CALORIMETRY OF ACTIVE GALACTIC NUCLEUS JETS: TESTING PLASMA COMPOSITION IN CYGNUS A

M. Kino1, N. Kawakatu2, and F. Takahara3

1 National Astronomical Observatory of Japan, 2-21-1 Osawa, Mitaka, Tokyo 181-8588, Japan
2 Graduate School of Pure and Applied Sciences, University of Tsukuba, 305-8571 Tsukuba, Japan
3 Department of Earth and Space Science, Osaka University, 560-0043 Toyonaka, Japan

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ABSTRACT
We examine plasma composition of jets in active galactic nuclei through the comparison of the total pressure (P) with partial pressures of electrons and protons in a cocoon. The total pressure is estimated from the analysis of expanding cocoon dynamics. We determine the average kinetic energy per particle for several representative cases of particle energy distribution such as one- and two-temperature thermal plasmas and non-thermal electrons by evaluating the dissipation of the total kinetic energy of the jet into the internal energy of cocoon plasma. The number density of the total electrons/positrons ($n_{\pm}$) in the cocoon is constrained by using the particle supply from hot spots and the absence of thermal bremsstrahlung emission from radio lobes. By inserting $P$, $n_{\pm}$, and the particle energy of each population into the equation of state, the number density ($n_e$) and pressure ($P_p$) of protons in the cocoon can be constrained. Applying this method to Cygnus A, we find that (1) electron/positron ($e^\pm$) pairs always dominate in terms of number density, but that (2) either an “$e^\pm$-supported cocoon (i.e., $P_{e} > P_p$)” or a “proton-supported one (i.e., $P_{\pm} < P_p$)” is possible.

Key words: galaxies: individual (Cygnus A) – magnetic fields – radiation mechanisms: non-thermal – radio continuum: galaxies – X-rays: galaxies

1. INTRODUCTION
Elucidating the formation mechanism of relativistic jets in active galactic nuclei (AGNs) is one of the greatest challenges in astrophysics of this century (e.g., Blandford & Znajek 1977; McKinney 2006; Komissarov et al. 2007; McKinney et al. 2012). Plasma composition of jets is a fundamental but difficult issue (see Begelman et al. 1984 for review), because emission timescales of the bulk population such as low-energy electrons/positrons and protons are too long. To examine plasma composition, discrete blobs in blazar jets have been utilized over the years. So far, three approaches have been pursued. The first is based on the synchrotron self-absorption limit combined with total kinetic powers of jets (Reynolds et al. 1996; Hirotani et al. 1999, 2000; Hirotani 2005). The literature indicates the existence of $e^\pm$ pair plasma in M 87, 3C 279, and 3C 345. The second is the constraint by the detection of circular polarization. Wardle et al. (1998) and Homan et al. (2009) examined the case of 3C 279 and found that the minimum Lorentz factor of non-thermal electrons/positrons should be much larger than unity for electron–proton (hereafter $e/p$) content. They rather favored an alternative possibility of dominant $e^\pm$ pair content with a small minimum Lorentz factor of non-thermal electrons/positrons (see, however, Ruszkowski & Begelman 2002). The third approach is the constraint from the absence of bulk-Compton emission in flat spectrum radio quasars (Sikora & Madejski 2000; Ghisellini & Tavecchio 2010), and it has been observationally tested for PKS 1510−089 and SWIFT J0746.3+2548 (Kataoka et al. 2008; Watanabe et al. 2009). The same approach has also been applied to the kiloparsec-scale knots in PKS 0637−752 (Georganopoulos et al. 2005; Uchiyama et al. 2005; Mehta et al. 2009). They claim that jets contain more $e^+e^-$ pairs than protons, but that jets are dynamically dominated by protons. However, it should be noted that the estimate of a total kinetic power $L_k$ of each blob is difficult because of the existence of invisible components such as low-energy electrons/positrons and protons. Therefore, the assumption of constant $L_k$ was made, and the $L_k$ are inferred from non-thermal emissions. Since plasma composition is sensitive to $L_k$, a better estimate of $L_k$ is essential. Regarding the estimate of $L_k$, it is essential to take into account the thermal component (e.g., Kino & Takahara 2008).

Cocoon associated with Fanaroff-Riley I and II (FR I and FR II) radio galaxies are also known to be good tools for exploring plasma composition. In contrast to blobs in blazars, investigations using cocoon dynamics allow us to better estimate the energy injection into the cocoon. The total pressure $P$ can be estimated with fewer uncertainties based on the dynamic interaction between jets and the intracluster medium (ICM), and $P$ involves the contributions of invisible components (e.g., Rawlings & Saunders 1991; Fabian et al. 2002). For FR I radio galaxies, many authors have discussed the ratio of $P$ to that of non-thermal electrons ($P_{NT}$) for various sources based on observed non-thermal emissions (e.g., Dunn et al. 2005; Croston et al. 2005; Rafferty et al. 2006; De Young 2006; Birzan et al. 2008). First of all, we should emphasize that these studies indicate that the total pressure $P$ tends to be greater than that of non-thermal electrons, i.e., $P > P_{NT}$, which means that the finite pressure of low-energy electrons/positrons and/or protons is required in these sources. The derived $P/P_{NT}$ values in the previous work extend over a wide range from the order of unity to thousands (e.g., Birzan et al. 2008; Cavagnolo et al. 2010). For FR I sources, however, an entrainment process of surrounding medium via the jet boundary layer could play a role (e.g., De Young 1993; Bicknell 1984; Rossi et al. 2008), and the process makes jets heavier. Therefore, jets in FR I sources could undergo severe proton loading during their propagations, and this could cause the large scatter of $P/P_{NT}$.

