Quantum superpositions of the speed of light

Sabine Hossenfelder
Nordita, Roslagstullsbacken 23, 106 91 Stockholm, Sweden

While it has often been proposed that, fundamentally, Lorentz-invariance is not respected in a quantum theory of gravity, it has been difficult to reconcile deviations from Lorentz-invariance with quantum field theory. The most commonly used mechanisms either break Lorentz-invariance explicitly or deform it at high energies. However, the former option is very tightly constrained by experiment already, the latter generically leads to problems with locality. We show here that there exists a third way to integrate deviations from Lorentz-invariance into quantum field theory that circumvents the problems of the other approaches. The way this is achieved is an extension of the standard model in which photons can have different speeds without singling out a preferred restframe, but only as long as they are in a quantum superposition. Once a measurement has been made, observables are subject to the laws of special relativity, and the process of measurement introduces a preferred frame. The speed of light can take on different values, both superluminal and subluminal (with respect to the usual value of the speed of light), without the need for Lorentz-invariance violating operators and without tachyons.

We briefly discuss the relation to deformations of special relativity and phenomenological consequences.

I. INTRODUCTION

The speed of light plays an important role for the Lorentz-group and the physics of special relativity (SR). It is the only speed that remains invariant under a change of reference frame, and it determines the causal structure of spacetime. Most importantly, the speed of light is the asymptotic limit of the speed of accelerated massive bodies, and information cannot be transmitted any faster. The derivation of these properties from the symmetries of Minkowski-space is straight-forward and SR has been experimentally confirmed to high precision. However, Lorentz-symmetry might not be respected by the yet-to-be found theory of quantum gravity, and in fact deviations from Lorentz-symmetry are the so far most promising route to make contact between theoretical approaches to quantize gravity and observation [1].

The maybe most obvious way that deviations from Lorentz-invariance can make themselves noticeable is a breaking of Lorentz-invariance by the existence of a preferred frame. The preferred frame defines a timelike vector field, and one expects this field to couple to other fields of the standard model (SM). Such a breaking of Lorentz-invariance in extensions of the SM is very strongly constrained already [2]. The introduction of a fundamental preferred frame also leaves open the question why, if not exact, Lorentz-symmetry is still approximately exact to high precision, in the sense that relevant operators and operators of dimension 5 that couple to the timelike vector field are so strongly suppressed. Therefore, Lorentz-invariance violating operators in the SM face both experimental and theoretical challenges.

An alternative to the introduction of a preferred frame is to make different values of the speed of light compatible with observer-independence by modifying the action of the Poincaré-group. This enables the observer-invariance of an energy-dependent speed of light, and has been developed in an approach known as “Deformed Special Relativity” (DSR) [10–13]. Such deformations of special relativity are intimately related to non-commutative geometry, an idea that dates back to Snyder in 1947 [3], and that has received a lot of attention since it was shown to arise by quantum deformations of Poincaré symmetry [4]. The relation to DSR was made in [5] and has given rise to many related works that have entered the literature under the names of modified commutation relations, minimal length deformed quantum mechanics, or generalized uncertainty. These frameworks all explicitly or implicitly make use of deformations of special relativity.

DSR has been motivated by Loop Quantum Gravity, though no rigorous derivation exists to date. There are however non-rigorous arguments that DSR may emerge from a semiclassical limit of quantum gravity theories in the form of an effective field theory with an energy dependent metric [6], or that DSR (in form of the κ-Poincaré algebra) may result from a version of path integral quantization [7]. In addition it has been shown that in 2+1 dimensional gravity coupled to matter, the gravitational degrees of freedom can be integrated out, leaving an effective field theory for the matter which is a quantum field theory on κ-Minkowski space-time, realizing a particular version of DSR [8]. Recently, it has also been suggested that DSR could arise via Loop Quantum Cosmology [9]. Originally formulated in momentum space, it has however proven difficult to extend the formalism of DSR to position space, and the so-far pursued attempts lead to macroscopic non-localities. The interpretation and relevance of these non-localities is subject of an ongoing discussion. (More on this in section [V].)

It has been proposed [14, 15] that DSR is a classical relic of the quantum gravitational regime in the following sense. To modify the structure of momentum space and the action of the Lorentz-group on it, one needs a constant of dimension mass that one can identify with the Planck mass, m_p. One does not however need a constant of dimension length. It now happens to be the case that in four dimensions one can send both Newton’s constant G and ℏ to zero, while keeping the ratio ℏ/ G = m_p^2 fixed. This corresponds to a limit with m_p finite and ℏ = 0 that, while not actually being quantum gravitational, may still capture deviations from SR that originated in Planck scale effects.

We will look here into an entirely different approach to modified Lorentz-invariance; an approach that circumvents both the bounds on Lorentz-invariance violations and the difficulties with locality, and therefore offers an intriguing new so-
olution to these open problems. Knowing that deformations as classical relics of quantum gravitational effects have been difficult to reconcile with local field theory, we will consider instead a modification that is a pure quantum effect. And instead of introducing a fundamental preferred frame on the level of the action, we introduce a preferred frame only through the process of measurement. The $h$ is thus instrumental but, at least for the purpose of this paper, we will not aim to describe gravitational effects and restrict ourselves to flat space.

