One loop gauge couplings in AdS$_5$

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Abstract

We calculate the full 1-loop corrections to the low energy coupling of bulk
gauge boson in a slice of AdS$_5$ which are induced by generic 5-dimensional
scalar, Dirac fermion, and vector fields with arbitrary $Z_2 \times Z'_2$ orbifold boundary conditions. In supersymmetric limit, our results correctly reproduce the
results obtained by an independent method based on 4-dimensional effective supergravity. This provides a nontrivial check of our results and assures the
regularization scheme-independence of the results.
I. INTRODUCTION

Models with extra dimension have provided a new insight on the large scale hierarchy between the Weak scale $M_W \sim 10^{2}$ GeV and the Planck scale $M_{Pl} \sim 10^{18}$ GeV. In this regard, the Randall-Sundrum model (RS1) is particularly interesting as it explains the Weak to Planck scale ratio using the warped 5D geometry [1]:

$$ds^2 = G_{MN}dx^Mdx^N = e^{-2kR|y|}g_{\mu\nu}dx^\mu dx^\nu + R^2dy^2,$$

where $-\pi \leq y \leq \pi$, $k$ is the AdS curvature and $R$ is the orbifold radius. In this spacetime background, 4-dimensional (4D) graviton is localized near the UV brane at $y = 0$ whose cutoff mass scale $M_{UV}$ is of order the 5D Planck scale. On the other hand, in the original RS1 model, all the standard model (SM) fields are assumed to be confined on the IR brane at $y = \pi$ whose cutoff scale $M_{IR} \sim e^{-\pi kR}M_{UV}$. Then with a moderately large value of $kR$ ($\sim 12$), the model can generate the large scale hierarchy $M_{Pl}/M_W \sim M_{UV}/M_{IR} \sim 10^{16}$ without any severe fine tuning of the fundamental parameters.

An apparent drawback of the original RS1 model is that one has to abandon the attractive possibility that the SM gauge couplings $g_a^2$ ($a = 1, 2, 3$) are unified at high energy scale through the quantum corrections calculable within the model. Experimental data show that $g_a^2$ at $M_W$ differ from each other by order unity:

$$\frac{1}{g_a^2(M_W)} - \frac{1}{g_b^2(M_W)} = \mathcal{O}(1) \quad (a \neq b).$$

On the other hand, the size of quantum corrections to $1/g_a^2$ which are calculable within the RS1 model is

$$\Delta \left(\frac{1}{g_a^2}\right) = \mathcal{O} \left(\frac{1}{8\pi^2} \ln(\frac{M_{Pl}^2}{M_W^2})\right) = \mathcal{O} \left(\frac{1}{8\pi^2}\right),$$

so the RS1 model does not give any insight on why the SM gauge couplings at $M_W$ differ from each other by order unity.

It has been noted recently [2–7] that one can achieve the gauge unification, while still solving the hierarchy problem, within the 5D effective field theory on AdS$_5$ if the SM gauge bosons propagate in 5D bulk spacetime. In such case, the size of quantum corrections calculable within the model is

$$\Delta \left(\frac{1}{g_a^2}\right) = \mathcal{O} \left(\frac{1}{8\pi^2} \ln(\frac{M_{Pl}^2}{M_W^2})\right) = \mathcal{O}(1),$$

as in the case of conventional 4D grand unified theories (GUT). This allows that the observed differences of gauge couplings are explained in terms of quantum corrections which are calculable within the model.

Calculation of the 1-loop corrections to gauge coupling in AdS$_5$ was first attempted in [2] for a GUT model in which all gauge-charged matter fields are confined on the UV brane. The computation involves a Pauli-Villars regulator with regulator mass $\Delta_{PV} \ll k$, so could catch only the corrections at scales significantly below $k$. In [3], a momentum cutoff depending on the position in 5-th dimension was proposed to regulate the 1-loop corrections.
Though intuitively sensible, it is difficult to isolate the regulator-independent part from the regulator-dependent total corrections in this regularization, which makes the interpretation of the results unclear. In [4,7], the 1-loop corrections have been computed for generic supersymmetric gauge theory on AdS\textsubscript{5} using the gauged $U(1)\textsubscript{R}$ symmetry and chiral anomaly in 5D supergravity (SUGRA) and also the known properties of gauge couplings in 4D effective SUGRA. In this approach, one could obtain the 1-loop corrections (including those from scales between $k$ and the 5D cutoff scale $\Lambda > k$) in obviously regulator-independent manner. In [5,6], 1-loop corrections in 5D scalar QED on AdS\textsubscript{5} have been computed (using dimensional regularization and also Pauli-Villars regularization) and the results are nicely interpreted in terms of AdS/CFT correspondence.

In this paper, we present the full 1-loop corrections to the low energy coupling of bulk gauge boson in a slice of AdS\textsubscript{5} which are induced by generic 5D scalar, Dirac fermion and vector fields with arbitrary $Z\times Z'$ orbifold boundary condition. To be explicit, we adopt dimensional regularization [8], but the results should be independent of the used regularization scheme as they correspond to the scheme-independent corrections calculable within 5D effective field theory. When applied to supersymmetric case [9,10], our results correctly reproduce the expressions which are obtained in a completely independent approach based on 4D effective SUGRA. This provides a nontrivial check of our results, and also assures the scheme-independence of the results. We also note that the subtraction scales of log divergences at two orbifold fixed points, i.e. $y = 0$ and $\pi$, differ by the warp factor $e^{-\pi kR}$. This is physically expected, and can be confirmed by comparing the results with those of Pauli-Villars regularization as well as with the results of 4D SUGRA calculation.

The organization of this paper is as follows. In section II, we set up the notations for 5D gauge theory on a slice of AdS\textsubscript{5} including the Kaluza-Klein (KK) analysis for generic 5D scalar, Dirac fermion, and vector fields with arbitrary $Z\times Z'$ orbifold boundary condition. The lagrangian is given by

$$\int d^4 x dy \sqrt{-G} \left[ -\frac{1}{4g_{5a}^2} F^{aMN} F_{aMN} - \frac{1}{2} D_M \phi D^M \phi - \frac{1}{2} m_\phi^2 \phi^2 - i \bar{\psi} (\gamma^M D_M + m_\psi) \psi \right],$$

where $D_M$ is the covariant derivative containing the gauge connections as well as the spin connection of AdS\textsubscript{5}. We parametrize the masses of scalar and fermion fields as

$$m_\phi^2 = A^2 k^2 + \frac{2k}{R} \left[ B_0 \delta(y) - B_\pi \delta(y - \pi) \right], \quad m_\psi = C k \epsilon(y),$$

where $\epsilon(y) = y/|y|$, $B_0$ and $B_\pi$ are the brane mass parameters at $y = 0$ and $y = \pi$, respectively, and $c$ is the fermion kink mass parameter. The 5D fields in the model can have arbitrary $Z\times Z'$ orbifold boundary condition.
\begin{align}
\phi(-y) &= Z_\phi \phi(y), \\
\psi(-y) &= Z_\psi \gamma_5 \psi(y), \\
A^a_\mu(-y) &= Z_a A^a_\mu(y),
\end{align}
(7)

with $Z_\phi = \pm 1$ and $Z'_{\phi} = \pm 1$ for $\Phi = \{\phi, \psi, A^a_\mu\}$ and $y' = y - \pi$. Though we are interested in the low energy coupling of $A^a_\mu$ having $Z_a = Z'_a = 1$, there can be 5D vector fields having other $Z_2 \times Z'_2$ parity which are charged for the gauge fields with $Z_a = Z'_a = 1$. Note that the brane mass of scalar field at $y = 0$ ($y = \pi$) is relevant only when $Z_\phi = 1$ ($Z'_\phi = 1$).