In this work, we focus on FR II radio galaxies (Figure 1) from the viewpoint of the important advantage they represent. Contrary to FR I sources, we know from relativistic hydrodynamic simulations that no significant entrainment appears for FR II sources (Scheck et al. 2002; Mizuta et al. 2004). Therefore, a plasma composition test for FR II radio galaxies would allow
The allocation of partial pressure of each plasma population is the central concern of this paper. In general, \( P \) is decomposed to

\[
P = P_{\text{e}} + P_{\text{p}} + P_B = p_T + p_T^{\text{NT}} + p_T^{\text{NT}} + p_T^{\text{NT}} + p_T^{\text{NT}} + p_B,
\]

where \( P_T^\text{NT}, P_T^\text{NT}, P_T^\text{NT}, P_T^\text{NT}, P_T^\text{NT}, \) and \( P_B \) are the partial pressures of thermal (T) electrons, thermal positrons, thermal protons, non-thermal (NT) electrons, non-thermal positrons, non-thermal protons, and the magnetic pressure, respectively. We also define \( P_B = P_{\text{e}} + P_{\text{p}} \) as the sum of the total pressures of electrons and positrons. Throughout this work, we do not include the magnetic pressure because it is sub-dominant in \( P \). Isobe et al. (2005) summarize the energy density of energetic electrons as typically being 10 times larger than that of magnetic fields in various radio lobes (e.g., Isobe et al. 2002; Tashiro et al. 1998, 2009; Hardcastle & Croston 2010), and it also holds in Cygnus A (Yaji et al. 2010).

2.1. Basic Idea of the Method

The essence of our method is as follows: First, the total pressure in the cocoon \( P \) is determined through dynamic considerations following I08, where they obtained \( P \) via the comparison of the expanding cocoon model with radio observations. Second, average energy per particle in the cocoon is evaluated. It is essential that our formulation is based on the basic conservation laws of mass, momentum, and energy in the cocoon. Since it depends on coupling of protons to the electrons/positrons, we examine several representative cases with different equations of state. Third, \( n_{\text{e}} \) can be partially constrained using the absence of thermal bremsstrahlung emission from the cocoon and the supply rate of electrons from the hot spots. Finally, \( n_{\text{e}} \) and \( n_p \) can be obtained by inserting the obtained quantities into the equation of state (EOS).

2.2. On Particle Distribution Functions

Since observational data at low frequencies below GHz are quite limited, it is hard to explore the properties of low-energy electrons (including positrons). Bearing this difficulty in mind, we pick up plausible cases of electron distribution function. As in the canonical case referred to as case (a), we consider two-temperature thermal plasmas, where protons and electrons have different temperatures and contributions of non-thermal components to the total pressure are negligible. As an alternative, we also examine case (b), in which protons and electrons take the same temperature without non-thermal components.

We further explore two cases, (c) and (d), in which the non-thermal population makes a dominant contribution to the total pressure with negligible pressure from the thermal population. For the non-thermal population, we assume the power-law distribution functions

\[
n_{\text{NT}}(\gamma_{pe}) \propto \gamma_{pe}^{-\nu_{\text{min}}} \leq \gamma_{pe} \leq \gamma_{pe,\text{max}} \quad \text{and} \\

n_{\text{NT}}(\gamma_p) \propto \gamma_p^{-\nu_{\text{min}}} \leq \gamma_p \leq \gamma_p,_{\text{max}}
\]

### Figure 1

Schematic of a powerful FR II radio galaxy. A pair of jets are ejected from the core, and they are decelerated via strong shocks. The shocks are identified as the hot spots and the remnant of decelerated jets envelop the overall jet system; this is identified as a cocoon. Part of the cocoon is normally observed as radio lobes. The cocoon head and the hot spots advance at a speed \( \gamma_{\text{hs}} \). Swept-up ambient matter becomes a shell and surrounds the cocoon. The projected linear size is denoted as LS in this work.
for case (c) with $s_p = s_e > 2$. Observations of the spectral index in the radio lobe of Cygnus A suggest $s_e > 2$ (e.g., Carilli et al. 1991; Yaji et al. 2010).

Last, we set case (d), in which the number spectrum of non-thermal electrons is given by a broken power law:

$$n_p^{NT}(γ_\text{p}) ≅ \begin{cases} \gamma_\text{p}^{-s_1} & (γ_\text{p,min} ≤ γ_\text{p} ≤ γ_\text{p, crit}), \\ \gamma_\text{p}^{-s_2} & (γ_\text{p, crit} ≤ γ_\text{p} ≤ γ_\text{p,max}), \end{cases}$$

where $s_1 < 2$ and $s_2 > 2$ are satisfied. This model is based on Stawarz et al. (2007), who suggested that observed spectra at the jet termination shock (hot spot) of FR II jets (Cygnus A) can be explained by the break at non-thermal electron energy (hereafter $γ_\text{p,crit}$). This type of spectra may be due to the absorption of electromagnetic waves emitted at the harmonics of cyclotron frequency of cold protons, as discussed by Hoshino et al. (1992) and Amato & Arons (2006). Some observations for other FR II sources could also be compatible with this picture (e.g., Perlman et al. 2010 for 3C445).

For cases (c) and (d), the minimum energy of non-thermal electrons/positrons ($γ_\text{p,min} m_e c^2$) and protons ($γ_\text{p,max} m_p c^2$) are generally assumed as

$$γ_\text{p,min} ≈ γ_\text{p,min} ≈ Γ_j,$$

which is expected when protons and electrons/positrons are separately heated and accelerated at termination shocks. On the other hand, the values of the maximum energy of non-thermal pairs ($γ_\text{p,max} m_e c^2$) and protons ($γ_\text{p,max} m_p c^2$) are largely uncertain. While $γ_\text{p,max} m_e c^2$ may be significantly affected by radiative coolings, $γ_\text{p,max} m_p c^2$ may reach the range of highest energy cosmic rays (e.g., Takahara 1990; Rachen & Biermann 1993). It is reasonable to suppose that $γ_\text{p,max} ≫ γ_\text{p,min} and γ_\text{p,max} ≫ γ_\text{p,min}$.

