Almost all traditional methods of advertisement distribution have been concerned only with primary information distribution via certain kinds of media. However, the rapid growth of the Internet and interactive media have demonstrated the power and efficiency of secondary information distribution of information by consumers such as words of mouth and free-mail. However, an advertisement distribution model which can be used to analyze and measure the effectiveness of such secondary distribution has never been discussed. Therefore, in this paper, we propose an advertisement distribution model and show how to use the model to analyze both primary and secondary information distributions. Our experiment and analytical results are also discussed. The experimental result shows that the proposed model can be used to measure and analyze the effectiveness of advertisement distribution.

1. Introduction

In general, there are two kinds of information flows in an information distribution environment such as advertisement distribution; one is the primary information distribution, and the other is the secondary information distribution. The primary information distribution is the distribution done by providers or broadcasters to consumers through certain kinds of media such as television, newspaper, etc. The secondary information distribution is the distribution done by consumers to consumers. Traditionally, the secondary distribution of advertisements has been done by “power of mouth”. However, the rapid growth of the Internet and interactive media devices have resulted in the secondary information distribution performing effectively in various ways such as email, etc. Therefore, we certainly cannot ignore this kind of distribution power.

In recent years, much research has been conducted to address marketing and advertising on specific media particularly on mobile phones. For example, Dimitris and George\(^1\) attempt to develop a model identifying the factors influencing the effectiveness of a mobile messaging advertisement. Chittenden and Rettie\(^2\) discuss an evaluation of e-mail marketing, and the research suggests that e-mail marketing is growing rapidly and should be integrated into the overall communication in order to increase the effectiveness of the whole marketing. Rettie, et al.\(^3\) investigate the effectiveness of text messaging advertisement or SMS to user responses and branding effects. Haghirian, et al.\(^4\) study how to increase the advertising value of mobile marketing or mobile advertising by investigating various factors such as the characteristics of mobile advertising, the characteristics of advertising contents and mobile consumers’ characteristics. Those researches\(^1\)\(^–\)\(^4\) also analyze the factors which increase the response rate from users, but they do not discuss the secondary information distribution. There are some researches\(^5\),\(^6\) which have discussed secondary information distribution. For instances, Ratsimor, et al.\(^5\) propose a framework for intelligent marketing in the mobile environment called eNcentive. Garyfalos and Almeroth\(^6\) develop applications and services using the idea of coupons for wireless ad hoc networks. The basic ideas of those researches are to provide an incentive and a framework for distributing information among users. But there are no proposals for analyzing the effect caused by the incentive or the framework.

With regard to the above mentioned works, the research does not concern the measurement and analysis methods for the effectiveness of advertisement distribution in both primary and secondary information distribution. The measurement methods of the effectiveness of advertisement distribution, at present, have only been done by consumer rating surveys, questionnaires and a few other researches\(^7\),\(^8\). Nevertheless, these methods and researches\(^7\),\(^8\) can measure only the effectiveness of primary information distribution. Moreover, those methods have not taken into account that contents may...
be freely copied by anyone in certain circumstance, which is a basic concept of the Superdistribution\textsuperscript{9}).

In order to find what will be the next generation of advertisement distribution and analyze it, we should measure and analyze both primary distribution and also secondary distribution. However, an advertisement distribution model which can be used to analyze and measure the effectiveness of secondary information distribution has never been investigated. Therefore, we are proposing an advertisement distribution model which can be used to analyze and measure the entire circulation including both primary and secondary information distributions.

In the next section, we explain in detail the proposed model and the definition. Sections 3 and 4 describe the analysis method of the proposed model, and some examples are illustrated in applying the model. Section 5 describes our experiment and analytical results, and our consideration of the proposed model and the analysis is discussed in Section 6. Finally, Section 7 shows our conclusion and future works.

2. The Proposed Advertisement Distribution Model

In general, the advertisement distribution life cycle is composed of “Distributed”, “Consumed” and “Discarded” states. Our study shows that contemplating and analyzing such states are not sufficient to produce an efficient and accurate analysis. For example, an advertisement can be consciously and unconsciously redistributed by consumers and such distribution can occur repeatedly by push and/or pull methods. Note that, in the push distribution method, the contents are transmitted to consumers in a sequence and at a rate which are both determined by senders. For the pull distribution method, the contents are delivered in response to some particular consumer’s requests. In order to determine and analyze the factors for realizing such redistribution, we must analyze the secondary information distribution as well. Hence, we propose an advertisement distribution model which includes the different states for primary and secondary information distributions as shown in Fig. 1. In this figure, the left and right hand sides represent the life cycle of primary and secondary information distributions, respectively. This model is composed of 6 states and 11 operations. The definition of each state is clarified as follows:

- **Ready for Primary Distribution**
  It represents the state in which advertisers or sponsors are ready to distribute their advertisements by push and/or pull methods of distribution. For examples, handbills are printed and prepared. Web advertisements are stored in Web servers and are ready for distribution.

