Chiral two-body currents in nuclei: Gamow-Teller transitions and neutrinoless double-beta decay

J. Menéndez,1,2 D. Gazit,3 and A. Schwenk2,1
1Institut für Kernphysik, Technische Universität Darmstadt, 64289 Darmstadt, Germany
2ExtreMe Matter Institute EMMI, GSI Helmholtzzentrum für Schwerionenforschung GmbH, 64291 Darmstadt, Germany
3Racah Institute of Physics, The Hebrew University, 91904 Jerusalem, Israel

We show that chiral effective field theory (EFT) two-body currents provide important contributions to the quenching of low-momentum-transfer Gamow-Teller transitions, and use chiral EFT to predict the momentum-transfer dependence that is probed in neutrino-less double-beta (0νββ) decay. We then calculate for the first time the 0νββ decay operator based on chiral EFT currents and study the nuclear matrix elements at successive orders. The contributions from chiral two-body currents are significant and should be included in all calculations.

PACS numbers: 23.40.-s, 12.39.Fe, 23.40.Hc, 21.60.Cs

Weak interaction processes provide unique probes of the physics of nuclei and fundamental symmetries, and play a central role in astrophysics [1]. The structure of strongly interacting systems is explored with β decays and weak transitions. Superallowed decays allow high precision tests of the Standard Model, and 0νββ decays probe the nature of neutrinos, their hierarchy and mass. Weak processes mediate nuclear reactions that drive stellar evolution, supernovae and nucleosynthesis.

Surprisingly, key aspects of well-known decays remain a puzzle. In particular, when calculations of Gamow-Teller (GT) transitions of the spin-isospin-lowering operator $g_A \mathbf{σ} \mathbf{σ}^\dagger - \mathbf{σ} \mathbf{σ}^\dagger$ are confronted with experiment, some degree of renormalization, or “quenching” $q$, of the axial coupling $g_A^{\text{eff}} = q g_A$ is needed. Compared to the single-nucleon value $g_A = 1.2695(29)$, the GT term seems to be weaker in nuclei. This was first conjectured in studies of β-decay rates, with a typical $q \approx 0.75$ in shell-model (SM) calculations [2] and other many-body approaches [3]. In view of the significant effect on weak reaction rates, it is no surprise that this suppression has been the target of many theoretical works. It is also a major uncertainty for 0νββ decay nuclear matrix elements (NMEs), which are confronted with experiment, some dependence is due to loop corrections [4].

Although the importance of 2b currents is known from phenomenological studies [2], chiral currents and the consistency with nuclear forces have only been explored in the lightest nuclei [2,2,3,8]. In this Letter, we present first calculations for GT transitions and for the 0νββ decay operator based on chiral EFT currents. A preview of the NMEs (Fig. 2) and the quenching of $g_A$ (Fig. 3) shows the great importance of using chiral 2b currents in nuclei.

At low energies, the coupling to weak probes is given by the current-current interaction, $H_W = G_F \int d^3 \mathbf{r} e^{-i \mathbf{p} \cdot \mathbf{r}} J_{\mu L} J^{\mu L} + \text{h.c.}$, where $G_F$ is the Fermi constant, $\mathbf{p}$ the momentum transferred from nucleons to leptons, and $J_{\mu L}$ the leptonic current of an electron coupled to a left-handed electron neutrino $\nuL$. In chiral EFT, the nuclear current $J^{\mu L}_Q$ is organized in an expansion in powers of momentum $Q \sim m_\pi$ over a breakdown scale $\Lambda_b \sim 500$ MeV. Consistently with nuclear forces [3], we count the nucleon mass as a large scale, corresponding numerically to $Q/m_\pi \sim (Q/\Lambda_b)^2$, so that the leading relativistic $1/m$ corrections are of order $Q^2$, and $1/m^2$ terms of order $Q^4$. To order $Q^2$ (and also $Q^3$ in this counting), the 1b current, $J^{\mu L}_Q(r) = \sum_{i=1}^A \mathbf{r}_i \Gamma_Q^0 J^{\mu L}_{1b,i} \delta(r - \mathbf{r}_i)$, has temporal and spatial parts in momentum space [3]:

$$J^{0,1b}_Q(p^2) = gV(p^2) - g_A \frac{\mathbf{P} \cdot \mathbf{σ}_i}{2m} + g_P(p^2) \frac{E(p \cdot \mathbf{σ})}{2m},$$

