Metastable dark matter mechanisms for INTEGRAL 511 keV $\gamma$ rays and DAMA/CoGeNT events

James M. Cline, Andrew R. Frey, Fang Chen

Physics Department, McGill University, 3600 University Street, Montréal, Québec, Canada H3A 2T8
e-mail: fangchen, jcline, frey@physics.mcgill.ca

(Dated: August 10, 2010)

We explore dark matter mechanisms that can simultaneously explain the galactic 511 keV gamma rays observed by INTEGRAL/SPI, the DAMA/LIBRA annual modulation, and the excess of low-recoil dark matter candidates observed by CoGeNT. It requires three nearly degenerate states of dark matter in the 4-7 GeV mass range, with splittings respectively of order an MeV and a few keV. The top two states have the small mass gap and transitions between them, either exothermic or endothermic, can account for direct detections. Decays from one of the top states to the ground state produce low-energy positrons in the galaxy whose associated 511 keV gamma rays are seen by INTEGRAL. This decay can happen spontaneously, if the excited state is metastable (longer-lived than the age of the universe), or it can be triggered by inelastic scattering of the metastable states into the shorter-lived ones. We focus on a simple model where the DM is a triplet of an SU(2) hidden sector gauge symmetry, broken at the scale of a few GeV, giving masses of order $\lesssim 1$ GeV to the dark gauge bosons, which mix kinetically with the standard model hypercharge. The purely decaying scenario can give the observed angular dependence of the 511 keV signal with no positron diffusion, while the inelastic scattering mechanism requires transport of the positrons over distances $\sim 1$ kpc before annihilating. We note that an x-ray line of several keV in energy, due to single-photon decays involving the top DM states, could provide an additional component to the diffuse x-ray background. The model is testable by proposed low-energy fixed target experiments.

PACS numbers: 98.80.Cq, 98.70.Rc, 95.35.+d, 12.60Cn

I. INTRODUCTION

Annihilation of positrons near the galactic center gives rise to a narrow 511 keV gamma ray line that was first observed in 1972 [1], and which has been confirmed by numerous experiments since then, most recently by the SPI spectrometer aboard the INTEGRAL satellite [2]. The signal has two distinct components, one associated with the central region of the galaxy (bulge) and another with the disk. There is as yet no consensus as to a conventional astrophysical origin for these gamma rays [3,4], which evidently originate from excess positrons annihilating nearly at rest. The apparent axial symmetry of the bulge component is a point in favor for proposals of models of dark matter (DM) that decays or annihilates into low-energy positrons, since DM should be distributed symmetrically near the galactic center. However, early proposals of this sort were driven toward DM candidates that were nearly as light as the electron itself [5], since the injection energy of the positrons can be no greater than a few MeV [6] (see however [7]). Models of MeV scale dark matter that couples to $e^+e^-$ are highly constrained by low energy collider data, and are not (in our opinion) theoretically attractive. If dark matter is the source of 511 keV gamma rays, one will need to verify its properties by direct detection or other complementary means to make the explanation of the 511 keV signal convincing.

In this work we propose and revisit scenarios in which a long-lived excited state of DM with mass at the 10 GeV scale can scatter into a nearby unstable state, whose mass differs by only a few keV. The unstable state decays into the ground state with the emission of a low-energy $e^+e^-$ pair. The decay can be relatively fast, but the energy is only released after the slow process of inelastic DM-DM collisions occurs. (See however the purely decaying variant described below.) There are two qualitatively different ways to realize this, depending upon whether the metastable state is the middle one, requiring endothermic scattering, or the top one, leading to exothermic. The

![FIG. 1: Spectrum of states for metastable dark matter models. Curves with arrows indicate the sequence of transitions for the endothermic case, $\chi_2 \rightarrow \chi_3 \rightarrow \chi_1$ (left) and the exothermic one, $\chi_3 \rightarrow \chi_2 \rightarrow \chi_1$ (right).](image)

---

1 In addition, the east-west asymmetry in the disk component claimed by [3] is not confirmed by the more recent analysis of [6], using more accumulated data from INTEGRAL.
mass spectra and sequence of transitions are sketched in figure\footnote{The original idea of excited dark matter assumed that only the ground state was significantly populated, so that excitation through the $\gtrsim 5$ MeV mass gap must occur in galactic inelastic collisions\footnote{Earlier work on decaying DM as the source of 511 keV gamma rays can be found in\cite{16,24}.}. However more detailed computations showed that the collision rate is not high enough with such a large energy barrier to overcome \cite{13,14,10}.} The endothermic version, in the context of 500-1000 GeV DM, was first presented in\cite{10} to try to simultaneously explain INTEGRAL, PAMELA (Payload for Antimatter Matter Exploration and Light-nuclei Astrophysics)\cite{11} and ATIC (Advanced Thin Ionization Calorimeter)\cite{12} excess electron observations. It was subsequently discussed with applications to direct DM detection in\cite{13}, for $\sim 100$ keV values of the small mass splitting. The viability of the scenario for INTEGRAL was further explored in\cite{14}, but only in the heavy DM regime.\footnote{Except in equation (11), we will rescale the $\epsilon_i$ to be the mixing parameter of $B_i$ with the photon for notational convenience.}

It would clearly be interesting if the DM mechanism for the INTEGRAL observations was somehow tied to direct detection of the DM\cite{13,14}. Our exothermic mechanism is partly motivated by ref.\cite{18} which proposed a model involving only the states $\chi_2$ and $\chi_3$ (in our notation), as a means of explaining two indications of direct detection of dark matter, namely the long-standing DAMA/LIBRA annual modulation\cite{19}, and the more recent observation of excess low-recoil events by the CoGeNT (Coherent Germanium Neutrino Technology) experiment\cite{20}. Ref.\cite{18} showed that DM with a mass of $\sim 4$ GeV and mass splitting of a few keV could be consistent with these observations, using the exothermic nuclear scattering $\chi_3 N \rightarrow \chi_2 N$. Their observation is that the shape of recoil spectrum is sensitive to modulations of the local DM velocity when the scattering is exothermic, and this can explain the DAMA observations. Additionally, the overall rate for the same parameters is correct for explaining the excess CoGeNT events. The idea of ref.\cite{18} is related to the inelastic dark matter proposal\cite{21}, which however assumed the scatterings to be endothermic rather than exothermic. (See\cite{22} for another discussion of exothermic scatterings.)

There have been several proposals for DM in the 5-10 GeV mass range to explain the DAMA and CoGeNT observations\cite{23}. Most recently, ref.\cite{24} showed that elastic DM interactions could simultaneously explain the DAMA/LIBRA and CoGeNT observations if the dark matter mass is near 7 GeV and the cross section on nucleons around $2 \times 10^{-40}$ cm$^2$. We will argue that our endothermic scenario is close enough to being elastic, if the small mass splitting $\delta M_{23}$ is of order a few keV, so that the same analysis applies. Even though such a small splitting has little effect on DM-nucleus scattering, it is important for DM-DM scattering in the galaxy, in the present case where the DM is lighter than the nuclei in the direct detectors. Getting the observed rate of galactic positrons limits the maximum mass splitting in this case, to values somewhat lower than those that would strongly affect the direct detection rates.

An interesting variant of the above mechanisms is to assume that the unstable excited state is so long-lived that it still has a relic population in the galaxy, and so does not need to be produced by DM collisions.\footnote{This version has more freedom, in that the rate of producing positrons (via decays into $\chi_1 e^+ e^-$) does not depend upon the small mass splitting $\delta M_{23}$, whereas the rate of inelastic scattering $\chi_3 \chi_3 \leftrightarrow \chi_2 \chi_2$ is rather sensitive to $\delta M_{23}$.} This version has more freedom, in that the rate of producing positrons (via decays into $\chi_1 e^+ e^-$) does not depend upon the small mass splitting $\delta M_{23}$, whereas the rate of inelastic scattering $\chi_3 \chi_3 \leftrightarrow \chi_2 \chi_2$ is rather sensitive to $\delta M_{23}$.

Our proposals fit nicely within the framework of dark matter with a nonabelian gauge symmetry in a hidden sector, as suggested by\cite{17}, since such DM automatically consists of multiple states, and small mass splittings are radiatively generated if the gauge symmetry is spontaneously broken. The simplest example that contains three DM states is a hidden SU(2) gauge sector, where the DM is in the triplet representation. After the hidden SU(2) breaks, two colors of the dark gauge boson must acquire small kinetic mixing $\epsilon_i \sim 10^{-3} - 10^{-6}$ with the standard model hypercharge $Y$,

$$\mathcal{L}_{\text{mix}} = \sum_i \epsilon_i B_i^{\mu \nu} Y_{\mu \nu}$$

while the remaining one must have negligible mixing to keep the long-lived state stable against decays to $\chi_1 e^+ e^-$. The mixing $B_i$’s couple weakly to charged Standard Model (SM) particles, and mediate the scatterings with nucleons or decays into $e^+ e^-$.\footnote{In\cite{16,25} we also worked out the crucial features of these hidden sector gauge boson masses and couplings is that they are in the right range to be directly probed by new proposed fixed-target experiments\cite{20}.}

We will present our main results first, in section II. The remaining parts of the paper supply the many details leading to these results. Section III specifies the hidden sector SU(2) particle physics models we consider. The gauge coupling $\alpha_g$ of this SU(2) is calculated in section IV by the requirement of getting the right thermal relic density for the DM. There we also work out the crucial relative abundances of the excited states. Section V describes how the rate and angular distribution of 511 keV gamma rays are computed. Here we also summarize what is believed about the location of gaseous media in the galactic bulge where positron annihilation is supposedly to take place, in response to criticisms of DM interpretations.
of the INTEGRAL observations in ref. [4]. In section VII we explain how the gauge kinetic mixing parameter \( \epsilon_1 \) is constrained to get the desired rates for DAMA. Various astrophysical constraints are addressed in section VIII. We conclude in IX. The appendices give further details about the kinetic equilibrium of the DM with the SM, and the cross section for DM annihilation.

II. MAIN RESULTS

In this section we summarize our main results. The details leading up to them will be given in subsequent sections. The relevant parameter space is the average DM mass \( M_\chi \), the mass splitting \( \delta M_{23} \), and the masses \( \mu_i \) of the hidden sector gauge bosons \( B_i \) that mediate the interactions with the standard model. We fix the larger mass splitting to be \( \delta M_{12} = 1.1 \) MeV so that there is sufficient phase space for the decay into \( \chi_1^0 \) while insuring that the decay products are not very relativistic, as required by constraints on the injection energy of the low-energy positrons [8]. Larger values of \( \delta M_{12} \) tend to suppress the positron rate, and the direct detection rate for exothermic DM, because of greater depletion of the excited state abundance, but our results are not greatly sensitive to the exact value so long as it is less than a few MeV.

Because the nonabelian SU(2) gauge interactions take the form

\[
g \chi_1 B_2 \chi_3 + \text{cyclic permutations}, \tag{2}
\]

\( B_1 \) mediates the transition \( \chi_3 \to \chi_2 \), etc. The strength of the gauge coupling \( g \) is fixed by the requirement of getting the observed relic density of DM from thermal freeze-out,

\[
\alpha_g \equiv \frac{c_g}{x} \left( 1 - \frac{g^2}{M_\chi^2} \right)^{-1/4} \frac{M_\chi}{\text{GeV}} \tag{3}
\]

where \( \alpha_g = g^2/4\pi \) and \( \bar{\mu} \) is the average mass of the gauge bosons. The value of the constant \( c_g \) depends upon the number of hidden sector Higgs bosons that can be present in the final state of \( \chi \chi \to HH \) annihilations; it can lie in the range \( c_g \approx (1.7 - 2.5) \times 10^{-5} \) for the scenarios we consider. In the following, we assume the dark Higgs bosons are heavier than the DM, which yields the top value in this range, hence a larger rate of positron production. The value of \( c_g \) is derived in section IV.

A. Exothermic dark matter

We first consider the exothermic case where \( \chi_3 \) is the stable excited state. In figure 2 we plot contours of \( \log R_\epsilon / R_{\text{obs}} \), the predicted rate of positron production at the galactic center versus the measured rate, in the \( M_\chi - \delta M_{23} \) plane, where \( \delta M_{23} \) is the small mass splitting between DM states 2 and 3. The contours are superimposed upon the DAMA/LIBRA allowed region of ref. [18]. To illustrate the dependence on the gauge boson masses, each graph has a different value of \( \mu_2 \), the mass of \( B_2 \). In the exothermic case, \( B_2 \) is the color that has negligible mixing with the SM (to avoid \( \chi_3 \to \chi_1 \) decays), and so \( \mu_2 \) does not directly affect the rates of either direct detection nor galactic positron production. However, the class of models we describe in section III predicts relations between \( \mu_2 \) and the other gauge boson masses,

\[
\mu_1 \simeq \sqrt{\mu_2^2 - \mu_2^2} \tag{4}
\]

\[
\mu_3 = \frac{2}{\alpha_g} \delta M_{23} + \mu_2 \tag{5}
\]

The first condition (4) depends on details of how the Higgs mechanism in the hidden sector gives masses to
the gauge bosons; we take the inequality to be saturated, which helps to increase the rate of \( \chi_3 \chi_3 \rightarrow \chi_2 \chi_2 \) scatterings (since \( B_1 \) is the exchanged boson). The second condition arises because the mass difference \( m_3 - m_2 \) determines the radiatively generated splitting \( \delta M_{23} = -\frac{1}{2} \alpha_3 (m_3 - m_2) \).

From figure 2 one observes that larger values of \( m_2 \) help to achieve a large enough rate of positron production, up to some optimal value \( m_2 \sim 600 \text{ MeV} \), beyond which the rate starts to slowly fall with \( m_2 \). The rise at small \( m_2 \) occurs because increasing \( m_2 \) inhibits \( \chi_3 \chi_3 \rightarrow \chi_1 \chi_1 \) downscatterings in the early universe, lessening the depletion of the \( \chi_3 \) state. Figure 3 left panel, illustrates this more directly, where the relic abundance of the excited state \( \chi_3 \) relative to that of total DM abundance \( Y_{\text{tot}} \) is plotted as a function of \( M_\chi \) for several values of \( m_2 \). There is a saturation \( Y_3/Y_{\text{tot}} \sim 1/3 \) as \( m_2 \) approaches the GeV scale, for the fiducial value \( \delta M_{12} = 1.1 \text{ MeV} \) of the large mass splitting. (The right panel of fig. 3 indicates that this saturation would occur at higher values of \( m_2 \) if \( \delta M_{12} \) is increased. The rate of \( \chi_3 \chi_3 \rightarrow \chi_1 \chi_1 \) increases with \( \delta M_{12} \) due to the larger phase space.) Further increase of \( m_2 \) beyond the optimal point decreases the positron rate, because \( m_1 \) is an increasing function of \( m_2 \), and the rate of \( \chi_3 \chi_3 \rightarrow \chi_2 \chi_2 \) transitions goes like \( m_1^{-4} \).

It may seem surprising that the rate of positron production is a decreasing function of the mass splitting \( \delta M_{23} \), since the phase space for \( \chi_3 \chi_3 \rightarrow \chi_2 \chi_2 \) increases with \( \delta M_{23} \). However, so does the exchanged momentum that appears in the propagator of the virtual gauge boson, and this has the more important effect of suppressing the amplitude; see eq. 22.

