Isospin odd $\pi K$ scattering length

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Abstract

We make use of the chiral two–loop representation of the $\pi K$ scattering amplitude [J. Bijnens, P. Dhonte and P. Talavera, JHEP 0405 (2004) 036] to investigate the isospin odd scattering length at next-to-next-to-leading order in the SU(3) expansion. This scattering length is protected against contributions of $m_s$ in the chiral expansion, in the sense that the corrections to the current algebra result are of order $M_\pi^2$. In view of the planned lifetime measurement on $\pi K$ atoms at CERN it is important to understand the size of these corrections.

Key words: Chiral symmetries, Meson-meson interactions
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1 Introduction

In the sixties and seventies a set of experiments was performed on $\pi K$ scattering [1]. To obtain predictions for the low–energy parameters, the measured $\pi K$ phases had to be extrapolated using dispersion relations and crossing symmetry [2], since the region of interest is not directly accessible by scattering experiments. The most precise values for the $\pi K$ scattering lengths were obtained only recently from an analysis of Roy-Steiner equations [3,4]. Alternatively, particular combinations of $\pi K$ scattering lengths may be extracted from experiments on $\pi K$ atoms [5,6,7]. The $\pi K$ atom decays due to the strong interactions into $\pi^0 K^0$ and a lifetime measurement will allow one to determine the isospin odd S-wave $\pi K$ scattering length $a_0^- = 1/3(a_0^{1/2} - a_0^{3/2})$. Such a measurement is planned at CERN [8]. Particularly interesting about...
the isospin odd $\pi K$ scattering length is that there exists a low–energy theorem due to Roessl [9]. Based on SU(2) chiral perturbation theory (CHPT) [9,10,11,12], where the strange quark mass is treated as a heavy partner, it is valid to all orders in powers of $m_s$. It states that Weinberg’s current algebra result [13,14] receives corrections of order $M^2$ only,

$$a_0^\pi = \frac{M_\pi M_K}{8\pi F_\pi^2(M_\pi + M_K)} \left\{ 1 + O(M^2) \right\}.$$ (1)

Here $M_\pi$, $M_K$ and $F_\pi$ denote the physical meson masses and the physical pion decay constant. In view of this low–energy theorem, one would expect higher order corrections to the scattering length to be relatively small. These days, the $\pi K$ scattering amplitude is available at next-to-next-to-leading order [15,16,17,18] in SU(3) CHPT [19]. The one–loop corrections [15,16,17] to $a_0^\pi$ turn out as expected, they change the current algebra value at the 11% percent level. Surprisingly, for the two–loop corrections this seems not to be the case. According to the numerical study performed in Ref. [18], the scattering length $a_0^\pi$ receives at order $p^6$ a 14% correction. The aim of the present article is to understand the nature of these rather substantial contributions at two–loop order. Other recent work on $\pi K$ scattering makes use of resonance chiral Lagrangian predictions [20] together with resummations [21]. There were also earlier attempts at unitarisation of current algebra for this process, see Ref. [22] and references therein.

We use the chiral two–loop representation for the $\pi K$ amplitude [18] to investigate the order $p^6$ corrections to $a_0^\pi$. In Section 2, we extract the contributions from the low–energy constants and determine the double chiral logs as well as the log×$L^r_i$ terms by means of the renormalization group equations for the renormalized coupling constants [23]. Further, we specify the 1-loop ×$L^r_i$ terms in an expansion in powers of $M_\pi/M_K$. The numerical analysis is carried out in Section 3 and the results for the partial two–loop contributions are collected in Table 2.

## 2 ‘Low cost’ terms at two–loop order

The SU(3) chiral expansion of the isospin odd $\pi K$ scattering length looks as follows

$$a_0^\pi = \frac{M_\pi M_K}{8\pi F_\pi^2(M_\pi + M_K)} \left\{ 1 + \delta^{(2)} + \delta^{(4)} + O(p^6) \right\},$$ (2)

where $O(p^6) = \{ \hat{m}^3, \hat{m}^2 m_s, \hat{m} m_s^2 \}$. The scattering length is expressed in terms of the physical meson masses $M_\pi$ and $M_K$ and the physical pion decay constant $F_\pi$ [24]. The next-to-leading order contribution $\delta^{(2)}$ [16,17] depends on one
single low–energy constant \( L_0^\pi \) [19] only,

