Neutrino masses from the GSI anomaly

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We investigate the influence of the strong Coulomb field of a heavy nucleus on massive neutrinos, produced in the K-shell electron capture (EC) decays of the H–like \(^{140}\text{Pr}^{58+}\) and \(^{142}\text{Pm}^{60+}\) ions. The corrections to the neutrino masses due to virtually produced charged lepton W–boson pairs in the strong Coulomb field of a nucleus with charge Ze are calculated and discussed with respect to their influence on the period of the time–modulation of the number of daughter ions, observed recently in the EC–decays of the H–like \(^{140}\text{Pr}^{58+}\) and \(^{142}\text{Pm}^{60+}\) ions at GSI in Darmstadt. These corrections explain the 2.9 times higher difference of the squared neutrino masses obtained from the time–modulation of the EC–decays with respect to the value deduced from the antineutrino–oscillation experiments of KamLAND. The values of neutrino masses are calculated.

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The experimental investigation of the EC–decays of the H–like ions \(^{140}\text{Pr}^{58+}\) and \(^{142}\text{Pm}^{60+}\), i.e. \(^{140}\text{Pr}^{58+}\rightarrow^{140}\text{Ce}^{58+}+\nu_e\) and \(^{142}\text{Pm}^{60+}\rightarrow^{142}\text{Nd}^{60+}+\nu_e\), carried out in the Experimental Storage Ring (ESR) at GSI in Darmstadt \cite{1}, showed a modulation in time with periods \(T_{EC} \approx 7\text{s}\) of the rate of the number of daughter ions. Since the rate of the number of daughter ions is defined by

\[
\frac{dN_{EC} (t)}{dt} = \lambda_{EC} (t) N_m(t), \tag{1}
\]

where \(\lambda_{EC} (t)\) is the EC–decay rate and \(N_m(t)\) is the number of mother ions \(^{140}\text{Pr}^{58+}\) or \(^{142}\text{Pm}^{60+}\), the time–modulation of the rate of \(N_{EC}^d (t)\) implies a periodic time–dependence of the EC–decay rate \(\lambda_{EC} (t)\).

As has been proposed in \cite{2}, such a periodic dependence of the EC–decay rate can be explained by the mass–differences of the neutrino mass–eigenstates. The period of the time–modulation \(T_{EC}\) has been obtained as

\[
T_{EC} = \frac{4\pi\frac{e}{c}M_m}{\Delta m_{21}^2}, \tag{2}
\]

where \(M_m\) is the mass of the mother ion, \(\gamma = 1.43\) is the Lorentz factor of the H–like ions \cite{1} and \(\Delta m_{21}^2 = m_2^2 - m_1^2\) is the difference of the squared neutrino masses \(m_2\) and \(m_1\).

For \(T_{EC} = 7.06(8)\text{s}\) \cite{1}, measured for the H–like \(^{140}\text{Pr}^{58+}\) ion, we have got \((\Delta m_{21}^2)_{\text{GSI}} = 2.18(3) \times 10^{-4}\text{eV}^2\) \cite{2}, which is by a factor 2.9 larger than \((\Delta m_{21}^2)_{\text{KL}} = 7.59(21) \times 10^{-5}\text{eV}^2\) \cite{3}, used also for the global analysis of the solar–neutrino and KamLAND experimental data \cite{4} (see also \cite{5}). For the first time the value \((\Delta m_{21}^2)_{\text{GSI}} \approx 2.25 \times 10^{-4}\text{eV}^2\) has been obtained by Kleinert and Kienle within the neutrino–pulsating vacuum approach \cite{6}. The same estimate for \(\Delta m_{21}^2\) one can get by using the period of the time–modulation derived by Lipkin \cite{7}.

For the understanding of such a discrepancy we propose the following mechanism. In the EC–decay of a H–like heavy ion a daughter ion with electric charge Ze and a massive neutrino are produced. Since a characteristic energy scale

\[\text{Wien, Österreich,} \]
is of order of a few $10^{-15}$ eV \[2\], one possible solution of the discrepancy between $(\Delta m^2_{21})_{\text{GSI}}$ and $(\Delta m^2_{21})_{\text{KL}}$ is that a massive neutrino gets a correction to its mass, caused by its interaction with the strong Coulomb field of the daughter ion due to virtually produced \(\ell^- W^+\) pairs, where \(\ell^- = e^-, \mu^-\) and \(\tau^-\) is a negatively charged lepton and \(W^+\)-boson, as an intermediate state. The Feynman diagrams of the process are depicted in Fig. 1 with the Green functions of virtual charged leptons in the strong Coulomb field \[8,9\].

