Analysis of Modified Bolotin Method on maximum deflection of three stiffeners plates

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Abstract. The main goal of this study is to analyze the correlation between the detonation source and the time duration of blast loading and the plate’s dynamic response with the variation of peak pressures. The plate was modeled as a concrete orthotropic plate with semi rigid support at all its sides. Natural frequency can be found with Modified Bolotin Method and the numerical calculation is solved with Mathematica and Microsoft Excel. The results show that the longer the explosion distance, the maximum pressure due to the explosion is smaller and the time that the blast wave needed to reach the plate’s surface will become longer. When the peak pressure of blast loading become bigger, the plate’s dynamic response and deflection will become bigger.

Keywords: Modified Bolotin Method, Dynamic Response, Plate Stiffener

1. Introduction
Research on plate behavior has been carried out since 1766 and research on this topic is growing very rapidly in the world. Actions of terrorism in Indonesia that have attracted a lot of attention include the bomb explosion at the JW Marriott hotel in 2009. Since then, research on plate behavior due to blast loads has begun to develop in Indonesia. Usually, earthquake loads are taken into account in the design of building structures [1].

However, with the occurrence of a terrorist attack in the form of a bomb explosion, it is possible that the structural planner will also consider the effect of the blast load in anticipation of the strength of the building [2].

Based upon International perspectives, reference guidelines for protecting structures from terrorist attacks have been developed, such as those contained in the Reference Manual to Mitigate Potential Terrorist Attacks Against Building [3].

This research itself provides an overview through modeling of the movement of the plates when getting an explosion load with maximum pressure strength due to different blast loads [4].
2. General Description on Building Floor Plate Modelling
The floor plate of the building is modeled as an orthotropic plate with semi-rigid placement on all four sides, measuring 5.05 m x 7.4 m which is given three stiffeners in one direction of the axis in the form of joists measuring 30 cm x 60 cm [5, 6].

The aforementioned modeling is illustrated in Figure 1, and Table 1 in the following depiction.

![Figure 1. Plate with Three Stiffeners](image)

| Unit | Dimension          | Remark                          |
|------|--------------------|---------------------------------|
| ax   | 2.4 m              | Distance among plate baseline   |
| ex   | 0.16 m             | System Eccentricity            |
| k1   | 5.0E+06 Nm/rad     | Stiffness of x                  |
| k2   | 3.5E+07 Nm/rad     | Stiffness of y                  |
| G    | 1.0713E+10 N/m²    | Shifting Module                |
| E    | 2.61252E+10 N/m²   | Plate's Module Elasticity      |
| Dx   | 1.33614E+08 Nm     | Plate's Flexible Rigidity of x  |
| Dy   | 6.29258E+06 Nm     | Plate's Flexible Rigidity of y  |
| B    | 6.29258E+06 Nm     | Effective Rotating Rigidity     |

Subsequently, in term of Properties of Concrete Slabs and Sub Beams; the orthotropic plate motion equation can be stated as follows:

$$D_x \frac{\partial^4 w(x,y,t)}{\partial x^4} + 2B \frac{\partial^2 w(x,y,t)}{\partial x^2 \partial y^2} + D_y \frac{\partial^2 w(x,y,t)}{\partial y^4} + \gamma h \frac{\partial^2 w(x,y,t)}{\partial t^2} = p(x,y,t)$$

$$D_x = \frac{E' \cdot h^3}{12} + \frac{E \cdot b \cdot x \left\{ \left[ (hx-\frac{b}{2})^2 \right] - \left( (b-x)^2 \right) \right\}}{6ax}$$

$$D_y = \frac{E' \cdot h^3}{12}$$
The boundary conditions that apply to rectangular plates with semi-rigid placement on all four sides are:

1. Along the coordinate of \( x=0 \) dan \( x=a \):
   \[ W(x,y) = 0 \]
   \[ -D_x \left[ \frac{\partial^2 W(x,y)}{\partial x^2} + \nu \frac{\partial^2 W(x,y)}{\partial y^2} \right] = k_1 \frac{\partial W(x,y)}{\partial x} \]  
   (4)

2. Along the coordinate of \( y=0 \) dan \( y=b \):
   \[ W(x,y) = 0 \]
   \[ -D_y \left[ \frac{\partial^2 W(x,y)}{\partial x^2} + \nu \frac{\partial^2 W(x,y)}{\partial y^2} \right] = k_2 \frac{\partial W(x,y)}{\partial y} \]
   (5)

