Amplitude and phase characterization by diffracted beam interferometry: blind dbi

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Abstract. Diffracted beam interferometry is a self referenced method characterization technique whose operation principle is based on the reconstruction of the phase of a beam starting from the interference data between the beam and its diffracted copy. The phase is recovered indirectly by means of an iterative algorithm that relates the irradiances of the interfering beams and its phase difference. The first experimental demonstration of DBI was implemented on a Mach-Zehnder interferometer which incorporated an afocal imaging system in each arm, in order to form an image of a common object in different planes at the output of the interferometer. The irradiance data as well as the phase difference data were picked up from one of the image planes and they were introduced in the iterative algorithm. In this work we discuss a modification of the algorithm that allows to reconstruct simultaneously the amplitude and phase of the wavefront starting from, exclusively, the phase difference between the two waves that interfere in one of the image planes. This new algorithm improves the reconstruction process because the data acquisition process is faster and consequently the method is less influenced by environment disturbances. The method has been applied successfully to the characterization of phase plates and laser beams as well as to the local characterization of ophthalmic lenses.

1. Introduction
The measurement of an optical wavefront has a great variety of technological applications in optics and astronomy, such us beam control, optical testing, recovery of blurred images and so on. There exist several methods that can perform this task but among this methods interferometry offers the greatest resolution [1-4]. Unfortunately the wavefront reconstruction from interferometry presents some drawbacks related to its high cost, its sensitivity to mechanical vibrations and the need to control critically the reference beam. This drawbacks can be overcome for example with shearing interferometry. Other alternative method is diffracted beam interferometry (DBI) which is based on the coherent superposition of a light wave with a diffracted copy of itself [4-7]. In this case, the essential information for wavefront reconstruction is obtained from the interference pattern and the irradiances of both the light beam and its diffracted copy. However these data do not provide direct information about the wavefront (from the interference pattern we only extract information about the phase difference between the beams). So we need to apply an iterative numerical algorithm.
The method can be implemented with a very simple interferometric device. We have used a Mach-Zehnder interferometer with an afocal lens system at each arm, although other configurations are possible. If we combine DBI with spatial modulation techniques, the method requires to record only a single interferogram.

2. The basis of DBI

The experimental set-up that we have use is shown in figure 1 that corresponds to a Mach-Zehnder interferometer which incorporates an afocal lens system in each arm. Each telescope provides respectively an image of the observation plane at plane P\(_1\) and P\(_2\) with different magnifications. We consider afocal systems for imaging because in this case, the optical field in both images planes reproduce exactly the complex object field, except magnification. At the output of the interferometer another telescope in a Badal configuration images one of these two planes in a CCD sensor. If we consider that this plane is P\(_1\), from now on we will consider the wave travelling along arm1 as the signal beam (\(u_s\)) and the other as the reference beam (\(u_r\)).

The translation of the mirror in arm 1 introduces a tilt in the signal beam at the image plane P\(_1\). This tilt generates a spatial frequency carrier (\(F\)) in the interferogram and the corresponding irradiance distribution is:

\[
I(x, y) = |u_s(x, y) + u_r(x, y)e^{i2\pi F} + u_s^*(x, y)u_r(x, y)e^{-i2\pi F}|^2
\]

The field amplitudes \(u_s\) and \(u_r\) at P\(_1\) are related by means of the Fresnel diffraction integral:

\[
u_s(x, y) = C \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u_s\left(\frac{x' M_s}{M_r}, \frac{y' M_s}{M_r}\right) \exp\left\{\frac{ik\left[(x-x')^2 + (y-y')^2\right]}{2d}\right\} dx' dy'
\]

where \(C\) is a complex constant, \(M_s\) and \(M_r\) the magnification ratios in the signal and reference beam respectively and \(d\) is the distance between P\(_1\) and P\(_2\) and is equal to 2(\(f_1+f_2-f_3+f_4\)), being \(f_1-f_4\) the focal length of lens L\(_1\)-L\(_4\) respectively. We will use this relation between the complex amplitudes in our algorithm to characterize the beam.

![Figure 1 Experimental set-up](image)

The spatial modulation generated in the interference pattern allows us to extract the product \(u_s u_r^*\) by means of well known Fourier Techniques [8]. We name traditionally this product as complex amplitude modulation (CAM). If we noted it as \(I_{\text{cam}}\). The amplitude of the signal beam can be expressed as a function of the CAM term as:

\[
u_s(x, y) = \frac{I_{\text{cam}}(x, y)}{u_r(x, y)}
\]
By combining equations (2) and (3) we can develop an iterative procedure: first, \( u_r \) is taken as a constant and is inserted in equation (3) to calculate \( u_s \), next by means of equation (2) we calculate an estimation of \( u_r \) and \( u_r^* \); then this new estimation is inserted in (3) to obtain a new approach of \( u_r \). This procedure is repeated and after few iterations, the estimated signal field converges to the real one.

