Magnetohydrodynamic Accretion–Ejection: Jets Launched by a Nonisotropic Accretion-disk Dynamo. II. A Dynamo Tensor Defined by the Disk Coriolis Number

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Abstract

Astrophysical jets are launched from strongly magnetized systems that host an accretion disk surrounding a central object. Here we address the question of how accretion-disk magnetization and the field structure required for jet launching are generated. We continue our work from Mattia & Fendt (Paper I) by considering a noncollinear accretion-disk mean-field $\alpha^2 \Omega$ dynamo in the context of large-scale disk-jet simulations. We now investigate a disk dynamo that follows analytical solutions of the mean-field dynamo theory, essentially based on a single parameter, the Coriolis number. We thereby confirm the anisotropy of the dynamo tensor acting in accretion disks, allowing both the resistivity and mean-field dynamo to be related to the disk turbulence. Our new model recovers previous simulations by applying a purely radial initial field while allowing for a more stable evolution for seed fields with a vertical component. We also present correlations between the strength of the disk dynamo coefficients and the dynamical parameters of the jet that is launched, and discuss their implications for observed jet quantities.

Unified Astronomy Thesaurus concepts: Magnetohydrodynamics (1964); Astrophysical fluid dynamics (101); Plasma astrophysics (1261); Magnetohydrodynamical simulations (1966); Jets (870); High energy astrophysics (739); Plasma jets (1263); Accretion (14)

1. Introduction

Astrophysical jets are launched from a wide range of astrophysical objects such as young stellar objects, microquasars, or active galactic nuclei (AGNs). It is commonly accepted that these jets are launched from strongly magnetized systems that host an accretion disk surrounding a central object (Frank et al. 2014; Hawley et al. 2015; Pudritz & Ray 2019). Further agreement is on the key role of the large-scale magnetic field for jet acceleration and collimation.

As we have further detailed in Mattia & Fendt (2020, hereafter Paper I), the origin of the jet-launching disk magnetic field is still not completely understood. A promising model scenario that can provide such a large-scale disk magnetic field is that of an accretion-disk dynamo process.

Essentially, astrophysical dynamos are thought to be of turbulent, and thus small-scale, origin. On the other hand, one is interested in the dynamical effects of the generated magnetic field on these systems on large scales, i.e., the entire disk–jet system. The disk turbulence—providing both a turbulent dynamo effect and also a turbulent magnetic diffusivity—is generally thought to be generated by the magnetorotational instability (Balbus & Hawley 1991).

Given the nature of the problem—the combination of small-scale effects of turbulence and the need for a large-scale, and thus global, disk magnetic flux—two paths to modeling the dynamo effect have been pursued. These are (i) direct simulations, which study the natural amplification of the magnetic field by the turbulent dynamics of the medium (see, e.g., Gressel 2010; Bai & Stone 2013) and (ii) the so-called mean-field approach (see, e.g., Krause & Rädler 1980; Rüdiger et al. 1995; Bardou et al. 2001), by which a mean electromagnetic force is derived from averaging the turbulent motions of the medium that under certain conditions may give rise to a dynamo effect amplifying a weak seed magnetic field (for further references, we refer to our introduction in Paper I). The mean-field dynamo is usually designated as the $\alpha^2 \Omega$ dynamo, where the $\alpha$ stands for the field amplification (poloidal and toroidal fields) by the turbulence, while the $\Omega$ stands for the induction of the (toroidal) magnetic field by differential rotation.

In our work, we follow the second approach. In Paper I, we have applied various (ad hoc) choices for the three components of the dynamo tensor, $\alpha_\phi$, $\alpha_\theta$, and $\alpha_R$. We found that the toroidal magnetic field component is always amplified by the turbulent dynamo component $\alpha_\phi$ and by the $\Omega$ effect. The component $\alpha_\theta$ is strongly correlated to the amplification of the poloidal magnetic field, such that a stronger $\alpha_\phi$ results in a more magnetized disk, which then launches a faster, more massive, and more collimated jet.

In contrast, the amplification of the poloidal field depends substantially on the existence of dynamo-inefficient zones, which, subsequently, affect the overall disk-jet evolution, thus accretion and ejection. We found that both a stronger dynamo component $\alpha_\phi$ and also a nonisotropic radial component $\alpha_R$ lead to the formation of dynamo-inefficient zones. It became clear that the formation of dynamo-inefficient zones can also be triggered by the vertical component of the initial magnetic field, even for a weak dynamo component $\alpha_\phi$. A strong $\alpha_\phi$ component triggers the formation of dynamo-inefficient zones predominantly in the inner disk region.

Here, in Paper II, we expand on this, investigating an analytical model of turbulent dynamo theory (Rüdiger et al. 1995) that incorporates both the magnetic diffusivity and the turbulent dynamo term, connecting their strength and their amount of anisotropy by only one parameter, the Coriolis number $\Omega$.

This paper is organized as follows. In Section 2, we summarize the main features of our numerical setup, while for an extended presentation we refer to Paper I. In Section 3, we introduce the standard accretion-disk dynamo model and apply
it to large-scale disk-jet simulations. We summarize our paper in Section 4. In the Appendix, we present a resolution study demonstrating the quality of our approach.

2. Model Approach

We solve the time-dependent, resistive magnetohydrodynamic (MHD) equations by applying PLUTO code (Mignone et al. 2007) version 4.3 on a spherical grid (R, θ, φ) assuming axisymmetry. We refer to (r, z, φ) as cylindrical coordinates.

