Machine learning based iterative learning control for non-repetitive time-varying systems

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Abstract

The repetitive tracking task for time-varying systems (TVSs) with non-repetitive time-varying parameters, which is also called non-repetitive TVSs, is realized in this paper using iterative learning control (ILC). A machine learning (ML) based nominal model update mechanism, which utilizes the linear regression technique to update the nominal model at each ILC trial only using the current trial information, is proposed for non-repetitive TVSs in order to enhance the ILC performance. Given that the ML mechanism forces the model uncertainties to remain within the ILC robust tolerance, an ILC update law is proposed to deal with non-repetitive TVSs. How to tune parameters inside ML and ILC algorithms to achieve the desired aggregate performance is also provided. The robustness and reliability of the proposed method are verified by simulations. Comparison with current state-of-the-art demonstrates its superior control performance in terms of controlling precision. This paper broadens ILC applications from time-invariant systems to non-repetitive TVSs, adopts ML regression technique to estimate non-repetitive time-varying parameters between two ILC trials and proposes a detailed parameter tuning mechanism to achieve desired performance, which are the main contributions.

Key words: Iterative learning control; machine learning; linear regression; parameter estimation; time-varying systems.

1 Introduction

In recent years, a large portion of human power in manufacturing work has been replaced by robots. Various control methods were proposed in the literature to control these robotic systems. However, their accuracy level might not be within the tolerance for some particular applications, e.g., laser cutting. Iterative learning control (ILC) is first developed in Arimoto, Kawamura, and Miyazaki (1984) to improve the performance of systems executing tasks repetitively. Since the task specifications are the same for each trial, ILC can iteratively update its input signal to gradually reduce the tracking error by learning from the previous trials. ILC has a wide range of applications, such as, robotic manipulator (Norrlof, 2002), wafer stage (Oomen & Rojas, 2017), to name a few. Please see Bristow, Tharayil, and Alleyne (2006) for a background overview of ILC in detail.

The research on classical ILC has confirmed its ability of theoretically reducing the error to zero after enough executions of repetitive tracking tasks (RTTs) even when there exists a certain degree of model uncertainties. However, classical ILC naturally has six design postulates, i.e., fixed reference, fixed trial length, fixed initial state, fixed system dynamics, fixed input update law and system stability, which restrict its potential application range. To broaden the application range and enable more design freedom, research has been conducted to remove some design postulates to allow variable reference (Chen, Chu, & Freeman, 2020; Chi, Hui, Huang, Hou, & Bu, 2020; Freeman, Cai, Rogers, & Lewin, 2010; Hoelzle & Barton, 2016; Jin, 2018), variable initial state (Chi, Hou, & Xu, 2008), variable trial length (Jin, 2020; Li, Xu, & Huang, 2014; Shen, Zhang, & J.-X, 2016) and nominal model update law (Steinhauser & Swevers, 2018). However, limited work has been devoted on time-varying systems (TVSs) with non-repetitive time-varying parameters at each trial, which are called non-repetitive TVSs (NTVSs). Furthermore, there is no work which has ever attempted to eliminate the design postulate on the invariance of system dynamics along trials. Although some ILC works, e.g., Hao, Liu, and Zhou (2019), can handle RTTs with...
TVSs, the dynamics of TVSs, i.e., the input and output relationship, are time-invariant but repetitive over the trials. In addition, Meng and Moore (2016) and Altun, Willems, Oomen, and Barton (2017) can show the robustness of ILC algorithms for trial-varying systems, but have to assume the boundness of system parameters. However, the change of system dynamics of TVSs will accumulate along the trials, and possibly make the system dynamics diverge significantly from the constant nominal model. This issue significantly violates the robust convergence condition requiring bounded system parameters in classical ILC design, as it will cause unbounded model uncertainties that cannot be handled by ILC techniques in Altun et al. (2017); Meng and Moore (2016). The limitation of existing works motivates the design objective of this paper to create a new mechanism to remove the ILC design postulate on invariant system dynamics, i.e., develop ILC mechanisms eligible for TVSs.

It is worth noting that TVSs also appear in practice. One example involving such systems is described in Freeman (2016) as the stroke rehabilitation process, in which the arm response rate of a patient gradually reduces along the time horizon due to muscle fatigue. Moreover, it is reported in Kim (2010) that capacity fade phenomenon happens within lithium battery applications, which slowly affects the system dynamics. For these TVSs, it is not appropriate to apply existing ILC algorithms with fixed nominal models, since significant model uncertainties may be accumulated along the trials and hence, may violate their robust convergence performance.

To achieve this objective, the nominal model for TVSs is introduced and will be updated at each trial to adapt to the changing trend of TVSs, such that the model uncertainties remain relatively small. Note that the classical system identification methods, e.g., frequency response method and least square method, mainly focus on figuring out the model of a time-invariant system via trial and error. It is generally hard to characterize an NTVS using these methods due to its time-varying system dynamics. Therefore, the idea in this paper is to utilize a linear regression technique from machine learning (ML), which is a data-driven technique that extracts meaningful information from historical data and replicates the input and output relationship, to update the nominal model to approach the NTVS.

