Widths of tetraquarks with hidden charm

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Abstract

The relativistic four-quark equations are found in the framework of coupled-channel formalism. The dynamical mixing of the meson-meson states with the four-quark states is considered. The four-quark amplitudes of the tetraquarks with hidden charm including $u, d, s$ and charmed quarks are constructed. The poles of these amplitudes determine the masses and widths of tetraquarks.

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I. Introduction.

The observation of the $X(3872)$ [1 – 4], the first of the $XYZ$ particles to be seen, brought forward the hope that a multiquark state has been received. Maiani et al. advocate a tetraquark explanation for the $X(3872)$ [5, 6]. Belle Collaboration observed the $X(3940)$ in double-charmonium production in the reaction $e^+e^- \rightarrow J/\psi + X$ [7]. The state, designated as $X(4160)$, was reported by the Belle Collaboration in Ref. 8.

The discovery of the $X(3872)$ has triggered intensive theoretical studies to understand the structure of this state [9 – 14]. One of the main issues is to clarify the quantum numbers, especially, the spin and the parity, which are key properties to understand the abnormally small width. Several review papers, as for example [15, 16], discuss the difficulty of interpreting these resonances as charmonium states.

In our papers [17 – 19] relativistic generalization of the three-body Faddeev equations was obtained in the form of dispersion relations in the pair energy of two interacting particles. The mass spectra of $S$-wave baryons including $u, d, s, c$ quarks were calculated by a method based on isolating of the leading singularities in the amplitude.

We searched for the approximate solution of integral three-quark equations by taking into account two-particle and triangle singularities, all the weaker ones being neglected. If we considered such an approximation, which corresponds to taking into account two-body and triangle singularities, and defined all the smooth functions of the subenergy variables in the middle point of the physical region of Dalitz-plot, then the problem was reduced to the one of solving a system of simple algebraic equations.

In the recent papers [20 – 22] the relativistic three-quark equations for the excited baryons are found in the framework of the dispersion relation technique. We have used the orbital-spin-flavor wavefunctions for the construction of integral equations. We
calculated the mass spectra of $P$-wave single, double and triple charmed baryons using the input four-fermion interaction with quantum numbers of the gluon [22].

In this paper the relativistic four-quark equations are found in the framework of coupled-channel formalism. The dynamical mixing between the meson-meson states and the four-quark states is considered [23 – 25]. Taking the $X(3872)$ and $X(3940)$ as input we predict the masses and widths of $S$-wave tetraquarks with hidden charm (Table I).

After this introduction, we discuss the four-quark amplitudes, which contain the two charmed quarks (Sect. II). In Sect. III, we report our numerical results (Tables I and II).

II. Four-Quark Amplitudes for the Tetraquarks with Hidden Charm.

We derive the relativistic four-quark equations in the framework of the dispersion relations technique.

The correct equations for the amplitude are obtained by taking into account all possible subamplitudes. It corresponds to the division of complete system into subsystems with the smaller number of particles. Then one should represent a four-particle amplitude as a sum of six subamplitudes

$$A = A_{12} + A_{13} + A_{14} + A_{23} + A_{24} + A_{34}. \tag{1}$$

This defines the division of the diagrams into groups according to the certain pair interaction of particles. The total amplitude can be represented graphically as a sum of diagrams.

We need to consider only one group of diagrams and the amplitude corresponding to them, for example $A_{12}$. We shall consider the derivation of the relativistic generalization of the Faddeev-Yakubovsky approach [26, 27] for the tetraquark.

We shall construct the four-quark amplitude of $c\bar{c}u\bar{u}$ meson, in which the quark amplitudes with quantum numbers of $0^{-+}$ and $1^{--}$ mesons are included. The set of diagrams associated with the amplitude $A_{12}$ can further be broken down into five groups corresponding to subamplitudes: $A_1(s, s_{12}, s_{34})$, $A_2(s, s_{23}, s_{14})$, $A_3(s, s_{23}, s_{123})$, $A_4(s, s_{34}, s_{234}$), $A_5(s, s_{12}, s_{123})$, if we consider the tetraquark with $J^{pc} = 2^{++}$.

