High-Fidelity Large-Signal Order Reduction Approach for Composite Load Model

Zixiao Ma, Zhaoyu Wang, Dongbo Zhao, and Bai Cui

Abstract—With the increasing penetration of electronic loads and distributed energy resources (DERs), conventional load models cannot capture their dynamics. Therefore, a new comprehensive composite load model is developed by Western Electricity Coordinating Council (WECC). However, this model is a complex high-order nonlinear system with multi-time-scale property, which poses challenges on stability analysis and computational burden in large-scale simulations. In order to reduce the computational burden while preserving the accuracy of the original model, this paper proposes a generic high-fidelity order reduction approach and then apply it to WECC composite load model. First, we develop a large-signal order reduction (LSOR) method using singular perturbation theory. In this method, the fast dynamics are integrated into the slow dynamics to preserve the transient characteristics of fast dynamics. Then, we propose the necessary conditions for accurate order reduction and embed them into the LSOR to improve and guarantee the accuracy of reduced-order model. Finally, we develop the reduced-order WECC composite load model using the proposed algorithm. Simulation results show the reduced-order large signal model significantly alleviates the computational burden while maintaining similar dynamic responses as the original composite load model.

I. INTRODUCTION

POWER system load modeling is important in stability analysis, optimization, and controller design [1]. Although this topic has been widely studied, it is still a challenging problem due to increasing diversity of load components and lack of detailed load information and measurements.

Load models can be classified into static and dynamic ones. Static load models such as static constant impedance-current-power (ZIP) model and exponential model have simple model structures [2][3]. However, they cannot capture the dynamic load behaviors [4]-[10]. Motivated by the 1996 blackout of the Western Systems Coordinating Council (WSCC), a widely-used dynamic composite load model was developed [11]. The model consists of a ZIP and a dynamic induction motor (IM). It was designed to represent highly stressed loading conditions in summer peak hours. However, this interim load model was unable to capture the fault-induced delayed voltage recovery (FIDVR) events [7]. A preliminary WECC composite load model (WECC CLM) was proposed by adding an impedance representing the electrical distance between substation and end-users, an electronic load and a single-phase motor [12]-[14]. After a series of improvements, the latest WECC composite load model (CMPLDWG) is developed as shown in Fig. 1. The electrical distance between the substation and end-users is represented by a substation transformer, a shunt reactance, and a feeder equivalent. The model consists of three three-phase motors, one aggregate single-phase AC motor, one static load, one power electronics component, and one distributed energy resource (DER). The DER in CMPLDWG is currently represented by the PVD1 model [15]. However, PVD1 has 5 modules, 121 parameters, and 16 states, which is as complex as the CMPLDW itself. Therefore, the Electric Power Research Institute (EPRI) has developed a simpler yet more comprehensive model to replace PVD1, which is named as DER_A model [15].

The above WECC CMPLDW + DER_A model is a complex high-order nonlinear dynamical system with multi-time-scale property, which means the state vector is high-dimensional and the transient velocity of each state varies significantly. These characteristics result in two main challenges. Firstly, it increases the difficulty of dynamic stability analysis due to the numerous state variables. Secondly, it makes simulation studies of a high-order power system computationally demanding or even infeasible. There are two main reasons for this high computational burden. One reason is the shear dimensionality of the problem. The other comes from the two-time-scale property of the model. This makes solving the model a stiff ordinary differential equation (ODE) problem, which requires small time steps to calculate the fast dynamics, and consequently results in long computational time to capture slow dynamics. The fast dynamics are often introduced by the intentionally added inductance and capacitance, moment of inertia, and parasitic elements inherent in the system [16]. However, simply neglecting the fast dynamics may lead to modeling inaccuracies in dynamic response and stability property. In order to accelerate computation while maintaining the accuracy and faithful stability property of the original load model, it is imperative to develop a high-fidelity reduced-order load model. To our best knowledge, this is the first paper on dynamic order reduction of WECC composite load model especially containing the DER_A model.

The existing model reduction methods usually project the higher dimensional counterpart into a lower dimensional subspace where dynamic features of the original model dominate. Singular perturbation is the kind of method which considers the fast dynamics as boundary-layers and includes their solutions into slow dynamics. Singular perturbation method is suitable for analyzing two-time-scale problems and is widely
used in power systems analysis. Previous applications include the derivation of reduced-order modeling of synchronous machines [17], microgrids [18], and distribution grid-tied systems with wind turbines [19]. However, these papers fall short of guaranteed accuracy and cannot be directly applied to the WECC composite load model due to the different system characteristics.

Therefore, this paper develops a novel accuracy assessment theorem which takes into account the impact of external inputs on the accuracy of reduced system. By embedding the theorem, we propose a high-fidelity order reduction approach for WECC composite load model. The derived high-fidelity reduced-order model can replace the original model in power system simulations for stability analysis and control applications with less computational complexity. Specifically, we improve the accuracy from two aspects. Firstly, without any simplification or linearization, we adopt the large-signal order reduction (LSOR) method based on singular perturbation theory to maintain all the dynamic characteristics of the original system. Secondly, we propose necessary conditions for accurate order reduction, and then integrate them into the LSOR method to theoretically guarantee the high accuracy of the reduced-order model. Note that this proposed approach is general and can be applied to various dynamic models.

