Constraints on two-lepton, two quark operators

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Abstract

Physics from beyond the Standard Model, such as leptoquarks, can induce four fermion operators involving a quark, an anti-quark, a lepton and an anti-lepton. We update the (flavour-dependent) constraints on the coefficients of such interactions, arising from collider searches for contact interactions, meson decays and other rare processes. We then make naive estimates for the magnitude of the coefficients, as could arise in texture models or from inverse hierarchies in the kinetic term coefficients. These “expectations” suggest that rare Kaon decays could be a good place to look for such operators.

1 Introduction

Various arguments, such as the “hierarchy problem” and the existence of Dark Matter, suggest New Physics (NP) with a mass scale $m_{NP} \gtrsim G_F^{-1/2}$. The new interactions could manifest themselves, at energies below $m_{NP}$, as small deviations from expected Standard Model rates, or as new processes, absent in the Standard Model (SM). If $m_{NP}$ is accessible to the LHC, the new particles could be discovered soon.

To identify New Physics produced at colliders, it helps to have input from low energy precision experiments. For instance, $G_F$ and $\alpha_{em}$ were important inputs for the electroweak fit to LEP data. In flavour physics, precision low energy experiments searching for suppressed or forbidden SM modes, are sensitive to New Physics scales $\gg$ TeV. These bounds can be compatible with TeV mass NP, provided there is some “similarity” between the flavoured interactions of the new particles and of the SM (such as Minimal Flavour Violation [1, 2]). It is therefore interesting to study the interplay between flavour and high energy experiments, both current and future, as was recently done in the series of workshops “Flavour in the era of the LHC” [3].

Contact interactions [4] involving two quarks and two leptons are particularly relevant because the LHC can have $q\bar{q}$ in the initial state. This paper compiles bounds on the coefficients of four fermion interactions involving a quark, an anti-quark, a lepton and an anti-lepton. These will be referred to as “two quark, two lepton” interactions in the following. The coefficients are assumed to be flavour-dependent, and the bounds are set assuming the presence of one interaction, of given flavour indices, at a time. An update of these constraints is timely, to take into account the improved limits from B factories.

We consider four fermion interactions which are induced by dimension six gauge invariant operators. The bound on the coefficient of a particular gauge invariant operator can be obtained by consulting the tables of bounds on all the four fermion interactions it induces, and selecting the most stringent one. We present bounds on the coefficients of (non gauge-invariant) four fermion interactions, rather than of gauge invariant operators, to allow constraints to be extracted for an arbitrary basis of gauge invariant operators. This is an artifact of setting bounds by allowing one interaction at a time; we could constrain gauge invariant operators if we fit simultaneously to all the coefficients.

Various processes can constrain a given four fermion interaction. We attempt to find the best bound on each coefficient, and list it in tables [3-13]. In the case of SM-allowed processes which are observed, the bound is obtained by requiring that the SM-New Physics interference contribute less than $2\sigma$. In the case of unobserved decay rates, the New Physics is bounded to contribute below the 90% confidence level experimental limit. The bounds should be correct at the factor of two level, but not to the two significant figures quoted.

The limits compiled here are mostly obtained from observables to which the two quark two lepton interactions contribute at tree level: contact interaction searches at colliders [5, 6, 7], and various low energy

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processes which are rare in the Standard Model: leptonic and semi-leptonic meson decays [11], semi-leptonic τ decays [11], and μ → e conversion [12]. The four fermion operators can also contribute in loops, for instance one obtains a penguin by closing two legs and attaching a gauge boson. However, such loop bounds depend on the New Physics inducing the contact interaction, as discussed in section 4.2. So we only include estimated bounds from Z decay observables at LEP1 [13], because these were the only constraints we could find on some higher generation operators.

Constraints have previously been compiled [14] on two quark and two lepton contact interactions [15, 16, 17] as could arise in strongly coupled models, and in other New Physics scenarios such as leptoquarks [20, 21, 22, 23, 24] and Z’s [24, 33, 34, 35, 36]. As reviewed in section 2, the most natural basis of four fermion operators depends somewhat on the New Physics scenario. Following the lepton chapter of the CERN Workshop report [15] (which gives bounds on contact interactions involving four leptons, and on some two lepton and two quark interactions), we use operators where the leptons and quarks are contracted, in most cases, among themselves.

Meson factories, such as Belle and NA62 [37], will be studying rare decays with great precision in the upcoming years. They could search, for instance, for lepton flavour changing but generation diagonal decays such as $K^+ \rightarrow \pi^+ \mu^+ \mu^-$ or $B^+ \rightarrow K^+ \tau^+ \mu^+$, which would be “natural” for leptoquarks with generation diagonal couplings. It would be useful to have some phenomenologically motivated “expectations” for whether NP should appear in second or third generation rare decays. So in the last section of this paper, we estimate the relative rates of different processes, which allows to compare the sensitivity of different processes to the underlying New Physics mass scale.

2 Operator Basis in the Effective Lagrangian

There are a large number of gauge invariant two quark two lepton operators at dimension six, not all of which are independent from each other: equations of motion give relations between operators, and Fiertz transformations rearrange the Lorentz contractions. In this section, we review the four fermion operator basis which are independent from each other: equations of motion give relations between operators, and Fiertz transformations rearrange the Lorentz contractions. In this section, we review the four fermion operator basis which are independent from each other: equations of motion give relations between operators, and Fiertz transformations rearrange the Lorentz contractions.

If New Physics is present at the scale $m_{NP}$, it can be added in the SM as terms in an effective Lagrangian $\mathcal{L}^{SM}_{eff}$, which can be written, at energies below $m_{NP}$ as an expansion in $1/m_{NP}$:

$$\mathcal{L}^{SM}_{eff} = \mathcal{L}_0 + \frac{1}{m_{NP}} \mathcal{L}_1 + \frac{1}{m_{NP}^2} \mathcal{L}_2 + \frac{1}{m_{NP}^3} \mathcal{L}_3 + \cdots$$

where $\mathcal{L}_0$ is the renormalizable SM Lagrangian, and the $\mathcal{L}_n$ contain dimension $d = 4 + n$ operators, constructed out of Standard Model fields, and invariant under the SM $SU(3) \times SU(2) \times U(1)$ gauge symmetry.

The SM fermions are written as:

$$q_i = \begin{pmatrix} u_{Li} \\ d_{Li} \end{pmatrix}, \quad \ell_i = \begin{pmatrix} \nu_{Li} \\ e_{Li} \end{pmatrix} u_{Ri}, \quad d_{Ri}, \quad e_{Ri}$$

where $i$ is the family/generation index. (Notice that, in what follows, $\ell$ stands for the lepton doublet, and $l$ for any charged lepton.) Then the basis of operators used in [15] for the dimension six $\mathcal{L}_2$ contains the $V\pm A$ two lepton-two quark operators:

$$O^{ijkn}_{(1)\ell q} = (\bar{\ell}_i \gamma^\mu P_L \ell_j) (\bar{q}_k \gamma_\mu P_L q_n),$$

$$O^{ijkn}_{(2)\ell q} = (\bar{\ell}_i \gamma^\mu \nu_j + \bar{\ell}_i \gamma^\mu P_L \ell_j) (\bar{u}_k \gamma_\mu P_L u_n + \bar{d}_k \gamma_\mu P_L d_n),$$

$$O^{ijkn}_{(3)q} = (\bar{\ell}_i \tau^I \gamma^\mu P_L \ell_j) (\bar{q}_k \gamma^\mu P_L q_n),$$

$$O^{ijkn}_{(4)q} = (\bar{e}_i \gamma^\mu P_R e_j) (\bar{q}_k \gamma_\mu P_L q_n),$$

$$O^{ijkn}_{ed} = (\bar{\ell}_i \gamma^\mu P_R e_j) (\bar{d}_k \gamma_\mu P_R d_n),$$

$$O^{ijkn}_{eu} = (\bar{e}_i \gamma^\mu P_R e_j) (\bar{u}_k \gamma_\mu P_R u_n).$$

1 For additional pre-1995 references, see the references of [28].
\[ O_{ijkn}^{tu} = (q_i \gamma^\mu P_L q_j)(\bar{q}_k \gamma_\mu P_R u_n) \]
\[ O_{ijkn}^{id} = (q_i \gamma^\mu P_L q_j)(\bar{d}_k \gamma_\mu P_R d_n) \]  
(5)

where the Lorentz, colour and SU(2) are contractions are implicit within the parentheses, \( \tau^i \) are the Pauli matrices \((\tau^3 = diag\{1, -1\})\), and generation indices are subscripts. With the same notation (but now SU(2) contraction between the parentheses), the \( \pm \) operators are:

\[ O_{ijkn}^{q_S} = (\bar{q}_i \epsilon_j)(\bar{q}_k u_n) , \]
\[ O_{ijkn}^{q_{dc}} = (\bar{q}_i \epsilon_j)(\bar{d}_k q_n) . \]  
(6)

See [15] for the expansion of these operators on the component fields which participate in observable interactions (e.g. \( \nu_L \) and \( e_L \) rather than \( \ell \)). A similar expansion, for the four fermion operators induced by scalar leptoquarks, can be found in [71].

After Electroweak Symmetry Breaking, an operator constructed with electroweak doublets, for instance \( O_{1\ell q} \) of eqn (3), will induce four fermion interactions among the doublet components (see eqn (4)). The data bound the coefficients of these four fermion interaction among chiral fields \((u_L, d_R, e_L, \text{ etc})\), so in our tables, we quote bounds on the coefficients of four-fermion interactions. The best bound on the coefficient of a gauge invariant operator will be the most restrictive limit to be found on all the four fermion interactions it induces.

We quote bounds on the coefficients of four fermion interactions, rather than those of gauge invariant operators, to minimise the dependence on operator basis choice. To motivate this, consider the alternative gauge invariant operator \( O_{alt} \), that could be induced by a scalar leptoquark:

\[ O_{alt} = (\bar{q} \tau_\ell)(\ell \tau_\ell^c) \]
\[ = \frac{1}{2} [ (\bar{q} \tau^\mu P_L u)(\tau_\mu \gamma_\mu P_L e) + (\bar{d} \tau^\mu P_L d)(\bar{e} \gamma_\mu P_L L) + (\bar{u} \gamma^\mu P_L u)(\bar{d} \gamma_\mu P_L d) + (\bar{d} \gamma^\mu P_L d)(\bar{e} \gamma_\mu P_L e) ] \]  
(7)

where \( \varepsilon \) gives the antisymmetric SU(2) contraction. Using the identity \( \delta_{ab} \delta_{cd} + \sum_i [\sigma_i]_{ab} [\sigma_i]_{cd} = 2\delta_{ad} \delta_{bc} \) for the SU(2) contraction between \( q \) and \( \bar{q} \ell \), the operator 1 can be rewritten in terms of \( O_{1\ell q} \) and \( O_{3\ell q} \). However, since the bounds are computed by allowing the coefficient of one operator at a time to be non-zero, the constraints on the coefficients of \( O_{alt} \) cannot be obtained easily from the bounds on the coefficients of \( O_{1\ell q} \) and \( O_{3\ell q} \). We therefore quote bounds on the more “physical” four fermion interactions, which allows bounds on any choice of gauge invariant dimension 6 operators to be extracted from our tables.

We choose the factors of 2 in the normalisation of the operators such that the coefficient \( C_{ijkn}/m_N^2 \) replaces \(-4G_F/\sqrt{2}\) as the coupling constant in the Feynman rules. So the hermitian operators of eqn (11) are included as

\[ L = ... + \frac{1}{2} \sum_{i,j,k,n=1}^3 \frac{C_{ijkn}}{m_N^2} O_{ijkn}^{q} + h.c. \]  
(8)

where the 1/2 compensates the +h.c.. The generation indices of the \( \pm P \) operators also are summed 1..3, and there is a +h.c. but no 1/2 because the operators are not hermitian.

In order to set bounds on dimensionless quantities, the coefficients of the two lepton-two quark operators in (5) and (6) are normalized relative to the Fermi interactions:

\[ \frac{C_{ijkn}^{q_S}}{m_N^2} = -\frac{4G_F}{\sqrt{2}} C_{ijkn}^{q_S} \]
\[ \frac{C_{ijkn}^{q_{dc}}}{m_N^2} = -\frac{4G_F}{\sqrt{2}} C_{ijkn}^{q_{dc}} \]
\[ \frac{C_{ijkn}^{\ell q}}{m_N^2} = -\frac{4G_F}{\sqrt{2}} C_{ijkn}^{\ell q} \]
\[ \frac{C_{ijkn}^{\ell q_S}}{m_N^2} = -\frac{4G_F}{\sqrt{2}} C_{ijkn}^{\ell q_S} \]
\[ \frac{C_{ijkn}^{\ell q_{dc}}}{m_N^2} = -\frac{4G_F}{\sqrt{2}} C_{ijkn}^{\ell q_{dc}} \]  
(9)

Notice that the first two indices, \( ij \), are always lepton indices, and the last two, \( kn \), are always quarks.
3 Constraints from Rare Decays

A multitude of precision experiments searching for rare meson decays provide stringent bounds on the generation-changing interactions of New Physics. We use the experimental bounds compiled and averaged by the Particle Data Group [11]. Constraints on New Physics have been obtained from this data in the papers mentioned in the introduction, and in more recent years by [10, 11, 12, 13], including CP violation effects [44], asymmetries in final state distributions [15], and discrepancies between the data and the SM [17, 18].

3.1 Leptonic meson decays

In the presence of New Physics, the leptonic decay rate of a charged pseudoscalar meson $M_{kn}$ (for instance $\pi^+$), made of constituent quarks $\bar{q}_k q_n$ (for instance $\bar{d} u$), can be written:

$$\Gamma (M_{kn} \to l^+l^-) = \frac{k G_F^2}{\pi m_M^2} \left\{ (\epsilon_{S+P}^{ijkn})^2 \tilde{P}^2 \left( m_M^2 - m_i^2 - m_j^2 \right) + \left[ V_{kn} - \epsilon_{S+P}^{ijkn} \right]^2 \tilde{A}^2 \left[ (m_M^2 - m_i^2 - m_j^2)(m_i^2 + m_j^2) + 4m_i^2 m_j^2 \right] + 2 \left[ \epsilon_{S+P}^{ijkn} \epsilon_{S+P}^{ijkn} - V_{kn} \epsilon_{S+P}^{ijkn} \right] \tilde{A} \tilde{P} m_j \left( m_M^2 + m_i^2 - m_j^2 \right) \right\}. \quad (13)$$

The expectation values of quark currents are taken to have their current algebra values:

$$\tilde{A}^{P\mu} = \frac{1}{2} \langle 0 |\gamma^\mu \gamma^5 q | M \rangle = \frac{f_M P^{\mu}}{2}, \quad \tilde{P} = \frac{1}{2} \langle 0 |\gamma^5 q | M \rangle = \frac{f_M m_M}{m_k + m_n}. \quad$$

These formulae are used for pions, kaons, $D$ [3] and $B$ mesons, with meson decay constants $f_M$ taken from [11], and from [49] for the $B_s$ (we take $f_{B_s} = f_{B^+} = 195$ MeV, and $f_{B_s} = 243$ MeV). $P^{\mu}$ is the momentum of the meson, and $k$ is the magnitude of the lepton 3-momentum in the center-of-mass frame:

$$k^2 = \frac{1}{4m_M^2} \left[ \left( m_M^2 - (m_i + m_j)^2 \right) \left( m_M^2 - (m_i - m_j)^2 \right) \right]. \quad (14)$$

In the Standard Model, charged pseudoscalar mesons decay to light leptons via $s$-channel $W$ boson exchange. This amplitude is suppressed by the lepton mass, due to angular momentum conservation: in the relativistic limit, where helicity $\simeq$ chirality, the chirality of one of the left-handed leptons must be flipped. This suppression can be seen in the bracket multiplying $\tilde{A}^2$ in eqn (13), and is precisely measured in the $R_\pi$ [21] and $R_K$ ratios

$$R_\pi \equiv \frac{\Gamma (\pi^+ \to e^+\nu)}{\Gamma (\pi^+ \to \mu^+\nu)} \quad \Gamma (\pi^+ \to e^+\nu) \bigg|_{\text{theory}} = 8.22 \times 10^{-4} \quad (15)$$

(we also use eqn (13) to describe the Flavour Changing Neutral Current (FCNC) decays of neutral mesons (such as $K_L \to \mu e$, or $B \to \mu \nu$) induced by the effective operators of eqn (5) and (6). The SM contribution in these decays is very small, so to use eqn (13), the term proportional to the CKM matrix element $V_{kn}$, is set to zero. These processes give bounds on the $V \pm A$ type interactions $(\epsilon_{ij}^{\nu\mu} P_{L,R} e_j)(\bar{u}_k\gamma_\mu P_{L,R} u_n)$ and $(\bar{e}_i\gamma_\mu P_{L,R} e_j)(d_k\gamma_\mu P_{L,R} d_n)$.

