Optimal Power Flow With State Estimation in the Loop for Distribution Networks

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Abstract—In this article, we propose a framework for running optimal control-estimation synthesis in distribution networks. Our approach combines a primal-dual gradient-based optimal power flow solver with a state estimation feedback loop based on a limited set of sensors for system monitoring, instead of assuming exact knowledge of all states. The estimation algorithm reduces uncertainty on unmeasured grid states based on certain online state measurements and noisy “pseudomeasurements.” We analyze the convergence of the proposed algorithm and quantify the statistical estimation errors based on a weighted least-squares estimator. The numerical results on a 4521-node network demonstrate that this approach can scale to extremely large networks and provide robustness to both large pseudomeasurement variability and inherent sensor measurement noise.

Index Terms—Distribution networks and power systems, feedback control, large-scale networks, optimal power flow (OPF), state estimation (SE), voltage regulation.

I. INTRODUCTION

The increasing penetration of distributed energy resources (DERs) has provided an opportunity to explore the benefits of advanced smart grid technologies in distribution networks. As the heterogeneous control strategies of grid-connected elements dominate distribution networks, many customers will become active and motivated end users to optimize their own power usage via optimal power flow (OPF) methods [1], [2], [3], [4], [5], [6], [7], [8]. This requires the power system control scheme to have real-time knowledge about the structure and state of the distribution network (e.g., operation states, net-loads variation, device dynamics, network topology, etc.), and to provide the corresponding real-time responses (e.g., optimal control inputs, set-points of DERs, etc.) for safe and efficient operation. However, the current distribution network control paradigm cannot satisfy the above requirement due to lack of real-time knowledge about the system state, and high expense of real-time state measurement. Future distribution systems will require more sophisticated and tightly integrated control, optimization, and estimation methods for these issues.

Most OPF methods for distribution networks in the literature assume complete availability of network states to implement various optimal control strategies [9], [10], [11], [12], [13]. However, in practice, network states must be estimated with a monitoring system from noisy measurements, which itself is a challenging problem due to the increasingly complex, extremely large-scale, and nonlinear time-varying nature of emerging networks. To address these issues, the authors in [14] proposed a hierarchical feedback-based OPF framework for extremely large-scale networks. The authors in [15] and [16] proposed an online feedback-based primal-dual algorithm to regulate the voltage in a distribution network. A unified real-time feedback-based algorithm was proposed in [17], which leverages a time-varying bilevel problem formulation to capture various performance objectives and engineering constraints. Two online voltage control frameworks were proposed for unbalanced distribution networks based on the projected newton method [18] and the first-order Taylor expansion of the branch flow model [19]. In addition, a feedback-based reactive power control framework was proposed in [20] to regulate the voltage in a distribution network. Overall, these recently proposed OPF frameworks [14], [15], [16], [17], [18], [19], [20] leverage real-time measurement feedback-based optimization methods to loop the physical measurement information back to controllers, which adapt the decisions to real-time data to mitigate the effects of inherent disturbances and modeling errors. In fact, it is unrealistic to have real-time physical measurements of system states at every corner of distribution networks due to heavy communication burdens, end-user privacy concerns, and high costs.

In transmission systems, the operational architecture for optimal dispatch, monitoring, and interlinks has been well developed for safe and reliable power delivery [21], [22]. A variety of
functions, including voltage regulation, economic dispatch, automatic generation control, and fault detection, can be achieved based on a sufficient communication network. However, it is unrealistic to have a well-established communication network for an extremely large-scale distribution network. Inspired by [21] and [22], in this article, we propose a more general framework than the existing OPF approaches, which tightly integrates state estimation (SE) techniques [23], [24], [25], [26], [27], [28] into online OPF control algorithms for distribution networks. The benefit of having SE in the loop in a distribution network is to analyze the tradeoffs between performance OPF solvers, costs of sensor deployment, and communication burden. The proposed OPF with SE in the loop framework allows us to utilize a limited set of real-time measurements together with a power system state estimator instead of exact knowledge of network states. Instead of deploying sensors at all nodes in a distribution network for exact knowledge of network states, this OPF with SE in the loop framework allows us to utilize a limited number of real-time sensor measurements of the electrical quantities of an operator’s interest (e.g., voltage magnitudes and current injections) and pseudomeasurements of all the power injections, together with a power system state estimator to solve a classic OPF problem. The power system state estimator, which may include data from the Supervisory Control and Data Acquisition (SCADA) system, phasor measurement units (PMUs), topology processor, and pseudomeasurements, provides the best available information about network states [29], [30], [31], [32] and in turn enables implementation and enhances the performance of OPF controllers.

Our approach allows OPF decisions to adapt to real-time time-varying stochastic DERs and loads, and compensates for disturbances and modeling errors, since SE results utilize measurement data from the actual nonlinear system response. In contrast to [33] using OPF solutions to enhance the estimation results considering different distribution network configurations, our work leverages the estimation results to support the primal-dual gradient-based OPF solvers for optimal dispatch decisions of DERs. A preliminary version of this work appeared in [34], and here, we significantly expand the work in several directions into the present paper. Our main contributions are summarized as follows.

1) We formulate a general convex OPF problem subject to power flow equations and network-wise coupling constraints. To integrate OPF with SE in the loop, we propose a primal-dual gradient-based OPF algorithm with SE feedback. Instead of requiring full knowledge of all system states, the controller utilizes at every gradient step real-time monitoring information from SE results to inform control decisions. Although real-time measurement feedback-based OPF and SE problems for distribution networks have been widely discussed. The crucial issues of feedback-based algorithms have been overlooked: in reality, current distribution networks do not have sufficient communication infrastructures and enough deployed real-time sensor measurements to measure adequate electrical quantities of interest (e.g., voltage magnitude, current injections, etc.) as inputs of the OPF solvers. Here, we are closing the loop between OPF and SE in large-scale distribution networks by utilizing state estimates [11], [35]. This allows us to react to real-time information of system states with a limited number of deployed sensors. In principle, the proposed framework is compatible with a variety of SE methods and control strategies in distribution networks. Here, we illustrate the approach through a voltage regulation problem, with voltage magnitude estimation in the loop.

