Precision Tests of the Standard Model at LEP

Barbara Mele

I.N.F.N., Sezione di Roma, Italy and
Dipartimento di Fisica, Università “La Sapienza”,
P.le Aldo Moro 2, I-00185 Rome, Italy

Abstract

Recent LEP results on electroweak precision measurements are reviewed. Line-shape and asymmetries analysis on the Z⁰ peak is described. Then, the consistency of the Standard Model predictions with experimental data and consequent limits on the top mass are discussed. Finally, the possibility of extracting information and constrains on new theoretical models from present data is examined.

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1 e-mail address: MELE@ROMA1.INFN.IT
1. Introduction

Four years of LEP operation, from the machine starting in 1989, have yielded an impressive amount of new data describing physics at the $Z^0$ peak. In the years '89-'92, the four LEP experiments (ALEPH, DELPHI, L3 and OPAL) have collected about 5 million of $Z^0$ events which have already been analysed [1]. These data have allowed up to now the measurements of many observables in the Electro-Weak (EW) sector of the Standard Model to a much higher precision than before the LEP era. The running of the machine during two further years will yield by the end of '94 about four times the present statistics, and will improve even more the present achievements.

In my talk, I would like to review the beautiful LEP results in the EW sector of the Standard Model, concentrating on precision tests of the SU(2)×U(1) theory at $\sqrt{s} = M_Z$. I will describe the main strategies to extract precision measurements from LEP experimental data, through the $Z^0$ line-shape analysis and asymmetries determination on the $Z^0$ peak. Then, I will discuss the experimental consistency of the Standard Model and the present status of precision tests of the purely EW sector of the theory. Limits on the top mass will be discussed as well. Finally, I will briefly report on the results of model independent analysis of the precision EW data, and consequent constraints that can be put on possible extensions of the Standard Model starting from present LEP data. My discussion will be based mostly on LEP results recently presented at the Europhysics International Conference of Marseilles [1, 2] and at the XVI International Symposium on Lepton-Photon Interactions at Cornell University, N.Y. [3, 4].

2. Experimental analysis

Due to the $Z^0$ peak, one observes at LEP a bump in the cross section at $\sqrt{s} \approx M_Z$ with a peak value of about 35 nb for visible (that is hadronic and charged-lepton) events. For instance, the hadronic cross section is about 750 times what one would expect by extrapolating the low-energy purely photonic contribution up to $\sqrt{s} \approx M_Z$. The corresponding visible-event rate is about $3.5 \cdot 10^6$ for an effective year of running of $10^7$s, with a luminosity of $10^{31}\text{cm}^{-2}\text{s}^{-1}$ (that is about twice the present typical LEP luminosity). With such a high statistics, LEP gives a unique opportunity to test the fine structure of the Standard Model at the $M_Z$ energy scale.
In order to fully exploit the wealth of data for precision measurements, one has to keep under control systematic errors in the experimental analysis. To this end, two basic issues are the luminosity monitoring and the beam energy calibration of the machine. The LEP luminosity $\mathcal{L}$ is monitored through the measurements of Bhabha $e^+e^−\rightarrow e^+e^−$ scattering. Hence, the total cross section for production of the fermion pair $\bar{f}f$ is obtained by the expression

$$\sigma_{\bar{f}f} = \frac{N_{\bar{f}f}}{\varepsilon_{\bar{f}f}} \cdot \frac{1}{\int \mathcal{L} dt} = \frac{N_{\bar{f}f}}{\varepsilon_{\bar{f}f}} \cdot \frac{\varepsilon_{\text{Bhabha}}}{N_{\text{Bhabha}}} \cdot \sigma_{\text{Bhabha}}^{\text{theory}} \quad (2.1)$$

where $N$ is the observed number of events and $\varepsilon$ is a factor including acceptance and efficiency effects. The main limitation in the luminosity measurement comes from the theoretical error in the prediction of the Bhabha cross section $\sigma_{\text{Bhabha}}^{\text{theory}}$. As a consequence, $\mathcal{L}$ is presently determined with a relative error of about 1%, whose main effect reflects in limiting the accuracy of the peak hadronic cross section $\sigma_h^0$ to about 0.3%[2]. On the other hand, a very good beam energy calibration is now reached through the resonant spin depolarization method, that exploits the transverse polarization of the initial beams.

By the way, it is interesting to recall that, in depolarization measurements, some energy spread has been observed which is correlated with tidal effects, that deform the earth’s surface. As a consequence, due to the variation of the collider circumference by a few $10^{-8}$, a periodic change (of amplitude about 9.6 MeV) is observed in the beam energy measured by resonant depolarization [4]. Such a good energy calibration has allowed to get systematic errors on the $Z^0$ mass $M_Z$ and width $\Gamma_Z$ of only 6.3 MeV and 4.5 MeV, respectively.

The precision determination of various EW observables at LEP is obtained by elaborating two main kinds of primary measurements: 1) cross sections and 2) asymmetries. In what follows, I will describe the main strategies to measure these quantities.

