Color Ferromagnetic Quark Matter in Neutron Stars

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Color superconducting state has been known as a possible phase of quark matter with sufficiently large baryon number density so as for perturbative analysis to be valid. We point out that a color ferromagnetic state is another possible phase of such a sufficiently dense quark matter. Furthermore, we show that the color ferromagnetic phase is energetically more favored than the color superconducting phase in the quark matter with smaller baryon number density. We expect that increasing baryon density in neutron stars transforms nuclear matter into the quark matter of the color ferromagnetic phase, not color superconducting phase. We find that a critical mass of the neutron star with such an internal structure is about $1.6M_\odot$.

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Neutron stars have quite dense nuclear matter\textsuperscript{1} at their inner core. The density is expected to reach at least several times the normal nuclear density ($\approx 2.8 \times 10^{14} \text{g/cm}^3$). Observationally\textsuperscript{2}, the average density of the neutron stars is about twice the normal nuclear density, when their mass is $1.4M_\odot$ and their radius is 10km. (Even if we take the radius as 13km, the average density exceeds the normal nuclear density.) Although their radii have not yet been determined definitely, this naive estimation implies that the hadronic matter at the center of the neutron stars is much more dense than the normal nuclear density.

It is natural to expect that the neutron stars involve quark matter at their center. As the density of the hadronic matter increases, nucleons begin to overlap each other at about three times the normal nuclear density. This suggests that a transition between hadrons and quarks occurs at a few times the normal nuclear density and hence the quark matter may be realized at a core of the neutron stars. Thus, it is quite important to analyze the quark matter in understanding properties of the neutron stars.

So far, several phases of the quark matter have been studied: normal quark gas state, color superconducting state\textsuperscript{3} and color ferromagnetic state\textsuperscript{4, 5} which we have recently pointed out. In the normal quark gas state, quarks interact weakly with each other and the SU(3) gauge symmetry remains exact. This state is naiveely expected to be realized in quark matter with sufficiently large baryon number density so as for the gauge coupling constant to be much small. But this expectation does not hold in the quark matter with low temperature. Such a quark matter has been shown to form di-quark condensates and form the color superconducting states; color flavor locked (CFL) state involves three flavors in the condensates, while two-flavor superconducting (2SC) state does only two light flavors in them. Owing to these condensates, the SU(3) gauge symmetry is broken fully or partially in the color superconducting states. On the other hand, the color ferromagnetic state (CF) is such a state that a color magnetic field arises spontaneously due to gluon’s dynamics and quarks occupy Landau levels under the color magnetic field. Because of the magnetic field, the gauge symmetry is partially broken also in this state.

The color ferromagnetic state was firstly discussed by Savvidy\textsuperscript{6} as a more stable vacuum than the perturbative one, namely, as a possible candidate for quark matter with vanishing baryon number density. This argument is based on the fact that a nonzero color magnetic field minimizes the one-loop effective potential of the magnetic field. Since the one-loop approximation is valid in a system with small gauge coupling constant, this CF state may be realized in the quark matter with sufficiently large baryon number density, not in the quark matter with vanishing baryon number density. Since the energy density of quarks under the color magnetic field is lower than that in the absence of the field, the nontrivial minimum remains in the effective potential including the effects of the quarks. Therefore, the CF is a possible phase of the dense quark matter as well as the color superconducting states.

There is, however, a naive argument that the color magnetic field does not arise because magnetic mass of gluons

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could be produced in the dense quark matter; owing to the mass, the magnetic field is expelled or screened. We would like to mention in this point that although the electric mass, i.e. Debye mass, of the gluons can be generated in the dense quark matter, the magnetic mass can be hardly produced just as the magnetic mass of photons in electron gas. Only the exception is the color superconducting quark matter\(^2\), where the magnetic field is suppressed.

This nontrivial state, however, was shown\(^2\) to be unstable in the vacuum soon after the work by Savvidy. There exist unstable modes of gluons which possess imaginary mass. We have shown\(^4, 5\) that the instability can be cured (see also\(^3\) in the dense quark matter by the formation of fractional quantum Hall states of the unstable gluons. Therefore, there are two possible phases of the dense quark matter at low temperature; the color superconducting phase without the color magnetic field and the stable ferromagnetic phase with the color magnetic field. In the color superconducting phase, the energy density of quarks is smaller than that of the normal quark gas due to the di-quark condensates. Similarly, in the color ferromagnetic phase, their energy density is smaller than that of the normal quark gas due to the presence of the magnetic field. Obviously, these two phases are incompatible with each other.

