Research on Brand Competition and Decisions in Supply Chains Under Manufacturer Information Referral

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ABSTRACT To study the impact of manufacturer information referral on the decision-making of supply chain members, a Stackelberg model is constructed based on a manufacturer, a professional retailer and a store brand retailer. The optimal price and profit of the manufacturer and both retailers when manufacturer refers consumers to the professional retailer or not are solved and compared. Besides, the effect of the manufacturer's information referral on the members of the supply chain is analyzed. The results show that the referral of manufacturer benefits both manufacturer and the referred retailer if the referral level meets certain conditions, otherwise it may reduce the profit of both parties, and the referral level is affected by the market size of the professional retailer. Meanwhile, under the influence of the professional retailer’s market size and the intensity of market competition, the store brand retailer will continue to operate only when the referral level stays in a certain range, but the profit of its store brand is damaged. However, if the referral level exceeds a certain threshold, the store brand product’s profit will increase with free riding.

INDEX TERMS Supply chain, information referral level, brand competition, Stackelberg game.

I. INTRODUCTION

The development of e-commerce has not only changed the shopping habits of consumers, it has also changed the commercial behavior of manufacturers and retailers. To avoid competition with retailers, some manufacturers choose to set up official websites without sales functions to display product information for consumers to access while recommending retailer purchasing channels through website links. This model of manufacturers indirectly selling products through the network is called “manufacturer referral,” Wu et al. [26]. For example, Panasonic’s (China) official website includes a “How to Buy” option, where it recommends purchasing channels to consumers. A large food company, Wu Fang Zhai, and the skin-care brand owned by Johnson & Johnson, Aveeno, also engage in referral behavior. Of course, consumers can buy the same products from retailers other than those recommended by the manufacturer, such as Wal-Mart or Watsons. Famous brand manufacturers, such as Pechoin, L’Oreal Paris, Ultimate Ears, and Rene Furterer, have chosen to set up their official websites and promote authorized retailers to consumers through their website links. Manufacturers integrate retail functions through official website platform promotion. Supply chain management involves understanding and meeting the needs of consumers, which is not only a way to optimize the value of consumers but is also a way to improve the returns of all stakeholders in the chain (Fearne,1996). Traditionally, the research on commercial market value is mainly focused on evaluating how suppliers create value for consumers and understanding how consumers perceive superior value. Creating value for consumers is the key to success in any specific market, Vieira et al. [25]. At the same time, consumer behavior will affect the performance of producers and the supply chain itself, Taghikhah et al., [23]. Through manufacturer referrals, on the one hand, consumers can understand the purchasing
channels and maintain product goodwill. On the other hand, manufacturers can also maintain cooperative relationships with retailers.

As the retail market evolves, a growing number of retailers aim to extract more profit from the vertical structure by developing private brands. This is a threat that manufacturers have to face. These retailers not only distribute the manufacturers’ products but also fully understand consumers’ needs through sales data or establish their own production bases or entrusted-related factories to produce products with retailers’ brands, Zhang et al. [30] and Ru et al.[22]. Interestingly, these retailers maintain cooperative relationships with manufacturers while developing their private brands to compete with manufacturers, such as Walmart’s Great Value, the TV brand “ONN”, Watson’s Skin Advanced, etc., Liu et al. [17] and Neebe [20]. Amazon, the world’s second-largest internet company, began offering its own-brand products to consumers in 2009. According to market research firm TJI Research, Amazon has more than 120 private-label brands covering batteries, mothers and babies, clothing, pets and health products, a ninefold increase from the private-label brand products offered in 2016, according to SunTrust Robinson Humphrey. It expects Amazon’s private label sales to reach $25 billion by 2020, Faherty et al. [8]. Private brands have much room for development and will be a trend in the retail industry. However, manufacturers must re-examine the relationship between private brand owners in their supply chains. Based on the above observations, whether to adopt a referral strategy has become a question worth considering for manufacturers. In addition, it is also worth exploring how the manufacturer’s referral strategy will affect the pricing decisions of members in the supply chain.

This paper considers a supply chain system consisting of a manufacturer, a professional retailer and a store brand retailer. The manufacturer refers consumers who intend to buy their products to a professional retailer through its own website. In the scenario where a manufacturer recommends a traditional retailer, the following three questions are discussed: How can supply chain members achieve optimal pricing strategies? How will the profits of the companies in the supply chain change when the manufacturer recommends a traditional retailer? Can referrals effectively prevent the intrusion of retailers’ own brands? We construct a Stackelberg game model to explore the effects of different information referral levels on the manufacturer’s profit, the professional retailer’s profit and the store brand retailer’s decisions.

Following Wu et al. [26] and Zhang et al. [29], this study contributes to the research on manufacturer referrals and supply chain members’ decision-making. The findings show that the higher the manufacturer’s referral level is, the more conducive it is to increasing the wholesale price and the retail price of all products, but it has different impacts on the referral sales and original sales of different products. When the market size and referral level meet certain conditions, the manufacturer’s referral to a traditional retailer will harm the profits of both firms. In addition, the manufacturer can prevent the retailer from developing its own brand by adjusting the referral level. However, in contrast to previous research conclusions, this paper finds that under the combined effect of market competition intensity, market size, and referral level, the harmonious development of the manufacturer’s products and the retailer’s own products can be achieved.

This paper is organized as follows. Section II reviews the related literature. In Section III, we formalize the problem, propose the basic hypothesis, and construct the model based on the description. We consider two problem scenarios, that is, the manufacturer provides or does not provide referral information regarding its professional retailer on its official website. The impact of the referral level on manufacturers and the impact on retailers (including the professional retailer and the store brand retailer) is analyzed in Section IV. Finally, we offer managerial implications and conclude this research in Section V.

II. LITERATURE REVIEW

This study is related to the following three streams of the literature: manufacturer referral, private brands by retailers and supply chain competition.

There are some theoretical bases regarding referral sales model of referrals, but there are relatively few recent studies on manufacturer referrals. Ghose et al. [10] were the first to study manufacturer referrals. By comparing manufacturer referrals with third-party referrals, they pointed out that in some cases, manufacturers can take advantage of wholesale prices to reap third-party profits. Then, on this basis, Wu et al. [26] constructed a Stackelberg game model with a manufacturer and two heterogeneous retailers, which focused on the manufacturer’s referral strategy when faced with retailers with different costs. They revealed the minimum threshold of the manufacturer’s referral level and found that the manufacturer is more willing to refer customers to retailers with cost advantages. They also found that the qualitative results are robust even if there was price discrimination among consumers, referral position disparity, local consumers, and asymmetric referral market sizes. Li et al. [16] studied the selection of referral strategies for manufacturers under linear production costs and diseconomies of scale. Compared with the linear cost case, the manufacturer prefers to refer consumers to official stores in the diseconomies of scale cost case. Subsequently, Li et al. [17] analyzed the impact of the risk aversion characteristics of retailers and manufacturers on manufacturers’ referral strategies and proposed the optimal referral strategies of manufacturers under different types of risk aversion. However, none of the above studies took into account the recommendation behavior of the manufacturer when the retailer had its own brand. However, in practice, many retailers have their own brands, which changes the brand competition structure of the supply chain and has an important impact on the manufacturer’s decision to provide recommendations.
They discovered that when the manufacturer increases its manufacturer and a retailer with value-added service competition. Dan et al. explored the precursors and consequences of manufacturers’ reliance on private brand retailers, focusing on the relationship between manufacturers and retailers. The results show that manufacturers’ reliance on private brand retailers directly leads to the growth of private brand sales. Cheng et al. built a game-theoretic model of price competition between a national brand manufacturer and a retailer that also sells its own private label. Amrouche et al. used a utility function model to analyze consumers’ choice of products from manufacturers and private brands. The study showed that both the manufacturer and the retailer could achieve Pareto improvements in advertising. Li et al. examined a retailer’s Stackelberg supply chain, in which the retailer sells a product from two brands, its own private brand and a national brand supplied by the manufacturer, and investigated the return policy choice problem and the effects of the competition between the two brands. Bauner et al. consider an economy with one manufacturer, one retailer, and a unit mass of consumers. They analyzed couponing behavior by manufacturer brands and private brands. They found that both quality and feature differentiation are important determinants of pricing and couponing behavior. Huang and Feng investigated the money back guarantee choice problem in a dual-channel supply chain consisting of one manufacturer and one retailer with the introduction of a private brand using a game model. The study found that money back guarantee policies can help retailers develop their private brands, which are viewed by consumers as low-end alternatives to national brands. Moreover, we found that the manufacturer could provide coordinate contracts for the retailer to improve its performance so that a win-win outcome could be reached for both the manufacturer and the retailer. Nevertheless, Cheng et al. discovered that the more powerful a manufacturer is, the greater damage the retailer would do to itself by introducing a private brand. The above studies discussed the strategies manufacturers use to deal with private brand dealers from the aspects of private brand positioning, advertising and channels. However, few studies have studied manufacturer referrals as a coping strategy.

