Fermion Dark Matter with Scalar Triplet at Direct and Collider Searches

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Abstract

Fermion dark matter (DM) as an admixture of additional singlet and doublet vector like fermions provides an attractive and allowed framework by relic density and direct search constraints within TeV scale, although limited by its discovery potential at the Large Hadron Collider (LHC). An extension of the model with scalar triplet can yield neutrino masses and provide some cushion to the direct search constraint of the DM through pseudo-Dirac mass splitting. This in turn, allow the model to live in a larger region of the parameter space and open the door for detection at LHC, even if slightly. The model however can see an early discovery at International Linear Collider (ILC) without too much of fine-tuning. The complementarity of LHC, ILC and direct search prospect of this framework is studied in this paper.

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I. INTRODUCTION

The existence of dark matter (DM) on a larger scale (> a few kpc) is irrefutably shown by many evidences, such as galaxy rotation curve, gravitational lensing, existence of large scale structure of the Universe, cosmic microwave background etc (See for a review \[1-4\]). In fact, the satellite borne experiments, such as WMAP \[5\] and PLANCK \[6\], which study the temperature fluctuations in the cosmic microwave background, precisely measure the current relic density of DM in terms of a dimensionless parameter $\Omega_{DM}h^2 = 0.1199 \pm 0.0027$, where $\Omega_{DM} = \rho_{DM}/\rho_c$; $\rho_c$ being the critical density of the Universe and $h \approx 0.73$ is a parameter which defines the current Hubble scale of expansion $H_0 = 100h$ km/s/Mpc. However, the above mentioned evidences are based on gravitational interaction of DM and pose a challenge for particle physicists to probe it on an earth-based laboratory where the DM density is extremely low in comparison to baryonic matter. Of many possibilities, a weakly interacting massive particle (WIMP) \[1, 7\] is an elusive candidate for DM \[1\]. Due to the additional weak interaction property, WIMPs can interact with the standard model (SM) particles at a short distance and can thermalise in the early Universe at a temperature above its mass scale. As the Universe expands and cools down, the WIMP density freezes out at a temperature below its mass scale. In fact, the freeze-out density of WIMP matches to a good accuracy with the experimental value of relic density obtained by PLANCK.

The weak interaction property of WIMP DM is currently under investigation at direct search experiments such as LUX \[10\], PANDA \[11\], XENON1T \[12\] as well as collider search experiments such as \[13, 14\].

At present the SM of particle physics is the best theory to describe the fundamental particles and their interactions in nature. After the Higgs discovery, the particle spectrum of the SM is almost complete. However, the SM does not possess a candidate that can mimic the nature of DM inferred from astrophysical observations. Moreover, the SM does not explain the sub-eV masses of the active left-handed neutrinos which is required to explain observed solar and atmospheric oscillation phenomena \[15\]. Therefore, it is crucial to explore physics beyond the SM to incorporate at least non-zero masses of active neutrinos as well as dark matter content of the Universe. It is quite possible that the origin of DM is completely different from neutrino mass. However, it is always attractive to find a simultaneous solution for non-zero neutrino mass and dark matter in a single platform with a minimal extension of the SM \[16, 17\].

Till date, the only precisely measured quantity related to DM known to us is its relic density. \[1\] The other possible candidates for DM may also come from feebly interacting massive particle (FIMP) \[8\], or strongly interacting massive particle (SIMP) \[9\] with limited experimental probe.
The microscopic nature of DM is hitherto not known. Amongst many possibilities to accommodate DM in an extension of SM, a simple possibility is to extend the SM with two vector-like fermions: $\chi^0(1,1,0)$ and $\psi(1,2,−1)$, where the numbers inside the parentheses are the quantum numbers under the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. The lightest component in the mixture of the neutral component of the doublet $\psi$ and the singlet $\chi^0$ gives rise to a viable DM candidate. The stability of the lightest component can be ensured by an added $Z_2$ symmetry. The singlet-doublet mixing, defined by $\sin \theta$, plays an important role in probing the DM at direct and collider search experiments. A large singlet-doublet mixing ($\sin \theta > 0.1$) introduces a larger doublet component and hence strongly constrained by the $Z$-mediated DM-nucleon scattering at direct search experiments, while small mixing ($\sin \theta < 10^{-5}$) leads to over production of DM after big bang nucleosynthesis (BBN) by the decay of the next-to-lightest-stable particle (NLSP) $\psi^\pm$, the charged component of doublet $\psi$. Therefore, the singlet-doublet mixing in a range: $10^{-5} < \sin \theta < 0.05$ is appropriate to give rise to correct relic density of the DM while being compatible with the latest bound from direct search experiments such as. It is important to note that due to the small mixing, the annihilation cross-section of the DM is not enough to acquire correct relic density, which requires contribution from co-annihilation with NLSP resulting to a small mass splitting between NLSP and DM. The collider search of such a framework is therefore narrowed down to only a displaced vertex signature of the NLSP: $\psi^\pm$.

In this paper we study the detector accessibility of the singlet-doublet DM in presence of a scalar triplet $\Delta(1,3,2)$, where the quantum numbers are with respect to the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. We demand that the scalar triplet should not acquire any explicit vacuum expectation value (vev) as in the case of type-II seesaw. However, after the electroweak phase transition the $\Delta$ can acquire an induced vev of sub-GeV order in order to be compatible with precision electroweak data $\rho \simeq 1$ in the SM. As a result the symmetrical coupling of $\Delta$ with the SM lepton doublet $L$ can give rise to sub-eV Majorana masses for the active neutrinos. Moreover, we show that the scalar triplet widens up the allowed parameter space through pseudo-Dirac splitting of the DM, which makes the direct search through inelastic $Z$ mediation harder. Aided by that, the model can acquire correct density and still obey direct search constraints for larger singlet doublet mixing as well as with larger mass splitting between NLSP and DM. This can yield leptonic signature excess through hadronically quiet opposite sign dilepton (OSD) at LHC. The model also has the advantage of searching for the NLSP $\psi^\pm$ decaying to DM through the same OSD channel at the ILC. The Complementarity of the discovery potential of the model at the LHC and the ILC, in comparison to that of direct search, is analyzed in detail in this paper.
The paper is organized as follows: in Sec. II, we discuss the important aspects of the model. Sec. III deals with the constraints on the model parameters. Then we discuss the DM phenomenology in Sec. IV where we demonstrate the model parameter space compatible with the observed relic density and latest direct search experiments. Sec. V is then devoted to find relevant collider signatures. In section VI, we discuss the Complementarity of the discovery potential of the model at the LHC and the ILC while being compatible with DM constraints. Finally we conclude in Sec. VII.

II. THE MODEL

A. Fields and interactions

We extend the Standard Model (SM) by introducing two vector like fermions (VLF): one singlet ($\chi^0$) and a doublet $\psi$. In addition to that we introduce a scalar triplet ($\Delta$) with hypercharge $Y = 2$. A discrete $Z_2$ symmetry is imposed on top of the SM gauge symmetry, under which the VLFs are odd, while other fields, including $\Delta$, are even to stabilize the DM from decay. The charges of the new particles as well as that of the SM Higgs under $SU(3)_c \times SU(2) \times U(1)_Y \times Z_2$ are given in Table I. The Lagrangian for this model is given as:

$$L = L_{SM} + L_f + L_s + L_{yuk},$$  \hspace{1cm} (1)$$

where $L_f$ is the Lagrangian for the VLFs, $L_s$ involves the SM doublet and the additional triplet scalar, and $L_{yuk}$ contains the Yukawa interaction terms. The interaction Lagrangian for the VLFs is given by [19, 20]:

$$L_f = \bar{\psi} \not{D} \psi + \bar{\chi}^0 \not{D} \chi^0 - M_{\psi} \bar{\psi} \psi - M_{\chi} \bar{\chi}^0 \chi^0,$$  \hspace{1cm} (2)$$

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
Particles & $SU(3)_c$ & $SU(2)$ & $U(1)_Y$ & $Z_2$
\hline
$\psi^T : (\psi^0, \psi^-)$ & 1 & 2 & -1 & -1
\hline
$\chi^0$ & 1 & 1 & 0 & -1
\hline
$\Delta$ & 1 & 3 & 2 & +1
\hline
$H$ & 1 & 2 & 1 & +1
\hline
\end{tabular}
\caption{Relevant particle content of the model and their charges under SM $\times Z_2$.}
\end{table}
where $D_\mu$ is the covariant derivative under $SU(2) \times U(1)$ and is given by:

$$D_\mu \psi = \partial_\mu \psi - ig \frac{\sigma^a}{2} W^a_\mu \psi + i \frac{g'}{2} B_\mu \psi,$$

(3)

where $g$ and $g'$ are the gauge couplings corresponding to $SU(2)$ and $U(1)_Y$ and $a = 1, 2, 3$, for the generators of $SU(2)$. $W_\mu$ and $B_\mu$ are the gauge bosons corresponding to SM $SU(2)$ and $U(1)_Y$ gauge groups. Lagrangian of the scalar sector involving SM Higgs doublet $(H)$ and the additional scalar triplet $(\Delta)$ can be written as [24]:

$$L_s = (D^\mu H)^\dagger (D_\mu H) + Tr \left[ (D^\mu \Delta)^\dagger (D_\mu \Delta) \right] - V(H, \Delta).$$

(4)

The covariant derivatives of the scalars are:

$$D_\mu H = \partial_\mu H - ig \frac{\sigma^a}{2} W^a_\mu H - i \frac{g'}{2} Y \Delta B_\mu H,$n

$$D_\mu \Delta = \partial_\mu \Delta - i g \left[ \frac{\sigma^a}{2} W^a_\mu , \Delta \right] - i \frac{g'}{2} Y \Delta B_\mu \Delta.$$

(5)

$\Delta$ is written in the adjoint representation of $SU(2)$ as follows:

$$\Delta = \begin{pmatrix} \Delta^+ / \sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+ / \sqrt{2} \end{pmatrix}.$$

(6)

The most general scalar potential for this model with scalar triplet $(\Delta)$ of hypercharge $Y = 2$ can be written as [24]:

$$V(H, \Delta) = -\mu^2 H^\dagger H + \lambda \left( \frac{1}{4} (H^\dagger H)^2 + \mu^2 Tr \left( \Delta^\dagger \Delta \right) + \left[ \mu \left( H^\dagger \sigma^2 \Delta^\dagger H \right) + h.c. \right] \\
+ \lambda_1 \left( H^\dagger H \right) Tr \left( \Delta^\dagger \Delta \right) + \lambda_2 \left( Tr \left[ \Delta^\dagger \Delta \right] \right)^2 + \lambda_3 \left( Tr \left[ \left( \Delta^\dagger \Delta \right)^2 \right] \right) + \lambda_4 \left( H^\dagger \Delta \Delta^\dagger H \right).$$

(7)

Finally, the Yukawa interaction is given by [19]:

$$-L_{yuk} = \frac{1}{\sqrt{2}} \left[ (y_L)_{ij} L^c_i i \sigma^2 \Delta L_j + y_\psi \psi^c i \sigma^2 \Delta \psi + h.c. \right] + \left( Y \tilde{\psi} \tilde{H} \chi^0 + h.c. \right),$$

(8)

where in the first parenthesis we have the interaction between the triplet scalar $(\Delta)$ with the SM lepton doublet $(L)$ proportional to $y_L$ where the indices $(i, j)$ run over three families and also the Yukawa interaction with the VLF doublet $(\psi)$ proportional to $y_\psi$. In the second parenthesis we have the VLF-SM Higgs Yukawa interaction proportional to the coupling strength $Y$, where $\tilde{H} = i \sigma^2 H^\ast$. 


