THOMSON SCATTERING OF COHERENT DIFFRACTION RADIATION BY AN ELECTRON BUNCH.

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Abstract

The paper considers the process of Thomson scattering of coherent diffraction radiation (CDR) produced by the preceding bunch of the accelerator on one of the following bunches. It is shown that the yield of scattered hard photons is proportional to $N_e^3$, where $N_e$ is the number of electrons per bunch. A geometry is chosen for the CDR generation and an expression is obtained for the scattered photon spectrum with regard to the geometry used, that depends in an explicit form on the bunch size. A technique is proposed for measuring the bunch length using scattered radiation characteristics.

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1. Introduction

The process of Compton backscattering (CBS) of the infra-red or visible photons by the relativistic electrons was used widely for obtaining X-ray - and $\gamma$ - beams with the energy from $\sim 10^6$eV up to $\sim 10^{10}$eV [1-4].

The development of laser technologies within recent years has brought up a suggestion to use the CBS process for electron bunch diagnostics [5-7]. The authors of an experiment [7] used a femtosecond near infrared terawatt laser as a source of radiation which was scattered on a bunch of electrons with the energy $E = 50$ MeV. They proposed to use this process for the measurement of electron bunch characteristics (longitudinal and transverse bunch sizes, divergence, etc.) The longitudinal bunch structure, for instance, was measured via the dependence of the scattered hard photon yield on the time delay between the electron and photon bunches.

It is clear that the accuracy of such measurements relies on the reproducibility and controllability of characteristics of a powerful laser, which is a rather complicated problem.

In further works [8,9] it was proposed to measure the bunch length through such characteristics of coherent transition radiation (i.e. the transition radiation with a wavelength comparable with the bunch length), as the radiation spectrum and the autocorrelation function. In the latter cases one is free from the errors associated with the laser. However,
the methods so far proposed are not non-destructive (viz. the electron beam crosses the foil target).

This paper considers a possibility of electron beam diagnostic using Thomson scattering of CDR from the preceding bunch on the following one. Diffraction radiation is produced when a charged particle moves close to a conducting target. The effects of the target on beam characteristics could be reduced to an acceptable level by choice of the distance between the beam and target. Thus, the method proposed here is nondestructive as are the methods involving the use of laser emission, nonetheless, characteristics of the scattered hard radiation are determined only by the electron beam parameters.

2. Thomson scattering of radiation by a moving bunch.

During the interaction of an incident photon with a moving electron the scattered photon energy is to be derived using the conservation laws:

\[ \omega_2 = \omega_1 \frac{1 - \beta \cos \theta_1}{1 - \beta \cos \theta_2 + \frac{\omega_1}{E}\{1 - \cos(\theta_1 - \theta_2)\}}. \]  

(1)

Here \( \omega_1, \omega_2 \) and \( E \) are the energies of the incident and scattered photons and that of the electron, respectively, \( \beta = v/c \), \( v \) is the electron velocity, the angles between the electron momentum and the incident and scattered photons \( \theta_1, \theta_2 \) are the same as in [6]. If the primary photon energy and that of the electron satisfy the conditions

\[ \gamma = E/mc^2 \gg 1, \quad \gamma \omega_1 < mc^2, \]  

(2)

the scattering photon energy (1) is linearly dependent on that of the incident photon:

\[ \omega_2 = \omega_1 \frac{1 - \beta \cos \theta_1}{1 - \beta \cos \theta_2} \approx \omega_1 \frac{2\gamma^2(1 - \beta \cos \theta_1)}{1 + (\gamma \theta_2)^2}, \]  

(3)

where the outgoing photon angle \( \theta_2 \sim \gamma^{-1} \).

In a frame where the electron is at rest (ERF), the energy of the photon scattered, is, according to (2), sufficiently less than the electron mass. The photon scattering then occurs virtually without any frequency changing and, therefore, the scattering process may be described in terms of classical electrodynamics (Thomson scattering).

