Multiple M0-brane equations in eleven dimensional pp-wave superspace and BMN matrix model

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We obtain the Matrix model equations in the background of the maximally supersymmetric
pp-wave solution of the 11D supergravity and discuss its relation with the Berenstein–Maldacena–
Nastase (BMN) model.

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I. INTRODUCTION

Matrix model [1] was proposed 15 years ago and all these years was an important tool for studying M-theory [2], see e.g. [3–12]. It was conjectured to provide a non-perturbative description of M-theory in some limit and, to stress this, the name M(atrix) theory was often used. Although this M(atrix) theory was considered to be eleven dimensional, the Lagrangian staying beyond it according to [1] was just a dimensional reduction of D=10 SYM down to d=1 (which for the gauge group U(N) is believed to provide a low energy description of the system of N nearly–coincident D0-branes); the sym-

metry enlargement to include D=11 Lorentz group was discussed in [1] and later papers. However it was not clear how to write the action for Matrix model in 11D super-

gravity background. This is why Matrix model action (and equations of motion) are known for a few particular supergravity backgrounds only, and for these they were rather guessed than derived. In particular, the action for Matrix model in maximally supersymmetric pp-wave background was proposed by Berenstein, Maldacena and Nastase in 2002 [3] and is known under the name of BMN (matrix) model. The other example is the so-called Ma-

trix Big Bang background [10, 11].

A natural way to resolve the problem was to obtain invariant action, or covariant equations of motion, for multiple M0-brane system. Indeed, the Matrix model originally was conjectured to be the theory of nearly co-

incident multiple D0–branes (mD0) [1] so that, as far as single D0-brane can be obtained by dimensional re-

duction of M0–brane [13], it is natural to expect that multiple M0–brane system (mM0) stays beyond the con-

jectured enlargement of the 10D Lorentz group symmetry of mD0 till 11D Lorentz group SO(1,10).

However, to write Lorentz invariant and supersymmetric action for mM0 system was not so easy: it was a par-

ticular case of the problem which in D=10 type II case was known as a search for supersymmetric non-Abelian Born-Infeld action [14]. The progress towards solution of this problem is only partial, although impressive. We refer on our previous papers [15, 17] for the elaborated de-

scription of the results of refs. [18–24] (the list of which is certainly incomplete), but here restrict our discussion by stating what is known about multiple M0-brane (mM0) system. The purely bosonic action for mM0 was pro-

posed in [24]. However, as far as this was the straight-

forward counterpart of the ‘dielectric brane action’ pro-

posed by Myers for multiple Dp-branes [18], it did not possess neither supersymmetry nor 11D Lorentz symme-
try (nor complete diffeomorphism symmetry), so that it was not clear how to introduce the coupling to the 11D supergravity background in such an action.

To resolve the problem of covariant and supersymmetric description of multiple mM0 system, the superembedding approach to this system was proposed in [15]. It was shown in [15] that, for the case of flat target super-

space, the equations of relative motion of the multiple M0–brane constituents, which follows from the proposed superembedding approach equations, coincide with the Matrix model equations of [1]. The superembedding approach to the mM0 system in an arbitrary curved 11D supergravity superspace was developed in [16, 17], where the equations of motion, which can be treated as equations of Matrix model coupled to an arbitrary supergrav-

ity background, were obtained.

The natural application of the results of [16, 17] is to obtain the Matrix model equations in particular interesting M-theoretical background, like AdS4 × S7 and AdS5 × S4 and to apply them in context of the AdS/CFT correspondence. It happens however that this is not so easy even for bosonic backgrounds: as we will see below on a relatively simple example, the final result cannot be reached by just substituting a particular bosonic solution of the spacetime 11D supergravity equations in the gene-

ral Matrix model equations of [16, 17], but requires first to lift of that bosonic solutions to a curved superspace solving the superspace supergravity constraints. Further-

more, then we have to find certain information about the

1 Superembedding approach for superstrings and eleven dimensional supermembrane was proposed in [27] developing the line of the so-called STV (Sorokin-Tkach-Volkov) approach to superparticle [28] and superstring [29] (see [30] for more references on STV approach), and was generalized to Dirichlet branes (Dp-branes) in [31] (see also [32]) and to M5-brane in [32], respectively. See [20, 33] for reviews and more references.
Thus it is natural to begin the program of specifying the Matrix model equations in an arbitrary supergravity background. In Sec. IVB we construct a particular worldline superspace $W_{10}^{(11|16)}$ describing the ground state motion of the center of energy of the mM0 system, for which all the associated bosonic and fermionic Goldstone fields are equal to zero. We use that in Sec. IVC to obtain the equations of relative motion for the mM0 constituents in 11D pp-wave background for the case when the center of energy motion is described by the above mentioned ground state configuration. These equations coincide with the BMN equations up to the fact that they are obtained for traceless, $su(N)$ valued matrices, rather then for $u(N)$ valued matrix fields like in the case of BMN model. Then we show that the complete set of the BMN equations describes the general motion of mM0 system in pp-wave background in the leading order on the center of energy Goldstone fields. Particularly, we show in Sec. IVD that the equations for Goldstone fields which follows from the superembedding approach to mM0 system coincide with the trace part of the BMN equations. Sec. V contains conclusions and discussion.

II. MATRIX MODEL EQUATIONS IN AN ARBITRARY SUPERGRAVITY BACKGROUND

A. Equations for matrix fields

The Matrix model equations in an arbitrary supergravity background were obtained in [16,17] as equations for multiple M0-brane (mM0) system in an arbitrary 11D supergravity superspace by using superembedding description of mM0 proposed in [15]. The set of these equations splits naturally on two subsets: first describing the center of energy motion and second describing the relative motion of the constituents of mM0 system. The latter subset contains the equations for matrix fields, namely, the traceless $N \times N$ matrix fields $X^i(\tau)$ and $\Psi_q(\tau)$ depending on one proper time variable $\tau$.

Here $N$ is the number of M0-branes forming the mM0 system, $X^i$ is bosonic, takes values in the vector representation of $SO(9)$, so that $i = 1, \ldots, 9$, and carries the $SO(1,1)$ weight 2, $X^i = X^i_\# := X^i_{++}$, while $\Psi_q$ is the fermionic, takes values in the spinor representation of $SO(9)$, so that $q = 1, \ldots, 16$, and carries the $SO(1,1)$ weight 3, $\Psi_q = \Psi_q := \Psi_{+++q}$.

Already at this stage one can guess that in our discussion we will use the nine-dimensional Dirac matrices $\gamma_{qp}$. These are real, symmetric $\gamma_{qp}^i = \gamma_{qp}^i$ and obey $\gamma_q^i \gamma^j + \gamma_q^j \gamma^i = \delta^{ij} I_{16\times16}$.

The set of the equations of relative motion of mM0 constituents contains the fermionic equation

$$
\nabla_{\mu} \Psi_q = \gamma_{qp}^i \frac{\partial X^i}{\partial x_p} + \frac{1}{2N} \epsilon_{abcde} \gamma^{a+b+c+d+e} \Psi_q.
$$

This equation is similar to the BMN equations, but includes the superfield formalism characteristic for superembedding approach, but rather describing the component worldline equations. Then, however, we see that some information on the worldline superfield formalism is necessary to specify the general equations for particular supergravity background, including for purely bosonic ones. Also the supergravity fields necessarily enter the general mM0 equations through the pull-back of superfields projected with the use of moving frame and spinor moving frame adapted to the center of energy motion of the mM0 system. As a result, an important part of the present paper is devoted to the superfield formalism.

