How to retrieve priced data

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Abstract

Databases are an indispensable resource for retrieving up-to-date information. However, curious database operators may be able to find out the users' interests when the users buy something from the database. For these cases, if the digital goods have the identical prices, then a $k$-out-of-$n$ oblivious transfer protocol could help the users to hide their choices, but when the goods have different prices, this would not work. In this paper, we propose a scheme to help users to keep their choices secret when buying priced digital goods from databases.

1. Introduction

I am quite sure that all readers are familiar with digital libraries, such as the digital libraries of ACM, IEEE and SIAM. These libraries provide researchers with a comprehensive resource of published papers, and users can easily retrieve their desired papers by visiting these libraries. Recall how we retrieve data from digital libraries: we log in the system, select our desired ones and download them. If one does not own a membership of a database, he would have to pay for the papers he reads, perhaps according to the length of the publication. This process is convenient, but we undertake the risk of revealing our private research interests to the database operators.

If every paper has the same price, this problem can be resolved perfectly: suppose that there are $n$ publications in the library and we are interested in $k$ of them, then by $k$-out-of-$n$ oblivious transfer, we pay for some $k$ publications while revealing nothing about our choices. In other word, the operator learns nothing but $k$, and could get the payment by adding up the prices for the $k$ sold items.

However, it is naive to assume that all publications have the same price. Nowadays, most papers are priced according to their lengths, perhaps one dollar per page. A more scientific way (although not perfect) is to price the data according to the number of bits it contains. More formally, here we may view the database as a binary string $x = x_1x_2 \cdots x_n$ of length $n$, and every bit has the same weight. Then the we could still use oblivious transfer to buy our desired bits from the library, leaking nothing but the number of bits we pay for.
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This scheme is not efficient (since the number of bits can be very large), and a more serious problem is that we should not assume every bit to have the same value. Instead of finding a method to assign prices for different goods, we would rather let the database operator to assign the value herself: a two-page communication may cost you 100 dollars, while a 200-page review may cost only 1 dollar. After all, we let the operator to assign the prices herself.

Now we consider the general problem: the database has \( n \) items, namely \( m_1, m_2, \ldots, m_n \). Each item \( m_i \) \( (1 \leq i \leq n) \) has its own weight \( p_i \). Let \( \sigma_1, \sigma_2, \ldots, \sigma_k \), a subset of \( \{1, 2, \ldots, n\} \), be the choices of a user, the goal is to leak \( m_{\sigma_j} \) \( (1 \leq j \leq k) \) to the user while revealing nothing but \( \sum_j p_{\sigma_j} \) to the database operator. This is a special case of oblivious transfer, we denote it weighted oblivious transfer.

**Organization.** In the rest of this section we discuss in more detail traditional oblivious transfer and weighted oblivious transfer. Section 2 presents two protocols for weighted oblivious transfer. Section 3 concludes the paper.

### 1.1. A short review of oblivious transfer

Oblivious Transfer (OT) refers to a kind of two-party protocols where at the beginning of the protocol one party, the sender, has an input, and at the end of the protocol the other party, the receiver, learns some information about this input in a way that does not allow the sender to figure out what it has learned [1]. Oblivious transfer is one of the key components of many cryptographic protocols and a fundamental primitive for cryptography and secure distributed computation [2, 3, 4]. The concept of oblivious transfer was proposed by Rabin [5], since then, many flavors of oblivious transfer were introduced and analyzed [5, 6, 7, 8, 9]. Now oblivious transfer is one of the most remarkable achievements in foundation of cryptography. The main flavors of oblivious transfer are as follows:

- **Original oblivious transfer (OT)**[5]. For OT, the sender has only one secret, \( m \), and would like to have the receiver obtain \( m \) with probability 0.5. On the other hand, the receiver does not want the sender to know whether it gets \( m \) or not.

- **1-out-of-2 oblivious transfer**((OT\(^2\)))[6]. For OT\(^2\), the sender has two secrets, \( m_1 \) and \( m_2 \), and would like to give the receiver one of them at the receiver’s choice. Again, the receiver does not want the sender to know which secret it chooses.

