Finite Element Sound Field Analysis of Diffuseness in Reverberation Rooms

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Abstract

Standard deviations of sound pressure levels and spatial correlation functions of sound fields in reverberation rooms are calculated from sound pressures obtained by sound field analyses based on the authors' finite element method. Distinct differences are found among the standard deviations under four sound source conditions in a regularly shaped reverberation room, while there is not much difference among them for an irregularly shaped reverberation room. Difference between standard deviations of sound fields under the best and worst conditions was found to be 1.0 dB within 1/3 octave band in the regularly shaped reverberation room. Next, spatial correlation function values obtained by the finite element sound field analysis are compared with values in the perfectly diffuse sound field. Differences between actual values in reverberation rooms and theoretical values in the perfectly diffuse sound field (i.e. \( \sin kr / kr \)) are also compared with standard deviations of sound pressure levels. As a result, both correlation functions and standard deviations are greatly affected by differences in room shape and sound source positions; in contrast, standard deviations are not affected by absorbing material on their boundaries.

Keywords: finite element method; sound field analysis; diffuse sound field; spatial correlation function; sound pressure distribution

1. Introduction

Reverberation rooms are essential for measurements in many kinds of building acoustics: for instance, the absorption of material, sound power of noise source, and transmission loss. All measurements in reverberation rooms presume a diffuse sound field. There remain issues caused by differences of sound fields or diffuseness in rooms used for measurements (Sharp, 1999; Jacobsen and Roisin, 2000). In particular, although the Japanese Industrial Standard (JIS) approves use of both regularly and irregularly shaped reverberation rooms for these measurements (e.g., JIS A 1416, 2000), it cannot be inferred that their characteristics of diffuseness are same.

Diffuseness of sound fields in reverberation rooms has been widely investigated by experiments and statistical considerations. In these studies, various descriptors were applied: sound intensity (Hanyu et al., 1995; Ikeda et al., 1996), modal density (Samejima et al., 1998), the coherence function (Jacobsen and Roisin, 2000), and so on. Brief reviews of descriptors are presented by Nelisse and Nicolas (1997) and by Jacobsen and Roisin (2000). Cook et al. (1955) derived the spatial correlation function of a perfectly diffuse sound field and showed good agreement between the function and spatial correlation values of sound fields in actual reverberation rooms for high-frequency. Koyasu and Yamashita (1971) noted that it is important to observe correlation function values for all directions in the field. It is also pointed out that quantitative assessments of sound fields are difficult by the correlation function (Nelisse and Nicolas, 1997). Spatial uniformity of sound fields is utilized in standards (e.g. ISO 140-3, 1995) and many studies (Yousri and Fahy, 1972; Toyama et al., 1989; Imai and Konishi, 1996; Imai, 2000). In rectangular rooms, modal analysis is widely applied to calculation of these descriptors (Yousri and Fahy, 1972; Chu, 1981; Toyama et al., 1989; Imai and Konishi, 1996; Nelisse and Nicolas, 1997; Imai, 2000).

On the other hand, numerical analyses based on the wave equation have been intensively used to explore many kinds of acoustic problems (Craggse, 1979; Choi and Tachibana, 1993; Ikeda et al., 1996; Sakuma et al., 1998). Among the analyses, the finite element method (FEM) is advantageous in its broad range of adaptability. The method can be applied successfully to such sound fields as those with complex absorbent walls and materials, those with temperature distribution, and so on. The authors presented a finite elemental procedure in preceding papers (Otsuru and Tomiku, 2000); the procedure enables us to estimate resulting accuracy of sound field analyses by the FEM with...
regularly shaped elements. The authors have also proven that sound pressure distributions in an irregularly shaped reverberation room obtained by the authors’ FE-analysis were in good agreement with those of measurements on various conditions caused by absorbent materials (Tomiku and Otsuru, 2002).

