Fate of charmed mesons near chiral symmetry restoration in hot matter

Chihiro Sasaki$^{1,2}$

$^1$Frankfurt Institute for Advanced Studies, D-60438 Frankfurt am Main, Germany
$^2$Institute of Theoretical Physics, University of Wroclaw, PL-50204 Wroclaw, Poland

(Dated: November 10, 2014)

Chiral thermodynamics of charmed mesons is formulated at finite temperature within a 2+1+1-flavored effective Lagrangian incorporating heavy quark symmetry. The charmed-meson mean fields act as an extra source which breaks the chiral symmetry explicitly. This leads to effective interactions between the light and heavy-light mesons, which intrinsically depend on temperature. Effective masses of the scalar and pseudoscalar charmed-mesons tend to approach each other as increasing temperature, so that the splitting between the chiral partners is reduced. These chiral splittings are shown to be less sensitive to the light-quark flavors, attributed to the underlying heavy quark symmetry. Consequently, chiral symmetry restoration is more manifest for the strange charmed-mesons than for the strange light mesons. The effective masses are also compared with the results in the one-loop chiral perturbation theory. A substantial difference is found at a relatively low temperature, $T \sim f_s$.

PACS numbers: 14.40.Lb, 12.39.Fe, 12.39.Hg

1. INTRODUCTION

Heavy flavors, charm and beauty, are produced at the initial stage of the high-energy heavy-ion collisions, so that they are expected to carry the dynamical history of a created matter, the Quark-Gluon Plasma (QGP) (see, e.g. [1] for a review). Recent experimental observations have revealed that charm quarks are thermalized [2–5], contrary to earlier anticipation. Charge fluctuations have revealed that charm quarks are thermalized [17], forming the lowest spin partners. Their low-energy dynamics is dominated by interactions with Nambu-Goldstone (NG) bosons, pions [18–21]. Introducing the multiplet including $D$ and $D^*$ states fill in the same multiplet, $H$, forming the lowest spin partners. Their low-energy dynamics is dominated by interactions with Nambu-Goldstone (NG) bosons, pions [18–21]. Introducing the multiplet including $D$ and $D^*$ states fill in the same multiplet, $H$, forming the lowest spin partners. Their low-energy dynamics is dominated by interactions with Nambu-Goldstone (NG) bosons, pions [18–21]. Introducing the multiplet including $D$ and $D^*$ states fill in the same multiplet, $H$, forming the lowest spin partners. Their low-energy dynamics is dominated by interactions with Nambu-Goldstone (NG) bosons, pions [18–21]. Introducing the multiplet including $D$ and $D^*$ states fill in the same multiplet, $H$, forming the lowest spin partners. Their low-energy dynamics is dominated by interactions with Nambu-Goldstone (NG) bosons, pions [18–21]. Introducing the multiplet including $D$ and $D^*$ states fill in the same multiplet, $H$, forming the lowest spin partners. Their low-energy dynamics is dominated by interactions with Nambu-Goldstone (NG) bosons, pions [18–21].

In-medium modifications of the charmed mesons have been extensively studied by using QCD sum rules [7–9] and effective theories [10–16]. In constructing effective Lagrangians for the heavy-light mesons, besides spontaneous chiral symmetry breaking, heavy quark symmetry [6] is a vital ingredient [17]. The pseudo-scalar mesons are deconfined together with light-flavor mesons [4]. Given those observations, comprehensive exploration for the chiral aspects of the heavy-light hadrons is important input to disentangle the transport properties of hadronic matter and QGP.

In-medium modifications of the charmed mesons have been extensively studied by using QCD sum rules [7–9] and effective theories [10–16]. In constructing effective Lagrangians for the heavy-light mesons, besides spontaneous chiral symmetry breaking, heavy quark symmetry [6] is a vital ingredient [17]. The pseudo-scalar mesons are deconfined together with light-flavor mesons [4]. Given those observations, comprehensive exploration for the chiral aspects of the heavy-light hadrons is important input to disentangle the transport properties of hadronic matter and QGP.

2. EFFECTIVE LAGRANGIAN

To describe the light-quark sector, we take the standard linear sigma model Lagrangian with three flavors:

$$\mathcal{L}_L = \bar{q} \left( i \gamma_\mu \partial^\mu - g T^a (\sigma^a + i \gamma_5 \pi^a) \right) q + tr \left[ \partial_\mu \Sigma^\dagger \cdot \partial^\mu \Sigma \right] - V_L (\Sigma), \quad (2.1)$$

where the potential, including $U(1)_A$ breaking effects, is

$$V_L = m^2 tr \left[ \Sigma^\dagger \Sigma \right] + \lambda_1 \left( tr \left[ \Sigma^\dagger \Sigma \right] \right)^2$$
$$+ \lambda_2 tr \left[ (\Sigma^\dagger \Sigma)^2 \right] - c \left( det \Sigma + det \Sigma^\dagger \right)$$
$$- tr \left[ h \left( \Sigma + \Sigma^\dagger \right) \right], \quad (2.2)$$

with the chiral field $\Sigma = T^a \Sigma^a = T^a (\sigma^a + i \pi^a)$ as a 3 x 3 complex matrix in terms of the scalar $\sigma^a$ and the pseudoscalar $\pi^a$ states. The last term with $h = T^a h^a$ breaks the chiral symmetry explicitly.

