Potential analysis of holographic Schwinger effect in the magnetized background

Zhou-Run Zhu,1,* De-fu Hou,1,† and Xun Chen1,‡

1Institute of Particle Physics and Key Laboratory of Quark and Lepton Physics (MOS), Central China Normal University, Wuhan 430079, China

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Abstract

We study the magnetic field effect on the Schwinger effect using the AdS/CFT correspondence. The potential analysis of particle pairs transverse and parallel to the magnetic field is performed in this paper. Firstly, we calculate separating length of the particle pairs at finite temperature with magnetic field. It is found that the maximum value of separating length decreases with the increase of magnetic field and/or temperature, which can be inferred that the virtual electron-positron pairs become real particles more easily. In the further investigation of the effect of magnetic field and temperature, we find the magnetic field and temperature reduce the potential barrier, thus favor the Schwinger effect. When particle pairs are transverse to the magnetic field, the effect of the magnetic field on the Schwinger effect is slightly larger than the parallel case. The difference between the transverse and parallel case becomes smaller with the increase of temperature. It indicates that the high temperature would reduce the anisotropic effect induced by the magnetic field.

* zhuzhourun@mails.ccnu.edu.cn
† houdf@mail.ccnu.edu.cn
‡ chenxunhep@qq.com
I. INTRODUCTION

The virtual electron-positron pairs can be materialized under the strong electric-field in quantum electrodynamic (QED). This non-perturbative phenomenon is known as the Schwinger effect\[1\]. This phenomenon is not unique to QED, but has a general feature of vacuum instability in the presence of the external field. The production rate in the weak-coupling and weak-field case was put forward in \[1\] and was extend to the arbitrary-coupling and weak-field case\[2\]:

\[
\Gamma \sim exp\left(-\frac{\pi m^2}{eE} + \frac{e^2}{4}\right),
\]

where m, e represent the mass and charge of the particle pairs, respectively. E is the external electric-field. There exists a critical value \(E_c\) of the electric field when the exponential suppression vanishes.

In string theory, there also exists a critical value \(E_c\) which is proportional to the string tension \[3, 4\]. By utilizing the AdS/CFT correspondence\[5–8\], the duality between the string theory on \(AdS_5 \times S_5\) space and the \(N = 4\) super Yang-Mills (SYM) theory, one can study the Schwinger effect in this holographic method. In order to realize the \(N = 4\) SYM system coupled with an U(1) gauge field, one can break the gauge group from \(U(N + 1)\) to \(SU(N) \times U(1)\) by using the Higgs mechanism. In the usual studies, the test particles are assumed to be heavy quark limit. To avoid pair creation suppressed by the divergent mass, the location of the probe D3-brane is at finite radial position rather than at the AdS boundary. The mass of the particles is finite so that the production rate can make sense\[9\]. Therefore, the production rate can be given as

\[
\Gamma \sim exp\left[-\sqrt{\frac{\lambda}{2}} (\sqrt{\frac{E_c}{E}} - \sqrt{\frac{E}{E_c}})^2\right],
\]

with a critical field

\[
E_c = \frac{2\pi m^2}{\sqrt{\lambda}},
\]

which agrees with the result from the Dirac-Born-Infeld (DBI) action and \(\lambda\) is the ’t Hooft coupling.

Following the holographic step, the potential analysis was performed in the confining theories in \[10, 11\]. The potential barrier can be regarded as a quantum tunneling process. The virtual particle pairs need to get enough energy from an external electric field. When reaching to a critical value \(E_c\) the potential barrier will vanish. Then the real particles
pairs production are completely uncontrolled and the vacuum turns into totally instability. The potential analysis provide a new perspective to study the Schwenger effect. A lot of research work have been studied by using the AdS/CFT correspondence. The production rate in the confining theories was discussed in [12–14]. The universal nature of holographic Schwinger effect in general confining backgrounds was analyzed in [15]. The Schwinger effect also has been investigated in the AdS/QCD models [16, 17]. The potential analysis in non-relativistic backgrounds[18] and a D-instantons background [19] were discussed. The holographic Schwinger effect in de Sitter space has been studied in [20]. Other important research results can be seen in [21–31].