We have further detailed our model approach in Paper I. Here, for convenience, we provide a summary of the most essential points. The resistive, time-dependent MHD equations considering a mean-field dynamo are

\[
\begin{align*}
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) &= 0, \\
\frac{\partial \mathbf{v}}{\partial t} + \nabla \left[ \mathbf{v} \cdot \mathbf{v} + \left( \mathbf{B} \cdot \mathbf{B} + \frac{\rho}{2} \right) \right] = -\rho \nabla \Phi - \frac{\partial \mathbf{B}}{\partial t}, \\
\frac{\partial \mathbf{B}}{\partial t} &= \mathbf{J} \\
\frac{\partial e}{\partial t} + \mathbf{v} \cdot \nabla \left[ e + \frac{\mathbf{v} \cdot \mathbf{v}}{2} + \rho \Phi \right] &= \frac{\partial \mathbf{B}}{\partial t} \cdot \mathbf{v} + \eta \mathbf{J} \times \mathbf{B}
\end{align*}
\]

where the primitive variables (\(\rho, \mathbf{v}, p, \mathbf{B}\)) are, respectively, the gas density, velocity and pressure, and the magnetic field, while \(e\), whose dependence on \(\rho\) and \(p\) is defined in the equation of state, is the internal energy. The tensors \(\mathbf{\tau}_{\text{dyn}}\) and \(\mathbf{\eta}\) describe the \(\alpha\) effect of the mean-field dynamo and the magnetic diffusivity. As in Paper I, for the sake of simplicity, we set the cooling term to be equal to the ohmic heating.

The length and timescales, as the primitive MHD variables, are normalized to their value at the inner disk radius \(R_\text{in}\) (i.e., the time unit is given as \(t_\text{in} = R_\text{in}/v_{\text{K, in}}\)).

The computational domain has a range of \(R = [1, 100]R_\text{in}\) in the radial direction, where a stretched grid is applied, and a range of \(\theta = [10^{-3}, \pi/2 - 10^{-3}] \approx [0, \pi/2]\) in the angular direction, where a uniform grid is applied. The numerical resolution is \([N_R \times N_\theta] = [512 \times 128]\) grid cells, which allows the initial disk height \(H = 0.2r\) with 16 cells to be resolved.

For the resolution study (see the Appendix), we applied a resolution of \([N_R \times N_\theta] = [1024 \times 256]\) and \([N_R \times N_\theta] = [256 \times 64]\) grid cells, namely 32 and 8 cells per disk height, respectively.

As the MHD equations are scale free, our normalized variables can be scaled to a variety of jet sources. We apply the same scaling as in previous works (see Table 1 in Paper I).

The numerical algorithms are the piecewise parabolic interpolation method (PPM; see Mignone 2014) for the spatial integration, a third-order Runge–Kutta scheme for the time integration, and a Harten–Lax–van Leer Riemann solver (Toro 2009). We apply the method of upwind constrained transport (Londrillo & del Zanna 2004) in order to preserve the divergence of the magnetic field. We choose a Courant–Friedrichs–Lewy time stepping with \(CLF = 0.4 < 1/\sqrt{N_\text{dim}}\).

2.1. Initial and Boundary Conditions

The initial and boundary conditions are identical to those of Paper I. Here we summarize them for convenience.

The initial state of the disk structure is obtained as a solution of the hydrostatic equilibrium, assuming self-similarity and ignoring the weak initial seed magnetic field. Thus, as given by the initial self-similarity, every (initial) characteristic speed will scale as the Keplerian velocity, \(\propto R^{-1/2}\). We assume a polytropic gas, \(P \propto \rho^{\gamma}\). We set the ratio between the isothermal sound speed and the Keplerian velocity at the disk midplane of the inner radius to be \(\epsilon = c_\text{s}/v_{\text{K, in}} = \pi/2 = 0.1\).

Outside the disk, we define a hydrostatic corona,

\[
\rho_c = \frac{\rho_{\text{c, in}} R^{1/(1-\gamma)}}{P_c} = \frac{\gamma - 1}{\gamma} \rho_{\text{c, in}} R^{\gamma/(1-\gamma)},
\]

with \(\rho_{\text{c, in}} = 10^{-3}\rho_{\text{in}}\). If not specified otherwise, we set the initial magnetic field as purely radial and exponentially decreasing in the vertical direction, with a maximum plasma beta of \(10^{-5}\).

The boundary conditions are identical to Paper I and are summarized in Table 1. Along the rotational axis an equatorial plane, standard symmetry conditions are applied. The inner radial boundary is divided into two different areas, (i) the disk accretion are (\(\theta > \pi/2 - 2\epsilon\)) and (ii) the inner coronal area (\(\theta < \pi/2 - 2\epsilon\)). Across the inner and outer boundaries, both the density and pressure are extrapolated using a power law.

Along the inner coronal boundary, we prescribe \(B_r = 0\), while we adopt a power law for the boundary area toward the inner disk and along the outer boundary. For \(v_\phi\), we prescribe a power law across the inner boundary, while the standard PLUTO outflow (zero-gradient) conditions are applied along the outer boundaries for all three velocity components (\(v_r, v_\theta\) and \(v_\phi\)). We also require the radial velocity to be nonpositive at the outer disk boundaries and nonnegative at the outer coronal boundaries. The component \(v_\theta\) is set to 0 at the inner boundary, while the radial component follows a power law in the inner disk boundary and a weak inflow into the domain of \(v_r = 0.2\) along the coronal boundaries.

Because we are using constrained transport, only the component \(B_\theta\) needs to be defined, while \(B_r\) is recovered from the solenoidal condition. Across the outer boundaries \(B_\theta\) follows a power law, while at the inner boundaries we prescribe the poloidal magnetic field inclination, choosing an angle

\[
\varphi = 70^\circ \left[ 1 + \exp\left( -\frac{\theta - 45^\circ}{15^\circ} \right) \right]^{-1},
\]
where $\varphi$ is the angle between the magnetic field and the initial disk surface. The radial component of the magnetic field is then computed by the code through the divergence-free condition of the magnetic field.

2.2. The Model for Diffusivity and Dynamo

For a thin disk, the nondiagonal components of the mean-field dynamo and the magnetic diffusivity tensors are negligible. We apply a dynamo tensor derived in the theoretical analysis of Rüdiger et al. (1995), Rekowski et al. (2000), and Bardou et al. (2001),

$$\bar{\alpha}_{\text{dyn}} = (\alpha_R, \alpha_\delta, \alpha_\phi) = -\bar{\alpha}_0 c_\alpha F_\alpha(z),$$

(4)

where $c_\alpha$ is the adiabatic sound speed at the disk midplane and $F_\alpha(z)$ is a profile function,

$$F_\alpha(z) = \begin{cases} \sin \left( \frac{z}{H} \right), & z \leq H, \\ 0, & z > H \end{cases},$$

(5)

with $H$ being the initial disk pressure scale height that confines the dynamo action within the accretion disk. For a thin disk, the nondiagonal components of the mean-field dynamo tensor are negligible.

