ReNets: Toward Statically Optimal
Self-Adjusting Networks

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Abstract

This paper studies the design of self-adjusting networks whose topology dynamically adapts to the workload, in an online and demand-aware manner. This problem is motivated by emerging optical technologies which allow to reconfigure the datacenter topology at runtime. Our main contribution is ReNet, a self-adjusting network which maintains a balance between the benefits and costs of reconfigurations. In particular, we show that ReNets are statically optimal for arbitrary sparse communication demands, i.e., perform at least as good as any fixed demand-aware network designed with a perfect knowledge of the future demand. Furthermore, ReNets provide compact and local routing, by leveraging ideas from self-adjusting datastructures.

1 Introduction

Modern datacenter networks rely on efficient network topologies (based on fat-trees \cite{1}, hypercubes \cite{2,3}, or expander \cite{4} graphs) to provide a high connectivity at low cost \cite{5}. These datacenter networks have in common that their topology is fixed and oblivious to the actual demand (i.e., workload or communication pattern) they currently serve. Rather, they are designed for all-to-all communication patterns, by ensuring properties such as full bisection bandwidth or $O(\log n)$ route lengths between any node pair in a constant-degree $n$-node network. However, demand-oblivious networks can be inefficient for more specific demand patterns, as they usually arise in practice: Empirical studies show that traffic patterns in datacenters are often sparse and skewed \cite{8}, featuring much (spatial and temporal) locality.

This paper investigates algorithms for demand-aware networks (DANs): networks which provide shorter average route lengths by accounting for locality in the demand and locating frequently communicating node pairs.
DANs come in two flavors: fixed and self-adjusting. Fixed DANs can exploit spatial locality in the demand. It has recently been shown that a fixed DAN can provide average route lengths in the order of the (conditional) entropy of the demand [7], which can be much lower than the $O(\log n)$ route lengths provided by demand-oblivious networks for specific demands. However, fixed DANs require a priori knowledge of the demand.

Self-adjusting DANs do not require such knowledge and can additionally exploit temporal locality, by adapting to the demand in an online manner. The vision of such self-adjusting networks is enabled by emerging optical technologies which allow us to reconfigure the topology over time [8, 9, 10, 11, 6].

However, the design of self-adjusting DANs is challenging: while more frequent reconfigurations allow to adapt the topology to the demand in a more fine-grained manner, such reconfigurations also come at a cost. Hence, an optimal tradeoff between the benefits and the costs of such reconfigurations has to be found. Further challenges are introduced by the online nature of the problem and the lack of a priori knowledge about the demand.

Ideally, a self-adjusting network provides an optimal performance even in...
hindsight: despite the lack of information on the demand, the performance is at least as good as the (fixed) demand-aware network, for sufficiently long demands. This property is called static optimality. Static optimality (and the related notion of regret minimization) is a strong notion of optimality and frequently used to evaluate algorithms based on limited information, for example in the context of coding (e.g., dynamic Huffman codes [12]), self-adjusting datastructures (e.g., splay trees [13]), or repeated games and machine learning [14, 15, 16, 17].

1.1 Analogy to Coding and Datastructures

Our vision of self-adjusting networks, and in particular, the difference between demand-oblivious and demand-aware networks (fixed and self-adjusting), can be explained by an analogy. This analogy will also provide an intuition why the entropy of the demand is an important metric for self-adjusting networks.

In fact, we will provide two analogies: one to coding, and one to datastructures, and in particular binary search trees (BSTs). Figure 1 illustrates the analogy and evolution of self-adjusting systems, from fixed demand-oblivious, over fixed demand-aware, to self-adjusting demand-aware.

Simple, oblivious coding schemes which do not rely on any knowledge on the input are based on “fixed length” coding: each symbol is encoded using the same code length (i.e., a logarithmic number of bits). Without further information on the input, this is also optimal. However, the performance of such an oblivious coding scheme may be far from optimal for a specific input. In contrast to input-oblivious fixed-length coding, coding schemes such as Huffman coding [18] account for the frequency of the communicated symbols: frequent symbols are assigned short codes, infrequent symbols long codes. Indeed, Huffman coding can result in much shorter codes for specific inputs: code lengths are proportional to the entropy in the input. A main drawback of such fixed approaches is that they require a priori knowledge about the (future) input, which may not be available. Furthermore, they only allow to leverage spatial locality, but not temporal locality, i.e., change over time in the input patterns/distribution. This can render such approaches inefficient under dynamic inputs, or even impractical. These limitations can be overcome by adaptive coding approaches, such as adaptive Huffman codes or arithmetic coding [19, 22]: these codes adjust to the input, in an online manner, leveraging spatial and temporal locality over time.

Similar tradeoffs arise in the design of network topologies (cf. Figure 1). Traditional communication network designs are demand-oblivious and fixed, which can be suboptimal under specific demand patterns. In contrast, fixed
DANs are optimized toward a specific demand, which is assumed to be known. Accordingly, the network can be designed in such a way that traffic between frequently communicating node pairs is routed along shorter paths, while traffic between node pairs communicating infrequently is routed along longer paths. In other words, the network topology accounts for the entropy in the communication demand. However, the required knowledge about the (future) demand may not be available, which makes adaptive (i.e., self-adjusting) DANs, like the one we present in this work, an attractive alternative: these networks can learn and adjust to the demand in an online manner.

Similar examples exist in the context of datastructures, and in particular, Binary Search Trees (BSTs). As we will see, BSTs play an important role in this paper in general. Traditional BSTs are (demand-)oblivious: items are stored at distance $O(\log n)$ from the root (on average), uniformly and independently of their frequency. Demand-aware, fixed BSTs (a.k.a. biased search trees) such as [20, 21, 22, 23] account for the frequency of the accessed items: frequent items are stored close to the root, infrequent items are lower in the tree. Finally, self-adjusting BSTs, or self-adjusting demand-aware BSTs, such as splay trees [13] allow to adapt to the workload over time and account for spatial as well as temporal locality.

1.2 Our Contributions

The main contribution of this paper is a self-adjusting demand-aware network called ReNet which: (1) is provably statically optimal under sparse communication patterns, and therefore, as we will show, provides entropy-proportional route lengths, without requiring any knowledge of future demands; (2) is scalable in that it is of constant degree and features compact routing (i.e., constant-size forwarding tables); (3) supports local routing, allowing us to reconfigure networks seamlessly while (4) relying on arbitrary addresses.

The ReNet network relies on ideas from self-adjusting datastructures. In particular a ReNet is based on a set of trees, called ego-trees, which are (dynamically) optimized for individual nodes. A ReNet is then a union of all the ego-trees of individual nodes, using algorithmic manoeuvres to make sure that the degree (and routing tables) remain constant at any time. The ego-tree of a given node stores the working set of that node: the set of its recent communication partners.

More specifically, the working set of each of these nodes is organized as a self-adjusting binary search tree (BST). While different types of such ego-trees can be used (e.g., Huffman trees, tango trees [24], etc.), ReNet uses splay trees [13]. As we will see, this will result in desirable properties, such
as compact and local routing.

1.3 Organization

The remainder of this paper is organized as follows. We introduce our model and identify desirable properties for self-adjusting demand-aware networks in Section 2. Subsequently, we present our algorithm (Section 3) and its analysis (Section 4). After reviewing related work in Section 5, we conclude and discuss future work in Section 6. Some proofs and additional examples are deferred to the Appendix.