Heavy-light meson fields with negative and positive
parity are introduced as \[ H = \frac{1 + \gamma_5}{2} \left[ P^* \gamma^\mu + i P \gamma_5 \right], \quad (2.3) \]
and chiral eigenstates are given from those parity eigenstates via
\[ \mathcal{H}_{L,R} = \frac{1}{\sqrt{2}} (G \pm i H \gamma_5). \quad (2.5) \]
The field operators are transformed under the chiral and heavy quark symmetries as
\[ \mathcal{H}_{L,R} \rightarrow S \mathcal{H}_{L,R} g^i_{L,R}, \quad (2.6) \]
\[ \Sigma \rightarrow g_L \Sigma g_R, \quad (2.7) \]
with the group elements \( g_{L,R} \in SU(3)_{L,R} \) and \( S \in SU(2)_{Q=c}. \)
The lowest-order Lagrangian to first order in \( \Sigma \) and to zeroth order in \( 1/m_Q \) up to two heavy-light fields is
\[ \mathcal{L}^{\text{kin}} = \frac{1}{2} \text{Tr} \left[ \mathcal{H}_L iv \cdot \partial \mathcal{H}_L + \mathcal{H}_R iv \cdot \partial \mathcal{H}_R \right], \quad (2.8) \]
\[ V^{(2)}_{HL} = \frac{m_Q}{2} \text{Tr} \left[ \mathcal{H}_L \mathcal{H}_L + \mathcal{H}_R \mathcal{H}_R \right] \]
\[ + \frac{g_{\pi}}{4} \text{Tr} \left[ \Sigma^i \mathcal{H}_L \mathcal{H}_R + \Sigma \mathcal{H}_R \mathcal{H}_L \right] \]
\[ - \frac{g_A}{2F_\pi} \text{Tr} \left[ \gamma_5 \phi \Sigma^i \cdot \mathcal{H}_L \mathcal{H}_R - \gamma_5 \phi \Sigma \cdot \mathcal{H}_R \mathcal{H}_L \right], \quad (2.9) \]
where traces are taken over Dirac and light-flavor indices. A self-consistent calculation for the \( \mathcal{H}_{L,R} \) requires a further contribution beyond \( V^{(2)}_{HL} \). Minimal extension for this is to add terms including four heavy-light fields. Following the given transformation properties, one finds in the same order
\[ V^{(4,0)}_{HL} = c_1 \text{Tr} \left[ \mathcal{H}_L \mathcal{H}_L \mathcal{H}_L \mathcal{H}_L + \mathcal{H}_R \mathcal{H}_R \mathcal{H}_R \mathcal{H}_R \right] \]
\[ + c_2 \left( (\text{Tr} \left[ \mathcal{H}_L \mathcal{H}_L \right])^2 + (\text{Tr} \left[ \mathcal{H}_R \mathcal{H}_R \right])^2 \right) \]
\[ + c_3 \text{Tr} \left[ \mathcal{H}_L \mathcal{H}_L \mathcal{H}_R \mathcal{H}_R \right] \]
\[ + c_4 \text{Tr} \left[ \mathcal{H}_L \mathcal{H}_L \mathcal{H}_L \mathcal{H}_R \right] \], \quad (2.10) \]
\[ V^{(4,1)}_{HL} = \kappa_1 \text{Tr} \left[ \Sigma^i \mathcal{H}_L \mathcal{H}_L \mathcal{H}_R \mathcal{H}_R + \Sigma \mathcal{H}_R \mathcal{H}_L \mathcal{H}_L \mathcal{H}_L \right] \]
\[ + \kappa_2 \text{Tr} \left[ \Sigma^i \mathcal{H}_L \mathcal{H}_L \mathcal{H}_L \mathcal{H}_R + \Sigma \mathcal{H}_R \mathcal{H}_L \mathcal{H}_L \mathcal{H}_L \right] \]
\[ + \kappa_3 \text{Tr} \left[ \Sigma^i \mathcal{H}_L \mathcal{H}_L \mathcal{H}_R \mathcal{H}_R \right] \text{Tr} \left[ \mathcal{H}_R \mathcal{H}_R \right] \]
\[ + \kappa_4 \text{Tr} \left[ \Sigma^i \mathcal{H}_L \mathcal{H}_L \mathcal{H}_R \mathcal{H}_R \right] \text{Tr} \left[ \mathcal{H}_R \mathcal{H}_L \right] \]
\[ + \text{Tr} \left[ \Sigma \mathcal{H}_R \mathcal{H}_L \right] \text{Tr} \left[ \mathcal{H}_R \mathcal{H}_R \right] \], \quad (2.11) \]

