Extension algorithm for generic low-voltage networks

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Abstract.
Distributed energy resources (DERs) are increasingly penetrating the energy system which is driven by climate and sustainability goals. These technologies are mostly connected to low-voltage electrical networks and change the demand and supply situation in these networks. This can cause critical network states. Network topologies vary significantly and depend on several conditions including geography, historical development, network design or number of network connections. In the past, only some of these aspects were taken into account when estimating the network investment needs for Germany on the low-voltage level. Typically, fixed network topologies are examined or a Monte Carlo approach is used to quantify the investment needs at this voltage level. Recent research has revealed that DERs differ substantially between rural, suburban and urban regions. The low-voltage network topologies have different design concepts in these regions, so that different network topologies have to be considered when assessing the need for network extensions and investments due to DERs. An extension algorithm is needed to calculate network extensions and investment needs for the different typologies of generic low-voltage networks. We therefore present a new algorithm, which is capable of calculating the extension for generic low-voltage networks of any given topology based on voltage range deviations and thermal overloads. The algorithm requires information about line and cable lengths, their topology and the network state only. We test the algorithm on a radial, a loop, and a heavily meshed network. Here we show that the algorithm functions for electrical networks with these topologies. We found that the algorithm is able to extend different networks efficiently by placing cables between network nodes. The main value of the algorithm is that it does not require any information about routes for additional cables or positions for additional substations when it comes to estimating network extension needs.

1. Background and objective
Due to climate and sustainability goals, distributed energy resources (DERs) like photovoltaic (PV)-systems are increasingly contributing to the electricity generation in Germany. One goal for German society is to install 2.5 GW additional PV-power every year [7]. Furthermore, plug-in electric vehicles (PEVs) are an option to reduce green house gases, if these vehicles are charged by renewable energy sources. Therefore, the federal government implemented a buyer’s premium for PEVs [3]. Economic incentives for PEVs and PV-systems could lead to an increasing market penetration for these technologies in Germany in the future [8, 22]. These technologies are mostly connected to low-voltage electrical networks and their operation can lead to high loads there. While load peaks can cause critical network states, distribution system operators (DSOs) are responsible for a stable and reliable network operation [15, 17, 19, 20]. In the long term DSOs respond to
critical network states by investing into network assets. This is done by replacing or adding new cables, overhead lines or voltage transformation substations to existing networks. Due to the long life time cycles of these network assets certain planning aspects are being taken into consideration. The planning of electrical networks has a long history [11, 14, 16]. In general all planning methods focus on certain aspects like the substation locations and capacities, or the feeder system switching and routing. But not all aspects of network planning can be taken into account [21].

DSOs still require tools for better investment decisions under uncertainty, which also contribute to an affordable, sustainable and reliable electric energy supply [12]. Therefore, optimal distributed generation placement algorithms have been developed to avoid critical network states in the future [13, 23]. In order to find optimal solutions for network planning methods like mixed integer programming or ant colony algorithms are used [1, 9, 10]. In Germany, more than 800 DSOs are responsible for the stable and secure operation of roughly 1 million kilometres of low-voltage networks [4, 5]. Network topologies vary significantly and depend on several conditions including geography, historical development, network design or the number of network connections. Due to a large number of different networks where the local conditions are also often unknown, not every single network can be planned by these algorithms to estimate the network investment needs on a low voltage level for the following years. Therefore, in the past, when it comes to estimating these investment needs, fixed topologies or a Monte Carlo approach is used [2, 6]. For example, if voltage range deviations occur networks are extended by an additional cable from the transformer to the last third of the overloaded line. Or a new cable is added between a transformer and the middle of the overloaded line if a thermal overload occurs [2]. In order to get more precise investment estimations for the future, this method should be formalize for arbitrary extension points at generic network topologies. Therefore, we introduce in section 2 a new extension algorithm which is capable of extending existing generic networks based on thermal overloads and voltage range deviations at these networks. In section 3 we set up a case study to test the algorithm on networks of radial, loop, or meshed topology. Finally in section 4, we draw conclusions for the algorithm and the case study.

2. Extension algorithm based on voltage range deviations and thermal overloads

2.1. Algorithm overview

The algorithm presented in this paper adds cables to the parts of an existing network topology which faces the strongest overload. The length of the added cables and consequently the network extension can be set by adjusting a relief factor $F_{\text{relief}}$ with $0 > F_{\text{relief}} \geq 1$. The algorithm checks the currents in all cables of the network $C$ and the voltages at all nodes $N$ of the network. Similar to [2] we differentiate between voltage range deviations and thermal overloads. Thermal overloads are a result of currents exceeding rated values in either cables or transformers. Voltage range deviations occur if the voltage at a given node crosses a threshold. For both situations, the algorithm attempts to bring the values back to the nominal value by adding a new cable to the network topology. The algorithm is designed to use the shortest path between the connections of a newly added cable. After a network extension, the network state is tested again for overloads and voltage range deviations. If an overload occurs new cables are added to the network until a network state without overloads is reached.