2 Preliminaries and Model

We consider a set $V$ of $n$ nodes $V = \{1, \ldots, n\}$ with unique but otherwise arbitrary addresses. The communication demand among these nodes is described as a (finite or infinite) sequence $\sigma = (\sigma_0, \sigma_1, \ldots)$ of communication requests where $\sigma_t \subseteq V \times V$ is a set of source-destination communication pairs $(u, v) \in V \times V$ which communicate simultaneously at time $t$. The communication demand is revealed in an online manner and can be adversarial.

In order to serve this demand, the nodes $V$ must be inter-connected by a DAN $N$, defined over the same set of nodes. In case of a self-adjusting DAN, $N$ can also change over time, and we denote by $N_t$ the network at time $t$. For scalability reasons and since reconfigurable links may be costly and consume space, the DAN must be chosen from the family of degree-bounded topologies: the networks $N$ considered in this paper are required to be of constant degree at most $\Delta$.

The route length to serve a request $\sigma_t = (u, v)$ on the DAN, is given by the hop distance $d_{N_t}(u, v)$ from $u$ to $v$, along the routing path chosen by the algorithm over $N_t$. If not specified otherwise, we assume shortest path routing.

We are interested in the fundamental tradeoff between the benefits of self-adjusting algorithms (i.e., shorter routes) and their costs (namely reconfiguration costs). Let $A$ be an algorithm that given the request $\sigma_t$ and the network $N_t$ at time $t$, transforms the current network to $N_{t+1}$ at time $t+1$. The cost of the network reconfiguration at time $t$ is given by the number of link changes performed to change $N_t$ to $N_{t+1}$; when $A$ is clear from the context, we will simply write this cost as $\text{adj}(N_t, N_{t+1})$. Recall that an algorithm incurs a communication cost to serve request $\sigma_t = (u, v)$, which depends on the hop distance $d_{N_t}(\sigma_t) = d_{N_t}(u, v)$ from $u$ to $v$ in $N_t$. 

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Definition 1 (Average and Amortized Cost). Given an algorithm $A$, an initial network $N_0$, a distance function $d_{N}(\cdot)$, and a sequence $\sigma = (\sigma_0, \sigma_1 \ldots \sigma_{m-1})$ of communication requests over time, we define the (average) cost incurred by $A$ as:

$$\text{Cost}(A, N_0, \sigma) = \frac{1}{m} \sum_{t=0}^{m-1} (d_{N_t}(\sigma_t) + \text{adj}(N_i, N_{i+1}))$$ (1)

The amortized cost of $A$ is defined as the worst possible cost of $A$ over all initial networks $N_0$ and all sequences $\sigma$, i.e., $\max_{N_0, \sigma} \text{Cost}(A, N_0, \sigma)$.

In addition to the DAN, nodes can also communicate over a demand-oblivious network: reconfigurable datacenter networks are usually hybrid [6], connecting fixed (electric) switches with reconfigurable (optical) switches. The demand-oblivious network plays a minor role in this paper, and is only used to exchange control information (e.g., discover new neighbors). Route lengths on the demand-oblivious networks cost $D$ per request, where $D$ is a parameter: e.g., $D$ is the diameter of the demand-oblivious network, $D = \Theta(\log n)$ in constant-degree networks.

A main challenge faced by self-adjusting DANs is that information about (future) demand may not be available. Our goal is to design algorithms for statically optimal networks:

**Property 1 (Static Optimality).** Let $\text{STAT}$ be an optimal static algorithm with perfect knowledge of the demand $\sigma$, and let $\text{ON}$ be an online algorithm producing a sequence of degree-bounded networks (i.e., the maximum degree is at most $\Delta$). We say that $\text{ON}$ is statically optimal if, for sufficiently long communication patterns $\sigma$:

$$\rho = \max_{\sigma} \frac{\text{Cost}(\text{ON}, \emptyset, \sigma)}{\text{Cost}(\text{STAT}, N^*, \sigma)}$$

is constant. Here, $N_0 = \emptyset$ is the empty network from which $\text{ON}$ starts, and $N^*$ is the statically optimal degree-bounded network for $\sigma$. In other words, $\text{ON}$’s cost is at most a constant factor higher than $\text{STAT}$’s in the worst case.

We conclude this section with some definitions. We first note that we can think of the entire sequence $\sigma$, or a subsequence $\sigma' \subseteq \sigma$, as a directed and weighted demand graph $G(\sigma') = (V(\sigma'), E(\sigma'))$. Here, the node set $V$ of $G$ is given by the set of nodes participating in $\sigma'$, i.e., $V(\sigma') = \{v : v \in \sigma'\}$, and the set of directed edges $E$ is given by $E(\sigma') = \{\sigma'_i : \sigma'_i \in \sigma'\}$. The
weight \( w(e) \) of each directed edge \( e = (u, v) \in E \) is the frequency \( f(u, v) \) of the request from \( u \) to \( v \) in \( \sigma' \), where \( \sum_{u,v \in V(\sigma')} f(u,v) = 1 \). We are interested in sparse communication patterns:

**Definition 2 (\((c,\delta)\)-sparse Communication).** We call a communication demand \( \sigma \) \((c,\delta)\)-sparse if and only if any subsequence \( \sigma' \) of \( \sigma \) of length \(|\sigma'| \leq \delta \), involves no more than \( c \cdot n \) unique communication pairs where \( c \) is a constant and \( \delta \) is a function of \( n \). That is, \( \sigma' \) implies a sparse demand graph \( G(\sigma') = (V,E(\sigma')) \) of average degree \( 2|E(\sigma')|/n \leq 2c \).

Note that for \( \delta = \infty \), the entire communication pattern \( \sigma \) needs to be sparse. For \( \delta \leq cn \), the constraint is trivial.

We define the *entropy of the demand* \( \sigma = (\sigma_1, \sigma_2, \ldots, \sigma_m) \) to be served by a communication network. Recall that \( \sigma_i = (\hat{X},\hat{Y}) \) describes a source-destination pair (e.g., a client \( x \) communicating to a server \( y \)). We will interpret \( \sigma \) as a joint empirical frequency distribution \((\hat{X},\hat{Y})\), where \( \hat{X} \) is the empirical frequency of the sources and \( \hat{Y} \) is the empirical frequency of the destinations. In particular, in the following, the term *entropy* will refer to the *empirical entropy* of \( \sigma \), i.e., the entropy implied by the communication frequencies.

More formally, let \( \hat{X}_\sigma = \{f(x_1,\cdot),\ldots,f(x_m,\cdot)\} \) be the empirical frequency distribution of the communication sources (origins) occurring in the communication sequence \( \sigma \), i.e., \( f(x_i,\cdot) \) is the frequency with which a node \( x_i \) appears as a source in the sequence: \( f(x_i,\cdot) = (\#x_i \text{ is a source in } \sigma)/m \), when \( m = |\sigma| \) is the length of \( \sigma \). We omit \( \sigma \) in \( \hat{X}_\sigma \) when it is clear from the context. The empirical entropy \( H(\hat{X}) \) is then defined as \( H(\hat{X}) = -\sum_{i=1}^n f(x_i) \log_2 f(x_i) \), where \( f(x_i) \) is used as a shorthand for \( f(x_i,\cdot) \). Similarly, we define the empirical entropy of the communication destinations \( H(\hat{Y}) \): we consider \( \hat{Y}_\sigma = \{f(\cdot,y_1),\ldots,f(\cdot,y_m)\} \) where \( f(\cdot,y_j) \) is the frequency with which a node \( y_j \) appears as a destination in the sequence. We use the normalization \( f(x|y) = f(x,y)/f(\cdot,y) \). The empirical joint entropy \( H(\hat{X},\hat{Y}) \) is defined as \( H(\hat{X},\hat{Y}) = -\sum_{i,j} f(x_i,y_j) \log_2 f(x_i,y_j) \) and the empirical conditional entropy \( H(\hat{X}|\hat{Y}) \) which measures spatial locality as \( H(\hat{X}|\hat{Y}) = -\sum_j f(y_j) \sum_i f(x_i \mid y_j) \log_2 f(x_i \mid y_j) \). We may simply write \( H \) for the entropy if the usage is given by the context. By default, we will denote by \( H \) the entropy computed using the binary logarithm; if a different logarithmic basis \( \Delta \) is used to compute the entropy, we will explicitly write \( H_\Delta \).

