Collective magnetic skyrmion gyrotrropic modes in a dot chain

Zukhra V Gareeva and Konstantin Y Guslienko

1 Institute of Molecule and Crystal Physics, Ufa Federal Research Centre, Russian Academy of Sciences, 450075 Ufa, Russia
2 Departamento de Física de Materiales, Universidad del País Vasco, UPV/EHU, E-20018 San Sebastián, Spain
3 IKERBASQUE, the Basque Foundation for Science, E-48013 Bilbao, Spain
E-mail: gzv@anrb.ru

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Abstract
We investigate the collective gyrotrropic modes in a linear chain of ferromagnetic nanodots in a skyrmion magnetization state. Individual skyrmions in the dots (with thickness about of 1 nm and radius about of 100 nm) are assumed to be stabilized by interplay of isotropic exchange and Dzyaloshinskii-Moriya exchange interactions, magnetic anisotropy and magnetostatic interactions. We consider the dipolar and quadrupolar interactions between moving skyrmions in the dots as prevailing mechanism responsible for dynamic interdot magnetostatic coupling. The frequency dispersion relations of the collective skyrmion gyrotropic excitations are calculated for Bloch and Neel skyrmion configurations and related differences in the frequencies are discussed. The accounting of magnetostatic interactions between the skyrmions removes phase degeneracy of the gyrotrropic frequency existing for isolated skyrmions and allows distinguishing the Bloch and Neel skyrmionic configurations by their dynamic response. The interdot magnetostatic coupling and frequency bandwidth are maximal for the large-radius Bloch skyrmions in the relatively thick dots.

1. Introduction
Patterned magnetic materials controlling propagation of spin waves (SW) received considerable attention due to their unique physical properties and prospect implementations in information technologies, data processing, logic, spintronics, etc. Nowadays magnonics, the branch of magnetism exploiting SW excitations or magnons, rapidly develops. Spin waves have a number of advantages important for SW based technologies: the wide frequency range (GHz–THz), adjustable SW length regulated by the exchange and dipolar interactions acting on nanoscopic and microscopic length scales, non-linear effects allowing realization of magnon–magnon interaction, etc. Confined magnetic structures allow tuning SW properties that open possibilities for realization of magnonic crystals, artificial magnetic materials whose properties vary in the space with periodicity comparable to the SW length. Propagation of SW in periodic patterned nanostructures results in appearance of the propagation/stop frequency bands that can be tailored by varying physical and geometrical properties of constituent materials or by external drives such as current, magnetic and electric fields [1, 2]. Although a variety of patterned magnonic structures are currently known, a lot of expectations are related to magnetic skyrmion arrays [3] that can be created artificially on the base of individual skyrmion arrangement in 1D or 2D periodic nanostructures.

Magnetic skyrmions represent topologically non-trivial magnetization configurations on the nanoscale that can be created individually by electric current or magnetic field or arranged in 2D hexagonal lattices. Initially the skyrmion lattices stabilized by Dzyaloshinskii-Moriya exchange interaction (DMI) were observed in B20 cubic compounds of MnSi-type at low temperatures and a high magnetic field that prevents their applications in nanodevices [3]. Recently it was shown that individual skyrmions can be stabilized at room temperature in ultrathin Co/Pt and Pt/Co/Ir multilayer films and dots by an interface DMI [4–6]. DMI and out-of-plane magnetic anisotropy are important for individual skyrmion stabilization, the skyrmion size varies in a range of 10 nm–100 nm, and topological charge is $N = 1$. These features distinguish skyrmions from magnetic vortices.
We consider periodically arranged circular nanodots in the linear chain according to their symmetry stabilized in soft magnetic material with no DMI and \( N = 1/2 \). The skyrmions in thin films/dots are classified according to their symmetry [3]: ‘Neel’ skyrmions stabilized in ultrathin nanostructures by DMI and ‘Bloch’ skyrmions stabilized by the magnetostatic interaction in thicker films/dots. Two approaches were suggested to create artificial crystals of the individual Neel/Bloch skyrmions stable at room temperature without bias magnetic field in patterned films: (1) combining the soft and hard layers [7, 8] with no DMI and (2) periodical arrays of the skyrmion state thin magnetic dots with DMI [9].

