Constraint on the solar $\Delta m^2$ using 4000 days of short baseline reactor neutrino data

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There is a well-known $2\sigma$ tension in the measurements of the solar $\Delta m^2$ between KamLAND and SNO/Super-Kamiokande. Precise determination of the solar $\Delta m^2$ is especially important in connection with current and future long baseline $CP$ violation measurements. Seo and Parke [Phys. Rev. D 99, 033012 (2019)] points out that currently running short baseline reactor neutrino experiments, Daya Bay and RENO, can also constrain solar $\Delta m^2$ value as demonstrated by a GLoBES simulation with a limited systematic uncertainty consideration. In this work, the publicly available data, from Daya Bay (1958 days) and RENO (2200 days) are used to constrain the solar $\Delta m^2$. Verification of our method through $\Delta m^2_{ee}$ and $\sin^2\theta_{13}$ measurements is discussed in Appendix A. Using this verified method, reasonable constraints on the solar $\Delta m^2$ are obtained using above Daya Bay and RENO data, both individually and combined. We find that the combined data of Daya Bay and RENO set an upper limit on the solar $\Delta m^2$ of $18 \times 10^{-5} \text{ eV}^2$ at the 95% C.L., including both systematic and statistical uncertainties. This constraint is slightly more than twice the KamLAND value. As this combined result is still statistics limited, even though driven by Daya Bay data, the constraint will improve with the additional running of this experiment.

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I. INTRODUCTION

Evidence that neutrinos are massive and mix is well established by a significant number of experiments. In this paper, we are interested in the mass squared difference, $\Delta m^2_{21}$; the mass squared difference of the two mass eigenstates that have the greatest fraction of electron neutrino, $\nu_1$ and $\nu_2$. This mass splitting is responsible for the neutrino flavor transformations that occur inside the Sun (hence the name the solar mass squared difference), and for the antineutrino oscillations observed at an $L/E \sim 15 \text{ km/MeV}$.

In this paper, we use publicly available data to follow up a recent paper [1], that Daya Bay [2] and RENO [3], the short baseline ($\sim 1.5 \text{ km}$) reactor antineutrino experiments currently running, have enough data already collected to constrain $\Delta m^2_{21}$. The combined constraint by Daya Bay and RENO, gives an important consistency check of the standard three neutrino paradigm as well as adding addition information to the size of $\Delta m^2_{21}$. The $\sim 2\sigma$ tension between the combined Super-Kamiokande (SK) [4] & Sudbury Neutrino Observatory (SNO) [5] solar neutrino measurements and KamLAND [6] reactor experiment ($L/E \sim 50 \text{ km/MeV}$) is not directly addressed by this constraint. However such a combined Daya Bay plus RENO constraint is at a different $L/E$ range ($\sim 0.5 \text{ km/MeV}$) than the above mentioned measurements as well as JUNO [7]. Moreover, the ratio of $\Delta m^2_{21}$ to $\Delta m^2_{31}$, at an $L/E \sim 0.5 \text{ km/MeV}$, is required for the precision measurement of leptonic $CP$ violation parameter, by NOvA [8], T2K [9] and future Long Baseline (LBL) experiments.

Currently there are two measurements of the solar mass squared difference, $\Delta m^2_{21}$. One measurement comes from a
combined measurement by SNO and SK using the observation of a day-night asymmetry by SK and the nonobservation of the low energy up turn of the $^8$B neutrino survival probability by SNO and SK. This combined result is

$$\Delta m_{21}^2 = 5.1^{+1.3}_{-1.0} \times 10^{-5} \text{ eV}^2,$$

(1)

from SNO and SK. Similar results are obtained by Nu-Fit [10]. The other measurement is from KamLAND, the long baseline reactor antineutrino experiment, see [6], at

$$\Delta m_{31}^2 = 7.50^{+0.20}_{-0.20} \times 10^{-5} \text{ eV}^2,$$

(2)

If CPT invariance is a good symmetry of nature then the $\Delta m_{31}^2$ measured from solar neutrinos and reactor antineutrinos is required to give the same value. Currently this important parameter for neutrino physics suffers from a $2\sigma$ level tension. This tension could come from new physics, some error in the analysis of one or more of the experiments or a statistical fluctuation.

