Analytical prediction of initiation of ferroresonance modes

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Abstract. This research focuses on the impact of basic circuit parameters to the initiation of characteristic ferroresonance modes, which is a non-linear phenomenon, in single-phase power transformers. This non-linear phenomenon is investigated using an equivalent circuit of the transformer together with a circuit simulation. The basic circuit parameters investigated were iron core loss, circuit capacitance, and the source voltage. A mathematical method is used to calculate the critical boundary values of each parameter to avoid the occurrence of ferroresonance in the system. In this paper, an analytical method was used to predict the occurrence of ferroresonance while Matlab/Simulink simulation was used to determine the different ferroresonance modes in the power transformers. By varying the circuit parameters, different ferroresonance modes are observed and discussed.

1. Introduction
The phenomena of ferroresonance in power transformers occurs when there is a sudden unintentional jump in the output signal amplitude due to a change in the input of the system [1]. The implication of ferroresonance impacted on power system can cause severe damage and power problems due to the highly distorted over-voltages and/or under-voltages. These magnitudes may vary depending on the type of initial condition and the different parameters of the transformer. Based on the waveforms of these voltages, different types or modes of ferroresonance can be categorised [2]. Although many approaches, i.e. analytical approaches [1-11] and simulation approaches [12-17], have been carried out to mitigate the occurrence of ferroresonance, more effort are still needed to further understand its initiation and also to prevent it from occurring in the system. In view of the uncertainty and the random behaviour of the occurrence of ferroresonance phenomena, the main goal of this research is to formulate a mathematical model so that a thorough understanding of the onset of different ferroresonant modes can be comprehended. In addition, the analytical formulation will also be used to predict the safe boundary region by varying the circuit parameters in the circuit model. The predictions on the onset of different modes of ferroresonance are analysed thoroughly and the results obtained are verified using MATLAB/Simulink models and MATCONT model in graphical representation. MATCONT is a graphical MATLAB software package for the interactive numerical study of dynamical systems [18].

2. Ferroresonant Mode
There are four different types or modes of ferroresonance, namely the fundamental mode, subharmonic mode, quasi-periodic mode and chaotic mode. Each type can be identified by the behaviour of its frequency spectrum or the stroboscopic image. Figure 1 shows all the different types of ferroresonance waveforms and their corresponding frequency spectrums [19]. In this research, the frequency spectrums are used to identify the different types of ferroresonance.
Figure 1. (a) Fundamental, (b) Subharmonic, (c) Quasi-periodic, (d) Chaotic ferroresonance modes

3. Modelling

The procedure used in modelling the types of ferroresonant modes can be summarised as

(i) using mathematical formulation developed in [9] to assess the boundaries conditions for the values of the series capacitor, $C_T$, the supply voltage, $e_T$ and the core losses, $R_m$ of the magnetising circuit,

(ii) using MATLAB/Simulink/Simscape to develop a graphical user interface model to determine the types of ferroresonant modes and also to validate the accuracy of the results obtained in part (i),

(iii) developing a state space equations model in MATCONT to validate the results obtained in part (ii).

Figure 2 shows the single-phase ferroresonant equivalent circuit model, which consists of a voltage source, $e_T$, a series capacitor, $C_T$, and a transformer model under no-load or light load condition. Under no-load or light load condition, the magnetising branch of the transformer is modelled with a core loss resistance, $R_m$, and nonlinear inductor, $L_m$. In order to study ferroresonance in transformers, a mathematical formulation developed in [9] and the circuit parameters studied in [20] are employed. Table 1 shows the initial parameters of the circuit employed for the ferroresonance studies.

Figure 2. Thevenin's equivalent circuit of a transformer: $e_T$ is source voltage, $i$ is the total current, $C_T$ is the series capacitance, $v_{CT}$ is the voltage across $C_T$, $L_p$ is the leakage inductance, $r_p$ is the winding resistance and $V_T$ is the voltage across the transformer.
The nonlinear inductor characteristic of the magnetising branch of the transformer is as shown in Figure 3. The magnetising characteristic of the transformer, i.e. the current flows through the inductor, \( i_{Lm} \), can be approximated by Equation (1).

\[
i_{Lm} = a\lambda + b\lambda^n
\]

where \( n \) is the degree of saturation, \( \lambda \) is the magnetic flux-linkage and \( a \) and \( b \) are arbitrary constants.