In the ERF the classical cross section of scattering of an electromagnetic wave by a free charge [10] is not controlled by its frequency and is given by the expression:

\[ \frac{d\sigma}{d\Omega'} = \frac{r_o^2}{2} \left(1 + \cos^2 \theta'\right). \]  

(4)

In (5), \( r_o = 2.82 \cdot 10^{-13} \text{ cm} \) is the classical radius of an electron, and the primes denote the angles in the ERF. Transforming these to the laboratory system, we have:

\[ \cos \theta' = \frac{\cos \theta_2 - \beta}{1 - \beta \cos \theta_2}, \]  

(5)
\[ d\Omega' = \frac{1 - \beta^2}{(1 - \beta \cos \theta_2)^2} d\Omega \] (6)

From (5) and (6) we obtain the classical cross section for the ultrarelativistic case:

\[ \frac{d\sigma}{d\Omega} = 4 \gamma^2 \frac{r_0^2}{1 + (\gamma \theta_2)^2} \] (7)

The total cross section derived through integrating expression (7) with respect to angles is the Thomson cross section:

\[ \sigma_T = \frac{8}{3} \pi r_0^2 \] (8)

The yield of secondary photons upon scattering, e.g. of incident laser photons, on a moving electron bunch is to be determined not only by the cross section of the process but also by the overlapping of the laser and electron beams in space and time, which is characterized by luminosity \( L \):

\[ \frac{dN_2}{dt} = L \sigma_T . \] (9)

Let us consider the head-on collision of electron and photon bunches. Luminosity in this case is defined as follows:

\[ L = c N_e N_{ph} F \int \int \int dx dy dz dt f_{ph}(x, y, z + ct) f_e(x, y, z - \beta ct). \] (10)

Here \( N_e, N_{ph} \) are the number of particles in the electron and photon bunches, \( f_e, f_{ph} \) are the corresponding normalized electron and photon distributions and \( F \) is the collision frequency of the bunches. For the monodirected beams with a Gaussian distribution in both transversal and longitudinal directions:

\[ f_e = \frac{2}{(2\pi)^{3/2} \sigma_e^2 l_e^2} \exp\left\{ -\frac{r^2}{\sigma_e^2} - \frac{(z - \beta ct)^2}{2l_e^2} \right\} , \]
\[ f_{ph} = \frac{2}{(2\pi)^{3/2} \sigma_{ph}^2 l_{ph}^2} \exp\left\{ -\frac{r^2}{\sigma_{ph}^2} - \frac{(z + ct)^2}{2l_{ph}^2} \right\} , \] (11)
\[ r^2 = x^2 + y^2 , \]

the luminosity is readily calculated

\[ L = N_e N_{ph} F \frac{1}{2\pi (\sigma_e^2 + \sigma_{ph}^2)} . \] (12)

In (11), \( \sigma_e^2, \sigma_{ph}^2 \) are the variables characterizing the transversal and \( l_e^2, l_{ph}^2 \) are those for the longitudinal distributions. For the head–on collisions it follows from (12) that the luminosity is governed solely by the transverse dimensions of the electron and photon bunches. The number of the photons scattered through collision of single bunches can be estimated from (9) and (12):

\[ N_2 = \frac{1}{2} N_{ph} \frac{N_e \sigma_T}{S_e + S_{ph}} , \] (13)
where $S_e, S_{ph}$ are the cross-sections of the electron and photon bunches. The value $S = \frac{1}{2} N_e \frac{\sigma_T}{S_e + S_{ph}}$ can be treated as the reflectivity of the electron bunch. For the electron numbers and bunch size attainable this value is considerably small. Therefore, one typically uses radiation of a powerful laser as a primary beam.

However, effective overlapping of the laser and accelerator bunches is a difficult task, while linear dependence of the scattered beam intensity (8) on the number of electrons in the bunch poses natural restrictions on the intensity of the resulting X-ray or $\gamma$-beam. If a beam of incident photons is to be generated by one of the preceding electron bunches of the accelerator, then the temporal and longitudinal structures of the colliding bunches will be similar.

