Mathematical model of interaction of a rolling elastic wheel with elastic-visco-plastic support base taking into account the history of loading

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Abstract. One of the main methods to predict permeability is a simulation of the motion of wheeled vehicles in different operating conditions, which are based on mathematical models of interaction between elastic tires with deformable irregularities of the support base. Currently in the theoretical study of the interaction of propellers with the ground formed two main directions: analytical method involving a mathematical description of the process under study and the finite element method based on computer simulation. The aim of this study is to develop a mathematical model of rolling elastic wheels on deformable rough reference base, taking into account the deformation of the contact patch at each end of the elementary playground. A mathematical model of the curvilinear rolling of the elastic wheel on an uneven elastic-visco-plastic support base is developed, taking into account the deformation of the contact spot in each finite elementary site; the influence of ground hooks on the parameters of the wheel movement; coupling properties of the support base in the horizontal direction under the action of a vertical load; change in the direction of radial and tangential reactions; elastic-visco-plastic properties of the support base;" history " of loading of each elementary platform of the support base; condition of the support base on its physical and mechanical characteristics.

1. Introduction
One of the main directions of economical growth in Russia remains to be a speedy development of north and northeast regions that are the constituents of the 60 percent of the country territory.

The further development of these territories requires new methods and technologies for solving transport and technological problems when off-road transportation of cargoes and people is conducting. In this way the movement of vehicle is difficult and impossible in some cases.

The existing models of wheeled, tracked and rotary screw vehicles don’t correspond to present functional, efficiency, reliability and ecological requirements for wheeled propulsion devices when operating in north regions of the country where supporting surfaces are weak-bearing. In this way there are technical, economic and social need in creating and using vehicles with pneumatic wheeled propulsion devices including ultra-low pressure devices that satisfy the requirements.

It should be noted that our country doesn’t have necessary collection of energy efficient off-road vehicles currently. The existing off-road machines performed according to old traditional schemes and produced serially by industry don’t meet the requirements defining the efficiency and environmental
friendliness of wheeled propulsion devices in difficult climatic operating conditions. In current situation the problem of wheeled vehicles patency prediction of movement over supporting surface with weak-bearing properties is relevant [1,2].

One of the fundamental methods of patency prediction is simulation method of wheeled vehicle movement in different operational conditions that is based on mathematical models of interaction the elastic tire with deformable support base unevenness.

Established [3, 4] that the traction patency wheeled vehicle mutually affect both the deformation properties of tire and physical and mechanical characteristics of soil: normal deflection of the tire and the depth gauge; the change in the reference area of the contact patch depending on load and air pressure in the tire; the presence of hysteresis losses in the material of the tire that affect rolling resistance caused by friction in the contact patch on the ground; creating a tangent reactions of the soil over the entire area of contact.

Along with the definition of the dependencies to describe "load-deformation of the ground " and "load-deformation mover", not less important task is the selection of modelreset vertical deformations of soil in contact with the pneumatic tire. On the correct choice of the type and kind of approximating dependence is largely determined by both qualitative and quantitative aspects of General solution of the problem of contact interaction of elastic mover with deformable ground [5].

Currently in the theoretical study of the interaction of propellers with the ground formed two main directions: analytical method involving a mathematical description of the process under consideration [6 – 8] and finite element method (FEM), based on computer simulation [9 – 13]. In the study of the interaction of wheel elements with the soil, an analytical method has found wide application. Developed mathematical models of the interaction of the tyre with the ground allow to solve various tasks. These models are used as in the study of processes of interaction of a single wheel mover with the soil array and at research of dynamic models of mobile machines operating in specific road-soil (RSC) and climatic conditions. One of the most significant shortcomings of these models is the description of the interaction of the wheel with a smooth deformable base, while the real profile of the support surface has a significant height of irregularities is comparable with the radius of the wheel.

The study of the processes occurring in the soil mass under the influence of the wheel mover, the finite element method are relatively new, currently the largest application. This method is better than others provided numerical procedures for the study of mathematical models of objects. The most important advantage is the availability of implicit methods of integration of systems of differential equations. The application of this method is the most accurate to describe the process of interaction of wheel elements with the soil, to determine the stresses in the soil mass deformation of tire and soil compaction. In contrast to analytical methods form the contact patch of the tyre with an elastic support surface is the result of the simulation taking into account the independent soil characteristics and the constructive and operational parameters of the engine. However, a significant disadvantage of this method is its high computational complexity, therefore, with currently available computers to use these models as part of the overall model of the motion of multi-wheeled vehicles difficult.

