Dark-matter detection by elastic and inelastic LSP scattering on $^{129}$Xe and $^{131}$Xe

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**Article info**

**Abstract**

We calculate the nuclear matrix elements involved in the elastic and inelastic scattering of the lightest supersymmetric particle (LSP) on the $^{129}$Xe and $^{131}$Xe dark-matter detector nuclei. This is the first time when both channels are addressed within the same unified microscopic nuclear framework, namely we perform large-scale shell-model calculations with a realistic two-body interaction to produce the participant nuclear wave functions. These wave functions successfully reproduce the spectroscopic data on the relevant magnetic moments and M1 decays. The tested wave functions are used to produce annual detection rates for both the elastic and inelastic channels. It is found that the inelastic channel has great detection potential for $^{129}$Xe if the LSP is heavy and stems from a SUSY model that enhances the spin-dependent scattering.

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The recent observations on cosmic microwave background suggest that the universe is flat and a notable part of the energy density of the universe (some 30%) is in a form of cold dark matter (CDM). Additional information comes from the rotational curves of galaxies [1]. The most likely candidates for constituents of the CDM are the weakly-interacting massive particles (WIMPs). They could make up a major component of the dark matter in our own galactic halo.

The only way to access the nature of dark matter are the direct detection experiments in the laboratory. These experiments look for recoil signals of the nucleus on which the WIMP has scattered elastically or inelastically. Many experiments have reported look for recoil signals of the nucleus on which the WIMP has scattered elastically or inelastically. Many experiments have reported...
SUSY model [21]. The coefficients $D_n$ are folded with the velocity distribution of the LSPs and contain all the information about the nuclear structure. They are defined as

$$
D_1 = \int \int \int G(\psi, \xi) F_{00}(u) \Omega_0^2 d\xi d\psi du,
$$

$$
D_2 = \int \int \int G(\psi, \xi) F_{01}(u) \Omega_0 \Omega_2 d\xi d\psi du,
$$

$$
D_3 = \int \int \int G(\psi, \xi) F_{11}(u) \Omega_0^2 d\xi d\psi du,
$$

$$
D_4 = \int \int \int G(\psi, \xi) F(\psi) \psi^2 d\xi d\psi du.
$$

Here $F_{\mu\nu}(u)$, $\mu, \nu = 0, 1$, are the usual spin structure functions, $\Omega_\nu$ the static spin matrix elements and $F(u)$ the nuclear form factor.\textsuperscript{1} The expression for the modulation function $G(\psi, \xi)$ is given in [14].

The limits in the integrals of Eqs. (2)–(5) are different for the elastic and inelastic channels. For the elastic channel they are given in [14] in terms of the threshold energy of the dark-matter detector. For the inelastic channel $D_4$ of (5) vanishes and we assume that the detector threshold is zero since the coincidence signal of the emitted gamma quantum can be used to reduce the threshold. For the inelastic channel the limits of the integrals (2)–(4) are given by

$$
\psi_{\text{min}} = \sqrt{T},
$$

$$
\psi_{\text{max}} = -\lambda \xi + \sqrt{\lambda^2 \xi^2 + 9.0891 + 0.135 \cos \alpha},
$$

$$
u_{\text{min}} = \frac{1}{2} b^2 \mu_0^2 \frac{v_0^2}{c^2} [1 - \sqrt{1 - \Gamma^2/\psi^2}],
$$

$$
u_{\text{max}} = \frac{1}{2} b^2 \mu_0^2 \frac{v_0^2}{c^2} [1 + \sqrt{1 - \Gamma^2/\psi^2}],
$$

where

$$
\Gamma = \frac{2E^* c^2}{\mu_0 c^2 v_0^2}.
$$

Above $E^*$ is the nuclear excitation energy of the recoiling daughter nucleus, $\mu_0$ is the reduced mass of the LSP-nucleus system, $c$ is the light velocity, and $v_0 = 220$ km/s. Angle $\alpha$ represents the phase of the Earth [15], $b = b(A)$ is the harmonic-oscillator size parameter for a target nucleus of mass $A$, and $\lambda = v_E/v_0$, where $v_E$ is given in [14].

The average kinetic energy ($T$) of the LSP can be obtained from the approximate expression [22]

$$
\langle T \rangle = 40 \text{ keV} \frac{m_e}{100 \text{ GeV}}.
$$

Hence, for heavy LSPs the scattering can well be inelastic, leading to the first excited states in $^{129}$Xe and $^{131}$Xe, at energies 39.6 keV and 80.2 keV, respectively. For $^{129}$Xe the transition is 1/2$^+_g \rightarrow$ 3/2$^+_g$ and for $^{131}$Xe it is 3/2$^+_g \rightarrow$ 1/2$^+_g$. The corresponding nuclear matrix elements are not suppressed much relative to the elastic channel, as discussed later in this work.

