EoS for strange quark matter: linking the NJL model to pQCD

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Neutron star constraints and ab initio pQCD evaluations require the EoS representing cold quark matter to be stiff at intermediate baryonic densities and soft at high-\(n_B\). Here, it is suggested that the three flavor NJL model with a density dependent repulsive coupling, \(G_V(\mu)\), can generate an EoS which interpolates between these two regimes. Such an interpolation requires repulsion to start decreasing with the chemical potential just after chiral transition takes place. The conjecture behind this mechanism is that repulsion should be necessary only as long as the quark condensates, which dress the effective masses, have non-vanishing values. This assumption guarantees that an initially hard EoS suffers a conspicuous change of slope at \(E \simeq 0.7\) GeVfm\(^{-3}\) converging to the pQCD results at higher energy densities. Then, the speed of sound naturally reaches a non-conformal maximum at \(n_B = 3.23 n_0 = 0.52\) fm\(^{-3}\) while the trace anomaly remains positive for all densities, in agreement with recent investigations. These non-trivial results cannot be simultaneously obtained when \(G_V\) vanishes or has a fixed value. Therefore, the simple model proposed here may help us to link the (non-perturbative) region of intermediate densities to the region where pQCD becomes reliable.
I. INTRODUCTION

Effective quark models, such as the Nambu–Jona-Lasinio model (NJL) \cite{1} and the MIT bag model \cite{2,3,4} capture some of the most representative characteristics of quantum chromodynamics (QCD), like confinement and chiral symmetry respectively \cite{5}. As a consequence, they are widely used to describe the thermodynamics of strongly interacting matter in regions of the phase diagram which are currently unaccessible to \textit{ab initio} evaluations. Nowadays, the corner of low densities and high temperatures can be well described by first principle evaluations based on lattice QCD simulations (LQCD). However, due to the well documented sign problem, LQCD is not yet in position to describe the corner of low temperatures and finite baryonic densities which concerns neutron stars (NSs). In this case the QCD equation of state (EoS) describing cold and dense strongly interacting matter can be reliably evaluated only in regimes where the baryonic density ($n_B$) is very low or extremely high. In the limit of low densities, chiral effective theory (CET) \cite{6,7} provides an accurate EoS up to about $n_B \lesssim 2n_0 \equiv n_{\text{CET}}$ ($n_0 = 0.16\,\text{fm}^{-3}$) so that the region composed by hadronic matter may be well described. At the other extremum, perturbative QCD (pQCD) \cite{8,9,10} gives a reliable equation for $n_B \gtrsim 40n_0 \equiv n_{\text{pQCD}}$, when quarks and gluons represent the relevant degrees of freedom \cite{11}. However, at the intermediate range $2n_0 \lesssim n_B \lesssim 8n_0$, which concerns NSs, the α-s is still high so that non-perturbative techniques and/or model approximations are generally employed. Within this region the presence of quark matter in massive NSs was recently found \cite{12} to be linked to the behavior of the speed of sound, $V_s$. The investigation performed in Ref. \cite{12} suggests that if the conformal bound $V_s^2 \leq 1/3$ is not strongly violated massive neutron stars should have sizable quark-matter cores. Moreover, the recent discovery of NSs whose estimated masses are about twice the value of the solar mass \cite{13,14,15} and the theoretical predictions on the maximum (gravitational) mass performed in Refs. \cite{16,17,18,19,20} favor a stiff EoS with $V_s^2 > 1/3$ at $n_B > n_0$. In this case, recent simulations \cite{21,22,23,24} indicate that $V_s^2$ is a non-monotonic function of $n_B$, which in turn suggests the existence of at least one local maximum where $V_s^2 > 1/3$. Together, all of these findings constrain the EoS to be initially stiff (so that $V_s^2 > 1/3$) before softening, at intermediate densities, to finally meet the pQCD predictions at high-$n_B$. As it is well known, when effective quark models are being employed the inclusion of a repulsive vector channel, parametrized by $G_V$, generates a harder EoS in most cases \cite{24,25}. However, a drawback is that such an equation remains stiff at higher densities so that the conformal limit, observed by pQCD, cannot be attained. On the other hand, when $G_V = 0$, asymptotic convergence to pQCD is observed but the EoS is far too soft to cope with NSs constraints at lower densities.