### 3. TOTAL PRESSURE $P$

In this section, we briefly describe the basic idea of estimating the total pressure $P$. In Figure 1 we show a schematic of the interaction of the jet and ICM. Heating and acceleration processes work at hot spots and those particles are injected into cocoons. The cocoon model was proposed by Begelman & Cioffi (1989), in which the dissipated energy of jet bulk motion is the origin of the total pressure of cocoon, and a cocoon of FR IIs is expected to be overpressured against ICM pressure ($P_{\text{ICM}}$) with a significant sideways expansion. Therefore, the assumption of $P = P_{\text{ICM}}$ is not valid. We have proposed a method of dynamic constraint of $P$ by comparison of the cocoon model with the actually observed morphology of the cocoons (Kino & Kawakatu 2005; I08), and this method is applied to various radio lobes (e.g., Machalski et al. 2010). We use this model in the present work. The reliability of the expanding cocoon model is well examined in Kawakatu & Kino (2006). The results of relativistic hydrodynamic simulations of Schek et al. (2002) and Peruchot & Martí (2007) support the above analytic model. The mass and energy injections from the jet into the cocoon, which govern the cocoon pressure $P$ and mass density $ρ$ averaged by the source age ($t_{\text{age}}$), are written as

$$\frac{\dot{r}}{r - 1} \frac{\rho V}{t_{\text{age}}} = 2 T_j^{01} A_j = 2 L_j, \quad T_j^{01} = ρ_j c^2 r_j^2 v_j,$$

where $\dot{r}$, $V$, $A_j$, $T_j^{01}$, $J$, $ρ_j$, and $Γ_j$ are the adiabatic index of the plasma in the cocoon, the volume of the cocoon, the cross-sectional area, the total energy flux, the rest mass flux, the rest mass density, and the bulk Lorentz factor of the jet, respectively. The term $V$ is evaluated as $V = 2(π/3)R^2 L S^3$, where $LS$ and $R$ are the linear size of the cocoon along the jet axis and the aspect ratio of the cocoon, respectively. Here, we denote physical quantities of the jet with the subscript $j$. Throughout this work, we focus on a relativistic jet. Correspondingly, the shocked plasma has relativistic energy; thus, we take $\dot{r} = 4/3$. The $P \dot{V}$ work done by the cocoon against ICM is taken into account in the energy equation (6) following I08. For a given $ρ_{\text{ICM}}$, we can dynamically estimate total pressure $P$ by measuring $LS$, $R$, and the head cross-sectional area of the cocoon. Here, the relations of LS = $ρ_{\text{ICM}} c t_{\text{age}}$ and $R = l_c / LS < 1$ hold, where $l_c$ and $ρ_{\text{ICM}}$ are the lateral size of the cocoon and the advance velocity of the hot spot, respectively. Since $R$ and $ρ_{\text{ICM}}$ have some uncertainties, actual $P$ is bounded by maximum and minimum values

$$P_{\text{min}} ≤ P ≤ P_{\text{max}}.$$  

Thus, we can obtain the total pressure of cocoon $P$, which includes the partial pressures of non-radiating particles. The estimate of $P$ has actually been done by I08 for some FR II sources, and we adopt $P$ values in I08 in this work.

### 4. PRESSURE AS A FUNCTION OF $η$

In this section, we express $P$ as a sum of the partial pressures and represent it as a function of $η$ for respective cases.

#### 4.1. Case (a)

First, we examine the canonical case of two-temperature thermal plasma. Here, we assume that $P_{\text{NT}} = p_{\text{NT}} = p_{\text{p}} = 0$ and $n_e = n_p = 0$. The EOS in the cocoon filled with relativistic plasma is given by

$$P \approx P_{\text{p}}^{\pm} + P_{\text{p}}^{\pm} = (n_e^T + n_p^T)kT_\pm + n_p^T kT_p,$$

where $T_\pm$ and $T_p$ are the electron/positron temperature and proton temperature, respectively. Hereafter, we adopt $T_\pm = T_{\text{e}} = T_{\text{p}} = T_{\text{e}}$, where $T_{\text{e}}$ and $T_{\text{p}}$ are the temperatures of the electrons and positrons, respectively. Following Kino et al. (2007), we can obtain $T_\pm$ and $T_p$ from Equations (6), (7), and (8):

$$kT_\pm = \frac{Γ_j m_e c^2}{4}, \quad kT_p = \frac{Γ_j m_p c^2}{4},$$

which are typically given by $kT_\pm = 1.3(G/10)\text{MeV}$ and $kT_p = 2.3(G/10)\text{GeV}$. Here, we assume the limit of inefficient $e/p$-coupling, i.e., protons and electrons are separately thermalized so that $kT_p = (m_e/m_p)kT_p$, since plasma number densities in large scale jets are conservatively expected to be too diluted to achieve efficient $e/p$-coupling (e.g., Kino et al. 2007 and references therein). The emission timescale is so long that radiative cooling is negligible. It is worth noting that the
geometric factors in Equations (6) and (7) are completely canceled out and \( kT_\pm \) and \( kT_P \) are governed only by \( \Gamma_\gamma \).

Inserting Equation (10) into Equation (9), we rewrite the total pressure in the cocoon \( P \) as

\[
P(\eta) = 2.05 \times 10^{-6} n_T^2 \left[ (2 - \eta) + \frac{\eta m_p}{m_e} \right] \left( \frac{\Gamma_\gamma}{10} \right) \text{erg cm}^{-3},
\]

(11)

where the first and second terms in the square brackets correspond to the partial pressure of pairs and protons, respectively.

