Comments on “Theoretical search for the nested quantum Hall effect of composite fermions” by Mandal and Jain, Phys. Rev. B 66,155302 (2002); cond-mat/0210181.

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We find that a large number of parameters are used to create the correct fraction. The parameters are $\nu$, $1-\nu$, $\nu^*$, $\bar{n}$, $n$, $p$ and $\bar{p}$. Therefore, the predicted fractions need not be having the correct origin. The wave function describes a composite fermion which has $2p$ (even number) of flux quanta attached to one electron. We find that it requires “decomposite” fermion which is the electron in an orbit from which the magnetic field has been detached. This kind of detachment (attachment) of flux quanta from (to) the electron is not consistent with the electromagnetic theory of light and violates Biot and Savart’s law, as well as theory of relativity. If flux quanta are to be attached to the electron, we should solve the bound-state equation and determine the binding energy but bound-state has not been solved. The wave function given is not a solution of the bound-state equation. Therefore, Mandal and Jain’s composite fermion (CF) model is incorrect.

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1. Introduction

The quantum Hall effect is dominated by the observation of plateaus in the transverse resistivity, $\rho_{xy} = \hbar/\nu e^2$ at integer values of $\nu$. In fact $\nu$ need not be an integer. The fractional values of $\nu$ such as $1/3$ also occur. The claim of the composite fermion (CF) model\(^1\) is that even number of flux quanta are attached to the electron which gives rise to the plateaus in the quantum Hall effect. The CF model proposes several series of fractions,

$$\frac{n}{2pn \pm 1}, 1 - \frac{n}{2pn \pm 1}, \frac{n}{4pn \pm 1}, 1 - \frac{n}{4pn \pm 1}, \ldots \text{etc.} \quad (1)$$

In place of $2p$, $4p$, $6p$, $8p$, etc. “even number” is proposed without any upper limit. In fact many of these fractions are in good agreement with the experimental data. Thus CF model is considered to be a strong candidate for the explanation of the quantum Hall effect. It is possible to make use of this “even number” in the Laughlin’s wave function so that one can obtain the fraction as well as the wave function of the quasiparticle which gives the quantum Hall effect. The expression (1) is neither a solution of the bound-state equation nor that of the Schrödinger equation.

In this paper, we wish to point out that the assertion that “even number of flux quanta are attached to the electron” is incorrect. Hence any agreement with the experimental data is because of the fact that too much force has been put to get the experimental values of the fractional charge.

2. The fractional charge

(a) Abundance.

The resistivity is given by $\rho = \hbar/\nu e^2$ so that $\nu$ gives the effective charge,

$$e_{\text{eff}} = \nu e. \quad (2)$$

When we take all of the prescribed values of the fraction, we find that a lot of values of (1) are the same as in experimental data. However, the series (1) give many more fractions than are observed in the data. At large values of the Landau level quantum number, the values given by (1) are far more than observed. Therefore, the abundance of fractions obtained by (1) is not in agreement with the data.
(b) Parameters.

The charge is calculated using the parameters,

\[ \nu, 1 - \nu, \nu^*, \bar{n}, n, p, \bar{p}, \text{etc.} \]  

Therefore, there are far too many parameters to find the experimentally measured fractions of charges.

(c) Particle-hole symmetry.

In the paper of Mandal and Jain, \[ \nu = \frac{n}{2pn \pm 1}. \]  

In addition, \[ \nu = 1 - \frac{n}{2pn \pm 1} \]  

has been introduced. If the first value is 1/3, then the second value is 2/3. However, if the wave function for 1/3 is antisymmetric, then it will be symmetric for 2/3. Therefore, 1/3 will be a fermion and 2/3 will be a boson. In the CF model, there are always even number of flux quanta attached to an electron and hence quasiparticles are fermions. Therefore particle-hole symmetry does not occur in the CF model. It is clear that there is internal inconsistency in the CF model. In the case of Laughlin’s wave function 1/3 is produced and next value is 1/5 but not 2/3 because 1/3 and 1/5 are antisymmetric and 2/3 is symmetric. There is no way for Laughlin to have an even numerator with odd denominator.

(d) Flux quanta attachment.

