Factor Analysis and Mathematical Modeling in Determining the Quality of Coal
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http://doi.org/10.29227/IM-2020-01-24
Submission date: 30-11-2019 | Review date: 28-03-2020

Abstract
The separation of coal material of three types of coals originating from three various Polish hard coal mines (types 31, 34.2 and 35, according to Polish nomenclature, which were steam coal, semi-coking coal and coking coal) into particle size fractions and then into particle density fractions was done and then the following parameters were measured for each particle size-density fraction: combustion heat, ash contents, sulfur contents, volatile parts contents, analytic moisture. In this way a 7-dimensional vector of data was created. Using methods of factor analysis the important features of coal were selected, which decide about their membership to individual types. To evaluate the appropriateness of the applied method the Bartlett's sphericity test as well coefficient of Kaiser-Mayer-Olkin (KMO) were used. To select important factors the Kaiser criterion and Cattell's scree test were used. The obtained results were compared with the results obtained in previous works by means of observation tunnels method. The results showed which particular features are crucial to define the type of coal what is also important to select appropriate method of its enrichment. Furthermore, the construction of a mathematical model presenting the relations between these properties and particle size and density is presented. Because of the fact that particles of certain size or density may occur in neighboring fractions three sorts of relations were examined basing on regression analysis. The analysis was conducted for all three coal types. Because of the fact that the models contain various amounts of independent variables $R^2$ coefficient, mean squared error (MSE) and Mallow's statistics $C_p$ were applied to evaluate and compare obtained results.

Keywords: coal, multidimensional statistical analysis, factor analysis, quality of coal, particle size, particle density

1. Introduction
Mineral raw materials which are beneficiated in purpose of their using characterize with many factors describing their features. In case of coal, these features are among others ash contents, sulfur contents, combustion heat, volatile parts contents or analytic moisture. The features mentioned above decide about coal quality also in economical aspect. Because of that the preciseness of determining values of these features is very important.

The most often researched properties of the coal are combustion heat, ash contents, sulphur contents, volatile parts contents and moisture. These features are very often highly correlated but also can occur independently. The selection of the necessary factors which influence on individual properties is the goal of the paper. To this purpose three types of coal (according to Polish nomenclature – coal types 31 (steam coal), 34.2 (gas-coking coal) and 35 (orto-coking coal)) were selected to the investigation which were divided into particle size and density fractions. The classification of coals is presented in Table 1.

| Size-Density Fraction | 1 | 2 | 3 | 4 | 5 |
|-----------------------|---|---|---|---|---|
| X₁ | combustion heat [cal] |
| X₂ | ash contents [%] |
| X₃ | sulfur contents [%] |
| X₄ | volatile parts contents [Va] |
| X₅ | moisture [Wa] |

Knowledge about these features can serve also to evaluate beneficiation process (Brożek, 1984; Dobosz, 2001; Foszcz et al., 2016; Glowiak 2019a; b; Niedoba, 2013a; Stanisz, 2007; Stępiński, 1964; Tumidajski and Saramak, 2009). The ash contents, sulfur contents and volatile parts contents were investigated dependably on particle size and particle density also by means of kriging method (Niedoba, 2013a). The application of non-conventional statistical methods can be very beneficial in getting precise information (Foszcz et al., 2018; Jamróz, 2009; 2014a; b; c; Jamróz et al., 2016; 2017; Jamróz and Niedoba, 2014; 2015a; b; Niedoba, 2009; 2011; 2013a; b; 2014; 2015; Niedoba and Surowiak, 2012; Niedoba et al., 2018; Öney, 2019; Pięta et al., 2018; Surowiak 2007, 2014). The presented work is an attempt of constructing new mathematical model describing relation between ash contents and particle size and density.

