LACK OF ANGULAR CORRELATION AND ODD-PARITY PREFERENCE IN COSMIC MICROWAVE BACKGROUND DATA

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ABSTRACT
We have investigated the angular correlation in the recent cosmic microwave background data. In addition to the known large-angle correlation anomaly, we find the lack of correlation at small angles with high statistical significance. We have investigated various non-cosmological contamination as well as the Wilkinson Microwave Anisotropy Probe (WMAP) team’s simulated data. However, we have not found a definite cause. In the angular power spectrum of WMAP data, there exists anomalous odd-parity preference at low multipoles. Noting the equivalence between the power spectrum and the correlation, we have investigated the association between the lack of large-angle correlation and the odd-parity preference. From our investigation, we find that the odd-parity preference at low multipoles is, in fact, a phenomenological origin of the lack of large-angle correlation. Further investigation is required to find out whether the origin of the anomaly is cosmological or due to unaccounted systematics. The data from the Planck surveyor, which has systematics distinct from WMAP, will greatly help us to resolve its origin.

Key words: cosmic background radiation – early universe – methods: data analysis – methods: statistical

Online-only material: color figures

1. INTRODUCTION
Over the past several years, there have been great successes in the measurement of cosmic microwave background (CMB) anisotropy by ground and satellite observations (Jarosik et al. 2011; Reichardt et al. 2009; Pryke et al. 2009; Tauber et al. 2010). Since the release of the Wilkinson Microwave Anisotropy Probe (WMAP) data (Hinshaw et al. 2007, 2009; Jarosik et al. 2011), there have been reports on various anomalies (Cruz et al. 2005, 2006, 2007, 2008; de Oliveira-Costa et al. 2004; Copi et al. 2004, 2006, 2007, 2009, 2010; Schwarz et al. 2004; Land & Magueijo 2005a, 2005b, 2007; Rakic & Schwarz 2007; Park 2004; Chiang et al. 2003; Eriksen et al. 2004; Hansen et al. 2009; Hoftuft et al. 2009; Kim & Naselsky 2010a, 2010b, 2010c; Gruppuso et al. 2011; Bennett et al. 2011). In particular, there are reports on the lack of angular correlation at large angles, which are observed in COBE-DMR data and subsequently in WMAP data (Hinshaw et al. 1996; Spergel et al. 2003; Copi et al. 2007, 2009, 2010, 2011). In order to figure out the cause of the anomaly, we have investigated non-cosmological contamination as well as the WMAP team’s simulated data. However, we have not found a definite cause, which makes us believe that the anomaly is produced by unknown systematics or may indeed be cosmological.

In the angular power spectrum of WMAP data, anomalous odd-parity preference exists at low multipoles (Land & Magueijo 2005b; Kim & Naselsky 2010a, 2010b, 2010c; Gruppuso et al. 2011). Noting the equivalence between power spectrum and correlation, we have investigated the association between odd-parity preference and lack of large-angle correlation. From our investigation, we find that the odd-parity preference at low multipoles is, in fact, a phenomenological origin of the lack of large-angle correlation. Even though it still leaves the fundamental question of its origin unanswered, the association between seemingly distinct anomalies will help the investigation of whether the underlying origin is cosmological or due to unaccounted systematics.

The outline of this paper is as follows: In Section 2, we briefly discuss the statistical properties of CMB anisotropy. In Section 3, we investigate the angular correlation anomalies of WMAP data and show the lack of correlation at small angles in addition to that at large angles. In Section 4, we investigate non-cosmological contamination and the WMAP team’s simulated data. In Section 5, we show that the odd-parity preference at low multipoles is a phenomenological origin of the lack of the large-angle correlation. In Section 7, we summarize our investigation.

