Heat Transfer and Flow Characteristics of Pseudoplastic Nanomaterial Liquid Flowing over the Slender Cylinder with Variable Characteristics

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Abstract: The present article investigates heat transfer and pseudoplastic nanomaterial liquid flow over a vertical thin cylinder. The Buongiorno model is used for this analysis. The problem gains more significance when temperature-dependent variable viscosity is taken into account. Using suitable similarity variables, nonlinear flow equations are first converted into ordinary differential equations. The generating structure is solved by the MATLAB BVP4C algorithm. Newly developed physical parameters are focused. It is observed that the heat transfer rate and the skin friction coefficient is increased remarkably because of mixing nano-particles in the base fluid by considering $\gamma_b = 1, 2, 3, 4$ and $\lambda = 1, 1.5, 2, 2.5, 3$. It is found that the temperature field increases by inclining the values of thermophoresis and Brownian motion parameters. It is also evaluated that the velocity field decreases by increasing the values of the curvature parameter, Weissenberg number and buoyancy ratio characteristics.

Keywords: pseudoplastic nanofluid; heat transfer; thermophoresis and Brownian motion features; slender cylinder; computational approach; variable viscosity

1. Introduction

A fluid that does not obey the viscosity law of Newton is known as a non-Newtonian fluid. Similar to many typically observed materials such as honey, starch, toothpaste and many salt solutions are non-Newtonian fluids. Non-Newtonian fluid drift has provided favourable results in fluid mechanics as it is common in the biological sciences and industry. Non-Newtonian fluids include polymer solutions, blood float, heavy lubrication oil and grease. The study of mass and heat transfer has important packages in various fields of engineering and technology such as milk production, engineering devices, blood oxygenators, dissolution processes, mixing mechanisms and many more. A nanomaterial liquid is a liquid that contains particles of nanometre size known as nanoparticles. The most impactful reason for adding nanoparticles to the base fluid reveals a remarkable increment of base fluid thermal properties. The nanoparticles that are usually used in nanofluids are carbides, metals, oxides and carbon nanotubes. Water and oil are common base fluids. Buongiorno model is utilized in the investigation of Brownian movement and thermophoresis impact.
on mass, flow, and transport of heat from the considered surface. The concept of nanofluids was initiated by Choi [1] similar to that of nanoparticles. The truth that nanofluids have higher thermal conductivity than ordinary fluids due to their nanostructure has fascinated many theoretical and engineering scientists. Kuznetsov et al. [2] introduced the influence of nanomaterials liquid on the flow of natural convection through a flat surface. They reveal that decreasing the Nusselt number is a reduction feature of each of the succeeding characteristics: Brownian motion characteristic and buoyancy ratio characteristic. In addition, Prasher et al. [3] confirmed that convection is a motive for increasing the thermal conductivity of nanomaterials liquid due to the Brownian motion of the nanoparticles. Wang et al. [4] confirmed that the thermal conductivity growth dependence could be very vulnerable due to Brownian motion. Lee et al. [5] later located that with the particle volume fraction, the thermal conductivity of the nanomaterial liquid would enhance linearly. The slender cylinder is a special type of cylinder upon which, due to its slimness, we can easily research the liquid’s boundary layer flow. Nadeem et al. [6] worked on cylinder and studied viscous nanofluid’s heat transport and flow analysis. In [7–13], researchers paid attention to the mass and heat transport investigation by assuming different geometries such as a vertical cone, stretching sheet, stretching cylinder and circular cylinder under the thermal radiations and magnetohydrodynamic effects. Analysis of nanofluid flow problems is presented in [14–29] under boundary layer effects.

From the above analysis and discussion, we conclude that this is an important area of fluid mechanics; therefore, we decided to study the Carreau-Yasuda nanofluid flow over a vertical slender cylinder. By using the boundary layer concept and equivalence transformations, the model equations are simplified. MATLAB BVP4C algorithm is used to find the solution. The Buongiorno model [30] is applied for this investigation. The Graphical behaviour and expressions for temperature, velocity and concentration are calculated. We obtained results for various parameters, i.e., curvature, Prandtl number, thermophoresis, buoyancy ratio, Weissenberg number, Brownian motion and Lewis number on flow.