2. Materials and methods
The considered types of coal originated from three various Polish coal mines and all of them were initially screened on a set of sieves of the following sizes: -1.00, -3.15, -6.30, -8.00, -10.00, -12.50, -14.00, -16.00 and -20.00 mm. Then, the size fractions were additionally separated into density fractions by separation in dense media using zinc chloride aqueous solution of various densities (1.3, 1.4, 1.5, 1.6, 1.7, 1.8 and 1.9 g/cm³). The fractions were used as a basis for further consideration and additional coal features were determined by means of chemical analysis. In purpose of appropriate identification
of coal type many parameters are being measured which describe coal quality. For each density-size fraction such parameters as combustion heat, ash contents, sulfur contents, volatile parts contents and analytical moisture were determined, making up, together with the mass of these fractions, seven various features for each coal.

The example of obtained data is presented in Table 2.

In purpose of selecting significant factors influencing on individual variables, the factor analysis method was applied. To evaluate adequacy of applying factor analysis to this problem two criteria were used: Bartlett’s test and Kaiser-Mayer-Olkin coefficient (KMO) (Comrey, 1973; Dobosz, 2001; Kline, 1994; Lawley and Maxwell, 1971; Tumidajski and Saramak, 2009).

The reduction of variables is done through the Cattell’s scree criteria and criterion of sufficient proportion which suggest to apply such number of factors that they explain together at least 85% of variance of all observed variables [Stanisz, 2007].
### Tab. 3. Influence of factors on properties of coal, type 31 by particle size fractions

| Feature | 0.5–1 | 1–3.15 | 3.15–6.3 | 6.3–8 | 8–10 | 10–12.5 | 12.5–14 | 14–16 | 16–20 |
|---------|-------|--------|----------|-------|------|---------|---------|-------|-------|
|         |       |        |          |       |      |         |         |       |       |
|         |       |        |          |       |      |         |         |       |       |
|         |       |        |          |       |      |         |         |       |       |
|         |       |        |          |       |      |         |         |       |       |

|         |       |        |          |       |      |         |         |       |       |

### Tab. 4. Influence of factors on properties of coal, type 34.2 by particle size fractions

| Feature | 0.5–1 | 1–3.15 | 3.15–6.3 | 6.3–8 | 8–10 | 10–12.5 | 12.5–14 | 14–16 | 16–20 |
|---------|-------|--------|----------|-------|------|---------|---------|-------|-------|
|         |       |        |          |       |      |         |         |       |       |
|         |       |        |          |       |      |         |         |       |       |
|         |       |        |          |       |      |         |         |       |       |
|         |       |        |          |       |      |         |         |       |       |

|         |       |        |          |       |      |         |         |       |       |

### Tab. 5. Influence of factors on properties of coal, type 35 by particle size fractions

| Feature | 0.5–1 | 1–3.15 | 3.15–6.3 | 6.3–8 | 8–10 | 10–12.5 | 12.5–14 | 14–16 | 16–20 |
|---------|-------|--------|----------|-------|------|---------|---------|-------|-------|
|         |       |        |          |       |      |         |         |       |       |
|         |       |        |          |       |      |         |         |       |       |
|         |       |        |          |       |      |         |         |       |       |
|         |       |        |          |       |      |         |         |       |       |

|         |       |        |          |       |      |         |         |       |       |

### 3. Results

#### 3.1. Factor analysis

Applying Bartlett’s test it occurred that for all researched cases the value of the test was significantly higher than the critical values on significance level being equal to $\alpha = 0.0005$. The lowest value of the test $U$ was obtained for coal, type 35 in particle density fraction (1.9–2.0) and was equal to $84.74$, while the critical value on this level is equal to $31.42$. It can
be said then that zero hypothesis (that correlation matrix is a unit matrix) should be rejected for all particle size and density fractions.

Furthermore, it can be noticed that in almost all cases the value of KMO coefficient was higher than 0.5. Only for density fraction lower than 1.3 g/cm³ for coal, type 34.2 and density fraction (1.6-1.7) for coal, type 35 it occurred to be slightly lower than 0.5. That means that the results of Bartlett’s test and the values of KMO coefficient gave strong basis to apply factor analysis.

In the work, the reduction of variables is done through the Cattell’s scree criteria and criterion of sufficient proportion which suggest to apply such number of factors that they explain together at least 85% of variance of all observed variables [22].

