Neutralization of excited antiprotonic helium ion in collisions with helium atoms

Grigory Ya. Korenman\textsuperscript{a} and Sergey N. Yudin\textsuperscript{b}

Skobeltsyn Institute of Nuclear Physics, Lomonosov Moscow State University, Moscow, Russia 119991

Received 5 May 2020 / Accepted 6 January 2021 / Published online 17 February 2021
© The Author(s), under exclusive licence to EDP Sciences, SIF and Springer-Verlag GmbH Germany, part of Springer Nature 2021

Abstract. We consider the neutralization of an excited antiprotonic helium ion which is accompanied by antiproton transitions to lower states during collisions with helium atoms \((\bar{p}\text{He}^{2+})_{nl} + \text{He} \rightarrow [(\bar{p}\text{He}^{2+})_{n'l'}f']_{1s} + \text{He}^+\) in a low-temperature medium. Interactions in the input and output channels are taken to be similar to the potentials of the \(H^+ - \text{He} (X^1\Sigma^+)\) and \(H - \text{He}^+ (A^1\Sigma^+)\) systems, respectively. The potentials in the output channels are shifted by an amount equal to the deexcitation energy of the antiproton. Therefore, the potentials in the input and output channels can intersect for the same antiproton quantum numbers leading to appreciable cross sections of the transitions. We have found that this process can increase the effective annihilation rates of highly excited antiprotonic helium ions in helium targets of low atomic density and temperature that were used in recent experiments.

1 Introduction

The study of the metastable states in antiprotonic helium has provided a large amount of data on various properties of this exotic system, especially due to the ASACUSA collaboration at Antiproton Decelerator at CERN (see [1–3] for reviews). High precision measurements of the laser-induced transitions between the metastable states and of the microwave-induced transitions between hyperfine structure sublevels provide precise information about the fundamental properties of antiprotons.

Other data, such as the lifetimes and populations of the metastable states and shift and broadening of the spectral lines, are connected with the mechanisms of the formation and subsequent interactions of antiprotonic helium with the surrounding ordinary He atoms, impurity atoms and molecules.

Among many other results, the ASACUSA collaboration observed cold long-lived antiprotonic helium ions \((\bar{p}\text{He}^{2+})\) in the states with quantum numbers \((n = 28–32, l = n - 1)\) [4]. It was found that these states have lifetimes \(\tau \sim 100\) ns against annihilation, and rates of annihilation \(\lambda = \tau^{-1}\) that increase roughly as a linear function of target density at \(\rho < (5 - 10) \cdot 10^{17}\) \(\text{cm}^{-3}\) and temperature \(T \simeq 10\) K. The antiproton annihilation occurs from the states with small angular momenta; therefore, the observed lifetimes have to be attributed to the time of transitions from the initial \((n, l)\)-state to \(s, p, d\)-states. Radiative transitions from the circular orbits \((l = n - 1)\) at large \(n\) are strongly suppressed, so the main contribution to the effective rate of annihilation \(\lambda\) arises from collisional transitions. In particular, collisional Stark transitions

\[
(\bar{p}\text{He}^{2+})_{nl} + \text{He} \rightarrow (\bar{p}\text{He}^{2+})_{n'l'} + \text{He}
\]

lead to a reduction in the population of the circular orbits and to the subsequent radiative transitions from the states with \(l' < l\). The observed annihilation rates \(\lambda\) (at given \(\rho\) and \(T\)) imply an effective cross section for the initial antiprotonic helium ionic states \(\sigma_{q} \sim (4 - 10) \cdot 10^{-15}\) \(\text{cm}^{2}\). This value is an order of magnitude larger than the theoretical cross sections for collisional Stark transitions obtained in the previous calculations [5–7] that were related to ions with a higher kinetic energy \(E = 1\) eV corresponding to a temperature of \(T \simeq 10^4\) K. In addition to the huge quenching cross sections, the experiment reveals an isotope effect \((^{3}\text{He}/^{4}\text{He})\) and an unexpected \(n\)-dependence for the \(^{3}\text{He}\) effective annihilation rates. These results also cannot be explained by the mentioned calculations.