One way to circumvent this problem is to assume that $G_V$ is density dependent as recently proposed in Ref. \cite{26}, where the two flavor NJL model has been considered. There, it has been suggested that the repulsion among (dressed) quarks is important only up to the point where the chiral transition occurs so that repulsion among (bare) quarks should be negligible. In Ref. \cite{26}, the running of $G_V$ was modeled by a simple ansatz which interpolates between a regime where repulsion is high (the EoS is stiff) and a regime where repulsion low (the EoS is soft). Thanks to this property the two-flavor NJL model with a running $G_V(\mu)$ predicted \cite{26} a non-monotonic behavior for $V_s^2$ implying that the existence of a peak, at $n_B \approx 3.25n_0$, can be conciliated with pQCD predictions at asymptotically high baryonic densities. Physically, these results indicate that repulsion should be necessary only as long as the quark condensates (which are directly related to the NJL quark self energies) exist.

Since strangeness may play an important role when describing more realistic situations the present work contemplates an extension to the case where this degree of freedom is present. With this purpose, the three flavor NJL model with a repulsive channel will be considered here as a prototype to describe cold strange quark matter. As we shall see, also in this case a density dependent $G_V$ allows us to obtain a non-conformal bump in $V_s^2$ at $n_B \approx 3.23n_0 = 0.52\,\text{fm}^{-3}$ (in agreement with Ref. \cite{22}) while the trace anomaly remains positive. This rather non-trivial result supports a recent claim \cite{27} which states that the presence of a non-conformal peak in $V_s^2$ is not necessarily in tension with the trace anomaly being positive for all densities. Concerning the EoS another important result obtained here predicts a prominent change of slope taking place at $\mathcal{E} \approx 0.7\,\text{GeVfm}^{-3}$, in agreement with what is observed in Refs. \cite{12,22}. These predictions indicate that the modified three flavor NJL model discussed in this investigation may contribute to describe the QCD EoS at intermediate baryonic densities.

The paper is organized as follows. In the next section the basic results for the three flavor NJL model are reviewed. The possible density dependence of the repulsive vector interaction is presented in Sec. \textbf{IV}. Numerical results associated with the relevant thermodynamical quantities are generated and discussed in Sec. \textbf{V}. The conclusions are presented in Sec. \textbf{VI}.

II. THE $N_f = 2 + 1$ NJL MODEL: STANDARD RESULTS

In the presence of a repulsive vector channel the standard three-flavor version of the NJL model can be written as \cite{24,25,28}.
\( \mathcal{L} = \bar{\psi}(i\gamma_\mu \partial^\mu - m)\psi + G_S \sum_{a=0}^{8} \left[ (\bar{\psi} \lambda^a \psi)^2 + (\bar{\psi} i\gamma_5 \lambda^a \psi)^2 \right] - K \left\{ \det_f(\bar{\psi}(1 + \gamma_5)\psi) + \det_f(\bar{\psi}(1 - \gamma_5)\psi) \right\} - G_V(\bar{\psi} \gamma^\mu \psi)^2, \)

