Generalized AdS-CFT Correspondence
for Matrix Theory in the Large N limit

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Abstract

Guided by the generalized conformal symmetry, we investigate the extension of AdS-CFT correspondence to the matrix model of D-particles in the large $N$ limit. We perform a complete harmonic analysis of the bosonic linearized fluctuations around a heavy D-particle background in IIA supergravity in 10 dimensions and find that the spectrum precisely agrees with that of the physical operators of Matrix theory. The explicit forms of two-point functions give predictions for the large $N$ behavior of Matrix theory with some special cutoff. We discuss the possible implications of our results for the large $N$ dynamics of D-particles and for the Matrix-theory conjecture. We find an anomalous scaling behavior with respect to the large $N$ limit associated to the infinite momentum limit in 11 dimensions, suggesting the existence of a screening mechanism for the transverse extension of the system.

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1. Introduction

The so-called AdS-CFT correspondence \cite{1} \cite{2} \cite{3} is originated from the comparison of two different but dual descriptions of D-branes, namely as classical BPS solutions of effective supergravity in the low-energy limit of closed superstring theory and a direct treatment of Dirichlet branes as collective degrees of freedom in open superstring theory, whose low-energy limit are effectively some conformal field theories describing the lowest modes of open strings. From the viewpoint of closed string theories or supergravity, the system is a field theory in the bulk space-time, while in the viewpoint of the world-volume theory of Dirichlet branes the system is regarded as a field theory defined on a boundary of the bulk space-time.

In the most typical case of $\text{AdS}_5 \times S^5$, the correspondence has been used to make various predictions for the behavior of 3+1 dimensional super conformal Yang-Mills theory in the large $N$ limit in its strong coupling regime $g_{\text{YM}}^2 N \gg 1$. For example, the masses of field excitations around the conformal invariant AdS background are related \cite{2} \cite{3} to the conformal dimensions of the physical operators of super Yang-Mills theory. Indeed the agreement between them is regarded as a strongest piece of evidence for the validity of the AdS-CFT correspondence.

It should be stressed that the conformal symmetry plays a crucial role for establishing the connection between the correlation functions based on the ansatz proposed in \cite{2} and \cite{3}, where Yang-Mills theory is assumed to live on the boundary of the AdS space-time. In the original discussion in \cite{1}, use of Yang-Mills theory is justified as the description of D-brane interactions whose distance scales are much smaller than the string scale $\ell_s$, by fixing the energy $U = r/\ell_s^2$ of open strings in the limit of small $\ell_s$, where, as is now well known, the higher string modes can be neglected. On the side of supergravity, this allows us to take the ‘near-horizon’ limit which approximates $1 + g_s N \ell_s^4 / r^4 = 1 + g_s N / \ell_s^4 U^4$ by $g_s N / \ell_s^4 U^4$. However, in the prescriptions of \cite{2} and \cite{3}, we have to consider the ‘boundary’ of the near horizon region by taking the limit $U \to \infty$. Obviously, this tacitly assumes that we can extend the correspondence of both descriptions up to the region where the near-horizon approximation begins to lose its justification, namely $g_s N / \ell_s^4 U^4 \sim 1$ or $r \sim (g_s N)^{1/4} \ell_s \gg \ell_s$. This length scale exceeds the naive region of validity of the Yang-Mills
description for the dynamics of open strings. A possible support for this extension of the region seems to be the conformal symmetry: Once the correspondence is established at sufficiently small distance scales, one can enlarge it to larger distance scales to the extent that the conformal symmetry is valid on both sides. The condition \( r < (g_s N)^{1/4} \ell_s \) places the limit for the validity of this assumption from the side of supergravity. Thus the basic assumption behind the AdS-CFT correspondence is that the correspondence between classical supergravities and Yang-Mills matrix models (or any appropriate conformal field theories) is extended to the whole ‘conformal region’ characterized by the near-horizon condition. Assuming this feature, the limit \( \ell_s \to 0 \) is not essential and we can adopt any convenient unit of length for discussing the AdS-CFT correspondence.

The conformal symmetry can also be used to constrain the dynamics of (probe) D-branes themselves in the AdS background. It has been shown that the conformal symmetries on supergravity side and on super Yang-Mills side can be directly related from this point of view. For example, the characteristic scale \((g_s N)^{1/4} \ell_s\) is shown to be obtained from the side of Yang-Mills theory without relying upon the correspondence with supergravity. The deeper meanings behind these conformal symmetries have been discussed from different viewpoints. For instance, the space-time uncertainty relation has been the motivation for extending the conformal symmetry to general D-branes in ref. 

It has been discussed that the correspondence between supergravity and super Yang-Mills theories may be extended to other Dirichlet branes of different dimensions which are not necessarily described by conformally invariant field theory. From the viewpoint of the conformal symmetry, it was argued in that a certain extended version of conformal symmetry exists for general D-branes. The extended symmetry indeed is shown to be as effective for constraining the dynamics of probe D-branes in the background of heavy D-brane sources as in the case of ordinary conformally invariant theory. Some aspects of the generalized conformal symmetry have been studied from a variety of different viewpoints.

The aim of the present work is first to substantiate the previous discussions of the generalized conformal symmetry by establishing the correspondence between the excitation spectrum in supergravity around the background of a heavy D0 source and the physical
operators in supersymmetric quantum mechanical model describing D-particles, namely, Matrix theory. We confirm that they behave as expected from the generalized conformal symmetry. Secondly, and more importantly, supergravity guided by the generalized conformal symmetry enables us to predict the explicit forms for the correlators of Matrix theory operators in a certain special regime of the large $N$ limit. We will discuss the implications of our results for the dynamics of many D-particle systems and for Matrix theory from both the viewpoints of discrete light-cone quantization for finite and fixed $N$ and of the large $N$ infinite momentum frame. We find some unexpected anomalous behaviors in the scaling properties in the large $N$ limit.

It might be worthwhile to add a remark here. One of the unsolved problems in Matrix theory is to establish whether the model is consistent with 11 dimensional supergravity which is regarded as the low-energy description of M-theory. Since 10 dimensional IIA supergravity is the dimensional reduction of 11 dimensional supergravity, one might think that invoking supergravity-matrix model correspondence almost amounts to assuming the result. But that is not correct. The Yang-Mils matrix model is intrinsically defined only in 10 dimensional space-time, and therefore it is not at all evident what is the appropriate interpretation of the theory from the viewpoint of 11 dimensional M-theory. We can in principle test the idea of Matrix theory by checking whether or not the behavior of the matrix model in the large $N$ limit predicted by the generalized AdS-CFT correspondence supports the interpretation of the model as being defined at the infinite momentum frame in 11 dimensional space-time.

The paper is organized as follows. In section 2, we briefly summarize the M-theory interpretation of the D0-brane matrix model and review the generalized conformal symmetry: Although most of the points there have already been discussed in previous works [7, 4, 10], we hope that our discussion is useful for the purpose of clarification. We then analyze the possible regime of validity in which we can expect the correspondence between supergravity and the Yang-Mills description of D-particles in the large $N$ limit. In particular, we emphasize that we have to set an infrared cutoff of the order $(g_s N)^{1/7} \ell_s$ in the strong coupling region $g_s N > 1$ of the large $N$ limit in applying the correspondence. In section 3, we present the results of a complete harmonic analysis of the bosonic excitations around the classical D-particle solution in type IIA supergravity in 10 dimen-
sions. In section 4, using the results obtained in the previous section, we discuss the correspondence of the matrix model operators with the supergravity fluctuations, and the possible implications of the results from the viewpoint of Matrix-theory interpretation. In Appendix, we give a brief summary of the definitions and basic properties of the general tensor harmonics.

2. Generalized conformal symmetry and the large $N$ limit in D0-matrix model

Let us start from the standard matrix model action for D-particles

$$S = \int dt \, \text{Tr} \left( \frac{1}{2g_s\ell_s} D_t X_i D_t X_i + i\theta D_t \theta + \frac{1}{4g_s\ell_s^5} [X_i, X_j]^2 - \frac{1}{\ell_s^2} \theta \Gamma_i[\theta, X_i] \right)$$

(2.1)

where $X_i$ ($i = 1, 2, \ldots, 9$) are the $N \times N$ hermitian matrices representing the collective modes of $N$ D-particles coupled with the lowest modes of open strings. In the present paper, we will use the $A = 0$ gauge. The diagonal elements of the coordinate matrices $X_i$ are interpreted as the 9 dimensional spatial positions of the D0-branes in 10 dimensional space-time. Alternatively, the same action can be interpreted [12] as the effective action for the lowest KK-mode of the graviton supermultiplet in 11 dimensional space-time in the infinite-momentum frame where the light-like momentum $P_-=\frac{1}{2}(P_0-P_{10})$ is quantized by the unit $1/R$ with $R = g_s\ell_s$ being the compactification radius along the 11th (space-like) dimension. For the 11 dimensional interpretation, it is more natural to rewrite the Lagrangian as

$$L = \text{Tr} \left( \frac{1}{2R} D_t X_i D_t X_i + i\theta^T D_t \theta + \frac{R}{4\ell_P^6} [X_i, X_j] - \frac{R}{\ell_P^3} \theta^T \gamma_i[\theta, X_i] \right)$$

(2.2)

and also the corresponding Hamiltonian as

$$H (=-2P^-) = R \text{Tr} h = \frac{N}{P_-} \text{Tr} h$$

(2.3)

$$h = \frac{1}{2} \Pi^2 - \frac{1}{4\ell_P^6} [X^i, X^j]^2 + \frac{1}{2\ell_P^3} [\theta_\alpha, [X^k, \theta_\beta]] \gamma^{k}_{\alpha\beta}$$

(2.4)

using the 11 dimensional Planck length $\ell_P = g_s^{1/3}\ell_s$. For any fixed finite $g_s$, the infinite momentum limit $P_- \to \infty$ requires to take the large $N$ limit. The form of the Hamiltonian implies that for the infinite momentum limit for finite $g_s$ to be meaningful, the
spectrum of the operator $\text{Tr} h$ in the large $N$ limit must scale as $1/N$ in the low-energy (near threshold) region. In other words, the time must scale as $N$. Therefore it is important to analyze the large-time behavior of appropriate correlators for the study of the Matrix-theory conjecture. The usual discussion \cite{13} for justifying Matrix theory at finite $N$ \cite{14} only deals with the (spatial) length scale smaller than the string scale. For example, such an argument is insufficient to explain the results \cite{15} \cite{16} of the explicit computations of D-particle scattering beyond one-loop approximation, in which the perturbative approximation becomes better and better for larger distance scales. One of the most crucial issues of Matrix theory is to understand whether the theory can describe gravity consistently at large distance scales either in the finite $N$ or in the large $N$ limit.

The space-time scaling property of D-particles \cite{17} can be qualitatively summarized into an uncertainty relation in space-time \cite{8}

$$\Delta T \Delta X \sim \ell_s^2,$$

between the minimum uncertainties $\Delta T$ and $\Delta X$ with respect to time and space, respectively. This relation is invariant under the opposite scaling of time and space

$$X_i(t) \rightarrow X'_i(t') = \lambda X_i(t), \quad t \rightarrow t' = \lambda^{-1} t.$$  \hspace{1cm} (2.6)

The matrix model action is invariant under these scaling transformations if the string coupling is scaled as

$$g_s \rightarrow g'_s = \lambda^3 g_s.$$  \hspace{1cm} (2.7)

The scaling of the string coupling is just equivalent to the fact that the characteristic spatial and time scales of the theory are $g_s^{1/3} \ell_s = \ell_P$ and $g_s^{-1/3} \ell_s$, respectively, apart from dimensionless but string-coupling independent proportional constant. This can be derived by combining the space-time uncertainty relation with the ordinary quantum mechanical uncertainty relations. The space-time uncertainty relation may be regarded as a simple but universally valid underlying principle \cite{8} governing the short distance space-time structure of string/M theory. In \cite{4}, we have pointed out that the symmetry of the model and the space-time uncertainty relation can be extended to the special conformal transformation,

$$\delta_K X_i = 2t X_i, \delta_K t = -t^2, \delta_K g_s = 6tg_s.$$  \hspace{1cm} (2.8)
These symmetries are appropriate to be called as ‘generalized conformal symmetry’.