Most existing well-known ML application areas, e.g., computer vision (Krizhevsky, Sutskever, & Hinton, 2012) and fault diagnosis (Tao, Wang, Chen, Stojanovic, & Yang, 2020), are related to classification problems. This paper adapts ML to address a regression problem to identify the TVS model. The time-varying regression problem for system parameter estimation has been studied in the literature using various approaches. For example, kernel adaptive filtering in Vaerenbergh, Lazaro-Gredilla, and Santamaria (2012) estimates parameters in an adaptive sense. However, no historical information is used and the on-line computing load is relatively high, which may cause slow response to dynamic changes. As a result, compared to traditional time-varying regression methods, since nominal model changes during each trial for TVSs, an off-line data driven method, i.e., ML using historical data between two consecutive ILC trials (see Fig. 1), is proposed.

In this paper, an ILC framework is proposed to achieve high performance of RTTs for TVSs. Within this framework its nominal TVS model is updated by ML at each trial, as shown in Fig. 1. Specifically, at the end of the current trial, the system state should be initialized to the same initial value (e.g., \( x_0 \) in (1) in this paper) for the next trial. This initialization period, firstly, ML is performed to obtain an appropriate nominal model which best describes the NTVS at the current trial. With the assumption that the NTVS dynamics between adjacent trials does not change dramatically, the information of above calculated nominal model, current trial ILC trajectory tracking error and current trial input signal will be used in our proposed optimal ILC update law to compute the input signal for the next trial. By using this newly proposed ML nominal model update mechanism with the above assumption, the model uncertainties can stay within an acceptable tolerated bound according to the ILC robust convergence condition along the trials. As a consequence, our proposed ILC update law can make the tracking error of TVSs converge to a relatively small range. To authors’ best knowledge, this is the first work to propose ILC algorithms to deal with NTVSs. The main contributions are listed as follows:

- The ILC design postulate on the invariance of system dynamics is broken, thus, the ILC application range is further broadened to handle the control tasks for TVSs while works in Altun et al. (2017); Hao et al. (2019); Meng and Moore (2016) cannot.
- The non-repetitive time-varying parameters at each trial are estimated off-line by an ML mechanism during the state initialization process, thus keeping the model uncertainties bounded and significantly reduces the demand of on-line computation load during task executions.
- How to tune ML/ILC parameters to achieve desired performance is provided.

![Fig. 1. The proposed algorithm work flow diagram.](image-url)
2 Preliminaries

Notation: \(\mathbb{N}\) is the set of non-negative integers; \(\mathbb{R}^n\) and \(\mathbb{R}^{n \times m}\) denote the sets of \(n\) dimensional real vectors and \(n \times m\) real matrices respectively; \(L^2([a, b])\) denotes the space of \(\mathbb{R}^n\) valued Lebesgue square-summable sequences defined on an interval \([a, b]; \langle x, y \rangle\) is the inner product of \(x\) and \(y\) in some Hilbert space. \(\text{diag}(\cdot)\) is a diagonal matrix. \(\partial G_k = G_{k+1} - G_k\) denotes the change of a given trial-variant matrix \(G_k\) between adjacent trials.

2.1 Knowledge for the general ILC problem

An \(\ell\) input and \(m\) output discrete time linear system defined on a finite time interval \([0, N]\) has a state space

\[
x(t + 1) = Ax(t) + Bu(t), \quad x(0) = x_0,
\]

where \(x(t) \in \mathbb{R}^n\) is the state at time \(t\), \(u(t) \in \mathbb{R}^\ell\) is the input signal at time \(t\), and \(y(t) \in \mathbb{R}^m\) is the output/observed signal. Matrices \(A, B\) and \(C\) are of appropriate dimensions. The set of output signals \(y = [y(1), y(2), \ldots, y(N)]^T \in \mathbb{R}^{mN}\) as a function of the set of input signals \(u = [u(0), u(1), \ldots, u(N-1)]^T \in \mathbb{R}^{\ell N}\) and the initial state \(x_0\) is written in a matrix form as

\[
y = Gu + Hx_0.
\]

The system matrix \(G \in \mathbb{R}^{mN \times \ell N}\) and the initial condition matrix \(H \in \mathbb{R}^{mN \times n}\) are naturally derived from the model (1) as

\[
G = \begin{bmatrix}
CB & 0 & \cdots & 0 \\
CAB & CB & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
CAN^{-1}B & CAN^{-2}B & \cdots & CB
\end{bmatrix}, \quad H = \begin{bmatrix}
CA \\
CA^2 \\
\vdots \\
CA^N
\end{bmatrix}.
\]

The input and output Hilbert spaces \(\mathbb{R}^{\ell N}\) and \(\mathbb{R}^m\) are defined using induced norms as follows:

\[
\|u\|_R^2 = \sum_{i=0}^{N-1} u^T(i)Ru(i), \quad \|y\|_Q^2 = \sum_{i=1}^{N} y^T(i)Qy(i),
\]

where \(u \in \mathbb{R}^{\ell N}\) and \(y \in \mathbb{R}^{mN}\) can be any signals in the input and output spaces; the weighting matrices \(R \in \mathbb{R}^{\ell \times \ell}\) and \(Q \in \mathbb{R}^{m \times m}\) are positive definite (not necessarily symmetric).