Collisional Stark transitions of a \((\bar{p}\text{He}^{2+})_{nl}\) ion in a low-temperature target \((T \simeq 10\) K) were considered [8,9] using the close coupling approach taking into account the states with angular momenta \(l\) from \(n - 1\) to 0 at a fixed \(n\)-value. The interaction between the two colliding subsystems were modeled using an adiabatic potential \(V_{0}(R)\) of the interaction between the He atom and the unit charge of the \((\bar{p}\text{He}^{2+})\) ion. The Stark transitions between the states of the \((\bar{p}\text{He}^{2+})_{nl}\) ion are pro-
duced by the interaction of the electric dipole and the quadrupole momenta of the ion with the electric field $E(R) \sim -\nabla_R V_0(R)$ of the polarized He atom. This model gives cross sections of Stark transitions from circular orbits with $n \approx 30$ at $E = 10$ K that are of the same order, or greater than the effective quenching cross section $\sigma_q$ estimated from the experimental data [4]. The model also shows an isotope effect and an increase in the transition rates with $n$ that agree with the experiment. Thus, the model may allow us to qualitatively understand the observed data. The Stark transition rates averaged over thermal motion are comparable with the observed effective annihilation rates. Nevertheless, the magnitudes of the obtained Stark rates are not large enough to provide a quantitative agreement of the whole cascade times with the experimental data on the effective annihilation times for the initial states $n \sim 30$, $l = n - 1$.

Using the potential energy surface for the two-electron system $\bar{p}He^{2+} - He$ instead of the model potential can increase the calculated Stark transition rates up to 20% [10]. On the other hand, the further understanding and better quantitative description of the observed data may be connected with some additional mechanisms of collisional quenching of antiprotonic helium ions in the cold target. One subset of such mechanisms can be the antiproton transitions that are accompanied by the transfer of kinetic energy to an electron of a He atom, such as

\begin{align}
(\bar{p}He^{2+})_{nl} + He &\to (\bar{p}He^{2+})_{n_f l_f} + e + He^+, \quad (1) \\
(\bar{p}He^{2+})_{nl} + He &\to [(\bar{p}He^{2+})_{n_f l_f}, e]_{1s} + He^+. \quad (2)
\end{align}

The two processes may seem very similar, differing only by the final state of the electron. The first process is usually referred to as an external Auger transition. The second process is the neutralization of the antiprotonic helium ion, or equivalently the electron transfer from the neutral He atom to the ion.

The external Auger transitions during low temperature collisions are strongly suppressed by the Coulomb repulsion of the two positive heavy ions in the final state. Moreover, the Auger process is also suppressed due to the highly multipolarity $\Delta l = l - l_f$ of the antiproton transitions that arise from the energy conservation condition. The kinetic energy of the colliding subsystems in the low-temperature target is negligibly small, and so the energy-allowed Auger transition $n, l \to n_f = n - \Delta n, l_f$ must satisfy the condition

$$-\frac{\mu Z^2}{2n^2} - I_0 \geq -\frac{\mu Z^2}{2(n - \Delta n)^2} + \varepsilon_e,$$  \quad (3)

where $\mu = m_p M_e / (m_p + M_e)$ denotes the reduced mass of the antiproton-nucleus system, $Z = 2$ the charge of the helium nucleus, $I_0 \approx 0.9036$ a.u. the ionization energy of the He atom, $\varepsilon_e \geq 0$ the electron energy of the final state. We use atomic units ($\hbar = e = m_e = 1$) unless otherwise specified. The minimum multipolarity of Auger transitions from the circular orbits coincides with the change in the principal quantum number $\Delta n$. For nearly circular orbits with the radial quantum number $n_r = n - l - 1$, the multipolarity diminishes as $\Delta l = \Delta n - n_r$ at $n_r \leq \Delta n$. Equation (3) implies that Auger transitions from the $n \geq 29$ states require the condition $\Delta n \geq 4$, which correspond to the additional constraints $\Delta l = 4$ and $\Delta l = 3$ for the orbits $l = n - 1$ and $l = n - 2$, respectively.

