Active Disturbance Rejection Control with Sensor Noise Suppressing Observer for DC-DC Buck Power Converters

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Abstract—The class of active disturbance rejection control (ADRC) algorithms has been shown in the literature to be an interesting alternative to standard control methods in power electronics devices. However, their robustness and stability are often limited in practice by the high-frequency measurement noise, common in industrial applications. In this article, this problem is addressed by replacing the conventional high-gain extended state observer (ESO) with a new cascade observer structure. The presented experimental results, performed on a DC-DC buck power converter system, show that the new cascade ESO design has increased estimation/control performance compared to the standard approach, while effectively suppressing the detrimental effect of sensor noise over-amplification.

Index Terms—noise suppression, power converter, high-gain observer, extended state observer, ADRC

I. INTRODUCTION

The majority of modern power electronic circuits are supplied with switching-mode power converters, like pulsewidth modulated DC-DC converters. A buck converter, for example, is responsible for stepping down voltage, while stepping up current, from its input (supply) to its output (load). Its relatively high efficiency (often higher than 90%) and low price, makes a buck converter a choice of practitioners in various power systems. With the constant pursuit of technological improvement, finding more effective control methods for power converters is an active research topic.

Practically appealing results on buck converter control using the idea of active disturbance rejection control (ADRC) were recently reported in [1]–[3]. Interestingly, some motor control companies attracted by the ADRC-based solutions (e.g. Texas Instruments), have embedded this approach in their selected commercial products [4]. The key element in any ADRC scheme is the extended state observer (ESO), which is responsible for estimating the system state vector and reconstructing the overall disturbance (also referred to as total disturbance) affecting the controlled variable [5].

However, since the conventional form of ADRC uses a high-gain observer (HGO) to estimate selected signals, its capabilities are intrinsically limited by the presence and severity of high-frequency sensor noise, as shown in [6]–[8]. The HGO-based ADRC design and tuning often comes down to a forced compromise between speed/accuracy of signals reconstruction and sensitivity to noise [9]. Same compromise can be seen in the ADRC works for buck converters in which the measured system output (voltage) is oftentimes corrupted with high-frequency noise [10]. Several solutions were proposed to solve the problem of attenuating the effects of measurement noise in high-gain observers. They mainly address the problem by: employing nonlinear [11], [12] or adaptive techniques [13], redesigning the local behavior by combining different observers [14], employing low-power structures [15]–[17], or modifying standard low-pass filters [18].

Motivated by the above problem, a new cascade ESO-based ADRC solution is introduced. It is based on a virtual decomposition of the total disturbance present in the DC-DC buck converter system, allowing to design a cascade structure of ESO, where each level of the observer cascade is responsible for handling a particular type and frequency range of estimated signal. The proposed topology enhances conventional state/disturbance estimation performance while avoiding over-amplification of sensor noise. The user-defined number of cascade levels allow to customize the overall control system structure to meet certain disturbance rejection requirements. Although a multi-level cascade observer is proposed, a straightforward design and implementation methodology is given, together with an intuitive tuning rules. The proposed ADRC with cascade ESOs is validated in this work using hardware experiments conducted on a DC-DC buck converter laboratory testbed.

Notation. Within this article, we treat $\mathbb{R}$ as a set of real numbers, $\mathbb{R}^+ = \{x \in \mathbb{R} : x > 0\}$ as a set of positive real numbers, $\mathbb{Z}$ as a set of integers, $\lambda_{\text{min}}(A)$ and $\lambda_{\text{max}}(A)$ are respectively the minimal and maximal eigenvalues of matrix $A$, while $A > 0$ means that matrix $A$ is positive definite. Function $f(x) : \mathbb{R} \rightarrow \mathbb{R}$ belongs to class $\mathcal{K}$ when it is strictly...
the reference signal and its specific time-derivatives satisfy
increasing and $f(0) = 0$. The expression $L_{\infty} := \lim_{t \to \infty} \sup_{t \geq 0} v_o(t)$ was used for the sake of notation compactness.

