Supporting Information for

Ubiquitous Coexisting Electron-Mode Couplings in High-temperature Cuprate Superconductors

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1. Summarized ARPES observations of the electron-mode couplings in Bi2201, Bi2212 and Bi2223

Table S1 shows the summarized ARPES observations of the electron-mode couplings in Bi2201, Bi2212 and Bi2212 from the previous reports[1–15] and our present work.

2. Samples and doping levels

Figure S1 shows the magnetic measurement results of the Bi2201, Bi2212 and Bi2223 single crystal samples used in the present ARPES measurements. High-quality single crystals of these Bi-based cuprate superconductors were grown by the traveling solvent floating zone method[16–19]. We changed the doping level of Bi2201 by varying the La content in Bi$_{2}$Sr$_{2−x}$La$_{x}$CuO$_{6+δ}$ (La-Bi2201) or Pb concentration in Bi$_{2.12−x}$Pb$_{x}$Sr$_{1.88}$CuO$_{6+δ}$ (Pb-Bi2201), combined with the post-annealing of the samples in different atmospheres at different temperatures[16, 17, 20] (Fig. S1a). The doping level of the Bi2212 samples was changed by post annealing in different atmospheres at different temperatures[21] (Fig. S1b). A heavily overdoped Bi2212 sample was prepared in situ in the ARPES chamber by annealing in ozone atmosphere (Fig. S2). The Bi2223 sample is obtained by annealing the as-grown sample in flowing air at 550°C (Fig. S1c). The doping level of the underdoped and optimally doped Bi2201 samples is estimated from the $T_c$–doping level correspondence established before[22]. The doping level of the overdoped Bi2201 samples is determined directly from the Fermi surface area measured by ARPES[20]. The doping level of the Bi2212 samples is estimated from the empirical relation, $T_c = T_{c,max}[1−82.6(p−0.16)^2]$, with a $T_{c,max}=91$ K[23].
3. Experimental geometry

Figure S3 shows the experimental geometry of the present ARPES measurements. The analyzer slit is along the X axis, and the incident light is in the X-Y plane. The angle between the analyzer lens axis and the incident light is 50 degrees. We change angle $\theta$ and $\Phi$ to get the band structures in different momentum regions. In the s polarization, the electric vector of the incident laser light is perpendicular to the X-Y plane. In the p polarization, the electric vector of the incident laser light lies in the plane. The photon flux is set between $10^{13}$ and $10^{14}$ photons/second and the acquisition time is $\sim$20 minutes for each measurement.

4. The second derivative method to reveal the electron coupling with two coexisting sharp modes in the band structures along the nodal direction in Bi2212

Figure S4 shows the observation of two coexisting mode-couplings in the band structures measured in Bi2212 along the nodal direction. Fig. S4a shows the band structure of the Bi2212 OD73K sample along the nodal direction at 17K. Fig. S4b shows a typical EDC from Fig. S4a, where a main peak (peak), a low energy hump (LH), a high energy hump (HH), a low energy dip (LD) and a high energy dip (HD) can be observed. Fig. S4c shows the second derivative of the EDC in Fig. S4b. The sign of the second derivative EDC in Fig. S4c is reversed to get the curve in Fig. S4d which shows a direct correspondence to the original EDC in Fig. S4b. In this way, the subtle features in the original data become more pronounced in the second
derivative EDC, and the peak (peak), dip (LD and HD) and hump features (LH and HH) in the second derivative data (Fig. S4d) correspond well to those in the original data (Fig. S4b). For this reason, we will use this method throughout the paper. Correspondingly, two types of second derivative images can be obtained. One is to show the absolute magnitude of the negative values of the second derivative EDC in Fig. S4d, as shown in Fig. S4e, which highlights the peak and hump features. The other is to show the absolute magnitude of the positive values of the second derivative EDC in Fig. S4d, as shown in Fig. S4f, which highlights the dip features. Four features can be observed in the second derivative image in Fig. S4e below the Fermi level marked by LLH (left-low energy-hump), LHH (left-high energy-hump), RLH (right-low energy-hump) and RHH (right-high energy-hump). Four features can also be observed in the other second derivative image in Fig. S4f below the Fermi level marked by LLD (left-low energy-dip), LHD (left-high energy-dip), RLD (right-low energy-dip) and RHD (right-high energy-dip).

5. Simulations of the electron-mode coupling in the normal state

We simulated the single-particle spectral functions $A(k, \omega)$ in the normal state at zero temperature[24]. The ARPES intensity $I(k, \omega)$ is expressed as:

$$I(k, \omega, T) = f(\omega, T)A(k, \omega)$$  \hspace{1cm} (1)

where $f(\omega, T)$ is the Fermi-Dirac distribution function. The spectral function $A(k, \omega)$ is expressed as:

$$A(k, \omega) = -\frac{1}{\pi} \frac{\Sigma''(k, \omega)}{[\omega - \epsilon(k) - \Sigma'(k, \omega)]^2 + [\Sigma''(k, \omega)]^2}$$  \hspace{1cm} (2)

in which $\epsilon(k)$ represents the bare band. The real part ($\Sigma'(k, \omega)$) and imag-
inary part \((\Sigma''(\mathbf{k}, \omega))\) of the electron self-energy \(\Sigma(\mathbf{k}, \omega)\) can be expressed as:

\[
\Sigma'(\mathbf{k}, \omega) = \int_{E_s}^{0} d\Omega \alpha^2 F(\Omega) \ln \left| \frac{\Omega + \omega}{\Omega - \omega} \right| \tag{3}
\]

\[
\Sigma''(\mathbf{k}, \omega) = \Sigma''_{\text{imp}} + \beta \omega^2 - \pi \int_{E_s}^{0} d\Omega \alpha^2 F(\Omega) \tag{4}
\]

Here \(\alpha^2 F(\Omega)\) is the Eliashberg function and \(E_s\) is the cutoff of the Eliashberg function. The first term of \(\Sigma''(\mathbf{k}, \omega)\) is from impurity scattering, the second term of \(\Sigma''(\mathbf{k}, \omega)\) is from the electron-electron interaction in the Fermi liquid picture and the third term comes from the electron-boson coupling.

The electron-boson coupling strength, \(\lambda\), is defined as:

\[
\lambda = -2 \int_{E_s}^{0} \frac{d\Omega}{\Omega} \alpha^2 F(\Omega) \tag{5}
\]

In order to simulate the electron coupling with sharp modes, we assume a Lorentzian form of the Eliashberg function for each mode:

\[
\alpha^2 F(\Omega) = \frac{C(\Gamma/2)^2}{(\Omega + \omega_0)^2 + (\Gamma/2)^2} \tag{6}
\]

Here \(\omega_0\) is the mode energy, \(\Gamma\) is the linewidth of the Lorentzian peak (full width at half maximum, FWHM) and \(C\) is a constant.

We first simulated the spectral function of the electron coupling with one mode as shown in Fig. S6a. To make the simulation comparable to the band structure measured along the nodal direction in Bi2212 (Fig. 1a1), we take the bare band \(\epsilon(k) = 2.4k\), the coefficient of electron-electron interaction \(\beta = -1\) and \(\Sigma''_{\text{imp}} = 0.017\) eV. The unit of \(\epsilon(k)\), \(\omega\) and \(\Omega\) is eV, and the unit of \(k\) is \(\pi/a\), where \(a\) (3.82 Å) is the distance of the two nearest Cu atoms in the CuO$_2$ plane. The mode energy is 70 meV, its linewidth is 15 meV and the electron-boson coupling strength \(\lambda = 0.5\). Fig. S6b and S6c are the
second derivative images of Fig. S6a in two different ways. Fig. S6d shows the EDCs obtained from the spectral function in Fig. S6a at three different momentum points. Their corresponding second derivative EDCs are shown in Fig. S6e. When only one mode is involved in the mode-coupling, only one dip and one hump structures can be produced in EDCs.