The correlation matrix of the factor Z with variable X is obtained by creation of matrix Z, which elements are numbers

$$z_{ij} = \sqrt{\rho_{ij}}, \quad i, j = 1, 2, ..., 5. \quad (1)$$

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Tab. 6. Influence of factors on properties of coal, type 31 by particle density fractions

| Feature | <1.3 | 1.3-1.4 | 1.4-1.5 | 1.6-1.7 | 1.7-1.8 | 1.8-1.9 | 1.9-2.0 |
|---------|-------|---------|---------|---------|---------|---------|---------|
| Z₁      | 86.99 | 87.47   | 35.10   | 84.97   | 59.66   | 28.46   | 87.94   | 75.69   |
| Z₂      | 60.40 | 8.15    | 20.53   | 63.98   | 27.82   |         |         |         |
| Z₃      |       | 9.15    |         |         |         |         |         |         |

Ash contents

| Z₁      | 92.42 | 94.03   | 82.88   | 70.94   | 83.86   | 82.04   | 75.15   | 51.62   |
| Z₂      | 1.33  | 25.38   | 11.19   | 0.02    | 9.01    | 36.33   |         |         |
| Z₃      | 3.12  | 17.05   | 0.01    |         |         |         |         |         |
| Z₄      |       | 15.30   |         |         |         |         |         |         |

Sulfur contents

| Z₁      | 7.82  | 17.61   | 35.58   | 64.03   | 36.52   | 18.13   | 1.60    | 40.24   |
| Z₂      | 56.73 | 80.64   | 48.87   | 4.86    | 36.97   | 67.04   | 87.51   | 54.30   |
| Z₃      | 34.85 | 14.49   | 17.92   | 23.87   |         |         |         |         |

Volatile parts contents

| Z₁      | 89.88 | 87.01   | 73.80   | 68.22   | 21.16   | 71.84   | 18.36   | 74.33   |
| Z₂      | 0.03  | 1.26    | 43.08   | 13.03   | 42.04   | 16.73   |         |         |
| Z₃      | 0.06  | 6.83    | 35.45   | 1.06    | 38.69   |         |         |         |
| Z₄      | 24.75 | 23.66   |         |         |         |         |         |         |

Moisture

| Z₁      | 1.87  | 79.85   | 63.42   | 46.36   | 93.10   | 60.04   | 39.66   | 66.11   |
| Z₂      | 67.24 | 0.09    | 5.97    | 37.93   | 5.84    | 34.95   | 0.07    |         |
| Z₃      | 30.74 | 18.61   | 27.06   | 15.37   | 31.75   | 0.06    | 37.06   |         |
| Z₄      |       | 24.35   |         |         |         |         |         |         |

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Tab. 7. Influence of factors on properties of coal, type 34.2 by particle density fractions

| Feature | <1.3 | 1.3-1.4 | 1.4-1.5 | 1.6-1.7 | 1.7-1.8 | 1.8-1.9 | 1.9-2.0 |
|---------|-------|---------|---------|---------|---------|---------|---------|
| Z₁      | 68.90 | 21.37   | 75.81   | 52.56   | 73.37   | 99.60   | 91.83   | 83.37   |
| Z₂      | 13.54 | 70.82   | 1.03    | 28.72   | 17.60   |         | 7.68    |         |
| Z₃      | 14.35 | 20.48   | 13.13   |         |         |         |         |         |

Ash contents

| Z₁      | 80.94 | 83.39   | 89.18   | 73.41   | 34.26   | 13.14   | 87.25   | 6.51    |
| Z₂      | 6.02  | 10.66   | 3.69    | 43.21   | 23.72   |         | 86.19   |         |
| Z₃      | 18.01 | 4.79    | 9.90    | 14.92   | 46.22   |         |         |         |
| Z₄      |       | 5.54    |         |         |         |         |         |         |

Sulfur contents

| Z₁      | 18.36 | 83.26   | 63.64   | 53.96   | 30.73   | 51.60   | 37.05   | 95.39   |
| Z₂      | 64.78 | 11.55   | 8.70    | 0.06    | 56.11   | 48.00   | 52.91   |         |
| Z₃      | 16.30 | 23.41   | 31.76   |         |         |         |         |         |

