A computational study on effect of pitch difference in pure plunging tandem wings

M R Rasani, M Z Nuawi, W M F W Mahmood and Z Harun
Jabatan Kejuruteraan Mekanik dan Bahan, Fakulti Kejuruteraan dan Alam Bina, Universiti Kebangsaan Malaysia, 43600 Bangi, Malaysia
E-mail: rasidi@ukm.edu.my

Abstract. Flapping wing in tandem configuration may offer enhanced aerodynamic performance at low Reynolds number, in which micro air vehicles operate. The present study aims to investigate the effect of fore-hind wing pitch difference on the aerodynamic performance of tandem wings. To that end, two-dimensional, laminar flow around two thin flat airfoils that are sinusoidally plunging in phase with each other, were computationally simulated at a Reynolds number of 10000, using a flow solver in an Arbitrary Lagrangian-Eulerian framework. The fore wing pitch angle was fixed to 10°, while the hind wing pitch angle was varied between −10°, 0°, 10° and 20°. Numerical results shows that aerodynamic performance of the fore wing may be affected by the hind wing pitch angle and that tandem wings may offer improved lift to drag efficiency at some optimal fore-hind wing pitch difference compared to twice the results of a similar single wing case. In addition, the complex fore-hind wing vortex interaction is also affected by the hind wing pitch angle.

1. Introduction
Research of flapping wings or airfoils have generated much interest in recent years, especially in development of bio-inspired micro air vehicles (MAV). Micro air vehicles and nano air vehicles are flying vehicles no larger than 15 cm in any of its dimensions and operate at low Reynolds numbers, over which flow is laminar and viscous [1, 2]. At these low Reynolds numbers, insects and small birds have been shown to generate enhanced aerodynamic performances by exploiting leading and trailing edge vortices in their flapping motion [3–5], which is advantageous for micro or nano air vehicles.

Aerodynamics of wings flapping in tandem have also attracted much attention [3, 6–8] and may offer improved aerodynamic efficiencies [4, 9] under certain configurations. Increased combined thrust were found when the fore and hind wings were flapping in phase with each other [2] or within 0 ± 50° phases of each other [3], which is suitable for rapid forward flights. Previous investigations also indicate that improved efficiency may be obtained at some optimal fore-hind wing phase difference [4], which may be efficient during cruising or hovering flights. In terms of tandem wing spacing, Broering and Lian showed that maximum thrust was achieved when spacing is 0.5 of chord length and that propulsive efficiency may be increased by 40% compared to a single wing configuration [10]. More recently, Kok et al. [11] investigated their insect-inspired micro air vehicle using blade element analysis and showed increased aerodynamic efficiency when asymmetrical flapping occurs at a frequency below the natural frequency of the system.
From the literature review, the effect of flapping phase and spacing between fore-hind wing on aerodynamic performance of tandem wing have been investigated. Moreover, their flapping kinematics have typically involved plunging in combination with pitching of both wings. However, the effect of pitching has not been fully investigated and therefore, the present study aims to analyze the effect of fore-hind wing pitch difference on aerodynamic performance of tandem wings. Towards that end, we simulate unsteady flow around two tandem thin airfoils under pure in-phase fore-hind wing plunging motion with constant wing spacing, to isolate the effects of pitching angle. Both unsteady aerodynamic forces and vortex interactions at various pitch angles were analyzed using computational fluid dynamics, at Reynolds number $Re = 10000$ and flapping frequency of 10 Hz.

2. Methodology

2.1. Flow equations

In order to simulate this problem, the two-dimensional, incompressible and unsteady Navier-Stokes and continuity equations were solved. Although at $Re = 10000$ the flow may not be entirely laminar, a laminar flow is assumed, considering previous studies indicating negligible force differences at $Re$ below 50000 [7, 12]. Furthermore, the present study begins by considering a two-dimensional model, as primary vortex interaction in tandem wings have been shown to be two-dimensional [9]. In order to account for fluid mesh deformation that follows tandem wing motions, the Navier-Stokes and continuity equations are expressed in an Arbitrary Lagrangian-Eulerian (ALE) description [13], as shown in equations (1) and (2):

$$
\frac{\partial u_i}{\partial t} + (u_j - \tilde{u}_j) \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right]
$$

$$
\frac{\partial (u_i - \tilde{u}_i)}{\partial x_i} = 0
$$

where $u_i$, $\tilde{u}_j$ and $p$ are the fluid velocities, fluid grid velocities and pressure respectively ($i, j = 1, 2$ represent the 2-D cartesian directions), $\rho = 1.185$ kg/m$^3$ is the constant air density used and $\mu = 1.831 \times 10^{-5}$ Pa s represents the air dynamic viscosity used in all simulations. The fluid grid velocity $\tilde{u}_j$ is computed using a Laplacian Diffusion model, which smooths the grid velocity from a value matching the wing motion near the fluid-wing interface, to zero at the fluid domain boundaries. Time advancement was accomplished using a second order backward euler scheme, while a high-resolution scheme was employed to solve the advective terms.