Figure 2 shows some overlap between the desired rate of positron production and the DAMA allowed region for \( m_2 \gtrsim 200 \text{ MeV} \). For each point in the \( m_\chi - \delta M_{12} \) plane, we have adjusted the value of \( \epsilon_3 \) to obtain the DAMA detection rate assumed by ref. 15. \( \epsilon_3 \) is taken to be \( \lesssim 10^{-3} \); the results shown are insensitive to the exact value. Concerning \( \epsilon_3 \), an intriguing prediction of our model is that each direct detection of the process \( \chi_3 N \rightarrow \chi_2 N' \) must be accompanied by the subsequent production of \( e^+ e^- \) through the decay \( \chi_2 \rightarrow \chi_1 e^+ e^- \) (whose rate scales as \( \epsilon_3^2 \)), so in principle one could look for the positron in coincidence. However, the lifetime for the decay cannot be much less than \( 10^4 \text{ s} \), as we will show in section VII C. Since the speed of DM in the galaxy is of order \( 10^{-3} \text{c} \), this occurs too far from the experiment to detect the \( e^+ e^- \) pair. In fact this lifetime is much longer than the age of the universe for \( \epsilon_3 \lesssim 10^{-8} \), leading to an alternative possibility for explaining the 511 keV signal via decays of primordial \( \chi_2 \), more about which in section VII C. The rate of positron production through DM excitations is sensitive to the density profile \( \rho \) of the DM halo; it scales like \( \rho^2 \) evaluated near the galactic center. We parametrize the shape using the Einasto profile

\[
\rho = \rho_\odot \exp \left( -\frac{2}{\alpha} \left( \frac{r}{r_\odot} \right)^{\alpha} - \left( \frac{r}{r_\odot} \right)^{\alpha} \right)
\]

A set of values that are often considered to be standard are \( \alpha = 0.17, r_\odot = 20 \text{ kpc}, \rho_\odot = 0.3 \text{ GeV/cm}^3, r_\odot = 8.5 \text{ kpc} \). These values for \( \alpha \) and \( r_\odot \) are based upon pure dark matter N-body simulations that do not not take into account the effects of baryons in the central region.
To obtain fig. 2. This could still be considered a conservative choice, since ref. [31] argues for $\rho_\odot = 0.43(11)(10)$ GeV/cm$^3$. With these error estimates, one might reasonably consider $\rho_\odot = 0.6$ GeV/cm$^3$. This allows for some reduction of the cuspiness of the halo with very similar results, to $\alpha = 0.08$, $r_s = 7.5$ kpc for example. Moreover, we can still achieve reasonable consistency using the same cuspy profile while keeping $\rho_\odot = 0.42$ GeV/cm$^3$; see fig. 4.

B. Endothermic dark matter

If $\chi_2$ is the stable state, then the transitions $\chi_2 \rightarrow \chi_3$ are endothermic. The energy barrier would tend to reduce the rate of such transitions compared to the exothermic case, but there are other differences that also affect the rate. Most importantly, even though eqs. (4,5) are unchanged, the roles of the gauge bosons $B_2$ and $B_3$ become interchanged relative to exothermic DM. $\mu_3$ now controls the rate of $\chi_2 \rightarrow \chi_1 \chi_1$ downscattering in the early universe, hence the relic density of $\chi_2$. Because $\mu_3$ is naturally the heaviest of the three gauge boson masses in our model (see section [1]), this means that the endothermic scenario leads to a significantly larger rate of galactic positrons than the corresponding exothermic one. We thus adopt a less cuspy halo profile in this case, 

$$\alpha = 0.08, \; r_s = 8 \; \text{kpc}, \; \rho_\odot = 0.42 \; \text{GeV/cm}^3$$

(8)

TABLE I: Characteristics of the density profiles of the haloes in the Aquarius galaxy formation simulations of ref. [24]. Column 1 gives the name of each halo. Columns 2-3 list $\alpha$, $r_s$, the parameters of the best fitting Einasto model, in the inner region of the galaxy.

| Galaxy  | $\alpha$ | $r_s$ (kpc) |
|---------|----------|-------------|
| Aq-A-5  | 0.065    | 5.3         |
| Aq-B-5  | 0.145    | 15.6        |
| Aq-C-5  | 0.115    | 10.2        |
| Aq-D-5  | 0.102    | 14.7        |
| Aq-E-5  | 0.098    | 11.1        |
| Aq-F-5  | 0.112    | 15.6        |
Our findings for the 511 keV signal for endothermic DM are illustrated in figure 6. We have the freedom to choose even less cuspy profiles if desired, with some accompanying decrease in the value of $\delta M_{12}$, as shown in fig. 6, using the more moderate parameter values $\alpha = 0.12$ and $r_s = 12$ kpc. Fig. 7 shows how the relative abundance of the stable state, $Y_2/Y_{\text{tot}}$, depends upon the masses $M_\chi$, $\mu_2$ and mass splitting $\delta M_{12}$. For the examples shown, $\delta M_{12}$ should be $\lesssim 4$ keV to match the direct detection rate corresponding to fig. 8 in the allowed $M_\chi$ region that is shaded in figs. 5, 6.

Similarly to the exothermic case, we fix the value of $\epsilon_1$ to get the desired direct detection rate, while $\epsilon_2$, which controls the rate of decay $\chi_2 \to \chi_1 e^+e^-$, is a free parameter. We assumed $\epsilon_2 = 10^{-3}$ in fig. 6. In contrast to the exothermic case, the results are somewhat sensitive to this choice: taking much smaller values of $\epsilon_2$ mildly suppresses the rate due to its effect on the relic abundance $Y_2$, as we will further discuss in section IV C.

Ideally, the analysis of ref. 24 should be redone for our slightly inelastic scattering to see how the overlap of the DAMA and CoGeNT allowed regions of fig. 8 might be modified. (For this reason we display our results in the same range of DM masses as in fig. 8.) We leave such an investigation to future work.

C. Decaying DM scenario

If the gauge mixing parameter $\epsilon_2$ or $\epsilon_3$ is sufficiently small, then the excited state $\chi_2$ or $\chi_3$ (depending upon whether the DM is exothermic or endothermic) which we have referred to as “unstable” can be as long lived as the universe. Let us denote the stable and “unstable” excited states by $\chi_s$ and $\chi_u$. Instead of being produced in $\chi_s\chi_s \to \chi_u\chi_u$ scattering, $\chi_u$ can have a significant relic density and produce $e^+e^-$ from its slow decays. In section V B we show that the correct lifetime for producing the observed rate of positrons results from taking

$$\epsilon_{2,3} \sim 10^{-11}$$

The exact expression depends upon other parameters and is given by eq. 41. In particular, the dependence upon the DM halo profile is much weaker for decays than for...
the inelastic scattering scenarios discussed above. We are no longer constrained to consider profiles such as (7).

It is intriguing that for reasonable choices of the halo profile, the decaying DM scenario can explain the morphology of the 511 keV signal without the need to invoke propagation of the positrons before annihilation. This is in contrast to the inelastic scattering mechanisms which localize the positron production much closer to the galactic center, as we next discuss.

### D. Angular profile of 511 keV signal

If positrons annihilate before propagating, the predicted intensity \( I_{e^+} \) of the 511 keV signal as a function of angle is just a reflection of the DM density profile, through a line-of-sight integral, whose form depends upon whether the positrons were created through scattering or decay:

\[
I_{e^+}(\hat{x}) = \int_{\text{l.o.s.}} dx \left\{ \frac{1}{2} (\sigma v) \frac{\rho_{s,u}}{M_{\chi}} \text{ scattering} \right. + \left. \frac{1}{2} \frac{y_{\chi}}{\tau_{\chi}} \text{ decay} \right\} \tag{10}
\]

The integral is taken along the \( \hat{x} \) direction, where \( \rho_{s,u} \) is the density of the stable or unstable excited DM component \( \chi_{s,u} \), proportional to the total density \( \rho \), and \( \tau_{\chi} \) is the lifetime of \( \chi_{\alpha} \).

For the Einasto profile (7) we considered for scatterings, \( \rho^2 \) is practically a delta function, and so the signal would appear to come from a point source. \( \rho^2 \) falls to \( e^{-8} \) of its maximum value at a distance of \( r_8 = r_s(2\alpha)^{1/\alpha} \). Even for the standard profile with \( \alpha = 0.17 \) and \( r_s = 20 \) kpc, \( r_8 \) is only 35 pc, which subdents an angle of 0.2°. However the observed signal subtends at least 8° (8); see fig. 9. Therefore if the scattering explanation is correct, all of the observed width must be due to propagation.6

The distance corresponding to 8° is 1.1 kpc, which may be astrophysically reasonable, depending upon the structure of the galactic magnetic field and the injection energy of the positrons.

Apart from astrophysical mechanisms of positron transport (33, 34), which strongly depend upon the poorly constrained magnetic field of the inner galaxy, our model suggests another way in which this widening could occur due to the streaming of \( \chi_3 \) (\( \chi_2 \) in exothermic case) before it decays. If the gauge mixing parameter for \( B_{2,3} \), the hidden gauge boson mediating the \( \chi_{3,2} \rightarrow \chi_1 e^+ e^- \) decay, is sufficiently small, \( \varepsilon_{2,3} \sim 10^{-7} \), then \( \chi_{3,2} \) is so long-lived that it will travel approximately 1 kpc before decaying.

For the decaying DM scenario, it is possible to fit the observed angular distribution without any smoothing from positron diffusion. Figure 9 shows several examples with Einasto parameters \( \alpha = 0.11 \) and \( \alpha = 0.13 \) that pass through all the error bars. These examples are close to the ones given in figure 11 and so could be considered realistic in light of baryonic compression of the inner part of the DM halo.

### III. PARTICLE PHYSICS MODELS

The simplest example of a nonabelian hidden sector model consistent with the observations we discuss has a dark SU(2) gauge group under which the DM transforms as a triplet. The most general form of the Lagrangian that we will need is

\[
\mathcal{L} = \frac{1}{2} \bar{\chi}_a (i \slashed{D}_a - M_\chi \delta_{ab}) \chi_b - \frac{1}{2} \chi_{ab} Y_{ab} - \frac{1}{2} \chi_{ab} \Sigma \bar{\chi}_a \chi_b + \mathcal{L}_{\text{Higgs}}(\Delta^{(i)}, \Sigma) \tag{11}
\]

Here \( \Delta^{(i)} \) and \( \Sigma \) are triplets and a quintuplet respectively of the hidden SU(2). Two such triplets are needed in order to get the kinetic mixing \( \epsilon_i \) required for the direct detection signal and galactic positron production. The mixing parameters \( \epsilon_i = (\Delta_i)/\Lambda_i \) arise when the triplets acquire VEVs. These mixing parameters lead to a coupling \( e \epsilon_i \cos \theta_W \) of the electric current to \( B_1 \); in the remainder of this paper, we rescale \( \epsilon_i \) to remove the Weinberg angle from this coupling. A third triplet is required to get the right pattern of mass splittings for exothermic DM. The VEV of the quintuplet \( \Sigma_{ab} \) gives the large \( \sim \) MeV mass splitting.

We studied this class of models previously in ref. (35). It is convenient to take the triplet VEVs to be mutually orthogonal \( \langle \Delta_1^{(i)} \rangle \equiv \delta_\alpha \Delta_\alpha \). Ref. (35) shows that it is easy to construct a potential leading to this pattern. We further restrict the traceless symmetric tensor \( \Sigma_{ab} \) to have VEVs only on the diagonal,

\[
\langle \Sigma \rangle = \text{diag}(A - B, 2B, -A - B), \tag{12}
\]

This alignment can be accomplished by suitable small interactions between \( \Sigma \) and the triplets. With these VEVs, the masses of the gauge bosons are given by

\[
\begin{align*}
\mu_1^2 &= g^2 (\Delta_1^2 + \Delta_3^2 + 2(A + 3B)^2) \\
\mu_2^2 &= g^2 (\Delta_1^2 + \Delta_3^2 + 8A^2) \\
\mu_3^2 &= g^2 (\Delta_1^2 + \Delta_3^2 + 2(A - 3B)^2) \tag{13}
\end{align*}
\]

The corresponding mass shifts in the \( \chi_\alpha \) states relative to the average mass \( M_\chi \) are given by

\[
\begin{align*}
\delta M_1 &= -\frac{1}{2} \alpha_g (\mu_2 + \mu_3) + y(A - B) \\
\delta M_2 &= -\frac{1}{2} \alpha_g (\mu_1 + \mu_3) + y(2B) \\
\delta M_3 &= -\frac{1}{2} \alpha_g (\mu_1 + \mu_2) + y(-A - B) \tag{14}
\end{align*}
\]

---

5 In the case of inelastic scattering, there is some mild \( r \)-dependence of \( \langle \sigma v \rangle \) due to the \( r \)-dependence of the velocity dispersion, which we neglect here.

6 Our results differ somewhat from those of ref. (32), which assumed a less cuspy halo.
We have introduced the Yukawa coupling contribution in order to explain two different scales of mass splittings: \( \delta M_{23} \sim \text{keV} \), and \( \Delta M_{12} \sim \delta M_{13} \sim \text{MeV} \). This can occur if the quintuplet VEVs satisfy \( A = -3B \); then the Yukawa term only contributes to the large mass splittings and not to \( \delta M_{23} \). Let us assume this to be the case; we will presently show how it can come about. Then and not to the diagonal, and which might perturb \( A_{SU(2)} = 0 \). (In the absence of the triplet VEVs, this would be irrelevant.

For endothermic.

Now we return to the question of why the quintuplet VEVs should satisfy the seemingly fine-tuned relation \( A = -3B \). Interestingly, the desired VEVs can arise from the simple renormalizable potential

\[
V(\Sigma) = \lambda_{\Sigma}(\text{tr} \Sigma^2 - \nu^2)^2 + \mu \det \Sigma
\]

which has three degenerate minima at \( A = \pm 3B \) and \( A = 0 \). (In the absence of the triplet VEVs, this would leave one of the three gauge bosons massless, breaking \( SU(2) \rightarrow U(1) \)). We assume that it is possible to design small interactions with the triplets that align \( \langle \Sigma \rangle \) along the diagonal, and which might perturb \( A \) slightly away from \(-3B\). For simplicity we take \( A = -3B \) in the following, so that \( \delta M_{12} \approx 6\mu B \).

Considering the smaller mass splitting, if \( M_3 \sim 5 \text{ GeV} \), the gauge coupling is of order \( \alpha \sim 10^{-3} \), and the difference in gauge boson masses should be of order \( \delta M_{23}/\alpha g \sim 100 \text{ MeV} \). This is consistent with triplet VEVs at the scale of \( \sim 30 \text{ GeV} \), since the gauge coupling is \( g \sim 0.035 \). To get the correct sign for the mass difference, \( M_3 > M_2 \) only requires that \( \Delta_2 > \Delta_3 \) (given our assumption \( A = -3B \)), so that \( \mu_3 > \mu_2 \).

IV. RELIC DENSITY

In ref. [35], a first attempt was made to compute the value of the gauge coupling \( \alpha_g \) corresponding to the observed DM density through thermal freeze-out. In this section we correct and refine that result, taking into account some subleading effects, including extra annihilation channels into dark Higgs bosons, dependence of the Born cross section on the DM velocity, and Sommerfeld enhancement [36] at the time of freeze-out. Moreover we estimate the amount of dilution of the excited states due to downscattering in the early universe.

