Optimal scheduling of energy storage resources

James Cruise∗ and Stan Zachary†

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Abstract

It is likely that electricity storage will play a significant role in the balancing of future energy systems. A major challenge is then that of how to assess the contribution of storage to capacity adequacy, i.e. to the ability of such systems to meet demand. This requires an understanding of how to optimally schedule multiple storage facilities. The present paper studies this problem in the cases where the objective is the minimisation of expected energy unserved (EEU) and also a form of weighted EEU in which the unit cost of unserved energy is higher at higher levels of unmet demand. We also study how the contributions of individual stores may be identified for the purposes of their inclusion in electricity capacity markets.

1 Introduction

In electricity systems there is a need to keep supply and demand carefully balanced at all times. However, the increasing penetration of renewable generation means that future systems are likely to characterised by much greater variability and uncertainty on the supply side, while patterns of demand will continue to vary considerably according to the time of day. It is thus likely that electricity storage will play a significant role in balancing such future systems—see [8] and, for a recent analysis of the impact of storage in European markets, see [12]. The problem of the optimal operation and control of storage may be viewed in several ways. For the storage operator much of the value of storage may be realised in price arbitrage, i.e. in buying electricity when it is cheap and selling it when it is expensive (see [3, 16] and the references therein) and in the provision of buffering and ancillary services (see [1, 4, 5, 10, 14]). However, for society and for the electricity system operator, a major concern is that of capacity adequacy, i.e. of ensuring that there is sufficient available supply to be able to meet demand in all but the most exceptional circumstances. Here, as indicated above, storage facilities may be used to cover periods of what would otherwise be shortfall in supply, and it is necessary to assess the contributions of such storage facilities and to value them individually for inclusion in capacity markets. For example, in Great Britain such valuation of individual storage facilities

∗Heriot-Watt University. Research supported by EPSRC grant EP/I017054/1
†University of Edinburgh. Research supported by EPSRC grant EP/I017054/1
is now considered by the system operator, National Grid plc, in its annual electricity capacity reports, see [11].

It is typically the case that continuous periods of what would otherwise be shortfall and which are to be covered by storage are well separated in time, so that stores may be fully recharged between such periods. (In Great Britain and in many other countries, for example, the system is presently only significantly at risk during a single evening peak period, and there is ample time for overnight recharging.) Thus in analysing the contributions of given storage facilities, it is typically sufficient to consider single periods of possible shortfall during which no recharging of stores may take place. However, within any such analysis there now arises the problem of the optimal scheduling of multiple stores, with the objective of minimising, for example, unserved energy. This problem arises because individual stores are subject to both capacity and rate constraints, and there is a danger that, if the rates at which these stores are used are not well coordinated over time, one may arrive at some time at which there is sufficient remaining energy in the stores to meet future needs, but that this energy is contained in too few stores which cannot between them supply that energy at the required rates.

As discussed above, it is only recently that the problem of optimal scheduling of multiple stores has become important in the analysis of electricity capacity adequacy, and perhaps for this reason it does not appear to have received much attention in the literature. However, there is some recent work by Evans et al. [9] which we discuss in Section 3. The present paper further considers this optimal scheduling problem. In Section 2 we present the underlying model, which is that of a nonnegative demand process over some given continuous period of time, together with a set of stores, each with given rate and capacity constraints, which may not be recharged during that period of time and which are to be used to serve that demand process as far as possible. The demand process might be a process of remaining energy demand after initial demand had been met as far as possible from sources such as generation. This process may be sufficiently well known in advance as to be capable of being modelled as deterministic. Alternatively it may be necessary to treat it as stochastic.

We consider also possible objective functions for the scheduling of the stores. In Section 3 we give a simple necessary and sufficient condition for a given demand process to be completely satisfiable by a given set of stores as described above. In Section 4 we study the problem of scheduling stores for the minimisation of expected energy unserved. It turns out that in this case the optimal decision at any time may be made independently of the demand process subsequent to that time, so that it is possible to construct a solution which is optimal in both a deterministic and a stochastic environment. In Section 5 we study more briefly the important variation of the scheduling problem in which the cost of unserved energy is higher at higher levels of unmet demand; here in a stochastic environment the solution of the problem is much more difficult. Finally, in Section 6 we give a number of illustrative examples.
2 Model

In this section we give our model for a demand process over some given period of time, together with a set of stores whose energy is to be used to meet that demand as far as possible in accordance with some criterion. (As previously remarked, the demand process corresponds to that energy demand which requires specifically to be met from storage.) Thus the model is defined by the following two components:

(a) a nonnegative demand process \((d(t), t \in [0, T])\) defined over some continuous period of time \([0, T]\); this demand process may be deterministic or stochastic;

(b) a set \(S\) of stores, where each store \(i \in S\) is characterised by its rate (power) constraint \(P_i\) and capacity (energy) constraint \(E_i\); these are such that the store \(i\) may serve energy at any rate \(r_i(t)\) for each time \(t \in [0, T]\) subject to the constraints

\[
0 \leq r_i(t) \leq P_i, \quad t \in [0, T],
\]

and

\[
\int_0^T r_i(t) \, dt \leq E_i.
\]

Thus it is assumed in particular, from (2), that stores may not recharge energy during the time period \([0, T]\).