The even number of flux quanta must be attached to the electron to obtain the effective field inside the sample. Therefore, the field is, \[ B^* = B - 2np\phi_o \]  

where \( n \) is the number of electrons per unit area, \( p \) is an integer and \( \phi_o = hc/e \) is the unit flux. In order to attach even number of flux quanta, let us detach the flux from the
electron. The electron going in a circular orbit produces a field as shown in Fig.1. Let us see whether it is allowed to detach the field from the electron. This process produces an electron current and a field. What is left after detaching B is called decomposite fermion (DF). The object (DF) will not be having the magnetic field but it has a charge. Hence Maxwell equations will not be obeyed. The DF will not emit light nor will it absorb. Hence it will be completely invisible. Such objects are not known to exist in the real world. They surely do not exist in GaAs. Let us try to attach two flux quanta to the electron. One electron gives one field B, two electrons give $2B$ as shown in Fig.2. The $2B$ can not be given by one $e$, otherwise we are left with a DF. Such a DF without field is an unphysical object.

When we put $B^* =0$, $B$ is found to be quantized and when we put $B=0$, then $B^*$ is quantized. That seems to be quite alright. However, in superconductors, field outside the sample is continuous and inside the sample, in the mixed state, is quantized. Therefore, it is not clear why both fields $B$ and $B^*$ get quantized. The density of the CF is the same as that of the electrons or there is only one density in the CF model. The CF are much larger objects than the electrons. Therefore, their density can not be the same as that of the electrons\textsuperscript{2}. The density of the CF is therefore not internally consistent\textsuperscript{3}. The energy calculation has been performed for $N=12, 16, 20, 24$, etc. but these values are too small to give any realistic results.

In the CF model, the fractional charge of $1/3$ was produced by inventing the series $n/(2pn \pm 1)$. Thus using three parameters, $p$, $n$ and $\pm 1$, to produce $1/3$ introduces too much arbitrariness and hence $n/(2pn\pm1)$ is not a model at all. The experimentalists\textsuperscript{4} have been misguided\textsuperscript{5} to identify the experimental data with the CF. In fact the experimental data is not related to CF.

3. Electromagnetic theory

A charged particle emits radiation which is composed of electric and magnetic field vectors, transverse to the direction of motion of the energy. The Maxwell equations determine the dynamics of the electric $\vec{E}$ and magnetic $\vec{H}$ vectors with charge density as
a constant. Thus $\vec{E}$ and $\vec{H}$ are strongly coupled. The formation of a DF decouples $\vec{H}$ from the charge and hence from $\vec{E}$. At the present time the $\vec{E}$ and $\vec{H}$ are not decoupled. Similarly, extra $\vec{H}$ can not be attached to the charge. Therefore, CF formation is not permitted in the usual Maxwell equations. However, new type of Maxwell equations may be written but classical form of electromagnetic theory is in agreement with the experiments, and hence there is no need of new type of Maxwell equations. The formation of CF is therefore not expected to agree with the present day experiments. The use of the same density for CF as for the electron makes the CF internally inconsistent. Therefore, at this time there is no hope for the CF model.

4. Conclusions.

The attachment of two flux quanta to one electron is not correct. It violates the classical electrodynamics. The use of only one density for the quantization of $B$ as well as $B^*$ is internally inconsistent. Therefore, the field expression $B^* = B - 2n\rho_0\phi_o$, much needed to produce experimentally observed fractional charge, is not internally consistent. The same number of small and large objects can not fill the same area. The even number of flux quanta produce the Laughlin type wave functions but 2/3 can not come, so there is inconsistency. The classical electromagnetic theory is not uniquely produced\(^6\). It is clear that the CF model lacks in good theoretical foundation\(^7\) and its various elements are inconsistent\(^8\). The field expression which attaches flux is not correct\(^9\).

Since Laughlin’s wave function is limited to antisymmetry of the wave functions, which leads to odd number in the density, it may be thought that this wave function is not a theory of the quantum Hall effect. The CF model is not free of troubles with the fundamentals. Therefore, the readers may be interested in a calculation which explains the experimental data without difficulty with fundamentals. Thus, a theory is given in a recent book\(^10\). About 200 references of recent prl articles have been examined to check the applicability of the theory\(^11\) and in all cases, the given theory agrees with the data. It may be mentioned that the particle-hole symmetry\(^12\) around $\nu = 1/2$ has been observed by Willett et al\(^13\) but the interpretation given in ref.[13] is not satisfactory.
5. References

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Fig.1: The electron in a circular orbit produces a field $B$. If this field is detached from the electron current we are left with two objects, one is a charge called DF and the other is a field $B$.

Fig.2: One electron current produces a field of $B$ and another electron also produces another field $B$. Now detach the field of one electron so that DF is left and attach it to another electron so that CF is formed.
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