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ABSTRACT

Numerical researches on the impact of a drop onto a bounded liquid bath are carried out using GERRIS code. A particular region of delayed bubble entrapment is presented. During a typical impact, a first crater, a liquid column and a second crater appear successively. It is found that gravity, restricted bath diameter and impact velocity play important roles in the whole process. With a proper bath diameter, the outward propagating waves at the liquid surface can be reflected back to the center in time and lead to a liquid column. The delayed bubble can be entrapped when the second crater is deep enough. When the bounded liquid bath is too narrow or too broad, no delayed bubble is formed, as the liquid column breaks before the second crater becomes deep enough. The mechanism of delayed bubble entrapment is explained through surface profiles and velocity fields.

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I. INTRODUCTION

Various phenomena come out after a liquid droplet impacts onto a liquid surface, such as floating, bouncing, coalescence, jetting, splashing, and bubble entrapment. These phenomena are associated with a lot of applications, such as metallurgical processes, spray cooling, spray printing, and dissolved oxygen increment. When a droplet impacts onto a broad deep bath, phenomena vary with the Weber number $We$ and the Froude number $Fr$. They are defined as

$$We = \frac{\rho_L V^2}{\sigma},$$

$$Fr = \frac{V}{\sqrt{gD}},$$

where $\rho_L$ is the liquid density, $D$ the drop diameter, $V$ its velocity, $\sigma$ the surface tension and $g$ is the acceleration due to gravity. With a low impact velocity ($Fr < 7$), coalescing, floating and bouncing occur without bubble entrapment or jet. Primary bubble occurs in higher velocity region ($36.2 Fr^{0.186} < We < 48.3 Fr^{0.247}$), together with a high-speed jet. Oguz and Prosperetti explained the formation of bubble was determined by a delicate balance between the times at which the outward motion of the crater walls was reversed at different positions. Large bubble is another bubble phenomenon whose size is as large as the droplet. Thoraval et al. used experiments and simulations to show that a concentrated vortex-ring controlled the formation process. At even higher velocities close to the terminal velocity, a crown rises high and close, forming a canopy bubble that floats at the surface.

Besides the impact velocity, restrictions of liquid zone, including bath depth in vertical direction and surrounding walls in horizontal direction, play important roles on the impact process. Based on the dimensionless ratio $h^*$, defined as

$$h^* = h/D,$$

where $h$ is the bath depth, the vertical restriction can be divided into three regions: deep baths ($h^*>>1$), shallow baths ($h^*<<1$) and thin films ($h^*<<1$). Vander Wal et al. investigated liquid baths with $0.1<h^*<10$, they found that thin films promoted splashing while shallow and deep baths inhibited it. Zou et al. found that the success probability of drop bounce almost remained...
constant, as the depth exceeded five times the maximum crater depth.

Nevertheless, restriction in horizontal direction is rarely studied, until Zou et al. did the pioneer works. The behaviors of drops impacting on bounded liquid baths with various \( D_b^* \) were studied using a high-speed video camera. The nondimensional bath diameter \( D_b^* \) is defined as

\[
D_b^* = \frac{D_t}{D},
\]

where \( D_t \) is the bath diameter. Different from the broad surface, the outwards propagating waves can be reflected back to the center and promote the flow processes. The outcomes, such as bouncing, large bubble entrapment and canopy, are dependent on the distance between the surrounding walls and the impact point. For the bouncing phenomenon, if the reflected waves reach the impact point before the drop rebounds from the liquid surface, the restitution coefficient will increase to about four times than that on broad surface. The canopy phenomenon can be observed frequently in small liquid baths, as the boundary walls stop the expansion of the crater and squeeze liquid into the crown. A unique phenomenon named delayed bubble was first observed by Zou et al. It is quite similar with the primary bubble, which is formed after the collapse of the first crater. However, the delayed bubble is formed after the crater-column-crater oscillation. Both the oscillation and the delayed bubble phenomenon only happen when a droplet impact onto a bounded liquid bath, and the mechanism is still unclear.

In this paper, a study of the dynamics of a drop impact onto a bounded liquid bath is presented. A numerical method is used to analyze the underlying mechanism. An approximate region of delayed bubble entrapment is carried out. Effect of gravity, diameter of the liquid bath, and impact velocity are investigated in detail. The liquid behaviors can be characterized by two nondimensional parameters: the position of the bath at the wall boundary \( H_b^* \), and the height of the surface at the center \( H_c^* \). They are defined as

\[
H_b^* = \frac{H_b}{D}, \quad \text{(5)}
\]

\[
H_c^* = \frac{H_c}{D}, \quad \text{(6)}
\]

where \( H_b \) is the position of the bath at the wall boundary, and \( H_c \) the height of the surface at the center. At the initial status we have \( H_b^* = 0 \), \( H_c^* = 1.05 \).