There are many achievements in the supply chain competition research. Dan et al. studied warranty service strategies in a dual-channel supply chain composed of a manufacturer and a retailer with value-added service competition. They discovered that when the manufacturer increases its warranty service level, the value-added service competition will be weakened, and when the warranty service level is high enough, there is no value-added service competition. Tang et al. concentrated on the impacts of simultaneous disruption demand and cost on the pricing, production, and coordination of a dual-channel supply chain with one manufacturer and one retailer. The study deduced the optimal price and production quantity in the event of disruption and found that the appropriate changes to and improvements in revenue sharing contracts can help coordinate the dual-channel supply chain with disruptions. In a dual-channel supply chain system consisting of a manufacturer and two retailers, Pi et al. studied pricing and service strategies with competition and cooperation of retailers by using a Stackelberg-Nash equilibrium game in which the manufacturer sells a single product through an online direct channel and two retail channels. Zhang and Hezarkhani developed a model to analyze the channel selection decisions of manufacturers using the following three channel strategies: a direct channel strategy, a retail channel strategy, and a dual channel strategy consisting of direct and retail channels. The results show that the channel preferences of manufacturers not only depend on the channel operating costs and consumers’ channel preferences but also depend on the channel strategies of competitors. He et al. studied logistics service sharing (LSS) and the resulting competition issues in a dual channel e-commerce supply chain composed of manufacturers and retailers. The findings show that the two firms can be Pareto improved by LSS if the price is within a certain range. When the price of the shared logistics service falls below a threshold, the social welfare of LSS will be greater than that of a society without LSS. However, few scholars have conducted a study based on the complex relationship of cooperation and competition between the manufacturer and the retailer.

From the above review, it can be seen that with regard to the impact that retailers introducing their own brands has on manufacturers, most studies focus on product characteristics, channel strategies, advertising, etc. However, little attention has been given to manufacturers’ referral strategies regarding retailers with private brands in the channel. How does the manufacturer’s referral strategy affect the retailer’s decision in the channel? How do supply chain members make decisions when the manufacturer and retailer have a complex relationship of cooperation and competition? These issues will be the focus of this paper. Based on the introduction of private brands by retailers in previous studies, this paper considers manufacturer referrals and investigates the impacts of different levels of referral information on the profit of the manufacturer, the profit of the professional retailer, and the decisions of the store brand retailer. We hope to provide theoretical guidance regarding manufacturers’ referral strategies and supply chain decision-making.

III. MODEL DESCRIPTION

There is a supply chain system consisting of a manufacturer (M), two retailers (R1, R2) and consumers, in which manufacturer M produces a single type of product X and sells X to
two retailers R1 and R2 at the wholesale price w. Assume that R1 is a retailer who specializes in selling the product (hereinafter referred to as the professional retailer), and determines its retail price \( p_1 \) according to the manufacturer’s wholesale price w. Retailer R2 has its own brand (hereinafter referred to as the store brand retailer) and sells both product X and substitutable product Y, which is from its private brand. R2 decides the retail price \( p_2 \) for X and the retail price \( p_3 \) for Y based on the manufacturer’s wholesale price w. Since R1 is the manufacturer’s professional retailer, the manufacturer can refer consumers who browse its products to R1 through its own web links and image promotion. The relationship between the manufacturer and the professional retailer is called a referral alliance. See FIGURE 1 for the supply chain system structure in this paper.

Referring to the research of Wu et al. (2015), Ghose et al. (2007) and Yang et al. (2017), the consumer market is divided into the following two independent parts: the original market and the manufacturer referral market (hereinafter referred to as the referral market). In the original market (indicated by subscript o), consumers are familiar with the products sold by the two retailers and can choose to buy. However, in the referral market (indicated by the subscript r), consumers may be loyal customers of the manufacturer. They are not familiar with these two retailers, and the manufacturer’s referral information may lead consumers to purchase at retailer R1. Assume that the overall size of the manufacturer’s product X in the original market is 1, and if the market size of retailer R1 is represented by \( s \) \((0 < s < 1)\), then the original market size of retailer R2 is \( 1 - s \). With reference to the symmetrical setting of Cai et al. [3] and Choi [5], the market size of product X sold by retailer R2 is \( \frac{1-s}{2} \), so the market size of product Y sold by retailer R2 is also \( \frac{1-s}{2} \).

Furthermore, suppose that the increased market size of professional retailer R1 after the manufacturer provides referral information is \( \alpha_r \) \((\alpha_r > 0)\), and this parameter also indicates the manufacturer’s referral information level. Without loss of generality, suppose that the production costs of the manufacturer and retailer R2 are both 0, and the market competitive intensity of the two retailers is \( \theta \). Retailer R1’s pricing is the same for consumers in both the original market and the referral market Wu et al. [26], Ghose et al. [10] and Yang et al. [27].

Referring to the research of Cai et al. [3] and considering that both product competition and channel competition exist between the two retailers, the consumer utility function in this paper is given as follows:

\[
U = sq_1 + \frac{1-s}{2}q_2 + \frac{1-s}{2}q_3 - \theta q_1q_2 - \theta q_2q_3 - \theta q_1q_3 - \frac{q_1^2}{2} - \frac{q_2^2}{2} - \frac{q_3^2}{2} - p_1q_1 - p_2q_2 - p_3q_3.
\]  (1)

Note that \( U \) represents consumer utility in the original market, \( q_1 \) and \( q_2 \) represent the sales volumes of product X sold by retailers R1 and R2, respectively, and \( q_3 \) represents the sales volume of product Y sold by retailer R2.

For comparison and analysis, two scenarios are examined in this paper, that is, the manufacturer does not provide referral information on its official website or that it provides the referral information to consumers regarding professional retailer R1. The situation where the manufacturer does not provide referral information is represented by the superscript NR, while the situation where the manufacturer provides referral information is represented by the superscript R.

### A. THE MANUFACTURER DOES NOT PROVIDE REFERRAL INFORMATION

Taking the model with no manufacturer referral as the benchmark, when there is no referral and only the original market exists, the corresponding demand function can be obtained by maximizing utility function (1) as follows:

\[
q_{1o}^{NR} = \frac{s - \theta + 2s\theta - p_1^{NR} - \theta p_1^{NR} + \theta p_2^{NR} + \theta p_3^{NR}}{(1 - \theta)(1 + 2\theta)},
\]

\[
q_{2o}^{NR} = \frac{1 - s - 2s\theta + 2\theta p_1^{NR} - 2p_2^{NR} - 2\theta p_2^{NR}}{2(1 - \theta)(1 + 2\theta)} + 2\theta p_3^{NR},
\]

\[
q_{3o}^{NR} = \frac{1 - s - 2s\theta + 2\theta p_1^{NR} + 2\theta p_2^{NR} - 2p_3^{NR} - 2\theta p_3^{NR}}{2(1 - \theta)(1 + 2\theta)}.
\]

Note that \( q_{no}^{NR}, n = 1, 2, 3 \) represents the sales volumes of products X and Y in the original market in the scenario where the manufacturer does not provide referral information. \( p_n^{NR}, n = 1, 2, 3 \) represents the retail prices of products X and Y when the manufacturer does not provide referral information.

Therefore, when the manufacturer does not provide referral information, the profit functions of the manufacturer and two retailers are given as follows:

\[
\pi_M^{NR} = w^{NR} (q_1^{NR} + q_2^{NR}),
\]

\[
\pi_{R1}^{NR} = q_1^{NR} (p_1^{NR} - w^{NR}),
\]

\[
\pi_{R2}^{NR} = (p_2^{NR} - w^{NR})q_2^{NR} + p_3^{NR}q_3^{NR}.
\]

Note that \( \pi_i^{NR}, i = M, R1, R2 \) represents the profits of the manufacturer, retailer R1 and retailer R2 when there is no referral information from the manufacturer.