The classical ILC design objective proposed in Arimoto et al. (1984) is to find an iterative update law \(u_{k+1} = F(u_k, e_k)\), starting from the initial input signal \(u_0\), such that \(u_k\) converges to a unique value and \(y_k\) converges to a predefined reference \(r \in \mathbb{R}^m\), i.e.,

\[
\lim_{k \to \infty} u_k = u^*, \quad \lim_{k \to \infty} y_k = y^* = r,
\]

where the ILC tracking error at the \(k^{th}\) trial is

\[
e_k = r - y_k.
\]
where the system matrix $G_k \in \mathbb{R}^{mN \times tN}$ and the initial condition matrix $H_k \in \mathbb{R}^{mN \times n}$ can be derived based on (3), and $u_k = [u_k(0), u_k(1), \ldots, u_k(N - 1)]^T \in \mathbb{R}^{tN}$, $y_k = [y_k(1), y_k(2), \ldots, y_k(N)]^T \in \mathbb{R}^{mN}$.

**Remark 1** In classical ILC, the initial state $x_0$ is an unchanged constant value for each trial, and can be absorbed into the reference profile $r$ (in Eq. (5)) to give $x_0 = 0$. However, the initial effort $H_kx_0$ has to be considered into the control design phase in this paper as the plant (10) varies for each trial. The incorporation of $H_kx_0$ explicitly increases the control design complexity comparing to classical ILC framework.

### 3.2 ILC design framework and problem formulation

Since the TVS dynamics (10) is non-repetitive and time-varying ($G_k$ and $H_k$ vary along the trials), but the reference to be tracked is a constant because of the case for ILC, so the input (constructed by the system dynamics) must change at each trial to track this constant reference as the system dynamics change. The asymptotic convergence of input $u_k$ means input $u_k$ will not change at last, which means the reference cannot be tracked. Therefore, the input and output signals in (5) cannot converge at the same time, and the asymptotic convergence design objective of $u_k$ has to be removed. In fact, $u_k$ should be adaptive to the TVS in order to achieve the perfect tracking of $r$ at each trial, i.e., $G_ku_k + H_kx_0 = y_k = r$ as $k \to \infty$ from (11) and (5). To achieve that, a trial-variant nominal model is considered as

$$
\begin{align*}
\hat{x}_k(t + 1) &= \hat{A}_k\hat{x}_k(t) + \hat{B}_k\hat{u}_k(t), \quad \hat{x}_k(0) = x_0, \\
\hat{y}_k(t) &= \hat{C}_k\hat{x}_k(t),
\end{align*}
$$

where the symbol $\hat{\cdot}$ associates with the nominal model, and $\hat{x}_k$ and $\hat{y}_k$ are the numerically computed state and output using (12) at trial $k$. Matrices $\hat{A}_k$, $\hat{B}_k$ and $\hat{C}_k$ are constant and do not change during each single ILC trial. Hence this model has the matrix form

$$
\hat{y}_k = \hat{G}_k\hat{u}_k + \hat{H}_kx_0,
$$

where $\hat{y}_k = [\hat{y}_k(1), \hat{y}_k(2), \ldots, \hat{y}_k(N)]^T \in \mathbb{R}^{mN}$. The analytic expressions of corresponding matrices $\hat{G}_k \in \mathbb{R}^{mN \times tN}$ and $\hat{H}_k \in \mathbb{R}^{mN \times n}$ are derived from the nominal state space model (12) as in (3).

Using $\hat{G}_k$ and $\hat{H}_k$, a model-based ILC update law

$$
u_{k+1} = \mathcal{F}(u_k, e_k, \hat{G}_k, \hat{H}_k),
$$

can be obtained as a unified function at each trial. As $\hat{A}_k$, $\hat{B}_k$ and $\hat{C}_k$ are constant during each trial, i.e., $\hat{G}_k \neq G_k$ and $\hat{H}_k \neq H_k$, the nominal model does not perfectly match the TVS during each trial. However, ILC inherently incorporates robustness against certain level of model uncertainties

$$
\Delta_k^G = G_k - \hat{G}_k, \quad \Delta_k^H = H_k - \hat{H}_k.
$$

In other words, the ILC convergence can be still achieved if the model uncertainties $\Delta_k^G$ and $\Delta_k^H$ are within given ranges.

**Problem 1** ILC Design Problem for NTVSs aims at designing an ILC update law (14) as well as a nominal model update method to update $u_k$, $\hat{G}_k$ and $\hat{H}_k$ at each trial from initial values $u_0$, $\hat{G}_0$ and $\hat{H}_0$, such that there exists a sufficiently small positive scalar $\varepsilon$ and the tracking error satisfies

$$
\lim_{k \to \infty} e_k = e^* \leq \varepsilon.
$$

Due to the existence of NTVS dynamics with which classical ILC cannot deal, the design objective does not require the error to converge to zero, but still makes sense in practice as it contains the error to a relatively small bounded interval.

**Assumption 1** There exist sufficiently small non-negative scalars $\kappa^G, \kappa^H$ and the change of the NTVS dynamics between adjacent trials satisfies $\|\partial G_k\| \leq \kappa^G$, and $\|\partial H_k\| \leq \kappa^H$.

Assumption 1 claims that the change of system dynamics, $\partial G_k$ and $\partial H_k$, between two adjacent trials should not be too large, i.e., the system dynamics change smoothly along the trials.

This paper adopts the ML linear regression technique for updating the nominal model parameters to prevent the occurrence of unbounded model uncertainties for NTVSs while performing RTTs using ILC.

