Optimizing On-Line Advertising

Fabrizio Caruso
Neodata Group, Catania, Italy
e.mail: fabrizio.caruso@neodatagroup.com

Giovanni Giuffrida
Dept. of Social Sciences, University of Catania, Italy
e.mail: ggiuffrida@dmi.unict.it

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Abstract

We want to find the optimal strategy for displaying advertisements e.g. banners, videos, in given locations at given times under some realistic dynamic constraints. Our primary goal is to maximize the expected revenue in a given period of time, i.e. the total profit produced by the impressions, which depends on profit-generating events such as the impressions themselves, the ensuing clicks and registrations. Moreover we must take into consideration the possibility that the constraints could change in time in a way that cannot always be foreseen.

Keywords: web advertisement, linear programming, data mining, machine learning.

1 Introduction

We want to find the optimal strategy for displaying (delivering) advertisements (“creatives”) in order to achieve different goals (maximum total profit, maximum average profit per advertisement, maximum visibility of the campaign) under some realistic constraints.

More specifically we need to find the optimal number of “impressions” (display of advertisements at a given time and at a given “location”) under some realistic dynamic constraints that both limit the possibility of certain creatives and limit the number of impressions in certain locations and/or moments in time. A location can represent a place where an advertisement can be displayed. It could also include information on the user or user’s category (it could be a combination of a location and user category, location and a user, location and a set of keywords inserted into the web-page, etc...). Hence the model we are considering can be used to target users by their categories. Similar optimization problems have already been treated in the scientific literature ([LNA+ 99, AN99, Nak02]).
We are given certain “creatives” (advertisements to be displayed, e.g. banners, videos, etc.), “campaigns” (sets of related creatives), certain “locations” and a period of time (set of “time frames”). At a given moment in time we have an expected profit for each creative of a given campaign in a given time frame and location. Usually the profit generated by 1000 impressions, called “eCPM” (effective cost per mille) is considered because the profit of one impression is very small. For the purpose of this paper we will only consider the profit of a single impression.

The profit of the web-page’s owner depends on the profit-generating events that have been agreed upon by the advertiser and the web-page’s owner. These events can be the impression itself, a click on the advertisement or a registration of any sort (e.g. registration into the advertised site, purchase of the advertised item, etc...), or any combinations of these events.

We denote the expected profit of a single “impression” as the “impression profit”. The impression profit is therefore the sum of the profits obtained by all the profit-generating events such as impressions themselves, their ensuing clicks and registrations of all types (“steps”). In such a way we can avoid keeping track of click-through rates and different registration rates. This choice is a compromise between performance and generality, since it makes our model less precise and slightly less general: we are not considering campaigns with separate budgets for different events; we cannot estimate the expected profit of an impression as precisely as when different rates for different events are considered.

The number of impressions (“supply”) on a given location at a given time is limited by the traffic of the corresponding webpage. It also depends on time in a way that can be only partially predicted. Moreover the maximum profit for a given campaign (“demand”) could be limited by a predefined budget.

Our primary goal is to maximize our expected revenue which is given by the expected total price paid. A secondary goal is to maximize the profit of a single impression, i.e. obtain the maximum revenue with the minimum effort (minimum number of impressions). Therefore we wish to maximize a weighted sum of all all expected profits obtained in all locations in the period of time under consideration. An additional goal which is considered in the constraints is to maximize the visibility of the campaigns.

Taking into account supply and demand constraints makes our model a special instance of a “transportation problem” for which very efficient solutions exist (see [Dan63] and [BBG77]).

The complexity of the model brings up the additional problem of deciding between simplifying the model and considering smaller problems, i.e. optimizing more locally.

In order to apply our optimization we need to make a projection of the future supply and a projection of the impression profits onto our period. Impressions are only possible on the points allowed by the scheduling of the campaigns.

The projection of the impression profits should also try to “guess” how the profit of an impression changes in time.