When using the backward induction process, it is necessary to verify whether the profit function of each member has a maximum value, and the proof is as follows:
The Hessian matrix is $H^N = \frac{-2(1+\theta)}{(1-\theta)(1+2\theta)} \begin{pmatrix} 20 & -2(1+\theta) \\ -2(1+\theta) & 2 \end{pmatrix}$, where the first-order principal minor satisfies $\frac{-2(1+\theta)}{(1-\theta)(1+2\theta)} < 0$ and the second -order principal minor satisfies $\frac{-2(1+\theta)}{(1-\theta)(1+2\theta)} > 0$. This satisfies the negative definite requirement, that is, there is a maximum value of the profit function value for store brand retailer.

Therefore, the optimal decisions are obtained as follows when the manufacturer does not provide referral information. (5)–(8), as shown at the bottom of the page.

In addition, to ensure that the above model and equilibrium solution are meaningful, we need to solve $w^N \geq 0, p^N_i \geq 0, p^N_3 \geq 0, p^N_3 \geq 0, q^N_r \geq 0, q^N_3 \geq 0, q^N_3 \geq 0, p^N_1 - w^N \geq 0$ and $p^N_1 - w^N \geq 0$ to obtain the feasible region, i.e., $s, s \leq 0$. Note that the expressions of $s$ and $s$ are as follows: $s = -2\theta^3 - 5\theta + 2 + s(12\theta^5 - 18\theta^4 - 46\theta^3 + 25\theta^2 + 57\theta + 18)$, $s = -2\theta^3 - 5\theta + 2 + s(12\theta^5 - 18\theta^4 - 46\theta^3 + 25\theta^2 + 57\theta + 18)$.

Therefore, when the manufacturer provides referral information, the profit functions of the manufacturer and two retailers are given as follows.

$$\pi^R_{R1} = \begin{cases} w^R(q^R_1 + q^R_2) + w^R q^R_1, & \alpha_r > \alpha_r^{(q_1 r)} \\ w^R(q^R_1 + q^R_2), & \alpha_r \leq \alpha_r^{(q_1 r)} \end{cases}$$

$$\pi^R_{R2} = \begin{cases} \alpha_r^{(q_2 r)} \end{cases}$$

$$\pi^R_{R2} = \begin{cases} \alpha_r^{(q_2 r)} \end{cases}$$

The sequence of the game is as follows: There is a Stackelberg game between the manufacturer and the two retailers, where the manufacturer, as the leader, decides the wholesale price $w$. There is a Nash game between the two retailers. After observing the wholesale price, both set their own retail prices simultaneously, that is, retailer R1 decides to charge $p_1$ and retailer R2 decides to charge $p_2$ for product X and $p_3$ for Y. Finally, consumers make purchases based on the maximum utility, and all parties obtain the corresponding benefits.

The equilibrium solution is obtained by backward induction, see Appendix 1. In the context of the manufacturer providing referral information, the referral level can be adjusted to ensure the nonnegativity of the equilibrium solution and the practical significance of the model. In addition, by analyzing the first derivative of the equilibrium solution (sales and price) with respect to the referral level, Theorem 1 can be obtained as follows.

**Theorem 1:** In the scenario where the manufacturer provides referral information, the optimal prices and order quantities satisfy the following properties.

1. $\frac{\partial w^R_s}{\partial \alpha_r} > 0, \frac{\partial p^R_1}{\partial \alpha_r} > 0, \frac{\partial p^R_3}{\partial \alpha_r} > 0, \frac{\partial q^R_1}{\partial \alpha_r} > 0$;
2. $\frac{\partial q^R_2}{\partial \alpha_r} < 0, \frac{\partial p^R_2}{\partial \alpha_r} < 0, \frac{\partial q^R_3}{\partial \alpha_r} > 0$.

From Theorem 1(1), it can be observed that the wholesale price and the retail price of product X will increase as the level of referral information provided by the manufacturer increases. In particular, the retail price $p_3$ of its alternative

$$w^N = \frac{(2\theta^3 - 3\theta - 1)s + 2\theta^2 - 1}{6(3\theta^5 - 18\theta^4 - 46\theta^3 + 25\theta^2 + 57\theta + 18)}$$

$$p^N_1 = \frac{6\theta^4 - 3\theta^3 - 16\theta^2 - 5\theta + 2 + s(12\theta^5 - 18\theta^4 - 46\theta^3 + 25\theta^2 + 57\theta + 18)}{12\theta^5 - 12\theta^4 - 46\theta^3 + 12\theta^2 - 8}$$

$$p^N_2 = \frac{12\theta^5 - 12\theta^4 - 46\theta^3 + 12\theta^2 - 8}{6(3\theta^5 - 18\theta^4 - 46\theta^3 + 25\theta^2 + 57\theta + 18)}$$

$$p^N_3 = \frac{-(6\theta^5 - 23\theta^4 - 35\theta^3 + 33\theta^2 + 5\theta + 16)}{4(6\theta^5 - 18\theta^4 - 46\theta^3 + 12\theta^2 + 8)}$$
product Y also increases with the increase in the referral level. In addition, Theorem 1(2) demonstrates that the level of referral information provided by the manufacturer has different effects on the referral sales volume compared with the original sales volume. In the referral market, with the continuous improvement of the referral information level, sales will continue to increase. However, in the original market, the higher the referral information level, the lower the sales volume of product X, while the sales volume of product Y increases. That is, although the manufacturer only refers consumers to the professional retailer, the store brand retailer obtains a free ride so that the sales volume and the retail price of its own product Y increase. As \( \frac{\partial \pi_R}{\partial \alpha} < \min\{\frac{\partial \pi_R}{\alpha_c}, \frac{\partial \pi_R}{\alpha_c}\} \), the price of product Y responds slowly to the level of manufacturer’s referral information: thus, it obtains a relative advantage in price and helps to increase its sales.

IV. DECISION ANALYSIS

A. IMPACT OF THE REFERRAL LEVEL ON THE MANUFACTURER

This section will discuss the impact that providing referral information has on the manufacturer’s decision by comparing the changes in the manufacturer’s profit under two situations, namely, providing referrals and not providing referrals.

The profit difference of the manufacturer in two different scenarios is obtained through Equations (2) and (6), which is represented by \( \Delta \pi_M = \pi_M^R - \pi_M \). If \( \Delta \pi_M \geq 0 \), then it is better for the manufacturer to provide referrals than not. Solving for \( \Delta \pi_M > 0 \), it can be concluded that under the condition \( \alpha_r < \alpha_r^{(q2)} \), the threshold that the manufacturer’s referral level needs to meet is \( \alpha_r < \alpha_r^{(m1)} \); and under the condition \( \alpha_r > \alpha_r^{(q1)} \), the threshold is \( \alpha_r > \alpha_r^{(m2)} \).

By comparing the four thresholds \( \alpha_r^{(m1)}, \alpha_r^{(q1)}, \alpha_r^{(m2)}, \) and \( \alpha_r^{(q2)} \) of the referral level, it is found that there is a threshold \( s_m \) in the definition domain, which is the critical market size value that satisfies \( \alpha_r^{(m1)} = \alpha_r^{(q1)} = \alpha_r^{(m2)} \). In addition, \( s_m \) also meets the following conditions:

1. When \( -s < s < s_m, \alpha_r^{(m1)} < \alpha_r^{(q1)} < \alpha_r^{(m2)} \).
2. When \( s_m < s < -s, \alpha_r^{(m2)} < \alpha_r^{(q1)} < \alpha_r^{(m1)} \).

That is to say, under the condition of \( -s < s < s_m \), if \( \alpha_r^{(m1)} < \alpha_r < \alpha_r^{(m2)} \), then \( \Delta \pi_M < 0 \); and when \( s_m < s < -s, \Delta \pi_M > 0 \). Thus, proposition 1 is obtained as follows.