In the experiment [11] a incident beam of infra-red radiation ($\lambda = 3.5\div 7$ mcm) was generated by the electrons with the energy $E = 50$ MeV ($\gamma \sim 100$) in an undulator with $\sim 4$ m length. The electron beam parameters satisfied the gain mode of the free electron laser.

It seems possible that one can use a beam of coherent radiation of a short electron bunch as a primary beam of soft photons. In this case, the radiation intensity in the wavelength region $\lambda_1$, comparable with the bunch length, is quadratically dependent on the number of electrons in the bunch [12], which compensates for the low reflectivity of the electron bunch. Instead of a laser source, coherent diffraction radiation (CDR), i.e. the radiation produced while a short bunch of electrons is passing close to a metal target [13], can be taken as a source of primary radiation.

Fig. 1 shows a potential experimental scheme. Electron bunches pass through a circular opening of the radius R in a metal target, which results in generation of CDR in the wavelength region $\lambda_1 \geq l_e$, the electrons are deflected by a bending magnet BM, while CDR is reflected and focused by a thin concave mirror CM on one of the following bunches. The scattered photons with the energy corresponding to the X-ray region are extracted through the center hole of the mirror CM, suffering but a small loss. The distance between the center hole of the mirror and the target, $L_o$, is selected from the condition

$$2L_o = \frac{L_B}{\beta} \cdot m, \quad m = 1, 2, 3..., \quad (14)$$

where $L_B$ is the distance between the bunches.

The spectrum of the photons backscattered by a single electron may be calculated in the following manner:

$$\frac{dN^0_2}{d\omega_2} = const \int \int d\Omega_2 d\omega_1 \frac{dN_1}{d\omega_1} \frac{d\sigma}{d\Omega_2} \delta(\omega_2 - \omega_1) \frac{4\gamma^2}{1 + (\gamma^2 \theta_2^2)}.$$  

Here $\frac{dN_1}{d\omega_1}$ is the spectrum of the incident photon beam. Integration in (15) should be carried out with respect to all the spectral region of the initial radiation and the exit aperture $\Delta\Omega_2$. 

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The yield of photons scattered by an electron bunch is described by a more complicated formula:

\[
\frac{dN_B^2}{d\omega_2} = \int \int d\Omega_2 d\omega_1 \frac{dN_1}{d\omega_1} \frac{d\sigma}{d\Omega_2} \frac{N_e}{2\pi(\sigma_e^2 + \sigma_{ph}^2)} \delta(\omega_2 - \omega_1) \frac{2\gamma^2}{1 + (\gamma\theta_2)^2}.
\] (16)

3. Spectrum of coherent diffraction radiation.

DR spectrum may be calculated numerically using the results of work [14] for the spectral-angular density of the energy radiated from a single electron passing through a circular opening with the radius \( R \) in an ideal conductor:

\[
\frac{d^2W}{dx d\Omega} = \frac{\alpha\omega_c}{\pi^2} \frac{\sin^2\theta}{(\sin^2\theta + \gamma^{-2})^2} J_0^2\left(\frac{x}{2\gamma\sin\theta}\right) K_1^2\left(\frac{x}{2}\right),
\] (17)

where \( \alpha \) is the fine structure constant, \( \omega_c = \frac{\gamma}{2R} \) is the characteristic energy of DR, \( \theta \) — outgoing photon angle, \( \omega_1 \) is the energy of emitted photon, \( x = \omega_1/\omega_c \) is the dimensionless energy variable. From here up to the end of paper there will be used the system of units \( \hbar = m = c = 1 \).

In expression (17) \( J_0(x) \) is the Bessel function of the zeroth order, \( K_1(x) \) is the modified Bessel function. From (17) one may obtain the DR intensity spectrum \( \frac{dW}{dx} \) after integration with respect to the solid angle covered by the reflected mirror. Calculated spectra for apex angles \( \theta_{1m} = k_1/\gamma \) (\( k_1 = 5, 10 \)) are shown in Fig. 2.

