Charmed and Bottom Baryons from Lattice NRQCD

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The mass spectrum of charmed and bottom baryons has been computed on anisotropic lattices using quenched lattice nonrelativistic QCD. Masses are extracted by using mass splittings which are more accurate than masses obtained directly by using the nonrelativistic mass-energy relation. Of particular interest are the mass splittings between spin-1/2 and spin-3/2 heavy baryons, and we find that these color hyperfine effects are not suppressed in the baryon sector although they are known to be suppressed in the meson sector. Results are compared with those obtained in a previous NRQCD calculation and with those obtained from a Dirac-Wilson action of the D234 type.

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I. INTRODUCTION

A comprehensive knowledge about the mass spectrum and spin splittings of heavy baryons is important for our understanding of quantum chromodynamics. However, except for singly heavy charmed baryons and only one singly heavy bottom baryon (Λ\textsubscript{b}), most of the heavy baryon masses have not yet been measured experimentally\cite{1}. On the theoretical side there are many results on heavy baryon masses from different models including, for example, a number of quark model variations\cite{2,3,4}. Using lattice QCD, substantial work has been done in the heavy meson sector. However, so far only very few results have been reported for heavy baryons\cite{5,6,7,8}, and there is only one work\cite{5} where heavy (bottom) quarks are treated nonrelativistically. A further study of charmed and bottom heavy baryons on the lattice using nonrelativistic QCD (NRQCD) therefore seems worthwhile.

Extraction of the experimentally observed mass splittings between vector and pseudoscalar mesons remains a challenging problem in lattice QCD: quenched calculations have so far only been able to extract the observed mass splittings\cite{9,10}, and unquenched studies have not resolved the issue\cite{11}. Therefore, it is natural to ask whether lattice results for baryon mass splittings also exhibit similar suppression compared to experiment.

Empirically, spin splittings in baryons are smaller than those in the meson sector. Moreover, in a lattice simulation the correlators for baryons, particularly for spin 3/2 states, are noisier than those for mesons, and thus, by using lattice QCD it is comparatively difficult to extract a reliable mass spectrum for heavy baryons. In this work we report on the charmed and bottom baryon mass spectrum and mass splittings by using a nonrelativistic heavy quark action and an improved light quark action on anisotropic lattices.

In section II we summarize different charged and bottom heavy baryons with their relevant quantum numbers and discuss our choice for interpolating fields. Section III presents numerical simulation details. For heavy quarks, we use the nonrelativistic action from Ref.\cite{12}, while a tadpole improved gauge action and an improved Dirac-Wilson action of the D234 type\cite{13} are used for light quarks. Since these actions were previously detailed elsewhere\cite{14,15}, we will describe them only in an appendix.

The calculations are done on two different anisotropic lattices with the same gauge configurations as were used in Ref.\cite{16} at $\beta = 2.1$ and $\beta = 2.3$.

In section IV we present our results. Masses are calculated using two methods; the first uses the standard NRQCD relation between mass and energy while the second employs mass splittings to calculate masses. As mass splittings can be estimated more accurately than masses, errors in the second method are smaller than those obtained from the first one. The overall systematic uncertainty is estimated by including scale uncertainty, uncertainty due to the choice of a time window for fitting correlation functions, error due to extrapolation to the physical light quark masses, uncertainty in fixing charm and bottom masses and uncertainty from our determination of the lattice anisotropy.

Spin splittings are discussed in section V. From our results, along with other published results, we conclude that the suppression of mass splittings is not present in the baryon sector in the same way as it is in the meson sector. Over the whole mass range where data are available, quenched lattice QCD simulations yield mass differences between spin 3/2 and spin 1/2 baryons which are comparable to or larger than experimental values.

II. CHARMED AND BOTTOM BARYONS

Singly and doubly charmed and bottom baryons are summarized in Tables I and II respectively. Table II also includes doubly heavy states containing two different heavy quarks (charmed and bottom quarks together). Quark content, as well as the spin-parity $J^P$, the isospin $I$, and $s_l$ which identifies the total spin of the light quarks (also spin-flavor symmetry: $s_l = 0$ is symmetric while
TABLE I: Summary of singly heavy baryons, showing valence quark content (q ≡ u, d and Q ≡ c, b), spin-parity, isospin and mass (in GeV). The quantity \( s_1 \) is the total spin of the light quark pair. The experimental values are from Ref. [1].

