SEIR Model and Simulation for Typhus Disease

S Side1*, N Badwi2, M Abdy1, and W Sanusi1

1Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Negeri Makassar, Jl Daeng Tata, Parang Tambung, Tamalate, Kota Makassar, Sulawesi Selatan 90224. Telp (62411)864936
2Department of Geography, Faculty of Mathematics and Natural Sciences, Universitas Negeri Makassar, Jl Daeng Tata, Parang Tambung, Tamalate, Kota Makassar, Sulawesi Selatan 90224. Telp (62411)864936

*e-mail: syafruddin@unm.ac.id

Abstract. The objectives of this research are to obtain a mathematical model for the spread of typhoid by including the factor of flies and food as variables in the model; analyze and simulate models that will provide predictions and status of typhoid. The data used are secondary data on the number of Typhus cases in Makassar City, analysis of the model is done using the Lyapunov function method, while the simulation uses the MatLab model. The results of this study are the SEIR model for typhoid spread, then the simulation results of the model using secondary data on the number of typhoid cases in the city of Makassar provide the number of typhus sufferers and the status of typhoid in Makassar city that is disease free or endemic. Prediction of the number of typhoid cases which can be a reference in the early steps of typhoid cases in Makassar.

Keywords: SEIR model, Typhus disease; Model simulation

1. Introduction
Typhoid Fever is an infectious disease caused by Salmonella typi bacteria which is a pathogenic germ. Typhoid is a systemic infectious disease with a picture of a long-standing fever, the presence of bacteremia that can relieve inflammation that can damage the intestine and liver. S. Typis bacteria can cause severe disease in one area but only cause mild disease in another region, meaning there is a relationship between regional differences with the severity of the disease [1].

Typhoid fever is a contagious disease that is spread throughout the world, and until now it is still the biggest health problem in developing and tropical countries such as Southeast Asia, Africa and America Latin. The incidence of this disease is still very high and an estimated 21 million cases with more than 700 cases ended in death [1].

In Indonesia, the incidence of typhoid fever is estimated at around 300-810 cases per 100,000 population per year, meaning the number of cases ranges from 600,000-1,500,000 per year. This relates to the level of individual hygiene, environmental sanitation and the spread of germs from careers or typhoid sufferers. In endemic areas where sanitation and health are well maintained, typhoid fever appears as a sporadic case. Based on the results of a household health survey (1986) typhoid fever caused deaths of 3% of all deaths in Indonesia. The average cases of death and complications of typhoid fever always change between different endemic areas [1]. Makassar as the capital of South
Sulawesi province is one of the cities with the largest number of typhoid fever cases in South Sulawesi. In 2011, two hospitals namely Haji and Labuan Baji Hospital reported 596 and 165 patients with typhoid fever, then increased in 2012 to 1,115 and 178 typhoid patients [2].

Mathematical modeling of infectious and non-communicable diseases such as dengue fever, tuberculosis, diabetes mellitus and typhus has been carried out like [3-12]. This mathematical modeling becomes one of the solutions offered in controlling and guarding because the simulation model can predict the number of cases of the disease, that the government can make prevention and treatment earlier. Research on typhus has been carried out by [6] and [7], but has not included flies and food as variables, in this study a SEIR model will be built by paying attention to food factors and flies as carriers of typhoid bacteria, then analyzing and simulating using data on the number of cases typhus in Makassar city, in the second year a numerical solution will be determined from the typhus distribution model in Makassar city.

2. Method
The first part of this research is to build a SEIR model for the spread of typhoid by including food and fly factors as variables. Furthermore, the model is analyzed and simulated using secondary data on the number of typhus in Makassar. Then a numerical solution is determined from the model and the simulation uses Maple programming to predict the number of typhus cases in the following months.

3. Result and Discuss
3.1. SEIR Model for Typhus Disease
This research discussed the model of SEIR model based on the assumptions. Furthermore, the equilibrium point of the model is then analyzed, which is further interpreted in real life problems. In this model, the population divided into four classes, the Susceptible class (S) states the population of susceptible individuals affected by the disease, Exposed class (E) states the population of individuals detected by disease but not yet infected, Infected (I) class indicates the population of individuals who have been infected with the disease, and class Recovered (R) states the individual population who is cured of the disease. After recovered, the individual be immunity to the disease.

