Analysis of problem-solving difficulties at limits of sequences

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Abstract. Every year, most mathematics education students have difficulty learning real analysis courses on the material of sequences and series, especially in the sub material of the limit of sequences. Difficulties in learning the concept of limits include difficulties in understanding the limit concept's definition and visualization. The purpose of this article is to analyze the difficulties of mathematics education students of Pancasakti University in the Tegal academic year 2019/2020 in problem solving at limits of sequences and providing appropriate Scaffolding to overcome these difficulties. This research is descriptive qualitative research. The research subjects were selected using the purposive sample technique, selected two students with high, medium, and low abilities. Two students chose each capacity. Interview with research subjects to inform the results of the test using the aloud method. Students experience difficulty thinking when solving the limit of sequences problem due to the lack of initial analysis and student analogy skills. When the procedure is rechecked, students have difficulty thinking too. This is due to the lack of mathematical connection skills of students. To overcome these student difficulties, the lecturer must provide Scaffolding.

1. Introduction
The Real Analysis course is one of the terms under the objectives of mathematics in Higher Education and compulsory subject in the Mathematics Education Study Program. Real analysis course is a very important subject. This course is axiomatic deductive. Most mathematics education students have difficulty learning Real Analysis [1,2]. Because the material in these courses is abstract, so students experience difficulties. The Real Analysis course is the first course that students learn to practice reasoning and prove mathematics formally. Students' difficulties are lack of mastery of proof techniques, lack of understanding of premises, inability to understand the use of definitions, theorems, existing problems, lack of ability to write sentences in a language and mathematical notation, and lack understanding of material requirements [3].

One of the considered problematic materials in the Real Analysis course is the material of Sequences and Series [2]. The point is to prove a convergent sequence and a convergent series in the material of sequences and series. [1] However, proving the convergent sequence, the theorem used varies greatly. Its use is under the pattern or characteristics of the sequence. This condition makes students confused about the exact theory used in proving the next theorem and the problem. One of the sub-materials of Sequences and Series, namely Limit of Sequences. [4] Limit of Sequences is a sub material that has a high difficulty level. Because there are many definitions and theorems in the limitsof sequences, this definition and theorem are widely used in its application. Considering that students consider the limit of
sequences to be a problematic sub-material, the lecturer provides a solution, namely by giving Scaffolding.

Scaffolding is an interaction process by providing assistance or guidance to students by lecturers or friends to understand knowledge or skills that cannot be achieved without help. Scaffolding is one of the strategies teachers can use to teach and develop students' abilities [5]. In learning mathematics Scaffolding is an aid to solving problems, as well as helping to build concrete mathematical concepts and increase students' self-confidence. Scaffolding can be given by a discussion process between students and lecturers and between students and students. Scaffolding used in this study refers to research [6], namely level 1 (environmental provisions), level 2 (explaining, reviewing, and restructuring), and level 3 (developing conceptual thinking).

The purpose of this article is to analyze the difficulties of mathematics education students of Pancasakti University in the Tegal academic year 2019/2020 in problem solving at limits of sequences and providing appropriate scaffolding to overcome these difficulties.

2. Method
2.1 Types and Research Approaches
The approach used in this study is qualitative. According to [7] a qualitative approach is an approach that builds knowledge. The preliminary test was carried out at the beginning of the study in the observation activity to determine the student's difficulty in learning the limits of sequences. This observation is preparation for scaffolding in dealing with different student difficulties. Based on the results of the initial test and student scores, the research subjects were determined. The research subjects were selected using the purposing sample technique, selected two students with high, medium, and low abilities. Two students chose each ability. Interview with research subjects to inform the results of the test using the aloud method. Then the research subject is given scaffolding for the difficulties experienced. If, after scaffolding, students can improve their answers, then an analysis is carried out. However, if the students have not been able to improve or do not respond after the scaffolding, then the next scaffolding is carried out.

2.2 Research Instruments
The main instrument in qualitative research is the researcher. Researchers carry out planners, implementers, data collectors, analysts, data interpreters, and research results. Test questions are a supporting instrument used by researchers. Test questions are given to students to get initial ability data and describe the student's difficulties in solving the limit of sequences problem before obtaining scaffolding from the researcher.

