The chemical composition of the cosmic radiation around the ankle and the related spectral indices

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Some recent measurements of the chemical composition of the cosmic radiation indicate that at the energy of $3 \times 10^{18}$ eV, around the ankle, light cosmic ions dominate the spectrum as it occurs in the preknee energy region. Taking advantage of a recent theory of cosmic radiation which provides a quantitative explanation of the knee, the second knee and the ankle, the chemical composition of cosmic radiation is explicitly calculated giving individual ion spectra and ion fractions from $10^{12}$ eV to $5 \times 10^{19}$ eV. The calculation assumes two components of the cosmic radiation feeding the ion flux at Earth: one originated in the disc volume and another one, called extradisc component, which from the disc boundaries traverses the Galaxy reaching the solar system. Data above $10^{17}$ collected during half century of experimentation by Auger, HiRes, Agasa, Akeno, Fly’ s Eye, Yakutsk, Haverah Park and Volcano Ranch experiments are reviewed, examined and compared with the theoretical $<\ln(A)>$. The comparison between computed and measured $<\ln(A)>$ exhibits a good global accord up to $2 \times 10^{19}$ eV except with the HiRes experiment and an excellent agreement in the range $10^{15}$-$10^{17}$ eV with Kascade, Eas-top, Tunka and other experiments. The accord requires a flux of the extradisc component of $1.8 \times 10^{14}$ particles/(m$^2$ sr s eV$^{1.5}$) at $10^{19}$ eV, twice that generated by disc sources.

1. Introduction

The spectrum [1] of the cosmic radiation between the knee and the ankle has been recently calculated according to the Theory of Constant Spectral Indices [1-4]. The galactic mechanisms and the parameters generating the knee are the same at the origin of the ankle: this circumstance constitutes a major characteristic of the solution of the knee and ankle problem. Another notable aspect of this solution is the presence of a structure at $(5 - 7) \times 10^{17}$ eV which is called the second knee, just a direct consequence of the theory [5]. Figure 1 shows an example of excellent quantitative accord between computed and measured cosmic-ray spectrum [4].

The Theory of Constant Spectral Indices determines the energy spectra of individual ions, or group of ions, in the interval $10^{11}$-$5 \times 10^{19}$ eV with only one normalization point for the particle flux. As a consequence, once the spectral indices and the abundances at a given energy are assigned, the chemical composition of the cosmic radiation is known at any energy. Fig. 2 shows that individual ion spectra in the knee energy region are also in good agreement with the theory. The simplicity of the theory and its anchorage to the empirical parameters governing the cosmic ion motion in the Galaxy are not minor aspects besides the excellent quantitative agreement with ion spectra exhibited in figures 1 and 2.

The chemical composition expressed by $<\ln(A)>$ has been also calculated with all cosmic-ray sources placed in the disc [6], thus ignoring any extragalactic component. Figure 3 reports the results of the calculation. The profiles of $<\ln(A)>$ measured by the Kaskade and Tunka experiments along with that extracted with the measurements of $X_{\text{max}}$ of the Auger Collaboration are shown in figure 3. There is an evident disagreement above $10^{17}$ eV between theory and data. All other experiments above $10^{17}$ eV have been purposely omitted from figure 3 to render...
the disagreement between theory and data more evident. According to the above quoted measurements the theoretical chemical composition, as generated by the galactic cosmic-ray sources above $4 \times 10^{17}$ eV, is incompatible with the data, primarily because it is too heavy, and secondly, because above $2 \times 10^{19}$ it has a flat trend, contrary to the measured $<\ln(A)>$, which increases with energy.

Notice that the Auger Collaboration disposes today (2009) of a powerful and redundant apparatus and a number of recorded atmospheric cascades superior to that of any other experiment.

The purpose of this work is to calculate the chemical composition of the cosmic radiation at Earth, not only assuming cosmic-ray sources in the disc, but also postulating a component that penetrates the disc volume from the exterior reaching the Earth. This cosmic-ray component is called here *extradisc* being a particular component of the more vast class of the extragalactic cosmic rays.

Both the extradisc cosmic rays and their flux will be denoted by the same symbol $I_{ed}$ ($ed$ for extradisc). Let us anticipate here that *extradisc component* reconciles the results of the theory with the Auger data on $<\ln(A)>$ shown in figure 3, still preserving a quantitative accord with the knees, the ankles and the second knee.

The structure of this paper preassumes the existence of the *Theory of Constant Spectral Indices* [1-4] and the companion paper [6] where the chemical composition of the cosmic radiation is evaluated under the restricted assumption that all cosmic-ray sources feeding the particle flux at Earth are placed in the disc volume. Section 2 presents a brief survey of the bases of the calculation and a flash of the cultural background of the theory. Section 3 is devoted to a survey on cosmic-ion energy spectra useful for the normalization of the theory at $10^{12}$ eV. Section 4 summarizes the calculation of the chemical composition obtained with the galactic sources only. Section 5 reports a comparison between data and theory under the assumption that all cosmic ray sources
Figure 3. Comparison of the $\langle \ln(A) \rangle$ derived from the Theory of Constant Indices with the Tunka [13], Kascade [14] and Auger [15] data. A significant discrepancy between this calculation and the Auger data appears above $4 \times 10^{17}$ eV.

The theoretical evaluation of $\langle \ln(A) \rangle$ (blue line) assumes that all cosmic-ray sources are placed in the disc volume, with no extragalactic component.

Section 11.

2. Bases and foundation of the calculation

A prerequisite inherent this calculation states that cosmic rays generated in the disc volume of the Milky Way might migrate into the halo, and eventually, into the intergalactic space [6]. The disc of Milky Way is regarded as standard, typical disc volume. All these cosmic rays escaped from the disc volume populating the extradisc space, constitute the extradisc component. This calculation admits that a fraction of the extradisc component existing at the outskirts of the disc volume, by displacement predominantly shaped by the magnetic field and gas density, can reach the solar cavity (the Earth): its intensity will denoted by $I_{ed}$.

In order to perform practical calculation it is necessary to specify some properties and features of the extradisc cosmic rays present outside the disc:

Table 1

Parameters of the theory denoted Low Energy (LE) and High Energy (HE) ion blends. Fluxes are multiplied by $E^{2.5}$ and expressed in units of $(m^{-2}sr^{-1}s^{-1}eV^{1.5})$.

| Blend        | LE      | HE       |
|--------------|---------|----------|
|              | $10^{12}$ eV | $10^{14}$ eV |
| % $\gamma$  | % $\gamma$ |
| H            | 42.4    | 33.5     |
| He           | 26.5    | 27.3     |
| CNO          | 11.9    | 13.6     |
| Ne-S         | 9.2     | 10.9     |
| Ca(17-20)    | 1.2     | 2.8      |
| Fe(21-28)    | 8.7     | 12.0     |

| Flux $\gamma$ | Flux $\gamma$ |
|---------------|---------------|
| H             | 1.15 $10^{17}$ | 2.77  | 3.93 $10^{16}$ | 2.67 |
| He            | 7.19 $10^{16}$ | 2.64  | 3.20 $10^{16}$ | 2.64 |
| CNO           | 3.24 $10^{16}$ | 2.68  | 1.60 $10^{16}$ | 2.65 |
| Ne-S          | 2.50 $10^{16}$ | 2.67  | 1.28 $10^{16}$ | 2.63 |
| Ca(17-20)     | 3.14 $10^{15}$ | 2.67  | 3.27 $10^{15}$ | 2.63 |
| Fe(21-28)     | 2.36 $10^{16}$ | 2.59  | 1.41 $10^{16}$ | 2.62 |
| Total         | 2.71 $10^{17}$ | 2.70  | 1.18 $10^{17}$ | 2.64 |
Figure 4. Energy spectra of 11 individual ions measured by balloon and satellite detectors at energies below $10^{15}$ eV suggesting and founding the postulate of the constant spectral indices (see ref. [16] and references therein).

(A) Cosmic rays emitted by the sources placed in the disc and escaping from the disc volume undergo alterations of their abundances as released at the sources, due to nuclear interactions with interstellar gas. It will be shown that this alteration depend both on the energy and the type of the ion.

(B) The spectral index of any ion released at the sources in the disc volume is constant in the energy interval $10^{11} - 5 \times 10^{19}$ eV. The plausibility of this fundamental hypothesis is rooted on the measurements of the spectral indices collected in more than half a century by balloon and satellite detectors (see fig. 4, 5 and 6). This hypothesis has been converted in 2006 into a postulate called Postulate of Constant Spectral Indices [1].