II. PRELIMINARIES

A. Simplified plant model and control objective

Following [3], an average dynamic model of a DC-DC buck converter, depicted in Fig. 1, can be written as:

$$\begin{align*}
\dot{v}_o(t) &= \frac{1}{C} \left[ i_L(t) - \frac{1}{CR} v_o(t) \right], \\
\dot{i}_L(t) &= \frac{1}{L} \left[ \mu(t) + d(t) \right] - \frac{1}{L} v_o(t), \\
y_o(t) &= v_o(t) + n(t),
\end{align*}$$

where $\mu \in [0, 1]$ is the duty ratio, $v_o[V]$ is the measured system output that consists of the average capacitor voltage $v_o[V]$ and the sensor noise $n[V]$, $i_L[A]$ is the average inductor current, $R[\Omega]$ is the load resistance of the circuit, $L[H]$ is the filter inductance, $C[F]$ is the filter capacitance, $V_{in}[V]$ is the input voltage source, and $d(t)$ represents the unknown (possibly time-varying and nonlinear) external disturbance.

The control objective is to force $v_o(t)$ to follow a reference capacitor output voltage trajectory $v_r(t)[V]$ by manipulating $\mu(t)$ with following assumptions applying.

**Assumption 1:** Following the limitations resulting from the physical properties of the considered electronic circuit, we may assume that the values of voltage and current are bounded, and belong to some compact set such that $\sup_{t \geq 0} v_o(t) < R$ and $\sup_{t \geq 0} i_L(t) < r_{L}$. $\sup_{t \geq 0} n(t) < n$.

**Assumption 2:** Output voltage $v_o(t)$ is the only measurable signal and is additionally corrupted by bounded, high-frequency measurement noise $\sup_{t \geq 0} |n(t)| < n$.

**Assumption 3:** The unknown external disturbance signal $\sup_{t \geq 0} |d(t)| < r_d$ is bounded and has bounded first time-derivative $\sup_{t \geq 0} |\dot{d}(t)| < r_{\dot{d}}$.

**Assumption 4:** There exists a positive constant $r_v$, such that the reference signal and its specific time-derivatives satisfy inequality $\sup_{t \geq 0} \left\{ |\dot{v}_r(t)| \right\} \leq r_v$, for $i \in \{0, 1, 2, 3\}$.

B. Application of the ADRC principle

Following the standard ADRC design, system [1] need to be reformulated, emphasizing the system input-output relation:

$$\dot{v}_o(t) = a_1 v_o(t) + a_2 \dot{v}_o(t) + b \mu(t) + d(t).$$

Combining the uncertain (or unknown) terms of (2), including the imperfect identification of the input gain, results in a following form of the output voltage dynamics:

$$\dot{v}_o = a_1 v_o + a_2 \dot{v}_o + b \mu + \dot{d} = F(\cdot) + \dot{d},$$

where $\dot{d}$ is an estimate of the input gain $d$ from (2) and $F(\cdot)$ represents the matched total disturbance of system [3].

Since $v_r(t)$ and its derivatives may not be known a priori, which may lead to possible inability of constructing the feed-forward term in the controller $\mu$, let us reformulate dynamics [3] into an error-domain as

$$\ddot{e} = \ddot{v}_r - \dot{v}_o = F(\cdot) - \dot{d} = F^*(\cdot) - \dot{d},$$

where $F^*(\cdot)$ is the total disturbance in the error-domain. In this article, we utilize a standard form of the ADRC controller

$$\mu = \hat{b}^{-1}(F^* + \mu_0),$$

which is constructed to simultaneously compensate the influence of disturbance using the estimated value of total disturbance $F^*$ and to stabilize system [4] in a close vicinity of the equilibrium point $e = 0$ using the output-feedback stabilizing controller $\mu_0$.

**Assumption 5:** Stabilizing controller $\mu_0$ has a structure that guarantees the boundedness of $\mu_0(\cdot)$ and $\dot{\mu}_0(\cdot)$. Although this assumption may seem conservative, it is relaxed with the previously introduced Assumptions [1] [3] and [4].

