Orbital migration and Resonance Offset of the Kepler-25 and K2-24 systems

C. Charalambous\textsuperscript{1}, X. S. Ramos\textsuperscript{1*}, P. Benítez-Llambay\textsuperscript{2}, and C. Beaugé\textsuperscript{1}

\textsuperscript{1} Instituto de Astronomía Teórica y Experimental, Observatorio Astronómico, Universidad Nacional de Córdoba, Laprida 854, X5000BGR, Córdoba, Argentina
\textsuperscript{2} Niels Bohr International Academy, The Niels Bohr Institute, Blegdamsvej 17, DK-2100, Copenhagen Ø, Denmark

E-mail: charalambous@oac.unc.edu.ar

Abstract. Based on the model described in [1], we present an analytical+numerical study of the resonance capture under Type-I migration for the Kepler-25 [2] and K2-24 [3] Kepler systems, both close to a 2/1 mean-motion resonance. We find that, depending on the flare index and the proximity to the central star, the average value of the period-ratio between two consecutive planets show a significant deviation with respect to the resonant nominal value, up to values well in agreement with the observations.

1. Introduction

Most of the planets found in Kepler multi-planetary systems lie outside mean-motion resonances (MMRs). This seems incompatible with a formation process strongly affected by planet migration and may either indicate that planets formed in-situ (e.g. [4–6]) or that planetary migration occurred in a turbulent environment, where resonance capture is not guaranteed (e.g. [7, 8]). However, the Kepler population also shows the existence of a statistically significant number of planetary pairs close to resonances, where the orbital period ratios is usually larger than the nominal value. It is generally believed that these systems are near-resonant and located outside the libration domain.

The origin of these near-resonant systems is a dilemma. Planetary migration in a turbulent disk [9] can lead to near-resonant configurations, although it is not clear whether this mechanism can reproduce the observed near-resonant distribution (e.g. [10]). This mechanism only seems to work for sub-Jovian bodies, and therefore, larger than the expected size for most of the Kepler systems.

In [1] we presented an analytical model that allow us to reproduce the general trend of the resonance offset, as the disk is assumed significantly flared and with a small scale height. This model strongly depends on the planetary masses, values which are not usually known for the Kepler systems, thus we performed Monte Carlo statistical analysis with no direct application to a given planetary system. Even so, the method has proved promising and able of reproducing the offset distribution around both the 2/1 and 3/2 MMRs, as well as predicting an increase in this value for planets closer to the central star.

From all the Kepler systems near the 2/1 MMR, just two cases have fairly credible estimations for the masses: Kepler-25 and K2-24, indicated as green and black circles in the left hand frame of Figure 1. Red circles correspond to systems detected by transits or Transit Time Variations (TTV). Bodies
Figure 1. **Left:** Distribution of orbital period ratios, in the vicinity of the 2/1 resonance, as a function of the orbital period of the inner planet. Red circles identify planets detected by transits or TTV, while those discovered by other methods are depicted with open circles. The green circle indicates the location of Kepler-25 while the black one corresponds to K2-24. Data was obtained from exoplanet.eu. **Right:** Dynamical map of max(Δe) for two-planet systems with $m_1 = 0.05m_{\text{Jup}}$ and $m_2 = 0.10m_{\text{Jup}}$ orbiting a central star of mass $m_0 = 1M_\odot$, in the vicinity of the 2/1 MMR. The orbit of the outer planet was initially circular with $a_2 = 1$ AU, and all the angular variables where chosen equal to zero. The black continuous line marks the location of the zero-amplitude ACR-type librational solutions, estimated from the simple analytical model (eq. (4)). White dots are the result of three N-body simulations of resonance trapping.

Discovered by any other method are identified by open circles. This distribution shows increasing $\Delta_{2/1}$ for planets closer to the star, indicating a possible smooth trend which, if confirmed, would indicate that the distribution found in different populations belong to the same functional form, and just differing from the distance to the star. It also shows little correlation with either the detection method or the stellar/planetary masses. Here, we apply the model of [1] to these two systems and attempt to constraint the properties of the protoplanetary disk that are consistent with their observed location and deviation from the exact resonance.

