π⁺ Emission from Hypernuclei and the Weak $\Delta I = 3/2$ Transitions

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Abstract

Low energy $\pi^+$ emission in hypernuclear weak decays is studied in the soft pion limit. It is found that the $\pi^+$ decay amplitude is dominated by the $\Delta I = 3/2$ part of nonmesonic weak decays according to the soft pion theorem. The ratios, $R(\pi^+/\text{soft } \pi^-)$, for light hypernuclei are estimated in the direct quark mechanism, which predicts violation of the $\Delta I = 1/2$ rule, and found to be about $1/3$ for $^\Lambda_4 He$.

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Recent experimental and theoretical studies of weak decays of hypernuclei have generated renewed interest on nonleptonic weak interactions of hadrons. A long standing problem is the dominance of $\Delta I = 1/2$ amplitudes in the strangeness changing transitions. The decays of kaons, and $\Lambda, \Sigma$ hyperons are dominated by the $\Delta I = 1/2$ transition but it is not clear whether this dominance is a general property of all nonleptonic weak interactions. In fact, the weak effective interaction which is derived from the standard model including the perturbative QCD corrections contains a significantly large $\Delta I = 3/2$ component [1]. It is therefore believed that nonperturbative QCD corrections, such as hadron structures and reaction mechanism are responsible for suppression of $\Delta I = 3/2$, and/or enhancement of $\Delta I = 1/2$ transition amplitudes.

From this viewpoint, decays of hyperons inside nuclear medium provide us with a unique opportunity to study new types of nonleptonic weak interaction, that is, two- (or multi-) baryon processes, such as $\Lambda N \rightarrow NN$, $\Sigma N \rightarrow NN$, etc. These transitions consist the main branch of hypernuclear weak decays because the pionic decay $\Lambda \rightarrow N\pi$ is suppressed due to the Pauli exclusion principle for the produced nucleon.

A conventional picture of the two-baryon decay process, $\Lambda N \rightarrow NN$, is the one-pion exchange between the baryons, where $\Lambda N\pi$ vertex is induced by the weak interaction [2]. In $\Lambda N \rightarrow NN$, the relative momentum of the final state nucleon is about 400 MeV/c, much higher than the nuclear Fermi momentum. The nucleon-nucleon interaction at this momentum is dominated by the short-range repulsion due to heavy meson exchanges and/or to quark exchanges between the nucleons. It is therefore expected that the short-distance interactions will contribute to the two-body weak decay as well. Exchanges of $K, \rho, \omega, K^*$ mesons and also correlated two pions in the nonmesonic weak decays of hypernuclei have been studied [3,4] and it is found that the kaon exchange is significant, while the other mesons contribute less [4].

Several studies have been made on effects of quark substructure [5–7]. In our recent analyses [7,8], we employ an effective weak hamiltonian for quarks, which takes into account one-loop perturbative QCD corrections to the $W$ exchange diagram in the standard model.
It was pointed out that the $\Delta I = 1/2$ part of the Hamiltonian is enhanced during downs- 
scaling of the renormalization point in the renormalization group equation. Yet a sizable $\Delta I = 3/2$ component remains in the low energy effective weak Hamiltonian. We proposed to evaluate the effective Hamiltonian in the six-quark wave functions of the two baryon systems and derived the “direct quark” weak transition potential for $\Lambda N \to NN$ [7,8]. Our analysis shows that the direct quark contribution largely improves the discrepancy between the meson-exchange theory and experimental data for the ratio of the neutron- and proton- 
induced decay rates of light hypernuclei. It is also found that the $\Delta I = 3/2$ component of the effective Hamiltonian gives a sizable contribution to $J = 0$ transition amplitudes [6]. Unfortunately, we cannot determine the $\Delta I = 3/2$ amplitudes unambiguously from the present experimental data [9].

In this paper, we would like to show that the $\Delta I = 3/2$ two-baryon transition amplitudes are directly related to the S-wave $\pi^+\pi^-$ emission from hypernuclei. The relation of these two amplitudes is derived from the soft-pion theorem and is a result of the chiral structure of the weak interaction.