Section III contains some necessary details on supersymmetric pp-wave solutions of 11D supergravity (sec. IIIA) and on its superfield description (sec. IIIB) by a particular 11D superspace $\sigma_{pp-w}^{(11|12)}$. Sec. IVA contains a brief review of superembedding approach to M0-brane in general 11D supergravity background. In Sec. IVB we construct a particular worldline superspace $W_{10}^{(11|16)}$ describing the ground state motion of the center of energy of the mM0 system, for which all the associated bosonic and fermionic Goldstone fields are equal to zero.
\[ D\#\Psi_{q} = -\frac{1}{4} \hat{\gamma}_{i}^{q} [X^{i}, \Psi_{p}] + \frac{1}{24} \hat{F}\#_{ijk} \hat{\gamma}_{q}^{jk} \Psi_{r} - \frac{1}{4} X^{j} \hat{T}_{\#i+q} \]  \hspace{1cm} (2.1)

the Gauss constraint

\[ [X^{i}, D\#X^{i}] = 4i \{\Psi_{q}, \Psi_{q}\} \]  \hspace{1cm} (2.2)

and the bosonic equation

\[ D\#D\#X^{i} = \frac{1}{n} [X^{i}, [X^{j}, X^{i}]] + i\hat{\gamma}_{i}^{q} \{\Psi_{q}, \Psi_{p}\} + \frac{1}{2} X^{j} \hat{R}_{\#j \#i} + \frac{1}{8} \hat{F}\#_{ijk} [X^{i}, X^{k}] - 2i\Psi_{q} \hat{T}_{\#i+q}. \]  \hspace{1cm} (2.3)

The left hand sides (l.h.s.’s) of these equations involve the covariant time derivative \( D\# \) which we describe below (see sec. IIC); it contains derivative with respect to the proper time variable \( \tau \) \((D\# = \mathcal{E}_{\#}(\tau)\partial_{\tau} + ... \)) but in general case other contributions are also present.

The r.h.s.’s of Eqs. (2.1) and (2.3) involve the following projections of the pull−backs of the superfield supergravity ’fluxes’ \( (i.e. \text{of the field strengths superfields}) \)

\[ \hat{F}\#_{ijk} := F^{abcd}(\hat{Z})u_{a}^{i}u_{c}^{j}u_{d}^{k}, \] \hspace{1cm} (2.4)

\[ \hat{R}_{\#i+\#j} := R_{dc}^{ba}(\hat{Z})u^{d}u^{c}u^{b}u^{a} = \] \hspace{1cm} (2.5)

\[ T_{\#i+q} := T_{ab}^{i}(\hat{Z})u_{aq}u_{ub}. \] \hspace{1cm} (2.6)

Here \( F^{abcd} = F^{[abcd]}(Z), T_{ab}^{i} = T_{ab}^{\#i} \) and \( R_{dc}^{ba} = R_{dc}^{ba}(Z) \) are superfield generalizations of the field strength of the 3-rd rank antisymmetric tensor gauge field, of the gravitino field strength and of the Riemann tensor of the 11D supergravity. We discuss their properties in the next subsection [IIC]. In Eqs. (2.4), (2.5) and (2.6) these flux superfields depend on bosonic and fermionic coordinate functions \( \hat{Z}^{M}(\tau) = (\hat{x}^{m}(\tau), \hat{\theta}^{\alpha}(\tau)) \) which define the embedding of the worldline \( W^{1} \), on which the matrix fields \( X^{i}(\tau) \) and \( \Psi_{q}(\tau) \) are defined, in the target 11D superspace.

\[ W^{1} \in \Sigma^{[11]}: Z^{M} = \hat{Z}^{M}(\tau) = (\hat{x}^{m}(\tau), \hat{\theta}^{\alpha}(\tau)) \] \hspace{1cm} (2.7)

This worldline \( W^{1} \) is naturally associated with the movement of the center of energy of the mM0 system so that its local coordinate \( \tau \), can be called center of energy proper time. The center of energy motion resembles a motion of a particle or a single 0-brane (we will discuss this below). Then, using the experience of studying single (super)particle or (super-)p-brane we can state that the (not pure gauge part of the) coordinate functions \( \hat{Z}^{M}(\tau) = (\hat{x}^{m}(\tau), \hat{\theta}^{\alpha}(\tau)) \) are bosonic and fermionic Goldstone fields corresponding to the translation symmetry and supersymmetry which are broken spontaneously due to the presence of the the effective worldline \( W^{1} \) describing the center of energy motion of the mM0 system.

Furthermore, as \( D\# \) in Eqs. (2.1), (2.2) and (2.3) is a covariant time derivative on \( W^{1} \) defined in subsection [IIC], in general case it contains the contributions from the coordinate functions \( \hat{x}^{m}(\tau) \) and \( \hat{\theta}^{\alpha}(\tau) \) (see subsection [IIC]).

Finally, the worldline fields \( u_{a}^{\varphi} = u_{a}^{\varphi}(\tau) \), \( u_{a}^{\varphi} = u_{a}^{\varphi}(\tau) \) and \( v_{a}^{\varphi} = v_{a}^{\varphi}(\tau) \), which are used in Eqs. (2.4), (2.5) and (2.6) define the moving frame adapted to the center of energy motion of the mM0 system and its spinorial representation (spinor moving frame). There indices \( a = 0, 1, ..., 9 \) and \( \alpha = 1, ..., 32 \) correspond to the vector and spinor representations of the eleven-dimensional Lorentz group \( SO(1,10) \), while \( -1 \) denotes the scaling dimension \( -1 \) for \( v_{a}^{\varphi} \) (and \( -2 \) for \( u_{a}^{\varphi} := u_{a}^{\varphi} \)) with respect to \( SO(11) \) subgroup of \( SO(1,10) \), and \( i = 1, ..., 9 \) and \( q = 1, ..., 16 \) are vector and spinor indices of \( SO(9) \subset SO(1,10) \).

As we discuss below, these moving frame variables can be used (instead of the coordinate functions \( \hat{Z}^{M} \) or together with these) to describe the center of energy motion of the mM0 system. It is useful to keep in mind the simplest case, when the frame is not actually moving, which is described by constant \( u_{a}^{\varphi} = \delta_{a}^{0} = \delta_{a}^{10} \), \( u_{a}^{\varphi} = \delta_{a}^{i} \) and \( v_{a}^{\varphi} = \delta_{a}^{-q} \). Actually this simplest frame is sufficient for the major part of our study here; only in the second part of sec. IVB we use a more complicated moving frame.

### B. Flux superfields (superfield generalizations of the field strength) of D=11 supergravity

The flux superfields \( F^{abcd} = F^{[abcd]}(Z), T_{ab}^{i} = T_{ab}^{\#i} \) and \( R_{dc}^{ba} = R_{dc}^{ba}(Z) \), entering Eqs. (2.4), (2.5) and (2.6), satisfy the superfield generalization of the 11D supergravity equations of motion, the set of which includes Einstein equations

\[ R_{ab} = -\frac{1}{3} F_{ac_1c_2c_3} F_{b}^{c_1c_2c_3} + \frac{1}{36} \eta_{ab} F_{c_1c_2c_3c_4} F_{c_1c_2c_3c_4}, \] \hspace{1cm} (2.8)

and the Rarita-Schwinger equations \( T_{bc}^{i} \Gamma_{\beta c}^{abc} = 0 \). It is convenient to write this latter in the equivalent form of

\[ T_{ab}^{\beta} \Gamma_{\beta c}^{\alpha c} = 0 \] \hspace{1cm} (2.9)

Here and below \( \Gamma_{\beta c}^{\alpha c} = \Gamma_{\beta c}^{\alpha c} = \Gamma_{\alpha c}^{\beta c} = C_{\gamma}^{\alpha} \) where \( \Gamma_{\alpha c}^{\beta c} \) are eleven dimensional Dirac matrices obeying \( \Gamma^{(\alpha} \Gamma^{\beta)} := \frac{1}{2} (\Gamma^{a} \Gamma^{b} + \Gamma^{b} \Gamma^{a}) = \eta^{ab} I_{32 \times 32} \), and \( C_{\gamma}^{\alpha} = -C_{\gamma}^{\beta} \) is the 11D charge conjugation matrix, \( \Gamma^{ab} = \Gamma^{[a} \Gamma^{b]} := \frac{1}{2} (\Gamma^{a} \Gamma^{b} - \Gamma^{b} \Gamma^{a}) \).
and \( \Gamma^\alpha \), \( \Gamma^{abc} = \Gamma^{[a} \Gamma^{b} \Gamma^{c]} \), etc. See e.g. [17, 33] for more details on the properties and the explicit representations of the 11D \( \Gamma \) matrices.

