Chaos synchronization in SMIB power system and its application to secure communication

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Abstract. This paper proposes a secure communication scheme for the master-slave framework using the concept of equivalent control for a class of chaotic SMIB power systems. For a given chaotic master system, a sliding mode controller introduced to the slave system can be constructed to synchronize the master system. Also the hidden message can be recovered directly by the concept of equivalent control. Theoretical analysis and numerical simulation verify the effectiveness of the proposed scheme.

1. Introduction

Chaos is an interesting phenomenon of nonlinear systems. A deterministic chaotic system has some remarkable characteristics, such as system evolution sensitive to the change in the initial condition, broad spectrum of Fourier transform, and fractal properties of the motion in phase space [1].

Since the pioneering work of Pecora and Carroll [2], synchronization of chaos has aroused much interest. Many synchronization schemes have been proposed and pursued [3 – 9]. In particular, the application of chaotic synchronization to secure communication has become an area of active research. There, an information signal is transmitted using a chaotic signal as hidden, and the synchronization is necessary to recover the information at the slave system. Inspired by previous works [10-15], this paper also considers the synchronization and secure communication problem in the master-slave framework. For a given chaotic master system, a sliding mode controller can be designed for the slave system. With the help of the sliding mode control, the slave system can globally synchronize the master system, and the hidden message can be approximately recovered. Lyapunov stability ensures the global stability of synchronization, which further results in the recovery of message by the equivalent control. Numerical simulation of SMIB power system verifies the effectiveness of this scheme.

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2. Problem statement

Consider the classical SMIB power system [16], called swing equation

\[ M \frac{d^2 \theta}{dt^2} + D \frac{d\theta}{dt} + P_{\text{max}} \sin \theta = P_m \]  

which can be written as a system of first order equations

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -cx_2 - \beta \sin x_1 + f \sin \omega t
\end{align*}
\]

(2)

where

\[ x_1 = \theta, \quad x_2 = \frac{d\theta}{dt}, \quad c = \frac{D}{M}, \quad \beta = \frac{P_{\text{max}}}{M}, \quad f = \frac{A}{M} \]

(3)

Here \( M \) is the moment of inertia, \( D \) is the damping constant, \( P_m \) is the power of the machine, \( P_{\text{max}} \) is the maximum power of generator, and \( P_m = A\sin \omega t \).

In this paper, our problem undertaken here is to synchronize the master-slave SMIB power systems via sliding mode control. And then, propose a new secure communication scheme using the concept of equivalent control. For the SMIB power system (1), the master and slave systems are defined below, respectively.

Master system:

\[ \begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -cx_2 - \beta \sin x_1 + f \sin \omega t + m(t)
\end{align*} \]

(4)

Slave system:

\[ \begin{align*}
\dot{y}_1 &= y_2 \\
\dot{y}_2 &= -cy_2 - \beta \sin y_1 + f \sin \omega t + u(t)
\end{align*} \]

(5)

where \( u(t) \in \mathbb{R} \) is the sliding mode controller such that the two SMIB power systems can be synchronized. \( m(t) \in \mathbb{R} \) is the message signal. In order to recover message \( m(t) \), we must make the following assumption:

**Assumption A1.** The message \( m(t) \) is bounded by a known positive constant \( \delta \), namely \( |m| \leq \delta \).

Next, we define the error signal as

\[ \begin{align*}
e_1 &= y_1 - x_1 \\
e_2 &= y_2 - x_2
\end{align*} \]

(6)

From Eq.(6), we have the following error dynamics:

\[ \begin{align*}
\dot{e}_1 &= e_2 \\
\dot{e}_2 &= -ce_2 - 2\beta(c y_1 + x_1) \sin \frac{e_1}{2} - m(t) + u(t)
\end{align*} \]

(7)

In sequence, for achieving the synchronization of master-slave SMIB power systems in the sense that \( \|e(t)\| \rightarrow 0 \) as \( t \rightarrow \infty \), where \( e(t) = [e_1, e_2] \), there exist two major phases. First, selecting an appropriate switching surface such that the sliding motion on the sliding mode is stable and ensures \( \lim_{t \to \infty} \|e(t)\| = 0 \); Second, establishing an control law which guarantees the existence of the sliding mode \( S(t) = 0 \).
3. Switching surface and controller design

In order to ensure the asymptotical stability of the sliding mode, a novel PI switching surface $S(t)$ is selected as follows:

$$S(t) = e_2 - \int_0^t (k_1 e_1(\tau) + k_2 e_2(\tau)) d\tau$$

(8)

where $S(t) \in R$ and $k_1$ and $k_2$ are design parameters which can be easily determined later. As well known, when the system operates in the sliding mode, it satisfies the following equations

$$S(t) = \dot{S}(t) = 0$$

(9)

From (9), one has

$$\dot{S}(t) = \dot{e}_2 - (k_1 e_1 + k_2 e_2) = 0 \quad \Rightarrow \dot{e}_2 = k_1 e_1 + k_2 e_2$$

(10)

Therefore, the following sliding mode dynamics can be obtained as

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ k_1 & k_2 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = Ae$$

(11)

Obviously, when all eigenvalues of matrix $A$ are with negative real parts, the stability of (11) is surely guaranteed, that is $\lim_{t \to \infty} \|e(t)\| = 0$ in the sliding mode. Thus the values of $k_1, k_2$ can be easily decided according to above rule.

