Calculation of diffractive optical elements for the formation of thin light sheet

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Abstract. In the work, the calculation and study of diffractive optical elements (DOE) for the formation of a diffraction-free beam in the form of a thin light sheet, which can be used in planar microscopy, were performed. The calculation of phase DOEs is made on the basis of an iterative algorithm, taking into account the quantization of the phase function.

1. Introduction

The term "diffraction-free beams" was introduced to refer to laser beams propagating along the optical axis without changing the transverse distribution, i.e. without the influence of diffraction effects. The best known among the diffraction-free beams are the Bessel modes [1-3], which are solutions of the Helmholtz equation in cylindrical coordinates. In addition, Mathieu beams [4] for an elliptical coordinate system and parabolic beams [5] in a parabolic coordinate system, as well as various generalized beams [6-8] are known.

One of the ways to form diffraction-free beams is to use a narrow annular diaphragm superimposed on the lens [3, 9, 10]. However, this scheme is ineffective, because only a small part of the incident beam energy passes through the narrow annular gap. This problem can be solved with the use of an additional lens and axicon, which allow us to convert all the energy of the incident beam into a ring [11]. The formation of Bessel beams with the help of axicons [12] or diffractive optical elements (DOE) [13, 14] is energetically much more advantageous [15-17]. In papers [18, 19], a simple method for the energy-efficient formation of various diffraction-free laser beams using partial diaphragm of the annular spatial spectrum was considered.

Increased interest in the development and formation of new types of diffraction-free beams [20, 21] is associated with their enormous success in various applications [22-28], including optical manipulation, information coding, metrology, and microscopy. Diffraction beams play a special role in fluorescence microscopy based on light sheets (planar microscopy) [29-32, 33-35]. This type of microscopy provides extremely high image processing speed, good signal-to-noise ratio, low level of photo-bleaching, and good optical depth of penetration. This unique combination allows you to successfully apply this technology to the study of living microorganisms in real time. The extremely low toxicity of the system allows you to view and differentiate groups of cells without causing damage to the samples. Such a system is specifically designed for research in marine and cellular biology, as well as in plant physiology. Light sheet fluorescence microscopy (LSFM) uses a thin front of light to obtain an optical cut of a sample of transparent samples - cultures of cells, tissues and organisms containing fluorescent molecules (Figure 1) [36].
Figure 1. Schematic actions representation of Light sheet microscopy, where 1 – microorganisms, 2 – beam source (Δ - radiation thickness), 3 – collected emission.

To obtain a high-quality image, such characteristics of the “light sheet” as its length, uniform intensity, minimal thickness and absence of side lobes are important. Some requirements are mutually exclusive, and some are difficult to achieve. Therefore, in order to achieve some compromise, various types of diffraction-free beams and their superposition are considered [37-39].

In this work, for these purposes, when calculating phase (energy efficient) DOEs that form diffraction-free beams in the form of a thin light sheet, we use an iterative algorithm similar to that previously discussed [39]. At the same time, modifications of the algorithms are considered, which provide for the optimization of the properties of diffraction-free beams, as well as the consideration of the technological possibilities of manufacturing the DOE.

2. Theoretical foundations

The task is to form an extended light sheet with uniform intensity, which will be suitable for radiography of larger living organisms, i.e. the problem is reduced to the calculation of a thin light sheet with diffraction-free properties in paraxial domain.

For this, the iterative algorithm [39] considered earlier was modified as follows: at the first iteration, the wave front phase was randomly set in the range from 0 to 2\(\pi\), then the wave front was cut off by a ring with a ring thickness \(R = R_2 - R_1\) (\(R_2\) - outer radius of the ring, \(R_1\) - inner ring radius) and propagated through the lens using the Fourier transform, further in the focal plane, the distribution was truncated by a narrow slit of size \(\Delta\); on subsequent iterations, the algorithm was iteratively reproduced, with the exception of the first item — the phase was not set randomly, but was set as the distribution at the end of the previous iteration. Thus, a diffraction relief of a DOE was obtained for the formation of a “thin light sheet”. Below is a picture showing the operation of the algorithm (Figure 2).