As it was shown in [34, 35], the terms of higher order in the decomposition of these superfields on Grassmann coordinates are expressed through their leading components and supergravity potentials so that no new degrees of freedom appear [34, 35] (see also [36] reviewing this in the present notation). In particular,

\[
D_a F_{abcd} = -6 T_{[ab} \Gamma_{cd]a \beta} \alpha \beta, \quad (2.10)
\]

\[
D_a T_{ab} = -\frac{1}{4} R_{ab} \gamma_{a \beta} \Gamma_{cd} \alpha \beta - 2 (D_{[a} t_{b]} + t_{[a} b) \alpha \beta, \quad (2.11)
\]

where \( t_{a \alpha} \beta \) is expressed through \( F_{abcd} \) by

\[
t_{a \alpha} \beta := \frac{1}{\tau} \left( F_{abcd} \Gamma_{a \beta}^{\alpha} + \frac{1}{2} F^{bcd} \Gamma_{abcd} \alpha \beta \right). \quad (2.12)
\]

The pull–backs of the superfields in Eqs. (2.7), (2.9) and (2.6) are obtained by substituting the center of energy coordinate functions \( Z^M = (\hat{x}^m(\tau), \hat{\theta}^\alpha(\tau)) \) for superfields coordinates \( Z^M = (\hat{x}^m(\tau), \hat{\theta}^\alpha(\tau)) \). As a result, in general case, the r.h.s.’s of Eqs. (2.7) and (2.8) contain, besides \( \hat{x}^m(\tau) \), also the contributions from the fermionic coordinate functions \( \hat{\theta}^\alpha(\tau) \). In particular, taking into account (2.10) and using the Wess–Zumino gauge for superfield supergravity one finds that

\[
\hat{F}_{abcd} = F_{abcd}(\hat{x}(\tau)) - 6 \hat{\theta}^\alpha(\tau) \Gamma_{[ab} \alpha \beta T_{cd] \beta}(\hat{x}(\tau)) + \alpha \times \hat{\theta}^\alpha(\tau) \hat{\theta}^\beta(\tau). \quad (2.13)
\]

Thus, generically the structure of Eqs. (2.7), (2.8) and (2.6) seems to be quite complicated. It simplifies for the target supergravity superfields describing the purely bosonic supergravity solutions described by supermanifolds with vanishing gravitino field strength superfield, \( T_{ab} \alpha \beta (Z) = 0 \). The expression for pull–back of bosonic fluxes entering the r.h.s.’s of Eqs. (2.7) and (2.8) in these cases do not contain the center of energy Goldstone fermion contributions (for instance (2.13) reduces to \( \hat{F}_{abcd} = F_{abcd}(\hat{x}(\tau)) \)). The completely supersymmetric \( AdS_4 \otimes S^7 \), \( AdS_4 \otimes S^7 \) and pp-wave superspaces are examples of such backgrounds characterized by constant and covariantly constant 4-form field strength \( F_{abcd} \) and Riemann tensor \( R_{abcd} \).

However, even for the cases of purely bosonic supergravity solutions the structure of \( \hat{D}_\# \) is not so simple.

### C. Covariant derivative \( \hat{D}_\# \)

The equations of the relative motion of m0 constituents, Eqs. (2.7), (2.8), (2.9), are obtained in the frame of superembedding approach, which provides a superfield description of the dynamics of m0 system. This implies that the matrix fields are leading components of some matrix superfields

\[
X^i(\tau, \eta) = X^i(\tau) + \alpha \eta^p, \\
\Psi^a(\tau, \eta) = \Psi^a(\tau) + \alpha \eta^p, \quad (2.14)
\]

which we denote by the same symbol. These depend not only on the bosonic variable \( \tau \), but also on 16 fermionic variables \( \eta^p \) (obeying \( \eta^p \eta^q = -\eta^q \eta^p \)), this is to say, they are functions on a (generically curved) superspace \( \mathcal{W}^{[11]} \) with one bosonic and 16 fermionic directions,

\[
\mathcal{W}^{[11]} = \{ \{ C^M \} = \{ (\tau, \eta^p) \} , \\
\eta^q \eta^p = -\eta^p \eta^q , \quad q, \bar{p} = 1, ..., 16. \quad (2.15)
\]

This superfield can be associated with the center of energy motion of the m0 system and can be called center of energy worldline superfield. The covariant derivative \( \hat{D}_\# \), which enters Eqs. (2.7), (2.8), (2.9), is defined as a leading component of the bosonic covariant derivative of \( \mathcal{W}^{[11]} \),

\[
\hat{D}_\# X^i(\tau, \eta) = (\hat{D}_\# X^i(\tau, \eta)|_{\eta^p = 0}\quad (2.16)
\]

As far as this latter appears in the decomposition of the covariant differential \( \hat{D} \) on the supervielbein of \( \mathcal{W}^{[11]} \),

\[
\hat{E}^A = (\hat{E}^A, \hat{E}^q) = d\tau \hat{E}^A(\tau, \eta) + d\eta^p \hat{E}^A(\tau, \eta) , \quad (2.17)
\]

\[
\hat{D} := \hat{E}^A \hat{D}_A + \hat{E}^q \hat{D}_q = d\tau \hat{D}_\tau + d\eta^q \hat{D}_q \quad (2.18)
\]

\[
d = d\tau \hat{D}_\tau + d\eta^q \hat{D}_q = d\tau \frac{\partial}{\partial \tau} + d\eta^q \frac{\partial}{\partial \eta^q} \quad (2.19)
\]

the \( D_\# \) in Eqs. (2.7), (2.8), (2.9) contains, besides the term with \( \hat{\theta} \), and SO(1, 1) \( \times \) SO(9) \( \times \) SU(N) connection, also the contribution from the ‘worldline gravitino’ \( \hat{E}^q \),

\[
\hat{E}^q(\tau) = -\hat{E}^q \hat{E}^p \hat{E}^q / \hat{E}^q \quad (2.21)
\]

and \( \hat{E}^q = (1 - \hat{E}^p \hat{E}^q \hat{E}^r) / \hat{E}^q \) as well as \( \hat{E}^q \) and \( \hat{E}^q \hat{E}^p \) are elements of the inverse supervielbein matrix.

As it was shown in [13], the fermionic covariant derivative of the bosonic matrix superfield \( \hat{X}^i \) is expressed through the fermionic superfield \( \Psi^a \) by

\[
D_\# \Psi^a(\tau, \eta) = 4 i \eta^q \hat{\gamma}^q \hat{\psi}^a \quad (2.22)
\]

so that \( D_\# \hat{X}^i(\tau, \eta) \) entering Eqs. (2.7), (2.8), (2.9) reads

\[
D_\# \hat{X}^i(\tau, \eta) = (\hat{E}^A \hat{D}_A(\tau, \eta) + \hat{E}^q \hat{D}_q(\tau, \eta)|_{\eta^p = 0} - \\
4 i \eta^q \hat{E}^p \hat{E}^q \hat{E}^r \hat{E}^s \hat{\psi}^a(\tau, \eta) \hat{\gamma}^q(\tau) \hat{\psi}^a(\tau) / \hat{E}^q \quad (2.23)
\]

and contains, besides \( \propto \hat{D}_\# X^i(\tau) \) term, the contribution \( \propto \hat{\psi}^a(\tau) \). Similarly, the expression for \( D_\# \Psi^a(\tau, \eta) \) in Eq. (2.21) contains, besides \( \hat{D}_\# \Psi^a \), also \( \propto \hat{X}^i(\tau) \) contributions

\[\text{as shown in [13, 17],} \quad D_\# \Psi^a(\tau, \eta) = \frac{1}{2} \Gamma_{\eta^q} D_\# \hat{X}^i + \frac{1}{16} \Gamma_{\eta^q} [\hat{X}^i, \hat{X}^j] - \frac{1}{16} \hat{X}^i \frac{i}{2} \bar{F}^{}_{jk l}(\delta^{j l} \eta^k \eta^l) + \frac{1}{16} \bar{F}^{}_{jk l}(\eta^j \eta^k \eta^l).\]
The expression for $D_{\mu}x^\nu$, which can be found in [16, 17], then so is the expression for $D_{\mu}x^\nu$ in Eq. (2.21).

Notice that the expressions for $D_\nu X^i$ and $D_\nu \Psi_\sigma$ simplify essentially when the worldline gravitino vanishes,

$$\mathcal{E}^+_{\tau} = 0 \Rightarrow \left\{ \begin{array}{l} D_\tau X^i = \mathcal{E}^+_{\tau}(\tau) D_\tau X^i(\tau) \\ D_\tau \Psi_\sigma = \mathcal{E}^+_{\tau}(\tau) D_\tau \Psi_\sigma(\tau) \end{array} \right. ,$$

(2.24)

As we show below, this happens for the mM0 system in pp-wave superspace when the center of energy Goldstone fermion is set to zero.

The general conclusion which we have caught in this and previous subsections is that some knowledge on the worldline superspace formalism is still necessary to extract the Matrix model equation in a particular 11D supergravity background from the equations of motion of mM0 system in an arbitrary supergravity superspace obtained in [16, 17].