2) We leverage linear approximations to the ac power flow equations to facilitate scalable and computationally efficient OPF formulation for SE feedback integration [15], [16], [36]. The voltage profile estimation uses a weighted least-squares (WLS) estimator. Convergence of the proposed gradient-based algorithm with SE feedback is analyzed. Additionally, we quantify the statistical estimation errors of the WLS estimator and show how the errors influence the theoretical results and convergence proofs in [34]. This analysis provides a measure of quality of the SE feedback associated with a particular allocation of sensors across the network.

3) The effectiveness, scalability, flexibility, and robustness of the proposed algorithm are demonstrated on a 4521-node multiphase unbalanced distribution network with 1043 (aggregated) net loads. With only 3.6% voltage measurement deployment, the integrated OPF controller with SE feedback effectively regulates the network voltage. The distributed algorithm using linearized distribution flow enables scaling to extremely large networks. The numerical results also indicate that the proposed OPF controller with SE feedback is robust to the inherent measurement noise and estimation errors.

The rest of this article is organized as follows. Section II discusses the general concept of OPF with SE in the loop for distribution networks. Section III formulates an OPF problem and introduces gradient algorithm with SE feedback. Section IV demonstrates numerical results on voltage regulations, and finally, Section V concludes this article.

II. OPF WITH SE IN THE LOOP

In this section, we propose an OPF solver with SE feedback. We first pose a general problem to highlight the overall approach, and in subsequent sections, we detail the model, objectives, constraints and state estimator for certain control and monitoring purposes.

Consider an OPF problem for distribution networks

\[
\min_{p, q} \sum_{i \in N} C_i(p_i, q_i) + C_0(p, q) \quad (1a)
\]

s.t. \[ g(r(p, q)) \leq 0 \quad (1b) \]

\[ (p_i, q_i) \in \mathbb{Z} \forall i \in N \quad (1c) \]
where $C_0(p,q)$ is a cost function capturing system objectives (e.g., cost of deviation of total power injections into the substation from preferred values) and the local objective functions $C_i(p_i,q_i)$ capture (a weighted mix of) generation costs, ramping costs, active power losses, renewable curtailment penalty, auxiliary service expenses, reactive compensation (comprising a weighted sum thereof), etc., at node $i \in \mathcal{N}$. We then define a vector $r(p,q) \in \mathbb{R}^{N_C}$ collecting the electrical quantities of an operator’s interest (e.g., voltage magnitudes, current injections, power injection at the substation, etc.), which depends on nodal power injections $p := \begin{bmatrix} p_1, \ldots, p_N \end{bmatrix}^T$ and $q := \begin{bmatrix} q_1, \ldots, q_N \end{bmatrix}^T$ through power flow equations. The function $g : \mathbb{R}^{N_C} \rightarrow \mathbb{R}^{N_C}$ models network constraints, including those on voltage magnitudes and angles, current injections, and line flows. The nodal power injections $(p,q)$ are constrained within convex and compact feasible sets $\mathcal{Z}_i$. Problem (1) is typically solved assuming that all needed system states are available. However, in practice, there is generally a lack of reliable measurement devices and communication infrastructure in distribution networks, rendering most system states directly unavailable, and thus, the conventional OPF approaches unimplementable. Therefore, the main challenges for solving (1) in practice lie in how to best integrate the estimates of current system states $r$ and understand tradeoffs between sensor deployment cost, SE performance, and OPF controller performance. We will tackle these challenges by integrating an SE feedback loop with a limited number of sensor measurements into OPF solvers and analytically characterize the overall performance thereof. This allows the OPF controller to respond to real-time information and update control decisions despite nodes without measurement in the grid. The overall approach is illustrated in Fig. 1.

In our general framework, the SE techniques may determine the system states using any or all of SCADA measurements, real-time voltage magnitude measurements, pseudomeasurements, and topology information to reduce estimation uncertainty. How to fuse different sources of information into OPF formulations remains largely an open question under exploration. We aim to indicate that there are many possibilities and research direction to potentially improve control and optimization in distribution networks through tight integration with SE. The increasing penetration of renewable energy resources and distributed generators enable the distribution networks with smart features, such as demand response and distributed automation. This allows the networks to turn to a more active and complex system with fast system response. An efficient, real-time monitoring of distribution networks should be looped into OPF controllers. In the rest of this article, we take the voltage regulation problem with voltage estimation as an illustrative example, which is based on a few voltage magnitude measurements and net-loads pseudomeasurements.

### III. GRADIENT-BASED OPF SOLVER WITH SE FEEDBACK

#### A. System Modeling

Consider a distribution network denoted by a directed and connected graph $G(\mathcal{N}_0, \mathcal{E})$, where $\mathcal{N}_0 := \mathcal{N} \cup \{0\}$ is a set of all “buses” or “nodes” with substation node 0 and $\mathcal{N} := \{1, \ldots, N\}$, and $\mathcal{E} \subset \mathcal{N} \times \mathcal{N}$ is a set of “links” or “lines” for all $(i,j) \in \mathcal{E}$. Let $V_i := |V_i| e^{j \angle V_i}$ and $I_i := |I_i| e^{j \angle I_i} \in \mathbb{C}$ denote the phasor for the line-to-ground voltage and the current injection at node $i \in \mathcal{N}$. The absolute values $|V_i|$ and $|I_i|$ denote the signal root-mean-square values and $\angle V_i$ and $\angle I_i$ corresponding to the phase angles with respect to the global reference. We collect these variables into complex vectors $v := [V_1,V_2,\ldots,V_N] \in \mathbb{C}^N$ and $i := [I_1,I_2,\ldots,I_N] \in \mathbb{C}^N$. We denote the complex admittance of line $(i,j) \in \mathcal{E}$ by $y_{ij} \in \mathbb{C}$. The admittance matrix $Y \in \mathbb{C}^{N \times N}$ is given by