The $\pi^+$ emission from light hypernuclei, for instance, $^4\Lambda H e$, has puzzled us for a long time. A few experimental data suggest that the ratio of $\pi^+$ and $\pi^-$ emission from $^4\Lambda H e$ is about 5% [10]. This small ratio is expected because the free $\Lambda$ decays only into $n\pi^-$ and $p\pi^0$. The $\pi^+$ emission requires an assistance of a proton, i.e., $\Lambda + p \to n + n + \pi^+$. Several microscopic mechanisms for the $\pi^+$ emission have been considered in literatures [11–13]. The most natural one is $\Lambda \to n\pi^0$ decay followed by $\pi^0 p \to \pi^+ n$ charge exchange reaction (Fig. 1(a)). It was evaluated for realistic hypernuclear wave functions and found to explain only 1.3% (according to Table II of ref. [12]) for the $\pi^+/\pi^-$ ratio [14,12]. Another possibility is to consider $\Sigma^+ \to \pi^+ n$ decay after the conversion $\Delta p \to \Sigma^+ n$ by the strong interaction such as pion or kaon exchanges (Fig. 1(b)). It was found, however, that the free $\Sigma^+$ decay which is dominated by $P$-wave amplitude, gives at most 0.2% for the $\pi^+/\pi^-$ ratio. Indeed it is clear that the $\Sigma^+$ mixing and its free decay is not the main mechanism, for experimental data suggest that the $\pi^+$ emission is predominantly in the S-wave with the
energy less than 15 MeV. Recently, it was proposed that a two-body process $\Sigma^+ N \rightarrow nN\pi^+$ must be important in the $^4\Lambda He$ decay [13]. But its microscopic mechanism is not specified.

We here propose to apply the soft-pion theorem to the $\pi^+$ emission, i.e., in the limit of zero pion four-momentum $q$. The soft-pion theorem for the process $\Lambda p \rightarrow nn\pi^+(q \rightarrow 0)$ reads [14]

$$\lim_{q \rightarrow 0} \langle nn\pi^+(q)|H_W|\Lambda p\rangle = -\frac{i}{\sqrt{2}f_\pi} \langle nn|[Q_5^-, H_W]|\Lambda p\rangle$$

where $Q_5^-$ is the axial charge operator, and $H_W$ is the weak hamiltonian which describes a strangeness changing transition. Because there is no contribution of the neutral current, $H_W$ consists only of the left-handed currents and the flavor-singlet right-handed currents which come from the penguin-type QCD corrections. Then the commutation relation in eq.(1) is given in terms of the isospin lowering operator $I_-$ as

$$[Q_5^-, H_W] = -[I_-, H_W]$$

and therefore can be evaluated by using the isospin property of $H_W$.

As $H_W$ changes the third component of the isospin by $-1/2$ when it converts $\Lambda p$ ($I_3 = +1/2$) to $nn\pi^+$ ($I_3 = 0$), it may contain $H_W(\Delta I = 1/2, \Delta I_z = -1/2)$ and $H_W(\Delta I = 3/2, \Delta I_z = -1/2)$. Now it is easy to see that $\Delta I = 1/2$ part vanishes in (2) as

$$[I_-, H_W(\Delta I = 1/2, \Delta I_z = -1/2)] = 0$$

$$[I_-, H_W(\Delta I = 3/2, \Delta I_z = -1/2)] = \sqrt{3}H_W(\Delta I = 3/2, \Delta I_z = -3/2)$$

We then obtain

$$\lim_{q \rightarrow 0} \langle nn\pi^+(q)|H_W|\Lambda p\rangle = \frac{i\sqrt{3}}{\sqrt{2}f_\pi} \langle nn|H_W(\Delta I = 3/2, \Delta I_z = -3/2)|\Lambda p\rangle$$

Thus we conclude that the soft $\pi^+$ emission in the $\Lambda$ decay in hypernuclei is caused only by the $\Delta I = 3/2$ component of the strangeness changing weak hamiltonian. In other words, the $\pi^+$ emission from hypernuclei probes the $\Delta I = 3/2$ transition of $\Lambda N \rightarrow NN$.

It is worth mentioning that the soft $\pi^+$ is emitted from the external lines of the $\Lambda p \rightarrow pn$ weak transition (fig. 2) as the vertex factor $q^\mu$ is to be canceled by the propagator of the
external line $\simeq 1/q$ in the $q \to 0$ limit. The reason why the $\Delta I = 1/2$ transition is not allowed can be understood from the fact that the $\pi^+$ emission from the initial proton, Fig. 2(a), and that from the final proton, Fig. 2(b), cancel each other for the $\Delta I = 1/2$ hamiltonian.