The aim of this study is to develop a mathematical model of rolling elastic wheels on deformable rough reference base, taking into account the deformation of the contact patch.

2. Mathematical model of deformable support base

The solution of problems associated with the action of short-term loads on the deformable support base is based on model representations of the properties of the continuous medium on which the load acts [14]. One of the approaches in solving this problem is to replace the real spatial system "load-deformable half-space" with an idealized mechanical system, the parameters of which correspond to the physical and mechanical characteristics of the support base [15,16]. Analysis of various computational schemes and mathematical models of support bases showed that to determine their complete and residual deformations under the action of short-term load it is advisable to use a mechanical system with one degree of freedom, the movement of which is determined by the model of a visco-elastic medium devoid of inertia (figure 1).
As presented in figure 1, the design scheme in parallel with a spring with elasticity modulus $E_{\text{elast}}$ connected to the damper with viscosity $\eta$.

The differential equation that determines the behavior of the support base under uniaxial compression, according to the Kelvin-Voigt model, has the form

$$
\dot{h}_{gr} = \frac{h_{gr}}{\eta} + \frac{p(t)}{\eta E_{\text{elast}}},
$$

where $h_{gr}$ – current deformation (draft) of the support base.

The diagram of the ultimate deformation $h_{gri}$ of the $i$-th platform of the support base is established on the basis of the Bernstein-Letoshnev formula [9]

$$
p_{0zi} = c h_{gri}^{\mu}, \text{MPa},
$$

Where $p_{0zi}$ – limiting pressure of the $i$-th platform of the support base at the draft $h_{gri}$; $c$ – the coefficient of deformation of the soil; $\mu$ – the density of the soil.

The meaning of the strain limit diagram is as follows: at $p_{i}(t) \leq p_{0zi}$ of soil sediment does not occur.

When re-increasing the load, when the pressure on the $i$-th support platform of the support base $p_{i}(t) > p_{0zi}$, the soil sediment $h_{gr}$ begins to grow according to equation (1).

In connection with the non-linear character of the dependence (2) the modulus of elasticity $E_{\text{elast}}$ of the $i$-th platform of the support base is a variable value $E_{\text{elast}} = \frac{dp_{0zi}}{dh_{gri}} = \mu c h_{gri}^{\mu-1}$. Increasing the modulus of elasticity $E_{\text{elast}}$ with increasing precipitation $h_{gri}$ allows to take into account the effect of soil compaction.

3. Mathematical model of interaction between elastic tires with the irregularities of deformed support base

The proposed model uses two different coordinate systems (figure 2) due to the structure and form of the equations of motion of the object.

First, the stationary coordinate system (SCS)$O_2X_2Y_2Z_2$ is used to simulate the specified ground traffic conditions. The origin of the system of point $O_2$ coincides with the beginning of the simulated track.

To determine the forces acting on the car from the ground we introduce a micro mobile coordinate system (MMCS) under which we understand the system $O_TX_TY_TZ_T$, the center of which $O_T$ coincides with the geometric center of rotation, the axis of $O_TX_T$ coincides with the projection of the longitudinal axis of symmetry of the wheel on the support surface.

Key assumptions:
1) The normal pressure in the contact spot is distributed proportionally to the deflection of the tire on each elementary section of the contact platform.
2) The force of interaction of the wheel with its support base directed in the opposite direction from the slip speed. Consider the scheme of movement of the wheel in figure 2.

Figure 2. A design scheme of wheel rolling over uneven sub-base:
1 – strain profile of soil; 2 – deformed soil profile; 3 – the undeformed profile of the soil; O – the center of the wheel; \( Z_{20}, X_{20} \) – the coordinates of the center wheel SCS; \( \omega_k \) – the angular velocity of rotation of the wheel; \( M_k \) – torque applied to the wheel; \( V_{0x} \) – longitudinal velocity of the wheel center; \( P_Z, P_X \) – vertical and longitudinal forces applied to the center of the wheel from the wheeled vehicle axis; \( R_r, R_t \) – radial and tangential projection of the wheel interaction reaction with the support base; \( R_g \) – wheel rolling drag force; \( r_k \) – free radius of the wheel; \( Z_{2i}, X_{2i} \) – the coordinates of a point under the support base of the \( i \)-th point of the undeformed profile wheels; \( \alpha_i \) – the angle between the vertical axis and the \( i \)-th point of the undeformed profile wheels; \( \alpha_{eqv} \) – the equivalent angle of the application point of the total reaction; \( d_{ri} \) – radial deflection of the tyre for the \( i \)-th point of the undeformed wheel profile; \( h_{gi} \) – the rut depth under the \( i \)-th point of the undeformed profile wheels.

