remanufactured products of manufacturer $m$ |
| $b_m$ | The unit recycling price specified by the recycler for the used products needed by the manufacturer $m$ |
| $\eta_m$ | The unit price at which manufacturer $m$ repurchases used products from recyclers |
Table 3. Definition of Parameters

| Parameter | Description |
|-----------|-------------|
| $R$       | Number of recyclers |
| $c_{mn}$  | The unit cost required by the manufacturer $m$ to manufacture a new product |
| $c_{mz}$  | The unit cost required for manufacturer $m$ to remanufacture the product |
| $\omega_m$ | Manufacturer $m$ uses the used products for remanufacturing cost savings |
| $\sigma$  | Remanufacturing ratio of recycled used products. It reflects the quality of recycled used products, $0 \leq \sigma \leq 1$ |
| $s$       | The government subsidies for recycler to recycle every unit of used products |
| $c_d$     | The unit cost for recyclers to dispose of used products |
| $c_q$     | The unit cost for recyclers to dispose of other used products that cannot be used for remanufacturing |
| $\delta_r$ | Economies of market scale for recyclers $r$, $0 < \delta_r < 1$ |
| $\lambda$ | The intensity of competition between two manufacturers that can replace new products, $0 < \lambda < 1$, competition, the more intense the competition among manufacturers. |
| $\mu$     | Market capacity, $\mu > 0$ |
| $\psi$    | When the recycling price is zero, the number of used products voluntarily returned by the consumer market which reflect consumers' environmental awareness, $\psi > 0$ |
| $\kappa$  | The intensity of recycling competition between two used products, $0 < \kappa < 1$ |

2.3. Model assumptions. For the accuracy of the supply chain model, we make the following reasonable assumptions.

**Assumption 1.** In the recycling and remanufacturing supply chain, the pricing decisions made by the two manufacturers and multiple recyclers in game theory are rational, and aim to maximize the profits.

**Assumption 2.** Recyclers recycle used products from the consumer market. Regardless of whether the recycling products can be used for remanufacturing by manufacturers, recyclers can receive government subsidies.

**Assumption 3.** Remanufacturing can reduce costs. Manufacturer $m$ can remanufacture used products to save costs, $\omega_m = c_{mn} - c_{mz} > 0$.

**Assumption 4.** The consumer market has a certain demand function $q_m$ for new and remanufactured products of manufacturer $m$. The products manufactured by the two competing manufacturers are different and have certain substitutability. In this way, the market demand function is related to the product price and the manufacturer’s competition intensity. The market demand function is as follows [27]:

$$q_m = \mu - p_m + \lambda_{pj}, \text{ where } m, j \in \{1, 2\} \text{ and } m \neq j.$$  

**Assumption 5.** In the case of asymmetric economies of scale in the recyclers market, the factors of the economies of scale in the recyclers market are different. Recycler $r$ reprices the used products needed by manufacturer $m$ under different market economies of scale. Therefore, there are differences in the repricing $x_{rm}$. The functional relationship is as follows:

$$x_{rm} = b_m(1 - \delta_r), \text{ where } r \in \{1, 2, ..., R\} \text{ and } m \in \{1, 2\}.$$  

**Assumption 6.** The recycling of used products by recyclers is the Cournot Competition. The recycling price and the intensity of competition will affect their recycling quantity. The recycler $r$’s recycling quantity of used products required by
manufacturer \( m \) is as follows\[12\]:

\[
 t_{rm} = \psi + x_{rm} - \kappa x_{rj}, \text{ where } m, j \in \{1, 2\} \text{ and } m \neq j; r \in \{1, 2, \ldots, R\}.
\]

**Assumption 7.** All games are the complete information games [15].

**Assumption 8.** Only consider a single operating cycle. It does not consider the impact of the previous cycle or the next cycle.

**Assumption 9.** The total market capacity of the two competing manufacturers is equal and never changed.

**Assumption 10.** Newly manufactured and remanufactured products are of the same quality and sold at the same unit price.

**Assumption 11.** The recycling and remanufacturing of used products has certain economic feasibility, namely:

\[
 p_m > c_{mn} > \eta_m + c_{mz}; s + \eta_m > b_m + c_d + c_q.
\]

### 2.4. Profit function in recycling and remanufacturing supply chain.

According to parameter definition and model assumptions, the profit function of manufacturer \( m \) is \( f_m, m \in \{1, 2\} \). The profit function of manufacturer 1 is \( f_1 \), the profit function of manufacturer 2 is \( f_2 \):

\[
 f_m = (p_m - c_{mn})q_m + (\omega_m - \eta_m)T_m\sigma_m \tag{1}
\]

The total amount of used products recycled by the recyclers for manufacturer \( m \) is \( T_m \):

\[
 T_m = \sum t_{rm}; m \in \{1, 2\}
\]

The total profits function of multiple recyclers is \( f_R \),

\[
 f_R = \sum_{m=1}^{2} (s - c_d - c_q + \sigma c_q)T_m + \sum_{m=1}^{2} \eta_m T_m \sigma - \sum_{m=1}^{2} \sum_{r=1}^{R} x_{rm} t_{rm} \tag{2}
\]

### 3. Model solution and analysis.

#### 3.1. Manufacturer-recycler cooperation game decision model (MRC model).

In the MRC model, two competing manufacturers and multiple recyclers do not make decisions independently, but jointly determine the pricing strategy. The decision variables for the recycling and remanufacturing supply chain system are two price factors. One is that recyclers recycle used products, and the other is that manufacturers sell new and remanufactured products. The price at which manufacturers repurchase used products from recyclers only affects the profits among the distribution members, and it will not affect the overall profit of the supply chain system. In the decision-making process, two manufacturers and multiple recyclers jointly determine the sale price of the product and the recycling price of the used product. Their purpose is to maximize the overall profit of the supply chain system. The overall profit function of the supply chain system is \( f \):

\[
 f = \sum_{m=1}^{2} (p_m - c_{mn})q_m + \sum_{m=1}^{2} (s - c_d - c_q + \sigma c_q)T_m
 + \sum_{m=1}^{2} \sigma \omega_m T_m - \sum_{m=1}^{2} \sum_{r=1}^{R} x_{rm} t_{rm} \tag{3}
\]
The decision variable in the overall profit function $f$ of the supply chain system is $p_1, p_2, b_1, b_2$. The corresponding Hessian matrix is $H_1$:

$$H_1 = \begin{bmatrix}
-2 & 2\lambda & 0 & 0 \\
2\lambda & -2 & 0 & 0 \\
0 & 0 & -2\sum_{r=1}^{R} (1 - \delta_r)^2 & -2\kappa \sum_{r=1}^{R} (1 - \delta_r)^2 \\
0 & 0 & -2\kappa \sum_{r=1}^{R} (1 - \delta_r)^2 & -2\sum_{r=1}^{R} (1 - \delta_r)^2
\end{bmatrix}$$

(4)

So $H_1$ is a negative definite matrix. The maximum value is obtained when the first derivative is zero. Get the first-order partial derivatives of $p_1, p_2, b_1$ and $b_2$ in $f$ and set the derivatives to zero.

$$\begin{align*}
\frac{\partial f}{\partial p_1} &= \mu - 2p_1 + 2\lambda p_2 + c_{1n} - \lambda c_{2n} = 0 \\
\frac{\partial f}{\partial p_2} &= \mu - 2p_2 + 2\lambda p_1 + c_{2n} - \lambda c_{1n} = 0 \\
\frac{\partial f}{\partial b_1} &= [(1 - \kappa)(s - c_d - c_q + \sigma c_q) + (\omega_1 - \kappa \omega_2)\sigma - \psi] \sum_{r=1}^{R} (1 - \delta_r) \\
\frac{\partial f}{\partial b_2} &= [(1 - \kappa)(s - c_d - c_q + \sigma c_q) + (\omega_2 - \kappa \omega_1)\sigma - \psi] \sum_{r=1}^{R} (1 - \delta_r)
\end{align*}$$