The heavy ion collisions at RHIC and LHC experiments produce strong electro-magnetic fields. As a result, studying the Schwinger effect in the strong magnetic field $\mathcal{B}(0.01GeV^2 -0.25GeV^2)$ ceated by RHIC and LHC[32–36] is the main motivation of this paper. The strong magnetic fields may provide some different views for the vacuum structure and we expect the Schwinger effect may be observed through the heavy-ion collisions experiments. Thence, we study the holographic Schwinger effect in the 5-dimensional Einstein-Maxwell system with a proper magnetic field range [37–39] which corresponds to the experimental results of the RHIC and LHC. This may give us some inspiration for studying the Schwinger effect through the experimental results.

The production rate of Schwinger effect with the presence of electric and magnetic fields was discussed in [24]. One way to turn on magnetic fields is considering a circular Wilson loop under the parallel electric and magnetic fields. Another way is to utilize circular Wilson loop solutions depending on additional parameters which are related to the magnetic fields. Anyway, these methods of adding magnetic field don’t affect the geometry of background. It seems to be more nature to incorporate a magnetic field of the background in this paper and we find the behavior of critical electric in magnetized Einstein-Maxwell system is not consistent with theirs.

Holographic effects of the magnetic field on the Schwinger effect by using the AdS/CFT correspondence were studied in this paper. The organization of the paper is as follows. In Sec. II, we introduce the 5-dimensional Einstein-Maxwell system with the magnetic field. In Sec. III, we study the potential analysis in the magnetized background. The discussion and conclusion are given in Sec. IV.
II. BACKGROUND GEOMETRY

The gravity background with magnetic field was introduced into the 5-dimensional Einstein-Maxwell system by using the AdS/QCD model \[37\], and the action is

$$S = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} (R - F^{MN} F_{MN} + \frac{12}{L^2}),$$  \hspace{1em} (4)

where $g$ is the determinant of metric $g_{MN}$. $R$, $G_5$, $F_{MN}$ are the scalar curvature, 5D Newton constant and the U(1) gauge field, respectively. $L$ is the AdS radius and we set it to 1.

As discussed in \[38\], turning on a bulk magnetic field in the $x_3$-direction and the metric of the black hole takes the form

$$ds^2 = r^2(-f(r)dt^2 + h(r)(dx_1^2 + dx_2^2) + q(r)dx_3^2) + \frac{dr^2}{r^2f(r)},$$  \hspace{1em} (5)

with

$$f(r) = 1 - \frac{r_h^4}{r^4} + \frac{2B^2}{3r^4} \ln\left(\frac{r_h}{r}\right),$$  \hspace{1em} (6)

$$h(r) = 1 + \frac{1}{3}B^2 \frac{\ln(r)}{r^4},$$  \hspace{1em} (7)

$$q(r) = 1 - \frac{2}{3}B^2 \frac{\ln(r)}{r^4},$$  \hspace{1em} (8)

where $r$ denotes the radial coordinate of the 5th dimension. The magnetic field breaks the rotation symmetry and allows us to analyze the anisotropic cases because the element $q(r)$ is not equal to $h(r)$ and the anisotropy was induced by the magnetic field \[40, 41\]. The anisotropic direction is along $x_3$-direction in this article.

The perturbative solutions of this black hole metric can work well when $B \ll T^2$. Following \[38\], Ref.\[39, 42\] has confirmed the perturbative solution has good convergency and can be good approximation in the region $B \leq 0.15 \text{ GeV}^2$. Note that the physical magnetic field $\mathfrak{B}$ is related with the magnetic field $B$ by the equation $\mathfrak{B} = \sqrt{3}B$. Hence the physical magnetic field $\mathfrak{B} \leq 0.26 \text{ GeV}^2$ which corresponds to the experiment result of the RHIC and LHC.