Define also \(r_S(t) = \{r_i(t), i \in S\}\) to be the set of rates at which energy is served by the stores in \(S\) at each time \(t \in [0, T]\). We shall say that a policy for the management of the set \(S\) of stores over the time period \([0, T]\) is a process \((r_S(t), t \in [0, T])\) of such rates such that the constraints (1) and (2) are satisfied and additionally, without loss of generality, is such that

\[
\sum_{i \in S} r_i(t) \leq d(t), \quad t \in [0, T].
\]

When the demand process \((d(t), t \in [0, T])\) is deterministic we require that any policy \((r_S(t), t \in [0, T])\) for the use of the stores as above should also be deterministic. When the demand process \((d(t), t \in [0, T])\) is stochastic then we allow that, at each time \(t\), the set of rates \(r_S(t)\) may depend on the realised value \((d(u), u \in [0, t])\) of the demand process to time \(t\).

Given any policy \((r_S(t), t \in [0, T])\) for the management of the stores as above, let the function \(w\) defined on the positive real line be such that \(w(d)\) is the cost per unit time associated with a given level \(d\) of the residual demand process \((d(t) - \sum_{i \in S} r_i(t), t \in [0, T])\). Then the problem of optimally scheduling the use of the stores typically becomes:

\[
\textbf{P}: \text{choose a policy } (r_S(t), t \in [0, T]) \text{ to minimise}
\]

\[
\mathbf{E} \int_0^T w\left(d(t) - \sum_{i \in S} r_i(t)\right) \, dt,
\]

where \(\mathbf{E}\) denotes the expectation operator. (The latter is only required when the demand process \((d(t), t \in [0, T])\) is stochastic.) The case \(w(d) = d\) corresponds to the minimisation of expected energy unserved (EEU). A complete solution to the
optimal scheduling problem in this case, valid in both deterministic and stochastic environments, is given in Section 4.

However, it is often the case that the unit cost of unserved energy is higher at higher levels of unmet demand; for example, modest levels of unmet demand may often be dealt with by actions such as voltage reduction, while, at the other extreme, the highest levels of unmet demand may result in major blackouts and consequent societal disruption. Thus it may sometimes be natural to consider a function \( w \) which, instead of being linear, is more generally increasing and convex with \( w(0) = 0 \).

Here the optimal scheduling problem \( P \) corresponds to the minimisation of a form of weighted EEU in which the marginal cost of unserved energy is an increasing function of the level of residual demand. The solution to this problem is considerably more complicated and is discussed in Section 5.

3 Storage and demand profiles

In this section we consider the question of whether any given demand process \((d(t), t \in [0, T])\) may be completely satisfied by some given set \( S \) of stores, i.e. whether there exists a policy \((r_S(t), t \in [0, T])\) (satisfying the constraints (1) and (2)) such that

\[
\sum_{i \in S} r_i(t) = d(t), \quad t \in [0, T].
\]  

