Two dimensional vortex structures with multi scale interactions in thin film superconductors

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Abstract

Interaction of particles of many systems can be effectively approximated by multiscale interaction potentials. Such potentials are widely used for investigation of colloidal systems and colloid-polymer mixtures, complex liquids (for instance, water) and other system. Also, it is of particular interest that effective interaction of vortices in a thin superconducting film can be approximated by a multiscale potential. The subject matter of the present paper is a system like this. Based on a multiscale potential we have shown that this system demonstrated a large number of different phases. It is particularly important that we analyze the influence of a long-range interaction of the system properties and show that properly taking into account long-range forces results in a dramatic change in the system phase diagram.

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I. INTRODUCTION

Two dimensional (2d) systems are of great interest for both fundamental science and technological applications. In many aspects 2d systems are basically different from three dimensional (3d) ones. First of all 2d melting is a much more complex phenomenon than the melting of 3d crystals. While 3d crystals always melt via the first order phase transition, 2d systems can melt via at least three different scenarios [1].

Another question of interest in 2d systems is their crystalline structure. Until recently, only a simple triangular crystal which is a close packed structure in 2d was known experimentally. The situation got a lot of attention when graphene was discovered [2]. This discovery gave impetus to further experimental investigation of possible ordered structures of 2d systems and some of them were really discovered. For example, in Ref. [3] a square phase of water (square ice) was observed when water was strongly confined between two graphene sheets. The square crystal was also observed in one atom thick film of iron confined between graphene sheets [4].

Another example of systems where complex 2d structures can be found is 2d colloidal suspensions. For example, in Ref. [5] a system of colloidal particles in a magnetic field was studied. The authors changed the system density intending to find the same sequence of phases which was obtained in their model system from Refs. [6, 7] and clearly observed a square phase in their colloidal system.

An important class of 2d systems is the systems of vortices in superconducting films predicted by Abrikosov in 1957 [8] and experimentally observed for the first time by Essmann and Trauble in 1967 [9]. These important results gave rise to a novel field in the studies of superconductivity. Numerous theoretical and experimental works are devoted to the lattices of vortices in superconductors. Like atomic and colloidal particles, the vortices usually form a triangular lattice. However, there are some experimental works where other types of ordered structures were observed (see, for instance, Ref. [10] for chains of vortices and [11] for the square phase of vortices).

One can see that attempts to find non-triangular one-component 2d structures have been made in different classes of systems. However, the progress is not very fast. At the same time a theoretical search for novel structures in 2d systems is very active and numerous different structures have been found. Most of the theoretical studies are concerned with
so called core-softened systems, i.e. systems with the negative curvature of the potential. Such systems can serve to describe effective interactions in many real systems, such as colloidal systems and colloid-polymer mixtures, water, liquid metals, some molecular liquids, etc. This potential is characterized by the presence of two length scales. The competition between these length scales leads to complex structure formation in the system. In the above mentioned publications Refs. \[6, 7\] exactly the core-softened potential was used which allowed the authors of those works to find several unusual crystalline structures such as square crystal and Kagome lattice. A cluster liquid was also observed. Many other authors studied other core-softened systems and found the formation of numerous phases (see, for instance, \[12, 22, 24\] and references therein). It was shown that 2d systems could form even quasi-crystalline phases (see, for instance, \[15, 21, 25\]). Some works reported particularly interesting phases \[26\] such as clusters (clump phase in \[26\]), holes (anti-clump in \[26\]) and ribbon-like stripes. Such phases as clusters or stripes were also observed in another system with an interaction similar to the one which was expected between the vortices in a superconductive film \[27\].

The interaction potential of vortices in a superconductive film is usually rather simple \[28, 31\]. However, much more complex vortex interactions can exist in systems consisting of layers of different superconductors, in particular, bilayers of type-I - type-II superconductors, which are widely discussed in the literature now. For example, in Refs. \[32, 33\] the interaction consists of long-range logarithmic repulsion and short-range attraction between the vortices. The authors report that this system can form different complex phases, such as circular clusters, ribbons, etc. In several publications potentials are used which consist of combinations of Bessel functions \[27, 34, 37\]. In those systems such very complex structures as clusters, labyrinths, etc were also found.