Right frame of Figure 1 shows a dynamical map for the 2/1 commensurability. We integrated series of two-planet systems with initial conditions in a grid defined in the $(P_2/P_1, e_1)$ plane and specifically chose $m_2/m_1 > 1$ to guarantee symmetric fixed points for the resonant angles (e.g. [11, 12]). The color code corresponds to the maximum value of $|e_1(t) - e_1(t = 0)|$ (denoted as max(Δe)) attained during 10$^3$ years integration time. Darked (lighter) tones are associated to small (large) variations in the eccentricity of the inner planet. Although this indicator does not measure chaotic motion, it is an important tool to probe the structure of resonances and identify the locus of stationary solutions (so-called ACR solutions, see [11, 13]). It also helps to identify the separatrix delimiting the librational from the circulation domains (e.g. [14]). The black line shows the approximate location of the family of zero-amplitude ACR solutions characterized by the simultaneous libration of the both resonant angles.

2. **The Ramos et al. Model**

In the following we will assume two planets of masses $m_1$ and $m_2$ orbiting a star $m_0$, orbital periods $P_1 < P_2$ and in the vicinity of a first-order $(p+1)/p$ MMR. We define the resonance offset $\Delta_{(p+1)/p}$ as

$$\Delta_{(p+1)/p} = \frac{P_2}{P_1} - \frac{(p + 1)}{p},$$

whose value indicates the distance from the exact resonance.
Different values of $\Delta_{(p+1)/p}$ are attained in different parts of the disk. In order to study this, we use a resonant Hamiltonian neglecting secular perturbations to estimate the resonance offset as function of $e_i$, as well as a relation between the eccentricities of both planets:

$$\Delta_{(p+1)/p} = C_1(\alpha) \frac{m_2}{m_0} \frac{1}{\epsilon_1} ; \quad e_2 = C_2(\alpha) \frac{m_1}{m_2} e_1,$$

(see [15, 16]), where the coefficients $C_i$ depends solely on $\alpha = a_1/a_2$ ($C_1 \simeq 1.5$ and $C_2 \simeq 0.29$). For a given resonance, very low $e_i$ are necessary to obtain a significant deviation from the exact resonance. However, since $e_i$ does not attain zero for the ACR solution, the singularity at $e_1 = 0$ is never reached.

If the disk-driven planetary migration is sufficiently slow and smooth, we expect the orbital evolution to follow the pericentric branch into the librational domain and exhibit low-amplitude oscillations of the resonant angles. In such an ideal scenario, the final eccentricities and resonant offset $\Delta_{(p+1)/p}$ will depend on the relative strength between the eccentricity damping and orbital migration timescales ([11, 17]) $\tau_{e_i}$ and $\tau_{a_i}$, respectively. Thus, the final outcome of a resonance trapping will depend on the ratios $K_i = \tau_{a_i}/\tau_{e_i}$, which we denote as the $K$-factors.

We performed 3 N-body simulations including an ad-hoc external acceleration (e.g. [18]), set the values of $r_{ao}$ at certain prefixed amounts, varied $K_i$ and analyzed its effects on the resonance offset (white dots in the right frame of Figure 1). We chose planetary masses $m_1 = 0.05m_{Jup}$ and $m_2 = 0.10m_{Jup}$, and $1M_\odot$ for the central star. The outer planet was initially at $a_2 = 1$ AU, in circular orbit and all angles equal to zero. The initial conditions were integrated for $10^5$ yrs. All values agree with the ACR loci given by expression (2) deduced from the analytical resonance model. Although these simulations indicate that is possible to obtain large values for the offset, they only appear attainable for $K$-factors of the order of $10^4$, much higher than predicted by linear models of Type-I disk-planet migration (e.g. [18–20]). In [1], we found that it is possible to overcome this problem assuming a significant flare for the disk (of the order of $f \simeq 0.25$) in addition to a relatively small value for the disk aspect ratio ($H_0 \simeq 0.03$). This combination, in addition to moderately low values for the mass ratios of the planets, generates large deviations from exact resonance even with $K$-factors of the order of $10^2$, well within the classical limits.