The Hawking temperature is

$$T = \frac{r_h}{\pi} \frac{B^2}{6\pi r_h^3},$$  \hspace{1em} (9)

where $r_h$ is the black-hole horizon. In this article, we will use this Einstein-Maxwell system and expand it to study the holographic effect of magnetic field on the Schwinger effect.
III. POTENTIAL ANALYSIS IN THE MAGNETIZED BACKGROUND

Since the magnetic field is along \(x_3\)--direction, it may be reasonable to consider the test particles are transverse to the magnetic field and parallel to the magnetic field. From this point of view, we perform the potential analysis with the two cases in the magnetized background.

A. Transverse to the magnetic field

We study the potential analysis with the test particle pairs separated in the \(x_1\)--direction first, which means the particle pairs are transverse to the magnetic field. The coordinates are parameterized by

\[
t = \tau, \quad x_1 = \sigma, \quad x_2 = x_3 = 0, \quad r = r(\sigma).
\]  

(10)

By utilizing the Euclidean signature, the Nambu-Goto action is given as

\[
S = T_F \int d\sigma d\tau \mathcal{L} = T_F \int d\sigma d\tau \sqrt{\det g_{\alpha\beta}},
\]

(11)

where \(g_{\alpha\beta}\) represents the determinant of the induced metric. \(T_F = \frac{1}{2\pi a'}\) is the string tension and

\[
g_{\alpha\beta} = g_{\mu\nu} \frac{\partial X^\mu}{\partial \sigma^\alpha} \frac{\partial X^\nu}{\partial \sigma^\beta},
\]

(12)

where \(g_{\mu\nu}\) denote the brane metric and \(X^\mu\) is target space coordinates.

Then the induced metric is

\[
g_{00} = r^2 f(r), \quad g_{11} = r^2 h(r) + \frac{1}{r^2 f(r)} \dot{r}^2, \quad g_{10} = g_{01} = 0,
\]

(13)

with \(\dot{r} = \frac{dr}{d\sigma}\).

The Lagrangian density is given as

\[
\mathcal{L} = \sqrt{\det g_{\alpha\beta}} = \sqrt{r^4 f(r) h(r) + \dot{r}^2},
\]

(14)

and \(\mathcal{L}\) does not rely on \(\sigma\) explicitly. The conserved quantity is obtained by

\[
\mathcal{L} - \frac{\partial \mathcal{L}}{\partial \dot{r}} \dot{r} = C,
\]

(15)
which leads to
\[
\frac{r^4 f(r)h(r)}{\sqrt{r^4 f(r)h(r) + r^2}} = C. \tag{16}
\]

By using the boundary condition
\[
\frac{dr}{d\sigma} = 0, \quad r = r_c \ (r_h < r_c < r_0), \tag{17}
\]
where the D3-brane located at finite radial position \( r = r_0 \). The conserved quantity \( C \) can be expressed as
\[
C = r^2_c \sqrt{f(r_c)h(r_c)}. \tag{18}
\]

Plugging Eq. (18) into Eq. (16), one gets
\[
\dot{r} = \frac{dr}{d\sigma} = r^2 \sqrt{h(r)f(r)[r^4 h(r)f(r) - r^4_c h(r_c)f(r_c) - 1]}. \tag{19}
\]

By integrating Eq. (19), one can get the separate length \( x_\perp \) of the test particle pairs
\[
x_\perp = \frac{2}{ar_0} \int_1^{\frac{1}{a}} dy \frac{1}{y^2 \sqrt{f(r)h(r)[y^4 f(r)^h(r_c) - f(r_c)h(r_c)]}}. \tag{20}
\]
with the dimensionless parameter
\[
y \equiv \frac{r}{r_c}, \quad a \equiv \frac{r_c}{r_0}. \tag{21}
\]

By using Eq. (14) and Eq. (19), the sum of the Coulomb potential and static energy can be given as
\[
V_{(CP+SE)\perp} = 2T_F \int_0^{x_\perp} d\sigma \mathcal{L}
= 2T_F a r_0 \int_1^{\frac{1}{a}} \frac{dy}{y^2 \sqrt{y^4 f(r)h(r) - f(r_c)h(r_c)}}. \tag{22}
\]

