Low-complexity Successive Detection Method for OFDM Systems Over Doubly Selective Channels

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Abstract: Orthogonal frequency division multiplexing (OFDM) system is robust against inter-symbol interference (ISI) by increasing the symbol duration. However, for high-speed application, time variation of a multipath channel result in inter-carrier interference (ICI). It leads to the performance degradation. This works propose a low-complexity successive detection algorithm combined with the techniques of Newton’s iterative matrix inversion method and successive detection method according to signal to noise ratio (SNR) in OFDM system. With the assumption of linearly varying channel impulse responses under fast fading conditions, the proposed algorithm reduces the complexity from $O(N^3)$ of conventional zero-forcing algorithm to $O(N^2)$, meantime it mitigates the noise enhancement problem. In all, it surpasses the performance bound of zero-forcing (ZF) equalization.

1. INTRODUCTION

In recent years, the demand for real-time and high-rate multimedia services increase rapidly. To satisfy the demand, some advanced broadband communication techniques have been proposed. In particular, Orthogonal Frequency Division Multiplexing (OFDM) technique is widely adopted in current and next-generation communication systems.

In OFDM systems, a high-rate serial data stream is split into many low data-rate parallel data streams, each is modulated by a prescribed subcarrier orthogonal to all the other subcarriers. By adding a cyclic prefix (CP) to the beginning of each OFDM symbol, the intersymbol interference (ISI), and intercarrier interference (ICI) caused by delay spread would be overcome.

However, when a channel response is time-varying, the channel variation within an OFDM symbol will destroy the orthogonality between subcarriers and result in ICI. The ICI effect will degrade the system performance. As a result, the error floor of bit-error rate (BER) would become more severe than that of a time-invariant channel.

To overcome the ICI problems due to time-varying channels, several data equalization algorithms have been developed, such as the zero-forcing (ZF) and minimum mean square error (MMSE) equalization methods. Unfortunately, both algorithms require matrix inversion operations which are computationally intensive. Besides, ZF equalization method will result in noise enhancement problem, while MMSE equalization method requires the second-order statistic of channels, which is difficult to obtain in practice.

In order to reduce the computational complexity, the authors in [1]-[2] proposed several methods. However, the proposed techniques still have the noise enhancement problem. On the other hand, the method in [3] has good performance but with high computation cost. Thus, the objective of this work is improving the performance and decreasing the computational complexity simultaneously, over the current equalization techniques.

Specifically, we propose a low-complexity and high-performance equalization algorithm by combining Newton’s matrix inversion method with a successive detection scheme [3].

The rest of this paper is organized as follows. Section 2 describes the OFDM system model and ICI analysis. In section 3, we will introduce the proposed successive detection algorithm. Its simulation results are provided in Section 4. Finally, Section 5 is the conclusion.

2. SYSTEM MODEL AND ICI ANALYSIS

In [4], Weinstein suggested that the modulators in the transmitter and the matched filters in the receiver for the OFDM systems can be implemented by IDFT and DFT, respectively. Figure 1 shows the discrete-time OFDM system model. The symbol index $i$ is dropped in the following discussion. The modulated signal can be written as

$$x(n) = \sum_{k=0}^{N-1} X(k)e^{j2\pi kn/N}, \quad 0 \leq n \leq N-1,$$ 

(1)
where $X(k)$ is the transmitted data at the $k$-th subcarrier. Then, the modulated signal is preceded with a CP and delivered over the air through a time-varying multipath fading channel. Therefore, the received signal can be represented as

$$y(n) = \sum_{l=0}^{L-1} h(n,l)x(n-l) + w(n)$$

where $h(n,l)$ is the $l$-th channel path at time instant $t = nT_s$, $T_s = \frac{T}{N}$ is sampling period, $T$ is the symbol duration, $L$ is the number of channel taps, $(\cdot)_l$ represents the modulo $N$ operation, and $w(n)$ is the sampled additive white complex Gaussian noise with variance $\sigma^2$. The received signal after DFT at the $k$-th subcarrier is

$$Y(k) = \sum_{n=0}^{N-1} y(n)e^{-j2\pi kn/N}$$

(3)