| Baryons | quark content | \( J^P \) | \( I \) | \( s_1 \) | Mass(c) | Mass(b) |
|---------|---------------|----------|------|------|--------|--------|
| \( \Lambda_Q \) | \( udQ \) | \( \frac{3}{2}+ \) | 0 | 0 | 2.285(1) | 5.624(9) |
| \( \Xi_Q \) | \( qsQ \) | \( \frac{1}{2}+ \) | \( \frac{1}{2} \) | 0 | 2.468(2) |
| \( \Sigma_Q \) | \( q\bar{q}Q \) | \( \frac{1}{2}+ \) | \( \frac{1}{2} \) | 1 | 2.453(1) |
| \( \Xi'_Q \) | \( q\bar{s}Q \) | \( \frac{1}{2}+ \) | \( \frac{1}{2} \) | 1 | 2.575(3) |
| \( \Omega_Q \) | \( ssQ \) | \( \frac{1}{2}+ \) | 0 | 1 | 2.704(4) |
| \( \Sigma'_Q \) | \( q\bar{q}Q \) | \( \frac{1}{2}+ \) | \( \frac{1}{2} \) | 1 | 2.518(2) |
| \( \Xi_Q \) | \( q\bar{s}Q \) | \( \frac{1}{2}+ \) | \( \frac{1}{2} \) | 1 | 2.645(2) |

\( s_1 = 1 \) is antisymmetric) are shown. Notice that masses for many singly heavy states are not measured yet and there are no data at all on masses for doubly heavy states.

To project out heavy baryon states we use the same interpolating operators as were used in Ref. [8]. For \( \Sigma \)-like baryons we choose

\[
\Sigma : \quad e^{abc}[q_a^T C\gamma_5 Q_b]q_c. \quad (1)
\]

where \( q \) is a light quark field and \( Q \) is a heavy quark field. Here \( a, b, c \) are color indices whereas Dirac indices have been suppressed. For \( \Sigma_Q, q \) is \( u \) or \( d \) and for \( \Omega_Q, q \) is \( s \). For doubly heavy \( \Sigma \)-like baryons with equal heavy masses, we interchange the role of light and heavy fields, i.e., to get \( \Xi_QQ \), we change \( q \rightarrow Q \) and \( Q \rightarrow u \) or \( d \). Similarly, for \( \Omega_QQ \), the change is \( q \rightarrow Q, Q \rightarrow s \).

The \( \Xi_Q \) is \( \Sigma \)-like but it contains two different light flavors so it is considered separately as

\[
\Xi' : \quad \frac{1}{\sqrt{2}} \left\{ e^{abc}[q_{a1}^T C\gamma_5 Q_{b1}]q_{c1} + e^{abc}[q_{a2}^T C\gamma_5 Q_{b2}]q_{c2} \right\}, \quad (2)
\]

with \( q = u \) or \( d \) and \( q' = s \).

The \( \Lambda \)-like baryons involve three distinct flavors. A simple choice is the heavy lambda:

\[
\Lambda : \quad e^{abc}[q_a^T C\gamma_5 Q_b]q_c. \quad (3)
\]

where for \( \Lambda_Q, q = u, q' = d, \) and for \( \Xi_Q, q = u, q' = s \). A more symmetrical choice would be the octet lambda

\[
\Lambda_o : \quad \frac{1}{\sqrt{6}} e^{abc} \left\{ 2[q_{a1}^T C\gamma_5 Q_{b1}]q_{c1} + [q_{a2}^T C\gamma_5 Q_{b2}]q_{c2} \right\} - [q_{a3}^T C\gamma_5 Q_{b3}]q_{c3}, \quad (4)
\]

with the same flavor assignment as for the heavy lambda. One can use either of these \( \Lambda \) states as they give consistent results [8]. We choose the octet-lambda (\( \Lambda_o \)) for this work. For spin 3/2 baryons we choose the following interpolating field:

\[
\Sigma^* : \quad e^{abc}[q_a^T C\gamma_{\mu} Q_{b}]q_c. \quad (5)
\]

TABLE II: Summary of doubly heavy baryons, showing valence quark content (q ≡ u, d and Q ≡ c, b), spin-parity, isospin and \( S_{QQ} \), the total spin of the heavy quark pair.