Here $s_{ik}$ is the two-particle subenergy squared, $s_{ijk}$ corresponds to the energy squared of particles $i, j, k$ and $s$ is the system total energy squared.

In order to represent the subamplitudes $A_1(s, s_{12}, s_{34})$, $A_2(s, s_{23}, s_{14})$, $A_3(s, s_{23}, s_{123})$, $A_4(s, s_{34}, s_{234})$ and $A_5(s, s_{12}, s_{123})$ in the form of dispersion relations it is necessary to define the amplitudes of quark-antiquark interaction $a_n(s_{ik})$. The pair quarks amplitudes $q\bar{q} \rightarrow q\bar{q}$ are calculated in the framework of the dispersion $N/D$ method with the input four-fermion interaction [28 – 30] with quantum numbers of the gluon [31].

The regularization of the dispersion integral for the $D$-function is carried out with the cutoff parameter $\Lambda$. The four-quark interaction is considered as an input [31]:

$$g_V (\bar{q} I_f \gamma_\mu q)^2 + g^{(s)}_V (\bar{q} I_f \gamma_\mu q) (\bar{s} \lambda \gamma_\mu \lambda_s) + g^{(ss)}_V (\bar{s} \lambda \gamma_\mu s)^2. \tag{2}$$

Here $I_f$ is the unity matrix in the flavor space ($u, d$). $\lambda$ are the color Gell-Mann matrices. Dimensional constants of the four-fermion interaction $g_V$, $g^{(s)}_V$ and $g^{(ss)}_V$ are parameters of the model. At $g_V = g^{(s)}_V = g^{(ss)}_V$ the flavor $SU(3)_f$ symmetry occurs. The strange quark violates the flavor $SU(3)_f$ symmetry. In order to avoid an additional violation parameters, we introduce the scale shift of the dimensional parameters [31]:

$$\text{dimensional}_V = g_V^\text{dimensional} = \frac{g_V}{g^{(ss)}_V}.$$
\[ g = \frac{m^2}{\pi^2} g_V = \frac{(m + m_s)^2}{4\pi^2} g_V^{(s)} = \frac{m_s^2}{\pi^2} g_V^{(ss)}. \]  

(3)

\[ \Lambda = \frac{4\Lambda(ik)}{(m_i + m_k)^2}. \]  

(4)

Here \(m_i\) and \(m_k\) are the quark masses in the intermediate state of the quark loop. Dimensionless parameters \(g\) and \(\Lambda\) are supposed to be constants which are independent of the quark interaction type. The applicability of Eq. (2) is verified by the success of De Rujula-Georgi-Glashow quark model \[32\], where only the short-range part of Breit potential connected with the gluon exchange is responsible for the mass splitting in hadron multiplets.

We use the results of our relativistic quark model \[31\] and write down the pair quarks amplitude in the form:

\[ a_n(s_{ik}) = \frac{G_n^2(s_{ik})}{1 - B_n(s_{ik})}, \]  

(5)

\[ B_n(s_{ik}) = \int_\frac{(m_i + m_k)^2}{2}^{\infty} \frac{d\rho(s_{ik}) g_n(s_{ik}) G_n^2(s'_{ik})}{\pi (s'_{ik} - s_{ik})}. \]  

(6)

Here \(G_n(s_{ik})\) are the quark-antiquark vertex functions. The vertex functions are determined by the contribution of the crossing channels. The vertex functions satisfy the Fierz relations. All of these vertex functions are generated from \(g_V\), \(g_V^{(s)}\) and \(g_V^{(ss)}\). \(B_n(s_{ik})\), \(\rho_n(s_{ik})\) are the Chew-Mandelstam functions with cutoff \(\Lambda\) and the phase spaces, respectively.