For the magnetic diffusivity tensor, we adopt an $\alpha$ prescription (Zanni et al. 2007; Sheikhi-khazemi et al. 2012),

$$\eta = (\eta_{gr}, \eta_\theta, \eta_\phi) = \eta_0 \alpha_{s}\alpha_{s} H F_\eta(z),$$

(6)

where $\alpha_{s\phi}$ is the dimensionless parameter of turbulence (Shakura & Sunyaev 1973). The profile function that confines the diffusivity within the disk region is

$$F_\eta(z) = \begin{cases} 1, & z \leq H \\ \exp \left[ -2 \left( \frac{z-H}{H} \right) \right], & z > H \end{cases}.$$  

(7)

Here, as in Paper I, we apply the so-called strong diffusivity model that we have previously invented (Stepanovs & Fendt 2014; Stepanovs et al. 2014),

$$\alpha_{s\phi} = \frac{2}{\gamma} \left( \frac{\mu_0}{\mu_\Omega} \right)^2,$$

(8)

with $\mu_0 = 0.01$ and $\mu_\Omega$ being defined as the ratio between the average total magnetic field (vertically averaged at a certain radius) in the disk and the gas pressure at the disk midplane (Stepanovs et al. 2014). As demonstrated in Stepanovs et al. (2014) and Fendt & Gaßmann (2018), this approach allows a stable evolution of the disk-jet structure to be performed over very long simulation times (up to 500,000 inner disk rotations).

2.3. Dynamo Number and Dynamo Quenching

We define a dynamo number as in Paper I,

$$D = \frac{\alpha_\phi \Omega H^3}{\eta_{\text{disk}}^2}$$

(9)

(Rüdiger et al. 1995; Rekowski et al. 2000), where the quantity $\alpha_\phi$ is computed at $z = H/2$.

This definition of the dynamo number is the product of the azimuthal magnetic Reynolds number, $R_{\Omega} = |\Delta\Omega| H^2/\eta_{\text{disk}}$, based on the shear of the flow $\Delta\Omega$, and the magnetic Reynolds number $R_\alpha = \alpha_\phi H/\eta_{\text{disk}}$, based on the $\alpha$ effect (considering $\alpha_\phi$ as the strongest dynamo contribution in disks).

For a diffusivity profile almost constant with radius, the dynamo number $D$ would scale almost linearly with the radius. However, this is just a first estimate, as the disk diffusivity does not follow a constant profile. Moreover, the disk orbital velocity and the midplane sound speed do undergo small changes through temporal evolution. The dynamo number also strongly depends on the turbulent viscosity $\alpha_{s\phi}$. We thus expect, as the magnetic diffusivity grows because of the strong diffusivity model, the dynamo number to decrease to a subcritical value, at which point the amplification of the magnetic field fades.

For the quenching of the dynamo effect, we apply the model of Stepanovs et al. (2014). This basically involves quenching by diffusivity through the strong feedback of the disk magnetization on the magnetic diffusivity. The study of physically more self-consistent feedback models for dynamo quenching will be the subject of our future work.

In Paper I, we elaborated that the dynamo number is not always a useful parameter to characterize the mode of amplification of the magnetic field. In particular, we found that the initial critical dynamo number depends on several factors, e.g., the number of grid cells or the magnetic field configuration. For this reason, an initial critical dynamo number is very hard to find, and it may not follow an unambiguous prescription (see also Stepinski & Levy 1988, 1990; Torkelsson & Brandenburg 1994).

This holds in particular in the presence of a dynamo-inefficient zone. The dynamo number has a strong dependence on the disk diffusivity, while dynamo-inefficient zones are characterized by a low diffusivity (compared to the rest of the accretion disk). Therefore, the dynamo number is a useful characteristic to spot dynamo-inefficient zones within the accretion disk, but here it does not imply any amplification of the magnetic field within such zones.

Nevertheless, the dynamo number still remains a key parameter in order to understand the evolution and saturation of the dynamo action.

3. Simulations of an Accretion-disk Dynamo

In Paper I, we considered an anisotropic mean-field dynamo tensor as a toy model for a realistic accretion-disk dynamo. In this section, we put this on more physical grounds, considering a dynamo tensor that follows from analytical dynamo theory. In particular, we now model the magnetic diffusivity $\eta$ and the mean-field dynamo $\bar{\alpha}_{\text{dy}}$ by applying the mean-field theory of Rüdiger et al. (1995) and Bardou et al. (2001). Here, the strength and distribution of the tensor components of both diffusivity and dynamo are constrained by the mean-field theory of turbulence.

The basic assumptions made are that the accretion disk is sufficiently ionized and that the effects of rotation on turbulence can be described by the Coriolis number

$$\Omega^* = 2\Omega\tau_c,$$

(10)

where $\Omega$ is the basic rotation frequency and $\tau_c$ is the turbulence correlation time.

The latter variable cannot be recovered from large-scale simulations, and it is a key parameter in order to connect the disk scale and the turbulent time and length scales. Direct
3.1. The $\alpha$ Tensor

An essential assumption for the $\alpha$ tensor is that we are considering a thin disk. In this case, the nondiagonal components of the dynamo tensor are negligible (Bardou et al. 2001). The explicit form of the dynamo term is described by Equation (4). The strength of the respective components of $\alpha$ tensor in cylindrical coordinates is

$$
\begin{align*}
\alpha_{0,r} &= \frac{1}{2\Omega^3} \left( \Omega^2 + 6 - 6\Omega^{\delta^2} - \Omega^{\delta^4} \operatorname{arctan} \Omega^* \right) \\
\alpha_{0,\theta} &= \frac{1}{2\Omega^3} \left( -10\Omega^{\delta^2} + 12 \Omega^2 + 12 \operatorname{arctan} \Omega^* \right) \\
\alpha_{0,\phi} &= \alpha_r,
\end{align*}
$$

(Rüdiger et al. 1995). These components are plotted in the left panel of Figure 1. We notice that for larger $\Omega^*$, the horizontal component $\alpha_r$ overcomes the vertical component $\alpha_\theta$. Moreover, the vertical component changes sign around $\Omega^* \approx 1.0$.