The KK spectrum of bulk fields on a slice of AdS$_5$ has been discussed in detail in [10]. It is rather straightforward to generalize the analysis of [10] to the field with arbitrary $Z_2 \times Z'_2$ parity. A generic 5D field $\Phi$ can be decomposed as

$$\Phi(x,y) = \sum \Phi_n(x) f_n(y),$$

where the KK wavefunction $f_n$ satisfies

$$\left[-e^{skR|y|} \frac{\partial}{\partial y} (e^{-skR|y|} \partial_y) + R^2 k^2 \tilde{M}^2_{\phi}\right] f_n = R^2 e^{2skR|y|} m_n^2 f_n$$

for the KK mass eigenvalue $m_n$. Here

$$s = \{2, 4, 1, 1\}$$

and the bulk mass parameters

$$\tilde{M}^2_{\phi} = \{0, A^2, C(C + 1), C(C - 1)\}$$

for

$$\Phi = \{A_\mu, \phi, e^{-2kR|y|} \psi_L, e^{-2kR|y|} \psi_R\} \quad (\psi_{L,R} = \frac{1}{2}(1 \pm \gamma_5) \psi).$$

This determines $f_n$ to be

$$f_n(y) = e^{skR|y|/2} \left[ J_\alpha \left( \frac{m_n}{k} e^{kR|y|} \right) + b_\alpha \left( m_n \right) Y_\alpha \left( \frac{m_n}{k} e^{kR|y|} \right) \right],$$

where

$$\alpha = \sqrt{(s/2)^2 + \tilde{M}^2_{\phi}}.$$ 

To determine the corresponding KK mass spectrum, one needs to impose the orbifold boundary condition. Parity-even condition under the reflection at $y = 0$ or $\pi$ leads to

$$\frac{df_n}{dy} = r k R f_n \quad \text{at } y = 0 \text{ or } \pi,$$

where

$$r = \{0, B_0 \text{ or } B_\pi, -C, C\}$$

(14)
for

\[ \Phi = \{ A_\mu, \phi, e^{-2kR|y|}\psi_L, e^{-2kR|y|}\psi_R \} . \]

Then using Eqs. (11) and (13), one finds

\[ b_\alpha(m_n) = \frac{(s/2 - r)J_\alpha \left( \frac{m_n}{k} e^{kR\tilde{y}} \right) + \frac{m_n}{k} e^{kR\tilde{y}} J'_\alpha \left( \frac{m_n}{k} e^{kR\tilde{y}} \right)}{(s/2 - r)Y_\alpha \left( \frac{m_n}{k} e^{kR\tilde{y}} \right) + \frac{m_n}{k} e^{kR\tilde{y}} Y'_\alpha \left( \frac{m_n}{k} e^{kR\tilde{y}} \right)} \],

where \( \tilde{y} = 0 \) or \( \pi \). Parity-odd condition under the reflection at \( y = 0 \) or \( \pi \) leads to

\[ f_n = 0 \quad \text{at} \quad y = 0 \text{ or } \pi , \]

yielding

\[ b_\alpha(m_n) = -\frac{J_\alpha \left( \frac{m_n}{k} e^{kR\tilde{y}} \right)}{Y_\alpha \left( \frac{m_n}{k} e^{kR\tilde{y}} \right)} . \]

With the above results, the KK spectrum of 5D field \( \Phi \) can be determined by the so-called \( N \)-function \( N(q) = N(-q) \) which has simple zeros at \( q = \pm m_n \neq 0 \):

\[ N(m_n) = 0 . \]

If there exists a massless mode, \( N \) has a double zero at \( q = 0 \). For later use, here we summarize the \( N \)-functions for all \( Z_2 \times Z_2 \) boundary conditions of the corresponding 5D field. Let \( r_0 \) and \( r_\pi \) denote the mass parameters at \( y = 0 \) and \( \pi \), respectively, given by

\[ r_0 = \{ 0, B_0, -C, C \}, \quad r_\pi = \{ 0, B_\pi, -C, C \} \]

for

\[ \Phi = \{ A_\mu, \phi, e^{-2kR|y|}\psi_L, e^{-2kR|y|}\psi_R \} . \]

The \( N \)-function for \( (Z_\Phi, Z'_\Phi) = (+, +) \) is given by

\[ N_{++}(q) = -\left\{ \left( \frac{s}{2} - r_0 \right) J_\alpha \left( \frac{q}{T} \right) + \frac{q}{k} J'_\alpha \left( \frac{q}{k} \right) \right\} \left\{ \left( \frac{s}{2} - r_\pi \right) Y_\alpha \left( \frac{q}{T} \right) + \frac{q}{T} Y'_\alpha \left( \frac{q}{T} \right) \right\} \]

\[ + \left\{ \left( \frac{s}{2} - r_\pi \right) J_\alpha \left( \frac{q}{k} \right) + \frac{q}{T} J'_\alpha \left( \frac{q}{T} \right) \right\} \left\{ \left( \frac{s}{2} - r_0 \right) Y_\alpha \left( \frac{q}{k} \right) + \frac{q}{k} Y'_\alpha \left( \frac{q}{k} \right) \right\} \]

where \( T = ke^{-\pi kR} \). As for the fields with other boundary conditions, i.e. \((Z_\Phi, Z'_\Phi) = (+, -), (-, +), (-, -), \) we find

\[ N_{+-}(q) = -Y_\alpha \left( \frac{q}{k} \right) \left[ \left( \frac{s}{2} - r_0 \right) J_\alpha \left( \frac{q}{k} \right) + \frac{q}{k} J'_\alpha \left( \frac{q}{k} \right) \right] \]

\[ + J_\alpha \left( \frac{q}{T} \right) \left[ \left( \frac{s}{2} - r_0 \right) Y_\alpha \left( \frac{q}{k} \right) + \frac{q}{k} Y'_\alpha \left( \frac{q}{k} \right) \right] , \]

\[ N_{-+}(q) = J_\alpha \left( \frac{q}{k} \right) \left[ \left( \frac{s}{2} - r_\pi \right) Y_\alpha \left( \frac{q}{T} \right) + \frac{q}{T} Y'_\alpha \left( \frac{q}{T} \right) \right] \]

\[ - Y_\alpha \left( \frac{q}{k} \right) \left[ \left( \frac{s}{2} - r_\pi \right) J_\alpha \left( \frac{q}{T} \right) + \frac{q}{T} J'_\alpha \left( \frac{q}{T} \right) \right] , \]

\[ N_{--}(q) = J_\alpha \left( \frac{q}{k} \right) Y_\alpha \left( \frac{q}{T} \right) - J_\alpha \left( \frac{q}{T} \right) Y_\alpha \left( \frac{q}{k} \right) . \]

\[ (21) \]
As we will see in the next section, one can choose an appropriate gauge fixing to make
that the KK spectrum of $A_5$ is determined by the $N$-function of 5D scalar field $\phi$ with a
specific mass:

$$N_{A_5} = N_\phi \quad \text{for} \quad m_\phi^2 = -4k^2 + \frac{4k}{R}(\delta(y) - \delta(y - \pi)).$$

(22)

In fact, one needs to know the asymptotic behaviors of these $N$-functions at $|q| \to \infty$ to
regulate the UV divergence and also the behaviors at $|q| \to 0$ to find the 1-loop couplings
in the IR limit. Some properties of the $N$-functions including those asymptotic behaviors are
summarized in Appendix A.

III. ONE LOOP EFFECTIVE COUPLINGS

In this section, we calculate the 1-loop effective coupling of gauge field zero mode in
AdS$_5$ using the background field method [11] with dimensional regularization [8]. Let us
first describe the calculation scheme. We split the gauge field as

$$A_5^a = \bar{A}_5^a + \tilde{A}_5^a,$$

(23)

where $\bar{A}_5^a$ denotes the background gauge field in the gauge $\bar{A}_5^a = 0$ and $\tilde{A}_5^a$ is the quantum
fluctuation. We choose the gauge fixing term

$$-\frac{1}{2g_{5a}^2} \int d^5 x \sqrt{-G} \left[ e^{2kR|y|} g^{\mu\nu} D_\mu \bar{A}_5^a + \frac{e^{2kR|y|}}{R^2} \partial_y (e^{-2kR|y|} \bar{A}_5^a) \right]^2$$

(24)

where $D_\mu$ is defined by the background gauge field $\bar{A}_5^a$. The corresponding ghost action is given by

$$\int d^5 x \sqrt{-G} \left[ e^{2kR|y|} \bar{\xi}^a D^2 \xi^a + \frac{e^{2kR|y|}}{R^2} \bar{\xi}^a \partial_y (e^{-2kR|y|} \partial_y \xi^a) \right],$$

(25)

where $D^2 = g^{\mu\nu} D_\mu D_\nu$. It is then straightforward to find the following gauge-fixed action which are quadratic in $\bar{A}_5^a$, $\tilde{A}_5^a$ and $\xi^a$:

$$\int d^5 x \left[ -\frac{1}{4g_{5a}^2} \left( -2R \bar{A}_5^a D^2 \bar{A}_5^a + 4R f_{abc} F_\mu^a \bar{A}_5^b \bar{A}_5^c - \frac{2}{R} \bar{A}_5^a \partial_y (e^{-2kR|y|} \partial_y) \bar{A}_5^a \right) \right.$$

$$\left. - \frac{2}{R} e^{-2kR|y|} \bar{A}_5^a D^2 \bar{A}_5^a - \frac{2}{R^2} e^{-2kR|y|} \bar{A}_5^a \partial_y (e^{-2kR|y|} \bar{A}_5^a) \right]$$

$$+ e^{-2kR|y|} R \left\{ \bar{\xi}^a D^2 \xi^a - \frac{1}{R^2} \bar{\xi}^a \partial_y (e^{-2kR|y|} \partial_y \xi^a) \right\}.$$

(26)

The action of scalar and fermion fields can be written as

$$\int d^5 x \left[ e^{-2kR|y|} R \frac{1}{2} \phi (D^2 + \frac{1}{R^2} e^{2kR|y|} \partial_y e^{-4kR|y|} \partial_y - e^{-2kR|y|} m_\phi^2) \phi \right.$$

$$- e^{-3kR|y|} R (\bar{\psi}_L i\gamma^\mu D_\mu \psi_L + \bar{\psi}_R i\gamma^\mu D_\mu \psi_R) - e^{-4kR|y|} (\bar{\psi}_L i\gamma^5 \partial_y \psi_R + \bar{\psi}_R i\gamma^5 \partial_y \psi_L)$$

$$- iRe^{-4kR|y|} m_\psi (\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L) \right]$$

(27)
Note that the quadratic action of $\tilde{A}_\mu^a$ has the same form as the action of 5D real scalar $\phi$ with $m_\phi^2 = -4k^2 + 4kR^{-1}(\delta(y) - \delta(y - \pi))$, justifying the relation (22).