Moreover, the ratio of $\Delta m_{31}^2$ to $\Delta m_{21}^2$ is required for the determination of the CP phase, $\delta$, in the long baseline neutrino oscillation experiments (NOvA, DUNE [11], T2K, T2HK [12], T2HKK [13]) as the size of the CP violation is proportional to $\Delta m_{31}^2$ to $\Delta m_{21}^2$, as well as the Jarlskog invariant. At $L/E \sim 500 \text{ km/GeV} = 0.5 \text{ km/MeV}$, the first oscillation peak in vacuum, for $\nu_\mu \to \nu_e$

$$P(\bar{\nu}_e \to \nu_e) - P(\nu_\mu \to \nu_e) \approx \pi J(\Delta m_{21}^2 / \Delta m_{31}^2)$$

(3)

where the Jarlskog invariant, $J$, is $J = \sin 2\theta_{12} \sin 2\theta_{13} \cos \theta_{13} \sin 2\theta_{23} \sin \delta \approx 0.3 \sin \delta$.

In the bievent plane for T2K, see Fig 44 of [14],

$$N(\nu_\mu \to \nu_e) = 37 \quad \text{and} \quad N(\bar{\nu}_\mu \to \bar{\nu}_e) = 4$$

is outside the allowed region (by about $1\sigma$). This can be well accommodated by a $\Delta m_{21}^2$ value, approximately twice the KamLAND value. Again, this is probably a statistical fluctuation but with only the KamLAND precision measurement of $\Delta m_{31}^2$, other possibilities are still viable.

The future medium baseline, $L/E \sim 15 \text{ km/MeV}$, reactor experiment JUNO will measure to better than 1% precision $\Delta m_{31}^2$ and $\sin^2 \theta_{12}$, see [7]. JUNO experiment is currently under construction and their precision measurements of $\Delta m_{21}^2$ and $\sin^2 \theta_{12}$ will not be available until approximately 5 years from now. Later next decade, the proposed experiments Hyper-K & DUNE will also give us precision measurements of $\Delta m_{21}^2$ using $^8$B solar neutrinos, see [15,16] respectively.

In Sec. II, we briefly discuss in detail the effects of increasing $\Delta m_{31}^2$ on the $\bar{\nu}_e$ survival probability. Then in Sec. III Daya Bay and RENO data sets used in this work are discussed followed by Secs. IV, V, and VI for methods and systematic uncertainties, results, and conclusion, respectively.

II. SURVIVAL PROBABILITY

In vacuum, the electron antineutrino survival probability is

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - P_{12} - P_{13}$$

with

$$P_{12} = \sin^2 2\theta_{12} \cos^4 \theta_{13} \sin^2 \Delta_{21},$$

$$P_{13} = \sin^2 2\theta_{13} (\cos^2 \theta_{12} \sin^2 \Delta_{31} + \sin^2 \theta_{12} \sin^2 \Delta_{32}),$$

(4)

where the kinematic phases are given by $\Delta_{jk} = \Delta m_{jk}^2 L/(4E)$ and $\theta_{13} \approx 8^\circ$ and $\theta_{12} \approx 33^\circ$ are the reactor and solar mixing angles respectively. The $P_{12}$ term is associated with the solar oscillation scale of 15 km/MeV and the $P_{13}$ term is associated with the atmospheric oscillation scale of 0.5 km/MeV. To excellent fractional precision,$^2$ the $P_{13}$ term can be approximated by

$$P_{13} \approx \sin^2 2\theta_{13} \sin^2 \Delta_{ee}$$

(5)

where $\Delta m_{ee}^2 \equiv \cos^2 \theta_{12} \Delta m_{31}^2 + \sin^2 \theta_{12} \Delta m_{32}^2$ [17,18], interpreted as the $\nu_e$ average of $\Delta m_{31}^2$ and $\Delta m_{32}^2$.

Using the fit values given in [10], and an $L/E$ range around the first oscillation minimum ($L/E \sim 0.5 \text{ km/MeV}$), $P_{12}$ and $P_{13}$ is well approximated by:

$$P_{12} \approx 0.002 \left( \frac{L/E}{0.5 \text{ km/MeV}} \right)^2 \left( \frac{\Delta m_{21}^2}{7.5 \times 10^{-5} \text{ eV}^2} \right)^2$$

(6)

$$P_{13} \approx 0.08 \sin^2 \left( \frac{\pi}{2} \left( \frac{L/E}{0.5 \text{ km/MeV}} \right) \right).$$

(7)

The $P_{12}$ term is almost negligible for all $L/E < 1 \text{ km/MeV}$, if $\Delta m_{21}^2$ = $7.5 \times 10^{-5} \text{ eV}^2$. For Daya Bay and RENO this covers the full $L/E$ range.