### 4.2. Case (b)

As an opposite extreme to case (a), here we consider the case of one-temperature plasma. In this example, some of the proton energy is somehow transferred to electrons/positrons to achieve an efficient \( e/p \)-coupling. Then, hotter electrons/positrons and colder protons are produced. From the condition \( kT_\pm = kT_P \), and Equations (6) and (7), we obtain

\[
kT_\pm = kT_P = \frac{\Gamma_\gamma m_e c^2}{8} \left[ (2 - \eta) + \frac{\eta m_p}{m_e} \right].
\]

(12)

In this case, each population (i.e., \( p/e^-/e^+ \)) has the same kinetic energy. The total pressure is given by Equation (11), the same as case (a). The essential difference from case (a) is that \( kT_\pm \) in case (b) is much higher than that in case (a).

### 4.3. Case (c)

For comparison with the canonical case (a), we examine case (c), in which the cocoon pressure is dominated by non-thermal particles. Case (c) concerns a situation in which the spectral indices of non-thermal particle energy distributions satisfy \( s_p = s_e > 2 \) as some theoretical work on relativistic shocks suggests (e.g., Bednaryz & Ostrowski 1998; Kirk et al. 2000; Achterberg et al. 2001; Spitkovsky 2008; Sironi & Spitkovsky 2011) and as the radio lobes of Cygnus A show \( s_e > 2 \) (e.g., Carilli et al. 1991; Yajie et al. 2010). In this case, electrons and protons with the lowest energies are the main carriers of energy. Then, the evaluation of partial pressures of non-thermal plasma is basically the same as in case (a) when we replace \( kT_\pm \) with \( \gamma_{\pm,min} m_e c^2 \) and \( kT_P \) with \( \gamma_{p,min} m_p c^2 \). Then, \( P \) is given by

\[
P(\eta) = \frac{\Gamma_\gamma n_{NT}^2}{3} \left[ \frac{s_e - 1}{s_e - 2} (2 - \eta) + \frac{\eta m_p}{m_e} \right] m_e c^2.
\]

(13)

From this, it is clear that we can appropriately evaluate \( \eta \) for case (c) by replacing \( n_T \) with \( n_{NT} \) in the same way as is done in case (a).

### 4.4. Case (d)

Here, we examine the pressure of non-thermal electrons when they follow a broken power-law spectrum (Equation (4)). Stawarz et al. (2007) indicated \( \gamma_{\pm, crit} \sim m_p/m_e \) for the hot spots in Cygnus A. The energy of the electron component is governed by those with break energy, while the number is dominated by those with lowest energies. Since \( s_p > 2 \) is satisfied, lowest-energy protons carry most of the energy. Therefore, the total pressure \( P \) is expressed as

\[
P(\eta) = \frac{\Gamma_\gamma n_{NT}^2}{3} \left[ \frac{s_e - 1}{s_e - 2} (2 - \eta) + \frac{\eta m_p}{m_e} \right] m_e c^2.
\]

(14)

where \( A_\pm = (\gamma_{\pm, crit}/\gamma_{\pm, min})^{-\gamma_e} m_e c^2 \) is the partial pressure of pairs and protons, respectively.

### 5. TESTING PLASMA COMPOSITION

We explain the method for constraining plasma composition of AGN jets for thermal plasma cases (a) and (b) in 5.1, 5.2, and 5.3. The application to non-thermal plasma cases (c) and (d) can be readily understood and is explained in 5.4.

#### 5.1. Characteristic Pressures

First, we define characteristic pressures that divide the number-density/pressure plane into several regions as shown in Figure 2. As a preparation, here we define \( n_{eq} \) as follows:

\[
\eta_{eq} \equiv \frac{2}{m_p/m_e - 1} = 1.1 \times 10^{-3} \quad (P_\pm = P_p).
\]

(15)

The partial pressure of proton-associated electrons is implicitly neglected since it is subdominant in the case of inefficient \( e/p \)-coupling. The line with \( n_\pm = 1 \times 10^3 n_p \) divides the pair-supported and proton-supported cocoons in the limit of inefficient \( e/p \)-coupling plasma. By definition, the cocoon with \( \eta > \eta_{eq} \) is proton-supported (dark gray region in Figure 2), while the cocoon with \( \eta < \eta_{eq} \) is pair-supported (light gray region in Figure 2). When \( n_\pm \) is bounded by \( n_{\pm, min} \) and \( n_{\pm, max} \) as argued in the next subsection, the allowed region of \( n_\pm \) is segmented by some characteristic pressures by the characteristic values of \( n_\pm \) and \( \eta \), i.e., \( n_\pm, min \rightarrow n_\pm, max \), \( \eta = 0 \), \( \eta = \eta_{eq} \), and \( \eta = 1 \). Here, we define six characteristic pressures as follows:

\[
P(\eta = 0; n_\pm = n_{\pm, min}) \equiv P_{0,\min},
\]

\[
P(\eta = \eta_{eq}; n_\pm = n_{\pm, min}) \equiv P_{eq,\min},
\]

\[
P(\eta = 0; n_\pm = n_{\pm, max}) \equiv P_{0,\max},
\]

\[
P(\eta = \eta_{eq}; n_\pm = n_{\pm, max}) \equiv P_{eq,\max},
\]

\[
P(\eta = 1; n_\pm = n_{\pm, min}) \equiv P_{1,\min},
\]

\[
P(\eta = 1; n_\pm = n_{\pm, max}) \equiv P_{1,\max}.
\]

(16)

Then, by definition, we have the following relations:

\[
P_{0,\min} : P_{eq,\min} : P_{0,\max} : P_{eq,\max} : P_{1,\min} : P_{1,\max} = 1 : 2 : \frac{n_{\pm, max}}{n_{\pm, min}} : \frac{m_p}{m_e} : \frac{m_p n_{\pm, max}}{m_e n_{\pm, min}},
\]

(17)
where we approximate $2 - n_{eq} \approx 2$. To evaluate these pressures, we estimate $n_{-\min}$ and $n_{-\max}$ in the next subsection.