It was recently shown that the conditional entropy of the demand, and in particular \( \max(H_\Delta(\hat{Y}|\hat{X}),H_\Delta(\hat{X}|\hat{Y})) \) is a lower bound for the average route
length in any (constant) degree-Δ bounded, fixed network [7]. This bound can be (asymptotically) matched if the demand is sparse and the demand $\sigma$ is known a priori, before designing the network. In contrast, in this work, we are interested in solutions that match the conditional entropy lower bound, but for a demand $\sigma$ that is unknown a priori. This makes the task much more challenging.

3 Statically Optimal Self-Adjusting Networks

This section presents statically optimal algorithms for ReNets, self-adjusting networks of bounded degree which support the following additional desirable properties. We first discuss desirable properties of such self-adjusting networks, then present algorithmic building blocks, and finally describe ReNets’ forwarding tables in details.

3.1 Desirable Properties of Self-Adjusting Networks

In order to ensure scalability, each node in a self-adjusting network should not only rely on at most a constant number of reconfigurable links, we would like to have an even stronger property: namely that the forwarding tables are of constant size, i.e., compact [25, 26].

Property 2 (Compact Routing). A network supports compact routing if the sizes of the nodes’ forwarding tables are constant, i.e., independent of the network size.

A key challenge in the design of self-adjusting networks is that topological reconfigurations may negatively affect routing. A particularly attractive (but seemingly difficult to achieve) property for routing in dynamic networks is local routing:

Property 3 (Local Routing). A network provides local routing if packets can be forwarded based on local knowledge only. In particular, topological changes should not entail the global re-computation of routes.

Furthermore, we do not want the compact and local routing properties to depend on any specific addressing scheme. In particular, addresses can be flat and location-independent.

Property 4 (Arbitrary Addressing). Nodes can have arbitrary (but unique) addresses.

\footnote{We note that the result in [7] is stated for the entropy and not the empirical entropy, however, the claim follows directly.}
3.2 Outline and Algorithmic Building Blocks

We first describe the main ideas and building blocks of ReNets. Ideally, each node \( u \in V \) in ReNet connects \textit{directly} to all its communication partners in \( G(\sigma) \), achieving an ideal average route length of 1. However, this is infeasible, as (1) the communication partners are not known to \( u \) \textit{a priori} and (2) a node \( u \) may have \textit{many} communication partners (even in an otherwise sparse demand graph), which would result in a high degree and large forwarding tables. To overcome this, ReNet leverages several key concepts:

**Concept 1 - Working Set.** Each node \( u \) in ReNet keeps track of its recent active communication partners, i.e., the so-called \textit{working set} \( W(u) \), hoping to exploit temporal locality as they are also likely to be relevant in the near future. The working set will be defined over the recent subinterval of \( \sigma \) that will be defined later.

**Concept 2 - Small and Large Nodes.** A node \( u \) in ReNet pursues one of two different approaches to communicate with its recent communication partners, depending on its working size. Towards this end, we define the \textit{size} of a node \( u \) to be the cardinality \( |W(u)| \) of \( u \)'s working set. We say that a node \( u \) is \textit{small} if the size of \( u \) is smaller than a parameter \( \theta \) and otherwise, a node is called \textit{large}. For now assume \( \theta \) is a constant which depends on the sparsity of the communication sequence (i.e., Definition 2), we will discuss the details later. A small node will communicate to its communication partners \textit{directly}; a large node \textit{indirectly}, by forwarding the traffic along its \textit{ego-tree}, which we explain next.

**Concept 3 - Ego-Tree/Ego-BST.** For large nodes \( u \), establishing links or storing forwarding rules for each communication partner in \( W(u) \) is infeasible as it would result in forwarding tables of non-constant size. Thus, in ReNet, a large node \( u \) organizes its communication partners \( W(u) \) in a self-adjusting \textit{ego-tree}: a \textit{(tree) network} optimized for just this source. In particular, we propose to use a self-adjusting \textit{binary search tree} for the ego-tree of a large node \( u \), short \textit{ego-BST}(\( u \)). An important property of BSTs (fixed and self-adjusting) is that they naturally supports local and compact routing for messages to or from the \textit{(root)} of the tree. In particular, the (self-adjusting) \textit{ego-BST}(\( u \)), is used to efficiently \textit{store} and \textit{lookup} (i.e., forward to) neighbors \( v \in W(u) \). Each node that belongs to such a \textit{ego-BST}(\( u \)) support the following interface:

- \texttt{ego-BST(}\( u \)).\texttt{insert(}\( v \))\texttt{: insert} \( v \) \texttt{to} \texttt{ego-BST(}\( u \))
- \texttt{ego-BST(}\( u \)).\texttt{forward(}\( v \))\texttt{: forward packet toward} \( v \)
Figure 2: Design principles of a (fixed and adaptive) ReNet: (a) The sparse demand graph $G(\sigma)$. Nodes are divided into large (gray, e.g., $u, v, w$) and small (white, e.g., $h$) nodes. (b) Hierarchical representation of the demand graph. Problematic edges are edges between two large nodes (e.g., $(u, v)$). (c) ReNet: every large node $x$ has an ego-BST($x$) connecting its working set $W(x)$. Every large-large edge, is routed with the help of a small node (acting as relay between ego-BSTs, black). E.g., $h$ is the helping node for edge $(u, v)$ and participates in ego-BST($u$) as a relay node toward $v$ and in ego-BST($v$) as a relay toward $u$. The resulting network has bounded degree. (d) Forwarding table for a small node, see text.

- **ego-BST($u$).adjust():** local update of tree network datastructure

**Concept 4 - Self-Adjustments.** ReNets perform two types of self-adjustments. First, ReNets are configured with self-adjusting versions of the underlying ego-BST, in particular, splay trees. In order to update the neighborhood structures and optimize the network after a routing request, a node $u$ makes use of the adjust() operation. For example in splay trees, we issue a splay operation on the tree network. Second, ReNets keep track of the total size of the nodes’ working sets. Once the total size, $\sum_v W(v)$, exceeds $n \cdot \theta/2$, all working sets are cleared (in the spirit of flush-when-full or marking techniques known from competitive paging [27]). Such reset operations are necessary to follow temporal locality, allowing the nodes to update the working sets and hence be able to adjust to changing demand patterns.

