Supplementary Materials for

100,000-spin coherent Ising machine

Toshimori Honjo*, Tomohiro Sonobe, Kensuke Inaba, Takahiro Inagaki, Takuya Ikuta, Yasuhiro Yamada, Takushi Kazama, Koji Enbutsu, Takeshi Umeki, Ryoichi Kasahara, Ken-ichi Kawarabayashi, Hiroki Takesue*

*Corresponding author. Email: toshimori.honjo.ht@hco.ntt.co.jp (T.H.); hiroki.takesue.km@hco.ntt.co.jp (H.T.)

Published 29 September 2021, Sci. Adv. 7, eabhl0952 (2021)
DOI: 10.1126/sciadv.abh0952

This PDF file includes:

Sections S1 to S10
Figs. S1 to S6
Tables S1 and S2
References
S1 Details of experimental setup

Figure S1 shows the detailed setup of the coherent Ising machine (CIM). A continuous-wave (CW) laser light at a wavelength of 1558.998 nm is split into two lights using a coupler, and a portion of the coupler output is amplified by an erbium-doped fiber amplifier (EDFA3) and input to a balanced homodyne detector (BHD) as a local oscillator light. The other portion of the CW light is modulated into a pulse train whose pulse width and repetitive frequency are 30 ps and 5 GHz, respectively, using an optical intensity modulator (IM1). The pulse train is then amplified by EDFA1 and separated into two paths with a coupler, where one of the outputs is used as injection pulses that convey the feedback signal from a field-programmable gate array (FPGA) system (see the main text). The other portion is input into IM2, with which we control the pump amplitude for the signal and training pulses described in the main text. The pulses output from IM2 are again amplified, this time by EDFA2, and launched into a periodically poled lithium niobate (PPLN) waveguide module, where a 779.5-nm pulse train is generated via a second-harmonic generation (SHG). The SHG pulses are then input into another PPLN waveguide module that works as a phase-sensitive amplifier (PSA) in a fiber cavity. The fiber cavity also contains two 90:10 couplers, an optical bandpass filter (BPF), a 5-km polarization maintaining fiber (PMF) spool, and a piezo-based fiber strecher for stabilizing the cavity. The cavity round-trip time is 24.7 $\mu$s. For long-term stability of CIM operation, the 5-km PMF spool is put inside a thermally insulated box, in which the temperature is stabilized within $\pm 0.05$ °C using a Peltier temperature controller (explained in detail in Section S3). In the fiber cavity, the squeezed vacuum pulses generated in the PSA undergo repetitive phase sensitive amplification by synchronizing the pump pulses with the pulses circulating in the cavity, and consequently, we obtain degenerate optical parametric oscillator (DOPO) pulses. As stated in the main text, we undertake measurement feedback (MFB) while DOPO pulse amplitude grows from a vacuum.
The estimated power consumption of the system is approximately 7 kW, of which \( \sim 6 \) kW is consumed by the FPGA system in the MFB. The details of the FPGA system are described in Section S5.

Fig. S 1: Experimental setup. The FPGA system includes 56 FPGAs in total (the details are explained in Section S5).
S2 Improvement of PPLN waveguide module for PSA

In our previous CIM reported in (18), we used a 1-km fiber cavity with 1-GHz repetition pump pulses, while the present CIM uses a 5-km fiber cavity and 5-GHz repetition pump pulses. This means that the cavity loss increases due to the excess loss of the fiber, while pump peak power decreases to one-fifth of that in the previous experiments. Accordingly, we need to increase the efficiency of the nonlinear interaction in the PPLN waveguide. To obtain high nonlinearity, we improved the PPLN waveguide fabrication process based on high-resolution photolithography and dry etching techniques (55). The use of the etching process enabled us to precisely control the waveguide core width along the waveguide axis, resulting in a significant reduction of waveguide loss. We also redesigned the fiber coupling modules to improve the coupling efficiency between the fiber and PPLN waveguide, while increasing the threshold pump power that damages the modules. The new PPLN fiber module with the improved waveguide exhibited a second-harmonic-generation efficiency of 600%/W for the module, which is about three times larger than the one used in our previous experiments (18). Together with the increased tolerable input pump power owing to the improvement of the fiber coupling, we can generate DOPO pulses at a 5-GHz repetition frequency with a 5-km fiber cavity.
S3  Stabilization of 5-km fiber bobbin