### Table 1. Simulation parameters

| Parameter | Value     |
|-----------|-----------|
| \( e_T \) | 123 kV    |
| \( f \)   | 50 Hz     |
| \( C_T \) | 17.045 nF |
| \( L_p \) | 2.7 \( \mu \)H |
| \( R_m \) | 92 M\( \Omega \) |
| \( r_p \) | 7490 \( \Omega \) |

In Equation (1), the former term corresponds to the linear region of the magnetisation curve and the latter term corresponds to the approximation of the saturated region or non-linear region [21].

![Figure 3. Magnetising characteristics of the transformer](image)

The degrees of saturation, \( n \), is chosen as 5 for modelling transformer core nonlinearity. The coefficient values of \( a \) and \( b \) are determined using Curve Fitting Tool [22] in MATLAB and their values are determined as 0.005109 and 2.184e-09, respectively.

### 3.1. Modelling of Ferroresonant Circuit using MATLAB Simulink

A single phase ferroresonant circuit model as shown in Figure 2 has been developed in MATLAB Simulink/Simscape [23-24] as shown Figure 4. The MATLAB Simulink/Simscape model is used to determine the types of ferroresonant modes. A Voltage Sensor, which is connected across the transformer terminal, is used for measuring the voltage across the transformer. To obtain the flux linkage of the transformer core, the output voltage of the transformer is mathematically integrated using an Integrator block since the induced voltage is the rate of change of the magnetic flux [25]. PS-Simulink block is used to convert the Simscape signals to Simulink signals so that the output can be transferred into the MATLAB workspace.
3.2. Modelling of Ferroresonant Circuit using Ritz Method of Harmonic Balance

The Ritz Method is used to predict the safe boundary values for each parameter to avoid the occurrence of ferroresonance [9]. Using this method, the boundary values of the solution for the equivalent circuit as given in Equation (2) and Equation (3) are calculated and plotted.

\[ z_1 = P_1 \lambda^2 - P_0 \]  
\[ z_2 = P_2 \lambda^{(n+1)} - \lambda^{2n} \]  

where,

\[ P_0 = \frac{\omega_s^2 E^2}{(k_1 \omega_s^2)^2} \]  
\[ \omega_2 = \sqrt{\frac{b}{C_T}} \]  
\[ \omega_3 = 2\pi f \]  
\[ k_1 = (-1)^{n-1} \left( \frac{n}{2} \right) \]  
\[ e_T = E \sin(\omega_3 t) \]

In this study, the iron-core losses, \( R_m \), source voltage, \( e_T \), and capacitor, \( C_T \), values are varied. The degree of saturation will also be varied to observe and analyse the differences. The points of intersection between the functions, \( z_1 \) and \( z_2 \), as shown in Figure 5, is the solution to the differential equation of the ferroresonant circuit in Figure 2.

In Figure 5, the plot of \( z_1 \) is a linear or straight-line graph while \( z_2 \) is a curved or non-linear graph. The intersections that occur at the right-hand side of the \( z_2 \) curve indicate that ferroresonance will occur. The intersections that occur at left-hand side of \( z_2 \) plot is in the linear steady state. The intersections that occur at the middle of the curve \( z_2 \) indicate that the solution is unstable because a small perturbation will result in the solution moving to either linear steady state or ferroresonant state.
[9]. Figure 5 also shows the graphs of $z_1$ and $z_2$ with varying magnitude of the source voltage with the rest of the circuit parameters kept constant at the initial condition. From Figure 5, it is shown that with increasing source voltage magnitude will drive the transformer further into ferroresonant state. However, even with a lower source voltage, it does not necessarily eliminate the possibility of ferroresonance as shown in the figure. It is found that it is almost impossible to decrease the source voltage magnitude to a level where the graph of $z_1$ does not meet the left-hand side $z_2$ curved to eliminate the possibility of ferroresonance. Therefore, with the initial circuit parameters used, it is safe to assume that any source voltage magnitude will drive the transformer into ferroresonant state.

To investigate the boundary limit of ferroresonance with respect to the iron-core losses, $R_m$, all other circuit parameters are kept constant at the initial condition while $R_m$ is varied. Figure 6 shows the plots of $z_1$ with different values of iron core losses, $R_m$. The function $z_2$ is independent of $R_m$ and therefore the plot of $z_2$ does not change with $R_m$. By referring to the plot, it can be concluded that ferroresonance will occur when $R_m > 0.25 \text{ MΩ}$. When $R_m$ is smaller than 0.25 MΩ, the transformer remains in linear steady state operation for the given initial condition of $C_T$ and $e_T$.