**Concept 5 - Helper Nodes.** The problem with the approach described so far is that while nodes in a single BST are of degree at most three (parent, left child, right child), a large node $v$ can still appear in multiple trees beside it’s own tree if he has large nodes in its working set. Combined, these trees can
induce a large forwarding table on \( v \), and hence, an additional mechanism is needed to bound the degree. To this end, \( ReNets \) leverage small nodes to help two communicating large nodes keep the forwarding table small. Concretely, as long as the average node size is smaller than \( \theta/2 \), a \( ReNet \) exploits small nodes (of size below \( \theta \)) as helper nodes. For example a small node \( h \) may serve as relay between two communicating large nodes \( v \) and \( u \). Node \( h \) will appear in both trees networks \( ego-BST(v) \) and \( ego-BST(u) \), in \( ego-BST(v) \), \( h \) will serve as a forwarder toward \( u \) and in \( ego-BST(u) \), as a forwarder toward \( v \). See Figure 2 for example. As we will discuss later, this not only allows us to bound the size of the forwarding table, but also to preserve local routing.

**Concept 6 - Centralized Bookkeeping and Coordination.** While reconfiguration is decentralized, bookkeeping and coordination is centralized in \( ReNet \). This avoids complexities due to possible inconsistencies and is efficient: a network coordinator (e.g., an arbitrary node in the network) only needs to keep track of which nodes are large and which nodes are small. That is, nodes inform the coordinator when they need to add a new partner to their working sets. Given this information, the coordinator can assign helper nodes upon request, in an event-driven manner. If no helper nodes are left, the coordinator schedules a reset() operation that clears all working sets for all nodes, and set their size to small. Such a reset can be done using a spanning tree, at linear cost.

Figure 2 illustrates some of the concepts introduced above, such as small and large nodes, working set, BST, and helping nodes.

### 3.3 Details of Forwarding Table and Reconfiguration Algorithm

With these intuitions in mind, we now present the network reconfiguration and forwarding algorithms underlying \( ReNets \) in details.

#### 3.3.1 Forwarding Table

Each node \( u \in V \) maintains the following forwarding table of given constant size \( 6\theta \) (details later) whose content may change over time. If \( u \) is small, \( u \)'s forwarding table contains (Figure 2(d)):

- A set \( S(u) = \{v_1, v_2, \ldots\} \) of small neighbors of \( u \). For each \( v_i \), \( u \) has a direct, physical link (port) toward \( v_i \).
- A set \( L(u) = \{ego-BST(w_1), ego-BST(w_2), \ldots\} \) of \( ego-BSTs \) of large
neighbors of \( u \). In each of these trees, say \( \text{ego-BST}(w) \), \( u \) will participate and forward messages toward/from the root of the tree, \( w \). Each such tree requires three entries in the forwarding table, and three \textit{physical ports} (i.e., direct links): (1) a forward entry to the parent of \( u \) in \( \text{ego-BST}(w) \), (2) a forward entry to the left child of \( u \) in \( \text{ego-BST}(w) \), and (3) similarly for the right child in \( \text{ego-BST}(w) \).

- A set \( H(u) = \{(x_1, y_1), (x_2, y_2), \ldots\} \) of pairs of \textit{large} nodes \( x_i, y_i \) for which \( u \) acts as a \textit{helper}. Helping such a \textit{large-large} connection, requires six entries in the forwarding table and six ports: three entries and ports for each tree, \( \text{ego-BST}(x_i) \) and \( \text{ego-BST}(y_i) \).

If \( u \) is \textit{large}, \( u \)'s forwarding table is simpler and contains:

- A (physical) link to the current root of \( \text{ego-BST}(u) \).
- A set of virtual roots to improve the performance of \( \text{ego-BST}(u) \).

### 3.3.2 Roles

The algorithms underlying \textit{ReNets} involve four different node roles:

- **The Source** (Algorithm 1): Let \( u \) be the source of a communication request \( (u, v) \). In case \( u \) is a large node, it will simply forward the request to the root of \( \text{ego-BST}(u) \) (or directly to \( v \) if it is one of the virtual roots of \( \text{ego-BST}(u) \)). In case \( u \) is a small node then: if \( v \in S(u) \) then it will forward it directly to \( v \); else, if \( u \) participates in \( \text{ego-BST}(v) \) then \( u \) will forward it to its parent in \( \text{ego-BST}(v) \). Else, If \( v \) is a new communication partner, \( u \) will notify the coordinator and request being connected to \( v \).

- **The Forwarder** (Algorithm 2): A forwarder \( x \) is a node which is neither the source nor the destination of a communication request \( (u, v) \), i.e., \( u \neq x \neq v \). It acts as an inner node in the ego-tree network and may also be a helper (see below). By our construction, node \( x \) must hence either be part of \( \text{ego-BST}(u) \) or \( \text{ego-BST}(v) \), or both (if it is a helper). If \( x \) is part of \( \text{ego-BST}(v) \) of the destination, it needs to forward the request toward the root \( v \); else if \( x \) is part of \( \text{BST}(u) \) of the source, it needs to forward

\footnote{2The use of virtual roots is a practical optimization. In a traditional self-adjusting BST, the root changes over time, depending on the demand: accessed elements are moved to the root. In \textit{ReNet}, a node \( u \) uses a set of virtual pointers to implement the root of \( \text{ego-BST}(u) \): the root is implemented using a \textit{constant} set of nodes (all at distance 1), managed in a first-come-first-serve queue, evicting the least-recently used (lru) root. However, this optimization does not affect the asymptotic performance of our network.}
the request to the correct child based on the ID of $v$. If $x$ is a helper, it belongs to both $ego-BST(u)$ and $ego-BST(v)$, and additionally needs to initiate $ego-BST(u).adjust()$ to update the tree.

- **The Destination (Algorithm 3)**: The behavior of the destination $v$ of a given communication request $(u, v)$ is simple: it delivers the request to the upper layer and if needed, triggers an adjust() operation on the $ego-BST(w)$, for which the packet was delivered: this optimizes the network to account for recent communications.

- **The Coordinator (Algorithm 4)**: The coordinator keeps track of which nodes are small, which nodes are large, and which small nodes have room in their forwarding table to help large-to-large edges. To serve an $addRoute(u \rightarrow v)$ request, the coordinator distinguishes between different cases, potentially resetting the forwarding tables (using $reset()$, see below), adding helper nodes where needed or rendering the source and/or destination node large (using $makeLarge()$, see below). In the simplest case, both $u$ and $v$ are small and the coordinator can instruct the two nodes to connect directly. If one node is large and one small, the route request is served by inserting one node in the other node’s $ego-BST$. Only if both nodes are large, the coordinator finds a helper node, which is used to relay between the two $ego-BSTs$, which must already exist.

When the coordinator learns that a node $u$ needs to become large, it invokes the $makeLarge(u)$ method, which instructs the creation of $ego-BST(u)$. On this occasion, the coordinator iterates over the working set of $u$: in case of a small neighbor $v$, $v$ is inserted into the $ego-BST(u)$ directly; otherwise, a helper node is used.

The coordinator also instructs the nodes to reset their working sets if no more helper nodes are available, i.e., if the total sizes of the working sets is $nθ/2$ and the network is, what we call, full. Concretely, using the $reset()$ method, the coordinator instructs all nodes to clear their forwarding tables (i.e., working sets).