In this study, data obtained by the authors’ FE-analysis are applied to calculate descriptors of diffuseness of sound fields in regularly and irregularly shaped reverberation rooms. This study shows effectiveness of the method for quantitative evaluations of sound fields. Spatial correlation function of the sound field and the standard deviation of sound pressure levels are utilized as descriptors because these are widely employed in many studies and are easily calculated from results obtained by FEM. The two descriptors are calculated by use of sound pressures at more than 10,000 points; then, diffuseness is compared among several conditions of sound fields in reverberation rooms, e.g., room shapes, frequencies, bandwidths, sound source locations, and absorptions on their boundaries.

2. Sound Field Analysis by Finite Element Method

2.1 Formulation of Analysis

Following the standard finite element procedure, acoustic element matrices are given as follows: first, let complex sound pressure, \( p \), at an arbitrary point in an element be expressed by

\[
p = \{N\}^T \{p\},
\]

where \( \{N\} \) is a shape function. This paper employs a 27-node element using spline polynomials. Characteristics of the function and the element are discussed in the authors’ former papers (Otsuru and Tomiku, 2000; Tomiku and Otsuru, 2002).

With kinetic, potential and dissipated energy in an element and work done by external force, the following discrete formula can be obtained by applying the minimum energy principle:

\[
[M] \{\dot{p}\} + [C] \{p\} + [K] \{p\} = -j \rho \omega \nu_e \{W\}.
\]

In that equation, \( \{W\} \) denotes the distribution vector in an element. The matrices used here, \( [K] \), \( [M] \), and \( [C] \), are defined as

\[
[K] = \iiint \frac{\partial^2 \{N\} \partial^2 \{N\} \partial^2 \{N\}}{\partial x \partial y \partial z} \, dx \, dy \, dz.
\]

\[
[M] = \iiint \{N\}^T \{N\} \, dx \, dy \, dz.
\]

\[
[C] = \frac{1}{c} \iiint \frac{1}{\varepsilon} \{N\}^T \{N\} \, dx \, dy \, dz.
\]

In this paper, the locally reactive model (Tomiku and Otsuru, 2002) is assumed for dissipation; then, \( [C] \) can be reassembled into a diagonal matrix. Global matrices and the global matrices equations can be assembled with these elemental matrices; the global matrices equation can be written as

\[
[M] \{\dot{p}\} + [C] \{p\} + [K] \{p\} = -j \rho \omega \nu_e \{W\}.
\]

2.2 Accuracy of Analysis

In the authors’ former paper (Tomiku and Otsuru, 2002), good agreement was found between sound pressure levels obtained by the authors’ FE-analyses and those of measurements in the room with a sound source on one wall. If sound pressures are accurately calculated at each point used by FE-analysis, accurately calculated statistical values of sound field characteristics can be expected. However, the sound source of analysis in the case, computed sound pressure levels by the FEM are compared with those of a measurement on the condition that a source is located in the middle of the room.

A room to be analyzed and FEM settings are the same as the paper written above. Sound source location and 378 receiving points are illustrated in Fig. 1. A point-source is assumed to be located in the field, which radiates white-noise filtered by 200 Hz in 1/3 octave band and in a steady state condition. An omnidirectional speaker is employed for measurement. Figure 2 shows a comparison of measured values with computed sound pressure level distribution obtained at

![Fig.1. Measured and Computed Points and a Sound Source Location](image)

![Fig.2. Comparison of Relative Sound Pressure Levels; Measurement vs FEM \((h=1.20 \text{ m}, 378 \text{ points})\)](image)
378 points. The average residual of computed sound pressure values to measured values is 0.83 dB. Thus, it can be said that results obtained by the authors’ finite element sound field analysis in frequency domain agrees well with measured data in this case.

3. Standard Deviations of Sound Pressure Levels

3.1 Settings of Finite Element Analysis

Figures 3 and 4 show schematic drawings of the irregularly shaped reverberation room (I.R.) and the regularly shaped reverberation room (R.R.) under consideration. The I.R. (Volume = 165 m³, Surface area = 180 m²) was analyzed in the former chapter. Dimensions of the R.R. (Volume = 220 m³, Surface area = 227 m²) were determined by considering the literature (Sakamoto et al., 1999). Table 1 gives finite element settings in the analysis below.