The same scaling properties exist on the side of supergravity solution too. From the point of view of 10 dimensional type IIA theory, the solution is expressed as

$$ds^2_{10} = -e^{-2\phi/3} dt^2 + e^{2\phi/3} dx_i^2,$$

(2.9)

$$e^\phi = g_s e^{\tilde{\phi}},$$

(2.10)

$$e^{\tilde{\phi}} = (1 + \frac{q}{r^7})^{3/4}$$

(2.11)

$$A_0 = -\frac{1}{g_s} \left( \frac{1}{1 + \frac{q}{r^7}} - 1 \right),$$

(2.12)

and the charge $q$ is given by $q = 60\pi^3 (\alpha')^{7/2} g_s N$ for $N$ coincident D-particles. In the near horizon limit $q/r^7 \gg 1$ where the factor $1 + q/r^7$ is replaced by $q/r^7$, the metric, dilaton and the 2-form field strength $F_{i0} dx_i \wedge dt \propto 7 r^6 dr \wedge dt / g_s^2$ are all invariant under the scale and the special conformal transformations

$$r \rightarrow \lambda r, \quad t \rightarrow \lambda^{-1} t, \quad g_s \rightarrow \lambda^3 g_s,$$

(2.13)

$$\delta_K t = -\epsilon (t^2 + \frac{2q}{5r^5}), \quad \delta_K r = 2\epsilon tr, \quad \delta_K g_s = 6\epsilon tg_s$$

(2.14)

which together with time translation form an $SO(1, 2)$ algebra. The additional term in the special conformal transformation does not affect the space-time uncertainty relation, but plays an important role \[7\] in constraining the effective action of a probe D-particle in the background metric (2.9). The mechanism how the additional term $\frac{2q}{5r^5}$ emerges in the bulk theory was clarified in refs. \[4, 10\] for general case of Dp-branes from the point of view of matrix models, namely, from the boundary theory. It should be emphasized that the new scale $q^{1/7} \propto (g_s N)^{1/7} \ell_s$ which characterizes the radius of the near horizon region around the system of the source D-particles was thus derived entirely within the logic of Yang-Mills matrix models. This may be regarded as independent evidence for the dual correspondence between supergravity and Yang-Mills matrix models.

As noted in previous works, the combination $g_s / r^3$ is invariant under the generalized conformal transformation, and hence the D0 space-time metric can be regarded as a sort of ‘quasi’ AdS$_2 \times$ S$^8$ with a variable but conformally invariant radius proportional to
\[ \rho = (g_s N \ell_s^2 / r^3)^{1/4} \ell_s \propto (q e^{\tilde{\phi}})^{1/7}. \]
Equivalently, the D0 metric in the near horizon limit is related by a Weyl transformation to the true AdS$_2 \times S^8$ as
\[
\begin{align*}
\text{ds}_{10}^2 &= (q e^{\tilde{\phi}})^{2/7} \left[ -\left(\frac{2}{5}\right)^2 \frac{dt^2}{z^2} - \frac{dz^2}{z^2} + d\Omega_8^2 \right] \equiv e^{2\tilde{\phi}/7} \text{ds}_{AdS}^2, \\
z &\equiv 2q^{1/2} r^{-5/2} / 5. 
\end{align*}
\]
This representation will be technically useful for performing the harmonic analysis in the next section. However, we have to be careful in considering the generalized conformal transformation using this metric, since the coordinate transformation \(2.16\) involves \(q\) which is not constant under the conformal transformation. In particular, the symmetry is not the isometry under any coordinate transformation. For example, taking derivative and performing the generalized conformal transformation are not commutative. To avoid possible confusions caused by this subtlety, it is safe to return to the original coordinate \(r\) whenever we discuss the conformal transformation of the fields. Note that in this picture the Weyl noninvariance of the theory, or the nonconformal nature of the theory, is canceled by the transformation of the string coupling constant. In other words, the Weyl factor itself is treated as being invariant under the generalized conformal transformation.

Whether the transformations being accompanied by the change of the string coupling constant should be called as a symmetry transformation may perhaps be a matter of debate. Our viewpoint is that the above transformations must be interpreted in the whole moduli space of the vacua of perturbative string/M theory. The change of the string coupling (namely, the asymptotic value of the dilaton expectation value or the asymptotic value of the compactification radius along the 11th dimension) is interpreted as a change of the vacuum at infinity. The conformal transformation is a symmetry characterizing the short distance property of the theory, but it must be accompanied by a change of vacuum at large distance asymptotic region, depending on the D-brane states we are considering. Note that the linear time dependence of the dilaton to its first order is an allowed deformation of the vacuum in the flat space-time. Ultimately, the string coupling should be eliminated from the fundamental nonperturbative formulation of the theory.

It is also worthwhile to point out that the scaling transformation of the generalized conformal symmetry is related to the discrete light-cone interpretation of the model with
fixed $N$ in the sense that the transformation is equivalent with the boost transformation,
$t \rightarrow \lambda^{-2} t, R \rightarrow \lambda^2 R, X_i \rightarrow X_i, \ell_P \rightarrow \ell_P$ and $\ell_s \rightarrow \lambda^{-1} \ell_s$, when we use the unit such that the 11 dimensional Planck length is fixed by changing the unit of length globally. Alternatively, we can use the unit such that the time is not changed. This leads to the transformation $t \rightarrow t, R \rightarrow \lambda^4 R, X_i \rightarrow \lambda^2 X_i, \ell_P \rightarrow \lambda^2 \ell_P$ and $\ell_s \rightarrow \lambda \ell_s$. The latter transformation is nothing but the redefinition proposed in [14].

Given now that there exists the same generalized conformal symmetry on both sides of the matrix model and supergravity, we can define the conformal dimensions of operators on both sides and consider the correspondence between the spectra of both sides by following the familiar prescription [2] [3] of computing the correlators of the matrix model using supergravity. Let us analyze the region of validity for the correspondence.

Throughout the present paper, we will use the unit of length in which the string scale $\ell_s$ is fixed, instead of the familiar convention of fixing the energy of the open strings stretched among D-branes. First for the supergravity approximation to be good, the curvature radius $\rho$ must be larger than the string scale giving

$$r \ll (g_s N)^{1/3} \ell_s,$$

while the near horizon condition gives

$$r \ll (g_s N)^{1/7} \ell_s$$

(2.17)

(2.18)

If we further require that the effective string coupling which can be determined locally by the dilaton is small, we must have also

$$g_s^{1/3} N^{1/7} \ell_s = g_s^{A/21} (g_s N)^{1/7} \ell_s \ll r$$

(2.19)

which essentially demands that the effective local compactification radius along 11th dimensions must be smaller than the string scale. This is necessary as long as we use the 10 dimensional supergravity picture. Since the last two conditions (2.18) and (2.19) both have the same behavior with respect to $N$, the existence of finite region of validity for the correspondence always require that the string coupling constant must be very small $g_s \ll 1$. This is especially true if one wishes to justify the Yang-Mills description of open strings which requires to go to much shorter distances than the string scale. Note
that the characteristic length scale of elementary processes of D-particle interactions is always expected to be $g_s^{1/3} \ell_s$. On the other hand, the first two conditions tell us that the extent to which the region of validity extends in the large distances depends crucially on whether $g_s N > 1$ or $g_s N < 1$. In the former strong coupling region, we have to put the infrared cutoff at $r \sim (g_s N)^{1/7} \ell_s$, while in the latter weak coupling region that is $r \sim (g_s N)^{1/3} \ell_s$. From the point of view of Matrix theory in which we wish to investigate the large-$N$ dynamics for fixed $g_s$, the former restriction is problematical, since it is expected from a general argument [22] based on the virial theorem that the typical extension of the system of $N$ D-particles is $(g_s N)^{1/3} \ell_s \gg (g_s N)^{1/7} \ell_s$ for large $g_s N$. We will come to this question later. To summarize, we must be in the region of small string coupling constant and large $N$ such that $g_s N \gg 1$, in order to go beyond the string scale $r > \ell_s$ on the basis of the AdS-CFT type correspondence between supergravity and Yang-Mills matrix models. It is also important to recall the remark which was already stressed in the Introduction, that we consider the whole near-horizon region (or conformal region) till its limit characterized by $(g_s N)^{1/7} \ell_s \gg \ell_s$. This is also necessary for studying the Matrix-theory conjecture. On the other hand, as far as we remain in the weak coupling region $g_s N < 1$, the region of validity of the supergravity-matrix model correspondence is restricted in the region below the string scale even if we take the large $N$ limit.

If we take the viewpoint of 11 dimensional M-theory and fix the Planck length $\ell_P = g_s^{1/3} \ell_s$, the near horizon condition is $r \ll g_s^{-4/21} N^{1/7} \ell_P$, which contains the small curvature region $r \ll N^{1/3} \ell_P$ for sufficiently small string coupling for any fixed $N$. However, in the large $N$ limit, these two infrared conditions can compete depending on the magnitude of $g_s N$ as explained above. Remember that in the case of D3 brane the infrared cutoff associated with the small curvature condition and that coming from the near horizon condition are governed by the same scale $r \sim (g_s N)^{1/4} \ell_s \gg \ell_s$, in sharp contrast to the case of D-particles. In passing, the condition that the nonvanishing components of 11 dimensional Riemann tensor ($\sim \partial_i \partial_j (q/r^7)$) around the D0-metric are small compared to the characteristic curvature $\ell_P^{-2}$ is

$$g_s^{5/27} N^{1/9} \ell_s \ll r \quad \text{(2.20)}$$

which is weaker than the 10 dimensional condition (2.19) only when $N$ is bigger than
\( \sim g_s^{-14/3} \). Therefore we have to keep in mind that unless the latter condition is satisfied it could be dangerous to elevate the classical 10 dimensional supergravity to classical 11 dimensional supergravity. In particular, this shows that in the \( \text{'t Hooft limit} \) in which we fix \( g_sN \) in taking the limit \( N \rightarrow \infty \), we cannot lift up the theory to 11 dimensions, while, if we take the large \( N \) limit with a fixed but small \( g_s \), the 11 dimensional picture enables us to go beyond the 10 dimensional lower bound (2.19). This will be relevant for the discussion of the Matrix-theory conjecture at the end of the present paper.

3. Harmonic analysis on D-particle background in type IIA supergravity

The idea behind the prescription [2] [3] for computing the correlators of the Yang-Mills theory using supergravity is as follows. Suppose we study the system of a heavy source of \( N \) D-branes by putting a light D-brane far away (i.e., outside the near horizon region) from the source as a probe. Thus we are considering the U\((N + 1)\) Yang-Mills theory broken down to U\((N) \times \text{U}(1)\). On the side of supergravity, the effect of the probe can be treated as a perturbation around the background of the source D-branes. For the inner region near to the horizon, information on a given state of the probe can be encoded into the boundary condition \( \{h_i\} \) for the independent set of the perturbing fields at the boundary of the near horizon region. From the Yang-Mills viewpoint, on the other hand, the perturbing fields produced by the probe should be coupled at the boundary to some independent set of operators of the Yang-Mills theory describing the source D-branes. This leads to the ansatz for the correlators of the Yang-Mills operators in Euclidean metric, as

\[
e^{-S_{SG}[h]} = \langle \exp \{\int dt \sum_i h_i(t)O_i(t)\} \rangle_{U(N)}, \tag{3.1}
\]

where \( S_{SG}[h] \) is the supergravity action as the functional of the boundary value of the perturbing fields. Although it is not literally correct to say that either the source or probe D-branes do reside at the boundary, we can interpret the relation (3.1) as if the U\((N)\) Yang-Mills theory describing the source D-branes lives on the boundary. The conformal symmetries on both sides allow us to diagonalize the operators and the perturbing fields with respect to the conformal dimensions.

In this section, we perform the diagonalization of the perturbing fields on the side of
supergravity by carrying out the harmonic analysis of the linearized perturbations around the D0 background. The calculation follows essentially the same method as in the case of non-dilatonic branes. See [18] for the typical case of AdS$_5 \times S^5$. In the present paper, we only treat the bosonic perturbations. The reader who cannot be patient in following the details of the harmonic analysis might want to skip the rest of this section and directly go to section 4 where we discuss the correspondence between the spectrum of the fluctuations and the Matrix model operators and its implications.

Let us start from the standard IIA supergravity action in 10 dimensions using the string frame ($\kappa^2 \sim g_s^2 \ell_s^8$),

$$S = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g} \left[ e^{-2\phi} \left( R + 4(\partial \phi)^2 - \frac{1}{2} |H_3|^2 \right) - \frac{g_s^2}{2} |F_2|^2 - \frac{g_s^2}{2} |\tilde{F}_4|^2 \right] - \frac{g_s^2}{4\kappa^2} \int B_2 \wedge F_4 \wedge F_4$$

(3.2)

where we have written down only the bosonic part of the action and $H_3 = dB_2, F_4 = dA_3, \tilde{F}_4 = F_4 - A_1 \wedge H_3, F_2 = dA_1$. Note also that $|F_p|^2 = F_{\mu_1 \mu_2 \cdots \mu_p} F^{\mu_1 \mu_2 \cdots \mu_p} / p!$. The nontrivial background fields exist for the metric, dilaton and RR 1-form fields. Although this is not compulsory, it is convenient for the harmonic analysis to make the Weyl transformation (2.15) for the metric. We denote the metric of the transformed frame by $g^{(total)}_{\mu \nu}$ and decompose the fields as $g^{(total)}_{\mu \nu} = g_{\mu \nu} + h_{\mu \nu}$. Namely, we denote the background metric in the AdS frame by $g_{\mu \nu}$ and the fluctuation by $h_{\mu \nu}$. For other fields similarly, we denote the background fields by using the original notation as $\tilde{\phi}, A_\mu$ and the fluctuations around them by putting 'hat' such as $\hat{\phi}, \hat{A}_\mu$. After the Weyl transformation the action takes the form

$$S = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g} \left[ e^{-2\tilde{\phi}} \left( R + \frac{16}{49} \partial_\mu \phi \partial^\mu \phi \right) - \frac{1}{12} e^{-\frac{10}{7} \tilde{\phi}} H_{\mu \nu \rho} H^{\mu \nu \rho} 
- \frac{g_s^2}{4} e^{\frac{8}{7} \phi} F_{\mu \nu} F^{\mu \nu} 
- \frac{g_s^2}{48} e^{\frac{2}{7} \phi} F_{\mu \nu \rho \sigma} F^{\mu \nu \rho \sigma} 
- \frac{g_s^2}{2 \cdot 24!4!} \epsilon^{\mu_1 \cdots \mu_{10}} B_{\mu_1 \mu_2} F_{\mu_3 \mu_4 \mu_5 \mu_6} F_{\mu_7 \mu_8 \mu_9 \mu_{10}} \right].$$