In the present paper we consider the phenomenological interactions introduced in Refs. \[38, 40\]. These interactions are characterized by core-softened potentials, and therefore one can expect these systems to demonstrate some complex phases. Indeed in Refs. \[38, 40\] the ground states of these systems were studied and they were found to form a set of very unusual structures.

Let us consider the particular system from Refs. \[38, 40\]. In these publications it is assumed that the interaction potential between the vortices consists of three parts which are responsible for intervortex repulsion and for core overlaps. The particular interaction
potential, as suggested in Refs. [38–40] has the following form:

\[ V(r)/V_0 = e^{-r/\lambda} - c_2 e^{-r/\xi} + c_3 \lambda \frac{\tanh[a(r - b)] + 1}{r + \delta} + \frac{q^2}{r}. \]  

(1)

Parameter \( \lambda \) sets the unit of length and is chosen to be unity. Coefficients \( c_2, c_3 \) and \( \delta \) are the phenomenological constants. Parameter \( \xi \) is the coherence length.

Refs. [38–40] give very elaborate studies of the ground state structures of the systems with the potential (1). In particular, a very interesting phase was obtained with parameters \( c_2 = 0.2, c_3 = 0.1, \delta = 0.1 \) and \( \xi/\lambda \) from zero up to 10 (see Ref. [40]). This phase consists of big circular clusters. The authors show that in small systems all particles collapse in a single cluster while if one increases the number of particles the system splits into several circular clusters with several hundred of particles each.

This cluster phase looks very unusual and is worth studying in more detail. In particular, it is interesting to study the influence of a long-range interaction on this phase. In Refs. [38–40] the potential is considered to extend up to half the simulation box. However, the potential from formula (1) tends to \( 1/r \) as \( r \to \infty \), i.e. it becomes Coulomb-like at greater distances. As is known from studies of coulombic systems, simple cutting of the potential at a finite distance can only be considered as a first approximation to the solution [41]. In the present paper we perform calculations of system behavior with a simple cut-off of the interaction potential with more precise treatment of the long-range interaction and compare the results. We show that the results basically change if the long-range interactions are taken into account.

II. SYSTEM AND METHODS

Here we investigate the influence of long-range interactions on system cluster phase behavior with the potential from (1). Following Ref. [40] we used \( c_2 = 0.2, c_3 = 0.1, \delta = 0.1, a = 2.5, b = 0.5 \) and \( \xi/\lambda = 4.5 \).

In order to take into account the long-range nature of the potential (1) we do the following: add and subtract the term \( q^2/r \):

\[ V_{1}(r)/V_0 = \left( e^{-r/\lambda} - c_2 e^{-r/\xi} + c_3 \lambda \frac{\tanh[a(r - b)] + 1}{r + \delta} - \frac{q^2}{r} \right) + \frac{q^2}{r}. \]  

(2)

Parameter \( q^2 = 2c_3 = 0.2 \) is selected based on \( q^2/r = \lim_{r \to \infty} V(r)/V_0 \).
The term in brackets is considered as a short-range interaction, while the last term is calculated by the Particle-Particle/Particle-Mesh (PPPM or P3M) method, which is similar in sense but computationally more efficient than Ewald summation [41]. The simplest method of performing Ewald summation in 2d systems is to add slabs of vacuum (see the chapter 'Ewald Summation in a Slab Geometry' of the book [41]). In this method one adds slabs of vacuum at the top and the bottom of the system and considers it as a three dimensional one with periodic boundary conditions. The thickness of the vacuum slab is 10 from both top and bottom. Importantly, the system appears to be not "charge neutral". So one should consider it as a "charged" system in a neutralizing background. The presence of this background gives an additional term in the system energy: $E_{\text{neut}} = \frac{\pi q_{\text{tot}}^2}{2 V \alpha}$, where $q_{\text{tot}}$ is the total charge of the system and $\alpha$ is a parameter of Ewald summation. This term does not depend on the position of the particles and therefore does not affect the equilibrium structure of the system.