This paper is organized as follows. In the next section we introduce the basic idea and its formalism, and in section III lay out the physics of interactions. Section IV is dedicated to locality and causality, and in section V we discuss some phenomenological consequences, though the details shall be left for a future work. We discuss assumptions made and questions left open in section VI before concluding in VII.

The signature of the metric is Lorentzian signature. We equip it with quantum properties and make the point that the usual speed of light. Its position does not matter, and even how small is a question of constraints from available data. The spectrum of the operator $\hat{g}$ should actually be derived from a theory of quantum gravity. In the absence of such a theory, we treat it as input for the model that will be described here. Our aim is to parameterize the possible effects. Of course the restriction to flat space is a very special case, but it is a good starting point to develop the idea.

To every $\eta(\cdot)$ there is a Lorentz-group with transformations $\Lambda(\cdot)$ that keep $\eta(\cdot)$ invariant. The operator $\hat{g}$ is invariant under the appropriate application of the corresponding $\Lambda(\cdot)$ to the subspace spanned by the eigenvector $\eta(\cdot)$. That is, the action of a unitary representation $U(\Lambda)$ of a Lorentz transformation $\Lambda$, specified by its group parameters, is given via the eigenfunctions as

$$\hat{g}'(\eta(\cdot)) = U(\Lambda) \hat{g} \eta(\cdot) = \Lambda(\cdot)^T \eta(\cdot) \Lambda(\cdot) \eta(\cdot) = \eta(\cdot) \eta(\cdot)$$

The above equality is fulfilled even if the generators are different for different values of $\cdot$. This means that in principle we can perform a different Lorentz-transformation on each subspace. For one distinct transformation, we have to match the generators for different subspaces suitably together. We do this by choosing the same generators of rotations for each subspace, and for the boosts we rescale the velocity so that, in the subspace belonging to $\cdot$, the velocity $v$ that parameterizes the boost is related to the velocity $\eta_v$ in the $\cdot$-subspace by $\eta_v/c_\cdot = v/c$. In other words, we match the transformations so that the commonly used quantities $\beta = v/c$ and $\gamma^{-1} = 1 - \beta^2$ remain the same in all subspaces.

The transformations $\Lambda(\cdot)$ for different values of $\cdot$ are equivalent representations of the Lorentz-group, i.e. there exists a matrix $S$ that fulfills

$$S \Lambda(\cdot) S^{-1} = \Lambda(\cdot')$$

for each $\cdot, \cdot'$. The matrix $S$ is $\Lambda(\cdot')$. The Lorentz-symmetry that we invoke here thus is not new in the sense that we use only the well-known representations and actions of the Lorentz-group. Normally however, the parameter $\cdot$ is considered fixed by experiment to one particular value, and the other,
equivalent, Lorentz-transformations are not regarded as physically interesting. The novel idea here is to relax this restriction on the value of \( c \) and consider the whole set of transformations.

While this symmetry transformation combines the symmetries of all the subspaces, it is not particularly useful for maintaining a space-time picture. That is because with these transformations a change of coordinates is performed differently in each \( c \)-subspace so that one obtains a whole set of coordinates. If we chose the coordinates to be the same for each \( c \)-subspace in one frame \( \Sigma \), then a transformation to a frame \( \Sigma' \), moving with some relative velocity, would have different results depending on \( c \). Keeping in mind that the coordinates are the ones constructed with Einstein’s synchronization procedure (sending light signals back and forth), this is what one expects: The coordinates no longer agree after a boost because different subspaces use different speeds of light for synchronization. And while this transformation behavior is what one gets if one has an observer for every \( c \), each of which has a different notion of simultaneity, we want instead to restrict ourselves to observers in the \( c \)-subspace and transform coordinates according to his, i.e. our normal, \( c \)-transformations.

The consequence is then that the transformation behavior of the elements of the other subspaces, and functions defined on these, has to be adjusted.

To see how this works, let us leave aside quantum mechanics for a moment, and consider a classical particle with (constant) momentum \( \mathbf{p} \) moving on a trajectory \( y(t) \) with tangential vector \( \mathbf{v} \). We endow the particle with a modified transformation behavior by requiring that \( f(t) = p^c_\nu t^\nu = 0 \) is fulfilled in all reference frames and \( f(t) \) transforms like a scalar function from \( \Sigma \) to \( \Sigma' \), so that \( f'(t') = f(t) \). The momentum \( p^c_\nu \) in addition be lightlike with respect to \( c \), i.e. \( p^c_\nu p^\nu_\kappa \eta^{c_\kappa} = 0 \) in all frames.

The momentum of the particle then transforms under \( \Lambda_{c_\nu}(c_\kappa) \) and \( y \) under its inverse. The interpretation of this is clear so long as we are talking about a curve: It is just a curve with an unusual transformation behavior. But now instead of a curve let us consider a scalar field whose value we want to know for different \( \lambda_\nu \), where \( \lambda_\nu \) are our usual coordinates transformed with \( \Lambda_{c_\nu}(c_\kappa) \). Then we have to adjust the transformation behavior of the defining equation, such that \( f'(t') = f((\Lambda_{c_\kappa})^{-1}\Lambda_{c_\nu}x) \). In \( \Sigma' \) is then correctly \( f'(t') = p^c_\nu x^\nu = p\Lambda_{c_\nu}(c_\kappa)\Lambda_{c_\kappa}^{-1}x \). The additional factor in the transformation of the scalar function ensures that the contraction of the vector \( x \) that transforms differently than \( \mathbf{p} \) remains invariant. Now, to return to quantum mechanics, if \( |\psi\rangle \) is a state of a scalar field, then \( \langle \eta_\kappa|\psi\rangle = f(\lambda) \) is a scalar function which has to obey the same transformation behavior.