2.2. Computational model

The computational domain spans 4 chord lengths upstream of the fore wing and 20 chord lengths downstream of the fore wing, with another 4 chord lengths separating respectively, top and bottom walls from both wings. Figure 1 shows the computational domain and boundary conditions imposed on the present flow problem, where a uniform velocity $U$ was specified at the inlet and atmospheric (zero gauge) pressure condition was defined at the outlet boundary. No-slip wall conditions are imposed on both fore and hind wings. Both wings are idealized as thin, flat airfoils with a chord length of 50 mm and a thickness of 2 mm. Inlet velocity $U$ was set to 3 m/s, giving a chord-based Reynolds number $Re = 10000$.

The flow domain was discretized using predominantly 8-noded hexahedral elements, with finer mesh surrounding both wings and finer 8-noded prismatic elements in regions close to wing surfaces, to better resolve flow and vortex patterns in the vicinity of both wings as shown in figure 2(a), yielding a total of 75168 elements. In order to maintain grid quality in regions close to both wings (as they move according to prescribed wing motion), separate domain for these regions (as shown by shaded area in figure 2(a)) were considered, so that grids in both regions...
Computational domain and conditions at respective boundaries used in this study. C represents chord length of fore or hind wing. For a single wing configuration, similar computational model and domain is employed but without hind wing.

**Figure 2.** Discretization of flow computational domain: (a) tandem wing, with shaded regions representing separate domain containing fore and hind wings (b) single wing configuration, where shaded region represents separate domain containing fore wing only.

may move uniformly together. Flow variables were conserved across the domain interfaces using an implicit and conservative grid interface algorithm [14].

**2.3. Case studies**

In the present study, a fore-hind wing spacing equal to 2 chord lengths and a pure sinusoidal plunging motion of the form $y(t) = y_0 \sin(2\pi ft)$, where plunging amplitude $y_0$ was set to 0.5 chord length and plunging frequency $f$ was set to 10 Hz were considered, giving a Strouhal number of approximately 0.17. In addition, we focus our attention to in-phase plunging between the fore and hind wing. Four cases were simulated, where the fore wing pitch angle was fixed to $\alpha_1 = 10^\circ$ in all cases, while the hind wing pitch angles were varied between $\alpha_2 = -10^\circ, 0^\circ, 10^\circ$ and $20^\circ$. An additional single wing, plunging with a pitch angle of $10^\circ$, was also simulated for comparison purposes. Considering the flapping frequency, a timestep size of 0.001 s was chosen in all cases and simulations were run to a total time of 1 s, giving 10 complete flapping cycles. Results reported herein were extracted from the final 10th cycle.
3. Results and discussion

3.1. Preliminary analysis

We first present aerodynamic force results between three different grid resolutions i.e. the baseline grid that was used in this study (total 75168 elements), a finer grid resolution where the grid is refined everywhere, especially in regions surrounding and between both wings (resulting in total 237648 elements) and a coarser grid where the grid size was approximately doubled the baseline grid (resulting in total 216568 elements). Comparison of the lift coefficient history at fore and hind wing, respectively shown in figures 3 and 4, suggest small difference between the baseline grid and the finer grid. Therefore, the baseline grid was used in the present simulation.

Table 1. Comparison of time-averaged lift and drag coefficients of tandem wings between present model with experimental results adapted from [15, 16].

|            | \( \bar{C}_L \) | \( \bar{C}_D \) |
|------------|-----------------|-----------------|
| exp. [15]  | 0.39            | 0.10            |
| present model | 0.34           | 0.07            |
| exp. [16]  | 0.06            |                 |

\(^a\) based on single wing data in [16]

In addition, lift and drag coefficients from a fixed tandem wing configuration experiment by Jones et al. [15] at Reynolds number \( Re = 100000 \) and 10\(^\circ\) angle of attack, is presented in table 1 for comparison. Present model with similar configuration shows reasonable agreement with experimental results. Discrepancy in drag coefficient is likely due to the squared leading and trailing edge geometry employed in [15] compared to a circular edge in the present model. Further comparison considering single wing experimental data taken from more rounded geometry edges in Pelletier and Mueller [16], suggest reasonable agreement in drag coefficient and discrepancy is likely due to difference in airfoil thickness used (4\% of chord in present model vs. 2\% of chord in [16]).

