The standard method of count data modeling is Poisson distribution, which has the assumption of equidispersion, as identified by the same mean and variance values. The modelling of count data frequently causes the emergence of over-dispersion which has a higher variance than mean itself. Many research could be found especially for handling over-dispersion problem such as Negative Binomial, Zero Inflated Poisson and Quasi approach. However, a few method in research could be fitted under-dispersion problem, such as Generalized Poisson which able to handle both problems, but has limited range of under-dispersion values. In this paper, a review of Conway-Maxwell-Poisson (COM-Poisson) distribution for count data is delivered. The COM-Poisson distribution is not only a generalization of the Poisson distribution, but also the distribution of Bernoulli and Geometric. Furthermore, we compare the performance of Negative Binomial, Generalized Poisson, and COM-Poisson models through its application to real data and simulations on overcome over- and under-dispersion problem.

1. Introduction
Frequently-operated method in modelling connection between response variable and predictor is well known as regression model. Data modeling uses ordinary linear regression using linearity to describe the relationship between the mean of response variable and the set of explanatory variables. With assuming that the respond distribution is normal. When, the data do not meet linearity and normality assumptions, a solution is required to handle the problem. Generalize linear model (GLM) is an extension of the linear regression model to handle the distribution of non-normal responses and possible nonlinear functions for the mean. GLM is defined by the set of independent random variables \( Y_1, \ldots, Y_n \) respectively with the distribution form of the exponential family and has 3 important components such as random components, linear predictors, and the relationship function.

The model which is capable of modelling the relationship of both response variable raised from Poisson distribution and predictor variable is assumed as equal-dispersion, in which the variance and mean are identical \( \mathbb{E}(Y_i) = \text{Var}(Y_i) \). Although, its applying operation repeatedly raises the occurrence of over-dispersion \( \mathbb{E}(Y_i) > \text{Var}(Y_i) \), and sometimes under-dispersion \( \mathbb{E}(Y_i) < \text{Var}(Y_i) \). This violates the assumption of equidispersion that must be satisfied by Poisson regression. Many models can be used to handle overdispersion cases, however, there are still few models that can handle underdispersion. So, it needs a development model to handle it. One way of handling overdispersion from count data is by mixed model, a mixed sample of Poisson and Gamma distribution, which is
Assumed by $y_i|u \sim poisson(u)$, $U \sim Gamma(\alpha, \beta)$, $y_i \sim Negative \ Binomial (u)$ – in which Poisson variable is counted on mean $\mu_i$ with prior U Gamma distribution. It is so then acquired into marginal distribution $y_i \sim Negative \ binomial (u)$, which is related to exponential set with canonically-linked function $log \mu_i = X'\beta$.

In addition, the mixed distribution among $\xi \sim degenerate(0), X \sim Poisson(\lambda), \xi$ and X are independent. Random variable $Y$ is noted as: $Y \sim ZIP(\phi, \lambda)$. So that, the distribution of ZIP can be regarded as the amalgam of denegerated distribution (0) and Poisson ($\lambda$). when $\phi = 0$, the distribution of ZIP is reduced into Poisson. Other method to construct ZIP distribution $Y \sim ZIP(\phi, \lambda)$ can be conducted by getting Bernouli dan Poisson involved simultaneously. For instance $Bernoulli (1 - \phi), X \sim Poisson(\lambda)$, where in Z and X independent. The model of Zero-inflated (Mullahy 1986; Lambert 1992) is considered as a rank of model which can take in hand from counted data of bigger zero-frequency percentage. Negative binomial and ZIP, however, are just restricted on over-dispersion data. Therefore, the reason of restriction is required to be a method that is coped with over-dispersion and under-dispersion nuisance.

The alternative model is able to deal over and under-dispersion through the relevance of restricted generalized Poisson regression (RGPR) [3], which is utilized as canonically-linked function $log \mu_i = X'\beta$. It is so-called restricted due to dispersion parameter $\alpha$ that is reduced at $1 + a\mu_i > 0$ and $1 + a\mu_i > 0$ Cui, Kim dan Zhu [1]. If it is addressed at $\alpha = 0$, it is then called Poisson distributin. If $\alpha > 0$ is identified as Over-dispersion, and $-2/\mu_i < \alpha < 0$ is identified as Under-dispersion, it can handle over and under-dispersion simultaneously. but range of under-dispersion on RGPR is merely restricted on $-2/\mu_i < \alpha < 0$. Consequently, it is necessary to extend a model within more widely-under-dispersed region.

Research on the handling of overdispersion and underdispersion problems has been done as [11] Binomial Negative regression to analyze the rate of death of patients infected with AIDS infection to overcome overdispersion, [12] Zero-Inflated Poisson and Negative Binomial regression for technological analysis, [13] Generalized Poisson regression And Binomial Negative to overcome the overdispersion, [14] a mixture of Generalized Poisson and GARCH regression models to overcome the problem of counting data that is influenced by time.

