COORDINATION CONTRACTS FOR A DUAL-CHANNEL SUPPLY CHAIN UNDER CAPITAL CONSTRAINTS

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Abstract. Manufacturers often face capital constraints when opening up online channel, at this time external financing and internal financing are usually considered. Previous literature has shown that internal financing, turns out to be a better option. To figure out how trade credit financing discount contract affects operations and performances of supply chain, this paper studies the pricing decision of a retailer-dominant dual-channel supply chain with manufacturer’s capital constraints. The Stackelberg game models under centralized decision and decentralized decision are constructed. Moreover, this paper conducts research about the effects of revenue-sharing (RS) contract, direct channel price discount (DP) contract and retail channel price discount (RP) contract on the performance of supply chain. Numerical examples are provided to explore the comparison of the optimal pricing strategies and total profits under different contracts. The results show that the retailer prefers RS and DP contracts to RP contract. Among them, RS contract has a broader scope of coordination, while DP contract can achieve a higher profit. The results can serve as insights for decision-makers to choose the most appropriate financial discount contract.

1. Introduction. As is known, with the development of economic globalization, capital constraint may have a destructive impact on the survival and development of Small and Medium Enterprises (SMEs). A survey in 2015 showed that only 47% of the 3459 SMEs in the United States applied for financing, while only half of the other companies that did not apply for financing were adequately funded (Barkley et al. [3]). Especially when manufacturers open their network channels, they often face capital problems. Capital constraints will have a serious impact on the operation of enterprises, so they are considered as the biggest obstacle to the development of enterprises (Ayyagari et al. [2]). When confronted with financial

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pressure, the general practice is to borrow from banks (external financing), or to encourage well-funded retailers to pay in advance (internal financing). Among them, banks financing requires credit and guarantees, and repayment of loan principal and interest will cause the profits loss of the supply chain. Therefore, the advance payment financing by retailers is more welcomed by manufacturers with capital constraints.

As supply chain members become more and more closely related, the financial constraints of one party will directly influence the operation of the other party. For example, in the auto parts industry, Delphi, the largest supplier in the United States, filed for bankruptcy protection in October 2005, causing the supply crisis of General Motors, which accounts for 51% of Delphi’s sales. The manufacturer’s capital problem will be more likely to leave the downstream retailers or distributors at the peril of facing supply disruption, thus the operation of the whole supply chain will be affected. Therefore, if a manufacturer with dual-channel needs to borrow money from a retailer for short-term production and compensate the retailer with price discount; the retailer with sufficient funds is willing to provide financial assistance to the manufacturer to avoid the risk of channel shortage.

In reality, the main financing mode chosen by retailers is advance payment. The International Monetary Fund (2008) showed that advance payment accounted for 19-22% of global trade finance (i.e. $3-3.5 trillion). In 2013, Huawei arranged advance payment to high-quality suppliers, totaling nearly 1 billion dollars, in order to achieve long-term win-win cooperation in the industrial chain. In addition, Midea Group dealers provided advance partial payments to the manufacturer in the off seasons to guarantee a stable supply of goods in the peak seasons.

To the best of our knowledge, there is little work in the literature studying the choice of advance payment discount contracts in a retailer-dominant dual-channel supply chain. In practice, more and more retailers dominate the supply chain, such as Wal-Mart and Carrefour. They all succeeded in their respective supply chains. The previous literature (Kouvelis and Zhao [12]; Wu et al. [24]) has shown that internal financing is more effective than external financing. To stimulate retailers to pay in advance, manufacturers will give retailers a price discount, such as direct channel price discount and retail channel price discount (Xu et al. [29]); or revenue-sharing (Moon et al. [19]), which are common in real life.

As far as we know, this research, which takes into account of the impact of different advance payment discount contracts on the performance of supply chain is silent on the extant literature. Our work aim to bring in a better understanding of the following questions: how do the channel members make their pricing decisions when facing manufacturer’s capital constraints? Which financing contract strategy (RS, DP and RP contracts) is more beneficial to the performance of supply chain? Therefore, the model proposed in this paper is different from the previous models in three aspects: Firstly, although we focus on the traditional dual-channel pricing problem, we innovatively consider the manufacturer’s capital constraints in the retailer-dominant supply chain. Secondly, previous literature has studied the comparison between internal financing and external financing. We pay more attention to the comparison of internal financing contracts. Thirdly, research shows that RS contract has a broader scope of coordination, while DP contract can achieve higher supply chain profits. Therefore, manufacturers should choose different contracts for different retailers.
2. **Literature review.** Two research fields related to the study are presented in this paper: supply chain financing and dual-channel supply chain.

2.1. **Supply chain financing.** In recent years, supply chain financing has increasingly become a hot issue in academic research. However, most of the current research is based on the traditional retail channel environment, and there is little literature on dual-channel supply chain financing. Generally speaking, most literature focuses on the comparison between internal financing and external financing. Some literature points out that trade credit has more advantages than bank loans, as trade credit can alleviate double marginalization (Tang et al. [22]; Yang et al. [33]; Yang and Birge [34]). Kouvelis and Zhao [12] showed that trade credit is one of the important short-term financing sources among enterprises. Compared with bank financing, retailers always prefer supplier financing. Yang et al. [32] found that a retailer prefers trade credit financing when trade credit financing’s risk-free interest rate is lower than bank interest rate. Wu et al. [24] studied the operation and financing decision of a green supply chain. It is concluded that when both bank financing and trade credit financing are available, a retailer with capital constraints prefers trade credit financing. The above literature proves that internal financing is more effective than external financing.