We will also need to describe the center of energy dynamics of mM0 system in pp-wave background. In the next section we review the equations of the mM0 center of energy motion in general supergravity background proposed in [16, 17].

D. Equations of the center of energy motion, moving frame and spinor moving frame

It is natural to formulate the equations of center of energy motion of the mM0 system in terms of the coordinate functions $Z^m(\tau) = (\hat{x}^m(\tau), \hat{\theta}^\alpha(\tau))$ of Eq. (2.27). However, it can be also described with the use of moving frame variables which can be considered as elements of the $SO(1,10)$ valued moving frame matrix

$$U_b^{(a)} = \left( \begin{array}{c} u_b^+ + u_b^- \\ u_b^+ - u_b^- \end{array} \right) \in SO(1,10) ,$$

(2.25)

$$a, b = 0, 1, \ldots, 9, 10 \ , \ i = 1, \ldots, 9 .$$

The above statement of Lorentz group valuedness of the moving frame matrix is tantamount to saying that the moving frame vectors obey the constraints [37]

$$u_a^+ u_a^- = 0 \ , \ u_a^+ u_a^i = 0 \ , \ u_a^+ u_a^{\#} = 2 ,$$

(2.26)

$$u_a^+ u_a^{\#} = 0 , \ u_a^+ u_a^{\#} = 0 , \ u_a^+ u_a^i = 0 ,$$

(2.27)

$$\delta_{ij} u_a^{\#} = -\delta_{ij} .$$

(2.28)

Notice that the light-like moving frame vector $u_b^+$ has already appeared in Eqs. (2.6), (2.29), (2.5).

In massless superparticle model the moving frame variables appear as a counterpart of the momentum (see [42] and refs therein) in the sense that the pull–back of the bosonic supervielbein form to the worldline is written as

$$\hat{E}^a := d\tau \hat{E}^a := d\hat{Z}^M(\tau) E^a_{\hat{M}}(\hat{Z}) = \frac{1}{2} \mathcal{E}^\# u^a = \frac{1}{2} \mathcal{E}^\# (\tau) u^a(\tau) ,$$

(2.29)

$$\hat{E}^a := \partial_\tau \hat{Z}^M(\tau) E^a_{\hat{M}}(\hat{Z}) = \frac{1}{2} \mathcal{E}^\# (\tau) u^a(\tau) ,$$

(2.29)

$$\hat{E}^a := \partial_\tau \hat{Z}^M(\tau) E^a_{\hat{M}}(\hat{Z}) = \frac{1}{2} \mathcal{E}^\# (\tau) u^a(\tau) ,$$

(2.29)

$$\hat{E}^a := \partial_\tau \hat{Z}^M(\tau) E^a_{\hat{M}}(\hat{Z}) = \frac{1}{2} \mathcal{E}^\# (\tau) u^a(\tau) ,$$

(2.29)

which is a double covering of the moving frame matrix. This latter statement implies, in particular, that $v_0^{-\alpha} = v_0^{-\alpha}(\tau)$ can be considered as square root of the light–like vector $u_a^-$ in the sense of that it obeys the following constraint

$$\delta_{qp} u_{a}^{-} = v_{q}^{-\alpha} \Gamma_{ab}^{a} v_{p}^{-\beta} .$$

(2.33)

The fermionic equations of the center of energy motion of the mM0 system, which in the superembedding approach of [15–17] coincide with the fermionic equation of motion of a single M0-brane, can be written as

$$\hat{E}^a_{\#} v_{\alpha p} = 0$$

(2.34)

(2.34)

$$(\hat{E}^a_{\#} = \mathcal{E}^\# \partial_\tau \hat{\theta}^\alpha + \ldots .)$$

To make their form more standard, one can use Eqs. (2.33) and (2.30) to show that (2.34) implies

$$\hat{E}^a_{\#} \Gamma_{aa' \beta} \hat{E}^a_{\#} = 0 .$$

(2.35)

When the worldline gravitino vanishes, this is equivalent to

$$\hat{E}^a_{\#} \Gamma_{aa' \beta} \hat{E}^a_{\#} = 0 .$$

(2.36)

More details on the moving frame and spinor moving frame variables can be found in [17] and in refs. therein. Here we will need to know only a few of their properties, in particular that, on the shell of the equations of the mM0 center of energy motion the $SO(1,10) \times SO(1,1) \times SO(9)$ covariant derivatives of $u_a^-$ is expressed in terms of $u_a^-$ and the covariant derivative of $u_a^-$ vanishes. Actually

$$D u_a^+ = \frac{1}{2} u_a^+ \Omega^+, \quad D u_a^- = 0 , \quad D u_a^+ = u_a^+ \Omega^+ ,$$

(2.37)
with the same one form coefficient $\Omega^{#i}$ from $Du^i_a$ enter
the expression for $Du^i_a$ and $Dv^+_q$, while $v^{-}\alpha$ is covariantly constant,
\[ Dv^-_\alpha = 0 , \quad Dv^+_q = -\frac{1}{2} \Omega^{#i} v^-_\alpha \gamma^i_{pq} . \tag{2.38} \]
Notice that the above relation also define the SO(1, 1) $\times$ SO(9) connection induced by (super)embedding and that their worldline superfield generalizations are also valid, see \[15, 17].

E. Fluxes allowing center of energy motion preserving 1/2 of the target space supersymmetry

An important observation is that with the above consequences of the center of energy equations of motion, the derivative acting on the projections of the pull–backs of fluxes of Eqs. (2.37), (2.38) and (2.4) reduce to the projections of the pull–backs of the targets superspace derivatives of the fluxes,
\[ DF_{\#ijk} : = u^a_u^b u^c u^d D F^{abcd}(Z)|_{Z=\hat{Z}} , \]
\[ DR_{\#i#} : = u^d u^c i u^b u^a = DR_{dcba}(Z)|_{Z=\hat{Z}} , \]
\[ DT_{\#i+q} : = v_{i\#} u^- u^b D T_{ab}(Z)|_{Z=\hat{Z}} . \tag{2.39} \]

Then, the above mentioned fact that the superfield generalizations of Eqs. (2.37), (2.38) are also valid allows \[17] to deduce the supersymmetry transformations of the projections of the pull–backs of the fluxes from the superfield supersymmetry of target superspace. In particular, as it was found in \[17], the center of energy motion of mM0 system in superspaces describing purely bosonic solutions of supergravity (i.e. superspaces with $T_{abc}^{\alpha\beta} = 0$) can preserve 1/2 of the target (super)space supersymmetry if the
\footnote{ Our notation here are close to \[6] where one might also find a number of references on pp-wave solutions of supergravity equations.}

projections of the pull–backs of the bosonic fluxes to the center of energy worldline (superspace) obey a worldline superfield generalization of the following relations \[3\]
\[ \hat{R}_{\#i\#} - \frac{1}{6} \hat{F}_{\#ik}\hat{F}_{\#klj} - \frac{1}{94} \delta^{ij}(\hat{F}_{\#ikl})_{2} = 0 , \tag{2.40} \]
\[ D\hat{F}_{\#ijk} = 0 , \tag{2.41} \]
\[ \hat{F}_{\#ijk[k]_3} = 0 . \tag{2.42} \]

Notice also that the projection \[2.3\] of the Riemann tensor is symmetric (due to the Bianchi identities $R_{[abc]d} = 0$). Furthermore, its trace (on SO(9) vector indices) is expressed through the product of the projections \[2.4\] of the 4-form fluxes by
\[ \hat{R}_{\#j\#} + \frac{1}{3} (\hat{F}_{\#ijk})^2 = 0 , \tag{2.43} \]
which is the $u^- u^b$ projection of the pull–back of the supergravity Einstein equation \[2.8\] to $\mathcal{W}^1$.

In the remaining part of this paper we will consider mM0 system in the maximally supersymmetric 11D pp-wave background.