- **1-out-of-\( n \) oblivious transfer**((OT\(^1_n\)))[7]. OT\(^1_n\) is a natural extension of OT\(^2\) to the case of \( n \) secrets, in which the sender has \( n \) secrets \( m_1, m_2, \ldots, m_n \) and is willing to disclose exactly one of them to the receiver at its choice.

- **\( k \)-out-of-\( n \) oblivious transfer**((OT\(^k_n\)))[10]. For OT\(^k_n\), the receiver can receive only \( k \) messages out of \( n \) messages sent by the sender. In general, one thinks that OT\(^k_n\) is exten-
sion of $OT_{n}^1$. It is obvious that a trivial $OT_{n}^k$ protocol can be obtained by performing $OT_{n}^1$ protocol $k$ times.

Essentially, all these flavors are equivalent in the information theoretic sense [11], but their functions vary, intuitively, we may use the following relation to describe the relation among all four flavors:

$$OT \subseteq OT_{2}^1 \subseteq OT_{n}^1 \subseteq OT_{n}^k.$$ (1)

There are many ways to construct an efficient oblivious transfer protocol. Classical oblivious transfer protocols are based on discrete logarithm [12, 13], the hardness of the decisional Diffie-Hellman problem [14] etc.

### 1.2. Definition of weighted oblivious transfer

To define the requirements of a weighted oblivious transfer protocol, we simply apply the requirements of general $OT_{n}^1$ protocol (with minor revisions) [15] to it: for convenience, let $m_1, m_2, \cdots, m_n$ to be items and $p_i$ be the weight of $m_i$.

**Definition 1** A $(k$-out-of-$n)$ weighted oblivious transfer should meet the following requirements:

- **Correctness.** The protocol achieves its goal if both the receiver and the sender behave properly. That is, if both the receiver and the sender follow the protocol step by step, the receiver gets $m_{\sigma_i}$’s after executing the protocol with the sender, where $\sigma_i$’s are the receiver’s choices, and the sender learns $\sum_{i=1}^{k} p_{\sigma_i}$ (i.e., the whole price of the goods).

- **Receivers’ Privacy-indistinguishability.** The transcripts corresponding to the receiver’s different choices $\{\sigma_{ai}\}$ and $\{\sigma_{bi}\}$, $\{\sigma_{ai}\} \neq \{\sigma_{bi}\}$, are computationally indistinguishable to the sender if the following equation is satisfied:

$$\sum p_{\sigma_{ai}} = \sum p_{\sigma_{bi}}.$$ (2)

If the transcripts are identically distributed, the choice of the receiver is unconditionally secure.

- **Sender’s Privacy-compared with ideal model.** We say that the sender’s privacy is guaranteed if, for every possible malicious $R$ which interacts with $S$, there is a simulator $R'$ (a probabilistic polynomial time machine) which interacts with $T$ such that the output of $R'$ is computationally indistinguishable from the output of $R$.

**Remark.** The weighted oblivious transfer also relies on the intractability of subset subproblem. The protocol implies that by the total price of the sold items, the sender cannot tell
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which items the receiver bought. Although subset sum problem is known to be NP-complete, sometimes it is still solvable (consider the case where the prices are $1, 2, 4, 8, \cdots, 2^{n-1}$, then the binary representation of the total price would betray the receiver’s choice). However, for this case, even if we apply a trusted third party $T$, the problem still exists. This problem is solvable when the database is stored by more than one servers (recall PIR), but this is out of the scope of this paper.

1.3. Comparing to priced oblivious transfer

Perhaps the idea that of weighted oblivious transfer is similar to [16]. In [16], the notion of “priced oblivious transfer” is proposed. Informally, assume that a buyer first deposits a pre-payment at the hands of a vendor. The buyer should then be able to engage in a virtually unlimited number of interactions with the vendor in order to obtain digital goods (also referred to as items) at a total cost which does not exceed its initial deposit amount. After spending all of its initial credit, the buyer should be unable to obtain any additional items before depositing an additional pre-payment. For priced oblivious transfer, unlimited number of interactions and prepayment is required, while these requirements relax in this paper. However, for weighted oblivious transfer, the receiver would disclose how much, when to the sender, since we do not assume that the receiver interacts with the sender many times. Comparing to priced oblivious transfer, weighted oblivious transfer is used when the receiver would like to buy the desired items once at the same time.

