Competition and interplay between topology and quasi-periodic disorder in Thouless pumping of ultracold atoms

Shuta Nakajima1,6, Nobuyuki Takei1,6, Keita Sakuma1, Yoshito Kuno1,3, Pasquale Marra4,5 and Yoshiro Takahashi1

Robustness against perturbations lies at the heart of topological phenomena. If, however, a perturbation such as disorder becomes dominant, it may cause a topological phase transition between topologically non-trivial and trivial phases. Here we experimentally reveal the competition and interplay between topology and quasi-periodic disorder in a Thouless pump realized with ultracold atoms in an optical lattice, by creating a quasi-periodic potential from weak to strong regimes in a controllable manner. We demonstrate a disorder-induced pumping in which the presence of quasi-periodic disorder can induce a non-trivial pump for a specific pumping sequence, whereas no pump is observed in the clean limit. Our highly controllable system, which can also straightforwardly incorporate interatomic interaction, could be a unique platform for studying various disorder-related effects in a wide range of topological quantum phenomena.

Historically, the robustness of the topological invariant Chern number against perturbations was first discussed by Niu and Thouless12, who investigated the effects of spatial potential disorder and inter-particle interaction on topological Thouless pumps13. A Thouless pump is the quantum transport of a fermionic gas in a one-dimensional periodic potential driven in adiabatic cycles. The charge pumped per cycle is equal to the Chern number defined over a two-dimensional Brillouin zone with one spatial and one temporal dimension: therefore the Thouless pump can be regarded as a (1+1)-dimensional counterpart of the quantum Hall effect. Niu et al. derived an effective Chern number even under perturbations by introducing twisted boundary conditions5. As a result, they revealed that the pumping amount corresponding to the Chern number does not change unless the gap between the associated energy bands is closed. The experimental realization of a Thouless pump in clean systems using ultracold atoms in optical lattices17–20 and photonic waveguides21–22 triggers various interesting theoretical investigations on the effect of perturbations such as on-site static23–26 and dynamic27 potentials as well as interatomic interaction28–30.

Here, by using a Thouless pump realized with ultracold fermions in a dynamical optical lattice, we experimentally reveal the competition and interplay between topology and quasi-periodic disorder by generating a controllable quasi-periodic potential. We successfully demonstrate a disorder-induced pumping, that is, a trivial phase with no pump in the clean limit that is driven to a non-trivial phase with finite pump owing to the quasi-periodic disorder. Moreover, our experimental observations quantitatively reveal the degree of robustness of the Thouless pumps against quasi-periodic disorder. Our measurement further suggests that the quasi-periodic disorder induces a topological phase transition from non-trivial to trivial phases.
Experimental set-up of Thouless pumping with quasi-periodic disorder

We experimentally implement Thouless pumping in the presence of quasi-periodic disorder with precisely controllable strength by using an ultracold non-interacting Fermi gas of ytterbium (\(^{171}\text{Yb}\)) atoms in a two-dimensional (2D) array of continuous Rice–Mele (cRM) lattices (Methods). Each lattice is a dynamically controlled optical superlattice that consists of a dynamical interferometric lattice with the lattice constant \(d = 532\) nm (‘long lattice’) and a stationary lattice with constant \(d/2\) (‘short lattice’). We superimpose another optical lattice at wavelength \(\lambda_s\) tilted by 45° with respect to the pumping direction \(x\) (Fig. 1a). Hereafter, we refer to this lattice as the ‘disorder lattice’. Owing to strong confinement along the \(y\) and \(z\) axes provided by other optical lattices, the following time-dependent one-dimensional (1D) superlattice is created:

\[
V(x, t) = \begin{cases} 
V_S \cos^2 \left( \frac{2\pi x}{\lambda_S} \right) - V_L \cos^2 \left( \frac{\pi x}{\lambda_L} - \phi(t) \right) 
- V_D \cos^2 \left( \frac{\pi x}{\lambda_D} + \frac{\pi}{4} \right), 
\end{cases}
\]

