INFORMATION IN BLACK HOLE RADIATION*

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Abstract

If black hole formation and evaporation can be described by an $S$ matrix, information would be expected to come out in black hole radiation. An estimate shows that it may come out initially so slowly, or else be so spread out, that it would never show up in an analysis perturbative in $M_{\text{Planck}}/M$, or in $1/N$ for two-dimensional dilatonic black holes with a large number $N$ of minimally coupled scalar fields.

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Hawking’s calculation of thermal emission from a stationary classical black hole \[1, 2\] soon led to a major unresolved puzzle concerning quantum mechanics and gravity: what happens to a pure quantum state that collapses to form a black hole which emits approximately thermal radiation? Hawking proposed \[2\] that the black hole would eventually disappear completely and that the resulting state of radiation, like a precisely thermal state, would be mixed. In other words, information would be permanently lost down the black hole, and there would be no \(S\) matrix to take an initial pure state to a final pure state.

It was soon objected \[3, 4\] that this conclusion is not justified by the classical or semiclassical approximation for the black hole used to derive it, and that, in its original form at least, it violates a strong form of \(CPT\) invariance \[4\]. A number of alternative possibilities were given \[4\]. The main options now under active investigation seem to be that either most of the information comes out with the bulk of the radiation to give an \(S\) matrix \[4, 5, 6, 7\], or most of the information goes into a long-lived \[8, 9\] or absolutely stable remnant \[10\], or else information is lost from our universe as Hawking proposed \[4\]. For recent reviews of the problem, see \[9, 11, 12, 13, 14, 15\].

The first of these options is in some sense the most conservative, and I have advocated it as the most productive to pursue \[4\]. However, a number of arguments have been given against this possibility. Hawking’s original proposal of a loss of information \[4\] argued that the semiclassical approximation (which would give a loss of information) is valid until the black hole gets down near the Planck mass, and then there is not enough energy left to carry the information (implicitly assuming no long-lived or stable remnants). Giddings and Nelson \[16\], and later Giddings alone \[12\], gave a more detailed version of this argument for a more tractable model of two-dimensional dilatonic black holes with \(N\) minimally coupled scalar fields \[17\].

In this two-dimensional model, the classical equations can be solved exactly \[17\], and the semiclassical equations can be solved numerically \[18\], though a full quantum solution for this model or any realistic variant is still out of reach. The semiclassical approximation appears to be good until the black hole reaches the strong-coupling regime (the analogue of the Planck mass for four-dimensional black holes), by which time the black hole has emitted most of its energy if its initial thermodynamic entropy \(s_h\) is large compared to \(N\). At least the semiclassical analysis seems to be valid until then for certain aspects of the problem, such as the average emission rate, though perhaps not for the information, as I shall argue below.

Assuming the validity of the semiclassical analysis until the weak-coupling approximation breaks down, Giddings and Nelson \[16\] conclude, “The above arguments therefore strongly suggest that within the present model information does not escape until the black hole is very small. Making these rigorous will therefore rule out one suggested resolution of the black-hole information problem, namely, that the information escapes over the course of black-hole evaporation if the effects of the back
reaction are included" [16]. Giddings later stated this more cautiously, that “working order-by-order in \(1/N\), it is probable that one can construct an argument...analogous to stating that the information doesn’t come out of four-dimensional black holes until they reach the Planck scale” [14].

Here I wish to object that if the information does indeed come out gradually over the entire emission process, it appears likely that the rate of information outflow may initially be so low that it would not show up in an order-by-order (perturbative) analysis, and that the information in the entire emission would be so spread out that it would require too many measurements to be found or excluded by a perturbative analysis.

An extreme example that shows how this is at least theoretically possible is the two-dimensional moving mirror model analyzed by Carlitz and Willey and by Wilczek [19]. In this model the early Hawking radiation is exactly thermal, in a maximally mixed state with no information, but then is entirely correlated with the late radiation, so that the total state of all the radiation is pure, containing all the initial information. An order-by-order analysis of the early radiation would reveal no information, but the tempting conclusion that the information cannot escape until the black hole gets near the Planck mass would be invalidated by the nonanalytic change in the information rate at the beginning of the late radiation that is correlated with the early radiation. Only by accurately measuring a huge number of correlations between the early and late radiation could one hope to find the information.