Suppose that $\Delta m_{21}^2$ is 3 times larger than KamLAND value, i.e., $22.5 \times 10^{-5} \text{ eV}^2$, then

$^2$The fractional precision is better than 0.05% for $L/E < 1 \text{ km/MeV}$. Also, in this $L/E$ range, the exact $P_{13}$ is very insensitive to mass ordering provided the value of $|\Delta m_{21}^2|$ is the same for both mass orderings.
\[ P_{12} \approx 0.02 \left( \frac{L/E}{0.5 \text{ km/MeV}} \right)^2 \left( \frac{\Delta m^2_{21}}{22.5 \times 10^{-5} \text{ eV}^2} \right)^2. \]  

Now \( P_{12} \) is now no longer tiny compared to \( P_{13} \) at \( L/E = 0.5 \text{ km/MeV} \), oscillation minimum, and as \( L/E \) gets larger than 0.5 km/MeV, \( P_{12} \) gets bigger, whereas \( P_{13} \) is getting smaller. At an \( L/E = 1 \text{ km/MeV} \), \( P_{12} \) would be approximately equal to \( \sin^2 2\theta_{13} \) (0.08) for this value of \( \Delta m^2_{21} \). It is this quadratic rise in \( P_{12} \) as \( \Delta m^2_{21} \) increases that we exploit to place an upper limit on \( \Delta m^2_{21} \). For further details on the survival probability as \( \Delta m^2_{21} \) increases see [1].

### III. DAYA BAY AND RENO DATA SETS

In this work, 1958 days of Daya Bay data [19] and 2200 days of RENO data [20] are used, where Daya Bay has about five times more inverse beta decay (IBD) events than RENO in their far detectors. Daya Bay data including background estimation, energy response function, and systematic uncertainties are taken from the supplementary material in [19]. RENO data and background estimation are extracted from Fig. 1 in [20] and systematic uncertainties are also taken from [20]. Table I shows summary of the basic parameters, i.e., \( L_{\text{eff}}, \) IBD rate, and background rate, for near and far detectors of Daya Bay and RENO used in this analysis. Note that there are two near detectors in different sites for Daya Bay.

### IV. METHODS AND SYSTEMATIC UNCERTAINTIES

Best fit values on \( \Delta m^2_{21} \) and \( \sin^2 2\theta_{13} \) are obtained by finding minimum \( \chi^2 \) values between data and predictions for all possible combination of the two parameters. Far-to-near ratio method is employed in this \( \chi^2 \) analysis to avoid the spectral shape anomaly around 5 MeV region [21] as well as to reduce systematic uncertainties.

The \( \chi^2 \) formalism as written below contains a covariance matrix \( \mathbf{V}_{\text{stat},ij} \) to include statistical uncertainty and pull parameters \( \xi_{\alpha} \) to include systematic uncertainties.

\[
\chi^2 = \sum_{i,j}^N \left( D_{i}^{F/N} - P_{i}^{F/N} \right) \mathbf{V}_{\text{stat},ij}^{-1} \left( D_{j}^{F/N} - P_{j}^{F/N} \right) + \sum_{\alpha}^{N_{\text{sys}}} \left( \frac{\xi_{\alpha} - 1}{\sigma_{\alpha}} \right)^2,
\]

where, \( D_{i}^{F/N} \equiv \frac{O_{i}^{F} - B_{i}^{F}}{O_{i}^{N} - B_{i}^{N}}, \) \( P_{i}^{F/N} \equiv \frac{X_{i}^{F}}{X_{i}^{N}}, \) and \( F(N) \) and \( i (j) \) represent the Far (Near) detector and \( i \)th \( (j) \)th prompt energy bin, respectively. Being \( O \) the observed number of IBD candidate events, \( B \) the estimated background number of events and \( X \) the expected number of events for a given \( \Delta m^2_{21} \) and \( \sin^2 2\theta_{13} \) pair. A total of 26 energy bins \( (N_{\text{bins}}) \) is used for RENO from 1.2 to 8.4 MeV. The same number of energy bins are used for Daya Bay from 0.7 to 12 MeV but two near detectors are taken into account in the \( \chi^2 \) formalism by replacing \( N_{\text{bins}} \) to \( 2N_{\text{bins}} \) where for \( 1 \leq i \leq N_{\text{bins}}, \) \( F = \text{EH3} \) and \( N = \text{EH1}, \) and for \( N_{\text{bins}} + 1 \leq i \leq 2N_{\text{bins}}, \) \( F = \text{EH3} \) and \( N = \text{EH2}. \)

For both Daya Bay and RENO, systematic uncertainties on the relative detection efficiency, relative energy scale, and the main background contributions are taken into account as summarized in Table II.