4 Analysis of Static Optimality

This section formally proves the properties and performance of ReNets. To improve readability, some lemmas and proofs are deferred to the appendix. We call a communication sequences $σ$ sparse if it is $(c, δ)$-sparse for a constant $c$ and $δ = \Omega(cnD)$ (cf Definition 2), where $D$ is the routing cost on the
Algorithm 1 Source $u$, upon request $u \rightarrow v$

1: if $u$ is large then
2: forward to root of ego-BST($u$)
3: else
4: (* small node *)
5: if $v \in S(u)$ then
6: forward directly to $v$
7: else if ego-BST($v$) \in L($u$) then
8: ego-BST($v$).forward($v$) (to parent)
9: else
10: (* new partner *)
11: notify coordinator: addRoute($u \rightarrow v$)

Algorithm 2 Forwarder $x$, for request $u \rightarrow v$

Require: by definition $x \in$ ego-BST($u$) and/or $x \in$ ego-BST($v$)
1: if $x \in$ ego-BST($v$) then
2: ego-BST($v$).forward($v$) (to parent)
3: if $x \in$ ego-BST($u$) then
4: (* $x$ is an helper to $(u,v)$ *)
5: ego-BST($u$).adjust()
6: else
7: (* $x \in$ ego-BST($u$) *)
8: if $\exists$ child $x$ toward $v$ then
9: ego-BST($u$).forward($v$) (to child)
10: else
11: notify coordinator: addRoute($u \rightarrow v$)

Algorithm 3 Destination $v$, upon request $u \rightarrow v$

1: process packet
2: if request received on some ego-BST($w$) then
3: ego-BST($w$).adjust()

demand-oblivious network (henceforth usually assumed to be $\Theta(\log n)$, the minimum possible diameter for a scalable, constant-degree network).

We now show that forwarding in ReNets does not require a global routing algorithm and can use arbitrary addressing, and its size is bounded by a constant $\Delta = 6\theta = 24c$.

Theorem 1. For any sequence $\sigma$, ReNets provide $\Delta$-compact and local
Algorithm 4 Coordinator

**addRoute**(*u → v*):
1: if the total size of all working sets is $\theta n/2$ then
2: (* network is full *)
3: reset()
4: (* available helper $x$ must exist *)
5: add $v$ to the working set of $u$, $W(u)$
6: if $u$ is small but $|W(u)| = \theta + 1$ then
7: **makeLarge**(u)
8: add $u$ to the working set of $v$, $W(v)$
9: if $v$ is small but $|W(v)| = \theta + 1$ then
10: **makeLarge**(v)
11: (* available room in both tables to add edge ($u \rightarrow v$) *)
12: cases:
13: if $u$ and $v$ small:
14: $u$ connects directly to $v$, update $S(u), S(v)$
15: if $u$ small and $v$ large:
16: ego-BST(v).insert(u), update $L(u)$
17: if $u$ is large and $v$ is small:
18: ego-BST(u).insert(v), update $L(v)$
19: if $u$ is large and $u$ is large:
20: find a helper node $x$
21: ego-BST(u).insert($x$ as $v$)
22: ego-BST(v).insert($x$ as $u$

**makeLarge**(u):
1: create ego-BST(u)
2: for each $v \in W(u)$ do
3: if $v$ is small then
4: ego-BST(u).insert($v$)
5: if $u$ is large then
6: find a helper node $x$
7: ego-BST(u).insert($x$ as $v$)
8: ego-BST(v).insert($x$ as $u$

**reset**():
1: for each $u \in V$ do
2: inform to clear $S(u), L(u), W(u)$
3: set $u$ to small
routing, as well as arbitrary addressing.

Proof. Local routing. The proof of the local routing property is by construction and the states of the source \( u \) and the destination \( v \). For routing a request from a small node \( u \) to a small node \( v \), the packet is directly forwarded to the destination \( v \). For routing a request from a small node \( u \) to a large node, the packet is forwarded to \( v \) by traversing \( ego-BST(v) \) from parent to parent, until the root of \( ego-BST(v) \) is reached, and from there directly to \( v \). For routing a request from a large node \( u \) to a small node \( v \), the packet is forwarded to \( v \) by traversing \( ego-BST(u) \) from the root of \( ego-BST(u) \) downward to \( v \), similar to a classic search on a binary search tree. For routing a request from a large node \( u \) to a large node \( v \), the packet is forwarded to \( v \) in two steps. By construction, there must exist a helper node \( x \) that participates both in \( ego-BST(u) \) and \( ego-BST(v) \). First the request is forwarded on \( ego-BST(u) \) downward to \( x \) (which is stored in the tree as \( v \)). Then, \( x \) notes that the destination is \( v \) and forwards the packet upward to \( v \), on \( ego-BST(v) \). Since all binary search trees in the system are maintained locally using the adjust() method, no global routing algorithm is needed.

Compact routing. We set the threshold \( \theta \) to be twice the largest possible average degree in a window of size at most \( \delta \), i.e., \( \theta = 4c \), so every node with working set size less than \( \theta \) is small, and otherwise, large. Let \( \Delta = 6\theta \) (a constant) and we set the forwarding table to size \( \Delta \), so a ReNet supports compact routing. We need to show that the forwarding table does not exceed the size \( \Delta \). As long as the coordinator did not call reset(), for a large node the forwarding table is by design at most \( \Delta \), it contains one link to its ego-BST root and a set of links to at most \( \Delta - 1 \) virtual roots. For a small node we prove this in Lemma 3 that we state later.

Arbitrary addressing. The support for arbitrary addressing follow by design, since the search operation in binary search trees can support it naturally. \( \square \)

We can now state our main result:

**Theorem 2.** For any \((c, \delta)\)-sparse communication sequence \( \sigma \), where \(|\sigma| \geq \delta = \Omega(nD)\), there is a constant \( \Delta \) for which ReNets relying on statically optimal ego-BSTs (e.g. splay trees [13]), are statically optimal for \( \Delta \)-degree bounded networks.

**Proof.** We again set the threshold to be twice the average degree \( \theta = 4c \), let \( \Delta = 6\theta \) (a constant), and set the forwarding table to size \( \Delta \). Let \( N^* \)
be the optimal \( \Delta \)-degree bounded network used by the optimal static algorithm \( \text{STAT}(\sigma) \). From \cite{7} it follows that the average cost of \( \text{STAT} \) is lower bounded by:

\[
\text{Cost}(\text{STAT}, N^*, \sigma) \geq \Omega \left( \max(H_\Delta(\hat{Y}_\sigma \mid \hat{X}_\sigma), H_\Delta(\hat{X}_\sigma \mid \hat{Y}_\sigma)) \right)
\]  

(2)

We will prove static optimality of \( \text{ReNets} \) in two steps. First, we will show that the routing cost of a \( \text{ReNet} \) is optimal and proportional to its trees adjusting cost. Second, we will bound the cost of the operations and messages that are related to the coordinator in the \( \text{ReNet} \). Overall, we will show that the amortized cost of a \( \text{ReNet} \) is

\[
\text{Cost}(\text{ReNet}, \emptyset, \sigma) \leq O \left( H(\hat{Y}_\sigma \mid \hat{X}_\sigma) + H(\hat{X}_\sigma \mid \hat{Y}_\sigma) \right)
\]

(3)

making it order optimal since \( \Delta \) is constant (recall that \( N_0 = \emptyset \) is an empty initial network).

We divide \( \sigma \) into subsequences, \( \sigma^{(i)} \), separated by the \( i \)th call to the \text{reset()} operations announced by the coordinator. If no \text{reset()} is called then \( \sigma^{(1)} = \sigma \). If \text{reset()} was called \( k \) times then the last (partial) subsequence is denoted by \( \sigma^{(k+1)} \). We start with the analysis of a single “window”, \( \sigma^{(1)} \), which is the subsequence of \( \sigma \) from the start until the first \text{reset()} operation. The length of \( \sigma^{(1)} \) is \( \Omega(cnD) \) by assumption on sparsity and it contains exactly \( cn \) unique requests (see the Coordinator algorithm, Algorithm 4). We claim the following:

Claim 1. ReNet is statically optimal on \( \sigma^{(1)} \).

