SI, CGSG, and $c\hbar$ units: metrology and special relativity

L.B. Okun
ITEP, Moscow, Russia

Abstract

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The text consists of seven sections:
1. Great importance of SI.
2. The necessity of updating SI.
3. Special relativity: four-dimensional quantities.
4. Maxwell equations in vacuum in CGSG units.
5. Maxwell equations in vacuum in SI.
6. $c\hbar$ units.
7. Systems CGSG and $c\hbar$ should be legalized.

1 Great importance of SI

The SI units are of great importance for science, industry, business, trade, jurisdiction, for the knowledge-based society as a whole. SI forms the language of modern metrology with its world-wide network of national metrological institutes. Therefore any improvements should be introduced into SI with maximal caution and conservatism.

2 The necessity of updating SI

On the other hand the successes of fundamental science and its role in the knowledge-based society necessitate the evolution of SI, whose basic elements were laid down in XIXth century. Since the middle of the XXth century SI
has been internationally accepted as the only legal unit system to be used in textbooks for pupils and students. According to the legal documents of SI, other systems of units might be exceptionally allowed in research papers, but, as was stressed by Thibaudal Damour, as nobody learns anymore how to use such systems, new generations of scientists are fully SI dominated. In particularly they are CGSG (CGSG = CGSGauss) ignorant and their knowledge of special relativity is non-adequate.

3 Special relativity: four-dimensional quantities

In CGSG units Maxwell equations in vacuum are expressed in terms of coordinate four-vector $x^i$, of four-vector potential $A_i$, four-vector current $j_i$ and four-tensor of electromagnetic field $F_{ik}$.

Contravariant and covariant expressions:

$$\begin{align*}
x^i &= x^0, x^1, x^2, x^3 = (ct, \mathbf{x}) \\
x_i &= (ct, -\mathbf{x}) \\
A_i &= A_0, A_1, A_2, A_3 = (\varphi, -\mathbf{A}) \\
A^i &= A^0, A^1, A^2, A^3 = (\varphi, +\mathbf{A}) \\
A^l &= g_{lm} A_m \\
\dot{j}_i &= (\rho, \mathbf{j}) \\
F_{ik} &= \frac{\partial A_k}{\partial x^i} - \frac{\partial A_i}{\partial x^k} \\
F^{lm} &= \frac{\partial A^m}{\partial x^l} - \frac{\partial A^l}{\partial x^m} \\
\tilde{F}_{ik} &= \frac{1}{2} \varepsilon_{iklm} F^{lm}
\end{align*}$$

where $\varepsilon_{iklm}$ is fully antisymmetric four-tensor ($\tilde{F}_{ik}$ is dual with respect to $F_{ik}$).

4 Maxwell equations in vacuum in CGSG units

In terms of $F_{ik}$ and $\dot{j}_i$ the two Maxwell equations are simple and beautiful:

$$\frac{\partial F_{ik}}{\partial x^k} = -\frac{4\pi}{c} \dot{j}_i$$
\[ \frac{\partial \vec{F}_{ik}}{\partial x^k} = 0 \]

They express the gist of classical electrodynamics and together with Lorentz transformations present one of the most remarkable manifestations of special relativity.

The difference between CGS and CGSG is that the former has three base units (that of length, time, and mass), while in the latter a fourth base (electromagnetic) unit is added (that of electric charge, or current, or electric permittivity). In this respect CGSG is similar to SI.

5 Maxwell equations in vacuum in SI

According to SI, Maxwell equations cannot be presented as two four-dimensional equations, but as four three-dimensional equations for four three-dimensional vectors \( \mathbf{E}, \mathbf{D}, \mathbf{B}, \mathbf{H} \) with dimensional coefficients \( \mu_0 = 1/\pi 10^{-7} \text{ N} \text{A}^{-2}, \varepsilon_0 = 1/\mu_0 c^2 = 8.854... \cdot 10^{-12} \text{ Fm}^{-1} \), which are called magnetic permeability and electric permittivity of vacuum and have no direct physical meaning. They acquire such meaning when vacuum is compared with material media for which \( \mu \) and \( \varepsilon \) are very important. Both \( \mu_0 \) and \( \varepsilon_0 \) originate from the obsolete concept of ether. The SI form of Maxwell equations hides their beauty and the genuine physical meaning which are seen so clearly in the CGS units.

This side of SI inevitably leads to lack of understanding of special relativity and to deterioration of general culture in the community of physicists and engineers, which cannot be tolerated in a knowledge-based society.

If in CGSG \( \varepsilon_0 \) is chosen as the unit of permittivity, the Coulomb law and Maxwell equation in general look identical in CGSG and CGS. But this choice does not reduce the number of base units in CGSG from four to three. For a similar discussion concerning \( c \) see next section.

6 \( c\hbar \) units

In Quantum Field Theory, combining Quantum Mechanics and Special Relativity, the \( c, \hbar \) units are widely used. In these units the velocity of light is the unit of velocity, while quantum \( h \) is the unit of action (and of angular momentum). Quite often this system is referred to as “\( c, \hbar = 1 \) units”. However this name is a kind of theoretical “stenography”. When using \( c \) and \( \hbar \) as units one naturally can replace in equations factors \( c \) and \( \hbar \) by \( c/c = 1 \) and \( \hbar/\hbar = 1 \). But this does not mean that unit of speed \( c \) and unit of action \( h \) are equal to unity. Such statements are much worse than “stenography”, they
are examples of confusing theoretical “jargon”. Unit, which is a dimensional quantity, cannot be equal to 1 or to any other dimensionless number.

The crucial role played by $c$ in relativity and its extraordinary stability and reproducibility have led to its nominalization (without experimental uncertainties). However neither nominalization, nor using $c$ as the unit of speed have led to reduction of number of units or to reducing space to time. The use of atomic clocks for defining distances does not reduce space to time. Astronomers have a long tradition of using light years as a measure of large distances. But they never claimed that time interval abolishes distance. Space and time would be “the same” in Euclidean world, with metric $s^2 = t^2c^2 + x^2$, but our world is Minkowskian ($s^2 = t^2c^2 - x^2$). Hence space is space and time is time. They are not “the same”.

7 Systems CGSG and $c\hbar$ should be legalized

The French Academy of Sciences should recommend to the International Committee on definition of units to explicitly allow in the legal SI documents the use of CGS based Gaussian units not only in research papers, but also in the textbooks and other educational texts.

Moreover it should address SUNAMCO of IUPAP with a recommendation on necessity to have CGSG as a part of basic electrodynamical courses. (SUNAMCO = Symbols, Units, Atomic Masses and Fundamental Constants, IUPAP = International Union of Pure and Applied Physics.)

Similarly $c\hbar$-system in which the unit of velocity is $c$ and the unit of action and angular momentum is $\hbar$ should be legalized and allowed for the textbooks on fundamental physics both theoretical and experimental.

A certain knowledge of CGSG as well as absolute units, $c\hbar$-units, atomic and Planck units should be a legal requirement (based on SI) on licenses for engineers, school teachers of physics, for university and college diplomas for physicists.

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