(1)

where \(V_S\), \(V_L\), and \(V_D\) are the depths of the short, long and disorder lattices, respectively. \(\lambda_D = \lambda_D/\sqrt{2}\) is the lattice constant of the disorder lattice along the pumping direction. When \(\lambda_s \neq d\), that is, \(\lambda_s \neq 752\) nm, the quasi-periodic disorder is produced, and its strength can be controlled by adjusting the depth \(V_S\) (Fig. 1b). \(\phi\) is the phase difference between the short and long lattices and \(\alpha\) is the phase difference between short and disorder lattices. Note that this phase \(\alpha\) takes different values for different 1D superlattices owing to our configuration in Fig. 1a (Methods). Hereafter, we use the lattice constant \(d\) as the length unit and the recoil energy \(E_R = \hbar^2/(8md^2)\) as the energy unit, where \(\hbar\) denotes Planck’s constant and \(m\) the atomic mass of \(^{171}\text{Yb}\). We load ytterbium atoms into the 2D array of 1D cRM lattices in the staggered phase (\(\phi = 0\)), first by ramping up the long lattice and then by simultaneously ramping up short and disorder lattices adiabatically, ensuring that they occupy the lowest energy band, and then we slowly sweep \(\phi\) over time. The lattice potential returns to its initial configuration whenever \(\phi\) changes by \(\pi\), thereby completing a pumping cycle. The typical length of a 1D superlattice is about 12 unit cells around the centre of the atomic cloud.

Our Thouless pump under quasi-periodic disorder can be approximately described by the tight-binding Rice–Mele (tRM) model\(^{1,12}\) with on-site quasi-periodic disorder:

\[
\hat{\mathcal{H}} = \sum_i \left( -(I + \delta(t)) \hat{a}_i^\dagger \hat{b}_i - (I - \delta(t)) \hat{a}_i \hat{b}_i + \h.c. \right) + \Delta(t) (\hat{a}_i^\dagger \hat{a}_{i+1} - \hat{b}_i^\dagger \hat{b}_{i+1}) + \Delta_0^a (\hat{a}_i^\dagger \hat{a}_i) + \Delta_0^b (\hat{b}_i^\dagger \hat{b}_i),
\]

(2)

where \(\hat{a}_i^\dagger\) (\(\hat{a}_i\)) and \(\hat{b}_i^\dagger\) (\(\hat{b}_i\)) are fermionic creation (annihilation) operators in the two sublattices of the \(i\)th unit cell, \(I \pm \delta(t)\) is the tunnelling amplitude within and between unit cells, \(\Delta(t)\) is the staggered on-site energy offset \((\max|\Delta(t)| = \Delta_0)\), \(\Delta_0^a, \Delta_0^b\) is the on-site quasi-periodic disorder of sublattice \(a\) (\(b\)) of the \(i\)th unit cell \((\max|\Delta_0^a, \Delta_0^b| = \Delta_D/2\) (see Supplementary Section 1 for details) and h.c. denotes Hermitian conjugate. Figure 1c represents this system in the corresponding cRM model in the clean limit \(\Delta_D^a, \Delta_D^b = 0\) (or \(V_S = 0\)). By sweeping the long lattice phase \(\phi(t)\), dynamical parameters \(\delta(t)\) and \(\Delta(t)\) change adiabatically and draw a closed trajectory in a \(\delta - \Delta\) space (Fig. 1d). See Methods for details about parameter mapping between the cRM and tRM models.
Effect of quasi-periodic disorder on Thouless pump

We begin by considering the effects of quasi-periodic disorder added to our cRM version of the Thouless pump. Niu and Thouless showed that the pump is robust against disorder as long as the bandgap remains open, but it is still unclear how much quasi-periodic disorder is needed to close the gap. Although, infinitely small random disorder induces localization in 1D systems, a finite amount of quasi-periodic disorder is needed to induce the localization. Our system has three energy scales that might be relevant for gap closing: the Anderson localization transition point \( V_{\text{AL}} \), the minimum bandgap \( \Delta E \), and the on-site offset \( \Delta_0 \) of the staggered potential. In our cRM lattice, \( V_{\text{AL}} \) is comparable to the maximum on-site offset \( 2\Delta_0 \) in the tRM model. By performing the experiments we can answer which energy scale is truly relevant. Figure 2a shows the results of cRM pump under the quasi-periodic disorder potential with \( \Delta_0 = 798 \text{ nm} \) for two sets of superlattice parameters, \( (V_s, V_l) = (20, 14)E_F \) (set A, red circles) and \( (V_s, V_l) = (30, 20)E_F \) (set B, blue squares) as a function of the disorder strength \( V_{\text{pr}} \). The measured results are in reasonable agreement and are also consistent with the numerical results in ref. 26. The numerical calculations and measured results are in reasonable agreement and are also consistent with the numerical results in ref. 26. In general, the Chern number corresponds to the average over the quasi-periodic disorder phase \( \alpha \).

