Classical tests of general relativity: Brane-world Sun from minimal geometric deformation

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Abstract – We consider a solution of the effective four-dimensional brane-world equations, obtained from the general relativistic Schwarzschild metric via the principle of minimal geometric deformation, and investigate the corresponding signatures stemming from the possible existence of a warped extra-dimension. In particular, we derive bounds on an extra-dimensional parameter, closely related with the fundamental gravitational length, from the experimental results of the classical tests of general relativity in the Solar system.

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Introduction. – Brane-world (BW) models [1] represent a well-known branch of contemporary high-energy physics, inspired and supported by string theory. These models are indeed a straightforward 5D phenomenological realisation of the Horava-Witten supergravity solutions [2], when the hidden brane is moved to infinity along one extra-dimension, and the moduli effects from the remaining compact extra-dimensions may be neglected [3]. The brane self-gravity, encoded in the brane tension $\sigma$, is one of the fundamental parameters appearing in all BW models, with $\sigma^{-1/2}$ playing the role of the (5D) fundamental gravitational length scale\textsuperscript{1}. In this work, we shall explicitly study the observational effects determined by the parameter $\beta \simeq (\sigma^{-1/2}/R)^2$, which describes a candidate for the modified 4D geometry surrounding a star of radius $R$ in the BW. This particular geometry will be obtained as an exact minimal geometric deformation (MGD) [4] of the Schwarzschild solution to the field equations in general relativity (GR). The MGD approach ensures, by construction, that this BW solution smoothly reduces to the GR Schwarzschild metric in the limit $\sigma^{-1} \to 0$, thus allowing us to analyse variations from GR predictions for small values of the deforming parameter $\beta$. This parameter controls the corrections and is related to the brane tension, the radius of the star, and will also be shown to depend on the compactness of the star.

Minimal geometric deformation. – The effective Einstein equations on the brane take the form [5]

$$G_{\mu\nu} = -\tilde{T}_{\mu\nu} - \Lambda g_{\mu\nu},$$

(1)

where $\tilde{T}_{\mu\nu} = T_{\mu\nu} + \Delta \frac{6}{\sigma} S_{\mu\nu} + \frac{1}{\sigma^2} \mathcal{E}_{\mu\nu}$ denotes the effective energy-momentum tensor, with $T_{\mu\nu}$ the stress tensor of brane matter, and $\mathcal{E}_{\mu\nu}$ and $S_{\mu\nu}$ the non-local and high-energy Kaluza-Klein corrections. If BW matter is a perfect fluid with 4-velocity $u^\mu$, and $h_{\mu\nu} = g_{\mu\nu} - u_{\mu} u_{\nu}$ are the components of the metric tensor orthogonal to the fluid lines, then

$$\mathcal{E}_{\mu\nu} = \frac{6}{\sigma} \left[ \mathcal{U} \left( u_\mu u_\nu + \frac{1}{3} h_{\mu\nu} \right) + \mathcal{P}_{\mu\nu} + \mathcal{Q}_{(\mu} u_{\nu)} \right],$$

(2)

where $\mathcal{U}$ denotes the bulk Weyl scalar, $\mathcal{P}_{\mu\nu}$ is the anisotropic stress and $\mathcal{Q}_{\mu}$ the energy flux.

Solving the effective 4D Einstein equations in the BW is a hard task and, already in the simple case of a spherically symmetric metric,

$$\text{d}s^2 = e^\nu \text{d}t^2 - e^\lambda \text{d}r^2 - r^2 \text{d}\Omega^2,$$

(3)

only a few “vacuum” solutions are known analytically [6–10]. Moreover, for static matter distributions

\textsuperscript{1}We shall mostly use units with the four-dimensional Newton constant $G = c = 1$, unless otherwise specified.
$Q_\mu = 0$ and $P_{\mu\nu} = \mathcal{P} \left( r_0, r_1, + \frac{1}{2} h_{\mu\nu} \right)$ [3]. For stellar systems, the quest for BW solutions becomes even more intricate, mainly due to the presence of extra terms, non-linear in the matter fields, which emerge from high-energy corrections [3,5,11]. Nonetheless, two approximate analytical solutions have been found in the MGD approach. It is worth emphasizing that these metrics are exact solutions of the effective equations (1), although they are not complete solutions of the full 5D equations [12,13]. This approach also yields physically acceptable interior solutions for stars [14], relates the exterior tidal charge found in ref. [10] to the ADM mass, and let us study (micro) black holes [15,16], elucidates the role of exterior Weyl stresses from bulk gravitons on compact stellar distributions [17] and shows the existence of BW stars with Schwarzschild exterior without energy leaking into the bulk [18]. Moreover, both the associated 5D solutions and black strings were obtained in various contexts [19–24], models for the quasar luminosity variation induced by BW effects [25,26].

