The written mathematical communication ability of junior high school students in solving set problems

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Abstract. Written mathematical communication helps students in the learning process, helps teachers to assess students’ understanding, and becomes the link between the known and the written information with the understood information. The concept of understanding or procedures will be more meaningful if the students can communicate their ideas well. The study aims to describe and analyze students’ written mathematical communication abilities. This research is a qualitative descriptive study. The researchers employ tests and interviews in collecting data. The validity of the data uses the triangulation method. The results reveal that the students who have high communication ability can reach written communication indicators even though not all indicators are achieved correctly. The students who don’t have high communication ability face difficulty in expressing ideas in the form of mathematical symbols/notations so that they use meaningless notation. Meaningless notations are written based on the known information in the task and the results of students' thinking when they experience difficulties. These students also have not been able to use algebraic expressions and draw conclusions correctly. They directly write the results of the answers without the procedure. They state that concluding is unnecessary. Overall, the students who have high, medium, and low ability can draw Venn diagrams even though the diagram is improper.

1. Introduction
Communication is a way to convey information and understanding. Good communication must be posed by someone, so that misunderstanding does not occur in interpreting information [1]. Communication ability is important for students in facing industrial revolution 4.0 and educational challenges in the 21st century. Literature states that the quality of learning depends on the teachers’ tasks, whether the tasks demand students’ cognitive achievement or the tasks encourage them to learn independently [2]. The selection of learning objectives is very significant for successful communication in the classroom [3]. The role of teachers is needed to guide the students so they can communicate and convey their ideas. Besides, the teachers must deliver materials clearly so the students understand the material.

Mathematical communication is an important part of learning mathematics. It is an activity to understand mathematics. If students have good communication skills, they directly have a good mathematical understanding [4]. Students who have good communication skills will certainly have a deep understanding of mathematical concepts. Mathematical communication helps students in exploring and consolidating thoughts and knowledge related to mathematics in solving problems [5]. Mathematical communication is a way to convey and share ideas, and clarify an understanding [6]. Students must be free to explore speaking, writing and explaining their thoughts [7].
Mathematics education develops into a more mature field of study with more serious attention to theories, problems, concepts, and also communication [8]. In learning mathematics, students are expected to be able discuss in class and improve oral and written communication [9]. Students need to have mathematical communication ability to help them to capture, filter ideas, and clarify understanding. When students are trained and expected to communicate their ideas to others, learning becomes clear. The teacher not only provides information orally but also writing activities, then students will take notes. Writing in mathematics is part of communication. Writing can help students combine thoughts because it is a reflection of students’ work and an explanation of ideas developed in learning.

There are written and oral mathematical communication [10]. Writing is used as a communication tool that needs increased attention in mathematics classrooms [11], because writing can develop learning content and skills in dealing with students' problems [12]. Written communication is important for students because writing is a communication tool that reflects students' mathematical understanding. Writing can be considered an effective assessment used by teachers to find out students’ understanding [13]. Written communication helps students in the process of learning and understanding material, helps teachers to assess students’ knowledge and understanding. The important role of writing is as a link between the known and written information with the understood information. The mathematical written ability has a function as a learning process or problem-solving and reflects students’ various ideas in solving mathematical problems [14].

Mathematical writing is an activity that helps students to represent reasoning, analyze data, compare, differentiate problems and interpret. Students who have high written mathematical communication have more understanding of deep mathematical concepts than those who do not have these abilities. Mathematical writing can organize students' thinking; there by it helps students when they face difficult questions [15]. Therefore, the process of writing can encourage the students to form abstract into a concrete understanding. The focus of studying mathematics is to interpret concepts, graphs, mathematical objects, and understand mathematical concepts [16].

In the mathematics education literature, mathematical communication is closely related to the use of symbols, diagrams, graphs and others [17]. Based on research [18], it shows that difficulties in explaining ideas in algebraic form because they are too abstract, making communication difficult. Other research is related to the communication process that occurs in the classroom between the teacher and students or students and other students [19]. As a result, the form of mathematical communication both written or oral appears in various ways such as algebraic expressions, arithmetic or in drawing such as graphics, diagrams, and tables [20]. While research according [21], which analyzes mathematical communication at the student level in solving problems.