(5)

The equilibrium solution in the model can be solved as:

$$\begin{align*}
p_1^{MRC} &= \frac{\mu + c_{1n}(1 - \lambda)}{2(1 - \lambda)} \\
p_2^{MRC} &= \frac{\mu + c_{2n}(1 - \lambda)}{2(1 - \lambda)} \\
b_1^{MRC} &= \frac{[(s - c_d - c_q + \sigma c_q + \omega_1 \sigma)(1 - \kappa) - \psi] \sum_{r=1}^{R} (1 - \delta_r)}{2(1 - \kappa) \sum_{r=1}^{R} (1 - \delta_r)^2} \\
b_2^{MRC} &= \frac{[(s - c_d - c_q + \sigma c_q + \omega_2 \sigma)(1 - \kappa) - \psi] \sum_{r=1}^{R} (1 - \delta_r)}{2(1 - \kappa) \sum_{r=1}^{R} (1 - \delta_r)^2}
\end{align*}$$

(6)

The maximum overall profit of the supply chain system is:

$$f^{MRC} = \sum_{m=1}^{2} \left[ \frac{\mu - (1 - \lambda)c_{mn}}{4(1 - \lambda)} \right]^2 + R\psi \sum_{m=1}^{2} (s - c_d - c_q + \sigma c_q + \omega_{mn}\sigma) \\
+ \sum_{m=1}^{2} \left[ \frac{[(s - c_d - c_q + \sigma c_q + \omega_{mn}\sigma)(1 - \kappa) - \psi] \sum_{r=1}^{R} (1 - \delta_r)}{4(1 - \kappa) \sum_{r=1}^{R} (1 - \delta_r)^2} \right]^2$$

(7)
Proposition 1. In the MRC model, when $\lambda$ increases, the recycling price and quantity of recycling will not be affected. However, the sale price and the market demand for manufacturers to manufacture new and remanufactured products have increased.

Proof. From the optimal solution in the MRC model, we can get:

$$T_1^{MRC} = R\psi - \frac{\left[(s - c_d - c_q + \sigma c_q)(1 - \kappa) + (\omega_1 - \kappa \omega_2)\sigma - \psi\right] \sum_{r=1}^{R} (1 - \delta_r)^2}{2 \sum_{r=1}^{R} (1 - \delta_r)^2}$$

$$T_2^{MRC} = R\psi - \frac{\left[(s - c_d - c_q + \sigma c_q)(1 - \kappa) + (\omega_2 - \kappa \omega_1)\sigma - \psi\right] \sum_{r=1}^{R} (1 - \delta_r)^2}{2 \sum_{r=1}^{R} (1 - \delta_r)^2}$$

(8)

$$q_1^{MRC} = \frac{\mu - c_{1n} + \lambda c_{2n}}{2}$$

$$q_2^{MRC} = \frac{\mu - c_{2n} + \lambda c_{1n}}{2}$$

(9)

Therefore:

$$\frac{\partial p_1^{MRC}}{\partial \lambda} > 0, \frac{\partial p_2^{MRC}}{\partial \lambda} > 0, \frac{\partial q_1^{MRC}}{\partial \lambda} > 0, \frac{\partial q_2^{MRC}}{\partial \lambda} > 0$$

(10)

The sale price and market demand of new and remanufactured products by manufacturers are increasing functions of $\lambda$. When $\lambda$ increases, the sale price and market demand increase. The recycling price and quantity of recycling will not be affected.

Proposition 1 shows that the competitive effect is gradually transferred to the downstream consumer market with the increasement of the intensity of horizontal competition between two manufacturers, and both manufacturers will obtain greater profits by increasing sale price and expanding market demand.

Proposition 2. In the MRC model, when $\lambda$ increases, the maximum overall profit of the supply chain system will increase.

Proof. From the optimal solution in the MRC model, we can get:

$$\frac{\partial f^{MRC}}{\partial \lambda} = \frac{\sum_{m=1}^{2} [\mu - (1 - \lambda)c_{mn}] [\mu + (1 - \lambda)c_{mn}]}{4(1 - \lambda)^2}$$

$$= \frac{\sum_{m=1}^{2} \mu [1 - (1 - \lambda)\frac{c_{mn}}{\mu}] [\mu + (1 - \lambda)c_{mn}]}{4(1 - \lambda)^2}$$

(11)

According to the assumptions, it can be derived that $\mu > c_{mn}$, the maximum profit of the entire recycling and remanufacturing supply chain system is an increasing function of $\lambda$. When $\lambda$ increases, the maximum overall profit will increase.

Proposition 2 shows that when the recycling and remanufacturing of used products have certain economic feasibility, the horizontal competition intensity of products between manufacturers will be improved combined with Proposition 1, and the market demand for new products and remanufactured products will be expanded.
This mode benefits the recycling and remanufacturing of used products, and it has a positive role in improving the overall profit of the supply chain system.

**Proposition 3.** In the MRC model, when \( \kappa \) increases, the sale price will not be affected, and the recycling price will be reduced.

**Proof.** From the optimal solution in the model MRC, we can get:

\[
\frac{\partial b_1^{MRC}}{\partial \kappa} = \frac{\partial b_2^{MRC}}{\partial \kappa} < 0 \tag{12}
\]

Therefore, the recycling price is a decreasing function of \( \kappa \). When \( \kappa \) is increased, the recycling price will be decreased.

**Proposition 4.** In the MRC model, when the intensity of recycling competition \( \kappa \) between two used products is increased, the maximum overall profit of the supply chain system will be decreased.

**Proof.** From the optimal solution in the MRC model, we can get:

\[
\frac{\partial f^{MRC}}{\partial \kappa} = \frac{\left[ \sum_{r=1}^{R} (1 - \delta_r) \right]^2}{4(1 - \kappa)^2 \sum_{r=1}^{R} (1 - \delta_r)^2} \left( \psi - (1 - \kappa)(\sigma \omega_m + s - c_d - c_q + \sigma c_q) \right) \left( \psi + (1 - \kappa)(\sigma \omega_m + s - c_d - c_q + \sigma c_q) \right) \tag{13}
\]

Based on assumptions,

\[
\psi - (1 - \kappa)(\sigma \omega_m + s - c_d - c_q + \sigma c_q) < 0
\]
\[
\psi + (1 - \kappa)(\sigma \omega_m + s - c_d - c_q + \sigma c_q) > 0
\]

\[
\frac{\partial f^{MRC}}{\partial \kappa} < 0 \tag{14}
\]

Therefore, the maximum overall profit of the supply chain system is a decreasing function of \( \kappa \). When \( \kappa \) is increased, the maximum overall profit will be decreased.

Propositions 3 and 4 show that increasing recycling competition can effectively reduce the recycling price, meanwhile it will discourage consumers from participating in recycling activities of used products, and reduce the numbers of recycling. In this way, the cost of manufacturing products by manufacturers will be increased. The recycling and remanufacturing activities of the entire supply chain will be affected, and the maximum overall profit of the supply chain system will be reduced.

**Proposition 5.** In the MRC model, when \( \sigma \) increases, it means that the quality of recycled used products improves, and the sale prices of new and remanufactured products manufactured by manufacturers are not affected. The recycling price will be increased accordingly.

