Propagating Properties of a Partially Coherent Flat-Topped Vortex Hollow Beam in Turbulent Atmosphere

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Using coherence theory, the partially coherent flat-topped vortex hollow beam is introduced. The analytical equation for propagation of a partially coherent flat-topped vortex hollow beam in turbulent atmosphere is derived, using the extended Huygens-Fresnel diffraction integral formula. The influence of coherence length, beam order \( N \), topological charge \( M \), and structure constant of the turbulent atmosphere on the average intensity of this beam propagating in turbulent atmosphere are analyzed using numerical examples.

Keywords: Partially coherent flat-topped vortex hollow beam, Atmospheric turbulence, Laser propagation

OCIS codes: (010.1300) Atmospheric propagation; (010.3310) Laser beam transmission; (140.3295) Laser beam characterization

I. INTRODUCTION

Recently, much attention has been paid to the propagation properties of a laser beam in turbulent atmosphere [1]. It is found that the intensity and spreading of a laser beam are affected by atmospheric turbulence [2-8], and the laser beam with a vortex has been widely studied, due to its potential applications in free-space laser communication. In past years, Wang et al. studied the focusing properties of a Gaussian Schell-model vortex beam in experiments [9]. Zhou et al. studied the partially coherent hollow vortex Gaussian beam through a paraxial ABCD optical system in turbulent atmosphere [10]. Wang and Qian studied the spectral properties of a random electromagnetic partially coherent flat-topped vortex beam in turbulent atmosphere, based on the extended Huygens-Fresnel principle [11]. Gu studied the transverse position of an optical vortex upon propagation through atmospheric turbulence [12]. Zhou and Ru studied the angular momentum density of a linearly polarized Lorentz-Gauss vortex beam [13]. Huang et al. studied the intensity distributions and spectral degree of polarization of partially coherent electromagnetic hyperbolic-sine-Gaussian vortex beams through non-Kolmogorov turbulence using numerical examples [14]. Wu et al. developed an expression for the wandering of random electromagnetic Gaussian-Schell model beams propagating in atmospheric turbulence, and studied the properties of the beams [15]. Recently, a new dark hollow beam called the partially coherent flat-topped vortex hollow beam has been proposed, which has advantages over a flat-topped hollow beam, and which has potential applications in free-space wireless laser communication. However, to the best of our knowledge, the propagation properties of a partially coherent flat-topped vortex hollow beam in turbulent atmosphere have not been reported.

In this work, we first introduce the partially coherent flat-topped vortex hollow beam based on the theory of coherence, and then investigate its propagation properties in turbulent atmosphere.

II. PROPAGATION OF A PARTIALLY COHERENT FLAT-TOPPED VORTEX HOLLOW BEAM IN TURBULENT ATMOSPHERE

In the Cartesian coordinate system with the \( z \)-axis set as the axis of propagation, a circular or elliptical flat-topped vortex hollow beam in the source plane can be described as [16]

\[
E(r_0,0) = \sum_{n=0}^{N} \frac{(-1)^{n}}{N} \binom{N}{n} \exp \left[ -n \left( \frac{x_n^2}{w_x^2} + \frac{y_n^2}{w_y^2} \right) \right] \left( \frac{x_n}{w_x} + i \frac{y_n}{w_y} \right)^{M} \tag{1}
\]

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where $N$ is the order of the elliptical flat-topped vortex hollow beam, $M$ is the topological charge, $w_x$ and $w_y$ are the beam width in the $x$ and $y$ directions, respectively, and $\binom{N}{n}$ denotes the binomial coefficient.

Based on the theory of coherence, a fully coherent flat-topped vortex hollow beam can be extended to a partially coherent flat-topped vortex hollow beam. The second-order correlation properties of an electromagnetic beam can be characterized by the cross-spectral density function introduced by Wolf [17],

$$W(r_1, r_2, z) = \left< E(r_1, z) E^*(r_2, z) \right>$$

$$= \sqrt{I(x_1, y_1, z) I(x_2, y_2, z)} g(x_1 - x_2, y_1 - y_2)$$

(2)

where $g(x_1 - x_2, y_1 - y_2)$ is the spectral degree of coherence, assumed to have a Gaussian profile, and

$$g(x_1 - x_2, y_1 - y_2) = \exp \left[ -\frac{(x_1 - x_2)^2 + (y_1 - y_2)^2}{2\sigma^2} \right]$$

(3)

where $\sigma$ is the transverse coherence length.

