Imagine a scenario in which the dark energy forms via the condensation of dark matter at some low redshift. The Compton wavelength therefore changes from small to very large at the transition, unlike quintessence or metamorphosis. We study CMB, large scale structure, supernova and radio galaxy constraints on condensation by performing a 4 parameter likelihood analysis over the Hubble constant and the three parameters associated with $Q$, the condensate field: $\Omega_\gamma$, $w_f$ and $z_t$ (energy density and equation of state today, and redshift of transition). Condensation roughly interpolates between $\Lambda$CDM (for large $z_t$) and sCDM (low $z_t$) and provides a slightly better fit to the data than $\Lambda$CDM. We confirm that there is no degeneracy in the CMB between $H$ and $z_t$ and discuss the implications of late-time transitions for the Lyman-$\alpha$ forest. Finally we discuss the nonlinear phase of both condensation and metamorphosis, which is much more interesting than in standard quintessence models.

98.80.Es, 04.62.+v, 98.80.Cq

1. INTRODUCTION

The observational evidence that the universe is accelerating today [1,2] changes the study of inflation from cosmic archaeology to real-time ghost hunting and implies that our universe is dominated by energy with negative pressure or that Einstein’s General Relativity is incorrect on large scales.

In this paper we discard the second possibility, but consider the idea that the dark energy comes from the condensation of dark matter (which we take to be CDM). We show that the current data does not particularly favour the simplest model of condensation over the standard dark energy candidate: quintessence [3]. Nevertheless it has a very interesting nonlinear picture of the universe, as we describe in section V, and may serve as an archetype for other scenarios for dark energy.

On a more general note, our analysis is perhaps the first detailed comparison with current data of a dark energy model with a Compton wavelength, $\lambda_c \equiv (V/m)^{-1/2}$, and speed of sound, $c_s^2 = \delta p/\delta \rho$, which change radically with time. By contrast, quintessence and metamorphosis [4] both involve scalar fields that are light at all times, though k-essence [5] by way of contrast, has a speed of sound which is low at decoupling, which may be observable in the CMB [6]. General issues with regard to the measurement of the dark energy speed of sound were discussed in [7].

The idea that acceleration might be induced by a condensate has been discussed by several authors. It has been suggested that a scalar condensate might arise in an effective field theory description of gravity [8]. Condensates have also been invoked to explain inflation in earlier work [9,10]. These condensates would naturally form at very high energies, but from condensed matter theory we are also familiar with condensates forming at very low temperatures.

An example is a super-conductive material where, below a threshold temperature, a phase transition takes place and the electrons (which are fermions) form Cooper pairs. Those Cooper pairs are collectively described by a boson field. In our cosmological scenario, when the temperature of the Universe reaches a suitable value, the cold dark matter, which could be associated to a fermion (e.g. the neutralino), might undergo a similar phase transition where a condensate of fermion-fermion pairs would emerge. Such pairs would then be described by an effective scalar field theory. Initially, the cold dark matter (CDM) scales as dust ($w_0 = 0$). If after the transition the final equation of state of the condensate $w_f < -1/3$, then the scalar field dynamics will drive the universe into an accelerating phase.

The condensation mechanism has also been advocated recently in order to explain the late acceleration of the universe. In particular, in [19] it has been observed that a cosmological constant with the suitable energy scale ($A \sim (10^{-3}\text{eV})^4$) can be explained by neutrino condensation. Furthermore, in [20] it has been found that in supersymmetric, non-Abelian gauge theories with chiral/antichiral matter fields the condensation of those chiral fields in the low-temperature/strong-coupling regime yields a scalar field which has an inverse power-law potential and therefore is a possible quintessence field candidate. Similar ideas are explored in [21].

In an earlier paper we studied vacuum metamorphosis which also involves a late-time transition. We found that
mass of the field is large before the transition \((z > z_t)\). We choose \(m > 1 GeV\) for \(z > z_t\) as appropriate for CDM, with a compton wavelength corresponding to nuclear scales, \(\lambda_C \lesssim 1 \text{fm}\). Other choices, such as warm dark matter \((m \sim 1 keV)\) are certainly possible. After the transition \((z < z_t)\) the scalar field becomes very light, with a Compton wavelength larger than 1000Mpc. We will show that the data are rather sensitive to these resulting differences in perturbation dynamics.

There are at least four reasons why it is worth analysing a condensation mechanism for the origin of quintessence: Firstly, it can provide a link between dark energy and dark matter through a known physical phenomenon. This minimalist philosophy has been recently exploited in trying to obtain acceleration and clustering from the same source field \([12]\) with a scale-dependent equation of state. It is also the basic driving idea behind the use of the Chaplygin gas which has equation of state \(p \propto -\rho^{-\alpha}, \alpha > 0\) and which interpolates between dark matter and dark energy \([13–15]\), quite similar to the class of models which we study here. But it appears to have problems with age estimates of high-\(z\) objects \([16]\).

Even more severe is that a recent analysis of fluctuations in Chaplygin-gas models finds that all models which differ significantly from effective \(\Lambda CDM\) are ruled out by the matter power spectrum \([17]\) or the CMB \([18]\). Sandvik \textit{et al} find that if the sound velocity of the dark matter is not zero, its fluctuations are either unstable towards collapse \((c_s^2 < 0)\) or start to oscillate \((c_s^2 > 0)\). Although the equation of state of the Chaplygin gas and our condensation model are quite similar, it must be pointed out, that the condensation model is immune to this problem. The Compton wavelength of the dark energy becomes very large just at the transition (when the sound speed starts to differ from zero), and the small-wavelength fluctuations in this component are suppressed and cannot grow out of control.