The short range part of the potential is cut off at distance $r_c = 14$.

Since we add a slab of vacuum the effective particles can leave the plane and the system will not be two-dimensional any more. In order to avoid it we introduce two repulsive walls in the system. The walls are located at height $-0.5$ and $0.5$ and repel the particles with Lennard-Jones 9-3 potential:

$$V_{\text{wall}}(r) / V_0 = \frac{2}{15} \left( \frac{\lambda}{r} \right)^9 - \left( \frac{\lambda}{r} \right)^3 . \quad (3)$$

In the present study three types of systems are investigated. Firstly, we study two-dimensional (2d) systems with the potential from (1) in order to compare the results with the ones of Ref. [40]. Secondly, we study a quasi two-dimensional system (q2d) with the same potential. So these systems have slabs of vacuum and walls at the top and bottom. Finally, we study systems with a long-range interaction (the potential from (2)) (coul), slabs of vacuum and walls. At the first step (from 2d to q2d) we make sure that transition from a two-dimensional to a quasi two-dimensional system does not affect its behavior. The second step (from q2d to coul) is necessary to monitor the influence of long-range forces on the cluster phase.

We simulated a system of 3200 particles in a box with periodic boundary conditions. The system had a square shape in the x-y plane. We performed molecular dynamics simulations. The Lammps package [42] was used. Firstly, we simulated all three kinds of the systems at
temperature $T = 0.001$ which is rather low and the results should be close to the ones at zero temperature. The densities $\rho = N/A$ where $A$ is the surface of the system (two-dimensional volume) studied were from $\rho = 0.01$ up to 2.6. The following densities were considered: $\rho = 0.01, 0.05, 0.1, 0.12, 0.14, 0.16, 0.18, 0.2$. Then the densities up to $\rho = 2.6$ with step $\Delta \rho = 0.2$ were simulated. After that we also monitored the influence of temperature by studying the system with $\rho = 0.01$ and $\rho = 0.2$ up to temperature $T = 0.5$. The time step is $dt = 0.0001$. Totally, $5 \cdot 10^6$ steps were performed for system equilibration. In order to see size effects we repeated the same procedure with a larger system of 50000 particles at some densities.

The system structure was analyzed using several methods. We calculated the radial distribution functions (RDF), order parameters $\psi_6$ which will be described below and the distribution of the nearest neighbors in the systems. Two types of the RDFs were considered: vortex-vortex (vv) and cluster-cluster (cc). The cc-RDFs were defined as the radial distribution functions of the centers of mass of clusters.

A. Low densities

We started with investigation of the structure of three types of systems: 2d, q2d and coul at $T = 0.001$. Let us first consider the system structure at low densities $\rho < 0.18$. Fig. shows snapshots of the systems at several densities. Two major conclusions follow from these snapshots. First, the structures of 2d and q2d are qualitatively identical. Below we will address this question in more detail. This conclusion confirms that transformation of a 2d system into q2d does not influence the structure. The second conclusion indicates that the structure of a coul system can be substantially different from 2d and q2d. Combining it with the first one we conclude that the changes of the structure are related to the long-range interaction in the coul system.

Let us discuss in more detail the structures of the coul system at different densities. At very low density $\rho = 0.01$ the system forms numerous small clusters. In Ref. [26] it was proposed to call this phase a clump phase. Fig. 2 enlarges part of the snapshot from Fig. One can see that the clusters can have different internal structures which also depend on cluster size. The small clusters mostly have triangular symmetry. The larger one can have not so well defined crystal-like ordering. Fig. 2 (b) shows the distribution of the size of the
clusters in the system. One can see that the smallest clusters consist of 4 particles while the largest one of 27. The most probable cluster size is 10 or 12 vortices (two peaks of the distribution functions). The vv-RDF demonstrates the first extremely high peak at $r = 4$, the second peak is located at $r = 7.46$ the third small peak at $r = 10.57$ (Fig. 2 (c)). At a distance of about $r = 14$ a gap with an almost zero value of $g(r)$ is observed, so this distance can be considered as the average size of the clusters. The next peak of the vv-RDF coincides with the peak of the cc-RDF, i.e. here it already determines the correlation between the particles of different clusters. The clusters form a kind of liquid.

