PROPOSITION EFFECTS IN MAGNETIZED TRANSRELATIVISTIC PLASMAS

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ABSTRACT

The transfer of polarized radiation in magnetized and nonmagnetized relativistic plasmas is an area of research with numerous flaws and gaps. The present paper is aimed at filling some gaps and eliminating the flaws. Starting from a Trubnikov's linear response tensor for a vacuum wave with \(|k| = \omega/c\) in thermal plasma, the analytic expression for the dielectric tensor is found in the limit of high frequencies. The Faraday rotation and Faraday conversion measures are computed in their first orders in the ratio of the cyclotron frequency \(\Omega_B\) to the observed frequency \(\omega\). The computed temperature dependencies of propagation effects bridge the known nonrelativistic and ultrarelativistic limiting formulae. The fitting expressions are found for high temperatures, where the higher orders in \(\Omega_B/\omega\) cannot be neglected. The plasma eigenmodes are found to become linearly polarized at much larger temperatures than thought before. The results are applied to the diagnostics of the hot interstellar medium, hot accretion flows, and jets.

Subject headings: magnetic fields — polarization — radiative transfer

Online material: color figures

1. INTRODUCTION

We learn much of our information about astrophysical objects by observing the light they emit. Observations of the polarization properties of light can tell us the geometry of the emitter, strength of the magnetic field, density of the plasma, and temperature. The proper and correct theory of optical activity is essential for making accurate predictions. While the low-temperature propagation characteristics of plasma are well established (Landau & Lifshits 1981), the theory of relativistic effects has not been fully studied. In this paper I discuss the propagation effects through a homogeneous magnetized relativistic plasma. A nonmagnetized case emerges as a limit of the magnetized case. The discussion is divided into three separate topics.

Two linear plasma propagation effects are Faraday rotation and Faraday conversion (Azzam & Bashara 1987). Traditionally, these effects are considered in their lowest orders in the ratio \(\beta\) of the cyclotron frequency \(\Omega_c\) to the circular frequency of light \(\omega\), i.e., in a high-frequency approximation. The distribution of particles is taken to be thermal,

\[
dN = \frac{n}{4\pi m^3 k_B^2 (T^{-1})} d^3 p, \tag{1}
\]

with the dimensionless temperature \(T\) in the units of the particle rest-mass temperature \(mc^2/k_B\). The Faraday rotation measure RM and conversion measure are known in nonrelativistic \(T \ll 1\) and ultrarelativistic \(T \gg 1\) limits (Melrose 1997c). I derive a surprisingly simple analytic expression for arbitrary temperature \(T\).

The smallness of \(\beta = \Omega_c/\omega, \beta \ll 1\), in the real systems led some authors (Melrose 1997a) to conclude that the high-frequency approximation will always work. However, there is a clear indication that it breaks down at high temperatures, \(T \gg 1\). It was claimed that the eigenmodes of plasma are linearly polarized for high temperatures \(T \gg 1\) (Melrose 1997c), because the second-order term \(\sim \beta^2\) becomes larger than the first-order term \(\sim \beta\) due to the \(T\) dependence. The arbitrarily large \(T\)-factor may stand in front of higher order expansion terms in \(\beta\) of the relevant expressions. I find the generalized rotation measure as a function of \(\beta\) and \(T\) without expanding in \(\beta\) and compare the results with the known high-frequency expressions. The high-\(T\) behavior of the plasma response is indeed significantly different.

Plasma physics involves complicated calculations. This led to a number of errors in the literature (Melrose 1997c), some of which have still not been fixed. In the article I check all the limiting cases numerically and analytically and expound on all the steps of derivations. Thus, I correct the relevant errors and misinterpretations made by previous authors, hopefully not making new mistakes. The analytical and numerical results are obtained in the Mathematica 6 system. It has an enormous potential in these problems (Marichev 2008).

The paper is organized as follows. The formalism of plasma response and calculations are described in § 2. Several applications to observations can be found in § 3. I conclude in § 4 with a short summary and future prospects.

