Renormalization of tensor self-energy in Resonance Chiral Theory

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We study the problems related to the renormalization of propagators in Resonance Chiral Theory, concentrating on the case of vector 1−− resonances in the antisymmetric tensor formalism. We have found that renormalization of the divergences of the self-energy graphs needs new type of kinetic counterterms with two derivatives which are not present in the original leading order Lagrangian. The general form of the propagator for antisymmetric tensor fields could then contain not only poles corresponding to the original 1−− resonance states but also to the additional states with opposite parity which decouple in the free field limit. In some cases, these dynamically generated additional states might be negative norm ghosts or tachyons.

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I. INTRODUCTION

The use of effective field theories for the description of the dynamics of hadrons has made considerable progress in recent years. In the low energy region $E < \Lambda_H = 1\,\text{GeV}$, the dynamics of the lowest lying states (the pseudoscalar mesons) is effectively described by Chiral Perturbation Theory ($\chi$PT) based on the spontaneous symmetry breaking of chiral symmetry of QCD.

In the intermediate energy region ($1\,\text{GeV} \leq E \leq 2\,\text{GeV}$) one uses the Resonance Chiral Theory ($R\chi T$). $R\chi T$ is based on large-$N_C$ QCD which partially shares a lot of interesting properties with the physical $N_C = 3$ case. In the leading order of the $1/N_C$ expansion, the QCD spectrum contains infinite towers of meson resonances with residual interaction suppressed
by powers of $1/\sqrt{N_C}$. Their dynamics at the leading order in $1/N_C$ can in principle be described in terms of tree level diagrams within an effective theory with an infinite number of fields. Such a theory is not known from first principles; however, it can be basically constructed on symmetry grounds and its free parameters can be fixed by phenomenology. The flavour group of large-$N_C$ QCD is $U(N_f)_L \times U(N_f)_R$ (because of the absence of the axial anomaly in the large-$N_C$ limit) that is spontaneously broken to $U(N_f)_{V}$.

$\chi_T$ is the approximation of the large-$N_C$ QCD when only a finite number of resonances in each channel is included. This approach is well-established, because for example the $O(p^4)$ coupling constants of the $\chi$PT Lagrangian [5] are successfully predicted. In addition, there are some developments in saturation of the $O(p^6)$ coupling constants [13] with a lot of phenomenological consequences, see e.g. [10, 11, 14].

In this paper we want to show that $\chi_T$ might contain some problems and features of inner inconsistency when we go behind leading order (quantum loops in $\chi_T$ were already studied in [15, 16]). The more detailed treatise of the discussed problem will be published in [17].

In the following we are interested only in the sector of vector resonances $1^{--}$ but the results of more general discussion do not differ from this special case.

II. ANTISYMMETRIC TENSOR FORMULATION OF $\chi_T$

The standard description of vector resonances is provided by the vector or antisymmetric tensor fields. It was shown that Lagrangians of these two formulations are not equivalent unless some contact terms are included (it was also proved in [7] that in the general case an infinite number of such terms is necessary). Another possibility of the description of spin-1 resonances is the so-called first-order formalism investigated in [7], where both types of fields are used. For illustrative purposes we restrict our discussion in the following to the antisymmetric tensor case.

The nonet of vector resonances $1^{--}$ can be represented by the antisymmetric tensor fields collected in the $3 \times 3$ matrix $R_{\mu\nu}$

$$R_{\mu\nu} = \begin{pmatrix}
\frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_0 & \rho^+ & K^{*+} \\
-\frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_0 & \rho^- & K^{*0} \\
\frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_0 & K^{*-} & -\frac{2}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_0
\end{pmatrix}_{\mu\nu}. \quad (1)$$