If the density is increased to $\rho = 0.05$ then the coul system forms a cluster phase with larger clusters (Fig. 1). Fig. 3 (a) enlarges part of the snapshot of the system. Panel (b) shows cluster size distribution. The smallest cluster consists of 50 vortices while the largest one of 98. The highest peak corresponds to 68 particles in a cluster. From the RDFs of this system (Fig. 3 (c)) we can conclude that the internal structure of the clusters does not show any kind of crystalline order observed in some lower density clusters. Importantly, the outer shell of a cluster is separated from the other particles of the same cluster by slightly greater distance compared with the internal particles. This shell is responsible for the second vv-RDF peak at $r = 4.79$ (see the inset of Fig. 3 (c)). The first cc-RDF peak is located at $r = 40$. Since the size of the clusters is comparable to the distance between them the vv-RDF does not go to zero at distances exceeding the cluster size. We can roughly estimate the size of the clusters as a minimum in the vv-RDF which is located at $r = 20.1$. Importantly, at this density a triangular superstructure of clusters appears which is visible from the snapshots and cc-RDF. However, larger systems are required for detailed investigation of this superstructure.

In the case of density $\rho = 0.1$ the structure of the system changes (Fig. 1). At this density we observe a single cluster percolating through the whole system. We propose to call this phase a "Nazca phase" due to its similarity to the nazca lines in South America. Fig. 4 (a) enlarges part of the snapshot of the nazca phase. One can see that the internal part of the cluster has a triangular structure. At the same time, as in the clusters at $\rho = 0.05$ the outer shell of the vortices is located at greater distance from the internal layers than the lattice constant of the crystal. Fig. 4 (b) demonstrates the vv-RDF of this system (There is no cc-RDF because the whole system is a single cluster). One can see modulations on the RDF. The inset shows the low r part of the RDF. The first peaks appear at distances 2.4,
FIG. 1: Snapshots of configuration of the 2d, q2d and coul systems at $T = 0.001$ and densities up to $\rho = 0.14$. Further figures give more close views of these structures.
FIG. 2: (a) An enlarged snapshot of a coul system at $T = 0.001$ and $\rho = 0.01$; (b) the distribution of the cluster size in the same system; (c) the vv- and cc- RDFs of the same system. The inset enlarges the part of the plot at low $r$.

4.21, 4.8 and 6.43, which corresponds to a triangular solid.

A further increase in density leads to inverting of the structure. While at low densities we observe clustering of the system, at density $\rho = 0.12$ the coul system demonstrates a structure with holes which in Ref. [26] was called anti-clump. We propose to call it a ”cheese phase”. A similar structure was observed in Ref. [40] for the same system, but with slightly
FIG. 3: (a) An enlarged snapshot of the coul system at $T = 0.001$ and $\rho = 0.05$; (b) the distribution of the cluster size in the same system; (c) the vv- and cc-RDFs of the same system. The inset enlarges the small r part of the vv-RDF.

different parameters of the potential and without consideration of the long range interaction. The vortices of the cheese phase are ordered in the triangular phase which can be seen from the RDF shown in Fig. 5. At the same time it seems that the holes also form a triangular superstructure. However, this question requires further investigation with larger systems.

We finish the discussion of the coul system at low densities by investigation of the system
FIG. 4: (a) An enlarged snapshot of the coul system at $T = 0.001$ and $\rho = 0.1$; (b) the vv- RDFs of the same system. The inset enlarges the low $r$ part of the RDF.

at $\rho = 0.14$. From the snapshot in Fig. 1 one can see that this system forms a usual triangular lattice. This conclusion is supported by the vv-RDFs of the system given in Fig. 6.