At this point, it is useful to note that the partial derivative \( \partial/\partial x^\nu \) of \( f \) transforms under \( \Lambda_{c_\kappa}(c_\nu) \), i.e. it transforms like the momentum. If one wants to evaluate a \( c \)-momentum in the \( c_\nu \)-background, one takes \( p^\nu_\kappa p^\nu_\kappa \eta^{c_\kappa} \). Since this contraction, while well-defined, is not invariant, the location of the indices matters. We take the contravariant momentum vector because otherwise the momentum would not be parallel to the tangential vector of the curve \( y \), which does not make physical sense. A lightlike \( c \)-momentum in the \( c \)-background then appears spacelike iff \( c > c_\kappa \) and timelike iff \( c < c_\kappa \), as one expects.

We usually do not measure \( g_{00} \), except possibly for gravitational waves, which however so far have not been directly detected. Instead, we measure the speed of particles in some spacetime background. We are therefore interested in a composite system

\[
|\Phi\rangle = |g, \Psi\rangle,
\]

where \( |\Phi\rangle \) is the complete wavefunction including the background and \( \Psi \) describes a particle in that background. If the particle is characterized by quantum numbers collectively called \( q, c \) with eigenstates \( |q, c\rangle \), its expansion is

\[
|\Phi\rangle = \sum_c \int dq\, \alpha(c)|\eta_{(c)}\rangle|q, c\rangle.
\]

In a more general case, \( \alpha \) could be a function also of \( q \), but for now we will not consider this dependence. We will come back to this possibility in the discussion.

To address the question of the invariance of the speed of light, we want to describe the propagation of the wavefunction. To that end, we start with the Klein-Gordon equation for a massless particle in the position representation, such that the quantum numbers \( q \) are the particle’s three momenta \( \mathbf{p} \).

We generalize the box operator so that it takes into account that the metric is now also an operator

\[
\hat{\Box} = \partial^\nu\partial_\nu g^{\nu\rho}.
\]

It is then

\[
\hat{\Box}|\Phi\rangle = \sum_c \int d^3p\, \alpha(c)\partial_\mu\partial^\mu\eta^{c_\mu}|\mathbf{p}, c\rangle
= \sum_c \int d^3p\, \alpha(c)\partial^\mu\partial_\mu\eta^{c_\mu}|\mathbf{p}, c\rangle.
\]

This expansion will fulfill the Klein-Gordon equation when

\[
|\mathbf{p}, c\rangle = :v_{\beta, c}(x) = \alpha e^{-i(E-t-\hat{\beta}\cdot\mathbf{x})} \text{ with } \delta(E-pc),
\]

where \( p = |\mathbf{p}| \). For momentum eigenstates, this is the usual solution, but every momentum now corresponds to a superposition of different energies, depending on the value of \( c \). This becomes clearer if we pick one momentum eigenstate \( \hat{\beta} \).

\[
|\Phi\rangle = \sum_c \alpha(c) e^{-i(p\cdot\mathbf{x} - \hat{\beta}\cdot\mathbf{x})}|\eta_{(c)}\rangle.
\]

We reproduce the standard limit for \( \alpha(c) = \delta c \), in which case the background spacetime is in an eigenstate to \( c \), and we can deal with just the field \( |\Psi\rangle \) in that background, where it fulfills the usual Klein-Gordon equation.

The eigenmodes are Lorentz-invariant in the way discussed above for the scalar function. We could either render \( |\Phi\rangle \) Lorentz-invariant by making the transformations on \( x^\nu = (t, \mathbf{x}) \) \( c \)-dependent, then we would get

\[
|\Phi\rangle = \sum_c \int d^3p\, \alpha(c)e^{-i(E' t' - \mathbf{p}' \cdot \mathbf{x}')}|\eta_{(c)}\rangle.
\]
with \( p' = p\Lambda, \) and \( x' = \Lambda^{-1}x \) and \( c \) remains invariant. But, as previously discussed, that is not useful because the meaning of these coordinates is ambiguous. Instead, we want to keep coordinates that transform all as \( x' = \Lambda^{-1}x, \) and then we transform the eigenmodes as \( \psi_{\vec{p}}(x') = \psi_{\vec{p}}((\Lambda^{-1})^{-1}\vec{\Lambda}, x). \)

To give a mass to the scalar field, one uses the operator \( \square - c^2m^2|f, \) where \( m \) is the (measured) mass of the particle in the \( c_s \)-background and \( f|\eta(c)\rangle \) may return any dimensionless function of \( c \) and \( m, \) as eigenvalues to eigenvectors \( |\eta(c)\rangle. \)

While \( f(c) = \rho/c^2 \) suggests itself, there is a priori no obvious relation between the masses in the different \( c_s \)-subspaces, because, for what the symmetry is concerned, not only \( E^2 - c^2p^2 \) is invariant and of the proper dimensionality, but so is its product with any dimensionless function of \( c. \) Thus, without more insights into the mechanism of mass generation, we have to treat the particle’s mass in another \( c_s \)-subspace as a parameter of the model. In the following, to slim down notation, we will write \( m^2 = m^2f \) and keep in mind that \( m \) is an operator and its value in some \( c_s \)-subspace not necessarily the measured mass of the particle.