\[
\delta^{(2)} = \frac{M_\pi^2}{32\pi^2 F_\pi^2} \left[ 256\pi^2 L_5^\pi - 3 \ln \frac{M_K^2}{\mu^2} - \frac{3(2M_K^2 - M_\pi^2)}{M_K^2 - M_\pi^2} \ln \frac{M_\pi^2}{M_K^2} \right. \\
\left. - \frac{4M_K^2 - M_\pi^2}{2(M_K^2 - M_\pi^2)} \ln \frac{4M_K^2 - M_\pi^2}{3M_\pi^2} + \frac{M_\pi M_K}{3F_\pi^2} \right] \times \left[ J(s_\text{thr}, M_K^2, \frac{1}{3}(4M_K^2 - M_\pi^2)) - J(u_\text{thr}, M_K^2, \frac{1}{3}(4M_K^2 - M_\pi^2)) \right],
\]

(3)

where \( s_\text{thr} = (M_\pi + M_K)^2 \), \( u_\text{thr} = (M_K - M_\pi)^2 \) and the function \( J \) is defined as follows

\[
J(p^2, m_1^2, m_2^2) = J(p^2, m_1^2, m_2^2) - J(0, m_1^2, m_2^2), \\
J(p^2, m_1^2, m_2^2) = -i \int \frac{d^dq}{(2\pi)^d} (m_1^2 - q^2)^{-1} (m_2^2 - (p + q)^2)^{-1}.
\]

(4)

Note that at the order considered it makes a difference whether we represent \( \delta^{(2)} \) as a function of the physical pion, kaon and \( \eta \) masses or express one of them through the other two. In Eq. (3), we choose to describe \( \delta^{(2)} \) in terms of the physical pion and kaon mass only, because this ensures that both \( \delta^{(2)} \) and \( \delta^{(4)} \) are independently scale invariant.

The two–loop order correction can be decomposed as

\[
\delta^{(4)} = \delta_{L_i = C_i = 0}^{(4)} + \delta_{1–\text{loop}}^{(4)} + \delta_{L_i L_j}^{(4)} + \delta_{C_i}^{(4)}.
\]

(5)

The first term contains the two–loop functions, the second one–loop functions with insertions of \( \mathcal{O}(p^4) \) coupling constants and the last two terms consist of counter term contributions. Some of the two–loop functions in \( \delta_{L_i = C_i = 0}^{(4)} \) are very demanding to analyze analytically. For the moment, we thus restrict ourselves to the chiral double logs,

\[
\delta_{L_i = C_i = 0}^{(4)} = \delta_{\text{log}^2}^{(4)} + \delta_{\text{rem}}^{(4)}.
\]

(6)

and neglect the remainder \( \delta_{\text{rem}}^{(4)} \) which is given numerically in Table 2. In a first step, we extract the contributions from the \( p^6 \) low–energy constants \( C_i^\pi \) [23,25] from the representation of the \( \pi K \) scattering amplitude in Ref. [18],

\[
\delta_{C_i}^{(4)} = \frac{16M_\pi^2}{F_\pi^2} \left[ -2M_K^2 \left( C_1^\pi - 2C_3^\pi - 4C_4^\pi + C_6^\pi \right) + 2C_2^\pi \\
-2C_5^\pi - C_6^\pi + 2C_9^\pi \right] + M_\pi^2 \left( C_{15}^\pi + 2C_{17}^\pi \right).
\]

(7)

\footnotesize
\textsuperscript{1} This will generate a correction proportional to \( \Delta_{\text{GMO}} = (4M_K^2 - M_\pi^2 - 3M_\eta^2)/(M_\eta^2 - M_\pi^2) \) [19] which contributes to \( \delta^{(4)} \).
\normalsize
as well as products of two $p^4$ constants ($L_i^r \times L_j^r$),

$$\delta_{L_i^r L_j^r}^{(4)} = \frac{64 M^2_{\pi} L^r_5}{F^2_{\pi}} \left\{ M^2_{\pi} [2(L^r_4 - 2L^r_6) - L^r_5] + M^2_{\pi} [L^r_4 - 2L^r_6 + 2(L^r_5 - L^r_8)] \right\}. \quad (8)$$

