Research Article

Structural Dynamic Load and Parameter Identification Based on Dummy Measurements of Displacement

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1.Introduction

Structural load and parameter identification are the essential contents in the field of structural dynamics, and some studies pay attention to the coupled recognition of uncertain structure parameters as well as unacquainted loads. Gillijns and De Moor presented the Kalman-type filter, which was used in the work for coupled identification as the unabbreviated form of GDF. However, it has been demonstrated that it is unstable for the extended GDF (EGDF) method, drifting in the identified unacquainted loads as well as displacement, just like most previous identification methods based on the least-square algorithm. In order to deal with this unstable issue, this paper applied the dummy measurements of displacements on a position level and modifies the EGDF algorithm using the information integration method about the accelerated measurements with dummy measurements. Numerical example of a truss is used for validating the applicability of the method in the work, and an influence of covariance matrices in dummy displacements is also considered.

1. Introduction

Structural external load is vital to structural optimization design, failure diagnosis, and health monitoring [1, 2]. Since the external load is difficult to measure by equipment instruments, various deterministic methods have been proposed for load identification. However, structural parameters of the system are uncertain [3], such as uncertain stiffness and damping, which makes the load identification not credible. In recent years, the uncertain methods for structural load and parameter identification have been attracted increasing attention in research.

The simultaneous identification of the parameters of set membership and a load was developed. Meanwhile, it is unusual for the approach needing given states in the applications of engineering [4]. Aiming at the problem of state/input/parameter coupling recognition in structure damage recognition [5], they proposed an EKF (extended Kalman filter algorithm), named EKF-UI, based on unacquainted input load. In [6], EKF-UI was derived by the least-squares estimation algorithm, indicating that complicated mathematical reasoning was used to obtain the recursive solution. The above simplified EKF-UI in [6] was also applied to identify nonlinear structure parameters. Nevertheless, derivations in the simplified method may bring on the confusion of the Bayesian framework when the prior probability density functions (PDFs) at $t+1$ were regarded as followed PDFs [7]. Naets et al. [8] presented another algorithm called the A-DEKF (augmented discrete extended Kalman filter) to recognize coupled parameter/input/state. The A-DEKF algorithm looks like the developed KF algorithm [9–11], wherein parameters, uncertain loads, and structural states constitute the extensive state vector in high dimension. More importantly, the A-DEKF algorithm requires a matched hypothesis about. A similar method has been used for the offshore wind turbine for the simultaneous identification of the system parameters based on stiffness, states, and the hydrodynamic load in [12]. Lately, the approach of DKF-UKF is proposed for the coupling recognition of states, unknown loads, and parameters in [13, 14]. A dual KF (Dual
KF) is adopted to estimate the load. Besides, the so-called augment states are estimated by the UKF (unscented KF), consisting of the structural states and the unknown parameters. Nevertheless, the research does not mention the convergence analysis, observability, and stability of the DKF-UKF method. Additionally, the DKF algorithm requires the assumption of the load covariance, and the assumption value strongly influences the Bayesian filters’ identification quality.

We developed an approach in terms of the GDF algorithm towards the coupling recognition topic, which is called as the extended GDF (EGDF) algorithm [15]. The EGDF algorithm is proposed based on the EKF algorithm’s linearization. Undetermined parameters are added to the structural states forming the extensive ones for recognition. New equations of measurement and state transmission become nonlinear, and the first-order Taylor expansions are used for linearizing these two equations. The EGDF algorithm and the standard GDF algorithm have a uniform structure, containing updates of time and measurement and load recognition. Their primary difference refers to the state-space formulas’ sensitivity matrices. Nevertheless, the conventional GDF method has been demonstrated to be inherently unstable based solely on limited accelerated measurements. In this situation, it results in the false drift of low frequency in recognizing shift and load [16]. In [15], the information integration of acceleration response and the measurement equations of the system state are changed into the following discrete-time forms taking the sampling frequency of 1/Δt as

\[
\begin{align*}
\mathbf{z}_{k+1} &= f_k(\mathbf{z}_k, \mathbf{u}_k) + \mathbf{w}_k, \quad k = 1, 2, \ldots, T, \\
\mathbf{y}_k &= h_k(\mathbf{z}_k) + \mathbf{D}_k \mathbf{u}_k + \mathbf{v}_k, \quad k = 1, 2, \ldots, T.
\end{align*}
\]

