Structure of exotic nuclei in the $sd$-$pf$ shell region and its relation to the effective interaction

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Abstract.
We investigate the shell evolution in unstable nuclei with large-scale shell-model calculations. Since the tensor force has been pointed out to be promising in accounting for the shell evolution, we construct a new interaction in the $sd$-$pf$ shell region whose tensor force in the cross-shell part is determined with attention. The resulting strength is much stronger than the Millener-Kurath (MK) interaction which has been often used as the cross-shell interaction, giving rise to a significant difference from that by MK in the proton shells in the $N = 28$ region and the nuclear structure of $^{42}$Si.

1. Introduction
Exploration of nuclei far from stability (i.e., exotic nuclei) has recently reached near the drip line in light mass regions such as the $p$- and $sd$-shell regions. The study has been clarifying several features characteristic of exotic nuclei, among which are the evolution of the nuclear shell structure (called shell evolution). The shell evolution stands for that the shell gap, even the shell ordering in some cases, is not stable but is significantly varying as the number of nucleons changes. The shell evolution can be caused by weakly binding nature of the exotic nuclei, where nucleons near the Fermi surface are much influenced by the diffuseness of the nuclear surface. In that case, it is likely that the evolution takes place in a monotonous way as neutrons are added toward the drip line.

On the other hand, another mechanism exists to bring about the shell evolution: the nuclear force, whose relation to the nuclear structure is one of the main subjects discussed in the symposium. Since the nuclear force is dominated by two-body interaction, what gives rise to the shell evolution due to the nuclear force is predominantly the dependence of mean force on the orbits to be occupied. If each nucleon is assumed to occupy an orbit whose radial form is unchanged against the change of the nucleon number, the situation becomes simplified: only the mean interaction between the valence orbits is responsible for the shell evolution. This mean interaction is referred to as the monopole interaction in terms of the shell model.
The importance of the shell evolution due to the interaction has been investigated in the structure of exotic nuclei around $N = 20$ in the context of the so-called disappearance of the $N = 20$ magic number. It was pointed out that such an exotic structure appeared without a significant reduction of the $N = 20$ shell gap by the conventional picture “island of inversion” [1]. That conjecture indeed succeeded in explaining the disappearance of the $N = 20$ magic structure for typical nuclei not less than $N = 20$, such as the large deformation in $^{32}$Mg [2]. However, performing large-scale shell-model calculations by the Monte Carlo shell model (MCSM) [3], we presented that the disappearance of the magic number occurs earlier than the prediction by “island of inversion” [4] (see also [5]). We also claimed that this earlier onset of the disappearance is actually the manifestation of the narrowing $N = 20$ shell gap for exotic nuclei.

The narrowing $N = 20$ shell gap should be less dominated by the weak binding because the two-neutron separation energy is as large as $\gtrsim 5$ MeV in the relevant region. In terms of the nuclear force, it was pointed out that the $N = 20$ shell evolution can be explained by the spin-isospin dependence of the nucleon-nucleon interaction [6]. This effect was generalized to be a strong $T = 0$ monopole interaction between $j_>$ and $j_<$ orbits, accounting for not only the $N = 20$ case but also other similar phenomena including the disappearance of the $N = 8$ magic number. The study on the shell evolution has recently progressed in a more microscopic and quantitative direction: the dominance of the tensor force [7]. The tensor force leads to the similar shell evolution to that given by the spin-isospin dependent central interaction in [6] when only orbits with the same orbital angular momentum are involved. But the tensor force also gives rise to a significant attractive interaction between $j_>$ and $j'_<$ orbits when $j$ and $j'$ are different. It is thus of great interest to find the shell evolution caused by the strong attraction between $j_>$ and $j'_<$ orbits because that is a fingerprint of the significance of the tensor force in the formation of the shell structure.

In the present study, we perform shell-model calculations in the $sd$-$pf$ shell region with a new effective interaction, focusing on the role of the tensor force in the shell evolution. In Sec. 2, we show the construction of the interaction, paying attention to the proper strength of the tensor force. In Sec. 3, we show some results and discuss the shell- and nuclear-structure evolution around $N = 28$ related to the tensor force. Finally, we summarize this study.

2. Construction of a new $sd$-$pf$ shell interaction focusing on the tensor shell evolution

2.1. Tensor force in the effective interaction

As an important ingredient of the interaction, we first examine what is the proper strength of the tensor interaction. In Ref. [7], it was shown that the tensor force of the $\pi + \rho$ one boson exchange potential is quite effective in explaining the shell evolution in the medium-heavy mass region. The shell evolution in [7] was evaluated simply with the effective single-particle energy, so that it is not clear whether or not that of the shell model is similar.