Proof. Let \( H_{\text{con}}^{(1)} = \max(H(\hat{Y}_{\sigma^{(1)}} \mid \hat{X}_{\sigma^{(1)}}), H(\hat{X}_{\sigma^{(1)}} \mid \hat{Y}_{\sigma^{(1)}})) \) be the maximum of the conditional entropies. We need to show that \( \text{ReNet} \)'s cost on \( \sigma^{(1)} \) is \( O(H_{\text{con}}^{(1)}) \) to prove its optimality. We separate the cost into two groups:

Routing and BST adjustment cost: For analytical reasons, we consider a symmetric version of \( \sigma^{(1)} \), named \( \tilde{\sigma}^{(1)} \), which keeps the total number of requests between pairs the same but divides them half-half: for each (directed) request \((u, v)\), we consider that half of the requests went the other direction, \((v, u)\), making the number of requests for a pair \((v, u)\) equal in both directions. This makes the frequency matrix (a matrix representation of the pair’s frequencies in the demand) of \( \tilde{\sigma}^{(1)} \) symmetric. Theorem 4 that we prove later, states that \( H(\hat{Y}_{\tilde{\sigma}^{(1)}} \mid \hat{X}_{\tilde{\sigma}^{(1)}}) = H(\hat{X}_{\tilde{\sigma}^{(1)}} \mid \hat{Y}_{\tilde{\sigma}^{(1)}}) \leq H_{\text{con}}^{(1)} + 1 \).

We consider the cost by node type. For a small node, if it connects to another small node, then the two nodes have a direct connection that
starts from the first request and stays active for the whole $\sigma^{(1)}$ (unless the node becomes large, which we address later). The amortized cost for such a request is one. If a small node connects to a large node, we charge the cost for routing and the cost for adjusting the network to the large node, which we discuss now. Each large node $w$ maintains an ego-BST($w$) for its communication partners. Since ego-BST($w$) is assumed to be a statically optimal datastructure [13] on all requests for which $w$ is the source or destination (recall that since $\bar{\sigma}^{(1)}$ is symmetric, the frequency distribution of destinations from $w$, $Y_w$, and sources to $w$, $X_w$, are the same), it follows that the cost of these requests (routing plus adjustments) is $O(H(Y_w)) = O(H(X_w))$ (see Lemma 2). This cost includes all ego-BST($w$).forward() and ego-BST($w$).adjust() operations. Since each routing request involves at most one forwarding operation by a helping node between two trees (for large-large edges), the (amortized) cost of routing and tree adjustment is at most $H(\hat{Y}_{\bar{\sigma}^{(1)}} | \hat{X}_{\bar{\sigma}^{(1)}}) + H(\hat{X}_{\bar{\sigma}^{(1)}} | \hat{Y}_{\bar{\sigma}^{(1)}})$, as required.

**Coordinator messages cost**: We discuss the coordinator functions one-by-one:

**reset()**: Happens once during the window $\sigma^{(1)}$. The cost is $n$, to broadcast the reset message to all nodes on the fixed network (using a broadcast tree).

**makeLarge()**: Happens at most once to each node during the window.

When makeLarge is executed at node $u$, we are guaranteed to have enough helper nodes if needed (since the network is not full yet, see Lemma 3), and since we do not add new edges to $u$, we only replace a constant number of existing edges $(u, v)$ (direct or via tree ego-BST($v$)), with a new connection via the newly created ego-BST($u$) of constant size. In each call of makeLarge, ego-BST($u$).insert() is amortized (accounting for in the adjustment cost above). The only additional cost is to notify helpers, but the number of helpers is bounded by $cn$ and sending a message is at most $O(D)$, so the total cost is $O(cnD)$.

**addRoute()**: Happens exactly $cn$ times during the window $\sigma^{(1)}$. The cost of ego-BST($u$).insert() and/or ego-BST($v$).insert() are amortized. The only cost is to notify the helper node which is at most $O(D)$. The cost during the window is therefore $O(cnD)$.

Summing up the total cost of the coordinator messages gives $O(cnD)$. Since the number of requests in the window is $\delta = \Omega(cnD)$, the amortized cost per coordinator request is $O(c')$, for a constant $c'$. To this we need to add
for each ego-BST\((w)\) its amortized cost for routing and adjusting, but this as mention is proportional to \(O(H(Y_w)) = O(H(X_w))\). Therefore the total amortized cost for the window (including routing, adjusting and coordinator messages) is \(O(H^{(1)}_{\text{con}})\).

To conclude the proof of Theorem 2, we divide \(\sigma\) into subsequences \(\sigma^{(i)}\), separated by reset() operations. A lower bound for Stat(\(\sigma\)) is:

\[
\text{Cost(Stat, } N^*, \sigma) \geq \sum_i \Omega(H^{(i)}_{\text{con}})
\]

While the cost for ReNet is:

\[
\text{Cost(ReNet, } \emptyset, \sigma) \leq \sum_i O(H^{(i)}_{\text{con}})
\]

which makes ReNet statically optimal.

Note that the coordination cost of the last subsequence (which may be shorter than \(\delta\)), can be amortized by coordination cost of \(\sigma^{(1)}\) so its amortized cost is also a constant.

Observe that the cost of ReNet could be much lower than the cost of Stat, since Stat is also lower bounded by the conditional entropy of the whole demand \(\sigma\), and not only the sum of entropies of the windows.

**Theorem 3.** The amortized cost of ReNet can be up to \(\log n\) times lower than the cost of Stat.

**Proof.** Consider for example a demand \(\sigma\) that is the concatenation of \(n\) demand subsequences \(\sigma^{(1)}, \sigma^{(2)}, \ldots, \sigma^{(n)}\). Each demand \(\sigma^{(i)}\) is of length \(\Theta(n \log n)\), is sparse and has a demand graph which is a (different) two-dimensional grid. Therefore the amortized cost of ReNet for each \(\sigma^{(i)}\) is constant. But if the \(\sigma^{(i)}\) is different each time (e.g., selected round robin), then \(\sigma\) could be made to be uniform, where overall each source communicates to all destinations with equal frequency (over the entire \(\sigma\)). This will force a lower bound on \(\Omega(\log n)\), the entropy of the uniform distribution, for Stat.,

**5 Related Work**

Datacenter networks have become a critical infrastructure and especially the popularity of online services \([5]\) (e.g., web search, social networks, storage,
financial services, multimedia, etc.) has led to a fast increase of datacenter traffic \cite{28,29}. Many applications (such as scatter-gather and batch computing applications) generate much internal datacenter traffic, and consequently, the traffic staying inside the datacenter is often much larger than the traffic entering or leaving the datacenter \cite{5}. It is hence not surprising that the design of effective and efficient (also in terms of cost and cabling) datacenter networks has received much interest over the last years \cite{11,31,1,2,31,32,33,31,35,36}. The situation has recently been compared to the early 1980s, when many new interconnection network designs were proposed \cite{37}, not for datacenters, but for parallel computers.