![Fig. S 2: Histogram of continuous oscillation time of DOPO with (orange) and without temperature control (blue) of 5-km fiber bobbin](image)

Although short-term stability of the cavity has been achieved by using a phase-locked loop circuit based on a dither-and-lock scheme \( (56) \), slow changes in the fiber delay time caused by temperature fluctuations in the laboratory severely alter the oscillation condition of the DOPOs. Since the temperature coefficient of the group index of silica is on the order of \( 10^{-5} \), a 1 degree change in temperature induces a change in group delay as large as \(~250\) ps in a 5-km fiber. Although we can compensate for the delay change by adjusting the repetition frequency of the pump pulses or by changing the optical delay line inserted into the cavity, such changes in the physical parameters result in the large change in oscillation condition of the DOPOs, even with fixed operational parameters (pump amplitude, pump turn-on schedule, coupling coefficient etc.). To suppress the long-term instability caused by the temperature sensitivity of optical fiber, we developed a compact system that can stabilize the temperature of spooled fiber very
precisely. In this system, a 5-km fiber bobbin (diameter: 25 cm) is placed in a round container with a thick hollow wall. The wall is filled with water, which works to stabilize the temperature in the container. Several thermistors and Peltier devices are attached on the outside of the container. By controlling the temperature of the water inside the container wall, we can stabilize the temperature of the fiber bobbin in the container. With this system, we were able to stabilize the temperature of the entire fiber spool to within $\pm 0.05 \, ^\circ C$. The container is surrounded by Styrofoam material, and the whole system is placed inside a compact aluminum box. The temperature-controlled water container covered by Styrofoam is also effective for suppressing environmental acoustic noise. As a result, we were able to maintain operation of the CIM with the 5-km fiber cavity for a long time without having to adjust any operational parameters.

We performed a measurement to examine the effectiveness of the temperature control system. We generated DOPO pulses without applying the pump turn-on schedule to pump pulses. To concentrate on the evaluation of the stability of the cavity, we did not inject any feedback pulses to the DOPO pulses. The phase of the cavity was locked by using a dither-and-lock scheme, and the repetition frequency of the pump pulses was occasionally optimized to maximize the DOPO power. We measured the temporal change of the DOPO power for 4 s with 1-ms temporal resolution using a photodetector followed by a digital oscilloscope. We repeated this measurement 25 times for the DOPO pulses with and without the temperature control system, which means the total measurement time was 100 s. When we did not use the system, the 5-km fiber bobbin was covered with urethane foam and placed in a Styrofoam box, and the box was placed on an optical table with a vibration isolation system. We considered that the DOPOs were oscillating if the power level was larger than 50% of the peak value in each 4-s measurement, and evaluated how long the DOPO continuously oscillated. The histograms of continuous oscillation time are shown in Fig. S2, which indicate that the oscillation times were significantly improved with the temperature control. The maximum continuous oscillation
times were 282 ms and 4 s (which were limited by the span of the measurement) for the setup without and with the temperature control, respectively. The average continuous oscillation time was 14 and 362 ms for the setup without and with the temperature control, respectively. Thus, the introduction of the temperature control system for the fiber was one of the crucial factors enabling us to realize the 100,000-spin CIM.
S4 Pump turn-on schedule