Therefore, we mainly focus on the impact of internal financing on supply chain performance. Some literature studied operational decisions under trade credit financing (Chen and Teng [6]; Zhang et al. [36]). Some studies have shown that trade credit as a way of coordination can effectively coordinate the supply chain with capital constraints (Lee and Rhee [14]; Xiao et al. [25]). Jiang and Hao [10] studied how well-funded manufacturers can alleviate supplier’s financial constraints by prepayment. Yan and Sun [31] discussed the retailer’s financial constraints and bankruptcy risk, and made a comparison of two internal financing modes based on order and capital-gap. The results showed that the manufacturer prefers order-based financing mode while the retailer favors the financing mode based on capital-gap. Li et al. [16] studied how a manufacturer combines penalty and financial assistance to deal with the supplier facing financial difficulties due to supply disruptions. Peng and Pang [20] proposed an advance payment mechanism with risk compensation to alleviate the supplier’s financial difficulties under uncertain productivity. Xiao and Zhang [27] hold that it is advisable for the manufacturer to pre-sell products to the retailer at discount prices for the purpose of gaining advance payment, thereby raising cash and alleviating the capital pressure. Jin et al. [11] compared bank financing, trade credit financing and bank financing with the supplier’s guarantee considering that both the supplier and the retailer are subject to capital constraints. Hua et al. [8] studied the ordering and financing decision of a supply chain with a retailer confronting capital constraints. The results showed that the retailer should tend to finance from the supplier when the retailer is at the risk of bankruptcy. All the above articles discussed how trade credit as an internal financing mode affects supply chain decision. However, they did not go further to study trade credit financing in a dual-channel environment.

2.2. **Dual-channel supply chain.** With the rapid development of e-commerce, more and more manufacturing enterprises are opening online direct channel other than the traditional retail channel (Yan et al. [30]). It is beneficial for manufacturers to open up online direct channel to increase market demand thereby improving efficiency and changing operation modes (Lu and Liu [18]). At present, most of the
research on dual-channel supply chain focuses on the price competition and coordination (Huang and Swaminathan [9]; Yoo and Lee [35]; Xie et al. [28]; Dzyabura and Jagabathula [7]; Lai et al. [13]).

Chen et al. [5] studied the pricing decisions of a manufacturer in a dual-channel supply chain, and proposed the conditions under which both sides of the supply chain prefer to open up online channel. They also found that some contracts with complementary agreements can coordinate the dual-channel supply chain and achieve a win-win situation. Xiao and Shi [26] discussed the situation that stochastic benefits would lead to supply shortage, and studied the pricing and channel priority strategy of dual-channel supply chain. Chen et al. [4] studied the pricing and quality decision of supply chain under a retail channel, an online channel and a dual-channel supply chain respectively, and found that the quality of products could be improved under dual-channel supply chain. Tang et al. [21] firstly proposed a revenue-sharing contract to coordinate the dual-channel supply chain, and then carried out research on how to change the contract to coordinate the supply chain when both demand and cost were interrupted. Xu et al. [29] designed an improved revenue-sharing contract to coordinate the dual-channel supply chain considering low-carbon preferences and channel substitution. The improved revenue sharing contract can effectively enhance the efficiency of supply chain members. Wang et al. [23] studied the pricing of complementary products in a dual-channel supply chain, provided that market demand and manufacturing cost are fuzzy variables. Aslani and Heydari [1] probed into the pricing of a green dual-channel supply chain under channel disruption and proposed a transshipment contract to coordinate the supply chain.

However, as a common practice phenomenon, capital constraints are seldom studied in a dual-channel supply chain. Thus far, the literature reveals that only Li et al. [15] investigated a dual-channel supply chain that consists of a supplier and a capital-constrained manufacturer. Our study differs from that of Li et al. (2019) in that we focus on the comparison of trade credit financing contracts, while Li et al. [15] laid emphasis on three financing strategies, namely, trade credit, bank loan, and hybrid financing (i.e., combined use of bank loan and equity financing). We extend the current literature to dual-channel supply chain by making comparisons of manufacturers’ financing contracts and fill in the research gap between academia and practice.

3. Model assumptions and notations. Figure 1 depicts the supply chain model studied in this paper. A two-level dual-channel supply chain is provided, in which the retailer (e.g., Wal-Mart) is the leader and the manufacturer is the follower. The manufacturer sells a certain type of product through retail and direct channels. The notations used in the paper and their descriptions are shown in Table 1.