- **Primary Pushed**
  In this state, the distributed advertisement is being distributed to consumers from advertisers. For instance, newspapers and magazines are being displayed in a store, a television commercial is being broadcast by a broadcaster, posters are put up in

![Fig. 1 The proposed advertisement distribution model.](image-url)
public places, and email is being transmitted. Note that another reason to define this state is to distinguish pull and push methods of advertisement distribution.

- Consumed
  In this state, advertisements are consumed consciously or unconsciously by consumers. All consumption methods such as watching, listening and reading are considered to be the same method (action) which leads to this state.
  Unlike the primary distribution and secondary distribution, the consumed state can be defined as a single state, because consumers are not interested in where the advertisement comes from but are interested in the content of the advertisement. It means that the Consumed state does not relate to who sends the advertisement. In other words, the probabilities of redistributing, discarding and storing the received advertisement are the same for both.

- Ready for Secondary Distribution
  This represents the state in which consumers are ready to redistribute consumed advertisements to third persons by push and/or pull methods. For example, they try to forward advertisements by email, or put the advertisements in their own servers for anyone to access.

- Secondary Pushed
  In this state, the consumed advertisement is being distributed to other consumers by the consumers. In other words, this state is similar to the “Primary Pushed” state but it is done by the consumers not by advertisers or sponsors. The reason for defining this state is the same as the “Primary Pushed” state discussed above.

- Discarded
  The distribution of the advertisement in this state is terminated, and no one consumes it anymore.

The definition of each operation is clarified as follows:

- Primary Push (P-Push)
  Advertisers or sponsors distribute advertisements to consumers by push methods.

- Primary Pull (P-Pull)
  Consumers access advertisements such as web advertisements to acquire advertisement information.

- Secondary Push (S-Push)
  Consumers redistribute the consumed advertisement to other consumers by push methods such as forwarding email.

- Secondary Pull (S-Pull)
  Consumers access other consumers’ sources of information to acquire advertisements prepared by other consumers.

- Primary Consume (P-Consume)
  Consumers consume advertisements distributed by advertisers or sponsors.

- Secondary Consume (S-Consume)
  Consumers consume advertisements redistributed by other consumers.

- Discard
  Consumers or advertisers discard advertisements physically or forget them. In addition, unsuccessful delivery of advertisements in the “Primary Pushed” and “Secondary Pushed” state is also considered as “discard”. As a result of this action, the distribution is ended.

- Wait
  Advertisements are waiting to be distributed by both push and/or pull methods.

- Leave
  Advertisements are still in the process of being distributed. For instance, newspapers are being kept in a book store.

- Store
  After having consumed them, consumers record or store the advertisements.

- Prepare
  Consumers make the secondary distribution of advertisements possible.

3. The Analysis Method of the Proposed Model

3.1 Advertisement Analysis
As illustrated in Fig. 2, “Ready for Primary Distribution”, “Primary Pushed”, “Consumed”, “Secondary Distribution”, “Secondary Pushed” and “Discarded” are defined as $S_1, S_2, S_3, S_4, S_5$ and $S_6$, respectively. In addition, an action from $S_i (1 \leq i \leq 6)$ to $S_j (1 \leq j \leq 6)$ is defined as $a_{ij}$. In this case, $S_i$ is to be considered as a state of an advertisement, and $a_{ij}$ is to be considered as a transition probability of an advertisement from the state $S_i$ to the state $S_j$ where $\sum_{j=1}^{6} a_{ij}$ is to be 1. Therefore, the Markov Chain Model and its theory\(^{10,11}\) can be used to analyze the advertisement transition among the 6 states. In other words, we define
Fig. 2 The proposed advertisement distribution model.

![Diagram](image)

Fig. 3 The state transition probability matrix.

![Matrix]

a vector of probabilities of each state at time \( t \) as \( P_t \) shown in Formula (1), and when we define the state transition probability matrix as \( A \) shown in Fig. 3, the next vector of probabilities of each state at time \( t + 1 \) can be obtained by Formula (2).

\[
P_t = \begin{bmatrix} S_1 & S_2 & S_3 & S_4 & S_5 & S_6 \end{bmatrix}
\]

\[
P_{t+1} = P_t \times A
\]

The detailed reason is explained in Section 4, but in our model, we call the value of \( S_i \) "quantity" rather than "probability" because the value may be more than 1.

3.2 Analyzing Advertisements being Consumed

In order to analyze the effectiveness of advertisement distribution, the quantity of advertisements being consumed at a particular time shall be observed. Using the defined state transition, this value is computed based on the following Formula (3).

\[
Y_t = \sum_{i=1}^{t} Y'_i
\]

\[
Y'_i = (X_1)_t + (X_2)_t + (X_4)_t + (X_5)_t - (X_6)_t
\]

\[
(X_1)_t = (S_1)_{t-1} \times a_{13}
\]

Fig. 4 The quantity of advertisement being consumed.