$$ Y_{ij} = \begin{cases} \sum_{l \sim i} y_{il} + y_{ii} & \text{if } i = j \\ -y_{ij} & \text{if } (i,j) \in \mathcal{E} \\ 0 & \text{if } (i,j) \notin \mathcal{E} \end{cases} \quad (2) $$

where $l \sim i$ indicates connection between node $l$ and node $i$, and $y_{ii}$ is the self-admittance of node $i$ to the ground.

Node 0 is modeled as a slack bus. The other nodes are modeled as PQ buses for which the injected complex power is specified. The admittance matrix can be partitioned as

$$ \begin{bmatrix} I_0 \\
0 \\
y \mathbf{Y} \end{bmatrix} = \begin{bmatrix} I_0 \\
0 \\
y \mathbf{Y} \end{bmatrix} \begin{bmatrix} V_0 \\
v \\
0 \\
0 \end{bmatrix}. $$

The net complex power injection then reads

$$ s = \text{diag}(v) (Y^*(v)^* + \vec{y} (V_0)^*) \quad (3) $$

where superscript $(\cdot)^*$ indicates the element-wise conjugate of complex vector $v$.

To facilitate computational efficiency using convex optimization, here, we leverage a linearization of (3) as follows:

$$ r = Ap + Bq + r_0 \quad (4) $$

where the parameters $A$, $B$, and $r_0$ can be attained from various linearization methods, e.g., [37] and [38], which also depend on the choice of the electrical quantities of operators’ interests.
We want to emphasize that formulating OPF and SE problem with linearized ac power flow equations but including the SE feedback loop is a way to tradeoff computational efficiency and feasibility. An explicit definition of the linearization coefficients in a voltage regulation problem will be introduced in Section IV.

In the rest of this article, the linearized coefficient matrices $A$ and $B$ are fixed over times for simplification. From now on, we limit $r(p, q)$ to the aforementioned linearized power flow model (4).

Recall that $r \in \mathbb{R}^{N_i}$ represents certain electrical quantities of an operator’s interest (e.g., voltage magnitudes, current injections, power injection at the substation, and so on).

### B. OPF Formulation and Primal-Dual Gradient Algorithm

In this section, we introduce a general OPF problem and the pertinent gradient algorithm with idealized measurement feedback\(^3\) from nonlinear power flow to reduce modeling errors. The feasible operating regions $Z_i$ depend on the terminal properties of various dispatchable devices, e.g., inverter-based distributed generators, energy storage systems, or small-scale diesel generators. We make the following assumptions.

**Assumption 1 (Slater’s Condition):** There exists a strictly feasible point within the operation region $(p, q) \in Z$, where $Z := Z_1 \times \ldots \times Z_N$, so that
\[
g(r(p, q)) < 0.
\]

**Assumption 2:** A set of local objective functions $C_i(p_i, q_i) \forall i \in N$ is continuously differentiable\(^4\) and strongly convex as functions of $(p_i, q_i)$, and their first-order derivatives are bounded within their operation regions indicated as $(p_i, q_i) \in Z_i, \forall i \in N$; The system-wise objective function $C_0(p, q)$ is continuously differentiable and convex with its first-order derivative bounded. Furthermore, the constraint function $g$ is continuously differentiable and convex with bounded derivatives on its domain.

**Assumption 2** guarantees decent and convenient properties in performance analysis and is usually satisfied in real-world implementations, e.g., quadratic cost functions and linear constraints.

We write the regularized Lagrangian $\mathcal{L}$ for (1) as follows:
\[
\mathcal{L} = \sum_{i \in N} C_i(p_i, q_i) + C_0(p, q) + \mu^\top g(r(p, q)) - \frac{\eta}{2} \| \mu \|^2_2
\]
where $\mu \in \mathbb{R}^{N_{\mu}}$ is the dual variable vector associated with the general inequality constraints and we keep the feasible regions $\mu \in \mathbb{R}^{N_{\mu}}$, and $(p, q) \in Z$ implicit. To facilitate proof of convergence, the Lagrangian (5) includes a Tikhonov regularization term $-\frac{\eta}{2} \| \mu \|^2_2$ with a prescribed small parameter $\eta$ that introduces bounded discrepancy. The upshot of having a regularized term in Lagrangian (5) is that the gradient-based approaches can be applied to (5) to find an approximate solution of the original Lagrangian with improved convergence properties. The discrepancy due to the regularized term is discussed and quantified in [39].