### 4 ILC update law for NTVSs

In this section, to address Problem 1, a modified norm optimization problem is given. A feedforward ILC update law shown in Fig. 2 is proposed to solve the above problem optimally. Moreover, the robust convergence performance of the above law is proposed in an explicit mathematical form to guarantee an upper bound on tracking error.
At the end of trial $k$, $u_{k+1}$ should be updated using (14) for the next trial. Inspired by Amann (1996), the authors optimize the following performance index

$$
\|u_{k+1} - u_k\|_R^2 + \|e_{k+1}\|_Q^2,
$$

(17)

which is the combination of the tracking error and the input difference between adjacent trials (trial $k$ and trial $k + 1$), which aims at minimizing the error for the next trial with a minimum change of input signal, thus maintaining robustness. The values of $R$ and $Q$ represent the weightings of robustness and error reduction. Using the nominal model (13), $e_{k+1}$ in (17) can be numerically approached as $r - \hat{G}_{k+1}u_{k+1} - \hat{H}_{k+1}x_0$. To formulate an optimization problem, the variable $u$ is used to denote any feasible option for $u_{k+1}$, which gives rise to

$$
\min_u \left\{ \|u - u_k\|_R^2 + \|r - \hat{G}_{k+1}u - \hat{H}_{k+1}x_0\|_Q^2 \right\}. \tag{18}
$$

Problem (18) can be solved to yield appropriate ILC update laws in form of (14).

In the following of this section, the authors assume knowing the nominal model (12) which will be estimated in Section 5 by the ML technique.

4.1 Feedforward ILC update law

The problem (18) has an analytical solution, which is shown in the following lemma.

**Lemma 1** The solution of problem (18) is

$$
u_{k+1} = u_k + \hat{G}_{k+1}(I + \hat{G}_{k+1}\hat{G}_{k+1}^*)^{-1} \left( \dot{e}_k - \partial \hat{G}_k u_k - \partial \hat{H}_k x_0 \right), \tag{19}
$$

where $\dot{e}_k = r - \hat{y}_k$, $\partial \hat{G}_k = \hat{G}_{k+1} - \hat{G}_k$ and $\partial \hat{H}_k = \hat{H}_{k+1} - \hat{H}_k$. The matrix $\hat{G}_{k+1}^*$ is the adjoint of $\hat{G}_{k+1}$ and it maps $z \in \mathbb{R}^{mN}$ to $v \in \mathbb{R}^{tN}$ with analytic expression

$$v(t) = R^{-1}\hat{B}_k^Tp_{k+1}(t), \tag{20}
$$

$$p_{k+1}(t) = A_T^{k+1}p_{k+1}(t + 1) + \hat{G}_{k+1}^*Qz(t + 1), \tag{21}
$$

where $p_{k+1}(t)$ is computed in reverse time in (21) for $t \in [0, N]$ with the boundary condition $p_{k+1}(N) = 0$.

**PROOF.** See Appendix A.

Note that $\dot{e}_k$ in (19) is the numerically computed error at trial $k$ using the nominal model (13), and should be replaced by the measured error $e_k$ in practical implementation to embed real system information of RTTs for robustness, as shown in the proposition below.

**Proposition 1** The problem (18) gives rise to the feedforward ILC update law

$$
u_{k+1} = u_k + Le_k, \tag{22}
$$

with the learning operator $L : \mathbb{R}^{tN} \to \mathbb{R}^{mN}$ is denoted as

$$L e_k = \hat{G}_{k+1}^*(I + \hat{G}_{k+1}\hat{G}_{k+1}^*)^{-1}(e_k - \partial \hat{G}_k u_k - \partial \hat{H}_k x_0). \tag{23}
$$

**PROOF.** The numerical computed error $\dot{e}_k$ in (19) is replaced by the measured error $e_k$ to yield (22).

The solution (19) in Lemma 1 purely depends on the nominal model information and does not employ any real system information, which lacks robustness. To address this issue, Proposition 1 introduces real system information $e_k$ to provide the feedforward ILC update law (22). Compared to the standard norm optimal ILC update law in Amann (1996), the extra term $(\partial \hat{G}_k u_k + \partial \hat{H}_k x_0)$ in (22) is used as an offset to adapt the change of system dynamics, $\partial \hat{G}_k$ and $\partial \hat{H}_k$, between adjacent trials (trial $k$ and trial $k + 1$).

4.2 Analysis on robust convergence performance

The robust convergence performance of the feedforward ILC law (22) in Proposition 1 is proposed as follows.

**Theorem 1** Under Assumption 1, if $\hat{G}_{k+1}$ has full row rank, the model uncertainty $\Delta^G_{k+1}$ and adjoint $\hat{G}^*_{k+1}$ satisfy

$$
\|I - \Delta^G_{k+1}\hat{G}^*_{k+1}\| < 1, \tag{24}
$$

and the current trial error $e_k$ satisfies

$$
\|e_k\| \geq \{ \rho_{k+1} \|G\| + (\rho_{k+1} + 1) \|\partial \Delta^G_{k+1}\| \|u_k\| \}
$$

$$
+ \{ \rho_{k+1} \|H\| + (\rho_{k+1} + 1) \|\partial \Delta^H_{k+1}\| \|x_0\| \}/(1 - \rho_{k+1}), \tag{25}
$$

where $\rho_{k+1}$ is the spectral radius of $(I + \hat{G}_{k+1}\hat{G}^*_{k+1})^{-1}$, then, the ILC update law (22) monotonically reduces the error for the next trial, i.e.,