The distribution of Conway-Maxwell-Poisson (CMP) is the development and generalization of previously known distributions of Poisson, Bernouli and Geometry distributions as a particularly flexible case in describing overdispersion and underdispersed data with a wider dispersion range. The more advanced the science, the more complex the problem of available data, especially the problem of count data that do not meet the assumption of equidispersion. So, with the need of solving the problem, we will do research on data modeling over overdispersion and underdispersion cultivation using Conway-Maxwell Poisson (COM-Poisson) distribution. The probability function for discrete random variable $Y$, where in $Y \sim Poisson (\theta)$ is [10]

$$f(y, \lambda) = \frac{\lambda^y e^{-\lambda}}{y!} \quad y = 0,1,2,...$$

One of the properties of the Poisson distribution is the ratio of the linear sequential probabilities to $x$.

$$\frac{P(Y=y-1)}{P(Y=y)} = \frac{\lambda}{y} \quad (1)$$

In some cases, (1) may occur not linearly decreasing in $x$, i.e., distributions having thicker and thinner tails, or

$$\frac{P(Y=y-1)}{P(Y=y)} = \frac{x^\nu}{\lambda} \quad (2)$$

The distribution is calculated in equation (2) called as the distribution of Conway-Maxwell Poisson (COM-Poisson). $\nu$ is dispersion parameter in which $\nu = 1$ is named equidispersion, $\nu > 1$ under.
dispersion, and \( v < 1 \) over-dispersion. The Distribution of COM-Poisson fits into exponential relatives, that offers a benefit for parameter estimation, conjugate prior, and so forth.

In section 2, we describe the COM-Poisson distribution, sufficient statistics, COM-Poisson moment generating function, COM-Poisson regression formula, maximum likelihood estimation, and dispersion test. In section 3, and 4, we describe on the COM-Poisson regression application with some real and simulated data cases and then compare COM-Poisson with some alternative models such as binomial negative (NB), generalized Poisson (GP) to see goodness of fit of the model. While, in section 5 we state the conclusion.

2. Conway-Maxwell-Poisson

This section is divided into two sections, the first Section discusses the characteristics and properties of the Conway-Maxwell-Poisson distribution, sufficient statistics, moment generating functions, the second section discuss the COM-Poisson regression models, parameter estimates and dispersion test.

2.1. Conway-Maxwell-Poisson (COM-Poisson) Distribution

Conway-Maxwell-Poisson distribution (CMP) is initially instituted by Conway and Maxwell (in 1962) in the context of linear system, which is then recognized as generalized Poisson distribution including Bernoulli and Geometric as particular and flexible case to control under-dispersed and over-dispersed. random variable \( Y \sim \text{COM} - \text{Poisson}(\lambda, v) \) and parameter \( \lambda > 0 \) and \( v \geq 0 \), the form of COM-Poisson probability function is explained as follow [4]:

\[
P(Y = y|\lambda, v) = \frac{\lambda^y}{(y!)^v Z(\lambda v)} \quad y = 0,1,2, \ldots \quad \lambda > 1, v \geq 1
\]

Wherein

\[
Z(\lambda, v) = \sum_{s=0}^{\infty} \frac{\lambda^s}{(s!)^v}
\]

\( \lambda \) : location parameter,

\( v \) : dispersion Parameter Dispersi.

\( Z(\lambda, v) \) : constant normalization.

in which \( v > 1 \) underdispersi and \( v < 1 \) overdisperi

From equation (3), in \( \frac{\lambda^s}{(s!)^v} \) that is convergent to a series of \( \lambda > 0, v > 0 \), due to sub-square of (3), that is

\[
\frac{\lambda}{s^v}
\]

That tends to approach 0 if \( s \to \infty \).

COM-Poisson is each of generalized discrete distributions that is observable like:

- Poisson : in \( (v = 1) \), as the result of
  \[
  Z(\lambda, v) = \exp(\lambda) \to Y \sim \text{Poi}(\lambda)
  \]
  \[
  P(Y = y|\lambda, 1) = \frac{\lambda^y}{y! \ e^\lambda}
  \]

- Geometric: in \( (v = 0, \lambda < 1) \), as the result of
  \[
  Z(\lambda, v) = \sum_{s=0}^{\infty} \frac{\lambda^s}{s^v} = \frac{1}{1 - \lambda} \to Y \sim \text{Geo}(1 - \lambda)
  \]
  \[
  P(Y = y|\lambda < 1, v = 0) = \lambda^y (1 - \lambda) , x = 1,2,3, \ldots
  \]

- Bernoulli : in \( (v \to \infty) \), as the result of
  \[
  Z(\lambda, v) = 1 + \lambda \to Y \sim \text{Ber}(\frac{\lambda}{1 + \lambda})
  \]
  \[
  P(Y = 1|\lambda, v \to \infty) = \frac{\lambda}{1 + \lambda}
  \]
Meanwhile \( v = 0, \lambda \geq 1, Z(\lambda, v) \) does not appear convergent as the consequence of being unidentified.

