MULTIPION SYMMETRIZATION EFFECTS ON THE SOURCE DISTRIBUTION

Q. H. ZHANG

Institut für Theoretische Physik, Universität Regensburg, D-93040 Regensburg, Germany
E-mail: qinghui.zhang@physik.uni-regensburg.de

Any-order pion interferometry formulas for fixed pion multiplicity events and for mixed events are given. Multipion Bose-Einstein correlation effects on the two-pion interferometry and source distribution are studied. It is shown that generalized pion interferometry formula should depends on pion multiplicity distribution. Pion condensate is also discussed in the paper.

1 Introduction

Multipion Bose-Einstein (BE) correlations has now aroused great interests among physicists. Among others, Lam and Lo were the first to suggest BASER concept in high energy physics. Zajc was the first to use Monte-Carlo methods to study multipion BE correlations effects on two-pion interferometry for fixed pion multiplicity event. Pratt first suggested that multipion BE symmetrization effects may lead to BE condensate in high energy heavy-ion collisions. Detail derivation of multipion BE correlation effects on single particle spectrum and two-pion interferometry was given by Chao, Gao and I in Ref. Recently Zimányi and Csörgő suggested a new class density matrix and study multipion BE correlations and wavepacket effects on two-pion interferometry. In Ref. I have derived any-order pion interferometry formula for a special class density matrix. In Ref. generalized any-order pion interferometry formulas are given which depends not only on the correlator as assumed previously but also on the pion multiplicity distribution which was neglected in the previous studies. Multipion BE correlation effects on the source distribution was studied in Ref. This paper is arranged as follows: In section 2, any-order pion interferometry formula for fixed pion multiplicity event are given. In section 3, any-order pion interferometry formula for mixed events are derived. In section 4, methods on multipion BE correlation simulation are discussed. Conclusions are given in section five.
2 Any-order pion interferometry formulas for fixed pion multiplicity events

Assume the source is totally chaotic. It is easily checked that the n-pion inclusive distribution in n-pion event can be expressed as

\[ P_n^n(p_1, \cdots, p_n) = \sum_{\sigma} \prod_{j=1}^{n} \rho_{j, \sigma(j)} \]  

with

\[ \rho_{ij} = \int d^4x g(x, p_i + p_j) \exp(i(p_i - p_j) \cdot x). \]  

Here \( g(x, k) \) is a Wigner function which can be explained as the probability of finding a pion at point \( x \) with momentum \( p \). \( \sigma(j) \) denotes the \( j \)-th element of a permutations of the sequence \( \{1, 2, \cdots, n\} \), and the sum over \( \sigma \) denotes the sum over all \( n! \) permutations of this sequence. Then the normalized \( k \) pion inclusive distribution in \( n \) pion events can be expressed as

\[ P_n^k(p_1, \cdots, p_k) = \frac{\int P_n^n(p_1, \cdots, p_n) \prod_{j=k+1}^{n} dp_j}{\int P_n^n(p_1, \cdots, p_n) \prod_{j=1}^{n} dp_j}. \]  

It is easily checked that \( P_n^k(p_1, \cdots, p_k) \) can be re-written as

\[ P_n^k(p_1, \cdots, p_k) = \frac{1}{n(n-1) \cdots (n-k+1)} \omega(n) \sum_{i=k}^{n} \frac{1}{i-(k-1)} \sum_{m_1=1}^{i} \sum_{m_2=1}^{i-m_1} \frac{1}{i-m_1-m_2} \frac{1}{i-m_1-m_2-m_3} \cdots \frac{1}{i-m_1-m_2-m_3-\cdots-m_{k-1}-1} \sum_{\sigma} G_{m_1}(p_1, p_{\sigma(1)}) G_{m_2}(p_2, p_{\sigma(2)}) \cdots G_{m_{k-1}}(p_{k-1}, p_{\sigma(k-1)}) G_{i-m_1-\cdots-m_{k-1}}(p_k, p_{\sigma(k)}) \omega(n-i) \]  

with

\[ \omega(n) = \frac{1}{n} \sum_{i=1}^{n} \omega(n-i) \int dp G_i(p, p), \quad \omega(0) = 1 \]  

and

\[ G_i(p, q) = \int \rho(p, p_1) dp_1 \rho(p_1, p_2) \cdots dp_{i-1} \rho(p_{i-1}, q). \]  

Here \( \sigma(i) \) denotes the \( i \)-th element of a permutations of the sequence \( \{1, 2, \cdots, k\} \), and the sum over \( \sigma \) denotes the sum over all \( k! \) permutations of this
sequence. Then the $k$-pion ($k \leq n$) interferometry formula for n-pion events can be expressed as

$$C_n^k(p_1, \cdots, p_k) = \frac{P_n^k(p_1, \cdots, p_k)}{\prod_{j=1}^{k} P_n(p_j)}. \quad (7)$$