Magnetic skyrmions exhibit non-trivial dynamics. Three low frequency (GHz range) excitation modes (clockwise, counterclockwise rotating modes, and breathing mode) were found in the collective spectra of the skyrmion lattices [10–13], and several gyrotropic and SW modes were found in the spectra of individual skyrmions in magnetic nanodots [14–18]. The physical origin of these modes is still actively discussed [14, 16]. Currently the most of researchers converge in opinion that the number of gyrotropic and high frequency SW modes depends on the skyrmion dimensions and physical mechanisms favoring the skyrmion state. Namely, one gyrotropic mode is realized in the skyrmions stabilized by DMI in ultrathin dots [14, 16–18]. It was shown that in FeGe circular dots [14] the high intensity mode with lowest frequency is the gyrotropic mode. Whereas, two gyrotropic modes appear in the spectra of bubble skyrmions stabilized by the magnetostatics and uniaxial magnetic anisotropy in thick dots [19, 20].

The complex excitation spectra of individual skyrmions should manifest themselves in the collective mode properties of the skyrmions arrays. While the collective modes of the coupled magnetic vortices in the dot lattices [21–25], dot clusters [26, 27] and nanopillars [28, 29] are sufficiently well explored, the investigation of collective excitations of the coupled skyrmions in patterned nanostructures is in a beginning stage. Recent publications on this subject concern dynamic magnonic crystals [30], gyrotropic and SW modes of the skyrmion dot chains [31], and coupled gyration modes of the skyrmions periodically arranged in nanostrips [32]. Simulation of the high frequency dynamics on the background of the periodic array of current induced skyrmions [30] showed drastic changes of the spin wave dispersion. The investigation of collective SW and gyration mode spectra of skyrmions in the linear chain of nanodots [31], and skyrmions periodically ordered in the linear chains in nanostrips [32] confirmed existence of the frequency band structure. The wide frequency bands of the oscillating skyrmions in a chain of touching nanodots with radii \( R \sim 15–50 \) nm was found in [31]. Authors of [32] considered magnetic stripes containing chains of 5 or 25 skyrmions with period \( \sim 27 \) nm–38 nm and simulated the gyrotropic frequency bandwidth of 0.5 GHz–1.5 GHz due to essential exchange and magnetostatic coupling between the skyrmion magnetization configurations excited by a pulsed magnetic field.

On the scales about of 100 nm magnetostatic interaction plays an essential role, and it is important to consider its impact analyzing the skyrmion excitation spectra. 1D and 2D arrays of the skyrmion state magnetic dots referred as 1D, 2D magnonic crystals open new route for traditional magnonics providing low frequency (~1 GHz) operating regime required for the functional microwave components in nanodevices. In this connection, investigation of the dynamics of the coupled skyrmion state nanodots is of practical and fundamental importance and understanding of the collective skyrmion dynamics is required.

In this article we report on the collective skyrmion gyrotropic modes excited in the magnetostatically coupled circular magnetic dots arranged in a linear chain. Implementing Thiele equation of motion we calculate the collective gyration frequencies and analyze the mode dispersion relations. We show that the frequencies of the skyrmion excitations depend on the dot material parameters, distance between neighboring dots, dot dimensions, and the spatial magnetization distribution inside the skyrmions.

2. Model

We consider periodically arranged circular nanodots in the linear chain (see figure 1) with the period \( D \). Each of the dots has radius \( R \), and height \( h \). Interdot separation is assumed to be of order of several nm. Therefore, there is no direct exchange coupling between the dots, however the long-range interdot magnetostatic interaction is quite essential.

We suppose that static skyrmion magnetic configurations are stabilized in the dots by the Dzyaloshinskii-Moriya exchange interaction. Both Bloch and Neel skyrmion configurations can be realized in magnetic nanodots. The conditions required for formation of a definite state are determined by the competition between magnetic interactions (primarily DMI asymmetric exchange, magnetostatics and uniaxial magnetic anisotropy). The existence of the Bloch and Neel skyrmions in thin dots was experimentally confirmed. The Bloch skyrmions were detected in thick CoB/Pt multilayer dots [20] (as expected in the materials with relatively weak perpendicular magnetic anisotropy, \( q = K_u/2\pi M_s^2 < 1 \)) and Neel skyrmions were found in ultrathin Pt/Co/Ir, Co/Pt, etc films and dots [4–6] with large interface DMI and high uniaxial magnetic anisotropy \( (q > 1) \). Recent micromagnetic simulations performed in [16] allowed identify the set of parameters at which soliton-like
states (vortex, Bloch and Neel skyrmions) are realized in thin magnetic nanodots. In our further consideration we appeal to the material parameters and dot sizes at which the Bloch or Neel skyrmions are stable or metastable.