Secondly, condensation of CDM into a scalar field may give rise to scaling solutions characterised by a fixed fraction of the condensate relative to the CDM independent of redshift. Additionally the characteristic temperature of a condensation process is usually low, so such mechanisms naturally explains why a quintessence field would dominate only at low redshifts, although a condensation occurring in the future cannot be ruled out. In general a condensation mechanism for late-time acceleration would provide a partial solution to the coincidence problem.

Thirdly, it is an example of a model where the effective mass of the dark energy field changes strongly. It allows us to study the influence of such a mechanism on CMB and LSS data, which probe the fluctuations as well as the background geometry.

Finally, one interesting feature of condensation is its nonlinear origin. Hence, CDM condensation may give rise to significant modifications for the \textit{background cosmology} in regions of high density (such as clusters of galaxies) as compared to voids. Therefore, there exists

\*In this paper we will denote the dark energy field by \(Q\), whether it be quintessence, metamorphosis or condensation. Since the context should always be clear this should not lead to confusion.

---

FIG. 1. A schematic figure showing the difference between condensation and metamorphosis & quintessence. **Top:** the Compton wavelength \(\lambda_C \equiv (V'')^{-1/2}\). **Bottom:** The equation of state \(w(z)\). In quintessence and metamorphosis \(w(z)\) has some slowly varying value during radiation and matter domination. In condensation, \(w(z)\) only vanishes on averaging over many oscillations, as for standard dark matter. After the transition at \(z = z_t\) condensation becomes very similar to both metamorphosis and quintessence. For this example we chose \(w_f = -1\). CMB, LSS and SN-Ia data prefer a transition around \(z \sim 1.5 – 2\) \([11]\). One of the aims of this paper is to check whether the data simply prefer a late-time transition in the background cosmology (which controls the onset of acceleration) or are also fundamentally sensitive to the nature of the perturbations. To this end we compare metamorphosis and condensation.

Within our phenomenological descriptions both of these have identical background dynamics. However, the perturbation physics is fundamentally different. In metamorphosis the scalar field is very light \((m_Q \sim 10^{-33} eV)\) at all times\(^*\), both before and after the transition at \(z_t\) where \(w(z)\) changes. In condensation by contrast, the
the possibility that condensation may be relevant to the core-cusp problem of CDM [23], an idea recently explored in [24].

The aim of this paper is to analyse in detail how the condensation mechanism can be tested/falsified through its effects on CMB anisotropy, the matter power spectrum and the supernova luminosity distance. We also compare this class of models with standard and non-standard quintessence models such as vacuum metamorphosis [4]. The outline of the paper is the following: in the next section we describe the class of phenomenological models which we have adopted for the condensation and how condensation of cold dark matter into a scalar field affects CMB anisotropies and the matter power spectrum. In section III we test the model predictions with CMB, large scale structure and supernova data. In section IV we discuss how condensation differs from conventional quintessence and metamorphosis. We go on to examine how further constraints could be made with large scale structure analysis, angular-diameter distance to radio sources measurements and detection of fluctuations in the dark energy. In section V we discuss the non-linear evolutionary phase of the cosmologies. Section VI contains our conclusions and discussion.

II. THE PHENOMENOLOGICAL MODELS

In this paper we will compare and contrast three phenomenological models, particularly focusing on the first two:

- **Metamorphosis** [4] – a light scalar field undergoes a transition in its equation of state from *exactly pressure free* dust to $w_f < -0.3$ at some redshift $z_t$. This is appropriate for approximately describing many quintessence models [25].

- **Condensation** – A heavy scalar field with quadratic potential (e.g. CDM, *dust on average*) undergoes a transition in $w(z)$ to $w_f < -0.3$ and a very light mass $\sim 10^{-33} eV$ at $z_t$. Before the transition there are no light scalar fields or cosmological constant.

- **Evaporation** – The inverse of condensation. At high redshifts $z > z_t$, there is no CDM. There is only a light scalar field and baryons. At $z_t$ the scalar field condensate evaporates to yield the CDM, while $w(z)$ becomes negative to drive acceleration at late times.

We will now discuss in more detail these three cosmic scenarios.

A. Metamorphosis

The archetype of metamorphosis is the model proposed by Parker and Raval [4] in which non-perturbative quantum effects drive a transition in the equation of state of the scalar field. Metamorphosis was studied in detail in [11] where it was found that the data prefers a late-time transition. The equation of state which we use to parametrise both metamorphosis and condensation is:

$$ w(z) = w_0 + \frac{(w_f - w_0)}{1 + \exp((z - z_t)/\Delta)} $$

where we assume that $w_0 = 0$ and $\Delta = 30z_t$ where $\Delta$ controls the width of the transition. This equation of state is then fed into a modified Boltzmann code which uses the reconstructed scalar field potential along the background trajectory of the field, $Q(t)$ (see [11]). For an analysis of the effects of $\Delta$ on the CMB see [28]. For condensation this equation holds exactly for $z < z_t$ but only on average for $z > z_t$.

In terms of the scalar field dynamics, this class of models is characterised by three free parameters: the redshift $z_t$ at which the transition takes place, the fraction $\Omega_Q$ of the total energy density today due to the scalar field and the final value $w_f$ of the equation of state of the scalar field.

We note that in fact this metamorphosis describes rather accurately a wide range of standard quintessence models [25], particularly the Albrecht-Skordis model [27].