3. Explosion Load and Analysis

The explosion load is elaborated in this paper [7] and this load is due to a bomb explosion, according to Saikov, that can be described as follows, according to Figure 2

![Figure 2](image_url)  

**Figure 2.** Relationship between Explosive Pressure and Time

To obtain a total system solution, it is necessary to define the load function that preliminarily works in the system which can be stated as follows:

\[ p(x,y,t) = P(x(t),y(t),t) = P(t) \delta(x-xo) \delta(y-yo) \]

(6)

Where as

- \( x(t) \) = function of movement for applied load in x direction
- \( y(t) \) = function of movement for applied load in y direction
- \( x(t) = xo \) and \( y(t) = yo \) = load position on the time of \( t = to \) (xo,yo)
- \( P(t) \) = impulse applied load on the time \( t \)

\[ P(t) = P_0 \left( 1 - \frac{t}{td} \right) \]

(7)

In this situation, \( P_{max} \) is depicted in the magnitude of 1.3 MPa and applied on the working position at premises of the following matrix \( \begin{bmatrix} a & b \\ 4 & 4 \end{bmatrix} \).
If the aforementioned equation is substituted to the prior equation, then it results in the following equation of:

\[ P(x,y,t) = P_0 \left(1 - \frac{t}{\tau_d}\right) \delta(x-x_0) \delta(y-y_0) \]  \hspace{1cm} (8)

As the analysis part, the following discourse are surrounding within the plate free vibration, with the buffer stage, in which \( \gamma = 0 \). It is interpreted as the transversal deflection on plate can be expressed in the following equation:

\[ W(x,y,t) = W(x,y) \sin \omega t \]  \hspace{1cm} (9)

In the prior equation (9), it can be interpreted as:

- \( W(x,y) \) = spatial function
- \( \omega \) = natural frequency system.

If equation (9) is substituted into equation (1), subsequently the new equation is resulted in the following equaiton:

\[
\begin{align*}
& D_x \frac{\partial^4 W}{\partial x^4} + 2B \frac{\partial^4 W}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 W}{\partial y^4} - \rho h \omega^2 W = 0 \\
& \frac{\partial^4 W}{\partial x^4} \frac{\partial^4 W}{\partial y^4}
\end{align*}
\]  \hspace{1cm} (10)

In order to both equation (4) and (5) are complying with the threshold of equation (10), then, transversal plate deflection has to be conveyed in the Double Fourier, within Navier solution of the following equation:

\[ W_{mn} = A_{mn} \sin \left(\frac{mnx}{a}\right) \sin \left(\frac{mny}{b}\right) \]  \hspace{1cm} (11)

In which,

- \( A_{mn} \) = determined frequency amplitude within initial condition
- \( mn \) = wave number in x direction
- \( a \) = integer index, within vibration pattern at x mode
- \( b \) = integer index, within vibration pattern at y mode
- \( a \) = Plate length at x direction
- \( b \) = Plate length at y direction

If the equation (11) is substituted into equation (10), then the plate system’s natural frequency is obtained with the allocation of all four joints of:

\[ \omega_{mn}^2 = \frac{n^4}{\rho h} \left[ D_x \left(\frac{mn}{a}\right)^4 + 2B \left(\frac{mn}{ab}\right)^2 + D_y \left(\frac{m}{b}\right)^4 \right] \]  \hspace{1cm} (12)

The natural frequency of the system for a rectangular plate with clamped, free, semi-rigid placement can be found by analogizing the rectangular plate as a plate having joint placement on all four sides [7].

The x (m) direction is replaced with p and the y (n) direction is replaced by q, where p and q are real numbers obtained from the transcendental equation.