![Diagram](image)

\[
(X_2)_t = (S_2)_{t-1} \times a_{23}
\]

\[
(X_4)_t = (S_1)_{t-1} \times a_{43}
\]

\[
(X_5)_t = (S_5)_{t-1} \times a_{36}
\]

\[
(X_6)_t = (S_3)_{t-1} \times a_{36}
\]

\((X_1)_t, (X_2)_t, (X_3)_t, (X_4)_t\) and \((X_5)_t\) are the quantities of advertisements being consumed at time \( t \), that come from \( S_1, S_2, S_4, S_5 \) to \( S_3 \), respectively, as shown in Fig. 4. The \( Y'_t \) is the quantity of advertisements being consumed at time \( t \). \( Y_t \) is the summation of \( Y'_t \) from the beginning of the advertisement distribution until time \( t \). In other words, \( Y_t \) shows the total quantities of consumed advertisements and represents the effectiveness of its distribution in the model.

4. Verification of the Model

In order to verify the use of this model, four simple cases of advertisement distributions using handbills, emails and Web servers are shown in this section; they are (1) "primary distribution only", (2) "primary and secondary distributions", (3) "primary and secondary distributions with copy" and (4) "primary distribution by pull method".

4.1 Primary Distribution Only (Example 1)

In this example, we assume that 100 advertisement handbills are distributed. 100 consumers receive all the distributed handbills. After that, 3/4 of consumers do nothing with the handbills, but 1/4 of consumers throw them away and/or forget them. In this case, the transition probability matrix \( A \) is to be defined as shown in Fig. 5. And the initial quantity of each state is to be defined as \( P_0 \) in Fig. 5 be-
cause all the handbills are in $S_1$ at the beginning that is to be represented as the quantity 1 as a normalized value. The actual number of advertisements of each state is able to be calculated from the normalized state value in $P_t$ by multiplying the initial absolute value (in short, $IAV$) at anytime of $t$, afterwards. In this example, the $IAV$ is 100. In the same manner, $Y'_t$ and $Y_t$ are represented in the normalized values, and the actual number of advertisements of $Y'_t$ and $Y_t$ are able to be calculated from $IAV$.

Then, according to the Formula (2), the next quantity of each state is to be calculated. Table 1 shows the quantities of $S_i$ from time 1 to 6, as well as $Y'_t$ and $Y_t$.

This example shows that the handbills being consumed decreases to 0 as shown in Fig. 6 if the secondary information distribution is not performed. Note that, in this example, $a_{44}$, $a_{55}$ and $a_{66}$ in $A$ of Fig. 5 have no impact on the analysis because there has been no secondary distribution. They are defined as 1 to keep $A$ as a Markov Chain transition matrix.

### 4.2 Primary and Secondary Distributions (Example 2)

100 handbills are distributed. 100 consumers received all the distributed handbills. After that, 7/20 consumers do nothing with the handbills. 5/20 consumers throw the handbills away and/or forget them. And 8/20 consumers redistribute the handbills to their friends. In this case, the initial state quantity ($P_0$) and transition probability matrix ($A$) are shown in Fig. 7. Table 2 shows the quantities of $S_i$ from time 1 to 6, as well as $Y'_t$ and $Y_t$.

This example shows that the handbills being consumed increases gradually as shown in Table 2 and Fig. 8 if the secondary information distribution is performed. The total number of handbills being consumed is exceeding 100. This is because the consumers who received handbills still remember the advertisement for some time. And this is also the reason why we call the state quantity rather than the state probability.

### 4.3 Primary and Secondary Distribution with Copy (Example 3)

In this example, 100 emails are distributed. After that, 7/20 consumers do nothing with the emails. 5/20 consumers delete and/or forget the received emails. And 8/20 consumers forward two emails to their friends. In other words, the number of advertisements being redistributed in the secondary distribution is dou-

---

**Table 1** Quantity transition in Example 1.

| Time | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S_5$ | $S_6$ | $Y'_t$ | $Y_t$ |
|------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1    | 0     | 1     | 0     | 0     | 0     | 0     | 0     | 0     |
| 2    | 0     | 0     | 1     | 0     | 0     | 0     | 0     | 0     |
| 3    | 0     | 0     | 0.75  | 0     | 0     | 0.25  | −0.25 | 0.75  |
| 4    | 0     | 0     | 0.5625| 0     | 0.4375| −0.1875| −1.4063| 0.5625|
| 5    | 0     | 0     | 0.421875| 0     | 0.578125| −0.14063| 0.421875|
| 6    | 0     | 0     | 0.316406| 0     | 0.683594| −0.10547| 0.316406|