2. Fluid Model

Carreau-Yasuda fluid’s Cauchy stress tensor is

\[ T = -pI + \left[ \mu_{\infty} - (\mu_0 - \mu_{\infty}) \right \{1 + (\Gamma \dot{\gamma})^{\frac{d}{2-n}}} \right \} A_1, \]  

where \( \mu_{\infty} \) shows the infinite shear rate, \( \mu_0 \) shows the zero shear viscosity rate, \( d, n \) and \( \Gamma \) are fluid characteristics of Carreau-Yasuda and \( \dot{\gamma} \) is defined as

\[ \dot{\gamma} = \sqrt{2 \ trace (D^2)} . \]

where

\[ D = \frac{1}{2} [\text{grad } v + (\text{grad } v)^t]. \]

Here, we assume \( \mu_{\infty} = 0 \), Then

\[ T = -pI + \mu_0 \left \{1 + (\Gamma \dot{\gamma})^{\frac{d}{2-n}}} \right \} A_1. \]

First Rivlin Erickson tensor \( A_1 \) is

\[ A_1 = \nabla v + (\nabla v)^t. \]

3. Statement

We consider a nanofluid incompressible flow along with a vertical slender cylinder of radius \( r_o \). Coordinates \((x, r)\) will be used along the cylinder surface.
The equations of mass conservation, momentum, energy transfer and nanoparticles concentration are

\[ \frac{w}{r} + w_r + u_z = 0, \quad (6) \]

\[ wu_r + uu_x = -\frac{1}{\rho} p_x + v\left(\frac{u_x}{r} + u_{rr}\right) + v\left[\frac{u_x}{r} + (d + 1)u_r\right] \frac{\rho - 1}{\rho} \Gamma^4 u_r^2 + \left[(\rho^2 - \rho)(\varphi - \varphi_\infty) + (1 - \varphi_\infty)(T - T_\infty)\right] \beta_{th} g_y, \quad (7) \]

\[ wT_r + uT_x = a \left(\frac{1}{r} T_r + T_{rr}\right) + \frac{\rho C_p b}{\rho C_p} \left(\frac{D_T}{T_\infty} T_r^2 + D_B \varphi_T T_r\right), \quad (8) \]

\[ w\varphi_r + u\varphi_x = \left(\frac{1}{r} \varphi_r + \varphi_{rr}\right) D_B + \left(\frac{1}{r} T_r + T_{rr}\right) D_T \frac{T}{T_\infty}, \quad (9) \]

The boundary conditions for the problem are given below [31].

\[
\begin{align*}
    u(x, r_o) &= 0, \quad \text{as } r \to r_o, \\
    w(x, r_o) &= V_x, \quad \text{as } r \to r_o, \\
    T(x, r_o) &= T_w(x), \quad \text{as } r \to r_o, \\
    \varphi(x, r_o) &= \varphi_w(x), \quad \text{as } r \to r_o, \\
    u(x, r) &= U(x), \quad \text{as } r \to \infty, \\
    T(x, r) &= T_\infty, \quad \text{as } r \to \infty, \\
    \varphi(x, r) &= \varphi_\infty, \quad \text{as } r \to \infty.
\end{align*}
\]

where \( V \) is the constant velocity of injection (\( V > 0 \)) or suction (\( V < 0 \)). The similarity transformation is defined as follows:

\[ u = \frac{U_\infty x}{l} F'(\eta), w = -\frac{r_o}{l} \left(\frac{v U_\infty}{l}\right)^{\frac{1}{2}} F(\eta), \quad (11) \]

\[ \theta = \frac{T - T_\infty}{T_w - T_\infty}, \psi = \frac{\varphi - \varphi_\infty}{\varphi_w - \varphi_\infty}, \eta = \frac{r^2 - r_o^2}{2r_o^2} \left(\frac{U_\infty}{vl}\right)^{\frac{1}{2}}, \quad (12) \]