These fields transform under the nonlinear realization of the $U(3)_L \times U(3)_R$. 
Let us start with the following Lagrangian
\[ \mathcal{L} = -\frac{1}{2} \langle \partial_\mu R^{\mu\nu} \partial^\nu R_{\mu\nu} \rangle + \frac{1}{4} M^2 \langle R_{\mu\nu} R^{\mu\nu} \rangle + \mathcal{L}_{\text{int}}, \] (2)
where the brackets denote the trace over group indices. In general the full two-point 1PI Green function has the form
\[ \Gamma^{(2)}_{\mu\nu\alpha\beta}(p) = \frac{1}{2} (M^2 + \Sigma^T(p^2)) \Pi^T_{\mu\nu\alpha\beta} + \frac{1}{2} (M^2 - p^2 + \Sigma^L(p^2)) \Pi^L_{\mu\nu\alpha\beta}, \] (3)
with the projectors
\[ \Pi^T_{\mu\nu\alpha\beta} = \frac{1}{2} (P^T_{\mu\alpha} P^T_{\nu\beta} - P^T_{\nu\alpha} P^T_{\mu\beta}), \] (4)
\[ \Pi^L_{\mu\nu\alpha\beta} = \frac{1}{2} (g_{\mu\alpha} g_{\nu\beta} - g_{\nu\alpha} g_{\mu\beta}) - \Pi^T_{\mu\nu\alpha\beta}, \] (5)
where \( P^T_{\mu\nu} = g_{\mu\nu} - p_\mu p_\nu / p^2 \). Inverting (3) we get for the propagator
\[ \Delta_{\mu\nu\alpha\beta}(p) = \frac{-2}{p^2 - M^2 - \Sigma^L(p^2)} \Pi^L_{\mu\nu\alpha\beta} + \frac{2}{M^2 + \Sigma^T(p^2)} \Pi^T_{\mu\nu\alpha\beta}. \]
This propagator has two types of (generally complex) poles \( s_V \) and \( s_{\tilde{V}} \). The first one satisfies the equations
\[ s_V - M^2 - \Sigma^L(s_V) = 0 \] (6)
and (assuming \( s_V = M_V^2 > 0 \)) we have for \( p^2 \to M_V^2 \)
\[ \Delta_{\mu\nu\alpha\beta}(p) = \frac{Z_V}{p^2 - M_V^2} \sum_\lambda u^{(\lambda)}_{\mu\nu}(p) u^{(\lambda)}_{\alpha\beta}(p)^* + O(1), \]
where
\[ Z_V = \frac{1}{1 - \Sigma^L(M_V^2)}. \]
The wave function \( u^{(\lambda)}_{\mu\nu}(p) \) is expressed in terms of the spin-one polarization vectors \( \varepsilon^{(\lambda)}_{\nu}(p) \) as
\[ u^{(\lambda)}_{\mu\nu}(p) = \frac{i}{M_V} \left( p_\mu \varepsilon^{(\lambda)}_{\nu}(p) - p_\nu \varepsilon^{(\lambda)}_{\mu}(p) \right). \]
Therefore, under the conditions \( M_V^2 > 0 \) and \( Z_V > 0 \) such a pole corresponds to the spin-one state \( |p, \lambda, V\rangle \) which couples to \( R_{\mu\nu} \) as
\[ \langle 0 | R_{\mu\nu}(0) | p, \lambda, V \rangle = Z_V^{1/2} u^{(\lambda)}_{\mu\nu}(p). \] (7)

\(^1\) Here and in what follows we omit the group indices and trivial group factors \( \delta^{ab} \) for simplicity.
One of the solutions of equation \((6)\) is perturbative
\[ s_V = M^2 + \delta M^2_V, \quad Z_V = 1 + \delta Z_V, \]
where \(\delta M^2_V\) and \(\delta Z_V\) are small corrections vanishing in the free field limit. This solution corresponds to the original degree of freedom described by the free part of the Lagrangian \(L_0\). The other possible non-perturbative solutions of \((6)\) decouple in the limit of vanishing interaction.