The projection algorithms should take into account different periodicities
(daily, weekly, etc.). More precise projections can be achieved through machine learning techniques in that the weights of different periodicities are computed during a training phase, possibly with different strategies for different nodes or other features (time of the day, day of the week, etc.).

Moreover we cannot assume the immutability of the constraints of the problem in the period of time under consideration. For this reason we have to decide how globally or locally we want to optimize the problem and we have to continuously readjust to new constraints, new expected impression profits and new expected supply.

2 Notation

We denote by $C_i$ the $i$-th campaign (set of creatives) and by $B_{i,j}$ its $j$-th creative, by $L_l$ the $l$-th location, by $T_k$ the $k$-th time frame. We denote by $x_{i,j,k,l}$ the (“impression value”) number of impressions of $B_{i,j}$ at time frame $T_k$ and at location $L_l$.

We denote by $p_{i,j,k,l}$ the “impression profit”, the profit of one single impression of creative $B_{i,j,k,l}$ at position $L_l$ and a time $T_k$.

For the sake of simplicity we will be omitting indices in our expression whenever our objects do not depend on them.

3 Configurations

We can consider our problem as the problem of finding the optimal impression values for the entries in a tridimensional matrix, i.e. for all points in a tridimensional discrete finite space given by a grid defined by couples (campaign, creative), time and location.

We refer to a single point in this tridimensional discrete finite space as an “impression-event” (or simply an “impression” when this is clear from the context). We call any choice for the values of all the impression-events as a “configuration”. An impression is in fact characterized by a couple (campaign, creative), location and time. Our goal is to choose the optimal delivery of each possible impression, i.e. an optimal configuration. We will simply refer to the number of impressions of an impression-event as the “impression value” or its “value”.

Moreover some of our constraints restrict the possible points in such a grid. We can see these restriction as an additional trivial constraints of the form $x_{i,j,k,l} = 0$ on such points. We refer to the points that do not contradict any constraints as “possible points”, points that are not forced to have a zero value by the constraints. A subset of the possible points is the set of points that are allowed by the schedule of the campaigns. We refer to such points as to “admissible points”.

Each admissible point in this space describes a dimension of our optimization problem. The worst case in our problem is produced when all points inside the cube of size given by the number of couples (campaign, creative), the number
of time frames, the number of locations, are admissible. Our problem lives in a space whose number of dimensions is given by the number of all admissible points. Hence in the worst case the number of dimensions is the product of the number of couples (campaign, creative), the number of locations and the number of time frames considered.

**Remark 1.** In practise we do not know the supply and cannot decide in advance how many impressions of a given creative should be delivered. Thus we need to translate the number of impressions \(x_{i,j,k,l}\) in terms of probability of delivery. We transform a configuration into a map that associates a couple \((k,l)\) with the probability of delivery of \(B_{i,j}\) for all possible corresponding couples \((i,j)\).

Given a set of \(t\)-uples \(C\), we introduce the following notation for subsets of \(t-1\)-uples:

\[
C[i \to \alpha] = \{(x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_t) \mid (x_1, \ldots, x_{i-1}, \alpha, x_{i+1}, \ldots, x_t) \in C\};
\]
\[
C[i \to \ast] = \{(x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_t) \mid \exists v \mid (x_1, \ldots, x_{i-1}, v, x_{i+1}, \ldots, x_t) \in C\}.
\]

i.e. we are considering respectively

- \((t-1)\)-uples obtained from \(t\)-uples in \(C\), where \(i\)-th component is \(\alpha\), in which the \(i\)-th component has been removed, and
- \((t-1)\)-uples obtained from \(t\)-uples in \(C\) where the \(i\)-th component has been removed independently of its value.

In the same way, if more components are removed in parallel, we introduce the notation: \(C[i_1 \to \alpha_1, \ldots, i_n \to \alpha_n]\) for \((t-n)\)-uples, where \(\alpha_j \in \mathbb{N} \cup \{\ast\}\) and \(i_j \in \mathbb{N}\) for \(j \in \{1, \ldots, n\}\).