2. Weighted oblivious transfer

The idea of our first protocol is straightforward. The sender locks the ever item $m_i$ with $p_i$ different locks. In this way, only with all $p_i$ locks can the receiver get $m_i$. This implies that the sender needs to generate $\sum_{i=1}^{n} p_i$ keys, and with $\sum_{i=1}^{k} p_{\sigma_i}$ locks and keys, the receiver could unlock the locks for $m_{\sigma_i}$’s. By using a $\sum_{i=1}^{k} p_{\sigma_i}$-out-of-$\sum_{i=1}^{n} p_i$ oblivious transfer, the sender could leak the corresponding keys to the receiver without knowing the chosen ones, thus unable to figure out the receiver’s choices. For simplicity, the best (i.e., most efficient) lock should be symmetric key encryption scheme.

Protocol 1

In this protocol, the sender has $\sum_{i=1}^{n} p_i$ pairs of (different) keys, denoted by $K_{ij}(i \in \{1, 2, \cdots, n\}, j \in \{1, 2, \cdots, p_i\})$. Intuitively, the sender locks $m_i$ with $K_{ij}$’s.
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Input: The receiver’s input is composed of \( k \) numbers \( \sigma_1, \sigma_2, \ldots, \sigma_k \), which is a subset of \( \{1, 2, \ldots, n\} \), and the sender’s input is composed of \( n \) priced items \( m_1, m_2, \ldots, m_n \), the weight (price) of item \( m_i \) is \( p_i \).

Output: The receiver’s outputs are \( m_{\sigma_1}, m_{\sigma_2}, \ldots, m_{\sigma_k} \), and the sender’s output is \( \sum_{i=1}^{k} p_{\sigma_i} \).

- **Step 1** The sender encrypts the items \( m_1, m_2, \ldots, m_n \) with the encryption keys. For \( m_i \), this is done by computing \( E_{K_{i1}}(E_{K_{i2}}(\cdots(E_{K_{ip_i}}(m_i)) \cdots) \). That is, \( m_i \) is encrypted by \( p_i \) locks: \( K_{i1}, \ldots, K_{ip_i} \) respectively.

- **Step 2** The sender sends all the ciphertexts to the receiver.

- **Step 3** By \( \sum_{i=1}^{k} p_{\sigma_i} \)-out-of-\( \sum_{i=1}^{n} p_i \) oblivious transfer, the sender reveals the keys for all \( m_{\sigma_i} \)'s, while learning nothing about the receiver’s choices.

- **Step 4** With the keys, the receiver easily decrypts and learns all \( m_{\sigma_i} \)'s (and nothing else).

This protocol is straightforward, and we do not prove that it is actually a weighted oblivious transfer in a formal way. Informally, assume the security of \( \sum_{i=1}^{k} p_{\sigma_i} \)-out-of-\( \sum_{i=1}^{n} p_i \) oblivious transfer protocol used in step 3, the sender leaks nothing but \( \sum_{i=1}^{k} p_{\sigma_i} \) during step 3, the only communication from the receiver to the sender. Also, the receiver could unlock no more than \( \sum_{i=1}^{k} p_{\sigma_i} \) locks, thus learns no more than what “costs” \( \sum_{i=1}^{k} p_{\sigma_i} \).

Although the protocol is not efficient enough, it is the cornerstone of the next protocol.

**Remark.** When the protocol is applied by databases, the first two steps are done before transactions. That is, the database publishes the encrypted items online and everyone could download them. When interested in some of the items, the user interacts with the database operator and completes the last two steps. In this way, they would not need to communicate the whole encrypted data, which turns out to be huge. Also, the items are encrypted only once for all users.

### 2.1. Making our protocol efficient

It is not hard to show that protocol 1 needs \( O(\sum_{i=1}^{n} p_i) \) encryptions, this number is clearly impractical, at least sometimes. In this subsection, we propose a very efficient protocol which only needs \( O(n) \) encryptions.