While the tensors for the alpha dynamo and the magnetic diffusivity are given in various forms (compare Equations (11) and (12)), we have transformed all tensor components to the spherical coordinate system we apply for all the simulations discussed here (because the dynamo equations of Rüdiger et al. 1995 are given in cylindrical coordinates). So, once the cylindrical components of the dynamo vector are computed, they are also rotated in order to recover the components in the spherical coordinates.

3.2. The Diffusivity Model

The magnetic diffusivity tensor follows the same general structure as the dynamo tensor (diagonal, and therefore treated as a vector). For the time evolution of the diffusivity, we again adopt the model described in Equations (6) and (8). However, the quantity $\eta_0$ that determines the strength and the anisotropy of the diffusivity tensor is computed following Rüdiger et al. (1995),

$$
\begin{align*}
\eta_{0,R} &= \frac{3}{4\Omega^4} \left[ 1 + \left( \Omega^* - 1 \right) \frac{\Omega^*}{\Omega^*} \right] \\
\eta_{0,\theta} &= \frac{\eta_{0,R}}{2} \\
\eta_{0,\phi} &= \frac{3}{2\Omega^2} \left[ 1 + \left( \Omega^* + 1 \right) \frac{\Omega^*}{\Omega^*} \right].
\end{align*}
$$

We note that, contrary to the dynamo prescription, the magnetic diffusivity is computed directly in spherical coordinates. The reason is the way the $\eta_//_{\perp}$ are computed in Rüdiger et al. (1995). The latter can be directly transformed in spherical coordinates, while the dynamo is computed in cylindrical coordinates. However, in the thin-disk approximation (which is the case in this paper), the spherical and cylindrical components are only a little different.

The right panel of Figure 1 shows the different components of the magnetic diffusivity as a function of the Coriolis number $\Omega^*$. If the turbulence is weak, $\Omega^* < 1$, the magnetic diffusivity is basically isotropic (Rüdiger et al. 1995). For strong turbulence, the diffusivity becomes highly anisotropic. Overall, the turbulence has a major impact on both the dynamo action and the diffusivity. We point out that the ratio between $\eta_\|$/ $\eta_\perp$ in the limit of fast rotation and high turbulence ($\Omega^* \approx 10$) is comparable to the one used previously (Stepanovs & Fendt 2014; Stepanovs et al. 2014).

3.3. A Reference Simulation

The main aim of this paper is to investigate jet launching by a mean-field dynamo based on a physical model of dynamo theory (Rüdiger et al. 1995). In our new approach, the parameter that governs both the mean-field dynamo and the magnetic diffusivity is the Coriolis parameter $\Omega^*$. We will discuss below simulations applying different Coriolis numbers in the range $\Omega^* \in [0, 10]$, therefore changing the strength of the dynamo and the diffusivity.

For a reference simulation, we have chosen a Coriolis number of $\Omega^* = 10$ while the other parameters (see above) are taken from Fendt & Gäßmann (2018). Our reference simulation is mainly used to provide a link to the toy models discussed above and which prescribe certain combinations of the dynamo tensor. In this section, we therefore also link the toy model to the physical theory of Rüdiger et al. (1995).

A Coriolis number of $\Omega^* = 10$ may be considered to be high (Gressel 2010; Gressel & Pessah 2015); this magnitude has commonly been used, for example, in studies of a direct dynamo (Rüdiger et al. 1995; Rekowski et al. 2000) in order to...
describe rotating disks for which turbulence has a major effect on the mean-field dynamo.

The run time of our reference simulation (denoted as OM10 from now on) is \( t_R = 10,000 \), corresponding to \( \approx 1500 \) inner disk rotations. This is the time needed to reach a quasi-steady state across the majority of the domain. As for Stepanovs et al. (2014), this time is not dictated by numerical issues but chosen in order to save CPU time, as the configuration of the accretion–ejection system does not really change afterwards.

In Figure 2, we show the temporal evolution of the reference simulation. Again, the initial setup consists of a weak radial magnetic field confined within the accretion disk. While the poloidal magnetic field is (if absent, i.e., \( B_\phi \)) generated and amplified only through a dynamo effect, the toroidal magnetic field is generated by the differential disk rotation and then amplified through the mean-field dynamo. As discussed in Paper I, the dynamo component \( \alpha_\phi \) provides the only mechanism that is able to amplify the poloidal magnetic field from the toroidal magnetic field.

Essentially, the reference model evolves very similarly to the scalar model of Paper I; we detect hardly any differences. The magnetic field is most rapidly amplified in the innermost disk region \( t \lesssim 500 \). As a consequence, super-Alfvénic and superfast (in the outer domain we reach \( v_{\text{jet}} \approx 1.5v_A \), where \( v_A \) is the Alfvén speed) outflows emerge from this part of the accretion disk, very similar to our toy model and to the literature (Stepanovs et al. 2014; Fendt & Gaßmann 2018), while in the outer regions the magnetic field is amplified on a longer timescale (\( t \approx 5000 \)).

Also, the inclination of the dynamo-generated magnetic field is favorable for the Blandford–Payne magnetocentrifugal acceleration mechanism (Blandford & Payne 1982; Pelletier & Pudritz 1992), just as in the scalar dynamo simulations. The jet is ejected from the inner radii of the accretion disk, \( R \lesssim 10 \). Its opening angle decreases as it moves away from the disk; thus, the jet becomes collimated. Because the disk is magnetically diffusive, the magnetic field structure is able to rearrange, leading to a loop structure in the disk without dynamo-inefficient zones (see also Paper I). This loop structure is swept outward during the long-term temporal evolution for \( t \approx 5000 \).

In Figure 3, we again display the evolution of the disk poloidal and toroidal magnetic energy as a main signature of the mean-field dynamo, although here derived from a physical model of the dynamo tensor. The field amplification works on a very short timescales—naturally for a dynamo effect, with the dynamo working much faster in the inner part of the disk.