We assume the source distribution as

$$g(r, t, p) = n_0 \cdot \left(\frac{1}{2\pi R^2}\right)^{3/2} \exp\left(-\frac{r^2}{2R^2}\right) \delta(t) \left(\frac{1}{2\pi \Delta^2}\right)^{3/2} \exp\left(-\frac{p^2}{2\Delta^2}\right). \quad (8)$$

Here $n_0$ is a parameter. Then the multipion BE correlation effects on two-pion interferometry for fixed events are shown in Fig.1. It is easy to see that as pion multiplicity increases, multipion BE correlations effects on the two-pion interferometry effects becomes stronger. In the limit $n \to \infty$ we have $C_n^2 = 1$.

### 3 Any-order pion interferometry formula for mixed events

In the experimental analyses, one normally mixed all events to study pion interferometry. The $k$-pion inclusive distribution can be expressed as

$$N_k(p_1, \cdots, p_k) = \sum_{n=k}^{\infty} P_n \cdot n \cdot (n-1) \cdots (n-k+1) P_n^k(p_1, \cdots, p_k). \quad (9)$$

Here $P_n$ is the normalized pion multiplicity distribution. It is easily checked that

$$\int N_k(p_1, \cdots, p_k) \prod_{j=1}^{k} dp_j = \langle n(n-1) \cdots (n-k+1) \rangle, \quad (10)$$

\[\text{Figure 1: Multipion BE correlation effects on two-pion interferometry. } R=3.5 \text{ fm and } \Delta = 0.14 \text{ GeV. The total momentum of the two pions is zero.}\]

\[\text{Figure 2: Multipion BE correlation effects on source distribution in momentum space. } R=3.5 \text{ fm and } \Delta = 0.16 \text{ GeV.}\]
then the $k$-pion interferometry formulas can be expressed as

$$C_k(p_1, \cdots, p_k) = \frac{N_k(p_1, \cdots, p_k)}{\prod_{j=1}^{k} N_1(p_j)}. \quad (11)$$

The discussion about the definition of two-pion interferometry formulas can be found in the paper by Miskowiec and Voloshin \cite{23} and can also be found in Ref. \cite{21}. Using Eq.(4), the $k$-pion inclusive distribution can be expressed as \cite{11}

$$N_k(p_1, \cdots, p_k) = \sum_{n=k}^{\infty} P_n \frac{\omega(n-i)}{\omega(n)} \sum_{i=k}^{n} \sum_{m_1=1}^{i-(k-1)} \cdots \sum_{m_{k-1}=1}^{i-m_1-\cdots-m_{k-2}-1} \sum_{\sigma} G_{m_1}(p_1, p_{\sigma(1)}) \cdots G_{m_{k-1}}(p_{k-1}, p_{\sigma(k-1)}) G_{i-m_1-\cdots-m_{k-1}}(p_k, p_{\sigma(k)}) \quad (12)$$

with

$$h_{m_1+\cdots+m_k} = \sum_{n=m_1+\cdots+m_k}^{\infty} P_n \frac{\omega(n-m_1-\cdots-m_k)}{\omega(n)}. \quad (13)$$

It is interesting to notice that if $P_n = \frac{\omega(n)}{\sum_n \omega(n)}$ as assumed in Ref. \cite{10, 7, 4}, we have the following modified $k$-pion inclusive distribution \cite{10}:\cite{4}

$$N_k(p_1, \cdots, p_k) = \sum_{\sigma} H_{1\sigma(1)} \cdots H_{k\sigma(k)} \quad (14)$$

with

$$H_{ij} = H(p_i, p_j) = \sum_{n=1}^{\infty} G_n(p_i, p_j). \quad (15)$$

One interesting property about Eq.(15) is: Eq.(15) is very similar to the pure $n$-pion inclusive distribution (Eq.(1)) but with a modified source distribution $S(x, K)$ which satisfies

$$H(p_i, p_j) = \int S(x, \frac{p_i + p_j}{2}) \exp(i(p_i - p_j) \cdot x) d^4x. \quad (16)$$

Based on Eq.(16), we will study multi-pion BE correlations effects on the source distributions \cite{23}. From Eq.(16), the single particle spectrum distribution $P_1^{(n)}(p)$, can be expressed as:

$$P_1^{(n)}(p) = \frac{N_1(p)}{\langle n \rangle} = \int S(x, p) d^4x \langle n \rangle, \quad \langle n \rangle = \int N_1(p) dp = \int S(x, p) d^4x dp. \quad (17)$$
Similarly the source distribution in coordinate space, \( P_1^{(n)}(r) \), can be expressed as

\[
P_1^{(n)}(r) = \int S(x,p)dpdt \langle n \rangle.
\]  

(18)