Additional type of poles, given by the solutions of
\[ M^2 + \Sigma^T(s_V) = 0, \quad (8) \]
is of a non-perturbative nature. For \(s_V = M^2_V > 0\) and \(p^2 \to M^2_V\) we get
\[ \Delta_{\mu\nu\alpha\beta}(p) = \frac{Z_{\bar{V}}}{p^2 - M^2_V} \sum_{\lambda} \tilde{u}^{(\lambda)}_{\mu\nu}(p) \tilde{u}^{(\lambda)*}_{\alpha\beta}(p) + O(1), \]
where
\[ Z_{\bar{V}} = \frac{1}{\Sigma^T(M^2_V)}, \]
\[ \tilde{u}^{(\lambda)}_{\mu\nu}(p) = \frac{1}{2} \varepsilon^{\alpha\beta\mu\nu} u^{(\lambda)}_{\alpha\beta}(p). \]
Assuming therefore that \(M^2_V > 0\) and \(Z_{\bar{V}} > 0\), such a pole corresponds to the spin-one particle states \(|p, \lambda, \bar{V}\rangle\) (with opposite parity w.r.t. \(|p, \lambda, V\rangle\)) which couple to the antisymmetric tensor field as
\[ \langle 0| R_{\mu\nu}(0)|p, \lambda, \bar{V}\rangle = Z^{1/2}_{\bar{V}} \tilde{u}^{(\lambda)}_{\mu\nu}(p). \quad (9) \]
In the free field limit \(\Sigma^T(p^2) = 0\) and the additional degrees of freedom are frozen.

The previous discussion suggests that the general form of the interaction Lagrangian can cause a dynamical generating of additional degrees of freedom at the one loop level. However, the general picture is a little bit more subtle. The point is, that the poles described above might correspond to negative norm ghosts (for \(Z_V, Z_{\bar{V}} < 0\)) or tachyons (for \(M^2_V, M^2_{\bar{V}} < 0\)) (see \cite{17} for details).

As a toy example let us assume a simple “interaction” Lagrangian of the form
\[ L_{\text{int}} = \frac{\alpha}{4} (\partial_\alpha R^{\mu\nu} \partial^\alpha R_{\mu\nu}), \quad (10) \]
which represents actually another type of kinetic term. It generates contribution to both self-energies \(\Sigma^{L,T}(p^2)\)
\[ \Sigma^T(p^2) = \Sigma^L(p^2) = \alpha p^2. \quad (11) \]
Fig. 1: The one loop correction to tensor self-energy. The double line stands for resonance fields, the single line stands for Goldstone bosons. The two graphs on the r.h.s. represent the loop and counterterm contributions respectively.

The two solutions of the equations (6–8) are therefore the perturbative one

\[
M_V^2 = M^2 (1 + \alpha + \ldots), \quad Z_V = (1 + \alpha + \ldots)
\]

and the non-perturbative one

\[
M_V^2 = -\frac{M^2}{\alpha}, \quad Z_V = \frac{1}{\alpha}.
\]

Thus for \( \alpha < 0 \) the additional negative norm ghost is propagated (tachyon for the case \( \alpha > 0 \)).

Note that, the “interaction” term (10) is not present at the tree level, however, it can be generated as a counterterm in the renormalization procedure as we will see in the next section.

### III. ONE LOOP CONTRIBUTION

In order to avoid lengthy expressions, let us concentrate on the effect of just one special term of the interaction Lagrangian\(^2\) with two resonance fields

\[
\mathcal{L}_{\text{int}} = d_1 \epsilon_{\mu\nu\alpha\sigma} \langle D_\beta^\alpha u^\sigma \{ R^\mu_{\nu\rho}, R^\rho_{\nu\beta} \} \rangle + \ldots
\]

The most general result will be published in [17] but it does not differ in essence from what follows.

