Comments on the Quasi-Steady-State Cosmology

Edward L. Wright
UCLA Astronomy, Los Angeles CA 90024-1562

ABSTRACT

The Quasi-Steady-State Cosmology as proposed by Hoyle, Burbidge and Narlikar does not fit the observed facts of the Universe. In particular, it predicts that 75-90% of the radio sources in the brightest sample that shows steeper than Euclidean source counts should be blueshifted.

1. Introduction

The quasi-steady-state cosmology (QSSC) of Hoyle, Burbidge & Narlikar (1994, hereafter HBN) is based on a spatially flat Universe with a scale factor having an exponential growth multiplying a sinusoidal oscillation. Since the observed expansion of the Universe is primarily due to the oscillation, this QSSC model provides 200 Gyr in which to produce the cosmic microwave background (CMB), instead of the $0.25/H_0 \approx 4$ Gyr allowed in the Steady-State cosmology. In addition, the large volume of the Universe during its oscillatory maxima provides a mechanism for steeper than Euclidean source counts. In this paper I show that it is still difficult, if not impossible, to produce the CMB in the QSSC model proposed by HBN; and that producing the CMB is incompatible with HBN’s model for the faint blue galaxies. The large millimeter wave optical depth required to blacken the CMB makes the millimeter wave luminosity of the QSO BR 1202-0725 the highest known luminosity. I further show that steeper than Euclidean source counts are always accompanied by a redshift distribution that is dominated by blueshifts.

2. The CMB Power Problem

In the QSSC model, the CMB is produced by dust absorbing the diffuse extragalactic background light (EBL), and re-emitting the energy in the millimeter region. The large millimeter opacity required is produced by iron whiskers. There is a relation between the ratio of EBL to CMB energy densities and the visible optical depth through an oscillatory minimum. In this section I will consider a simple model, with a single frequency bin for the “optical” light that
heats the dust which makes the CMB, and a second frequency bin for the CMB. If I assume a
grey opacity in the optical, then the “optical” energy density $U_{EBL}$ satisfies the equation
\[ d[a^4U_{EBL}]/dt = 4\pi a^4 j - \kappa ca^4U_{EBL} \]  
(1)

where $4\pi j$ is the luminosity density, $\kappa$ is the optical depth per unit length, and $a$ is the scale factor
of the Universe. In the QSSC model $u$, $j$ and $\kappa$ are periodic functions of time with period $Q$. The
scale factor follows
\[ a(t) \propto (1 + \alpha \cos(2\pi t/Q)) \exp(t/P) \]  
(2)

where HBN gives values $\alpha = 0.75$, $Q = 40$ Gyr, $P/Q = 20$, and $t_0/Q = 0.85$. I will now assume
that the luminosity density and dust density follow $j = j_1/A^3$ and $\kappa = \kappa_1/A^3$. With these
assumptions it is easy to find periodic solutions for $U_{EBL}(t)$. Define
\[ F(t) = \exp(-\kappa_1 c \int A^{-3}(t)dt) \]  
(3)

and then
\[ U_{EBL} = a^{-4} F(t) \left( C_1 \int F^{-1}(t')A(t') \exp(4t'/P) dt' + C_0 \right) . \]  
(4)

The ratio of $C_1$ to $C_0$ can be adjusted to give a periodic solution.

The “CMB” energy density $U_{CMB}$ satisfies the equation
\[ d[a^4U_{CMB}]/dt = +\kappa ca^4U_{EBL} \]  
(5)

where $\kappa$ is again the visual opacity. Here I assume that any CMB radiation absorbed by dust is
reradiated as CMB radiation, and the net effect on the energy density cancels out. The solution
to this equation is
\[ U_{CMB} = a^{-4} \left( \int \kappa_1 cA(t') \exp(4t'/P)U_{EBL} dt' + U_0 \right) \]  
(6)

and I choose the constant of integration $U_0$ to make $U_{CMB}$ a periodic function.