Figure 5. Graphical plot of $z_1$ and $z_2$ with varying source voltage. A = 15 kV, B = 60 kV, C = 80 kV, D = 100 kV, E = 123 kV, F = 140 kV, G = 155 kV.

Figure 6: Graphical plot of $z_1$ and $z_2$ with varying iron core losses. Curves A = 0.15 MΩ, B = 0.2 MΩ, C = 0.25 MΩ, D = 0.35 MΩ, E = 0.5 MΩ, F = 1.0 MΩ, G = 92 MΩ.

Figure 7: Graphical plot of $z_1$ and $z_2$ with varying series capacitance.
Figure 7 shows the same graphs with varying series capacitance, \( C_T \). The graphs of \( z_2 \) change with varying capacitance unlike the previous graphs. This is because the series capacitance changes both the Equation (2) and Equation (3). The graph shows that ferroresonance will occur regardless of the series capacitance used. From the results, it is shown that the \( z_1 \) graph will always meets the \( z_2 \) graph at the right-hand side of \( z_2 \) graph which is the ferroresonant region. Since the gradient of the \( z_2 \) graph becomes more positive with increasing capacitance, a larger capacitance might reduce the likelihood of ferroresonance. However, the magnetic flux-linkage at the transformer also increases with increasing capacitance which increase the likelihood ferroresonance. Therefore, as a whole, changing the series capacitance has very little effect on changing the transformer state.

3.3. Modelling of Ferroresonant Circuit using MATCONT

To verify these calculated values discussed in the previous section, those parameters are fed into both MATCONT and Simulink models. The results from the simulations and calculations should show that ferroresonance will only occur outside the boundary values of each parameter calculated using the Ritz method. To develop the model for MATCONT, which based on state space equations, the voltage across the transformer and the flux-linkage at the transformer need to be derived [9]. Equations (12) to (14) show the derived state space equations from [20].

\[
\begin{align*}
\frac{dx_1}{dt} &= x_2 \\
\frac{dx_2}{dt} &= \frac{R_m}{L_p} \left[ e_T - v_{CT} - r_p \left( \frac{1}{R_m} \frac{d\lambda_m}{dt} + a\lambda_m + b\lambda_m^n \right) - \frac{d\lambda_m}{dt} \right] \\
\frac{dx_3}{dt} &= \frac{1}{C_T} \cdot i = \frac{1}{C_T} \left( \frac{1}{R_m} \frac{d\lambda_m}{dt} + a\lambda_m + b\lambda_m^n \right)
\end{align*}
\]

(12) to (14)

where \( x_1 \) is the flux-linkage, \( x_2 \) is the change in magnetic flux-linkage which is the voltage across the transformer, \( v_m \), and \( x_3 \) is the voltage across the series capacitance, \( v_{CT} \).

4. Results

In this section, the results of the simulations are presented. First, the Simulink model is verified and then the results of varying the different parameters are produced and analysed. Only the key results will be plotted and presented while the other results are shown in a tabular form.

4.1. Model Verification

Figure 8(a) and (b) show the voltage across the transformer as calculated in Simulink and MATCONT, respectively, using the parameters specified in Table 2. The maximum voltage is lower in Simulink than in MATCONT with a percentage error of 0.5%. Both simulations showed that it reached maximum voltage after 0.019 seconds with a value of about 320 kV. The voltage then continues into a steady-state with a peak value of about 170 kV.

Figure 8(c) shows the Fast Fourier Transformer (FFT) spectrum of Figure 8(a). It shows that the frequency components of the voltage across the transformer are of mainly 50 Hz, 150 Hz and 250 Hz. Based on Figure 1, it is verified that the type of ferroresonance is the fundamental ferroresonant mode.

Figure 9 show the flux-linkage at the transformer calculated in Simulink and MATCONT, respectively. In Simulink, the flux-linkage at the transformer reached a maximum of 756 Wb-T after 0.021 seconds. The steady-state peak value was 655 Wb-T after 0.06 seconds. In MATCONT, the maximum flux-linkage was 757 Wb-T after 0.02 seconds which was comparable to the results in Simulink.
The results as shown in Figure 9(a) and (b) verified the Simulink model since the graphs from Simulink and MATCONT are closely matched with an acceptable percentage error. The results also show that the Ritz method correctly predicts the occurrence of ferroresonance for this case.