We give a simple, and readily testable, necessary and sufficient condition for this to be possible. This makes it easy to check ahead of time whether a given set of stores is adequate to meeting a given demand process, and, if necessary, to appropriately adjust that set. Central to establishing the sufficiency of the condition is the use of a particular policy for the prioritisation of the use of stores at any time. Let \( E_i(t) \) be the residual energy in each store \( i \in S \) at each time \( t \); note that \( E_i(0) = E_i \); define also the residual time of each store \( i \) at each time \( t \) as \( E_i(t)/P_i \) (this is the length of further time for which the store \( i \) could supply energy at its maximum rate). Suppose that at time \( t \) it is desired to serve energy at a total rate \( \hat{d}(t) \), where \( \hat{d}(t) \leq \sum_{i \in S: E_i(t) > 0} P_i \) (the rate \( \hat{d}(t) \) might be the demand \( d(t) \) to be met at time \( t \) or might be some lesser rate); group the stores according to their residual times \( E_i(t)/P_i \) at that time (so that two stores belong to the same group if and only if their residual times are equal); rank the groups in descending order of their residual times, and select in this order just sufficient groups of stores such that, using these stores at their maximum rates (i.e. each selected store \( i \) is used at a rate \( r_i(t) = P_i \), the required total rate \( \hat{d}(t) \) is met; the exception to this rule is that, in order to exactly meet the total rate \( \hat{d}(t) \), the stores in the last group thus selected may only require to be used at some fraction \( \lambda \) of their maximum rates, so that each store \( i \) in this last group is required to supply energy at rate \( r_i(t) = \lambda P_i \) for some common \( \lambda \leq 1 \). We shall refer to a policy in which, at each time \( t \), the use of stores is prioritised as above as a longest residual time first (LRTF) policy. It is uniquely determined given the total rate \( \hat{d}(t) \) at which energy is served at each time \( t \in [0, T] \). This policy is also considered in [9] which shows that any process \((\hat{d}(t), t \in [0, T])\) which can be entirely satisfied by a given set of stores can be satisfied by supplying
energy via the corresponding LRTF policy. Thus in particular we may check whether a given demand process \((d(t), t \in [0, T])\) may be thus satisfied by simply checking numerically whether the LRTF policy succeeds in doing so. However, a simpler and more readily checkable condition is desirable.

The effect of any LRTF policy, as time \(t\) progresses, is to gradually equalise the residual lifetimes \(E_i(t)/P_i\) over stores; further once the residual lifetimes of any set of stores have become equal they remain so. As also noted in [9], this latter property has the immediate consequence of preserving through time the ordering of stores by their residual times, i.e. under an LRTF policy, for any pair of stores \(i, j \in S\) and for any time \(t\),

\[
E_i(t)/P_i \geq E_j(t)/P_j \quad \Rightarrow \quad E_i(t'/P_i \geq E_j(t'/P_j \quad \text{for all times } t' > t. \quad (6)
\]

Suppose now that a given demand process \((d(t), t \in [0, T])\) is served via the use of the unique greedy LRTF policy, where by this it is meant that the total rate \(\sum_{i \in S} r_i(t)\) at which demand is served at each time \(t\) is given by

\[
\sum_{i \in S} r_i(t) = \min\left(d(t), \sum_{i \in S: E_i(t) > 0} P_i\right), \quad (7)
\]

and this demand is then served using the corresponding LRTF policy. (Note that this policy is available whether the demand process is deterministic or stochastic, the latter since the rates \(r_S(t)\) at each time \(t\) do not depend on the demand process subsequent to time \(t\).) Let \(S_e\) and \(S_{ne}\) be respectively the sets of stores which, under this policy, are empty and nonempty at time \(T\). It is an immediate consequence (which we subsequently require) of the definition of the greedy LRTF policy—in particular of (6) and (7)—that, again under this policy, the use of the stores in the set \(S_{ne}\) is prioritised over those in the set \(S_e\) throughout the entire time period \([0, T]\) so that, for all \(t\),

\[
\sum_{i \in S_{ne}} r_i(t) = \min\left(d(t), \sum_{i \in S_{ne}} P_i\right). \quad (8)
\]

We now consider whether any given demand process (which we here treat as if it were known in advance) may be totally satisfied by a given set of stores. In doing so it is clear that we may reorder the succession of time instants within the period \([0, T]\) so that this demand process is (weakly) decreasing over time. Thus, given any demand process \((d(t), t \in [0, T])\), it is convenient to define its corresponding demand profile \((d^*(t), t \in [0, T])\) (often referred to in the engineering literature as the load duration curve, see, e.g. [2]) as the given demand process reordered as above, i.e. \((d^*(t), t \in [0, T])\) is the unique nonincreasing nonnegative process on \([0, T]\) such that, for all \(d' \geq 0\),

\[
m\{t : d^*(t) \leq d'\} = m\{t : d(t) \leq d'\}, \quad (9)
\]

where the function \(m\) applied to any set of times within the interval \([0, T]\) defines the total length of that set.\(^1\)

\(^1\)Formally, the function \(m\) denotes Lebesgue measure, and there is an assumption that any demand process is Lebesgue measurable. Since, in applications, time is usually considered as a succession of discrete intervals on each of which everything is constant, there are no practical difficulties here.
Similarly, given the set $S$ of stores as described in Section 2, define its storage profile as the nonnegative function $(s_S(t), t \geq 0)$ defined on the positive half-line, such that, for all $t \geq 0$, $s_S(t)$ is the rate at which energy is supplied at time $t$ when, starting at time 0, every store is discharged continuously at its maximum rate, i.e.

