ABSTRACT

We present the results of several collisional-radiative models describing optically-thin emissivities of the main lines in neutral helium formed by recombination, for a grid of electron temperatures and densities, typical of H II regions and Planetary Nebulae. Accurate emissivities are required for example to measure the helium abundance in nebulae and as a consequence its primordial value. We compare our results with those obtained by previous models, finding significant differences, well above the target accuracy of one percent. We discuss in some detail our chosen set of atomic rates and the differences with those adopted by previous models. The main differences lie in the treatment of electron and proton collision rates and we discuss which transitions are least sensitive to the choice of these rates and therefore best suited to high precision abundance determinations. We have focused our comparisons on the case B approximation where only He and He$^+$ are considered, but also present results of full models including the bare nuclei, photo-excitation and photo-ionisation and either blackbody or observed illuminating spectrum in the case of the Orion nebula, to indicate which spectral lines are affected by opacity. For those transitions, accurate radiative transfer calculations should be performed. We provide tables of emissivities for all transitions within $n_e \leq 5$ and all those between the $n \leq 5$ and $n' \leq 25$ states, in the log $T_e [K]=10^{3.0(0.1)4.6}$ and log $N_e [cm^{-3}]=10^{2.0(5)8}$ ranges, and a FORTRAN code to interpolate to any $T_e$, $N_e$ within these ranges.

Key words: atomic data – atomic processes – ISM: atoms – ISM: clouds – H II regions

1 INTRODUCTION

Helium lines in the visible and near infrared are particularly important for nebular astrophysics, for example being routinely used to measure the helium abundance in different astrophysical sources, and then extrapolating the results to obtain a measurement of its primordial abundance (see, e.g. Peimbert et al. [2007]; Izotov et al. [2014]; Aver et al. [2013], and references therein). Such measurements are very important as they provide constraints on e.g. Big Bang Nucleosynthesis models and galactic chemical evolution.

As spectral line intensities can be measured within an uncertainty of 1% or so, a similar accuracy has been sought in atomic modelling. As we describe below, a significant effort was put in place by various groups to try to achieve such accuracy, by improving the modelling and the basic atomic rates. However, significant differences (up to 50% or so) are found in the different calculations.

Recently, we constructed a new model for the level populations of, and resulting line emission from, helium in the relatively high-temperature, high-density plasma of the solar corona (Del Zanna et al. [2020]). We reviewed the basic atomic rates and found some shortcomings in the rates adopted by previous authors. In the present paper we extend that model to predict helium line emissivities in the relatively low-temperature, low-density photoionized plasma typical of nebulae, and provide some comparisons with the previous models.

In the next Section we provide a brief overview of some of the most widely-used previous models. In the following Section we present a summary of the rates adopted and the various atomic models we have built. A sample of results and comparisons with previous models is presented in Section 4, while Section 5 draws the conclusions.

2 PREVIOUS MODELS

Various models of helium emission in the conditions prevailing in photoionized plasma were developed in the 1950s and 1960s by several authors, including Mathis (1957); Seaton (1960); Robbins (1970). Most of the early theory and methods adopted by subsequent authors are due to Seaton and Burgess, see e.g. Burgess & Seaton (1960); Seaton (1962); Pengelly & Seaton (1964).

The details of the various atomic models are not always en-
tirely clear. However, we now summarise the main assumptions and rate coefficients adopted by the various authors.

The first detailed model of the He recombination spectrum is that of Brocklehurst (1972) (B72). He built a model with the rates available at the time, and provided emissivities in the case A and B approximations (see Baker & Menzel 1938). For the l-changing collisional excitation (CE) rates, the semi-classical impact parameter approximation of Seaton (1962) (S62) and Pengelly & Seaton (1964) (PS64) was used. He used the non-degenerate formulation (S62, hereafter IP) for \( l \leq 5 \), and the degenerate formulation (PS64) for higher \( l \). A few shortcomings in the model were pointed out by subsequent authors, the main one being that Brocklehurst (1972) neglected the metastability of the \( 2^1S, 2^3S \) states. The approach was to solve the statistical balance equations in terms of departure coefficients \( b \) from Saha-Boltzmann level populations (the so-called \( b \)-factors). The calculations treated the singlet and triplet series separately and considered first \( n \)-resolved levels, \( b_n \) and then the \( b_{nl} \) for the terms (in \( LS \) coupling) were calculated for a lower set.

Almog & Netzer (1989) (AN89) built a model with \( LS \) resolved states up to \( n = 10 \) (singlets) and \( n = 12 \) (triplets), and added four collapsed levels. Burgess & Summerlin (1969) to mimic the presence of levels up to \( n = 100 \), with the usual assumption that the collapsed states contain levels that are statistically populated. As the authors state, this model is a good assumption for high densities only, which was the main topic of that paper. Indeed collisional ionization was added into the model, as it is important for higher densities.

Smith (1996) (S96) built an \( LS \) model, improving and correcting some errors in his previous models. Some details of the model can be found in Smith (1991) (S91): many rates were calculated with the hydrogenic approximation. The method followed B72 and first calculated the \( b \)-factors for the \( n \)-resolved states up to \( n = 496 \). A matrix condensation technique (Burgess & Summerlin 1969) was employed to reduce the number of levels to 100. The \( b_{nl} \) factors were then calculated for a set of \( LS \)-resolved states up to \( n = 50 \) (2549 levels). The author then provided line emissivities calculated in case A and B for low densities, with the assumption that levels above \( n = 50 \) are statistically redistributed. As the author pointed out, this assumption is not always valid. The \( b_{nl} \), at \( n = 50 \) are not equal to the \( b_n \), for example a density of 100 cm\(^{-3} \). Smith argued that errors introduced by this assumption are not large, but without actually quantifying the statement. We will return to this issue below, as we have built several models and are able to assess this assumption.

S96 used for the \( n \leq 9 \) states the A-values obtained from the oscillator strengths calculated by Kono & Hattori (1984). For higher states, the Coulomb approximation was used for \( l \leq 2 \) states, and for the others a scaled hydrogenic approximation was adopted. CI rates were not included. The radiative recombination (RR) rate coefficients for lower \( n \) states were obtained from the OP photoionization (PI) cross-sections of Fernley et al. (1978). For higher \( n \), scaled hydrogenic rates were used. S91 states that CE rates within the \( n = 2, 3 \) levels are taken from the \( R \)-matrix calculations of Berrington & Kingston (1987) (BK87). They were used to find the populations of the \( n = 2, 3 \) levels, to include the metastability of the \( 2^1S, 2^3S \), which was not included by Brocklehurst (1972). The rest of the CE rates were taken from Brocklehurst (1972), i.e. using the impact parameter approximation Smith (1996) noted that transfer of population between singlet and triplet states by electron collisions would be included if \( R \)-matrix CE rates for the lower states were used. This mixing was included in his model only for the \( n = 2 \) states. Spin-orbit and other relativistic interactions between \( ^3L_J \) and \( ^1L_J \) states for high \( L \) are also real effects not taken into account by Smith (1996). Both effects could reduce the populations of the triplets, compared to the singlets. However, Smith (1996) noted that comparisons with observations indicated an opposite trend, i.e. some of the predicted intensities of the triplets were lower than observations.

Drawing on the high precision intermediate coupling variational calculations of helium structure and radiative processes by Drakel (1998), Bauman et al. (2005) built an \( LS \) model of helium in the low-density limit (i.e. no collisions), and concluded that singlet-triplet mixing has a negligible effect on the total intensities of the lines within a multiplet. However, it is still unclear what effects collisional processes linking singlets and triplets have. As discussed below, we include these processes in our model up to \( n = 5 \) states. Bauman et al. (2005) also pointed out that individual intensities within a multiplet would be affected, but are generally not observable as line widths are larger than the separation of lines. A simpler \( LS \) model is therefore equivalent to an \( LS \) model. Benjamin et al. (1999) (B99) built a case B \( LS \) model with only the lower 29 states (up to \( n = 5 \)), using the Hummer & Storey (1998) (HS98) were adopted to calculate the \( \text{He}_2^+ \) contribution from the higher states was estimated accounted for. To improve it, ‘cascading’ (described as an indirect recombination) contribution from the higher states was estimated so as to match the S96 populations within 2% for densities lower than 10\(^6 \). However, as we show below, much larger differences between B99 and S96 are actually present in the line emissivities.