| Baryons | quark content | \( J^P \) | \( I \) | \( S_{QQ} \) |
|---------|---------------|----------|------|--------|
| \( \Xi_{QQ} \) | \( qQQ \) | \( \frac{1}{2} \) | \( \frac{1}{2} \) | 1 |
| \( \Omega_{QQ} \) | \( sQQ \) | \( \frac{1}{2} \) | \( \frac{1}{2} \) | 0 |
| \( \Xi^*_{QQ} \) | \( qQQ \) | \( \frac{3}{2} \) | \( \frac{1}{2} \) | 1 |
| \( \Omega^*_{QQ} \) | \( sQQ \) | \( \frac{3}{2} \) | \( \frac{1}{2} \) | 0 |
| \( \Xi_{bc} \) | \( qbc \) | \( \frac{1}{2} \) | \( \frac{1}{2} \) | 0 |
| \( \Omega_{bc} \) | \( sbc \) | \( \frac{1}{2} \) | \( \frac{1}{2} \) | 0 |
| \( \Xi^*_{bc} \) | \( qbc \) | \( \frac{3}{2} \) | \( \frac{1}{2} \) | 1 |
| \( \Omega^*_{bc} \) | \( sbc \) | \( \frac{3}{2} \) | \( \frac{1}{2} \) | 0 |

where for \( \Sigma^*_Q \), \( q = q' \) is \( u \) or \( d \) and for \( \Omega^*_Q \), \( q = q' \) is \( s \). To get \( \Xi_{QQ} \), one needs to consider \( q = u \) or \( d \) and \( q' = s \). Similarly, to get the doubly heavy states with equal heavy masses one needs to interchange the role of light and heavy fields. For example, to get \( \Xi^*_{QQ} \), one needs to change \( q, q' \rightarrow Q \) and \( Q \rightarrow u \) or \( d \), whereas \( \Omega^*_{QQ} \) requires \( q, q' \rightarrow Q \) and \( Q \rightarrow s \).

The operator in Eq. (3) has both spin 1/2 and spin 3/2 states. At zero momentum the corresponding correlation function can be written as [3]:

\[
C_{ij}(t) = \left( \delta_{ij} - \frac{1}{3} \gamma_{ij} C_{3/2}(t) + \frac{1}{3} \gamma_{ij} C_{1/2}(t) \right), \quad (6)
\]

where \( i, j \)'s are spatial Lorentz indices and \( C_{3/2(1/2)} \) are the spin projections for spin 3/2 (1/2) states. By choosing different Lorentz components the spin 3/2 part, \( C_{3/2}(t) \), is extracted and used to calculate the mass of the spin 3/2 baryons.

Operators for baryons with two unlike heavy flavors may be constructed from the above interpolating operators by interchanging the role of heavy and light fields. For example, \( \Xi_{QQ} \) and \( \Omega_{QQ} \) can be obtained from Eq. (5) by letting \( q, q' \rightarrow Q, Q' \) and \( Q \rightarrow q \) with \( q = u \) or \( d \) and \( q' = s \) respectively. For \( \Xi^*_{QQ} \) and \( \Omega^*_{QQ} \) we use the symmetrical form again, as given by Eq. (3), making the same replacements i.e., \( q, q' \rightarrow Q, Q' \) and \( Q \rightarrow q \) with \( q = u \) or \( d \) and \( q' = s \) respectively. Finally, \( \Xi_{QQ} ^* \) and \( \Omega_{QQ} ^* \) are the doubly heavy analogs of \( \Lambda \) and \( \Lambda_o \) and they can be obtained from Eqs. (3) and (4) as previously.

III. NUMERICAL SIMULATION

A. Actions

The gauge action as well as the heavy quark NRQCD action used for this work are described in detail in Ref. [1]. The gauge action is tadpole improved and the leading classical error is quartic in lattice spacing. The
Hamiltonian corresponding to the NRQCD action is complete to $\mathcal{O}(1/M^3)$ in the classical continuum limit. For light quarks we use a Dirac-Wilson action of the D234 type [12] which has been used previously and detailed in Refs. [7, 8]. Its leading classical errors are cubic in lattice spacing. All these actions are summarized in the appendix.

**B. Simulation Details**

This work is done with two sets of quenched gauge configurations (at $\beta = 2.1$ and 2.3) on anisotropic lattices with a bare aspect ratio $a_s/a_t = 2$, where the spatial lattice spacing varies from about 0.22 to 0.15 fm.

