Back Reaction of Cosmological Perturbations and the Cosmological Constant Problem

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Abstract. The presence of cosmological fluctuations influences the background cosmology in which the perturbations evolve. This back-reaction arises as a second order effect in the cosmological perturbation expansion. The effect is cumulative in the sense that all fluctuation modes contribute to the change in the background geometry, and as a consequence the back-reaction effect can be large even if the amplitude of the fluctuation spectrum is small. We review two approaches used to quantify back-reaction. In the first approach, the effect of the fluctuations on the background is expressed in terms of an effective energy-momentum tensor. We show that in the context of an inflationary background cosmology, the long wavelength contributions to the effective energy-momentum tensor take the form of a negative cosmological constant, whose absolute value increases as a function of time since the phase space of infrared modes is increasing. This then leads to the speculation that gravitational back-reaction may lead to a dynamical cancellation mechanism for a bare cosmological constant, and yield a scaling fixed point in the asymptotic future in which the remnant cosmological constant satisfies $\Omega_\Lambda \sim 1$. We then discuss how infrared modes effect local observables (as opposed to mathematical background quantities) and find that the leading infrared back-reaction contributions cancel in single field inflationary models. However, we expect non-trivial back-reaction of infrared modes in models with more than one matter field.

1 Motivation

It is well known that gravitational waves propagating in some background space-time affect the dynamics of the background. This back-reaction can be described in terms of an effective energy-momentum tensor $\tau_{\mu\nu}$. In the short wave limit, when the typical wavelength of the waves is small compared with the curvature of the background space-time, $\tau_{\mu\nu}$ has the form of a radiative fluid with an equation of state $p = \rho/3$ (where $p$ and $\rho$ denote pressure and energy density, respectively).

In most models of the early Universe, scalar-type metric perturbations are more important than gravity waves. Here, we report on two studies of the back reaction problem for scalar gravitational perturbations. The first approach [1, 2] is based on defining an effective energy-momentum tensor $\tau_{\mu\nu}$ which describes the back-reaction, and applying the results to an inflationary background cosmology. The second approach [3] focuses on evaluating the
back-reaction of fluctuations on an observable measuring the local Hubble expansion rate. Both studies are in the context of Einstein gravity coupled to scalar field matter.

Our studies are closely related to work by Woodard and Tsamis [4, 5] who considered the back-reaction of long wavelength gravitational waves in pure gravity with a bare cosmological constant, and to work of Abramo and Woodard [6, 7] who initiated the study of back-reaction of infrared modes on local observables.

In the following, we first review the derivation of the effective energy-momentum tensor $\tau_{\mu\nu}$ which describes the back-reaction of linear cosmological fluctuations on the background cosmology (Section 2), and summarize the evaluation of this tensor in an inflationary cosmological background (Section 3). We find that contribution of long wavelength (i.e. super-Hubble-scale) scalar metric fluctuations to $\tau_{\mu\nu}$ acts as a negative cosmological constant whose absolute value increases in time since the phase space of infrared modes is increasing. This gives rise to the speculations of Section 4 that back-reaction of infrared modes may lead to a dynamical cancellation mechanism for the bare cosmological constant. In Section 5 we address some major deficiencies in this first approach to computing back-reaction. Many of these objections were first raised by Unruh [8]. In Section 6 we then summarize the present status of an improved approach to studying gravitational back-reaction based on the calculation of corrections to an observable measuring the local expansion rate. We find that the leading infrared back-reaction effects vanish in single matter field models. However, based on results concerning the parametric amplification of super-Hubble cosmological fluctuations during inflationary reheating [9, 10, 11, 12] we argue that in multi-field models the back-reaction of infrared modes will be important.

2 Framework

The method of analyzing gravitational back-reaction [1] based on the computation of an effective energy-momentum tensor for fluctuations is related to early work on the back-reaction of gravitational waves by Brill, Hartle and Isaacson [13], among others. The idea is to expand the Einstein equations to second order in the perturbations, to assume that the first order terms satisfy the equations of motion for linearized cosmological perturbations [14] (hence these terms cancel), to take the spatial average of the remaining terms, and to regard the resulting equations as equations for a new homogeneous metric $g_{\mu\nu}^{(0,br)}$ which includes the effect of the perturbations to quadratic order:

$$G_{\mu\nu}(g_{\alpha\beta}^{(0,br)}) = 8\pi G \left[ I_{\mu\nu}^{(0)} + \tau_{\mu\nu} \right]$$

where the effective energy-momentum tensor $\tau_{\mu\nu}$ of gravitational back-reaction contains the terms resulting from spatial averaging of the second order metric
and matter perturbations:

\[ \tau_{\mu\nu} = < T^{(2)}_{\mu\nu} - \frac{1}{8\pi G} G^{(2)}_{\mu\nu} > , \]  

(2)

where pointed brackets stand for spatial averaging, and the superscripts indicate the order in perturbation theory.