### 2.1. Cross sections

By energy scanning around the $Z^0$ peak (from about $\sqrt{s} = 88\text{GeV}$ up to about 94 GeV), one measures (through eq.(2.1)) the cross section $\sigma_{\bar{f}f}(s)$, for production of the $f$ fermion pair, versus the c.m. energy. A Breit-Wigner resonant shape around $M_z$ is obtained. There are four different main cases corresponding to the three charged-lepton ($f = e, \mu, \tau$) and to hadron ($f = \sum q_i$) production. In order to derive from
the measured cross sections a measurement of relevant quantities (\(Z^0\) mass, total and partial \(Z^0\) widths and peak cross sections), one has first to subtract the effect of initial-state photon radiation [4]. This is a rather large effect, that causes a reduction of about 25% in the peak cross section. It can be accounted for through a radiator function \(G(z, s)\) that can be deconvoluted from the measured \(\sigma_{\bar{f}f}(s)\)

\[
\sigma_{\bar{f}f}(s) = \int dz \ G(z, s) \hat{\sigma}_{\bar{f}f}(zs)
\]

where \(z\) is the fraction of the c.m. energy squared \(s\) left after the photon radiation to the collision, and \(\hat{\sigma}_{\bar{f}f}(s)\) is defined by eq.(2.2) as a reduced cross section deconvoluted from initial radiation effects. \(G(z, s)\) can be theoretically predicted to a good accuracy through renormalization-group methods, that resums the effect of large terms of the order \(\alpha \pi \log(M^2_{Z}/m^2_{\text{light}})\).

At this point, \(M_Z\) and \(\Gamma_Z\) are defined through the expression

\[
\hat{\sigma}_{\bar{f}f}(s) = \sigma^o_{\bar{f}f} \cdot \frac{s\Gamma^2_{Z}}{(s - M^2_{Z})^2 + (s^2\Gamma^2_{Z})/M^2_{Z}} + (\gamma \text{exchange} + \text{Inter's})
\]

where the last term in brackets is small and takes into account the photon exchange contribution and its interference with the \(Z^0\) amplitude. It is theoretically evaluated and subtracted from \(\hat{\sigma}_{\bar{f}f}(s)\), in order to isolate the Breit-Wigner \(Z^0\) contribution. The peak cross section \(\sigma^o_{\bar{f}f}\) is connected to the \(Z^0\) partial widths for \(Z \rightarrow ee\) and \(Z \rightarrow f\bar{f}\), \(\Gamma_e\) and \(\Gamma_f\), through the expression

\[
\sigma^o_{\bar{f}f} = \frac{12\pi \Gamma_e \Gamma_f}{M^2_{Z} \Gamma^2_{Z}}
\]

(2.4)

Of particular relevance for the LEP data analysis are the peak hadronic and leptonic cross sections. According to eq.(2.4), the former is directly related to the \(Z^0\) hadronic width \(\Gamma_{\text{had}}\) by

\[
\sigma^o_{\text{had}} = \frac{12\pi \Gamma_{\text{had}}}{M^2_{Z} \Gamma^2_{Z}}
\]

(2.5)

The latter is replaced by the ratio of the hadronic and leptonic \(Z^0\) widths

\[
R_\ell \equiv \frac{\sigma^o_{\text{had}}}{\sigma^o_{\ell \ell}} = \frac{\Gamma_{\text{had}}}{\Gamma_{\ell}}
\]

(2.6)

The use of the ratio’s \(R_\ell\), with \(\ell = e, \mu, \tau\), parametrizes the \(Z^0\) leptonic couplings, avoiding the systematic uncertainties connected to the measurement of cross sections.
Table 1: Primary measurements (apart from asymmetries).

| Parameter                  | Value               |
|----------------------------|---------------------|
| $M_Z$ (GeV)                | 91.187 ± 0.007      |
| $\Gamma_Z$ (GeV)           | 2.489 ± 0.007       |
| $\sigma^0_{\text{had}}$ (nb) | 41.55 ± 0.14     |
| $R_\ell = \Gamma_{\text{had}}/\Gamma_\ell$ | 20.77 ± 0.05 |
| $R_b = \Gamma_b/\Gamma_{\text{had}}$ | 0.2191 ± 0.0027 |

In the Standard Model $\Gamma_\ell$ can be expressed in terms of the vector and axial-vector lepton coupling constants, $g_{V\ell}$ and $g_{A\ell}$, by

$$\Gamma_\ell = \frac{G_F M_Z^3}{6\pi\sqrt{2}} (g_{V\ell}^2 + g_{A\ell}^2) \left(1 + \frac{3}{4\pi} \alpha\pi\right)$$

(2.7)

where the last term in brackets takes into account electromagnetic corrections.