Thus the natural frequency of a system with asymmetrical placement can be stated as follows:

\[ \omega_{mn}^2 = \frac{p^4}{\rho h} \left[ D_x \left(\frac{pq}{a}\right)^4 + 2B \left(\frac{pq}{ab}\right)^2 + D_y \left(\frac{q}{b}\right)^4 \right] \]  \hspace{1cm} (13)

In which

- \( pq \) = wave number at x direction
- \( a \)
\[ q_{п} = \text{wave number at y direction} \]
\[ p, q = \text{real numbers (for positioning not joints on both opposite sides) which can be solved by the Levy-type problem of the two auxiliary equations. This method is also known as the Modified Bolotin Method [8, 9].} \]

4. Results

The plate dynamic response can be found using the separation of variable method [10, 11,12]. Thus the particular solution of equation (1) can be expressed as:

\[ W(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} X_{mn}(x)Y_{mn}(y)T_{mn}(t) \] (14)

In which

\[ T_{mn} (t) = \text{the result of time function through further analysis, in particular through differential equation with coefficient function of } T_{mn} (t) \]

\[ T_{mn} (t) + 2\omega_{mn} \ddot{T}_{mn}(t) + \omega_{mn}^2 T_{mn}(t) = \frac{1}{\rho h Q_{mn}} \int_{0}^{a} \int_{0}^{b} X_{mn}(x)Y_{mn}(y) \rho(x,y,t) dx dy \] (15)

In which, \( Q_{mn} \) is normalization factor.

The result of particular solution in \( T_{mn} (t) \) can be written in integral form [13]

\[ T_{mn} (t) = \frac{\rho h Q_{mn}}{2} \int_{0}^{a} \int_{0}^{b} X_{mn}(x)Y_{mn}(y) \rho(x,y,t) dx dy \left[ e^{\frac{\omega_{mn} (t-t)}{\sqrt{1 - \gamma^2}} \sin \left( \sqrt{1 - \gamma^2} \omega_{mn} (t-t) \right) \right] dt \] (16)

Furthermore, the result of General solution system due to transversal dynamic load, can be represented by the following equation.

\[ W(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left( X_{mn}(x)Y_{mn}(y) \rho(x,y,t) e^{\frac{\omega_{mn} (t-t)}{\sqrt{1 - \gamma^2}}} \sin \left( \sqrt{1 - \gamma^2} \omega_{mn} (t-t) \right) \right) \frac{1}{\rho h Q_{mn}} \sqrt{1 - \gamma^2} \omega_{mn} \] (17)

The system natural frequency is calculated for the first 5 modes in the x direction and the second 5 modes in the y direction [14, 15] as illustrated in Table 2 and 3.
Table 2. Price of Natural Frequency for Various Modes of Three Stiffeners

| m  | n  | p          | q      | $\omega_{mn}$ | $\omega_{mn}$ (rad/det) |
|----|----|------------|--------|---------------|-------------------------|
| 1  | 1  | 0.999398  | 1.26149| 75237.422     | 274.2944082             |
| 1  | 2  | 0.999459  | 2.38509| 100387.15     | 316.8393056             |
| 1  | 3  | 0.999533  | 3.44566| 165322.3      | 406.5984482             |
| 1  | 4  | 0.999609  | 4.48299| 301970.88     | 549.5187697             |
| 1  | 5  | 0.999678  | 5.51145| 553454        | 743.9448908             |
| 2  | 1  | 1.99986   | 1.1393 | 1127992.4     | 1062.069881             |
| 2  | 2  | 1.99986   | 2.25388| 1184331.6     | 1088.270005             |
| 2  | 3  | 1.99986   | 3.33931| 1300152.5     | 1140.2423               |
| 2  | 4  | 1.99987   | 4.40286| 1507248.6     | 1227.700526             |
| 2  | 5  | 1.99988   | 5.4525 | 1848970.4     | 1359.768513             |
| 3  | 1  | 2.99994   | 1.09938| 5661637.1     | 2379.419484             |
| 3  | 2  | 2.99994   | 2.19162| 5769409.9     | 2401.959591             |
| 3  | 3  | 2.99994   | 3.27277| 5970360.5     | 2443.432111             |
| 3  | 4  | 2.99994   | 4.34248| 6296024       | 2509.187908             |
| 3  | 5  | 2.99994   | 5.40241| 6789923.8     | 2605.748217             |
| 4  | 1  | 3.99996   | 1.08417| 17845446      | 4224.387085             |
| 4  | 2  | 3.99996   | 2.16548| 18025554      | 4245.611185             |
| 4  | 3  | 3.99996   | 3.24183| 18346924      | 4283.331001             |
| 4  | 4  | 3.99996   | 4.31215| 18841068      | 4340.629906             |
| 4  | 5  | 3.99997   | 5.37631| 19551852      | 4421.747666             |
| 5  | 1  | 4.99998   | 1.08182| 43517408      | 6596.77256              |
| 5  | 2  | 4.99998   | 2.16191| 43793065      | 6617.632908             |
| 5  | 3  | 4.99998   | 3.23888| 44273665      | 6653.845916             |
| 5  | 4  | 4.99998   | 4.31134| 44990636      | 6707.505913             |
| 5  | 5  | 4.99998   | 5.37888| 45987408      | 6781.401627             |

