Reduction of Nonlinear Mode Coupling in Mode Division Multiplexing Systems Based on Wavelength Interleaving Method

E K Hamed, J K Hmood* and M A Munshid

Laser and Optoelectronics Engineering Department, University of Technology, 10066, Baghdad, Iraq
140011@uotechnology.edu.iq, 140046@uotechnology.edu.iq

Abstract: Mode division multiplexing (MDM) based few-mode fiber (FMF) transmission scheme is considered as one of the promising choices when one trying to achieve high bit rates in cascade to reduced cost per bit and upgrading the fiber capacity. Nonlinear mode coupling over FMFs can be addressed as the main degradation issue that can limit the efforts to reach their full capacity. In this work, we propose a new method to reduce nonlinear modal coupling by suppressing phase matching between different fiber modes using wavelength interleaving (WI) technique. This technique relays on interleave the optical carrier wavelength in a specific mode from those in adjacent ones. The five modes MDM system is analytically modeled and numerically demonstrated. Each spatial mode is modulated by a 4-ary quadrature amplitude modulation (4QAM) format with a symbol rate of 20 Gsymbol/s. The results reveal that the nonlinear phase (NLP) noise is reduced and the signal to noise ratio (SNR) is increased when applying the WI method. Additionally, it turned out that analytical and numerical results are in good agreement with each other.

1. Introduction

MDM based FMF transmission scheme has considered as one of an innovative technology that increases the bit rates in cascade with reduced cost per bit and upgrading the fiber capacity [1-3]. The idea behind MDM is to use an optical fiber that permits the propagation of several spatial modes through a single core [4-6]. All spatial modes in one fiber are orthogonal with respect to each other [7]. Theoretically, capacity per fiber can be multiplied by the number of spatial modes [8]. In fact, several detrimental effects are produced with MDM systems. For long haul transmission, intermodal nonlinear coupling over FMFs can be considered as one of the main degradation issues that can minimize transmission distance and limit the efforts in order to reach their full capacity [1, 9, 10]. Nonlinear coupling among fiber modes is produced due Kerr effects inside optical fiber that induce NLP noise [11, 12]. The NLP noise interacts with spontaneous noise inside optical amplifier, causing a random noise that degrade the received signals [13, 14]. These impairments are strong between adjacent spatial channels where effective index difference, $\Delta n_{\text{eff}}$ is small, and becomes much weaker for larger $\Delta n_{\text{eff}}$ [15-17]. In the previous literatures, several approaches are used to counteract or suppress the effect of nonlinear effects in MDM communication systems.
Phase-conjugated twin wave technique [18], optical shaping the envelopes of different fiber modes that carry mQAM signals to lower fiber nonlinearities by reducing the effective intensity and interference time between modes [9], or by designing fibers with specific features to reduce nonlinear effects inside optical fiber [16, 17, 19] are some of them. In this work, nonlinear modal coupling is reduced by suppressing phase matching between fiber modes by using WI technique. This technique relays on interleave the optical carrier wavelength in a specific mode from those in adjacent ones.

2. Analytical representation of nonlinear phase distortions:
Once optical signals propagate inside FMF in multi-span MDM system, SPM and XPM noise are created. The phase noise due to SPM and XPM at end of \( M \) spans can be expressed as [1, 20]:

\[
\theta_{\text{SPM}} (ML) = \gamma_{pppp} L_{pppp} \sum_{\mu=1}^{M} \left| E_{\mu} \right|^2 + \sum_{\mu}^{M} n_{pk}(t),
\]

\[
\theta_{\text{XPM}} (ML) = 2 \sum_{l=1}^{M} \sum_{n=1}^{M} \gamma_{lunn} L_{lunn} \left| E_{l} \right|^2 + \sum_{\mu}^{M} n_{lk}(t)
\]

(1)

where \( \theta_{\text{SPM}} \) and \( \theta_{\text{XPM}} \) are the NLP that added due to \( p \)-th mode propagation over FMF by SPM and XPM phenomena, respectively, \( E_{\mu} \) is the slowly varying optical signal, \( N \) is number of modes, \( M \) is number of spans, \( \gamma_{lunn} \) is nonlinear coefficients, and \( n_{pk}(t) \) denotes the amplifier noise at \( p \)-th mode and \( k \)-th span. \( L_{lunn} \) signifies effective length of fiber which can be expressed as [12]:

\[
L_{lunn}(z) = \frac{1-\exp(-\alpha + j\Delta\beta_{lunn} z)}{\alpha + j\Delta\beta_{lunn}}
\]