One might counter that this extreme case of exactly thermal local radiation, with correlations only between the first half and the second half, is not at all plausible. Therefore, here I shall here examine a more natural model, in which the black hole and its surrounding radiation are two subsystems of a combined system which is assumed to be in a random pure state. Tracing over the black hole subsystem gives a statistical state (i.e., a state represented by a density matrix) for the radiation subsystem that generically is mixed (i.e., with the density matrix having more than one nonzero eigenvalue, not a pure state with only one nonzero eigenvalue). Then one can ask what the typical information is in the radiation subsystem at various stages of the black hole evaporation.

To control the dimensions of the Hilbert spaces involved, imagine forming the black hole from a pure state of radiation in a box. For simplicity, suppose the radiation is initially in a superposition of energy and angular momentum eigenstates with eigenvalues clustered near \(E\) and 0 respectively (so that the initial state is essentially one pure state out of a microcanonical ensemble with zero angular momentum). Let the box volume \(V\) initially give \(E^5/V\) large enough in Planck units, so that once a black hole forms, it would be semiclassically stable in the microcanonical ensemble, with the black hole having more than 4/5 of the total energy [20]. (One could imagine that the black hole forms either by having the radiation initially aimed inward
to collapse, or else by squeezing the box until the radiation suffers Jeans collapse.)
In the spirit of the hypothesis that no information is lost in black hole formation and evaporation, assume that the radiation subsystem has dimension $m \sim e^{s_r}$, where $s_r$ is the thermodynamic radiation entropy, and the black hole subsystem has a Hilbert space dimension $n \sim e^{s_h}$, where $s_h = A/4$ is the semiclassical Hawking entropy \[1,20\] of a black hole of area $A$. Since the total angular momentum in the box is assumed to be zero, most of the black hole states would have little rotation and would be nearly Schwarzschild with mass $M$, so $s_h \approx 4\pi M^2$. Assuming the box is much larger than the hole, so $E^3 \ll V$, most of the spacetime within the box will be nearly flat. If the radiation is in semiclassical equilibrium with the black hole of Hawking temperature $\sim (8\pi M)^{-1}$, the energy and thermodynamic entropy of the radiation would be

$$E - M \approx a(8\pi M)^{-4}V,$$

$$s_r \approx \frac{4}{3}a(8\pi M)^{-3}V,$$  

where $a$ is the radiation constant for the species of massless particles present, assuming a negligible contribution from massive particles.

The idea now is that the radiation and the black hole are subsystems of a total system of Hilbert space dimension $mn$. Although the total system is in a pure state with density matrix $\rho_{rh} = \rho_{rh}^2$, each subsystem is in a mixed state,

$$\rho_r = \text{tr}_h \rho_{rh}, \quad \rho_h = \text{tr}_r \rho_{rh},$$

with a von Neummann or entanglement entropy

$$S_r = -\text{tr}(\rho_r \ln \rho_r) = S_h = -\text{tr}(\rho_h \ln \rho_h)$$

and information (deviation of the entanglement entropy from maximum)

$$I_r = \ln m - S_r \approx s_r - S_r, \quad I_h = \ln n - S_h \approx s_h - S_h.$$  

We would like to know how much information $I_r$ to expect in the radiation at various stages of the black hole evaporation. Without a full quantum analysis, we cannot really answer this question definitely. However, a reasonable first guess would be that $I_r$ is near the average information in a subsystem of dimension $m$ when the total system, of dimension $mn$, is in a random pure state. For $m \leq n$ (which is the case for a locally stable black hole in a box), this average appears to be \[21\]

$$I_{m,n} = \ln m + \frac{m - 1}{2n} - \sum_{k=n+1}^{mn} \frac{1}{k},$$

and for $1 \ll m \leq n$, it can more definitely be shown to be \[21\]

$$I_{m,n} \approx \frac{m}{2n} \sim e^{s_r - s_h}.$$  

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For \( m \geq n \), the fact that \( S_r = S_h \) implies that Eqs. (5)-(7) give

\[
I_{m,n} = \ln m - \ln n + I_{n,m} = \ln m + \frac{n - 1}{2m} - \sum_{k=m+1}^{mn} \frac{1}{k} \sim \ln m - \ln n + \frac{n}{2m}.
\]