Assumption 1. The demand function for retail channel and direct channel in this study can be expressed as follows, respectively:

\[ D_r = sa - p_r + \lambda p_d \]  
\[ D_d = (1-s)a - p_d + \lambda p_r \]

where, \( D_r \) is the customer demand for the retail channel, and \( D_d \) is the customer demand for the direct channel. The demand for two channels is positive, and
Assumption 2. The manufacturer has limited capital $B$ and needs to offer discounts to attract retailer pay in advance with the advance payment period $N$. The financing capital is $F = c(D_r + D_d) - B$. Figure 2 presents the sequence of events and decisions.

Assumption 3. We assume that $B \leq c(D_r + D_d) \leq wD_r + B$, which means that retailer can solve the manufacturer’s financial problems while meeting their retail channel needs.

Assumption 4. Assume lead time is zero and shortages are not allowed. Supply chain members are completely rational, so they make decisions based on profit maximization.

4. Models without coordination contracts. This section focuses on exploring the equilibrium solutions for the supply chain members in two cases without contracts: decentralized decision and centralized decision. In this case, the retailer will finance the manufacturer through advance payment without discount contract to obtain stable supply and avoid the risk of supply interruption.

4.1. Decentralized decision with financing ($S$). When making decentralized decisions, both sides of the supply chain make decisions based on maximizing his/her profit without considering the profit of another member. The profits of the manufacturer and retailer can be modeled as follows:

$$\pi_{m}^{S} = (w - c)D_r + (p_d - c)D_d + I_mN[c(D_r + D_d) - B]$$  
(3)

$$\pi_{r}^{S} = (p_r - w)D_r - I_rN[c(D_r + D_d) - B]$$  
(4)

For the manufacturer, the first part represents the profit in the retail channel, and the second part represents the profit in the direct channel. The last one represents the capital opportunity cost brought by the retailer’s advance payment. For the
Table 1. Notations and descriptions used in the paper.

| Parameters   | Description                                                                 |
|--------------|-----------------------------------------------------------------------------|
| $c$          | The unit production cost of products, $/\text{unit}$.                       |
| $w$          | The unit wholesale price of products, $/\text{unit}$.                       |
| $I_i$        | The interest charged per dollar per year, $/\text{year}, i = m, r$          |
| $N$          | The length of advance payment period offered by the retailer in years.      |
| $a$          | Market primary demand, $a > 0$                                              |
| $\lambda$    | The cross price sensitivity and $0 < \lambda < 1$, which can reveal         |
|              | that the effect of ownership price is greater than that of cross-price.     |
| $s$          | The degree of customer loyalty to the retail channel and $0 < s < 1$, and $1 - s$ represents the degree of customer loyalty to the direct channel. |
| $L$          | Retailer’s advance payment to manufacturer as a function of $c, D_r, D_d$ and $B$. $L = c(D_r + D_d) - B$, where $B \leq c(D_r + D_d) \leq wD_r + B$. |
| $\pi_i$      | The supply chain member’s annual profit, $i = m, r, sc$ refer to the manufacturer, the retailer and the supply chain separately.  |
| $p^*_d$      | Direct price decided by the manufacturer, $$/\text{unit}, with$ $p_d > w > c$. |
| $p^*_r$      | Retail price decided by the retailer, $$/\text{unit}, with$ $p_r > w > c$.  |

* Represents the optimal value of a decision variable.

Decision variables

- Direct price decided by the manufacturer, $$/\text{unit}.$ with $p_d > w > c.$
- Retail price decided by the retailer, $$/\text{unit}.$ with $p_r > w > c.$

Superscript

- The Stackelberg game with the retailer as the leader
- The centralized decision
- The revenue-sharing contract
- The retail channel price discount contract
- The case with green products in the dual-channel supply chain

Subscript

- Manufacturer
- Retailer
- Supply chain

retailer, the first part represents the profit in the retail channel and the second part represents the capital opportunity cost lost by the retailer.

We use the backward induction approach to solve the retailer-Stackelberg game of each case. Similar solutions can be found in Yan et al. [30], Chen et al. [4] and Xu et al. [29]. The retailer firstly determines the selling price in the retail channel; then, the manufacturer determines the selling price in the direct channel. Proposition 1 can be obtained.

**Proposition 1.** The optimal decision variables of supply chain under decentralized decision are given by

$$p^*_r = \frac{\lambda a + sa(2 - \lambda) + 2w + \lambda c(1 - \lambda) + w\lambda^2 + c(1 - \lambda)(2NI_r - \lambda NI_m)}{4 - \lambda^2}$$

$$p^*_d = \frac{2a + sa(\lambda - 2) + 3\lambda w + c(1 - \lambda)(NI_r \lambda - 2NI_m) + 2c(1 - \lambda)}{4 - \lambda^2}$$

(5) 

(6)
The proof of Proposition 1 and other Propositions and Corollaries are given in the Appendix.

From Proposition 1, Corollary 1 is deduced as follows:

**Corollary 1**

1. Higher values of $I_r$ and $w$ cause a higher value of $p_r$ while a higher value of $I_m$ cause a lower value of $p_r$.
2. Higher values of $I_r$ and $w$ cause a higher value of $p_d$ while a higher value of $I_m$ cause a lower value of $p_d$.
3. Under the condition that $s > \frac{1}{2}$ is satisfied, we can get that $p_{r}^{*} > p_{d}^{*}$.