Let us start by revisiting the MGD approach, which is built on the requirement that GR must be recovered in the low-energy limit $\sigma^{-1} \to 0$. In particular, by solving the effective 4D equations (1), the radial component of the metric is deformed by bulk effects and can be written as [16]

$$e^{-\lambda} = \mu + f,$$

where

$$f = e^{-\lambda} \left( \beta + \int_{r_0}^{r} \frac{e^{\lambda}}{r^2 + \frac{2}{\sigma}} \left( H + \frac{1}{\sigma} \left( \rho^2 + 3 \rho p \right) \right) \right) - \mu \left( \frac{\rho}{2} + \frac{\rho}{2} + \frac{\rho}{2} + \frac{\rho}{2} \right),$$

and

$$\mu = \begin{cases} 1 - \frac{2M}{r}, & \text{for } r > R, \\ 1 - \frac{2M}{r}, & \text{for } r \leq R, \end{cases}$$

where $m$ denotes the standard GR interior mass function. The constant $M$ depends in general on the brane tension $\sigma$ and must take the value of the GR mass $M_0 = m(R)$ in the absence of BW effects, namely $M_0 = M |_{\sigma^{-1} = 0}$. The function $H$ in (4) is given by

$$H(p, \rho, \nu) \equiv 24 \pi p - \left[ \mu \left( \frac{\nu'}{2} + \frac{1}{\nu} \right) \right] + \mu \left( \frac{\nu''}{2} + \frac{2\nu'}{r} + \frac{1}{r^2} - \frac{1}{r^3} \right),$$

and encodes anisotropic effects due to the bulk gravity on the pressure $p$, matter density $\rho$ and the metric function $\nu$. Finally, the parameter $\beta$ in (5) depends on the brane tension $\sigma$, the radius $R$ and the mass $M$ of the self-gravitating system, and must be zero in the GR limit. In the interior, $r < R$, the condition $\beta = \beta(\sigma, R, M) = 0$ must hold in order to avoid singular solutions at $r = 0$ (since the integral in (6) would diverge for $r_0 \to 0$). However, for a vacuum solution, or more properly, in the region $r > R$ where there is only a Weyl fluid surrounding the spherically symmetric star, the parameter $\beta$ can differ from zero.

The crucial point is that, any given perfect fluid solution in GR yields $H(p, \rho, \nu) \equiv 0$, which provides the foundation for the MGD approach. In fact, every perfect fluid solution in GR can be used to produce a minimal deformation on the radial metric component (4), in the sense that all the deforming terms in eq. (5) are removed, except for (a) those produced by the density $\rho$ and pressure $p$, which are always present in a realistic stellar interior (where $\beta = 0$ for $r < R$), and (b) the one proportional to the parameter $\beta$ in a vacuum exterior (with $p = \rho = 0$ for $r > R$).

It is worth emphasizing that the condition $H = 0$ holds for any BW solution obtained by the MGD approach, and corresponds to a minimal deformation in the sense explained above. Moreover, $H$ may not be negative when a perfect fluid is used as the gravitational source on the brane, since [4] $H(p, \rho, \nu) \equiv 24 \pi p - (2G^2 + G^1) |_{\sigma^{-1} = 0}$ and the components of the Einstein tensor $G^1 = G^2 = 8 \pi p$ for a spherically symmetric perfect fluid, so that the condition $H = 0$ always holds.

In order to obtain a deformed exterior geometry, we then start by inserting the spherically symmetric Schwarzschild metric

$$e^{\nu} = e^{-\lambda} = 1 - \frac{2M}{r},$$

in the expression (4) for $r > R$, where $p = \rho = 0$. Since (5) is a GR solution, $H(r > R) = 0$ and the correction in eq. (5) will thus be minimal,

$$f^+(r) \equiv f(r) \mid_{p = \rho = H = 0} = \beta e^{-\lambda}.$$

The outer radial metric component (4) will read

$$e^{\nu} = e^{-\lambda} = 1 - \frac{2M}{r} + \beta(\sigma, R, M) e^{-\lambda},$$

which clearly represents a BW solution different from the GR Schwarzschild metric, with $\beta$ equal to the extra-dimensional correction to the GR vacuum evaluated at the star surface, that is $\beta = f^+(r = R)$, and containing a “Weyl fluid” for $r > R$ [17].

