Intermittency in the solar wind turbulence through probability distribution functions of fluctuations

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Abstract
Intermittency in fluid turbulence can be emphasized through the analysis of Probability Distribution Functions (PDF) for velocity fluctuations, which display a strong non–gaussian behavior at small scales. Castaing et al. (1990) have introduced the idea that this behavior can be represented, in the framework of a multiplicative cascade model, by a convolution of gaussians whose variances is distributed according to a log–normal distribution. In this letter we have tried to test this conjecture on the MHD solar wind turbulence by performing a fit of the PDF of the bulk speed and magnetic field intensity fluctuations calculated in the solar wind, with the model. This fit allows us to calculate a parameter $\lambda^2$ depending on the scale, which represents the width of the log–normal distribution of the variances of the gaussians. The physical implications of the obtained values of the parameter as well as of its scaling law are finally discussed.

Introduction
The statistics of turbulent fluid flows can be characterized by the Probability Distribution Function (PDF) of velocity differences over varying scales (Frisch, 1995, and references therein). At large scales the PDF is approximately Gaussian, as the scale decreases, the wings of the distribution become increasingly stretched, so that large deviations from the average value are present. This phenomenon, usually ascribed to intermittency, has been observed and deeply investigated in fluid flows (Frisch, 1995, and references therein), and recently also in Magneto-hydrodynamic (MHD) flows (see for example Biskamp, 1993; Marsch and Tu, 1997). Intermittency in MHD flows has been analyzed mainly by using satellite measurements of solar wind fluctuations (Burlaga, 1991; Marsch and Liu, 1993; Carbone et al., 1995, 1996; Ruzmaikin et al., 1995; Horbury et al., 1997), or by using high resolution 2D numerical simulations (Politano et al., 1998) and Shell Models (Biskamp, 1993, Carbone, 1994). All these analysis deal with the scaling exponents of structure functions, aimed to show that they follow anomalous scaling laws which can be compared with the usual energy cascade models for turbulence.

The non gaussian nature of PDF in MHD solar wind turbulence has been evidenced by Marsch and Tu (1994). In order to investigate the properties of intermittency through the analysis of non gaussian character of PDF, it would be necessary to quantify the departure of PDF from gaussian statistics and to analyze how this departure depends on the scale. Because of the idea of self–similarity underlying the energy cascade process in turbulence, Castaing and co–workers (Castaing et al., 1990) introduced a model which tries to characterize the behavior of the PDF’s through the scaling law of a parameter describing how the shape of the PDF changes in going towards small scales (Vassilicos, 1995). In its simpler form the model can be introduced by saying that the PDF of the increments $\delta \psi$ (representing here both velocity and magnetic fluctuations) at a given scale $\tau$, is made by a convolution of the
typical Gaussian distribution \(P_G\), with a function \(G_\tau(\sigma)\) which represents the weight of the gaussian distribution characterized by the variance \(\sigma\)

\[
P_\tau(\delta\psi) = \int G_\tau(\sigma) P_G(\delta\psi, \sigma) \, d\sigma
\]

In the usual approach where the energy cascade is introduced through a fragmentation process, \(\sigma\) is directly related to the local energy transfer rate \(\epsilon\). In a self–similar situation, where the energy cascade generates only a scaling variation of \(\sigma = \langle \delta\psi^2 \rangle^{1/2}\) according to the classical Kolmogorov’s picture (Frisch, 1995), \(G_\tau(\sigma)\) reduces to a Dirac function \(G_\tau(\sigma) = \delta(\sigma - \sigma_0)\). In this case from eq. (1) a Gaussian distribution \(P_\tau(\delta\psi) = P_G(\delta\psi, \sigma_0)\) is recast. On the contrary when the cascade is not strictly self–similar, the width of the distribution \(G_\tau\) is different from zero. In this way the scaling behavior of the width (which takes into account the height of the PDF’s wings) can be used to characterize intermittency. In the present paper we will try to see if the departure from the gaussian statistics can be described within the framework of the cascade model (1).

Solar Wind Observations

The satellite observations of both velocity and magnetic field in the interplanetary space, offer us an almost unique possibility to gain information on the turbulent MHD state in a very large scale range, say from 1 AU (Astronomical Units) up to \(10^8\) km. Since the aim of this letter is essentially to show that the PDF of solar wind fluctuations can be represented by the model (1), we limit to analyse only plasma measurements of the bulk velocity \(V(t)\) and magnetic field intensity \(B(t)\). The detailed analysis of single velocity and magnetic field components fluctuations is left for a more extended work.