Table 3. The Effect Result and its Variety of Threshold

| The Effect Result | Glass Minor cuts | Threshold injuries open or buildings | Potentially Lethal Injuries | Threshold, concrete column fail |
|-------------------|------------------|--------------------------------------|-----------------------------|--------------------------------|
| Detonation Distance R (m) | 121.92 | 45.72 | 24.384 | 2.4384 |
| Weigh of TNT W (kg) | 45 | 45 | 45 | 45 |
| Z | 34.28 | 12.85 | 6.86 | 6.86 |
| Pso (KPa) | 3 | 11 | 28 | 3100 |
| td (ms) | 24.9 | 17.78 | 14.23 | 1.6 |

The further the detonation distance, the smaller the maximum explosion pressure will be (as shown at figure 3). It can be expressed in the equation $y = 12935 x ^ (-1.81)$, where $y$ is the maximum burst pressure (KPa) and $x$ is the detonation distance (m).
The longer the duration of the explosion (td), the smaller the maximum explosion pressure will be (as shown at figure 4). This relationship can be expressed in the equation $y = -1220 \ln(x) + 3597.5$, where $y$ is the maximum burst pressure (KPa) and $x$ is the duration of the explosion (m).

**Figure 4.** Relationship between Pso and td
Table 4. Price of Maximum Deflection and Dynamic Responses of Plate with Three Stiffeners

| The Effect Results | Glass Minor cuts | Threshold injuries open or buildings | Potentially Lethal Injuries | Threshold, concrete column fail |
|--------------------|------------------|--------------------------------------|-----------------------------|-----------------------------|
| Maximum Deflection Value (m) | 0.000194013 | 0.0014606 | 0.0030009 | 0.0933488 |
| In time function t | 1301.67 | 2719.61 | 13104.7 | 273495 |
| In y axis | 6927.2 | 35194.7 | 124245 | 3792294.1 |
| In x axis | 4564.37 | 25057 | 24231.9 | 2178548.3 |
| In time function t | 1046.76 | 2469.71 | 19171.9 | 512707 |
| Mx (Nm) | 3744.71 | 9068.05 | 27786 | 200637 |
| In y axis | 1972.01 | 1377.55 | 4936.64 | 289537 |
| In x axis | 782.28 | 3317.59 | 9031.11 | 211356 |
| In time function t | 487.48 | 2945.01 | 8250.11 | 190268 |
| My (Nm) | 782.28 | 3317.59 | 9031.11 | 211356 |
| In y axis | 487.48 | 2945.01 | 8250.11 | 190268 |
| In x axis | 1067.35 | 44869.7 | 159339 | 3335071.1 |
| In time function t | 650.26 | 1836.22 | 6527.17 | 161488 |
| Qx (N) | 4888.62 | 18110.6 | 34678.3 | 3006848.6 |
| In time function t | 1067.35 | 44869.7 | 159339 | 3335071.1 |
| In y axis | 650.26 | 1836.22 | 6527.17 | 161488 |
| Qy (N) | 4888.62 | 18110.6 | 34678.3 | 3006848.6 |
| In x axis | 169.29 | 40.97 | 2062 | 36551.4 |

5. Conclusions
Based on the simulation of the dynamic response movement of the floor of the orthotropic rectangular building slab, it can be concluded that the further the detonation distance, the smaller the maximum explosion pressure will be.

In further elaboration, this relationship can be expressed in the equation $y = 12935 x^{-1.81}$, where $y$ is the maximum burst pressure (KPa) and $x$ is the detonation distance (m).

The longer the duration of the explosion (td), the smaller the maximum explosion pressure will be. This relationship can be expressed in the equation $y = -1220 \ln(x) + 3597.5$, where $y$ is the maximum burst pressure (KPa) and $x$ is the duration of the explosion (m).

The greater the maximum pressure force due to a bomb explosion, the more will the deflection and dynamic response on the plate

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