Volatile parts contents

| Z₁      | 82.88 | 71.58   | 56.23   | 11.26   | 52.25   | 1.46    | 75.65   | 87.32   |
| Z₂      | 15.10 | 0.06    | 0.01    | 81.28   | 34.98   | 72.72   | 19.64   |         |
| Z₃      | 26.70 | 34.32   | 14.75   |         |         |         |         |         |

Moisture

| Z₁      | 49.75 | 47.22   | 2.97    | 43.19   | 19.22   | 43.02   | 40.90   | 64.67   |
| Z₂      | 25.45 | 42.22   | 86.52   | 8.72    | 2.11    | 32.11   | 48.26   | 24.86   |
| Z₃      | 20.63 | 39.77   | 76.65   | 4.38    |         |         |         |         |
| Z₄      |       | 20.13   |         |         |         |         |         |         |
where $\lambda_i$ - $i$th eigenvalue of correlation matrix; $a_{ji}$ - elements of matrix $A$ which fulfills the condition $A^T = R$, where $R$ is correlation matrix of variables $X_j$.

The square of number $z_{ij}$ is the percentage of variance changeability explained by the factor $Z_j$. For example, considering coal, type 31 from the particle size fraction (10-12.5) it is obtained that matrix $Z$ is in form

$$
Z = \begin{bmatrix}
-0.9813 & 0.1331 & -0.0962 & 0.0676 & 0.0747 \\
0.9828 & -0.1017 & 0.1145 & -0.0767 & 0.0700 \\
-0.0667 & -0.0963 & -0.0484 & 0.0246 & 0.0033 \\
-0.9793 & -0.0651 & -0.0297 & 0.1893 & -0.0019 \\
-0.9620 & -0.1035 & 0.2487 & 0.0063 & -0.0029
\end{bmatrix}
$$

The plot of scree is presented on Figure 1.

On the basis of the presented Cattell's scree plot only these factors remain which are located to the left from the point in which a mild decline of eigenvalues is observed. In this case these are factors $Z_1$ and $Z_2$.

The group of factors ($Z_1$, $Z_2$) explain 98.07% of changeability of combustion heat, 97.12% of changeability of ash contents, 99.71% of changeability of sulfur contents, 96.33% of changeability of volatile parts contents and 93.62% of changeability of moisture.

It is obtained then that factor $Z_1$ is responsible for variables $\{X_1, X_2, X_4, X_5\}$ and factor $Z_2$ for variable $X_3$.

Let consider the particle density fraction (1.6-1.7) of coal, type 34.2. The matrix $Z$ is in form

$$
Z = \begin{bmatrix}
-0.8566 & -0.4196 & -0.1506 & -0.1824 & 0.3643 \\
-0.5054 & -0.6574 & 0.3863 & 0.1622 & 0.1400 \\
-0.5544 & -0.7491 & -0.2733 & 0.1082 & -0.2073 \\
-0.7229 & -0.5915 & -0.1727 & 0.2615 & -0.0447 \\
-0.4385 & 0.1454 & -0.0755 & 0.0956 & 0.0961
\end{bmatrix}
$$

The eigenvalues of correlation matrix in this case are numbers $\lambda_1=3.8177$; $\lambda_2=1.3055$; $\lambda_3=0.0875$; $\lambda_4=0.0488$; $\lambda_5=0.0105$.

The plot of scree is presented on Figure 2.

On the basis of the presented Cattell's scree plot only these factors remain which are located to the left from the point in which a mild decline of eigenvalues is observed. In this case these are factors $Z_1$, $Z_2$ and $Z_3$.

The group of factors ($Z_1$, $Z_2$, $Z_3$) explain 93.25% of changeability of combustion heat, 92.41% of ash contents, 94.32% of sulfur contents, 90.33% of volatile parts contents and 97.99% of moisture.

