An experimentally testable proof of the discreteness of time

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By proposing a paradox between the impossibility of superluminal signal transfer and the normalization condition of wavefunctions, we predict that when a change happens to the conditions that determining the status of a quantum system, the system will show no response to this change at all, until after a certain time interval. Otherwise either special relativity or quantum mechanics will be violated. As a consequence, no physical process can actually happen within Planck time. Therefore time is discrete, with Planck time being the smallest unit. More intriguingly, systems with a larger size and a slower speed will have a larger unit of time. Unlike many other interpretations of the discreteness of time, our proof can be tested, at least partly, by experiments. Our result also sets a limit on the speed of computers, and gives instruction to the search of quantum gravity theories.

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I. INTRODUCTION

How does time flow? This mystery puzzled people for centuries. In the age of Newton’s classical mechanics, time was regarded as a continuous coordinate that flows independently of any other object and event. With the advance of modern physics, people started to aware that time cannot exist solely without involving any physical process. That is, time can only be sensed and measured when changes occur to the status of objects. As indicated by quantum mechanics, there should be a smallest unit of time called Planck time, which is the limit of time that can be measured due to the uncertainty principle. Thus physics cannot reason in a meaningful way what happens within a time interval shorter than Planck time. Here we show that it is not only the limit of measurement. Instead, we give a simple proof that no physical process can actually happen within Planck time, otherwise either special relativity or quantum mechanics will be violated. Therefore it is proven that time is discrete instead of continuous.

II. A PARADOX BETWEEN SPECIAL RELATIVITY AND QUANTUM MECHANICS

According to the theory of special relativity, no signal can travel faster than the speed of light. On the other hand, quantum mechanics claims that any physical system is completely described by a wavefunction, which has to be normalized. Now let us show that there is a paradox between the two. Suppose that we want to induce a change on the wavefunction of a quantum system. Then we need to make a change on the elements which determine the wavefunction of the system, e.g., the potential, the status of the boundary, etc.. How fast will the wavefunction show a change after these elements changed?

For simplicity, let us consider the state of a particle in a one-dimensional finite square well with a potential $V_0$ as shown in Fig. 1(a). Denote the normalized wavefunction of the particle in this case as $\psi_0$. At time $t_1$, the potential at point $A$ suddenly changes to $V_1$, as illustrated in Fig. 1(b). Suppose that $\psi_1$ is the normalized wavefunction satisfying the current value of the potential. Then what is the minimal time for the state to change from $\psi_0$ to $\psi_1$?

It is important to note that even though such kinds of questions seem quite usual in quantum mechanics, in literature they were always solved under nonrelativistic approximation, despite that this was not clearly stated most of the time (see e.g., Refs. [1, 2]). That is, it is assumed that at any given time $t$, the wavefunction $\psi(t)$ satisfies the Schrödinger equation of the same $t$. This actually means that the change to the wavefunction occurs in the whole space instantaneously when the Hamiltonian changes. But this will violate special relativity as shown below. Suppose that two people Alice (located at point $A$) and Charlie (located at point $C$, where the distance between points $A$ and $C$ is $l$) want to communicate. They prepared $N$ ($N$ is sufficiently large) copies of the system shown in Fig. 1(a) beforehand. At time $t_1$, if Alice wants to send the bit 1, she makes the potential $V_0$ of the first $N/2$ systems at point $A$ change to $V_1$ simultaneously, while leaving the last $N/2$ systems unchanged. Else if she wants to send the bit 0, she keeps all the $N$ systems unchanged. At time $t_1 + \Delta t$ Charlie measures all the $N$ systems at point $C$. If the probability of finding the particle at point $C$ in the first $N/2$ systems can be considered equal to that of the last $N/2$ systems within the variation range allowed by statistical fluctuation, he assumes that the bit sent by Alice is 0. Else if the probabilities of finding the particle at point $C$ look much different in the two halves of the systems, he assumes that the bit is 1. With this method, a superluminal signal can be transferred from point $A$ to $C$ if $l > c\Delta t$, where $c$ denotes the speed of light. Thus special relativity is violated if we assume that the wavefunction can change from $\psi_0$ to $\psi_1$ in the whole space.

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instantaneously at time $t_1$.