Porter et al. (2005) (P05) constructed an \( LS \) model up to \( n^8 \), with an extra ‘collapsed’ \( n^9 + 1 \) level describing the missing states between \( n^8 \) and the continuum, and presented emissivities in the case B approximation. They noted that with \( n^9 \) = 100, the corrections due to the collapsed level are negligible for low densities. Transition probabilities from the nearly exact calculations of Drakel (1996) (up to \( n = 10 \)) were used. PI cross-sections from Hummer & Storey (1998) (HS98) were adopted to calculate the RR rates, while the CE rates of Bray et al. (2000) were used. For the l-changing collisions, they used the IP method of S62 for the s.p.d states, and the semi-classical theory of Bruntsson & Flannery (2001) (VF01) for higher \( l \).

Porter et al. (2007) (P07) built a similar \( LS \) model up to \( n = 40 \), and added collapsed \( n \)-resolved levels between \( n = 41 \) and \( n = 100 \), i.e. similar to our coronal model for helium. They applied this model to study the helium abundance in the Orion nebula.

Porter et al. (2012) (P12) presented an updated case B model, with the full set of HS98 PI cross sections (up to \( n = 25 \)) and associated RR rates. For higher \( n \) states, hydrogenic RR rates were used. The model included \( LS \) states up to \( n = 100 \), and a single collapsed \( n = 101 \) line. The paper reports that a code error in their earlier calculations (P05,P07) was uncovered, which affected mostly the 5876, 6678 Å lines, the decays from the singlet and triplet 3d levels. Fixing the code error mostly increased the recombination coefficients into the 3d levels. Other differences were due to a different
implementation of the semi-classical Vriens & Smeele (1980) collisional excitation rates.

The P05, P07, P12 models were built within the various improved versions of the He model within the CLOUDY (Ferland et al. 2017) photoionization code. The semi-classical theory of VF01 has been discussed by Guzmán et al. (2017) who show that due to the truncation of the cross-sections at energies that neglect the quantum mechanical tail, it grossly underestimates the collision rates, by a factor of six at $n = 30$, for example. Guzmán et al. (2017) also show that the PS64 method, on the other hand, gives rates close to those obtained from a quantum mechanical treatment.

Another point worth noting is that the IP proton rates as shown in Guzmán et al. (2017) and calculated within CLOUDY are also incorrect, as we noted in our coronal model paper (Del Zanna et al. 2020). We also found that the Bray et al. CE rate file in CLOUDY inverted by mistake the values for the transitions from the metastable $2\,^3S$ to the $4\,^3S$ and $4\,^1P$ levels (12,14), thus affecting somewhat the emissivities of the main decays from these two levels, the 3188 and 4713 Å lines. Thus all the previous helium recombinations models have apparent defects, in terms of using rates coefficients that are now considered not accurate or the best available at present. The purpose of this paper is therefore to investigate whether correcting these shortcomings has any impact on the emissivities of the spectroscopically important transitions.

### 3 MODELS

The present models are an extension of our previous coronal models described in Del Zanna et al. (2020). Among them, the most extended neutral He model for low-temperature ($T = 20000$ K) plasma was a set of $L\,S$-resolved states up to $n = 40$, and a set of $n$-resolved levels up to $n = 100$. Considering the behaviour of the $b$ factors, this model was deemed sufficient for electron densities higher than $N = 10^6$ cm$^{-3}$.

For the present paper we have built a model for neutral He with $L\,S$-resolved states up to $n = 100$ and $n$-resolved levels up to $n = 500$. This is a much larger model than the previous ones, and especially larger than the largest model ($L\,S$-resolved states up to $n = 50$) produced by S91 and subsequently used by S96 and B99. For He$^+$ we adopted the $J$-resolved CHIANTI (Dere et al. 2019) model. We create a collisional-radiative model, with matrices that contain all the main rate coefficients affecting the bound levels, and obtain the level populations in equilibrium by direct inversion.

We did not include dielectronic recombination (DR), the key process in our coronal model, as it is negligible at the low temperatures of interest here. For RR and $L \leq 3$, we use rates obtained by numerical integration of the photo-ionization cross-sections calculated by Hummer & Storey (1998) in the R-matrix approximation with a target that accounted for the dipole and quadrupole polarisabilities of the He$^+$ ground state. Hummer & Storey (1998) showed that their calculated bound-free cross-sections agree within 1% with the bound-bound radiative data of Drake (2000) when extrapolated to the series limit. Hydrogenic values were used for RR for $L > 3$.

For the $L\,S$ levels up to $n = 10$ we have used the energies and $A$-values of Drake & Morton (2007). The energies for the higher levels were obtained from the quantum defects using the updated coefficients of Drake (2000). The $A$-values for the higher $L\,S$ states were obtained either from fits to the results of Drake & Morton (2007), as described by Hummer & Storey (1998), or using the methods of Bates & Damgaard (1949), van Regemorter et al. (1979), and the hydrogenic approximation using the code RADZI (Storey & Hummer 1991). The $A$-values for the $n$-resolved states to the lower $n = 2, 3$ states were obtained from statistically weighted averages of the extrapolated Drake & Morton (2007). The $A$-values for the $n$-resolved states to the lower $n = 4, 40$ states were obtained by averaging hydrogenic values. The $A$-values between the $n$-resolved states were obtained as in our previous coronal model, using the hydrogenic analytical formulation and tabulated Gaunt factors.

For the electron CE rates of states up to $n = 5$ we use the $R$-matrix results of Bray et al. (2000), although we have also experimented with other rates. The lowest temperature for the Bray et al. (2000) rates is 5623 K. To provide estimates to lower temperatures, we have proceeded as follows. For the CE rates within the $n=2,3$ states, we have taken the $R$-matrix rates from BK87, which were calculated down to 1000 K. As some small differences between the two sets of rates are present, we have interpolated the BK87 rates to 5623 K, and scaled them so they agree at this temperature with the Bray et al. values. For the $n=4$ states, we have scaled the SB93 rates in a similar way, and then extrapolated them down to 1000 K in the Burgess & Tully (1992) scaled domain, with a linear fit to the first two points (SB93 CE rates were calculated for 2000 and 5000 K). For the $n=5$ states, we have linearly extrapolated the Bray et al. CE rates in the Burgess and Tully scaled domain, considering again the first two points. The CE rates were stored in the scaled domain, where the interpolation in temperature is carried out, as done within the CHIANTI software.

Note that the collision strength calculations of Sawey & Berrington (1993) and of Bray et al. (2000) both employ the $R$-matrix method but the latter calculation is to be preferred because it makes allowance for the presence of continuum target states with the result that their collision rates between bound states are almost always smaller than those of Sawey & Berrington (1993) due to flux lost to the continuum states. It should be noted, however, that the allowance for continuum states gives rise to some resonance features near threshold that, although they represent a physical process, are not necessarily correct in detail. Thus a measure of uncertainty is still attached to the CE between low-lying states that is difficult to quantify. Finally, we point out that extrapolating the rates to temperatures lower than 1000 K is feasible but the values would be quite uncertain.

To connect the states with $n \leq 5$ to higher states, $m > 5$, we extrapolated the cross-sections of Bray et al. (2000) for $n < m \leq 5$ assuming they vary as $m^{-3}$. Attempts to directly calculate CE rates to the higher levels proved unreliable, as discussed in Del Zanna et al. (2020). For the other states we used the IP method for $\Delta n = 1$ transitions. For $\Delta n = 2, 3, 4$ transitions within the $n$-resolved levels, we adopted the the Percival & Richards (1978) approximation.

