Currently epsilon near zero materials (ENZ) have become important for controlling the propagation of light and enhancing by several orders of magnitude the Kerr and other nonlinearities. Given this advance it is important to examine the quantum electrodynamic processes and information tasks near ENZ materials. We study the entanglement between two two-level systems near ENZ materials and compare our results with the case where the ENZ material is replaced by a metal. It is shown that with ENZ materials substantial entanglement can be achieved over larger distances than for metal films. We show that this entanglement over large distances is due to the fact that one can not only have large emission rates but also large energy transmission rates at the epsilon-near-zero wavelength. This establishes superiority of ENZ materials for studying processes specifically important for quantum information tasks.

PACS numbers:

I. INTRODUCTION

A large number of problems in physics and chemistry require very significant dipole-dipole interaction which includes fundamental interactions such as van der Waals forces and vacuum friction [1,2], Förster (radiative) energy transfer (FRET) [3,4], radiative heat transfer [2,5], quantum information protocols like the realization of CNOT gates [6,7], pairwise excitation of atoms [9,11], and Rydberg blockade [12,13]. In the last decades numerous plasmonic and metamaterial platforms have been developed to enhance the dipole-dipole interaction significantly. For example for FRET it could be shown theoretically and experimentally that when two atoms placed in the vicinity of 2D or 2D-like plasmonic structures as graphene sheets [14,16] and metal films [17,21] can persist over long distances due to the plasmon assisted energy transfer. More astonishingly is that one can even find a significant energy transfer across metal films [22] due to the interaction with the coupled surface plasmons which can be highly improved by replacing the metal film by a hyperbolic meta-material [23]. This effect of a long-range energy transfer across a hyperbolic metamaterial can be regarded as one form of the so-called super-Coulombic atom-atom interaction [24].

Currently epsilon near zero materials (ENZ) have become important for controlling the propagation of light [25] and enhancing by several orders of magnitude the Kerr and other nonlinearities [26]. Given this advance it is important to examine the quantum electrodynamic processes and information tasks near ENZ materials. The ENZ media can also be used to increase tunneling electromagnetic energy through subwavelength channels [27],

Figure 1: Sketch of the considered configuration.
where the \( \mathbf{e}_i \) is the general complex valued unit vector pointing in the direction of the dipole moment of TLS \( i \), and introducing the free space emission rate of a single TLS \( \gamma_0 \)

\[
\gamma_0 := \frac{2|p|^2\omega_0^3}{3c^3\hbar} \frac{1}{4\pi \epsilon_0}
\]

we can express \( \gamma_{ij} \) and \( \Omega_{ij} \) as

\[
\gamma_{ij} := 6\pi \gamma_0 \frac{c}{\omega_0} \text{Im} \left( \mathbf{e}_i \cdot \mathbf{G}(\mathbf{r}_i, \mathbf{r}_j, \omega_0) \cdot \mathbf{e}_j^* \right),
\]

\[
\Omega_{ij} := -6\pi \gamma_0 \frac{c}{\omega_0} \text{Re} \left( \mathbf{e}_i \cdot \mathbf{G}(\mathbf{r}_i, \mathbf{r}_j, \omega_0) \cdot \mathbf{e}_j^* \right).
\]

If we consider a symmetric configuration as for example depicted in Fig. 1 we further have

\[
\gamma_{12} = \gamma_{21} =: \gamma_c,
\]

\[
\gamma_{11} = \gamma_{22} =: \gamma_s,
\]

\[
\Omega_{12} = \Omega_{21} =: \Omega_c,
\]

\[
\Omega_{11} = \Omega_{22} =: \Omega_s
\]

where the indices stand for 'single' and 'collective'. In this case we find the dynamical equations

\[
\dot{\rho}_{cc} = -4\gamma_s \rho_{cc},
\]

\[
\dot{\rho}_{eg} = -2\rho_{eg}[\gamma_s + i(\omega_0 + \Omega_s)],
\]

\[
\dot{\rho}_{ss} = -2(\rho_{ss} - \rho_{ee})(\gamma_s + \gamma_c),
\]