where \( \psi = (u, d, s)^T \) denotes a quark field with three flavors (and three colors), and \( m = \text{diag}(m_u, m_d, m_s) \) is the corresponding mass matrix. Setting \( m_u = m_d \equiv m \neq m_s \) implies that isospin symmetry is observed while the \( SU(3) \) flavor symmetry is explicitly broken. The eight Gell-Mann matrices are represented by \( \lambda^a \) (\( a = 1, ..., 8 \)) and \( \lambda^0 = \sqrt{2/3} I \). In \( 3+1 \) d the NJL is composed by irrelevant operators so that the couplings \( G_S \) and \( K \) respectively have canonical dimensions [-2] and [-5] implying that the model is non-renormalizable. Here, the (ultra violet) divergent integrals will be regularized by a sharp non-covariant cut-off, \( \Lambda \), whose numerical value is set by phenomenological inputs. For the numerical analysis let us adopt the parameter values of Ref.\[29\] which are \( m = 5.5 \text{MeV}, m_s = 140.7 \text{MeV}, G\Lambda^2 = 1.835, KA^5 = 12.36, \) and \( \Lambda = 602.3 \text{MeV} \). Then, at \( T = 0 \) and \( \mu_f = 0 \), one reproduces \( f_\pi = 92.4 \text{MeV}, m_\pi = 135 \text{MeV}, m_K = 497.7 \text{MeV}, \) and \( m_\rho = 960.8 \text{MeV} \). For the quark condensates one obtains \( \sigma_u = \sigma_d = -(241.9 \text{MeV})^3, \) and \( \sigma_s = -(257.7 \text{MeV})^3 \). Fixing \( G_V \) poses and additional problem since this quantity should be determined by considering the \( \rho \) meson mass which, in general, happens to be higher than the maximum energy scale set by \( \Lambda \). In this situation, most authors adopt values between 0.25\( G_S \) and 0.5\( G_S \) (see Ref.\[30\] for more details). Here, the value \( G_V = G_S/3 \) will be adopted when dealing with a fixed vector coupling \[30, 31\]. Note that to assure rotational invariance only the zeroth component of the vector channel contributes so that, at the mean field level, the chemical potential gets shifted as \[5, 24, 25\]

\[ \tilde{\mu}_f = \mu_f - 2G_V \sum_f n_f, \]

with \( n_f \) representing the quark number density per flavor \[5, 24, 25, 30\]. At \( T = 0 \), a standard mean field approximation (MFA) evaluation yields the following result \[5, 30\]

\[ n_f = \frac{N_c}{3\pi^2} p_{F,f}^3, \]

where the effective Fermi momentum is just \( p_{F,f} = \sqrt{\tilde{\mu}_f^2 - M_f^2} \). The quark effective masses are given by \[5\]

\[ M_f = m_f - 4G_S \sigma_f + 2K \sigma_f \sigma_k, \]

where \( \sigma_f = \langle \bar{\psi} \psi \rangle_f \) represents the quark condensate for a given flavor

\[ \sigma_f = -\frac{N_c}{2\pi^2} M_f \left[ A_p \Lambda_f - M_f^2 \log \left( \frac{\Lambda + p_{A,f}}{M_f} \right) \right] + \frac{N_c}{2\pi^2} M_f \left[ \tilde{\mu}_f p_{F,f} - M_f^2 \log \left( \frac{\tilde{\mu}_f + p_{F,f}}{M_f} \right) \right], \]

where \( p_{A,f} = \sqrt{\Lambda^2 + M_f^2} \). The effective Fermi momentum, \( p_{F,f} \), is then determined by solving Eqs. \[2\] and \[4\] simultaneously.

Having the quark number density, \( n = \sum_f n_f \), one can express the squared speed of sound in terms of the baryonic number susceptibility, \( \chi_B = \frac{dn_B}{d\mu_B} \), as

\[ V_s^2 = \frac{\frac{n_B}{\mu_B \chi_B}}{\mu_B \chi_B}, \]

where \( \mu_B = \sum_f \mu_f \) and \( n_B = n/3 \). For simplicity, the present application concerns the case of symmetric strange quark matter only so that one can now set \( \mu_u = \mu_d = \mu_s \equiv \mu \).