Sound pressure responses in the sound field can be computed by Eq. (6) in the frequency domain ($p_{FEM}(f)$); values of $\lambda/d$ ($d$: nodal distance) of all elements exceed 4.4. In both rooms, all walls, floor and ceiling surfaces are smooth and flat without neither absorbent material nor diffuser. In FE-analysis, complex admittance of all boundaries is given by considering reverberation time measured in the I.R.

Assuming that the sound field has a band of frequency ranging from $f_1$ to $f_2$, standard deviation of sound pressure levels at M-points is defined as

$$SD(f) = \left[ \frac{1}{M-1} \sum_{i=1}^{M} \left( L_{p,i}(f) - \bar{L}_p(f) \right)^2 \right]^{\frac{1}{2}},$$

(7)

where

$$L_p(f) = 10 \log \left( \sum_{i=1}^{N} (p_{FEM}(f)_i)^2 \right),$$

(8)

and $\bar{L}_p(f)$ is the mean $L_p(f)$.

The M-points are chosen to avoid the reflection effect from walls onto acoustic pressure, they are placed more than 1.0 m from the nearest wall. Consequently, in the case of R.R., a total of 11 745 points are selected.

3.2 Results Obtained by Finite Element Analysis

Imai and Konishi (1996) investigated diffuseness in relations with sound source conditions in reverberation rooms both statistically and experimentally. Sound

![Fig.3. Schematic Drawing of an Irregularly Shaped Reverberation Room and Finite Element Division (13*13*11)](image1)

![Fig.4. Schematic Drawing of a Regularly Shaped Reverberation Room and Finite Element Division (10*16*17)](image2)

![Fig.5. Standard Deviation of Sound Pressure Levels Averaged in Each Sound Source Conditions (1/3 oct. band and 1/12 oct. band)](image3)
source conditions were studied for the "corner", "ridge", "wall", and in the "field".

Figure 5 shows comparisons of \(SD(f)\) within both 1/3 and 1/12 octave bands when the sound source condition or the bandwidth is changed. The number of sound source locations per one condition is the same as Imai's former study (=4); \(SD(f)\) values are averaged in each condition. In R.R., better sound field diffuse-ness is found in the order of "corner", "ridge", "wall", and "field". In contrast, four conditions show the same diffuseness of the sound field in the I.R. These results show the same tendency as Imai's experiments, which utilize sound power variance. It is also observed that the \(SD(f)\) difference between "corner" and "field" within 1/3 octave band is 1.0 dB in R.R.

4. Spatial Correlation Function of the Sound Field

4.1 The Plane Wave Model in the Perfectly Diffuse Field

When sound pressures at two arbitrary points of a sound field are expressed as \(p_1\) and \(p_2\), the spatial cross-correlation function of sound pressure between two points in the sound field, or simply the spatial correlation function, is defined as

\[
SC = (p_1 \cdot p_2) = \frac{1}{\sqrt{|p_1|} \cdot |p_2|} \cdot \langle p_1 \rangle \cdot \langle p_2 \rangle
\] (9)

In that equation, \(\langle \rangle\) shows the average time. When \(p_1\) and \(p_2\), vibrate at the same frequency, but with phase differing by \(\phi\), then \(SC = \cos \phi\) stands. Assuming that a plane wave of wavelength \(\lambda\) passes over points \(A_1\) and \(A_2\), \(\theta\) denotes incidence angle of the plane wave and \(r\) denotes distance between \(A_1\) and \(A_2\), as shown in Fig. 6; then

\[
SC(r, f) = \cos(2\pi \cdot \frac{r \cos \theta}{\lambda}) = \cos(kr \cos \theta) .
\] (10)

An integration of Eq. (10) over all directions must be performed to obtain the spatial correlation function for a diffuse sound field. Using polar coordinates \(r\), \(\theta\), and \(\phi\) gives the following result in three dimensions (Cook, 1955).

\[
SC(r, f) = \left[ \frac{\int d\phi}{2\pi} \int \cos(kr \cos \theta) \sin \theta d\theta \right] / 4\pi = (\sin kr) / kr .
\] (11)

It has been noted that if one or some correlation function values of the sound field correspond with Eq. (11), the field is not ensured to be perfectly diffused. This remark has been pointed out in former papers (Koyasu and Yamashita, 1971; Nelisse and Nicolas, 1997; Jacobsen and Roisin, 2000); the simplest case is Eq. (10) = Eq. (11).