$$
\|e_{k+1}\| < \|e_k\|. \tag{26}
$$

**PROOF.** See Appendix B.

Theorem 1 confirms that Proposition 1 can still reduce the tracking error between adjacent trials even when there exist model uncertainties and the system dynamics vary at each trial. Since the NTVS dynamics vary between adjacent trials, the tracking error is not guaranteed to converge to zero for NTVS as that for classical ILC even if the model uncertainties are zero. The upper bound of the tracking error is illustrated as the right hand side of the inequality (25). One can see that (24) is a condition for the validity of Theorem 1. It is worth noting that this condition also appears in Owens and Chu (2014); Owens, Freeman, and Chu (2013, 2014).
Remark 2 The ILC update law in Amann (1996) can be included into (22) in this paper as a special case where the system dynamics are repetitive and the nominal model is not updated over the trials, i.e., $\kappa^G = \kappa^H = \partial \Delta_k^G = \partial \Delta_k^H = 0$. In this case, the condition (25) is always satisfied, and the tracking error can theoretically converge to zero if (24) holds.

Remark 3 In (24) and (25), the only tuning variables are the weighting matrices $R$ and $Q$ (affecting $G_k^+$ and $p_k+1$) and the model uncertainties $\Delta_k^G$ and $\Delta_k^H$. From (24), if the model uncertainties are kept to a relative small bound, the spectral radius of $Q/R$ is allowed for a large value giving rise to a small value of $p_k+1$, and the error upper bound (the right hand side of (25)) becomes small enough to satisfy (16). The comparison simulations about different values of $Q/R$ is also presented in Section 7.2.

5 ML estimating mechanism for NTVSs
In the previous section, the authors assume knowing the trial-varying nominal model (12). Therefore, it is targeted to realize this assumption in this section.

5.1 Description of the ML scenario

ML is applied to produce an accurate estimate $\hat{\sigma}_k$ for the non-repetitive time-varying parameters in $A_k$, $B_k$ and $C_k$ in (10) to reduce the model uncertainties. At the end of each trial, ML analyzes the relationship among the input $u_k$, output $y_k$ and state $x_k$ to update the nominal model for the next trial by creating a data set $\varphi_k = (\varphi_k^1, \varphi_k^2, \ldots, \varphi_k^N)$. This data set can be obtained by recording the input, output and state values at each sample time index along an ILC trial, which generates the elements

$$\varphi_k^i = (x_k(i), x_k(i-1), u_k(i-1), y_k(i-1)), \quad (27)$$

for $i = 1, \ldots, N$, to update the nominal model for the next trial. Using this data set, the ML problem is defined as below.

Definition 1 ML based Nominal Model Update Problem: choose an initial estimate $\hat{\sigma}^0_{k+1}$ and then find an iteratively update procedure of $\hat{\sigma}_{k+1}$ to minimize the loss function

$$L(\hat{\sigma}_{k+1}, \varphi_k^i), \quad i = 1, \ldots, N, \quad (28)$$

where $N$ is the finite trial length from (10) and the definition of $\hat{\sigma}$ is in Sec. 2.2.

In ML, the design objective is to make the loss (28) be reduced to a sufficiently small value. This objective makes sense in practice, since the nominal model can generate almost the same control performance as the real plant when the loss is relatively small.

5.2 Gradient descent ML update mechanism

The gradient descent method is used to update the estimate $\hat{\sigma}_{k+1}$. Specifically, at current ILC trial $k$, for each training epoch $j$, each element $\varphi_{k+1}^j \in \varphi_k^{train}$, $i = 1, \ldots, \eta$, updates the estimate $\hat{\sigma}_{k+1}$ using

$$\hat{\sigma}_{k+1}^j = \hat{\sigma}_{k+1}^j - \gamma \nabla \hat{\sigma}_{k+1}^j \cdot L(\hat{\sigma}_{k+1}^j, \varphi_k^i), \quad (29)$$

such that the loss with respect to $\varphi_k^i$ is reduced, i.e.,

$$L(\hat{\sigma}_{k+1}^j, \varphi_k^i) \leq L(\hat{\sigma}_{k+1}^{j-1}, \varphi_k^i), \quad (30)$$

where $\gamma$ is the learning rate (to be specified later), $\hat{\sigma}$ is the index number of the gradient descent update at each training epoch and $\nabla \hat{\sigma}_{k+1}^j \cdot L(\hat{\sigma}_{k+1}^j, \varphi_k^i)$ contains the scaled partial derivatives of (28) defined as

$$\lim_{\delta \to \infty} \frac{|\hat{\sigma}_{k+1}^0(z)|}{|\hat{\sigma}_{k+1}^0(z) - L(\hat{\sigma}_{k+1}^j, \varphi_k^i)|/\delta}, \quad (31)$$

where $|\hat{\sigma}_{k+1}^0(z)|$ is the magnitude of the $z$th initial estimated parameter. It is used here to rescale the magnitudes of the different parameters to a certain range, so that the convergence rates of all parameters can be similar with each other, which improves the convergence performance. When $j = \eta j_{max}$, $\hat{\sigma}_{k+1}^j$ becomes $\hat{\sigma}_{k+1}$, which will be used in ILC law (22) to update the input signal $u_{k+1}$ of trial $k + 1$.

