Encrypting Majorana Fermions-\textit{qubits} as Bound States in the Continuum

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We theoretically investigate a topological Kitaev chain symmetrically connected to a double quantum-dot (QD) setup hybridized with metallic leads. In this system, we observe the emergence of two striking phenomena: i) a decrypted Majorana Fermion (MF)-\textit{qubit} recorded over a single QD, which is detectable by means of conductance measurements due to the asymmetrical MF-leaked state into the QDs; ii) an encrypted \textit{qubit} recorded in both QDs when the leakage is symmetrical. In such a regime, we have a cryptography-like manifestation, since the MF-\textit{qubit} becomes bound states in the continuum, which is not detectable in conductance experiments.

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\textbf{Introduction}.—It is well known that Majorana fermions (MFs) zero-modes\cite{1, 2} are expected to appear bounded to the edges of a topological Kitaev chain\cite{3–7}. Interestingly enough, by approaching the Kitaev chain to a quantum dot (QD), the MF state leaks\cite{8} into it and manifests itself as a zero-bias peak (ZBP) in conductance measurements. The latter reveals experimentally the MF-\textit{qubit} recorded over the QD. Indeed, such a phenomenon was experimentally confirmed in a QD hybrid nanowire made by InAs/Al\cite{9} with huge spin-orbit interaction\cite{15, 16}, similarly to magnetic atomic chains covering superconductors with magnetic fields, being the nanowire placed close to an $s$-wave superconductor. It is worth mentioning that MFs can also emerge in the fractional quantum Hall state with filling-factor $\nu = 5/2$\cite{10}, in three-dimensional topological insulators\cite{11}, at the core of such exotic excitations. BICs were proposed by von Neumann and Wigner in 1929\cite{18, 19} as quantum states with measurable invisibility feature of the BICs. Hence, for the sake of simplicity, we label by cryptography of the MF-\textit{qubit}.

In this work, we show that the employment of two QDs, as depicted in Fig.1(a), enables the cryptography of the MF-\textit{qubit} state. Our main theoretical findings rely on the interplay between the leakage effect and the so-called bound states in the continuum (BICs)\cite{18, 19}. In this context, it is worth recalling the underlying Physics of such exotic excitations. BICs were proposed by von Neumann and Wigner in 1929\cite{18} as quantum states with localized square-integrable wave functions, but surprisingly within the domain of the energy continuum region. Noteworthy, such states trap particles indefinitely. BICs constitute a current topic of broad interest\cite{20}, appearing in several physical systems like graphene\cite{21–23}, optics and photonics\cite{24–27}, arrangements exhibiting singular chirality\cite{28} and Floquet-Hubbard states due to A.C. fields\cite{29, 30}. Moreover, BICs assisted by MFs enable applications like the storage of \textit{qubits}\cite{31} and the electrical current switch\cite{32} as well. It should be mentioned that electrons trapped at BICs are prevented to decay into the energy continuum of the environment. Once BICs are undetectable by electrical conductance and accounting for the leakage effect, we benefit of such a remarkable invisibility feature of the BICs. Hence, for the sake of simplicity, we label by cryptography of the MF-\textit{qubit} when the ZBP disappears as a BIC, turning itself undetectable via conductance measurements. As it will
be discussed below, we also find an asymmetrical leakage of the MF-qubit. In such a situation, the ZBP is visible in the conductance and we call such a regime decrypted MF-qubit, since the MF state from the Kitaev chain edge leaks solely into a single QD of the proposed setup (Fig.1). Equivalently, the qubit is recorded over this QD. Hence, the encrypted qubit is achieved when the recording is symmetrical over the QDs, but with an invisible ZBP in the conductance. In this regime, the MF-leaked state at zero-bias is split into the QDs, thus becoming BICs.