After a rapid amplification, the magnetic energy slightly decreases over time. This is caused by the new model for the dynamo tensor, which now depends on the midplane adiabatic sound speed and therefore is not constant in time. Although the sound speed shows no significant change throughout the temporal evolution, it decreases with time due to the mass loss from the disk by accretion and ejection. We find this behavior in both scalar and vector dynamo simulations as a consequence of the decrease in dynamo efficiency (sound speed) together with the high diffusivity (diffusive quenching).

As for the toy model, we considered the dynamo number \( \mathcal{D} \) as a key parameter to determine the stability and evolution of the system (see Figure 4). In the inner disk region, the diffusive quenching acts on a very rapid timescale, saturating the magnetic field and decreasing the dynamo number critically.
Figure 4. Dynamo number \( D \) as a function of time and radius for the reference simulation. The left panel shows the evolution of the dynamo number in log scale for all radii. The right panel shows the dynamo number at \( t = 10,000 \) within an area of steady state. The lines denote the dynamo number \( D \) (black) and the power-law approximations (dashed, see text).

below 10 in the very early stages of the evolution. As we move farther out in radius, the mean-field dynamo leads to a slower and weaker field amplification. The disk magnetization, and thus the critical dynamo number, defined as the magnitude of the dynamo number at which the disk has reached a stable configuration, is reached on a longer timescale. We find that the critical dynamo number is \( D \approx 10 \), which is similar to the magnitude\(^5\) found in the literature (see, e.g., Brandenburg & Subramanian 2005).

In quasi-steady state, the local dynamo number grows with radius (see Figure 4, right panel). Interestingly, we may fit this dependence with a broken power law. Thus, after saturation, we can divide the domain of dynamo action into two parts. We find an inner part with \( R \in [1, 20] \) that is best reproduced with a power-law exponent \( \approx 0.25 \), while for the outer part, for \( R > 20 \), a square-root dependence is the best fit.

As a physical reason for the broken power law, we disentangled the evolution of the disk diffusivity, in particular the dependence on the magnetization provided by \( \alpha_{st} \) (see Equation (8)). In the inner region, a power-law approximation of the disk magnetization suggests a power-law index of \( -0.07 \) (blue dashed–dotted line), while in the outer region a power index of \( -0.17 \) is preferred (green dashed–dotted line).

Physically, this indicates that the accretion disk is pressure dominated, although very close to a magnetization constant in radius. For this reason, a linear approximation (red dashed line) also provides a reasonably good fit without the need to separate the steady-state disk regions into two parts. Essentially, even if a linear approximation is simpler, the split into two power laws is (i) more accurate and can also be (ii) related to the disk physics.

3.4. A Parameter Survey

In order to understand in more detail how the magnetic field evolution is correlated with a different dynamo tensor, we performed simulation runs applying a different Coriolis number \( \Omega' \) ranging within \([0,10]\); see Table 2. We stress again that the Coriolis number compares the effects of rotation to those of turbulence, with turbulence being responsible for amplifying a poloidal field with rotation amplifying the toroidal field.

We first look at the dynamo coefficients and diffusivity coefficients (see Figure 1). We see that the \( \alpha_z \) component of the dynamo tensor changes sign and vanishes at \( \Omega' \approx 1 \). However, this component of the dynamo tensor becomes effectively relevant only for low Coriolis numbers. This is the limit of low rotation. In the limit \( \Omega' \rightarrow 0 \), all of the dynamo components tend to vanish, and the magnetic diffusivity becomes isotropic.

3.4.1. Amplification of the Magnetic Field

As for the toy dynamo model, the primary effect of the mean-field dynamo is the amplification of the disk magnetic field. We first compare the magnetic field amplification for different Coriolis numbers (see Figure 5). Because the dynamo component \( \alpha_{st} \) depends monotonously on the Coriolis number (see Equation (11)), one would expect a higher \( \Omega' \) to result in a stronger magnetic field. However, the critical dynamo number discussed in Paper I is no longer applicable, because the Coriolis number also has a strong effect on the disk diffusivity.

What we find is that for \( \Omega' \lesssim 0.15 \), the dynamo amplification of the magnetic field is sufficiently efficient in order to generate a collimated outflow, corresponding to a maximum (absolute) value of \( \alpha_{crit} \approx 0.005 \). Note that this value \( \approx 10 \) times larger than the one recovered by Fendt & Gaßmann (2018) and almost twice as large as the value that we recovered for our toy model above.

This discrepancy is related to the model for magnetic diffusivity, which is now self-consistently determined by the Coriolis numbers, similar to the dynamo alpha. In fact, for the critical strength of the dynamo, the diffusivity level is now also higher than in Fendt & Gaßmann (2018) and also higher than for the toy model discussed above. For \( \Omega' \approx 0.1 \), thus slightly below its critical magnitude, the dynamo process is also able to amplify the poloidal field; however, we do not find collimated outflows from the resulting magnetic field configuration.

We note that a correlation between the profile of disk magnetization and jet collimation has already been proposed by Fendt (2006), such that a high degree of collimation requires a flat magnetization profile, thus also a sufficient magnetization for larger disk radii. This is what we seem to observe in our dynamo simulations, as the magnetization of case \( OM01 \) is lower for larger radii.

We therefore disentangle the following correlations. A higher \( \Omega' \) implies a large dynamo efficiency \( \alpha_z \) that leads to a larger disk magnetization (stronger field, as the disk gas pressure remains similar), which finally supports jet collimation. For \( \alpha_z > \alpha_{crit} \approx 0.005 \) the poloidal magnetic field is amplified to different magnitudes and also on different timescales. Naturally,

| Run ID | \( \Omega' \) | \( t_f \) | Comment |
|-------|-------|-------|--------|
| OM01  | 0.1   | 10    | No jet collimation |
| OM04  | 0.4   | 10    | Dynamo-inefficient zones present |
| OM1   | 1.0   | 10    | Dynamo-inefficient zones present |
| OM5   | 5.0   | 10    | Dynamo-inefficient zones absent |
| OM10  | 10.0  | 10    | Reference simulation |

Note. The sole dynamo parameter is now the Coriolis number \( \Omega' \). The run time of the simulations is \( t_f \) in units of 1000.
In particular, we see that the poloidal magnetic energy increases and the dynamo number evolve with respect to our main simulation because the toroidal field only to lower strength, the poloidal disk magnetic energy increases rapidly before \( t = 500 \), and after a strong amplification, the saturation state is reached on a later timescale.