We then simulated the spectral function of the electron coupling with two modes as shown in Fig. S6f. Here the same simulation parameters are used except that two modes are considered in the simulation. The two mode energies are 70\,\text{meV} and 40\,\text{meV}, both the linewidths are 15\,\text{meV} and the electron-boson coupling strength $\lambda$ is 0.5. Fig. S6g and S6h are the second derivative images of Fig. S6f in two different ways. Fig. S6i shows the EDCs obtained from the spectral function in Fig. S6f at three different momentum points. Their corresponding second derivative EDCs are shown in Fig. S6j. When two sharp modes are involved in the mode-coupling, two dip and two hump features can be produced in EDCs.

In order to find out the mode energy when the mode involved in the electron-boson coupling has a finite linewidth, we simulated the spectral functions by considering different mode linewidth for the single mode case (Fig. S7(a-d)) and two modes case (Fig. S7(e-h)). In the single mode case, the positions of the hump and dip change with the mode linewidth as shown in Fig. S7d. But the middle point of the hump and dip features is very close to the energy of the mode. These results indicate that, when the involved mode has a finite linewidth, the energy position of the dip in EDCs no longer represents the mode energy. It is the middle point between the dip and hump features that represents the mode energy. The same conclusion holds when
two modes are involved in the electron-boson coupling as seen in Fig. S7h.

When the bandwidth is comparable or smaller than the involved mode energy, the spectral function of the electron-boson coupling can still be simulated by the formula (1)-(6). To make the simulation comparable to the band structure measured near the antinodal region in Bi2212 (Fig. 2a1), we take the bare band $\epsilon(k) = -0.035 + 7.12(k - 0.07011)^2$, the coefficient of electron-electron interaction $\beta = -1$ and $\Sigma^{''}_{imp}=0.017 \text{ eV}$. The two mode energies are 70 meV and 40 meV, both the linewidths are 15 meV and the electron-boson coupling strength $\lambda$ is 0.5. The simulated spectral function is shown in Fig. S5a2.

6. Simulated spectral functions involving two mode-couplings and their comparison with the measured data

Figure S5 shows the simulated single-particle spectral functions by involving electron-boson coupling with two modes at 70 meV and 40 meV in the normal state. Fig. S5a1 shows the simulated spectral function along the nodal direction which is the same as Fig. S6f. The corresponding second derivative images are presented in two different ways with one to highlight the peak and hump features (Fig. S5b1) and the other to highlight the dip features (Fig. S5c1). Fig. S5d shows the EDCs of the spectral function in Fig. S5a1 at different momentum points. The two hump (marked as LH and HH) and two dip (marked as LD and HD) features can be clearly observed in the EDCs. The corresponding second derivative EDCs are shown in Fig. S5e where the two hump and two dip features can be observed more clearly.

Figure S5a2 shows the simulated spectral function near the antinodal
region in the normal state. Fig. S5b2 and S5c2 show the corresponding second derivative images to highlight the hump features and dip features, respectively. Fig. S5f shows the EDCs of the antinodal spectral function at different momentum points. The two hump (marked as LH and HH) and two dip (marked as LD and HD) features can also be clearly observed in the EDCs. These fine structures can be seen more clearly in the corresponding second derivative EDCs in Fig. S5g. These simulated results indicate that, when electrons are coupled with two modes, in addition to the main peak, coexisting double hump and double dip structures are produced in EDCs over a wide range of momentum space. It is clear that the simulated results (Fig. S5) show strong resemblance to the measured results in Bi2212 and Bi2201 (Fig. 1 and Fig. 2).

7. Observation of the \( \sim 40 \)-meV kink in the band structure measured at 17 K near the antinodal region in Bi2212

Figure S8a shows the \( \sim 40 \)-meV kink in the band structure of the Bi2212 OD73K sample measured at 17 K near the antinodal region. The bonding band (BB) and antibonding band (AB) can be observed simultaneously. The black line and red line are the dispersions for the right antibonding band (AB\(_R\)) and left bonding band (BB\(_L\)), respectively, obtained by fitting MDCs at different energies. A kink with the energy of \( \sim 40 \) meV can be observed clearly in both the dispersions. For comparison, we show the second derivative image of Fig. S8a in Fig. S8b and S8c. Four features marked as LLH, LHH, RLH and RHH can be observed in Fig. S8b which represent the hump features in EDCs, and two features marked as LHD and
RHD can be seen in Fig. S8c which represent the dip features in EDCs. The dip and hump features in the second derivative images indicate that ∼40-meV and ∼70-meV mode-couplings coexist in the band structure. The ∼40-meV kink in the band structure in Fig. S8a corresponds to the ∼40-meV mode-coupling obtained from the second derivative method in Fig. S8b and S8c.

8. Momentum dependence of the two coexisting mode-couplings in Bi2212 and Bi2201

Figure S9a shows the band structures of the Bi2212 OD73K sample measured along six momentum cuts at 17 K. They are the original band structures of the second derivative images in Fig. S3a. Figure S9b shows the band structures of the Bi2201 OD17K sample measured along different momentum cuts at 20 K. They are the original band structures of the second derivative images in Fig. S3b. Fig. S9c shows the real parts of the electron self-energy of the Bi2212 OD73K sample measured along five momentum cuts at 17 K. Two energy scales (marked as LScale and HScale) are observed and the low energy scale (LScale) becomes pronounced with the momentum cuts changing from the nodal to near the antinodal region. The two energy scales observed are plotted in Fig. 3e and they show a good agreement with the results obtained from the second derivative method. Fig. S9e shows the electron self-energy of the Bi2201 OD17K sample measured along different momentum cuts at 20 K. Two main energy scales can also be observed with the low energy scale (LScale) getting relatively pronounced and becoming dominant when the momentum cuts change from the nodal direction to the antinodal region. The
two energy scales determined from the self-energy in Fig. S9e are plotted in Fig. 3i; they are also consistent with those results obtained from the second derivative method. These observations demonstrate that there are two coexisting mode-couplings over the entire momentum space in Bi2212 and Bi2201.

9. Extraction of the real part of the electron self-energy

Figure S10 shows the extraction of the real part of the electron self-energy from the measured band structures. Fig. S10a and S10b show the band structures of the Bi2212 OD73K sample measured along the nodal direction and near the antinodal region, respectively, at 17 K. The band dispersions obtained by fitting the MDCs at different energies from Fig. S10a and S10b are shown in Fig. S10c and S10d, respectively, by the black lines. In order to extract the real part of the electron self-energy, we take the bands from the band structure calculations[25] as the bare bands, shown as the red lines in Fig. S10c and S10d. The extracted real parts of the electron self-energy from Fig. S10c and S10d are shown in Fig. S10e and S10f, respectively.