Besides, other researches have shown that students' mathematical communication ability in solving geometry problems are relatively low, especially in the written text indicator reaching 10.00% compared to mathematical expression, and drawing indicators reaching 13.33% and 23.33% [22]. There are other researches related to the mathematical communication ability, namely the effect of students' mathematical communication from the teachers’ point of view when teaching geometry material and the written ability of prospective mathematics teachers in mathematics learning which contains the way of proof [23]. Based on previous researches, the researchers do not explain written mathematical communication of students in solving problems deeply. Therefore, written communication in mathematics learning requires special attention to help students in understanding mathematical concepts and convey their ideas.
2. Method
The objective of the study is to describe and analyze students' written communication abilities viewed from each written communication indicator. This is a descriptive qualitative study, qualitative research is a method for exploring and understanding meaning by a number of individuals or groups [24]. This research was conducted in one of the junior high schools in Sragen regency with 32 students in seventh grade. The researchers employ purposive sampling in selecting the subject, where the chosen subject fulfills the adequacy and has the knowledge to solve the problem. The chosen subjects in this study are 6 students with good, medium, and low categories. Furthermore, in-depth interviews are conducted after the students answer questions in writing. The research instruments are mathematical communication tests in the form of structured questions and unstructured interviews. The essay task is arranged to measure the written mathematical communication ability of the subject. The research obtain data from students' answers in completing set tasks and then they are interviewed to clarify the written answers. To validate the data use triangulation method.

3. Result and Discussion
Mathematical communication can be in the form of writing and oral. In this study, the researchers focus on the written communication of students with 6 students. Written mathematical communication indicators are developed based on some criteria [19], namely (1) Using mathematical notations/symbols in stating situations, (2) Stating ideas through one's language (3) explaining ideas in writing through algebraic expressions and drawing conclusions, and (4) explaining ideas in the form of drawings and diagrams. Then, the researchers provide tests that consist of three essay tasks. The achievement of each written communication indicator that is converted into a percentage can be seen in Table 1.

| No | Indicator                                                                 | Percentage |
|----|---------------------------------------------------------------------------|------------|
| 1  | Using mathematical notations/symbols in stating the situation             | 30,00 %    |
| 2  | Expressing ideas using their language                                     | 32,50 %    |
| 3  | Explaining ideas in writing through algebraic operations and drawing conclusions | 37,50%   |
| 4  | Explaining ideas in the form of pictures and diagrams                     | 51,25 %    |

In detail, it shows that the indicator (1), Using mathematical notation/symbols in stating the situation is 30%. It means that the students are still low in using symbols properly. In indicator (2), expressing ideas using their own language/sentence is 32.25%. Indicator (3), explaining ideas in writing through algebraic operations and drawing conclusions only get a percentage of 37.50% and in indicators (4), explaining ideas in the form of pictures and diagrams that is equal to 51.25%. Furthermore, the indicators are seen in detail with a qualitative approach to compare and describe the students’ answers and it is explained in depth based on the results of students' answers and students' opinions in interviews. After that, the students fill the writing task and take interviews as supportive data.

3.1. The Answer to The Problems Number One
For the Problem number 1, the students are expected to solve story problems by mentioning and presenting the sets using the notation/symbol and giving arguments using their language concerning members of the sets.
Problem 1
On the admission of new students in Darul Falah Ihsan Junior High School, the distribution is based on the results of task 1 and task 2.

| Name   | Test I | Test II |
|--------|--------|---------|
| Ahmad  | Failed | Pass    |
| Zainal | Pass   | Pass    |
| Fikria | Pass   | Pass    |
| Saiful | Pass   | Failed  |
| Aryani | Failed | Pass    |
| Merlina| Pass   | Failed  |
| Anisya | Pass   | Pass    |
| Rahma  | Failed | Pass    |
| Nurul  | Pass   | Failed  |
| Ratna  | Failed | Pass    |

The students who pass the first test and second test will be placed in class VII-A, while the students who only pass the first test will be placed in class VII-B, and the students who only pass the second test will be placed in class VII-C. The results of the 15 student tests are shown in the following table.

If A is the sets of students who passed the first test and B is the set of students who passed the second test.

a. Determine the members of the set A and the members of the set B.
b. Make students’ data according to their class and explain.
c. Draw the Venn diagrams of sets A and B.