With the above definitions, hadronic and leptonic line-shape data are analyzed in a model-independent way (separately by each of the four LEP experiments), assuming $M_Z$, $\Gamma_Z$, $\sigma^0_{\text{had}}$, $R_\ell$ and the asymmetries $A^\ell_{FB}$ (defined in the next section) as the set of most independent parameters. A combined fit to the line-shapes and $A_{FB}$'s is made, based on a $\chi^2$-minimization that takes into account the full covariant error matrix of the data, and the experimental and theoretical correlations between different channels. About leptonic data, two different assumptions are made. Assuming lepton universality, one has $R_\ell = R_\mu = R_\tau = R_\ell$ and $A^e_{FB} = A^\mu_{FB} = A^\tau_{FB} = A^\ell_{FB}$ and fits the data to the 5-parameters $M_Z, \Gamma_Z, \sigma^0_{\text{had}}, R_\ell$ and $A^\ell_{FB}$. Releasing this assumption, one makes a 9-parameter fits on $M_Z, \Gamma_Z, \sigma^0_{\text{had}}, R_\ell, R_\mu, R_\tau, A^e_{FB}, A^\mu_{FB}, A^\tau_{FB}$. In Table 1 [2], the values obtained from a 5-parameter fit, after combining the results from the four LEP experiments, are presented. One can remark the impressive precision obtained on $M_Z$. Its relative error is about $8 \cdot 10^{-5}$ and is going to improve even further after the '93 data. The improvement in energy calibration will lower the present systematic error of ±0.006 down to ±0.0025. For $\Gamma_Z$ the energy-scale error will go from ±0.0045 down to ±0.002, allowing an accuracy better than the present 0.3%. The main systematics for $\sigma^0_{\text{had}}$ comes instead from the luminosity monitoring and, therefore, from theoretical uncertainties on $\sigma^{\text{theory}}_{\text{Bhabha}}$.

From the measurement of the primary quantities $M_Z, \Gamma_Z, \sigma^0_{\text{had}}$ and $R_\ell$, one can straightforwardly obtain a measure of some derived quantities, that are the $Z^0$ leptonic...
and hadronic widths, $\Gamma_\ell$ and $\Gamma_{\text{had}}$, and the $Z^0$ invisible width $\Gamma_{\text{INV}}$. The last is directly connected with the number of light neutrino species. From eqs. (2.3) and (2.4) (assuming from lepton universality $\Gamma_e = \Gamma_\ell$), starting from the measured $M_Z, \Gamma_Z, \sigma^0_{\text{had}}$, and $R_\ell$, the combined four LEP experiment results are [2]

$$
\Gamma_\ell = (83.79 \pm 0.28) \text{MeV} \quad \Gamma_{\text{had}} = (1740 \pm 6) \text{MeV}
$$

(2.8)

The invisible $Z^0$ width is defined as the difference between the total $Z^0$ width and the sum of all the $Z^0$ visible decay widths

$$
\Gamma_{\text{INV}} = \Gamma_Z - \Gamma_{\text{had}} - 3\Gamma_\ell
$$

(2.9)

Assuming that only Standard Model $\nu$’s contribute to $\Gamma_{\text{INV}}$, the value of $\Gamma_{\text{INV}}$ is proportional to $N_\nu$, the number of neutrino’s lighter than $M_Z/2$. Actually, the Standard Model gives a cleaner prediction for the ratio $\Gamma_{\text{INV}}/\Gamma_\ell$, since EW corrections in each single width, that are dependent on the unknown $m_t$ and $m_H$, largely cancel in the ratio. From the Standard Model, one has [3]

$$
\frac{\Gamma_{\text{INV}}}{\Gamma_\ell} = (1.994 \pm 0.003) N_\nu
$$

(2.10)

On the other hand, the above ratio can be experimentally determined from primary measurements through

$$
\frac{\Gamma_{\text{INV}}}{\Gamma_\ell} = \sqrt{\frac{12\pi R_\ell}{M_Z^2 \sigma^0_{\text{had}}} - R_\ell - 3}
$$

(2.11)

where eqs. (2.5) and (2.6) have been used in eq. (2.9). Assuming a central value for the coefficient in eq. (2.10), one gets, combining the four-experiment results

$$
N_\nu = 2.980 \pm 0.027
$$

(2.12)

LEP confirms with a remarkable (and unprecedented) accuracy the minimal Standard Model prediction of 3 families of light $\nu$’s.

A derived quantity of different nature from EW data is the measurement of the QCD strong coupling constant at the $M_Z$ scale, $\alpha_s(M_Z)$. Since QCD corrections affect considerably $\Gamma_{\text{had}}$, the values of both $R_\ell$ and $\Gamma_Z$ are sensitive to $\alpha_s(M_Z)$. $R_\ell$ is the most sensitive variable. Its dependence on $\alpha_s(M_Z)$ has been calculated up to the third order in the perturbative expansion

$$
R_\ell = R_0^\ell \left\{ 1 + 1.05 \left( \frac{\alpha_s(M_Z)}{\pi} \right) + (0.9 \pm 0.1) \left( \frac{\alpha_s(M_Z)}{\pi} \right)^2 - 13 \left( \frac{\alpha_s(M_Z)}{\pi} \right)^3 \right\}
$$

(2.13)
From the $R_t$ measurement alone, one gets a value

$$\alpha_s(M_Z) = 0.123 \pm 0.008 \quad (2.14)$$

while a combined fit of all EW observables to $\alpha_s(M_Z)$ gives (cf. section 3)

$$\alpha_s(M_Z) = 0.120 \pm 0.007 \quad (2.15)$$

Comparing this determination with its accuracy to the results of hadronic-event-shape analysis ($\alpha_s(M_Z) = 0.123 \pm 0.006$), one finds perfect agreement and gets a nice consistency check of the SU(3)$\times$SU(2)$\times$U(1) theory. The accuracy in eq.(2.15) is essentially limited by our ignorance of $m_t$ and $m_H$, which enter the EW corrections that must be subtracted before fixing QCD corrections.

