Stability study of magnetoelastic damper of high-speed machines

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Abstract. The article describes a new device for magnetoelastic damper of high-speed machines. The operation of the device is based on the strength of magnetic levitation and magnetoelastic interaction of the opposite parts. The stability of the damper by the Lyapunov criterion is analyzed. When using the known criterion, particular criteria for the stability of the device were added. Due to the poor convergence of numerical calculations, analytical research methods for specific individual expressions were used. Achieving the goal is solved using the analytical mathematical apparatus.

1. Introduction

The current level of technological development is characterized by a tendency to increase the speeds of rotation of the working details, an increase in static and shock loads, which increases the likelihood of more frequent failure of bearings. In the process of operation of high-speed machines, one of the main causes of wear is friction and vibration of bearing assemblies [1].

The trend towards an increase in technological operating modes requires the development of more reliable component parts of the system. Classical varieties of rolling and sliding bearings do not allow for minimal energy losses [2]. The physical principle of operation of the sliding and rolling bearings limit the speed of the equipment [3-4]. Magnetic dampers have a tendency to stable operation in all operating modes, including when critical speeds are reached [5].

2. Relevance

The development and implementation of permanent magnet magnetic bearings with an adaptive effect is an urgent task. Magnetic bearing is one of the main components in various technical and electromechanical products, for example, turboexpander, gas turbine units, electric motors, etc. [6-7]. The development of self-regulating damping systems allows automation of the process of monitoring the operation of equipment [8].

The use of magnetoelastic supports avoids most of the main problems during the operation of the units and expand the scope of their application. In [9-12], levitation devices were studied, but these models do not have the adaptability principle. The aim of the work is to study the stability of the model of magnetic bearings with permanent magnets using the Lyapunov criterion.
3. Damper device and research methodology
The device in question is an axial adaptive damper with a passive magnetic force regulator that performs the function of a magnetic support. The device is shown in figure 1.

![Figure 1](image1.png)

**Figure 1.** Adaptive damper: 1 – shaft; 2, 3 – permanent magnets; 4, 5 – bevels; 6 – framework; 7, 8 – bodies; 9 – groove.

In the initial idle state, the elastic parts of the damper mounted on the shaft 1 are disconnected. During start-up and operation, with increasing axial vibrations, the thrust bearing 6 with the elastic part installed in it moves towards the rotating elastic part. As a result of this, magnets 2 and 3 located on the contacting cones 4 and 5 are connected sequentially from the periphery to the center, depending on the amplitude of the oscillations. In this case, magnetic forces appear and gradually increase, counteracting axial displacement. The order of inclusion in the work of magnets installed in four rows in a checkerboard pattern occurs sequentially when crossing the allowable vibration limits.

Upon receipt of axial disturbance, the forces are distributed as shown in figure 2.

![Figure 2](image2.png)

**Figure 2.** The distribution of forces in the developed device.

For clarity, the interaction of permanent magnets in the bearing, the magnetic model is replaced by an equipotential mechanical model (figure 3), used in calculating the forces according to the Maxwell tension tensor.
4. System stability studies
The main criterion for the continued performance of any system is stability. We prove the stability of the magnetoelastic damper using the Lyapunov theorem [13].

We use the following assumptions:

- permanent magnets are considered as coaxially located solenoids;
- the magnetic moment of the solenoid is equal to the magnetic moment of permanent magnets;
- permanent magnets of the same brand are used;
- angular displacements of the rotor are not taken into account;
- ambient temperature conditions are constant throughout the entire working process.

The magnetization is known to be determined by the following expression

$$ J = \frac{B_r}{\mu_0} $$  \hspace{1cm} (1)

where: \( B_r \) is the residual induction of permanent magnets; \( \mu_0 \) is the magnetic constant.

For the equation of motion along the axis of motion of the magnets \( z \), the following equality is fair

$$ m \left( \frac{d^3 z}{dt^3} + \frac{d^2 z}{dt^2} \right) = \frac{I_1^2}{2} (\delta_2 - z) \frac{dL}{dz} - \frac{I_1^1}{2} (\delta_1 - z) \frac{dL}{dz} - \frac{B_r^2 S^2 \delta_s^2}{\mu_0} $$ \hspace{1cm} (2)

where: \( I_1, I_2 \) – currents of interacting magnets; \( \delta_1, \delta_2 \) – gaps between the magnets and the axis of the system; \( \delta_s \) is the average diameter of the air gap; \( S \) is the area of interaction of the magnets (the magnets are assumed to be the same); \( L \) is the inductance of the magnets; \( m \) is the mass of the shaft.

We introduce the notation

$$ Q = \frac{I_2^2}{2} (\delta_2 - z) \frac{dL}{dz} - \frac{I_1^1}{2} (\delta_1 - z) \frac{dL}{dz} - \frac{B_r^2 S^2 \delta_s^2}{\mu_0} $$ \hspace{1cm} (3)

We integrate the expression (2)

$$ \frac{d^3 z}{dt^3} + \frac{d^2 z}{dt^2} = \frac{1}{m} \int Q dt $$ \hspace{1cm} (4)

Given the total damping coefficient and the displacement coefficient, we represent the law of motion of the system in the form

$$ I = \alpha z + \beta \frac{dz}{dt} $$ \hspace{1cm} (5)
where: \(\alpha\) – coefficient taking into account the movement; \(\beta\) – damping coefficient of oscillations in the system.

Multiplying expressions (4) and (5), as well as making elementary transformations, we obtain

\[
m \cdot \frac{dz}{dt} \left( \alpha z + \beta \frac{d^2 z}{dt^2} \right) = \left( a z + \beta \frac{dz}{dt} \right) \cdot \left[ Q dt - m \left( az \frac{dz}{dt} - \left( \beta \frac{d^2 z}{dt^2} \right) \right) \right]
\]

(6)

Using the already known transformations of similar expressions for the suspension system can be represented as

\[
\frac{d}{dt} \left( \beta \frac{dz}{dt} \right)^2 + \frac{a(z)^2}{2} + az \frac{dz}{dt} = \left( a - \beta \right) \left( \frac{dz}{dt} \right)^2 + \frac{1}{m} \left( z - \beta \frac{dz}{dt} \right) \cdot Q dt
\]

(7)

The integrand is a negative definite function. The left side of differentiable expression (7) is a Lyapunov function \([14]\). Its right side is its derivative. The left side is a definitely positive function, and the right side is its derivative with the opposite sign, which, according to Lyapunov's theorem, indicates the stability of the system.

### 5. Conclusion

For reliable retention of the rotor when receiving axial vibrations, it is advisable to use magnetoelastic dampers with permanent magnets.

The adaptability property of the device in question makes it possible to increase the stability region in critical equipment coasts situations.

The advantage of the adaptability of the device is in the versatility of use for various operating conditions of the equipment.

The use of the Lyapunov function allows one to prove the stability of the equilibrium position of the magnetoelastic damper system. This method is relevant for studying the stability of various differential equations and systems.

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