Magnitudes of $q\bar{q}$ jet and $q\bar{q}g$-jet at $Z^0$-pole in OPAL BEC
— Application of the second conventional formula with two-component (CFII) —

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Abstract

We investigate the contribution of 2-jet and 3-jet at $Z^0$-pole in OPAL Bose-Einstein correlation (BEC) by using the second conventional formula with two-component, as well as introducing random variables for the MINUIT method. It is found that the magnitude of the 3-jet is $1.29 \pm 0.27$ fm, which is about 34% higher than that of the 2-jet ($0.96 \pm 0.01$) fm. The fraction of 3-jet to all-jet is $0.18 \pm 0.03$, which is consistent with an empirical value by the OPAL collaboration.

1 Introduction

We are interested in a paper on BEC at $Z^0$-pole by the OPAL collaboration in 1991 [1], because we would like to know whether the data on BEC contains any information on QCD. In fact, we know that the multiplicity distribution (MD) contains the 2-jet and 3-jet events [2,3]. For our purpose, we have to adopt an extended theoretical formula for analyses of the BEC [4–8]. Before discussing them, however, we would like to mention some information about OPAL BEC.

In Fig. 1 the data on OPAL BEC at $Z^0$-pole are shown. They are analyzed by the following conventional formula, which is named the first conventional formula

$$CF_1 = [1.0 + \lambda E_{BE}(R, Q)] \cdot LRC,$$

where $E_{BE}$ is the exchange function between two identical pions and $Q = -\sqrt{-(p_1 - p_2)^2}$ is the magnitude of the momentum transition squared between them. Typically, $E_{BE}$ is given by the Gaussian distribution and/or the exponential function. The degree of coherence is expressed by $\lambda$. The following long-range correlation (LRC) is frequently used:

$$LRC_\delta = C(1 + \delta Q)$$

As far as the OPAL BEC at $Z^0$-pole, the OPAL collaboration used the following LRC

$$LRC_\delta(\varepsilon) = C(1 + \delta Q + \varepsilon Q^2)$$

| LRC  | $R$ (fm)(G) | $\lambda$ | $C$ | $\delta$ (GeV$^{-1}$) | $\varepsilon$ (GeV$^{-2}$) | $\chi^2$/ndf |
|------|-------------|----------|-----|----------------------|-------------------|--------------|
| $\delta$ | $1.12 \pm 0.03$ | $0.78 \pm 0.04$ | $0.73 \pm 0.00$ | $0.17 \pm 0.00$ | —               | $264/62$     |
| $\delta, \varepsilon$ | $0.95 \pm 0.02$ | $0.88 \pm 0.04$ | $0.63 \pm 0.01$ | $0.49 \pm 0.03$ | $-0.13 \pm 0.01$ | $114/61$     |
As is seen in Table 1 and Fig. 1, when \( LRC_{(\delta, \varepsilon)} \) (i.e., Eq. (3)) is utilized in the analysis, we obtain a better \( \chi^2 \) value than that using Eq. (2). This fact may suggest us that Eq. (3) reflects some physical meaning.

However, we proposed the following second conventional formula (named as \( CF_{II} \)) in analysis of CMS BEC at 0.9 and 7 TeV, because the multiplicity distributions (MD) contain the non-diffractive dissociation (ND), the single diffractive dissociation (SD) and the double diffractive dissociation (DD):

\[
CF_{II} = [(1.0 + \lambda_1 E_{BE_1}(R_1, Q) + \lambda_2 E_{BE_2}(R_2, Q)] \cdot LRC, \tag{4}
\]

where the second \( \lambda \) (\( \lambda_2 \)) and the second exchange function (\( E_{BE_2} \)) are introduced to describe the BEC data at the LHC. A detailed derivation and analysis with Eq. (2) (i.e., \( LRC_{(\delta)} \)) are presented in Refs. [6–8]. For our purposes, the analytic \( LRC_{(\delta, \varepsilon)} \) is also necessary in the \( CF_{II} \).

In this paper, for the \( e^+ e^- \) annihilation process [1, 10–12] we assign the contributions of the \( q\bar{q} \)-jet and \( qg \)-jet to the second and third terms in Eq. (4). See Fig. 2. That picture is taken from Ref. [12]. In our analysis, we assume that the 3-jet contribution includes the 4-jet effect. For the LRC we adopt Eq. (3) in the second and third sections.

Figure 1: Analysis of the OPAL BEC at the \( Z^0 \)-pole by Eq. (1) with Eqs. (2) and (3).

Figure 2: Jet-production in \( e^+ e^- \)-annihilation.
MINUIT method \cite{14}. In the third section, our results are presented. In the fourth section, we investigate the possibility of different LRC from Eq. (3). In the final section, concluding remarks are presented.