(C) Ion abundances as a function of energy of the extradisc particles traversing the disc and arriving at Earth are evaluated assuming the same parameters adopted for the disc component $I_d$. Though the present calculation ignores the mechanism that accelerate cosmic rays in the Milky Way, two constraints have to be obeyed by the accelerator, whatever it might be, in the range $10^{11} - 5 \times 10^{19}$ eV: (1) spectral indices generated at the sources placed in the disc volume are independent of energy; (2) the cosmic-ray sources are uniformly distributed in the disc volume at any energy [12].
It has been demonstrated by full simulation of cosmic-ray motion that the displacement of the cosmic rays from the sources to any point in the Galaxy has a modest effect on the spectral indices released at the sources. For example, the propagation of helium in the disc entails an index modification of 0.04 in the energy band $10^{12}$-$10^{14}$ eV being at the sources 2.65. While for most calculations this index modification is a negligible effect, for the evaluation of the chemical composition around the ankle is of quite notable importance.

The interval of application of the Theory of Constant Spectral Indices is $10^{11}$-$5 \times 10^{19}$ eV. The upper energy extreme is determined by the realization that the characteristics of the knee and the ankle of the complete spectrum (all-particle spectrum) are quantitatively explained by the theory. The theory predicts the exact position where the ankle is observed, at $(4 - 5) \times 10^{18}$ eV and that of the knee as well, and many other features. The quantitative characteristics of the ankle of the complete spectrum derive from those of the ankles of the individual ions. The proton ankle is at the lowest energy while the iron ankle is at the maximum energy, which turns out to be $5 \times 10^{19}$ eV.

The theory distinguishes an ankle in the grammage versus energy and an ankle in the cosmic-ray intensity versus energy. The knees are intimately related by the theory to the ankles of any ions, and since there exist an excellent quantitative agreement with numerous experimental data, the maximum energy involved in the knee-ankle relationship is the Fe ankle energy. The Fe ankle energy in the grammage is at $5 \times 10^{19}$ eV, and as a consequence, it delimits the range of application of this theory. Above the energy of $5 \times 10^{19}$ eV the results reported in this paper and the companion one [6] have to be regarded as numerical exercises. In fact, physical phenomena having empirical evidence operate above $5 \times 10^{19}$ eV but they are not incorporated in the Theory of Constant Spectral Indices as it is presently formulated [1,2,3,4].

The results of the calculations depend on the geographic site of the sources generating the extradisc component. For instance, if the sources of the extradisc component would be the ensemble of the spiral galaxies, due to the similarity of the Milky Way Galaxy, only minor modifications of the calculations would be necessary to deter-
mine $<\ln(A)>$. If the sources of the extradisc cosmic rays would be placed in the local Supercluster of galaxies, the present calculations are not sufficiently precise for a number of reasons. If the extradisc component are just reentrant particles escaped from the disc of the Milky Way, as already suggested [2], the calculations are exact. Total kinetic energy of particles is used everywhere in this paper.

3. Measurements of the spectral indices and ion abundances

Let us preliminarily notice that the interpolation of the energy spectra of individual ions with a constant spectral index in a restricted energy band it is an established tradition at energies below $10^{15}$ eV. An index slightly depending on energy would probably conform better to some data, in some energy intervals. This refinement, however, it is still uncommon, and unnecessary in the present work, and consequently omitted here.

The fundamental hypothesis of the present calculation incorporated in the Theory of Constant Spectral Indices is that all ions at the sources have constant spectral indices.

This simple hypothesis predominantly concurs in the prediction of the complete spectrum leading to the excellent agreement with experimental data as shown, for example, in figure 1.

Figure 4 reports the energy spectra of 11 different ions in the interval $10^{10}$-$10^{15}$ eV. The spectra form a grid of parallel straight lines in the flux units shown. A posteriori the regular forms of these spectra with no dips and no spikes justify the postulate of Constant Spectral Indices. Though the evidence for constant indices is solid and irrefutable below $5 \times 10^{16}$ eV there exist in Nature magnificent and universal phenomena that modify indices released at the sources. The ensemble of the processes referred to as solar modulation occurring in all stars is an example of violation of the Postulate of Constant Spectral Indices at low energy. Thus, local phenomena evidently violates the postulate. In order to ascertain if additional local phenomena in the Galaxy, besides solar modulation and the knees, in the interval $10^{11.5}$-$10^{19}$ eV alter the indices, a careful analysis of the complete spectrum is required.

![Figure 4: Energy spectra of 11 different ions in the interval $10^{10}$-$10^{15}$ eV.](image)

Table 2

Parameters of the theory at $10^{12}$ eV denoted universal and $HE4$ ion blends. Fluxes are multiplied by $E^{2.5}$ and expressed in units of $(m^{-2}sr^{-1}sec^{-1}eV^{1.5})$.

| Element | HE4 | Universal |
|---------|-----|-----------|
| H       | 24.4 | 37.2      |
| He      | 26.5 | 26.4      |
| CNO     | 11.9 | 13.9      |
| Ne-S    | 9.2  | 10.1      |
| Ca(17-20)| 1.2 | 0.15      |
| Fe(21-28)| 8.7 | 10.8      |

Total 3.89E+17 2.31E+17
Fraction

Energy (eV)

HE4 ion blend (thin lines) LE ion blend (thick curves)

light ions (p + He)

medium ions (CNO + Ne-S)

heavy ions (Ca + Fe)

Figure 9. Relative amounts of light, intermediate and heavy ions versus energy for the LE (thin curves) and HE (thick curves) ion blends. Notice the minimum of the light component around $3 \times 10^{17}$ eV in both ion blends, followed by a stable increase of the light ion fraction up to $4 \times 10^{18}$ eV.

examination of the experimental data is mandatory.

A compilation of indices of a number of nuclides from Hydrogen to Nickel is shown in figure 5. This compilation based on measurements of the indices mainly between $10^{10}$-$10^{15}$ eV is referred to as Wiebel-Sooth compilation [17] and the corresponding parameters incorporated in the theory are called LE ion blend (Low Energy). Another compilation of spectral indices collecting measurements at higher energies [16] between $10^{12}$-$10^{15}$ eV is also shown in the same figure 5 (blue open dots) and it is referred to as HE ion blend (High Energy). The spectral indices shown in figure 5 accumulate around a common value of 2.65 between $10^{11}$-$10^{15}$ eV. It is also evident from the experimental data on the indices shown in figure 5 (red dots) that higher the atomic mass harder the index. This correlation between indices and atomic mass is more pronounced for LE than HE ion blend.

Notice that the parameters of the Wiebel-Sooth Compilation incorporated in the LE ion blend based on the interval $10\times Z(GeV)$ up to $10^{12}$ eV are three orders of magnitudes from the knee energy region (e.g. $10^{15}$-$10^{18}$ eV) while those of the HE ion blend are closer. If the postulate of the constant spectral indices is exactly described by a straight line (in logarithmic scales of intensity and energy) either the LE or HE blend would suffice to correctly calculate the chemical composition.

In this circumstance the differences in the chemical composition at any energy are only due to the ion abundances at the normalization energy of the theory. If quite small violations of the postulate take place, then, the differences originating from the two ion blends become significant.

Recent experimental data between 40 and 400 GeV indicates that the proton index changes from $2.77 \pm 0.02$ to $2.67 \pm 0.03$ in the interval $4 \times 10^{14}$-$10^{15}$ eV taking into account the data of Atic2 [18] and Cream experiments [19]. The hardening of the index seems to affect the helium spectrum as well. In fact, the index of $2.70 \pm 0.02$ measured at low energy below 400 GeV decreases to the value of $2.62 \pm 0.05$ above 400 GeV up to the maximum energy explored to day of $4 \times 10^{14}$ eV [20]. The alteration of the H and He indices in the preknee energy region would entail a change in the H/He flux ratio which has been proved to be constant at low energy, $10^{13}$ eV [21]. The H/He flux ratio versus energy has been examined in another paper [16] where it has been shown that the problem of the H/He flux ratio above $10^{13}$ eV, is, at the present times, inextricable. The recent data of the Cream experiment [22] do not resolve the problem of the H/He flux ratio below $10^{15}$ eV due to the large error bars of the experiment at high energy, in the decade $10^{11}$-$10^{15}$ eV.