Unlike process (1), there is no obvious reason why process (2) should be suppressed. There is no Coulomb repulsion between the heavy particles in the final state. In addition, as will be seen in the next section, the process (2) of the types $l = n - 1$ and $l \leq n - 2$ has minimum multiplicities of $\Delta l = 2$ and 1, respectively. These minimum multiplicities are smaller than the corresponding ones for Auger transitions. So, we can expect that the neutralization process (2) in a cold environment will have a much larger cross section than the external Auger process (1).

In this paper, we consider the neutralization process (2) of excited antiprotonic helium ions colliding with helium atoms in low-temperature media, which is accompanied by antiproton transitions to lower-energy states. Interactions in the input and output channels are taken to be similar to the potentials of the $H^+ - He$ ($X^1\Sigma^+$) and $H - He^+$ ($A^1\Sigma^+$) systems, respectively. But the potential in the output channels is shifted down due to a change of the antiproton energy. Therefore, the potentials corresponding to the input and output channels can intersect for some combinations of antiproton quantum numbers, which leads to appreciable transition cross sections for these cases. We have found that this process can provide an increase in the effective annihilation rates for the highly excited antiprotonic helium ion in the low density and temperature targets which have been used in the experiments.

A general description of the process of Eq. (2) at low energy is outlined in Sect. 2. Approximations for the cross sections and transition rates of the neutralization are formulated in Sect. 3. Results of calculations are given in Sect. 4. Short concluding remarks are given in Sect. 5.

### 2 General description of neutralization process

The neutralization process (2) leads to the formation of neutral antiprotonic helium atoms with the quantum numbers $n_f = n - \Delta n \leq 30$, $l_f \leq n_f - 1$ in final state. The rate of this process depends on the minimum multipolarity $\Delta l = l - l_f$ of the antiproton transition. To determine the value of $\Delta l$, it is necessary to obtain the energy $E_{n_f l_f}$ of the final state. The energy levels of the $\bar{p}He^+$ system were calculated with high precision [11,12] for high-lying metastable states with $n > 30$ related to the laser-induced transitions. Unfortunately, these results cannot be used in the problem under consideration, because the final states after the neutralization will have smaller quantum numbers $n_f$. 

\[\text{Springer}\]
In order to make a rough estimation, we can begin from the simple model [13] of hydrogen-like wave functions with effective charges \( Z_\text{\bar{p}} \) for the antiproton and \( Z_e \) for the electron. The mean radii of the central and circular antiprotonic orbits are \( \bar{r}_e = 3/(2Z_e) \) and \( \bar{r}_{\bar{p}} = n(n + 1/2)/(\mu Z_\bar{p}) \), respectively. Variational calculations for circular orbits within this model show [13] that \( 1.95 < Z_\bar{p} < 2, 1 < Z_e \leq 1.15 \) at \( n \leq 30 \), hence \( \bar{r}_{\bar{p}} \leq 0.3, \bar{r}_e \simeq 1.5 \), and \( r_{\bar{p}}^2 \ll r_e^2 \). Therefore, we can simplify the model considering the system at \( n \leq 30 \) in a zeroth approximation as the antiproton residing in a Coulomb potential of nucleus charge +2 and the electron in a Coulomb potential of charge +1, which is positioned in the center-of-mass of the antiproton-nucleus system (see Fig. 1).