$$s_S(t) = \sum_{i \in S: E_i/P_i \geq t} P_i.$$  

(10)

For example, in the case of a single store with capacity constraint $E$ and rate constraint $P$, its storage profile is the function $s_S$ given by $s_S(t) = P$ for $t \in [0, E/P]$ and $s_S(t) = 0$ for $t > E/P$. Note that storage profiles are additive, i.e. the storage profile of the union of two disjoint sets of stores is the sum of their individual storage profiles. In general, the storage profile $(s_S(t), t \geq 0)$ of a set $S$ of stores is (weakly) decreasing, and it is clear (and is formally a consequence of Theorem 1 below) that, in terms of the model of the present paper, two sets of stores have equivalent capabilities if and only if their storage profiles are the same. Notably, in the case of a single store as above, a given demand process $(d(t), t \in [0, T])$ may be completely served by that store if and only if $d(t) \leq P$ for all $t \in [0, T]$ and $\int_0^t d(t) dt \leq E$. A demand process satisfying these conditions may similarly be served by a set $S$ of stores such that $E_i/P_i$ is the same for all $i \in S$, and $\sum_{i \in S} P_i = P$ and $\sum_{i \in S} E_i = E$: it is only necessary to ensure that at every time all stores are used at rates proportional to their maximal rates, so that the residual lifetimes $E_i(t)/P_i$ are kept equal at all times $t$ and so no store empties before the final time $T$. The following result generalises this observation and give a simple necessary and sufficient condition for a given demand process to be capable of being satisfied by a given set of stores.

**Theorem 1.** A given demand process $(d(t), t \in [0, T])$ may be completely satisfied by a given set $S$ of stores if and only if, for all $t \in [0, T]$,

$$\int_0^t s_S(u) du \geq \int_0^t d^*(u) du,$$

(11)

where, as defined above, $(d^*(t), t \in [0, T])$ is the demand profile (load duration curve) corresponding to the given demand process.

**Proof.** As discussed above, the demand process $(d(t), t \in [0, T])$ may be satisfied by the set $S$ of stores if and only if its corresponding demand profile $(d^*(t), t \in [0, T])$ may be so satisfied. For the latter the energy to be served in any time interval $[0, t]$ is given by the integral on the right side of (11). That it is necessary that the condition (11) holds for all $t \in [0, T]$ now follows from the observation that, for each such $t$, the integral on the left side of (11) is the maximum energy which can be served by the stores in the time interval $[0, t]$.

To show sufficiency suppose now that the condition (11) holds for all $t \in [0, T]$. Suppose also that that the original demand process $(d(t), t \in [0, T])$ is served (as far as possible) using the greedy LRTF policy. Recall that $S_e$ and $S_{ne}$ are then defined to be the set of stores which are respectively empty and nonempty at time $T$. It follows from the earlier observation (8) that, at each time $t \in [0, T]$, the total rate at which energy is then served by the stores in the set $S_{ne}$ is given by $\min(d(t), \hat{P})$, where
\[ \dot{P} = \sum_{i \in S_{\text{se}}} P_i. \] Thus the residual demand process \( (d_e(t), t \in [0, T]) \) to be served by the stores in the set \( S_e \) is given by \( d_e(t) = \max(0, d(t) - \dot{P}) \) for all \( t \in [0, T] \). Since \( d_e(t) \) is an increasing function of \( d(t) \), the residual demand process \( (d_e(t), t \in [0, T]) \) is similarly given by

\[ d^*_e(t) = \max(0, d^*(t) - \dot{P}), \quad t \in [0, T], \quad (12) \]

while the storage profile defined by the stores in the residual set \( S_e \) is given by

\[ s_{S_e}(t) = \max(0, s_S(t) - \dot{P}), \quad t \geq 0. \quad (13) \]

Let \( T' = \sup\{t: d^*(t) \geq \dot{P}\} \); note that \( T' \leq T \). Then

\[
\int_0^T s_{S_e}(t) \, dt \geq \int_0^{T'} s_{S_e}(t) \, dt \\
\geq \int_0^{T'} s_S(t) \, dt - T' \dot{P} \\
\geq \int_0^{T'} d^*(t) \, dt - T' \dot{P} \\
= \int_0^{T} d^*_e(t) \, dt \\
= \int_0^{T} d_e(t) \, dt,
\]

where the second inequality above follows from (13), the third inequality follows from the condition (11), and the immediately succeeding equality follows from the condition (12). Thus the initial energy in the set of stores \( S_e \) is at least as great as the residual demand \( \int_0^T d_e(t) \, dt \) to be met by those stores. Since, by definition, the stores in the set \( S_e \) are empty at time \( T \), it follows that the residual demand process \( (d_e(t), t \in [0, T]) \) is entirely served by those stores by time \( T \).

