Research on the method of determining satellite state vector based on Kepler orbital parameters

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Abstract: to simplify the process of satellite orbit determination, can use limited conditions known to the calculation of satellite orbit dynamic state vector to determine satellite position to implement the standardization of satellite orbit, this paper introduces the satellite state vector at any time using the Kepler orbital parameters method, the calculation method of Kepler orbit parameters are mostly only for satellite without disturbing force movement. This paper is based on the three-point method which is used to calculate the orbit parameters of Kepler satellite in the case of satellite unphotographed.

1. Introduction
Kepler orbit parameter is an important concept in satellite orbit theory. The position and velocity of the satellite at any time can be determined by Kepler orbital parameters, namely, the state vector of the satellite, which represents the running state of the satellite at time T. The orbit state vector of synthetic aperture radar (SAR) satellite represents the operation state of the satellite at any time, that is, the position and velocity of the satellite. It not only affects the precision of SAR image geometric correction and stereo mapping, but also has an important influence on the precision of SAR image interferometry. The state vector of several time satellites is provided in the metadata of radar data products, but it is still a difficult problem to reconstruct the orbit of satellites by using the limited orbital information provided in radar metadata. Especially for the case that the orbital state vector data provided in the metadata is too widely spaced. In view of this phenomenon, this paper introduces a method to determine the state vector of the satellite at any time by Kepler orbital parameters, and makes an experiment to calculate the state vector of the satellite by orbital root number.

2. Kepler orbit parameters determination of satellite state vector method
The actual orbit of the satellite is affected by the perturbation force as well as the gravitation of the center of mass. The orbit is not fixed but changes with time, which can be regarded as a function of time [1-4]. The state vector of the satellite can take four Kepler orbital parameters $\rho, \Omega, I, \omega$ to describe.

1) The distance from the earth's core of a satellite.
2) the ascending node right ascension $\Omega$: satellite across equatorial plane in the northern hemisphere from the equatorial plane intersection point.
3) Orbital inclination $I$: the Angle between the positive orbital direction (the direction of satellite movement) and the positive equatorial direction at the ascending intersection.
4) perigee Angle distance: from the point of intersection to the diameter calculated from the counterclockwise rotation to the perigee to the diameter through the Angle.

The Kepler orbital parameters are expanded to the second order, as follows:
\[
\begin{align*}
\rho(t) &= \rho_0 + \rho_1 \cdot t^2 \\
I(t) &= I_0 + I_1 \cdot t \\
\omega(t) &= \omega_0 + \omega_1 \cdot t \\
\Omega(t) &= \Omega_0 + \Omega_1 \cdot t \\
\end{align*}
\] (1)

The solution of Kepler orbit parameters by state vector involves the calculation of inverse trigonometric function, and the final value of the inverse trigonometric function can only be finally determined by judging the direction limit of the Angle. The calculation model [5-7] has been mentioned, but the expression is not clear. Now the calculation formula is derived. You can solve the problem one by one by following the following steps:

Distance from the satellite to the center of the earth:
\[
\rho = \sqrt{X_s^2 + Y_s^2 + Z_s^2}
\]

\[V = \left( V_x, \ V_y \right) \] (2)

\[N = \left( N_x, N_y, N_z \right) = \frac{\overrightarrow{R_s} \cdot \overrightarrow{V_s}}{|\overrightarrow{R_s} \cdot \overrightarrow{V_s}|} \]

Some constants are abbreviated as follows:
\[
\begin{align*}
C_x &= Y_s \cdot V_s z - Z_s \cdot V_s y \\
C_y &= Z_s \cdot V_s x - S_x \cdot V_s z \\
C_z &= X_s \cdot V_s y - Y_s \cdot V_s x \\
C &= \sqrt{C_x^2 + C_y^2 + C_z^2} \] (3)

From the geometric meaning of N, it is easy to figure out the orbital inclination
\[I = \arccos \left( \frac{C_z}{C} \right) \] (4)

Construct unit vectors from vector N
\[
\begin{align*}
V_x &= \frac{-N_y}{\sqrt{N_x^2 + N_y^2}} \\
Y_y &= \frac{-N_x}{\sqrt{N_x^2 + N_y^2}} \\
X_r &= \begin{bmatrix} 100 \\
0 \cos(-I) - \sin(-1) \\
0 \sin(-1) \cos(-I) \end{bmatrix} \\
Y_r &= \begin{bmatrix} \cos(-\Omega) - \sin(-\Omega)0 \\
-\sin(-\Omega) \cos(-\Omega)0 \\
001 \end{bmatrix} \] (5)

using have I, \ Omega following the coordinates of the rotary press type calculation state vector:
\[
\begin{align*}
X_s &= \rho \left[ \cos \omega \cdot \cos \Omega - \sin \omega \cdot \sin \Omega \cdot \cos I \right] \\
Y_s &= \rho \left[ \cos \omega \cdot \sin \Omega + \sin \omega \cdot \cos \Omega \cdot \cos I \right] \\
Z_s &= \rho \left[ \sin \omega \cdot \cos I \right]
\end{align*}
\]