Let us consider nth skyrmion in the dot chain (figure 1). To describe the skyrmion gyroscopic motion we employ the skyrmion rigid model [18] valid for the systems with dominating exchange interaction that is the case of ultrathin dots (the dot with the thickness $h$ about of 1 nm and $h \ll R$). In the frame of the rigid skyrmion model the reduce magnetization components $m_x + im_y = \frac{1}{2}(x + iy)$, $m_z = 1 - |f(z)|^2$ ($\mathbf{m} = \mathbf{M}/M_s$) can be expressed via the complex function $f(z) = e^{i\alpha}(z - s)/c$, where $z = (x + iy)/R$. $c = R / R_s$ is the reduced skyrmion radius, $\Phi_0$ is the skyrmion phase, $\Phi_0 = C\pi / 2$ ($C = \pm 1$ is the skyrmion chirality) for Bloch skyrmion or $\Phi_0 = 0$, $\pi$ for Neel skyrmion, and $s = \mathbf{X} / R$, $s = s_x + is_y$ is the skyrmion center position in the dot plane. We consider the skyrmion small amplitude oscillations $|\mathbf{X}_n| \ll R$ in each n-dot near the equilibrium position $\mathbf{X}_n = 0$ in the dot center $\rho = 0$.

To describe the skyrmion dynamics in the nth dot we use the Thiele equation of motion

$$G_n \times \mathbf{X}_n - \nabla \mathbf{X}_n U = 0,$$

(1)

where $G_n = G_m z$, $G_m = -p_n |G_n|$ are the $z$-projection and absolute value of the gyrovector, $z$ is the unit vector perpendicular to the dot plane (figure 1), $p_n = \pm 1$ is the nth skyrmion core polarization, $\mathbf{X}_n$ is the nth skyrmion center position, $U$ is the potential energy of the skyrmions displaced from the equilibrium positions, and the overdot denotes the derivative with respect to time. The gyrovector component $G_m$ of the centered nth skyrmion ($\mathbf{X}_n = 0$) is proportional to the skyrmion topological charge [18] and can be calculated using the definition $G_m = (M_s h / \gamma) \left( \int d^2 \mathbf{m} \times [\partial_{x} \mathbf{m} \times \partial_{y} \mathbf{m}] \right)$, where $\mathbf{m} = \mathbf{M}_n / M_s$ is the reduced dot magnetization.

Hereinafter to calculate the skyrmion chain gyrotropic eigenfrequencies we assume identical dots, $p_n = 1$ and zero damping. The Thiele equation allows determining the gyrotropic frequency of isolated skyrmion in thin magnetic nanodot as a function of the nanodot size and material parameters [18]

$$\omega_G = \omega_M \frac{c^2}{(1 + c^2)^2} \left[ (1 - q)(1 + c^2) + \frac{1}{\pi} \frac{1}{r^2} \left| \frac{1}{2} c \right| d_{DM} - 1 \right].$$

(2)

where $L_e = \sqrt{2A / M_s}$, $d_{DM} = I_D L_e / A$, $\omega_M = \gamma 4\pi M_s$, $q = K_u / 2\pi M_s^2$, $M_s$ is the saturation magnetization, $A$ is the exchange stiffness, $r = R / L_e$ is the reduced dot radius, and the reduced equilibrium skyrmion radius $c = R_s / R$ is a function of the dot sizes and magnetic parameters $A, I_D, K_u, M_s$. $R_s$ is the radius of skyrmion defined by the equation $m_e(R_e) = 0$. The DMI strength $I_D$ is determined by an interface DMI for the Neel skyrmions ($w_0 = I_D |m_x (\nabla \cdot \mathbf{m}) - (\mathbf{m} \cdot \nabla) m_x|$) or bulk DMI for the Bloch skyrmions ($w_0 = I_D (\mathbf{m} \cdot rot \mathbf{m})$). The magnetostatic energy renormalizes the effective magnetic anisotropy constant within ultrathin dot limit, $K_u \rightarrow K_u - 2\pi M_s^2$. The absolute value of the skyrmion gyrovector is determined as $|G| = (4\pi M_s h / \gamma) / (1 + c^2)$.