**Proof.** From the optimal solution in the MRC model, we can get:

\[
\frac{\partial b_1^{MRC}}{\partial \sigma} > 0, \quad \frac{\partial b_2^{MRC}}{\partial \sigma} > 0 \tag{15}
\]

The recycling price is an increasing function of \( \sigma \). When \( \sigma \) increases, the recycling price increases accordingly.
Proposition 5 shows that when the quality of recyclers’ recycled used products increases, the recycling price increases. For effective resources utilization and environmental protection, recyclers should encourage more consumers to participate in recycling activities of used product and recycle high-quality used products. Recyclers can take measures to motivate consumers to participate in recycling activities such as increasing the recycling price of high-quality products. □

Proposition 6. In the MRC model, when $\sigma$ increases, it means that the quality of recycled used products improves, the maximum overall profit of the supply chain system increases.

Proof. From the optimal solution in the MRC model, we can get:

$$\frac{\partial f_{MRC}}{\partial \sigma} = R\psi \sum_{m=1}^{2} (\omega_m + c_q) \left[ \sum_{r=1}^{R} (1 - \delta_r) \right] \frac{2(1 - \kappa) \sum_{r=1}^{R} (1 - \delta_r)^2}{2(1 - \kappa) \sum_{m=1}^{R} \{ (\omega_m + c_q)(1 - \kappa) - \psi \}}$$

(16)

According to assumptions:

$$(\omega_m + c_q) \left[ (\omega_m \sigma + c_q \sigma + s - c_d - c_q)(1 - \kappa) - \psi \right] > 0$$

Thus, the maximum overall profit of the supply chain system is an increasing function of $\sigma$. When $\sigma$ increases, the maximum profit increases accordingly.

Combined with Proposition 5, Proposition 6 shows that when $\sigma$ increases, the recycling price increases. Under the stimulation of high recycling price, more consumers are willing to participate in recycling activities. From the perspective of saving costs of product manufacturing, manufacturers are more willing to participate in remanufacturing activities. As the sale price of products is not affected by $\sigma$, the maximum overall profit of the supply chain system will be increased. □

3.2. Manufacturer-led Stackelberg game decision model (MS model). In the MS model, the manufacturer and the recycler are not partner in joint decision-making and game theory. The manufacturer is the market leader in this mode, and the recycler is the follower of the manufacturer. The manufacturer has the right to make decisions first. The dynamic game can be divided into two stages, and the sequence of the game is:

1. The manufacturer $m$ determines the sale price $p_m$ and the repurchase price $\eta_m$;
2. Recycler $r$ determines the recycling price $b_r$.

$$f_1 = (p_1 - c_{1n})(\mu - p_1 + \lambda p_2) + (\omega_1 - \eta_1)T_1 \sigma$$

$$f_2 = (p_2 - c_{2n})(\mu - p_2 + \lambda p_1) + (\omega_2 - \eta_2)T_2 \sigma$$

$$f_R = \sum_{m=1}^{2} (s - c_d - c_q + \sigma c_q)T_m + \sum_{m=1}^{2} \eta_m T_m \sigma - \sum_{m=1}^{2} \sum_{r=1}^{R} x_{rm} t_{rm}$$

(18)
The equilibrium solution in the MS model is:

\[ p_1^{MS} = \frac{\mu(\lambda + 2) + \lambda c_{2n} + 2c_{1n}}{4 - \lambda^2} \]

\[ p_2^{MS} = \frac{\mu(\lambda + 2) + \lambda c_{1n} + 2c_{2n}}{4 - \lambda^2} \]

\[ \eta_1^{MS} = \frac{1}{\sigma(\kappa - 2)} \left[ \frac{2R\psi \sum_{r=1}^{R} (1 - \delta_r)^2}{\sum_{r=1}^{R} (1 - \delta_r)} \right] + (1 - \kappa)(s - c_d - c_q + \sigma c_q - \psi) - \frac{\sigma(2\omega_1 + \kappa \omega_2)}{\kappa + 2} \]

\[ \eta_2^{MS} = \frac{1}{\sigma(\kappa - 2)} \left[ \frac{2R\psi \sum_{r=1}^{R} (1 - \delta_r)^2}{\sum_{r=1}^{R} (1 - \delta_r)} \right] + (1 - \kappa)(s - c_d - c_q + \sigma c_q - \psi) - \frac{\sigma(2\omega_2 + \kappa \omega_1)}{\kappa + 2} \]

\[ b_1^{MS} = \frac{R\psi}{(\kappa - 2) \sum_{r=1}^{R} (1 - \delta_r)} - \frac{[s - c_d - c_q + \sigma c_q + (1 - \kappa)\psi] \sum_{r=1}^{R} (1 - \delta_r)}{2(\kappa - 2) \sum_{r=1}^{R} (1 - \delta_r)^2} \]

\[ b_2^{MS} = \frac{R\psi}{(\kappa - 2) \sum_{r=1}^{R} (1 - \delta_r)} - \frac{[s - c_d - c_q + \sigma c_q + (1 - \kappa)\psi] \sum_{r=1}^{R} (1 - \delta_r)}{2(\kappa - 2) \sum_{r=1}^{R} (1 - \delta_r)^2} \]

\[ b_1^{MS} = \frac{b_1}{2(1 - \kappa) \sum_{r=1}^{R} (1 - \delta_r)^2} \]

**Proof.** Use backward induction to solve this problem, \( f_R \) is on the Hessian matrix \( H_2 \) of \( b_1, b_2 \):

\[ H_2 = \begin{bmatrix} -2 \sum_{r=1}^{R} (1 - \delta_r)^2 & 2\kappa \sum_{r=1}^{R} (1 - \delta_r)^2 \\ 2\kappa \sum_{r=1}^{R} (1 - \delta_r)^2 & -2 \sum_{r=1}^{R} (1 - \delta_r)^2 \end{bmatrix} \]

\( H_2 \) is a negative definite matrix. The maximum value will be obtained when the first derivative is zero. We can get:

\[ b_1^{MS} = \frac{[(s - c_d - c_q + \sigma c_q + \sigma \eta_1)(1 - \kappa) - \psi] \sum_{r=1}^{R} (1 - \delta_r)}{2(1 - \kappa) \sum_{r=1}^{R} (1 - \delta_r)^2} \]
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\[ b_2^{MS} = \frac{[(s - c_d - c_q + \sigma c_q + \sigma \eta_2)(1 - \kappa) - \psi] R \sum_{r=1}^{R} (1 - \delta_r)}{2(1 - \kappa) \sum_{r=1}^{R} (1 - \delta_r)^2} \]  \hspace{1cm} (21)

Substitute the solved \( b_1^{MS}, b_2^{MS} \) into \( f_1, f_2 \), and then find the Hessian matrix \( H_3 \) about \( p_1, p_2, \eta_1, \eta_2 \):

\[
H_3 = \begin{bmatrix}
-2 & \lambda & 0 & 0 \\
\lambda & -2 & 0 & 0 \\
0 & 0 & -\sigma^2 \left[ \frac{\sum_{r=1}^{R} (1 - \delta_r)}{R} \right]^2 & \kappa \sigma^2 \left[ \frac{\sum_{r=1}^{R} (1 - \delta_r)}{R} \right]^2 \\
0 & 0 & \kappa \sigma^2 \left[ \frac{\sum_{r=1}^{R} (1 - \delta_r)}{R} \right]^2 & -\sigma^2 \left[ \frac{\sum_{r=1}^{R} (1 - \delta_r)}{R} \right]^2 \\
0 & 0 & 2 \sum_{r=1}^{R} (1 - \delta_r)^2 & \kappa \sigma^2 \left[ \frac{\sum_{r=1}^{R} (1 - \delta_r)}{R} \right]^2 \\
0 & 0 & \kappa \sigma^2 \left[ \frac{\sum_{r=1}^{R} (1 - \delta_r)}{R} \right]^2 & 2 \sum_{r=1}^{R} (1 - \delta_r)^2 \\
\end{bmatrix}
\]  \hspace{1cm} (22)