Remark 4 Although the obtained estimate $\hat{\sigma}_{k+1}$ is a direct approximate of the non-repetitive parameters $\sigma_k$ at the current trial, it can still approach $\sigma_{k+1}$ at the next trial if the change rate of $\sigma_k$ between the adjacent trials is not too fast (see time flow shown in Fig. 1). Also, the total training loss $\sum_{i=1}^{\eta} \bar{L}(\hat{\sigma}_k, \varphi_i^i)$ where $\bar{L}(\hat{\sigma}_k, \varphi_i^i)$ is defined in (7) and the total testing loss (8) do not converge to zero, since there always exists a certain level of model mismatch between the nominal model and the NTVS.

The ML update procedure is performed during the state initialization step of each trial, since it aims at making the model uncertainties small enough to guarantee the tracking error converging to its upper bound as shown in (25) of Theorem 1. As the technique of ML is closely related to computational statistics, there does not exist a general theoretical convergence proof for ML as for the control theory, e.g., ILC. The testing data in $\varphi_k^{test}$ is employed to evaluate the training performance for cross-validation.
6 Proposed algorithm

In this section, the ILC update law (22) with the ML nominal model update mechanism (29) are combined to yield a comprehensive algorithm in Sec. 6.1. How to tune the whole algorithm parameters is also discussed in Sec. 6.2.

6.1 Algorithm description

The proposed ILC for NTVSs algorithm is summarized in Algorithm 1 of which the work flow is shown in Fig. 1. Specifically, at the end of trial \( k \), the system state is initialized to \( x_0 \) for RTTs, where \( x_0 \) is defined in (10). Meanwhile, Algorithm 1 fully utilizes this time period to perform the ML training procedure (i.e., Algorithm 2) to estimate the non-repetitive time-varying parameters using the data recorded at trial \( k \), which is completely off-line. Algorithm 2 then provides ILC with a reliable estimate of the system dynamics for trial \( k + 1 \) to guarantee the desired performance of RTTs within NTVSs. Algorithm 1 can be applied efficiently on physical systems as it does not require a high on-line computational load.

Algorithm 1 ILC for NTVSs

1: Input: Initial estimate \( \hat{x}_0 \), initial input \( u_0 \), predefined reference \( r \), training set number \( \eta \), the total testing loss precision \( \varepsilon > 0 \) and \( k_{\text{max}} \).
2: initialization: Set \( k = 0 \).
3: Run \( u_0 \) to the system plant (10) and record the output trajectory \( y_0 \) as well as the error \( e_0 \) to construct data \( \varphi_0 \).
4: while \( k \leq k_{\text{max}} \) do
5: ML Algorithm 2 using \( \eta, \varepsilon, \sigma_k \) and \( \varphi_k \) to obtain \( \hat{x}_{k+1} \).
6: Apply feedforward ILC law (22) to obtain \( u_{k+1} \) using \( \sigma_k \) and \( \varphi_k \).
7: Apply \( u_{k+1} \) to the system (10); then record \( y_{k+1} \) and \( e_{k+1} \) to construct data \( \varphi_{k+1} \).
8: \( k \leftarrow k + 1 \)
9: end while

Algorithm 2 Gradient Descent ML

1: Input: Previous estimate \( \hat{x}_k \), training set number \( \eta \), the total testing loss precision \( \varepsilon > 0 \) and data \( \varphi_k \).
2: initialization: Set \( \hat{x}_0 = 0 \) and \( x(0) = x_0 \).
3: Evaluate \( e_k^\varphi \) (total testing loss in (8)) using \( \varphi_k^{\text{test}} \).
4: while not \( e_k^\varphi \leq \varepsilon \) do
5: for \( i = 1 : \eta \)
6: Perform (29) to obtain \( \hat{x}_{k+1} \) (note \( \hat{x}_{k+1}^0 = \hat{x}_k \)).
7: end for
8: Evaluate \( e_k^\varphi \) using (8) based on the data in \( \varphi_k^{\text{test}} \).
9: end while

Note that the initial estimate \( \hat{x}_{k+1}^0 \) for (29) is considered as the estimate \( \hat{x}_k \) of the previous ILC trial as shown in Step 6 (in Algorithm 2), and the total testing loss precision \( \varepsilon > 0 \) is a sufficiently small scalar representing the accuracy criteria of the ML procedure.

6.2 Parameter tuning methods

In addition, parameter values within the algorithm should be set to a reasonable range to be adaptive to the exact task description to provide desired convergence rates. Therefore, based on experiences, we suggest the choice of \( R \) and \( Q \) for ILC law (22) as

\[
0.1/\|\hat{G}_k\|^2 \leq \|Q/R\| \leq 10/\|\hat{G}_k\|^2.
\]

For (29), an appropriate value of \( \gamma \) is chosen according to Bertsekas (1976) as

\[
\gamma = \alpha^2 \gamma_0,
\]

to achieve a maximum step size and guarantee the loss reduction, where \( 0 < \alpha < 1 \) affects the decreasing speed of the learning rate \( \gamma \) (a smaller \( \alpha \) value gives a faster \( \gamma \) decreasing speed), \( \beta \) is the smallest non-negative integer number such that (30) holds, and \( \gamma_0 > 0 \) is the initial learning rate. Note that in this paper, the authors propose the initial learning rate \( \gamma_0 \) according to the obtained data (gradient and nominal model estimate in (29)) as follows:

\[
\gamma_0 = \|\hat{x}_{k+1}^j\|/\|\nabla_{\hat{x}_{k+1}^j} \mathcal{L}(\hat{x}_{k+1}^j, \varphi_{k}^1)\|.
\]

In this way, \( \gamma_0 \) is not too small or too large such that the gradient decent calculation is more calculation-effective.