(3.3)

3.1 The fluctuations of metric, dilaton and RR 1-form fields

We first consider the fluctuations of the metric, dilaton and RR 1-form field, since other fields without the nontrivial background fields are decoupled from them. The linearized

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\footnote{We have explicitly confirmed that the final results for the dimensions and correlators are the same as those obtained using the metric without making the Weyl transformation.}
equations for the fluctuations are listed below.

\[ h_{\mu}^{\ \
u,\nu} - h_{\mu,\nu}^{\ \
u} + \frac{8}{21} (2 h_{\mu}^{\ \mu} - h_{\mu,\mu}^{\ \
u}) \partial_{\nu} \bar{\phi} \]

\[ + h_{\nu}^{\ \
u} \left\{ R_{\mu}^{\ \nu} + \frac{16}{21} D_{\mu} \partial_{\nu} \bar{\phi} - \frac{48}{147} \partial_{\mu} \bar{\phi} \partial_{\nu} \bar{\phi} + \frac{1}{2} g_{s}^{2} e^{2 \bar{\phi}} F_{\mu \nu} F_{\nu}^{\mu} \right\} - \frac{16}{21} D_{\mu} \partial_{\nu} \bar{\phi} + \frac{96}{147} \partial_{\mu} \bar{\phi} \partial_{\nu} \bar{\phi} \]

\[ + \frac{6}{7} \phi \left\{ R + \frac{16}{21} D_{\mu} \partial_{\nu} \phi - \frac{48}{147} \partial_{\mu} \phi \partial_{\nu} \phi - \frac{1}{2} g_{s}^{2} e^{2 \phi} F_{\mu \nu} F_{\nu}^{\mu} \right\} - \frac{1}{2} g_{s}^{2} e^{2 \phi} \hat{F}_{\mu \nu} F_{\nu}^{\mu} = 0 \] (3.4)

\[ -\frac{1}{2} (h_{\rho \nu}^{\mu,\mu} \rho - h_{\rho \nu}^{\mu,\nu} \rho - h_{\rho}^{\mu,\mu} \rho) + \frac{1}{2} g_{s}^{2} (h_{\rho}^{\sigma,\sigma} - h_{\rho}^{\mu,\mu} - h_{\nu}^{\nu,\nu}) \partial_{\sigma} \bar{\phi} \]

\[ - \frac{3}{7} g_{s}^{2} (2 h_{\rho}^{\sigma,\rho} \sigma - h_{\rho}^{\mu,\mu}) \partial_{\sigma} \bar{\phi} + h_{\nu}^{\mu} \left\{ \frac{1}{2} R - \frac{6}{7} D_{\rho} \partial_{\nu} \phi - \frac{4}{7} \partial_{\sigma} \bar{\phi} \partial_{\rho} \bar{\phi} + \frac{1}{8} g_{s}^{2} e^{2 \phi} F_{\rho \sigma} F_{\nu}^{\rho \sigma} \right\} \]

\[ + h_{\nu}^{\rho} \left\{ R_{\rho}^{\ \nu} + \frac{6}{7} D_{\rho} \partial_{\nu} \phi - \frac{20}{49} \partial_{\rho} \bar{\phi} \partial_{\nu} \bar{\phi} - \frac{1}{2} g_{s}^{2} e^{2 \phi} F_{\rho \sigma} F_{\nu}^{\rho \sigma} \right\} \]

\[ - \frac{1}{2} g_{s}^{2} e^{2 \phi} \bar{\phi} h_{\rho \sigma}^{\mu} F_{\mu}^{\sigma} F_{\nu}^{\sigma} + g_{s}^{2} h_{\rho}^{\sigma} \left\{ \frac{1}{2 \rho} R_{\sigma}^{\rho} - \frac{6}{7} D_{\sigma} \partial_{\rho} \phi + \frac{4}{7} \partial_{\nu} \bar{\phi} \partial_{\rho} \bar{\phi} + \frac{1}{4} g_{s}^{2} e^{2 \phi} F_{\rho \sigma} F_{\nu}^{\rho \sigma} \right\} \]

\[ - \frac{6}{7} D_{\rho} \partial_{\nu} \phi + \frac{6}{7} g_{s}^{2} D_{\rho} \partial_{\nu} \phi + \frac{20}{49} \partial_{\rho} \bar{\phi} \partial_{\nu} \bar{\phi} + \frac{20}{49} \partial_{\mu} \bar{\phi} \partial_{\nu} \bar{\phi} - \frac{8}{7} g_{s}^{2} e^{2 \phi} \partial_{\rho} \bar{\phi} \partial_{\nu} \bar{\phi} \]

\[ + \frac{6}{7} \phi (R_{\mu}^{\nu} + \frac{6}{7} D_{\mu} \partial_{\nu} \phi - \frac{20}{49} \partial_{\mu} \bar{\phi} \partial_{\nu} \bar{\phi} + \frac{1}{2} g_{s}^{2} e^{2 \phi} F_{\mu}^{\nu \rho} F_{\nu}^{\rho}) \]

\[ + \frac{1}{7} \phi g_{s}^{2} \left\{ \frac{1}{4} \hat{F}_{\rho}^{\nu} F_{\nu}^{\rho} + \frac{1}{2} \hat{F}_{\rho}^{F_{\mu}^{\rho}} F_{\nu}^{\mu} - \frac{1}{4} g_{s}^{2} e^{2 \phi} \hat{F}_{\rho} F_{\rho} F_{\nu}^{\sigma} \right\} = 0 \] (3.5)

\[ - h_{\rho}^{\mu,\nu} F_{\rho}^{\mu} + \frac{1}{2} h_{\rho}^{\mu,\nu} F_{\rho}^{\mu} - h_{\rho}^{\mu,\nu} F_{\rho}^{\mu} + h_{\rho}^{\mu} \left\{ - D_{\rho} F_{\mu}^{\nu} - \frac{6}{7} \partial_{\rho} \bar{\phi} F_{\mu}^{\nu} \right\} \]

\[ + \frac{6}{7} \partial_{\rho} \bar{\phi} F_{\mu}^{\nu} + D_{\rho} \hat{F}_{\mu}^{\nu} + \frac{6}{7} \partial_{\rho} \bar{\phi} F_{\mu}^{\nu} = 0 \] (3.6)

They are obtained by the variations of dilaton, metric, and RR 1-form fields, respectively.

From now on unless otherwise specified, we use the following conventions for denoting the tensor indices: \( \mu, \nu, \ldots \) for full 10 dimensional contractions. \( i, j, \ldots \) for the metric on the sphere \( S^{8} \). Thus, for example, the covariant derivative \( D_{\mu} \) is defined using only the metric of \( S^{8} \). The convenience of the AdS frame metric \( ds^{2}_{AdS} \) is that the \( S^{8} \) metric is completely decoupled from that of AdS2. Note however that our background is never the AdS\(_{2}\times S^{8}\) itself: the Weyl factor induces complicated couplings between various modes.
in contrast to the case of ‘non-dilatonic’ D-branes where we can link the computations
to the representation theory of (super) conformal algebra, and makes our analysis fairly
nontrivial. Unfortunately, no analysis has been done on the group-theoretic aspect of the
generalized (super) conformal symmetry.

To analyze the spectrum it is necessary to fix the gauge. We adopt the following gauge
conditions anticipating the simplicity of the harmonic expansion,

\[ D_i (h^i_j - \frac{1}{8} \delta^i_j h^k_k) = 0, \]
\[ D^i h^0_i = D^i h^z_i = 0, \]
\[ D^i \hat{A}_i = 0. \]  

The reader should refer to Appendix for the connection of these gauge conditions and
the harmonic expansion. The number of the fields are originally 1(dilaton) +55 (metric)+10(RR 1-form). There are 10(metric)+1(RR 1-form) constraints coming from the
field equations. The gauge conditions (3.7) eliminate 8 + 2 + 1 = 11 components. Thus
we have 66 − 2 × 11 = 44 physical components. The result of the harmonic analysis on
S^8 will show that for generic value of angular momentum there is one radial degree of
freedom for the symmetric divergenceless and traceless tensor, two for the divergenceless
vector and three for scalar, which indeed sum up to 44 total degrees of freedom.

Using the standard general theory of tensor spherical harmonics, we can expand the
fluctuations as

\[ h^0_0(x^\mu) = \sum b^0_0(t, z) Y(x^i), \quad h^0_z(x^\mu) = \sum b^0_z(t, z) Y(x^i), \]
\[ h^z_z(x^\mu) = \sum b^z_z(t, z) Y(x^i), \quad h^i_i(x^\mu) = \sum b^i_i(t, z) Y(x^i), \]
\[ \hat{A}_0(x^\mu) = \sum a_0(t, z) Y(x^i), \quad \hat{A}_z(x^\mu) = \sum a_z(t, z) Y(x^i), \]
\[ \hat{\phi}(x^\mu) = \sum \varphi(t, z) Y(x^i), \]  

\[ h^0_i(x^\mu) = \sum b^0_i(t, z) Y_i(x^i), \quad h^z_i(x^\mu) = \sum b^z_i(t, z) Y_i(x^i), \]
\[ \hat{A}_i(x^\mu) = \sum a_i(t, z) Y_i(x^i), \]  

\[ h^i_j(x^\mu) - \frac{1}{8} \delta^i_j h^k_k(x^\mu) = \sum b(t, z) Y_{ij}(x^i). \]  

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We note that the gauge conditions (3.7) eliminate the possible contributions coming from the derivatives of the harmonics with lower number of tensor indices. Here, $Y, Y_i$ and $Y_{ij}$ are the scalar, vector and symmetric-traceless tensor harmonics of $S^8$, respectively. Note also that we have suppressed the angular momentum index $\ell$ in the harmonic expansion. See Appendix.

It is appropriate to classify the coefficient functions in these expansions into the following three categories: (1) scalar components (3.8), (2) vector components (3.9) and (3) tensor components (3.10). Since these harmonic functions are mutually independent (orthogonal to each other), the linearized equations can be separated into the coefficient equations for each harmonic functions. There arise 16 coefficient equations. The equation for tensor components $b$ is a single equation which is the symmetric traceless tensor part of the equation (3.5)

\[ \left[ -\frac{25}{8} z^2 \partial_0 \partial_0 b + \frac{25}{8} z^2 \partial_z \partial_z b - \frac{45}{8} z \partial_z b + \left( \frac{\lambda_T}{2} - 1 \right) b \right] Y_{jk} = 0, \quad \ell \geq 2. \quad (3.11) \]

For the vector components $(b^0, b^z, a)$, we have four equations; three of them come from (3.5) and one from (3.6), given as

\[ \frac{25}{8} \partial_z \partial_z b^0 + \frac{25}{8} \partial_0 \partial_0 b^z - \frac{95}{8} \frac{1}{z} \partial_z b^0 - \frac{45}{8} \frac{1}{z} \partial_0 b^z 
+ \left( \frac{\lambda_V}{2} + 21 \right) \frac{1}{z^2} b^0 - \frac{7}{2} \left( \frac{2}{5} \right) \frac{1}{q} g_s q^{-\frac{2}{5}} \frac{z}{z^2} \partial_z a = 0, \quad \ell \geq 1, \quad (3.12) \]

\[ -\frac{25}{8} \partial_z \partial_0 b^0 - \frac{25}{8} \partial_0 \partial_0 b^z + \frac{50}{8} \frac{1}{z} \partial_0 b^0 
+ \left( \frac{\lambda_V}{2} + \frac{7}{2} \right) \frac{1}{z^2} b^z + \frac{7}{2} \left( \frac{2}{5} \right) \frac{1}{q} g_s q^{-\frac{2}{5}} \frac{z}{z^2} \partial_0 a = 0, \quad \ell \geq 1, \quad (3.13) \]

\[ -\frac{1}{2} \partial_0 b^0 - \frac{1}{2} \partial_z b^z + \frac{19}{10} \frac{1}{z} b^z = 0, \quad \ell \geq 2, \quad (3.14) \]

\[ \frac{7}{g_s} \left( \frac{2}{5} \right) \frac{q}{q^2} \frac{z}{z^2} \left\{ \partial_z b^0 + \partial_0 b^z - \frac{2}{5} b^0 \right\} 
+ \frac{25}{4} \partial_0 \partial_0 a - \frac{25}{4} \partial_z \partial_z a - \frac{45}{4} \frac{1}{z} \partial_z a + \left( -\lambda_V + 7 \right) \frac{1}{z^2} a = 0, \quad \ell \geq 1. \quad (3.15) \]