One proceeds similarly for spinors. First, one generalizes the \( \gamma \)-matrices to
\[
\{\gamma^{c}, \gamma^{c'}\} = \eta^{J, J} \gamma^{c}_{J} \gamma^{c'}_{J},
\]
where quantities with capital Latin indices have the speed of light normalized to one, i.e. \( \eta^{J, J} = \text{diag}(1, -1, -1, -1), \)
\[
e^{c}_{J} = \text{diag}(1, c, c, c), \quad \eta^{c}_{J} = \eta^{c}_{J} e^{c}_{J} e^{c}_{J} \eta^{J, J}.
\]
Then, in the Dirac equation, one replaces the \( \gamma^c \) with \( \gamma^c \) that has eigenstates with the property
\[
\gamma^c \eta(c) = \gamma^c \eta(c),
\]
to obtain
\[
(\vec{\gamma} \cdot \partial_v - c^2m)|\Phi\rangle = 0.
\]
As in the scalar case, the solution to this equation is a superposition of the solutions to the Dirac-equation for the subspaces of the eigenvectors, i.e. different values of \( c. \)

One can use the \( e^{c}_{J} \) to convert the transformation behavior of tensors (similar to the way one uses the vierbein to convert from coordinate transformation to a local transformation behavior). If \( V^{\nu} \) transforms under \( \Lambda(c) \), then \( e^{c}_{\mu} e^{c}_{\nu} V^{\mu} \) transforms under \( \Lambda(c). \) One can use this to define a new momentum for the \( c \)-particles \( \vec{p}^{c} = e^{c}_{\mu} p^{\mu} c \) that transforms like a normal Lorentz-vector. However, one then gets a factor \( c/c, \) in the phase of wavefunctions, and the wave-velocity is not given by \( \vec{p}/|\vec{p}|, \) though one could work instead with this quantity and carry around the factors.

So far, we have considered only the evolution of the quantum state, now we will look at the measurement. We will assume that the process of measurement produces an observable that henceforth transforms under the \( c_s \)-representations. The process of measurement also picks out one particular restframe that plays the rôle of a preferred frame once the measurement has been made. One may add this additional rôle of the measurement as an axiom to the standard interpretation of quantum mechanics, but it comes about naturally if the collapse is replaced by environmentally induced decoherence, where the environment selects the frame.

We will thus assume that a measurement of observable \( \hat{O}, \) represented by a hermitian operator, performed in a frame \( \Sigma \) collapses the state to
\[
|\Phi\rangle \rightarrow |O\rangle_{\Sigma},
\]
where \( |O\rangle \) is an eigenstate of \( \hat{O} \) and the index \( \Sigma \) means it is expressed in the measurement’s frame. If decoherence is induced by entanglement with a thermal bath, as it is typically assumed, then the frame is the restframe of the bath.

The probability for the measurement outcome is as usual. The additional assumption here is that the measurement outcome \( O \) is a classical quantity (a number on an LCD screen) that transforms under the Lorentz group \( \Lambda(c) \). This means that the measurement reduces the extended Lorentz-symmetry with the additional parameter \( c \) to the usual one along with the transition from quantum to classical.

The expectation value of the momentum operator \( \hat{p}_v = i\partial_v \) is
\[
\langle\Phi|\hat{p}_v|\Phi\rangle_{\Sigma} = \alpha(c)\alpha(c')\hat{p}_{v}\langle\eta(c')|\eta(c)\rangle = \hat{p}_{v},
\]
which is indeed the momentum of the momentum eigenstate. If one prefers to use the \( x^c_{(c)} \), one uses the operator
\[
P_v = i\sum_{c} \hat{p}^c v \quad \text{with} \quad \hat{p}^c v = \frac{\partial}{\partial x^c_{(c)}}.
\]
For the energy \( E = \hat{p}_0 \) one has similarly
\[
\langle\Phi|E|\Phi\rangle_{\Sigma} = p_v \sum_c c\alpha(c)\alpha^*(c) = p_v c_s.
\]
If we do the transformation into a different restframe \( \Sigma' \) with relative velocity \( v \) before measurement, we obtain according to the above for each energy value in the sum
\[
E_s = \frac{1}{\sqrt{1 - (v^2/c^2)(c/c_s)^2}} (E_v - v c/c_s p_v)
\]
\[
= \sqrt{\frac{v - c_s}{v + c_s}} E_v,
\]
which is just the usual relativistic Doppler redshift! In particular it does not depend on \( c \) and can be pulled out of the sum. Since \( c \) is invariant, this means in that case it does not matter in which reference frame one calculates the expectation value, and one can omit the index \( \Sigma \) since it transforms under the usual SR transformation anyway.