The renormalized anisotropy is obtained from

$$\xi = \frac{a_s}{a_t} = \frac{a_s V (r_2) - a_s V (r_1)}{a_t V (r_2) - a_t V (r_1)}, \quad (7)$$

where $V (r)$ is the potential between a static quark-antiquark pair with separation $r$, and is extracted from an exponential fit to a sequence of Wilson loops. In the numerator of Eq. (7), the sequence of Wilson loops extends in a coarsely-spaced direction, and in the denominator the sequence extends in the finely-spaced direction. The separation $r$ may be along a lattice axis or off-axis, and various possibilities were included in the calculation. However, the separation $r$ never includes the separation $r$.

We used fixed time boundaries to construct quark propagators, and gauge fields were generated using a pseudo-heat-bath Monte Carlo algorithm with 400 ($\beta = 2.1$) to 800 ($\beta = 2.3$) sweeps between saved configurations. For $\beta = 2.1$, we use 720 configurations and for $\beta = 2.3$ the number of configurations is 442. Two sets of bare masses are used for each heavy quark while four sets of hopping parameters are used for the light one. Bare masses for heavy quarks are chosen to surround the physical value so that an interpolation can be used. For example, at $\beta = 2.1$, the charm mass is in between 1.2 and 1.5 and the bottom mass is in between 5.0 and 6.0. The charm mass is fixed by setting the $\eta_c$ mass to its experimental value, whereas the $B^0$ mass is used to fix the bottom mass. The hopping parameter corresponding to the strange quark is fixed from the $D_s$ meson mass. The temporal lattice spacing and correspondingly the scale is fixed by setting the $\rho$-meson mass to its experimental

![FIG. 1: Effective Mass $M(t)$ versus $t$ for singly heavy \(\Sigma\)-like baryons for different combinations of light and heavy quark mass (denoted by hopping parameter $\kappa$ and bare mass $m$, respectively). Open symbols are for calculations with a correlation function with local source and sink, filled symbols are for local source and smeared sink.](image-url)
value. Summaries of lattice parameters as well as hopping parameters for heavy and light fields are given in Tables III and IV respectively.

Correlation functions are calculated using interpolating operators in local form at both source and sink. In addition to that we use a gauge invariant smearing for quark propagators at the sink using the smearing function from Eq. (13) of Ref. [4]. These local and sink-smeared correlators are fitted simultaneously to obtain hadron masses. The required correlations among different quantities are taken into account by covariant matrices obtained from singular value decomposition, and the statistical error is estimated from bootstrapping the fitting procedure. As in Ref. [4], local correlators are fitted with two exponential functions \( A \exp(-m_1 x) + B \exp(-m_2 x) \), while the sink-smeared correlation function is fitted with a single exponential \( C \exp(-m_1 x) \). The mass parameter for the sink-smeared fit is constrained to be the same as the lowest mass of the fit to the local correlator. The time window for the fit is chosen in a way such that the ending time is large and the fit is stable under variation of both starting and ending time by a few time steps. Light quark extrapolation is done by extrapolating the hadron masses extracted at four light quark masses with the form \( c_0 + c_2 m_\pi^2 + c_3 m_\pi^4 \), where \( m_\pi \) is the pion mass. In most of the cases the cubic \( m_\pi^3 \) contributions are small and they are included only to get systematic errors.

Figs. 1–3 show some representative examples of our simulation results. We plot the effective mass for different heavy baryons versus time \( t \), where the effective mass is defined to be \( M(t) = \ln(g(t)/g(t+1)) \) with \( g(t) \) being the zero-momentum time correlation function of baryon fields. Open symbols in these figures are for calculations with a correlation function with local source and sink, filled symbols are for local source and smeared sink.

**TABLE III**: Summary of lattice parameters. The quantity \( a_t^{-1} \) is the inverse of the temporal lattice spacing while \( u_s \) and \( u_t \) are the tadpole improvement factors for spatial and temporal links respectively.

| \( \beta \) | size | configurations | \( a_t^{-1} \) (GeV) | \( u_s \) | \( u_t \) |
|---|---|---|---|---|---|
| 2.1 | \( 12^3 \times 32 \) | 720 | 1.803(42) | 0.7858 | 0.9472 |
| 2.3 | \( 14^3 \times 38 \) | 442 | 2.210(72) | 0.8040 | 0.9525 |