10. Doping dependence of the two coexisting mode-couplings in Bi2201, Bi2212 and Bi2223

Figure S11 shows the doping dependence of the two mode-couplings in Bi2201, Bi2212 and Bi2223 measured along the nodal direction. Fig. S11a shows the nodal band structures of Bi2201 over a wide doping range from underdoped all the way to the heavily overdoped non-superconducting samples. Fig. S11b shows the nodal band structures of the underdoped, optimally doped and overdoped Bi2212, including the heavily overdoped sample
with a $T_c$ of $\sim 36$ K. Fig. S11c shows the nodal band structure of the Bi2223 sample with a $T_c$ of 108 K. The corresponding second derivative EDCs obtained from the band structures in Fig. S11(a-c) are shown in Fig. S11e. The energy positions of the two energy scales obtained from the second derivative EDCs in Fig. S11e are summarized in Fig. S11f. The two coexisting mode-couplings are observed in all the measurements along the nodal direction in Bi2201, Bi2212 and Bi2223 with different doping levels. The energy scales of the two mode-couplings, within the experimental uncertainty, fall into two categories, the high energy one lies at $(79 \pm 10)$ meV while the low energy one lies at $(46 \pm 10)$ meV. The two energy scales show an overall slight decrease in their energy with the increasing doping level on the order of 5$\sim$8 meV.

Figure S12 shows the doping dependence of the two mode-couplings in Bi2212 and Bi2201 near the antinodal region. Fig. S12(a-c) shows the band structures of the optimally doped and overdoped Bi2212, including the heavily overdoped sample with a $T_c$ of $\sim 36$ K. Fig. S12(d-g) show the band structures of Bi2201 from underdoped to the heavily overdoped non-superconducting samples. The corresponding second derivative EDCs obtained from the band structures in Fig. S12(a-g) are shown in Fig. S12j. The energy positions of the two energy scales obtained from the second derivative EDCs in Fig. S12j are summarized in Fig. S12h. The two coexisting mode-couplings are observed in all the measurements near the antinodal region in Bi2212 and Bi2201 with different doping levels. The energy scales of the two mode-couplings, within the experimental uncertainty, also fall into two categories, the high energy one lies at $(83 \pm 10)$ meV while the low energy one lies at $(47 \pm 10)$ meV. The two energy scales also show an overall slight de-
crease in their energy with the increasing doping level on the order of 10 meV.

11. Observation of two coexisting mode-couplings above and below $T_c$ in overdoped Bi2212

Figure S13 shows the observation of two coexisting mode-couplings in the band structures measured above and below $T_c$ in Bi2212. Fig. S13(a-c) shows the band structures of the Bi2212 OD36K sample measured near the antinodal region at 20 K, 45 K and 60 K. Two types of EDC second derivative images obtained from Fig. S13(a-c) are shown in Fig. S13(d-f) and Fig. S13(g-i). Two flat band-like features can be observed in the second derivative images in Fig. S13(d-f) marked by LH and HH which correspond to the two hump features in EDCs. Two features can also be observed in the other second derivative images in Fig. S13(g-i) marked by LD and HD that correspond to the two dip features in EDCs. Fig. S13j shows the EDCs at the Fermi momentum, $k_{FR}$, measured at different temperatures. Fig. S13k shows the corresponding symmetrized EDCs. There is a superconducting gap opening in the symmetrized EDC measured at 20 K with a gap size of $\sim 6.5$ meV. There are no energy gap openings in the symmetrized EDCs measured at 45 K and 60 K. This indicates that the pseudogap temperature $T^*$, if it exists, is below 45 K. This is consistent with the temperature dependent measurement in Fig. S2. Fig. S13l shows the typical EDCs measured at different temperatures where two humps (LH and HH) and two dips (LD and HD) can be observed in each EDC. Fig. S13m shows the corresponding second derivative EDCs obtained from the EDCs in Fig. S13l where the two hump (LH and HH) and two dip (LD and HD) features can be observed more
clearly.

Figure S14a shows the band structures of the Bi2212 OD73K sample measured along the nodal direction at different temperatures. They are the original band structures of the second derivative images in Fig. 4a. Figure S14b shows the band structures of the Bi2212 OD73K sample taken near the antinodal region at different temperatures. They are the original band structures of the second derivative images in Fig. 4b. Figure S14c shows the band structures of the Bi2201 OD17K sample measured near the antinodal region at different temperatures. They are the original band structures of the second derivative images in Fig. 4c.

Figure S15 shows the observation of the two coexisting mode-couplings at different temperatures in the Bi2212 OD73K sample. Fig. S15a shows the real parts of the electron self-energy measured along the nodal direction at different temperatures in the Bi2212 OD73K sample. The high energy scale (marked by HScale) can be observed clearly up to 100 K. The high energy scale does not show obvious change with increasing temperature. The low energy scale (marked by LScale) can also be observed at different temperatures, as shown in Fig. S15c, and it also does not show obvious change with increasing temperature. Fig. S15b shows the real parts of the electron self-energy measured near the antinodal region at different temperatures in the Bi2212 OD73K sample. The low energy scale can also be seen up to 100 K and the corresponding energy position also does not vary with increasing temperature. The pseudogap temperature $T^*$ of the Bi2212 OD73K sample is $\sim 89$ K[26]. These observation demonstrate that the energy scales of the two mode-couplings do not show obvious change across the superconducting
temperature $T_c$ or the pseudogap transition $T^*$ in the Bi2212 OD73K sample.

12. Observation of the two coexisting mode-couplings under different photon polarizations in Bi2212 and Bi2201

Figure S16 shows the observation of the two coexisting mode-couplings under different photon polarizations in Bi2212 and Bi2201. In the s polarization, the electric vector of the incident laser light is perpendicular to the plane formed by the laser light and the axis of the electron energy analyzer. In the p polarization, the electric vector of the incident laser light lies in the plane. Fig. S16a1 shows the band structure of the Bi2212 OD36K sample measured under the s polarization near the antinodal region at 20 K. Two types of second derivative images obtained from Fig. S16 are shown in Fig. S16b1 and Fig. S16c1. Two flat band-like features can be observed in the second derivative image in Fig. S16b1 marked by LH and HH which correspond to the two hump features in EDCs. Two features can be observed in the other second derivative image in Fig. S16c1 marked by LD and HD that correspond to the two dip features in EDCs. Fig. S16d1 shows a typical EDC where two humps (LH and HH) and two dips (LD and HD) can be observed. Fig. S16e1 shows the corresponding second derivative EDC obtained from the EDC in Fig. S16d1 where the two hump (LH and HH) and two dip (LD and HD) features can be observed more clearly. When the band structure is measured under the p polarization (Fig. S16a2), the two coexisting mode-couplings can also be clearly observed in Fig. S16b2-e2. The same is true in the measurements of the Bi2201 OD17K sample under the s polarization (Fig. S16a3-c3) and under the p polarization (Fig.
S16(a4-e4)). In both cases, the two coexisting mode-couplings can be clearly observed. These results indicate that the observation of the two coexisting mode-couplings is not affected by the photon polarization because it is an intrinsic part of the single-particle spectral function.

13. Simulations of the mode-coupling in the superconducting state of Bi2212

We also carried out the simulation of the single particle spectral function in the superconducting state of Bi2212 by including electron coupling with one boson mode. The main purpose is to check on the effect of the superconducting transition on the spectral function and related mode-coupling. The simulations are performed by using the finite temperature Eliashberg equations[27, 28] and we put in the boson mode with a finite linewidth.