![Graph showing voltage across transformer](image1)

![Graph showing flux-linkage at transformer](image2)

**Figure 8.** $R_m = 92 \text{ MΩ}$, Voltage across transformer (a) Simulink (b) MATCONT (c) FFT Spectrum

**Figure 9.** Flux-linkage at transformer (a) Simulink (b) MATCONT

4.2. Varying the Core Losses, $R_m$

The core losses parameter, $R_m$, was varied to analyse the different output voltages. Figure 10 shows the voltage outputs and its FFT spectrum for $R_m = 0.25 \text{ MΩ}$. The maximum voltage reached 117 kV and
the steady state peak voltage is 113 kV. The fundamental frequency was 50 Hz and there were no harmonic present. Therefore, there was no ferroresonance occurring.

Figure 11 shows the output voltages and its FFT spectrum when $R_m$ is changed to 0.35 MΩ. The maximum voltage reached is 175 kV and the steady-state peak voltage is 162 kV. The steady state output voltages show similarity to the voltage graph Figure 1(a) which indicates a likelihood of fundamental ferroresonant mode.

To confirm the ferroresonant mode, the FFT of Figure 11(a) is plotting in Figure 11(b). The graph further confirms that the ferroresonance is indeed a fundamental mode i.e. it consists of higher frequencies harmonics with decreasing amplitudes.

**Table 2. Summary of results with varying iron core loss**

| Iron core loss, $R_m$ (MΩ) | Maximum voltage (kV) | Simulation Results | Theoretical Ritz Method |
|----------------------------|----------------------|--------------------|------------------------|
| 0.15                       | 79                   | No ferroresonance  | No ferroresonance      |
| 0.2                        | 98                   | No ferroresonance  | No ferroresonance      |
| 0.25                       | 117                  | No ferroresonance  | No ferroresonance      |
| 0.35                       | 175                  | Fundamental        | Ferroresonance occurs  |
| 0.5                        | 198                  | Fundamental        | Ferroresonance occurs  |
| 1.0                        | 257                  | Fundamental        | Ferroresonance occurs  |
| 92                         | 315                  | Fundamental        | Ferroresonance occurs  |

shows the summary of the above results and the other results using different iron core losses. The simulation results in the table show the type of ferroresonance that occurred. The theoretical results determine whether the ferroresonance has been predicted using Ritz method. It can be shown that increasing the iron core loss will increase the tendency of ferroresonance occurrence. The initial iron core loss was 92 MΩ.

**Figure 10.** $R_m = 0.25$ MΩ (a) voltage across transformer (b) FFT Spectrum
Figure 11. R_m = 0.35 MΩ (a) Voltage across transformer (b) FFT spectrum

Table 2. Summary of results with varying iron core loss

| Iron core loss, R_m (MΩ) | Maximum voltage (kV) | Simulation Results | Theoretical Ritz Method |
|--------------------------|----------------------|--------------------|-------------------------|
| 0.15                     | 79                   | No ferroresonance  | No ferroresonance       |
| 0.2                      | 98                   | No ferroresonance  | No ferroresonance       |
| 0.25                     | 117                  | No ferroresonance  | No ferroresonance       |
| 0.35                     | 175                  | Fundamental        | Ferroresonance occurs   |
| 0.5                      | 198                  | Fundamental        | Ferroresonance occurs   |
| 1.0                      | 257                  | Fundamental        | Ferroresonance occurs   |
| 92                       | 315                  | Fundamental        | Ferroresonance occurs   |

4.3. Varying the Source Voltage Amplitude

Figure 12 shows the voltage across the transformer and its FFT spectrum as the source voltage amplitude is increased to 60 kV. The maximum voltage reached 113 kV while the steady-state peak voltages are well below 100 kV. It is shown that some harmonics are present in the FFT graph. Based on Figure 1(b), this type of response can be identified as subharmonic ferroresonance. Also, by referring to Figure 5, the Ritz method correctly predicts this occurrence of ferroresonance.
Figure 12. $e_T = 60$ kV (a) Voltage across transformer (b) FFT spectrum

Figure 13. $e_T = 155$ kV (a) Voltage across transformer (b) FFT spectrum

Figure 13 shows the voltage across the transformer and its FFT spectrum after the source voltage amplitude was further increased to 155 kV. It can be clearly seen that there is ferroresonance occurring. The maximum voltage reached 316 kV. The steady-state peak voltage was around 180 kV after 0.17 seconds. From the FFT spectrum, it confirmed that fundamental ferroresonant mode is observed.