(8)

This average information, along with the average entanglement entropy \( S_{m,n} = \ln m - I_{m,n} \), is plotted versus the subsystem thermodynamic entropy \( \ln m \) in Fig. 1 for the case \( mn = 291600 \), whose 105 integer divisors are taken to be the values of \( m \).

**FIG. 1.** Average von Neumann or entanglement entropy \( S_r = -\text{tr}_r(\rho_r \ln \rho_r) \) and information \( I_r = \ln m - S_r = s_r - S_r \) of a radiation subsystem of Hilbert-space dimension \( m \) versus its thermodynamic entropy, here defined to be \( s_r = \ln m \). The radiation is assumed to be coupled to another subsystem (e.g., a black hole) of dimension \( n \) (and hence thermodynamic entropy \( s_h = \ln n \)), such that the two subsystems form a combined system in a random pure state in its product Hilbert space of fixed total dimension \( mn \), here taken to be \( 2^43^65^2 = 291600 \sim e^{4\pi} \) (about the number of states very na"ively expected for a black hole near the Planck mass).
Eq. (7) means that for a typical pure quantum state of a joint system, the smaller subsystem is very nearly maximally mixed, showing little sign that the total system is pure. For example, when the radiation that has been emitted from a black hole has a smaller Hilbert-space dimension than that of the hole that remains, the radiation would typically have very little information in it. Alternatively, consider the case in which the black hole has emitted most of its energy, so that the radiation has the larger dimension. If one then examines only part of the radiation at a time, so that each part has a smaller dimension than that of the rest of the system, one would expect to see in the separate parts only a tiny amount of the information. The total information is instead mostly encoded in the correlations between all the parts.

As a black hole evaporates, the dimension \( n \) of the Hilbert space of black hole states of energy near the actual (decreasing) black hole mass decreases, and the effective dimension \( m \) of radiation states macroscopically near the actual radiation state increases. Therefore, by Eq. (6)-(8), the expected information in the radiation (in correlations spread throughout it) also increases.

For example, a black hole in a box would evaporate adiabatically as the box is slowly expanded. This would keep the total semiclassical entropy, \( s \equiv s_r + s_h \), constant. Under the assumption that the box is much larger than the hole, which is itself large in Planck units, so \( 1 \ll E^3 \ll V \), but not necessarily \( V \ll E^5 \), one can conveniently parametrize the adiabatic expansion by the monotonically growing ratio of the radiation energy to the black hole energy,

\[
x \equiv \frac{E-M}{M} \simeq \frac{aV}{(8\pi)^4 M^5},
\]

which must be less than \( \simeq 1/4 \) for locally stable equilibrium [20]. Then as a function of \( x \) and of the energy \( E_0 \) when \( x \) is negligibly small (which is when \( V \ll E^5 \)), the parameters of the hole and radiation in adiabatic equilibrium at constant \( s \simeq 4\pi E_0^2 \) vary as

\[
E \simeq E_0(1 + x)(1 + 8x/3)^{-1/2},
\]

\[
M \simeq E(1 + x)^{-1} \simeq E_0(1 + 8x/3)^{-1/2},
\]

\[
T \simeq (8\pi M)^{-1} \simeq (8\pi E_0)^{-1}(1 + 8x/3)^{1/2},
\]

\[
V \simeq (8\pi)^4 a^{-1} E_0^5(1 + 8x/3)^{-5/2},
\]

\[
s_r \simeq s \frac{8x}{3 + 8x} \simeq 4\pi E_0^2 \frac{8x}{3 + 8x},
\]

\[
s_h \simeq s \frac{3}{3 + 8x} \simeq 4\pi E_0^2 \frac{3}{3 + 8x},
\]

\[
I_r \sim \exp(4\pi E_0^2 - 8\pi M^2) \sim \exp(-4\pi E_0^2 \frac{3}{3 + 8x}).
\]
During the adiabatic evaporation stage, when the box is expanded slowly and $x \lesssim 1/4$, the rate at which information comes out of the black hole is roughly

$$\frac{dI}{dt} \sim \exp\left(-4\pi E_0^2 \frac{3-8x}{3+8x}\right) \frac{dx}{dt}. \quad (17)$$