3. Experimental results

We applied the technique to the local characterization of ophthalmic lenses and also to the global characterization of photoresists phase plates. The parameters of the interferometric sensor, that is the Mach-Zehnder interferometer, are the focal length of the lenses that determine the magnification at the image planes. When we choose these parameters we must take into account two requirements: one is that the reference beam must overlap completely the signal beam, and second the diffraction of reference beam between the image planes \( P_1 \) and \( P_2 \) must be significant.

In the first experiment, we set the diffraction distance \( d=21 \text{cm} \) and the ratio between magnifications \( M_r/M_s = 2.5 \). In figure 2 we show the results for an astigmatic lens with +0.25D and +0.5D along each principal meridian and in figure 3 the results for an astigmatic lens with +0.25/-0.25D. For comparison, we have also represented the irradiance of the signal and reference beam which have been measured separately. We must note that in both cases the diffracted beam exhibit an elliptical shape as a consequence of lens astigmatism and the retrieved reference beam reproduce this shape with the proper orientation. This result is improbable if we deal with wrong phases. As a test, we have changed the sign of the CAM term and executed the algorithm. In this case the algorithm converges into a beam but with the opposite phase curvature, and the irradiance of the diffracted beam has a different (and wrong) ellipticity and orientation. This result is very important because it means that DBI avoids an ambiguity present in many phase retrieval method that is the sign of the phase curvature.

![Figure 2: Amplitude and phase reconstruction of an astigmatic lens of +0.25/0.5 D at its main meridians. (a) signal beam irradiance, (b) reference beam irradiance and (c) signal phase. (d) and (e) are respectively the measured signal and reference irradiances.](image-url)
Figure 3: Amplitude and phase reconstruction of an astigmatic lens $o +0.25/-0.25D$ at its main meridians. (a) signal beam irradiance,(b) reference beam irradiance and (c) signal phase. (d) and (e) are respectively the measured signal and reference irradiances.

A second set of experiments concern the global characterization of phase plates in photoresist [9]. The first phase plate reproduces a Zernicke polynomial that simulates a secondary trefoil. In this case the diffraction distance was $d=5$cm and the relative magnification is 1.5625. In figure 4 we plot the results of the beam reconstruction. We observe that the retrieved beam irradiances are noisier than the original ones but this has little effect on the retrieved phase that maintains the appearance of a secondary trefoil. We repeat the experiments but changing the magnification ratio to 2.5 that reduce the diffraction effects in the reference beam and we compare the results of the characterization in figure 5. The reconstruction is better in the first case where the reference wave suffers strong diffraction effects while the second case the configuration approach to a radial shearing set-up. In figure 6 we apply the procedure to characterize another phase plate whose phase reproduces a Zernicke polynomial. Note that in spite of the noise present in the signal amplitude, the algorithm retrieves accurately the phase if we compare with the simulated one except where the irradiance drops to zero.

4. Conclusions

We have shown that DBI is a useful tool to realize a simultaneous characterization of the amplitude and phase of optical beams. By combining DBI with spatial phase modulation techniques the measurement can be performed in a single shot. It is important to stress that DBI requires that the reference beam overlaps completely the signal beam, otherwise the algorithm does not work. It must be noted that the role of the signal and the reference beams can be interchanged in equation (2), so the iterative algorithm can be easily modified to be adapted to this context.
The method as it is has some limitations. One of them is related to the amplitude division by 0 in equation (3)\[5,6\]. When the irradiance approach to the noise level of the camera it can induce significative errors. The second limitation is related to the Fourier approach when, for example, the spatial filtering can remove significant terms when the object includes high frequency components [7]. To avoid these problems the spatial carrier frequency must be greater that one-half the sum of the background and the CAM bandwidth. Therefore, Fourier processing may induce errors in beams that exhibit sharp edges or wavefronts with step-like jumps.

Finally, we have also shown that if we deal only with phase characterization, the signal and reference amplitudes can be used as a powerful test of the validity of reconstruction.

Figure 4 Amplitude and phase reconstruction of a phase plate (a) signal beam irradiance,(b) reference beam irradiance (c) signal phase. (d) and (e) are respectively the measured signal and reference irradiances.

Figure 5. Reconstructed phase of the phase plate of figure 5. (a) d=5cm and the magnification ratio is 1.5625, (c) d=21cm and magnification ratio is 2.5. (b) numerical simulation of a secondary trefoil.
Figure 6 Signal beam amplitude (a) and phase reconstruction (b) of a second photoresist phase plate. The parameters are the same as in figure 4. (c) synthetic phase

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