3.2. Aerodynamic forces

Table 2 summarizes time-averaged lift and drag coefficients (\( \bar{C}_L \) and \( \bar{C}_D \)) over a single flapping period for all tandem cases, together with a single wing configuration. Although the fore wing was maintained at pitch angle \( \alpha_1 = 10^\circ \) (similar to the single wing case), both its \( \bar{C}_L \) and \( \bar{C}_D \)
Table 2. Summary of time-averaged lift and drag coefficient for single and all tandem cases. Total single wing values were based on twice its lift and drag coefficients.

| $\alpha_2$ | fore wing | hind wing | single wing | total tandem | total single |
|-----------|-----------|-----------|-------------|--------------|--------------|
|           | $C_L$ | $C_D$ | $C_L$ | $C_D$ | $C_L$ | $C_D$ | L/D | $C_L$ | $C_D$ | L/D |
| −10       | 1.03 | 0.14 | −0.57 | 0.02 | - | - | 0.46 | 0.16 | 2.9 | - | - |
| 0         | 1.14 | 0.16 | 0.15 | −0.08 | - | - | 1.30 | 0.08 | 16.0 | - | - |
| 10        | 1.23 | 0.18 | 0.77 | 0.04 | 1.21 | 0.18 | 2.00 | 0.21 | 9.3 | 2.41 | 0.35 |
| 20        | 1.33 | 0.20 | 1.40 | 0.40 | - | - | 2.74 | 0.60 | 4.6 | - | - |

Figure 5. Variation of time-averaged lift coefficient against hind wing pitch angle $\alpha_2$ (fore wing pitch angle fixed at $\alpha_1 = 10^\circ$).

Figure 6. Variation of time-averaged drag coefficient against hind wing pitch angle $\alpha_2$ (fore wing pitch angle fixed at $\alpha_1 = 10^\circ$).

Figure 7. Variation of aerodynamic efficiency (L/D) against hind wing pitch angle $\alpha_2$ (fore wing pitch angle fixed at $\alpha_1 = 10^\circ$).

shows increasing trend with increasing hind wing pitch angle $\alpha_2$, which is better illustrated in figures 5 and 6. At $\alpha_2 = 20^\circ$, fore wing $C_L$ is higher than the single wing, but its $C_D$ is also increased compared to similar single wing configuration, indicating that hind wing $\alpha_2$ may
influence aerodynamic performance of the fore wing. In contrast, at $\alpha_2 = 10^\circ$, the hind wing does not generate as much lift and drag to the similar single wing case, which may be explained by its interaction with the fore wing.

Table 2 also presents total or combined force coefficients from both fore and hind wings, as these represent the total lift and drag experienced by a tandem wing flyer. In order to fairly compare the performance of combined fore-hind wing to those of a single wing, the equivalent total force for a single wing flyer were based on twice its lift and drag coefficients, as shown in Table 2. Summing $C_L$ and $C_D$ from both fore and hind wings, the combined lift and drag for tandem cases increases as the hind wing $\alpha_2$ is increased. More importantly, the combined lift to drag ratio in Table 2 and illustrated in Figure 7, shows a non-linear trend with increasing $\alpha_2$, suggesting an optimal phase difference may be found for the best lift to drag efficiency. This is likely due to non-linearity in the hind wing drag coefficient, which is minimal at low angles of attack where drag is mainly frictional, and increases at higher pitch angles as greater induced drag profile is generated. In addition, although the tandem case where both fore and hind wing are pitched at $10^\circ$ shows lower combined lift than twice the lift of similar single wing configuration, their combined lift to drag ratio suggest better performance in comparison to ratio of twice the lift and drag, of similar single wing configuration.

3.3. Vortex interactions

![Figure 8](image1)

**Figure 8.** Contour of vorticity slightly after start of upstroke (80% of flapping cycle) for: (a) hind wing pitch $\alpha_2 = -10^\circ$ (b) hind wing pitch $\alpha_2 = 0^\circ$ (c) hind wing pitch $\alpha_2 = 10^\circ$ and (d) hind wing pitch $\alpha_2 = 20^\circ$.

![Figure 9](image2)

**Figure 9.** Contour of vorticity at mid of upstroke (0% of flapping cycle) for: (a) hind wing pitch $\alpha_2 = -10^\circ$ (b) hind wing pitch $\alpha_2 = 0^\circ$ (c) hind wing pitch $\alpha_2 = 10^\circ$ and (d) hind wing pitch $\alpha_2 = 20^\circ$.