Then, at finite chemical potential and zero temperature, the pressure versus chemical potential relation for quark matter can be obtained from \[25, 32\]

\[ P(\mu) = P(0) + \int_0^\mu n(\nu) d\nu, \]

where \( P(0) \) is the vacuum pressure. From \( P(\mu) \) one can determine the energy density, \( E = -P + \mu_B n_B \), the trace anomaly, \( \Delta = E - 3P \), as well as the conformal measure, \( \mathcal{C} = \Delta/\mathcal{E} \).
III. INTERPOLATING BETWEEN SOFTNESS AND STIFFNESS

Let us now discuss how to tune $G_V(\mu)$ so as to obtain an EoS which interpolates between the stiff and the soft regimes. When dealing with symmetric quark matter one can further simplify the notation by setting $M_u = M_d \equiv M$ as previously done for $m_u$ and $m_d$. Using these definitions and taking $G_V = 0$ one can write the Fermi momentum for a light flavor, in symmetric matter, as $\sqrt{\mu^2 - M^2}$. As compression increases the quark condensates decrease and the chiral transition sets in ($M \to m$) so that the Fermi momentum changes as $\sqrt{\mu^2 - M^2} \to p_F^0 = \sqrt{\mu^2 - m^2}$, where $p_F^0$ represents the case of free (bare) quarks considered within pQCD. Now, when $G_V$ is fixed chiral symmetry (partial) restoration implies that $\sqrt{\tilde{\mu}^2 - M^2} \to \sqrt{\tilde{\mu}^2 - m^2}$ and since the quark number density grows with $\mu$ the Fermi momentum $p_F^0$ cannot be reached, preventing the NJL results to converge to the pQCD predictions at arbitrarily high baryonic densities. Nevertheless, as proposed in Ref. [26], one can assure $\tilde{\mu} \to \mu$ (and $\sqrt{\tilde{\mu}^2 - M^2} \to p_F^0$) by requiring $G_V(\mu) \to 0$ after the chiral transition takes place according to

$$G_V(\mu) = \frac{G_V(0)}{1 + e^{(\mu - \mu_0)/\delta}},$$

where $G_V(0) = G_S/3$ [26, 30-31]. Considering the parametrization adopted here one has $M(0) = 367.7\text{ MeV}$ [5] and $\Lambda = 602.3\text{ MeV}$ so that $\mu_0 = [M(0) + \Lambda]/2 = 485\text{ MeV}$. The “thickness” $\delta = 10\text{ MeV}$ assures that the drop starting at $\mu = M(0)$ terminates at $\mu = \Lambda$ just as in the $N_f = 2$ case [26]. It is obvious from Eq. (8) that such running coupling interpolates between the two extrema, $G_V = 0$ and $G_V = G_S/3$, which respectively give a softer and a stiffer EoS [5, 24, 25]. Fig. 1 shows the running of $G_V(\mu)$ and also illustrates how it affects the Fermi momentum. From the physical point of view it is important to notice that the ansatz assumes that after chiral symmetry gets
(Light flavor) quark condensate, $\sigma_{u,d}(\mu)$ normalized by $\sigma_{u,d}(0)$, as a function of $\mu/\mu_c$. For the cases $G_V = G_S/3$ and $G_V(\mu)$ the coexistence quark chemical potential is $\mu_c = 0.368\,\text{GeV}$ while for $G_V = 0$ it reads $\mu_c = 0.361\,\text{GeV}$.

Effective Fermi momentum for light quarks, $p^*_F = \sqrt{\mu^2 - M^2}$, normalized by $p^0_F = \sqrt{\mu^2 - m^2}$, as a function of $\mu/\mu_c$. For the cases $G_V = G_S/3$ and $G_V(\mu)$ the coexistence quark chemical potential is $\mu_c = 0.368\,\text{GeV}$ while for $G_V = 0$ it reads $\mu_c = 0.361\,\text{GeV}$.