In the case in question, the interacting quarks do not produce a bound state; therefore, the integration in Eqs. (7) – (11) is carried out from the threshold \((m_i + m_k)^2\) to the cutoff \(\Lambda(i, k)\). The coupled integral equation systems (the meson state \(J^{PC} = 2^{++}\) for the \(c\bar{c}u\bar{u}\)) can be described as:

\[ A_1(s, s_{12}, s_{34}) = \frac{\lambda_1 B_1(s_{12}) B_1(s_{34})}{[1 - B_1(s_{12})][1 - B_1(s_{34})]} + 4\hat{J}_2(s_{12}, s_{34}, 1, 1) A_3(s, s'_{23}, s'_{123}), \]  

(7)

\[ A_2(s, s_{23}, s_{14}) = \frac{\lambda_2 B_1(s_{23}) B_1(s_{14})}{[1 - B_1(s_{23})][1 - B_1(s_{14})]} + 2\hat{J}_2(s_{23}, s_{14}, 1, 1) A_4(s, s'_{34}, s'_{234}) \]  

\[ + 2\hat{J}_2(s_{23}, s_{14}, 1, 1) A_5(s, s'_{12}, s'_{123}), \]  

(8)

\[ A_3(s, s_{23}, s_{123}) = \frac{\lambda_3 B_2(s_{23})}{[1 - B_2(s_{23})]} + 2\hat{J}_3(s_{23}, 2) A_1(s, s'_{12}, s'_{34}) \]  

\[ + \hat{J}_1(s_{23}, 2) A_4(s, s'_{34}, s'_{234}) + \hat{J}_1(s_{23}, 2) A_5(s, s'_{12}, s'_{123}), \]  

(9)

\[ A_4(s, s_{34}, s_{234}) = \frac{\lambda_4 B_2(s_{34})}{[1 - B_2(s_{34})]} + 2\hat{J}_3(s_{34}, 2) A_2(s, s'_{23}, s'_{14}) \]  

\[ + 2\hat{J}_1(s_{34}, 2) A_3(s, s'_{23}, s'_{234}), \]  

(10)
where \( \lambda_i, \ i = 1, 2, 3, 4, 5 \) are the current constants. They do not affect the mass spectrum of tetraquarks. Here \( n = 1 \) corresponds to a \( qq' \)-pair with \( J^{pc} = 1^- \) in the \( 1_c \) color state, and \( n = 2 \) defines the \( qq' \)-pairs corresponding to tetraquarks with quantum numbers: \( J^{pc} = 0^{++}, 1^{++}, 2^{++} \). We introduce the integral operators:

\[
\hat{J}_1(s_{12}, l) = \frac{G_l(s_{12})}{[1 - B_l(s_{12})]} \int_{(m_1 + m_2)^2}^{(m_1 + m_2)^2} \frac{d s'_{12}}{\pi} \frac{G_l(s'_{12}) \rho_l(s'_{12})}{s'_{12} - s_{12}} \int_{-1}^{+1} \frac{d z_1}{2}, \\
\hat{J}_2(s_{12}, s_{34}, l, p) = \frac{G_l(s_{12}) G_p(s_{34})}{[1 - B_l(s_{12})][1 - B_p(s_{34})]} \int_{(m_1 + m_2)^2}^{(m_1 + m_2)^2} \frac{d s'_{12}}{\pi} \frac{G_l(s'_{12}) \rho_l(s'_{12})}{s'_{12} - s_{12}} \\
\times \int_{(m_3 + m_4)^2}^{(m_3 + m_4)^2} \frac{d s'_{34}}{\pi} \frac{G_p(s'_{34}) \rho_p(s'_{34})}{s'_{34} - s_{34}} \int_{-1}^{+1} \frac{d z_3}{2} \int_{-1}^{+1} \frac{d z_4}{2}, \\
\hat{J}_3(s_{12}, l) = \frac{G_l(s_{12}, \Lambda)}{[1 - B_l(s_{12}, \Lambda)]} \frac{1}{4 \pi} \int_{(m_1 + m_2)^2}^{(m_1 + m_2)^2} \frac{d s'_{12}}{\pi} \frac{G_l(s'_{12}, \Lambda) \rho_l(s'_{12})}{s'_{12} - s_{12}} \\
\times \int_{-1}^{+1} \frac{d z_1}{2} \int_{-1}^{+1} \frac{d z}{2} \int_{z_2}^{z_2^+} \frac{d z_2}{\sqrt{1 - z^2 - z_1^2 - z_2^2 + 2 z z_1 z_2^2}}, \\
\]

here \( l, p \) are equal to 1 or 2.