2 Introduction of random variables set for MINUIT method

Here, we analyze OPAL BEC data by means of Eqs. (4) and (3). Table 1 provides estimated values which are used by Eqs (1) and (3) in this analysis. Furthermore, we must assume the following inequalities for 2-jet \((R_1, \lambda_1)\) and 3-jet \((R_2, \lambda_2)\), taking into account of Fig. 2 and estimated values in Table 1.

\[
\begin{aligned}
R_1 < R_2 < 3 \text{ fm, and } \lambda_2 < \lambda_1 < 1.0, \\
\lambda_1 + \lambda_2 \leq 1.0, \text{ and } \chi^2 \leq 113.82.
\end{aligned}
\] (5)

Out method is shown in Fig. 3.

Accepting values by CF\(_1\)(G)\times LRC in Table 1

Preparing sets of random numbers for initial values \(R_1(G), R_2(E), \lambda_1, \text{ and } \lambda_2\) for application of MINUIT method.

\[N_{\text{trial}} = 0.7M\]

Applying inequalities and a cutoff: \(R_1 < R_2 < 3.0 \text{ fm, } \lambda_2 < \lambda_1 < 1.0, \lambda_2 + \lambda_1 \leq 1.0, \text{ and } \chi^2 < 111.\)

\[N_{\text{selected}} = 177\]

Make distribution of \(dN/d\chi^2\). Chose the ensemble with minimum \(\chi^2 < 109.\)

\[N_{\text{final}} = 34\]

Calculate averaged values of \(R_1, R_2, \lambda_1, \lambda_2, C, \delta, \text{ and } \varepsilon.\)

Figure 3: Flow chart for use of CF\(_{II}\)(G + E)\times LRC.

As seen in \(dN/d\chi^2\) in Fig. 4, the minimum \(\chi^2\) ensembles are located at \(\chi^2 = 107\) and 108. The total number of sets in those ensembles is 34. They are shown in Table 2.

3 Our analysis based on CF\(_{II}\)\times LRC

Our analysis of the OPAL BEC at \(Z^0\)-pole by means of averaged values in Table 2 is shown in Fig. 5. Indeed, \(\chi^2/\text{ndf} = 108/62\) is improved more than \(\chi^2/\text{ndf} = 114/61\) in Table 1.

However, there are discrepancies between data and theoretical values in the region, \(Q \leq 0.1 \text{ GeV. The data points are more than theoretical ones.}\)

To explain that phenomena we have to introduce other LRCs. This problem is investigated in the next section.

4 Possibility of different LRC

Before making a concrete calculation for the different LRCs, we review the statements made by OPAL collaboration in Ref. 11.
Figure 4: $dN/d\chi^2$ distribution after the selection for G+E scheme. Here we consider a combined ensemble with $\chi^2 = 107$ and 108, because of the small number 8 sets in the ensemble at $\chi^2 = 107$.

Figure 5: Analysis of the OPAL BEC at the $Z^0$-pole by Eq. (4) with Eq. (3). Average values in Table 2 are used.

Table 2: Results in ensembles with $\chi^2 < 109$, their mean values (mean) and standard deviations (S.D.). $X_{ran1,2,3,4}$ mean the random variables for $R_1$, $R_2$, $\lambda_1$, and $\lambda_2$, respectively. They are necessary at the initial time for the MINUIT method. The lines No.4~31 are skipped.

| No. | $R_1$ (G) | $R_2$ (E) | $\lambda_1$ | $\lambda_2$ | $C$ | $\delta$ (GeV$^{-1}$) | $\varepsilon$ (GeV$^{-2}$) | $X_{ran1}$ | $X_{ran2}$ | $X_{ran3}$ | $X_{ran4}$ | $\chi^2$ |
|-----|-----------|-----------|-------------|-------------|-----|---------------------|-------------------|------------|------------|------------|------------|---------|
| 1   | 0.976     | 1.289     | 0.821       | 0.168       | 0.624       | 0.505   | -0.130              | 0.27         | 1.15       | 0.97       | 0.67       | 108.5   |
| 2   | 0.954     | 1.128     | 0.781       | 0.210       | 0.611       | 0.540   | -0.139              | 0.59         | 2.59       | 0.60       | 0.78       | 108.0   |
| 3   | 0.972     | 1.014     | 0.851       | 0.145       | 0.611       | 0.543   | -0.142              | 1.39         | 0.04       | 0.68       | 0.39       | 108.9   |
| ... | ...       | ...       | ...         | ...         | ...        | ...     | ...                 | ...          | ...        | ...        | ...        | ...     |
| 32  | 0.974     | 1.087     | 0.852       | 0.140       | 0.615       | 0.533   | -0.140              | 0.53         | 0.14       | 0.29       | 0.29       | 108.8   |
| 33  | 0.988     | 1.065     | 0.808       | 0.181       | 0.617       | 0.522   | -0.134              | 0.21         | 2.10       | 0.51       | 0.86       | 108.9   |
| 34  | 0.973     | 1.178     | 0.814       | 0.161       | 0.624       | 0.502   | -0.129              | 0.16         | 2.81       | 0.71       | 0.81       | 108.8   |
| mean| 0.960     | 1.287     | 0.808       | 0.177       | 0.618       | 0.524   | -0.137              |              |            |            |            |         |
| S.D.| 0.013     | 0.269     | 0.026       | 0.027       | 0.007       | 0.021   | 0.007               |              |            |            |            |         |
Table 3: By making use of average values in Table 2, we estimate $\chi^2 = 108.0$ in Fig. 5. The error bars and the standard deviations are calculated by 34 sets.