The ARPES intensity is obtained by multiplying the spectral weight, $A(k, \omega)$, by the Fermi-Dirac distribution function, $f(\omega, T)$.

$$I(k, \omega, T) = f(\omega, T)A(k, \omega)$$  \hspace{1cm} \text{(7)}$$

where

$$A(k, \omega) = -\frac{1}{\pi} Im \frac{\omega - \Sigma(k, \omega) + \xi(k) + \chi(k, \omega)}{(\omega - \Sigma(k, \omega))^2 - (\xi(k) + \chi(k, \omega))^2 - \phi^2(k, \omega)}$$  \hspace{1cm} \text{(8)}$$

The self-energies, $\Sigma$ and $\chi$, can be obtained by self-consistent Eliashberg equations:

$$\Sigma(k, \omega) = \sum_{k'} \int_{-\infty}^{\infty} d\varepsilon \int_{-\infty}^{\infty} d\varepsilon' K(\varepsilon, \varepsilon', \omega) A_N(k', \varepsilon) \alpha^2 F(k - k', \varepsilon')$$  \hspace{1cm} \text{(9)}$$

$$\chi(k, \omega) = \sum_{k'} \int_{-\infty}^{\infty} d\varepsilon \int_{-\infty}^{\infty} d\varepsilon' K(\varepsilon, \varepsilon', \omega) A_\chi(k', \varepsilon) \alpha^2 F(k - k', \varepsilon')$$  \hspace{1cm} \text{(10)}$$
where $\alpha^2 F(k - k', \varepsilon')$ is the Eliashberg function and

$$A_N = -\frac{1}{\pi} \text{Im} \frac{\omega - \Sigma(k, \omega)}{(\omega - \Sigma(k, \omega))^2 - (\xi(k) + \chi(k, \omega))^2 - \phi^2(k, \omega)}$$

(11)

$$A_\chi = -\frac{1}{\pi} \text{Im} \frac{\xi(k) + \chi(k, \omega)}{(\omega - \Sigma(k, \omega))^2 - (\xi(k) + \chi(k, \omega))^2 - \phi^2(k, \omega)}$$

(12)

$$K(\varepsilon, \varepsilon', \omega) = \frac{f(\varepsilon) + n(-\varepsilon')}{\varepsilon + \varepsilon' - \omega - i\delta}$$

(13)

Here $n(-\varepsilon')$ is Boson-Einstein distribution function. We do not leave out the particle-hole asymmetric part of the self-energy, $\chi$, for more realistic calculation. We simply assumed a $d$-wave superconducting gap:

$$\Delta(k) = \Delta_0 (\cos(k_x) - \cos(k_y))/2$$

(14)

with $\Delta_0$ being the maximum gap at the antinodal region. We use $\Delta_0=20$ meV. The pairing self-energy, $\phi$, can then be obtained by:

$$\phi(k, \omega) = \Delta(k) Z(k, \omega) = \Delta(k)(1 - \Sigma(k, \omega)/\omega)$$

(15)

In the calculation, we choose the Eliashberg function as:

$$\alpha^2 F(q, \omega) = \alpha_0 \left( \frac{(\Gamma/2)^2}{(\omega - \omega_b)^2 + (\Gamma/2)^2} - \frac{(\Gamma/2)^2}{(\omega + \omega_b)^2 + (\Gamma/2)^2} \right)$$

(16)

Here we separate the energy dependence and momentum dependence parts of the Eliashberg function. It has a single peak at $\omega_b=40$ meV and its width can be controlled by $\Gamma$. We use $\Gamma=15$ meV. For the momentum dependence of the Eliashberg function, we adopt the Holstein phonon mode where $\alpha_0$ is a constant.

We set the value of the $\lambda$ defined as below to be 0.5 by adjusting $\alpha_0$.

$$\lambda = \int_{-\infty}^{\infty} d\omega \int d\mathbf{q} N(\omega) \frac{\alpha^2 F(q, \omega)}{\omega}$$

(17)
where $N(\omega)$ is the density of state.

The bare dispersion, $\xi(k)$, is given by:

$$\xi(k) = -2t_1(\cos(k_x) + \cos(k_y)) + 4t_2\cos(k_x)\cos(k_y) - 2t_3(\cos(2k_x) + \cos(2k_y)) + \mu$$

(18)

where $t_1 = 0.395$, $t_2 = 0.084$, $t_3 = 0.042$ and $\mu = 0.474$. Here, we set the saddle point energy to be 30 meV. The self-energy effect lifts the saddle point energy by 10 meV so that we have 20 meV of the renormalized saddle point energy.

Figure S17 shows the simulated spectral functions of the electron-boson coupling in Bi2212 along the nodal direction and the related energy scales. We have carried out the simulations for three cases: 100 K in the normal state (Fig. S17a), 1 K in the normal state (Fig. S17b) and 1 K in the superconducting state (Fig. S17c). Fig. S17d shows the band dispersions obtained by fitting MDCs at different energies from the band structure in Fig. S17(a-c), where we can observe clearly a kink structure in each dispersion. Fig. S17f shows the energy difference by subtracting a straight line from the dispersion in Fig. S17d. We can see that the kink structure keeps at the same position of 40 meV for the 100 K and 1 K cases in the normal state. But it shifts to a higher energy of $\sim$54 meV for the 1 K case in the superconducting state.

14. Ubiquitous “peak-double dip-double hump” structure in Bi2212 and Bi2201

Figure S18 shows EDCs at different momentum points, at different temperatures, in Bi2212 and Bi2201 samples with different doping levels. The “peak-double dip-double hump” structure can be commonly observed in all
these EDCs.

[1] P. V. Bogdanov, A. Lanzara, S. A. Kellar, X. J. Zhou, E. D. Lu, W. J. Zheng, G. Gu, J. I. Shimoyama, K. Kishio, H. Ikeda, R. Yoshizaki, Z. Hussain, and Z. X. Shen. Evidence for an energy scale for quasiparticle dispersion in Bi$_2$Sr$_2$CaCu$_2$O$_8$. *Physical Review Letters*, 85(12):2581–2584, 2000.

[2] P. D. Johnson, T. Valla, A. V. Fedorov, Z. Yusof, B. O. Wells, Q. Li, A. R. Moodenbaugh, G. D. Gu, N. Koshizuka, C. Kendziora, Sha Jian, and D. G. Hinks. Doping and temperature dependence of the mass enhancement observed in the cuprate Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$. *Physical Review Letters*, 87(17):177007, 2001.

[3] A. Kaminski, M. Randeria, J. C. Campuzano, M. R. Norman, H. Fretwell, J. Mesot, T. Sato, T. Takahashi, and K. Kadowaki. Renormalization of spectral line shape and dispersion below $T_c$ in Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$. *Physical Review Letters*, 86(6):1070–1073, 2001.

[4] A. Lanzara, P. V. Bogdanov, X. J. Zhou, S. A. Kellar, D. L. Feng, E. D. Lu, T. Yoshida, H. Eisaki, A. Fujimori, K. Kishio, J. I. Shimoyama, T. Noda, S. Uchida, Z. Hussain, and Z. X. Shen. Evidence for ubiquitous strong electron–phonon coupling in high-temperature superconductors. *Nature*, 412:510–514, 2001.

[5] X. J. Zhou, T. Yoshida, A. Lanzara, P. V. Bogdanov, S. A. Kellar, K. M. Shen, W. L. Yang, F. Ronning, T. Sasagawa, T. Kakeshita, T. Noda, H. Eisaki, S. Uchida, C. T. Lin, F. Zhou, J. W. Xiong, W. X. Ti, Z. X. Zhao, A. Fujimori, Z. Hussain, and Z. X. Shen. Universal nodal Fermi velocity. *Nature*, 423(6938):398–398, 2003.