$P_t = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0.75 & 0 & 0 & 0.25 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

$A = \begin{bmatrix} 0 & 0 & 0.35 & 0.4 & 0 & 0.25 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$
Table 2 Quantities transition in Example 2.

| Time | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S_5$ | $S_6$ | $Y'_t$ | $Y_t$ |
|------|-------|-------|-------|-------|-------|-------|-------|-------|
| 0    | 0     | 1     | 0     | 0     | 0     | 0     | 0     | 0     |
| 1    | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 1     |
| 2    | 0     | 0     | 0.35  | 0.4   | 0     | 0.25  | −0.25 | 0.75  |
| 3    | 0     | 0     | 0.1225| 0.14  | 0.4   | 0.3375| −0.0875| 0.6625|
| 4    | 0     | 0     | 0.442875| 0.049 | 0.14  | 0.368125| 0.369375| 1.031875|
| 5    | 0     | 0     | 0.295006| 0.17715| 0.049 | 0.478844| 0.029281| 1.061156|

Table 3 Quantity transition in Example 3.

| Time | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S_5$ | $S_6$ | $Y'_t$ | $Y_t$ |
|------|-------|-------|-------|-------|-------|-------|-------|-------|
| 0    | 0     | 1     | 0     | 0     | 0     | 0     | 0     | 0     |
| 1    | 0     | 0     | 1     | 0     | 0     | 0     | 0     | 1     |
| 2    | 0     | 0     | 0.35  | 0.4   | 0     | 0.25  | −0.25 | 0.75  |
| 3    | 0     | 0     | 0.1225| 0.14  | 0.8   | 0.3375| −0.0875| 0.6625|
| 4    | 0     | 0     | 0.84288| 0.049 | 0.28  | 0.36813| 0.769375| 1.431875|
| 5    | 0     | 0     | 0.57501| 0.33715| 0.098 | 0.57884| 0.06928| 1.501155|

Fig. 8 Handbills being consumed in Example 2.

Fig. 9 Email being consumed in Example 3.

4.4 Primary Distribution by Pull Method (Example 4)

In this example, an advertisement website is accessed. And there is no redistribution of advertisements done by consumers. In other words, the secondary distribution is not performed in this example. In pull distribution methods such as Web, the probability of $a_{13}$ and the $IAV$ are not obviously obtained unlike the previous examples. Therefore, we propose the following method to estimate $a_{13}$ and $IAV$ so that our proposed model is also able to apply for any pull access methods.

First we define $t$ as a certain period of observation, which can be 1 day, 6 hours or 1 hour, and observe the number who have accessed to the Web site. Suppose from $t = 1$ to $n$, we obtain $W_t$ which is a number who have accessed the website from $t − 1$ to $t$. In this case, $a_{13}$ is to be estimated according to the following Formula (5) where $MAX$ is a function to get the maximum value from arguments.

$$a_{13} = \frac{\sum_{t=1}^{n} W_t}{MAX(W_1, \ldots, W_n)} \times n$$

The denominator defines the possible maximum number accessing the Web site within the observation period, that can be regarded as a maximum capacity or power of the advertisement to attract consumers. While the numerator defines the total number accessing the Web site, that can be regarded as an averaged capacity or power of the advertisement. Therefore, $a_{13}$ is defined as an average accessing rate of the Web site within the observation period against the maximum capacity.
Table 4 Number of hitting website in Example 4.

| Day   | Number of Accessing the Web site |
|-------|----------------------------------|
| First | 150                              |
| Second| 210                              |
| Third | 80                               |
| Fourth| 270                              |
| Fifth | 100                              |
| Sixth | 90                               |
| Seventh| 100                             |

\[
P_t = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0.4709 & 0 & 0.5291 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0.25 & 0 & 0 & 0.75 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}
\]

\[
A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\]

Fig. 10 The initial state quantity and transition probability of Example 4.

Assuming that we define \( t \) as one day and the observed accessing the Web site is as shown in Table 4, \( a_{13} \approx 0.5291 \) is obtained. So, \( a_{11} \) is to be defined as 0.4709 in this case. We also assume that 3/4 of the consumers forget the advertisement after accessing it while the rest can remember. Hence, \( a_{33} \) and \( a_{36} \) are 0.25 and 0.75, respectively. Consequently, the initial system state quantity \( (P_0) \) and the transition probability matrix \( (A) \) are to be defined as shown in Fig. 10.