In Eqs. (12) and (14) \( z_1 \) is the cosine of the angle between the relative momentum of particles 1 and 2 in the intermediate state and the momentum of the particle 3 in the final state, taken in the c.m. of particles 1 and 2. In Eq. (14) \( z \) is the cosine of the angle between the momenta of particles 3 and 4 in the final state, taken in the c.m. of particles 1 and 2. \( z_2 \) is the cosine of the angle between the relative momentum of particles 1 and 2 in the intermediate state and the momentum of the particle 4 in the final state, is taken in the c.m. of particles 1 and 2. In Eq. (13) \( z_3 \) is the cosine of the angle between relative momentum of particles 1 and 2 in the intermediate state and the relative momentum of particles 3 and 4 in the intermediate state, taken in the c.m. of particles 1 and 2. \( z_4 \) is the cosine of the angle between the relative momentum of particles 3 and 4 in the intermediate state and that of the momentum of the particle 1 in the intermediate state, taken in the c.m. of particles 3 and 4.

We can pass from the integration over the cosines of the angles to the integration over the subenergies.

The solutions of the system of equations are considered as:

\[
\alpha_i(s) = F_i(s, \lambda_i)/D(s),
\]

(15)
where zeros of $D(s)$ determinants define the masses of bound states of tetraquarks. $F_i(s, \lambda_i)$ determine the contributions of subamplitudes to the tetraquark amplitude.

III. Calculation results.

The model in consideration has only two parameters: the cutoff $\Lambda = 10.0$ and the gluon coupling constant $g = 0.794$. These parameters are determined by fixing the tetraquark masses for the $J^{pc} = 1^{++} X(3872)$ and $J^{pc} = 2^{++} X(3940)$ [25]. The quark masses of model $m_{u,d} = 385 \text{MeV}$ and $m_s = 510 \text{MeV}$ coincide with the ordinary meson ones in our model [18, 19]. In order to fix anyhow $m_c = 1586 \text{MeV}$, we use the tetraquark mass for the $J^{pc} = 2^{++} X(3940)$. The masses and widths of meson-meson states with the spin-parity $J^{pc} = 0^{++}, 1^{++}, 2^{++}$ are considered in Table I. In our paper we predicted the tetraquark $s\bar{s}c\bar{c}$ with the spin-parity $J^{pc} = 2^{++}$ and mass $M = 4160 \text{MeV}$. Belle Collaboration observed the similar state, designated as $X(4160)$, with mass $M = (4156 \pm 29) \text{MeV}$ and total width of $\Gamma = 139^{+113}_{-79} \text{MeV}$ [8]. We predicted also the scalar tetraquark $u\bar{u}c\bar{c}$ with mass $M = 3708 \text{MeV}$, which was considered by Gamermann et al. [33]. They used the extension of SU(3) Lagrangian to SU(4), with the appropriate SU(4) flavor symmetry breaking and described the structure of charmed resonances as dynamically generated states. The plans to produce this particle at BEPC-II, open up the possibility to see the $X(3700)$ resonance as a narrow peak around $50 \text{MeV}$ in the photon spectrum from the radiative decay $\psi(3770)$ into $X(3700) \gamma$. The tetraquark $X(3700)$ with $J^{pc} = 0^{++}$ can decay to $J/\psi \omega, \eta_c$ and $\eta_c'$. The functions $F_i(s, \lambda_i)$ (Eq. (15)) allow us to obtain the overlap factors $f$ for the tetraquarks. We calculated the overlap factors $f$ and the phase spaces $\rho$ for the reactions $X \rightarrow M_1M_2$ (Table II). We calculated the widths of the tetraquarks with hidden charm (Table I). We considered the formula $\Gamma \sim f^2 \times \rho$ [34], there $\rho$ is the phase space.