[6] A. D. Gromko, A. V. Fedorov, Y. D. Chuang, J. D. Koralek, Y. Aiura, Y. Yamaguchi, K. Oka, Y. Ando, and D. S. Dessau. Mass-renormalized electronic excitations at $(\pi, 0)$ in the superconducting state of Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$. *Physical Review B*, 68(17):174520, 2003.

[7] T. K. Kim, A. A. Kordyuk, S. V. Borisenko, A. Koitzsch, M. Knupfer, H. Berger, and J. Fink. Doping dependence of the mass enhancement in (Pb,Bi)$_2$Sr$_2$CaCu$_2$O$_8$ at the antinodal point in the superconducting and normal states. *Physical Review Letters*, 91(16):167002, 2003.

[8] T. Sato, H. Matsui, T. Takahashi, H. Ding, H. B. Yang, S. C. Wang, T. Fujii, T. Watanabe,
A. Matsuda, T. Terashima, and K. Kadowaki. Observation of band renormalization effects in hole-doped high-$T_c$ superconductors. *Physical Review Letters*, 91(15):157003, 2003.

[9] T. Cuk, F. Baumberger, D. H. Lu, N. Ingle, X. J. Zhou, H. Eisaki, N. Kaneko, Z. Hussain, T. P. Devereaux, N. Nagaosa, and Z. X. Shen. Coupling of the $B_{1g}$ phonon to the antinodal electronic states of Bi$_2$Sr$_2$Ca$_{0.92}$Y$_{0.08}$Cu$_2$O$_{8+\delta}$. *Physical Review Letters*, 93(11):117003, 2004.

[10] A. A. Kordyuk, S. V. Borisenko, V. B. Zabolotnyy, J. Geck, M. Knupfer, J. Fink, B. Büchner, C. T. Lin, B. Keimer, H. Berger, A. V. Pan, Seiki Komiya, and Yoichi Ando. Constituents of the quasiparticle spectrum along the nodal direction of high-$T_c$ cuprates. *Physical Review Letters*, 97(1):017002, 2006.

[11] W. T. Zhang, G. D. Liu, L. Zhao, H. Y. Liu, J. Q. Meng, X. L. Dong, W. Lu, J. S. Wen, Z. J. Xu, G. D. Gu, T. Sasagawa, G. L. Wang, Y. Zhu, H. B. Zhang, Y. Zhou, X. Y. Wang, Z. X. Zhao, C. T. Chen, Z. Y. Xu, and X. J. Zhou. Identification of a new form of electron coupling in the Bi$_2$Sr$_2$CaCu$_2$O$_8$ superconductor by laser-based angle-resolved photoemission spectroscopy. *Physical Review Letters*, 100(10):107002, 2008.

[12] J. F. He, W. T. Zhang, J. M. Bok, D. X. Mou, L. Zhao, Y. Y. Peng, S. L. He, G. D. Liu, X. L. Dong, J. Zhang, J. S. Wen, Z. J. Xu, G. D. Gu, X. Y. Wang, Q. J. Peng, Z. M. Wang, S. J. Zhang, F. Yang, C. T. Chen, Z. Y. Xu, H. Y. Choi, C. M. Varma, and X. J. Zhou. Coexistence of two sharp-mode couplings and their unusual momentum dependence in the superconducting state of Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ revealed by laser-based angle-resolved photoemission. *Physical Review Letters*, 111(10):107005, 2013.

[13] N. C. Plumb, T. J. Reber, H. Iwasawa, Y. Cao, M. Arita, K. Shimada, H. Namatame, M. Taniguchi, Y. Yoshida, H. Eisaki, Y. Aiura, and D. S. Dessau. Large momentum-dependence of the main dispersion ‘kink’ in the high-$T_c$ superconductor Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$. *New Journal of Physics*, 15(11):113004, 2013.

[14] T. Kondo, Y. Nakashima, W. Malaeb, Y. Ishida, Y. Hamaya, T. Takeuchi, and S. Shin. Anomalous doping variation of the nodal low-energy feature of superconducting (Bi, Pb)$_2$(Sr, La)$_2$CuO$_{6+\delta}$ crystals revealed by laser-based angle-resolved photoemission spectroscopy. *Physical Review Letters*, 110(21):217006, 2013.

[15] H. Anzai, M. Arita, H. Namatame, M. Taniguchi, M. Ishikado, K. Fujita, S. Ishida, S. Uchida, and A. Ino. A new landscape of multiple dispersion kinks in a high-$T_c$ cuprate superconductor. *Scientific Reports*, 7(1):4830, 2017.
[16] J. Q. Meng, G. D. Liu, W. T. Zhang, L. Zhao, H. Y. Liu, W. Lu, X. L. Dong, and X. J. Zhou. Growth, characterization and physical properties of high-quality large single crystals of $\text{Bi}_2(\text{Sr}_{2-x}\text{La}_x)\text{CuO}_{6+\delta}$ high-temperature superconductors. *Superconductor Science and Technology*, 22(4):045010, 2009.

[17] L. Zhao, W. T. Zhang, H. Y. Liu, J. Q. Meng, G. D. Liu, W. Lu, X. L. Dong, and X. J. Zhou. High-quality large-sized single crystals of Pb-doped $\text{Bi}_2\text{Sr}_2\text{CuO}_{6+\delta}$ high-$T_c$ superconductors grown with traveling solvent floating zone method. *Chinese Physics Letters*, 27(8):087401, 2010.

[18] S. Y. Liu, W. T. Zhang, L. Zhao, H. Y. Liu, Y. Wu, G. D. Liu, X. L. Dong, and X. J. Zhou. Growth and characterization of high-quality single crystals of Ni- and Zn-doped $\text{Bi}_2\text{Sr}_2\text{Ca}(\text{Cu}_{2-x}\text{M}_x)\text{O}_{8+\delta}$ ($\text{M} = \text{Ni}$ or Zn) high-temperature superconductors. *Chinese Physics Letters*, 29(8):4, 2012.

[19] A. Maljuk and C. T. Lin. Floating zone growth of $\text{Bi}_2\text{Sr}_2\text{Ca}_2\text{Cu}_3\text{O}_y$ superconductor. *Crystals*, 6(5):62, 2016.

[20] Y. Ding, L. Zhao, H.-T. Yan, Q. Gao, J. Liu, C. Hu, J.-W. Huang, C. Li, Y. Xu, Y.-Q. Cai, H.-T. Rong, D.-S. Wu, C.-Y. Song, H.-X. Zhou, X.-L. Dong, G.-D. Liu, Q.-Y. Wang, S.-J. Zhang, Z.-M. Wang, F.-F. Zhang, F. Yang, Q.-J. Peng, Z.-Y. Xu, C.-T. Chen, and X. J. Zhou. Disappearance of superconductivity and a concomitant Lifshitz transition in heavily overdoped $\text{Bi}_2\text{Sr}_2\text{CuO}_{6+\delta}$ superconductor revealed by angle-resolved photoemission spectroscopy. *Chinese Physics Letters*, 36(1):017402, 2019.

[21] Y. X. Zhang, L. Zhao, G. D. Gu, and X. J. Zhou. A reproducible approach of preparing high-quality overdoped $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ single crystals by oxygen annealing and quenching method. *Chinese Physics Letters*, 33(6):067403, 2016.