We extract bounds from $D_s^0$ decays, although there is a few $\sigma$ discrepancy among these decay and lattice results, which has been discussed in leptoquark models [22].

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2 We extract bounds from $D_s^0$ decays, although there is a few $\sigma$ discrepancy among these decay and lattice results, which has been discussed in leptoquark models [22].

3 So the $\epsilon_{ij}^{LL}$ interference term, which is included to make the decay rate self-consistent, will not be used to set bounds.
3.2 Semi-leptonic decays

Semi-leptonic decays of mesons and hadrons can occur at tree level in the SM, via charged current interactions, and the good agreement between theory and data can be used to set bounds on New Physics in charged currents (see e.g. [54] for flavour diagonal bounds). We will use the \( K^+ \) form factors and the observed unitarity of the CKM matrix to set bounds on charged current two quark, two lepton interactions. We start by discussing bounds from generation changing, “neutral current” decays, which are forbidden at tree level in the SM.

3.2.1 Neutral current decays

Semileptonic meson decays can be used to set bounds on generation changing neutral current operators of the \( V \pm A \) -type, by using various isospin symmetries to relate the New Physics operator to SM-induced charged current processes. We use the following approximations:

\[
\langle K^+ | \bar{O} | \pi^0 \rangle \approx \frac{1}{\sqrt{2}} \langle K^+ | \bar{O} | \pi^+ \rangle \tag{16}
\]

\[
\langle D^+ | \bar{u} \gamma^\mu P_c | \pi^+ \rangle \approx \langle D^0 | \bar{d} \gamma^\mu P_c | \pi^- \rangle \quad \text{with } P = P_L, P_R \tag{17}
\]

\[
\langle B^+ | \bar{d} \gamma^\mu \gamma^5 b | \pi^+ \rangle \approx \langle B^+ | \bar{u} \gamma^\mu \gamma^5 b | \pi^0 \rangle \tag{18}
\]

\[
\langle B^0 | \bar{u} \gamma^\mu \gamma^5 b | \pi^- \rangle \approx \sqrt{2} \langle B^+ | \bar{u} \gamma^\mu \gamma^5 b | \pi^0 \rangle \tag{19}
\]

where \( \bar{O} \) is some isospin 1/2 operator.

The constraints arise from generation changing decays of \( K, D \) or \( B \) mesons, such as \( K^+ \to \pi^+ e^+ \nu \) or \( K^+ \to \pi^+ \nu_1 \bar{\nu}_j \) [46]. We estimate the decay rates for such processes, induced by the New Physics \( V \pm A \) operator, as the squared ratio of the NP to SM quark matrix elements, times the measured rate for a SM allowed process (for instance, \( K^+ \to \pi^0 e^+ \nu \)). In the case of \( K \) decays, this has errors due to lepton mass effects. For semi-leptonic \( B \) decays, the errors are more significant. For instance, we approximate

\[
\frac{\Gamma(B^+ \to K^+ \tau^\pm \mu^\mp)}{\Gamma(B^+ \to D^0 e^+ \nu)} \approx \frac{|\epsilon^{\mu b s}|^2}{|V_{cb}|^2} \tag{20}
\]

3.2.2 \( K_{13}^+ \) and (pseudo)scalar operators

Following [29], the decays \( K_{13}^+ = K^+ \to \pi^0 e^+ \nu \) and \( K_{13}^+ \) can be used to constrain the \( S \pm P \) operators \( O_{13} = (\bar{e} e)(\bar{q} u) \) (see eqn (4)). The bound arises from the experimental limit [52] on \( f_{exp}/f_{\tau}(0) \), where \( f_{exp} \) is the scalar decay constant of the kaon, and \( f_{\tau} \) one of the \( V \pm A \) decay constants, both defined as follows:

\[
\mathcal{M} = G_F V_{us} \left\{ - (\bar{\nu}_L^i \gamma_{\mu L}^j f^L_{+})(f^+ p^\mu + f^- q^\mu) + 2m_K (\bar{\nu}_L^i f^L_{L R}^j) f_{exp} \right\} \tag{21}
\]

The matrix element induced by the SM and \( S \pm P \) New Physics, is:

\[
\mathcal{M} = -G_F V_{us} (\bar{\nu}_L^i \gamma_{\mu L}^j) (f^+ p^\mu + f^- q^\mu) + \frac{1}{\sqrt{2}} \frac{C_{ijLq}^{\alpha \gamma}}{m_L^2} m_K f_{NP}^L (\bar{\nu}_L^i f_{LR}^j) \tag{22}
\]

with \( p^\mu = (p_K + p_\tau)^\mu \) and \( q^\mu = (p_K - p_\tau)^\mu \). This implies

\[
f_{exp} = \frac{C_{ijLq}^{\alpha \gamma}}{m_L^2} \frac{\sqrt{2}}{4G_F V_{us}} f_{NP} \tag{23}
\]

With the current algebra identity:

\[
f_{NP} = \frac{1}{2m_K} \frac{m_k^2 - m_\pi^2}{m_s - m_u} f^+ + \frac{q^2}{(m_s - m_u) 2m_K} f^- \tag{24}
\]

we obtain

\[
\frac{C_{ijLq}^{\alpha \gamma}}{m_L^2} \frac{\sqrt{2}}{4G_F V_{su}} \frac{1}{2m_K} \frac{m_k^2 - m_\pi^2}{m_s - m_u} \leq \left[ \frac{f_{NP}}{f_{exp}(q^2 = 0)} \right] \tag{25}
\]

\footnote{Due to the \( u_R \), these operators are not strictly constrained by the rare neutral kaon decays like \( K_L \to \bar{\nu}_e \).}
3.2.3 CKM Unitarity

We can obtain constraints on the coefficient $\epsilon^{ijkn}$ of the charged current four fermion interaction $(\bar{v}_i \gamma^\mu P_L e_j)$, from the observed unitarity of the CKM matrix $V$. We allow $\epsilon^{ijkn}$ for only one combination $ijkn$, and impose that CKM remain unitary within the $2\sigma$ uncertainties on its elements. So for instance, $\epsilon^{\nu_c e u d}$ would contribute to nuclear $\beta$ decay like a shift $V_{ud} \rightarrow V_{ud} + \epsilon^{\nu_c e u d}$. So we obtain

$$1 - |V_{ud}|^2 = 2|V_{ud}|^2 \epsilon^{\nu_c e u d} + |\epsilon^{\nu_c e u d}|^2 \pm 4V_{us} \sigma_{us}$$

(26)

where $V_{us} \pm \sigma_{us} = 0.2255 \pm 0.0019$, and we neglect the experimental uncertainty on $V_{ud} = 0.97418 \pm 0.00027$. The left hand side of this equality is $\approx 10^{-4}$, which gives the bounds quoted in the tables. Comparing $\tau^- \rightarrow \pi^- \nu$ to $\pi \rightarrow \mu \nu$ determines the ratio of the leptonic couplings to the $W$: $g_\tau/g_\mu = 0.996 \pm 0.005$ [55], which we translate to $2\epsilon^{\nu_c \tau du}, |\epsilon^{\nu_c \tau du}|^2 < 2 \times 0.1$. Unitarity of the first row of $V$ also gives $2V_{us} \epsilon^{\nu_c l u s u}, |\epsilon^{\nu_c l u s u}|^2 < 4V_{us} \sigma_{us}$, where $j \neq l$ and $l = e$ or $\mu$. For $l = \tau$, bounds on $\epsilon^{\nu_c \tau su}$ can be obtained from the strange decays of $\tau$s, which give $V_{us}$ with an uncertainty [54] $\sigma_{us} = 0.0027$. Similarly, unitarity of the first column of $V$, gives $2V_{cd} \epsilon^{\nu_c l d c}, |\epsilon^{\nu_c l d c}|^2 < 4V_{cd} \sigma_{cd}$, where $j \neq l$ and $l = e$ or $\mu$. $V_{cd}$ can be determined from $D \rightarrow \pi \nu$ decays ($\sigma_{cd} = 0.024$), and is most accurately determined in $\nu_\mu$ scattering ($\sigma_{cd} = 0.011$) [11]. The small mass difference $m_D - m_\tau$ makes $\epsilon^{\nu_c \tau dc}$ less easy to constrain; by comparing the upper bound [11] $BR(D^- \rightarrow \tau \bar{\nu}) < 1.2 \times 10^{-3}$ and $BR(D^- \rightarrow \mu \bar{\nu}) = 3.38 \times 10^{-4}$ to eqn (26), one can require $\epsilon^{\nu_c \tau dc} < V_{cd}$. Finally, unitarity also implies that contributions of $\epsilon^{\nu_c lsc}$ should fit in the uncertainty in $V_{cd} = 1.04 \pm 0.06$, which is determined from decays with $l = e, \mu$ and $\tau$. Since the uncertainty is from the lattice QCD determination of the form factor, we use it for all lepton generations.

It is more difficult to use unitarity to constrain the charged current interactions involving $c b$, and $u b$. Instead, we require $\epsilon^{\nu_c l x b} \lesssim V_{x b}$, for $x = c$ and $u$, and all charged leptons $l_i$ (since $V_{x b}$ are measured in decays to all lepton flavours).

3.3 Tau decays

Constraints on New Physics from rare tau decays, including loop processes, have been studied in [58, 19, 56]. Here we consider tree level $\tau$ decays, such as the lepton flavour violating $\tau \rightarrow l M$, where $l$ is $e$ or $\mu$ and $M$ is a meson lighter than the tau ($\pi$ and $K$). The quark matrix elements and kinematic factors are estimated by assuming that the decay rate was the appropriate $|\epsilon|^2 \times$ the measured charged current SM process. For instance, a constraint can be obtained from the experimental upper bound on $\tau \rightarrow \mu \pi^0$, by comparing it to $\tau \rightarrow \nu_\tau \pi^-$. This means that the mass of final state leptons is neglected, and the matrix elements are assumed to satisfy [28]:

$$\langle 0 | \bar{u} \gamma^\mu \gamma^5 u | \pi^0 \rangle \approx \langle 0 | \bar{u} \gamma^\mu \gamma^5 d | \pi^- \rangle$$

(27)

$$\langle 0 | \bar{d} \gamma^\mu \gamma^5 s | K^0 \rangle \approx \langle 0 | \bar{u} \gamma^\mu \gamma^5 s | K^- \rangle$$

(28)

4 Other Processes

4.1 Contact Interactions and Atomic Parity Violation

The non-observation of “contact interactions” at colliders implies bounds on flavour diagonal four fermion operators. The constraints are quoted as lower bounds on a scale $\Lambda$ (in TeV), which, in the case where two of the fermions are electrons, is defined via the Lagrangian

$$L = \frac{4\pi}{\Lambda^2} \left[ \eta_{LL} (\bar{e} \gamma^\mu P_L e)(\bar{\psi} \gamma^\mu P_L \psi) + \eta_{RR}(\bar{e} \gamma^\mu P_R e)(\bar{\psi} \gamma^\mu P_R \psi) + \eta_{LR}(\bar{e} \gamma^\mu P_L e)(\bar{\psi} \gamma^\mu P_R \psi) + \eta_{RL}(\bar{e} \gamma^\mu P_R e)(\bar{\psi} \gamma^\mu P_L \psi) \right]$$

(29)

where the $\eta$s are $\pm 1$ or 0, and $\psi$ can be any of $\{\mu, \tau, u, d, s, c, b\}$. Bounds are quoted on the $\Lambda$s with the notation $\Lambda^+_{LL} \equiv \Lambda$ for $\{\eta_{LL}, \eta_{RR}, \eta_{LR}, \eta_{RL}\} = (\pm 1, 0, 0, 0)$.

The CCFR neutrino experiment [6], which scattered mostly muon neutrinos off nuclei, sets bounds on operators of the form $(\bar{\nu}_\mu \gamma^\mu \nu_\mu)(\bar{q}_i \gamma_q q_i)$, for $q_i$ a first generation quark. The most recent bounds that we found

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5We here follow the notation of [6], who put the electron current before the quark in the four fermion interaction; notice that the Tevatron experiments invert the order, so that $\Lambda_{LR}$ from LEP is $\Lambda_{RL}$ from the Tevatron.
on operators of the form $(\bar{\mu}\gamma\mu)(\bar{q}_i\gamma q_j)$, are from D0 \cite{8}. For operators of the form $(\bar{e}\gamma e)(\bar{s}\gamma s)$ and $(\bar{e}\gamma e)(\bar{c}\gamma c)$, we use the bounds from LEP \cite{6}, and for first generation quark operators $(\bar{e}\gamma e)(\bar{q}_i\gamma q_j)$ we follow \cite{24} in combining the bounds from LEP \cite{6}, the Tevatron \cite{8} HERA \cite{10} and Atomic Parity Violation. The LEP bounds assume the contact interaction is simultaneously present for all the light quarks $\{u,d,s\}$. Since we wish to constrain the four fermion interactions “one quark flavour at a time”, we divide the LEP $\Lambda$s by $\sqrt{3}$ to obtain our bounds. The D0 \cite{8} and HERA \cite{10} bounds assume the same contact interaction of the leptons with $u$ and $d$ quarks. So for up quarks, we use the given bounds on $\Lambda$, and for $d$ quarks, we estimate $\Lambda \sim \frac{1}{3} \Lambda_{D0,HERA}$, and $\Lambda \sim \frac{1}{\sqrt{3}} \Lambda_{D0,HERA}$ for $d_L$ quarks.

 Atomic Parity Violation (APV) experiments\cite{62,61,11} measure the weak charge of nuclei:

$$Q_W = -2 \left[ C_1^{eu}(2Z + N) + C_1^{ed}(Z + 2N) \right]$$

(30)

where $C_1^{eq}$’s is the dimensionless coefficient of the chiral four fermion interaction $-\sqrt{2}G_F(\bar{\psi}\gamma\psi)(\bar{q}\gamma q)$, so can be replaced by any of our $q$. Experiments with cesium \cite{61} agree with the SM to within $\sim 1\sigma$, and give a data-theory uncertainty \cite{11} of $\Delta(C_1^{eu} + C_1^{ed}) \approx 0.0008 \pm 0.0013$, $\Delta(C_1^{eu} - C_1^{ed}) \approx 0.0015 \pm 0.0015$. Since we wish to set bounds separately on the coefficients of four fermion interactions involving $u$ or $d$, we take the $(2\sigma)$ bound from APV to be $|\epsilon| < .03$.