The rest of the paper is organized as follows. Section II proposes high-fidelity order reduction approach in a general form. Section III introduces mathematical representation of WECC composite load model. Section IV derives the reduced-order model using the proposed method. Section V shows the simulation results and analysis. Section VI concludes the paper.

II. HIGH-FIDELITY ORDER REDUCTION METHOD

Accurate load modeling is essential to power system stability analysis, optimization and control. To solve the challenges raised by high-order characteristics of WECC composite load model, we propose a general approach for high-fidelity order reduction in this section. We first introduce the LSOR method based on singular perturbation theory. A novel accuracy assessment theorem is then derived and embedded into the LSOR to guarantee the accuracy of reduced-order model.

A. LSOR Based on Singular Perturbation Theory

Consider a standard singular perturbation model as follows,

\[ \dot{x} = f(x, z, u, \varepsilon), \quad \varepsilon \dot{z} = g(x, z, u, \varepsilon), \]

where \( x \in \mathbb{R}^n \) represents slow state vector, \( z \in \mathbb{R}^m \) denotes fast state vector, \( u \in \mathbb{R}^p \) denote external input vector, and \( \varepsilon \in [0, \varepsilon_0] \); \( f \) and \( g \) are Lipshitz continuous functions.

Remark 1. Selecting the perturbation coefficient \( \varepsilon \) for real physical systems is challenging. In most cases, we pick it based on our knowledge of the real system. In cases where it is unclear which parameter is small, we can locally linearize the system around the equilibrium point and use modal decomposition to identify the slow and fast dynamics.

When \( \varepsilon \) is small, the fast transient velocity \( \dot{z} = g/\varepsilon \) can be much larger than that of the slow transient \( \dot{x} \). To solve this two-time-scale problem, we can set \( \varepsilon = 0 \), then equation (2) degenerates to the following algebraic equation,

\[ 0 = g(x, z, u, 0). \]

Assuming that equation (3) has at least one isolated real root, and satisfies the implicit function theorem, then for each argument, we can obtain the quasi-steady-state (QSS) solution in a local vicinity around the isolated root,

\[ z = h(x, u). \]

Substituting equation (4) into equation (1) and setting \( \varepsilon = 0 \), we obtain the QSS model,

\[ \dot{x} = f(x, h(x, u), u, 0). \]

We call the QSS system (5) the reduced-order model since its order drops from \( n + m \) to \( n \). The slow states can be obtained by solving the reduced-order model (5), whereas the fast states are represented by equation (4). However, (4) only gives approximate solution unless \( \varepsilon \) is zero. To quantify the error between approximate and actual fast states, we denote the error as \( y = z - h(x, u) \). Then in the fast-time-scale \( \tau = t/\varepsilon \), the dynamics of \( y \) are governed as follows,

\[ \frac{dy}{d\tau} = G(x, y, u, \varepsilon) = g(x, h(x, u) + y, u, \varepsilon) - \varepsilon \left[ \frac{\partial h}{\partial x} f(x, h(x, u) + y, u, \varepsilon) + \frac{\partial h}{\partial u} \dot{u} \right]. \]

Let \( \varepsilon = 0 \), we obtain the boundary-layer model:

\[ \frac{dy}{d\tau} = g(x, y + h(x, u), u, 0). \]

Note that the exact fast states are \( z = y + h(x, u) \), but we do not know \( (x, y) \). Therefore, if we can guarantee the accuracy of reduced-order model and boundary-layer model, then we can use their solutions \( (\dot{x}, \dot{y}) \) instead of \( (x, y) \). However, these models are exact only when \( \varepsilon \) is exactly zero, which is obviously not the case for the studied system. Thus, we need to quantitatively assess the accuracy of reduced-order model when \( \varepsilon \) is small yet nonzero. This motivates the next subsection.
B. High-Fidelity LSOR with Accuracy Assessment

Before deriving the performance guarantee of the proposed high-fidelity order reduction approach, we first introduce a few technical definitions and assumptions:

**Definition 1.** Class $\mathcal{K}$ function $\alpha : [0, t) \rightarrow [0, \infty)$ is a continuous strictly increasing function with $\alpha(0) = 0$.

**Definition 2.** Class $\mathcal{KL}$ function $\beta : [0, t) \times (0, \infty) \rightarrow [0, \infty)$ is a continuous function satisfying: for each fixed $s$, the function $\beta(r, s)$ belongs to class $\mathcal{K}$; for each fixed $r$, the function $\beta(r, s)$ is decreasing with respect to $s$ and $\beta(r, s) \rightarrow 0$ for $s \rightarrow \infty$.

