$N\bar{N}, \Delta N, \Delta \bar{N}$ excitation for pion propagator in nuclear matter

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Abstract

The particle - hole and delta - hole excitations are well known elementary excitation modes for pion propagator in nuclear matter. But, the excitation also involves antiparticles, namely, nucleon-antinucleon, antidelta-nucleon and delta-antinucleon excitations. These are important for high energy-momentum as well, and not studied before. In this paper, we give both the formulae and the numerical calculations for the real and the imaginary parts of these excitations.

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The excitations caused by the meson propagation in nuclear matter, or the polarization
insertion of the meson propagators, is very elementary and important, related to many fields
in nuclear physics [1, 2], e. g., the meson propagators are indispensable to the calculation of
the binding energy of nuclear matter in the RPA approximation and in relativistic many-body
field theory [4]. Taking the pion as another example, its propagator has a decisive effect on
the dilepton production rate in heavy ion collisions [3]. But in the calculation of the dilepton
production rate, the pion propagator used has been nonrelativistic. The validity of using
a nonrelativistic pion propagator for the rho meson production energy-momentum region is
obviously suspicious. A relativistic description of the particle-hole (ph) excitation for the meson
propagator was not available until our last work [5, 6]. In that work, we pointed out that the
ph and particle-antiparticle (N̅N) contributions could not be split by using the traditional
method [7, 8]. We put forth a new method and calculated the dimensonic function [5] and pion
propagator [6] with an analytical expression for relativistic ph excitation. We have extended
this formalism to study the pion propagator in nuclear matter [9]. Besides ph excitation, the
delta-hole (∆h) excitation channel will open and play a dominant role for pion propagator. In
all these calculations, traditional or new formalism, the N̅N excitation antidelta-nucleon (∆N),
and delta-antinucleon (∆N̅) excitations were ignored, and nobody has ever studied them.

In this paper, we will study these excitations with antiparticles involved, by calculating the
imaginary part of the polarization insertion of pion propagator. The nucleon and ∆-isobar
propagators in nuclear matter $S(p), S_{\mu\nu}(p)$ can be expressed in terms of a particle-antiparticle
formalism as follows:

\begin{equation}
S(p) = S_p(p) + S_h(p) + S_{\bar{p}}(p)
\end{equation}

\begin{equation}
S_p(p) = (1 - n_p) \frac{1}{2E_N(p)} \frac{\gamma_0 E_N(p) - \vec{\gamma} \cdot \vec{p} + \tilde{m}_N}{p_0 - E_N(p) + i\epsilon},
\end{equation}

\begin{equation}
S_h(p) = \frac{n_p}{2E_N(p)} \frac{\gamma_0 E_N(p) - \vec{\gamma} \cdot \vec{p} + \tilde{m}_N}{p_0 - E_N(p) - i\epsilon},
\end{equation}

\begin{equation}
S_{\bar{p}}(p) = -\frac{1}{2E_N(p)} \frac{-\gamma_0 E_N(p) + \vec{\gamma} \cdot \vec{p} + \tilde{m}_N}{p_0 + E_N(p) - i\epsilon},
\end{equation}

where $n_p$ is the nucleon distribution function and $S_p, S_h, S_{\bar{p}}$ are the particle, hole, antiparticle
propagators. Also,

\begin{equation}
S_{\mu\nu}(p) = S_{\mu\nu}^\Delta(p) + S_{\mu\nu}^{\bar{\Delta}}(p),
\end{equation}

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\begin{equation}
S_\Delta^{\mu\nu}(p) = \frac{\tilde{m}_\Delta}{E_\Delta(p)} \cdot \frac{\Lambda_+^{\mu\nu}(p)}{p_0 - E_\Delta(p) + i\epsilon}, \tag{6}
\end{equation}

\begin{equation}
S_\Delta^{\mu\nu}(p) = -\frac{\tilde{m}_\Delta}{E_\Delta(p)} \cdot \frac{\Lambda_-^{\mu\nu}(-p)}{p_0 + E_\Delta(p) - i\epsilon}, \tag{7}
\end{equation}