(partially) restored the repulsion among the (bare) quarks decreases as the density increases. In other words, it is assumed that quarks with large effective masses tend to strongly repel each other as compression increases and the quark condensates decrease. After the chiral transition occurs, and the effective masses tend to their bare values, quarks can be further compressed without repelling each other indicating that repulsion should be necessary only as long as the quark condensates, $\sigma_f$, are non-zero. A pictorial representation of the physical process driving the running of $G(\mu)$ is presented in Fig. 2. Also, remark that $\delta$ was chosen so as to give a smooth transition within a narrow 10 MeV width since taking $\delta \to 0$ could lead to discontinuities in $V_2$ which do not seem to be observed in the simulations of Refs. 21–23. Note that in order for $\sqrt{\mu^2 - M^2} \to \sqrt{\mu^2 - m^2}$ it is not compulsory that $G_S$ and $K$ run with $\mu$ since the quark condensates, multiplying these parameters in Eq. (4), naturally decrease with $\mu$. In summary, for a given flavor $i$, $G_S$ and $K$ always appear in combinations such as $G_S\sigma_i$ and $K\sigma_j\sigma_k$ (see Eq. (4)) which tend to vanish at high-$\mu$ while $G_V$ appears in combinations such as $G_Vn$ (see Eq. (2)) which always give a finite high-$\mu$ contribution when $G_V$ is fixed.

Finally, it must be pointed out that the idea of considering $G_V$ to depend on a control parameter, as proposed here, is not new. A similar course of action was originally taken by Kunihiro 32, who considered $G_V$ to be temperature dependent in order to evaluate quark susceptibilities at high-$T$ (see also Ref. 34).

**IV. NUMERICAL RESULTS**

Let us now analyze the effect of $G_V(\mu)$ on some relevant thermodynamical observables starting with the quark condensates for the light flavors. Fig. 3 shows the results for $\sigma_{u,d} = \langle \bar{u}u \rangle = \langle \bar{d}d \rangle$ obtained with different $G_V$ values. Around $\mu = 1.3\,\mu_c$ the $G_V(\mu)$ curve, which initially agrees with the $G_V = G_S/3$ result, converges towards the one predicted by using $G_V = 0$. In the same spirit Fig. 4 compares the dressed Fermi momentum for light quarks,
MS renormalization scale varies from the central

\[ n_B \approx 3 \mu \]

FIG. 5. Baryonic number density, in units of \( n_0 = 0.16 \text{fm}^{-3} \), as a function of the baryonic chemical potential, \( \mu_B = 3\mu \). The \( G_V(\mu) \) result interpolates between those predicted by \( G_V = 0 \) and \( G_V = G_S/3 \). The chiral first order phase transition takes place at \( \mu_B = 1.083 \text{GeV} \) for \( G_V = 0 \) and at \( \mu_B = 1.104 \text{GeV} \) for the other two cases.

\[ V_s^{2} \approx 1/3 \]

FIG. 6. Speed of sound (squared) as a function of the baryonic number density, \( n_B \). The running coupling predicts a non-conformal peak, \( V_s^{2} = 0.38 \), at \( n_B = 3.23 n_0 \). The light band corresponds to the pQCD results \[35\] when the \( \overline{\text{MS}} \) renormalization scale varies from the central value, \( 2\mu \) (bottom edge), to \( 4\mu \) (top edge). The thin dotted line represents the conformal result, \( V_s^{2} = 1/3 \).

\[ p_F = \sqrt{\mu^2 - M^2} \]

\[ p_F^0 = \sqrt{\mu^2 - m^2} \]

reproducing the pictorial view (shown in Fig. 2) from a quantitative perspective. Next, let us examine the baryonic number density, \( n_B \), which in the present work represents the fundamental thermodynamical quantity. The result obtained with \( G_V(\mu) \) is presented in Fig. 5 together with the predictions from the \( G_V = 0 \) and \( G_V = G_S/3 \) cases. The results from \( G_V(\mu) \) and \( G_V = G_S/3 \) agree up to \( \mu_B \approx 1.4 \text{GeV} \) when the former starts to converge towards the \( G_V = 0 \) curve. Fig. 6 shows the speed of sound squared for the three relevant cases. The \( G(\mu) \) curve peaks at \( n_B \approx 3.23 n_0 = 0.52 \text{fm}^{-3} \) (corresponding to \( \mathcal{E} = 0.59 \text{GeVfm}^{-3} \)) producing the non-conformal result \( V_s^{2} \approx 0.38 \) (note these numerical values are consistent with those reported in Ref \[22\]).