In these equations, \(\mathbf{z}_k\) is the state estimation, \(\mathbf{y}_k\) is the measurement, \(f(\cdot)\) and \(h(\cdot)\) are the two nonlinear functions associated with the augment state vector. The vectors \(\mathbf{v}_k\) and \(\mathbf{w}_k\) represent the vectors of measurement and system noises, which are supposed to be zero-mean, irrelevant, and white stochastic signal with the given covariance matrices \(\mathbf{G}_k\) and \(\mathbf{R}_k\) separately.

It is assumed that \(\mathbf{u}_k\) and \(\mathbf{z}_{k|k}\) represent the posterior estimates of \(\mathbf{u}_k\) and \(\mathbf{z}_k\) separately, according to the observed vector \((\mathbf{y}_0, \mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_k)\). The state variance is denoted as \(\mathbf{P}_{z|k} = E[(\mathbf{z}_k - \mathbf{z}_{k|k})(\mathbf{z}_k - \mathbf{z}_{k|k})^T]\), and \(\mathbf{z}_{0|0}\) and \(\mathbf{P}_{0|0}\) are the initial unbiased state estimate vector and the corresponding variance matrix, respectively, which are supposed to be given. Also, the EGDF formula works requiring the two following conditions: (i) the number of measured signals needs to be more significant compared with that of unacquired loads. (ii) Since the weighted least-squares estimation is used to calculate the unacquired input load inversely, the accelerated response signal at the unacquired input position should be determined.

Based on the above declaration, the EGDF algorithm can be obtained by the following three steps.

(i) Input identification step is

\[
\mathbf{R}_k = \nabla \mathbf{h}_k \cdot \mathbf{P}_{z, k|k-1} \cdot (\nabla \mathbf{h}_k)^T + \mathbf{R}_k,
\]

\[
\mathbf{J}_k = (\mathbf{D}_k^T \mathbf{R}_k^{-1} \mathbf{D}_k) - \mathbf{D}_k^T \mathbf{P}_{z, k|k-1}^{-1} \mathbf{D}_k,
\]

\[
\mathbf{u}_k = \mathbf{J}_k (\mathbf{y}_k - h_k(\mathbf{z}_{k|k-1})),
\]

\[
\mathbf{P}_k' = (\mathbf{D}_k^T \mathbf{R}_k^{-1} \mathbf{D}_k)^{-1}.
\]

(ii) Measurement update step is

\[
\mathbf{z}(t) = (\mathbf{p}(t) \quad \dot{\mathbf{p}}(t) \quad \mathbf{a})^T,
\]

\[
\mathbf{u}(t) \text{ indicates the external load vector; } \mathbf{H}_t \text{ is the impact matrix related to the external load } \mathbf{u}(t). \text{ Usually, the structure’s mass can be obtained accurately. Thus, the mass matrix } \mathbf{M} \text{ is given.}
\]

In structure system equation (1), it is assumed that \(\mathbf{C}\) and \(\mathbf{K}\) include the unacquired parameters \(\mathbf{a}\) to be identified; thus, the initial structural status vector is rewritten as the extensive one.
\[ K_k = P_{k|k-1}^z (V_z h_k)^T \tilde{R}_k^{-1}, \]  
\[ z_{k|k} = z_{k|k-1} + K_k (y_k - h_k(z_{k|k-1} - D_k \tilde{u}_k)), \]  
\[ P_{k|k} = P_{k|k-1}^z - K_k (\tilde{R}_k - D_k P_{k|k}^u D_k^T) K_k^T, \]  
\[ P_{k+1|k}^u = (P_{k|k}^u)^T = -K_k D_k P_{k|k}^u. \]  