A. Annihilation cross section; determination of \( \alpha_g \)

Ref. [35] derived the annihilation cross section for \( \chi \chi \rightarrow BB \) by separately considering \( \chi_1 \chi_1 \rightarrow B_j B_j \) (with \( j = 2, 3 \)) and \( \chi_1 \chi_j \rightarrow B_1 B_j \) (again with \( j = 2, 3 \)), and explicitly averaging over the initial state and summing over the final state colors. In this paper, we make several improvements to the previous calculation as well as minor corrections. Details of our calculation are given in appendix A.

First, we now include the process where \( \chi \chi \) goes to two hidden sector Higgs bosons through exchange of a virtual \( B \) in the \( s \)-channel. Furthermore, as pointed out in [37], velocity-dependence (including Sommerfeld enhancement) may make significant contributions to the cross section in some models. Therefore, we include corrections to order \( \nu^2 \) in the tree-level cross section as well as the leading contribution from Sommerfeld enhancement.

Moreover, in the present application, the dark matter is sufficiently light that its annihilation may take place in the broken phase of the theory. Therefore, we have computed the cross section for \( \chi \chi \rightarrow BB \) taking into account the gauge boson masses. The main effect comes simply from the reduction in phase space, which is a factor of \( (1 - \mu_i^2/M_X^2)^{1/2} \) in the cross section, where \( \mu_i \) is the mass of the gauge boson in the final state. Annihilation to light Higgs bosons is also modified only by this factor (of course, Higgs bosons heavier than the DM are not annihilation products). For simplicity we replace \( \mu_i \) by the average mass \( \bar\mu \) of the gauge bosons and light Higgs bosons in this part of the calculation.

The full cross section for annihilation into gauge bosons, \( N_3 \) Higgs triplets, and \( N_5 \) Higgs quintuplets, including leading velocity dependence, is

\[
\sigma v_{\text{rel}} = \frac{\pi}{12} \frac{\alpha_g^2}{M_X^2} \left[ \left( \frac{25}{2} + 2N_3 + 10N_5 \right) \left( 1 + \frac{\pi\alpha_g}{v_{\text{rel}}} \right) + \left( \frac{317}{48} \frac{5N_3}{12} + \frac{25N_5}{12} \right) v_{\text{rel}}^2 \right] \left( 1 - \frac{\bar\mu^2}{M_X^2} \right)^{1/2}
\]

in the center of momentum frame. The factor \( 1 + \pi\alpha_g/v_{\text{rel}} \) incorporates Sommerfeld enhancement neglecting the masses of the gauge bosons. This neglect is valid for large \( M_X \), such that the freezeout temperature \( \sim M_X/20 \) is above the symmetry breaking scale. For smaller \( M_X \), this factor is roughly an upper bound on

\[\text{VII A} \] though, for further discussion of some subtleties.
Sommerfeld enhancement (except very close to a resonance) and furthermore $\alpha_g$ is sufficiently small that Sommerfeld enhancement is an unimportant correction during freezeout. Conversion to the rest frame of the cosmic fluid introduces an additional correction of order $V^2$ for center of momentum velocity $V$.

To compute the relic density, one needs the thermal average $\langle \sigma v_{\text{rel}} \rangle$. Using the Maxwell-Boltzmann distribution, we find

$$\langle \sigma v_{\text{rel}} \rangle = \frac{\pi \alpha_g^2}{12 M_X^2} \left( \frac{25}{2} + 2N_3 + 10N_5 \right) \times \left( 1 + \alpha_g \sqrt{\frac{\pi M_X}{T}} - \frac{1}{2\pi M_X} \right) +$$

$$\left( \frac{317}{8} - \frac{5N_3}{2} - \frac{25N_5}{2} \right) \frac{T}{M_X^2} \left( 1 - \frac{\bar{\mu}^2}{M_X^2} \right)^{1/2}.$$  

(17)

Annihilations go out of equilibrium at a temperature given by $M_X/T_f = x_f \equiv \ln \xi - \frac{1}{2} \ln \ln \xi$ with $\xi = 1.0 \times 10^{12}(M/\text{TeV})$ for triplet DM, giving $x_f = 20.4$ for $M_X = 5 \text{ GeV}$ and $x_f = 23.3$ for $M_X = 100 \text{ GeV}$. We should equate (13) at the $T_f$ with the cross section needed to match current constraints on the DM density. This varies mildly with $M_X$ as $\langle \sigma v \rangle_0 \approx (3.2 - 0.24 \log(M/\text{GeV})) \times 10^{-26} \text{ cm}^3/\text{s}$.[40] However, the latter expression assumes the usual particle content of the standard model at the time of freezeout, whereas in our model there are three additional gauge bosons and extra dark Higgs bosons. This increases both the Hubble rate and redshifting between freezeout and the present. Thereby, the extra particle content decreases $\langle \sigma v \rangle_0$ by a factor of $\sqrt{1 + (6 + 3N_3 + 5N_5)}/61.75$. Details are given in appendix A. For two Higgs triplets and one quintuplet, we find that a good approximation is given by

$$\alpha_g \approx \frac{1.7 \times 10^{-5} (M_X/\text{GeV})}{(1 - \bar{\mu}^2/M_X^2)^{1/4} \sqrt{1 + 7.7\alpha_g}}$$

(18)

For the range of $M_X$ we are interested in, $\alpha_g$ is so small that the Sommerfeld enhancement factor $\sqrt{1 + 7.7\alpha_g}$ can be neglected. Further results for other light Higgs states are given in [A20]

B. Relative density of excited state

Let us denote the stable excited state by $\chi_s$, and the unstable one as $\chi_u$. At the freezeout temperature, all three DM states are equally populated, but if the rate of downscattering transitions $\chi_s \chi_s \to \chi_i \chi_i$ remains larger than the Hubble rate at temperatures below the mass splitting $\delta M_{1s}$, the density $n_1$ of the excited state gets suppressed relative to $n_1$ of the ground state. The rate of $\chi_s \chi_s \to \chi_u \chi_u$ transitions in the galaxy at the present epoch scales with $(n_u/n_{\text{tot}})^2$ (for $n_{\text{tot}} = n_1 + n_s + n_u$). We must therefore compute this ratio to accurately predict the rate of positron production. The direct detection rate for $\chi_s N \to \chi_u N'$ similarly scales like $(n_s/n_{\text{tot}})$.

To compute the dilution of $\chi_s$ from downscattering, we solve the Boltzmann equation for the abundance $Y_s = n_s/s$, where $s$ is the entropy density. Defining $z = \delta M_{1s}/T$, it can be cast in the form [39]

$$\frac{dY_s}{dz} = -\frac{\lambda}{z^2} \left( Y_s^2 - Y_1^2 \left( \frac{\gamma_{\text{eq}}}{\gamma_1} \right)^2 \right)$$

(19)

where $\lambda$ is related to the cross section $\sigma_\chi$ for $\chi_s \chi_s \to \chi_1 \chi_1$ downscattering by

$$\lambda = \langle \sigma v \rangle \left[ \frac{s}{\cal H} \right]_{z=1}$$

(20)

except (as we shall describe below) the multiplicity factors $g_s$ and $g_{s*}$ that appear in $\lambda$ should retain their $z$-dependence (only explicit factors of $T$ get replaced by $\delta M_{1s}$).

We can simplify this by assuming that the abundance of the ground state does not change significantly during the depletion of $\chi_s$, so $Y_1$ is just a constant. Furthermore $Y_{2s}^\text{eq}/Y_{1s}^\text{eq} \approx e^{-z}$ to a good approximation if the DM is in kinetic equilibrium with the standard model particles (we will discuss this caveat below). Defining the fraction $f = Y_s/Y_1$, (19) becomes

$$\frac{df}{dz} = -\frac{\bar{\lambda}}{z^2} \left( f^2 - e^{-2z} \right)$$

(21)

where $\bar{\lambda} = \lambda Y_1$. (However the $e^{-2z}$ will be modified when we take into account kinetic decoupling effects; see next subsection.) To explicitly compute $\lambda Y_1$, let us parametrize the DM ground state density as $n_1 = (g_{s*}/g_{s*0}) \xi T^3$, where the $g_{s*}$ factors take into account the dilution of $n_1$ as a function of temperature due to entropy production after freezeout. Then

$$\bar{\lambda} = \frac{g_{s*} \xi \delta M_{1s} M_p}{1.66 \sqrt{g_{s*0}} \langle \sigma v \rangle}$$

(22)

where $M_p = 1.22 \times 10^{19} \text{ GeV}$ and $\xi = 7 \times 10^{-10} \text{ GeV}/M_X$ to match the observed DM density.

The cross section $\sigma_\chi$ is straightforward to compute, since it is similar to $e^- e^-$ scattering, with just two diagrams, exchange of a gauge boson in the $t$ and $u$ channels. In the low-velocity limit, we obtain

$$\langle \sigma v \rangle = 4 \pi \alpha_g^2 \frac{M_X^2 v_t}{(\mu_i^2 + M_X^2 v_t^2)^2}$$

(23)

where $v_t = \sqrt{2\delta M_{1s}/M_X}$ is the velocity of $\chi_1$ at threshold (when the incoming $\chi_s$ particles are at rest) and $\mu_i$ is the mass of the exchanged gauge boson. For the exothermic DM model, where $\chi_s = \chi_i$, $i = 2$, while for the endothermic case where $\chi_s = \chi_i$, $i = 3$. Using this constant cross section is actually a conservative estimate, as it is near the maximum value of the full velocity dependent cross
section given in equation (26) for \( \delta M_{13} \sim 1.1 \text{ MeV} \) and our typical values of \( M_\chi, \mu_1 \).

To solve the Boltzmann equation, we first tried to employ the semianalytic technique popularized in Kolb and Turner [40]. Namely, one writes \( f = e^{-z^2} + \Delta \) and linearizes the equation in \( \Delta \) for the early time behavior, giving \( \Delta = z^2/(2\lambda) \), while \( \Delta' = -\lambda z^{-2}\Delta^2 \) at late times. Integrating the latter equation between the \( z \) of freezeout, \( z_f \), and infinity gives the final abundance \( Y_{\chi_f} = \Delta_\infty = z_f/\lambda \). The trick then is to appropriately determine the value of \( z_f \). One does this by assuming that

\[
\Delta(z_f) = \frac{z_f^2}{2\lambda} = c e^{-z_f} \quad (24)
\]

and then finding the value of \( c \) for which this procedure best reproduces the full numerical solution. However we find that this procedure is not sufficiently accurate for the regime we are interested in, where \( Y_s/Y_1 \approx 0.1 \) rather than the exponentially small values of interest for \( Y_1 \) itself. There is no fixed value of \( c \) that accurately gives the same as the numerical result as \( Y_s/Y_1 \) ranges between 0.1 and 1. Therefore we numerically solve the Boltzmann equation in all cases.

To relate \( Y_s/Y_1 \) to the fractional abundance of the stable state to the total dark matter population at the present time, we must remember that the unstable state \( \chi_u \) is also kept in equilibrium with \( \chi_1 \) until a similar freezeout temperature (which is the same in the limit that \( \mu_2 = \mu_3 \)). Only at much later times \( > 10^9 \, \text{s} \), \( \chi_u \) decays to the ground state. The total abundance of dark matter is then \( Y_1 + Y_s + Y_u \). The fractional abundance of \( \chi_s \) is therefore

\[
\frac{Y_s}{Y_{\text{tot}}} \approx \frac{Y_s/Y_1}{1 + Y_s/Y_1 + Y_u/Y_1} \quad (25)
\]

where \( Y_{s,u}/Y_1 \) denotes the values at freezeout, from solving the Boltzmann equation. We compute \( Y_u/Y_1 \) in exactly the same way as \( Y_s/Y_1 \). The only difference is the exchange of \( \mu_2 \leftrightarrow \mu_3 \) for the gauge boson mass appearing in the propagator of the cross section (23). An astute reader may wonder whether \( \chi_3 \chi_3 \rightarrow \chi_2 \chi_2 \) scatterings change the ratio \( Y_u/Y_s \) additionally; however, the cross section (23) is greatly reduced for \( \delta M_{23} \sim 1 - 10 \, \text{keV} \), and we find like [18] that this process freezes out at temperatures well above the mass splitting \( \delta M_{23} \).

### C. Kinetic equilibrium

The preceding discussion of the Boltzmann equation assumed that the DM is in kinetic equilibrium until the freezeout of downscattering. If this is not the case, the relic density of \( \chi_s \) will be smaller than estimated there. The reason is that the equilibrium density depends upon the kinetic temperature \( T_k \) and this redshifts with the expansion of the universe \( T_k \sim 1/a^2 \), in contrast to the temperature of particles that are still coupled to the thermal bath, \( T \sim 1/a \). To get some sense of the size of the effect, we can follow the analytic procedure for an approximate result, even though in the end we solve the Boltzmann equation numerically.

If \( T_d \) is the kinetic decoupling temperature, then \( T_k = T^2/T_d \) for \( T < T_d \). Let \( z_d = \delta M_{13}/T_d \). Then the term \( e^{-z_d} \) in (19) must be replaced by \( \exp(-2\max(z, z_d^2/z_d)) \). Following the semianalytic approach described above, one finds that eq. (24) is replaced by

\[
\Delta(z_f) = \frac{z_f^3}{c\lambda z_d} = e^{-z_f^2/z_d} \quad (26)
\]

which can be rewritten as \( z_f = (z_d \ln(c\lambda z_d/z_f^2))^{1/2} \). As a consequence the relic abundance of \( \chi_s \) is suppressed by \( \sqrt{z_d} \) in this case. Thus it is preferable for kinetic decoupling to occur after the chemical freezeout of \( \chi_s \), for maximizing its relic density.

The principal interaction for maintaining kinetic equilibrium with the SM is the electron scattering diagram shown in fig. 10. The rate for this process is computed in appendix B. The decoupling temperature as a function of \( \epsilon \) (the kinetic mixing parameter for whichever gauge boson is exchanged) is shown in fig. 11 for the case \( M_\chi = 5 \, \text{GeV} \) and \( \mu = 100 \, \text{MeV} \). This can easily be generalized to other DM and gauge boson masses by noticing that the rate scales like \( \alpha_\epsilon\epsilon^2/\mu_4 \) and \( \alpha_\epsilon \) is proportional to \( M_\chi \). Hence the scaling of \( \epsilon \) in fig. 11. For lower values of \( \epsilon \) than shown in the figure, the relation extrapolates to a power law,

\[
\frac{T}{10 \, \text{MeV}} \approx \left( \frac{\epsilon}{1.2 \times 10^{-6}} \right)^{-2/3} \quad (27)
\]

In reality there are two transitions with two different mass splittings that can maintain kinetic equilibrium, since we also have the \( \chi_2 e^+ \leftrightarrow \chi_3 e \) reaction with the small mass splitting \( \delta M_{23} \). We compute the decoupling temperature for both reactions and take the smaller of the two as the true \( T_d \). Roughly speaking, only the larger of the two \( \epsilon \)'s is therefore relevant for kinetic equilibrium. Figure 11 shows that there is a weak dependence upon \( \delta M \) with the large mass gap giving a bigger effect. There is also dependence upon the gauge boson masses.
For the exothermic model, the coupling $\epsilon_1$ is large enough so that $\epsilon_2$ is practically irrelevant for kinetic equilibrium. This is illustrated in figure 12 (left panel), which shows that contours of $\log R_+/R_{\text{obs}}$ hardly change between $\epsilon_2 = 10^{-1}$ and $10^{-10}$. (The example shown is for $\mu_2 = 250$ MeV; for larger $\mu_2$ the dependence is even weaker.) For the endothermic model, $\epsilon_1$ is smaller and so $\epsilon_3$ can have a bigger impact. The right panel of fig. 12 shows that $\delta M_{23}$ must decrease by about 1 keV in the INTEGRAL/DAMA-allowed region to compensate the effect of making $\epsilon_3$ arbitrarily small.