In the case of skyrmions chain the interaction energy between the skyrmions in equation (1) should be accounted along with the single dot energy

$$U = \frac{1}{2} \sum_n \kappa_n \mathbf{f}_n \mathbf{f}_n + \frac{1}{2} \sum_{n,n'} U_{nn'} (n, n').$$

(3)

We consider the magnetostatically coupled magnetic nanodots with identical gyrating skyrmions and determine the magnetostatic interdot interaction energy in equation (3) by Coulomb—like law.

Figure 1. Schematic illustration of the linear chain of skyrmion state circular nanodots.
\[ U_{\text{int}}(n, n') = \int \int \frac{\rho(r_n) \rho(r_{n'})}{|r_n - r_{n'} - R_{mn}|} dV dV', \]

where \( R_{mn} \) is the radius vector connecting \( n \)-th and \( n' \)-th dot centers \(|R_{mn}| = (n - n')D, D = Rd \) (figure 1), and \( \rho(r_n) \), \( X_n \) are the magnetic 'charges' of the \( n \)-th nanodot with local coordinates \( r_n \) generated by moving skyrmions described by \( X_n(t) \).

Implementing the approach elaborated in [23] we expand the dot pair \((n, n')\) interaction energy (4) via the dot multipole moments \( Q_{lm}^m \):

\[ U_{\text{int}} = \sum \frac{U_l(\varphi_r)}{L} R_{mn}^{-\frac{l+1}{2}}, \quad U_l(\varphi_r) = (-1)^l S_{LM} \left( \frac{L + M}{l + M} \right)^{\frac{1}{2}} \left( \frac{L - M}{l - M} \right)^{\frac{1}{2}} e^{-iM\varphi_r} Q_l^m Q_{L-M}^m, \]

\[ S_{LM} = (-1)^{(l+M)/2} \frac{|(L + M - 1)!(L - M)!|}{(L + M)!}\left(\frac{L - M}{L + M}\right)\left|\varphi_r\right|^2, \quad Q_l^m = \frac{4\pi}{2(l + 1)} \int dV \rho(r) r^l Y_l^m(r), \]

where \( Y_l^m(r) \) are the normalized to unit spherical functions [23], \( l = 1, 2, ..., m = M \).

The interdot magnetostatic interaction is mostly determined by the dot dipolar \((l = 1)\) and quadrupolar \((l = 2)\) moments. The dipolar and quadrupolar moments of the Bloch and Skyrmionskyrmions accounting the domain period \((\text{div} \text{m})\), face and side surface magnetic charges \((\text{m} \cdot \text{n})\), \( (\text{n} = \pm \hat{z}, \hat{p} \text{ for the face and side surface charges}) \) are determined as follows:

\[ Q_1^0(n) = M_s V \left( \mu_z^0 - 2 \frac{e^2}{(1 + c^2)^2} s_\text{z} \right), \quad Q_1^1(n) = M_s V e^{i\varphi_n} \sqrt{2\pi} e^{i\xi_n} |s_n| \frac{c}{1 + c^2} \]

\[ Q_2^\pm(n) = \mp M_s V \sqrt{6} \left| s_n \right| e^{i\xi_n} c \left( \ln \left( 1 + \frac{1}{c^2} \right) - \frac{1}{1 + c^2} \right) \]

where \( \mu_z^0 = 2e^2 \ln \left( 1 + \frac{1}{c^2} \right) - 1 \) is the average dot z-magnetization component, \( V = \pi R^2 h \) is the dot volume, and the dimensionless complex variable is \( s_n = X_n / R, s_n = s_{nx} + i s_{ny} = |s_n| e^{i\xi_n}, s_n = s_{nx} - i s_{ny} \).