To determine the strength of the tensor force in the realistic interaction, we take the GXPF1 interaction [8]. The GXPF1 interaction is an interaction whose matrix elements are determined, starting with a microscopic interaction, in an empirical way to reproduce experimental energies in the region. Its applicability is not restricted only to stable nuclei but does spread over unstable nuclei. Thus, since its spin-isospin property should be rather reliable, GXPF1 deserves a good measure of the tensor strength.

The tensor part is extracted from GXPF1 by using the spin-tensor decomposition [9] which enables to do even from a shell-model interaction given as a set of matrix elements in numbers. Table 1 compares the monopole centroids between $pf$-shell interactions of GXPF1, $\pi + \rho$ and Millener-Kurath (MK) [10]. Here, in the $\pi + \rho$, the cutoff at $0.7$ fm is assumed similarly to [7]. The result clearly shows that the tensor force in GXPF1 is very similar to the $\pi + \rho$ interaction.
Table 1. \( T = 0 \) monopole centroids between \( i \) and \( j \) orbits for the tensor part compared between GXPF1, \( \pi + \rho \) and MK. The unit is MeV. Only the monopole centroids associated with \( 0f_{7/2} \) are listed here.

| \( i \)   | \( j \)   | GXPF1 | \( \pi + \rho \) | MK  |
|---------|---------|-------|-----------------|-----|
| \( 0f_{7/2} \) | \( 0f_{7/2} \) | 0.223 | 0.210 | 0.080 |
| \( 0f_{7/2} \) | \( 1p_{3/2} \) | 0.036 | 0.035 | 0.013 |
| \( 0f_{7/2} \) | \( 0f_{5/2} \) | -0.335 | -0.315 | -0.120 |
| \( 0f_{7/2} \) | \( 1p_{1/2} \) | -0.073 | -0.070 | -0.026 |

Together with the good description by \( \pi + \rho \) tensor force in the medium-heavy mass region [7], the proper tensor strength in the effective interaction may be rather universal independent of the shell to be taken. On the other hand, the tensor force in MK, which has been widely used as the cross-shell interaction in some regions, is as weak as one third of \( \pi + \rho \). It is intriguing to look over in which nucleus the difference in the tensor force appears.

2.2. Other parts of the interaction and shell-model calculations

Since the present study aims predominantly to investigate the importance on the tensor shell evolution in the \( sd-pf \) shell region, we now let the interaction chosen as simply as possible. For the \( sd \)-shell and the \( pf \)-shell parts, USD [11] and GXPF1A (with a small modification by Honma et al. to GXPF1) are adopted without any further changes, respectively.

The cross-shell part is up to dated. As the tensor interaction, \( \pi + \rho \) is used for \( T = 0 \), and no tensor force for \( T = 1 \). Here, the vanishing \( T = 1 \) tensor force is assumed because in this part there is no clear relation between the empirical and the the microscopic interactions. This may be due to a large effect from the core polarization in the realistic interaction that is more dominant in the \( T = 1 \) part. Anyway, there is still some ambiguity in the tensor force from a very quantitative point of view, but it is not probably so weak as that of MK. As the spin-orbit part, we adopt the M3Y interaction [12]. For the central part, we construct a new interaction based on the MK: the same functional form but renewed strengths. The strength is determined so as to roughly simulate the central part of GXPF1 for the \( pf \) shell, and the same parameters are also adopted in the calculation of the cross-shell interaction. We find that relative strength among different \( l \)'s is improved by introducing a density dependent interaction such as the FPD6 interaction [13].

The single-particle energies on top of the \( ^{16}\text{O} \) are determined in a natural way as follows. For the \( sd \)-shell part, they are the same as those taken in USD. For the \( pf \)-shell part, they are fixed so that the single-particle structure around \( ^{48}\text{Ca} \), such as its \( 2^{+}_{1} \) energy and low-lying levels of \( ^{47,49}\text{Ca} \), is reproduced as well as that obtained by the original GXPF1A interaction.

In the present study, we focus on the shell evolution from near \( N = 20 \) to \( N = 28 \) for isotopes having systematic experimental data. For these isotope chains, we can assume that the effect of the excitation from the \( N = 20 \) shell gap is not so large. Here, it is a good approximation to truncate the model space not to allow the excitation across the \( N = 20 \) shell gap. The model space with this truncation can be dealt with modern shell-model codes, so that we perform the shell-model calculation with the code MSHELL [14].