Following [12] one may write the spectrum of CDR emitted by a bunch of \( N_e \) electrons as below:

\[
\frac{dN_B^0}{d\omega_1} = N_e(1 + f(\lambda_1)N_e) \frac{dN_B^0}{d\omega_1} \approx N_e^2 f(\lambda_1) \frac{dN_0^0}{d\omega_1}, \lambda_1 \geq l_e.
\] (18)

Here \( \lambda_1 \) is the wavelength of DR and \( f(\lambda_1) \) is the bunch form factor defined as the squared Fourier transform of longitudinal distribution of electron density in a bunch. For the Gaussian distribution (11) we have:

\[
f(\lambda_1) = \frac{1}{\sqrt{2\pi l_e}} \int \exp\left\{-\frac{2\pi^2 z^2}{2l_e^2}\right\} \exp\left(-i\frac{2\pi z}{\lambda_1}\right) dz \approx \exp\left(-\frac{4\pi^2 l_e^2}{\lambda_1^2}\right) = \exp\left(-\omega_{ch2}^2\right).
\] (19)

The photon DR spectrum may be easily derived from the DR intensity spectrum:

\[
\frac{dN_0^0}{d\omega_1} = \frac{1}{\omega_1} \cdot \frac{dW}{d\omega_1} = \frac{1}{\omega_1} \cdot \omega_c \cdot d\omega_c dx.
\] (20)

It is clear that there are two energies characterizing the spectrum (18):

\[
\omega_{ch1} \sim \omega_c = \frac{\gamma}{2R}, \omega_{ch2} \sim \frac{1}{l_e}.
\] (21)
one of them $\omega_{ch1}$ connected with the DR spectrum from a single electron and the other
$(\omega_{ch2})$-- with the collective emission from the bunch.

For an ultrarelativistic electron beam the transversal and longitudinal sizes of a bunch
may be less than $1$ mm. In a similar case, one may use a hole with the radius $R$ about a
few millimeters. So, we may consider the case

$$\frac{\gamma}{2R} \gg \frac{1}{l_x}. \quad (22)$$

It means that the coherent effects are significant in the region $\omega_1 \ll \omega_c$ where the DR
intensity spectrum may be taken as a constant (see Fig. 2). In the limit $\omega_1 \to 0 (x \to 0)$
we have

$$\frac{dW}{d\omega_1} \approx \frac{\alpha}{\pi} \left\{ \ln(1 + k_1^2) - \frac{k_1^2}{1 + k_1^2} \right\} = \frac{\alpha}{\pi} C \parallel. \quad (23)$$

After all substitutions one may obtain:

$$\frac{dN^B}{d\omega_2} = 2 \frac{\pi}{\alpha^2} \alpha r_0^2 N_e^3 C \parallel \int d\omega_1 d\Omega_2 \frac{\gamma^2 [1 + (\gamma \theta_2)^2]}{\omega_1 [1 + (\gamma \theta_2)^2]} \left[ \frac{\omega_1}{\sigma_e^2 + \sigma_{ph}^2} \right] \times
\delta \left( \omega_2 - \omega_1 \frac{4\gamma^2}{1 + (\gamma \theta_2)^2} \right). \quad (24)$$

In formula (24) the denominator has the value $\sigma_{ph}^2$ characterizing the radius of the
focused photon beam in the interaction point. Due to the diffraction limit the size of the
light spot cannot be less than $\lambda_1/2\pi$. So, for estimations we shall use the latter value
instead of $\sigma_{ph}$.

As one may see from (24) the scattered yield has the cubic dependence on the number
of electrons per bunch.

Other authors [15,16] considered electromagnetic radiation produced by the collision
of short electron bunches and also arrived at a cubic dependence of the photon yield with
the energy $\omega < \frac{4\gamma^2}{l_x}$ during collision of identical bunches.

Roughly speaking, the works mentioned earlier studied scattering of the field of virtual
photons of one bunch on the other, while this paper deals with the process where real
photons emitted by the preceding bunch are scattered on one of the following bunches.