Figure 2. The n and n+1 iteration of the thin light sheet formation algorithm.
It was proposed to consider several modifications of the algorithm in which the mask was superimposed on the intensity distribution at each iteration, like a ring with a variable split thickness $\Delta$. The criterion for the quality of a diffraction-free beam is propagation invariance in free space. To analyze the generated image, compare the cross sections of the intensity distribution at some distances.

3. Numerical simulation

| Table 1. Algorithm for the synthesis of the proposed DOE. |
|---------------------------------------------------------|
| **First iteration**                                       | **Subsequent $N-1$ iterations** |
| $\varphi_1(x) = \text{rand}(0, 2\pi)$                  | $\varphi_n(x) = F^{-1}\{\Psi_{n-1}(u)\}$ |
| $\psi_1(x) = \begin{cases} \exp[\varphi_1(x)], R_1 \leq r \leq R_2 \\ 0, \quad \text{else} \end{cases}$ | $\varphi_n(x) = \begin{cases} \varphi_n(x), R_1 \leq r \leq R_2 \\ 0, \quad \text{else} \end{cases}$ |
| $\Psi_1(u) = F\{\psi_1(x)\}$                           | $\psi_n(x) = \varphi_n(x)$ |
| $\hat{\Psi}_1(u) = \Psi_1(u), |u| < \Delta/2$       | $\Psi_n(u) = F\{\psi_n(x)\}$ |
|                                                         | $\varphi_n(u) = \begin{cases} \varphi_n(u), |u| < \Delta/2 \\ 0, \quad \text{else} \end{cases}$ |
|                                                         | endfor. |

Table 2 shows the amplitude and phase distributions for each step from Table 1: $n = 1, 4,$ and 16 iterations. In a number of experiments, it was revealed that 4 iterations are sufficient for synthesis of amplitude-phase DOE.

Given the complexity of manufacturing the amplitude phase DOE, it is necessary to calculate the phase element. To do this, at the end of each iteration, we divide the resulting complex array into nonzero amplitude values. As a result, a diffraction pattern was obtained for the phase physical realizable diffractive element.

| Table 2. Illustration of algorithm steps for DOE synthesis (1.4 and 16 iterations). |
|-----------------------------------|
| $n=1$                            | $n=4$ | $n=16$ |
| $\varphi_n(x)$                   | $\psi_n(x)$ | $\Psi_n(u)$ |
| Random phase ($n=1$)             | Wavefront | Fourier image |
| Fourier preimage ($n=1$)         | $\Psi_n(u)$ | $\hat{\Psi}_n(u)$ |
| $\varphi_{n+1}(x)$              | Fourier image (truncated) | $\varphi_n(x)$ |
| Fourier preimage                 | $\psi_n(x)$ | $\Psi_n(u)$ |
Table 3 presents the images of the light sheet. In a number of experiments, it was revealed that the algorithm has a stagnant character, i.e. with an increase in the number of iterations (n = 64), the structure and the diffractionless properties of the beam do not improve. In addition, it is shown that the variation of the parameter $\Delta$ leads to the creation of a thinner or thicker light sheet.

Table 3. Fourier image of a “thin light sheet” with the parameter $\Delta$ equal to 20, 10 and 5 mm.

|       | n=16 | n=64 |
|-------|------|------|
| $\Delta$=20 | ![Image](#) | ![Image](#) |
| $\Delta$=10 | ![Image](#) | ![Image](#) |
| $\Delta$=5 | ![Image](#) | ![Image](#) |

From table 3 it can be seen that the distribution obtained has a non-uniform structure. Such a structure may make it difficult to study objects with small elements, but significantly improve the analysis of larger microorganisms.

Wavefront propagation through a lens can be modeled using an optical Fourier transform (F), which looks like this in a 2D representation:

$$F(u, v) = \frac{1}{\lambda f} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \exp \left[ -i \frac{2\pi}{\lambda f} (ux + vy) \right] \, dx \, dy,$$

where $f(x, y)$ - Fourier preimage, $\lambda$ - wavelength; $f$ - lens focal length; $F(u, v)$ - Fourier image.
Figure 3. Amplitude (a) and phase (b) of the phase DOE to form a “thin light sheet”.

It is known [8] that, for simplicity and accuracy in the manufacture of phase DOEs, it is possible to carry out phase quantization. In the framework of this work, quantization was carried out for 2 and 3 values, uniformly distributed over the interval from 0 to 2π.