But note that even for \( \alpha(c, \vec{p}) = \alpha(c) \) the eigenvalue of a single measurement, if \( c \neq c^s \), does no longer transform the same way before and after measurement. Consider we have measured the specific value \( \vec{c} \) with probability \( \alpha^2(\vec{c}). \) Then, a Lorentz-transformation after measurement gives
\[
E_s = \frac{1 - v^2/c^2}{\sqrt{1 - v^2/c^2}} E_s,
\]

whereas a Lorentz-transformation before measurement would have resulted in \( \psi \). In particular, the velocity \( \tilde{c} \) itself transforms after measurement under the usual addition law and is no longer invariant.

To summarize this section, we have seen that we can extend Lorentz-symmetry so that it accommodates different invariant speeds of light, so long as the state is in a quantum superposition. We assumed that the process of measurement does not only reduce the superposition to an eigenstate, but does at the same time reduce the symmetry to the normal Lorentz-symmetry. As a result the probability distribution over different values of the speed of light is invariant, but the outcome of any one measurement no longer is.

III. QUANTUM FIELD THEORY AND INTERACTIONS

With these prerequisites from quantum mechanics, we can now look at the 2nd quantization. We expand the field as

\[
\phi(x) = \sum_c \int d^3 p \hat{a}_{c,\vec{p}} \psi_c(x) + \hat{a}_{c,\vec{p}}^\dagger \psi_c^\ast(x),
\]

(22)

where the \( \psi_c(x) \) are the solutions (5) to the free particle wave equation, and

\[
a_{p,c} |\eta_{c'}\rangle = 0, \quad a_{p,c}^\dagger |\eta_{c'}\rangle = \delta_{c,c'} |\tilde{p},c\rangle |\eta_{c'}\rangle,
\]

(23)

and repeated action of creation operators produce multi-particle states in the \( c \)-background. For a scalar field the annihilation and creation operators fulfill the commutation relation

\[
[a_{\tilde{p},c}, a_{p',c'}^\dagger] = \delta_{c,c'} \delta(\tilde{p} - p').
\]

(24)

For spinor fields, one takes the appropriate spinor coefficient functions and anticommutation relations.

To proceed, we now have to investigate which products of fields we can construct invariantly in order to find out which interaction terms are allowed. For the gauge fields, \( A_\nu \), we use the Lagrangian

\[
L_g = -\frac{e^2}{4} g^{\mu\nu} F_{\mu\nu} F_{\nu\lambda},
\]

(25)

where \( F \) is the field strength tensor as usual, \( e \) is the coupling constant, and the \( c \)-value of the fields is determined by the \( c \)-subspace of \( \hat{\mathbf{g}} \). This means that four-boson vertices in the non-abelian case cannot mix different \( c \)-values.

In the Lagrangian for fermions

\[
L_f = \overline{\psi} (i\gamma^\mu \partial_\mu - m) \psi + cc',
\]

(26)

the \( c \)-value of the fermion is determined by the \( c \)-subspace of \( \hat{\gamma} \). The interaction term takes the form

\[
L_{\text{int}} = e M \overline{\psi} \gamma^\mu A_\mu \psi,
\]

(27)

with the transition matrix

\[
\langle \eta_{c'}|M|\eta_{c}\rangle = M_{cc'},
\]

(28)

and \( M = M^\ast \). \( M \) is not necessarily diagonal because the \( c \)-subspace of the fermions does not need to be the same as that of the gauge field. This is because, as noted earlier, we can put in a factor \( e_{c(c')} e_{c(c')} \) to adjust the transformation behavior of the \( \gamma's \) (that are contracted with the partial derivative acting on the \( c \)-spinor and produce a \( c \)-momentum) to that of the \( A_\mu \).

Or, in other words, the relevant property characterizing the symmetry of the gauge field is the transformation of the phase and not the transformation of the polarization vector. \( M \) is a matrix that, when projected on the \( c \)-eigenspaces, encodes the coupling between different \( c \)-sectors. These vertices then mix different \( c \)-values of fermions and gauge bosons.

If one inserts the field expansion in such a Lagrangian, the Feynman rules in momentum space are then the normal ones with the following additions:

- Every vertex obtains a factor \( M_{cc'} \), one \( c \) for the fermions, one for the gauge boson.
- The external ingoing lines belong to the same \( c \)-subspace. External outgoing lines also belong to the same \( c \)-subspace, but not necessarily the same as the ingoing ones.
- Vertex indices must be matched to the coupling particles’ transformation behavior.
- Sum over all \( c \)'s of virtual particles.

And, as laid out in the previous section, the momenta transform under the respective \( \Lambda(c) \) until measurement, after which they become a \( c \), four vector.

The amplitudes then have as usual a \( \delta \)-function for conservation of the four-momentum. Note that the argument of this \( \delta \)-function differs from one frame to the next. It does not differ by a coordinate transformation, it is actually a different argument. It is only the measurement that one selects one. As we have seen in the previous section, the outcome depends on the frame of the measurement and disagreements are unobservable.