The electroweak symmetry breaking (EWSB) occurs when the SM Higgs acquires a VEV \((v_d)\) given by:

\[
\langle H \rangle = \begin{pmatrix} 0 \\ v_d \sqrt{2} \end{pmatrix}.
\]  

(9)

We assume that \(\Delta\) does not acquire any explicit vev. However, the vev of SM Higgs induces a small vev to the scalar triplet \(\Delta\) \((v_t)\) given by:

\[
\langle \Delta \rangle = \begin{pmatrix} 0 \\ v_t \sqrt{2} \\ 0 \end{pmatrix}.
\]  

(10)

The alignment of the two vevs may not be same. Therefore, it is convenient to define \(v = \sqrt{v_d^2 + 2v_t^2} = 246\) GeV. After minimization of the potential in Eq. 7, one arrives at the following necessary conditions [24]:

\[
\mu^2_\Delta = \frac{2\mu v_d^2 - \sqrt{2} (\lambda_1 + \lambda_4) v_d^2 v_t - 2\sqrt{2} (\lambda_2 + \lambda_3) v_t^3}{2\sqrt{2} v_t},
\]

\[
\mu^2_H = \frac{\lambda v_d^2}{4} - \sqrt{2} \mu v_t + \frac{(\lambda_1 + \lambda_4) v_t^2}{2}.
\]  

(11)

B. Mixing of the doublet and triplet scalar

In the scalar sector, masses of the doubly and singly-charged fields corresponding to the triplet can be found in [24] and are as follows:

\[
m^2_{H^\pm} = \frac{\sqrt{2} \mu v_d^2 - \lambda_4 v_d^2 v_t - 2\lambda_3 v_t^2}{2v_t}, \quad m^2_{H}\frac{\lambda v_d^2}{4} - \sqrt{2} \mu v_t + \frac{(\lambda_1 + \lambda_4) v_t^2}{2},
\]  

(12)

The neutral scalar sector consists of CP-even and CP-odd mass matrices as:

\[
M^2_{CP\ even} = \begin{pmatrix} P & Q \\ Q & R \end{pmatrix}, \quad M^2_{CP\ odd} = \begin{pmatrix} 2v_t & -v_d \\ -v_d & v_d^2/2v_t \end{pmatrix},
\]  

(13)

where

\[
P = \frac{\lambda}{2} v_d^2, \quad Q = v_d \left(-\sqrt{2} \mu (\lambda_1 + \lambda_2) v_t\right) \quad \text{and} \quad R = \frac{\sqrt{2} \mu v_t^2 + 4 (\lambda_2 + \lambda_3) v_t^3}{2v_t}.
\]  

(14)

The CP-even mass matrix is diagonalized using the orthogonal matrix:

\[
U = \begin{pmatrix} \cos \theta_0 & \sin \theta_0 \\ -\sin \theta_0 & \cos \theta_0 \end{pmatrix},
\]  

(15)
where $\theta_0$ is the mixing angle. Upon diagonalization, we end up with the following physical CP-even eigenstates:

\[ H_1 = \cos \theta_0 h + \sin \theta_0 \zeta^0, \quad H_2 = \sin \theta_0 h + \cos \theta_0 \zeta^0, \]

where $h$ and $\zeta^0$ are the real parts of $H^0$ and $\Delta^0$ fields, shifted by their respective VEVs as:

\[ H^0 = \frac{1}{\sqrt{2}} (v_d + h + i\eta_1), \quad \Delta^0 = \frac{1}{\sqrt{2}} (v_t + \zeta + i\eta_2). \]  

As it is evident from Eq. (16) under small mixing approximation, $H_1$ acts like SM Higgs, while $H_2$ behaves more like a heavy Higgs. We call $H_2$ heavy as we have not observed any such neutral scalar in experiments yet and is therefore limited by a lower mass limit as we discuss next in the constraints section. The mixing angle in the CP-even scalar sector is given by:

\[ \tan 2\theta_0 = \frac{2Q}{P - R}. \]

The CP-odd mass matrix, on diagonalization, gives rise to a massive physical pseudoscalar ($A_0$) with mass:

\[ m^2_{A_0} = \frac{\mu (v_d^2 + 4v_t^2)}{\sqrt{2}v_t}, \]

and another massless Goldstone boson. Therefore, after EWSB, the scalar spectrum contains seven massive physical Higgs bosons: two doubly charged ($H^{\pm \pm}$), two singly charged ($H^{\pm}$), two CP-even neutral Higgs ($H_1, H_2$) and a CP-odd Higgs ($A_0$). All the couplings, which can be casted in terms of the physical masses appearing in the scalar potential are listed in A 2.

C. Mixing of the VLFs

The neutral components of the doublet ($\psi^0$) and singlet ($\chi^0$) mix after EWSB thanks to the Yukawa interaction (Eq. 8). The mass matrix can be diagonalized in the usual way using orthogonal rotation matrix to obtain the masses in the physical basis ($\psi_1, \psi_2^T$):

\[
\begin{pmatrix}
M_{\psi_1} & 0 \\
0 & M_{\psi_2}
\end{pmatrix}
= \mathcal{U}^T
\begin{pmatrix}
M_{\psi} & m \\
m & M_{X}
\end{pmatrix}
\mathcal{U},
\]
where the non-diagonal mass term is obtained by $m = Y v_d / \sqrt{2}$, from Eq. 8 and the rotation matrix is given by $U = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$. The mixing angle can be related to the mass terms as:

$\tan 2\theta = \frac{2m}{M_\psi - M_\chi}$.

(21)

Therefore, the physical eigenstates (in mass basis) are the linear superposition of the neutral weak eigenstates and are given in terms of the mixing angle:

$\psi_1 = \cos \theta \chi^0 + \sin \theta \psi^0$, $\psi_2 = -\sin \theta \chi^0 + \cos \theta \psi^0$.

(22)

The lightest electromagnetic charge neutral $Z_2$ odd particle is a viable DM candidate of this model and we choose it to be $\psi_1$. The charged component of the VLF doublet $\psi^\pm$ acquires a mass as (in the small mixing limit):

$M_{\psi^\pm} = M_{\psi_1} \sin^2 \theta + M_{\psi_2} \cos^2 \theta \approx M_{\psi_2}$.

(23)

From Eq. 21 we see that the VLF Yukawa is related to the mass difference between two physical eigenstates and is no more an independent parameter:

$Y = \frac{(M_{\psi_2} - M_{\psi_1}) \sin 2\theta}{\sqrt{2}v_d} = \frac{\Delta M \sin 2\theta}{\sqrt{2}v_d}$.

(24)

Therefore, to summarize the model section, we see that the model provides with a fermion DM ($\psi_1$) which is an admixture of the doublet and singlet VLFS, with additional charged and neutral heavy fermions which all have Yukawa and gauge interactions with SM. On the other hand, the scalar sector is more rich with the presence of additional triplet which not only provides additional charged and neutral heavy scalar fields but also, have interactions to the dark sector through the Yukawa coupling. The model has several independent parameters and they are as follows:

$\{ M_{\psi_1}, \Delta M, \sin \theta, y_L, y_\psi, m_{H_2}, m_A, m_{H^\pm}, m_{H^{\pm \pm}}, \sin \theta_0 \}$

(25)

We vary some of these relevant parameters to find relic density and direct search allowed parameter space of the model to proceed further for discovery potential of the framework at collider.

III. CONSTRAINTS ON MODEL PARAMETERS

In this section we will discuss the possible constraints appearing on the parameters of this model from various theoretical and experimental bounds.
Stability

In order the potential to be bounded from below, the quartic couplings appearing in the potential must satisfy the following co-positivity condition $[24, 25]$:

$$
\lambda > 0, \quad \lambda_2 + \lambda_3 > 0, \quad \frac{\lambda_2 + \lambda_3}{2} > 0
$$

$$
\lambda_1 + \sqrt{\lambda (\lambda_2 + \lambda_3)} > 0, \quad \frac{\lambda_1 + \sqrt{\lambda (\lambda_2 + \lambda_3)}}{2} > 0
$$

$$
(\lambda_1 + \lambda_4) + \sqrt{\lambda (\lambda_2 + \lambda_3)} > 0, \quad \frac{(\lambda_1 + \lambda_4) + \sqrt{\lambda (\lambda_2 + \lambda_3)}}{2} > 0
$$

(26)

Perturbativity

The quartic couplings ($\lambda_i$) and the Yukawa couplings appearing in the theory need to satisfy the following conditions in order to remain within perturbative limit:

$$
|\lambda_i| < 4\pi, \quad |y_\psi| < \sqrt{4\pi}, \quad |Y| < \sqrt{4\pi},
$$

(27)

where $\lambda_i = \lambda, \lambda_1, \lambda_2, \lambda_3, \lambda_4$.

Electroweak precision observables (EWPO)

$T$-parameter puts the strongest bound on the mass splitting between $m_{H^{\pm\pm}}$ and $m_{H^{\pm}}$, requiring: $|m_{H^{\pm\pm}} - m_{H^{\pm}}| \lesssim 50$ GeV $[26]$. Here we have assumed a conservative mass difference of 10 GeV.

Experimental bounds

Since the addition of scalar triplet can modify the $\rho$-parameter, hence a bound on the triplet Higgs VEV can appear from the measurement of the $\rho$ parameter $\rho = 1.0008^{+0.0017}_{-0.0010}$ $[15]$. Theoretically this can be expressed as:

$$
\rho \simeq 1 - \frac{2v_t^2}{v_d^2} = 1 + \delta \rho,
$$

(28)

which further translates into: $v_t \leq 3$ GeV assuming $v = \sqrt{v_d^2 + 2v_t^2} = 246$ GeV, which enters into the expression for the known SM gauge boson masses. For a small triplet VEV $v_t \lesssim 10^{-4}$ GeV,
stringent constraint on \( m_{H^\pm} \) has been placed by CMS searches: \( m_{H^\pm} > 820 \) GeV at 95 \% C.L. \[27\] and also by ATLAS searches: \( m_{H^\pm} > 870 \) GeV at 95 \% C.L. \[28\]. For \( v_t \lesssim 10^{-4} \) GeV, direct search bound from LHC also constrains other non-standard Higgs masses: \( m_{H^+} > 365 \) GeV and \( m_{H_2,A_0} > 150 \) GeV \[29\]. For a larger triplet VEV, however, these constraints are significantly loosened. In our analysis we have kept \( v_t = 0.1 \) GeV, where all these bounds can be overlooked \[30\]. We have still maintained a particular mass hierarchy amongst different components of the triplet:

\[
m_{H^\pm} > m_{H^\pm} > m_{H_2,A_0},
\]

which is dubbed as “Negative scenario” \[26\]. The mixing between the CP-even scalar states is also constrained from Higgs decay measurement. As obtained in \[19\], \( \sin \theta \lesssim 0.05 \) is consistent with experimental data of \( H_1 \to WW^* \) with \( m_{H_1} = 125 \) GeV.