[22] Y. Ando, Y. Hanaki, S. Ono, T. Murayama, K. Segawa, N. Miyamoto, and S. Komiya. Carrier concentrations in $\text{Bi}_2\text{Sr}_{2-\delta}\text{La}_\delta\text{CuO}_{6+\delta}$ single crystals and their relation to the Hall coefficient and thermopower. *Physical Review B*, 61(22):R14956–R14959, 2000.

[23] M. R. Presland, J. L. Tallon, R. G. Buckley, R. S. Liu, and N. E. Flower. General trends in oxygen stoichiometry effects on $T_c$ in Bi and Tl superconductors. *Physica C: Superconductivity*, 176(1):95–105, 1991.

[24] G. Grimvall. *The Electron-Phonon Interaction in Metals*. North-Holland, New York, 1981.

[25] R. S. Markiewicz, S. Saharakorpi, M. Lindroos, Hsin Lin, and A. Bansil. One-band tight-
binding model parametrization of the high-$T_c$ cuprates including the effect of $k_z$ dispersion. *Physical Review B*, 72(5):054519, 2005.

[26] T. Kondo, W. Malaeb, Y. Ishida, T. Sasagawa, H. Sakamoto, T. Takeuchi, T. Tohyama, and S. Shin. Point nodes persisting far beyond $T_c$ in Bi2212. *Nature Communications*, 7699:8, 2015.

[27] A. W. Sandvik, D. J. Scalapino, and N. E. Bickers. Effect of an electron-phonon interaction on the one-electron spectral weight of a d-wave superconductor. *Physical Review B*, 69(9):094523, 2004.

[28] J. M. Bok, J. J. Bae, H.-Y. Choi, C. M. Varma, W. T. Zhang, J. F. He, Y. X. Zhang, L. Yu, and X. J. Zhou. Quantitative determination of pairing interactions for high-temperature superconductivity in cuprates. *Science Advances*, 2(3):e1501329, 2016.

[29] Y. He, M. Hashimoto, D. Song, S. D. Chen, J. He, I. M. Vishik, B. Moritz, D. H. Lee, N. Nagaosa, J. Zaanen, T. P. Devereaux, Y. Yoshida, H. Eisaki, D. H. Lu, and Z. X. Shen. Rapid change of superconductivity and electron-phonon coupling through critical doping in Bi-2212. *Science*, 362(6410):62–65, 2018.
TABLE S1. Summarized ARPES observations of the electron-mode couplings in Bi2201, Bi2212 and Bi2223. “√” represents “observed” while “×” represents “not observed”.