Figure 4. The phase of the quantized phase DOE for the formation of a "thin light sheet ": (a) - 0 and π; (b) - 0, π/2, π, 3π/2.

To visualize the diffraction-free properties in the far-field zone, we will conduct a Fresnel test for the calculated “thin light sheet”, i.e. let us pass the plane wave front through the calculated DOE and fix the Fresnel image in planes at some distances. The Fresnel transform is expressed by formula:

$$U(u, v, z) = -\frac{ik}{2\pi z} \exp\left[\frac{ik}{2z}(u^2 + v^2)\right] \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \exp\left[\frac{ik}{2z}((x-u)^2 + (y-v)^2)\right] dxdy,$$

provided that $\sqrt{(x-u)^2 + (y-v)^2} \ll z$, where $k = \frac{2\pi}{\lambda}$, $\lambda$ – wavelength, $z$ – distance over which the wavefront is propagated.

In the framework of this work, the Fresnel transform is implemented through the Fourier transform (F) as follows:

$$U(u, v, z) = \exp\left[\frac{ik}{2z}(u^2 + v^2)\right] \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ f(x, y) \exp\left[\frac{ik}{2z}(x^2 + y^2)\right] \right\} \exp[-2\pi i(xu + yv)] dxdy =$$

$$= \exp\left[\frac{ik}{2z}(u^2 + v^2)\right] F\left\{ f(x, y) \exp\left[\frac{ik}{2z}(x^2 + y^2)\right] \right\}.$$

In a number of experiments, it was found that a change in the parameter $\Delta$ leads to a change in the thickness of the light sheet. Thus, the presented algorithm for the synthesis of DOE allows you to create such a phase element with which you can reproduce a beam of any physically possible thickness. And there is a clear relationship between the parameter $\Delta$ and the energy focusing - the smaller the parameter $\Delta$, the more energy is focused on the working area of the light sheet.
Figure 5. Fresnel image of a “thin light sheet” at distances of 30, 60, 120 mm - (a) and (d), (b) and (e), (c) and (f) respectively in increasing x1 and x4.

Figure 6. Fresnel image of a “thin light sheet” with the parameter Δ equal to 1 mm(a) and 2 mm(b).

In addition, as part of this work, a test was carried out with a lens for the calculated “thin light sheet”. The Fresnel image was simulated near the focal plane. It turned out that the proposed diffraction-free beam before the passage of the focal plane of the lens and after it behaves the same, i.e. retains its structure. Figure 7 presents the results of numerical simulation of the propagation of the light sheet before and after the focal plane of the lens.

Propagation predetermined field through spherical lens can be described by the following transformation

\[ U(u, v, z) = \frac{1}{2\pi z} \exp[ikz] \exp\left[\left(\frac{ik}{2z}\right)(u^2 + v^2)\right] \times \]

\[ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \exp\left[ik\left(x\left(\frac{1}{z} - \frac{1}{f}\right) + y\left(\frac{1}{z} - \frac{1}{f}\right)\right)\right] \exp\left[-\frac{ik}{z}(xu + yv)\right] dx dy \]  

(4)

4. Conclusion
An algorithm for calculating diffraction optical elements was shown in the paper, and the resulting distribution can be used as a “light sheet”. An analysis was conducted of the possibility of forming quantized phase DOEs. In addition, it was found that the size of the ring radius does not significantly affect the diffractionless properties of the beam, but at the same time affects the energy focusing in the working region of the light sheet. As a result, a diffraction relief was calculated for the phase physical realizable diffractive element, and the diffraction-free properties of the calculated “thin light sheet” in the far zone were shown.
In addition, as part of this work, a test with a spherical lens was performed. It was found that the distribution retains its structure and shape before and after the focal plane. A comparison was also made with a diffraction-free beam obtained by propagating a Gaussian beam through a cylindrical lens. It turned out that the change in beam span in the focal plane and after it is much larger than that of the proposed thin sheet.

Thus, a quantized diffraction relief of an optical element was calculated for the energy-efficient formation of a diffraction-free beam, which can be used in various applications, including optical manipulation, information coding, metrology and microscopy.

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