Rearrangement in folding potentials with density-dependent nucleon-nucleon interaction

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Abstract

We discuss optical potentials for the nuclear elastic scattering from a variational viewpoint. Density-dependence in the effective $N-N$ interaction leads to density rearrangement terms, in addition to the conventional folding term. Effects of the rearrangement on the $N-A$ optical potential are illustrated in the nuclear-matter limit. Closely relating to consistency with the saturation, the rearrangement appreciably improves the isoscalar optical potential depth over the previous folding model calculations. The rearrangement gives stronger effects as the density grows. We also present rearrangement terms in the $A-A$ double-folding potential. Since the rearrangement terms are relevant to the nuclear structure but should be handled within the reaction model, $N-N$ effective interactions applicable both to structure models and to the folding model will be desired for unambiguous description of the nuclear elastic scattering.

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The optical potential for elastic scattering is basic to describing various nuclear reactions \[1\]; the direct reactions including inelastic scattering, charge exchange and nucleon transfer reactions, which are treated in the distorted-wave Born approximation (DWBA), as well as the compound nuclear reactions. The folding model has successfully been applied to obtain the optical potentials, their real parts in particular, based on effective $N-N$ interactions $\[2\].$

In regard to the effective $N-N$ interaction, density-dependence is required to reproduce the saturation of nuclear densities and energies, as qualitatively disclosed by the Brueckner theory $\[3\]$. However, Brueckner’s $g$-matrix does not reproduce the nuclear saturation quantitatively (as long as we apply it with two-body interaction within the non-relativistic framework). In most nuclear structure problems we use phenomenological effective interactions in which density-dependence is contained so as to reproduce the saturation $\[4\]$. Density-dependence has also been introduced in the effective $N-N$ interaction for the folding model $\[5\]$. Efforts have been made to connect the folding-model interaction to the saturation $\[6, 7\]$. It seems hopeful to unify the folding model at relatively low incident energies and nuclear structure models such as the mean-field approximation $\[8, 9\]$. However, there exists inconsistency between them in treating the density-dependence. In this article we reconsider the folding model from a variational viewpoint, and show where the inconsistency comes from and how it should be resolved.

We here express an effective $N-N$ interaction in the following form:

$$\hat{v} = \sum_n C_n[\rho] \hat{w}_n,$$

where $\hat{w}_n$ is a two-body interaction operator, $C_n[\rho]$ its density-dependent coupling constant. As in Refs. $\[3, 6, 7\]$, the coupling constants are assumed to depend on the total density $\rho$. Although there could also be energy-dependence in the coupling constants, it does not play an essential role in the arguments below.

We first consider the $N-A$ scattering. The system has $(A+1)$ nucleons, with the total wave-function $|\Psi_{A+1}\rangle$ under the total Hamiltonian $\hat{H}_{A+1}$. We decompose $|\Psi_{A+1}\rangle$ as

$$|\Psi_{A+1}\rangle \approx |\Psi_A\rangle \otimes |\psi_N\rangle,$$

where $|\Psi_A\rangle$ is the wave-function of the target nucleus $A$, and $|\psi_N\rangle$ denotes the wave-function of the incident and scattered nucleon. In the folding model, we customarily assume $|\Psi_A\rangle$ to be the ground-state function $|\Psi_A^{(0)}\rangle$, which satisfies $\hat{H}_A|\Psi_A^{(0)}\rangle = E_A^{(0)}|\Psi_A^{(0)}\rangle$. This assumption is practically equivalent to the frozen density approximation. With the decomposition $\[2\]$, the Schrödinger equation for $\psi_N$ (or the Dirac equation in relativistic approaches, with appropriate subtraction of the mass) is derived by the variational equation,

$$\frac{\delta}{\delta \psi_N^*} \left[ \langle \Psi_{A+1}|(\hat{H}_{A+1} - E)|\Psi_{A+1}\rangle \right] = 0,$$
which can be written as

$$\hat{h}_N \psi_N = E_N \psi_N; \quad \hat{h}_N = \hat{K}_N + \hat{U}, \quad E_N = E - E_A,$$