Figure 6 reports the ion fluxes for the 2 compilations defining the parameters of the LE and HE blends. The fluxes are almost equal for the two blends as shown in fig. 6. The flux of the complete spectrum based on the Wiebel-Sooth compilation (LE blend) is $2.71 \times 10^{17}$ particles/m$^2$ s sr eV$^{1.5}$ at $10^{12}$ eV. The HE blend at $10^{12}$ is normalized at the same flux which at $10^{14}$ eV becomes $1.07 \times 10^{17}$ particles/m$^2$ s sr eV$^{1.5}$. The complete spectrum has been measured in the region $10^{12}$, $10^{15}$ giving a spectral index of 2.74 and an extrapolated intensity at $10^{12}$
around $2 \times 10^{17}$ particles/$m^2 s sr eV^{1.5}$ \cite{23}. In the interval $10^{15}$-$10^{17}$ eV recent flux measurements of the complete spectrum by Kascade give (Sibyll) $7.68 \times 10^{16}$ and (QGSjet) $7.48 \times 10^{16}$ particles/$m^2 s sr eV^{1.5}$ at the reference (and arbitrary) energy of $2 \times 10^{15}$ eV. Statistical errors of these measurements are about 7 per cent. Note that the difference in the complete spectra resulting from QGSjet and Sibyll algorithms is negligible (see fig. 1). Differences in flux measurements resulting from QGSjet and Sybll algorithms become quite significant for individual ions amounting to factors 2-10 for some ions in some energy intervals as pointed out elsewhere (see Section 9 of ref. \cite{11}).

The Kascade data suggest that the spectral indices of heavy ions are constant in any preknee energy region. Empirically, they are expected to be constant on the basis of balloon and satellite data on proton and helium spectra collected below $10^{15}$ eV, provided that the H and He knees are regarded as experimentally observed, around $2 \times 10^{15}$ eV and $6.7 \times 10^{15}$ eV, respectively. If so, heavy ion spectra are expected to behave similarly.

The heavy ion spectra measured by the Kascade experiment do have constant spectral indices close to 2.65 \cite{24,25}. Any methods of analysis (QGSjet, Sibyll or deconvolution method) yield indices of about 2.65 or harder but not softer. When harder indices are observed a contamination between nearby ions is suspected or demonstrated (see Section 7 of ref. \cite{11} devoted to this argument). With the appropriate ion groupings of contaminated adjacent ion samples and the correct guide from the theory \cite{1-4}, a constant common index for heavy ion spectra in the fundamental energy region $10^{15}$-$10^{17}$ emerges \cite{11}.

Though the present calculation does not depend on any acceleration mechanism, a theory of cosmic-ion acceleration in pulsar atmospheres having constant spectral indices up to $5 \times 10^{19}$ eV has been conceived in 1969 \cite{26}.

The findings on the indices of heavy ion spectra of the Kascade experiment consolidate the Postulate of Constant Spectral Indices from $10^{15}$ eV up to about $10^{17}$ eV (unobserved Fe knee). As far as the Postulate of Constant Indices is at stake, the outcomes of the Kascade experiment on heavy ion spectra in the interval $10^{15}$-$10^{17}$ eV have anticipated, probably by more than half a century, those plausibly expected by future balloon and satellite experiments, which identify individual ions with superior discrimination power but with a very few nuclei.

The data shown in figure 5 are included in the theory with 6 indices and 6 fluxes at the normalization energy of $10^{12}$ eV. The values of these parameters are given in Table 1.

In the same Table 1 are given the parameters of the HE blend tuned to the data at higher energy. In the following, in order to appreciate differences and similarities with previous calculations, the parameters of 2 additional ion blends referred to as universal and HE4 blends are given in Table 2. In the universal ion blend all indices are equal with a value of 2.65 except for helium set at 2.64 which takes into account the recent trend of the measurements suggesting an index harder than that of other ions. The HE4 blend is a variant of the HE blend where the dominant nuclei, e.g. protons and helium, H and He have the classical value of 2.74 \cite{23} and 2.72 \cite{24} believed for many years to be standard, reliable measurements. The parameters of the HE4 blend are used to determine the chemical composition around the ankle (see fig. 6). The HE4 blend is normalized at $10^{12}$ eV with a flux of $3.79 \times 10^{17}$ particles/$m^2 s sr eV^{1.5}$. This value is inspired from the Kascade flux measurement which is $7.58 \times 10^{17}$ particles/$m^2 s sr eV^{1.5}$ at $2 \times 10^{15}$ eV (mean value of QGSjet and Sibyll algorithms) \cite{7}. The HE4 flux normalization at $10^{12}$ eV is larger than that of the LE blend (Wiebel-Sooth flux compilation) by a factor 1.4.

Because of the ion propagation in the disc volume the indices of the original spectra released by the sources modify (see fig. 6 of ref. \cite{3}).

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2 In the paper: The Transition from Tortuous to Rectilinear Cosmic Ray Trajectories is at the Origin of the Knee by A. Codino and F. Plouin \cite{4}, the theoretical Fe spectrum is compared with that measured by the Kascade experiment, which has an index of 2.64 ± 0.06. In this comparison heavy ions include nuclei from Silicon to Nickel. Different groupings result in similar indices but always in overabundant fluxes compared to the theory (see figs. 16 and 17 of ref. \cite{5}).
It may be useful to subdivide the index of any ion spectrum measured at the solar cavity in two parts: the propagation index and the accelerator index at the sources. The propagation indices due to ion displacement in the disc volume for the 6 ions H, He, N, Si, Ca and Fe are, respectively, 0.060, 0.055, 0.045, 0.045, 0.040 and 0.040. Taking into account the accelerator indices of the universal blend (see Table 2) the global index become: 2.585, 2.585, 2.605, 2.605, 2.610 and 2.610 for the same ion sequence just mentioned.

4. Chemical composition of the cosmic radiation generated by cosmic-ray sources placed in the disc

The energy spectra of 6 ions derived from the Theory of Constant Spectral Indices with the LE and HE4 blends are, respectively, in figure 7 and 8. The complete spectrum which is the sum of the partial spectra of individual ions is also shown in the same figures 7 and 8 for both blends resulting in a flux of $8.2 \times 10^{16}$ particles/$m^2 s sr eV^{1.5}$ for the HE4 ion blend at $10^{15}$ eV. The ion abundances derived from the energy spectra reported in figures 7 and 8 are shown in figure 9 for the interval $10^{12}-5 \times 10^{15}$ eV. The light ion fraction (H+He) decreases with a gentle slope going from 68 per cent at $10^{12}$ eV to 55 per cent at $1.5 \times 10^{15}$ eV (HE4 ion blend). Above this energy the light ion fraction descends abruptly reaching a minimum of 11 per cent at the energy of $4 \times 10^{17}$ eV, quite evident in figure 9. A similar behaviour is exhibited by the LE blend but with higher light ion fractions.

The slight increase in the (Ca+Fe) fraction in the interval $10^{12.4} \times 10^{15}$ eV for the HE4 blend is caused by the tiny difference in the Ca and Fe slopes (2.66 for Fe and 2.650 for Ca, see Table 2) with respect to the light ion slopes (2.74 for H and 2.72 for He). The strong increase of the (Ca+Fe) fraction between $4 \times 10^{15}$ eV and $7 \times 10^{17}$ eV is due not only to the difference in the indices (2.70 light ions against 2.60 heavy ions) but predominantly to the high fall of the light ion flux (light ion knees) above the nominal knee of the complete spectrum at $3 \times 10^{15}$ eV. The fall is quite notable for protons and helium in the interval $2 \times 10^{15}-2 \times 10^{17}$ eV but insignificant for heavy ions (Ca+Fe) at least up to $4 \times 10^{17}$ eV. The processes causing this abrupt fall of intensity (knees) have been analyzed and described elsewhere.

5. The $<\ln(A)>$ of cosmic-ray disc component compared with the data

It is instructive to compare the theory with the experimental data on $<\ln(A)>$ under the restricted assumption that all cosmic-ray sources are in the disc volume with no extragalactic component.

![Figure 10](image-url)
Figure 11. Comparison of the theoretical $\langle \ln(A) \rangle$ for the LE and HE4 blends with that measured by Kascade (black triangles) [28] using the deconvolution method and that measured by Eas-top (red dots) [29].

resulting from 3 different ion blends: LE, HE4 and the universal blend. The differences in the LE and HE4 blends are quite negligible in the entire interval $10^{12.5} \times 10^{19}$. There are significant similarities but also some differences. The 3 profiles are similar in the band $2 \times 10^{15.2} \times 10^{17}$ eV, they all increase by about 2 units of $\langle \ln(A) \rangle$ and, in the range $5 \times 10^{17.5} \times 10^{18}$, they all descend by about one unit. Differently, the $\langle \ln(A) \rangle$ profile of the universal blend is flat from the initial energy at $10^{12}$ up to $2 \times 10^{15}$ eV, then it increases reaching a maximum of 3.05 at $2.97 \times 10^{17}$ eV, and a more pronounced descent at high energy, and then, above $5 \times 10^{18}$ eV, a modest decreasing trend dominates. The form of the $\langle \ln(A) \rangle$ profile of the HE4 blend increases up to a maximum value of 3.33 at $3.94 \times 10^{17}$ eV, then it decreases rapidly up to $(4-5) \times 10^{18}$ eV leveling off at higher energies around 2.8. As explained elsewhere (see fig. 4 of ref. [1]) the spectra of heavy ions (Ca and Fe) attain the low asymptotic plateaux (see fig. 7 and 8) at higher energies than the light ions. Low asymptotic plateaux reflect also the quasi rectilinear propagation in the Milky Way.