At densities $\rho = 0.16$ and $\rho = 0.18$ the coul system preserves a triangular lattice. Because of this we do not show the results for these densities.

Now we turn to the description of 2d and q2d systems at the same densities. As mentioned above, the structure of these systems appears to be identical which can be seen from Fig. 1. At the same time the behavior of these systems is more simple compared with the case of the coul one. 2d and q2d systems form clusters at all densities $\rho < 0.18$. As the density increases the size of the clusters also increases. At density $\rho = 0.01$ the clusters are small and some of them demonstrate a triangular internal structure. At higher densities the clusters consist of a triangular core and several layers above it. We will consider the internal structure of these clusters below. Importantly, the 2d and q2d systems demonstrate unusual dynamics of vortices. The systems rapidly split into fast and slow parts. Some of the vortices move
FIG. 5: The vv-RDFs of the coul system at $\rho = 0.12$ and $T = 0.001$. The inset enlarges the low $r$ part of the plot.

FIG. 6: The vv-RDFs of the coul system at $\rho = 0.14$ and $T = 0.001$. 


extremely actively, while others quickly condense into clusters and move slowly. As soon as
the clusters grow the fraction of slow particles will increase. However, some of the particles
do not fall into clusters for very long time. Because of this we observe many single vortices
in the cluster phases of 2d and q2d systems. We expect that in the limit of infinite time
all vortices will belong to clusters. It is also worth noting that such a cluster phase was
observed in Refs. [38–40], but the authors of these publications considered only the ground
state of the system and therefore they did not observe such peculiar dynamic behavior of
vortices.

B. Intermediate and high densities

Fig. 7 shows snapshots of 2d, q2d and coul systems at five different densities. Here we
show only the densities at which some changes in the structure take place.

We start the discussion again from the coul system. This system preserves a triangular
crystal phase up to density $\rho = 1.2$ where it becomes disordered. At density $\rho = 1.6$ the
system forms a stripe liquid which was earlier observed in several core-softened systems
[6, 7, 12, 13] and in the experiment with colloidal films in a magnetic field [5]. Finally, at
density $\rho = 2.0$ the stripe phase collapses into another disordered phase. Apparently, the
high density limit of this phase should be a triangular lattice, therefore higher densities are
out of the scope of the present work.

Let us now turn to 2d and q2d systems. The densities studied in this part of the paper
correspond to the range of densities investigated in Ref. [40], therefore we can make a
comparison of the results. However, one should remember that in Ref. [40] only the ground
state of the system was studied, while we perform simulations at finite temperatures.

From Fig. 7 one can see that the structures of 2d and q2d systems are again very
similar. In agreement with Ref. [40] the systems form a circular cluster at density 0.2 which
transforms to a ribbon at $\rho = 1.2$. It is important that the cluster at $\rho = 1.2$ does not occupy
the whole simulation box. The whole system can be described as a ribbon surrounded by
a vacuum. However, at $\rho = 1.6$ we observe a stripe phase. This is in contradiction to
Ref. [40] where a cluster phase is reported for densities up to $\rho = 2.5$. This contradiction
can be related to the difference in temperature: while in Ref. [40] only ground states were
considered we perform investigations at finite temperature.
FIG. 7: Snapshots of the configuration of 2d, q2d and coul systems at $T = 0.001$ and densities up to $\rho = 2.0$. The different colors mark different clusters in the systems. In some cases the number of clusters exceeds the number of colors used and different clusters can have the same color. Periodic boundary conditions are used, therefore the clusters can partially be located at different edges of the image.
We again see that the contribution of long-range interactions completely changes the structure of the system. The origin of this effect can be explained by exploration of the RDFs of 2d, q2d and coul systems $g(r)$. Fig. 8 shows $g(r)$ of all three types of systems at $\rho = 0.2$ and $T = 0.001$.