| Materials | Bi2201 | Bi2212 | Bi2223 |
|-----------|--------|--------|--------|
| Momentum  | Nodal  | Antinodal | Nodal | Antinodal | Nodal |
| Temperature | Below $T_c$ | Above $T_c$ | Below $T_c$ | Above $T_c$ | Below $T_c$ | Above $T_c$ |
| Modes     | 70 meV | 40 meV | 70 meV | 40 meV | 70 meV | 40 meV | 70 meV | 40 meV | 70 meV | 40 meV | 70 meV | 40 meV | 70 meV | 40 meV |
| P. V. Bogdanov et al. [1] | √ | √ | | | | | | | | | | | | |
| P. D. Johnson et al. [2] | √ | √ | | | | | | | | | | | | |
| A. Kaminski et al. [3] | √ | √ | √ | | | | | | | | | | | | |
| A. Lanzara et al. [4] | √ | √ | | | | | | | | | | | | |
| X. J. Zhou et al. [5] | √ | √ | | | | | | | | | | | | |
| A. D. Gromko et al. [6] | √ | √ | | | | | | | | | | | | |
| T. K. Kim et al. [7] | | | | | | | | | | | | | | |
| T. Sato et al. [8] | √ | √ | | | | | | | | | | | | |
| T. Cuk et al. [9] | √ | | | | | | | | | | | | | |
| A. A. Kordyuk et al. [10] | | | | | | | | | | | | | | |
| W. T. Zhang et al. [11] | √ | | | | | | | | | | | | | |
| J. F. He et al. [12] | √ | | | | | | | | | | | | | |
| N. C. Plumb et al. [13] | √ | | | | | | | | | | | | | |
| T. Kondo et al. [14] | ̅√ | ̅√ | ̅√ | | | | | | | | | | | | |
| H. Anzai et al. [15] | √ | ̅√ | ̅√ | | | | | | | | | | | | |
| H. T. Yan et al. Present work | ̅√ | ̅√ | ̅√ | ̅√ | ̅√ | ̅√ | ̅√ | ̅√ | ̅√ | ̅√ | ̅√ | ̅√ | ̅√ | ̅√ | ̅√ |
FIG. S1. Magnetic measurements of Bi2201, Bi2212 and Bi2223 samples. (a) Magnetic measurements of Bi2201 single crystals including La-Bi2201 and Pb-Bi2201. All the Bi2201 samples are labeled by the nominal composition of the La or Pb content. (b) Magnetic measurements of Bi2212 single crystals. (c) Magnetic measurement of the Bi2223 single crystal.
FIG. S2. Estimation of $T_c$ of a Bi2212 sample annealed in situ in ozone atmosphere. The temperature dependent in-gap spectral weight (integrated within 4-meV energy window of $E_F$) obtained from EDCs measured near the antinodal region at different temperatures is shown as the red dotted line. Its temperature derivative is shown as the black dotted line. $T_c$ is estimated to be $\sim 36$ K from the largest slope in the spectral weight and the maximum in the first derivative curve [29].
FIG. S3. *Experimental geometry of ARPES measurements.* The detection of different momentum ranges is realized by changing $\theta$ and $\Phi$. 
FIG. S4. The second derivative method to reveal the electron coupling with two co-existing sharp modes in the band structures along the nodal direction in Bi2212. (a) Band structure of the Bi2212 OD73K sample taken at 17 K along the nodal direction. The location of the momentum cut is shown in the bottom-right inset by the red line. (b) A typical EDC from (a) at the momentum point k1 with a main peak (peak), a low energy hump (LH), a high energy hump (HH), a low energy dip (LD) and a high energy dip (HD). (c) Second derivative of the EDC in (b). (d) Negative of the EDC intensity second derivative in (c). It shows a good correspondence to the original EDC in (b). The peak, dip and hump structures become more pronounced. (e) Second derivative image of (a) with respect to energy showing the absolute magnitude of the negative values of the second derivative EDC intensity in (c). It highlights the peak and hump structures in the original data in (a). In addition to the main band, four features are observed and marked as LLH, RLH, LHH and RHH. (f) Second derivative image of (a) with respect to energy showing the absolute magnitude of the positive values of the second derivative EDC intensity in (c). It highlights the dip structures in the original data in (a). Four features are observed marked as LLD, RLD, LHD and RHD.
FIG. S5. Simulated spectral functions of mode-coupling involving two modes. (a1) Simulated spectral function of the mode-coupling along the nodal direction in the normal state. Two modes at 40 meV and 70 meV are used in the simulation, as shown in the inset of (b1). The location of the momentum cut is shown in the bottom-right inset by the red line. (b1) Second derivative image of (a1) with respect to energy showing the absolute magnitude of the negative values. Four features can be observed marked as LLH, RLH, LHH and RHH. (c1) Second derivative image of (a1) with respect to energy showing the absolute magnitude of the positive values. Four features can be observed marked as LLD, RLD, LHD and RHD. (a2-c2) Same as (a1-c1) but for the simulated spectral function along a momentum cut near the antinodal region. (d) Typical EDCs at different momentum points in (a1). The corresponding momentum range is represented by the top horizontal red line with an arrow in (a1). Two dip features are marked as HD and LD and two hump features are marked as HH and LH. (e) Corresponding second derivative EDCs obtained from (d). (f-g) Same as (d-e) but for the simulated spectral function in (a2). Two dip features are marked as HD and LD, and two hump features are marked as HH and LH.
FIG. S6. The simulated spectral functions of mode-coupling involving one mode and two modes. (a) Simulated spectral function of the mode-coupling involving one mode with an energy of 70 meV. The mode used in the simulation is shown in the bottom-right inset in (b). We take a straight line as the bare band that is similar to the calculated band of Bi2212 along the nodal direction in order to make a comparison with the measured results. (b) Second derivative image of (a) with respect to energy showing the absolute magnitude of the negative values. In addition to the main band, two features appear marked as LHH and RHH. (c) Second derivative image of (a) with respect to energy showing the absolute magnitude of the positive values. Two features are observed marked as LHD and RHD. (d) EDCs from the spectral function in (a) at three momentum points, k1, k2 and k3. In addition to the EDC peaks near the Fermi level, on their left side, one dip (HD) and one hump (HH) features can be observed. The red vertical dashed line indicates the position of the mode energy at 70 meV. (e) Second derivative EDCs obtained from (d). The HD feature in (e) corresponds to the dip structure in (d) and (LHD, RHD) features in (c). The HH feature in (e) corresponds to the hump structure in (d) and (LHH, RHH) features in (b). The red vertical dashed line indicates the position of the mode energy at 70 meV. (f-j) Same as (a-e) but for the simulated spectral function involving two modes with energies of 40 meV and 70 meV. Different from the one mode-coupling case where only one dip and one hump features are produced, in the two modes case, in addition to the EDC peaks near the Fermi level, two dip features and two hump features can be observed.
FIG. S7. Determination of the energy scales from the hump and dip structures in the simulated spectral function with different mode widths. (a) Simulated spectral function of the mode-coupling involving one mode with an energy of 40 meV and a FWHM of 1 meV. (b) EDCs from the spectral functions simulated with different mode linewidths at the momentum point k1 shown in (a). The bottom-left inset shows the mode used in the simulations with the linewidth marked. A dip (LD) and a hump (LH) features can be observed in these EDCs. The red vertical dashed line indicates the position of the mode energy at 40 meV. (c) Second derivative EDCs obtained from (b). The LD feature corresponds to the dip structure in (b) and the LH feature corresponds to the hump structure in (b). (d) The effect of the mode linewidth on the energy position of the dip feature (blue circles), hump feature (red circles) and the middle point between the dip and the hump features (black circles) obtained from the second derivative EDCs in (c). The horizontal dashed line marks the position of the mode energy at 40 meV. (e-h) Same as (a-d) but for the simulated spectral functions involving two modes with energies of 40 meV and 70 meV. The two horizontal dashed lines mark the positions of the mode energies at 40 meV and 70 meV.
FIG. S8. Observation of the $\sim$40-meV kink in the band structure measured at 17 K near the antinodal region in Bi2212. (a) Band structure of the Bi2212 OD73K sample taken at 17 K near the antinodal region. The location of the momentum cut is shown in the bottom-right inset by the blue line where the bonding (BB) and antibonding (AB) Fermi surface sheets are both depicted. The dispersions obtained by fitting MDCs at different energies are shown as a black line for the right antibonding band (AB$_R$) and a red line for the left bonding band (BB$_L$). The red arrows mark the kink positions of the AB$_R$ and BB$_L$ bands at $\sim$40 meV. (b) Second derivative image of (a) with respect to energy showing the absolute magnitude of the negative values. In addition to the main bands, four features are observed and marked as LLH, RLH, LHH and RHH. (c) Second derivative image of (a) with respect to energy showing the absolute magnitude of the positive values. Two features are observed marked as LHD and RHD.
FIG. S9. Momentum dependence of the band structures and real parts of the electron self-energy of Bi2212 and Bi2201. (a) Band structures of the Bi2212 OD73K sample measured along different momentum cuts at 17 K. The locations of momentum cuts are shown by the blue lines in (d). (b) Same as (a) but for the Bi2201 OD17K sample taken at 20 K. The locations of momentum cuts are shown by the blue lines in (f). (c) Real parts of the electron self-energy of the Bi2212 OD73K sample obtained from (a). For clarity, the data are offset along the vertical axis with the offset values marked on the right side of each curve. Two energy scales are observed marked by HScale and LScale. (d) The locations of the momentum cuts used in (a). (e) Same as (c) but for the Bi2201 OD17K sample obtained from (b). (f) The locations of the momentum cuts used in (b).