2. ANGULAR CORRELATION OF CMB ANISOTROPY
CMB anisotropy over a whole sky is conveniently decomposed in terms of spherical harmonics:

$$\tilde{T}(\hat{n}) = \sum_{l,m} a_{lm} Y_{lm}(\hat{n}),$$

where $a_{lm}$ and $Y_{lm}(\hat{k})$ are a decomposition coefficient and a spherical harmonic function, respectively. In most inflationary models, decomposition coefficients of CMB anisotropy follow the Gaussian distribution of the following statistical properties:

$$\langle a_{lm} a_{m^*}^\ast \rangle = \delta_{ll} \delta_{mm^*} C_l,$$

where $\langle \ldots \rangle$ denotes the average over an ensemble of universes and $C_l$ denotes the CMB power spectrum. Given CMB anisotropy data, we can estimate two-point angular correlation:

$$C(\theta) = \tilde{T}(\hat{n}_1) \cdot \tilde{T}(\hat{n}_2),$$

where $\theta = \cos^{-1}(\hat{n}_1 \cdot \hat{n}_2)$. Using Equations (1) and (2), we can easily show that the expectation value of the correlation is given by Padmanabhan (1993):

$$\langle C(\theta) \rangle = \sum_l \frac{2l + 1}{4\pi} W_l C_l P_l(\cos \theta),$$

where $\theta$ is a separation angle, $W_l$ is the window function of the observation, and $P_l$ is a Legendre polynomial. From Equation (4), we may easily see that the angular correlation $C(\theta)$ and power spectrum $C_l$ possess some equivalence.
Figure 1. Angular correlation of CMB anisotropy. Solid lines denote the angular correlation of WMAP data. The dotted line and shaded region denote the theoretical prediction and 1σ correlation of WMAP (Spergel et al. 2003; Copi et al. 2007, 2009, 2010, 2011).

(A color version of this figure is available in the online journal.)

3. LACK OF/angular correlation in the WMAP DATA

In Figure 1, we show the angular correlation of the WMAP seven-year data, which are estimated, respectively, from the WMAP team’s Internal Linear Combination (ILC) map and the foreground-reduced maps of the V and W bands. In the angular correlation estimation, we have excluded the foreground-contaminated region by applying the WMAP KQ75 mask, as recommended for non-Gaussianity study (Gold et al. 2011). In the same plot, we show the angular correlation of the WMAP concordance model (Komatsu et al. 2011), where the dotted line and shaded region denote the mean value and 1σ ranges, respectively, of Monte Carlo simulations at the V band. For simulation, we have made $10^4$ realizations with the same configuration with the WMAP data (e.g., a foreground mask, beam smoothing, and instrument noise). In order to include WMAP noise in our simulation, we have subtracted one piece of Differences Assembly (D/A) data from another and added it to simulations.

As shown in Figure 1, non-negligible discrepancy exists between the data and the theoretical prediction. Most noticeably, angular correlation of the WMAP data nearly vanishes at angles larger than ~60°, which were previously investigated by Hinshaw et al. (1996), Spergel et al. (2003), and Copi et al. (2007, 2009, 2010). In the previous investigations, the lack of large-angle correlation was assessed by the following statistic (Spergel et al. 2003; Copi et al. 2007, 2009, 2010):

$$S_{1/2} = \int_{-1}^{1/2} (C(\theta))^2 d(\cos \theta).$$

(5)

The investigation shows that the $S_{1/2}$ estimated from WMAP data is anomalously low, which requires the chance $\lesssim 10^{-3}$ (Spergel et al. 2003; Copi et al. 2007, 2009, 2010, 2011). Besides the lack of correlation at large angles, we can see from Figure 1 that the correlation at small angles tends to be smaller than the theoretical prediction. Noting this, we have investigated the small-angle correlation with the following statistic:

$$S_{\sqrt{3}/2} = \int_{\sqrt{3}/2}^{1} (C(\theta))^2 d(\cos \theta),$$

(6)

where the square of the correlation is integrated over small angles ($0^\circ \leq \theta \leq 30^\circ$).