There are remaining 11 coefficient equations for 7 scalar components $(b^0_0, b^z_0, b^z_z, b^z_i, a_0, a_z, \phi)$; the first of them comes from the dilaton equation (3.4).
From the metric equation (3.5) we have the following 7 equations
\[
\begin{align*}
-6 \left(\frac{2}{5}\right)^{-\frac{10}{9}} g s q^{-\frac{2}{5}} c^2 (\partial_0 a_z - \partial_z a_0) + \frac{\lambda s}{z^2} \left\{ \frac{3}{4} b_i^0 - \frac{3}{4} \frac{1}{2} b_i^0 + \frac{6}{7} b_i \right\} = 0, \quad \ell \geq 0. \quad (3.16)
\end{align*}
\]

From the metric equation (3.3) we have the following 7 equations
\[
\begin{align*}
-45 \frac{1}{4} \partial_z b_i^0 + 45 \frac{1}{4} \partial_z b_i^0 + 45 \frac{1}{4} \partial_z b_i^0 + 28 \frac{1}{z^2} b_i - 25 \partial_z \partial_0 b_i^0 + 5 \frac{1}{2} \partial_z b_i^0 - 7 \frac{1}{2} \frac{1}{z^2} b_i^0 + 75 \frac{1}{14} \partial_z \partial_z b_i^0 - 135 \frac{1}{14} \partial_z \partial_0 b_i^0 - 21 \frac{1}{z^2} b_i^0
\end{align*}
\]

\[
\begin{align*}
-45 \frac{1}{4} \partial_0 b_i^0 + 45 \frac{1}{4} \partial_0 b_i^0 + 45 \frac{1}{4} \partial_0 b_i^0 + 28 \frac{1}{z^2} b_i - 25 \partial_0 \partial_z b_i^0 + 5 \frac{1}{2} \partial_0 b_i^0 - 7 \frac{1}{2} \frac{1}{z^2} b_i^0 + 75 \frac{1}{14} \partial_0 \partial_0 b_i^0 - 135 \frac{1}{14} \partial_0 \partial_z b_i^0 - 21 \frac{1}{z^2} b_i^0
\end{align*}
\]

\[
\begin{align*}
-45 \frac{1}{4} \partial_0 b_i^0 + 45 \frac{1}{4} \partial_0 b_i^0 + 45 \frac{1}{4} \partial_0 b_i^0 + 28 \frac{1}{z^2} b_i - 25 \partial_0 \partial_z b_i^0 + 5 \frac{1}{2} \partial_0 b_i^0 - 7 \frac{1}{2} \frac{1}{z^2} b_i^0 + 75 \frac{1}{14} \partial_0 \partial_0 b_i^0 - 135 \frac{1}{14} \partial_0 \partial_z b_i^0 - 21 \frac{1}{z^2} b_i^0
\end{align*}
\]

\[
\begin{align*}
-45 \frac{1}{4} \partial_0 b_i^0 + 45 \frac{1}{4} \partial_0 b_i^0 + 45 \frac{1}{4} \partial_0 b_i^0 + 28 \frac{1}{z^2} b_i - 25 \partial_0 \partial_z b_i^0 + 5 \frac{1}{2} \partial_0 b_i^0 - 7 \frac{1}{2} \frac{1}{z^2} b_i^0 + 75 \frac{1}{14} \partial_0 \partial_0 b_i^0 - 135 \frac{1}{14} \partial_0 \partial_z b_i^0 - 21 \frac{1}{z^2} b_i^0
\end{align*}
\]
\[ \frac{1}{2}b_0^0 + \frac{1}{2}b_1^1 + \frac{3}{8}b_i^j - \frac{6}{7} \varphi = 0, \quad \ell \geq 2. \] (3.23)

Finally, the RR 1-form equation (3.6) gives the following 3 equations:

\[ - \frac{175}{8} \partial_z b_0^0 - \frac{175}{8} \partial_z b_1^1 + \frac{175}{8} \partial_z b_i^j + \frac{75}{2} \partial_z \varphi - \left( \frac{2}{5} \right) g_s q \sqrt{2} z \ \frac{10}{7} \left\{ \frac{25}{4} \partial_z (\partial_0 a_z - \partial_z a_0) + \frac{95}{4} \frac{1}{z} (\partial_0 a_z - \partial_z a_0) - \frac{\lambda_s a_0}{z^2} \right\} = 0, \quad \ell \geq 0, \] (3.24)

\[ - \frac{175}{8} \partial_0 b_0^0 - \frac{175}{8} \partial_0 b_1^1 + \frac{175}{8} \partial_0 b_i^j + \frac{75}{2} \partial_0 \varphi - \left( \frac{2}{5} \right) g_s q \sqrt{2} z \ \frac{10}{7} \left\{ \frac{25}{4} \partial_0 (\partial_0 a_z - \partial_z a_0) - \frac{\lambda_s a_0}{z^2} \right\} = 0, \quad \ell \geq 0, \] (3.25)

\[ - \frac{25}{4} \partial_0 a_0 + \frac{25}{4} \partial_z a_z + \frac{45}{4} \frac{1}{z} a_z = 0, \quad \ell \geq 1. \] (3.26)

In these equations, \( \lambda_T, \lambda_V \) and \( \lambda_S \) are the eigenvalues of the scalar Laplacian on \( S^8 \) on the tensor, vector and scalar harmonics respectively,

\[ \lambda_a = -\ell (\ell + 7) + n_a, \] (3.27)

with \( n_T = 2, n_V = 1 \) and \( n_S = 0 \).

Let us now analyze the spectrum using these equations. First, we treat the symmetric traceless tensor mode \( b \). The equation is already diagonalized as given in (3.11) which is solved by modified Bessel functions. The solution in the Euclidean space-time \((t \rightarrow -i\tau, \omega_M \rightarrow i\omega)\) which is regular at the origin is

\[ b = z^7 K_{\ell(2\ell+7)}(\omega z) e^{-i\omega \tau} \quad (\ell = 2, 3, \ldots). \] (3.28)

For the vector modes, we shall first analyze the special case \( \ell = 1 \). We have 3 equations (3.12), (3.13) and (3.15) for 3 variables \( b^0, b^i, a \). However, there is a residual gauge symmetry corresponding to the Killing vector on \( S^8 \)

\[ \xi_0 = \xi_z = 0, \quad \xi_i = \zeta Y_i^{(\ell=1)}, \]

by which the fields are transformed as

\[ \delta b^0 = -z^2 \partial_0 \zeta, \quad \delta b_i = z^2 \partial_i \zeta, \quad \delta a = 0. \]
One of the three variables can be gauged away using this residual symmetry and another one is determined by the constraint equation, so we expect one physical degree of freedom. By examining the equations (3.12) and (3.13), we first note that they are rewritten as

$$\partial_z \{ \hat{b} + 7\hat{a} \} = 0, \quad \partial_0 \{ \hat{b} + 7\hat{a} \} = 0,$$

where $\hat{a}$ and $\hat{b}$ are defined by

$$\hat{a} = g_5 a,$$

$$\hat{b} = -q_5^2 z^{-\frac{2}{5}} \left( 2 z \right)^{\frac{3}{5}} \left\{ \partial_0 \partial_z b^0 + \partial_z \partial_0 b^0 - \frac{2}{z} b^0 \right\}.\quad (3.29)$$

Thus, we can set

$$\hat{b} + 7\hat{a} = C \quad \text{(constant).}$$

Using (3.31) we can eliminate $\hat{b}$ from (3.15) and derive

$$\partial_z \partial_z \hat{a} - \partial_0 \partial_0 \hat{a} + \frac{9}{5} z \partial_z \partial_0 \hat{a} + \frac{252}{25} \frac{1}{z^2} (\hat{a} + C) = 0 \quad (3.32)$$

which is solved as

$$\hat{a} = z^{-\frac{3}{5}} K_{16}(\omega z) e^{-i\omega \tau}. \quad (3.33)$$

We have set the constant $C = 0$ by invoking the boundary condition (see section 4) that the action evaluated with the solution have no contribution from the boundary at the origin ($z \to \infty$). This condition which we always assume in the present work essentially amounts to requiring that the self-adjointness of the kinetic operator is not violated at the core, i.e., inside boundary $z \to \infty$. That is in general the natural boundary condition at points where the classical solutions are singular. For such an analysis of linear perturbations around singular classical solutions, we would like to refer the reader to [19].

Now for the generic case $\ell \geq 2$, there are four equations for three variables. First it is easy to check that the set of the equations are consistent by showing that (3.14) is derived from (3.12), (3.13) and (3.15). Since two of them (3.13) and (3.15) contain time derivative of second order, we note that there are two physical degrees of freedom. To obtain the diagonalized excitations, we make a linear combination of the derivatives of (3.12) and (3.13) which reads

$$\partial_z \partial_z (\hat{b} + 7\hat{a}) - \partial_0 \partial_0 (\hat{b} + 7\hat{a}) + \frac{9}{5} z \partial_z (\hat{b} + 7\hat{a}) + \frac{4}{25} \left\{ -\ell (\ell + 7) + 8 \right\} \frac{1}{z^2} \hat{b} = 0. \quad (3.34)$$
The coupled equations (3.15) and (3.34) can be diagonalized by defining
\[ \hat{a}_1 = \hat{a} - (\ell + 1)\hat{b}, \] (3.35)
\[ \hat{a}_2 = \hat{a} + \frac{1}{\ell + 6}\hat{b}, \] (3.36)
and the solutions are given by
\[ \hat{a}_1 = z^{-\frac{2}{5}} K_{\frac{2}{5}(\ell+7)}(\omega z) e^{-i\omega \tau}, \quad (\ell = 1, 2, \ldots) \] (3.37)
\[ \hat{a}_2 = z^{-\frac{2}{5}} K_{\frac{2}{5}}(\omega z) e^{-i\omega \tau}, \quad (\ell = 2, 3, \ldots) \] (3.38)
where we have included also the \( \ell = 1 \) mode.

Let us next turn to the scalar modes. We shall start by examining the modes with lowest angular momentum \( \ell = 0 \). There are 7 equations for 7 variables \( \varphi, b_0^0, b_z^0, b_z^i, a_0 \) and \( a_z \). As in the vector case, we have to examine the residual gauge symmetry for the equations. There are two kinds of residual gauge symmetries. One is the two dimensional diffeomorphism for which the gauge parameter is of the form
\[ \xi_0 = \zeta_0 Y^{(\ell=0)}, \quad \xi_z = \zeta_z Y^{(\ell=0)}, \quad \xi_i = 0, \]
which leads to
\[ \delta b_0^0 = 2 \partial_0 \zeta^0 - 2 \frac{1}{z} \zeta^z, \quad \delta b_z^0 = - \partial_0 \zeta^z + \partial_z \zeta^0, \quad \delta b_z^z = 2 \partial_z \zeta^z - 2 \frac{1}{z} \zeta^z, \quad \delta b_z^i = 0, \]
\[ \delta \varphi = \frac{21}{10} \frac{1}{z} \zeta^z, \quad \delta a_0 = - \frac{1}{g_s} \left( \frac{2}{25} \right)^{\frac{14}{5}} q^2 z^{-\frac{14}{5}} \left\{ \partial_0 \zeta^0 - \frac{14}{5} \frac{1}{z} \zeta^z \right\}, \quad \delta a_z = - \frac{1}{g_s} \left( \frac{2}{25} \right)^{\frac{14}{5}} q^2 z^{-\frac{14}{5}} \partial_z \zeta^0. \]
The other is the RR 1-form gauge transformation
\[ \Lambda = \lambda Y^{(\ell=0)}, \]
which leads to
\[ \delta a_0 = \partial_0 \lambda, \quad \delta a_z = \partial_z \lambda. \]
As in the vector case, 3 variables can be gauged away using this gauge freedom and another 3 variables will be determined by the constraints. We thus expect only one physical degree of freedom. For \( \ell = 0 \), we note that (3.24) and (3.25) can be rewritten, respectively, as
\[ \partial_z \left\{ - \frac{7}{2} b_0^0 - \frac{7}{2} b_z^z + \frac{7}{2} b_z^i + 6 \varphi - \left( \frac{2}{5} \right)^{\frac{10}{3}} g_s q^2 z^{-\frac{10}{3}} (\partial_0 a_z - \partial_z a_0) \right\} = 0, \] (3.39)
\[ \partial_0 \left\{ - \frac{7}{2} b_0^z - \frac{7}{2} b_z^z + \frac{7}{2} b_i^z + 6 \varphi - \left( \frac{2}{5} \right)^{\frac{10}{4}} g_s q^{-\frac{2}{5}} z^{\frac{10}{4}} (\partial_0 a_z - \partial_z a_0) \right\} = 0. \] (3.40)

Therefore we can set the quantity in the parenthesis to be a constant \((C_1)\)

\[ - \frac{7}{2} b_0^z - \frac{7}{2} b_z^z + \frac{7}{2} b_i^z + 6 \varphi - \left( \frac{2}{5} \right)^{\frac{10}{4}} g_s q^{-\frac{2}{5}} z^{\frac{10}{4}} (\partial_0 a_z - \partial_z a_0) = C_1 \] (3.41)

which enables us to eliminate \((\partial_0 a_z - \partial_z a_0)\). (3.19) can now be written as

\[ \partial_0 \left\{ - \frac{9}{4} z b_z^z - \frac{5}{4} \partial_z b_i^z - \frac{5}{4} b_i^z + \frac{15}{7} \partial_z \varphi \right\} = 0 \] (3.42)

and, using (3.41), (3.17) can be written as

\[ \partial_z \left\{ z^{-\frac{2}{5}} \left( - \frac{9}{4} z b_z^z - \frac{5}{4} \partial_z b_i^z - \frac{5}{4} b_i^z + \frac{15}{7} \partial_z \varphi \right) \right\} = 0. \] (3.43)

Thus we have

\[ - \frac{9}{4} z b_z^z - \frac{5}{4} \partial_z b_i^z - \frac{5}{4} b_i^z + \frac{15}{7} \partial_z \varphi = C_2 z^2 \] (3.44)

We can set the constants \(C_1 = C_2 = 0\) for the same reason as in the vector case. Combining the equations (3.16), (3.18), (3.20) and using the constraints (3.41), (3.44), we obtain the diagonalized equation for a physical mode \(b_i^z\)

\[ - \partial_0 \partial_0 b_i^z + \partial_z \partial_z b_i^z - \frac{9}{5} \partial_z b_i^z - \frac{392}{25} \frac{1}{z^2} b_i^z = 0. \] (3.45)

All the other variables can be gauged away or determined by the constraints. Indeed, if we adopt the gauge conditions \(b_0^0 = b_z^z = a_0 = 0\), the rest of the variables \(a^z, b_z^z, \varphi\) are determined by (3.41), (3.44), and another constraint equation which follow from (3.16), (3.18), (3.20), (3.41) and (3.44)

\[ \frac{5}{2} \partial_z \partial_0 b_z^0 - \frac{5}{2} \partial_z b_z^0 - \frac{5}{4} \partial_z b_i^0 - \frac{19}{2} \frac{1}{z^2} b_i^0 = 0. \]

It is easy to check the consistency of the result by seeing that the set of the solutions obtained above actually satisfies the original equations.