As analyzed in detail in [1, 2], the back-reaction equation (1) is covariant under linear space-time coordinate transformations even though \( \tau_{\mu\nu} \) is not invariant \(^1\). In the following, we will work in longitudinal gauge (see e.g. [14] for a review of the theory of cosmological perturbations).

For simplicity, we shall take matter to be described in terms of a single scalar field. In this case, there is only one independent metric perturbation variable which we denote by \( \phi (x, t) \), and in longitudinal gauge the perturbed metric can be written in the form

\[ ds^2 = (1 + 2\phi) dt^2 - a(t)^2 (1 - 2\phi) \delta_{ij} dx^i dx^j , \]  

(3)

where \( a(t) \) is the cosmological scale factor. The energy-momentum tensor for a scalar field is

\[ T_{\mu\nu} = \varphi_{,\mu}\varphi_{,\nu} - g_{\mu\nu} \left[ \frac{1}{2} \varphi^{,\alpha}\varphi_{,\alpha} - V(\varphi) \right] . \]  

(4)

By expanding the Einstein tensor and the above energy-momentum tensor to second order in the metric and matter fluctuations \( \phi \) and \( \delta\varphi \), respectively, it can be shown that the non-vanishing components of the effective back-reaction energy-momentum tensor \( \tau_{\mu\nu} \) become

\[ \tau_{00} = \frac{1}{8\pi G} \left[ +12H\langle \phi\dot{\phi} \rangle - 3\langle (\dot{\phi})^2 \rangle + 9a^{-2}\langle (\nabla\phi)^2 \rangle \right] 
+ \frac{1}{2}\langle (\delta\phi)^2 \rangle + \frac{1}{2}a^{-2}\langle (\nabla\delta\varphi)^2 \rangle 
+ \frac{1}{2}V''(\varphi_0)\langle \delta\varphi^2 \rangle + 2V'(\varphi_0)\langle \phi\delta\varphi \rangle , \]  

(5)

and

\[ \tau_{ij} = a^2\delta_{ij} \left\{ \frac{1}{8\pi G} \left[ (24H^2 + 16H)\langle \phi^2 \rangle + 24H\langle \dot{\phi}\phi \rangle \right] 
+ \langle (\dot{\phi})^2 \rangle + 4\langle \phi\ddot{\phi} \rangle - \frac{4}{3}a^{-2}\langle (\nabla\phi)^2 \rangle \right\] 
+ \frac{1}{2}\langle (\delta\phi)^2 \rangle - \frac{1}{6}a^{-2}\langle (\nabla\delta\varphi)^2 \rangle - 4\varphi_0\langle \delta\phi\phi \rangle 
- \frac{1}{2}V''(\varphi_0)\langle \delta\varphi^2 \rangle + 2V'(\varphi_0)\langle \phi\delta\varphi \rangle \right\} , \]  

(6)

where \( H \) is the Hubble expansion rate.

\(^1\)See [8], however, for important questions concerning the covariance of the analysis under higher order coordinate transformations.
3 Application to Inflationary Cosmology

The metric and matter fluctuation variables $\phi$ and $\delta \phi$ are linked via the Einstein constraint equations, and hence all terms in the above formulas for the components of $\tau_{\mu \nu}$ can be expressed in terms of two point functions of $\phi$ and its derivatives. The two point functions, in turn, are obtained by integrating over all of the Fourier modes of $\phi$, e.g.

$$\langle \phi^2 \rangle \sim \int_{k_i}^{k_u} dk k^2 |\phi_k|^2,$$

where $\phi_k$ denotes the amplitude of the $k$’th Fourier mode. The above expression is divergent both in the infrared and in the ultraviolet. The ultraviolet divergence is the usual divergence of a free quantum field theory and can be “cured” by introducing an ultraviolet cutoff $k_u$. In the infrared, we will discard all modes $k < k_i$ with wavelength larger than the Hubble radius at the beginning of inflation, since these modes are determined by the pre-inflationary physics. We take these modes to contribute to the background.