The main contrast between metamorphosis and condensation is that in the metamorphosis case the Compton wavelength of the field $Q$ is very large at all times, despite having $w = 0$ at early times. In condensation, as mentioned earlier, before the transition ($w = 0$ only), the Compton wavelength is very short until the transition, after which the field is very similar to the metamorphosis case; see fig. (1).

B. Condensation

The distinctive feature of condensate models is that the dark energy emerges from a condensation process which occurs when the universe reaches a certain critical temperature. This implies that above the redshift $z_t$ at which the condensation occurs the dynamics of the background is determined by photons, baryons, CDM, and neutrinos, while below the redshift $z_t$ cosmological dynamics is also determined by the dark energy field $Q$, its energy density $\rho_Q$ and its equation of state $w(z)$.

In general the details of the condensation will depend on the precise model used. We are not even guaranteed that such a condensate process is possible for realistic

\[\text{\textsuperscript{1}}\text{The parametrisation of } w(z) \text{ in eq. (2.1) is interesting in terms of the statefinder pair } \{r, s\} \text{ introduced in [29]. For a rapid transition, } \{r, s\} = \{1, 0\} \text{ for } z > z_t \text{ and } \sim \{1, 0\} \text{ for } z < z_t \text{ if } w_f \sim -1. \text{ The pair differ strongly from these values only near } z = z_t \text{ where } w(z) \text{ is large.}\]
beyond-the-standard-model physics. Hence for simplicity and due to our ignorance of the detailed microphysics we shall assume that the condensation occurs instantaneously on a constant energy hypersurface. Furthermore we impose that the total energy density is conserved through the transition. Since the transition is instantaneous this implies that the total CDM-$Q$ density perturbation is conserved.

\[
\dot{\rho}_{CDM} = \dot{\rho}_{CDM}^+ + \dot{\rho}_Q^+ \quad (2.2)
\]

\[
\delta \rho_{CDM} = \delta \rho_Q^+ + \delta \rho_{CDM}^+ \quad (2.3)
\]

The fraction $F$ of CDM energy density which is transferred to the scalar field $Q$ ($\rho_Q = F \rho_{CDM}$) can be easily inferred from Eq. (2.2)

\[
F = \frac{\Omega_Q}{\Omega_{CDM}} \frac{\Psi_w(z_1)}{\Psi_w(z)} \frac{1}{1 + \frac{\Omega_Q}{\Omega_{CDM}} \Psi_w(z_1)} \quad (2.4)
\]

where $\Omega_Q, \Omega_{CDM}$ are respectively the energy densities of the scalar field and CDM in units of the critical energy density today. The function $\Psi_w(z)$ is given by

\[
\Psi_w(z) = e^{\int_{z_f}^z \frac{dz}{(1+z)^3 w Q(z)}} \quad (2.5)
\]

In the following we shall consider a particular profile for the equation of state of the scalar field which coincides with that used for metamorphosis, i.e. equation (2.1).

Using Eq. (2.4,2.5) it is straightforward to see how dark energy domination is addressed within condensation. To achieve $\Omega_Q/\Omega_{CDM} = O(1)$ implies that

\[
F \sim e^{\int_{z_f}^z \frac{dz}{(1+z)^3 w Q(z)}} \quad (2.6)
\]

If we assume that $w(z) < 0$ for $z < z_f$, condensation at high redshift has to produce a small fraction of scalar field energy density. Similarly, the more negative the equation of state of the scalar field is, the less efficient the condensation process has to be.

In order to analyse the imprint of the condensation process on the CMB anisotropies and on the matter power spectra we need to solve the evolution equation for the scalar field fluctuations $\delta Q$. The simplest initial conditions for the perturbations of the scalar field $Q$ are $\delta Q = \dot{\delta Q} = 0$. However, since we are considering an instantaneous transition, this choice of initial conditions may not preserve adiabaticity \(^\dagger\). In particular a discontinuity in the time-derivatives of the scalar field $Q$ could generate a non-zero value of the intrinsic entropy perturbation of the scalar field, $S_{QQ}$:

\[
S_{QQ} = \frac{\delta Q}{Q} - \frac{\delta \dot{Q}}{Q} \quad (2.7)
\]

This always vanishes on large scales \([44]\) but could alter sub-Hubble dynamics if not treated carefully.

In order to fix the initial conditions for the scalar field perturbations, we follow the method developed in \([45]\) and we match the fluctuations of the scalar field $Q$ along a constant energy density hypersurface, which appears the most appropriate for a condensation process. In particular, the matching conditions for the perturbations are obtained by requiring the continuity of the induced metric and the extrinsic curvature. In the synchronous gauge and in a frame comoving with the CDM fluid, this implies that, for each Fourier mode $\delta Q_k$ the quantity $\theta_Q$ defined as

\[
(\rho_Q(1 + w_Q)\theta_Q \equiv -k^2 \dot{Q} \delta Q_k \quad (2.8)
\]

must be continuous at the transition. Using the conditions (2.2,2.3) we have $\theta_Q = \delta Q = 0$ across the transition and $\delta Q$ can be determined in terms of $Q$ and $\dot{\rho}_{CDM}$ by requiring $S_{QQ} = 0$. From then on, the evolution of the perturbations is calculated using the same modified Boltzmann code approach as for the metamorphosis model \([11]\).

