Numerical Simulation and Experimental Study of Three-Dimensional Density-Stratified Flow

Ming Yang, Ming Wang, Fangxiu Zhang and Ruixun Lai
Sediment Research Centre, Yellow River Institute of Hydraulic Research, YRCC, MWR, No. 45 Shunhe Rd. Zhengzhou, China

Abstract. The numerical relationship between vertical turbulence P/S $\sigma_z$, turbulence and stratification feature parameters is established by analyzing the flux expression of turbulent stress/flux algebraic mode. Corresponding density change rate parameters reflecting changes in temperature and sediment concentration are introduced to correct the vertical diffusion coefficient of turbulent. A three-dimensional turbulence model with multi-phase coupling of water flow, temperature and sediment is established on this basis. The established standard Johnson water channel model is adopted for experimental study on the density-stratified flow model of temperature and sediment concentration and the experimental results are compared with the calculation results of three-dimensional density-stratified flow mathematical model.

1. Introduction
Density-stratified flow is a common natural phenomenon. For example, after a reservoir is built, due to the wide water surface and large water body in the dam area, the reservoir shows a special water-temperature stratified structure in a state of water temperature-stratified flow; the turbid density current moving downstream along the riverbed due to fine sediment mixed in water sinking in the bottom of the reservoir is also a sediment density-stratified flow.

Turbulence is a relatively complex physical phenomenon. If there is simultaneous influence of temperature, sediment and other density stratification, the mixing mechanism of turbulence will be more complicated and the simulation will be very difficult.

In this study, under the consideration of effects of temperature and sediment stratification on the turbulent diffusion coefficient, the numerical relationship among vertical turbulence P/S, turbulence and stratification feature parameters is established by analyzing the flux expression of turbulent stress/flux algebraic mode, and corresponding density change rate parameters reflecting changes in temperature and sediment concentration are introduced to correct the vertical diffusion coefficient of turbulent. On this basis, a three-dimensional turbulence model with multi-phase coupling of water flow, temperature and sediment is established, and the established standard Johnson water channel model is adopted for experimental study on the density-stratified flow model of temperature and sediment concentration. The experimental results are compared with the calculation results of three-dimensional density-stratified flow mathematical model.

2. Governing Equation
Continuity equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_j}{\partial x_j} = 0$$  \hspace{1cm} (1)
Momentum equation
\[ \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = \Gamma_{\phi} \frac{\partial^2 u_i}{\partial x_j^2} - \frac{\partial p}{\partial x_i} + f_i [1 - \beta(T_0)T + \beta(T)T + \eta(T, S)S] \]  

(2)

Temperature diffusion equation:
\[ \frac{\partial T}{\partial t} + \frac{\partial (u_i T)}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \Gamma_T \frac{\partial T}{\partial x_j} \right] + \frac{J}{\rho C_p} \]

(3)

Sediment motion equation:
\[ \frac{\partial S_i}{\partial t} + \frac{\partial u_i S_j}{\partial x_j} - \alpha_L \frac{\partial S_L}{\partial z} = \frac{\partial}{\partial x_j} \left( \Gamma_S \frac{\partial S_i}{\partial x_j} \right) \]

(4)

Riverbed deformation equation:
\[ \gamma' \frac{\partial \zeta_{bl}}{\partial t} = \omega_l \left( S_{bl} - S_{bl}^* \right) \]

(5)

Where \( f_i \) is mass force; \( P \) is pressure (\( N/m^2 \)); \( \Gamma_{\phi} \) is turbulent diffusion coefficient (\( kg/m/s \)). It is necessary to introduce a turbulence model to solve it. \( T_0 \) is reference temperature. The reference temperature is taken to be ambient water temperature. \( \rho_0 \) is water density of clean water at temperature of \( T_0 \). \( \beta(T) \), \( \eta(T, S) \), \( \rho_0 \) are the followings respectively:
\[ \beta(T) = \beta(T) / \rho_0, \eta(T, S) = \eta(T, S) / \rho_0, \rho_0 = \rho_0 + \tilde{\beta}(T_0) \cdot T_0 \]

