On the $W$-algebra in the Calegero-Sutherland model using the
Exchange operators

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Abstract

We study the $W_\infty$ algebra in the Calegero-Sutherland model using the exchange operators. The presence of all the sub-algebras of $W_\infty$ is shown in this model. A simplified proof for this algebra, in the symmetric ordered basics, is given. It is pointed out that the algebra contains in general, nonlinear terms. Possible connection to the nonlinear $W_\infty$ is discussed.

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In recent times there is a revival of interest in the Calegari-Sutherland model [1] due to its relevance in various fields like Quantum Hall Effect [2], Spin Chains [3], Fractional Statistics [4], Quantum Chaos [5], 2D gravity [6], 2D QCD [7]. Apart from its application to these and other fields, this model is interesting in its own right due to its classical and quantum integrability. Also this one dimensional many-body system, is the only known example for which certain dynamic correlation functions can be exactly computed [8]. Despite these remarkable developments in recent years, the model is not completely solved, in the sense that the complete excited eigenstates are not known explicitly. Another interesting, perhaps related, problem is with regards to the symmetry algebra in this model. Recently, using Quantum Inverse Scattering Method (QISM) [9] and earlier, by collective field method [10], \( W_\infty \) algebra [11] was shown to be present in this model. In this paper we study the \( W_\infty \) algebra in this model, using the recently developed exchange operator formulation [12]. As will be shown later, this formalism is well suited to study the \( W_\infty \) algebra.

In this formalism, ordinary derivative is replaced by a ‘covariant derivative’ (known as Dunkl derivative [13]) with the gauge field part containing an exchange operator \( M_{ij} \), with the following properties:

\[
M_{ij} \phi_j = \phi_i M_{ij} \quad (i, j = 1 \cdots N) \quad (1a)
\]

for any operator \( A_i \) (including \( M_{ij} \) itself)

\[
M_{ij} \phi_k = \phi_k M_{ij} \quad (i, j \neq k) \quad (1b)
\]

\[
M_{ij} = M_{ji} \quad (1c)
\]

\[
M_{ij}^2 = 1 \quad (1d)
\]

This covariant derivative, is explicitly given as \([h = m = 1]\)

\[
\pi_i = p_i + i \lambda \sum_j \frac{1}{x_i - x_j} M_{ij} \quad (2)
\]

where \( \lambda \) is a constant and prime over the summation \( j \), indicates the absence of \( j = i \) term. This derivative \( \pi_i \), obeys the zero curvature condition:
\[ [\pi_i, \pi_j] = 0 \]  \hspace{1cm} (3)

The Hamiltonian for the Calogero model with the common harmonic confinement (of frequency \( \omega = 1 \)), is given as

\[ H = -\frac{1}{2} \sum_{i=1}^{N} \pi_i^2 + \frac{1}{2} \omega^2 \sum_{i=1}^{N} x_i^2 \]  \hspace{1cm} (4)

Note that this is diagonal in the particle index and formally resembles that of harmonic oscillator. This Hamiltonian is the same as that of the Calogero Hamiltonian

\[ H_c = \frac{1}{2} \sum_{i=1}^{N} p_i^2 + \frac{1}{2} \sum_{i>j}^{N} \lambda (\lambda - 1) (x_i - x_j)^2 + \frac{1}{2} \omega^2 \sum_{i=1}^{N} x_i^2 \]  \hspace{1cm} (5)

for the physical states obeying the Bosonic condition,

\[ M_{ij}|\psi> = +|\psi> \]  \hspace{1cm} (6)

Due to the formal similarity of the Hamiltonian (4) with that of the harmonic oscillators, define a generalised annihilation and creation operators \((a_i, a_i^\dagger)\) as

\[
a_i = \frac{\pi_i - i x_i}{\sqrt{2}} \\
a_i^\dagger = \frac{\pi_i + i x_i}{\sqrt{2}} \]  \hspace{1cm} (7)

using which (4) can be expressed as

\[ H = \frac{1}{2} \sum_{i=1}^{N} (a_i a_i^\dagger + a_i^\dagger a_i) \]  \hspace{1cm} (8)

The commutation relation between \(a_i\) and \(a_i^\dagger\) can easily shown to be

\[
[a_i, a_j^\dagger] = \delta_{ij} (1 + \lambda \sum_k M_{ik}) - (1 - \delta_{ij}) \lambda M_{ij} \]  \hspace{1cm} (9)

\[
[a_i, a_j] = [a_i^\dagger, a_j^\dagger] = 0 \]  \hspace{1cm} (10)

