Passive earth pressure under the log-spiral failure mechanism

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Abstract. Under either convex or concave log-spiral failure mechanism, the passive earth pressure may be overestimated. This paper develops a new formula satisfying the moment limit equilibrium condition, which combines a new procedure to determine the passive earth pressure. The procedure could consider convex and concave log-spiral failure mechanisms simultaneously and figure out which failure mechanism is critical. The present method can investigate the influence of the inclination of wall, the slope of backfill or the wall friction on the shape of the critical slip surface. Additionally, the comparisons illustrate that the critical slip surface is concave log-spiral under certain conditions.

1. Introduction
The passive earth pressure plays an indispensable role in the engineering practice of foundations and retaining walls. From the point of view of safety design, the reasonable estimate of the passive earth pressure on those structures is therefore particularly important. When those structures occurs the passive failure, according to the shape of the critical slip surface, there are usually three failure mechanisms, i.e., planar, log-spiral, and composite failure mechanisms. The last one here has such critical slip surface combining a log-spiral and a straight line.

For the planar failure mechanism, the passive earth pressure got from Coulomb’s theory is the most widely accepted among practicing engineers [1]. However, Coulomb’s theory will result in the overestimate of the passive earth pressure when designing rough retaining walls [2]. Many experiments for earth pressure problems, moreover, have shown that the critical slip surface is log-spiral or composite [3]. In other words, the determination of the passive earth pressure is more reasonably based on the log-spiral or the composite failure mechanism. For these two failure mechanisms, the passive earth pressure is calculated, some by the limit equilibrium (LE) method [4-8], others by the limit analysis (LA) [9-13].

In the analysis of the passive earth pressure under the composite failure mechanism, the concave slip surface is occasionally involved [4, 6, 10]. This kind of the slip surface has been in reality found in the experiment and numerical analysis [14-15]. Nevertheless, the critical slip surface is the convex or concave log-spiral that is hard to determine. Current analysis under the log-spiral failure mechanism is thus often considering either the convex log-spiral slip surface only [5, 12, 16] or just the concave one [17], which will lead to an overestimation of the passive earth pressure.

In this paper, a new passive earth pressure formula that satisfies the moment LE condition is developed. The proposed formula based on the log-spiral failure mechanism, can take into account the effects of the inclination of wall, the slope of backfill, the wall friction and the internal friction of
backfill. A new procedure is then put forward to figure out which of convex and concave log-spiral failure mechanisms is critical one to calculate the passive earth pressure.

2. Formulation

2.1 Log-spiral failure mechanism

Figure 1 illustrates the parameters appeared in the convex and concave log-spiral failure mechanisms, respectively, to determine the passive earth pressure. To facilitate the formulation, the assumptions are therefore made as follows: 1) the backfill is homogenous and cohesionless; and 2) D is equal to H/3.

Figure 1. Log-spiral failure mechanism for passive earth pressure problem with (a) convex and (b) concave log-spiral slip surfaces

Similar to the derivation process by Baker and Garber [18], the radius vector for the log-spiral slip surfaces in pole coordinates, R, can be expressed as follows:

\[ R = Ae^{-\psi\beta} \] (1)

The formulation of A can be deduced by two different formulas of Y(E) (i.e., the ordinate value of Point E in the Cartesian coordinates, see Fig.1) which are Formula I: \[ Y(E) = H + [R_2 \sin \phi - R_1 \sin \phi] + H \tan \phi \] and Formula II: \[ Y(E) = R_1 \cos \phi - R_2 \cos \phi \]. Combining with Formulas I and II, we can obtain the formula for A that is as follows:

\[ A = \frac{H(1 + \tan \phi \tan \alpha)}{e^{-\psi\beta} (\cos \phi_1 + \sin \phi_1 \tan \alpha) - e^{-\psi\beta} (\cos \phi_2 + \sin \phi_2 \tan \alpha)} \] (2)

2.2 Passive earth pressure coefficient formula

The passive earth pressure coefficient, KP, can be defined herein as:

\[ K_P = \frac{2P_e}{\gamma H^2} \] (3)
Writing the moment limit equilibrium equation about the pole of the slip surface, \((X_C, Y_C)\), and rearranging the terms to match the format of Eq. (3), one can get

\[
K_p = \frac{2}{H} \int_{\beta_1}^{\beta_2} (Ae^{-\omega'} \sin\beta - Ae^{-\omega'} \sin\beta')(Ae^{-\omega'} \cos\beta)(\sin\beta + \omega \cos\beta)
\]

\[
+ \frac{1}{H^2} (Ae^{-\omega'} \sin\beta - Ae^{-\omega'} \sin\beta')^2 \tan\omega (Ae^{-\omega'} \sin\beta)
\]

\[
+ \frac{Ae^{-\omega'} \sin\beta - Ae^{-\omega'} \sin\beta}{3} - (\tan\omega + \tan\omega \tan^2\omega) (Ae^{-\omega'} \sin\beta)
\]

\[
- \frac{H \tan\omega}{3} \cos(\omega - \delta)
\]

\[
\text{Eq. (4) can be solved via a simple minimization process. This process is analogous to finding the minimum safety factor in an unstable slope that is based on the log-spiral failure mechanism [18]. Due to convex and concave log-spiral failure mechanisms having different ranges of } \beta - [\beta_1, \beta_2], \text{ the minimization process will thus yield two values of } K_p \text{ which one is from convex log-spiral failure mechanism, and the other is from concave log-spiral failure mechanism. The less } K_p \text{ is the true value, and then the critical log-spiral failure mechanism will be figured out accordingly.}
\]