Scaffolding sheets are another instrument used by researchers. The scaffolding sheet distribution aims to analyze the difficulty of solving problems at the limit of sequences as experienced by students, thus helping and directing students to correct the errors they have experienced. The scaffolding guide sheet contains questions, directions, and statements by researchers to students. The causes of these errors will be investigated by asking the items in question. It is hoped that the mistakes experienced by students can be overcome by giving scaffolding. Some of the scaffolding carried out can be seen in Table 1.

3. Results and Discussion
Based on the test questions and interview results, it was found that students had difficulty solving problems in the Limit of sequences. Student difficulties in solving problems occur when understanding the problem, preliminary analysis, carrying out evidence or solving, and examining the proving process and the complete results. Students' difficulties include difficulty thinking due to the lack of students' initial analytical knowledge, difficulties due to the lack of students' logical abilities, and difficulties due to the lack of students' mathematical connection skills.

Difficulty thinking due to the student's initial analyst's lack of knowledge occurs at the stage of understanding the concept of the problem, planning completion, and carrying out proof/completion. Students' thinking difficulties and types of initial analysis knowledge can be seen in Table 2.
### Table 1. Scaffolding Activities

| Scaffolding Activities | Activities performed |
|------------------------|----------------------|
| **Environmental Provisions** | Make assignment sheets, prepare another picture when students do not understand the problem verbally |
| **Explaining** | Students must explain the definition of sequence real numbers according to what is already known. Then students must read the problem repeatedly on the assignment sheet that has been given, then direct and explain to students to understand the problem |
| **Reviewing** | Students must reflect on the answers found so that students can find out their mistakes, then students must correct them. |
| **Restructuring** | Provide directions to students in the form of questions to find existing problems, and then students can answer them again with a better design. |
| **Developing Conceptual Thinking** | Asking students to solve problems in other alternative ways, giving directions in the form of questions that can make students find other concepts related to existing problems. |

### Table 2. Types of students’ initial abilities and thinking difficulties

| Initial ability type | Student Thinking Difficulties | The type of scaffolding done |
|----------------------|------------------------------|-----------------------------|
| Infinite function limit | Students have difficulty solving the limit of the infinite function on the division | Reminiscent of the concept of infinite limits when studying calculus |
| Real number sequence definition | Students still use arithmetic sequences and geometric sequences | Recalls the definition of real number sequences and provides some examples |
| Definition at limit of sequences | Students forget the definition of sequence limits and cannot explain | Recall the definition of sequence limits with visualization and provide examples of problems |
| Convergent sequence definition | Students forget the definition of a convergent sequence and cannot visualize it | Recall the definition of convergent sequence with visualization and provide some sample problems |
| The Pinch Theorem | Students forget to solve the problem with the pinch theorem | It reminds you of the wedge theorem and gives some example problems |
| Row ratio | Students forget the concept of sequence ratio | Recalls the concept of sequence ratios and provides some sample problems |

The difficulty in thinking of students due to students' lack of logical ability occurs at the planning stage of completion, proving/completing, and re-examining the process of proof and solution. Students' thinking difficulties and the types of logic abilities can be seen in Table 3.
### Table 3. Types of students' mathematical logic abilities and thinking difficulties

| Logic type                  | Description of students' thinking difficulties                                                                 | The type of scaffolding done                                                                 |
|-----------------------------|---------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------|
| Convergent and Divergent Sequences | Students forget the definition of Convergent and Divergent sequences | Reminiscent of the definition of convergent and divergent sequences with visualization |
| Determination (K)           | Students cannot find the K value as well as in changing the form of inequality is wrong                       | Remind students and students to correct it according to the definition in determining (K) and students to solve inequality problems in fractions |
| The Pinch Theorem           | Cannot prove the use of the fix theorem                                                                       | Reminds back to using the fixed theorem and gives some examples of problems and their solutions |
| Row Ratio                   | Students forget to determine the comparison of the ranks                                                      | Recalling the Sequence ratio theorem, giving examples of problems and solutions               |

The difficulty of thinking due to the lack of mathematical connection skills of students occurs at the stage of completing and re-checking procedures and solutions. The problem of students' mathematical connection ability in succession can be seen in Table 4.