**Definition 3.** $|f| = O(\varepsilon)$ is equivalent to $|f| \leq k_\varepsilon$.

**Assumption 1.** The functions $f, g$, and their first partial derivatives are continuous and bounded with respect to $(x, z, u, \varepsilon)$; $h$ and its first partial derivatives $\partial h/\partial x, \partial h/\partial u$ are locally Lipschitz; and the Jacobian $\partial g/\partial z$ has bounded first partial derivatives with respect to its arguments.

**Assumption 2.** The reduced-order model (5) is input-to-state stable with Lyapunov gain $\alpha$ as follows,

$$\hat{x} \leq \beta(\|x(0)\|, t) + \alpha(\|u\|),$$

where $\hat{x}$ is a solution of (5), $\beta$ is a function of class $\mathcal{KL}$, $\alpha$ is a class $\mathcal{K}$ function, and $\|\cdot\|$ denotes any $p$-norm.

**Assumption 3.** The origin of the boundary-layer model (7) is a uniformly globally exponentially stable equilibrium and the solution $\hat{y}$ of (7) follows that

$$\|\hat{y}(\tau)\| \leq k_1 e^{-\alpha \tau}, \forall \tau \geq 0,$$

where $k_1$ and $\alpha$ are positive constants.

**Remark 2.** Equation (7) means that when Assumption 1 and 3 are satisfied, we can use $h + \hat{y}$ to accurately represent the solution of fast dynamics for $\varepsilon \in [0, \varepsilon^*]$ and bounded inputs. However, it requires solving the boundary-layer model. Further, (12) means that if $\varepsilon \leq \varepsilon^* < \varepsilon^*$, the solution of fast transient can be estimated by only $h(t, \hat{x}(t))$ after $T > 0$. This result significantly simplifies the order reduction.

**Proof.** From Assumption 2, we know that the solution of reduced-order model is bounded for bounded inputs. Therefore, we can expect that $x$ is also bounded if $\|x - \hat{x}\| = O(\varepsilon)$. However, we cannot use this inequality since it has not been proven yet. Therefore, we exploit signal truncation as Lemma 1 to show that $x$ is in a compact set. Then by Assumption 1, we have that the origin of $f(x, z, u, \varepsilon)$ is compact. Since $f$ is continuous, it follows that $f$ is bounded, i.e., $|f| \leq k_0$, and $x(t)$ is Lipschitz.

Then using Assumption 1, 3 and Lemma 9.8 in [16], we conclude that there exists a Lyapunov function $V_y(x, y, u)$ and positive constants $b_1, b_2, \ldots, b_6$ and $\rho_0$ satisfying

$$b_1 \|y\|^2 \leq V_y(x, y, u) \leq b_2 \|y\|^2,$$

$$\frac{\partial V_y}{\partial y} G(x, y, u, 0) \leq -b_3 \|y\|^2,$$

$$\left\| \frac{\partial V_y}{\partial y} \right\| \leq b_4 \|y\|; \quad \left\| \frac{\partial V_y}{\partial x} \right\| \leq b_5 \|y\|^2; \quad \left\| \frac{\partial V_y}{\partial u} \right\| \leq b_6 \|y\|^2,$$

for all $y \in \{\|y\| < \rho_0\}$ and all $(x, u) \in \mathbb{R}^n \times \mathbb{R}^p$.

To assess the accuracy of solutions of fast dynamics, we define the estimation error as

$$\sigma_y(\tau, \varepsilon) = y(\tau, \varepsilon) - \hat{y}(\tau).$$

Differentiate both sides of equation (16) and abbreviate $x(t_0 + \varepsilon \tau, \varepsilon), y(\tau, \varepsilon), u(t_0 + \varepsilon \tau, \varepsilon), \sigma_y(\tau, \varepsilon)$ as $x, y, u, \sigma_y$, respectively, then we have

$$\frac{\partial \sigma_y}{\partial \tau} = G(x, y, u, \varepsilon) - G(x_0, \hat{y}, u_0, 0)$$

$$= G(x, y, u, 0) + \Delta G,$$

where $\Delta G = G(x, y, u, \varepsilon) - G(x, \sigma_y, u, 0) - G(x_0, \hat{y}, u_0, 0)$. Utilizing the Lipschitz conditions of $G$ and $x$, and the condition of Lemma 1, we have

$$\|\Delta G\| \leq k_2 \|\sigma_y\|^2 + \varepsilon l_1 + (k_3 \|\sigma_y\| + l_2 \|u - u_0\| + l_3 \|x - x_0\|) \|\bar{y}\|$$

$$\leq k_2 \|\sigma_y\|^2 + k_3 \|\sigma_y\| e^{-\alpha \tau} + \varepsilon l_1 + \varepsilon (l_2 \mu \tau + l_3 k_4 + l_3 k_5) e^{-\alpha \tau}$$

$$\leq k_2 \|\sigma_y\|^2 + k_3 \|\sigma_y\| e^{-\alpha \tau} + \varepsilon k_5,$$

(18)
for some nonnegative constants \(a, k_i, i = 1, \ldots, 5\) and nonnegative Lipschitz constants \(l_j, j = 1, \ldots, 3\), where \(k_5 = l_1 + k_1 \max\{(l_1k_1, l_2 + l_3k_0) \times \max\{1, 1/a\}\).