Figure 1. Answer of subject (FK)

Based on Figure 1, it shows that the student can present and register the set members of A and B appropriately. She can use mathematical symbols in presenting sets. She can form a set of A and B equipped with a notation in presenting the set correctly. Besides, she can explain the answer using their language that the VIIA set is a set that passes the results of tests 1 and test 2, the VIIB set is obtained from students who only passed the test 1, and the VIIC consists of students who only passed test 2. This shows that the student can use mathematical notations/symbols correctly and express arguments using their language.

Figure 2. Answer of subject (AB)
The student’s answer in Figure 2 shows that the student can mention the members of the set correctly and explain her arguments using her language in making the set A, set B, and set C. However, she does not use mathematical notations/symbols correctly in grouping members of the set and she does not write down arguments in her sentences to form groups of sets. She faces difficulty in presenting members using the parentheses symbol. The use of symbols aims to present the set correctly. She writes the members of the set by registering downward the members of the association with the use of meaningless symbols. The researchers suspect that the student’s answer is caused by the presentation of the set members by registering members downward which same as in the tasks that are presented in tabular form. The student has difficulty using mathematical symbols. This is in line with the statement of the student who says that "in my opinion, it is correct to present the set like this because I am confused in presenting the set members using the notation".

The following is a dialogue between the subject (AB) and the researcher (P) in an interview:

P : “What is the asked information for task number 1?”
AB : "Are they determining and presenting the set A and set B, mom?"
P : "Is it correct how to create an association and register members of such sets?"
RA : "In my opinion, it is correct to present the set like this, because I am confused in presenting the set members using notation. So I made it like the table in the task".

Based on the results of written tests and interviews, it shows that the (AB) subject does not understand how to present the set correctly and she does not use symbols to present the set. This case is indicated by the mistakes of the student in S1 and S2. In these answers, she uses meaningless notation, namely "{" and "," notation. She presents the members of the set by registering downwards. In number 1, she is expected to write down arguments to form a set, but she does not write down the reasons. The researchers suspect that the student does not pay attention to the questions in the task.

The results of the student’s written answers indicate that she has difficulty in using mathematical symbols correctly in forming the set and registering the members of the set. Writing mathematical notation/symbol is a statement of a situation or idea of thought in the form of mathematical language. Writing ideas into symbols is considered difficult for a student who has a low ability. Besides, writing symbols is difficult because it writes from what has thought abstractly in concrete form. The mathematical symbol is also an abstract thing. This discussion supports previous research which reports that not all students can express ideas using mathematical symbols [26]. Mathematical symbols are statements of situations or ideas of thought in the form of mathematical language [27]. Based on the explanation and student’s answer, it can be concluded that the student with subject (AB) cannot express ideas using mathematical symbols and she cannot explain ideas using her language through words.

3.2. The Answer to The Problems Number Two
In this task, the students are expected to be able to solve story problem related to the daily activities, explain ideas in written way through algebraic expression and drawing conclusion. This task is in line with written communication indicator of the students. The following are tasks and the aswers of the students with (MS) subject.

**Problem 2**

After recording 50 children about the types of popular sports, 32 children like volleyball, 40 children who like soccer, and 25 children who like both.

a. Can you present the data into a Venn diagram? If so, present it in the Venn diagram!

b. How many children do not like volleyball or soccer? Write down the working procedure!
a. Yes, I can

Like volleyball = 7
Like soccer = 15
Like both = (7 + 15) = 22
Children who do not like volleyball or soccer = 50 - (7 + 25 + 15) = 50 - 47 = 3
So, children who do not like volleyball or soccer is 3

Figure 3. Answer of subject (MS)

Writing and drawing are a way to convey ideas in solving problems correctly. Based on the results of students' answers in Figure 3, it shows that the student understands the purpose of the task to present the set in the form of a correct Venn diagram. She conveys ideas using algebraic expressions to determine that the students who like volleyball are 7, the students who like soccer are 15. This can be seen in the students' answers using a clear calculation by determining the students who like both are (7 + 15) = 22 and determining the students who are not found of volleyball and soccer are= 50 - (7 + 25 + 15) = 50 - 47 = 3. MS subject uses and conveys ideas through systematic algebraic calculation operations. In the picture, it can be seen that MS can conclude by using the conjunction "so" to clarify the answer that the students who don't like volleyball and soccer are 3.