The advent of technologies for reconfigurable (a.k.a. malleable \cite{38}) networks introduces an additional degree of freedom to the datacenter network design problem \cite{38,6,39,40,8,11,41,42,43}. By relying on movable antennas \cite{42}, mirrors \cite{11,41}, and “disco-balls” \cite{6}, novel technologies in the context of optical circuit switches \cite{8,9,40,10}, 60 GHz wireless communication \cite{41,44}, free-space optics \cite{11,6}, provide unprecedented topological flexibilities, allowing to adapt the topology to traffic demands.

Indeed, the physical topology is the next frontier in an ongoing effort to render communication networks more flexible and reconfigurable. Over the last years, reconfigurable technologies (typically software) already enabled various innovations in important domains such as traffic engineering \cite{45,46}, load-balancing \cite{47,48}, and switching \cite{49,50}.

While the discussions on the benefits and limitations of such technologies are still ongoing \cite{38}, the community has identified a number of benefits of more flexible networks. While full bisection bandwidth allows for a flexible placement and scale-out of applications across clusters \cite{28,32,11,51}, the cost of (and energy consumed in) traditional networks is high, and today many datacenters are over-subscribed (e.g., fat-trees or folded Clos networks use a subset of possible roots only). Some empirical evaluations show that for certain workloads, a demand-aware network can achieve a performance similar to a demand-oblivious full-bisection bandwidth network at 25-40% lower cost \cite{11,6}. Empirical studies also confirm that communication patterns are often sparse and of low entropy, which can be exploited in demand-aware networks: in \cite{6}, it is shown that a high percentage of rack pairs does not exchange any traffic at all, while less than 1% of them account for 80% of the total traffic. In general, most bytes are delivered by large flows \cite{32,52,53}. Furthermore, the need for reconfigurability is motivated by empirical studies showing the difficulty of estimating traffic matrices and predicting the future \cite{54,55,56}.

We also note that the study of reconfigurable networks is not limited
to datacenter networks. Interesting use cases also arise in the context of wide-area networks \cite{39, 57} and, more traditionally, in the context of overlays \cite{58, 59, 60}.

Existing self-adjusting demand-aware network designs typically rely on some estimate or snapshot of the traffic demands, from which an optimized network topology is (re)computed periodically (often using exact algorithms or heuristics) \cite{61, 38, 34}. In contrast, inspired by self-adjusting datastructures, we in this paper present a more refined model, accounting also for the reconfiguration costs, and allowing us to study (within our model) the tradeoff between the costs and benefits of reconfigurations. We presented a survey and taxonomy of the problem space in \cite{62}, and adopt the terminology introduced there. However, in contrast to the current paper, no technical results were derived in the survey paper. Moreover, in contrast to existing algorithms relying on mixed integer programming, our algorithms are efficient (polynomial-time), and in contrast to existing algorithms relying on heuristics, our approach comes with provable guarantees, even over time. Moreover, existing solutions to provide on-demand bandwidth between communicating servers or top-of-the-rack switches are sometimes limited to (or at least prioritize in their opportunistic network \cite{6}) direct connections \cite{34}. We believe that local routing (as supported by ReNets) is an intriguing property that can simplify multihop routing, by supporting topological adjustments, without the need for global route recomputations.

In terms of formal guarantees, an upper bound on what can be achieved in terms of statically optimized demand-aware networks is due to Avin et al. \cite{7}, who build upon initial insights in \cite{59, 63}. We in this paper leverage the degree reduction technique of \cite{7}, however, to derive a very different result. The fixed demand-aware network designs by Avin et al. \cite{7} have recently also been extended to optimize for load, in addition to route lengths \cite{64}.

SplayNets \cite{59, 63} also rely on splay trees to adjust the network, and dynamically adapt to changing traffic patterns. However, besides their convergence properties under specific fixed demands, these networks do not provide any optimality but only heuristic guarantees. In fact, as we discussed, this is an inherent limitation, as static optimality is impossible to achieve based on single tree networks. Indeed, to the best of our knowledge, so far, no result existed on how to actually match the lower bound provided in \cite{59}, without perfect knowledge of the demand.

We also note that our approach of reconfiguring network topologies to reduce communication costs, is orthogonal to optimization approaches changing the traffic matrix itself (e.g., \cite{65}) or migrating communication endpoints on a fixed topology \cite{66}.
6 Conclusion

This paper presented the first self-adjusting network which provides entropy-proportional (and hence statically optimal) route lengths and reconfigurations, constant-sized forwarding tables and local routing. Our approach leveraged an intriguing connection to self-adjusting datastructures, leveraging self-adjusting BSTs as building blocks (i.e., per-source “ego-trees”), and combining them to a network.

We believe that our work opens several interesting directions for future research. In particular, an intriguing open question from our work regards the design of self-adjusting DANs which optimize metrics related to temporal locality, such as working sets, or even achieve dynamic optimality in specific settings. More generally, we believe that self-adjusting networks can be of interest beyond the datacenter context considered here; for example, it may be interesting to explore self-adjusting peer-to-peer overlays.

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A Intuition and Examples

This section establishes the connection between entropy of demand and route lengths in fixed demand-aware networks, which lies at the heart of the static optimality of the self-adjusting DANs presented in this paper. While a formal
proof appeared in [7], the objective of this section is to provide intuition and examples, as well as first empirical results.

In the following, the route length to serve request $\sigma_t = (u, v)$, is given by the hop distance $d_{N_t}(u, v)$ from $u$ to $v$, along the routing path chosen by the algorithm over $N_t$. If not specified otherwise we assume that routes are along shortest paths. Furthermore, we will sometimes consider certain time intervals of the request sequence, and we will denote the requests occurring at any time $t$ where $t_1 \leq t \leq t_2$ for two times $t_1 \leq t_2$ by $\sigma[t_1, t_2]$. (Hence $\sigma = \sigma[0, \infty]$.) Moreover, we can again think of a subsequence $\sigma' \subseteq \sigma$, as a directed and weighted demand graph (or guest graph) $G(\sigma') = (V(\sigma'), E(\sigma'))$. To give an example, Figure 3 (a) and (b) show two different demand graphs, each for a different time interval, $t_1$ and $t_2$ receptively. Since the communication traffic is changing over time, the demand graph also changes (the edges and their weights).

### A.1 Changing Demands Require Reconfigurations

If the communication patterns are sufficiently different, it can make sense to reconfigure the adaptive demand-aware network, $N_t$, as well. In general and ideally, each $N_t$ should be optimized toward the request graph of the future. In this example, for simplicity, we set the maximum allowable degree of $N_t$ to be two (i.e., we only have cycles and lines). Note that the network should also support multihop routing. For example in Figure 3 (b), node 1 communicates with three partners but its degree is bounded by two, so it must also know where to forward messages toward node 4, for example.

### A.2 Demand-Oblivious vs Demand-Aware Networks

First, we note that the route lengths in demand-oblivious networks (such as state-of-the-art expander networks [67, 4]) cannot be proportional to the conditional entropy and hence cannot provide the desired solution (even in the presence of traffic engineering flexibilities [34]). To give a simple example (for the sake of simplicity and clarity we leave some of the details out), consider a workload describing a communication pattern $\sigma$ whose demand graph $G(\sigma)$ forms a two-dimensional square grid, of size $\sqrt{n} \times \sqrt{n}$, see Figure 4 (a). For this sequence $\sigma$, $H(\hat{X})$ and $H(\hat{Y})$ are of order $\log n$, since the frequency of sources and destination is uniform which results in the maximum possible entropy. Embedding this workload on a static expander in an demand-oblivious way (i.e., random, or arbitrary) will result in an average route length also in the order of $\log n$, which is the diameter of
Figure 3: The need for reconfigurations. In the upper part, two (directed, weighted) demand graphs are shown for two different times: (a) $G(\sigma[t'_1, t_1])$ and (b) $G(\sigma[t'_2, t_2])$. In the lower part, their corresponding demand-aware networks $N_{t_1}$ and $N_{t_2}$ of bounded degree two are shown. (a) In $N_{t_1}$ every node is a direct neighbor of its communication partner. (b) In $N_{t_2}$ multihop routing and changing forwarding tables are needed.