V. POSITRON PRODUCTION RATE AND ANGULAR PROFILE

A. Rate from inelastic collisions

The most recent determination of the observed positron annihilation rate in the bulge is $1.1 \times 10^{13}/s$. This value depends upon the assumed distance between the sun and the galactic center; consistently with [8] we take $r_\odot = 8.5$ kpc [11]. For the predicted rate, we have

$$R_+ = \frac{1}{2} \left( \frac{Y_\odot}{Y_{\text{tot}}} \right)^2 \int d^3x \langle \sigma v \rangle \frac{\mu^2}{M^2}$$

where $\sigma$ is the cross section for $\chi_6 \chi_8 \rightarrow \chi_6 \chi_8$ (recall that $\chi_{s,u}$ are the stable and unstable excited states). The 1/2 is to avoid double-counting, and the abundance factor $Y_\odot/Y_{\text{tot}}$ is given by [28]. We integrate over a region of radius 1.5 kpc, corresponding to an angular diameter of approximately $10^\circ$. The observed profile, fig. 4 suggests that the signal falls below the sensitivity of INTEGRAL near this angle.

The phase space average of $\sigma v$ is given by

$$\langle \sigma v \rangle = \int d^3v_1 d^3v_2 f(v_1) f(v_2) \sigma |\vec{v}_1 - \vec{v}_2|$$

We take a Maxwellian velocity distribution

$$f(v) = N e^{-v^2/\sqrt{2}} \theta(v - v_{\text{esc}})$$

cut off at the escape velocity

$$v_{\text{esc}}^2(r) = 2v_0^2(r) [2.39 + \ln(10 \text{ kpc}/r)]$$

and having velocity dispersion

$$v_0(r)^3 \propto r^x \rho(r)$$

with $\chi = 1.64$, and the normalization such that $v_0(r_\odot)$ is 220-230 km/s. This form of $v_0$ is suggested by $N$-body simulations that include the effects of baryonic contraction [24]. Our choice of $v_{\text{esc}}$ follows ref. [22]; see appendix C of that paper.

In our previous work, the major challenge was to compute $\sigma$ since we were concerned with DM at the TeV scale, implying gauge couplings $\alpha_g$ larger than the average DM velocity $v$. In this case a nonperturbative calculation of $\sigma$ was necessary, since multiple gauge boson exchanges occur when $v < \alpha_g$, similarly to the Sommerfeld enhancement in DM annihilation [26]. However in the present situation $\alpha_g \ll v$ and a perturbative treatment suffices.

We define some kinematic variables to facilitate the presentation of the cross section:

$$v_t^2 = 2 \frac{\delta M_{23}}{M_\chi}, \quad \Delta = \frac{v^2}{v_t^2}$$

where $\delta M_{23}$ is the small splitting between the two excited states, $v$ is the DM velocity in the center of mass frame, and $v_t$ is the threshold velocity for $\chi_2 \chi_2 \rightarrow \chi_3 \chi_3$ excitations. The cross section for excitations can be expressed as

$$\sigma_{t} v_{\text{rel}} = 4\pi \alpha_g^2 \sqrt{\Delta - 1} \frac{v_t M_\chi^2}{D^2} \left( \frac{2}{1 - \eta^2} - \frac{1}{2\eta} \ln \frac{1 + \eta}{1 - \eta} \right)$$

where

$$D = M_\chi^2 v_t^2 (2\Delta - 1) + \mu^2; \quad \eta = 2 \frac{M_\chi^2 v_t^2 \sqrt{\Delta(\Delta - 1)}}{D}$$

Notice that $\sigma_{t} v_{\text{rel}}$ vanishes at threshold, $\Delta = 1$. The related cross section for $\chi_3 \chi_3 \rightarrow \chi_2 \chi_2$ de-excitations is

$$\sigma_{i} v_{\text{rel}} = 4\pi \alpha_g^2 \sqrt{\Delta + 1} \frac{v_t M_\chi^2}{D^2} \left( \frac{2}{1 - \eta^2} - \frac{1}{2\eta} \ln \frac{1 + \eta}{1 - \eta} \right)$$

where

$$D = M_\chi^2 v_t^2 (2\Delta + 1) + \mu^2; \quad \eta = 2 \frac{M_\chi^2 v_t^2 \sqrt{\Delta(\Delta + 1)}}{D}$$

As expected, $\sigma_{i} v_{\text{rel}}$ can be obtained from $\sigma_{t} v_{\text{rel}}$ by changing $\delta M_{23} \rightarrow -\delta M_{23}$, which implies $v_t^2 \rightarrow -v_t^2$ and $\Delta \rightarrow -\Delta$ (notice that $v_t \rightarrow \cdots = \sqrt{v_t^2} \cdots$). In the limit $\Delta \rightarrow 0$, and substituting $\delta M_{23} \rightarrow \delta M_{1s}$ and $\mu_1 \rightarrow \mu_1$, we recover the zero-velocity cross section for downscattering through the large mass gap, [23].
B. Rate from decaying DM

We consider the scenario where the “unstable” state \( \chi_u \) is so long lived that it is already present in the galaxy due to its relic density, and decays with a lifetime \( \tau_u \) greater than the age of the universe. Assuming that the 511 keV gamma rays observed by INTEGRAL come from a central region of radius \( r_c \), the rate of positrons is

\[
R_{e^+} = \frac{4\pi}{M_\chi} \int_0^{r_c} dr \, r^2 \rho_u(r)
\]

\[
\equiv 4\pi \zeta \frac{\rho_\odot \text{kpc}^3}{M_\chi \tau_u} \left( \frac{Y_u}{Y_{\text{tot}}} \right)
\]

(38)

where \( \rho_u \) is the density of \( \chi_u \) and \( Y_u/Y_{\text{tot}} \) is the abundance of \( \chi_u \) relative to the total DM population. We assume the Einasto profile to obtain the dimensionless factor

\[
\zeta = \left( \frac{r_s}{\text{kpc}} \right)^3 e^{(2/\alpha)(r_\odot/r_s)\alpha}
\]

\[
\times \frac{1}{\alpha} \left( \frac{\alpha}{2} \right)^{3/\alpha} \left[ \Gamma \left( \frac{3}{\alpha} \right) - \Gamma \left( \frac{3}{\alpha} - \frac{2}{\alpha} \left( \frac{r_c}{r_s} \right)^\alpha \right) \right]
\]

(39)

Matching \( R_{e^+} \) to the observed rate \( 3.4 \times 10^{12}/s \), we find that the lifetime of \( \chi_u \) relative to the age of the universe (here we define \( \tau_U \equiv 10^{10} \text{y} \) rather than the actual age of the universe)

\[
\frac{\tau_u}{\tau_U} = 2.1 \times 10^5 \left( \frac{\zeta}{30} \right) \left( \frac{3Y_u}{Y_{\text{tot}}} \right) \left( \frac{5 \text{ GeV}}{M_\chi} \right) \left( \frac{\rho_\odot}{\bar{\rho}_\odot} \right)
\]

(40)

where \( \bar{\rho}_\odot = 0.3 \text{ GeV/cm}^3 \). The factor \( \zeta \) is plotted over a wide range of Einasto parameters in fig. 13, showing that it is between 25 and 75 for reasonable profiles. With \( Y_u/Y_{\text{tot}} \sim 1/3 \) this gives \( \tau_u \sim 10^5 \) times \( \tau_U \).

To see how small \( \epsilon_{2,3} \) this corresponds to, we can rescale the bound (40) to be derived below from demanding that similar decays of the “stable” excited state must take longer than \( \tau_U \). It implies that

\[
\epsilon_{2,3}^2 \cong \left( 7 \times 10^{-12} \right)^2 \left( \frac{10^{-4}}{\alpha_g} \right) \left( \frac{\mu_{2,3}}{1 \text{ GeV}} \right)^4 \left( \frac{0.1 \text{ MeV}}{\delta M_{1s-}} \right)^3
\]

\[
\times \left( \frac{50}{\zeta} \right) \left( \frac{Y_{\text{tot}}}{3Y_u} \right) \left( \frac{M_\chi}{5 \text{ GeV}} \right) \left( \frac{\bar{\rho}_\odot}{\rho_\odot} \right)
\]

(41)

where \( \delta M_{1s-} = \delta M_{1s} - 2m_e \) is the energy available for the decay. It is theoretically easy to achieve the desired rate of positron creation just by adjusting \( \epsilon_{2,3} \) to this small value, since there is no other constraint on \( \epsilon_{2,3} \).
C. Angular distribution

In this section we elaborate on the angular profile of the 511 keV signal in the case of scatterings only, since only there is it definitely necessary to consider the effects of positron propagation. The intensity of the signal as a function of angle is found by computing the line-of-sight integral \[ \int d\Omega f(\hat{x} \cdot \hat{x}') I_{e^+}(\hat{x}) \]
where the line is oriented along the direction \( \hat{x} \) specified by angles \( \theta, \phi \) relative to the galactic center. This expression assumes that the positrons decay at the same position where they were created. To model the effects of propagation before decay, we smear the angular distribution by averaging \( \hat{x} \) weighted by some function \( f(\cos \theta) \),

\[
\bar{I}_{e^+}(\hat{x}') = \int d\Omega f(\hat{x} \cdot \hat{x}') I_{e^+}(\hat{x})
\]

The integral over solid angle can be combined with the integral over the line of sight and rewritten in terms of a volume integral, \( d\Omega d\Omega = d^3x / x^2 \), with the origin of coordinates at the earth. Now, given that \( \rho^2 \) is strongly peaked near the galactic center, we can write \( \rho^2 \sim \delta(3)(x - x_0) \), where \( x_0 \) is the position of the galactic center. Then we find that

\[
\bar{I}_{e^+}(\theta) \sim f(\cos \theta)
\]

The intensity has the same shape as the smoothing function. As argued in section IIIA this is a good approximation for the DM halo profiles that we are considering for the inelastic scattering mechanism.

It is interesting to notice that even though \( \rho^2 \) looks like a delta function with respect to the measure \( d^3x / x^2 \), not so for the usual volume measure \( d^3x \). Indeed, the function \( r^2 \rho^2(r) \) has a maximum near \( r = \sqrt{2} r_0 \) even in the limit \( \alpha \to 0 \). Therefore the total rate of positron production in the galaxy gets significant contributions away from the galactic center, although these are not counted in the observations of the bulge component upon which we focus in this paper, since only near the center the intensity high enough to be detected.

D. Regions of positron annihilation

In the above discussion we have assumed that positrons are able to migrate to the regions where positronium forms and where they can subsequently annihilate. It is known from fitting the observed \( \gamma \) ray spectrum that \( \sim 97\% \) of the positrons indeed form positronium before annihilating \[ [34, 44] \]. This is because orthopositronium decays to three photons, and comparison of the 511 keV line flux with the continuum level is consistent with nearly all annihilations coming from positronium rather than positrons encountering free electrons. The spectral shape also shows that most annihilations take place in warm (\( \sim 8000 \) K) \[ 45 \] regions, which may be mostly ionized \[ 45 \] or else a combination of neutral and ionized regions \[ 44 \].

Efforts have been made to independently map out the positions of the warm regions in the galactic bulge (GB); doing so could provide a consistency check on the above determinations, since then the morphology of the INTEGRAL detection of the galactic bulge 511 keV gamma rays should match the position of the warm regions. Ref. \[ 46 \] has modeled the spatial distribution of molecular gas in the GB based on CO emission data \[ 47 \] for the central molecular region (CMZ) in the inner 150 pc, and borrowing an older model \[ 48 \] based on HI observations for the “holed GB disk” region extending to radii of \( \sim 1 \) kpc. It has been suggested that the warm neutral or ionized regions relevant for positronium ionization coincide with these molecular gas clouds \[ 49 \]. In figure 14 we have transposed an outline of the CMZ and holed disk regions (fig. 4 of ref. \[ 46 \]) on the most recent INTEGRAL 511 keV intensity map \[ 6 \].

From fig. 14 it is clear that the INTEGRAL/SPI instrument does not have sufficient spatial resolution to test whether positron annihilation really comes from the molecular gas regions. Furthermore, the assumption that these regions coincide with the warm ionized or neutral + ionized regions of positronium annihilation is questionable. The direct measurements of the ionized component are based upon pulsar observations \[ 50 \], which suggest the existence of warm \( H^+ \) regions with similar morphology to the molecular gas. But this is not considered to be a very reliable measurement of the ionized gas den-
sity in the GB due to the scarcity of pulsars in this region \[51\]. Thus we do not know with a high level of confidence where the warm regions of positronium annihilation are really located. It is possible that they extend beyond the molecular gas regions identified by \[40\].

Because of the lack of very reliable information as to the spatial distribution of the warm ionized (+ neutral) regions, an uncertainty that is acknowledged in careful studies such as \[46\], it is possible that decays of DM lead to positronium production in the vicinity of the initially produced positrons, so that the INTEGRAL signal could be a reflection of the underlying DM distribution. On the other hand if the positrons result from DM scattering, it is possible that they extend beyond the regions, an uncertainty that is acknowledged in careful studies such as \[46\].

In figure 15 (left panel) we plot contours of \(\delta M = 4\) GeV, \(\delta M2 = 4.5\) keV, where the INTEGRAL and DAMA rates best fit simultaneously, \(\bar{\epsilon}_1 \sim 10^{-3.27}\), significantly larger than the generic estimate \(10^{-5}\) given in ref. \[18\]. This is due to the \(A/Z\) correction, the fact that we need \(\mu_1\) to be heavier than the nominal 100 MeV value assumed in \[18\], and that \(Y_s/Y_{\text{tot}}\) can be significantly less than 1 in our model.

### VI. DIRECT DETECTION RATES

In our computation of the 511 keV rate, we fixed the value of gauge kinetic mixing parameter \(\epsilon_1\) so as to match the direct detection rates determined respectively by references \[18\] and \[24\] for the exothermic and endothermic DM models. Although \(\epsilon_1\) does not directly affect the rate of \(\chi_2\chi_3 \leftrightarrow \chi_3\chi_3\) transitions, it does so indirectly, because of its influence (through kinetic decoupling) on the relic density of the stable excited state. Here we give details on the determination of \(\epsilon_1\) in these two cases. We note briefly that the nuclear form factor is trivial for collisions studied here, so we will ignore it.