To solve (1), we come to the following saddle-point problem:
\[
\max_{\mu \in \mathbb{R}^{N_{\mu}}} \min_{(p, q) \in Z} \mathcal{L}(p, q, \mu) \tag{6}
\]
which can be solved by an iterative primal-dual gradient algorithm to reach the unique saddle-point of (6) as
\[
\begin{align*}
r^k &= Ap^k + Bq^k + r_0, \tag{7a} \\
p^{k+1} &= p^k - \epsilon \nabla_p \mathcal{L}(p^k, q^k, \mu^k) \\
q^{k+1} &= q^k - \epsilon \nabla_q \mathcal{L}(p^k, q^k, \mu^k), \tag{7b}
\end{align*}
\]
\[
\mu^{k+1} = [\mu^k + \epsilon \nabla_\mu \mathcal{L}(x^k, \mu^k)]_{\mathbb{R}^{N_{\mu}}}, \tag{7c}
\]
so that (7) can be rewritten as
\[
x^{k+1} = [x^k - \epsilon \Phi(x^k)]_{\mathbb{R}^{N_{p}} \times Z} \tag{8}
\]
where $x^k := [(p^k)^\top, (q^k)^\top, (\mu^k)^\top]^\top$. Under Assumption 2, it can be shown [14] that $\Phi$ is strongly monotone and Lipschitz continuous, i.e., it satisfies for all feasible points $x_1$ and $x_2$ and for some constants $M > 0$ and $L > 0$ we have
\[
(\Phi(x_1) - \Phi(x_2))^\top (x_1 - x_2) \geq M \| x_1 - x_2 \|_2^2
\]
\[
\| \Phi(x_1) - \Phi(x_2) \|_2^2 \leq L^2 \| x_1 - x_2 \|_2^2.
\]
We now have the following convergence results.

**Lemma 1 ([14, Th. 1]):** Under Assumptions 1 and 2, if the step size $\epsilon$ satisfies
\[
0 < \epsilon < 2M/L^2 \tag{11}
\]
then algorithm (7) converges to the unique saddle point of (6).

### C. Feedback-Based Implementation

The optimization problem (1) and gradient algorithm (7) are based on a linearized power flow to guarantee its convexity and prove convergence to the saddle point. However, linearization errors cause the solution of (7) to be suboptimal or even infeasible for the system with nonlinear power flow. To address this issue, feedback-based online optimization methods [15], [36], [40] have been leveraged to reduce the effects of modeling error.

\(^3\)The “idealized” refers to the full measurement of vector $r$ without noise.

\(^4\)The continuous differentiability of objective functions is not necessary to have the bound in Theorem 2, but it contributes to the convergence performance in a large-scale network system.

\(^5\)We use the projection operator $[\cdot]_Z$ in primal updates instead of $[\cdot]_{Z_p}$ and $[\cdot]_{Z_i}$, since the feasible sets of active and reactive power are not independent, but correlated based on the terminal apparent power limits.
In particular, by replacing (7a) with the following nonlinear power flow model:

$$\mathbf{\hat{r}}^k = f(p^k, q^k),$$ (12)

a more accurate measurement $\mathbf{\hat{r}}^k$ can be used instead of an approximate model, to update the dual variables in (7c). This facilitates a real-time implementation to track the time-varying grid response, with proofs of convergence to a bounded region surrounding the optimum [15], [16].

To this end, the following information availability and communication structure are usually applied to conduct (7) in practice. A network coordinator is responsible for maintaining operational constraints by actively monitoring the values of $r$ and updating $\mu$ according to (7c). The network coordinator usually does not know the exact values of $p$ and $q$ at local DERs, so the updating of (7b) is executed by DERs in a distributed way.

However, another crucial issue of such feedback-based algorithms has been largely overlooked: in practice, there are too few monitoring devices in distribution systems to measure all components of $r$, and therefore, it is not possible to directly implement feedback-based algorithms to solve the problem (1). Our preliminary results [34] demonstrated that limited knowledge of system states can lead the OPF controller to cause constraint violations.

To enable an implementation of feedback-based OPF algorithms in distribution networks, and also to improve performance of algorithms that make use of “pseudomeasurements,” we integrate an SE algorithm based on a sparse set of available measurements, before performing the dual variable update (7c). This allows us to utilize improved information on the network state to make decisions, specifically improving information at nodes without measurement of the grid where there are no direct measurements. Fig. 2 illustrates the proposed OPF framework with SE in the loop.

### D. SE in the Loop

We define $\mathbf{z}^k = ([p^k]^\top, [q^k]^\top]^\top$ as the system states at iteration $k$, and the grid measurement model

$$\mathbf{y}^k = \mathbf{h}(\mathbf{z}^k) + \xi^k$$ (13)

where $\mathbf{y}^k \in \mathbb{R}^{N_y}$ is a measurement vector received at iteration $k$ comprising raw noisy measurements from sensors (e.g., voltage magnitude and angles) and pseudomeasurements (e.g., real and reactive power injections at load buses), and measurement function $\mathbf{h} : \mathbb{R}^{N_x} \rightarrow \mathbb{R}^{N_y}$. The vector $\xi^k \in \mathbb{R}^{N_y}$ models measurement errors, which are assumed to be independently and identically distributed Gaussian noise $N(0, \Sigma)$ where $\Sigma \in \mathbb{R}^{N_y \times N_y}$ is a diagonal matrix. The pseudomeasurements are modeled as sensor measurements corrupted by high-variance Gaussian noise based on historical data (e.g., customer billing data and typical load profile) that provide rough information about variations in the state of the grid [35].

To estimate grid states from the available measurements, we consider the WLS estimator [11], [35], [41] as

$$\mathbf{\hat{z}}^k_{SE} = \arg \min_{\mathbf{z}^k} \frac{1}{2} (\mathbf{y}^k - \mathbf{h}(\mathbf{z}^k))^\top \mathbf{W} (\mathbf{y}^k - \mathbf{h}(\mathbf{z}^k))$$ (14)

where the weight matrix is defined as $\mathbf{W} = \Sigma^{-1}$. Again, we consider the estimated active and reactive power injections $\mathbf{\hat{z}}^k_{SE} = ([\mathbf{p}^k_{SE}]^\top, [\mathbf{q}^k_{SE}]^\top]^\top$ as the primary states of a distribution network. The static operation of a distribution system is determined given the information of nodal power injections. The estimated electrical quantities of interest $r$ is then calculated from those primary estimations using the power flow model. In this article, we solve the OPF problems with SE in the loop by having one gradient update of the OPF problem with the input of a fully solved SE result. Hence, we used the same superscript $k$ in (13) and (14) as the iteration counter of the primal-dual gradient algorithm in (7).