Also, it can be shown,

\[ [H, a_i(a_i^\dagger)] = -a_i(-a_i^\dagger) \]  \hspace{1cm} (11)

similar to that of harmonic oscillator.
The problem we address here, is about the $W$ algebra in this model using such a formulation. The exchange operator method, discussed above, is well suited for this purpose, due to its formal resemblance to the oscillator problem. It is known that the generators of $W_\infty$ algebra can be expressed in terms of the oscillator algebra. In one of the basis [14],

$$W_{nm} = a_o^n A^m \quad n, m \geq 1$$

where $A \equiv a_o a_0^\dagger$, and $a_o, a_0^\dagger$ satisfies $[a_o, a_0^\dagger] = 1$.

The commutation relation of the differential operators, is

$$[a_o^n f(A), a_o^m g(A)] = a_o^{n+m} (f(A-m)g(A) - f(A)g(A-n)) \quad (12)$$

where $f$ and $g$ are polynomials.

This has as the sub-algebra,

(a) (centreless) Virasoro algebra generated by $L_n = -a_o^n A$

$$[L_n, L_m] = (n-m) L_{nm} \quad (13a)$$

(b) $[L_o, W_{nm}] = -n W_{nm} \quad (13b)$

(c) $[W_{ok}, W_{ol}] = 0$ (Cartan-sub algebra) \quad (13c)

One can also, consider the basis [15], in which

$$W_n^{(s)} \equiv a_o^{s-n-1} a_o^\dagger s + n - 1 \quad (14a)$$

obeys the algebra

$$[W_n^{(s)}, W_m^{(t)}] = [n(t-1) - m(s-1)] W_{n+m}^{s+t-2} + \cdots \quad (14b)$$

where $\cdots$ denotes the lower order terms, due to quantum correction, and its coefficients depend on the ordering in the definition of $W_n^{(s)}$.

Due to the realisation of $W_\infty$ algebra in terms of oscillator algebra, it is pertinent to ask, if the oscillator algebra is replaced by the modified one given by (10), does it give rise to the
same $W$ algebra? This is the question, we attempt, in this paper, Bergshoeff and Vasilev [16], have shown Virasaro algebra, in the same formulation. The quantum integrability of this model, shown by Polychronakos [12], implies the presence of the Cartan sub-algebra (13c). Ujino and Wadati [9], have used the Lax operators to show $W_{\infty}$ algebra, after long and tedious calculations. In contrast, we find that the exchange operator formalism simplifies the proof enormously. Also, we show that, in contrast to Ujino and Wadati [9], there are non-linear terms in the algebra. This paper is organised as follows: Section I shows the sub-algebra’s mentioned in (13) using the oscillator defined in (10). Section II, deals with the symmetric ordered form of the basis in (14a). Finally, we end with the conclusion.

Section I:

In this we show the presence of the sub-algebras of $W_{\infty}$ in the basis given in (13). The presence of Virasavo algebra, in the basis (13), was shown by Bergshoeff and Vasiliev [16]. Due to the restriction $n, m > 0$, central term is absent. Next we show the sub-algebra (13b) with $a_o, a_o^\dagger$ replaced by $a_i, a_i^\dagger$ obeying (9),

$$[L_o, W_{nm}] = \left[\sum_{i=1}^{N'} A_i, \sum_{j=1}^{N} a_j^n A_j^m\right]$$

where the prime over the summation implies $i = j$ term is excluded. Using the commutation rules (9) and (10) and the Leibnitz rule, we get

$$= -n \sum_{i=1}^{N} a_i^n A_i^m - \lambda \sum_{i=1}^{N} \sum_{\beta=0}^{n-1} a_i^{\beta+1} M_{ij} a_i^{n-\beta} A_j^m$$

$$+ \lambda a_i \sum_{\beta=0}^{n-1} a_j^\beta M_{ij} a_j^{n-\beta} A_j^m + \lambda \sum_{i,j} a_j^n [A_i, A_j] M_{ij} A_j^m$$

The last term in the above equation follows from using,

$$[A_i, A_j] = \lambda (A_i - A_j) M_{ij}$$

which follows from using (9) and (10) in the definition of $A_i$. Next we show that all terms dependent on $\lambda$ cancel with each other, leading to the desired result. Collecting only $\lambda$
dependent terms, separating the sum over $\beta$ in the second term (and the third term) as
$\beta = n - 1$ ($\beta = 0$) and the rest, and also using the property (2.1) of $M_{ij}$, we get

$$\lambda \sum_{ij} \left[ -a_i^n A_j^m - \sum_{\beta=0}^{n-2} a_i^{1+\beta} a_j^{n-1-\beta} A_j^m + a_i^n A_i^m + \sum_{\beta=1}^{n-1} a_i^{n-\beta} a_j^\beta A_i^m \\
+ \sum_{\beta=0}^{m-1} a_j^\beta A_i^m - a_j^n \sum_{\beta=0}^{m-1} A_j^{\beta+1} A_i^{m-1-\beta} \right] M_{ij}$$