Explicit \( SU(3) \) symmetry breaking is encoded in the potential with the following replacement,
\[ \Sigma \rightarrow \lambda \hbar \Sigma, \quad (2.12) \]
with \( \lambda \) denoting a set of constant parameters. The entire potential is thus
\[ V_{HL} = V^{(2)}_{HL} + V^{(4,0)}_{HL} + V^{(4,1)}_{HL} + V^{\text{exp}}_{HL}. \quad (2.13) \]

For our thermodynamic calculations, we employ the mean field approximation. The \( SU(2) \) isospin violation is also neglected, which leads to \( \sigma_0 \) and \( \sigma_s \) as non-vanishing condensates. Those fields contain both non-strange and strange components. Thus, it is convenient to transpose them into pure non-strange and strange parts, via
\[ \begin{pmatrix} \sigma_q \\ \sigma_s \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} \sqrt{2} & 1 - \sqrt{2} \end{pmatrix} \begin{pmatrix} \sigma_0 \\ \sigma_s \end{pmatrix}. \quad (2.14) \]
The effective quark masses in this base are
\[ M_q = \frac{g}{2} \sigma_q, \quad M_s = \frac{g}{\sqrt{2}} \sigma_s. \quad (2.15) \]
The partially conserved axial current (PCAC) hypothesis relates \( \sigma_q, \sigma_s \) with the weak decay constants for pions and kaons:
\[ \langle \sigma_q \rangle = f_\pi, \quad \langle \sigma_s \rangle = \frac{1}{\sqrt{2}} (2f_K - f_\pi). \quad (2.16) \]
The scalar charmed-mesons are accommodated in the multiplets,
\[ D = (D_q, D_q, D_s), \quad (2.17) \]
where due to the isospin symmetry \( D_u = D_d = D_q \). In-medium masses are calculated from the potential via
\[ \Delta M_{D_i} = M_{D_i} - m_c = \frac{1}{2} \frac{\partial^2 V_{HL}}{\partial D_i^2}, \quad (2.18) \]
with \( i = u, d, s \) and the charm quark mass \( m_c \). From Eq. (2.13), one finds
\[ \Delta M_{D}(0^+) = m_0 + \frac{1}{4} g_\pi^2 \sigma_q + 2k_0 (4D_q^2 + D_s^2) \]
\[ + 6k_0 \sigma_q D_q^2, \quad (2.19) \]
\[ \Delta M_{D_0}(0^+) = m_0 + \frac{1}{4} g_\pi^2 \sigma_q + 2k_0 (2D_q^2 + 3D_s^2) \]
\[ + 6\sqrt{2} k_s \sigma_s D_s^2, \quad (2.20) \]
\[ \Delta M_{D}(0^-) = m_0 - \frac{1}{4} g_\pi^2 \sigma_q + 2k_0 (4D_q^2 + D_s^2) \]
\[ - 6\sqrt{2} k_s \sigma_s D_s^2, \quad (2.21) \]
\[ \Delta M_{D_0}(0^-) = m_0 - \frac{1}{4} g_\pi^2 \sigma_q + 2k_0 (2D_q^2 + 3D_s^2) \]
\[ - 6\sqrt{2} k_s \sigma_s D_s^2, \quad (2.22) \]
where \( k_0, g_\pi^2, g_\sigma^2, k_q, k_s \) are functions of the parameters \( \epsilon \)'s, \( \kappa \)'s and \( \lambda \)'s.
In the mean field approximation, thermodynamics of this system is described by the following potential:

\[ \Omega = \Omega_q + V_L + V_{HL}, \]

\[ \Omega_q = 6T \sum_{f=u,d,s} \int \frac{d^3p}{(2\pi)^3} \left[ \ln (1 - n_f) + \ln (1 - \bar{n}_f) \right], \]

(2.23)

(2.24)

with the Fermi-Dirac distribution functions \( n_f, \bar{n}_f = 1/(1 + e^{E_f/T}) \) and the quasi-quark energies \( E_f = \sqrt{p^2 + M_f^2} \). By minimizing the thermodynamic potential, the four mesonic mean-fields are determined self-consistently at a given \( T \) and \( \mu_f \) via

\[ \frac{\partial \Omega}{\partial \sigma_q} = \frac{\partial \Omega}{\partial \sigma_s} = \frac{\partial \Omega}{\partial D_q} = \frac{\partial \Omega}{\partial D_s} = 0. \]