Because a weaker dynamo can amplify the poloidal magnetic field only to lower strength, the poloidal disk magnetic energy does not increase immediately in the case of \( \Omega^* \approx 0.1 \). This is simply due to the evolution of the magnetic diffusivity, which follows a faster timescale than the dynamo \( \alpha_R \). However, because the toroidal field is amplified from the initial field by the \( \Omega \) effect, the poloidal field is eventually amplified as well.

**3.4.2. Magnetic Diffusivity and Dynamo Number**

We now investigate how the magnetic diffusivity and the dynamo number evolve with respect to our main simulation parameter, the Coriolis number. In Figure 5 (middle panel), we show the disk magnetic diffusivity profile for different Coriolis numbers at \( t = 5000 \). We identify three different evolutionary characteristics.

For (i) high Coriolis numbers, \( \Omega^* \gtrsim 3 \), the diffusivity profile is very similar to the one for the reference simulation with \( \Omega^* = 10 \) (blue curve). The diffusivity profile remains somewhat constant for \( 10^{-2} < \eta < 10^{-1} \). Here, the magnetic field amplification leads to a quite rapid increase of diffusivity (diffusive quenching), and a steady state is reached soon at \( t \lesssim 500 \) in the inner disk region.

For (ii) lower Coriolis numbers, dynamo-inefficient zones are formed (one or more) within the accretion disk, due to the low \( \alpha_R \). These dynamo-inefficient zones are clearly visible in Figure 5 as zones where the magnetic disk diffusivity sharply decreases. This behavior can be seen for simulations applying \( 0.4 < \Omega^* < 1.0 \).

For (iii) even lower Coriolis numbers, e.g., for \( \Omega^* = 0.1 \), the magnetic field amplification remains low. Therefore, in addition to the emerging magnetic loops, the dynamo in outer regions of the disk is not able to amplify the magnetic field. Again, as discussed above, because of the weak magnetic field, magnetic diffusivity remains low as well. Still, the inner disk has a substantial magnetic field and also a high diffusivity.

In order to understand if and where the amplification of the magnetic field is saturating, we have a look at the dynamo number at \( t = 10,000 \) (Figure 5, lower panel). For larger \( \Omega^* \), e.g., \( \Omega^* \gtrsim 3 \), the magnetic field (both poloidal and toroidal) has been amplified in all areas of the accretion disk at this time (but not in the dynamo-inefficient zones). As we know, the actual amplification of the magnetic field plays a key role in the diffusive quenching model (see Equation (8)). Therefore, for the Coriolis numbers considered, the dynamo number, which directly depends on the magnetic diffusivity, falls under a critical magnitude for dynamo action.

This does not apply for the dynamo-inefficient zones. Although these zones are characterized by a large dynamo number, they are not correlated with the amplification of the magnetic field. With a lower Coriolis number, the magnetic field amplification occurs on longer timescales, especially for the outer disk. For this reason, besides the dynamo-inefficient zones, the dynamo number also remains over its critical magnitude in the outer disk regions, for which just more time would be required in order to reach magnetic field saturation. Moreover, for \( \Omega^* \lesssim 0.1 \), the dynamo number is not a good measure for the mean-field dynamo, as it is no longer connected to the process of field amplification.

**3.5. Dependence on the Initial Seed Field**

Mean-field dynamo action is expected to be independent of the initial seed field, due to the exponential growth by the dynamo amplification. However, we discovered that the second-order effect of the initial evolution may also affect the long-term evolution of the system.

In Paper I, we discussed the impact of the dynamo component \( \alpha_0 \) in the toy model. We found that when applying a vertical initial magnetic field, the scalar dynamo model may lead to a nonphysical hydrodynamical evolution, mainly caused by low-density zones forming in the proximity of the inner radial boundary. The origin of these numerical issues seems to be the formation of dynamo-inefficient zones in the very inner part of the accretion disk. Because for the toy model there are
no a priori constraints on the dynamo tensor components, we also tested the effects of an initial vertical seed field with a reduced strength of $\alpha_\theta$ ($\psi = 0.1$), just in order to avoid the formation of the dynamo-inefficient zones in the inner disk.

In the analytical model of Rüdiger et al. (1995), the anisotropy of the tensor component $\alpha_\theta$ is introduced naturally on physical grounds, and it does not require any additional constraint. We performed a simulation with $\Omega^* = 10$ and a vertical initial magnetic field (applying a vector potential $A_\phi = 10^{-5}$. Indeed, the results are comparable with the simulations run in Paper I (see Figure 6).

Here, the component $\alpha_\theta$ is suppressed, as directly inferred from analytical dynamo theory, and no ad hoc assumption of anisotropy is required. Therefore, the effects of shear between the rotating disk and the steady-state corona are not amplified by the dynamo as they were in the scalar dynamo model.

As demonstrated in Paper I, the amplification of the poloidal disk magnetic field occurs on different timescales depending on the distance from the central object. Although during early stages the field amplification looks different from the case of an initially radial initial field (see e.g., Figure 3 for a comparison), at $t = 4000$ the poloidal magnetic energy that is dynamo-amplified is comparable.

The saturation of the magnetic field amplification toward the same magnitude is evidence for the ongoing action of the mean-field dynamo, which is able to generate a magnetic field regardless of the initial magnetic field configuration. The fact that the two panels of Figure 6 are basically indistinguishable from Figure 3 indicates how much the component $\alpha_\theta$ is overestimated in the scalar dynamo model when a nonradial initial magnetic field is present. This is a clear advantage of the tensor model, because it allows the different dynamo components to be suppressed without adding additional constraints.

A substantial difference between simulations applying an initially radial or vertical initial field, respectively, is the formation of dynamo-inefficient zones even for $\Omega^* = 10$. This implies that antialigned magnetic loops can also form in case of a high Coriolis number.

Overall, the evolution of dynamo-inefficient zones may also depend on the quenching model and the diffusion model.