In order to determine the chiral double logs and the log $\times L_i^r$ terms, we consider the renormalization group equations of the renormalized order $p^4$ and $p^6$ low–energy constants [23],

$$\mu \frac{dL_i^r(\mu)}{d\mu} = -\frac{1}{(4\pi)^2} \Gamma_i, \quad \mu \frac{dC_i^r(\mu)}{d\mu} = \frac{1}{(4\pi)^2} \left[ 2\Gamma_i^{(1)} + \Gamma_i^{(L)}(\mu) \right]. \quad (9)$$

The coefficients $\Gamma_i^{(L)}$ are linear combinations of $p^4$ constants which satisfy the following differential equations,

$$\mu \frac{d\Gamma_i^{(L)}(\mu)}{d\mu} = -\frac{\Gamma_i^{(2)}}{8\pi^2}, \quad (10)$$

in accordance with Weinberg’s consistency conditions [10]. The coefficients $\Gamma_i^{(1)}$, $\Gamma_i^{(2)}$ and $\Gamma_i^{(L)}(\mu)$ are listed in Table II of Ref. [23]. The solutions of the renormalization group equations read [26]

$$L_i^r(\mu) = L_i^r(\mu_0) - \frac{\Gamma_i}{2} L(\mu/\mu_0),$$

$$C_i^r(\mu) = C_i^r(\mu_0) - \frac{1}{4} \Gamma_i^{(2)} L(\mu/\mu_0)^2 + \frac{1}{2} \left[ 2\Gamma_i^{(1)} + \Gamma_i^{(L)}(\mu_0) \right] L(\mu/\mu_0), \quad (11)$$

with the chiral logarithm

$$L(\mu/\mu_0) = \frac{1}{(4\pi)^2} \ln \frac{\mu^2}{\mu_0^2}. \quad (12)$$

As a two–loop order quantity $\delta^{(4)}$ consists of

$$\delta^{(4)} = \hat{a}(\mu) + \sum_i b_i C_i^r(\mu) + \sum_{i,j} b_{ij} L_i^r(\mu) L_j^r(\mu), \quad (13)$$

where $\hat{a}(\mu)$ is scale dependent and contains one–loop functions with insertions of $p^4$ constants as well as two–loop functions. In order to extract the double log and log $\times L_i^r$ contributions from $\hat{a}(\mu)$, we insert the solutions for the renormalized coupling constants into the latter equation,

$$\delta^{(4)} = \hat{a}(\mu_0) + \sum_i b_i C_i^r(\mu_0) + \sum_{i,j} b_{ij} L_i^r(\mu_0) L_j^r(\mu_0),$$

$$\hat{a}(\mu_0) = \hat{a}(\mu) - \frac{1}{4} L(\mu/\mu_0)^2 \left[ b_i \Gamma_i^{(2)} - b_{ij} \Gamma_i \Gamma_j \right] + \frac{1}{2} L(\mu/\mu_0) \left[ b_i \left( 2\Gamma_i^{(1)} + \Gamma_i^{(L)}(\mu_0) \right) - 2b_{ij} \Gamma_i \Gamma_j \right]. \quad (14)$$
Now, the scale dependence of $\hat{a}(\mu_0)$ becomes apparent and we may read off the wanted $\log^2$ and $\log \times L_i^r$ terms. The solutions of the renormalization group equations thus allow us to determine the double log and $\log \times L_i^r$ contributions from Eqs. (7) and (8).

The double chiral logs ($\log^2$) amount to

$$\delta^{(4)}_{\log^2} = \frac{M^2_\pi}{F^2_\pi} \left[ \frac{37}{8} M^2_K + \frac{59}{24} \right] L(M_\chi/\mu)^2,$$

while the single logarithms times $p^4$ constants ($\log \times L_i^r$) yield

$$\delta^{(4)}_{\log L_i} = -\frac{2 M^2_\pi}{3 F^2_\pi} \left\{ M^2_K \left[ 84 L_1^r + 114 L_2^r + 53 L_3 - 96 L_4^r - 28 L_5^r \right. \right.$$  
$$\left. + 48 (3 L_6^r + L_7 + 2 L_8^r) - M^2_\pi \left[ 12 L_1^r + 30 L_2^r + 19 L_3 - 64 L_5^r + 24 (2 L_7 + L_8^r) \right] \right\} L(M_\chi/\mu).$$

Here $M_\chi$ stands for a characteristic meson mass.