**Neutrino mass constraint**

Light neutrino mass is generated due to the coupling of the SM leptons with the scalar triplet through Yukawa interaction. As the triplet gets a non-zero VEV, one can write from Eq. 8 \[31\]:

\[
(m_\nu)_{ij} = \frac{1}{2} (y_L)_{ij} \langle \Delta \rangle \simeq (y_L)_{ij} \frac{\mu v_t^2}{2 \sqrt{2} \mu_{\Delta}},
\]

(29)

where \( \{i,j\} = \{1,2,3\} \) are the family indices. We can then generate small neutrino masses through a small value of triplet VEV, i.e. by having a large triplet scalar mass through type II seesaw. Interestingly, the triplet scalar also interacts with the VLFs via Yukawa coupling \( y_\psi \) as described in Eq. 8. Thus, the VEV of \( \Delta \) induces a Majorana mass term \( (m) \) for the VLFs on top of the Dirac mass term as follows:

\[
m = \frac{1}{2} y_\psi \sin^2 \theta \langle \Delta \rangle.
\]

(30)

If we trade \( \langle \Delta \rangle \) from Eq. 29 then from Eq. 30 we obtain the following relation between light neutrino mass and Majorana mass term for the DM:

\[
(m_\nu)_{ij} = \left( \frac{(y_L)_{\alpha\beta}}{y_\psi \sin^2 \theta} \right) m.
\]

(31)
Now, due to the introduction of the Majorana mass, the Dirac state $\psi^0$ splits into two pseudo-Dirac states with a mass difference $\delta = 2m$. This plays a very important role in direct search of the DM, which we shall explore further in subsec. IV B 1. We shall show, in order to avoid $Z$-mediated direct detection of the DM, $\delta \gtrsim O(100) \text{ keV}$. Therefore, if we consider, the light neutrino mass $\sim O(0.1 \text{ eV})$ and the Majorana mass $\sim O(100 \text{ keV})$ to forbid $Z$-mediation, then from Eq. 31 we immediately get:

$$\mathcal{R} = \left( \frac{(y_L)_{\alpha \beta}}{y_0 \sin^2 \theta} \right) \lesssim 10^{-6}. \quad (32)$$

This shows that the coupling of the scalar triplet to the SM sector is highly suppressed compared to the DM sector. Although we have chosen $y_0 = 1$ for our analysis in order to have contribution from the triplet, but the constraint from Eq. 32 has also been followed in order to ensure that the model also addresses correct neutrino mass. It is important to note that unlike the usual type-II seesaw scenario, where the correct neutrino mass predicts very heavy triplet scalars beyond any experimental reach, the presence of VLFs alter the situation significantly by allowing the triplet scalar within experimental search while addressing correct light neutrino masses.

**Relic abundance constraint**

The PLANCK-observed relic abundance puts a stringent bound on the DM parameter space as it suggests, for CDM: $\Omega_{DM} h^2 = 0.1199 \pm 0.0027$ [32]. the effect of this constraint on the parameter space of the model will be explored in detail in our analysis.

**Invisible decay constraints**

When the DM mass is less than half of Higgs or $Z$ Boson mass, they can decay to a pair of the VLF DM ($\psi_1$). Higgs and $Z$ invisible decays are however well constrained at the LHC [15, 33], which therefore constrains our DM model in such a mass limit. Both Higgs and $Z$ invisible decay to DM is proportional to VLF mixing angle $\sin \theta$ (These have been explicitly calculated and tabulated in A 1). We will show later that DM direct search constraint limits the mixing to small $\sin \theta$ regions which therefore naturally evade the invisible decay width limits.
IV. DARK MATTER PHENOMENOLOGY

As mentioned earlier, $\psi_1$ is the DM candidate in this model and in the following subsections we shall analyze the parameter space allowed by observed relic abundance of DM and also from direct detection bounds. Relic density and direct search outcome of the VLF DM as an admixture of singlet-doublet has already been studied elaborately before [18]. The case in presence of scalar triplet has also been studied briefly [19]. We would therefore elaborate on the effect of scalar triplet in the DM scenario.

A. Relic abundance of DM

Relic abundance of $\psi_1$ DM is determined by its annihilation to SM particles and also to scalar triplet, if the DM is heavier than the triplet. Such processes are mediated by SM Higgs, gauge bosons and scalar triplet. As the dark sector has charged fermions ($\psi^{\pm}$) and a heavy neutral fermion ($\psi_2$), the freeze-out of the DM will also be affected by the co-annihilation of the additional dark sector particles. This important feature makes this model survive the strong direct search limits, as we will demonstrate. All the Feynman graphs for freeze-out are shown in A 3. Relic density can then be calculated by:

$$\frac{dn}{dt} + 3Hn = -\langle \sigma v \rangle_{eff} \left( n^2 - n_{eq}^2 \right),$$

(33)

where

$$\langle \sigma v \rangle_{eff} = \frac{g_1^2}{g_{eff}^2} \langle \sigma v \rangle_{\psi_1 \psi_1} + \frac{g_1 g_2}{g_{eff}^2} \langle \sigma v \rangle_{\psi_1 \psi_2} \left( 1 + \frac{\Delta M}{M_{\psi_1}} \right)^{\frac{3}{2}} e^{-2\Delta M \frac{M_{\psi_1}}{M_{\psi_1}}} +$$

$$+ \frac{g_1 g_1}{g_{eff}^2} \langle \sigma v \rangle_{\psi_1 \psi^-} \left( 1 + \frac{\Delta M}{M_{\psi_1}} \right)^{\frac{3}{2}} e^{-2\Delta M \frac{M_{\psi_1}}{M_{\psi_1}}} +$$

$$+ \frac{g_2 g_1}{g_{eff}^2} \langle \sigma v \rangle_{\psi_2 \psi^-} \left( 1 + \frac{\Delta M}{M_{\psi_1}} \right)^{\frac{3}{2}} e^{-2\Delta M \frac{M_{\psi_1}}{M_{\psi_1}}} +$$

$$+ \frac{g_2^2}{g_{eff}^2} \langle \sigma v \rangle_{\psi_2 \psi_2} \left( 1 + \frac{\Delta M}{M_{\psi_1}} \right)^{\frac{3}{2}} e^{-2\Delta M \frac{M_{\psi_1}}{M_{\psi_1}}} +$$

$$+ \frac{g_3^2}{g_{eff}^2} \langle \sigma v \rangle_{\psi^+ \psi^-} \left( 1 + \frac{\Delta M}{M_{\psi_1}} \right)^{\frac{3}{2}} e^{-2\Delta M \frac{M_{\psi_1}}{M_{\psi_1}}},$$

(34)

with $n = n_{\psi_1} + n_{\psi_2} + n_{\psi^\pm}$. In above equation, $g_{eff}$ is defined as effective degrees of freedom, given by:

$$g_{eff} = g_1 + g_2 \left( 1 + \frac{\Delta M}{M_{\psi_1}} \right)^{\frac{3}{2}} e^{-\Delta M \frac{M_{\psi_1}}{M_{\psi_1}}} + g_3 \left( 1 + \frac{\Delta M}{M_{\psi_1}} \right)^{\frac{3}{2}} e^{-\Delta M \frac{M_{\psi_1}}{M_{\psi_1}}}.$$
where $g_1$, $g_2$ and $g_3$ are the degrees of freedom of $\psi_1$, $\psi_2$ and $\psi^-$ respectively and $x = x_f = \frac{M_{\psi_1}}{T_f}$, where $T_f$ is the freeze out temperature. For the numerical analysis we implemented the model in LanHEP \cite{34} and the outputs are then fed into MicrOmegas \cite{35} to obtain relic density.

In the top panel of Fig. 1 we have shown how the relic abundance of the DM varies with its mass for some chosen singlet-doublet VLF mixings. In the LHS of the top panel, $\Delta M$ is fixed at 10 GeV, while in the RHS it is kept fixed at a larger value 500 GeV. First of all we see three different kinds of resonance drops: one at half of the $Z$ mass $\sim 45$ GeV, the second at half of the Higgs mass $\sim 62.5$ GeV and the third at the half of the triplet scalar mass $\sim 150$ GeV (the triplet scalar masses are kept fixed around $\sim 300$ GeV). The first resonance is prominent, the second one is mild, while the third one is only visible for smaller $\sin \theta$ and large $\Delta M$ (right hand side of the top panel). Finally at around 300 GeV, a new annihilation channel to the triplet scalar opens up and correspondingly we observe a drop in relic density. Importantly, for small $\Delta M$, co-annihilation plays an important role. This can be seen on the top left panel, where the relic density drops, particularly for small $\sin \theta$, while for large $\Delta M$ such effect is subdominant. With the increase in DM mass, the relic density finally increases suggesting decrease in annihilation cross section due to unitarity. Note that the relic density decreases, *i.e.* the annihilation cross-section rises with larger $\sin \theta$ (for a fixed $\Delta M$) due to larger gauge ($Z$) mediated contribution. We have kept $y_\psi = 1, v_t = 0.1$ GeV for plots in the top panel, while for all the plots in Fig. 1 other physical masses are kept fixed at: $m_{H^{\pm \pm}} = 310$ GeV, $m_{H^{\pm}} = 300$ GeV and $m_{A,H_2} = 280$ GeV. In the middle panel of Fig. 1 we have illustrated how the relic abundance behaves with the triplet-VLF coupling $y_\psi$ for a fixed $\sin \theta = 0.1$ and $\Delta M$ (5 GeV in the left panel and 100 GeV in the right panel). The effect of $y_\psi$ is only observed in the annihilation to triplet final state (i.e. for DM mass $> \sim$ triplet mass which is kept at 300 GeV). As we increase $y_\psi$, more annihilation to triplet state is expected, which causes the relic density to further decrease. Again the effect of co-annihilation is apparent for small $\Delta M$ in the left panel where relic density drops due to such effects, which, for large $\Delta M$ is not visible in the right hand panel. Lastly, we show the effect of triplet VEV $v_t$ as a function of DM mass in the bottom panel of Fig. 1 for two different choices of $\Delta M$. Again, the effect can be realised for DM annihilation to triplet final states and therefore lies in the region where DM mass $\gtrsim$ triplet mass. As the triplet final state (charged or neutral) diagrams are proportional to $(y_\psi/v_t)^2$ (see A 2), for a fixed $y_\psi = 1$, increasing $v_t$ reduces the annihilation cross-section, resulting in over-abundance.

Now, once we have identified the important physics aspects of the variation of relic abundance with different parameters, we are in a position to find the relic density allowed parameter space. The independent DM parameters that we vary for this model are:
FIG. 1. **Top Left:** Variation of relic abundance of $\psi_1$ with its mass $M_{\psi_1}$ for different singlet-doublet mixing: $\sin \theta = 0.05$ (blue), $\sin \theta = 0.1$ (orange) and $\sin \theta = 0.5$ (green) keeping $\Delta M = 10$ GeV. **Top Right:** Same with $\Delta M = 500$ GeV. In both cases $y_{\psi} = 1.0$ and $v_t = 0.1$ GeV. **Middle left:** Variation of relic abundance with DM mass for three different choices of the Yukawa $y_{\psi} : \{0.01, 0.1, 1.0\}$ in red, green and blue respectively for $\Delta M = 5$ GeV and $\sin \theta = 0.1$. **Middle right:** Same with $\Delta M = 100$ GeV and $\sin \theta = 0.1$. **Bottom Left:** Variation of relic abundance of the DM with DM mass for different choices of the VLF-triplet Yukawa coupling $y_{\psi}$ with the triplet VEV $v_t = 0.1$ GeV. **Bottom Right:** Same for three different values of the triplet VEV for $y_{\psi} = 1.0$. In each case the black dashed line shows the right order of observed relic abundance.
\{M_{\psi_1}, \Delta M, \sin \theta\}, \quad (36)

while the effects of triplet scalar parameters like \(M_\Delta, y_\psi, v_t\) are also important, which we have kept at fixed values. We have scanned the relic density allowed parameter space in the following region:

\[M_{\psi_1} : \{10 - 1000\} \text{ GeV, } \Delta M = \{1 - 1000\} \text{ GeV, } \sin \theta = \{0.01 - 0.5\}.\] \quad (37)

We would like to remind once more that, other parameters are kept fixed throughout the scan at the following values:

\[y_\psi = 1.0, v_t = 0.1 \text{ GeV, } m_{H^\pm \pm} = 310 \text{ GeV, } m_{H^\pm} = 300 \text{ GeV, } m_{A,H_2} = 280 \text{ GeV},\]

which evade the constraints discussed in Sec. III.