For the calculation of the diagrams in Fig. 1 we use the weak leptonic interaction \[10\]

\[
\mathcal{L}_W = - \frac{G_F}{\sqrt{2}} \sum_{j\neq'} \sum_{\ell \ell'} U_{j\ell} U_{j'\ell'}^* \left[ \bar{\psi}_{\nu_j}(x) \gamma^\mu \left(1 - \gamma^5\right) \psi_{\nu_{j'}}(x) \right] \\
\quad \times \left[ \bar{\psi}_{\nu_j}(x) \gamma^\mu \left(1 - \gamma^5\right) \psi_{\nu_{j'}}(x) \right],
\]

(3)

defined by the W–boson exchange, where \(x = (t, \vec{r})\), \(G_F\) is the Fermi constant, \(\psi_{\nu_j}(x)\) and \(\psi_{\nu_j}(x)\) are operators of the neutrino \(\nu_j\) and lepton fields \(\ell = e^-, \mu^-\) and \(\tau^-\), respectively, and \(U_{ij}\) are the elements of the unitary neutrino–flavour mixing matrix \(U\). In our analysis neutrinos \(\nu_j (j = 1, 2, 3)\) are Dirac particles with masses \(m_j (j = 1, 2, 3)\), respectively.5

A correction \(\delta m_j\) to the neutrino mass, induced by the interaction of the neutrino \(\nu_j\) with a strong Coulomb field of a nucleus, is defined by

\[
\delta m_j(r) = \sum \mathcal{M}_\ell(r),
\]

(4)

where we have denoted

\[
\mathcal{M}_\ell(r) = \left[ \frac{e^2}{4\pi} \int_0^{\infty} \frac{dE}{2\pi} \Re \{G_\ell(\vec{r}, \vec{r}; E) \gamma^0}\right].
\]

(5)

Here \(G_\ell(\vec{r}, \vec{r}; E)\) is the energy–dependent Green function of the negatively charged leptons \(\ell^-\) in a strong Coulomb field, produced by a positive electric charge Ze \[8,9\].

Using the results, obtained in \[9\], we get

\[
\mathcal{M}_\ell(r) = \sqrt{2} G_F m_\ell \sum_{n=1}^{\infty} \int_0^\infty \int_0^\infty dx dt \\
\times e^{-m_\ell r \sqrt{x^2 + 1}} \left[ 2 Z\alpha \cosh t \cos \left( \frac{2 Z\alpha x t}{\sqrt{x^2 + 1}} \right) \right] \\
\times \left[ 2 Z\alpha r I_{2n+1}(2 Z\alpha r \sqrt{x^2 + 1}) + \frac{2\nu x}{\sqrt{x^2 + 1}} \right],
\]

(6)

where \(\nu = \sqrt{n^2 - (Z\alpha)^2}\) and \(I_\nu(z)\) is a modified Bessel function \[11\], \(I_{2n+1}(z) = I_{2n+1}(z) - I_{2n+1}(z)\) and \(I_{2n}(z) = I_{2n}(z) - I_{2n}(z)\). We would like to notice that at \(Z\alpha \to 0\) the corrections to the neutrino masses vanish as \(\mathcal{M}_\ell(r) \to 0\). Hence, a non–vanishing correction to the massive neutrino mass appears only due to the Coulomb field. Since at \(r \to \infty\) the corrections introduced by Eq. (9) vanish rapidly, so that in the subsequent interactions \[12\] the neutrino \(\nu_j\) should be with a proper mass \(m_j\). The very rapid vanishing (see Fig. 2) of the \(\delta m_j(r)\) with \(r\) makes it reasonable to take into account the influence of the correction only at the nuclear surface \[13\] just after the production of the massive neutrino \(\nu_j\) and the daughter ion\[6\].

At the nuclear radius \(r = R = 5.712\) fm \[14\],

\[
M(m \to d\nu_j)(t) \propto \int d^3 x \Psi_j^*(r) \Psi_{\nu}(r) \psi_{\nu_j}(r) e^{i E_{\nu_j}(r) t} = e^{i E_{\nu_j}(R) t} \langle \Psi_{\nu_j}^{(Z)} | \mathcal{M}_{\nu_j} | \Psi_{\nu_j}^{(Z)} \rangle,
\]

where \(E_{\nu_j}(r) = \sqrt{k_r^2 + (m_j + \delta m_j(r))^2}\). Using an analogy between the Fermi–Dirac distribution function and the Woods–Saxon shape of the nuclear density \[14\] and following \[15\] one can show that the relation \(\langle \Psi_{\nu_j}^{(Z)} | e^{i E_{\nu_j}(R) t} \rangle = \langle \Psi_{\nu_j}^{(Z)} | e^{i E_{\nu_j}(R) t} \rangle\) is fulfilled with an accuracy better than 1%. For the confirmation of the validity of this relation we refer also on \[13\].
we get

\[ M_{e^-}(R) = -2.02 \times 10^{-3} \text{ eV}, \]
\[ M_{\mu^-}(R) = -5.16 \times 10^{-4} \text{ eV}, \]
\[ M_{\tau^-}(R) = -3.88 \times 10^{-5} \text{ eV} \]  