We will first put the focus on precise and on-line estimation of perturbation term $F^*(\cdot)$, crucial for proper active disturbance rejection. To calculate $F^*$, we first need to define the extended state $z = [z_1, z_2, z_3]^T \in D_z$, where $D_z = \{ z \in \mathbb{R}^3 : \|x\| < r_z \}$ for some $r_z \in \mathbb{R}^3$. The dynamics of state vector $z$ can be expressed, upon [4], as a state-space model

$$\begin{align*}
\dot{z} &= Az - b\dot{\mu} + F^*, \\
y &= e - n = c^T z - n,
\end{align*}$$

where $A = [0^2 \times 1, r_2 \times 2, r_0 \times 2], d \in [0 1 0]^T, c \in [1 0 0]^T, \text{and } b \in [0 0 1]^T$. Given (6), the output of this system $y$ corresponds to the control error $e$ which, according to Assumption 2, is influenced by the measurement noise $n$.

**Remark 1:** Control error $e$, together with its derivative $\dot{e}$ are bounded according to the Assumptions [1] [3] and [4] and the specific form of system dynamics [1].

**Remark 2:** Under the Assumptions [1] [3] and [4], function $F^*(\cdot)$ is continuously differentiable, and thus, there exist bounded continuous functions $\Psi_{F}, \Psi_{\dot{F}}$, such that $\sup_{t \geq 0} |F^*(\cdot)| < \Psi_{F}(e, \dot{e}, v_r, \dot{v}_r, \dot{\mu})$, $\sup_{t \geq 0} |\dot{F}^*(\cdot)| < \Psi_{\dot{F}}(e, \dot{e}, v_r, \dot{v}_r, \dot{\mu}, \ddot{\mu})$, for all $[e, \dot{e}] \in \mathbb{R}^2$. Both practical and theoretical justifications of lumping selected components as parts of $F^*(\cdot)$, including control signal an state-dependent variables, has been thoroughly discussed in [5].
III. MAIN RESULT: PROPOSED CASSE ESO ADRC

To calculate the estimated value of extended state vector \( z \), let us now introduce a novel \( p \)-level structure of a cascade observer \( (p \in \mathbb{Z} \text{ and } p \geq 2) \) in a following form

\[
\dot{\hat{x}}_1(t) = A_1 \hat{x}_1(t) - d_1 \mu(t) + l_1 \left[ g(t) - c^T \hat{x}_1(t) \right],
\]

\[
\dot{\hat{x}}_i(t) = A_i \hat{x}_i(t) + d \left( -b \mu(t) + b \sum_{j=1}^{i-1} \hat{x}_j(t) \right) + l_i c^T \left[ \hat{x}_{i-1}(t) - \hat{x}_i(t) \right], \quad i \in \{2, \ldots, p\},
\]

(7)

where \( \hat{x}_j = [\hat{x}_{j,1}, \hat{x}_{j,2}, \hat{x}_{j,3}]^T \in \mathbb{R}^3 \) is the state of particular observer cascade level, \( l_j = [3\omega_{o1} 3\omega_{o2} \omega_{o3}]^T \in \mathbb{R}^3 \) is the observer gain vector with design parameter \( \omega_{o} \in \mathbb{R}^3 \) for \( \alpha > 1 \), \( \omega_{o1} \in \mathbb{R}^+ \), and \( j \in \{1, \ldots, p\} \). The estimate of \( z \), resulting from the observer (7) can be expressed as

\[
\hat{z} = [\hat{z}_1 \hat{z}_2 \hat{z}_3]^T = \hat{\xi}_p + b b^T \sum_{j=1}^{p-1} \xi_j \in \mathbb{R}^3.
\]

(8)

Remark 3: It is worth noting, that if we reduce the observer to a single level \( (p = 1) \), we would obtain a standard form of a linear high-gain ESO, as seen in (19). An introduction of the subsequent cascade levels allows us to keep the same observation quality with smaller values of \( \omega_{o1} \), resulting in the decrease of the measurement noise amplification, see (7). This effect will become visible in the upcoming experiments.

Remark 4: The idea of the cascade observer structure proposed in (7) is based on a specific choice of the first level observer bandwidth \( \omega_{o1} \), which should be large enough to guarantee a precise estimation of the first element of the extended state vector \( z \), and low enough to filter out the measurement noise. The latter elements of the extended state vector, i.e. \( z_2 \) and \( z_3 \), usually have faster transients, and thus, are not estimated precisely with the first level observer. The consequent observer levels are introduced to improve the estimation quality of \( z_2 \) and \( z_3 \) using higher observer bandwidths \( \omega_{o2} \) for \( i > 1 \). The following observer levels are using the state vectors of previous observer levels instead of a measured signal, and thus, should result in lower noise amplification than a single-level ESO with high bandwidth value.