Pictorially, we could see a single configuration \(C = (x_{i,j,k,l})_{i,j,k,l}\) as a tridi-dimensional matrix:
4 Realistic Model

We want to consider a realistic model in which several constraints of different nature are taken into account.

4.1 The Constraints

We distinguish between the primary (physical) constraints of the problem, the secondary ones (commercial and optional) and the learning constraints (required by the learning phase if it is included in the mathematical model).

4.2 Primary constraints

The primary constraints are given by the desired scheduling of the campaigns, (the impossibility of certain creatives at certain times and locations), by the limited supply of impressions and by a (possibly) limited demand (campaign’s budget):

1. The scheduling of the campaigns limits the admissible points: certain creatives \( B_{i,j} \) are only possible at certain time frames and locations. Typically a campaign (set of creatives) begins and ends at certain times and its creatives are limited to certain locations, hours of the day, days of the week, etc. . .

2. Any location at a given time receives a limited supply of impressions (“location supply”), which solely depends on the traffic of its page (more precisely on the portion of traffic given to the ad-serving optimizer);

3. For any given campaign a given total profit may not be exceeded (“campaign demand”) because only a finite campaign budget can be available.

Remark 2. A campaign can have an unbounded budget, e.g. a campaign that only pays the client for an actual purchase (highest step of a registration).

For the sake of simplicity from now on we will assume implicitly that the first primary constraint is always satisfied.

4.3 Secondary constraints

The secondary constraints may be of a commercial nature and depend on the conditions in the contract between the web-site’s owner, the advertiser and possibly the ad-serving company. These constraints could be enforced, up to a certain extent, at real time while monitoring the delivery, although having them as constraints is better for the optimality of the solution.

They are necessary to increase the visibility of a certain campaign/creative:

1. Any given creative/campaign should not last less than a given period, e.g. the period in which the campaign is scheduled. We enforce this by setting a minimum for the number of impressions for each possible time frame.
2. We would like to avoid having only one creative at a given location and time frame when more than one choice is available.

### 4.4 Learning constraints

Moreover, if the learning phase on the performance of new creatives and new locations is to be included in the mathematical model, additional constraints should be considered. One way to force the system to learn on new creatives and new nodes can be a constraint of this form: for each new couple \((\text{creative}, \text{node})\) we must have a minimum number of impressions in all (or some initial) possible time frames.

### 4.5 The Goal

We want to maximize our expected revenue, which is given by the total profit received in a given configuration.

### 5 A linear programming model

Under the false but nevertheless mild hypothesis that the impression profit is constant with respect to its value we can assume that our constraints are linear. This assumption is not true in general because there is no linear dependence between the total profit generated by an impression-event and an impression value, i.e. displaying the same advertisement \(x\) times on the same node, possibly more than once to the same user, does not necessarily produce \(x\) times the profit produced by one single display.

Since we are ultimately interested in the probability of delivery and since integer linear programming is computationally unfeasible (NP-hard), a possible approach to this problem could be real linear programming: we approximate our discrete problem with a continuous one and we do not mind considering a real number of impressions.

#### 5.1 Formalized constraints

The points that do not contradict the first primary constraint will form the unknowns of our model. In such a way we can avoid to include inequalities for the first primary constraint.

##### 5.1.1 Primary constraints

We do not include the first primary constraints for the reasons given above and assume that in our expressions all indices run over points that do not contradict the first primary constraints.

Supply and demand are formalized as follows:
Second primary constraint:

\[ \forall i,k \sum_{j,l} x_{i,j,k,l} \leq S_{i,k}; \quad \text{(Supply)} \tag{1} \]

where \( S_{i,k} \) is the supply at location \( L_l \) and at time \( T_k \).

Third primary constraint:

\[ \forall i \sum_{j,l,k} p_{i,j,k,l} x_{i,j,k,l} \leq D_i; \quad \text{(Demand)} \tag{2} \]

where \( D_i \) is the budget of the \( i \)-th campaign.