FIG. S10. **Extraction of the real part of the electron self-energy.** (a) Band structure of the Bi2212 OD73K sample measured along the nodal direction at 17 K. The location of the momentum cut is show in the bottom-right inset in (b) by a red line marked as Cut1. (b) Same as (a) but for the momentum Cut2 near the antinodal region. (c) Band dispersion (black line) obtained by fitting MDCs at different energies from (a). The red line represents the band dispersion from the band structure calculations[25]. It is used as the bare band to extract the real part of the electron self-energy. (d) Same as (c) but obtained from the band structure in (b). (e) The real part of the electron self-energy obtained from (c). (f) The real part of the electron self-energy obtained from (d).
FIG. S11. **Doping dependence of the two mode-couplings in Bi2201, Bi2212 and Bi2223 along the nodal direction.** (a) Band structures of Bi2201 with different doping levels taken along the nodal direction at 14~20 K. The location of the momentum cut is shown in (d) by a blue line. These are the second derivative images with respect to energy showing the absolute magnitude of the negative values. (b) Same as (a) but for Bi2212 with different doping levels measured at 17~20 K. (c) Same as (a) but for the Bi2223 sample with a $T_c$ of 108 K measured at 18 K. The two features observed in (a, b and c) are marked by LH (pink dashed line) and HH (blue dashed line). (d) The location of the momentum cuts used in (a-c). (e) Second derivative EDCs obtained from the band structures in (a, b and c). The corresponding momentum positions are shown by arrows in each panel of (a, b and c). The two hump features are labeled as HH and LH, and the two dip features are labeled as HD and LD. For clarity, the data are normalized by the intensity difference between the HH and LD features, and are offset along the vertical axis. (f) Summarized energy positions of the two energy scales obtained from (e) for Bi2201 (black and gray symbols), Bi2212 (red symbols) and Bi2223 (blue symbols). The energy positions of the high energy scale is marked by the purple shaded region and the energy positions of the low energy scale is marked by the pink shaded region. The blue dashed line indicates a linear fit of energy position of the high energy scale while the red dashed line indicates a linear fit of energy position of the low energy scale.
FIG. S12. Doping dependence of the two mode-couplings in Bi2212 and Bi2201 near the antinodal region. (a-c) Band structures of Bi2212 with different doping levels, including optimally doped ($T_c = 91$ K) (a, Cut1), overdoped ($T_c = 73$ K) (b, Cut5) and heavily overdoped ($T_c = 36$ K) (c, Cut4), taken along the momentum cuts near the antinodal region at $17 \sim 20$ K. The locations of the momentum cuts are shown in (i) by the blue lines. These are the second derivative images with respect to energy showing the absolute magnitude of the negative values. (d-g) Same as (a-c) but for Bi2201 with different doping levels, including underdoped ($T_c = 28$ K) (d, Cut3), optimally doped ($T_c = 32$ K) (e, Cut3), overdoped ($T_c = 17$ K) (f, Cut5) and heavily overdoped (non-superconducting) (g, Cut2), measured at $15 \sim 20$ K. The two features observed in (a-g) are labeled as LH (pink dashed line) and HH (blue dashed line). (h) Summarized energy positions of the two energy scales obtained from (j) for Bi2201 (black and gray symbols) and Bi2212 (red symbols). The energy positions of the high energy scales are marked by the purple shaded region and the energy positions of the low energy scales are marked by the pink shaded region. The blue dashed line indicates a linear fit of the energy positions of the high energy scales while the red dashed line indicates a linear fit of the energy positions of the low energy scales. (i) The location of the momentum cuts used in (a-g). (j) Second derivative EDCs obtained from the band structures in (a-g). The corresponding momentum positions are shown by arrows in each panel of (a-g). The two hump features are marked by HH and LH, and the two dip features are marked by HD and LD. For clarity, the data are normalized by the intensity difference between the HH and LD features, and are offset along the vertical axis.
FIG. S13. Observation of two coexisting mode-couplings in the band structures measured above and below $T_c$ in the Bi2212 OD36K sample near the antinodal region. (a-c) Band structures measured at 20 K (a), 45 K (b) and 60 K (c) near the antinodal region. The location of the momentum cut is shown in the bottom-right inset by the red line in (a). (d-f) Second derivative images of (a-c) with respect to energy showing the absolute magnitude of the negative values. Two features are marked by LH (pink dashed line) and HH (blue dashed line). (g-i) Second derivative images of (a-c) with respect to energy showing the absolute magnitude of the positive values. Two features are marked by LD (pink dashed line) and HD (blue dashed line). (j) EDCs obtained from (a-c) at the Fermi momentum $k_{FR}$, marked by the arrow in (a). (k) Symmetrized EDCs obtained from the EDCs in (j). (l) EDCs obtained from (a-c) at the momentum point $k_1$, marked by the arrow in (a). Two dip features are marked by HD and LD while two hump features are marked by HH and LH. (m) Second derivative EDCs obtained from (l) with the two dip features (HD and LD) and two hump features (HH and LH) marked.
FIG. S14. Temperature dependence of the band structures of Bi2212 and Bi2201. (a) Band structures of the Bi2212 OD73K sample taken at different temperatures along the momentum Cut1. The location of the momentum Cut1 is shown in the bottom-right inset by the red line. (b) Same as (a) but taken along the momentum Cut2. (c) Same as (a) but for the Bi2201 OD17K sample measured along the momentum Cut3.
FIG. S15. **Observation of two coexisting mode-couplings in the band structures measured at different temperatures in Bi2212.** (a) Real parts of the electron self-energy of the Bi2212 OD73K sample taken along the momentum Cut1 at different temperatures. For clarity, the data are offset along the vertical axis with the offset values marked on the right side of each curve. The high energy scales can be observed clearly marked by HScale. (b) Same as (a) but along the momentum Cut2. The Low energy scales can be observed clearly marked by LScale. (c) Energy difference between the measured dispersions and a straight line connecting two points at the Fermi level and -0.06 eV for the Bi2212 OD73K sample taken along the nodal direction at different temperatures. The Low energy scales can be seen marked by LScale. (d) The locations of the momentum cuts used in (a-c).
FIG. S16. Photon polarization dependence of the two coexisting mode-couplings in the band structures measured in Bi2212 and Bi2201. (a1) Band structure of the Bi2212 OD36K sample taken at 20K near the antinodal region under the s polarization. The location of the momentum cut is shown in the bottom-right inset by the red line. (b1) Second derivative image of (a1) with respect to energy showing the absolute magnitude of the negative values. (c1) Second derivative image of (a1) with respect to energy showing the absolute magnitude of the positive values. (d1) EDC at the momentum point k1 as marked in (a1). (e1) Second derivative EDC obtained from (d1). (a2-e2) Same as (a1-e1) but taken under the p polarization. (a3-e3) Same as (a1-e1) but for the Bi2201 OD17K sample measured under the s polarization. (a4-e4) Same as (a3-e3) but measured under the p polarization. In (b1,b2,b3,b4), two features appear marked as LH and HH. In (c1,c2,c3,c4), two features are observed marked as LD and HD. In (d1,d2,d3,d4) and (e1,e2,e3,e4), the observed features are marked as HH, LH, HD and LD.
FIG. S17. Simulated spectral functions of the electron-boson coupling in Bi2212 along the nodal direction and related energy scales. The simulation involves one mode with an energy of 40 meV and a linewidth of 15 meV. The location of the involved momentum cut is shown in (e) by the blue line. (a) Simulated spectral function at 100 K in the normal state. (b) Simulated spectral function at 1 K in the normal state. (c) Simulated spectral function at 1 K in the superconducting state with a superconducting gap of 20 meV. (d) Band dispersions obtained by fitting MDCs at different energies from the band structures in (a), (b) and (c). The pink dashed straight line is subtracted from the dispersions to get the energy differences in (f). (e) The location of the momentum cuts in (a-c). (f) Energy differences by subtracting a straight line from the dispersions in (d). The vertical dashed line indicates the position of the mode energy at 40 meV.
FIG. S18. Ubiquitous “peak-double dip-double hump” structure in Bi2212 and Bi2201. (a1-a2) EDCs from the band structure of Bi2212 OD36K sample measured along the nodal direction at two momentum points k1 (Fermi momentum, a1) and k2 (a2). The measured band structure is shown in the inset of (a2) where the location of the two momentum points k1 and k2 is marked by the arrows. The location of the momentum cut is shown in the inset of (a4) by a blue line marked as Cut1. Two EDCs are shown in (a1) and (a2) that are measured at 20 K (black curve) and 45 K (gray curve). (a3-a4) Same as (a1-a2) but for the EDCs at two momentum points k3 (Fermi momentum, a3) and k4 (point at the (0, 0)-(0, π) line, a4) near the antinodal region. The measured band structure is shown in the inset of (a3) where the location of the two momentum points k3 and k4 is marked by the arrows. The location of the momentum cut is shown in the inset of (a4) by the blue line marked as Cut2. Two EDCs are shown in (a3) and (a4) that are measured at 20 K (black curve) and 60 K (gray curve). (b1-b4) Same as (a1-a4) but for the Bi2212 OD73K sample measured at 17 K and 100 K. (c1-c4) Same as (a1-a4) but for the Bi2212 OP91K sample measured at 20 K and 108 K. (d1-d4) Same as (a1-a4) but the Bi2212 UD64K sample measured at 18 K and 75 K. (e1-e4) Same as (a1-a4) but for the Bi2201 ODNSC sample measured at 16 K and 100 K in (e1-e2) and at 25 K and 100 K in (e3-e4). (f1-f4) Same as (a1-a4) but for the Bi2201 OD17K sample measured at 20 K in (f1-f2) and at 13 K and 100 K in (f3-f4). (g1-g4) Same as (a1-a4) but for the Bi2201 OP32K sample measured at 15 K and 80 K. (h1-h4) Same as (a1-a4) but for the Bi2201 UD28K sample measured at 15 K.