To explain the experimental results in Fig. 2a, we numerically calculate the Chern number of the tRM model with the on-site quasi-periodic disorder. In general, the Chern number corresponds to the total pumped charge per pumping cycle if the bandgap is kept open during the pumping cycle. Here, owing to the presence of quasi-periodic disorder, the tRM model does not have translational invariance and the conventional definition of Chern number cannot be applied. Instead, we calculate the Chern number by introducing twisted phase boundary conditions and by using the coupling matrix method (Supplementary Section 3). Figure 2b shows the results of numerical calculations for a superlattice with 20 unit cells. The obtained Chern number takes non-integer values around the transition point (moderate disorder strength \( V_{\text{pr}}/V_l \) ≈ 1). Although non-adiabatic processes shift the suppression point toward weaker disorder, they result in only a small underestimation of the suppression point and do not play the dominant role. Our experiment shows that the topological transition occurs well in the regime where all states are Anderson-localized and cannot be regarded as a delocalization–localization transition of instantaneous Hamiltonian eigenstates. Instead, the observed breakdown of quantized pumping is regarded as a manifestation of a delocalization–localization transition of Floquet eigenstates.

Fig. 2 | Breakdown of Thouless pump under quasi-periodic disorder.

a, CoM shift per cycle averaged after three cycles and plotted as a function of the disorder lattice depth \( V_{\text{pr}} \) for cRM lattice with \( (V_s, V_l) = (20, 14)E_F \) (set A, red circles) and \( (V_s, V_l) = (30, 20)E_F \) (set B, blue squares). Vertical red (blue) arrows indicate three energies: \( V_{\text{AL}}^s = 0.3E_F \) (\( V_{\text{AL}}^l = 0.07E_F \)) for Anderson localization transition point \( V_{\text{AL}} \), \( \Delta E^s = 3.2E_F \) (\( \Delta E^l = 3.38E_F \)) for minimum bandgap of the bare cRM model \( \Delta E \), and \( V_{\text{AL}}^s = 20E_F \) (\( V_{\text{AL}}^l = 30E_F \)) for the depth of long lattice corresponding to the maximum on-site offset \( 2\Delta_0 \). The inset shows the same data as a function of a normalized disorder strength \( V_{\text{pr}}/V_l \) for each parameter set. Error bars denote the 1σ confidence bound derived from more than 40 CoM measurements. Red and blue curves are guides to the eye. b, Numerical calculation of the Chern number in the presence of quasi-periodic disorder based on our tight-binding model. The result was obtained by averaging over the quasi-periodic disorder phase \( \alpha \).
gap closes. Here, we investigate the gap closing between the first and second bands by the presence of quasi-periodic disorder with a band-mapping technique (Supplementary Section 5). Figure 3a shows the fraction of atoms excited to the second band with respect to the total number of atoms, after three cycles of cRM pumping with the parameter set A, namely \((V_d, V_c) = (20, 14)E_R\). We evaluate the population of the second Brillouin zone of the basic cRM lattice. To this end, we remove the disorder lattice by adiabatically ramping down its strength \(V_d\) before the band-mapping measurements, so that the fraction of atoms in the second band under quasi-periodic disorder is adiabatically transferred to the corresponding one in the clean limit. The experimental result shows that the excitation to the second band initially increases as disorder increases, reaches the maximum at \(V_d \approx V_c\) and then decreases as the disorder further increases. Such an increase in the excitation to the second band around \(V_d \approx V_c\) suggests that the gap that initially opened in a clean limit closes around \(V_d \approx V_c\) and then reopens. The bandgap changes dynamically during the pump cycle, and Landau–Zener-like non-adiabatic transitions could occur at the gap closing point. Note that, for a strong disorder, the gap may close during the removal of the disorder lattice performed at \(\phi = 0\) (Fig. 3b). However, if the gap remains closed even in the region of \(V_d/V_c \gg 1\), a decrease in the second-band fraction in the strong disorder region is not expected; the fraction decrease is instead observed in the experiment.