$$J^{1,1b}_Q(p^2) = g_A(p^2) \mathbf{σ}_i - g_P(p^2) \frac{\mathbf{P} \cdot \mathbf{σ}_i}{2m} + i(g_M + g_V) \frac{\mathbf{σ}_i \times \mathbf{P}}{2m} - g_V \frac{\mathbf{P}}{2m},$$

where $E = E_i - E'_i$, $\mathbf{p} = \mathbf{p}_i - \mathbf{p}'_i$, and $\mathbf{P} = \mathbf{p}_i + \mathbf{p}'_i$; and vector ($V$), axial ($A$), pseudo-scalar ($P$), and magnetic ($M$) couplings, $g_V(p^2)$, $g_A(p^2)$, $g_P(p^2)$, and $g_M(p^2)$ [9].

In chiral EFT, the $p$ dependence is due to loop corrections and pion propagators, to order $Q^2$: $g_{V,A}(p^2) = g_{V,A}(1 - \frac{M^2}{p^2})$ 

FIG. 1: Chiral 2b currents and 3N force contributions.
FIG. 2: (Color online) Nuclear matrix elements $M^{0\nu\beta\beta}$ for $0\nu\beta\beta$ decay. At order $Q^0$, the NMEs include only the leading $p = 0$ and vector and axial 1b currents. At the next order, all $Q^2$ 1b-current contributions not suppressed by parity are taken into account. At order $Q^3$, the thick bars are predicted from the long-range parts of 2b currents ($c_D = 0$). The thin bars estimate the theoretical uncertainty from the short-range coupling $c_D$ by taking an extreme range for the quenching (see text). For comparison, we show the SM results of Ref. [12] based on phenomenological 1b currents. The inset (representative for $^{136}$Xe) shows that the GT part, $M^{0\nu\beta\beta}_{GT} = \int dp C_{GT}(p)$, is dominated by $p \sim 100$ MeV.

With $g_V = 1$, $A_V = 850$ MeV, $A_A = 2\sqrt{3}/r_A = 1040$ MeV, $g_p(p^2) = \frac{2g_{gam}F_\pi}{m_\pi^2 + p^2} - 4g_A(p^2)\frac{m_\pi^2}{m_A^2}$ and $g_m = \mu_p - \mu_n = 3.70$, with pion decay constant $F_\pi = 92.4$ MeV, $m_\pi = 138.0$ MeV, and $g_{epm} = 13.05$ [11].

At leading order $Q^0$, only the momentum-independent $g_A$ and $g_V$ terms contribute. They give rise to $p \lesssim 1$ MeV GT and Fermi ($\tau^-$) single- and 2b $\nu\beta\bar{\nu}$ decay. On the other hand, when studying processes that probe larger momentum transfers, such as $0\nu\beta\beta$ decay with $p \sim 100$ MeV, terms of order $Q^2$ need to be included. For $0\nu\beta\beta$ decay, the $Q^0$ terms are still most important and the axial term dominates. In SM calculations, one has $M^{0\nu\beta\beta}_{Q^0,axial}/M^{0\nu\beta\beta} \approx 1.20$ and $M^{0\nu\beta\beta}_{Q^0,vector}/M^{0\nu\beta\beta} \approx 0.15$ compared to the final $M^{0\nu\beta\beta}_{Q^0}$ [12]. Other calculations yield similar ratios, also for the $Q^2$ terms, except the vector term can be significantly larger [13, 14].

Among the $Q^2$ terms, form-factor-type (ff) contributions and the $g_p$ part of $J_{1b}$ dominate. The pseudoscalar term is important, because $pg_p(p^2) \approx 7.9$ for $p \sim 100$ MeV in $0\nu\beta\beta$ decay. They reduce the NMEs: $M^{0\nu\beta\beta}_{ff}/M^{0\nu\beta\beta} \approx -0.20$ and $M^{0\nu\beta\beta}_{g_p}/M^{0\nu\beta\beta} \approx -0.20$ [12].