(7)

\[
\begin{align*}
V_{sx} &= \frac{dX_s}{dt} \\
V_{sy} &= \frac{dY_s}{dt} \\
V_{sz} &= \frac{dZ_s}{dt}
\end{align*}
\]

(8)

3. Experiment

The ers-1 image of track no. 8050 is now used as experimental data. The five satellite point state vectors provided in the image header file and their related data are shown in Table 1.

| Dot number | Satellite position vector /m | Satellite velocity vector /m |
|------------|-------------------------------|-----------------------------|
| 1 | -2278222.47 | 5479754.23 | 4005048.86 | 55.34572 | 4479.90806 | -6080.46200 |
| 2 | -2277973.64 | 5497858.17 | 3980375.00 | 67.95587 | 4455.85055 | -6098.06701 |
| 3 | -2277672.74 | 5515864.44 | 3955630.03 | 80.55065 | 4431.70637 | -6115.56322 |
| 4 | -2277320.84 | 5533772.71 | 3930814.38 | 93.12980 | 4407.47598 | -6132.95030 |
| 5 | -2276918.00 | 5551582.63 | 3905928.50 | 105.69302 | 4383.15980 | -6150.22794 |

Image acquisition time: 1993-01-29
First satellite moment: 03:23:03.968
Interval time per satellite point: 4.052

Calculate the values at each satellite point according to the formula given above \( \rho, \Omega, I, \omega \) The value. The time of the first satellite point is known, the time interval of every two consecutive satellite points is known, and the time of other satellites can be calculated. Taking the acquisition time of the first line of the image as the starting point, the least square method was used to solve the coefficients of each equation. The orbital parameters of the satellite Kepler are calculated as follows.

\[
\begin{align*}
\rho(t) &= 7159503.959922 + 9.793741691282 \cdot t + 0.00134608721557 \cdot t \cdot t \\
I(t) &= 1.76642965448 + 6.60170161877 e - 005 \cdot t \\
\omega(t) &= 2.53528498629 + 0.001046000281452 \cdot t \\
\Omega(t) &= 4.972258825745 + 4.588973197918e - 005 \cdot t
\end{align*}
\]

(9)

In order to verify the accuracy of the above formula (9), the time of each ephemeris point was substituted into the formula and the state vector of each satellite point was inversely calculated by using the fitting orbit equation. The reverse calculation results are shown in Table 2.

| Dot number | Inverse satellite position vector /m | Inverse satellite velocity vector /m/s |
|------------|-------------------------------------|---------------------------------------|
| 1 | -2278214.302 | 5479756.817 | 4005050.538 | Vxs and ΔVxs | Vys and ΔVys | Vzs and ΔVzs |
|   | 50.80982306 | 4478.641311 | -6081.321813 |
Analysis of calculated results with the change of the reference time $t$, the ascending node right ascension $\Omega$ changing situation is as follows (figure 1). As can be seen from Figure 1, with the advance of observation time $T$, the overall trend of right ascension at the ascending intersection gradually decreases, but the value is always around 184°. Since the given known data are not completely without perturbation force, the right ascension of the ascending point obtained by calculation may fluctuate, so the situation shown in Figure 1 is also reasonable.

| $t$ | $-9.168$ | $-2.586999999$ | $-1.678$ | $4.53589694$ | $1.266749$ | $0.859813$ |
|-----|----------|----------------|--------|-----------|--------|--------|
| 2   | -2277978.286 | 5497856.855 | 3980374.153 | 65.68375459 | 4455.223383 | -6098.513968 |
|     | 4.646    | 1.314999999   | 0.847   | 2.27211541 | 0.627167 | 0.446958 |
| 3   | -2277682.002 | 5515861.834 | 3955628.332 | 80.55700403 | 4431.721648 | -6115.593693 |
|     | 9.262    | 2.606000001   | 1.698   | -0.00635403 | -0.015278 | 0.030473 |
| 4   | -2277325.454 | 5533771.414 | 3930813.534 | 95.42921703 | 4408.136507 | -6132.560686 |
|     | 4.614    | 1.296         | 0.846   | -2.29941703 | -0.660527 | -0.389614 |
| 5   | -2276908.646 | 5551585.26   | 3905930.213 | 110.3000392 | 4384.468363 | -6149.414647 |
|     | -9.354   | -2.63         | -1.713  | -4.6070192  | -1.308563 | -0.813293 |