(2.25)

In the thermal model applied in heavy-ion collisions, temperature \( T \) and baryon chemical potential \( \mu_B \) are independent parameters. On the other hand, the strange and charm chemical potentials are fixed via strange and charm number conservation \[24\]. The chemical potential for a particle \( i \) is introduced as

\[ \mu_i = \mu_B B_i + \mu_u S_i + \mu_c C_i, \]

(2.26)

with the baryon number \( B_i \), strangeness \( S_i \) and charm \( C_i \) quantum numbers. In term of the particle number density,

\[ n_i = g_i \frac{1}{2\pi^2} \int_0^\infty dp \frac{p^2}{e^{(E_\mu_i)/T} + 1}, \]

(2.27)

with the degeneracy factor \( g_i \), the conservation conditions are give as

\[ V \sum_i n_i S_i = 0, \quad V \sum_i n_i C_i = 0, \]

(2.28)

where \( V \) denotes the volume of a system. In the following, we will take \( \mu_B = 0 \), so that the strange and charm conservation are fulfilled trivially, \( \mu_s = \mu_c = 0 \).

3. CHIRAL THERMODYNAMICS IN THE MEAN-FIELD APPROXIMATION

Conventionally model parameters are fixed at zero temperature so as to reproduce the meson masses and decay constants. However, regarding effective models as an approximation of QCD in low energy, the parameters in the effective Lagrangian vary with temperature. Such intrinsic thermal effects are carried by the higher-lying hadrons, and formally introduced via functional integration in deriving Green’s functions. At finite temperature, a reliable way to extract those effective parameters is to match them with the observables in lattice QCD.

![FIG. 1: Thermal expectation values of the mean fields, \( \sigma_q \) and \( \sigma_s \). The pseudo-critical temperature fixed from the chiral susceptibility is \( T_{pc} = 154 \text{ MeV} \).](image)

In the light-flavor sector, the sigma meson mass \( m_\sigma \) is often treated as an adjustable parameter. In the vacuum the nature of the lowest-lying scalar state remains not fully understood yet due to a strong mixing to other states with the same quantum number, e.g. a tetra-quark state \[25\]. Thus, we use \( m_\sigma \) as a parameter to adjust the pseudo-critical temperature of chiral symmetry restoration to the lattice result, \( T_{pc} = 154 \text{ MeV} \) extracted from the chiral susceptibility \( \partial(q\bar{q})/\partial m_q \[26\]. The resultant choice in the linear sigma model is \( m_\sigma = 400 \text{ MeV} \), we will use other parameters fixed in the vacuum \[27\], summarized in Table [I].

The parameters in the heavy-light sector at zero temperature are determined as in Table [II] where the following input was used: \( m_e = 1.27 \text{ GeV} \), \( M_D(0^-) = 1.868 \text{ GeV} \), \( M_{D_s}(0^-) = 1.969 \text{ GeV} \), \( M_{D_s}(0^+) = 2.318 \text{ GeV} \), and the pion and kaon decay constants \( f_\pi = 92.4 \text{ MeV} \), \( f_K = 113 \text{ MeV} \[28\]. The current experimental value for the \( D(0^+) \) mass by the Particle Data Group has a rather large error, \( M_{D_{PDG}}(0^+) = 2318 \pm 29 \text{ MeV} \). With the given parameters, the model predicts \( M_D(0^+)/M_{D_{PDG}}(0^+) = 0.96 \). The coupling constant \( g_\pi^D \) is extracted also from the decay mode, \( D(0^+) \rightarrow D(0^-)\pi \), yielding \( g_{\pi_{PDG}}^D = 3.6 = 0.95 \times g_\pi^D \).

In Fig. [I] we show thermal expectation values of the \( \sigma_q \) and \( \sigma_s \) in the mean field approximation. One readily sees that the chiral crossover is extremely broad; at \( T_{pc} \) the order parameter \( \sigma_q \) reaches \( \sim 0.73 \times \sigma_q(T = 0) \), whereas in lattice QCD the bilinear quark condensate drops more rapidly to almost a half of its vacuum value \[29\]. Also, the \( \sigma_s \) is accompanied to a large extent by the non-strange condensate \( \sigma_q \), which is again inconsistent with the lattice observation.