The widths of the tetraquarks are fitted by the fixing width $\Gamma_{2^{++}} = (39 \pm 26) \text{MeV}$ [35] for the tetraquark $X(3940)$. The results of calculations allow us to consider the three tetraquarks as the narrow resonances. The calculated width $X(3872)$ tetraquark with the mass $M = 3872.2 \text{MeV}$ [35] and the spin-parity $J^{pc} = 1^{++}$ is about $\Gamma_{1^{++}} = 2.1 \text{MeV}$. In our model we neglected with the mass distinction of the $u$ and the $d$ quarks. We used the sum of the masses of mesons $J/\psi$ and $\rho_0$ ($m_{J/\psi} + m_{\rho_0} = 3871.8 \text{MeV}$) [35] for the calculation of phase space of the decay $\rho_0 J/\psi \rightarrow \pi^+\pi^- J/\psi$. We believe that this mass is very nearly equal to sum of the masses of $D^0$ and $D^{*0}$ mesons ($m_{D^0} + m_{D^{*0}} = 3871.81 \pm 0.36 \text{MeV}$) for the tetraquark with the spin-parity $J^{pc} = 1^{++}$. This sum was recently been precisely measured by the CLEO Collaboration [36]. The experimental value of total width of the tetraquark $X(3872)$ with the spin-parity $J^{pc} = 1^{++}$ is less than $2.3 \text{MeV}$ [35]. The total width of the tetraquark $X(4160)$ is predicted $\Gamma_{1^{++}} = 25 \text{MeV}$. The tetraquark $X(3700)$ with the spin-parity $J^{pc} = 0^{++}$ decays to $\eta_c$. The calculated width of this state is equal to $\Gamma_{0^{++}} = 143 \text{MeV}$. The tetraquarks with the spin-parity $J^{pc} = 0^{++}, 1^{++}$ $s\bar{s}c\bar{c}$ have only the weak decays.

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Table I. Masses and widths of tetraquark with hidden charm.

Parameters of model: quark masses $m_{u,d} = 385 \text{ MeV}$, $m_s = 510 \text{ MeV}$, $m_c = 1586 \text{ MeV}$, cutoff parameter $\Lambda = 10.0$, gluon coupling constant $g = 0.794$.

| Tetraquark          | $J^{PC}$ | Mass (MeV) | Width (MeV) |
|---------------------|----------|------------|-------------|
| $X(3700)$           | 0$^{++}$ | 3708       | 143         |
| $(c\bar{c})(u\bar{u})$ |          |            |             |
| $(u\bar{c})(c\bar{u})$ |          |            |             |
| $X(3872)$           | 1$^{++}$ | 3872.2     | 2.1         |
| $(c\bar{c})(u\bar{u})$ |          |            |             |
| $(u\bar{c})(c\bar{u})$ |          |            |             |
| $X(3940)$           | 2$^{++}$ | 3940       | 39          |
| $(c\bar{c})(u\bar{u})$ |          |            |             |
| $(u\bar{c})(c\bar{u})$ |          |            |             |
| $X(4160)$           | 2$^{++}$ | 4160       | 25          |
| $(s\bar{s})(c\bar{c})$ |          |            |             |
| $(s\bar{c})(c\bar{s})$ |          |            |             |

Table II. Overlap factors $f$ and phase spaces $\rho$ of tetraquark with hidden charm.

| Tetraquark (channels) | $J^{PC}$ | $f$   | $\rho$  |
|-----------------------|----------|-------|---------|
| $X(3700)$ $\eta_c$   | 0$^{++}$ | 0.421 | 0.233  |
| $X(3872)$ $J/\psi$ $\rho$ | 1$^{++}$ | 0.211 | 0.0137 |
| $X(3940)$ $J/\psi$ $\rho$ | 2$^{++}$ | 0.274 | 0.149  |
| $X(4160)$ $J/\psi$ $\varphi$ | 2$^{++}$ | 0.238 | 0.124  |
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