FIG. 2. Left: Parameter space allowed by relic density in \(M_{\psi_1}-\Delta M\) plane for different choices of the singlet-doublet VLF mixing: \(\sin \theta : \{0.01 - 0.1\} \) (red), \(\sin \theta : \{0.1 - 0.2\} \) (green), \(\sin \theta : \{0.2 - 0.3\} \) (blue) and \(\sin \theta : \{0.4 - 0.5\} \) (magenta). Right: In the same plane the underabundant (green) and overabundant (red) regions are shown together with observed relic density (blue) region for \(\sin \theta = 0.2\).

LHS of Fig. 2 shows the parameter space allowed by PLANCK observed relic density for a range of \(\sin \theta\) varying: \{0.01-0.5\} (shown in different colours). Both DM mass and \(\Delta M\) have been varied up to 1 TeV for the scan. Now, the plot shows several features. In order to understand the patterns, let us choose \(\sin \theta = 0.2\) as shown in the right panel. As before, there are two resonance drops at \(M_{\psi_1} = \frac{M_{H_1}}{2}\) and \(M_{\psi_1} = \frac{M_{H_2}}{2}\) corresponding to SM Higgs and the triplet Higgs mediation.
At $M_{\psi_1} \sim 300$ GeV the relic abundance becomes independent of the choice of $\Delta M$, which is more prominent for larger $\sin \theta$. This is because of the fact that at $\sim 300$ GeV the annihilation channel to triplet Higgs opens up, providing additional freedom to choose the VLF-Higgs Yukawa coupling $Y$ to produce the correct relic abundance. As we move beyond 300 GeV, channels corresponding to final states with the charged Higgs open up, making the annihilation cross-section even larger. This causes underabundance of the DM and as a result in the range $M_{\psi_1} : \{300 - 400\}$ GeV we find no points that satisfy correct relic abundance. 400 GeV onward the annihilation cross-section again starts decreasing due to $\frac{1}{m_{DM}^2}$ suppression. As a result we again get a region with observed relic abundance. But as we move to higher DM mass, for example beyond $\sim 800$ GeV the suppression becomes even larger and results in overabundance. This feature is clear from the RHS of Fig. 2 where we have shown the under-abundant regions in green, overabundant regions in red, along with the regions that provide right relic density in blue for a fixed mixing $\sin \theta : \{0.2\}$. We also note that right relic density in the vicinity of $Z$ resonance i.e. $M_{\psi_1} = \frac{M_Z}{2}$ can only be observed for small $\sin \theta$ (red and green points in the left plot) which vanishes for large $\sin \theta \geq 0.2$ to yield under abundance. This is also clear from the right panel and can be attributed to larger gauge mediation in the large $\sin \theta$ limit.

![Graph](image)

**FIG. 3.** The figure shows the effect of triplet-VLF Yukawa coupling in $M_{\psi_1} - \Delta M$ plane satisfying relic density constraint for three different choices of $y_{\psi}$: $\{0.01, 0.1, 1.0\}$ shown in blue, red and green respectively. The triplet scalar VEV is fixed at $v_t = 0.1$ GeV.

Another noteworthy feature is in Fig. 3 where have shown how the relic density allowed parameter space changes pattern for different choices of the VLF-triplet Yukawa coupling $y_{\psi}$ (in Eq. 8) for $0.01 \leq \sin \theta \leq 0.1$. For $y_{\psi} = 0.01$, there is almost no contribution from the triplet scalar. In
that case, co-annihilation plays vital role in producing the correct relic abundance and hence one needs to resort to smaller $\Delta M$, as shown by the blue curve. For larger DM mass the curve bends down due to $1/M_{\psi_1}^2$ suppression coming from the cross-section (unitarity). As $y_\psi$ is increased to 0.1, the triplet starts playing role. This can be understood by the rise of the red and green curves at $M_{\psi_1} \sim 300$ GeV. Now, as the triplet gets into the picture, it provides enough annihilation channels and as a result co-annihilation plays a sub-dominant role here. This is again evident from the larger values of $\Delta M$ for both $y_\psi = 0.1$ and $y_\psi = 1.0$ curves. The drop in the high DM mass region is again due to unitarity.

B. Direct search of DM

In this section we shall investigate the effect of spin-independent direct search constraints on the DM parameter space. Our goal is to find how much of the parameter space, satisfied by PLANCK-observed relic density, is left after imposing the upper limit from XENON1T. The pivotal role in this regard is played by the triplet scalar. As we shall see in the following subsection, due to the presence of the triplet, the Z-mediated inelastic direct search is forbidden for $\sin \theta < \sim 0.1$ for DM mass upto 1 TeV.

1. Emergence of pseudo-Dirac states and its effect on direct search

The presence of the triplet scalar plays a decisive role in determining the fate of this model in direct search experiment as discussed in [19]. Since the VEV of the neutral component of the triplet scalar induces a Majorana mass term (as seen from Eq. 8), it splits the Dirac spinor $\psi_1$ into two pseudo-Dirac states $\psi_1^{\alpha,\beta}$ with mass difference proportional to the VLF-mixing angle and VEV of $\Delta^0$ (already mentioned in [11]):

$$\delta = 2m = y_\psi \sin^2 \theta \langle \Delta^0 \rangle.$$  (38)

Now, the Z-mediated direct detection interaction of the DM is given as:

$$\mathcal{L} \supset i \bar{\psi}_1 (\bar{\phi} - ig_\mu Z^\mu) \psi_1,$$  (39)

where $g_\mu = \frac{\sin \theta}{2 \cos \theta_w} \sin^2 \theta$, $\theta_w$ being the Weinberg angle. In presence of the pseudo-Dirac states, this interaction takes the form:
\[ \mathcal{L} \supset \bar{\psi}_1^\alpha i \gamma_\mu \psi_1^\alpha + \bar{\psi}_1^\beta i \gamma_\mu \psi_1^\beta + g_s \bar{\psi}_1^\alpha \gamma_\mu \psi_1^\beta Z^\mu. \]  

(40)

As one can notice, the \( Z \)-interaction is off-diagonal, \( i.e., \) \( Z \) is coupled to \( \psi_1^\alpha \) and \( \psi_1^\beta \), unlike the diagonal kinetic terms. This therefore induces inelastic \( Z \) mediated scattering for the fermion DM in presence of triplet. Such an inelastic scattering is kinematically allowed if [36]:

\[ \delta_{max} < \frac{\beta^2}{2} \frac{M_\psi M_N}{M_\psi + M_N}, \]  

(41)

where \( \beta_c = v_{DM} \) can be within: 220 km/s \( \lesssim \beta_c < 650 \) km/s, where the lower limit corresponds to the DM velocity in the local DM halo and the upper limit refers to the escape velocity \( (v_{esc}) \) of DM particles in the Milky Way, and \( M_N \) is the nucleus mass. Now, the present strongest bound on spin-independent direct detection cross-section comes from XENON1T, which we abide by for the available parameter space of the model. Then, using Xe nucleus mass \( M_N = 130 \) amu and following Eq. 41 we can have an upper limit on \( \delta_{max} \) as a function of DM mass, below which \( Z \)-mediated inelastic scattering is allowed. This is shown in the upper left panel of Fig. 4 where the shaded region allows such inelastic scattering. The solid and black dashed lines show the limit beyond which \( Z \)-mediated inelastic scattering is disallowed corresponding to the lower and upper limit of DM velocity \( \beta_c \). As one can see, \( Z \)-mediated cross-section is forbidden for \( \delta \gtrsim 240 \) keV for DM mass of \( \sim 1 \) TeV corresponding to the upper limit on \( \beta_c \). This constraint can be viewed also in a different way. The minimum velocity of the DM which produces a recoil energy \( E_R \) in the detector through inelastic scattering takes the form [36]:

\[ v_{min} = \sqrt{\frac{1}{2 M_N E_R} \left( \frac{M_N E_R}{\mu_r} + \delta \right)}, \]  

(42)

where \( \mu_r \) is the reduced mass of the DM-nucleus system. Eq. 42 will also yield a similar constraint on \( \delta \) (as obtained in top left figure of Fig. 4) but for a given recoil energy \( (E_R) \) specific to a detector used for the DM direct search. For \( E_R \sim 30 \) keV, the conclusions are roughly the same.

If this constraint on \( \delta \) (derived from Eq. 41) is implemented in our model, we can have a relation between the mixing \( \sin \theta \) and the triplet Yukawa \( y_\psi \) from Eq. 38. This is depicted in the top right panel of Fig. 4 where we have shown the \( Z \)-mediation forbidden region of the parameter space in
FIG. 4. **Top left:** The grey region is where inelastic scattering of the DM via Z-mediation is allowed as derived from Eq. [41]. The solid black line corresponds to DM velocity $\beta = 220$ km/s, while dashed black line corresponds to $\beta = v_{\text{esc}} \simeq 650$ km/s. **Top right:** The purple region shows the choices of $\sin \theta$ and $y_\psi$ which shall forbid the Z-mediated direct detection for $v_t = 0.1$ GeV as obtained from Eq. [38], the blue region underneath is the same for $v_t = 1$ GeV assuming $\beta = v_{\text{esc}}$. **Bottom left:** Same as top right but for a particular $v_t = 0.1$ GeV with two regions corresponding to the lower (light green region) and upper (green region) limit on DM velocity. **Bottom right:** Z-mediation allowed region in $\delta$ vs. $\sin \theta$ plane satisfying Eq. [38] for DM mass of 300 GeV, $v_t = 0.1$ GeV and $y_\psi = 1$. We use $v_{DM} \leq 650$ km/sec. 

$\sin \theta - y_\psi$ plane for two different choices of the triplet VEVs: $v_t = \{0.1, 1\}$ GeVs shown in purple and pale blue respectively. As the splitting is proportional to $v_t$, larger the $v_t$, larger is the $Z$-forbidden region. For this plot we have used a liberal limit of maximum possible DM velocity of 650 km/sec to avail the maximum splitting $\delta$. We can see from the top right figure that with $y_\psi < 1$, in order to avoid $Z$-mediated direct search, one has to choose $\sin \theta \gtrsim 0.05$ for DM mass of 1 TeV. The bound on $\sin \theta$ is even more conservative to allow $Z$ mediation ($\sin \theta \gtrsim 0.02$) for $v_t = 1$ GeV.
(shown by the pale blue region). A similar plot as in top right panel, is plotted in the bottom left panel to show the \( Z \) forbidden region for the minimum and maximum permissible DM velocities for \( v_t = 0.1 \) GeV. Lastly, in the bottom right panel of Fig. 4, we have illustrated a situation (following Eq. 38) where \( Z \)-mediated inelastic scattering is possible for a fixed DM mass of \( M_{\psi} = 300 \) GeV. If we choose \( v_{DM} \leq 650 \) km/s, this yields a bound on \( \delta \) (following top left figure) and is shown by the red solid line below which \( Z \)-mediation is possible. Once we choose a specific \( y_{\psi} = 1 \) and \( v_t = 0.1 \) GeV, a bound on \( \sin \theta \) is also obtained, and is shown by black dashed line. On the left side of this line \( Z \)-mediation is possible. If we now consider the splitting that the model can generate following Eq. 38 for the chosen values of \( y_{\psi} = 1 \) and \( v_t = 0.1 \) GeV, we obtain a specific relation between the splitting \( \delta \) to \( \sin \theta \), shown by the diagonal solid black line. To summarize, the olive coloured region allows \( Z \) mediated interaction, and the part of the black line within this can be realized in our model framework. We would however be interested to work in the parameter space where \( Z \) mediation is forbidden, which crucially alters the direct search allowed parameter space of the model in presence of scalar triplet.

