Annihilation Type Radiative Decays of $B$ Meson in Perturbative QCD Approach

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With the perturbative QCD approach based on $k_T$ factorization, we study the pure annihilation type radiative decays $B^0 \to \phi \gamma$ and $B^0 \to J/\psi \gamma$. We find that the branching ratio of $B^0 \to \phi \gamma$ is $(2.7^{+0.3+1.2}_{-0.6-0.6}) \times 10^{-11}$, which is too small to be measured in the current $B$ factories of BaBar and Belle. The branching ratio of $B^0 \to J/\psi \gamma$ is $(4.5^{+0.6+0.7}_{-0.5-0.6}) \times 10^{-7}$, which is just at the corner of being observable in the $B$ factories. A larger branching ratio $BR(B^0 \to J/\psi \gamma) \approx 5 \times 10^{-6}$ is also predicted. These decay modes will help us testing the standard model and searching for new physics signals.

PACS numbers: 13.25.Hw, 12.38.Bx

$B$ meson rare decays are interesting for testing the standard model and searching for new physics. However, due to our poor knowledge of non-perturbative QCD, predictions for many interesting exclusive decays are polluted by large hadronic uncertainties. The two body radiative $B$ decays involve simpler hadronic dynamics with only one hadron in the final states, so they suffer much less pollution than non-leptonic decays. The radiative decays such as $B \to K^\ast \gamma, \rho(\omega)\gamma$ thus attract much attention [1]. The isospin breaking effects between the charged $B^\pm$ and neutral $B^0$ in these modes are mainly due to contributions from the annihilation type diagrams [1-5].

The importance of the annihilation type diagrams can also be shown from the pure annihilation radiative $B$ decays. The color suppressed $B^0 \to J/\psi \gamma$ and $B^0 \to \phi \gamma$ modes are of this kind. The former is tree dominant while the latter is a pure penguin flavor changing neutral current decay. Despite the fact that they are annihilation type decays, these decay amplitudes can be factorized as the $B$ meson to photon transition form factor $\langle \gamma|\bar{q}q(1-\gamma_5)b|B \rangle$ times the decay constant $\langle \phi|\bar{s}t\mu\nu|0 \rangle$ or $\langle J/\psi|\bar{c}r\mu\nu|0 \rangle$ in the naive factorization approach.

Recently, the vertex corrections for the four quark operators have been performed in the so called QCD factorization approach [6], utilizing the light-cone wave functions. The branching ratios turn out to be one order of magnitude different from the naive factorization approach [7, 8]. Such a large contribution from next-to-leading order corrections implies that the hadronic uncertainty in this kind of decays is as large as other hadronic annihilation type decays [9]. Recent study of soft collinear effective theory [10] also shows that the naive factorization contribution is not the only dominant contribution, which contradicts to QCD factorization claims. More theoretic study is needed before one can claim new physics effects in these decays.

In this paper, we will use an alternative approach- the perturbative QCD approach (PQCD) [11] to calculate the pure annihilation type decays $B^0 \to \phi \gamma$ and $B^0 \to J/\psi \gamma$. Based on $k_T$ factorization, the PQCD approach has been proposed and applied to calculate two body non-leptonic $B$ decays such as $B \to K\pi$ [12], $\pi\pi$ [13], $\rho\rho$ [14], the radiative decays $B \to K^\ast\gamma$ [4], $\rho(\omega)\gamma$ [5], etc. and the results are consistent with experimental data. In this approach, the quark transverse momentum $k_T$ is kept in order to kill the end-point singularity. Because of inclusion of transverse momenta, double logarithms from the overlap of two types of infrared divergences, soft and collinear, are generated in radiative corrections. The resummation of these double logarithms leads to a Sudakov form factor, which suppresses the long-distance contribution.

For convenience, we work in the light-cone coordinate, where the $B$ meson momentum in its rest frame, is

$$ P_B = (P_{Bx}^+, P_{B-}, \vec{P}_{B\perp}) = \frac{M_B}{\sqrt{2}}(1, 1, 0\perp). \quad (1) $$