IV. LOCALITY AND CAUSALITY

Locality can become a problematic concept in theories in which the speed of light can take different values but still observer independence should be fulfilled. The reason is that with the requirement that different values of speeds remain invariant, space-time points have no well-defined transformation behavior: The location of a point after a change of reference frame depends on which speed is kept invariant. In particular, a point defined in one reference frame by different means though intersecting curves can, after a change of reference frame, split up into various points. For the case of DSR this has been shown in \([16, 17]\). Recent suggestions for how to address the problem have built up to a new ‘Principle of Relative Locality’ \([15, 16, 21]\). This approach accepts the arising nonlocality and aims to show it is not problematic after all. (For some discussion, see also \([22, 24]\).)

We too encountered in the previous section the need to transform coordinates depending on the value of \( c \), reflected
in the set of transformations $\Lambda_{(c)}$. But our approach offers an entirely new solution for the problem. The observer-independence of the speed of light is now a fundamental property of the evolution of a quantum state, but each single measurement outcome depends on the restframe in which the measurement was made. In particular, a speed of photons different from the average value $c$, will after measurement no longer be an invariant of the transformation, but transform under normal Lorentz-transformations $\Lambda_{(c)}$. Thus, disagreements in different observers’ definitions of a point due to different invariant speeds are never reflected in actual observables.

The ‘Box-problem’ discussed in reference [16,17] is circumvented because observers never disagree on the outcome of the measurement. In DSR the particle’s worldline transforms under a non-standard Lorentz-transformation that is energy-dependent. As a consequence, the statement whether three lines meet or do not meet in one point depends on the reference frame, and (to some precision) the question whether they meet is a requirement for local interactions to take place. In the scenario discussed here, in contrast, making the measurement in one frame fixes the eigenvalue of the speed and a different observer would interpret the speed to be the normal Lorentz transformation of the speed measured. The measurement is either made in the laboratory frame, in which case the bomb blows up and the observer in the satellite agrees, or it is made in the satellite frame, in which case the bomb does not blow up and the observer in the lab agrees. The situation is entirely symmetric as long as both frames represent identical measuring processes with some relative velocity.

Solving the problem with locality does however not solve the problems with causality that superluminal information exchange creates. Indeed, it seems one has to give one up for the other. One creates a problem with locality if there exist curves with different transformation behaviors because their intersections can be used to define points. If one does not accept these non-localities, the need to reproduce SR in the limit of non-quantum objects means that everything that we can plausibly refer to as an observer transforms under normal Lorentz-transformation. But a normal Lorentz-transformation can turn a superluminal curve into one going backwards in time. In DSR on the other hand, the modified transformation behavior of the speed-of-light allows it to remain in the upper, $t > 0$, part of each reference frame.

To be more precise, the problem with causality is not that a curve that in one reference frame is superluminal seems to be going backwards in time in another frame, because the curve itself does not have a direction. Both observers could interpret the particle as moving forward in time but into opposite spatial directions. Closed curves in flat Minkowski-space then are not a problem fundamentally; one just has to demand consistency. This means if a particle at some time $t_0$ could affect its own earlier curve at $t_1 < t_0$, this would have been taken into account at the time $t_0$ already.

As an example, consider the closed curve in Figure 1 top, and interpret each corner as a scattering event. One could read this curve as a particle that propagates freely from $A$ to $B$, scatters in $B$ and produces a particle with superluminal momentum that then moves towards $C$ which, in the chosen reference frame is earlier in time than $B$. In $C$ and in $D$ the particle scatters again, produces an outgoing particle that then intersects with the original particle’s previous curve in $A$. Now on the level of elementary particles, this would just mean that the state of the particle at $B$ would have had to take into account event $A$ already, because it was always there. We can also read the curve the other way round which (in the chosen example) would correspond to some particle going from $C$ to $D$, scattering twice and creating to superluminal particles going to $A$ and $C$ respectively.

![FIG. 1. Closed curves. Top: Without arrow of time. 2nd from top: With inconsistent arrows of time, generating the possibility for grandfather paradox. 2nd from bottom and bottom: With consistent arrow of time. Closed curves could be constructed with only 3 straight lines. We have included a fourth to show that taking into account the finite amount of time necessary to process information does not remove the problem.](image)

However, this does no longer work once we take into account that, for better or worse, our world is evidently not time-reversal invariant and we do have an arrow of time that points somewhere we can for lack of a better word call ‘forward.’ Problems with causality can no longer simply be solved by
demanding consistency in this simple form when we consider macroscopic objects that display an arrow of time because it becomes possible to create paradoxes (that have been extensively used and abused in the science-fiction literature).

To see the difference, consider in A you fill out a lottery ticket. On the way to B you learn that you didn’t win but write down the winning numbers and send them to your friend at C. Your friend then, at D, sends the numbers back to your earlier self at A. Consistency would now demand that you already knew the numbers all the time, in which case you would have won the lottery already and still there was no avoiding that you will send the numbers to your friend. Alternatively, your friend is not able to send you the numbers, or you will not be able to receive them. But apart from that being what consistency demands, it is difficult to see exactly what cosmic conspiracy would prevent you or your friend from sending the numbers back and forth and creating a paradox.

The difference to the case of elementary particles is that this story has a direction of information flow and a notion of ‘learning’. The process of ‘sending’ is different to the process of ‘receiving’ because the sender previously knew of the information whereas the receiver does not. If we time-reverse this process it looks very different. (One does not untype an email when one receives it.) For macroscopic objects thus the curve would have to be endowed with arrows indicating a direction, depicted in Fig. 1, 2nd from top, that inevitably have to run backwards in time somewhere (in any reference frame). That is what creates the problem.