The dipole-dipole \((l = 1 + l' = 2; l, l' = 1, 2)\) and quadrupole \((l = 2 + l' = 2)\) interaction energy between two coupled skyrmion dots can be written in the form:

\[ U_{\text{int}}^{d-d} = \frac{1}{2d^2 R^4} \left[ 2Q_1^0(n)Q_1^0(n') + (Q_1^1(n)Q_1^1(n')) + 3(Q_1^{-1}(n)Q_1^{-1}(n')) \right] \]

\[ - 3(Q_1^{-1}(n)Q_1^{-1}(n')) \]

\[ U_{\text{int}}^{d-q} = \frac{2i\sqrt{3}}{3d^2 R^4} \left[ Q_1^{-1}(n)Q_2^{-1}(n') + Q_1^{1}(n)Q_2^{1}(n') + Q_1^{-1}(n)Q_2^{1}(n') + Q_1^{1}(n)Q_2^{-1}(n') \right] \]

\[ - 2(Q_1^{-1}(n)Q_2^{1}(n') + Q_1^{1}(n)Q_2^{-1}(n')) \]

where \( Q_1^{\pm,+1}(n), Q_2^{\pm,+1}(n) \) are the dipole and quadrupole moments of the 1-st and 2-nd skyrmion state dots, respectively.

In the case of linear skyrmion chain composed by \( N \) skyrmions the interaction energies (8), (9) can be rewritten in the form

\[ U_{\text{int}}^{d-d} = \frac{1}{4} \sum_{m' \neq m} \frac{1}{R_{mn}^{m'}} \left[ 2Q_1^0(n)Q_1^0(n') + (Q_1^{-1}(n)Q_1^{1}(n')) + 3(Q_1^{-1}(n)Q_1^{-1}(n')) \right] \]

\[ - 3(Q_1^{-1}(n)Q_1^{-1}(n')) \]

\[ U_{\text{int}}^{d-q} = \frac{2i\sqrt{3}}{3} \sum_{m' \neq m} \frac{1}{R_{mn}^{m'}} \left[ (Q_1^{-1}(n)Q_2^{1}(n') + Q_1^{1}(n)Q_2^{-1}(n')) \right] \]

\[ - 2(Q_1^{-1}(n)Q_2^{1}(n') + Q_1^{1}(n)Q_2^{-1}(n')) \]

where \( Q_1^{\pm,+1}(n), Q_2^{\pm,+1}(n) \) are the dipole and quadrupole moments of the skyrmion state dot occupying \( n \)-th position in the linear chain, \( R_{mn} = (n - n')D \) is the distance between \( n \)-th and \( n' \)-th dots, and \( D = Rd \) is the chain period (figure 1).

Accounting equations (6), (7), (10), (11) we determine the interaction energy of the linear chain of the Bloch or Neel skyrmions in quadratic approximation on the small oscillation amplitudes \( |s_n| \ll 1 \)

\[ U_{\text{int}}^{d-d} = f(\epsilon) \sum_{m' \neq m} \sigma_{mn}^{m'} \left( 3s_{n} s_{n'}^* + \frac{3}{2} (e^{-i2\xi_n} \bar{s}_{n}^* s_{n'} + s_{n} s_{n'} e^{i2\xi_n}) \right), \]
We calculated the magnetic skyrmion gyrotropic dynamics of the coupled 1D nanodot array (chain) with the Bloch or Neel skyrmion configurations in the ground state assuming that the dot thickness is about of 1 nm and dot radius is about of 100 nm. The interdot magnetostatic coupling results in formation of the skyrmion gyrotropic frequency bands. The individual skyrmion excitation frequencies and the frequency bands are calculated as a function of the skyrmion equilibrium radius, dot radius, dot lattice period, and the dot magnetic parameters. The frequency dispersion, the band gaps and widths of allowed bands can be manipulated and controlled by use of enumerated parameters. Excitation of the gyrations in the skyrmion chain can be triggered by equation
by applying spin polarized electric current or variable magnetic field. The skyrmion gyrotrropic modes correspond to the skyrmion central position motion around equilibrium in the dot center. These gyrotrropic modes are in 0.1–1.0 GHz frequency range for the typical dot parameters and might be exploited in magnonic and spin-torque nano-oscillator devices.