Mathematical writing helps students to express their ideas into writing. Task 2 expects students to be able to use ideas through algebraic expressions to solve story problems and clarify answers by writing conclusions well.

b. Like soccer = 15
Like both = (7 + 15) = 22
Children who do not like volleyball or soccer = 50 - (7 + 25 + 15) = 50 - 47 = 3
So, children who do not like volleyball or soccer is 3

Figure 4. The answer of subject (NV)

The answer of the student who has low communication skills is shown in Figure 4. It reveals that the student does not use algebraic expressions in determining the asked information of the task and he cannot draw conclusions. This is seen in S3. The student does not know how to determine the rest who are not fond of volleyball and soccer. He tends to write directly to make a diagram of the known information on the problem.

The answer of the student who has low communication skills is shown in Figure 4. It reveals that the student does not use algebraic expressions in determining the asked information of the task and he cannot draw conclusions. This is seen in S3. The student does not know how to determine the rest who are not fond of volleyball and soccer. He tends to write directly to make a diagram of the known information on the problem.
To answer task 2, it requires written communication ability to use algebraic expressions systematically and to clarify answers to conclude. Concluding helps students in clarifying the written solution. NV subjects find it difficult to solve these problems. The following are excerpts from interviews with NV students to reinforce the answers that students cannot explain ideas through algebraic expressions and cannot conclude.

\[ P : \text{“How do you determine the number of students who only like volleyball and soccer?”} \]
\[ NV : \text{“I don’t know how to look for a lot of students who like to play volleyball and who only like football. I just write it down and present it in the Venn diagram?”} \]
\[ P : \text{”What conclusions are obtained to determine students who are not fond of both?”} \]
\[ NV : \text{“Yes, like this, so it’s simpler and it doesn’t need an explanation”}. \]

The results of the answers and interviews show that NV has difficulty in solving problems. The student cannot determine the number of students who love to play volleyball and soccer. The researchers know that the student feels confused in writing answers. This can be seen from the answers of 2b, the student only writes the number 20 without knowing where the answers were obtained. The Venn diagram drawn is wrong although the student has tried to convey their ideas through writing and drawing.

Writing conclusions requires verbal communication ability into the written text because writing text is a form of embodiment of communication in written form. Based on the explanation, it shows that NV has difficulty in explaining ideas through algebraic expressions and she cannot draw conclusions.

### 3.3. The Answer to The Problems Number Three

In problem number 3, the students are expected to solve problems related to intersection and register the members of the sets and present them in the form of a Venn diagram. When registering a member of the set, it is necessary to use good symbols and the students are also instructed to present mathematical ideas in the form of pictures or Venn diagrams. The following are the task and the students’ answers with DA and SS subjects.

#### Problem 3
Is known :
- \( A = \{\text{natural number less than 20}\} \)
- \( B = \{\text{natural number even less than 15}\} \)
- \( C = \{\text{odd natural number less than 10}\} \)
- \( D = \{\text{natural number more than 7 and less than 15}\} \)

a. Determine the members of the set \( A, B, C \) and \( D \)

b. Assign members from \( B \cap C, B \cap D, \) and \( C \cap D \)

c. Draw the Venn diagram!

\[ a. \text{ Set } A, B, C \text{ and } D \]
\[ A = \{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19\} \]
\[ B = \{2,4,6,8,10,12,14\} \]
\[ C = \{1,3,5,7,9\} \]
\[ D = \{8,9,10,11,12,13,14\} \]
b. \[ B \cap C = \{ \} \]
\[ B \cap D = \{ 8, 10, 12, 14 \} \]
\[ B \cap C = \{ 9 \} \]

c. Diagram venn

\[ B \cap C = \{ \} \]
\[ B \cap D \]
\[ B \cap C = \{ 9 \} \]