(19)

Note that $\beta = 0$ ($\beta = m - 1$) in the fifth term (sixth term) cancels with the first (third) term in (19). Upon interchanging $i, j$ in the second term, in (19), we find all of them cancels. More explicitly,

$$\lambda \sum_{ij} \left[ \left(- \sum_{\beta=1}^{n-1} a_i^{\beta} a_j^{n-\beta} + \sum_{\beta=1}^{n-1} a_j^{\beta} a_i^{n-\beta} \right) A_j^m \\
+ a_j^m \left( \sum_{\beta=1}^{m-1} A_j^{\beta} A_i^{m-\beta} - \sum_{\beta=1}^{m-1} A_j^{\beta} A_i^{m-\beta} \right) \right] M_{ij} = 0$$

(20)

Thus we get

$$[L_o, W_{nm}] = -nW_{nm}$$

The Cartan sub-algebra follows from the quantum integrability of the model. It was shown by Polychronokos, $\sum_{i=1}^{N} A_i^n \equiv I_n$, obey $[I_n, I_m] = 0$. It is interesting to note that all the sub-algebras’ are satisfied as an operator relation, without having to use the physical state condition (6), for only when the latter holds, this is equivalent to Calugero-Sutherland model.

**Section II**

In order to study the full $W_\infty$ algebra, it is found convenient to consider the basis, in which generator is a symmetric combination of arbitrary powers of $a^n$ and $a^{\dagger m}$.

$$O_{nm} \equiv \frac{n!m!}{(n + m)!} \sum_{i=1}^{N} \left( a_i^n a_i^{\dagger m} + a_i^{n-1} a_i^{\dagger m} a_i + \cdots + a_i^{\dagger m} a_i^n \right)$$

(21)

This is the basis used by Ujino and Wadati to show $W_\infty$ in Calugero mode. Note that the normalisation $(n + m)!/n!m!$ gives the number of terms in the symmetric combination.

First we show that

$$[O_{nm}, a_i(a_i^{\dagger})] = -mO_{n,m-1}(i)(nO_{n-1,m}(i))$$

(22)
A similar equation with $a_i$ and $a_i^\dagger$ replaced by the Lax operators, called as a ‘generalised Lax equation’ was derived by Ujino and Wadati after a lengthy algebra, and this formed the basis for their proof of $W_\infty$ algebra in this model. We show that, in contrast, this can be arrived with great ease, in the exchange operator formulation.

First note that the symmetric combination can be generated by the generating function

$$D(i) \equiv \alpha a_i + \beta a_i^\dagger$$

by defining $O_{nm}$ to be

$$O_{nm} = \left. \frac{1}{(n+m)!} \partial_\alpha^n \partial_\beta^m \sum_{j=1}^N D^{n+m}(j) \right|_{\alpha=\beta=0}$$

As an example,

$$O_{23} = \left. \frac{1}{5!} \partial_\alpha^2 \partial_\beta^3 \sum_{j=1}^N (\alpha a_j + \beta a_j^\dagger)^5 \right|_{\alpha=\beta=0}$$

Due to the condition $\alpha = \beta = 0$, only those terms with coefficient $\alpha^2 \beta^3$ contribute,

$$O_{23} = \frac{2!3!}{5!} \sum_{i=1}^N \left( a_i^2 a_i^3 + \cdots + a_i^3 a_i^2 \right)$$

Using (9) and (10), with (23) inserted in (24), we get

$$= \left. \frac{1}{(n+m)!} \partial_\alpha^n \partial_\beta^m \sum_{j=1}^N D^{n+m}(j), a_i \right|_{\alpha=\beta=0}$$

$$= \left. \frac{1}{(n+m)!} \partial_\alpha^n \partial_\beta^m \sum_{j=1}^N \sum_{l=0}^{n+m-1} D^l(j) [D(j), a_i] D^{n+m-l-1}(j) \right|_{\alpha=\beta=0}$$

$$= \frac{1}{(n+m)!} \partial_\alpha^n \partial_\beta^m \sum_{l=0}^{n+m-1} \left[ D^l(i) \left( -\beta + \lambda \beta \sum_{j \neq i} M_{ij} \right) - D^l(j) \beta \lambda M_{ij} D^{n+m-l-1}(j) \right]_{\alpha=\beta=0}$$