On the other hand, assuming that the sound field has a band of frequency with a unique weighting, ranging from \(k_1\) to \(k_2\), then the spatial correlation function becomes

\[
SC(r, f) = \frac{\int_0^{\pi} \sin u du}{\pi} (k r - k r) .
\] (12)

Figure 7 presents comparison among numerically integrated values of Eq. (12) within 1/1, 1/3, 1/12 octave and Eq. (11). It can be observed that \(SC(r, f)\) within 1/3 and 1/12 octave bands are in good agreement with Eq. (11) in this range of \(kr\). However, discrepancies between \(SC(r, f)\) in 1/1 octave band and Eq. (11) can be found above \(kr = \pi\). Furthermore, \(SC(r, f)\) approaches 0 above \(kr = 2\pi\).

4.2 Results Obtained by Finite Element Analysis

FEM settings and rooms to be analyzed are the same as the former chapter. Then, assuming that sound pressures at two arbitrary points of the sound field obtained by FEM are expressed as \(p_{1,FEM}(f)\) and \(p_{2,FEM}(f)\); spatial correlation function values can be calculated as follows by Eq. (9):

\[
SC(r, f)_{FEM} = \frac{\text{Re}[p_{1,FEM}(f)] \cdot \text{Re}[p_{2,FEM}(f)] + \text{Im}[p_{1,FEM}(f)] \cdot \text{Im}[p_{2,FEM}(f)]}{|p_{1,FEM}(f)|^2 + |p_{2,FEM}(f)|^2} .
\] (13)

In that equation, \(\text{Re}[]\) and \(\text{Im}[]\) denote real and imaginary parts, respectively. If the sound field has a band of frequency ranging from \(f_1\) to \(f_2\), \(SC(r, f)_{FEM}\) values are averaged in several octave bands as

![Fig.6. Correlation in Plane Wave](image1)

![Fig.7. Comparisons of Spatial Cross Correlation](image2)
5. Comparisons between Spatial Correlation Functions and Standard Deviations of Sound Pressure Levels

5.1 Influence of Room Shape, Sound Source Location, Frequency, and Bandwidth

FEM settings and rooms to be analyzed are the same as in former sections. In this section, as shown in Fig.10, three cases of a sound source location were employed in each room: "SP1", a point sound source at one corner; "SP2", a point sound source on one side-wall; and "SP3", a point source in the middle of the room.

To characterize sound fields in rooms using the spatial correlation function, difference between Eq. (11) and $SC(r, f)_{FEM}$ can be calculated as

$$D_{sc}(f) = \frac{1}{M} \sum SC(r, f)_{FEM} - \left( \frac{\sin kr}{kr} \right).$$

(15)

The descriptor includes several $kr$ in addition to several directions, which was emphasized by Koyasu and Yamashita (1971).

The $D_{sc}(f)$ and the $SD(\overline{f})$ are given to show diffuseness difference in Figs. 11 and 12 when room shapes, sound source locations, frequencies, or bandwidths are changed. Both results correspond to each other in the following aspects: 1) Clear difference can be found between results using SP1 and those using SP3 in R.R. while little difference can be found in the I.R.; 2) When bandwidth is identical, I.R. results fall on intermediate points between results using SP1 and results using SP3 in R.R.; 3) Worst characteristics of diffuseness in each room are shown when using SP3 within 125 Hz 1/12 octave band in R.R. and using SP1 within 100 Hz 1/12 octave band in I.R. Pearson's correlation coefficient between $SD(\overline{f})$ and $D_{sc}(\overline{f})$ in 108 conditions (2 rooms x 3 sound source locations x 3 frequency bandwidths x 6 center frequencies) is 0.92.