### 5.1.2 Secondary constraints

The secondary constraints are formalized as follows:

First secondary constraint:

\[ \forall i,k \sum_{j,l} x_{i,j,k,l} \geq \mu_{i,k}; \quad \text{(lasting)} \tag{3} \]

where \( \mu_{i,k} \) is the desired minimum delivery of impressions of the \( i \)-th campaign at time \( T_k \).

Second secondary constraint:

\[ \forall l,k \in \mathcal{D} \forall i,j x_{i,j,k,l} \leq P_{l,k} \cdot S_{l,k}; \quad \text{(no overflow)} \tag{4} \]

where \( P_{l,k} \in [0, 1] \) (usually close to 1) defines how much a single creative can occupy a location at a given time frame and where \( \mathcal{D} \) is the set of indices corresponding to locations and time frames where at least 2 different creatives are possible.

**Remark 3.** The second secondary constraints \( \tag{4} \) should only be limited to those cases in which at a given location and time frame more than one pair of campaign and creative is possible because otherwise the constraint in \( \tag{4} \) would prevent the node from being filled with impressions even when this could be possible.

### 5.1.3 Learning constraints

If the learning phase is included in the model a constraint should force the new creatives and new locations to have a minimum number of impressions:

\[ \forall_{\text{new } j,l} \forall i,k x_{i,j,k,l} \geq \lambda_{i,j,k,l}. \tag{5} \]

We are also implicitly assuming that the unknowns are non-negative, i.e.

\[ \forall i,j,k,l x_{i,j,k,l} \geq 0. \tag{6} \]
6 Existence of a solution

We see that there is no guarantee of consistency once the secondary and learning constraints are introduced, even if we exclude the first primary constraints. In general we need to solve the system of inequalities in order to make sure that there is indeed a solution. Nevertheless if we carefully choose $\lambda_{i,j,k,l}$ and $\mu_{i,k}$ we can avoid that the first secondary or learning constraints contradict the second and third primary constraints.

In particular we have the following facts

**Fact 1.** If we choose

$$
\mu_{i,k} \leq \min \{ \frac{D_i}{|C[1 \to i, 2 \to *, 4 \to *]|M}, \min_{t \in C[1 \to i, 2 \to *, 3 \to k]} \frac{S_{k,t}}{|C[2 \to *, 3 \to k, 4 \to t]|} \} \tag{7}
$$

then the semi-algebraic set defined by the inequalities (7), (8), (9) and (10) is not empty.

**Proof.** The second secondary constraint (8) and our choice allow to have

$$
\sum_{j,l} x_{i,j,k,l} \leq \frac{D_i}{|C[1 \to i, 2 \to *, 4 \to *]|M}. \tag{8}
$$

Therefore for any campaign $C_i$ we have

$$
\sum_{j,k,l} p_{i,j,k,l} x_{i,j,k,l} \leq M \sum_{j,k,l} x_{i,j,k,l} \leq M \sum_{k \in C[1 \to i, 2 \to *, 4 \to *]} \sum_{j,l} x_{i,j,k,l} \leq M \sum_{k \in C[1 \to i, 2 \to *, 4 \to *]} \frac{D_i}{|C[1 \to i, 2 \to *, 4 \to *]|M} = D_i. \tag{9}
$$

Moreover we can take

$$
\sum_{j,l} x_{i,j,k,l} \leq \min_{t \in C[1 \to i, 2 \to *, 3 \to k]} \frac{S_{k,t}}{|C[2 \to *, 3 \to k, 4 \to t]|}. \tag{10}
$$