Assume the input \( g(x,p) \) as Eq.(8), multipion BE correlation effects on the source distribution are shown in Fig.2 and Fig.3. It is clear that as \( \langle n \rangle \) increases, multipion BE correlations effects on the source distribution becomes stronger. Multipion BE correlation make pions concentrate in momentum and coordinate space. It is easily checked that the half width of the source distribution in momentum space and coordinate space \( \Delta_{eff} \) and \( R_{eff} \) satisfies the following relationship:

\[
\sqrt{\Delta^2} \leq \Delta_{eff} \leq \Delta, \quad \sqrt{R^2} \leq R_{eff} \leq R.
\]  

(19)

If we take \( \langle n \rangle \rightarrow \infty \), we have \( R_{eff} = \sqrt{\frac{R^2}{\Delta^2}} \) and \( \Delta_{eff} = \sqrt{\frac{\Delta^2}{\Delta^2}} \). The source size satisfy the minimal Heisenberg relationship. That is all pions are concentrated in a single phase space and pion condensate occurs. But the pion multiplicity distribution is Boson form: \( P_n = \langle n \rangle^n \langle n \rangle + 1 \).

In general, pion interferometry formula depends not only on the correlator \( \rho_{i,j} \) but also on \( P_n \) as shown in Eq.(12). The later property was neglected in the previous studies. Recently, Egger, Lipa and Buschbeck results seem support the conclusion presented here. Previous studies based on pure multipion interferometry formulas have shown that the difference between three-pion interferometry and two-pion interferometry are not so larger. Certainly, there is also another possibility that the there are some correlation among the emitted pions which was not include in the above derivation. So one needs more studies about the two-pion and higher-order pion interferometry!

If we assume the pion state as

\[
|\phi\rangle = \sum_n a_n |n>.
\]  

(20)

Here \( a_n \) is a parameter which connected with pion multiplicity distribution. \( |n> \) is the non-normalized pion \( n \) pion state which can be expressed as

\[
|n\rangle = \left( \int dp j(p) a^+(p) \right)^n |0>.
\]  

(21)

Here \( a^+(p) \) is the pion creation operator and \( j(p) \) is the pion probability
amplitude. Then the pion multiplicity distribution $P_n$ can be expressed as

$$P_n = \frac{|a_n|^2 \omega(n)}{\sum_n |a_n|^2 \omega(n)}. \quad (22)$$

The two-pion interferometry results for different $P_n$ (different $a_n$) is shown in Fig. 4. One can see clearly that there are big differences among the two-pion interferometry results for different $P_n$.

4 Monte-Carlo simulation of multipion interferometry

To put two-pion BE correlations in event generator has been studied by different authors. To put multipion BE correlations in event generator is a very difficult task as shown in Ref. Recently Fialkowski et al. used the methods of Bialas, Krzywicki and Wosiek and implemented approximately multipion BE correlation in JETSET/PYTHIA to study multipion BE correlation effects on the W mass. Due to the fact that there is no space-time picture in the model JETSET, so they had to put $\rho_{ij}$ by hand. On the other hand for model which has space-time picture, one can construct $g(x, p)$ according to the following method. $D(x, p)$ is a total classical function which is the output of the event generator and can be written down as:

$$D(x, p) = \sum_i \delta(x - x(i)) \delta(p - p(i)). \quad (23)$$

Here $x(i)$ and $p(i)$ are the $i$th particle coordinate and momentum. The natural and easiest way to construct $g(x, p)$ the semiclassical function is to replace the above delta function with a Gaussian wavepacket

$$g(y, k) = \sum_i \prod_{j=0}^3 \exp\left(-\frac{(y_j - x(i)_j)^2}{\delta x_j^2}\right) \exp\left(-\frac{k_j - p(i)_j}{\delta p_j^2}\right). \quad (24)$$

In general $\delta x_j$ ($\delta p_j$) should not be the same. But for simplicity one can set them the same value and take $\delta p_j = \frac{1}{\delta x_j}$. The same discussion can be found in the Ref. Unfortunately the calculating work for the above methods to put multipion BE correlation in event generator is so large that a new method which enable us to implement quickly multipion correlation in event generator is a debate for physics in this field.
5 Conclusions

In the above, we mention nothing about the effects of flow and resonance on the pion interferometry. As our main interests is the BE statistical effects and the flow and resonance can be included in the function $g(x, p)$. Certainly the multipion coulomb effects is still a open question in this field. Energy constraint effects on the pion interferometry can be studied by the method presented in Ref. In this paper, we have derived any-order pion interferometry formula for fixed multipion events and for mixed events. In general pion interferometry formulas should depends on not only on the correlator but also on the pion multiplicity distribution which was neglected in previous studies. It is shown that as the pion multiplicity becomes very larger all pions concentrate in a single phase space and pion condensate occurs. But the pion multiplicity distribution is BE form. The methods to put multipion BE correlations in the event generator is discussed.