Therefore, it seems natural to assume that the response of the wavefunction corresponding to the change of the potential at point $A$ should propagate along the $x$ axis with a finite speed $v$ ($0 < v \leq c$). However, this will cause trouble to the normalization of the wavefunction. Suppose that at time $t_1$ the potential at point $A$ changes, and at time $t_1 + l/v$ the change of the wavefunction from $\psi_0$ to $\psi_1$ occurs to the locations between points $A$ and $C$, while the wavefunction at the right side of $C$ has to remain strictly unchanged due to the impossibility of superluminal signal transfer. Such a wavefunction is plotted as the solid blue curve in Fig. 1(c). Then we can immediately see from the figure that the resultant wavefunction at this moment is no longer normalized.

There is also other possible way of evolution of the wavefunction which can keep the normalization condition unbroken. However, as shown in the appendix, such a solution will not satisfy the basic equations of quantum mechanics, i.e., quantum theory will be violated too.

Thus we found a paradox between the impossibility of superluminal signal transfer and the normalization condition of wavefunction. Though it was well-known that the theories of relativity and quantum mechanics do not go well with each other, the current paradox reveals yet another conflict between the theories which does not seem to have been reported before. Also, it does not involve the transformation between reference frames, so it cannot be solved simply by replacing Schrödinger equation with Klein-Gordon or Dirac equation. Therefore it differs by nature from previously known conflicts between the two theories, and put forward a new challenge to our understanding on the quantum world.

### III. THE DISCRETENESS OF TIME

Intriguingly, we find that this paradox can be solved if we adopt the bizarre idea that time is discrete. As shown above, relativity does not allow the change of wavefunction to occur before the time $t_1 + l/v$ for any location whose distance from point $A$ is larger $l$. Meanwhile, quantum mechanics does not allow the wavefunction to change part by part. As a consequence, to obey both theories simultaneously, logically the wavefunction has to evolve in the following way. After the potential at point $A$ changed at time $t_1$, the wavefunction should show no response at all during a period of time $\tau$. Then at time $t_1 + \tau$ or some point later, the wavefunction in the whole space changes simultaneously to keep the normalization condition unbroken. Here

$$\tau = L'/v \geq L/v, \quad (1)$$

where $L$ is the distance between points $A$ and $B$, which can be regarded as the effective size of the system. $L'$ is the distance between points $A$ and $B'$ or points $A$ and $A'$, depending on which one is larger. The location of points $A'$ and $B'$ are determined by the wavefunctions, in such a way that the difference between $\psi_0$ and $\psi_1$ at the left side of $A'$ and the right side of $B'$ is completely drowned by statistical fluctuation, so that it will not lead to any detectable superluminal signal from point $A$ to these regions when the wavefunction changes from $\psi_0$ to $\psi_1$.

In the spirit of Newton’s first law, any physical system will persist in its state of motion unless being applied with an inducement. Meanwhile, since quantum mechanics is recognized as the complete description of the physical world, any physical process can be viewed as the change of the wavefunction of the system under a certain inducement on a certain point. Therefore Eq. (1) sets a limit on how fast any physical process can occur. That is, when any inducement is applied on a system with size $L$, no change can happen to the state of the system within the time $\tau$. This conclusion covers all systems including any object we want to measure, as well as all apparatus we use as timekeepers or detectors to measure other objects. Now consider the lower bound of $\tau$ for any system. Due to the uncertainty principle of quantum mechanics, the minimal size of any physical system that can be reasoned in a meaningful way is Planck length $l_P \approx 1.616 \times 10^{-35} \text{meter}$. Meanwhile, the theory of relativity requires $v \leq c$. Thus we have

$$\tau_{\text{min}} \geq l_P/c \equiv t_P \approx 5.39 \times 10^{-44} \text{sec}. \quad (2)$$

Here $t_P$ is known as Planck time, which was already recognized as the minimum of time that can be measured. Our result suggests that the significance of $t_P$ is more than that. Any physical change can only happen after a time which is not less than $t_P$. Within a time interval of $t_P$, any physical system simply persists in its previous state. Therefore, according to the modern understanding of time, nothing happens within $t_P$ so that there is no further division of time possible in this range. In this sense, time is discrete, with $t_P$ being the minimal unit. Because the value of $t_P$ is so small, it is not surprising that the discreteness of time is less noticeable in practice, and previous nonrelativistic treatment of quantum mechanical problems [1, 2] seems fine in most cases.