**TABLE IV**: Hopping parameters and bare masses. Four \( \kappa \) values were used in simulations at each \( \beta \). \( \kappa_s \) is the hopping parameter for the strange quark, and \( c \) and \( b \) are the charmed and bottom bare masses respectively.

| \( \beta \) | \( \kappa \) | \( \kappa_s(\phi) \) | bare mass |
|---|---|---|---|
| 2.1 | 0.229,0.233,0.237,0.240 | 0.2338 | 0.9525 |
| 2.3 | 0.229,0.233,0.237,0.240 | 0.2371 | 1.041.24 |

FIG. 2: Effective Mass \( M(t) \) versus \( t \) for doubly heavy \( \Sigma \)-like baryons for different combinations of light and heavy quark mass (denoted by hopping parameter \( \kappa \) and bare mass \( m \), respectively). Open symbols are for calculations with a correlation function with local source and sink, filled symbols are for local source and smeared sink.
calculated with a correlation function with local source and sink, filled symbols are for local source and smeared sink. There is good agreement between local and smeared results at large times.

It should be noted that the actual fits to determine the masses are performed directly with the correlation functions, and not on the effective masses plotted in Figs. 1-3, but the plots provide an indication of the quality of our data. Although the sink-smeared results appear to be somewhat noisy they are quite helpful in constraining the two-exponential fit of the local correlation function.

C. Mass Extraction

The kinetic mass of a nonrelativistic state can be extracted from the usual NRQCD relation \[11\]

\[ M_{\text{kin}} = \frac{2\pi^2}{N_s^2 \xi^2 a_t [a_t (E_p - E_0)]}, \]  

(9)

which is derived from \( E = \frac{p^2}{2M_{\text{kin}}} \). Here \( N = L a_s \), with \( L \) being the lattice size and \( a_s \) the spatial lattice spacing. \( \xi = a_t/a_s \) is the anisotropy whereas \( E_0 \) and \( E_p \) are simulation energies corresponding to the ground state and the state with momentum \( p = \frac{2\pi}{L a_t} \), respectively. Mass differences between two states (\( H_1 \) and \( H_2 \)) with the same heavy quark can be obtained by taking the difference of their zero momentum simulation energies:

\[ M_{H_1} - M_{H_2} = E^1_{\text{sim}}(0) - E^2_{\text{sim}}(0), \]  

(10)

which follows from the lattice NRQCD expression for the hadron mass

\[ M_H = E_{\text{sim}}(0) + Z M_Q - E_{\text{shift}}, \]  

(11)

where \( E_{\text{sim}} \) is the simulation energy at zero momentum and the last two terms represent the renormalized heavy quark mass. The bare quark mass \( M_Q \) has both a multiplicative \( (Z) \) and additive \( (E_{\text{shift}}) \) renormalization \[15\] which should be independent of hadronic state. A more precise result is obtained for heavy hadron masses with the heaviest light quark (\( \kappa = 0.229 \)) rather than with a lighter light quark (\( \kappa = 0.233 \) and higher).

Moreover, mass differences (Eq. 10) can be calculated more precisely than masses (Eq. 9). Therefore, for example, one can calculate a meson mass from the relation

\[ M(q_l, Q) = M(q_h, H) - \Delta M = M(q_h, H) - \Delta E, \]  

(12)

where

\[ \Delta M = \Delta E = E(q_h, Q) - E(q_l, Q). \]  

(13)

Here \( q_h \) and \( q_l \) denote the heaviest light quark and a lighter one respectively, and \( M(q_h, H) \) is extracted by
where $\Delta E_{sh}$ and $\Delta E_{dh}$ are the energy differences between the states

\begin{align}
M(q_1q_2, Q) &= M(q_1q_2, Q) - \Delta E_{sh}, \\
M(q_1, QQ) &= M(QQ) - \Delta E_{dh},
\end{align}

using Eq. (9). Eq. (13) is valid as long as $Z$ in Eq. (11) is the same \textit{i.e.}, both states consist of the same heavy quark $Q$.

Similarly, masses of singly and doubly heavy baryons can be extracted from meson masses by using

\begin{align}
M(q_1q_2, Q) &= M(q_1q_2, Q) - \Delta E_{sh}, \\
M(q_1q_2, QQ) &= M(q_1q_2, QQ) - \Delta E_{dh},
\end{align}

For example, the $\Sigma_{c(b)}$ mass is extracted by taking its difference (at each $\kappa$) with the $D(B)^0$ mass ($m$) at $\kappa = 0.229$ and then subtracting that from $m$. Masses extracted by using Eq. (9) and Eqs. (12-17) are consistent with each other. However, errors in the second method are smaller than the previous one.