For the $l$-changing ($\Delta l = 0, \Delta l = 1$) collisions for electrons and protons we used the the IP approximation among the non-degenerate levels with lower $l$ and the Pengelly & Seaton (1964) (PS64) for the remainder. The switching was applied when the difference in energy reached $10^{-4}$ cm$^{-1}$. We note that an improved PS64 method has been developed by Badnell (2021, in press). We have checked that differences with the PS64 rates are negligible. We assumed a fixed proton to electron ratio of 0.91, which results from assuming that the helium abundance is 10% by number that of hydrogen.

We included collisional ionization (CI) and three-body recombination as described in our previous model for the lowest states,
but use the semiclassical CI rates of [Vriens & Smeets (1980)] instead of those developed by A. Burgess (ECIP). We recalculated the rates for the low temperatures considered here. We note that CI has a negligible effect at the low densities considered here, as the model is driven by photoionization (PI) and resulting RR cascading. For the PI cross-sections we have used the [Hummer & Storey (1998)] results, instead of the semi-classical Kramers hydrogenic formula and the Gaunt factors from [Karas & Latter (1961)], as in our previous paper. The key factors are the RR rates, the CE rates within the lower levels (for higher densities), the set of A-values, and the cascading effects from the higher to the lower states. Further details on other specific rates are given in [Del Zanna et al. (2020)].

The emissivities are defined as

\[ E = \frac{4\pi j}{N_e N(He^+)} = \frac{h \nu_{ji} N(He) A_{ji} N(He)}{N_e N(He^+)} \text{ erg cm}^{-3} \text{ s}^{-1} \]  

where the first definition is how the emissivities are usually indicated in the literature, and the second one is how we calculated them: \( N_j \) is the population of the upper level \( j \), relative to the total population of He, \( N(He) \), i.e. \( \text{He}^0 \) and \( \text{He}^+ \); \( A_{ji} \) is the radiative transition probability, and \( h \nu_{ji} \) is the energy of the photon.

We calculate the emissivities in case A (optically thin plasma) and case B, which is an approximation to model the real plasma emission when the strongest singlet lines, decaying to the ground state, are re-absorbed on the spot. To obtain case B, we have set to zero the RR to the ground state and all the A-values of the singlet (above \( n=2 \)) decays to the ground state.

Clearly, detailed modelling allowing for radiative transfer of line and continuum photons is needed to study specific sources. Here, we are mostly interested in showing how line emissivities are affected by the use of different models and atomic rates.

Our general model includes all three ionization stages of He but to make a direct comparison to earlier tabulated emissivities, we make the simplifying assumption adopted in previous models, where only neutral He and \( \text{He}^+ \) (and not the bare nuclei) are considered, to mimic the conditions in the \( \text{He}^+ \) zone of a photoionized nebula. Various options are available to achieve this, forcing the relative numbers of neutral He and \( \text{He}^+ \); \( A_{ji} \) is the radiative transition probability, and \( h \nu_{ji} \) is the energy of the photon.

To assess how reliable a reduced model (similar to that of S96) is for nebular densities, we had initially considered the earlier coronal model, with \( LS \)-resolved states up to \( n = 40 \) and \( n \)-states up to \( n=100 \). However, for the lowest densities considered here (10^2 \( \text{cm}^{-3} \)), we found that the \( b \)-factors reach unity only around \( n=200 \), so we have therefore built a new model, which we call PB40, with \( LS \)-resolved states up to \( n = 40 \) and \( n \)-states up to \( n=300 \). The rates for this model are essentially the same as those of the full \( n = 500 \) model. We used the same black-body flux to photoionize the ground state to establish a reasonable ionization balance. Also, to show the effects that different CE rates can have, we have also modified the PB40 model by replacing the Bray et al. CE rates with those from [Sawey & Berrington (1999)] for states up to \( n = 4 \), and IP rates for the \( n = 5 \) levels, to approximate the rates used by B99 (although some differences are present). We call this model SB93.

Finally, we have also built another case B model (which we call PB+PE+PI) where we have considered the level balance between He, \( \text{He}^+ \) and \( \text{He}^{++} \) still using the same black-body flux, and including both PI and PE. Photoexcitation (and de-excitation) was included for all allowed transitions. PI was included for all states up to \( n=25 \) using the PI cross sections from [Hummer & Storey (1998)], although we note that only those for the ground state and the metastable are relevant here. These cross-sections are close (within a few percent) of those in TOPBASE. The photon energies have been adjusted to the observed thresholds. As shown below, for sufficiently low dilutions, the results of this model are close to those of the PB model.

4 RESULTS

We present a sample of results, for a temperature \( T = 10000 \text{ K} \) and three electron densities, \( N_e = 10^6, 10^7, 10^8 \text{ cm}^{-3} \). Figure 1 shows the \( b \) factors obtained from the model PB, \( LS \)-resolved states up to \( n = 100 \) and \( n \)-resolved levels up to \( n = 500 \) in case B. \( b \) factors only up to \( n = 200 \) are shown, as they have already reached near unity for \( n = 100 \). For example, at \( 10^7 \text{ cm}^{-3} \), the \( b \) factors are 0.9968 at \( n=200 \) and 0.998 at \( n=300 \). The figures indicate a smooth behaviour in transitioning from the \( LS \)-resolved to the \( n \)-resolved states. The assumption that the \( n \)-resolved states are in statistical equilibrium is validated.

The resulting emissivities for all the strongest He lines in the visible/near infrared are shown in the last two columns of the emissivity Tables 1, 2 and 3 below. Additional Tables are provided in the Appendix. Our final results are in the penultimate column, designated PB. The final column displaying the effect of adding photoionization and photoexcitation of excited states will be discussed later.

The emissivities with the reduced \( n = 40 \) model (PB40) are also given in the Tables. There is generally excellent agreement with the PB results, with the largest difference of 1.7% for the lowest temperatures and densities. This largely confirms the suggestion by S91 that their model (which we recall was similar to our PB40 calculation in that it included \( LS \)-resolved states up to \( n = 50 \)) would provide reasonably accurate emissivities even at \( 10^7 \text{ cm}^{-3} \) for the most prominent optical lines.

Earlier results are shown in previous columns. Despite the simplified rates used in the first complete model built by [Brooke (1972)], and the neglect of the metastability of the \( 2^2S \), \( 2^2S \) states, relatively good agreement for many lines can be seen. We expected close agreement with the other models, but that is not the case. There are also surprising large differences for some lines between the S96 and B99 models, contrary to the B99 statement that agreement was present at the 2% level.

We experimented with changing various rates, and found that at these plasma \( T_e, N_e \) the electron CE rates among the lower levels have a significant effect, as already pointed out in previous literature. The values of the CE rates are relatively uncertain compared to radiative rates, so we begin by comparing the different calculations at the lowest density, where CE is much less significant.
At 10000K, we find average absolute differences of 1.6% (maximum 5.0%) compared to S96, 1.5% (maximum 11.1%) compared to B99, 1.1% (maximum 4.1%) compared to P05 and 0.45% (maximum 1.8%) compared to P12. All four of these calculations used highly accurate bound-bound radiative transition probabilities, although from different authors, and differ primarily in their treatment of radiative recombination. As in the current work, the results of P05 and P12 derive radiative recombination rates from the photoionization cross-sections of Hummer & Storey (1998) which are to be preferred to the Opacity Project cross-sections used by S96 and B99.