Here, \( U(x) = \left(\frac{l}{x}\right) U_\infty \) is the mainstream velocity, \( v \) is called the kinematic viscosity and is denoted as \( v = \left(\frac{\nu}{\rho}\right) \). Here, \( \rho \) denotes the fluid density. The temperature of the slender cylinder surface is \( T_w(x) \) with the form \( T_w - T_\infty = \Delta T \left(\frac{x}{l}\right) \) and concentration of the slender cylinder surface is \( \varphi_w(x) \) with the form \( \varphi_w - \varphi_\infty = \Delta \varphi \left(\frac{x}{l}\right) \), where \( l \) is a characteristic length, \( U_\infty \) is the characteristic velocity, the temperature characteristic is \( \Delta T \) and the nanoparticle concentration characteristic is \( \Delta \varphi \). Using the above transformations, Equation (6) is satisfied automatically and Equations (7)–(9) take the following form

\[ (2\eta \gamma_\beta + 1) F'' + 2n - 1 W_e (2\eta \gamma_\beta + 1)^{\frac{3}{2}} F'' + 3(n - 1) W_e \gamma_\beta (2\eta \gamma_\beta + 1)^{\frac{3}{2}} F'' + 2 \gamma_\beta F'' + FF'' + Fr^2 + 1 + \lambda(1 - \varphi_\infty)(\theta + N_p \psi) = 0, \quad (13) \]

\[ (2\eta \gamma_\beta + 1) \theta'' + 2 \gamma_\beta \theta'' - Pr(F'\theta - F\theta') + (2\eta \gamma_\beta + 1) \left(N_p \theta' + N_b \theta' \psi'\right) = 0, \quad (14) \]

\[ (2\eta \gamma_\beta + 1) \psi'' + 2 \gamma_\beta \psi'' - Le Pr \left(F' \psi - F \psi'\right) + \frac{N_p}{N_b} \left[(2\eta \gamma_\beta + 1) \theta'' + 2 \gamma_\beta \theta''\right] = 0, \quad (15) \]

in which the \( N_p = \frac{\rho \gamma_\beta D_p (T_w - T_\infty)}{\rho C_p T_\infty} \) is the thermophoresis parameter, \( \gamma_\beta = \left(\frac{l}{U_\infty r_o}\right)^{\frac{1}{2}} \) is the curvature characteristic, \( \lambda = \frac{\varphi_\infty \Delta T}{U_\infty} \) is the buoyancy characteristic, \( N_b = \frac{\rho \gamma_\beta D_p (\varphi_w - \varphi_\infty)}{\rho \gamma_\beta D_p (\varphi_w - \varphi_\infty)} \) is the Brownian movement characteristic, \( Pr = \frac{l}{2} \) is the Prandtl number, \( N_r = \frac{\rho \gamma_\beta \gamma_\beta D_p (\varphi_w - \varphi_\infty)}{\rho \gamma_\beta D_p (\varphi_w - \varphi_\infty)} \) is the Lewis number.
The non-dimensional form of boundary conditions are
\[
\begin{align*}
F'(\infty) &= 1, \quad \text{as} \eta \to \infty \quad \text{and} \quad F(0) = c_o, \quad F'(0) = 0, \quad \text{as} \eta \to 0. \\
\theta(\infty) &= 0, \quad \psi(\infty) = 0, \quad \text{as} \eta \to \infty \quad \text{and} \quad \theta(0) = 1, \quad \psi(0) = 1, \quad \text{as} \eta \to 0. 
\end{align*}
\] (16)
where \(c_o\) is any constant. The expression for the skin friction coefficient and the Nusselt number are defined as
\[
\frac{N_a}{K_e^{1/2}} = -\theta'(0), \quad \frac{1}{2} C_f R_e^{1/2} = F''(0) + (n - 1) W_e F''(0)
\] (17)