III. SUPERSYMMETRIC 11D PP-WAVE SOLUTION AND ITS SUPERFIELD DESCRIPTION

A. Supersymmetric pp-wave solution of the 11D supergravity equations

The completely supersymmetric pp-wave solution of $D = 11$ supergravity equations is characterized by the 11D spacetime interval above, can be seen from Eq. (5.7) of \[17].

\[ ds^2 = dx^- dx^+ + \left[ \left( \frac{\mu}{3} \right)^2 x^I x^J + \left( \frac{\mu}{6} \right)^2 x^I x^J \right] dx^I dx^J , \quad \{ I = 1, 2, 3, \quad J = 4, 5, 6, 7, 8, 9, \tag{3.1} \]

and by the constant 4 form flux \[4\]
\[ F_{abcd} = 2\mu \delta_{[a}^{++} \delta_{b}^{L} \delta_{c}^{J} \delta_{d]}^{K} \epsilon_{1JK} . \tag{3.2} \]

The dual seven form flux is then given by
\[ F_{c_1...c_7} = \frac{1}{4!} \epsilon_{c_1...c_7} b_{1...b_4} F_{b_1...b_4} = \]
\[ - \frac{7\mu}{2} \eta^{++} \epsilon_{c_1} \delta_{J_1} \cdots \delta_{J_6} \epsilon_{J_1...J_6} . \tag{3.3} \]
The corresponding vielbein one-forms and nonvanishing components of the spin connection read
\[ e^{-} = dx^{-} + \left[ \left( \frac{m}{3} \right) x^{i} x^{j} + \left( \frac{m}{6} \right)^{2} x^{i} x^{j} \right] dx^{++}, \]
\[ e^{++} = dx^{++}, \quad e^{l} = dx^{l}, \quad e^{j} = dx^{j}, \quad (3.4) \]
\[ \omega^{J} = 2 \left( \frac{m}{3} \right)^{2} e^{++} x^{j} = 2 \left( \frac{m}{3} \right)^{2} dx^{++} x^{j}, \]
\[ \omega^{J} = 2 \left( \frac{m}{6} \right)^{2} e^{++} x^{j} = 2 \left( \frac{m}{6} \right)^{2} dx^{++} x^{j}. \quad (3.5) \]
As a result the only nonvanishing components of the \( SO(1,10) \) curvature 2-form are
\[ R^{-l} = -2 \left( \frac{m}{3} \right)^{2} e^{++} \wedge e^{l}, \]
\[ R^{-J} = -2 \left( \frac{m}{6} \right)^{2} e^{++} \wedge e^{j}, \quad (3.6) \]
so that the only nonvanishing components of the Riemann curvature tensor are
\[ R_{++IJ++} = \frac{1}{4} R_{---J---} = - \left( \frac{m}{3} \right)^{2} \delta^{l} J, \]
\[ R_{++IJ++} = \frac{1}{4} R_{---J---} = - \left( \frac{m}{6} \right)^{2} \delta^{l} J. \quad (3.7) \]
Using (3.7) and (3.8) one can easily check that the Einstein equation (2.8) is satisfied.

B. Supersymmetric pp-wave solution of the 11D superspace supergravity constraints

The 11D superspace representing the completely supersymmetric pp-wave solution of the 11D supergravity, which we denote by \( \Sigma^{(11|32)} \), was described e.g. in \[7\]. It can be defined through the following Maurer-Cartan equation (reduction of the 11D supergravity constraints from \[31\] to \[33\])
\[ T_{ab} = - i E^a \wedge E^b \Gamma^a_{ab}, \quad T^a = - E^a \wedge E^b T_{ab} \gamma^a, \quad (3.8) \]
\[ R_{ab} = \frac{1}{2} E^a \wedge E^b R_{ab} + \frac{1}{4} E^d \wedge E^c R_{d}^{ab}, \quad (3.9) \]
with constant \( T_{ab} \gamma^a \) and \( R_{ab} \gamma^a \) which are constructed from the constant flux \( \Sigma_{2} \) as
\[ T_{ab} \gamma^a = \frac{i}{18} (F_{a[3]} \Gamma^{[3]} + \frac{1}{8} F_{a[4]} \Gamma^{[4]} \gamma_{a} \), \quad \gamma_{a}. \quad (3.10) \]
\[ R_{ab} \gamma^a = \left( - \frac{2}{3} F_{ab} \Gamma_{[2]} + \frac{2i}{3} \right) (\ast F)^{ab} \Gamma^{[3]} \gamma_{a} \quad (3.11) \]
Fixing the Wess–Zumino (WZ) gauge
\[ i_\theta E^a := \Theta^\beta E^a_{\beta} (Z) = \Theta^a, \]
\[ i_\theta E^a := \Theta^\beta E^a_{\beta} (Z) = 0, \]
\[ i_\theta \omega^{ab} := \Theta^\beta \omega^{ab}_{\beta} = 0 \quad (3.12) \]
one can solve the above constraints and find the complete expressions for the supervielbein and spin connection (see \[29\] and refs therein)
\[ E_{ab} (x, \Theta) = e^a (x) - 2i D \Theta^b \sum_{n=0}^{15} \frac{1}{(2n + 2)!} ((\Theta \Theta M)^n) \Gamma_{ab} \Theta^b, \quad (3.13) \]
\[ E^a (x, \Theta) = D \Theta^a + D \Theta^b \sum_{n=1}^{16} \frac{1}{(2n + 1)!} ((\Theta \Theta M)^n) \gamma^a, \quad (3.14) \]
\[ \omega^{ab} (x, \Theta) = \omega^{ab} (x) + D \Theta^b \sum_{n=0}^{15} \frac{1}{(2n + 1)!} ((\Theta \Theta M)^n) \gamma^a R_{\gamma a} \gamma^a \Theta^a, \quad (3.15) \]
with
\[ (\Theta \Theta M)_{\alpha}^a := 2i (\Theta \Gamma)_{\alpha} \Theta \Gamma T_{a} \gamma - \frac{1}{4} \Theta \gamma R_{\gamma a} \gamma^a \Theta^a. \quad (3.16) \]
Here \( e^a (x) = E_{ab} (x, 0) \) are the bosonic vielbein forms \[2,4\] and \( \omega^{ab} (x) = \omega^{ab} (x, 0) = dx^{a} \omega^{ab} (x) \) is the purely bosonic limit of the \( SO(1,9) \) ('spin') connection, the nonvanishing components of which are given by Eq. (3.3). These are used to define \( D \Theta^a \),
\[ D \Theta^a := D \Theta^b - e^a (x) \Theta^a T_{a} \gamma^b - d \Theta^b - \Theta^a \sum_{n=0}^{15} \frac{1}{(2n + 1)!} ((\Theta \Theta M)^n) \gamma^a R_{\gamma a} \gamma^a \Theta^a. \quad (3.17) \]
Notice that \( (\Theta \Theta M)_{\alpha}^a \) in (3.16) obeys \( \Theta^a (\Theta \Theta M)_{\alpha}^a = 0 \) so that the WZ gauge conditions (3.12) are satisfied.
We will use the above expressions to study the pp-wave superspace, but notice that they can be also used to determine the supervielbein and spin connection forms of $AdS_4 \times S^7$ and $AdS_4 \times S^7$ superspaces. To be more specific, we substitute (3.2) and (3.3) and find that, for pp-wave superspace $\Sigma^{pp-w}$,

$$E^{\alpha} T_{a \beta} = \frac{iu}{6} E^{++} \left( I + \frac{1}{4} (\Gamma^{-1} + \Gamma^{++}) \Gamma^{123} \right) \alpha \beta + \frac{iu}{6} E^I (\Gamma^{++} \Gamma^I \Gamma^{123}) \alpha \beta + \frac{iu}{12} E^I (\Gamma^{++} \Gamma^I \Gamma^{123}) \alpha \beta ,$$

(3.18)

$$R_{a \beta} \Gamma_{ab \gamma} = -\frac{4u}{3} (\Gamma^{++} \Gamma^I \Gamma^{123}) \alpha \beta \delta - \frac{iu}{3 \cdot 5} \epsilon^{IJL} (\Gamma^I \Gamma^L \Gamma^{123}) \alpha \beta \delta - \frac{4u}{3} (\Gamma^{++} \Gamma^I \Gamma^{123}) \alpha \beta \delta - \frac{iu}{6} \epsilon^{IJL} (\Gamma^{++} \Gamma^I \Gamma^{123}) \alpha \beta \delta .$$

(3.19)

These expressions shall be used to specify (3.10). However, this gives a quite complex expression which does not result in essential simplification of the series in Eqs. (3.14) -(3.15).

The hope now is that, taking into account for the equations defining the center of energy motion of the mM0 system, or using a particular solution of these equations, one can simplify the expressions for pull–backs of the forms (3.14, 3.15, 3.16, 3.18, 3.19) to $W^{[16]}$ in such a way that the equations for the relative motion of mM0 constituents, (2.1), (2.2) and (2.3) become manageable.

