Reflective Inquiry: Why Area is Never Negative?

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Abstract. Reflective Thinking is an important process for an educator to be more professional. Educator can have reflective thinking with doing reflective inquiry. The reflective inquiry has a doctrine, that we not receive immediately something into information, but it’s requires examined and proven to be accepted as information. Information that examine in this inquiry is statement about area is never negative. A deep inquiry to prove that area is never negative lead to a re-construction of the belief system within self, thereby generating new perspective and providing alternative solution to answer why area is never negative? Although it’s not clearly defined that area is never negative, why people accept it as a common sense? What underlies that area is never negative, even teacher itself don’t know the reason? Doesn’t teaching without understanding is consider as irresponsible action to student? Worried with this condition researcher try to figure it out and prove area is never negative with relation between measure, metric, dimension, length, and area in real set. Measure is foundation in this relation, with measure defined non-negative, causes metric and dimension defined with non-negative value. Resulting length as measure in dimension one is non-negative and proved that area is never negative.

1. Introduction
It begins with the curiosity about negative number and it is operation. There is negative number in real set, the part of real set that definite the value of something in real set, as area is part of real set \cite{1} why it can’t be definite by negative number? Why it has to be non-negative? Why it can be like that? How to explain it? If we can prove that area it is not negative or can be definite in negative, how to promote it? How to teach it to other people? Drive by those question we try to took preliminary study in public Senior High School in Cimahi City, SMAN 2 Cimahi, into 71 students. The result reveal that they don’t care about why area have to be non-negative, and mostly they just realize the fact at the moment. They just care about how to use formula to calculating the area, and have understanding to be proficient in mathematics mean how many formula\textsuperscript{s} that you memorize. if they care and try to inquiry the fact by them self, it can help them to understand how to learn mathematics without always memorize the formula.

They lost the essences of learning mathematics, it is because the learning methods and book that teacher use based on recommendation from government is deliver learning in practical ways \cite{2} that is not foster learning by meaning. Learning without meaning means the student just follow what teacher give without thinking, and it is not learning \cite{3}. We try to change this condition with improve how to learn mathematics with meaning. That help the student to not just memorize the formula, but get the thinking, get the logic, the analysis, and realize calculating it is not everything in learning mathematics.
Mathematics itself is knowledge that we use to solve daily problem [4]. How we can solve the daily problem if we just memorize the formula without meaning and implement the knowledge in our way?

The purpose of this research is to find a reason and a new understanding that can be evidence to answer why the value of area is never negative, and lead into new learning design to promote the answer that we get. With hope it can help teacher, student, and other people to understand why area is never negative, as the first step to learning with meaning.

2. Method
Because the purpose of this research is to find deep understanding that can be answer and reason to explain why area is never negative, the right methods that we use is Reflective Inquiry. This research also become an example of research to use Reflective Inquiry as methods. Reflective inquiry is an investigation or research using reflective thinking, so that the subject used is development in my mind, causing the instrument used in this research to be human instrument. Reflective inquiry consists of two words, reflective and inquiry. Inquiry itself is interpreted as an official process in finding out the cause of something [5], in this research to find the cause of why area is never negative. Reflective as thinking has the same meaning as reflection [6], reflection is awareness that something is believed or not, not only based on how someone is directed to get it, such as being told or taught, but that person needs to find evidence, witness, foundation, and orders that can support something to be believed or refuted [7]. So that reflective inquiry is an active and persistent activity of someone in looking for evidence that can support something to be believed, and aims to rearrange the belief system within [7]. Because this research is an act of reflection, the research is carried out in three levels, called the three levels of reflective thinking [8] as follows:

2.1. Implicit Reflection
When a person faced with a matter or a problem, the person does not immediately accept it as an information, but researched and interpreted it to seek understanding and solutions, the search done first is referring to the experience and knowledge that is self-owned, so the search is carried out into self (implicit), and this level is called implicit reflection.

2.2. Explicit Reflection
However, often due to limited knowledge mastered by oneself, the matter yet be proven or the problem yet be found a solution, so then we doing a search performed out of oneself (explicit). Search is done by referring to knowledge outside the self-based on books, discussion with other parties, and explicit knowledge. This level called explicit reflection.

2.3. Critical Reflection
In certain cases, even though there have been implicit and explicit reflection, evidences and solutions have not been found, the person is in critical condition (critical). So that by using human creative power he will try to create a new understanding that can be evidence or solution to the problem at hands. Even though it is said to be new, the understanding that is formed does not have to be obtained from zero, the understanding and actions obtained from the combination of some expert views and summing it up with one is own view can be said to be a new understanding, because in critical processes involves the result of implicit and explicit processes as well. Critical reflection is also referred to as learning in adulthood, where in adulthood a person begins to reassess the orientation in understanding, feeling, and actions he has had and has ever done [9]. There are two possible outcomes from the critical reflection, first the discovery of new understanding and solution, or the second the formation of new question leading to further research.
3. Result and Discussion

We discover two approaches to answer why area is never negative, but first we have to explain the relation between area and length to understand our approaches and it is part of implicit reflection section.