**Definition 1 (Full Observability)** [25, 42]: A state-to-output system $\mathbf{y} = \mathbf{h}(\mathbf{z})$ is fully observable if $\mathbf{z} = 0$ is the only solution for $\mathbf{h}(\mathbf{z}) = 0$, which allows a unique solution to (14).

**Assumption 3**: The system (13) is fully observable by having pseudomeasurements of power injections at all the nodes of a distribution network.

Since distribution networks typically have only a sparse set of real-time sensor measurements of the electrical quantities of the operators’ interest, we use pseudomeasurements of power injections over all nodes to ensure full observability. The combinations of real-time sensor measurements and pseudomeasurements have been observed to be effective in [41] and [43]. The ultimate goal of this setting is to significantly reduce the cost of real-time measurements (e.g., deployment and communication) while ensuring full observability of the SE problem. Overall, the distribution network SE methods have been widely discussed and shown to be accurate and computationally efficient under nominal operating conditions of distribution networks [44].

Fig. 2 and Algorithm 1 illustrate and describe the proposed OPF controller with the SE feedback loop.

Note that the step 2 in Algorithm 1 is not implemented in the proposed OPF controller, but instead is a physical response of the power system to the up-to-date nodal power injections $(\mathbf{p}^k, \mathbf{q}^k)$. We utilize SE in the loop to compute a state estimate $\mathbf{\hat{r}}^k$, which then contributes to the update of dual variables $\mu^{k+1}$ in step 6. Our numerical experiments in Section IV compare

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6This definition should be distinguished from observability of linear dynamical systems. Here, we limit the definition of observability to power system static state estimation problems [27] throughout this manuscript.
Algorithm 1: OPF With SE in the Loop.

Require: net-loads initialization \((p^0, q^0)\) and \(\mu^k\)
1: for \(k = 0 : K\) do
2: \(\hat{r}^k \leftarrow \) nonlinear power flow \((p^k, q^k)\)
3: receive system measurement \(y^k\)
4: estimate the system states \(\hat{z}^k_{SE}\) based on \(y^k\) and calculate the electrical quantities of interest \(r^k(\hat{z}^k_{SE})\)
5: \[\begin{bmatrix} p^{k+1} \\ q^{k+1} \end{bmatrix} = \begin{bmatrix} p^k - \alpha \nabla_p \mathcal{L}(p^k, q^k, \mu^k) + \alpha \nabla_p C_0(p^k, q^k) \\ q^k + A^T \nabla_r g(r^k(\hat{z}^k_{SE})), \mu^k) \end{bmatrix}\]
6: \(\mu^{k+1} = [\mu^k + \epsilon \nabla_\mu \mathcal{L}(r^k(\hat{z}^k_{SE})), \mu^k)]_{\mathbb{R}^N_{\mu}}\)
7: end for

this approach with the direct use of noisy measurements and pseudomeasurements without any estimation scheme.

E. Convergence Analysis

To analyze the impact of the power flow model adopted by the proposed framework, we first define \(\tilde{\Phi}(\cdot)\) as the gradient mapping based on a nonlinear power flow model

\[\tilde{\Phi} : \{p^k, q^k, \mu^k\} \mapsto \begin{bmatrix} \nabla_p \mathcal{L}(p^k, q^k, \mu^k) \\ \nabla_q \mathcal{L}(p^k, q^k, \mu^k) \\ -\nabla_\mu \mathcal{L}(r^k(p^k, q^k, \mu^k)) \end{bmatrix}\]

where different from the mapping \(\Phi\) defined for (7), the linearized power flow \(r^k(p^k, q^k)\) in (7a) is replaced with a more accurate nonlinear model \(r^k(p^k, q^k) = f(p^k, q^k)\).

The computations and updates in steps 4–6 of Algorithm 1 are written more explicitly as

\[\hat{z}^k_{SE} = \arg\min_{z^k} \frac{1}{2} (y^k - h(z^k))^\top W (y^k - h(z^k))\]

\[\begin{bmatrix} p^{k+1} \\ q^{k+1} \end{bmatrix} = \begin{bmatrix} p^k - \epsilon (\nabla_p C(p^k, q^k) + \nabla_p C_0(p^k, q^k) \\ q^k + A^T \nabla_r g(r^k(\hat{z}^k_{SE}), \mu^k) \\ + B^T \nabla_r g(r^k(\hat{z}^k_{SE}), \mu^k)) \end{bmatrix}\]

\[\mu^{k+1} = [\mu^k + \epsilon (g(r^k(\hat{z}^k_{SE})) - \eta \mu^k)]_{\mathbb{R}^N_{\mu}}\]

where \(C(p^k, q^k) := \sum_{i \in \mathcal{X}} C_i(p_i, q_i)\). With that we define the gradient mapping with SE in the loop as \(\bar{\Phi}(\cdot)\), where the dual variables \(\mu\) are updated using the SE result from (15a) as follows:

\[\bar{\Phi} : \{p^k, q^k, \mu^k\} \mapsto \begin{bmatrix} \nabla_p \mathcal{L}(p^k, q^k, \mu^k) \\ \nabla_q \mathcal{L}(p^k, q^k, \mu^k) \\ -\nabla_\mu \mathcal{L}(r^k(\hat{z}^k_{SE}), \mu^k) \end{bmatrix}\]

Assumption 4: The squared error in the gradient mapping caused by SE has a uniform bound, i.e., there exists \(\alpha > 0\) such that

\[\mathbb{E} \left[\|\Phi(x^k) - \bar{\Phi}(x^k)\|^2\right] = \sigma^2_\Phi \leq \alpha \quad \forall x^k.\]  