![Figure 1. Flow Chart of the SEIR Mathematical Model for Typhus Transmission](image-url)
The definition of variables and parameters used in mathematical modeling of Typus disease is presented in Table 1 below:

Table 1. Definition of Variable and parameter for Typus

| No | Variable/Parameter | Definition                                                                 |
|----|--------------------|-----------------------------------------------------------------------------|
| 1  | $S_h$              | Humans have the potential to be infected with salmonella typhosa bacteria   |
| 2  | $E_h$              | Humans who show symptoms are infected by salmonella typhosa bacteria.       |
| 3  | $I_h$              | Humans who have been infected with salmonella typhosa bacteria.             |
| 4  | $R_h$              | Humans who have recovered.                                                 |
| 5  | $S_f$              | Flies that have the potential to be infected with salmonella typhosa bacteria |
| 6  | $I_f$              | Flies that have been infected with salmonella typhosa bacteria              |
| 7  | $S_F$              | Foods that have the potential to be infected with salmonella typhosa bacteria|
| 8  | $I_F$              | Foods that have been infected with salmonella typhosa bacteria              |
| 9  | $a$                | Levels or rate of time of transmission in the individual body.              |
| 10 | $b$                | Levels or time rates for humans infected by the transmission of salmonella typhosa bacteria. |
| 11 | $c$                | Human levels recovering from the transmission of the virus due to the care given. |
| 12 | $d$                | Potential human levels to be infected with salmonella typhosa bacteria (direct contact from infected humans). |
| 13 | $e$                | Eating lots of food that has been infected with salmonella typhosa bacteria |
| 14 | $f$                | Levels of time the food is potentially infected with salmonella typhosa bacteria. |
| 15 | $h$                | the number of flies infected with salmonella typhosa bacteria perch on food so that food becomes infected. |
| 16 | $u$                | Levels or time rates for flies infected by the transmission of salmonella typhosa bacteria. |
| 17 | $\gamma_h$        | Deaths caused by typhus                                                     |
| 18 | $\mu_h$           | Human death naturally or not caused by salmonella typhosa bacteria (typhoid disease) |
| 19 | $\mu_L$           | Death of flies naturally or not caused by salmonella typhosa bacteria       |
| 20 | $\mu_F$           | Food is stale or discarded by humans because it is not eaten                |

Based on the flow diagram in Figure 2, the SEIR model equation for typhoid disease is obtained:

\[
\begin{align*}
\frac{dS_h}{dt} &= \mu_h N_h + e(I_F) + d(I_h) - (\mu_h + a)(S_h) \\
\frac{dE_h}{dt} &= a(S_h) - (\mu_h + \gamma_h + b)(E_h) \\
\frac{dI_h}{dt} &= b(E_h) - (\mu_h + \gamma_h + c + d)(I_h) \\
\frac{dR_h}{dt} &= c(I_h) - \mu_h(R_h)
\end{align*}
\]  

(1)
ii. Vector Population
\[
\frac{dS_L}{dt} = A - (\mu_L + u)(S_L)
\]
\[
\frac{dI_L}{dt} = u(S_L) - (\mu_L + h)(I_L)
\] (2)

iii. Food Population
\[
\frac{dS_F}{dt} = B + h(I_L) - (\mu_F + f)(S_F)
\]
\[
\frac{dI_F}{dt} = f(I_F) - (e + \mu_F)(I_F)
\]

3.2. Analysis and Simulation of SEIR Model for Typhus Disease
The results of the simulation of the SEIR mathematical model using the initial values of the number of suspected, exposed, infected and recovered typhus cases, and also the values of the parameters that have been given as in Table 2 using Maple are presented in Figure 2:

Let:
\[
p = S_h \quad s = I_L \quad r = I_h
\]
\[
q = E_h \quad t = S_F \quad u = I_F
\]
then the system in equations (1), (2) and (3) becomes the following equation system (4):
\[
\frac{dp}{dt} = -(\mu_h + a)p + dr + eu
\]
\[
\frac{dq}{dt} = ap - (\mu_h + \gamma_h + b)q
\]
\[
\frac{dr}{dt} = bq - (\mu_h + \gamma_h + c + d)r
\]
\[
\frac{ds}{dt} = -(\mu_L + h)s
\]
\[
\frac{dt}{dt} = hs - (\mu_F + f)t
\]
\[
\frac{du}{dt} = ft - (e + \mu_F)u
\] (4)

Finding eigenvector values:
\[
\begin{bmatrix}
-\mu_h - a - \lambda & 0 & d & 0 & 0 & e \\
\mu_h - \gamma_h + b - \lambda & 0 & 0 & 0 & 0 & 0 \\
0 & -\mu_h - \gamma_h - c - d - \lambda & 0 & 0 & 0 & 0 \\
0 & 0 & -\mu_L - h - \lambda & 0 & 0 & 0 \\
0 & 0 & 0 & h & -\mu_L - h - \lambda & 0 \\
0 & 0 & 0 & 0 & -e - \mu_F - \lambda & f
\end{bmatrix} = 0 (5)
Table 2. Initial values of variables and parameters of the SEIR model for typhus

| Variable/Parameter | Value  | Parameter | Value |
|-------------------|--------|-----------|-------|
| $S_h(0)$          | 7675406| b         | 0.165 |
| $E_h(0)$          | 7675893| f         | 0.256 |
| $I_h(0)$          | 7675893| h         | 0.1428|
| $I_L(0)$          | 0.052  | $\mu_L$  | 0.032 |
| $S_F(0)$          | 0.035  | D         | 0.32  |
| $I_F(0)$          | 0.02   | E         | 0.45  |
| a                 | 0.76   | $\gamma_h$| 0.002054|
| $\mu_h$          | 0.000054| C         | 0.4361|
| $\mu_F$          | 0.0004 |