We can repeat the similar analysis for \(\ell = 1\). In this case, there are 10 equations for 7 variables. We first check that 3 of them are consequences of the other equations: for \(\ell \geq 1\), (3.17) + (3.19) + (3.23) \(\rightarrow\) (3.21), (3.17) + (3.18) + (3.13) + (3.16) + (3.24) \(\rightarrow\) (3.22), (3.24) + (3.25) \(\rightarrow\) (3.26). There is a residual gauge symmetry associated with the
conformal Killing vector on $S^8$, (the so-called 'conformal diffeomorphism' [18]). It is the
diffeomorphism whose parameters take the form

$$
\xi_0 = \partial_0 \eta Y^{(\ell = 1)}, \quad \xi_z = \partial_z \eta Y^{(\ell = 1)}, \quad \xi_i = -\eta D_i Y^{(\ell = 1)}.
$$

which leads to

$$
\delta b_0^0 = -z^2 \partial_0 \partial_0 \eta - z \partial_z \eta, \quad \delta b_z^0 = -z^2 \partial_0 \partial_z \eta - z \partial_0 \eta, \quad \delta b_z^z = z^2 \partial_z \partial_z \eta + z \partial_z \eta, \quad \delta b_i^i = \frac{32}{25} \eta.
$$

We find two physical degrees of freedom for $\ell = 1$ mode. Since the final result, however,
is more conveniently summarized by using the dynamical modes for generic $\ell$, let us now
turn to the case $\ell \geq 2$.

First, there are 11 equations for 7 variables. We can check that 4 of them are conse-
quences of the other equations. In addition to the relations mentioned in the $\ell = 1$ case
above, we have (3.21) + (3.22) + (3.26) $\rightarrow$ (3.23) for $\ell \neq 1$. Only 3 of the remaining 7
equations are the equations of motion. The combinations of the 3 dynamical components
which diagonalize the equations of motion are given as

$$
s_1 = z^{-7/5}\left(-\frac{7(-\ell + 7)}{16} b_i^i + f\right),
$$

$$
s_2 = z^{-12/5}\left(\frac{\ell(\ell + 7)}{4} b_z^z + \frac{5\ell(\ell + 7)}{21} \partial_\varphi + \frac{5\left(-\ell(\ell + 7) + 49\right)}{64}\partial_\partial b_i^i - \frac{5}{8} z \partial_0 f\right),
$$

$$
s_3 = z^{-7/5}\left(-\frac{7(\ell + 14)}{16} b_i^i + f\right),
$$

where

$$
f = g_s \left(\frac{5}{2}\right)^{19/5} q^{-2/5} z^{19/5} (\partial_0 a_z - \partial_z a_0).
$$

It can be checked that $s_3$ is a gauge mode for $\ell = 1$. That is, we can proceed in the same
way as $\ell \geq 2$ case if we impose (3.23) using the residual gauge symmetry of the $\ell = 1$
mode. However, there still remains a gauge symmetry which enables us to gauge away
$s_3$. Also, note that the result agrees with the analysis for $\ell = 0$. In this case, we impose
(3.21), (3.22), (3.26) and (3.23) using the residual gauge symmetries. From the constraint
and the gauge condition (3.23), we can see that $s_2$ and $s_3$ vanish for $\ell = 0$. The final results for the equations and their solutions are summarized as

$$
-\partial_0 \partial_0 s_1 + \partial_z \partial_z s_1 + \frac{1}{z} \partial_z s_1 - \frac{(2\ell + 21)^2}{25} \frac{1}{z^2} s_1 = 0,
$$

$$
-\partial_0 \partial_0 s_2 + \partial_z \partial_z s_2 + \frac{1}{z} \partial_z s_2 - \frac{(2\ell + 7)^2}{25} \frac{1}{z^2} s_2 = 0,
$$

$$
-\partial_0 \partial_0 s_3 + \partial_z \partial_z s_3 + \frac{1}{z} \partial_z s_3 - \frac{(2\ell - 7)^2}{25} \frac{1}{z^2} s_3 = 0,
$$

(3.49)

$$
\begin{align*}
s_1 &= K_{\frac{5}{2}(2\ell+21)}(\omega z)e^{-i\omega \tau}, \quad \text{for } \ell \geq 0, \\
s_2 &= K_{\frac{5}{2}(2\ell+7)}(\omega z)e^{-i\omega \tau}, \quad \text{for } \ell \geq 1, \\
s_3 &= K_{\frac{5}{2}(2\ell-7)}(\omega z)e^{-i\omega \tau}, \quad \text{for } \ell \geq 2.
\end{align*}
$$

(3.50)

3.2 The fluctuations of NS-NS 2 form and RR 3-form fields

Let us next study the fluctuations whose background fields are trivial; $B_{\mu\nu}^{(total)} = \hat{B}_{\mu\nu}$ and $A_{\mu\nu\rho}^{(total)} = \hat{A}_{\mu\nu\rho}$. The equations of motions for NSNS 2 form and RR 3 form fields are

$$
D^\sigma \left( e^{-\frac{10}{9}\hat{\phi}} H_{\sigma\mu\nu} \right) + g_\mu^2 D^\sigma \left( A^\rho e^{\frac{1}{2}\hat{\phi}} \tilde{F}_{\rho\mu\nu} \right) = 0,
$$

(3.51)

$$
D^\sigma \left( e^{\frac{10}{9}\hat{\phi}} \tilde{F}_{\sigma\mu\nu\rho} \right) = 0,
$$

(3.52)

respectively. The natural gauge conditions for performing the harmonic analysis are

$$
\begin{align*}
D^i \hat{B}_{0i} &= D^i \hat{B}_{z i} = D^i \hat{B}_{ij} = 0 \\
D^i \hat{A}_{0zi} &= D^i \hat{A}_{0ij} = D^i \hat{A}_{z ij} = D^i \hat{A}_{ijk} = 0
\end{align*}
$$

(3.53)

which make us possible to expand as

$$
\begin{align*}
\hat{B}_{0z} &= \beta_0 z Y, \quad \hat{B}_{0i} = \beta_0 Y_i, \quad \hat{B}_{z i} = \beta_2 Y_i, \quad \hat{B}_{ij} = \beta Y_{[ij]} \\
\hat{A}_{0zi} &= \alpha_0 z Y_i, \quad \hat{A}_{0ij} = \alpha_0 Y_{[ij]}, \quad \hat{A}_{z ij} = \alpha z Y_{[ij]}, \quad \hat{A}_{ijk} = \alpha Y_{[ijk]}
\end{align*}
$$

(3.54)

where $Y_i$, $Y_{[ij]}$ and $Y_{[ijk]}$ are called 1-form, 2-form and 3-form harmonics respectively whose definitions and properties are summarized in Appendix. For notational brevity, we abbreviated the sum over $\ell$. 

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We first comment on the counting of the degrees of freedom. The number of the fields are originally $45 (\hat{B}_{\mu \nu}) + 120 (\hat{A}_{\mu \nu \rho})$. There are $8 (\hat{B}_{\mu \nu}) + 28 (\hat{A}_{\mu \nu \rho})$ constraints coming from the field equations, and the gauge conditions (3.53) eliminate $9 + 36$ components. Thus we have $84$ physical components. In terms of the expansion by spherical harmonics, we will see that for the generic case ($\ell \geq 1$) there is no radial degree of freedom for the scalar, one for 1-form, two for divergenceless 2-form and one for divergenceless 3-form, which indeed sum up to 84 degrees of freedom.

As in the previous subsection, let us start from the case of lowest angular momentum. From the definition, there is no $\ell = 0$ contribution for $p$-form harmonics with nonzero $p$. Therefore, only possibility for $\ell = 0$ mode is the $S^8$ scalar component $\beta_{0z}$. However, it is easy to see that the scalar component $\beta_{0z}$ can be gauged away using the residual gauge symmetry for $\hat{B}_{\mu \nu}$ with parameter

$$
\Lambda_0 = \lambda_0 Y^{(\ell=0)}, \quad \Lambda_z = \lambda_z Y^{(\ell=0)} \quad \Lambda_i = 0.
$$

which leads to

$$
\delta \beta_{0z} = \partial_0 \lambda_z - \partial_z \lambda_0.
$$

Thus there is no $\ell = 0$ mode in the physical spectrum.

Now, we shall analyse the generic $\ell \geq 1$ case. The equations listed in the rest of this section are valid for $\ell \geq 1$. The component equations for the scalar harmonics are

$$
\beta_{0z} = 0, \quad \partial_z \beta_{0z} - \frac{1}{z} \beta_{0z} = 0, \quad \partial_0 \beta_{0z} = 0,
$$

which are solved trivially by

$$
\beta_{0z} = 0.
$$

The equations for 1-form harmonics are

$$
\partial_z \partial_z \beta_0 - \partial_z \partial_0 \beta_z - \frac{1}{z} \partial_z \beta_0 + \frac{1}{z} \partial_0 \beta_z - \frac{4}{25} (\ell + 6)(\ell + 1) \frac{1}{z^2} \beta_0 = 0,
$$

$$
\partial_z \partial_0 \beta_0 - \partial_0 \partial_0 \beta_z - \frac{4}{25} (\ell + 6)(\ell + 1) \frac{1}{z^2} \beta_0 = 0,
$$

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\[ \partial_0 \beta_0 - \partial_z \hat{\alpha}_0 z + \frac{3}{z} \hat{\alpha}_0 z = 0, \]  
(3.61)  
\[ \hat{\alpha}_0 z - \beta_z = 0, \]  
(3.62)  
\[ \partial_z (\hat{\alpha}_0 z - \beta_z) - \frac{11}{5} z (\hat{\alpha}_0 z - \beta_z) = 0, \]  
(3.63)  
\[ \partial_0 (\hat{\alpha}_0 z - \beta_z) = 0, \]  
(3.64)

where we have defined
\[ \hat{\alpha}_0 z = g_s \left( \frac{2}{5} \right) - \frac{\frac{14}{5}}{z} q^{-\frac{1}{4}} z^{\frac{14}{5}} \alpha_0 z. \]  
(3.65)

There are six equations for three variables. The consistency of the equations is proved by noticing that (3.61) is derived by (3.59) and (3.60), and that (3.63) and (3.64) are consequences of (3.62). Using the similar arguments as in the previous subsection, this shows that there is only one physical degree of freedom. The radial equation is obtained by combining \( z \)-derivative of (3.59) and \( 0 \)-derivative of (3.60)
\[ \partial_z \partial_z u + \frac{1}{z} \partial_z u - \partial_0 \partial_0 u - \frac{(2\ell + 7)^2}{25} \frac{1}{z^2} u = 0, \]  
(3.66)

where
\[ u = \partial_z \beta_0 - \partial_0 \beta_z. \]  
(3.67)

The solution is
\[ u = K_{\frac{1}{2}(2\ell+7)}(\omega z) e^{-i\omega r}. \]  
(3.68)

For the 2-form modes, there are four equations for three variables \( \beta, \alpha_0 \) and \( \alpha_z \).
\[ -\partial_0 \partial_0 \hat{\beta} + \partial_z (\partial_z \alpha_0 - \partial_0 \alpha_z) - \frac{11}{5} z \partial_z \alpha_0 - \partial_0 \alpha_z - \frac{4}{25} (\ell + 5)(\ell + 2) \frac{1}{z^2} \alpha_0 = 0, \]  
(3.69)  
\[ -\partial_z \partial_z \hat{\beta} - \frac{27}{5} \frac{1}{z^2} \partial_z \hat{\beta} + \frac{4}{25} (\ell + 9)(\ell - 2) \frac{1}{z^2} \hat{\beta} \]  
\[ + \partial_z (\partial_z \alpha_0 - \partial_0 \alpha_z) + \frac{13}{5} z \partial_z \alpha_0 - \partial_0 \alpha_z - \frac{4}{25} (\ell + 5)(\ell + 2) \frac{1}{z^2} \alpha_0 = 0, \]  
(3.70)  
\[ -\partial_0 \partial_0 \hat{\beta} - \frac{14}{5} \frac{1}{z^2} \partial_0 \hat{\beta} + \partial_0 (\partial_z \alpha_0 - \partial_0 \alpha_z) - \frac{4}{25} (\ell + 5)(\ell + 2) \frac{1}{z^2} \alpha_z = 0, \]  
(3.71)  
\[ -\partial_0 \hat{\beta} + \partial_0 \alpha_0 - \partial_z \alpha_z - \frac{3}{5} \frac{1}{z} \alpha_z = 0, \]  
(3.72)

where
\[ \hat{\beta} = \frac{1}{g_s} \left( \frac{2}{5} \right) - \frac{\frac{14}{5}}{z^2} q^{-\frac{1}{4}} z^{\frac{14}{5}} \beta. \]  
(3.73)
The consistency of the equations can be proved by checking that (3.72) is derived from (3.70) and (3.71). There are two physical components.