Proposition 1: Comparing the manufacturer’s profit changes in the two scenarios of providing referrals and not providing referrals, the following conclusions are drawn:

1. When \( -s < s < s_m, \alpha_r^{(m1)} < \alpha_r < \alpha_r^{(m2)} \), then \( \Delta \pi_M < 0 \); and if \( 0 < \alpha_r < \alpha_r^{(m1)} \) or \( \alpha_r \geq \alpha_r^{(m2)} \), then \( \Delta \pi_M \geq 0 \).
2. When \( s_m < s < -s, \alpha_r > 0 \), then \( \Delta \pi_M \geq 0 \).

Proposition 1 demonstrates that the market size of the professional retailer is the main factor affecting the level of referrals provided by the manufacturer. So set \( \theta = 0.1 \) and plot FIGURE 2, with \( s \) as the horizontal axis and \( \alpha_r \) as the vertical axis, to show the change of \( \Delta \pi_M \). The white area in FIGURE 2 indicates that \( \Delta \pi_M \geq 0 \), while the gray area indicates that \( \Delta \pi_M < 0 \). That is, when the market size of R1 is relatively small (\( -s < s < s_m \)) and the referral level satisfies \( \alpha_r^{(m1)} < \alpha_r < \alpha_r^{(m2)} \), the manufacturer is more profitable without any referral, which suggests that under these conditions the manufacturer’s referral regarding the professional retailer hurts its own profit. However, in other remaining areas, providing referral information is beneficial to the manufacturer itself.

When \( \theta = 0.1 \) and \( s_m = 0.175 \), FIGURE 3(a) (sets \( \theta = 0.1 \) and \( s = 0.17 \)) and FIGURE 3(b) (sets \( \theta = 0.1 \) and \( s = 0.53 \)) illustrate the influence of the referral level on the manufacturer’s profit under the different market scales in FIGURE 2. Since the decision regarding the manufacturer’s referral level is not affected by \( \theta \), the following analysis is suitable for \( \forall \theta \in (0,1) \).

FIGURE 3(a) shows that when the market size of R1 is relatively small, namely when \( -s < s < s_m \), the difference
in the manufacturer’s profit between the two scenarios of giving referral information and not giving referral information decreases first and then increases. The manufacturer’s profit may be lower in the referral scenario. Define the area where no referral sales are generated due to the low referral level as I. In this area, the manufacturer’s profit is higher with referral than without referral mainly because referrals increase competition among the two retailers, causing them to cut prices and the original sales increased. Although the profit of the referral market is 0 and retailers cutting prices will force the manufacturer to lower its wholesale price, the increase in original sales can compensate for the loss of the wholesale price, ensuring that the manufacturer will make more profits in the referral scenario. However, with the increase in the referral level, the profit difference gradually narrows. This is because the professional retailer gradually gains the advantage of a “commercial reputation”, alleviating price competition with R2 and causing the original sales increment to gradually decrease. When the referral level satisfies \( \alpha_{r}^{(m1)} < \alpha_{r} < \alpha_{r}^{(m2)} \), making a referral is not beneficial to the manufacturer and this area is defined as II, i.e., the gray shaded area in FIGURE 2. FIGURE 3(c) illustrates the changes in the manufacturer’s profit within the region in detail. The manufacturer cut its wholesale prices to ease price-cutting competition between the two retailers. However, as the referral level increases, the original sales volume gradually decreases and either no referral sales are generated or the referral volume is too small to compensate for the loss of the wholesale price. This results in the manufacturer’s profit in the referral scenario being lower than the profit without the referral. When the referral level increases to satisfy \( \alpha_{r} > \alpha_{r}^{(m2)} \), the referral sales will also increase, and the manufacturer will gradually increase the wholesale price to obtain more profits in the referral market. If the original market sales volume is not 0, it is defined as area III; if the original market sales volume drops to 0, the area is defined as IV. In regions III and IV, although the high referral level damages the original market profit, the profit of the manufacturer in the referral market is higher, which is sufficient to guarantee the profit of the manufacturer in the referral scenario.

By comparing FIGURE 3(a) and (b), it is found that when the market size of R1 is large and satisfies \( s_{m} < s < s_{m} \), the manufacturer’s profit is always higher in the referral scenario. Similar to FIGURE 3(a), the part where the profit difference is reduced is defined as area I. In this region, the profit of the manufacturer is higher under the referral scenario mainly because of the increase in original sales, but the gradual decrease in the profit difference is also caused by the gradual decrease in the original sales volume increment. When the manufacturer’s referral level is further improved referral sales are generated, and the manufacturer can still obtain the original sales, it is defined as area II. Under the high referral level, when the original sales volume drops to 0, it is defined as area III. In these two regions, the sales from referrals are gradually increasing, and the manufacturer will set a higher wholesale price to obtain more profit, which is sufficient to compensate for the loss caused by the original sales reduction. This ensures that it is better to provide referrals and causes the profit difference to be larger.

Based on the above analysis, it can be observed that when the manufacturer’s referral level is larger than \( \alpha_{r}^{(q1r)} \), the referral sales gradually increase, causing the manufacturer to gain more profits in the referral market to ensure that the referral situation is better than the nonreferral situation. While \( \alpha_{r}^{(q1r)} \) is affected by the intensity of market competition and market size Theorem 2 is obtained from the analysis.

**Theorem 2**: The threshold value \( \alpha_{r}^{(q1r)} \) of the referral level generating the referral sales volume meets \( \frac{\partial q_{1}}{\partial s} > 0 \), \( \frac{\partial q_{1}}{\partial m} < 0 \).

Theorem 2 shows that \( \alpha_{r}^{(q1r)} \) gradually decreases with decreasing of market size and weakening competitive intensity. This indicates that when the original market size of retailer R1 is small and the market competition is fierce, the manufacturer’s referral is likely to have an effect on the referral market. Therefore, manufacturers need to grasp the opportunity to form a referral alliance with R1 and gain more profits.

**B. IMPACT OF THE REFERRAL LEVEL ON RETAILERS**

1) IMPACT OF THE REFERRAL LEVEL ON THE PROFESSIONAL RETAILER

According to the previous analysis, referrals may damage the manufacturer’s profits. As the leader of the game, the manufacturer can choose a reasonable referral level to maximize its profits. This section will further discuss what impact the manufacturer selecting the corresponding referral level will have on the profit of another main body in the referral alliance, namely, professional retailer R1. By calculating Equations (3) and (7), \( \Delta \pi_{1} = \pi_{R1}^{P} - \pi_{R1}^{NR} \) is obtained to represent the profit difference of retailer R1 in different scenarios. If \( \Delta \pi_{1} \geq 0 \), then the manufacturer’s referral is beneficial to R1; otherwise, R1 will be more profitable in the scenario without referral.

Solving for \( \Delta \pi_{1} > 0 \), the referral level threshold obtained under the condition of \( \alpha_{r} < \alpha_{r}^{(q1r)} \) is \( \alpha_{r}^{(1)} \); and under the condition of \( \alpha_{r} > \alpha_{r}^{(q1r)} \), the referral level threshold is \( \alpha_{r}^{(2)} \). By comparing \( \alpha_{r}^{(q1r)} \), \( \alpha_{r}^{(1)} \), and \( \alpha_{r}^{(2)} \), we find that when \( s < s_{m} < \min \{ s_{1}, -s_{1} \}, \alpha_{r}^{(1)} < \alpha_{r}^{(2)} < \alpha_{r}^{(q1r)} \); and when \( \min \{ s_{1}, -s_{1} \} < s < s_{m} \), \( \alpha_{r}^{(1)} < \alpha_{r}^{(q1r)} < \alpha_{r}^{(2)} \) where, \( s_{1} \) both the critical value of the existence of threshold \( \alpha_{r}^{(1)} \) and the market size critical value of \( \alpha_{r}^{(2)} \) are obtained as \( \alpha_{r}^{(1)} \) and \( \alpha_{r}^{(2)} \). Combined with proposition 1, it is found that when \( s_{m} < s_{1} \) and \( -s < s < s_{m} \), then \( \alpha_{r}^{(1)} < \alpha_{r}^{(m1)} \). Therefore, proposition 2 is obtained.