Figure 1B in the main text shows the schedule of the normalized pump amplitude $p$, which is defined as pump amplitude normalized by the pump threshold of a single isolated DOPO pulse. Three pump turn-on schedules were used in our experiments. Schedule 1 in Fig. 1B is the schedule designed to minimize the time to reach the reference score. To increase the amplitude of DOPOs rapidly, we started at a pump amplitude with a relatively large value (0.5) and then increased it by 0.1 every 100 circulations until it reached 1.0 at the 500th circulation. After the 600th circulation, the pump amplitude was set at zero for 400 circulations to eliminate the DOPO lights in the cavity. As explained in the main text, the score reached the SG score after the first several tens of circulations in the experiments. Schedule 2 and 3 in Fig. 1B show the two schedules for obtaining scores with better accuracy. To obtain better scores, we increased $p$ more slowly than for Schedule 1 by increasing the number of circulations to 900 in both schedules. In Schedule 2, we started with $p = 0.15$, and gradually increased $p$ until it reached 0.9 at the 800th circulation. Then, we set $p$ to 0 at the 900th circulation and kept this value for 100 circulations. With this schedule, the DOPO amplitudes increased gradually, and they saturated with high probability because of the sudden pump increase in the last 200 circulations. On the other hand, Schedule 3 is intended to operate the DOPOs at closer to the oscillation threshold. Here, $p$ was increased from 0.1 to 0.3 during 900 pulse circulations, leaving most of the DOPO amplitudes unsaturated for the whole operation. The final DOPO amplitudes were read out at the 890th circulation for evaluation of cut values. Intuitively, the pump for the DOPO stays at around the oscillation threshold longer with Schedule 3. Consequently, the DOPO pulses undergo fluctuations at the critical point longer than with Schedule 2, resulting in the larger relative standard deviation (RSD) of the DOPO pulses’ absolute amplitudes mentioned in Section S9.
One of the key elements for realizing the 100,512-spin CIM is an FPGA system for the MFB. To implement MFB for each circulation, the FPGA system needs to complete a multiplication of a $100,512 \times 100,512$ matrix and a 100,512-element vector within the round-trip time in the 5-km fiber cavity (25 ms). This means that it should possess the calculation performance of 367.5 tera multiply accumulate operations (MAC) per second. High memory bandwidth is also required. Even though the values of the elements in the $J_{ij}$ matrix are limited to $\{-1, 0, 1\}$, which can be expressed with 2 bits in the current implementation, the total size of the $J_{ij}$ matrix is as large as 2.53 giga bytes. All elements of the $J_{ij}$ matrix have to be readout every circulation, which corresponds to 91.9 tera-bytes per second. To satisfy the above requirements, we designed and implemented a custom FPGA system. We used state-of-the-art commercial FPGAs (Xilinx UltraScale+ XCVU13P-2FHGB2104I). We implemented a custom board on which two FPGAs can be installed. We fabricated 28 FPGA boards, each of which had 64 high-speed communication channels (12.5Gbps $\times$ 64). Among the boards, one was used to accommodate an analog-digital converter (ADC) and a digital-analog converter (DAC), and the others were used to calculate the feedback signals. All the FPGA boards were cascaded with the high speed communication channels. The analog output from a balanced homodyne detector was digitized by a 5.0 GSPS ADC (Abaco Systems FMC126), and the digitized signal was sent to the following FPGA boards for calculation of the feedback signals, where in total 54 FPGAs were accommodated and serially connected. Each FPGA undertook the multiplication of a $1,872 \times 100,512$ sub-matrix and a 1,872-element vector, and the calculation result together with the results obtained by the preceding FPGAs were sent to the next FPGA. This procedure was repeated in the 53 FPGAs, and the 54th FPGA undertook the remaining $1,296 \times 100,512$ sub-matrix calculation. To achieve high memory bandwidth to retrieve the $J_{ij}$ elements, a
Xilinx UltraRAM, a large and lightweight memory block, was used. Each FPGA had 45Mbyte UltraRAM, which was enough to store $J_{ij}$ elements for a $1,872 \times 100,512$ matrix. In addition, 512MB DDR3 SDRAM was installed on each board to record the data from the ADC. The final calculation results were transferred to another FPGA where the feedback signals were created using the results from the submatrix calculations performed by the previous FPGAs, and the signal was converted into analog signal by a 12-bit DAC (Euvis FMC1662) operated at 5.0 GSPS. The total time from the ADC to DAC was less than 25 microseconds. A gigabit Ethernet interface on each FPGA board was mainly used to set $J_{ij}$ elements and read the recorded data via a PC.
S6  Maximum cut problem