This is traced back to a rather strong mixing between the light and heavy-light mean fields. The charmed-meson mean fields act as an extra source which breaks the chiral symmetry explicitly. The effective symmetry
In the light-flavor sector, a hierarchy lying among up, down and strange flavors is spoiled since the heavy quark symmetry over-influences the light-flavored quark condensates. Therefore, the correct tendency needs to be restored by controlling the effective interaction between the light and heavy-light mesons. Practically, we need to constrain the size of $h_q^*/h_s^*$ within a certain range. To this end, we dictate in-medium changes to the parameters in the Lagrangian with the use of a reliable set of $\sigma_{q,s}(T)$. Possible profiles for the sigma fields are as given in Fig. 2 which are consistent with the quark condensates calculated in the $N_f = 2 + 1$ lattice QCD [26]. In order to control a relative size of $\sigma_q$ to $\sigma_s$, we will introduce thermal modifications into the parameters in the strange sector, $g_{q}^s$ and $k_s$, whereas the non-strange parameters are kept to be the values fixed at $T = 0$ #2. The effective couplings $g_{q}^s(T)$ and $k_s(T)$ are determined by solving the gap equations (2.25) with the given profiles for the $\sigma_{q,s}$. In Fig. 3 one finds that those interactions decrease their strengths as the system approaches the chiral symmetry restoration, so that the overall symmetry breaking $h_s^*$ is not reduced significantly. This is shown in Fig. 4.

The charmed-meson masses (2.22) are now calculated consistently to the lattice result in the light-flavor sector, as summarized in Fig. 5. The parity partners approach each other as temperature is increased both in the non-strange and strange sector, in consistent with the chiral restoration. The two pseudo-scalar states have the same trend that their masses are increasing with temperature, although the non-strange meson mass exhibits a rather weak modification. On the other hand, the two scalar states drop significantly; the non-strange meson mass by ∼200 MeV and the strange meson mass by ∼100 MeV. The mass splittings between the non-strange and strange states are around 200 MeV above $T_{pc}$ due to the fact that

#2 Alternatively, one can use $g_{q}^s$ and $k_s$ being their vacuum values and introduce $g_{q}^s(T)$ and $k_s(T)$. This does not alter our main conclusion.

### Table I: Set of parameters in the light sector with $m_s = 400$ MeV [27].

| Parameter | Value |
|-----------|-------|
| $m_0$ [GeV] | 1.04 |
| $g_q^s$ | 3.78 |
| $g_s^s$ | 2.61 |
| $k_0$ [1/GeV$^2$] | -(1/0.74)$^2$ |
| $k_q$ [1/GeV$^3$] | -(1/0.44)$^3$ |
| $k_s$ [1/GeV$^3$] | -(1/0.53)$^3$ |

### Table II: Set of parameters in the heavy-light sector.

### Fig. 2: Assumed profiles of thermal expectation values of $\sigma_q$ and $\sigma_s$ with $T_{pc} = 154$ MeV.

\[
\begin{align*}
    h_q^* &= h_q - D_q^2 \left( \frac{1}{2} g_q^2 + 2 k_q D_q^2 \right), \\
    h_s^* &= h_s - \frac{1}{\sqrt{2}} D_s^2 \left( \frac{1}{2} g_s^2 + 2 k_s D_s^2 \right).
\end{align*}
\]
the chiral symmetry in the strange sector is not restored yet. Nevertheless, the chiral mass splittings between the scalar and pseudo-scalar states are almost of the same size,

\[ \delta M_D(T_{pc}) \sim \delta M_{\Delta s}(T_{pc}) \sim 200 \text{ MeV}, \]

i.e. the chiral mass differences in the heavy-light sector are blind to the light flavors. This is a striking difference from the chiral properties of the light mesons, and is attributed to the heavy quark symmetry possessed by the leading-order Lagrangian in \( 1/m_Q \) expansion. In contrast, the chiral \( SU(4) \) model, where the charmed mesons are treated on the equal footing to the non-strange and strange mesons, yields a qualitatively different result from Eq. (3.3): \( \delta M_D \) is much smaller than \( \delta M_{\Delta s} \), similar to the light meson masses \[10\].

The effective coupling \( g_\pi(T) \) also affects the hadronic decays involving the \( D_s \) states. Those decay modes violate the isospin symmetry and thus they are suppressed. This is followed dominantly via the \( \pi^0 - \eta \) mixing \[29\]. The decay width of \( D_s(0^+) \to D_s(0^-) + \pi^0 \) is

\[ \Gamma \simeq \frac{(g_\pi^2)^2}{4\pi} \bar{p}_\pi \delta_{\pi^0 \eta}^2, \]

where \( \bar{p}_\pi = |\bar{p}_\pi| \) is the three-momentum of the pion in the rest frame of the decaying particle, and the \( \pi^0 - \eta \) mixing is given by

\[ \delta_{\pi^0 \eta} = \frac{2m_{\pi}^2(m_u - m_d)}{(m_\eta^2 - m_\pi^2)(m_u + m_d)} \delta_{\pi^0 \eta}. \]

We recall that the \( g_\pi^2 \) decreases with increasing temperature as given in Fig. 3 i.e. the \( D_s \) meson tends to be decoupled from the light-flavor sector. On top of the small isospin breaking, this decay mode becomes more suppressed in approaching \( T_{pc} \), due to the \( g_\pi^2(T) \) which controls the onset of chiral symmetry restoration. The decay process, \( D_s(1^+) \to D_s(1^-) + \pi^0 \), is quenched as well.