In this letter we determine which phase is realized in the quark matter with sufficiently large baryon density so as for the perturbative approximation or loop approximation to be valid. For the purpose, we compare the free energy of the phases at zero temperature in terms of baryon chemical potential and find the state with the lowest free energy. The free energy, \(\Omega\), of quark matter has contributions from quarks and gluons,

\[
\Omega = \Omega_q + \Omega_{gl}.
\]

where \(\Omega_q\) represents the free energy of quarks, while \(\Omega_{gl}\) does vacuum energy in which main contributions come from gluon’s loops (and gluon’s condensation in the case of CF phase).

We first estimate the vacuum energies, \(\Omega_{gl}\), in each phase of sufficiently dense quark matter. In the one-loop approximation, the gluon vacuum energy \(4, 5\), \(\text{Re} V(gB) = \frac{9}{32\pi^2}(gB)^2(\log(gB/\Lambda^2) - 1/2)\) vanishes at \(gB = 0\) in the color superconducting phases, while it takes \(\text{Re} V(gB = \Lambda^2) = \frac{9}{64\pi^2}(gB)^2\) in CF phase; we have included loop effects of quarks with three flavors in \(V(gB)\). \((g\) is a gauge coupling constant.) We should also take into account of the condensation energy of gluons, which stabilize the color magnetic field by forming a quantum Hall state in the CF phase. The energy is given by \(-2B^2 = -2(gB)^2/g^2\). Thus, the free energies, \(\Omega_{gl}\), are given by

\[
\Omega_{gl} = 0 \quad \text{for CS}, \quad \Omega_{gl} = -\frac{9}{64\pi^2}(gB)^2 - 2\frac{(gB)^2}{g^2} \quad \text{for CF}
\]

respectively.

Secondly, we wish to estimate the free energies, \(\Omega_q\) of quarks. Before doing so, we give a brief review of the color ferromagnetic phase (see\(^5\) for more details). As has been shown, one-loop effective potential of color magnetic field, \(B\), shows not only a non-trivial minimum, but also the presence of imaginary part for \(gB \neq 0\), \(V(gB) = \frac{9}{32\pi^2}(gB)^2(\log(gB/\Lambda^2) - 1/2) - \frac{1}{8}(gB)^2\). The real part of \(V(gB)\) shows the spontaneous generation of the color magnetic field. (We may assume that the direction of the field in the color space points to \(\lambda_3\).) But the presence of the imaginary part of \(V(gB)\) indicates that the state is unstable. In other words the state is not true minimum in energy. Actually, there exist unstable modes of gauge fields with \(\lambda_3\) color charges. These unstable modes occupy the lowest Landau level \((n = 0)\) with spins parallel to \(B\) and have spectrum, \(E^2_g = k^2 + 2gB(n+1/2) - 2gB = k^2 - gB < 0\), where \(k\) represents a momentum parallel to the spatial direction of the magnetic field. Among them, the modes with \(E_g(k = 0)\) whose amplitudes increase most rapidly, make a stable vacuum. As we have shown in previous papers, the unstable modes of the gauge fields with \(k = 0\), which are spatially two-dimensional ones, condense to make stable fractional quantum Hall states: Repulsive self-interaction of the gluons occupying the Lowest Landau level leads to the quantum Hall state even if the interaction is small. This formation of the quantum Hall state\(^6\) is similar to the case that two-dimensional electrons under strong magnetic field form fractional quantum Hall states due to the Coulomb repulsion. Since the condensate has the color charges, the formation of the quantum Hall states is possible only when the quark matter is present; the quarks supply the color charges for the stable vacuum. Therefore, the color ferromagnetic phase is a possible state of the quark matter with sufficiently large baryon number density so as for the loop approximation to be valid. The gluon condensations leading to the quantum Hall states produce a sufficiently large magnetic mass to the unstable modes to stabilize the color ferromagnetic state. In this way, it turns out that the state in true minimum of the effective potential is a stable CF state which gains the condensation energy of the gluons, \(-2B^2\).

Here, we note that there is one unknown dimensional parameter in QCD associated with a renormalization scale, \(\Lambda\) in \(V(gB)\). In our case \(gB\) is such a parameter. Although the reliable value of \(gB\) has not yet been determined phenomenologically, we expect that it has a typical scale of QCD. Here, we assume that the value of \(gB\) is around \((200\text{MeV})^2\).