### 2.2 ICI analysis

According to the system model depicted in Section 2.1, the received signal after DFT at the $k$-th subcarrier is obtained by substituting equations (1) and (2) into (3)

$$Y(k) = G(k,k)X(k) + \sum_{m=0}^{N-1} G(k,m)X(m) + W(k)$$

(4)

where

$$G(k,m) = \frac{1}{N} \sum_{n=0}^{N-1} \sum_{l=0}^{L-1} h(n,l)e^{-j2\pi kn/N}$$

and

$$W(k) = \sum_{n=0}^{N-1} w(n)e^{-j2\pi kn/N}, \quad 0 \leq m, k \leq N-1.$$  

Since the second term on the right hand side of equation (4) is not zero, the desired signal suffers ICI when the channel is time-varying. For further ICI analysis, let’s first define the time average of channel impulse response, $h(n,l)$, as

$$h_{\text{ave}}(l) = \frac{1}{N} \sum_{n=0}^{N-1} h(n,l)$$

(5)

and the variation $\Delta h(n,l)$ of $h(n,l)$ as

$$\Delta h(n,l) = h(n,l) - h_{\text{ave}}(l)$$

(6)

Therefore,

$$G(k,k) = \sum_{l=0}^{L-1} h_{\text{ave}}(l)e^{-j2\pi kn/N}$$

(7)

and

$$G(k,m) = \frac{1}{N} \sum_{n=0}^{N-1} \sum_{l=0}^{L-1} (\Delta h(n,l))e^{-j2\pi kn/N}$$

(8)

If the channel impulse response is time-invariant within an OFDM symbol, $\Delta h(n,l)$ will be zero and $G(k,m) = 0$, $m \neq k$. Generally, the channel impulse response will be time-varying, especially for a mobile user. Fortunately, the variation of a channel path can be described by a linear function [1] under the conditions of slow fading up to moderately fast fading, particularly when the normalized Doppler frequency (NDF) $f_d$ is roughly less than 0.1, where NDF is defined as $f_d = f_sT$, and $f_s$ is the maximum Doppler frequency. This paper focuses on the effect of ICI within an OFDM symbol.

Specifically, the variation of each channel path can be approximated as [1,5,6]

$$\Delta h(n,l) = (n - \frac{N-1}{2})\alpha_l, \quad 0 \leq n \leq N-1, \quad 0 \leq l \leq L-1$$

(9)

where $\alpha_l$ is the slope of the $l$-th channel path. Given (6), the response of the $l$-th channel path at the $n$-th time instance can be written as

$$h(n,l) = h_{\text{ave}}(l) + (n - \frac{N-1}{2})\alpha_l$$

(10)

The received signal after DFT can be rewritten as the following matrix form

$$Y = \frac{1}{N} QH^H X + W$$

(11)

where

$$Y = [Y(0), \cdots, Y(N-1)]^T,$n

$$X = [X(0), \cdots, X(N-1)]^T,$n

$$W = [W(0), \cdots, W(N-1)]^T,$n

$$H(n,m) = h(n, (n-m)_{\text{mod}N}),$$n

and $Q$ is an $N$-point DFT matrix with its elements $Q(n,m) = e^{-j2\pi nm/N}$.

According to (10), we can express the received frequency-domain signal as

$$Y = \frac{1}{N} Q(H_{\text{ave}} + Da)Q^H X + W$$

$$= \frac{1}{N} (QH_{\text{ave}}Q^H + QDQ^H aQ^H) X + W$$

(12)

where matrix $D$ is a diagonal matrix with diagonal elements of $[-\frac{N-1}{2}, (1-\frac{N-1}{2}), \cdots, (N-\frac{N-1}{2}), \cdots, (\frac{N-1}{2})]$, $H_{\text{ave}}$ and $a$ are circular matrices which contain the static terms and variation slopes of the channel, respectively. Furthermore, $H_{\text{ave}}$ and $a$ are diagonal matrix. For convenience, $G$ is defined as $G = H_{\text{ave}} + QDQ^H a$. Thus, the received signal after DFT can be written as
\[ Y = \frac{1}{N} GX + W \] \hspace{1cm} (13)

where \( w_k = \frac{G'(k,k)}{\sum_{m=0}^{M-1}|G(k,m)|^2} \).