The residual interaction

\[
V_{\text{res}} = -\frac{2}{|\mathbf{r} + \nu \mathbf{r}|} + \frac{1}{|\mathbf{r} - \lambda \mathbf{r}|} + \frac{1}{\bar{r}_e} \tag{4}
\]

will be taken into account in the first-order perturbation theory. Here, we use the definitions \( \lambda = M_e/(M_a + m_\bar{p}) \), \( \nu = m_\bar{p}/(M_a + m_\bar{p}) \). The vector \( \mathbf{r} \) denotes the antiproton coordinates relative to the helium nucleus, and \( \mathbf{r}_e \) the coordinate vector relative to the center-of-mass of the antiproton-nucleus system. In this approximation, the energy of the neutral antiprotonic atom is estimated as

\[
E_{\text{n}l_\text{f}} = -\frac{\mu Z^2}{2n_f^2} - \epsilon_{1s} - \Delta \epsilon_{1s}(n, l_\text{f}) \tag{5}
\]

where \( \epsilon_{1s} = 1/2 \) denotes the binding energy of the electron in 1s-state, and \( \Delta \epsilon_{1s}(n, l) = -\langle 1s, nl|V_{\text{res}}|1s, nl \rangle \) is a correction to \( \epsilon_{1s} \) due to the residual interaction defined by Eq. (4). Under the condition \( \bar{r}_e^2 \gg \bar{r}_{\bar{p}}^2 \), we obtain

\[
\Delta \epsilon_{1s}(nl) \simeq \frac{\lambda^2 - 2\mu}{3(\mu Z)^2} n^2 [5n^2 + 1 - 3l(l + 1)]. \tag{6}
\]

Now, the condition for energy-allowed transitions in the neutralization process (2) can be written as

\[
-\frac{\mu Z^2}{2n^2} - I_0 \geq -\frac{\mu Z^2}{2(n - \Delta n)^2} - (\epsilon_{1s} + \Delta \epsilon_{1s}) \tag{7}
\]

This differs from the expression of Eq. (3) by the negative electron energy \(-(\epsilon_{1s} + \Delta \epsilon_{1s})\) in the final state, which allows transitions with \( \Delta n = 2 \) at \( n_f \leq 30 \) to occur. The minimum multipolarity of the neutralization transitions was \( \Delta l = 2 \) and \( \Delta l = 1 \) for initial circular and nearly circular \((l \leq n - 2)\) orbits, respectively. This is in contrast to the \( \Delta l = 4 \) minimum polarity for external Auger transitions. The selection rules allow monopole \( \Delta l = 0 \) transitions from ionic states with \( l \leq n - 3 \). These transitions, however, may be suppressed due to the small overlap between the initial and final radial wave functions of the antiproton, in the same way that monopole transitions of various other processes are suppressed.

Consider the interaction \( V_i(R) \) of the colliding subsystems in the input channel of the reaction of Eq. (2). At large and intermediate distances \((R \gg n^2/\mu Z \simeq 0.3)\), the interaction \( V_i(R) \) can be approximated as the interaction between a heavy structureless particle of unit charge +1 with the He atom which corresponds to the \( X^1\Sigma^+ \) state of the \((p - \text{He})\) system. We take the potential energy curve (PEC) \( U_{X^1\Sigma^+}(R) = V_{\text{H-He}}(R) \) as the potential in the input channel

\[
V_i(R) = V_{\text{H-He}}(R). \tag{8}
\]

The data for this PEC obtained by ab initio calculations were taken from the paper [14] (see also [15]). The energy of the \( X^1\Sigma^+ \) term relative to the ground state of the He atom tends to zero as \(-\alpha/(2R^4)\) at large distances, where \( \alpha \) denotes the polarizability of the He atom.