3.1. Implicit Reflection

When we want to know about area of the plane, we have to know the length of sides of the plane in advance. Why it should be like that? We try to understand it using Riemann Partition idea. Riemann Partition idea is we search an area of plane with partitioning the plane into polygons [1]. The smaller polygons partitions will lead into more accurate result to area of the plane. The we assume if we part the plane into partition polygons, the smallest polygon that we get will be a line of the plane. An idea to reverse the proses of Riemann Partition arise: “What if we stacked up those smallest polygons called lines? Guaranteed by Density Theorem, without gap between the line. Wouldn’t it build a plane?”

This assumption explain why we have to know about length first before calculate the area, because plane is like lines which experience dimension transformation from dimension one into dimension two, and lines can build plane guaranteed by Density Theorem, as the line and plane is part of real set. Then to know why area is never negative we have to find if there is any length that negative?

But we realize there is same things between area and length, it is dimension and measure. length is definite as measure in dimension one, and area is definite as measure in dimension two [10]. The to determine if there is length and area that negative, we have to find the value of dimension and measure. but dimension itself is topological measure of the size of covering properties in object [11]. Because dimension is a measure, the key to determine the value of length, area, and dimension is measure. but because our knowledge is not enough to determine the value of measure, we did the next step of this inquiry, called explicit reflection.

3.2. Explicit Reflection

We have the key to determine the value, that is measure, so we try to find what measure is, and the measure is defined as an extended real valued, non-negative, and countably additive functions [12]. More clearly if \((X, \Sigma)\) is a measure-able space, the measure is a function that denoted by \(\mu(\mu)\), with \(\mu: \Sigma \rightarrow [0, +\infty)\) said to be a measure on \((X, \Sigma)\) if:

\[
\mu(\emptyset) = 0 \\
\mu\left(\bigcup_{n=1}^{\infty} A_n\right) = \sum_{n=1}^{\infty} \mu(A_n)
\]

for all sequences \((A_n)\) of pairwise disjoint sets which are contained in \(\Sigma\) [13].

As we read from two definition above, measure is a function that maps each member of a set to interval \([0, +\infty)\). To understand it we give example, because we search about area, the example we give it is about area. A line in area can be consider have zero value, because it is not build space inside like area. It can be seen in the following Figure 1:

![Figure 1](image)

**Figure 1.** Example to understand definition of measure as function.

A line is map into 0 because it have zero value in area/dimension 2. A unit box is map into 1 because it have one value in dimension 2. A box that build from two unit box is map into 2, and a box that build from three unit box is map into 3, and et cetera.

There is an agreement in the form of definition that measure is definite in non-negative, make length, area, dimension as part of measure have non-negative value. After we find the solution. We began to
ask how to explain it/ how to promote it to other people? That can help them in some way. We find two approaches to explain it, that will be detailed in critical reflection section.

3.3. Critical Reflection

We can seen the relation between measure, length, area, and dimension in the following Figure 2:

![Figure 2. Relation between Area, Length, Dimension, and Measure.](image)

From this relation we find two approaches based on learning experience in each individual. The student, teacher, and people can choose their right approach for them. The approaches can be seen in the following Figure 3:

![Figure 3. Two Approaches to understand Why Area is Never Negative.](image)

There are two approaches:

3.3.1. Shortcut. The Shortcut approach have fewer content, it is measure, length, and area. Because it immediately takes the meaning of both length and area as measure, and continue it into definition of measure. it is easier to understand, and can lead into learning design for student at school.

3.3.2. Deep understanding. The Deep understanding approach contain measure, topology, dimension, length, area. It is for someone who have more advance learning experience, because we have to understand about topology and dimension first. This approach can lead into more complex learning design and challenge our self.

4. Conclusion

We found the reason why area is never negative it is because there is an agreement in the form of definition that definite the value of measure is never negative, makes area, length, and dimension as part of measure can’t have negative value. There are two approaches to understand why area is never
negative based on learning experience in each individual, so an individual can choose the approaches that right for them, this is the two approaches: **Shortcut: Measure → Length → Area** and **Deep Understanding: Measure → Topology → Dimension → Length → Area**

This research is an example of research using Reflective Inquiry Methods, so we want to share our experience in using this method in the form of argument. This reflective Inquiry Methods, is methods that face the problem by continuously reflecting with the knowledge formed within self. A deep understanding, a deep mastering of the problem and the solution is formed, build strong foundation in our belief system, and assure to assess something to be accepted or rejected. We can see a continuous inseparable relation between materials and knowledges. We felt this to keep looking and looking an evidence, understanding, explanation, and statement that can answer our curiosity. The result of this methods is not certainly one, there is two or more result. It can be certainly solution, or another question, or both solution and question. In our case the result that we get is the both. We get the answer why area is never negative with it is approaches, but another question a rise “What if there is Negative Dimension? A dimension that have same value with positive dimension, except all denoted in negatively and can accommodate area with negative value?”. This method itself is interesting, with our argument we recommendation this method of inquiry needs to be cultivated.

5. **References**

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