The ratio of the optical light now (at $t_0/Q = 0.85$) to the CMB now is shown as a function of
the total visible optical depth through an oscillation in Figure 1. Even for $\tau_{opt} >> 1$ the required
optical power remains higher than the observed limits on the EBL. An EBL brightness of $1 S_{10}$
in the visible gives $U_{EBL}/U_{CMB} \lesssim 0.01$, while the QSSC requires $U_{EBL}/U_{CMB} \geq 0.05$ even for $\tau_{opt} = 8$.

The quantity $U_{CMB} + \tau_{opt}U_{EBL}/\tau_{mm}$ is $\propto T_{eq}^4$, where $T_{eq}$ is the equilibrium temperature of
the dust, and $\tau_{mm}$ is the millimeter wave optical depth through the oscillatory cycle. But dust
observed at a redshift $1 + z = 1/a$ has an apparent temperature of $T_{ap} = T/(1 + z)$, which is
\[ \propto ((U_{CMB} + \tau_{opt}U_{EBL}/\tau_{mm})a^4)^{0.25} . \]  
This quantity is plotted versus $t/Q$ for several values of $\tau_{opt}$ while letting $\tau_{mm} \to \infty$ in Figure 2. For larger values of $\tau_{opt}$ the 5% jump in $T_{ap}$ slightly before
the oscillatory minimum instead of at the minimum. This slightly reduces the millimeter optical
depth required to adequately blacken the CMB.
For a grey opacity in the millimeter range, it is easy to show that the required opacity is $\tau \gtrsim 4$ between now and the first 5% temperature step. The spectrum is approximately a superposition of blackbodies: $T_0$ with weight $1 - e^{-\tau}$ and $0.95T_0$ with weight $e^{-\tau}$. The resulting $\text{var}(T)/T^2 = e^{-\tau}/20^2$. But since $\text{var}(T)/T^2 = 2y < 5 \times 10^{-5}$ according to Mather et al. (1994), one needs $e^{-\tau} < 2y(P/Q)^2 = 0.02$, or $\tau > 4$ to the oscillatory minimum. Numerical integration of $\text{var}(T)/T^2$ for a grey millimeter opacity shows that $\tau_{mm} \geq 6$ gives a sufficiently small $y$ if $\tau_{opt}$ is small, but $\tau_{mm} \geq 4$ suffices if $\tau_{opt} >> 1$. The opacity curve given in HBN’s Figure 5 is far from grey, but this calculation agrees well with the HBN estimate of $\tau \approx 10$ through the minimum for adequate blackening of the CMB.

3. Millimeter Point Sources at High Redshift
The observations of McMahon et al. (1994) essentially rule out the published version of the QSSC, since they have observed millimetric fluxes from quasars with $z > 4$. As discussed above, the QSSC requires that the millimeter transmission between $t_o$ and the oscillatory minimum be very close to zero, and these high redshift quasars are at or beyond the oscillatory minimum. The ratio of the optical depth to $z = 4.69$ for an observed wavelength of 1.25 mm to the 1.4 GHz optical depth through a full oscillation is 63. This calculation is based on the opacity vs. $\lambda$ from Figure 5 of HBN. Since the 1.4 GHz optical depth should be $> 0.125$ to give enough CMB flux at 1.4 GHz, the quasars observed by McMahon et al. must have gone through $\tau > 8$. Thus the QSSC requires that these sources be $> 2000$ times more luminous in millimeter waves than one would normally assume. This raises $\nu L_\nu$ for BR 1202-0725 to $5 \times 10^{15} \ L_\odot$. If 3C273 had this $\nu L_\nu$ in the V band its magnitude would be $V = 4.5$. 

4. Blueshifts
Fig. 3.— Source counts and blueshifted fraction for the power law luminosity function used by HBN. The dashed curves show the effect of radio absorption with $\tau = 0.125$ at 21 cm.

