The influence of reference position deviation on industrial robots positioning precision

Andrei Luncanu and Stan Gheorghe
“Vasile Alecsandri” University of Bacău, Department of Industrial Engineering, Calea Mărăşeşti Street, No. 157, 600115, Bacău, Romania

Email: andreluncanu@yahoo.com

Abstract: Due to industrial robot’s higher precision and better repeatability that human workforce, they have been chosen to replace human workload in all types of industrial activities. Due to the increasing positioning precision and repeatability needed by the industrial operations, it is necessary to use industrial robots that comply with the required technical specifications. For this reason, research on new methods to increase positioning accuracy is necessary. Some of the positioning errors are caused by the reference positioning deviations present in the industrial robot structure. Due to the systematic nature of reference position deviations, by entering the values in a compensation method, the positioning error is permanently minimized. In the paper are presented new methods of reference positioning deviations determination. In the paper is presented a mathematical model based on direct kinematic modeling method, used for calculating the position of the industrial robot end-effector. By applying these method’s, we can determine the reference positioning deviations and by introducing them into a specialized calculation system we can simulate the impact of these deviations. By applying these methods on a six axis articulated industrial robot and introducing the industrial robot’s parameters in a specialized calculating programs the positioning errors are determined and shown in the paper.

1. Introduction

Ever since the first primitive cartesian industrial robots, maintaining the reference position of the industrial robot joints has been problematic. Because of the different factors that are present in the joints, angular deviations usually occur, implicitly deviations occur from the reference position of the joints. By examining the reference positioning deviations in specialized books [1] the parallelism and perpendicularity deviations are the main types of deviations. By determining the parallelism deviation and perpendicularity deviation throw the methods shown in the article, the angular values of these deviations are obtained. The values of the reference position deviations obtained by analyzing the kinematic structure of the industrial robots are introduced into a mathematical model [2 - 7] and used to calculate the positions of the end-effector based on the homogenous transfer matrixes of the direct kinematic model. In order to understand how the reference position deviations, affect the precision of the end-effector and the precision of the industrial robot, the values obtained throw measurement are introduced in the direct kinematic model and used to calculate of the position of the end-effector. By using the simulation methods of positional errors based on specialized matrix programs [8 - 10] the positioning errors of the industrial robot are obtained and by using the compensation methods we can see where the theoretical position is.
2. Methods of determining the reference position deviations

By investigating the factors that cause reference position deviations in specialized books [1] a number of factors that are affecting the kinematic structures of the industrial robots have been determined. These factors include the perpendicular deviations of the joints and the parallelism deviations from the linking element. Due to reference position deviations present in the kinematic structure of the industrial robot positioning errors occur. For this reason, it is necessary to research and develop methods of determining perpendicularity deviations and parallelism deviations and after introducing methods of compensation.

2.1 Method of determining the perpendicular deviation

Due to the inclined surfaces between the assembly of the industrial robot between the mounting flange of the joint and the mounting element between the joints, the perpendicular deviation occurs. In order to determine the angle value of the perpendicularity deviation between the joint used in the industrial robot structure, we can use the method shown in figure 1.

**Figure 1.** Mechanical assembly between the industrial robot joint and the linking element.

In Figure 1 is shown the mechanical assembly to determine the deviation and it is formed by assembling the joints C1, C2 and the linking element. The discontinued lines symbolize the real position of the joint and the kinematic link. By analyzing the elements used by the assembly shown in figure 1 and using the graphical analysis model of the perpendicular deviation the method is obtained and it is shown in figure 2. The perpendicularity deviation value shown in Figure 2 is obtained by comparing the theoretical projected axis and the current axis of the kinematic assembly and the value is written as $e_{pp}$.