Since the equation (4) that does not provide close form of the moment so in [10] has statement that explains that it is equally assessed as asymptotic, it is

\[
Z(\lambda, v) = \frac{\exp(v \lambda)}{\lambda^{1/2} (2\pi)^{(v-1)/2} \sqrt{v}} 
\]

(6)

From the equation (6) is acquired that the equation (3) turns into

\[
P(Y = y | \lambda, v) = \frac{\lambda^{y} v^{-1} \lambda \exp(y) (2\pi)^{(v-1)/2} \sqrt{v}}{(y!)^v \exp(v \lambda)} 
\]

(7)

2.1.1 Sufficient Statistic. Likelihood function of COM-Poisson by creating observable sets of \( n \) \( y_1, y_2, \ldots, y_n \) independent dan identical is calculated as follow:

\[
L(y_1, y_2, \ldots, y_n | \lambda, v) = \frac{\prod_{i=1}^{n} \lambda^{y_i} v^{-1} \lambda \exp(y_i) Z^n(\lambda, v)}{(\prod_{i=1}^{n} y_i!)^v} 
\]

\[
= \lambda^{\sum_{i=1}^{n} y_i} \exp(-v \sum_{i=1}^{n} \log(y_i)) Z^n(\lambda, v) 
\]

\[
= \exp(\sum_{i=1}^{n} y_i \log \lambda - v \sum_{i=1}^{n} \log(y_i)) Z^n(\lambda, v) 
\]

\[
= \exp(k_1 \log \lambda - v k_2) Z^n(\lambda, v) 
\]

(8)

wherein \( k_1 = \sum_{i=1}^{n} y_i \) dan \( k_2 = \sum_{i=1}^{n} \log(y_i) \). using factorization theorem, \( (k_1, k_2) \) is regarded to be sufficient statistic along with parameter \( (\lambda, v) \) for \( y_1, y_2, \ldots, y_n \). Hence, the equation (8) points out that COM-Poisson distribution is also included into exponential family.

2.1.2 Moment-generating function on COM-Poisson. COM-Poisson moment can be determined by lending recursive formula. Shmueli explains in [4] that

\[
E(Y^{k+1}) = \begin{cases} 
\lambda E(Y + 1)^{1-v} & k = 0 \\
\frac{d}{d \lambda} \frac{\lambda}{\lambda} E(\lambda) E(Y^k) & k > 0 
\end{cases} 
\]

is figured out by making use of asymptotic approximation in \( Z(\lambda, v) \) to the equation (7). Therefore, according to Minka’s statement [10], moment of asymptotic join sufficient statistic of parameter \( r (\lambda, v) \):

\[
E[Y] = \lambda \frac{\partial \log Z(\lambda, v)}{\partial \lambda} 
\]

\[
E[\log(Y^k)] = - \frac{\partial \log Z(\lambda, v)}{\partial v} \approx \frac{1}{2v^2} \log \lambda + \frac{\lambda^{1/v}}{2v} \left( \frac{\log \lambda}{v} - 1 \right) 
\]

The approximation is accurate to be \( v \leq 1, \lambda > 10^v \).

Variance value and Mean \( Y \) is estimated by breaking down \( Z(\lambda, v) \) against \( \log(\lambda) \). Consequently, it is marked in the point;

\[
E(Y) = \lambda \frac{\partial \log Z(\lambda, v)}{\partial \lambda} \approx \lambda^{1/v} - v - \frac{1}{2v} 
\]

\[
Var(Y) = \frac{\partial E(Y)}{\partial \log \lambda} \approx \frac{1}{v} \lambda^{1/v} 
\]

2.2. COM-Poisson Regression Formulas

Ordinary linear regression models use linearity to describe the relationship between mean of response variables and the set of explanatory variables. With assuming that the response distribution is normal. Generalize linear model (GLM) is an extension of the standard linear regression model for handling the distribution of non-normal responses. And possible nonlinear functions for the mean. GLM is
defined by the set of independent random variables \( Y_1, \ldots, Y_n \) respectively with the distribution form of the exponential family and has 3 components as follows:

- Random component : distribution of every \( Y_i \) has canonical form and depends on one parameter \( \theta_i \) (\( \theta_i \) unnecessary equation).
- Linier predictor: for parameter vector \( \beta = (\beta_1, \beta_2, \ldots, \beta_n)^T \) and nxp Matrix. Model X which contains the value of \( p \) explanatory variable and \( n \) observable, linier predictor \( X\beta \).
- Link function: \( g \) is any of correlated functions that is monotonous, it means \( g \) correlated function in association to the function \( E(Y_i) \) with its predictor.

\[
g(\mu_i) = x_i^T \beta
\]
\[
\mu_i = E(Y_i)
\]

Since COM-Poisson is derived from the exponential family that has been proven in equation (8), with the link function \( \log(\lambda) \) for the COM-Poisson regression equation, which has a wider range for its underdispersion region. Seller and Shmueli [5] introduces a compliant COM-Poisson formula for dispersion. By using the link function

\[
\log(\lambda) = \beta + \sum_{j=1}^{p} \beta_j X'_j = X_i \beta
\]
\[
\log(\nu) = \gamma + \sum_{j=1}^{p} \gamma_k G'_k = G_i \gamma
\]

In which

- \( G_k \) : Dummi variable that is correlating to each of groups \( K \) from the data
- \( \beta, \gamma \) : coefficient is estimated by the extension of log-likelihood.