2. CALCULATIONS

2.1. Geometry of the Problem

I assume the traditional geometry depicted on Figure 1 has the following:

1. Euclidean basis \((\vec{e}_1, \vec{e}_2, \vec{e}_3)\),
2. magnetic field along the third axis \(\vec{B} = (0, 0, B)^T\),
3. a wavevector of the wave \(\vec{k} = k(\sin \theta, 0, \cos \theta)^T\) with an angle \(\theta\) between \(\vec{k}\) and \(\vec{B}\).

The basis is rotated from \((\vec{e}_1, \vec{e}_2, \vec{e}_3)\) to \((e^1, e^2, e^3)\), so that the wave propagates along \(\vec{k} = (0, 0, k)^T\) in the new basis. The transformation has the form

\[
e^1 = \vec{e}^1 \cos \theta - \vec{e}^3 \sin \theta, \quad e^2 = \vec{e}^2, \quad e^3 = \vec{e}^1 \sin \theta + \vec{e}^3 \cos \theta, \tag{2}
\]

which can be conveniently written as

\[
e^{\mu} = \vec{e}^{\mu} S^{\alpha \mu}, \quad S^{\alpha \mu} = \begin{pmatrix}
\cos \theta & 0 & \sin \theta \\
0 & 1 & 0 \\
-\sin \theta & 0 & \cos \theta
\end{pmatrix}. \tag{3}
\]
Vectors and tensors then rotate according to

\[ A^\mu = (ST)^{\mu \nu} A^\nu, \quad \alpha^{\mu \nu} = (ST)^{\mu \rho} \varepsilon^{\rho \sigma \nu} S^\sigma. \]  \hfill (4)

### 2.2. Linear Plasma Response

The propagation of weak electromagnetic (EM) waves in a homogeneous magnetized plasma can be fully described by the response tensor \( \alpha^{\mu \nu} \). It expresses the linear proportionality between the induced current density and the vector potential \( j^\mu = \alpha^{\mu \nu} A^\nu \). The spatial projection of a so-defined four-dimensional tensor \( \alpha^{\mu \nu} \) is equal to the three-dimensional tensor \( \alpha_{ij} \) defined by \( j = \alpha_{ij} A^j \).

I consider Trubnikov's form of the response tensor (Trubnikov 1958; Melrose 1997a). I work in a low-density regime, where the plasma response is calculated for a vacuum wave with \( |k| = \omega/c \). I take the tensor \( \varepsilon^{\mu \nu} \) from the first-hand derivations (Trubnikov 1958; Melrose 1997a), make the transformation from equation (4), and take the first and second components in both indices. Thus, the projection onto the \( (e^1, e^2) \)-plane in cgs units is

\[ \alpha^{\mu \nu}(k) = \frac{i e^2 n_0 \rho^2}{cm \Omega_0} \int_0^\infty dr \left[ \frac{t^{\mu \nu}_r(k)}{r^2} - R^\mu \tilde{R}^\nu(k) \right], \]  \hfill (5)

\[ t^{\mu \nu}_r = \left( \cos^2 \theta \cos \Omega_0 \xi + \sin^2 \theta \eta \cos \theta \sin \Omega_0 \xi \right), \]  \hfill (6a)

\[ R^\mu = \frac{\omega \sin \theta}{\Omega_0} \left[ \cos \theta (\cos \Omega_0 \xi - \Omega_0 \xi) - \eta (1 - \cos \Omega_0 \xi) \right], \]  \hfill (6b)

\[ \tilde{R}^\nu = \frac{\omega \sin \theta}{\Omega_0} \left[ \cos \theta (\sin \Omega_0 \xi - \Omega_0 \xi) \eta (1 - \cos \Omega_0 \xi) \right], \]  \hfill (6c)

\[ r = \left[ \rho^2 - 2 \Omega_0 \xi \rho + \frac{\omega^2 \sin^2 \theta}{\Omega_0^2} (2 - \Omega_0^2 \xi^2 - 2 \cos \Omega_0 \xi) \right]^{1/2}, \]  \hfill (8)

where \( \eta \) is the sign of the charge and \( K_n(r) \) is the nth Bessel function of the second kind.\(^1\) The quantity \( \rho \) is the dimensionless inverse temperature,

\[ \rho = T^{-1} = \frac{m c^2}{k_B T_p}, \]  \hfill (9)

where \( T_p \) is the actual temperature of particles. The response of plasma is usually characterized by the dielectric tensor. Its projection onto the \( (e^1, e^2) \)-plane is

\[ \varepsilon^{\mu \nu} = \delta^{\mu \nu} + \frac{4 \pi e^2}{\omega^2} \alpha^{\mu \nu}. \]  \hfill (10)