The remaining $Q^2$ terms are odd under parity, so they require either a $P$-wave electron (whose phase space is suppressed [10] by $0.03 - 0.06$ for $0\nu\beta\beta$ decay candidates) or another odd-parity term to connect $0^+$ states. Therefore, the $P$ and $E$ terms in Eqs. (1) and (2) can be neglected, and only the term with the large $g_M + g_V = 4.70$ is kept, leading to a small $\approx 5\%$ contribution [12].

At order $Q^3$, 2b currents enter in chiral EFT [3]. These include vector spatial, axial temporal, and axial spatial parts [13]. The first two are odd under parity, and therefore can be neglected. Consequently, for the cases studied here, the dominant weak 2b currents only have an axial spatial component, $J^{\text{axial}}_{2b} = \sum_{i<j} J_{ij}$, with [3]

$$J_{12} = -\frac{g_A}{F_\pi} \left[ 2d_1 (\sigma_1 - \sigma_2) + d_2 (\sigma_3 - \sigma_4) \right],$$

$$-\frac{g_A}{2F_\pi} \left( \frac{1}{m_\pi^2 + k^2} \left[ (c_4 + \frac{1}{4m_0}) k \times (\sigma_4 \times k) \right] \sigma_4^\prime - \frac{i}{2m_0} k \cdot (\sigma_4 - \sigma_2) q \tau^- \right),$$

where $\tau^- = (\tau_1 - \tau_2)^-$ and the same for $\sigma_4$, $k = \frac{1}{2} (p_2 - p_1 + p_3)$, and $q = \frac{1}{2} (p_1 + p_3 - p_2)$. Equation (3) includes contributions from the one-pion-exchange $c_3, c_4$ parts and from the short-range couplings $d_1, d_2$, where due to the Pauli principle only the combination $d_1 + 2d_2 = c_D/(g_{A\Lambda_\chi})$ enters [with $\Lambda_\chi = 700$ MeV].

We study the impact of chiral 2b currents in nuclei at the normal-ordered 1b level by summing the second nucleon over occupied states in a spin and isospin symmetric reference state or core. $J^{\text{eff}}_{1b,2b} = \sum_{i} (1 - P_{ij})J_{ij}$, where $P_{ij}$ is the exchange operator. The normal-ordered 1b level is expected to be a very good approximation in medium-mass and heavy nuclei, because of phase space arguments for normal Fermi systems at low energies [16]. This approximation has also been explored for chiral 2b currents in nuclear matter [17], but limited to long wave-lengths and without connecting 2b currents and nuclear forces. Taking a Fermi gas approximation for the core and neglecting tensor-like terms $(k \cdot \sigma k - \frac{1}{3} k^2 \sigma) \tau^-$, we obtain the normal-ordered 1b current:

$$J^{\text{eff}}_{1b,2b} = -\frac{g_A}{F_\pi} \sum_{i<j} \rho \left[ \frac{c_D}{g_{A\Lambda_\chi}} + \frac{2}{3} \frac{c_3}{2} \frac{p^2}{4m_\pi^2 + p^2} + I(\rho, P) \left( \frac{1}{3} (2c_4 - c_3 + \frac{1}{6m}) \right) \right],$$

where $\rho = 2k_3^3/(3\pi^2)$ is the density of the reference state, $k_F$ the corresponding Fermi momentum, and $I(\rho, P)$ is due to the summation in the exchange term,

$$I(\rho, P) = 1 - \frac{3m_\pi^2}{2k_F^2} + \frac{3m_\pi^2}{2k_F^2} \arccot \left[ \frac{m_\pi^2 + \frac{p^2}{2m_\pi^2}}{k_F^2} \right] + \frac{3m_\pi^2}{4k_F^2} \left[ k_F^2 + m_\pi^2 - \frac{p^2}{2} \right] \log \left[ \frac{m_\pi^2 + (k_F - \frac{p}{2})^2}{m_\pi^2 + (k_F + \frac{p}{2})^2} \right].$$

The contributions from 2b currents depend on the density of the reference state. We consider a typical range for nuclei $\rho = 0.10...0.12$ fm$^{-3}$, which corresponds to average nucleon momenta $Q \sim 150 - 200$ MeV, so that 2b currents are expected to be more important compared to very light nuclei. For these densities, we have $I(\rho, P) = 0.64...0.66$, using the Fermi-gas mean-value...
TABLE I: Quenching $q$ of the $p = 0$ GT operator predicted by the long-range parts of 2b currents ($c_D = 0$) for given $c_3$, $c_4$ couplings. Dots correspond to a density range $\rho = 0.10 \ldots 0.12$ fm$^{-3}$, and all $c_3$, $c_4$ values are in GeV$^{-1}$. In addition, we give the short-range coupling $c_D$ obtained by requiring that 2b currents lead to a quenching of $q = 0.74$ or $q = 0.96$ over this density range. For given $q$, the * values correspond to the weakest/strongest $p$ dependence in Fig. 3 (smallest/largest $|c_3|\rho$), which yield the thin bars in Fig. 2.