### 2.2.1 Estimation of regression parameters.

A few methods that can be performed in the parameter estimation of distribution is Maximum Likelihood estimation. In random variable \( Y_1, \ldots, Y_n \) and \( Y_i \sim COM - Poisson (\lambda, \nu) \), that is acquired, log function of likelihood distribution is then estimated;

\[
\log L(\lambda, \nu) = \sum_{i=1}^{n} (y_i \log \lambda - \nu \log(\nu) - \log Z(\lambda, \nu))
\]

In \( \bar{Y} = \frac{1}{n} \sum_{i=1}^{n} y_i \) dan \( \bar{\log(Y)} = \frac{1}{n} \sum_{i=1}^{n} \log(y_i) \)

which is acquired as

\[
\log L(\lambda, \nu) = n \ \bar{Y} \log \lambda - \nu \log(\nu) - n \log Z(\lambda, \nu)
\]

wherein likelihood is a function of sufficient statistic \( \bar{Y} \) and \( \log(Y!) \), for example, \( \theta = (\log(\lambda), \nu) \). Parameter Estimation can be determined by applying Newton Rhapson method. Log-likelihood gradient is measured as

\[
\nabla L(\theta) = n \begin{bmatrix} \frac{\bar{Y} - E[X]}{\log(Y!)} \\ -(\log(Y!)) \end{bmatrix}
\]

and the second derivation is:

\[
\nabla^2 L(\theta) = n \begin{bmatrix} -\text{var}(Y) & -\text{cov}(Y, \log(Y!)) \\ -\text{cov}(Y, \log(Y!)) & \text{var}(\log(Y!)) \end{bmatrix}
\]

Newton Rhapson formula is estimated as

\[
\theta^{t+1} = \theta^t - (\nabla^2 L(\theta))^{-1} \nabla L(\theta)
\]

Initial value probably appears for iteration that MLE from poisson is acquired as \( (\lambda = \bar{Y}, \nu = 1) \) so \( \theta = (\log(Y), 1) \).
2.2.2 Dispersion observation. To examine dispersion of data, it is required to conduct a test. Hypotheses $H_0: \nu = 1$ vs $H_1: \nu \neq 1$, for example, deals with Poisson Regression ahead of COM-Poisson by statistical observation:

$$C = -2\log A = -2\left[\log L(\hat{\beta}(0), \hat{\nu} = 1) - \log L(\hat{\beta}, \hat{\nu})\right]$$

Wherein $A$ is statistical ratio observation of likelihood, $\hat{\beta}(0)$ is maximum likelihood estimation that is acquired in the hypotheses $H_0: \nu = 1$ as (based on Poisson estimation), and $(\hat{\beta}, \hat{\nu})$ maximum likelihood estimation that is under-assumption of COM-Poisson. With assumption of hypotheses $H_0: \nu = 1$, $C$ is approximated $\chi^2$ into $db$ 1. If $\hat{\nu} > 1$ can be identified that data is underdispersion and and $\hat{\nu} < 1$ is overdispersion.

3. Application for COM-Poisson Modeling

An example of regression application based on COM-Poisson Model will involve Simulation and real data. In the calculation, this research uses software R 3.3.3 by using COMPoissonReg Package for COM-Poisson regression calculation with glm.cmp() function, package CompGLM with glm.comp() function, package MASS for Negative binomial with glm.nb function(), And the VGAM package for Generalized Poisson with the function vglm().

3.1 Simulation data

The application of COM-Poisson regression method in the simulation data is divided into 2 cases, the simulation data with parameter values $(\beta, \nu)$ is determined and the simulation data with parameter value $(\lambda, \nu)$ is determined.

3.1.1 Case 1: Simulated Data by given parameter regression value $(\beta, \nu)$. We will weigh up numerous simulated data, wherein random variable $Y$ is resurrected based on. Then, various data is largely employed such as $n=20,50,100,500$, determining parameter value $\beta_0, \beta_1 = 1$, and the difference of various dispersed provided provided $\nu = 0.1, 0.75, 1, 2$, to represent over-dispersion and under-dispersion. Furthermore, it is conducted 100-time-simulation. The calculation value, is aimed at examining regression COM-Poisson performance compared to Negative Binomial and Generalized Poisson.