4. Numerical Solution

By using BVP4C, the non-linear differential Equations (12)–(14) are solved numerically. We assume
\[
\begin{align*}
y_1 &= F, \quad y_4 = \theta, \quad y_6 = \psi, \\
y_2 &= F', \quad y_5 = \theta', \quad y_7 = \psi', \\
y_3 &= F'', \quad y_5' = \theta'', \quad y_7' = \psi'', \\
y_3' &= F'''.
\end{align*}
\] (18)
The equivalent equations become
\[
y_5' = \frac{-\{3(n-1)W_e \gamma_b (2\eta^2 \gamma_b + 1)^{1/2} y_5^2 + 2 \gamma_b y_3 + y_1 y_3 - y_2^2 + 1 + \lambda (1 - \varphi_0) (\theta + N_t y_6)\}}{(2\eta^2 \gamma_b + 1) + 2(n - 1) W_e (2\eta^2 \gamma_b + 1)^{1/2} y_3},
\] (19)
\[
y_5' = \frac{-\{2 \gamma_b y_5 - Pr (y_2 y_4 - y_1 y_5) + (2 \eta^2 \gamma_b + 1) (N_t y_5 y_7 + N_t y_5^2)\}}{(2\eta^2 \gamma_b + 1)}
\] (20)
\[
y_7' = \frac{-\{2 \gamma_b y_7 - Pr Le (y_2 y_6 - y_1 y_7) + \frac{N_t}{N_e} (2 \eta^2 \gamma_b + 1) y_5^2 + 2 \gamma_b y_3\}}{(2\eta^2 \gamma_b + 1)},
\] (21)
with conditions
\[
\begin{align*}
y_1(0) &= c_o, \quad y_2(0) = 0, \quad y_2 \to 1 \quad \text{as} \eta \to \infty, \\
y_4(0) &= 1, \quad y_6(0) = 1, \quad y_4 \to 0, \quad y_6 \to 0 \quad \text{as} \eta \to \infty.
\end{align*}
\] (22)

5. Graphical Results and Discussion

The nonlinear partial differential equations of nanofluid heat transfer and the boundary layer flow over a vertical cylinder are shown. Figure 1 represents the geometry of the fluid flow problem. The governing equations are articulated by applying similarity transformations. Figures 2a–4a provide the behaviour of the velocity profile for the specific characteristic concerned. Figure 2a shows the behaviour of the curvature parameter \(\gamma\) on the field of velocity. It is shown that by increasing the values of the curvature parameter, the velocity field decreases. Figure 2b describes the behaviour of \(N_e\) on the field of velocity. The velocity profile declines by inclining the values of the buoyancy ratio. Figure 3a shows the influence of \(W_e\) on the velocity field. The Weissenberg number differentiates the elastic forces from the viscous forces and it is the ratio of specific processes of time and time relaxation of fluid; therefore, by enlarging the values of the Weissenberg number, the specific process time decreases, and the velocity distribution also decreases. Figure 3b exhibit the impact of Lewis number on velocity distribution. It is easily observed that velocity profile expands by enlarging the values of Lewis number. Figure 4a indicates the influence of \(N_t\) on the field of temperature. Temperature distribution rises through the growing amount of \(N_t\). Figure 4b shows the increasing result of temperature profile for \(N_t\). Figure 5a exhibits the behaviour of the Prandtl number towards temperature distribution. The increase in the Prandtl number is the main reason for the slow rate of thermal diffusion; therefore, it has been found that the field of temperature declines by enlarging values of \(Pr\). Figure 5b describes the impact of the Lewis number on the temperature field. The profile of temperature first decreases and it later increases by enlarging the values of the Lewis number. Figure 6a
express the behaviour of the Lewis number on the nanoparticle concentration profile. The Lewis number is the ratio between thermal and mass diffusivity, and results show that the concentration profile declines and it inclines by enhancing the values of the Lewis number. Figure 6b shows the impact of $\gamma_b$ over $\frac{1}{2}C_f R_e^2$ against the buoyancy parameter $\lambda$. Therefore, skin friction coefficient has increasing levels of behaviour for these parameters. Figure 7 shows the impact of $\gamma_b$ over $\frac{N_r}{R_e^2}$ against the values of $\lambda$. Therefore, the Nusselt number increases in magnitude by increasing the values of the buoyancy parameter. Table 1 expresses the value of the Nusselt number for distinct characteristics $\gamma_b$, $L_e$, $Pr$, $N_b$ and $N_t$. The Nusselt number expands for $\gamma_b$ but decrease for $Le$, $N_b$ and $N_t$. Table 2 exhibits the values of the skin friction coefficient on distinct characteristics $\gamma_b$, $We$, $n$, $\lambda$, $\varphi_{\infty}$ and $N_r$. The skin friction coefficient expands by enlarging the amount of $\gamma_b$ and $\lambda$, but declines for $We$, $n$, $\varphi_{\infty}$ and $N_r$. Table 3 displays the skin friction coefficient’s numerical values for different values of $\lambda$ vs. $\gamma_b$. With an increase in these parameters, the skin friction coefficient inclines, whereas Table 4 expresses the Nusselt number’s numerical values for different values of $\lambda$ vs. $\gamma_b$. Therefore, by increasing the values of $\gamma_b$ and $\lambda$, the values of the Nusselt number also rise.

