V$_{cs}$ from Pure Leptonic Decays of $D_s$ with Radiative Corrections

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Abstract

The radiative corrections to the pure leptonic decay $D_s^{-}\rightarrow\ell\nu\ell$ up to one-loop order is presented. We find the virtual photon loop corrections to $D_s^{-}\rightarrow\tau\nu\tau$ is negative and the corresponding branching ratio is larger than $3.51\times10^{-3}$. Considering the possible experimental resolutions, our prediction of the radiative decay $D_s^{-}\rightarrow\tau\nu\tau\gamma$ is not so large as others, and the best radiative channel to determine the $V_{cs}$ or $f_{D_s}$ is $D_s^{-}\rightarrow\mu\nu\mu\gamma$.

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The pure-leptonic decay $D_s^{-}\rightarrow\ell\nu\ell$ can be used to determine the decay constant $f_{D_s}$ if the fundamental Cabibbo-Kobayashi-Maskawa matrix element $V_{cs}$ of Standard Model (SM) is known. Conversely if we know the value of decay constant $f_{D_s}$ from other method$^{[1,2,3]}$, these process also can be used to extract the matrix element $V_{cs}$. But there are the well known effect of helicity suppression we can see it by factor of $m_{\ell}^2/m_{D_s}^2$: \[ \Gamma(D_s^{-}\rightarrow\ell\nu\ell) = \frac{G_F^2}{8\pi} |V_{cs}|^2 f_{D_s}^2 m_{D_s}^3 \frac{m_{\ell}^2}{m_{D_s}^2} \left( 1 - \frac{m_{\ell}^2}{m_{D_s}^2} \right)^2, \] (1)

Of them only the process $D_s^{-}\rightarrow\tau\nu\tau$ is special, it does not suffer so much from the helicity suppression, and its branching ratio may reach to $4.5\%$ in SM. However the produced $\tau$ will decay promptly and one more neutrino is generated in the cascade decay at least, thus it makes the decay channel difficult to be observed. For the channels $D_s^{-}\rightarrow\ell\nu\ell$ and $D_s^{-}\rightarrow\mu\nu\mu$, besides the small branching ratios, there are only one detected final state, the measurement of such channels are very difficult.

Fortunately, having an extra real photon emitted in the leptonic decays, the radiative pure leptonic decays can escape from the suppression$^{[4,5,6]}$, furthermore, as pointed out in Ref.$^7$, with the extra photon to identify the decaying pseudoscalar meson $D_s$ in experiment from the backgrounds has advantages, since one more particle can be detected in the detector. Although
the radiative corrections are suppressed by an extra electromagnetic coupling constant \( \alpha \), it will not be suppressed by the helicity suppression. Therefore, the radiative decay may be comparable, even larger than the corresponding pure leptonic decays [4, 5, 6].

The radiative pure leptonic decays, theoretically, have infrared divergences and will be canceled with those from loop corrections of the pure leptonic decays. In all the existing calculation of radiative decays [4, 5, 6], this part is ignored, since they do not include the radiative corrections of the pure leptonic decays. In this paper, we are interested in considering the radiative decays\([4, 5, 6]\), this part is ignored, since they do not include the radiative corrections of the pure leptonic decays. Since the process \( D_s \to \tau \nu \) do not suffer the helicity suppression and has a large branching ratio, the corresponding loop correction (virtual photon) to these process should has a considerable larger branching ratio, at least comparing with the radiative decay, and can not be ignored.

The contributions of the radiative decays are corresponding to the four diagrams in Fig. 1. According to the constituent quark model which is formulated by Bethe-Salpeter (B.-S.) equation, the amplitude turns out to be the four terms \( M_i (i = 1, 2, 3, 4)\):

\[
M_1 = \text{Tr} \left[ \int \frac{d^4q}{(2\pi)^4} \chi(p, q) i \left( \frac{G_F m_{w}^2}{\sqrt{2}} \right)^{\frac{3}{2}} \gamma_{\mu} (1 - \gamma_5) V_{cs} \right] \times \frac{i \left(-g^{\mu \nu} + \frac{p^{\nu} p^{\nu}}{m_w^2}\right)}{p^2 - m_{w}^2} \chi, 
\]

\[
M_2 = \text{Tr} \left[ \int \frac{d^4q}{(2\pi)^4} \chi(p, q) i \left( \frac{G_F m_{w}^2}{\sqrt{2}} \right)^{\frac{3}{2}} \gamma_{\mu} (1 - \gamma_5) V_{cs} \right] \frac{i \left(-g^{\mu \sigma} + \frac{(p-k)^{\sigma} (p-k)^{\nu}}{m_w^2}\right)}{(p-k)^2 - m_{w}^2} \frac{ig}{2\sqrt{2}} \gamma_{\sigma} (1 - \gamma_5) \nu_{\ell},
\]