**Figure 2.** The method of determining the perpendicular deviation.

2.2 Method of Determining the Parallelism Deviation

Due to the deformations of the linking element produced by various factors, an additional angle is formed in the kinematic structure of the industrial robot. In figure 3 it is shows the deformation of the linking element through which the $e_{pr}$ angle is formed between the original axial position of the kinematic coupling $c_2$ and the affected axial positioning of the joint $c_2$. This is the parallelism deviation which affects the kinematic structure. By analyzing elements in figure 3 and using the graphical modeling methods of the joints we obtain the parallelism deviation graphical determination model shown in figure 4.

![Diagram of parallelism deviation](image-url)
3. Determination of the influences caused by the reference position deviations on the positioning precision of the articulated industrial robot

In order to determine the influences of reference position deviations on the positioning accuracy of the articulated industrial robot it is necessary to determine the mathematical model of the industrial robot. For this determination, an six-axis articulated industrial robot is used. By using the structural modeling methods investigated in articles [2 - 6] the kinematic structure of the six-axis articulated industrial robot is shown in Figure 5.

By applying the methods for determining the referente position deviations on the kinetic structure of the six axes articulated robot shown in figure 5, the referente position deviations of each kinematic link are obtained. By using the methods for determining deviations from the reciprocal position on the support formed between the support of the hinged industrial robot and the joint 1, the model of the embodiment of figure 6, in which the deviation from perpendicularity is presented, is obtained.
By applying the determination method on the ansambly between joints 1 and 2 shown in figure 7, the reference position deviations are $e_{pr2}$ and $e_{pp2}$. By applying the determination method on the ansambly between joints 2 and 3 shown in figure 8, the reference position deviations are $e_{pr3}$ and $e_{pp3}$.

By applying the determination method on the ansambly between joints 3 and 4 shown in figure 9, the reference position deviations are $e_{pr4}$ and $e_{pp4}$. By applying the determination method on the ansambly between joints 4 and 5 shown in figure 10, the reference position deviations are $e_{pr5}$ and $e_{pp5}$.

By applying the determination method on the ansambly between joints 5 and 6 shown in figure 11, the reference position deviations are $e_{pr6}$ and $e_{pp6}$. By applying the same determination method to the assembly between joint 6 and the end-effector, the model is shown in figure 12 and the reference position deviation is in with the $e_{pp7}$. By using the reference position deviations obtained throw the previous analysis and the six-axis articulated industrial robot dimensional and kinematic parameters
from figure 5 we can determine the direct kinematic model of the industrial robot shown in equation (1).

\[ T_{07} = L_{01} \cdot R_{01epp1} \cdot R_{12} \cdot R_{12epp2} \cdot R_{12epp2} \cdot L_{12} \cdot R_{23} \cdot R_{23epp3} \cdot R_{23epp3} \cdot L_{23} \cdot R_{34} \cdot R_{34epp4} \cdot R_{34epp4} \cdot L_{34} \cdot R_{45} \cdot R_{45epp5} \cdot R_{45epp5} \cdot L_{45} \cdot R_{56} \cdot R_{56epp6} \cdot R_{56epp6} \cdot L_{56} \cdot R_{67} \cdot R_{67epp7} \cdot L_{67} \]  

(1)

By using the direct kinematic model and the homogeneus transfer matrixes of both the industrial robot’s parameters and the reference positioning deviations the articulated industrial robots movement matrix is obtained. Because of the high number of elements that are used by the mathematical model only the first 6 rows of the resulted positioning equations are shown, but it is enough to show that these deviations affect the positioning equations and as a result affect the positioning precision of the articulated industrial robot. By using the homogenous transfer matrixes determined throw the analysis of the kinematic structure shown in figure 5 and the homologous transfer matrixes of the reference position, the positioning equations are needed and are shown below:

\[ x = a_1 \cdot (\cos(e_{pp5}) \cdot \cos(e_{pp2}) \cdot \sin(q_1) \cdot \sin(q_3)) + \sin(e_{pp3}) \cdot \sin(q_2) \cdot \cos(e_{pp2}) \cdot (\cos(e_{pp5}) \cdot \cos(e_{pp2}) \cdot \cos(e_{pp6}) \cdot \cos(q_1) - \sin(e_{pp3}) \cdot \sin(q_2) \cdot \cos(e_{pp2}) \cdot (\cos(e_{pp5}) \cdot \cos(e_{pp2}) \cdot \cos(q_1) + \sin(e_{pp3}) \cdot \sin(q_2) \cdot \cos(e_{pp2})) + \sin(e_{pp3}) \cdot \sin(q_2) \cdot \cos(e_{pp2}) \cdot (\cos(e_{pp5}) \cdot \cos(e_{pp2}) \cdot \sin(q_1) + \cos(q_1) \cdot \sin(e_{pp3}) \cdot \sin(q_2)) - \sin(e_{pp2}) \cdot (\sin(e_{pp3}) \cdot \sin(q_1) - \cos(e_{pp2}) \cdot \cos(q_1) - \cos(e_{pp2}) \cdot \sin(q_1) + \cos(q_1) \cdot \sin(e_{pp3})) - \cos(e_{pp1}) \cdot \cos(q_1)) \]  

\[ y = a_2 \cdot (\sin(e_{pp0}) \cdot \cos(e_{pp6}) \cdot \sin(q_2) \cdot \cos(e_{pp5}) \cdot \cos(q_1) + \sin(e_{pp3}) \cdot \sin(q_2) \cdot \cos(e_{pp5}) \cdot \cos(e_{pp6}) \cdot \cos(q_1) - \sin(e_{pp3}) \cdot \sin(q_2) \cdot \cos(e_{pp5}) \cdot \cos(e_{pp6}) \cdot \sin(q_1) - \cos(e_{pp5}) \cdot \sin(e_{pp3}) \cdot \sin(q_2) - \cos(e_{pp5}) \cdot \cos(e_{pp6}) \cdot \cos(q_1)) + \sin(e_{pp2}) \cdot (\sin(e_{pp3}) \cdot \cos(e_{pp5}) \cdot \cos(q_1) + \sin(e_{pp3}) \cdot \cos(e_{pp5}) \cdot \sin(q_2) + \cos(q_1) \cdot \sin(e_{pp3}) \cdot \sin(q_2) + \cos(e_{pp5}) \cdot \sin(e_{pp3}) \cdot \sin(q_2)) + \sin(e_{pp2}) \cdot (\cos(e_{pp5}) \cdot \sin(q_1) + \cos(q_1) \cdot \sin(e_{pp3}) \cdot \sin(q_2) + \cos(q_1) \cdot \sin(e_{pp3}) \cdot \sin(q_2)) \]  

(2)

\[ z = a_7 + a_8 + a_9 \cdot (\sin(e_{pp7}) \cdot \cos(e_{pp3}) \cdot \cos(q_2) \cdot \cos(e_{pp5}) \cdot \cos(q_1) + \sin(e_{pp2}) \cdot (\cos(e_{pp5}) \cdot \cos(q_1) + \sin(e_{pp3}) \cdot \cos(q_2) + \cos(q_1) \cdot \sin(e_{pp3}) \cdot \sin(q_2)) + \sin(e_{pp4}) \cdot (\cos(q_1) \cdot \sin(e_{pp1}) \cdot \sin(q_2) \cdot \cos(e_{pp3}) \cdot \cos(q_2) + \cos(e_{pp3}) \cdot \cos(q_2) + \cos(q_1) \cdot \sin(e_{pp3}) \cdot \sin(q_2)) - \cos(e_{pp3}) \cdot \sin(e_{pp2}) \cdot (\cos(e_{pp5}) \cdot \cos(q_1) + \sin(e_{pp3}) \cdot \cos(q_2) + \cos(q_1) \cdot \sin(e_{pp3}) \cdot \sin(q_2)) + \cos(e_{pp3}) \cdot \sin(q_1) + \cos(q_1) \cdot \sin(e_{pp3}) \cdot \sin(q_2) + \cos(e_{pp3}) \cdot \sin(q_1) + \cos(q_1) \cdot \sin(e_{pp3}) \cdot \sin(q_2)) \]  