One-loop effective action of the gauge field zero mode can be obtained by integrating out all quantum fluctuation fields at 1-loop order. This procedure yields

$$S_{\text{eff}} = \int d^4x \left( -\frac{\pi R}{4g_5^2} F^{a\mu\nu} F_{a\mu\nu} \right) + \Gamma_\phi[A_\mu] + \Gamma_\psi[A_\mu] + \Gamma_A[A_\mu], \quad (28)$$

where the first term is obviously the tree level action, and $\Gamma_\phi$, $\Gamma_\psi$ and $\Gamma_A$ represent the 1-loop corrections due to the loops of $\phi$, $\psi$, and $A_\mu^a$ (and also the ghost fields $\xi^a$, $\bar{\xi}^a$), respectively:

$$i\Gamma_\phi = -\frac{1}{2} \text{Tr}_\phi \ln \left( -D^2 + M^2(\phi) \right),$$

$$i\Gamma_\psi = \frac{1}{2} \text{Tr}_\psi \ln \left( -D^2 + M^2(\psi) + F_{\mu\nu} J_{1/2}^{\mu\nu} \right),$$

$$i\Gamma_A = -\frac{1}{2} \text{Tr}_A \ln \left( -D^2 + M^2(A_\mu) + F_{\mu\nu} J_{1}^{\mu\nu} \right),$$

$$-\frac{1}{2} \text{Tr}_A \ln \left( -D^2 + M^2(A_5) \right) + \text{Tr}_\xi \ln \left( -D^2 + M^2(\xi) \right). \quad (29)$$

Here we replace the background gauge field $\tilde{A}_\mu^a$ by un-barred $A_\mu^a$, and $M^2(\Phi)$ is the mass-square operator whose eigenvalues $m_n^2$ are determined by the zeros of the corresponding $N$-function. $J_{j}^{\mu\nu}$ is the 4D Lorentz spin generator normalized as $\text{tr}(J_{j}^{\mu\nu} J_{j'}^{\sigma\rho}) = C(j)(g^{\mu\rho} g^{\sigma\nu} - g^{\mu\sigma} g^{\rho\nu})$ where $C(j) = (0, 1, 2)$ for $(j = 0, 1/2, 1)$.

The above 1-loop effective action is divergent, so need to be regulated. As in the case of flat 5D orbifold, the UV divergence structure of 5D gauge theory on AdS$_5$ is given by

$$-\int d^5x \sqrt{-G} \left[ \frac{\gamma_a}{96\pi^3} \Lambda F_{aMN}^a F^a_{MN} + \frac{\ln \Lambda}{32\pi^2} \left( \lambda_0 \frac{\delta(y)}{\sqrt{G_{55}}} + \lambda_\pi \frac{\delta(y - \pi)}{\sqrt{G_{55}}} \right) F_{a\mu\nu}^a F^{a\mu\nu} \right] \quad (30)$$

where the coefficient of linear divergence ($\gamma_a$) is highly sensitive to the used regularization scheme, while those of log divergences at fixed points ($\lambda_{0,\pi}$) are scheme-independent. In dimensional regularization, $\gamma_a = 0$, however this does not have any special physical meaning. As for the coefficients of log divergences, it is straightforward to find [12]

$$\lambda_0 = \frac{1}{24} \left[ T_a(\phi_{++}) + T_a(\phi_{+-}) - T_a(\phi_{-+}) - T_a(\phi_{--}) \right],$$

$$-\frac{23}{24} \left[ T_a(A_{++}^M) + T_a(A_{+-}^M) - T_a(A_{-+}^M) - T_a(A_{--}^M) \right],$$

$$\lambda_\pi = \frac{1}{24} \left[ T_a(\phi_{++}) - T_a(\phi_{+-}) + T_a(\phi_{-+}) - T_a(\phi_{--}) \right],$$

$$-\frac{23}{24} \left[ T_a(A_{++}^M) - T_a(A_{+-}^M) + T_a(A_{-+}^M) - T_a(A_{--}^M) \right], \quad (31)$$

where $T_a(\Phi) = \text{Tr}(T^a_\Phi)$ for the gauge group representation given by $\Phi$, $\phi_{z,z'}$ ($z, z' = \pm$) is 5D real scalar field with $Z_2 \times Z'_2$ parity ($z, z'$), and $A_{z,z'}^M$ is 5D real vector field.

With the UV divergences given by (30), the low energy effective gauge coupling can be written as

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\[ \frac{1}{g_a^2(p)} = \left[ \frac{1}{g_{5a}^2(\Lambda)} + \frac{\gamma_a \Lambda}{24\pi^3} \right] \pi R + \frac{1}{g_{0a}^2(\Lambda)} + \frac{1}{g_{\pi a}^2(\Lambda)} + \frac{1}{8\pi^2} \Delta_a(p, A, B_0, B_\pi, C, k, R, \ln \Lambda) + O(1/\Lambda) \quad (32) \]

where \( p \) is the 4D momentum of the external gauge boson zero mode, \( g_{0a}^2(\Lambda) \) and \( g_{\pi a}^2(\Lambda) \) denote the bare brane gauge couplings at the orbifold fixed points \( y = 0 \) and \( y = \pi \), respectively, and \( O(1/\Lambda) \) stands for the part suppressed by \( 1/\Lambda \). Here the log-divergent piece of (30) and also the conventional momentum running and finite KK threshold corrections are all encoded in \( \Delta_a \). The bare brane couplings \( g_{0a}^2(\Lambda) \) and \( g_{\pi a}^2(\Lambda) \) can be interpreted as the Wilsonian brane couplings at \( \Lambda \) in the metric frame of \( G_{MN} \) (see Eq. (1)). However, when measured in the metric frame of 4D massless graviton \( g_{\mu\nu} = e^{2kR|y|} G_{\mu\nu} \), they should be interpreted as the Wilsonian couplings at different scales, \( g_{0a}^2 \) at the scale \( \Lambda \) and \( g_{\pi a}^2 \) at the rescaled scale \( e^{-\pi kR}\Lambda \). One can then assume that \( g_{0a}^2 \) and \( g_{\pi a}^2 \) are of order \( 8\pi^2 \) [13], so

\[ \frac{1}{g_a^2(p)} = \frac{\pi R}{g_{5a}^2} + \frac{1}{8\pi^2} \Delta_a(p, A, B_0, B_\pi, C, k, R, \ln(\Lambda)) + O \left( \frac{1}{8\pi^2} \right), \quad (33) \]

where

\[ \frac{1}{g_{5a}^2} = \frac{1}{g_{0a}^2} + \frac{\gamma_a \Lambda}{24\pi^3} \]

are the bare bulk couplings which are not calculable within 5D effective field theory. In the low momentum limit \( p \ll m_{KK} \) where \( m_{KK} \) is the KK threshold scale which corresponds to the lowest nonzero KK mass, the calculable one-loop correction \( \Delta_a \) can be written as

\[ \Delta_a(p, A, B_0, B_\pi, C, k, R, \ln(\Lambda)) = \Delta_a(A, B_0, B_\pi, C, k, R, \ln(\Lambda)) + b_a \ln \left( \frac{\Lambda}{p} \right) + O \left( \frac{p^2}{m_{KK}^2} \right), \quad (34) \]

where \( b_a \) are the 4D one-loop beta function coefficients determined by the zero mode spectrum. In AdS\(_5\) background, \( A^M_\perp \) gives a massless 4D vector, \( A^M_- \) a massless 4D real scalar, and \( \psi_{zz} (z = \pm) \) a massless 4D chiral spinor for any values of \( k, R \) and \( C \). However 5D scalar field \( \phi_{zz} \) can give a zero mode for any value of \( R \) only when \( z = z' = + \) and its bulk and brane masses satisfy

\[ B_0 = B_\pi, \quad \sqrt{4 + A^2} = |2 - B_0|. \quad (35) \]

Then \( b_a \) are given by

\[ b_a = -\frac{11}{3} T_a(A^M_\perp) + \frac{1}{6} T_a(A^M_-) + \frac{1}{6} T_a(\phi^{(0)}_{++}) + \frac{2}{3} T_a(\psi_{++}) + \frac{2}{3} T_a(\psi_{--}), \quad (36) \]

where \( \phi^{(0)}_{++} \) denotes 5D real scalar field having a zero mode. Note that the conditions of (35) are automatically satisfied in supersymmetric theories as it should be. In the following, we compute \( \Delta_a \) induced by generic 5D scalar, Dirac fermion and vector fields with arbitrary \( Z_2 \times Z_2' \) boundary condition.