Figure 8 to Figure 11 presents contours of vorticity (where red contours represent positive (counterclockwise) vorticity and blue contours represent negative (clockwise) vorticity) for all tandem cases, from start of upstroke to the middle of downstroke. In the beginning of the upstroke, a trail of counterclockwise (CCW) vortex from the trailing edge of the fore wing is about to bisect the hind wing, as shown in Figure 8. At larger hind wing pitch angle, the strength of the trailing vortex that bisects the leading edge of the hind wing does not appear as
pronounced as at lower pitch angles (comparing figure 8(d) to figure 8(a)). This may explain the decreasing size of counterclockwise leading edge vortex (LEV) forming underneath the hind wing with increasing $\alpha_2$, as indicated in figure 9, which may then generate lower negative lift or downforce during the upstroke.

![Figure 10. Contour of vorticity nearing end of upstroke (20% of flapping cycle) for: (a) hind wing pitch $\alpha_2 = -10^\circ$ (b) hind wing pitch $\alpha_2 = 0^\circ$ (c) hind wing pitch $\alpha_2 = 10^\circ$ and (d) hind wing pitch $\alpha_2 = 20^\circ$.](image)

Consequently, with smaller CCW LEV at larger $\alpha_2$, it is hypothesized that the incoming clockwise LEV shed from the fore wing is less deflected away from the hind wing, allowing stronger interaction with the leading edge of the hind wing at larger $\alpha_2$ during downstroke. This may generate larger LEV at the hind wing for larger $\alpha_2$, as indicated in figure 11, which may generate larger lift during the downstroke. Unlike a single wing or the fore wing, the hind wing generates LEV during both upstroke and downstroke that leads to a lower time-averaged lift coefficient in a single flapping cycle.

![Figure 11. Contour of vorticity in the mid of downstroke (50% of flapping cycle) for: (a) hind wing pitch $\alpha_2 = -10^\circ$ (b) hind wing pitch $\alpha_2 = 0^\circ$ (c) hind wing pitch $\alpha_2 = 10^\circ$ and (d) hind wing pitch $\alpha_2 = 20^\circ$.](image)

4. Conclusion
A computational study on the effect of fore-hind wing pitch difference to aerodynamic performance of tandem wings at $Re = 10000$ was presented. Although, in the present tandem configuration, their combined lift may not better twice the lift of similar single wing case, tandem wing may offer improved lift to drag efficiency at some optimal fore-hind wing pitch difference. In addition, vortex interaction between fore and hind wings may also explain the variation in lift at different hind wing pitch angles. As the present study had limited its attention to a fixed set of configuration, further investigation is recommended to examine effect of pitch difference but at smaller wing spacing, larger Strouhal numbers and different fore-hind wing phasing.

Acknowledgments
Acknowledgments The authors would like to gratefully acknowledge the financial support provided by Universiti Kebangsaan Malaysia under project grant GGPM 2013-092.
References
[1] Platzer M F, Jones K D, Young J and S Lai J C 2008 AIAA Journal 46 2136–2149
[2] Broering T, Lian Y and Henshaw W 2010 Aerospace Sciences Meetings (American Institute of Aeronautics and Astronautics) AIAA 2010-865
[3] Warkentin J and DeLaurier J 2007 Journal of Aircraft 44 1653–1661
[4] Usherwood J R and Lehmann F O 2008 Journal of The Royal Society Interface 5 1303–1307
[5] Sirohi J 2013 Proc. SPIE: Bioinspiration, Biomimetics, and Bioreplication vol 8686 86860U
[6] Kumar A and Hu H 2011 Fluid Dynamics and Co-located Conferences (American Institute of Aeronautics and Astronautics) AIAA 2011-3120
[7] Broering T M 2013 The Characterization of Wing-Wing Vortex Interactions of A Tandem Flapping Wing Configuration and Its Relationship to the Phase Angle and Wing Spacing. Electronic theses and dissertations paper 157. University of Louisville
[8] Gravish N, Peters J M, Combes S A and Wood R J 2015 Phys. Rev. Lett. 115(18) 188101
[9] Rival D, Schönweitz D and Tropea C 2011 Bioinspiration & Biomimetics 6 016008
[10] Broering T and Lian Y 2010 Fluid Dynamics and Co-located Conferences (American Institute of Aeronautics and Astronautics) AIAA 2010-4385
[11] Kok J M, Lau G K and Chahl J S 2016 Journal of Aircraft 53 800–810
[12] Young J, S Lai J C and Germain C 2008 AIAA Journal 46 918–924
[13] Donea J, Huerta A, Ponthot J P and Rodriguez-Ferran A 2004 Arbitrary Lagrangian-Eulerian Methods Encyclopedia of Computational Mechanics vol 1 ed Stein E, Borst R D and Hughes T J R (United Kingdom: John Wiley & Sons) chap 14
[14] ANSYS Inc 2014 ANSYS CFX Solver Theory Guide - Release 14.5
[15] Jones R, Cleaver D J and Gursul I 2015 Experiments in Fluids 56 1–25
[16] Pelletier A and Mueller T J 2000 Journal of Aircraft 37 825–832