On the occurrence of close frequency pairs in selected δ Scuti stars

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Abstract. Amplitude variability is a common feature in δ Scuti stars. These variations can be explained by beating of close frequencies or by true amplitude variability. Observations have shown that the occurrence of amplitude variability follows regularities in the frequency spectra. In the present work we compute the expected incidence of close frequency pairs in pulsation models of a selected sample of δ Scuti stars. Different stellar evolutionary stages are considered. We estimate how many close frequency pairs are predicted and examine the regularities of close frequency pairs in the frequency spectra.

1. Introduction
Beating effects of close frequencies and true amplitude variability can be distinguished by their different phasing behaviour. Two close frequencies beating with each other show the largest phase change near the times of minimum amplitude, when assuming an unresolved single frequency. This test requires extensive amounts of data and has already successfully been applied to explain the nature of selected modes in FG Vir [1] and other δ Scuti stars [2]. However, the cause for the amplitude variability of a large number of modes has not yet been revealed. It is therefore necessary to estimate the expected occurrence of close frequency pairs from theoretical computations to assess the importance of beating compared to other reasons for amplitude variability. Moreover, observations show that the incidence of close frequencies is high in specific frequency regions (see Fig. 1). The reason for this needs to be explained.

2. Theoretical approach to estimate the occurrence of close frequencies
The commonly used assumption that photometry detects only modes with ℓ ≤ 3 may not be correct. As shown in [3], small-amplitude peaks of about 1 mmag can be caused by modes of higher spherical degrees. In this investigation we use a similar theoretical approach to compute the visibility of modes.

For three well-studied δ Scuti Stars (FG Vir, 44 Tau and 4 CVn) we compute nonadiabatic frequencies of all modes with ℓ ≤ 10. Rotational effects are considered up to second order. For each mode of the predicted pulsation spectrum the amplitude in the Strömgren y filter has to be estimated. The observed amplitude of a mode depends on its intrinsic amplitude, ϵ, geometrical cancellation (derived from ℓ, m) and the inclination angle, i. Within the framework of linear theory the intrinsic mode amplitude cannot be determined. However, relying on the assumption
Figure 1. Observed spectrum of all independent frequencies in FG Vir. Note the logarithmic amplitude scale.

that $\epsilon$ is the same for all modes, it is possible to derive its value by comparing the computed amplitudes in a specific filter to the observed amplitudes (see [3] for details on this approach). Finally, we scan the computed frequency spectrum for close frequency pairs which fulfill two criteria:

- the frequency separation is smaller than 0.01 c/d
- the Strömgren $y$ amplitude of one mode is at least 0.4 mmag while the amplitude of the second mode should exceed 1 mmag.

The second condition is necessary to ensure the reliable photometric detectability of amplitude variability. For each star shown below, the estimated incidence of close frequency pairs is given for two cases: (1) considering only trapped modes and (2) taking into account all unstable modes. These two cases can be considered as upper and lower limits of our estimates. All models were computed with OPAL opacities extended with the tables from [4]. The GN93 element mixture [5] was used assuming a standard chemical composition of X=0.70 and Z=0.02. Furthermore, we made use of the Vienna grid of model atmospheres ([6], [7]). The parameters of all computed models are given in Table 2.

3. FG Vir (main sequence)

With several seasons of data [8] 67 independent frequencies ranging between 5.7 - 44.3 c/d were detected in FG Vir. The lowest amplitudes are about 0.2 mmag. 7 close frequency pairs with a separation smaller than the adopted limit of 0.01 c/d are present in the data. Furthermore, 7 other frequencies show an annual amplitude variability of at least 0.8 mmag in the $y$ filter. These modes may also represent unresolved close frequency pairs. Consequently, the fraction of close frequency pairs ranges between 10 % and 20 % of the total number of detected frequencies.

Our theoretical estimate is based on the same pulsation model as described in [1]. Considering only trapped modes the predicted occurrence of close frequency pairs is around 5 %. If we include all unstable modes this value increases to 9 %. Thus beating of close frequencies cannot be the only cause for amplitude variability in FG Vir.