The wave equation for transverse waves in terms of \( \varepsilon^{\mu \nu} \) is

\[ (n^2 \varepsilon^{\mu \nu} - \varepsilon^{\rho \sigma} \left( \frac{E_1}{E_2} \right)) = 0, \]  \hfill (11)

where \( E_1 \) and \( E_2 \) are the components of the electric field along \( e^1 \) and \( e^2 \), respectively, and \( n^2 = k^2 c^2 / \omega^2 \) (Swanson 2003).

#### 2.3. High-Frequency Limit

Let me first calculate the limiting expression for \( \alpha^{\mu \nu} \) in the high-frequency limit \( \Omega_0 / \omega \). I denote

\[ \alpha = \omega \xi, \quad \beta = \frac{\Omega_0}{\omega}, \]  \hfill (12)

substitute the definitions from equation (12) into equation (5), and expand the response tensor \( \alpha^{\mu \nu} \) in \( \beta \). I retain only up to the second order of the expansion, which gives the conventional generalized Faraday rotation (Melrose 1997c). The first terms of the series of \( r, r^2 \), and \( R^\mu \tilde{R}^\nu \) read

\[ r = r_0^2 + \delta r^2, \quad r_0^2 = \rho^2 - 2 i \alpha \rho, \quad \delta r^2 = -\frac{\sin^2 \theta}{12} \beta^2 \alpha^4, \]  \hfill (13)

\[ t^{\mu \nu}_r = \left( 1 - \cos^2 \theta (\alpha^2 \beta^2 / 2 \right) \begin{pmatrix} \alpha \beta \eta \cos \theta \\ -\alpha \beta \eta \cos \theta \end{pmatrix}, \]  \hfill (14)

\[ R^\mu \tilde{R}^\nu = -\frac{\alpha^2 \beta^2}{4} \sin^2 \theta \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \]  \hfill (15)

Melrose (1997c) used the approximation \( r^2_0 = -2 i \alpha \rho \) instead of the expansion from equation (13) and obtained the approximate high-\( F \) expressions as his final answers.

However, one can take the emergent integrals, if one considers the exact expansions from equations (13), (14), and (15). Three terms appear in the expanded expression for \( \alpha^{\mu \nu} \),

\[ \int_0^\infty d\alpha \left[ t^{\mu \nu}_r \frac{K_2(r)}{r_0^2} \right], \]  \hfill (16a)

\[ \int_0^\infty d\alpha \left[ t^{\rho \sigma}_r \frac{K_3(r) \rho^2 r^2}{r_0^2} \right], \]  \hfill (16b)

\[ \int_0^\infty d\alpha \left[ R^\mu \tilde{R}^\nu \frac{K_3(r)}{r_0^2} \right]. \]  \hfill (16c)

The second term, equation (16b), originates from the expansion of \( K_2(r) \rho^2 r^2 \) to the first order,

\[ \frac{K_2(r)}{r^2} \frac{K_2(r_0)}{r_0^2} = \frac{\delta r^2}{2} \frac{K_3(r_0)}{r_0^2}. \]  \hfill (17)