(2)

where \( \Delta\beta_{lunn} \) is the phase mismatch parameter and \( \alpha \) fiber attenuation coefficient. The propagated fiber modes have dissimilar group velocities and different propagation constants, which means that phase matching can be broken through a suitable choice of the wavelengths for different excited modes [21]. The phase mismatch parameter is given by [22, 23]:

\[
\Delta\beta_{lunn} = -\beta_i(\omega_l) + \beta_m(\omega_m) + \beta_n(\omega_n) - \beta_p(\omega_p)
\]

(3)

where \( \beta_{i}(\omega_{i}) \) is the modal propagation constant for \( i \)-th spatial mode at frequency \( \omega_{i} = c/\lambda_{i} \). Here \( c \) and \( \lambda_{i} \) are the speed of light and wavelength of \( i \)-th mode, respectively. \( \beta_{i}(\omega_{i}) \) is calculated by expanding the propagation constant of each mode in a Taylor series around a reference frequency \( \omega_{b} \) [24-26]:

\[
\beta_{i}(\omega_{i}) = \beta_{i}(c) + \beta_{i,1}(\omega_{i} - \omega_{b}) + \frac{\beta_{i,2}}{2}(\omega_{i} - \omega_{b})^{2} + \frac{\beta_{i,3}}{6}(\omega_{i} - \omega_{b})^{3} + \ldots
\]

(4)

Here, \( \beta_{i,0}, \beta_{i,1}, \beta_{i,2} \) and \( \beta_{i,3} \) represent the propagation constant in vacuum, the inverse of the group velocity, chromatic dispersion and third-order dispersion, respectively. Let us suppose that we have four signals \( l, m, n, p \) with an equal wavelength \( \lambda_{i} \), \( \beta_{0} \) is cancelled out, ignoring \( \beta_{3} \), and higher order terms for simplicity then:

\[
\Delta\beta_{lunn} = -\beta_{i}(\Omega_{i}) + \beta_{m}(\Omega_{m}) + \beta_{n}(\Omega_{n}) - \beta_{p}(\Omega_{p}) - \frac{\beta_{i,2}}{2}(\Omega_{i})^{2}
\]

\[
+ \frac{\beta_{i,3}}{2}(\Omega_{i})^{3} + \frac{\beta_{i,2}}{2}(\Omega_{m})^{2} - \frac{\beta_{i,2}}{2}(\Omega_{n})^{2} - \frac{\beta_{i,2}}{2}(\Omega_{p})^{2}
\]

(5)

where \( \Omega_{i} = \omega_{i} - \omega_{b} \) is the frequency detuning from a reference.
In our proposed technique, the modes are launched at different wavelengths with constant spacing of $\Delta \lambda$. The phase matching between the excited modes can be expressed as follows:

\[
\Delta \beta_{\text{err}} = -\beta_{\text{in}} (\Omega_i) + \beta_{\text{in}} (\Omega_i + \Delta \omega) + \beta_{\text{in}} (\Omega_i + 2\Delta \omega) - \beta_{\text{in}} (\Omega_i + 3\Delta \omega) - \left(\frac{\beta_{\text{in}}}{2}\right) (\Omega_i)^2
\]

\[
+ \left(\frac{\beta_{\text{in}}}{2}\right) (\Omega_i + \Delta \omega)^2 + \left(\frac{\beta_{\text{in}}}{2}\right) (\Omega_i + 2\Delta \omega)^2 - \left(\frac{\beta_{\text{in}}}{2}\right) (\Omega_i + 3\Delta \omega)^2
\]

where $\Delta \omega$ is the frequency spacing between successive modes. In contrast to Eq. (5), Eq. (6) shows that the phases mismatch among the propagated modes increases due to adding a wavelength separation between transmitted modes. On the other hand, the inline amplifiers that added to mitigate the attenuation add amplified spontaneous emission (ASE) noise, which at some time, initiates the amplifiers saturation. As a result, signal power reduces while noise power keeps on growing; resulting in SNR degradation [27]. Therefore, ASE induced a random fluctuations with variances of [9]:

\[
\sigma^2_{p,\text{SPM}}(ML) = \frac{M (M + 1)}{2} \gamma_{pppp}^2 L_{pppp}^2 (P) \left| A_p \right|^2 \sigma_p^2
\]

\[
\sigma^2_{p,\text{XPM}}(ML) = 2M (M + 1) \gamma_{pppp}^2 L_{pppp}^2 (P) \sum_{l=1}^{N} \sum_{j=1}^{N} \left| A_l \right|^2 \sigma_l^2
\]

where $P$ is the optical power, and $A_p$ is the complex amplitude corresponding to the $p^{th}$ mQAM symbol. Additional nonlinear effect is called four-wave mixing (FWM), which is produced due to interaction four modes inside the FMF. The interacted modes have different group velocity and dispersions that effect of the amount of phase-matching. FWM can cause instability during long transmission distances when phase matching occurs between optical channels [28] with variance given by [1]:

\[
\sigma^2_{p,\text{FWM}}(ML,t) = \frac{2M \sigma_p^2}{P} \sum_{n=1}^{N} \sum_{p=1}^{N} \sum_{m=1}^{N} \gamma_{pppp}^2 L_{pppp}^2 (\mu L)
\]

\[
\times \left( \left| A_p \right|^2 \left| A_p \right|^2 \sigma_p^2 + \left| A_p \right|^2 \left| A_p \right|^2 \sigma_m^2 + \left| A_p \right|^2 \left| A_p \right|^2 \sigma_n^2 \right)
\]

3. System Model

In this section, we design and simulate an MDM-FMF system of five spatial linearly polarized (LP) modes (LP01, LP11a, LP11b, LP21a, and LP21b). Figure 1, illustrates setup of MDM based FMF. It comprises an MDM transmitter, an optical fiber link, and an MDM receiver. At the transmitter, five laser diodes (LD) for each spatial mode are used. The mQAM modulator uses complex modulator to create 4QAM signals, which individually modulates each laser beam. The input data is mapped as in-phase (I) and quadrature-phase (Q) components that drive the complex modulator. Finally, 4QAM signals have been combined and conveyed through multi-span optical link. Each optical link contains FMF, dispersion compensating fiber (DCF), and a few-mode optical amplifier (OA). The FMF is modeled with an attenuation coefficient of 0.2 dB/km, and fiber length of 50 km. The OA with noise figure of 6 dB is located at the end of each span for totally compensating the accumulated losses (assumed the same for each spatial mode). The dispersion is fully compensated by adopting few-mode DCFs with 5 km length. At the end of the optical link, the signals are separated by few-mod demultiplexer. Then, the signals are severally filtered by band pass filter and recovered by coherent 4QAM receiver. Table 1 displays the main parameters of our designed FMF.
Figure 1. Setup of MDM based FMF transmission system.

Table 1. SI-FMF parameters at wavelength of 1550 nm.

| Mode    | Effective index $n_{\text{eff}}$ | Delay (ns) | $A_{\text{eff}}$ (μm) | Dispersion ps/(km-nm) |
|---------|----------------------------------|------------|------------------------|-----------------------|
| LP01    | 1.451292                         | 4.9077     | 220.9915               | 23.4                  |
| LP11a   | 1.450034                         | 4.9101     | 316.807                | 24.3                  |
| LP11b   | 1.450034                         | 4.9101     | 316.807                | 24.3                  |
| LP21a   | 1.448447                         | 4.9123     | 359.5131               | 23                    |
| LP21b   | 1.448447                         | 4.9123     | 359.5131               | 23                    |

4. Results and discussion
The impact of WI method on the reduction the nonlinear coupling between modes in long haul 4QAM MDM system is investigated in this section. In order to examine the proposed technique, five modes LP01, LP11a, LP11b, LP21a, and LP21b are simultaneously excited. Each spatial mode carries a 4QAM signal at symbol rate of 20 Gsymbol/s over specific optical carrier. Table 2 lists the wavelength of optical carriers used for each propagated modes. The NPL noise is predicted regarding to the analytical model in Section 2 by Matlab. Additionally, the performance of WI-4QAM and 4QAM MDM systems is numerically examined. All signals are transmitted over a SI-FMF described in Table 1. The validity of the analytical model is assessed through comparison with the numerical results.
Table 2. Carrier wavelengths for each mode.

