Calculation of the Deflection of Light Ray near the Sun with Quantum-corrected Newton’s Gravitation Law

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Abstract

The deflection of light ray passing near the Sun is calculated with quantum-corrected Newton’s gravitation law. The satisfactory result suggests that there may exist other theoretical possibilities besides the theory of relativity.

The deflection of light ray near the Sun was a successful prediction of Einstein’s theory of relativity, to which classical mechanics is futile. That was one of the best experimental supports to Einstein’s theory. But as many people know, the successful synthesis of Einstein’s theory with quantum theory has not emerged. In the attempt to construct the theory of quantum gravity, some scientists even thought that one of the two theories was temporary[1]. Though research on such a theory of quantum gravity has made no convincing breakthrough, some features of it can be seen[2]. One of the crucial points is the quantization of spacetime.

To get a theory which is compatible with quantum mechanics and at the same time preserves the successful conclusions of the theory of relativity as much as possible, I proposed a theoretical framework in recent years[3-5]. It has given new insights to problems like EPR paradox. Also, consideration of space quantization in this framework gives naturally a correction to Newton’s Gravitation Law[6]:

\[ F = G \frac{Mm}{r(r - \delta)} \]  

(1)

where \( \delta \) is the space quantum. We have used this formula in the calculation of planetary precession of their perihelions and got quite good result[7]. Here in this paper, we shall see it also gives satisfactory explanation to the deflection of the light ray passing at the edge of the Sun.
The orbital equation in classical physics for centered force is [8]

\[ h^2 u^2 \left( \frac{d^2 u}{d\theta^2} + u \right) = \frac{F}{m} \]  \hspace{1cm} (2)

where \( r = \frac{1}{u} \) and \( \theta \) are the polar coordinates of the photo, \( h = cR \), \( c \) is the speed of light and \( R \) is the radius of the Sun. Substituting (1) into (2), we get

\[ h^2 u^2 \left( \frac{d^2 u}{d\theta^2} + u \right) = \frac{GMu^2}{1 - \delta u} \]  \hspace{1cm} (4)

This in the first order turns into

\[ h^2 u^2 \left( \frac{d^2 u}{d\theta^2} + u \right) = GMu^2(1 + \delta u) \]  \hspace{1cm} (5)

It follows that

\[ \frac{d^2 u}{d\theta^2} + (1 - D\delta)u = D \]  \hspace{1cm} (6)

where \( D = \frac{GM}{R^2} \). It is straightforward to verify that the solution of equation (6) is

\[ u = A \cos \sqrt{1 - D\delta \theta} + B \sin \sqrt{1 - D\delta \theta} \]  \hspace{1cm} (5)

in which the constants \( A \) and \( B \) can be determined in the following consideration. When \( \theta = 0 \), \( r = \infty \), so that \( u = 0 \). This leads to \( A = - \frac{D}{1 - D\delta} \). From \( y = r \sin \theta \) and (5) we have

\[ \frac{1}{y} = D \frac{1 - \cos \sqrt{1 - D\delta \theta}}{1 - D\delta} + B \frac{\sin \sqrt{1 - D\delta \theta}}{\sin \theta} \] \hspace{1cm} (6)

when \( \theta \to 0 \), \( y \to R \). It follows that \( B = \frac{1}{R \sqrt{1 - D\delta}} \). Thus finally the orbital equation is

\[ u = \frac{D}{1 - D\delta} \left( 1 - \cos \sqrt{1 - D\delta \phi} \right) + \frac{1}{R \sqrt{1 - D\delta}} \sin \sqrt{1 - D\delta \phi} \] \hspace{1cm} (7)

Designating the final polar angle of the light ray as \( \phi \), when \( r \to \infty \), the deflection angle may be expressed as \( \Delta \theta = \phi - \pi \). Thus the equation for \( \phi \) is

\[ \frac{D}{1 - D\delta} \left( 1 - \cos \sqrt{1 - D\delta \phi} \right) + \frac{1}{R \sqrt{1 - D\delta}} \sin \sqrt{1 - D\delta \phi} = 0 \] \hspace{1cm} (8)

This gives

\[ \phi = \frac{2}{\sqrt{1 - D\delta}} \left[ \arctan \left( \frac{- \sqrt{1 - D\delta}}{RD} \right) + m\pi \right] \] \hspace{1cm} (9)
Table 1: Deflection of Light Ray near the Sun (unit: second. m=1)

| $\delta$ | 1R   | 1.3R  | 2R   | observation |
|----------|------|-------|------|-------------|
| $\triangle \theta$ | 1.563 | 1.769 | 2.250 | 1.775±0.019 |

Table 2: Defl. at different values of m. $a = 10^6$ (unit: second. $\delta = 1.3R$)

| m | -7 | -6 | -5 | -4 | -3 | -2 | -1 | 0  | 1  |
|---|----|----|----|----|----|----|----|----|----|
| $\triangle \theta$ | -10.0a | -9.0a | -7.8a | -6.5a | -5.2a | -3.9a | -2.6a | -1.3a | 1.769 |
| m | 9  | 8  | 7  | 6  | 5  | 4  | 3  | 2  | 1  |
| $\triangle \theta$ | 10.0a | 9.0a | 7.8a | 6.5a | 5.2a | 3.9a | 2.6a | 1.3a | 1.769 |

where $m$ is any integer.

In determining the constant $B$, we suppose $\theta \to 0$. This is equivalent to presuming $R \to 0$. In this arithmetical process, the space quantum $\delta$ has been specified, somehow inadvertently. This is in complete accordance with the physical meaning of the space quantum: the unmeasurable quantity presupposed to be zero in the problem. This has great exemplary significance in determining the uncertainty quantum which is crucial in our theoretical framework. In Table 1 we show our calculation for $\triangle \theta$ with $\delta = R$, 1.3R and 2R, together with experimental observation [9]. It is easily seen that our calculation gives quite satisfactory explanation to the deflection of the light ray.

Another interesting and perhaps also important discovery in the calculation is that $m = 1$ is only integer giving reasonable $\triangle \theta$ value. From Table 2 it is easy to find that other values for $m$ produce incredibly large values which, oddly enough, are symmetrical relative to $m = 1$, where there occurs a sudden dramatical fall in the order of magnitude. I believe this is a profound reflection of its innate quantum nature of the problem, and therefore, an indication that our framework is reasonable.

It is one of the most important topics in modern physics to preserve the quantum feature of quantum mechanics while keeping the theoretical competence of the theory of relativity. Our research indicate that there may exist theoretical possibilities other than the theory of relativity.

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