The algorithm is available at GitHub\(^1\) and implemented within the programming language Java. Nevertheless, the algorithm works independent of the programming language.

\(^1\) https://github.com/ChrisOlk/GenericLVExtensionAlgorithm
2.2. Network extension due to thermal overload

If a thermal overload occurs, the algorithm adds a new cable between two nodes in the network parallel to overloaded components. The algorithm finds that sequence of cables \( S_c \) which faces the thermal overloaded. To determine the extent of the overload of a cable, the actual current \( I(c_i) \) is compared to the rated current of the cable \( I_{\text{rated}}(c_i) \). In a second step, two nodes in \( S_c \) are found based on \( F_{\text{relief}} \). Finally, at these nodes the network is extended.

2.2.1. Find the cable sequence with the highest thermal overload. A search algorithm is deployed to find the serial sequence of cables \( S_c \) with the strongest thermal overload. Cables are added to \( S_c \) on the basis of their thermal overload \( \left( \frac{I(c_i)}{I_{\text{rated}}(c_i)} \right) \) and the voltage at their connected nodes \( V(n) \). First the cable \( c_{i,max} \) is selected which holds the highest relative thermal overload \( \left( \frac{I(c_i)}{I_{\text{rated}}(c_i)} \right) \). The search for other cables starts from that selected cable \( c_s \).

\[
c_s = c_{i,max} = \arg \max_{c_i \in \mathcal{C}} \left( \frac{I(c_i)}{I_{\text{rated}}(c_i)} \right) \tag{1}
\]

Due to Ohms law one node \( n_{c_s,\text{high}} \) connected to \( c_s \) holds a higher voltage \( V(n_{c_s,\text{high}}) \) than the other node \( n_{c_s,\text{low}} \) connected to \( c_s \). Starting at the node \( n_{c_s,\text{high}} \) and \( n_{c_s,\text{low}} \) the search algorithm analyses the network. In the following the search in the direction of \( n_{c_s,\text{low}} \) is described. The analogous search in the direction of \( n_{c_s,\text{high}} \) is indicated in brackets.

First all, cables \( C_{n_{c_s,\text{low}}} \left( C_{n_{c_s,\text{high}}} \right) \) connected to \( n_{c_s,\text{low}} \left( n_{c_s,\text{high}} \right) \) are checked for the strongest thermal overload and whether the voltage at the other end of the cable \( V(n_{c_s,\text{low}}) \left( V(n_{c_s,\text{high}}) \right) \) is lower (higher) than the voltage at \( n_{c_s,\text{low}} \left( n_{c_s,\text{high}} \right) \). The second condition ensures that the voltage along \( S_c \) monotonously decreases and guarantees a unidirectional current flow. The cable fulfilling both conditions is added to \( S_c \). Afterwards, the node \( n_{c_s,\text{low}} \left( n_{c_s,\text{high}} \right) \) is set to the new \( n_{c_s,\text{low}} \left( n_{c_s,\text{high}} \right) \). Starting at the new node, the process is repeated until no cable fulfilling the condition can be found \( 2 \left( 3 \right) \). This end point is either a local voltage minimum (maximum) and/or a transformer is connected there.

\[
c_{s,\text{low}} = \arg \max_{c_i \in C_{n_{c_s,\text{low}}}} \left( \frac{I(c_i)}{I_{\text{rated}}(c_i)} \right) \land V(n_{c_s,\text{low}}^*) < V(n_{c_s,\text{low}}) \tag{2}
\]

\[
c_{s,\text{high}} = \arg \max_{c_i \in C_{n_{c_s,\text{high}}}} \left( \frac{I(c_i)}{I_{\text{rated}}(c_i)} \right) \land V(n_{c_s,\text{high}}^*) > V(n_{c_s,\text{high}}) \tag{3}
\]

2.2.2. Find two nodes for a network extension cable. Hereafter \( S_c \) and \( F_{\text{relief}} \) are used to determine two nodes to which the new cable is added to the network. The search for these two nodes starts at \( V_{\text{max}} \) and \( V_{\text{min}} \) separately and analogously. Here we describe the search at \( V_{\text{max}} \).