### 4. RESULTS IN CHIRAL PERTURBATION THEORY

At zero temperature, the composition of the lowest scalar meson is nontrivial because of a strong mixing between the conventional quarkonium and tetra-quark states. The scalar state around 1 GeV is a good candidate for the lowest \( \bar{q}q \)-dominated meson. Thus, as long as a characteristic temperature is lower than the chiral crossover, the sigma meson in the linear sigma model can be integrated out since it lies well above the pion mass scale. The resultant Lagrangian contains only the NG bosons and the chiral symmetry is non-linearly realized. The basic building block is the 1-form \( \alpha_\perp \),

\[ \alpha_\perp^\mu = \frac{1}{2i} \left( \partial^\mu \xi \cdot \xi^\dagger - \partial^\mu \xi^\dagger \cdot \xi \right), \]
where the NG bosons $\pi$ are embedded in $\xi$ as $\xi = e^{i\pi/T}$. The interaction to the heavy-light meson fields (2.4) is quantified by the following Lagrangian [30]:
\[
L_{\text{int}} = k \left( \text{Tr} \left[ H \gamma_{\mu} \gamma_5 \gamma^\mu \bar{H} \right] + \text{Tr} \left[ G \gamma_{\mu} \gamma_5 \gamma^\mu \bar{G} \right] \right) - i \text{Tr} \left[ G_{\perp \mu} \gamma^\mu \gamma_5 \bar{H} \right] + i \text{Tr} \left[ H_{\perp \mu} \gamma^\mu \gamma_5 \bar{G} \right],
\]
with the coupling constant $k = 0.59$ extracted from the decay $D^* \to D\pi$. At finite temperature, thermal corrections to the charmed-meson masses are induced from the self-energy. The major temperature dependence is carried by the NG bosons. Therefore, we approximate the in-medium propagator of the D mesons to their vacuum forms. The pion propagator is replaced as
\[
\frac{1}{p^2 - m_\pi^2 + i\epsilon} \to \frac{1}{p^2 - m_\pi^2 + i\epsilon} - \frac{2\pi i}{e^{\beta\omega} - 1} \delta(p^2 - m_\pi^2).
\]

Substituting those propagators to the self-energy at one loop [30], one obtains the following thermal corrections to the scalar ($\Pi^S$) and the pseudo-scalar ($\Pi^P$) states:
\[
\Pi^S = C_2(N_f) \frac{k^2}{f_\pi^2} \int \frac{d^3p}{(2\pi)^3} n_\pi(m_\pi, T) \left( M_P + M_S + \frac{m_\pi^2}{M_S} \right) A_0^S(m_\pi, T) - \left( M_P + M_S + \frac{m_\pi^2}{M_S} \right) A_0^P(m_\pi, T) - \frac{M_P^2 - m_\pi^2}{M_P} A_0^P(m_\pi, T) - M_P A_P^P(m_\pi, T),
\]
\[
\Pi^P = C_2(N_f) \frac{k^2}{f_\pi^2} \int \frac{d^3p}{(2\pi)^3} n_\pi(m_\pi, T) \left( M_P + M_S + \frac{m_\pi^2}{M_S} \right) A_0^S(m_\pi, T) - \left( M_P + M_S + \frac{m_\pi^2}{M_S} \right) A_0^P(m_\pi, T) - \frac{M_P^2 - m_\pi^2}{M_P} A_0^P(m_\pi, T) - M_P A_P^P(m_\pi, T),
\]
where $M_{S,P}$ represents the scalar (pseudo-scalar) charmed meson mass, and the functions $A_0^S$ and $A_0^P$ are introduced in terms of the Bose-Einstein distribution function $n_\pi$ as
\[
n_\pi = \frac{1}{e^{\omega/T} - 1}, \quad \omega = \sqrt{p^2 + m_\pi^2},
\]
\[
A_0^S = \int \frac{d^3p}{(2\pi)^3} \frac{n_\pi(\omega)}{\omega},
\]
\[
A_0^P = \int \frac{d^3p}{(2\pi)^3} \frac{n_\pi(\omega)}{\omega + M_{S,P}}.
\]
The group factor $C_2(N_f)$ is defined as $(T^n)_{ij}(T^n)_{ji} = C_2(N_f)\delta_{ij}$. The chiral mass difference is thus
\[
\delta M = C_2(N_f) \frac{k^2}{f_\pi^2} m_\pi^2 \left( \frac{1}{M_P} + \frac{1}{M_S} \right) A_0^S(m_\pi) + \frac{1}{M_S} A_0^S(m_\pi) - \frac{1}{M_P} A_0^P(m_\pi),
\]
which is less sensitive to a temperature rise. If the mass parameters $M_{S,P}$ carry a certain temperature dependence so that $M_S$ approaches $M_P$ as temperature is increased, the chiral partners tend to be degenerate. Such non-trivial intrinsic thermal effects must show up beyond the standard one-loop perturbative method.