**Corollary 1 (1) and (2)** can be explained as follows. When the interest charged per dollar per year and the wholesale price are higher, the retailer’s cost increases. In order to increase profit, the retailer’s retail price will increase. The manufacturer also chooses to increase direct channel price in order to gain a higher profit to compete with retail channel. When the manufacturer’s interest charged per dollar per year is higher, the manufacturer’s capital opportunity cost increases, so the manufacturer will choose to reduce the direct channel price to expand channel demand. The retailer also chooses to lower retail price in order to grab demand.

**Corollary 1 (3)** can be explained as follows. When customer loyalty to retail channel is greater than 1/2, more customers are willing to buy products through retail channel. Therefore, the retailer will increase the retail price, so that the retail price is higher than the direct price, in order to obtain more profits.

### 4.2. Centralized decision-marking with financing ($C$)

When making centralized decisions, the supply chain makes decisions based on maximizing the total profit of the supply chain, which is the sum of the manufacturer’s profit and retailer’s profit. The total profit of the supply chain can be modeled as follows:

$$
\pi_{sc}^C = (p_r - c)D_r + (p_d - c)D_d + (I_m - I_r)N[c(D_r + D_d) - B] 
$$
The backward induction approach turns to be the optimal solutions, by which the proposition 2 can be obtained.

**Proposition 2.** The optimal decision variables of supply chain under centralized decision are given by

\[ p^C_d = \frac{sa(\lambda - 1) + a + (1 + \lambda)(c - \lambda c - cN(1 - \lambda)(I_m - I_r))}{2(1 - \lambda^2)} \]  

(8)

\[ p^C_r = \frac{sa(1 - \lambda) + \lambda a + (1 + \lambda)(c - \lambda c - cN(1 - \lambda)(I_m - I_r))}{2(1 - \lambda^2)} \]  

(9)

From Proposition 2, Corollary 2 is deduced as follows:

**Corollary 2** Under the condition that \( I_r \geq I_m \), we can get that higher values of \( I_r \) and \( c \) cause higher values of \( p_r \) and \( p_d \) while a higher value of \( I_m \) cause lower values of \( p_r \) and \( p_d \).

**Corollary 2** can be explained as follows. When the interest charged by the retailer increases, the interest expense on the supply chain will increase. Therefore, the supply chain will choose to increase retail price and direct price to increase sales revenue. When the interest charged by the manufacturer increases, the interest income of the supply chain will increase. So the supply chain will choose to lower retail price and direct price to increase demand.

In centralized decision, the optimal values of the decision variables are calculated to optimize \( \pi^C_{sc} \). While under decentralized decision, \( p^S_d \) is determined to maximize \( \pi^S_m \) and \( p^S_r \) is calculated to optimize \( \pi^S_r \). Therefore, centralized decision can achieve a higher supply chain profit than decentralized decision. To encourage the retailer to pay in advance for financing and ensure that the supply chain members are willing to participate in the cooperation, three discount contracts have been proposed. In the proposed model, we consider mathematical models with three contracts separately and analyze which financing contract strategy is more efficient and should be chosen.

5. **Mathematical model with contracts.** This section proposes three mathematical models with three contracts, i.e., direct channel price discount contract, retail channel price discount contract, and revenue sharing contract.

5.1. **Direct channel price discount contract (DP).** This section focuses on a price discount contract in which the wholesale price is a discount on the direct channel price to increase the profits of supply chain members. The direct channel price discount can motivate the retailer to finance. Therefore, the wholesale price is denoted by \( w = \xi p_d \). The profits of the retailer and the manufacturer can be formulated as following:

\[ \pi^D_m = (\xi p_d - c)D_r + (p_d - c)D_d + I_m N[c(D_r + D_d) - B] \]  

(10)

\[ \pi^D_r = (p_r - \xi p_d)D_r - I_r N[c(D_r + D_d) - B] \]  

(11)
The relevant explanations of the formula and the decision process are similar to those in Section 4.1. The backward induction approach turns to be the optimal solutions, by which the proposition 3 can be obtained.

**Proposition 3.** Under the DP contract, the optimal decision variables are given by

\[
p_{DP}^* = \frac{-(a - as + c - 2p_d^{DP*} + \xi(\mu - 1))}{(\lambda - \xi)} \tag{12}
\]

\[
p_d^{DP*} = \frac{2p_r^{DP*} - as + cI_r N(\lambda - 1)}{\lambda + \xi} \tag{13}
\]

From Proposition 3, Corollary 3 is deduced as follows:

**Corollary 3** A higher value of \(\xi\) cause higher values of \(p_r\) and \(p_d\).

Corollary 3 can be explained as follows. The bigger the direct channel price discount coefficient is, the higher the wholesale price the manufacturer offers to the retailer. Therefore, the retailer will raise the retail price to increase profit due to the retailer bears high cost. To increase the revenue of direct channel, the manufacturer will also choose to increase direct price.