**II. EXPERIMENTAL RESULTS**

The delayed bubble was first discovered by Zou et al. They investigated a droplet impact upon a bounded liquid surface. Highly purified water was held by glass tubes of inner diameters \( D_t = 7.9 \text{mm}, 12.0 \text{mm}, 17.0 \text{mm} \) and \( 26.0 \text{mm} \), while droplets with uniform diameter \( D = 2.64 \text{mm} \) were produced with a stainless steel needle. A high speed video camera (FASTCAM-ultima APX, USA) and a Nikkor 60-mm micro lens are employed to capture the images. The speed of the high-speed camera is 2000fps (frames per second) and the resolution is 68μm/pixel. Flickerless backlighting is produced by a high-intensity LED lamp with a thin sheet of drafting paper as a diffuser. The environment pressure was kept at 101.3kPa. The viscosity and surface tension coefficient of the liquid were 0.8937 × 10^{-3} \text{ Pa·s} and 0.07197N/m at temperature of 25°C, respectively. The static contact angle between the tube wall and the liquid was 60.8° with uncertainty of 1°, which was measured from the images. The impact velocity was measured from the images with an uncertainty of 0.04m/s. The maximum standard deviation of drop diameter is 0.4mm. The error of Weber numbers is estimated to be less than 0.2. Each experimental point was performed several times (3 times as a minimum) to ensure the repeatability.

A typical case with a bath diameter \( D_b^* = 4.54 \) \((D_t = 12.0 \text{mm})\) and \( \text{We} = 73.3 \) is shown in Fig. 1. After the impact, a first crater, a liquid column and a second crater appear in turn, which can be described as crater-column-crater oscillation. Here we name the jet which is totally above the surface as liquid column. Then a necking process (from 65ms to 70ms) occurs during the collapse of the second crater, and finally entraps a delayed bubble at 71ms.