**4 Minimisation of EEU**

Suppose now that the objective function \( w \) of the optimal scheduling problem \( P \) is given by \( w(d) = d \), so that the problem becomes that of the optimal use of such storage as is available for the minimisation of EEU. In the case where the demand process \( (d(t), t \in [0, T]) \) is deterministic, the optimisation problem \( P \) is then formally a linear programme. However, the inequality nature of the constraints (1) and (2) ensures that this problem has a particularly simple solution, which we discuss below. Further this solution continues to be optimal when the demand process \( (d(t), t \in [0, T]) \) is stochastic.

In identifying optimal policies in either a deterministic or a stochastic environment, it is sufficient to consider those which are greedy, i.e. those in which, as previously, at each successive time \( t \), the total rate \( \sum_{i \in S} r_i(t) \) at which energy is served by the stores in the set \( S \) is given by (7), so that any residual demand at the time \( t \) is reduced as far as possible. Essentially the reason for this is that, when the objective is the minimisation of unserved energy, nothing is to be gained by withholding for later
possible use energy which could have been used to reduce further residual demand at any time $t$; further, when the demand process $(d(t), t \in [0, T])$ is stochastic, there is the risk that any such withheld energy may turn out to be not needed subsequently. It follows that, in the case of a single store, it is clearly always optimal to follow the policy of using the energy of the store as quickly as possible without wasting it—see also Edwards et al. [7] for further analysis of this case.

In the case of multiple stores Edwards et al. [7] consider various heuristic greedy policies, but show that none of these is in general optimal. However, the above simple and obvious result for a single store is a special case of the following more general result for multiple stores.

**Theorem 2.** In either a deterministic or a stochastic environment, the minimisation of EEU over the period $[0, T]$ is achieved by the unique greedy LRTF policy.

**Proof.** Recall from Section 3 that the unique greedy LRTF policy may be followed in either a deterministic or a stochastic environment, and that $S_{ne}$ (which is random when the demand process $(d(t), t \in [0, T])$ is stochastic) is then the set of stores which are nonempty at time $T$ under this policy. As in the proof of Theorem 1, it follows from (8) that at each time $t \in [0, T]$, the total rate at which energy is served by the stores in the set $S_{ne}$ is then given by $\min(d(t), \sum_{i \in S_{ne}} P_i)$. Hence, for every realisation of the demand process $(d(t), t \in [0, T])$, the stores in the set $S_{ne}$ make the maximum contribution of which they are capable to the reduction of unserved energy over the period $[0, T]$. Again under the greedy LRTF policy, the stores in the complementary set $S_e$ are all empty at time $T$ and hence also make the maximum contribution of which they are capable to the reduction of unserved energy over the period $[0, T]$. Thus collectively the stores in the set $S$ minimise EEU over the period $[0, T]$. □

**Remark 1.** In the case where the demand process $(d(t), t \in [0, T])$ is deterministic and hence assumed known, the greedy LRTF policy is typically not the only policy which minimises EEU. For example, the greedy LRTF policy might be applied in reverse time to also minimise EEU. Other policies may also be available in a deterministic environment—see the further discussion of Section 5. However, such alternatives require the demand process to be known in advance and hence are typically not available when the latter is stochastic.

**Remark 2.** Observe that Theorem 2 gives a very simple proof of the earlier result of [9] that any demand process $(d(t), t \in [0, T])$ which can be entirely satisfied by a given set of stores (so that the corresponding EEU is zero) can be satisfied by supplying energy via the corresponding LRTF policy.

### 4.1 Contributions of individual stores

In applications it is often necessary to determine the contribution of any individual store $i \in S$ to the overall reduction in EEU achieved by the stores in the set $S$. We define this contribution to be $EEU(S \setminus \{i\}) - EEU(S)$ where, for any set of stores $S' \subseteq S$, the quantity $EEU(S')$ denotes the minimised EEU achieved by the
use of the stores in the set $S'$. It follows from Theorem 2 that this minimisation may be achieved via the use of the unique greedy LRTF policy. Consideration of that policy as in the proof of that theorem, or indeed standard linear programming theory, shows that if a store $i \in S$ belongs to the set $S_{ne}$ of stores which, under that policy, are nonempty at time $T$, then the contribution of store $i$ to the minimisation of EEU is the same as if it were not capacity constrained. If, on the other hand, the store $i$ belongs to the complementary set $S_e$, then it is easy to see that its contribution to the minimisation of EEU is given by its energy constraint $E_i$. This again follows from the consideration of the greedy LRTF policy: if the store $i \in S_e$ is removed from the set $S$ of available stores, then the set $S_{ne}$ remains as before and, from (8), the stores in that set contribute the same amount of energy as before; similarly each remaining store $j \in S_e$ continues to contribute its total energy $E_j$.