(The prefactor in front of the exponential has been dropped, since it is less relevant than corrections to the large negative exponent.) At the beginning of the black hole emission, when (in Planck units) $M \simeq E_0 = 1/y$ in terms of the small perturbative parameter

$$y = M_{\text{Planck}}/E_0, \quad (18)$$

the initial rate of information outflow appears to be roughly

$$\frac{dI}{dt} \sim e^{-4\pi/y^2}. \quad (19)$$

This is not analytic in $y$ at $y = 0$, so we would never find it by an order-by-order (perturbative) analysis in four dimensions analogous to the one that Giddings and Nelson propose $[16, 12]$ in two dimensions. Therefore, even if it can be proved that the information is not emitted at any finite order of a perturbation series in $y$, it would be consistent with the most naïve expectation of what would happen if the information were indeed coming out, and so it would not be evidence against that conservative possibility.

One may also apply this argument directly to the toy model $[17]$ of two-dimensional dilatonic black holes actually analyzed by Giddings and Nelson $[16, 12]$. They propose to rule out a gradual emission of the information by a perturbative analysis in $1/N$ for large $N$, the number of minimally coupled scalar fields.

When the quantum corrections are small so that the classical equations of $[17]$ provide good approximations for the various thermodynamical quantities, rewriting the null coordinates therein as

$$x^- = \sqrt{M/\lambda^3} \ u, \quad x^+ = \sqrt{M/\lambda^3} \ v, \quad (20)$$

gives the classical black hole metric $[22]$

$$ds^2 = \frac{-\lambda^{-2}du \ dv}{1 - uv} = -\tanh^2(\lambda r)dt^2 + dr^2, \quad (21)$$

where the last expression applies only outside the horizon, for $u = -\sinh(\lambda r)e^{-\lambda t} < 0$, $v = \sinh(\lambda r)e^{\lambda t} > 0$. This metric depends only on the cosmological constant $\lambda^2$ (which sets the scale) and is independent of the mass $M$. Therefore, it is not surprising that the Hawking temperature at $r = \infty$ (where $g_{tt} = -1$),

$$T = \frac{\kappa}{2\pi} = \frac{1}{2\pi} \frac{d}{dr} \tanh(\lambda r)|_{r=0} = \frac{\lambda}{2\pi}, \quad (22)$$
is independent of $M$ (in the classical limit).

In the classical solution, the mass $M$ occurs only in the expression for the dilaton,

$$e^{-2\phi} = \lambda^{-1} M (1 - uv) = \lambda^{-1} M \cosh^2(\lambda r).$$  \hspace{1cm} (23)

That is,

$$M = \lambda e^{-2\phi_H}$$  \hspace{1cm} (24)

in terms of the value $\phi_H$ of the dilaton at the horizon, $uv = 0$ or $r = 0$.

The first law then gives the thermodynamic entropy of the two-dimensional black hole as

$$s_h = \int dM/T = 2\pi M/\lambda = 2\pi e^{-2\phi_H},$$  \hspace{1cm} (25)

up to a constant of integration that I shall assume is negligible when $s_h$ is large.

The quantum-corrected equations [17] are valid outside the horizon for

$$e^{-2\phi_H} \gtrsim N/24 \text{ or } s_h \gtrsim \pi N/12,$$  \hspace{1cm} (26)

so the minimum thermodynamic entropy of a two-dimensional black hole is bounded below by a constant (of order unity) times the number $N$ of scalar fields, at least if we stay in the regime where the $1/N$ expansion is valid.