To solve the equations, we define the combination \( \hat{\alpha}_1 = z(\partial_z \alpha_0 - \partial_0 \alpha_z - \partial_z \hat{\beta}) \). From (3.69) and (3.70), we find

\[
- \frac{1}{2} \partial_0 \partial_0 \hat{\beta} + \frac{13}{5} \frac{1}{z} \partial_z \hat{\beta} - \frac{4}{25} (\ell + 9)(\ell - 2) \frac{1}{z^2} \hat{\beta} - \frac{14}{5} \frac{1}{z} \partial_0 \hat{\alpha}_1 = 0
\]  
(3.74)

and, from (3.70) and (3.71),

\[
- \partial_0 \partial_0 \hat{\alpha}_1 + \frac{13}{5} \frac{1}{z} \partial_z \hat{\alpha}_1 - \frac{4}{25} (\ell + 5)(\ell + 2) \frac{1}{z^2} \hat{\alpha}_1 - \frac{14}{5} \left\{ - \partial_0 \partial_0 \hat{\beta} + \partial_z \partial_z \hat{\beta} + \frac{13}{5} \frac{1}{z} \partial_z \hat{\beta} \right\} = 0.
\]  
(3.75)

The coupled equations (3.74) and (3.75) can be diagonalized by defining

\[
v_1 = z^4 \left( \hat{\alpha} - \frac{2}{5} (\ell + 9) \hat{\beta}_1, \right)
\]  
(3.76)

\[
v_2 = z^4 \left( \hat{\alpha} + \frac{2}{5} (\ell - 2) \hat{\beta}_1, \right)
\]  
(3.77)

which lead to

\[
- \partial_0 \partial_0 v_1 + \frac{1}{z} \partial_z v_1 - \frac{4}{25} \ell^2 \frac{1}{z^2} v_1 = 0,
\]  
(3.78)

\[
- \partial_0 \partial_0 v_2 + \frac{1}{z} \partial_z v_2 - \frac{4}{25} (\ell + 7)^2 \frac{1}{z^2} v_2 = 0.
\]  
(3.79)

The solutions are

\[
v_1 = K_{\xi \ell}(\omega z) e^{-i\omega \tau},
\]  
(3.80)

\[
v_2 = K_{\xi(\ell+7)}(\omega z) e^{-i\omega \tau}.
\]  
(3.81)

Finally, for 3-form modes, there is only one equation for one variable

\[
- \partial_0 \partial_0 \alpha + \frac{3}{5} \frac{1}{z} \partial_z \alpha - \frac{4}{25} (\ell + 4)(\ell + 3) \frac{1}{z^2} \alpha = 0,
\]  
(3.82)

which is solved as

\[
\alpha = z^4 K_{\xi(2\ell+7)}(\omega z) e^{-i\omega \tau}.
\]  
(3.83)

We have completed the harmonic analysis of bosonic fluctuations around the D0 background.
4. Supergravity-Matrix theory correspondence

We now try to establish the correspondence between 10 dimensional supergravity and Matrix theory in the large \( N \) limit using the results of the harmonic analysis. We will follow the specific prescription in Euclidean metric given in [2] by assuming that the ‘boundary’ on the side of supergravity is located at the limit of the region of validity for the near horizon condition, namely, at \( z = q^{\frac{1}{7}} \) \( (r \propto q^{1/7}) \). Remember, as emphasized in section 2, that this is compulsory because we are interested in the region \( g_s N > 1 \).

Since the perturbing fields are diagonalized, we can discuss each diagonalized components separately.

First we consider the traceless-symmetric tensor mode as a simple example and then present the general result. The relevant part of the action is

\[
S = \frac{1}{8\kappa^2} \int d\Omega_8 Y_{ij} Y_{i'j'} g^{ij} g^{i'j'} \int d\tau \int dz \left( \frac{2}{9} \right) \frac{2}{5} q^{\frac{8}{5}} z^{-\frac{2}{5}} [\partial_z b^I b^J + \partial_0 b^I \partial_0 b^J]
+ \frac{4}{25} \ell (\ell + 7) \frac{1}{z^2} b^I b^J \right) \tag{4.1}
\]

where we have introduced the indices \( I, J \), labeling the harmonics that have been suppressed in the last section. The action evaluated for the classical solution which we call \( K \) can be expressed in terms of the boundary value of the field,

\[
K = \frac{1}{8\kappa^2} C \delta^{IJ} \int d\tau \left( \frac{2}{9} \right) \frac{2}{5} q^{\frac{8}{5}} \left[ z^{-\frac{2}{5}} b^I b^J \right]_q \tag{4.2}
\]

The harmonics are normalized as \( \int d\Omega_8 Y_{ij} Y_{i'j'} g^{ij} g^{i'j'} = C \delta^{IJ} \) with \( C \) being a numerical constant independent of \( I, J \) and of \( q \propto g_s N \). In what follows we always suppress the indices \( I \). The solution \( b \) satisfying the boundary condition \( b(q^{1/7}, \tau) = \int d\omega e^{-i\omega \tau} f_\omega \) is

\[
b(\tau, z) = \int d\omega e^{-i\omega\tau} \tilde{b_\omega}(z) f_\omega \tag{4.3}
\]

where \( \tilde{b_\omega}(z) \) is the solution of the radial equation normalized to 1 at the boundary \( (z \to q^{\frac{1}{7}}) \),

\[
\tilde{b_\omega}^I(z) = \frac{z^{\frac{2}{5}} K^{\frac{1}{2}(2\ell+7)}(\omega z)}{q^{\frac{1}{5}} K^{\frac{1}{4}(2\ell+7)}(\omega q^{1/7})} \quad \text{(for} \quad \ell = 2, 3, \ldots) \tag{4.4}
\]
Then the action is evaluated as
\[
K = -\frac{\pi}{4\kappa^2} C \left(\frac{2}{3}\right) \frac{q}{\lambda} \int d\omega f_\omega f_{-\omega} \left[ -\frac{2}{5} \ell + (\text{terms analytic in } \omega) \right]
\]
\[
+ (q^2 \omega)^{\frac{2}{3}(2\ell+7)} \left\{ -2^{-\frac{47}{10}} \frac{\Gamma(\frac{2}{5} \ell + \frac{7}{5})}{\Gamma(\frac{2}{5} \ell + \frac{2}{5})} \right\} + (\text{terms analytic in } \omega) \right\} .
\] (4.5)

The connected part of the two-point function of the operator of Matrix theory which couples to \( b \) is now given by the second variation of (4.5) with respect to \( f_\omega \). The leading part in the long-time region is
\[
\langle O(\tau)O(\tau') \rangle_c = \int d\omega \int d\omega' e^{i\omega \tau} e^{i\omega' \tau'} \langle O(\omega)O(\omega') \rangle_c = -\int d\omega \int d\omega' e^{i\omega \tau} e^{i\omega' \tau'} \frac{\delta}{\delta f_\omega} \frac{\delta}{\delta f_{\omega'}} K[\lambda]
\]
\[
= 2^{-\frac{47}{10}(2\ell+7)} \frac{p_2}{\ell + \frac{7}{5}} \left( \frac{2}{5} \right) \frac{\Gamma(\frac{2}{5} \ell + \frac{19}{5})}{\Gamma(\frac{2}{5} \ell + \frac{7}{5})} \right\} \frac{1}{|\tau - \tau'|^{\frac{47}{10} + \frac{19}{5}}}
\] (4.6)

where we have ignored the short time delta-function singularities coming from the analytic terms in \( \omega \) as usual.\(^8\) This result shows that the scaling dimension of the operator under the generalized scaling transformation \( \tau \rightarrow \lambda^{-1} \tau, g_s \rightarrow \lambda^3 g_s \) is
\[
\Delta = 1 + \frac{4}{7} \ell .
\] (4.7)

The final results of all the modes including this case can succinctly be expressed in a unified manner as follows.

The linearized action, up to a total derivative term which does not contribute to the large time behavior of the correlators, is
\[
S = \frac{1}{8\kappa^2} \sum_{IJ} \int d\tau \int dqz \left[ \partial_\tau g^I \partial_\tau g^J + \partial_0 g^I \partial_0 g^J + \nu^2 \frac{1}{z^2} g^I g^J \right]
\] (4.8)

where \( g^I \) denotes each diagonalized field and \( C_{I^J} \) comes from the normalization of each kind of the spherical harmonics. The diagonalized fields \( g^I \) are redefined by making suitable scalings by powers of \( z \) such that they obey the Bessel equations without any pre-factor of \( z \). We also note that, in terms of \( g^I \) the boundary condition at the inside boundary, \( r = 0 \) or \( z \rightarrow \infty \), corresponding to the self-adjointness of the kinetic operator which enables us to extract the correlators from the near-horizon boundary, is \( z g^I \partial_\tau g^I \rightarrow \)

\(^8\)While we were preparing the present manuscript, we came to know that a similar result (\( \ell = 0 \)) for D1-brane has been given in a recent paper [23] for a certain particular mode of metric fluctuations.
0. The constant $\nu$ is the order of the Bessel function. The leading part of the correlator, omitting numerical proportional constant, is

$$\langle \mathcal{O}(\tau)\mathcal{O}(\tau') \rangle_c = \frac{1}{\kappa^2 q^{1+\tilde{\nu}}} \frac{1}{|\tau - \tau'|^{2\nu+1}},$$

(4.9)

giving the general formula for the scaling dimension of the operator $\mathcal{O}$

$$\Delta = -1 + \frac{10}{7}\nu.$$  

(4.10)

The results are summarized in the tables below.

| Table 1 |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| SUGRA fields    | $h^i_j$         | $h^0_i, h^z_i, A_i$ | $\phi, h^0_0, h^0_z, h^z_0, A_0, A_z$ |
| physical modes  | $b$             | $a_1$           | $a_2$           | $s_1$           |
| order $\nu$     | $\frac{1}{3}(2\ell + 7)$ | $\frac{2}{7}(2\ell + 21)$ | $\frac{1}{7}(2\ell + 7)$ | $\frac{1}{7}(2\ell - 7)$ |
| dimensions of $\mathcal{O}$ | $1 + \frac{4}{7}\ell$ | $3 + \frac{4}{7}\ell$ | $-1 + \frac{4}{7}\ell$ | $5 + \frac{4}{7}\ell$ |
| regions of $\ell$ | $\ell \geq 2$ | $\ell \geq 1$ | $\ell \geq 2$ | $\ell \geq 0$ |

| Table 2 |
|-----------------|-----------------|-----------------|
| SUGRA fields    | $B_{0i}, B_{zi}$ | $B_{ij}, A_{0ij}, A_{zij}$ | $A_{ijk}$ |
| physical mode   | $a$             | $v_1$           | $v_2$           | $\alpha$ |
| order $\nu$     | $\frac{1}{3}(2\ell + 7)$ | $\frac{2}{7}(2\ell + 21)$ | $\frac{1}{7}(2\ell + 7)$ | $\frac{1}{7}(2\ell - 7)$ |
| dimensions of $\mathcal{O}$ | $1 + \frac{4}{7}\ell$ | $3 + \frac{4}{7}\ell$ | $-1 + \frac{4}{7}\ell$ | $1 + \frac{4}{7}\ell$ |
| regions of $\ell$ | $\ell \geq 1$ | $\ell \geq 1$ | $\ell \geq 1$ | $\ell \geq 1$ |

We note that, up to a total derivative term, the above linearized action (4.8) is equivalent to the s-wave part of the following special (Euclidean) action for a massive scalar field $\psi$ after making a scaling transformation $\psi \rightarrow \exp(2\tilde{\phi}/3)\psi$,

$$S = \frac{1}{2} \int d^{10}x \sqrt{-g} e^{-2\phi} [g^{\mu\nu} \partial_\mu \psi \partial_\nu \psi + m^2 e^{-2\phi/7} \psi^2]$$

(4.11)

where the mass is related to the order $\nu$ by

$$\tilde{m}^2 + \frac{49}{25} = \nu^2,$$  

(4.12)
with $25\tilde{m}^2/4 = \left(\frac{2}{ny}\right)^{2/7}m^2$. The total derivative term, being analytic in $\omega$, does not contribute to the long-time behavior of the correlation function. Thus the relation between the mass and the generalized conformal dimension is

$$
\Delta = -1 \pm \frac{10}{7} \sqrt{\tilde{m}^2 + \frac{49}{25}}. \quad (4.13)
$$

For example, for the traceless-symmetric tensor mode, we have $\tilde{m}^2 \to \frac{4\ell(\ell+7)}{25}$ by choosing the branch of the square root such that $\Delta$ has the positive coefficient with respect to $\ell$. It is an interesting question whether the above mass can be related to the Casimir operator in the representation of the generalized conformal symmetry.