(4)

where $E_A = \langle \Psi_A | \hat{H}_A | \Psi_A \rangle (\approx E_A^{(0)})$, $\hat{K}_N$ stands for the kinetic energy operator and $\hat{U}$ is regarded as the optical potential. In applying Eq. (2) to (3), one should notice that $\rho_{A+1}$ obtained from $|\Psi_{A+1}\rangle$ should be used in $C_n[\rho]$ of $\hat{H}_{A+1}$, while $\hat{H}_A$ contains $\rho_A$ which is calculated from $|\Psi_A^{(0)}\rangle$. Having certain contribution to $\hat{U}$, $\delta \rho = \rho_{A+1} - \rho_A = \psi_N^* \psi_N$ is not negligible though infinitesimal at each position. We then obtain

$$\hat{U} \psi_N = \sum_n \left\{ C_n[\rho_A] \cdot \langle \Psi_A | \hat{w}_n \left( |\Psi_A\rangle \otimes |\psi_N\rangle \right) + \left\langle \frac{\delta C_n}{\delta \rho} \delta \left( \frac{r_i + r_j}{2} - r_N \right) \hat{w}_n \right\rangle_A \cdot |\psi_N\rangle \right\}.$$  

(5)

Here $\langle O \rangle_A \equiv \langle \Psi_A | O | \Psi_A \rangle$, and $\delta \left( \frac{r_i + r_j}{2} - r_N \right)$ equates the center-of-mass position of the interacting nucleons in $A$ to the position of the scattered nucleon. The first term on the rhs in Eq. (5) is the usual folding potential. It is remarked that the density rearrangement term (DRT) comes out as the second term. While such rearrangement terms are known in structure models and in some reaction approaches (e.g. the one within the time-dependent Hartree-Fock approximation), the DRT has not been taken into account in the folding model. We call $\hat{U}$ in Eq. (5) renormalized folding potential (RFP), because of the correspondence to the renormalized Brueckner-Hartree-Fock approach [10] as discussed later. The RFP differs from the conventional folding potential because of the presence of the DRT.

To illustrate significance of the DRT, we consider the nucleon scattering on the uniform nuclear matter with equal numbers of protons and neutrons. This nuclear-matter folding potential corresponds to depth of the isoscalar optical potential in the $N$-$A$ scattering. As far as contribution of the dynamical polarization is negligibly small, the RFP is

$$U(k; k_F) = \sum_n \left\{ C_n[\rho] \cdot \frac{\Omega}{(2\pi)^3} \sum_{\sigma, \tau} \int_{|k| \leq k_F} d^3 k' \langle k\sigma \tau k' \sigma' \tau' | \hat{w}_n | k\sigma \tau k' \sigma' \tau' \rangle + \frac{1}{\Omega} \frac{\delta C_n[\rho]}{\delta \rho} \langle \hat{w}_n \rangle \right\},$$

(6)

where $k_F$ denotes the Fermi momentum, and $\Omega$ is the volume of the nuclear matter. We have $\rho = (2/3\pi^2)k_F^3$, as usual. Note that $U$ becomes independent of the spin and isospin component $\sigma$ and $\tau$ in the symmetric nuclear matter, as well as of direction of the momentum $k$. Obviously $U(k; k_F)$ is connected to the single-particle (s.p.) energy. The s.p. energy is defined by variation of the nuclear matter energy $E$ with respect to the occupation number $n_{k\sigma\tau}$, which is unity for $k \leq k_F$ and vanishes for $k > k_F$. The nuclear matter energy is expressed by

$$E = \frac{\Omega}{(2\pi)^3} \sum_{\sigma, \tau} \int d^3 k \frac{k^2}{2M} n_{k\sigma\tau}$$

$$+ \frac{1}{2} \sum_n C_n[\rho] \frac{\Omega^2}{(2\pi)^6} \sum_{\sigma, \tau} \int d^3 k d^3 k' \langle k\sigma \tau k' \sigma' \tau' | \hat{w}_n | k\sigma \tau k' \sigma' \tau' \rangle n_{k\sigma\tau} n_{k'\sigma'\tau'},$$

(7)
and therefore
\[ \varepsilon(k; k_F) = \frac{\delta E}{\delta n_{k_F}} = \frac{k^2}{2M} + U(k; k_F). \] (8)

This \( \varepsilon(k; k_F) \) is equivalent to \( E_N \) in Eq. (4) in the scattering problems, manifesting similarity of the folding model to the mean-field picture. Let us now recall the Hugenholtz-van Hove (HvH) theorem [11]. The saturation point is obtained as the minimum of \( \mathcal{E} \equiv E/A = E/\rho \Omega \).