We check this gap closing and opening around \(V_d \approx V_c\) numerically. Figure 3b shows a numerical calculation of the energy gap \(E_{\text{gap}}\), for the set A as functions of \(V_d\) and \(\phi\) based on a continuous model [14] in the presence of a quasi-periodic superlattice with half-filling and a total length \(198d\) (see Supplementary Section 4 for details). The energy gap averaged over \(\phi\) becomes zero around \(V_d \approx 20E_R\) (Fig. 3c). It qualitatively supports our experimental observations that the pumping is suppressed at disorder strength \(V_d/V_c \approx 1\). The disappearance of the bulk energy gap also can be seen in the density of states (DoS) calculation and the band calculation (Fig. 3d,e). Here, with increasing \(V_d\), some in-gap states are formed and the bulk bandgap observed in the clean limit gradually disappears. Conversely, numerical calculations do not show such a reopening of the gap. However, the energy difference between ground vibrational level (local first band) and first excited vibrational level (local second band) within one lattice site would become large in the strong (deep) disorder lattice regime. This large local (on-site) energy gap in large \(V_d\) suppresses again excitations to the second band during pumping sequence as is observed in our experiment. Our consideration of the sliding and disorder lattices (Supplementary Section 7) indicates that the gap should be open locally on each site for \(V_d \gg V_c\). We also numerically confirmed that the gap closing and reopening can be seen more clearly in the case of shallower lattice parameter sets (see Supplementary Section 4 for details). Consequently, the observed pumping suppression due to quasi-periodic disorder should indicate a topological phase transition in which the non-trivial Chern number changes into a trivial one via gap closing as the disorder strength increases.

We also examined the dependence of the pumping suppression on the wavelength of disorder lattice \(\lambda_s\) (Supplementary Section 6). We find that the pump is suppressed at \(V_d \approx V_c\) regardless of the disorder lattice wavelength. This can be understood intuitively by considering a sliding lattice, which is topologically equivalent to our cRM pump[1], superimposed on a disorder lattice (Supplementary Section 7).

**Fig. 3** | Gap closing and opening induced by quasi-periodic disorder. a. Second-band fraction measured after three pumping cycles for cRM lattice with the parameter set A, namely \((V_d, V_c) = (20, 14)E_R\), plotted as a function of disorder lattice depth and represented by blue squares. Error bars denote the 1σ confidence bound derived from more than 10 band-mapping measurements. As a reference, we show the corresponding data of charge pumping with red circles, which are the same as the red circles in Fig. 2a. The population reaches a maximum around the region of the breakdown of the pumping. Negative values at weak disorder are due to fluctuations in the number of atoms. b. 2D plot of energy gap as functions of superlattice phase and disorder strength \(V_c\) calculated based on the continuous model under half-filling (one atom per unit cell) with the parameter set A. c. Energy gap averaged over \(\phi\) as a function of the disorder strength \(V_c\). d. DoS as a function of the disorder lattice strength calculated using a continuous model (with arbitrary units) averaged over the phase \(\phi\). e. Band structures with different disorder lattice strengths (from top to bottom for \(V_d = 0, 10E_R\) and \(40E_R\)) calculated using a continuous model.
Disorder-induced pumping