In the remaining part of this section, we investigate Roessl’s low–energy theorem [9] at next-to-next-to-leading order in SU(3) CHPT. More precisely, we specify the order $M^2_\pi$ and order $M^4_\pi$ corrections to Eq. (1). To approach the SU(2) chiral expansion, we regard the kaon mass as heavy and expand $a_0^-$ in powers of $M_\pi/M_K$,

$$a_0^- = \frac{M_\pi M_K}{8 \pi F^2_\pi (M_\pi + M_K)} \left\{ 1 + M^2_\pi c_2 + M^4_\pi c_4 + O(M^6_\pi) \right\}. \quad (17)$$

Again, the quantities $M_\pi$, $M_K$ and $F_\pi$ stand for the physical masses and the physical pion decay constant [24]. At next-to-leading order in SU(3) CHPT, the coefficient $c_2$ depends on $L_5^r$ [16,17],

$$c_2 \left|_{1\text{-loop}} = \frac{1}{F^2_\pi} \left\{ 8 L_5^r - \frac{1}{32 \pi^2} \left[ 3 \ln \frac{M^2_K}{\mu^2} + 4 \ln \frac{M^2_\pi}{M^2_K} \right. \right.$$  
$$\left. + \frac{1}{144 \pi^2} \left\{ -12 + 10 \sqrt{2} \arctan \sqrt{2} - 7 \ln \frac{4}{3} \right\} \right\}, \quad (18)$$

while the one–loop contributions to $c_4$ do not contain any low–energy constants and can safely be neglected numerically. At next-to-next-to-leading order in the chiral SU(3) expansion, the contributions from counter terms, double chiral logs and $\log \times L_i^r$ terms to the coefficients $c_2$ and $c_4$ are specified in Eqs. (7), (8), (15) and (16). In addition, we list the expansion of the one–loop functions
with insertions of $p^4$ couplings in powers of $M_\pi/M_K$. We have

$$c_2 \mid_{\text{1-loop}L_1} = \frac{M_K^2}{12\pi^2 F_\pi^4} \left\{ \frac{1}{2} [84L_1^r + 114L_2^r + 53L_3 - 96L_4^r - 28L_5^r] 
+ 48 (3L_6^r + L_7 + 2L_8)] \ln \frac{M_K^2}{\mu^2} 
- \frac{4}{27} L_3 \left[ 56\sqrt{2} \arctan\sqrt{2} - 5 \ln \frac{4}{3} \right] \right. $$

$$- \frac{1}{3} [L_5^r - 6(2L_7 + L_8)] \left[ 13\sqrt{2} \arctan\sqrt{2} + 2 \ln \frac{4}{3} \right] + 93L_1^r 
+ \frac{189}{2} L_2^r + \frac{2045}{36} L_3 - 16 [L_5^r + 6(L_4^r - L_6^r + L_7)] \right\},$$

(19)

and

$$c_4 \mid_{\text{1-loop}L_1} = \frac{1}{8\pi^2 F_\pi^4} \left\{ \frac{1}{3} [12L_1^r + 30L_2^r + 19L_3 - 64L_5^r] 
+ 24(2L_7 + L_8)] \ln \frac{M_\pi^2}{M_K^2} + 4[8L_1^r + 12L_2^r + 6L_3 - 8L_4^r] 
- 9L_5^r + 6(2L_6^r + L_8)] \ln \frac{M_\pi^2}{M_K^2} - \sqrt{2} \left[ \frac{1840}{81} L_3 - \frac{1415}{18} L_5^r \right] 
+ 45(2L_7 + L_8)] \arctan\sqrt{2} + \frac{4}{9} \left[ \frac{2}{9} L_3 - 17L_5^r \right] \right. $$

$$+ 18(2L_7 + L_8)] \ln \frac{4}{3} - \frac{1}{4} \left[ 8L_1^r + 4L_2^r - \frac{410}{27} L_3 
+ \frac{323}{6} L_5^r - 67(2L_7 + L_8) \right] \right\},$$