Now we understand why the previous attempts to explaining the $\pi^+ / \pi^-$ ratio failed. Both the charge exchange process and the $\Sigma^+$ decay are suppressed as they are induced by the $\Delta I = 1/2$ part of the hamiltonian. Eq.(1) tells us that they vanish in the soft-pion limit or are cancelled by other diagrams. In fact, the suppression of the free-space $S$-wave $\Sigma^+ \to n\pi^+$ decay can also be explained from the soft-pion theorem. In the same way, two-body $\Sigma^+$ decay will be suppressed for low-energy $\pi^+$, unless it is induced by a $\Delta I = 3/2$ weak interaction. Recently, Shmatikov proposed a new contribution, in which the weak $\Lambda \to n\pi^+\pi^-$ vertex is followed by $\pi^-$ absorption by the rest of the nucleus [15]. There the interferences between this “absorption” diagram and other diagrams of the same order (in the chiral perturbation theory) are neglected. It is, however, important to consider all the diagrams of the same order to realize the suppression due to the soft-pion theorem. A consistent study in the chiral perturbation theory is underway [16].

In our previous study [7,8], we found that the direct quark mechanism supplemented by the one-pion exchange transition account for the non-mesonic weak decays of the $S$-shell hypernuclei fairly well. There the direct quark amplitudes contain significant $\Delta I = 3/2$ contribution especially in the $J = 0$ transitions. Now the two-baryon matrix elements in eq.(3) are directly related to the non-mesonic decay amplitudes of hypernuclei by the Wigner-Eckert formula. Indeed all the two-body matrix elements can be expressed in terms of three types of reduced matrix elements: $M_0 \equiv \langle NN(I = 0)||H_W(\Delta I = 1/2)||\Lambda N \rangle$, $M_1 \equiv \langle NN(I = 1)||H_W(\Delta I = 1/2)||\Lambda N \rangle$, and $M_2 \equiv \langle NN(I = 1)||H_W(\Delta I = 3/2)||\Lambda N \rangle$. Table 1 shows possible reduced transition amplitudes for the transition of the relative $S$-wave $\Lambda N$ initial state to the $S, P, D$-wave $NN$ final states. Now the rates of the soft $\pi^+$ emission from $\Lambda p$ and similarly those of the soft $\pi^-$ emission from $\Lambda p$ and $\Lambda n$ are given in terms of the reduced matrix elements by
\[ \Gamma^+_p \equiv \Gamma(\Lambda p(J = 0) \rightarrow nn\pi^+) = A \frac{3}{8}(a_2^2 + b_2^2) \]
\[ \Gamma^+_n \equiv \Gamma(\Lambda p(J = 1) \rightarrow nn\pi^+) = A \frac{3}{8}f_2^2 \]
\[ \Gamma^-_p \equiv \Gamma(\Lambda p(J = 0) \rightarrow pp\pi^-) = A \frac{1}{6}\left((a_1 + a_2)^2 + (b_1 + b_2)^2\right) \]
\[ \Gamma^-_n \equiv \Gamma(\Lambda p(J = 1) \rightarrow pp\pi^-) = A \frac{1}{6}(f_1 + f_2)^2 \]
\[ \Gamma^0_n \equiv \Gamma(\Lambda n(J = 0) \rightarrow pn\pi^-) = A \frac{1}{12}\left((a_1 - 2a_2)^2 + (b_1 - 2b_2)^2\right) \]
\[ \Gamma^1_n \equiv \Gamma(\Lambda n(J = 1) \rightarrow pn\pi^-) = A \frac{1}{12}(f_1 - 2f_2)^2 + A \frac{1}{4}(c_0^2 + d_0^2 + e_0^2) \]

where \( A \) is a common kinematical factor.