As there are several experiments which set bounds of similar magnitude on some of the first generation contact interactions, the combined bound is more restrictive. Such a combination was performed several years ago by Cheung\cite{24}, and despite the improvements in the data, the bounds of \cite{24} are still the most restrictive. Instead, for $u$ and $d$ quarks separately, we combine the limits from the four experiments as

$$\epsilon_{com} = \frac{1}{w} \sum_x \frac{\epsilon_x}{\sigma_x^2} \pm \sqrt{\frac{1}{w}}$$

(31)

where $\epsilon_x \pm \sigma_x$ is the 95\% CL range allowed by an experiment, and $w = \sum_x 1/\sigma_x^2$. The central values $\epsilon_{com}$ are $< \sqrt{1/w} \sim .01$, so they are all approximated as zero. The resulting limits are listed in the tables.

4.2 Decays of the Z

New Physics which induces two lepton two quark operators at tree level, can also mediate penguin/dipole operators at one loop. In general, we avoid bounds from processes where the New Physics contributes in loops, because they are more sensitive to the New Physics details. However, in this section, we estimate bounds from the $Z$ vertex, because these are the only constraints we could find on certain higher generation operators.

We estimate the contribution of the contact interactions to the two-fermion-$Z$ boson operators by closing two fermion legs, and attaching a gauge boson (see figure 11). Notice however, that this contact interaction part may not be the full contribution: in the case of leptoquarks \cite{13}, there are two diagrams in the full theory, with the gauge boson attached to either the leptoquark or the fermion. Only the fermion diagram is included in the effective theory estimate we will make here. We make these estimates for the $Z$ vertex, rather than the photon, because closing the quark legs of the $V \pm A$ operators of eqns (31), and attaching a photon, gives no contribution to the lepton-leptoquark-photon dipole. This is an artifact of our effective field theory approach and operator basis choice, since there can be bounds \cite{24,58} from dipole operators $(\mu \to e\gamma, b \to s\gamma)$ on leptoquarks and $Z'$s.

Four fermion operators of the form $(\bar{q}_k\gamma^\alpha P_q q_k)(\bar{l}_l\gamma^\alpha P_l l_j)$, where $l_i$ and $l_j$ are charged leptons, can contribute at one-loop to the decays $Z \to l_i l_j$ via the diagram of figure 11. For $i = j$, the effective operator would interfere with the SM amplitude, and contribute linearly to \cite{11}

$$R_{ij} = \frac{\Gamma(Z \to \text{hadrons})}{\Gamma(Z \to l_i l_j)} = \begin{cases} 20.804 \pm .050 & l = e \\ 20.785 \pm .033 & l = \mu \\ 20.764 \pm .045 & l = \tau \end{cases}$$

(32)

\footnote{In the case where the coefficients of the operators are flavour independent, constraints have been calculated more recently in \cite{60}. This analysis allows the coefficients of 21 dimension six operators to be simultaneously non-zero.}
For $i \neq j$, the loop amplitude squared would induce lepton flavour changing $Z$ decays, whose branching ratios are bounded

$$BR(Z \to l_i \bar{l}_j) < \begin{cases} 1.7 \times 10^{-6} & ij = e\mu \\ 9.8 \times 10^{-6} & ij = e\tau \\ 1.2 \times 10^{-5} & ij = \mu\tau \end{cases}$$

These data gives bounds of order $\epsilon \lesssim 1$, which can be interesting for higher generation quarks and/or leptons.

Figure 1: One loop contribution to $Z \to l_i \bar{l}_j$ from an effective vertex $2\sqrt{2}e^{ijk}G_F (\bar{q}_k \gamma^\alpha P Q q_k)(\bar{l}_i \gamma^\alpha P Z l_j)$. $l_i$ are charged leptons. Combined with the LEP1 Z data, such a diagram can set bounds on operators involving two top quarks, and/or two $\tau$s, and/or charged leptons of different flavour.

If the $Z$ coupling to fermions $f_j$ is written

$$\mathcal{L} = -\frac{e}{2s_W c_W} T_j c^\alpha (g_L^i P_L + g_R^i P_R) f_j Z_{\rho}$$

then the loop of figure 1 modifies the couplings $g_{Z}^i$ by adding a contribution $\delta g_{Z}^{ij}$, where the subscript $Z = L, R$ is the chirality of the final state leptons. Formulae for the vacuum polarisation loop can be found, for instance, in chapter 21 of [59]. An operator $2\sqrt{2}e^{ijk}G_F (\bar{q}_k \gamma^\alpha P Q q_k)(\bar{l}_i \gamma^\alpha P Z l_j)$ gives

$$\delta g_{Z}^{ij} \approx \frac{N_c 2\sqrt{2}e^{ijk}G_F}{8\pi^2} \left\{ \begin{array}{l} g_Q^k \frac{m_{t}^2}{m_{Q}^2} \ln \left( \frac{m_{t}^2}{m_{Q}^2} \right) \quad \text{massless } q_k \\ g_L^i m_t^2 \ln \left( \frac{m_{t}^2}{m_{Q}^2} \right) \quad q_k = t \end{array} \right. \quad (35)$$

where $Q = L, R$ is the chirality of the quark in the loop. For light quarks, this gives $\delta g \sim 6 \times 10^{-3}$. Recall that $g_L = \pm 1 - 2s_W^2 Q_{em}$, where $Q_{em}$ is the electric charge of the quark, and $g_R = 2s_W Q_{em}$. For a top quark in the loop, the $Z$ can always couple to the $t_L$ with $g_L \approx 2/3$, because in the case where $t_R$ participates in the four fermion vertex, the chirality flip can be provided by $m_t$ without suppressing the loop.

To obtain the bounds in the tables, we require for $i = j$

$$\frac{2g_Q^j \delta g_{Z}^{jj}}{(g_L^j)^2 + (g_R^j)^2} < 2 \frac{\delta R_{l_j}}{R_{l_j}}$$

(36)

(at “two $\sigma$”), and and for $i \neq j$

$$\frac{1}{2.5GeV} \frac{\sqrt{2}G_F m_{t}^3}{6\pi} (\delta g^{ij})^2 < BR(Z \to l_i \bar{l}_j)$$

(37)

### 4.3 Neglected Constraints

The constraint arising from $\mu - e$ conversion [63] is reproduced from [15], because the experimental value [12] has not changed. Constraints can also be obtained from other processes, such as loop contributions to $g-2$ [64] and neutrino interactions [65, 66, 67]. We neglect bounds from oblique parameters [68], because these depend on the leptoquark couplings to the Higgs, which we do not consider.
We quote bounds on four fermion interactions of chiral fields. Bounds on the coefficients of gauge invariant operators an be extracted by expanding the operators as a sum of four fermion interactions, then identifying from the tables the most restrictive bound on the various four fermion interactions. In the table 1 are listed the tables whose bounds apply to the coefficients of the gauge invariant operators of eqns 3 to 5.

Table 2 contains bounds which apply to $V \pm A$ generation-off-diagonal interactions involving charged leptons and ups of any chirality: $((\bar{l}_i \gamma^\mu P_{L,R} l_j)(\bar{u}_k \gamma^\mu P_{L,R} u_n))$. Table 3 is the same for interactions involving $d$-type quarks: $((\bar{l}_i \gamma^\mu P_{L,R} l_j)(\bar{d}_k \gamma^\mu P_{L,R} d_n))$. Notice that $u$ and $d$ can be SU(2) singlets or doublets. Table 14 gives bounds which apply to all the $S_{\pm}P$ operators. Many of the bounds in tables 2, 3 and 14 arise from leptonic meson decays. In tables 2, 3 and 14, all the flavour off-diagonal index combinations are listed. In the case where the bound depends on the chirality of the operator (e.g. $Z \rightarrow l_i \bar{l}_j$), the process is listed, but the bounds are in the chirality-specific tables 4, 5, 7, 6, 8, 9, 10, 11, and 12, 13.

In the tables 4, 5, 7, 6, 8, 9, 10, 11 and 13, are listed all the generation diagonal coefficients $\epsilon^{ijk}_{kk}$, and other chirality dependent bounds, again on the coefficients of four fermion interactions, rather than the coefficients of gauge invariant operators. Tables 4 to 11 involve two charged leptons, whereas tables 12 and 13 constrain respectively interactions containing $(\bar{\nu} \gamma \nu)$, and charged current interactions. The bounds on four fermion interactions involving $(\bar{\nu} \gamma \nu)$ are incomplete (see, e.g. [69] for a more complete analysis), because we are interested in dimension six contact interactions. At dimension six, any gauge invariant operator inducing $(\bar{\nu} \gamma \nu)(f \gamma f)$, where $f \in \{q, u, d\}$ also induces charged current, or charged lepton neutral current interactions, whose coefficients are usually more strictly constrained. So table 12 only contains a few interesting constraints.

The first column of each table gives the generation indices $ijkn$ of the operator. The bounds are ordered from first to third generation in each index, so they start with $ijkn = 1111$, then 1112 and so on to 3333. The second column is the numerical bound on $\epsilon^{ijkn}$, defined in eqn (12), arising from the observation listed in the third column. In the last column, we give the numerical value of the data we used; in the cases where the bound can be expressed $|\epsilon|^2 < BR_{\text{exp}} \times \text{constant}$, then this allows to rescale the bounds on $\epsilon$ when the data improves.

5 The tables

All the four fermion interactions except those of table 13 are hermitian, so all bounds apply under simultaneous permutation of lepton and quark indices. In many cases, the experimental bounds apply under permutation on one pair of indices; for instance, the bound from $D^+ \rightarrow \pi^+ e^\pm \mu^\pm$ applies to $\epsilon^{\mu\nu\mu\nu}$ and $\epsilon^{\nu\mu\nu\mu}$. We assume that bounds on the decays $D^+ \rightarrow X$ and $B^+ \rightarrow X$ apply also to the CP conjugate processes $D^- \rightarrow \bar{X}$ and $B^- \rightarrow \bar{X}$.

Several operators involving a top quark are unconstrained, although they would contribute to tree level top decay (Therefore our tables are often missing the rows corresponding to top quarks.). One could hope for a rough bound $\epsilon < V_{tb}$ from top data at the Tevatron. However, we did not find a Tevatron analysis of $t$ decay via a four-fermion operator, where the kinematics will be different from the expected $t \rightarrow Wb$. 

| Operator | tab 2 | tab 3 | tab 4 | tab 5 | tab 6 | tab 7 | tab 8 | tab 9 | tab 10 | tab 11 | tab 12 | tab 13 |
|----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| $O^{(1)}_{\ell q}$ | x     | x     | x     |       |       |       |       |       |       |       |       |       |
| $O^{(3)}_{\ell q}$ | x     | x     | x     |       |       |       |       |       |       |       |       |       |
| $O_{ed}$         | x     | x     | x     |       |       |       |       |       |       |       |       |       |
| $O_{eu}$         | x     | x     | x     |       |       |       |       |       |       |       |       |       |
| $O_{ld}$         | x     | x     | x     |       |       |       |       |       |       |       |       |       |
| $O_{lu}$         | x     | x     | x     |       |       |       |       |       |       |       |       |       |

Table 1: Operators whose coefficients are constrained by the bounds in tables 2 to 13.
| $\langle \bar{e}_i \gamma^\mu P_{L,R} e_j \rangle \langle \bar{d}_k \gamma^\mu P_{L,R} d_n \rangle$ | Constraint on $\epsilon^{ijkl}$ | Observable | Experimental value |
|---|---|---|---|
| $eed_5$ | $5.7 \times 10^{-3}$ | $BR(K_S^0 \to ee)$ | $9.0 \times 10^{-12}$ |
| $eedb$ | $2.0 \times 10^{-4}$ | $BR(B^+ \to \pi^0 \mu^+ \mu^-)$ | $< 1.8 \times 10^{-7}$ |
| $eessb$ | $1.8 \times 10^{-4}$ | $BR(B^+ \to K^0 \pi^\mp)$ | $< 1.34 \times 10^{-7}$ |
| $e\mu d_5$ | $8.5 \times 10^{-7}$ | $\mu - e$ conversion on Ti | $< 4.3 \times 10^{-12}$ |
| $e\mu d_5$ | $3.0 \times 10^{-7}$ | $BR(K_S^0 \to e\mu)$ | $< 4.7 \times 10^{-12}$ |
| $e\mu d_5$ | $2.0 \times 10^{-4}$ | $BR(B^+ \to \pi^0 \mu^+ \mu^-)$ | $< 1.7 \times 10^{-7}$ |
| $e\mu s_5$ | $8 \times 10^{-5}$ | $BR(B^+ \to K^0 \pi^\mp)$ | $< 1.34 \times 10^{-7}$ |
| $e\mu s_5$ | $8 \times 10^{-5}$ | $BR(B^+ \to \pi^0 \mu^+ \mu^-)$ | $< 9.1 \times 10^{-8}$ |
| $e\tau d_5$ | $8.4 \times 10^{-4}$ | $BR(\tau^+ \to \pi^0 \pi^- \mu^-)$ | $< 10.91 \times 10^{-8}$ |
| $e\tau d_5$ | $4.9 \times 10^{-4}$ | $BR(\tau^+ \to \pi^0 \pi^- \pi^-)$ | $< 8 \times 10^{-8}$ |
| $e\tau d_5$ | $4.1 \times 10^{-3}$ | $BR(B^+ \to \pi^0 \mu^+ \mu^-)$ | $< 1.1 \times 10^{-8}$ |
| $e\tau s_5$ | | $Z \to e\tau$ | |
| $e\tau b$ | | $Z \to e\tau$ | |
| $\mu d_5$ | $7.8 \times 10^{-6}$ | $BR(K_S^0 \to \mu^+ \mu^-)$ | $6.84 \times 10^{-9}$ |
| $\mu d_5$ | $1.3 \times 10^{-4}$ | $BR(B^+ \to \pi^0 \mu^+ \mu^-)$ | $< 6.9 \times 10^{-8}$ |
| $\mu b_5$ | $7.0 \times 10^{-4}$ | $\Gamma(B_s \to \mu^+ \mu^-)$ | $< 1.3 \times 10^{-5}$ |
| $\mu d_5$ | $9.8 \times 10^{-4}$ | $\Gamma(B_s \to \mu^+ \mu^-)$ | $< 1.1 \times 10^{-7}$ |
| $\mu d_5$ | $5.4 \times 10^{-4}$ | $BR(\tau^+ \to \mu^+ \mu^-)$ | $< 0.10 \times 10^{-8}$ |
| $\mu d_5$ | $2.1 \times 10^{-2}$ | $BR(B^+ \to \mu^+ \mu^-)$ | $< 2.2 \times 10^{-3}$ |
| $\mu s_5$ | | $Z \to \mu\tau$ | |
| $\mu s_5$ | | $Z \to \mu\tau$ | |
| $\mu b_5$ | | $Z \to \mu\tau$ | |
| $\tau d_5$ | | $Z \to \mu\tau$ | |
| $\tau d_5$ | | $Z \to \mu\tau$ | |
| $\tau d_5$ | | $Z \to \mu\tau$ | |
| $\tau s_5$ | | $Z \to \mu\tau$ | |
| $\tau s_5$ | | $Z \to \mu\tau$ | |