We assume a laminar disk with surface density $\Sigma(r) = \Sigma_0 r^{-\alpha}$ and aspect-ratio $H(r) = H_0 r^\beta$, where $r$ is the distance to the central star in astronomical units. We will consider $H_0$, $\alpha$ and $\beta$ as unknown parameters that will be estimated in accordance with the observed dynamical characteristics of the planetary systems.

Following [21] and [20], orbital migration and eccentricity damping timescales are approximated as

$$\tau_{a_i} = Q_a \frac{t_{wave}}{H_{r_i}^2} ; \quad \tau_{e_i} = Q_e \frac{t_{wave}}{0.780} ; \quad t_{wave} = \frac{m_0}{m_i} \frac{m_0}{\Sigma(a_i)a_i^2} \frac{H_{r_i}^4}{\Omega(a_i)}.$$

(3)

In these expressions, $H_{r_i}$ is the disk aspect-ratio in the position of each planet and $\Omega(a_i)$ their orbital frequency. $Q_a$ is a constant introduced by [22] in order to reproduce the eccentricity damping rates from hydro-dynamical simulations, while $Q_e = Q_a(\alpha)$ is a function of the surface density profile. Finally, $t_{wave}$ is the typical timescale of planetary migration.

Recently, the classical Type-I migration models were revised by [23], who considered the contribution of eccentricity damping to changes in the semimajor axis associated to (partial) conservation of the angular momentum. They found that the effective characteristic timescale for the orbital evolution should actually be given by $\tau_{a_{eff}} = (\tau_{a_i}^{-1} + 2\beta e_i^2 \tau_{e_i})^{-1}$, where $\tau_{a_i}$ and $\tau_{e_i}$ maintain the same form as equations (3) and $\beta$ is a factor that quantifies the fraction of the orbital angular momentum preserved during the migration. This modified migration timescale changes the K-factor, leading to a new “effective” form. This revised migration model, together with the analytical resonant Hamiltonian led in [1] to a relation between the disk properties and resonance offset in the form:

$$\Delta_{(p+1)/p}^2 = \frac{2}{D} \left( C_1 \frac{m_2}{m_0} \right)^2 \frac{\left( (1 - D)(1 + \beta)K_2 \left( C_2 \frac{m_1}{m_2} \right)^2 + (B + D\beta)K_1 \left( \frac{t_{a_i}}{\tau_{e_i}} \right) \right)}{1 - \left( \frac{t_{a_i}}{\tau_{e_i}} \right)}.$$

(4)
Figure 2. Results of Monte Carlo simulations for Kepler-25 (upper frames) and K2-24 (bottom panels) for different density slopes values. In the left we show $\alpha = 0.5$ and in the right $\alpha = 1.5$. Red colors indicate more possible pairs $(H_0, f)$ for an observed $\Delta_{2/1}$, than the blue values.

where the parameters $B$ and $D$ depend both on planetary mass ratio and the resonance under consideration

$$D = \frac{1}{(p + 1)} \left( 1 + \frac{a_1 m_2}{a_2 m_1} \right)^{-1}; \quad B = \frac{m_1 n_2 a_2}{m_2 n_1 a_1} + D.$$  \hspace{1cm} (5)

The importance of expression (4) lies in the fact that it shows that large values of the offset may be obtained if the denominator is sufficiently small, independently of the K-factors. Since the ratio $\tau_{a_2}/\tau_{a_1}$ depends on the planetary mass ratio (as well as on parameters $f$ and $\alpha$), it is possible to obtain values of $\Delta_{(p+1)/p}$ consistent with the observed planetary systems, as long as the parameters lie within certain values.

3. Application to Kepler-25 and K2-24 systems

Of 165 planetary pairs lying in the vicinity of the 2/1 resonance and with $P_1 \leq 100$ days, only in 18 cases have the masses of both bodies have been measured or estimated with some accuracy. Of these, only 10 have orbital period ratios in the interval $P_2/P_1 \in [2.0, 2.10]$, and may be thus cataloged as members of the (near)-resonant region. This number continues to decline as we note that 7 planetary pairs have at least one of its members with $m_i > 50m_{\oplus}$, more than sufficient to open a gap in the disk and to have
migrated following a Type-II scenario. Since our model is based on analytical prescriptions for laminar Type-I migration, these systems are beyond the scope of our work.