Note that even if the minimal size limit $l_P$ could be somehow broken in the future, it is still natural to believe that as long as a system exists physically, its size has to be a finite non-vanishing value. Therefore according to Eq. (1), time still cannot be made continuous, though the exact value of the minimal unit might differs from $t_P$.

The above analysis is based on the assumption that the change of the potential from $V_0$ to $V_1$ is completed instantly. Some may wonder how this can be possible if $t_P$ is the minimal unit of time. Also, it would be interesting to ask what will happen if the potential changes more than once within $t_P$. We believe that these problems should be understood as follows. Even if there exists a change of the potential (or any other inducement) which could be so fast that it occurred and completed instantly, our result above showed that the response of any system
to this change cannot occur within \( t_P \). Therefore the response will surely take more time to occur if the change takes a finite time to complete, so that it will not conflict with the conclusion that no physical process can occur within a time interval less than \( t_P \). Also, any change of the potential has to be made by a certain physical apparatus, which is also limited by \( t_P \). Once the potential has a change at time \( t_1 \), no physical process can change it again before time \( t_1 + t_P \). That is, the change should also be considered as discrete instead of continuous. Therefore there does not exist the case where the system encounters a series of changes within \( t_P \).

### IV. EXPERIMENTAL TEST

Now back to the system with a size \( L >> t_P \). It is interesting to notice from Eq. (1) that the system has its own minimal unit of time \( \tau \). The larger the size \( L \) is, the slower the system can evolve. Note that for complicated systems containing more than one particle, \( L \) should be understood as the minimal localization length of the particles in the system, rather than the overall size. Therefore macroscopic systems, e.g., human bodies or planets, do not mean having a tremendous \( \tau \), because they contain plenty of particles which are highly localized on the microscopic scale. But for a system which is relatively large while having a simple structure, if we can keep all the particles on extended states whose localization length is comparable with the size of the system, then it may become possible to observe a larger discreteness of time. Therefore, though we cannot test directly our above interpretation of the discreteness of time with systems having the size of Planck length because we cannot find detectors smaller then they do, we can perform indirect experimental test with larger systems.

For example, we can stimulate the device in Fig. 1 with cold atoms or quantum dots. We also use a detector to measure whether the particle inside the well can be found within a fixed region inside the potential well. This region serves as point \( C \) in Fig. 1(c). To test whether time is discrete, first we repeat this experiment many times to get an estimation of the probability \( p_0 \) of finding the particle around point \( C \) when the potential is set to \( V_0 \). Secondly, we re-initialize the system, i.e., prepare such a system again with the potential \( V_0 \) and keep it unmeasured. Then change the potential at point \( A \) from \( V_0 \) to \( V_1 \). After the change we wait for a time interval \( t_x < 1/c \) where \( l \) is the distance between points \( C \) and \( A \). Now we measure whether the particle can be found at point \( C \). Repeat this for many times too, so we can get an estimation of the probability \( p_x \) of finding the particle around point \( C \) at time \( t_x \) after the potential changed. Third, we re-initial the system again. Change the potential from \( V_0 \) to \( V_1 \), and wait for a time interval \( t_y (l/c < t_y < L/c) \). Here \( L \) is the width of the well. Then we measure whether the particle can be found at point \( C \). Repeat this also for many times and we can get an estimation of the probability \( p_y \) of finding the particle around point \( C \) at time \( t_y \) after the potential changed. Finally we compare whether \( p_0, p_x, \) and \( p_y \) are equal within the variation of statistical fluctuation. Then we can have the following conclusion.

1. If \( p_0 \neq p_x \), then it seems to enable superluminal signal transfer and thus violates the theory of relativity.
2. Else if \( p_0 = p_x \neq p_y \), then the theory of relativity is obeyed but quantum mechanics seems to be violated.
3. Else if \( p_0 = p_x = p_y \), then it proves that our above interpretation of the discreteness of time is correct.

For a more rigorous test on the validity of the normalization condition, we can keep changing the location of the detector (i.e., point \( C \)), and measure the probabilities of finding the particle at different positions at a given time after the potential changed from \( V_0 \) to \( V_1 \). If for a period of time after the potential changes, the measured value of the probability at every position remains unchanged, and at a later time we find that all in a sudden, the probability at every point shows difference from its previous value, then we can conclude that the wavefunction in the whole space indeed changes simultaneously. Else if we find that at a given time, the probabilities change at some positions while remain unchanged at the others, then we can deduce the form of the current wavefunction and check whether the normalization condition could be broken.