It is clear that, the manufacturer and the retailer will only accept this coordination contract if his/her position does not worsen when moving from decentralized decision to the coordinated scenario. Therefore, it is necessary to ensure that the profits of the manufacturer and the retailer of the supply chain after coordination are higher than those before the coordination, and the following conditions need to be met: \(\pi_{DP*}^m \geq \pi_{S*}^m\), \(\pi_{DP*}^m \geq \pi_{S*}^m\). According to these inequalities, the validity of the contract is analyzed by a numerical example in section 6.

5.2. Retail channel price discount contract \((RP)\). This section focuses on a price discount contract in which the wholesale price is a discount on the retail price to increase the profits of supply chain members. The wholesale price is denoted by \(w = \mu p_r\). The profits of the retailer and the manufacturer can be formulated as following:

\[
\pi_{m}^{RP} = (\mu p_r - c)D_r + (p_d - c)D_d + I_m N[c(D_r + D_d) - B] \tag{14}
\]

\[
\pi_{r}^{RP} = (p_r - \mu p_r)D_r - I_r N[c(D_r + D_d) - B] \tag{15}
\]

The relevant explanations of the formula and the decision process are similar to those in Section 4.1. The backward induction approach turns to be the optimal solutions, by which the proposition 4 can be obtained.

**Proposition 4.** Under the RP contract, the optimal decision variables are given by

\[
p_{r}^{RP*} = \frac{-(a - as + c + \lambda c - 2p_d^{RP*} + cI_m N(\lambda - 1))}{(\lambda - \mu)} \tag{16}
\]

\[
p_d^{RP*} = \frac{-[p_{r}^{RP*} - (p_r^{*} - as)(\mu - 1) - \mu p_r^{RP*} + cI_r N(\lambda - 1)]}{\lambda(\mu - 1)} \tag{17}
\]

From Proposition 4, Corollary 4 is deduced as follows:

**Corollary 4** A higher value of \(\mu\) cause higher values of \(p_r\) and \(p_d\).

The explanation of Corollary 4 is similar to that of Corollary 3. The bigger the retail channel price discount coefficient is, the higher the wholesale price the manufacturer offers to the retailer. So the retailer will choose to increase the retail
price in order to increase sales revenue. In order to compete with retail channel, the manufacturer will also choose to increase direct price.

To ensure that the profits of the manufacturer and the retailer after coordination is better than those before the coordination, so the following conditions need to be met: \( \pi_{r}^{RS*} \geq \pi_{r}^{S*}, \pi_{m}^{RS*} \geq \pi_{m}^{S*} \). According to these inequalities, the validity of the contract is analyzed by a numerical example in section 6.

5.3. Revenue sharing contract (RS). In this contract, the manufacturer and retailer make an agreement that the manufacturer shares a fraction \( \kappa \) of his sales revenue with the retailer. Through the profit transfer between the manufacturer and retailer, the decision variables can be changed to achieve a win-win situation. So the profits of the retailer and manufacturer can be formulated as following:

\[
\pi_{m}^{RS} = (1 - \kappa)(wD_{r} + p_{d}D_{d} + I_{m}N[c(D_{r} + D_{d}) - B]) - c(D_{r} + D_{d}) \quad (18)
\]

\[
\pi_{r}^{RS} = (p_{r} - w)D_{r} - I_{r}N[c(D_{r} + D_{d}) - B] + \kappa(wD_{r} + p_{d}D_{d} + I_{m}N[c(D_{r} + D_{d}) - B]) \quad (19)
\]

The relevant explanations of the formula and the decision process are similar to those in Section 4.1. The backward induction approach turns to be the optimal solutions, by which the proposition 5 can be obtained.

**Proposition 5.** Under the RS contract, the optimal decision variables are given by

\[
p_{r}^{RS*} = -\frac{[\kappa - 1](a - as + \lambda w - 2p_{d}^{RS*} + cI_{m}N(\lambda - 1)) + c(\lambda - 1)]}{\lambda(\kappa - 1)} \quad (20)
\]

\[
p_{d}^{RS*} = \frac{2p_{r}^{RS*} - w - as + \kappa[w - cI_{m}N(\lambda - 1) + cI_{r}N(\lambda - 1)]}{\lambda + \kappa\lambda} \quad (21)
\]

To ensure that the profits of the manufacturer and the retailer after coordination are higher than those before the coordination, the following conditions need to be met: \( \pi_{r}^{RS*} \geq \pi_{r}^{S*}, \pi_{m}^{RS*} \geq \pi_{m}^{S*} \). According to these inequalities, the validity of the contract is analyzed by a numerical example in section 6.

6. Numerical example and analysis. In this section, we use some numerical examples and sensitivity analysis to further verify our results and obtain some managerial insights. Referring to Liu et al. [17], the parameters are assumed as following: \( c = $10/\text{unit} \), \( w = $25/\text{unit} \), \( I_{m} = $0.05/\text{unit/year} \), \( I_{r} = $0.25/\text{unit/year} \), \( N = 0.5\text{year} \), \( \lambda = 0.5 \), \( a = 100 \), \( s = 0.4 \) for numerical study.