**III. NUMERICAL RESULTS**

**A. Method and settings**

We perform numerical simulations to explore drop impact on a bounded liquid bath using the GERRIS code. It is an open source software, and it has been validated by researchers on similar drop
impact phenomena.\textsuperscript{9,18} The governing equations of incompressible
Newtonian fluids can be written\textsuperscript{9,18}
\begin{equation}
\rho(\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u}) = -\nabla p + \nabla \cdot (2\mu \mathbf{D}) + \sigma \kappa \delta \mathbf{n},
\end{equation}
\begin{equation}
\partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0,
\end{equation}
\begin{equation}
\nabla \cdot \mathbf{u} = 0,
\end{equation}
with \( \mathbf{u} \) the fluid velocity vector, \( p \) the pressure, \( \rho \equiv \rho(x,t) \) the density, 
\( \mu \equiv \mu(x,t) \) the dynamic viscosity, \( \sigma \) the surface tension coefficient, 
\( \mathbf{D} \) the deformation tensor defined as \( \mathbf{D} = (\partial_i u_j + \partial_j u_i)/2 \). Note that 
\( x \) is the position vector. The Dirac distribution function \( \delta \) expresses 
the fact that surface tension is concentrated on the interface, \( \kappa \) and 
\( \mathbf{n} \) the curvature and normal to the interface. For two-phase fluid, the 
density and dynamic viscosity can be defined as
\begin{equation}
\rho = \tilde{\rho} c_1 + (1 - \tilde{c}) \rho_2,
\end{equation}
\begin{equation}
\mu = \tilde{\mu} c_1 + (1 - \tilde{c}) \mu_2,
\end{equation}
where \( \rho_1, \rho_2 \) and \( \mu_1, \mu_2 \) the densities and viscosities of the first and 
second fluids, respectively. \( c(x,t) \) is the liquid volume fraction of the 
first fluid (VOF method), defined as
\begin{equation}
c(x,t) = \begin{cases}
1 & \text{the first fluid exits at } x \\
0 & \text{the first fluid not exits at } x, 
\end{cases}
\end{equation}
while \( 0 < c(x,t) < 1 \) represents the interface. Field \( \tilde{c} \) is either identical 
to \( c \) or is constructed by applying a smoothing spatial filter to \( c \). 
The density advection equation can be replaced by an equivalent 
equation for the volume fraction
\begin{equation}
\partial_t c + \nabla \cdot (c \mathbf{u}) = 0.
\end{equation}
For the original continuum-surface-force \( \sigma \kappa \delta \mathbf{n} \), it can be calculated 
approximately by the following method
\begin{equation}
\sigma \kappa \delta \mathbf{n} = \sigma \kappa \nabla \tilde{c},
\end{equation}
\begin{equation}
\kappa \approx \nabla \cdot \mathbf{n} \equiv \nabla \cdot \frac{\nabla \tilde{c}}{|\nabla \tilde{c}|}.
\end{equation}
A staggered in time discretization of the volume fraction/density and 
pressure leads to the following formally second-order accurate time 
discretization
\begin{equation}
\rho_{n+\frac{1}{2}} \left[ \frac{\mathbf{u}_{n+\frac{1}{2}} - \mathbf{u}_{n-\frac{1}{2}}}{\Delta t} + \mathbf{u}_{n-\frac{1}{2}} \cdot \nabla \mathbf{u}_{n+\frac{1}{2}} \right] = -\nabla p_{n+\frac{1}{2}} + \nabla \cdot \left[ \mu_{n+\frac{1}{2}} (\mathbf{D}_{n} + \mathbf{D}_{n+1}) \right] + (\sigma \kappa \delta \mathbf{n})_{n+\frac{1}{2}} \mathbf{u}_{n+\frac{1}{2}},
\end{equation}
\begin{equation}
c_{n+\frac{1}{2}} - c_{n-\frac{1}{2}} \Delta t + \nabla \cdot (c_n \mathbf{u}_n) = 0,
\end{equation}
\begin{equation}
\nabla \cdot \mathbf{u}_n = 0.
\end{equation}
Space is discretized using a graded quadtree partitioning (octree 
in three dimensions). The detail numerical methods can be found in 
Stephane Popinet’s works.\textsuperscript{16,17} GERRIS code solves the Navier-
Stokes equations with a volume-of-fluid method for two-phase flow, 
by a dynamic adaptive grid refinement. We use axisymmetric simu-
lation method in the GERRIS code, the computational domain is 
shown in a symmetrical form in Fig. 2, only the right half is for calcu-
lation. The left boundary is the axisymmetric axis. The top boundary 
is set outflow \((p = 0, \frac{\partial p}{\partial y} = 0)\). The right side and bottom boundaries are 
set no-slip solid walls. Concerning the contact angle makes the prob-
lem too complicated, which cannot be completed with the GERRIS 
code. It should deserve a separate study and is therefore out of the 
scope of the present research. Consequently, we use the default con-
stant 90° contact angle in GERRIS code. A spherical droplet with

![Figure 2](https://scitation.aip.org/content/aip/journal/adv/9/10/10.1063/1.5124537)

**FIG. 2.** Computational domain for impact of a droplet onto a bounded liquid bath, shown in a symmetrical form, only the right half is for calculation.

| TABLE I. Four types of grid, with different refinements, for \( D_i^* = 4.54 \). |
| --- |
| coarsest grid level | coarsest grid cell size | finest grid level | finest grid cell size |
| (I) | 5 | 0.071D | 7 | 0.018D |
| (II) | 6 | 0.036D | 8 | 0.009D |
| (III) | 7 | 0.018D | 9 | 0.004D |
| (IV) | 8 | 0.009D | 10 | 0.002D |
FIG. 3. Comparison of grid (I) on the left and grid (III) on the right, at \( t = 6 \text{ms}, \ We = 73.3, \ D_t^* = 4.54 \).

FIG. 4. Typical time sequence of drop impact on a bounded liquid bath, \( We = 73.3, \ D_t^* = 4.54 \), using grid (I) shown in Fig. 3 on the left.