The Model.—Below we describe theoretically the setup outlined in Fig.1(a) with a topological Kitaev chain coupled to a double QD setup hybridized with metallic leads[33]. The oversimplified sketch of such a system is depicted in Fig.1(b), which is ruled by the Hamiltonian

\[
\mathcal{H}_{\text{Full}} = \sum_{\alpha} \xi_{\alpha} c_{\alpha}^\dagger c_{\alpha} + \sum_{\alpha} \xi_{\alpha} d_{\alpha}^\dagger d_{\alpha} + T_c (d_L^\dagger d_R + \text{H.c.}) + V \sum_{\alpha} (\xi_{\alpha} d_{\alpha}^\dagger + \text{H.c.}) + \mathcal{H}_{\text{MFS}},
\]

where the electrons in the lead \( \alpha = L, R \) are described by the operator \( c_{\alpha}^\dagger, c_{\alpha} \) for the creation (annihilation) of an electron in a quantum state labeled by the wave number \( k \) and energy \( \xi_{\alpha} = \xi_k - \mu_\alpha \), with \( \mu_\alpha \) as the chemical potential. For the QDs coupled to the leads, \( d_{\alpha}^\dagger (d_{\alpha}) \) creates (annihilates) an electron in the state \( \xi_{\alpha} \), which is gate tunable. The left-right QD coupling is \( T_c \), while \( V \) stands for the hybridization between these QDs and the leads. Additionally, the QDs couple symmetrically to the Kitaev chain with tunneling amplitude \( \lambda \), respectively for the left and right QDs as follows

\[
\mathcal{H}_{\text{MFS}} = i \xi_M \Psi_1 \Psi_2 + \lambda (d_R - d_L^\dagger) \Psi_1 + \lambda (d_L - d_R^\dagger) \Psi_2, \tag{1}
\]

where \( \Psi_1 = \Psi_1^\dagger \) and \( \Psi_2 = \Psi_2^\dagger \) account for the MFs lying on the edges of the chain with overlap term \( \xi_M \sim e^{-L/\xi} \), wherein \( L \) and \( \xi \) designate respectively, the size of the Kitaev chain and the superconducting coherence length.

We stress that, for a sake of simplicity, by employing the following substitutions \( d_L = (\cos \theta) \tilde{d}_L - (\sin \theta) \tilde{d}_R^\dagger \), \( d_R = (\sin \theta) \tilde{d}_L^\dagger + (\cos \theta) \tilde{d}_R \), \( \xi_{kL} = (\cos \theta) \xi_k + (\sin \theta) \xi_{kL} \) and \( \xi_{kR} = (\sin \theta) \xi_k^\dagger + (\cos \theta) \xi_{kL}^\dagger \) into the Hamiltonian of Eq.(1), in particular at the zero-bias regime (\( \mu_\alpha = 0 \equiv \text{Fermi level of the leads} \)), we obtain

\[
\mathcal{H}_{\text{Full}} = \sum_{k,\sigma} \tilde{\xi}_k \tilde{c}_k^\dagger \tilde{c}_k + \sum_{\sigma} \xi_{d\sigma} \tilde{d}_\sigma^\dagger \tilde{d}_\sigma + V \sum_{k,\sigma} (\xi_{k\sigma} \tilde{d}_\sigma^\dagger + \text{H.c.}) + \mathcal{H}_{\text{MFS}}, \tag{2}
\]

which mimics an effective single QD coupled to an unique lead both exhibiting an artificial spin degree of freedom \( \sigma = \pm 1 \) (\( \uparrow, \downarrow \)) (see Fig.1(c) for such a representation).

We call attention that from now on, we label the aforementioned variable by pseudo-spin, wherein \( \cos(2\theta) = \sqrt{4(T_c)^2 + (\Delta \epsilon)^2} \), \( \Delta \epsilon = \epsilon_L - \epsilon_R \) as the detuning of the original spinless QDs, the pseudo-Zeeman gap \( \epsilon_{d\uparrow} - \epsilon_{d\downarrow} \), with

\[
\epsilon_{d\sigma} = \frac{(\epsilon_L + \epsilon_R)}{2} + \frac{\sqrt{4(T_c)^2 + (\Delta \epsilon)^2}}{2}
\]

and

\[
\mathcal{H}_{\text{MFS}} = \xi_M \eta_\uparrow^\dagger \eta_\uparrow - \frac{1}{2} \sum_\sigma \xi_\sigma \tilde{d}_\sigma \eta_\sigma^\dagger + \tilde{d}_\sigma \eta_\sigma + \text{H.c.}, \tag{4}
\]