The values of the amplitudes given in Table 1 are computed from those for the nonmesonic decay of the \( S \)-shell hypernuclei [7]. We consider the direct quark, the one-pion exchange and the one-kaon exchange processes. Among them, we assume that \( \pi \) and \( K \) exchanges contribute only to the \( \Delta I = 1/2 \) amplitudes, \( a_1, b_1, f_1, c_0, d_0, \) and \( e_0 \), while the \( \Delta I = 3/2 \) parts, \( a_2, b_2 \) and \( f_2 \), are purely from the direct quark mechanism. From these values we estimate the ratios of the \( \pi^+ \) emission to the soft \( \pi^- \) emission in \( ^4\Lambda He \) as

\[
R(\pi^+/\text{soft } \pi^-) = \frac{\Gamma^+_p + 3\Gamma^+_n}{\Gamma^-_p + 3\Gamma^-_n + 2\Gamma^0_n} \approx 0.34
\]

where we have assumed that the kinetic factors such as the phase space integral and the final state distortions are common to all the channels. If we omit the kaon exchange, the DQ + OPE gives \( 0.31 \) for the same ratio.

This value cannot be compared with the observed ratio \( R(\pi^+/\text{all } \pi^-) \) because the emitted \( \pi^- \) comes predominantly from the one-body “quasi-free” decay, \( \Lambda \rightarrow p\pi^- \). It is therefore necessary to estimate the ratio of the soft \( \pi^- \) and “quasi-free” \( \pi^- \). If we take the above ratio for the \( \pi^+ \) emission and the observed value \( R(\pi^+/\text{all } \pi^-) \approx 5\% \), then we conclude that about 85\% of \( \pi^- \) emission from \( ^4\Lambda He \) should come from the “quasi-free” \( \Lambda \rightarrow p\pi^- \) decay. Or, we may directly compare the low energy part \( (E_\pi < 20 \text{ MeV}) \) of the \( \pi^+ \) and \( \pi^- \) spectra. Such experimental data are most preferable.

A natural question is whether other hypernuclei emit \( \pi^+ \) as well. Experimentally, the confirmed \( \pi^+ \) emissions are mostly attributed to \( ^4\Lambda He \) and none to \( ^5\Lambda He \) or \( ^4\Lambda H \). However,
similar calculations as above lead to the $\pi^+ / \pi^-$ ratio, 0.35 for $^5\Lambda He$ and 0.77 for $^4\Lambda H$, respectively. This discrepancy can be accounted for by considering the “shell effect” or the “Pauli blocking” in the final states, which is known to be important in the pionic decays of, for instance, $^{12}C$ [17].

$^5\Lambda He$ has the following decay modes with the corresponding Q values,

$$
^5\Lambda He \rightarrow ^4He + p + \pi^- + 35\text{MeV}
\rightarrow ^4He + n + \pi^0 + 38\text{MeV}
\rightarrow ^3He + p + n + \pi^- + 14\text{MeV}
\rightarrow ^3H + n + n + \pi^+ + 12\text{MeV}
$$

One sees that the Q value for the $\pi^+$ emission is much smaller than that for $^4He + p + \pi^-$ decay. Furthermore, two neutrons emitted in $^3H + n + n + \pi^+$ decay are blocked by the Pauli exclusion principle as we have four neutrons in the final state [11]. Therefore we expect that $\pi^+$ decay is strongly suppressed in the $^5\Lambda He$ decay. However, this suppression does not happen in the $^4\Lambda He$ decays,

$$
^4\Lambda He \rightarrow ^3He + p + \pi^- + 35\text{MeV}
\rightarrow ^3H + n + \pi^+ + 34\text{MeV}
$$

Although the outgoing $p$ and $n$ are Pauli blocked, the Q values are the same and the $\pi^+ / \pi^-$ ratio is not affected by the Pauli blocking. For $^4\Lambda H$, the situation is similar to $^5\Lambda He$, as the $\pi^+$ decay has to break $^3H$ nucleus as

$$
^4\Lambda H \rightarrow 4n + \pi^+ + 25\text{MeV} \quad (13)
$$

Again the Q value is smaller and two of the final state neutrons are blocked by the Pauli principle. On the other hand,

$$
^4\Lambda H \rightarrow ^4He + \pi^- + 56\text{MeV} \quad (14)
$$
has a large Q-value and also has an extra enhancement factor due to the antisymmetrization. Thus the calculation without considering the phase space and the Pauli blocking corrections must seriously overestimate the $\pi^+ / \pi^-$ ratios for $\frac{3}{2}^+ H e$ and $\frac{1}{2}^+ H$.