### 3.6. Accretion and Ejection

A difference in the magnetic field structure plays a key role in the dynamics of the accretion disk and the outflow. This holds for the toy model for the dynamo tensor as well as for the physical model of the tensor components. In this section, we want to discuss the dynamical evolution of the accretion–ejection structure for the model of Rüdiger et al. (1995) and compare the results for different Coriolis numbers $\Omega^*$.

In fact, as a first general result, we do not see significant differences between the scalar toy model and the reference simulation OM10. This nice agreement validates the model approach described in Paper I in the context of large-scale jet-launching simulations.

We now compare further simulation runs. We first consider the accretion and ejection rates in Figure 7. The accretion rate increases with the Coriolis number, meaning it increases as well with the strength of the mean-field dynamo. This is because a stronger field amplification, implying a higher disk magnetization, leads to a higher diffusivity and therefore facilitates accretion. In addition, a stronger magnetic field is also more efficient in angular momentum removal. When dynamo-inefficient zones are present (see Figure 5), they effectively enhance the difference between accretion and ejection rates as we already discussed in Paper I.

The ejection rate increases with the Coriolis number, similar to the accretion rate. In general, the ejection–accretion ratio is higher for lower dynamo efficiency, in agreement with
previous simulations (Stepanovs et al. 2014) and with the toy dynamo model, as it depends on the dynamo components $\alpha_\phi$ and $\alpha_R$.

We also notice a slow decrease over time in the ejection rates, which we understand are due to subtle changes in the disk dynamics. Such variations could be triggered by the disk mass loss, which in turn affects the dynamo tensor components, as they are parameterized by the sound speed at the disk midplane.

Before reaching the quasi-steady state, the accretion—ejection rate, defined as $M_{\text{ej}}/M_{\text{acc}}$ (see Appendix B in Paper I), may exceed unity. The reason for such a high ejection efficiency in early evolutionary stages is due to the timescales of the processes involved. In fact, accretion requires more time to establish and to saturate, while ejection operates on a faster timescale.

A reason why a more turbulent state of the accretion disk evolves is the magnitude of $\alpha_\phi$, which changes as well with the Coriolis number. As shown before, for a lower strength of $\Omega^*$, magnetic loops are formed in the disk, implying a more turbulent evolution. A peculiar case is when $\alpha_\phi < \alpha_{\text{crit}}$ (e.g., for OM01). Here, the magnetic field is amplified, but not to a sufficient strength in order to collimate the jet. In this case the accretion rate—correlated to the magnetic diffusivity—is almost negligible; however, we still find some slight ejection in the form of uncollimated disk winds.

The differences in the mass loading and in the magnetic field are reflected on the jet speed and kinematics. As for the toy model (see Paper I), we expect the jet speed to increase with the magnetization, which is strictly correlated with the Coriolis number $\Omega^*$.

The correlation between poloidal disk magnetization and jet speed is shown in Figure 8. The increase in the jet speed as a function of the disk magnetization shows a nice agreement with Stepanovs & Fendt (2016) and with the toy model. We find that for $\Omega^* \gtrsim 1$, the jet speed reaches the Keplerian velocity at the inner disk radius, which is a well-known result for jet formation simulations (see, e.g., Ouyed & Pudritz 1997; Krasnopolsky & Li 1999) and decreases for lower values of the Coriolis number.

Another observable is the jet collimation, which shows the impact of the disk dynamo on the jet dynamics. Using the same definition of collimation used in Paper I, we see from Figure 9 how the Coriolis number (and therefore the dynamo tensor) affects the jet collimation. As shown in Section 3.4.2, it is possible to find three different outcomes. For high Coriolis numbers ($\Omega^* \gtrsim 3$), we find a highly collimated jet. For $\Omega^* \lesssim 3$, the evolution is characterized by the formation of dynamo-inefficient zones, which play a key role in the jet speed and collimation. The structure of the poloidal magnetic field is more turbulent, which implies a less collimated jet. In addition, a lower value of the Coriolis number also means a weaker $\alpha_\phi$ component, which leads to weaker disk magnetization (see Figure 8) and therefore, in agreement with Fendt (2006), a less collimated jet. Below the critical Coriolis number ($\Omega^* < 0.15$) the amplification of the poloidal field does not occur, and therefore, the outflow is not collimated. We also see that the toroidal field is not able to expand through the domain, and it remains confined in the inner regions of our domain. These results are a combination of the two main results found in Paper I, i.e., the strength of the component $\alpha_\phi$ and the formation of the dynamo-inefficient zones.

Here we may close the loop to the observed jet quantities. Overall, we find that magnetic fields generated by a disk dynamo can well launch outflows and accelerate and collimate them into jets. In particular, this holds for an anisotropic dynamo of a thin disk, which can produce a disk magnetization that is able to eject strong jets.

However, we also find that, outside the thin accretion disk approximation, the dynamo is also influenced by other tensor components. Those lead to more unstable, more structured, but slower outflows, which may potentially not survive on the observed spatial scales. We find a variation in the jet speed between 0.3 and 1.1 times the Keplerian speed at the inner disk orbit.

We propose that the variety of observed jet structures may thus reflect the underlying variation of accretion disks, both coupled by the disk-dynamo-generated magnetic field.

### 4. Conclusions

We presented MHD dynamo simulations in the context of large-scale jet launching. Essentially, a magnetic field that is amplified by a mean-field disk dynamo is able to drive a high-speed jet. All simulations were performed in axisymmetry, treating all three vector components for the magnetic field and velocity. We applied the resistive code PLUTO 4.3 (Mignone et al. 2007); however, we extended it by implementing an additional term in the induction equation that considers the mean-field dynamo action.

Extending our approach from Paper I where we applied (ad hoc) choices for the dynamo tensor components, here we consider an analytical model of turbulent dynamo theory (Rüdiger et al. 1995) that incorporates both the magnetic diffusivity and the turbulent dynamo term, connecting their module and anisotropy by only one parameter, the Coriolis number $\Omega^*$.

In particular, we obtained the following results:

1. The prime advantage of the tensor dynamo model is the reduced number of parameter space, in combination with the physically more consistent approach for the dynamo. Both the dynamo and diffusivity tensor can be fully recovered from one single parameter—the Coriolis number $\Omega^*$. Another significant advantage of the tensor model is the physical constraint for the different dynamo components. Applying a nonradial seed magnetic field, the tensor model naturally suppresses the
dynamo action by the component $\alpha_\phi$, which plays a key role in the presence of a nonradial initial magnetic field.

