2D Extreme–Point Symmetric Mode Decomposition Method

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Abstract. This paper proposes a kind of 2D extension of Extreme–Point Symmetric Mode Decomposition Method. The 2D-sifting process is performed by 4 steps: 1 Extrema points detection (by neighboring window). 2 Delaunay triangulation is done over the grid of all extrema points. 3 Mid points of the triangulation are obtained. 4 Thin plate splines are used to interpolate the mid points. 5 Sifting processing is carried out. Finally, experiments are done with real images to show the efficiency of this 2D Extreme–Point Symmetric Mode Decomposition Method.

Keywords: Extreme-point Symmetric Mode Decomposition (ESMD); Intrinsic Mode Function (IMF); 2D EMD; Scattered Data Interpolation; Delaunay Triangulation.

1. Introduction
In recent years, the empirical mode decomposition (EMD) and HHT technique are developed to analyze time-frequency distribution of nonlinear and non-stationary data[1-4]. EMD method is a self-adaptive decomposition method which can decompose complicated signal into its intrinsic mode functions (IMFs) and provide well defined instantaneous and local frequency information about a signal. Recently, beside the widely discussed EMD method, some other similar and improved and sometimes more efficient methods are also developed. For example, by carrying forward the methodology of EMD, Jinliang Wang[5] developed an “extreme point symmetric mode decomposition (ESMD)” method which can be seen as a new alternative of EMD and proved to be more effective in many cases. In EMD, each mode function is supposed to be symmetric about its upper and lower envelopes interpolated by the local maxima and minima points. However, in ESMD, differing from the EMD approach, Dr. Wang et. al. choose other approaches to do the decomposition. The ESMD algorithm is designed to make each mode to be symmetric about its own maxima and minima points. Instead of constructing two outer envelopes, the ESMD sifting process is carried out by using inner curves which are the interpolation of the midpoints of the line segments connecting the local maxima and minima points. Experiments show that ESMD method is superior to EMD in many cases. Like the 2D extension of EMD method, in this paper, the authors carry on the spirit of 1-dimension ESMD method to put forward a kind of 2D extension of ESMD.

2. 2D ESMD Algorithm

2.1. Extreme Detection
In this section, the authors carry forward the 2D ESMD algorithm. The authors detect extrema points by using neighboring window method which employs a $3 \times 3$ window. In general, a $3 \times 3$ window can get an optimum extrema map for a given 2D data. In this process, the authors move the $3 \times 3$ window all over the 2D data points. If the pixel corresponding to the center of the window has a value
that is strictly higher than all of its neighboring pixels within the window, then it is identified as a local maximum; in the same way, if the same pixel has a value that is lower than all its neighboring pixels, then it is considered a local minimum. The authors deal with the corner and boundary pixels in the same way despite that each corner pixel has only three neighbors and each boundary pixel has only five neighbors to be considered.

**Stopping Criteria for Each Single IMF**
Similar to 1-dimension ESMD, the authors set up stop criterion for each single IMF. The sifting process for each IMF stops if \( D - L \) is done and \( |L| < E \) (E is a preset error) or the sifting times attain a preset maximum number K.

**Stopping Criteria for The Entire Decomposition**
In the 2D ESMD, the entire sifting process is supposed to stop at the moment that the number of the extreme points in the remaining data is less than 4. And the variance of the remaining data is calculated when each single IMF is subtracted. At last, select the remaining data \( R \) with the least variance as the last residual data which is actually an optimal AGM curved surface.

2.2. 2D-ESMD Algorithm
Denote the original 2D data (most times can be seen as an image) with \( A \), a 2D IMF with \( F \), and the final residue with \( R \). In the decomposition process, the \( i \)-th IMF \( F_i \) is obtained from its source image \( A_i \), where \( A_i = A_{i-1} - F_{i-1} \) and \( A_1 = A \). It usually requires many time iterations to obtain \( F_i \), where the temporary state of an IMF (TS-IMF) in \( j \)-th iteration can be denoted as \( FT_j \). Using the above-mentioned definitions, the algorithm of the 2D ESMD decomposition process can be summarized as below (see Figure 1 for reference).

**Step 1:** Set \( i = 1 \) and \( A_i = A \).

**Step 2:** Set \( j = 1 \) and \( FT_j = A_i \).

**Step 3:** Obtain the local maxima and minima points of \( FT_j \) denoted with \( MAX_j \) and \( MIN_j \) respectively, e.g., extreme detection. If the number of extreme points of \( MAX_j \) and \( MIN_j \) is less than 4, go to step 11.

**Step 4:** Do Delaunay triangulation \( DT_j \) over the \((MAX_j \cup MIN_j)\).

**Step 5:** Find the mid point for each triangle in \( DT_j \) and put all these mid points to form the set \( MID_j \).

(see the illustration in fig. 1, the points marked with circles are local minima points and the points marked with squares are local maxima points and pentagons are mid points).

**Step 6:** Check out if each triangle in \( DT_j \) is composed of either 3 points from \( MAX_j \) or 3 points from \( MIN_j \), if it is the case, drop its mid point from \( MID_j \).

**Step 7:** Interpolate \( MID_j \) using TPS to get interpolate curved surface \( MS_j \). (see fig 1, there can be 16 mind points for the 13 triangles and TPS Interpolate surface is therefore done through these points)

**Step 8:** Calculate \( FT_{j+1} \) where \( FT_{j+1} = FT_j - MS_j \).

**Step 9:** Check if \( FT_{j+1} \) follows the stopping criteria for each single IMF.

**Step 10:** If \( FT_{j+1} \) meets the the stopping criteria for each single IMF, then take \( F_i = FT_{j+1} \), set \( A_{i+1} = A_i - F_i \) and calculate the variance \( \sigma_{i+1}^2 \) of \( A_{i+1} \), and let \( i = i + 1 \); go to step 2. Otherwise, set \( j = j + 1 \), go to step 3 and continue up to step 10.

**Step 11:** The entire sifting process is complete. Select the remaining data with the least variance as the last AGM residual and the entire decomposition result is \( F_1, F_2, \ldots, F_k, \) and \( R \).
3. Numerical Experiments For 2D ESMD Method

In this section, the authors do numerical experiments to show the efficiency of 2D ESMD. 2D ESMD is used to decompose real hurricane image (see figure 2). The authors make the comparison between 2D ESMD and 2D EMD which was proposed by J.C.Nunes [6] with the above image and list the result in the Table1. One can see in terms of time cost and the variance of the remaining data, 2D ESMD is more efficient.

Table 1. Comparison between 2D ESMD and 2D EMD

|          | 2D ESMD |          | 2D EMD |
|----------|---------|----------|--------|
| Time Cost| 8 Min   | Time Cost| 15 Min |
| Variance | 78      | Variance | 92     |
4. Conclusion
2D ESMD is a kind of 2-dimension generalization of 1-dimension extreme point symmetric mode decomposition (ESMD) method. Compared with the current widely used 2D EMD, it has three advantages, first, 2D ESMD is faster in terms of time cost simply because 2D EMD compute only one interpolation surface while 2D EMD computes two. Second, 2D ESMD output optimal AGM remaining data which is more meaningful in image process. Third, the concept of extreme-point symmetry is wider than the envelop symmetry. From the viewpoint of material movement, the oscillation occurs around the equilibrium which may also shift during this process. So the extreme point symmetry actually reflects the local symmetry about itself. Thus the IMFs obtained through 2D ESMD reflect more precisely the real picture of the 2D data.

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