FIG. 5. Typical time sequence of drop impact on a bounded liquid bath, \( We = 73.3, \ D_t^* = 4.54 \), using grid (III) shown in Fig. 3 on the right.
diameter $D$ is initialized set above the liquid bath ($0.05D$), with a certain impact velocity $V$. We add several calculating boxes (each box is a square with a side length of $0.5D_t$) in vertical direction to form a rectangle computational domain, whose width and vertical height are $0.5D_t$ and $L_1+L_2$, respectively. The height above liquid bath $L_1$ is large enough ($>5D$, higher than the maximum column height) to capture the whole process and the bath depth $L_2$ is large enough ($>10D$) to avoid the influence of the bottom. Hobbs and Osheroff studied the collision of a droplet onto liquid pools with different depths. They found that with depth of 25mm or larger, the sequence of results is the same as that of splashing on a deep pool. Considering the calculating time, we set the pool depth as $L_2>10$, note that, $25mm/2.64mm=9.47$. Two tracers are used in the simulations, a main tracer to identify the air-water interface, and a passive tracer to identify the liquid originating from the drop. The two tracers are independent of each other.

### B. Grid dependence

GERRIS code have the dynamic adaptive grid refinement function, by setting the coarsest grid level and finest grid level. The case $D_t^*=4.54, We=73.3$ is calculated using four kinds of grid to test the grid dependence, which are shown in Table I. The grid cell size can be calculated as $0.5D_t/2^n$, $n$ is the refinement level, $0.5D_t$ is the side length of the calculating box. The comparison of grid (I) and (III) is shown in Fig. 3, the left side is the axisymmetric boundary and the right side is the wall boundary. The grid is refined near the air/water interface, the passive tracer interface, and in the regions of high vorticity. It is observed a coarse grid could not capture the physics of the process very well, the delayed bubble phenomenon is not observed as shown in Fig. 4 for grid (I). However, with fine grid (III) (shown in Fig. 5) and (IV), the delayed bubble can be captured well. As a matter of fact, it is the grid cell size ($0.5D_t/2^n$) that influences the simulation. Thus we choose different refinement levels $n$ for different $D_t$ to keep the grid cell sizes approximately equal to that in grid (III), which gives consideration to both precision and computation time, shown in Table II. The finest grid cell size is $11\mu m$ approximately the same as $15\mu m$ in the work of Deka et al.²⁰

### C. Validation

A typical time evolution of delayed bubble entrapment ($D_t^*=4.54, We=73.3$) using grid (III), is shown in Fig. 5. The numerical results are in good agreement with the experimental results. A thin air sheet is formed and become a little bubble (shown in Fig. 5 at 5ms) during the coalescence of the drop and the bath, which is called the Thoroddsen bubble.²¹ The sequence of crater-column-crater oscillation is identical to that in experiment, and finally a delayed bubble is entrapped. The delayed bubble is entrapped at 71ms for experiment and 77ms for numerical simulation. This difference may be caused by the neglect of contact angle, which affect the liquid structure close to the boundary walls.

### TABLE II. Grid refinement levels for different bath diameter $D_t^*$.  

| $D_t^*$  | 3.0 | 4.0 | 4.54 | 5.0 | 6.0 | 7.0 | 8.0 | 10.0 | 15.0 | 20.0 |
|----------|-----|-----|------|-----|-----|-----|-----|------|------|------|
| coarsest grid level | 7  | 7  | 7  | 7  | 8  | 8  | 8  | 9  | 9  | 9  |
| finest grid level | 9  | 9  | 9  | 9  | 10 | 10 | 10 | 11  | 11  | 11  |

![FIG. 6. Numerical (solid symbols) and experimental (open symbols) results of the parameter space.](image-url)
FIG. 7. Typical time sequence of drop impact on a bounded liquid bath, experimental results, a) $We=73.3$, $D_t^*=4.54$, b) $We=94.8$, $D_t^*=4.54$, c) $We=115.0$, $D_t^*=4.54$. The scale bars are 2 mm long.

FIG. 8. Velocity fields of drop impact on a bounded liquid bath, numerical results, a) $We=73.3$, $D_t^*=4.54$, b) $We=105.6$, $D_t^*=4.54$, c) $We=127.1$, $D_t^*=4.54$. The red color represents upward velocity and the blue color represents downward velocity.
D. Parameter space

We explore the effect of $D_t^*$ and velocity on delayed bubble entrapment. The $We-D_t^*$ parameter space is shown in Fig. 6. Numerical results are generally in accordance with the experimental results. The dash oval represents an approximate region of delayed bubble. The delayed bubble forms in the range from $D_t^*=4.0$ to $D_t^*=5.0$, with proper impact velocities.