where \(S_\Delta^{\mu\nu}(p), S_\Delta^{\mu\nu}(p)\) are the delta and antidelta isobar propagator, respectively, and \(\Lambda_\pm^{\mu\nu}(p)\) is the corresponding projection operator:

\begin{equation}
\Lambda_+^{\mu\nu}(p) = -\frac{\gamma \cdot p + \tilde{m}_\Delta}{2\tilde{m}_\Delta} \cdot \left[ g^{\mu\nu} - \frac{1}{3} \gamma^\mu \gamma^\nu - \frac{2}{3\tilde{m}_\Delta^2} p^\mu p^\nu + \frac{1}{3\tilde{m}_\Delta} (p^\mu \gamma^\nu - p^\nu \gamma^\mu) \right], \tag{8}
\end{equation}

\begin{equation}
\Lambda_-^{\mu\nu}(p) = -\frac{\gamma \cdot p + \tilde{m}_\Delta}{2\tilde{m}_\Delta} \cdot \left[ g^{\mu\nu} - \frac{1}{3} \gamma^\mu \gamma^\nu - \frac{2}{3\tilde{m}_\Delta^2} p^\mu p^\nu - \frac{1}{3\tilde{m}_\Delta} (p^\mu \gamma^\nu - p^\nu \gamma^\mu) \right]. \tag{9}
\end{equation}

Here and in eqs. (2-4), \(\tilde{m}_N, \tilde{m}_\Delta\) is effective mass of nucleon and \(\Delta\)-isobar in nuclear matter, and \(E_{N(\Delta)}(p) = \sqrt{p^2 + \tilde{m}_{N(\Delta)}^2}\).

The polarization with the nucleon and nucleon-delta loop insertions for the pion propagator can be calculated in the standard field theory, and it reads:

\begin{equation}
\Pi(q) = \Pi_N(q) + \Pi_\Delta(q), \tag{10}
\end{equation}

\begin{equation}
\Pi_N(q) = -2i \frac{f_{\pi NN}^2}{m_\pi^2} q_\mu q_\nu \int \frac{dp}{(2\pi)^4} Tr[\gamma_5 \gamma^\mu S(p) \gamma_5 \gamma^\nu S(p + q)], \tag{11}
\end{equation}

\begin{equation}
\Pi_\Delta(q) = -\frac{8i}{6} \frac{f_{\pi N\Delta}^2}{m_\pi^2} q_\mu q_\nu \int \frac{dp}{(2\pi)^4} Tr[S(p) S^{\mu\nu}(p + q)] + (q \to -q), \tag{12}
\end{equation}

where \(\Pi_N(q), \Pi_\Delta(q)\) are due to nucleon and nucleon-delta loop contributions, respectively. We have used the pseudo-vector coupling for \(\pi NN\) and \(\pi N\Delta\) interacting vertices, \(f_{\pi NN}\) and \(f_{\pi N\Delta}\) are the corresponding coupling constants. \(f_{\pi N\Delta} = 2 f_{\pi NN}\) with \(f_{\pi NN} = 0.988\) [8]. In ref. [9], we have explained why the arbitrary parameter \(\xi\) that appeared in ref. [8] does not appear in our formalism.

Substitute eqs. (2-4, 5-9) into eqs. (11, 12), and the polarization insertion can be expressed in terms of nothing but the \(ph, N\bar{N}, \Delta h, \bar{\Delta} N\) and \(\Delta \bar{N}\) excitation contributions:

\begin{equation}
\Pi_N(q) = \Pi_{ph}(q) + \Pi_{N\bar{N}}(q), \tag{13}
\end{equation}

\begin{equation}
\Pi_\Delta(q) = \Pi_{\Delta h}(q) + \Pi_{\Delta N}(q) + \Pi_{\bar{\Delta} N}(q), \tag{14}
\end{equation}