Recently, the implementation of a micro-vertex detector for $b$-tagging at LEP, has allowed a rather accurate measurement of the total cross section for $Z \to \bar{b}b$ at $\sqrt{s} = M_Z$, and in particular of the ratio of the $b$ over the hadronic $Z^0$ width

$$R_b \equiv \frac{\sigma_{\bar{b}b}^0}{\sigma_{\text{had}}^0} = \frac{\Gamma_b}{\Gamma_{\text{had}}} \quad (2.16)$$

This ratio has the advantage of being rather sensitive to top-quark vertex corrections ($t$ enters the 1-loop $Z \to \bar{b}b$ diagrams without Cabibbo-Kobayashi-Maskawa suppression) and, at the same time, less sensitive than the individual $\Gamma_b$ and $\Gamma_{\text{had}}$ to the exact value of $\alpha_s(M_Z)$ that heavily affects all the hadronic widths through QCD radiative corrections. In the Standard Model, one has

$$R_b \simeq R_d \left\{ 1 - \frac{20}{13} \frac{\alpha}{\pi} \left( \frac{m_t^2}{M_Z^2} + \frac{13}{6} \log \left( \frac{m_t^2}{M_Z^2} \right) \right) \right\} \quad (2.17)$$

where $R_d$ is the analogous of $R_b$ for $d$ quarks (for which top-loop effects are negligible). If $m_t = 150\text{GeV}$, eq.(2.17) gives a 2% effect. Therefore, if $R_b$ is measured with an accuracy better than about 1%, one can constrain $m_t$. Contrary to other EW observables at LEP, this effect is rather clean, since it is almost independent on $m_H$ and other $Z^0$ propagator effects. The measured value of $R_b$ (reported in Table 1) puts an upper limit of 221 GeV on $m_t$ at 95% of confidence level (using also the present Tevatron lower limit on $m_t$ of 113 GeV)\[2\]. This strategy for limiting the top mass is thus becoming competitive with the results of global fits of LEP data to the Standard Model (cf. section 3).
2.2. Asymmetries

There are different kinds of asymmetries measured at LEP\[6\]: forward-backward asymmetries for charged fermions (leptons and heavy quarks), \(A_{FB}^f\), \(t\)-polarization asymmetries, \(P_t\), \(t\)-polarization forward-backward asymmetries, \(P_{FB}^t\), charge forward-backward asymmetries, \(Q_{FB}\). The main goal of determining these quantities is an accurate measurement of the ratio of vector and axial \(Z^0\) couplings to fermions and, as a consequence, of the value of \(\sin^2\theta_{\text{eff}}\). This quantity is a basic one in order to check the radiative-correction pattern of the Standard Model. In practice, all these asymmetries can be expressed as functions of the quantities \(A_f\)'s, that express the unbalance in the left- and right-handed fermion couplings to the \(Z^0\)

\[
A_f \equiv \frac{g_{L}^2(f) - g_{R}^2(f)}{g_{L}^2(f) + g_{R}^2(f)} = \frac{2g_{V}g_{A}}{g_{V}^2 + g_{A}^2} = \mathcal{F}\left(\frac{g_{V}}{g_{A}}\right)
\]  

(2.18)

where \(f = e, \mu, \tau, c, b\) for the experimentally interesting cases, and \(g_V\) and \(g_A\) enters the \(Z f \bar{f}\) coupling \(\gamma^\mu(g_{V} - g_A\gamma_5)\). In the following, whenever \(g_V\) and \(g_A\) are reported without a suffix \(f\), they refer to the leptonic couplings (assuming lepton universality for \(e, \mu\) and \(\tau\)). In fact, leptonic asymmetries are the easiest to determine experimentally due to the cleaner reconstruction of leptonic final states.

In the Standard Model the vector and axial-vector couplings for leptons at the \(Z^0\) peak can be parametrized in the following way

\[
g_A(M_Z) = -\frac{1}{2}\sqrt{\rho_{eff}}
\]

(2.19)

\[
g_V(M_Z) = g_A(M_Z)(1 - 4\sin^2\theta_{w}^{eff})
\]

(2.20)

By measuring the ratio \(g_V/g_A\) from various asymmetries, one determines \(\sin^2\theta_{w}^{eff}\) through eq.(2.20)

\[
\sin^2\theta_{w}^{eff} = \frac{1}{4}(1 - \frac{g_V}{g_A})
\]

(2.21)

Eqs.(2.19) and (2.20) trade \(g_V\) and \(g_A\) for \(\sin^2\theta_{w}^{eff}\) and \(\rho^{eff}\). In the Standard Model, at tree level, their respective expressions are

\[
\rho^{eff}(\text{tree}) = 1 \quad \sin^2\theta_{w}^{eff}(\text{tree}) = \frac{e}{g}
\]

(2.22)

(\(e\) is the electric charge and \(g\) the \(SU(2)\) weak charge), but they acquire computable radiative corrections depending, at 1-loop, linearly on \(m_t^2\) and logarithmically on \(m_H\) \[6\]. In order to check the Standard Model predictions for these corrections, one should
know the values of $m_t$ and $m_H$. For the moment, we have a lower limit on them. From top search at Tevatron, one gets $m_t > 113\text{GeV}$ (95% of confidence level). Higgs direct search at LEP yields a limit $m_H > 63.5\text{GeV}$ (95% of confidence level)\footnote{8}

On the other hand, we can see that LEP limits on possible deviations from the Standard Model in the values of different observables give to $m_t$ an upper limit (cf. previous section and section 3).