As in Example 3 in the previous subsection, we also need a value to compensate \( S_1 \) in this case; i.e. the quantity of \( S_1 \) shall be always kept as 1 because a kind of copy-and-delivery of advertisements happened while accessing the Web site and it does not take any resources away from it. Therefore, \( P_t \) is to be calculated by the Formula (6).

\[
P_{t+1} = P_t A + P_c 
\]

where \( P_c = \begin{bmatrix} c_1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \)

In this example, \( c_1 = a_{13} \) to make \( S_1 \) always 1.

\[
Y_t \times 270
\]

Fig. 11 Web advertisement being consumed in Example 4.

The next step is to obtain the \( IAV \). In order to do so, we propose the following Formula (7) to be used to define it.

\[
IAV = \frac{\sum_{t=1}^{n} W_t}{\sum_{t=1}^{n} \{(S_1)_{t13}\}} 
\]

Fig. 11 Web advertisement being consumed in Example 4.

The denominator defines the total quantity of advertisements flowing from \( S_1 \) to \( S_3 \), while the numerator defines the total number of consumers accessing the Web site during observation period. Thus, \( IAV \) is to be defined as an absolute number of website accesses corresponding to the quantity “one”. However, according to Formula (5) where \( a_{13} \) is defined as a constant and \( S_1 \) is compensated as always 1, in this particular case, Formula (7) can be simplified into Formula (8) as below.

\[
IAV = \text{MAX}(W_1, \ldots, W_n) 
\]

(8)

It means that \( IAV \) in Formula (8) is defined as the maximum number accessing the Web site within the observation period in order to prevent \( S_1 \) from becoming a negative value. Thus, each quantity of state can be calculated by multiplying \( IAV \) to \( P_t \) as same as in Example 1, 2 and 3. In this example, according to the Formula (8), \( IAV \equiv 270 \) is obtained.

Finally, using the parameters we obtain, we are able to calculate the quantity transition of each \( S_i \). Table 5 shows the quantities of \( S_i \) from time 1 to 6, as well as \( Y'_t \) and \( Y_t \). Figure 11 shows the number of Web advertisement being consumed day by day, and it shows that...
tools for supporting secondary distribution and the count function can count only some parts of Web and email distribution. Nevertheless, the count function can count only some parts of Web and emails distribution. For instance, forgetting and discarding advertisement which are defined as $a_{36}$ cannot be counted by this count function.

The period of our experiment is 21 days. To clarify conditions of our experiment, Table 6 is shown.

| Experiment Place | Waseda University, Honjo Campus |
|------------------|---------------------------------|
| Primary Advertisement Distribution Media | Handbill, Web |
| Experiment Period | 21 days |
| Tools for Supporting Secondary Distribution | E-mail service provided in the website |
| Tool for Gathering Distribution Result | Questionnaires, Count function |

5. The Experiment and Its Analysis

To verify whether the proposed model can be used in real-life advertisement distribution, we conducted a practical experiment of advertisement distribution at Global Information and Telecommunication Studies (GITS), Waseda-Honjo Campus.

In our experiment, we set out handbills and the Web as the primary advertisement distribution media. In our handbills, there is some useful information for living in Saitama Honjo and information inviting consumers to access our Web site called EZHONJO. On our Web site, there is also a lot of useful information for living in Saitama Honjo.

Questionnaires are designed to collect information related to consumers receiving information via handbills, emails, our Web site or word-of-mouth. Then what consumers react to the received information. Such feedback from consumers is used to define the transition probability matrix ($A$) of the handbills and word-of-mouth distribution.

We also provide an email service as a tool for distributing the information in the Web site. In other words, consumers can use this service to send emails to their friends easily. In the Web site, we also create functions for counting and recording the number of consumers accessing our Web site by consumers; and number of transmitting emails from our email service. This counting of consumers’ responses is used to determine transition probability matrix ($A$) of Web and email distribution. Nevertheless, the count function can count only some parts of Web and emails distribution. For instance, forgetting and discarding advertisement which are defined as $a_{36}$ cannot be counted by this count function.

The period of our experiment is 21 days. To clarify conditions of our experiment, Table 6 is shown.

We distributed 50 handbills to students attending the entrance ceremony at GITS, Waseda University Honjo-Campus. Then three weeks after distributing the handbills, we distributed 50 questionnaires to anyone in the campus randomly. There are 38 responses to the questionnaires; 7 out of 38 questionnaires returned state that they received the handbills at the Entrance ceremony. 2 out of the 7 redistributed the advertisement information (in the handbills received) to 9 persons. 4 out of 7 stated that they stored the handbills. The remaining one stated that he/she discarded his/her handbill.