At any time $t$ we can separate the integral in (7) into the contribution of infrared and ultraviolet modes, the separation being defined by setting the physical wavelength equal to the Hubble radius. Thus, in an inflationary Universe the infrared phase space is continually increasing since comoving modes are stretched beyond the Hubble radius, while the ultraviolet phase space is either constant (if the ultraviolet cutoff corresponds to a fixed physical wavelength), or decreasing (if the ultraviolet cutoff corresponds to fixed comoving wavelength). In either case, unless the spectrum of the initial fluctuations is extremely blue, two point functions such as (7) will at later stages of an inflationary Universe be completely dominated by the infrared sector. In the following, we will therefore restrict our attention to this sector, i.e. to wavelengths larger than the Hubble radius.

In order to evaluate the two point functions which enter into the expressions for $\tau_{\mu \nu}$, we need to know the time evolution of the linear fluctuations $\phi_k$, which is given by the linear theory of cosmological perturbations [14]. On scales larger than the Hubble radius, and for a time-independent equation of state, $\phi_k$ is constant in time. The Einstein constraint equations relating the metric and matter fluctuations give

$$\dot{\phi} + H \phi = 4\pi G \dot{\phi}_0 \delta \phi .$$

If the background scalar field $\varphi_0$ is rolling slowly, then $\dot{\varphi}_0 \simeq -\frac{V'}{M^2}$, where a prime denotes the derivative with respect to the scalar matter field. Thus,

$$\delta \phi = -\frac{2V}{V'} \phi .$$

Hence, in the expressions (5) and (6) for $\tau_{\mu \nu}$, all terms with space and time
derivatives can be neglected, and we obtain
\[ \rho_{br} \equiv \tau_0^0 \approx \left( 2 \frac{V''V^2}{V'^2} - 4V \right) < \phi^2 > \] (10)

and
\[ p_{br} \equiv -\frac{1}{3} \tau_i^i \approx -\rho_{br} , \] (11)

The main result which emerges from this analysis is that the equation of state of the dominant infrared contribution to the energy-momentum tensor \( \tau_{\mu\nu} \) which describes back-reaction takes the form of a negative cosmological constant
\[ p_{br} = -\rho_{br} \text{ with } \rho_{br} < 0 . \] (12)

The second crucial result is that the magnitude of \( \rho_{br} \) increases as a function of time. This is due in part to the fact that, in an inflationary Universe, as time increases more and more wavelengths become longer than the Hubble radius and begin to contribute to \( \rho_{br} \).

How large is the magnitude of back-reaction? The basic point is that since the amplitude of each fluctuation mode is small, we need a very large phase space of infrared modes in order to induce any interesting effects. In models with a very short period of primordial inflation, the back-reaction of long-wavelength cosmological fluctuations hence will not be important. However, in many single field models of inflation, in particular in those of chaotic inflation type [15], inflation lasts so long that the infrared back-reaction effects can build up to become important for the cosmological background dynamics.

To give an example, consider chaotic inflation with a potential
\[ V(\varphi) = \frac{1}{2} m^2 \varphi^2 . \] (13)

In this case, the values of \( \varphi_k \) for long wavelength modes are well known (see e.g. [14]), and the integral in (7) can be easily performed, thus yielding explicit expressions for the dominant terms in the effective energy-momentum tensor. Comparing the resulting back-reaction energy density \( \rho_{br} \) with the background density \( \rho_0 \), we find
\[ \frac{\rho_{br}(t)}{\rho_0} \approx \frac{3}{4\pi} \frac{m^2 \varphi_0^2}{M_P^4} \left[ \frac{\varphi_0(t)}{\varphi_0(t_i)} \right]^4 . \] (14)

Without back-reaction, inflation would end [15] when \( \varphi_0(t) \sim M_P \). Inserting this value into (14), we see that if
\[ \varphi_0(t_i) > \varphi_{br} \sim m^{-1/3} M_P^{4/3} , \] (15)

then back-reaction will become important before the end of inflation and may shorten the period of inflation. It is interesting to compare this value with
the scale \( \varphi_0(t_i) \sim \varphi_{sr} = m^{-1/2} M_P^{3/2} \) above which the stochastic terms in the scalar field equation of motion arising in the context of the stochastic approach to chaotic inflation [16, 17] are dominant. Notice that since \( \varphi_{sr} \gg \varphi_{br} \) (recall that \( m \ll M_P \)), back-reaction effects can be very important in the entire range of field values relevant to stochastic inflation.