C. Evaporation

Since we have considered the possibility that dark matter condenses at some low temperature into dark energy, it seems natural to consider the inverse process. This would be the evaporation of a condensate to yield all or part of the dark matter of the universe. Less severe forms of evaporation have been studied within the context of Affleck-Dine baryogenesis \([22]\) where a scalar condensate decays to give the required baryon asymmetry.

In its extreme form (no dark matter before evaporation) and if the evaporation occurs suddenly and completely at $z_t$ then a clear disadvantage of evaporation is that the universe does not accelerate today since $\Omega_Q = 0$. It therefore makes sense to study evaporation models in which the transition has not completed by today and has a large width, $\Delta$ or not all the scalar field evaporates into dark matter. Can we put limits on such models?

Let us consider the simplest model in which the there is no dark matter before the transition redshift $z_t$. For $w_f < -0.5$ and for $z \gg z_t$ the universe is essentially a baryon-dominated flat universe since the condensate energy density is negligible at high redshifts \(^5\). This places large constraints on the model since it is extremely difficult to reconcile CMB data, big bang nucleosynthesis (BBN), a flat universe and SN-Ia data (the incompatibility is $3 \sigma$ or more \([58]\) in a $\Lambda$ baryon dominated model). In addition, the large baryon content of

\(^\dagger\)We assume the primordial spectrum of fluctuations is purely adiabatic.

\(^5\)Though note that if $w = 0$ and $\Omega_Q$ is not negligible at decoupling this will not be true.
the universe tends to cause very prominent oscillations in the matter power spectrum that are not strongly observed in current data. This latter test is less straightforward to apply to evaporation since the nascent dark matter might tend to smooth out the oscillations if it is warm enough to have a sufficiently large free-streaming length.

A very interesting point is that evaporation would lead to a strongly time-dependent bias around the transition redshift, an effect which naively links to the work of Magliochetti et al [50] who found a strong redshift-dependence of the bias.

From the results of Griffiths et al [58] we can conservatively infer that, for \( w_f \approx -1 \), a low \( z_t \) is unacceptable for evaporation unless \( w_0 \) is close to 0. In fact, since the secondary acoustic peaks are sensitive to the baryon and dark matter content, it is likely that best fit to the CMB would occur for \( z_t > 1000 \).

Such values would be disfavoured from SN-Ia observations unless the transition were extremely wide so that there still remained a significant component with negative equation of state at low redshifts. As such, only "glacier evaporation" models would seem to be observationally viable. However, the evaporation process of the condensate in such a slow manner is physically realistic and evaporation has some interesting nonlinear implications which we discuss in section V. We will not, however, explicitly compare evaporation models with current data, leaving that to future work.

III. COMPARISON WITH OBSERVATIONS

A. Data and analysis method

In order to constrain/falsify our phenomenological models of condensation, we compare its predictions with a number of observations. In particular we consider three different kinds of datasets: CMB anisotropy, large scale structure (LSS) and type-1a supernova (SN-Ia) data. For the CMB anisotropies, we use the decorrelated COBE-DMR data [33] and the recent data from the DASI [34], MAXIMA [35], BOOMERanG [36] and CBI [37] experiments. The total number of data points is \( n_{\text{CMB}} = 58 \), with \( l \) ranging from 2 to 1235 for the first four sets, and the mosaic data from CBI extends this to 1900. In the analysis, we take into account calibration uncertainties for DASI, MAXIMA, BOOMERanG and CBI.

As far as LSS data are concerned, we use the matter power spectrum estimated from the following surveys: 2dF 100k redshift survey [38], IRAS PSCz 0.6 Jy [39], and Abell/ACO cluster survey [40]. In order to avoid possible effects due to non-linear contaminations, we fix a cut-off \( k_{\text{max}} = 0.2h/\text{Mpc} \). The total number of data points is \( n_{\text{LSS}} = 48 \). Finally, we use the redshift-binned SN-Ia data [42] from HZT and SCP (\( n_{\text{SN}} = 7 \)). Additionally we also studied the angular-diameter distance to radio sources [41]. This method is potentially able to probe a large range of redshifts, current data already covers \( 0.1 < z < 4 \). All of our models as well as \( \Lambda \)CDM are perfectly consistent with current data, which provides a valuable test. Unfortunately we will have to wait for larger samples before we can constrain our additional parameters significantly.

We performed a likelihood analysis by varying the three parameters directly related to the scalar field (\( \Omega_Q, w_f, z_t \)) and the Hubble parameter, \( H_0 \equiv 100h_0 \text{ km s}^{-1} \text{Mpc}^{-1} \). For the other parameters we chose the following values: \( \omega_b \equiv \Omega_b h^2 = 0.02 \), \( n_s = 1 \), \( n_t = 0 \), \( \tau = 0 \), \( \Omega_{\text{tot}} = 1 \), where \( \tau \) is the reionisation optical depth and \( n_{s,t} \) are the spectral indices for the scalar and tensor perturbations respectively. We also include the "standard" effective number number of neutrinos (3.04) and we fix the cold dark matter density as \( \Omega_{\text{CDM}} = 1 - \omega_b/(h_0^2) - \Omega_Q \). In our analysis we set the amplitude of the tensor perturbations to zero.