In Table 1, we show $S_{1/2}$ and $S_{\sqrt{3}/2}$ of the WMAP seven-year data. Note that the slight difference between the V and W bands is due to the distinct beam size, and simulations are made accordingly for each band. In the same table, we show the $p$-value, where the $p$-value denotes the fraction of simulations as low as those of the WMAP data. As shown in Table 1, the WMAP data have unusually low values of $S_{1/2}$ and $S_{\sqrt{3}/2}$, as indicated by their $p$-values. Note that the $p$-value of $S_{\sqrt{3}/2}$ corresponds to very high statistical significance, even though it may not be as low as that of $S_{1/2}$. Since $S_{\sqrt{3}/2}$ and $S_{1/2}$ correspond to the integrated power at small and large angles, respectively, we find anomalous lack of correlation at small angles in addition to large angles.

In Figure 2, we show $S_{1/2}$ and $S_{\sqrt{3}/2}$, which are estimated from the WMAP three-, five-, and seven-year data, respectively. As shown in Figure 2, the $S$ statistics of WMAP seven-year data are lowest, while WMAP seven-year data are believed to have more accurate calibration and less foreground contamination than earlier releases (Hinshaw et al. 2009; Jarosik et al. 2011; Gold et al. 2011). Therefore, we may not readily attribute the anomaly to calibration error or foregrounds.

We have also slightly varied the partition of $S$ within $\pm 5^\circ$. The $p$-value of $S_{\sqrt{3}/2}$ stays the same when the bound of the partition is set to $25^\circ - 32^\circ$ and increases slightly when the bound is $35^\circ$. For $S_{1/2}$, the $p$-value almost stays the same and decreases even further when the bound of the partition is set to
62°–64°. Therefore, we find that our results are robust to the slight variations in the partition, and the enhancement on the statistical significance by the posteriori choice of the partition is not significant.

4. NON-COSMOLOGICAL CONTAMINATION

The WMAP data contain contamination from residual Galactic and extragalactic foregrounds, even though we have applied the conservative KQ75 mask (Gold et al. 2011). In order to investigate residual foregrounds, we have subtracted the foreground-reduced W-band map from that of the V band. This difference map mainly contains residual foregrounds at the V- and W-band maps with a slight amount of CMB. Note that the CMB signal is not completely canceled out, because the beam sizes at the V and W band differ from each other. From the difference map V(n) − W(n), we have obtained $S_{1/2} = 0.31$ and $S_{\sqrt{2}/2} = 31.36$. By comparing these values with those in Table 1, we can see that residual foregrounds at the V and W band are too small to affect the correlation power of the WMAP data.

There is instrument noise in the WMAP data. 1/f noise, when coupled with WMAP scanning pattern, may result in less accurate measurements at certain angular scales (Hinshaw et al. 2003, 2007; Rieke 2002). In order to investigate the association of noise with the anomaly, we have produced noise maps of WMAP seven-year data by subtracting one D/A map from another of the same frequency channel. In Table 2, we show $S_{1/2}$ and $S_{\sqrt{2}/2}$ estimated from the noise maps. Comparing Table 1 with Table 2, we can see that the noise is not significant enough to cause the correlation anomalies of the WMAP data.

In Figure 3, we show the values of $S_{1/2}$ and $S_{\sqrt{2}/2}$ for each year and D/A data set. As shown in Figure 3, we find that the anomaly is not associated with a particular D/A channel or a particular year’s data, but is present at all years and D/A channels.

Besides the contamination discussed above, there are other sources of contamination, such as sidelobe pickup. In order to investigate these effects, we have investigated simulations produced by the WMAP team. According to the WMAP team, time-ordered data (TOD) have been simulated with realistic noise, thermal drifts in instrument gains and baselines, smearing of the sky signal due to finite integration time, transmission imbalance, and far-sidelobe beam pickup. Using the same data pipeline used for real data, the WMAP team have processed simulated TOD and produced maps for each D/A and each year. From the simulated maps, we have estimated $S_{1/2}$ and $S_{\sqrt{2}/2}$, which are plotted in Figure 4. As shown in Figure 4, $S$ statistics of simulated data are significantly higher than those of WMAP data. Therefore, the anomaly may be produced by unknown systematics or may indeed be cosmological.