Having \( \hat{z} \), the application of control action (5) to the system (4) results in following second-order error dynamics

\[
\dot{e} = F^* - \hat{F}^*-\mu_0,
\]

(9)

where \( \hat{F}^* \in \mathbb{R} \) is the residue of the total disturbance resulting from the imperfect observation of \( F^* \) by observer (7). The introduction of consecutive cascade level of the observer can be interpreted as an attempt to estimate the total disturbance residue, based on the signals returned on the output of previous cascade level, and its inclusion in the overall estimate of the extended state vector (8).

A block diagram of the proposed ADRC with cascade ESO for the DC-DC buck power converter is shown in Fig. 2.

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**Fig. 2:** Proposed ADRC with sensor noise suppression via cascade ESO structure for the DC-DC buck power converter.

**Theorem 1:** Under Assumptions [23] and by taking a stabilizing proportional derivative controller

\[
\mu_0 = k_p (z - n) + k_d \dot{z}_2, \quad k_p, k_d > 0,
\]

(10)

the observation errors of the extended state obtained with the \( p \)-level cascade observer, defined as

\[
\dot{\tilde{z}}_p = z - \tilde{z}_p = z - \xi_p - b b^T \sum_{j=1}^{p-1} \xi_j \in \mathbb{R}^3,
\]

(11)

together with the control error \( e \), described with the dynamics (9), are bounded. In other words:

\[
\forall \omega_{o1}, k, \alpha > 0 : \text{Is}_{\infty} \left[ \tilde{z}_p(t) \right] < \delta_\varepsilon \wedge \text{Is}_{\infty} \left[ e(t) \right] < \delta_c.
\]

Remark 5: To keep the notational conciseness of the following theoretical analysis, we propose to tune the stabilizing controller (10) with a single parameter \( k_p > 0 \), setting the values of proportional and derivative gains, respectively, as \( k_p = k^2 \) and \( k_d = 2k \). Chosen tuning procedure places the poles of control error dynamics (9) at the value of \(-\sqrt{k} \). Similar controller parameterization was used in (19).

**Proof of Theorem 1.** The dynamics of the observation error defined for a particular cascade level, i.e. \( \dot{\tilde{z}}_i = z - \tilde{z}_i - b b^T \sum_{j=1}^{i-1} \xi_j \in \mathbb{R}^3 \) for \( i \in \{1, \ldots, p\} \), can be expressed (after some algebraic transformations) as

\[
\dot{\tilde{z}}_1 = (A - l_1 c^T) \tilde{z}_1 - l_2 \tilde{z}_1 + b \tilde{F}^*,
\]

\[
\dot{\tilde{z}}_i = (A - l_i c^T) \tilde{z}_i + (l_i c^T - b b^T l_{i-1} c^T) \tilde{z}_{i-1} - b b^T l_1 \tilde{z}_1 + b \tilde{F}^* - b b^T \sum_{j=1}^{i-2} (l_j c^T - l_{j+1} c^T) \tilde{z}_{j}, \quad i \in \{2, \ldots, p\}.
\]

(13)

Equations (13) allow us to write the dynamics of the aggregated observation error \( \dot{\tilde{\zeta}} = [\tilde{z}_1^T \ldots \tilde{z}_p^T]^T \in \mathbb{R}^{3p} \) in a form

\[
\dot{\tilde{\zeta}} = H_\zeta \tilde{\zeta} + \delta \tilde{F}^* + \gamma n,
\]