This bound always exists and is realistic due to the physical limits of network states. It measures the influence of noisy measurements on the SE in the loop framework, which will be incorporated in our stochastic convergence analysis of Algorithm 1. At the saddle point of (6), the squared error caused by SE is

\[\sigma^2 := \mathbb{E} \left[\|\Phi(x^*) - \Phi(x^*)\|^2\right].\]

Assumption 5: The squared distance between the gradient mapping with SE in-the-loop and that with the nonlinear power flow model in (12) has a uniform bound, i.e., there exists \(\rho > 0\) such that

\[\|\Phi(x^k) - \Phi(x^k)\|^2 \leq \rho \quad \forall x^k.\]

Theorem 1: Suppose the step size \(\epsilon\) satisfies the condition (11) from Lemma 1. Under Assumptions 4 and 5, the sequence \(\{x^k\}\) generated by Algorithm 1 satisfies

\[\limsup_{k \to \infty} \mathbb{E} \left[\|x^{k+1} - x^k\|^2\right] = \frac{\rho + 3\alpha}{2M/\epsilon - L^2}\]

where \(x^* = [(p^*)^\top, (q^*)^\top, (\mu^*)^\top]^\top\) is saddle point of \(L\) in (6).

Proof: See Appendix.

The result (18) from Theorem 1 provides an upper bound on the expected squared distance between the sequence \(\{x^k\} | x^k := [(p^k)^\top, (q^k)^\top, (\mu^k)^\top]^\top, k \leq K, K \to \infty\) generated by our proposed OPF with SE in-the-loop algorithm (15) and the saddle point \(x^*\) of (6). Note that the final result of Algorithm 1 might have a bias; namely, the difference in (18) will statistically converge to a positive constant instead of zero. This is due to the SE and the power flow linearization errors. We will visualize the biased result of Algorithm 1 in Section IV and discuss potential solutions to eliminate it. The analytical bound on this bias also indicates that our proposed approach has a robust performance to the linearization approximation errors and the inherent measurement noises.

1) Inherent Measurement Noise: The online measurements are typically within 1%–2% of the actual values. The pseudomeasurements of active and reactive power can be regarded as a rough initialization to time-varying actual values of power (with up to 50% variations). These errors can be reduced through the estimation phase in (14), which improves the decisions of the OPF controller with SE feedback (15a), improving robustness to measurement noise and power variability;

2) Linearization Approximation Errors: The update of \([p, q]\) in (15b) and maybe also the state estimator (15a) utilize the linearized power flow model (4) to promote computational efficiency. The discrepancy between linearized and nonlinear power flow model is quantified in (17) by \(\rho\), when the electric quantities of interest \(r\) are realized by the nonlinear power flow of the physical network.

To summarize, the discrepancy in (18) depends on the following:

1) the step size of gradient update, the monotonicity, and Lipschitz coefficients of objective and constraint functions;
2) the difference between nonlinear and linearized power flow models;
3) a general bound to the error in gradient mapping caused by SE.
Our result reveals that the SE errors will not be accumulated with the increasing number of algorithm iterations. Moreover, the bound characterized in (18) can be made arbitrarily small by adopting sufficiently small $\epsilon$.

Remark 1 (Feedback-Based OPF): For an OPF problem in a large-scale distribution network, our method can effectively reduce the computational complexity by adopting the linearized power flow model in the problem formulation, and can improve accuracy by compensating for the modeling error with measurements from the physical network. SE bridges the gap between the theoretical design where all the quantities of interest are measured and the reality where only a few nodes are equipped with measuring devices. Analyses for the model-based feedback algorithms to solve OPF can be found in a few recent works [16], [36]. Extending the convergence analysis to the nonlinear power flow model will be pursued as a future research effort.

F. Estimation Error Analysis

In this subsection, we analytically quantify the errors of the SE algorithm under a linearized measurement model as

$$y^k = Hz^k + \xi^k$$

where $H \in \mathbb{R}^{N_z \times N_y}$ is a measurement matrix that could be obtained by linearizing the nonlinear measurement function around a given nominal operating point.

The closed-form solution to the linear WLS problem

$$\min_{x^k} \frac{1}{2} (y^k - H z^k)^\top W (y^k - H z^k)$$

is $\hat{z}_{\text{SE}}^k = (H^\top W H)^{-1} H^\top W y^k$. When the matrix $H^\top W H$ is nonsingular (which occurs when $W$ is positive definite and $H$ has full column rank), the estimate can be expressed as

$$\hat{z}_{\text{SE}}^k = (H^\top W H)^{-1} H^\top W H z^k + (H^\top W H)^{-1} H^\top W \xi^k$$

$$= z^k + (H^\top W H)^{-1} H^\top W \xi^k.$$  \(18\)

The WLS estimator is unbiased (since $E[\hat{z}_{\text{SE}}^k] = z^k$ with zero-mean noise $\xi$), and the variance is given by $\text{Var}(\hat{z}_{\text{SE},j}^k) = \sum_{i=1}^n \Gamma_{ji}\sigma_i^2$, where $\Gamma_{ji}$ denotes the $j$th element of $\Gamma = (H^\top W H)^{-1} H^\top W$, and $\sigma_i^2$ is the $i$th diagonal element of the measurement covariance matrix $\Sigma$. The confidence interval for the state estimate $\hat{z}_{\text{SE}}^k$ can be constructed as

$$\hat{z}_{\text{SE},j}^k \pm c \sqrt{\text{Var}(\hat{z}_{\text{SE},j}^k)} = \hat{z}_{\text{SE},j}^k \pm c \sqrt{\sum_{i=1}^n \Gamma_{ji}\sigma_i^2}$$

where $c$ can be chosen based on the prescribed confidence level. In addition to the bound in Theorem 1, the confidence interval provides a numerical performance metric with respect to the SE errors. In the next section, we will use this metric to quantify estimation errors caused by noisy voltage measurements and net-loads pseudomeasurements.