where we have used \( \Psi_1 = \frac{1}{\sqrt{2}} (\eta_\uparrow^\dagger + \eta_\downarrow) \) and \( \Psi_2 = \frac{i}{\sqrt{2}} (\eta_\uparrow - \eta_\downarrow) \) in order to build the qubit \( \eta_\uparrow \) composed by the MFs, namely the MF-qubit, with \( \lambda = \frac{\sqrt{2}}{\sqrt{2}} (\cos \theta + \sigma \sin \theta) \) as a pseudospin-dependent amplitude. As a result, the pseudo-Zeeman gap becomes dressed by such an interaction, i.e., \( \tilde{\epsilon}_{d\uparrow} - \tilde{\epsilon}_{d\downarrow} \), which will be addressed later on.

In what follows, we use the Landauer-Büttiker formula for the zero-bias conductance \( \mathcal{G} \) to analyze the transport through the QDs, which is

\[
\mathcal{G} = \frac{e^2}{h} \int d\epsilon \left( \frac{d \rho_{T}}{d\epsilon} \right) T_{\text{Total}}, \tag{5}
\]

where \( f_\epsilon \) stands for the Fermi-Dirac distribution, \( T_{\text{Total}} = \sum_j T_{j\uparrow} + \sum_{j\downarrow} T_{j\downarrow} \) encodes the system total transmittance with \( j = \xi, R \) respectively for \( j = R, L \) to correlate distinct QDs, in which \( T_{j\uparrow} = T_{j\uparrow} + T_{j\downarrow} + T_{j\uparrow} + T_{j\downarrow} \) dictates the transmittance through the channels \( l, j = \xi, R \) in terms of the coefficients \( T_{\sigma\delta} \) for the pseudospin representation.

Furthermore, \( T_{j\uparrow} = \pi \Gamma_{j\uparrow} \) depends upon the Anderson broadening \( \Gamma = \pi V^2 \sum_k \delta (\epsilon - \xi_k) \) [35] and \( \rho_{j\uparrow} = (-1/\pi) \text{Im}(\tilde{G}_{d\uparrow},d\downarrow) \), the densities of states for the spinless QDs from the Hamiltonian of Eq.(1) in terms of the retarded Green’s functions \( \tilde{G}_{d\sigma}, d\downarrow \), which are given by

\[
\rho_{LL} = -\frac{1}{\pi} \text{Im} \{ \cos^2 \theta \tilde{G}_{d\uparrow},d\uparrow + \sin^2 \theta \tilde{G}_{d\downarrow},d\downarrow \}, \tag{6}
\]

\[
\rho_{RR} = -\frac{1}{\pi} \text{Im} \{ \sin^2 \theta \tilde{G}_{d\uparrow},d\uparrow + \cos^2 \theta \tilde{G}_{d\downarrow},d\downarrow + \sin \theta \cos \theta (\tilde{G}_{d\uparrow},d\uparrow + \tilde{G}_{d\downarrow},d\downarrow) \}, \tag{7}
\]

\[
\rho_{RL} = -\frac{1}{\pi} \text{Im} \{ \sin \theta \cos \theta (\tilde{G}_{d\uparrow},d\uparrow - \tilde{G}_{d\downarrow},d\downarrow) + \cos^2 \theta \tilde{G}_{d\uparrow},d\uparrow - \sin^2 \theta \tilde{G}_{d\downarrow},d\downarrow \}, \tag{8}
\]

and

\[
\rho_{LR} = -\frac{1}{\pi} \text{Im} \{ \sin \theta \cos \theta (\tilde{G}_{d\uparrow},d\uparrow - \tilde{G}_{d\downarrow},d\downarrow) - \sin^2 \theta \tilde{G}_{d\uparrow},d\uparrow + \cos^2 \theta \tilde{G}_{d\downarrow},d\downarrow \}, \tag{9}
\]

here written as functions of the retarded Green’s functions \( \tilde{G}_{d\sigma}, d\downarrow \) within the mapping on the pseudospin degree. To evaluate \( \tilde{G}_{d\sigma}, d\downarrow \), we should employ the equation-of-motion method[34] by using Eqs.(3) and (4) as follows:
the linear system:

\[
(\varepsilon - \epsilon_{ds} - \lambda^2 K + i\Gamma)\tilde{G}_{ds,ds} = -\lambda^2 K\tilde{G}_{ds,ds} + \lambda^2 K\tilde{G}_{ds,ds} = 0,
\]

and

\[
+ (\varepsilon + \epsilon_{ds} - \lambda^2 K + i\Gamma)\tilde{G}_{ds,ds} = 0.
\]

where \(\sigma\) is the opposite of \(\sigma\) and \(K = (\varepsilon + \epsilon_{M})^{-1} + \varepsilon_M^{-1}\). To perform the analysis of the model in the next section, we make explicit that we have solved the current system numerically.