(iii) Time update step is
\[ z_{k+1|k} = f_k(z_{k|k}, \tilde{u}_k), \]
\[ P_{k+1|k}^z = \begin{bmatrix} V_z f_k & V_u f_k \end{bmatrix} \begin{bmatrix} P_{k|k}^z & P_{k}^{zu} \\ P_{k}^{zu} & P_{k}^u \end{bmatrix} \begin{bmatrix} (V_z f_k)^T \\ (V_u f_k)^T \end{bmatrix} + G_k, \]

where \( V_z f_k, V_u f_k, \) and \( V_z h_k \) are the sensitivity matrices as
\[ V_z f_k = \frac{\partial f(z, u)}{\partial z} \bigg|_{z = \tilde{z}_k, u = \tilde{u}_k}, \]
\[ V_u f_k = \frac{\partial f(z, u)}{\partial u} \bigg|_{z = \tilde{z}_k, u = \tilde{u}_k}, \]
\[ V_z h_k = \frac{\partial h(z, u)}{\partial z} \bigg|_{z = \tilde{z}_k, u = \tilde{u}_k}. \]

It is found that the EGDF algorithm is actually dependent on the EKF method. Using the first-order Taylor division to linearize formula (4) of nonlinear state measurement and transmission, the normal GDF formula is appropriate to identify the augmented states and unacquainted offered burden of an approximately linear power system. In addition, it is proved that edge-type algorithms in terms of the limited accelerated measurements are naturally labile and produce the false displacement of low frequency during identifying input loading and shifts [19]. The accelerated insensitivity to quasi-static components of external loads causes this displacement [16]. The author presented the extension of the EGDF algorithm to recognize the extrinsic loads and structure parameters in terms of fusing the date of shift and acceleration [15]. However, displacement transducers have a large volume and are not suitable for placement in some situations, which also may alter the system properties. It is necessary to develop a novel algorithm based solely on acceleration measurements in engineering applications.

### 3. The EGDF Algorithm Based on Dummy Measurements

In this paper, data fusion of the accelerometers and dummy measurements of displacements is developed based on the EGDF algorithm for structural dynamics load and parameter identification. The dummy measurements approach is motivated by the one proposed by Chatzi and Fuggian [19, 20] to make the monitoring for civil structure stable. Usually, the deformation of a structural system is bounded to a limited range. Besides, an order of magnitude for deformation is used as a priori estimate that may be implemented in simulation. The work regards the boundary of transformation as the uncertainty of virtual measurement and points out that the assumed transformation is zero. Analog measurements of displacement has the following measurement equation:
\[ H_{dm} p + v_{dm} = 0, \]

where \( H_{dm} \) is the impact matrix related to the dummy displacements. The formula expresses that, under the unacquainted \( v_{dm} \), the covariance \( R_{dm} \), the location variation of the degrees of freedom is 0. The matrix \( R_{dm} \) can be used for the tuning of the KF to obtain the desired results, and it ought to choose a higher order of magnitude than the practical movement of the structural systems. The high covariance based on virtual measurements cannot appropriately limit the drifting in recognition. If the covariance is chosen too small, the estimates will be constrained to generate erroneous results.