$P^2 = 6k^2/5$, and $I(\rho, P) = 0.58 \ldots 0.60$ for $P = 0$, so the total-momentum dependence is very weak.

The effective 1b current $J_{1b}^{\text{eff}}$ only contributes to the GT operator and can be included as a correction to the $g_A(p^2)$ part of the 1b current, Eq. 2. This result is general for a spin/isospin symmetric reference state. Beyond the Fermi-gas evaluation, only the momentum dependence will be replaced by a state/orbital dependence.

For $c_D = 0$, the leading 2b currents are fully determined by the couplings $c_3$, $c_4$ from nuclear forces. We take $c_3$, $c_4$ from the N$^3$LO NN potentials of Ref. 18 (EM) or Ref. 19 (EGM), as well as from a NN partial wave analysis (PWA) extraction 20. Because the $c_3$, $c_4$ are large in chiral EFT without explicit Deltas, we also explore typical changes expected at higher order $\delta c_3 \approx 1$ GeV$^{-1}$. The different $c_3$, $c_4$ are listed in Table II. As shown, the corresponding long-range parts of chiral 2b currents predict a quenching of the GT operator at $p = 0$ ranging from $q = 0.85 \ldots 0.66$ for densities $\rho = 0.10 \ldots 0.12$ fm$^{-3}$. The largest quenching is obtained from the largest $c_4$, $c_3$ coupling, which is closest to the single-Delta excitation with $c_4$, $c_3 \approx 15$ GeV$^{-1}$. The density dependence of the 2b-current contributions for $c_D = 0$ is also shown as inset in Fig. 3.

This demonstrates that chiral 2b currents naturally contribute to the quenching of GT transitions. A reduction of $g_A$ in the currents is also expected considering chiral 3N forces as density-dependent two-body interactions 21. We can constrain the short-range coupling $c_D$ by requiring to reproduce an empirical quenching $q = 0.74$ needed in many-body calculations 2. This leads to values $|c_D| < 1$ in Table II which is compatible with independent determinations from 3N forces, especially in the best studied EM case 6, 22 (the comparison should be made for large 3N cutoffs); e.g., $c_D$’s obtained from the $^3$H half-life fit 3 lead to $q = 0.75 \ldots 0.67$. Because part of the quenching may be due to truncations in the many-body basis, we can ask what values of $c_D$ are needed to reproduce a smaller quenching $q = 0.96$ (based on an extraction $q^2 = 0.92 \pm 0.11$ from GT strength functions to high energies 23). As shown in Table II the resulting $c_D$ are more negative, which seems compatible with 3N force fits only for the EGM and PWA cases 22. Because of these uncertainties, we consider a conservative range from empirical to small quenching. The resulting quenching of $g_A$ is shown in Fig. 3 versus density. Chiral 2b currents will universally affect $p \approx 0$ GT lifetimes and strength functions by $q^2$ and $2\nu\beta$ lifetimes by $q^4$.

The momentum-transfer dependence is predicted in chiral EFT and shown in Fig. 3 for the 2b-current contributions. This $p$ dependence is determined by the density and the $c_4$ coupling in Eq. 1, and is stronger for larger $|c_3|\rho$ values. With increasing $p \sim m_\pi$, the impact of 2b currents, and thus the quenching, is reduced and the GT 1b operator could even be somewhat enhanced in the smaller quenching case. A reduction is consistent with muon capture studies, which probe $p \sim m_\mu \sim 100$ MeV, where GT quenching is not needed 24.