IV. MULTIPLE M0-BRANES IN SUPERSYMMETRIC PP-WAVE BACKGROUND AND BMN MODEL EQUATIONS

In (13–17) the center of energy motion of the mM0 system is described by the same equations as the motion of a single M0-brane. More precisely, the embedding of the center of energy superspace $W^{[16]}$ into the target 11D supergravity superspace is assumed to be described by the same superembedding equations as the embedding of the worldline superspace of a single M0. Consequently, to study the mM0 system in the 11D pp-wave background it is necessary to find a(t least a particular) solution of the superembedding equation describing embedding of $W^{[16]}$ into the 11D pp-wave superspace.

A. Worldline superspace $W^{[16]}$ and superfield equations of M0-brane

In the superembedding description the coordinate functions of M0–brane (or of the center of energy of the mM0 system) are superfields $\hat{W}(\tau, \eta^a) = (\hat{W}(\zeta), \hat{B}(\tau, \eta^a))$ which determine the embedding of the (center of energy) worldline superspace $W^{[16]}$, Eq. (2.15), into the target 11D superspace $\Sigma^{[11]}$.

$$W^{[16]} \in \Sigma^{[11]}; \ Z^M = \hat{W}(\tau, \eta^a).$$

(4.1)

Using the gauge symmetries one can also specify the form of the coordinate superfields

$$\hat{Z}^M(\tau, \eta^a) = (\hat{X}^+(\tau, \eta^a), \hat{\Theta}^a(\tau, \eta^a)).$$

(4.2)

In particular, the fermionic coordinate function can be presented in the form

$$\hat{\Theta}^a(\tau, \eta^a) = \eta^a v^+ - \Theta^a(\tau, \eta^a) v^-,$$

(4.3)

which separates explicitly the 16 component (SO(9) spinor) Goldstone fermion superfield $\Theta^a(\tau, \eta^a)$ and the moving frame superfield by the equation

$$\hat{\Theta}^a(\tau, \eta^a) = \eta^a v^+ + \Theta^a(\tau, \eta^a) v^-.$$

(4.4)

Notice that in the Wess-Zumino gauge (4.12) the index of the fermionic coordinate of superspace is identified with the $Spin(1,10)$ index, which makes possible to write Eq. (4.3) in a generic curved supergravity superspace. Similarly, after fixing the gauge (4.3), the worldline superspace fermionic coordinate $\eta^a$ is transformed nontrivially under the $SO(1,1)$ transformations acting on spinor moving frame superfields $v^\pm$.
and by constant moving frame vectors
worldline superspace
ing energy motion of the mM0 system, but rather chose a
to find general solution of the equations of the center of
energy superspace
implies that the above equations are used to describe the
tions of motion of the M0-brane. This
This gives the superfield generalization of the fermionic equa-
tions of motion of the single M0-brane. This
Actually, the M0–brane equations of motion can be
of mM0 system.
As far as our main interest here is in relation with
Following [15–17], we describe the center of energy mo-
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Our
Following [15–17], we describe the center of energy motion of
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This supports the above statement on that the M0–brane
dynamics can be completely described in terms of the
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moving frame superfields.

\[
\delta \mathcal{E}^{+q}(\tau, \eta) + d\rho^p \mathcal{E}^{+q}_{\rho}(\tau, \eta) \quad \text{and is expressed through the pull–back of the fermionic supervielbein to } \mathcal{W}_0^{[16]}, \quad \hat{E}^\alpha := E^\alpha(\hat{Z}), \quad \text{and spinor moving frame superfields by}
\]

\[
\hat{E}^\alpha := d\hat{Z}^M(\tau, \eta) E_M^\alpha(\hat{Z}(\tau, \eta)) = \mathcal{E}^{+q} v_q^{-\alpha}(\tau, \eta),
\]

\[
\mathcal{E}^{+q} = \hat{E}^\alpha v_{\alpha q}^+.
\]

The other projection,
\[
\hat{E}^\alpha v_{\alpha q}^- (\tau, \eta) = 0,
\]
gives the superfield generalization of the fermionic equations
of motion of the M0-brane.

Actually, the M0–brane equations of motion can be
expressed by stating the covariant constancy of the light–
like vector \( u^a = \) \( \ldots \)
\[
Du^a(\tau, \eta):=du^a = -u^a, \quad 2u^a = \omega(0)(\tau, \eta) = 0.
\]

This supports the above statement on that the M0–brane
dynamics can be completely described in terms of the
moving frame superfields.

Following [15–17], we describe the center of energy motion of
the mM0 system by the equations which coincide
with the equations of motion of a single M0-brane. This
implies that the above equations are used to describe the
center of energy superspace \( \mathcal{W}_0^{[16]} \) of mM0 system.

As far as our main interest here is in relation with
the BMN matrix model, for simplicity we will not try
to find general solution of the equations of the center of
energy motion of the mM0 system, but rather chose a
particular solution of the equations of the superembed-
ding approach, specifying the embedding of a particular
worldline superspace \( \mathcal{W}_0^{[16]} \) in the pp-wave superspace
\( \Sigma_{pp–w}^{(11|32)} \) described in the previous section.

\section{Worldline superspace \( \mathcal{W}_0^{[16]} \) describing a
particular 1/2 BPS state of M0-brane in \( \Sigma_{pp–w}^{(11|32)} \)}

Our \( \mathcal{W}_0^{[16]} \) superspace is characterized by vanishing of the Goldstone fermion superfield in \( \mathcal{E}^q(\tau, \eta) \), so that
\[
\hat{\Theta}^\alpha(\tau, \eta) = \Theta^+ q(\tau, \eta) v^{-\alpha}_q,
\]

\[
\text{by vanishing of nine bosonic Goldstone superfields}
\]

\[
\hat{x}^I(\tau, \eta) = 0, \quad \hat{x}^J(\tau, \eta) = 0,
\]

and by constant moving frame vectors
\[
\begin{align*}
\omega^a_{\alpha} &= \delta^{++}_a \equiv \delta^0_a - \delta^{10}_a, \\
\omega^a_{\beta} &= \delta^+_{a} \equiv \delta^0_a + \delta^{10}_a, \\
\omega^a_{\mu} &= \delta^a_\mu.
\end{align*}
\]

This \( \mathcal{W}_0^{[16]} \) describes a particular supersymmetric
ground state of a center of energy of the mM0-system (or
of the single M0–brane) in the 11D pp-wave background.

Eq. \( (4.12) \) implies that the spinor moving frame superfields are also constant,
\[
v^{-\alpha}_q = v^{\alpha}_q = \delta^{\alpha}_q,
\]

obeying
\[
\begin{align*}
v^{\alpha}_q \Gamma^{++}_{\alpha \beta} &= 0, \\
v^{\alpha}_q \Gamma^{+–}_{\alpha \beta} &= 2v^{\gamma \beta}_q, \\
v^{\alpha}_q \Gamma^{–+}_{\alpha \beta} &= \gamma^q v^{\beta +}_q, \\
v^{\alpha}_q \Gamma^{––}_{\alpha \beta} &= 0, \\
v^{\alpha}_q \Gamma^{ij}_{\alpha \beta} &= \gamma^q v^{\beta \mu}_{q} v^{\mu +}_q.
\end{align*}
\]

Notice that the (constant) frame \( \mathcal{E}_p \) is oriented in such
a way that
\[
\mathcal{E}^{+q}_p = -6\hat{x}^{++}_q (\hat{x}^{++})^{123} F_{++123} = -3\mu \theta_1 \theta_2 \theta_3.
\]

Using these equations one can easily check that the BPS
conditions \( (2.40) \) and \( (2.41) \) are obeyed. Below we will also see that
Eq. \( (2.41) \) is satisfied so that our particular solution of the superembedding approach equations
preserves 1/2 of the target space supersymmetry.