|    | $R_1$ (fm) (G) | $R_2$ (fm) (E) | $\lambda_1$ | $\lambda_2$ | $\delta$ (GeV$^{-1}$) | $\varepsilon$ (GeV$^{-2}$) | $\chi^2$/ndf |
|----|----------------|----------------|-------------|-------------|----------------------|------------------------|-------------|
|    | 0.96 ± 0.01    | 1.29 ± 0.27    | 0.81 ± 0.03 | 0.18 ± 0.03 | 0.52 ± 0.02          | -0.14 ± 0.01           | 108.0/59    |

“For 15% of pion like-sign pions, at least one of the charged tracks arose from the decay chain $\eta' \to \pi^+\pi^-\eta$ followed by $\eta \to \pi^+\pi^-\pi^0$. The data points in the range $Q < 0.1$ GeV lie somewhat above the curve (i.e., the exchange function $E_{BE}$), probably because of pions from $\eta'$ and $\eta$ decays.”

For a new LRC, we require the condition $LRC = C$ at both $Q = 0$ and $\infty$. To explain that behavior in $Q < 0.1$ GeV in Fig. 5, we would like to adopt a fractional expression (see Ref. [9]). As a plausible solution, we propose the following expression for the new LRC.

$$ LRC_{\text{fraction}} = \frac{C}{1 + \alpha Q \exp(-\beta Q)} = \begin{cases} 
C & \text{at } Q \to 0 \\
\text{Plausible subtraction of contamination due to } \eta' \text{ and } \eta \\
\text{decays in } Q \leq 0.1 \text{ GeV} \\
\text{Reproduction of LRC à la OPAL in } 0.25 \leq Q < 2.0 \text{ GeV} \\
C & \text{at } Q \to \infty
\end{cases} \quad (6) $$

Concrete behaviors are seen in Fig. 6.

Figure 6: LRC of the OPAL BEC at the $Z^0$-pole by Eq. (4) with Eqs. (3) and (6). Numerical values are presented in Tables 3 and 5.

Before analysis of BEC by $CF_1 \times LRC_{\text{fraction}}$, the results by $CF_1 \times LRC_{\text{fraction}}$ are presented in Table 4.

Taking into account the analysis in Table 4, we combine Eq. (4) and Eq. (6) and analyze OPAL BEC at $Z^0$-pole in Fig. 7.

To analyze data on OPAL BEC at $Z^0$-pole by combining Eq. (4) and Eq. (6), the similar procedure shown in Fig. 3 is used. The inequalities are the same as Fig. 3. But the cutoff factor is $\chi^2 \leq 76.5$ in this
Table 4: Analysis of the OPAL BEC at the $Z^0$-pole by Eq. (1) with Eq. (6).

| $E_{BE}$ | $R$ (fm) | $\lambda$ | $C$ | $\alpha$ (GeV$^{-1}$) | $\beta$ (GeV$^{-1}$) | $\chi^2$/ndf |
|----------|----------|-----------|-----|------------------------|-----------------------|--------------|
| E        | 1.16 ± 0.20 | 1.00 ± 0.04 | 0.93 ± 0.00 | 3.60 ± 0.81 | 3.66 ± 0.23 | 76.7/61      |
| G        | 0.93 ± 0.04 | 0.52 ± 0.03 | 0.94 ± 0.00 | 2.24 ± 0.14 | 3.15 ± 0.11 | 91.3/61      |

The number of select $N_{select} = 300$ is calculated. They are shown in Table 5 and Fig. 7. In fact, there is a correlation among data and theoretical amounts.