We based our analysis on plasma measurements as recorded by the instruments on board Helios 2 during its primary mission in the inner heliosphere. The analysis period refers to the first 4 months of 1976 when the spacecraft orbited from 1 AU, on day 17, to 0.29 AU on day 108. The original data were collected in 81 s bins and we choose a set of subintervals of 2 days each. The subintervals were selected separately within low speed regions and high speed regions. Fast wind was chosen having care of selecting a two–day interval within the trailing edge of each high speed stream. The choice was such that the average value of the wind speed was never below 550 km/sec for all the ”fast” intervals. Slow wind was selected picking up two–day intervals just before the stream–stream interface having care that the average speed value was never above 450 km/sec for each interval. For each subinterval we calculated the velocity and magnetic increments at a given scale \(\tau\) through \(\delta V_\tau = V(t+\tau) - V(t)\) and \(\delta B_\tau = B(t+\tau) - B(t)\), which represent characteristic fluctuations across eddies at the scale \(\tau\). Then we normalize each variable to the standard deviation within each subinterval \(\delta V_\tau = \delta V_\tau/\langle (\delta V_\tau)^2 \rangle^{1/2}\) and \(\delta B_\tau = \delta B_\tau/\langle (\delta B_\tau)^2 \rangle^{1/2}\) (brackets being average within each subinterval at the scale \(\tau\)). Then we get two data sets: a set containing both the normalized velocity and magnetic field fluctuations for the low speed streams (each variable is made by 10890 samples), and a different set containing the same quantities for the high speed streams (each variable made of 13068 samples). We calculate the PDF’s at 11 different scales logarithmically spaced \(\tau = \Delta t \, 2^n\), where \(n = 0, 1, ..., 10\) and \(\Delta t = 81\) s. We collect the number of events within each bins by using 31 bins equally spaced in the range within 3 times the standard deviation of the total sample. Before we mixed the different subperiods belonging to a given class (high or low speed streams), we tested for the fact that thegross features of PDF’s shape does not change in different subintervals. Then our results for high and low speed streams are representative of what happens at the PDF’s.

The results are shown in figures 1 and 2 where we report the PDF’s of both velocity and magnetic intensity for the high speed streams (the same figures can be done for the slow speed streams). At large scales the PDF’s are almost Gaussian, and the wings of the distributions grow up as the scale becomes smaller. This is true in all cases, say for both types of wind. Stronger events at small scales have a probability of occurrence greater than that they would have if they were distributed according to a gaussian function. This behavior is at the heart of the phenomenon of intermittency as currently observed in fluid flows (Frisch, 1995) and in the solar wind turbulence (Marsch and Tu, 1997). As a characteristic it is worthwhile to note that for the magnetic intensity, the PDF’s wings at small scales are more ”stretched” with respect to the corresponding PDF’s calculated for velocity. This is true both in slow and fast wind.
Results and Discussion

In order to make a quantitative analysis of the energy cascade leading to the process described in the previous section, we have tried to fit the distributions by using the log-normal ansatz (Castaing et al., 1990).

\[
G_\tau (\sigma) d\sigma = \frac{1}{\lambda(\tau)\sqrt{2\pi}} \exp \left[ -\frac{\ln^2(\sigma/\sigma_0)}{2\lambda^2(\tau)} \right] d(\ln \sigma)
\]

(2)
even if also other functions gives rise to results not really different. The parameter \(\sigma_0\) represents the most probable value of \(\sigma\), while \(\lambda(\tau) = \langle (\Delta \ln \sigma)^2 \rangle^{1/2}\) is the width of the log-normal distribution of \(\sigma\).

We have fitted the expression (1) on the experimental PDF’s for both velocity and magnetic intensity, and we have obtained the corresponding values of the parameter \(\lambda\). The values of the parameters \(\sigma_0\), which do not display almost any variation with \(\tau\) are reported in the Table. Our results are summarized in figures 1 and 2, where we plot, as full lines, the curves relative to the fit. As can be seen the scaling behavior of PDF’s in all cases is very well described by the function (1), thus indicating the robustness of the cascade model. From the fit, at each scale \(\tau\), we get a value for the parameter \(\lambda^2(\tau)\), and in figures 2 we report the scaling behavior of \(\lambda^2(\tau)\) for both high and low speed streams. Starting from \(\lambda^2 \simeq 10^{-3}\) at the large scales (about 1 day), the parameter increases abruptly to \(\lambda^2 \simeq 10^{-1}\) at about 2 hours, and finally a scaling law starts to become evident up to \(\Delta t = 81\) sec. In this last range, which corresponds roughly to what is usually called the "Alfvénic range", we fitted the parameter with a power law \(\lambda^2(\tau) = \mu \tau^{-\beta}\). The values of \(\mu\) and \(\beta\) obtained in the fitting procedure and the corresponding range of scales, are reported in the Table.
Table 1. We report the values of the parameters $\sigma_0$, and the values of $\mu$ and $\beta$ obtained in the fitting procedure for $\lambda^2(\tau)$. We also report the range of scales where the fit has been done.