Of particular interest in energy applications is the determination of the equivalent firm capacity (EFC) of any store $i \in S$. This is now used, for example in Great Britain, in determining the value of storage (and also variable generation) within electricity capacity markets—see [11]. A formal definition of EFC is given, for example, in [17]. For present purposes, the EFC $efc_i$ of any store $i$ may be defined as that level of firm capacity (ability to supply energy at any rate up to a given constant) which is able to substitute for the store $i$ within the set $S$ while achieving the same value of minimised EEU. We consider the case where the contributions of individual stores to the overall reduction in unserved energy are marginal, i.e. relatively small. This is commonly the situation in electricity capacity markets—see [13]. For any $S' \subseteq S$ define $LOLE(S')$ to be the loss-of-load expectation (see [2], and also [6] for a description of the use of this metric in GB capacity adequacy analysis) associated with the minimised EEU achieved by (the sole use of) the set of stores $S'$, when the stores in this set are scheduled via the use of the greedy LRTF policy. This is the expectation of the total length of time for which the use of the set of stores $S'$ in accordance with the greedy LRTF policy fails to meet fully the demand process $(d(t), t \in [0,T])$. We allow that the set $S'$ may be random, as is the case for the set $S_{ne}$ when the demand process $(d(t), t \in [0,T])$ is stochastic. We then have the following result

**Theorem 3.** The EFC $efc_i$ of any store $i \in S$ contributing marginally to the reduction of EEU is given by

$$efc_i = \frac{EEU(S' \setminus \{i\}) - EEU(S)}{LOLE(S_{ne})}. \quad (14)$$

**Proof.** For any $S' \subseteq S$ and for any $z \geq 0$, let $EEU(S', z)$ denote the minimised EEU achieved by the use of the set of stores $S'$ supplemented by firm capacity equal to $z$ (note that the latter makes the same contribution as a store able to supply energy at rate $z$ and with very large or unlimited capacity). The function $EEU(S', z)$ is easily seen to be convex in $z$; let $EEU'(S')$ denote its (right) derivative with respect to $z$ at $z = 0$. We then have in particular that

$$EEU'(S) = -LOLE(S_{ne}). \quad (15)$$

To see this, suppose that the set $S$ is supplemented by firm capacity $\delta z > 0$ where $\delta z$ is small. Under the greedy LRTF policy the latter may be considered to be
scheduled as an additional store with rate $\delta z$ and no capacity constraint. Since $\delta z$ is small, under this policy the stores in the set $S_{ne}$ also continue to behave as if they were capacity unconstrained and their use continues to be completely prioritised over that of the stores in the set $S_e$. Thus, taking expectations and noting that no additional problems are caused by the fact that in a stochastic environment the set $S_{ne}$ is random, we have

$$\text{EEU}(S_{ne}, \delta z) = \text{EEU}(S_{ne}) - \delta z \times \text{LOLE}(S_{ne}).$$

(16)

Further, again since $\delta z$ is small, under the greedy LRTF policy the stores in the remaining set $S_e$ continue to contribute all their energy, so that, from (16)

$$\text{EEU}(S, \delta z) = \text{EEU}(S) - \delta z \times \text{LOLE}(S_{ne}).$$

The claimed result (15) now follows on dividing by $\delta z$ and letting $\delta z \to 0$.

It now follows from the definition of $\text{EEU}'(S)$ that, the equivalent firm capacity $efc_i$ of any small store $i \in S$ is given by the solution of

$$\text{EEU}(S) - \text{EEU}(S \setminus \{i\}) = \text{EEU}'(S) \times efc_i,$n

(17)

where we are here assuming that the contribution of the store $i$ is sufficiently small that we may effectively identify $\text{EEU}'(S \setminus \{i\})$ and $\text{EEU}'(S)$. The required result now follows from (15) and (17).

Remark 3. In the above theory, and in particular in the statement of Theorem 3, it is important to note that the EFC of any store is defined independently of the exact way in which the use of any set of stores is scheduled, provided that scheduling is such that EEU is minimised. However, the quantity $\text{LOLE}(S_{ne})$ in equation (14) is defined in terms of the outcome of the greedy LRTF policy. (Other optimal scheduling policies are possible and would result in a different value of this quantity, and it would then be the case that the equation (15) no longer held.)