Substituting Eq. (1) into Eq. (2), the partially coherent flat-topped vortex hollow beam can be written as

$$W_{\rho_0}(r_{10}, r_{20}, 0) = \sum_{n=0}^{N} \sum_{m=0}^{N} \left( -1 \right)^{m+n} \binom{N}{n} \binom{N}{m}$$

$$\exp \left[ -\frac{m}{w_x^2} \left( \frac{x_0^2}{w_x^2} + \frac{y_0^2}{w_y^2} \right) \right] \exp \left[ -\frac{n}{w_y^2} \left( \frac{x_0^2}{w_x^2} + \frac{y_0^2}{w_y^2} \right) \right]$$

$$\times \left( \frac{x_{10}w_{10} - y_{10}w_{20} + i \cdot x_{10}w_{10} - y_{10}w_{20}}{w_x w_y} \right)^M$$

$$\exp \left[ -\frac{(x_{10} - x_{20})^2 + (y_{10} - y_{20})^2}{2\sigma^2} \right]$$

(4)

where $r_{10} = (x_{10}, y_{10})$ and $r_{20} = (x_{20}, y_{20})$ are the position vectors at the source plane $z = 0$.

According to the extended Huygens-Fresnel principle, the spectral density of a laser beam propagating through turbulent atmosphere can be expressed as follows [1-9]:

$$W(r, r', z) = \frac{k^2}{4\pi^2 z^2} \exp \left[ -\frac{ik}{2z} (r - r_0)^2 + \frac{ik}{2z} (r' - r_0)^2 \right]$$

$$\times \exp \left[ -\frac{(r - r_0)^2}{\rho_0^2} \right] \frac{M!^l}{l!} \frac{1}{w_x} \left( \frac{1}{w_y} \right)^l$$

$$\sum_{s=0}^{M} \frac{M!^l}{l! (M-s)!} \frac{1}{w_x} \left( \frac{1}{w_y} \right)^s I_x I_y$$

(5)

where $k = 2\pi/\lambda$ is the wave number; $\psi(x_0, y_0, x, y)$ is the solution to the Rytov method that represents the random part of the complex phase (the asterisk denoting complex conjugation), and $r = (x, y)$ and $r_0 = (x_0, y_0)$ are respectively the position vectors at the output plane $z$ and the input plane $z=0$. The ensemble average in Eq. (5) can be expressed as [5]
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with

\[ I_x = (M - l)! \left( \frac{\pi}{a_1} \right)^{M-1} \exp \left[ -\frac{1}{a_1} \left( \frac{k}{2z} x_1 - \frac{y_1 - y_2}{2\rho_0^2} \right)^2 \right] \]

\[ \times \sum_{d_1=0}^{[M-2d_1]} \frac{1}{d_1! (M - l - 2d_1)!} \frac{a_1}{4} \frac{1}{t_2} \left( \frac{2\sigma^2 + 1}{\rho_0^2} \right)^{2^{d_1-2d_1}} \times i^{2d_1} \exp \left( \frac{c_1^2}{b_1^2} \right) \frac{\pi}{b_1} \left( \frac{1}{\sqrt{b_1}} \right) H_{M-1}(i) \left( -i \frac{c_2}{\sqrt{b_2}} \right) \]

\[ I_y = l! \left( \frac{\pi}{a_2} \frac{1}{a_2} \right)^{y_2} \exp \left[ -\frac{1}{a_2} \left( \frac{k}{2z} y_1 - \frac{y_1 - y_2}{2\rho_0^2} \right)^2 \right] \]

\[ \times \sum_{d_2=0}^{l-2d_2} \frac{1}{d_2! (l - 2d_2)!} \frac{a_2}{4} \frac{1}{t_2} \left( \frac{2\sigma^2 + 1}{\rho_0^2} \right)^{2^{d_2-2d_2}} \times i^{2d_2} \exp \left( \frac{c_2^2}{b_2^2} \right) \frac{\pi}{b_2} \left( \frac{1}{\sqrt{b_2}} \right) H_{l-d_2}(i) \left( -i \frac{c_2}{\sqrt{b_2}} \right) \]

and

\[ a_1 = \frac{n}{w_x^2} + \frac{ik}{2z} + \frac{1}{2\sigma^2} + \frac{1}{\rho_0^2} \] (14a)

\[ b_1 = \frac{m}{w_x^2} - \frac{ik}{2z} + \frac{1}{2\sigma^2} + \frac{1}{\rho_0^2} - \frac{1}{a_1} \left( \frac{2\sigma^2 + 1}{\rho_0^2} \right) \] (14b)