(20)

where we have checked that the log-$L_1^r$ terms agree with Eq. (16). Here both the contributions to $M_\pi^2 c_2$ and $M_\pi^4 c_4$ are numerically sizeable, see Table 2.
In the following, we present the numerical results for the partial $p^6$ corrections to $\delta^{(4)}$. The pion and kaon mass in the isospin symmetry limit are identified with their charged masses $M_\pi \doteq M_{\pi^+}$ and $M_K \doteq M_{K^+}$. To be consistent with the numerical analysis performed in Ref. [18], we use for the pion decay constant \(^2 F_\pi = 92.4\) MeV. In Table 1, we list the various numerical results for $a_\pi$ available in the literature. The first row contains the current algebra value, the next number is the SU(2) prediction at next-to-leading order [9], row three and four display the order $p^4$ [16] and order $p^6$ [18] SU(3) predictions and the last value is based on a phenomenological analysis from Roy-Steiner equations [4]. As can be read off, the SU(3) prediction at order $p^6$ is in good agreement with the Roy-Steiner value. The SU(3) chiral expansion of the scattering length $a_\pi$ looks as follows

$$
\frac{8\pi F_\pi^2 (M_\pi + M_K)}{M_K M_\pi} a_\pi = 1 + \delta^{(2)} + \delta^{(4)} + \cdots
= 1 + 0.11 + 0.14 + \cdots
$$

(21)

The one–loop contribution $\delta^{(2)}$ changes the current algebra result at the 11% level, while the two–loop contributions $\delta^{(4)}$ amount to a 14% correction. The aim was to understand this rather large order $p^6$ correction and our insights are collected in Table 2 which contains a splitting up of the various contributions at two–loop order.

For the low–energy constants $L_i^r$ at the scale $\mu = 770\) MeV ($M_\rho$), we use fit

\(^2\) Recently, a new value was obtained $F_\pi = 92.2 \pm 0.2\) MeV [27].
\[ L_i = C_i = 0 \quad 1\text{-loop } L_i \quad L_i L_j \quad C_i \]

\[
\Delta \delta^{(4)}_a = -0.03 \quad 0.02 \quad -0.01 \quad 0.02
\]

Table 3:
Variations of the partial \( p^6 \) contributions to \( \delta^{(4)} \) for \( M_\eta \leq \mu \leq 770 \text{ MeV} \) \( (M_\rho) \). More precisely, we display the difference \( \Delta \delta^{(4)}_a = \delta^{(4)}_a \mid_{\mu=\eta} - \delta^{(4)}_a \mid_{\mu=\rho} \). For the notation, see Table 2.

10 of Ref. [28]. The double chiral logs are evaluated for a characteristic meson mass\(^3\) \( M_\chi = M_K \) and the size of the remainder \( \delta^{(4)}_{\text{rem}} \) is estimated by the use of Eq. (6). Row two and three of Table 2 contain the partial order \( p^6 \) corrections to the coefficients \( c_2 \) and \( c_4 \), respectively. Note that for the double chiral logs as well as for the products of \( p^4 \) constants their contribution to \( c_4 \) can be neglected while for the one-loop functions with insertions of \( L_i \)'s, both \( M_\pi^2 c_2 \) and \( M_\pi^4 c_4 \) are numerically sizeable. The enhancement of the coefficient \( c_4 \) is mainly due the contributions proportional to \( \ln M_\pi/M_K \), see Eq. (20).

As one can read off from Table 2, more than half of the contributions to \( \delta^{(4)} = 0.14 \) stem from the resonance estimate for the \( p^6 \) constants which includes effects of the lowest-lying vector and scalar resonances [18]. We checked that with this procedure the meson resonance exchange contributions to \( C_{15}' \) and \( C_{17}' \) vanish which implies that \( c_4 \mid_{C_1} \) is equal to zero. Further, for the combination of \( p^6 \) constants occurring in \( c_2 \mid_{C_1} \), the contributions from scalar resonances do not play a dominant role: They amount to 0.03 of the 0.08 generated by the \( C_i' \)'s in total. It would be instructive to see whether these features persist in an improved estimate for the \( p^6 \) constants which respects the constraints that follow by imposing the proper asymptotic behaviour for massless QCD [29].

The splitting of the order \( p^6 \) contributions in Table 2 is scale dependent. Table 3 displays the scale dependence of the various contributions to \( \delta^{(4)} \). The values for the 1-loop \( \times L_i \), \( L_i \times L_j \) and \( C_i' \) terms at the scales \( \mu = 770 \) MeV and \( \mu = M_\eta \) allow us to read off the scale dependence of the pure loop contributions \( \delta^{(4)}_{L_i=C_1=0} \).

