Ambivalence of the anisotropy of the vortex lattice in an anisotropic type-II superconductor.

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We present a geometry-based discussion of possible vortex configurations in the mixed state of anisotropic type-II superconductors. It is shown that, if energy considerations assign six nearest neighbors to each vortex, two distinct modifications of the vortex lattice are possible. It is expected that certain conditions lead to a first order phase transition from one modification of the vortex lattice to the other upon varying the external magnetic field.

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In anisotropic type-II superconductors the upper critical field $H_{c2}$, the magnetic-field penetration depth $\lambda$, and the coherence length $\xi$ depend on the direction with respect to crystallographic axes of the material and these dependencies may schematically be represented by ellipsoids. We consider the case where the mixed state is created by an external magnetic field directed along one of the principal axes of these ellipsoids, say the $y$-axis. To keep the discussion simple, the above mentioned quantities are assumed to be isotropic in the $xy$-plane and $H_{c2}^{(xy)} > H_{c2}^{(z)}$. We also assume that in the fully isotropic case, the energetically most favorable arrangement assigns six nearest neighbors to each vortex (triangular lattice).

We use the commonly accepted notation where the anisotropy is expressed as

$$\gamma = \frac{H_{c2}^{(xy)}}{H_{c2}^{(z)}} = \frac{\xi^{(xy)}}{\xi^{(z)}} = \frac{\lambda^{(z)}}{\lambda^{(xy)}}$$

(1)

In this case, $\lambda^{(z)}$ is the screening length corresponding to screening currents oriented along the $z$-axis. The anisotropies of $\lambda$ and $\xi$ in the plane perpendicular to the magnetic field, i.e., the $xz$-plane, may both be represented by ellipses with the same eccentricities and elongated along the $x$-axis, as is shown in Fig. 1.

In magnetic fields well below $H_{c2}$, it is the magnetic interaction between the vortices that provides the main contribution to the free energy of the mixed state. This means that the distances between one chosen vortex line and its six nearest neighbors should all be equal in units of $\lambda$, i.e., the nearest neighbors are situated along ellipses that reflect the anisotropy of $\lambda$. As was shown in Ref. 3, if $H < H_{c2}$ is oriented along the $xy$-plane, all such vortex arrangements have the same free energy, independent of the orientation of the triangular unit cell in the $xz$-plane. It is well known, however, that part of the free energy arises from the interaction of Abrikosov vortices with the crystal lattice. This is why we argue that only the two most symmetrical configurations, shown in Fig. 1, are of practical interest. These lattices differ by their orientations with respect to the $x$- and the $z$-axis. Although lattice 1 and lattice 2 appear as significantly different, both of them correspond to the same average vortex density and the distances between the central vortex line ($a$) and its nearest neighbors ($b$) are also the same in units of $\lambda$.

The vortex lattice arrangements that follow from the two configurations shown in Fig. 1 are displayed in Fig. 2. While the vortex lattice 1 is indeed strongly anisotropic and its geometrical appearance is as it might be expected intuitively, lattice 2, on the other hand, looks much less anisotropic. In real space the nearest neighbors of the vortex line $a$ are the vortices $b$ and $c$. These vortices form a pattern with a close to triangular symmetry, which is emphasized by the dashed-line ellipse in the bottom panel of Fig. 2. At the same time, if we consider the magnetic interaction between the vortices, the distances must be measured in units of $\lambda$ and in this renormalized space, the nearest neighbors of vortex $a$ are the vortices $b$ and $c$.

![FIG. 1: Two possible configurations of vortices for $H$ oriented along $y$-axis and assuming $\gamma = 3.5$. The small ellipses represent single vortices, their shape indicates the anisotropy of $\xi$.](image-url)
and \( b' \), as is illustrated by the solid-line ellipse in the bottom panel of Fig. 2. Thus, although both vortex lattices shown in Fig. 2 correspond to the same anisotropy of \( \lambda \), the anisotropies of the corresponding vortex lattices in real space are quite different. The real space anisotropy of vortex lattice 2 may be characterized by the eccentricity \( \varepsilon(2) \) of the ellipse drawn through the nearest neighbors of one chosen vortex line (dashed-line ellipse in Fig. 2). A simple calculation reveals that if for the vortex lattice 1, \( \varepsilon(1) = \gamma \), the eccentricity for the lattice 2 is \( \varepsilon(2) = \gamma/3 \). This means that, if, e.g., the original anisotropy is \( \gamma = 3 \), the vortex lattice 2 is perfectly isotropic in real space.