3. Results and comparisons

3.1 Critical slip surfaces

3.1.1 Effect of the inclination of wall

![Critical slip surfaces under different inclination of wall](image)

**Figure 2.** Critical slip surfaces under different inclination of wall

When \(\alpha = 0, \delta = 0, \phi = 35^\circ\), and \(\omega = -20^\circ, -10^\circ, 0, 10^\circ, 20^\circ\), \(K_p\) is equal to 6.78, 4.85, 3.69, 2.97, and 2.53, respectively. In the Fig.2, the critical slip surface is convex log-spiral for negative \(\omega\), but is concave log-spiral for positive \(\omega\).

3.1.2 Effect of the slope of backfill
When $\omega = 0$, $\delta = 0$, and $\phi = 35^\circ$, $\alpha = 20^\circ$, $10^\circ$, $0$, $-10^\circ$, $-20^\circ$, $K_P$ is equal to 6.78, 5.08, 3.69, 2.59, and 1.75, respectively. In the Fig.3, the critical slip surface is convex log-spiral for positive $\alpha$, but is concave log-spiral for negative $\alpha$.

### 3.1.3 Effect of the wall friction

When $\omega = 0$, $\alpha = 0$, $\phi = 35^\circ$, and $\delta/\phi = 2/3$, $1/3$, $0$, $-1/3$, $-2/3$, $K_P$ is equal to 7.95, 5.44, 3.69, 2.48, and 1.63, respectively. In the Fig.4, the critical slip surface is convex log-spiral for positive $\delta$, but is concave log-spiral for negative $\delta$.

From Fig. 2 to Fig.4, we can see that when $\omega = 0$, $\alpha = 0$, $\phi = 35^\circ$, and $\delta/\phi = 0$, the critical slip surface degenerates to the planar surface, and the $K_P$ is equal to the value calculated by the Coulomb’s theory or the method based on current log-spiral and composite failure mechanisms [11-12]. It can also be said that the planar failure mechanism is exceptional case in comparison with the log-spiral or composite failure mechanism.

### 3.2 Comparisons

Though the upper-bound LA (ULA) is equivalent to the variational LE method when determining the passive earth pressure under the log-spiral failure mechanism, the current analysis ignores the case of exiting concave slip surface [5, 12]. The comparison is to show the difference between ULA under the convex log-spiral failure mechanism and the moment LE (MLE) method under both convex and concave log-spiral failure mechanisms when calculating the value of $K_P$, which is illustrated in Table 1.

| $\phi$ | $\alpha/\phi$ | $\delta/\phi = 0$ | $\delta/\phi = 1/3$ | $\delta/\phi = 2/3$ |
|--------|---------------|-------------------|-------------------|-------------------|
| 25     |               | ULA               | MLE               | ULA               | MLE               | ULA               | MLE               |
| 0      | 2.46          | 2.46              | 0                 | 3.07              | 3.07              | 0                 | 3.76              | 3.76              | 0                 |
| -1/3   | 1.94          | 1.92              | -1.03             | 2.32              | 2.32              | 0                 | 2.77              | 2.77              | 0                 |
| -2/3   | 1.47          | 1.42              | -3.40             | 1.65              | 1.63              | -1.21             | 1.88              | 1.88              | 0                 |
When $\phi$ ranges from 25° to 40°, $\delta \phi=0$, and $\alpha \phi=1/3$ or $-2/3$, $K_p$ by MLE compared by ULA decreases more than 1%, even up to 7.19%. When $\phi$ ranges from 25° to 40°, $\delta \phi=1/3$, and $\alpha \phi=-2/3$, $K_p$ by MLE compared by ULA decreases more than 1%, even up to 2.40%. This is because MLE in the paper considers the concave log-spiral failure mechanism. In the other words, the critical slip surface here is concave log-spiral, not convex log-spiral. In addition, the value of $K_p$ by MLE is equal to that by ULA, that is to say, the critical slip surface is convex log-spiral or planar.

4. Conclusions

A moment LE approach is presented to determine the passive earth pressure under the log-spiral failure mechanism. This approach can ascertain which slip surface (plane, convex or concave log-spiral) is critical. The results show that the positive value of the inclination of wall, or the negative value of the slope of backfill or the wall friction tends to produce the concave log-spiral slip surface, on the contrary, that is inclined to bring the convex one. It is also found that the passive earth pressure will be broadly overestimated based on the planar failure mechanism in virtue of which is not general failure mechanism. Compared with the upper-bound LA under the convex log-spiral failure mechanism, the present method illustrates that the concave log-spiral slip surface, in some cases, is critical instead of the convex one.

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