(2) Our new approach confirms previous results of dynamo simulations, as they are included in the new modeling as a limiting case (e.g., Stepanovs et al. 2014; Fendt & Gaßmann 2018). Essentially, the tensor dynamo model shows very good agreement with previous studies and the toy model described in Paper I, recovering very similar results, thereby supporting the approach of the toy model. Looking at different Coriolis numbers, we can distinguish between high values ($\Omega^* \gtrsim 3$), where the disk shows no dynamo-inefficient zones, and a low $\Omega^* \lesssim 3$, where the evolution of the disk is affected by the formation of one or more dynamo-inefficient zones. For even lower $\Omega^* \lesssim 0.15$, dynamo-inefficient zones form and the disk magnetization does not saturate at large radii—both effects affect the jet collimation on the simulation timescales considered.

(3) We studied the evolution of the launching process and also the properties of the ejected jet flow for the different Coriolis numbers $\Omega^*$ that affect the dynamo process. We find that a higher $\Omega^*$ leads to a stronger amplification of the magnetic field. This result is in agreement with previous (scalar) mean-field dynamo simulations but is now put on a more physical ground as it is connected to a more physical disk dynamo model.

(4) We further extended the correlation found by Stepanovs & Fendt (2016) and in Paper I between the accretion-disk magnetization and the jet speed, linking the former quantity to the mean-field dynamo. In particular, we found that higher values of the Coriolis number $\Omega^*$ lead to a stronger magnetization within the accretion disk and therefore to a faster jet. If the Coriolis number (and therefore the dynamo) is not strong enough to amplify the poloidal magnetic field, we find an uncollimated outflow in the form of a slow disk wind.

(5) We investigated the formation of the so-called dynamo-inefficient zones for different values of the Coriolis number and their effect on the disk–jet connection. We find that for small Coriolis numbers $\Omega^* \lesssim 3$, dynamo-inefficient zones are formed in the accretion disk.

(6) We investigated the detailed physical interaction of the dynamo with the field structure by applying a vertical seed magnetic field following the initial evolution of the field amplification by the dynamo tensor component $\alpha_\phi$, which is naturally overestimated in the scalar dynamo model (for disk dynamos). Essentially, we find that a nonisotropic dynamo leads to more stable evolution of the disk–jet system, as the component $\alpha_\phi$ (leading to a magnetic field substructure) is naturally suppressed without any additional constraints.

(7) We finally emphasize the astrophysical relevance of our findings. First, dynamo-generated magnetic fields can well launch outflows and accelerate and collimate them into jets. This holds in particular for a turbulent, anisotropic disk dynamo, which can produce strong jets. Second, thick accretion disks are also influenced by other dynamo tensor components that lead to more unstable, more structured, but slower outflows, which may potentially not survive on the observed spatial scales. We find a variation in the jet speed between 0.3 and 1.1 times the Keplerian speed at the inner disk orbit. Third, the observed variety of jet structures thus may reflect the
underlying variety of accretion disks, which are coupled to the outflows via the disk-dynamo-generated magnetic field.

So far, we have not looked for an unsteady jet-launching process, which can lead to the pulsed ejection that is observed in most jet sources. The model investigated does not have any direct feedback of the magnetic field on the dynamo term. Future simulations should include more physical feedback (e.g., quenching) models and will be presented in a forthcoming paper.

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Appendix

Resolution Study

A numerical study is incomplete without presenting a resolution study. This is done in the following where we discuss how our physical results depend on the numerical resolution applied. We compare our reference simulation (resolution $[512 \times 128]$) of the tensor model (Section 3.3) with two simulation runs applying exactly the same physical parameters, but different resolutions. We choose $[1024 \times 512]$ for a higher-resolution run and $[256 \times 64]$ for a lower-resolution run. The results are displayed in Figure A1, where we show the density and poloidal magnetic field distribution and the evolution of the dynamo-generated poloidal magnetic energy.

First of all, we notice that the reference resolution shows very small differences with the high-resolution case, and this is mostly in the initial evolutionary stages. The open field lines, favorable for the launching, in the inner disk region and the magnetic loops in the outer disk are present in all simulations, with almost no difference (see Figure A1). This holds in particular for the evolution of the disk poloidal magnetic energy. On the other hand, for the low-resolution run, the differences persist also in the later stages, although the qualitative temporal evolution is the same as in the reference case (see Figure A1).

The differences in the evolution of the magnetic field are mostly related to the different numerical diffusivity, which is higher for lower resolution. Before the dynamo quenching by diffusivity has taken place, we believe that the numerical diffusivity contributes to the low-resolution case, leading to a damping of the magnetic field amplification (a higher diffusivity lowers the dynamo number). However, at later times, the physical magnetic diffusivity (which is triggered by the disk magnetization) becomes dominant and therefore the poloidal magnetic energy saturates around the same level (see Figure A1).

Numerical diffusivity plays a key role in the dynamics of the disk–jet connection, e.g., in the efficiency of the accretion process, and also for the mass loading of the disk wind. Because in the low-resolution case the field amplification is slower, the saturation of the diffusivity level that allows the disk matter from the outer disk to be replenished (by accretion) happens on a longer timescale as well. Therefore, the disk accretion rate decreases for the lower-resolution setup.

In summary, our simulation results are not completely resolution independent. However, the results of our reference simulation are very close to a higher-resolution study, so a higher resolution would not lead to any improvement. In contrary, a lower resolution would affect the hydrodynamics of the system as well as the evolution of the magnetic field. Thus, we conclude that the resolution we chose is in fact appropriate to capture the essential physics while keeping the computational costs low.

Figure A1. Resolution study. Density distribution (color, in log scale) and poloidal magnetic field (white lines) at $t = 4000$ for simulations applying the reference parameters, but for different resolutions (three left panels). Temporal evolution of the poloidal disk magnetic energy (fourth panel) integrated from $R = 5$ to the outer boundary, $R = 100$, and the accretion rate (right panel) computed at $R = 5$. 
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