\( H_3 \) is a negative definite matrix, when the first derivative is zero, we can get:

\[
p_1^{MS} = \frac{\mu(\lambda + 2) + \lambda c_{2n} + 2c_{1n}}{4 - \lambda^2} \\
p_2^{MS} = \frac{\mu(\lambda + 2) + \lambda c_{1n} + 2c_{2n}}{4 - \lambda^2} \\
\eta_1^{MS} = \frac{1}{\sigma(\kappa - 2)} \left[ \frac{2R\psi \sum_{r=1}^{R} (1 - \delta_r)^2}{\sum_{r=1}^{R} (1 - \delta_r)^2} + (1 - \kappa)(s - c_d - c_q + \sigma c_q - \psi) - \frac{\sigma(2\omega_1 + \kappa \omega_2)}{\kappa + 2} \right] \\
\eta_2^{MS} = \frac{1}{\sigma(\kappa - 2)} \left[ \frac{2R\psi \sum_{r=1}^{R} (1 - \delta_r)^2}{\sum_{r=1}^{R} (1 - \delta_r)^2} + (1 - \kappa)(s - c_d - c_q + \sigma c_q - \psi) - \frac{\sigma(2\omega_2 + \kappa \omega_1)}{\kappa + 2} \right] 
\]  \hspace{1cm} (23)

Substituting, it is proved.

From the above formulas, the profits of manufacturer 1 and manufacturer 2 in the MS model will be obtained:

\[
T_1^{MS} = \left\{ \frac{(\kappa^2 - 2)\sigma \omega_1 + \kappa \sigma \omega_2}{\kappa^2 - 4} - \frac{2R\psi \sum_{r=1}^{R} (1 - \delta_r)^2}{(\kappa - 2) \sum_{r=1}^{R} (1 - \delta_r)^2} \\
- \frac{(1 - \kappa)(s - c_d - c_q + \sigma c_q - \psi)}{\kappa - 2} \right\} \left[ \frac{\sum_{r=1}^{R} (1 - \delta_r)^2}{2 \sum_{r=1}^{R} (1 - \delta_r)^2} \right] 
\]
\[ T_{2}^{MS} = \left\{ \frac{(\kappa^2 - 2)\sigma\omega_2 + \kappa\sigma\omega_1}{\kappa^2 - 4} - \frac{2R\psi \sum_{r=1}^{R} (1 - \delta_r)^2}{(\kappa - 2) \left[ \sum_{r=1}^{R} (1 - \delta_r) \right]^2} \right. \\
\left. - \frac{(1 - \kappa)(s - c_d - c_q + \sigma c_q - \psi)}{\kappa - 2} \right\} \sum_{r=1}^{R} (1 - \delta_r)^2\]

\[ f_{1}^{MS} = \left\{ \frac{(\kappa^2 - 2)\sigma\omega_2 + \kappa\sigma\omega_1}{\kappa^2 - 4} - \frac{2R\psi \sum_{r=1}^{R} (1 - \delta_r)^2}{(\kappa - 2) \left[ \sum_{r=1}^{R} (1 - \delta_r) \right]^2} \right. \\
\left. - \frac{(1 - \kappa)(s - c_d - c_q + \sigma c_q - \psi)}{\kappa - 2} \right\} \sum_{r=1}^{R} (1 - \delta_r)^2\]

\[ f_{2}^{MS} = \left\{ \frac{(\kappa^2 - 2)\sigma\omega_2 + \kappa\sigma\omega_1}{\kappa^2 - 4} - \frac{2R\psi \sum_{r=1}^{R} (1 - \delta_r)^2}{(\kappa - 2) \left[ \sum_{r=1}^{R} (1 - \delta_r) \right]^2} \right. \\
\left. - \frac{(1 - \kappa)(s - c_d - c_q + \sigma c_q - \psi)}{\kappa - 2} \right\} \sum_{r=1}^{R} (1 - \delta_r)^2\]

\[ f_{R}^{MS} = (s - c_d - c_q + \sigma c_q) \left\{ 2R\psi + (1 - \kappa)(b_1^{MS} + b_2^{MS}) \sum_{r=1}^{R} (1 - \delta_r) \right\} \]

\[ - \left[ (b_1^{MS})^2 + (b_2^{MS})^2 - 2\kappa b_1^{MS} b_2^{MS} \right] \]

\[ \sum_{r=1}^{R} (1 - \delta_r)^2 - (b_1^{MS} + b_2^{MS}) \psi \sum_{r=1}^{R} (1 - \delta_r) \]

\[ + \left[ \eta_1^{MS} \omega_1 + \eta_2^{MS} \omega_2 - (\eta_1^{MS})^2 -(\eta_2^{MS})^2 \right] \frac{\sigma^2 \left[ \sum_{r=1}^{R} (1 - \delta_r) \right]^2}{2 \sum_{r=1}^{R} (1 - \delta_r)^2} \]

\[ f^{MS} = (s - c_d - c_q + \sigma c_q + \sigma \omega_1) \left[ R\psi + (b_1^{MS} - \kappa b_2^{MS}) \sum_{r=1}^{R} (1 - \delta_r) \right] \\
+ (s - c_d - c_q + \sigma c_q + \sigma \omega_2) \left[ R\psi + (b_2^{MS} - \kappa b_1^{MS}) \sum_{r=1}^{R} (1 - \delta_r) \right] \]

(24)
Proof. From the optimal solution in the MS model, we can get:

\[-\psi(b_1^{MS} + b_2^{MS}) \sum_{r=1}^{R} (1 - \delta_r) + \left[ \frac{\lambda^2 c_{2n} + \lambda (\mu + c_{1n}) + 2(\mu - c_{2n})}{4 - \lambda^2} \right]^2 + \left[ \frac{\lambda^2 c_{1n} + \lambda (\mu + c_{2n}) + 2(\mu - c_{1n})}{4 - \lambda^2} \right]^2 - \left( (b_1^{MS})^2 + (b_2^{MS})^2 - 2\kappa b_1^{MS} b_2^{MS} \right) \sum_{r=1}^{R} (1 - \delta_r)^2 \]

The expressions of \( f_R^{MS} \) and \( f^{MS} \) are too complicated and will be a further work in numerical analysis.

**Proposition 7.** In the MS model, when \( \lambda \) increases, the recycling price and the repurchase price are not affected. However, the sale price and market demand for manufacturers to manufacture new and remanufactured products will be increased.

**Proof.** From the optimal solution in the MS model, we can get:

\[ q_1^{MS} = \frac{\lambda^2 c_{1n} + \lambda (\mu + c_{2n}) + 2(\mu - c_{1n})}{4 - \lambda^2} \]
\[ q_2^{MS} = \frac{\lambda^2 c_{2n} + \lambda (\mu + c_{1n}) + 2(\mu - c_{2n})}{4 - \lambda^2} \]
\[ \frac{\partial q_1^{MS}}{\partial \lambda} > 0, \frac{\partial q_2^{MS}}{\partial \lambda} > 0, \frac{\partial R^{MS}}{\partial \lambda} > 0, \frac{\partial q_2^{MS}}{\partial \lambda} > 0 \]  

The sale price and the market demand of new and remanufactured products by manufacturers are increasing functions of \( \lambda \). When \( \lambda \) increases, the sale price and the market demand of new and remanufactured products by manufacturers increase accordingly. The recycling price and repurchase price are not affected by \( \lambda \). □

**Proposition 8.** In the MS model, when \( \lambda \) increases, the profits of manufacturer 1 and manufacturer 2 will increase, the total profits of multiple recyclers will not be affected, and the maximum overall profit of the supply chain system will increase.

