We predict the emergence and quantization of the valley Hall current of indirect excitons residing in a noncentrosymmetric two-dimensional material. This current appears due to the anomalous group velocity of exciton center-of-mass motion with an account of the Berry phase of individual electrons and holes in the exciton within a given valley and in the presence of the quasi-condensate. In equilibrium, the net exciton flux vanishes. However, under the action of an external electromagnetic field, the excitons pushed out from the ground state contribute to stationary nonequilibrium valley Hall current, which exhibits a quantized behavior as a function of electromagnetic field frequency.

Anomalous transport of carriers of charge in low-dimensional structures is a captivating fundamental and a simultaneously application-oriented subject of research, which is recently attracting growing attention [1]. One of the conceptual problems here is the study of the anomalous Hall effect, resulting from the topological properties of charge carriers [2] due to the emergence of the Berry phase associated with closed trajectories in the parameter space [3]. It turns out that the Berry curvature plays a central role in various transport phenomena in condensed matter physics [4–6], one of the most intriguing of which is the valley Hall effect (VHE) [7–10].

In intuitively apprehensible quasiclassical language, the origin of the anomalous Hall transport lies in a specific term in the expression of the particle group velocity, \( \mathbf{v} = \partial_{\mathbf{p}} \varepsilon / \partial_{\mathbf{p}} - \mathbf{p} \times \mathbf{\Omega}_{\mathbf{p}} \), where \( \varepsilon(\mathbf{p}) \) is the dispersion relation and \( \mathbf{\Omega}_{\mathbf{p}} \) is the Berry curvature, expressed via the periodic amplitude of the Bloch wave function \( |\mathbf{u}(\mathbf{p})| \) (\( h = 1 \)): \( \mathbf{\Omega}_{\mathbf{p}} = \partial_{\mathbf{p}} \times \langle \mathbf{u} | i \partial_{\mathbf{p}} | \mathbf{u} \rangle \). For a charged particle subject to an electric field \( \mathbf{E}(t) \), the quasiclassical equation of motion reads \( \dot{\mathbf{p}} = e \mathbf{E}(t) \), giving the anomalous velocity term \( -e \mathbf{E}(t) \times \mathbf{\Omega}_{\mathbf{p}} \) acting as a pseudomagnetic field in momentum space. This very term is responsible for the anomalous Hall currents.

We can pose a question: Is there any influence of the Berry curvature on dynamics of (overall) neutral compound particles like bosonic excitations in semiconductor crystals, called excitons (electron-hole pairs)? For the first glance, it seems to vanish due to the insensitivity of their center-of-mass motion to external uniform electromagnetic fields. In the meantime, transport of neutral particles is of great interest, and also it would be highly beneficial to map the topological properties of fermions on bosons. Excitons not only play an important role in optical phenomena occurring in low-dimensional structures [11–12] but also they can be widely used in applications [13]. One of the advantages of excitonic devices is the possibility of exciton-photon coupling, which is essential for the manipulation of optical signals, including the opportunity to form coherent quasi-condensate states with the vanishing sensitivity to a disorder present in any nanostructure.

Excitons can form from electrons and holes sharing a common quantum well (direct excitons) or separated from each other, thus residing in a double quantum well (indirect excitons). The main feature of indirect excitons is that they possess a built-in dipole moment. The most well-studied system of materials for double quantum wells and thus for the study of optical and transport properties of excitons is GaAs and its alloys with aluminum and indium. The discovery of novel purely 2D materials, such as monomolecular layers of transition metal dichalcogenides (TMDs), benchmarked new chapter in both the direct [13–18] and indirect [19–22] exciton physics. These monolayers have a complex Brillouin zone [5], containing two inequivalent valleys located at the opposite edges. Due to sizeable intervalley distance in momentum space, the quantum number indicating the valley is robust against external perturbations, thus creating an additional degree of freedom similar to spin or charge of the particles. This paradigm forms the basis of valleytronics.

Another property of indirect excitons, which distinguishes them from their direct counterparts, is a long lifetime, typically ranging from nanoseconds to microseconds. The lifetime is long due to the weak overlap of spatially separated electron and hole wave functions. Fast energy relaxation allows for cooling indirect excitons down to \(~0.1\) K within their lifetime, that is well below the temperature of quantum degeneracy of exciton gas, thus opening an experimental way to form a coherent exciton state – the excitonic quasi-condensate. Experiments show typical temperatures of exciton condensation \(1–5\) K in GaAs nanostructures. Recently there have been reported reasonable arguments to expect a much higher condensation temperature for indirect exciton in TMD monolayers such as MoS\(_2\) [11].