In the case of $\rho = 0.2$ the difference in the radial distribution functions of 2d (or q2d) and coul systems is dramatic. Importantly, the first peak appears at $r_1 = 0.55$ in the 2d system, $r_1 = 0.58$ in the q2d and $r_1 = 2.4$ in the coul one. It means that a long range interaction changes the characteristic distances in the system. Indeed the nature of the interaction is different in the case of short and long-range ones. In the former case only several particles which are within the cut-off distance to the one under consideration interact with it. In the latter case each particle interacts with the whole system, which can substantially shift the energy and the characteristic distances in the system.

The difference in $g(r)$ becomes less pronounced as density increases. The first $g(r)$ peaks of 2d, q2d and coul systems become closer to each other, and for densities above 1.8 the RDFs of all three types of systems become indistinguishable which is illustrated in Fig. 9. It means that while a long range interaction determines the structural properties at low densities it becomes much less significant at high densities.

Let us consider the cluster phase of 2d systems in more detail. Fig. 10 shows a snapshot of the 2d system with $\rho = 0.2$. The particles are colored according to the number of neighbors they have. The number of neighbors was defined as the number of particles which were closer
FIG. 9: The radial distribution functions of 2d, q2d and coul systems at $T = 0.001$ and (a) $\rho = 0.8$; (b) $\rho = 1.2$ (c) $\rho = 1.8$.

to a given one than the first minimum of $g(r)$. One can see that the number of neighbors can have values from zero to 6. From the snapshot one can see that close to the center of the cluster the particles are arranged in a triangular lattice. However, going from the cluster center to the edge the distance between particles becomes greater. The border particles
FIG. 10: A snapshot of a cluster at $\rho = 0.2$ and $T = 0.001$. The particles are colored according to their coordination number. The inner circle denotes the boundary of the inner crystalline core of the cluster. The outer circle bounds the region where the distance to neighboring particles exceeds the first RDF minimum.

of the cluster form a circle with radius $R = 23.718$. In order to characterize the internal structure of the cluster we calculate the radial distributions of density, order parameter $\psi_6$ and number of nearest neighbors.

Parameter $\psi_{6,i}$ where $i$ is the particle number is defined as

$$\psi_{6,i} = \sum_{j=1}^{N_{N_i}} \exp(6i\theta_{ij}),$$

(4)

where the $j$ is one of the nearest neighbors of $i$ and $\theta_{ij}$ is the angle between vector $\mathbf{r}_{ij}$ and an arbitrary direction. The global order parameter in some region is defined as

$$\psi_6 = \sum_i \psi_{6,i},$$

(5)

where $i$ denotes particles belonging to the region of interest.

The radial distributions of different quantities are defined as follows. From analysis of the cluster outer shell we find that it has a circular shape. If we place the origin of the coordinate frame in the center of this circle we can determine the circular layer bounded by concentric circles with radius $r$ and $r + dr$. The area of such a layer is $S(r + dr, r) = \pi(r + dr)^2 - \pi r^2$. We define the radial density of a cluster as the number of particles inside such a layer divided by area $S(r)$. The sum of $\psi_{6,i}$ for all particles in the layer divided by this number of particles is the radial distribution of $\psi_6$. The radial distribution of the nearest neighbors is calculated in the same way.
FIG. 11: (a) The radial density distribution of the 2d phase cluster at density $\rho = 0.2$ and $T = 0.001$. (b) The radial distribution of order parameter $\psi_6$ and the number of nearest neighbors NN at the same point. NN is divided by 10 in order to place both curves on the same plot.

The radial density has a very sharp peak at the origin. Two effects are responsible for this phenomenon. First, there are many particles next to the center of the circle and, second, the area of the layer from zero to $dr$ is extremely small. The distribution of order parameter $\psi_6$ shows a plateau at radiiuses from zero up to 8. After that it decays monotonically and becomes negligible at $r = 16.3$. We define the crystalline segment of the cluster as a part with $\psi_6 \geq 0.4$. In this case the radius of the crystalline part is $r_{cr} = 13.93$. The outer layers where the particles have no nearest neighbors extends from $r = 16.3$ up to 23.718, i.e. the width of this outer shell is $\Delta = 7.418$.