3.1.2 Case 2: Simulated data by given Parameter value $(\lambda, \nu)$. The second instance on simulated data, we will also take into account the various simulated-data, wherein $Y$-random variable is selected based on the distribution of COM-Poisson. The variety that is largely employed worth $n=20,100,500$, then we predict the value of lamda or mu = 1,2 and 7, and a variety of dispersed value worth $\nu = 0.1, 0.5, 0.75, 1, 5, 10, 20, 30$ and 40, to represent the instance of over-dispersion dan under-dispersion. By the given value earlier, it aims at taking a consideration of the performance achieved by COM-Poisson compared to Negative Binomial and Generalized Poisson.

3.2 Real Data

Real data that is used in application of COM-Poisson model is The case of IPB student’s failure to pass one of subjects, statistical method (Over-dispersion dan Under-dispersion cases). This study was taken from data of IPB students on 2008/2009 and 2011/2012 statistical method score that was obtained from Statistic department, including additional data obtained from directory of joint preparation level (TPB) IPB. Response variable $Y$ is in accordance with the total number of students—having passed this statistical Method. With the predictor variable $X_1$ that is the student's graduation variable on the statistical method subject which consists of two categories "pass the course of Statistics method" and "not pass the course of Statistics method", with the graduation criteria above the value of 65 or With quality lettersof C. predictor variable $X_2$ Lecturer of the statistical method, namely "lecturer of the department Statistiaka" and "outside lecturer of the department of Statistics. Observable parallel class in 2008-2009 research is recorded as many as 30 classes, whereas 2011/2012 is recorded as many as
31. Every department generally owns one parallel class, instead of two classes on 2011/2012 agribusiness department, two classes on public development, and three classes on veterinary.

### 4. Results

The result of some case in experimental design sub-bab is described below, with three kinds of sub-bab including result of case 1, result of case 2, and result of real data:

#### 4.1 Result of Case 1.

The result of simulation is enclosed in Table 1, estimated as parameter regression and standard error for each parameter regression estimator. Table 2, which describes AIC and BIC of three models, are designed to estimate the quality of model from three models.

| N  | V | $\beta_0$=1 | $\beta_1$=1 |
|----|---|---|---|
| 20 | 0.1 | 1.60 1.93 0.42 1.33 1.35 0.40 0.04 | 0.50 0.70 0.20 0.50 0.70 0.15 0.08 |
| 0.75 | 1.27 1.32 1.24 1.20 1.156 1.35 1.20 | 0.40 0.30 1.02 0.30 0.30 0.68 0.50 |
| 1 | 1.01 1.00 1.46 1.03 1.07 1.33 1.00 | 0.52 0.5 0.96 0.53 0.5 0.70 0.61 |
| 2 | 0.31 0.21 0.90 0.44 0.49 0.81 1.73 | 0.40 0.50 0.60 0.40 0.60 0.50 0.90 |
| 50 | 0.1 | 0.30 0.54 0.10 0.20 0.20 0.10 0.01 | 0.21 0.25 0.02 0.23 0.28 0.02 0.05 |
| 0.75 | 1.24 1.29 1.12 1.15 1.16 1.08 0.88 | 0.32 0.32 0.51 0.31 0.31 0.35 0.27 |
| 1 | 0.97 0.97 1.08 0.98 0.98 1.04 1.10 | 0.25 0.23 0.51 0.26 0.25 0.34 0.39 |
| 2 | 0.50 0.35 1.17 0.69 0.71 1.08 2.30 | 0.31 0.36 0.62 0.26 0.34 0.49 0.75 |
| 100 | 0.1 | 3.27 5.52 1.00 2.34 3.58 0.99 0.10 | 0.15 0.21 0.21 0.16 0.22 0.15 0.03 |
| 0.75 | 1.32 1.38 1.09 1.20 1.21 1.06 0.81 | 0.14 0.13 0.33 0.15 0.15 0.21 0.24 |
| 1 | 1.01 1.00 1.05 1.01 1.01 1.04 1.04 | 0.22 0.22 0.38 0.21 0.22 0.27 0.29 |
| 2 | 0.46 0.33 1.04 0.69 0.73 1.04 2.14 | 0.19 0.22 0.40 0.17 0.23 0.32 0.54 |
| 500 | 0.1 | 3.44 5.78 1.18 2.47 3.85 1.10 0.13 | 0.06 0.08 0.10 0.07 0.09 0.07 0.01 |
| 0.75 | 1.20 1.34 1.00 1.18 1.16 1.00 0.76 | 0.08 0.08 0.15 0.08 0.09 0.09 0.01 |
| 1 | 1.00 1.00 1.02 1.00 1.00 1.01 1.02 | 0.08 0.08 0.16 0.08 0.09 0.11 0.13 |
| 2 | 0.49 0.34 1.00 0.69 0.71 0.99 2.01 | 0.09 0.10 0.18 0.08 0.10 0.14 0.20 |

It is proved from Table 1 that values $n$, $v$ dan $\beta$ which are given in advance, are estimated in which the overall simulated calculation demonstrates estimation $\hat{v}, \hat{\beta}$ come closer or even equal to $v$ dan $\beta$ which is given by method of COM-Poisson. Meanwhile, by the method of NB and GP.
Values of $\hat{v}, \hat{\beta}$ have variety of estimations, in which estimated value more than the point of $\hat{\beta}$ whether or not it is predictive. For that reason, by the levels of dispersed-parameter, simulated data of COM-Poisson typically corresponds to deal with a range of variant data.