At typical nebular temperatures, collisional excitation from the ground state is negligible compared to recombination but excitation from the \(^2\)P \(- \to \:^2\)S\(^+\) metastable is significant as temperature and/or density increases. It is instructive to consider the emissivity of the \(^2\)P \(- \to \:^2\)S\(^+\) transition is also important because it is particularly useful in constraining the helium abundance, as shown e.g. by Izotov et al. (2014) and subsequent authors. The emissivities for \(^2\)P \(- \to \:^2\)S\(^+\) are summarised in Table 4 at three densities and three temperatures. The fuller tables of emissivities for nebular astrophysics
The contribution of collisional excitation from the metastable 2 states for temperatures of 5000K and 20000K are in the Appendix.

Table 3. Emissivities (10^{-26} erg cm^3 s^{-1}) of the strongest He lines, for T_e=10000 K and N_e=10^6 cm^{-3} in case B.

| \( \lambda \) (Å) | levels | Earlier work | Present work |
|------------------|--------|--------------|--------------|
|                  |        | B72 | S96 | B99 | P05 | P12 | SB93 | PB40 | PB | PB+PI+PE |
| 2945             | T 5p-2s | 2.72 | -   | 2.79 | 2.96 | 2.97 | 2.92 | 2.92 | 2.92 | 2.92 |
| 3188             | T 4p-2s | 5.66 | 5.72 | 6.32 | 6.51 | 6.48 | 6.40 | 6.24 | 6.24 | 6.24 |
| 3889             | T 3p-2s | 14.0 | 13.9 | 17.9 | 18.3 | 18.1 | 18.3 | 17.9 | 17.9 | 17.9 |
| 3965             | S 4p-2s | 1.45 | 1.42 | 1.47 | 1.54 | 1.59 | 1.51 | 1.51 | 1.51 | 1.51 |
| 4026             | T 3d-2p | 2.93 | 2.89 | 2.89 | 3.18 | 3.14 | 3.09 | 3.09 | 3.09 | 3.09 |
| 4388             | S 5d-2p | 0.79 | 0.77 | 0.76 | 0.83 | 0.85 | 0.81 | 0.81 | 0.81 | 0.81 |
| 4471             | T 4d-2p | 6.15 | 6.20 | 6.70 | 6.81 | 6.76 | 6.78 | 6.60 | 6.60 | 6.60 |
| 4713             | T 4s-2p | 0.56 | 0.63 | 1.07 | 0.91 | 0.90 | 1.07 | 0.98 | 0.98 | 0.98 |
| 4922             | S 4d-2p | 1.68 | 1.65 | 1.73 | 1.80 | 1.85 | 1.75 | 1.73 | 1.73 | 1.73 |
| 5016             | S 3p-2s | 3.63 | 3.54 | 3.87 | 4.04 | 4.13 | 3.96 | 3.93 | 3.93 | 3.93 |
| 5876             | T 3d-2p | 16.8 | 16.6 | 19.9 | 20.2 | 20.85 | 20.1 | 19.9 | 19.9 | 19.9 |
| 6678             | S 3d-2p | 4.80 | 4.73 | 5.15 | 5.22 | 5.77 | 5.21 | 5.14 | 5.14 | 5.14 |
| 7065             | T 3s-2p | 2.0 | 2.86 | 7.34 | 7.17 | 7.00 | 7.25 | 7.00 | 7.00 | 7.00 |
| 7281             | S 3s-2p | 0.85 | 0.89 | 1.42 | 1.36 | 1.38 | 1.42 | 1.34 | 1.34 | 1.34 |
| 10830            | T 2p-2s | 27.1 | 320 | 267 | 255 | 247 | 259 | 247 | 247 | 247 |
| 18685            | T 4f-3d | - | - | 2.24 | 2.37 | 2.47 | 2.26 | 2.24 | 2.24 | 2.24 |
| 20387            | S 2p-2s | - | 8.54 | 8.15 | - | 8.25 | 8.09 | 7.97 | 7.97 | 7.97 |

Table 4. Emissivities (10^{-26} erg cm^3 s^{-1}) of the \( \lambda \)10830 line as a function of temperature (T_e) electron density (N_e) in case B.

| T[K] | Calc. N_e (cm^{-3}) |
|------|---------------------|
|      | 10^2 | 10^4 | 10^6 |
| 5000 | PB   | 50.6 | 130 | 189 |
|      | P12  | 50.8 | 140 | 213 |
|      | P05  | 49.9 | 140 | 214 |
|      | B99  | 50.7 | 152 | 234 |
| 10000| PB   | 33.5 | 188 | 247 |
|      | P12  | 33.4 | 185 | 247 |
|      | P05  | 33.2 | 188 | 255 |
|      | B99  | 34.0 | 205 | 267 |
|      | PB40 | 33.5 | 188 | 247 |
|      | SB93 | 33.8 | 196 | 259 |
| 20000| PB   | 23.6 | 204 | 256 |
|      | P12  | 23.8 | 181 | 215 |
|      | P05  | 23.4 | 188 | 237 |
|      | B99  | 24.6 | 207 | 253 |

densities were collisional excitation of \( \lambda \)10830 is dominant. The SB93 emissivities are larger by 4.3% at density 10^4 cm^{-3} and 4.9% at density 10^6 cm^{-3}. Sawey & Berrington (1993) and Bray et al. (2000) tabulate their effective collision strengths on different temperature grids which only coincide at 10000K, where we find that the Sawey & Berrington (1993) effective collision strengths for the 2 P \rightarrow 2 S excitation is larger than that of Bray et al. (2000) by 4.2% showing that changes to the collision strengths result in commensurate changes to the emissivity, as one might expect. Benjamin et al. (1999) used the Sawey & Berrington (1993) CE rates and their results agree better with our SB93 model than with PB40.

There are, however, significant differences between our final results and those of P12, even though we used the same CE rates of Bray et al. (2000). As noted above, agreement is very good at the lowest density where radiative processes dominate and CE is not significant but not at the higher densities for some temperatures. The exception is the temperature of 10^4K where the P12 emissivity is 1.6% lower than ours at 10^4 cm^{-3} and agrees within 0.4% at 10^6 cm^{-3}. But at 5000K the differences are 7.7% and 13.4% at those densities, while at 20000K the differences are -11.7% and -16.3%. Bray et al. (2000) tabulate effective collision strengths on a logarithmic, base ten, mesh with interval 0.25 with the result that the temperature of 10000K is the only one of the three temperatures under discussion which corresponds to a tabulated value in their paper. The observation that P12 agrees well with the present results only at 10000K suggests that P12 have used a different approach to the interpolation of the tables of Bray et al. (2000) or have not used their results for all temperatures.

The features that we have seen when comparing our results to those of P12 for the \( \lambda \)10830 transition are present in most transitions in the emissivity tables at 5000, 10000 and 20000 K. Agreement is good at the lowest density with average absolute differences of 1.3% or less. With increasing density the differences become larger, reaching +5.5% at 5000 K and 16.1% at 20000 K at density 10^6 cm^{-3}. These average differences conceal some very large individual differences between our work and P12. For example at 20000 K and a density of 10^6 cm^{-3}, the P12 values for the...
Figure 1. \( b \) factors obtained with our full case B model, \( LS \)-resolved states up to \( n = 100 \) and \( n \)-resolved levels up to \( n = 500 \). Only values up to \( n = 200 \) are shown.

singlets are all larger than ours by a maximum of 66%, this value being for \( \lambda 6678 \), and all smaller than ours by as much as 16.3% for the triplets. A similar trend but of smaller magnitude is seen at \( 10^2 \) cm\(^{-3} \). Given that collisional excitation from the metastable is increasingly important for higher densities and temperatures, and despite the fact that P12 used Bray et al. (2000) for collisional excitation, as did we, these differences are likely to be attributable to unidentified differences in the way that we and P12 calculated the rates for collisional processes among the lower levels.

The emissivities of the earlier P05 model are similar to those of P12, except for the S876, 6678 Å lines, which were the most affected by the code error previously mentioned (see also Aver et al. 2013).

