The chiral cosmological models with two components

I V Fomin
Department of Physics, Bauman Moscow State Technical University,
2-nd Baumanskaya street, 5, Moscow, Russia
E-mail: ingvor@inbox.ru

Abstract. The purpose of this paper is to consider cosmological models, containing two scalar fields. The method of exact solutions constructions of the scalar field’s dynamical equations on the second stage of the accelerated expansion of the universe with matter on the basis on models including two fields without interactions was considered. A cosmological models are associated with nonlinear sigma models in this work.

1. Introduction
The theory of cosmological inflation successfully describes the state of the accelerated expansion of the universe in the early stages of evolution. Also, inflationary cosmology explains the origin of primary inhomogeneities and predicts their spectrum [1, 2, 3, 4].

The mechanism of the initial inflationary scenario and the second accelerated expansion [5] can be described under the assumption of the existence of a scalar field, or an inflaton \( \phi \) specifying the stage of inflation and the quintessence field, which determines the observed accelerated expansion of the universe [6]. Different cosmological scenarios differ in the choice of effective potential \( V(\phi) \) of scalar field.

In four-dimensional space-time with a single scalar field there were obtained the restrictions on state parameter for quintessence \(-1 < w < -1/3\) and for phantom scalar field \( w < -1 \), where \( w \) is the state parameter in the equation of state \( p = w \rho \), \( p \) is the pressure and \( \rho \) is the energy density [7].

To describe the stage of cosmological inflation and re-accelerated expansion in the framework of the Einstein gravity theory, models with nontrivial kinetic energy or the \( k \)-essence models [8, 9] and models with two scalar fields [10, 11, 12, 13, 14, 15] were investigated. Within the framework of the approach, based on the chiral cosmological models, various types of kinetic and potential interaction between the field of the inflaton and a scalar field simulating dark energy at the inflation stage, as well as interactions of dark matter and dark energy in the subsequent stages of the evolution of the Universe are possible [13, 14]. The cosmological perturbations in the chiral models of inflation were considered in the work [15].

In present paper, the simple way of exact solutions construction for cosmological models with two scalar fields and matter field on the basis of known solutions for models with two noninteracting scalar fields is considered. This approach is associated with nonlinear sigma models that combine kinetic and potential type interactions between scalar fields and lead to Friedmann cosmological models [13, 14, 15].
2. Chiral cosmological models

Nonlinear sigma models with a potential of interaction (chiral cosmological models) are widely used to describe various epochs of the evolution of the universe. In such models there is an internal space, the metric of which determines the interaction between scalar fields.

Let us write the action for the nonlinear sigma model with additional material sources [15]

\[ S = \int dx^4 \sqrt{-g} \left( \frac{R}{2} + \frac{1}{2} h_{ij} \partial_\mu \psi^i \partial_\nu \psi^j g^{\mu \nu} - V(\phi, \chi) \right) + S_m, \]  

where \( R \) is Ricci scalar, \( V(\phi, \chi) \) is the potential of scalar fields, \( g^{\mu \nu} \) is the metric tensor of space, \( h_{ij} \) is the metric tensor of the fields space or the target (chiral) space, \( \psi^i, \psi^j \) are the coordinates of the fields space, \( \phi, \chi \) is the potential of scalar fields, and \( S_m \) is the action for material fields.

The metric of the field space is defined as follows

\[ ds^2 = h_{ij}(\psi^k) d\psi^i d\psi^j. \]  

A system of Einstein’s equations in a flat Friedman-Robertson-Walker (FRW) space for a sigma model with two scalar fields and a material field corresponding to a barotropic perfect fluid in the system of units \( 8\pi G = c = 1 \) can be written as follows

\[ 3H^2 = \frac{1}{2} h_{11}(\chi) \dot{\phi}^2 + h_{12}(\phi, \chi) \dot{\phi} \dot{\chi} + \frac{1}{2} h_{22}(\phi) \dot{\chi}^2 + V(\phi, \chi) - \frac{\rho_m}{a^n}, \]  

\[ -\dot{H} = \frac{1}{2} h_{11}(\chi) \dot{\phi}^2 + h_{12}(\phi, \chi) \dot{\phi} \dot{\chi} + \frac{1}{2} h_{22}(\phi) \dot{\chi}^2 - \frac{\rho_m}{a^n}, \]  

where \( \rho_m = \rho_m a^{-n} \) is the energy density of the barotropic perfect fluid, which effectively describes the material components of the universe (dark matter and baryon matter) at the stage of the second accelerated expansion, with following equation of state \( p_m = \rho_m(n - 3)/3 \), where \( 0 \leq n \leq 6 \) and \( p_m \) is the pressure of fluid. Radiation does not have a significant effect on the dynamics of the universe in this era [16]. The components of metric tensor of fields space \( h_{11} \) and \( h_{22} \) determine the potential interaction between the scalar fields, and \( h_{12} = h_{21} \) determines the kinetic interaction. Also, positive components \( h_{11}, h_{22} \) correspond to canonical real scalar fields with positive kinetic energy \( h_{11}(\chi) \dot{\phi}^2 / 2 > 0 \), for example, and negative components correspond to phantom fields with negative kinetic energy.