If the two-dimensional black hole plus radiation outside can be treated as a coupled joint quantum system with unitary evolution, then it would be reasonable to suppose that the number of quantum states of the black hole of mass near $M$ would be roughly

$$n \sim e^{s_h} \gtrsim e^{\pi N/12}.$$  \hspace{1cm} (27)

If the black hole has radiated an effective Hilbert-space dimension $m \sim e^{s_r} < n$ of radiation with thermodynamic entropy $s_r < s_h$, then the information in the radiation would be expected to be

$$I_r \sim I_{m,n} \sim \frac{m}{2n} \sim e^{s_r-s_h} \lesssim e^{s_r-\pi N/12} = e^{s_r-\pi/(12z)},$$  \hspace{1cm} (28)

where now the small parameter is $z = 1/N$.

Thus we see that for a fixed effective dimension of radiation states (e.g., Hawking evaporation at the $M$-independent temperature for a fixed time), the inequalities (26)-(28) imply that the information expected in the radiation would not be analytic in $z = 1/N$ at $z = 0$. Therefore, even if the information were coming out in nonthermal corrections throughout the Hawking radiation, one would not expect to see it by the order-by-order (perturbative) analysis that Giddings and Nelson advocate. That is, even if they succeed in their goal of proving that the information does not come out at any finite order of the perturbation, it would not be a convincing argument that the information is not actually coming out in a nonperturbative way, since that seems to be the typical behavior for a random joint pure state of a black hole plus radiation.
Giddings has argued [23] that the $1/N$ expansion should be valid until $s_h$ gets down to be of order $N$, by which time $s_r$ can be much larger. That is, now the radiation would be the larger subsystem, with dimension $m > n$. Then the information in the radiation would, by Eq. (8), be expected to be

$$I_r \sim \ln m - \ln n + \frac{n}{2m} \sim s_r - s_h + \frac{1}{2} e^{s_h - s_r},$$  \hspace{1cm} (29)$$

which would not be exponentially small as Eq. (28) gave for $s_r < s_h$.

However, to test whether or not the information is there, one would expect to have to measure most of the $m^2 - 1$ independent real parameters of the density matrix of the radiation of Hilbert-space dimension $m$. This would presumably require a number of measurements at least of order $m^2 \sim e^{2s_r} > n^2 \sim e^{2s_h} \gtrsim e^{\pi N/6}$, which is thus at least exponentially large in $N$. Therefore, this task becomes enormously more difficult at $N$ is increased, despite the greater accuracy of the prediction of each measurement by the $1/N$ expansion.

For example, suppose that for large $N$ the rms error of the prediction for each measurement could in principle be made smaller than any finite power of $1/N$ in the perturbative expansion. But when one squares and sums the errors for the more than $e^{\pi N/6}$ measurements typically necessary to determine the information, one does not have a result that can be controlled by making $N$ large. Therefore, this perturbative analysis apparently could not say whether the information is there or not.

Of course, my argument that the hypothesis of the gradual emission of the information apparently cannot be disproved by perturbative analyses does not prove the truth of the hypothesis either. We do not even yet know any really plausible mechanisms for getting the information out from inside what classically appears to be a black hole. (One might consider [3, 24, 8, 6], but in my opinion it is too early to say positively that any of them is yet very plausible.) Another idea, described briefly at the end of my review article in [13], is that the information is brought out from near the center of the black hole to near the (apparent) horizon by wormholes, threads, tubes, or energy conduits, which can be described as topologically nontrivial or trivial narrow regions where the metric and causal structure are much different from that of the surrounding spacetime, due to quantum fluctuations. However, there is not space to describe this speculative idea here in more detail.

In conclusion, if all the information going into gravitational collapse escapes gradually from the apparent black hole, it would likely come at initially such a slow rate or be so spread out (requiring so many measurements) that it could never be found or excluded by a perturbative analysis. No really plausible nonperturbative mechanisms are known for bringing out the information, but one can speculate about how it might conceivably be brought out from behind the apparent horizon.
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