4. 44 Tau (post-main sequence)

In [9] 13 frequencies were found of which all four $\ell = 1$ modes show very strong amplitude variability. The reason for this is still unknown. In total, 6 frequencies show amplitude variability
Table 1. Observed fundamental parameters for the selected δ Scuti stars.

| Star     | $T_{\text{eff}}$ | Observed frequency range | $L/L_\odot$ | Inclination |
|----------|------------------|--------------------------|-------------|-------------|
| FG Vir   | 7400 ± 200 K     | 5-45 c/d                | 1.170 ± 0.055 | 20 ± 5°    |
| 44 Tau   | 6900 ± 100 K     | 6-13 c/d                | 1.340 ± 0.065 | 60 ± 25°   |
| 4 CVn    | 6800 ± 100 K     | 4-9 c/d                 | 1.550 ± 0.065 | unknown    |

Table 2. Parameters of the pulsation models. Standard chemical composition and no convective overshooting from the core were assumed. $V_{\text{rot},0}$ denotes the rotational velocity at the ZAMS.

| Star     | M/M_\odot | $T_{\text{eff}}$ | $L/L_\odot$ | log $g$ | $V_{\text{rot}}$ [km/s] | $V_{\text{rot},0}$ [km/s] | $\alpha_{\text{MLT}}$ |
|----------|-----------|------------------|-------------|--------|-------------------------|--------------------------|------------------|
| FG Vir   | 1.80      | 3.8658           | 1.120       | 3.980  | 62.5                    | 70.0                     | 0.5              |
| 44 Tau   | 1.875     | 3.8422           | 1.360       | 3.671  | 4.2                     | 5.0                      | 0.2              |
| 4 CVn    | 2.400     | 3.8320           | 1.760       | 3.320  | 82.0                    | 120.0                    | 1.0              |

exceeding the adopted limit of 0.8 mmag. If these variations are caused by beating of close frequencies the fraction of frequency pairs is 46 %. The measured rotational velocity of 44 Tau is only $3 \pm 2$ km/s [10]. The rotational splitting is therefore expected to range between 0.006 and 0.03 c/d. Consequently, rotational splitting may be small enough to cause amplitude variability due to beating of modes with different $m$. Here we examine a model taken from [11] rotating with 4.2 km/s. This case can be considered a lower limit of the incidence of close frequencies since the rotational splitting is larger than the adopted limit of 0.01 c/d. For trapped modes 25 % and for all unstable modes 60 % of the observed modes are expected to be a component of a close pair. It is therefore likely that the amplitude variability of many frequencies detected in 44 Tau is caused by beating.

5. 4 CVn (late post-main sequence)

4 CVn is a special case because it is one of the most evolved δ Scuti stars known so far. As discussed in [12] almost any independent mode shows amplitude variability with a typical time scale of years. Some rapid variations also occur. An average annual amplitude variability of 12 % for different pulsation modes was determined. Relying on the model parameters suggested in [2] we derive that for trapped modes the fraction of close pairs to the total number of frequencies should be around 27 %. The unlikely situation that all unstable modes are excited would indicate that virtually all modes should be components of close frequency pairs. Since the inclination of 4 CVn is unknown, our estimates were derived by averaging the results for different inclinations.

6. Distribution of close frequency pairs

Close pairs are often observed at frequencies which are close to the theoretical frequencies of radial modes [1]. This observation can be explained by mode trapping in the stellar envelope. Not all of the predicted unstable frequencies are excited to observed amplitudes, but only modes with low kinetic energies. These modes are nonradial analogs of radial modes.

As shown in Fig. 2, for higher spherical degrees only trapped modes are unstable in δ Scuti models. On the top of the diagram, the spectrum of the sum of all spherical degrees is shown. It is clearly visible that several dense regions do exist and are separated by regions with a lower number of modes.
7. Conclusions

We estimated the occurrence of close frequency pairs for three $\delta$ Scuti stars in different evolutionary stages (FG Vir, 44 Tau and 4 CVn). For all pulsation modes with $\ell \leq 10$ the expected amplitudes in the Strömgren $y$ filter were computed following the approach outlined in [3]. Our results show that it is justified to consider only modes up to a spherical degree of 10. The amplitudes of the modes with $\ell \geq 7$ were too small to ensure a reliable detectability of amplitude variability. Nevertheless, we also confirm the results of [3] who showed that high-degree modes are indeed likely to explain low-amplitude peaks ($A < 1 \text{ mmag}$) in photometric data. It is shown that the incidence of close frequencies increases as the star evolves. This is mainly the effect of the denser frequency spectrum of oscillations in evolved stars. Furthermore, it is shown that only a fraction of the observed amplitude variability can be explained by close frequency pairs. However, we have to remember that these results are only a crude estimate based on several assumptions within the framework of linear theory. Finally, we find indications that the observed affinity of close frequency pairs to radial modes may be related to mode trapping in the stellar envelope.

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Figure 2. Unstable modes in a $\delta$ Scuti star model.
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