The coordinate function \( \hat{x}^{++} \) can be identified with the
(generalization of the) particle proper time,
\[
\hat{x}^{++}(\tau, \eta) = \tau,
\]
while the remaining bosonic Goldstone superfield
\( \hat{x}^{–+}(\tau, \eta) \) can take an arbitrary constant value,
\( \partial_\tau \hat{x}^{–+}(\tau, \eta) = 0 \). The nonvanishing components \( \Theta^+ q \) of the Grassmann superfield \( \Theta^\alpha \) are related to the Grass-
mann coordinates \( \eta^+ q \) of the worldvolume superspace
\( \mathcal{W}_0^{[16]} \) by
\[
\hat{\Theta}^+ q(\tau, \eta) = \eta^{+p} \left( exp \left( \frac{\mu}{6} \hat{x}^{++ + 123} \right) \right)_{pq}.
\]

One can check that this configuration solves the equations of the superembedding approach and, hence,
describes a particular solution of the equations of motion of
single M0–brane in \( \Sigma_{pp–w}^{(11|32)} \). Indeed, it is characterized by
\[
e^I(\hat{x}) = 0, \quad e^J(\hat{x}) = 0, \quad e^{++}(\hat{x}) = \hat{d}^{++}, \quad e^{–+}(\hat{x}) = \hat{d}^{–+},
\]

so that, at zero order in Grassmann coordinate \( \eta^+ q \), the
equations of motion \( (4.9) \) are satisfied.

Then, Eqs. \( (4.20) \) and \( (3.18) \) result in
\[
\hat{d}^{\alpha} T_{a \beta}^{\alpha} = -\frac{\mu}{6} d^{++} (\Theta^+ q^{123}) v^{–\alpha}_q,
\]

so that calculating the pull–back of \( \hat{d} \hat{\Theta}^\alpha \) in \( (3.17) \), one finds
\[
\hat{d} \hat{\Theta}^\alpha = d \eta^{+p} \left( exp \left( \frac{\mu}{6} \hat{x}^{++ + 123} \right) \right) v^{–\alpha}_q.
\]
which implies
\[ \tilde{D} \tilde{\Theta}^a v_{aq}^- = 0, \]  
(4.23)\] 
\[ \tilde{D} \tilde{\Theta}^a v_{aq}^+ = d\eta^p \left( \exp \left\{ -\frac{\mu}{6} \tilde{x}^{+q} \right\} \right)_{pq}. \]  
(4.24)\]

At this stage it is important to notice that the pull–back to \( W_0^{(1)16} \) of \( \Theta \Theta , M \), defined in (5.10), (5.11) and (5.19), has the following block-diagonal structure
\[ (\hat{\Theta} \hat{\Theta} M)^{\alpha}_\beta = v_{\beta p}^+ (S^T (\eta \eta M)^{++} S)_{pq} v_q^{-a} + v_{\beta p}^- (S^T (\eta \eta M)^{++} S)_{pq} v_q^{+a}. \]  
(4.25)\]

The explicit form of the matrices \( S, (\eta \eta M)^{+-} \) \( (\eta \eta M)^{--} \) \( (\eta \eta M)^{++} \) (which can be found in the Appendix, Eqs. (A.45), (A.46) and (A.47)) is not needed to check that Eqs. (4.23) and (4.24) imply that \( E^a v_{aq}^- \propto \tilde{D} \tilde{\Theta}^a v_{aq}^- \). Then the (superfield generalization of the) fermionic equations of motion, Eq. (4.33) are obeyed as a consequence of (4.23),
\[ \tilde{E}^a v_{aq}^- = \tilde{D} \tilde{\Theta}^a v_{aq}^{-} \left( \delta_{pq} + \sum_{n=1}^{8} \frac{(\eta \eta M)^{++})^n_{pq}}{(2n + 1)!} \right) = 0. \]  
(4.26)\]

Furthermore, using (4.23) one finds that the superembedding equations is satisfied to all orders in \( \eta^q \), this is to say that \( \tilde{E}^I = 0, \tilde{E}^J = 0 \) and \( \tilde{E}^{--} = 0 \).

The fermionic and bosonic supervielbein forms of the worldvolume superspace read
\[ \mathcal{E}^+ = d\tilde{x}^{++} - 2i\eta^q \eta^{+q} - 4i\eta^q (\eta \eta \mathcal{K})_{pq} \eta^{+q}, \]  
(4.27)\]
\[ \mathcal{E}^{-+} = d\eta^q \Xi^p q (\tau, \eta), \]  
(4.28)\]
(see Eqs. (A.48) and (A.49) in the Appendix for the explicit form of \( (\eta \eta \mathcal{K})_{pq} \) and \( \Xi^p q (\tau, \eta) \)). This implies that the covariant derivatives for scalar superfields are
\[ \nabla^# = \tilde{\partial}^+_+, \]  
\[ \nabla^-q = (L^{-1})^q p (D^-p + 4i(\eta \eta \mathcal{K})_{pq} \eta^{+q} \tilde{\partial}^-q), \]  
(4.29)\]
where
\[ D^-q = \tilde{\partial}^-q + 2i\eta^q \tilde{\partial}^-q, \quad \tilde{\partial}^-q := \frac{\partial}{\partial \eta^{+q}} \]  
(4.30)\]
\[ \tilde{\partial}^+_+ := \frac{\partial}{\partial \tilde{x}^{++}} = \frac{1}{\partial_x^{++} + \partial_t} \]  
are derivatives covariant with respect to the flat superspace supersymmetry.

The pull–back of the spin connection forms to \( W_0^{(1)16} \) is characterized by
\[ \omega^{--}_I (\tilde{x}, \tilde{\theta}) = 0, \quad \omega^{++}_I (\tilde{x}, \tilde{\theta}) = 0, \]  
(4.31)\]
\[ \omega^{--}_J (\tilde{x}, \tilde{\theta}) = 0, \quad \omega^{++}_J (\tilde{x}, \tilde{\theta}) = 0, \]  
(4.32)\]
\[ \omega^{++}_I (\tilde{x}, \tilde{\theta}) = d\eta^q \omega^{++}_I (\tilde{x}, \tilde{\theta}), \]  
(4.33)\]
(see Eqs. (A.50) and (A.51) in Appendix for explicit form of \( \omega^{++}_I (\tilde{x}, \tilde{\theta}) \) and \( \omega^{++}_I (\tilde{x}, \tilde{\theta}) \) so that it is easy to check that the (superfield generalization of the) M–brane bosonic equation, Eq. (4.9), is satisfied.

Furthermore, using the fact that the moving frame vectors characterizing our solution are constant, \( du^-q = 0, du^q = 0, du^p = 0 \), together with Eqs. (2.37) and (4.31)–(4.33), we find that \( \Omega^{Iq} = 0, (\Omega^{Iq} = 0 \text{ is equivalent to } (4.35)) \) as well as the explicit form of the SO(9) and SO(1,1) connection. These are characterized by
\[ \Omega^{(0)} = 0, \quad A^{Iq} = 0, \]  
(4.34)\]
which imply \( \Omega^{Iq} = 0 \) and \( A^{Iq} = 0 \) so that
\[ D^# = \nabla^# = \tilde{\partial}^+_+ . \]  
(4.35)\]

Now we see that the third BPS equation (2.41) is satisfied just because \( F_{#ijk} \) in Eq. (4.16) is a constant. This completes the proof of the fact that our particular solution of the superembedding approach equations preserves 1/2 of the supersymmetry of the 11D pp-wave background. This 16 preserved supersymmetry of the target superspace are identified with the supersymmetry of the worldline superspace \( W_0^{(1)16} \) described by this solution so that the equations of the relative motion of the mM0 system, defined on this superspace in a manifestly covariant way, are invariant under this 16 parametric supersymmetry by construction.

C. Multiple M–brane equations in 11D pp-wave background and BMN matrix model equations

Now we are ready to write explicitly the equations of the relative motion of the constituents of multiple M–brane system the center of energy of which moves in the pp–wave superspace in the above described particular manner. These read
These equations coincide with the ones which can be obtained by varying the BMN action \[6\] up to the fact that they are formulated for the traceless matrices.