Acknowledgments

Q.H.Z. thanks Drs. U. Heinz, J. Zimányi, T. Csörgő, R. Lednicky, Y. Sinyukov, D. Miśkowiec, H. Egger, B. Buschbeck, P. Scotto and Urs. Wiedemann for helpful discussions. Part of the talk is based on the work with U. Heinz and P. Scotto. Q.H.Z was supported by the Alexander von Humboldt Foundation.

References
1. S. Pratt, in Quark-Gluon-Plasma 2, ed. R. C. Hwa (World Scientific Publ. Co. Singapore 1995)p. 700; I.V. Andreev, M. Plümer and R.M. Weiner, Int. J. Mod. Phys. 8, 4577 (1993).
2. C.S. Lam and S.Y. Lo, Phys. Rev. Lett. 52, 1184 (1984).
3. W. A. Zajc, Phys. Rev. D 35, 3396 (1987).
4. S. Pratt, Phys. Lett. B301, 159 (1993).
5. W.Q. Chao, C.S. Gao and Q.H. Zhang; J. Phys. G21, 847 (1995);
6. Q.H. Zhang, W.Q. Chao, and C.S. Gao, Phys. Rev. C52, 2064 (1995).
7. J. Zimányi and T. Csörgő, hep-ph/9705432
8. T. Csörgő and J. Zimányi, Phys. Rev. Lett. 80, 916 (1998), T. Csörgő, Phys. Lett. B 409, 11 (1997),
9. Q. H. Zhang, Phys. Rev. C57, 877 (1998); Phys. Lett. B406, 366 (1997); Nucl. Phys. A634,190 (1998); J. Phys. G24, 175 (1998).
10. Q.H. Zhang, hep-ph/9804388, Phys. Rev. C58, R18 (1998).
11. Q. H. Zhang, hep-ph/9805499.
12. Y. Sinyukov and B. Lorstand, Z. Phys. C61,587 (1994),
13. N. Suzuki, M. Biyajima and I. V. Andreev, Phys. Rev. C56, 2736 (1997).
14. R. L. Ray, Phys. Rev. C57, 2523 (1998).
15. A. Bialas and A. Krzywicki, Phys. Lett. B354, 134 (1995);
16. J. Wosiek, Phys. Lett. B399, 130 (1997).
17. K. Fialkowski, R. Wit and J. Wosiek, hep-ph/9803399; K. Fialkowski and R. Wit, Euro. Phys. C 2, 691 (1998).
18. A. Bialas and K. Zalewski, hep-ph/9803408 and hep-ph/9806435.
19. Urs. Wiedemann, nucl-th/9801009.
20. J. Zimányi, this proceeding, Y. Sinyukov, this proceeding.
21. Q.H. Zhang, P. Scotto and U. Heinz, nucl-th/9805046.
22. N. Amelin and R. Lednicky, SUBATECH-95-08;SUBATECH-95-09.
23. D. Miskowiec and S. Voloshin, nucl-ex/9704006.
24. H.C. Eggers, P. Lipa, and B. Buschbeck, Phys. Rev. Lett. 79 197 (1997).
25. U.Heinz and Q.H.Zhang, Phys. Rev.C56, 426 (1997); J.Cramer and K.Kadija,ibid. 53 908 (1996); Q.H.Zhang, Phys. Rev. D57, 287 (1998).
26. P. Seyboth this proceedings; B. Lörstad, this proceedings.
27. W.Q. Chao, C.S. Gao and Q. H. Zhang, Nucl. Phys. A573, 641 (1994); S. Pratt et. al, Nucl. Phys. A566, 103c (1994).
28. Q. H. Zhang et. al., Phys. Lett. B407, 33 (1997); U.A. Wiedemann et. al., Phys. Rev. C56, R614 (1997).
29. S. S. Padula, M. Gyulassy and S. Gavin, Nucl. Phys. B329, 357 (1990);
30. U. Heinz, Nucl. Phys. A610, 264c (1996); T. Csörgő and B. Lörstad, Phys. Rev. C54, 1390 (1996);Q. H. Zhang et. al., in the proceedings of the international workshop on quark-gluon structure of hadron & nuclei
at Shanghai, May 28-June 1(1990) China. P201, ed: L.S. Kisslinger and Xijun Qiu; B. R. Schlei et. al., Phys. Lett. B293, 275 (1992).

31. T. Csörgo and B. Lörstad and J. Zimányi, Z. Phys. C71, 491 (1996); S. Padula and M. Gyulassy, Nucl. Phys. B339, 378 (1990); F. Cannata, J. P. Dedonder and M. P. Locher, Z. Phys. A 358 275 (1997); Urs. Wiedemann and U. Heinz, Phys. Rev. C56, 3265 (1997).