**Proposition 2**: When the manufacturer chooses the referral level, the profit change of the professional retailer satisfies the following:

1) When \( -s < s < s_{m} \), if \( 0 < \alpha_{r} < \alpha_{r}^{(1)} \), then \( \Delta \pi_{1} < 0 \); and if \( \alpha_{r}^{(1)} < \alpha_{r} < \alpha_{r}^{(m2)} \) or \( \alpha_{r} \geq \alpha_{r}^{(m2)} \), then \( \Delta \pi_{1} > 0 \).
The relationship between referral level and market size.

(2) When $s_m < s < s_1$, if $0 < \alpha_r < \alpha_r^{(1)}$, then $\Delta \pi_1 < 0$; and if $\alpha_r \geq \alpha_r^{(1)}$, then $\Delta \pi_1 \geq 0$.

(3) When $s_1 < s < -s$, if $0 < \alpha_r < \alpha_r^{(2)}$, then $\Delta \pi_1 < 0$; and if $\alpha_r \geq \alpha_r^{(2)}$, then $\Delta \pi_1 \geq 0$.

Proposition 2 shows that the market size of R1 is still the main factor affecting the role of the manufacturers’ referral level. Therefore, to illustrate the effects of the market size and referral level on the profit of the manufacturer and the profit of the professional retailer, FIGURE 4 ($\theta = 0.1$) is drawn based on FIGURE 2. The blank area in the figure indicates that the profits of both the manufacturer and the professional retailer have been improved with the referral, and the newly added gray shaded area indicates that the referral is hurtful for the professional retailer. That is, when $-s < s < s_1$ and $0 < \alpha_r < \alpha_r^{(1)}$, or $s_1 < s < -s$ and $0 < \alpha_r < \alpha_r^{(2)}$, then $\Delta \pi_1 < 0$, namely, the manufacturer’s referral hurts the professional retailer’s profit.

Next, we will analyze in detail the changes in the profit of the professional retailer after the manufacturer chooses the referral level to maximize its profit. To specify the profit change of professional retailer R1 for a given manufacturer referral level, when the market size satisfies $-s < s < s_m$, $s_m < s < s_1$, and $s_1 < s < -s$, set $s = 0.17$, $s = 0.3$, $s = 0.53$ respectively and plot FIGURE 5(a), FIGURE 5(b), and FIGURE 5(c) based on FIGURE 4 when $\theta = 0.1$, $s_m = 0.175$, $s_1 = 0.456$. The dotted line represents the change in the manufacturer’s profit, and the solid line represents the profit change of R1.

In FIGURE 5(a), when the referral level is lower than $\alpha_r^{(1)}$ (i.e., the gray shaded area $\Box$ in FIGURE 4), then $\Delta \pi_1 < 0$, which suggests that the profit of R1 is worse in the referral scenario. This is because the manufacturer’s referral level is too low to attract the attention of consumers in the referral market, i.e., sales $q_{1r}^M = 0$. Although R1 forms a referral alliance with the manufacturer, it does not gain profit from the referral market. However, the referral caused price competition between retailers R1 and R2, which severely damaged R1’s original market profit, so the referral actually harmed R1’s profit. However, in combination with proposition 1, it can be known that at this time, the manufacturer can rely on price-cutting competition among two retailers to increase the original sales volume and obtain a higher profit, as shown by the dotted line that indicates that $\Delta \pi_M > 0$.

When the referral level satisfies $\alpha_r > \alpha_r^{(1)}$, then $\Delta \pi_1 > 0$, which means that the referral is beneficial to R1. When the referral level satisfies $\alpha_r^{(1)} < \alpha_r < \alpha_r^{(q1r)}$, although R1’s profit in the referral market is still 0, due to the increase in the manufacturer’s referral level, retailer R1’s commercial reputation continues to improve and its relative advantage is gradually highlighted, which alleviates the price war and increases the profit in the original market in the referral scenario. Therefore, the profit of R1 in the referral scenario is higher than that in the nonreferral scenario. When the referral level exceeds $\alpha_r^{(q1r)}$, the manufacturer will raise the wholesale price due to the increase in the referral sales, and the retailer will raise the selling price accordingly, resulting in the decrease in the sales volume in the original market, and the profit will be reduced or may even be 0. However, R1 will always achieve greater profits in the referral situation because of the higher profit in the referral market.
It is also worth noting that when the market size is small and meets \( s < s < s_m \), referrals may damage the interests of the manufacturer, so the rational manufacturer will avoid the referral level range \( (\alpha_r^{(1)}, \alpha_r^{(m2)}) \). Therefore, the interval that guarantees the referral to R1 at this time is \( \alpha_r^{(1)} < \alpha_r < \alpha_r^{(m1)} \) or \( \alpha_r \geq \alpha_r^{(m2)} \).

Corollary 1: The conditions for realizing the Pareto optimization of the referral alliance are as follows:

When \( s < s < s_m \), the referral level satisfies \( \alpha_r^{(1)} < \alpha_r < \alpha_r^{(m1)} \) or \( \alpha_r \geq \alpha_r^{(m2)} \);

When \( s_m < s < s_1 \), the referral level satisfies \( \alpha_r \geq \alpha_r^{(1)} \);

When \( s_1 < s < -s \), the referral level satisfies \( \alpha_r \geq \alpha_r^{(2)} \).

At this time, in the referral scenario, the profits of both the manufacturer and the professional retailer are higher than the profits under the nonreferral scenario, as shown in the blank area in FIGURE 4. Both the rational manufacturer and professional retailer will choose the referral level in Corollary 1, so further research will be conducted on the premise of Corollary 1.

2) IMPACT OF THE MANUFACTURER’S REFERRAL ON RETAILER R2

Different from Retailer R1, R2 is a retailer with its own private brand and competes with the manufacturer, so R2 cannot interfere with the manufacturer’s referral level decision. When the manufacturer provides the referral to R1, R2 will continue to sell product X as long as it is profitable. However, as the owner of private brand product Y, R2 pays more attention to the profit changes of product Y before and after the referral and makes management decisions by comprehensively considering the profit changes of products X and Y. Obviously, if the profit of product X in the referral scenario is 0 and the referral damages the profit of product Y, then retailer R2 will exit the market of product X. Moreover, the manufacturer has a cooperative relationship with R2. After the manufacturer and R1 form a referral alliance, to maintain the market competition structure of product X, the manufacturer also needs to pay attention to R2’s operation performance, which is also a problem that will be studied in depth in this section.

First, the profit change of product X in the referral situation is studied after the manufacturer chooses the appropriate referral level maximize the profits of all parties in the referral alliance. Second, the profit change of product Y under different situations is studied. The profit changes of products X and Y are comprehensively considered to discuss the influence of the manufacturer’s referral on retailer R2.

Let \( \pi_{X0}^R = (p_2^R - w_R^2)q_{X0}^R \) denote the income of retailer R2 from selling product X in the original market under the referral scenario; R2 will continue to sell product X as long as \( \pi_{X0}^R \geq 0 \). Solve for \( \pi_{X0}^R = 0 \) to obtain the referral thresholds level \( \alpha_r^{(2o)} \) and \( \alpha_r^{(3o)} \). If \( \alpha_r > \alpha_r^{(2o)} \), then the sales volume of product X in the referral scenario is 0, i.e., \( q_{X0}^R = 0 \). If \( \alpha_r > \alpha_r^{(2o-w)} \), then \( p_2 - w < 0 \) exists in the referral scenario, and R2 will exit the market because of the exorbitant wholesale price. Let \( \Delta\pi_{Y0} = p_3^R q_{Y0}^R - p_{NR}^R q_{Y0}^R \) represent the profit difference of product Y in the original market under the referral scenario and nonreferral scenario.

Solve for \( \Delta\pi_{Y0} = 0 \) to obtain the referral level threshold \( \alpha_r^{(AY)} \), and \( \Delta\pi_{Y0} < 0 \) exists when \( \alpha_r < \alpha_r^{(AY)} \), indicating that the manufacturer’s referral hurts the profit of product Y.

To analyze the profit change of product X in the referral scenario and the profit change of product Y in different scenarios, we first need to compare \( \alpha_r^{(2o)} \) with \( \alpha_r^{(2-w)} \); then, we need to compare \( \alpha_r^{(2o)} \) with \( \alpha_r^{(AY)} \) and calculate \( \alpha_r^{(2o)} - \alpha_r^{(AY)} = 0 \) to obtain the market size threshold \( s_2 \). Finally, need to compare \( \alpha_r^{(2-w)} \) with \( \alpha_r^{(AY)} \) to obtain \( s_3 \).