In these experiments, we’re mostly interested on whether the probabilities measured at different time or positions are equal or not. The exact values of the probabilities are not very important as long as we do not need to check the normalization condition in exact numbers. Therefore it does not matter how much the detection efficiency of the detector is. As long as the efficiency remains stable during the experiments, the result will be valid.

But we have to notice that both \( t_x \) and \( t_y \) are very small time intervals. For instance, the size \( L \) of quantum dots are usually hundreds of nanometers, so \( L/c \) is at the order of magnitude of femoseconds. Thus it will be hard to measure \( t_x \) and \( t_y \) precisely with current technology. Nevertheless, Eq. (1) shows that the minimal time interval \( \tau \) of a system is determined by \( L/v \) instead of \( L/c \). If \( v \) is significantly smaller than \( c \), then we can expect a much longer time interval \( t_x > L/c > t_y \), during which the system still does not evolve as long as \( t_x < L/v \). Since the exact value of \( v \) is unknown to us so far, in experiments we can measure the probability \( p_x \) of finding the particle around point \( C \) at time \( t_x \) after the potential changes, where \( t_y \) is the shortest time interval we can achieve with current technology. If we find \( p_x = p_0 \) within the variation of statistical fluctuation, then it proves our above interpretation of the discreteness of time, while also indicates that \( v < L/t_x \). By increasing \( t_x \) gradually and repeating the experiment until we find \( p_x \neq p_0 \), the speed \( v \) can be more rigorously determined.

Here we would like to discuss the speed \( v \) a little further. \( v \) describes how fast a system responds to the change of the status of the boundary at one side, there-
fore its value is related with the understanding on how the particle in the system “knows” the status of the boundary. This is a question of quantum interpretation theory which is beyond the standard framework of quantum mechanics. Indeed, previous quantum mechanical calculations all settled with the nonrelativistic approximation (e.g., Refs. [1, 2]) where \( v \) is treated as infinite. Therefore these theories are insufficient for calculating the value of \( v \) without the help of quantum interpretation. In the square well problem we considered here, however, different quantum interpretation theories may have different understanding on how the particle “knows” the status of the boundary of the well, so they may predict different values of \( v \). Thus it will be very useful if we could measure the speed \( v \) experimentally. While it may not be easy to implement the above experimental proposal to measure \( v \) precisely with current technology, a very feasible scheme was proposed in Ref. [3], which can measure a similar speed in double-slit interference using state-of-the-art technology. Though that experiment has not been carried out yet, it was also shown in the same reference that the speed it measures may very probably equal to the classical speed \( v_0 \) of the particle in the system if the mainstream quantum interpretation theories are correct. This is because in double-slit interference, the mainstream interpretations believe that the particle “knows” the status of the slits by reaching them by itself. Therefore, it is reasonable to believe that the speed \( v \) in our current case also equals to the speed \( v_0 \) of the particle. If so, we can expect a very low \( v \) from systems at low energy levels. Then it will be easier to observe a larger \( \tau \) in the above experiments. Moreover, \( v = v_0 \) can also explain why the system needs to wait a time interval \( \tau \) before its wavefunction starts to evolve. As we know, the wavefunction (e.g., \( \psi_1 \) in Fig. 1(b)) is not determined merely by the status of the potential at one side of the well. Instead, the potential on anywhere of the whole system has its share of contribution. Therefore, the particle needs time \( \tau \geq L/v_0 \) to travel from one side of the well to the other, so that it “knows” the status of the potential and the boundary at every point of the system before it “decides” how the wavefunction should evolve to. Of course, all quantum interpretation theories are still in need of more experimental support at the present moment. Therefore it is still too early to reach any conclusion theoretically, before the above experimental proposals are implemented and prove whether \( v = v_0 \) or not.

V. APPLICATIONS

As a corollary of the discreteness of time, there exists a limit for the speed of all kinds of computers, either classical or quantum ones. Since the state of a register of the computer cannot evolve within \( \tau \), every step of the instruction on the register needs at least a time interval of \( \tau \) to complete. Thus the maximum operational speed on a single register is \( 1/\tau \text{ IPS} \) (Instructions per second). If \( 1/\tau \) is indeed the minimal unit of time, then the maximum speed is \( 1/1_{\text{IPS}} \approx 1.86 \times 10^{43} \text{IPS} \) per register. Of course, a computer can contain many registers that operate in parallel. Therefore the total speed will rise with the increase of the number of the registers.