Hence for any couple $(k, l)$ of location and time frame we have

$$
\sum_{i,j} x_{i,j,k,l} \leq \sum_{s} \sum_{i,j} x_{i,j,k,s} \leq \sum_{s} \sum_{i,k} x_{i,j,k,s} \leq \sum_{t \in C[1 \to i, 2 \to *, 3 \to k]} \min_{i} \frac{S_{k,t}}{|C[1 \to i, 2 \to *, 3 \to t]|} \leq \sum_{i} \frac{S_{k,l}}{|C[2 \to *, 3 \to k, 4 \to k]|} = S_{k,l}. \tag{11}
$$
Fact 2. If we choose 
\[ \lambda_{i,j,k,l} \leq \min \left\{ \frac{D_i}{|C[1 \to i]|M}, \frac{S_{k,l}}{|C[3 \to k, 4 \to l]|} \right\} \] (12)
then the semi-algebraic set defined by the inequalities (1), (2), (4), (6) is not empty.

Proof. The learning constraint (3) and our choice allow to have 
\[ x_{i,j,k,l} \leq \frac{D_i}{|C[1 \to i]|M}. \] (13)
from which it follows that for any campaign \( C_i \):
\[ \sum_{j,k,l} p_{i,j,k,l} x_{i,j,k,l} \leq M \sum_{j,k,l} x_{i,j,k,l} \leq M \sum_{j,k,l} \frac{D_i}{|C[1 \to i]|M} = D_i. \] (14)
Moreover we can also take 
\[ x_{i,j,k,l} \leq \frac{S_{k,l}}{|C[3 \to k, 4 \to l]|}. \] (15)
from which it follows that for any couple \((l, k)\) of location and time we have 
\[ \sum_{i,j} x_{i,j,k,l} \leq \sum_{i,j} \frac{S_{k,l}}{|C[3 \to k, 4 \to l]|} = S_{k,l}. \] (16)

Theorem 1. The system of inequalities formed by second and third primary constraints, first secondary constraints and learning constraints has at least a solution.

Proof. This follows from Fact 1 and Fact 2.

Remark 4. A slightly simpler model in which only primary constraints (including the first ones) are considered is an instance of the “Hitchcock’s style transportation problem” [F.L41].

6.1 The objective function

We want to maximize our expected revenue, which is given by the sum of all expected profits received in a given configuration.

Hence we can estimate the revenue by taking the weighted sum of the expected revenue in a given configuration \( C \). Such a sum will be our “objective function”:
\[ F(c) = \sum_{i,j,k,l} p_{i,j,k,l} x_{i,j,k,l} \] (17)
where \( p_{i,j,k,l} x_{i,j,k,l} \) is the expected profit generated by \( B_{i,j} \) at location \( L_l \) and at time \( T_k \).
7 Simplified models: transportation problems

The standard algorithm for solving linear programming problems is the well-known simplex algorithm. The simplex algorithm, although fast in many practical situations, is exponential in the worst case and does not scale well enough for the size of problems we want to consider. Moreover, no general linear programming algorithm is known to be strongly polynomial.

An important speed-up can be achieved by simplifying the model we have considered. In particular, we could consider a simpler model that could fall into the category of “transportation problems”. For such problems, very efficient algorithms are known such as the “stepping stone algorithm” (see [Dan63] and [BBG77]).

The classical transportation problem is a linear programming problem whose constraints describe demands $d_i$ to be met and supplies $s_j$ to be delivered:

$$\forall j \sum_i x_{i,j} = s_j; \quad \forall i \sum_j x_{i,j} = d_i.$$ (18)

We say that a transportation problem is “balanced” if the total demand equals the total supply. Taking into consideration only balanced problems is not a real restriction because one can always put oneself into this case by adding an extra dummy supply or extra dummy demand with zero cost/gain.

In our case, demands could describe a required number of creatives and supplies the number of impressions that can be shown in given locations and time frames.

A more general transportation problem goes under the name of “Hitchcock’s style transportation problem” ([F.L41]) that better approximates our problem, in that the demand is obtained by multiplying the impression values by a factor (representing their profit). The constraints are of the following kind:

$$\forall j \sum_i x_{i,j} = s_j; \quad \forall i \sum_j c_{i,j} x_{i,j} = d_i,$$

where $c_{i,j}$ is the value (cost or profit) associated to $x_{i,j}$.