Figures 11 and 3 show data on $\langle \ln(A) \rangle$ of the Kascade experiment which separates all nuclei in 5 groups along with the corresponding theoretical curves (LE and HE4). The Eas-top data on $\langle \ln(A) \rangle$ are also shown in figure 11. Though there are discrepancies in the ion spectra in figure 3 obtained by QGSjet and Sibyll algorithms of the Kascade experiment [11], the resulting $\langle \ln(A) \rangle$ profiles show the correct general trends also observed by other experiments. By correct trend is meant an average increase of 1.5 units of $\langle \ln(A) \rangle$ in the interval $10^{15.5} - 10^{17}$ eV from an initial value of 1.8 at $10^{15}$ eV dictated by the extrapolation of balloon and satellite data at lower energies. The forms of the $\langle \ln(A) \rangle$ profiles in figure 11 obtained by the method of deconvolution [?] do not disagree with those of QGSjet and Sibyll algorithms shown in figure 3, though the $\langle \ln(A) \rangle$ in some energy bands have notable differences for the 3 methods. The agreement of the theoretical $\langle \ln(A) \rangle$ profiles for the LE and HE4 blends with the Kascade data has been described in detail elsewhere [6].

At energies larger than $10^{17}$ eV the determination of chemical composition of the cosmic radiation becomes more involved because ions are not resolved individually nor in restricted groups. In order to determine the $\langle \ln(A) \rangle$ from the observables of the atmospheric showers recorded by
detectors, accurate and detailed models of the nucleus-air interactions are required. Traditionally, the depth in g/cm$^2$ of the maximum of the atmospheric showers (i.e. number of particles versus cascade axis) is denoted $X_{\text{max}}$ or equivalently elongation.

A number of methods have been devised to measure $X_{\text{max}}$: (1) muon number and muon density detected at ground; (2) the mean width of the $X_{\text{max}}$ distribution denoted in short $\sigma(X_{\text{max}})$; (3) Cherenkov light generated in the cascades; (4) fluorescence light generated in the cascades; (5) curvature of the cascade front measured by time-of-flight techniques. Apparata exploiting Cherenkov and fluorescence light can be equipped with additional detectors.

Figure 12 gives the $X_{\text{max}}$ versus energy evaluated by Heck [30] for two models of nuclear interactions denoted in short QGSjet and Sibyll. It results that the depth of the maximum of the atmospheric showers induced by primary cosmic proton goes from 500 g/cm$^2$ at $10^{14}$ eV to 850 g/cm$^2$ at $10^{15}$ eV. In the same energy interval the Fe elongation goes from 350 g/cm$^2$ at $10^{14}$ eV to 750 g/cm$^2$ being the total thickness of the air about 1000 g/cm$^2$.

Above $10^{14}$ eV the gap between the proton $X_{\text{max}}$ profiles of Sibyll and QGSjet regularly enlarges reaching 40 g/cm$^2$ at $10^{20}$ eV (see fig. 12).

According to some simplifications the elongation is given by:

$$X_{\text{max}} = D((\ln E/E_0) - \ln(A)) + C$$

where $X_{\text{max}}$ is the atmospheric depth in g/cm$^2$, for primary particles of energy $E$, $E_0$ is a reference energy, $<\ln(A)>$ the chemical composition and $C$ and $D$ two appropriate functions. Let us note that in the present calculation, unlike others, $C$ and $D$ vary with energy. In the following, in the comparison with the experimental data, both $X_{\text{max}}$ and $<\ln(A)>$ are maintained since the two variables are affected by small differences. The function $X_{\text{max}}^A(E,E_0)$ is the elongation for the nucleus $A$ with the energy $E$ normalized at the energy $E_0$.

The function $X_{\text{max}}^A(E,E_0)$ obtained by cascade simulations depends on the hadronic models used...
Figure 15. Chemical composition in terms of $\langle \ln (A) \rangle$ extracted by the $X_{\text{max}}$ measured by Casa-Blanca [34] and Auger experiments. The $\langle \ln (A) \rangle$ profiles of each experiment curiously exhibit prominent depressions at $4 \times 10^{15}$ eV and $(2-3) \times 10^{18}$ eV. The Casa-Blanca depression is probably due to an instrumental effect as argued in the text.

To describe the interactions of cosmic nuclei with the air. Once the experimental value of the elongation $X_{\text{exp}}^{\text{H}}$ has been determined, the value of $\langle \ln (A) \rangle$ is obtained in the superposition model by the equation:

$$\langle \ln (A) \rangle = \ln (56) \times \frac{(X_{\text{max}}^{\text{H}} - X_{\text{max}}^{\text{Fe}})}{(X_{\text{max}}^{\text{H}} - X_{\text{max}}^{\text{Fe}})}$$

being $X_{\text{max}}^{\text{H}}$ and $X_{\text{max}}^{\text{Fe}}$ the elongations simulated by Monte Carlo for protons and Fe nuclei, respectively. Using the above equation, the $\langle \ln (A) \rangle$ derived from the theory can be converted into $X_{\text{max}}$. The result is shown in figure 12 with the blue and red curves which are the average value of $X_{\text{max}}^{\text{H}}$ and $X_{\text{max}}^{\text{Fe}}$ profiles obtained by QGSjet and Sibyll codes.

The Yakutsk [32] and Fly’s Eye [31] data on $\langle \ln (A) \rangle$ are shown in figure 13 and they represent an example of accord between theory and data. Note that at energies above $10^{18}$ eV the theoretical $X_{\text{max}}$ profile has a substantial gap with the data. The accord below $10^{18}$ eV and the discrepancy at high energy have been discussed in the companion paper [6].

Figure 14 reports examples of disagreement. The $X_{\text{max}}$ measured by Auger above $10^{17}$ eV exhibits a large discrepancy between data and theory in the huge interval $4 \times 10^{17} - 10^{19}$ eV. Subsequent elaborations of the Auger data samples [?] mark the stability of the $X_{\text{max}}$ except for the last data point which decreases by about 5 g/cm$^2$. The Casa-blanca and Space2/Vulcan data points in figure 14 have opposite deviations with respect to the theory, in different energy intervals.

The measurements of the atmospheric depths of the cosmic radiation made by Auger has been converted into $\langle \ln (A) \rangle$ and shown in figure 15. According to these measurements cosmic rays around $4 \times 10^{18}$ eV mainly consist of 37 per cent of heavy ions. This figure comes from the partition of the cosmic nuclei in two groups, light and heavy (see subsequent Section 8). Besides the Auger experiment, a notable structure in the $\langle \ln (A) \rangle$ profile, shown in figure 15, has been also observed by the Casa-Blanca experiment in the interval $10^{14} - 4 \times 10^{16}$ eV. The depression of $\langle \ln (A) \rangle$ measured by Casa-Blanca in the energy band $(3-9) \times 10^{15}$ eV would imply the disappearance of heavy elements. This depression corresponds to the decreasing difference between the measured $X_{\text{max}}$ profile and the theoretical $X_{\text{max}}^{\text{H}}$ in the interval $2 \times 10^{14} - 3 \times 10^{15}$ eV as shown in figure 14. As a consequence, the depression of the Casa-Blanca experiment shown in figure 15 is not a spurious effect in the conversion of $X_{\text{max}}$ into $\langle \ln (A) \rangle$. This circumstance favours an instrumental effect and not a physical cause for the origin of the depression. The conclusion is corroborated by the measured by the Kascade Collaboration with 3 methods (fig. 3 and 11) and others experiments which, contrary
6. Ion abundances outside the disc volume

In order to determine the ion abundances in the space outside the disc volume it is necessary to calculate the probability of escaping from the disc, \( P_E \), for any individual ions at all energies. At very high energy, as ions propagate almost rectilinearly, this calculation is extremely simple.