The introduction of strong quasi-periodic disorder suppresses the topological charge pumping as described previously. However, the interplay between topology and quasi-periodic disorder could lead to more counter-intuitive phenomena such as TAT\(^{-}\). Here, we demonstrate the phenomenon of disorder-induced pumping, realized with a newly configured pumping sequence. We designed a pumping sequence in which the charge pumping is implemented in three stages: using lattice parameters \((V_0, V_\gamma) = (36, 24)E_\text{F}\) in the first stage, then using different parameters \((V_0, V_\gamma) = (15, 10)E_\text{F}\) with pumping in the reverse direction in the second stage, and finally using the lattice parameters of the first stage to obtain a closed pumping cycle. This sequence is depicted in Fig. 4a, in the tRM model. In the clean regime, the gap closes only at \((\delta, \Delta) = (0, 0)\) and opens in the other region, so that the outer loop yields the Chern number \(C_{\text{outer}} = 1\) and the inner loop \(C_{\text{inner}} = -1\). Thus, the overall pumping amount in this sequence can be given by \(C_{\text{outer}} + C_{\text{inner}} = 0\). Theoretically, if the two loops are far apart, a quantized pump is expected under moderate disorder (Supplementary Section 3). We added the disorder lattice to this basic sequence and measured the pumping amount per cycle (Fig. 4a). The pumping amount was obtained by evaluating the CoM shift after three cycles for each disorder strength. At zero disorder strength, the pumping amount is zero, as discussed previously. However, as the disorder strength increases, the pumping amount becomes finite. Subsequently, the pumping amount reaches its maximum and then is reduced to zero. This clearly demonstrates the observation of disorder-induced pumping in the Thouless pump.

Our findings can be explained intuitively in the following manner. As discussed in Fig. 2a,b, the pumping is suppressed at disorder strengths of the order of \(V_\gamma\). Namely, in Fig. 4a, the pumping is expected to be suppressed around \(V_\gamma = 36E_\text{F}\) in the outer and inner trajectories, respectively. This indicates that in the intermediate region \(15E_\text{F} \lesssim V_\gamma \lesssim 36E_\text{F}\), the pumping during the inner and outer trajectories do not cancel each other, resulting in a non-zero total pumping.

We quantitatively reproduced this behaviour by numerical calculations (Fig. 4b). For each circular trajectory, we calculated the Chern number using the quasi-periodic disordered Harper–Hofstadter–Hatsugai model (see Supplementary Section 3 for details). For all datasets, we averaged over 60 samples corresponding to different values of \(\alpha\). Numerical results for each trajectory are shown in Fig. 4b. The data display \(\Delta_0\)-dependence (not \(2\Delta_0\), scale) of the Chern number. Furthermore, Fig. 4c shows the sum of the Chern numbers for the \(C_{\text{outer}}\) and \(C_{\text{inner}}\) trajectories. The result clearly captures the presence of a non-trivial pump between \(10E_\text{F} \lesssim V_\gamma \lesssim 30E_\text{F}\), quantified by the imperfect cancellation between the Chern numbers for the \(C_{\text{outer}}\) and \(C_{\text{inner}}\) trajectories. Note that the experimental trajectory surely yields the disorder-induced pumping; however, this does not necessarily mean that this pump is topological. It could be topological if we adjust appropriately the lattice parameters including the phase connecting the outer and inner trajectories.

Discussion

Our unique experimental platform provides us with interesting opportunities for studying a wide range of topological quantum phenomena with disorder. By introducing a disorder lattice with \(\lambda_D \approx 1,217\) nm, it is possible to add a quasi-periodic disorder in which \(d_\gamma/d\) approximates the golden ratio, which is often studied in the Aubry–André model, or to add a genuinely random (non-quasi-periodic) disorder by using a speckle pattern. Moreover, in this study we connect two non-trivial pump trajectories to create a trivial trajectory in the clean limit and then add disorder to observe disorder-induced pumping. The method presented here allows the realization of disorder-induced pumping and will give a suggestion or guideline for the future study of a wide range of disorder-induced topological phases. Our highly controllable optical lattice system loaded with ultracold fermions can straightforwardly incorporate interatomic on-site interaction with reasonably large strengths compared with tunnelling energy. Consequently, we can study the effect of disorder on a Thouless pump in strongly correlated regimes and especially in many-body localization regimes, which may protect topological phenomena up to higher excited states. The pumping can be extended to higher dimensions, which is possible only for a Floquet system and revealed by non-adiabatic pumping. Our experimental set-up can also enable us to study the effect of non-adiabaticity.
Moreover, the effect of disorder on higher-order topological phenomena \cite{45-47} is an interesting new research direction \cite{48}.