After the electron transfer, the interaction between the two subsystems in the output channel at large \( R \) becomes similar to the \((\text{H} - \text{He}^+)\) interaction. We take the potential energy curve as \( U_{A^1\Sigma^+}(R) = V_{\text{HHe}^+}(R) + I_0 - \epsilon_{1s} \), where the potential \( V_{\text{HHe}^+}(R) \) of the \((\text{H} - \text{He}^+)\) interaction was taken from the paper [16].\(^1\) The potential energy curve \( U_{A^1\Sigma^+}(R) \) goes to \( I_0 - \epsilon_{1s} \) at \( R \to \infty \) and does not cross the potential curve \( U_{X^1\Sigma^+}(R) \). However, the inner energy of the two subsystems \(([p\text{He}^{2+}]_{n_{1s}}|\epsilon_{1s}^\text{He}^{2+}]\) is shifted to lower energies due to the change of the the antiproton state and a correction \( \Delta \epsilon_{1s} \) to the electronic binding energy. Therefore, we take the interaction potential in the output channel as

\[
V_f(R) = V_{\text{HHe}^+}(R) + E_{n_{1f}} + \frac{\mu Z^2}{2n^2} + I_0, \tag{9}
\]

where \( E_{n_{1f}} \) is given by Eq. (5). The potential curves \( V_f(R) \) and \( V_i(R) \) can intersect for some combinations of quantum numbers.

Some typical examples for such curve crossings are shown in Fig. 2. It is seen from the figure that tran-
ions with $\Delta n = 1$ at thermal collisions are forbidden by energy conservation. Potentials $V_f(R)$ corresponding to transitions of the type $\Delta n = 2, 3, 4$ cross $V_i(R)$ in the classically allowed region of $R > 1$ and $V_i(R) < 0$. At $\Delta n \geq 5$; however, the possible crossing points are located in the repulsive region $R < 1$ which is classically forbidden for the input channel at thermal energy. Higher $\Delta n$ transitions require higher multipo- 
larities that suppress the transition probabilities.

3 Approximations for cross-sections and transition rates

The main contribution to the transition probability during low-velocity collisions occurs at interatomic distances near the crossing point of the initial and final potentials. Therefore, using the Landau-Zener approximation [17, 18], we can estimate the cross section of the electron transfer of Eq. (2) by the equation

$$\sigma_x(E) = 2\pi \int_0^{b_m(E)} P_{fi}(b, E) \left[ 1 - P_{fi}(b, E) \right] b \, db,$$  

(10)

where $P_{fi}(b, E)$ denotes the transition probability during a single passing of the system through the crossing point. This probability can be written as

$$P_{fi}(b, E) = \exp[-\zeta(b, E)],$$  

(11)

$$\zeta(b, E) = \frac{2\pi |W_{fi}(R_x)|^2}{v(R_x)|V'_i(R_x) - V'_f(R_x)|},$$  

(12)

where $W_{fi}(R)$ denotes the transition matrix element of the interaction that couples the channels, $R_x$ is a crossing point between potentials $V_i(R)$ and $V_f(R)$, $v(R) = v_0\sqrt{1 - V_i(R)/E - b^2/R^2}$ is the classical velocity of the relative motion at the point $R$; $v_0$ denotes the asymptotic velocity, $E$ the kinetic energy and $b$ the impact parameter of the collisions. The upper limit $b_m(E)$ of the integration in Eq. (10) corresponds to the maximum impact parameter that allows the system to approach the crossing point that corresponds to the condition at the rightmost classical turning point $R_t(E, b_m) = R_x$ along the $R$-axis.

According to the approximation we adopted for $V_i(R)$ and $V_f(R)$, the initial and final wave functions of the system can be written as

$$|i\rangle = \Phi_{nlm}(r)\Psi_{X^1\Sigma^+}(r_1, r_2; R),$$  

(13)

$$|f\rangle = \Phi_{nl'm'}(r)\Psi_{A^1\Sigma^+}(r_1, r_2; R),$$  

(14)

