The effect of axis ratio on the vortex-induced vibration of an elliptical cylinder

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Abstract. A numerical investigation is conducted to study the vortex-induced vibration (VIV) of an elliptical body subjected to transverse oscillation with the non-dimensionless mass (\(m_r\)) fixed to be 10 and Reynolds number set to be 100. Three sets of axis ratio (\(\Lambda\)) are considered in the analysis where, \(\Lambda\) is defined as the ratio of the major and minor axis, with minor axis aligned parallel to the flow. Spectral/hp element-based software, Nektar++ is employed for the solution of incompressible flow equations. The computations are conducted for reduced velocity ranging from \(2 \leq U^* \leq 9\). The amplitude response is directly proportional to the increase in axis ratio in association with the pre-initiation of lock-in regime for higher axis ratio. It is also noticed that the initial branch gets widened, whereas the lower branch is constricted when the axis ratio of the cylinder is increased. However, the vortex shedding pattern in all the branches is observed to be 2S and its various modes.

1. Introduction

Vortex induced vibration of a cylindrical structure are of interest because of enormous richness of flow dynamics aspects and a broad range of engineering practices (like marine structures, chimney stacks and bridges, etc.). Extensive research has been carried out in the recent decades to study the fundamental characteristics and physics of flow behind the bluff structures which can be found in the exhaustive reviews by (Bearman 1984), (Williamson and Govardhan 2004), (Williamson and Govardhan 2008) and (Sarpkaya 2004).

Among different cylindrical structures, the circular cylinder has been considered as a generic model to study the VIV characteristics because of its symmetricity and enriched flow characteristics. Comprehensive work done by ((Bearman 2011) and (Singh and Mittal 2005), few to be mentioned) provides fundamental physics to understand different characteristics of VIV around such an isolated cylinder. In contrast to this, the flow around an elliptical cylinder has not caught similar attention. To understand the flow characteristics behind such cross-section, (Radi, Thompson et al. 2013) studied the Strouhal number variation for the elliptical cylinder on varying the axis ratio. (Johnson, Thompson et al. 2001, Johnson, Thompson et al. 2004) analysed the wake patterns and low-frequency unsteadiness associated to elliptical cylinders with different cross-sections. Similarly, many previous studies
including, (Thompson, Radi et al. 2014) and (Faruquee, Ting et al. 2007) have shown how greatly the wake parameters, fluid forces, pressure and velocity distribution get influenced as the function of axis ratio. However, most of the investigations related to elliptical cross-section are focussed mainly on the stationary case, and only a few researches have been carried out to study the vortex-induced vibration (VIV) of an elliptical cylinder. An experimental investigation was done for cylinder with different axis ratio by (Zhao, Hourigan et al. 2019) to show the effect of lack of afterbody on the vibration response and synchronisation regime. (Leontini, Griffith et al. 2018) carried out the numerical simulation to study the effect of asymmetry on the flow patterns and dynamic response of cylinder with \( \Lambda \)-1.5 by varying the angle of attack. Very rich wake behaviour was observed, and it was also found the maximum amplitude of oscillation occurred when the cylinder is placed perpendicular to flow stream.

The above literature review indicates that where there are many investigations carried out for circular cylinders and stationary elliptical body, the study of the VIV phenomenon of an elliptical structure is still lacking. This study, therefore, presents the fluid dynamics behind such bluff structures by examining the effect of variation of axis ratio on the response characteristics and aerodynamic forces which will help to provide some contribution to the less explored elliptical cylindrical structure.

The article is outlined as follows: The methodology and governing equation are discussed in Section 2. This is followed by the problem setup and validation test in Section 3. Similarly, in Section 4, results and discussions are presented. The major findings are finally summarised in Section 5.