Regularizing a field theory on compact space involves the regularization of the KK summation. It is then convenient to convert the KK summation into an integral by introducing
a pole function $P(q)$ [8] having the following properties: (i) $P(q)$ has poles at $q = m_n$, (ii) each pole has the residue 1, (iii) there exists $\delta > 0$ such that $P \to B$ for $|\text{Re}(q)| \to \infty$ and $\text{Im}(q) > \delta$, while $P \to -B$ for $|\text{Re}(q)| \to \infty$ and $\text{Im}(q) < -\delta$, where $B$ is an imaginary constant. These conditions uniquely determine the pole function. In our case, it is given by

$$P(q) = \frac{N'(q)}{2N(q)},$$

for which

$$\sum_{m_n} \int d^4p f(p, m_n) = \int \frac{dq}{2\pi i} \int d^4p P(q)f(p, q),$$

where $\Rightarrow$ denotes the contour depicted in Fig. 1.

To obtain the 1-loop effective action of gauge field zero mode, one needs to compute

$$\text{Tr} \ln \left(-D^2 + M^2(\Phi) + F_{\mu\nu}J^{\mu\nu}_j\right)$$

which contains the following two-point amplitude:

$$\int \frac{dq}{2\pi i} P(q) \int \frac{d^4p}{(2\pi)^4} A_\mu(-p)A_\nu(p)T_a(\Phi) \times \left[ d(j) \int \frac{d^4k}{(2\pi)^4} \frac{g^{\mu\nu}((p+k)^2 + q^2) - \frac{1}{2}(p+2k)^\mu(p+2k)^\nu}{(k^2 + q^2)((p+k)^2 + q^2)} \right.$$

$$-2C(j)\left(p^2g^{\mu\nu} - p^\mu p^\nu\right) \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 + q^2)((p+k)^2 + q^2)} \left.$$

$$\equiv i \int \frac{d^4p}{(2\pi)^4} G_a(p)A_\mu(-p)\left(p^2g^{\mu\nu} - p^\mu p^\nu\right)A_\nu(p),$$

where $d(j) = (1, 4, 4)$ and $C(j) = (0, 1, 2)$ for $j = (0, 1/2, 1)$. For the computation of the above integral, it is convenient to split the pole function into two parts:

$$P(q) = \tilde{P}(q) + P_\infty(q),$$

where $\tilde{P} \to O(q^{-2})$ at $|q| \to \infty$. Then $P_\infty$ can be written as

$$P_\infty(q) = -\frac{A}{q} - B\epsilon(\text{Im}(q)),$$

where $\epsilon(x) = x/|x|$ and $A$ and $iB$ are some real constants, which gives

$$\tilde{P}(q) = \frac{N'(q)}{2N(q)} + \frac{A}{q} + B\epsilon(\text{Im}(q)).$$

With the decomposition (41), all UV divergences appear in the contribution from $P_\infty$ in a manner allowing simple dimensional regularization, while the contribution from $\tilde{P}$ is finite.

The 4D momentum integral $d^4p$ in (40) exhibits a branch cut on the imaginary axis of $q$. For the contribution from $\tilde{P}$, one can change the contour as in Fig. 2 since the contribution
from the infinite half-circle vanishes. After integrating by part, we find that the part of $\mathcal{G}_a$ from $\tilde{P}$ is given by

$$
\Delta \mathcal{G}_a = \frac{T_a(\Phi)}{8\pi^2} \left[ \left( \frac{1}{6} d(j) - 2C(j) \right) \mathcal{F}(q) \right]_{q \rightarrow i\infty} - \frac{1}{8\pi^2} \int_0^1 dx \left( \frac{1}{2} d(j)(1 - 2x)^2 - 2C(j) \right) \mathcal{F}(q) \Bigg|_{q = i\sqrt{x(1-x)p^2}},
$$

(44)

where

$$
\mathcal{F}(q) = \frac{1}{2} \ln N + A \ln(-iq) + Bq.
$$

The contribution from $P_\infty$ includes the log divergence from the pole term $1/q$. This can be regulated by the standard dimensional regularization of 4D momentum integral, $d^4p \rightarrow d^Dp$, yielding a $1/(D - 4)$ pole. On the other hand, the step-function contribution from $\epsilon(\text{Im}(q))$ involves a 5D momentum integral which is linearly divergent, but it simply gives a finite result in dimensional regularization. Adding the divergent contribution from $P_\infty$ to the finite part from $\tilde{P}$, we obtain

$$
\mathcal{G}_a = \frac{T_a(\Phi)}{8\pi^2} \left[ \left( \frac{1}{6} d(j) - 2C(j) \right) \mathcal{F}(q) \right]_{q \rightarrow i\infty} + \int_0^1 dx \left( -\frac{1}{2} d(j)(1 - 2x)^2 + 2C(j) \right) \left( \frac{1}{2} \ln N \right) \Bigg|_{q = i\sqrt{x(1-x)p^2}} + A \int_0^1 dx \left( -\frac{1}{2} d(j)(1 - 2x)^2 + 2C(j) \right) \left( \frac{1}{D - 4} \right).
$$

(45)

In fact, the values of $A$ and $\mathcal{F}(q)$ at $q \rightarrow i\infty$ depend only on the $Z_2 \times Z'_2$ parity of the corresponding 5D field, not on the spin of the field. We then find

$$
A = (-1/2, 0, 0, 1/2)
$$

for $Z_2 \times Z'_2$ parity ($Z_\Phi, Z'_\Phi$) = $(++, +-, --, --)$ and

$$
\mathcal{F} \Bigg|_{q \rightarrow i\infty} = \left( \frac{1}{4} \pi kR - \frac{1}{2} \ln k, -\frac{1}{4} \pi kR, \frac{1}{4} \pi kR, -\frac{1}{4} \pi kR + \frac{1}{2} \ln k \right)
$$

for the same $Z_2 \times Z'_2$ parity.

In order to get a physical result from (45), we still need to subtract the $1/(D - 4)$ pole. When written in the position space of 5-th dimension, $1/(D - 4)$ term in (45) eventually leads to a term $\propto (\lambda_0 \delta(y) + \lambda_\pi \delta(y - \pi)) F_{\mu \nu}^a F^{a \mu \nu}/(D - 4)$ in the 1-loop effective action. (See Eqs. (30) and (31) for the definition of $\lambda_0$ and $\lambda_\pi$.) Then the subtraction procedure should take into account that the cutoff scales at $y = 0$ and $\pi$ differ by the warp factor $e^{-\pi kR}$. The correct subtraction scheme is to add a counter term

$$
\int d^4x dy \sqrt{G} \frac{1}{32\pi^2} \left[ \lambda_0 \left( \frac{1}{(D - 4)} - \ln(\Lambda) \right) \frac{\delta(y)}{\sqrt{G_{55}}} + \lambda_\pi \left( \frac{1}{(D - 4)} - \ln(\Lambda e^{-\pi kR}) \right) \frac{\delta(y - \pi)}{\sqrt{G_{55}}} \right] F_{\mu \nu}^a F^{a \mu \nu},
$$

(46)
which gives an extra $R$-dependent contribution $\propto \lambda_n \pi k R$ to the low energy gauge coupling. This can be considered in principle as a different choice of the bare IR brane coupling $g_{\pi a}^2(\Lambda)$. However if the 5D orbifold field theory is regulated in $R$-independent manner, which is the most natural choice in view of that $R$ is a dynamical field in 5D theory, this extra piece should be considered as a part of calculable correction. Also the strong coupling assumption on the bare brane couplings [13], $g_{\alpha a}^2(\Lambda) \approx g_{\pi a}^2(\Lambda) = \mathcal{O}(8\pi^2)$, applies for the $R$-independent part. As we will see in the next section, our subtraction scheme correctly reproduces the results in supersymmetric case which can be obtained by a completely independent method based on 4D effective SUGRA whose regulator mass is $R$-independent. We also explicitly show in Appendix B that our subtraction scheme gives the precisely same result as the $R$-independent Pauli-Villars regularization for the case of 5D scalar QED.