As can be seen in Fig. 6, the liquid column breaks when the Weber number is too high. For instance, when $D_t^*=4.54$ the critical Weber number is 94.8 in the experiment (shown in Fig. 7(b)) and 105.6 in the simulation (shown in Fig. 8(b)). Note that the bubble in Fig. 7(b) is the primary bubble. When the Weber number is above this critical $We$, the column-break phenomenon always occurs. The delayed bubble can also be entrapped with column-break phenomenon, as shown in Fig. 7(c). However, in our simulation, the delayed bubble disappears when the critical $We$ is exceeded, as shown in Fig. 8(c). This may be due to the neglect of contact angle which causes capillary forces at the boundary. In experiments, the upward capillary forces restrain the rising of the liquid column and help to break it. In simulations, there are no capillary forces due to the $90^\circ$ contact angle. The critical Weber number increases and the column-break time delays ($We=94.8$ at 48ms in the experiment, and $We=105.6$ at 63ms in the simulation). Comparing Fig. 7(c) at 43ms to Fig. 8(c) at 50ms, it can be found that the column is much thicker in the simulation, which leads to quite different crater dynamics. The outside liquid keeps rising (Fig. 8(c) at 68ms) but not descending and converging to the center (Fig. 8(a) at 74ms), no necking process is formed. This indicates that our simulations are not competent column-break phenomenon. Consequently, this research focuses on the region below the critical $We$.

IV. DISCUSSION

A. Role of gravity

The role of gravity in the entrapment process is studied and presented as follow. It can be simply accomplished in numerical simulations by setting gravity to 0. Typical sequences are shown in Fig. 9 and time evolutions of center and boundary are shown in Fig. 10. Although both cases have the similar process, delayed bubble is not formed without gravity.
The Froude number of droplet ($D=2.64\text{mm}$, $We=73.3$), $Fr = \frac{V}{\sqrt{gD}} = 9$ indicates that gravity can be neglected in the first few milliseconds, which is similar to large bubble entrainment. This can be evidenced by the coherence of boundary and center movements before 12 ms, shown in Fig. 10. However, as time goes on, gravity may play an important role in the later process. After 12 ms, the curves of the center and boundary begin to bifurcate. Without gravity, only surface tension dominates the oscillation, which causes a longer oscillation period and a larger amplitude. The second crater is in V-shape instead of U-shape. Finally the crater bottom rebounds before side walls retract, consequently no delayed bubble is entrapped.

### B. Propagation of energy

After a drop impacts onto a bounded liquid bath, the impact kinetic energy $E_k$ is converted into gravitational potential energy $E_{pg}$ and surface energy $E_{ps}$, also portion of the energy dissipates. As we use the axisymmetric simulation method, the velocity can be written as $u=(u,v)$, $u$ and $v$ are the radial and axial velocities, respectively. The liquid energy can be defined in the cylindrical coordinates as

\[
E_k = \iint \frac{1}{2} c_p (u^2 + v^2) 2\pi r dr dz,
\]

\[
E_{pg} = \iint c_p g 2\pi r dr dz - E_{pg0},
\]
FIG. 14. Horizontal crest positions of the front wave and the swell wave, varying with time, \( D_t^* = 20.0 \), \( We = 73.3 \).

\[
E_{pu} = \sigma (S - S_0), \tag{21}
\]

\[
E_m = E_k + E_{pg} + E_{ps}, \tag{22}
\]

where \( E_k \) is the kinetic energy, \( E_{pg} \) the gravitational potential energy, \( E_{ps} \) the surface energy, \( E_m \) the mechanical energy, \( \psi \) the volume fraction of liquid phase. \( x \) and \( y \) are the radial and axial coordinates, respectively. The initial gravitational potential energy \( E_{pg0} \) can be written as

\[
E_{pg0} = \int \int c \rho \underbrace{g \pi x y dx dy }. \tag{23}
\]

The kinetic energy \( E_k \) and the gravitational potential energy \( E_{pg} \) are calculated by the "sum function" in the GERRIS code. For the surface energy \( E_{ps} \), \( S \) is the liquid-air interface area and \( S_0 \) is the initial interface area. The interface profile line \( y = f(x) \) is first extracted, then the area \( S \) and \( S_0 \) can be simply calculated by

\[
S = \int_0^{D_t/2} 2\pi x \sqrt{1 + (f')^2} dx, \tag{24}
\]

\[
S_0 = \int_0^{D_t/2} 2\pi x \sqrt{1 + (f')^2} dx. \tag{25}
\]