Finally, we apply chiral EFT currents to the $0\nu\beta\beta$ decay NME, which is given by the nuclear currents 10:

\[ M^{0\nu\beta\beta} = \left< 0^+ \right| \frac{R}{g_A^2} \sum_{i,j} \tau_i^- \tau_j^- \int \frac{d\mathbf{p}}{2\pi^2} e^{i\mathbf{p} \cdot \mathbf{r}_i} \frac{J_{\beta\mu}^* \mu \mathbf{p}}{p(p + E_1)} \left| 0^+ \right> \]

Here, a size scale $R = 1.2 A^{1/3}$ fm is introduced so that $M^{0\nu\beta\beta}$ is dimensionless, and the closure approximation was used with an average intermediate-state energy $E_1$.

Constructing the $0\nu\beta\beta$ decay operator in chiral EFT,
the nuclear currents are given by the $1b$ currents $J^p_i$, $J^h_i$, and $J^3_i$, of Eqs. (4) and (5), and we include $2b$ currents at the normal-ordered $1b$ level $J^\text{eff}_{i,1b}$ of Eq. (4). With these, the transition operator can be decomposed into $GT$, Fermi ($F$), and tensor ($T$) terms that receive contributions from the $V, A, P, M$ components in the currents $[13]$

$$J^p_i J^h_m = h^{GT} (p^2) \sigma_1 \cdot \sigma_2 - h^F (p^2) - h^T S_{12}(\hat{p}),$$

$$h^{GT} (p^2) = h^{AA}_{\text{GT}} + h^{AP}_{\text{GT}} + h^{GT}_{\text{PP}} + h^{GT}_{\text{MM}}.$$  

At leading order $Q^0$, only the $p$ axial-axial and vector-vector terms $h^{GT}_{\text{AA}}(0)$ and $h^{VT}_{\text{VV}}(0)$ contribute. At order $Q^2$, this includes the $p^2$ terms in $g_{V,A}(p^2)$ and the other terms in the $1b$ currents. The $Q^2$ terms are also important in phenomenological currents, as they lead to short-range correlation effects being small $[22]$, which is consistent with chiral EFT. At order $Q^3$, $2b$-current contributions enter in $h^{GT}_{\text{AA}}(0)$ and $h^{GT}_{\text{VV}}(0)$. The $h^{\text{AA}}$ and $h^{\text{VV}}$ parts are small (less than 1%, except in $^{48}\text{Ca}$ $[12, 14]$), consistent with neglecting $(k \cdot \sigma k - \frac{1}{2} k^2 \sigma)$ terms from the $2b$ currents.

We then calculate $M^{0\nu\beta\beta}$ based on chiral EFT currents for $^{48}\text{Ca}$, $^{76}\text{Ge}$, $^{82}\text{Se}$, $^{124}\text{Sn}$, $^{130}\text{Te}$ and $^{138}\text{Xe}$. These are challenging calculations and require approximations. Here we perform state-of-the-art SM calculations using the coupled code NATHAN $[26]$, ideal for $0^+$ states. The details of the SM spaces and interactions are given in Ref. $[12]$. Because $2b$ currents are independent of the many-body spaces, they should be included in all calculations, and we focus on relative changes.

Our results are shown in Fig. 2 for successive orders in chiral EFT. At order $Q^2$, the NMEs are similar to calculations based on phenomenological $1b$ currents only, with differences mainly due to the $Q^2$ terms compared to phenomenological dipole form factors, as well as different $\Lambda_A$ values used. At order $Q^3$, the contributions from chiral $2b$ currents range from a 10% increase to a 35% reduction of $M^{0\nu\beta\beta}$. This range is obtained for the couplings of Table $[1]$ and overlaps with the prediction based on the long-range parts of $2b$ currents. The changes at order $Q^3$ agree with the $p \sim 100$ MeV estimate from Fig. 3. The lower $M^{0\nu\beta\beta}$ values correspond to the quenching of $g_A$ in SM calculations of $p \approx 0$ single-$\beta$ and $2\nu\beta\beta$ decays, but the $p$ dependence weakens the quenching compared to the naive GT $q^2 \approx 0.74^2 \approx 0.55$ or 45% reduction. Finally, we note that in Delta-full EFT, the long-range single-Delta part of $2b$ currents would contribute at order $Q^2$, which would shift the $Q^2$ range results more into the $Q^3$ range, improving the order-by-order expansion in Fig. 2.