We used a maximum cut (MAX CUT) problem to benchmark the performance of the CIM. The objective of MAX CUT is to divide a set of nodes of a graph into two subsets, such that the sum of the weights having one endpoint in each is maximized. Here, a cut is a partition of the vertices \( V \) into two disjoint subsets \( \{S_1, S_2\} \) and the size of the cut is the total weight of edges \( w_{ij} \) with one vertex \( i \) in \( S_1 \) and the other \( j \) in \( S_2 \). The size of the cut can be counted by assigning the binary spin values to express which subset the vertex \( i \) belongs to \( \sigma_i \in \pm 1 \):

\[
C(\sigma) = \sum_{i \in S_1, j \in S_j} w_{ij} = \sum_{i<j} w_{ij} (1 - \sigma_i \sigma_j) = \frac{1}{2} \left( \sum_{i<j} w_{ij} - H(\sigma) \right),
\]

where \( H \) is an Ising Hamiltonian defined in Eq. (1) in the main text with \( J_{ij} = -w_{ij} \). The above equation indicates that the MAX CUT problem is equivalent to the Ising problem except for the constant factor. A MAX CUT problem can be easily converted into the ground state search of the corresponding Ising model that the CIM can directly treat.
S7 Graph generation

The graphs we used in the present experiment were generated using a general graph generator called Rudy (35). Rudy is a machine-independent graph generator invented by Giovanni Rinaldi. To generate a K100000 graph, which is a fully connected 100,000-node graph with 9,999,900,000 undirected edges, which is randomly weighted by \{-1, +1\}, we used the following options.

```
rudy -clique <size> -random 0 1 55555 -times 2 -plus -1
```

The clique option specifies the generation of a complete graph of \(<size>\) node. 55555 is a seed for random number generation. The random option specifies the weight of 0 or 1. With the last two options, weight is converted from \{0,1\} to \{-1, +1\}.

100,000-node graphs with densities of 1, 10, 50, and 100% were generated as follows. First, we generated a fully connected 100,000-node graph with 9,999,900,000 undirected edges, which was randomly weighted between -100000 and +100000 by Rudy using the following options.

```
rudy -clique 100000 -random 0 200000 55555 -plus -100000
```

Depending on the density, the weight was converted to \{-1, 0, +1\} based on the following rule: when the absolute value of the weight was larger than \((100 - density) \times 1000\), the weight was converted to \(sign(weight)\). Otherwise it was set to 0.
S8 Additional information for “Scaling in problem size”

Figure S3 shows the score trajectories obtained in the experiments described in the main text, where top 10 results in 500 CIM runs and the average score trajectories are plotted. The average times to reach the SG scores (red dotted lines) obtained from this data are divided by the number of embeddings for each graph to obtain the effective time to reach the SG scores shown in Fig. 4 in the main text.

We estimated the number of circulations to reach the SG score for K100000 using c-number stochastic differential equations (CSDE) based on the truncated Wigner approach (40) implemented on an Intel(R) Xeon(R) CPU E5-2697 v2 (12 cores, 24 threads, 2.70 GHz). Assuming complete stability of all the optical parts, we ran the simulations 50 times for solving MAX CUT of K100000. The obtained evolutions of the cuts are shown in Fig. S4. Here, we assumed that the pump amplitude rapidly increased in the first 50 circulations to saturate the DOPO as quickly as possible. The average number of circulations to reach the SG score was 24.3 ± 0.6, indicating that the time to reach the reference score can be well stabilized if we can realize a stable optical setup. The average CSDE computation time to obtain the SG score was 861 ± 22 [s]. However, in the real experiments, the number of circulations where the DOPO amplitudes started to rise tended to fluctuate because of experimental noise in the optical setup. Since the number of circulations to reach the reference score was as small as 20 to 30, such a fluctuation would result in larger variance in the CIM computation time. Therefore, to show the potential of the computation speed of the CIM, we took the data where the amplitude started to rise immediately after turning on the pump.

According to a numerical calculation in our previous work (21), the average time to obtain the exact solutions for MAX CUT problems of relatively small graphs (up to 200) was proportional to exp(O(N^{1/2})), when the injection amplitudes were optimized for each size. In the
experiment described in the main text, we tried to optimize the injection signal amplitude to decrease the time to reach the SG score for each graph, but did not observe significant changes.