\[
\dot{\rho}_{aa} = -2(\rho_{as} - \rho_{ee})(\gamma_s - \gamma_c),
\]

\[
\dot{\rho}_{as} = -2\rho_{as}(\gamma_s - i\Omega_c)
\]

for the states \( |e\rangle = |e_1e_2\rangle, |g\rangle = |g_1g_2\rangle, |s\rangle = (|e_1g_2\rangle + |g_1e_2\rangle)/\sqrt{2} \) and \( |a\rangle = (|e_1g_2\rangle - |g_1e_2\rangle)/\sqrt{2} \). These equations correspond to Eqs. (15.27) in Ref. [34]. Accordingly, the solutions of the dynamical equations are

\[
\rho_{cc}(t) = \rho_{cc}(0)e^{-4\gamma_s t},
\]

\[
\rho_{eg}(t) = \rho_{eg}(0)e^{-2(\gamma_s + i(\omega_0 - \Omega_s)) t},
\]

\[
\rho_{ss}(t) = \rho_{ss}(0)e^{-2(\gamma_s - \gamma_c) t},
\]

\[
\rho_{as}(t) = \rho_{as}(0)e^{-2(\gamma_s + \gamma_c) t}
\]

from these equations the dynamics of the system of TLS coupled by an arbitrary environment can be studied. Here we are interested in the entanglement which is measured by the concurrence function \( C(t) \) introduced by Wootters [35]. This function is defined by

\[
C = \max(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4})
\]

where the \( \lambda_i \) \((i = 1, \ldots, 4)\) are the eigenvalues of the matrix \( \hat{\rho} \hat{\rho} \) and can be defined by means of the Pauli matrix \( \sigma_y \) by

\[
\hat{\rho} = \sigma_y \otimes \sigma_y \rho^* \sigma_y \otimes \sigma_y.
\]

The conclusions of our study are given in Sec. V.
values between \([0,1]\) giving 0 for unentangled states and 1 for maximally entangled states. When starting at \(t = 0\) with the initially unentangled state \(|\phi_1, g_2\rangle\) then the concurrence function \(C(t)\) is given by

\[
C(t) = e^{-2\gamma_c t} \sqrt{\sin^2(2\gamma_c t) + \sin^2(2\Omega_c t)}. 
\]

(22)

It can be seen that \(C(t = 0) = 0\) as expected, but for times \(t \to 0\) it can have values larger than zero which means that due to the coupling of the two TLS via the environment an entanglement of the states of the two TLS is produced.

### III. DYADIC GREEN’S FUNCTION

The goal is now to study the entanglement measured by the concurrence function for the two TLS when they are coupled by a thin film as depicted in Fig. 1. To this end, it is necessary to determine \(\gamma_c, \gamma_s\) and \(\Omega_c\) which means that we have to determine the corresponding Green’s function for that configuration. For an initially excited TLS at \(r_1 = (0, 0, z_1)\) with \(z_1 < 0\) and a second TLS which is initially in the ground state at \(r_2 = (0, 0, z_2 = d + |z_1|)\) the Green’s function in Weyl’s representation is given by

\[
G(r_1, r_2) = \int \frac{d^2\kappa}{(2\pi)^2} G(\kappa, z) 
\]

(23)

where \(\kappa = (k_x, k_y)^t\) and

\[
G(\kappa, z) = \frac{i e^{i k_z \text{vac} (d + |z_1|)}}{2 k_{z, \text{vac}}} \sum_{i=s,p} t_i a_i^+(k_0) \otimes a_i^+(k_0) 
\]

(24)

introducing the vacuum wavevector in \(z\)-direction \(k_{z, \text{vac}} = \sqrt{k_0^2 - \kappa^2}\) and \(k_0 = \omega_0/c\). Here \(t_s\) and \(t_p\) are the amplitude transmission coefficients and \(a_{s,p}\) are the polarization vectors defined by

\[
a_s^+(k_0) = \frac{1}{\kappa} \begin{pmatrix} k_y \\ -k_x \\ 0 \end{pmatrix}, \quad (25)
\]

\[
a_p^+(k_0) = \frac{1}{\kappa k_0} \begin{pmatrix} -k_x k_{z, \text{vac}} \\ -k_y k_{z, \text{vac}} \\ \kappa^2 \end{pmatrix}. \quad (26)
\]