By choosing the coordinate frame where the vector meson moves in the “−” direction and photon in the “+” direction, the momenta of final state particles are

$$ P_V = (P_{V+}, P_{V-}, \vec{P}_{V\perp}) = \frac{M_B}{\sqrt{2}}(r^2, 1, 0\perp), \quad (2) $$

$$ P_\gamma = (P_{\gamma+}, P_{\gamma-}, \vec{P}_{\gamma\perp}) = \frac{M_B}{\sqrt{2}}(1-r^2, 0, 0\perp), \quad (2) $$

where $r = M_V/M_B$. The momentum of the light quark in $B$ meson is:

$$ k_1 = (k_{1x}^+, k_1^-, \vec{k}_{1T}) = (\frac{M_B}{\sqrt{2}}x_1, 0, k_{1T}^-). \quad (3) $$

For the final state vector meson, we set the momentum of $q(x = s,c)$ as

$$ k_2 = (k_2^+, k_2^-, \vec{k}_{2T}) = (\frac{M_B}{\sqrt{2}}x_2r^2, \frac{M_B}{\sqrt{2}}x_2, k_{2T}^-). \quad (4) $$

In above functions, $x_1$ and $x_2$ are momentum fractions of the quarks.

In PQCD approach, the decay amplitude is factorized into the convolution of the mesons’ wave functions, the
where the formalism can be written as:

\[ A = (\epsilon_V \cdot \epsilon_\gamma^*) M^S + \frac{i}{P_V \cdot P_\gamma} \epsilon_{\mu\nu\rho\sigma}\epsilon_\gamma^{\mu\nu} \epsilon_V^{\rho\sigma} P_V^\mu P_\gamma^\sigma M^P, \]

where \( P_V \) and \( P_\gamma \) are the momenta of vector meson and photon, respectively. \( \epsilon_V^* \) and \( \epsilon_\gamma^* \) are the relevant polarization vectors. The matrix element \( M^{S(P)} \) can be calculated in the PQCD approach.

In principal, the leading order contributions for \( B^0 \to V\gamma \) (\( V = J/\psi, \phi \)) decays involve only four-quark operators plus a photon emitted from any of the quark line. The effective weak Hamiltonian is formed by the 12 four-quark operators and the corresponding QCD corrected Wilson coefficients [15]. Very recently the electromagnetic penguin operator \( O_{7\gamma} \) contribution through \( B^0 \to \gamma\gamma \) with one photon connecting to the \( \phi \) meson is studied in ref.[16]. The branching ratio for \( B \to \phi \gamma \) is found to be \( 1 \times 10^{-11} \) which is larger than the four quark operator contribution from QCD factorization approach [7, 8]. In PQCD language, the contribution from \( O_{7\gamma} \) is next-to-leading order. Its contribution is still smaller than other contributions in PQCD approach which will be shown later. The contribution of this kind of operator to the \( B^0 \to J/\psi\gamma \) decay is negligibly small.

The lowest order Feynman diagrams of \( B^0 \to \phi \gamma \) in PQCD are shown in Fig.1. In principle, the photon can be emitted from any quark line of the four-quark operator. However, the contribution of Fig.1(c) is canceled exactly by that of (d) because of topology symmetry. Thus, only the decay amplitudes from Fig.1(a) and (b) are left as:

\[ M^S = \frac{1}{\sqrt{3}} G_F e M_B^3 r f_V V_{\text{CKM}} \int_0^1 dx_1 \int_0^\infty db_1 b_1 \phi_B(x_1, b_1) \left[ (1 - r^2) C(t^a) K_0(b_1 A_a) e^{-S_\phi(t^a)} - (1 - r^2) C(t^b) K_0(b_1 B_a) e^{-S_\phi(t^b)} \right], \]

\[ M^P = \frac{1}{\sqrt{3}} G_F e M_B^3 r f_V V_{\text{CKM}} \int_0^1 dx_1 \int_0^\infty db_1 b_1 \phi_B(x_1, b_1) \left[ C(t^a) K_0(b_1 A_a) e^{-S_\phi(t^a)} + C(t^b) K_0(b_1 B_a) e^{-S_\phi(t^b)} \right], \]

with

\[ A_a^2 = (1 + x_1 - r^2) M_B^2, B_a^2 = x_1(1 - r^2) M_B^2, \]

\[ t^a = \max(A_a, 1/b_1), t^b = \max(B_a, 1/b_1). \]

\( K_0(x) \) is the modified Bessel function which results from Fourier transformation of the quark propagator. \( e^{-S_\phi(t)} \) is Sudakov form factor, \( f_V \) is vector meson decay constant and \( V_{\text{CKM}} \) denotes CKM matrix elements. In above calculations, the \( B \) meson is treated as a heavy-light system, whose wave function is defined as:

\[ \phi_B = \frac{i}{\sqrt{6}} (P_B + M_B) \gamma_5 \phi_B(x_1, b_1), \]

and the expression of distribution amplitude \( \phi_B \) is shown in ref.[12, 13] with parameter \( \omega_b = 0.4 \text{GeV} \), which is normalized as

\[ \int_0^1 dx_1 \phi_B(x_1, b_1) = \frac{f_B}{2\sqrt{6}}, \]

where \( f_B \) is the decay constant of the \( B \) meson. Since we do not need the \( \phi \) or \( J/\psi \) wave functions in the calculations of Feynman diagrams Fig.1(a) and (b), we need not show them here.