### 3. PROPOSED SUCCESSIVE DETECTION METHOD

First, we will introduce the well-known Newton’s iterative matrix inversion method [7]. Let \( U_v \) be an approximation to \( G^{-1} \) at the \( v \)-th iteration and the residual matrix is defined as \( R_v = I - GU_v \). Furthermore, the Newton’s iteration can be written in the following form

\[ U_{v+1} = U_v (2I - GU_v) \] \hspace{1cm} (14)

Here, let \( U_{in} \) be the initial matrix, the result of the first iteration is

\[ U_1 = U_{in} (2I - GU_{in}) = 2U_{in} - U_{in} GU_{in} \] \hspace{1cm} (15)

In fact, it follows immediately from (14) and (15) that

\[ \sum_{n=1}^{m} c(n) (U_{in} G)^{(n-1)} U_{in} \]

where \( c(1) = 4 \), \( c(2) = -6 \), \( c(3) = 4 \) and \( c(4) = -1 \). Thus, for the \( v \)-th iteration, \( U_v \) can be represented as:

\[ U_v = \sum_{n=1}^{m} c(v,m) (U_{in} G)^{(n-1)} U_{in} \] \hspace{1cm} (17)

In order to obtain the \( c(v,m) \), the coefficients can be generated iteratively, through a simple proposed real-time architecture as shown in Figure 2. Besides, one can also obtain the coefficients off-line and store at memory devices for lower computational complexity.

The initial value for Figure 2 is \( C(1,m) = [2, -1, 0, 0, \ldots, 0] \). It is very easy to verify the coefficient generator by expanding (17). Through this iteration process, the coefficients, \( c(v,m) \), are acquired.

Clearly, the equalized result is

\[ \tilde{Y} = U_v Y = \sum_{n=1}^{\infty} c(v,m) (U_{in} G)^{(n-1)} U_{in} Y \]

For convenience, define \( \tilde{s}_n \) as \( \tilde{s}_n = (U_{in} G)^{(n-1)} U_{in} Y \).

By observing (18), one can show that \( \tilde{s}_n \) can be recursively calculated easily by the equation

\[ \tilde{s}_{n+1} = (U_{in} G) \tilde{s}_n + QDQ^H \tilde{s}_n \]

and the initial matrix [2] is

\[ U_{in} = \text{diag}([w_0, w_1, \ldots, w_{M-1}]) \]

After Newton’s iterative matrix inversion, the equalized result can be obtained by (18).

From now on, in order to utilize the time diversity to resist the noise enhancement problem and the residual interference during the successive detection process, we detect the data according to signal to noise ratio (SNR) from high to low.

Recall the results in (13) and (18). The equalized data is

\[ \tilde{Y} = U_v Y = \frac{1}{N} U_v GX + U_v W \]

Thus, one can determine the maximum SNR as follows,

\[ \arg \max_k SNR_k = \arg \max_k \frac{\left| u_{v,k} g_k \right|^2}{\sigma^2 \left| u_{v,k} \right|} \]

\[ = \arg \max_k \frac{1}{\sigma^2 \left| u_{v,k} \right|} = \arg \min_k \left| u_{v,k} \right| \]

where \( u_{v,k} \) is the row vector of the equalizer matrix \( U_v \), and \( g_k \) is the column vector of the matrix \( G \). For further
reduction of the complexity, one can obtain the same result as (22) through the following optimization process
\[
\arg \max_k \text{SNR}_k = \arg \min_k \|u_{m,k}\| = \arg \min_k \|v_k\|
\]
This result is based on the fact that the norm of \(U_{ini}\) term will dominate the norm of \(U_v\), because the \(\{(U_{ini}G)^{m-1}\}\) term will be less significant in \(U_v\) for larger \(m\) as described by (17).