Algorithm 1 couples the ML update procedure with the ILC update law to provide sufficiently small model uncertainties. In this sense, the convergence condition (24) in Theorem 1 holds at each trial, and the upper bound of the tracking error as shown in (25) can be reduced according to Remark 3.

Remark 5 Algorithm 1 differs from classical ILC algorithms as it allows the modifications of the nominal model along the trials, which breaks the design postulate on the invariance of system model and improves the tracking performance for NTVSs.

Remark 6 A neural network model with two layers can be used to address this problem when system state is unobservable. The output of the first layer is the state and the output of the second layer is the output of the system. In this sense, the same gradient descent training procedure (29) solves the problem using only the input-output data of the system. However, this paper focuses on designing ILC algorithms for NTVS dynamics, and this part will be considered as part of future work.
7 Numerical simulation case study

A numerical case study is performed to check the performance of Algorithm 1 to show its robustness and reliability, and a comparison is made with other control methods.

7.1 Task specifications

A gantry robot model addressed in Ratcliffe (2005) with three axes (x-axis, y-axis and z-axis) is considered here, and the non-repetitive time-varying parameters are further embedded into its state space form as

\[
\begin{align*}
A_k(t) &= \text{diag}(a_1(t + kN), a_2(t + kN), a_3(t + kN)), \\
B_k(t) &= \text{diag}(b_1(t + kN), b_2(t + kN), b_3(t + kN)), \\
C_k(t) &= \text{diag}(0.22, 0.22, 0.2016),
\end{align*}
\]

where the time-varying parameters within \( A_k(t) \) and \( B_k(t) \) are defined as shown in Fig. 7. The tracking task is specified as repetitively following a path shown in Fig. 4 defined by

\[
r(t) = \begin{bmatrix}
0.005 \cos\left(\frac{-2\pi t}{N}\right) + 0.005 \\
0.005 \sin\left(\frac{-2\pi t}{N}\right) \\
0.01t/N
\end{bmatrix}, \quad t \in [0, N],
\]

within 2s at 0.01s sample time (\( N = 200 \)). Due to the ILC design requirements, the initial path position should be the same as that of the system. The nominal model is chosen as

\[
\begin{align*}
A_k &= \text{diag}(\hat{a}_1(kN), \hat{a}_2(kN), \hat{a}_3(kN)), \\
B_k &= \text{diag}(\hat{b}_1(kN), \hat{b}_2(kN), \hat{b}_3(kN)), \\
C_k &= \text{diag}(0.22, 0.22, 0.2016).
\end{align*}
\]

The feedforward ILC update law (22) is considered in Step 11 of Algorithm 1. The initial input signal and state are set as \( u_0 = 0 \) and \( x_0 = 0 \). For ILC update, the trial number is set as \( k_{\text{max}} = 100 \), the weighting matrices are set as \( R = I \) and \( Q = 1,000,000I \) based on (32). For ML based nominal model update, the initial estimate \( \hat{\sigma}_0 \) is

\[
\hat{\sigma}_0 = (\hat{a}_1(0), \hat{a}_2(0), \hat{a}_3(0), \hat{b}_1(0), \hat{b}_2(0), \hat{b}_3(0)) = (0.93, 0.84, 0.95, 0.0023, 0.0022, 0.00125).
\]

A proportion of 80% elements from the data set \( \varphi_k \) is chosen to form the training set, i.e., \( \eta = 160 \). The parameters in (33) are set as \( \alpha = 0.6 \) and \( \beta \) is chosen from 0 and gradually increases along the training time line.

7.2 Simulation results

The mean absolute tracking error of Algorithm 1 along the ILC trials is plotted as the magenta curve in Fig. 3 where the error monotonically converges at the first few trials, and then begins to oscillate around certain level of accuracy, i.e., between \( 10^{-1}\text{mm} \) and \( 10^{-2}\text{mm} \). This is because of the fact that the error reaches its upper bound in (25) of Theorem 1. The ILC update law cannot further reduce the error below this bound, but it can reduce the error at the next trial once it exceeds this bound, which leads to the oscillation phenomenon in the figure.

This result follows from Theorem 1 that the proposed algorithm can reduce the tracking error to a certain accuracy. The convergence results show that this algorithm has a certain level of robustness against model uncertainties and non-repetitive parameters. For comparison, the classical ILC algorithm proposed in Amann (1996) and the feedback control method with a proportional gain 3,000 are also implemented to perform the same task, and the corresponding results are plotted in Fig. 3. One can see those two methods can only achieve an accuracy level between \( 10^0\text{mm} \) and \( 10^{-1}\text{mm} \), which is much higher than that of Algorithm 1. Their corresponding tuning parameters have been applied, but there are no significant improvement on their accuracy. Therefore, Algorithm 1 outperforms these control methods in terms of higher accuracy. Further note that its converged error range (10\( \mu \text{m} \) – 100\( \mu \text{m} \), or \( 10^{-1}\text{mm} - 10^{-2}\text{mm} \)) is close to the optical encoder resolution around 16 – 32\( \mu \text{m} \) as stated in Ratcliffe (2005), which means it can approach the perfect task performance under the given hardware.
Fig. 5. The mean absolute error along the trials for different values of $Q$ with $R = I$.