$$= \frac{1}{(n+m)!} \partial_\alpha^n \partial_\beta^m \left[ -\beta(n+m)D^{n+m-1}(i) - \lambda \sum_{j \neq i} \sum_{l=0}^{n+m-1} D^l(i) D^{n+m-l-1}(j) - D^l(j) D^{n+m-l-1}(i) \right]_{\alpha=\beta=0}$$

$$= \left. \frac{1}{(n+m)!} \partial_\alpha^n \partial_\beta^m \sum_{j \neq i} \sum_{l=0}^{n+m-1} D^l(i) D^{n+m-l-1}(j) - D^l(j) D^{n+m-l-1}(i) \right]_{\alpha=\beta=0}$$
Note that the $\lambda$ dependent terms can be expressed as

$$\lambda \sum_{j \neq i}^{n+m-1} \sum_{l=0}^{n-1} [D^l(i), D(j)^{n+m-l-1}]$$

which vanishes on using

$$[D(i), D(j)] = 0$$

Thus we get

$$[O_{nm}, a_i] = -\frac{1}{(n+m-1)!} \beta^n \alpha^m (\alpha^n \beta^m) \left[ a_i^n a_i^{m-1} + \cdots + a_i^{m-1} a_i^n \right]$$

$$= -mO_{n,m-1}(i) \quad (29)$$

Similarly, it can be shown that

$$[O_{nm}, a_i^\dagger] = nO_{n-1,m}(i)$$

Similar relation with $a_i, a_i^\dagger$ replaced by the Lax operators, was obtained by Ujino and Wadati [9]. In contrast to their approach, here the similar result has been obtained with great ease.

$$W_n^{(s)} \equiv O_{s-n-1, s+n-1}$$

$$[W_n^{(s)}, W_m^{(t)}] = \left[ W_n^{(s)}, \frac{(s-n-1)! (s+n-1)!}{(2s)!} \sum_{i=1}^{N} a_i^{s-n-1} a_i^{s+n-1} + \cdots \right]$$

Using (22),

$$[W_n^{(s)}, W_m^{(t)}] = 2(n(t-1) - m(s-1))W_n^{(s+t-2)} + \cdots \quad (30)$$

This defines the $W_\infty$ algebra. The dotted terms, are obtained when the commutation relation (9) and (10) are used to express the leading term in (30) as $W_\infty$ generator with that structure constant. In general, there will be terms (apart from linear lower order $\lambda$ independent terms) which are non-diagonal in $i$ and $j$. These, in general, will be $\lambda$ dependent, with $M_{ij}$ relating the $i$ and $j$ indices, and the summation over all $i, j$ with $i \neq j$. Explicitly we will have a generic form,

$$\lambda^p \sum_i (a_i^{n_i} a_i^{m_i}) \sum_{j \neq i} (a_j^{n_j})^{m_j} M_{ij} \quad (31)$$
with \( p \geq 1, n_i, m_i(i = 1, 2) \geq 0 \). Since the equivalence of the Hamiltonian (4) to the usual Calegero-Sutherland model is valid only on the physical states, obeying the condition (6), on a physical states, \( M_{ij} \) drops out (This some times leads to \( N \) dependence). Then in the terms \( \sum_{j \neq i} a_j^{1n_2}a_j^{m_2} \), replacing it with

\[
\sum_{j=1}^{N} \left( a_j^{1n_2}a_j^{m_2} - (a_i^{1})^{n_2}(a_i^{m})^{m_2} \right),
\]

the former appears to be non-linear in the generator (after symmetrization). The latter is diagonal in the particle index. Thus in general a \( \lambda \)-dependent non-linear algebra results. But the linear term also can have \( \lambda \) dependent coefficient. Even in the QISM [9], such non-linear terms in the \( W \) algebra are, in general, possible, although, it was not explicitly stated in their work.