The forward-backward asymmetry, $A_{FB}^f$ is defined through the angular distribution observed for the fermion $f$ by the expression

$$\frac{d\sigma}{d\cos\theta} = c\{1 + \cos^2\theta + \frac{8}{3}A_{FB}^f\cos\theta\} \quad (2.23)$$

where $c$ is a normalization constant. For $f = \ell$, $\theta$ is the angle between the initial $e^-$ and the final negative lepton $\ell^-$. $A_{FB}^\ell$ takes into account the angular asymmetry coming from the parity-violating $Z^0$ coupling. At the $Z^0$ peak, it can be expressed through the $A_f$’s by

$$A_{FB}^\ell = \frac{3}{4}A_eA_\ell \quad (2.24)$$

where $A_e$ and $A_\ell$ are defined by eq.(2.18). For leptons, $g_V/g_A$ is rather small and the lepton forward-backward asymmetry can be well approximated by

$$A_{FB}^\ell = \frac{3}{4}A_eA_\ell \simeq 3\left(\frac{g_V}{g_A}\right)^2 \quad (2.25)$$

Hence, also $A_{FB}^\ell$ is small. The quark forward-backward asymmetries are more attractive, since in the Standard Model $A_b, A_c > A_\ell$. However, hadronization and flavor-identification problems make the measurement of $A_{FB}^b$ and $A_{FB}^c$ much more delicate.

Combining eq.(2.7) and eq.(2.25), one can determine from a measure of $\Gamma_\ell$ (i.e. of $g_V^2 + g_A^2$) and $A_{FB}^\ell$ (i.e. $g_V/g_A$) the couplings $g_V$ and $g_A$ separately. Furthermore, by isolating informations relative to different leptonic species, one can test the compatibility of results for $e, \mu$ and $\tau$ leptons, that is lepton universality. Presently, $g_{V\ell}$ and $g_{A\ell}$ universality is well verified at LEP (within a 1$\sigma$ accuracy)\footnote{11}.

At LEP, two methods are used for measuring $A_{FB}^f$. One is by fitting eq.(2.23) to the observed experimental distribution. The other is by counting the number of events in the forward and backward hemisphere

$$A_{FB}^f = \frac{N_F - N_B}{N_F + N_B} \quad (2.26)$$
The final determination of $A_{FB}$ is given after a deconvolution of initial-state QED radiation effects (in a way analogous to eq. (2.2)) and other QED effects. Although the $A_{FB}^\ell$’s have been measured in a wide range of $\sqrt{s}$ around the $Z^0$ peak, in what follows we will concentrate on their values at $\sqrt{s} = M_Z$.

The four-experiment combined result for $A_{FB}^\ell$ is reported in Table 2 [2]. In spite of the smallness of $A_{FB}^\ell$, this quantity has been determined with an error better than 12%.

In principle, a much easier way to measure $A_{FB}^\ell$ is provided by $\tau$-polarization asymmetries, which depend linearly (and not quadratically as $A_{FB}^\ell$) on the small parameter $A_{FB}^\ell$. The $\tau$-polarization asymmetry $P_{\tau}$ is defined as

$$P_{\tau} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$$  \hspace{1cm} (2.27)

where $\sigma_{R(L)}$ is the cross section for the production of right(left)-handed $\tau^-$. Averaging over all production angles, $P_{\tau}$ is given by

$$P_{\tau} = -A_{\tau}$$  \hspace{1cm} (2.28)

$P_{\tau}$ is measured by fitting momentum distributions of $\tau$-decay products, which reflect their $P_{\tau}$-dependent angular distributions in the $\tau$ rest frame. The four-experiment combined result for $A_{\tau}$ is shown in Table 2.

Studying the angular dependence of $P_{\tau}$ gives further informations on $A_e$. In fact, one has

$$P_{\tau}(\cos \theta) = -\frac{A_{\tau}(1 + \cos^2 \theta) + 2A_e \cos \theta}{(1 + \cos^2 \theta) + 2A_e A_{\tau} \cos \theta}$$  \hspace{1cm} (2.29)
Therefore, the forward-backward $P_{\tau}$ asymmetry, $P_{\tau}^{FB}$ is given by

$$P_{\tau}^{FB} = -\frac{3}{4} A_e$$

(2.30)

The advantage of a measurement of $P_{\tau}^{FB}$ with respect to $P_{\tau}$ is that systematic errors from false $\tau$-polarization effects cancel in the forward-backward subtraction. The final result for $A_e$ from $P_{\tau}^{FB}$ is reported in Table 2. Although $P_{\tau}^{FB}$ is presently affected by a rather large error, its accuracy is expected to improve in the next future.

As is clear from eqs. (2.28) and (2.30) compared to eq. (2.25), $\tau$ polarization has, with respect to $A_{FB}^f$, also the advantage of providing a measurement of the relative sign of $g_V$ and $g_A$ and of overcoming correlations between $e, \mu$ and $\tau$ variables. In this way, one can check lepton universality [1].

Another direct and efficient determination of $A_e$ comes from the measurement of the left-right polarization asymmetry $A_{LR}$ at SLAC by the SLD experiment [10] (see Table 2). This is based on the longitudinal polarization of the initial $e^-$ beam. The accuracy of this result is presently penalized by the low statistics.