Although the two lattices shown in Fig. 2 are quite different in real space, in both cases the distances between the nearest neighbors in the renormalized space are the same in units of \( \lambda \). Furthermore, if Eq. (1) is exact, i.e., in frameworks of the applicability of the Ginzburg-Landau theory, the distances between the vortices are also equal in units of \( \xi \). This makes the free energies of the two configurations identical not only in low magnetic field limit but in the entire magnetic field range \( H_{c1} \leq H \leq H_{c2} \). In this case, minor contributions to the free energy depending, e.g., on the symmetry of the crystal lattice, may change the energy balance in favor of one or the other configuration. Our simple arguments cannot serve to answer the question, which of the two considered vortex lattices is stable. Depending on the particular chosen superconductor, any of the two may energetically be favored.

The situation is different if the Ginzburg-Landau theory is not quantitatively applicable, for instance, at temperatures well below \( T_c \). In such cases the anisotropies \( \gamma_{\xi} = \xi^{(xy)}/\xi^{(z)} \) and \( \gamma_{\lambda} = \lambda^{(z)}/\lambda^{(xy)} \) can be different. This circumstance changes the energy balance between the two considered vortex configurations in higher magnetic fields where overlapping of vortex cores has to be taken into account. It is easy to check that if \( \gamma_{\xi} > \gamma_{\lambda} \), lattice 1 has a lower free energy and vice versa. In the case of \( \gamma_{\xi} \neq \gamma_{\lambda} \), it is possible that the energy balance between the two vortex lattices depends on the applied magnetic field. If this is indeed the case, we expect a first order phase transition from one vortex configuration to the other with increasing magnetic field.

The main result of our consideration is that the symmetry of the vortex lattice does not provide unambiguous information about the anisotropy \( \gamma \) of the magnetic field penetration depth. It turns out that each vortex configuration is consistent with two unequal values of \( \gamma \), differing by a factor of 3. For instance, vortex lattice 1 equally well corresponds to \( \gamma = 3.5 \) and \( \gamma = 10.5 \), while lattice 2 reflects the anisotropies of 3.5 and 1.17, as it is shown by the solid and dashed-line ellipses in Fig. 2.

Recognizing the fact that the anisotropy of the vortex lattice in real space may be quite different from that of \( \lambda \) is rather important for the correct interpretation of experimental observations. With this in mind we recall the recent experimental observation of the vortex lattice in MgB\(_2\), invoked by magnetic fields oriented along the \( ab \)-planes of the hexagonal crystal lattice\(^4\). A hexagonal vortex lattice corresponding to an eccentricity \( \varepsilon = 1.19 \) was observed in these experiments. This value of \( \varepsilon \) is much smaller than the corresponding anisotropy \( \gamma \) of \( H_{c2} \). Although some theoretical justifications for the anisotropy of the magnetic field penetration depth \( \gamma_{H} = \lambda^{(z)}/\lambda^{(xy)} \) to be smaller than \( \gamma_{\xi} = H_{c2}^{(xy)}/H_{c2}^{(z)} = \xi^{(xy)}/\xi^{(z)} \) were offered in Ref.\(^4\), we argue that the observation of Ref.\(^4\) most likely reflects a \( \lambda \)-anisotropy \( \gamma_{\lambda} = 3\varepsilon \approx 3.6 \) (see the bottom panel of Fig. 2). This latter value is in fair agreement with the observed anisotropy of \( H_{c2} \).

In conclusion, we showed that for each value of the anisotropy of the magnetic field penetration depth \( \gamma_{\lambda} \) there are two possible arrangements of the hexagonal vortex lattice with corresponding eccentricities \( \varepsilon(1) = \gamma_{\lambda} \) and \( \varepsilon(2) = \gamma_{\lambda}/3 \). This is why an unambiguous determination of \( \gamma_{\lambda} \) from experimental observations of the vortex lattice is only possible if an approximate value of \( \gamma_{\lambda} \) is \textit{a priori} known.

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1. M. Tinkham, Introduction to Superconductivity, Ch. 9.2, McGraw-Hill, Inc., Singapore, 1996.
2. E. H. Brandt and U. Essmann, Phys. Status Solidi b 144,
3 L. J. Campbell, M. M. Doria, and V. G. Kogan, Phys. Rev. B 38, 2439 (1988).
4 M. R. Eskildsen, N. Jenkins, G. Levy, M. Kugler, Ø. Fischer, J. Jun, S. M. Kazakov, and J. Karpinski, preprint.
5 A. V. Sologubenko, J. Jun, S. M. Kazakov, J. Karpinski, and H. R. Ott, Phys. Rev. B 65, 180505 (2002).