Note that in the case of \( \rho_m a = k \), where \( k \) is the coefficient determining the curvature of the FRW space, and \( n = 2 \), the system (3)–(4) corresponds to the equations of the scalar field’s dynamics in the FRW space with nonzero curvature \( k = -1, 0, 1 \) without matter.

Now, we consider the models with the kinetic interaction \( h_{11} = h_{22} = 1 \) under the condition

\[ h_{12}(\phi, \chi) \dot{\phi} \dot{\chi} = \frac{\rho_m}{a^n}, \]  

and the system of equations (3)–(4) are

\[ 3H^2 = \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} \dot{\chi}^2 + V(\phi, \chi) \]  

\[ -\dot{H} = \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} \dot{\chi}^2 \]  

Thus, the system of equations (6)–(7) corresponds to the models with scalar fields without interactions. Further, we will consider exact solutions for polynomial and exponential potentials on the base of known solutions for two scalar field without interactions between them and without material fields.
3. Polynomial potential

Following the work [17], we consider the Hubble parameter

\[ H = H_0 + H_1 \phi + H_2 \chi, \]  

where

\[ H_0^2 = \frac{2}{9} \left( \frac{3}{2} A + E + D \right), H_1^2 = \frac{D}{3}, H_2^2 = \frac{E}{3} \]  

As a result, from the equations (6)–(7) one can obtain

\[ V(\phi, \chi) = A + B \phi + C \chi + D \phi^2 + E \chi^2 + F \phi \chi, \]  

where

\[ B^2 = \frac{8D}{3} \left( \frac{3}{2} A + E + D \right), C^2 = \frac{8E}{3} \left( \frac{3}{2} A + E + D \right), F^2 = 4D^2 E^2 \]  

with scalar fields

\[ \phi(t) = \phi_0 - 2H_1 t \]  
\[ \chi(t) = \chi_0 - 2H_2 t \]  

and scale factor

\[ a(t) = a_0 \exp \left[ \left( H_0 + H_1 \phi_0 + H_2 \chi_0 \right) t - (H_1^2 + H_2^2) t^2 \right] \]  

Using our approach we derive from equation (5) that

\[ h_{12}(t) = \frac{\rho_{m0}}{H_1 H_2 a_0^\alpha} = \frac{\rho_{m0}}{H_1 H_2 a_0^\alpha} \exp \left[ -n(H_0 + H_1 \phi_0 + H_2 \chi_0) t + n(H_1^2 + H_2^2) t^2 \right] \]  

or

\[ h_{12}(\phi, \chi) = \frac{\rho_{m0}}{H_1 H_2 a_0^\alpha} \exp \left[ \frac{n(H_0 + H_1 \phi_0 + H_2 \chi_0)(\phi - \phi_0)}{2H_1} + n(H_1^2 + H_2^2) \left( \frac{\chi - \chi_0}{2H_2} \right)^2 \right] \]  

\[ h_{12}(\phi, \chi) = \frac{\rho_{m0}}{H_1 H_2 a_0^\alpha} \exp \left[ \frac{n(H_0 + H_1 \phi_0 + H_2 \chi_0)(\chi - \chi_0)}{2H_1} + n(H_1^2 + H_2^2) \left( \frac{\phi - \phi_0}{2H_2} \right)^2 \right] \]  

Thus, from the solutions for models with two noninteracting scalar fields at the stage of inflation [17], new exact solutions were obtained for the stage of second accelerated expansion with a matter on the base of chiral cosmological models.