In order to compute the likelihood functions for the various parameters we follow the usual procedure. We compute the power spectra \( C_\ell \) of the CMB anisotropies and the CDM matter power spectra \( P(k) \)
for each set of parameters over the four-dimensional space \((\Omega_Q, z_t, w_f, H_0)\). We then evaluate the \(\chi^2\) at each grid point. The CMB anisotropy spectra and the matter power spectra are related through their respective amplitude normalisations. We found that the CMB data fixes quite well the overall amplitude for each model. As far as the LSS data are concerned we allowed a bias \(1/5 < b < 5\) for 2dF and PSCz and \(1/9 < b < 9\) for Abell/ACO since clusters are expected to be more biased than galaxies. We choose these large bias values since the biasing mechanism is still not very well understood especially in the context of non-standard dark matter models, and since (as argued in section V) the possibility of strongly non-linear physics enters on cluster scales.

We then finally determine the one-dimensional and two-dimensional likelihoods by integrating over the other parameters.

**B. Results**

Our main results are shown in figures (3) and (4). Both models prefer a energy density \(\Omega_Q\) close to 0.7 (0.7 for metamorphosis, 0.75 for condensation). Both models also prefer a final value of the equation of state \(w_f < -0.8\). A big difference is seen in the likelihood curve for \(z_t\). Whereas metamorphosis favours \(z_t \sim 1.5-2.5\), in condensation a late transition \((z_t < 1.5)\) is strongly disfavoured.

Overall the condensate model and the metamorphosis models both have slightly better best-fit \(\chi^2\) than the best \(\Lambda\)CDM model, although this is to be expected since we have extra parameters and is not highly significant. The condensation best fit \(\chi^2 = 79.3\), with parameters \(z_t = 4, w_f = -0.95\) and \(\Omega_Q = 0.75\), while the \(\Lambda\)CDM best fit \(\chi^2 = 84.9\) (for \(\Omega_{\Lambda} = 0.73\)). The best fit for metamorphosis has \(\chi^2 = 78.8\), with parameters \(z_t = 1.5, w_f = -1\) and \(\Omega_Q = 0.73\).

The following sections discuss in detail the CMB and LSS results. It should be noted that the supernova data only probes the background geometry, which is the same for both metamorphosis and condensation. Since it is also independent of the Hubble constant, the results of [11] are unchanged, except that we have updated the brightness of the supernovae at \(z \sim 1.7\) to account for lensing [43]. Namely, there is a slight preference for a low-\(z\) transition, but the relatively large error bars and lack of points at \(z > 1\) do not allow strong conclusions. Section IV C discusses more general aspects of using distance measurements to probe the background geometry of the universe.

**IV. COMPARISON OF CONDENSATION AND METAMORPHOSIS**

**A. CMB**

The likelihood plots for condensation and metamorphosis are very similar when considering \(\Omega_Q\) and \(w_f\). The major difference comes with the redshift of the transition \(z_t\). Although the background dynamics are identical, Metamorphosis favours a transition of \(z_t \simeq 2\), whereas condensation strongly disfavours \(z_t < 1.5\).

To explain this we need to take a closer look at the respective CMB angular power spectra, see fig. 7. Metamorphosis, like quintessence, has an almost trivial effect on the CMB relative to \(\Lambda\)CDM, simply through the ISW effect which changes the Sachs-Wolfe plateau and hence the relative height of the peaks through the CMB normalisation on large scales. The overall form of the CMB power spectrum of metamorphosis is not changed.

The effect of \(z_t\) on the acoustic peaks of condensation is more subtle. Although the total density of all species is unchanged, the phase-transition means that the change in the CDM density is not continuous. Changing \(z_t\) changes the CDM density at decoupling and the redshift of matter-radiation equality, even though its value, \(\Omega_{\text{CDM}} \simeq 1 - \Omega_Q\), is unchanged today. This alters the gravitational potential wells, although the effect is only significant for small \(z_t\), where the amount of CDM that is condensing out is at a maximum. Decreasing the amount of CDM changes the time of matter-radiation equality, which boosts the amplitude of all oscillations through an ISW effect. The third peak is moved to slightly higher \(\ell\) and thereby damped more, making it less high than the
second peak [47–49]. Overall, the condensation CMB power spectrum effectively interpolates between the two extreme cases of sCDM ($\Lambda = 0, \Omega = 1$) and ΛCDM.

For interest’s sake we also plot in fig (8) the predictions for the temperature-polarisation ($T \ E$) cross-power spectra for condensation and metamorphosis and compare them with the data from the first year of WMAP data [23]. Currently the data cannot distinguish between the models, especially as the curves are sensitive to reionisation history which will in turn depend on $z_t$ etc...

**B. LSS**

The large scale structure data also strongly disfavours any value of the transition redshift $z_t < 2$ for condensation. In contrast, the case of metamorphosis, $z_t$ seems to be almost independent of the LSS data, as shown in figs. (5,6). Why is this?

Comparing the transfer functions, $T(k)$, for the metamorphosis and condensation models with ΛCDM and sCDM (where the universe is flat and CDM dominated without cosmological constant so $\Omega_{\text{CDM}} = 1 - \Omega_0$) is informative. When normalised to give the correct COBE normalisation, sCDM gives a slightly smaller amount of power on large scales, but gives more power on small scales.

In condensation the Compton wavelength is small before the transition, and only becomes big after the transition. Clustering therefore does take place on all scales for $z > z_t$. For $z < z_t$, the growth of clustering on small scales ceases due to the large $\lambda_c$ but this also occurs on all scales since the universe starts to accelerate. The closer that transition is to today, the more clustering on

---

**FIG. 4.** The 2d-likelihood contours (Top: Condensation, bottom: Metamorphosis) for the scalar field parameters ($\Omega Q, z_t, w_f$) obtained combining all the CMB, LSS and SN-Ia data.