To analyze the effect of the referral (that is, the referral level satisfies corollary 1) on R2, the relationships between A, B, C, and D need to be compared. At the same time, the relationship between the referral level thresholds obtained by comparison is \( \alpha_r^{(1)} < min(\alpha_r^{(2o)}, \alpha_r^{(AY)}) \), \( \alpha_r^{(2)} < min(\alpha_r^{(2o)}, \alpha_r^{(AY)}) \). The detailed processed for the above comparative analysis is shown in the appendix.
However, if the market size of R1 is very large (region $z$), and the alliance referral level satisfies $\alpha_r^{(q_{20})} < \alpha_r < \alpha_r^{(\Delta Y)}$, then R2 exits the sales market of product X. This is because as the referral level of the alliance increases, the retail price continues to increase, causing the original sales volume of product X to drop to 0 and the sales volume of product Y in the referral scenario to be lower than that without referral. Therefore, R2 is not motivated to continue operating and therefore exits the sales market of product X. However, when the market size of R1 is small (the sum of the regions $\delta$ and $\xi$), due to $\frac{d\alpha_r^{(q_{20})}}{\partial x} < 0$, referrals will quickly increase the sales of product X (Theorem 2) when market competition is fierce, leading to R1 increasing the retail price. By combining Theorem 1 (the price change of product Y lags behind that of product X), it can be observed that the sales volume of alternative product Y will increase accordingly. Therefore, R2 will gain more profits by selling product Y in the referral scenario, $\alpha_r^{(\Delta Y)} < \alpha_r^{(q_{20})}$ guarantees that R2 can always benefit from the profit of selling product X or the referral to increase the profit of product Y; that is, the referral alliance will not cause R2 to leave the market.

The above analysis shows that under certain alliance referral levels, R2 selling product X is profitable in the referral scenario, while the profit of its own product Y may be lower in the referral scenario. Thus, proposition 4 is obtained. At this time, because the competition intensity of the market will also affect the role of the alliance referral level, on the basis of FIGURE 2, set $\theta = 0.1$ and $\theta = 0.57$ to draw FIGURE 7(a) (involving regions $\delta$, $\xi$, $\eta$, and $\zeta$) and FIGURE 7(b) (involving regions $\theta$, $\tau$, and $\nu$) to illustrate the influence of the referral level and market size on the manufacturer, retailer R1 and retailer R2 under different market intensities. The blank area in FIGURE 7 indicates that when the manufacturer refers consumers to the professional retailer, both of them will make a profit. The blank area above the referral level $\alpha_r^{(\Delta Y)}$ means that the product Y of the private-brand retailer is free-riding and will gain more profits in the referral scenario. Only in the blank area below $\alpha_r^{(q_{20})}$ can it be guaranteed that the sales of product X at R2 are not 0 in the referral scenario. Compared with FIGURE 4, the new gray shaded area in FIGURE 7(a) indicates that R2 refuses to sell product X due to the high wholesale price.

Proposition 4: When the referral level meets the following conditions, the referral alliance is an effective strategy for the manufacturer to deal with store brand retailer R2.

(1) In region $\delta$ and $\xi$, $\alpha_r^{(1)} < \alpha_r < \alpha_r^{(m1)}$ or $\alpha_r^{(m2)} < \alpha_r < \alpha_r^{(\Delta Y)}$; (2) In region $\eta$ and $\nu$, $\alpha_r^{(1)} < \alpha_r < \alpha_r^{(\Delta Y)}$; (3) In region $\zeta$, $\alpha_r^{(3)} < \alpha_r < \alpha_r^{(\Delta Y)}$; (4) In region $\theta$, $\alpha_r^{(2)} < \alpha_r < \alpha_r^{(q_{20})}$; (5) In region $\tau$ and $\nu$, $\alpha_r^{(1)} < \alpha_r < \alpha_r^{(\Delta Y)}$.

As shown in the slanted shaded area in FIGURE 7, the manufacturer’s referral at this time can effectively reduce the profit of R2’s own product Y while obtaining the profit.
from R2’s sales of product X. When the market size of R1 is small (the sum of the regions ①, ③, ②, ⑧, and ③), it is difficult to prevent R2’s own product Y from free-riding, so the alliance referral level cannot be higher than \( \alpha_{r}^{(AY)} \). Otherwise, product Y will get a free ride, and the sales volume of the manufacturer’s product X at R2 may be damaged.

\[
\begin{align*}
s_m &= \frac{-1}{(1 + 2\theta)} \cdot \frac{(40^3 - 8\theta^2 - \theta + 3) \sqrt{-A_1A_2B_1B_4} - (2\theta^2 - 1) C_1}{(40^3 - 10\theta^2 + 3\theta + 5) \sqrt{-A_1A_2B_1B_4} - (\theta^2 - \theta - 1) C_1}, \\
s_1 &= \frac{1}{C_2} \left[ \frac{C_3}{1 + 2\theta} - (\theta - 1)^2 A_1A_2C_4C_6 \right], \\
s_2 &= \frac{C_9 + 2(1 - \theta)(240^5 - 820^4 + 370^3 + 800^2 - 400\theta - 32) A_1A_2 \sqrt{C_9}}{(1 + 2\theta) C_{10}}, \\
s_3 &= \frac{C_{11} - 2(1 - \theta) A_1A_2C_2 \sqrt{C_{13}}}{(1 + 2\theta) C_{14}}, \\
C_1 &= 112\theta^6 - 300\theta^5 - 112\theta^4 + 575\theta^3 + 115\theta^2 - 328\theta - 128, \\
C_2 &= \left( \begin{array}{c} 5184\theta^{19} + 34176\theta^{19} - 45216\theta^{17} - 121344\theta^{17} \\
+257724\theta^{16} + 117064\theta^{15} - 110180\theta^{14} \\
-240136\theta^{13} - 168676\theta^{12} + 121868\theta^{11} \\
+525011\theta^{10} - 2322190\theta^9 - 808532\theta^8 + 1519586\theta^7 \\
+732119\theta^6 + 561236\theta^5 - 362772\theta^4 \\
-1210464\theta^3 + 661320\theta^2 + 443392 \theta + 69632 \\
\end{array} \right), \\
C_3 &= (1 + \theta), \\
C_4 &= 6\theta^4 - 12\theta^3 - 5\theta^2 + 16\theta + 8, \\
C_5 &= 2\theta^2 - \theta - 2, \\
C_6 &= \sqrt{(1 + \theta)(B_1 - 2B_4)C_8}, \\
C_7 &= -64\theta^6 + 168\theta^5 + 58\theta^4 - 305\theta^3 - 57\theta^2 + 168\theta + 64, \\
C_8 &= \left( \begin{array}{c} 122112\theta^{19} - 686592\theta^{18} - 12624\theta^{17} + 4605576\theta^{16} \\
+1155316\theta^{15} - 2161960\theta^{14} - 1538910\theta^{13} + 6941283\theta^{12} \\
+6742155\theta^{11} - 13175016\theta^{10} - 16377852\theta^9 + 12558500\theta^8 \\
+23138255\theta^7 - 16397632\theta^6 - 16953408\theta^5 - 68891136\theta^4 \\
+37787392\theta^3 + 40167424\theta^2 + 12709888\theta + 1441792 \\
\end{array} \right), \\
C_9 &= \left( \begin{array}{c} 173376\theta^{16} - 516768\theta^{15} - 1337228\theta^{14} \\
+3797204\theta^{13} + 6076037\theta^{12} - 1127536\theta^{11} - 18927215\theta^{10} \\
+13924772\theta^9 + 35503748\theta^8 + 2128256\theta^7 - 3248412\theta^6 \\
-21028096\theta^5 + 5737664\theta^4 + 12961536\theta^3 + 6676736\theta^2 \\
+15749120 + 147456 \\
\end{array} \right), \\
C_{10} &= \left( \begin{array}{c} 115200\theta^{19} - 584064\theta^{18} - 642816\theta^{17} \\
+6375384\theta^{16} + 249220\theta^{15} - 28469190\theta^{14} - 2316953\theta^{13} \\
+75590140\theta^{12} + 33615309\theta^{11} - 124919048\theta^{10} - 118003213\theta^9 \\
+106514520\theta^8 + 191577028\theta^7 + 4332166\theta^6 - 145638272\theta^5 \\
-75349248\theta^4 + 28358912\theta^3 + 40437760\theta^2 + 14553088\theta + 1835008 \\
\end{array} \right).
FIGURE 7. Impact of referral on private brand product Y.