8 Maximizing the value of impressions

A related problem to the one of maximizing the revenue is that of minimizing the number of used impressions under some constraints on the revenue generated by each campaign. This is equivalent to the problem of maximizing the profit generated by a single impression.

In other words, we want to maximize the revenue and secondly minimize the number of impressions such that the maximum revenue is achieved, i.e.

$$\sum_i x_{i,j} = 1$$
want to maximize the average profit of impressions, provided that the maximum revenue is achieved.

If we exclude secondary and learning constraints, this can be formalized as a special instance of the “Hitchcock’s style transportation problem” (see [F.L41]).

Given the constraints
\[ \forall k,l \sum_{i,j} x_{i,j,k,l} \leq s_{k,l} \]  \hspace{1cm} (19)
\[ \forall i \sum_{j,k,l} p_{j,k,l} x_{i,j,k,l} = d_i \]  \hspace{1cm} (20)

we want to minimize the following objective function
\[ \sum_{i,j,k,l} x_{i,j,k,l} \]  \hspace{1cm} (21)
i.e. the total number of impressions.

9 Projections

In order to apply our optimization algorithms we need to have at least a projection of the supply and a projection of the expected profit of all impressions allowed by the first primary constraint. The supply and impression profits could be estimated by taking a proper weighted average from the historical data. The projection should take into account different factors: episodic factors and possibly different periodicities (daily, weekly, yearly, etc...).

9.1 Projecting the profit

The profit of an impression-event may depend on the periodicity of its campaign and of its location. Since the eCPM tends to change slowly in time, it can be predicted better than the supply. If we want to determine the expected profit for an impression-event \((B_{i,j}, T_k, L_l)\) we can take some average profit from our historical data on “similar” events. Our strategy is to use the most accurate and recent available information in the historical data.

A simplified version of our algorithm can be described by the following procedure (more subcases are considered by the actual algorithm):

1. Try to find enough impression-events of the form \((B_{i,j}, T, L_l)\) where \(T\) is a similar time (possibly same day of the week and same hour) starting from the closest dates first.

2. Try to find enough impression-events of the form \((B, T, L_l)\) and \((B_{i,j}, T, L)\) and take a weighted average of the two averages, where \(L\) is any location, \(B\) is any creative, \(T\) is a similar time.

3. Try to do the same as in the previous step but with \((B, T, L_l)\) and \((B_{i,k}, T, L)\) where \(B_{i,k}\) is a different creative belonging to the same campaign.

4. Try to find enough impression-events (only on same the campaign, only the same node, etc...).
9.2 Projecting the supply

A model for the projection of the supply should take into consideration the periodicity of the location, i.e. some sites are more often visited in particular periods of the year, day, hours, etc. More periodicities may concur, e.g. a site may be visited more often in a specific day of the week and at a specific hour of the day, and may also have an episodic surge in the number of visitors for a short period for some unpredictable event. A mathematical model that could describe the concurrent effects of different periodicities could be that of superimposing waves, where each wave describe a different factor, e.g. a weekly factor and a contingent factor.

9.2.1 Weighted average

In many practical cases it is enough to consider a weighted average of the supply in the previous two weeks and at similar hours in a similar fashion as to procedure 9.1 used for the projection of the profit.

9.2.2 Machine learning

Regression analysis through machine-learning techniques such as support vector machines can be a viable approach for the problem of properly choosing the weights of the average of the different “features” (e.g. periodicities). Non-linear kernels could also be taken into considerations if they perform significantly better for the data sets under consideration.

10 Dynamic and stochastic nature of our problem

In reality this approach has a serious drawback: we are making a very false assumption because by applying linear programming we are assuming that in the period under consideration the expected impression profits $p_{i,j,k,l}$, the expected supplies $S_{l,k}$, and our constraints do not change. We are also erroneously assuming that $p_{i,j,k,l}$ is a constant with respect to $x_{i,j,k,l}$.