As indicated in equation 4 (5) out of all cables in \( S_c \) the cable \( c_{s,V_{\text{max}}} \left( c_{s,V_{\text{min}}} \right) \) connected to the node with the highest (lowest) voltage is selected.

\[
c_{s,V_{\text{max}}} = \arg \max_{n_i \in S_c} \left( V(n_i) \right) \tag{4}
\]

\[
c_{s,V_{\text{min}}} = \arg \min_{n_i \in S_c} \left( V(n_i) \right) \tag{5}
\]

Finally, all cables which are an element of \( S_c \) are ordered by decreasing (increasing) nodal voltages. In a decreasing (increasing) order the cable \( c_s \) out of \( S_c \) is checked for condition 6 until the condition is fulfilled.
\[
\left( \frac{I(c_s)}{I_{rated}(c_s)} \right) \geq \left( \frac{I(c_{I,max})}{I_{rated}(c_{I,max})} \right) (1 - F_{relief}) \tag{6}
\]

If condition 6 is fulfilled, the connection point for a new cable is the node holding the higher (lower) voltage at \( c_s \).

2.3. Network extension due to voltage range deviations

Here we describe how the algorithm reacts to a voltage range deviation. Based on the relief factor \( F_{relief} \) introduced in section 2 and the voltages at all nodes of the network \( V(n_i) \), two nodes are found to which a new cable is added. The goal of the algorithm is to reduce voltage range deviations efficiently. First a sequence of nodes \( S_n \) with monotonously increasing voltages is found containing the node with the strongest voltage range violation. Afterwards the nodes are compared using \( F_{relief} \) to find two connection points for a new cable.

2.3.1. Find the node sequence with the highest voltage range deviation

If a voltage range deviation occurs, the node \( n_{\Delta V_{max}} \) in the network with the highest deviation from nominal voltage \( |V(n_i) - V_{rated}| \) is found. Therefore, the node voltage \( V(n_i) \) at all nodes in the network \( \mathcal{N} \) is compared to the rated network voltage \( V_{rated} \). Furthermore, the node with the highest voltage deviation \( n_{\Delta V_{max}} \) is selected as \( n_s \) where the search algorithm starts (see equation 7).

\[
n_s = n_{\Delta V_{max}} = \arg \max_{n_i \in \mathcal{N}} (|V(n_i) - V_{rated}|) \tag{7}
\]

By definition \( V(n_s) \) has to be the global maximum or minimum of the network voltage. Here we describe the case where \( n_s \) holds the minimum of the network voltage. The procedure in case that the voltage \( V(n_s) \) is a global maximum is indicated in brackets. First the algorithm checks all cables \( C_n \) connected to \( n_s \) for the cable \( c_i \) where the voltage at the other end of \( V(n_s^*) \) holds the highest (lowest) voltage. Additionally, the node \( n_s^* \) is selected as the new \( n_s \) and added to the sequence of nodes \( S_n \) (see equation 8 (9)).

\[
n_s = \arg \max_{c_i \in C_n} (V(n_s^*)) \land V(n_s^*) > V(n_s) \tag{8}
\]

\[
n_s = \arg \min_{c_i \in C_n} (V(n_s^*)) \land V(n_s^*) < V(n_s) \tag{9}
\]

Voltages at nodes are compared, selected and added to \( S_n \) until no cable is connected to \( n_s \) with a higher (lower) voltage at the other end \( n_s^* \) (see equation 10 (11) for the stopping criterion).

\[
V(n_s) > \arg \max_{c_i \in C_n} (V(n_s^*)) \tag{10}
\]

\[
V(n_s) < \arg \min_{c_i \in C_n} (V(n_s^*)) \tag{11}
\]

2.3.2. Find two nodes in the sequence of nodes to add a new cable between

After \( S_n \) is complete, the algorithm searches for two nodes within \( S_n \) where a new cable is added to the network. First all nodes which are an element of \( S_n \) are compared by their voltage \( V(n_i) \). The starting point for the search is that node \( n_s \) whose voltage is closed to the rated voltage \( V_{rated} \) of the network (Equation 12).

\[
n_s = \arg \min_{n_i \in S_n} (|V(n_i) - V_{rated}|) \tag{12}
\]
Independently of whether a voltage range deviation is caused by a low or a high voltage, two searches are performed starting at $n_s$. One search is executed to nodes holding decreasing lower voltages than $n_s$, the other search leads towards nodes holding increasing voltages. Furthermore, all nodes have to be in $S_n$ with the highest voltage. Here we describe the search towards the node with the highest voltage $n_{V_{max}}$. The analogous search in the direction of the node with the lowest voltage $n_{V_{min}}$ in $S_n$ is provided in brackets.