![Analysis of the OPAL BEC at the $Z^0$-pole by Eq. (1) with Eq. (6).](image)

**Figure 7:** Analysis of the OPAL BEC at the $Z^0$-pole by Eq. (1) with Eq. (6).

Table 5: Analysis of the OPAL BEC at the $Z^0$-pole by Eq. (4) with Eq. (6). $C = 0.93$. The error bars and standard deviations are calculated by using $N_{\text{final}} = 100$ sets.

| $R_1$ (fm) (E) | $R_2$ (fm) (G) | $\lambda_1$ | $\lambda_2$ | $\alpha$ (GeV$^{-1}$) | $\beta$ (GeV$^{-1}$) | $\chi^2$/ndf |
|----------------|----------------|-----------|-----------|------------------------|-----------------------|--------------|
| 0.94 ± 0.07    | 1.19 ± 0.04    | 0.84 ± 0.04 | 0.14 ± 0.03 | 4.14 ± 0.28 | 3.75 ± 0.04 | 75.9/59      |

## 5 Concluding remarks

**C1)** In the CFII $\times$ LRC analysis of OPAL BEC at $Z^0$-pole by Eq. (1) and Eq. (3), we must introduce inequalities and cutoff in Fig. 3. In addition, when applying the MINUIT method to OPAL BEC, we must use the random variables expressed as $X_{\text{ran}1,2,3,4}$ for initial values.

**C2)** When we use Eq. (4) with (3) in analysis of OPAL BEC at $Z^0$-pole, we obtain the following values,

\[
\begin{align*}
R_1(G) &= 0.96 \pm 0.01 \text{ fm, and } \lambda_1 = 0.81 \pm 0.03, \\
R_2(E) &= 1.29 \pm 0.27 \text{ fm, and } \lambda_2 = 0.18 \pm 0.03.
\end{align*}
\]

The ratio of 3-jet to all-jet is approximately $0.18 \pm 0.03$, which is comparable with the empirical value of $0.17 \pm 0.01$ by OPAL collaboration [15–19], because $\lambda_1 + \lambda_2 \cong 1.0$. 


C3) We have proposed the new LRC expressed by the fractional expression, which is shown in Fig. 6. It shows the local minimum point near $Q = 0.45$ GeV. Provided that Eq. (6) was correct, we obtain the following values:

$$\begin{align*}
R_1(E) &= 0.94 \pm 0.07 \text{ fm}, \quad \lambda_1 = 0.84 \pm 0.04, \\
R_2(G) &= 1.19 \pm 0.04 \text{ fm}, \quad \lambda_2 = 0.14 \pm 0.03.
\end{align*}$$

D1) In Tables 3 and 5, two sets of four variables ($R_1$, $R_2$, $\lambda_1$, and $\lambda_2$) are almost identical. The degree of coherence ($\lambda_1$ and $\lambda_2$) become the weight factors for the first and second contributions to the BEC, because $\lambda_1 + \lambda_2 \approx 1.0$. Furthermore, we believe that Eq. (6) is a plausible solution for decay chain due to $\eta'$ and $\eta$ decays mentioned by OPAL collaboration.

D2) The geometrical combinations (main G + E) and (main E + G) in CF II formulas are interchanged in Tables 3 and 5. They depend on LRC’s. We should investigate various relationships among CF I, CF II, and LRCs in the future.

D3) It is worthwhile to mention the magnitudes of 2-jet and 3-jet at $Z^0$-pole by L3 collaboration. Since their data on 2-jet and 3-jet [9,11] are separated, applying CF I $\times$ LRC($\delta, \varepsilon$), i.e., Eqs. (1) and (3) to them, we obtain the following magnitudes,

$$\begin{align*}
R_{2\text{-jet}}(E) &= 1.00 \pm 0.02 \text{ fm} \ (\lambda = 0.83 \pm 0.02, \ \chi^2/\text{ndf} = 128/95), \\
R_{3\text{-jet}}(E) &= 1.18 \pm 0.02 \text{ fm} \ (\lambda = 1.0 \pm 0.00, \ \chi^2/\text{ndf} = 167/95).
\end{align*}$$

They seem to be compatible with our results mentioned above.

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   Therein, for $N^{BG}$, an identical separation between two ensembles with $\alpha_1$ and $\alpha_2$ is assumed.
   When there is no separation between them, the following formula is obtained:
   $$N^{(2+2-)} / N^{BG} = 1 + (a_1/s)(2/k_1)E_1^2 + (a_2/s)(2/k_2)E_2^2,$$
   where $s = a_1 + a_2 = \alpha_1 \langle n_1 \rangle^2 + \alpha_2 \langle n_2 \rangle^2$ (see succeeding Refs. [7,8]).
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