2. Governing equation and numerical method

The governing equations of the flow field for two dimensional and incompressible fluid are the Navier-Stokes equation and continuity equation which can be expressed in cartesian system as

\[
\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = - \frac{\partial p}{\partial x_i} + \frac{1}{\text{Re}} \frac{\partial^2 u_i}{\partial x_i \partial x_j} \ldots \ldots (1)
\]

\[
\frac{\partial u_i}{\partial x_i} = 0 \ldots \ldots \ldots (2)
\]

Here, \( u_i \) (or \( u_j \)) represents the flow velocity in \( x_i \) (or \( x_j \)) coordinate direction and \( p \) is the dynamic pressure. Similarly, another related parameter is Reynolds number \( \text{Re} \) which is based on the inlet free stream velocity, \( U_\infty \), characteristics length, \( D \), and kinematic viscosity, \( \nu \), and defined as \( \text{Re} = U_\infty D / \nu \).

The transverse dynamic response of a cylinder mounted on elastic support can be expressed by the following equation

\[
\ddot{Y} + 4\pi f_p \zeta \dot{Y} + (2\pi f_p)^2 Y = C_l / 2m \ldots \ldots (3)
\]

Here, \( Y \) is the transverse displacement, \( f_p \) denotes the natural frequency of the elliptical body and \( \zeta \) is the structural damping ratio considered to be zero in the present investigation. \((U^* = U_\infty / f_p D)\) represents the reduced velocity and \((m= m / p D^2)\) is the non-dimensional mass, where \( m \) is the mass of the body. Similarly, \( C_D \) and \( C_L \) are the instantaneous drag and lift coefficients defined as, \( C_L = 2F_L / p U_\infty^2 D \) and \( C_D = 2F_D / p U_\infty^2 D \) respectively, where, \( F_L \) and \( F_D \) are the force components acting in the crossflow and inline directions.

The computational domain is discretised using a quadrilateral spectral element which is internally sub-divided with high order Jacobian basis polynomial. Similarly, velocity correction scheme is used for the temporal discretisation employing a second-order time-splitting scheme. Spectral/hp element method based software, NEKTAR++((Karniadakis and Sherwin 2013) and (Cantwell, Moxey et al. 2015)) is then employed for solving the coupled fluid-structure interaction problem.

3. Description of the problem
3.1. Simulation parameter
In the present study, three sets of axis ratios ($\Lambda$-1.0, 1.33 and 2.0) (defined based on Fig 1(a)) free to vibrate in transverse direction only is considered with a non-dimensional mass of 10.0. The simulations are carried at Reynolds number, 100 and for reduced velocities ranging from ($U^* = 2.0$ - 9.0). C-type computational domain is considered in current simulation, with downstream boundary positioned at 50$D$. The boundary condition at the inlet, outlet and lateral boundaries are specified, as shown in Fig 1(b).

3.2. Code validation test
The current methodology is validated against (Singh and Mittal 2005) where the transverse oscillation parameter of an isolated circular cylinder is taken as a reference to check its accuracy. The results presented in Fig 2. shows a remarkable agreement which confirms the accuracy of the current simulation.

4. Result and discussion
4.1. Amplitude response
Fig. 3 presents the variation of rms (root mean square) value of transverse vibration response of cylinder as the function of $U^*$ for different $\Lambda$. It is observed that the amplitude response and its associated branches are altered as the elliptical ratio is changed. We can clearly notice that as the axis ratio of cylinder is increased the transverse oscillation is magnified and for $\Lambda$=2.0, the peak amplitude is almost 1.5 times that of a circular cylinder. This suggests that on reducing the afterbody, the vibration response gets amplified. The experimental study by (Zhao, Hourigan et al. 2019) also shows resemblance to the
inference drawn by the present study. It is also apparent from the Figure that on increasing the axis ratio of the cylinder, the range of the initial branch gets widened whereas the lower branch is constricted. This phenomenon observed in the response can be well clarified on further discussion in the upcoming section.

