Transport through a double barrier in Large Radius Carbon Nanotubes with a transverse magnetic field

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Abstract. We discuss the Luttinger Liquid behaviour of Large Radius Carbon Nanotube e.g. the Multi Wall ones (MWNT), under the action of a transverse magnetic field \( B \). Our results imply a reduction with \( B \) in the value of the bulk critical exponent, \( \alpha_{\text{bulk}} \), for the tunneling density of states, which is in agreement with that observed in transport experiments. Then, the problem of transport through a Quantum Dot formed by two intramolecular tunneling barriers along the MWNT, weakly coupled to Tomonaga-Luttinger liquids is studied, including the action of a strong transverse magnetic field \( B \). We predict the presence of some peaks in the conductance \( G \) versus \( B \), related to the magnetic flux quantization in the ballistic regime, at a very low temperature \( T \), and also at higher values of \( T \), where the Luttinger behaviour dominates. The temperature dependence of the maximum \( G_{\text{max}} \) of the conductance peak according to the Sequential Tunneling follows a power law, \( G \propto T^{\gamma_e - 1} \) with \( \gamma_e \) linearly dependent on the critical exponent, \( \alpha_{\text{end}} \), strongly reduced by \( B \).

PACS. 05.60.Gg Quantum transport – 71.10.Pm Fermions in reduced dimensions – 73.63.-b Electronic transport in nanoscale materials and structures – 71.20.Tx Fullerenes and related materials; intercalation compounds

1 Introduction

In a recent paper \cite{1} we discussed the transport through a double barrier for interacting quasi one-dimensional electrons in a Quantum Wire (QW), in the presence of a transverse magnetic field. Here we want to extend the results obtained there to an analogous device based on Large Radius Carbon Nanotubes (LRCNs), such as the Multi Wall ones (MWNT). This aim is not trivial to pursue, because of the geometry-dependent electronic properties of the Carbon Nanotubes (CNs) and the effects of many subbands crossing the Fermi level in LRCNs.

Transport in 1 Dimension — Electronic correlations have been predicted to dominate the characteristic features in quasi one dimensional (1D) interacting electron systems. This property, commonly referred to as Tomonaga-Luttinger liquid (TLL) behaviour \cite{2}, has recently moved into the focus of attention by physicists, also because in recent years several electrical transport experiments for a variety of 1D devices, such as semiconductor quantum wires ~3 (QWs) and carbon nanotubes (CNs) \cite{4} have shown this behaviour.

In a 1D electron liquid Landau quasiparticles are unstable and the low-energy excitations take the form of plasmons (collective electron-hole pair modes); this is known as the breakdown of the Fermi liquid picture in 1D. The LL state has two main features: (i) the power-law dependence of physical quantities, such as the tunneling density of states (TDOS), as a function of energy or temperature; (ii) the spin-charge separation: an additional electron in the LL decays into decoupled spin and charge wave packets, with different velocities for charge and spin. It follows that 1D electron liquid are characterized by the power-law dependence of some physical quantities, as a function of the energy or the temperature. Thus, the tunneling conductance \( G \) reflects the power law dependence of the DOS in a small bias experiment \cite{5}

\[
G = \frac{dI}{dV} \propto T^{\alpha_{\text{bulk}}}
\] (1)

for \( eV_b \ll k_B T \), where \( V_b \) is the bias voltage, \( T \) is the temperature and \( k_B \) is Boltzmann’s constant.

The power-law behaviour characterizes also the thermal dependence of \( G \) when an impurity is present along the 1D devices. The theoretical approach to the presence of obstacles mixes two theories corresponding to the single particle scattering (by a potential barrier \( V_B(\mathbf{r}) \)) and the TLL theory of interacting electrons. The single particle scattering gives the transmission probability, \(|t|^2\), depending in general on the single particle energy \( \varepsilon \). Hence,
following reference [6], the conductance, $G$, as a function of the temperature and $|t|$ can be obtained

$$G \propto |t(\varepsilon, T)|^2 \equiv |t(\varepsilon)|^2 T^{2\alpha_{\text{end}}},$$ (2)

where we introduced a second critical exponent, $\alpha_{\text{end}}$.

**Intrinsic Quantum Dot** — Experiments [7,8] show transport through an intrinsic quantum dot (IQD) formed by a double barrier within a 1D electron system, allowing for the study of the resonant or sequential tunneling. The linear conductance typically displays a sequence of peaks when, at the gate voltage, $V_g$, increases. Thus, also the double-barrier problem has attracted a significant amount of attention among theorists [9–16], in particular the case of two identical, weakly scattering barriers at a distance $d$. In general, the transmission is non-zero for particular values of the parameters corresponding to a momentum $k_F$.