From this expression we can determine \(\gamma_c\) and \(\Omega_c\). On the other hand, if we want to determine \(\gamma_s\) we have to determine the the Green’s function \(G(r_1, r_1)\) evaluated solely at the position of the TLS which is initially in the excited state. In this case, we have

\[
G(r_1, r_1) = \int \frac{d^2\kappa}{(2\pi)^2} G_{\text{single}}(\kappa, z_1) 
\]

(27)

with

\[
G_{\text{single}}(\kappa, z_1) = \frac{i}{2 k_{z, \text{vac}}} \sum_{i=s,p} \left[a_s^+(k_0) \otimes a_s^+(k_0) + e^{i k_z \text{vac} |z_1|} r_i a_i^+(k_0) \otimes a_i^+(k_0)\right]. 
\]

(28)

where \(r_s\) and \(r_p\) are the amplitude reflection coefficients.

In order to see how the entanglement is affected by a plasmonic structure like a simple metal film or a hyperbolic meta-material we only need the appropriate expressions for the transmission and reflection coefficients. These are well known and for a in general uni-axial material with the optical axis oriented along the surface normal the transmission and reflection coefficients are given by

\[
t_s = \frac{4 k_{z,o} k_{z, \text{vac}} e^{i(k_{z,o} - k_{z, \text{vac}})d}}{(k_{z,o} + k_{z, \text{vac}})^2 - (k_{z,o} - k_{z, \text{vac}})^2 e^{2ik_{z,o}d}} \quad (29)
\]

\[
t_p = \frac{4 e^{i k_{z,e} k_{z, \text{vac}} e^{i(k_{z,e} - k_{z, vac})d}}}{(k_{z,e} + \epsilon \| k_{z, \text{vac}})^2 - (k_{z,e} - \epsilon \| k_{z, vac})^2 e^{2ik_{z,e}d}} \quad (30)
\]

and

\[
r_s = \frac{R_s}{1 - e^{2ik_{z,o}d}}, \quad (31)
\]

\[
r_p = \frac{R_p}{1 - e^{2ik_{z,e}d}}, \quad (32)
\]

where we have introduced the reflection coefficients of a single interface

\[
R_s = \frac{k_{z, \text{vac}} - k_{z,o}}{k_{z, \text{vac}} + k_{z,o}}, \quad (33)
\]

\[
R_p = \frac{k_{z, \text{vac}} \epsilon \perp - k_{z,e}}{k_{z, \text{vac}} \epsilon \perp + k_{z,e}} \quad (34)
\]

and the \(z\) components of the wavevector for the ordinary and extraordinary modes

\[
k_{z,o} = \sqrt{k_0^2 \epsilon \perp - \kappa^2}, \quad (35)
\]

\[
k_{z,e} = \sqrt{k_0^2 \epsilon \parallel - \kappa^2 \epsilon \perp \epsilon \parallel}. \quad (36)
\]

The permittivities \(\epsilon \perp\) and \(\epsilon \parallel\) are the permittivities perpendicular and parallel to the optical axis which is here the \(z\) axis, i.e. the optical axis is along the surface normal.