The same analysis is performed for the other clusters of the 2d system for densities 0.4, 0.6, 0.8 and 1.0 (not shown in the text). Interestingly, the cluster size appears to be independent of the density: the radial densities for all densities of the system are identical except $\rho = 1.0$ where the radial density is a bit higher. Also the sizes of the cluster crystalline core and
outer zero neighbor shell appear to be identical for all the cases except $\rho = 1.0$. To be more specific, the radius of the cluster is from $R = 23.704$ to $R = 23.74$, the inner crystalline core is $r_{cr} = 13.93$ and the outer shell $r_{shell} = 7.4$ for the densities from 0.2 up to 0.8 and $R = 25.775$, $r_{cr} = 13.9$, $7.415$ for $\rho = 1.0$.

For densities $\rho = 1.2$ and 1.4 the system forms a ribbon-like cluster instead of a circular one. However, the cluster internal structure looks very similar to the circular case. The clusters consist of an internal crystalline ribbon, a transition region and an outer shell with a zero number of the nearest neighbors. For the case of $\rho = 1.2$ the crystalline core extends from $x = 14.52$ to $x = 27$ and the outer shells - from $x = 0.0$ to $x = 7$ and $x = 34 - x = 40.0$. In the case of $\rho = 1.4$ the crystalline core is from $x = 11.17$ to 27.0 and the outer shells are from $x = 0.0$ to $x = 4.18$ and $x = 34$ to $x = 38.8$.

One can see that at low densities cluster size increases with density while at intermediate ones the whole system forms a single cluster. However, it can be a finite size effect. Also, in Ref. [40] it was noticed that while small systems formed a single cluster, larger ones could split into several clusters. In order to check the influence of system size we simulate much larger systems of 50000 particles.

Fig. 12 shows snapshots of configurations of large 2d systems at three densities: $\rho = 0.2$, 0.6 and 1.2. At $\rho = 0.2$ the system apparently splits into numerous clusters. However, the dynamics of the system is very slow. Even at $\rho = 0.2$ the system has not come into an equilibrium state yet. One can see that small clusters are still included into larger ones. The dynamics of the systems at larger densities is even slower. During the period of $1 \cdot 10^7$ steps we do not observe any substantial changes in the structure. However, the snapshots of both $\rho = 0.6$ and $\rho = 1.2$ resemble the initial period of the system splitting into numerous spherical clusters. From this observation we can conclude that the equilibrium state at densities $\rho = 1.2$ and 1.4 should rather be a state of circular clusters, than a ribbon. We can suppose that the number of vortices in circular clusters at these densities exceeds the number of vortices in a small system (3200) which leads to the formation of a ribbon-like cluster. For instance, at density $\rho = 1.2$ the system has already split into 8 clusters which means 6250 vortices in a cluster on average.

One should be careful to ensure that the system has reached a state of equilibrium. For instance, the stripe and labyrinth phases of Ref. [27] (see Fig. 7 of this reference) look qualitatively similar to the phases of Fig. 12. Due to this we suppose that these phases are
FIG. 12: Snapshot of large 2d systems at $\rho = 0.2$ (top), $\rho = 0.6$ (middle) and $\rho = 1.2$ (bottom).

FIG. 12: Snapshot of large 2d systems at $\rho = 0.2$ (top), $\rho = 0.6$ (middle) and $\rho = 1.2$ (bottom).
really not completely equilibrated and finally they will change to the cluster phase.