2. Spin-independent direct detection constraint

From the previous section, we see that for a moderate choice of \( y_{\psi} \simeq 1 \), the \( Z \) mediated inelastic scattering for the DM will have no contribution if we choose \( \sin \theta \gtrsim 0.05 \) limit (as seen from Fig. 4). Therefore the DM particles can recoil against the nucleus, giving rise to direct search signature as shown in Fig. 5 only through Higgs (\( H_{1,2} \)) mediation. The spin-independent (SI) direct detection cross section per nucleon is given by [37]:

\[
\sigma^{SI} = \frac{1}{\pi A^2} \mu^2 |\mathcal{M}|^2, \quad (43)
\]

where \( A \) is the mass number of the target nucleus, \( \mu = \frac{M_{\psi_1} M_N}{M_{\psi_1} + M_N} \) is the DM-nucleus reduced mass and \( |\mathcal{M}| \) is the DM-nucleus amplitude, which reads:

\[
\mathcal{M} = \sum_{i=1,2} \left[ Z f^i_p + (A - Z) f^i_n \right]. \quad (44)
\]

The effective couplings in Eq. 44 are:

\[
f^i_{p,n} = \sum_{q=u,d,s} f^{p,n}_{T_q} \alpha_q \frac{m_{p,n}}{m_q} + \frac{2}{27} f^{p,n}_{T_G} \sum_{Q=c,t,b} \alpha_Q \frac{m_{p,n}}{m_Q}, \quad (45)
\]
with

$$\alpha_1^q = \frac{Y \sin 2\theta \cos \theta_0^2 m_q}{m_{H_1}^2} \frac{m_{H_1}}{v} \tag{46}$$

$$\alpha_2^q = -\frac{Y \sin 2\theta \sin \theta_0^2 m_q}{m_{H_2}^2} \frac{m_{H_2}}{v}. \tag{47}$$

FIG. 5. Feynman graph showing scattering of DM particle against the nucleus. This can be mediated both by the SM Higgs $H_1$ and the triplet Higgs $H_2$.

Different coupling strengths between the DM and the light quarks are given by [38]:

$$f_{pT_u} = 0.020 \pm 0.004, \ f_{pT_d} = 0.026 \pm 0.005, \ f_{pT_s} = 0.118 \pm 0.062, \ f_{nT_u} = 0.014 \pm 0.004, \ f_{nT_d} = 0.036 \pm 0.008, \ f_{nT_s} = 0.118 \pm 0.062. \tag{48}$$

The coupling of the DM with the gluons (through one loop graphs) in the target nuclei is taken into account by the effective form factor:

$$f_{p,nT_G} = 1 - \sum_{q=u,d,s} f_{p,nT_q}. \tag{49}$$

Upper panel of Fig. 6 shows the parameter space allowed by the spin-independent (SI) direct detection cross section in $M_{\psi_1}\sigma_{DD}$ plane. As one can see, the allowed region of parameter space that lies below the exclusion limit of present XENON1T data corresponds to $\sin \theta : \{0.015 - 0.2\}$ (shown in red and green). In the bottom left panel we have shown the net parameter space satisfied by both relic abundance and direct search. One should note here, large $\Delta M \gtrsim 100 \text{ GeV}$ can be achieved for small $\sin \theta : \{0.015 - 0.1\}$ near Higgs and $Z$ resonance region and also for $M_{\psi_1} \sim 300 \text{ GeV}$. Why such regions are available from relic density constraint, has already been
FIG. 6. Top: Relic density allowed parameter space satisfied by spin-independent direct detection in direct search plane. Different coloured regions correspond to different singlet-doublet VLF mixings: $\sin \theta = \{0.01 - 0.1\}$ in red, $\sin \theta = \{0.1 - 0.2\}$ in green, $\sin \theta = \{0.2 - 0.3\}$ in blue and $\sin \theta = \{0.4 - 0.5\}$ in magenta. The black dashed line corresponds to exclusion limit from XENON1T. Bottom left: Net parameter space left after satisfying relic density and direct detection constraints (color codes are same as that of left figure). Bottom right: Parameter space allowed by relic abundance and XENON1T exclusion limit but without the triplet scalar included.

elaborated before. The reason, that they are not forbidden by direct search can be attributed to forbidden $Z$ mediation, which is possible only when the triplet scalar is present in the model. The case of relic density and direct search allowed parameter space for the DM model without the scalar triplet is shown in the right side of bottom panel of Fig. 6. Here we can see, the maximum splitting one may achieve is $\Delta M \sim 10$ GeV with small $\sin \theta$ satisfying both relic density and direct search bounds. This serves as a crucial ingredient to discover such a model at the upcoming Large Hadron Collider (LHC).

Before moving on to the collider section, we shall choose a few benchmark points (BP) which satisfy relic density, direct detection exclusion bound and all the constraints mentioned in Sec. III.
TABLE II. Choices of the benchmark points used for collider analysis. Masses, mixings, relic density and direct search cross-sections for the DM candidate are tabulated. These are tabulated in Tab. II, where the input parameters and also relic density and direct search outcomes have been mentioned. The BPs are chosen based on different choices of $\Delta M$, where large $\Delta M$ can be probed at the LHC, while small $\Delta M$ are better suited for ILC search as we demonstrate. BP1-BP4 can therefore be probed at the LHC because of large $\Delta M$. Due to small $\Delta M$, BP5 and BP6 can be seen at very early run of ILC, while BP7 and BP8 can only be probed at ILC with $\sqrt{s} = 1$ TeV. Note that, a lower limit on pair-produced charged heavy vector-like leptons have been set by LEP: $m_L > 101.2$ GeV at 95% C.L. for $L^\pm \to \nu W$ final states \cite{39}. So, our benchmark points are safe from LEP bounds.

V. COLLIDER PHENOMENOLOGY

In this section we shall discuss the possibility of probing the model at the ongoing and future collider experiments. As we have already seen, due to the presence of the scalar triplet, large $\Delta M$ is allowed by relic abundance and direct detection bounds in the vicinity of the $Z$ and Higgs resonance. Apart from that, moderate $\Delta M$ can also be achieved near the triplet resonance and at $M_{\psi_1} \simeq M_{H_2}$. We shall see, in the following sections, large $\Delta M$ (and hence larger missing energy) is always favorable at the LHC, au contraire, ILC search is more favoured for smaller $\Delta M$ regions (missing energy peaks at small values). Thus, due to the presence of the triplet scalar, this model provides a scope of being probed both at LHC and ILC searches, which correspond to complementary $\Delta M$ regions of the parameter space. As we have examined, in order to unveil this model at the collider experiments, a high luminosity is required for LHC, while the model
may show up in the early runs of ILC at a much lower luminosity. In subsection V A we have elaborated the LHC analysis with important kinematic distributions and event rates for both signal (BP1-BP4) and dominant SM backgrounds for $\sqrt{s} = 14$ TeV. In subsection V B the same is done from ILC perspective for both $\sqrt{s} = 350$ GeV, corresponding to an early ILC run and $\sqrt{s} = 1$ TeV, corresponding to future prediction.

A. Sensitivity of the signal at the LHC

The charged companion of the VLF doublets can be produced at the LHC via $Z, \gamma$ mediation. These charged particles can decay to DM ($\psi_1$) via $W^\pm$, producing missing energy in the final state. Note here, for the BPs chosen for LHC (i.e, BP(1-4)), the decay happens on-shell as $\Delta M > m_W$. The charged $W$-bosons further decays into leptons and jets, which are registered in the detector, and also to neutrinos which escape the detector and adds to missing energy. The model, in general, can give rise to three different final states:

- Hadronically quiet Opposite sign dilepton (OSD) with missing energy ($\ell^+\ell^- + E_T$).
- Single lepton, with two jets plus missing energy ($\ell^\pm + jj + E_T$).
- Four jets plus missing energy ($jjjj + E_T$).

As the hadronic final states are infested with SM background, particularly at LHC, while leptonic channels are cleaner, we shall only analyze the OSD final states with missing energy (Fig. 7).

![Diagram](image)

FIG. 7. OSD+$E_T$ final state at the LHC.
1. Object reconstruction and simulation strategy at the LHC

We have used LanHep [34] to implement the model framework and used CalcHEP [40] in order to generate the parton level events. These events then showered through PYTHIA [41] for hadronization. All events have been simulated at a center of mass energy of $\sqrt{s} = 14$ TeV, using CTEQ6l [42] as the parton distribution function. To mimic the collider environment, the leptons and jets are re-constructed using the following criteria:

- **Lepton ($l = e, \mu$):** Leptons are identified with a minimum transverse momentum $p_T > 20$ GeV and pseudorapidity $|\eta| < 2.5$. Two leptons are isolated objects if their mutual distance in the $\eta - \phi$ plane is $\Delta R = \sqrt{(|\Delta \eta|^2 + (\Delta \phi)^2) \geq 0.2}$, while the separation between a lepton and a jet has to satisfy $\Delta R \geq 0.4$.

- **Jets ($j$):** All the partons within $\Delta R = 0.4$ from the jet initiator cell are included to form the jets using the cone jet algorithm PYCELL built in PYTHIA. We demand $p_T > 20$ GeV for a clustered object to be considered as jet. Jets are isolated from unclustered objects if $\Delta R > 0.4$. Although our signal events (hadronically quiet OSD) do not carry jets, the definition of jet turns out to be important in order for the signal to be identified with zero jet veto.

- **Unclustered Objects:** All the final state objects which are neither clustered to form jets, nor identified as leptons, belong to this category. All particles with $0.5 < p_T < 20$ GeV and $|\eta| < 5$, are considered as unclustered. Again, unclustered objects do not enter into our signal definition, but is important in identifying missing energy of the event.

- **Missing Energy ($E_T$):** The transverse momentum of all the missing particles (those are not registered in the detector) can be estimated from the momentum imbalance in the transverse direction associated to the visible particles. Missing energy (MET) is thus defined as:

$$E_T = -\sqrt{(\sum_{\ell,j}p_x)^2 + (\sum_{\ell,j}p_y)^2},$$

(49)

where the sum runs over all visible objects that include the leptons, jets and the unclustered components.

- **Invariant dilepton mass ($m_{\ell\ell}$):** We can construct the invariant dilepton mass variable for two opposite sign leptons by defining:

$$m_{\ell\ell}^2 = (p_{\ell^+} + p_{\ell^-})^2.$$  

(50)
Invariant mass of OSD events, if created from a single parent, peak at the parent mass, for example, Z boson. As the signal events (Fig. 7) do not arise from a single parent particle, invariant mass cut plays a crucial role in eliminating the Z mediated SM background.

- $H_T$: $H_T$ is defined as the scalar sum of all isolated jets and lepton $p_T$'s:

$$H_T = \sum_{\ell,j} p_T.$$  \hspace{2cm} (51)

Of course, for our signal, the sum only includes the two leptons that are present in the final state.

It is very important for collider analysis to estimate the SM background that mimic the signal. All the dominant SM backgrounds have been generated in MadGraph [43] and then showered through PYTHIA.

2. Event rates and signal significance at the LHC

![Graph showing variation of production cross section $\sigma_{pp\rightarrow\psi^+\psi^-}$ at LHC with $\Delta M$ for $\sqrt{s} = 14$ TeV. DM mass is varied between $M_{\psi_1} : \{1-65\}$ GeV. Different benchmark points (BP1-BP4, see Tab. III) are also indicated in blue. BP2 and BP3 are superimposed on each other as they have almost the same production cross-section. LEP limit on charged fermion mass is also shown by the shaded region.]

We have shown the variation of production cross-section $\sigma_{pp\rightarrow\psi^+\psi^-}$ at LHC for $\sqrt{s} = 14$ TeV with $\Delta M$ for different DM masses ranging between $M_{\psi_1} : \{1-65\}$ GeV in Fig. 8. As expected, with
FIG. 9. Top: Missing energy distribution for OSD+\(\vec{E}_T\) final state for the benchmark points are shown in red. Those of the dominant SM backgrounds are also shown with different colours. Bottom: \(H_T\) distribution for the same. The simulation is done assuming LHC with \(\sqrt{s} = 14\) TeV.

larger \(\Delta M\) the cross-section for \(\psi^+\psi^-\) falls due to phase space suppression with \(M_{\psi^\pm} = M_{\psi_1} + \Delta M\). The production cross-section for the benchmark points (BP1-BP4), relevant for the LHC search are also indicated in the same plot. We see that, BP2 and BP3 fall on each other as they have almost equal production cross-section. LEP exclusion for the charged fermion is also shown by the shaded grey region (\(M_{\psi^\pm} > 101.2\) GeV).