Let us again address the quasiclassical picture, now from the point of view of excitons. The dipole moment of an exciton \( p = ed (e > 0) \), where \( d \) is the interlayer...
In the vicinity of the conduction and valence band edges, \( P \) and \( R \) govern the transport of an individual exciton, where \( P \) and \( R \) are the electron and hole, respectively. To describe their Coulomb-interacting Dirac particles, we will follow the method suggested in Ref. 24. The Hamiltonian describing an exciton in the dynamics we will follow the method suggested in Ref. 24. The Hamiltonian describing an exciton in the Coulomb-interacting Dirac particles. To describe their Coulomb-interacting system, the current density becomes quantized in the presence of two types of indirect excitons. If an electron and a hole of the top layer inhabit the same valley, such an exciton is considered to be direct-rect in momentum space (we will refer to it as DMSE). The presence of the Berry phase results in an anomalous contribution to the exciton center of mass motion due to the change of the electric field along the layers \( E_z(R) \) [13]. The presence of the Berry phase effect is introduced via the gauge transformation

\[
\alpha \rightarrow \alpha + \frac{1}{2} \Omega_\alpha(\mathbf{p}_\alpha) \times \mathbf{p}_\alpha, 
\]

where \( \alpha = e, h \) and \( \Omega_\alpha \) is a Berry curvature of an individual electron or a hole.

Substituting (3) in (2) and then in (1), one finds the equation of \( \text{VMSE} \) exciton motion (see the Supplemental Material [27] for the details of calculation),

\[
\dot{\mathbf{R}} = \mathbf{P}/M - \frac{1}{8} [\mathbf{F}_0 \times \Omega_{\text{ex}}],
\]

where \( \mathbf{F}_0 = \mathbf{F}(0) \) is the in-plane static force acting on the exciton center-of-mass and \( \Omega_{\text{ex}} = 2\Omega_\alpha \) is the exciton Berry curvature, taken at the band edges. The corresponding exciton current reads

\[
j = \sum_{\mathbf{P}} \dot{\mathbf{R}} f(\mathbf{P}, \mathbf{R}, t). 
\]

In equilibrium, the distribution function is an even function of the center-of-mass momentum \( f_0(-\mathbf{P}) = f_0(\mathbf{P}) \). Therefore only the second term in (4) if substituted in (5) gives a non-vanishing contribution. If condensed, the excitons occupy the lowest energy state with zero momentum, \( f_0(\mathbf{P}) = n_c \delta_{\mathbf{P}, 0} \), where \( n_c \) is a condensate density. The resulting equilibrium Hall current reads

\[
j_{eq} = -\frac{n_c}{8} [\mathbf{F}_0 \times \Omega_{\text{ex}}].
\]

Under the action of an external electromagnetic field the exciton distribution acquires the nonequilibrium correction, \( f(\mathbf{P}, \mathbf{R}, t) = f_0(\mathbf{P}) + \delta f(\mathbf{P}, \mathbf{R}, t) \), describing

\[
H = H_0(\mathbf{p}_e) + H_0(\mathbf{p}_h) + u(\mathbf{r}_h - \mathbf{r}_e) + U(\mathbf{r}_e, \mathbf{r}_h).
\]

In the vicinity of the conduction and valence band edges, both the \( H_0 \) terms can be approximated by the parabolic term \( H_0(\mathbf{p}) = \frac{p^2}{2m} \). In the framework of the two-band Dirac model, which describes well the low-energy spectrum of a TMD monolayer, the electron and hole have equivalent effective masses. The account of the higher energy bands results in a difference of the particle masses. However, for our purpose, which is the proof of principle, the two-band model is sufficient.

The second term in Hamiltonian (2) describes the Coulomb interaction between the electron and hole, and \( U(\mathbf{r}_e, \mathbf{r}_h) \) term corresponds to the exciton energy shift due to the voltage applied across the layers. The Berry-phase effect is introduced via the gauge transformation

\[
\alpha \rightarrow \alpha + \frac{1}{2} \Omega_\alpha(\mathbf{p}_\alpha) \times \mathbf{p}_\alpha, 
\]

where \( \alpha = e, h \) and \( \Omega_\alpha \) is a Berry curvature of an individual electron or a hole.