\[ c_1 = \frac{1}{a_1} \left( \frac{ik}{2z} x_1 - \frac{x_1 - x_2}{2\rho_0^2} \right) + \frac{1}{2\sigma^2} + \frac{1}{\rho_0^2} \] (14c)

\[ a_2 = \frac{n}{w_y^2} + \frac{ik}{2z} + \frac{1}{2\sigma^2} + \frac{1}{\rho_0^2} \] (15a)

\[ b_2 = \frac{m}{w_y^2} - \frac{ik}{2z} + \frac{1}{2\sigma^2} + \frac{1}{\rho_0^2} - \frac{1}{a_2} \left( \frac{2\sigma^2 + 1}{\rho_0^2} \right) \] (15b)

\[ c_2 = \frac{1}{a_2} \left( \frac{ik}{2z} y_1 - \frac{y_1 - y_2}{2\rho_0^2} \right) + \frac{1}{2\sigma^2} + \frac{1}{\rho_0^2} - \frac{ik}{2z} y_2 + \frac{y_1 - y_2}{2\rho_0^2} \] (15c)

Eqs. (11)~(15) make up the main analytical expression for a partially coherent circular or elliptical flat-topped vortex beam propagating in turbulent atmosphere. Using the derived equations we can investigate the propagation and transformation of a partially coherent circular or elliptical flat-topped vortex hollow beam in turbulent atmosphere.

The degree of coherence of the laser beam is written as [19]

\[ \mu(r_1, r_2, z) = \frac{W(r_1, r_2)}{W(r_1, r_2) W(r_2, r_2)} \] (16)

and the position of coherence vortices at the propagation L is expressed as [20]

\[ \text{Re} \mu(r_1, r_2, z) = 0 \] (17a)

\[ \text{Im} \mu(r_1, r_2, z) = 0 \] (17b)

where \text{Re} and \text{Im} are respectively the real and imaginary parts of \( \mu(r_1, r_2, z) \).

III. NUMERICAL EXAMPLES AND ANALYSIS

In this section we study the propagation properties of a partially coherent flat-topped vortex hollow beam in turbulent atmosphere. In this work, the calculation parameters \( \lambda \) and \( w_0 \) throughout the text are set to be \( \lambda = 1064 \text{ nm} \) (Nd:YAG laser) and \( w_0 = 20 \text{ nm} \).

Figures 1 and 2 show the normalized average intensity and corresponding contour graphs of, respectively, partially coherent circular and elliptical flat-topped vortex hollow beams propagating in turbulent atmosphere; the calculation parameters are \( C_n^2 = 10^{-14} \text{m}^{-2/3/3}, N=2, M=1, \sigma=10 \text{ mm} \) and \( w_y = 40 \text{ mm} \) (Fig. 2). As can be seen from Figs. 1 and 2, a partially coherent circular or elliptical flat-topped vortex hollow beam can keep its original intensity pattern over a short propagation distance (Figs. 1(a) and 2(a)), and with increasing propagation distance either beam loses its initial dark, hollow centre, and the flat-topped vortex dark hollow beam evolves into a flat-topped beam (Figs. 1(b), 1(c), 2(b), and 2(c)). The partially coherent circular and elliptical flat-topped vortex hollow beams eventually evolve respectively into circular and elliptical Gaussian beams in the far field, due to the influence of the coherence length.

Figure 3 shows the cross section (y=0) of normalized average intensity for a partially coherent circular flat-topped
FIG. 1. Normalized average intensity of a partially coherent circular flat-topped vortex hollow beam propagating in turbulent atmosphere with $C_{n}^{2}=10^{-14} m^{-2/3}$, $N=2$, and $M=1$, $w_{x}=w_{y}=20 \text{ mm}$, and $\sigma=10 \text{ mm}$. (a) $z=100 \text{ m}$, (b) $z=300 \text{ m}$, (c) $z=600 \text{ m}$, (d) $z=2000 \text{ m}$.

FIG. 2. Normalized average intensity of a partially coherent elliptical flat-topped vortex hollow beam propagating in turbulent atmosphere with $C_{n}^{2}=10^{-14} m^{-2/3}$, $N=2$, and $M=1$, $w_{x}=20 \text{ mm}$, $w_{y}=40 \text{ mm}$, and $\sigma=10 \text{ mm}$. (a) $z=100 \text{ m}$, (b) $z=300 \text{ m}$, (c) $z=600 \text{ m}$, (d) $z=2000 \text{ m}$.