Online content
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Methods
Preparation of a degenerate Fermi gas of $^{171}\text{Yb}$. Because $^{212}\text{Yb}$ atoms have a very small s-wave scattering length of $-0.15\text{fm}$ (ref. 1), they can be regarded as non-interacting. Therefore, we use sympathetic evaporative cooling with $^{173}\text{Yb}$ to obtain the degenerate Fermi gas of $^{171}\text{Yb}$. The number of atoms for each spin is typically 3,500. A typical atom temperature before lattice loading is $T/T_F \approx 0.24$ for the measurements in Figs. 2 and 3 and Supplementary Fig. 8, and $\approx 0.19$ for Fig. 4 at the end of the evaporation with the trap frequencies of the far-off resonant optical trap of $(\omega_x', \omega_y', \omega_z, 2\pi) = (51.2, 21.5, 153)\text{Hz}$, where $T_F$ is the Fermi temperature and the $x'$ and $y'$ axes are tilted from the lattice axes ($x$ and $y$) by 45°. The estimated sizes of our atomic clouds (full width at half maximum) are about 4.4 μm, 21 μm, and 3.4 μm for the $x'$, $y'$ and $z$ directions, respectively. The system size along the pumping direction ($x$) is about 6.3 μm, which corresponds to about 12 unit cells around the centre of the atomic clouds. The entire atomic system consists of about 60 × 6 1D lattices along the $y$ and $z$ axes, or a total of about 360 lattices.

Set-up for the crM superlattice and quasi-periodic disorder lattice. An incommensurate optical lattice potential created by a retro-reflected laser beam (wavelength at $\lambda_0$) and tilted by 45° from the pumping direction is superimposed on a 2D array of a 1D crM superlattice. This superimposed optical lattice (disorder lattice) creates a periodic potential with lattice constant $d_0 = \lambda_0 / \sqrt{2}$ in the pumping direction of the crM; this incommensurate periodic potential acts as a quasi-periodic disorder. In our set-up, the wavelength $\lambda_0$ can range from 776 nm to 820 nm, as realised by a Ti:Sapphire laser, and the relative incommensurate lattice constant $d_0/d$ ranges from 1.03 to 1.09, respectively. Because the ratio of this wavelength is relatively close to 1, this quasi-periodic disorder does not take the same structure in the trap region. Also, because this lattice is tilted by 45°, the phase of the quasi-periodic disorder shifts by 3.05, 2.96 and 2.88 rad for $\lambda_0 = 776$, 798 and 820 nm, respectively, in the adjacent array (Fig. 1a).

Parameter mapping between the crM model and tRM model. The parameters $(J, \xi(t), \Delta(t))$ in the tRM model are determined by matching the band structures of the tRM and crM models that correspond to the points $(\phi = 0, \pi/2, \pi, 3\pi/2)$ of the actual pumping sequences. For example, when $(V_x, V_y) = (20, 14) E_r$, and for $\phi = 0$, we get $(J, \xi, \Delta) = (0.861, 0, 0.65) E_r$, whereas for $\phi = \pi/2$, we get $(J, \xi, \Delta) = (0.861, 0.852, 0) E_r$. We then sweep $(\xi(t), \Delta(t))$ along the circular path in Fig. 1d. Such mapping does preserve the topological properties of the system. Note that the actual experiment is only approximately described by the tRM model, and therefore we use the circular trajectory only as a pictorial description of pumping sequences.

Pumping speed and adiabaticicity. An essential requirement of the topological Thouless pump is adiabaticity. This is necessary to avoid adiabatic Landau–Zener transitions to the higher band during the pumping process, at least in the clean limit. The pumping times per cycle are 130 ms on average for the normal charge pumping (Figs. 2 and 3 and Supplementary Fig. 8) and 460 ms for the disorder-induced pumping (Fig. 4), in particular 100 ms for outer and inner trajectories and 130 ms for the connecting region (two times), respectively. These pumping times are sufficiently longer than the Landau–Zener transition timescales, which are estimated to be ~1 ms based on the maximum and minimum bandgaps during the pumping cycles.

Data availability
The data that support the plots within this paper and other findings of this study are available from the corresponding author upon reasonable request.

Code availability
The codes used for the numerical simulations within this paper are available from the corresponding author upon reasonable request.

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Author contributions
S.N., N.T. and K.S. carried out experiments and the data analysis. Y.K. and P.M. carried out the theoretical calculation. Y.T. conducted the whole experiment. All the authors contributed to the writing of the manuscript.

Competing interests
The authors declare no competing interests.

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