### IV. METAL VS HMM FILMS

In the following we will consider a single silver film described by the Drude model

\[
\epsilon_{\text{Ag}} = \epsilon \parallel = \epsilon \perp = \epsilon_\infty - \frac{\omega_p^2}{\omega(\omega + i\tau_\text{Drude})}. 
\]

(37)

The parameters from [27, 38] are \(\epsilon_\infty = 3.7, \omega_p = 1.4 \cdot 10^{16} \, \text{rad/s}, \tau = 4 \cdot 10^{-14} \, \text{s}\). As in Ref. [23] we use a much smaller relaxation time of \(\tau = 0.45 \cdot 10^{-14} \, \text{s}\) which accounts for the increased collision frequency found in thin metal films [39]. The surface plasmon resonance wavelength is in this case given by \(\lambda_{\text{SP}} = 291 \, \text{nm}\) and the ENZ wavelength is \(\lambda_{\text{ENZ}} = 259 \, \text{nm}\).
In Fig. 2 we show some examples for the concurrence function $C(t)$ for silver films of different thickness. Throughout the paper we choose $z_1 = 10$ nm. It can be seen that for very thin films with thickness $d = 10$ nm one can find a relatively large entanglement for $\lambda > \lambda_{SP}$ of the two TLS due to the coupling via the coupled surface plasmons inside the metal film. On the other hand for thicker films with $d = 60$ nm the entanglement for $\lambda > \lambda_{SP}$ becomes already very small and for $d = 120$ nm it is practically not existing. To get more inside why this happens we have plotted $\gamma_c$, $\gamma_s$ and $\Omega_c$ in Fig. 3. As is clear from the expression for the concurrence function in Eq. (22) for $\gamma_c |t| \gg 1$ or $\Omega_c |t| \ll 1$ we have

$$C(t) \approx \frac{1}{2} e^{2(\gamma_c |t| - \gamma_s |t|)}.$$ 

(38)

Obviously, in this limit the concurrence can only have a maximum value of 0.5 and there can only be a noticeable entanglement if the collective damping rate $\gamma_c$ is on the same order as the single-atom emission rate $\gamma_s$. Since the energy transmission rate is proportional to $|G_{xx/zz}(r_1, r_2)|^2 = \gamma_c^2 + \Omega_c^2$ this means that we can find a noticeable entanglement if loosely speaking the ‘energy transmission rate’ $\gamma_c$ is on the same order of magnitude as the single-atom spontaneous emission rate $\gamma_s$ of the initially excited TLS.

From Fig. 3 it becomes apparent that the spontaneous emission rate $\gamma_s$ is very large around $\lambda_{SP}$ as expected. Recently such changes in the spontaneous emission have also been studied for three-level systems on meta-surfaces and hyperbolic materials [40, 41]. In particular, the effect of the coupling of the surface plasmons in the thin silver film $d = 10$ nm can be nicely seen. The energy transmission rate is also very large for wavelengths around $\lambda_{SP}$ as expected, but it drops rapidly when the film thickness is increased. This is so because the coupling between the surface plasmons on both interfaces becomes very small when $d$ is increased due to the evanescent nature of the surface plasmon polariton modes. This leads to a less efficient coupling of both TLS and therefore to very small entanglement for thicker metal films. Note, that at the ENZ wavelength of 259nm there is no significant effect of increased entanglement. The relatively large entanglement which can be seen in Fig. 2 for thick films is in the transparency region of the silver film for $\lambda < 259$ nm.

In order to contrast the results obtained for metal films, we consider as a second structure a multilayer hy-
Figure 3: $\gamma_s$, $\gamma_0$ and $\Omega_c$ for two TLS separated by a silver film of thicknesses $d = 10$ nm and 60 nm normalized to the free space emission rate $\gamma_0$. Left column is for x-orientation of the dipole moments and right column for the z-orientation. The dashed vertical line marks the surface plasmon wavelength $\lambda_{SP} = 291$ nm.

A hyperbolic meta-material of alternating Ag and TiO$_2$ layers. The effective permittivities are for this structure given by

$$\epsilon_\perp = f \epsilon_{Ag} + (1 - f) \epsilon_{TiO_2},$$  
$$\epsilon_\parallel = \frac{\epsilon_{Ag} \epsilon_{TiO_2}}{f \epsilon_{TiO_2} + (1 - f) \epsilon_{Ag}},$$

where $f$ is the filling fraction of silver and $\epsilon_{Ag}/\epsilon_{TiO_2}$ are the permittivities of the both constituents of the multilayer structure. For silver we use the Drude model in Eq. (37). TiO$_2$ is transparent in the visible regime. Its permittivity $\epsilon_{TiO_2}$ is nearly constant in that regime and can be well described by the formula [42]