where $\Phi_{nlm}(r)$ and $\Psi_{X^1\Sigma^+}(r_1, r_2; R)$ are wave functions of the antiproton and of the two electrons, respectively. The wave function $\Psi_{A^1\Sigma^+}(r_1, r_2; R)$ is dependent on the distance between the two subsystems. Indexes $X^1\Sigma^+$ and $A^1\Sigma^+$ indicate wave functions that correspond to the ground state of He$^+ +$H and He$^+ +$He systems, respectively. The process (2) under consideration involves electron transfer and a simultaneous antiproton transition to a lower orbit so that the transition operator has to depend on both electronic and antiproton variables. The transition operator can be obtained from the Coulomb interaction between the electron and antiprotonic ion constituents,

$$W = \sum_{i=1,2} \left[ \frac{1}{|R - \lambda r + r_i|} - \frac{2}{|R + \nu r + r_i|} \right].$$  

(15)

A detailed calculation of the transition matrix element $W_{fi}$ is complicated because of the requirement of knowing an extensive set of parameters for the initial and final electronic wave functions at various values of $R$. However, we can estimate this matrix element using the electronic $A^1\Sigma^+ \rightarrow X^1\Sigma^+$ transition dipole moment $D(R)$ tabulated in Ref. [16] as a function of $R$. For this aim, we approximate the non-diagonal part of $W$ by the interaction of the electronic dipole with the antiprotonic dipole and quadrupole momenta,

$$W \Rightarrow d_e \cdot \left\{ (\lambda + 2\nu)[r - 3n(r \cdot n)]/R^3 \right.$$  

$$+ \frac{3}{2}(\lambda^2 - 2\nu^2)[nr^2 + 2r(r \cdot n) - 5n(r \cdot n)^2]/R^4 \right\},$$  

(16)

where $d_e = \sum_i r_i$ and $n = R/R$. The matrix element of the vector $d_e$ is directed along $n$ due to axial symmetry of the $^1\Sigma^+$ terms,

$$\langle A^1\Sigma^+ | d_e | X^1\Sigma^+ \rangle = D(R) n,$$  

(17)

therefore we can write

$$W_{fi}(R) \approx D(R) \left[ \frac{2}{R^3}(\lambda + 2\nu)(l'm' | \cos \theta | lm) I_1(nl, n'l') \right.$$  

$$+ \frac{3}{R^4}(\lambda^2 - 2\nu^2)(l'm' | P_2(\cos \theta) | lm) I_2(nl, n'l') \right],$$  

(18)
where \( \cos \theta = (\mathbf{n} \cdot \mathbf{r})/r \), \( P_2(\cos \theta) \) is a Legendre polynomial,

\[
I_k(nl, n'l') = \int_0^\infty R_{nl}(r)R_{n'l'}(r)r^{k+2} dr, \tag{19}
\]

and \( R_{nl}(r) \) are hydrogen-like radial wave functions of the antiproton.

It should be noted that in addition to the last term in Eq. (18), there exist one more term with the same \( R^{-4} \) dependence corresponding to the interaction between the transition electronic quadrupole and the antiproton dipole momenta. Due to the lack of data on the electronic quadrupole moment, we omit this term keeping the second term in Eq. (18) in order to estimate the contribution from antiproton quadrupole transitions accompanied by electronic dipole transitions.

We have seen (see Fig. 2) that due to energy conservation, the neutralization of an antiprotonic helium ion having \( n \sim 30 \) requires a minimum change in the principal quantum number of at least \( \Delta n = 2 \). This implies minimum multipolarities \( \Delta l = 2 \) and \( \Delta l = 1 \) for initial circular orbits and \( l \leq n - 2 \) allowed transitions, respectively. In this paper, we calculate neutralization cross sections only for transitions with \( \Delta n = 2 \), \( \Delta l = 1 \) or 2.