Besides the systematic uncertainties, additional systematic paddings (adjustment factors) are added in our work to match Daya Bay and RENO results on \( \theta_{13} \) and \( \Delta m^2_{ee} \) measurements. For Daya Bay a 1.3 adjustment factor to the relative energy scale and Li-He background uncertainties is added. Whereas in RENO a 1.4 adjustment factor is added to the relative detection efficiency uncertainty. More details on the validation of our method and expected event description can be found in Appendices A and B. The RENO predictions are computed using the Daya Bay detector response function and the relative far-to-near normalization is computed comparing our total number of expected events with the total number of expected events in the RENO Far detector. In order to match the best fit values of \( \theta_{13} \) and \( \Delta m^2_{ee} \) a 0.984 adjustment factor is added to this normalization of a total event rate for RENO.

### TABLE II. Relative systematic uncertainties used in this work for Daya Bay and RENO, taken from [19,20] respectively.

| Source                     | Daya Bay | RENO  |
|----------------------------|----------|-------|
| Detection efficiency       | 0.13     | 0.21  |
| Energy scale               | 0.2      | 0.15  |
| Li-He background           | 30       | 5–8   |
| Fast neutron background    | 13–17    | …     |
| Accidental background      | 1        | …     |
V. RESULTS

A 2-dimensional scan over $\Delta m^2_{21}$ and $\sin^22\theta_{13}$ is performed to find the best fit value pair at the minimum value of $\chi^2$ described earlier, where in the oscillation probability, the parameter $\theta_{12}$ is fixed at $\sin^2\theta_{12} = 0.310$. The $\Delta m^2_{ee}$ parameter is constrained with a pull parameter, allowing it to vary within a $2\sigma$ range of a prior $\Delta m^2_{ee}$ value with a penalizing term

$$ \left( \frac{\Delta m^2_{ee,prior} - \Delta m^2_{ee}}{\sigma} \right)^2 $$

The prior $\Delta m^2_{ee}$ value and its uncertainty are taken to be

$$ \Delta m^2_{ee} = 2.45 \pm 0.15 \times 10^{-3} \text{ eV}^2 \quad (9) $$

which is inferred from the combined measurement on $\Delta m^2_{\mu\mu}$ by current long baseline neutrino experiments in [10] through $\Delta m^2_{\mu\mu} \approx \Delta m^2_{ee} \pm \cos 2\theta_{12} \Delta m^2_{21}$, see [17], where the $+/-$ comes from the unknown mass ordering (NO/IO) and ignoring terms proportional to $\sin\theta_{13}\Delta m^2_{31}$. The unknown mass ordering is treated as an additional uncertainty ($4\%$) to $\Delta m^2_{\mu\mu}$ uncertainty ($4\%$) for the $\Delta m^2_{ee}$ uncertainty which, therefore, becomes about 6%.

The best fit, 1, 2, and 3$\sigma$ allowed regions of $\Delta m^2_{21}$ vs $\sin^22\theta_{13}$ are shown in Fig. 1 with (solid lines) and without (dashed lines) systematic uncertainties for Daya Bay and RENO, separately and combined. Daya Bay’s result is better than RENO’s due to about five time more statistics at the far detector, see Table I.

Figure 2 shows the $\chi^2$ projection over $\Delta m^2_{21}$, obtained by minimizing over $\sin^22\theta_{13}$, for the Daya Bay plus RENO combined analysis. The upper bounds on $\Delta m^2_{21}$, including systematic uncertainties, are 12.3, 18.3 and $22.3 \times 10^{-5} \text{ eV}^2$ at 1, 2 and 3$\sigma$ C.L., respectively. Current upper bounds are limited by statistics.