Table 2: Constraints on the dimensionless coefficient $\epsilon^{ijkl}$, of the four-fermion interaction $2\sqrt{2}G_F \langle \bar{e}_i \gamma^\mu P_{L,R} e_j \rangle \langle \bar{d}_k \gamma^\mu P_{L,R} d_n \rangle$. $P_{L,R}$ can be $P_L$ or $P_R$. The bounds collected in this table are flavour-changing, and apply to many of the operators; see the table 1. The generation indices $ijkl$ are given in the first column, and the best bound in column two. It arises from the observable of column 3, and the experimental value we used is given in column 4. All bounds apply under permutation of the lepton and/or quark indices.
| $(\bar{e}i\gamma^{i}P_{L,R}e_j)(\bar{u}k\gamma_{\mu}P_{L,R}u_n)$ | Constraint on $\epsilon^{ijkn}$ | Observable | Experimental value |
|--------------------------------|------------------------|-----------|-------------------|
| $eeuc$ | $7.9 \times 10^{-3}$ | $BR(D^+\rightarrow\pi^+e\bar{e}) / BR(D^0\rightarrow\pi^-e\bar{e})$ | $< 7.4 \times 10^{-6}$ / $2.83 \times 10^{-5}$ |
| $eekt$ | $eeut$ | $eect$ | $0.092$ | $Z \rightarrow \bar{e}e$ | $BR < 1.7 \times 10^{-6}$ |
| $f\mu\nu u$ | $8.5 \times 10^{-7}$ | $\mu - e$ conversion on Ti | $\sigma(\mu^{-}\rightarrow e^{-}\nu_{\mu}\tau^{+}) / \sigma(\mu^{-}\rightarrow e^{-}\nu_{\mu}) < 4.3 \times 10^{-12}$ |
| $e\mu\nu u$ | $1.7 \times 10^{-2}$ | $BR(D^+\rightarrow\pi^+\mu\bar{\nu}_{\mu}) / BR(D^0\rightarrow\pi^-\mu\bar{\nu}_{\mu})$ | $< 3.4 \times 10^{-5}$ / $2.83 \times 10^{-5}$ |
| $e\nu uu$ | $8.4 \times 10^{-4}$ | $BR(\tau^{+}\rightarrow\pi^+\nu_{\tau}) / BR(\tau^{-}\rightarrow\pi^-\nu_{\tau})$ | $< 8 \times 10^{-7}$ |
| $e\nu uu$ | $e\nu uu$ | $e\nu uu$ | $0.2$ | $Z \rightarrow \bar{e}\tau$ | $BR < 9.8 \times 10^{-6}$ |
| $\mu\mu uu$ | $6.1 \times 10^{-3}$ | $BR(D^+\rightarrow\pi^+\bar{\mu}\mu) / BR(D^0\rightarrow\pi^-\bar{\mu}\mu)$ | $< 3.9 \times 10^{-6}$ / $2.83 \times 10^{-5}$ |
| $\mu\mu uu$ | $\mu\mu uu$ | $\mu\mu uu$ | $0.061$ | $Z \rightarrow \bar{\mu}\mu$ | $R_{\mu} = 20.785 \pm 0.033$ |
| $\mu\tau uu$ | $9.8 \times 10^{-4}$ | $BR(\tau^{+}\rightarrow\pi^+\bar{\mu}) / BR(\tau^{-}\rightarrow\pi^-\bar{\mu})$ | $< 1.1 \times 10^{-7}$ / $10.91 \times 10^{-8}$ |
| $\mu\tau uu$ | $\mu\tau uu$ | $\mu\tau uu$ | $1$ | $Z \rightarrow \tau\bar{\mu}$ | $BR < 12 \times 10^{-6}$ |
| $\mu\tau uu$ | $\mu\tau uu$ | $\mu\tau uu$ | $0.086$ | $Z \rightarrow \tau\bar{\tau}$ | $R_{\tau} = 20.764 \pm 0.045$ |

Table 3: Flavour-changing constraints on the dimensionless coefficient $\epsilon^{ijkn}$, of the four-fermion interaction $2\sqrt{2}GF_{\epsilon} (\bar{e}i\gamma^{i}P_{L,R}e_j)(\bar{u}k\gamma_{\mu}P_{L,R}u_n)$. $P_{L,R}$ can be $P_L$ or $P_R$. The bounds collected in this table apply to many of the operators; see the table. The generation indices $ijkn$ are given in the first column, and the best bound in column two. It arises from the observable of column 3, and the experimental value we used is given in column 4. All bounds apply under permutation of lepton and quark indices.
| $({\bar{e}}i\gamma^\mu P_L e_j) (\bar{u}_k\gamma^\mu P_L u_n)$ | Constraint on $c_{ijkn}^{\epsilon_{(n)/q}}$ | Observable | Experimental value |
|-----------------------------------------------|-----------------------------------------------|-------------|-------------------|
| $eeuu$                                        | $> 1 \times 10^{-2}$                          | $\Lambda_{eeuu,LL}^+$                          | see sec 4.1       |
| $eecc$                                        | $< 1 \times 10^{-2}$                          | $\Lambda_{eecc,LL}^+$                          | $> 4.4, 5.6$ TeV  |
| $eett$                                        | $0.092$                                       | $Z \rightarrow e\bar{e}$                      | $R_e = 20.804 \pm 0.050$ |
| $eucc$                                        | $0.6$                                         | $Z \rightarrow \bar{e}\mu$                    | $BR < 1.7 \times 10^{-6}$ |
| $etcc$                                        | $1.4$                                         | $Z \rightarrow \bar{e}\tau$                   | $BR < 9.8 \times 10^{-6}$ |
| $\mu\muuu$                                   | $>-2.0 \times 10^{-2}$                       | $\Lambda_{\mu\muuu,LL}^+$                     | $> 2.9$ TeV       |
| $\muucc$                                      | $0.4$                                         | $Z \rightarrow \mu\bar{\mu}$                 | $R_\mu < 20.785 \pm 0.033$ |
| $\mu\mu\tau\tau$                             | $0.54$                                       | $Z \rightarrow \tau\tau$                     | $R_\tau < 20.765 \pm 0.045$ |
| $\mu\mu\tau\tau$                             | $0.54$                                       | $Z \rightarrow \tau\tau$                     | $R_\tau < 20.765 \pm 0.045$ |

Table 4: Flavour diagonal constraints on the dimensionless coefficient $c_{ijkn}^{\epsilon_{(n)/q}}$, of the four-fermion interaction $2\sqrt{G_F} (\bar{e}_i\gamma^\mu P_L e_j) (\bar{u}_k\gamma^\mu P_L u_n)$. The indices $ijkn$ are given in the left column, the best calculated constraints are in column 2, arising the observable of column 3, with the experimental value in column 4. All bounds apply under permutation of lepton and/or quark indices.

| $({\bar{e}}i\gamma^\mu P_R e_j) (\bar{u}_k\gamma^\mu P_R u_n)$ | Constraint on $c_{ijkn}^{\epsilon_{(n)/q}}$ | Observable | Experimental value |
|-----------------------------------------------|-----------------------------------------------|-------------|-------------------|
| $eeuu$                                        | $1 \times 10^{-2}$                           | $\Lambda_{eeuu,RR}^+$                          | see sec 4.1       |
| $eecc$                                        | $>-9 \times 10^{-2}$                         | $\Lambda_{eecc,RR}^+$                          | $> 1.5$ TeV       |
| $eett$                                        | $< 1.8 \times 10^{-2}$                       | $\Lambda_{eett,RR}^-$                          | $> 3.8$ TeV       |
| $eucc$                                        | $0.092$                                       | $Z \rightarrow e\bar{e}$                      | $R_e < 20.804 \pm 0.050$ |
| $etcc$                                        | $1.2$                                         | $Z \rightarrow e\mu$                          | $BR < 1.7 \times 10^{-6}$ |
| $\mu\muuu$                                   | $2.8$                                         | $Z \rightarrow \mu\tau$                      | $BR < 9.8 \times 10^{-6}$ |
| $\muucc$                                      | $0.79$                                        | $Z \rightarrow \mu\bar{\mu}$                 | $R_\mu < 20.785 \pm 0.033$ |
| $\mu\mu\tau\tau$                             | $0.061$                                       | $Z \rightarrow \mu\bar{\mu}$                 | $R_\mu < 20.785 \pm 0.033$ |
| $\mu\mu\tau\tau$                             | $3$                                           | $Z \rightarrow \mu\tau$                      | $BR < 12 \times 10^{-6}$ |
| $\tau\tau\tau\tau$                           | $1.1$                                         | $Z \rightarrow \tau\tau$                     | $R_\tau = 20.764 \pm 0.045$ |
| $\tau\tau\tau\tau$                           | $1.1$                                         | $Z \rightarrow \tau\tau$                     | $R_\tau = 20.764 \pm 0.045$ |
| $\tau\tau\tau\tau$                           | $0.086$                                       | $Z \rightarrow \tau\tau$                     | $R_\tau = 20.764 \pm 0.045$ |

Table 5: Flavour diagonal constraints on the dimensionless coefficient $c_{ijkn}^{\epsilon_{(n)/q}}$, of the four-fermion interaction $2\sqrt{G_F} (\bar{e}_i\gamma^\mu P_R e_j) (\bar{u}_k\gamma^\mu P_R u_n)$. The indices $ijkn$ are given in the left column, the best calculated constraints are in column 2, arising the observable of column 3, with the experimental value in column 4. All bounds apply under permutation of lepton and/or quark indices.
| $(\bar{e}_i \gamma^\mu P_{Re_j})(\bar{u}_k \gamma^\mu P_{Lu_n})$ | Constraint on $\epsilon_{eq}^{ijkn}$ | Observable | Experimental value |
|---|---|---|---|
| $eeuu$ | $1.2 \times 10^{-2}$ | $\Delta_{eeuuLR}^+$ | see sec 4.1 |
| $eecc$ | $4.2 \times 10^{-2}$ | $\Delta_{eeecLR}^-$ | $\gtrsim 3$ TeV |
| $eett$ | 0.092 | $Z \rightarrow ee$ | $BR < 20 \pm 0.08$ |
| $ejcc$ | 0.6 | $Z \rightarrow e\mu$ | $BR < 1.7 \times 10^{-6}$ |
| $etcc$ | 1.4 | $Z \rightarrow e\tau$ | $BR < 9.8 \times 10^{-6}$ |
| $\mu\muuu$ | $< 1.5 \times 10^{-2}$ | $\Lambda_{\mu\muuu, RL}^+$ | $\gtrsim 5.2$ TeV |
| $\mu\mucc$ | 0.40 | $Z \rightarrow \mu\mu$ | $R_\mu < 20.785 \pm 0.033$ |
| $\mu\muut$ | 0.061 | $Z \rightarrow \mu\tau$ | $BR < 12 \times 10^{-6}$ |
| $\mu\taucc$ | 1.6 | $Z \rightarrow \mu\tau$ | $BR < 12 \times 10^{-6}$ |
| $\tau\tauuu$ | 0.54 | $Z \rightarrow \tau\tau$ | $R_\tau = 20.764 \pm 0.045$ |
| $\tau\taucc$ | 0.54 | $Z \rightarrow \tau\tau$ | $R_\tau = 20.764 \pm 0.045$ |
| $\tau\tault$ | 0.086 | $Z \rightarrow \tau\tau$ | $R_\tau = 20.764 \pm 0.045$ |

Table 6: Flavour diagonal constraints on the dimensionless coefficient $\epsilon_{ijkn}$ of the four-fermion interaction $2\sqrt{G_F} (\bar{e}_i \gamma^\mu P_{Re_j})(\bar{u}_k \gamma^\mu P_{Lu_n})$. The indices $ijkn$ are given in the left column, the best calculated constraints are in column 2, arising the observable of column 3, with the experimental value in column 4. All bounds apply under permutation of lepton and/or quark indices.

| $(\bar{e}_i \gamma^\mu P_{Le_j})(\bar{u}_k \gamma^\mu P_{Lu_n})$ | Constraint on $\epsilon_{eq}^{ijkn}$ | Observable | Experimental value |
|---|---|---|---|
| $eeuu$ | $1.4 \times 10^{-2}$ | $\Delta_{eeuuLR}^+$ | see sec 4.1 |
| $eecc$ | $> -8.6 \times 10^{-2}$ | $\Lambda_{eeecLR}^+$ | $> 2.1$ TeV |
| | $< 2.6 \times 10^{-2}$ | $\Lambda_{eeecLR}^-$ | $> 3.4$ TeV |
| $eett$ | 0.092 | $Z \rightarrow ee$ | $R_\tau < 20.804 \pm 0.050$ |
| $ejcc$ | 2.4 | $Z \rightarrow e\mu$ | $BR < 1.7 \times 10^{-6}$ |
| $etcc$ | 2.8 | $Z \rightarrow e\tau$ | $BR < 9.8 \times 10^{-6}$ |
| $\mu\muuu$ | $1.4 \times 10^{-2}$ | $\Lambda_{\mu\muuu, LR}^+$ | $> 5.2$ TeV |
| $\mu\mucc$ | 0.79 | $Z \rightarrow \mu\mu$ | $R_\mu < 20.785 \pm 0.033$ |
| $\mu\muut$ | 0.061 | $Z \rightarrow \mu\tau$ | $BR < 12 \times 10^{-6}$ |
| $\mu\taucc$ | 3 | $Z \rightarrow \mu\tau$ | $BR < 12 \times 10^{-6}$ |
| $\tau\tauuu$ | 1.1 | $Z \rightarrow \tau\tau$ | $R_\tau = 20.764 \pm 0.045$ |
| $\tau\taucc$ | 1.1 | $Z \rightarrow \tau\tau$ | $R_\tau < 20.764 \pm 0.045$ |
| $\tau\tault$ | 0.086 | $Z \rightarrow \tau\tau$ | $R_\tau = 20.764 \pm 0.045$ |

Table 7: Flavour diagonal constraints on the dimensionless coefficient $\epsilon_{ijkn}$ of the four-fermion interaction $2\sqrt{G_F} (\bar{e}_i \gamma^\mu P_{Le_j})(\bar{u}_k \gamma^\mu P_{Lu_n})$. The indices $ijkn$ are given in the left column, the best calculated constraints are in column 2, arising the observable of column 3, with the experimental value in column 4. All bounds apply under permutation of lepton and/or quark indices.
(e_i\gamma^\mu P_L e_j) (\bar{d}_k\gamma^\mu P_L d_n) \) | Constraint on $\epsilon_{ijkn}^{(n)}$ | Observable | Experimental value
--- | --- | --- | ---
$e e d d$ | $1.3 \times 10^{-2}$ | $\Lambda_{e e d d}^{LL}$ | see sec 4.1
$e e s s$ | $1.5 \times 10^{-2}$ | $\Lambda_{e e s s}^{LL}$ | $> 9.7, 8.0$ TeV
$e e b b$ | $>-4.3 \times 10^{-3}$ | $\Lambda_{e e b b}^{LL}$ | $> 9.4$ TeV
$< 1.5 \times 10^{-2}$ | $\Lambda_{e e b b}^{LLL}$ | $> 4.9$ TeV
$e \mu s s$ | $0.5$ | $Z \rightarrow e\mu$ | $BR < 1.7 \times 10^{-6}$
$e \mu b b$ | $0.5$ | $Z \rightarrow e\mu$ | $BR < 1.7 \times 10^{-6}$
$e \tau s s$ | $1$ | $Z \rightarrow e\tau$ | $BR < 9.8 \times 10^{-6}$
$e \tau b b$ | $1$ | $Z \rightarrow e\tau$ | $BR < 9.8 \times 10^{-6}$
$\mu \mu d d$ | $>-6.4 \times 10^{-2}$ | $\Lambda_{\mu \mu d d}^{LL}$ | $> 4.2$ TeV
$< 2.3 \times 10^{-2}$ | $\Lambda_{\mu \mu d d}^{LLL}$ | $> 7.0$ TeV
$\mu \mu s s$ | $0.32$ | $Z \rightarrow \mu\mu$ | $R_\mu < 20.785 \pm .033$
$\mu \mu b b$ | $0.32$ | $Z \rightarrow \mu\mu$ | $R_\mu < 20.785 \pm .033$
$\mu \tau s s$ | $1$ | $Z \rightarrow \mu\tau$ | $BR < 12 \times 10^{-6}$
$\mu \tau b b$ | $1$ | $Z \rightarrow \mu\tau$ | $BR < 12 \times 10^{-6}$
$\tau \tau d d$ | $0.43$ | $Z \rightarrow \tau\tau$ | $R_\tau = 20.764 \pm .045$
$\tau \tau s s$ | $0.43$ | $Z \rightarrow \tau\tau$ | $R_\tau = 20.764 \pm .045$
$\tau \tau b b$ | $0.43$ | $Z \rightarrow \tau\tau$ | $R_\tau = 20.764 \pm .045$