Figure 1. The configuration of the problem.

Figure 2. (a) Influence of $\gamma_b$ over $F_{\infty}$. (b) Effects of $N_b$ over $F_{\infty}$. 

(a)
Figure 2. (a) Influence of $\gamma_b$ over $F'$. (b) Effects of $N_{\text{f}}$ over $F'$.

Figure 3. (a) Impact of $We$ over $F'$. (b) Impact of $Le$ over $F'$.

Figure 4. (a) Impact of $N_{\text{f}}$ over $\theta$. (b) Impact of $N_{\text{r}}$ over $\theta$. 

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Figure 3. (a) Impact of $W_e$ over $F_{\infty}$. (b) Impact of $L_e$ over $F_{\infty}$.

Figure 4. (a) Impact of $N_\tau$ over $\theta$. (b) Impact of $N_b$ over $\theta$.

Figure 5. Cont.
Figure 5. (a) Impact of Pr over θ. (b) Impact of Pr over θ.

Figure 6. (a) Influence of Le over ψ. (b) Influence of γb over 1/2CfRe^1/2 against λ.
Figure 6. (a) Influence of $L$ over $\psi$. (b) Influence of $\gamma$ over $\phi$. $\psi$ against $\lambda$.

Figure 7. Influence of $\gamma$ over $\frac{N_u}{R^2}$ against $\lambda$.

Table 1. The effect of different parameters on the Nusselt number.

| $Pr$ | $Le$ | $N_t$ | $N_b$ | $\gamma_b$ | $\frac{N_u}{R^2} = -\theta'(0)$ |
|------|------|------|------|------------|----------------------------------|
| 1    | 1    | 1    | 1    | 1          | 0.7774                           |
| 2    | 1    | 1    | 1    | 1          | 1.3430                           |
| 3    | 1    | 1    | 1    | 1          | 1.9899                           |
| 1    | 2    | 1    | 1    | 1          | 0.7774                           |
| 2    | 2    | 1    | 1    | 1          | 0.7205                           |
| 3    | 2    | 1    | 1    | 1          | 0.6932                           |
| 1    | 3    | 1    | 1    | 1          | 0.7774                           |
| 2    | 3    | 1    | 1    | 1          | 0.4571                           |
| 3    | 3    | 1    | 1    | 1          | 0.2927                           |
| 1    | 1    | 2    | 1    | 1          | 0.7774                           |
| 2    | 2    | 2    | 1    | 1          | 0.3671                           |
| 3    | 3    | 2    | 1    | 1          | 0.1660                           |
| 1    | 1    | 3    | 1    | 1          | 0.7774                           |
| 2    | 2    | 3    | 1    | 1          | 0.8421                           |
| 3    | 3    | 3    | 1    | 1          | 0.9184                           |

Table 2. The impact of distinct characteristics on the coefficient of skin friction.