We should note that the above analysis is based on the assumption that both special relativity and quantum mechanics are valid on any scale. This assumption seems to be valid on most scales, even down to a few nanometers. Therefore our above analysis and experimental proposal are valid for systems with a larger size \( L \), e.g., quantum dots and cold atoms. But currently there is no proof that special relativity and quantum mechanics must remain valid in the range of Planck length and Planck time, despite that neither counterexample was found so far. If either of the theories fails on this scale, then there may be physical processes happening within the range of Planck time. Ironically, if the above interpretation on the discreteness of time is truth, then as we mentioned, all timekeepers and detectors are bounded by the limit of the discreteness too. Thus we cannot perform experimental test directly on this scale despite that we can perform indirect test on larger scales. Nevertheless, it does not mean that our above analysis is futile even on the scale where special relativity or quantum mechanics becomes invalid. Currently there are many attempts to develop new theories (e.g., loop quantum gravity theory) trying to describe the events within the scale of Planck time, e.g., the first few moment of our universe just after it was born from the Big Bang. Our current result indicates that if there indeed exists a theory capable of handling the physical processes in any small time interval, i.e., time is treated as continuous, then it should be better incompatible with either the impossibility of superluminal signal transfer or the normalization condition of wavefunction (or even both). Or it should find an even smaller unit as a replacement for Planck time to describe the discreteness of time. Otherwise it will have a hard time solving the paradox between relativity and quantum mechanics we proposed above. This is in agreement with a recent proposal [4], which prefers an indefinite causal structure of the theory.

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APPENDIX A: ANOTHER SOLUTION THAT WILL VIOLATE QUANTUM MECHANICS

Alternatively, we find that there exists another solution to the above paradox between the impossibility of superluminal signal transfer and the normalization condition of wavefunctions, which does not require the discreteness
of time. That is, when the potential of the well changed from $V_0$ to $V_1$ at time $t_1$, the wavefunction evolves from $\psi_0$ to $\psi_1$ in the following way. At time $t_1 + \delta t$, the wavefunction $\psi_{\delta}$ varies from $\psi_0$ with a wave-like shape in a small region around point $A$ only. At one side of point $A$ the wavefunction rises a little, while at the other side of point $A$ it drops the same amount, so that the overall wavefunction is still normalized. An example of such a wavefunction is illustrated in Fig. 2. The width $d$ of the varied region at each side of point $A$ grows as $\delta t$ increases, but for any given $\delta t$, it always satisfies $d \leq c\delta t$ so that no superluminal signaling occurs. Thus the above paradox is avoided.

Nevertheless, we must notice that even though such an evolution of the wavefunction does not violate the normalization condition, it may still violate quantum mechanics since $\psi_{\delta}$ is not the solution to Schrodinger (or Klein-Gordon/Dirac) equation with a Hamiltonian corresponding to $V_0$ or $V_1$. That is, standard quantum mechanical formulas alone are insufficient to describe the behavior of such a wavefunction. We will have to find new formulas and even new postulations to explain why the system should take such a wavefunction, how it will finally evolve to $\psi_1$, and what determines the shape of $\psi_{\delta}$ (e.g., the phase, amplitude, and speed of the wave-like variation between $\psi_\delta$ from $\psi_0$), which seem to exceed the framework of standard quantum mechanics. Therefore, the existence of such a solution does not conflict with the conclusion that the discreteness of time should be the solution if we want to keep both special relativity and quantum mechanics unbroken.

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FIG. 1: The wavefunctions in a square well. (a) The wavefunction $\psi_0$ when the potential is $V_0$. (b) The wavefunction $\psi_1$ (solid blue curve) when the potential is $V_1$ (solid green curve). (c) The wavefunction at time $t_1 + t'/v$ (solid blue curve) as a mix of $\psi_0$ and $\psi_1$. 
FIG. 2: An example of the variation between the wavefunction $\psi_0$ (solid blue curve) and $\psi_0$ (dashed blue curve) at time $t_1 + \delta t$. 

$\psi_0$