(14)

where matrix \( H_\zeta \) is lower triangular and its eigenvectors \( \lambda_i \in \{-\omega_{o1}, -\alpha \omega_{o1}, \ldots, -\alpha^p \omega_{o1}\} \) for \( i \in \{1, \ldots, 3p\} \), vector
\[ \delta = [b^T \ldots b^T]^T, \quad \gamma = [(1 \ldots 1) b^T]^T. \] Introducing the transformation \( \tilde{\chi} = \Lambda_ch \) for \( \Lambda_ch = \text{blkdiag}(L_1, \ldots, L_p) \in \mathbb{R}^{3p \times 3p} \) where \( L_i = \text{diag}\{(\alpha^i \omega_1)^{-2}, (\omega_1)^{-1}, 1 \} \in \mathbb{R}^{3 \times 3} \) for \( i \in \{1, \ldots, p\} \in \mathbb{R}^{p \times p} \), we can rewrite (14) to a form
\[
\dot{\chi} = \Lambda_ch^T Hc + \Lambda_ch^T \delta \dot{F}^* + \Lambda_ch^T \gamma n
= \omega_1 \Lambda_ch^T Hc + \delta \dot{F}^* + \gamma n,
\]
where \( Hc \) is dependent only on parameter \( \alpha \) and its eigenvalues \( \lambda_i \in \{-1, -\alpha, \ldots, -\alpha^p\} \) for \( i \in \{1, \ldots, 3p\} \). To conduct a stability analysis of the observation subsystem, let us introduce a Lyapunov function candidate \( Vc = \chi^T P \chi : \mathbb{R}^{3p} \to \mathbb{R} \) limited by \( \lambda_{\min}(Pc) |\chi|^2 \leq Vc \leq \lambda_{\max}(Pc) |\chi|^2 \), where \( P > 0 \) is the solution of Lyapunov equation \( Hc^T Pc + PcHc = -I \).

The derivative of \( Vc \), based on (15), can be written down as
\[
\dot{Vc} = -\omega_1 \chi^T \chi + 2 \chi^T P \delta \dot{F}^* + \gamma n
= -\omega_1 |\chi|^2 + 2 |\chi| \lambda_{\max}(Pc) \sqrt{\beta (|\dot{F}^*| + \omega_1 n)}
\]
and holds
\[
\dot{Vc} \leq -(1 - \nu_\chi) \omega_1 |\chi|^2 \quad \text{for}
|\chi| \geq \frac{2 \lambda_{\max}(Pc)}{\omega_1 \nu_\chi} \sqrt{\beta (|\dot{F}^*| + \omega_1 n)} + \frac{2 \lambda_{\max}(Pc) \sqrt{\beta \omega_1 n}}{\nu_\chi},
\]
where \( \nu_\chi \in (0, 1) \) is a chosen majorization constant. The lower bound of \( [\chi] \) is a class \( K \) function with respect to the perturbations \( |\dot{F}^*| \) and \( |n| \), so according to the Remark 2 and Assumption 2, system (15) is input-to-state stable (ISS), and according to (20), satisfies
\[
\rho_\chi \leq \lambda_{\max}(Pc) \sqrt{\beta (|\dot{F}^*| + \omega_1 n)} + \frac{2 \lambda_{\max}(Pc) \sqrt{\beta \omega_1 n}}{\nu_\chi},
\]
for \( \rho_\chi = \sqrt{\lambda_{\max}(Pc) / \lambda_{\min}(Pc)} \). Since \( \lambda_{\max}(A) = \max\{1, (\nu_\chi^{-1} \omega_1)^{-2}\} \) and \( \tilde{z}_p \) is a subvector of \( \tilde{z} \), we may write down that \( |\tilde{z}_p| \leq \tilde{\chi} \leq \lambda_{\max}(A) |\chi| \) and thus the asymptotic relation
\[
\rho_\chi \leq \lambda_{\max}(A) \leq \rho_{\tilde{z}} = \lambda_{\max}(A) \leq \lambda_{\max}(Pc) \sqrt{\beta (|\dot{F}^*| + \omega_1 n)} + \frac{2 \lambda_{\max}(Pc) \sqrt{\beta \omega_1 n}}{\nu_\chi},
\]
which completes the proof of the observer part of (12).

Remark 6: Upon the result (18), we can see that in the nominal conditions, when \( n(t) = 0 \), the asymptotic relation \( \rho_{\tilde{z}} = \lambda_{\max}(A) |\chi| \to 0 \) as \( \omega_1 \to \infty \) resulting in the possibility of getting an arbitrarily small value of \( \delta_{\tilde{z}} \).

