Quantum cryptographic three party protocols

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I. INTRODUCTION

In a multi party protocol a set $P$ of players wants to correctly compute a function $f(a_1, \ldots, a_n)$ which depends on secret inputs of $n$ players. Some players might collude to cheat in the protocol as to obtain information about secret inputs of the other players or to modify the result of the computation. Possible collusions of cheaters are modelled by adversary structures

Definition 1 An adversary structure is a monotone set $A \subseteq 2^P$, i.e., for subsets $S' \subseteq S$ of $P$ the property $S \in A$ implies $S' \in A$.

The main properties of a multi party protocol are:

1. A multi party protocol is said to be $A$-secure if no single collusion from $A$ is able to obtain information about the secret inputs of other participants which cannot be derived from the result and the inputs of the colluding players.

2. A multi party protocol is $A$-partially correct if no possible collusion can let the protocol terminate with a wrong result.

3. A multi party protocol is called $A$-fair if no collusion from $A$ can reconstruct the result of the multi party computation earlier than all honest participants together. No collusion should be able to run off with the result.

We will be more strict here and demand robustness even against disruptors.

2. A multi party protocol is $A$-correct whenever no single collusion from $A$ can abort the protocol, modify its result, or take actions such that some player gets to know a secret value.

A protocol is called $A$-robust if it has all of the above properties. Note that we will allow only one collusion to cheat, but we think of every single player as being curious, i.e., even if he is not in the collusion actually cheating he will eavesdrop all information he can obtain without being detected cheating.

With oblivious transfer all multi party protocols can be realized with perfect security if all players are cooperating. But a collusion of players can abort the calculation, see next section.

Classically one can avoid this problem only by introducing a new cryptographic primitive which is more powerful than oblivious transfer.

This paper analyzes three party quantum protocols and how they can cope with the problem of disruption. We prove that there are situations where a quantum channel is strictly more powerful than oblivious transfer. Together with the results of [10,8,7] we can conclude that the cryptographic power of a quantum channel is incomparable to the power of oblivious transfer.

II. IMPOSSIBILITY OF CLASSICAL THREE PARTY PROTOCOLS

To clearly show the advantage of quantum protocols we restate the following impossibility result of [1].

Lemma 2 Let $P$ be a set of players for which each pair of players is connected by a (private) oblivious transfer channel and each player has access to an authenticated broadcast channel. Then $A$-robust multi party computations are possible for all functions if and only if no two sets of $A$ cover $P \setminus \{P_i\}$ for a player $P_i \in P$ or $|P| = 2$.

The basic idea to prove this impossibility result of [2] is to have two possible collusions $A_1, A_2$ covering $P \setminus \{P_i\}$ (for a player $P_i$) where either all players from $A_1$ or all players from $A_2$ refuse to cooperate with the players of the other possible collusion. Then the single player $P_i$ has to assist all other players. In [6] it is proven that one cannot avoid that the player $P_i$ learns a secret.

Especially three party protocols cannot necessarily be realized robustly if every player is possibly cheating.

III. THREE PARTY PROTOCOLS

If all players in a three party protocol cooperate we can use the protocols of [1] to implement $A$-robust quantum multi party protocols. Hence we focus on the situation where three players (Alice, Bob, and Helen) want to perform three party protocols and Alice and Bob are in conflict. One of the two is refusing to cooperate with the other and it is unclear for Helen who is cheating.

As a first step we will introduce a bit commitment protocol for Alice and Bob. The idea is that Alice sends her quantum states via Helen and Bob does not know which quantum data is coming from Helen and which data is just forwarded by Helen. Hence Bob cannot complain
without reason or he risks to be detected cheating by Helen. Forwarding information as if it were ones own without being able to eavesdrop is impossible classically. One further advantage of the protocol below is that Alice can forward all information via Helen and Bob cannot know whose information it is: An anonymous quantum channel. This way Bob cannot distinguish between commitments of Alice and “pretended” commitments of Helen. Later we want to follow this idea with larger protocols containing this bit commitment protocol as a subprotocol. Then we let Alice forward all her information via Helen.

**Commit**($b$)

**FOR** $i \in \{1, \ldots, t\}$ **DO**

1. Alice gives a random string $r$ of qubits encoded in random bases $s \in \{+, \times\}$ to Helen.
2. Helen sends a substring to Bob interleaved with quantum states of her own. Helen tells Alice which quantum states are hers without revealing information about which substring she forwarded.
3. Bob announces to have received all quantum states. With a probability of $1/2$ he publishes all his measurement results.
4. Alice opens to Helen the bases she used. Now Alice is bound to the parity bit of $r$.