|        | B (Fast) | B (Slow) | V (Fast) | V (Slow) |
|--------|----------|----------|----------|----------|
| $\sigma_0$ | 0.85 ± 0.05 | 0.90 ± 0.05 | 0.90 ± 0.05 | 0.95 ± 0.05 |
| $\mu$   | 0.90 ± 0.03 | 0.75 ± 0.03 | 0.54 ± 0.03 | 0.38 ± 0.02 |
| $\beta$ | 0.19 ± 0.02 | 0.18 ± 0.03 | 0.44 ± 0.05 | 0.20 ± 0.04 |
| Scales  | $\tau \leq 0.72$ hours | $\tau \leq 0.72$ hours | $\tau \leq 1.44$ hours | $\tau \leq 1.44$ hours |

*Figure 1.* The scaling behavior of the PDF for $\delta v_\tau$ as calculated from the experimental data (white symbols) in the fast streams. The full lines represent the fit obtained through the model as described in the text.

*Figure 2.* The scaling behavior of the PDF for $\delta b_\tau$ as calculated from the experimental data (white symbols) in the fast streams. The full lines represent the fit obtained through the model as described in the text.

Looking at Figure 3, it can be seen that both in fast and in slow streams magnetic field intensity is more
intermittent than bulk speed (values of $\lambda^2$ are at least two times larger for magnetic field intensity than for velocity). This has also been reported by Marsch and Tu (1994), and the same indications comes from 2D MHD direct simulations (Politano et al., 1998), and in analysis of solar wind intermittency performed using different techniques (Veltri and Mangeney, 1999). The values of $\lambda^2(\tau)$ are more or less the same for magnetic field intensity both in fast and in slow wind. This is perhaps related to the fact that magnetic field intensity fluctuations are related to compressive fluctuations, which should have the same nature in both types of wind. The bulk velocity fluctuations on the contrary are more intermittent at small scales (81 sec) in the fast wind and at large scale ($\simeq 1$ hour) in the slow wind. This result is due to the different values of $\beta$ for fast and slow wind. From the Table it appears that the value of $\beta$ is not universal, a result which has also been found in fluid flows (Castaing et al., 1990) being close to $\beta \simeq 0.2$ for magnetic field intensity in both fast and slow wind and for the velocity field in slow wind, while in fast wind the value of $\beta$ for the bulk velocity fluctuations is $\beta \simeq 0.44$.

**Figure 3.** We show the scaling behavior of $\lambda^2(\tau)$ vs. $\tau$ for both fast (black symbols) and slow (open symbols) streams. Circles refer to the magnetic field intensity, squares refer to the bulk velocity.

In the framework of the cascade model, Castaing et al. (1990) give an interpretation of the parameter $\beta$ as the co-dimension of the more intermittent structures in a 1D cut of the turbulent field. If one believes to this interpretation, our results show that singular structures which are responsible for intermittency of the bulk velocity, look different for both type of winds. In particular structures in fast wind appears to lie on set with higher co-dimension. The fact that the value of $\beta$ for the bulk velocity fluctuations in slow wind is the same as the value of $\beta$ for magnetic field intensity suggests that intermittent structures in slow wind are perhaps mainly associated with compressive fluctuations. On the contrary the different value of $\beta$ found in fast wind evidence a different nature of velocity fluctuations in fast wind, perhaps related to the fact that such fluctuations are mainly incompressible. This result is in agreement with what has been recently found by Veltri and Mangeney (1999); these authors found that in fast wind the more intermittent structures are tangential discontinuities with almost no variation in magnetic field intensity, while in slow wind the most intermittent structures are shock waves, which display the same behavior in bulk velocity and magnetic field intensity.

**Acknowledgments.** We are grateful to H. Rosenbauer and R. Schwenn for making the Helios plasma data available to us.

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November 17, 1998; revised January 29, 1999; accepted March 16, 1999.