The trace parts of the matrices in \[6\] should be related with the center of energy motion of the mM0 system. In our approach that is described separately by the geometry of the embedding of the center of energy worldvolume superspace \(\mathcal{Y}^{(1|10)}\) into \(\mathcal{X}^{(1|12)}\). Thus to find the equations of motion for the center of energy coordinate functions (which are essentially center of energy Goldstone bosons and fermions), one should go beyond the ground state solution of the superembedding equation, which we have used above. This will be the subject of the forthcoming paper. Here we will restrict ourself by showing that the complete set of the BMN equations is reproduced by the mM0 equations in the leading approximation on the center of energy Goldstone fields.

\[\begin{align*}
\partial_{++} \Psi_q + \frac{\mu}{4} \gamma_{qp}^{123} \Psi_p &= -\frac{1}{4} \gamma_{qp} \left[ X^i, \Psi_p \right], \\
\left[ X^i, \partial_{++} X^i \right] &= 4i \left\{ \Psi_q, \Psi_q \right\} \\
\hat{\partial}_{++} \hat{\partial}_{++} X^i + \left( \frac{\mu}{3} \right)^2 X^i &= \frac{1}{16} \left[ X^i, [X^j, X^k] \right] + i \gamma_{qp} \left( \Psi_q, \Psi_p \right) - \frac{\mu}{8} \varepsilon^{ijk} \left[ X^j, X^K \right], \\
\hat{\partial}_{++} \hat{\partial}_{++} X^J + \left( \frac{\mu}{6} \right)^2 X^J &= \frac{1}{16} \left[ X^i, [X^i, X^J] \right] + i \gamma_{qp} \left( \Psi_q, \Psi_p \right).
\end{align*}\]

Now, let us turn back to the relative motion equations \((4.30), (4.37), (4.38), (4.39)\) which are written for traceless matrices. The BMN action produces the same equations but for tracefull matrices. As trace parts of the matrices do not contribute to the commutator terms, the traceless parts of BMN equations coincide with \((4.36), (4.37), (4.38), (4.39)\).

The trace part of the BMN counterparts of equations \((4.38)\) and \((4.39)\) coincides with Eqs. \((4.42)\). The trace part of the BMN version of fermionic equations \((4.36)\) seems to be different from \((4.43)\): it describes a relativistic fermion of mass \(\frac{\mu}{4}\) rather than \(\frac{\mu}{2}\) in \((4.43)\). However, in \(d = 1\) field theory the fermionic mass is a matter of convenience in the sense that the value of mass can be changed by an appropriate (coordinate dependent) field redefinition. Indeed the field \(\psi^{-q} := \theta^{-p} \exp \{i \left( \frac{\mu}{16} \hat{\partial}_{++} \right) \} \frac{\gamma^{123}}{p q}\) obeys the equation for massive fermion of mass \(\mu/4\),

\[\hat{\partial}_{++} \psi^{-q}(\tau) = \frac{\mu}{4} \psi^{-p}(\tau) \gamma^{123}_{pq}\]

which coincides with the trace part of the BMN version of Eq. \((4.36)\).

This allows us to conclude that the mM0 equations in general 11D supergravity background obtained in \([16, 17]\) when specified for the case of completely supersymmetric pp-wave solution, does reproduce the equations of BMN model as an approximation. This is a leading approximation in the decomposition on powers of the center of energy Goldstone fields (bosonic \(\hat{\epsilon}^i, \hat{x}^i\) and fermionic \(\theta^{-q}\)) which is made over the supersymmetric vacuum solution of the equations of center of energy motion described by Eqs. \((4.11), (4.12), (4.18), (4.19)\).

\section{Conclusion}

In this paper we have begun the program of the studying and developing applications of the Matrix model equations in general 11D supergravity background \([16, 17]\) by specifying them for particular 11D background which are interesting in M-theoretical perspective. We have begun by the case of completely supersymmetric 11D pp-wave background, which is natural as far as a Matrix model in this background have been proposed nine...
years ago by Berenstein, Maldacena and Nastase [6]. So the good check of the multiple M0-brane equations of [16, 17] as equations for matrix model in an arbitrary 11D supergravity background is to check whether they can reproduce the equations of BMN model [6].

We have shown that, when the Goldstone fields of the center of energy motion of multiple M0-brane (mM0) system are set to zero, and the center of energy motion preserves 1/2 of the supersymmetries of the pp-wave background, the equations describing relative motion of the mM0 constituents coincide with the BMN equations, but for written for traceless matrix fields. Furthermore, we have shown that the complete mM0 equations in pp-wave superspace actually reproduce the BMN equations in the leading approximation on Goldstone bosons and fermions describing the center of energy motion (when these are defined as excitations over the above mentioned 1/2 BPS vacuum solution).

The complete accounting of the contribution of the center of energy Goldstone fields $\hat{x}^I$, $\hat{x}^J$ and $\theta^{-}(\tau)$ into the equations of relative motion of mM0 constituents requires a complete description of an arbitrary center of energy superspace $\mathcal{W}^{(I)}[16]$, this is to say it requires a more general solution of the superembedding approach equations in the case of pp-wave target superspace $\Sigma_{pp-wave}^{11|32}$. Then the equation of relative motion could get modified by contributions of these center of energy Goldstone fields and thus deviate from the BMN equations. We hope to turn to this issue in the future publication.

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Appendix.

In this Appendix we collect some technical details.

The explicit form of the matrices entering Eq. (4.25) is given by

\[
S_{pq} := \left( \exp \left\{ -\frac{\mu}{6} \hat{x}^{++} \gamma^{123} \right\} \right)_{pq},
\]

(\eta \eta \mathcal{M})^{-+}_{pq}^{+} = \frac{2i\mu}{3} \eta^{+p} (\eta^{+} \gamma^{123})_{q} - \frac{i\mu}{3} \epsilon^{JK} (\eta^{+} \gamma^{J})_{p} (\eta^{+} \gamma^{K})_{q} - \frac{i\mu}{12} \frac{1}{4!} \epsilon_{IJJ_{1}...J_{q}} (\eta^{+} \gamma^{J_{1}}...\gamma^{J_{q}})_{p} (\eta^{+} \gamma^{j})_{q},
\]

(\eta \eta \mathcal{M})^{-+}_{pq}^{-} = -\frac{2i\mu}{3} (\eta^{+} \gamma^{J})_{p} (\eta^{+} \gamma^{J})_{q} - \frac{2i\mu}{3} (\eta^{+} \gamma^{J})_{p} (\eta^{+} \gamma^{J})_{q} - \frac{i\mu}{3} (\eta^{+} \gamma^{J})_{p} (\eta^{+} \gamma^{J})_{q} -
\]

- \frac{i\mu}{6} \frac{1}{5!} \epsilon_{IJJ_{1}...J_{q}} (\eta^{+} \gamma^{J_{1}}...\gamma^{J_{q}})_{p} (\eta^{+} \gamma^{j})_{q}.
\]

The matrices entering Eqs. (4.27) and (4.28) read

\[
(\eta \eta \mathcal{K})_{pq} := \sum_{n=1}^{7} \frac{1}{(2n+2)!} (\eta \eta \mathcal{M})_{pq}^{-+}^{n},
\]

\[
\Xi^{pq}(\tau, \eta) := \left( \delta_{pp'} + \sum_{n=1}^{8} \frac{1}{(2n+1)!} (\eta \eta \mathcal{M})_{pqp'}^{-+}^{n} \right) \left( \exp \left\{ \frac{\mu}{6} \hat{x}^{++} \gamma^{123} \right\} \right)_{p'q}.
\]

The nonvanishing components of the pull–back of pp-wave superspace spin connection to $\mathcal{W}^{(1)}_{0}$ (see Eqs. (4.32) and (4.33)) are

\[
\omega^{IJ}(\hat{x}, \hat{\Theta}) = \frac{8i\mu}{3} \epsilon^{JK} d\eta^{+q} \sum_{n=1}^{7} \frac{1}{(2n+2)!} (\eta \eta \mathcal{M})_{pq}^{-+}^{n} \gamma^{K} \eta^{+q},
\]

\[
\omega^{IJ}(\hat{x}, \hat{\Theta}) = \frac{\mu}{6} \frac{1}{4!} \epsilon_{IJJ_{1}...J_{q}} d\eta^{+q} \sum_{n=1}^{7} \frac{1}{(2n+2)!} (\eta \eta \mathcal{M})_{pqp'}^{-+}^{n} \gamma^{J_{1}}...\gamma^{J_{q}}.
\]
The $SO(1,1)$ connection on $\mathcal{W}_0^{(1\ell)}$ and of the connection on the normal bundle over $\mathcal{W}_0^{(1\ell)}$ read

$$\Omega^{(0)} = 0, \quad \Omega^\# I = 0 = \Omega^\# J = 0,$$

$$A^{IJ} = 0, \quad A^{I J} = \frac{8 \mu}{3} \epsilon^{IJK} \frac{d\eta}{\gamma^p} (\gamma^K \eta^p), \quad A^{I J} = \frac{\mu}{6} \frac{1}{4!} \epsilon^{I J I_1 \ldots I_k} \frac{d\eta}{\gamma^p} (\gamma^{I_1} \ldots \gamma^{I_k} \eta^p),$$

where $\Xi^p$ is the invertible matrix presented in Eq. (A.49).

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