The curve then dives into the sub-conformal region reaching \( V_s^{2} \approx 0.08 \) at \( n_B \approx 7.50 n_0 \) before converging to the conformal result as \( n_B \) further increases. The onset of strangeness occurs at \( n_B \approx 5.50 n_0 \) so that the three EoS become softer due to the inclusion of one more degree of freedom. As a consequence of this softening the \( G_V = G_S/3 \) also predicts a non-conformal peak at \( n_B \approx 5.50 n_0 \) and \( V_s^{2} \approx 0.43 \) while \( G_V = 0 \) predicts a peak at \( n_B \approx 5.50 n_0 \) and \( V_s^{2} \approx 0.33 \). However, at higher \( n_B \) values the use of a fixed coupling prevents convergence towards to conformal result, as the figure shows. This result is not unexpected since, as already discussed, the Fermi momentum for this case does not converge to its pQCD counterpart, \( p_F^0 \). It is important to remark that the conjectured coupling running predicts that after peaking at the super-conformal region, \( V_s^{2} \) approaches the conformal value from below, like pQCD. It should be also emphasized that the shape of the curve generated with \( G_V(\mu) \) resembles some of those recently predicted in Refs. \[24,25\].

The NJL pressure together with the pQCD results for the \( N_f = 2 + 1 \) case, obtained from Ref. \[35\], is displayed in Fig. 7. The pQCD results were obtained by varying the \( \overline{\text{MS}} \) renormalization scale from \( \mu \) to \( 4\mu \) while the Fermi-Dirac
FIG. 7. Pressure, normalized by $P_0$ (see text), as a function of $\mu_B$. The gray band represents the region where $\mu = \mu_B/3 > \Lambda$. The light band corresponds to the pQCD results when the $\overline{\text{MS}}$ renormalization scale covers the range from $\mu$ (bottom edge), to $4\mu$ (top edge). The dotted line represents the pQCD predictions at the central $\overline{\text{MS}}$ scale, $2\mu$.

FIG. 8. EoS for the three cases considered. The light band corresponds to the pQCD results when the $\overline{\text{MS}}$ renormalization scale varies from $\mu$ (bottom edge) to $4\mu$ (top edge). The softening of the $G_V(\mu)$ curve takes place at $\mathcal{E} \approx 0.7 \text{ GeV fm}^{-3}$ in agreement with Refs. [12, 22]. For completeness, the region which concerns CET has also been indicated.

FIG. 9. Conformal measure, $C = \Delta/\mathcal{E}$, as a function of $n_B/n_0$. The running coupling predicts a change of slope of high amplitude, at $n_B = 4 - 10 n_0$. A fixed $G_V$ leads to a negative $C$ at $n_B/n_0 \approx 14$. The light band corresponds to the pQCD results when the $\overline{\text{MS}}$ renormalization scale covers the range from $\mu$ (top edge) to $4\mu$ (bottom edge).
limit for free massless quarks, used to normalize the pressure in Fig. 7 reads

\[ P_0 = \frac{N_c N_f}{12\pi^2} \left( \frac{\mu_B}{3} \right)^4. \]

Fig. 8 displays the NJL EoS as well as the pQCD result (generated from Ref. [35]). The figure clearly shows that the predictions coming from \( G_V(\mu) \) and \( G_V = 0 \) agree with pQCD at high energies while \( G_V = G_S / 3 \) does not. Of utmost importance is the fact that at \( \mathcal{E} \approx 0.7 \text{GeVfm}^{-3} \) a sudden change of slope takes place producing the softening of the EoS produced by \( G_V(\mu) \), in accordance with Refs. [12, 22]. A second change of slope happens at \( \mathcal{E} \approx 2 \text{GeVfm}^{-3} \). At the same time, our running coupling predicts a change of slope of high amplitude, at \( n_B = 4 - 10 n_0 \), preventing \( \mathcal{C} \) from becoming negative. In summary, \( G(\mu) \) shifts the high-\( n_B \) behavior of the trace anomaly which then approaches zero while remaining positive, supporting the hypothesis advanced in Ref. [27].