Equation (17) can be viewed as the perturbation of
\[
\frac{\partial y}{\partial \tau} = G(x, \sigma, y, u, 0).
\]

Using (13)-(15) and (18), the derivative of Lyapunov function \(V_t(x, \sigma, u)\) along the trajectories of (17) can be calculated as
\[
\dot{V}_t = \frac{\partial V_t}{\partial x} f + \frac{1}{\varepsilon} \frac{\partial V_t}{\partial \sigma} G + \frac{\partial V_t}{\partial u} u \\
\leq b_0k_0||\sigma_y||^2 - \frac{b_3}{\varepsilon} ||\sigma_y||^2 + b_6\mu||\sigma_y||^2 \\
+ \frac{b_4}{\varepsilon} ||\sigma_y||^2 (k_2||\sigma_y||^2 + k_1k_3||\sigma_y||e^{-a\tau} + \varepsilon k_3) \\
- \frac{b_3}{\varepsilon} e^{-a\tau}||\sigma_y||^2 + b_5k_3||\sigma_y|| \\
\leq -\frac{2}{\varepsilon} (\xi_1 - \xi e^{-a\tau})V_t + 2\xi_3 \sqrt{V_t},
\]
(20)

for 0 < \(\varepsilon \leq \varepsilon^*\) and \(||\sigma_y|| \leq b_3/(4b_2k_2)\), where \(\xi_1 = b_3/(4b_2), \xi_2 = b_4k_3/(2b_2), \xi_3 = b_5k_3/(2b_3),\) and \(\varepsilon^* = b_3/(3b_2k_0\mu).\)

Let \(W_0 = \sqrt{V_t}\) and use the comparison lemma, we have
\[
W_0(\tau) \leq \phi(\tau, 0)W_0(0) + \xi \int_0^\tau \phi(\tau, s) ds,
\]
(21)
\[
|\phi(\tau, s)| \leq e^{-f_s(\xi_1 - \xi e^{-a\tau})} ds \leq \xi_4 e^{-\bar{a}(\tau-s)},\]
(22)

for some positive constants \(\xi_4\) and \(\bar{a}\). Since \(\sigma_y(0) = O(\varepsilon)\), it follows that \(\sigma_y(\tau) = O(\varepsilon)\) for all \(\tau \geq 0\). Then we can conclude that (22) holds \(\forall \varepsilon \leq \varepsilon^*\) and \(\forall \tau \geq 0\).

Moreover, from (9), we have \(e^{-at/\varepsilon} \leq \varepsilon, \forall at \geq \varepsilon \ln(1/\varepsilon)\), then the term \(\bar{y}(t/\varepsilon)\) will be \(O(\varepsilon)\) on \([T, \infty)\) for \(\varepsilon \in [0, \varepsilon^*]\), where \((\varepsilon^*, T)\) is a pair of solution of
\[
\varepsilon \ln \left(\frac{1}{\varepsilon}\right) = aT.
\]
(23)

Now we have proved the accuracy of the solutions of fast dynamics. To show the conditions for accurate solutions of slow dynamics, we can define \(\sigma_x(t, \varepsilon) = x(t, \varepsilon) - \hat{x}(t)\). Following the similar procedure as (13)-(22), it can be verified that if Assumption 1-3 are satisfied, then (10) holds for \(\varepsilon \in [0, \varepsilon^*]\) and all \(t \in [0, \infty)\).

Remark 3. Note that \(\varepsilon^*\) is a function of the bound of input signals and it follows that
\[
\lim_{\mu \rightarrow 0} \varepsilon^* = \frac{b_3}{b_5k_0} \quad \text{and} \quad \lim_{\mu \rightarrow +\infty} \varepsilon^* = 0.
\]
(24)

This means when inputs are zero, the upper bound of \(\varepsilon\) is equal to that of its autonomous system; while when the inputs are unbounded, \(\varepsilon\) must be exactly zero to guarantee the accuracy of the reduced-order model. This result reflects the impact of external inputs on the accuracy of the reduced-order model.

The overall algorithm of this proposed high-fidelity order reduction method can be concluded as Algorithm 1.