### 5.2. Estimation of $n_-$

Here, we constrain the number density of electrons in the cocoon $(n_-)$. We denote the lower and upper limits of $n_-$ as $n_{-\min}$ and $n_{-\max}$, respectively. The values of $n_{-\min}$ and $n_{-\max}$ are independently constrained; we show them below.

#### 5.2.1. Lower Limit of $n_-$

Here, we estimate the lower limit of $n_-$ and examine the case in which the number density of thermal electrons is greater than that of non-thermal electrons $n_T^\alpha \geq n_{NT}^\alpha$, since non-thermal electrons are partially injected from the background thermal electrons. (Later, the extreme cases of $n_T^\alpha \leq n_{NT}^\alpha$ will also be discussed, as they are identical to cases (c) and (d)). Since the shocked plasma at hot spots expands sideways and is injected into the cocoon, we can estimate $n_{NT}^\alpha$ by using $n_{hs}^\alpha$, where $n_{hs}^\alpha$ is the number density of non-thermal electrons in a hot spot. We stress that $n_{hs}^\alpha$ is well constrained by observed non-thermal emissions of hot spots for FR II sources (see, e.g., Harris & Krawczynski 2006 for review). From this, we can density from the jet to the cocoon based on Equation (7) and shock conditions along the jet axis shown in Kino & Takahara (2004, hereafter KT04), we obtain

$$n_{-\min} = \frac{n_{hs}^\alpha A_{LS}}{2V_{hs}^\alpha}. \quad (18)$$

In general, number density of non-thermal electrons with power-law distribution $n_{hs}^\alpha \propto \int_{\gamma_{hs,min}}^{\gamma_{hs,max}} \gamma_{hs}^\alpha d\gamma_{hs}$ can be given by

$$n_{hs}^\alpha \propto \gamma_{hs,min}^{-\alpha+1}. \quad (19)$$

We assume the standard value of $\gamma_{hs} \approx 2$ and $\gamma_{hs,min} \approx \Gamma_j$.

#### 5.2.2. Upper Limit of $n_-$

The upper limit of $n_-$ can be constrained by the absence of thermal bremsstrahlung from hot electrons in the cocoon/lobes viewed in X-ray observations (Wilson et al. 2000, 2006). The observed X-ray emissions associated with radio lobes are non-thermal emissions, and there is no evidence of thermal X-ray emission from cocoons/lobes (see Harris & Krawczynski 2006 for review). From this, we can safely use the condition of $L_X_{obs} > L_{brem}(n_T^\alpha, T_\pm)$, where $L_{brem}/V = \alpha r_f^3 m_e c^3 (n_T^\alpha)^2 F_\pm(\Theta_\pm) \text{erg cm}^{-1} \text{s}^{-1}$, $F_\pm(\Theta_\pm) = 48\Theta_\pm^2 (\ln 1.1\Theta_\pm^\pm/5/4)$, and $\Theta_\pm = kT_\pm/m_e c^2$, for bremsstrahlung at relativistic temperature (Equation (22) in Svensson 1982), and $\alpha$ and $r_f$ are the fine structure constant and the classical electron radius, respectively. From this, we obtain the maximum $n_-$ as follows:

$$n_{-\max} = \left(\frac{L_{brem}}{V\alpha r_f^3 m_e c^3 F_\pm(\Theta_\pm)}\right)^{1/2}. \quad (20)$$

It is worth commenting on the availability of constraining the upper limit of $n_-$ by the analysis of the internal depolarization of the radio lobes. Relativistic plasma makes a smaller contribution to Faraday rotations since electron inertia increases for the relativistic regime, and it suppresses rotations of the polarization angle (e.g., Ichimaru 1973; Melrose 1997; Quataert & Gruzinov 2000; Huang & Shcherbakov 2011). Therefore, it is not effective to use the constraint by rotation measure (RM) in the present work.

### 5.3. Estimation of $n_p$

Once $n_-$ is estimated, the proton number density $n_p$ can be determined as

$$n_p = \eta n_-. \quad (21)$$

by definition. Here, of course, the conditions of $0 \leq \eta \leq 1$ and $n_{-\min} \leq n_- \leq n_{-\max}$ are imposed. In Figure 2, the allowed region of $n_p$ is added to that of $n_-$ shown in Figure 2. In the same way as in Figure 2, the plane is divided into five regions.

Finally, the allowed regions of $n_j$ and $n_-$ can be obtained by adjoining the range of $P$. The allowed regions drawn in Figure 3 are bound by Equation (8). Thus, we can obtain the definitive allowed regions of $n_p$ and $n_-$.

### 5.4. Application to Cases (c) and (d)

In Sections 5.1, 5.2, and 5.3, we considered physical quantities associated with thermal plasma in cases (a) and (b). But those can also be applied to non-thermal plasma by the proper replacements of number densities and average energies of particles. With regard to average energies, we have already explained the replacements in the previous section. As for $n_{-\max}$, the estimate shown in Section 5.2.1 can be applied for both thermal and non-thermal plasmas. As for $n_{-\max}$, the estimate shown in Section 5.2.2 can be applied only for thermal plasma. Therefore, we do not use $n_{-\max}$ for the cases (c) and (d). Thus, we can properly estimate $\eta$ for cases (c) and (d).