#### A. Exothermic dark matter

Ref. \[18\] determined the elastic limit of the DM-nucleon cross section needed to get the right rate of DAMA transitions:

\[
\sigma_{n,\text{el}} = \frac{\mu_n^2}{4\pi A^4},
\]

where \(\mu_n = m_n M_\chi/(m_n + M_\chi)\) is the reduced mass and \(A = 340\) GeV.\(^9\) In our model, the coupling is to protons only, and the analogous quantity is given by

\[
\sigma_{p,\text{el}} = 16\pi\epsilon_1^2\alpha\alpha_g \frac{\mu_n^2}{\mu_1^4}.
\]

To determine the value of \(\epsilon_1\) needed to match the observed rates, we must account for the coupling to protons only since the rate is proportional to \(A(Z f_p + (A - Z) f_n)^2\) for atomic number and mass \(Z, A\) and relative strengths of couplings to protons and neutrons \(f_p, f_n\). In ref. \[18\], the couplings were assumed to be \(f_p = f_n = 1\), but we have \(f_p = 1, f_n = 0\). Moreover, we have a different local density of the excited state than that assumed by \[18\] because of the abundance factor \(Y_s/3Y_1\) (which also appears in the positron rate \[28\]), and because we allow the local DM density to vary with respect to the fiducial value \(\rho_0 = 0.3\) GeV/cm\(^3\). The result is

\[
\epsilon_1 = \frac{\mu_1^2 A}{8\pi A^2 Z} \left[ \frac{1}{\alpha\alpha_g} \frac{Y_{\text{tot}}}{Y_s} \frac{\rho_0}{\rho_\odot} \right]^{1/2}
\]

If \(\epsilon_{2,3} \ll \epsilon_1\) so that \(\epsilon_1\) determines the kinetic decoupling temperature of the DM, then \(Y_s\) depends implicitly on \(\epsilon_1\) and \[40\] must be solved numerically. The factor with \(A/Z\) depends upon which nucleus we are talking about, and is given by 2.4 and 2.28 respectively for I and Na. As \[18\] notes, scattering from Na nuclei is preferentially detected in our region of parameter space, so we choose the latter number.

#### B. Endothermic dark matter

Ref. \[24\] finds that DM with a mass of approximately 7 GeV and cross section on nucleons

\[
\sigma_n = 2 \times 10^{-4} \text{ pb}
\]

\(^9\) Note that taking this elastic limit negates the need to average over DM speeds as in \[19\].
can simultaneously fit the DAMA/LIBRA annual modulation and the CoGeNT low-recoil events. Their allowed regions of $\sigma_n$ versus $M_\chi$ are reproduced in figure 8. The logic for matching our cross section to theirs is similar to the exothermic case, except for the fact that endothermic scatterings are kinematically blocked if the DM velocity is below the threshold value

$$v_t = \sqrt{2v_{23}/\mu_N}$$

where $\mu_N = m_N M_\chi/(m_N + M_\chi)$ is the nucleus-DM reduced mass. We take this into account by doing the phase space average of $\sigma v$. The phase space factor in $\sigma v$ that is sensitive to the threshold is $\sqrt{v^2 - v_t^2}$. For elastic scattering, this factor would be $v$. Therefore we match the quantity $\delta M$ that also appears in our slightly inelastic cross section to 

$$\sigma_{p,el} = \frac{\langle v \rangle}{\langle v^2 - v_t^2 \rangle} \left( \frac{A}{Z} \right)^2 \frac{\rho_0}{\rho} \frac{3Y_1}{Y_s} \sigma_n,$$

where the averages over velocity are performed with the Maxwellian distribution function $f = N e^{-v^2/v_0^2}$ cut off at the escape velocity $v_{esc}$. Since we are comparing with ref. [24], we use their values $v_0 = 230$ km/s and $v_{esc} = 600$ km/s for this part. Once again, scatterings from Na are preferentially detected, so $A/Z = 2.28$. Similarly, we take the threshold velocity for sodium in the above.

Figure 10 (right panel) shows contours of $\epsilon_1$ for the case of $\mu_2 = 500$ MeV. In the overlap region for INTEGRAL and DAMA, $\epsilon_1 \approx 10^{-5}$. This is smaller than required in the exothermic model because the corresponding value of $\mu_1$ is smaller, and also the cross section [17] is approximately 0.15 that in [14]. Kinetic equilibrium of the DM with the SM in the early universe is not as efficiently maintained by $\chi_2 \leftrightarrow \chi_3$ transitions in this case. This is why the relic density of $\chi_2$ is sensitive to the value of $\epsilon_2$ the other nonvanishing kinetic mixing parameter for the endothermic model, whereas it is practically insensitive in the exothermic case.

**VII. ASTROPHYSICAL CONSTRAINTS**

In this section we address the astrophysical and cosmological constraints on our proposal that are complementary to the 511 keV and direct detection signals, as well as to laboratory constraints from electron beams.

**A. Lifetime of metastable state**

We need to insure that the “stable” excited state $\chi_s$ is either truly stable or else sufficiently long-lived. The most dangerous process is $\chi_s \rightarrow \chi_1 \pm \pm e^-$. At the phenomenological level, we suppress this by setting the kinetic mixing of the gauge boson that mediates this process to a sufficiently small value. The rate for this decay can be computed analytically with the approximation that $\delta M_{13-} = \delta M_{13} - 2m_e$ is sufficiently small for the final state particles to be nonrelativistic. Then

$$\Gamma_{\chi_s} \approx 2\alpha g \alpha^2 m_\chi^2 \delta M_{13-}^2\mu^{-4}$$

Demanding that $\tau_s$ exceed $10^{10}$ yr requires that

$$\epsilon < 2 \times 10^{-9} \left( \frac{10^{-4}}{\alpha_g} \right)^{1/2} \left( \frac{\mu}{1\text{ GeV}} \right)^2 \left( \frac{0.1 \text{ MeV}}{\delta M_{13-}} \right)^{3/2}$$

In the endothermic model, we can set $\epsilon_3 = 0$ at tree level by removing the $\Delta_3$ Higgs boson, but there seems to be no symmetry to ensure that $\epsilon_3$ is not generated by loops if the other two mixing parameters are nonzero.
Nonetheless, we are not able to find an example of a loop diagram that generates nonzero $\epsilon_3$; any that superficially seem promising vanish because of Furry’s theorem. Instead, we find a one-loop process where $B_3$ acquires a magnetic moment coupling to the electron, $\mu_B B^\mu \bar{\sigma}_{\mu\nu} e$. The decay of $\chi_2$ proceeds by $B_3$ exchange in the one-loop diagrams of [14]. These diagrams would cancel exactly if $\mu_2 = \mu_1$, so the magnetic moment can be estimated as

$$\mu_B \sim g_\alpha \epsilon_1 \epsilon_2 \frac{(\mu_2 - \mu_1)}{4\pi \bar{\mu}^2} \ln \frac{\Lambda}{\bar{\mu}}$$

(52)

where $\bar{\mu} = \frac{1}{2}(\mu_1 + \mu_2)$ and $\Lambda$ denotes the hidden SU(2) symmetry breaking scale, above which the kinetic mixing of $B_{1,2}$ is replaced by an interaction with the triplet Higgs fields. The squared matrix element of fig. [14] can be estimated as $|\mathcal{M}|^2 \sim g^2 \mu_2^2 M_1^2 m_c^2 \delta M_{12} \delta \mu_3^4$, and the decay rate in the limit of small $\delta M_{12} = \delta M_{12} - 2m_c$ is

$$\Gamma_{\chi_2 \rightarrow \chi_1 e^+ e^-} \sim \frac{\alpha g}{32\pi} \mu_B^2 m_c^3 \delta M_{12}^2 \delta M_{23}^2 - \frac{\mu_3^4}{3}$$

(53)

For $\alpha g \sim 10^{-4}$ and $\epsilon_1 \sim \epsilon_2 \sim 10^{-3}$, $\delta \mu \sim \mu_3 \sim \bar{\mu} \sim 1$ GeV, $\delta M_{12} \sim 0.1$ MeV, $\Lambda \sim 10$ GeV, we find a lifetime of $10^{26}$ s, much larger than the age of the universe. Therefore it seems technically natural to neglect the dangerous kinetic mixing term and assume the “stable” state is sufficiently long-lived. As it turns out, a careful calculation is even more suppressed; see appendix [C].

![FIG. 16: Decay of metastable $\chi_2$ state due to nonzero $\epsilon_{1,2}$.](image)

The $\chi_3 \rightarrow \chi_2 X$ decay in the exothermic model is not problematic, since the mass splitting is much smaller and the only available decay channels are with $X = \gamma$, $X = \nu\bar{\nu}$, and $X = 3\gamma$. These have been studied previously [13, 22, 35]. The single-photon decay has a lifetime longer than the age of the universe for the value of $\epsilon_1$ required for direct detection; since it could be observed, we discuss it in more detail in section VII B below.

The partial width for the $\nu\bar{\nu}$ final state is easy to estimate in analogy with [56]. For this channel, there is an additional suppression in the kinetic mixing. First, the mixing of $B_3$ with the $Z$ boson current has an extra factor of $\mu_3^2/\mu_2^2$ [23], and, second, the SM $Z$ boson mixes with the $B_1$ current with opposite sign such that the two mixings nearly cancel at small energy-momentum transfer (see appendix [B] for discussion of the same cancellation in $\chi\nu$ scattering). The ensuing bound on $\epsilon_1$ is much weaker than that on $\epsilon_2$; practically speaking there is no constraint.

The decay $\chi_3 \rightarrow \chi_2 + 3\gamma$ is due to the operator $\sim (\alpha g^2/90 m_2^2) B_1^{\mu\nu} F_{\mu
u} F^2$ induced by an electron loop, similar to the Euler-Heisenberg $F^4$ interaction in QED. (Furry’s theorem forbids a term of the form $B_1 F^2$ and $B_1 F$ mixing is already taken into account by diagonalizing the kinetic terms.) The rate is suppressed by $\delta M_{23}^4$ [13, 22], leading to lifetimes that far exceed the age of the universe for the small $\sim 5$ keV splittings relevant to our exothermic model.

### B. Single-photon decays of metastable state

The exothermic version of our proposal faces the challenge that the excited state can decay by emission of a single photon, via $\chi_3 \rightarrow \chi_2 \gamma$. The origin of this decay was pointed out in [25]: the nonabelian term in the field strength $B_1^{\mu\nu}$ leads to interactions of the form $\epsilon_1 B_2^{\mu\nu} B_3 F_{\mu\nu}$ with the photon, from the gauge kinetic mixing operators [11]. This can be put into a loop diagram which results in a transition magnetic moment $\delta \mu_{\chi_2}$,

$$\mu_{\chi_2} \approx \frac{\epsilon_1 g^2}{128\pi^2 M_\chi} \left( \ln \frac{M_\chi}{\mu} - 1 \right)$$

(54)

where $\mu$ is of order $\mu_2, \mu_3$. Therefore, there is a decay channel $\chi_3 \rightarrow \chi_2\gamma$. The rate is

$$\Gamma_{\gamma} = \frac{\mu_{\chi_2}^2}{8\pi} (\delta M_{23})^2$$

(55)

For $\epsilon_1 \approx 10^{-5}$, $M_\chi \approx 4.5$ GeV and $\delta M_{23} \approx 5$ keV, the lifetime is $4 \times 10^{19}$ s which is much longer than the age of the universe. However, this is not necessarily enough because such photons could be visible in astronomical searches.

Of the various instruments that could be sensitive to low-energy $\gamma$ rays, INTEGRAL/SPI comes the closest. Ref. [52] gives limits on the intensity of gamma ray lines that could come from such decays in the galaxy; however INTEGRAL’s sensitivity cuts out below 20 keV, making our scenario just out of reach. Interestingly limits on the diffuse gamma ray background put a $\delta M_{23}$-dependent lower limit on the partial lifetime times the mass $\chi_3$ of approximately

$$\tau_{\gamma, M_\chi} > 1 \times 10^{20} \left( \frac{3 M_\chi}{Y_{\text{top}}} \right) \left( \frac{\delta M_{23}}{10 \text{ keV}} \right)^{1.2} \text{ GeV s}$$

(56)

for $\delta M_{23} > 10$ keV. Again because of INTEGRAL’s energy sensitivity, data is not given for lower photon energies. Nevertheless, extrapolating the bound to $\delta M_{23} = 5$ keV gives $4.3 \times 10^{19}$ GeV s, which is not even five times less than our nominal value $1.8 \times 10^{20}$ GeV s, assuming $M_\chi = 4.5$ GeV. Therefore an instrument sensitive to these lower energies might detect this low-energy photon, which is in the x-ray part of the spectrum.

In fact, observations of the galactic center by the Chandra x-ray telescope [54] may rule out this particular...
model. Observations are presented for a region of size 35 arcmin$^2$ that is 7.5 arcmin away from the GC. No evidence of an unidentified line is observed in the 1–8 keV band (fig. 17), and the continuum seen there is modeled by thermal sources with a flux of $6 \times 10^{-12}$ erg cm$^{-2}$ s$^{-1}$. We can compute the expected flux by integrating over the line of sight and the solid angle ($d\Omega = d\phi d\cos \psi$) of the observed region:

$$F_{\text{th}} = \frac{Y_3}{4\pi M_\chi r_\gamma} \int d\Omega \int dl \rho(l - r_\odot) = \frac{\rho^2 r_s}{2 M_\chi r_\gamma Y_{\text{tot}}} I$$  \hspace{1cm} (57)

where $I = e^{(2/\alpha)y^\alpha} \int d\cos \psi \int dl e^{-(2/\alpha)(y^2 + l^2 - 2yl \cos \psi)^{\alpha/2}}$, $y = r_\odot/r_s$ and $l = l/r_s$. We numerically integrate over an annular region of similar area and displacement from the GC to the observed one, using the Einasto parameters (17) to find $I \approx 10^{-4}$. Using the value of $M_\chi r_\gamma$ determined above, this gives $F_{\text{th}} \approx 0.001$ photons cm$^{-2}$ s$^{-1}$ (for the maximal ratio $Y_3/Y_{\text{tot}}$). The corresponding energy flux for a 5 keV mass difference is $10^{-11}$ erg cm$^{-2}$ s$^{-1}$, not quite 2 times greater than the observed continuum flux.

In the above estimate we did not take account of absorption of the decay signal, which would help to soften the discrepancy, especially if the photon energy is somewhat lower. It may also be possible to evade the problem by extending the gauge group to SU(2)$\times$U(1) and replace the kinetic mixing of $B_1$ by that of the extra U(1) gauge boson; this would remove the $\mu_3$ transition magnetic moment. Notice that this problem does not affect the endothermic model because $\chi_3 \to \chi_1 e^+ e^-$ proceeds much more quickly, as we discuss in section VII C below.

### C. Lifetime of unstable state

In passing, we can also estimate the decay rate for the unstable excited state into $e^+ e^-$ using (50). It has the same form, except for the substitutions of $\epsilon$ and $\mu$ by the corresponding quantities for $B_3$, in the exothermic model; for the endothermic model, (50) applies as written, to the unstable excited state. Laboratory experiments constrain the appropriate $\epsilon_i$ to be $\lesssim 10^{-3}$, so the lifetime could be $10^{10}$ times shorter than the above estimate using $\epsilon \sim 10^{-8}$, thus on order of 1 y. This assumes the large mass splitting is only 1.1 MeV. With a 2.1 MeV splitting one gains a factor of $10^3$ in the rate due to the larger phase space, giving a lifetime of several hundred seconds. It cannot be significantly smaller in our model.