We mentioned earlier the reasons for preferring the rates of Bray et al. (2000) over those of Sawey & Berrington (1993) and the fact that the Bray et al. (2000) results are also not without weaknesses. In view of this, we consider that it is useful to view the differences between our two \( n=40 \) models, PB40 and SB93, that differ only in that they use these different CE rates, as a measure of the uncertainty due to this choice of rate coefficients, and as a way to identify lines that are relatively insensitive to that choice. For example Table A1 shows that the results of PB40 and SB93 for the strongest lines all agree within 1% at \( 10^2 \) cm\(^{-3} \), whereas in Table A2 at \( 10^4 \) cm\(^{-3} \), eight of the lines, mostly but not exclusively among triplet states, differ by more than 1%

The final column in the emissivity tables (PB+PE+PI) shows the result of adding photoionization and photoexcitation of excited states by a \( 100,000 \) K diluted black-body radiation field, in case B. With a dilution factor, \( d=10^{-16} \), we can see that the emissivities of the main lines are very close to those of the PB model. We note that an appropriate dilution factor 0.1 pc from a typical white dwarf would be \( \approx 5 \times 10^{-17} \) at which level PE and PI have a negligible effect on the emissivities.

The results of our Case B calculation, PB, are available from the CDS on a grid of electron temperatures and densities, \( \log_{10} N_e[cm^{-3}]=2.0(0.5)6.0 \) and \( \log_{10} T_e[K]=3.0(0.1)4.6 \), for all transitions with upper principal quantum number \( \ell = 2-25 \) and lower principal quantum number \( \ell = 2-5 \). We also provide a small FORTRAN program to make two-dimensional interpolation of the emissivity tables to any desired density and temperature within the tabulated ranges.

### 4.1 Infrared lines

It is also interesting to compare our PB rates with those calculated by S96 for weaker infrared lines, some of which were observed by Rubin et al. (1998). Table 5 shows such a comparison, for the lines listed by Rubin et al. The authors provide the S96 emissivities (case B) relative to the 4471 Å reference line, calculated for \( T=10000 \) K and \( N=10000 \) cm\(^{-3} \). The emissivity of the reference line in our PB case is about 4% higher than the S96 value. Regarding the infrared lines, in most cases the relative intensities are within 1% of our values. However, there are a few notable deviations, particularly the 5d-4f transitions.

As we have previously mentioned, the rates for the electron and proton induced \( l \)-changing collisions used by P05 and P12 are rather uncertain in that their calculations are based on CLOUDY but Guzmán et al. (2017) present proton rates that are stated to come from CLOUDY, which are too small by a factor of approximately forty. It is not clear whether these rates were used in the P05 and P12 calculations. It is clear, however, that those calculations did use the semi-classical methods of VF01 which grossly under-
estimate collision rates due to neglect of the quantum mechanical contribution from large impact parameters.

As already shown by Guzmán et al. (2017), the emissivities of the main optical lines change by less than 0.1%, when different electron rates are used, with the exception of the case for a very low $T = 100$ K and a high density of $10^6$ cm$^{-3}$. The same authors showed that variations of about 5% are present in several cases for weaker transitions.

To assess how much $l$-changing collisions affect the infrared lines in the Table, we have run our case B model decreasing the electron collision rates by a factor of $l$ and also by decreasing the proton ones by a factor of forty, consistent with the values in Figure 1 of Guzmán et al. (2017). The results, shown in the last column of Table 5 (PBi), indicate however, transitions from higher principal quantum numbers are significantly affected. The effect of dramatically reducing the proton collision rates is to reduce the coupling between the populations of the higher-$l$ states and the lower-$l$ states, for a given principal quantum number. Since the low-$l$ states have the largest radiative decay rates this leads to larger departure coefficients in the reduced rate case for the higher-$l$ states and smaller departure coefficients for the lower-$l$ states. These effects only appear at intermediate principal quantum number, once the $l$-changing collision rates begin to dominate over radiative rates. As principal quantum number increases, these effects appear first in the high-$l$ states. For example, for $N_e=10000$ cm$^{-3}$ and $T_e=10000$K and when $l=n−1$ the reduced rates lead to an 8% population increase at $n=10$ and an 9% increase at $n=20$, while for $l=0$ decreases of a similar magnitude only appear for $n=40$ and greater. The increases in the high-$l$ state populations are already apparent for $n=5$ in the 5g-4f transitions in Table 5.

| $\lambda$ (microns) | levels $S96$ | PB | PBF |
|----------------------|-------------|----|-----|
| 2.855020 T 5p – 4s  | 0.00171     | 0.00171 | 0.00171 |
| 3.330581 S 5p – 4s  | 9.264 $10^{-4}$ | 9.32 $10^{-4}$ | 9.30 $10^{-4}$ |
| 3.703571 T 5d – 4p  | 0.005595   | 0.00563 | 0.00562 |
| 4.006400 S 5p – 4d  | 4.287 $10^{-4}$ | 4.32 $10^{-4}$ | 4.30 $10^{-4}$ |
| 4.037735 T 5f – 4d  | 0.02739    | 0.0268 | 0.0270 |
| 4.040934 S 5f – 4d  | 0.000913   | 0.000777 (4%) | 0.000882 |
| 4.049014 T 5g – 4f  | 0.07561    | 0.0737 (3%) | 0.0761 (3%) |
| 4.049034 S 5g – 4f  | 0.02520    | 0.0245 (3%) | 0.0253 (3%) |
| 4.055060 S 5d – 4f  | 4.74 $10^{-5}$ | 4.90 $10^{-5}$ (53%) | 4.89 $10^{-5}$ |
| 4.056346 T 5d – 4f  | 2.01 $10^{-4}$ | 1.84 $10^{-4}$ (9%) | 1.84 $10^{-4}$ |
| 4.122730 S 5d – 4p  | 0.00222    | 0.00221 | 0.00221 |
| 4.244067 T 5p – 4d  | 0.00312    | 0.00312 | 0.00311 |
| 4.600661 S 5s – 4p  | 7.479 $10^{-4}$ | 7.83 $10^{-4}$ (5%) | 7.85 $10^{-4}$ |
| 4.694980 T 5s – 4p  | 0.001581   | 0.00187 (16%) | 0.00188 |
| 0.4471 T 4d–2p      | 6.16        | 6.44 (4%) | 6.43 |

### 4.2 The Orion nebula model

The case B emissivities usually used to measure the helium abundance are susceptible to opacity effects, as e.g. discussed by Robbin (1968), Porter et al. (2007), Blagrave et al. (2007). To illustrate this issue, we consider as an example the Orion nebula.

In real nebulae, the stellar incident spectrum would produce PI and PE within all the levels in neutral He. The neutral helium recombination spectrum is obtained by a balance between He$^0$, He$^+$, and He$^{++}$. To approximate the case of a real nebula, and see which spectral lines are more sensitive to the parameters of a real model, we have therefore considered PE within all levels and PI from all levels up to $n=25$, although the results are the same if only PI from the ground state and the metastable are considered.

For case B, we have set the photo-excitation and de-excitation to the ground state to zero. We do not model the effects of optical depth in lines or continuum.

We have adopted a modelled photo-ionising spectrum obtained from the grid of O-star atmospheres by Lanz & Hubeny (2003) to represent the dominating ionizing flux, from $\Theta^1$ Ori C, an O6 star a radius about 9.4 $R_\odot$. We use the model with an effective temperature of 40 000 K and log $g=4.0$.

We also experimented with a widely-used line-blanketed LTE atmosphere model spectrum from Kurucz, calculated with solar abundances, and surface gravity log $g=4.5$. The results are nearly the same, as this spectrum is very similar to the above, although with much lower spectral resolution. We note that in the visible and infrared the spectra are close to that of a black-body of $T=40,000$ K, but the He ground state photoionizing flux below 504 Â is very different. We also note that the photons below the 2600 Å threshold for photoionizing the metastable $2s \ 3S$ are important in driving the population of this level, which in turns affects the populations of all the triplets (see, e.g. Clegg & Harrington 1989). We have extended the input spectrum above 90 microns with that of a black-body of $T=40,000$ K.