| $\lambda$ | $N_r$ | $\gamma_b$ | $W_e$ | $n$ | $\phi_{\infty}$ | $F'(0) + (n - 1)W_eF'(0)$ |
|------------|------|------------|------|----|-----------------|----------------------------|
| 1          | 1    | 1          | 0.2  | 2  | 0.05            | 2.1121                     |
| 2          | 1    | 1          | 0.2  | 2  | 0.05            | 1.7856                     |
| 3          | 1    | 1          | 0.2  | 3  | 0.05            | 1.6072                     |
| 1          | 1    | 2          | 0.2  | 2  | 0.05            | 2.3202                     |
| 2          | 1    | 2          | 0.2  | 2  | 0.05            | 2.6707                     |
| 3          | 1    | 2          | 0.2  | 3  | 0.05            | 3.0244                     |
| 1          | 1    | 3          | 0.2  | 2  | 0.05            | 2.6809                     |
| 2          | 1    | 3          | 0.2  | 3  | 0.05            | 2.6975                     |
| 3          | 1    | 3          | 0.2  | 4  | 0.05            | 3.6115                     |
| 1          | 1    | 1          | 0.2  | 2  | 0.05            | 3.3132                     |
| 2          | 1    | 1          | 0.2  | 3  | 0.05            | 3.3065                     |
| 3          | 1    | 1          | 0.2  | 4  | 0.05            | 3.3098                     |
Table 3. $\frac{1}{2}C_fR_e^2$ for $\lambda$ versus $\gamma_b$.

| $\gamma_b$ | $\lambda$ | 1  | 2  | 3  | 4  |
|------------|------------|----|----|----|----|
| 1          | 3.1322     | 3.92376 | 4.53894 | 5.51579 |
| 2          | 4.18662    | 4.71277 | 5.24436 | 5.7809  |
| 3          | 5.11698    | 5.58784 | 6.06337 | 6.54327 |
| 4          | 6.09744    | 6.52795 | 6.96226 | 7.40022 |

Table 4. $N_u/R_e^2$ for $\lambda$ versus $\gamma_b$.

| $\gamma_b$ | $\lambda$ | 1  | 2  | 3  | 4  |
|------------|------------|----|----|----|----|
| 1          | 0.777472   | 0.785111 | 0.79240 | 0.79939 |
| 2          | 0.842123   | 0.84633  | 0.85043 | 0.85443 |
| 3          | 0.918447   | 0.921069 | 0.92364 | 0.92618 |
| 4          | 0.996596   | 0.998363 | 1.00011 | 1.00184 |

6. Conclusions

The impact of significant parameters on mass and heat transport characteristics are examined. The analysis made in this article exhibits that the mass and heat transport rate was found to be improved in the flow of pseudoplastic non-Newtonian nanomaterial liquid. The pseudoplastic nanomaterial liquid is applicable in all electronic gadgets for increasing their cooling or heating rate. Furthermore, pseudoplastic nanomaterial liquids are also applicable in reducing the skin friction coefficient. The fundamental conclusions received from the above evaluation are indexed below.

1. The temperature distribution decreases through a rise in the amount of $Pr$.
2. The field of temperature inclines by increasing the values of $N_t$ and $N_b$.
3. The velocity field decreases by enhancing the values of $N_r$, $We$ and $\gamma_b$.
4. The temperature profile first decreases and it increases by enlarging the values of the Lewis number.
5. Profile of velocity increases by expanding the values of Lewis number.
6. The nanoparticle concentration distribution declines and it increases by increasing the values of $Le$.
7. The skin friction coefficient increases by expanding the amount of $\gamma_b$ and $\lambda$.
8. The heat transfer rate increases by enlarging the amount of $\gamma_b$ and $\lambda$.

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Nomenclature

c_p  Specific heat  \mu_o  Zero shear rate
F    Dimensionless velocity function  \mu_\infty  Infinity shear rate
P    Pressure  \eta  Dynamic viscosity
A_1  Rivlin Erickson tensor  \nu  Kinematic viscosity
\gamma  Curvature parameter  \lambda  Buoyancy parameter
\rho  Density  \theta  Dimensionless temperature function
\gamma  shear rate  T  Fluid temperature
Pr   Prandtl number  T_\infty  Ambient temperature
r_0  Radius of cylinder  N_l  Thermophoresis parameter
\phi  Concentration of nanoparticles  N_b  Brownian motion parameter
\Gamma  n, d  Fluid parameters  L_e  Lewis number
c_f  Skin friction coefficient  \psi  Dimensionless concentration function
Nu   Nusselt number  x, r  Coordinates
We   Weissenberg number  U_\infty  Free stream velocity

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