10.1 Non-linear and dynamic problem

Our problem is in fact non-linear and dynamic. To make things worse its state depends on external factors that cannot always be forecast (e.g. new campaigns can come into play). Hence we are forced to continuously readapt to the new constraints. Thus we can expect a good performance if our expected impression profit and constraints do not change too much in the period of time under consideration.
10.2 Learning phase

An additional problem comes from the fact that the system has to learn how new creatives and new locations perform. The corresponding theoretical problem goes under the name of “exploration-exploitation trade-off”, i.e. the challenge of deciding between learning how some resources perform versus exploiting the ones that have so far performed better. In practice we need to decide between displaying advertisements that help the system learn about their performance versus displaying those that generate a more immediate reward. This problem has been addressed in [AN99] where a technique based on the Gittins index has been used.

10.3 How far into the future

This also poses the problem of deciding how fast we want to update our information and how far in time we want our optimization to “see” our problem (i.e. how globally we want to solve the problem). A global solution could be a very bad one if the conditions of the problem were to change too quickly. We might want to give a different weight to an expected profit far in time in order to compensate for possible changes and so limit the risk.

We must also take into consideration the stochastic nature of our data. For example we might use historical data to extract the standard deviation for the profit of the impression and use it to better assess the risk.

A possibility could be to have an adaptive or semi-automatic system in which the time span given to the optimizer is adaptively/manually adjusted when there is a high probability of a significant change in the constraints, e.g. a new campaign is likely going to come into play, the estimation of profit of an impression is not stable enough, etc.

11 Targeting users

The approach we have so far presented optimizes the delivery of advertisements in both space (nodes) and time (time frames). It does not explicitly take users’ profiles into consideration.

Nevertheless the very same algorithms and code can be used to take into account users’ profiles by encapsulating the profile information into the node information. Therefore we should simply store a pair \((\text{node}, \text{profile})\) into a single “extended node”. The result would be that

- the supply are projected onto triples \((\text{node}, \text{profile}, \text{time})\);
- the eCPM and the delivery are computed for quintuples \((\text{campaign}, \text{creative}, \text{node}, \text{profile}, \text{time})\).

No modification of the code is necessary.
12 Computational considerations

The large number of unknowns and constraints in this general approach can pose a serious problem to its computable feasibility.

12.1 Reducing dimensions and constraints

We could reduce the number of dimensions by clustering similar attributes, i.e. combinations of locations and time frames (see [AN99] for an approach to this problem) or by simplifying our model. For example we could simplify our model as follows:

- We could restrict our optimization problem to periods of time in which the time constraints do not change. This greatly reduces the number of unknowns but could also bring us to suboptimal solutions.
- We could avoid considering the secondary and learning constraints within the model and have them enforced during the delivery.
- We can use a time horizon, beyond which all the time frames are considered jointly.
- We could use similar attributes as one single attribute, e.g. an impression at location \( \lambda_1 \) at time \( t_1 \) could perform similarly to a location \( \lambda_2 \) at some other time \( t_2 \).
- Projecting the “impression profits” is a costly operation because of the sheer number of points to be considered. This operation can be sped up by assuming that similar points produce the same profit.

12.2 Other optimization algorithms

The simplex algorithm is not the only known efficient algorithm for linear programming. Interior-point algorithms provide a valid alternative and are polynomial (more specifically “weakly polynomial”). Totally different approaches to the optimization could be possible, e.g. gradient based, genetic, etc. Unfortunately these other approaches are suboptimal because they are intrinsically local. Moreover they do not exploit the linear nature of the constraints and of the objective function.

\[ 1 \] An algorithm is “strongly polynomial” if and only if

- the number of operations in the arithmetic model of computation is bounded by a polynomial in the number of integers in the input instance; and
- the space used by the algorithm is bounded by a polynomial in the size of the input.