Because of the massive number of element’s that are used by the mathematical model only the first 6 rows of the resulted positioning equations are shown, but it is enough to show that these deviations affect the positioning equations and as a result affect the positioning precision of the articulated industrial robot. By using the homogenous transfer matrixes determined throw the analysis of the kinematic structure shown in figure 5 and the homologous transfer matrixes of the reference position deviations the end-effector positioning equations can be computed. Because of the high number of elements used in this mathematical model the equations of the end-effector positions (2) are cut in order to save space in this article.

\[ \sin(e_{pp3}) + \ldots \]  

(3)

As can be seen from the end-effector positioning equations (2), all the equations are affected by the reference positions deviations.

4. Results and discussions

Considering the six-axis articulated industrial robot dimensional parameters: \( a_1 = 500 \) mm; \( a_2 = 1300 \) mm; \( a_3 = 600 \) mm; \( a_4 = 425 \) mm; \( a_5 = 290 \) mm; \( a_6 = 300 \) mm; \( a_7 = 600 \) mm; \( a_8 = 445 \) mm. By applying the reference positions deviation determination methods the values of these deviations are: \( e_{pp2} = 0.00057^\circ, e_{pp3} = 0.00024^\circ, e_{pp4} = 0.000075^\circ, e_{pp5} = 0.00038^\circ, e_{pp6} = 0.00067^\circ, e_{pp1} = 0.00012^\circ, e_{pp2} = \)
0.00068˚, $e_{pp4} = 0.00035˚$, $e_{pp5} = 0.00093˚$, $e_{pp6} = 0.00014˚$, $e_{pp7} = 0.00044˚$. In order to simulate and determine the positioning error caused by the reference positions deviations a series of displacements are needed, shown in table 1.

### Table 1. The values of the kinematic coupling displacements.

| Number of displacements | q1(°) | q2(°) | q3(°) | q4(°) | q5(°) | q6(°) |
|-------------------------|-------|-------|-------|-------|-------|-------|
| 1                       | -180  | -20   | -40   | -30   | -100  | 60    |
| 2                       | -150  | -10   | -20   | -15   | -75   | 70    |
| 3                       | -120  | 0     | 0     | 0     | -50   | 80    |
| 4                       | -90   | 10    | 20    | 15    | -25   | 90    |
| 5                       | -60   | 20    | 40    | 30    | 0     | 100   |
| 6                       | -30   | 30    | 60    | 45    | 25    | 110   |

By introducing the values of the displacements in the simulation program [8 – 10] the positioning precision of the six-axis articulated robot can be determined. The results of the simulations are presented in table 2.

### Table 2. Table of the positioning errors of the articulated robot caused by reference position deviations.

| Positioning errors | 1. | 2. | 3. | 4. | 5. | 6. |
|--------------------|----|----|----|----|----|----|
| $e_x$ (mm)         | 0.0063 | 0.00077 | -0.0013 | -0.0023 | -0.00099 | -0.00025 |
| $e_y$ (mm)         | 0.0665 | -0.00019 | 0.00074 | -0.00025 | -0.00057 | 0.00094 |
| $e_z$ (mm)         | 0.0142 | 0.0022 | 0.0018 | -0.00022 | -0.002 | -0.0021 |

Table 2 shows that there is a positioning error in any end-effector displacement. By introducing the values of the reference positioning deviations in a compensation system these errors are permanently minimized.

### 5. Conclusion

By researching methods for determining the reference position deviations, it has been demonstrated that they affect the positioning prediction of the end-effector mounted on the last joint of the industrial robot. Through the research on the methods for determination of the reference positioning deviations done and presented in this article, a graphical method for determining these deviations has been determined and by implementation of a mathematical modeling method of compensation of these deviations can be used. By introducing the measured values of these deviations in the mathematical model of the six-axis articulated industrial robot and the use of a specialized program it has been demonstrated that the reference position deviations present in the industrial robot structure affect the positioning precision of the articulated industrial robot.

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