In Fig. 6 the results are compared with the effective masses of the non-strange charmed mesons discussed in the previous section. The pion loop yields a monotonic decrease with temperature for the scalar and pseudo-scalar states. A difference from the mean-field theory starts to appear at rather low temperature, $T \sim f_\pi \sim 0.67 m_\pi$. This temperature cannot be fixed solely by the symmetries, but relies on the dynamics of the theories, in particular the onset of chiral criticality. This is absent in the standard chiral perturbation theory, whereas present in the mean-field theory considered in the previous section via a self-consistent prescription. The agreement with the self-consistent result will be expected at a higher temperature when higher loops and/or more resonances are included.
in lattice QCD. The coupling of the strange charmed meson can be extracted from the chiral condensates calculated depending on temperature are introduced, which can be extracted from the chiral condensates calculated in lattice QCD. The coupling of the strange charmed meson to the sigma meson, \( g_s^s \), becomes quenched as temperature is increased toward the chiral pseudo-critical point \( T_{pc} \). Our main result is that the chiral mass splittings are essentially insensitive to the light-quark flavors, in spite of a non-negligible explicit breaking of the chiral \( SU(3) \) symmetry. This “blindness” of the charm quark to the light degrees of freedom is dictated by the heavy quark symmetry. In contrast, the kaon and its chiral partner masses become degenerate at a higher temperature than \( T_{pc} \), indicating a delay of the \( SU(3) \) symmetry restoration. In particular, the dissociation of the charmed mesons needs to be reexamined.

Application of our approach to a dense system requires further implementation of (i) strange and charm number conservation, and (ii) reliable constraint(s) on the effective interaction between the light and heavy-light mesons. At zero density, the latter has been provided by the lattice chiral condensates. Alternatively, a more microscopic framework will enable us to derive the intermediate coupling as a function of temperature and density. This becomes an essential input especially for the study of various charge fluctuations \[33\].

5. CONCLUSIONS

In this paper we have formulated a chiral mean-field theory for the light and heavy-light mesons at finite temperature based on the heavy quark symmetry. In order to avoid an unrealistically strong mixing between the light-flavor and the charmed meson sector, effective interactions depending on temperature are introduced, which can be extracted from the chiral condensates calculated in lattice QCD. The coupling of the strange charmed meson to the sigma meson, \( g_s^s \), becomes quenched as temperature is increased toward the chiral pseudo-critical point \( T_{pc} \). Our main result is that the chiral mass splittings are essentially insensitive to the light-quark flavors, in spite of a non-negligible explicit breaking of the chiral \( SU(3) \) symmetry. This “blindness” of the charm quark to the light degrees of freedom is dictated by the heavy quark symmetry. In contrast, the kaon and its chiral partner masses become degenerate at a higher temperature than \( T_{pc} \), indicating a delay of the \( SU(3) \) symmetry restoration. In the heavy-light sector, on the other hand, the strange charmed meson captures the onset of chiral symmetry restoration more strongly than the strange light meson does. The quenched \( g_s^s \) leads also to a strong suppression of the scalar \( D_s \) decay toward \( T_{pc} \), on top of the suppression due to the small isospin violation. The same should be carried over to the \( B \) and \( B_s \) mesons with which the heavy quark symmetry is more reliable.

Although the present model does not enable to handle a confinement/deconfinement transition, it reliably captures the chiral aspects of the charmed mesons constrained by the heavy flavor symmetry. Given the lattice QCD observations that non-strange and strange hadrons seem to be resolved to their constituents around the chiral crossover, the system contains low-lying mesons and the chiral fermions (quarks) when the baryon chemical potential is sufficiently small. The main result shown in this work will be robust in hadronic phase up to the chiral restoration point as long as there is no strong first-order transition.

Those medium modifications may yield some consequences on the nuclear modification factor and the elliptic flow in high-energy heavy-ion collisions, on top of the effects discussed in \[31\] where the scalar \( D \) and \( D_s \) states are chirally unmodified. Scenarios of charm-quark suppression \[32\] will certainly be affected by the modified heavy-light mesons around the chiral symmetry restoration. In particular, the dissociation of the charmed mesons needs to be reexamined.