Let us define control error vector \( e = [e \ e]^T \in \mathbb{R}^2 \). The application of feedback controller (10) to dynamics (9) gives
\[
\dot{e} = \begin{bmatrix} 0 & 1 \\ -k & -2k \end{bmatrix} e + \begin{bmatrix} 0 & 0 \\ 0 & 2k \end{bmatrix} \dot{z}_p - \begin{bmatrix} 0 \\ k \end{bmatrix} n,
\]
which can be transformed with substitution \( e = \Lambda_c e \), where \( \Lambda_c = \text{diag}\{k^{-1}, 1\} \), into
\[
\dot{e} = \Lambda_c^{-1} K \Lambda_c e + \Lambda_c^{-1} Z \tilde{z}_p - \Lambda_c^{-1} \kappa n
= k \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} e + Z \tilde{z}_p - \kappa n.
\]

Let us now introduce a Lyapunov function candidate \( \dot{Vc} = e^T P \dot{e} : \mathbb{R}^2 \to \mathbb{R} \) limited by \( \lambda_{\min}(Pc) |e|^2 \leq \dot{Vc} \leq \lambda_{\max}(Pc) |e|^2 \), where \( P > 0 \) is the solution of Lyapunov equation \( Hc^T Pc + PcHc = -I \).

The derivative of \( \dot{Vc} \), based on (15), can be written down as
\[
\dot{Vc} = -\kappa |e|^2 + 2 \kappa |e| \lambda_{\max}(Pc) \sqrt{\beta (|\dot{\rho}^*| + \omega_1 n)}
\]
and holds
\[
\dot{Vc} \leq -(1 - \kappa) \omega_1 |e|^2 \quad \text{for}
|e| \leq \frac{2 \lambda_{\max}(Pc) \sqrt{\beta \omega_1 n}}{\nu_\chi},
\]
where \( \nu_\chi \in (0, 1) \) is a chosen majorization constant. The lower bound of \( [e] \) is a class \( K \) function with respect to the perturbations \( |\dot{\rho}^*| \) and \( |n| \), so according to the Remark 2 and Assumption 2, system (15) is ISS and satisfies
\[
ls_{\infty} [e(t)] \leq m_k \rho_{\tilde{z}} = m_k \rho_{\tilde{z}} + 2 \kappa |e| \lambda_{\max}(Pc) \sqrt{\beta (|\dot{\rho}^*| + \omega_1 n)} + \frac{2 \lambda_{\max}(Pc) \sqrt{\beta \omega_1 n}}{\nu_\chi},
\]
for \( m_k = \frac{2 \kappa |e|}{\nu_\chi} \).

Remark 7: Similarly to the comment made in Remark 5 in the case of \( \rho_\chi \equiv 0 \) and upon the result (25), we can say that \( ls_{\infty} [e(t)] \to 0 \) as \( \omega_1 \to \infty \) and \( k \to \infty \), making it possible to get an arbitrarily small value of \( \delta_{\tilde{z}} \).

Remark 8: Upon the result (25), we may observe that the increasing gains of both observer and controller are amplifying measurement noise, thus, it is not recommended to use extremely high values of \( \omega_1 \) and \( k \) in practice.

IV. HARDWARE EXPERIMENT

A. Tested description

The experimental setup used for the study is seen in Fig. 3. The output voltage was measured by a Hall effect-based sensor and converted through a 16-bit A/D converter in the dSPACE platform. The output was recorded by a digital oscilloscope.
and dedicated PC-based software. The sampling period was set to $T_s = 10^4$ Hz. The physical parameters of the DC-DC converter, described with $\{1\}$, were $V_{in} = 20\text{V}$, $L = 0.01\text{H}$, $C = 0.001\text{F}$, and $R = 50\Omega$. This allowed to straightforwardly calculate the system gain in $\{3\}$ as $\bar{b} = V_{in}/(CL) = 2 \times 10^6$. The tested control algorithm was first implemented in a Matlab/Simulink-based model, from which a C code program was generated and run on the dSPACE controller in real-time.

