Producing ultra-strong magnetic fields in neutron star mergers

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We report an extremely rapid mechanism for magnetic field amplification during the merger of a binary neutron star system. This has implications for the production of the short class of Gamma-Ray Bursts, which recent observations suggest may originate in such mergers. In detailed magnetohydrodynamic simulations of the merger process, the fields are amplified via Kelvin-Helmholtz instabilities beyond magnetar field strength and may therefore represent the strongest magnetic fields in the Universe. The amplification occurs in the shear layer which forms between the neutron stars and on a time scale of only 1 millisecond, i.e. long before the remnant can collapse into a black hole.

The orbital decay of a neutron star binary system due to the emission of gravitational waves is one of the prime targets of gravitational wave detectors such as LIGO \cite{1} or GEO600 \cite{2}. Moreover, the long-suspected connection of neutron star binaries to Gamma-Ray Bursts (GRBs), the most luminous explosions in the universe, has received solid support from the first detections of afterglows from the short class of GRBs \cite{3, 4, 5}: unlike their long-duration
cousins (which are associated with the deaths of massive stars) short GRBs occur systematically at lower redshifts, both in galaxies with and without star formation and are not accompanied by a supernova explosion. The millisecond variability observed in the lightcurves of short GRBs suggests that a compact object, either a neutron star or a stellar mass black hole acts as the central engine.

The observed cosmological distances imply that large energies are involved and therefore, to avoid the so-called “compactness problem” (6), relativistic outflows with Lorentz-factors of several hundreds are required. To reach such extreme velocities a large amount of energy has to be deposited per rest mass, for example by the annihilation of neutrino-antineutrino pairs, $\bar{\nu}_i + \nu_i \rightarrow e + e^+$, or via magnetic mechanisms (7,8). Therefore, strong magnetic fields have been suggested as being important in producing GRBs (9,10,11,12,13), but the question of what field strengths can actually be reached in a merger remnant before it collapses to a black hole, has so far remained unanswered. Recently, a very energetic giant flare from the magnetar SGR 1806-20 has been observed (14, 15). Would it have been further away, but within 40 Mpc, its initial spike would -both on grounds of duration and spectrum- have been interpreted as a short gamma-ray burst. The lack of excess events from the direction of the Virgo cluster, however, suggests that only a small portion of previously observed short bursts could have been giant magnetar flares. Nevertheless, the similarity in physical properties may point to a common or similar mechanism behind both phenomena.

Although computer simulations of binary neutron star mergers have reached a good degree of realism (16,17,18,19), none has so far been able to take magnetic fields into account, primarily due to the numerical challenge posed by simulating even the hydrodynamics of the merger process. Here we present global neutron star merger simulations that follow the evolution of the magnetic field. Our main result is that the existing neutron star magnetic fields ($10^{12}$ G) become amplified by several orders of magnitude within the first millisecond after the merger, which is
long before the collapse to a black hole can proceed. Our robust lower limit on the field that can be reached is $2 \times 10^{15}$ G, but is it highly probable that much stronger fields are realized in nature.

Our simulations are three-dimensional computer simulations of two neutron stars that coalesce due to the emission of gravitational waves. The equations of hydrodynamics are solved with a Lagrangian particle scheme (Smoothed Particle Hydrodynamics, SPH) [for a review see (20)] that is coupled to a temperature- and composition-dependent nuclear equation of state (21, 17). We include the effects of cooling and the change in matter composition due to neutrino-producing weak interactions. As the debris material covers the full range from completely opaque to completely transparent to neutrinos, we have to incorporate opacity effects. Thus on an additional grid, we calculate for each fluid parcel the opacities for each neutrino species and take them into account in the emission process (18). The Newtonian self-gravity of the neutron star fluid is evaluated efficiently using a binary tree algorithm. In addition we apply forces that emerge due to the emission of gravitational waves (17) - these are the forces that drive the binary towards coalescence. The new physics employed in these simulations is the inclusion of magnetic fields. To ensure the robustness of our results we apply two different methods: one using a recently developed algorithm for “Smoothed Particle Magnetohydrodynamics” (22) and one using a method where the magnetic field is calculated from the so-called “Euler potentials”, $\alpha$ and $\beta$ that are advected with each fluid particle (23). The magnetic field is calculated from these potentials according to

$$B = \nabla \alpha \times \nabla \beta.$$  \hspace{1cm} (1)

This prescription has the advantage that the divergence constraint (“no monopoles condition”) on the magnetic field is satisfied by construction. Apart from this difference, both methods yield similar results. The computational costs are dominated by the calculation of self-gravity – the
costs for the magnetic fields, the equation of state and the neutrino physics are negligible by comparison.