The eigenvalue equation based on equation (5) is:

$$1.0000000083 \lambda^6 + 2.564952670 \lambda^5 + 2.548770657 \lambda^4 + 1.20452959 \lambda^3 + 0.27903470 \lambda^2 + 0.029579540 \lambda + 0.001091368152$$

(6)

Equation (6) gives the following eigenvalues:

$\lambda_1 = -0.0773; \lambda_2 = -0.172; \lambda_3 = -0.261; \lambda_4 = -0.447; \lambda_5 = -0.804 + 0.239i; \text{and} \lambda_6 = -0.804 - 0.238i$

Equation (6) also gives the value of the basic reproduction number $R_0 = 0.0011$ or $R_0 \leq 1$ sehingga 1 so that Typhus disease in Makassar is at an alarming status, but preventive measures are still needed.

Figure 2. Prediction of the number of Suspected, Exposed, Infected and Recovered Typhus

Figure 2 that resulted from MATLAB for the SEIR model on the spread of typhus can be described as follows: The number of individuals suspected of typhus in the city of Makassar in the first month is quite high, then continues to decrease, then the number of exposed individuals in the city of Makassar increases sharply in the fourth to sixth months, then decreases. The number of individuals infected...
with typhus in the first month is quite high, then decreased even though the decrease was not too high and constant with the number of typhus sufferers around 200 individuals. The number of flies that cause typhoid in the first month is also quite high, then continues to decrease, whereas food causing typhoid is also quite high in the first month, then decreases in the following months.

4. Conclusion
The results of the analysis of the SEIR model of typhoid show that the type of stability in the model is stable and flashlight, this shows that typhoid disease continues to exist and is centered on a particular place or area. The results of the simulation of the SEIR model on the spread of typhoid showed that the number of suspect and infected typhus in the city of Makassar in the first month tended to be high, but then decreased, the same thing happened in the fly population as carriers of typhoid viruses, as well as food factors. While the number of exposed populations tends to increase, this is because individuals who have been infected with typhus tend not to immediately check their health and choose to rest at home.

Acknowledgements
The authors gratefully acknowledge the financial support received from PDUPT DIKTI with project number No:131/UN.36.9/PL/2019 and also thanks to Universitas Negeri Makassar for supporting this paper.

References
[1] Yatmina, 2011. Bakteri Salmonella Typhi Dandemamtifoid. Jurnal Kesehatan Masyarakat. Vol. 6, No. 1
[2] Hasan Basri, 2013. Penderita Demam Thypoid Masuk di Rumah Sakit Meningkat. Tribun Timur, Makassar
[3] Rangkuti, Y. M., Side, S., Noorani, M.S.M., 2014. Numerical analytic solution of SIR model of dengue fever diseasein South Sulawesi using homotopy perturbation method and variational iteration method. Journal of Mathematical and Fundamental Sciences. 46A(1), 91-105.
[4] Syafruddin S. et. all. 2013. Lyapunov functions of SIR and SEIR model for transmission of dengue fever disease. International Journal Simulation and Process Modeling: Inderscience Publishers 8(2,3):177-184.
[5] Syafruddin S., 2015. A Susceptible-Infected-Recovered Model and Simulation for Transmission of Tuberculosis. Advanced Science Letters. 21 (2):137-139.
[6] Muh. Altaf K, et. all., 2015. Mathematical Analysis of Typhoid Model with Saturated Incidence Rate. Advanced Studies in Biology, Vol. 7(2), 65 – 78.
[7] J. K. Nthiiri, et. all. 2016. Mathematical Modelling of Typhoid Fever Disease Incorporating Protection against Infection. British Journal of Mathematics & Computer Science 14(1): 1-10.
[8] Syafruddin S. et. all. 2016. A SEIR Model for Transmission of Tuberculosis . AIP Proceedings. 1830 (020004)
[9] S Side, W Sanusi, MK Aidid, S Sidjara, 2016. Global stability of SIR and SEIR model for Tuberculosis disease transmission with Lyapunov function method. Asian. J. Appl. Sci 9, 87-96
[10] Poerwanto, B, et. all. 2018. A matlab code to compute prediction of survival trends in patients with DHF. Journal of Physics: Conference Series. Vol (1028)
[11] Sanusi, W, et. all. 2017. Comparison of the methods to estimate missing values in monthly precipitation data. International Journal on Advanced Science, Engineering and Information Technology. Vol. 7(6): 2168-2174.
[12] S Side, Irwan, U Mulbar, W Sanusi, 2017. SEIR model simulation for Hepatitis B. AIP Conference Proceedings 1885 (1), 020198