Results and Discussion.---In the simulations below the temperature \(T = 0\) is assumed and \(\Gamma = 40\mu\text{eV}\) [8, 35] as the energy scale. The topological Kitaev chain, for a sake of simplicity, is treated as very large, which imposes \(\epsilon_M \to 0\). Thus, in order to make explicit the phenomenon of MF-qubit cryptography, we begin discussing the picture requested for the emergence of such in Fig. 2. Fig. 2(a) accounts for \(\epsilon_L = -2\Gamma\), \(\lambda = 5\Gamma\), \(T_L = 1\Gamma\) and \(\epsilon_L = 1\Gamma\), where we verify a ZBP with amplitude of 1/4 in \(T_{\text{Total}}\) of Eq.(5) as a function of \(\varepsilon\). This detectable resonance represents the leakage of the MF-qubit \(\eta_1\) into the double QD setup. Additionally, it also encodes the recording of a decrypted MF-qubit over the left QD, which will be elucidated later on via Figs. 3 and 4. On this ground, let us consider the sequence of panels from (b) to (d), which describes the qubit cryptography itself: by changing just \(\epsilon_L\), we notice that the ZBP amplitude becomes reduced progressively up to entire quenching in Fig. 2(d). In this case, solely a couple of peaks stay visible denoting the dressed pseudo-Zeeman gap \(\epsilon_{\text{ZBP}} - \epsilon_{\text{dL}}\). Indeed, we will clarify that the ZBP becomes BICs, being undetectable by \(T_{\text{Total}}\). It means that if the ZBP is not perceived, we have the accomplishment of the MF-qubit cryptography, which appears addressed in detail by Figs. 3 and 4.

Fig. 3 exhibits the density plots for \(T_{\text{Total}}, T_{LL}\) and \(T_{RR}\) spanned by the axis \(\varepsilon\) and \(\varepsilon_L\) for fixed \(\epsilon_R = -2\Gamma\), \(\lambda = 5\Gamma\) and \(T_L = 1\Gamma\). It is worth noticing that all panels in Figs. 3(a)-(c) present a ZBP structure. However, each one reveals different aspects on the leakage effect. For instance, in Fig. 3(a) we highlight the upper region marked by a yellow dashed ellipse: it gives the domain where the MF-qubit cryptography is allowed, once the ZBP is absent. Figs. 3(b) and (c) contain the asymmetrical leakage into the QDs and the decrypted MF-qubit left recording as well. Notice that in the latter, nearby \(\epsilon_L = 1\Gamma\), the right QD decouples from the setup, due to \(T_{RR} = 0\). This region is then identified by white dashed ellipses in panels (a)-(c) of the same figure. As a result, the MF state is recorded solely at the left QD as Fig.3(b) ensures. Concerning the satellite arcs aside the ZBP in Figs. 3(a)-(c), they account for the dressed pseudo-Zeeman gap \(\epsilon_{\text{ZBP}} - \epsilon_{\text{dL}}\). These arcs are predominantly absent, as we can see, at the lower region of Fig. 3(a). This points out that BICs away from the ZB limit are also reliable in this device.