Table 2
The $S$ Statistics of WMAP Instrument Noise in ($\mu$K$^2$)

| Data   | $S_{1/2}$ | $S_{\sqrt{2}/2}$ |
|--------|-----------|------------------|
| V1–V2  | 0.25      | 83.94            |
| W1–W2  | 2.49      | 587.45           |
| W1–W3  | 2.18      | 664.26           |
| W1–W4  | 2.24      | 625.27           |
| W2–W3  | 2.72      | 808.32           |
| W2–W4  | 4.39      | 764.96           |
| W3–W4  | 4.39      | 764.96           |

Figure 3. $S$ statistics of WMAP data at each D/A and year.
(A color version of this figure is available in the online journal.)

5. ODD-MULTIPOLE PREFERENCE IN CMB POWER SPECTRUM DATA

Without the loss of generality, we may consider the CMB anisotropy field as the sum of even- and odd-parity functions:

$$T(\hat{n}) = T^+(\hat{n}) + T^-(\hat{n}),$$  \hspace{1cm} (7)

where

$$T^+(\hat{n}) = \frac{T(\hat{n}) + T(-\hat{n})}{2},$$  \hspace{1cm} (8)

$$T^-(\hat{n}) = \frac{T(\hat{n}) - T(-\hat{n})}{2}. \hspace{1cm} (9)$$

Using Equation (1) and the parity property of spherical harmonics $Y_{lm}(\hat{n}) = (-1)^l Y_{lm}(-\hat{n})$ (Arfken & Weber 2000),
we may show

\[ T^+(\hat{n}) = \sum_{lm} a_{lm} Y_{lm}(\hat{n}) \cos^2 \left( \frac{l \pi}{2} \right), \]  
\[ T^-(\hat{n}) = \sum_{lm} a_{lm} Y_{lm}(\hat{n}) \sin^2 \left( \frac{l \pi}{2} \right). \]  

\[ (C(\theta)|_{\epsilon > 0})^2 < (C(\theta)|_{\epsilon = 0})^2 \quad (60^\circ \leq \theta \leq 180^\circ). \]  

From Equation (14), we can see that the odd-parity preference (i.e., \( \epsilon > 0 \)) leads to the lack of large-angle correlation power.

We emphasize that the lack of large-correlation is associated with the odd-parity preference at low multipoles (i.e., power excess at even multipoles and power deficit at odd multipoles). On the other hand, simple suppression of overall low multipole power does not necessarily lead to the lack of large-angle correlation.
correlation. For instance, suppressing octupole power, which mitigates the odd-parity preference, instead increases the large-angle correlation power. In Figure 7, we show $S_{1/2}$ of the WMAP team’s ILC map, where the octupole components are multiplied by the suppression factor $r$ to the quadrupole component of the map. From Figure 7, we can see that the large-angle correlation power increases, as the octupole component is more suppressed.

6. POSSIBLE COSMOLOGICAL ORIGIN

As discussed previously, we have not found a definite non-cosmological cause for the discussed anomaly. Therefore, in this section, we consider possible cosmological origins. Since primordial fluctuations, which were once on sub-Planckian scales, are stretched to the observable scales by inflation, trans-Planckian effects may leave imprints on a primordial power spectrum. Decomposition coefficients are related to primordial perturbation as follows:

$$a_{lm} = 4\pi (-i)^l \int \frac{d^3k}{(2\pi)^3} \Phi(k) g_l(k) Y_{lm}^*(\hat{k}),$$

where $\Phi(k)$ is the primordial perturbation in Fourier space and $g_l(k)$ is a radiation transfer function. Using Equation (15), we can show that the decomposition coefficients of CMB anisotropy are given by

$$a_{lm} = \frac{(-i)^l}{2\pi^2} \int_0^\infty dk \int_0^\pi d\theta_k \sin \theta_k \times \int_0^\pi d\phi_k g_l(k) Y_{lm}^*(\hat{k}) \Phi(k) + (-1)^l \Phi^*(k),$$