Blagrave et al. (2007) report an HST STIS observation (STIS-SLITIC), for which they derive an approximate temperature and density of $T_e=8000$ K and $N_e=2500$ cm$^{-3}$. Table 6 presents in the first column the observed fluxes corrected for reddening and relative to the 4471 Å line. The second and third columns give the results of the CLOUDY model $M$ and the case B (forced model) predictions from Porter et al. (2005) (P05), as reported by Blagrave et al. (2007). As pointed out by the above-mentioned previous authors, opacity effects related to the population of the metastable $2s \ 3S$ are present. In fact, the decays to this state such as the 3889 and 3188 are significantly over-predicted by the case B approximation (compared to observations), indicating self-absorption. As a consequence, the decays to the 2p $^3P$ (as the 7065 and 4713 Å lines) are strongly under-predicted. As shown by Blagrave et al. (2007), the CLOUDY model, which takes into account (in a simplified way) opacity effects, provides in general emissivities closer to observation. Similar effects (although much reduced in size) are also present in the singlets.

Returning to our model, we list in column six of Table 6 the emissivities calculated with case B, column (PB). As we have seen in previous tables, there is a general agreement with the P05 results, although not at the level that one might expect (we note that the 5876 and 6678 Å lines were the most affected by code errors).

We have then run our Case B model with PI and PE and various distances from the ionizing source. Baldwin et al. (1991) estimated that various regions of the Orion nebula range in distance from 6.1 Ori C between $3 \times 10^{17}$ and $10^{18}$ cm, which result in di-
Table 6. Emissivities of a selection from the strongest He optical lines. The values are emissivity ratios relative to the 4471 Å line. The third column gives the FOS-15W, STIS-SLIT1c observed values, corrected for reddening; the fourth and fifth columns give the results of the CLOUDY model M and the case B predictions from B"urger et al. (2003), as reported by Blagrave et al. (2007). Column PB gives our case B solution, while the following ones give the results of the full model with P1 and PE, for different dilutions d. All model emissivities in columns six, seven and eight have been calculated for $T_e$=8000 K and $N_e$=2500 cm$^{-3}$. Column nine (MD09) lists VLT observed intensities from Mesa-Delgado et al. (2009) and column ten (PB) our Case B results calculated for $T_e$=8180 K and $N_e$=2890 cm$^{-3}$. Columns eleven (MD21) and twelve (PB) contain corresponding results from the VLT observations of Mendez-Delgado et al. (2021) calculated for $T_e$=8360 K and $N_e$=5650 cm$^{-3}$. For our calculations, we provide the emissivities of the 4471 Å line in 10$^{-26}$ erg cm$^{-2}$ s$^{-1}$ in brackets.

| $\lambda$ (Å) | levels | STIS-SLIT1c | CLOUDY | P05 | PB | d=10$^{-12}$ | d=10$^{-14}$ | MD09 | PB | MD21 | PB |
|--------------|--------|-------------|--------|-----|----|------------|----------|------|----|------|----|
| 2945         | T 5p-2s | 0.26±0.01 | 0.290  | 0.414 | 0.412 | 0.605 | 0.415  | -    | 0.415 | -    | 0.418 |
| 3188         | T 4p-2s | -          | 0.441  | 0.878 | 0.862 | 1.197 | 0.866  | 0.802 | 0.875 | 0.471 | 0.886 |
| 3355         | S 7p-2s | -          | 0.034  | 0.038 | 0.056 | 0.038 | 0.056  | 0.038 | 0.056 | 0.038 | 0.056 |
| 3448         | S 6p-2s | -          | 0.056  | 0.061 | 0.079 | 0.061 | 0.079  | 0.061 | 0.079 | 0.061 | 0.079 |
| 3554         | T 10d-2p| -          | 0.052  | 0.054 | 0.061 | 0.054 | 0.061  | 0.054 | 0.061 | 0.054 | 0.061 |
| 3587         | T 9d-2p | -          | 0.077  | 0.075 | 0.081 | 0.075 | 0.081  | 0.075 | 0.081 | 0.075 | 0.081 |
| 3614         | S 5p-2s | -          | 0.101  | 0.108 | 0.089 | 0.108 | 0.089  | 0.108 | 0.089 | 0.108 | 0.089 |
| 3634         | S 8d-2p | -          | 0.099  | 0.106 | 0.080 | 0.106 | 0.080  | 0.106 | 0.080 | 0.106 | 0.080 |
| 3705         | T 7d-2p | -          | 0.150  | 0.162 | 0.159 | 0.162 | 0.159  | 0.162 | 0.159 | 0.162 | 0.159 |
| 3820         | T 6d-2p | -          | 0.262  | 0.262 | 0.251 | 0.262 | 0.251  | 0.262 | 0.251 | 0.262 | 0.251 |
| 3889         | T 3p-2s | -          | 0.780  | 2.315 | 2.24  | 2.778 | 2.238  | 1.561 | 2.29  | 1.150 | 2.36 |
| 3965         | S 4p-2s | -          | 0.206  | 0.221 | 0.219 | 0.214 | 0.219  | 0.211 | 0.221 | 0.194 | 0.222 |
| 4026         | T 5d-2p | -          | 0.466  | 0.472 | 0.470 | 0.471 | 0.470  | -    | 0.471 | 0.462 | 0.471 |
| 4144         | S 4d-2p | -          | 0.065  | 0.069 | 0.070 | 0.069 | 0.070  | 0.069 | 0.070 | 0.069 | 0.070 |
| 4388         | S 5d-2p | -          | 0.120  | 0.125 | 0.124 | 0.122 | 0.125  | 0.122 | 0.124 | 0.119 | 0.124 |
| 4471         | T 4d-2p | -          | 1      | 1     | (7.64)| (7.81)| (7.64) | 1    | (7.38)| 1    | (7.48) |
| 4713         | T 4s-2p | -          | 0.153  | 0.103 | 0.102 | 0.115 | 0.102  | 0.133 | 0.102 | 0.142 | 0.107 |
| 4922         | S 4d-2p | -          | 0.204±0.005 | 0.258 | 0.272 | 0.269 | 0.264 | 0.270 | 0.278 | 0.269 | 0.262 | 0.268 |
| 5016         | S 3p-2s | -          | 0.47±0.01 | 0.517 | 0.566 | 0.558 | 0.544 | 0.558 | 0.548 | 0.563 | 0.495 | 0.567 |
| 5876         | T 3d-2p | -          | 2.90±0.04 | 2.815 | 2.789 | 2.88  | 2.842 | 2.880 | 3.014 | 2.87 | 3.033 | 2.89 |
| 6678         | S 3d-2p | -          | 0.74±0.01 | 0.737 | 0.784 | 0.811 | 0.791 | 0.811 | 0.805 | 0.804 | 0.761 | 0.802 |
| 7065         | T 3s-2p | -          | 1.46±0.02 | 1.714 | 0.612 | 0.560 | 0.615 | 0.560 | 1.247 | 0.593 | 1.527 | 0.671 |
| 7281         | S 3s-2p | -          | 0.148±0.002 | 0.155 | 0.151 | 0.146 | 0.140 | 0.147 | 0.138 | 0.151 | 0.136 | 0.159 |
| 10028        | T 7f-3d | -          | 0.046  | 0.048 | 0.050 | 0.048 | 0.050  | 0.048 | 0.050 | 0.048 | 0.050 |
five lines we find an observed value of 6.75 from the MD09 observations and 6.73 from our Case B calculations, which is excellent agreement. The corresponding results for the MD21 observations are 6.32 and 6.91 but the large difference between observation and theory for the 4p-2s $\lambda$3188 transition indicates that this transition is showing strong self-absorption and even higher members of np-2s series would need to be included to make a valid comparison with theory.