Now, we turn to calculate the free energies of quarks in the CF phase. Quarks in SU(3) gauge theory are in a color triplet; \(q = (q_t, q_6, q_8)\). Among them, quarks, \(q_t\) and \(q_6\), occupy Landau levels under the color magnetic field, \(B \propto \lambda_3\).
Spectra of such quarks are given by $E_q^2 = m_f^2 + k^2 + gB(n + 1/2 + s_z)$ where $s_z = \pm 1/2$ represents a spin contribution and $m_f$ is the quark mass with flavor, $f$. We take $m_{u,d} = 0$ and $m_s = 100\text{MeV} \sim 300\text{MeV}$. Each Landau level has the degeneracy per unit area being given by $gB/4\pi$. The spectra of the quarks, $q_u$ and $q_s$, are identical to each other. Hence, the number density, $n_f$, and the energy density, $\rho_f$, of each flavor of quarks are given by

$$n_f = \frac{gB}{4\pi^2} \sqrt{\mu_f^2 - m_f^2} + \cdots, \quad \rho_f = \frac{gB}{8\pi^2} \left( \mu_f \sqrt{\mu_f^2 - m_f^2} + m_f^2 \ln \frac{\mu_f + \sqrt{\mu_f^2 - m_f^2}}{m_f} \right) + \cdots,$$

(3)

where we have explicitly shown only the contributions of the lowest Landau level; we should understand that contributions of higher Landau levels are present in the above formulae. $\mu_f$ denotes the chemical potential of quarks, with the color types, $q_u$ and $q_s$. We should note that the chemical potential of $q_s$ is the same as the one of $q_u$, since we have an exchange symmetry between $q_u$ and $q_s$. On the other hand, spectra of the quark, $q_b$, are given by $\sqrt{m_b^2 + k^2}$.

Namely, the quarks do not couple with the magnetic field. Then, the number density, $\tilde{n}_f$, and the energy density, $\tilde{\rho}_f$, of the quark with the color type, $q_b$, are given, respectively, by

$$\tilde{n}_f = \frac{2(\mu_f^2 - m_f^2)^{3/2}}{3\pi^2}, \quad \tilde{\rho}_f = \frac{2\mu_f^2 - m_f^2}{8\pi^2} \ln \frac{\mu_f + \sqrt{\mu_f^2 - m_f^2}}{m_f},$$

(4)

where $\tilde{\mu}_f$ is the chemical potential of the quarks, $q_b$.

In the following, we should take into account three important conditions [11] which must be satisfied by the quarks and electrons in a realistic situation, namely, in neutron stars. These are the conditions of color neutrality, electric neutrality and beta-equilibrium,

$$n_u + n_d + n_s = \tilde{n}_u + \tilde{n}_d + \tilde{n}_s \quad \text{(color neutrality)},$$

(5)

$$\frac{2}{3}(2n_u + \tilde{n}_u) - \frac{1}{3}(2n_d + 2n_s + \tilde{n}_d + \tilde{n}_s) - n_e = 0 \quad \text{(electric neutrality)},$$

(6)

$$\mu_u = \mu_d = \mu_e \quad \text{and} \quad \tilde{\mu}_s = \tilde{\mu}_d = \tilde{\mu}_u + \mu_e \quad \text{(beta-equilibrium)},$$

(7)

where $n_e = \mu_e^2/3\pi^2$ and $\mu_e$ denote the number density of electrons and their chemical potential, respectively. The color neutrality condition comes from the average of each color charge vanishing, $\langle \lambda_3 \rangle = \langle \lambda_8 \rangle = 0$. The electric neutrality condition comes from the fact that the total electric charge density in neutron star should vanish. (Here we do not consider a possibility of mixing phase.) Note that the number density of each flavor $n_f$ is given by, $n_f = 2n_f + \tilde{n}_f$.

Furthermore, the beta-equilibrium condition comes from $u$, $d$ and $s$ quarks transforming into each other in neutron stars such as $d \rightarrow u + e$, or its inverse process due to the weak interactions. These conditions reduce many independent variables to only one such as the baryon chemical potential.