Where \( a_0 = 999.842594 kg/m^3 \). The \( \beta(T) \) is equivalent to the negative value of the thermal expansion coefficient, and \( \beta \) has a nonlinear relationship with \( T \).
\[ \Gamma_T = K_j + \frac{\nu}{P_n} \] is turbulent heat diffusion coefficient. \( K_j = \lambda / \rho_0 C_p \cdot C_p \) is specific heat at constant pressure and \( \lambda \) is fluid heat transfer coefficient.

\( J \) is the net heat absorbed by water (W/m\(^2\)), which mainly includes radiation, evaporation and conduction. The heat flux entering water body through water surface is:
\[ J = \varphi_{sn} + \varphi_{an} - \varphi_{br} - \varphi_c - \varphi_e \]

Where \( S_i \) is group sediment concentration (\( kg/m^3 \)); \( l \) is sediment particle size group number; \( \omega_l \) is sediment sinking speed (\( m/s \)) and \( \Gamma_S \) is sediment diffusion coefficient. \( \zeta_b \) is bed elevation (\( m \)); \( \gamma' \) is the dry density of sludge near the riverbed surface (\( kg/m^3 \)), and \( S_{bl} \) and \( S_{bl}^* \) are the sediment concentration and sedimentation force near the riverbed surface (\( kg/m^3 \)).

\( k \sim \varepsilon \) correction equation:

Turbulent diffusion coefficient in water flow equation is \( \Gamma = \nu + \frac{\nu_t}{\sigma_{u_0}} \).
\[
\frac{\partial (\rho k)}{\partial t} + \frac{\partial (\rho u_j k)}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \Gamma_k \frac{\partial k}{\partial x_j} \right) + S_k \\
\frac{\partial (\rho \varepsilon)}{\partial t} + \frac{\partial (\rho u_j \varepsilon)}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \Gamma_\varepsilon \frac{\partial \varepsilon}{\partial x_j} \right) + S_\varepsilon
\]

(6)

Where

\[\Gamma_k = \mu + \frac{H}{\sigma_k} ; \Gamma_\varepsilon = \mu + \frac{H}{\sigma_\varepsilon} ; S_k = G - \rho \varepsilon ; S_\varepsilon = c_{1\varepsilon} \frac{\varepsilon}{k} G - c_{2\varepsilon} \rho \frac{\varepsilon^2}{k} ; G = \mu_t \frac{\partial u_i}{\partial x_j} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \]

Where \(\sigma_k\) and \(\sigma_\varepsilon\) are Prandtl numbers corresponding to the turbulent kinetic energy \(k\) and turbulent dissipation rate \(\varepsilon\), and they are 1.0 and 1.3; \(c_{1\varepsilon}\), \(c_{1\varepsilon}\) and \(c_{2\varepsilon}\) are the empirical constants and they are 0.09, 1.44 and 1.92; \(G\) is the generation item of turbulent kinetic energy due to average velocity gradient.

The expansion equation of corresponding \(G\) is

\[G = \mu_t \frac{\partial u_i}{\partial x_j} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) = \mu_t \left( \frac{\partial u_i}{\partial x_j} \right)^2 + \frac{1}{2} \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i} \]

(7)

Where Turbulent Prandtl (Ratio of viscosity coefficient to diffusion coefficient, the same below) turbulent pulsation) is \(\mu_t = \frac{\mu}{\rho \varepsilon^2}\), the turbulent kinetic energy \(k\) and turbulent dissipation rate \(\varepsilon\) are obtained by solving the standard \(k - \varepsilon\) equation.