After the highest SNR determination process, one can detect data on the subcarrier with the highest SNR. Then, the interference coming from the said subcarrier will be cancelled from the frequency-domain received signal \(Y\).

\[
Y_{new} = Y_{original} - g_d \tilde{X}
\]
where \(\tilde{X}\) is the decided data. Next, the second highest SNR should be determined to cancel the interference coming from the second highest-SNR subcarrier, and so on. For clear interpretation, Figure 3 shows the whole process.

1. \(n = 1\)
2. \(i_n = \arg \min_{k} \|w_k\|\)
3. Loop
4. \(\tilde{Y} = \sum_{m=1}^{V} c(v,m) s_n(i_m)\)
5. \(\tilde{X}(i_n) = \text{slice}(\tilde{Y}(i_n))\)
6. \(Y = Y - g_n \tilde{X}(i_n)\)
7. \(G = [g_0, \ldots, g_{n-1}, 0, g_{n+1}, \ldots, g_{N-1}]\)
8. \(i_{n+1} = \arg \min_{k \in \{0, \ldots, N\} \setminus \{i_n\}} \|w_k\|\)
9. if \(n \neq N\)
10. \(n = n + 1\)
11. else
12. END

Figure 3 - Proposed successive detection method

4. SIMULATION RESULTS

Here, we simulate an OFDM system assuming \(N = 64\) sub-carriers, \(L_p = 16\), QPSK modulation and \(v = 3\). The simulation channel is a Rayleigh fading channel described in [8]. The power delay profile is chosen as the “Vehicular A” channel model defined by ETSI for the evaluation of UMTS radio interface proposals [9]. By applying the Newton’s iterative matrix inversion method to the data equalization process, one can reduce computational complexity from \(O(N^3)\) of conventional zero-forcing algorithm to \(O(N^2)\).

Because all of the operations in (18) and (19) are vector multiplications or vector additions, the computational complexity can be reduced significantly. The total complexities of the proposed and conventional zero-forcing methods are summarized in Table 1.

![Figure 4 - BER vs. SNR at normalized Doppler frequency 0.05](image)

![Figure 5 - BER vs. SNR at normalized Doppler frequency 0.08](image)

Furthermore, the bit error rate (BER) performance of the proposed detection algorithm surpasses the performance bound of the zero-forcing equalization, because the new technique mitigates the noise enhancement problem by utilizing successive detection. Figure 4 and Figure 5 show the BER simulation results of the proposed method and the zero-
forcing method, under two different normalized Doppler frequency settings of $f_{nd} = 0.05$ and $f_{nd} = 0.08$. As shown, the proposed method has better performance than the zero-forcing (ZF) method in both simulation environments.

|                | No of ADDs     | No. of MPYs     | No of DIVs     |
|----------------|---------------|----------------|---------------|
| Zero-forcing   | $11N^3 - 3N^2$| $12N^3 + 6N^2 - 10N$ | $2N^2 - 2N$   |
| Proposed method| $(12\times2^2 + 8)N^2 - 2N$ | $(17\times2^2 + 12)N^2$ | $2N^2$        |

*Table 1 – Complexity comparison between the proposed and conventional zero-forcing methods*

Besides, Figure 5 shows that the performance of the proposed method is significantly better than conventional zero-forcing algorithm in higher NDF, than in the case of lower NDF as shown in Figure 4.

5. CONCLUSIONS

This paper presents a successive detection method with low computational complexity. Especially, simulation results show better performances and lower computational complexity than conventional zero-forcing approach. Hence, the proposed method can be effectively applied to OFDM systems with time-varying and multi-path Rayleigh fading channel such as WLAN, DVB-H and WiMax systems, and etc.

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