\[
M_3 = \text{Tr} \left[ \int \frac{d^4q}{(2\pi)^4} \chi(p, q) i \left( \frac{G_F m_{w}^2}{\sqrt{2}} \right)^{\frac{3}{2}} \gamma_{\mu} (1 - \gamma_5) V_{cs} \right] \frac{i \left(-g^{\mu \sigma} + \frac{(p-k)^{\sigma} (p-k)^{\nu}}{m_w^2}\right)}{(p-k)^2 - m_{w}^2} \frac{ig}{2\sqrt{2}} \gamma_{\sigma} (1 - \gamma_5) \nu_{\ell},
\]

\[
M_4 = \text{Tr} \left[ \int \frac{d^4q}{(2\pi)^4} \chi(p, q) \left( \frac{2e}{3} \right)^{\phi} \frac{ig}{p - q - k - m_s} \left( \frac{G_F m_{w}^2}{\sqrt{2}} \right)^{\frac{3}{2}} \gamma_{\mu} (1 - \gamma_5) V_{cs} \right] \times \frac{i \left(-g^{\mu \sigma} + \frac{(p-k)^{\sigma} (p-k)^{\nu}}{m_w^2}\right)}{(p-k)^2 - m_{w}^2} \frac{ig}{2\sqrt{2}} \gamma_{\sigma} (1 - \gamma_5) \nu_{\ell},
\]

where \( \chi(p, q) \) is Bethe-Salpeter wave function of the meson \( D_s \); \( p \) is the momentum of \( D_s \); \( \epsilon, k \) are the polarization vector and momentum of the emitted photon.
As the $D_s$ meson is a nonrelativistic S-wave bound state in nature, higher order relativistic corrections may be small, being the first order approximation for a S-wave state, we ignore $q$ dependence in amplitude and in the $D_s$ wave-function and write the wave function of the meson $D_s$ as:

$$\int \frac{d^4q}{(2\pi)^4} \chi(p, q) = \frac{\gamma_5 (p+m)}{2\sqrt{m}} \psi(0).$$

Here $\psi(0)$ is the wave function at origin in the coordinate space, and by definitions it connects to the decay constant $f_{D_s}$:

$$\psi(0) = 2\sqrt{m} f_{D_s},$$

where $m$ is the mass of $D_s$ meson. Moreover we note that for convenience we take unitary gauge for weak boson to do the calculations throughout the paper.

There is infrared infinity when performing phase space integral about the square of matrix element at the soft photon limit. It is known that the infrared infinity can be cancelled completely by that of the loop corrections to the corresponding pure leptonic decay $D_s \rightarrow \ell \nu$.

If Feynman gauge for photon is taken, the amplitude of loop corrections corresponding to the box diagrams (a), (b) can be written as:

$$M_{(2)}(a) = \frac{2}{3} eA \int \frac{d^4l}{(2\pi)^4} \left[ \frac{-4i e^\alpha \beta \gamma_\mu p_\alpha l_\beta - 4(p_\mu l_\nu - p_\nu l_\mu) + \frac{8m_c}{m_s+m_c} p_\mu p_\nu}{l^2(l^2 - 2p \cdot l - m_s^2)((l^2 - 2m_c p \cdot l)(l^2 - 2l \cdot (p-k_2))} \right]$$

where $\ell$, $k_2$ denote the momenta of the loop and the neutrino respectively. These two terms also have infrared infinity when integrating out the loop momentum $l$.

After doing the on-mass-shell subtraction, the terms corresponding to vertex and self-energy diagrams (c), (d), (e), (f) can be written as:

$$M_{(2)}(c + d + e + f) = \frac{i eA}{4\pi^2} \bar{p}(1-\gamma_5)\nu \times \left[ \ln(4) - \frac{8}{9} + \frac{2m_s-m_c}{9m_s+m_c} \ln \left( \frac{m_s}{m_c} \right) \right]$$

where $A$ is:

$$A = f_{D_s} \left( \frac{G_F m_s^2}{\sqrt{2}} \right)^{\frac{3}{2}} V_{cs} e^{\frac{2}{\sqrt{2}}} = f_{D_s} \left( \frac{G_F m_s^2}{\sqrt{2}} \right) V_{cs} e^{\frac{2}{\sqrt{2}}}.$$
In these loop diagrams, we can ignored their contributions safely. Furthermore we should note that in our calculations throughout the paper, the dimensional regularization to regularize both infrared and ultraviolet divergences is adopted, while the on-mass-shell renormalization for the ultraviolet divergence is used.