The angular matrix elements of \( \cos \theta \) and \( P_2(\cos \theta) \) in Eq. (18) can be calculated in terms of Clebsch–Gordan coefficients, and then the averaged square of the transition matrix element

\[
|W_{fi}(R)|^2 = \frac{1}{2l+1} \sum_{m m'} |W_{fi}(R)|^2 \tag{20}
\]

is reduced to the form

\[
|W_{fi}(R)|^2 = \frac{4}{3} D^2(R)(\lambda + 2\nu)^2 \\
\langle l 0 1 0 | l' 0 0 \rangle^2 I^2_1(nl, n'l')/R^6 \\
+ \frac{9}{5} D^2(R)(\lambda^2 - 2\nu^2) \langle l 0 2 0 | l' 0 0 \rangle^2 I^2_2(nl, n'l')/R^8, \tag{21}
\]

where the first and second terms correspond to dipole and quadrupole antiproton transitions. Of course, an interference term is absent here due to the parity selection rule.

### 4 Neutralization rates

Using the above-mentioned approximations, we have calculated the cross sections and transition rates of the neutralization process (2) for an antiprotonic helium ion populating the initial state with \( n \)-values between 28 and 32, and various \( l \)-values \( l \leq n - 1 \) at the thermal energies \( T = 10 \) K used in the experiment [4]. In order to compare the neutralization process with the radiative decay of the antiprotonic helium ion, we calculated the transition rates

\[
\lambda(i \to f) = \rho \sigma(i \to f)v \tag{22}
\]

The typical energy dependence of the transition rate is shown in Fig. 3 for the antiproton dipole transition \((30, 28 \rightarrow 28, 27)\) at the target density \( \rho = 5 \times 10^{17} \) cm\(^{-3} \). It was found that this rate has a weak dependence on the kinetic energy, changing by less than 1% when the energy increases from 2 to 10 K. We therefore did not average over thermal motion, and instead used the value of the transition rate at kinetic energy \( E = T \).

Figures 4, 5 and 6 show the calculated neutralization rates for the initial states \( n = 28, 30, 32 \) depending on initial angular momentum \( l \) at a target temperature \( T = 10 \) K and density \( \rho = 5 \times 10^{17} \) cm\(^{-3} \). For comparison, the rates of radiative transitions and the total rates of Stark transitions [8,9] from the state \((n, l)\) to all states \((n', l') < l\) are also shown superimposed on the same figures. The rates of radiative
Fig. 5 Rates of transitions from the states with \( n = 30 \) and various orbital angular momentum quantum numbers. The definitions of the curves are the same as those of Fig. 4

Fig. 6 Rates of transitions from the states with \( n = 32 \) and various orbital angular momentum quantum numbers. The definitions of the curves are the same as those of Fig. 4

Transitions were calculated using the analytical expression for hydrogen-like atoms [19] with the scaling by the antiproton reduced mass \( \mu \) and the nuclear charge \( Z = 2 \). The shown neutralization rates correspond to dipole antiprotonic transitions from non-circular orbits. As for circular orbits, the dipole transitions are forbidden by energy conservation, whereas quadruple ones are suppressed at least by an order of magnitude; therefore, they are not shown in the figures.

In addition to the neutralization of \( (\bar{p}^4\text{He}^{2+}) \), we have also considered a similar process for \( (\bar{p}^3\text{He}^{2+}) \). Figure 7 shows the neutralization rates of the two isotopes as a function of the initial orbital angular momentum quantum number \( l \) of the \( n=30 \) ionic states, which were calculated for targets with temperature \( T = 10 K \) and density \( \rho = 5 \times 10^{17} \text{ cm}^{-3} \). The sign of the isotope effect coincides with the experimental data, i.e., the rates of transitions for \( ^3\text{He} \) are greater than for \( ^4\text{He} \), although the isotopic effect is rather small, especially for nearly circular states. The isotope effect slowly increase for states of lower \( l \), and approaches several percent for \( l = 20 \).