Using these number densities and energy densities, we can calculate the free energy of the quark matter in the color ferromagnetic phase,

$$\Omega_q^{\text{CF}} = 2\rho_u + \tilde{\rho}_u + 2\rho_d + \tilde{\rho}_d + 2\rho_s + \tilde{\rho}_s + \rho_e - \sum_{f=u,d,s} (2\mu_f n_f + \tilde{\mu}_f \tilde{n}_f) - \mu_e n_e$$

$$= 2\rho_u + \tilde{\rho}_u + 2\rho_d + \tilde{\rho}_d + 2\rho_s + \tilde{\rho}_s + \rho_e - \mu_B n_B$$

(8)

where the second line of the equation has been derived using the conditions, eqs. [4]. $\mu_B$ and $n_B$ are the baryon chemical potential and the baryon number density. ($\rho_e = \mu_e^2/4\pi^2$ is the energy density of electrons.) These can be represented such that,

$$\mu_B = \frac{2(\mu_u + \mu_d + \mu_s) + \tilde{\mu}_u + \tilde{\mu}_d + \tilde{\mu}_s}{3}$$

$$n_B = \frac{\tilde{n}_u + \tilde{n}_d + \tilde{n}_s}{3}$$

(9)

(10)

As we have mentioned, this free energy as well as the other physical quantities can be expressed simply in terms of the baryon chemical potential, $\mu_B$, using the above three conditions.

In this way, we can calculate the free energy of the quarks in the color ferromagnetic phase. When the baryon number density becomes larger, quarks occupy higher Landau levels. We can find numerically that as quarks start to occupy higher Landau levels than the lowest Landau level, the free energy rapidly approaches to that of the normal
quark gas with no magnetic field; \( \Omega^\text{CF}_q/\Omega^\text{NG}_q \to 1 \) as \( \mu \to \infty \), as we have shown in the previous paper[4]. This is because the ratio depends only on the dimensionless quantity \( g B/\mu^2 \) in the limit.

In order to compare this free energy with those of the color superconducting states (CFL and 2SC) and the normal quark gas state (NG), we write down the free energies of these states[11],

\[
\Omega^\text{CFL}_q = -\mu_B^4 + 9\mu_B^2 m_s^2 + \frac{m_s^4}{16\pi^2} \left(1 - 12 \ln \frac{3m_s}{2\mu_B} \right) - \frac{\mu_B^2 \Delta^2_{\text{CFL}}}{3\pi^2} \\
\Omega^\text{2SC}_q = -\mu_B^4 + 9\mu_B^2 m_s^2 + \frac{m_s^4}{16\pi^2} \left(5 - 12 \ln \frac{3m_s}{2\mu_B} \right) - \frac{\mu_B^2 \Delta^2_{\text{2SC}}}{9\pi^2} \\
\Omega^\text{NG}_q = -\mu_B^4 + 9\mu_B^2 m_s^2 + \frac{m_s^4}{16\pi^2} \left(7 - 12 \ln \frac{3m_s}{2\mu_B} \right)
\]

where \( \Delta_{\text{CFL}} \) and \( \Delta_{\text{2SC}} \) are the gap energies in the color superconducting phases, CFL and 2SC, respectively. The formulae have been obtained up to second order in \( m_s^2/\mu_B^2 \), with the assumption that \( m_s^2 \) is of the same order as \( \mu_B \Delta_c/3 \) (\( c = \text{CFL or 2SC} \)). Then, the last two terms in \( \Omega^\text{CFL}_q \) and \( \Omega^\text{2SC}_q \) become of the same order of the magnitude. It turns out that the di-quark pairing effects in the superconducting states reduce the energies by \( (\mu_B \Delta_c/3)^2 \sim m_s^4 \) from the energy of NG.

We should mention that the free energies obtained above are reliable only for the quark matter with the small gauge coupling constant, in other words, with sufficiently large baryon number density. Quarks interact weakly with each others. Hereafter we apply the free energies for quark matters with realistic baryon number density such as several times of normal nuclear density. It is expected that such quark matters are present in neutron stars.

Theoretically, the values of the gap parameters have much ambiguity, so that we consider a range of the parameter such as \( 10 \text{MeV} < \Delta_c < 100 \text{MeV} \), supposing \( \Delta_{\text{CFL}} \approx \Delta_{\text{2SC}} \). We also consider a range of the strange quark mass such as \( 100 \text{MeV} < m_s < 300 \text{MeV} \).