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Under the action of an external electromagnetic field the exciton distribution acquires the nonequilibrium correction, \( f(\mathbf{P}, \mathbf{R}, t) = f_0(\mathbf{P}) + \delta f(\mathbf{P}, \mathbf{R}, t) \), describing
Here $\omega$ is the rate of exciton transitions to the final state $|n, P\rangle$. In the stationary regime, the current of photoexcited excitons is determined by the time-averaged distribution function $\delta f_n(P) = \delta f(P, R, t)$, yielding

$$j = -\frac{1}{8} \{ F_0 \times \Omega_{ex} \} \sum_{n, P} \delta f_n(P),$$

where $\delta f_n(P) = \tau R$, $\tau$ is the exciton relaxation time in the excited state,

$$R = \frac{2\pi}{hS} |\langle n, P | \hat{W} \rangle_{\text{BEC}}|^2 \times \delta (E_{\text{BEC}}(n, P) - E_{\text{BEC}} - \omega)$$

is the rate of exciton transitions to the final state $|n, P\rangle$ from the equilibrium Bose-condensed state $|\text{BEC}\rangle$ (per second per unit area). Here $\omega$ is the frequency of external electromagnetic field, $E_{\text{BEC}}$ is the energy of exciton system in BEC regime and $E_{\text{BEC}}(n, P)$ is the energy of the excited state consisting of the condensate and a photoexcited exciton. We use a constant $\tau$ assuming the scattering of excited excitons via the short-range impurities. The Hamiltonian describing the exciton interaction with the external uniform electromagnetic field $E$ is taken in the dipole approximation, $W(r) = -e(rE)$, where $r = r_n - r_e$ is the relative in-plane electron-hole coordinate.

The exciton energy is characterized by the momentum $P$ and discrete energy levels $\epsilon_n$ of internal exciton motion, $E_n(P) = P^2/2M + \epsilon_n$. We assume that the condensate occurs at the energy state with zero center-of-mass momentum $P = 0$ and at the lowest energy level of the internal exciton motion $\epsilon_{n=0} = 0$. According to the Bogoliubov model of a weakly-interacting Bose gas, the low-energy properties of a Bose condensate can be characterized by the excitations having the dispersion $\omega_k = sk/\sqrt{1 + (k\xi)^2}$, where $s = \sqrt{g_n/M}$ is a phase velocity, $\xi = 1/2M$ is a healing length, and $g$ is the strength of exciton-exciton interaction. Following the Bogoliubov theory, the Bose-condensate state reads

$$\Psi_{\text{BEC}} = \frac{a_0}{\sqrt{S}} \psi_0(r) + \psi_0(r) \sum_{P \neq 0} \left( u_P a_P + v_P a_P^\dagger \right) \exp(iPR)/\sqrt{S}.$$  \hspace{1cm} (9)

Here $a_0 = \sqrt{n_cS}$, the first term describes the macroscopically-occupied BEC state, whereas the second term corresponds to the fluctuations with the dispersion $\omega_P$, and $u_P, v_P$ are the Bogoliubov transformation coefficients. The function $\psi_0(r)$ is the eigen function of internal exciton motion, while the excited states with energies $E_n(P)$ have the eigen functions $\Psi_n(R, r) = \psi_n(r) \exp(iPR)/\sqrt{S}$.

Due to the presence of two terms in Eq. (9), the matrix element of external field $W(r)$ also consists of two terms describing two types of processes, schematically shown in Fig. 3. The matrix element of the first-type processes reads \cite{27}

$$M_n(P) = a_0 W_{n,0} (\frac{2\pi\hbar}{S})^2 \delta(P),$$

$$W_{n,0} = \int d\psi_{n}^\dagger(r) W(r) \psi_{0}(r).$$

It corresponds to direct exciton transitions from the condensate to a non-condensed state with the excitation of an internal degree of freedom of individual exciton. The corresponding photoexcited Hall current reads

$$j^{(1)} = [\Omega_{ex} \times F_0] \frac{n_c \tau^2}{4\hbar^2} \sum_{n \neq 0} \frac{|W_{n,0}|^2}{1 + (\hbar\omega - \epsilon_n)^2 \tau^2/\hbar^2}.$$  \hspace{1cm} (11)

It has a resonant structure and is proportional to the condensate density.