**FIG. 5.** Condensation: the 1d-likelihood functions for the scalar field parameters ($\Omega Q, z_t, w_f$). Left-column: CMB. Middle column: LSS. Right column: SN-Ia. The thick solid orange curves are the likelihoods obtained by introducing a Gaussian prior on $H_0$ ($h_0 = 0.72 \pm 0.08$).

**FIG. 6.** Metamorphosis: the 1d-likelihood functions for the scalar field parameters ($\Omega Q, z_t, w_f$). Left-column: CMB. Middle column: LSS. Right column: SN-Ia. The thick solid orange curves are the likelihoods obtained by introducing a Gaussian prior on $H_0$ ($h_0 = 0.72 \pm 0.08$).
small scales that can occur, resulting in a larger mass dispersion, $\sigma_s$. Again, the power spectrum interpolates between sCDM ($z_1 \rightarrow 0$) and $\Lambda$CDM ($z_t \gg 1$). Metamorphosis, in contrast, is a light scalar field before and after the transition, with little change in the Compton wavelength and so the large scale structure is only weakly dependent on the transition redshift, as discussed in [11].

But why does the LSS data disfavour a low $z_t$ for condensation? We have not computed $\sigma_s$ for all these models (we do so below for specific parameter values), so the reason cannot be that. Similarly, our bias upper limits of 5 and 9 for galaxy and clusters respectively are unlikely to have a real impact in excluding low-$z_t$ models.

The key lies in the change to the redshift of the matter-radiation equality, $z_{eq}$, which changes significantly with $z_t$ for $z_t < 5$ and provides the dominant effect on the shape of the power spectrum. Comparing the LSS data (fig. III A) and the condensation $T(k)$ (fig. IV B) we see that the sCDM power spectrum turns over around $k \sim 0.1 h Mpc^{-1}$ while for $z_t = 0.5$ the turnover starts around $0.05 h Mpc^{-1}$. In comparison, the data do not show any definite sign of a turnover, even out to $k = 0.01 h Mpc^{-1}$.

Hence both the CMB and the LSS disfavour condensation with $z_t < 1.5$. The same conclusion is likely if one does include $\sigma_s$ constraints.

The overall normalisations of the power spectra are not trivial, since they are linked to the COBE normalisation of the CMB power spectrum. We can read it off the high-$k$ region of fig. 9. While condensation lies between $\Lambda$CDM and sCDM as expected, we have to remember that the ISW effect of metamorphosis changes the normalisation strongly and leads to a low value of $\sigma_s$ [28].

The differences in perturbation evolution mentioned earlier can also be seen in fig. 10, which shows $\sigma_s$ (relative to the one of a $\Lambda$CDM universe) as a function of redshift for $z_t = 1.5$ and $w_f = -0.95$. Standard CDM starts to produce more structure, especially at low redshifts where a cosmological constant would start dominating. Condensation behaves for $z > z_t$ like sCDM, but its behaviour changes at $z_t$ roughly when the universe begins to accelerate, thereby forcing the linear perturbations to stop growing.

The redshift dependence of $\sigma_s$ and the bias factor can therefore be used to distinguish these models. Magliocchetti et al [50] found a strong signal of a redshift-dependent $\sigma_s$ for $z > 2$ and redshift dependent bias. The
stant. In particular, (6) with and without the HST prior on the Hubble constant.

leave this interesting issue to future work.

sudden change in the dark matter characteristics. We since the baryons will take some time to respond to the redshift dependent bias $b(z)$ is expected in our models since the baryons will take some time to respond to the sudden change in the dark matter characteristics. We leave this interesting issue to future work.

As a final point we note the strong differences between the CMB 1d likelihood curves in figs. (5) and (6) with and without the HST prior on the Hubble constant. In particular, $w_f$ changes drastically reflecting the well-known CMB degeneracy between $H$ and $w_f$.

C. Distance measurements

One important tool to constrain the equation is state is the determination of distances as a function of redshift, e.g. with supernovae. The basic building block is the comoving distance (for a flat universe)

$$d_C(z) \equiv c(\eta - \eta(z)) = \frac{c}{a_0} \int_0^z \frac{dz}{H(z)} \quad (4.1)$$

where $\eta(z)$ is the conformal time and

$$H(z) = H_0 \left[ \sum_i \Omega_i (1 + z)^{3(1+w_i)} \right]^{1/2} \quad (4.2)$$

$i$ indexes the constituents of the energy density in the universe, eg. radiation ($w = 1/3$), matter ($w = 0$), curvature ($w = -1/3$) and quintessence type contributions, where $w$ can vary as a function of $z$ like in our models. The apparent distance to a bright object such as a supernova is then determined by the luminosity distance $d_L = (1+z)d_C$, while the distance to an object of a given physical length (like radio sources, see [41]) is described by the angular diameter distance $d_A = d_C/(1+z)$ **.

It is also possible to use the CMB in a similar way: the angle subtended by the sound horizon at recombination gives us a fixed scale, and we can use it to determine the geometry between $\eta_{rec}$ and today. Since the horizon size is a comoving quantity and not a physical quantity, it is $d_C(\eta_{rec})$ which enters in this case. (See e.g. [59] and references therein for a detailed discussion.)

While it seems at a first glance that the distance measures depend on the value of the Hubble constant $H_0$, this is not so: all data available compares two lengths and thereby divides out any dependence on $H_0$. In the CMB case, the size of the sound horizon at last scattering depends linearly on $1/H_0$ just like $d_C$; in measuring the luminosity distance to SN-Ia, the normalisation of the distance ladder is treated as a nuisance parameter and effectively determined in units of $1/h$, as is the case for the physical reference length when working with the angular size of radio sources.