We next consider the general matching conditions between a general interior MGD metric (for $r < R$),

$$ds^2 = e^{\nu} (r) dt^2 - \frac{dr^2}{1 - \frac{2m(r)}{r} + f^-(r)} + r^2 d\Omega^2,$$

where $f^-$ is also given by eq. (5) with $H = 0$, and the above exterior metric (for $r > R$), which can be written

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like (11) by replacing $-$ with $+$. Continuity of the metric at the star surface $\Sigma$ of radius $r = R$ yields

$$
\nu_R^+ = \nu_R^- + \frac{2M}{R} = \nu_R^- + \left( f_R^+ - f_R^- \right),
$$

where $f_R^+ \equiv F(r \to R^+)$ for any function $F$. Continuity of the second fundamental form on $\Sigma$ likewise provides the expression $[G_{\mu\nu}]^+ = 0$, where $r_n$ denotes a unit radial vector and $[f]_\Sigma \equiv \hat{f}(r \to R^+) - f(r \to R^-)$. On using the effective 4D equations (1), this condition becomes

$$
\left[ p + \frac{1}{\sigma} \left( 2U + \frac{\rho^2}{2} + \rho \sigma \right) + 4 \frac{\sigma}{\sigma} \right] = 0.
$$

Since the star is assumed to be only surrounded by a Weyl fluid described by $U^+$ and $\sigma^+$ (and $p = p = 0$) for $r > R$, this matching condition takes the final form

$$
\sigma p_R + 4 \sigma^+ + 2U_R + \frac{\rho^2}{2} + \rho p p R = 2 (2 \sigma^+ + U_R),
$$

with $p_R \equiv p_R^-$ and $\rho_R \equiv \rho_R^-$. The limit $\sigma^+ \to 0$ in eq. (14) leads to the well-known GR matching condition $p_R = 0$ at the star surface. Equations (12) and (14) are the necessary and sufficient conditions for the matching of the interior MGD metric to a spherically symmetric “vacuum” filled by a BW Weyl fluid [27].

**BW star.**— BW effects on spherically symmetric stellar systems have already been extensively studied (see, e.g., refs. [28] for some recent results). Let us now investigate in detail the MGD function $f^+(r)$ produced by the Schwarzschild solution (8). By inserting it into eq. (9), we obtain

$$
f^+(r) = \beta(\sigma, R, M) \frac{b}{r} \left( 1 - \frac{2M}{r} \right),
$$

where $b$ is a length given by $b \equiv R(1 - \frac{3M}{2R})/(1 - \frac{2M}{R})$ and the deformed exterior metric components read

$$
e^{\nu^+} = 1 - \frac{2M}{r}, \quad e^{-\lambda^+} = \left( 1 - \frac{2M}{r} \right) \left[ 1 + \frac{\beta(\sigma, R, M)}{1 - \frac{2M}{R}} \right],
$$

matching the vacuum solution found in ref. [27] when $\beta b = K/\sigma$, with $K > 0$. The corresponding Weyl fluid is described by [17]

$$
\frac{\sigma}{\sigma} = \frac{\beta b}{9r^3 (1 - \frac{3M}{2R})^2} \left( \frac{9r^3 (1 - \frac{3M}{2R})^2}{\sigma} \right) = \frac{-\beta b M}{12 r^4 \left( 1 - \frac{3M}{2R} \right)}.
$$

We can now obtain the parameter $\beta = \beta(\sigma, R, M)$, depending on the interior structure, by employing the deformed Schwarzschild metric (16a) and (16b) in the matching conditions (12) and (14). Equation (12) just becomes $e^{\nu} = 1 - \frac{2M}{R}$, whereas eq. (14) yields

$$
p_R + \frac{f_R}{R} \left( \nu_R + \frac{1}{\nu_R} \right) = -\frac{f_R^+}{R^2},
$$

where $\nu_R \equiv (\nu^-)'\vert_{r=R}$. These are the necessary and sufficient conditions for matching the two minimally deformed metrics given by eqs. (11), (16a) and (16b). If $M$ in eq. (8) were the GR mass $M_0$, one would have $f_R^+ = f_R$ (see eq. (12)), which is an unphysical condition, according to eq. (18). In fact, the interior deformation $f = f^-(r)$ is positive, but the matching condition (18) shows that the exterior deformation must be negative at the star surface, $f_R^+ < 0$, or else a negative pressure $p_R < 0$ would appear. Hence, according to eq. (15), the deformation $f^+(r > R)$ is negative for $\beta < 0$ [18].