As an example, \([W_{5/2}^{1/2}, W_{5/2}^{5/2} - 1/2]\) is calculated, using the Eqns. (9,10) and (29) in the above basis,

\[
[W_{1/2}^{5/2}, W_{5/2}^{1/2} - 1/2] = -3W_o^3 - \frac{1}{2}\lambda^2 \sum_{ij} M_{ij}^2 + \frac{1}{2} \sum_{i} [a_i, a_i^\dagger] \\
= -3W_o^3 - \frac{1}{2}\lambda^2 N(N - 1) + \frac{1}{2}(N + \lambda \sum_{ij} M_{ij})
\]

(32)

Acting on the physical states due to (9), the last give \( N(N - 1) \)

\[
= -3W_o^3 + \frac{1}{2}N + \frac{\lambda(1 - \lambda)}{2} N(N - 1)
\]

Since

\[
\sum_{i=1}^{N} O_{oo} = W_{oo}^{1} = N
\]

we get

\[
[W_{5/2}^{1/2}, W_{5/2}^{5/2} - 1/2] = -3W_o^3 + \frac{1}{2}W_o^1 + \frac{\lambda(1 - \lambda)}{2} W_o^2(W_o - 1)
\]

(33)

Thus there are nonlinear terms also.

Similarly another example is given in the basis, generated by \( a_i \), and \( A_i \), discussed earlier. With the definition of \( W_{nm} = \sum_{i=1}^{N} a_i^{n}A_i^{m} \) consider the commutator \([W_{11}, W_{12}]\). After straightforward algebra, using (9,10) and (18), we get
\[ [W_{11}, W_{12}] = W_{22} - W_{21} - 2\lambda \sum_{ij} a_i^2 A_j M_{ij} \]

\[ + \lambda \sum_{ij} a_j^2 A_j M_{ij} + \lambda^2 \sum_{i \neq j \neq k} a_i^2 A_i M_{ik} M_{ij} - \lambda^2 \sum_{i \neq j \neq k} a_i^2 A_j M_{kj} M_{ij} \]

Using the physical state condition (2.14a), terms with \( \lambda^2 \) as coefficient cancels, and the rest are given by,

\[ = W_{22} - W_{21} - 2\lambda \sum_{ij} a_i^2 A_j + \lambda (N - 1) \sum_{ij} a_j^2 A_j \]  

(34)

In the third term which is non-diagonal in \( i \) and \( j \), express it as

\[ = 2\lambda \sum_i a_i^2 \left( \sum_{j \neq i} A_j \right) \]

\[ = 2\lambda \sum_i a_i^2 \left( \sum_j A_j - A_i \right) \]

\[ = 2\lambda (W_{20} W_{01} - W_{21}) \]

Using \( N = W_{00} \), the final commutator is

\[ [W_{11}, W_{12}] = W_{22} - W_{21} - 2\lambda (W_{20} W_{01} - W_{21}) + \lambda (W_{00} W_{21} - W_{21}) \]

\[ = W_{22} - W_{21} - 2\lambda W_{20} W_{01} + \lambda W_{00} W_{21} + \lambda W_{21} \]  

(35)

Thus, this also displays non-linear terms in the algebra.

**Discussion:**

In this paper, we have studied the symmetry algebra in the quantum Calegero-Sutherland model, using the exchange operator formalism. In this, operators \( a_i, a_i^\dagger \) are introduced (8), which obey algebra like that of oscillator (11). Since \( W \) algebra generators can be expressed in terms of ordinary (\( \lambda \to 0 \) case of (9)), oscillator we ask about the \( W_\infty \) algebra, involving the modified oscillator (7), obeying the relation (9). We showed that the sub-algebras of \( W_\infty \) are obeyed, as an operator relation. We also gave a simpler proof, compared with Ref.(9), for \( W \) algebra in this model. But in contrast to them, existence of terms non-linear in the generator, in the commutation algebra is pointed out, when the physical state condition is (6) imposed. The non-linear term are always \( \lambda \) dependent. It should be interesting to
see if the $W$ algebra present here is isomorphic to the non-linear $\hat{W}_\infty$ algebra proposed by Yu and Wu [17]. The latter is also centreless and is a two-parameter deformation of $W_\infty$. One parameter signifies the non-leading terms and the other non-linear terms. The Calegero model has two parameters $\lambda$ and $\bar{h}$ (when the latter is restored), but appearing in the combination $\lambda \bar{h}$. It remains to be seen, if by suitable scaling and taking appropriate linear combination of its generators, a connection between these two algebras can be made.

Such a connection, will be of interest, as the $\hat{W}_\infty$ is known to provide the second Hamiltonian structure to $KP$ hierarchy [17]. Such a connection should also be useful to prove integrability of the continuum Calegero-Sutherland model [18]. This may also shed light on the relationship between Calegero-Sutherland model and two-dimensional physics. Apart from these, if should be possible to extend the results of this work to spin generalization of Calegero model, and to lattice integrable systems [19].

**Note added:** When this work was completed, we received the ref. [20], which also points out the non-linear terms in $W$-algebra in the Calegero model. We thank Ujino, for correspondence and for sending us the ref. (20).

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