The critical field is obtained by the DBI action in the Lorentzian signature. The DBI action is
\[
S_{DBI} = -T_{D3} \int d^4x \sqrt{-\text{det}(G_{\mu\nu} + \mathcal{F}_{\mu\nu})}, \tag{23}
\]
with a D3-brane tension
\[
T_{D3} = \frac{1}{g_s (2\pi)^3 \alpha'^2}. \tag{24}
\]

From Eq. (5), the induced metric \( G_{\mu\nu} \) reads
\[
G_{00} = -r^2 f(r), \quad G_{11} = G_{22} = r^2 h(r), \quad G_{33} = r^2 q(r). \tag{25}
\]
Then considering $F_{\mu\nu} = 2\pi\alpha' F_{\mu\nu}$ [43] and the electric field $E$ is along $x_1$ direction[11], one gets

$$G_{\mu\nu} + \mathcal{F}_{\mu\nu} = \begin{pmatrix} -r^2 f(r) & 2\pi\alpha'E & 0 & 0 \\ -2\pi\alpha'E & r^2 h(r) & 0 & 0 \\ 0 & 0 & r^2 h(r) & 0 \\ 0 & 0 & 0 & r^2 q(r) \end{pmatrix},$$

which leads to

$$\det(G_{\mu\nu} + \mathcal{F}_{\mu\nu}) = -r^4 h(r) q(r) [r^4 f(r) h(r) - (2\pi\alpha')^2 E^2].$$

(27)

By plugging Eq.(27) into Eq.(23), one gets

$$S_{DBI} = -T_{D3} \int d^4x \sqrt{r_0^4 h(r_0) q(r_0) \sqrt{r_0^4 f(r_0) h(r_0) - (2\pi\alpha')^2 E^2}}.$$  

(28)

where $r = r_0$ is the location of the D3-brane. To avoid Eq.(28) being ill-defined,

$$r_0^4 h(r_0) f(r_0) - (2\pi\alpha')^2 E^2 \geq 0. \tag{29}$$

The critical field $E_c$ is obtained by

$$E_c = T_F r_0^2 \sqrt{f(r_0) h(r_0)}.$$  

(30)

In Eq.(30), one can see that the critical field is related to the magnetic field. By introducing a dimensionless parameter $\alpha \equiv \frac{E}{E_c}$, the total potential $V_{tot(\perp)}$ is

$$V_{tot(\perp)} = V_{(CP+SE)(\perp)} - E x_{\perp}$$

$$= 2T_F a r_0 \int_1^{\frac{1}{\alpha}} dy \frac{y^2 \sqrt{f(r) h(r)}}{\sqrt{y^4 f(r) h(r) - f(r_c) h(r_c)}}$$

$$- \frac{2T_F a r_0}{a} \int_1^{\frac{1}{\alpha}} dy \frac{\sqrt{f(r_0) h(r_0) \sqrt{f(r_c) h(r_c)}}}{y^2 \sqrt{f(r) h(r) [y^4 f(r) h(r) - f(r_c) h(r_c)]}}. \tag{31}$$

B. Parallel to the magnetic field

We consider the test particle pairs separated in the $x_3$-direction which means the particle pairs are parallel to the magnetic field. The coordinates are parameterized by

$$t = \tau, \quad x_3 = \sigma, \quad x_1 = x_2 = 0, \quad r = r(\sigma). \tag{32}$$
FIG. 1. The separate length $x$ versus the parameter $a$. (a) for $T = 0$, $B = 0$, (b) for $T = 0.1 \, GeV$, and (c) for $T = 0.15 \, GeV$. The red line, black line, blue line in (b) and (c) denote $B = 0.05 \, GeV^2$, $0.1 \, GeV^2$, $0.15 \, GeV^2$, respectively. The solid line in (b) and (c) indicates the particle pair is parallel to the magnetic field direction, and the dashed line is perpendicular to the magnetic field direction.