5 Minimisation of weighted EEU

We now consider more briefly the case where the objective function $w$ of the optimal scheduling problem $P$ is a more general convex increasing function (with $w(d) = 0$ as before). As discussed in Section 2 this corresponds to the situation where the unit economic cost of unserved energy is higher at higher levels of unmet demand. This is something which is often considered to be the case in applications (see [11]), even if in the absence of storage it is rarely formally modelled (presumably because there is relatively little opportunity to schedule generation so as to target particularly those times at which demand is highest).

The optimal scheduling problem $P$ is now a nonlinear constrained optimisation problem. In its solution it makes sense to concentrate such storage resources as are available at the times of highest demand. Thus in general it is no longer the case that an optimal policy will be greedy, since at times of low demand, storage resources may be better withheld for later use at those times at which they may be used more
effectively. The optimal decision at each successive time $t$ now generally depends, in a deterministic environment, on the entire demand process $(d(t), t \in [0, T])$ and, in a stochastic environment, on the entire distribution of this demand process.

Consider now the case where the demand process $(d(t), t \in [0, T])$ is deterministic and there is a single store with rate constraint $P$ and capacity constraint $E$. For any $k \geq 0$, consider also the rate process $(r(k)(t), t \in [0, T])$ for the service of energy by the store given, for each $t \in [0, T]$, by

$$ r(k)(t) = [d(t) - k]^P_0, \quad (18) $$

where by the expression on the right side of (18) is meant $d(t) - k$ truncated below at 0 and above at $P$. It is clear that provided the above rate process is feasible for the use of the store, i.e. is such that

$$ \int_0^T r(k)(t) \, dt \leq E, \quad (19) $$

then there is an optimal solution $(r(t), t \in [0, T])$ to the problem $P$ such that $r(t) \geq r(k)(t)$ for all $t \in [0, T]$. The reason for this is that the process $(r(k)(t), t \in [0, T])$ serves as much demand above the level $k$ as is possible, and it follows from the convexity of the function $w$ that any switching of energy resource from serving demand above the level $k$ to serving demand below the level $k$ cannot lead to an improved solution to $P$. The optimal solution to the problem $P$ is thus given by taking $r(t) = r(k)(t)$ for all $t \in [0, T]$ where $k \geq 0$ is chosen as small as possible consistent with the constraint (19) being satisfied. Observe that this solution is independent of the precise specification of the “weighting” or objective function $w$ for the problem $P$, subject only to its convexity. Thus it also solves the problem in the “unweighted” case considered in Section 4 where $w(d) = d$ for all $d$ and where the objective is the minimisation of EEU. (However, in the latter case a greedy policy may be preferred in applications as the demand process $(d(t), t \in [0, T])$ is rarely precisely known in advance.) When the set $S$ consists of multiple stores, the optimal solution of the problem $P$ is more complex and we do not pursue this here.

When the demand process $(d(t), t \in [0, T])$ is stochastic the solution of the problem $P$ (when the function $w$ is nonlinear) is complex and, as remarked above, depends on the distribution of this entire process. Stochastic dynamic programming provides a possible approach, but this requires that the distribution of the demand process $(d(t), t \in [0, T])$ is sufficiently known. In practice it may be desirable to take a more heuristic approach. For example, at each time $t \in [0, T]$ one might make a deterministic estimate of the future demand process, and then make the optimal decision as to how much energy to serve at that time on the basis of that estimate (for example, in the case of a single store this might be determined as described earlier); a new estimate of the future demand process might then be made at each subsequent time and the amount of energy to be then served correspondingly re-optimised. Such deterministic re-optimisation or rolling intrinsic techniques are robust and known to work well in stochastic environments where the underlying probability distributions are themselves uncertain—see, for example, [15] and the references therein.
6 Examples

In this section we give two fairly simple examples which between illustrate much of the theory of the present paper.