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the subscript denoting the corresponding excitation. The $\phi h$ and $\Delta h$ excitations have been studied extensively in previous work \cite{6, 9}, so we will pay more attention to antiparticle excitation in this paper. The imaginary part of these polarization insertions can be written as follows:

\begin{align*}
Im \Pi_{\phi h}(q) &= 4 \frac{f_{\pi NN}^2}{m_\pi^2} \tilde{m}_N^2 q^2 \pi \int \frac{dp}{(2\pi)^3 E_N(p)E_N(p + q)} (1 - n_p)n_{p+q} \cdot [\delta(E_N(p) - E_N(p + q) - q_0) + \delta(E_N(p) - E_N(p + q) + q_0)], \\
Im \Pi_{NN}(q) &= -4 \frac{f_{\pi NN}^2}{m_\pi^2} \tilde{m}_N^2 q^2 \pi \int \frac{dp}{(2\pi)^3 E_N(p)E_N(p + q)} (1 - n_p) \cdot [\delta(E_N(p) + E_N(p - q) - q_0) + \delta(E_N(p) + E_N(p + q) - 2q)], \\
Im \Pi_{\Delta h}(q) &= \frac{4 f_{\pi NN}^2}{9 m_\pi^2} [\tilde{m}_N^2 + \tilde{m}_\Delta^2 - q^2] \cdot [q^2 - \frac{1}{4\tilde{m}_\Delta^2} (\tilde{m}_\Delta^2 - \tilde{m}_N^2 + q^2)^2] \cdot \\
& \quad \pi \int \frac{dp}{(2\pi)^3 E_\Delta(p)E_N(p + q)} n_{p+q} \cdot [\delta(E_\Delta(p) - E_N(p + q) - 3q_0) + \delta(E_\Delta(p) + E_N(p + q) + q_0)], \\
Im \Pi_{\Delta N}(q) &= -\frac{4 f_{\pi NN}^2}{9 m_\pi^2} [\tilde{m}_N^2 + \tilde{m}_\Delta^2 - q^2] \cdot [q^2 - \frac{1}{4\tilde{m}_\Delta^2} (\tilde{m}_\Delta^2 - \tilde{m}_N^2 + q^2)^2] \cdot \\
& \quad \pi \int \frac{dp}{(2\pi)^3 E_\Delta(p)E_N(p + q)} (1 - n_p) \cdot [\delta(E_\Delta(p) + E_N(p + q) - q_0) + \delta(E_\Delta(p) + E_N(p + q) + q_0)].
\end{align*}

The nucleon distribution function and its combination correctly indicates the excitation properties of each excitation above. But this is not the case for traditional approach\cite{6, 8}. The variables in the $\delta$ functions in the integrands gives the thresholds and the boundary of the energy-momentum for the corresponding excitations. In Fig. 1, we show the imaginary part as a

\begin{center}
Figure 1
\end{center}
function of energy $\omega$ for fixed momentum $q = 2.5k_F$, where $k_F = 1.42 fm^{-1}$. The effective mass of nucleon is derived from the calculation of the binding energy of nuclear matter in the relativistic Hartree approximation [10]. That is $x \equiv \frac{m_N}{m_N} = 0.72$ $(m_N$ is nucleon mass in free space). We set $x_\Delta \equiv \frac{\bar{m}_\Delta}{m_\Delta} = 1$ $(m_\Delta$ is the $\Delta$-isobar mass in free space) for this calculation. The thresholds and the magnitudes for antiparticle excitation are larger than $ph$ and $\Delta h$ excitations. For $ph$ and $\Delta h$ excitations, they are nonvanishing only in a very limited region of energy. But for antiparticle excitations, once the energy is larger than the thresholds, no matter how large it is, the corresponding excitation will not vanish. The $\Delta \bar{N}$ and $\bar{N} N$ excitations, which start from the same threshold, are hard to distinguish from each other. We also find that all these excitations depend on the effective mass of nucleon and delta very strongly. In ref. [11], we calculated the binding energy with pion and $\Delta$-isobar degree of freedom explicitly included, and we found $x = 0.905, x_\Delta = 0.928$ at $k_F = 1.42 fm^{-1}$. The imaginary parts are calculated again by use of these parameters and shown in Fig. 2.

Comparing with Fig. 1, we can see that the $\Delta h$ excitation moves to the left but those of antiparticles move to the right apparently. The strength of the $ph$ and $\Delta h$ are also stronger than those in Fig. 1.