Considering the above parameters of the utilized testbed and the controller/observer structures introduced in $\{5\}$, $\{10\}$, and $\{7\}$, we can derive the transfer-function-based relation

$$U(j\omega) = G_{uy}(j\omega) E(j\omega) = N(j\omega) Y(j\omega),$$  \hspace{1cm} \text{(26)}

where $U(j\omega)$, $E(j\omega)$, $N(j\omega)$, and $Y(j\omega)$ correspond respectively to signals $\mu(t)$, $e(t)$, $n(t)$, and $y(t)$ after Laplace transformation. The amplitude Bode diagram of $G_{uy}(j\omega)$ obtained for the observer levels $p \in \{1, 2, 3\}$ and tuned with the nominal parameters utilized in the experiment is presented in Fig. 4. The vertical dashed lines represent the chosen controller bandwidth $k$, which is the range we expect the closed-loop system to operate in, and the experiment sampling frequency $\omega_s$. The green area represents the frequency range, where CESO ($p = 2$ and $p = 3$) should react more rapidly than standard ESO, and red area is the range where only CESO ($p = 2$) should provide quicker response with respect to control errors. The points at the intersection of $\omega_s$ and observers graphs indicate the amplification factors of high frequency signals (e.g. measurement noise) within signal $\mu(t)$. Consequently, in the following experiments, we can expect the measurement noise to be least amplified in CESO $p = 3$, followed by CESO $p = 2$ and finally standard ESO.

### Table I: Used bandwidth parameterization of CESOs.

| Bandwidth       | Cascade level | $p = 1$ | $p = 2$ | $p = 3$ |
|-----------------|---------------|--------|--------|--------|
| 1st level ESO ($\omega_{o1}$) | $\lambda$ | $\frac{\lambda}{\alpha}$ | $\frac{\lambda}{\alpha^2}$ | |
| 2nd level ESO ($\omega_{o2}$) | $\lambda$ | $\lambda$ | $\lambda$ | |
| 3rd level ESO ($\omega_{o3}$) | $\lambda$ | $\lambda$ | $\lambda$ | |

### Table II: Integral quality criteria for experiment E1.

| Observer type | $\int |e(t)|dt$ | $\int |u(t)|dt$ | $\int |i(t)|dt$ |
|---------------|---------------|---------------|---------------|
| Standard ESO ($p = 1$) | 0.2310 | 0.5368 | 315.58 |
| Cascade ESO ($p = 2$) | 0.0467 | 0.5496 | 113.23 |
| Cascade ESO ($p = 3$) | 0.0381 | 0.5545 | 29.11 |

\textbf{Fig. 3:} Laboratory setup, with a - buck converter, b - dSPACE controller, c - input voltage, d - oscilloscope, e - voltage sensor, f - A/D converters, and g - PC with control software.

\textbf{Fig. 4:} Bode diagram representing the module of $G_{uy}(j\omega)$.

\textbf{B. Test methodology and results}

Two following sets of experiments were conducted to test the ADRC scheme with the proposed cascade ESO (CESO): E1: Comparison with standard ESO (i.e. CESO with $p = 1$).

E2: Influence of parameters $\omega_{o1}$ (E2a), $k$ (E2b), and $\alpha$ (E2c).

\textbf{Remark 9:} Please note that a standard, single ESO is synonymous with the CESO with cascade level $p = 1$.

The control objective in both tests was to track a smooth voltage trajectory despite the presence of a varying input-additive external disturbance shown in Fig. 5. Reference trajectory was designed as a filtered and biased square signal with the bias equal to 7V, the amplitude of square signal equal to 6V, and period 1s. The filtering transfer function applied to the square signal was $G_f(s) = \frac{4}{0.025s^2 + 0.6s + 1}$.

The results of E1 are gathered in Fig. 6. The observer bandwidth for the standard ESO ($p = 1$) was $\omega_{o1} = 3600\text{rad/s}$, which was close to the maximum that could be obtained for a 10kHz sampling without observing any undesirable effects. For the comparison, only CESOs with $p = 2$ and $p = 3$ levels were utilized to maintain legibility of the results while not loosing their generality. In order to provide a systematic tuning methodology across tested observers, bandwidths of the CESOs were parameterized and set according to Table I with $\alpha = 3$ and $\lambda = 3600\text{rad/s}$. The controller gains from (10) were set to $k_p = 6400$ and $k_d = 160$ in each case, which corresponds to controller bandwidth $k = 80$, introduced in Remark 5.