FIG. 1 Change curve of right ascension over time at the intersection of liter

With the change of reference time $T$, the orbital plane inclination $I$ changes as follows (FIG. 2). Orbital plane Angle $I$ with observation time $t$ change curve, we can conclude that Angle $I$ show sine function change, change cycle is about 24 observation time interval, which is 6 h, and in keeping the changing trend of sine function also is accompanied by increased at the same time, but I numerical orbit plane Angle in roughly 53.797°.
FIG. 2 Change curve of orbital plane inclination $I$ with observation time

With the change of reference time $T$, the change of Angle distance of perigee is as follows (FIG. 3). It can be found that the Angle distance of perigee changes periodically with the observation time $T$, and the specific value fluctuates approximately in the interval $(191.9^\circ,191.4^\circ)$.

FIG. 3 Change of Angle distance of perigee with observation time
As the change of reference time $t$, the change of the elliptical orbit eccentricity $e$ as follows (figure 4) by comparing observation found that eccentricity $e$ along with the change of observation time curve and perigee angular distance curve along with the change of the observation time is roughly same, omega eccentricity $e$ range between 5 (0.0104, 0.010), so if you keep two decimal places, then the numerical stability of the eccentricity $e$ in 0.01.

![Variation of satellite orbital eccentricity E with observed time](image)

**FIG. 4** Variation of satellite orbital eccentricity E with observed time

4. CONCLUSION

Were introduced in this paper using the Kepler orbital parameters at any time of the satellite position and velocity vector calculation method, is given when the discrete time interval is small, high precision orbit state vectors, the experiments show that using the Kepler orbital parameters to determine the satellite state vector precision is high, the study on radar image geometric processing, as well as other relevant processing has basic significance, the method can well describe the satellite orbit.

**REFERENCES**

[1] Weng, C. (2010). Method and device for predicting gnss satellite trajectory extension data in mobile apparatus.
[2] Jessica, Agarwal, and, Michael, Müller, & and 等. (2010). The dust trail of comet 67p/churyumov-gerasimenko between 2004 and 2006. Icarus.
[3] Barai, P., Gies, D. R., Choi, E., Das, V., Deo, R., & Huang, W., et al. (2004). Mass and angular momentum transfer in the massive algol binary ry persei. Astrophysical Journal, 608(2), 989.
[4] Howard, C. D., Rich, R. M., Reitzel, D. B., Koch, A., De Propris, R., & Zhao, H. S. (2008). The bulge radial velocity assay (brava). i. sample selection and a rotation curve. The Astrophysical Journal, 688(2), 1060-1077.
[5] Postman, & Marc. (2012). Advanced technology large-aperture space telescope: science drivers and technology developments. Optical Engineering, 51(1), 1007.
[6] Levanon, N. (2000). Passive position determination using two low-earth orbit satellites.

[7] Chen, S., Du, L., Zhang, Z., Danzeng, Q., Wang, R., & Wang, H., et al. (200). Orbit Determination of Lunar Probe Brake Course Based on Compensation to Dynamic Parameters. Conference of spacecraft TT&C technology in China. Information Engineering University, Zhengzhou 450052, China.

[8] Burak, I., Hepburn, J. W., Sivakumar, N., Hall, G. E., Chawla, G., & Houston, P. L. (1987). State-to-state photodissociation dynamics of trans-glyoxal. Journal of Chemical Physics, 86(3), 1258-1268.

[9] Green, D., Santos, L., & Chamon, C. (2010). Isolated flat bands and spin-1 conical bands in two-dimensional lattices. Physical review, 82(7), p.075104.1-075104.7.

[10] Bigelow, M. S., Zerom, P., & Boyd, R. W. (2004). Breakup of ring beams carrying orbital angular momentum in sodium vapor. Physical Review Letters, 92(8), 083902.

[11] A Bahring, I V Hertel, E Meyer, W Meyer, N Spies, & H Schmidt. (1999). Excitation of laser state-prepared na*(3p) to na*(3d) in low-energy collisions with na+: experiment and calculations of the potential curves of na2+. Journal of Physics B Atomic & Molecular Physics, 17(14), 2859.