where we use the reality condition $\Phi(-k) = \Phi^*(k)$ and $Y_{lm}(-\hat{n}) = (-1)^l Y_{lm}(\hat{n})$. Using Equation (16), it is trivial to show, for the odd number multipoles $l = 2n - 1$,

$$a_{lm} = -\frac{(-i)^{l-1}}{\pi^2} \int_0^\infty dk \int_0^\pi d\theta_k \sin \theta_k \times \int_0^\pi d\phi_k g_l(k) Y_{lm}^*(\hat{k}) \text{Im} [\Phi(k)],$$

and, for even number multipoles $l = 2n$,

$$a_{lm} = \frac{(-i)^l}{\pi^2} \int_0^\infty dk \int_0^\pi d\theta_k \sin \theta_k \int_0^\pi d\phi_k g_l(k) Y_{lm}^*(\hat{k}) \text{Re} [\Phi(k)].$$

It should be noted that the above equations are simple reformulations of Equation (15) and are exactly equal to it. From
Equations (16) and (17), we can see that the observed odd-parity preference might be produced, provided

$$|\text{Re}(\Phi(k))| \ll |\text{Im}(\Phi(k))| \quad (k \lesssim 22/\eta_0),$$

where $\eta_0$ is the present conformal time. Taking into account the reality condition $\Phi(-k) = \Phi^*(k)$, we can show that the primordial perturbation in real space is given by

$$\Phi(x) = 2 \int_0^\infty dk \int_0^\pi d\theta_k \sin \theta_k \int_0^\pi d\phi_k (\text{Re}(\Phi(k))) \cos(k \cdot x)$$

$$- \text{Im}(\Phi(k)) \sin(k \cdot x).$$

Noting Equations (18) and (19), we find that our primordial universe may possess odd-parity preference on large scales $(2/\eta_0 \lesssim k \lesssim 22/\eta_0)$. This explanation requires the violation of the large-scale translational invariance, putting us at a special place in the universe. However, it is not in direct conflict with the current data on the observable universe (i.e., WMAP CMB data), and the invalidity of the Copernican Principle such as our living near the center of void has already been proposed in a different context (Alexander et al. 2009; Clifton et al. 2008).

Independently, some theoretical models exist that predict a parity-odd local universe (Urban & Zhitnitsky 2011; Zhitnitsky 2011). In these models, some level of non-zero temperature and B mode polarization (TB) and E and B mode polarization (EB) correlations are predicted as well (Urban & Zhitnitsky 2011).

Depending on the type of cosmological origins, distinct anomalies are predicted in the polarization power spectrum and correlations (e.g., TB, EB). Therefore, polarization maps of large-sky coverage (i.e., low multipoles) will allow us to remove degeneracy and determine a cosmological origin, provided the odd-parity preference is indeed cosmological.

7. DISCUSSION

We have investigated angular correlation in the recent CMB data. In addition to the well-known correlation anomaly at large angles, we find a lack of correlation at small angles with high statistical significance.

In the angular power spectrum of WMAP data, anomalous odd-parity preference exists at low multipoles (Land & Magueijo 2005b; Kim & Naselsky 2010a, 2010b, 2010c; Gruppuso et al. 2011). The angular power spectrum and angular correlation possess some equivalence. Noting this, we have investigated the association between the lack of correlation and the odd-parity preference. We find that the odd-parity preference is, in fact, a phenomenological origin of the correlation anomaly (Kim & Naselsky 2010a, 2010b; Gruppuso et al. 2011).

We have investigated non-cosmological contamination and the WMAP team's simulated data. However, we have not found a definite cause. The Planck surveyor data possesses wide frequency coverage and systematics distinct from WMAP. Therefore, it may allow us to resolve its origin. Most of all, Planck's polarization data, which have low noise and large-sky coverage, will greatly help us to understand the underlying origin of the anomaly.

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