But this analysis of the problem also contains the seed for its resolution. Since there is no point denying the existence of an arrow of time, we have to take it into account consistently. Recall that in our framework it is only after measurement that the curve of a superluminal particle is subject to SR transformations. All we have to do is to endow the process of measurement with an arrow of time which comes naturally through the framework of environmentally induced decoherence. Thus, there is an environment that creates an arrow of time, which is a vector field timelike in the measurement frame.

It may be of interest to the reader that Geroch [27] has argued on very general grounds that the existence of different causal cones is possible without being in conflict with SR. In Geroch’s work it was however not investigated the transformation behavior of these cones and their invariance in particular.

V. POSSIBLE PHENOMENOLOGICAL CONSEQUENCES

The standard model, and quantum electrodynamics (QED) in particular, are extremely well tested theories. This raises the question how tightly existing experiments constrain the possibility discussed here. While we will leave details of the phenomenology to be studied in a future work, here we want to discuss some general properties.

Since we have never noticed a photon moving with anything else than $c$, (to some precision) we should expect the probability for photons to propagate in other $c$-subspaces to be small, $\ll 1$. Let us consider a very simplified case, that in which there is only one other value $c_1$ in addition to $c$. With the convention that $M_{c_1c} = 1$ (if not, the factor can be absorbed in the coupling constant), we have two remaining free parameters: $M_{c_1} = \lambda \ll 1$ and $M_{c_1c_1} = \mu$ in addition to the masses of the $c_1$-fermions which, as we noted earlier, stand in no obvious relation to their masses in the $c_1$-subspace.

There can be no modification to Compton scattering, because both photons and fermions are in the ingoing state. The amplitude for Bhabha scattering between $c_1$-electrons obtains an additional contribution of order $\lambda^2$ and the amplitude for $c_1$-electrons in the outgoing state has a factor $\mu$. Note however that the same factor comes in again through the amplitude for the detection cross-section. That is to say, if the particles are difficult to produce, they are also difficult to detect.

A not detected $c_1$-photon or electron would result in missing energy and momentum that does not fulfill the condition $E^2 - c_2^2 p^2 = m_2^2 c_4^2$. Instead, it would appear to have an apparent mass $m_{app}$ of

$$m_{app} = m \sqrt{1 + \frac{p^2 c_4^2}{m^2 c_1^4} \left(1 - \frac{c_1^2}{c_2^2}\right)^2}, \quad (29)$$

where $p^2$ is the (square of the) three-momentum in the measurement frame. Note that this apparent mass can be imaginary if $c_1 > c$. This apparent mass is the mass that we would assign to the particle within special relativity. The particle’s actual mass, which appears in the Lagrangian, remains positive and real valued.
The more general case would be that between the particle in some background and the background. It is an intrinsic property of the particle.

If \( \alpha(c, q) \) is an invariant and intrinsic property of the particle, then it may be a function of the momentum. This allows us to relate the here proposed model to DSR. For a photon, the only way to obtain a non-trivial (on shell) momentum dependence that is also Lorentz-invariant is to use a modified version of transformations in momentum space. Then, the speed of light obtained from the expectation values of energy and momentum may become a function of the particle’s energy that in the low energy limit reproduces non-Lorentz-symmetry, thereby connecting the here proposed model to DSR. However, in this case \( c \) would need to have a continuous spectrum (since the energy can be continuously redshifted). Such a version of the model would still be different from DSR in that the different speeds of light can exist only in superpositions.

VI. DISCUSSION

Let us summarize the assumptions made here and discuss their relevance.

First, we have assumed that \( c \) takes on discrete values. While the formalism presented here can easily be extended to continuous \( c \)-values, it seems more plausible that, if there exist indeed superpositions of different \( c \)'s, the values are discrete, at least in the vicinity of \( c_* \). If they are not, one would expect the probability to be smooth in the vicinity of \( c_* \). Then, in the absence of a gap in the spectrum, one loses the rationale to use only one particular \( c \)-value, namely \( c_* \) for the measurement outcome.

We have further, in section II, considered a product state between the particle in some background and the background. The more general case would be that \( \alpha \) is a function also of the quantum numbers of the particle \( \alpha(c, q) \). For one, this would be the outcome of some scattering process in which case \( \alpha \) would generically depend on all the momenta of scattering particles. But one may also consider the possibility that \( \alpha(c, q) \) is an intrinsic property of the particle.

We have shown here that, next to Lorentz-invariance breaking and deformations of special relativity, there exists a novel third way how departures from Lorentz-invariance that may arise in quantum gravitational effects can make themselves noticeable.

The departure from Lorentz-invariance proposed here arises from superpositions of different metrics on the same background manifold, so that for each of the metrics the maximally possible and invariant speed of massless particles takes on a different value. The process of measurement produces observables that obey the laws of special relativity. To preserve causality, we have assumed that the measurement does introduce a preferred frame. This modification of Lorentz-invariance makes it unnecessary to introducing Lorentz-invariance violating operators, and does not create problems with locality or causality; at least in flat space.

The here proposed model has phenomenological consequences for particle physics that need to be further explored to find out how tightly the parameters of the model are constrained by available data already.