A partially coherent circular flat-topped vortex hollow beam propagating in turbulent atmosphere for various values of the coherence length $\sigma$, with $C_{n}^{2}=10^{-14} m^{-2/3}$, $N=2$, and $M=1$. It can be seen from Fig. 3 that a partially coherent circular flat-topped vortex hollow beam spreads...
FIG. 3. Cross section ($r=0$) of the normalized average intensity for a partially coherent circular flat-topped vortex hollow beam propagating in turbulent atmosphere with $C_{n}^{2}=10^{-14} m^{-2/3}$, $N=2$, and $M=1$. (a) $z = 100 \ m$, (b) $z = 300 \ m$, (c) $z = 600 \ m$, (d) $z = 2000 \ m$.

FIG. 4. Cross section ($r=0$) of the normalized average intensity for a partially coherent circular flat-topped vortex hollow beam propagating in turbulent atmosphere with $\sigma=10 \ mm$, $N=2$, and $M=1$. (a) $z = 100 \ m$, (b) $z = 300 \ m$, (c) $z = 600 \ m$, (d) $z = 2000 \ m$. 
more rapidly than a fully coherent beam (\(\sigma=\infty\)), with the initial coherence length \(\sigma\) decreasing during propagation, and that a partially coherent beam with a small coherence length will evolve into a Gaussian beam faster than a beam with a large coherence length in the far field (Fig. 3(d)).

Figure 4 presents the cross section (\(y=0\)) of normalized average intensity for a partially coherent circular flat-topped vortex hollow beam propagating in turbulent atmosphere for various values of \(C_{n}^{2}\), with \(\sigma=10\) mm, \(N=2\), and \(M=1\).

It can be seen that a beam propagating in turbulent atmosphere and free space (\(C_{n}^{2}=0\)) can almost keep its initial dark hollow profile over a short distance (Figs. 4(a) and (b)), and with increasing propagation distance a partially coherent flat-topped vortex hollow beam loses its initial dark hollow profile faster with increasing structure constant \(C_{n}^{2}\) in the far field (Fig. 4(d)).

Figure 5 depicts the cross section (\(y=0\)) of the normalized average intensity for a partially coherent circular flat-topped

![Figure 5](image1)

FIG. 5. Cross section (\(y=0\)) of the normalized average intensity of a partially coherent circular flat-topped vortex hollow beam propagating in turbulent atmosphere for different \(M\) and \(N\) with \(\sigma=10\) mm. (a) \(z=100\) m, (b) \(z=1000\) m, (c) \(z=100\) m, (d) \(z=1000\) m.

![Figure 6](image2)

FIG. 6. The curves for \(\text{Re } \mu=0\) and \(\text{Im } \mu=0\) for a partially coherent circular flat-topped vortex hollow beam propagating in turbulent atmosphere with \(\sigma=10\) mm, \(N=2\) and \(M=1\). (a) \(z=100\) m, (b) \(z=500\) m.
vortex hollow beam propagating in turbulent atmosphere for different values of $M$ and $N$ with $\sigma=10$ mm and $C_{n}^2=10^{14}$ m$^{-5/3}$. As can be seen, a beam with higher order $M$ (Figs. 5(a) and (b)) loses its initial dark hollow center more slowly, while a beam propagating in turbulent atmosphere with different orders $N$ has similar evolution properties with increasing propagation distance.

Figure 6 presents the curves for Re $\mu=0$ and Im $\mu=0$ for a partially coherent flat-topped vortex hollow beam propagating in turbulent atmosphere with $C_{n}^2=10^{14}$ m$^{-5/3}$, $\sigma=10$ mm, $N=2$, and $M=1$, and $r_2=(10$ mm, 20 mm). From Fig. 6(a) it can be seen that the beam propagation at a distance of $z=10$ m has a coherent vortex, and with increasing propagation distance the beam at a distance of $z=500$ m has two coherent vortices. Thus a partially coherent flat-topped vortex hollow beam will experience a change in its number of coherent vortices with increasing propagation distance in turbulent atmosphere.

IV. CONCLUSION

In this paper the partially coherent flat-topped vortex hollow beam is introduced, and then the propagation Eq. for a partially coherent flat-topped vortex hollow beam in turbulent atmosphere is derived. The average intensity of a beam propagating in turbulent atmosphere is examined using numerical examples. It is found that a partially coherent flat-topped vortex hollow beam will evolve into a Gaussian beam in the far field, and that a beam propagation in turbulent atmosphere with small coherence length or large structure constant $C_{n}^2$ will evolve into a Gaussian beam more rapidly. We also find that a beam with higher order $M$ loses its initial dark, hollow center more slowly, while beams of different order $N$ have similar evolution properties.

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