**Proof.** From the optimal solution in the MS model, we can get:

\[ \frac{\partial f_1^{MS}}{\partial \lambda} = 2 \left[ \frac{\lambda^2 c_{1n} + \lambda (c_{2n} + \mu) + 2(\mu - c_{1n})}{4 - \lambda^2} \right] \left[ \frac{(2\lambda c_{1n} + \mu + c_{2n})(4 - \lambda^2) + 2 \lambda \left[ \lambda^2 c_{1n} + \lambda (c_{2n} + \mu) + 2(\mu - c_{1n}) \right]}{(4 - \lambda^2)^2} \right] > 0 \]
\[ \frac{\partial f_2^{MS}}{\partial \lambda} = 2 \left[ \frac{\lambda^2 c_{2n} + \lambda (c_{1n} + \mu) + 2(\mu - c_{2n})}{4 - \lambda^2} \right] \left[ \frac{(2\lambda c_{2n} + \mu + c_{1n})(4 - \lambda^2) + 2 \lambda \left[ \lambda^2 c_{2n} + \lambda (c_{1n} + \mu) + 2(\mu - c_{2n}) \right]}{(4 - \lambda^2)^2} \right] > 0 \]  

So \( f_1^{MS} \), \( f_2^{MS} \) increase when the \( \lambda \) increases; since \( b_1^{MS}, b_2^{MS}, q_1^{MS}, q_2^{MS} \) are not affected by \( \lambda \), \( f_R^{MS} \) is not affected by \( \lambda \), and \( f^{MS} \) increases when the \( \lambda \) increases. □

**Proposition 9.** In the MS model, when \( \kappa \) increases, the sale price of new and remanufactured products by manufacturers will not be affected; The recycling price decreases, the repurchase price increases, and the total profits of multiple recyclers...
Proposition 10. In the MS model, when $\sigma$ increases, it means that the quality of recycled used products improves, the sale price of new products and remanufactured products manufactured by manufacturers are not affected; The recycling price increase, and the repurchase price of manufacturers will increase; The overall profit of the supply chain system and the profit of each node will increase.

3.3. Comparative analysis of MRC model and MS model. After comparing and analyzing the optimal results in the MRC model and the MS model:

Proposition 11. The relationship between the optimal sale price, recycling price and the overall profit of the supply chain system in the MRC model and the MS model:

1) $p_{m_{MRC}} > p_{m_{MS}}$, $q_{m_{MRC}} < q_{m_{MS}}$; (2) $b_{m_{MRC}} > b_{m_{MS}}$; (3) $f_{MRC} > f_{MS}$

Proposition 11 shows: (1) the sale price in the MRC model is higher than the sale price in the MS model, while the market demand in the MRC model is lower than the market demand in the MS model. In the MS model, depending on their dominant role in the market, the two competing manufacturers determine the sale price of their products and affect market demand, which helps maximize their profits. Therefore, in the recycling and remanufacturing supply chain, the MS model is the most beneficial for manufacturers. (2) The recycling price in the MRC model is higher than the recycling price in the MS model. In the MRC model, recyclers can proactively determine the recycling price to maximize their profits. From the perspective of recovery rate of used product and environment protection, the best decision is the MRC model. In the MRC model, the recycling price is the highest, so the quantity of recycling is the largest, which can increase the reuse rate of resources, help save resources and protect the environment. (3) In the MRC model, the overall profit of the supply chain system is higher than that of MS model.

4. Improved coordination mechanism. When two competing manufacturers recycle used products by integrating multiple recyclers, analysis shows that there will be a double marginal effect in the MS model, which reduces the efficiency of the supply chain system. Meanwhile the system cannot reach the Pareto optimal solution. Two main optimum objects cannot be realized, namely the overall profit of the system and the profit of manufacturers and recyclers. The purpose of establishing a coordination mechanism is mainly to improve the enterprises profits of the supply chain nodes and the overall profit of the supply chain system in the MS model. This paper applies the recycling cost sharing contract to coordinate the closed-loop supply chain. Since two competing manufacturers are in a dominant position in the supply chain, they can rely on their own influence to sign related contracts with multiple recyclers. In this way, the profits of the manufacturers and recyclers under the contract can be increased. Under the contract, based on the optimal solution in the MS and MRC models, the sale price of the two manufacturers and the recycling price specified by the recycler for the used products of the manufacturer $m$ are specified. According to the extended producer responsibility system, manufacturers should help recyclers share $1-\theta$ times. The recycling costs that can be used for remanufacturing, and recyclers share $\theta$ times. The recycling costs that can be used for remanufacturing, among them $0 < \theta < 1$. After adopting
the coordination mechanism, the profits of the two manufacturers are \( f_{1}^{\text{MST}} \) and \( f_{2}^{\text{MST}} \) respectively.

\[
\begin{align*}
  f_{1}^{\text{MST}} &= (p_{1} - c_{1n})(\mu - p_{1} + \lambda p_{2}) + (\omega_{1} - \eta_{1})T_{1}\sigma - (1 - \theta) \sum_{r=1}^{R} x_{1r}t_{r1} \\
  f_{2}^{\text{MST}} &= (p_{2} - c_{2n})(\mu - p_{2} + \lambda p_{1}) + (\omega_{2} - \eta_{2})T_{2}\sigma - (1 - \theta) \sum_{r=1}^{R} x_{2r}t_{r2}
\end{align*}
\]

The total profits function of multiple recyclers is \( f_{R}^{\text{MST}} \):

\[
\begin{align*}
  f_{R}^{\text{MST}} &= \sum_{m=1}^{2} [s - c_{d} - c_{q} + \sigma c_{q}] T_{m} + \sum_{m=1}^{2} \eta_{m}T_{m}\sigma - \sum_{m=1}^{2} \sum_{r=1}^{R} \theta x_{rm}t_{rm}
\end{align*}
\]

On the above basis:

\[
\begin{align*}
  \theta b_{1}^{\text{MST}} &= \frac{[(s - c_{d} - c_{q} + \sigma c_{q} + \sigma\eta_{1})(1 - \kappa) - \psi]}{2(1 - \kappa)} \sum_{r=1}^{R} (1 - \delta_{r}) \sum_{r=1}^{R} \frac{(1 - \delta_{r})}{2}
\end{align*}
\]

Since \( b_{1}^{\text{MST}} = b_{1}^{\text{MRC}} \), we have:

\[
\begin{align*}
  \eta_{1}^{\text{MST}} &= \frac{(\theta - 1)(s - c_{d} - c_{q} + \sigma c_{q}) + \theta\sigma c_{q}}{\sigma} \sum_{r=1}^{R} (1 - \delta_{r}) \sum_{r=1}^{R} \frac{(1 - \delta_{r})}{2}
\end{align*}
\]

Similarly:

\[
\begin{align*}
  \eta_{2}^{\text{MST}} &= \frac{(\theta - 1)(s - c_{d} - c_{q} + \sigma c_{q}) + \theta\sigma c_{q}}{\sigma} \sum_{r=1}^{R} (1 - \delta_{r}) \sum_{r=1}^{R} \frac{(1 - \delta_{r})}{2}
\end{align*}
\]

After the coordination mechanism is adopted, the sales prices of the two manufacturers are consistent with the sales prices in the MS model, and then:

\[
\begin{align*}
  p_{1}^{\text{MST}} &= \frac{\mu(\lambda + 2) + \lambda c_{2n} + 2c_{1n}}{4 - \lambda^{2}} \\
  p_{2}^{\text{MST}} &= \frac{\mu(\lambda + 2) + \lambda c_{1n} + 2c_{2n}}{4 - \lambda^{2}}
\end{align*}
\]