$$\epsilon_{TiO_2} = 5.913 + \frac{0.2441}{\lambda^2 - 0.0803}.$$  

As shown in Ref. [23] when choosing $f = 0.35$ this multilayer structure has a type I hyperbolic band [$\text{Re}(\epsilon_\parallel) < 0$ and $\text{Re}(\epsilon_\perp) > 0$] at wavelengths below the epsilon-near-pole wavelength $\lambda_{ENP} = 395$ nm and a type II hyperbolic band [$\text{Re}(\epsilon_\parallel) > 0$ and $\text{Re}(\epsilon_\perp) < 0$] above the epsilon-near-zero wavelength $\lambda_{ENZ} = 551$ nm. For $\lambda_{ENP} < \lambda < \lambda_{ENZ}$ the multilayer structure behaves like a normal uniaxial dielectric [$\text{Re}(\epsilon_\parallel) > 0$ and $\text{Re}(\epsilon_\perp) > 0$].

In Fig. 4 we show the concurrence function for the HMM as a function of time and wavelength for $d = 60$ nm and 120 nm. It can be seen that for $d = 60$ nm the entanglement is especially large at the ENZ and ENP wavelengths. For $d = 120$ we have still a relatively large entanglement especially close to the ENZ wavelength and in the normal dielectric region with $\lambda_{ENP} < \lambda < \lambda_{ENZ}$. As can be seen in Fig. 5 this is in agreement with the large transmission at the ENZ wavelength which has been studied in very much detail in Ref. [23]. The difference to the silver film is that here we have propagating modes inside the hyperbolic material with large wavevectors allowing a strong coupling between the two TLS even for relatively thick films. The main limiting factor for this strong coupling is the damping of these propagating modes inside the hyperbolic materials. Apart from the fact that we can have large entanglement for relatively thick films hyperbolic materials have the advantage that the position of the ENP and ENZ wavelength can be engineered by the combination of different materials and by changing the filling fraction of the metal part of the
structure so that the wavelength for which a strong coupling is needed can be adapted at will.

V. CONCLUSION

To summarize, we have studied the entanglement of two TLS separated by a thin film using the master-equation approach and the concurrence function as entanglement measure. We have compared the entanglement as function of the film thickness of the intermediate layer for a silver film and a multilayer Ag/TiO$_2$ hyperbolic meta-material. Our main finding is summarized in Fig. 6 where the concurrence function is plotted for the silver film and the hyperbolic material close to the ENZ wavelength at 550nm for different film thicknesses. At this wavelength the single-atom emission rates and the energy transmission rates for the TLS in presence of the hyperbolic material are very large compared to the vacuum value. By changing the filling fraction of the silver layers in the Ag/TiO$_2$ hyperbolic meta-material one can shift this important frequency. It can be seen in Fig. 6 that for the hyperbolic material one can find the same entanglement as for the metallic film but for twice as thick layers. At the ENZ wavelength of the metallic film we find in most cases a relatively small entanglement as is shown in Fig. 6 so that for operation at the ENZ wavelengths hyperbolic metamaterial are much more advantageous than metal films. We believe that by optimizing the vertical and horizontal positions of the TLS and by optimizing the multilayer layout one can achieve substantial entanglement even for thicker films.

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Appendix

Appendix A: Derivation of $G_{xx}$ and $G_{zz}$

If the dipole moments of the TLS are oriented both in $z$ or $x$ directions. In this case the above expressions can be further simplified.