We are now ready to discuss the correspondence of the spectrum of supergravity to the Matrix-theory operators. Various currents (more precisely $x^-$-integrated currents) of Matrix theory have been identified in the work [20] from the results of perturbative calculations for the interactions between pairs of general background configurations of Matrix theory. Let us quote their results below using their convention. We will only present the parts of the definitions to the extent that are needed in order to read off their generalized conformal dimensions. These operators have definite dimensions under the generalized conformal transformations. Note that the generalized conformal dimensions are not identical to the ‘engineering dimensions’. For full expressions of these operators, we refer the reader to [20]. Anticipating the correspondence with 11 dimensional supergravity, we will use their notations using the 11 dimensional light-cone indices. Corresponding to 11 dimensional metric, we have

\[
\begin{align*}
T^{++} &= \frac{1}{R} \text{STr}(1), \\
T^{+i} &= \frac{1}{R} \text{STr}(\dot{X}_i), \\
T^+- &= \frac{1}{R} \text{STr}\left(\frac{1}{2} \dot{X}_i \dot{X}_i + \cdots\right), \\
T^{ij} &= \frac{1}{R} \text{STr}(\dot{X}_i \dot{X}_j + \cdots), \\
T^{-i} &= \frac{1}{R} \text{STr}\left(\frac{1}{2} \dot{X}_j \dot{X}_j + \cdots\right), \\
T^{--} &= \frac{1}{4R} \text{STr}(F_{ab}F_{bc}F_{cd}F_{da} + \cdots).
\end{align*}
\]
Corresponding to 11 dimensional 3-form, we have

\[
J^{+ij} = \frac{1}{6R} \text{Str}(F_{ij}),
\]

\[
J^{-i} = \frac{1}{6R} \text{Str}(F_{ij} \dot{X}_j + \cdots),
\]

\[
J^{ijk} = \frac{1}{6R} \text{Str}(-\dot{X}_i F_{jk} - \dot{X}_j F_{ki} - \dot{X}_k F_{ij} + \cdots),
\]

\[
J^{-ij} = \frac{1}{6R} \text{Str}(\dot{X}_i \dot{X}_k F_{kj} - \dot{X}_j \dot{X}_k F_{ki} + \cdots).
\]

(4.15)

We have omitted the 6-form current since the supergravity fluctuations do not contain it directly. The convention is that the indices \(i,j,\ldots\) run over 9 spatial dimensions, and the indices \(a,b,\ldots\) do over the 10 dimensions (=time + 9 spatial dimensions). The field strength \(F_{ab}\) thus consists of 2 part, \(F_{0i} = \dot{X}_i\) and \(F_{ij} = [X_i, X_j]/\ell_s^2\). To avoid possible confusion, we remark that the light-cone indices on these operators must be interpreted as being for the current densities before integration. There is implicitly a hidden integration over \(x^-\) which is not manifest in Matrix theory. The generalized conformal dimensions of these operators are given in Tables 3 and 4.

**Table 3**

| currents | \(T^{++}\) | \(T^{+i}\) | \(T^{--}\) | \(T^{ij}\) | \(T^{-i}\) |
|----------|----------|----------|----------|----------|----------|
| dimensions | -3       | -1       | 1         | 1         | 3         | 5         |

**Table 4**

| currents | \(J^{+ij}\) | \(J^{+-i}\) | \(J^{ijk}\) | \(J^{-ij}\) |
|----------|----------|----------|----------|----------|
| dimensions | -1       | 1         | 1         | 3         |

Comparing the tables 3 and 4 with the tables 1 and 2, respectively, it is natural to make the identifications listed in tables 5 and 6 below between the Matrix-theory operators and the supergravity fluctuations at the boundary.
Table 5

| Matrix operators | $T_{\ell}^{++}$ | $T_{\ell}^{++}$ | $T_{\ell}^{+}$ | $T_{\ell}^{+}$ | $T_{\ell}^{-}$ | $T_{\ell}^{-}$ |
|------------------|----------------|----------------|-------------|-------------|-------------|-------------|
| SUGRA modes      | $s_{1}^{\ell}$ | $a_{2}^{\ell}$ | $s_{2}^{\ell}$ | $b^{\ell}$ | $a_{1}^{\ell}$ | $s_{1}^{\ell}$ |

Table 6

| Matrix operators | $J_{\ell}^{ij}$ | $J_{\ell}^{ij}$ | $J_{\ell}^{ijk}$ | $J_{\ell}^{-ij}$ |
|------------------|----------------|----------------|-----------------|-----------------|
| SUGRA modes      | $v_{2}^{\ell}$ | $u^{\ell}$     | $\alpha^{\ell}$ | $v_{1}^{\ell}$  |

The indices $\ell$ in these tables denote the $\ell$-th component in the harmonic expansion. On the side of Matrix theory, the corresponding operators up to possible ordering ambiguity are, *e. g.*, 

\[
T_{\ell,i_{i_{1}}i_{2}...i_{\ell}}^{++} = \frac{1}{R} \text{STr}(\bar{X}_{i_{1}}\bar{X}_{i_{2}}...\bar{X}_{i_{\ell}} + \cdots), \quad (\ell \geq 2)
\]

\[
T_{\ell,i_{i_{1}}i_{2}...i_{\ell}}^{+i} = \frac{1}{R} \text{STr}(\bar{X}_{i}\bar{X}_{i_{1}}\bar{X}_{i_{2}}...\bar{X}_{i_{\ell}} + \cdots), \quad (\ell \geq 2)
\]

etc,

where the coordinate matrices corresponding to the nonzero orbital angular momentum are normalized by dividing by the radial distance $q^{1/7}$ of the boundary, $\bar{X}_{i} \equiv X_{i}/q^{1/7}$ which accounts for the coefficient $4/7 = 1 - 3/7$ of $\ell$ in the formula of the generalized conformal dimensions (4.7). The noncontracted spatial indices of the currents here only take the values from the 8 dimensional space $S^{8}$ instead of the full 9 dimensional space. Note also that the trace part of the noncontracted indices should be subtracted, corresponding to the 8 dimensional harmonic expansion. For example, the operator $T_{\ell}^{ij}$ in the Table 5 must be traceless with respect to the symmetric tensor indices $ij$ and to the orbital part, separately. The tilde on the operator $\tilde{T}_{\ell}^{++} = T_{\ell}^{++} + cT_{\ell}^{ji}$ indicates that this operator can mix [21] with the trace part of $T_{\ell}^{ij}$ whose coefficient $c$ cannot be predicted from our results alone.

On the supergravity side, the components which have indices along the radial direction and also the ones with lower angular momentum than the restriction indicated in Tables

\[\text{We would like to thank W. Taylor for a comment on this.}\]
1 and 2 are either pure gauge modes or not independent physical propagating modes in the bulk.

The agreement of the generalized conformal dimensions between the fluctuations of supergravity and the Matrix-theory operators is not surprising if we consider it from 11 dimensional viewpoint of discrete light-cone quantization (DLCQ). The reason is (see the second paper in ref. [6]) that the scaling transformations (2.6) and (2.7) are equivalent, as was already mentioned in section 2, to the following boost-like transformation (Minkowski metric)

\[ t \rightarrow \lambda^{-2} t, \quad R \rightarrow \lambda^{2} R, \quad X_i \rightarrow X_i. \]  

Note that we have shifted from the original string unit to the new system of unit in which the 11 dimensional Planck length \( g_s^{1/3} \ell_s \) is kept invariant by making the global scaling transformation \( X_i \rightarrow \lambda^{-1} X_i, \quad t \rightarrow \lambda^{-1} t, \quad \ell_s \rightarrow \lambda^{-1} \ell_s. \) This leads to \( q/r^7 \sim \ell_p^6/R^2 r^7 \rightarrow \ell_p^6/\lambda^2 R^2 r^7. \) Then the 11 dimensional metric for small \( R, \)

\[ ds_{11}^2 = 2dx^+ dx^- + \frac{q}{r^7} dx^- dx^- + dx^i dx^i, \]  

\((x^- = x_{10} - t, x^+ = (x_{10} + t)/2)\) corresponding to the classical D-particle solution is invariant under the boost \( x^+ \rightarrow \lambda^{-2} x^+, \quad R \rightarrow \lambda^2 R, \quad X_i \rightarrow X_i, \) which is almost equivalent to (4.16) except for the identification of the time variable, provided that we interpret the compactification radius \( R \) to be along the \( x^- \) direction \( x^- \sim x^- + 2\pi R. \) Although the reduction to 10 dimensions in general \textit{breaks} the boost invariance by setting the dilaton to be of the form (2.10), the symmetry is recovered in the near-horizon limit. In fact, it is easy to check that taking the near horizon limit in 10 dimensions is equivalent to modify the 11 dimensional metric for small \( R, \)

\[ ds_{11}^2 = 2dt dx^- + \frac{q}{r^7} dx^- dx^- + dx^i dx^i, \]  

which is indeed invariant under (4.16). Remember that the 10D and 11D metrics are related by \( ds_{11}^2 = e^{-2\phi/3} ds_{10}^2 + e^{4\phi/3} (dx_{10} - A_0 dt)^2. \) Thus we expect that the fluctuations around the D-particle solution in the near horizon limit is classified by the transformation property corresponding to the boost if we reinterpret the time as the light-cone time \( t \rightarrow x^+ \) which is understandable in the limit of small compactification radius \( R \) along the \( x^- \) direction. The dimensions indicated in the tables 3 and 4 agree precisely with
those expected from the boost transformation after making the shift corresponding to the change of unit: The resulting dimensions are uniformly shifted by one unit from the generalized conformal dimensions and are given by the formula $2(n_- - n_+) + 2 + 4\ell/7$. The additional factor 2 corresponds to the hidden integration over $x^-$. Thus the agreement of the dimensions on both sides is just as it should be. This provides strong evidence for the consistency of Matrix theory with the DLCQ interpretation for large $N$, conforming to the results of perturbative calculations [15] [16] at fixed $N$.

However, in general, the knowledge of the dimensions is not sufficient to fix the form of the correlators even for 2- and 3-point functions, in contrast to the case of usual conformal symmetry. Once the dependence on the coupling $g_s$ is given, we would be able to fix the scaling behavior with respect to the time differences. Namely, the coupling constant dependence and the scaling behavior with respect to time are only simultaneously determined. We should recall here that in the usual conformal case, the coupling constant is invariant under the conformal transformation, so that we cannot fix the coupling constant dependence of the correlators by conformal symmetry. We again need an explicit computation to determine it. Thus the strength of the constraint that the generalized conformal transformation puts on Green functions is not less than the one we had in the case of the ordinary conformal symmetry. For example, even though the Wilson loop in the case of $\text{AdS}_5 \times S^5$ exhibits the Coulomb behavior, AdS-CFT correspondence predicts a very nontrivial behavior $(g^2_{YM} N)^{1/2}$, instead of $g^2_{YM} N$ of the free theory, for the effective (charge)$^2$, suggesting the existence of a screening effect of factor $1/g_s N$ due to complicated large $N$ dynamics. In a similar sense, our result (4.9) for the correlators gives nontrivial predictions for the 2-point correlators of Matrix theory in the large $N$ limit.

We remark that in the particular problem treated in the present paper, it is actually possible to predict the $g_s$ (and $N$) dependence of two point correlators from the generalized conformal dimension $\Delta$ as

$$g_s^{(\Delta + \Delta_e - 3)/5} = \frac{g_s^{\Delta_e - 1}}{g_s^{\Delta}}$$

where $\Delta_e = -1$ is the engineering dimension of the operator, using the fact that, apart from the Newton constant $\kappa^2$, $g_s$ only appears in the combination $q \propto g_s N \ell_s^T$.

From a purely 10 dimensional viewpoint, we can consider the non-extremal black hole.
solution corresponding to D0-branes \[25\], whose near horizon geometry is described by

\[
ds^2 = -e^{-2\tilde{\phi}/3}(1 - \left(\frac{R_0}{r}\right)^7)dt^2 + e^{2\tilde{\phi}/3}(1 - \left(\frac{R_0}{r}\right)^7)^{-1}dr^2 + e^{2\tilde{\phi}/3}r^2d\Omega_8^2.
\]

(4.18)

The Hawking temperature and the entropy is given, up to numerical coefficients, by

\[
T_H \sim (g_s N)^{-1/2} \left(\frac{R_0}{\ell_s}\right)^{5/2} \ell_s^{-1}, \quad S \sim N^2 (g_s N)^{-3/5} \left(\ell_s T_H\right)^{9/5},
\]

(4.19)

within the range of validity of the 10 dimensional picture \(1 \ll g_s N(T_H \ell_s)^{-3} \ll N^{10/7}\).