Considering that the horizontal diffusion is less affected by density gradient, while the vertical diffusion is greatly affected by it, the horizontal turbulent P/S \(\sigma_k\) is taken to be a constant (=0.9), while the vertical turbulence P/S \(\sigma_\varepsilon\) is considered to be related to the turbulence and stratification feature parameters based on the analysis of the flux expression of turbulent stress/flux algebraic mode. The following formula is obtained through simulation calculation of measured data of the thermal stratified flow:

\[\sigma_\varepsilon = 0.54308 + 0.00075 g k^2 \frac{T}{\varepsilon^2} \left( \beta \frac{\partial T}{\partial z} + \eta \frac{\partial S}{\partial z} \right) \]

(8)

Where \(\beta\) and \(\eta\) are density change rates corresponding to changes in temperature and sediment concentration respectively.

\[\beta = - \frac{1}{\rho_o v} \frac{1}{d\bar{T}} \]
\[\eta = \frac{1}{\rho_o} \left( 1 - \frac{\rho_o}{\rho_s} S \right) \]

(9)

(10)

Where \(v\) is the viscosity coefficient of water flow movement \((m^2/s)\).

3. Boundary Conditions

The boundary conditions of the solution area include inlet boundary, outlet boundary and wall boundary.

(1) Upstream inlet boundary. The flow rate, temperature, and sediment concentration are given as essential boundary conditions.

(2) Downstream outlet boundary. Control is performed based on free pressure outflow.

(3) Free surface boundary.
The VOF method is adopted to calculate the free surface and the symmetrical boundary conditions are adopted as the hydrodynamic variables of the free surface, that is, the normal gradient of horizontal velocities $u$, $v$, $k$ and $\varepsilon$ is 0, and the vertical velocity of them is $w = 0$.

In the case of surface heat exchange, the definition of the temperature boundary is treated separately for different situations.

(4) Lower surface boundary and side boundary. In this study, the wall boundary is treated by the convection wall function method.

4. Numerical Method

The SIMPLE-C method is adopted to solve the governing equations. The specific process is as follows:

1) Give the initial predicted value of the pressure field $p^*$.  

2) Obtain $u^*$, $v^*$ and through the momentum discrete equation. Firstly, the coefficients $A_p$, $A_i$, and the source term $S_\phi$ are solved; secondly, $A_p/\alpha_p$ is solved and the source term is $S_{pu} + \phi_p(1-\alpha_p)A_p/\alpha_p$; finally, $u^*$, $v^*$ and $w^*$ are obtained through the momentum discrete equation, and they are the solution after the sub-relaxation.

3) The pressure correction equation is adopted to solve the pressure correction value $p'$. Based on $u^*$, $v^*$ and $w^*$, the interface control coefficients $a_E$, $a_W$, $a_N$, $a_S$, $a_f$ and $a_g$ and source term $S_\phi$ are obtained; finally the pressure correction equation is introduced and $p'$ is obtained.

4) The corrected pressure and flow rate are calculated. $u'$, $v'$ and $w'$ are obtained, and then the corrected pressure flow rate is obtained based on $p = p^* + p'$, $u = u^* + u'$, $v = v^* + v'$ and $w = w^* + w'$.

5) The corrected $p$, $u$, $v$ and $w$ are taken as new initial values to calculate the coefficients and source terms of the momentum discrete equation. Go back to the second step and repeat the whole process until a convergent solution is obtained.

5. Model Application Calculation

5.1. Basic Situation

The built model is applied to the Johnson water channel model to conduct an experimental study on the density-stratified flow of temperature and sediment concentration.

The Johnson water channel model layout is shown in Figure 1. The model is 24.39m long and the depth gradually changes from 0.3m at the inlet to 0.91m at the dam site. According to the shape change of the section, the model reservoir can be divided into two sections. The first section is a section with length of 6.1 m, horizontal bottom and width varying linearly from 0.3 m to 0.91 m. The second section is a section with a unchanged width of 0.91m and bottom elevation lower than the length of 18.29 m by 0.61 m.
Figure 1. Standard Johnson water channel schematic diagram

5.2. Calculating Conditions:
(1) Water temperature density-stratified flow test:
The reservoir was filled with water at 21.44 °C first, and then the cold water at 16.67 °C was introduced into the reservoir from the porthole 0.15 m high at inlet near the bottom. The outlet of the dam is located at the site 0.15 m above the bottom. The hole height is 0.15 m, and the inflow velocity is 0.014 m/s.