For \( B^0 \to \phi \gamma \) decay, only penguin operators can contribute, and the CKM matrix elements are \( V_{\text{CKM}} = V_{td}V_{td}^\ast \). The combination of the Wilson coefficients in eq.(7, 8) are:

\[ C_{\phi\gamma}^P = C_3 + \frac{1}{3} C_4 + C_5 + \frac{1}{3} C_6 - \frac{1}{2} C_7 - \frac{1}{6} C_8 - \frac{1}{2} C_9 - \frac{1}{6} C_{10}. \]
The decay width of $B$ operators is $\rho$ of color suppressed tree contribution. The $\rho$ parameters in Wolfenstein parametrization [18], and $\bar{C}$ Again, we consider the hard scale very important uncertainty for rare decays. To estimate f cients. The missing next-to-leading order correction is a should be large uncertainty since we work only at leading order. The Wilson coefficient of dominant QCD penguin operator $C_3$ cancels much with $C_4/3$, and $C_5$ cancels with $C_6/3$. This is a result of color suppression in this decay, since at least three gluons are needed for a vector $\phi$ meson produced from penguin diagram [17]. For $B \to J/\psi \gamma$, both tree and penguin operators give contribution, and corresponding conclusions of the Wilson coefficients are:

$$C_{\psi \gamma}^T = C_1 + \frac{1}{3} C_2,$$

$$C_{\psi \gamma}^{P} = C_3 + \frac{1}{3} C_4 + C_5 + \frac{1}{3} C_6 + C_7$$

$$+ \frac{1}{3} C_8 + C_9 + \frac{1}{3} C_{10}. \quad (13)$$

Again, $C_1$ cancels much with $C_2/3$ which is also a result of color suppressed tree contribution. The $V_{CKM}$ in tree (penguin) operators is $V_{U3}V_{cd}$ ($V_{U6}V_{td}$).

With the amplitudes $M^S$ and $M^P$ defined in Eq.(6), the decay width of $B^0 \to V \gamma$ is given by

$$\Gamma = \frac{|M^S|^2 + |M^P|^2}{8\pi M_B}(1 - r^2). \quad (14)$$

In our numerical calculations, the input parameters are summarized in Table I, where $\lambda$, $A$, $\rho$ and $\bar{\eta}$ are CKM parameters in Wolfenstein parametrization [18], and $\bar{\rho} = \rho(1 - \frac{1}{3} \Lambda^2)$, $\bar{\eta} = \eta(1 - \frac{1}{3} \Lambda^2)$. Their values can be found in Review of Particle Properties [19].

At the leading order, the main uncertainty for decay branching ratios comes from the $B$ meson wave function. But it is constrained by the measured exclusive hadronic decays, like $B \to K \pi$ [12], $B \to \pi \pi$ [13] with parameter $\omega_B$ from 0.3GeV to 0.5GeV. On the other hand, there should be large uncertainty since we work only at leading order in $\alpha_s$ for the hard part and also for the Wilson coefficients. The missing next-to-leading order correction is a very important uncertainty for rare decays. To estimate it, we consider the hard scale $t$ at a range of

$$\text{max}(0.75 A_{6}, \frac{1}{b_1}) < t^a < \text{max}(1.25 A_{6}, \frac{1}{b_1}), \quad (15)$$

$$\text{max}(0.75 B_{6}, \frac{1}{b_1}) < t^b < \text{max}(1.25 B_{6}, \frac{1}{b_1}). \quad (16)$$

With the above major uncertainties from $B$ meson wave function parameter $\omega_B$ and the different scale $t$, respectively, we give out the branching ratio of $B^0 \to \phi \gamma$:

$$\text{BR}(B^0 \to \phi \gamma) = (2.7_{-0.6}^{+0.5 + 1.2}) \times 10^{-11}. \quad (17)$$

There are many other uncertainties in our calculation such as decay constants and CKM matrix elements. However, the uncertainty induced by above factors is not more than 10%[20]. With such a small branching ratio (17), this decay is too rare to be measured at the running $B$ factories or even in future LHC-b experiment. Our result is still much smaller than the recent upper limit $8.5 \times 10^{-7}$ from experiments [21]. If some new physics particles enhance this ratio through tree or loop effects [7], it may be measurable in near future experiments.