5 Concluding remarks

We have considered the process of the antiprotonic helium ion neutralization accompanied by antiproton transitions to lower states during collisions with helium atoms

\[
(\bar{p}\text{He}^{2+})_{nl} + \text{He} \rightarrow [(\bar{p}\text{He}^{2+})_{n'l'}e]_{1s} + \text{He}^+
\]

in low-temperature helium targets. This process can also be considered as a charge exchange between the ion and atom. A simple model of the process is developed, allowing the estimation of the cross sections and transition rates. It is shown that the neutralization gives a remarkable contribution to the total rates of antiprotons deexciting from the states with \( n = 28-32 \) to lower-lying states. The rates of this process are up to 50% of the radiative transition rates. The isotopic effect on the process has the same sign as in the experiment, but its value is rather small.

Note that the developed model can be used to calculate similar processes with ions of other hadronic atoms, in particular, of pionic helium, for which data on long-lived states were recently obtained [20].

As a general conclusion, it was shown that the neutralization process has to be taken into account along with the radiative transitions in the cascade calculations of the effective annihilation rates for the antiprotonic helium ion.

Acknowledgements One of the authors (G.K.) thanks Dr. M. Hori for his interest in the work and comments on the experimental data.

Author contributions

GK conceived this research, and SY provided computer codes and numerical results. Both authors contributed
to the work presented here and reviewed the final text of the manuscript.

Data Availability Statement  This manuscript has no associated data or the data will not be deposited. [Authors’ comment: Any data can be obtained directly from the first author.]

References

1. T. Yamazaki, N. Morita, R.S. Hayano, E. Widmann, J. Eades, Phys. Rep. 366, 183–329 (2002)
2. R.S. Hayano, M. Hori, D. Horváth, E. Widmann, Rep. Prog. Phys. 70, 1995–2065 (2007)
3. M. Hori, Philos. Trans. R. Soc. Lond. Ser. A 376, 2017.00 (2018)
4. M. Hori, J. Eades, R.S. Hayano et al., Phys. Rev. Lett. 94, 065301 (2005)
5. G. Reifenröther, E. Klempt, R. Landua, Phys. Lett. B 203, 9 (1988)
6. R. Landua, E. Klempt, Phys. Rev. Lett. 48, 1722 (1982)
7. T.S. Jensen, V.E. Markushin, Eur. Phys. J. D 19, 165 (2002)
8. G.Ya. Korenman, S.N. Yudin, J. Phys. Conf. Ser. 88, 012060 (2007)
9. G.Ya. Korenman, S.N. Yudin, In Proceedings of International Conference on Muon Catalyzed Fusion and Related Topics (MuCF-07), Dubna, 2007, edited by L.N. Bogdanova et al. (JINR, Dubna, 2008), p. 191
10. S.N. Yudin, I.V. Bodrenko, G.Ya. Korenman, arXiv:1612.03874v3 [nucl-th]
11. V.I. Korobov, Phys. Rev. A 54, R1749 (1996)
12. Y. Kino, M. Kamimura, H. Kudo, Nucl. Phys. A 631, 649 (1998)
13. J.E. Russel, Phys. Rev. A 1, 721 (1970)
14. M. Juřek, V. Špirko, W.P. Kraemer, Chem. Phys. 193, 287 (1995)
15. J. Loreau, J. Liévin, P. Palmeri et al., J. Phys. B At. Mol. Opt. Phys. 43, 065101 (2010)
16. W.P. Kraemer, V. Špirko, M. Juřek, Chem. Phys. Lett. 236, 177 (1995)
17. N.F. Mott, H.S.W. Massey, The Theory of Atomic Collisions (Springer, Berlin, 1965)
18. E.E. Nikitin, S.Ya. Umansky, Theory of Slow Atomic Collisions (Springer, Berlin, 1984)
19. H.A. Bethe, E.E. Salpeter, Quantum Mechanics of One- and Two-Electron Atoms (Springer, Berlin, 1957)
20. M. Hori, H. Aghai-Khozani, A. Sótér et al., Nature 581, 37 (2020)