3. Some results and discussions

It is known that the proton shell structure concerning the \( sd \) shell visibly evolves as valence neutrons occupy the \( pf \) shell: the gap between \( 0d_{3/2} \) and \( 1s_{1/2} \) is considerably reduced when
approaching $N = 28$, as exemplified in the inversion of these single-particle levels in $^{47}\text{K}$. Figure 1 presents this shell evolution compared between experiment and shell-model calculation with the present interaction. The experimental shell evolution is fairly reproduced, deviating from the calculation by so small as $\sim 300\text{ keV}$ as seen in the spectra of $^{47}\text{K}$ and $^{43}\text{P}$.

This reduction between $0d_{3/2}$ and $1s_{1/2}$ is rather sensitive to the tensor force. Whereas the tensor force between $0f_{7/2}$ and $1s_{1/2}$ always vanishes, that between $0f_{7/2}$ and $0d_{3/2}$ does not and gives an attractive monopole interaction with a reasonable force. Thus, it pulls down the proton $0d_{3/2}$ orbit as neutrons occupy $0f_{7/2}$. With the $\pi + \rho$ tensor force, since the relevant $T = 0$ monopole centroid is $-0.31\text{ MeV}$, the tensor force roughly accounts for that shell evolution by $-0.31 \times 8 \times \frac{1}{2} = -1.24\text{ MeV}$ that is almost half of the total value.

While the tensor force reduces the gap between $0d_{3/2}$ and $1s_{1/2}$, it also reduces that between $1s_{1/2}$ and $0d_{5/2}$ inevitably due to the identity given in [7]. Hence, this should produce a different behavior around $Z = 14$, where $0d_{5/2}$ is fully occupied, when approaching $N = 28$.

Figure 2 shows the systematics of the $2^+_1$ levels in $N = 28$ isotones going down from $Z = 20$ to 14. The calculation gives a good agreement with the experimental $2^+_1$ energy levels for the isotones having experimental data, suggesting a good $N = 28$ and proton shell structures for these isotopes. Although there is some deviations for the $B(E2)$ values, it may be remedied by tuning the effective charges that must be determined with a more systematic study.
At $Z = 14$, we see an interesting phenomenon: the $2^+_1$ energy is located rather low although there is a certain $Z = 14$ shell gap. Indeed, that isotope was reported to appear a new magic nucleus based on two-knockout reaction [16], and similarly it was forecast by shell-model calculations [17]. In the present calculation, the structure of the $^{42}$Si nucleus is very sensitive to the tensor force reducing the proton gap between 1$s_{1/2}$ and 0$d_{5/2}$. With the MK interaction having a weak tensor force, the $2^+_1$ level is calculated to lie at around 2 MeV which is close to those of previous shell-model studies [17].

To understand the situation in terms of the deformation, the potential energy surface (PES) is useful even in the shell-model calculation [18]. In Fig. 3, the PES is drawn with the constrained Hartree-Fock method for $^{42}$Si. A competition is seen between the spherical minimum and the oblate local minimum connected by a low barrier in between. At the oblate local minimum, the valence protons are excited from 0$d_{5/2}$ by about 1.5 on average, so that the reduction of the shell gap, traced back to the tensor force, is very sensitive to the competition. We should note that Azaiez and Dombrádi reported the $2^+_1$ energy level lower than 1 MeV obtained by a recent experiment [19]. Although the present calculation still overestimates it, a significantly improved value lower than 1 MeV can be obtained with a small tuning of the tensor force to adjust the proton shell evolution (see Fig. 1 and related discussions) because of the high sensitivity of the shell gap on the competition of deformation.

4. Summary

We have investigated the importance of the tensor force in the shell evolution and its impact on the nuclear structure in exotic nuclei using large-scale shell-model calculations. This is motivated by the previous studies that the need for the shell evolution originating from the effective interaction has been suggested in the disappearance of the $N = 20$ magic number around sodium region and that the tensor force accounts for its tendency. As a starting point of a unified $sd$-$pf$ shell interaction, we have constructed a new interaction with the cross-shell interaction having a proper tensor strength. The strength of the tensor force as the effective interaction seems to be close to that of the $\pi + \rho$, compared to that of the GXPF1 interaction. Adopting the $T = 0$ $\pi + \rho$ interaction as the tensor force of the new interaction, shell-model calculations from $N = 20$ to 28 have been performed. The tensor force carries a major part of the proton shell evolution in moving from $N = 20$ to 28, and the present interaction reproduces it well. The interaction affects the shell gap between 1$s_{1/2}$ and 0$d_{5/2}$ simultaneously, leading to a considerable deformation in $^{42}$Si which was reported in the present symposium.

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