Our initial neutron stars are cold and have masses of 1.4 solar masses ($M_\odot$) each. The two stars are placed at an initial separation of 48 km with velocities corresponding to a circular orbit around their common centre of mass. As the inspiral dynamics only allows for a short time of tidal interaction, the neutron stars cannot be spun up substantially (24), therefore we start our calculations with non-spinning neutron stars. We choose uniform magnetic fields of the typical strength of neutron stars ($10^{12}$ Gauss) as initial conditions (note that we are modelling the interior fields, not the exterior dipole-like fields). The field orientation is parallel to the orbital angular momentum for one star and opposite for the other. Test calculations with different orientations yield maximum field strengths that are very similar.

The global dynamical evolution is shown (Fig. 1 and as an animation in the supporting online material): the two stars merge into a single object within about one orbital period ($\sim 2$ms). Subsequently, excess angular momentum is transported outward in spiral arms which quickly spread into a thick accretion disk around the central object. When the stars come into contact a shear interface forms, across which the tangential velocity exhibits a jump (Fig. 2). In such a shear layer even infinitesimally small perturbations are unstable to the Kelvin-Helmholtz (KH) instability and will grow, in this case causing the interface to curl up into vortex rolls. An analogous effect occurs when wind blows across the surface of a lake, curling up the surface into waves. The initial growth rate, $\sigma_\lambda$, of the KH instability may be calculated analytically for the inviscid, incompressible case (both properties are good approximations for neutron stars). In the linear regime the growth rate is $\sigma_\lambda = \pi v / \lambda$ (25), where $\lambda$ is the wavelength of the growing mode and $v$ the velocity across the jump. As the shortest modes grow fastest, the numerical results can only be lower limits on the growth realized in nature. Inserting the smallest length that we can numerically resolve for $\lambda$ and the simulation value for $v$ into the above equation...
yields growth rates consistent with the simulations.

In all cases we find that the field in the vortex rolls is amplified within $\sim 1$ millisecond by orders of magnitude (Figs. 3 and 4). This time scale should be compared to the 50-100 milliseconds which are estimated for the newly formed, differentially rotating central object to collapse into a black hole. The high field strength material produced in the shear instability between the stars is subsequently advected with the matter to cover the surface of the central merger remnant (Fig. 1).

As the length scales we can resolve numerically are larger than the physical lengths that will trigger the KH instability in nature, our numerical results represent robust lower limits on the true magnetic field strengths. This is demonstrated by our numerical resolution study (Fig. 4). Each time we double the numerical resolution (increase the particle number by a factor of 10), the peak field strength increases by a factor of a few. The highest numerical resolution that we can currently afford (2 $\times$ 10$^6$ fluid particles), yields a field strength beyond 2 $\times$ 10$^{15}$ Gauss, i.e. larger than the largest magnetic fields that have been observed in magnetars. The growth is likely to saturate when the magnetic field becomes strong enough to feed back on the fluid or alternatively when the field becomes buoyant. In either case the field strength reached will be comparable to the equipartition field strength (here 10$^{16}$-10$^{18}$ G), where the magnetic pressure becomes comparable to the gas pressure.

While we can only speculate about the field strengths that will be actually produced in nature, it is clear that the strong magnetic fields that have been conjectured in earlier work, occur naturally in the initial shear phase even before possible dynamo mechanisms could have set in. If a fraction $\epsilon$ of the rotational energy of the central object of the remnant, 8 $\times$ 10$^{52}$ erg, is channelled into the magnetic field, the field strength averaged over the central object will be

$$B = 1.2 \times 10^{17} G(\epsilon/0.1)^{1/2}(E_{\text{kin}}/8 \times 10^{52}\text{erg})^{1/2}(15\text{km}/R_{\text{co}})^{3/2},$$

where $R_{\text{co}}$ is the radius of the central object. Locally, the field strength could be even higher. Near equipartition matter blobs
in high field pockets (such as the vortices seen in Figure 3) will become buoyant, float up and produce a relativistic blast as they break through the surface of the central object (12). This could be the variable, relativistic outflow that is required to produce a GRB far from the central engine. In this case the millisecond substructures would be determined by the fluid instabilities in the central object, but the overall duration would be set by the time it takes the central object to collapse or to consume its rotational energy. If we use the magnetic dipole spin down time as an order of magnitude estimate and insert typical numbers from the simulation, we find

\[ \tau = 0.14s \left( \frac{B}{10^{17} G} \right)^{-2} \left( \frac{P}{1.5\text{ms}} \right)^2 \left( \frac{15\text{km}}{R_{co}} \right)^4 \left( \frac{M_{co}}{2.5M_\odot} \right), \]  

where \( P \) is the rotational period and \( M_{co} \) is the mass of the central object. This timescale is close to the typical duration of a short GRB. Note also that the high-field strength matter is transported by the fluid motion to the remnant surface. The sudden appearance of \( 10^{17} \)-G-material at the surface of the neutron-star-like central object will very plausibly launch magnetized blasts similar to those described in (27). Similar processes involving buoyant magnetic fields, although at lower field strengths (\( \sim 10^{14} - 10^{16} G \)), may also be at work in the accretion torus. It is also worth pointing out the somewhat speculative possibility that such a merger could produce magnetars.