### D. Single-photon decays of unstable state

The decay mechanism discussed in section VII B was originally conceived for the decay of the unstable state in ref. [35]. This goes through the MeV-scale mass gap, so the photon in this case is a gamma ray. For definiteness let us consider the endothermic model, so $\chi_3$ is the unstable state and the relevant decay is $\chi_3 \to \chi_1 \gamma$ via the $\mu_3$ transition magnetic moment, which is proportional to $\epsilon_2$, in analogy to (54). The partial decay rate is the obvious generalization of (55). The branching ratio for the single photon decay relative to that into $e^+ e^-$ is

$$\text{BR}_\gamma = \frac{\alpha^2}{8192\pi^2} \frac{\mu^4 (\delta M_{13})^3}{M_\gamma^2 (\delta M_{13-})^2 (\delta M_{13+})^2} \ln^2 \frac{M_\chi}{e \mu}$$  \hspace{1cm} (58)

where $\mu \sim \mu_1, \mu_3$, $\delta M_{13\pm} = \delta M_{13} \pm 2m_e$ and $e = 2.71828\ldots$. The resulting photon might be observed by INTEGRAL in the diffuse gamma-ray background. In ref. [35], a bound was derived, which however overestimated the sensitivity of INTEGRAL to the signal. We therefore reconsider it here.

The analysis of ref. [52] is particularly relevant for us, since they searched for line sources from the galactic center region, having a spatial distribution similar to that of the 511 keV line. They limit the flux of such a line, for energies between 1 and 2 MeV, to less than $3 \times 10^{-5}$ cm$^{-2}$ s$^{-1}$. This is to be compared to the flux from positrons, $\sim 3.6 \times 10^{-3}$ cm$^{-2}$ s$^{-1}$. Therefore $\text{BR}_\gamma$ should not be greater than about 0.01. However, using the typical values of parameters of interest for our present application, we find $\text{BR}_\gamma \sim 10^{-7}$, far below the sensitivity of current searches.

### E. Cosmic ray and CMB constraints

DM annihilations can occur even after freezeout, with the production of gamma rays or charged particles that can have an observable effect. In the class of models we consider, the DM annihilates directly into hidden sector gauge bosons, which in turn decay into any charged SM particles that are sufficiently light. Gamma rays emerge only as secondary products of these charged particles.
Their contribution to the diffuse gamma ray background can potentially give interesting constraints \cite{56}, but currently the uncertainties from details of structure formation do not allow one to draw firm conclusions. The production of antiprotons in the galaxy gives more definite constraints, which can be quite stringent \cite{57}. To avoid them, we need to assume that the gauge bosons which mix with the photon are lighter than 2\(m_p\), so that \(pp\) pairs are not produced.

Charged particles that are produced around the time of recombination reionize the plasma and change the optical depth to the surface of last scattering, a quantity that affects the Doppler peaks of the cosmic microwave background \cite{58,59}. The effect is particularly strong for DM with mass \(M_\chi \lesssim 10\) GeV, as in the exothermic proposal for DAMA. Ref. \cite{58} shows that such DM is marginally ruled out if it decays exclusively into \(e^+e^-\), while it is marginally allowed if it decays into heavier charged particles (which decay into electrons that are less energetic than if they were primary products). The relevant bounds are reproduced in fig. 18. In most of our examples, the gauge bosons can decay into muons and charged pions, so the branching ratio into electrons will be suppressed and the bound should be somewhere between the two cases shown in fig. 18. Ref. \cite{58} also derives a bound from excess heating of the interstellar gas, which is more stringent than the CMB bound in this small \(M_\chi\) region, but which is also more subject to uncertainty because of its dependence upon assumptions about the details of structure formation.

**F. Long-lived gauge bosons and nucleosynthesis**

It is interesting to consider possible effects of the hidden sector gauge bosons in the early universe. Decays around the time of big bang nucleosynthesis or later can be deleterious, although it is also possible to improve the predictions of big bang nucleosynthesis (BBN), notably for lithium \cite{60}. In our models, \(B_1\) is usually the lightest gauge boson, and it couples to electrons (and muons and pions) with \(\epsilon_1 \sim 10^{-5}\), fixed by the rate of direct DM detection. Its decay rate is therefore of order \(\alpha\epsilon_1^2\mu_1\), which for \(\mu_1 \sim 500\) MeV leads to a lifetime of \(10^{-10}\) s, which is clearly harmless.

On the other hand, if \(\epsilon_2 \lesssim 10^{-10}\) in the exothermic model, where we have the constraint \(\epsilon_2 \lesssim 10^{-8}\), then \(B_2\) can have a lifetime greater than 1 s and possibly be relevant for nucleosynthesis. The question is whether its relic density is large enough to have an effect. We have computed the cross section for \(B_2B_2 \rightarrow B_1B_1\) using FeynCalc \cite{61}. The cross section as \(v \rightarrow 0\) can be expressed as

\[
\sigma v = \frac{\pi\alpha_3^2}{2\mu_3^2} f(x)
\]

where \(x = 1 - \mu_2/\mu_3 = 2\delta M_{23}/\alpha_3\mu_3\) and \(f\) has a minimum value of 18 at \(x = 0\) (treating (4) as an equality to eliminate \(\mu_1\)). This neglects dark Higgs exchange in the s-channel, but we have checked that including it makes no dramatic change unless the virtual Higgs goes on shell. For typical values we find that the standard relic abundance calculation gives a freezeout temperature around 8 MeV for \(B_2\), and an abundance \(10^{-2}\) times smaller than that of the baryon asymmetry. This is too small to have any effect on BBN.

Ref. \cite{60} point out that a more likely candidate for giving interesting effects is the dark Higgs bosons. In particular, if there exist a Higgs boson that is lighter than the gauge bosons, it would decay into 4 leptons through emission of two virtual gauge bosons, with a rate suppressed by \(\sim \alpha_3^2m_h/\mu^3\). The annihilation cross section is suppressed for similar reasons. This can more naturally give long-lived relics (on the time scale of BBN) that could solve the lithium problem.

**VIII. LABORATORY SEARCHES**

**A. Beam dump experiments**

An interesting feature of the class of models we consider is that they can be tested in proposed low-energy laboratory experiments. A beam dump on an absorbing target can produce the weakly interacting \(B\) bosons that mix with the photon. These can decay into \(e^+e^-\) or other charged particles before reaching the detector, providing a signal not present in the standard model.

In our scenario, two of the three colors of bosons should mix with the photon: \(B_1\), with strength \(\epsilon_1 \sim 10^{-3} - 10^{-5}\) to get the right rate of direct detection, and either \(B_3\) or \(B_2\), depending upon whether \(\chi_2\) or \(\chi_3\) is the stable excited state. Let us denote the corresponding mixing parameter by \(\epsilon_{3,2}\). We noted in section IV.C (figure 12) that this parameter is essentially unconstrained. If \(\epsilon_{3,2} \gtrsim 10^{-6}\) then the effects of \(B_{3,2}\) could be discovered.
in laboratory searches. But since we have more definite predictions for $B_1$, we will focus here on its discovery potential. Moreover we have argued that there are certain advantages to having very small values of $\epsilon$ which could make laboratory detection of $B_{3,2}$ impossible for the present.

The authors of ref. \cite{62} has recently summarized the current experimental constraints in the $\epsilon_i - \mu_i$ plane (where $\mu_i$ is the mass of the relevant gauge boson), and they have also proposed strategies for new experiments that can cover more of the still-allowed region in this plane. Fig. 19 reproduces some of their results. On top of these we plot several examples of predictions from our endothermic and exothermic models (circles containing “n” or “x” respectively), corresponding to those shown in figures 6-8 (see also fig. 15). Almost all of these points are contained within the contours denoting the reach of feasible new experiments suggested by ref. \cite{62}. The solid (blue) line denoting the high resolution, high rate trident spectrometer, and the dashed (red) one for the thin-target with double arm spectrometer. It is suggested that such experiments would be feasible at several existing laboratories, including Jlab (Thomas Jefferson National Accelerator Facility), SLAC (Stanford Linear Accelerator Center), ELSA (Electron Stretcher and Accelerator), and MAMI (Mainzer Mikrotron). We see that only one of our examples (the right-most “x”) would lie outside of the reach of the proposed experiments. This corresponds to the extreme case where $\mu_2 = 2 \text{ GeV}$ in fig. 9. The more typical models would therefore be in the discoverable region.

### IX. CONCLUSIONS

If the anomalous 511 keV gamma rays from the galactic center are truly distributed in an axisymmetric manner, as suggested by the INTEGRAL observations, this provides strong motivation to seriously consider DM decays or scatterings as their source, rather than localized sources such as supernovae or x-ray binaries. A new measurement with higher spatial resolution would be very desirable to help settle this question. In the meantime, it seems worthwhile to explore possible DM interpretations, especially if they can explain more than just the 511 keV signal. In the present work we have shown how a three-component DM model with two mass splittings and a hidden SU(2) gauge boson might address both the 511 keV signal. In the present work we have shown how a three-component DM model with two mass splittings and a hidden SU(2) gauge boson might address both the 511 keV signal. In the present work we have shown how a three-component DM model with two mass splittings and a hidden SU(2) gauge boson might address both the 511 keV signal. In the present work we have shown how a three-component DM model with two mass splittings and a hidden SU(2) gauge boson might address both the 511 keV signal. In the present work we have shown how a three-component DM model with two mass splittings and a hidden SU(2) gauge boson might address both the 511 keV signal.

In the approximation $m_Z$ is much greater than the masses of the decay products. Demanding that this be less than the experimental error on the invisible $Z$ width, 1.5 MeV \cite{63}, we find that $\Lambda > 1.3 \text{ TeV}$. For $\langle \Delta \rangle \sim 10 \text{ GeV}$, this leads to the bound $\epsilon \lesssim 10^{-2}$, which is less stringent than other constraints shown in fig. 19. For the $Z \rightarrow B_j B_k$ channel, the partial width is of order $\alpha g^2 m_Z$. This leads to a weaker bound on $\epsilon$ than does the $Z \rightarrow \Delta_i B_i$ channel.
though still consistent with examples from N-body simulations that take into account compression by baryons.

We have highlighted two distinct mechanisms for getting the 511 keV signal: either inelastic $\chi_{2,3}\chi_{2,3} \rightarrow \chi_{1}\chi_{1}e^+e^-$ decays, or the decay process by itself when $\chi_{3,2}$ has a lifetime of order $10^3$ times the age of the universe. Whereas the first mechanism requires some mutual adjustments of the particle physics and DM halo parameters to get the right rate, the second is more easily arranged by just tuning the gauge kinetic mixing parameter $\epsilon_{2,3} \sim 10^{-11}$ that controls the decay rate. The decay mechanism points to the exciting possibility that the angular profile of the 511 keV signal is actually a picture of the DM halo profile in the inner part of the galaxy, if positron diffusion is a negligible effect. The scattering mechanism on the other hand requires significant positron diffusion, or else propagation of the excited DM state before decay, since otherwise it predicts too narrow angular profile. It is interesting that our model can naturally explain such long-distance travel of the excited DM prior to its decay, by tuning $\epsilon_{2,3} \sim 10^{-7}$.

A very encouraging aspect of these proposals is their testability in low-energy electron beam dump experiments. The kinetic mixing parameter $\epsilon_1 \sim 10^{-4}$ and the mass $\mu_1 \lesssim 1$ GeV of the gauge boson mediating the direct detection scatterings are in prime territory for their discovery by such experiments, which could be done at existing laboratories. The models presented here are also potentially rich in consequences for cosmic rays, the diffuse x-ray or gamma-ray backgrounds, the CMB, and big bang nucleosynthesis.

Acknowledgment. We thank Celine Boehm, Laurent Bouchet, Marco Cirelli, Neal Dalal, Malcolm Fairbairn, Jonathan Feng, Katia Ferriere, Ben Grinstein, Pierre Jean, Manoj Kaplinghat, Vicky Kaspi, Jonathan McDowell, Guy Moore, Nicolas Prodeut, Gerald Skinner, and Tracy Slatyer for helpful correspondence or discussions. Our work is supported by the Natural Sciences and Engineering Research Council (NSERC) of Canada. JC thanks the CERN theory group and the Perimeter Institute for their hospitality during part of this work.

Appendix A: Annihilation amplitudes and rates

In this appendix, we derive the invariant amplitudes (squared) for annihilation of DM particles $\chi$ to both gauge and Higgs bosons, including the lowest order corrections due to dark matter velocity, which we use to find the relation between the thermal relic density and the dark gauge coupling $\alpha_3$ in section 4.1. Including annihilation to Higgs bosons extends and corrects the results listed in [35]; in addition, we correct the final state polarization and color sums carried out in [35]. To keep the final result simple, we will first assume that symmetry breaking occurs at a lower temperature than DM freeze-out, so the gauge and Higgs bosons may be treated as massless. The effects of symmetry breaking are discussed at the end. For reference, we will consider a general gauge group and general representations for both DM and the Higgs.

We consider first DM annihilation to Higgs bosons. This process is mediated by t- and u-channel diagrams involving Yukawa couplings at each vertex, s-channel diagrams with an intermediate Higgs particle connecting a Yukawa coupling at one vertex to a scalar potential vertex, and a s-channel diagram with an intermediate gauge boson coupling to the DM and Higgs particles at either end. Since we are concerned in this paper with either parametrically small or vanishing Yukawa couplings, we assume that the s-channel diagram with an intermediate gauge boson dominates.

Consider incoming DM states $\chi$ in representation $R$ and outgoing Higgs states $\Delta_i$ in representation $R'$; the incoming momenta are $p_i$ and outgoing momenta $q_i$. The matrix element is

$$|\mathcal{M}|^2 = \frac{1}{d_R} \frac{g^4}{d_{adj} d_{adj}} \text{tr} \left[ (\eta_i + M_i \chi) (\eta_j - M_j \chi) (\eta_j - M_j \chi) (\eta_j - M_j \chi) \right] \times T_{1i} T_{2j} T_{3j} T_{4j} ,$$

where $d_R$ is the dimension of representation $R$. The color sums both take the form

$$\text{tr}_R T^a T^a = (d_R/d_{adj}) C_2(R) \delta^{ab} ,$$

where $C_2$ is the quadratic Casimir; the sum over the adjoint indices gives $\delta^{ab} \delta^{cd} = d_{adj}$. For nonrelativistic dark matter at center-of-mass velocity $v = v_{rel}/2$ and scattering angle $\theta$, the amplitude becomes

$$|\mathcal{M}|^2 = \left( \frac{d_R}{d_{adj} d_{adj}} C_2(R) C_2(R') \frac{g^4}{2} (1 - v^2 \cos^2 \theta) \right) ,$$

Amnihilations to gauge bosons (of colors $a, b$ and momenta $p_{a,b}$) receives contributions from s-, t-, and u-channels. The amplitudes for each channel are (as in [35])

$$|\mathcal{M}_s|^2 = \frac{g^2}{s} \frac{1}{d_R} \frac{g^4}{d_{adj} d_{adj}} \text{tr} \left[ (\eta_i + M_i \chi) (\eta_j - M_j \chi) (\eta_j - M_j \chi) (\eta_j - M_j \chi) \right] \times T_{1i} T_{2j} T_{3j} T_{4j} ,$$

$$|\mathcal{M}_t|^2 = \frac{g^2}{t - M_i^2} \frac{1}{d_R} \frac{g^4}{d_{adj} d_{adj}} \text{tr} \left[ (\eta_i + M_i \chi) (\eta_j - M_j \chi) (\eta_j - M_j \chi) (\eta_j - M_j \chi) \right] \times T_{1i} T_{2j} T_{3j} T_{4j} ,$$

$$|\mathcal{M}_u|^2 = \frac{g^2}{u - M_i^2} \frac{1}{d_R} \frac{g^4}{d_{adj} d_{adj}} \text{tr} \left[ (\eta_i + M_i \chi) (\eta_j - M_j \chi) (\eta_j - M_j \chi) (\eta_j - M_j \chi) \right] \times T_{1i} T_{2j} T_{3j} T_{4j} .$$

10 We use a mostly plus metric with Dirac algebra $\{\gamma^\mu, \gamma^\nu\} = -2\gamma^{\mu\nu}$. 


We need to account for both direct squares and cross terms in the amplitudes. After some tedious algebra including Dirac traces, we find the following results for the (color and spin summed and averaged) square amplitude:

\[
|\mathcal{M}_s|^2 = \frac{g^4}{d_R} \text{tr}_R T^c T^d f^{abc} f^{abcd} \left( -\frac{19}{4} + 1 + \frac{1}{8} v^2 (11 - 5 \cos 2\theta) \right),
\]

\[
|\mathcal{M}_{t,u}|^2 = 2 \frac{g^4}{d_R} \text{tr}_R (T^a T^b T^c T^d) (1 + v \cos \theta + v^2),
\]

\[
\mathcal{M}_s \mathcal{M}_t = i g^4 \frac{d_R}{d_R} \text{tr}_R (T^{c\mu} T^{\nu a}) (2 - v^2 \sin^2 \theta),
\]

\[
\mathcal{M}_s \mathcal{M}_u = -i g^4 \frac{d_R}{d_R} \text{tr}_R (T^{c\mu} T^{\nu b}) (2 - v^2 \sin^2 \theta),
\]

\[
\mathcal{M}_t \mathcal{M}_u = \frac{2 g^4}{d_R} \text{tr}_R (T^a T^b T^c T^d) v^2.
\]

Here we have expanded around zero DM velocity as for annihilation to Higgs bosons.