![Image](image_url)

**Fig 3:** Variation of rms value of transverse vibration of the cylinder with $U^*$ for different axis ratios

### 4.2. Force coefficient:

The rms value of the lift and drag coefficient is presented in Fig 4(a and b). We can see that as $A$ is increased, the range of $U^*$ for maximum lift contribution is found to be earlier. The lift force is further decomposed into viscous ($C_{LV}$) and pressure ($C_{LP}$) force to provide further insight of force component acting on the body. We can observe that viscous contribution for higher $A$ is more as observed in Fig 4(c) which is obvious because as the axis ratio is increased the gradient of the boundary causes the viscous force to act towards the lift direction. Though the maximum lift force generated for $A$-1.33 is higher, the amplitude response for $A$-2.0 is observed to be more. This can be made clear with the help of lock-in regime, which will be described in Section 4.3. Similarly, we can see that the drag coefficient has the trend similar to that of amplitude response (Refer Fig 4(b)) and for higher $A$ the drag is more because of large oscillation and relatively abrupt flow separation.
Fig 4: Variation of the rms value of hydrodynamic force coefficient with reduced velocity (a) Total lift, (b) Total drag, (c) Viscous lift and (d) Pressure lift

4.3. Frequency ratio

The variation of reduced frequency of vortex shedding and transverse oscillation of cylinder with different $A$ against $U^*$ is presented in Fig. 5. Here, $f_n = 1/U^*$ is the structural natural frequency of the cylinder, $f_l$ being the vortex shedding frequency, $f_y$ is the transverse oscillation frequency. Wide range of lock-in can be observed such that the $f_y$ is almost equal to $f_l$. It can also be noticed from the Figure that for higher axis ratio, the onset of lock-in is earlier compared to that of lower axis ratio. Similarly, it is noticed that the soft lock-in phenomenon is observed for $A$-1.33 and 1.0 whereas perfect lock-in takes place for $A$-2.0 in the initial branch. This is the primary reason why we account a relatively large amplitude of oscillation for a cylinder with $A$-2.0. Another interesting fact is noticed when $f_l$ reaches
back to the vortex shedding frequency of stationary cylinder (marked by an asterisk). We can observe that as the axis ratio is lowered the range of $U^*$ for which the synchronisation persist gets widened which is the major reason for the wider lower branch as the axis ratio of cylinder is decreased. In the desynchronisation state, however, the cylinder with all the axis ratios switches back to that of vortex shedding frequency of the stationary elliptical cylinder as seen in Fig 5.

4.4. Vortex shedding patterns
The spanwise vorticity at an instant shows a 2S mode and its variants for all axis ratios. On the initiation of the initial branch, $C_{NW} (2S)$ mode (Yogeswaran, Sen et al. 2014) is observed in which the vortices of same sign merge in the near wake which causes an increase in the lift force (Refer Fig 6(a)). Similarly, parallel vortex shedding occurs in the lock-in region corresponding to large vibration for $A$ -2.0, which interacts at farther downstream to form a longer wavelength vortex street as seen in

![Image](a)

![Image](b)

![Image](c)

![Image](d)

Fig. 6: Instantaneous vorticity field for the representative case for different axis ratio at various $U^*$ (a) $A$ -2.0 $U^*$=3.0, (b) $A$ -2.0, $U^*$=4.3, (c) $A$ -1.33, $U^*$=5.0 and (d) $A$ -1.33, $U^*$=9.0

Fig. 6(b) whereas C(2S) mode is encountered for other $A$ (See Fig 6(c)). In the desynchronization state, 2S mode (Fig. 6(d)) is observed for all $A$, but for $A$ -2.0, this 2S vortices arrange themselves to a two-layered zone at a certain distance downstream.

5. Conclusions:
This numerical study has been conducted to study the transverse vortex-induced vibration of the elliptical cylinder for axis ratios ($A$ =1.0, 1.33, 2.0) using Nektar++, a spectral/hp element-based software. The simulations are carried at Reynolds number of 100 with non-dimensional mass ($m_r$) of cylinder fixed to be 10. It is found that with the increase in axis ratio, the peak amplitude of vibration is increased. Moreover, the onset of lock-in is also found to be earlier. However, the extent of the lock-in range is found to be truncated, and its effect is prominent in the lower branch. The initiation of the initial branch is characterised by $C_{NW}(2S)$ mode causing intensification in the lift force. With the increase in $U^*$, C(2S) regime is observed in lock-in region for $A$-1.33 and 1.0, but due to high amplitude of vibration, parallel vortex shedding mode is also noticed for $A$ -2.0.

This work can be further extended to study the energy that can be harvested considering multiple cylinders with tandem arrangements.

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