The exterior geometry given by eqs. (16a) and (16b) may seem to have two horizons, namely $r_h = 2M$ and $r_t = 3M/2 - \beta b$. However, since $\beta$ must be proportional to $\sigma^{-1}$ in order to recover GR, the condition $r_t < r_h$ must hold, and the outer horizon radius is given by $r_h = 2M$. The specific value $\beta = -M/2$ would produce a single horizon $r_h = r_t = 2M$, but the limit $\sigma^{-1} \to 0$ does not reproduce the Schwarzschild solution, as seen from the condition $M_0 = M(\sigma^{-1}=0)$. On the other hand, $f_R^+ < 0$ implies that the deformed horizon radius $r_h = 2M$ is smaller than the Schwarzschild radius $r_H = 2M_0$, as it can be clearly realised from eq. (12). This general result shows that 5D effects weaken the strength of the gravitational field produced by the self-gravitating stellar system.

Finally, when (15) is considered in the matching condition (18), we obtain

$$
\beta = f_R^+ = -R^2 \left[ \nu_R + \left( \frac{1}{\nu_R} + \nu_R \right) \frac{f_R}{R^2} \right],
$$

showing that $\beta$ is always negative and (interior) model-dependent through $\nu_R$. In particular, we can find $\beta$ by considering the exact interior BW solution of ref. [12], that is

$$
f^+ = \frac{32C}{49\sigma} \left[ \frac{240 + 589Cr^2 - 25C^2r^4 - 41C^3r^6 - 3C^4r^8}{3(1 + Cr^2)^4(1 + 3Cr^2)} \right] - \frac{80}{1 + Cr^2} \frac{\arctan(\sqrt{Cr})}{(1 + Cr^2)^2},
$$

where $C$ denotes a constant (with the same dimensions of $\sigma$) given by $C R^2 = \frac{\sqrt{M}}{2} \equiv \alpha$, and $\nu^+ = 8\sqrt{R} (1 + Cr^2)^{-1}$. Using the explicit form of $f^-(R)$ and $p_R = 0$ in eq. (19) yields

$$
\beta(\sigma, R) = f_R^+ = -\frac{C_0}{R^2 c_0},
$$

where $C_0 \approx 1.35$ is a (dimensionless) constant. The exterior deformation is finally obtained by using eq. (21) in
The observed difference \( \delta \phi - \delta \phi_{\text{GR}} = 0.13 \pm 0.21 \) arcsec/century [30] can thus be ascribed to BW effects, according to eq. (25). Observational data [29,30] yield the bound \( f(\beta) \leq (1.89 \pm 2.33) \times 10^{-8} \), which constrains

\[ \beta \lesssim (2.80 \pm 3.45) \times 10^{-11}. \]

**Light deflection.** A similar procedure describes photons on null geodesics, with the equation of motion that can be written as

\[ \left( \frac{du}{d\phi} \right)^2 + u^2 = \frac{1}{c^2} \left( \frac{GM}{L^2} \right)^2 e^{-\nu - \lambda} + g(u) u^2 \equiv p(u), \]

which therefore implies \( \frac{du}{d\phi} + u = \frac{1}{2} \frac{dp(u)}{du} \). In the lowest approximation, the solution is \( u = \frac{\cos \phi}{R_0} \), where \( R_0 \) is the distance of the closest approach to the mass \( M \). It can be iteratively employed in the above equation, yielding \( \frac{du}{d\phi} + u = \frac{1}{2} (\frac{dp(u)}{du})^{1/2} \). The total deflection angle of the light ray is given by \( \delta = 2\pi \) [29].

For the geometry (16a) and (16b), eq. (23) leads to

\[ g(u) = (2GM/c^2) u, \]

resulting in

\[ p(u) = \frac{\beta_0}{(2 - 4GM/c^2)^2} \left( -u^2 + \frac{GMu}{c^2} \left( \frac{9GM}{c^2} - u - 11 \right) + 3 \right) - 2u^2 + \frac{3GMu^2}{c^2}. \]

The total deflection of light is finally given by

\[ \delta \phi = \frac{4GM}{c^2 R_0} + \beta_0 \frac{E^2 R_0}{c^2 L^2} + \frac{18\pi^2}{G M R_0}, \]

in the limit \( \left( \frac{GM}{c^2 R_0} \right)^2 \ll 1, \frac{M}{T} \ll 1 \) and \( \frac{E^2}{c^2} - 1 \ll 1 \), which implies the bound

\[ \beta \lesssim (1.07 \pm 4.28) \times 10^{-10}. \]