1. Dipoles in $z$-direction

If the dipole moments are oriented in $z$-direction we find

$$[a^+_k(k_0) \otimes a^+_k(k_0)]_{zz} = 0,$$

$$[a^+_k(k_0) \otimes a^+_k(k_0)]_{zz} = \frac{\kappa^2}{k_0^2}$$

and therefore

$$G_{zz}(r_1, r_2) = \int \frac{d\kappa}{2\pi} \frac{\kappa^2}{2k_z,_{\text{vac}} k_0^2} I_p \frac{\kappa^2}{k_0^2}$$

and

$$G_{zz}^{\text{single}}(r_1, r_2) = \int \frac{d\kappa}{2\pi} \frac{\kappa^2}{2k_z,_{\text{vac}} k_0^2} \left(1 + r_p e^{2i k_z,_{\text{vac}} |z_1|}\right).$$

From this expression we can retrieve the results for the case where the TLS are only coupled by vacuum by setting $I_p = 1$ and $r_p = 0$ so that

$$G_{zz}^{\text{vac}}(r_1, r_2) = \int \frac{d\kappa}{2\pi} \frac{\kappa^2}{2k_z,_{\text{vac}} k_0^2}$$

and

$$G_{zz}^{\text{single, vac}}(r_1, r_1) = \int \frac{d\kappa}{2\pi} \frac{\kappa}{2k_z,_{\text{vac}}}.$$

The emission rate for the single atom in vacuum is in this case

$$\gamma_{zz}^{\text{vac}} = \frac{6\pi\gamma_0}{k_0} \text{Im}(G_{zz}) = \frac{6\pi\gamma_0}{k_0} \frac{1}{4\pi} k_0^2 \frac{2}{3} = \gamma_0.$$
If the dipole moments of the TLS are oriented in x-direction we find
\[
[a_s^+(k_0) \otimes a_s^+(k_0)]_{xx} = \frac{k_s^2}{\kappa_s^2}, \quad (A8)
\]
\[
[a_p^+(k_0) \otimes a_p^+(k_0)]_{xx} = \frac{k_p^2 k_s^2}{\kappa_s^2 k_0^2}. \quad (A9)
\]
Introducing polar coordinates for \( \kappa = \kappa(\cos \theta, \sin \theta)^t \) we find therefore
\[
G_{xx}(r_1, r_2) = \int \frac{d\kappa}{2\pi} \frac{e^{i k_z, \text{vac}(d+2|z_1|)}}{2k_z, \text{vac}} \frac{1}{2} \left( t_s + t_p \frac{k_z, \text{vac}}{k_0^2} \right)
\]
(A10)
and
\[
G_{xx}^\text{single}(r_1, r_2) = \int \frac{d\kappa}{2\pi} \frac{i}{2k_z, \text{vac}} \left[ k_0^2 + k_z, \text{vac}^2 \right] \left[ k_0^2 \frac{k_z, \text{vac}}{k_0^2} \right]
\]
(A11)
Again we can retrieve the relations for the case where both TLS are coupled by vacuum by setting \( t_p = 1 \) and \( r_p = 0 \) so that
\[
G_{xx}^\text{vac}(r_1, r_2) = \int \frac{d\kappa}{2\pi} \frac{e^{i k_z, \text{vac}(d+2|z_1|)}}{2k_z, \text{vac}} \frac{1}{2} \left( 1 + \frac{k_z, \text{vac}^2}{k_0^2} \right)
\]
(A12)
and
\[
G_{xx}^\text{single, vac}(r_1, r_1) = \int \frac{d\kappa}{2\pi} \frac{k_0^2 + k_z, \text{vac}^2}{2k_0^2}
\]
(A13)

The emission rate for the single atom in vacuum is in this case again
\[
\gamma, \text{vac} = \frac{6\pi \gamma_0}{k_0} \text{Im}(G_{xx}) = \frac{6\pi \gamma_0}{4\pi} \frac{1}{k_0^2} \frac{2}{3} = \gamma_0.
\]
(A14)

2. Dipoles in x-direction

If the dipole moments of the TLS are oriented in x-direction we find
\[
\begin{align*}
[a_s^+(k_0) \otimes a_s^+(k_0)]_{xx} &= \frac{k_s^2}{\kappa_s^2}, \\
[a_p^+(k_0) \otimes a_p^+(k_0)]_{xx} &= \frac{k_s^2 k_z^2}{\kappa_s^2 k_0^2}.
\end{align*}
\]

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