When the referral level further increases beyond \( \alpha_r^{(q_2o)} \), then \( q_r^{(q_2o)} = 0 \). When the market size of R1 is large (the sum of regions \( \Theta, \Theta, \Theta, \Theta, \Theta, \) and \( \Upsilon \)), it can effectively prevent product Y from free-riding, but the referral level of the alliance cannot exceed \( \alpha_r^{(q_2o)} \). Otherwise, the sales volume of product X sold by R2 in the original market will be 0, which will cause R2 to withdraw from the sales market of product X.

It is worth noting that under the above conditions, the self-owned product Y is simply free-riding, that is, the referral increases the profit of the product Y but reduces the sales volume of product X sold by R2 to 0.

V. CONCLUSION

This paper studies the brand competition and decision-making of a supply chain under manufacturer information referrals. The digital economy has become a huge driving force promoting economic development. An increasing number of manufacturing enterprises are attempting to participate in online markets. However, considering their professional fields and channel conflicts, some manufacturers choose to set up official websites to display product information for consumers to access and then refer consumers to authorized retailers to make purchases Ghose et al. [10]. Some enterprises recommend on their official websites that customers visit the official malls and retailers to buy their products, such as Panasonic, Johnson & Johnson, Haier, PlayStation, etc., Liu et al. [17]. However, some traditional retailers

\[
C_{11} = \begin{pmatrix}
49152 \theta^{19} - 562944 \theta^{18} + 2749120 \theta^{17} - 5526848 \theta^{16} \\
-4321668 \theta^{15} + 35872918 \theta^{14} - 25478448 \theta^{13} + 8025690 \theta^{12} \\
+104200497 \theta^{11} + 134788435 \theta^{10} - 176618095 \theta^9 - 160112788 \theta^8 \\
+162729436 \theta^7 + 151447424 \theta^6 - 71105664 \theta^5 - 95260672 \theta^4 \\
-2746624 \theta^3 + 26544128 \theta^2 + 11137024 \theta + 1441792
\end{pmatrix},
\]

\[
C_{12} = -64 \theta^5 + 16 \theta^4 - 5 \theta^3 - 152 \theta^2 + 8 \theta + 32,
\]

\[
C_{13} = \begin{pmatrix}
4096 \theta^{16} + 57216 \theta^{15} - 258192 \theta^{14} + 399192 \theta^{13} \\
+1874633 \theta^{12} + 1729414 \theta^{11} - 5953231 \theta^{10} - 5793116 \theta^9 \\
+9258996 \theta^8 + 12262896 \theta^7 - 4875548 \theta^6 - 13244736 \theta^5 \\
-3768128 \theta^4 + 4633856 \theta^3 + 4096256 \theta^2 + 1280000 \theta \\
+147456
\end{pmatrix},
\]

\[
C_{14} = \begin{pmatrix}
82176 \theta^{18} - 665600 \theta^{17} + 1669024 \theta^{16} - 718464 \theta^{15} \\
-2281506 \theta^{14} + 5156867 \theta^{13} - 14777336 \theta^{12} + 5518065 \theta^{11} \\
+56863628 \theta^{10} - 3968486 \theta^9 + 103000424 \theta^8 + 56139588 \theta^7 \\
+117861248 \theta^6 - 23364480 \theta^5 - 80391936 \theta^4 + 13969152 \theta^3 + 22734848 \theta^2 \\
+12193792 \theta + 1835008
\end{pmatrix}.
\]
transform themselves into private brand retailers by producing and selling their own products, such as Great Value, a private brand of Walmart, Neebe, [20]. Thus, when there is a complex cooperative and competitive relationship between manufacturers and retailers, the following question arises: how do supply chain members make decisions? To explore...
To reduce dependence on upstream manufacturers and improve profits, retailers introduce private brands, Bontemps et al. [31]. However, the opening of online channels by manufacturers and the introduction of private brands by retailers may put both sides in a prisoner’s dilemma Li et al. [16]. As far as we know, the existing literature has found that manufacturers have incentives to provide recommendations to all retailers because it keeps profits from flowing from the supply chain to third-party referrals.

\[
D_{17} = (1 + \theta) \begin{pmatrix}
5184\theta^{19} - 28416\theta^{18} + 23856\theta^{17} \\
+156048\theta^{16} - 367188\theta^{15} - 181576\theta^{14} \\
+1293421\theta^{13} - 278247\theta^{12} - 2481047\theta^{11} \\
+8368150\theta^{10} + 3143898\theta^{9} \\
-583230\theta^8 - 2643655\theta^7 - 238233\theta^6 \\
+1260808\theta^5 + 500588\theta^4 \\
-189472\theta^3 - 179328\theta^2 - 46080\theta - 4096
\end{pmatrix}.
\]

\[
D_{18} = \left(6\theta^5 - 9\theta^4 - 30\theta^3 + 10\theta^2 + 40\theta + 16\right)^2,
\]

\[
D_{19} = \begin{pmatrix} 67526^{10} - 27448\theta^9 + 55200\theta^8 \\
+87802\theta^7 - 480146\theta^6 - 120555\theta^5 \\
+49422\theta^4 + 90596\theta^3 - 69442\theta^2 \\
-30464\theta - 7680
\end{pmatrix},
\]

\[
D_{20} = \begin{pmatrix} -13408\theta^{10} + 60184\theta^9 - 42160\theta^8 \\
-131388\theta^7 + 129495\theta^6 \\
+146172\theta^5 - 110046\theta^4 - 105832\theta^3 \\
+24608\theta^2 + 36608\theta + 7680
\end{pmatrix},
\]

\[
D_{21} = \begin{pmatrix} 57600\theta^{20} - 482304\theta^{19} + 1758592\theta^{18} \\
-4200704\theta^{17} + 3203616\theta^{16} \\
+25477344\theta^{15} - 63228952\theta^{14} \\
-57955296\theta^{13} + 23433430\theta^{12} \\
+11127598\theta^{11} - 55456510\theta^{10} \\
-24691731\theta^9 + 495845197\theta^8 \\
+405590184\theta^7 - 230792468\theta^6 \\
-349682848\theta^5 - 51465344\theta^4 \\
+103478272\theta^3 + 67862528\theta^2 \\
+17170432\theta + 1638400
\end{pmatrix},
\]

\[
D_{22} = \begin{pmatrix} 261888\theta^{19} - 3136896\theta^{18} \\
+13552448\theta^{17} - 10107968\theta^{16} \\
-78374000\theta^{15} + 136104936\theta^{14} \\
+217395612\theta^{13} - 463112616\theta^{12} \\
-454631172\theta^{11} + 810055218\theta^{10} \\
+790736462\theta^9 - 740393038\theta^8 \\
-96507768\theta^7 + 201885272\theta^6 \\
+650310848\theta^5 + 187681536\theta^4 \\
-144185344\theta^3 - 118194176\theta^2 \\
-3205856\theta - 3276800
\end{pmatrix},
\]

\[
D_{23} = \begin{pmatrix} 1292352\theta^{18} - 7977792\theta^{17} \\
+5581648\theta^{16} + 48853344\theta^{15} \\
-6443556\theta^{14} - 149947440\theta^{13} \\
+204259304\theta^{12} + 319307588\theta^{11} \\
-318108516\theta^{10} - 496259460\theta^9 \\
+226146689\theta^8 + 509439360\theta^7 \\
+14234092\theta^6 - 283818656\theta^5 \\
-121083008\theta^4 + 45487104\theta^3 \\
+50888704\theta^2 + 15335424\theta + 1638400
\end{pmatrix},
\]