![Fig. 1. The attractor of classical SMIB power system.](image)

Now a proper sliding surface has been decided. It followed by designing a sliding mode control scheme to drive the error system trajectories onto the switching surface $S(t) = 0$. To ensure the occurrence of the sliding mode, a sliding mode control scheme is proposed as

$$u(t) = -\gamma \eta(t) \text{sign}(S)$$

(12)

where

$$\gamma > 1, \quad \eta(t) = -c e_2 - 2\beta (\cos \frac{\gamma t}{2} + \sin \frac{\gamma t}{2}) - (k_1 e_1 + k_2 e_2) + \delta$$

(13)

**Theorem 1.** The reaching condition of expression (14) of the sliding mode is satisfied, if the control $u(t)$ is given by (12).
To prove this theorem, we need the following lemma.

**Lemma 1.** The motion of the sliding mode is asymptotically stable, if the following reaching condition is held

\[ S(t) \dot{S}(t) < 0 \quad (14) \]

**Proof.** Let \( V(t) = 0.5S^2(t) \) be the Lyapunov function. Differentiating \( V(t) \) with respect to time yields

\[ \dot{V}(t) = S(t) \dot{S}(t) \quad (15) \]

Therefore, according to the Lyapunov stability theorem, we known that if \( S(t) \dot{S}(t) < 0 \), then equilibrium at the origin is asymptotically stable; i.e. the vector \( S(t) \) will decay to zero.

**Proof of Theorem 1:** Substituting (8) and (12) into the term of \( S(t) \dot{S}(t) \), it yields

\[
\begin{align*}
S(t) \dot{S}(t) &= S[\dot{e}_2 - (k_1 e_1 + k_2 e_2)] \\
&= S[-ce_2 - 2 \beta \left( \cos \frac{y_1 + x_1}{2} \sin \frac{e_1}{2} \right) - m + u(t) - (k_1 e_1 + k_2 e_2)] \\
&= S[-ce_2 - 2 \beta \left( \cos \frac{y_1 + x_1}{2} \sin \frac{e_1}{2} \right) - m - (k_1 e_1 + k_2 e_2) - \gamma \eta(t) \text{sign}(S)] \\
&\leq |S| \eta(t) - \gamma \eta(t)|S| \\
&\leq (1 - \gamma) \eta(t)|S|
\end{align*}
\]

![Fig. 2. The time responses of \( x_1 \) & \( y_1 \).](image)

Since \( \gamma > 1 \) has been selected in (13), one can conclude that reaching condition \( (S(t) \dot{S}(t) < 0) \) is always satisfied. Thus the proof is achieved completely.

Summing up the above proof, an ideal sliding motion takes place on \( S \) in a finite time. The switch surface \( S \) can be attained in a finite time. We conclude that \( e_i = e_z = 0 \) and \( \dot{e}_1 = \dot{e}_2 = 0 \). It follows from (7) that

\[ 0 = 0 - 0 - m(t) + u_{eq}(t) \quad (17) \]

where \( u_{eq} \) stands for the equivalent control action during the sliding motion. Because of the exponential stability of error dynamics (7), we conclude that
\[
\lim_{t \to -\infty} (m - u_{eq}) = 0
\]  
(18)

which means that the message \( m(t) \) can be approximated by the equivalent control \( u_{eq} \). This equivalent control signal represents the average behavior of the discontinuous component \( u(t) \) and the effort necessary to maintain the motion on the sliding surface. As mentioned in [14], it can be approximated by the following continuous function:

\[
u_{eq}(t) = -\gamma \eta(t) \frac{S}{[S] + \sigma} \]

(19)

where \( \sigma \) is a sufficiently small positive constant. For sufficiently small choice of \( \sigma \), continuous action (19) can approach discontinuous action (12) very well. Therefore, the message \( m(t) \) can be recovered by the approximate equivalent control (19).

Fig. 3. The time responses of \( x_2 \) & \( y_2 \).

Fig. 4. Synchronization errors between the master-slave systems.

4. Numerical example

In this section, to verify and demonstrate the effectiveness of the proposed method, we discuss the simulation results for master-slave SMIB power systems. In simulation experiments, values of swing equation are chosen as follows: \( c = 1, \ \beta = 3, \ f = 5, \ \omega = 1 \). The hidden message \( m(t) = \sin(3t) \). Based on (11), \( k_1 = -2 \) and \( k_2 = -3 \) can be obtained. We choose \( \gamma = 2 > 1, \ \delta = 1.5 \geq |m(t)|, \ \sigma = 0.001 \) for sliding
mode controller. The initial states of the master system (4) are \( x_1(0) = 1, \ x_2(0) = -0.5 \) and initial states of the slave system (5) are \( y_1(0) = 0.5, \ y_2(0) = 0.2 \). The simulation results are shown in Fig.1-Fig.5. The chaotic attractor of classical SMIB power system is shown in Fig.1. The time responses of \( x_1 \) & \( y_1 \) and \( x_2 \) & \( y_2 \) are shown in Fig.2 and Fig.3, respectively. Fig.4 shows the error state between master and slave systems. Finally, the most important result in this paper is shown in Fig.5. Fig.5 shows the hidden and recovered messages. From the simulation results, it reveals that the hidden message can be recovered approximately.

\[ \text{Fig. 5. The hidden and recovered messages} \]

5. Conclusions
In this paper, we investigate the synchronization problem of SMIB power systems and propose a new secure communication scheme using the concept of equivalent control. A numerical simulation is provided to show the effectiveness of our method.

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