IV. NUMERICAL RESULTS

We conduct numerical experiments to close the loop between OPF and SE. In particular, we explore the tradeoff between the sensing and communication overhead and the performance of OPF controllers in a large network.

We use a modified three-phase unbalanced 4521-node distribution network (which was reduced from the 11 000-node network in Fig. 3 by merging the secondary loads into transformers) to demonstrate the effectiveness and scalability of the proposed OPF solver with SE in the loop. This extremely large system is divided into five clusters, and then, we utilized a spatially distributed optimization algorithm for computational affordability. The details of multiphase power flow modeling and the feasibility of distributed algorithm were discussed in [14]. Our simulations are conducted on a desktop with AMD Ryzen 7 2700X Eight-Core Processor CPU at 3.7 GHz, 64-GB RAM, Python 3.7, and Windows 10.

We consider a voltage regulation problem where the electrical quantities of interests are voltage magnitudes. In particular, we specify the vector $r$ as the voltage magnitude $r := [v_1, \ldots, v_N]^\top \in \mathbb{R}^N$ and consider

$$\text{OPF} - V : \min_{p, q, r} \sum_{i \in N} C_i(p_i, q_i) + C_0(p, q)$$

s.t.

$$|v| = Ap + Bq + |v_0|$$

$$(p_i, q_i) \in Z_i \forall i \in N.$$  \(19\)

The inequality constraints enforce the safety bounds $(\nu, \tilde{v})$ to voltage magnitudes. In particular, we linearize the ac power flow in the aforementioned voltage regulation problem around the voltage magnitude at the point of common coupling (PCC), which is commonly used in the literature. The coefficient matrices $(A, B)$ and constant vector $|v_0|$ can be attained based on the ac power flow linearization in rectangular coordinates [45]. Note that we did not observe large differences in the results when choosing different values for the linearization points. The
gradient-based OPF controller (15) utilizes the online voltage magnitude measurement and voltage estimation to make the system converge. For the SE problem in the loop, the primary estimation variables are the active and reactive power injections at all nodes, and then, the estimated voltage magnitude \( v \) is calculated by the nonlinear ac power flow model. We consider a cost function \( C_i(p_i, q_i) = (p_i - p_i^0)^2 + (q_i - q_i^0)^2 \) that minimizes the deviation of the power setpoints \( (p_i, q_i) \) from their nominal/preferred level \( (p_i^0, q_i^0) \) for node \( i \). The term \( C_0(p) = \alpha (P_0(p) - P_00)^2 \) penalizes the deviation of the total active power injection \( P_0(p) \) at the substation from its preferred values \( P_00 \). We choose a small weighting factor \( \alpha = 0.0005 \) to focus on voltage regulation.

The default voltage profile without any control\(^7\) is calculated by OpenDSS [46] and shown by blue dots in Fig. 4. The voltage limits \( \mathbf{v} \) and \( \mathbf{v} \) are set to 1.05 and 0.95 p.u. This particular network has a significant undervoltage situation. We implement Algorithm 1 to solve the aforementioned voltage regulation problem, while minimizing the objective function. We consider DERs with box constraints. The default net-loads settings can be found in [46] and [47]. The proposed framework can also be adapted to incorporate more complicated models and objectives for energy storage devices, photovoltaics, distributed diesel generators, etc.

Most literature assumed that the full knowledge of real-time voltage is available to a system operator, which requires unrealistic sensor deployment, heavy communication, and huge investment. To balance information availability and measurement overhead, we randomly gather voltage magnitude measurements at 3.6% of the nodes (i.e., about 160 nodes) with zero-mean Gaussian noise of 1% standard deviation.\(^8\) We also have pseudomeasurements of all the nodal power injections with zero-mean and huge 50% standard deviation, which guarantees full observability for SE. The estimated active and reactive power injections are the primary estimations in the proposed framework. The estimated voltage magnitudes are then calculated from those primary estimations using a nonlinear power flow model. The detailed model for voltage estimation can be found in our prior work [41]. We implement Algorithm 1 with step-size \( 7 \times 10^{-4} \) for primal updates and \( 1 \times 10^{-3} \) for dual updates.

Fig. 4 visualizes the voltage profile regulated by the proposed SE in the loop method with orange dots. In order to prevent voltage from failing below 0.95 p.u., the net loads must be curtailed based on the SE feedback information. The voltage magnitudes at most nodes are bounded within \([0.95, 1.05]\). On the left side of Fig. 4, there are a few nodes whose voltage magnitudes slightly violate the safety bound. This indicates that the final result of Algorithm 1 might have a bias due to the power flow linearization and estimation errors. After we utilize a tighter bound \([0.96, 1.05]\) to compensate for the inherent errors of the SE in the loop, the voltage profile on the right of Fig. 4 then meets the constraint. Here, as shown in Fig. 4, we only included a potential but simple solution to eliminate this bias by increasing the lower bound of the voltage constraint. For future work, we will develop a stochastic reformulation to improve the feasibility of the proposed framework by taking the estimation errors into account.

The average and maximum voltage estimation errors are shown in Fig. 5 for SE in the loop. For comparison, we also test a scenario where the OPF solvers directly utilize raw noisy measurements of all the voltage magnitudes. The noises of those raw measurements are subjected to zero-mean Gaussian distribution with 1% standard deviation of their true values. From Fig. 5, we observe that having SE in the loop will significantly reduce the estimation errors, compared to the direct usage of raw measurements. Moreover, we utilize the analysis in Section III-F to get Fig. 6, where the average errors under SE in-the-loop are almost always bounded by the analytic 99% confidence interval over 1000 OPF iterations.