Adopting the idea of the dummy measurements, the conventional EGDF algorithm can be extended with (18). Equations (5), (10), and (17) are changed into

\[ \tilde{R}_k = V_z h_k \cdot P_{k|k-1}^z \cdot (V_z h_k)^T + \begin{bmatrix} R_{dm} & 0 \\ 0 & R_k \end{bmatrix}, \]
\[ z_{k|k} = z_{k|k-1} + K_k \begin{bmatrix} 0 \\ y_k \end{bmatrix} - \begin{bmatrix} H_{dm} P_{k|k-1} \\ H_{dm} \end{bmatrix}, \]
\[ V_z h_k = \begin{bmatrix} H_{dm} & 0 \\ 0 & 0 \end{bmatrix}, \]
\[ -H_s M^{-1} K - H_s M^{-1} C \frac{\partial K}{\partial \alpha} p(k) - H_s M^{-1} \frac{\partial C}{\partial \alpha} \dot{p}(k). \]
And also, the matrix $D_k$ in (6) is changed as

$$D_k = \begin{bmatrix} 0 \\ H_a M^{-1} H_u \end{bmatrix},$$

(20)

where $H_a$ represents the influence matrix related to the extrinsic loading, and, from (20), it can been seen that the matrix $D_k$ is a constant matrix in this paper.

They do not benefit quick estimation needed for load estimation because of great uncertainty about the virtual measurement. However, they can stop long-term displacement caused through accelerated measurements and stabilize the approximated filter covariance. If the constant location of the given structural system is not zero, this behavior need to be taken into account for the dummy measurements to obtain a more accurate identification.
4. Numerical Simulation

A numeric example of a plane truss is expressed to estimate the modified EGDF formula in the work based on the dummy measurements. The plane truss structure is supported at two ends, modeling through thirty-one finite elements of the plane truss, the lateral and longitudinal degrees of freedom of free nodes (See Figure 1). All horz and vert members have been with the lengths of 2 and $\sqrt{2}$ m, respectively. Besides, the cross-sectional area of all elements is $8.95 \times 10^{-5}$ m$^2$, and assuming Rayleigh damping, the related coefficients are $\beta = 4.6503 \times 10^{-4}$ and $\alpha = 0.1523$. The specific mass is $7.85 \times 10^3$ kgm$^{-3}$; moreover, the elastic coefficient is $2 \times 10^7$ Pa. First, inherent frequencies of the structure are 0.15, 0.41, 0.86, 1.02, 1.39, 1.77, 2.14, and 2.29 Hz, respectively, based on the finite element (FE) computation. At nodes 4 and 9, we apply two external loads, respectively. Load $v_1$ is sin excitation.

$$v_1 = 20 \sin(24\pi t) + 20 \sin(48\pi t).$$

And the other load $u_3$ is an impact input. Assume the rigidities for truss elements 17, 15, 14, 10, 7, and 5 are unacquainted parameters for identification, with their original values of 633.0, 759.5, 1,163.5, 1,342.5, 633.0, and 759.5 N/m separately. Thus, the extended status vector refers to $[p_1, p_2, \ldots, p_{30}, k_3, k_7, k_{10}, k_{14}, k_{15}, k_{17}]^T$.

\[ $u_1$ \]
Seven accelerated signals, determined for recognition, are the horizontal ones at node 9, as well as the vertical ones at nodes 10, 9, 7, 5, 4, and 3. At the same time, the measurement signals was used for simulation by adding 4% ambient noise. Original augmented status covariance $P_{i0}^0$, is composed of three parts which are the covariance vector of the displacement, velocity $(P_{0j}^d, P_{0j}^v)$, and unknown stiffness $(P_{0j}^z)$. The system noise $R_{dm}$ is considered as

$$
\begin{align*}
P_{0j}^d &= \text{diag}\{10^{-16}, 10^{-16}, \ldots\}_{1\times \text{ndof}}, \\
P_{0j}^v &= \text{diag}\{10^{-8}, 10^{-8}, \ldots\}_{1\times \text{ndof}}, \\
\tilde{P}_{0j}^z &= \text{diag}\{10^{14} \times 0.5^2, 10^{14} \times 0.5^2, \ldots\}_{1\times 6},
\end{align*}
$$

in which $\text{ndof}=30$ represents the amount of trussed structures. The covariance matrices of the system noise $w_i$ and the measured noise $\psi_i$ have been assumed as $R_w = 0.01I_6$ and $G_k = \text{diag}\{0, 0, \ldots\}_{1\times \text{ndof}}$, $\text{diag}\{10^6, 10^6, \ldots\}_{1\times 6}$ separately. In terms of the proposed formula, two external loads have been recognized with recognized curves (see Figures 2 and 3), comparing the active curves of loads. Both identified loads have the drift problem compared with the real values. Structure shift and speed are recognized together. Figure 4 shows the recognized outcome of vertical displacement and velocity at node 7 and theoretic values. Recognized velocity is precise; however, recognized displacement has the drift problem in low frequency due to the sole acceleration measurements.