If for any \( m_i \), if there is way to divide \( m_i \) into pieces such that it is easily reconstructable from \( p_i \) pieces, but even complete knowledge of \( p_i - 1 \) pieces reveals absolutely no information about \( m_i \), then we can propose a new protocol. This is really easy: let

\[
m_i = \bigoplus_{j=1}^{p_i} m_{ij} = m_{i1} \oplus m_{i2} \oplus \cdots \oplus m_{ip_i}
\]
Then $m_i$ could only be recovered with all $p_i m_{ij}$’s. However, this division scheme is not very efficient since $m_i$ can be very long. Also, for this case, instead of downloading the encrypted data from the website, the receiver has to learn all he needs from the sender. So we slightly revise our idea and comes up with protocol 2:

**Protocol 2**

In this protocol, the sender has $n$ pairs of different keys, denoted by $K_i (i \in \{1, 2, \cdots, n\})$. Intuitively, she intends to encrypt $m_i$ with $K_i$.

*Input:* The receiver’s input is composed of $k$ numbers $\sigma_1, \sigma_2, \cdots, \sigma_k \in \{1, 2, \cdots, n\}$, and the sender’s input is composed of $n$ items $m_1, m_2, \ldots, m_n$, the weight of item $m_i$ is $p_i$.

*Output:* The receiver’s outputs are $m_{\sigma_1}, m_{\sigma_2}, \ldots, m_{\sigma_k}$.

- **Step 1** The sender encrypts the items $m_1, m_2, \cdots, m_n$ with the encryption keys and obtains $E_{K_1}(m_1), E_{K_2}(m_2), \cdots, E_{K_n}(m_n)$.

- **Step 2** The sender sends all the ciphertexts to the receiver.

- **Step 3** For every $i \in \{1, 2, \cdots, n\}$, the sender divides $K_i$ into $p_i$ parts. This is done by finding $K_{i1}, K_{i2}, \cdots, K_{ip_i}$ such that $\bigoplus_{j=1}^{p_i} K_{ij} = K_i$.

- **Step 4** Using oblivious transfer, the sender leaks $K_{\sigma_1}, \cdots, K_{\sigma_k}$ to the receiver while learning nothing about $\sigma_i$’s. This is done by revealing $\sum_{i=1}^{k} p_{\sigma_i}$ parts (i.e., the parts of key $K_{\sigma_i}$’s) out of all $\sum_{i=1}^{n} p_i$ parts (i.e., for every $\sigma_i$, reveal $K_{\sigma_1}, \cdots, K_{\sigma_{p_{\sigma_i}}}$).

- **Step 5** The receiver recovers the keys for all $m_{\sigma_i}$’s by exclusive-oring $m_{\sigma_i j}$ and decrypts them.

Similarly, when the protocol is applied by databases, the first three steps are done before transactions. That is, the database publishes the encrypted items online and everyone could download them. When interested in some of the items, the user interacts with the database operator and completes the last two steps.

Now we show that protocol 2 is indeed a weighted oblivious transfer: If both parties behave properly, then the receiver would learn all parts of $K_{\sigma_i}, i \in \{1, \cdots, k\}$, and by XOR operations he learns $K_{\sigma_i}$, thus able to learn $m_{\sigma_i}$.

The scheme takes only three rounds. This is almost optimal since at least the receiver has to choose $\{\sigma_1, \cdots, \sigma_k\}$’s and let the sender know and the sender has to respond to the receiver’s request.

For computation, the receiver needs $k < n$ decryptions and $\sum_{i=1}^{k} p_{\sigma_i}$ XOR operations. The sender needs $n$ encryptions, $n$ XOR operations and choosing $\sum_{i=1}^{n} (p_i - 1)$ random numbers.
**Lemma 1** For protocol 2, the receiver’s choice is unconditional secure, assuming that the oblivious transfer used in step 4 is secure.

Proof. For any choices \( \{\sigma_{ai}\} \), if \( \sum p_{\sigma_{ai}} = \sum p_{\sigma_i} \) holds, in step 4 the receiver and the sender still perform \( \sum p_{\sigma_i} \)-out-of-\( \sum p_i \) oblivious transfer, then the security of oblivious transfer used in step 4 shows that the sender cannot learn anything—it cannot tell \( \{\sigma_{ai}\} \) from \( \{\sigma_i\} \). Since the receiver sends nothing else, the sender cannot tell what the receiver’s choice is. \( \square \)

**Lemma 2** For protocol 2, if the receiver is semi-honest, it gets no information about \( m_i, i \notin \{\sigma_1, \cdots, \sigma_k\} \), assuming the security of the encryption scheme and oblivious transfer.