6.1. Effects of \( I_{r} \) on optimal solutions for the centralized and decentralized decision. Figure 3 shows the effects of the interest charged of the retailer on the decision variables and profits for the centralized decision and decentralized decision. The following can be deduced from Figure 3:

Figure 3(a) shows that, in decentralized decision and centralized decision, as the increase of retailer’s interest charged, the retail price to increase her profit. Furthermore in order to compete with retail channel, the manufacturer will choose to increase the direct price. From Figure 3 (a), we can find that the retail price is higher than direct price, which verifies corollary 1 and corollary 2. Figure 3 (a) also implies that centralized decision can achieve a higher direct price and retail price than that under the decentralized decision.

Figure 3(b) shows that, in decentralized decision and centralized decision, as the increase of retailer’s interest charged, the retailer’s profit decreases while the manufacturer’s profit slowly increases, resulting in a reduction trend in the total profit of
the supply chain. Therefore the retailer would prefer to finance the manufacturer when the interest charged of the retailer is low to get more profit. Figure 3 (b) implies that the centralized decision is better in the total profit of supply chain. And with the increase of the retailer’s interest charged, the gap of total profit between centralized decision and decentralized decision is bigger, so the manufacturer and retailer are more willing to cooperate in financing production under the condition of lower the interest charged of the retailer.

6.2. Effects of \( I_m \) on optimal solutions for the centralized and decentralized decision. Figure 4 shows the effects of the interest charged of the manufacture on the decision variables and profits for the centralized decision and decentralized decision. The following can be deduced from Figure 4:
Figure 4(a) shows that, in decentralized decision and centralized decision, along with the interest charged of the manufacturer increases, interest income for the manufacturer will increase, so the manufacturer will lower the direct price to increase demand. At the same time, the retailer will decrease retail price to compete with direct channel. From Figure 4(a), it shows that the retail price is higher than direct price, which verifies corollary 1 and corollary 2. Figure 4(a) also implies that centralized decision can achieve a higher direct price and retail price than decentralized decision.

Figure 4(b) shows that in decentralized decision and centralized decision, along with the interest charged of the manufacturer increases, the retailer’s profit slowly decreases while the manufacturer’s profit increases, resulting in an increasing trend in the profit of the entire supply chain. Therefore the retailer would prefer to finance the manufacturer when the interest charged of the manufacturer is lower to get more profit. However, when the interest charged by the manufacturer is large, the manufacturer and the retailer can get a higher total profit when they cooperate. Figure 4(b) implies that centralized decision can achieve a higher total profit than decentralized decision, which is the same conclusion as in Figure 3. The difference is that the total profit of supply chain increases with the interest charged of the manufacturer, so the supply chain members are more willing to cooperate in financing production under the condition of higher the interest charged of the manufacturer.

6.3. Effects of $\xi$ on optimal solutions for direct channel price discount contract. Figure 5 shows the effects of the contract coefficient on the decision
variables and profits of the manufacturer and the retailer for the DP contract. The following can be deduced from Figure 5:

As can be seen from Figure 5(a), when the DP contract coefficient increases, the direct and retail prices will increase and this confirms corollary 3. Compared with decentralized decision, we can get that when the contract coefficient is smaller, the direct and retail prices under the contract are lower than those of decentralized decision. When the contract coefficient is greater than the intersection point in the Figure 5(a), the direct and retail prices under the contract are higher than those of decentralized decision.

As can be seen from Figure 5 (b), the manufacturer’s profit with coordination is less than that before coordination when the contract coefficient is smaller. While when the contract coefficient is greater than the intersection point in the Figure 5(b), the manufacturer can benefit from coordination under this coordination mechanism. From the manufacturer’s perspective, the larger the value of contract coefficient is, the higher the direct price decided by the manufacturer and the higher the profit of the manufacturer. Hence, the manufacturer wants to increase the contract coefficient to increase the direct price so as to increase profit.

As can be seen from Figure 5(c), the retailer’s profit after coordination is greater than that before coordination when the contract coefficient is smaller than the intersection point, which shows the retailer can benefit from coordination under this coordination mechanism. However, when the contract coefficient is large, the retailer’s profit after coordination will be less than that before coordination. From the retailer’s perspective, the larger the value of contract coefficient is, the higher the retail price and the smaller the profit of the retailer. Hence, the retailer wants to decrease the contract coefficient to decrease the wholesale price so as to increase profit.

Figure 5(d) shows the effect of contract coefficient on the total profit of the supply chain. When contract coefficient lies in a certain range, the total profit is greater than that before coordination. Thus, the coordination mechanism can coordinate the supply chain for greater efficiency and increase the total profit of the supply chain. Moreover, from Figure 5(b) and (c), we can get this contract can achieve a win-win situation when contract coefficient lies in a certain range. When contract coefficient approaches 0.6, the total profit can achieve maximum, which can provide some practical suggestions for the selection of enterprise contract coefficient.

6.4. Effects of $\mu$ on optimal solutions for retail channel price discount contract. Figure 6 shows the effects of the contract coefficient on the decision variables and profits of the manufacturer and the retailer for the RP contract. The following can be deduced from Figure 6:

As can be seen from Figure 6(a), when the RP contract coefficient increases, the direct and retail prices will increase, which confirms corollary 4. Compared with decentralized decision, we can get that the direct price under the contract is greater than that of decentralized decision in a certain range and the retail price under the contract are lower than that of decentralized decision, which is different from DP contract.