Therefore, local dummy measurement shifts have been introduced to the measurement accelerations. Vertical shifts of nodes 4 and 9 have been taken as dummy measurements together with the above seven accelerations, and $R_{dm} = \text{diag}\{10^{-4}, 10^{-4}\}$. Figure 5 shows the comparisons of the identified results of load $\mu_1$, and Figure 6 shows the amplified segment (14–14.5 s) in Figure 5 for better illustration. Figures 7 and 8 show the identified load $\mu_2$, shift, and speed. Spurious drift reduces obviously using the proposed EGDF based on dummy measurements of displacement. Table 1 shows the identified stiffness values. Recognized relative error (RE) is defined as follows:

$$
\text{RE} = \frac{\| \tilde{k}_{\text{identified}} - \tilde{k}_{\text{exact}} \|}{\| \tilde{k}_{\text{exact}} \|} \times 100%. \tag{23}
$$

The recognized parameters are precise.

As in the above description, the covariance matrix $R_{dm}$ influences the estimations much. The two following cases demonstrate the effect of $R_{dm}$ value. One is a smaller value as $R_{dm} = \text{diag}\{10^{-6}, 10^{-6}\}$, and the other is a larger value as $R_{dm} = \text{diag}\{10^{-2}, 10^{-2}\}$. The identified results are shown in Figures 9–11. It is found in Figures 9 and 10 that the load is not identified accurately, especially the low-frequency components, and the identified displacement is smaller compared with the accurate value due to the smaller $R_{dm}$. From Figures 11 and 12, it is found that the identified load is not accurate as the actual load, and the identified displacement has the low-frequency drift problem due to the larger $R_{dm}$.

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**Table 1: Comparison of the stiffness parameters of the identified truss elements.**

| Element no. | 5    | 7    | 10   | 14   | 15   | 17   |
|-------------|------|------|------|------|------|------|
| $k_i$ (N/m) exact | 1265.9 | 1265.9 | 895  | 895  | 1265.9 | 1265.9 |
| $k_i$ (N/m) identified | 1265.3 | 1260.6 | 900.878 | 889.758 | 1277.2 | 1257.7 |
| RE (%)     | 0.05 | 0.42 | 0.66 | 0.59 | 0.89 | 0.65 |

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**Figure 8:** (a) Theoretic and determined vertical shift at node 7 through dummy measurements. (b) Theoretic and determined vertical speed at node 7.
Figure 9: Identification of external load $u_2$ with smaller $R_{dm}$.

Figure 10: (a) Theoretically and determined vertical shift at Node 7 with smaller $R_{dm}$. (b) Theoretically and determined vertical velocity at node 7.

Figure 11: Identification of external load $u_2$ in the case of a smaller $R_{dm}$. 
5. Conclusions

The work proposed the modified formula by fusing the data about determining acceleration and dummy shift reaction for coupled dynamic load and parameter identification. It reduces the false displacement of low frequency in estimating loads and shifts effectively, which can be seen in the conventional EGDF algorithm using acceleration measurements solely. The algorithm is useful depending on the covariance matrix of dummy displacements. The covariances are selected to be an order of magnitude greater compared with the system’s real movement for a suitable estimate. A numerical example is presented for proving the formula in the work, including the influences of different levels of dummy displacement covariance. Additionally, the algorithm is suitable for the modal domain of identification.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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