The algorithm checks in an increasing (decreasing) order whether the local voltage $V(n_s)$ is higher (lower) than the nominal voltage plus (minus) an acceptable voltage deviation which depends on $F_{relief}$ (see equation 13 (14)).

$$V(n_s) \geq V_{rated} + (F_{relief} (V(n_{V_{max}}) - V_{rated}))$$ (13)

$$V(n_s) \leq V_{rated} - (F_{relief} (V_{rated} - V(n_{V_{min}})))$$ (14)

If equation 13 (14) is fulfilled, $n_s$ is the node to which a new cable extends the network. If not, the node in $S_n$ with the next higher (lower) voltage is selected. This process is repeated until a node has been found which fulfils equation 13 (14). After both nodes are found this way, the new cable connects these nodes along the shortest possible path.

3. Case study

Here we describe how the algorithm responds to voltage range deviations and thermal overloads on three networks using a given relief factor $F_{relief}$. We set up test cases for one radial, one loop, and one heavily meshed network (see section 3.1). Afterwards, the algorithm’s response to these test cases is described (see section 3.2).

3.1. Test cases for a radial, a loop and a meshed network

The test case is defined for networks with three different topologies and a rated phase to phase voltage $V_{rated}$ of 400 V. Each cable is of type NAYY-J 150 mm$^2$ with a maximal allowed one phase cable current of 275 A. The first topology is based on [18] and consists of two symmetric radial lines. Each line holds six nodes. The end of each line is connected to a slack node $n_0$ which represents a transformer (see Figure 1a). For the second topology we add an additional 60 m long cable between node $n_6$ and node $n_{12}$ to the radial topology. This leads to the loop topology from Figure 1b. The third network is also based on the radial topology and is extended by five diagonal cables. These cables connect node $n_1$ with $n_8$, $n_2$ with $n_9$, $n_3$ with $n_{10}$, $n_4$ with $n_{11}$ and $n_5$ with $n_{12}$. Each diagonal cable is 85 m long (Figure 1c).

![Network topologies](image)

Figure 1: Networks topologies

Here we allow a voltage range derivation of $\pm 4 \%$. In order to test for the thermal response at each network every node $n_1$ to $n_6$ is connected to one 20 kW load. Each node from $n_7$ to $n_{12}$ connects one 35 kW load. Afterwards, for the voltage range deviation response we reduce the load on node $n_7$ to $n_{12}$ from 35 kW to 30 kW at each network. If an overload occurs, a new cable of type NAYY-J 150 mm$^2$ is added to the network. The release factor $F_{relief}$ is set to 0.8 for the voltage range derivations and thermal overload response. The network parameters are summarized in table 1.
Table 1: Parameters for the radial, the loop and the meshed network topology

| Transformer (slack node) | \( n_0 \) |
|--------------------------|----------|
| No. of nodes             | \( 12 + n_0 \) |
| Cable type               | NAYY-J 150 mm$^2$ |

| Cable length between nodes: | |
|-----------------------------|------|
| Radial grid                 | 60 m |
| Closing cable for loop grid | 60 m |
| Diagonal cables at meshed grid | 85 m |

| Load at \( n_1 \) to \( n_6 \) | 20 kW (\( \cos(\phi) = 1 \)) |
|-------------------------------|-------------------|
| Thermal response:             |                   |
| Load at \( n_7 \) to \( n_{12} \) | 35 kW (\( \cos(\phi) = 1 \)) |
| Voltage response:             |                   |
| Load at \( n_7 \) to \( n_{12} \) | 30 kW (\( \cos(\phi) = 1 \)) |

3.2. Results

3.2.1. Thermal overload response  
In the case that 35 kW loads are connected at \( n_1 \) to \( n_7 \) the thermal overload response triggers at all three networks. For the radial network the cable with the strongest thermal overload \( c_{I,\text{max}} \) is the cable between node \( n_0 \) and \( n_7 \). The current on that cable \( |I(c_{I,\text{max}})| \) is 318 A and therefore higher than the rated current of 275 A of that cable. The extension algorithm responds to the overload by placing a new cable between \( n_0 \) and \( n_9 \). This reduces the maximal current on that cable by 43 %. At the loop network the strongest overloaded cable is also the cable between node \( n_0 \) and \( n_7 \). Here the current is 278 A before and 185 A after the extension algorithm responds. In this case a new cable is added between \( n_0 \) and \( n_9 \). The cable with the strongest thermal overload at the meshed grid is the cable between \( n_0 \) and \( n_1 \). The current on that cable is 286 A before the algorithm for thermal overloads responds. The algorithm builds a new cable between \( n_0 \) and \( n_2 \). This reduces the current at the cable between \( n_0 \) and \( n_1 \) by 35 % (see table 2).