### 6. APPLICATION TO CYGNUS A

Here, we apply the above method to Cygnus A ($z = 0.0562$), which is one of the best-studied FR II radio galaxies (e.g., Carilli & Barthel 1996; Steenbrugge et al. 2008, 2010; Yaji et al. 2010). The physical quantities of Cygnus A have been well constrained by previous works. To constrain the real values of $P$ and $n_-$, we carefully evaluate $R$, $\beta_{hs}$, and $\Gamma_j$. The term $R$ has an effect on $n_-$ via a cocoon volume $V$. The term $\Gamma_j$ is directly proportional to $P$. The term $\beta_{hs}$ controls the source age $\tau_{src}$, which governs the injection rates of mass and energy into the cocoon. These are summarized in Section 6.1. The resultant allowed region of $n_-$ and $n_p$ is summarized in Section 6.2.
6.1. Viable Ranges of Physical Quantities

We show adopted conditions of the model parameters for deriving the above results. We fix the cross section area of the jet as \( A_j = \pi R_0^2 = \pi (2\, \text{kpc})^2 \) (Wilson et al. 2000) and the number density of ICM just ahead of the hot spot as \( n_{\text{ICM}} = 0.5 \times 10^{-2} \, \text{cm}^{-3} \) (the shell No. 6 in Table 5 in Smith et al. 2002).

1. **Cocoon morphology \( \mathcal{R} \).** From images of the Cygnus A cocoon, we can directly constrain \( \mathcal{R} \). The upper limit \( \mathcal{R} \approx 0.5 \) is determined by the *Chandra* X-ray image (Wilson et al. 2000, 2006; Yaji et al. 2010). The lower limit \( \mathcal{R} \approx 0.25 \) is directly measured by the 330 MHz Very Large Array (VLA) image (see also Carilli et al. 1991; Lazio et al. 2006). Therefore, we set

\[
0.25 \leq \mathcal{R} \leq 0.5
\]

in the present work.

2. **Cocoon head velocity \( \beta_{\text{hs}} \).** Cocoon head velocity, which equals the hot spot advance velocity (\( \beta_{\text{hs}} \)), is well constrained by the synchrotron aging method. The estimated \( \beta_{\text{hs}} \) has some uncertainty due to the uncertainty of magnetic field strength in the cocoon. From the result of synchrotron aging diagnosis in Carilli et al. (1991), we adopt the allowed range of \( \beta_{\text{hs}} \) as

\[
0.01 \leq \beta_{\text{hs}} \leq 0.06.
\]

We emphasize that sufficiently large uncertainty is taken into account here. The adopted value of \( \beta_{\text{hs}} \) is quite typical for hot spots in FR II radio galaxies (e.g., Scheuer 1995).

3. **Lorentz factor of the jet \( \Gamma_j \).** It is difficult to determine the true velocity of the jet. At least we may say that apparent velocity of blobs obtained by very long baseline interferometry (VLBI) observations shows a minimum velocity of underlying flow. A fast apparent motion of a blob at the jet base (0.56 ± 0.28)c has been reported by VLBI observations (Bach et al. 2003). Furthermore, suggestions of superluminal motion have been made (Krichbaum et al. 1998; Bach et al. 2002) although they have not been clearly confirmed. On VLA scale, a clear asymmetry in brightness distribution of a kiloparsec-scale jet due to a relativistic motion is seen (Perley et al. 1984). Therefore, overall radio observations seem to indicate relativistic motion. Bearing this in mind, we assume that the jet is relativistic and the four-velocity of the jet \( \Gamma_j \beta_j \) is set as

\[
1 \leq \Gamma_j \beta_j \leq 30.
\]

Here, the upper limit is assumed to be \( \Gamma_j \approx 30 \) based on the statistical study of radio jets of MOJAVE sources (Lister et al. 2001, 2009; Kellermann et al. 2004).

4. **Cocoon pressure \( P \).** Using the value of \( V = 1 \times 10^{70} \mathcal{R}^2 \, \text{cm}^3 \), we can estimate the total pressure \( P \) as

\[
8 \times 10^{-11} \, \text{erg cm}^{-3} \leq P \leq 4 \times 10^{-9} \, \text{erg cm}^{-3}.
\]  

The lower limit equals the ICM pressure 8 \times 10^{-11} \, \text{erg cm}^{-3} measured by Arnaud et al. (1984) to satisfy the overpressed cocoon condition. Although the upper limit of \( P \) is basically adopted from I08, the value 4 \times 10^{-9} \, \text{erg cm}^{-3} is four times larger than the original estimate in I08. This is due to the change in minimum value of \( \mathcal{R} \) from 0.5 to 0.25 based on VLA’s 0.3 GHz image. It should be stressed that our adoption of the allowed range of \( P \) is sufficiently wide compared with all of the previous work (see, e.g., Carilli et al. 1998 for review). Note that Yaji et al. (2010) estimate that \( P_{\text{NT}} \) in the radio lobes is \( P_{\text{NT}} \approx (1-2) \times 10^{-9} \, \text{erg cm}^{-3} \) for \( \gamma_{\perp} \approx 1 \), which causes \( P_{\text{NT}} > P_{\text{min}} \). So, if \( P \) completely equals the radio lobe pressure, then the range \( P_{\text{min}} \leq P < P_{\text{NT}} \) is excluded and the allowed \( P \) range becomes narrower. The allowed example with \( P_{\text{min}} \leq P_{\text{NT}} \leq P \leq P_{\text{max}} \) is involved in cases (c) and (d).

5. **Non-thermal electron number density \( n_{\text{NT}} \).** The lower limit \( n_{\text{min}} \) largely depends on \( n_{\text{hs}} \). For \( \gamma_{\text{hs}} = 2 \), the number density of non-thermal electrons in the hot spot can be obtained from

\[
n_{\text{hs}}^{\text{NT}} \approx 1 \times 10^{-3} \left( \frac{\gamma_{\text{hs}, \text{min}}}{10} \right)^{-1} \, \text{cm}^{-3},
\]

via detailed comparisons of the SSC model with the observed broadband spectrum (Wilson et al. 2000; KT04; Stawarz et al. 2007) where \( \gamma_{\text{hs}, \text{min}} \approx \gamma_j \). We stress that these three independent papers derive similar values of \( n_{\text{hs}}^{\text{NT}} \), although Stawarz et al. (2007) adopt the different electron-distribution function shown in Equation (4). Furthermore, we note the importance of low-frequency radio spectra since it affects the estimate of \( n_{\text{hs}}^{\text{NT}} \). Regarding low-frequency radio observation, we briefly comment on the work of Lazio et al. (2006). They indicated spectral flattening and turnover at \( \sim 100 \, \text{MHz} \). However, it seems difficult to determine these accurately because the spot sizes are smaller than the VLA beam sizes at the above frequencies. The LOFrequency ARray (LOFAR) (http://www.lofar.org/) and Square Kilometer Array (SKA) (http://www.skatelescope.org/) will, in the future, tell us the real turnover frequency with sufficiently high resolution.