\[ \mu \]

### V. CONCLUSIONS

The three-flavor NJL model with a repulsive vector channel, parametrized by \( G_V \), has been considered in the evaluation of the EoS describing symmetric cold quark matter. The work extends the application performed in Ref. [20], where the two-flavor version has been considered in the presence of density dependent repulsive coupling, \( G_V(\mu) \). Here, it has been shown that the presence of strangeness does not affect the main physical properties displayed by key thermodynamical quantities evaluated with \( G_V(\mu) \). The advantage of such a model is that one is then able to interpolate between a regime where repulsion is high (the EoS is stiff) and a regime where repulsion low (the EoS is soft). In this way the NJL model can simultaneously observe astrophysical constraints, which require the EoS to be stiff at lower densities, while producing results which agree with pQCD at arbitrarily high densities. For instance, considering the moderate value \( G_V(0) = G_S / 3 \) this work shows that it is possible to describe a non-conformal peak at \( V_s^2 = 0.38 \) and \( n_B = 3.23 n_0 = 0.52 \text{fm}^{-3} \) (corresponding to \( \mathcal{E} = 0.59 \text{GeVfm}^{-3} \)). These numerical values are in good agreement with some of the values quoted in Ref. [22]. We have also seen that, as the density increases, the interpolating model predicts that \( V_s^2 \) approaches the pQCD (conformal) prediction, \( V_s^2 \rightarrow 1/3 \) from below, as expected. Another important result obtained here shows that our model can produce a noticeable change of slope in an initially hard EoS so that it will soften and join the pQCD predictions at higher energy densities. Interestingly enough this change happens at \( \mathcal{E} \approx 0.7 \text{GeVfm}^{-3} \), in conformity with predictions made in Refs. [12, 22, 36]. The model also allows us to conclude that a non-conformal peak in \( V_s^2 \) is not in tension with the trace anomaly being positive for all densities, a result which agrees with a scenario proposed in Ref. [27]. As explicitly shown here, these findings cannot be reproduced if one naively uses \( G_V = 0 \) (the EoS is far too soft at low-\( n_B \)), or if one fixes \( G_V \) to a finite value (the EoS is far too hard at high-\( \mu_B \)). At first sight it seems remarkable that with a simple modification the model is able to reproduce such highly non-trivial results, which were originally obtained through the use of more sophisticated approaches [12, 22, 27]. However, it should be clear that the simple modification encoded within the \( G_V(\mu) \) running has physical consequences which in turn imply that the fundamental concept of repulsion should be reviewed. More precisely, the results obtained here suggest that quarks with bare masses do not tend to repel each other when compressed, in opposition to the behavior displayed by quarks with effective masses. Obviously the simple \textit{ansatz} proposed here is not unique so that one is free to consider alternative forms (such as gaussian, skewed gaussian, etc) as well as the use of other parametrizations \textit{provided} that \( G_V \) \textit{decreases} with the density after the chiral transition takes place (keeping in mind that this is the main ingredient driving the crucial change of slope observed in the corresponding EoS). In principle, the mechanism described in this work can be generalized to any model which contains a repulsive channel. Possible extensions include the consideration of non symmetric quark matter in \( \beta \)-equilibrium, in order to describe quark stars, as well as the inclusion of a diquark interaction channel, in order to explore the high-density region of QCD, among others. In future applications one could also consider replacing the popular pQCD predictions with those furnished by the \textit{renormalization group optimized perturbation theory}, since this resummation technique generates results which are less sensitive to scale changes [37, 39].
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