As can be seen from Figure 6(b) and 6(c), the profit curve of supply chain members is similar to Figure 5(b) and 5(c). The manufacturer’s profit with coordination is greater than that before coordination when the contract coefficient is greater than the intersection point in the Figure 6 (b), hence, the manufacturer can benefit from coordination under this incentive mechanism. From the manufacturer’s point
of view, the manufacturer wants to increase the contract coefficient to increase the wholesale price so as to increase profit. Further, the retailer’s profit after coordination is greater than that before coordination when the contract coefficient is smaller than the intersection point in the Figure 6 (c), which shows the retailer can benefit from coordination under this coordination mechanism. From the retailer’s point of view, the retailer wants to decrease the contract coefficient to decrease the wholesale price so as to increase profit.

Figure 6(d) shows the effect of contract coefficient on the total profit of the supply chain. No matter how much contract coefficient is, the total profit is smaller than that before coordination. Thus, the coordination mechanism fails to coordinate supply chain. Moreover we can get this contract cannot achieve a win-win situation.

### 6.5. Effects of $\kappa$ on optimal solutions for revenue-sharing contract

Figure 7 shows the effects of the contract coefficient on the decision variables and profits of the manufacturer and the retailer for the RS contract. The following can be deduced from Figure 7:

As can be seen from Figure 7(a), when the RS contract coefficient increases, the direct and retail prices will increase. Compared with decentralized decision and centralized decision, we can get that the direct price under the contract is larger than that of decentralized decision. However the retail price only is larger than that of decentralized decision when the contract coefficient is larger than the intersection point in the Figure 7 (a).

As can be seen from Figure 7(b) and (c), the manufacturer’s profit with coordination is lower than that before coordination, hence, the manufacturer cannot benefit
from coordination under this coordination mechanism. From the manufacturer’s point of view, the manufacturer wants to decrease the RS contract coefficient so as to increase profit. Further, the retailer’s profit after coordination is greater than that before coordination as shown in the Figure 7(c), which shows the retailer can benefit from coordination under this coordination mechanism. From the retailer’s perspective, the retailer wants to increase the contract coefficient to increase profit.

Figure 7 (d) shows the effect of the contract coefficient on the total profit of the supply chain. No matter how much contract coefficient is, the total profit is greater than that before coordination. Thus, the coordination mechanism can coordinate the supply chain. Moreover we can get this contract cannot achieve a win-win situation. Hence, when the supply chain members decide to adopt revenue-sharing contract, transfer mechanism is needed to promote the manufacturer to accept the contract and achieve a win-win situation.

6.6. Comparison of the total profit of the three contracts. Figure 8 shows the comparison of the total profit of the supply chain under the three contracts. The total profits under decentralized decision and centralized decision are further given to show the effectiveness of three contracts. The following can be deduced from Figure 8:

Figure 8 shows that in the above three coordination, the RP contract is fail to achieve coordination, while the DP contract and RS contract can effectively increase the total profit compared with the decentralized decision. Therefore, when choosing
Figure 8. Comparison of the total profit of the three contracts

As can be seen from Figure 8, among the DP contract and RS contract, from the perspective of coordination scope, the RS contract has a broader coordination scope than the DP contract. And through the above analysis, we can get that the DP contract can achieve a win-win situation, while the RS contract is favorable to the retailer, but not to the manufacturer. Therefore, if the manufacturer and retailer choose the contract more considering that the contract can be coordinated in a wider scope, they are more inclined to choose the RS contract. However, it should be combined with other supplementary mechanisms such as transfer payment to achieve a win-win situation.

From the perspective of supply chain’s profit after coordinated, in a certain range, DP contract can maximize the total profit under the three contracts, even when contract coefficient is close to 0.56, the total profit after coordinated can reach the profit under centralized decision. Therefore, if the manufacturer and retailer choose the contract more considering that the contract can maximize the total profit, they are more inclined to choose the DP contract.

7. Conclusions. At present, how to solve the capital problem of SMEs has become a hot topic of current researches. This paper mainly studies the financing
contract selection of a retailer-dominated dual-channel supply chain under manufacturer’s capital constraints. A decentralized model and a centralized model are first formulated and compared, to be followed by the proposal of three contracts as compared through some numerical examples. The results show that, when the manufacturer and the retailer make centralized decisions, the supply chain can receive larger profit and higher pricing, in comparison with that under decentralized decision. Secondly, RP contracts cannot coordinate the supply chain. Comparing DP contract and RS contract, RS contract has a broader scope of coordination, and DP contract can achieve a greater total profit of supply chain. The results provide the following management sights.

First, if the manufacturer and retailer have a good relationship, the best way to solve the problem of capital is to make centralized decisions in cooperation with the retailer. Especially when the retailer’s interest charge is lower and manufacturer’s interest charge is higher, the supply chain can achieve a higher total profit. Secondly, the manufacturers should choose different contracts according to different retailer. If the retailer is extremely strong and the contract coefficient is difficult to agree on, then the RS contract is more stable. If the retailer’s credit is good, then choosing DP contract can achieve a higher supply chain profit. Thirdly, the dominant retailer should also consider the profit of the manufacturer to ensure that the manufacturer is willing to accept the contract. Both sides should endeavor to establish a long-term stable cooperative relationship, which is conducive to achieving a win-win situation.