C. Cluster phase thermal stability

Finally we estimated the thermal stability of the cluster phase in three system types. In order to do this we increased the temperature at two different densities: \( \rho = 0.01 \) and \( \rho = 0.2 \). Small systems of 3200 vortices were used in this part of the work. Fig. 13 shows the RDFs of 2d, q2d and coul systems at \( \rho = 0.01 \) and different temperatures. One can see that at low temperature all systems demonstrate an extremely high first peak, which corresponds to the distance between particles in the clusters. When the temperature increases the peak becomes smaller and finally the RDFs become almost flat which can be considered as complete disappearance of ordering in the system. Moreover, at some temperature the peak height drops dramatically. This temperature can be considered as the "melting" temperature of clusters. This "melting" temperature can be estimated as \( T_{2d} = 0.008 \) for the 2d system, \( T_{q2d} = 0.06 \) for the q2d and \( T_{coul} = 0.04 \) for the coul one.

The same procedure was applied to the systems at density \( \rho = 0.2 \). The RDFs of 2d, q2d and coul systems at this density are given in Fig. 14. In this case 2d and q2d consist of a single cluster, while the coul system is in the triangular crystal phase. One can see that the melting temperatures of the 2d and q2d systems are extremely high compared with the coul one: \( T_{2d} = 0.4 \), \( T_{q2d} = 0.3 \) and \( T_{coul} = 0.01 \). Therefore, taking into account the long range interactions in the system dramatically changes not only its structure, but also thermodynamical properties.

III. CONCLUSIONS

In the present paper we investigated a system of vortices in a thin superconductive film interacting via multiscale potential. The following results were obtained.

1. In contrast with Refs. \[38-40\] we took into account long-range interactions in the system (Eq. (2)). In order to do this we introduced a coulombic term (by adding and subtracting) into the interaction potential. In order to perform calculations with model (2) two changes needed doing. Firstly we had to substitute the 2d system with a q2d one, i.e. the system became 3d, but the vortices were stuck to the plane by the LJ walls. Secondly,
FIG. 13: Temperature dependence of the RDF of (a) 2d, (b) q2d and (c) coul systems at density $\rho = 0.01$.

to evaluate the long-range forces in the coul system we employed the PPPM method \[41\].
We found that a change from a 2d to a q2d system did not affect the result, while the behavior of the coul system was entirely different from both 2d and q2d ones. Based on this we concluded that the changes in the system behavior were induced by the long-range interactions.
FIG. 14: Temperature dependence of the RDF of (a) 2d, (b) q2d and (c) coul systems at density $\rho = 0.2$.

2. We found that taking into account the long-range interaction in the system led to a substantial change in its phase diagram. In the 2d and q2d systems a phase of circular clusters existed with densities ranging from the lowest up to $\rho = 1.4$. A liquid phase appeared at higher densities. In the coul system the phase diagram was richer. The phase of circular clusters was observed at densities $\rho = 0.01$ and $\rho = 0.05$. At $\rho = 0.1$ a percolating cluster
was observed (Nazca phase). At $\rho = 0.12$ a phase with holes (a cheese or anti-clump phase) took place. At densities from $\rho = 0.14$ up to $\rho = 1.0$ the triangular crystal was stable. At higher densities the system was in a liquid state. Comparing the radial distribution functions of 2d, q2d and coul systems we saw that taking into account long-range interactions led to a substantial change in the characteristic length parameters in the system: the distance to the first RDF peak was more than four times greater than in the 2d and q2d systems.

3. We studied the finite size effects on the ribbon phase at $\rho = 1.2$ and 1.4. We found that large systems formed the phase of circular crystals while small ones - the ribbon phase. We concluded that ribbons were formed when system size was less than the average size of a circular cluster.

4. We investigated the thermal stability of 2d, q2d and coul systems at two densities $\rho = 0.01$ and $\rho = 0.2$. At the former density all systems demonstrated a phase of circular clusters. At the latter one the 2d and q2d systems were still in the phase of circular clusters while the coul system formed a triangular crystal. We found that upon heating, the internal structure of clusters broke first. On further heating the clusters spread and the system became isotropic. We also found that the melting temperature of the 2d system at $\rho = 0.2$ was 40 times higher than the melting temperature of the coul system triangular phase at the same density.

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