We can evaluate the group theory factors using \( f^{abc} f^{abcd} = C_2(\text{adj}) C^{cd} \) and \( T^{a \mu} T^{\nu b} \), antisymmetry of the structure constants, and the group algebra. We find

\[
|\mathcal{M}_s|^2 = \frac{g^4}{d_R} C_2(\text{adj}) C_2(R) \left( -\frac{19}{4} + 1 + \frac{1}{8} v^2 (11 - 5 \cos 2\theta) \right),
\]

\[
|\mathcal{M}_{t,u}|^2 = 2 \frac{g^4}{d_R} C_2(R)^2 \left( 1 + v \cos \theta + v^2 \right),
\]

\[
\mathcal{M}_s \mathcal{M}_t = \frac{g^4}{2 d_R} C_2(\text{adj}) C_2(R) (2 - v^2 \sin^2 \theta),
\]

\[
\mathcal{M}_s \mathcal{M}_u = \frac{g^4}{2 d_R} C_2(\text{adj}) C_2(R) (2 - v^2 \sin^2 \theta),
\]

\[
\mathcal{M}_t \mathcal{M}_u = \frac{2 g^4}{d_R} \left( C_2(R)^2 + \frac{i}{2} C_2(\text{adj}) C_2(R) \right) v^2.
\]

In the above discussion, we took the sum over gauge boson polarization vectors to give the metric for simplicity; this includes longitudinal and timelike polarizations as well as the transverse ones. In a nonabelian gauge theory, the unphysical polarizations do not automatically vanish when contracted in the amplitudes, so we must correct for their inclusion. We can do this by subtracting the squared amplitude for ghost production. The amplitude is

\[
\mathcal{M} = \frac{g^2}{s} \bar{u}_j T^c_{ji} g^a u_i f^{abc},
\]

so we find

\[
|\mathcal{M}|^2 = \frac{g^4}{8 d_R} C_2(\text{adj}) C_2(R) \left( 1 + v^2 (1 - \cos^2 \theta) \right).
\]

As an example of this effect, we can consider \( \chi_1 \chi_2 \rightarrow B_1 B_2 \) scattering (with fixed colors for SU(2) triplet DM). Taking just the kinematical factors, the amplitude for annihilation into gauge bosons is given by \( |\mathcal{M}_s + \mathcal{M}_u|^2 = 5 g^4/4 \), as in \( \text{(35)} \). The ghosts subtract \( g^2/8 \) for a total of \(|\mathcal{M}|^2 = 9 g^4/8\), in agreement with the massless limit of the amplitude in the symmetry breaking phase.

We are primarily interested in SU(2) triplet DM with triplet and quintuplet Higgs fields, and now specialize to that case, assuming \( N_3 \) Higgs fields in the triplet and \( N_5 \) in the quintuplet. The total squared amplitude, with gauge and Higgs boson final states added incoherently, is

\[
|\mathcal{M}|^2 = \frac{g^4}{3} \left[ \left( \frac{25}{2} + 2 N_3 + 10 N_5 \right) + \frac{63}{2} v^2 + \left( \frac{7}{2} - 2 N_3 - 10 N_5 \right) v^2 \cos^2 \theta \right].
\]

For identical nonrelativistic initial particles and identical massless final particles, the differential cross-section is

\[
\frac{d\sigma}{d\Omega} = \frac{1}{2} \frac{1}{64 \pi^2 s} \frac{\bar{q}_a}{\bar{q}_b} |\mathcal{M}|^2,
\]

where the factor of 1/2 is due to overcounting identical final states in the color sum. (Here we have assumed that the Higgs particles are in a real representation; if not, drop the factor of 1/2 for the Higgs final states.)

So far we have worked at tree-level and in the CM frame, but there are small corrections to both approximations. First, the annihilation cross section experiences Sommerfeld enhancement since the DM is nonrelativistic. Under the assumption that the gauge symmetry is not yet broken, the boost factor is

\[
S(v) = \frac{\pi \alpha_g/v}{1 - e^{-\pi \alpha_g/2v}} = \frac{g^2}{4 \pi}.
\]

In the parameter space appropriate to the thermal DM freezeout, \( \pi \alpha_g/v < 1 \), so we treat it as a small parameter and expand \( S(v) \sim 1 + \pi \alpha_g/2v \). Next, since the CM frame is not quite the rest frame of the cosmic fluid, we must include the Lorentz transformation of the cross section. For nonrelativistic center-of-mass velocity \( \vec{V} \), this correction takes \( \sigma \rightarrow \sigma (1 - V^2 \sin^2 \phi/2) \), where \( \phi \) is the angle between \( \vec{V} \) and \( \vec{v} \).

Finally, we must average the cross section over the DM velocity distribution, which is Maxwell-Boltzmann. In terms of the \( \vec{V} \) and \( \vec{v} \), this average takes the form
\[
\langle \sigma v_{\text{rel}} \rangle = \left( \frac{M_X}{2\pi T} \right)^3 \int d^3V d^3\bar{V} (\sigma v_{\text{rel}})(v) \left( 1 - \frac{1}{2} V^2 \sin^2 \phi \right) e^{-M_X(v^2+V^2)/T} \\
= \frac{\pi \alpha_g^2}{12 M_X^2} \left[ \left( \frac{25}{2} + 2 N_3 + 10 N_5 \right) \left( 1 + \alpha_g \sqrt{\frac{\pi M_X}{T}} - \frac{1}{2\pi M_X} \right) T \right] + \left( \frac{317}{8} - \frac{5}{2} N_3 - \frac{25}{2} N_5 \right) \left( \frac{T}{M_X} \right),
\]

(A13)

where the relative velocity \( v_{\text{rel}} = 2v \). We have expanded the result to first order in the small parameters \( \alpha_g \sqrt{M_X/T} \) and \( T/M_X \). In the sequel, we will treat the ratio \( M_X/T \) as roughly independent of the DM mass and use an approximate value of \( M_X/T \sim 20 \).

We should also address the issue of symmetry breaking. We will assume that the gauge bosons are light compared to the DM and that Higgs bosons are either light compared to the DM or too heavy to be produced in DM annihilation. The annihilation amplitudes are slightly modified by the gauge boson masses (in a manner that does not respect the gauge symmetry, of course). For example, \( \chi_1 \chi_2 \rightarrow B_1 B_2 \) annihilation has

\[
\langle \sigma v \rangle = g^4 \frac{\mu_1^4 + 2 \mu_1^2 (5 \mu_2^2 - \mu_3^2) + (\mu_2^2 - \mu_3^2)^2}{8 \mu_1^2 \mu_2^2} \langle A \alpha \rangle \approx N \chi \langle \sigma v \rangle_0.
\]

at zero velocity and lowest order in the gauge boson masses. This deviates from the leading order massless result only in as much as the gauge boson masses differ from each other. Note that the Goldstone boson states become longitudinal gauge bosons in gauges such as the unitary gauge. The significant effect of symmetry breaking is to change the kinematical factors in the cross section. Using the average mass \( \bar{\mu} \) for all the light gauge bosons and Higgs states,

\[
|\mathcal{M}|^2 = g^4 \frac{\mu_1^4 + 2 \mu_1^2 (5 \mu_2^2 - \mu_3^2) + (\mu_2^2 - \mu_3^2)^2}{8 \mu_1^2 \mu_2^2} \langle A \alpha \rangle M_X^2 (1 + v^2/2 - \bar{\mu}^2/2) \quad (A14)
\]

For symmetry breaking at a small scale compared to the DM mass, this just renormalizes the cross section by a factor of \( (1 - \bar{\mu}^2/M_X^2)^{1/2} \).

We can now compare the cross section to that required for the correct relic density of dark matter. Under the normal assumption that only SM particles are lighter than the dark matter, the required cross section is approximately \( \langle \sigma v_{\text{rel}} \rangle_0 \sim 2.84 \times 10^{-26} \text{ cm}^3/\text{s} \), with a logarithmic dependence on DM mass. We will take this central value. However, our dark matter models contain extra light degrees of freedom, which affects the required cross section in two ways. First, the Hubble parameter is greater at a given temperature, which causes freezeout to occur earlier. Second, due to heating of photons by annihilation of these light degrees of freedom (see Big Bang nucleosynthesis constraints given in [25]), the universe expands more between freezeout and the present day, which means that the freezeout density of DM must be higher than in minimal DM models. As a result, the desired cross section satisfies

\[
\langle \sigma v_{\text{rel}} \rangle = \frac{\sqrt{g_*/g_*}}{(\sqrt{g_*} / g_*)_0} \langle \sigma v_{\text{rel}} \rangle_0.
\]

(A16)

For DM masses near \( M_X = 5 \text{ GeV} \), this is

\[
\langle \sigma v_{\text{rel}} \rangle = \frac{\langle \sigma v_{\text{rel}} \rangle_0}{\sqrt{1 + (6 + 3 N_3 + 5 N_5)/61.75}}.
\]

(A17)

Including all these corrections, we can write the desired cross section in terms of the gauge coupling as

\[
\langle \sigma v_{\text{rel}} \rangle = \frac{\langle \sigma v_{\text{rel}} \rangle_0}{(A + B \alpha_g)^{1/2}} M_X.
\]

(A18)

Our \( SU(2) \) models have

\[
\alpha_g \approx \frac{M_X}{\text{GeV}} \left( 1 - \frac{\bar{\mu}^2}{M_X^2} \right)^{-1/4}
\]

\[
\times \begin{cases} 
2.5 \times 10^{-5} & (N_3 = 0, N_5 = 0; \text{ all } m_b > M_X) \\
2.2 \times 10^{-5} & (N_3 = 2, N_5 = 0) \\
2.0 \times 10^{-5} & (N_3 = 3, N_5 = 0) \\
1.7 \times 10^{-5} & (N_3 = 2, N_5 = 1) \\
1.7 \times 10^{-5} & (N_3 = 3, N_5 = 1)
\end{cases}
\]

(A20)

In the end, corrections due to the initial velocity of the DM particles contribute at the \( 5 - 15\% \) level, while corrections from Sommerfeld enhancement contribute only 1 part in \( 10^4 \) due to their additional dependence on \( \alpha_g \) (since the coefficient \( B \) is of the same order as \( A \)).

Appendix B: Kinetic coupling to SM

In this appendix, we find the freezeout temperature of the dark matter kinetic coupling to the Standard Model. For convenience calculating phase space factors, we consider downscattering \( \chi_{2,3} f \rightarrow \chi_1 f \), labeling the \( \chi_j \) momenta as \( p_j \) and the initial and final energy and momenta of the SM fermions \( f \) as \( E_{i,f} \), and \( q_{i,f} \) respectively (similarly for other subscripts). Here, the relevant initial DM state is \( \chi_3 \) for the endothermic case and \( \chi_2 \) for exothermic. In this appendix, we consider the endothermic case.
but it should be clear that these results apply equally well in both scenarios. At temperatures under 100 MeV (and, in particular near the important scale of $\delta M_{13} \sim$ MeV), only scattering from $e^\pm$ will be important. Furthermore, if this process occurs roughly once per Hubble time for each of the more massive DM particles, it efficiently maintains the distribution of these two DM states given by the Boltzmann factor at the SM temperature. Also, if $\chi \chi$ scattering is still in equilibrium, this reaction can insure that all the DM states maintain a thermal velocity distribution at the SM temperature. (As we have seen in the main text, $\chi \chi$ scattering typically freezes out later than this process.) We note that the same calculations apply for scattering between the two top states $\chi_3 f \rightarrow \chi_2 f$ with the appropriate replacements of $\epsilon_2$, $\mu_2$, and $\delta M_{13}$. As shown in figure 11, the cross section is only slightly smaller for keV mass splittings.

The matrix element for the scattering process shown in figure 11 is (taking electrons for specificity)

$$\mathcal{M} = -i \frac{ge\epsilon_2}{(p_3 - p_1)^2 - \mu_2^2} \bar{u}_1 \gamma^\mu u_3 \bar{u}_f \gamma_\mu u_i . \quad (B1)$$

After the spin sum and average, to lowest order. Due to cancellations in [B2], we will need $s$ to the same (second) order in small quantities:

$$s = M_x^2 + 2M_x (\delta M_{13} + E_i) + \delta M_{13}^2 + 2\delta M_{13} E_i + m_e^2 + 2|\vec{q}_f|^2 . \quad (B6)$$

To lowest order in small quantities, we find

$$|\mathcal{M}|^2 = 16 \frac{(ge\epsilon_2)^2 M_x^2}{(\mu_2^2 - t)^2} \left[ E_i^2 + \delta M_{13} E_i + \frac{1}{4} t \right] . \quad (B7)$$

for the (spin averaged and summed) squared amplitude. Since $d\sigma/dt = (1/64\pi s)|\mathcal{M}|^2/|\vec{q}_f|^2$ and the relative velocity is dominated by the electron velocity, we find that

$$\sigma_{\text{rel}} = \frac{(ge\epsilon_2)^2}{4\pi} \frac{1}{E_i |\vec{q}_f|} \left[ (E_i^2 + \delta M_{13} E_i + \frac{1}{4} \mu_2^2) \left( \frac{1}{\mu_2^2 - t_+} - \frac{1}{\mu_2^2 - t_-} \right) - \frac{1}{4} \ln \left( \frac{\mu_2^2 - t_-}{\mu_2^2 - t_+} \right) \right] . \quad (B8)$$

We are especially interested in whether the DM and SM can maintain kinetic equilibrium at temperatures $T \lesssim m_e$, since those temperatures are relevant for $\chi - \chi$ downscattering. Then temperatures are parametrically less than $\mu_2$, so

$$\sigma_{\text{rel}} = \frac{(ge\epsilon_2)^2}{2\pi} \frac{1}{\mu_2^2 E_i} \left( 2m_e^2 + |\vec{q}_f|^2 + \delta M_{13} E_i \right) \times \left( \delta M_{13}^2 + |\vec{q}_f|^2 + 2\delta M_{13} E_i \right)^{1/2} . \quad (B9)$$

With the replacement $m_e \rightarrow m_p$, $\delta M_{13} \rightarrow \delta M_{23}$, and $\mu_2 \rightarrow \mu_1$, this is also the result for inelastic DM scattering off protons, which is relevant to direct detection experiments.

