Prediction of Interpolants in Zero Diluted Images

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Abstract — This paper provides an algorithmic procedure to predict interpolants of zero diluted images. Given a digital image, one can zero dilute it by right adjoining a column consisting of ‘0s’ to every column except the last column and inserting a row consisting of ‘0s’ below every row except the last row. This yields a new image with a size (2W-1)×(2H-1), where W is the width and H is the height of the original image. Another way of zero diluting an image is by right adjoining a column consisting of ‘0s’ to every column and inserting a row consisting of ‘0s’ below every row. This yields a new image with a size (2W)×(2H), where W is the width and H is the height of the original image. Alternatively, subsampling of an image is carried out by forcing pixel values in the alternate columns and rows to zero. Thus, the size of the subsampled image is reduced to half of the size of the original image. This means 75% of the information in the original image is lost in the subsampled image. On the other hand, zero dilution of an image does not cause loss of information but increases the possibility of predicting more information. The question that arises here is whether it is possible to predict more pixel values, which are called interpolants so that the reconstructed image is an enhanced version of the original image in resolution. In this paper, two novel interpolant prediction techniques, which are reliable and computationally efficient, are discussed. They are (i) interpolant prediction using neighborhood pixel value averaging and (ii) interpolant prediction using extended morphological filtering. These techniques can be applied to predict interpolants in a subsampled image also.

Index Terms — Ground Penetrating Radar, Zero Diluted Imaging, Targeted Buried Object Detection, Interpolant Prediction.

I. INTRODUCTION

Digital zoom-in increases the size of the pixels leaving a pixilated image that often is not even suitable for viewing on screen. Resizing of an image also causes similar kind of pixilation problem. Alternatively, one can enlarge a given image using the technique described below.

A. Concept of Zero Dilation

Fig. 1 shows an array of size 9×9 and its zero diluted version of size 17×17. One can also zero dilute the 9×9 array by adding one column at the right and one more row at the bottom of the array so that one gets zero diluted array of size 18×18, which is the double the size of given array. Depending on requirement, one may choose either of the methods.

![Image](324x406 to 531x559)

An array of size 9x9

| 5 5 5 5 5 5 5 5 5 |
|-------------------|
| 5 5 5 5 5 5 5 5 5 |
| 5 5 5 5 5 5 5 5 5 |
| 5 5 5 5 5 5 5 5 5 |
| 5 5 5 5 5 5 5 5 5 |
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| 5 5 5 5 5 5 5 5 5 |

9x9 array zero diluted resulting in an array of size 17x17

Fig. 1. An array of size 9x9 and its zero diluted form.

Fig. 2 shows a sample gold mine image of size 1177×891 and its zero diluted version of size 2353×1781. This is achieved using the technique demonstrated in Fig. 1. The 0’s are called the ‘interpolants’. The question that arises here is whether it is possible to predict suitable values for the interpolants so that the enlarged image does not show up pixilation effects. In this paper, two techniques of (i) interpolant prediction using neighborhood pixel value averaging and (ii) interpolant prediction using extended morphological filtering are briefly explained. Consider the sample gold mine image of size 1177×891 shown in Fig. 1. After applying the zero dilution algorithm to this image, one gets the zero diluted image of size 2353×1781, which is also shown in Fig. 1.

![Image](364x561 to 491x664)

Gold mine image (Size = 1177×891)

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In what follows, the image reconstruction is carried out using the two interpolant predicting techniques mentioned above.

II. INTERPOLANT PREDICTION TECHNIQUES

A. Interpolant Prediction Using Neighborhood Pixel Value Averaging

This is a computationally intensive process for interpolant prediction.

\[ \text{Scan the subsampled image with the 9-neighborhood window shown in Fig. 3. At every position, the interpolant is evaluated as the average of all available non-zero values using the prediction formula.} \]

\[ \frac{1}{n} \sum_{i=1}^{n} (x(i, j-1)+x(i, j)+x(i, j+1)+x(i-1, j-1)+x(i-1, j)+x(i+1, j-1)+x(i+1, j)+x(i+1, j+1)) \]

where \( n \) is the number of non-zero pixel values. The algorithm is applied to the whole image. Fig. 4 shows the sample gold mine image zero diluted and its pixel averaged version.

B. Interpolant Prediction Using Extended Morphological Filtering

Morphological filtering makes use of two fundamental operations of ‘dilation’ and ‘erosion’, which are defined as follows. Dilation: Let image to be dilated be \( A \) and the structuring element that dilates \( A \) be \( B \). Then the dilation of \( A \) by \( B \) is defined as the Minkowski addition \( D(A,B) = A \oplus B = \text{EXTSUP}_{(x,y)}[A_{x,y} + B(x,y)] \), where \( D_B \) is the domain of the image \( B \), and \( \text{EXTSUP} \) is an operation of supremum over the union of the domains. Erosion: Let image to be eroded be \( A \) and the structuring element that erodes \( A \) be \( B \). Then the erosion of \( A \) by \( B \) is defined as the Minkowski subtraction \( A \ominus B = \text{INF}_{(x,y)}[A_{x,y} - B(x,y)] \), where \( D_B \) is the domain of the image \( B \), and \( \text{INF} \) is an operation of infimum over the intersection of the domains. Following these definitions, morphological filtering operations of Closing and Opening are defined in the following manner. Closing of \( A \) by \( B \) is represented as \( A \circ B \) and defined as \( A \circ B = (A \oplus B) \ominus B \). Opening of \( A \) by \( B \) is represented as \( A \bullet B \) and defined as \( A \bullet B = (A \ominus B) \oplus B \). Same structuring element should be used for dilation and erosion in any morphological filtering. Now, \( (A \circ B) \circ (A \circ B) = A \circ B \) and \( (A \bullet A) \bullet (A \bullet B) = A \bullet B \), which means closing and opening are idempotent operators.