The correlator \(1.4\) for the traceless symmetric tensor part of the metric perturbation gives precisely the same \(N\) and \(g_s\) dependencies in the part which is independent of \(\ell_s\). The agreement may be regarded as evidence for the fact that the correlator corresponding to the energy-momentum tensor without mixing of other modes adequately counts the number of degrees of freedom in the low-energy regime of many D-particle dynamics.

Let us finally turn to the crucial problem, namely, the possible implications of our results on the Matrix theory conjecture as originally proposed in \[12\]. As summarized in section 2, the basic assumption behind this conjecture is that the infinite-momentum frame (IMF) is achieved by taking the large \(N\) limit with the compactification radius along the 11th spatial direction being kept fixed. If the conjecture is valid, the large \(N\) behavior must also be consistent with the different identification of the boost transformation in sending the system to IMF. Namely, the boost factor is proportional to \(N\),

\[
\tau \rightarrow N\tau, \quad P^- \rightarrow \frac{1}{N} P^-, \quad P^+ \rightarrow NP^+, \quad \ldots.
\]

(4.20)

Note that now \(R, \ell_p\) and the transverse coordinates \(X_i\) are all fixed. This scaling is indeed consistent with the form of the Hamiltonian \(2.4\) as already reviewed in section 2. This limit is, however, entirely different from the usual 't Hooft limit in which we keep \(g_s N \propto g_{YM}^2 N\) fixed and then \(4.9\) would be proportional to \(N^2\) as it should be. In the limit \(4.20\), on the other hand, the 2-point functions behave quite differently as

\[
\langle \mathcal{O}(\tau)\mathcal{O}(\tau') \rangle \rightarrow N^{-\frac{7}{5}} G(\tau - \tau').
\]

(4.21)

We parametrize the order \(\nu\) of Bessel function as

\[
\nu = \frac{7}{5}(1 + n_+ + n_-) + \frac{2\ell}{5}.
\]

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where \( n_\pm \) is the number of light-cone indices \( \pm \) of the corresponding Matrix operators. Using this result, we can define the effective boost dimensions of the operators by

\[
d_{IMF} = \frac{6}{5}(n_+ - n_- - 1) - \left(\frac{1}{5} + \frac{1}{7}\right) \ell.
\]

The factor \( 1/7 \) in the last parenthesis is canceled by the normalization factor in the harmonic expansion, while the factor \(-1\) in the first parenthesis should correspond to the hidden integration over \( x^- \). It is now possible to assign the dimensions \( N_{IMF} = N^{6/5} = N^{1/5} N^{1/5} \) to the upper light-cone indices \( \pm \) respectively, and \( N^{-1/5} \) to each orbital factor \( X_i \) along \( S^8 \) in the harmonic expansion. Namely, the dimension \( d_{IMF} \) is determined solely by the external space-time indices of the operator. This itself is a nontrivial phenomenon, suggesting that the large \( N \) limit is indeed connected with some space-time symmetry of Matrix theory. It implies the validity of a large \( N \) renormalization group equation of the following type

\[
\left[ N \frac{\partial}{\partial N} + \sum_{i=1}^{n} \left( \tau_i \frac{\partial}{\partial \tau_i} - d_{IMF,i} \right) \right] \langle \mathcal{O}_1(\tau_1)\mathcal{O}_2(\tau_2)\cdots\mathcal{O}_n(\tau_n) \rangle_c = 0
\]

for general \( n \)-point correlation functions of Matrix theory before making the scaling transformation \((4.20)\).

However, the usual kinematics would require that the scaling factor associated to the light-cone indices be \( N^{1/5} \) instead of \( N^{6/5} \). How to interpret the anomalous factor \( N^{1/5} \) is not clear to us. Is it correlated with the same power factor \( N^{-1/5} \) associated to the orbital factor along \( S^8 \)? In view of holography \([12]\), on the other hand, the latter behavior \( N^{-1/5} \to 0 \) is quite puzzling at least apparently, since the transverse size of the system should increase as the number of partons increases. It is not completely obvious to us, however, whether this implies an immediate contradiction with holography. It might indicate some kind of screening effect in the large \( N \) limit with respect to the effective sizes of the states for higher angular momentum as seen from physical operators. This is somewhat reminiscent of the behavior of the Wilson loop in the case of AdS\(_5\)-SYM correspondence as mentioned above. Remember that the increase of transverse size under the boost usually associated with holography is itself a very puzzling behavior and certain screening must be operative for its resolution.
Let us examine the range of validity of the above predictions on the large $N$ IMF. We have emphasized in section 2 that the range of validity of the generalized AdS-CFT correspondence for Matrix theory is limited by an infrared cutoff of order $(g_s N)^{1/7} \ell_s$ in the large $N$ limit for $g_s N > 1$. In the large $N$ limit with a small but fixed $g_s$, this cutoff is bigger than the transverse extension proportional to $N^{1/9}$ of the typical states derived in mean field approximation [12] (or $N^{11/81}$ in the Thomas-Fermi approximation [26]) in effective theory for diagonal components, but is smaller than the more reliable estimate $N^{1/3}$ obtained by use of the virial theorem [22], which explicitly takes account into the fluctuations of off-diagonal components. Therefore, it seems that the limitation in the infrared region is rather serious. This strongly suggests that our results should be regarded as predictions for the correlators of Matrix theory put in a ‘small’ box from the point of view of the Matrix-theory conjecture. The anomalous scaling behavior may then be interpreted as an artifact caused by the finite size effect. The latter effect may contribute to the lessening of the degrees of freedom $∥$.

On the other hand, in the short distance limit for the validity of the generalized AdS-CFT correspondence was (2.19), $r_{10} \equiv g_s^{1/3} N^{1/7} \ell_s < r$, in the 10 dimensional picture. For a small but fixed $g_s$ as required by the large $N$ IMF, the lower limit increases in the same order as the near-horizon limit. This certainly makes dubious our procedure in extracting the correlators from the near horizon boundary. Formally, however, the contribution at the inside boundary still vanishes exponentially in the large $N$ limit for fixed but sufficiently small $g_s$. Remember that we have normalized the solutions at the outer near-horizon boundary. It is also possible to take the limit such that both $g_s N \rightarrow \infty$ and vanishing of the ratio of two distances, $r_{10}/q^{1/7} \rightarrow 0$, are valid simultaneously in the limit of large $N$ by allowing that $g_s$ slowly changes with large $N$. This suggests that our result may be continued to the region of the large $N$ IMF at least in the short distance region. Furthermore, we can improve the situation by going to the 11 dimensional picture. In the latter, the short distance condition for the validity of the classical approximation is (2.20), $r_{11} \equiv g_s^{5/27} N^{1/9} \ell_s < r$. Since $r_{11}/q^{1/7} = g_s^{8/189} N^{-2/63} \rightarrow 0$ (or $z_{11}/q^{1/7} = g_s^{-20/189} N^{5/63} \rightarrow \infty$) for $N \rightarrow \infty$ for fixed $g_s$, our procedure of extracting the correlators from the near-horizon boundary in this picture is safer than in the 10 dimensional picture.

$∥$For a further discussion on this point, see ref. [27].
Do the limitations which we have discussed here explain the above anomalous behavior? Or is it related to a different limitation that we are considering the small coupling region $R \ll \ell_P$? Note that from the viewpoint of 11 dimensions the parameter $R$ only appears as an overall scale factor of the Hamiltonian (2.3). This seems to indicate that the scaling property might not be affected by the condition $R \ll \ell_P$. In any case, it is necessary to clarify these problems before drawing definite conclusions on the Matrix theory conjecture from our predictions on the correlation functions. We hope that our predictions will be useful for future investigations.

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Appendix

Spherical harmonics on $S^N$

Any function defined on $S^N$ can be expanded into the set of the irreducible representations of $SO(N+1)$, namely the spherical harmonics. We briefly summarize the definitions and the properties of the spherical harmonics. Details on the spherical harmonics in general dimensions can be found e.g. in [28].

A scalar function $\hat{\phi}(r, x^i)$, where $r$ is the radius of the sphere $S^N$ and $x^i \ (i = 1, ..., N + 1; x^i x^i = 1)$ are normalized Cartesian coordinates on the sphere, can be expanded as

$$\hat{\phi} = \sum \varphi(r) Y(x^i).$$

(A.1)
$Y$ is called the scalar harmonics whose explicit form is given by

$$Y = C_{m_1 \ldots m_\ell} x^{m_1} \cdots x^{m_\ell} \quad (\ell = 0, 1, \ldots) \tag{A.2}$$

where $C_{m_1 \ldots m_\ell}$ is totally symmetric and traceless in its indices. In what follows we set $N = 8$. We have suppressed the indices which label the harmonics here and in the text for the sake of brevity. Note that harmonics with a given $\ell$ transform irreducibly under $SO(9)$ and the number of independent harmonics is given by the dimension of the representation. Spherical harmonics are the eigenfunctions of the Laplacian on the sphere (which corresponds to the second order Casimir of $SO(9)$). The eigenvalue evaluated on the unit sphere $S^8$ is

$$D^i D_i Y = -\ell(\ell + 7)Y. \tag{A.3}$$

A vector function $\hat{A}_i$ on the sphere can be written as a sum of the divergenceless part and the derivative of a scalar

$$\hat{A}_i = \sum a(r) Y_i(x^i) + \sum \bar{a}(r) D_i Y(x^i). \tag{A.4}$$

Divergenceless vector $Y_i$ is called the vector harmonics. The explicit form is given by

$$Y_n = C_{nm_1 \ldots m_\ell} x^{m_1} \cdots x^{m_\ell} \quad (\ell = 1, 2 \ldots). \tag{A.5}$$

The coefficients $C_{nm_1 \ldots m_\ell}$ are antisymmetric under the exchange of the first two indices $(n, m_1)$, totally symmetric and traceless with respect to $m_1, \ldots, m_\ell$. The first condition is to ensure that the vector is tangent to the sphere. We need $\ell \geq 1$ to satisfy the conditions. The eigenvalue of the laplacian is

$$D^j D_j Y_i = [-\ell(\ell + 7) + 1] Y_i \tag{A.6}$$

If we impose the condition $D^i \hat{A}_i = 0$, the expansion must be

$$\hat{A}_i = \sum a Y_i. \tag{A.7}$$

which is easily verified using (A.3) and $D_i Y(\ell = 0) = 0$.

Symmetric traceless tensor on $S^8$ is written as the sum of divergenceless part, derivative of a divergenceless vector and second derivative of a scalar.

$$h_{ij} - g_{ij} h_k^k = \sum b(r) Y_{ij}(x^i) + \sum \bar{b}(r)(D_i Y_j + D_j Y_i)(x^i) + \sum \bar{\bar{b}}(r)(D_i D_j - g_{ij} D^k D_k) Y(x^i) \tag{A.8}$$
The explicit form of the tensor harmonics $Y_{ij}$ is given by

$$Y_{n_1n_2} = C_{n_1n_2m_1...m_\ell}x^{m_1}...x^{m_\ell} \quad (\ell = 2, 3...). \quad (A.9)$$

The coefficients $C_{n_1n_2m_1...m_\ell}$ must be antisymmetric under the exchange of $(n_1, m_1)$ or $(n_2, m_2)$ to ensure that the tensor indices are tangent to the sphere, symmetric under the exchange of $(n_1, n_2)$ to make the tensor indices symmetric, and totally symmetric and traceless with respect to $m_1...m_\ell$. We need $\ell \geq 2$ to meet the conditions. The eigenvalue of the Laplacian is

$$D^kD_kY_{ij} = [\ell(\ell + 7) + 2]Y_{ij}. \quad (A.10)$$

If we impose the condition $D^i(h_{ij} - g_{ij}h^k_k) = 0$, it can be proved that the expansion is reduced to

$$h_{ij} - g_{ij}h^k_k = \sum bY_{ij}. \quad (A.11)$$

Similarly, $p$-form $\hat{A}_{i_1...i_p}$ ($p = 1, 2, 3$) can be written as the sum of the divergenceless part and the exterior derivative of $(p - 1)$-form

$$\hat{A}_{i_1...i_p}(x^i) = \sum \alpha Y_{[i_1...i_p]}(x^i) + \sum \tilde{\alpha}D_{[i_1} Y_{i_2...i_p]}(x^i) \quad (A.12)$$

The explicit form of the $p$-form harmonics $Y_{[i_1...i_p]}$ is

$$Y_{[n_1...n_p]} = C_{n_1...n_pm_1...m_\ell}x^{m_1}...x^{m_\ell} \quad (\ell = 1, 2...). \quad (A.13)$$

The coefficients $C_{n_1...n_pm_1...m_\ell}$ are antisymmetric under the interchange of $m_1$ with any one of $n_1, ..., n_\ell$ and totally symmetric and traceless with respect to $(m_1...m_\ell)$. To meet the conditions, we need $\ell \geq 1$. Under the condition $D^{i_1}\hat{A}_{i_1...i_p} = 0$, the expansion becomes

$$\hat{A}_{i_1...i_p} = \sum \alpha Y_{[i_1...i_p]} \quad (A.14)$$

The eigenvalue of the Laplacian is

$$D^kD_kY_{[i_1...i_p]} = [-\ell(\ell + 7) + p]Y_{[i_1...i_p]} \quad (A.15)$$

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