(2) Sediment density-stratified flow test:
The inlet sediment particle size is d50=0.006mm and the sediment concentrations are set to 38kg/m³ and 100kg/m³.

5.3. Calculating Results
Figure 2 shows the temperature stratified flow condition. The longitudinal flow velocity distribution of mid-perpendicular line on the cross section 11.43m away from the inlet at t = 11 min in model calculation is compared with the actual measured distribution.

Figure 2. Longitudinal flow velocity distribution of mid-perpendicular line 11.43m away from the inlet at t = 11 min

According to the flow direction of longitudinal flow velocity, water body of the section can be divided into two parts: the subsurface flow layer and the upper water body. The flow velocity of the bottom cold water layer is large, the thickness of the subsurface flow layer is about 0.12 m, and the reverse vortex flow appears in the upper water body of the subsurface flow with small flow velocity.

The comparison of prediction results of the model and the measured results shows that submerged flow motion of cold water and reverse eddy current of the upper water body are reflected; the flow velocity distribution of them is basically the same; the calculated thickness of the submerged flow is in
good agreement with the actual measured thickness, and the longitudinal flow velocity along the water depth direction is basically the same.

Figure 3 compares the variation of the outflow water temperature obtained from model calculation and that from actual measurement. The measured results show that the outflow water temperature began to decrease at t=14.6min, and decreased relatively fast at the beginning and then slowed down; the water temperature dropped to 19.4 °C at t=25.6min. The analysis of calculation results of the model and the actual measurement shows that time delay of outflow cooling is 2 minutes, and the degree of temperature drop of the outflow water is small; the temperature of the outflow is about 19.8 °C at stable state, about 0.4 °C higher than the actual situation.

![Figure 3. Outflow water temperature changes with time](image1)

Figures 4 show the calculation results of the turbid water thickness along the density-stratified flow with 38kg/m³ sediment concentration.

![Figure 4. Turbid water thickness distribution along the sediment density-stratified flow (S=38kg/m³)](image2)

The calculation results show that the density-stratified flow phenomenon due to density difference is obvious, and the thickness of the density-stratified flow is also significantly different under different sediment concentration. It can also be seen from the figure that under the boundary conditions of this experiment, the location of the density-stratified flow plunging point and plunging depth under different sediment concentrations are basically the same, and with the transfer of density-stratified flow, the difference of thickness of the turbid water shows a trend of gradual increase. It can be seen from the above calculation results that the three-dimensional turbulence model established in this
study can simulate the flow pattern, water temperature and sediment transfer process well, and can provide good calculation effect.

6. Conclusion

The numerical relationship among vertical turbulence $P/S$ $\sigma_t$, turbulence and stratification feature parameters is established by analyzing the flux expression of turbulent stress/flux algebraic mode. Corresponding density change rate parameters reflecting changes in temperature and sediment concentration are introduced to correct the vertical diffusion coefficient of turbulent. A three-dimensional turbulence model with multi-phase coupling of water flow, temperature and sediment is established on this basis. The model reflects the basic physical phenomena of density-stratified flow well. And the adopted finite volume method and SIMPLEC algorithm have the characteristics of good convergence, fast calculation speed and stable calculation, and solve the computational efficiency problem of three-dimensional turbulent density-stratified flow well. The numerical model proposed in this paper can simulate the density stratification and sediment stratification of reservoirs, and it has certain application value for the operation and management decision about reservoirs.

Acknowledgments

This work was partially funded and supported by the National Natural Science Foundation of China (Grant Nos. 51479081) and the National Key Research and Development Program (2016YFC0402409-07).

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