The recycling prices are \( b_{1}^{\text{MST}} \) and \( b_{2}^{\text{MST}} \) :

\[
\begin{align*}
  b_{1}^{\text{MST}} &= \frac{[(s - c_{d} - c_{q} + \sigma c_{q} + \omega_{1}\sigma)(1 - \kappa) - \psi]}{2(1 - \kappa)} \sum_{r=1}^{R} (1 - \delta_{r}) \sum_{r=1}^{R} \frac{(1 - \delta_{r})}{2}
\end{align*}
\]

Manufacturers and recyclers will be willing to adopt a coordination mechanism only when the individual is under rational constraints. It means that the profits of each subject in the supply chain are greater than the profits before coordination. Therefore, the proportional parameter \( \theta \) must meet the following conditions:

\[
(1) f_{1}^{\text{MST}} > f_{1}^{\text{MS}} ; (2) f_{2}^{\text{MST}} > f_{2}^{\text{MS}} ; (3) f_{R}^{\text{MST}} > f_{R}^{\text{MS}}
\]
Because the above expression is more complicated, it will be discussed in numerical analysis.

**Proposition 12.** After adopting the coordination mechanism, the overall profit of the supply chain system is higher than the level of the MS model; Compared with the MS model, the profits of the two competing manufacturers and the total profits of multiple recyclers are improved.

Proposition 12 explains: After adopting the coordination mechanism, with the increase of profits, the enthusiasm of manufacturers and recyclers to participate in the supply chain is mobilized. Then the member enterprises of the whole supply chain can be coordinated effectively.

**Proposition 13.** After adopting the coordination mechanism,

\[
\begin{align*}
    p_{MST}^1 &= p_{MS}^1 < p_{MRC}^1, \\
    p_{MST}^2 &= p_{MS}^2 < p_{MRC}^2, \\
    b_{MST}^1 &= b_{MRC}^1 > b_{MS}^1, \\
    b_{MST}^2 &= b_{MRC}^2 > b_{MS}^2
\end{align*}
\]

Proposition 13 explains: (1) Compared with the MRC model, after coordination, the sales prices of two competing manufacturers have been reduced; (2) Compared with the MS model, after coordination, the recycling price has increased; (3) For the consumer market, when it comes to the optimal solution in the two models, after coordination, consumers can purchase products at a lower price, and they can recycle them to recyclers at a higher recycling price. Meanwhile the utility and enthusiasm of consumers are improved.

5. **Numerical analysis.** With decision model of a recycling and remanufacturing supply chain, the manufacturers and recyclers can get the optimal dynamic decision. Considering the complexity of the partial solution results, the relationship between the variables cannot be obtained intuitively. This section uses numerical analysis methods to further illustrate the correctness of the above conclusions and the effectiveness of the coordination contract. The main parameter values are shown in Table 4. From the previous discussion, and suppose that when the competitive intensity of manufacturer 1, 2 is \( \lambda = 0.4 \), the manufacturing rate available for remanufacturing is \( \sigma = 0.8 \), and the competitive intensity between the recycling of used products in the recycling market is \( \kappa = 0.3 \). Before adopting the coordination mechanism, the optimal solutions in the MRC model and the MS model are shown in Table 5.

According to the data in Table 5, the recycling price and the overall profit of the supply chain system in the MRC model are better than the MS model, which is good for the development of the recycling and remanufacturing supply chain.
Table 5. The optimal solutions in the MRC model and the MS model before the coordination mechanism is adopted

| Decision variables | p1 | p2 | b1 | b2 | η1 | η2 | f1 | f2 | fR | f  |
|--------------------|----|----|----|----|----|----|----|----|----|----|
| MRC                | 145| 140| 9.35| 6.87| -  | -  | -  | -  | -  | 14371|
| MS                 | 118| 114| 2.79| 1.71| 11.71| 9.52| 6207| 7072| 157| 13436|

5.1. The impact of \( \lambda \) and \( \kappa \) on supply chain pricing. This section analyzes the impact of manufacturer competition intensity \( \lambda \) and recycling competition intensity \( \kappa \) on supply chain pricing. Figure 2 and Figure 3 show that in the forward supply chain the larger \( \lambda \) brings to the higher sale price of the manufactured and remanufactured products by the two competing manufacturers, and also brings to the greater market demand. The main reason is that when two competing manufacturers compete horizontally, both manufacturers will strive for greater profits by increasing sale price and by expanding market demand. When \( \lambda \) is closer to the limit value 1, the sale price increases faster. When it is close to 1, the product of one manufacturer can completely replace the product of the other manufacturer, which leads to market monopoly; For the same manufacturer, the sale price in the MRC model is higher than the sale price in the MS model. This is mainly because that the market demand is positively correlated with \( \lambda \). For more profits, manufacturers should consider both the profit of a single product and the market demand. In the MS model, the manufacturer is in a dominant position and has priority in determining the sale price, and the manufacturer firstly considers the sale price of the product to maximize its profit; Both in the MRC model and in the MS model, the sale price of manufacturer 1 is higher than the sale price of manufacturer 2, and the demand for products manufactured by manufacturer 1 is lower than the demand for products manufactured by manufacturer 2. This is mainly because the cost of manufacturing and remanufacturing products of manufacturer 1 is higher than that of manufacturer 2. Manufacturer 1 should obtain more profits through higher sale price, however, consumers are more willing to choose lower-priced products. Combined with Figure 4, under the equal competition intensity, the profit of manufacturer 2 in the MS model is higher than the profit of manufacturer 1. This shows that in the competitive recycling and remanufacturing supply chain, manufacturers should improve their competitiveness by reducing manufacturing costs.

From Figure 4, it is found that the larger \( \lambda \) will lead to increase the overall profit of the supply chain system and the profit of each node both in the MRC model and in the MS model. In addition, the overall profit in the MRC model is higher than the overall profit in the MS model. (1) In the MRC model, each node of the recycling and remanufacturing supply chain system can more quickly and agilely respond to market changes, with less consumption among members in the system and higher efficiency of the entire supply chain. (2) More intensive competition between manufacturers is positive on the manufacturers themselves and the overall recycling and remanufacturing supply chain. Manufacturers should focus on improving the competitiveness of their products. From Figure 5-Figure 7, it is found that the recycling competition intensity \( \kappa \) between the two used products
will affect the recycling price, repurchase price, and the total profits of multiple recyclers. In the reverse supply chain, the larger $\kappa$ leads to a lower recycling price and a higher repurchase price. It shows that recyclers can increase the intensity of the recycling competition between the two used products. Recycling at low prices in the consumer market and transferring high prices to manufacturers, thus maximize the profits of the recyclers; On the other hand, when the intensity of recycling competition $\kappa$ is equal, the recycling price in the MRC model is higher than the recycling price in the MS model, and the MRC model can encourage more consumers to participate in the recycling activities of used products. It can increase the reuse rate of resources, and is good for constructing an environment friendly and resources-saving society.

From Figure 8-11, in the MRC and MS models, when the greater intensity of recycling competition $\kappa$ between the two used products become greater, the overall profit of the supply chain system and the profit of each node will be lower.

From Figure 12 and Figure 13, when $\lambda$ and $\kappa$ increase, it means that the competition intensity of the recycling and remanufacturing supply chain increases, the overall profit of the supply chain system in the MRC model and the MS model increases. Manufacturers increase the market demand by improving the competitiveness of their products, and then increase its own sale profit; Multiple recyclers reduce their own recycling costs by increasing the intensity of recycling competition.
between two used products. Under the combined effect of the two, the overall profit of the supply chain system can be maximized.