Since the Fermi energy is given by \( \varepsilon_F \equiv \varepsilon(k_F; k_F) = \frac{1}{\Omega} \left( \frac{\partial E}{\partial \rho} \right) \), we obtain
\[ \varepsilon_F = E + \rho \frac{\partial E}{\partial \rho}. \] (9)

This yields \( \varepsilon_{F0} \equiv \varepsilon(k_{F0}; k_{F0}) = E_{\text{min}} \) at the saturation point, where \( k_{F0} \) represents the Fermi momentum at the saturation and \( E_{\text{min}} \) the saturation energy. We then have [12]
\[ U(k_{F0}; k_{F0}) = E_{\text{min}} - \frac{k_{F0}^2}{2M}. \] (10)

Since the saturation is directly linked to the variation of \( \mathcal{E} \), this relation is essential to consistency with the saturation. Furthermore, the effective mass at the saturation point, \( M_0^* \), is given by
\[ \frac{k_{F0}}{M_0^*} \equiv \left. \frac{\partial \varepsilon(k; k_F)}{\partial k} \right|_{k=k_{F0}} = \frac{k_{F0}}{M} + \left. \frac{\partial U(k; k_F)}{\partial k} \right|_{k=k_{F0}}, \] (11)
which derives
\[ \left. \frac{\partial U(k; k_{F0})}{\partial E_N} \right|_{k=k_{F0}} = 1 - \frac{M_0^*}{M}. \] (12)

Here \( E_N = \varepsilon(k; k_F) \) is related to \( k \) via Eq. (8). Note that, even without energy-dependent coupling constants, non-locality in the interaction gives rise to \( k \)-dependence of \( U \), leading to \( E_N \)-dependence. For the scattering problems, \( E_N \) must be positive. However, with the extrapolation \( k \to k_{F0} \) (i.e. \( E_N \to E_{\text{min}} \approx -16 \text{ MeV} \)), \( U(k; k_{F0}) \) is constrained by Eqs. (10,12), as long as it is continuous.

Figure 1 shows \( U(k; k_F) \) of Eq. (6) as a function of \( E_N \) at \( \rho = 0.16 \text{ fm}^{-3} \). The calculations are implemented with several interactions used in the mean-field approaches as well as with a density-dependent interaction developed for the folding model, BDM3Y1 [6]. In the mean-field interactions, a \( \rho \)-dependent coupling constant is introduced for a contact term of the interaction. All of their results meet almost at the same point by extrapolation to \( E_N \approx -16 \text{ MeV} \), fulfilling the HvH theorem. They also give slopes close to one another at \( E_N \approx -16 \text{ MeV} \), corresponding to \( M_0^* \approx 0.7M \) [9,13]. However, as \( E_N \) increases, \( U(k; k_F) \) behaves differently among various effective interactions. This is not surprising because these mean-field interactions are determined from the structural information, which is not sensitive to \( U(k; k_F) \) far from the saturation point. The Skyrme interaction [4,14] necessarily gives linear dependence on \( E_N \). While D1S [15] does not give reasonable \( E_N \)-dependence, D1 [16] and M3Y-P2 [9] have \( E_N \)-dependence similar to that of the empirical depth of the optical
potential. There is a class of effective interactions in which the M3Y interaction is multiplied by a \( \rho \)-dependent factor. We here treat BDM3Y1 as a representative of them. For BDM3Y1, both the results with and without the DRT are presented. We do not use the factor giving additional energy-dependence (denoted by \( g(E_N) \) in Ref. [6]). Although BDM3Y1 is adjusted so as to reproduce \( k_{F0} \) and \( E_{\text{min}} \), the folding potential calculated with BDM3Y1 in Ref. [6] is contradictory to Eq. (11), because the DRT is discarded. If the DRT is added, depth of the folding potential with BDM3Y1 is much closer to the empirical values, and is almost indistinguishable from that with M3Y-P2. Thus the RFP improves the optical potential depth over the previous folding model approach to a substantial extent, due to the presence of the DRT. Even the additional \( E_N \)-dependent factor seems unnecessary for BDM3Y1 at \( E_N \lesssim 200 \text{ MeV} \). This result demonstrates significance of the constraint (10).