Example 1. We give a numerical example which, although simple, is nevertheless reasonably realistic in the case of the use of storage to cover what might otherwise be a significant period of shortfall in a country such as Great Britain. We take \( T = 8 \) and the demand process to be met from storage (after the exhaustion of all generation) to be given by \( (2, 2, 2, 2, 5, 5, 1, 1) \) on successive time intervals of unit length. If these time units were half-hours (these being the units used in Great Britain for the initial attempt in the balancing of supply and demand ahead of real time) and if the power units were 0.5 GW, this might correspond to a credible period of severe shortfall such as might occur during an early evening period of peak demand. (However, when as at present most electricity capacity is provided by generation, such a period of shortfall to be met by storage would be rare.) Recall from Section 3 that any set of stores with identical values of \( E_i / P_i \) are equivalent to a single larger store; a corollary of this is that, in choosing examples, there is no loss of generality for illustrative purposes in assuming all stores to have the same rate constraint. Thus consider 5 stores each with \( P_i = 1 \) and with \( E_i = 5, 4, 4, 3, 2 \) for \( i = 1, 2, 3, 4, 5 \). (This might correspond to a number of large batteries or small pumped storage facilities). It is easy to check that the greedy LRTF policy empties all the stores over the time period \([0, 6]\) and satisfies all the demand over that time period, serving a total of 18 units of energy and leaving unsatisfied all the demand during the period \([6, 8]\), thus giving a (residual) minimised EEU of 2 units of energy. However, this is not the only policy capable of minimising EEU. For example, the greedy LRTF policy applied to the demand process in reverse time satisfies all demand over the time period \([1, 8]\) and none of the demand occurring during the time period \([0, 1]\); however, such a “time-reversed” policy could not be attempted in a stochastic environment (see Section 4). Consider also the policy which uses the LRTF prioritisation of stores to serve as far as possible all demand in excess of some level \( k \). It turns out that for \( k = 0.25 \) all demand in excess of the level \( k \) may be thus served and that the given stores are then empty at the final time 8, so that this policy again minimises EEU. Arguments analogous to those in Section 5 show that this policy is also optimal when the objective is the minimisation of weighted EEU for any convex increasing objective function \( w \) (with \( w(0) = 0 \)) as described in that section. (We remark that in general, in the case of multiple stores, the solution to the problem of minimising weighted EEU does not take such a simple form) However, implementation of a policy of this form again requires a knowledge of the demand process ahead of the times at which decisions need to be implemented, and so again cannot be directly implemented in a stochastic environment.

Finally consider the heuristic greedy policy suggested in [7] in which stores are arranged in some order and, with respect to that order, earlier stores are completely prioritised over later ones. It is again easy to check that neither the above policy in which the stores are arranged in descending order of capacity, nor that in which they are arranged in ascending order of capacity, succeeds in emptying all the stores—the
remaining stored energy at time 8 under each of these two policies being respectively 1 and 2 units. Hence neither of these two policies succeeds in minimising EEU.

**Example 2.** We give an example to illustrate the additional difficulties when, as in Section 5, the objective function $w$ of the optimal scheduling problem $P$ is nonlinear—corresponding to the objective being the minimisation of a form of weighted EEU—and when additionally the demand process $(d(t), t \in [0, T])$ is stochastic. We take this objective function to be given by $w(d) = d^p$ for some $p \geq 1$. We take $T = 2$ and let the demand process $(d(t), t \in [0, 2])$ be given by $d(t) = 2$ for $t \in [0, 1]$ and $d(t) = k$ for $t \in [1, 2]$ where $k$ is a random variable which is uniformly distributed on $[0, 4]$. Finally we consider a single store with capacity $E = 2$ and rate constraint $P = 2$.

In the case $p = 1$ the optimisation problem is that of minimising EEU and here, by Theorem 2, the optimal solution is given by the greedy policy of serving all demand in the time period $[0, 1]$, thus emptying the store at time 1 and so serving no demand thereafter. That this solution is here unique follows since, if the store is not emptied by time 1, there is then a nonzero probability that it may not be possible to empty it by the final time $T = 2$.

We now consider the case $p > 1$, corresponding to the problem of minimising weighted EEU. If the random variable $k$ is replaced by its expected value of 2 then, as in Section 5, for all $p > 1$ the optimal solution is given by serving energy at a constant rate 1 throughout the entire time period $[0, 2]$. For the original stochastic problem, we assume that the constant value $k$ of the demand process over the period $[1, 2]$ is not known until time 1. Arguing as in Section 5, it is straightforward to see that the optimal policy is that of serving energy at some constant rate $x \leq 2$ during the time period $[0, 1]$ and then at constant rate $\min(k, 2 - x)$ during the time period $[1, 2]$. For given $x$ the objective function (weighted EEU) to be minimised is then given by

$$
(2 - x)^p + \frac{1}{4} \int_0^4 \max (0, (k + x - 2)^p) \, dk.
$$

(20)

The latter quantity is equal to $(2 - x)^p + \frac{(x+2)^{p+1}}{4(p+1)}$ and it is then routine to show that the optimal value of $x$ is unique and is a monotonic decreasing function of $p$—as might be expected—which tends to 2 as $p \to 1$ (corresponding to the earlier case $p = 1$ where a greedy policy minimises the risk of failing to empty the store by time 2) and which tends to 0 as $p \to \infty$ (corresponding to the very high penalty attached to high levels of residual demand when $p$ is large).

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