| modes   | λ(nm) |
|---------|-------|
| LP01    | 1556  |
| LP11a   | 1555.2|
| LP11b   | 1554.4|
| LP21a   | 1553.6|
| LP21b   | 1552.8|

Figure 2 shows the NLP variance versus mode power for three propagating modes: LP01, LP11a, and LP21a over FMF link with length of 500 km. In general, the effect of amplifier noise is dominant at low mode powers because the optical signal to noise ratio is low. In contrast with 4QAM transmission, NLP noise reduction is clearly observed specifically at high optical power. This due to the proposed technique is more efficient to reduce the noise originated from Kerr effects such as SPM, XPM and FWM. Also, the optimum powers of both schemes are the same. For WI-4QAM scheme, minimum NLP noise variances are $1.3 \times 10^{-3}$, $1.4 \times 10^{-3}$ and $1.2 \times 10^{-3}$ rad$^2$ for LP01, LP11a and LP21a modes, respectively. This represents that NLP noise is reduced by about 50%, 51.7% and 52% for LP01, LP11 and LP21 modes, respectively as compared with 4QAM signals. It can be concluded that the proposed method is able to reduce phase noise compared with that of 4QAM scheme. This is because that using different wavelengths for carrier signals can reduce phase matching between propagated signals.

Figure 3 illustrates the effect of WI method in reducing EVM. The results are obtained at transmission distance of 500 km (10 spans). In general, the proposed method can lower the EVM for all modes by noteworthy percent. In contrast with 4QAM MDM scheme, the minimum EVM is reduced from 11.32% to 5.99% in LP01 channel where as it reduced from 14.15% to 7.92% for LP11a channel and from 15.00% to 8.50% for LP21a channel when WI-4QAM MDM scheme is adopted. In addition, a good enhancement in SNR can be noticed as shown in Figure 4. The SNRs of all modes are enhanced by about 5.5 dB in comparing with the conventional MDM system without WI scheme. This phenomenon is resulted due to considerable reduction in nonlinear interaction when the modes data are carried by optical carriers with deferent wavelengths. The phase mismatch is elevated because dependency of refractive index on the wavelength, causing more difference in the dispersion and speed of the modes. From Figures 3 and 4, a good agreement between analytical and numerical results can be obtained.

In Figure 5, the impact of WI scheme on the achievable transmission reach is explored by comparing the obtained results with results in Ref [9]. SER versus transmission distance has been obtained at optimum powers for all modes. As expected, the proposed scheme increase the transmission reach at longer distance than conventional 4QAM-MDM. For LP01 channel, the transmitted signal reaches 1500 km at SER of $1 \times 10^{-5}$ whereas 4QAM is ranged to nearly 1150 km at the same SER as depicted in Figure 5(a). A greater enhancement in the transmission reach can be observed, Figure 5 (b), for LP11 mode. At SER of $1 \times 10^{-5}$, the distance is extended from 400 km to 1400 km. With transmitting the modes with different wavelengths, the phase matching can be broken, specifically; in the degenerated modes, resulting in lower coupling between them and hence the transmission distance is enlarged. The same observation can be seen in Figure 5 (c) for LP21a where the transmission distance is enlarged from 1700 km to 2400 km.
Figure 2. Total NLP versus mode power for 5-modes propagation: (a) LP01, (b) LP11a, and (c) LP21a.

Figure 3. EVM versus mode power for 5-modes propagation: (a) LP01, (b) LP11a, and (c) LP21a.
Figure 4. SNR versus mode power for 5-modes propagation: (a) LP01, (b) LP11a, and (c) LP21a.

Figure 5. SER versus transmission distance for 5-modes propagation: (a) LP01, (b) LP11a, and (c) LP21a.
5. Conclusions
In this study, an efficient way to reduce NLP noise produced due to modal coupling in MDM-FMF has been presented and examined. In this technique, the wavelengths of optical carriers that convey modes have been set at different wavelengths in order to minimize phase matching between modes and increase effective refractive index. Five modes MDM system has been analytically modeled and numerically demonstrated with and without WI method. Each spatial mode has been modulated by 4QAM format at symbol rate of 20 Gsymbol/s. The results have showed a nearly 50% reductions in NLPN and EVM for all channels and an increase in SNR by approximately 5.5 dB. An enhancement in the transmission reach can be also noted for all channels when applying the WI method. Finally, a good agreement between analytical and numerical results can be observed.

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