In Fig. 2 \( \rho \)-dependence of \( U(k; k_F) \) is depicted for several \( E_N \). Because of the \( E_N \)-dependence presented in Fig. 1, the folding potential becomes less attractive, or even repulsive, as \( E_N \) increases. As viewed in the BDM3Y1 results, effects of the rearrangement are
the stronger for the higher $\rho$. Therefore, although the rearrangement will not seriously influence highly peripheral reactions, it affects the volume integral of the potential. It will be important to reassess reaction calculations by applying the RFP. Whereas BDM3Y1 without the DRT gives attractive $U(k; k_F)$ even at twice the saturation density, the RFP comes repulsive at high $\rho$ for relatively high $E_N$. This $\rho$-dependence is qualitatively similar to that with M3Y-P2. Another interesting point suggested by Fig. 2 will be that the DRT tends to produce or enhance the so-called wine-bottle-bottom shape in the optical potential at a certain energy region, which significantly influences the analyzing power \[19\].

Whereas we have assumed the interaction form of Eq. (1), similar arguments hold for the
microscopic $N-N$ interaction obtained in terms of the $g$-matrix \cite{10,12,20,21,22}. From the $g$-matrix viewpoints, the density-dependence in the effective interaction arises from the Pauli principle, which affects the Pauli exclusion operator and the self-energy in the energy denominator. In the $N-A$ scattering, the projectile shifts the energy of the target nucleus because of the Pauli blocking; the s.p. state occupied by the projectile is blocked in addition to those occupied by the nucleons in the target nucleus. This additional blocking leads to a DRT, which is equivalent to the DRT in Eq. (5). The density rearrangement in the $g$-matrix was discussed in Refs. \cite{12,20}. In terms of the hole-line expansion, the density rearrangement is primarily represented by the 2nd-order diagrams (rearrangement for the Pauli exclusion operator and for the energy denominator) shown in Fig. \ref{fig:goldstone} \cite{20}, while the conventional folding takes account only of the 1st-order diagram. In this regard the RFP is compared to the renormalized Brueckner-Hartree-Fock approach in Ref. \cite{10}. Relevance of the rearrangement to the HvH theorem was also pointed out in Ref. \cite{20}. However, its effects and importance in the optical potential have not been recognized sufficiently so far. The present results indicate that the DRT is not negligible. The DRT does not give all the 2nd order terms in the hole-line expansion. It seems that, rather than the order in the hole-line expansion, the constraint of Eq. (10) \cite{10} plays an important role.

We next turn to the $A-A$ scattering. Let us denote the target nucleus and the projectile by $A_1$ and $A_2$, respectively. In this case we have the following DRTs in the RFP,

\begin{equation}
U^{\text{rearr}} = \sum_n \left\{ \langle \left( C_n [\rho_{A_1} + \rho_{A_2}] - C_n [\rho_{A_1}] \right) \hat{w}_n \rangle_{A_1} + \langle \left( C_n [\rho_{A_1} + \rho_{A_2}] - C_n [\rho_{A_2}] \right) \hat{w}_n \rangle_{A_2} \right\}.
\end{equation}

(13)
The first term on the rhs in Eq. (13) is evaluated only from $|\Psi_{A_1}\rangle$, with displacement of the $\rho$-dependent coupling constant in the interaction of Eq. (10); *vice versa* for the second term. As the $A_2$ nucleus approaches $A_1$, the density changes, causing the energy shift through the $\rho$-dependence in the interaction. Note that, in the $\rho_{A_2} \to 0$ limit, the first term recovers the DRT in Eq. (10).