![Figure 16. Probability of escaping from the disc \( P_E \) as a function of the energy for particles having cosmic-ray sources distributed uniformly in the disc volume \([12]\). The probability \( P_E \) is normalized to \( 2 \times 10^5 \) particles.](image)

Let \( E_A^d \) be the energy above which ions of mass \( A \) propagate almost rectilinearly in the disc. In the rectilinear propagation regime ions encounter a minimum amount of matter as evaluated elsewhere \([3]\).

The amount of matter experienced by ions while propagating in the disc is referred to as the grammage, \( g \). It is given by: \( g = mnL \), where \( m \) is the mean atomic mass in the disc, \( n \) is the average number of atoms per \( cm^3 \) in the interstellar space and \( L \) the trajectory length. The grammage depends on nuclear cross sections via trajectory length, \( L \). Since for \( E > E_A^d \) the grammage is only \( 0.006 \ g/cm^2 \), the number of nuclear collisions taking place in the disc is negligible, and consequently, ion abundances released by the cosmic-ray sources are almost unmodified when they are observed in the solar cavity.

![Figure 17. Number of nuclear collisions versus energy occurring in the disc volume for nuclei (H, He and Fe) having all the initial points of the trajectories placed in the solar cavity.](image)

Unlike the rectilinear propagation regime in the energy interval where \( E < E_A^d \), between the knee and the ankle energy region, e.g. \( 3 \times 10^{15} - 5 \times 10^{18} \) eV, the galactic magnetic field affects ion motion, bending and inverting efficiently ion directions. This effect of the magnetic field for each ion makes the average grammage encountered by cosmic rays dependent on particle momentum. For \( E < E_A^d \), the calculation of \( P_E \) and the probability for cosmic ions of entering (or re-entering) from the disc boundary to the solar cavity, \( P_R \), is more involved and difficult. In the following the
functions $P_R$ are determined.

The number of particles escaping from the disc $N_F$ versus energy for 6 ions or group of ions is given in figure 16. The quantity $N_F$ is normalized at the same number ($2 \times 10^5$) of emitted particles (becoming the probability $P_E$). All curves in figure 16 have a minimum at a particular energy $E_{e\text{min}}$ ($e$ is for escape). The characteristic energies where the minima of the 6 ions H, He, N, Si, Ca and Fe occur are, respectively : $1.1 \times 10^{15}$ eV, $2.6 \times 10^{15}$ eV, $1.2 \times 10^{16}$ eV, $2.4 \times 10^{16}$ eV, $3.9 \times 10^{16}$ eV, and $4.5 \times 10^{16}$ eV. The decreasing segment of $N_F$ from $10^{12}$ eV up to $E_{e\text{min}}$ is due to the increasing nuclear cross sections with energy while the rising segment above $E_{e\text{min}}$ is predominantly caused by the vanishing efficiency of the galactic magnetic field with rising energy to retain particles for long times.

Escaped particles populating the extradisc space may re-enter the disc in the scheme delineated in Section 2, in a variety of circumstances. Let be $P_R$ the mean probability for ions of energy $E$ of reaching the Earth from the disc boundary. A cosmic-ray trajectory consists of an initial point (the source) and a termination point, which is the location where the nuclear collision occurs within the disc volume. The calculation of $P_R$ exploits the approximate symmetry of the cosmic-ray trajectories contained in the disc volume between the initial point $X_I$ and the final point $X_F$ of the trajectory. Samples of $2 \times 10^5$ particle are injected from the solar system position and the trajectories reconstructed in the disc volume. Let $N_I$ be the number of ions stopping in the disc volume by nuclear collisions and $N_T$ the total number of trajectories (or injected particles). The function $N_I$ is of fundamental importance in the present calculation since it incorporates the effect of the magnetic field, the dimension of the disc, the variation of the nuclear cross sections with the energy and the position of the solar system in the Galaxy. The calculation is finally accomplished by taking advantage of the inversion of the particle trajectories, the interchange of $X_F$ with $X_I$, as explained in detail elsewhere [35].

Figure 17 reports the functions $N_I$ for 3 representative ions H, He and Fe taken as reference examples. The $N_I$ profile of any ion has a high plateau, a quite small rise controlled by nuclear cross sections, a rapid and high descend, a minimum, and finally, a modest increase up to the highest energies. For example, the number of He interactions in the disc, in the interval $5 \times 10^{11}$-$10^{15}$ eV, increases by 3.5 per cent. At $10^{12}$ eV the number of He particles interacting in the disc volume is 173000 out of 200000 injected from the Earth (hence, $N_T=2 \times 10^5$ and $N_I=1.73 \times 10^5$). At $5 \times 10^{15}$ eV where the minimum of $N_I$ for He nuclei occurs, the number of He interactions reduces to 2000 (hence $N_I=2000$). Therefore, the probability for He particles of reaching the solar cavity, having the initial points of the trajectories at the disc boundary, is : $P_R = 1 - (N_I/N_T)$. With the figures given above, the probability $P_R$ is 0.11 at $10^{12}$ eV and 0.99 at $10^{18}$ eV. By this procedure the 6 probability functions versus energy $P_R$ are obtained and shown in figure 18. The number of nuclear collisions taking place in the disc volume shown in figure 17 (see figure 6 of ref. [H]) is converted in terms of probability $P_R$ in figure 18. The normalization of the functions...
in figure 17 is explained in detail on the investigation on the origin of the knee (Section 4 of ref. [3]). Thus, the functions in figure 17 are closer to the simulation of the cosmic-ray trajectories and they facilitate the comprehension of the processes causing the ion knees [3], while those in figure 18 more specifically relate to the present study.

Figure 19. Particle fraction, $f$, versus energy of the extradisc component reaching the solar cavity from the disc boundaries (Halo of the Milky Way). The quantity $f$ is defined as $f = N_e T$ where $N_e$ is the number of particles escaped from the disc and $T$ the transmission probability (see fig. 18) from the disc boundaries to the solar cavity at a specified energy.

Figure 20. Small differences of $\pm 0.1 \%$ in the spectral indices at $10^{12}$ eV will reverberate large alterations of the ion abundances around the ankle energy region. The grid of red and blue curves, specifying H and Fe ion abundances versus energy, result from the change, by arbitrary steps of 0.02 of the H and Fe indices, with respect to the He index taken as reference value (see text for details). While the Fe and H indices are varied by small amounts of 0.02, 0.04, etc. with respect to $\gamma_{He}$, in order to map how ion abundances vary with energy. The result is displayed in figure 20 by the grid of red and blue curves in the interval $10^{12.5} \times 10^{19}$ eV. For example, with a change of 0.08 in the index, the H/Fe abundance ratio of 19.9 (arbitrary value) at $10^{12}$ eV becomes 0.23 at $10^{19}$ eV, a factor 86.5 minor. Such a figure represents a large variation of the chemical composition. Note that at very high energy, above $10^{19}$ eV, the abundance ratios outside the disc volume are the same existing at the cosmic-ion sources in the disc, since particle displacement is unaffected by nuclear collisions, due to the small grammage traversed.

Measurements of the spectral indices in the energy region $10^{11} - 10^{15}$ eV acquire a fundamental
importance because their values determine the chemical composition at very high energy. The postulate of *Constant Spectral Indices*, incorporating the balloon and satellite data below $10^{15}$ eV, relates the chemical composition at low energy to that at very high energy by logical necessity avoiding any *ad hoc* mechanisms, at small, arbitrary energy bands, to explain the chemical composition of the cosmic radiation.

7. The Chemical composition of the extradisc component

The chemical composition of the extradisc component is radically different from that of the disc component. Heavy ions, escaping from the disc volume and penetrating through the disc of the Milky Way from its exterior, pay twice a tribute to larger nuclear interaction cross sections with respect to light ions. Therefore, the heavy-to-light ion flux ratio inevitably augments above $10^{15}$ eV up to the ankle energy region.

Figures 21 and 22 report the energy spectra of the extradisc ions reaching the solar cavity for the two *LE* and *HE4* blends. Firstly, note that a simplified situation takes place above the maximum of the Fe energy spectrum located at $2 \times 10^{19}$ eV. The ion abundances above this energy coincide with those at $10^{12}$ eV reported in Table 1. Above $2 \times 10^{19}$ eV the grammage is so small that nuclear interactions in the interstellar medium do not alter the chemical composition of the extradisc component existing at the outskirts of the disc volume. The complete spectrum in figure 21 and that in figure 22 are normalized at the arbitrary energy of $10^{19}$ eV to the galactic flux of $5.35 \times 10^{14}$ (LE) and $3.91 \times 10^{14}$ (HE4) particles/m$^2$ sr s eV$^{1.5}$.