One can notice from Fig. 6 that, with the applied tuning methodology, all the tested controllers have realized the given task, however the standard ESO ($p = 1$) provided the worst performance in terms of tracking accuracy and noise suppression. On the other hand, with the increase of cascade level $p$ in CESO, better performance was achieved. This observation is supported with the calculated integral quality indices in Table II. Besides the improvement of control error performance, the transfer of sensor noise into the control signal has decreased with the increase of parameter $p$ thanks to the
lower values of $\omega_{01}$ related to the first level of CESO. This result is supported with the values of $\int |\dot{u}(t)|dt$ criterion in Table 4 [3] which represents the impact of rapid fluctuations of control signal, mostly caused by the amplified noise.

The initial premises formulated upon Fig. 4 have been confirmed with the results presented in Fig. 6. As expected, the control signal with the lowest content of measurement noise was obtained for CESO $p = 3$, then CESO $p = 2$, and finally the standard ESO.

Next, in order to provide potential CESO users with guidelines for its construction and tuning, the influence of its design parameters was investigated. To this effect, the results of E2 are seen in Fig. 7-9. In contrast to E1, here only control errors and control signals are presented to save space. However, the estimated total disturbance is part of the control signal (see (5)) so its influence is explicitly visible in the control signal.

The results of E2a are depicted in Fig. 7. In case of the standard ESO ($p = 1$), a well-known relation known from high-gain observers, discussed in the Introduction, can be observed. Namely, with the increase of observer bandwidth $\omega_{01}$, significant noise amplification occurs in the control signal. At the same time, a slight improvement of the control error was obtained. In case of the proposed CESO ($p = 2$ and $p = 3$), with the increase of $\omega_{01}$, the amplitude of the control signal increases but no visible improvement in the control accuracy can be observed. In other words, in engineering practice, at some point, due to multiple factors like maximum sampling frequency and noise characteristics, increasing the observer bandwidth $\omega_{01}$ will no longer provide better performance. We can conclude, that with CESO one can achieve better control performance for the wide range of $\omega_{01}$ values, comparing to the results of standard ESO ($p = 1$) in Fig. 7a.

The results of E2b are depicted in Fig. 8. In the case of the standard ESO ($p = 1$), it is clear that increasing the controller bandwidth $k$ improves the control accuracy while keeping a significant, undesired level of control signal and noise therein. In case of the proposed CESO ($p = 2$ and $p = 3$), increasing the controller bandwidth $k$ results in comparable control errors retaining similar level of the control signal. Due to the characteristics of CESO, it is possible to obtain better control performance for the wide range of $k$ values, comparing to the results obtained for standard ESO in Fig. 8a.

The results of E2c are depicted in Fig. 9. In case of the CESO ($p = 2$), increasing $\alpha$ improves both the tracking accuracy and noise suppression in the control signal. However, in case of the CESO ($p = 3$), increasing $\alpha$ keeps improving the noise suppression in the control signal but at some point deterioration in the tracking accuracy can be spotted. It results from a fact that, in this case, observer bandwidth $\omega_{01}$ is set too small, which makes the observer not providing a fast-enough and accurate-enough estimate of the first state variable of the extended state vector (see Remark 4).

V. CONCLUSION

An active disturbance rejection control with a novel cascade extended state observer (CESO) for DC-DC buck converters has been proposed. The validity of the new approach has been shown through a dedicated stability analysis and a set of hardware experiments. The comparison between the proposed cascade ESO-based ADRC and a standard single ESO-based ADRC showed that the former has stronger capabilities of sensor noise suppression and provides better control performance (understood as tracking accuracy and energy efficiency). The structure of the proposed ADRC is bulkier than the conventional one but in return provides an additional and practically appealing degree of freedom in shaping the influence of measurement noise on the observer/controller part.

VI. ACKNOWLEDGEMENT

The article was created thanks to participation in program PROM of the Polish National Agency for Academic Exchange. The program is co-financed from the European Social Fund within the Operational Program Knowledge Education Development, non-competitive project entitled International

Fig. 5: External disturbance applied in all of the experiments.

Fig. 6: Results of experiment E1.
scholarship exchange of PhD students and academic staff executed under the Activity 3.3 specified in the application for funding of project No. POWR.03.03.00-00-PN13 / 18. The work has been also supported by the Fundamental Research Funds for the Central Universities project no. 21620335.

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Fig. 9: Results of experiment E2c.