In Fig. 9 the MET and \(H_T\) distribution for the BPs (along with the SM dominant backgrounds) are shown in top and bottom panels respectively. The cross-section for all the SM backgrounds have been calculated upto next-to-leading order using appropriate \(K\)-factors \[^{[43]}\]. Since the background dominates over the signal, we have employed \(\vec{E}_T\) and \(H_T\) cuts to distinguish the signal region from the background. For the background the only source of missing energy is the SM neutrinos, while for the signal, along with the SM neutrinos, MET also comes from the DM produced during the decay of the charged VLFs. With larger \(\Delta M\) the MET distribution gets flattened as more \(p_T\) is being carried away by the DM. This is what is seen from the MET distributions, particularly we see that BP1 with least \(\Delta M\) is almost falling on top of SM background. So, the efficiency of using an MET cut to select the signal is also the least here. It is therefore obvious that the other BPs
like BP5-BP8 (not shown in this distribution) will not be able to survive any large MET cut. $H_T$ distributions are almost similar to that of MET. We have finally employed following cuts (with zero jet veto) in order to separate the signal from the background:

- $E_T > 300$ GeV is employed to kill all the backgrounds. Although, as it can be seen from Fig. 9, $E_T > 150$ GeV is good enough to separate the signal from the background, but the $W^+W^-$ background will still persist, hence we chose a hard cut on MET.

- $H_T > 100$ GeV is used to reduce the background further, without harming the signal events.

- Invariant mass cut over the $Z$-window $|m_z - 15| < m_{ll} < |m_Z + 15|$ is required to get rid-off the $ZZ$ background to a significant extent.

Next, we would like to see the number of signal and corresponding background events using the cuts mentioned above. In Tab. III we have tabulated the number of events for the signal at a future luminosity of $\mathcal{L} = 100 \text{ fb}^{-1}$ with all the cuts incorporated. The cross-sections are also quoted in each case and a set of two different MET cuts have been illustrated to demonstrate the cut-flow. With larger MET cut the number of final state signal events get diminished as expected. The effective number of events at a particular luminosity ($\mathcal{L}$) as has been mentioned in Tab. III is obtained from the simulated events in the following way:

$$N_{\text{eff}} = \frac{\sigma_p \times n}{N} \times \mathcal{L},$$

where $\sigma_p$ is production cross-section as shown in Fig. 8, $n$ is the number of events generated out of $N$ simulated events (after putting all the cuts and showering through PYTHIA) and $\mathcal{L}$ is the luminosity, which we have considered to be $100 \text{ fb}^{-1}$.

Tab. IV enlists the number of events coming from dominant SM backgrounds after using the same set of cuts mentioned before. Events from $t\bar{t}$ and $ZZ$ can be eliminated to a significant extent by demanding zero jet veto and putting a high MET cut (along with the $m_{ll}$ cut for $ZZ$ events in particular). The hard MET cut also helps to get rid off the $W^+W^-$ background. The only background that remains (although with only one event) is that from $W^+W^−Z$. But the cuts employed also eliminate some of the signal events, making the significance is low.

The discovery potential of hadronically quiet OSD signal for different BPs are shown in Fig. 10, as a function of luminosity. We have chosen $E_T > 300$ GeV and $H_T > 100$ GeV to compute the signal significance so that the SM background is minimum. As one can see from Tab. III the
Benchmark Point | $\sigma^{\psi^+\psi^-}$ (fb) | $E_T$ (GeV) | $\sigma^{\text{OSD}}$ (fb) | $N^{\text{OSD}}_{\text{eff}}@\mathcal{L} = 100 \text{ fb}^{-1}$
---|---|---|---|---
BP1 | 218.19 | > 200 | 0.13 | 13
 | | > 300 | 0.04 | 4
BP2 | 74.80 | > 200 | 0.15 | 15
 | | > 300 | 0.04 | 4
BP3 | 71.80 | > 200 | 0.17 | 17
 | | > 300 | 0.04 | 4
BP4 | 35.93 | > 200 | 0.13 | 13
 | | > 300 | 0.03 | 3

TABLE III. Signal events with $\sqrt{s} = 14$ TeV at the LHC for luminosity $\mathcal{L} = 100 \text{ fb}^{-1}$ for the benchmark points (BP1-BP4) in Tab. II.

| Backgrounds | $\sigma_{\text{production}}$ (pb) | $E_T$ (GeV) | $\sigma^{\text{OSD}}$ | $N^{\text{OSD}}_{\text{eff}}@\mathcal{L} = 100 \text{ fb}^{-1}$
---|---|---|---|---
$t\bar{t}$ | 814.64 | > 200 | <0.81 | 0
 | | > 300 | <0.81 | 0
$W^+W^-$ | 99.98 | > 200 | 1.99 | 199
 | | > 300 | <0.49 | <1
$W^+W^-Z$ | 0.15 | > 200 | 0.04 | 4
 | | > 300 | 0.01 | 1
ZZ | 14.01 | > 200 | <0.07 | 0
 | | > 300 | <0.07 | 0

TABLE IV. Events for dominant SM backgrounds with $\sqrt{s} = 14$ TeV at the LHC for luminosity $\mathcal{L} = 100 \text{ fb}^{-1}$. The cross-sections are quoted in NLO order by multiplying with appropriate $K$-factors (see text). The number of signal events left after imposing the cuts are more or less the same for all the benchmark points. This is also reflected in Fig. 10 where we can see all the BPs reach a 5$\sigma$ discovery at a luminosity $\mathcal{L} \sim 800 \text{ fb}^{-1}$. Here we would like to remind once more, the possibility of getting a signal excess in hadronically quiet OSD channel is due to the presence of the scalar triplet, without which the model would have failed to produce any such signature at the LHC. We will later discuss the possibility of seeing a displaced vertex signal and this adds to the Complementarity of the search strategy of this model.
FIG. 10. Signal significance for different BPs, where we have used $E_{T} > 300$ GeV and $H_{T} > 100$ GeV. The solid red and dashed red lines correspond to $3\sigma$ and $5\sigma$ discovery limits respectively.

B. Sensitivity of the signal at the ILC

The VLFs can also be produced at the ILC via gauge mediation as shown in Fig. 11. The model thus can be probed at the ILC in the same $\ell^{+}\ell^{-} + E_{T}^{\gamma}$ final state as that of the LHC. However, one may note that unlike LHC, jet rich final state signal at the ILC is not disfavored due to smaller SM background contribution due to absence of QCD processes like $t\bar{t}$. Therefore, we are still left with the SM gauge boson productions to potentially mimic our signal. One can still analyse the single lepton plus jet channel or dijet channel at the ILC, but to show the complementarity of the hadronically quiet dilepton final state signature at the LHC and the ILC, we analyze this particular channel in details here. The main goal is to show sensitivity of the signal for different choices of $\Delta M$ that can be probed at the ILC, which can not be probed at the LHC. We shall demonstrate, because of smaller $\Delta M$, BP5-BP8 are suitable for ILC searches. Of the four BPs, BP5 and BP6 can be probed at the early run of ILC with $\sqrt{s} = 350$ GeV, while BP7 and BP8 need higher $\sqrt{s}$.

1! Object reconstruction and simulation strategy at the ILC

As before, we have generated the parton-level signal events in CalcHEP and showered them through PYTHIA, while the relevant background events are generated via MadGraph. Now, for event reconstruction, we have used the following criteria [44]:

- Leptons are required to have $p_T(l) > 10$ GeV where $l = \mu, e$ with pseudorapidity $|\eta| < 2.4$.

Two leptons are said to be isolated if $\Delta R \geq 0.2$, while a lepton and a jet can be identified
FIG. 11. OSD+\(E_T\) signal at the ILC.

as separate objects if \(\Delta R \geq 0.4\).

- Jets are reconstructed using the cone jet algorithm in-built in PYTHIA. Objects with \(p_T(j) > 20\) GeV and \(|\eta| < 3.0\) are considered as jets. Again, this is required so that we select events for the desired signal with zero jet veto.

Now, ILC will be providing highly polarized electron beam (\(P_{e^-} : 80\) %) and moderately polarized positron beam (\(P_{e^+} : 20\) %) \[45\]. We have used + sign for right polarization and − for left polarization. In order to minimize the SM background, we have looked into three different polarizations of the incoming beam:

- 80 \% left polarized \(e^-\) and 20\% right polarized \(e^+\) beam (\([P_{e^-}, P_{e^+}] : [-80 \%, +20 \%]\)).
- 80 \% right polarized \(e^-\) and 20\% left polarized \(e^+\) beam (\([P_{e^-}, P_{e^+}] : [+80 \%, -20 \%]\)).
- Unpolarized incoming beams (\([P_{e^-}, P_{e^+}] : [0 \%, 0 \%]\)).

2. Event rates and signal significance at the ILC

Production cross-section of the dominant SM backgrounds with different beam polarizations are tabulated in Tab. \(\text{V}\) and Tab. \(\text{VI}\) for \(\sqrt{s} = 350\) GeV and \(\sqrt{s} = 1\) TeV respectively. One can notice that all the SM background cross-sections are minimum for (\(P_{e^-}, P_{e^+})=(+80\%, -20\%\)). This is because left handed particles form a doublet under \(SU(2)\), boosting the SM gauge boson production for dominantly left polarised beams. On the other hand, right handed electrons are singlet under \(SU(2)\) and therefore, dominantly right polarized beams will suppress the SM gauge
boson production. The case of unpolarised beam falls in between the two extreme cases described here. The signal cross-section will also change similarly due to the choice of beam polarization. However, the final state fermions being vector-like, the change will only appear at the SM vertex (left vertex of Fig. 11) due to change in polarization. Therefore, the change in cross-section for the signal due to change in polarization of the electron beam will be milder. The signal $\psi^+\psi^-$ production cross-section with the polarization of the beams is tabulated in Tab. VII and Tab. VIII for $\sqrt{s} = 350$ GeV and $\sqrt{s} = 1$ TeV respectively. We have therefore chosen dominantly right polarized beams i.e. $(P_{e^-}, P_{e^+}) = (+80\%, -20\%)$ for the maximum signal sensitivity of the model at ILC.

| $P_{e^-}$ | $P_{e^+}$ | $\sigma (W^+W^-)$ (pb) | $\sigma (W^+W^- Z)$ (pb) | $\sigma (ZZ)$ (pb) |
|-----------|-----------|------------------------|--------------------------|-------------------|
| -80%      | +20%      | 24.37                  | 0.026                    | 1.08              |
| +80%      | -20%      | 1.90                   | 0.002                    | 0.49              |
| 0%        | 0%        | 11.31                  | 0.012                    | 0.67              |

TABLE V. Dominant SM background cross-sections for different polarization of the $e^-e^+$ beams at $\sqrt{s} = 350$ GeV at the ILC.

It is important to note here, all the cross-sections, irrespective of the signal or the SM background, diminish significantly at higher center-of-mass energy with $\sqrt{s} = 1$ TeV. This is simply due to the fact that cross-section diminishes as $1/\sqrt{s}$. This is shown for $\psi^+\psi^-$ production cross-section with $M_{\psi^\pm} = 100$ GeV in Fig. 12.