Figure 23 shows ion fractions versus energy of the extradisc component for 3 groups of ions: light, intermediate and heavy being, respectively, (H+He), (CNO + Ne-S) and (Ca + Fe). This ion partition is adopted in some experiments. Let us now analyze the ion fraction profiles starting from the extreme high energy where the situation...
Figure 23. Relative amounts of light, intermediate and heavy ions versus energy for the LE (thin curves) and HE4 (thick curves) ion blends generated by the extradisc component.

is simpler.

Just above $2 \times 10^{19}$ eV ion abundances of the extradisc component reaching the solar system are equal to those of the cosmic-ray component present in the parent galaxy, because particle displacement through the disc (escaping and entering) do not alter ion abundances. Around $2 \times 10^{19}$ eV terminates the increase of 2 orders of magnitude of the (Ca+Fe) ion fraction. The (Ca+Fe) fraction above $5 \times 10^{18}$ eV does not surpass 17 per cent, even with an increase of a factor 66 from the low level flat fraction, below $5 \times 10^{16}$ eV. The intermediate ions, i.e. the sum of CNO and (Ne-S) group of nuclei, has a similar pattern with a step, between the low and high level, of a factor 25, which constitutes 31 per cent of the extradisc component above $5 \times 10^{18}$ eV.

From the profiles of the ion fractions shown in figure 23 or from the more detailed ion spectra shown in figures 21 and 22, the $\langle \ln(A) \rangle$ of the extradisc component is directly calculated; it is shown in figure 24 (pink curve, HE4 blend). The same figure 24 shows, for visual comparison, the corresponding $\langle \ln(A) \rangle$ resulting from the disk component (blue curve, blend HE4), which has a quite different pattern. The $\langle \ln(A) \rangle$ for the LE blend of $I_{ed}$ has a quite similar pattern (fig. 10). The two $\langle \ln(A) \rangle$ profiles of the disc and the extradisc components in figure 24 are normalized at the same cosmic-ray intensity at Earth.

The $\langle \ln(A) \rangle$ of the extradisc component exhibits a constant value below $5 \times 10^{16}$ eV, followed by a rising trend in the interval $5 \times 10^{16}$ - $2 \times 10^{19}$ eV with an approximate constant slope. Above $2 \times 10^{19}$ eV the $\langle \ln(A) \rangle$ stabilizes to the constant value of 2.1. The physical phenomena shaping this silhouette have already been discussed. A simplified, instant, qualitative comprehension might be depicted in terms of grammage encountered by the cosmic rays during the displacement in the Galaxy. When the grammage is large (see figure 16 ref. [3]) heavy nuclei are destroyed, when the grammage is minimum (0.006 g/cm²) heavy nuclei travel undisturbed like light
nuclei and the persists unchanged regardless of the energy. In the intermediate region $5 \times 10^{16}$- $2 \times 10^{19}$ nuclear cross sections and geometrical factors compete and interplay, forging the chemical composition with the stable, rising trend shown in figure 24.

When the disc and extradisc components are combined together for different values of $r=I_{ed}/I_d$, it results the surprising $<\ln(A)>$ profiles shown in figure 25. For $r=1$ the flux in the solar cavity is $1.8 \times 10^{14}$ particles/(m$^2$ sr s eV$^{1.5}$) at 10$^{19}$ eV for both disc and extradisc components. The surprising aspect of the $<\ln(A)>$ profiles in figure 25 resides in the fact that they thoroughly resemble to the profile of the experimental data on $<\ln(A)>$ extracted from the measurements of $X_{max}$ of the Auger experiment.

Flux measurements of the cosmic radiation at Earth imperatively constraint $r$, the maximum value of the extradisc component, as evident in figure 26. The sum of the fluxes of the disc and extradisc components is shown in figure 26 along with flux data of two experiments, taken as an example. Magnifying the extradisc component to match the Auger data on $<\ln(A)>$ would entail a conflict with the observed fluxes.

Notice that the theoretical spectra in figure 26 intrinsically exhibit the adequate change in the slope around $4 \times 10^{18}$ eV, from 3.2 to 2.7, which constitutes the ankle. Similarly, around $10^{15}$ eV, the slope of the complete spectrum changes from 2.7 to about 3.0 which is a characteristic features of the knee.

With the assumptions adopted in this paper any types of extradisc component generate a rather light chemical composition below $10^{17}$ eV due to the dominant role of nuclear cross sections suffered by heavy ions while propagating from periphery to disc core.
8. Comparison between theoretical and measured $<\ln(A)>$ with the disc and extradisc components

In the following the experimental data on $<\ln(A)>$ are compared with the profile derived from the theory in the condition $r=2$. Note that the $<\ln(A)>$ in the $r=1$ or $r=3$ would describe the experimental data as well, due to the large systematic uncertainties inherent to the hadronic models describing atmospheric showers. The value $r=2$ is regarded here as an adequate compromise between cosmic-ray intensity (see fig. 26) and the depth of the minimum of the $<\ln(A)>$ profile (see fig. 25).

In all the subsequent figures 27, 28, 29, 30, 31, 32, and 33, where high energy data on $<\ln(A)>$ above $10^{17}$ eV are examined and compared with the theory, the $<\ln(A)>$ determined by the Kascade experiment in the range $10^{15}$-$10^{17}$ eV is displayed as adequate, reference measurement.

From the Kascade Collaboration three different procedures to determine $<\ln(A)>$ are available called here Sybill algorithm, QGSjet algorithm and deconvolution method. Data of the first two methods are in figure 3 and 27 while those of the deconvolution method, which covers the largest range, i.e. $9 \times 10^{14}$-$1.4 \times 10^{17}$ eV, are in figure 28. The resulting $<\ln(A)>$ profiles from the three methods of measurement by Kascade are simultaneously shown elsewhere (see figure 24 of ref. [11]) exhibiting a coherent silhouette in the entire interval $10^{15}$-$10^{17}$ eV.

Recent measurements of $X_{\text{max}}$ of the Auger Collaboration converted into $<\ln(A)>$ by the reference profiles for H and Fe shown in figure 14 are given in figure 27. Two predicted $<\ln(A)>$ profiles according to the Theory of Constant Indices with $r=2$ and $r=10$ are shown in fig. 27. In the condition $r=2$ the Auger data in the interval $4 \times 10^{17}$-$10^{19}$ eV are still below the theoretical
The $\langle \ln(A) \rangle$ extracted from the $X_{\text{max}}$ measured by the Fly’s Eye experiment [31] is shown in figure 28. The increasing trend of $\langle \ln(A) \rangle$ in the interval $10^{17}$-6 $\times$ $10^{17}$ eV contrasts with the opposite trend of HiRes and Agasa-Akeno experiments (see figs. 29 and 32); it also disagrees with a flat silhouette observed by Yakutsk in the same energy interval, and with a value of 3.5 measured by Kascade. The behavior of $\langle \ln(A) \rangle$ in figure 28 is reminiscent of that observed by the Haverah Park experiment (see fig. 31) in the same energy interval, where a lack of uniformity in the detector acceptance is suspected.

The profile of $\langle \ln(A) \rangle$ extracted from the $X_{\text{max}}$ measured by the HiRes experiment is shown in figure 29. The HiRes data in figure 29 include HiRes Prototype [30], HiRes Stereo [37] and a data revision which takes into account acceptance correction and detector performance.

Figure 29. Comparison of the theoretical profile of $\langle \ln(A) \rangle$ (red curve) with the empirical one extracted from the data of HiRes experiment before (blue data points) and after (red data points) acceptance corrections.

Figure 30. Comparison of the theoretical profile of $\langle \ln(A) \rangle$ (red curve) with the empirical one extracted from the data of the Yakutsk experiment (blue small circles).

Around $4 \times 10^{18}$ eV data reported in figure 29 have a mean value of $\langle \ln(A) \rangle$ of about 1. This tiny value would correspond to the minimum observed by Auger at $(2−4) \times 10^{18}$ eV as far as a vague increase of $\langle \ln(A) \rangle$ would delineate in the band $5 \times 10^{18}$-4 $\times$ $10^{19}$ eV. The chemical composition extracted from the $X_{\text{max}}$ measured by HiRes is lighter than that resulting from the highest permissible values of the parameter $r$ (dominance of extradisc component) in the interval $10^{17}$-10$^{19}$ eV. It is also lighter, by about one unit of $\langle \ln(A) \rangle$, than that of Auger, Fly’s Eye, Haverah Park and Agasa experiments (see figure 4 of ref. [39].)
9. Comparison between theoretical and measured $<\ln(A)>$ in Haverah Park, Agasa and Volcano Ranch

The partition of the cosmic nuclei in two groups, light and heavy, with fractions $F_p$ and $F_{Fe}$ respectively, characterizes the chemical composition extracted from Haverah Park, Agasa and Volcano Ranch data samples. This bi-modal data analysis, though appropriate for some detectors, has a clear bias since it gives intrinsically higher $<\ln(A)>$ than realistic ion groupings as demonstrated below.