**Figure 5.** The answer of subject (NV)

The students' answer in Figure 5 shows that the students try to show the relationship of intersection by presenting it in the form of a Venn diagram of \( B \cap C = \{ \} \), \( B \cap D = \{ 8, 10, 12, 14 \} \), and \( B \cap C = \{ 9 \} \). Besides that, the students can use mathematical symbols/notations to register the members of the sets even though they are not completely perfect. This student is still having difficulty in presenting data into the Venn diagram. On the error shown in S3 that the students try to present the intersection of sets B and D. The next error is on S5, when the students describe the intersection of set C and D. At slice \( B \cap C = \{ \} \), the students draw with squares and blanks. The purpose of this task is that students are expected to be able to present a Venn diagram in one intersection of \( B \cap C \), \( B \cap D \), and \( B \cap C \) so that they are not separated one by one. The students find it difficult to present pictures, this is supported by students' statements stating that "I am confused to make a Venn diagram, for drawings B and C are empty sets, so I just draw an empty square, and this second picture is wrong, I mean intersection of B and D and finally intersection of C and D ".

Based on the answers and statements of students, it can be concluded that the students try to use notation to form sets and draw Venn diagrams. Using mathematical notations/symbols and drawing Venn diagrams are indicators of written mathematical communication.

\[ A = \{ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19 \} \]
\[ B = \{ 2, 4, 6, 8, 10, 12, 14 \} \]
\[ C = \{ 1, 3, 5, 7, 9 \} \]
\[ D = \{ 8, 9, 10, 11, 12, 13, 14 \} \]
\[ B \cap C = \{ \} \]
\[ B \cap D = \{ 8, 10, 12, 14 \} \]
\[ B \cap C = \{ 9 \} \]

**Figure 6.** The answer of subject (PN)
Based on Figure 8, the students can register the members of the sets, and explain the relationship of the intersection correctly. But the students cannot present the intersection of $B \cap C = \{ \}$, $B \cap D = \{ 8, 10, 12, 14 \}$ and $B \cap C = \{ 9 \}$. In the picture, it is seen that the students draw a Venn diagram but with the intention that the set $B = 7, C = 5$ and $D = 7$. The students try to present the Venn diagram into one without being separated but in drawing it is not quite right. The students find it difficult to draw Venn diagrams. The following excerpts from interviews with PN students to reinforce the answers that the students cannot present Venn diagrams.

\[
P : \ "Do you think this task is difficult? And what is the asked information in this matter? "
PN : \ "In my opinion, it is difficult when drawing data into Venn diagrams. This asked information is to register the members of the set and draw data into the Venn diagram about the intersection"
\[
P : \ "How can you draw this Venn diagram?"
PN : \ "I was confused to draw a Venn diagram, therefore I only drew a Venn diagram for $A = 20, B = 7$ was obtained from the members of the set"
\]

Both students' answers with both DA and PN subjects have difficulty in drawing Venn diagrams. They draw Venn diagrams without knowing the purpose. Drawing Venn diagrams is very important to clarify student ideas. Drawing is a form of students' written communication and mathematical ideas, such as drawing diagrams and graphs. This study supports the existence of previous studies [20] that some students cannot express mathematical ideas through pictures and diagrams. Based on the explanation, DA and PN students have not been able to present the relationship of intersection into Venn diagrams and they simply draw Venn diagrams as usual without knowing clear objectives.

4. Conclusion

Written communication is important for students because writing is a communication tool that reflects students’ understanding to express ideas. Mathematical communication is closely related to the use of symbols, diagrams, graphs, and others. This study aims to describe and analyze students' communication ability in writing so that it becomes a serious concern for improving students' written communication ability that has not been fully achieved. Students’ answers and interview results are validated using triangulation methods. The results reveal that the students who have high communication skills can achieve written communication indicators even though not overall indicators are achieved correctly. The students who do not have a high ability to face difficulty in expressing ideas in the form of mathematical symbols/notations so that they use meaningless notation. Meaningful notations are written based on the task and the results of students' thinking when they are experiencing difficulties. These students also have not been able to use algebraic expressions, and conclude correctly. They directly write the results of the answers without procedure and state if there is no need to conclude. Overall, students who have high, medium, and low abilities can draw Venn diagrams even though the diagram improper. They just draw a Venn diagram. For further researchers, it is expected to be able to contribute and provide solutions so that students can have written mathematical communication abilities as well as developing learning tools. Teachers are expected to develop learning models that can improve students' mathematical communication ability.

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