Table 8: Flavour diagonal constraints on the dimensionless coefficient $\epsilon_{ijkn}^{(n)}$ of the four-fermion interaction $2\sqrt{2}G_F (e_i\gamma^\mu P_L e_j) (\bar{d}_k\gamma^\mu P_L d_n)$. The indices $ijkn$ are given in the left column, the best calculated constraints are in column 2, arising the observable of column 3, with the experimental value in column 4. All bounds apply under permutation of lepton and/or quark indices.
Table 9: Flavour diagonal constraints on on the dimensionless coefficient $\epsilon^{ijkn}_{cd}$, of the four-fermion interaction $2\sqrt{2}G_F (\bar{e}_i\gamma^\mu P_L e_j) (\bar{d}_k\gamma_\mu P_L d_n)$. The indices $ijkn$ are given in the left column, the best calculated constraints are in column 2, arising the observable of column 3, with the experimental value in column 4. All bounds apply under permutation of lepton and/or quark indices.
| \((\bar{e}_i \gamma^\mu P_L e_j) (\bar{d}_k \gamma^\mu P_R d_n)\) | Constraint on \(\phi_{ij}^{eq}\) | Observable | Experimental value |
|------------------|------------------|---------|------------------|
| \((e\bar{e}dd)\) | \(2.7 \times 10^{-2}\) | \(\Lambda_{e\bar{e}dd,LR}^+\) | see sec 4.1 |
| \((e\bar{e}ss)\) | \(> -6.8 \times 10^{-2}\) | \(\Lambda_{e\bar{e}ss,LR}^+\) | 4.1 TeV |
| \((e\bar{e}bb)\) | \(< 4.3 \times 10^{-2}\) | \(\Lambda_{e\bar{e}bb,LR}^-\) | 5.2 TeV |
| \((e\mu ss)\) | \(> -2.5 \times 10^{-2}\) | \(\Lambda_{e\mu ss,LR}^+\) | 3.9 TeV |
| \((e\mu bb)\) | \(< 5.0 \times 10^{-2}\) | \(\Lambda_{e\mu bb,LR}^-\) | 2.8 TeV |
| \((e\tau ss)\) | \(6\) | \(Z \to \bar{e}\tau\) | \(BR < 9.8 \times 10^{-6}\) |
| \((e\mu dd)\) | \(1.2 \times 10^{-1}\) | \(\Lambda_{e\mu dd,LR}^-\) | 5.2 TeV |
| \((\mu\mu ss)\) | \(1.6\) | \(Z \to \mu\mu\) | \(R_{\mu} < 20.785 \pm 0.033\) |
| \((\mu\mu bb)\) | \(1.6\) | \(Z \to \mu\mu\) | \(R_{\mu} < 20.785 \pm 0.033\) |
| \((\mu\tau ss)\) | \(6\) | \(Z \to \mu\tau\) | \(BR < 12 \times 10^{-6}\) |
| \((\mu\tau bb)\) | \(6\) | \(Z \to \mu\tau\) | \(BR < 12 \times 10^{-6}\) |
| \((\tau\tau dd)\) | \(2.2\) | \(Z \to \tau\tau\) | \(R_{\tau} = 20.764 \pm 0.045\) |
| \((\tau\tau ss)\) | \(2.2\) | \(Z \to \tau\tau\) | \(R_{\tau} = 20.764 \pm 0.045\) |
| \((\tau\tau bb)\) | \(2.2\) | \(Z \to \tau\tau\) | \(R_{\tau} = 20.764 \pm 0.045\) |

Table 10: Flavour diagonal constraints on the dimensionless coefficient \(\phi_{ij}^{eq}\), of the four-fermion interaction \(2\sqrt{2}G_F (\bar{e}_i \gamma^\mu P_L e_j) (\bar{d}_k \gamma^\mu P_R d_n)\). The indices \(ij\) are given in the left column, the best calculated constraints are in column 2, arising the observable of column 3, with the experimental value in column 4. All bounds apply under lepton and/or quark index permutation.

| \((\bar{e}_i \gamma^\mu P_R e_j) (\bar{d}_k \gamma^\mu P_L d_n)\) | Constraint on \(\phi_{ij}^{eq}\) | Observable | Experimental value |
|------------------|------------------|---------|------------------|
| \((e\bar{e}dd)\) | \(2.2 \times 10^{-2}\) | \(\Lambda_{e\bar{e}dd,RL}^\pm\) | see sec 4.1 |
| \((e\bar{e}ss)\) | \(> -8.0 \times 10^{-2}\) | \(\Lambda_{e\bar{e}ss,RL}^+\) | 3.8 TeV |
| \((e\bar{e}bb)\) | \(< 3.2 \times 10^{-2}\) | \(\Lambda_{e\bar{e}bb,RL}^-\) | 6.0 TeV |
| \((e\mu ss)\) | \(> -6.6 \times 10^{-2}\) | \(\Lambda_{e\mu ss,RL}^+\) | 2.4 TeV |
| \((e\mu bb)\) | \(< 1.8 \times 10^{-2}\) | \(\Lambda_{e\mu bb,RL}^-\) | 4.6 TeV |
| \((e\tau ss)\) | \(0.5\) | \(Z \to \bar{e}\mu\) | \(BR < 1.7 \times 10^{-6}\) |
| \((e\tau bb)\) | \(0.5\) | \(Z \to \bar{e}\tau\) | \(BR < 9.8 \times 10^{-6}\) |
| \((e\mu dd)\) | \(1\) | \(Z \to \bar{e}\tau\) | \(BR < 9.8 \times 10^{-6}\) |
| \((\mu\mu ss)\) | \(1.2 \times 10^{-1}\) | \(\Lambda_{\mu\mu ss,RL}^\pm\) | 5.2 TeV |
| \((\mu\mu bb)\) | \(0.32\) | \(Z \to \mu\mu\) | \(R_{\mu} < 20.785 \pm 0.033\) |
| \((\mu\tau ss)\) | \(0.32\) | \(Z \to \mu\tau\) | \(BR < 12 \times 10^{-6}\) |
| \((\mu\tau bb)\) | \(1.8\) | \(Z \to \mu\tau\) | \(BR < 12 \times 10^{-6}\) |
| \((\tau\tau dd)\) | \(0.43\) | \(Z \to \tau\tau\) | \(R_{\tau} = 20.764 \pm 0.045\) |
| \((\tau\tau ss)\) | \(0.43\) | \(Z \to \tau\tau\) | \(R_{\tau} = 20.764 \pm 0.045\) |
| \((\tau\tau bb)\) | \(0.43\) | \(Z \to \tau\tau\) | \(R_{\tau} = 20.764 \pm 0.045\) |

Table 11: Flavour diagonal constraints on the dimensionless coefficient \(\phi_{ij}^{eq}\), of the four-fermion interaction \(2\sqrt{2}G_F (\bar{e}_i \gamma^\mu P_R e_j) (\bar{d}_k \gamma^\mu P_L d_n)\). The indices \(ij\) are given in the left column, the best calculated constraints are in column 2, arising the observable of column 3, with the experimental value in column 4. All bounds apply under lepton and/or quark index permutation.
| $(\bar{\nu}_i \gamma^\mu P_L \nu_j)$ | Constraint on $\epsilon^{ijkn}$ | Observable | Experimental value |
|-----------------------------------|-------------------------------|-------------|-------------------|
| $\nu_\mu \nu_\mu q_1 q_1$        | $7.3 \times 10^{-3}$          | $\Delta_{\nu_{\mu}\nu_{\mu};L_L}$ | $> 5.1 \text{ TeV}$ |
| $\nu_\mu \nu_\mu d s$            | $9.4 \times 10^{-6}$          | $BR(K \rightarrow \pi^+ \pi^0 \nu)$ | $1.8 \times 10^{-9}$ |
| $\nu_\mu \nu_\mu b d$            | $4.9 \times 10^{-2}$          | $BR(B^+ \rightarrow \pi^0 \pi^+ \nu)$ | $< 1.0 \times 10^{-4}$ |
| $\nu_\mu \nu_\mu b s$            | $1.0 \times 10^{-3}$          | $BR(B^0 \rightarrow \pi^+ \pi^+ \nu)$ | $< 1.4 \times 10^{-8}$ |

Table 12: Some constraints on the dimensionless coefficient $\epsilon^{ijkn}$, of the four-fermion interaction $2\sqrt{2}G_F (\bar{\nu}_i \gamma^\mu P_L \nu_j) (\bar{q}_k \gamma_\mu P_L q_n)$. The indices $ijkn$ are given in the left column, where is is specified whether $q$ can be a $u$-type or $d$-type quark. $q_1$ is a $u$ or $d$. See [69] for a more complete list of constraints; here are listed only those which can give more restrictive constraints than charged lepton interactions on the coefficients of gauge-invariant dimension six operators.