1. Extended morphological filters

These are essentially morphological filters except that the structuring element need not remain the same for dilation and erosion in any morphological filtering. With reference to Fig. 5, for example, one can use the structuring element E11337799 for dilation and subsequently E1379 for erosion, so that extended closing operation is carried out on a given image. The structuring elements of E11337799 and E1379 are shown in the respective dialog boxes. Extended morphological filtering is a computationally less intensive process for interpolant prediction. At every position, the interpolant is evaluated using the procedure given below. Scan the given subsampled image and dilate it with E-11337799 shown in Fig. 5. Subsequently, erode with E-1379 shown in Fig. 5. The resulting image is extended morphological filtered version, which is also the interpolant predicted version. By extended morphological filtering, one can reconstruct the original image from the subsampled version, which means the lost information is predicted. Let us take some sample images and do the process of
subsampling and extended morphological filtering. Fig. 8 shows a sample image of a gold mine, its subsampled version, the subsampled image dilated using structuring element E-11337799 and the dilated image eroded again using structuring element E-1379. Scan subsampled image with E-11337799. At every position, find the maximum image pixel value and assign it to the central pixel and thus dilation is carried out. Then, scan dilated version of subsampled image with E-1379. At every position, find the minimum image pixel value and assign it to the central pixel and thus erosion is carried out.

2. Information gain due to zero dilution and reconstruction using extended morphological filtering

Given an image of size 100×100, size of its zero diluted version turns out to be 199×199. The number of pixels in the zero diluted version is 39,601, whereas number of pixels in the original image is 10,000. This means 396.01% additional predicted information is added to the original image by zero diluting it followed by extended morphological filtering.

3. Comparative study of prediction using neighborhood pixel value averaging and extended morphological filtering

A comparative study is made on the two interpolant prediction techniques in terms of visual quality analysis. Visual quality parameters include Trade of Threshold (ToT), entropy and visual quality. ‘Statistical parameters’ is also another measure by which one can compare the two techniques. However, due to space restriction, this paper provides results of comparative study based on visual quality parameters only.

C. Visual Quality Parameters of Gold Mine Image, Its Subsampled Image, and Reconstructed Images Using Prediction Formula and Extended Morphological Filter

Visual quality increases as one decreases the entropy of an image. The entropy of an image refers to the degree of randomness in the given image. Noise adds to entropy but reduces visual quality and filtering reduces entropy but increases visual quality. For better visual quality and considerable entropy, one has to accept a trade-off between the two. Visual quality is generally improved by quantizing an image with a threshold value (Th). The number of segments in an image depends on threshold value (Th). More the threshold value, less the number of segmented regions in an image. Less the number of segments, more is the visual quality. The visual quality parametric values of the original gold mine image, its zero diluted and the reconstructed images using extended morphological filter are presented in Table 1.

| TABLE I: VISUAL QUALITY PARAMETERS OF THE ORIGINAL GOLD MINE IMAGE, ITS ZERO DILUTED IMAGE AND THE RECONSTRUCTED IMAGES USING PIXEL AVERAGING AND EXTENDED MORPHOLOGICAL FILTERING METHODS |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| Gold mine image | Gold mine image |
| (Size = 1177×891)| zero diluted    |
| (Size = 2353×1781)|                 |
| Th               | Visual Quality  | Entropy         | Th               | Visual Quality  |
|                 |                 |                 |                 |                 |
| 0                | 0.004390682     | 99.99566932    | 0                | 0.004390682     |
| 1                | 0.00239622      | 99.99876038    | 1                | 0.004390682     |
| 2                | 0.00239622      | 99.99876038    | 2                | 0.004390682     |
| 3                | 0.008200575     | 99.9917943     | 3                | 0.017371828     |
| 4                | 0.029464855     | 99.97035144    | 4                | 0.017371828     |
| 5                | 0.072356261     | 99.92724374    | 5                | 0.017371828     |
| 6                | 0.117891812     | 99.8216136     | 6                | 0.017371828     |
| 7                | 0.327164785     | 99.67385322    | 7                | 0.014496717     |
| 8                | 0.586150374     | 99.4134963     | 8                | 0.014496717     |
| 9                | 0.957274053     | 99.04272595    | 9                | 0.081060579     |
| 10               | 1.440345111     | 98.55985489    | 10               | 0.081060579     |

Gold mine image zero diluted. (Size = 2353×1781)
Fig. 7 shows graphical presentation of the values of ToT and HVQT of the original gold mine image of size 117x891, its zero diluted version of size 2352x1781, the reconstructed image by interpolant prediction using pixel averaging and the reconstructed image by interpolant prediction using extended morphological filtering.

Fig. 7. Graphs of visual quality parametric values of sample image, zero diluted version and the reconstructed forms.

Observations
1. ToT and HVQT in zero diluted image are very poor (ie., ToT = 170 and HVQT = 206).

2. ToT and HVQT in the reconstructed image using prediction formula is pretty much closer to the original image.

3. ToT and HVQT in the reconstructed image using extended morphological filtering is pretty much closer to the original image.

4. There is an increase in the contrast in the reconstructed images when compared to the original image.

III. CONCLUSIONS

Two methods for predicting interpolant pixel values of a zero diluted image (i) interpolant prediction using neighborhood pixel value averaging and (ii) interpolant prediction using extended morphological filtering are discussed briefly in this paper with a real time sample image. While both methods yield appreciable results, interpolant prediction using extended morphological filtering seems to be better than the interpolant prediction using neighborhood averaging, in the sense of prediction accuracy.

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