With the prescription to compute the regularized one-loop gauge coupling which has been discussed so far, it is now straightforward to compute $\Delta_a$ induced by generic 5D fields with arbitrary $Z_2 \times Z'_2$ boundary condition. The correction due to 5D scalar fields is given by

$$
\Delta_a(\phi) = \frac{1}{12} \left[ T_a(\phi_{++}) \left\{ \ln \left( \frac{\Lambda}{k} \right) - \frac{3}{2} \int_0^1 du F(u) \ln N_{\phi_{++}} \left( \frac{iu}{2} \sqrt{p^2} \right) \right\} 
-3T_a(\phi_{+-}) \int_0^1 du F(u) \ln N_{\phi_{+-}} \left( \frac{iu}{2} \sqrt{p^2} \right) 
-3T_a(\phi_{-+}) \int_0^1 du F(u) \ln N_{\phi_{-+}} \left( \frac{iu}{2} \sqrt{p^2} \right) 
-T_a(\phi_{--}) \left\{ \ln \left( \frac{\Lambda}{k} \right) + \frac{3}{2} \int_0^1 du F(u) \ln N_{\phi_{--}} \left( \frac{iu}{2} \sqrt{p^2} \right) \right\} \right] \tag{47}
$$

where the part with coefficient $T_a(\phi_{zz'})$ represents the contribution from the loops of 5D scalar field $\phi_{zz'}$ and

$$F(u) = u(1-u^2)^{1/2}.$$ 

Here $N_{\phi_{zz'}}$ ($z = \pm$, $z' = \pm$) are the $N$-functions of Eqs. (20) and (21) for

$$(Z_\phi, Z'_\phi, s, r_0, r_\pi, \alpha) = (z, z', 4, B_0, B_\pi, \sqrt{4+A^2}).$$

The 1-loop corrections due to 5D fermion and vector fields are similarly obtained to be

$$
\Delta_a(\psi) = \frac{1}{3} \left[ T_a(\psi_{++}) \left\{ 2 \ln \left( \frac{k}{p} \right) - \pi k R + \frac{3}{2} \int_0^1 du G(u) \ln N_{\psi_{++}} \left( \frac{iu}{2} \sqrt{p^2} \right) \right\} 
+T_a(\psi_{+-}) \left\{ -\pi k R + \frac{3}{2} \int_0^1 du G(u) \ln N_{\psi_{+-}} \left( \frac{iu}{2} \sqrt{p^2} \right) \right\} 
+T_a(\psi_{-+}) \left\{ \pi k R + \frac{3}{2} \int_0^1 du G(u) \ln N_{\psi_{-+}} \left( \frac{iu}{2} \sqrt{p^2} \right) \right\} 
+T_a(\psi_{--}) \left\{ 2 \ln \left( \frac{k}{p} \right) - \pi k R + \frac{3}{2} \int_0^1 du G(u) \ln N_{\psi_{--}} \left( \frac{iu}{2} \sqrt{p^2} \right) \right\} \right], \tag{48}
$$

$$
\Delta_a(A) = \frac{1}{12} \left[ T_a(A_{++}) \left\{ 23 \ln \left( \frac{p}{\Lambda} \right) + 21 \ln \left( \frac{p}{k} \right) + 22\pi k R + \int_0^1 du K(u) \ln N_{A_{++}} \left( \frac{iu}{2} \sqrt{p^2} \right) \right\} \right].
$$
\[ + T_a(A_{+-}) \left\{ -\pi kR + \int_0^1 du K(u) \ln N_{A_{+-}} \left( \frac{iu}{2} \sqrt{p^2} \right) \right\} \]
\[ + T_a(A_{-+}) \left\{ \pi kR + \int_0^1 du K(u) \ln N_{A_{-+}} \left( \frac{iu}{2} \sqrt{p^2} \right) \right\} \]
\[ + T_a(A_{--}) \left\{ 23 \ln \left( \frac{A}{k} \right) + 2 \ln \left( \frac{k}{p} \right) - \pi kR + \int_0^1 du K(u) \ln N_{A_{--}} \left( \frac{iu}{2} \sqrt{p^2} \right) \right\} , \]

where
\[
G(u) = u(1 - u^2)^{1/2} - u(1 - u^2)^{-1/2},
\]
\[
K(u) = -9u(1 - u^2)^{1/2} + 24u(1 - u^2)^{-1/2}.
\]

Here \( N_{\psi_{++}} , N_{\psi_{+-}} , N_{\psi_{-+}} \) and \( N_{\psi_{- -}} \) are the \( N \)-functions of Eqs. (20) and (21) for
\[
(Z_\Phi, Z'_\Phi, s, r, \alpha) = (-, -, 1, C, |C - 1/2|), (+, -, 1, -C, |C + 1/2|),
\]
\[
(-, +, -C, |C + 1/2|), (-, -, 1, -C, |C + 1/2|),
\]

where \( r = r_0 = r_\pi \), and \( N_{A_{++}}, N_{A_{+-}}, N_{A_{-+}} \) and \( N_{A_{--}} \) are the \( N \)-functions for
\[
(Z_\Phi, Z'_\Phi, s, r, \alpha) = (-, -, 4, 2, 0), (+, -, 2, 0, 1),
\]
\[
(-, +, 2, 0, 1), (-, -, 2, 0, 1).
\]

Note that \( N_{\psi_{++}} \) and \( N_{A_{++}} \) are given by \( N_{- -} \) in Eq. (21), not \( N_{++} \) in Eq. (20).

For a practical application of the above results, one may consider the low momentum limit \( p \ll m_{KK} \) where \( m_{KK} \) denotes the lowest KK mass which can be determined by the corresponding \( N \)-function. In such limit, one-loop gauge couplings can be written as
\[
\frac{1}{g_a(p)^2} = \frac{\pi R}{g_5^2} + \frac{1}{8\pi^2} \left[ \tilde{\Delta}_a(p, A, B_0, B_\pi, C, k, R, \ln \Lambda) + O(1) \right]
\]
\[
= \frac{\pi R}{g_5^2} + \frac{1}{8\pi^2} \left[ \Delta_a(A, B_0, B_\pi, C, k, R, \ln \Lambda) + b_a \ln (\Lambda/p) + O \left( \frac{p^2}{m_{KK}^2} \right) \right]. \quad (49)
\]

The results on \( \Delta_a \) are summarized in Table I. We also provide in Table II the expressions of \( \Delta_a \) induced by a scalar field with particular values of bulk and brane mass parameters, i.e. \( B_0 = B_\pi \) and \( \sqrt{4 + A^2} = |2 - B_0| \), which corresponds to the scalar field in supersymmetric theory.

**IV. 4D SUPERGRAVITY CALCULATION**

In [4,7], 1-loop low energy gauge couplings in AdS\(_5\) have been obtained in supersymmetric case using the gauged \( U(1)_R \) symmetry and chiral anomaly \[14\] in 5D SUGRA in AdS\(_5\) [9,10] and also the known properties of gauge couplings in 4D effective SUGRA \[15\]. In this section, we confirm that the results of the previous section correctly reproduce the SUGRA results when applied in supersymmetric case.

To proceed, let us briefly discuss supersymmetric 5D theory on AdS\(_5\). The theory contains two types of 5D supermultiplets other than the SUGRA multiplet, one is the hypermultiplet \( H \) containing two 5D complex scalar fields \( h^i \ (i = 1, 2) \) and a Dirac fermion \( \psi \), and
the other is the vector multiplet $V$ containing a 5D vector $A_M$, real scalar $\Sigma$ and a symplectic Majorana fermion $\lambda^i$. In supersymmetric model, all scalar fields have $B_0 = B_\pi \equiv B$ and $\sqrt{4 + A^2} = |2 - B|$ (see Eqs. (6) for the definitions of $B_0, \pi$ and $A$) and their superpartner fermion has a kink mass parameter $C = (3 - 2B)/2$. Also the $U(1)_R$ symmetry is gauged with the graviphoton $B_M$ in the following way:

$$D_M h^i = \partial_M h^i - i \left( \frac{3}{2} (\sigma_3)^i_j - C \delta^i_j \right) k \epsilon(y) B_M h^j + ...$$
$$D_M \psi = \partial_M \psi + i C k \epsilon(y) B_M \psi + ...$$
$$D_M \lambda^i = \partial_M \lambda^i - i \frac{3}{2} (\sigma_3)^i_j k \epsilon(y) B_M \lambda^j + ...,$$  

(50)

where $\psi$ has a kink mass $C k \epsilon(y)$ and the ellipses stand for the couplings with other gauge fields. Taking into account the $Z_2 \times Z_2'$ parity, the supermultiplet structure is given by

$$H_{zz'}(C) = \left( h^1_{zz'}(B = \frac{3}{2} - C), h^2_{zz'}(B = \frac{3}{2} + C), \psi^{\text{Dirac}}_{zz'}(C) \right),$$

$$V_{zz'} = \left( A^M_{zz'} = (A^a_{zz'}, A^5_{zz'}(B = 2)), \lambda^i = \lambda^{\text{Dirac}}_{zz'}(C = \frac{1}{2}), \Sigma_{zz'}(B = 2) \right),$$  

(51)

where the subscripts $z, z'$ denote the $Z_2 \times Z_2'$ parity, $\tilde{z} = -z, \tilde{z}' = -z'$, $B$ is the brane mass parameter and $C$ is the kink mass parameter.