By repeating the previous calculation, one can get the separate length $x_{\parallel}$

$$x_{\parallel} = \frac{2}{ar_0} \int_1^\frac{1}{2} dy \frac{1}{y^2 \sqrt{f(r)q(r)} \left[y^4 \frac{f(r)q(r)}{f(r_c)q(r_c)} - 1\right]}.$$  \hspace{1cm} (33)

The separate length $x$ versus the parameter $a$ is plotted in Fig. 1 and Fig. 2. First, we note that there are two possible U-shape string configurations same as heavy quark
FIG. 2. The separate length $x$ versus the parameter $a$ for different temperature. The red line, black line, blue line denote $T = 0.1 \text{ GeV}$, $0.15 \text{ GeV}$, $0.2 \text{ GeV}$, respectively. (a) for $B = 0.05 \text{ GeV}^2$, (b) for $B = 0.1 \text{ GeV}^2$, and (c) for $B = 0.15 \text{ GeV}^2$. The solid line indicates the particle pair is parallel to the magnetic field, and the dashed line is perpendicular to the magnetic field.

The U-shape string remains unchanged at vanishing temperature for all separate distance, while the U-Shape string exists only at large $a$ and become unstable at small $a$ for finite temperature case. So, we need take the stable branch, corresponding to large values of $a$ in the potential analysis. In our numerical computation, we set $T_F$ and $r_0$ as constants for simplicity. Next, from these two pictures, we can see that the maximum
value of distance is decreasing with the increasing temperature and magnetic field. In some sense, we infer that Schwinger effect happens easily at large temperature and magnetic field. When particle pairs are transverse to the magnetic field at high temperature, the effect of the magnetic field on the separate length is almost same as the parallel case. It indicates the high temperature may reduce the anisotropic effect induced by the magnetic field. To further confirm that point of view, we need to study the total potential.

The the sum of the Coulomb potential and static energy at the finite temperature in the magnetized background is

$$\begin{align*}
V_{(C_{P}+SE)(\parallel)} &= 2T_{F}ar_{0}\int_{1}^{\frac{1}{2}} dy \frac{y^{2}\sqrt{f(r)q(r)}}{\sqrt{y^{4}f(r)q(r) - f(r_{c})q(r_{c})}}.
\end{align*}$$

The the total potential $V_{tot}(\parallel)$ can be obtained as

$$\begin{align*}
V_{tot}(\parallel) &= V_{(C_{P}+SE)(\parallel)} - Ex_{\parallel} \\
&= 2T_{F}ar_{0}\int_{1}^{\frac{1}{2}} dy \frac{y^{2}\sqrt{f(r)q(r)}}{\sqrt{y^{4}f(r)q(r) - f(r_{c})q(r_{c})}} - \frac{2T_{F}ar_{0}}{a}\int_{1}^{\frac{1}{2}} dy \frac{\sqrt{f(r_{0})h(r_{0})\sqrt{f(r_{c})q(r_{c})}}}{y^{2}\sqrt{f(r)q(r)[y^{4}f(r)q(r) - f(r_{c})q(r_{c})]}}.
\end{align*}$$

The shapes of the total potential $V_{tot}$ with respect to the separate length $x$ for various $\alpha$ are plotted in Fig. 3. We can find that the potential barrier decreases with the external electric-field increasing and will vanish at a critical field. When $\alpha < 1$, the potential barrier is existent and the pairs production can be explained by the tunneling process. When $\alpha > 1$, the particles are easier to produce as the external electric-field increases. The vacuum becomes unstable extremely and the production of the pairs are explosive. The result agrees with the shapes of the potential for various values of $E_{c}$ in [11].