The power-law behaviour characterizes also the thermal dependence of $G$ in the presence of an IQD. A first theory about the transport through an IQD is known as uncorrelated sequential tunneling (UST), where an incoherent sequential tunneling is predicted. It follows the dependence of the peaks of the conductance according to the power law

$$G_{\text{max}} \propto T^{-\alpha_{\text{end}}-1}.$$ (3)

Some experiments [7,8] showed transport through an IQD formed by a double barrier within a Single Wall CN (SWCNT) allowing one to study the resonant or sequential tunneling. In order to explain the unconventional power-law dependencies in the measured transport properties of a CN, a mechanism was proposed [7,12], namely, correlated sequential tunneling (CST) through the island. The temperature dependence of the maximum $G_{\text{max}}$ of the conductance peak, according to the CST theory, yields the power law behaviour

$$G_{\text{max}} \propto T^{2\alpha_{\text{end}}-1} \propto T^{2\alpha_{\text{end}}-1}.$$ (3)

Recently, a lot of theoretical work has been carried out on the double impurity problem in TLL systems. In an intermediate temperature range $\varepsilon_c < k_B T < \Delta_{\text{dot}}$, where $\varepsilon_c$ is the Infra Red cut-off energy and $\Delta_{\text{dot}}$ is the level spacing of the dot, some authors [13,14] predict a behaviour according to the UST, while others [16] find results in agreement with the CST theory. In a recent paper [18] the authors discussed how the critical exponent can depend on the size of the dot and on the temperature, by identifying three different regimes, i.e. the UST at low $T$, a Kirchoff regime at intermediate $T$ ($G_{\text{max}} \propto T^{2\alpha_{\text{end}}}$) and a third regime for $T \gg \Delta_{\text{dot}}$, with $G_{\text{max}} \propto T^{-1}$. Thus, in their calculations, obtained starting from spinless fermions on the lattice model, no evidence of CST is present.

**Multi Wall Carbon Nanotubes** — An ideal Single Wall CN (SWCN) is a hexagonal network of carbon atoms (graphene sheet) that has been rolled up, in order to make a cylinder with a radius about 1 nm and a length about 1 $\mu$m. The unique electronic properties of CNs are due to their diameter and chiral angle (helicity) [19]. MWCNs, instead, are made by several (typically 10) concentrically arranged graphene sheets with a radius above 5 nm and a length which ranges from 1 to some hundreds of $\mu$m. The transport measurements carried out in MWNTs reflect usually the electronic properties of the outer layer, to which the electrodes are attached. Thus, in what follows we mainly discuss the LRCNs as a general class of CNs including also MWNTs. In general the LRCNs are affected by the presence of doping, impurities, or disorder, what leads to the presence of a large number of subbands, $N$, at the Fermi level [20]. It follows that the critical exponent has a different form, with respect to that calculated in reference [1].

The bulk critical exponent can be calculated in several different ways, e.g. see reference [21] where we obtained

$$\alpha_{\text{bulk}} \approx \frac{1}{4N} \left( K_N + \frac{1}{K_N} - 2 \right),$$ (4)

where

$$\frac{1}{K_N} \approx \sqrt{\frac{NU_0(q_c, B)}{(2\pi v_F)}}.$$ (5)

Here $v_F$ is the Fermi velocity, $U_0(p)$ corresponds to the Fourier transform of the 1D electron-electron interaction potential and $q_c = 2\pi/L$ is the infra-red natural cut-off due to the length of the CN, $L$. For a strictly 1D system, such as a CN in absence of magnetic field, $U_0(p)$ does not depend on the momenta of the interacting electrons. In general [22] we need to introduce two different couplings for two different forward scattering processes (with a small transferred momentum). The first term, $g_2$, is obtained by considering 2 scattered electrons with opposite momenta ($\pm k_F$). The second term, $g_4$, is obtained by considering 2 scattered electrons with (almost) equal momenta $(k_1 \sim k_2 \sim k_F)$. It follows that

$$K_N \approx \sqrt{\frac{2\pi v_F + N (g_4 - g_2)/2}{2\pi v_F + N (g_4 + g_2)/2}},$$ (6)

which corresponds to the previous formula when $g_2 = g_4 = U_0(q_c)$. As in reference [22], the presence of a magnetic field gives $g_2 \neq g_4$ because of the edge localization of the currents with opposite chiralities, and we need the $B$ dependent values of $g_2$ and $g_4$.

The value of $\alpha_{\text{bulk}}$ obtained in reference [21] is in agreement with the one obtained in reference [23], where also the end critical exponent was obtained as

$$\alpha_{\text{end}} \approx \frac{1}{2N} \left( \frac{1}{K_N} - 1 \right).$$ (5)