Let us assume that our 5D theory is compactified in a manner preserving $D = 4$ $N = 1$ supersymmetry. This allows the low energy physics to be described by 4D effective SUGRA whose action can be written as

$$S_{4D} = \int d^4 x \left[ \int d^4 \theta \left\{ -3 \exp \left( -\frac{K}{3} \right) \right\} + \left( \int d^2 \theta \frac{1}{4} f_a W^{aa} W^a_{\alpha} + \text{h.c.} \right) \right],$$  

(52)

where $W^a_{\alpha}$ is the chiral spinor superfield for the 4D gauge multiplet and we set the 4D gravity multiplet by their vacuum values. The Kähler potential $K$ can be expanded in powers of generic gauge-charged chiral superfield $Q$:

$$K = K_0(\mathcal{T}, \mathcal{T}^*) + Z_Q(\mathcal{T}, \mathcal{T}^*) Q^* e^{-V} Q + ...,$$  

(53)

where $\mathcal{T}$ denotes the radion superfield whose scalar component is given by

$$\mathcal{T} = R + iB_5,$$

and the gauge kinetic function $f_a$ is a holomorphic function of $\mathcal{T}$. Then the 1-loop gauge couplings in effective 4D SUGRA can be determined by $f_a$ containing the 1-loop threshold correction from massive KK modes and also the tree-level Kähler potential $K$ [15]:

$$\frac{1}{g_a^2(p)} = \text{Re}(f_a) + \frac{b_a}{16\pi^2} \ln \left( \frac{M_{Pl}^2}{e^{-K_{0}/3} p^2} \right)$$
$$- \sum_Q T_a(Q) \ln \left( e^{-K_{0}/3} Z_Q \right) + \frac{T_a(\text{Adj})}{8\pi^2} \ln \left( \text{Re}(f_a) \right),$$  

(54)
where \( b_a = \sum T_a(Q) - 3T_a(\text{Adj}) \) is the 1-loop beta function coefficient and \( M_{Pl} \) is the Planck scale of \( g_{\mu\nu} \) which defines \( p^2 = -g_{\mu\nu} \partial_{\mu} \partial_{\nu} \).

Let us consider the 4D effective SUGRA of a 5D theory which contains \( H_{++}, H_{+-}, H_{-+}, H_{--} \) as well as \( V_{++}, V_{+-}, V_{-+}, V_{--} \). The 5D vector multiplet \( V_{++} \) gives a massless 4D gauge multiplet containing \( A^a_{++} \) whose low energy couplings are of interest for us, while \( V_{--} \) gives a massless 4D chiral multiplet containing \( \Sigma_{++} + iA^5_{++} \). \( H_{++} \) and \( H_{--} \) also give massless 4D chiral multiplets containing \( h^1_{++} \) and \( h^2_{++} \), respectively, whose tree level Kähler metrics are required to compute the 1-loop gauge coupling (54). Other 5D multiplets, i.e. \( V_{-+}, V_{+-}, H_{-+} \) and \( H_{++} \) do not give any massless 4D mode. Let \( Y_Q = e^{-K_{0}/3} Z_Q \) where \( Z_Q (Q = H_{++}, H_{--}, V_{--}) \) denote the Kähler metric of the 4D massless chiral superfields coming from the 5D multiplets \( H_{++}, H_{--} \) and \( V_{--} \), respectively. Following Refs. [7,16], it is straightforward to find the tree level \( Z_{H_{++}}, Z_{H_{--}} \) and \( f_a \) containing the 1-loop threshold corrections from massive KK modes:

\[
M_{Pl}^2 = e^{-K_{0}/3} M_5^2 = \frac{M_5^2}{k} (1 - e^{-k\pi(T+\bar{T})}) ,
\]

\[
Y_{H_{++}} = \frac{M_5}{(\frac{k}{2} - C_{++})} (e^{(\frac{k}{2} - C_{++})\pi k(T+\bar{T})} - 1) ,
\]

\[
Y_{H_{--}} = \frac{M_5}{(\frac{k}{2} + C_{--})} (e^{(\frac{k}{2} + C_{--})\pi k(T+\bar{T})} - 1) ,
\]

\[
Y_{V_{--}} = \frac{k}{M_5 e^{\pi k(T+\bar{T})} - 1} ,
\]

\[
f_a = \frac{\pi T}{g_{5a}^2} + \frac{z'}{8\pi^2} \left( \frac{3}{2} \sum_{\bar{\nu}} T_a(V_{\bar{\nu}\nu}) - \sum_{H_{\nu}^{\nu'}} T_a(H_{\nu}^{\nu'}) \right) k\pi T ,
\]  

(55)

where \( M_5 \) is the 5D Planck scale, and \( C_{\nu\bar{\nu'}} \) is the kink mass of \( H_{\nu\bar{\nu'}} \). As was noted in [7], the KK threshold correction to \( f_a \) can be entirely determined by the chiral anomaly w.r.t the following \( B_5 \)-dependent phase transformation:

\[
\chi^{ai} \rightarrow \left( e^{3i(y|B_5s/2)} \right)^i j \chi^{aj} , \quad \psi \rightarrow e^{-iCk|y|B_5} \psi .
\]  

(56)

Using the above results, we find the one-loop gauge couplings at low momentum limit \( p \ll m_{KK} \):

\[
\frac{1}{g_a(p)^2} = \frac{\pi R}{g_{5a}^2} + \frac{1}{8\pi^2} \left[ (\Delta_a)_{\text{SUSY}} + (b_a)_{\text{SUSY}} \ln(\Lambda/p) \right] ,
\]  

(57)

where

\[
(\Delta_a)_{\text{SUSY}} = -T_a(H_{++}) \left[ \ln \left( \frac{\Lambda}{k} \right) + C_{++}\pi k R + \ln \left( \frac{e^{(1-2C_{++})\pi k R} - 1}{1 - 2C_{++}} \right) \right] \\
+ C_{+-} T_a(H_{+-}) \pi k R - C_{-+} T_a(H_{-+}) \pi k R \\
- T_a(H_{--}) \left[ \ln \left( \frac{\Lambda}{k} \right) - C_{--}\pi k R + \ln \left( \frac{e^{(1+2C_{--})\pi k R} - 1}{1 + 2C_{--}} \right) \right] \\
+ T_a(V_{++}) \left[ \ln (M_5\pi R) + \frac{3}{2}\pi k R \right]
\]
\[-\frac{3}{2} T_a(V_{+-}) \pi k R + \frac{3}{2} T_a(V_{++}) \pi k R \]
\[+ T_a(V_{-+}) \left[ \ln \frac{M_5}{k} + \frac{1}{2} \pi k R + \ln \left( \frac{1 - e^{-2 \pi k R}}{2} \right) \right] \quad (58)\]

and

\[(b_a)_{\text{SUSY}} = -3 T_a(V_{++}) + T_a(V_{+-}) + T_a(H_{++}) + T_a(H_{+-}).\]

A rough estimate of \(m_{KK}\) yields

\[m_{KK} \sim k e^{-\pi k R}\]

for the bulk fields other than \(H_{+-}\) or \(H_{-+}\). On the other hand, \(H_{+-}\) has

\[m_{KK} \sim k e^{-(\frac{1}{2} + C_{+-}) \pi k R} \quad (C_{+-} \geq 1/2) \quad \text{or} \quad k e^{-\pi k R} \quad (C_{+-} \leq 1/2),\]

while \(H_{-+}\) has

\[m_{KK} \sim k e^{-(\frac{1}{2} - C_{-+}) \pi k R} \quad (C_{-+} \leq -1/2) \quad \text{or} \quad k e^{-\pi k R} \quad (C_{-+} \geq -1/2).\]

The above result (58) obtained by 4D SUGRA analysis perfectly agrees with the result that one would obtain using the results of Tables I and II when \(M_5\) is replaced by \(\Lambda\). This provides a nontrivial check for the results obtained in the previous section and assures that our results are truly scheme-independent.

**V. CONCLUSION**

In this paper, we have calculated the full 1-loop corrections to the low energy coupling of bulk gauge boson in AdS\(_5\) induced by generic 5D scalar, fermion and vector fields with arbitrary \(Z_2 \times Z'_2\) orbifold boundary condition. The used calculation scheme is the background field method with dimensional regularization. We noted that the subtraction scale for the log divergence at the IR brane \((y = \pi)\) should be taken to be \(\Lambda e^{-\pi k R}\) where \(\Lambda\) is the subtraction scale for the UV brane \((y = 0)\). We also considered supersymmetric case to assure that our results correctly reproduce the results obtained by a completely independent method based on 4D effective supergravity analysis.