### Table 4. Types of students' mathematical connection ability and thinking difficulties

| Mathematical Connection Types | Description of Student's difficulty in thinking                                                                 | The type of scaffolding done                                                                 |
|-------------------------------|---------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------|
| Convergent and Divergent Sequences | Students cannot make a pattern of a sequence and their conclusions are wrong                                    | Remind members of the line one by one to form a line pattern                                |
| Limit of Sequences            | Students are wrong in determining the concept of sequence limits                                              | Reminiscent of the definition of sequence limits in real analysis, as well as with the initial analysis to determine the value of (K) |
| Proof of Limit                | Students cannot use the use of limit definitions in solving problems                                           | Reminded the definition of limit and asked students to improve it                            |
| Row ratio                     | Students cannot determine the rank comparison                                                                  | Reminiscent of the sequence ratio theory and students to correct the answer                 |
Students have difficulty thinking because there are six initial abilities in table 2. Students' initial ability in the proofing process is very necessary. [8] Initial analysts are needed in the proving process because they can develop students' abilities in the proof process to understand and validate arguments from mathematical statements. [9] Mathematical concepts are necessary for preliminary analytical knowledge because they can give meaning to the concept definition and related formal solutions. Procedural analysis needs to be done in the initial analysis because with procedural travel, and students can understand the proof [10]. The initial analysis is beneficial in the evidentiary process, including by building graphic arguments then building symbol-verbal evidence based on these graphical arguments. [11]. lack of knowledge of the initial analysis in proving the limits of sequences in Figure 1, Figure 2, and Figure 3

**Figure 1.** Initial analysis of subject one is proving the limits of sequences

**Figure 2.** Initial analysis of subject two is proving the limits of sequences

**Figure 3.** Initial analysis of subject three is proving the limits of sequences
It is difficult for students to think about solving the problem limits of sequences because of their lack of mathematical logic skills. These difficulties in Table 3 are found to be four. In the process of proving the need for logical ability, which has the consequence that the perfecting of the idea of equality will be obtained, [12]. Mathematical logic skills attempt to place all mathematics based on set theory and classical logic [13].

Then it is difficult to think about solving the limit of sequences problem because of the lack of mathematical connection skills. [14] The mathematical connection is students' ability to use related ideas in mathematics, connections with other disciplines, and apply mathematical ideas in the context of everyday life. They need mathematical connection skills in learning-related mathematical concepts [15]. The mathematical connection is the most important part that needs to be done in every level of education. Mathematical connection means the capacity above the information provided, with a critical attitude to evaluate something and to have metacognitive awareness and problem-solving skills [16].

In real analysis, it is necessary to make connections, as the ability to relate the proving task with definitions, theorems, multiple representations, and examples from the current where a student is, and the possible previous experiences of previous material. Within this category, it is necessary to consider making connections: between definitions/theorems, between representations, and between examples. Each of the subcategories is described below. [17] Between Definitions / Theorems To improve the ability to make connections, students must use the definitions/theorems previously discussed in possible terms from other materials.

Scaffolding is expected to play an essential role in developing students' thinking process skills during learning, [18]. Scaffolding is very helpful for lecturers in mapping the conceptual development of students' mathematical thinking so that lecturers can make informed instructional decisions [19]. With scaffolding, lecturers can identify the relevance of student reasoning for mathematical purposes and students' understanding of mathematical ideas related to these goals, [20]. scaffolding in mathematics learning is an aid to solving problems, as well as helping to build concrete mathematical concepts and increase students' self-confidence [21].

4. Conclusions

Based on the study results, it was found that students' difficulties in solving the limit of sequences were due to the lack of students' initial abilities, logical abilities, and mathematical connection skills. Lack of initial skills because students cannot understand the problem and do not find ideas in solving problems. Lack of mathematical connection skills in Karen students, they do not understand the difficulty in solving / proofing, and do not recheck answers, and do not understand the evidentiary procedures used. Lack of mathematical logic skills due to lack of understanding of the problem, lack of completion plan at the time of completion.

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