The vector mesons may be included in this estimate. Most literatures assume the octet ($\Delta I = 1/2$) dominance for the $\Lambda N$-meson couplings and therefore no contribution is expected to the $\pi^+$ emission, although a recent study [18] suggested that the $\rho$ meson exchange may break the $\Delta I = 1/2$ rule. It is not clear whether the direct quark mechanism and the vector meson exchange mechanism are related to each other. In the present study, we have not considered isospin symmetry breaking in the $A = 4$ hypernuclei either.

Validity of the soft-pion limit may be crucial in this study. As far as we consider low-energy $S$-wave pions, both its energy and momentum are about $100 - 150$ MeV and therefore are much smaller than the chiral scale $\simeq 1$ GeV. In general, this is the energy region where the soft-pion limit works [13]. The tree level chiral perturbation theory has been questioned in a similar pion production process (in the strong interaction) $pp \rightarrow pp\pi^0$ [21]. There the pion rescattering term calculated in the soft-pion limit largely underestimates the observed $\pi^0$ production at the threshold. The reason for this failure seems the large off-shellness of the exchanged pion. Indeed, as there is a large momentum mismatch in this process, the exchanged pion carries a significant four momentum. It should be noted that this does not apply to our case, because we do not use the soft-pion approximation for the exchanged pion in our approach. On the contrary, we claim that the short-distance effects represented by the direct quark diagrams are responsible for the $\pi^+$ emission from hypernuclei.

In conclusion, we have proved that the $\pi^+$ emission from hypernuclear weak decay is induced only by the $\Delta I = 3/2$ weak interactions in the soft-pion limit. Therefore the $\pi^+$ decay due to the $\Delta I = 1/2$ processes are suppressed for low-energy $\pi^+$ and the decay will be a clear signal of the $\Delta I = 3/2$ amplitudes of the strangeness-changing weak interactions. It has also been shown that the $\pi^+$ emission amplitudes are related directly to those of two-body weak matrix elements. We have evaluated the latter in the direct-quark mechanism of the non-mesonic decays of hypernuclei and have found the ratio, $R(\pi^+/\text{soft } \pi^-) \approx 1/3$ for
Further experimental efforts to measure $\pi^+$ emissions are strongly encouraged.

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**TABLE I.** Two-body reduced matrix elements of the weak hamiltonian for DQ + OPE + OKE / DQ + OPE in units of $10^{-9}$ MeV$^{-1/2}$.

| $\Lambda N \rightarrow NN$ | $\langle I = 0 || H_W (\frac{1}{2}) || \Lambda N \rangle$ | $\langle I = 1 || H_W (\frac{1}{2}) || \Lambda N \rangle$ | $\langle I = 1 || H_W (\frac{3}{2}) || \Lambda N \rangle$ |
|--------------------------|-------------------------------------------------|-------------------------------------------------|-------------------------------------------------|
| $^1S_0 \rightarrow ^1S_0$ | $-\phantom{0}$                                  | $a_1 \phantom{0}$                              | $a_2 \phantom{0}$                              |
| $^1S_0 \rightarrow ^3P_0$ | $-\phantom{0}$                                  | $b_1 \phantom{0}$                              | $b_2 \phantom{0}$                              |
| $^3S_1 \rightarrow ^3S_1$ | $c_0 \phantom{0} (-5.95/ -7.55)$                | $-\phantom{0}$                                  | $-\phantom{0}$                                  |
| $^3S_1 \rightarrow ^3D_1$ | $d_0 \phantom{0} (7.65/13.93)$                  | $-\phantom{0}$                                  | $-\phantom{0}$                                  |
| $^3S_1 \rightarrow ^1P_1$ | $e_0 \phantom{0} (-5.16/ -4.13)$                | $-\phantom{0}$                                  | $-\phantom{0}$                                  |
| $^3S_1 \rightarrow ^3P_1$ | $-\phantom{0}$                                  | $f_1 \phantom{0} (11.71/6.30)$                 | $f_2 \phantom{0} (0.37)$                        |
FIGURES

FIG. 1. Charge exchange mechanism (a) and virtual Σ excitation (b) for $\pi^+$ emission in the $\Lambda$ weak decay.

FIG. 2. Soft $\pi^+$ emissions in the two-body weak decay.