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**Note added:** While this work was in completion, we received [17,18] discussing the 1-loop gauge coupling renormalization due to 5D scalar loops in AdS\(_5\) background and its interpretation in the context of AdS/CFT correspondence and also [19] discussing the 1-loop renormalization in the context of deconstructed AdS\(_5\).
Appendix A. Some properties of the $N$-functions

In this appendix, we present some properties of the $N$-functions, $N_{zz'}$ ($z, z' = \pm$), given in Eqs. (20) and (21). Using $Y_\alpha(x) = (\cos \alpha \pi J_\alpha(x) - J_\alpha(x))/\sin \alpha \pi$ and also the fact that $N_{zz'}$ are antisymmetric under the exchange of $J_\alpha$ and $Y_\alpha$, one can rewrite the $N$-functions as

$$
N_{++}(q) = \frac{1}{\sin \alpha \pi} \left\{ \frac{(s - r_0)J_\alpha(q/k)}{T} + \frac{q}{k} J'_\alpha \left( \frac{q}{k} \right) \right\} \left\{ \frac{(s - r_\pi)J_{-\alpha}(q/T)}{T} + \frac{q}{T} J'_{-\alpha} \left( \frac{q}{T} \right) \right\} 
- \left\{ \frac{(s - r_\pi)J_\alpha(q/k)}{T} + \frac{q}{k} J'_\alpha \left( \frac{q}{k} \right) \right\} \left\{ \frac{(s - r_0)J_{-\alpha}(q/T)}{T} + \frac{q}{T} J'_{-\alpha} \left( \frac{q}{T} \right) \right\},
$$

$$
N_{+-}(q) = \frac{1}{\sin \alpha \pi} \left\{ \frac{(s - r_0)J_\alpha(q/k)}{T} + \frac{q}{k} J'_\alpha \left( \frac{q}{k} \right) \right\} \left\{ \frac{(s - r_\pi)J_{-\alpha}(q/T)}{T} + \frac{q}{T} J'_{-\alpha} \left( \frac{q}{T} \right) \right\},
$$

$$
N_{-+}(q) = \frac{1}{\sin \alpha \pi} \left\{ \frac{(s - r_\pi)J_\alpha(q/k)}{T} + \frac{q}{k} J'_\alpha \left( \frac{q}{k} \right) \right\} \left\{ \frac{(s - r_0)J_{-\alpha}(q/T)}{T} + \frac{q}{T} J'_{-\alpha} \left( \frac{q}{T} \right) \right\},
$$

$$
N_{-\cdot}(q) = -\frac{1}{\sin \alpha \pi} \left[ J_\alpha \left( \frac{q}{k} \right) J_{-\alpha} \left( \frac{q}{T} \right) - J_\alpha \left( \frac{q}{T} \right) J_{-\alpha} \left( \frac{q}{k} \right) \right],
$$

where $T = ke^{-\pi kR}$. Then using $J_\alpha(x) = x^\alpha f(x^2)$, one can easily see that all $N$-functions are even-functions:

$$
N_{zz'}(q) = N_{zz'}(-q).
$$

We already know $N_{zz'}(q)$ is analytic near $q = 0$, allowing an expansion around $q = 0$:

$$
N_{zz'}(q) = Q_{zz'} + \frac{q^2}{k^2}R_{zz'} + \mathcal{O}(q^4),
$$

where

$$
Q_{++} = \frac{1}{\pi \alpha} \left[ (-\alpha + r_\pi - \frac{s}{2})(\alpha - r_0 + \frac{s}{2})e^{-\alpha \pi kR} + (\alpha + r_0 - \frac{s}{2})(\alpha - r_\pi + \frac{s}{2})e^{\alpha \pi kR} \right],
$$

$$
Q_{+-} = \frac{1}{\pi \alpha} \left[ (\alpha - r_0 + \frac{s}{2})e^{-\alpha \pi kR} + (\alpha + r_0 - \frac{s}{2})e^{\alpha \pi kR} \right],
$$

$$
Q_{-+} = \frac{1}{\pi \alpha} \left[ (\alpha - r_\pi + \frac{s}{2})e^{\alpha \pi kR} + (\alpha + r_\pi - \frac{s}{2})e^{-\alpha \pi kR} \right],
$$

$$
Q_{-\cdot} = \frac{1}{\pi \alpha} \left[ e^{\alpha \pi kR} - e^{-\alpha \pi kR} \right],
$$

$$
R_{++} = -\frac{1}{4\pi} \left[ \frac{1}{\alpha(\alpha - 1)} \left\{ (2\alpha - r_0 + \frac{s}{2})(\alpha - r_\pi + \frac{s}{2})e^{\alpha \pi kR} \right. \right.
+ \left. (\alpha - r_\pi + \frac{s}{2})(\alpha - r_0 + \frac{s}{2})e^{(2-\alpha)\pi kR} \right\}
+ \frac{1}{\alpha(\alpha + 1)} \left\{ (-\alpha + r_\pi + \frac{s}{2})(2\alpha - r_0 + \frac{s}{2})e^{-\alpha \pi kR} \right. 
+ \left. (\alpha + r_0 - \frac{s}{2})(2\alpha - r_\pi + \frac{s}{2})e^{(\alpha + 2)\pi kR} \right\} \right],
$$

16
\[
R_{+-} = \frac{1}{4\pi} \left[ \frac{1}{\alpha(\alpha-1)} \left\{ (-2 + \alpha + r_0 - \frac{s}{2})e^{\alpha\pi k R} + (\alpha - r_0 + \frac{s}{2})e^{(2-\alpha)\pi k R} \right\} \\
- \frac{1}{\alpha(\alpha+1)} \left\{ (2 + \alpha - r_0 + \frac{s}{2})e^{-\alpha\pi k R} + (\alpha + r_0 - \frac{s}{2})e^{(2+\alpha)\pi k R} \right\} \right],
\]
\[
R_{--} = -\frac{1}{4\pi} \left[ \frac{1}{\alpha(1-\alpha)} \left\{ (-2 + \alpha + r_\pi - \frac{s}{2})e^{(2-\alpha)\pi k R} + (\alpha - r_\pi + \frac{s}{2})e^{\alpha\pi k R} \right\} \\
+ \frac{1}{\alpha(1+\alpha)} \left\{ (2 + \alpha - r_\pi + \frac{s}{2})e^{(2+\alpha)\pi k R} + (\alpha + r_\pi - \frac{s}{2})e^{-\alpha\pi k R} \right\} \right],
\]
\[
R_{--} = \frac{1}{4\pi} \left[ -\frac{1}{\alpha(\alpha-1)} \left\{ e^{-(\alpha-2)\pi k R} - e^{\alpha\pi k R} \right\} + \frac{1}{\alpha(\alpha+1)} \left\{ e^{-\alpha\pi k R} - e^{(\alpha+2)\pi k R} \right\} \right]. \tag{61}
\]

The KK mass eigenvalue \( m_n \) is determined by the zeros of \( N \)-function: \( N(m_n) = 0 \). Obviously a 5D field has a massless 4D mode iff \( Q_{\alpha} = 0 \). Generically, a nonzero KK mass eigenvalue starts to appear from \( m_n = \mathcal{O}(T) \). However in some special case, there can be nonzero mass eigenvalues much smaller than \( T = ke^{-\pi k R} \). For instance, if \( \alpha = \frac{s}{2} - r_0 \) and \( \alpha \) has a large value, \( Q_{-+} \sim e^{-\alpha \pi k R} \) and \( R_{--} \sim e^{\alpha \pi k R} \), giving a very light state of \( \Phi_{++} \) with \( m_n \sim ke^{-\alpha \pi k R} \). Similarly, if \( \alpha = r_\pi - \frac{s}{2} \), \( \Phi_{--} \) can also have a very small \( m_n \). However \( \Phi_{--} \) does have neither a massless state nor a very light state with \( m_n \ll ke^{-\pi k R} \).

The asymptotic behavior of \( N \)-function at \( |q| \to \infty \) is essential for regularizing the 1-loop gauge coupling. Using the asymptotic formulae of Bessel functions:
\[
J_\alpha(x) \to \sqrt{\frac{2}{\pi x}} \cos \left[ x - \left( \alpha + \frac{1}{2} \right) \right],
\]
\[
Y_\alpha(x) \to \sqrt{\frac{2}{\pi x}} \sin \left[ x - \left( \alpha + \frac{1}{2} \right) \right],
\]
we find
\[
N_{++}(q) \to \frac{2qe^{\pi k R/2}}{\pi k} \sin \left( \frac{(1 - e^{\pi k R})q}{k} \right),
\]
\[
N_{+-}(q) \to \frac{2}{\pi} e^{-\pi k R/2} \cos \left( \frac{(1 - e^{\pi k R})q}{k} \right),
\]
\[
N_{-+}(q) \to \frac{2}{\pi} e^{\pi k R/2} \cos \left( \frac{(1 - e^{\pi k R})q}{k} \right),
\]
\[
N_{--}(q) \to -\frac{2k}{\pi q} e^{-\pi k R/2} \sin \left( \frac{(1 - e^{\pi k R})q}{k} \right).\]

**Appendix B. Comparison with Pauli-Villars Regularization**

A natural regularization in 5D theory is to cut off 5D momentum in the 5D metric frame of \( G_{MN} = -G^{MN} \partial_M \partial_N \prec \Lambda^2 \). In AdS background, this would correspond to an effective \( y \)-dependent cut-off of 4D momentum in the 4D metric frame of \( g_{\mu\nu} \prec p^2 = -g^{\mu\nu} \partial_\mu \partial_\nu \prec e^{-2kR|y|}\Lambda^2 \). In dimensional regularization, such feature is not manifest, but can be taken into account by choosing the subtraction scale \( \sim \Lambda e^{-kR\bar{y}} \) where \( \bar{y} = 0 \) or \( \pi \) is the location...
of log divergence. On the other hand, such feature is rather manifest in Pauli-Villars (PV) regularization in which \( \Lambda \) corresponds to a 5D regulator mass. In this appendix, we compare our result using the dimensional regularization with the subtraction scheme (46) to the PV result for scalar QED. For simplicity, we consider the massless scalar QED with \( Z_2 \times Z_2^\prime \) parity (++).