5.2 Influence of Sound Absorption on Boundaries

To find relations between sound fields with absorption on boundary and $SD(\overline{f})$ and between fields and $D_{sc}(\overline{f})$, sound fields with glass-wool (10 m²) are analyzed by FEM. In the analysis, the following material constants are given to calculate acoustic impedance of glass-wool by using Miki's equation (1990): flow resistivity $R = 10000$ Pa·s/m², porosity $\Omega$. 
These impedance values are changed by increasing glass-wool thickness in the same frequency. The glass-wool and a sound source location, "SP4", a point source in the room near the corner, are assumed in this section are shown in Fig. 10.

Relations between sound absorption coefficients ($\alpha$) and material thicknesses are shown in Fig. 13. Using these data, $D_{ac}(\tilde{f})$ in 1/3 octave band obtained by FE-analysis can be found to relate with thickness ($l$) of a given absorbing material in Fig. 14. The $D_{ac}(\tilde{f})$ values are in good correspondence with $\alpha$ in I.R. at 160 Hz and 200 Hz, the $D_{ac}(\tilde{f})$ values of which are small in $l = 0.05$ m. However, $D_{ac}(\tilde{f})$ values remain large at 125 Hz in I.R. and at all frequencies in R.R. regardless of the difference of $\alpha$. It is well known that if sound field diffuseness is bad, absorption on boundaries affects diffuseness of the sound field less (Koyasu and Yamashita, 1971). Figure 15 shows $SD(\tilde{f})$ obtained by FE-analysis as a function of $l$. It is difficult to see a clear relation between $SD(\tilde{f})$ and $l$. Furthermore, while $D_{ac}(\tilde{f})$ values in R.R. are higher than those in I.R., $SD(\tilde{f})$ values in R.R. are equal to or less than those in I.R. Unlike the former section, both descriptors must be used together to ensure quality of diffuseness evaluation in such sound fields.
6. Conclusions
Standard deviation of sound pressure levels and the spatial correlation function of sound fields in reverberation rooms are calculated from sound pressures obtained by sound field analysis based on the authors’ finite element method. Standard deviations of sound fields were shown to relate to sound source conditions in reverberation rooms. Spatial correlation functions of sound fields in reverberation rooms are compared with the (sin kr)/kr for various frequencies and bandwidths conditions. It is confirmed that these descriptors obtained by the authors’ finite element sound field analysis agree well with experimental and statistical results that were presented in former studies. Furthermore, quantitative evaluation values including influence of whole sound fields can be calculated by these descriptors; diffuseness can be compared using evaluation values between several conditions of sound fields in reverberation rooms, such as room shapes, frequencies, bandwidths, sound source locations, and absorptions on their boundaries. Further investigations of evaluation values would provide information of relations between sound fields and results of measurements in reverberation rooms.

Symbols
- $c$: speed of sound
- $d$: nodal distance in a finite element method
- $D_{SC}(f)$: difference between spatial correlation values in reverberation room and (sin kr)/kr
- $f$: frequency
- $f_{p}$: square root of (-1)
- $k$: wavelength constant
- $K_{s}$: structure constant
- $L_{p}(f)$: sound pressure level at $f$/Hz
- $M$: total number of sampling points
- $\{N\}$: shape function
- $p$: sound pressure
- $\{p\}$: sound pressure vector
- $r$: distance between two arbitrary points
- $R$: flow resistivity
- $SD(f)$: standard deviation of sound pressure levels at $f$/Hz
- $SC(r, f)$: spatial correlation function value of sound field between two arbitrary points at distance $r$ at $f$/Hz
- $\{W\}$: distribution vector in an element
- $\lambda$: normal acoustic impedance ration at the surface
- $\delta$: wave length
- $\theta$: incidence angle of plane wave
- $\rho$: mass density of air
- $\omega$: angular frequency
- $\epsilon$: porosity
- $\{i\}$: vector
- $\{i\}_{e}$: elemental vector
- $\{\}$: matrix
- $\{\}$: elemental matrix
- $^T$: transpose of $[]$

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