Table 2. The comparison of AIC and BIC value from the distribution of GP, NB, and COM

| N   | $\nu$ | AIC  | BIC  |
|-----|-------|------|------|
|     |       | GP   | NB   | COM  |
|     |       | GP   | NB   | COM  |
| 20  | 0.1   | 35.97| 35.75| 32.25|
|     | 0.75  | 56.45| 56.88| 56.28|
|     | 1     | 54.81| 55.26| 54.57|
|     | 2     | 30.44| 31.42| 29.55|
| 50  | 0.1   | 35.81| 69.88| 27.60|
|     | 0.75  | 144.11| 144.35| 144.10| 149.85| 149.80|
|     | 1     | 133.80| 134.27| 133.53| 139.50| 139.20|
|     | 2     | 106.92| 109.82| 105.13| 112.66| 115.56| 110.80|
| 100 | 0.1   | 646.27| 628.32| 515.25| 654.08| 636.14| 523.00|
|     | 0.75  | 297.39| 297.42| 297.05| 305.20| 305.24| 304.80|
|     | 1     | 260.32| 260.58| 260.15| 268.13| 268.40| 267.90|
|     | 2     | 208.15| 212.55| 205.50| 215.69| 220.37| 213.30|
| 500 | 0.1   | 3178.6| 3081.3| 2462.0| 3191.2| 3094.0| 2474.0|
|     | 0.75  | 1411.3| 1438.9| 1435.7| 1424.0| 1451.0| 1448.0|
|     | 1     | 1325.2| 1325.8| 1325.2| 1337.9| 1338.5| 1337.0|
|     | 2     | 1021.5| 1037.6| 1010.8| 1034.1| 1050.3| 1023.0|

In general, the findings of simulated COM-Poisson provide the evidence of smaller values AIC and BIC compared to than other method in Table 2. The followings are findings that are required to consider the value of $\nu = 0.1 \ , n = 20, 50, 100, 500$, in which COM-Poisson illustrates the smallest of AIC and BIC in comparison to others, the value of $\nu = 0.75, 1 \ , n = 20, 50, 100, 500$, in which COM-Poisson illustrates the relatively-similar of AIC and BIC one each other, the value of $\nu = 2 \ , n = 20, 50, 100, 500$, in which COM-Poisson demonstrates the smallest of AIC and BIC among others. It is eventually concluded that the closer $\nu$-value approaches point 1, the more equal COM-Poisson, NB, and GP are relatively-intended in the modelling. The higher $\nu$-value is calculated, COM-Poisson will be much better in the modelling.

4.2 Result of Case 2
The calculation finding is made a list in Figure 1 up to Figure 3, which are regarding plot of various simulated data as information regarding AIC from three models for the quality standard.
Figure 1. AIC plot comparison for which \( n=20, \lambda = 1 \), \( \mu = 0.1, 0.5, 0.75, 1, 5, 10, 20, 30, 40 \)

Figure 2. AIC plot comparison for which \( n=20, \lambda (\mu) = 2 \), \( \nu = 0.1, 0.5, 0.75, 1, 5, 10, 20, 30, 40 \)

Figure 3. AIC plot comparison for which \( n=20, \lambda (\mu) = 7 \), \( \nu = 0.1, 0.5, 0.75, 1, 5, 10, 20, 30, 40 \)

Figure 4. AIC plot comparison for which \( n=100, \lambda = 1 \), \( \mu = 0.1, 0.5, 0.75, 1, 5, 10, 20, 30, 40 \)

Figure 5. AIC plot comparison for which \( n=100, \lambda (\mu) = 2 \), \( \nu = 0.1, 0.5, 0.75, 1, 5, 10, 20, 30, 40 \)

Figure 6. AIC plot comparison for which \( n=100, \lambda (\mu) = 7 \), \( \nu = 0.1, 0.5, 0.75, 1, 5, 10, 20, 30, 40 \)
From the simulation results obtained AIC values plot in the Figure 1 up to 9, overall for some types of lots of data \( n = 20,100,500, \mu = 1,2, \text{and} 7, \) and \( \nu = 0.1, 0.5, 0.75,1,5,10, 20,30,40, \) it was found that the value of AIC for the entire simulation result of COM-Poisson Model has the smallest value among Negative Binomial and Generalized Poisson models. COM-Poisson can handle very well in cases of overdispersion and underdispersion, followed by generalized Poisson and Binomial negative models. That means COM-Poisson is very flexible in handling overdispersion and underdispersive cases supported by a wider range of dispersion parameter values.