4. Dependence of characteristics of scattered photons
on electron bunch parameters.

Due to narrow angular distribution of the scattered photons decreasing as $(\gamma \theta_2)^{-4}$ for
large emission angle $\theta_2 \gg \gamma^{-1}$ eq.(24) may be simplified:

$$\frac{dN^B}{d\omega_2} = 2 \frac{\pi}{\alpha^2} \alpha r_0^2 N_e^3 \gamma^2 \Delta\Omega_2 C \parallel \int d\omega_1 \delta(\omega_2 - 4\gamma^2 \omega_1) \frac{\exp \left( -\frac{\omega_1^2 l_x^2}{\omega_1^2} \right)}{\omega_1 \left( \sigma_e^2 + \frac{1}{\omega_1^2} \right)}, \quad (25)$$
if the exit aperture $\Delta \Omega_2 = \pi \theta_{2\text{max}}^2$ is comparable with $\gamma^{-2}$:

$$\theta_{2\text{max}} = k_2 \gamma^{-1}, \ k_2 \sim 1.$$  

Using the well–known property of the $\delta$–function we may obtain:

$$\frac{dN_2}{d\omega_2} = \frac{2}{\pi} \alpha r_0^2 N_e^3 C_{\parallel}k_2^2 \exp\left[\frac{-(\frac{\omega_2 l_e}{4\gamma^2})^2}{\omega_2 \left[\sigma_e^2 + \left(\frac{4\gamma^2}{\omega_2}\right)^2\right]}\right] = \frac{2}{\pi} \alpha r_0^2 N_e^3 C_{\parallel}k_2^2 F_{\omega_2} \quad (26)$$

One may see from (26) that the yield of scattered photons does not depend on the electron energy, if the maximum outgoing angle $\theta_{2\text{max}}$ is measured in units $\gamma^{-1}$. Of course, the scale of transformation of the photon energy is defined by the electron energy (see Eq.(3)).

The spectrum (26) is shown in Fig. 3 for different ratios between $\sigma_e$ and $l_e$. There are the clear broad maxima whose positions are determined by the ratio $r = \sigma_e/l_e$. With this ratio decreasing spectral maximum shifts to the value

$$\omega_{2\text{max}} = \frac{1}{\sqrt{2}} \frac{4\gamma^2}{l_e}$$

and the intensity rises due to increasing luminosity. Let us estimate the photon yield at the maximum for following parameters: $N_e = 10^{10} e^-$/bunch; $\sigma_e = l_e=1\text{mm}$; $k_1 = 10(C_{\parallel} = 3.6)$; $k_2=3$; $\Delta \omega_2/\omega_2 = 10\%$.

Then

$$\Delta N_2^B = \frac{dN_2^B}{d\omega_2} \Delta \omega_2 = \frac{2\alpha}{\pi} \left(\frac{r_0}{l_e}\right)^2 N_e^3 C_{\parallel}k_2^2 F_{\omega_2} \Delta \omega_2 = 3.7 \cdot 10^4 \text{ph/bunch.}$$

For the electron energy $E = 1000$ MeV the photons scattered at the spectral maximum have the energy around $1.6$ keV.

However, the estimation of the yield obtained above is valid only if the focusing mirror is located at a large distance from the target.

$$L_0 \gg L_f.$$  

(27)

Here $L_f$ is the formation length that characterizes the distance at which the radiation of the wavelength $\lambda$, propagating at the angle $\theta$, is completely separated from the initiating charge:

$$L_f = \frac{\beta \lambda}{1 - \beta \cos \theta}$$  

(28)

For forward emission ($\theta_1 \sim \gamma^{-1}$) in the ultrarelativistic case ($\gamma \geq 10^2$) the CDR formation length

$$L_f \approx \frac{2\gamma^2 \lambda_1}{1 + \gamma^2 \theta_1^2}.$$  

(29)
can exceed tens of meters. In a real case the mirror CM (Fig. 1) can be placed at a distance \( L_0 \ll L_f \). Then the DR intensity (initial photon flux) is suppressed as \( (L_f/L_0)^2 \) [17]. For the case considered, the suppression factor may reach \( \sim 10^{-4} \div 10^{-5} \) for a distance between target and mirror about a few meters.