\textsuperscript{1} The form factors $F(u)$ and spin structure functions $F_{\mu\nu}(u)$ can be requested in numerical form from the corresponding author.

| State     | Exp. | Th. | $|S_1|/|S_2|$ | $|L_1|/|L_2|$ |
|-----------|------|-----|-------------|-------------|
| $^{129}$Xe(1/2$^+_g$) | −0.78 | −0.797 | 0.273 | −0.0019 |
| $^{129}$Xe(3/2$^+_g$) | 0.58 | 0.466 | −0.049 | −0.0034 |
| $^{131}$Xe(1/2$^+_g$) | 0.69 | 0.062 | −0.125 | −0.00069 |
| $^{131}$Xe(1/2$^+_g$) | −0.866 | 0.293 | −0.0034 | 0.095 | 0.116 |

Use of various theoretical methods has been made to compute nuclear matrix elements involved in the elastic WIMP-nucleus scattering. The most complete calculations have been done by using the nuclear shell model in [14,23,24]. In [14] the shell model results were compared with those calculated by the use of the microscopic quasiparticle–phonon model [25]. The shell model was found to better reproduce the magnetic moments of the ground states of $^{71}$Ga, $^{73}$Ge, and $^{127}$I, thus being in a position of providing a more reliable description of the LSP-nucleus scattering process than the MQPM.

For the inelastic channel there are only few estimates [22,26,27] for the scattering cross sections or event rates. None of these works actually calculates the needed nuclear wave functions in a reliable microscopic nuclear framework. To our knowledge the present work is the first to address both the elastic and inelastic event rates within a unified and complete nuclear scheme.

In the present work we perform large-scale shell-model calculations in a realistic model space with realistic effective two-body interactions. The calculations were made using the shell model code eiocode [28]. The ground states and the first excited states of $^{129}$Xe and $^{131}$Xe were computed in the valence space 2s1d0g7

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The last four columns show the calculated spin and orbital angular-momentum matrix elements for protons and neutrons.

The limits in the integrals of Eqs. (2)–(5) are different for the elastic and inelastic channels. For the elastic channel they are given in [14] in terms of the threshold energy of the dark-matter detector. For the inelastic channel $D_4$ of (5) vanishes and we assume that the detector threshold is zero since the coincidence signal of the emitted gamma quantum can be used to reduce the threshold. For the inelastic channel the limits of the integrals (2)–(4) are given by

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Above $E^*$ is the nuclear excitation energy of the recoiling daughter nucleus, $\mu_0$ is the reduced mass of the LSP-nucleus system, $c$ is the light velocity, and $v_0 = 220$ km/s. Angle $\alpha$ represents the phase of the Earth [15], $b = b(A)$ is the harmonic-oscillator size parameter for a target nucleus of mass $A$, and $\lambda = v_E/v_0$, where $v_E$ is given in [14].

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\textsuperscript{1} The form factors $F(u)$ and spin structure functions $F_{\mu\nu}(u)$ can be requested in numerical form from the corresponding author.

Table 1

| State     | Exp. | Th. | $|S_1|/|S_2|$ | $|L_1|/|L_2|$ |
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| $^{129}$Xe(3/2$^+_g$) | 0.58 | 0.466 | −0.049 | −0.0034 |
| $^{131}$Xe(1/2$^+_g$) | 0.69 | 0.062 | −0.125 | −0.00069 |
| $^{131}$Xe(1/2$^+_g$) | −0.866 | 0.293 | −0.0034 | 0.095 | 0.116 |
We also calculated $B(M1)$ values using the effective gyromagnetic factors. The experimental data for the relevant transitions are the following: $B(M1; 3/2^+_1 \rightarrow 1/2^+_g)$ exp. = $0.049(\mu_N/c)^2$ for $^{129}$Xe and $B(M1; 1/2^+_1 \rightarrow 3/2^+_g)$ exp. = $0.062(\mu_N/c)^2$ for $^{131}$Xe. The corresponding computed values are $0.033(\mu_N/c)^2$ for $^{129}$Xe and $0.043(\mu_N/c)^2$ for $^{131}$Xe. For the bare $g$ factors we obtain $0.042(\mu_N/c)^2$ for $^{129}$Xe and $0.059(\mu_N/c)^2$ for $^{131}$Xe. The use of effective gyromagnetic factors does not improve the quality of the $M1$ transitions for $^{129}$Xe or $^{131}$Xe. This notwithstanding, since the improvement in the magnetic moments was substantial we will use the renormalized spin operators for the LSP scattering calculations. We thus adopt the renormalization factors $r_p = 0.571$ for protons and $r_n = 0.881$ for neutrons.

The static spin matrix elements (SSME), present in Eqs. (2)–(4), are reviewed in Table 2 for both the elastic and inelastic scattering channels. Variations in the values of the $g$ factors induce variations in the values of the final computed SSMEs. To assess these variations we have included in Table 2 the results based on both the bare and effective $g$ factors. It is seen that the SSMEs of the two calculations deviate some 12% from each other. Numerical calculations show that the differences in the final computed $D$ coefficients (2)–(5) stem essentially from the $\Omega^2$ factors and are thus of the order of 20%. One can thus say that the manipulation of the $g$ factors causes a rough 20% variation in the values of the relevant observables listed later in this article.