**OD**

5. Alice is now bound to the Xor of all quantum states (which were not measured and published by Bob) Alice announces (via Helen if needed) if this parity bit is equal to the bit $b$ she originally wanted to commit to.

**Unveil**

1. Alice opens (via Helen if necessary) all choices she made.
2. Helen and Bob check consistency.

**Lemma 3** For $\mathcal{A} = \{\{\text{Alice}\}, \{\text{Bob}\}, \{\text{Helen}\}\}$ the above protocol realizes an $A$-robust bit commitment for Alice which binds her to Bob and to Helen even if Alice and Bob are in conflict.

**Proof:** We say the protocol has failed if many of the quantum states measured and published by Bob do not match what Alice sent or what Helen sent. If there are only very few cases with discrepancies the protocol is considered a success.

If the protocol does not fail then the protocol is concealing to Helen. Helen has sent some substrings to Bob and as she could not know which substrings would be measured and published by Bob there are some substrings which she forwarded, but which were not tested. Hence Helen cannot measure the overall parity bit $b$ even after getting to know the bases. Helen cannot measure before getting to know the bases as she will be detected cheating whenever Bob measures and publishes a quantum state she disturbed.

If the protocol did not fail it is concealing to Bob unless Helen and Bob collude. This is clear as Bob does not have the complete quantum state, and can hence not measure the parity bit.

If the protocol did not fail it is binding for Alice unless she colludes with Helen, which is impossible according to our assumption.

If the protocol failed there are two cases to be considered. First, Bob published a lot of measurement results which do not match what Alice sent, but Helen was not complaining about Bob, then it is clear for Alice that Helen and Bob collude somehow, but as this is not possible according to our assumption this will not happen. The second case is that Helen complains about Bob, then Bob is identified as a cheater as every player complains about Bob.

As Alice is by the above protocol bound to Bob and Helen we have bit commitment from Alice to Bob and from Alice to Helen and from Bob to Alice and from Bob to Helen. As the quantum channel between Alice and Helen and between Bob and Helen is working we even have oblivious transfer from Helen to Alice and from Helen to Bob by forcing honest measurements with bit commitment.

**Corollary 4** The above sketched oblivious transfer protocol between Alice and Helen and between Bob and Helen is $A$-robust and becomes $\tilde{A}$-robust after it terminated for $\mathcal{A} = \{\{\text{Alice}\}, \{\text{Bob}\}, \{\text{Helen}\}\}$ and $\tilde{\mathcal{A}} = \{\{\text{Alice}, \text{Bob}\}, \{\text{Helen}\}\}$.

**Proof:** The bit commitment used for forcing measurements need only be shortly binding. It need only be binding until the honest measurement is performed unconditionally. Hence the collusion $\{\text{Alice}, \text{Bob}\}$, which would be able to violate the binding condition but not the concealing condition of the above bit commitment, cannot cheat after the measurement is performed.

Next we have to define some notions which are important for multi party protocols. Details can be looked up in [4].

**Definition 5** A bit commitment with Xor (BCX) to a bit $b$ is a commitment to bits $b_{1L}, b_{1R}, \ldots, b_{mL}, b_{mR}$ such that for each $i$ $b_{iL} \oplus b_{iR} = b$.

The following result about zero knowledge proofs on BCX can be found in [1] and in references therein.

**Theorem 6** Bit commitments with Xor allow zero knowledge proofs of linear relations among several bits a player has committed to using BCX. Especially (in)equality of bits or a bit string being contained in a linear code.

Furthermore BCXs can be copied, as proofs may destroy a BCX.

In a multi party scenario it is necessary that a player should be committed to all other players. In our three party case this is given by our bit commitment protocol which binds one player (Alice or Bob) to the other two. For Helen the following [global bit commitment with Xor can be implemented by Corollary 4] and the techniques used in [1] as she is not in conflict with anyone.
A global bit commitment with Xor (GBCX) is a BCX commitment from a player Alice ∈ P to all other players such that all players are convinced that Alice did commit to the same bit in all the different BCX.

**Definition 7** A global bit commitment with Xor (GBCX) is a BCX commitment from a player Alice ∈ P to all other players such that all players are convinced that Alice did commit to the same bit in all the different BCX.

**Corollary 8** Zero knowledge proofs of linear relations among several GBCX are possible. Furthermore GBCX can be copied by copying the individual BCX.