5.1 The Analytical Result of Handbill and Word-of-Mouth Distribution

The result from the questionnaire shows the distribution of handbills and word-of-mouth because some consumers use word-of-mouth to distribute the advertisement as a secondary distribution. The detail of $a_{ij}$ acquired from the questionnaires gathered after 21 days is clarified in Table 7, and the initial state quantity
Table 7 Transition probability detail acquired using questionnaires.

| Transition Probability | Definition                                           | Value         |
|------------------------|------------------------------------------------------|---------------|
| $a_{12}$               | Number of Transmitted Handbills / Number of Printed Handbills | $1 \left( \frac{50}{50} \right)$ |
| $a_{23}$               | Number of Consumed Handbills / Number of Successfully Transmitted Handbills | $1 \left( \frac{50}{50} \right)$ |
| $a_{33}$               | Number of Stored Handbills / Number of Consumed Handbills | $0.5714 \left( \frac{4}{7} \right)$ |
| $a_{34}$               | Number of Consumers who Redistributed Advertisement / Number of Consumed Handbills | $0.2857 \left( \frac{2}{7} \right)$ |
| $a_{36}$               | Number of Discarded Handbills / Number of Consumed Handbills | $0.1429 \left( \frac{1}{7} \right)$ |
| $a_{45}$               | Number of Redistributed Advertisement / Number of Secondary Distribution Advertisement | $1 \left( \frac{9}{9} \right)$ |
| $a_{53}$               | Number of Consumed Redistributed Advertisement / Number of Redistributed Advertisement | $1 \left( \frac{9}{9} \right)$ |
| $a_{66}$               | This transition probability is always be 1            | 1             |

$P_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0.5714 & 0.2857 \\ 0 & 0 & 1 & 0 & 0.1429 \end{bmatrix}$

$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

Fig. 12 The initial state quantity and transition probability matrix of handbills and word-of-mouth distribution of 21 days.

($P_0$) and transition probability matrix ($A$) are shown in Fig. 12. Note that, the calculation of $a_{33}$, $a_{34}$, $a_{36}$, $a_{45}$ and $a_{53}$ is based on information from collected questionnaires.

In order to calculate $Y_t$, we use the same method discussed in Section 4.3. In this analysis, we define an instance time $t$ as one day. According to the questionnaire, some consumers copied and redistributed the handbills, and some consumers used word-of-mouth to redistribute advertisement information. Using word-of-mouth is also considered to be copying. We can find the average quantities of advertisement which are copied ($c_5$) and distributed in everyday ($t$) by using the Formula (10).

$$c_5 = \begin{cases} 0 & (t = 0, 1, 2, 3) \\ \frac{1}{\text{TNAC}} \times \text{ANCRDA} \times \frac{1}{\text{PO}} & \text{Otherwise} \end{cases} \quad (10)$$

In this analysis, when $t = 0, 1, 2, 3$ the quantity of $S_5$ is 0 because the advertisements are not firstly reached at this state. Therefore, $c_5$ is defined as 0 when $t = 0, 1, 2, 3$. “TNAC”, “ANCRDA” and “PO” are defined as “Total Number of Advertisements Corresponding to the normalized value”, “Average Number of Copied and Re-Distributed Advertisement” and “Period of Observation”, respectively.

With regard to our questionnaires, “TNAC”, “ANCRDA” and “PO” are 50, 64.285 and 21, respectively. Since we cannot know the exact ANCRDA in this experiment, the ANCRDA is estimated by using Formula (11) where as “NCRDA” is the Number of Copied and Re-Distributed Advertisements which gained from the questionnaires, “NPRH” is the Number of People who Received Handbills at the GITS entrance ceremony which acquired from the questionnaires, and “ANPRH” is the Actual Number of People who Received Handbills at GITS entrance ceremony. According to the experiment and its questionnaires, “NCRDA”, “NPRH” and “ANPRH” are 9, 7 and 50, respectively.

$$\text{ANCRDA} = \frac{\text{NCRDA}}{\text{NPRH}} \times \text{ANPRH} \quad (11)$$

The calculated “$c_5$” shows the result of 0.0612.

As discussed in Section 4, $IAV$ can be considered as “Numbers of Primary Distribution Advertisement” in push distribution methods, which is 50 in this experiment; because we firstly distributed 50 handbills. Moreover, $c_1$ and $c_4$ in Formula (9) are defined as 0 because there is no primary and secondary pull distri-
Table 8 Transition probability detail of three weeks acquired using count function.

| Transition Probability | Definition                                                                 | Value   |
|------------------------|---------------------------------------------------------------------------|---------|
| $a_{11}$               | From $\sum_{j=1}^{6} a_{ij}$ with $a_{12}, a_{16} = 0$                     | 0.6892  |
| $a_{23}$               | By Formula (5)                                                            | 0.3108  |
| $a_{33}$               | From $\sum_{j=1}^{6} a_{ij}$ with $a_{36} = 0$                            | 0.387   |
| $a_{34}$               | $\frac{\text{Number of Accessing to the Email Service Web Page}}{\text{Number of Access to the Advertisement Website}}$ | 0.6129  |
| $a_{45}$               | $\frac{\text{Number of Transmitted Email}}{\text{Number of Accessing to Email Service Web Page}}$ | 0.25 ($\frac{19}{76}$) |
| $a_{46}$               | From $\sum_{j=1}^{6} a_{ij}$ with $a_{44} = 0$                            | 0.75    |
| $a_{53}$               | $\frac{\text{Number of Consumed Transmitted Email}}{\text{Number of Successfully Transmitted Emails}}$ | 1 ($\frac{19}{19}$) |
| $a_{66}$               | This transition probability is always be 1                                | 1       |

Fig. 13 Handbills and word-of-mouth advertisements being consumed of 21 days.