For model development we use the results of [9, 17-20]. On the lower semicircle of the undeformed profile of the wheel will select a certain number of points, \( n \), the position of which will determine the angle \( \alpha_i \) between the vertical line dropped from the center of the wheel on the axis \( X_2 \), and a ray connecting a point of the undeformed profile of the wheel with its center (figure 2). The number of points is chosen based on a compromise between accuracy of the model and its performance. Define the coordinates \( X_{2i} \) and \( Y_{2i} \) of selected points of the profile at the SCS (figure 3).

\[
\begin{align*}
X_{2i} &= X_{20} + r_k \sin \alpha_i \cos(\theta + \beta); \\
Y_{2i} &= Y_{20} + r_k \sin \alpha_i \sin(\theta + \beta); \\
\frac{\pi}{2} \leq \alpha_i \leq \frac{\pi}{2},
\end{align*}
\]

where \( X_{20}, Y_{20} \) are the coordinates of the center of the wheel \( O \) SCS.

The vertical coordinate \( Z_{2i} \) of the undeformed profile of the wheel in the SCS, we define by the formula

\[
Z_{2i} = Z_{20} - r_k \cos \alpha_i,
\]

where \( Z_{20} \) is the vertical coordinate of the wheel center at the SCS.
Figure 3. Design scheme for determining the force of interaction between the wheel and the support surface: $\vec{V}_k$ – the vector of linear speed of the wheel center; $\vec{R}_k$ – reaction vector of the wheel interaction with the support base; $\theta$ – the angle between the longitudinal axis of the machine and the axis $X_2$ SCS; $\beta$ – the rotation angle of the steering wheel; $\gamma$ – the angle between the plane of symmetry of the wheel and the vector $\vec{V}_k$.

The formation of the longitudinal profile $Z_{2gri}$ is carried out according to the following algorithm

$$Z_{2gri} = Z_{2gri}^{undef} - h_{gri},$$

where $h_{gri}$ — vertical draft of soil under the $i$-th elementary platform of the contact spot; $Z_{2gri}^{undef}$ – the vertical coordinate of the undeformed profile of the gauge (simulated in advance by a known method described, for example, in [19]).

The $d_i$, deflection of the tire in the radial direction for the $i$-th point of the undeformed profile is determined from the following ratios

$$d_i = \begin{cases} 0, & Z_{2gri} \leq Z_i \\ (Z_{2gri} - Z_{2i}) \cos \alpha_i, & Z_{2gri} > Z_{2i} \end{cases},$$

where $Z_{2gri}$ – vertical coordinate of the profile of the support base for the $i$-th point of the wheel.

Thus, to determine the reactions of interaction of wheels with the supporting surface $R_x$ and $R_z$ in MMCS in the presence of several areas of "overlap" profile substructure undeformed contour of the wheel it is necessary to determine the equivalent angle $\alpha_{eqv}$ the point of application of the total radial reaction $R_r$ in the radial direction and the tangential reaction $R_\tau$ (figure 2).

Define $\alpha_{eqv}$ as a weighted average value

$$\alpha_{eqv} = \frac{\sum_{i=1}^{n} \alpha_i d_i}{\sum_{i=1}^{n} d_i}.$$ 

Radial reaction $R_r$ is the sum of two components: elastic $R_{ry}$ and damping $R_{rd}$: $R_r = R_{ry} + R_{rd}$. $R_{ry}$ depends on the equivalent deflection of the tire

$$d_{r_{eqv}} = \frac{\sum_{i=1}^{n} d_i}{n_k},$$

where $n_k$ is the number of points in the undeformed profile in contact with the support surface.

$R_{rd}$ depends on the speed of the deflection of the tire in the radial direction. We define the projection of velocity of points of the contour of the wheel on the axis $X_\tau$ and $Z_\tau$: 

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\[ V_{iX_T} = \omega_k (r_k - dr_i) \cos \alpha_i + V_{0X_T}; \]
\[ V_{iZ_T} = \omega_k (r_k - dr_i) \sin \alpha_i + V_{0Z_T}, \]

where \( \omega_k \) – angular velocity of rotation of the wheel; \( V_{0X_T} \) and \( V_{0Z_T} \) – projection of the velocity vector of the wheel center (point \( O \)) on the axis \( X_T \) and \( Z_T \) respectively.