Recent calculations (28, 29) in the GRB context have shown that the deposition of thermal energy above accretion disks, for example from neutrino annihilation, can drive relativistic outflows. Such outflows can be narrowly collimated, but in general a large spread in energies and opening angles depending on the specifics of the merging system is expected. In the light of the above presented results, it is hard to see how a signal from the strong magnetic fields -on top of the neutrino-annihilation driven outflows- can be avoided.

At the neutrino luminosities produced in the merger (\( > 10^{52} \) erg/s), neutrinos will, as in the case of newborn neutron stars, drive a strong baryonic wind (30). This material poses a
potential threat to the emergence of the required ultra-relativistic outflow. The central object is rather hot \((20 - 25 \text{ MeV}, \text{ where } 1 \text{ Mev} = 1.16 \times 10^{11} \text{K})\), but very opaque to neutrinos. It therefore only contributes moderately to the total neutrino luminosity, which is dominated by the inner shock-heated torus regions, where we expect the most of the wind material to come from. Directly after the merger the environment is of very low density and rising magnetic bubbles will, via magnetic pressure, help to keep the region above the central object relatively clean of baryons. But as the neutrino luminosity rises and the continuously braked central object takes longer and longer to reach buoyancy field strength, it will become increasingly difficult to launch relativistic outflows. The interaction between such magnetic bubbles and a baryonic wind will be very complicated and whether relativistic outflow develops or not may depend on the details of the merging system. The estimated double neutron star merger rate ranges from about \(4 - 220 \times 10^{-6}\) per year and galaxy \((7)\), and is thus comfortably two orders of magnitude larger than the rate required to explain short GRBs. Thus, even allowing for beaming and for a fraction of systems that could possibly fail to provide the right conditions (instead producing baryon-loaded X-ray or UV-flashes), the merger rate is still large enough to explain the observed short GRBs.

The two mechanisms - neutrino annihilation and magnetic processes - will show a different temporal evolution and they will also differ in the energies they can provide the burst with. The torus that dominates the neutrino emission takes a few dynamical time scales to form (in our simulations the neutrino luminosity peaks about 30 milliseconds after the stars have come in contact). The initial amplification of the magnetic field occurs on a much shorter time scale. Therefore, we expect the very early prompt emission to come from the magnetic field alone. The outflows driven by neutrino-annihilation contain typically \(10^{48}\) ergs \((7)\), magnetic mechanisms could easily provide \(10^{51}\) ergs or more \((8, 19)\). Very energetic short bursts would therefore have to be attributed to magnetic mechanisms. In any case, short GRBs that arise from the merger
of magnetized neutron stars will exhibit a large intrinsic diversity, a very complex temporal behavior and their observed properties will drastically depend on their orientation relative to the line of sight.

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Figure 1: Snapshots (left to right, top to bottom) of the coalescence of two magnetized neutron stars, showing magnetic field strengths in the material at and below the orbital plane. Dimensions in each panel are $\sim 140$ km from left to right. The stars move gradually towards each other and then merge in a “plunging phase” within about one orbital period ($\sim 2$ ms; first two snapshots). This object sheds mass into spiral arms that are subsequently wrapped around the central object (snapshots three to five) to form a hot torus (last snapshot). The magnetic field is amplified in the shear instability between the stars and subsequently advected with the matter to cover the surface of the central merger remnant.
Figure 2: Density and velocity field in the orbital plane at the moment when the stars come into contact (\(t=1.51\) ms). A shear interface forms between the stars, across which the tangential velocity exhibits a large jump. This interface is unstable to the Kelvin-Helmholtz instability and will curl up into vortex rolls (see next Figure).
Figure 3: Close-up of the central regions at t=2.65 milliseconds. The colour coding shows strength of the magnetic field whilst the arrows show the fluid velocity in the corotating frame (that is, with the dominant orbital velocity component removed). The shear interface shown in Figure 2 can be seen to have curled up into vortex rolls. In these vortices the field is strongly amplified to strengths exceeding $10^{15}$G. High-field material that has passed through these vortex rolls is subsequently spread across the surface of the central merger remnant (see first three panels of Figure 1).
Figure 4: Maximum magnetic field strength as a function of the number of fluid particles. All three runs are identical apart from the numerical resolution. The maximum field strength of the best resolved run, $2 \times 10^{15}$ G, is a strict lower limit on the magnetic field that can be reached in a neutron star merger.