5.2. The impact of $\sigma$ on supply chain pricing. From Figure 14 and Figure 15, when $\sigma$ increases, it means that the quality of recycled products is improved, as well the recycling price and the manufacturer’s repurchase price. Recyclers are
willing to recycle used products at higher recycling price, and manufacturers are willing to repurchase used products at higher repurchase price. The rate of change in the recycling price in the MRC model is higher than that in the MS model.

From Figure 16, it is found that improving the quality of recycled products will increase the overall profit of the supply chain system both in the MRC model and in the MS model; The overall profit change rate in the MRC model is higher than that of the MS model. This shows that: (1) Recycling high-quality used products is positive on promoting the recycling work in supply chain. Recyclers can take measures to encourage consumers to participate in recycling and recycle high-quality used products; manufacturers are willing to remanufacture used products to reduce production costs. (2) When adopting the MRC model, each node of the supply chain can respond more quickly and agilely to market changes, and the efficiency of overall supply chain system is higher.

5.3. Analysis of the coordination mechanism of the recycling and remanufacturing supply chain. First, calculate the value range of $\theta$ according to the individual rational constraints and the data given by the calculation example: $0.31 < \theta < 0.51$. At this time, take $\theta = 0.4$ and $\theta = 0.35$ separately recorded as
coordination mechanism 1 and coordination mechanism 2. The optimal pricing and the profit of the recycling and remanufacturing supply chain after the coordination mechanism is adopted are shown in Table 6.

**Table 6. The optimal solution under the coordination mechanism**

| Decision variables | $p_1$ | $p_2$ | $b_1$ | $b_2$ | $\eta_1$ | $\eta_2$ | $f_1$ | $f_2$ | $f_R$ | $f$ |
|--------------------|-------|-------|-------|-------|----------|----------|-------|-------|-------|-----|
| MS model           | 117.7 | 114   | 2.8   | 1.7   | 11.7     | 9.5      | 6207  | 7072  | 157   | 13436 |
| Coordination1      | 117.7 | 114   | 9.35  | 6.87  | 10.45    | 8.45     | 6241  | 7099  | 206   | 13546 |
| Coordination2      | 117.7 | 114   | 9.35  | 6.87  | 9.24     | 7.49     | 6257  | 7109  | 180   | 13546 |

Based on the analysis of the data in Table 6, it is found that after adopting the recycling cost sharing contract, the value can be set within the range from the individual rational constraint conditions, which can effectively regulate the competitive recycling and remanufacturing supply chain. When $\theta$ satisfies the individual rational constraints, the overall profit of the competitive supply chain system is higher than that of the MS model. At the same time, compared with the MRC model, after supply chain coordination, consumers can buy products at a lower price, and recyclers recycle used products at a higher recycling price, which improves the utility and enthusiasm of consumers. The coordination of the competitive recycling and remanufacturing supply chain is good for the development and improvement of the closed-loop recycling system, which also helps achieve a win-win situation for consumers and supply chain members.

**5.4. Changes in the profitability of each node after coordination.** From the Figure 17-Figure 19, after adopting the coordination mechanism, when $\theta \in (0.31, 0.51)$, the profit of manufacturer 1 is higher than the profit 6207 in the MS model. The profit of manufacturer 2 is higher than the profit 7072 in the MS model, and the total profits 157 of multiple recyclers is higher than that of the MS model. It shows that under the coordination mechanism each node of the
competitive recycling and remanufacturing supply chain system can reach a higher level: When \( \theta \) increases, the profits of manufacturer 1, 2 decrease, and the total profits of multiple recyclers increase accordingly; Comparing the change trend of the profit of manufacturer 1, 2, it can be found that the change trend of manufacturer 1 is faster than that of manufacturer 2, which is mainly because the manufacturer needs to share part of the recovery cost. In this way, the profit of the manufacturer is related to both total sale profit and recycling cost of the recyclers.

6. Conclusions. This paper mainly proposes the pricing and coordination of the supply chain system consisting of two competing manufacturers and multiple recyclers for a recycling and remanufacturing supply chain system. The insights of this paper can be concluded as follow.

(1) The quality of recycled used products will not affect the pricing decisions in the forward supply chain, but it is positively related to the recycling price, the recycling quantity, the repurchase price, and the overall profit of the supply chain system in the reverse supply chain. Recyclers should adopt an appropriate incentive system to encourage consumers to participate in activities of the used products recycling, meanwhile increase the remanufacturing rate of recycled products.

(2) In the MRC model and the MS model, along with growing of the competition intensity among manufacturers, the market demand for the new and remanufactured
products has increased, as well the overall profit of the supply chain system and the profit of its each node. Competition between manufacturers will damage the rights of consumers, however, it benefits the supply chain. This mode is good for robust and rapid development of the recycling and remanufacturing industry.

(3) In the MS model, more fierce recycling competition between two used products leads to the lower recycling price of the recyclers, the higher repurchase price, and more profit of the recyclers. Both in the MRC model and the MS model, more fierce recycling competition between the two used products will lead to reduce the overall profit of the supply chain system. Therefore, from the perspective of consumers, manufacturers, and supply chain system, the recycling competition strategies among used products should not be advocated.

(4) Comparing and analyzing the optimal solutions in the MRC model and the MS model, it is found that the overall profit of the supply chain in the MS model is low, as well the recycling price and the recycling quantity. There exists a double marginal utility in the MS model. On this basis, the recycling cost sharing contract is used to achieve the coordination of the supply chain. Afterward, compared with the MS model, the important index value (such as the profits, the recycling price, and the recycling quantities) obtained by two competing manufacturers and multiple recyclers have increased. The overall profit of the supply chain system is higher than the level in the MS model, which improves consumer’s utility and enthusiasm.

Based on the complex environment of multiple manufacturers and multiple recyclers, this paper considers the quality of recycled products, supply chain competition and supply chain coordination, which provides a reference for the long-term dynamic decision-making of the recycling and remanufacturing supply chain. Meanwhile improving the overall efficiency of the supply chain is improved and the dynamic optimization of the system is realized. However, the information of all parties in this paper is symmetric, and only the competition between manufacturers and recycled products is considered. Future research will consider the decision-making and coordination of closed-loop supply chain with multiple competitive retailers under asymmetric information. In addition, when considering the impact of the quality of recycled products on the supply chain, it rarely considers the impact of the quality of recycled products on government subsidies, which is also one of the open issues in the future.

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REFERENCES

[1] M. Abbas, K. Devika, E. H. Rebort et al., Evaluation of green and sustainable supply chain management using structural equation modelling: A systematic review of the state of the art literature and recommendations for future research, *J. Clean. Prod.*, 249 (2020), 119383.

[2] M. Arshad, Q. S. Khalid, J. Lloret et al., An efficient approach for coordination of dual-channel closed-loop supply chain management, *Sustainability*, 10 (2018), 3433.

[3] R. Bhattacharya, A. Kaur and R. K. Amit, Price optimization of multi-stage remanufacturing in a closed loop supply chain, *J. Clean. Prod.*, 186 (2018), 943–962.

[4] J. Chen, H. Zhang and Y. Sun, Implementing coordination contracts in a manufacturing stackelberg dual-channel supply chain, *Omega*, 40 (2012), 571–583.

[5] Q. W. Deng, S. M. Guo, Q. H. Ren et al., Research of policy on competitive closed-loop supply chain based on quality uncertainty, *Ind. Techno. Econ.*, 36 (2017), 137–146.
[6] B. C. Giri, A. Chakraborty and T. Maiti, Pricing and return product collection decisions in closed-loop supply chain with dual-channel in both forward and reverse logistics, *J. Manuf. Syst.*, 42 (2017), 104–123.