At the end of Table 2, also the average $A_\ell$, derived, assuming lepton universality, by the LEP $A_\tau$ and $A_e$ measurements and the SLD $A_{LR}$ determination is reported [2].

As for heavy $b$- and $c$-quark, the measurement of forward-backward asymmetries gives $g_{Vb}/g_{Ab}$ and $g_{Vc}/g_{Ac}$. The main experimental problem here comes from the difficulty of tagging the heavy quark in the hadronic background. The LEP outcome for $A_{FB}^b$ and $A_{FB}^c$ is shown in Table 2.

In Table 3, a summary of different determinations of $\sin^2 \theta_{w}^{eff}$ comings from various asymmetry measurements is shown [4]. One can note the beautiful agreement among all the determinations that leads to a final error of only 0.0006 on $\sin^2 \theta_{w}^{eff}$.

In Table 3, also the information coming from charge asymmetry in $Z^0$ decays in all hadronic states is reported. $Q_{FB}$ is a measurement of the forward-backward asymmetry in the charge flow in hadronic events. Indeed, the large value of $A_{FB}^f$ for quarks imply a non-zero average charge produced in the forward and backward hemispheres. This is given, summing over the five quark flavours, by

$$Q_{FB} = \sum_f 2q_f A_{FB}^f \frac{\Gamma_f}{\Gamma_{had}}$$

(2.31)

However, problems connected with hadronic event reconstruction makes this measurement not as solid as the leptonic ones.
Table 3: Different determinations of $\sin^2 \theta_{\text{eff}}$.

|       | $\sin^2 \theta_{\text{eff}}$ |
|-------|-------------------------------|
| $A_{FB}^e$ | $0.2316 \pm 0.0012$ |
| $P_\tau$ | $0.2327 \pm 0.0018$ |
| $P_{FB}^\tau$ | $0.2338 \pm 0.0031$ |
| $A_{FB}^b$ | $0.2322 \pm 0.0011$ |
| $A_{FB}^c$ | $0.2313 \pm 0.0036$ |
| $Q_{FB}$ | $0.2320 \pm 0.0016$ |
| $A_{LR}$ | $0.2378 \pm 0.0056$ |
| average | $0.2322 \pm 0.0006$ |

3. Global fits to the Standard Model

By considering the bulk of LEP results versus the time, one finds a constant progress in accuracy, joined to a continuous convergence towards the Standard Model predictions. For instance, previous little discrepancies with the Standard Model in the $Z^0$ leptonic width value has faded away in the last months.

A true check of consistency for the Standard Model predictions through the LEP EW data is made complicated by the presence of two unknown parameters, the top and the Higgs masses, that affect EW corrections to different observables. At 1-loop level, EW corrections can be classified in three main classes [6]:

- **Oblique** or vacuum polarization corrections of vector bosons, whose main effect is the running of the electromagnetic coupling constant $\alpha$ from the low scale value ($\alpha = 1/137$) up to
  \[ \alpha(M_Z) = (128.87 \pm 0.12)^{-1} \]  
  They include also smaller (and calculable) effects due to top and higgs loops.

- Vertex corrections, that are small and in general uninteresting. One exception is the $Zb\bar{b}$ vertex, which is sensitive to $m_t$.

- Box corrections, that are always very small and negligible.

As a result of the dependence of radiative corrections on the unknown $m_t$ and $m_H$, one single measurement is not enough to test the theoretical model, but several
and complementary (i.e. with a different sensibility to \( m_t \) and \( m_H \)) measurements are necessary.

In practice, what one does is to use as inputs the parameters that are measured with the best accuracy, that is \( \alpha, G_F \) and \( M_Z \). Then, the Standard Model predictions (including all the presently available radiative corrections) for the set of observables that are measured at LEP are computed, keeping \( m_t \) and \( m_H \) as free parameters. Finally, the consistency with the experimental values of these observables is checked, and ranges of \( m_t \) and \( m_H \) that give the best fit to them is computed. By the way, we will see that the present accuracy in LEP data does not yet allow to constrain \( m_H \). Indeed, while EW radiative corrections depend on \( m_t^2 \) linearly at the leading order, they are less sensitive to the exact value of \( m_H \), that enters only through \( \log m_H \).

The relevant observables for this analysis are \( \Gamma_Z, \sigma^0_{\text{had}}, R_\ell, R_b \) and \( g_V/g_A \), that is determined from all asymmetries. To these, it is very convenient to add two other high-precision EW data that are the ratio of \( W \) and \( Z \) masses, which is measured at hadron colliders by the CDF and UA2 groups \[11\]

\[
\frac{M_W}{M_Z} = 0.8798 \pm 0.0028 \tag{3.2}
\]

and the ratio of neutral- and charged-current cross sections in neutrino-nucleon scattering from CHARM, CDHS and CCFR \[12\]

\[
R_\nu \equiv \frac{\sigma_{\text{NC}}}{\sigma_{\text{CC}}} = 0.312 \pm 0.003 \tag{3.3}
\]

that gives

\[
\sin^2 \theta_W \equiv 1 - \frac{M_W^2}{M_Z^2} = 0.229 \pm 0.003 \pm 0.005 \tag{3.4}
\]

where the first error is experimental and the second comes from theoretical uncertainties. Note that the above definition of the Weinberg angle coincides with \( \sin^2 \theta_{\text{eff}}^w \) (defined by eq.(2.20)) only at tree level, while it is modified differently by radiative corrections \[3\].