It is not possible to determine several quantities from only one data point (e.g. the location of the CMB

**$d_L$ and $d_A$ are, of course, equivalent and related via the reciprocity relation.**
FIG. 11. The left figure shows the degeneracies for measuring $\Omega_m$ versus $\Omega_\Lambda$ with supernovae at $z = 1$ (dashed lines) and the position of the CMB peaks (solid lines). The degeneracies are clearly orthogonal and an accurate measurement of both variables is possible. The right figure shows the same for $w$ and $\Omega_Q$ and measurements at $z = 1$ (dotted), 3 (dashed) as well as the CMB peak position (solid). The likelihood is very flat in the $w$ direction, making it difficult to pinpoint it precisely, especially for a very negative equation of state. The models shown here have a constant equation of state ($z_t$ is very large).

FIG. 12. The left figure shows the degeneracies for a model with $w_f = -1$ and observations at $z = 0.5$ (dotted), $z = 3$ (dashed) and the CMB (solid). The right figure shows the same for $\Omega_Q = 0.5$. A priori it is necessary to bracket the phase transition. As the right plot shows, high-$z$ distance data can help finding the transition in certain cases, but does usually not improve much on the CMB data (which is effectively at $z \approx 1100$). Also, transitions at redshifts higher than 5 will be very hard to detect unambiguously using distance measurements.

peaks)\(^\dagger\). Measuring distances over a range of $z$ values as done for supernovae and radio galaxies, improves the situation somewhat, as does combining different data sets. Indeed, a low $z_t$ can be strongly constrained by data on both sides of the transition (see figure 12). A high $z_t > 5$ may be impossible to detect given the current limits on $w$, since the dark energy becomes irrelevant very quickly.

Figure 11 also reinforces what the full likelihood figures in section III have already shown: $\Omega_Q$ can be determined to a much better accuracy than $w$, since distance measurements depend only quite weakly on its value.

It must be emphasised at this point that especially the CMB degeneracies depend strongly on the quantities which are being kept constant. If we fix $\Omega_m h^2$ instead of the Hubble constant (and therefore vary $h$ as a function of $\Omega_m$), the curves look substantially different (see e.g. [57]). Nonetheless, the overall conclusions remain the same.

D. Change in the comoving number density of the Lyman-$\alpha$ absorbers

The number of Lyman-$\alpha$ absorption lines per unit redshift, $dn/dz$, has been of interest to observers for a long time. It is sensitive both to the background evolution and to details of the linear and nonlinear density perturbation evolution [51]. This makes it a possible probe of quintessence type models. Unfortunately, the non-gravitational effects make this instrument very difficult to analyse and interpret reliably and would require high-resolution hydrodynamic N-body simulations. Nevertheless we illustrate the underlying point with a simple model.

Observers use a comoving volume element with a fixed physical radius $r_p$ and comoving length $dr_c$ [52]. As $r_c = c\eta_p$, we find that $dr_c/dz = 1/H(z)$, and since $r_c = (1+z)r_p$ we find that

$$dV = dr_c^3 \propto (1+z)^2/H(z)dr_p^2dz. \quad (4.3)$$

If we naively assume that the comoving number density of Lyman-$\alpha$ absorbers is constant, so that they directly probe the volume as a function of redshift, we set $dn(z) = \rho_0 dv(z)$ and obtain

$$\frac{dn}{dz} = \rho_0 H_0 \frac{(1+z)^2}{H(z)}. \quad (4.4)$$

Observers use the parametrisation $dn/dz \propto (1+z)^\gamma$. A pure CDM universe would therefore have $\gamma = 1/2$, while a pure $\Lambda$ universe had a $\gamma = 2$. As observers find $\gamma > 2$ even at redshifts larger than 1 [53] we have to conclude that either the average equation of state is extremely negative at these redshifts, or that the non-gravitational evolution effects dominate.

If it became possible to study the cosmological evolution only, then the density of Ly-$\alpha$ absorbers would be an extremely useful tool since the change of physical density depends on $H(z)$ and not its integral (as is the case for distance measurements and the position of the first peak in the CMB, see section IV C). Unfortunately the actual change of the number density is strongly influenced by evolutionary effects. At the moment one can speculate

\(^\dagger\)Of course the full CMB power spectrum contains many more observables, thereby breaking some of the degeneracies.
whether it is possible to draw conclusions from a differential approach: figure 13 shows $dn/dz$ for a CDM model and for a universe with a transition at $z_t = 1$. The second case shows a break in the comoving number density, to a slope with a lower $\gamma$. This change in the shape of $dn/dz$ might be detectable even though the actual form of the curve is strongly modified by additional physics.

As a final comment we note that there is also evidence for a change in the Lyman-α optical depth at a redshift $z \sim 3.2$ [54].

V. THE NONLINEAR EVOLUTIONARY PHASE

Both the condensation and metamorphosis models have an extremely interesting nonlinear portrait. Since the exact time and location of the two transitions is sensitive to local conditions of density and temperature, areas of nonlinearity can significantly delay the onset of the transition.

Let us consider metamorphosis first. In metamorphosis the transition occurs when the Ricci scalar grows to be of order $-m^2/\xi_{-1/6}$ [11]. Now $R = -\kappa T$ where $T = \rho - 3p \sim \rho$ for CDM. Hence, in high-density regions which will separate from the Hubble flow and recollapse to form bound structures like galaxy clusters, the Ricci scalar is more negative than in the low-density inter-galactic medium (IGM).