Now, once we have chosen the right combination of the beam polarisation to suppress SM background, we are in a position to analyse a favourable cut flow for the signal events. We plot, the main kinematic variables: MET and $H_T$ distribution for all the BPs, along with the SM backgrounds in Fig. 13. This is done for both $\sqrt{s} = 350$ GeV in the upper panel and for $\sqrt{s} = 1$ TeV in the lower panel of Fig. 13. We see that our benchmark points (BP5-BP8) produce a sharp peak in MET and $H_T$ at lower values, while the SM background distribution is flatter. This is because, in signal events, the mass difference ($\Delta M$) between the charged fermions to that
TABLE VI. Dominant SM background cross-sections for different polarization of the $e^-e^+$ beams at $\sqrt{s} = 1$ TeV at the ILC.

| $P_{e^-}$ | $P_{e^+}$ | $\sigma(W^+W^-)$ (fb) | $\sigma(W^+W^-Z)$ (fb) | $\sigma(ZZ)$ (fb) |
|----------|----------|------------------|------------------|------------------|
| -80%     | +20%     | 5.87             | 0.12             | 0.23             |
| +80%     | -20%     | 0.43             | 0.009            | 0.11             |
| 0%       | 0%       | 2.65             | 0.05             | 0.15             |

TABLE VII. Variation of $\psi^+\psi^-$ production cross-section at $\sqrt{s} = 350$ GeV with different choices of polarization of the incoming beam at the ILC for benchmark points BP5 and BP6.

| $P_{e^-}$ | $P_{e^+}$ | $\sigma$(BP5) (fb) | $\sigma$(BP6) (fb) |
|----------|----------|------------------|------------------|
| -80%     | +20%     | 1225.7           | 1252.5           |
| +80%     | -20%     | 690.32           | 705.13           |
| 0%       | 0%       | 958.01           | 978.56           |

of the DM is small. This essentially dictates that momentum available for the DM or for those of the SM leptons are on the smaller side. On the other hand, due to large mass difference between the produced SM gauge boson and the SM leptons, the available momentum for the leptons can be much larger. Therefore, we can safely choose a judicious upper cut on MET and $H_T$ to retain such signals and diminish SM backgrounds further. In order to show this dependence of MET on $\Delta M$ explicitly, we have compared the MET distribution for BP3 (with $\Delta M = 272$ GeV) and BP7 (with $\Delta M = 47$ GeV) in Fig. 14. As already pointed out, due to larger $\Delta M$, BP3 produces larger missing energy and MET distribution becomes flatter and gets submerged into the SM background. BP7, with smaller $\Delta M$, peaks at lower end of the distribution. We choose therefore the following
| $P_e^-$ | $P_e^+$ | $\sigma$(BP5) (fb) | $\sigma$(BP6) (fb) | $\sigma$(BP7) (fb) | $\sigma$(BP8) (fb) |
|-------|-------|-------------------|-------------------|-------------------|-------------------|
| -80%  | +20%  | 156.44            | 155.48            | 136.9             | 155.17            |
| +80%  | -20%  | 90.46             | 90.41             | 79.16             | 89.55             |
| 0%    | 0%    | 123.45            | 123.48            | 108.27            | 122.44            |

TABLE VIII. Variation of $\psi^+\psi^-$ production cross-section at $\sqrt{s} = 1$ TeV with different choices of polarization of the incoming beam at the ILC for benchmark points BP5, BP6, BP7 and BP8.

FIG. 12. Variation of production cross section for the signal $\psi^+\psi^-$ with $\sqrt{s}$ at ILC. $M_{\psi^\pm} = 100$ GeV is chosen as an illustration.

Selection cuts for selecting signal events:

- MET cut of $E_T < \{100, 50\}$ GeV, which retains most of the signals while killing majority of the background for $\sqrt{s} = 1$ TeV, while for $\sqrt{s} = 350$ GeV the MET cut is even milder: $E_T < \{30, 20\}$ GeV.

- A $H_T$ cut of $H_T < 150$ GeV to reduce the background further for $\sqrt{s} = 1$ TeV. For $\sqrt{s} = 350$ GeV we employed: $H_T < 50$ GeV.

- An invariant mass cut around Z-window: $|m_Z - 15| < mll < |m_Z + 15|$ helps to get rid off
FIG. 13. Top Left: MET distribution for OSD+\( \not{E}_T \) final state at \( \sqrt{s} = 350 \) GeV for BP5 and BP6 (shown in red). Corresponding dominant SM backgrounds are also shown with different colors. Top Right: \( H_T \) distribution for the same. Bottom Left: MET distribution for OSD+\( \not{E}_T \) final state at \( \sqrt{s} = 1 \) TeV for BP7 and BP8 (in red). Corresponding dominant SM backgrounds are also shown with different colors. Bottom Right: \( H_T \) distribution for the same.

FIG. 14. Comparison of MET distribution of BP3 and BP7 at the ILC with \( \sqrt{s} = 1 \) TeV. the Z-dominated background in both cases.
We have finally tabulated the number of signal and background events at the ILC for both \(\sqrt{s} = 350\) GeV and \(\sqrt{s} = 1\) TeV for the chosen polarization \((P_{e^-}, P_{e^+})=(+80\%, -20\%)\). In Tab. IX and Tab. X, we have shown the variation in signal events with the cuts applied for \(\sqrt{s} = 350\) GeV and \(\sqrt{s} = 1\) TeV respectively. The same for the dominated SM background are also tabulated in Tab. XI and Tab. XII for \(\sqrt{s} = 350\) GeV and \(\sqrt{s} = 1\) GeV respectively. In order to find the discovery potential of such signals at the ILC we have again computed the signal significance. This is shown in Fig. 15. As one can see, for \(\sqrt{s} = 350\) GeV a 5\(\sigma\) discovery reach is possible at a very low luminosity (left panel of Fig. 15): \(L \sim 8\) fb\(^{-1}\) for BP5. For \(\sqrt{s} = 1\) TeV the same can be reached for BP7 at a luminosity \(L = 30\) fb\(^{-1}\) as shown in the right panel of Fig. 15. This tells us, there is a chance that this model might show up at a very early run of the ILC, compared to that of LHC which demands a much higher luminosity to be probed.

We conclude this section, by again pointing out that small \(\Delta M\), i.e. small mass difference between the charged fermions and DM, can only be probed at the ILC through hadronically quiet OSD events, thanks to the absence of \(t\bar{t}\) and Drell Yan type background events. While this has been established with some benchmark points (BP5-BP8) in presence of scalar triplet, the feature can also be captured for the same fermion DM model \[18\] in absence of scalar triplet. The scalar triplet rather paves the way for probing the model at higher \(\Delta M\) region at the LHC. The complementarity of the LHC and the ILC searches for the model is an interesting noteworthy feature of this analysis.

### Table IX. Signal events with \(\sqrt{s} = 350\) GeV at the ILC.

| Benchmark Point | \(\sigma^{\psi^+\psi^-}\) (fb) | \(E_T\) (GeV) | \(\sigma^{\text{OSD}}\) (fb) |
|-----------------|-------------------------------|---------------|-----------------------------|
| BP5             | 690.32                        | < 30          | 6.27                        |
|                 |                               | < 20          | 3.64                        |
| BP6             | 705.13                        | < 30          | 3.11                        |
|                 |                               | < 20          | 3.09                        |

### Table X. Signal events with \(\sqrt{s} = 1\) TeV at the ILC.

| Benchmark Point | \(\sigma^{\psi^+\psi^-}\) (fb) | \(E_T\) (GeV) | \(\sigma^{\text{OSD}}\) (fb) |
|-----------------|-------------------------------|---------------|-----------------------------|
| BP7             | 79.16                         | < 100         | 2.04                        |
|                 |                               | < 50          | 1.85                        |
| BP8             | 89.55                         | < 100         | 1.84                        |
|                 |                               | < 50          | 1.24                        |
Background $\sigma_{production}$ (pb) $E_T$ (GeV) $\sigma_{OSD}$ (fb)

|        |        |        |        |
|--------|--------|--------|--------|
| $W^+W^-$ | 1.90   | < 30   | 3.80   |
|        |        | < 20   | 1.88   |
| $W^+W^-Z$ | 0.002  | < 30   | 0.001  |
|        |        | < 20   | 0.009  |
| ZZ     | 0.49   | < 30   | 0.18   |
|        |        | < 20   | 0.11   |

TABLE XI. Events for dominant SM background with $\sqrt{s} = 350$ GeV at the ILC.

|        |        |        |        |
|--------|--------|--------|--------|
| $W^+W^-$ | 0.43   | < 100  | 4.97   |
|        |        | < 50   | 2.61   |
| $W^+W^-Z$ | 0.009  | < 100  | 0.03   |
|        |        | < 50   | 0.01   |
| ZZ     | 0.11   | < 100  | 0.13   |
|        |        | < 50   | 0.08   |

TABLE XII. Events for dominant SM background with $\sqrt{s} = 1$ TeV at the ILC.

FIG. 15. Left: Signal significance for BP5 and BP6 at the ILC for $\sqrt{s} = 350$ GeV. Right: Significance of BP7 and BP8 at $\sqrt{s} = 1$ TeV. In both the plots The solid red and dashed red lines correspond to $3\sigma$ and $5\sigma$ discovery limits respectively.
VI. DISPLACED VERTEX SIGNATURE AND COMPLEMENTARITY OF DIFFERENT SEARCH STRATEGIES

Finally, we would like to highlight the displaced vertex signature of this model, which is elaborated in [19]. If the mass difference between $\psi^\pm$ and $\psi_1$ is less than that of $W$-mass, then the charged fermions will decay via three body process. In such cases we can see a displaced vertex signature for our model at the LHC, provided the track length (which is inverse of the 3-body decay width) is $\sim O(1 \text{ mm})$. Now, the decay width is given by [19]:

$$\Gamma = \frac{G_F^2 \sin^2 \theta M_{\psi}^5}{24\pi^3} \xi,$$  (53)

where $G_F$ is the Fermi coupling constant and the function $\xi$ is given by:

$$\xi = \frac{1}{4} \sqrt{\alpha} (x^2, y^2) \zeta_1 (x, y) + 6 \zeta_2 (x, y) \ln \left( \frac{2x}{1 + x^2 - y^2 - \alpha^{1/2}} \right).$$  (54)

Here $\zeta_1$ and $\zeta_2$ are two polynomials of $x = M_1/M_{\psi}$ and $b = m_\ell/M_{\psi}$, where $m_\ell$ is the mass of charged leptons. Upto order $O(y^2)$, $\zeta_{1,2}$ are given as:

$$\zeta_1 (x, y) = (x^6 - 2x^5 - 7x^4(1 + y^2) + 10x^3(y^2 - 2) + x^2(12y^2 - 7) + 3y^2 - 1)$$

$$\zeta_2 (x, y) = (x^5 + x^4 + x^3(1 - 2y^2)),$$  (55)

where $\alpha = 1 + x^4 + y^4 - 2x^2 - 2y^2 - 2x^2y^2$ is the phase space. The length of the displaced vertex is given as $c\tau \equiv \frac{c}{\Gamma}$, where $\Gamma$ can be obtained from Eq. 53.

In Fig. 16 we have shown two different parametrisation for a realizable displaced vertex signature produced in this model. In the left panel, we plot the displaced vertex length $c\tau$ as a function of $\Delta M$ for a fixed $\sin \theta \sim 10^{-4}$. We illustrate three different choices of DM mass $M_{\psi_1} = \{100, 150, 200\}$ GeVs. The horizontal black dashed line corresponds to displaced vertex length of $10^{-4}$ cm (i.e, 1 $\mu$m). In the right panel, we show the limit on $\sin \theta$ for producing displaced vertex of $c\tau \geq 10^{-4}$ cm as a function of $\Delta M$ for two specific DM masses 100 and 150 GeVs. The upshot is, if we have to detect a measurable displaced vertex length at collider, $\sin \theta$ has to be extremely small. However, with small $\sin \theta$, the allowed parameter space behaves similar to that of $\sin \theta \lesssim 0.1$, which has to heavily rely on co-annihilation effects to obtain correct relic density and is allowed by direct search bounds. It is also important to note that the presence of triplet scalar do not at all alter the displaced vertex signature discussed before for the fermion DM alone [18].
FIG. 16. Left: Displaced vertex length \((c\tau \text{ in cm})\) versus \(\Delta M\) for three different choices of DM mass \(M_{\psi_1} = \{100, 150, 200\} \text{ GeVs}\) for \(\sin \theta = 10^{-4}\). The horizontal black dashed line corresponds to displaced vertex length of \(10^{-4}\) cm. Right: Limit on \(\sin \theta\) for producing displaced vertex of \(c\tau \geq 10^{-4}\) cm as a function of \(\Delta M\) for two specific DM masses 100 and 150 GeVs.