Acknowledgments

We acknowledge stimulating discussions with K. Redlich. This work has been partly supported by the Hessian LOEWE initiative through the Helmholtz International Center for FAIR (HIC for FAIR), and by the Polish Science Foundation (NCN) under Maestro grant 2013/10/A/ST2/00106.

\[1\] R. Rapp and H. van Hees, R. C. Hwa, X.-N. Wang (Ed.) Quark Gluon Plasma 4, World Scientific, 111 (2010).
\[2\] A. Adare et al. [PHENIX Collaboration], Phys. Rev. C 84, 044905 (2011).
\[3\] L. Adamczyk et al. [STAR Collaboration], arXiv:1404.0185 [nucl-ex].
\[4\] B. Abelev et al. [ALICE Collaboration], JHEP 1209, 112 (2012).
\[5\] B. Abelev et al. [ALICE Collaboration], Phys. Rev. Lett. 111, 102303 (2013).
\[6\] A. Bazavov, H.-T. Ding, P. Hegde, O. Kaczmarek, F. Karsch, E. Laermann, Y. Maezawa and S. Mukherjee et al., Phys. Lett. B 737, 210 (2014).
\[7\] A. Hayashigaki, Phys. Lett. B 487, 96 (2000).
\[8\] T. Hilger, R. Thomas and B. Kämpfer, Phys. Rev. C 79, 025202 (2009).
\[9\] Z. G. Wang and T. Huang, Phys. Rev. C 84, 048201 (2011).
\[10\] D. Roder, J. Ruppert and D. H. Rischke, Phys. Rev. D 68, 016003 (2003).
[11] A. Mishra, E. L. Bratkovskaya, J. Schaffner-Bielich, S. Schramm and H. Stoecker, Phys. Rev. C 69, 015202 (2004).
[12] M. F. M. Lutz and C. L. Korpa, Phys. Lett. B 633, 43 (2006).
[13] L. Tolos, A. Ramos and T. Mizutani, Phys. Rev. C 77, 015207 (2008).
[14] D. Blaschke, P. Costa and Y. L. Kalinovsky, Phys. Rev. D 85, 034005 (2012).
[15] R. Molina, D. Gamermann, E. Oset and L. Tolos, Eur. Phys. J. A 42, 31 (2009).
[16] S. Yasui and K. Sudoh, Phys. Rev. C 87, 015202 (2013).
[17] N. Isgur and M. B. Wise, Phys. Lett. B 232, 113 (1989); Phys. Lett. B 237, 527 (1990).
[18] H. Georgi, Phys. Lett. B 240, 447 (1990).
[19] M. B. Wise, Phys. Rev. D 45, 2188 (1992).
[20] G. Burdman and J. F. Donoghue, Phys. Lett. B 280, 287 (1992).
[21] T. M. Yan, H. Y. Cheng, C. Y. Cheung, G. L. Lin, Y. C. Lin and H. L. Yu, Phys. Rev. D 46, 1148 (1992) [Erratum-ibid. D 55, 5851 (1997)].
[22] M. A. Nowak, M. Rho and I. Zahed, Phys. Rev. D 48, 4370 (1993).
[23] W. A. Bardeen and C. T. Hill, Phys. Rev. D 49, 409 (1994).
[24] P. Braun-Munzinger and J. Stachel, Phys. Lett. B 490, 196 (2000).
[25] J. Beringer et al. [Particle Data Group Collaboration], Phys. Rev. D 86, 010001 (2012).
[26] A. Bazavov, T. Bhattacharya, M. Cheng, C. DeTar, H. T. Ding, S. Gottlieb, R. Gupta and P. Hegde et al., Phys. Rev. D 85, 054503 (2012).
[27] B. J. Schaefer and M. Wagner, Phys. Rev. D 79, 014018 (2009).
[28] S. Borsanyi, G. Endrodi, Z. Fodor, S. D. Katz, S. Krieg, C. Ratti, C. Schroeder and K. K. Szabo, PoS LATTICE 2011, 201 (2011).
[29] W. A. Bardeen, E. J. Eichten and C. T. Hill, Phys. Rev. D 68, 054024 (2003).
[30] M. Harada, M. Rho and C. Sasaki, Phys. Rev. D 70, 074002 (2004).
[31] M. He, R. J. Fries and R. Rapp, Phys. Rev. Lett. 110, no. 11, 112301 (2013).
[32] R. Rapp, D. Blaschke and P. Crochet, Prog. Part. Nucl. Phys. 65, 209 (2010).
[33] C. Sasaki and K. Redlich, in progress.