The result of a global fit of all LEP data to theoretical predictions reveals perfect consistency of the Standard Model with present data \[1\]. Leaving the strong coupling constant as a free parameter, and allowing \( m_H \) to vary between 60 and 1000 GeV, the values of \( m_t \) and \( \alpha_s(M_Z) \) that give the best fit are reported in the first column of Table 4, while the effect of including also non-LEP data from eqs.(3.2) and (3.4) in the analysis are shown in the second column. The second error quoted corresponds
|                | LEP           | LEP + COLLIDER + ν |
|----------------|---------------|-------------------|
| $m_t$(GeV)     | $166^{+17+19}_{-19-22}$ | $164^{+16+18}_{-17-21}$ |
| $\alpha_s(M_Z)$ | $0.120 \pm 0.006 \pm 0.002$ | $0.123 \pm 0.006 \pm 0.002$ |
| $\chi^2$/d.o.f. | 3.5/8        | 4.4/11            |
| $\sin^2 \theta_{eff}$ | $0.2324 \pm 0.0005^{+0.0001}_{-0.0002}$ | $0.2325 \pm 0.0005^{+0.0001}_{-0.0002}$ |
| $1 - \frac{M^2}{M^2_Z}$ | $0.2255 \pm 0.0019^{+0.0006}_{-0.0003}$ | $0.2257 \pm 0.0017^{+0.0004}_{-0.0003}$ |
| $M_W$(GeV)     | $80.25 \pm 0.10^{+0.02}_{-0.03}$ | $80.24 \pm 0.09^{+0.01}_{-0.02}$ |

Table 4: Fits to the Standard Model.

to the allowed $m_H$ variation. Also shown in each case is the (very good) value of the ratio of $\chi^2$ over the number of degrees of freedom in the analysis.

Although it is not yet possible with present experimental accuracies to get a significant upper bound on $m_H$ that is more stringent than the theoretical one of about 1 TeV, a $\chi^2$ analysis prefers low values for $m_H$ [1], (even lower than the direct 63.5 GeV limit in a combined two-variable $\chi^2(m_t, m_H)$ analysis [13]).

As already stressed in previous sections, the value obtained for $\alpha_s(M_Z)$ is in perfect agreement with the result $\alpha_s(M_Z) = 0.123 \pm 0.006$ from hadronic-event-shape analysis.

From the same fit, one also gets the best estimates for other observables, that are reported in Table 4, as well. It is very interesting to note that the error obtained on the $W$ mass (about 100 MeV) is much smaller than the present experimental error of about 250 MeV coming from $W$ studies at hadron colliders [13].

Even at the present non-conclusive stage, the EW LEP data are very constraining. In general, one can say, that the Standard Model has been tested with an accuracy of 0.5% or better. In the next section, we will discuss the real implications of this level of accuracy for a decisive test of the theory.

4. EW data versus Standard Model “Born” predictions

We want now to discuss the problem of establishing at which level the Standard Model radiative-correction pattern is tested by LEP data. In fact, what one really wants in order to check the Standard Model is to probe it beyond what is predicted
Table 5: Comparison between measured and “Born” values.

| Observable | Measured  | “Born”    |
|------------|-----------|-----------|
| $M_W/M_Z$  | 0.8798(29)| 0.8768(2) |
| $g_{Af}$   | 0.5008(8) | 0.5000    |
| $g_{Vf}/g_{Af}$ | 0.0712(28) | 0.0753(12) |
| $\Gamma_\ell$ ($MeV$) | 83.79(28) | 83.57(2)   |
| $\Gamma_{had}$ ($MeV$) | 1740(6) | 1741(5)    |
| $\Gamma_Z$ ($MeV$) | 2489(7) | 2490(5)    |
| $\sigma_{had}$ ($nb$) | 41.55(14) | 41.44(5)   |
| $R_\ell$   | 20.77(5)  | 20.84(6)  |
| $R_b$      | 0.2191(27)| 0.2197(1) |

by the tree-level theory implemented with QED radiative corrections, the last being the better established part of the theory. In other words, one would like to detect purely EW corrections. Although, the present 0.5% LEP accuracy seems to be close to this goal, we can see that this aim is not yet really accomplished.

Pure QED corrections are rather large but computable with good precision. Their main effects can be accounted for by substituting the fine structure constant $\alpha$ with its value at the $M_Z$ energy scale $\alpha \equiv \alpha(M_Z)$ (cf. eq.(3.1)) and by computing initial-state photon radiation effects (cf. section 2.1).

At this point, it is useful to define as “Born” predictions [14] the Standard Model results obtained by tree-level calculations, where one uses as input parameters $G_F$, $M_Z$ and $\alpha$ is substituted by $\alpha$. Accordingly, the tree-level Weinberg angle is replaced by

$$\sin^2 \theta_0 \cos^2 \theta_0 \equiv \frac{\pi \alpha}{\sqrt{2} G_F M_Z^2}$$

We will call genuine EW radiation any deviation from these results.

The goal of this procedure gets clear by looking at Table 5 [15]. Here, for various observables, the experimental value is reported in the first column, while the “Born” prediction is shown in the second. Perfect agreement is found between the two values, in each case, within errors (reported in brackets). No deviation from the “Born” approximation is observed experimentally. This means that genuine EW corrections
are small and in particular compatible with 0, within the present 1σ accuracy. It is important to stress that, on the other hand, a true Born calculation (which does not include QED effects) deviates by a few σ’s by the experimental values.