6. **Thermal electron number density \( n_T \).** Here, we comment on the difficulty of constraining \( n_T \). We use the absence of bremsstrahlung emission. The X-ray observations for Cygnus A show the flux upper limit as \( \sim 1 \times 10^{-13} \, \text{erg s}^{-1} \, \text{cm}^{-2} \) (e.g., Smith et al. 2002).

As already mentioned, the constraint from the intrinsic RM is not available, because plasma temperature is relativistic in the present work. Even worse, Cygnus A is known for its unusually large RM values; thus, it is not a good example from which to argue the intrinsic depolarization (Dreher et al. 1987; Garrington & Conway 1991). No evidence for intrinsic depolarization between 5 and 15 GHz is found, and the origin of the large RM is thought to be the external bow shock that surrounds the radio lobes (Dreher et al. 1987; Carilli et al. 1988). Hence, it is not appropriate to use the constraint from RM for Cygnus A.

6.2. Results

Below, we show the resultant allowed region of \( n_- \) and \( n_p \) for cases (a), (b), (c), and (d).

6.2.1. Case (a)

Considering the uncertainties of \( \Gamma_j \beta_j \) and \( \beta_{\text{hs}} \), we examine two limiting cases, with \( \Gamma_j \beta_j = 1 \) and \( \beta_{\text{hs}} = 0.01 \) being a High-\( n \) case, and \( \Gamma_j \beta_j = 30 \) and \( \beta_{\text{hs}} = 0.06 \) being a Low-\( n \) case. For the High-\( n \) case, \( n_- \) is about two orders of magnitude larger than that of the Low-\( n \) case.
In Figure 4, we show the allowed regions of $n_-$ and $n_p$ for the High-$n$ case. First, we find that $n_- > n_p$ always holds, and this satisfies $\eta \sim 10^{-2}$ at $P = P_{\text{max}}$. This implies that positron mixture is inevitable. In other words, $P_{1,\text{min}}$ is much larger than $P_{\text{max}}$ obtained by the Cygnus A cocoon calorimetry. (If we are forced to make $P_{1,\text{min}}$ smaller, then $n_\gamma$ becomes larger; such a case coincides with case (b).) The allowed regions of $n_-$ and $n_p$ are further divided by two regions. The pair of light-gray regions show the one in which $P_{\pm} > P_p$ is satisfied. On the contrary, the pair of dark-gray regions display the one in which $P_{\pm} < P_p$ holds. Interestingly, we find that the regions of $P_p < P_{\pm}$ and $P_{\pm} > P_p$ are both wide in the range of allowed $P$. In the range of $P \sim (3-6) \times 10^{-10}$ erg cm$^{-3}$, the pair dominance $P_p < P_{\pm}$ alone is permitted in the High-$n$ case.

Figure 5 displays the result for the Low-$n$ case. Similar to the High-$n$ case, $n_- > n_p$ always holds, and they satisfy $\eta \sim 10^{-1}$ at $P = P_{\text{max}}$. Due to the decrease in $n_{\text{min}}$, the number densities in allowed regions are about two orders of magnitude smaller than that for the High-$n$ case shown in Figure 4. Correspondingly, $P_{0,\text{min}}$, $P_{\text{eq, min}}$, and $P_{1,\text{min}}$ decrease. Since $P_{\text{eq, max}} < P_{\text{max}}$ is still satisfied, both of the regions with $P_p < P_{\pm}$ and that with $P_p > P_{\pm}$ are allowed in this case. In other words, the Low-$n$ case also qualitatively draws the same conclusion with the High-$n$ case. Quantitatively, the upper limit of $n_p$ becomes larger when $n_{\text{min}}$ becomes smaller; correspondingly, the maximum $\eta$ achieved at $P_{\text{max}}$ becomes larger by a factor of $\sim 10$ than that for the High-$n$ case.

In summarizing case (a), we find that $\eta < 1$ always holds in the allowed range of $P$. In other words, this indicates the existence of $e^\pm$ pairs in the cocoon. We find that (1) the $e^\pm$ pair is dominant in terms of number density, and (2) both the “pair-supported cocoon (i.e., $P_{\pm} > P_p$)” and the “proton-supported one (i.e., $P_{\pm} < P_p$)” are allowed. The pair-supported cocoon is different from the previously suggested one in which protons are dynamically dominated (e.g., De Young 2006).

6.2.2. Case (b)

For Cygnus A, we face a difficulty of realizing one-temperature plasma. First, let us consider the case of the same $n_{\text{min}}$ as in Figures 4 and 5. All of these thermal electrons should be heated to $kT_{\pm} \sim 10^4 m_e c^2$ and injected into the lobes in case (b). In the radio lobes, Yaji et al. (2010) evaluate the number density of non-thermal electrons as $\sim 10^{-7}$ cm$^{-3}$ at $\gamma \sim 10^4$. So, if we allow the existence of thermal plasma with the same $n_{\text{min}}$ in Figures 4 and 5 but with $kT_{\pm} = kT_p \sim 10^4 m_e c^2$, a big thermal bump at $\sim 10^8$ Hz should appear. However, there is no such bump in the observed spectra of the radio lobes. Therefore, we can exclude the case of the same $n_{\text{min}}$ with $kT_{\pm} = kT_p \sim 10^4 m_e c^2$.