In Fig. 3, we give the constraints on the three parameter fit, $\Delta m^2_{21}$, $\Delta m^2_{ee}$ and $\sin^22\theta_{13}$, without imposing any constrain on $\Delta m^2_{ee}$, using the combined Daya Bay and RENO data sets. Both statistical and systematic uncertainties are included in this plot. As before $\theta_{12}$ is fixed at $\sin^2\theta_{12} = 0.310$, see [1] for discussion on allowing $\sin 2\theta_{12}$ to also vary.

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A discussion on the effects of varying $\theta_{12}$ in this analysis can be found in [1].
Results with $\Delta m^2_{ee}$ fixed or free are obtained for each experiment and for when the data from both experiments are combined. These are described and given in Appendix C. It was found that the effect of free $\Delta m^2_{ee}$ is bigger than that of systematic uncertainty, but our representing results are based on constrained $\Delta m^2_{ee}$ since it is a reasonably well measured oscillation parameter using LBL experiments.

VI. CONCLUSION

Using the currently available public data from Daya Bay (1,958 days) and RENO (2,200 days), we have provided additional information on the solar $\Delta m^2$. A reasonable upper bound is obtained from a combined analysis of the Daya Bay and RENO data as $18 \times 10^{-5}$ eV$^2$ at 95% CL, where $\Delta m^2_{ee}$ was constrained using a pull parameter with input information from LBL experiments. Our combined analysis result is currently limited by statistics and, as expected, Daya Bay data drives the combined analysis results. Our analysis method was validated by reproducing the $\Delta m^2_{ee}$ and $\sin^2 2\theta_{13}$ contours for each experiment as discussed in Appendix A.

Given that the previous measurements by KamLAND and SK/SNO of the solar $\Delta m^2$ are in a 2$\sigma$ tension and the importance of solar $\Delta m^2$ for the determination of CP violation in LBL experiments, it is crucial that we understand the value of the solar $\Delta m^2$ better. It is expected by circa 2025 that the JUNO experiment will provide additional, important information on the value of the solar $\Delta m^2$.

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FIG. 3. Simultaneous three parameter fit for $\Delta m^2_{21}$, $\Delta m^2_{ee}$ and $\sin^2 2\theta_{13}$ using the combined Daya Bay (1,958 days, 487 K IBD events at Far) and RENO (2,200 days, 98 K IBD events at Far) data. The best fit point is found at $\Delta m^2_{21} = 3.3 \times 10^{-5}$ eV$^2$, $\Delta m^2_{ee} = 2.5 \times 10^{-3}$ eV$^2$, $\sin^2 2\theta_{13} = 0.088$.

APPENDIX A: VALIDATION OF OUR ANALYSES

Using the data and the $\chi^2$ formalism described in Secs. III and IV, our method reproduces the contours in

FIG. 4. Our validation on $\Delta m^2_{ee}$ vs $\sin^2 2\theta_{13}$ fit using the Daya Bay data (1958 days), including systematics and statistics uncertainties in red solid lines, and including statistics only in blue dashed lines, for 1, 2 and 3$\sigma$ allowed regions. The fit of the Daya Bay collaboration with 1958 days from[19] is represented in the solid black lines. The agreement between our analysis (solid red lines) and Daya Bay’s analysis (solid black lines) is excellent.
FIG. 5. Our validation on $\Delta m_{ee}^2$ vs $\sin^2 2\theta_{13}$ fit using the RENO data (2200 days), including systematics and statistics uncertainties in red solid lines, and including statistics only in blue dashed lines, for 1, 2 and $3\sigma$ allowed regions. The fit of the RENO collaboration with 2,200 days from [20] is represented in the solid black lines. The agreement between our analysis (solid red lines) and RENO’s analysis (solid black lines) is excellent.

The expected numbers of signal events in a detector $d$, in a prompt energy bin $i$, $X^d_i$, is computed as follows up to a common input (e.g., reactor power, total number of protons) which cancels when taking ratios in the $\chi^2$ computation.