\(^1\) Note that the analogous expression in Melrose (1997c) has an extra factor \( \Omega_0 \xi \) in the component \( t^{\mu \nu}_r \) and the opposite sign of the \( R^\mu \tilde{R}^\nu \) term by an error. The author has corrected his formulae in Melrose (2008).
The integrals from equations (16a), (16b), and (16c) can be evaluated knowing that
\[
\int_0^\infty d\alpha \left[ \frac{K_2(\sqrt{\alpha^2 - 2i\alpha})}{\alpha^2 - 2i\alpha} \right] = n!i^{n+1} \frac{K_{n-1}^2(0)}{\pi^2}. \tag{18a}
\]
\[
\int_0^\infty d\alpha \left[ \frac{K_3(\sqrt{\alpha^2 - 2i\alpha})}{(\alpha^2 - 2i\alpha)^{3/2}} \right] = n!i^{n+1} \frac{K_{n-2}(0)}{\pi^3}. \tag{18b}
\]

2.4. Components in High-Frequency Limit

I substitute the high-frequency expansions from equations (13), (14), and (15) into equation (10) for the projection of the dielectric tensor \( \epsilon^\mu_\nu \) with the projection of the response tensor \( a^\mu_\nu \) from equation (5) and take the integrals from equations (16a), (16b), and (16c) analytically. The components of the dielectric tensor (eq. [10]) in the lowest orders in \( \Omega_0/\omega \) are then
\[
\epsilon_1^1 = 1 - \frac{\omega_p^2}{\omega^2} \frac{K_1(\rho)}{K_2(\rho)} \left( 1 + \frac{\Omega_0^2}{\omega^2} \cos^2 \theta \right), \tag{19a}
\]
\[
\epsilon_2^2 = 1 - \frac{\omega_p^2}{\omega^2} \frac{K_1(\rho)}{K_2(\rho)} \left( 1 + \frac{\Omega_0^2}{\omega^2} \sin^2 \theta \right), \tag{19b}
\]
where the plasma frequency \( \omega_p \) in cgs units is
\[
\omega_p^2 = \frac{4\pi n q^2}{m}. \tag{21}
\]

The results reproduce the nonrelativistic limits for \( \rho \to +\infty,
\]
\[
\epsilon_1^1 = 1 - \frac{\omega_p^2}{\omega^2} \left( 1 + \frac{\Omega_0^2}{\omega^2} \cos^2 \theta \right), \tag{22a}
\]
\[
\epsilon_2^2 = 1 - \frac{\omega_p^2}{\omega^2} \left( 1 + \frac{\Omega_0^2}{\omega^2} \sin^2 \theta \right), \tag{22b}
\]
where all Bessel functions of \( \rho \) approach unity\(^2\) (Landau & Lifshits 1981; Trubnikov 1996; Swanson 2003; Bellan 2006). The corresponding relativistic limits \( \rho \to 0 \) of the same components are
\[
\epsilon_1^1 = 1 - \frac{\omega_p^2}{\omega^2} \left[ 1 + \frac{\Omega_0^2}{\omega^2} \cos^2 \theta \right] + \frac{\Omega_0^2 \sin^2 \theta}{\omega^2}, \tag{23a}
\]
\[
\epsilon_2^2 = 1 - \frac{\omega_p^2}{\omega^2} \left[ 1 + \frac{\Omega_0^2}{\omega^2} \sin^2 \theta \right] + \frac{\Omega_0^2 \sin^2 \theta}{\omega^2}, \tag{23b}
\]
\[
\epsilon_1^1 = 1 - \frac{\omega_p^2}{\omega^2} \sin^2 \theta, \tag{23c}
\]
consistent with Melrose (1997c) and Quataert & Gruzinov (2000).\(^3\) The ultrarelativistic nonmagnetized dispersion relation then reads
\[
\omega^2 = \frac{\omega_p^2}{2T} + c^2 k^2 = \frac{2\pi n q^2}{mT} + c^2 k^2 \tag{24}
\]
\(^2\) The nondiagonal term has a wrong sign in Melrose (1997c).
\(^3\) The diagonal plasma response is 2 times larger in Melrose (1997c) by an error.