On these commitments operates the committed oblivious transfer protocol, defined in [4] which forms the basis of our multi party protocols.

**Definition 9** Given two players Alice and Bob where Alice is committed to bits b₀, b₁ and Bob is committed to a bit a. Then a committed oblivious transfer protocol (COT) is a protocol where Alice inputs her knowledge about her two commitments and Bob will input his knowledge about his commitment and the result will be that Bob is committed to bₐ.

In a global committed oblivious transfer protocol all players are convinced of the validity of the commitments, i.e., that indeed Bob is committed to bₐ after the protocol.

As Helen is not actively cheating by assumption and Alice or Bob cannot complain about Helen without being expelled from the protocol we can realize GCOT from Helen to Alice and from Helen to Bob by following the protocol of [4]. To realize GCOT from Alice to Bob is more difficult. We will do this in two steps. First we realize a subprotocol which we call subGCOT and second we will observe that all other steps can be realized easily once one round of subGCOT was successful.

To realize subGCOT between Alice and Bob we carry out the first 7 steps of the GCOT protocol of [4] in a way that Bob cannot decide if the data comes from Alice or from Helen if he then complains without reason he risks to get in conflict with Helen, which would prove him cheating.

**subGCOT**

1. Alice randomly picks c₀, c₁ from a previously agreed on code C (for requirements on C see [4]) and commits to all bits of the codewords, and proves that the codewords fulfill the linear relations of C (for the zero knowledge technique used confer [4]).
2. Bob randomly picks I₀, I₁ ⊂ {1, . . . , M}, with |I₀| = |I₁| = σm (σ is a parameter of the code C), I₀ ∩ I₁ = ∅ and sets b₀ ← ̃b for i ∈ I₀ and b₁ ← ̃b for i /∈ I₀.
3. Alice runs OT(c₀, c₁)(b₀) with Bob (by [4]) and the above bit commitment which binds Bob to Helen and Alice who gets wᵢ for i ∈ {1, . . . , m}. Bob tells I₀ = I₀ ∪ I₁ to Alice who opens c₀ᵢ, c₁ᵢ for each i ∈ I.
4. Bob checks that wᵢ = cᵢ for i ∈ I₀ and wᵢ = cᵢ for i ∈ I₁, sets wᵢ ← cᵢ for i ∈ I₀ and corrects w using the code C’s decoding algorithm, commits to wᵢ for i ∈ {1, . . . , m}, and proves that w₁ . . . wₘ ∈ C.
5. All players together randomly pick a subset I₂ ⊂ {1, . . . , m} with |I₂| = σm, I₂ ∩ I = ∅ and opens c₀ᵢ and c₁ᵢ for i ∈ I₂.
6. Bob proves that wᵢ = cᵢ for i ∈ I₂.

Alice and Helen play subGCOT with Bob in a way that Alice plays via Helen using the above bit commitment protocol and the forcing measurement technique of [13] such that Bob cannot distinguish between commitments/quantum states of Alice and pretended commitments/quantum states of Helen, then Bob cannot distinguish between the subGCOT protocols he plays with Alice and those which are pretended by Helen. Two cases can occur:

1. After l trials a subGCOT protocol was successful from Alice to Bob and neither player complains. This subGCOT protocol can be used to perform GCOT from Alice to Bob, as all other steps of GCOT are not critical.

2. After l trials no subGCOT protocol between Alice and Bob was successful. Then, as Helen and Bob do not collude and Bob cannot distinguish between Alice’s and Helen’s data, it is clear that Alice is cheating if Bob only complained about her data and it is clear that Bob is cheating if he complained about Helen as well as Alice.

Once Alice and Bob were able to run one round of subGCOT they can complete this protocol to a GCOT protocol by the steps:

- Alice randomly picks and announces a privacy amplification h : {0, 1}ᵐ → {0, 1} such that a₀ = h(c₀) and a₁ = h(c₁) and proves a₀ = h(c₀₁, . . . , cᵢᵐ) and a₁ = h(cᵢ₁, . . . , c₁ᵐ).
- Bob sets a ← h(w), commits to a and proves a = h(w₁, . . . , wᵐ).

Alice and Bob give their proofs (following the procedure of [4]) in the above steps to Helen. Hence the proofs must be correct as no one can risk to get into conflict with Helen, also convincing Helen is enough as she is not colluding with Alice or with Bob.