The vector of linear speed of the \( i \)-th point of the undeformed profile of the wheel in the radial direction
\[ V_{ri} = V_{iX_T} \sin \alpha_i + V_{iZ_T} \cos \alpha_i. \]

The speed of profile deformation of the \( i \)-th point in the radial direction
\[ \frac{d}{dt} (dr_i) = \frac{dZ_{2gr_i}}{dt} \cos \alpha_i - V_{ri}. \]

Equivalent speed of deflection
\[ \frac{dr_{kib}}{dt} = \frac{\sum_{i=1}^{n} \left( \frac{d}{dt} (dr_i) \right)}{n_k}. \]

Further, knowing the elastic and damping characteristics of the tire in the radial direction, we find \( R_s \).

**Determination of the parameters of the contact spot and the magnitude of ground subsidence \( h_{kr} \) given the "history" of its loading**

The area of the contact patch \( F_{it} \) (figure 4) defined by the formula
\[ F_{it} = 2 \int_{\alpha_i}^{\alpha_i} b(\alpha) [r_k - dr(\alpha)] d\alpha, \]
\[ b(\alpha) = b_t \left( 1 + \frac{dr(\alpha)}{r_k} \right) \]

where \( b_t \) – the width of the tyre.

The discrete analogue of the formula (3) can be written as
\[ F_{it} = b_t \left( 2 + \frac{dr_{i+1} + dr_i}{r_k} \right) \left( r_k - \frac{dr_i + dr_{i-1}}{2} \right) (\alpha_{i+1} - \alpha_i) \]

**Figure 4.** Design scheme for determining the contact spot area.
Determine the normal pressure $p_{zi}$ in the $i$-th elementary area of the contact spot. Each such platform (figure 5) is formed by two planes passing through the axis of rotation of the wheel and rotated relative to each other at an angle $\alpha_i - \alpha_{i-1}$.

$$p_{zi} = \frac{P_z}{F_{ii} \cdot 10^6} \sum_{i=1}^{n} \frac{dr_i}{dr_i^* \cos \alpha_i}, \text{ MPa,}$$

where $P_z$ – vertical wheel load.

Then the draught of each elementary $i$-th site is calculated using equation (1).

The contact conditions of the tyre with the support base must be met:

a) if $h_{gri} > dr_i$, then $h_{gri} = dr_i$, which means that the punching of the soil in each elementary area of the contact spot can not be more than the elastic deflection of the tire in this area;

b) if the cumulative sediment of the soil on the $i$-th elementary site of the contact spot $h_{gri} \sum_{i=1}^{n}$, calculated by formula (1), allows to provide such its bearing capacity, which can withstand the pressure (4), the condition for further precipitation will be $\frac{P_z}{F_{ii} \sum_{i=1}^{n} dr_i} \cos \alpha > ch_{gri}^* \sum_{i=1}^{n} 10^6$

When $Z_{gri} < Z_{20} - r_i \cos \alpha_i$, then $h_{gri} = 0$, which means that the soil sediment in the $i$-th support area can not go beyond the contour of the undeformed tire.

The total length of the midline contact spot $L$ can be defined as the sum of the lengths of the elementary pads $L_i$

$$L = \sum_{i=1}^{n} L_i = \left( r_k - \frac{dr_i + dr_{i-1}}{2} \right) (\alpha_{i+1} - \alpha_i)$$

Influence of lugs on the parameters of the wheel movement
In the presence of the cleats must also calculate the shear forces in the areas of the projections and depressions, as well as the removal of soil from the zone of contact in case of intense slipping of the wheel.

Additional vertical penetration $dh_{gp}$ the center of the wheel in the ground caused by the excavation of soil from the area of contact is calculated by the formula [21]

$$dh_{gp} = \frac{t_g h_{g} \sum_{k}^{S_k}}{t_{gro} (1 - S_k)} - t_{gro} \frac{2 \pi r_k}{n_{gro}},$$

where $t_g$ – length trench lug; $h_{gro}$ — grouser height; $t_{gro}$ — step lug; $n_{gro}$ — the total number of lugs on the tyre circumference.

Condition of removal of soil from the contact zone of the wheel with the support base

$$r_k \int_{t_j}^{t_j-1} \omega dt < L,$$

where $t_j, t_{j-1}$ – the current and previous time points.

If the condition (5) is not met, then $dh_{gp} = 0$.