Thereby, in order to fully understand the underlying physics on the decrypted MF-qubit left recording versus the MF-qubit cryptography, we should consider Fig. 4. In Fig. 4(a) the analysis of \(T_{\text{Total}}\) shows that the leakage of the MF occurs only over the left QD. In this way, the decrypted MF-qubit situation is achieved: \(T_{\text{Total}}\) exhibits a ZBP with amplitude 1/4 in contrast to \(T_{\text{RR}}\). Thus in order to understand such an issue, we should focus on the insets. \(T_{\text{RR}}\) presents \(T_{\uparrow\uparrow} + T_{\uparrow\downarrow}\) perfectly phase shifted by \(\pi\) with respect to \(T_{\downarrow\uparrow} + T_{\downarrow\downarrow}\) [Fano dip] [36, 37], thus resulting in a decoupled QD from the setup. For \(T_{\text{LL}}, T_{\uparrow\uparrow} + T_{\uparrow\downarrow}\) and \(T_{\downarrow\uparrow} + T_{\downarrow\downarrow}\) interfere constructively. In Fig. 4(b) for \(T_{\text{RR}}\), the Fano dip in \(T_{\uparrow\downarrow} + T_{\downarrow\uparrow}\) is not perfect as previously and does not cancel \(T_{\downarrow\uparrow} + T_{\downarrow\downarrow}\) anymore. Particularly, the Fano dip found in \(T_{\text{RR}}, T_{\uparrow\downarrow} + T_{\downarrow\uparrow}\) interferes destructively and perfectly with the peak in \(T_{\text{LL}} + T_{\text{RR}}\). Finally, this yields the MF-qubit cryptography here proposed. In this way, the recording of the qubit is found secure at two apart sites and hidden as BICs, which are equally split into the

![Figure 2](image-url)

Figure 2. (Color online) \(T_{\text{Total}}\) as a function of \(\varepsilon\): (a) The ZBP gives the asymmetrical leakage of the MF-qubit \(\eta_1\) into the left QD (see also Fig. 3). (b)-(c) The increasing of \(\epsilon_R\) yields the process for encrypting this qubit, which is characterized by the quenching of the ZBP amplitude. (d) Here the ZBP (the MF-qubit) is hidden as BICs equally split into the QDs, where only the dressed pseudo-Zeeman gap is visible (see also Fig. 4).
Figure 3. (Color online) Density plots of: (a) $T_{\text{Total}}$, (b) $T_{LL}$ and (c) $T_{RR}$ spanned by $\varepsilon_L$ and $\varepsilon$, with $\varepsilon_R = -2\Gamma$, $\lambda = 5\Gamma$ and $T_c = 1\Gamma$. The ellipses depicted show at zero-bias: i) the region for the MF-qubit cryptography (yellow dashed ellipse) in (a) and ii) the corresponding for the decrypted MF-qubit left recording (white dashed ellipse) in (a) and (b), due to the right QD entirely decoupled from the system as panel (c) shows (white dashed ellipse).

Figure 4. (Color online) $T_{jl}$ in (a) characterizing the decrypted MF-qubit left recording. $T_{LL}$ shows a ZBP with amplitude $1/4$, while $T_{RR}$ does not: the inset reveals that $T_{RR}$ exhibits $T_{\uparrow\uparrow} + T_{\downarrow\downarrow}$ perfectly phase shifted by $\pi$ with respect to $T_{\uparrow\downarrow} + T_{\downarrow\uparrow}$ (Fano dip). As aftermath, this QD is disconnected from the system. In (b), we have the MF-qubit cryptography: in $T_{RR}$, the Fano dip is not perfect as before. However, a Fano dip in $T_{LR} + T_{RL}$ interferes destructively and exactly with $T_{LL} + T_{RR}$. It means that MF-qubit is hidden as BICs equally divided into the QDs.

Interestingly enough, the underlying physics of this cryptography assisted by BICs has a simple picture: the electronic waves traveling forth and back between the QDs ($T_{LR} + T_{RL}$), in particular at zero-bias, interfere destructively with those waves that only pass through these QDs ($T_{LL} + T_{RR}$) and as a result, the BICs within the latter emerge. Regarding the satellite arcs aside the ZBP in Figs.4(a) and (b), we should mention that they are also the result of interference processes in $T_{jl}$ as observed.

Conclusions.—In summary, we have found theoretically that the cryptography of the MF-qubit is feasible in the system of Fig.1(a). We have showed that the recording of the MF-qubit over a single QD is due to an asymmetrical leakage of the MF state into the QDs. The encrypted MF-qubit is performed when the leaking is symmetrical, wherein the MF-leaked state becomes BICs. Interestingly enough, the MF-qubit is hidden as BICs equally divided into the QDs.

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