### Table 8: Influence of factors on properties of coal, type 35 by particle density fractions

| Feature          | <1.5 | 1.5-1.4 | 1.4-1.5 | 1.5-1.6 | 1.6-1.7 | 1.7-1.8 | 1.8-1.9 | 1.9-2.0 |
|------------------|------|---------|---------|---------|---------|---------|---------|---------|
| **Combustion heat** |      |         |         |         |         |         |         |         |
| $Z_1$            | 36.72| 94.80   | 58.46   | 93.14   | 55.65   | 88.39   | 59.42   | 99.70   |
| $Z_2$            | 47.32| 0.17    | 43.09   | 18.46   |         |         |         |         |
| $Z_3$            | 13.34| 37.14   |         |         |         |         |         |         |

### Table 9: Ash contents by separation in accordance to particle size – coal, type 31

| Feature          | 1.75 | 2.69 | 5.38 | 7.62 | 9.81 |
|------------------|------|------|------|------|------|
| **Ash contents** |      |      |      |      |      |
| $Z_1$            | 15.03| 94.01| 71.14| 76.54| 77.59|
| $Z_2$            | 56.07| 23.93| 9.48 | 5.65 | 32.02|
| $Z_3$            | 13.34| 27.41|      |      |      |

### Table 10: Moisture of coal, type 31

| Feature          | 1.75 | 2.69 | 5.38 | 7.62 | 9.81 |
|------------------|------|------|------|------|------|
| **Moisture**     |      |      |      |      |      |
| $Z_1$            | 70.12| 23.27| 86.19| 7.81 | 27.96|
| $Z_2$            | 14.68| 1.38 | 33.79| 44.98| 68.92|
| $Z_3$            | 11.48| 72.67| 48.87| 20.76| 25.99|

where: $\lambda_i$ - $i$th eigenvalue of correlation matrix; $a_{ji}$ - elements of matrix $A$ which fulfills the condition $A^T = R$, where $R$ is correlation matrix of variables $X_j$. The square of number $z_{ij}$ is the percentage of variance changeability explained by the factor $Z_j$. For example, considering coal, type 31 from the particle size fraction (10-12.5) it is obtained that matrix $Z$ is in form

$$
Z = \begin{bmatrix}
-0.9813 & 0.1331 & -0.0962 & 0.0676 & 0.0747 \\
0.9828 & -0.1017 & 0.1145 & -0.0767 & 0.0700 \\
-0.0667 & -0.0963 & -0.0484 & 0.0246 & 0.0033 \\
-0.9793 & -0.0651 & -0.0297 & 0.1893 & -0.0019 \\
-0.9620 & -0.1035 & 0.2487 & 0.0063 & -0.0029
\end{bmatrix}
$$

The group of factors ($Z_1$, $Z_2$) explain 98.07% of changeability of combustion heat, 97.12% of changeability of ash contents, 99.71% of changeability of sulfur contents, 96.33% of changeability of volatile parts contents and 93.62% of changeability of moisture.
ture, while factor $Z_i$ is related to variables $X_1, X_2, X_3, X_4$; factor $Z_2$ to variables $X_2, X_3, X_4$ and factor $Z_3$ to variable $X_5$.

Another criterion of limiting number of factors is determination of amount of percent of total variance explained by chosen factors (most often it is required to not be lower than 85%). In this case, for coal type 31, factors $Z_1$ and $Z_2$ explain 93.14% of variation of variable $X_1$ (combustion heat), 96.65% of variation of variable $X_2$ (ash contents), 99.00% of variation of variable $X_3$ (sulfur contents), 91.14% of variation of variable $X_4$ (volatile parts contents) and 89.14% of variation of variable $X_5$. Finally, for coal type 35, these factors explain 98.21% of variation of variable $X_1$, 98.39% of variation of variable $X_2$, 99.87% of variation of variable $X_3$, 95.57% of variation of variable $X_4$ and 99.00% of variation of variable $X_5$.

The influences of individual factors on considered variables in all fractions of individual types of coal are presented in Tables 3-8. It was assumed that changeability of each feature should be explained by factors in at least 85%.

3.2. Mathematical modeling

On the basis of one- and multidimensional regressive analysis four models presenting relations between ash contents in certain particle size fraction (or density fraction), particle density (or particle size) and ash contents in neighboring size or density fractions.