Table 2: Electric currents of cables with the strongest thermal overload \( |I(c_{I,\text{max}})| \) and cables by which the networks are extended.

| Network | \( c_{I,\text{max}} \) | \( |I(c_{I,\text{max}})| \) Before ext. | After ext. | Extended cable |
|---------|-------------------|-----------------|----------|----------------|
| Radial  | \( n_0 \) \( n_7 \) | 318 A           | 182 A    | \( n_0 \) \( n_9 \) |
| Loop    | \( n_0 \) \( n_7 \) | 278 A           | 185 A    | \( n_0 \) \( n_9 \) |
| Meshed  | \( n_0 \) \( n_1 \) | 286 A           | 187 A    | \( n_0 \) \( n_2 \) |

3.2.2. Voltage range deviation response  
If the load at all nodes from \( n_1 \) to \( n_7 \) is reduced from 35 kW to 30 kW, the voltage range deviation response triggers at all networks. The highest voltage drop at the radial network occurs at node \( n_{12} = n_{\Delta V_{\text{max}}} \). At that node the voltage drops by 5 % to 380 V. At the radial network the algorithm reacts by placing a new cable between node \( n_0 \) and \( n_8 \). After the extension the maximal voltage drops to 385 V occurs at node \( n_{12} \). For the loop network the node with the highest voltage deviation \( n_{\Delta V_{\text{max}}} \) is also node \( n_{12} \). The voltage drops at this node to 383 V. Therefore, the algorithm extends the network by a new
cable between node $n_0$ and $n_8$. This leads to a maximal voltage deviation at node $n_{12}$ with a voltage drop of 3.5 %. At the meshed network node $n_6$ is the node with the highest voltage deviation $n_{\Delta V_{\text{max}}}$. At this network the highest voltage drop occurs at node $n_6$ by 4.3 % before the network extension. The algorithm responds by adding a new cable between node $n_0$ and $n_2$. After this extension the highest voltage deviation occurs at node $n_6$ with 387 V (see table 3).

Table 3: Voltages at the nodes with the highest voltage deviation $|V(n_{\Delta V_{\text{max}}})|$ before and after the voltage extension response for the radial, the loop and the meshed network.

| Network | Before ext. | After ext. | Extended cable |
|---------|-------------|------------|----------------|
| Radial  | $n_{12}$ 380 V | $n_{12}$ 385 V | $n_0$ $n_8$ |
| Loop    | $n_{12}$ 383 V | $n_{12}$ 386 V | $n_0$ $n_8$ |
| Meshed  | $n_6$ 383 V | $n_6$ 387 V | $n_0$ $n_2$ |

4. Conclusions
This paper introduces an extension algorithm which reacts to critical network states by placing additional cables to an existing network. The algorithm requires the network topology, the current and maximal current of every cable as well as the voltages at all nodes of the network. Furthermore, a maximal voltage deviation range and a release factor must be set. Here we test the algorithm in a case study with a radial, a loop, and a meshed electrical network. The algorithm distinguishes between voltage range deviations and thermal overloads. We found that the algorithm functions for each of these networks adequately and reliably. For all examined networks the algorithm is capable of eliminating thermal overloads and voltage drops by adding one cable to the given networks. Based on these findings it can be concluded that the suggested algorithm offers an easy-to-use approach to estimate investments for electric low voltage networks. Especially for generic low voltage networks where just the network topology is available, the algorithm is capable of estimating network extension needs. Compared to methods for optimal network planning, the algorithm does not need information about routes for additional cables or positions for additional substations. Due to the algorithm design, adding additional cables is the only response for network extensions. Additional extension options are not taken into consideration. Furthermore, no forecast for additional loads and their placement are taken into consideration when placing additional cables. Therefore, the main value of the work is that the algorithm is a flexible and an easy-to-use approach for calculating grid extension costs for large scale network costs assessments. The algorithm works sufficiently to estimate investment needs especially on generic networks of which no detailed information is available but the algorithm is not a tool for detailed network planning.

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The algorithm works well for more complex networks. Networks of any topology can be set up and examined as JUnit tests. These tests are available at the project homepage.

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1 The algorithm works for more complex networks. Networks of any topology can be set up and examined as JUnit tests. These tests are available at the project homepage.
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