Consequently, the number of circulations (or computational steps) to reach the reference score was nearly unchanged when we changed the problem size from 1,000 to as many as 100,000, resulting in an increased gap between the CIM and SA in computation time. We also analyzed our previous data using the 2,000-node CIM presented in (18), where we also put a significant effort into minimizing the computation time, and found the number of circulations to reach the SG score (30094) was 20, which nearly coincided with the number of circulations obtained in the present experiment. In (18), we used the score obtained with the semidefinite programming relaxation algorithm proposed by Goemans and Williamson (GW-SDP) as a reference (29619).
Fig. S 3: MAX CUT score as a function of computation time for graphs with (a) 1000, (b) 10,000, (c) 50,000 and (d) 100,000 nodes obtained in the experiment described in the main text. The thin lines correspond to the temporal change of scores obtained in the top 10 results in 500 runs, while the red thick lines show the average score trajectories. The red dotted lines correspond to the reference scores obtained with SG.
Fig. S 4: MAX CUT scores of K100000 graph as a function of the number of circulations obtained by numerical simulation using CSDEs based on the truncated Wigner approach. Here, the results of 50 simulations are overlaid. The dotted line indicates the score obtained by SG (10,759,955).
S9 Additional information for “Comparison of solution accuracy”

We describe the details of the filtering methods that we used in the experiment and investigation of DOPO amplitude mentioned in the main text.

Filtering methods

Despite the use of a large-scale fiber cavity that is vulnerable to various noises in the environment, the current CIM experimental setup can be stably operated for more than a week. However, there are still two factors that occasionally cause erroneous results in CIM computation. In our evaluation of the CIM, we eliminated the obviously erroneous results to evaluate the CIM’s potential. Here, we describe the details of the two filtering methods.

Phase check filter

Although the 5-km fiber cavity was well stabilized as explained above (Fig. S1), optical phases of injection pulses were not fully locked for a long-time measurement. A phase flip of the injection pulse ended up in solving a MAX CUT of a different graph where the signs of the edges were inverted. The phase check filter is intended to eliminate the results obtained with phase-flipped injection pulses. For this purpose, we implemented a 32-bit bipartite graph in the redundant 512 pulses in the signal pulses as phase-check bits (21). If the optical phase of an injection pulse is incorrect, we obtain a negative cut value from the phase-check bits. So by monitoring the sign of the cut value of the phase-check bits, we can extract the results presumably with the correct optical phase.
**Correlation filter**

As explained in Section S4, we turned off the pump for several hundred DOPO pulse circulations to eliminate the DOPO light so that we could start next computation with random phase states. However, when we checked the correlation between spin sets obtained in the two consecutive runs, although in most cases we observed only a small correlation ($< 0.1$), we sometimes observed a correlation close to 0.5, which indicates that the pump turn-off was insufficient and part of information of previous spin set survived. Such an imperfect pump turn-off may have occurred as a result of fluctuation of the DC bias applied to IM2 to modulate the pump pulses. With the correlation filter, we eliminated the results that showed a correlation of $\geq 0.1$ to obtain the spin sets where the information on the previous sets were considerably suppressed.

Figure S5 is the histogram of the spin correlation for the data shown in Fig. 6 in the main text. In obtaining the data in Fig. 6, we removed the data appearing to the right of the dotted line with the correlation filter. The rates that the phase-checked data were filtered out by the correlation filter was 0.3% for Schedule 2 and 8.0% for Schedule 3.

The overall failure rates in obtaining the data shown in Fig. 6, which we defined as the probability that a result would be removed with the phase and correlation filters, were 0.28 and 0.55 for Schedule 2 and 3, respectively.
Fig. S 5: Histogram of spin correlation between the two consecutive CIM runs for the data shown in Fig. 6 in the main text. The phase check filter had already been applied beforehand. The dotted line corresponds to 0.1 correlation, which is the threshold value used for filtering the data.
Relative standard deviation of DOPO amplitudes

To investigate the origin of the difference between Schedule 2 and 3, we plot the histograms of the relative standard deviation (RSD, denoted as $R$ hereafter) of the absolute amplitude in Fig. 6 in the main text. $R$ is defined as the standard deviation divided by the mean. A small $R (~0)$ means that the pulse amplitudes are nearly uniform as a result of amplitude saturation, while a large $R (\gtrsim 1)$ indicates that they have a large variation and the oscillation is unstable. According to Fig. S6, most of the data obtained with Schedule 2 had $R$ smaller than 0.3, while $R$ for the data with Schedule 3 was mostly larger than 0.3, which means that a slower increase in pump amplitude gave rise to larger variation in the DOPO amplitudes.