A bounded degree expander. However, since every node has at most four neighbors, the conditional entropy (both $H(\hat{Y}|\hat{X})$ and $H(\hat{X}|\hat{Y})$) is only 2: a gap of $\Theta(\log n)$. A demand-aware design could achieve this bound.

Another example introducing a large gap of $\Theta(\log n)$ between demand-oblivious and demand-aware networks is a demand graph $G(\sigma)$ which forms a star (with unbounded degree), see Figure 4(b): node pairs communicate at different frequencies (skewed distribution, as indicated by the thickness). For this demand, the conditional entropy could be much lower than $\log n$ which will be the cost of serving this demand on a demand-oblivious expander.

More generally, one can see that every sparse communication pattern which is embedded on a demand-oblivious expander, will result in average route lengths in the order of $\Omega(\log n)$, the diameter, regardless of the entropy
Figure 4: Expander networks do not achieve optimal average route lengths for sparse demand graphs. (a) Oblivious embedding of a 2-dimensional grid demand graph (upper graph) on a constant degree expander network (lower graph) will result in average route lengths of $\Omega(\log n)$, while the conditional entropy of the demand graph is less than two. (b) Oblivious embedding of a weighted star demand graph on a constant degree expander network will result in an average route length of $\Omega(\log n)$ while the conditional entropy of the demand graph could be much lower.

In order to illustrate the potential gap between the upper bound of entropy (e.g., $H(\hat{X})$) and lower bounds of conditional entropy (i.e., $H(\hat{X}|\hat{Y})$) with some concrete numbers, we plot in Figure 5 the empirical entropy as well as the conditional empirical entropy of 3M routing requests from a Facebook datacenter trace [68]. The demand $\sigma$ consists of $n = 13,748$ communication partners, note that $\log n = 13.74$ for this case (we consider the binary logarithm). The figure considers times $t$ that are multiplicatives of $100K$. For each time $t$, the measures are presented both for the full range $\sigma[1,t]$ (labelled $H$) as well as for a time window of the last $100K$ requests $\sigma[t-100K,t]$ (labelled $H_W$), to shed light on the temporal locality.
Clearly the conditional entropy is lower than the entropy, and in particular the conditional entropy of the window is much less than the entropy of $X$ and $Y$. This indicates that demand-aware designs could reduce the average route length in the network.

### A.3 Limitations of Tree Networks

In order to be statically optimal, a self-adjusting network hence needs to achieve the conditional entropy bound, without knowledge of $\sigma$, but using reconfigurations (which come at a cost), in an online manner. Note that while the lower bound only holds for fixed topology networks, a self-adjusting network can in principle perform much better.

The best upper bound known so far for self-adjusting networks is $O(H(\hat{X}) + H(\hat{Y}))$, where $H(\hat{X})$ and $H(\hat{Y})$ are the empirical entropies of sources and destinations in $\sigma$, respectively [59]. It is achieved by a self-adjusting tree network. While this is optimal for some distributions, in particular for product distributions, in general, it is far from optimal.

Moreover, it is important to note that, following the results in [59], if such an almost optimal self-adjusting network exists, then it cannot be a bounded degree tree. The design we present in this work is far from a tree, in fact it is based on a network which is a union of trees.
B Deferred Proofs

B.1 Deferred Analysis of Symmetric Matrix

Let $M = M(\sigma)$ be a joint (non-symmetric) frequency matrix resulting from $\sigma$. Let $H_M(Y \mid X)$ denote the conditional entropy of $Y$ given $X$ under the joint probability distribution $M$. By definition, $H_M(Y \mid X) = H(Y_\sigma \mid X_\sigma)$. Let $H^*_\text{con} = \max(H(\tilde{Y}_\sigma \mid \tilde{X}_\sigma), H(\tilde{X}_\sigma \mid \tilde{Y}_\sigma))$, the maximum of both possible conditional entropies. Let $\bar{M} = (M + M^T)/2$ be the symmetric version of $M$. The conditional entropies of the symmetric and non-symmetric distributions are related as stated in the following theorem:

**Theorem 4.** The conditional entropy of the symmetric matrix $\bar{M}$ cannot be much larger than the maximal conditional entropy of $M$.

$$H_{\bar{M}}(Y \mid X) = H_{\bar{M}}(X \mid Y) \leq H^*_\text{con} + 1 \quad (4)$$

The proof of the theorem mainly follows from the following Lemma that is based on the concavity of entropy [69] and simple entropies algebra.

**Lemma 1.** Let $\tilde{p}$ and $\tilde{q}$ be two probability (frequency) distributions for the same set. Let $H^* = \max(H(\tilde{p}), H(\tilde{q}))$. Then

$$\frac{1}{2} H^* \leq \frac{1}{2} H(\tilde{p}) + \frac{1}{2} H(\tilde{q}) \leq H\left(\frac{\tilde{p} + \tilde{q}}{2}\right) \leq H^* + 1 \quad (5)$$

B.2 Other Deferred Lemmas and Proofs

**Lemma 2.** Consider a node $u$ connected directly to the root of a statically optimal self-adjusting ego-BST($u$), serving only requests to and from $u$. If $\tilde{p}$ is the empirical frequency distribution of destinations and $\tilde{p}$ is the empirical frequency distribution of sources, then the amortized cost of routing in and adjusting ego-BST($u$) is $O(H(\tilde{p}))$.

**Proof.** A self-adjusting ego-BST($u$) is originally designed to serve requests from the root to internal nodes. If the empirical frequency distribution on destinations (searched items) is $\tilde{p}'$, then the amortized cost of ego-BST($u$) is $O(H(\tilde{p}'))$, which is optimal [13]. In our case, we also have routes from internal nodes in the tree toward the root. But for the self-adjusting ego-BST($u$), it does not matter if the request is $(u,v)$ or $(v,u)$: the adjustments are the same, hence we can assume that each route request $(v,u)$ is actually a $(u,v)$ request, i.e., all requests are from the root of the tree. The new empirical frequency distribution on destinations (when all requests are from root to destinations) is also $\tilde{p}$. Therefore the results holds. \qed
Lemma 3 (Helping Nodes). As long as the coordinator did not call reset(), the size of the forwarding table of small nodes is at most $\Delta = 6\theta$ and helping nodes are available if needed.

Proof. Only small nodes can be helper nodes. A small node has a maximum degree of $\theta$, so it may need $12c = 3\theta$ ports in its forwarding table for the working set. For how many edges can a helper node be used as a relay? Since the number of helper nodes is at least $n/2$ (otherwise more than half of the nodes have degree larger than twice the average degree, which leads to a contradiction) and since there are at most $cn$ large-large edges, each helper node needs to help at most $2c$ such edges. Each helper node requires 6 ports (3 for each tree), so in total it needs at most $3\theta$ ports. Since the size of the forwarding table is $6\theta$, there will always be a helper node while the number of edges is less than $cn$ which mean total size of all working sets is less than $2cn = \theta n/2$. \qed