If the profile line overtops, then it needs to be separated into several parts, each part will be calculated as above. All energy is nondimensionalized by the initial kinetic energy

\[
E_{k0} = \frac{\pi}{12} \rho L^3 V^2, \tag{26}
\]

\[
E_{m}^* = E_m / E_{k0}, E_k^* = E_k / E_{k0}, E_{pg}^* = E_{pg} / E_{k0}, E_{ps}^* = E_{ps} / E_{k0} - 1. \]

According to the Navier-Stokes equations, the energy dissipation equation for incompressible fluid without heat conduction can be derived as

\[
\frac{d \varepsilon}{dt} = \frac{1}{\rho} 2 \mu D : \nabla u, \tag{27}
\]

where \( \varepsilon \) is the internal energy. It can be found that, the mechanism energy dissipates and finally becomes internal energy. Time evolutions of energy are shown in Fig. 11, after drop impact with \( We = 73.3 \) onto a liquid bath (\( D_t^* = 4.54 \)). Generally, the mechanical energy \( E_m \) decreases constantly, while the kinetic energy, gravitational potential energy and surface energy exhibit oscillatory behaviors. Gravitational potential energy and surface energy are in phase, while in phase opposition with kinetic energy.

The oscillatory behaviors correspond to the crater-column-crater oscillation. This oscillation does not occur when it is an infinite surface. The time sequence and the center movement when \( We = 73.3 \) and \( D_t^* = 20.0 \) are shown in Fig. 12. This later value is

FIG. 15. (left) Comparison of \( t_{cf}, t_{cs}, t_{minf}, t_m \) varying with \( D_t^* \), \( We = 73.3 \). \( t_m \) and \( t_{minf} \) represent to column peak time for a bounded liquid bath and an infinite liquid surface, respectively. (right) The maximum heights of the liquid column varying with \( D_t^* \), \( We = 73.3 \). The red triangle \( H_{cmax}^* \) represents the maximum column height of infinite surface.
FIG. 16. Results of drop impact on a bounded liquid bath, with impact \( We=73.3 \). (a) \( D^*_t=4.54 \), (b) \( D^*_t=6.0 \), (c) \( D^*_t=8.0 \), (d) \( D^*_t=20.0 \). The scale bars are \( D \) long.

FIG. 17. Velocity fields of drop impact on the bounded liquid bath \( D^*_t=4.54 \). The red color represents upward velocity and the blue color represents downward velocity. The scale bars are \( D \) long.
large enough to be regarded as infinite surface. A high-speed thin jet is formed at 15ms. Under the action of gravity and surface tension, the thin jet gradually slows down and falls back to the bath. The maximum height is reached at 27ms, marked as $t_{\text{minf}}$, shown in Fig. 12 (right). No second crater is formed and no delayed bubble is entrapped.

Without boundary walls, the impact energy is propagated outside by waves without reflection after the impact. Outlines of first few milliseconds after the impact of $D_t^* \equiv 20.0$, $We=73.3$, is shown in Fig. 13. Two main kinds of waves can be found, small amplitude front wave (small arrows in Fig. 13) and large amplitude swell wave (large arrows in Fig. 13). The front wave is a disturbance formed during the initial coalescence of the droplet and the liquid bath, and the swell wave is formed during the expanding of the first crater. A tongue is created above the cavity at 3ms, it erects up and moves outward under the action of surface tension, which forms the swell wave. In certain conditions, this tongue can induce a large bubble. The horizontal crest motions of the front wave and the swell wave are shown in Fig. 14, whose velocity turns a constant after 3ms and $D_t^* \equiv 6.0$. The arrows show the motions of the crater side walls and the bottom. The scale bars are $D$ long.

C. Crater-column-crater oscillation

When the liquid bath is bounded, the waves will reflect back to the center with the energy. As shown in Fig. 13, the propagating energy is mainly carried by the swell wave, and it could promote the upward motion of center. According to $V_f$ and $V_s$, we can predict the time $t_f$ and $t_s$ when the front wave and swell wave reach the center. As shown in Fig. 15 (left), the reflected energy reaches the blue line (front wave $t_f$) and the red line (swell wave $t_s$), it delays when $D_t^* \equiv 6.0$, the reflected energy converges to the center before $t_{\text{minf}}$, which gives extra energy to the thin jet and finally forms a liquid column. This liquid column can be found in Fig. 16(a) at 43ms and Fig. 16(b) at 52ms. However, the thin jet fails to transform to liquid column when $D_t^* \equiv 6.0$, since the reflected energy reaches the center too late. Note that, when $D_t^* = 7.0$, very few energy reaches before $t_{\text{minf}}$ as the main energy are carried by the swell wave. The extra energy delays the column peak time from $t_{\text{minf}}$ to $t_m$, and increase the column peak height from $H_{\text{max}}^{\text{inf}}$ to $H_{\text{max}}^{\text{f}}$. With increasing $D_t^*$, both the peak time and the peak height tends to the value of infinite $D_t^*$.