Then the total vertical depth $h_k$ wheel center

$$h_k = h_r + dh_{gp}.$$

We assume that if $h_k \geq r_k$, you lose the mobility of the machine due to the hanging of the machine body on the ground.
To determine the additional shear forces $R_{\text{gro}}^{\text{tj}}$ in the areas of the projections and depressions of the cleats (figure 5) use the results obtained in [20]:

$$R_{\text{gro}}^{\text{tj}} = F_{\text{gro}} e_{s} 10^{6} \exp \left[ - \left( \frac{|e_{s}| - e_{sm}}{0.05e_{sm}} \right) \tan \phi_{s}^{*} \right],$$

$$e_{s} = S(\Delta t) - \int_{t_{1}}^{t_{2}} \omega_{k}(t) \left( r_{k} - dr_{eqv} \right) dt,$$

$$\Delta t = t_{2} - t_{1},$$

where $F_{\text{gro}}$ – frontal area of the projection of the lug; $e_{sm}$ – the maximum shift of the soil in which the cohesion of soil particles is not broken; $e_{s}$ – current shift of the soil; $S(\Delta t)$ is the path traversed by the center of the wheel during the time $\Delta t$; $\omega_{k}(t)$ is the current angular speed of rotation of the wheel.

Total tangent force $R_{\text{gxT}}^{\text{gro}}$ grsv projection on the axis $X_{T}$ MMSC for all for all $m$ grouser in the zone of contact of the wheel with its support base.

![Figure 5](image.png)

**Figure 5.** Calculation scheme for determination of the tangential forces $R_{\text{gro}}^{\text{tj}}$ in the areas of the projections and depressions of the cleats.

$$R_{\text{gxT}}^{\text{gro}} = \sum_{j=1}^{m} R_{\text{gro}}^{\text{tj}} \cos \beta_{j}, \quad m = \frac{L_{1} + L_{2}}{2\pi r_{e}} \cdot n_{\text{gro}}.$$

Then the expression for determining the longitudinal reaction $R_{\text{xT}}^{\text{xT}}$ wheels with a support base in a projection on the axis $X_{T}$ MMSC

$$R_{\text{xT}}^{\text{xT}} = R_{c} \cos \alpha_{eqv} - R_{c} \sin \alpha_{eqv} - R_{f} + R_{\text{gro}}^{\text{xT}}.$$

**Account the bearing capacity of the Foundation soil in horizontal direction under the action of vertical loads**

The magnitude of the horizontal reaction $R_{\text{xT}}^{\text{xT}}$ may be limited by two factors [21]: a wheel slip on the surface of the compacted soil after overcoming the forces of adhesion and loss of bearing capacity from the shear weight of the soil in the direction of action of the horizontal reaction. And if the coupling properties are accounted for by the factor $\mu_{s}$, the loss of bearing capacity can be assessed by comparing the existing shear stresses $\tau$ with the maximum allowable stresses $\tau_{\text{max}}$. 

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The maximum tangential stress in the contact spot is calculated using the Coulomb expression

\[ \tau_{\text{max}} = p_s \tan \varphi_s + c_s 10^6, \]

where \( c_s \) – the coefficient of cohesion (adhesion) of the soil, MPa.

The current tangent stress \( \tau \) redefine by the formula [20]:

\[
\tau = p_s \tan \varphi_s \left[ 1 - \exp \left( - \frac{e_s}{0.1e_s} \right) \right] + c_s 10^6 \exp \left[ - \frac{\left( e_s - e_{sm} \right)^2}{0.05e_{sm}} \right].
\]

Finally, the expression for \( R_{KX_T} \) takes the form

\[
R_{X_T} = k_\tau \left( R_{c, \cos \alpha_{eqv}} - R_{s, \sin \alpha_{eqv}} - R_{fr} + R_{gro} \right),
\]

\[
k_\tau = \begin{cases} 
1, & \text{if } \tau \leq \tau_{\text{max}} \\
\frac{\tau_{\text{max}}}{\tau}, & \text{if } \tau > \tau_{\text{max}} 
\end{cases}
\]

Conclusions
A mathematical model of the curvilinear rolling of an elastic wheel on an uneven elastic-visco-plastic support base is developed:
- deformation of the contact spot in each finite element area;
- influence of lugs on the parameters of the wheel movement;
- coupling properties of the support base in the horizontal direction under vertical load;
- change of direction of radial and tangential reactions;
- elastic-visco-plastic properties of the support base;
- "history" of loading of each elementary platform of the base;
- the state of the support base on its physical and mechanical characteristics.

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