4.3. Result of Real Data Case
For the purposes of data analysis using COM-Poisson regression model we need to check the value of variant and mean Y response variable.

Table 3. Table 3. Mean value and variable
Respon Data variance in 2008/2009 and 2011/2012 graduation

| Response     | Mean | Variance |
|--------------|------|----------|
| 2008/2009    | 7.5  | 23       |
| 2011/2012    | 7.7  | 19       |

From Table 3, mean value in 2008/2009 is more diminutive than variance between 7.5 and 23 respectively, so is mean value in 2011/2012. From the estimation, it is proved that equidispersion of Poisson assumption is declined, which indicate as overdispersion, in 2008/2009, based on residual deviance point, it is 3.0847 which is relatively equal to \( X^2_{(0.05,1)} = 3.841459. \) Therefore it is indicated that there is none of over-dispersion nor under-dispersion in 2011/2012 residual deviance is estimated.
at 0.51657, which is smaller than point $\chi^2_{0.05,1} = 3.841459$, which have indicated as underdispersion. Next, we will be done data analysis using COM-Poisson regression model, with regression model.

$$\log(\lambda) = \beta + \beta_1 X_1 + \beta_2 X_2$$

Table 4. Estimated coefficient and standard of error in 2008/2009 graduation

| Model          | $\hat{\beta}_0 (\hat{\sigma}_{\hat{\beta}_0})$ | $\hat{\beta}_1 (\hat{\sigma}_{\hat{\beta}_1})$ | $\hat{\beta}_2 (\hat{\sigma}_{\hat{\beta}_2})$ |
|----------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|
| Binomial       | 1.07                                          | 0.54                                          | 1.10                                          |
| Negative       | (0.42)                                        | (0.37)                                        | (0.41)                                        |
| Generalized    | 1.31                                          | 0.52                                          | 0.98                                          |
| Poisson        | (0.50)                                        | (0.30)                                        | (0.33)                                        |
| COM-Poisson    | 1.74                                          | 0.82                                          | 1.51                                          |
| ($\hat{\theta} = 1.542, \hat{\sigma}_{\hat{\theta}} = 1.1394$) | $\hat{\beta}_0 (\hat{\sigma}_{\hat{\beta}_0})$ | $\hat{\beta}_1 (\hat{\sigma}_{\hat{\beta}_1})$ | $\hat{\beta}_2 (\hat{\sigma}_{\hat{\beta}_2})$ |

To ensure the estimation, we calculate with dispersed assessment through hypotheses, in which $H_0: \nu = 1$ vs $H_1: \nu \neq 1$ is whether the data is appropriate for using Poisson regression or using COM-Poisson regression. From table 4, it is proved that point $\hat{\theta} = 1.542$ is higher than 1, indicating as under-dispersed. From Chi-squared-test of equal-dispersion, it is acquired estimation $\chi^2 = 0.2918$, df = 1, p-value = 0.58095. Since p-value > $\alpha$ is denoted as $H_0$ accepted, it is then drawn the conclusion that Poisson regression is reasonable to estimate data analysis. From table 3, it is estimated that $\hat{\nu} = 8.6944 > 1$, having indicated as Under-dispersed. In support of statement validity, it is essential to estimate dispersion from Chi-squared test for equal-dispersion $\chi^2 = 4.9348$, df = 1, p-value = 0.026321 < $\alpha$. It can be concluded that $H_0$ is declined—it means match with COM-Poisson Regression. From table 2 and 3 by applying negative Binomial, estimated regression coefficient will be equally calculated to Poisson. Generalized Poisson regression model in Table 5 is hardly estimated because the model is limited in range when Under-dispersed.

Table 5. Estimated coefficient and standard error of Estimated coefficient in 2011/2012

| Model            | $\hat{\beta}_0 (\hat{\sigma}_{\hat{\beta}_0})$ | $\hat{\beta}_1 (\hat{\sigma}_{\hat{\beta}_1})$ | $\hat{\beta}_2 (\hat{\sigma}_{\hat{\beta}_2})$ |
|------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|
| Binomial         | 1.07                                          | 0.54                                          | 1.10                                          |
| Negative         | (0.42)                                        | (0.37)                                        | (0.41)                                        |
| Generalized      | 1.31                                          | 0.52                                          | 0.98                                          |
| Poisson          | (0.50)                                        | (0.30)                                        | (0.33)                                        |
| COM-Poisson      | 1.74                                          | 0.82                                          | 1.51                                          |
| ($\hat{\theta} = 1.542, \hat{\sigma}_{\hat{\theta}} = 1.1394$) | $\hat{\beta}_0 (\hat{\sigma}_{\hat{\beta}_0})$ | $\hat{\beta}_1 (\hat{\sigma}_{\hat{\beta}_1})$ | $\hat{\beta}_2 (\hat{\sigma}_{\hat{\beta}_2})$ |