Fig. 6. The input signals at the final trial with $Q = 1,000,000I$.

Fig. 7. The comparison between the estimate and the time-varying parameters along the total sample time index to check the ML performance. One can see ML is capable of updating $\hat{A}_k$ and $\hat{B}_k$ to follow the changes of $A_k$ and $B_k$, despite the initial estimate is not accurate enough. This makes the model uncertainties between the nominal model and the NTVS within a tolerated bound for ILC update laws. Therefore, the comparison result verifies the reliability of the ML based nominal model update method. Note that the delay between the estimate and the benchmark values is due to the update procedure, since the information at the end of previous trial is used to obtain the nominal model for updating the input at beginning of next trial as stated in Remark 4. Also, the estimated values $\hat{b}_1$, $\hat{b}_2$ and $\hat{b}_3$ do not fully match the benchmark values. This is because that the ML procedure reduces the loss function rather than observe the time-varying parameter values.

Fig. 8. The average testing loss (defined in (9)) of the estimate along the training epochs at the final trial with different values of $\eta$ using (8).

To further exploit the performance of the ML update method, Fig. 8 demonstrates the training results of different values of $\eta$ to obtain the nominal model of the final trial ($k_{\text{max}} = 100$). The results confirm the reliability of the ML update method using the data from testing set. Note that the average testing losses converge to or oscillate around none-zero values, which is a proof of the fact stated in Remark 4. The average testing losses are small enough to ensure the ML procedure performs in a good manner and reduce when $\eta$ increases from 100 to 160, but rises for $\eta = 180$, which might be caused by the over-fitting problem. Therefore, an appropriate proportion of the training data, i.e., 80%, should be selected to provide a desired training performance.
8 Conclusion and future work

This paper presents the initial idea on the collaboration of ML and ILC to aim at handling repetitive tracking tasks for time-varying systems with non-repetitive parameters (NTVSs). It develops an ILC update law with a certain level of robust performance based on a norm optimal approach. Meanwhile, an ML based nominal model update mechanism is proposed to guarantee the model uncertainties within the acceptable bound. These two aspects are combined to yield a comprehensive algorithm with robustness and reliability, which is verified by a numerical simulation case study.

The next step of this paper is to establish a practical test platform embedding NTVSs to validate the practical tracking performance. In fact, there also exist potential extensions that can be made to enhance the impact of this technique. For instance, it can follow the ideas in Chen, Chu, and Freeman (2018, 2019) to address the point-to-point tracking task for NTVSs, and optimize the time allocation of the points to achieve extra practical benefits. In addition, it is an appropriate solution to the leader-follower time-varying formation control problem of multi-agent NTVSs based on the idea in Jiang, Wen, Peng, Huang, and Rahmani (2019), in which the system dynamics are somehow time-invariant.

A Proof of Lemma 1

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B Proof of Theorem 1

It follows from (6) and (22) that

\[ e_{k+1} = r - G_{k+1}u_{k+1} - H_{k+1}x_0 \]  
\[ = r - G_k u_k - H_k x_0 - \partial G_k u_k - \partial H_k x_0 + G_{k+1} \]  
\[ \times \hat{G}_{k+1}(I + \hat{G}_{k+1}\hat{G}_{k+1}^*)^{-1}(e_k - \partial \hat{G}_k u_k - \partial \hat{H}_k x_0) \]  
\[ = (I - \Delta^G_{k+1}\hat{G}_{k+1}^*)(I + \hat{G}_{k+1}\hat{G}_{k+1}^*)^{-1}(e_k - \partial \hat{G}_k u_k) \]  
\[ - \partial \hat{H}_k x_0) + (\partial \hat{G}_k - \partial G_k)u_k + (\partial \hat{H}_k - \partial H_k)x_0. \]

Since \( \hat{G}_{k+1} \) has full row rank, it is clear that

\[ \|(I + \hat{G}_{k+1}\hat{G}_{k+1}^*)^{-1}\| \leq \rho_{k+1} < 1, \]  
\[ \text{(B.2)} \]

which with the condition (24) gives rise to the inequality

\[ \|e_{k+1}\| \leq \rho_{k+1}\|I - \Delta^G_{k+1}\hat{G}_{k+1}^*\|\|e_k - \partial \hat{G}_k u_k - \partial \hat{H}_k x_0\| \]  
\[ + \|\partial \Delta^G_k u_k\| + \|\partial \Delta^H_k x_0\| \]  
\[ \leq \rho_{k+1}(\|e_k\| + \|\hat{G}_k\|\|u_k\| + \|\hat{H}_k\|\|x_0\|) + (\rho_{k+1} + 1) \]  
\[ (\|\partial \Delta^G_k\|\|u_k\| + \|\partial \Delta^H_k\|\|x_0\|) \]  
\[ = \rho_{k+1}\|e_k\| + \rho_{k+1}\|G_k\| + (\rho_{k+1} + 1)\|\partial \Delta^G_k\|\|u_k\| \]  
\[ + (\rho_{k+1}\|H_k\| + (\rho_{k+1} + 1)\|\partial \Delta^H_k\|\|x_0\|). \]  
\[ \text{(B.3)} \]

If the condition (25) holds, the inequality (B.3) gives rise to the monotonic convergence condition (26).

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