Next, we consider smaller $n_{\text{min}}$. Using the relation $n_{\text{min}} \propto \gamma_{\text{m.e.}}^{-1}$ in Equation (19), the increase in $\gamma_{\text{m.e.}}$ leads to the decrease in $n_{\text{min}}$ in Figures 4 and 5; basically, $\gamma_{\text{m.e.}} \sim 10^4$ is required at the hot spot (e.g., Harris et al. 2000; Hardcastle et al. 2001; Blundell et al. 2006; Godfrey et al. 2009). However, in the case of Cygnus A, the model spectra of the hot spots with $\gamma_{\text{m.e.}} \gg 2000$ conflict with the observed ones (KT04). Therefore, case (b) is not likely for Cygnus A.

6.2.3. Case (c)

Let us consider the case of dominant non-thermal pressures and a separate acceleration of electrons and protons with a steep power-law spectrum. This is almost identical to case (a). A slight difference between this case and case (a) is the evaluation of $n_{\text{min}}$. Since non-thermal pairs are dominated in this case, the allowed region would be limited around $n_- \approx n_{\text{min}}$ in Figures 4 and 5.

6.2.4. Case (d)

Let us consider case (d). The factor $A_{\pm} = (\gamma_{\pm, \text{cr.}} / \gamma_{\pm, \text{min}})^{\gamma+2}$ in Equation (14) is the only element to change the result from case (a). Since $\gamma_{\text{crit.}} = m_p / m_e$ is suggested by Stawarz et al. (2007), we can estimate $A_{\pm}$ as $A_{\pm} \approx 14(\rho/10)^{0.5}$ for $s_{\text{e,1}} = 1.5$. Therefore, a difference between this case and case (a) is the larger $P_{\pm}$ by a factor of $A_{\pm}$. Although the spectral break may be suggested from radio observations for case (d), $n_{\text{min}}$ is dominated by electrons at a break energy $\gamma_{\text{crit.}, \text{m.e.}} c^2$, and proton energies are not entirely transported to electrons. Therefore, results of case (d) are expected to be intermediate between cases (a) and (b).
7. SUMMARY AND DISCUSSIONS

In this work, we propose a new method for testing plasma composition of AGN jets using cocoon dynamics. In particular, we properly evaluate partial pressures of protons and composition of AGN jets using cocoon dynamics. In particular, plasma with a broken power-law electron spectrum. with their spectral indices harder than two, and non-thermal plasma, one-temperature thermal plasma, non-thermal plasma with their spectral indices harder than two, and non-thermal plasma with a broken power-law electron spectrum.

The three significant advantages of the present work compared with previous work are summarized as follows:

1. $P$ estimate is based on global cocoon dynamics. Since it is beaming-independent calorimetry of the true amount of energy released by the jet, the estimate of $P$ from cocoon dynamics has fewer uncertainties compared with blazar studies.

2. We focus on powerful FR II sources. Relativistic hydrodynamic simulations tell us that FR II sources have less entrainment phenomena than FR I sources. Therefore, FR IIs are better for testing genuine plasma composition of AGN jets.

3. We properly deal with the partial pressure of thermal electrons/positrons $P_{\pm}$. Although $P_{\pm}$ is a critically important finite quantity, most prior efforts assume $P_{\pm} = 0$ merely for simplicity.

Applying the method to the best-studied FR II source Cygnus A, we draw the following conclusions, which primarily indicate the existence of numerous $e^{\pm}$ pairs in the cocoon of Cygnus A:

1. Cases (a), (c), and (d), in which the average energy of electrons and positrons is significantly lower than that of protons ($\eta < 10^{-1}$ for Low-n case; $\eta < 10^{-2}$ for High-n case), are allowed without violating the observational constraints. The results in cases (a) and (c) are almost the same, except that the lowest energy electrons are thermal ones and non-thermal ones for cases (a) and (c), respectively. Cases (a) and (d) show similar results but for a $P_+\times$ in case (d) larger by a factor of ~14 than the one in case (a).

2. We can rule out case (b), in which electrons and positrons are heated to the proton temperature of $\sim 10^4 m_p c^2$, because there is no thermal bump due to the hot thermal plasma.

3. For cases (a), (c), and (d), we find that the number density of $e^\pm$ is larger than $n_p$ in any allowed $P$, and the obtained $n_+$ is always more than 10 times larger than $n_p$. We conclude that pure $e/p$ plasma is excluded and $e^{\pm}$–proton mixture composition is achieved in the Cygnus A jet. Therefore, further studies of the $e^{\pm}$ pair loading problem extending previous ones (e.g., Blandford & Levinson 1995; Li & Liang 1996; Thompson 1997; Beloborodov 1999; Yamasaki et al. 1999) will be important, and the study of its bulk acceleration of $e^{\pm}$ outflow (Iwamoto & Takahara 2002, 2004; Asano & Takahara 2007, 2009) will also be highly motivated.

4. We find that both $e/p$ plasma and $e^{\pm}$ pair pressure supported scenarios are permitted within the limits of current observational constraints. We quantitatively show the allowed regions of $P_+ > P_\pm$ and $P_+ < P_\pm$ by our new method (see Figures 4 and 5).

Last, we add a brief comment on $P_{p,NT}$. Recently, Atoyan & Dermer (2008) suggested the possibility of a secondary emission induced by high-energy protons at Cygnus A. The luminosity of the secondary emission depends on $P_{p,NT}$. If the emission is detected in the future, it will provide a new direct constraint on $P_{p,NT}$. It could also give a new constraint on cosmic-ray propagations influenced by the galactic magnetic field (Dermer et al. 2009).

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