Of the three remaining candidates, HD 219134 have recently been questioned. The first reference to this system appears in [24], who analyzed a total of 98 nightly averaged RV observations obtained with HARPS-N and found evidence of 4 low-mass planets. The outer member of the (near)-resonant pair was not detected. Later, [25] analyzed a total of 276 RV Doppler measurements and identified a total of 6 planets in this system. [26] found a substantial periodicity in the RV data due to stellar rotation with a period of 22.8 days, a value practically equal to $P_1/2$. Although the authors do not believe there is sufficient evidence to rule out the existence of the inner planet, the amplitude generated by the planet in the RV signal may be affected by stellar rotation and thus, the mass deduced for $m_1$ could in fact be substantially lower.

This leaves us with two systems, Kepler-25 and K2-24. Table 1 gives the masses and orbital periods of both systems, together with the stellar masses and the respective standard deviations. Both systems have nominal mass ratios larger than unity (i.e. $m_2/m_1 > 1$) and are thus candidates for resonant trapping in the $2/1$ commensurability. We therefore proceeded to analyze whether the observed value of the resonance offset $\Delta_{2/1}$ could be achieved using our model with the assumption of a laminar flared disk.

Since the value of $\Delta_{2/1}$ is known, we can invert expression (4) to obtain explicitly the value of $H_0$ as function of the masses and the disk flare:

$$H_0^2 = \frac{2}{D \Delta_{\text{obs}}^2} \left( \frac{c_1 m_2}{m_0} \right)^2 \left[ \left( 1 - D(1 + \beta)K^2 \right) \left( C_2 \frac{m_2}{m_0} \right)^2 + (B + D\beta)K^2 \left( \frac{a_2}{a_1} \right) \right] \left( \frac{H_0}{H_0^*} \right)^2 \left( \frac{a_2}{a_1} \right)^{2f}, \tag{6}$$

where $\Delta_{\text{obs}}$ is the observed value of the offset and $K_i^* = 0.78(Q_a/Q_c)a_i^{-2f}$ are $H_0$-normalized expressions for the K-factors of each planet. If the planetary masses are known with even some accuracy, it is then possible to estimate relations between $f$ and $H_0$ leading to the observed values of the offset. Since uncertainties in these values may be significant, we chose a statistical Monte Carlo approach incorporating the errors in the masses into the calculation.

For each system we ran a series of 1000 sets of $(m_1, m_2)$ from a normal distribution with mean and variance as depicted from Table 1. From the values of each run, we then determined the distribution of values of $(H_0, f)$ according to (6), for fixed values of $\alpha$. Results are shown in color scales in Figure 2, where the top (bottom) panels correspond to Kepler-25 (KE-24), respectively. Left plots were drawn assuming $\alpha = 0.5$ while the right graphs are the results obtained considering $\alpha = 1.5$. As can be noted, outcomes appear only weakly dependent on the surface density profile, such that we have plotted only the extreme results, and not for $\alpha = 1$, as it is the interpolated plot between $\alpha = 0.5$ and $\alpha = 1.5$. The color code corresponds to the possibility of a pair $(H_0, f)$ to be a solution of equation (6). Blue colors mean low possibilities while red is associated to higher frequency in the outcomes.

For Kepler-25, most of the positive results are occur along a broad curve with low values of $H_0$ and large values of the disk flare $f$. This is consistent with the Monte-Carlo simulations presented in [1] for the overall near-resonant population and systems with under-determined masses. Notice that the large uncertainty in $m_1$ does not significantly affect the result and the loci of values of $(H_0, f)$ consistent with the observed offset remains fairly restricted.