4 Speculations Concerning a Dynamical Relaxation Mechanism for \( \Lambda \)

Since the back-reaction of cosmological fluctuations in an inflationary cosmology acts (see (12)) like a negative cosmological constant, and since the magnitude of the back-reaction effect increases in time, one may speculate [18] that back-reaction will lead to a dynamical relaxation of the cosmological constant (see Tsamis & Woodard [4] for similar speculations based on the back-reaction of long wavelength gravitational waves).

The background metric \( g^{(0,br)}_{\mu\nu} \) including back-reaction evolves as if the cosmological constant at time \( t \) were

\[
\Lambda_{\text{eff}}(t) = \Lambda_0 + 8\pi G \rho_{br}(t)
\]

and not the bare cosmological constant \( \Lambda_0 \). Hence we propose to identify (16) with a time dependent effective cosmological constant. Since \( |\rho_{br}(t)| \) increases as \( t \) grows, the effective cosmological constant will decay. Note that even if the initial magnitude of the perturbations is small, eventually (if inflation lasts a sufficiently long time) the back-reaction effect will become large enough to cancel any bare cosmological constant.

Furthermore, we speculate that this dynamical relaxation mechanism for \( \Lambda \) will be self-regulating. As long as \( \Lambda_{\text{eff}}(t) > 8\pi G \rho_m(t) \), where \( \rho_m(t) \) stands for the energy density in ordinary matter and radiation, the evolution of \( g^{(0,br)}_{\mu\nu} \) is dominated by \( \Lambda_{\text{eff}}(t) \). Hence, the Universe will be undergoing accelerated expansion, more scales will be leaving the Hubble radius and the magnitude of the back-reaction term will increase. However, once \( \Lambda_{\text{eff}}(t) \) falls below \( \rho_m(t) \), the background will start to decelerate, scales will enter the Hubble radius, and the number of modes contributing to the back-reaction will decrease, thus reducing the strength of back-reaction. Hence, it is likely that there will be a scaling solution to the effective equation of motion for \( \Lambda_{\text{eff}}(t) \) of the form

\[
\Lambda_{\text{eff}}(t) \sim 8\pi G \rho_m(t).
\]

Such a scaling solution would correspond to a contribution to the relative closure density of \( \Omega_\Lambda \sim 1 \).
5 Criticism and Open Issues

There are important concerns about the above formalism, and even more so about the resulting speculations (some of these were first discussed in print in [8]). On a formal level, since our back-reaction effect is of second order in cosmological perturbation theory, it is necessary to demonstrate covariance of the proposed back-reaction equation (1) beyond linear order, and this has not been done. Next, it might be argued that by causality super-Hubble fluctuations cannot affect local observables. Thirdly, from an observational perspective one is not interested in the effect of fluctuations on the background metric (since what the background is cannot be determined precisely using local observations). Instead, one should compute the back-reaction of cosmological fluctuations on observables describing the local Hubble expansion rate. One might then argue that even if long-wavelength fluctuations have an effect on the background metric, they do not influence local observables. Finally, it is clear that the speculations in the previous section involve the extrapolation of perturbative physics deep into the non-perturbative regime.

These important issues have now begun to be addressed. Good physical arguments can be given [6, 7] supporting the idea that long-wavelength fluctuations can effect local physics. Consider, for example, a black hole of mass $M$ absorbing a particle of mass $m$. Even after this particle has disappeared beyond the horizon, its gravitational effects (in terms of the increased mass of the black hole) remain measurable to an external observer. A similar argument can be given in inflationary cosmology: consider an initial localized mass fluctuation with a characteristic physical length scale $\lambda$ in an exponentially expanding background. Even after the length scale of the fluctuation redshifts to be larger than the Hubble radius, the gravitational potential associated with this fluctuation remains measurable. On a more technical level, it has recently been shown that super-Hubble scale (but sub-horizon-scale) metric fluctuations can be parametrically amplified during inflationary reheating [9, 10, 11, 12]. This clearly demonstrates a coupling between local physics and super-Hubble-scale fluctuations.

These arguments, however, make it even more important to focus on back-reaction effects of cosmological fluctuations on local physical observables rather than on the mathematical background metric. This topic will be discussed in the following section. The results of that analysis will then determine the answer to the first of the concerns listed at the beginning of this section.