The general form of proposed models are:

98.21% of variation of variable $X_1$, 98.39% of variation of variable $X_2$, 99.87% of variation of variable $X_3$, 95.57% of variation of variable $X_4$ and 99.00% of variation of variable $X_5$.

The influences of individual factors on considered variables in all fractions of individual types of coal are presented in Tables 3-8. It was assumed that changeability of each feature should be explained by factors in at least 85%.

3.2. Mathematical modeling

On the basis of one- and multidimensional regressive analysis four models presenting relations between ash contents in certain particle size fraction (or density fraction), particle density (or particle size) and ash contents in neighboring size or density fractions.

The general form of proposed models are:
One-dimensional model
\[ y = ax_1 + b \]  

Two-dimensional models
\[ y = a_1x_1 + a_2x_2 + b \]  

\[ y = a_3x_3 + a_4x_4 + b \]  

Three-dimensional model
\[ y = a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 + b \]  

Where:
- \( y \) – ash contents in certain particle size (or particle density) fraction;
- \( x_1 \) – particle size or particle density;
- \( x_2 \) – ash contents in previous particle size (or density) fraction;
- \( x_3 \) – ash contents in following particle size (or density) fraction.

Because of the fact that during material separation process particles from other fractions transfer to the certain considered fraction in two- and three-dimensional models ash contents in neighboring fractions were taken into account and their influence was evaluated.

The analysis was conducted for all three types of coal. The results of analyzes were presented in Tables 9–14.

On the basis of the results the regressive analysis was conducted and the following models were obtained in accordance to the equations (4)–(7):

\[
\begin{align*}
\hat{y}_1 &= 2.505x_1 + 10.910 \\
\hat{y}_2 &= 1.397x_1 + 0.385x_2 + 10.307 \\
\hat{y}_3 &= 2.407x_1 + 0.166x_2 + 6.480 \\
\hat{y}_4 &= 1.979x_1 + 0.153x_2 + 0.145x_3 + 6.720
\end{align*}
\]

In this case the models created in accordance to equations (4)–(7) are in form:
\[ y_1 = 84.798x_1 - 105.185 \\
\]
\[ y_2 = 67.294x_1 + 0.227x_2 - 82.350 \\
\]
\[ y_3 = 84.577x_1 + 0.010x_2 - 105.096 \\
\]
\[ y_4 = 63.617x_1 + 0.268x_2 + 0.022x_2 - 77.993 \\
\]

3.3. Investigation of models quality

To evaluate the quality of models obtained by means of general formulas presented in equations (4)–(7) such factors as \( R^2 \) coefficient, mean squared error \( \text{MSE} \) and Mallow’s statistics \( C_p \) were calculated which are given by the following formulas (Stanisz, 2007; Tumidajski and Saramak, 2009), presented in equations (8)–(10):

\[
R^2 = 1 - \frac{\sum(y_j - \hat{y}_j)^2}{\sum(y_j - \bar{y})^2}, \quad j = 1, 2, 3, 4
\]

(8)

\[
\text{MSE} = \frac{\sum(y_j - \hat{y}_j)^2}{n - q - 1}
\]

(9)

where \( q \) is an amount of independent variables occurring in considered function

\[
C_p = \frac{\sum(y_j - \hat{y}_j)^2}{\text{MSE}_i}
\]

(10)

where \( \text{MSE}_4 \) is mean squared error calculated for \( y_4 \).

The obtained results of calculated errors are presented in Tables 15 and 16.

4. Conclusions

Because of the fact that the most often three factors occur in individual fractions and considering power of relations between individual properties the investigated variables can be divided into three subsets. First one contains combustion heat, ash contents and volatile parts contents, second one contains sulfur contents and the third one contains moisture. In scientific works [3, 4, 5, 6, 7, 8, 9, 12, 13, 14, 15, 16, 17, 18, 19, 20, 23, 24], through application of various visualization methods it was claimed that features being sufficient to identify coal type are sulfur contents, moisture and volatile parts contents. The conducted analysis confirms these results. The selection of variable \( X_j \) (volatile parts contents) occurs from the fact that this variable is explained by other factor than mutual factor with variables moisture and combustion heat.