![Fig. 2.—Multiplier \( f(X) \) for the difference of the diagonal components \( \epsilon_1 - \epsilon_2 \) for \( \theta = \pi/4 \). [See the electronic edition of the Journal for a color version of this figure.]]
Let me define \( X \) to be the following combination of the parameters

\[
X = T \sqrt{2 \sin \theta} \left(10^{-1} \frac{\Omega_0}{\omega}\right).
\]  

(27)

For the fiducial \( \Omega_0/\omega = 10^{-3} \) and \( \theta = \pi/4 \), the parameter \( X \) is just temperature, \( X = T \).

I first identify the boundaries, where the high-frequency limit is valid. Then I find a fit for the multipliers at higher \( X \). Equation (25) for the difference \( \varepsilon_{1} - \varepsilon_{2} \) is accurate to within 10% for \( X < 0.1 \) if we set \( f(X) = 1 \). Equation (26) for \( \varepsilon_{1} \) is accurate to within 10% for \( X < 30 \) if we set \( g(X) = 1 \). The accuracy depends on the parameter \( X \) rather than on the individual parameters \( T, \Omega_0/\omega, \) and \( \theta \). The expression

\[
f(X) = 2.011 \exp\left(-\frac{X^{1.035}}{4.7}\right) - \cos\left(\frac{X}{2}\right) \exp\left(-\frac{X^{1.2}}{2.73}\right) - 0.011 \exp\left(-\frac{X}{47.2}\right)
\]  

(28)

extends the applicability domain of equation (25) up to \( X \approx 200 \). Figure 4 shows the fit for \( f(X) \) in comparison with the numerical results. The expression

\[
g(X) = 1 - 0.11 \ln (1 + 0.035X)
\]  

(29)

extends up to \( X \approx 200 \) the domain of equation (26). Figure 5 shows the fit for \( g(X) \) in comparison with the numerical results.

2.6. Exact Plasma Response

The expression for the response tensor (eq. [5]) is written for a vacuum wave with \( |k|c = \omega \). In the real plasma, the wave is modified by the plasma response. A more general self-consistent response tensor should be used (Trubnikov 1958; Melrose 1997c). One needs to solve a dispersion relation similar to equation (11) to obtain the eigenmodes. Thus, the eigenmodes and the response tensor should be computed self-consistently. One should not forget about the anti-Hermitian and longitudinal components of the dielectric tensor \( \varepsilon_{\mu}^{\nu} \) that modify the dispersion relation.

2.7. Eigenmodes

The above calculation is applicable also to a nonmagnetized plasma. The dispersion relation of EM waves in a nonmagnetized plasma reads

\[
\omega^2 = k^2 \varepsilon^2 + \omega_p^2 \frac{K_1(T^{-1})}{K_2(T^{-1})}
\]  

(30)

in a high-frequency approximation \( \omega \gg \omega_p \). The opposite limit of \( kc \ll \omega \) was considered by Bergman & Eliasson (2001).

Now we turn to the magnetized case. Melrose (1997c) only considered the first terms of the expansion of \( \alpha_{\mu}^{\nu} \) in \( \beta \) to get the eigenmodes. I do the next step: consider the full expression in \( \beta \) in the low-density regime \( kc = \omega \), but consider only the Hermitian part of \( \alpha_{\mu}^{\nu} \) in computations. The ellipticity \( \Upsilon = (\varepsilon_{1}^\prime - \varepsilon_{2}^\prime) : [\varepsilon_{1}^\prime] \) determines the type of eigenmodes. If \( |\Upsilon| \gg 1 \), then the eigenmodes are linearly polarized unless \( \theta = 0 \). If \( |\Upsilon| \ll 1 \), then the eigenmodes are circularly polarized for \( \theta \) far from \( \pi/2 \). Let me consider the fiducial model with \( \Omega_0/\omega = 10^{-3} \) and \( \theta = \pi/4 \). Figure 6 shows the ratio \( \Upsilon \) calculated in a high-frequency approximation (dashed line; see § 2.3) and in a general...
low-density approximation (solid line; see § 2.5). The high-frequency approximation produces the linear eigenmodes already at $T \gtrsim 10$ consistently with Melrose (1997c). However, the general low-density limit produces the eigenmodes with $\gamma < 1$ up to very high temperatures $T \sim 50$. Unexpectedly, the sign of the diagonal difference $(e_1^2 - e_2^2)$ changes at about $T \approx 25$.