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**Algorithm 1: High-Fidelity Order Reduction**

1. Find the perturbation coefficients \(\varepsilon\). Identify the states with \(\varepsilon\) as fast states, while the others as slow states.
2. **Procedure** Reduced model derivation
   1. Let \(\varepsilon = 0\), solve the algebraic equation (5) to obtain the isolated QSS solutions \(z = \hat{h}(x, u)\).
   2. Derive the boundary-layer model using equation (7).
3. **End procedure**
4. **Procedure** Calculate the bound of \(\varepsilon\)
   1. Calculate \(\varepsilon^* = \frac{b_3}{b_5k_0 + b_6\mu}\).
   2. Calculate \(\varepsilon^*\) by solving equation (23).
5. **End procedure**
6. **Procedure** Accuracy assessment
   1. If \(\varepsilon^* \leq \varepsilon^*\) then
      1. \(z = \hat{h}(\hat{x}, u) + \hat{y}\) by solving (7).
      2. **End if**
   2. **Else**
      1. Return to Step 1 to re-identify slow/fast dynamics
      2. **End if**
   3. **End procedure**

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### III. Mathematical Representation of WECC Composite Load Model

To apply the singular perturbation theory, we need the mathematical representation of WECC composite load model, which can be found in our previous work [20]. Since our objective is to reduce the order of dynamic parts, the static ones such as single-phase motor (which is modeled as a performance model [14]), electronic loads [2] and static load are out of the scope of this paper. For brevity, only the mathematical representation of dynamic components are introduced in this section.

#### A. Three-Phase Motor Model

WECC composite load model uses three three-phase fifth-order induction motors, called motor A, B and C, to represent different types of dynamic components. These three-phase motors have the same structure but different parameter settings. The block diagram of the induction motor model is shown in Fig.\[\text{II}\]. There are four dynamic equations with respect to \(E_{q1}', E_{d1}', E_{q2}', E_{d2}'\). By adding the dynamic equation governing the slip \(s\), we can represent the complete fifth-order model as follows,
Fig. 2. The block diagram of three-phase motor adopted in the WECC composite load model [14].

\[
\dot{E}_q' = \frac{1}{T_{p0}} \left[ -E_q' - i_d (L_s - L_p) - E_d' \omega_0 \cdot s \cdot T_{p0} \right],
\]

(25)

\[
\dot{E}_d' = \frac{1}{T_{p0}} \left[ -E_d' + i_q (L_s - L_p) + E_q' \omega_0 \cdot s \cdot T_{p0} \right],
\]

(26)

\[
\dot{E}_q'' = \frac{T_{p0} - T_{pp0}}{T_{p0} T_{pp0}} E_q' - \frac{T_{pp0} (L_s - L_p) + T_{p0} (L_p - L_{pp})}{T_{p0} T_{pp0}} \cdot i_d
\]

(27)

\[
\dot{E}_d'' = \frac{T_{p0} - T_{pp0}}{T_{p0} T_{pp0}} E_d' - \frac{T_{pp0} (L_s - L_p) + T_{p0} (L_p - L_{pp})}{T_{p0} T_{pp0}} \cdot i_d
\]

(28)

\[
\dot{s} = \frac{p \cdot E_d'' \cdot i_d + q \cdot E_q'' \cdot i_q - T_L}{2H}.
\]

(29)

The algebraic equations are:

\[
T_L = T_{m0} \left( Aw^2 + B w + C_0 + D w^{\text{Etrig}} \right),
\]

(30)

\[
T_{m0} = p E_{d0} i_{d0} + q E_{q0} i_{q0},
\]

(31)

\[
i_d = \frac{r_s}{r_s^2 + L_{pp}^2} (V_d + E_d'') + \frac{L_{pp}}{r_s^2 + L_{pp}^2} (V_q + E_q''),
\]

(32)

\[
i_q = \frac{r_s}{r_s^2 + L_{pp}^2} (V_q + E_q'') - \frac{L_{pp}}{r_s^2 + L_{pp}^2} (V_d + E_d''),
\]

(33)

\[
P = V_d i_d + V_q i_q,
\]

(34)

\[
Q = V_q i_d - V_d i_q,
\]

(35)

where \(E_d', E_d '', E_q', E_q '', s\) are the five state variables; \(L_s, L_p\) and \(L_{pp}\) are synchronous reactance, transient and subtransient reactance, respectively; \(T_{p0}\) and \(T_{pp0}\) are transient and subtransient rotor time constants, respectively; and \(\omega_0\) is the synchronous frequency.

**B. DER_A Model**

Recently, EPRI developed a new model to represent aggregated renewable energy resources named DER_A which has fewer states and parameters than the previous PVD1 model. The dynamic model of DER_A is as follows,

\[
\dot{S}_0 = \frac{1}{T_v} (V_c - S_0),
\]

(37)

\[
\dot{S}_1 = \frac{1}{T_p} (S_8 - S_1),
\]

(38)

\[
\dot{S}_2 = \begin{cases} 0 \text{ if } \text{Pflag} = 0, \\ \frac{S_2}{T_{iq}} + \frac{Q_{\text{ref}}}{s_2} & \text{if } \text{Pflag} = 1, \\ \frac{S_2}{T_{iq}} + \tan(\text{pflag}) \times S_1 & \text{if } \text{Pflag} = 1, \\ \text{sat}_2\{S_2 + \text{sat}_3\{DBV(V_{\text{ref}0} - S_0) \cdot K_{uv}\}\} - S_3 \\ \frac{T_g}{T_g} & \text{if } \text{Vtripflag} = 0, \\ \text{sat}_2\{S_2 + \text{sat}_3\{DBV(V_{\text{ref}0} - S_0) \cdot K_{uv}\}\}S_4 - S_3 \\ \frac{T_g}{T_g} & \text{if } \text{Vtripflag} = 1, \\ \end{cases}
\]