4. Exponential potential

For an exponential potential of the form [17]

\[ V = V_0 \left( 1 - A_1 e^{-\alpha \phi} + A_2 e^{-2\alpha \phi} - B_1 e^{-\beta \chi} + B_2 e^{-2\beta \chi} + \frac{A_1 B_1}{2} e^{-\alpha \phi - \beta \chi} \right), \]  

where

\[ \alpha^2 = \frac{3}{2} - 6A_2 \frac{A_1}{A_1}, \beta^2 = \frac{3}{2} - 6B_2 \frac{B_1}{B_1} \]  

the Hubble parameter [17]

\[ H = \sqrt{\frac{V_0}{3}} \left( 1 - A_1 e^{-\alpha \phi} - B_1 e^{-\beta \chi} \right), H_0 = \sqrt{\frac{V_0}{3}} \]
Thus, one can obtain

$$\phi(t) = \frac{1}{\alpha} \ln[e^{\alpha \phi_0} - A_1 \alpha^2 H_0 t]$$ \hspace{1cm} (21)

$$\chi(t) = \frac{1}{\beta} \ln[e^{\beta \chi_0} - A_1 \beta^2 H_0 t]$$ \hspace{1cm} (22)

$$a(t) = a_0 \left(e^{\alpha \phi_0} - A_1 \alpha^2 H_0 t\right)^{\frac{1}{2n_2}} \left(e^{\beta \chi_0} - B_1 \beta^2 H_0 t\right)^{\frac{1}{2n_2}} e^{H_0 t}$$ \hspace{1cm} (23)

Ones again, applying our method, from equation (5) we derive

$$h_{12}(t) = \frac{\rho_{m0} \left(e^{\alpha \phi_0} - A_1 \alpha^2 H_0 t\right)^{-\frac{n_2}{2n_2}+1} \left(e^{\beta \chi_0} - B_1 \beta^2 H_0 t\right)^{-\frac{n_2}{2n_2}+1} e^{-H_0 t}}{\alpha \beta A_1 B_1 H_0^2 a_0^n}$$ \hspace{1cm} (24)

$$h_{12}(\phi, \chi) = \frac{\rho_{m0} \exp\left[(\alpha - \frac{n}{2n_2}) \phi + \left(\beta - \frac{n}{2n_2}\right) \chi\right]}{\alpha \beta A_1 B_1 H_0^2 a_0^n \exp\left[\frac{n(e^{\alpha \phi_0} - e^{\alpha \phi})}{A_1 \alpha^2}\right]}$$ \hspace{1cm} (25)

$$h_{12}(\phi, \chi) = \frac{\rho_{m0} \exp\left[(\alpha - \frac{n}{2n_2}) \phi + \left(\beta - \frac{n}{2n_2}\right) \chi\right]}{\alpha \beta A_1 B_1 H_0^2 a_0^n \exp\left[\frac{n(e^{\beta \chi_0} - e^{\beta \chi})}{B_1 \beta^2}\right]}$$ \hspace{1cm} (26)

These solutions also determine the possible background dynamics of the universe at the modern era with considering the material fields as barotropic perfect liquid.

5. Conclusion

Thus, the work proposed a method for constructing exact solutions in the context of chiral cosmological models for both: cosmological models with two scalar fields and matter, also, for the inflation stage, which involves only scalar fields with nonzero curvature of FRW space as a special case of general solutions. It should also be noted that by specifying the components of the fields space tensor $h_{11} = h_{11}(\chi)$ and $h_{22} = h_{22}(\phi)$ it is possible to obtain more complicated models, based on known solutions, with cross potential interactions between scalar fields.

References

1. Starobinsky A 1979 Phys. Lett. B 91 99
2. Guth A 1981 Phys. Rev. D 23 347
3. Albrecht A and Steinhardt P 1982 Phys. Rev. Lett. 48 1220
4. Mukhanov V, Feldman H and Brandenberger R 1992 Phys. Rep. 215 203
5. Perlmutter S et al. 1999 Astrophys. J. 517 565
6. Peebles P and Vilenkin A 1999 Phys. Rep. D 59 063505
7. Liddle A and Lyth D 1993 Phys. Rep. 231 1
8. Putter R and Linder E 2007 Astropart. Phys. 28 263
9. Fomin I 2015 (in Russian) Herald of the Bauman Moscow State Tech. Univ. Nat. Sci. 4 37 (Original Russian title: Vestnik MGITU im. N.E. Baumana)
10. Urena-Lopez M 2004 Phys. Rev. D 63 6 06350
11. Bohmer C et al. 2008 Phys. Rev. D 78 2 023505
12. Costa F et al. 2012 Phys. Rev. D 85 107302
13. Beesham A et al. 2009 Class. Quant. Grav. 26 075017
14. Chervon S et al. 2015 Russian Physics Journal 58 5 597
15. Chervon S 2013 Quantum Matter 2 71
16. Ade P et al. 2016 Astron. Astrophys. A13 594
17. Byrnes C and Tasinato G 2009 JCAP 16 0908