An algorithm which runs in polynomial time but which is not strongly polynomial is said to run in weakly polynomial time. The existence of a strongly polynomial algorithm for real linear programming is still an open problem.
12.3 Tuning the supply projection

Properly projecting the supply from historical data can be a hard task due to the fact that the available historical data might not correspond to the real traffic but only to a possible variable portion of the real traffic which is given to the ad-server. This problem might be impossible if no regularity is present in the data. A machine learning approach may better tackle it than taking a simple weighted average of selected profits at some previous periods with constant weights. For instance this is the case if an optimal method is non-linear.

13 Benchmarking linear solvers for ad-serving problems

In order to test the computational feasibility of two of the main linear programming solvers on our problems we have used the free and open-source lp_solve and glpk libraries. They both come as C libraries; both have wrapper interfaces in higher level languages such as Java. The result was a clear win for glpk at least for the type of problems under our consideration. Other alternative libraries are bpmip, scoplex, spx.

14 Software Implementation

We have implemented an ad-server optimizer in both C and in Java (see CCG). We have used the glpk library for solving the mathematical model and libsvm (see CCL) for automatically learning how to project future supply (Internet traffic).

14.1 Features

Our implementation has the following features:

- **Constraints:** We consider in our model the three types of primary constraints and provide as option the secondary and learning constraints.

- **Projection of the profit:** We perform a projection of the impression profits from historical data, by taking into account different periodicities (e.g. hourly, daily).

- **Projection of the supply:** We perform a projection of the supply from historical data, by both machine learning (support vector machine libsvm and/or by a fixed weighted average which both take into account different periodicities.

- **Time horizon:** We can set a time horizon in order to reduce the number of unknowns.
14.2 Extracting the data from the database

Our implementation requires as input:

1. historical data necessary for projecting the impression profits and the future supply,
2. campaign data (budgets for each campaign),
3. scheduling data (set of possible impression not contradicting the first primary constraints).

14.3 Main steps of the optimization

The algorithm can be roughly subdivided in the following macro steps:

1. Historical data is read.
2. The past supply is extracted from the historical data.
3. The future impression profits are projected from the historical data.
4. The future supply is projected from the past supply.
5. The mathematical model for the optimization problem is constructed.
6. The model is solved.
7. The solution of the problem is translated in terms of probabilities of delivery.
8. The delivery probability is only used on the very next time frame.
9. This procedure is repeated on the next time frame.

15 Results on real data provided by Neodata

Our prototype has been used on some real data used at Neodata and has been compared against the results produced by the optimizer currently used at Neodata, which uses a simple greedy algorithm:

1. after a learning phase;
2. if a campaign is achieving its target at the current rate, nothing is done; otherwise, the campaign is stopped in its less profit-generating nodes.

The data we used were the logs and schedules creatives used by two clients of Neodata, which, we call A and B. We have considered the data of April 2010 for both companies. We must remark that the percentage of traffic that is managed by Neodata, neither is the total traffic nor is it a constant percentage of
the traffic generated by the sites under consideration. This makes the problem of properly estimating the supply much harder (or even impossible).

The prototype achieved the following results: A was optimized equally well by the current optimizer and our prototype; whereas B was optimized better by a large margin (more than 20%) by our code. We do not know for sure why the data on A are not optimized equally well. Possible reasons are: there is no room for further improvement, the data on the supply cannot be used for the projection because it does no correspond to a constant percentage of the real traffic.

The data was used as follows: the initial portion of the month (e.g. the first 20 days) were used for training the system, i.e. projecting the supply (traffic) and the profits. The remaining part of the month was used as a schedule and was optimized.

16 Conclusion

Our prototype has shown that real data can be indeed optimized better than what a greedy algorithm does. There are still some open issues: how to correctly project the supply when the conditions of the problem change quickly and the data does not correspond to a constant percentage of the traffic.

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