Considering the mathematical models it must be said that during grained material separation (in this case – coal) into particle size or density fractions some of the particles from neighboring fractions \((j-1 \text{ or } j+1)\) occur in \( j \)th fraction it seems to be justified to consider this fact during construction of mathematical model describing ash contents by means of particle size or density.

In the paper four models are proposed:

- One-dimensional, which does not consider influences of neighboring fractions;
- Two-dimensional, which takes the influence of one of neighboring fractions into consideration – two models of such type;
- Three-dimensional, which takes the influence of both neighboring fractions.

The verification of these models was conducted on the basis of three factors: \( R^2 \) coefficient, mean squared error \( \text{MSE} \) and Mallow’s statistics \( C_p \).

Taking into consideration the \( R^2 \) coefficient it is visible that for all considered models the value of this factor is relatively high (above 0.9). It can be noticed that the \( R^2 \) achieves higher values when the separation is done in accordance to particle density than in case of particle size (apart from coal, type 34.2).

Furthermore, the value of mean squared error indicates that the models are well fitted, but (apart from coal, type 34.2) significantly better fitting to empirical results is achieved in case of separation done in accordance to particle density. To compare the models for various dimensions the Mallow’s statistics \( C_p \) was used, which suggests that the best model is the one which values of \( C_p \) is close to the value \( q+1 \), where \( q \) is a number of independent variables occurring in the model. Analyzing Tables 8 and 9 it can be stated that the best model is a three-dimensional one, but in some cases, as for coal, type 35 by separation done in accordance to particle size, the two-dimensional models have the value of \( C_p \) around \( q+1=3 \).

The analyzed cases indicate that despite satisfying results of one-dimensional approximation to obtain better models is worthy to consider also influences of the researched feature in neighboring fractions.

Acknowledgement

The paper is a result of AGH UST statutory project no. 11.11.100.276.
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Analiza czynnikowa i modelowanie matematyczne w określaniu jakości węgla
Wykonano rozdział trzech typów węgla o różnych charakterystykach, pochodzących z trzech różnych kopalń węgla kamiennego w Polsce (typy 31, 34.2 oraz 35, według Polskich norm, którymi były węgiel energetycznym, pół-koksujący oraz koksujący) na klasy ziarnowe a następnie na frakcje gęstościowe. Dla każdej otrzymanej w ten sposób frakcji wielkościowo-gęstościowej zmierzono następujące parametry: ciepło spalania, zawartość popiołu, zawartość siarki, zawartość części lotnych, wilgotność analityczna. W ten sposób otrzymano siedmiowymiarowy wektor danych. Za pomocą analizy czynnikowej wybrano istotne cechy węgla, które decydują o jego przynależności do określonego typu węgla. Aby ocenić prawidłowość zastosowanej metody wykorzystano test sferyczności Bartletta oraz współczynnik Kaisera-Mayera-Olkina (KMO). Otrzymane wyniki porównano z wynikami otrzymanymi w poprzednich pracach, które uzyskano metodą tuneli obserwacyjnych. Wyniki pokazywały, że cechy węgla są niezbędne do określenia typu węgla, co wpływa na dobór odpowiedniej metody jego wzbogacania. Ponadto, zaprezentowano model prezentujący relacje pomiędzy tymi cechami a wielkością i gęstością ziaren. Ponieważ ziarna określonej wielkości lub gęstości mogą występować w sąsiednich klasach lub frakcjach, wykonano trzy typy modeli, bazując na analizie regresji. Analiza została wykonana dla trzech typów węgla. Ponieważ modele zawierają różne ilości zmienionych niezależnych do oceny i porównania otrzymanych wyników zastosowano współczynnik determinacji $R^2$, błąd średniokwadratowy (MSE) oraz statystykę Mallowa $C_p$.

Słowa kluczowe: węgiel, wielowymiarowa analiza statystyczna, analiza czynnikowa, jakość węgla, wielkość ziarna, gęstość ziarna