(39)

\[
\dot{S}_3 = \begin{cases} 0 & \text{if } \text{Freqflag} = 0, \\ \text{sat}_8\{\text{sat}_7\{S_0\}\} & \text{if } \text{Freqflag} = 1, \\ \frac{1}{T_{\text{porf}}} (S_7 - S_8), & \text{if } \text{Freqflag} = 1, \\ \text{sat}_7\{\text{sat}_6\{DBV(V_{\text{ref}0} - S_0) \cdot K_{uv}\}\} + \text{sat}_6\{DBV(V_{\text{ref}0} - S_0)\} + \frac{K_{pp}}{T_p} S_1 \\ \text{sat}_5\{DBV(V_{\text{ref}0} - S_0)\} + \frac{K_{pp}}{T_p} S_1 \\
\end{cases}
\]

(40)

where \text{sat}_9(x), i = 1, \ldots, 9 are the saturation functions; DBV(x) and DBF(x) are deadzone functions with respect to voltage and frequency, respectively; and VP(x, V_{\text{frac}}) represents the voltage protection function, which is a piecewise algebraic function. The parameter definitions are given in Table II. Here we only summarize the dynamic equations that will be used in the order reduction. The complete detailed mathematical model can be found in [20].

**IV. REDUCED ORDER WECC COMPOSITE LOAD MODEL**

In this section, we will derive the reduced-order large-signal model of WECC composite load model using singular perturbation method. For the purpose of order reduction, we only focus on the dynamic components. These components are connected in parallel and we will reduce each individual component’s order.

**A. Reduced-Order Three-Phase Motors Model**

Each three-phase motor model has five states, \(\dot{x}_M = [E_{dq}, E_{dq}, E_{dq}, E_{dq}, s]\). When applying the Algorithm 1, the first step is to identify the slow and fast dynamics. Since the fast dynamics
are characterized by the small perturbation coefficient \( \varepsilon \), we rewrite the left-hand-side of the dynamic equations as

\[
[T_{p0} \dot{E}_q', T_{p0} \dot{E}_d', T_{pp0} \dot{E}_q'', T_{pp0} \dot{E}_d''] H \delta]^{T}. \tag{47}
\]

Given one set of parameter setting in Table II equation \(47\) becomes

\[
[0.1 \dot{E}_q', 0.1 \dot{E}_d', 0.0026 \dot{E}_q'', 0.0026 \dot{E}_d'', 0.1 s]^{T}. \tag{48}
\]

The smaller perturbation coefficients in equation \(48\) suggest that dynamic response velocities of \([E'_q, E'_d, \delta]^{T}\) are much slower than the rest of the states. This difference is also an evidence of the two-time-scale property of this model. Then the slow and fast dynamics are divided as \(\dot{x}_S = [\dot{x}_M, \dot{z}_M]^{T}\), where \(x_M = [E'_q, E'_d, \delta]^{T}\), \(z_M = [E''_q, E''_d]^{T}\). For consistency, denote the input voltages \([V_q, V_d]\) as \(U_M\). Following the singular perturbation method \(1\)–(5), we can obtain the reduced-order large-signal model of three-phase motor as

\[
\dot{x}_M = \frac{1}{T_{p0}} \left[ -x_M - i_d (L_n - L_p) - \omega_0 T_{p0} x_M x_{235} \right], \tag{49}
\]

\[
\dot{x}_{M2} = \frac{1}{T_{p0}} \left[ -x_{M2} + i_q (L_n - L_p) + \omega_0 T_{p0} x_M x_{35} \right], \tag{50}
\]

\[
\dot{x}_{M3} = \frac{T_L - p \cdot h_2(x_M) \cdot i_d - q \cdot h_1(x_M) \cdot i_q}{2H}, \tag{51}
\]

where the QSS solutions are

\[
h_1(x_M) = \frac{1}{r_s + L_p^2} \left[ (L_p L_{pp} + r_s^2) x_M - (L_p - L_{pp}) r_s x_{M2} - (L_p - L_{pp}) L_p U_1 - (L_p - L_{pp}) r_s U_2 \right], \tag{52}
\]

\[
h_2(x_M) = \frac{1}{r_s + L_p^2} \left[ (L_p - L_{pp}) r_s x_M - (L_p L_{pp} + r_s^2) x_{M2} + (L_p - L_{pp}) r_s U_1 - (L_p - L_{pp}) L_p U_2 \right]. \tag{53}
\]

Denote \(\bar{x}_M = 1 - x_M\). The other algebraic equations are

\[
T_{L} = T_{m0} \left[ A \bar{x}_{M3}^2 + B \bar{x}_{M3} + C_0 + D \bar{x}_{M3} \right], \tag{54}
\]

\[
T_{m0} = p \cdot h_2(x_M) \cdot i_d + q \cdot h_1(x_M) \cdot i_q, \tag{55}
\]

\[
i_q = \frac{r_s}{r_s^2 + L_p^2} (U_{M1} + x_{M1}) - \frac{L_p}{r_s^2 + L_p^2} (U_{M2} + x_{M2}), \tag{56}
\]

\[
i_d = \frac{L_p}{r_s^2 + L_p^2} (U_{M1} + x_{M1}) + \frac{r_s}{r_s^2 + L_p^2} (U_{M2} + x_{M2}). \tag{57}
\]