It has been argued [15] that the “smallness” of EW radiative corrections must be due to some *conspiracy* between the top quark and other particles (light quarks, Higgs, $W$ and $Z$ bosons), since they contribute with different sign to EW loop corrections. This makes the magnitude of these corrections smaller than their *natural* value, that is naively of the order of $\alpha W/\pi \sim \pi$. Furthermore, it is exactly this smallness that allows to put rather stringent limits on $m_t$.

Other two years of LEP running are expected to collect a further 50pb$^{-1}$ of integrated luminosity and to halve the present statistical errors. This could be sufficient to start disentangling genuine EW effects. Furthermore, once the top mass will be measured in a direct way through top observation at hadron colliders, one should be able to exactly compute the top contribution to radiative corrections and possibly separate Higgs loop effects. This could allow to get some more stringent experimental information on $m_H$, too.

### 5. Model independent analysis of LEP data

The high level of accuracy of present EW data can be also used to put some constrain on possible new physics. In order to do that, one has to analyze the experimental data in a model independent way. This problem has been by now extensively studied, following several different approaches [16, 17]. In general, one parametrizes the possible deviations from the Standard Model predictions in terms of a set of new variables. In the approach of ref.(17), one expresses all the basic EW observables as functions of the four variables $\varepsilon_1$, $\varepsilon_2$, $\varepsilon_3$ and $\varepsilon_b$, that vanish in the limit of the “Born” Standard Model approximation. The fact that, up to now, no observable deviates from the Standard Model “Born” predictions implies that presently these variables are all compatible with 0. Nonetheless, they can be extracted from data with a precise error, that gives the amount of experimentally acceptable deviation from Standard Model “Born” predictions. The variables $\varepsilon_1$, $\varepsilon_2$, $\varepsilon_3$ and $\varepsilon_b$ include all the top and Higgs effects. Hence, they can be extracted from data without referring to particular values of $m_t$ and $m_H$. In the Standard Model, they can be expressed as functions of $m_t$ and $m_H$, so that an experimental bound on the magnitude of the $\varepsilon_i$’s
can be translated in top and Higgs mass limits.

In order to study LEP constraints on new physics corresponding to some known theoretical model, one can accordingly calculate, starting from the same model, the non-standard contributions to the $\varepsilon$ parameters. The experimentally acceptable range for $\varepsilon_1$, $\varepsilon_2$, $\varepsilon_3$ and $\varepsilon_b$ can then either exclude or constrain the new model.

Consequences of the $\varepsilon_i$’s analysis on various fashionable extensions of the Standard Model have been reviewed in ref.([13]).

Concerning alternative mechanisms to the EW symmetry breaking, technicolour models seem to be disfavoured by present data, since they tend to contribute to $\varepsilon_1$, $\varepsilon_3$ and $\varepsilon_b$ more substantially than experimentally allowed [18]. On the other hand, this conclusion can be controverted by the fact that a true theory of technicolour and realistic technicolour models have not yet been found. Hence, theoretical predictions in this case are rather poorly defined.

A different situation is found for Supersymmetry. Here, we have a well definite theory which gives clear predictions, although these predictions depend on several new parameters. In the Minimal Supersymmetric Standard Model (MSSM), we find two qualitatively extreme situations [19]. The first is when all the masses of Susy partners are large. As far as the $\varepsilon_i$’s analysis is concerned, this case is equivalent to a Standard Model with a light Higgs ($m_H \sim 100$GeV) and, hence, is perfectly compatible with experimental data. The latter case is when some Susy partners are rather light and close to their present experimental bounds. For instance, if light gauginos and s-top quark exist in a range of masses that can be covered by LEP200 searches, they could produce a detectable deviation from the Standard Model $\varepsilon$ values.

Finally, the case of extended gauge group has been considered in the simple case of an extra $U(1)$ [20]. Present data are already constraining enough as to allow only for a very small amount of mixing ($\xi < 1\%$) between the Standard Model $Z^0$ and the new neutral vector boson associated to the extra $U(1)$.

6. Conclusions

After four years from its starting, LEP has produced a huge amount of data at the $Z^0$ peak. These data have allowed the determination of several EW observables with unprecedented accuracy. There is spectacular (and improving with time) agreement
with all the pattern of Standard Model predictions, although the present accuracy measurements, at the level of 0.5%, is not yet sufficient to disentangle purely EW radiative corrections. Nonetheless, the small errors on different observables already allow to considerably constrain the top mass in the range

$$m_t = (164 \pm 27) \, GeV$$  \hspace{1cm} (6.1)

Further improvements are foreseen after the '93 and '94 running completion. With about four times the present statistics, one will halve the today statistical errors. This hopefully will permit to distinguish genuine EW effects and, in case top will be directly observed at Tevatron, to disentangle Higgs-loop contributions.

On the other hand, by performing a model independent analysis of the data, one can already put severe constrains on possible extensions of the Standard Model.

In a pessimistic picture, where one assume that no effect from new physics will be observed directly in the next future, the comparison between more and more accurate experimental data and more and more precise theoretical predictions could be the only way to discover what is beyond the Standard Model.
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