The explicit calculation (using dimensional regularization\(^3\)) of the first Feynman diagram depicted in Fig. 1 with vertices corresponding to the interaction term (14) gives for the self-energies \( \Sigma_T(p^2) \) and \( \Sigma_L(p^2) \),

\[
\Sigma_{\text{loop}}^T(p^2) = \Sigma_{\text{loop}}^L(p^2) = \frac{5}{6} d_1^2 \left( \frac{M}{F} \right)^2 \frac{d - 2}{d^2} \mu^{d-4} \frac{2}{d - 4} + \gamma_E - \ln 4\pi - 1 + \ln \frac{M^2}{\mu^2} \right) (p^2 + M^2) + \ldots
\]

In order to cancel the UV divergences it is necessary to add to (14) the following counterterms

\[
\mathcal{L}_{\text{ct}} = \frac{1}{4} \delta M^2 \langle R^\mu_{\nu\rho} R^{\rho\mu\nu} \rangle + \frac{\alpha}{4} \langle D^\alpha R_{\mu\nu} R_\alpha R^{\mu\nu} \rangle + \frac{\beta}{2} \langle D^\alpha R_{\alpha\mu} D_\beta R^{\beta\mu} \rangle + \ldots
\]

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\(^2\) The complete list of terms in even intrinsic parity sector can be found in [11], the part of the basis for odd intrinsic parity sector is provided in [10].

\(^3\) In order to avoid the problems with \( d \)-dimensional Levi-Civita tensor, we use the simplest form of dimensional regularization (known as Dimensional Reduction) by means of performing first the four-dimensional tensor algebra and only then regularizing the remaining integrals. The infinite part of the result does not depend on this choice.
i.e. a mass term and two kinetic terms one of which was not present in the original leading order Lagrangian. These counterterms contribute to $\Sigma^T(p^2)$ and $\Sigma^L(p^2)$ as (cf. (3) and (11))

\[
\Sigma^T_{ct}(p^2) = \delta M^2 + \alpha p^2, \tag{17}
\]
\[
\Sigma^L_{ct}(p^2) = \delta M^2 + (\alpha + \beta)p^2, \tag{18}
\]

the infinite parts of which are fixed as

\[
\delta M^2 = -\frac{40}{3} d_1^2 M^2 \left( \frac{M}{F} \right)^2 \lambda_\infty + (\delta M^2)^r(\mu) + \ldots, \]
\[
\alpha = -\frac{40}{3} d_1^2 \left( \frac{M}{F} \right)^2 \lambda_\infty + \alpha^r(\mu) + \ldots, \]
\[
\beta = \beta^r(\mu) + \ldots,
\]

where

\[
\lambda_\infty = \frac{\mu^{d-4}}{16\pi^2} \left( \frac{1}{d-4} - \frac{1}{2}(-\gamma_E + \ln 4\pi + 1) \right).
\]

We see that the interaction Lagrangian in the antisymmetric formulation of Resonance Chiral Theory can lead to the nontrivial momentum dependence of $\Sigma^T(p^2)$, and therefore to the possible presence of additional poles which correspond to opposite parity asymptotic states or resonances or even negative norm ghosts or tachyons.

It can be shown that not only the antisymmetric formalism but also the vector formalism (the additional poles are spin-0 modes) and the first-order formalism (where the structure of states is much richer) suffer from this feature. In [17] the complete calculation in all three formalisms will be published with complete Lagrangians up to $O(p^6)$.

IV. CONCLUSION

In this article we have illustrated the problems connected with the one-loop renormalization of the propagators of spin-1 resonances within the antisymmetric tensor formulation of $R\chi T$. As we have shown by means of explicit calculation, the renormalization of the theory at one loop level needs counterterms including a new type of kinetic term connected with possible propagation of additional degrees of freedom. In some cases, these could correspond to negative norm states or tachyons. Analogous feature can be seen also in alternative formulations of $R\chi T$ with spin-1 resonances described by vector fields or by the first-order formalism [17] and can be understood as a manifestation of the well-known fact that, without gauge symmetry and Higgs mechanism, the
quantum field theory of massive spin-1 particles might suffer from internal inconsistencies (for a recent discussion see e.g. \[18\] and references therein).

In all cases, in order to vindicate RxT as a useful effective quantum field theory, we have to take into account this phenomenon. More detailed discussion will be published in \[17\].

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