In Fig.1 we show the free energy densities of the phases in terms of the baryon chemical potential, \( \mu_B \) relative to the free energy density of NG phase, i.e. \( \Delta \Omega = \Omega^\text{CF}_q - \Omega^\text{NG}_q \). We take \( g B = (200 \text{MeV})^2 \) and a large strange quark mass, \( m_s = 275 \text{MeV} \) along with a large gap parameter \( \Delta = 80 \text{MeV} \) to reduce the free energies of CFL or 2SC phases. (We have taken a value of the gauge coupling constant, \( g^2/4\pi = 0.3 \).) Obviously, the color ferromagnetic phase is the most favored in the region depicted in Fig.1. With the increase of the baryon chemical potential, higher Landau levels are occupied and the CF free energy approaches to that of the normal quark matter; \( \Omega^\text{CF}_q \approx \Omega^\text{NG}_q \sim -b(gB)^2 \). Eventually, CFL phase becomes energetically more favored than the ferromagnetic phase in the extremely large chemical potential as can be seen in Fig.1.

Physically, the energy density of the quarks (\( \rho^\text{CF}_q \sim -O(gB\mu^2) \)) in the magnetic field is much smaller than the energy density of the normal quark gas (\( \rho^\text{NG}_q \sim -O(\mu^2) \)), since all of quarks occupy the lowest Landau level in the limit of large magnetic field. Even if the pairing effects in the color superconducting states are taken into account, this situation is not altered. The pairing effects reduce the energy density of the normal quark gas only by the order of \( \Delta^2/\mu^2 \); \( \rho^\text{CFL}_q \approx \rho^\text{NG}_q - a\Delta^2/\mu^2 \) with a positive numerical factor \( a \). With increasing the quark chemical potential, \( \mu \), quarks become occupying higher Landau levels and the energy density approaches rapidly that of the normal quark.

![FIG. 1: The free energy of various phases as a function of the chemical potential \( \mu_B \), relative to the normal quark gas state.](image1)

![FIG. 2: Mass-radius relation of the compact stars in unit of the solar mass.](image2)
gas \cite{4}; $\Omega_{q}^{\text{CF}} \simeq \Omega_{q}^{\text{NG}} - b(gB)^2$ with a positive numerical factor, $b \simeq 0.055$. This is because the limit of $\mu \rightarrow \infty$, is equivalent to the limit of $gB \rightarrow 0$. Hence, in an intermediate value of $\mu$, $\Omega_{q}^{\text{CF}}$ becomes larger than $\Omega_{q}^{\text{CFL}}$. This general argument implies that as far as $b(gB)^2 > a\Delta^2\mu^2$, the color ferromagnetic phase is more favored, i.e. $\Omega^{\text{CF}} < \Omega^{\text{CFL}}$ for small chemical potential or large magnetic field.

In this way, the color ferromagnetic phase is more favored than the CFL phase or 2SC phase, if the chemical potential, $\mu$, satisfies, $b(gB)^2 > a\Delta^2\mu^2$, but is sufficiently large for the loop approximation to be valid. Then, an intriguing question we should ask is 'what is the critical baryon chemical potential between hadron phase and the quark phase?'. In order to answer the question, we need to know precisely the free energy of gluons $\Omega_{gl}$, in the color ferromagnetic phase. The critical point depends very sensitively on the value, $\Omega_{gl}$, which is of the order of $(100\text{MeV})^4$. Due to a forth power of an energy scale, its contribution to the free energy is quite sensitive to the energy scale so that it is difficult to obtain precisely the critical point.

Finally, in Fig.2 we show the mass-radius relation of a hybrid neutron star composed of a nuclear matter and the quark matter in the CF phase in its core. The relation has been obtained by solving the Tolman-Oppenheimer-Volkoff equation with the use of the equation of states of the quark matter as well as the nuclear matter. As a nuclear matter, we have used an equation of state by Akmal, Pandharipande, and Ravenhall (APR)\cite{12}. We have used tentatively $\Omega_{gl} = (180\text{MeV})^4$, $gB = (200\text{MeV})^2$, $\Delta_c = 80\text{MeV}$ and $m_s = 275\text{MeV}$. In Fig.2, the dotted line represents the relation for a neutron star containing only nuclear matter (NM). The solid line for the hybrid star involving the CF quark matter inside of the nuclear matter. For comparison, we also show the relation (dashed line) of a hybrid star composed of the CFL quark matter with the same $\Omega_{gl}$ inside of the nuclear matter. It turns out that the critical mass of the hybrid neutron stars discussed in the paper is consistent with observations\cite{2}. We will report the details in the near future.

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