This implies that the IGM will be the first place where the metamorphosis transition occurs. Inside rich clusters, it may even occur that the local nonlinear growth of density perturbation overcomes the average FLRW growth of $R$ towards zero, and causes it locally to begin decreasing again. In these regions the metamorphosis transition might never take place.

The first implication of this is radical - the perturbed FLRW formalism for describing the universe rapidly becomes invalid as the universe now consists of a fluid with an inhomogeneous background equation of state - negative in voids and positive in rich clusters.

This will lead rapidly to a strongly scale dependent bias since density perturbations in voids and in clusters will evolve very differently. On scales smaller than the inhomogeneity in the equation of state, the perturbed FLRW approximation is good. Inside clusters perturbations will grow as they would in a flat, unaccelerating CDM model while in voids the perturbations will feel the expansion.

This has important implications since the issue of cluster abundance versus redshift suddenly becomes an extremely complex problem. It is no longer clear that clusters will be assembled at low-redshifts as happens in open or $\Lambda$-dominated universes due to the freeze-out in perturbation growth. Hence our linear calculations of $\sigma_8$ versus redshift must be carefully examined and may be wrong due to significant non-linear corrections.

In this vein it is interesting to note that even $\Omega_Q \rightarrow 1$ is not strongly excluded in the condensation case. An
obvious test of this would be to examine galaxy rotation curves at low and high redshifts; one might expect the rotation curves at \( z = 0 \) to show the lack of dark matter. However, due to the nonlinear nature of the condensation, the galaxies might never undergo the condensation process and hence might well be dark matter dominated despite most or all of the dark matter in the universe having already condensed. This nicely illustrates the potential subtleties involved in condensation.

The first implication of these effects is the fact that metamorphosis and condensation (as we consider them) are likely to make the core-cusp problem of CDM worse.

On the other hand, a recent paper [56] discussing Chandra observations of the elliptical galaxy NGC 4636 finds no discrepancy between the Chandra data and the existence of a central dark matter cusp. The averaged central dark matter density inferred for this galaxy is about two orders of magnitude larger than for dwarf spirals and low-surface brightness galaxies (where the core-cusp problem seems most notable). Also intriguing, their fit significantly improved when using a two-component dark matter model rather than just a single component.

It would also be interesting to study the formation of halo substructure in condensation type models. As this is a highly non-linear process, N-body simulations are needed for reliable predictions.

A. Nonlinear effects in evaporation

Given that the condensation and metamorphosis models appear to make the core-cusp problems of standard \( \Lambda \)CDM worse due to the incomplete transition in regions of high density, it is attractive to consider evaporation models, as described in section (II.C).

At the critical phase boundary the condensate, which exists early in the universe, starts to evaporate. At high temperatures, the universe consists (in the simplest models) solely of baryons and the condensate in addition to the radiation.

In these models, the high dark matter densities inside clusters would encourage a return to the condensate form and hence, assuming that the condensate had a long Compton wavelength, would suppress the formation of strong core density cusps which is perhaps the biggest current problem for \( \Lambda \)CDM. Meanwhile the IGM would remain in the normal, CDM, phase.

VI. DISCUSSION AND CONCLUSIONS

In this paper we have quantitatively studied a number of issues in detail:

- We have considered a phenomenological model for dark energy which forms via condensation of dark matter, which we took to be CDM. By comparing with current CMB, LSS and SN-Ia data we show that we do find a better fit to the data than from simple \( \Lambda \)CDM, but not significantly so.
- The condensation model has a Compton wavelength which changes from very small to very large around \( z = z_t \) in contrast to quintessence or metamorphosis where the Compton wavelength is always large. Condensation therefore has growth of small-scale structure absent in the other models which impacts on the CMB and LSS. Nevertheless, the data slightly prefer metamorphosis, and strongly favour metamorphosis if \( z_t \) is forced to be small, \( z_t < 1.5 \).
- We have confirmed the results of [11] for metamorphosis even when we allow the Hubble constant to vary: current data favour a late-time transition in \( w(z) \) at \( z_t \sim 2 \) (i.e. there is no degeneracy with \( H \) as there is for \( w_f \)). The best-fit value of \( z_t \) for condensation is \( z_t = 4 \).

We have also qualitatively discussed the nonlinear evolution of condensation and metamorphosis which is significantly more complex than quintessence.

Supernova data can be used to study the evolution of the background geometry quite independently of the dark energy model. It already is able to set useful limits on the equation of state today. For our class of models we find \( w_f < -0.6 \). The supernova data also seems to prefer a late transition in \( w \), but more data is needed to draw strong conclusions.

Future work should consider more realistic models for the condensate and examine the non-linear features of both metamorphosis and condensation in more detail. Future data should improve the constraints on these models significantly. Eventually, the study of the properties of the dark energy might lead to important insights into the physics at high energies which cannot be probed directly otherwise.

VII. ACKNOWLEDGEMENTS

We thank Rob Lopez for part of the code used in our analysis. We thank Céline Boehm, Rob Crittenden, Joe Silk, James Taylor and David Wands for useful comments and discussions. We thank Leonid Gurvits for the radio data. MK acknowledges support from the Swiss National Science Foundation. BB and CU are supported in part by the PPARC grant PPA/G/S/2000/00115. Part of the analysis was performed using the ICG SGI Origin.

[1] P. de Bernardis, et. al., Nature, 404, 955 (2000).