Fig. 14 The initial state quantity and transition probability matrix of web and email distribution of 21 days.

Having applied Markov Chain and Formula (9), the result is shown in Fig. 13. This analytical result principally represents the distribution of the handbills and word-of-mouth advertisement. It shows that after 21 days, the handbills and word-of-mouth advertisements being consumed ($Y_{21} \times 50$) is 156,3157.

Since the distribution are be spread to outside our campus, we cannot find out the real numbers of handbill and word-of-mouth advertisement being consumed. Nevertheless, this result can show that the numbers of advertisements being consumed have increased approximately 3 times of $IAV$ after 21 days.

5.2 The Analytical Result of Web and Emails Distribution

The analytical result from our count function represents the distribution of Web and email advertisement. The detail of acquired information from the count function is presented in Table 8, and the initial state quantity ($P_0$) and transition probability matrix ($A$) is shown in Fig. 14. Note that, this transition probability is defined by using the results from the count function obtained during 21 days of observation. Moreover, we define $a_{36}$ as 0 because the count function in this experiment does not have the capability to count the numbers of advertisements which are forgotten or discarded. Hence, we assume that all consumers can remember the advertisement after having accessed the Web site. In addition, accessing the email service Web page is regarded as transiting to “Ready for Secondary Distribution” state.

Since, there are secondary information distributions done by emails in this experiment, Formula (10) is used to define this. According to our count function, $TNAC$ is 124 which is the numbers of consumers accessing our advertisement website. For $ANCRDA$, we use “Number
of Transmitted E-mail” which is 19; and the result of $c_5$ is 0.0073 ($t \geq 3$).

As discussed in Section 4.4, $c_1$ in Formula (9) is obtained as 0.3108. For the $IAV$, it is obtained by using Formula (8) and the result is 18.9986. And $c_4$ is defined as 0 because there is no secondary pull distribution in our experiment.

After getting those parameters, we use Markov Chain and Formula (9) to analyze the distribution. Consequently, we produce the result of “Web and Email Advertisement being Consumed” after 21 days ($Y_{21} \times 18.9986$) which is 158.9082 as shown in Fig. 15. The graph shows a constant-increasing line because we determine $a_{13}$ and $IAV$ as averaged values from the observation and $a_{36}$ as 0.

Note that the real number of advertisements being consumed after 21 day is 143. This number is obtained by totalling the number accessing the experimental Website during 3 weeks which is 124 and the total number who transmitted email during 3 weeks which is 19.

The analytical result of 158.9082 shows that it is not far from the real result which is 143. It shows that the proposed model can present and also predict the advertisement distribution rate of pull and push methods.

6. Consideration

6.1 On the Proposed Model

In the proposed model, the effectiveness of secondary information distribution can be measured and analyzed. We consider that one of the key factors for realizing future advertisement distribution is how to increase the effectiveness of secondary information distribution such as email and word-of-mouth, and make this distribution occur repeatedly. We believe that the proposed model can be used to find such factors. For instance, we can conduct simulations and analyze by using the proposed model so that not only the factors which increase the effectiveness of distribution but also the factors which reduce the effectiveness are to be observed and analyzed as well. And according to the analytical result described in Section 5.2 on the pull distribution, we can also consider that the proposed calculation method for pull distribution is effective.

Generally, there are two approaches to analyze the effectiveness of the advertisement distribution; analyzing advertisements by focusing on contents $^2$, and analyzing distribution quantity statistically without regard to the contents $^7$. Since the proposed model has taken the latter approach, the model can analyze the effectiveness of advertisement distribution in the statistical way, but not the effectiveness of advertising contents. Furthermore, in the real world of advertisement distribution, the parameters may be varied from time to time. And in some advertisement distributions, the push and pull are done simultaneously by multiple advertisement media. The experiment we have conducted cannot examine those issues. However, in this case, we may define appropriate $P$ and $A$ to represent the combination of advertisement distribution effectiveness on different media. The combination of media in advertisement distribution may give more effectiveness than distributing them separately. We will conduct an experiment in our future work to prove this consideration.

6.2 Parameters Impact

Each parameter of Formula (9) has a different impact for the effectiveness of advertisement distribution. In order to clarify the impact, here we focus on $c_5$ and $a_{34}$ because both parameters have impacts against the secondary distribution effectiveness and one of our goals is to analyze the factors to increase the effectiveness of the secondary distribution.