[7] B. C. Giri, C. Mondal and T. Maiti, Optimal product quality and pricing strategy for a two-period closed-loop supply chain with retailer variable markup, *RAIRO-Oper. Res.*, 53 (2019), 609–626.

[8] A. Goli, E. B. Tirkolaee and G. W. Weber, A perishable product sustainable supply chain network design problem with lead time and customer satisfaction using a hybrid whale-genetic algorithm, *Logistics Oper. Manag. Recycl. Reuse*, (2020), 99–124.

[9] Q. Gu and T. Gao, Management of two competitive closed-loop supply chains, *Int. J. Sustain. Eng.*, 5 (2012), 325–337.

[10] V. D. R. Guide and J. Li, The potential for cannibalization of new products sales by remanufactured products, *Dec. Sci.*, 41 (2010), 547–572.

[11] X. H. Han and S. J. Xue, Reverse channel decisions for competition closed-loop supply chain based on evolutionary game, *Comput. Interg. Manuf. Syst.*, 16 (2010), 1487–1493.

[12] C. He, X. F. Song and C. H. Feng, Research on double contracts selection with recyclers’ competition of closed-loop supply chain based on multi-agent model, *Chinese J. Manag. Sci.*, 23 (2015), 75–83.

[13] Q. He, N. Wang, Z. Yang et al., Competitive collection under channel inconvenience in closed-loop supply chain, *European J. Oper. Res.*, 275 (2019), 155–166.

[14] J. Heydari, K. Govindan and R. Sadeghi, Reverse supply chain coordination under stochastic remanufacturing capacity, *Int. J. Prod. Econ.*, 202 (2018), 1–11.

[15] G. W. Hua, S. Y. Wang and T. C. E. Cheng, Price and lead time decisions in dual-channel supply chains, *Eur. J. Oper. Res.*, 205 (2010), 113–126.

[16] S. K. Jena, and S. P. Sarmah, Price competition and cooperation in a duopoly closed-loop supply chain, *Int. J. Prod. Econ.*, 156 (2014), 346–360.

[17] J. Ji, Z. Zhang and L. Yang, Carbon emission reduction decisions in the retail-/dual-channel supply chain with consumers’ preference, *J. Clean. Prod.*, 141 (2017), 852–867.

[18] Y. Jing and C. Z. Li, Pricing strategy of recycling and remanufacturing under the corporate social responsibility, *Comput. Interg. Manuf. Syst.*, 25 (2019), 256–266.

[19] D. R. Liu, Research on products pricing non-cooperative game methods of supply chain considering service level, *J. Zhengzhou Univ. Aero.*, 38 (2020), 73–84.

[20] W. J. Liu, N. N. Shen, J. Zhang et al., Optimal pricing for remanufacturing closed-loop supply chain under different channel power structures and product dua differentiation, *Ind. Eng. J.*, 21 (2018), 54–63.

[21] Z. Ma, A. Prasad and S. P. Sethi, Strategic remanufacturing under competition, *Rev. Mark. Sci.*, 16 (2018), 85–107.

[22] Y. Z. Mehrjerdi and R. Lotfi, Development of a mathematical model for sustainable closed-loop supply chain with efficiency and resilience systematic framework, *Int. J. Sup. Oper. Manag.*, 6 (2019), 360–388.

[23] S. Mitra, Models to explore remanufacturing as a competitive strategy under duopoly, *Omega*, 59 (2016), 215–227.

[24] J. J. Nie and L. Zhong, A research on the remanufacturing models with green customers, *Ind. Eng. J.*, 21 (2018), 9–18.

[25] S. Panda and M. M. Nikunja, Coordinating a socially responsible closed-loop supply chain with product recycling, *Int. J. Prod. Econ.*, 188 (2017), 11–21.

[26] S. Rahman and N. Subramanian, Factors for implementing end-of-life computer recycling operations in reverse supply chains, *Int. J. Prod. Econ.*, 140 (2012), 239–248.

[27] R. C. Savaskan and L. N. V. Wassenhove, Reverse channel design: The case of competing retailers, *Manag. Sci.*, 52 (2006), 1–14.

[28] C. R. Shen, Z. K. Xiong and Z. Q. Peng, Decision and coordination research for remanufacturing closed-loop supply chain under patent protection and government subsidies, *Ind. Eng. Manag.*, 27 (2013), 132–137.

[29] C. Su, X. Liu and W. Du, Green supply chain decisions considering consumers’ low-carbon awareness under different government subsidies, *Sustainability*, 12 (2020), 2281.

[30] E. B. Tirkolaee, P. Abbasi and G. W. Weber, Sustainable fuzzy multi-trip location-routing problem for medical waste management during the COVID-19 outbreak, *Sci. Tot. Env.*, 756 (2021), 143607.
[31] N. Wan and D. Hong, The impacts of subsidy policies and transfer pricing policies on the closed-loop supply chain with dual collection channels, J. Clean. Prod., 224 (2019), 881–891.
[32] W. B. Wang, Q. Chen and Q. L. Da, Decision and analysis of closed-loop supply chain with manufacturer-led and manufacturer-compete based on the reward-penalty mechanism, Chinese J. Manag. Sci., 21 (2013), 57–63.
[33] J. Wang, Z. Zhou and M. Yu, Pricing models in a sustainable supply chain with capacity constraint, J. Clean. Prod., 222 (2019), 57–76.
[34] J. Wei, K. Govindan, Y. Li et al., Pricing and collecting decisions in a closed-loop supply chain with symmetric and asymmetric information, Comput. Oper. Res., 54 (2015), 257–265.
[35] H. Y. Wu and X. H. Han, Production decisions in manufacturer competing closed-loop supply chains under remanufacturing costs disruptions scenarios, Comput. Interg. Manuf. Syst., 22 (2016), 1129–1138.
[36] L. Wu, L. Liu and Z. Wang, Competitive remanufacturing and pricing strategy with contrast effect and assimilation effect, J. Clean. Prod., 257 (2020), 120333.
[37] M. Z. Xu and F. Tang, Coordination mechanism of dual-channel closed-loop supply chain based on third-party collection, Comput. Interg. Manuf. Syst., 19 (2013), 2083–2089.
[38] A.-T. Yang and L.-D. Zhao, Supply chain network equilibrium with revenue sharing contract under demand disruptions, Int. J. Ant. Comput., 8 (2011), 177–184.
[39] F. M. Yao and C. X. Tong, Decision and coordination for competitive closed-loop supply chains with third-party collector dominated by a retailer, J. Syst. Eng., 34 (2019), 93–101.
[40] A. Yenipazarli, Managing new and remanufactured products to mitigate environmental damage under emissions regulation, European J. Oper. Res., 249 (2016), 117–130.
[41] Y. Y. Yi and J. M. Liang, Coordination of remanufacturing closed-loop supply chain under premium and penalty mechanism, Comput. Interg. Manuf. Syst., 19 (2013), 841–849.
[42] K. F. Yuan, G. Q. Wu, H. Dong et al., Differential pricing and emission reduction in remanufacturing supply chains with dual-sale channels under CCT-mechanism, Sustainability, 12 (2020), 8150.
[43] Y. J. Zhang, C. X. Guo and L. C. Wang, Supply chain strategy analysis of low carbon subsidy policies based on carbon trading, Sustainability, 12 (2020), 3532.
[44] L. Zhang, J. Wang and J. You, Consumer environmental awareness and channel coordination with two substitutable products, European J. Oper. Res., 241 (2015), 63–73.
[45] J. Zhao, J. Wei and M. Li, Collecting channel choice and optimal decisions on pricing and collecting in a remanufacturing supply chain, J. Clean. Prod., 167 (2017), 530–544.

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