**Radar echo delay.** Another classical test of GR measures the time for radar signals to travel to, for instance, a planet [30]. The time for light to travel between two planets, respectively at a distance \( \ell_1 \) and \( \ell_2 \) from the Sun, is well known to be \( T_0 = \int_{-\ell_1}^{\ell_2} dx/c \). On the other hand, if light travels in the vicinity of the Sun, the time lapse \( \delta T = T - T_0 \) is given by [29]

\[ \delta T = \frac{1}{c} \int_{-\ell_1}^{\ell_2} \frac{e^{\lambda(x - \ell_1) - \nu(x + \ell_2)}}{\sqrt{1 - \frac{\nu^2}{c^2}} - \frac{\lambda}{c^2}} - 1 \left( \frac{2GM}{c^2 r^2} \right)^{-1} \left( \frac{2\beta_0}{3GM - 2c^2 r + 1} \right)^{1/2} \approx \frac{2GM}{c^2 r^2} - \frac{\beta_0}{3GM - 2c^2 r + 4\beta_0 GM c^2 r}, \]

where we used a first-order approximation based on eqs. (16a) and (16b). Equation (31), using the
approximations $R^2/l_i^2 \ll 1$ ($i = 1, 2$), and considering terms up to order $(GM/c^2R)^2$, hence reads

$$\delta T \simeq \frac{1}{c^3R} \frac{\beta \ell_0}{c^3R} \left[ \ln \left( \frac{4\ell_1\ell_2}{R^2} \right) - \frac{5\pi GM}{2} \right],$$

(33)

which reproduces the Schwarzschild radar delay $\delta T_{\text{GR}} = \frac{2GM}{c^3R} \ln \frac{4\ell_1\ell_2}{T_{\text{radar}}}$. When $\beta = 0$, and the second term imposes a constraint on BW models. Recent measurements of the frequency shift of radio photons both to and from the Sun, one obtains $\Delta t_{\text{radar}} = \Delta t_{\text{radar}}^{\text{GR}} (1 + \Delta_{\text{radar}})$, with $\Delta_{\text{radar}} \simeq (1.1 \pm 1.2) \times 10^{-5}$ [31]. In the BW geometry (16a) and (16b), measurements of the frequency shift of radio photons [29,31] finally yield the physical bound

$$\beta \lesssim \frac{5\pi G^2M^2}{2\ell_0 R \ln \left( \frac{4\ell_1\ell_2}{R^2} \right)} \simeq (3.96 \pm 4.30) \times 10^{-5}.$$  

(34)

This provides a bound on the MGD parameter $\beta$, which is the weakest one among those in our analysis.

**Concluding remarks.** – BW models can be confronted with astronomical and astrophysical observations at the solar-system scale. In this paper we have in particular considered the BW exterior solution (16a) and (16b) obtained by means of the MGD procedure, and compared its predictions with standard GR results. This exterior geometric contains a parameter $\beta$ and we were able to constrain it from the presently available observational data in the solar system. We found the strongest constraint is given by measurements of the perihelion precession, namely eq. (26).

Let us recall that limits for the brane tension in the DMPR and Casadio-Fabbri-Mazzacurati BW solutions have already been determined via the classical tests of GR [29]. Since bounds on the parameter $\beta$ imply lower bounds for the brane tension from eq. (21), we can conclude that the constraint (26) complies with the ones provided by such solutions of the effective 4D Einstein equations (1). In fact, the brane tension in the MGD framework is bounded according to

$$\sigma \geq \frac{9M_{\odot}c^2}{\pi R_{\odot}^3 \beta} \left( 1 - \frac{2GM_{\odot}}{c^2R_{\odot}} \right)^2 \left( 1 - \frac{3GM_{\odot}}{2c^2R_{\odot}} \right),$$

(35)

which implies that $\sigma \geq 5.19 \times 10^9 \text{MeV}^4$, when the bound (26) is taken into account (we omit errors here since we are solely interested in orders of magnitude). This bound is still much stronger than the cosmological nucleosynthesis constraint, however much weaker than the lower bound obtained from measurements of the Newton law at short scales. We can therefore conclude that the MGD geometry (16a) and (16b) is acceptable within the present measurements of BW high-energy corrections.

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**Additional remark:** After this work had been completed, we developed an extension of the MGD, which produces a rich but complex set of new exterior solutions [32], whose complete analysis is highly non-trivial. However, the solution used here represents the simplest non-trivial extension of the Schwarzschild solution within the extended MGD approach.

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