This presents the first study of chiral $2b$ currents for GT transitions and $0\nu\beta\beta$ decay in medium-mass nuclei. Compared to light nuclei, their contributions are amplified because of the larger nucleon momenta. For a spin and isospin symmetric reference state, the leading axial $2b$ currents contribute only to the GT operator. A quenching of $p \approx 0$ GT transitions is predicted by the long-range parts of $2b$ currents, and we have estimated the theoretical uncertainty from the short-range part assuming an empirical to small quenching range.

Chiral EFT predicts the momentum-transfer dependence of $2b$ currents that is probed in $0\nu\beta\beta$ decay. This first calculation of the $0\nu\beta\beta$ decay operator based on chiral EFT currents shows that $2b$ contributions to the NMEs range from $-35\%$ to $10\%$ and should be included in all calculations. Future directions are: going beyond the $1b$ approximation, investigating other electroweak transitions, developments of consistent interactions and operators (see also $[21]$), and astrophysics explorations.

We thank R. J. Furnstahl, T. R. Rodríguez and U. van Kolck for very useful discussions. This work was supported in part by the DFG through grant SFB 634 and the Helmholtz Alliance HA216/EMMI.

[1] E. M. Henley and W. C. Haxton, *Symmetries and Fundamental Interactions in Nuclei* (World Scientific, 1995).
[2] B. H. Wildenthal, M. S. Curtin and B. A. Brown, Phys. Rev. C 28, 1343 (1983); G. Martínez-Pinedo et al., Phys. Rev. C 53, R2602 (1996).
[3] M. Bender et al., Phys. Rev. C 65, 054322 (2002); T. R. Rodríguez and G. Martínez-Pinedo, Phys. Rev. Lett. 105, 252503 (2010).
[4] F. Epelbaum, H.-W. Hammer and U.-G. Meißner, Rev. Mod. Phys. 81, 1773 (2009).
[5] T. S. Park et al., Phys. Rev. C 67, 055206 (2003). Here relativistic corrections are counted as $Q/m \sim Q/\Lambda_n$.
[6] D. Gazit, S. Quaglioni and P. Navratil, Phys. Rev. Lett. 103, 102502 (2009).
[7] A. Arima et al., Adv. Nucl. Phys. 18, 1 (1987); I. S. Towner, Phys. Rept. 155, 263 (1987); B. A. Brown and B. H. Wildenthal, Nucl. Phys. A 474, 290 (1987).
[8] K. Kubodera and M. Rho, arXiv:1011.4919.
[9] The same structure is obtained phenomenologically based on symmetries $[10]$, with dipole-form couplings.
[10] T. Tomoda, Rep. Prog. Phys. 54, 53 (1991).
[11] V. Bernard, L. Elouadrhiri and U.-G. Meißner, J. Phys. G 28, R1 (2002).
[12] E. Caurier et al., Phys. Rev. Lett. 100, 052503 (2008); J. Menéndez et al., Nucl. Phys. A 818, 139 (2009).
[13] E. Caurier et al., Phys. Rev. C 67, 055502 (1999).
[14] F. Šimkovic et al., Phys. Rev. C 75, 051303 (2007).
[15] M. Kortelainen et al., Phys. Rev. C 75, 051303 (2007).
[16] K. Bogner et al., Phys. Rev. C 69, 055801 (2004).
[17] B. Friman and A. Schwenk, arXiv:1101.4858.
[18] T. S. Park, H. Jung and D. P. Min, Phys. Lett. B 409, 26 (1997).
[19] D. R. Entem and R. Machleidt, Phys. Rev. C 68, 041001(R) (2003).
[20] E. Epelbaum, W. Glöckle and U.-G. Meißner, Nucl. Phys. A 747, 362 (2005).
[21] E. Epelbaum, W. Glöckle and U.-G. Meißner, Nucl. Phys. A 747, 362 (2005).
[22] M. C. M. Rentmeester, R. G. E. Timmermans and J. J. de Swart, Phys. Rev. C 67, 024002 (2003).
[23] K. Bogner et al., Phys. Rev. C 69, 055801 (2004).
[24] M. Sasano et al., Phys. Rev. C 79, 024602 (2009).
[25] N. T. Zinner, K. Langanke and P. Vogel, Phys. Rev. C 74, 024326 (2006).
[25] F. Šimkovic et al., Phys. Rev. C 79, 055501 (2009).
[26] E. Caurier et al., Rev. Mod. Phys. 77, 427 (2005).