After similar calculations, we also give the branching ratio of $B^0 \to J/\psi \gamma$ decay:

$$\text{BR}(B^0 \to J/\psi \gamma) = (4.5_{-0.5}^{+0.6 + 0.7}) \times 10^{-7}. \quad (18)$$

Because this process is tree diagram dominated, it is much larger than that of $B^0 \to \phi \gamma$. This result is still not big enough to be measured at the $B$ factories but it is already around the corner of $B$ factories capability. Currently, there is only upper limit, which is $1.6 \times 10^{-6}$ at 90% confidence level [22].

From our calculation, we find that most of the contribution comes from Fig.1(b), which indicates that photon emission from the light quark of the $B$ meson is easier than that from the heavy quark. The reason is that the heavy quark is much difficult to become off-shell than the light quark, while a nearly on-shell quark rarely emits photons.

Within QCD factorization approach, the branching ratios of these two decays have been calculated in Ref.[7, 8]. The results are given as :

$$\text{BR}(B^0 \to J/\psi \gamma) = 7.65 \times 10^{-9}; \quad (19)$$

$$\text{BR}(B^0 \to \phi \gamma) = 3.6 \times 10^{-12}. \quad (20)$$

Compared with their results, we find that our results are one order of magnitude larger. In QCD factorization approach, some next-to-leading order contribution has been added in addition to the naive factorization contribution. It shows that these presumably order $\alpha_s$ corrections change the branching ratios by one order of magnitude. In fact these two decays are non-factorizable contributions of these two decays have been calculated in Ref.[7, 8].

| TABLE I: Summary of input parameters[19] |
|-----------------------------------------|
| CKM parameters and QCD constant         |
| $\lambda$ $A$ $\bar{\rho}$ $\bar{\eta}$ $\Lambda_{f=4}$ $\tau_{B^0}$ |
| 0.2196 0.819 0.20 0.33 250MeV 1.54ps   |
| Meson decay constants                   |
| $f_{B^0}$ $f_{B_s}$ $f_{\phi}$ $f_{J/\psi}$ |
| 216MeV 236MeV 254MeV 405 $\pm$ 14MeV   |
| Meson masses                            |
| $M_W$ $M_B$ $M_\phi$ $M_{J/\psi}$      |
| 80.41GeV 5.28GeV 1.02GeV 3.10GeV        |
The $B_s \rightarrow J/\psi \gamma$ decay is the same as $B^0 \rightarrow J/\psi \gamma$ decay within SU(3) symmetry. It is also a pure annihilation type decay. The decay formulas are exactly the same except replacing the corresponding CKM factor $V_{cd}$, $V_{td}$ by $V_{cs}$, $V_{ts}$, respectively. Taking into account the SU(3) breaking effect, we used $\omega_{B_s} = 0.5$ GeV [24], $f_{B_s} = 236$ MeV, we can get

$$BR(B_s \rightarrow J/\psi \gamma) \approx 5 \times 10^{-6},$$  

which should be easy to be measured in the future LHCb experiment.

In a summary, we calculate the branching ratios of pure annihilation type radiative decays $B^0 \rightarrow \phi \gamma$ and $B^0 \rightarrow J/\psi \gamma$ within the standard model in PQCD approach. We find the branching ratio of $B^0 \rightarrow \phi \gamma$ is at the order of $10^{-11}$. This small branching can not be detected in the running B factories of BaBar and KEK, unless some new physics enhance this results sharply. In future, this decay may be measured in LHCb experiment or other high luminosity experiments. For $B^0 \rightarrow J/\psi \gamma$ decay, the branching ratio is about $4 \times 10^{-7}$, which is just close to the $B$ factory experiment capability of measurement. The experimental measurements of these decays would be very useful for understanding various QCD methods, like QCD factorization and PQCD approaches.

**Acknowledgments**

This work was partly supported by the National Science Foundation of China. We thank Y.D. Yang, J.-X Chen, Y.-L Shen, W. Wang, X.-Q Yu and J. Zhu for useful discussions.

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