Under the condition described in Section 5.1, the total quantities of advertisements being consumed after 21 days ($Y_{21}$) are calculated using the different values of $c_5$ and $a_{34}$, which is shown in Fig. 16. In Fig. 16, the y-axis represents the total quantity $Y_{21}$, and x-axis represents the different values of $c_5$ and $a_{34}$ incremented by 0.1 against their original values. By increasing $a_{34}$, we have to adjust other $a_{3x}$ parameters so that the $\sum_{i=1}^{6} a_{3x}$ shall be always maintained as 1. In this consideration, we as-
sume $a_{33}$ is to be decreased against the $a_{34}$ increasing. It should be noted that the value of $c_5$ and $a_{34}$ are not in the same scale, but we can estimate how large the increments give impacts against $Y_{21}$ in a qualitative way. Consequently, Fig. 16 shows $Y_{21}$ rapidly and constantly increases when $c_5$ increases; whereas in case of $a_{34}$, $Y_{21}$ gradually increases. Moreover, the result from our simulation also shows that 0.2857, 0.3857, 0.4857, 0.5857 and 0.6857 of $a_{34}$ requires 2, 2.7, 3.4, 4.1 and 4.8 out of 7 consumers to make secondary distribution because this simulation is based on the data in Section 5.1, i.e. 0.2857 of $a_{34}$ is obtained by the questionnaires that 2 out of 7 consumers who received the handbill advertisement redistributed the advertisement information.

In general, to increase $a_{34}$ is more difficult and more expensive than to increase $c_5$. That is, increasing $a_{34}$ means increasing the number of consumers who are willing to redistribute advertisements, and that involves a higher cost; e.g. by conducting marketing campaigns. On the other hand, to increase $c_5$ is done more easily, for example, by providing tools which can be easily used for copying and distributing advertisement such as sending multiple emails at the same time.

**Figure 17** shows NCRDA done by 1 consumer with corresponding $c_5$ under the same condition as described in Section 5.1. The result in Fig. 17 shows that if we need 0.0612, 0.1612, 0.2612, 0.3612 and 0.4612 of the $c_5$, 1 consumer must copy and redistribute 4.4982, 11.8482, 19.1982, 26.5482 and 33.8982 advertisements, respectively. Note that, in order to find this number which is done by 1 consumer, NCRDA gained from Formula (11) must be divided by 2 because NCRDA in Section 5.1 are the total values done by 2 consumers. Using Formula (10) and (11) to find the number of copied and redistributed advertisement by 1 consumer, the Formula (12) is to be used.

$$c_5 \times TNAC \times PO \times \frac{NPRH}{ANPRH} \times \frac{1}{2} \quad (12)$$

Hence, using our proposed model, it is clearly shown that providing redistribution tools such as email which enable copying of advertisement information in the secondary distribution leads to the more effective proliferation of advertisements. In the actual situation there is almost no possibility that consumers will send more than 10 emails for 1 advertisement. In other words, it is quite difficult to make $c_5$ more than 0.1612. However, with regard to Fig. 16, even 0.1612 of $c_5$ gives higher $Y_{21}$ than 0.6857 of $a_{34}$. This result also represents that increasing number of consumer who will redistribute advertisement gives less distribution effect than increasing number of copied and redistributed advertisement in consumers even only a few numbers of consumer.

7. Conclusion and Future Works

Having conducted the experiment to verify the validity and usefulness of our model, we believe that our proposed advertisement distribution model can be used to analyze and measure the entire circulation of its distribution which includes primary and secondary information distributions. Our experiment shows that the model can present a part of the real world advertisement distribution, and analyze effectiveness of advertisements distribution. The proposed model can be also applied to analyze general information distribution such as an analysis of information awareness. By using the proposed model, we can analyze or estimate the factors which make the secondary distribution occur repeatedly in the entire distribution program. We believe that such factors are the
important keys for new kinds of advertisement distribution in the future.

However, there are some remaining issues in the proposed model because the experiment is done on simple cases of advertisement distribution. Complicated case of advertisement distribution will be investigated in our future works such as pull methods of secondary distribution and the combination of various advertisement distribution media. In addition, because of the rapid growth of mobile communication, new advertising models for making secondary information distribution such as incentive coupons have been being developed. Therefore, we will apply the proposed model to analyze the effectiveness of such incentives in mobile environments. Furthermore, some influencing factors for real-world advertisement distribution will be investigated to find the relation to and impacts on the parameters in the proposed model.

Another experiment will be conducted to confirm the validity of the proposed model, and dynamic change of transition probabilities will be investigated and applied to the proposed model in order to maximize its accuracy and effectiveness.

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