By solving equation \(23\), we can find a pair of solution \((T, \varepsilon^{**}) = (0.012, 0.035)\). Since \(\varepsilon_M = 0.0026 < 0.035\), the solutions of fast dynamics \(\dot{z}_M\) converge to \(h(\bar{x}_M)\) exponentially fast within time 0.012 sec which is short enough. Therefore, we can use only the QSS solution \(h(\bar{x}_M)\) to represent the solution of fast dynamics.

B. Reduced-Order DER_A model

The DER_A model has 10 states in total, \(\bar{x}_D = [S_0, S_1, \ldots, S_9]^{T}\). Different from the three-phase motor model, due to the existence of switches such as \(Pr_{t_{flag}}\) and \(PQ_{t_{flag}}\), the DER_A model is actually a switching system consisting of \(2^5 = 32\) subsystems. Each subsystem is determined when the switches are fixed. Since these switches are preset, we only need to derive the reduced-order model for each subsystem. For brevity, we give the reduced-order model for one of the subsystems to illustrate the model order reduction procedure. The reduced-order models for other subsystems can be obtained using the same method.

To find \(\varepsilon\), we rewrite the dynamics as

\[
[T_{pv} \dot{S}_0, T_p \dot{S}_1, T_{iq} \dot{S}_2, T_6 \dot{S}_4, T_r \dot{S}_5, T_{t_{ord}} \dot{S}_8, T_{g} \dot{S}_9]^{T}. \tag{58}
\]
Given the parameter setting in Table III, equation (58) becomes

$$\begin{bmatrix} 0.1 \dot{S}_0, 0.1 \dot{S}_1, 0.005 \dot{S}_2, 0.005 \dot{S}_3, 0.005 \dot{S}_4, 
0.1 \dot{S}_5, 0.1 \cdot 0.1 \dot{S}_6, \dot{S}_7, 0.005 \dot{S}_8, 0.005 \dot{S}_9 \end{bmatrix}^T. \quad (59)$$

The smaller perturbation coefficients in equation (59) suggest that dynamic response velocities of \([S_0, S_1, S_5, S_7] \) are much slower than other states. This difference is also an evidence of the two-time-scale property of this model. Then the slow and fast dynamics are divided as \( \dot{z}_D = \left[ \dot{z}_D \right]^T \), \( \dot{x}_D = \left[ \dot{x}_D \right]^T \), \( z_D = \left[ S_2, S_3, S_4, S_6, S_8, S_9 \right]^T \). Defining the terminal voltage and frequency \([V_t, \text{Freq}] \) as \( U_D \). Following the same procedure as above (equations (1)-(5)), we can derive the reduced-order large-signal model of DER_A as

$$\begin{align*}
\dot{x}_D &= \frac{1}{T_T} (U_D - x_D), \\
\dot{x}_D &= \frac{1}{T_P} (x_D - x_D), \\
\dot{x}_D &= \frac{1}{T_T} (U_D - x_D), \\
\dot{x}_D &= 0.
\end{align*} \quad (60) \quad (61) \quad (62) \quad (63)$$

To obtain the output power, we also need to calculate the output currents, which are identified as fast states. According to Algorithm 1, there are two options to represent the solutions...
of fast dynamics depending on the magnitude of \( \varepsilon \). For simplicity, it is better to use only the QSS solution to represent the fast states since it does not require solving the boundary-layer model. Let \( \varepsilon^{**} = 0.06 \) to make sure \( \max \{ \varepsilon_D \} < \varepsilon^{**} \), then solving equation (23), we obtain \( T = 0.242 \, \text{sec} \). This means if we use only the QSS solutions, the solution of fast dynamics is inaccurate within 0.242 sec. This time period is intolerably long for stability analysis. Therefore, we should use \( z = h + \hat{y} \) by solving the boundary-layer model. The \( d-q \) axis currents \( i_d \) and \( i_q \) are states \( S_3 \) (\( z_{D2} \)) and \( S_9 \) (\( z_{D6} \)), respectively. Their equations are

\[
\begin{align*}
    i_q &= \text{sat}_2 \left\{ \gamma(x_D) \right\} \times \text{VP} (x_{D1}, V_{\text{frac}}) + \hat{y}_{D2}, \\
    i_d &= \text{sat}_3 \left[ \frac{\text{sat}_7(x_{D4})}{\text{sat}_1(x_{D1})} \right] \times \text{VP} (x_{D1}, V_{\text{frac}}) + \hat{y}_{D6}, \\
    \gamma(x_D) &= \frac{\text{tan}(\text{p}	ext{ar}e\text{f})x_{D2}}{\text{sat}_1(x_{D1})} + K_{qv} \text{sat}_3 \left[ \text{DBV} (V_{\text{ref}} - x_{D1}) \right],
\end{align*}
\]