The estimator of regression parameters obtained can be modeled as follows:

$$\log(\lambda) = 13.2 - 0.52 \cdot (\text{Student graduation})_1 + 8.58 \cdot (\text{Supporting lecturer})_2$$
The particular model has the meaning that, many of the graduation of students to the course of statistics method (\( \lambda \)) is influenced by graduation of students and lecturers subjects. \( \hat{\beta}_0 = 13 \) means that the average number of students passed the statistical program without influenced by the passing variables and lecturers, for \( \hat{\beta}_1 = -0.52 \) means that the student passing variables have a negative effect on the number of graduation students, \( \hat{\beta}_2 = 1.1 \) means variable lecturer has a positive influence on log of the graduation students number on the course of statistics method.

**Table 6. Comparisson of Goodness of fit for regression Models in 2008/2009**

| Model                  | AIC   | BIC   |
|------------------------|-------|-------|
| Negative Binomial      | 25.38 | 22.93 |
| Generalized Poisson    | 25.15 | 22.69 |
| COM-Poisson            | 25.09 | 22.63 |

**Table 7. Comparisson of Goodness of fit for regression Models in 2011/2012**

| Model                  | AIC   | BIC   |
|------------------------|-------|-------|
| Binomial Negative      | 23.57 | 21.12 |
| Generalized Poisson    | -     | -     |
| COM-Poisson            | 18.64 | 16.18 |

In determining Goodness of fit the model which is used, in this research AIC and BIC are represented. For the quantity of graduation in 2008/2009 based on hypotheses, it points out that the data agrees with Poisson regression. In addition, in Table 6, it considers the quality standard of the model in 2008/2009, in which it is proved that AIC and BIC with the property of GP, NB and COM-Poisson result in the same value which is not too different to estimate from other AIC and BIC. For that reason, it meets the certainty that Poisson regression seems apposite for such a data. For the 2011/2012 data in accordance with hypotheses, it is estimated that the data is appropriate in the use of COM-Poisson. Additionally, in table 7, it weighs up the quality standard in 2011/2012. From the calculation of AIC dan BIC estimation, it seems that COM-Poisson regression contain the smallest value among others. For that reason, it is true to claim that COM-Poisson regression corresponds with the data.

**5. Conclusion**

Based on the above description can be taken some conclusions about the modeling using some of the above models, including:

- If the specified dispersion parameter values are greater, especially in the case of underdispersion, COM-Poisson performs very well in comparison with GP and NB models of the smallest AIC values obtained among others.

- From the Simulation results show that the generated data is not derived from some generalizations of the COM-Poisson distribution as for Poisson \( v = 1 \), Geometry, \( (v = 0, \lambda <1) \), Bernouli \( (v \rightarrow \infty) \). Thus, COM-Poisson is used to model it, and the AIC values are smaller than GP and NB models.

The COM-Poisson model is more flexible in dealing data with overdispersion and underdispersion problems. This is because the underdispersion value area is wider than the GP model.
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References
[1] Cui Y, Kim D, and Zhu J Genetics 174 2159-2172
[2] Dobson J A 2002 An Introduction To Generalized Linier Models 2nd Edition (London :A CRC Pres Company)
[3] Famoye F 1993 Comm. Statis. Theory Methods 22 1335-1354 MR1225247
[4] Shmueli G Minka T Borle S and Boatwright P 2005 J. of Roy Stat Soc. 54 127–42
DOI: 10.1111/j.1467-9876.2005.00474.x
[5] Sellers F K and Shmueli G 2011 Annals.of.App.Stat. 4 943-61
[6] Kadane J Shmueli G Minka T Borle and Boatwright P 2006 Bayes.anal. 1 363-374
[7] Kurniasari E 2013 Pemodelan Kasus Ketidak Lulusan Mahasiswa IPB pada Mata Kuliah Metode Statistika Thesis Institute Pertanian Bogor
[8] Sellers F K Borle and S Shmueli G 2010 J.App.Stoc.Models.in.Bus.andInd DOI: 10.1002/asmb.918
[9] Guikema S and Gofelt J 2008 Risk.anal. 28 213-223
[10] Minka T Shmueli G Kadane J Borle J and Boatwright P 2003 Tech.report.CMU.stat. department, http://www. stat.cmu.edu/tr/tr776/tr776 .html.
[11] Mohd A Aand Nyi N N 2012 IOSR.J.of Math (IOSR-JM), ISSN: 2278-5728. 3(6) 34-8
[12] Jong M K and Sunghae J 2016 Int.J.of Soft Eng and Its App 10(12) 431-48
[13] Ayunda M Yeni S Haris E Sista R and Purhadi 2013 Int.J.of Scientific & Tecchnology Research, 2(8) ISSN 2277-8616
[14] Fukang Z 2012 J. Math. Anal. Appl. 389 58–71