Proof. If the receiver is semi-honest, due to the security of oblivious transfer, it learns nothing about \( K_i, i \notin \{\sigma_1, \cdots, \sigma_k\} \). The security of the encryption scheme promises that \( E_{K_i}(m_i), i \notin \{\sigma_1, \cdots, \sigma_k\} \) is computational indistinguishable from \( E_{K_i}(r), i \notin \{\sigma_1, \cdots, \sigma_k\} \), where \( r \) is a randomly chosen sequence. \( \square \)

**Lemma 3** Protocol 2 meets the requirement of sender’s privacy assuming the security of oblivious transfer used in step 4.

Proof. For each malicious receiver \( R \) in the real run, we construct a simulator \( R' \) in the Ideal Model such that the outputs of \( R \) and \( R' \) are computationally indistinguishable.

As the oblivious transfer used in step 4 is secure, there exists a simulator \( R'' \) in the Ideal Model such that the outputs of \( R \) and \( R'' \) are computationally indistinguishable. Now let \( R' \) acts the same as \( R'' \), then \( R \) and \( R' \) are computationally indistinguishable. Since there is no other iteration with \( T \), we prove the theorem. \( \square \)

With these preparations, we come up with:

**Theorem 1** Protocol 2 is indeed weighted oblivious transfer.

Since the fact that \( OT^{k}_n \) can be achieved from weighted oblivious transfer is trivial, we see the equivalence between the two flavors. Moreover, all (existing) flavors of oblivious transfer are equivalent in the information theoretic sense.

### 2.2. Reducing the computation complexity

The computational complexity of the protocol is much more expensive than a traditional \( k \)-out-of-\( n \) scheme where every item has the same weight. This deficiency may, to some extent, affect the application of the protocol, but there are some ways to reduce the computational complexity.
If $p_1, \ldots, p_n$ share a greatest common divisor $q > 1$, then by dividing $q$ and paying $q$ times the money for each decryption, the weight of the $i$-th element becomes $p_i/q$, where $p_i$ is the original weight, and the times of encryptions and decryptions can be decreased to $1/q$ (of the original one). In the cases that the greatest common divisor of $p_1, \ldots, p_n$ is $1(q = 1)$, we introduce two additional methods:

**Method 1** Arrange the items into some certain categories, and the items of each category share the same weight. This is practical in everyday life. Assume that there are three categories, with weight one, two and three, then the computation is around $3n$.

**Method 2** Generally speaking, methods one could save most computation. In the cases where it is difficult to arrange the items into certain categories, we have following method:

First consider an example. Assume that there are four items with weights 105, 190, 307, 689. Then the greatest common divisor of the weights is $q = 1$, and we could do nothing with them. However, if the weights change a little into 100, 200, 300, 700, then $p = 100$, and the complexity can be greatly reduced by changing the weights into 1, 2, 3, 7.

More generally, suppose that the weight of each item is $p_i (i = 1, 2, \ldots, n)$. The two parties could decide a “greatest common divisor” $q$, and calculate the new weights-the closest integer of $p_i/q$. In this way, the complexity could also be greatly reduced.

### 3. Concluding remarks

This paper discusses weighted oblivious transfer, which can be used for selling priced digital goods. Two implementations of it was proposed and analyzed. The protocol is especially useful when the prices of the items are not very large, or the prices of digital goods fall in very limited categories. In this way, the computation can be done most efficiently.

However, weighted oblivious transfer also suffers from shortcomings. We assume that subset problem is hard to compute, but sometimes it is possible (recall the example that the prices are $1, 2, \ldots, 2^n - 1$). And this shortcoming is unsolvable even when we apply the trusted third party. In addition, sometimes the whole price of the digital goods itself can leak part (not all) of the choices, which would also not be secure. Similarly, the problem exists even a trusted third party is employed. This additional asks the sender to be careful when assigning the prices for the goods.

I think that the above problems are unsolvable in current settings. However, the idea can be used for SPIR, when there are more than one servers. Further we may consider the implementation of adaptive queries in the future.
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