| $(\bar{\nu}_i \gamma^\mu P_L e_j)$ | Constraint on $\epsilon^{ijkn}$ | Observable | Experimental value |
|-----------------------------------|-------------------------------|-------------|-------------------|
| $\nu_e e d u$                     | $1.0 \times 10^{-3}$          | unitarity $\delta V_{us} = 0.019$ | $\delta V_{us} = 0.019$ |
| $\nu_e e d u$                     | $4.5 \times 10^{-2}$          | unitarity $\delta V_{cd} = 0.049$ | $\delta V_{cd} = 0.024$ |
| $\nu_e e d c$                     | $4.8 \times 10^{-2}$          | unitarity $\delta V_{cd} = 0.024$ | $\delta V_{cd} = 0.024$ |
| $\nu_e e d c$                     | $1.5 \times 10^{-4}$          | unitarity $\delta V_{cd} = 0.019$ | $\delta V_{cd} = 0.019$ |
| $\nu_e e s u$                     | $4.0 \times 10^{-3}$          | unitarity $\delta V_{us} = 0.06$ | $\delta V_{us} = 0.06$ |
| $\nu_e e s u$                     | $3.0 \times 10^{-2}$          | unitarity $\delta V_{cb} = 0.06$ | $\delta V_{cb} = 0.06$ |
| $\nu_e e s c$                     | $1.2 \times 10^{-1}$          | unitarity $\delta V_{cs} = 0.06$ | $\delta V_{cs} = 0.06$ |
| $\nu_e e s c$                     | $2.4 \times 10^{-1}$          | unitarity $\delta V_{cs} = 0.06$ | $\delta V_{cs} = 0.06$ |
| $\nu_e e b u$                     | $3.9 \times 10^{-3}$          | $\lesssim V_{ub} < 3.93 \times 10^{-3}$ | $< 3.93 \times 10^{-3}$ |
| $\nu_e e b c$                     | $4.1 \times 10^{-2}$          | $\lesssim V_{cb} < 4.12 \times 10^{-2}$ | $< 4.12 \times 10^{-2}$ |
| $\nu_\mu \mu d u$                | $4.0 \times 10^{-3}$          | $R_c = (1.230 \pm 0.004) \times 10^{-4}$ | $R_c = (1.230 \pm 0.004) \times 10^{-4}$ |
| $\nu_\mu \mu d u$                | $9.0 \times 10^{-2}$          | $R_c = (1.230 \pm 0.004) \times 10^{-4}$ | $R_c = (1.230 \pm 0.004) \times 10^{-4}$ |
| $\nu_\mu \mu d c$                | $2.2 \times 10^{-2}$          | unitarity $\delta V_{cd} = 0.011$ | $\delta V_{cd} = 0.011$ |
| $\nu_\mu \mu d c$                | $1.0 \times 10^{-1}$          | unitarity $\delta V_{cd} = 0.011$ | $\delta V_{cd} = 0.011$ |
| $\nu_\mu \mu s u$                | $4.0 \times 10^{-3}$          | unitarity $\delta V_{us} = 0.022$ | $\delta V_{us} = 0.022$ |
| $\nu_\mu \mu s u$                | $3.0 \times 10^{-2}$          | unitarity $\delta V_{us} = 0.022$ | $\delta V_{us} = 0.022$ |
| $\nu_\mu \mu s c$                | $1.2 \times 10^{-1}$          | unitarity $\delta V_{cs} = 0.06$ | $\delta V_{cs} = 0.06$ |
| $\nu_\mu \mu s c$                | $2.4 \times 10^{-1}$          | unitarity $\delta V_{cs} = 0.06$ | $\delta V_{cs} = 0.06$ |
| $\nu_\mu \mu b u$                | $3.9 \times 10^{-3}$          | $\lesssim V_{ub} < 3.93 \times 10^{-3}$ | $< 3.93 \times 10^{-3}$ |
| $\nu_\mu \mu b c$                | $4.1 \times 10^{-2}$          | $\lesssim V_{cb} < 4.12 \times 10^{-2}$ | $< 4.12 \times 10^{-2}$ |
| $\nu_\tau \tau d u$              | $1.0 \times 10^{-2}$          | $\frac{g_T}{g_\mu} \bar{\mu}$ | $0.996 \pm 0.005$ |
| $\nu_\tau \tau d u$              | $2.0 \times 10^{-1}$          | $\frac{g_T}{g_\mu} \bar{\mu}$ | $0.996 \pm 0.005$ |
| $\nu_\tau \tau d c$              | $1.5 \times 10^{-1}$          | $\lesssim V_{cd} = 0.23$ | $V_{cd} = 0.23$ |
| $\nu_\tau \tau s u$              | $6.0 \times 10^{-3}$          | unitarity $\delta V_{us} = 0.027$ | $\delta V_{us} = 0.027$ |
| $\nu_\tau \tau s u$              | $7.7 \times 10^{-2}$          | unitarity $\delta V_{us} = 0.027$ | $\delta V_{us} = 0.027$ |
| $\nu_\tau \tau s c$              | $1.2 \times 10^{-1}$          | unitarity $\delta V_{cs} = 0.06$ | $\delta V_{cs} = 0.06$ |
| $\nu_\tau \tau s c$              | $2.4 \times 10^{-1}$          | unitarity $\delta V_{cs} = 0.06$ | $\delta V_{cs} = 0.06$ |
| $\nu_\tau \tau b u$              | $3.9 \times 10^{-3}$          | $\lesssim V_{ub} < 3.93 \times 10^{-3}$ | $< 3.93 \times 10^{-3}$ |
| $\nu_\tau \tau b c$              | $4.1 \times 10^{-2}$          | $\lesssim V_{cb} < 4.12 \times 10^{-2}$ | $< 4.12 \times 10^{-2}$ |
| $\bar{\nu}_i e_j$ | Constraint on $\epsilon_{ijkn}$ | Observable | Experimental value |
|----------------|--------------------------------|------------|-------------------|
| $\nu_{e,edu}$ | $6.3 \times 10^{-4}$ | $R_e$ | $(1.230 \pm 0.004) \times 10^{-4}$ |
| $\nu_{e,edu}$ | $1.6 \times 10^{-3}$ | $R_e$ | $(1.230 \pm 0.004) \times 10^{-4}$ |
| $\nu_{e,dc}$ | $1.4 \times 10^{-3}$ | $\Gamma(D^+ \to \bar{e} \nu)$ | $< 8.8 \times 10^{-6}$ |
| $\nu_{e,sw}$ | $2.3 \times 10^{-6}$ | $BR(K^+ \to \bar{e} \nu)$ | $(1.55 \pm 0.07) \times 10^{-6}$ |
| $\nu_{e,sw}$ | $1.5 \times 10^{-3}$ | $BR(K^+ \to \bar{e} \nu)$ | $(1.55 \pm 0.07) \times 10^{-6}$ |
| $\nu_{e,sc}$ | $4.9 \times 10^{-3}$ | $BR(D^+_c \to \bar{e} \nu)$ | $< 1.3 \times 10^{-4}$ |
| $\nu_{e,ba}$ | $1.8 \times 10^{-3}$ | $BR(B^+_c \to \bar{e} \nu)$ | $< 5.2 \times 10^{-6}$ |
| $\nu_{e,bc}$ | $9.7 \times 10^{-4}$ | $BR(K^+ \to \bar{\mu} \nu)$ | $< 4.0 \times 10^{-3}$ |
| $\nu_{e,dud}$ | $1.3 \times 10^{-4}$ | $R_e$ | $(1.230 \pm 0.004) \times 10^{-4}$ |
| $\nu_{e,dud}$ | $3.2 \times 10^{-3}$ | $R_e$ | $(1.230 \pm 0.004) \times 10^{-4}$ |
| $\nu_{e,dcd}$ | $7.6 \times 10^{-4}$ | $BR(D^+ \to \bar{\mu} \nu)$ | $(3.82 \pm 0.33) \times 10^{-4}$ |
| $\nu_{e,dcd}$ | $3.7 \times 10^{-3}$ | $BR(D^+ \to \bar{\mu} \nu)$ | $(3.82 \pm 0.33) \times 10^{-4}$ |
| $\nu_{e,swu}$ | $2.4 \times 10^{-4}$ | $\frac{I_{ij}^{(20)}}{I_{i(20)}}$ for $K^+_{\mu3}$ | $0.2 \times 10^{-2}$ |
| $\nu_{e,swu}$ | $3.0 \times 10^{-3}$ | $R_K$ | $(2.44 \pm 0.11) \times 10^{-10}$ |
| $\nu_{e,scd}$ | $4.3 \times 10^{-3}$ | $\frac{BR(D^0 \to \tau e \nu)}{BR(D^0 \to \mu e \nu)}$ | $(11.0 \pm 1.4 \pm 0.6)$ |
| $\nu_{e,scd}$ | $1.7 \times 10^{-2}$ | $\frac{BR(D^0 \to \tau e \nu)}{BR(D^0 \to \mu e \nu)}$ | $(11.0 \pm 1.4 \pm 0.6)$ |
| $\nu_{e,bud}$ | $1.0 \times 10^{-4}$ | $BR(B^+ \to \bar{\mu} \nu)$ | $< 1.7 \times 10^{-6}$ |
| $\nu_{e,bcd}$ | $4.5 \times 10^{-3}$ | $BR(\tau \to \pi^+ \nu)$ | $(10.91 \pm 0.07) \times 10^{-2}$ |
| $\nu_{e,bud}$ | $8.0 \times 10^{-2}$ | $BR(\tau \to \pi^+ \nu)$ | $(10.91 \pm 0.07) \times 10^{-2}$ |
| $\nu_{e,bcd}$ | $1.5 \times 10^{-1}$ | $BR(D^+ \to \tau \nu)$ | $< 1.2 \times 10^{-3}$ |
| $\nu_{e,bud}$ | $2.3 \times 10^{-2}$ | $BR(\tau \to K^+ \nu)$ | $(6.96 \pm 0.23) \times 10^{-4}$ |
| $\nu_{e,bud}$ | $2.2 \times 10^{-1}$ | $BR(\tau \to K^+ \nu)$ | $(6.96 \pm 0.23) \times 10^{-4}$ |
| $\nu_{e,bcd}$ | $7.2 \times 10^{-2}$ | $\frac{BR(D^0 \to \tau e \nu)}{BR(D^0 \to \mu e \nu)}$ | $(11.0 \pm 1.4 \pm 0.6)$ |
| $\nu_{e,bcd}$ | $2.9 \times 10^{-1}$ | $\frac{BR(D^0 \to \tau e \nu)}{BR(D^0 \to \mu e \nu)}$ | $(11.0 \pm 1.4 \pm 0.6)$ |
| $\nu_{e,bud}$ | $8.2 \times 10^{-4}$ | $BR(B^+ \to \tau e \nu)$ | $(1.4 \pm 0.4) \times 10^{-4}$ |
| $\nu_{e,bcd}$ | $3.1 \times 10^{-4}$ | $BR(B^+ \to \tau e \nu)$ | $(1.4 \pm 0.4) \times 10^{-4}$ |

Table 14: Constraints from “charged current” processes on $S \pm A$ operators. These apply to $\epsilon_{ijkn}$ and $\epsilon_{ijkn}^\dagger$. The first column is the index combination $ijkn$, the second is the constraints, which arise from the observable given in column 3. The experimental value used is the last column. $\nu_i$ is any flavour of neutrino.

| $(\bar{\nu}_i e_j) (\bar{q}_k u_n)$ | Constraint on $\epsilon_{ijkn}^{\dagger}$ | Observable | Experimental value |
|----------------|--------------------------------|------------|-------------------|
| $eeuc$ | $7.6 \times 10^{-4}$ | $BR(D^0 \to \bar{e} e)$ | $< 1.2 \times 10^{-6}$ |
| $\mu euc$ | $6.3 \times 10^{-4}$ | $BR(D^0 \to \bar{\mu} e)$ | $< 8.1 \times 10^{-7}$ |
| $\mu uuc$ | $7.9 \times 10^{-4}$ | $BR(D^0 \to \bar{\mu} \mu)$ | $< 1.3 \times 10^{-6}$ |

Table 15: Constraints on $\epsilon_{ijkn}^{\dagger}$ for the $ijkn$ index combination given in the first column. The bounds given in table 13 also apply. The second column is the constraint, which arises from the observable given in column 3. The experimental value used is the last column. These bounds are also valid under lepton and/or quark index permutation.
Table 16: Constraints on $\epsilon_{ijkl}^{qde}$ for the $ijkl$ index combination given in the first column. The bounds given in Table 14 also apply. The second column is the constraint, which arises from the observable given in column 3. The experimental value used is the last column. These bounds are also valid under lepton and/or quark index permutation.

| $(\ell_i e_j) (\bar{d}_k q_n)$ | Constraint on $\epsilon_{ijkl}^{qde}$ | Observable | Experimental value |
|--------------------------------|-------------------------------------|-------------|--------------------|
| $eeds$                         | $2.1 \times 10^{-8}$               | $BR(K_\ell^0 \to \bar{\ell}e)$ | $9.0 \times 10^{-12}$ |
| $eedb$                         | $3.2 \times 10^{-8}$               | $BR(B^0 \to \bar{\ell}e)$      | $< 1.13 \times 10^{-7}$ |
| $eesb$                         | $5.6 \times 10^{-4}$               | $BR(B^0 \to \bar{\ell}e)$      | $< 5.4 \times 10^{-5}$ |
| $\mu eds$                      | $9.0 \times 10^{-9}$               | $BR(K_\mu^0 \to \bar{\mu}e)$    | $< 4.7 \times 10^{-12}$ |
| $\mu edb$                      | $2.9 \times 10^{-6}$               | $BR(B^0 \to \bar{\mu}e)$      | $< 9.2 \times 10^{-8}$  |
| $\mu esb$                      | $6.1 \times 10^{-4} \times 10^{-3}$ | $BR(B^0 \to \bar{\mu}e)$      | $< 6.1 \times 10^{-6}$  |
| $e\tau ds$                     | $1.1 \times 10^{-3}$               | $BR(B^0 \to \bar{\ell}\tau)$   | $< 1.1 \times 10^{-4}$  |
| $e\tau db$                     | $5.9 \times 10^{-7}$               | $BR(K_\tau^0 \to \bar{\tau}e)$ | $6.84 \times 10^{-9}$   |
| $\mu \mu ds$                   | $1.2 \times 10^{-10}$              | $BR(B^0 \to \bar{\tau}e)$      | $< 1.5 \times 10^{-8}$  |
| $\mu \mu db$                   | $1.7 \times 10^{-7}$               | $BR(B^0 \to \bar{\tau}e)$      | $< 4.7 \times 10^{-8}$  |
| $\mu \mu ds$                   | $6.6 \times 10^{-4}$               | $BR(B^0 \to \bar{\mu}\tau)$    | $< 3.8 \times 10^{-5}$  |
| $\mu \tau ds$                  |                                     |                                        |                     |
| $\mu \tau sb$                  |                                     |                                        |                     |
| $\tau \tau ds$                 |                                     |                                        |                     |
| $\tau \tau db$                 | $8.0 \times 10^{-3}$               | $BR(B^0 \to \bar{\tau}\tau)$    | $< 4.1 \times 10^{-3}$  |
6 Expectations for flavour structure

In this section, we aim to make “motivated” guesses for the flavour structure of the two-quark, two lepton operator coefficients. We prefer not to use the predictions of Minimal Flavour Violation (MFV) [1,2] for two reasons: from a phenomenological perspective, there is not a unique extension to the lepton sector [70], and in a more model-dependent approach, defining MFV for New Physics such as leptoquarks is even more ambiguous [71].

Instead, we consider an alternative to MFV, which provides almost enough suppression of Flavour Changing Neutral Currents (FCNC) in the quark sector. Following Cheng and Sher [38], we assume NP couplings have a flavour hierarchy patterned on the Yukawa couplings: $\epsilon_{ij} \sim \sqrt{y_i y_j y_k y_n}$ where $y_i$ is the Yukawa couplings of fermion $i$. This can arise in models with extra dimensions [72], or can be described in 4 dimensions by inverse hierarchies of the $Z$ coefficients of the fermion kinetic terms (the hierarchy being imprinted on all interactions when the kinetic terms are canonically normalised [73]).

The Yukawa couplings of $u, c, t$ are $y_{u,c,t} = \frac{g_{W}}{2m_{W}}$, and for charged lepton and $d$-type quarks, a factor of $\tan \beta$ is included:

$$y_{d,s,b} = \tan \beta \frac{g_{md,s,b}}{2m_{W}}.$$ 

We include the rescaling parameter $\tan \beta$ to allow for the possibility that the underlying theory of flavour physics gives hierarchies of flavoured couplings. In this perspective, the largest eigenvalue of all matrices of flavoured couplings might be $\sim 1$, with the magnitude of dimensionful parameters like masses controlled by some other physics (such as Higgs vevs).

To obtain a flavour structure for the coefficients of two-lepton two quark operators, we assume that the scale of new physics is

$$\frac{1}{m_{N}^{2}} \approx \frac{4G_{F}}{\sqrt{2}}$$

and that the coefficient of an operator containing $u_{i}$ (or $d_{i}$, $e_{i}$), will contain a factor $\sqrt{y_{ui}}$ (or $\sqrt{y_{di}}, \sqrt{y_{ei}}$).

For the doublets, we assume that $q_{i}$ comes with a factor $\sqrt{y_{qi}}$ (because its larger), and $\ell_{i}$ with a factor $\sqrt{y_{ei}}$ (because we do not consider neutrino masses). This suggests a hierarchy of order

$$\epsilon_{ij}^{(1)} \ell_{q} \epsilon_{ij}^{(3)} \ell_{q} \epsilon_{ij}^{(1)} \ell_{u} \epsilon_{ij}^{(3)} \ell_{u} \epsilon_{ij}^{(1)} \ell_{q} \epsilon_{ij}^{(3)} \ell_{q} \epsilon_{ij}^{(1)} \ell_{u} \epsilon_{ij}^{(3)} \ell_{u} \epsilon_{ij}^{(1)} \ell_{q} \epsilon_{ij}^{(3)} \ell_{q} \epsilon_{ij}^{(1)} \ell_{u} \epsilon_{ij}^{(3)} \ell_{u} \epsilon_{ij}^{(1)} \ell_{q} \epsilon_{ij}^{(3)} \ell_{q}$$

$$\sim \sqrt{y_{i} y_{j} y_{k} y_{n}}$$

for the coefficients of the operators listed in equations (3) to (6). Since $m_{b} < m_{c}, m_{s} < m_{c}$ and $m_{d} \sim 2m_{u}$, we replace $(y_{b} y_{d})^{2}/y_{b} y_{u} \rightarrow y_{b} y_{d}$ in eqn (39), and use the estimate to the right of the arrow. The experimental contraints are weaker than our guesses, even with this optimistic approximation.

In tables 17 and table 18, we respectively list, in the first column, all the flavour index combinations $e_{i} e_{j} u_{k} u_{n}$ and $e_{i} e_{j} d_{k} d_{n}$. Then in the following columns we give the best bound we obtained for any chiral structure, the expected value of $\epsilon_{ij}^{(1)}$ for $\tan \beta = 1$, and the observable from which the bound is obtained. The bounds which arise from charged current processes are specifically labelled, because it is less clear whether the “expectation” would be $\propto \sqrt{y_{i} y_{j} y_{k} y_{n}}$ or $\propto \sqrt{y_{i} y_{j} y_{k} y_{n}}$.

The tables show that in most cases, for $\tan \beta = 1$, our naive expectations are far below the experimental sensitivity of the processes we have considered. (Recall, however, that we neglected most loop processes. There would be additional constraints, for instance from meson-anti-meson mixing recently studied by [44], on any new particles inducing our operators.) In bold face, we draw attention to some kaon observables, where the current bounds are close to our guesses. This suggests that more sensitive rare Kaon experiments could be a good place to look for two lepton, two quark operators.