\section*{IV. RESULTS}

The mass spectrum and spin splittings of heavy quark baryons have been computed on an anisotropic lattice using the NRQCD heavy quark action. Results are summarized in Table V, where the first error is the statistical error obtained from a bootstrap analysis with a bootstrap sample size equal to the configuration sample size. The second error is an overall systematic error due to scale and anisotropy uncertainties, the uncertainty due to choosing a time window, the light quark extrapolation error and the strange quark mass uncertainty. Mesons, singly heavy baryons and doubly heavy baryons are separated into different groups by horizontal lines. In Table VI we have compared our results with those obtained by using a relativistic (D234) heavy quark action and experimental numbers (where available). One can notice that the NRQCD results and D234 results are consistent
with each other. Results are also consistent with a previous NRQCD calculation \[8\]. As in Ref. \[8\], it is found that the suppression of spin splittings is not present in the baryon sector, although such a suppression is known to be characteristic of the heavy meson sector. One can also notice that the spin splittings for doubly heavy baryons are as large as their singly heavy counterparts.

**V. DISCUSSION AND SUMMARY**

In order to put the results of the present calculation into perspective it is useful to consider spin splittings over the whole range of available quark masses. We start with mesons where it has been known for a long time that the squared mass difference \( M_1^2 - M_2^2 \) for vector and pseudoscalar mesons is approximately constant for all mesons of the form \( \mathbb{Q} q \), where \( q \) is up or down and \( Q \) is any light or heavy flavor. This relation is illustrated in Fig. 4 for the mass pairs \((\rho, \pi), (K^*, K), (D^*, D)\) and \((B^*, B)\). Also shown are the results of lattice simulations including the present work. The tendency for quenched lattice QCD to underestimate the spin splittings relative to experimental values is clearly visible.

In Ref. \[8\] we showed that it is useful to consider the behavior of the spin splittings in the baryon sector as a function of quark mass also in terms of the mesonic average mass \((M_V + M_P)/2\). The results for the baryon pairs \((\Delta, N), (\Sigma^*, \Sigma), (\Sigma^*_c, \Sigma_c)\) and \((\Sigma^*_b, \Sigma_b)\) are shown in Fig. 5. It is a remarkable empirical fact that the baryon spin splitting scales almost exactly like the inverse of the average meson mass. The implication is that the ratio of meson to baryon spin splittings is almost constant. This was discussed in Ref. \[8\] and was to some extent anticipated by Lipkin \[18\] from the point of view of the quark model (see also Lipkin and O’Donnell \[19\]).

The results of quenched lattice calculations are also shown in Fig. 5. The suppression of spin splittings relative to experiment, visible for mesons, is not seen for baryons. The results of the present lattice NRQCD calculation support this conclusion in the charm and bottom sectors. It is clear that a definitive measurement of \( \Sigma_b \) and \( \Sigma_b^* \) masses would be highly desirable to extend the experimental comparison to larger mass values.

From the point of view of lattice NRQCD our results present an interesting challenge. As is well known, the spin splittings of both charmonium \[20\] and heavy-light mesons \[9, 11\] are clearly underestimated by quenched lattice QCD. NRQCD is used to describe charm baryons containing one or two heavy quarks using an effective quark model (see also Lipkin and O’Donnell \[19\]).

To summarize, we have calculated the masses of baryons containing one or two heavy quarks using quenched lattice QCD. NRQCD is used to describe charm and bottom quarks. In the charm sector the results of this work are compatible with those obtained previously where a Dirac-Wilson action of the D234 type was used for the heavy quark. No suppression of the spin splittings observed in lattice NRQCD simulations of heavy-light mesons is seen in the heavy baryon sector.
This and our previous work leave a number of difficult open questions. One would like to be able to improve the lattice calculations of baryons to reduce the uncertainties to the same level achievable in mesons. Also how (and whether) the addition of dynamical quarks to the simulations will solve the dilemma of spin splittings has yet to be understood. A phenomenological issue is to understand the remarkable constancy in the meson to baryon spin splitting ratio over the whole available quark mass range. On the experimental side it will be a significant challenge to extend baryon mass measurements in the bottom and doubly heavy sectors.