ACKNOWLEDGEMENTS

I thank Ben Koch, Jakub Mielczarek, Stefan Scherer and Lee Smolin for helpful feedback and discussions, and Sean Carroll for drawing my attention to reference [27].

[1] D. Mattingly, Living Rev. Rel. 8, 5 (2005) [arXiv:gr-qc/0502097].
[2] V. A. Kostelecky and N. Russell, “Data Tables for Lorentz and CPT Violation,” Rev. Mod. Phys. 83, 11 (2011) arXiv:0801.0287 [hep-ph].
[3] H. S. Snyder, “Quantized Space-Time,” Phys. Rev. 71, 38 (1947).
[4] S. Majid, H. Ruegg, “Bicrossproduct structure of kappa Poincare group and noncommutative geometry,” Phys. Lett. B334, 348-354 (1994). [hep-th/9405107].
[5] J. Kowalski-Glikman, S. Nowak, “Doubly special relativity and de Sitter space,” Class. Quant. Grav. 20, 4799-4816 (2003). [hep-th/0304101].
[6] G. Amelino-Camelia, L. Smolin and A. Starodubtsev, “Quantum symmetry, the cosmological constant and Planck scale phenomenology,” Class. Quant. Grav. 21, 3095 (2004) [arXiv:hep-th/0306134]; L. Smolin, “Could deformed special relativity naturally arise from the semiclassical limit of quantum gravity?,” [arXiv:0808.3765] [hep-th].
[7] J. Kowalski-Glikman and A. Starodubtsev, “Effective particle kinematics from Quantum Gravity,” Phys. Rev. D 78, 084039 (2008) [arXiv:0808.2613] [gr-qc].
[8] L. Freidel, J. Kowalski-Glikman and L. Smolin, “2+1 gravity and doubly special relativity,” Phys. Rev. D 69, 044001 (2004) [arXiv:hep-th/0307085].
[9] M. Bojowald, “Quantum geometry and quantum dynamics at the Planck scale,” AIP Conf. Proc. 1196, 62 (2009) [arXiv:0910.2936] [gr-qc].
[10] G. Amelino-Camelia, “Testable scenario for relativity with minimum-length,” Phys. Lett. B 510, 255 (2001) [arXiv:hep-th/0012238].
[11] J. Kowalski-Glikman, “Observer independent quantum of mass,” Phys. Lett. A 286, 391 (2001) [arXiv:hep-th/0102098].
[12] G. Amelino-Camelia, “Doubly special relativity,” Nature 418, 34 (2002) [arXiv:gr-qc/0207049].
[13] J. Magueijo and L. Smolin, “Generalized Lorentz invariance with an invariant energy scale,” Phys. Rev. D 67, 044017 (2003) [arXiv:gr-qc/0207085].

[14] J. Kowalski-Glikman, “Doubly special relativity: Facts and prospects,” In *Oriti, D. (ed.): Approaches to quantum gravity* 493-508. [gr-qc/0603022].

[15] L. Smolin, “Classical paradoxes of locality and their possible quantum resolations in deformed special relativity,” [arXiv:1004.0664] [gr-qc].

[16] S. Hossenfelder, “The Box-Problem in Deformed Special Relativity,” [arXiv:0912.0090] [gr-qc].

[17] S. Hossenfelder, “Bounds on an energy-dependent and observer-independent speed of light from violations of locality,” Phys. Rev. Lett. 104, 140402 (2010) [arXiv:1004.0418 [hep-ph]].

[18] U. Jacob, F. Mercati, G. Amelino-Camelia and T. Piran, “Modifications to Lorentz invariant dispersion in relatively boosted frames,” [arXiv:1004.0575] [astro-ph.HE].

[19] G. Amelino-Camelia, M. Matassa, F. Mercati and G. Rosati, “Taming nonlocality in theories with deformed Poincare symmetry,” [arXiv:1006.2126] [gr-qc].

[20] L. Smolin, “On limitations of the extent of inertial frames in non-commutative relativistic spacetimes,” [arXiv:1007.0718] [gr-qc].

[21] G. Amelino-Camelia, L. Freidel, J. Kowalski-Glikman and L. Smolin, “The principle of relative locality,” [arXiv:1101.0931] [hep-th].

[22] S. Hossenfelder, “Comments on Nonlocality in Deformed Special Relativity, in reply to [arXiv:1004.0664] by Lee Smolin and [arXiv:1004.0575] by Jacob et al.,” [arXiv:1005.0535] [gr-qc].

[23] S. Hossenfelder, “Reply to [arXiv:1006.2126] by Giovanni Amelino-Camelia et al.,” [arXiv:1006.4587] [gr-qc].

[24] S. Hossenfelder, “Comment on [arXiv:1007.0718] by Lee Smolin,” [arXiv:1008.1312] [gr-qc].

[25] J. Friedman, M. S. Morris, I. D. Novikov, F. Echeverria, G. Klinkhammer, K. S. Thorne and U. Yurtsever, “Cauchy problem in space-times with closed timelike curves,” Phys. Rev. D 42, 1915 (1990).

[26] M. Visser, “The Quantum physics of chronology protection,” [astro-ph/0204022].

[27] R. Geroch, “Faster Than Light?,” [arXiv:1005.1614] [gr-qc].