The chemical composition measured by the Haverah Park experiment is reported in figure 31. Ruling out the data points at the extreme energies, below $2 \times 10^{17}$ eV and above $10^{18}$ eV, where detector acceptance for atmospheric showers might have been deteriorated (See Section 3 ref. [40]), the agreement with the theoretical $<\ln(A)>$ profile in the condition $r=2$ is excellent. For protons and Fe nuclei the differences in the theoretical $X_{\text{max}}$ profiles shown in figure 12 for Sibyll and QGSjet codes are small in the interval $10^{17}-10^{18}$ eV, and so the agreement between theory and Haverah Park data is quite stable. It has been remarked [41] the critical role of hadronic models in the partition of all cosmic-ray nuclei in two classes, light ($F_p$) and heavy ($F_{Fe}$). Adopting the code $QGSjet98$ it results a proton fraction $F_p$ of .34 while the code $QGSjet01$ gives $F_{Fe} = 0.48$ [41].

Data samples collected by the Agasa Collaboration have been revisited in 2003 [42] in order to determine the $X_{\text{max}}$ of the cosmic radiation in the range $10^{17}$-$10^{20}$ eV. The chemical composition re-
Figure 33. Comparison of the theoretical profile of \(<\ln(A)\) (red curve) with the empirical one extracted from the data of Volcano Ranch experiment (blue cross in the rectangle). The data point corrected according to the Theory of Constant Spectral Indices is represented by a pink cross in the rectangle.

The partition of atmospheric showers in two groups according to the quoted re-analysis of the Agasa data \cite{42} has been here refined with 6 particles (or 6 ion groups) giving the outcome shown in figure 32 by pink square data points. In this refinement the Theory of Constant Spectral Indices with the HE4 blend has been used. The resulting \(<\ln(A)\) profile (pink squares in fig. 32) is systematically shifted downward by about one unit resulting from this data re-elaboration \cite{43} is shown in figure 32 by blue squares. The global profile of the empirical \(<\ln(A)\) in figure 32 would resemble to that measured by Auger in a significant aspect: \(<\ln(A)\) decreases above the energy of \(10^{17}\) eV up to \((2 - 3) \times 10^{18}\) eV, where a minimum is attained. In the interval \(2 \times 10^{18} - 2 \times 10^{19}\) eV, with some imagination, an increasing trend would faintly delineate, being in accord with the \(<\ln(A)\) profile of the Auger experiment above \(3 \times 10^{18}\) eV (see fig. 27).

Figure 34. Measurements of the chemical composition of the cosmic radiation by HiRes (pink dots) and Auger (black dots) experiments using the mean width of the longitudinal profile of the fluorescence light generated in atmospheric showers. The theoretical mean width in \(g/cm^2\) versus energy evaluated by Heck \cite{30} for protons and Fe nuclei with QGSJet and Sibyll algorithms is also shown as turquoise bands.

\footnote{Unfortunately, an heavy ion fraction \(F_{Fe}\) higher than 1 results in figure 4 of ref. \cite{42}, which signals a logical incoherence whatsoever in the data elaboration materializing an unphysical situation. The incoherence is not easy to be eliminated since reverberates elsewhere \cite{46}. Probably, an additional correction of a few \(g/cm^2\) in the \(X_{max}\) derived from the Mocca-Sibyll code would eliminate the Fe fractions higher than 1 in fig. 4 of the quoted reference \cite{42} or in fig. 4 of ref. \cite{44}. Note that this additional correction probably is unnecessary since the oversimplified data elaboration with only two ion groups (instead of 6 or more) entails a bias in \(<\ln(A)\) which is clearly demonstrated.}
Figure 35. Evidence for the present uncertainties in cosmic-ray flux measurements in different experiments using giant terrestrial cascades in air above $10^{17}$ eV. The spectrum (black curve) resulting from the Theory of Constant Spectral Indices is normalized at $10^{12}$ eV (black square) with a flux of $3.79 \times 10^{17}$ particles/($m^2$ sr s eV$^{1.5}$) and it joins the Kaskade data in an excellent accord.

A consistency verification of the chemical compositions derived from the Agasa-Akeno data [43] and that extracted by $X_{\text{max}}$ measured by Fly’s Eye has been performed in 1998 [44]. Both the radial muon density from the cascade core $\rho_{\mu}$ and the $X_{\text{max}}$ have been simulated with the same hadronic code, Mocca-Sibyll. Calculations have been adapted to Akeno (A1), Agasa (A100) and Fly’s Eye detectors including muon thresholds, trigger efficiencies and other instrumental constraints, which are significant. Grouping all nuclei in two fractions, light and heavy, so that $F_p + F_{Fe} = 1$, it results a fraction $F_{Fe}$ greater than 1 for the Fly’s Eye data around $10^{17}$ eV. In order to remove this incoherence a shift of $-30$ $g/cm^2$ at $10^{17}$ eV, and similarly at higher energies. Such a difference would alleviate, as a gratuity, the gap between the $<\ln(A)>$ predicted by the Theory of Constant Spectral Indices with $r=2$ and the Auger data (see fig. 27).

An elaboration and revision of atmospheric shower data of Volcano Ranch experiment operated in the period 1959-1964 has been performed in 1975 using a bi-modal analysis [45]. Figure 33 shows the theoretical profile (red curve) along with the data point of Volcano Ranch (blue rectangle) spanning the interval $5 \times 10^{17}-10^{19}$ eV. The resulting $<\ln(A)>$ has a mean value of 3, which indicates that the cosmic radiation at $10^{18}$ eV is still dominated by intermediate and heavy nuclei. If the same procedure of ion ponderation applied to the Agasa data is also applied to Volcano Ranch data with the $HE_4$ ion blend, the mean value of $<\ln(A)>$ decreases from 3 to 2.4 (pink cross in the rectangle) approaching the theoretical value of 2.5 as displayed in figure 33.

10. Reported measurements of the mean atmospheric depth of the cosmic radiation by fluorescence light in HiRes

Though the longitudinal profile of giant atmospheric showers reconstructed by the fluorescence light is rather insensitive to the hadronic models employed in the data analysis, it would seem that some instrumental unknowns or uncontrolled measurement procedures still plague the determination of $X_{\text{max}}$ by fluorescence light.

Preliminarily, notice that experiments taking advantage of the fluorescence light in the measurements of $X_{\text{max}}$ like HiRes do not agree one another. The $X_{\text{max}}$ observed by the HiRes experiment [38] is larger than that measured by Auger [15] which uses the fluorescence light as well. As an example, the Fly’s Eye data on $<\ln(A)>$ [31] in the energy decade $3 \times 10^{17}-3 \times 10^{18}$ eV shown in figure 28 differ from those of HiRes Prototype

in figure 32 or in figure 33 with the Volcano Ranch data.
(figure 29) by one unit of $<\ln(A)>$ which is quite large by any standard, theoretical, empirical or instrumental.

The measurements of $X_{\text{max}}$ by florescence light demands the interpolation of the longitudinal profile by an appropriate function $f_{\text{LP}}$ which has a characteristic width denoted here $\sigma(X_{\text{max}})$. The average value of the distribution of $\sigma(X_{\text{max}})$ at a given energy is a measurement of the chemical composition. The observed $\sigma(X_{\text{max}})$ may be cross-check method in the measurements of the chemical composition with the same data samples.

Presently (2009), the available measurements of $\sigma(X_{\text{max}})$ of the HiRes Collaboration \cite{47} disagree with the analogous data of the Auger experiment \cite{37} as shown in figure 34. Note that the theoretical $\sigma(X_{\text{max}})$ profiles shown in figure 34 derive from the same sample of the simulated cascades used to calculate the $X_{\text{max}}$ profiles of figure 12. The $\sigma(X_{\text{max}})$ profile measured by HiRes is rather flat around the value of 50 $g/cm^2$ in the interval $10^{18} - 5 \times 10^{19}$ eV while Auger data (full black dots) exhibit a clear decreasing profile from a maximum of 58 $g/cm^2$ down to a minimum of 22 $g/cm^2$. Imagine a linear scale of energy between $10^{18} - 5 \times 10^{19}$ eV in figure 34 and the disagreement between the two experiments dilates, since the initial agreement below $4 \times 10^{18}$ eV is confined in less than 5 per cent of the explored energy band.