![Fig. S6: Histogram of relative standard deviation values of DOPO amplitudes for the data shown in Fig. 6. The blue and orange columns represent the data for Schedule 2 and 3, respectively.](image)
S10 Implementation of SA

Here, we describe the details of our SA implementation. The implementation used in this paper is based on our previous work (18). In the current CIM experimental setup, $J_{ij}$ elements are limited to $\{-1, 0, 1\}$, and we used the complete graph for our evaluation. For a fair comparison, the implementation of SA should be also optimized for the complete graph weighted by $\{ -1, +1 \}$. Thanks to this weight limitation, we can represent a graph in a binary format, which allows us to apply swift bitwise operation in many SA procedures. Moreover, we can use single instruction multiple data (SIMD) operation (popcnt) to accelerate the calculation of the energy difference.

Compared with the original implementation, the sequence of the spin selection for the next flip was modified. Although the spin to be flipped was randomly chosen in the original implementation, to efficiently access the next spin, the spin in the present implementation is sequentially selected, i.e., the $(i + 1)$-th spin is chosen after the $i$-th spin, which can slightly improve the overall performance.

The scheduling is based on the following logarithmic function, which was also used in our previous work (18).

\[
\beta = \beta_0 \log(1 + t/T)
\]  

(1)

where $\beta_0$ and $T$ are the temperature and a time scaling factor, respectively. The next spin-flipped Ising state is accepted when energy difference $\Delta E$ is less than 0 or $\exp(-\beta \Delta E)$ is more than a random value sampled between 0 and 1.

$\beta_0$ and $T$ were optimized for each evaluation mentioned in the main text. To optimize these parameters for a shorter computation time or better solution accuracy, $\beta_0$ and $T$ were scanned from 5 to 100 by 5 and from 1,000 to 15,000 by 1,000, respectively.

Table S1 shows the parameters we used to evaluate the time to reach the SG scores shown
in Fig.3,4 and 5. Table S2 shows the parameters we used for the evaluation of solution accuracy shown in Fig.6. In addition, we compared the scheduling based on a logarithmic function with that based on a linear or exponential function (57). We confirmed that logarithmic function was the best to achieve the short calculation time for the fully connected 100,000-node graph.

| Graph size | $\beta_0$ | $T$  |
|------------|-----------|------|
| 1000       | 95        | 3000 |
| 10000      | 90        | 7000 |
| 50000      | 15        | 3000 |
| 100000     | 40        | 10000|

Table S1: $\beta_0$ and $T$ for the computation time.

| Time (ms)  | $\beta_0$ | $T$  |
|------------|-----------|------|
| 1000       | 30        | 14000|
| 500        | 65        | 11000|

Table S2: $\beta_0$ and $T$ for the solution accuracy.

The code was implemented in C++ language and compiled by a GNU C++ compiler (version 7.4.0) with ”-Ofast” optimization on Ubuntu 18.04. We also applied multithreading capabilities with OpenMP to improve the calculation time. In the SA, the initial energy calculation part and SA part can be parallelized, so we investigated whether the multithreading is useful for reducing the computation time or not. For the initial energy calculation part, by simply splitting the energy calculation into multiple threads, we successfully improved the calculation time. The optimized number of threads for K100000 was eight under our current environment. For the multithreaded SA part, each thread conducts the SA procedure independently, and the best solution is periodically shared between the threads. Unfortunately, due to the communication and synchronization costs, we did not observe any improvement in the SA part with the use of multithreading. From this investigation, we decided to apply multithreading only for the initial
energy calculation part in the SA evaluation in the main text.
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