7 Conclusion

We have compiled flavour dependent bounds on effective interactions between two leptons and two quarks, which could be induced by $SU(3) \times SU(2) \times U(1)$ invariant dimension six operators. The constraints are listed in tables 24-16 with the rows labelled by the fermion generations. The bounds are set assuming that only
| $ijkn$ | Constraint on $\epsilon_{ijkn}$ | Expectation | Observable |
|--------|----------------------------------|-------------|------------|
| $\nu_e \nu_e u_d$ | $1 \times 10^{-2}$ | $5 \times 10^{-11}$ | $\Lambda_{\nu_e \nu_e u_d}^{LL}$ |
| $\nu_e \nu_e u_d$ | $6 \times 10^{-7}$ | | |
| $\nu_e \nu_e u_s$ | $2 \times 10^{-4}$ | $1 \times 10^{-9}$ | $\text{BR}(D^0 \to \nu_e \nu_e)$ |
| $\nu_e \nu_e u_b$ | $1 \times 10^{-4}$ | $2 \times 10^{-8}$ | $\text{BR}(B^+ \to \nu_e \nu_e)$ |
| $\nu_e \nu_e u_c$ | $1 \times 10^{-2}$ | $2 \times 10^{-8}$ | $\Lambda_{\nu_e \nu_e u_c}^{LL}$ |
| $\nu_e \nu_e u_t$ | $9 \times 10^{-2}$ | $3 \times 10^{-6}$ | $Z \to \nu_e \nu_e$ |
| $\nu_e \nu_e u_d$ | $1.6 \times 10^{-5}$ | $3 \times 10^{-9}$ | $\text{BR}(\tau \to \pi^0 \nu_e \nu_e) / \text{BR}(\tau \to \pi^- \nu_e)$ |
| $\nu_e \nu_e u_s$ | $7 \times 10^{-2}$ | $2 \times 10^{-7}$ | $\text{BR}(B^+ \to \nu_e \nu_e)$ |
| $\nu_e \nu_e u_b$ | $8 \times 10^{-4}$ | $2 \times 10^{-6}$ | $Z \to \mu_e$ |
| $\nu_e \nu_e u_t$ | $1.8 \times 10^{-4}$ | $1 \times 10^{-5}$ | $\text{BR}(B^+ \to \nu_e \nu_e)$ |
| $\mu \mu u_t$ | $2 \times 10^{-4}$ | | |
| $\nu_e \nu_e u_s$ | $1.3 \times 10^{-4}$ | $2 \times 10^{-6}$ | $\text{BR}(D^0 \to \mu \mu)$ |
| $\nu_e \nu_e u_b$ | $1 \times 10^{-2}$ | $6 \times 10^{-10}$ | $\Lambda_{\mu \mu u_b}^{LR}$ |
| $\nu_e \nu_e u_c$ | $1 \times 10^{-2}$ | $2 \times 10^{-10}$ | $\text{BR}(D^0 \to \mu \mu)$ |
| $\nu_e \nu_e u_t$ | $1 \times 10^{-4}$ | $1 \times 10^{-7}$ | $\text{BR}(D^0 \to \mu \mu)$ |
| $\mu \mu u_t$ | $2 \times 10^{-4}$ | | |
| $\nu_e \nu_e u_s$ | $2 \times 10^{-4}$ | | |
| $\nu_e \nu_e u_b$ | $1 \times 10^{-2}$ | | |
| $\nu_e \nu_e u_c$ | $1 \times 10^{-2}$ | | |
| $\nu_e \nu_e u_t$ | $1 \times 10^{-2}$ | | |
| $\mu \mu u_t$ | $1 \times 10^{-2}$ | | |
| $\nu_e \nu_e u_s$ | $1 \times 10^{-2}$ | | |
| $\nu_e \nu_e u_b$ | $1 \times 10^{-2}$ | | |
| $\nu_e \nu_e u_c$ | $1 \times 10^{-2}$ | | |
| $\nu_e \nu_e u_t$ | $1 \times 10^{-2}$ | | |
| $\mu \mu u_t$ | $1 \times 10^{-2}$ | | |
| $\nu_e \nu_e u_s$ | $1 \times 10^{-2}$ | | |
| $\nu_e \nu_e u_b$ | $1 \times 10^{-2}$ | | |
| $\nu_e \nu_e u_c$ | $1 \times 10^{-2}$ | | |
| $\nu_e \nu_e u_t$ | $1 \times 10^{-2}$ | | |
| $\mu \mu u_t$ | $1 \times 10^{-2}$ | | |
| $\nu_e \nu_e u_s$ | $1 \times 10^{-2}$ | | |
| $\nu_e \nu_e u_b$ | $1 \times 10^{-2}$ | | |
| $\nu_e \nu_e u_c$ | $1 \times 10^{-2}$ | | |
| $\nu_e \nu_e u_t$ | $1 \times 10^{-2}$ | | |
| $\mu \mu u_t$ | $1 \times 10^{-2}$ | | |
| $\nu_e \nu_e u_s$ | $1 \times 10^{-2}$ | | |
| $\nu_e \nu_e u_b$ | $1 \times 10^{-2}$ | | |
| $\nu_e \nu_e u_c$ | $1 \times 10^{-2}$ | | |
| $\nu_e \nu_e u_t$ | $1 \times 10^{-2}$ | | |
| $\mu \mu u_t$ | $1 \times 10^{-2}$ | | |
| $\nu_e \nu_e u_s$ | $1 \times 10^{-2}$ | | |
| $\nu_e \nu_e u_b$ | $1 \times 10^{-2}$ | | |
| $\nu_e \nu_e u_c$ | $1 \times 10^{-2}$ | | |
| $\nu_e \nu_e u_t$ | $1 \times 10^{-2}$ | | |
| $\mu \mu u_t$ | $1 \times 10^{-2}$ | | |
| $\nu_e \nu_e u_s$ | $1 \times 10^{-2}$ | | |
| $\nu_e \nu_e u_b$ | $1 \times 10^{-2}$ | | |
| $\nu_e \nu_e u_c$ | $1 \times 10^{-2}$ | | |
| $\nu_e \nu_e u_t$ | $1 \times 10^{-2}$ | | |
| $\mu \mu u_t$ | $1 \times 10^{-2}$ | | |
| $\nu_e \nu_e u_s$ | $1 \times 10^{-2}$ | | |
| $\nu_e \nu_e u_b$ | $1 \times 10^{-2}$ | | |
| $\nu_e \nu_e u_c$ | $1 \times 10^{-2}$ | | |
| $\nu_e \nu_e u_t$ | $1 \times 10^{-2}$ | | |
| $\mu \mu u_t$ | $1 \times 10^{-2}$ | | |
| $\nu_e \nu_e u_s$ | $1 \times 10^{-2}$ | | |
| $\nu_e \nu_e u_b$ | $1 \times 10^{-2}$ | | |
| $\nu_e \nu_e u_c$ | $1 \times 10^{-2}$ | | |
| $\nu_e \nu_e u_t$ | $1 \times 10^{-2}$ | | |
| $\mu \mu u_t$ | $1 \times 10^{-2}$ | | |
| $\nu_e \nu_e u_s$ | $1 \times 10^{-2}$ | | |
| $\nu_e \nu_e u_b$ | $1 \times 10^{-2}$ | | |
| $\nu_e \nu_e u_c$ | $1 \times 10^{-2}$ | | |
| $\nu_e \nu_e u_t$ | $1 \times 10^{-2}$ | | |

Table 17: This table indicates the proximity of current experimental bounds to the hierarchical expectation: $\epsilon_{ijkn} \approx \sqrt{\beta \gamma}$ for $i, j, k, n$. The first column gives all the flavour index combinations $e_i e_j u_k u_n$. The second column is the best bound we obtained for any chiral structure, the third column is the expected value of $\epsilon_{ijkn}$, and the last is the observable from which the bound is obtained. Notice that $\epsilon \propto \tan \beta$, so it is straightforward to see which observables become interesting as $\tan \beta$ increases.
| $ijkn$ | Constraint on $\epsilon^{ijkn}$ | Expectation | Observable |
|-------|---------------------------------|-------------|------------|
| $eedd$ | $1.0 \times 10^{-2}$ | $9 \times 10^{-11}$ | $\Lambda_{eedd LL}$ |
| $eeds$ | $5.7 \times 10^{-3}$ | $3 \times 10^{-10}$ | $BR(K_L^0 \to \pi^+ \pi^- \mu^+ \mu^-)$ |
| $\nu_e \nu_e d_s$ | $9.4 \times 10^{-6}$ | $3 \times 10^{-10}$ | $BR(K^+ \to \pi^+ \pi^- \mu^+ \mu^-)$ |
| $eedb$ | $2.0 \times 10^{-4}$ | $3 \times 10^{-9}$ | $BR(B^+ \to D^{*+} \pi^0 \mu^+ \mu^-)$ |
| $eees$ | $1.5 \times 10^{-4}$ | $2 \times 10^{-9}$ | $\Lambda_{eees LL}$ |
| $eeeb$ | $1.8 \times 10^{-4}$ | $1 \times 10^{-8}$ | $BR(K^+ \to \pi^+ \pi^- \mu^+ \mu^-)$ |
| $eebb$ | $1.5 \times 10^{-2}$ | $8 \times 10^{-8}$ | $\Lambda_{eebb LL}$ |
| $emdd$ | $8.5 \times 10^{-7}$ | $1 \times 10^{-9}$ | $\mu \to e$ conversion |
| $emds$ | $9.0 \times 10^{-9}$ | $4 \times 10^{-9}$ | $BR(K_L^0 \to \mu e)$ |
| $emdb$ | $2.9 \times 10^{-8}$ | $4 \times 10^{-8}$ | $BR(B^+ \to \mu e)$ |
| $emss$ | $0.5$ | $2 \times 10^{-8}$ | $Z \to \mu \mu$ |
| $emsb$ | $8 \times 10^{-5}$ | $2 \times 10^{-7}$ | $BR(B^+ \to D^{*+} \mu^+ \mu^-)$ |
| $embb$ | $3$ | $1 \times 10^{-6}$ | $Z \to \mu \mu$ |
| $etdd$ | $8.4 \times 10^{-4}$ | $6 \times 10^{-9}$ | $BR(\pi^+ \to \pi^0 \pi^- \mu^+ \mu^-)$ |
| $etds$ | $4.9 \times 10^{-4}$ | $2 \times 10^{-8}$ | $BR(\tau^+ \to K^- \mu^+ \nu)$ |
| $\nu_e \nu_e \nu_e \nu_e$ | $9.4 \times 10^{-6}$ | $2 \times 10^{-8}$ | $BR(\tau^+ \to K^- \mu^+ \nu)$ |
| $etdb$ | $1.1 \times 10^{-4}$ | $2 \times 10^{-8}$ | $BR(B^+ \to \mu \tau)$ |
| $etss$ | $1$ | $1 \times 10^{-7}$ | $Z \to \tau \tau$ |
| $etsb$ | $1.0 \times 10^{-3}$ | $8 \times 10^{-7}$ | $BR(B^+ \to D^{*+} \mu^+ \mu^-)$ |
| $\nu_e \nu_e \nu_e \nu_e$ | $1$ | $5 \times 10^{-8}$ | $Z \to \tau \tau$ |
| $etbb$ | $3.2 \times 10^{-4}$ | $2 \times 10^{-8}$ | $R_K$ |
| $\mu dd$ | $4.3 \times 10^{-2}$ | $2 \times 10^{-8}$ | $\Lambda^+_\mu dd BB$ |
| $\mu ds$ | $5.9 \times 10^{-7}$ | $6 \times 10^{-8}$ | $BR(K_L^0 \to \mu \mu)$ |
| $\mu db$ | $1.2 \times 10^{-6}$ | $6 \times 10^{-7}$ | $BR(B^+ \to \mu \mu)$ |
| $\mu ss$ | $0.8$ | $4 \times 10^{-7}$ | $Z \to \mu \mu$ |
| $\mu sb$ | $3.0 \times 10^{-2}$ | $2 \times 10^{-6}$ | $BR(D^+_s \to \tau \mu)$ |
| $\mu bb$ | $1.7 \times 10^{-8}$ | $2 \times 10^{-6}$ | $BR(B^+_s \to \mu \mu)$ |
| $\mu \tau dd$ | $9.8 \times 10^{-4}$ | $6 \times 10^{-8}$ | $BR(\tau^+ \to K^- \pi^- \mu^+ \mu^-)$ |
| $\mu \tau ds$ | $5.4 \times 10^{-4}$ | $2 \times 10^{-7}$ | $BR(\tau^+ \to \mu K^+)$ |
| $\nu_e \nu_e \nu_e \nu_e$ | $9.4 \times 10^{-6}$ | $2 \times 10^{-6}$ | $BR(\tau^- \to \nu K^+)$ |
| $\mu \tau db$ | $6.6 \times 10^{-4}$ | $2 \times 10^{-6}$ | $BR(B^+ \to \mu \tau)$ |
| $\mu \tau ss$ | $1$ | $1 \times 10^{-6}$ | $Z \to \tau \mu$ |
| $\mu \tau sb$ | $4 \times 10^{-3}$ | $8 \times 10^{-6}$ | $BR(B^+ \to \mu \tau)$ |
| $\nu_e \nu_e \nu_e \nu_e$ | $1.0 \times 10^{-3}$ | $2 \times 10^{-6}$ | $BR(B^+ \to \mu \tau)$ |
| $\mu \tau bb$ | $1$ | $5 \times 10^{-9}$ | $Z \to \tau \tau$ |
| $\tau \tau dd$ | $0.8$ | $3 \times 10^{-7}$ | $Z \to \tau \tau$ |
| $\nu_e \nu_e \nu_e \nu_e$ | $9.4 \times 10^{-6}$ | $1 \times 10^{-5}$ | $BR(K_L^0 \to \pi^+ \pi^- \mu^+ \mu^-)$ |
| $\tau \tau db$ | $0.2$ | $1 \times 10^{-8}$ | $BR(B^+ \to \tau \tau)$ |
| $\tau \tau ss$ | $0.8$ | $6 \times 10^{-6}$ | $Z \to \tau \tau$ |
| $\tau \tau sb$ | $1.0 \times 10^{-3}$ | $4 \times 10^{-9}$ | $BR(B^+ \to K^+ \pi^- \mu^+ \mu^-)$ |
| $\tau \tau bb$ | $0.8$ | $3 \times 10^{-8}$ | $Z \to \tau \tau$ |

Table 18: This table indicates the proximity of current experimental bounds to the hierarchical “expectation” $\epsilon^{ijkn} \simeq \sqrt{V_{ij}^* V_{jk}^* V_{kn}^* V_{jn}^*}$. The first column gives all the flavour index combinations $e,e,d,d,n$. The following columns are the best bounds we obtained for any chiral structure, the expected value of $\epsilon^{ijkn}$ for $\tan \beta = 1$, and the observable of a given mode.

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one interaction is present at a time (so we neglect possible cancellations). For each possible combination of external leg flavours, the strongest bound is listed. Constraints were obtained from rare meson decays (leptonic or semi-leptonic), semi-leptonic tau decays, contact interactions at colliders, $Z$ decay data from LEP1, and $\mu - e$ conversion.

We also discussed, in section 6, a naive “guess” for the expected flavour structure of the operator coefficients. The expectations are always below the current experimental bounds (for $\tan \beta = 1$), as can be seen in tables 17 and 18. Some rare Kaon decay bounds are close to these “expectations”.

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References

[1] A. J. Buras, P. Gambino, M. Gorbahn, S. Jager and L. Silvestrini, Phys. Lett. B 500, 161 (2001) [arXiv:hep-ph/0007085]. R. S. Chivukula and H. Georgi, Phys. Lett. B 188 (1987) 99.

[2] G. D’Ambrosio, G. F. Giudice, G. Isidori and A. Strumia, Nucl. Phys. B 645 (2002) 155 [arXiv:hep-ph/0207036].

[3] See, for instance, http://flavlhc.web.cern.ch/flavlhc/.

[4] W. Buchmüller and D. Wyler, “Effective Lagrangian analysis of new Interactions and flavor conservation,” Nucl. Phys. B 268 (1986) 621.

[5] K. S. McFarland et al. [CCFR Collaboration and The E744 Collaboration and The E770 Collaboration], “A precision measurement of electroweak parameters in neutrino nucleon scattering,” Eur. Phys. J. C 1, 509 (1998) [arXiv:hep-ex/9701010].