At the energy of $10^{17}$ eV the profile of $<\ln(A)>$ measured by Kascade joins that of the HiRes experiment \cite{56} (fig. 29). The average value of the $<\ln(A)>$ of the Kascade experiment is about 3.1 (average value of 3 methods of measurements) while that of HiRes is close to 2.5. It is important to emphasize the disagreement, or the potential disagreement, between these two experiments on $<\ln(A)>$ around $10^{17}$ eV. The knees of the light ions have been observed by a number of experiments above $10^{15}$ eV \cite{19,30}. This fact necessarily implies an increase of $<\ln(A)>$ above $10^{15}$ eV, since the disappearance of light ions automatically enhances the heavy ion fraction. Knowing that at $10^{15}$ eV the value of $<\ln(A)>$ is certainly comprised between 1.74 and 1.78 as inferred by extrapolating balloon and satellite data \cite{52}, any estimate of $<\ln(A)>$ at $10^{17}$ eV leads to the value of 3.1 and not a lower value as HiRes Prototype data (2004) \cite{36} around $10^{17}$ eV in figure 29 would suggest.

As apparent in figure 29 the mean values of $<\ln(A)>$ of the HiRes experiment after the acceptance correction \cite{38} are globally displaced downward by some 0.5 units. The shift is surprisingly large compared to the reported error bars before and after acceptance corrections \cite{38}. The large excursions in $<\ln(A)>$ (blue dots) also signal that the global detector performance suffered from residual uncontrolled unknowns.

The correction of the detector acceptance \cite{38}, a routine effect applied after 3 years from the original measurement \cite{37} does not preclude future data re-analysis.

In synthesis, the HiRes data pertaining the chemical composition of the cosmic radiation disagree \cite{54} (1) with the Auger data on $X_{\text{max}}$; (2) with the Auger data on $\sigma(X_{\text{max}})$ (see figure 34 and the related text); (3) with those of all other experiments on $<\ln(A)>$ above $10^{17}$ eV (Volcano Ranch, Haverah Park, Akeno, Agasa, Fly’ s Eye and Yakutsk); (4) with the Kascade data at $10^{17}$ eV. Theoretically, a bias exists in the conversion of $X_{\text{max}}$ into $<\ln(A)>$ made by the HiRes Collaboration because, out of all current versions of

\footnote{The resonance of this conclusion tunes up with the criticism expressed by A. A. Watson \cite{51} on Fly’ s Eye and HiRes experiments: \&... By contrast, with florescence detectors the aperture continues to grow with energy and remains considerable uncertainty about the HiRes aperture. Extensive Monte Carlo calculations must be made to establish it and these make assumptions about the slope of the spectrum and about the primary mass, with protons being assumed on the basis of the claims from the Fly’ s Eye and HiRes experiments ignoring often contradictory evidence from other experiments without discussion \&.}
hadronic codes adopted to simulate nuclear interactions in air (e.g. QGSJet, Sibyll, etc.), those adopted by HiRes tend to generate protons and depress heavy ions (see fig. 8 of ref. 10 and fig. 4 ref. [39]).

Energy (eV)

| Heck evaluation | QGSjet | Sibyll | Auger |
|----------------|--------|--------|-------|
| QGSjet01 (2008)|        |        |       |
| Hörandel - QGSjet01 model 3 (modified) |    |        |       |
| model 3 |        |        |       |
| model 1 (original QGSjet) |    |        |       |

Figure 36. Theoretical difference of the atmospheric depth for protons and Fe nuclei according to a number of evaluations. The model QGSJet-03 (thick pink curve) [52] would reduce the discrepancy between Auger data on $<\ln(A)>$ and the corresponding profile derived from the Theory of Constant Indices.

11. Conclusions.

The comparison of the chemical composition of the cosmic radiation expressed by $<\ln(A)>$ derived from the Theory of Constant Indices with the observed $<\ln(A)>$ extracted from the Auger data on $X_{\text{max}}$ dictates the necessity of introducing an extradisc component of the cosmic radiation above $10^{17}$ eV. In Sections 1 and 2 it is explained why this component is better termed *extradisc component*, $I_{\text{sd}}$. The variable $r$ is a free parameter of the theory defined as the extradisc-to-disc flux ratio in the solar cavity at $10^{19}$ eV, e.g. $r=I_{\text{sd}}/I_{d}$.

From the ensemble of the profiles of $<\ln(A)>$ extracted from the measurements of $X_{\text{max}}$ of the Auger, Yakutsk, Fly’s Eye, Agasa, Akeno, Haverah Park and Volcano Range experiments it emerges a global accord with the theory.

In the energy interval $10^{15}$-$10^{17}$ eV there is an excellent agreement with the observations of Eas-top, Kascade, Tunka (see fig. 3) and other experiments. Below $10^{17}$ eV the extradisc component is a negligible fraction of the disc component $I_d$, and as a consequence, the theoretical and empirical analysis reported in the companion paper [8] remains valid.

Because of the unknowns entailed in the conversion of $X_{\text{max}}$ into $<\ln(A)>$ above $10^{17}$ eV, the accord between the theoretical profile and data in figures 27, 28, 30, 31, 32 and 33 is more than satisfactory. Values of $r=I_{\text{sd}}/I_d$ higher than 2 would alleviate the gap between the empirical profile of $<\ln(A)>$ and the theory, as shown in figure 27. Presently, values of $r$ higher than 2 are neither convincing nor necessary, due to large systematic errors in the experimental data. The last two figures 35 and 36 intend to illustrate this position.

Figure 35 depicts in a flash the present experimental uncertainties in flux measurements quoting arbitrarily data from Kascade, Yakutsk and Auger experiments. Fluxes from other experiments do not agree as well. Such uncertainties impede to identify a more precise and reliable value of the ratio $r$ in figure 26 on an empirical basis, being $r=2$ the present adequate global estimate.

Let be $X_{\text{cal}}^{\text{cal}}$ (H) the theoretical atmospheric depth for protons, $X_{\text{cal}}^{\text{cal}}$ (Fe) that for Fe nuclei and $D_{\text{max}}$ the difference between the two depths i.e. $D_{\text{max}} = X_{\text{cal}}^{\text{cal}}$ (H) - $X_{\text{cal}}^{\text{cal}}$ (Fe) at a given energy $E$. Error sources generating differences in $<\ln(A)>$ in theories and in experiments might
arise both from $X_{\text{cal}}^{\text{max}}(H)$ and $D_{\text{max}}$ as well. Ideally, different hadronic models may result in a combination of $X_{\text{cal}}^{\text{max}}(H)$ and $D_{\text{max}}$ having the same $<\ln(A)>$. Figure 36 reports the $D_{\text{max}}$ versus energy in some current hadronic models and, at the arbitrary energy of $10^{17.5}$ eV, the $X_{\text{cal}}^{\text{max}}(H)$. Figures 36 and 12 show that the $X_{\text{cal}}^{\text{max}}(H)$ and $D_{\text{max}}$ profiles used in this paper for the conversion of $X_{\text{max}}$ into $<\ln(A)>$ are quite similar to those adopted by others (for example, Auger).

Notice that the hadronic code called QGSjet model-03 [52], having a larger $D_{\text{max}}$ of 120-130 g/cm$^2$ in the relevant range $10^{17.5} \times 10^{19}$ eV and a deeper $X_{\text{cal}}^{\text{max}}(H)$ of 739 g/cm$^2$ at $10^{17.5}$ eV (see figure 36), it would shift upward the simulated $X_{\text{max}}$ profiles in figure 12 by 9 g/cm$^2$ at 4.12 \times 10^{17}$ eV alleviating the gap between Auger data and theory. As far as uncertainties in hadronic models above $10^{17}$ eV used in the analysis of giant atmospheric cascades remain large, the determination of a value of $r$ higher than 2 based on the $<\ln(A)>$ data would appear premature. This same conclusion is suggested by the flux measurements in figure 35.

Let us finally mention the systematic disagreement between the $<\ln(A)>$ extracted from the $X_{\text{max}}$ measured by the HiRes experiment and theory, amounting to about one unit in the entire interval $10^{17.5} \times 10^{19}$ eV (see fig. 29). The examination of the HiRes data on $<\ln(A)>$ made in Section 9 suggests a number of inconsistencies with the data of numerous experiments and with different versions of the HiRes experiment itself (HiRes Prototype, HiRes Stereo, data revision [35]).

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