I have computed source counts using the formulae given by HBN, which are the standard equations of relativistic cosmology once the non-standard quasi-periodic $a(t)$ is assumed. HBN have assumed that radio sources are uniformly distributed in physical volume, rather than the usual assumption of a constant comoving density. This assumption maximizes the steepness of the source counts, which is needed to fit the observed data, but it also increases the blueshifted fraction. The relevant equations are:

$$\frac{1 + z}{a(t_o)/a(t)} \quad (7)$$

and the comoving radius of our past light cone is

$$r = \int_t^{t_o} (1 + z)cdt. \quad (8)$$
The angular size distance is \( r_A = r/(1 + z) \) while the luminosity distance is \( r_L = (1 + z)r \). A source with luminosity \( L_\nu = L_\odot(\nu/\nu_\odot)^{-\gamma} \), where \( \nu_\odot \) is the observed frequency now, has a flux

\[
F(\nu_\odot) = \frac{(1 + z)^{1-\gamma}L_\odot}{4\pi r_L^2}.
\]  

The source counts are given by

\[
n(F) = \int_{-\infty}^{t_0} \phi(4\pi r_L^2(1 + z)^{-\gamma}e^\tau F) 4\pi r_L^2(1 + z)^{-\gamma}e^\tau r_A^2 cdt
\]  

where \( n(F)dF \) is the source count per steradian in the flux range \([F, F + dF]\), the luminosity function \( \phi(L_\odot)dL_\odot \) is the number density of sources with luminosities in the range \([L_\odot, L_\odot + dL_\odot]\), and the blueshifted source counts are

\[
n_B(F) = \int_{-\infty}^{t_0} P_B(z) \phi(4\pi r_L^2(1 + z)^{-\gamma}e^\tau F) 4\pi r_L^2(1 + z)^{-\gamma}e^\tau r_A^2 cdt
\]  

where \( P_B(z) = 1 \) if \( z < 0 \) and zero otherwise. Figure 3 shows the normalized cumulative source counts \( N(F)F^{1.5} = F^{1.5} \int_F^\infty n(F')dF' \) and the blueshifted fraction \( FB = N(F)^{-1} \int_F^\infty n_B(F')dF' \) for the luminosity function used by HBN: a one decade wide function going as \( L^{-2.1} \). While HBN did not specify a radio spectral index, this figure has been computed for sources with a \( L_\nu \propto \nu^{-0.75} \) spectrum. The solid curves were computed with \( \tau = 0 \), while the dashed curves in this figure show the effect of radio absorption when the optical depth at 1.4 GHz is \( \tau = 0.125 \) through the first oscillatory minimum. Note that the blueshifted fraction jumps to more than 90% for the brightest sources that show a steeper than Euclidean source count. Thus the 3CR sample, which has source counts that are significantly steeper than Euclidean, should be dominated by blueshifts if the QSSC were correct. While it is possible to have no blueshifts in the 3CR flux range by a suitable choice of the luminosity function, one then predicts source counts that are substantially less steep than Euclidean, producing a \( > 3\sigma \) discrepancy in source count slopes between the model and the data (Jauncey, 1975). At the peak of \( NF^{1.5} \), the blueshifted fraction predicted by the QSSC is \( > 75\% \). The 1 Jy sample of Allington-Smith et al. (1988) is at the peak of \( NF^{1.5} \), has a secure redshift completeness \( > 80\% \) (Rawlings et al., private communication), and no blueshifts. Since one can not fit a \( > 75\% \) blueshifted fraction into a \(< 20\% \) unidentified fraction, the QSSC prediction is false.

Another way of looking at this problem is shown in Figure 4. In this plot the integrated physical volume is shown as \( N \), with

\[
N = \int_t^{t_0} r_A^2 cdt.
\]  