It is obvious that even in models in which the perturbative back-reaction results of Sections 2 - 3 have locally measurable implications, the analysis has to be extended beyond perturbation theory to justify the speculations of Section 4. For some initial ideas in this direction, see [20, 21].
6 Back-Reaction on Local Observables

In this section we will be summarizing recent work [3] in which the leading infrared back-reaction effects on a local observable measuring the Hubble expansion rate were calculated. Note that initial work on gravitational back-reaction effects of infrared modes on local observables was done in [6, 7], using different methods and a different observable than the one used below.

If we consider a perfect fluid with velocity four vector $u^\alpha$ in an inhomogeneous cosmological geometry, the local expansion rate which generalizes the Hubble expansion rate $H(t)$ of homogeneous isotropic Friedmann-Robertson-Walker cosmology is given by $\frac{1}{3} \Theta$, where $\Theta$ is the four divergence of $u^\alpha$:

$$ \Theta = u^\alpha_{;\alpha}, $$

the semicolon indicating the covariant derivative. In [3], the effects of cosmological fluctuations on this variable were computed to second order in perturbation theory. To leading order in the infrared expansion, the result is

$$ \Theta = 3 \frac{a'}{a^2} (1 - \phi + \frac{3}{2} \phi^2) - 3 \frac{\phi'}{a}, $$

(19)

where the prime denotes the derivative with respect to conformal time. If we now calculate the spatial average of $\Theta$, the term linear in $\phi$ vanishes, and - as expected - we are left with a quadratic back-reaction contribution.

Superficially, it appears from (19) that there is a non-vanishing back-reaction effect at quadratic order which is not suppressed for super-Hubble modes. However, we must be careful and evaluate $\Theta$ not at a constant value of the background coordinates, but rather at a fixed value of some physical observable. For example, if we work out the value of $\Theta$ in the case of a matter-dominated Universe, and express the result as a function of the proper time $\tau$ given by

$$ d\tau^2 = a(\eta)^2 (1 + 2 \phi) d\eta^2 $$

(20)

instead of as a function of conformal time $\eta$, then we find that the leading infrared terms proportional to $\phi^2$ exactly cancel, and that thus there is no unsuppressed infrared back-reaction on the local measure of the Hubble expansion rate.

A more relevant example with respect to the discussion in earlier sections is a model in which matter is given by a single scalar field. In this case, the leading infrared back-reaction terms in $\Theta$ are again given by (19) which looks different from the background value $3H$. However, once again it is important to express $\Theta$ in terms of a physical background variable. If we choose the value of the matter field $\varphi$ as this variable, we find after easy manipulations that, including only the leading infrared back-reaction terms,

$$ \Theta(\varphi) = \sqrt{3 \sqrt{V(\varphi)}}. $$

(21)
Hence, once again the leading infrared back-reaction contributions vanish, as already found in the work of [7] which considered the leading infrared back-reaction effects on a local observable different than the one we have used, and applied very different methods.\footnote{For a different approach which also leads to the conclusion that there can be no back-reaction effects from infrared modes on local observables in models with a single matter component see [24].}

However, in a model with two matter fields, it is clear that if we e.g. use the second matter field as a physical clock, then the leading infrared back-reaction terms will not cancel in $\Theta$, and that thus in such models infrared back-reaction will be physically observable. The situation will be very much analogous to what happens in the case of parametric resonance of gravitational fluctuations during inflationary reheating. This process is a gauge artifact in single field models of inflation [10] (see also [22, 23]), but it is real and unsuppressed in certain two field models [11, 12]. In the case of two field models, work on the analysis of the back-reaction effects of infrared modes on the observable representing the local Hubble expansion rate is in progress.

7 Conclusions

We have summarized the present status of the work on the gravitational back-reaction of cosmological fluctuations. In an inflationary background cosmology, the perturbations generated during inflation are shown to contribute as an effective energy-momentum tensor of the form of a negative cosmological constant to the evolution of the background metric. In addition, the absolute value of this induced cosmological constant grows in time since the phase space of infrared modes is increasing. This leads to the intriguing speculation that gravitational back-reaction of fluctuations may provide a dynamical cancellation mechanism for the cosmological constant, leaving behind a remnant effective cosmological constant which at all sufficiently late times corresponds to $\Omega_{\Lambda} \sim 1$.

However, we have also shown that in single field models, the leading infrared terms in the back-reaction equation cancel when calculating the local Hubble expansion rate as a function of physical variables. We have argued why in two-field models we do not expect this cancellation to persist.

Obviously, a lot more work is required in order to be able to extend the present calculations, which show at - leading order in the cosmological perturbation expansion - the onset of dynamical relaxation of $\Lambda$, to higher orders and to a full nonlinear argument.

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