From Table 2 we notice the interesting feature that for $^{129}$Xe the SSMEs suppress by a factor of three the inelastic channel relative to elastic channel, whereas for $^{131}$Xe there is only very little suppression. In [22] it was found within a very simplified nuclear model that the SSMEs would even enhance the inelastic channel for $^{127}$I. It remains to be explored if a more complete shell-model calculation, like the present one, would reproduce this finding.

The spin structure functions of Eqs. (2)–(5) are given in Figs. 1 and 2 as functions of the momentum transfer $u$ (see footnote 1). For the elastic channel of Fig. 1 the $F_{pp'}(u)$ are smooth functions whereas for the inelastic channel of Fig. 2 they behave more irregularly. The form factor $F(u)$ of the coherent channel is absent from the inelastic channel. For the elastic channel it has a peaked structure as seen in Fig. 1 (the undulations beyond the first two peaks are masked by the scale of the figure).

There are not too many other calculations for the form factors of the Xe isotopes. In [31] a very rudimentary nuclear wave function for the ground state of $^{131}$Xe was used to compute the structure functions related to the elastic LSP-nucleus scattering. We can compare our spin structure functions $F_{pp'}(u)$ for the elastic scattering with the corresponding $S_{pp'}(q)$ functions of [31] by using the conversion formula (18) of [25]. In [25] this formula was used to compare Fig. 2 of [25] with Fig. 4 of [23] for the elastic scattering of an LSP on $^{73}$Ge. In the present case we obtain from the conversion formula and from the $\Omega$ factors of Table 2 for $^{131}$Xe:

$$S_{00}(q) = (6.5 - 7.8) \times 10^{-3} F_{00}(u), \quad (12)$$

$$S_{01}(q) = -(0.013 - 0.016) F_{01}(u), \quad (13)$$

$$S_{11}(q) = (6.4 - 7.7) \times 10^{-3} F_{11}(u). \quad (14)$$

From the above conversion formulae and from Fig. 3 of [31] and Fig. 1, lower panel, we notice that the present shell-model calculation gives factors 3–6 smaller spin structure functions than the very much simplified approach of [31]. This is due to our quite small $\Omega_0$ and $\Omega_1$ factors of Table 2 for $^{131}$Xe. The overall shape of the curves is similar for the two calculations.

The values of the $D_n$ coefficients in (2)–(5) depend on the LSP mass $m_X$, the detector threshold energy $Q_{thr}$ and the Earth’s phase $\phi$. We can average $D_n$ over $\phi$ to produce their annual average values $\bar{D}_n$. Let us denote by $d_n$ the $Q_{thr} = 0$ values of $\bar{D}_n$. Then $d_n$ depend only on $m_X$. In Table 3 we list these $d_n$ coefficients for the

| $g$ factors | Mode | $^{129}$Xe | | | $^{131}$Xe | | |
|-------------|--|----------|---|---|----------|---|---|
| Bare        | Elastic | $0.941$ | $-0.954$ | $-0.326$ | $0.322$ |
|             | Inelastic | $0.306$ | $-0.311$ | $0.236$ | $-0.224$ |
| Effective   | Elastic | $0.831$ | $-0.838$ | $-0.286$ | $0.284$ |
|             | Inelastic | $0.270$ | $-0.273$ | $0.206$ | $-0.199$ |
elastice channel in units of $y^{-1}\text{kg}^{-1}$ for selected values of $m_x$. The values computed by using the bare $g$ factors are some 20% larger, as discussed earlier. The $Q_{\text{thr}}$ dependence of $D_n$ can be fitted by the exponential

$$D_n(m_x, Q_{\text{thr}}) = e^{-\alpha_n - g_\mu(m_x) Q_{\text{thr}}}$$

for reasonably small values of $Q_{\text{thr}}$. Here the reduced mass $\mu_s$ of the nucleus-LSP system is given in units of GeV and $Q_{\text{thr}}$ in units of keV. The fitting was done for the ranges of 0 keV $\leq Q_{\text{thr}} \leq 30$ keV. Our parametrization (15) enables an easy extraction of $D_n$ for the wanted LSP mass and detector threshold energy.

From Table 3 we notice that the coherent channel, represented by $D_1$, is strong. The importance of this channel is enhanced further by the fact that it is proportional to $A^2$, as seen from Eq. (1). However, as noticed in [14], for certain parametrizations of the SUSY models the spin-dependent channel, represented by $D_2$, can overwhelm the coherent channel. The spin-dependent channel seems to be more important for $^{129}\text{Xe}$ than for $^{131}\text{Xe}$.

In Table 4 we present the computed annual averaged coefficients $D_n(m_x)$ for the inelastic scattering. As discussed earlier, the values computed by using the bare $g$ factors are some 20% larger. Here we have assumed that the detector threshold is zero. As seen from Table 4, this channel is far more important for $^{129}\text{Xe}$ than for $^{131}\text{Xe}$. We present a useful parametrization of the elastic detection rates in terms of the LSP mass and detector threshold energy. The inelastic rates are calculated assuming zero threshold. It is found that the inelastic channel has great detection potential for $^{129}\text{Xe}$ if the LSP is heavy and stems from a SUSY model that enhances the spin-dependent scattering. The obtained results are especially interesting for the ZEPLIN and XENON experimental programs.

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