Since we are interested in temperatures much less than the DM mass $M_x$ (and all other energy scales are also much less than $M_x$), we can work in center-of-momentum (CM) frame up to overall errors of order $\sqrt{T/M_x} \ll 1$ in the cross-section compared to the cosmic rest frame. To lowest order in $T/M_x$, the final electron energy is

$$E_f = E_i + \delta M_{13} , \quad (B3)$$

or

$$|\vec{q}_f|^2 = \delta M_{13}^2 + 2\delta M_{13} E_i + |\vec{q}_i|^2 . \quad (B4)$$

Then the Mandelstam $t$ ranges between $t_-$ and $t_+$ satisfying

$$t_\pm = -2|\vec{q}_i|^2 - 2\delta M_{13} E_i \pm 2|\vec{q}_i| \left[ \delta M_{13}^2 + 2\delta M_{13} E_i + |\vec{q}_i|^2 \right]^{1/2} . \quad (B5)$$

$$\sigma_{\text{rel}} = \frac{(ge\epsilon_2)^2}{4\pi} \frac{1}{E_i |\vec{q}_f|} \left( \left( E_i^2 + \delta M_{13} E_i + \frac{1}{4} \mu_2^2 \right) \left( \frac{1}{\mu_2^2 - t_+} - \frac{1}{\mu_2^2 - t_-} \right) - \frac{1}{4} \ln \left( \frac{\mu_2^2 - t_-}{\mu_2^2 - t_+} \right) \right) . \quad (B8)$$

$$\sigma_{\text{rel}} = \frac{(ge\epsilon_2)^2}{2\pi} \frac{1}{\mu_2^2 E_i} \left( 2m_e^2 + |\vec{q}_f|^2 + \delta M_{13} E_i \right) \times \left( \delta M_{13}^2 + |\vec{q}_f|^2 + 2\delta M_{13} E_i \right)^{1/2} . \quad (B9)$$
times corrections of relative order $t/\mu_2^2$ and $\sqrt{q_i/M_s}$, where $v_{rel}$ is the CM frame electron speed. At a fixed temperature $T$, we find the total scattering rate by integrating over the Fermi-Dirac distribution for marginally relativistic electrons (since $T \sim m_e$). Assuming a thermal origin for the dark matter abundance so $\alpha \propto M_s$, this total rate can be written in terms of a normalized rate $\tilde{\Gamma}$ as

$$\langle n_e \sigma v_{rel} \rangle \equiv \left( \frac{\mu_2^2}{\mu_2^2 - t} \right) \frac{\tilde{\Gamma}(\delta M_{13}, T)}{m_e^2} .$$

(B10)

In $n_e$, we include both spin states of electrons and positrons.

Therefore, the ratio $3H/\tilde{\Gamma}$ considered as a function of $T$ inverts to give the DM/SM decoupling temperature as a function of $\delta M_{13}^2/\mu_2^2$. This decoupling temperature is shown in figure 11 for several values of $\delta M_{13}$. Since we consider temperatures near the electron mass, we calculate the effective species number $g_*$ numerically. This includes heating of photons due to $e^\pm$ annihilation and also the neutrino density.

Since the kinetic mixing is between the $B^\mu_a$ bosons and SM hypercharge and therefore includes mixing with $Z_\mu$, we can ask if the DM comes into equilibrium with neutrinos through the weak force. Above the electron mass, scattering from electrons will always dominate scattering from neutrinos because $m_Z \gg \mu_2$ (and because the $B^\mu$ coupling to the weak current is suppressed by $\sim \mu_2^2/m_Z^2$). However, below the electron mass, there are many more neutrinos than electrons, so neutrino scattering is potentially important. Scattering from neutrinos progresses through two Feynman diagrams similar to figure 10; one with a $B$ propagator and one with a $Z$ propagator, but there is a relative sign between the two in the kinetic mixing. Therefore, to include both diagrams properly, we should replace

$$\frac{1}{(\mu_2^2 - t)^2} \rightarrow \left[ \frac{\mu_2^2/m_e^2}{\mu_2^2 - t} - \frac{1}{m_Z^2 - t} \right]^2 \sim \frac{t^2}{\mu_2^2 m_Z^2} ,$$

(B11)

in equation (B7) at low temperatures. Also, taking $\epsilon$ to be the $B$ coupling to the electric current, we should replace $\epsilon \rightarrow \epsilon \tan \theta_w$. Due to the fact that the two Feynman diagrams nearly cancel, the cross-section is highly suppressed. A straightforward estimate of the total scattering rate and comparison to the Hubble rate indicates that $\epsilon$ would need to be of order $10^5$ for $\chi - \nu$ scattering to equilibrate at temperatures below $m_e$!

**Appendix C: $\chi_2 \rightarrow \chi_1 e^+ e^-$ at one loop**

In this appendix, we give a careful derivation of the lifetime for $\chi_2 \rightarrow \chi_1 e^+ e^-$ decay at one loop level when $\epsilon_3 = 0$. This is the dominant decay process we have been able to find for this case.

Consider the loops in figure 10. As noted above the figure, the two diagrams nearly cancel due to the opposite signs of the nonabelian 3-gauge-boson couplings; in fact, they do not cancel completely only because the two gauge bosons in the loop have different masses. The complete amplitude can be written as

$$\mathcal{M} = ig \frac{\tilde{G}^\gamma_{\mu2} \tilde{G}^\nu_{\mu1} L_{\mu} v_e}{q^2 - \mu_3^2} ,$$

(C1)

where the spinors of the $\chi$ particles are labeled by their color (as their momenta $k_{2,1}$ will be), we take the outgoing momenta of the positron and electron to be $p_\pm$ respectively, and $q = k_2 - k_1 = p_+ + p_-$. The momenta running counter-clockwise around the loops are $l + \delta p$ on the electron line, $l - \bar{p}$ on the upper gauge boson line, and $l + \bar{p}$ on the lower gauge line, where $\bar{p} = q/2$ and $\delta p = (p_+ - p_-)/2$. With these conventions, the loop integrals are

$$L_\mu = ig e^2 \epsilon_1 \epsilon_2 \int \frac{d^4 l}{(2\pi)^4} \left[ \frac{\eta_{\nu\lambda}(3\bar{p} - l)_\lambda + \eta_{\nu\lambda}(2l)_\mu}{(l + \delta p)^2 - m_e^2} \right] \left[ \eta_{\mu\nu}(3\bar{p} - l)_\lambda + \eta_{\mu\lambda}(2l)_\mu - \eta_{\nu\lambda}(l + 3\bar{p})_\nu \right] \left[ \frac{1}{[(l + \bar{p})^2 - \mu_2^2][(l - \bar{p})^2 - \mu_1^2]} - (\mu_1 \leftrightarrow \mu_2) \right] .$$

(C2)

As usual, we can rewrite the denominators with Feynman parameters as

$$\int \frac{dx dy dz}{[l'^2 + l'^2 \cdot (\bar{p}(y - x) + \delta p z) + \bar{p}^2(x + y) + \delta p^2 z - \mu_2^2 x - \mu_2^2 y - m_e^2 z]^3} = \int \frac{dx dy dz}{[l'^2 + \delta p^2 z(1 - z) + \bar{p}^2(x(1 - x) + (1 - y) + 2xy) - \mu_1^2 x - \mu_2^2 y - m_e^2 z]^3} ,$$

(C3)

where $l' = l + \bar{p}(y - x) + \delta p z$ and we have used $\bar{p} \cdot \delta p = 0$. From the second form, it is clear that taking $\mu_1 \leftrightarrow \mu_2$ is
the same as swapping the Feynman parameters $x \leftrightarrow y$. Therefore, the only terms that survive taking the difference in (C2) must be antisymmetric in $x \leftrightarrow y$, and these must come from shifting $l$ to $l'$. In the end, we find

\[ L_{\mu} = -8 \left( \frac{\hat{p}_\mu \delta \psi + \delta p_\mu \bar{\psi}}{(-2\pi)^2} \right) \int_0^1 dx \int_0^{1-x} dy \int \frac{d^4l}{(2\pi)^4} \frac{(y-x)(1-x-y)}{[l^2 - \Delta(x,y)]^3} \]

\[ \frac{-i}{12\pi^2} \left( \frac{\hat{p}_\mu \delta \psi + \delta p_\mu \bar{\psi}}{\mu_1^2 - \mu_2^2} \right) \left[ \frac{1}{\mu_1^2 - \mu_2^2} + \frac{\mu_1^2 + \mu_2^2}{\mu_1^2 + \mu_2^2} \ln \left( \frac{\mu_1}{\mu_2} \right) \right], \quad (C4) \]

with $E_+ \pm \bar{p}_+$ the energy and 3-momentum of the $e^\pm$. Integrating over the nonrelativistic phase space, we find

\[ \Gamma = \frac{16}{9\pi} \frac{\alpha^2 \alpha^2 \alpha^2 m^2 E^2}{\mu_3^2} (\delta M_{12} - 2m_e)^3 (\delta M_{12} - m_e) \Lambda^2. \quad (C7) \]

Using the same estimated parameters as below equation (53), we find a lifetime of more than $10^{23}$ s.

[1] R. Diehl and M. Leising, arXiv:0906.1503 [astro-ph.HE].
[2] J. Knödlseder et al., Astron. Astrophys. 411, L457 (2003) [arXiv:astro-ph/0309442]; P. Jean et al., Astron. Astrophys. 407, L55 (2003) [arXiv:astro-ph/0309484]; J. Knödlseder et al., Astron. Astrophys. 441, 513 (2005) [arXiv:astro-ph/0506026]; L. Bouchet, E. Jourdain, J. P. Roques, A. Strong, R. Diehl, F. Lebrun and R. Terrier, arXiv:0801.2086 [astro-ph].
[3] W. Wang, C. S. J. Pun and K. S. Cheng, Astron. Astrophys. 446, 943 (2006) [arXiv:astro-ph/0509760]; M. Casse, B. Cordier, J. Paul and S. Schanne, Astrophys. J. 602, L17 (2004) [arXiv:astro-ph/0309824]; G. Bertone, A. Kusenko, S. Palomares-Ruiz, S. Pascoli and D. Semikoz, Phys. Lett. B 636, 20 (2006) [arXiv:astro-ph/0405005]; P. A. Milne, J. D. Kurfess, R. L. Kinzer and M. D. Leising, New Astron. Rev. 46, 553 (2002) [arXiv:astro-ph/0110442]; K. Ahn, E. Komatsu and P. Hoffich, Phys. Rev. D 71, 121301 (2005) [arXiv:astro-ph/0506126]; R. M. Bandyopadhyay, J. Silk, J. E. Taylor and T. J. Maccarone, Mon. Not. Roy. Astron. Soc. 392, 1115 (2009); arXiv:0810.3674 [astro-ph].
[4] J. Knödlseder et al., Astron. Astrophys. 411, L457 (2003) [arXiv:astro-ph/0309442]; P. Jean et al., Astron. Astrophys. 407, L55 (2003) [arXiv:astro-ph/0309484]; J. Knödlseder et al., Astron. Astrophys. 441, 513 (2005) [arXiv:astro-ph/0506026]; L. Bouchet, E. Jourdain, J. P. Roques, A. Strong, R. Diehl, F. Lebrun and R. Terrier, arXiv:0801.2086 [astro-ph].
[5] W. Wang, C. S. J. Pun and K. S. Cheng, Astron. Astrophys. 446, 943 (2006) [arXiv:astro-ph/0509760]; M. Casse, B. Cordier, J. Paul and S. Schanne, Astrophys. J. 602, L17 (2004) [arXiv:astro-ph/0309824]; G. Bertone, A. Kusenko, S. Palomares-Ruiz, S. Pascoli and D. Semikoz, Phys. Lett. B 636, 20 (2006) [arXiv:astro-ph/0405005]; P. A. Milne, J. D. Kurfess, R. L. Kinzer and M. D. Leising, New Astron. Rev. 46, 553 (2002) [arXiv:astro-ph/0110442]; K. Ahn, E. Komatsu and P. Hoffich, Phys. Rev. D 71, 121301 (2005) [arXiv:astro-ph/0506126]; R. M. Bandyopadhyay, J. Silk, J. E. Taylor and T. J. Maccarone, Mon. Not. Roy. Astron. Soc. 392, 1115 (2009); arXiv:0810.3674 [astro-ph].
[6] K. S. Cheng, D. O. Chernyshov and V. A. Dogiel, Astrophys. J. 645, 1138 (2006) [arXiv:astro-ph/0603659]; A. Calvez and A. Kusenko, arXiv:1003.0045 [astro-ph].
D. Varshalovich, Mon. Not. Roy. Astron. Soc. 357, 1377 (2005) [arXiv:astro-ph/0411351].

[44] P. Jean et al., Astron. Astrophys. 445, 579 (2006) [arXiv:astro-ph/0509298].

[45] N. Guessoum, P. Jean and W. Gillard, arXiv:astro-ph/0504186.

[46] K. Ferriere, W. Gillard and P. Jean, Astron. Astrophys. 467, 611 (2007) [arXiv:astro-ph/0702532].

[47] T. Sawada, T. Hasegawa, T. Handa and R. J. Cohen, Mon. Not. Roy. Astron. Soc. 349, 1167 (2004) [arXiv:astro-ph/0401286].

[48] H.S. Liszt, W.B. Burton, Astrophys. J. 236, 779 (1980).

[49] G. K. Skinner, arXiv:1009.2098 [astro-ph.GA].

[50] J. M. Cordes and T. J. W. Lazio, arXiv:astro-ph/0207156.

[51] V. Kaspi, private communication.

[52] B. J. Teegarden and K. Watanabe, Astrophys. J. 646, 965 (2006) [arXiv:astro-ph/0604277].

[53] H. Yuksel and M. D. Kistler, Phys. Rev. D 78, 023502 (2008) [arXiv:0711.2906 [astro-ph]].

[54] M. P. Muno et al., Astrophys. J. 613, 326 (2004) [arXiv:astro-ph/0402087].

[55] M. Baumgart, C. Cheung, J. T. Ruderman, L. T. Wang and I. Yavin, JHEP 0904, 014 (2009) [arXiv:0901.0283 [hep-ph]].

[56] C. Arina and M. H. G. Tytgat, arXiv:1007.2765 [astro-ph.CO].

[57] J. Lavalle, arXiv:1007.5253 [astro-ph.HE].

[58] S. Galli, F. Iocco, G. Bertone and A. Melchiorri, Phys. Rev. D 80, 023505 (2009) [arXiv:0905.0003 [astro-ph.CO]].

[59] T. R. Slatyer, N. Padmanabhan and D. P. Finkbeiner, Phys. Rev. D 80, 043526 (2009) [arXiv:0906.1197 [astro-ph.CO]].

[60] M. Cirelli, F. Iocco and P. Panci, JCAP 0910, 009 (2009) [arXiv:0907.0719 [astro-ph.CO]].

[61] M. Pospelov and J. Pradler, arXiv:1006.4172 [hep-ph].

[62] R. Mertig, M. Bohm and A. Denner, Comput. Phys. Commun. 64, 345 (1991).

[63] K. Nakamura et al., Journal of Physics G 37, 075021 (2010).