Detail cancellation of infrared divergence is given in Ref[8]. Here we simply show the results. The ‘whole’ leptonic decay branching ratios, i.e., the sum of the corresponding radiative decay branching ratios and the corresponding pure leptonic decay branching ratios with radiative corrections, and put them in Table (1). The reason we put the radiative decay and the pure leptonic decay with radiative corrections together is to make the branching ratios not to depend on the experimental resolution for a soft photon. For comparison, the branching ratios of each pure leptonic decay at tree level is also put in Table (1). The values for the parameters $\alpha = 1/132$, $|V_{cs}| = 0.974[8]$, $m_{D_s} = 1.9686$ GeV, $m_s = 0.5$ GeV, $m_c = 1.7$ GeV, $f_{D_s} = 0.24$ GeV[11] and the lifetime $\tau(D_s) = 0.469 \times 10^{-12}$ s[9].

| $B_e(10^{-5})$ | ‘whole’ | tree |
|----------------|---------|------|
| 2.56           |         | 0.0108 |
| $B_\mu(10^{-3})$ | 4.706   | 4.605 |
| $B_\tau(10^{-2})$ | 4.138   | 4.489 |

We can see that, the ‘whole’ decay branching ratios $Br_e$ and $Br_\mu$ are larger than the corresponding branching ratios of tree level, while the ‘whole’ $Br_\tau$ is smaller than the tree level one. It means the contributions of loop corrections are negative, the dominate contributions of first order corrections to the pure leptonic $D_s$ decays are radiative decays when the lepton is $e$ or $\mu$, and is loop corrections when the lepton is $\tau$. So, the loop contributions are important for the decays $D_s \to \mu \nu_\mu$ and $D_s \to \tau \nu_\tau$, especially for the later. Through Table (1), we obtained that the radiative decay has a branching ratio $Br(D_s \to \mu \nu_\mu \gamma) > 1.01 \times 10^{-4}$ and the loop correction to $D_s \to \tau \nu_\tau$ has a branching ratio $Br > 3.51 \times 10^{-3}$.

To see the contributions of the radiative decays precisely we present the radiative decay branching ratios with a cut of the photon energy, i.e., the branching ratios of the radiative decays $D_s \to l \nu_\gamma$ with the photon energy $E_\gamma \geq k_{min}$ as the follows: $k_{min} = 0.00001$ GeV, $k_{min} = 0.0001$ GeV, $k_{min} = 0.001$ GeV, $k_{min} = 0.01$ GeV and $k_{min} = 0.1$ GeV respectively in Table (2). We also show the existing results of other methods in the same table.

| $k_{min}$ | $Br_e$ | $Br_\mu$ | $Br_\tau$ |
|-----------|--------|---------|---------|
| GeV       | $10^{-5}$ | $10^{-4}$ | $10^{-6}$ |
| 0.00001   | 2.552  | 4.901  | 6.336  |
| 0.0001    | 2.552  | 3.908  | 4.597  |
| 0.001     | 2.551  | 2.915  | 2.868  |
| 0.01      | 2.549  | 1.927  | 1.217  |
| 0.1       | 2.475  | 0.971  | 0.727  |
| Ref[4]    | 10     | 1      | –      |
| Ref[5]    | 17     | 1.7    | –      |
| Ref[6]    | 7.7    | 2.6    | 320    |
Considering the possible experimental resolutions of photon, our prediction of the radiative decay branching ratios $Br(D_s \rightarrow e\nu e\gamma)$ and $Br(D_s \rightarrow \mu\nu\mu\gamma)$ are close to the values in Refs. [4, 5, 6], but our prediction of $Br(D_s \rightarrow \tau\nu\tau\gamma)$ is much smaller than the one in Ref. [6]. In our model, if we use a smaller cut $k_{min}$, then obtained a larger $Br(D_s \rightarrow \ell\nu\ell\gamma)$, because the decay widths depend on $\log(k_{min})[8]$, the change of branching ratios will be not so much on the selection of $k_{min}$, we can see this in Table (2), and for another example, if $k_{min} = 1.0 \times 10^{-10}$ GeV, then we obtain $Br(D_s \rightarrow e\nu e\gamma) = 3.59 \times 10^{-5}$, $Br(D_s \rightarrow \mu\nu\mu\gamma) = 1.38 \times 10^{-3}$, $Br(D_s \rightarrow \tau\nu\tau\gamma) = 2.11 \times 10^{-5}$, but so small a $k_{min}$, it is very difficult in experiment. We can conclude that the best radiative decay channel is easy to search in experiment is $D_s \rightarrow \mu\nu\mu\gamma$.

For the convenience to compare with experiments, we present the photon spectrum of the radiative decays in Fig.2 and Fig.3. In addition, we should note that the widths are quite sensitive to the decay constant $f_{D_s}$, and are sensitive to the values of the quark masses $m_s$ and $m_c$.

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Figure 1: **Diagrams for** $D_s \rightarrow ℓνγ$.

Figure 2: **Photon energy spectra of radiative decays** $D_s \rightarrow ℓνγ(ℓ = e, μ)$. 
Figure 3: Photon energy spectra of radiative decays $D_s \rightarrow \tau \nu \gamma$. 
\[ \frac{dT}{dE_\gamma}(10^{-15}GeV) \]