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APPENDIX A: DETAILS OF ACTIONS

1. NRQCD Action

The heavy quark action is nonrelativistic and is discretized to give the following Green’s function propagation:

\[ G_{\tau+1} = \left(1 - \frac{a_t H_B}{2} \right) \left(1 - \frac{a_t H_A}{2n} \right)^n \frac{U^\dagger}{u_s} \left(1 - \frac{a_t H_A}{2n} \right)^n \left(1 - \frac{a_t H_B}{2} \right) G_{\tau}, \]  \hspace{1cm} (A1)

The nonrelativistic Hamiltonian is complete to \( O(1/M^3) \) in the classical continuum limit:

\[ H = H_0 + \delta H, \]  \hspace{1cm} (A2)

\[ H_0 = -\frac{\Delta^{(2)} g}{2M}, \]  \hspace{1cm} (A3)

\[ \delta H = \delta H^{(1)} + \delta H^{(2)} + \delta H^{(3)} + O(1/M^4), \]  \hspace{1cm} (A4)

\[ \delta H^{(1)} = -\frac{c_1 g}{u_s^2 2M} \hat{\mathbf{E}} \cdot \hat{\mathbf{B}}, \]  \hspace{1cm} (A5)

\[ \delta H^{(2)} = \frac{c_2 g}{u_s^2 u_t^2} \frac{i}{8} \left( \Delta \cdot \hat{\mathbf{E}} - \hat{\mathbf{E}} \cdot \Delta \right) \]  \hspace{1cm} (A6)

\[ \delta H^{(3)} = -\frac{c_3 g}{u_s^2} \frac{a s (\Delta^{(2)})^2}{16n \xi M^2}, \]  \hspace{1cm} (A7)

Here a tilde signifies discretization errors have been removed. In particular,

\[ \tilde{E}_i = \tilde{F}_{4i}, \]  \hspace{1cm} (A8)

\[ \tilde{B}_i = \frac{1}{2} \epsilon_{ijk} \tilde{F}_{jk}, \]  \hspace{1cm} (A9)

\[ \tilde{F}_{\mu\nu}(x) = \frac{5}{6} F_{\mu\nu}(x) - \frac{1}{6u_s^2} U_\mu(x) F_{\mu\nu}(x + \vec{\mu}) U^\dagger_\mu(x), \]  \hspace{1cm} (A10)

\[ -\frac{1}{6u_s^2} U^\dagger_\mu(x - \vec{\mu}) F_{\mu\nu}(x - \vec{\mu}) U_\mu(x - \vec{\mu}) - (\mu \leftrightarrow \nu). \]  \hspace{1cm} (A11)

The various spatial lattice derivatives are defined as follows:

\[ a_s \Delta_i G(x) = \frac{1}{2u_s}[U_i(x)G(x + \vec{i}) \]  \hspace{1cm} (A12)

\[ -U^\dagger_i(x - \vec{i})G(x - \vec{i})], \]  \hspace{1cm} (A13)

\[ a_s \Delta_i^{(+)} G(x) = \frac{U_i(x)}{u_s} G(x + \vec{i}) - G(x), \]  \hspace{1cm} (A14)

\[ a_s \Delta_i^{(-)} G(x) = G(x) - \frac{U^\dagger_i(x - \vec{i})}{u_s} G(x - \vec{i}), \]  \hspace{1cm} (A15)

\[ a_s^2 \Delta_i^{(2)} G(x) = \frac{U_i(x)}{u_s} G(x + \vec{i}) - 2G(x) \]  \hspace{1cm} (A16)

\[ + \frac{U^\dagger_i(x - \vec{i})}{u_s} G(x - \vec{i}), \]  \hspace{1cm} (A17)

\[ \Delta_i = \Delta_i - \frac{a_s^2}{6} \Delta_i^{(+)} \Delta_i^{(-)}, \]  \hspace{1cm} (A18)

\[ \Delta_i^{(2)} = \frac{1}{12} \Delta_i^{(4)}; \]  \hspace{1cm} (A19)

\[ \Delta_i^{(4)} = \sum_i \left( \Delta_i^{(2)} \right)^2 \]  \hspace{1cm} (A20)