5 CONCLUSIONS

Having established in a previous work (Del Zanna et al. 2020) that earlier studies of the He recombination spectrum suffered from various shortcomings, we have built several collisional-radiative models and compared the emissivities of the main spectral lines in neutral He with some of the most widely-used values in the literature. We have focused the comparisons on the case B approximation, as in most previous work.

As the requirement on the accuracy of the predicted emissivities is stringent, of the order of 1%, we conclude that there are several problems in the previous studies. A detailed assessment on the reasons why significant (larger than 1%) differences among different models are present is difficult. The adoption of different CE rates clearly has an effect, but the differences between the S96 and B99 models are surprising.

Before the various implementations of different models in CLOUDY, the S96 was the largest recombination model for He. Comparing the results of a reduced model ($LS$-resolved states up to $n=40$) to a full model, we were able to partially validate the S96 assumption, showing that for most spectral lines within the low-lying states the emissivities are accurate (within 1%) with the reduced model. So for most cases, a larger model is not strictly necessary. Despite this, significant differences with the S96 results are found, especially for a selection of infrared lines.

The latest implementation within CLOUDY, described in P12, used more accurate rates than previous versions, but still included incorrect rates for $l$-changing collisions. Using our models we found that in reality such rates have little effect on the emissivities of the main optical transitions. A similar conclusion was found by Guzmán et al. (2017) when varying the electron collision rates. There is generally excellent agreement between our emissivities and the P12 ones at the lowest densities, irrespective of temperature, where radiative processes dominate the populations but significant discrepancies are present for several transitions for higher densities at all temperatures.

The principal remaining uncertainty in our atomic model is now the choice of CE rate coefficients from the $^3S$ metastable, which becomes increasingly significant as density increases. Of the two most accurate calculations of these coefficients by Sawey & Berrington (1993) and Bray et al. (2000) we have chosen to use the latter but with the caveat that both have weaknesses. We suggest that it is reasonable to view the differences in emissivities resulting from using one or other of these CE calculations as a measure of the uncertainty due to the CE rates. In particular, by comparing our results from the SB93 model with our PB40 model it is possible to identify transitions which are relatively insensitive to the choice of CE rates. Broadly, the relatively weaker intersystem CE rates means that transitions among singlet states are less sensitive than those within the triplets, although there are some CE effects even on the singlets for transitions with an $n=3$ upper state. Within the triplet states themselves, transitions are preferred where the upper state is not linked to the metastable by an an electric dipole transition, such as the nd-2p series. The 4f-3d and 5g-4f transitions in the IR are another such example where CE effects are expected to be very small. At the highest density we consider, the differences between emissivities from the SB93 and PB40 models reach 5% for the $\lambda$10830 transition, which is the one most affected by CE.

Aside from the above issues, there is a further major one which has often been overlooked in the literature. The tabulated He emissivities calculated with the un-physical case B forced solution are widely used in astrophysical codes (see, e.g. Luridiana et al. 2015) and in general within the literature to e.g. measure the helium abundance (see, e.g. Peimbert et al. 2007; Aver et al. 2013; Izotov et al. 2014; Aver et al. 2015; Peimbert et al. 2016, for some recent examples).

A full analysis should include observed or well-modeled photoionizing radiation fields, updated atomic data, all the PI and PE effects we have included, and solve the full radiative transfer problem including the nebular expansion, which have a significant effect as shown by Robbins (1965). In some cases, where optical depths are not too great, the sum of the intensities of a subset of the triplet lines can be a useful diagnostic.

However, for those cases where opacity and other effects are negligible, and the case B solution is an acceptable approximation we provide our results in electronic form, including all transitions within $n \leq 5$ and all those between the $n \leq 5$ and $n' \leq 25$ states. We also provide an interpolation program. We note that we have found significant differences between our emissivities and those calculated by S96 for a few infrared transitions, discussed by Rubin et al. (1998). We therefore recommend our emissivities for future studies, and to benchmark any new case B model.

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DATA AVAILABILITY

The results of our Case B calculation, PB, are available from the CDS on a grid of electron temperatures and densities, $\log_{10} N_e[cm^{-3}]=2.0(0.5)6.0$ and $\log_{10} T_e[K]=3.0(0.1)4.6$, for all transitions with upper principal quantum number $\leq 2-5$ and lower principal quantum number $\leq 2-5$. We also provide a FORTRAN program to make two-dimensional interpolation of the emissivity tables.

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APPENDIX A: OTHER CASES
The first column gives the wavelength (in air, except the last ones in vacuum), the second indicates if a line is between singlets (S) or triplets (T). B72 (A): Brocklehurst (1972) case A; B72 (B): Brocklehurst (1972) case B; S99 (A): Smits (1999), case A; S99 (B): Smits (1999), case B; B99 (B): Benjamin et al. (1999) case B; P05 (B): Porter et al. (2005) case B; P12 (B): Porter et al. (2012) case B; SB93(B): n=40 model with SB93 rates, case B; PB40 (B): n=40 model with the Bray et al. rates, case B; PB(A): full n=500 model case A; PB(B): full n=500 model case B.

### Table A1. Emissivities \((10^{-26} \text{ erg cm}^3 \text{ s}^{-1})\) of the strongest He lines, for \(T_e=20000 \text{ K}\) and \(N_e=10^6 \text{ cm}^{-3}\).

| \(\lambda\) (Å) levels | Earlier work | Present work |
|-------------------------|--------------|--------------|
|                         | B72 (A)      | B72 (B)      |
|                         | S96 (A)      | S96 (B)      |
|                         | B99 (B)      | P05 (B)      |
|                         | P12 (B)      | SB93(B)      |
|                         | PB40         | PB (A)       |
|                         | PB (B)       |
| 2945 T 5p-2s            | 1.66         | 1.66         |
| 3188 T 4p-2s            | 3.43         | 3.43         |
| 3889 T 3p-2s            | 8.30         | 8.30         |
| 3965 S 4p-2s            | 0.027        | 0.03         |
| 4026 T 5d-2p            | 1.45         | 1.45         |
| 4388 S 5d-2p            | 0.37         | 0.38         |
| 4471 T 4d-2p            | 2.98         | 2.98         |
| 4713 T 4s-2p            | 0.41         | 0.46         |
| 4922 S 4d-2p            | 0.78         | 0.80         |
| 5016 S 3p-2s            | 0.045        | 0.045        |
| 5876 T 3d-2p            | 7.62         | 7.62         |
| 6678 S 3d-2p            | 2.09         | 2.14         |
| 7065 T 3s-2p            | 1.40         | 1.40         |
| 7281 S 3s-2p            | 0.33         | 0.55         |
| 10830 T 2p-2s           | 14.9         | 14.9         |
| 18685 T 4f-3d           | -            | 1.23         |
| 20887 S 2p-2s           | 2.6 \times 10^{-4} | 6.28         |

### Table A2. Emissivities \((10^{-26} \text{ erg cm}^3 \text{ s}^{-1})\) of the strongest He lines, for \(T_e=20000 \text{ K}\) and \(N_e=10^5 \text{ cm}^{-3}\) case B.