In PV scheme, the UV divergence is regulated by a PV regulator with 5D mass \( \Lambda \) which has the same \( Z_2 \times Z_2^\prime \) boundary condition as \( \phi \) but opposite statistics:

\[
\sum_n \int \frac{d^4p}{(2\pi)^4} f(p, m_n) \longrightarrow \sum_n \left\{ \int \frac{d^4p}{(2\pi)^4} f(p, m_n) - \int \frac{d^4p}{(2\pi)^4} f(p, M_n) \right\},
\]

where \( M_n \) is the KK spectrum for the PV regulator. We convert the summation into an integral using the pole functions:

\[
P_\phi = \frac{N_\phi'}{2N_\phi}, \quad P_{PV} = \frac{N_{PV}'}{2N_{PV}},
\]

and then the regulated amplitude is given by

\[
\int \frac{dq}{2\pi i} P_{reg}(q) \int \frac{d^4p}{(2\pi)^4} f(p, q),
\]

where \( P_{reg}(q) \equiv P_\phi(q) - P_{PV}(q) \). Since \( N_\phi \) and \( N_{PV} \) are the same limiting behavior at \( |q| \to \infty \), \( P_{reg}(q) \) vanishes at infinity. After a partial integration along \( q \), we find

\[
\bar{\Delta}_{PV} = 8\pi^2 \int_C \frac{dq}{2\pi i} \left\{ \frac{1}{2} \ln N_\phi - \frac{1}{2} \ln N_{PV} \right\}
\]

\[
\times \int dx (1 - 2x)^2 \left\{ \frac{1}{(4\pi)^2} \ln (x(1-x)p^2 + q^2) \right\}
\]

\[
= -\frac{1}{4} \int dx (1 - 2x)^2 (\ln N_\phi - \ln N_{PV}) \bigg|_{q=i\sqrt{x(1-x)p^2}},
\]

where \( C \) is the contour line described in Fig. 2. For \( q \ll ke^{-\pi kR} \),

\[
N_\phi \approx \frac{q^2 e^{\pi kR}}{k^2} \left( \frac{e^{\pi kR} - e^{-\pi kR}}{\pi} \right),
\]

\[
N_{PV} \approx \frac{(\alpha - 2)(\alpha + 2)}{\pi \alpha} \left( e^{-\alpha kR} - e^{\alpha kR} \right),
\]

where \( \alpha = \sqrt{4 + \Lambda^2/k^2} \). For \( \Lambda \gg k \), \( \alpha \approx \Lambda/k \), so

\[
\ln N_{PV} \approx \Lambda \pi R + \ln(\Lambda/k).
\]

After subtracting the power-law divergent part which is regularization scheme dependent, we find

\[
\Delta_{PV} \equiv \bar{\Delta}_{PV} - b_{a} \ln(\Lambda/p) = -\frac{1}{12} \left[ \ln(\Lambda/k) + \pi kR + \ln \left( \frac{e^{\pi kR} - e^{-\pi kR}}{2\pi} \right) \right],
\]

which is precisely same as the result in Table II for a massless real \( \phi_{++} \) with \( A = B_0 = B_\pi = 0 \). In scalar QED, a charged scalar field should be complex, so gives a loop correction twice of the above result.
### TABLE I

One loop corrections for $p \ll m_{KK}$ where $m_{KK}$ is the lowest nonzero KK mass. One-loop gauge couplings are given by $g^2_{a} = \frac{2\pi}{g_{5a}} + \frac{1}{8\pi^2}[\Delta_a + b_a \ln(\Lambda/p)]$ where $b_a$ is the 4D one-loop beta function coefficient due to zero modes (see (36)). Here $\phi_{zz'}$ ($z, z' = \pm$) stand for real scalar fields which do not have zero mode since they have generic mass-squares given by $A^2k^2 + \frac{2k}{R}[B_0\delta(y) - B_\pi\delta(y - \pi)]$, while $\phi_{(0)}^{(0)}$ denotes a scalar field with zero mode, i.e. a scalar field with $(++)$ parity, $B_0 = B_\pi \equiv B$ and $\sqrt{4 + A^2} = |2 - B|$. Here $\alpha \equiv \sqrt{4 + A^2}$.

| Type | $(zz')$ | $\Delta_a(A, B_0, B_\pi, C, k, R, \ln \Lambda)$ |
|------|---------|--------------------------------------------------|
| real scalar | $(++)$ | $-\frac{1}{12}T_a(\phi_{++}^{(0)}) \left[ \ln(\Lambda/k) + \pi k R + \ln \left( \frac{e^{(1-B)\pi k R} - e^{-(1-B)\pi k R}}{2(1-B)} \right) \right]$ |
| | | $+ \frac{1}{12}T_a(\phi_{++}) \left[ \ln(\Lambda/k) - \ln \left( \frac{(\alpha + B_0 - 2)(\alpha - B_\pi + 2)e^{\alpha \pi k R} - (\alpha - B_0 + 2)e^{-\alpha \pi k R}}{2\alpha} \right) \right]$ |
| | $(+-)$ | $-\frac{1}{12}T_a(\phi_{+-}) \ln \left( \frac{(\alpha + B_0 - 2)e^{\alpha \pi k R} + (\alpha - B_\pi + 2)e^{-\alpha \pi k R}}{2\alpha} \right)$ |
| | $(-+)$ | $-\frac{1}{12}T_a(\phi_{-+}) \ln \left( \frac{(\alpha - B_\pi + 2)e^{\alpha \pi k R} + (\alpha + B_\pi - 2)e^{-\alpha \pi k R}}{2\alpha} \right)$ |
| | $(--)$ | $-\frac{1}{12}T_a(\phi_{--}) \left[ \ln(\Lambda/k) + \ln \left( \frac{e^{\alpha \pi k R} - e^{-\alpha \pi k R}}{2\alpha} \right) \right]$ |
| Dirac spinor | $(++)$ | $-\frac{2}{3}T_a(\psi_{++}) \left[ \ln(\Lambda/k) + \frac{1}{2}\pi k R + \ln \left( \frac{e^{(C-\frac{1}{2})\pi k R} - e^{-(C-\frac{1}{2})\pi k R}}{2(C-\frac{1}{2})} \right) \right]$ |
| | $(-+)$ | $\frac{2}{3}T_a(\psi_{+-}) C\pi k R$ |
| | $(+-)$ | $-\frac{2}{3}T_a(\psi_{-+}) C\pi k R$ |
| | $(--)$ | $-\frac{2}{3}T_a(\psi_{--}) \left[ \ln(\Lambda/k) + \frac{1}{2}\pi k R + \ln \left( \frac{e^{(C+\frac{1}{2})\pi k R} - e^{-(C+\frac{1}{2})\pi k R}}{2(C+\frac{1}{2})} \right) \right]$ |
| vector | $(++)$ | $\frac{1}{12}T_a(A_{++}^M) \left[ 21 \ln(\Lambda \pi R) + 22\pi k R \right]$ |
| | $(+-)$ | $-\frac{11}{6}T_a(A_{+-}^M) \pi k R$ |
$$\left(\begin{array}{c|c}
(\pm) & \frac{11}{6} T_a(A^{M,\pm}_- \pi k R) \\
\hline
(-) & \frac{1}{12} T_a(A^M_-) \left[ 21 \ln(\Lambda \pi R) - \pi k R + 21 \ln \left( \frac{e^{\pi k R} - e^{-\pi k R}}{2\pi k R} \right) \right] \\
\end{array}\right)$$

**TABLE II.** 5D scalar contribution for $p \ll m_{KK}$ when $B_0 = B_{\pi} \equiv B$ and $\alpha \equiv \sqrt{4 + A^2} = |2 - B|$. 

| (+) | $-\frac{1}{12} T_a(\phi_{++}) \left[ \ln(\Lambda/k) + \pi k R + \ln \left( \frac{e^{(1-B)\pi k R} - e^{-(1-B)\pi k R}}{2(1-B)} \right) \right]$ |
| (-) | $\frac{1}{12} T_a(\phi_{+-}) (2-B) \pi k R$ |
| (+) | $-\frac{1}{12} T_a(\phi_{-+}) (2-B) \pi k R$ |
| (-) | $-\frac{1}{12} T_a(\phi_{--}) \left[ \ln(\Lambda/k) + \ln \left( \frac{e^{(2-B)\pi k R} - e^{-(2-B)\pi k R}}{2(2-B)} \right) \right]$ |
FIG. 1. Contour $\equiv$ in the complex $q$-plane. Bold dots represent the mass poles.

FIG. 2. For the contribution from $\tilde{P}(q)$, the contour $\leftarrow$ can be deformed to the contour $C$ represented by the bold line since the contribution vanishes on the dotted infinite half circle. Hatched lines on the imaginary axis are logarithmic branch-cuts. After integrating by parts, the point $x$ where the branch-cut starts becomes a simple pole. Then the integral along $C$ is given by the values of integrand at the boundary of $C$ at infinity and the residue value at the point $x$. The integral along $\rightarrow$ can be similarly treated in the lower half plane.
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