\[
D_{24} = \begin{pmatrix} 13312\theta^{10} - 58816\theta^9 + 40444\theta^8 \\
+123812\theta^7 - 119376\theta^6 \\
-132715\theta^5 + 97631\theta^4 + 91896\theta^3 \\
-20192\theta^2 - 29952\theta - 6144
\end{pmatrix}.
\]
In addition, manufacturers prefer to recommend retailers with cost advantages Ghose et al. [10]. While we find that when the manufacturer provides a referral to the professional retailer, the profit of one party may worsen, there is a Pareto improvement area so that both parties can obtain positive profits from the referral. Wu et al. [26] constructed a Stackelberg game model with one manufacturer and two heterogeneous retailers. They revealed that a nonexclusive referral is the manufacturer’s equilibrium choice if the referral segment market size is sufficiently large; otherwise, an exclusive referral is the equilibrium choice. This paper considers a supply chain system consisting of a manufacturer, a professional retailer and a store brand retailer. The optimal solutions in the two scenarios of manufacturer referral and nonreferral are calculated. Li et al. [16] studied the referral strategy problem of dual-channel manufacturers considering different production costs. They established a model under a linear cost and considered the model under scale diseconomies. Compared with the linear cost case, the manufacturer prefers to refer consumers to official stores in the diseconomies of scale cost case. In this paper, optimal solutions in the two scenarios of manufacturer referral and nonreferral are calculated, the impact of referral on the profit of the manufacturer and the profit of the professional retailer is analyzed in turn, and the areas improved by the Pareto of both parties are obtained. Li et al. [17] analyzed the impact of the risk aversion characteristics of retailers and manufacturers on manufacturers’ referral strategies. When the market size is small, the manufacturer only recommends the official mall; when the recommendation market size is large, the manufacturer chooses the all promotion strategy. In addition, most studies on the impact of retailers’ private brands on manufacturers mainly focus on product characteristics, channel strategies, advertising, etc. Hezarkhani [29]; Pi et al. [21]. This paper also analyzes the impact of alliance referral levels on the store brand retailer. When the manufacturer’s referral level meets certain conditions, the store brand retailer’s enthusiasm to sell product X will be damaged, but the referral level within a certain range will increase the profit of the store brand product Y.

In conclusion, unlike the existing literature, this paper assumes that there is both a cooperative and competitive relationship between manufacturers and private brand retailers, which enriches the research on manufacturers’ referrals and retailers’ introduction of private brands. First, the optimal solutions in the two scenarios of manufacturer referral and nonreferral are calculated, and then the impacts of referral on the profit of the manufacturer and the profit of the professional retailer are analyzed in turn. The areas improved by the Pareto of both parties are obtained, that is, the conditions for the formation of the referral alliance. We also analyze the impact of alliance referral levels on the store brand retailer.

The results of this study show that (1) the manufacturer’s referral strategy is conducive to increasing the wholesale and retail prices of products, both the products of the manufacturer and the private brand retailer. However, it may damage the sales volume of the manufacturer’s products in the original market. In contrast, the sales volume of the retailer’s own products in the original market will increase due to free riding. (2) The manufacturer’s recommendation of the traditional retailer may lead to worse profits for one party. As the leader of the game, the manufacturer can choose a reasonable promotion level to maximize its profits. In fact, there are still Pareto improvement areas, which can cause both sides to obtain the profits from referrals. (3) By controlling the referral level, the manufacturer can incentivize the own-brand retailer to sell the manufacturer’s products while effectively preventing the own-brand retailer from developing its own brand.

VI. LIMITATIONS AND FUTURE RESEARCH

This paper studies manufacturer referrals and considers the competition and cooperation between the manufacturer and private brand retailer, which yields interesting results and enriches the relevant research, which is lacking. However, there are still some limitations, which may provide new directions for future research.

This paper takes the manufacturer and two retailers as the main research objects. However, consumers are integral to a B2C environment. Manufacturers provide consumers with product information, but linking consumers to the retailer’s website may cause consumers to give up the purchase because it increases the hassle cost of shopping, or it may reduce consumers’ trust in the website. In future research, relevant consumer factors can be considered, such as the degree of trust, the consumer response to the referral and the change of utility caused by the purchase of goods through the referral. Social factors can also be taken into account, which is more in line with the economics of the platform.

It is assumed that consumers have no prior preference for the manufacturer’s brand and the retailer’s private brand. However, some consumers tend to prefer manufacturers’ products because they are not familiar with the retailers’ private brands. Therefore, future research can be carried out based on the fact that consumers have different value preferences for manufacturer brands and retailer brands.

APPENDIX

A. USE BACKWARD INDUCTION TO OBTAIN THE FOLLOWING EQUILIBRIUM RESULTS

1) WITHOUT REFERRAL

\[
\begin{align*}
W^* &= \frac{1}{A_1} \left[ (2\theta^3 - \theta^2 - 3\theta - 1)s + 2\theta^2 - 1 \right], \\
P_1^* &= (\theta + 3 \frac{5}{2A_2})W^* - 4\theta^2 - 1 \frac{2A_1A_2}{2A_1} s - \frac{6\theta^2 + 8\theta - 3}{2A_1}, \\
P_2^* &= (\theta + 3 \frac{1}{4})W^* + \frac{A_3}{2A_1A_2}, \\
P_3^* &= (\theta + 3 \frac{3}{4})W^* + \frac{A_3}{2A_1A_2},
\end{align*}
\]
Note that,
\[ A_1 = 6\theta^3 + 3\theta^2 - 12\theta - 8, \quad A_2 = \theta^3 - 2\theta - 2, \]
\[ A_3 = (6\theta^2 + 5\theta + 1)s + (6\theta^5 - 3\theta^4 - 3\theta^3 - 17\theta^2 + 26\theta + 17). \]

2) WITH REFERAL
\[
\begin{align*}
\omega_{r}^s &= \frac{B_2}{B_1} [(4\theta + 3)\alpha_r + 2(\theta + 1)s + 1] + \frac{B_3}{B_1} (\alpha_r + s), \\
p_1^{R_s} &= \frac{16\theta^2 - 11\theta - 17}{2B_4} w_r - \frac{B_5}{2B_1 B_4}, \\
p_2^{R_s} &= \frac{64\theta^3 - 4\theta^2 - 100\theta - 41}{16B_4} w_r + \frac{B_6}{16B_1 B_4}, \\
p_3^{R_s} &= \frac{64\theta^3 - 44\theta^2 - 68\theta - 9}{16B_4} w_r + \frac{B_6}{16B_1 B_4}.
\end{align*}
\]

Note that,
\[ B_1 = 24\theta^4 - 42\theta^3 - 35\theta^2 + 40\theta + 24, \]
\[ B_2 = 2\theta^3 - 5\theta^2 + 2, \]
\[ B_3 = 8\theta^2 + \theta - 2, \]
\[ B_4 = 5\theta^2 - 46 - 4, \]
\[ B_5 = \begin{bmatrix} (18\theta^3 - \theta^2 - 13\theta - 4)\alpha_r \\ -(30\theta^4 - 39\theta^3 - 49\theta^2 + 36\theta + 26) \end{bmatrix}, \]
\[ B_6 = \begin{bmatrix} (10\theta^3 + 11\theta^2 - 97\theta + 36)\alpha_r \\ -(9600\theta^7 - 17283\theta^6 - 22406\theta^5 + 31404\theta^4 + 29740\theta^3) \\ -1300\theta^3 + 1740\theta^2 + 402\theta \\ +4806\theta^6 - 11045\theta^5 - 568\theta^4 + 1854\theta^3 + 499\theta^2 \\ -920\theta + 366 \end{bmatrix}. \]

By comparing the relationship between \( s_m, s_1, s_2, \) and \( s_3, \) it can be found that when \( 0 < \theta < 0.095, \) there is \( -s > s_3 > s_2 > s_1 > s_m > -s; \) when \( 0.095 < \theta < 0.528, \) there is \( -s > s_2 > s_1 > s_m > -s; \) when \( 0.528 < \theta < 0.586, \) there is \( -s > s > s_2 > s_m > -s; \) when \( 0.586 < \theta < 0.804, \) there is \( -s > s > s_m > -s; \) and when \( 0.804 < \theta < 1, \) there is \( -s > s_m > -s. \) At the same time, through comparison, the relationship between the threshold of referral level is \( \alpha_r^{(1)} < \min(\alpha_r^{(q_2)}, \alpha_r^{(\Delta Y)}), \]
\[ \alpha_r^{(2)} < \min(\alpha_r^{(q_2)}, \alpha_r^{(\Delta Y)}). \]

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