As follows from (28), for the emission angles \( \theta_1 \sim \pi/2 \) the formation length is comparable with the wavelength. For these large emission angles the mirror positioned at \( L_0 \gg \lambda_1 \) does not effect the DR intensity. Fig. 4 shows the schematic of a potential application of the proposed geometry. An electron beam passes in the vicinity of a metal target tilted at \( \theta = 45^\circ \) with respect to the electron momentum, CDR propagates at \( \theta_1 \approx 90^\circ \) to the beam (in a close analogy with backward transition radiation [18]).

Spectral-angular distribution of DR when a single charge passes near a tilted ideally conducting semi-plane was obtained in [19]. For the ultrarelativistic case, when we introduce the angles \( \theta_x, \theta_y \) measured from the direction of mirror reflection (the \( x \)-axis is oriented along the target edge), the spectral-angular distribution of DR is written in a simpler form [20]:

\[
\frac{d^2W}{d\omega_1d\Omega} = \frac{\alpha}{4\pi^2} \exp\left(-\frac{\omega_1}{\omega_c}\sqrt{1+\gamma^2\theta_x^2}\right) \frac{\gamma^{-2} + 2\theta_x^2}{(\gamma^{-2} + \theta_x^2)(\gamma^{-2} + \theta_x^2 + \theta_y^2)}
\]  

(30)

Here \( \omega_c = \frac{\gamma}{2a} \), \( a \) is the spacing between the particle trajectory and the edge of the target.

Fig. 5 shows the DR intensity spectrum, \( \frac{dW}{d\omega_1} \), obtained by integrating expression (30) with respect to the focusing mirror aperture \( \theta^2 = \theta_x^2 + \theta_y^2 \leq (k_1\gamma^{-1})^2 \) for \( k_1 = 5 \) and 10. In contradiction with the case of passing through the centre of the hole, the spectrum in the energy range \( \omega_1 \ll \omega_c \) will be aproximated by a linear dependence:

\[
\frac{dW_1}{d\omega_1} = \frac{\alpha}{\pi} C_{\perp} \left(1 - B(\theta_{1m})\frac{\omega_1}{\omega_c}\right), \quad C_{\perp} = \frac{\alpha}{2\pi} \left\{ \ln(1 + k_1^2) + \frac{1}{\sqrt{1 + k_1^2}} - 1 \right\}
\]  

(31)

where \( B(5\gamma^{-1}) \approx 2.6 \).

The luminosity for the 90° collision of bunches described by distributions (11) can also be calculated analytically:

\[
L = cN_eN_{ph}F \int \int \int dxdydzdt f_{ph}(x, y, z + ct)f_e(x, y + \beta ct, z) = 
\]

\[
= \frac{N_eN_{ph}F}{\pi\sqrt{\sigma_e^2 + \sigma_{ph}^2}(\sigma_e^2 + \sigma_{ph}^2 + 2l_{ph}^2 + 2l_e^2)}.
\]  

(32)

Using the same approximations as in deriving expression (26) we can arrive at:

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\[
\frac{dN_B^H}{d\omega_2} = \frac{4}{\pi} \alpha r_0^2 N_e^3 C_\perp k_2^2 \frac{\exp\left[-\left(\frac{\omega l_e}{2\gamma}\right)^2\right]}{\omega_2 \sqrt{\left[\sigma_e^2 + \left(\frac{2\gamma^2}{\omega_2}\right)^2\right] \left[\sigma_e^2 + \left(\frac{2\gamma^2}{\omega_2}\right)^2 + 4l_e^2\right]}} = \\
= \frac{4}{\pi} \frac{\left(\frac{r_0}{l_e}\right)^2}{\alpha} N_e^3 C_\perp k_2^2 \frac{\exp\left[-\left(\frac{\omega l_e}{2\gamma}\right)^2\right]}{\omega_2 \sqrt{\left[r^2 + \left(\frac{2\gamma^2}{l_e \omega_2}\right)^2\right] \left[r^2 + 4 + \left(\frac{2\gamma^2}{l_e \omega_2}\right)^2\right]}}.
\]

(33)

For the geometry considered the coefficient of frequency transformation is twice as small as compared with the head-on collision (see formula (3)).