For the correctness of the GCOT protocol we refer to the proof in [4]. We conclude:

**Lemma 10** Even if Alice and Bob are in conflict there exists an A-robust protocol for GCOT between Alice and Helen, Bob and Helen and Alice and Bob. This protocol becomes ̃A-robust after it terminated for A = \{ {Alice}, {Bob}, {Helen} \} and ̃A = \{ {Alice}, {Bob}, {Helen} \}.

**Proof:** By Corollary 8 we can have oblivious transfer from Helen to Alice and from Helen to Bob. As Helen cannot be in conflict with Alice or Bob (or a cheater can be identified) we can realize GCOT from Helen to any other player by the protocols of [4] with the security of the oblivious transfer channel of Corollary 8.

The above protocol for GCOT between Alice and Bob is still concealing for Helen even if Alice and Bob collude, but the resulting commitments need not be binding any more. This is no problem as we allow Alice and Bob to collude only after the termination of the protocol (See comment after Theorem 11).

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From a bit commitment which can bind one player to the other two players we can realize GBCX and together with a GCOT protocol working in at least one direction between every two players we can obtain all three party protocols, see [4] (and [1] for creating a distributed bit commitment, which is needed in [6], in the presence of conflicts). Hence we can conclude:

**Theorem 11** For three players Alice, Bob, and Helen all functions can be realized by quantum multi party protocols if two players are honest. The protocol becomes \{\{Alice\}, \{Bob\}, \{Helen\}\} \cup \{A\}-secure after its execution if there is a player outside of A nobody complained about.

Note that the bit commitment used during the protocol is not necessarily binding after Alice and Bob collude. If one wants to implement a long binding bit commitment one has to implement it as a multi party computation.

IV. A COMPLETENESS THEOREM FOR QUANTUM MULTI PARTY PROTOCOLS

In the paper [11] quantum multi party protocols were proposed which use secret sharing to force measurements to implement oblivious transfer. Then these protocols follow [1] to implement multi party computations with oblivious transfer.

There was an impossibility result in [11] that quantum multi party protocols for all functions become impossible if two possible collusions cover the set \{P\} of players. But due to the use of oblivious transfer this had to be weakened to the condition of Lemma 2. The impossibility result of [11] did not seem to be sharp. Now we can prove that the result is indeed sharp as we can implement multi party protocols even in the case not covered by [11]:

**Theorem 12** \(\mathcal{A}\)-robust quantum multi party protocols for all functions are possible if and only if no two collusions of \(\mathcal{A}\) cover the set \{P\} of players.

These protocols become \(\mathcal{A}\)-robust after termination for an adversary structure \(\mathcal{A}\) which may contain one and only one complement of a set of \(\mathcal{A}\).

**Proof:** If one looks at [11] one can see that the above theorem is proven there for all cases but one. The case left open is that two collusions \(A_1, A_2\), covering \(P \setminus \{P_i\}\) for a player \(P_i\), are in conflict such that no player from \(A_1\) can use the oblivious transfer to any player of \(A_2\) (and vice versa).

In this case we can proceed analogously to our three party protocols. All commitments are made via \(P_i\) (Helen) and equality of commitments is proven to Helen. This way we obtain GBCX in a way that whenever a player complains a cheater can be identified.

The GCOT protocol can be realized analogously to the above three party protocol. A player Alice \(\in A_1\) runs subGCOT via Helen with a player Bob \(\in A_2\) such that Bob cannot distinguish data from Helen and data from Alice. Again Bob cannot complain without either being detected cheating or proving that Alice cheats. On top of subGCOT GCOT can easily be realized. With GBCX and GCOT we can realize all multi party protocols if one keeps in mind that distributed bit commitments can be realized without problems even when conflicts are present.

The set \(A \in \mathcal{A}\), for which \(A^c\) may be contained in \(\mathcal{A}\), can in the above case be chosen to be any set containing Helen.

The set \(\tilde{\mathcal{A}}\) can even contain more than one complement of a set of \(\mathcal{A}\) provided Helen is not in the additional collusion. The protocol seems to be more secure than a quantum protocol where no complaints were present. This is true, due to the fact that Helen becomes a trustable third party in the way that we know she is not colluding with anyone. This shows that conflicts appearing during the protocol can yield additional information which can be exploited to increase the security.

One of the ideas of this paper, namely to anonymize oblivious transfer, can also be applied to classical protocols. With the primitive of anonymous oblivious transfer all multi party protocols become possible with perfect security, and whenever a player tries to abort the protocol this player is identified or the protocol terminates correctly [12].

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