where \( \hat{y}_{D2} \) and \( \hat{y}_{D6} \) are the solutions of boundary-layer model:

\[
\begin{align*}
    \dot{y}_{D1} &= -y_{D1}, \\
    \dot{y}_{D2} &= y_{D3} - y_{D2} - \text{VP} (x_{D1}, V_{\text{frac}}) \times \left\{ \text{sat}_2 \left\{ \gamma(x_D) \right\} + \text{sat}_2 \left[ \gamma(x_D) + \gamma(x_D) \right] \right\}, \\
    \dot{y}_{D3} &= -y_{D3}, \\
    \dot{y}_{D4} &= -T_{rH/D5}, \\
    \dot{y}_{D5} &= -y_{D5}, \\
    \dot{y}_{D6} &= -\text{sat}_9 \left[ \frac{\text{sat}_7(x_{D4})}{\text{sat}_1(x_{D1})} \right] - y_{D6} \times \text{VP} (x_{D1}, V_{\text{frac}}) + \text{sat}_9 \left[ \frac{\text{sat}_7(y_{D5} + x_{D4})}{\text{sat}_1(x_{D1})} \right] \times \left[ y_{D3} + \text{VP} (x_{D1}, V_{\text{frac}}) \right].
\end{align*}
\]

V. MODEL VALIDATION VIA SIMULATION

In this section, the reduced-order models of three-phase motors and DER_A are tested in Matlab using different solvers. We compare the performance of reduced-order model with original model to verify the effectiveness of the proposed high-fidelity order reduction approach. Moreover, we compare the computational time between two models using different solvers to show the reduction of computational burden.

A. Validation of Reduced-Order Three-Phase Motors

To verify the proposed reduced-order model of three-phase motor, we simulate the reduced and original model in Matlab with the same input voltage. Consequently, we can compare their output power and other states. Refer to an EPRI white paper [21] and [15], this paper tests a voltage sag benchmarking bus voltage input that is generated by (73). The parameters are set as Table II referring to [22].

\[
V(t) = \begin{cases} 
    a & \text{if } 1 \leq t < (1 + b/60) \\
    (1-d)(c+1-t) + 1 & \text{if } (1+b/60) \leq t < 1 + c \\
    1 & \text{otherwise}
\end{cases} \tag{73}
\]

Fig. 4. Dynamic responses of \( E_d' \) and \( E_q' \) of reduced/original models of three-phase motor A.

Fig. 5. Dynamic responses of \( E_d' \) and \( E_q' \) of reduced/original models of three-phase motor B.

Fig. 6. Dynamic responses of \( E_d' \) and \( E_q' \) of reduced/original models of three-phase motor C.
Fig. 7. Real/reactive powers of reduced/original models of three-phase motor A.

Fig. 8. Real/reactive powers of reduced/original models of three-phase motor B.

Fig. 9. Real/reactive powers of reduced/original models of three-phase motor C.

**TABLE IV**

| Power          | Motor A | Motor B | Motor C |
|----------------|---------|---------|---------|
| Real power     | 1.0509 × 10⁻⁴ | 1.1295 × 10⁻⁴ | 8.0264 × 10⁻⁵ |
| Reactive power | 1.1422 × 10⁻⁵ | 1.4294 × 10⁻⁵ | 2.1112 × 10⁻⁵ |

a reduced-order non-stiff problem while considerably reducing the computational time. This reduction will be more significant in large-scale system with multiple composite loads.

**B. Validation of DER_A Model**

Similar to the verification of three-phase motor, we simulate the original and reduced-order model of DER_A in Matlab. The voltage input is the same as (73). The frequency input is set to be 60 Hz. The parameter setting follows the reliability guideline in [23] as Table III.

Fig. 10 shows the dynamic responses of DER_A. The blue lines denote the output powers of original model, while the red lines represent those of reduced one. Fig. 11 shows filtered voltage $V_{\text{filt}}$, filtered generated power $P_{\text{genfilt}}$, and filtered current $i_q$ and $i_d$ of reduced and original model of DER_A. The mean square errors (MSE) of real and reactive power are $7.1363 \times 10^{-4}$ and $1.3045 \times 10^{-5}$, respectively. Further, the computational time of original and reduced-order model using ODE45 are 11.205 sec and 0.2074 sec, respectively; the computational time of original and reduced-order model using ODE15s are 2.0012 sec and 0.1598 sec, respectively.

**VI. CONCLUSION**

This paper proposes a high-fidelity large-signal order reduction approach for the latest WECC composite load model including DER_A. The derived reduced-order model has guaranteed high accuracy that can replace the original load...
model in high-order system simulation to perform stability analysis, optimization and controller design. This replacement can significantly reduce the difficulty of stability analysis and computational burden. The simulation results verify the accuracy and efficiency of the proposed algorithm.

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