Further analysis shall be conducted on relevant research extensions. In practice, it is necessary to study the coordination of multi-echelon supply chain consisting of several downstream and upstream enterprises. Competition among enterprises of similar size has an impact on their decision process. In addition, for different products in the supply chain, the manufacturer and the retailer will make decisions on pricing under trade credit. Furthermore, more challenging cases of stochastic demand shall be studied.

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Appendix.

Proof of Proposition 1. To determine $p_d$, with the first and second derivatives of $\pi_m^S$ regarding $p_d$, $\frac{d\pi_m^S}{dp_d} = -2p_d + \lambda p_r + a - sa + \lambda (w - c) + c + cNI_m(1 - \lambda)$ can be obtained. Thus $\frac{d^2\pi_m^S}{dp_d^2} = -2 < 0$, so is obtained as a concave function of $p_d$ and the optimal solution satisfies the condition $\frac{d\pi_m^S}{dp_d} = 0$, therefore, $p_d = \frac{\lambda p_r + a - sa + \lambda (w - c) + c + cNI_m(1 - \lambda)}{2}$. Can be obtained.

Bring $p_d$ into Eq. (4). With the first and second derivatives of $\pi_r^S$ regarding $p_r$, $\frac{d\pi_r^S}{dp_r} = \lambda a + s(2 - \lambda) + 2w + \lambda c(1 - \lambda) + w\lambda^2 + c(1 - \lambda)(2NI_r - \lambda NI_m) - p_r(4 - \lambda^2)$ can be obtained. Thus $\frac{d^2\pi_r^S}{dp_r^2} = -(4 - \lambda^2) < 0$, the optimal solution satisfies the condition $\frac{d\pi_r^S}{dp_r} = 0$ and bring $p_r$ into $p_m$, $p_m^{S+} = \frac{\lambda a + s(2 - \lambda) + 2w + \lambda c(1 - \lambda) + w\lambda^2 + c(1 - \lambda)(2NI_r - \lambda NI_m)}{4 - \lambda^2}$, $p_m^{S-} = \frac{2a + s(\lambda - 2) + 3\lambda w + c(1 - \lambda)(NI_r - 2NI_m) + 2c(1 - \lambda)}{4 - \lambda^2}$.

Proof of Corollary 1. Based on the optimal solutions for $p_r^{S+}$ and $p_d^{S+}$, Corollary 1 can be easily proved.
Proof of Proposition 2. To determine $p_d$ and $p_r$, with the first and second derivatives of $\pi_s$ regarding $p_d$ and $p_r$, so we can get
\[
\frac{d^2\pi_s}{dp_d dp_r} = -2p_r + 2\lambda p_d + sa - \lambda c + c - cN(I_m - I_r)(1 - \lambda) \frac{d^2\pi_s}{dp_r dp_d} = -2 < 0.
\]
\[
\frac{d^2\pi_s}{dp_r dp_d} = 2\lambda |H| = \frac{d^2\pi_s}{dp_r dp_d} \frac{d^2\pi_s}{dp_d dp_r} - \left(\frac{d^2\pi_s}{dp_r dp_d}\right)^2 = 4 - 4\lambda^2 > 0.
\]
So $\pi_s$ is obtained as a concave function of $p_d$ and $p_r$, and the optimal solution satisfies the condition $\frac{d\pi_s}{dp_r} = 0$ and $\frac{d\pi_s}{dp_d} = 0$, therefore we can obtain that
\[
p_{d}^{C^*} = \frac{sa(\lambda - 1) + a + (1 + \lambda)(c - \lambda c - cN(1 - \lambda)(I_m - I_r))}{2(1 - \lambda^2)}
\]
\[
p_{r}^{C^*} = \frac{sa(1 - \lambda) + \lambda a + (1 + \lambda)(c - \lambda c - cN(1 - \lambda)(I_m - I_r))}{2(1 - \lambda^2)}
\]
\]

Proof of Corollary 2. Based on the optimal solutions for $p_{r}^{S^*}$, $p_{d}^{S^*}$, $p_{d}^{C^*}$ and $p_{d}^{C^*}$, Corollary 4 can be easily proved.

Proof of Proposition 3. The proof of Proposition 3 follows in the same way of the proof of Proposition 1, so it can omit.

Proof of Corollary 3. Based on the optimal solutions for $p_{r}^{DP^*}$ and $p_{d}^{DP^*}$, Corollary 3 can be easily proved.

Proof of Proposition 4. The proof of Proposition 4 follows in the same way of the proof of Proposition 1, so it can omit.

Proof of Corollary 4. Based on the optimal solutions for $p_{r}^{RP^*}$ and $p_{d}^{RP^*}$, Corollary 4 can be easily proved.

Proof of Proposition 5. The proof of Proposition 5 follows in the same way of the proof of Proposition 1, so it can omit.

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