Finally, we sum up the various SU(3) one- and two-loop contributions to \( c_2 \) and \( c_4 \) and get for the expansion of \( a_0^{-} \) in powers of \( M_\pi/M_K \),

\[
\frac{8\pi F_\pi^2(M_\pi + M_K)}{M_\pi M_K} a_0^{-} = 1 + M_\pi^2 c_2 + M_\pi^4 c_4 + \cdots
\]

\[= 1 + 0.2 + 0.01 + \delta^{(4)}_{\text{rem}} + \cdots \quad (22)\]

\(^3\) The choice \( M_\chi = \sqrt{M_\pi M_K} \) leads to an unnatural large number for the double logs \( \delta^{(4)}_{\log^2} = 0.058 \), to be compared with the full pure loop corrections \( \delta^{(4)}_{L_i=C_1=0} = 0.05 \) [18]. For \( M_\chi = M_\pi \) the value becomes even more unreasonable.
Note that this decomposition is valid up to the contribution of $\delta_{\text{rem}}^{(4)} = 0.04$ only. Compared to the chiral SU(3) expansion in Eq. (21), the series in $M_\pi/M_K$ converges much more rapidly. The correction $M_\pi^2c_2$ consists of

$$M_\pi^2c_2 = \frac{M_\pi^2}{(4\pi F_\pi)^2} \left[ \alpha + \frac{M_K^2}{(4\pi F_\pi)^2} \beta + \cdots \right] ,$$

(23)

where the coefficients $\alpha$ and $\beta$ contain the one–loop and two–loop contributions, respectively. Numerically, we have $\alpha = 7.6$, where the dominant part stems from the term proportional to $\ln M_\pi/M_K$ in Eq. (18). The contributions from double logs, 1-loop×$L_i$ terms and $p^6$ constants to $\beta$ are listed in Table 2. Here the bulk part comes from the resonance estimate for the $p^6$ constants [18].

4 Conclusions

In the present work, we used the chiral two–loop representation for the $\pi K$ amplitude available in the literature [18] to investigate the isospin odd S-wave scattering length $a_0^-$. This scattering length differs from other low–energy parameters in $\pi K$ scattering in the sense that contributions of $m_s$ in the chiral expansion are suppressed by powers of $\hat{m}$. Based on SU(2) CHPT [9], there exists a low–energy theorem (1) which states that the current algebra result for $a_0^-$ receives corrections of order $M_\pi^2$ only. It was therefore expected that the one–loop result [15,16,17] in SU(3) CHPT represents a decent estimate for the scattering length. However, the dispersive analysis from Roy-Steiner equations [4] and the chiral two–loop calculation [18] are not in agreement with this expectation. In fact, the numerical analysis performed in Ref. [18] showed that the two–loop order corrections to $a_0^-$ are of the same order of magnitude as the one–loop contributions.

In order to understand this rather substantial next-to-next-to-leading order correction, we determined analytically the contributions containing $p^6$ constants (7), products of two $p^4$ constants (8), double chiral logs (15) and single logarithms times $p^4$ constants (16). We further expanded the one–loop functions with insertions of $p^4$ constants in powers of $M_\pi/M_K$, see Eqs. (19) and (20). The expansion of the pure two–loop functions in powers of $M_\pi/M_K$ was beyond the scope of this work. The numerical values of the partial $p^6$ contributions are collected in Table 2.

In the remaining part of this work, we investigated the low–energy theorem for $a_0^-$ at next-to-next-to-leading order in the SU(3) expansion. While it is true that the corrections are of order $M_\pi^2$, the chiral expansion of the accompanying coefficient proceeds in powers of $M_K$ and is not protected against
sizeable contributions. At two–loop accuracy in the SU(3) expansion, the order $M_\pi^2$ correction roughly amounts to about 20%, see Eq. (22). Note that this number depends on the resonance estimate [18] for the $p^6$ constants. If we compare this result with Roessl’s value [9], the SU(2) prediction for the scattering length $a_0^-$ seems to be underestimated. At first surprisingly, we have to keep in mind that the numerical estimates for the low–energy constants in SU(2) CHPT were obtained through matching the scattering amplitude with the corresponding SU(3) CHPT result at one–loop order. It would be very interesting to estimate these low–energy constants using a resonance saturation approach in the context of SU(2) CHPT with strangeness number 1.

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