Results for K2-24 appear less defined which implies that a wide range of disk parameters would lead to the observed resonance offset. In part this is due to the lower, and more easily achieved, value of $\Delta_{2/1}$

| System  | $m_1 [m_0]$   | $m_2 [m_0]$   | $P_1$ [d] | $P_2/P_1$ | $m_0[M_\odot]$ |
|---------|---------------|---------------|-----------|-----------|-----------------|
| Kepler-25 | 9.6 ± 4.2    | 24.6 ± 5.7    | 6.24      | 2.0390    | 1.22 ± 0.06    |
| K2-24   | 21.0 ± 5.4    | 27.0 ± 7.0    | 20.89     | 2.0284    | 1.12 ± 0.05    |
but also to particular planetary masses. On one hand, the individual masses are larger than for Kepler-25 which in itself leads to a wider resonance domain. On the other hand, the ratio of \(m_2/m_1\) is almost unity which, as seen from equation (4) also implies larger offset even for low flare values and/or large \(H_0\).

4. Conclusions
We present an application of the [1] model for resonance offset to two planetary systems (Kepler-25 and K2-24) close to a 2/1 MMR with significant observed offset and fairly established planetary masses. We find that the disk parameters necessary to explain the deviation from exact resonance under a laminar type-I migration are similar to those predicted by [1], namely a low disk scale-height and significant flare index. This agreement indicates that the proposed mechanism could indeed have played a dominant role in determining the observed distribution of (near)-resonant exoplanets.

Acknowledgments
This work has been supported by research grants from ANCyT, CONICET and Secyt-UNC. We are grateful to IA TE and CCAD (UNC) for extensive use of computational facilities.

References
[1] Ramos X S e a 2017 A&A 602 A101 (Preprint 1704.06459)
[2] Marcy G W e a 2014 ApJ S 210 20 (Preprint 1401.4195)
[3] Petigura E A e a 2016 ApJ 818 36 (Preprint 1511.04497)
[4] Hansen B M S and Murray N 2012 ApJ 751 158 (Preprint 1105.2050)
[5] Petrovich C, Malhotra R and Tremaine S 2013 ApJ 770 24 (Preprint 1211.5603)
[6] Chatterjee S and Tan J C 2014 ApJ 780 53 (Preprint 1306.0576)
[7] Rein H 2012 MNRAS 427 L21–4 (Preprint 1208.3583)
[8] Baruteau C and Papaloizou J C B 2013 ApJ 778 7 (Preprint 1301.0779)
[9] Paardekooper S J, Rein H and Kley W 2013 MNRAS 434 3018–29 (Preprint 1304.4762)
[10] Quillen A C, Bodman E and Moore A 2013 MNRAS 435 2256–67 (Preprint 1304.6124)
[11] Beaugé C, Michchentko T A and Ferraz-Mello S 2006 MNRAS 365 1160–70 (Preprint astro-ph/0404166)
[12] Michchentko T A, Beaugé C and Ferraz-Mello S 2008 MNRAS 391 215–27
[13] Beaugé C, Ferraz-Mello S and Michchentko T A 2003 ApJ 593 1124–33 (Preprint astro-ph/0210577)
[14] Ramos X S, Correa-Otto J A and Beaugé C 2015 CMDA 123 453–79 (Preprint 1509.03607)
[15] Ferraz-Mello S 1988 AJ 96 400–408
[16] Lee M H 2004 ApJ 611 517–527
[17] Lee M H and Peale S J 2002 ApJ 567 596–609
[18] Cresswell P and Nelson R P 2008 A&A 482 677–90 (Preprint 0811.4322)
[19] Papaloizou J C B and Larwood J D 2000 MNRAS 315 823–833
[20] Tanaka H and Ward W R 2004 ApJ 602 388–95
[21] Tanaka H, Takeuchi T and Ward W R 2002 ApJ 565 1257–74
[22] Cresswell P and Nelson R P 2006 A&A 450 833–53
[23] Goldreich P and Schlichting H E 2014 AJ 147 32 (Preprint 1308.4688)
[24] Motalebi F e a 2015 A&A 584 A72 (Preprint 1507.08532)
[25] Vogt S S e a 2015 ApJ 814 12 (Preprint 1509.07912)
[26] Johnson e a 2016 ArXiv e-prints (Preprint 1602.05200)