To discuss the density rearrangement in the $A-A$ scattering from the $g$-matrix viewpoints, we shall assume that the momentum is approximately a good quantum number in the s.p. states, for simplicity. If the $A_1$ nucleus is isolated, nucleons in $A_1$ occupy the s.p. levels of $k \leq k_{F,A_1}$, and this distribution gives rise to the Pauli blocking effects among the nucleons in $A_1$. When the $A_2$ nucleus is present nearby, the s.p. levels of $|k - K| \leq k_{F,A_2}$ are also blocked ($K$ is the relative momentum of $A_2$ to $A_1$). This additional blocking affects the $A_1$ energy, producing a DRT which corresponds to the first term in Eq. (13). We note that the Pauli blocking due to the two Fermi spheres in the $A-A$ collision was considered in Ref. [23], although relation to the folding model was not clarified. It is often a good approximation to replace the Pauli blocking effects by a function of the total density [2]. Under this approximation, we go back to the same arguments as those based on Eq. (10), which have yielded the DRTs in Eqs. (13).

The DRTs represent energy shift of the nuclei during the scattering process. They should be handled within the reaction model, because they affect the nuclear properties dynamically. Now the following question is raised: should the interaction $\hat{v}$ in Eq. (13) be the interaction in the structure model or that in the reaction model? Conventionally, separability of nuclear reaction problems from nuclear structure has been postulated, and different effective interactions have been used between the nuclear reaction theory and the nuclear structure theory. However, there are many cases in which reaction problems strongly couple to nuclear structure. Unified description using consistent interactions is desirable for total understanding of the phenomena. The above question suggests that the DRTs are located at an intersection of the structure theory and the reaction theory, exemplifying importance of unified description. To avoid ambiguity, consistent effective interactions should be applied to structure and reaction calculations. If we restrict ourselves to reactions at relatively low energies, it might be possible to develop an interaction applicable both to structure and reaction calculations. Analysis like that in Fig. 1 may be useful for first selection of effective interactions for that purpose.

In low-energy nuclear reactions, $\rho$-dependence of the effective interaction is important, and could somewhat be correlated with the saturation. In contrast, $\rho$-dependence is not so significant in high-energy reactions. If the incident energy is sufficiently high, the impulse approximation works well. In such energy regime, the phenomenological effective interactions employed in Figs. 1 and 2 will lose their validity. Furthermore, effects of the density rearrangement are expected to be weak. It will be interesting to estimate quantitatively by the $g$-matrix approach how large the DRTs are at varying $E_N$.

Many-body forces have similar effects to the density-dependent two-body interactions [4].
When a many-body force is introduced instead of the density-dependent interaction, we have DRTs again. For instance, in the $N$-$A$ scattering with a three-body force, the interaction among the projectile and two nucleons in the target nucleus gives rise to energy shift of the target nucleus in an effective manner, which produces a DRT as in Eq. (5).

Effective interactions for inelastic scattering will also be subject to the density rearrangement effects. Because the density-derivative of the optical potential is relevant to the inelastic scattering to collective states within the DWBA framework, terms up to $\delta^2 C_n[\rho]/\delta^2 \rho$ come into the issue of the $N$-$A$ process. This coincides the residual interaction in the random-phase approximation. Details will be discussed in a future publication.

The folding model using density-dependent interactions has been successful in describing nuclear reactions to a certain degree, without the DRTs. There could be several reasons for it. First, some reactions may be insensitive to the rearrangement effects, because, e.g., they are highly peripheral. Second, some of the density-dependent interactions that have been used in the folding model were fitted to the reaction data. This may mask some rearrangement effects. Furthermore, the dynamical polarization effects should be treated properly, for full understanding of the optical potential. Still it will be important to reinvestigate individual reactions by taking the DRTs into account. As stated already, this may open a way to unified description of the nuclear elastic scattering and the nuclear structure.

In summary, we have discussed the $N$-$A$ optical potential based on a variational equation. Density-dependence in the effective $N$-$N$ interaction gives a density rearrangement term, which has been discarded in the conventional folding-model calculations. We call the optical potential with the rearrangement term renormalized folding potential. If the rearrangement term is taken into account, depth of the $N$-$A$ folding potential is in better agreement with the experimental data than in the conventional folding model approach. We emphasize significance of a constraint due to the Hugenholtz-van Hove theorem, which assures consistency with the saturation. Effects of the rearrangement become stronger as the density increases. Rearrangement terms in the $A$-$A$ scattering are also discussed. The rearrangement terms seem to be located at an intersection of the nuclear structure models and the folding model. It is desired to explore effective $N$-$N$ interactions applicable to both of these models in a consistent manner, for unambiguous description of the low-energy nuclear elastic scattering.

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