Figure 2. The collective skyrmion gyrotrropic frequency in the linear dot chain with different periods $D = dR$ and the skyrmion radii $c = R_c/R$: (a) the Bloch skyrmions ($\phi_0 = \pi/2$), (b) the Neel skyrmions ($\phi_0 = 0$), (c) gyrotrropic frequency of the isolated skyrmion as function of its reduced radius $c$. The parameters are $\omega_p/2\pi = 43$ GHz, $L_x = 14.6$ nm, $R = 50$ nm, $d_{isol} = 2.91, 0.02, 1.05, 3.43$. Dashed lines stand for $c = 0.6 (\omega_p/2\pi = 0.18$ GHz), solid lines stand for $c = 0.7 (\omega_p/2\pi = 0.40$ GHz); Green color lines stand for $d = 2$, blue color lines stand for $d = 2.1$, and red color lines stand for $d = 2.4$, respectively.
The frequency bandwidth is proportional to the coupling frequencies $\omega_{dq}$, $\omega_p$. The interdot magnetostatic coupling and frequency bandwidth can be quite essential for the Bloch skyrmions appearing in the relatively thick dots and skyrmions whose reduced radii $c > 0.5$. The gyrotropic frequency bandwidth simulated in [31] changes from 0.1 GHz up to 0.5 GHz for the Neel skyrmion state ultrathin dots whose radius $R$ varies in the interval 25 nm–15 nm, correspondingly. The calculated in [31] group velocities can attain the values up to $30 \text{ m s}^{-1}$. The linear dependence between the bandwidth and dot dimensions determined by $\beta$ coefficient in equation (15) indicates the increase of the frequency bandwidth and group velocity when diameters of the dots decrease. It also follows from equation (15) that in the case of the touching dots ($d = 2$) considered in [31] the bandwidth and the group velocity attain the maximal values (figures 2, 3, $d = 2$). I.e., the numerical findings by Mruczkiewicz et al are confirmed by the analytical calculations of our work if we assume the limiting case of the touching dots ($d = 2$) and large dot aspect ratios ($\beta$) due to extremely small dot radii. However, fabrication of the high-quality dots with diameter 30 nm–50 nm is so far unreachable for lithography technique. In the case of the dots whose $2R > 50$ nm the bandwidth of the Neel skyrmion excitations essentially decreases. However, we predict that the frequency bandwidth and the corresponding group velocity are in 2–3 times bigger for the Bloch skyrmions as compared to the Neel skyrmions, for the same dot magnetic and geometrical parameters.

The Bloch skyrmion state can be stabilized in the nanodots in three cases: (1) the dots are made from the magnetic materials with the bulk DMI (FeGe, for instance), (2) the DMI has an interface origin like one in Co/Pt films/dots, but the dot thickness is not very small, (3) no DMI and $Q < 1$. In the first case, the dot thickness can be several tens of nm and the dot aspect ratio ($\beta$) is large resulting in the considerable large coupling frequencies $\omega_{dq}$, $\omega_p$ and the collective frequency bandwidths. However, the highest Curie temperature for B20 compounds (FeGe) supporting such Bloch skyrmions is about 270 K, below room temperature, that prevents practical applications. In the case of interfacial DMI, it gradually becomes ineffective increasing the dot thickness.

Figure 3. Group velocity for the collective skyrmion excitations propagating along the linear dot chain with different periods $D = dR$: (a) the Bloch skyrmions ($\Phi_0 = \pi/2$), (b) the Neel skyrmions ($\Phi_0 = 0$). The parameters are $\omega_{M} = 2\pi \cdot 43$ GHz, $L_s = 14.6$ nm, $R = 50$ nm, $|d_{dot}| = 2.91$, $\beta = 0.02$, $q = 1.05$, $\epsilon = 0.7$, $r = R/L_s = 3.43$. The frequency bandwidth is proportional to the coupling frequencies $\omega_{dq}$, $\omega_p$. The interdot magnetostatic coupling and frequency bandwidth can be quite essential for the Bloch skyrmions appearing in the relatively thick dots and skyrmions whose reduced radii $c > 0.5$. The gyrotropic frequency bandwidth simulated in [31] changes from 0.1 GHz up to 0.5 GHz for the Neel skyrmion state ultrathin dots whose radius $R$ varies in the interval 25 nm–15 nm, correspondingly. The calculated in [31] group velocities can attain the values up to $30 \text{ m s}^{-1}$. The linear dependence between the bandwidth and dot dimensions determined by $\beta$ coefficient in equation (15) indicates the increase of the frequency bandwidth and group velocity when diameters of the dots decrease. It also follows from equation (15) that in the case of the touching dots ($d = 2$) considered in [31] the bandwidth and the group velocity attain the maximal values (figures 2, 3, $d = 2$). I.e., the numerical findings by Mruczkiewicz et al are confirmed by the analytical calculations of our work if we assume the limiting case of the touching dots ($d = 2$) and large dot aspect ratios ($\beta$) due to extremely small dot radii. However, fabrication of the high-quality dots with diameter 30 nm–50 nm is so far unreachable for lithography technique. In the case of the dots whose $2R > 50$ nm the bandwidth of the Neel skyrmion excitations essentially decreases. However, we predict that the frequency bandwidth and the corresponding group velocity are in 2–3 times bigger for the Bloch skyrmions as compared to the Neel skyrmions, for the same dot magnetic and geometrical parameters.