FIG. 17. Summary of the available parameter space in \(M_{\psi_1} - \Delta M\) plane from relic density, direct search constraints and collider sensitivity. Red and green points correspond to PLANCK-observed relic abundance satisfying region (same as in LHS of Fig. 2); Black thick and black dashed lines correspond to XENON1T upper bound for \(\sin \theta = \{0.1, 0.2\}\). The blue shaded region can be probed at ILC, while the orange shaded region can potentially be probed at LHC. We have assumed the triplet scalar to have a mass \(\sim 300\) GeV.

Finally, all the searches and constraints for this DM model put together, allow us to visualize
how all these different searches are complementary to one another. Such a summary plot is shown in Fig. [17]. The green and red points correspond to observed relic abundance that are allowed by PLANCK data. XENON1T direct detection limit as shown in Fig. [5] are indicated by the solid and dashed black lines for $\sin \theta = \{0.1, 0.2\}$ respectively. The blue shaded region for $\Delta M < M_W$ can be probed at ILC, while the region above with $\Delta M > M_W$ shown by orange shaded region, can be probed at LHC. It is difficult to calculate the significance for the signal cross-section in this plane, and therefore the fading in the colour shades imply that the cross-section diminishes with large $M_{\psi_1}$ as well as with large $\Delta M$. From the figure, firstly we identify that for small $\sin \theta \leq 0.1$, there is a large region (red points) that fall below the direct search limit, which can be probed at future direct search experiments and they can will have small $\Delta M \lesssim 50$ GeV and spans the all of DM mass range. The small $\sin \theta$ region can generally be probed OSD signal excess at ILC, while the small $M_{\psi_1}$ region can also be probed by displaced vertex signature at LHC. The $Z$ and $H$ resonance regions for $\sin \theta \leq 0.1$ can only be probed through OSD signal excess at LHC. For larger $\sin \theta$, direct search allowed points again limit to $\Delta M \lesssim 50$ GeV and therefore can be probed dominantly at ILC. Displaced vertex signature (of $c\tau \lesssim 1$ mm) requires further suppressed values of $\sin \theta \sim 10^{-4}$ and therefore could not be shown in the plot.

VII. CONCLUSIONS

The paper focuses on a beyond SM (BSM) framework by introducing two vector-like fermions: a singlet $\chi$ and a doublet $\psi$, where the DM emerges as a lightest component, $\psi_1$, as an admixture of the neutral component of $\psi$ and $\chi$. The phenomenology of the model crucially dictates small singlet-doublet mixing ($\sin \theta$) to abide by the non observation of DM in direct search. Moreover, the observed relic density of DM restricts the mass splitting between DM and NLSP ($\Delta M$) to less than 10 GeV due to which the model can be probed at LHC only through displaced vertex signature of the NLSP. In this analysis we show that the ILC, however, can probe such model even in small mixing limit through the production and subsequent decays of the charged companion $\psi^\pm$ via hadronically quiet opposite sign dilepton (OSD) channels. This is possible due to primarily the nature of the missing energy distribution for signal and background events at the ILC, which allows to put an upper cut on missing energy for event selection, while the possibility of utilising the polarization of $\{e^-, e^+\} = \{80\%, -20\%\}$ reduces SM background significantly, retaining most of the signal events due to vectorlike fermion nature.

The presence of a scalar triplet of hypercharge 2 in the model can produce non-zero masses
for the active neutrinos, as required by solar and atmospheric oscillation data. This also alters the DM phenomenology crucially. In presence of the scalar triplet, the dark fermion $\psi^1_1$ splits into two pseudo-Dirac states $\psi^\alpha_1$ and $\psi^\beta_1$. As a result, the $Z$-mediated DM-nucleon scattering at direct search experiments become inelastic. Assuming the mass splitting between the two pseudo-Dirac states to be of order 100 keV, we showed that the DM-nucleon scattering through $Z$ mediation is forbidden. This helps to achieve larger singlet-doublet mixing in the DM state. In fact, we showed that the doublet component can be as large as 20%. Moreover, the mass splitting between the DM and NLSP can be chosen to be as large as a few hundred GeVs. These are the two key factors which paved a path for detecting the DM at the LHC through hadronically quiet OSD channel. However, the broadening of the mass splitting can not be obtained in all region of the parameter space, rather it is specific to the Higgs, $Z$ and triplet scalar resonance regions, as well as when the DM mass is equal or slightly larger than the triplet scalar. So, the LHC search for a signal excess can only be possible in such regions of the DM mass parameter with large $\Delta M$. On the contrary, if we embed the fermion DM model with a scalar singlet DM [20], the possibility of exploring such signal excess at LHC spans larger DM mass range.

It is easily understood that the displaced vertex signature of the NLSP not only requires small mixing, but also small mass splitting $\Delta M$. So, while we enhance the possibility of seeing a signal excess at LHC through enlarging the mass splitting (a result of adding scalar triplet), the displaced vertex signature gets washed off. On the other hand, while LHC favors large mass splitting between NLSP and DM for the signal to be segregated from SM background due to indomitable $t\bar{t}$ channel, the absence of such a channel at ILC will favour the cases of small mass splitting to yield a signal excess over background. Thus, the model has a complementarity in its variety of signatures that can be probed at upcoming experiments.

VIII. ACKNOWLEDGEMENTS

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Appendix A: Appendix

1. Invisible Higgs and Z-decay

Here we have shown that the BPs chosen for LHC analysis (Tab. II) are allowed by experimental bounds on invisible Higgs and Z-decays. The SM Higgs can decay to $\psi_1$ pairs. Now, the combination of SM channels yields an observed (expected) upper limit on the Higgs branching fraction of 0.24 at 95% CL [33] with a total decay width $\Gamma = 4.07 \times 10^{-3}$ GeV. On the other hand, SM Z boson can also decay to DM pairs and hence constrained from observation: $\Gamma_{Z_{inv}} = 499 \pm 1.5$ MeV [15].

So, if Z is allowed to decay into $\psi_1\psi_1$ pair, the decay width should not be more than 1.5 MeV.

| Benchmark Point | $B_{inv}^{higgs}$ | $\Gamma_{inv}^{Z}$ (MeV) |
|-----------------|-------------------|--------------------------|
| BP1             | $1.116 \times 10^{-3}$ | NA                       |
| BP2             | $654.86 \times 10^{-6}$ | NA                       |
| BP3             | $5.738 \times 10^{-3}$ | 1.201                    |
| BP4             | $80.24 \times 10^{-6}$ | NA                       |

TABLE XIII. Invisible Higgs branching ratio and invisible Z decay width for different benchmark points tabulated in Tab. II. NA stands for ‘Not Applicable’ for cases where $M_{\psi_1} > m_Z/2$.

Since $\Delta M > 100$ GeV for all the BPs, hence Higgs or Z can not decay to $\psi_2$’s. Therefore, the expressions for $H_1 \rightarrow \psi_1\psi_1$ and $Z \rightarrow \psi_1\psi_1$ decay widths are given by:

$$\Gamma_{inv}^{higgs} (H_1 \rightarrow \psi_1\psi_1) = \left(\frac{y_N^2 \sin^4 \theta \cos^2 \theta_W}{8\pi} \right) m_{H_1} \left(1 - \frac{4m_{\psi_1}^2}{m_{H_1}^2}\right)^{3/2}$$ (A1)

$$\Gamma_{inv}^{Z} (Z \rightarrow \psi_1\psi_1) = \frac{m_Z}{48\pi} \frac{e^2 \sin^4 \theta}{\sin^2 \theta_W \cos^2 \theta_W} \left(1 + \frac{m_{\psi_1}^2}{m_Z^2}\right) \sqrt{1 - \frac{4m_{\psi_1}^2}{m_Z^2}}.$$ (A2)

In Tab. XIII we have tabulated the Higgs branching ratio and Z-decay width for all the chosen benchmark points. Constraint from invisible Z-decay is only applicable for BP1 and BP5 which correspond to $M_{\psi_1} = 41$ GeV and $M_{\psi_1} = 45$ GeV respectively, while invisible Higgs decay constraint is applicable for all the benchmarks.
2. Lagrangian parameters

One can express all the couplings appearing in the scalar potential in terms of the physical masses. Apart from the parameters $\mu_H$ and $\mu_\Delta$ obtained through electroweak symmetry breaking condition (See Eq. 11), one can also determine the following parameters:

$$\lambda_1 = -\frac{2m_A^2}{v_d^2 + 4v_t^2} + \frac{4m_{H^\pm}^2}{v_d^2 + 2v_t^2} + \frac{\sin 2\theta_0 (m_{H_1}^2 - m_{H_2}^2)}{2v_d v_t},$$

$$\lambda_2 = \frac{1}{v_t} \left[ \frac{1}{2} \left( \sin^2 \theta_0 m_{H_1}^2 + \cos^2 \theta_0 m_{H_2}^2 \right) + \frac{1}{2} \frac{v_d^2 m_A^2}{v_d^2 + 4v_t^2} - \frac{2v_d^2 m_{H^\pm}^2}{v_d^2 + 2v_t^2} + m_{H^\pm}^2 \right],$$

$$\lambda_3 = \frac{1}{v_t} \left[ -\frac{v_d^2 m_A^2}{v_d^2 + 4v_t^2} + \frac{2v_d^2 m_{H^\pm}^2}{v_d^2 + 2v_t^2} - m_{H^\pm}^2 \right],$$

$$\lambda_4 = \frac{4m_A^2}{v_d^2 + 4v_t^2} \frac{4m_{H^\pm}^2}{v_d^2 + 2v_t^2},$$

$$\lambda = \frac{2}{v_d^2} \left( \cos^2 \theta_0 m_{H_1}^2 + \sin^2 \theta_0 m_{H_2}^2 \right),$$

$$\mu = \sqrt{2v_t m_A^2} \frac{v_d^2}{v_d^2 + 4v_t^2}.$$ 

3. Annihilation and co-annihilation in presence of Higgs triplet

Here we have gathered all the annihilation and co-annihilation graphs in presence of the triplet scalar.

![Diagram](FIG. 18. Annihilation ($i = j$) and co-annihilation ($i \neq j$) of vector-like fermion DM. Here ($i, j = 1, 2$).)
FIG. 19. Co-annihilation process of $\psi_i$ ($i = 1, 2$) with the charge component $\psi^-$ to SM particles.

FIG. 20. Co-annihilation process of charged fermions $\psi^\pm$ to SM particles in final states.

FIG. 21. Additional annihilation $\psi_i \bar{\psi}_i$, in presence of scalar triplet.
FIG. 22. Dominant annihilation ($\psi_i(\psi_i)^c$) and co-annihilation ($\psi^-(\psi_i)^c$, $\psi^-(\psi^-)^c$) processes of DM ($\psi_i$) to scalar triplet in final states.

FIG. 23. Co-annihilation channels of DM ($\psi_i$), with charged fermions $\psi^-$ in presence of scalar triplet.

FIG. 24. Co-annihilation processes involving only charged partner of DM, $\psi^\pm$ in presence of scalar triplet.
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