3. APPLICATIONS

The calculated transrelativistic propagation effects have far-reaching consequences in many topics of astronomy. Let me concentrate on four applications: propagation delay, Faraday rotation measure of light from the Galactic center (GC), circularly polarized light from the GC, and diagnostics of jets.

3.1. Dispersion Measure

Propagation delay is an important effect in pulsar dispersion (Phillips & Wolszczan 1992). The relativistic part of this delay can be obtained from the dispersion relation from equation (30). I retain only the first-order correction in $T$, since $T \ll 1$ in the interstellar medium (Cox & Reynolds 1987). Since $K_1(T^{-1})/K_2(T^{-1}) \approx 1 - 3T/2$ at low $T$, the nonrelativistic dispersion measure (DM) should be modified as

$$\text{DM}_{\text{rel}} = \text{DM}_{\text{nonrel}} \left(1 - \frac{3}{2}T\right).$$

This shows that the gas density is slightly underestimated, if the nonrelativistic formulae are used. However, the relativistic correction to the DM is small and can be neglected in most practical cases when $T \ll 1$. The effects in magnetized plasma are also relevant for pulsars.

3.2. Magnetized Radiative Transfer

3.2.1. General Formulæ

Relativistic plasmas exhibit a generalized Faraday rotation for a general orientation of the magnetic field (Azzam & Bashara 1987). One can decompose it into two effects: Faraday rotation and Faraday conversion. The former operates alone at $\theta = 0$ and $\pi$, the latter operates alone at $\theta = \pi/2$, and both should be considered together for the intermediate angles. The transfer equations (Mueller calculus) for the Stokes parameters $I$, $Q$, $U$, and $V$ were devised to treat together the propagation effects, emission, and absorption (Azzam & Bashara 1987; Melrose & McPhedran 1991). Good approximations for emission and absorption have been known (Trubnikov 1958; Rybicki & Lightman 1979; Melrose & McPhedran 1991; Wolfe & Melia 2006). Now one can combine them with the proper approximations of the propagation effects given by

$$\frac{d}{ds} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\rho_U & \rho_U & 0 \\ 0 & \rho_U & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix},$$

and do the radiative transfer calculations. Here $e_+^2$ stands for the Hermitian part given by equations (28) and (29) with the real multipliers $f(X)$ and $g(X)$. One of the most interesting objects for such calculations is our GC Sgr A*.

The transfer equations were recently solved for a simple time-independent dynamical model of the GC accretion (Huang et al. 2008). The authors treat the ordinary and extraordinary modes as linearly polarized. They assume these eigenmodes constitute a basis, where either $U$ or $Q$ components of emissivity and propagation coefficients vanish. Actually, $U$ components vanish ($\rho_U = 0$) already in the basis $(e_1^1, e_2^1)$, since the projection of the magnetic field onto $(e_1^1, e_2^1)$ is parallel to $e_1^1$ (see Melrose & McPhedran 1991, p. 184). As I have shown in § 2.7, plasma modes are far from being linearly polarized at temperatures $T \lesssim 10$ estimated for the GC (Sharma et al. 2007a). Thus, the propagation coefficients should be taken from equations (25) and (26). The Faraday conversion coefficient $\rho_Q$ cannot be defined via emissivities and the Faraday rotation coefficient $\rho_V$ as in Huang et al. (2008). The Faraday rotation measure was calculated from a simulated accretion profile in Sharma et al. (2007b). However, the paper considered only the Faraday rotation and did not carry out the self-consistent treatment of propagation. It is impossible to disentangle the effects of Faraday rotation and Faraday conversion in a relativistic plasma.

3.2.2. Faraday Rotation

The crucial part of any radiative transfer is the proper transfer coefficients. It allows one to estimate the electron density near the accreting object (Quataert & Gruzinov 2000; Shcherbakov 2008). Several formulæ were suggested for the temperature dependence of the component $e_2^2$ responsible for Faraday rotation. These formulæ were yet given for the high-frequency approximation (see § 2.3). Let me compare them with the exact temperature dependence (eq. [20]) $J = K_0(T^{-1})/K_2(T^{-1})$ and its limits. The limits are $J \rightarrow 1$ as $T \rightarrow 0$ and $J \rightarrow \ln(T)/2T^2$ as $T \rightarrow +\infty$. The results of this comparison are shown in Figure 7.