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However, the magnetostatic interaction increases with the dot thickness increasing and inevitable results in the stabilization of the Bloch skyrmions for relatively small values of the interface DMI. This effect was very recently simulated by Zelent et al [34]. It was shown that increasing the dot thickness the multilayer (Ir/Co/Pt)ₙ dot is in the Bloch skyrmion state for the number of the layer repeats ₙ ≥ 9 if ₁ < 0.5 nm m⁻² or, alternatively, the large-radius Bloch skyrmion state is realized for the single layer Co/Pt dot at ₁ = 0.95 nm m⁻² for the Co-thickness >3 nm (the dot radius was 250 nm). In the third case, the Bloch skyrmions are stable within the range ₀ < ₁ < ₀ < ₁ serving as intermediate states between the perpendicular single domain and vortex states.

As follows from equation (15), the frequency bandwidth and the group velocities are regulated by geometrical parameters (period of a chain, skyrmion and dot dimensions) included to the coupling frequencies ωₙ, ωₘ. The position of bandwidth depends on the gyrotropic frequency of isolated skyrmion and can be tuned by the change of the dot parameters (exchange length Lₑ = 2√2A / Mₑ, Dzyaloshinskii-Moriya interaction constant ₀ = ₁ / A₁, radius of the dot R). Using equation (15) and material parameters required for the existence of stable magnetic skyrmion in thin dots taken from the experiment [20] we get sufficiently high values of the frequency bandwidth for the coupled Bloch skyrmion state dots (~20%–30% of the gyrotropic frequency of the isolated dot). The group velocities 10–20 m s⁻¹ of the gyrating Bloch skyrmions are close to the velocities of the skyrmion translation motion measured in a magnetic nanostripe driven by a spin polarized current [3].

An interesting finding gained in our work is the featured dynamic response allowing to distinguish the Bloch and Neel skyrmions arranged in an array. We show that dynamic characteristics (frequency bandwidth, FMR frequency and group velocities, see figures 2, 3) depend on the spatial spin distribution inside the skyrmions. The bandwidth ₀ = ω(k = π/D) − ω(k = 0) is found to be sufficiently wider for the Bloch skyrmions (₀/₀ ~ 25% of the isolated dot gyrotropic frequency) and relatively narrow for the Neel skyrmions (₀/₀ ~8% of the gyrotropic frequency) (see figures 2(a), 2(b)). The relative bandwidth ₀/₀ depends on the sizes of skyrmions (ₚ), nanodots (ₚ, R), lattice period (ₚ) and can be tuned by the dot material quality factor ₚ = ₂ / ₚ / ₚ (₀/₀ increases when ₚ enhances within the model). The value of ₀/₀ increases drastically approaching the skyrmion stability border determined by the equation ₀(ₚ, ₚ, ₚ, ₀) = 0. For instance, we calculated ₀/₀ = 90 MHz, ₀/₀ = 0.5 for ₚ = 1.07, ₚ = 0.6, ₚ = 2 (the rest of the parameters are as in the caption to figure 2).