$$X^d_i = \sum_i \sum_{iso} \frac{d^d_i}{L_{rd}} \int_{E^\nu_{rec}} \int_0^{E^\nu_{rec}} \int_0^{\infty} dE_{\nu} \sigma(E_{\nu}) f^{iso}(E_{\nu})$$

$$\times P^{rd}_{\nu_{rec}}(E_{\nu}) R(E^\nu_{rec}, E_{\nu})$$

where, the indices $i, r, d$, and iso refers to the $i$th energy bin, $r$th reactor, $d$th detector, and a fissionable isotope ($^{235}$U, $^{239}$Pu, $^{238}$U, or $^{241}$Pu), respectively, and $d^d_i$ is the detector efficiency. $L_{rd}$ is the baseline between the reactor $r$ and the detector $d$. $E_\nu$ and $E^{rec}$ are the neutrino true energy and the reconstructed energy, both related by the detector response function $R(E^{rec}, E_\nu)$. The $\sigma(E_{\nu})$ is the IBD cross section computed performing the integral in $d\cos \theta$ of the differential cross section in [22] and the $f^{iso}$ is the averaged fission fraction and the $P^{iso}(E_{\nu})$ is the Huber-Mueller flux prediction [23,24]. $P^{rd}_{\nu_{rec}}(E_{\nu})$ is the oscillation probability from reactor $r$ to detector $d$ in the three neutrino oscillation paradigm.

The pull parameters accounting for detection efficiency ($\epsilon^d$) and relative energy scale ($\eta^d$) are included in the number of expected events as follows

$$X^d_i(\epsilon^d, \eta^d) = e^d_i \sum_i \sum_{iso} \frac{d^d_i}{L_{rd}} \int_{E^\nu_{rec}} \int_0^{E^\nu_{rec}} \int_0^{\infty} dE_{\nu}$$

$$\times \sigma(E_{\nu}) f^{iso}(E_{\nu}) P^{rd}_{\nu_{rec}}(E_{\nu}) R(E^\nu_{rec}, E_{\nu}).$$

For RENO, the efficiency pull parameter is included in the ratio.

The background pull parameters are included in background events $B^d_i$ used in $D^F/N$ as follows

$$B^d_i(B^d_{LH}, B^d_{acc}, B^d_{n}) = B^d_i + (b^d_{LH} - 1) B^d_{LH,i}$$

$$+ (b^d_{acc} - 1) B^d_{acc,i} + (b^d_{n} - 1) B^d_{n,i},$$

where $B^d_i$ ($B^d_{LH,i}$, $B^d_{acc,i}$ and $B^d_{n,i}$) represents the number of total (Li-He, accidental and fast neutron) background events in the $i$th prompt energy bin in the $d$th detector, and the small $b$ represents the corresponding pull parameter.

**APPENDIX C: FIXED VS FREE $\Delta m_{ee}^2$**

For the results in the main body of our paper we constrained $\Delta m_{ee}^2$ treating it as a pull parameter using LBL experiments input. In this section we show the impact of $\Delta m_{ee}^2$ fixed and set free. A 2-dimensional scan over $\Delta m_{ee}^2$ and $\sin^2 2\theta_{13}$ is performed to find the best fit value pair at the minimum value of $\chi^2$ described earlier, where in the oscillation probability $\theta_{12}$ is fixed as $\sin^2\theta_{12} = 0.310$ but $\Delta m_{ee}^2$ is set free within the range of $[1.55, 3.55] \times 10^{-3}$ eV$^2$. Results with a fixed $\Delta m_{ee}^2 = 2.45 \times 10^{-3}$ eV$^2$ are also obtained and compared to those with $\Delta m_{ee}^2$ set free. Figure 6, left and middle panels, shows the results of $\Delta m_{ee}^2$ fixed and free for Daya Bay and RENO. It is observed that the effect of floating $\Delta m_{ee}^2$ is bigger than adding systematic uncertainty for both Daya Bay and RENO. For floating $\Delta m_{ee}^2$ case, the corresponding

4Ideally we would have the information on the fission factions as a function of time in each reactor, but since we do not have this information we take the same averaged values for all the detectors. This means that any systematic uncertainty on the flux predictions will cancel when taking ratios of the expected events in different experimental sites.
The values for the minimum $\chi^2$ are found to be $2.50 \times 10^{-3}$ eV$^2$ for Daya Bay (RENO) and it is within 1σ uncertainty of each of their measurements. Figure 6, right panels shows the results with combined analysis. For floating $\Delta m_{ee}^2$ case, the corresponding $\Delta m_{ee}^2$ value for the minimum $\chi^2$ is found to be $2.54 \times 10^{-3}$ eV$^2$ and it is within 1σ uncertainty of the Daya Bay best fit value, i.e., $[2.52 \pm 0.07] \times 10^{-3}$.

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