(7)

with an electron \( e^- \), muon \( \mu^- \) and \( \tau^- \)-lepton in the intermediate state, respectively. The corrections to the neutrino masses are equal to

\[ \delta m_1(r) = \cos^2 \theta_{12} M_{e^-}(r) + \sin^2 \theta_{12} (\cos^2 \theta_{23} M_{\mu^-}(r) + \sin^2 \theta_{23} M_{\tau^-}(r)), \]
\[ \delta m_2(r) = \sin^2 \theta_{12} M_{e^-}(r) + \cos^2 \theta_{12} (\cos^2 \theta_{23} M_{\mu^-}(r) + \sin^2 \theta_{23} M_{\tau^-}(r)), \]  

(8)

where \( \theta_{12} \) and \( \theta_{23} \) are mixing angles. The corrections to the neutrino masses Eq. (8) are defined for \( \theta_{13} = 0 \) (see also [3]). Then, setting \( \theta_{12} = 34^\circ \) and \( \theta_{23} = 45^\circ \) we obtain

\[ \delta m_1(R) = -14.74 \times 10^{-4} \text{ eV}, \]
\[ \delta m_2(R) = -8.22 \times 10^{-4} \text{ eV}. \]  

(9)

The period of modulation is thus redefined as

\[ T_{EC} = \frac{4\pi \gamma M_m}{(m_2 + \delta m_2(R))^2 - (m_1 + \delta m_1(R))^2}. \]  

(10)

Neglecting the contributions of \( (\delta m_1(R))^2 \) we transcribe the denominator into the form

\[ \delta m_2(R) - \delta m_2^2(R) = (\Delta m_{21}^2)_{\text{GSI}} - (\Delta m_{21}^2)_{\text{KL}} + (\delta m_1(R))^2 - (\delta m_2(R))^2, \]  

(11)

where \( \delta m_2^2(R) = 2m_1 \delta m_1(R) \). Using the numerical values of the corrections Eq. (9), \( (\Delta m_{21}^2)_{\text{GSI}} = 2.20 \times 10^{-4} \text{ eV}^2 \), \( (\Delta m_{21}^2)_{\text{KL}} = 7.59 \times 10^{-5} \text{ eV}^2 \) and a relation \( m_2 - m_1 = (\Delta m_{21}^2)_{\text{KL}}/(m_2 + m_1) \) we solve Eq. (11) and get the following values for neutrino masses

\[ m_2 = 0.11 + 0.82 \times 10^{-4} \text{ eV}, \]
\[ m_1 = 0.11 + 4.26 \times 10^{-4} \text{ eV}. \]  

(12)

The mass \( m_3 \) of the neutrino \( \nu_3 \) is

\[ m_3 = 0.12 + 8.05 \times 10^{-4} \text{ eV}. \]  

(13)

We obtain it using Eq. (12) and the experimental value \( \Delta m_{21}^2 = 2.4 \times 10^{-3} \text{ eV}^2 \) [10]. The sum of neutrino masses amounts to

\[ \sum_{j=1,2,3} m_j = 0.34 \text{ eV}, \]  

(14)

which agrees well with the upper limit \( \sum_j m_j < 1 \text{ eV} \) [4].

We have shown that an interaction of virtually produced \( \ell^- W^+ \) pairs \( \nu_j \to \sum_{\ell} U_{j\ell} \ell^- W^+ \) of massive neutrinos \( \nu_j \) in the strong Coulomb field of the daughter ion can induce certain corrections to neutrino masses, which allow to reconcile the value \( (\Delta m_{21}^2)_{\text{GSI}} = 2.18(3) \times 10^{-4} \text{ eV}^2 \) [2], deduced from the period of the time–modulation of the rate of the number of daughter ions in the EC–decays of the H–like ions \(^{140}\text{Pr}^{58+}\) and \(^{142}\text{Pm}^{60+}\), with \( (\Delta m_{21}^2)_{\text{KL}} = 7.59(21) \times 10^{-5} \text{ eV}^2 \) [15], obtained as a best–fit of the global analysis of the solar–neutrino and KamLAND experimental data [4] (see also [5]). We would like to notice that for the calculation of the corrections to neutrino masses we have taken into account the contribution of the \( W^+ \)–boson exchange only. The contribution of the \( Z \)–boson exchange is proportional to the constant \( g_V = -0.040 \pm 0.015 \)
This means that the corrections to neutrino masses, caused by the Z–boson exchanges, are smaller compared with corrections, which can be caused by the experimental uncertainties of the mixing angles $\theta_{12} = 33.9^{+2.4}_{-2.2}$ degrees and $\theta_{23} \leq 45$ degrees \[4\].

The proposed change of the neutrino masses together with the experimental data on the time–modulation of the rate of the number of daughter ions in the EC–decays of the H–like ions and $(\Delta m^2_{21})_{KL} = 7.59 \times 10^{-5}$ eV$^2$ allows to estimate the values of neutrino masses $m_j \simeq 0.11$ eV agreeing well with the constraint on the sum of neutrino masses $\sum m_j < 1$ eV \[4\]. The value of the heaviest neutrino mass $m_3 = 0.12 + 8.05 \times 10^{-4}$ eV satisfies also the constraint $0.04 < m_3 < 0.40$ eV \[4\].

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