When $D_t^* \equiv 6.0$, the liquid column latter collapses and causes a second crater, as the liquid bath is confined in boundary walls. This second crater is a prerequisite for delayed bubble entrapment, as shown in Fig. 16(a) at 70ms and Fig. 16(b) at 82ms. When $D_t^* > 6.0$ no second crater occurs as no liquid column is formed, as shown in Fig. 16(c) at 50ms and Fig. 16(d) at 50ms.

D. Dynamics of necking process

After a drop impacts on a bounded liquid bath, a second crater could forms after the liquid column’s collapse. However, as shown in Fig. 6, the delayed bubble can only be entrapped in the red dash region of the phase diagram. The question is therefore why not all the second craters entrap a delayed bubble?

Numerical results of drop impact on the liquid bath $D_t^* \equiv 4.54$ with varying impact $We$ are shown in Fig. 17, the delayed
bubble occurs when $We=73.3$ and $84.1$. As shown in the second line of Fig. 17, for $We<105.6$, the outside liquid moves downwards and converges to the center during the second crater’s collapse. This convergence causes the necking process, shown in Fig. 18. As shown in Fig. 17, the liquid column turns higher and thinner with increasing $We$, and finally breaks at $We=105.6$. When the liquid column breaks, the second crater becomes too shallow to form a necking process, and also the detached drop disturbs the collapse of the second crater.

The collapse processes of the second crater are shown in Fig. 18 and the $H_{\text{min}}-We$ map is shown in Fig. 19. Generally, the center of second crater descends and then rebounds, while the side walls of the crater keep converging to the center. For $We<73.3$, the second crater could go deeper with increasing velocity (Fig. 19), which forms a sharper bottom at the time just before it rebounds (Fig. 18, $We=41.1, 69.8$). The bottom rebounds before the side walls reach each other, with no entrapment of delayed bubble. For $We\geq73.3$, the bottom of the second crater remains at the lowest position (Fig. 18, $We=73.3$) or still keeps moving down (Fig. 18, $We=84.1$) when the side walls collide with each other. A nipple can be seen at the bottom of the crater, then a delayed bubble is pinched off. The maximum depth decreases slightly when $We\geq73.3$ (Fig. 19), as the bottom’s downward movement is disturbed by the pinch-off phenomenon. When the impact velocity is too high (Fig. 17, $We=105.6$), the column breaks and the second crater becomes quite shallow. The bottom rebounds very early, before the convergence of side walls starts.

This relationship between the impact velocity, the maximum depth of second crater and entrapment of delayed bubble, can also be found in other $D_t^*$. The $H_{\text{min}}-We$ map of the second crater are presented in Fig. 20. For a constant $D_t^*$, before the delayed bubble is entrapped, the maximum depth generally increases with velocity. When the second crater is deep enough, marked by black circles in Fig. 20, the delayed bubble is entrapped. With higher impact velocity, the liquid column goes higher and finally breaks, marked by black triangles in Fig. 20, which causes a much shallower second crater and no bubble entrapment.

However, not all liquid baths can entrap the delayed bubble with a proper impact velocity. If the bath is too narrow ($D_t^*<4.0$ in Fig. 6) or too broad ($D_t^*>5.0$ in Fig. 6), no delayed bubble can be formed. If the surface is too narrow, the column breaks with a quite low Weber number (We=48.3 for $D_t^*=3.0$). Thus the liquid column cannot stored enough energy, which later cannot cause an enough deep second crater. If the liquid bath is too broad, the reflected energy is not as effective as narrower surfaces. This causes a shallower second crater in a certain velocity and cannot entrap the delayed bubble. Consequently, it needs more initial impact energy to form a deeper second crater. However, with such high velocity, the liquid column is too thin and easy to be pulled off, which causes energy loss and no delayed bubble can be formed.