Ballantyne et al. (2007)$^6$ divided the thermal distribution into ultrarelativistic and nonrelativistic parts as marked by the electron energy $\gamma_{\text{crit}} = 10$. They sum the contributions of both species with calculated densities. To make a plot, I take their effective temperature $\Theta$ of plasma above $\gamma_{\text{crit}}$ to be just temperature $\Theta = T$.

$^6$ The paper Ballantyne et al. (2007) has likely confused the three-dimensional projection of the four-dimensional response tensor $J = \alpha_{\mu}A_{\mu}$ (Melrose 1997c) with the three-dimensional response tensor $J = \alpha_{\mu}A$ that has the opposite sign.
1 THz is predicted by Huang et al. (2008). The phase of Faraday rotation is quite accurate.7

and not the average kinetic energy as Ballantyne et al. (2007) suggest. This brings Θ to lower values and decreases the rotation measure. Even with this decrease the rotation measure is severely overestimated at T ≈ 1. The convergence to the relativistic limit is not achieved even at T ≈ 30. Huang et al. (2008) found the simpler fitting formula that reproduces the limits. Their expression is quite accurate.7

3.2.3. Faraday Conversion

The increase in the circular polarization of Sgr A* at frequency 1 THz is predicted by Huang et al. (2008). The phase of Faraday conversion approaches unity and the destructive interference does not occur at this frequency. The result seems to be qualitatively correct regardless of the expression for the conversion measure, but the proper equations (25) and (26) should be used for quantitative predictions.

3.2.4. Jets

The better treatment of propagation effects may also play a role in observations of jets. As we saw in § 2.5, the propagation effects in thermal plasma cannot be described in the lowest orders in $\Omega_0/\omega_c$ if the temperature $T$ is sufficiently high. The power-law distribution of electrons can have a quite high effective temperature. Thus, the high-frequency limit (Sazonov 1969; Jones & Odell 1977; Melrose 1997b) may not approximate well the Hermitian part of the response tensor. Careful analysis of jet observations (Beckert & Falcke 2002; Wardle et al. 1998) may be needed. It should be based at least on the expressions for $\varepsilon^{\mu\nu}$ in a general low-density regime.

4. DISCUSSION AND CONCLUSION

This paper presents several new calculations and amends the previous calculations of propagation effects in a uniform magnetized plasma with thermal particle distribution equation (1). Equation (5) for the correct response tensor is given in a high-frequency approximation. The exact temperature dependence from equations (19) and (20) is found in first order in $\Omega_0/\omega$ in addition to the known highly relativistic and nonrelativistic results. The higher order terms may be important for relativistic plasmas in jets and hot accretion flows. The fitting equations (28) and (29) are found for the dielectric tensor components from equations (25) and (26) at relatively high temperatures.

The results of numerical computations are given only when the corresponding analytical formulae are found. One can always compute the needed coefficients numerically for every particular frequency $\omega$, plasma frequency $\omega_p$, cyclotron frequency $\Omega_0$, and distribution of electrons. However, the analytic formulae offer a simpler and faster way of dealing with the radiative transfer for a nonspecialist. The eigenmodes were not considered in much detail, since radiative transfer problems do not require a knowledge of eigenmodes. However, the knowledge of eigenmodes is needed to compute the self-consistent response tensor (see § 2.6).

The response tensor in the form of equation (5) can be expanded in $\Omega_0/\omega$ and $\omega_p/\omega$. This expansion is of mathematical interest and will be presented in a subsequent paper, as will the expressions for a power-law electron distribution. Propagation through nonmagnetized plasmas will also be considered separately.

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Fig. 7.—Temperature dependence of the Faraday rotation measure. [See the electronic edition of the Journal for a color version of this figure.]