| \(\lambda\) (Å) levels | Earlier work | Present work |
|-------------------------|--------------|--------------|
|                         | B72          | S96          |
|                         | B99          | P05          |
|                         | P12          | PB40         |
|                         | PB           |
| 2945 T 5p-2s            | 1.65         | 1.62         |
| 3188 T 4p-2s            | 3.41         | 3.41         |
| 3889 T 3p-2s            | 8.22         | 8.17         |
| 3965 S 4p-2s            | 0.82         | 0.81         |
| 4026 T 5d-2p            | 1.43         | 1.44         |
| 4388 S 5d-2p            | 0.38         | 0.38         |
| 4471 T 4d-2p            | 2.95         | 3.0          |
| 4713 T 4s-2p            | 0.41         | 0.46         |
| 4922 S 4d-2p            | 0.80         | 0.79         |
| 5016 S 3p-2s            | 2.03         | 1.98         |
| 5876 T 3d-2p            | 7.60         | 7.58         |
| 6678 S 3d-2p            | 2.15         | 2.14         |
| 7065 T 3s-2p            | 1.40         | 1.92         |
| 7281 S 3s-2p            | 0.55         | 0.58         |
| 10830 T 2p-2s           | 14.8         | 272.7        |
| 18685 T 4f-3d           | -            | 1.21         |
| 20887 S 2p-2s           | 4.95         | 4.56         |

### Table A3. Emissivities \((10^{-26} \text{ erg cm}^3 \text{ s}^{-1})\) of the strongest He lines, for \(T_e=20000 \text{ K}\) and \(N_e=10^6 \text{ cm}^{-3}\) case B.

| \(\lambda\) (Å) levels | Earlier work | Present work |
|-------------------------|--------------|--------------|
|                         | B72          | S96          |
|                         | B99          | P05          |
|                         | P12          | PB40         |
|                         | PB           |
| 2945 T 5p-2s            | 1.65         | 1.65         |
| 3188 T 4p-2s            | 3.40         | 3.40         |
| 3889 T 3p-2s            | 8.20         | 8.17         |
| 3965 S 4p-2s            | 0.80         | 0.81         |
| 4026 T 5d-2p            | 1.43         | 1.44         |
| 4388 S 5d-2p            | 0.38         | 0.38         |
| 4471 T 4d-2p            | 2.94         | 3.00         |
| 4713 T 4s-2p            | 0.41         | 0.46         |
| 4922 S 4d-2p            | 0.79         | 0.80         |
| 5016 S 3p-2s            | 2.03         | 1.97         |
| 5876 T 3d-2p            | 7.58         | 7.56         |
| 6678 S 3d-2p            | 2.14         | 2.18         |
| 7065 T 3s-2p            | 1.40         | 1.92         |
| 7281 S 3s-2p            | 0.58         | 0.61         |
| 10830 T 2p-2s           | 14.7         | 272.7        |
| 18685 T 4f-3d           | -            | 0.92         |
| 20887 S 2p-2s           | 2.24         | 2.24         |
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Table A4. Emissivities ($10^{-26}$ erg cm$^3$ s$^{-1}$) of the strongest He lines, for $T_e=5000$ K and $N_e=10^8$ cm$^{-3}$ case B.

| $\lambda$ (Å) levels | Earlier work | Present work |
|----------------------|--------------|--------------|
|                      | B72          | S96          |
|                      | B99          | P05          |
|                      | P12          | PB40         |
|                      | PB           |
| 2945 T 5p-2s        | 4.30 -       | 4.36 -       |
| 3188 T 4p-2s        | 9.04 9.10    | 9.18 9.59    |
| 3889 T 3p-2s        | 22.9 22.7    | 23.3 25.0    |
| 3965 S 4p-2s        | 2.41 2.36    | 0.95 2.53    |
| 4026 T 5d-2p        | 5.56 5.45    | 5.47 5.87    |
| 4388 S 5d-2p        | 1.51 1.45    | 1.46 1.57    |
| 4471 T 4d-2p        | 12.0 12.0    | 12.1 12.70   |
| 4713 T 4s-2p        | 0.77 0.90    | 0.98 0.95    |
| 4922 S 4d-2p        | 3.32 3.23    | 3.25 3.46    |
| 5016 S 3p-2s        | 6.20 6.03    | 6.11 6.50    |
| 5876 T 3d-2p        | 35.4 34.8    | 35.3 36.9    |
| 6678 S 3d-2p        | 10.2 10.0    | 10.1 10.5    |
| 7065 T 3s-2p        | 2.89 4.29    | 5.19 5.30    |
| 7281 S 3s-2p        | 1.39 1.37    | 1.46 1.50    |
| 10830 T 2p-2s       | 47.5 245.    | 234. 214     |
| 18685 T 4f-3d       | - -         | 4.97 5.41    |
| 20587 S 2p-2s       | - 10.7 10.6 | 10.7 10.1    |

Table A5. Emissivities ($10^{-26}$ erg cm$^3$ s$^{-1}$) of the strongest He lines, for $T_e=5000$ K and $N_e=10^4$ cm$^{-3}$ case B.

| $\lambda$ (Å) levels | Earlier work | Present work |
|----------------------|--------------|--------------|
|                      | B72          | S96          |
|                      | B99          | P05          |
|                      | P12          | PB40         |
|                      | PB           |
| 2945 T 5p-2s        | 4.12 -       | 4.20 -       |
| 3188 T 4p-2s        | 8.66 8.76    | 8.79 8.69    |
| 3889 T 3p-2s        | 21.9 21.8    | 22.0 22.5    |
| 3965 S 4p-2s        | - 2.27 2.28  |
| 4026 T 5d-2p        | 5.31 5.28    | 5.31 5.28    |
| 4388 S 5d-2p        | 1.41 1.42    | 1.44 1.43    |
| 4471 T 4d-2p        | 11.6 11.74   | 11.8 11.5    |
| 4713 T 4s-2p        | 0.76 0.88    | 0.89 0.90    |
| 4922 S 4d-2p        | - 3.16 3.17  |
| 5016 S 3p-2s        | - 5.81 5.85  |
| 5876 T 3d-2p        | 35.0 34.5    | 35.3 33.6    |
| 6678 S 3d-2p        | - 10.05 10.11|
| 7065 T 3s-2p        | 2.83 4.18    | 4.27 4.24    |
| 7281 S 3s-2p        | - 1.33 1.35  |
| 10830 T 2p-2s       | 46.0 50.6    | 50.7 49.9    |
| 18685 T 4f-3d       | - -         | 5.19 5.06    |
| 20587 S 2p-2s       | - 7.42 7.42  |

Table A6. Emissivities ($10^{-26}$ erg cm$^3$ s$^{-1}$) of the strongest He lines, for $T_e=5000$ K and $N_e=10^2$ cm$^{-3}$ case B.

| $\lambda$ (Å) levels | Earlier work | Present work |
|----------------------|--------------|--------------|
|                      | B72          | S96          |
|                      | B99          | P05          |
|                      | P12          | PB40         |
|                      | PB           |
| 2945 T 5p-2s        | 4.12 -       | 4.20 -       |
| 3188 T 4p-2s        | 8.66 8.76    | 8.79 8.69    |
| 3889 T 3p-2s        | 21.9 21.8    | 22.0 22.5    |
| 3965 S 4p-2s        | - 2.27 2.28  |
| 4026 T 5d-2p        | 5.31 5.28    | 5.31 5.28    |
| 4388 S 5d-2p        | 1.41 1.42    | 1.44 1.43    |
| 4471 T 4d-2p        | 11.6 11.74   | 11.8 11.5    |
| 4713 T 4s-2p        | 0.76 0.88    | 0.89 0.90    |
| 4922 S 4d-2p        | - 3.16 3.17  |
| 5016 S 3p-2s        | - 5.81 5.85  |
| 5876 T 3d-2p        | 35.0 34.5    | 35.3 33.6    |
| 6678 S 3d-2p        | - 10.05 10.11|
| 7065 T 3s-2p        | 2.83 4.18    | 4.27 4.24    |
| 7281 S 3s-2p        | - 1.33 1.35  |
| 10830 T 2p-2s       | 46.0 50.6    | 50.7 49.9    |
| 18685 T 4f-3d       | - -         | 5.19 5.06    |
| 20587 S 2p-2s       | - 7.42 7.42  |