Depicted in Fig. 6 is the scattered photon spectrum calculated following formula (33). Similar to the head-on collision the spectrum has a maximum in the region of energies

\[
\omega_{2m} \approx 0.5 \cdot \frac{2\gamma^2}{l_e}.
\]

Estimation of the scattering photon yield for the geometry considered here for the same conditions as before gives a close value:

\[
\Delta N_B^H = 2.9 \cdot 10^4 \text{ph/bunch}.
\]

Contrary to the geometry used previously, however, in this case the radiation forming length coincides with the wavelength (\(\lambda_1 \sim 1\)mm). Therefore, the focusing mirror positioned at a distance \(L_0 \gg \lambda_1\) would not cause any suppression of the DR yield, and the resulting expression (33) could be used for estimation of the hard photon yield when planning an experiment.

Notewortly is the fact that when calculating the luminosity (32) it was assumed that the centers of the photon and electron bunches pass the interaction point at the same time. Should the focusing mirror be placed with a certain error \(\Delta L_0\), then there would appear an additional term in expressions (32),(33):

\[
D(\Delta L_0) = \exp\left\{-\frac{\Delta L_0^2}{\sigma_e^2 + \sigma_{ph}^2 + 2(l_e^2 + l_{ph}^2)}\right\}.
\]

(34)

For the frequent case, \(\sigma_e < l_e\), one can get the information on the electron bunch length via measuring the scattered photon yield versus \(\Delta L_0\) (detuning curve), since \(l_{ph} = l_e\).

5. Conclusion.

As discussed above, the energy of scattered photons for the case of ultrarelativistic electrons (\(\gamma \geq 1000\)) corresponds to the X-ray region, while for moderate relativistic energies (\(\gamma \leq 100\)) the secondary photon spectrum would include the visible range. It is known that the common techniques for electron beam diagnostics based on detection of optical
transition radiation do not allow us to measure the length of submillimeter bunches. In this context, measurement of the detuning curve by mechanical displacement of the focusing mirror seems to offer a means for measuring even shorter bunches with the use of simpler equipment than a streak camera.

It should be noted that the CBS process of laser photons on an electron bunch was considered in $90^\circ$ geometry [21], and it was shown that for a certain geometry and bunch parameters the yield of scattered photons may be by 2-3 orders exceed that from scattering on $Ne$ electrons independent of each other. The enhancement factor, dictated by the coherent compton scattering, is proportional to $Ne\frac{\lambda_1}{\gamma^2}$. It is to be expect that during scattering of CDR on the following electron bunch the effect of coherence could be made manifest in as more pronounced fashion, since the wavelength of primary radiation is by 2-3 orders higher than laser emission wavelength and, secondary, the coherent Thomson scattering would involve the dependence of the number of secondary photons on the number of electrons per bunch to be proportional to $Ne^4$.

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Fig. 1 Scheme for Thomson scattering of CDR from circular aperture. T - conducting target, BM - bending magnet, CM - concave mirror.

Fig. 2 Diffraction radiation intensity spectrum from circular aperture (lower curve - for $k = 5$, upper curve - for $k = 10$).
Fig. 3 Spectrum of the scattered photons for the scheme shown in Fig. 1 (r=0.1, 1=5 - dashed-dotted line, r=1 - dotted line, r=0.2 - solid line).

Fig. 4 Scheme for Thomson scattering of CDR from the edge of tilted target.
Fig. 5 Diffraction radiation intensity spectrum from the edge of tilted target (lower curve - $k = 5$, upper curve - $k = 10$).

Fig. 6 Spectrum of the scattered photons for the scheme shown in Fig. 4 ($r = 5$ - dashed-dotted line, $r = 1$ - dotted line, $r = 0.2$ - solid line).