The effect of the magnetic field on the total potential is studied in Fig. 4. It is found that the potential barrier decreases with the magnetic field increasing. From Eq.(30), one can find that the magnetic field reduces the critical electric field $E_{c}$ by numerical analysis. The magnetic field decreases the potential barrier and enhances the Schwinger effect. When particle pairs are transverse to the magnetic field, the effect of the magnetic field on the Schwinger effect is slightly larger than parallel case.
FIG. 3. The total potential $V_{tot}$ with respect to the separate length $x$ with $T = 0.1$ GeV. The red line, black line, blue line, green line denote $\alpha = 0.8, 0.9, 1.0, 1.1$, respectively. (a) for $B = 0.05$ GeV$^2$ and (b) for $B = 0.15$ GeV$^2$. The solid line indicates the particle pair is parallel to the magnetic field, and the dashed line is perpendicular to the magnetic field.

In [16, 24] the magnetic field perpendicular and parallel to the electric field has been discussed and found $E_c$ increases with $B_\perp$. This result is different of our work. The reasons for the inconsistence between the two methods may be caused by the different ways of turning on the magnetic field. In this paper, the magnetic field may affect the geometry of background and we expect it could give us some inspiration in studying the Schwinger effect in a magnetized system.

The relationship between the total potential and the temperature is analyzed in Fig. 5. One can see that the potential barrier decreases with the temperature increasing. It is found that the temperature also reduces the critical electric field $E_c$ by numerical analysis and the temperature enhances the Schwinger effect. When particle pairs are transverse to the magnetic field, the effect of the magnetic field on the the Schwinger effect is slightly larger than the parallel case. The difference between the transverse case and the the parallel case decreases with the temperature increasing. It indicates the high temperature may reduce the anisotropic effect induced by the magnetic field.
FIG. 4. The total potential $V_{tot}$ against the separate length $x$ with $\alpha = 0.8$ for the different magnetic fields. The black line denotes $B = 0.05 \text{ GeV}^2$ and the red line indicates $B = 0.15 \text{ GeV}^2$. (a) for $T = 0.1 \text{ GeV}$ and (b) for $T = 0.15 \text{ GeV}$. The solid line indicates the particle pair is parallel to the magnetic field, and the dashed line is perpendicular to different magnetic field.
FIG. 5. The total potential $V_{\text{tot}}$ against the separate length $x$ with $\alpha = 0.8$ for different temperature. The black line denotes $T = 0.1 \text{ GeV}$ and the red line indicates $T = 0.15 \text{ GeV}$. (a) for $B = 0.1 \text{ GeV}^2$ and (b) for $B = 0.15 \text{ GeV}^2$. The solid line indicates the particle pair is parallel to the magnetic field, and the dashed line is perpendicular to the magnetic field.

IV. CONCLUSION AND DISCUSSION

In this paper, we study the potential analysis in the 5-dimensional Einstein-Maxwell system with the magnetic field which corresponds to the RHIC and LHC magnetic field range. Since the heavy ion collisions at RHIC and LHC experiments produce strong electromagnetic fields. The strong magnetic fields may provide some different views for the vacuum structure and we expect that the Schwinger effect could be observed through the heavy-ion collisions.

The separate length between test particle pairs by using a probe D3-brane at a finite radial position were discussed in this article. We consider the test particle pairs both transverse to the magnetic field and parallel to the magnetic field. We also found that the separating length $x$ decreases with the magnetic field and the temperature increasing, which is expected to be a signal of enhancing the Schwinger effect.

We calculated the critical electric field via the DBI action and derived the formula of the
total potential so that we can perform the potential analysis in the magnetized background. It is found that both the magnetic field and the temperature reduce the potential barrier and enhance the Schwinger effect. That means the magnetic field and the temperature increases the production rate of the real particle pairs. In addition, when particle pairs are transverse to the magnetic field, the effect of the magnetic field on the Schwinger effect and the separate length is slightly larger than the parallel case. The difference between the transverse case and the the parallel case is smaller at high temperature. The high temperature may reduce the anisotropic effect induced by the magnetic field.

We expect that the Schwinger effect in the magnetized background could provide some inspiration of QCD with a strong electric field. Moreover, the production rate in the Einstein-Maxwell-dilaton system in a holographic QCD model may be worth to be investigated [46–49]. We hope to report in these directions in future.

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