However, the skyrmion gyroscopic dynamics near the equilibrium state looses its sense too close to the border. The group velocities of the spin excitations transferred by the coupled gyrating Bloch skyrmions along the skyrmion dot chain are in 2–3 times higher than the velocities produced by Neel skyrmions (figures 3(a), 3(b)). Increase of the dot thickness and the dot aspect ratio ₀ leads to the bandwidth ₀ increase as well as to stabilization of the Bloch skyrmions in thin dots. FMR frequency ₀(k = 0) appear to be different for the Bloch and Neel skyrmions oscillating in a chain, the difference is of the order of 0.1 GHz (figures 2(a), 2(b)) that can be, in principle, experimentally detected, for example by using broadband ferromagnetic resonance technique. On the contrary, the isolated skyrmions are dynamically equivalent, the gyroscopic frequencies are equal for the isolated Bloch and Neel skyrmions for same dot magnetic and geometrical parameters (see equation (3) in [18]). The details of skyrmion magnetization configuration appear to be hidden in the gyroscopic frequency of isolated skyrmions. However, they can manifest themselves in a skyrmion ensemble via the skyrmion interaction energy, magnetostatic interaction energy in our case. The accounting of magnetostatic interactions between the skyrmions, particularly in the skyrmion chain, removes phase (θₙ) degeneracy of the gyroscopic frequency and allows distinguishing the dot skyrmionic configurations by their dynamic response. This effect exists solely due to the skyrmion dot dynamic quadrupolar Qₙ(ₙ) and higher order multipole moments. The same effect should be present in magnetization hysteresis loops if one applies an in-plane magnetic field to the coupled skyrmion dot array. We note that the magnetization damping (parameter α) in ultrathin films and dots of Co/Pt type is large [20] that prevents nowadays precise measurements of any magnetic resonance frequencies in such nanostructures by frequency domain as well as time domain techniques.

We can also compare the collective gyroscopic frequencies of the magnetic skyrmion dot arrays and vortex dot arrays [21–25]. The skyrmion as compared to the vortex has considerable z-magnetization component, both static and dynamic one. This leads to appearance of Qₐ(ₙ) components of the dot dipolar moments and Qₐ(ₙ) components of the dot quadrupolar moments, which were absent for magnetic vortices. The former results in the frequency shift independent on the wave vector, whereas the latter results in the dipole-quadrupole and quadrupole-quadrupole interdot coupling. As a consequence, the skyrmions reveal smaller dipole-dipole, but essentially higher dipole-quadrupole and quadrupole-quadrupole interdot coupling resulting in more rapid decrease of the skyrmion collective frequency bandwidths with the interdot distance d increasing in comparison with the vortex ones. The concepts of the dot-to-dot transfer time τ = π/₀ and efficiency of the microwave signal transfer ε = exp(−1/₀) ≤ 1 used for coupled magnetic vortices in [23, 24] (1 = α₀ is the relaxation frequency of the isolated dot) can be also applied to magnetic skyrmions ordered in a linear chain. The first dot in the chain is assumed to be excited by a microwave magnetic field with the frequency close to the dot gyroscopic frequency. The efficiency is determined by the ratio πα₀₀ / ₀ and can reach the value of exp(−3πα₁) for the
Bloch skyrmions in closely spaced dots ($d = 2–2.1$). Therefore, if the effective Gilbert damping parameter is $\alpha < 1/3\pi \approx 0.1$, then the efficiency $\varepsilon$ can be close to 1.

4. Conclusions

We present an approach that allows calculate the low frequency excitation spectrum of magnetostatically coupled skyrmions in a long chain of the circular nanodots. Skyrmionic configurations in the dots are stabilized by interplay of isotropic exchange and Dzyaloshinskii-Moriya exchange interactions, magnetic anisotropy and magnetostatic interactions, while external drivers induce the skyrmion gyrations. The approach enables to analyze collective dynamics of gyrating skyrmions in a linear chain for wide range of parameters and to tune the dynamic characteristics such as the group velocity and the bandwidth. We demonstrate the differences in the dynamic responses of the Bloch and the Neel skyrmions arranged in 1D array and suggest a method allowing recognize the type of a skyrmion (Bloch or Neel) on the base of measurements of FMR frequency of such patterned film. The interdot magnetostatic coupling and frequency bandwidth are maximal for the large-radius Bloch skyrmions in the relatively thick dots. Such skyrmions are stabilized at room temperature due to in-dot magnetostatic interaction and moderate out-of-plane magnetic anisotropy with weak or even zero Dzyaloshinskii-Moriya interaction.

Being one of the attractions of up-to-date magnetism, skyrmionics opens a new way for the different branches of spin-based technologies. In this connection, the understanding of the collective gyrotropic dynamics of a skyrmion chain, the kind of magnonic crystals, can be important for the development of magnonics.

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ORCID iDs

Zukhra V Gareeva  
https://orcid.org/0000-0003-3690-6720

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