The real and the imaginary part are related to each other by the dispersion relation:

$$Re \Pi(q_0, \mathbf{q}) = \frac{P}{\pi} \int_0^\infty d\omega \frac{Im \Pi(\omega, \mathbf{q})}{\omega^2 - q_0^2}. \quad (20)$$

For antiparticle excitations, $Re \Pi$ is divergent due to the fact that $Im \Pi \neq 0$ and is proportional to $\omega^2$ at least for $\omega \to \infty$; it is also nonrenormalizable for pseudo-vector coupling in the sense of the standard counter-term renormalization scheme. On the other hand, however, nucleon and $\Delta$-isobar are not bare particles, their structure are strongly energy-momentum dependent instead. This dependence is described by $\pi NN$ and $\pi \Delta N$ vertex functions, or form factors (see ref. [12] for example). For the space-like region, the form factor can be described by monopoles or dipoles which are widely used for baryon-baryon interactions [13, 14]. But in the time-like region, there are some problems with these multi-pole form factors especially for large energy-momentum transfer [15]. The exponential form factor is also a good candidate[4, 16]; it resembles multi-pole for low energy-momentum but converges more quicker for high energies.
It has the following form:

\[ F(q^2) = e^{-\frac{q^2}{\Lambda^2}}\theta(q^2) + e^{\frac{q^2}{\Lambda^2}}\theta(-q^2), \]  

(21)

where \( \Lambda \) is the cut-off parameter. Generally \( \Lambda \) it is about \( 7m_\pi \) [4, 17], but this was a too low value \( 2.7m_\pi \) in ref. [16]. We choose \( \Lambda = 6.8m_\pi \) in this paper. Substituting \( f_{\pi NN} \) with \( f_{\pi NN}F(q^2) \) in eqs. (16), \( f_{\pi N\Delta} \) with \( f_{\pi N\Delta}F(q^2) \) in eqs. (18, 19) respectively, the corresponding real parts calculated by eq. (20) are shown in Fig. 3. The other parameters

![Figure 3](image)

are the same as in Fig. 1. We can see that the magnitude of \( Re\Pi \) for \( N\bar{N} \) excitation is about \( 10^2 \) larger than \( \Delta N \) and \( \Delta \bar{N} \) excitations. This is due to the energy threshold for \( N\bar{N} \) excitation being smaller than the latter as shown in Fig. 1, resulting in large suppression by the form factor in the large energy region. We also find that instead of the exponential form factor (21), the monopole form factor [12, 13, 14] does not converge quickly enough to make eq. (20) finite, except for a very small cut-off parameter. This is the same case as mentioned in ref. [16]. That means that measurements and theoretical studies for the nucleon strong interaction form factor in the time-like region and in the larger energy transfer region are very necessary indeed, since it plays a key role in obtaining the real part of the polarization insertions.

In this paper, we neglect the natural width of the \( \Delta \)-isobar propagator, but this will be remedied in future work. Another related issue on the renormalization of the real part of the polarization insertion is the renormalization scheme. For nonrenormalizable pseudo-vector coupling, our scheme adopted in this paper is to use the dispersion relation with the introduction of a form factor with a cut-off parameter \( \Lambda \). An alternative scheme is to introduce a cut-off momentum in momentum space \( \Lambda_p \) [18]. The renormalized polarization insertion thus obtained depends on \( \Lambda_p \) also, so it will be very interesting to compare two schemes and study the difference. The work along line is in progress.

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Figure Captions

Figure 1. $-Im\Pi(\omega, q)$ (in unit of $m^2_\pi$) versus energy $\omega$ (in unit of $m_\pi$) for fixed momentum $q = 2.5k_F$. The meaning of the letters which are explained in the text, indicate the corresponding excitations.

Figure 2. The same as Fig. 1 but with effective mass of nucleon and delta different as described in the text.

Figure 3. The real part of the polarization insertion - $Re\Pi$ (in unit of $m^2_\pi$) versus energy $\omega$ (in unit of $m_\pi$) for fixed momentum $q = 2.5k_F$. The left axis is for $NN$ (solid line), the right axis is for $\Delta N$ (dotted line) and $\Delta \bar{N}$ (dashed line) excitations as indicated by the letters. The right axis is multiplied by factor $10^3$. 

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