V. CONCLUSIONS

In this paper, numerical simulations are performed to investigate delayed bubble entrapment after a droplet impact onto a bounded liquid bath. An approximate region of delayed bubble entrapment is presented. Effects of gravity, surrounding walls and impact velocity are analyzed. With surrounding walls, a crater-column-crater oscillation can occur due to the timely reflected energy. Both the surface tension and gravity restrain the liquid motion during the oscillation, absence of gravity will cause a longer oscillation period and a larger amplitude. When $4.0\leq D_t^*\leq5.0$, the delayed bubble can be entrapped with proper impact velocities. Lower or higher impact velocity will lead to a shallower crater and no bubble entrapment. When $D_t^*<4.0$ or $D_t^*>5.0$, the crater cannot be deep enough to entrap a bubble, as the liquid column breaks.

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REFERENCES

1. J. Zou, Y. L. Ren, C. Ji, X. D. Ruan, and X. Fu, “Phenomena of a drop impact on a restricted liquid surface,” Exp. Therm. Fluid Sci. 51, 332–341 (2013).
2. L. J. Leng, “Splash formation by spherical drops,” J. Fluid Mech. 427, 73–105 (2001).
3. M. Rein, “Phenomena of liquid drop impact on solid and liquid surfaces,” Fluid Dyn. Res. 12, 61–93 (1993).
4. A. L. Yarin, “Drop impact dynamics: Splashing, spreading, receding, bouncing,” Annu. Rev. Fluid Mech. 38, 159–192 (2006).
5. M. Rein, “The transitional regime between coalescing and splashing drops,” J. Fluid Mech. 306, 145–165 (1996).
6. D. C. BE (Hons) JCU.Ph.D., “The splashing morphology of liquid-liquid impacts,” James Cook University (2007).
7. H. C. Pumphrey and P. A. Elmore, “The entrainment of bubbles by drop impacts,” J. Fluid Mech. 220, 539–567 (1990).
8. H. N. Oguz and A. Prosperetti, “Bubble entrainment by the impact of drops on liquid surfaces,” J. Fluid Mech. 219, 143–179 (1990).
9. M. J. Thoraval, Y. F. Li, and S. T. Thoroddsen, “Vortex-ring-induced large bubble entrainment during drop impact,” Phys. Rev. E 93, 033128 (2016).
10. O. G. Engel, “Crater depth in fluid impacts,” J. Appl. Phys. 37, 1798 (1966).
11. A. Bisighini and G. E. Cossali, “Crater evolution after the impact of a drop onto a semi-infinite liquid target,” Phys. Rev. E 82, 036319 (2010).
12. P. V. Hobbes and T. Osheroff, “Splashing of drops on shallow liquids,” Science 158, 1184–1186 (1967).
13. R. L. Vander Wal, G. M. Berger, and S. D. Mozes, “Droplets splashing upon films of the same fluid of various depths,” Exp. Fluids 40, 33–52 (2006).
14. J. Zou, P. F. Wang, T. R. Zhang, X. Fu, and X. Ruan, “Experimental study of a drop bouncing on a liquid surface,” Phys. Fluids 23, 044101 (2011).
15. J. Zou, C. Ji, B. G. Yuan, Y. L. Ren, X. D. Ruan, and X. Fu, “Large bubble entrainment during drop impacts on a restricted liquid surface,” Phys. Fluids 24, 057101 (2012).
16. S. Popinet, “Gerris: A tree-based adaptive solver for the incompressible Euler equations in complex geometries,” J. Comput. Phys. 190, 572–600 (2003).
17. S. Popinet, “An accurate adaptive solver for surface-tension-driven interfacial flows,” J. Comput. Phys. 228, 5838–5866 (2009).
18. M. J. Thoraval, K. Takehara, and T. G. Etoh, “von Karman vortex street within an impacting drop,” Phys. Rev. Lett. 108, 264506 (2012).
19. B. Ray, G. Biswas, and A. Sharma, “Bubble pinch-off and scaling during liquid drop impact on liquid pool,” Phys. Fluids 24, 082108 (2012).
20. H. Deka, B. Ray, G. Biswas, A. Dalal, P. Tsai, and A. Wang, “The regime of large bubble entrainment during a single drop impact on a liquid pool,” Phys. Fluids 29, 092101 (2017).
21. S. T. Thoroddsen, T. G. Etoh, and K. Takehara, “Air entrainment under an impacting drop,” J. Fluid Mech. 478, 125–134 (2003).
22. G. D. Crapper, Introduction to Water Waves (Ellis Horwood, 1984).