The blueshifted region is labeled \( B \). The flux range in which blueshifted sources occur in the first oscillatory maximum occur is shown by the lower two horizontal line segments, which carry this flux range over to “normal” branch in the current half-cycle. In the region labeled \( R \) the redshifted sources have the same fluxes as the blueshifted sources. The range of redshifts during this interval is \( 3.12 < z < 3.53 \). But the observed physical volume in the interval \( R \) is 6000 times
smaller than the volume in the interval $B$. Thus for every source in the interval $R$ there should be 6000 blueshifted sources. The flux intervals are identical, so this multitude of blueshifted sources should definitely be present in radio surveys. The 1 Jy sample has at least 1 source in the interval $R$, a radio galaxy with redshift 3.4 (Lilly 1988), so it should contain 6000 blueshifted sources if the QSSC were correct. It is also possible that this galaxy is just past the oscillatory minimum, in the region $R'$. But this possibility only decreases the volume ratio to 1500, while adding a factor of $\geq 5$ flux ratio. Thus, if the galaxy were due to a source in $R'$, then one would expect 1500 blueshifted sources in a sample of sources brighter than 5 Jy at 408 MHz within the 1 Jy sample area of 0.11 sr. But this flux level corresponds to 9 Jy at 178 MHz for $F_\nu \propto \nu^{-0.75}$, so all of these sources would be in the 3C catalog. But Spinrad et al. (1985) have redshifts for 214 out of the 235 3CR sources with $|b| > 15^\circ$, a solid angle of $> 4$ sr, so this option requires that 50,000 of the 21 3CR sources without measured redshifts would have to be blueshifted. Of course, the radio galaxy could be even more distant, in the region marked $R''$. But the blueshifted volume in $B$ is 250 times larger than the volume in $R''$ and the flux ratio is $\geq 19$, so this explanation of the radio galaxy requires 250 blueshifted sources brighter than 19 Jy at 408 MHz in 0.11 sr, or 25,000 such sources.
over the whole high galactic latitude sky. Finally, the region $R''$ has a volume that is 135 times smaller than the blueshifted volume in $B$, and a flux ratio $\geq 34$. This possibility requires 13,500 blueshifted sources brighter than 34 Jy over the whole sky. Thus for each possibility the QSSC requires that there be many times more blueshifted sources than the total number of observed sources.

I note that the Hewitt & Burbidge QSO catalog (1993) contains roughly 130 QSOs with $3.12 < z < 3.53$ but no blueshifts.

It is this sudden addition of the sources from the last oscillatory maximum that creates the steep source counts in the QSSC model, but the brightest of these sources, and hence the easiest to see, are blueshifted. The QSSC cannot have both steep source counts and no blueshifts simultaneously, but the observed Universe does. Hence the QSSC is not a successful model for the Universe.

5. Conclusion

The QSSC is disproved by the same data that disproved the Steady State: counts of bright radio sources. This failure of the QSSC model, and the CMB power problem, were fully explained to both HBN and to the editor of MNRAS in a referee’s report dated July 1993, which included a more extensive version of Figure 3, but HBN proceeded to publish incorrect results.

The saddest part of this tale is that the QSSC can be compatible with the radio source counts, the QSO $\langle V/V_{\text{max}} \rangle$ test, and the CMB; by making $\alpha$ larger and $\tau$ large which eliminates the blueshifts. To get the steep source counts one adds a new factor $[(1 + \alpha \cos(2\pi t))/1 + \alpha \cos(2\pi t)]^n$ inside the integrals in Equations 10 and 11. This introduces density evolution into the QSSC, but in a periodic way. While the philosophy of the Steady State model did not allow evolution of the radio source population, the QSSC model certainly allows a periodic evolution of the source population. In fact, one might expect a high density of radio sources at the oscillatory minima. The only problem with this approach is that it eliminates the observational differences between the QSSC and the Big Bang, and there is nothing left to motivate the introduction of new physics. But eliminating these differences is necessary, since the Big Bang can fit the data, while HBN’s QSSC model does not.

REFERENCES

Allington-Smith, J. R., Spinrad, H., Djorgovski, S. & Liebert, J. 1988, MNRAS, 234, 1091-1103.
Hewitt, A. & Burbidge, G. 1993, ApJS, 87, 451.
Hoyle, F., Burbidge, G. & Narlikar, J. 1994, MNRAS, 267, 1007.
Jauncey, D. L. 1975, ARAA, 13, 23.
Lilly, S. J. 1988, ApJ, 333, 161.
Mather, J. C. et al. 1994, ApJ, 420, 439-444.
McMahon, R. G., Omont, A., Bergeron, J., Kreysa, E. & Haslam, C. G. T. 1994, MNRAS, 267, L9-L12.
Spinrad, H., Djorgovski, S., Marr. J. & Aguilar, L. 1985, PASP, 97, 932.