2. Gauge Field Action

The leading classical errors of the gauge field action are quartic in lattice spacing. The action is

\[ S_G(U) = \frac{5 \beta}{3} \sum_{ps} \left( 1 - \frac{1}{3} \text{ReTr} U_{ps} \right) \]  \hspace{1cm} (A21)

\[ -\frac{1}{20u_s^2} \sum_{rs} \left( 1 - \frac{1}{3} \text{ReTr} U_{rs} \right) \]  \hspace{1cm} (A22)

\[ + \frac{\xi}{u_s^2} \sum_{pt} \left( 1 - \frac{1}{3} \text{ReTr} U_{pt} \right) \]  \hspace{1cm} (A23)
\[-\frac{\xi}{20u_s^2u_t^2} \sum_{rst} \left( 1 - \frac{1}{3} \text{Re} \text{Tr} U_{rst} \right) \]
\[-\frac{\xi}{20u_s^2u_t^2} \sum_{rts} \left( 1 - \frac{1}{3} \text{Re} \text{Tr} U_{rts} \right) \] (A19)

where \( \xi \equiv \alpha_s/\alpha_t \) and \( \beta \) is the lattice gauge field coupling constant.

\( ps \): spatial plaquettes
\( rs \): spatial planar \( 1 \times 2 \) rectangles,
\( pt \): plaquettes in the temporal-spatial plane,
\( rts \): rectangles with the long side in a spatial/temporal direction.

3. Light Quark Action

For light quarks, we used a D234 action [8, 12] with parameters set to their tadpole-improved classical values. Its leading classical errors are cubic in lattice spacing and the action can be written as

\[
S_F(\bar{q}, q; U) = \frac{4\kappa}{3} \sum_{x,i} \left[ \frac{1}{u_s^2} D_{11}(x) - \frac{1}{8u_s^2} D_{22}(x) \right] + \frac{4\kappa}{3} \sum_{x} \left[ \frac{1}{u_t} D_{11}(x) - \frac{1}{8u_t^2} D_{22}(x) \right] + \frac{2\kappa}{3u_s^2} \sum_{x,i<j} \bar{\psi}(x)\sigma_{ij} F_{ij}(x) \psi(x) + \frac{2\kappa}{3u_s^2} \sum_{x,i} \bar{\psi}(x)\sigma_0 F_{00}(x) \psi(x) - \sum_x \bar{\psi}(x)\psi(x), \quad (A20)
\]

where

\[
D_{11}(x) = \bar{\psi}(x)(1 - \xi \gamma_i) U_i(x) \psi(x) + \bar{\psi}(x) + \hat{i}(1 + \xi \gamma_i) U_i^\dagger(x) \psi(x), \quad (A21)
\]
\[
D_{11}(x) = \bar{\psi}(x)(1 - \gamma_4) U_4(x) \psi(x) + \bar{\psi}(x) + \hat{i}(1 + \gamma_4) U_4^\dagger(x) \psi(x), \quad (A22)
\]
\[
D_{22}(x) = \bar{\psi}(x)(1 - \xi \gamma_i) U_i(x) U_4(x + \hat{i}) \psi(x + 2\hat{i}) + \bar{\psi}(x + 2\hat{i})(1 + \xi \gamma_i) U_i^\dagger(x) U_4^\dagger(x) \psi(x), \quad (A23)
\]
\[
D_{22}(x) = \bar{\psi}(x)(1 - \gamma_4) U_4(x) U_4(x + \hat{i}) \psi(x + 2\hat{i}) + \bar{\psi}(x + 2\hat{i})(1 + \gamma_4) U_4^\dagger(x) U_4^\dagger(x) \psi(x), \quad (A24)
\]

\[
g F_{\mu\nu}(x) = \frac{1}{2i} \left( \Omega_{\mu\nu}(x) - \Omega_{\mu\nu}^\dagger(x) \right), \quad (A25)
\]

\[
\Omega_{\mu\nu} = \frac{-1}{4} \left[ U_{\mu}(x) U_\nu(x + \hat{\mu}) U_\nu^\dagger(x + \hat{\nu}) U_\mu^\dagger(x) + U_\mu^\dagger(x + \hat{\mu}) U_\nu(x + \hat{\nu}) U_\nu^\dagger(x + \hat{\mu}) U_\mu(x) + U_\nu^\dagger(x + \hat{\nu}) U_\mu(x + \hat{\nu}) U_\nu(x) \psi(x + \hat{\nu}) + U_\nu^\dagger(x) U_\mu(x + \hat{\mu}) U_\nu(x) \psi(x + \hat{\mu}) \right]. \quad (A26)
\]

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