Table 3. Summary of results with varying source voltage

| Source voltage, $e_T$ (kV) | Maximum voltage (kV) | Simulation Results | Theoretical Ritz Method |
|---------------------------|----------------------|--------------------|------------------------|
| 15                        | 21                   | Subharmonic        | Ferroresonance occurs  |
| 60                        | 113                  | Subharmonic        | Ferroresonance occurs  |
| 80                        | 238                  | Fundamental        | Ferroresonance occurs  |
| 100                       | 291                  | Fundamental        | Ferroresonance occurs  |
| 123                       | 315                  | Fundamental        | Ferroresonance occurs  |
| 140                       | 319                  | Fundamental        | Ferroresonance occurs  |
| 155                       | 316                  | Fundamental        | Ferroresonance occurs  |
shows the summary of the results with different source voltages amplitudes. At lower voltages, ferroresonance still occurred and the system was unstable. It is clearly shown that increasing the source voltage amplitude will drive the transformer from subharmonic mode to fundamental mode. When the transformer enters into fundamental mode, the maximum peak voltage across the transformer was almost tripled the source voltage amplitude.

| Source voltage, $e_T$ (kV) | Maximum voltage (kV) | Simulation Results | Theoretical Ritz Method |
|----------------------------|----------------------|--------------------|------------------------|
| 15                         | 21                   | Subharmonic        | Ferroresonance occurs  |
| 60                         | 113                  | Subharmonic        | Ferroresonance occurs  |
| 80                         | 238                  | Fundamental        | Ferroresonance occurs  |
| 100                        | 291                  | Fundamental        | Ferroresonance occurs  |
| 123                        | 315                  | Fundamental        | Ferroresonance occurs  |
| 140                        | 319                  | Fundamental        | Ferroresonance occurs  |
| 155                        | 316                  | Fundamental        | Ferroresonance occurs  |

4.4. Varying the Series Capacitance, $C_T$

Figure 14 shows the voltage response and its FFT spectrum when the series capacitance was changed to 0.01 µF. The maximum voltage reached 291 kV and the steady-state peak voltage was around 150 kV after 0.22 seconds. The FFT plot shows that fundamental ferroresonant mode had occurred.
Figure 15 shows the voltage variations and its FFT spectrum when the series capacitance increases to 0.025 µF. The maximum voltage was 328 kV and the steady-state peak voltage was 193 kV after 0.07 seconds. From the FFT response, the occurrence of fundamental ferroresonant mode is observed.

**Error! Reference source not found.** shows the summary of results using different series capacitance. It shows that increasing the capacitance would lessen the effect of ferroresonance. The simulation showed that the ferroresonance changed from chaotic mode to fundamental mode when the capacitance was changed from 0.001 µF to 0.01 µF.

![Figure 15. C_T = 0.025 µF (a) Voltage across transformer (b) FFT spectrum](image)

| Series capacitance, C_T (µF) | Maximum voltage (kV) | Simulation Results | Theoretical Ritz Method |
|-----------------------------|----------------------|--------------------|-------------------------|
| 0.001                       | 164                  | Chaotic            | Ferroresonance occurs   |
| 0.01                        | 291                  | Fundamental        | Ferroresonance occurs   |
| 0.017045                    | 315                  | Fundamental        | Ferroresonance occurs   |
| 0.02                        | 322                  | Fundamental        | Ferroresonance occurs   |
| 0.025                       | 328                  | Fundamental        | Ferroresonance occurs   |
| 0.1                         | 1293                 | Fundamental        | Ferroresonance occurs   |

**5. Conclusion**

Using an equivalent ferroresonant circuit model, simulations were conducted using Simulink and MATCONT. The iron core loss, series capacitance and source voltage are varied to observe the occurrence of ferroresonance. Using the Ritz method of harmonic balance, the safe operating boundary can be predicted reliably. From the results, it is shown that only the core loss resistance can change the transformer from a ferroresonant state to a non ferroresonant state. Whereas changing other circuit parameters can only change the ferroresonant state from one mode to another. By comparing the results, the Ritz method predicted correctly the occurrence of ferroresonance for any given circuit parameter. One of the limitations of the method was that it cannot predict the type of ferroresonance.
occurring. However, using Ritz method coupled with Simulink or MATCONT simulation will provide an essential tool to predict the occurrence of ferroresonance and also determine the type of ferroresonant mode.

6. References

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