The processes of the second type are described by the second term in Eq. (9), reflecting the density fluctuations of the BEC (the Bogoliubov quanta). At zero (small) temperature this excitation branch is (nearly) empty thus the only term containing the creation operator $a_P^\dagger$ contributes to the optical transitions. This corresponds to the exciton excitation by the photon absorption accompanied by the emission of a Bogoliubov quasiparticle, as indicated in Fig. 3. Contrary to the processes of the first type, here the exciton is excited into the non-condensate state with a nonzero momentum $P \neq 0$ from the condensate state with $P = 0$. Such indirect transitions may occur only if the momentum $P$ is carried away by a “third
The Bose condensate here plays this very role, carrying away the momentum by means of the Bogoliubov excitations with the energy $\omega_{-p}$.

The matrix element describing these processes reads

$$M_n(P', P) = W_n(2\pi\hbar)^2 S^{-}\epsilon_p \delta(P' - P).$$

In the most interesting limit of linear dispersion of Bogoliubov excitations ($\omega_p \approx sp \gg P^2/2M$), we can carry out analytical calculations \cite{27} and find the corresponding contribution to the exciton Hall current:

$$j^{(2)} = |\Omega_{ex} \times F_0| \left( \frac{M\tau}{16\pi\hbar^3} \right) \sum_{n \neq 0} |W_{n,0}|^2 \left( \frac{\pi}{2} + \arctan \left( \frac{(h\omega - \epsilon_n)\tau}{\hbar} \right) \right).$$

Evidently, this term vanishes if we disregard the Berry phase effect (the term $j^{(1)}$ is also zero in this case). Note, that it is the arctangent function which ultimately determines the behavior of this component of the electric current. Another important property of (13) is that it does not explicitly depend on the condensate density, in contrast with (11), even though $j^{(2)}$ also vanishes in the absence of the condensate. The system response (13) is a unique characteristic of bosonic systems, making the QVHE here conceptually different from the conventional fermionic VHE. Thus, we conclude that condensation is the necessary condition for the current quantization in the absence of an external magnetic field.

In an experiment, one creates the exciton gas when the sample is illuminated by light with the frequency exceeding the material band gap. Due to the fast energy relaxation, the photo-excited electrons and holes cool down and form excitons in the monolayers. We can expect that both the valleys will be populated with DMSE and IMSE nearly equally. The Berry curvature of IMSE $\Omega_{ex} = 0$ since the signs of the Berry curvatures of electrons and holes located in different valleys are opposite. Thus, only DMSE will contribute to the effect. Due to the nearly equivalent population of the valleys, the equilibrium valley Hall current vanishes since $\Omega_{ex} = -\Omega_{ex}'$. However, the photoinduced part of the valley Hall current might be nonzero. Due to the axial symmetry of the Coulomb electron-hole interaction potential $u(r)$, the eigen states of the internal exciton motion are characterized by the principal (radial) quantum number and the projection of the angular momentum: $n = (n_r, m)$. The axial symmetry dictates the degeneracy of quantum states $(n_r, \pm m)$. As it is shown in Ref. 24, the Berry curvature influences these states resulting in their energy splitting of the order of tens of meV. Moreover, the splitting has opposite sign in different valleys, see Fig. 4.

If the exciton gas is exposed to a circularly polarized electromagnetic field, the intra-excitonic transitions occur in one valley due to the valley-dependent optical selection rules, simultaneously exciting the state $m = +1$ or $m = -1$ in one valley. Thus, the photoexcited valley Hall current occurs in the system resulting in the exciton accumulation at the sample edge across $F_0$, see Fig. 5. The latter can be detected at the sample boundary by measuring the polarization of luminescence. Due to the step-like behaviour of the exciton Hall current, the intensity of the luminescence should inherit analogous step-like behaviour since its intensity is proportional to the density of excitons at the sample edge, determined by the value of the Hall current.

The last issue we want to address is the applicability of the Bogoliubov model. Even though this model implies a weakly-interacting system, in the long-wavelength limit it works well even in the case of a strongly interacting Bose gas \cite{29}. This is due to the low-energy excitations
under strong interaction between the Bose particles also acquire linear (sound-like) dispersion (with some effective renormalized parameters).

**Conclusions.** We have considered indirect exciton transport in two parallel layers of noncentrosymmetric two-dimensional materials. We have shown that in the presence of the quasi-condensate, there emerges a quantized valley Hall current of excitons due to the anomalous group velocity of exciton center-of-mass motion under the action of an external electromagnetic field. This current experiences steplike behavior as a function of electromagnetic field frequency, mimicking the quantization of the internal exciton motion.

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