Our previous work [14] proposed the feedback-based OPF problem having similar setups but without the SE feedback loop. Having exact measurement feedback information as the input of the primal-dual gradient-based OPF solver results in strict feasibility of voltage magnitude constraints; see [14, Fig. 6]. In this work, we remove this strong assumption of having exact

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\(^7\)We disable all rule-based control of voltage regulators, local capacitors and low-voltage transformers, etc., and only solve the nonlinear power flow.

\(^8\)We assume that the received measurement data are of decent quality following the assumed probability distributions without considering bad data contamination.
measurements of all nodal voltage magnitudes and introduce an additional SE feedback loop, which comes to the biased final OPF result as shown in Fig. 4(a), but is more realistic from the practical implementation perspective. The bias of the OPF result can be eliminated by reformulating the classic OPF problem as a stochastic OPF taking the power flow linearization and estimation errors into account.

To resolve the voltage violations caused by estimation errors, we impose a tighter voltage lower bound 0.96 p.u. based on the analytic confidence interval. As shown in Fig. 7, the network curtails more net-loads to achieve a more conservative voltage profile, which leads to a higher operational cost. Note that the proposed SE feedback-based OPF with a tighter bound \( v = 0.96 \) results in a similar convergence performance compared to the OPF solver with the measurement-based feedback [14]. We emphasize that there is always a tradeoff between the cost of the measurement system (e.g., number of deployed sensors, communication infrastructure, etc.) and the OPF controller performance (e.g., robustness, feasibility, and optimality). In general, our proposed approach provides utilities and system operators a framework to systematically design OPF controllers under a limited set of sensor measurements.

In summary, numerical results conclude that the proposed OPF controller with SE feedback is able to systematically reduce voltage estimation errors at unmeasured nodes and achieve safe voltage regulation. The benefits of closing the loop between OPF controllers and SE can be clearly observed from the perspectives of effectiveness, robustness, and efficiency.

V. CONCLUSION

In this article, we proposed a general OPF controller with SE feedback to facilitate the operation of modern distribution networks. The controller depends explicitly on the SE results derived from system measurements. In contrast to existing works, our method utilizes a feedback loop to the OPF controller to estimate the system voltages from a limited number of sensors rather than making strong assumptions on full observability or requiring full state measurements. The performance of our design is analyzed and numerically demonstrated. The numerical results demonstrate the effectiveness, scalability, and robustness of the proposed OPF controller with SE in the loop.

Our work launched an initial step to close the loop between control and SE in power systems. There are several lines of future work to further explore the benefits and overcome the limitations of this idea, including but not limited to the following:

1) performance evaluation of various OPF formulations with different SE techniques in the loop;
2) optimal sensor placement with SE feedback for better OPF performance;
3) OPF and SE codesign considering the estimation errors for a more efficient communication structure in a real network;
4) extension of the proposed framework to include the devices with integer decision variables, such as the OLTC tap position, capacitor banks, unit commitment of DGs, tie lines, etc.;
5) convergence analysis based on the nonlinear power flow;
6) performance discussions with different distributed algorithms, e.g., alternating direction method of multipliers;
7) extension to the time-varying power flow linearization.

APPENDIX

Proof of Theorem 1

Proof: Now we are ready to show the convergence of Algorithm 1. The expected distance between the generated sequence \( \{x^{k+1}\} \) and the saddle point of \( \mathcal{L} \) can be characterized as

\[
\mathbb{E} \left[ \|x^{k+1} - x^*\|_2^2 \right] \\
\leq \mathbb{E} \left[ \|x^k - \bar{\Phi}(x^k) - x^* + \epsilon \Phi(x^*)\|_2^2 \right] \\
= \mathbb{E} \left[ \|x^k - \bar{\Phi}(x^k) + \epsilon \bar{\Phi}(x^k) - \bar{\Phi}(x^k) - x^* + \epsilon \Phi(x^*) + \epsilon \Phi(x^*) - \epsilon \Phi(x^*)\|_2^2 \right]
\]
\[
\begin{align*}
&\leq E \left[ ||x^k - \epsilon \Phi(x^k) - x^* + \epsilon \Phi(x^*)||^2_2 + \epsilon^2||\Phi(x^k) - \Phi(x^*)||^2_2 \right] \\
&= E \left[ ||x^k - \epsilon \Phi(x^k) + \epsilon \Phi(x^k) - \epsilon \Phi(x^k) - x^* + \epsilon \Phi(x^*) - \epsilon \Phi(x^*)||^2_2 + \epsilon^2||\Phi(x^k) - \Phi(x^*)||^2_2 \right] \\
&\leq E \left[ ||x^k - \epsilon \Phi(x^k) - x^* + \epsilon \Phi(x^*)||^2_2 \right] \\
&+ E \left[ \epsilon^2||\Phi(x^k) - \Phi(x^*)||^2_2 + 2\epsilon^2||\Phi(x^k) - \Phi(x^*)||^2_2 \right] \\
&+ \epsilon^2||\Phi(x^k) - \Phi(x^*)||^2_2
\end{align*}
\]

By applying the SE estimation variance bound in Assumption 4, (20a) is relaxed to (20b). Then, we have the step size chosen as \(0 < \epsilon \leq \bar{\epsilon} < 2M/L^2\) from Lemma 1, which leads to \(0 < \Delta \leq \epsilon^2L^2 - 2\epsilon M + 1 < 1\). As \(k \to \infty\), \(\Delta^{k+1}\) on the right-hand-side in (20b) will vanish. Given such \(\Delta\) and any feasible initial point \(x^0\), we let \(k\) approach infinity to get the expectation of this discrepancy as

\[
\limsup_{k \to \infty} E \left[ ||x^{k+1} - x^*||^2_2 \right] = \frac{\rho + 3\alpha}{2M/\epsilon - L^2},
\]

This concludes the proof. □

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