[6] S. Schael et al. [ALEPH Collaboration], “Fermion pair production in $e^+e^-$ collisions at 189-209-GeV and constraints on physics beyond the standard model,” Eur. Phys. J. C 49 (2007) 411 [arXiv:hep-ex/0609051].

[7] F. Abe et al. [CDF Collaboration], “Limits on quark-lepton compositeness scales from dileptons produced in 1.8 TeV $p\bar{p}$ collisions,” Phys. Rev. Lett. 79, 2198 (1997).

[8] http://www-d0.fnal.gov/Run2Physics/WWW/results/np.htm, D0 note 4922-CONF, D0 note 4552-CONF.

[9] C. Adloff et al. [H1 Collaboration], “Search for new physics in $e^\pm q$ contact interactions at HERA,” Phys. Lett. B 568, 35 (2003) [arXiv:hep-ex/0305015].

S. Chekanov et al. [ZEUS Collaboration], “Search for contact interactions, large extra dimensions and finite quark radius in $e p$ collisions at HERA,” Phys. Lett. B 591, 23 (2004) [arXiv:hep-ex/0401009].

A. Raval [H1 Collaboration and ZEUS Collaboration], “Search for contact interactions at HERA,” arXiv:0810.1420 [hep-ex].

[10] R. Ciesielski [H1 and ZEUS Collaborations], “Search for leptoquarks and contact interactions at HERA,” PoS E-HEP2009 (2009) 269.

[11] C. Amsler et al. “The Review of Particle Physics”, Particle Data Book, Physics Letters B 667 (2008), and 2009 partial update for the 2010 edition.

[12] C. Dohmen et al. [SINDRUM II Collaboration], “Test Of Lepton Flavor Conservation In Mu → E Conversion On Titanium,” Phys. Lett. B 317 (1993) 631.

[13] G. Bhattacharyya, J. R. Ellis and K. Sridhar, “Bounds on the masses and couplings of leptoquarks from leptonic partial widths of the $Z$,” Phys. Lett. B 336 (1994) 100 [Erratum-ibid. B 338 (1994) 522] [arXiv:hep-ph/9406354].

J. K. Mizukoshi, O. J. P. Eboli and M. C. Gonzalez-Garcia, “Bounds on scalar leptoquarks from $Z$ physics,” Nucl. Phys. B 443 (1995) 20 [arXiv:hep-ph/9411392].

R. Benbrik and C. K. Chua, “Lepton Flavor Violating $l \rightarrow l'\gamma$ and $Z \rightarrow ll'$ Decays Induced by Scalar Leptoquarks,” Phys. Rev. D 78 (2008) 075025 [arXiv:0807.4240 [hep-ph]].

[14] V. D. Barger, K. m. Cheung, K. Hagiwara and D. Zeppenfeld, “Global study of electron quark contact interactions,” Phys. Rev. D 57 (1998) 391 [arXiv:hep-ph/9707412].

[15] M. Raidal et al., “Flavour physics of leptons and dipole moments,” Eur. Phys. J. C 57 (2008) 13 [arXiv:hep-ph/0801.1826].

[16] A. E. Nelson, “Contact terms, compositeness, and atomic parity violation,” Phys. Rev. Lett. 78 (1997) 4159 [arXiv:hep-ph/9703379].
[17] E. Eichten, K. D. Lane and M. E. Peskin, “New Tests For Quark And Lepton Substructure,” Phys. Rev. Lett. 50 (1983) 811.

[18] M. K. Gaillard and B. W. Lee, “Rare decay modes of the K-Mesons in gauge theories,” Phys. Rev. D 10 (1974) 897.

[19] A. Pich and J. P. Silva, “Constraining new interactions with leptonic \( \tau \) decays,” Phys. Rev. D 52, 4006 (1995) [arXiv:hep-ph/9505327].

[20] A. Ibarra, E. Masso and J. Redondo, “Systematic approach to gauge-invariant relations between lepton flavor violating processes,” Nucl. Phys. B 715 (2005) 523 [arXiv:hep-ph/0410386].

[21] R. J. Cashmore et al., “Exotic Phenomena In High-Energy EP Collisions,” Phys. Rept. 122 (1985) 275.

[22] W. Buchmuller and D. Wyler, “Constraints on the universal contact interaction,” Phys. Lett. B 407 (1997) 147 [arXiv:hep-ph/9704317].

[23] N. Di Bartolomeo and M. Fabbrichesi, “Four-fermion effective interactions and recent data at HERA,” Phys. Lett. B 406 (1997) 237 [arXiv:hep-ph/9703375].

[24] K. m. Cheung, “Constraints on electron quark contact interactions and implications to models of leptoquarks and extra Z bosons,” Phys. Lett. B 517 (2001) 167 [arXiv:hep-ph/0106251].

[25] A. F. Zarnecki, “Global analysis of \( eeqq \) contact interactions and future prospects for high-energy physics,” Eur. Phys. J. C 11 (1999) 539 [arXiv:hep-ph/9904334].

[26] W. Buchmüller, R. Rückl and D. Wyler, “Leptoquarks in lepton quark collisions,” Phys. Lett. B 191 (1987) 442 [Erratum-ibid. B 448 (1999) 320].

[27] W. Buchmüller and D. Wyler, “Constraints on SU(5) Type Leptoquarks,” Phys. Lett. B 177 (1986) 377.

[28] S. Davidson, D. C. Bailey and B. A. Campbell, “Model independent constraints on leptoquarks from rare processes,” Z. Phys. C 61 (1994) 613 [arXiv:hep-ph/9309310].

[29] M. Herz, “Bounds on leptoquark and supersymmetric, R-parity violating interactions from meson decays. (In German),” [arXiv:hep-ph/0301079]

[30] M. Leurer, “Bounds on vector leptoquarks,” Phys. Rev. D 50 (1994) 536 [arXiv:hep-ph/9312341].

[31] M. Leurer, “A Comprehensive study of leptoquark bounds,” Phys. Rev. D 49 (1994) 333 [arXiv:hep-ph/9309266].

[32] J. Blumlein, “On the expectations for leptoquarks in the mass range of O (200-GeV),” Z. Phys. C 74 (1997) 605 [arXiv:hep-ph/9703287].

[33] R. N. Cahn and H. Harari, “Bounds On The Masses Of Neutral Generation Changing Gauge Bosons,” Nucl. Phys. B 176 (1980) 135.

[34] E. Salvioni, A. Strumia, G. Villadoro and F. Zwirner, “Non-universal minimal Z’ models: present bounds and early LHC reach,” JHEP 1003 (2010) 010 [arXiv:0911.1450 [Unknown]].

[35] X. G. He and G. Valencia, “D - \( \bar{D} \) mixing constraints on FCNC with a non-universal Z’,” Phys. Lett. B 651 (2007) 135 [arXiv:hep-ph/0703270].

[36] T. G. Rizzo, “Z’ phenomenology and the LHC,” [arXiv:hep-ph/0610104]

[37] See for instance, [http://na62.web.cern.ch/NA62/](http://na62.web.cern.ch/NA62/).

[38] T. P. Cheng and M. Sher, “Mass Matrix Ansatz and Flavor Nonconservation in Models with Multiple Higgs Doublets,” Phys. Rev. D 35 (1987) 3484.

[39] T.P. Cheng and L.F. Li, Gauge theory of elementary particle physics, Oxford: Clarendon Press (Oxford Science Publications) (1989).
H. K. Dreiner, M. Kramer and B. O’Leary, “Bounds on R-parity violation from leptonic and semi-leptonic meson decays,” Phys. Rev. D 75 (2007) 114016 [arXiv:hep-ph/0612278].

H. K. Dreiner, G. Polesello and M. Thorweiser, “Bounds on broken R-parity from leptonic meson decays,” Phys. Rev. D 65 (2002) 115006 [arXiv:hep-ph/0112228].

A. Matsuzaki, “General Analysis of B Meson Decay into Two Fermions,” Prog. Theor. Phys. 123 (2010) 499 [arXiv:0904.4375 [hep-ph]].

A. D. Smirnov, Mod. Phys. Lett. A 22 (2007) 2353 [arXiv:0705.0308 [hep-ph]].

J. P. Saha, B. Misra and A. Kundu, “Constraining Scalar Leptoquarks from the K and B Sectors,” arXiv:1003.1384 [Unknown].

C. S. Kim, J. Lee and W. Namgung, “CP violation in the semileptonic B(14) (B → D π l μ) decays: Multi-Higgs doublet model and scalar-leptoquark models,” Phys. Rev. D 59 (1999) 114006 [arXiv:hep-ph/9811396].

A. K. Alok, A. Dighe, D. Ghosh, D. London, J. Matias, M. Nagashima and A. Szynkman, “New-physics contributions to the forward-backward asymmetry in B → K* l l Decays,” JHEP 1002 (2010) 053 [arXiv:0912.1382 [Unknown]].

C. Bobeth, G. Hiller and G. Piranishvili, “Angular Distributions of B → K ll Decays,” JHEP 0712 (2007) 040 [arXiv:0709.4174 [hep-ph]].

F. Mescia, C. Smith and S. Trine, “K( L) → π0 e+ e− and K( L) → π0 µ+ µ−: A binary star on the stage of flavor physics,” JHEP 0608 (2006) 088 [arXiv:hep-ph/0606081].

E. Golowich, J. Hewett, S. Pakvasa and A. A. Petrov, “Implications of D0 - D̄0 Mixing for New Physics,” Phys. Rev. D 76 (2007) 095009 [arXiv:0705.3650 [hep-ph]].

C. Bernard et al., “B and D meson decay constants,” PoS LATTICE2008 (2008) 278 [arXiv:0904.1895 [hep-lat]].

B. A. Dobrescu and A. S. Kronfeld, “Accumulating evidence for nonstandard leptonic decays of Ds mesons,” Phys. Rev. Lett. 100 (2008) 241802 [arXiv:0803.0512 [hep-ph]].

R. Benbrik and C. H. Chen, “Leptoquark on P → ℓ+ν, FCNC and LFV,” Phys. Lett. B 672 (2009) 172 [arXiv:0807.2373 [hep-ph]].

A. Bazavov et al. [Fermilab Lattice and MILC Collaborations], “The Ds and D+ Leptonic Decay Constants from Lattice QCD,” PoS LAT2009 (2009) 249 [arXiv:0912.5221 [Unknown]].

S. Fajfer and N. Kosnik, “Leptoquarks in FCNC charm decays,” Phys. Rev. D 79 (2009) 017502 [arXiv:0810.4858 [hep-ph]].

I. Dorsner, S. Fajfer, J. F. Kamenik and N. Kosnik, “Can scalar leptoquarks explain the fDs puzzle?,” Phys. Lett. B 682 (2009) 67 [arXiv:0906.5585 [hep-ph]].
[53] O. P. Yushchenko et al., “High statistic study of the $K^- \rightarrow \pi^0 \mu^- \nu$ decay,” Phys. Lett. B 581 (2004) 31 [arXiv:hep-ex/0312004].

[54] E. Gamiz, M. Jamin, A. Pich, J. Prades and F. Schwab, “Theoretical progress on the $V_{us}$ determination from tau decays,” PoS KAO (2008) 008 [arXiv:0709.0282 [hep-ph]].

[55] A. Pich, “Theoretical overview on tau physics,” Int. J. Mod. Phys. A 21 (2006) 5652 [arXiv:hep-ph/0609138].

[56] S. Kanemura, T. Ota and K. Tsumura, Phys. Rev. D 73 (2006) 016006 [arXiv:hep-ph/0505191].

[57] R. Benbrik and C. K. Chua, “Lepton Flavor Violating $l \rightarrow l' \gamma$ and $Z \rightarrow l\bar{l}'$ Decays Induced by Scalar Leptoquarks,” Phys. Rev. D 78 (2008) 075025 [arXiv:0807.4240 [hep-ph]].

[58] E. Gabrielli, “Model independent constraints on leptoquarks from MU and TAU lepton rare processes,” Phys. Rev. D 62 (2000) 055009 [arXiv:hep-ph/9911539].

[59] M. E. Peskin and D. V. Schroeder, “An Introduction To Quantum Field Theory,” Reading, USA: Addison-Wesley (1995) 842 p

[60] Z. Han and W. Skiba, “Effective theory analysis of precision electroweak data,” Phys. Rev. D 71 (2005) 075009 [arXiv:hep-ph/0412166].

[61] C. S. Wood, S. C. Bennett, D. Cho, B. P. Masterson, J. L. Roberts, C. E. Tanner and C. E. Wieman, “Measurement of parity nonconservation and an anapole moment in cesium,” Science 275 (1997) 1759.

[62] R. D. Young, R. D. Carlini, A. W. Thomas and J. Roche, “Testing the Standard Model by precision measurement of the weak charges of quarks,” Phys. Rev. Lett. 99 (2007) 122003 [arXiv:0704.2618 [hep-ph]].

[63] O. U. Shanker, “Z Dependence Of Coherent Mu E Conversion Rate In Anomalous Neutrinoless Muon Capture,” Phys. Rev. D 20 (1979) 1608.

[64] K. Huitu, J. Maalampi, M. Raidal and A. Santamaria, “New constraints on R-parity violation from mu e conversion in nuclei,” Phys. Lett. B 430 (1998) 355 [arXiv:hep-ph/9712249].

[65] K. m. Cheung, “Muon anomalous magnetic moment and leptoquark solutions,” Phys. Rev. D 64 (2001) 033001 [arXiv:hep-ph/0102238]. A. Czarnecki and W. J. Marciano, “The muon anomalous magnetic moment: A harbinger for ‘new physics’,” Phys. Rev. D 64 (2001) 013014 [arXiv:hep-ph/0102122]. G. Couture and H. Konig, “Bounds on second generation scalar leptoquarks from the anomalous magnetic moment of the muon,” Phys. Rev. D 53 (1996) 555 [arXiv:hep-ph/9507263].

[66] M. A. Doncheski and R. W. Robinett, “Leptoquark production in ultrahigh-energy neutrino interactions revisited,” Phys. Rev. D 56 (1997) 7412 [arXiv:hep-ph/9707328].

[67] L. A. Anchordoqui, C. A. Garcia Canal, H. Goldberg, D. G. Dunn and F. Halzen, “Probing leptoquark production at IceCube,” Phys. Rev. D 74 (2006) 125021 [arXiv:hep-ph/0609214].

[68] Y. Grossman, “Nonstandard Neutrino Interactions And Neutrino Oscillation Experiments,” Phys. Lett. B 359 (1995) 141 [arXiv:hep-ph/9507344].

[69] M. Honda, Y. Kao, N. Okamura, A. Pronin and T. Takeuchi, “Constraints on New Physics from Long Baseline Neutrino Oscillation Experiments,” arXiv:0707.4545 [hep-ph].
[69] C. Biggio, M. Blennow and E. Fernandez-Martinez, “General bounds on non-standard neutrino interactions,” JHEP 0908 (2009) 090 [arXiv:0907.0097 [hep-ph]].

[70] V. Cirigliano, B. Grinstein, G. Isidori and M. B. Wise, “Minimal flavor violation in the lepton sector,” Nucl. Phys. B 728 (2005) 121 [arXiv:hep-ph/0507001].

M. B. Gavela, T. Hambye, D. Hernandez and P. Hernandez, “Minimal Flavour Seesaw Models,” JHEP 0909 (2009) 038 [arXiv:0906.1461 [hep-ph]].

S. Davidson and F. Palorini, “Various definitions of minimal flavour violation for leptons,” Phys. Lett. B 642 (2006) 72 [arXiv:hep-ph/0607329].

T. Feldmann and T. Mannel, “Minimal Flavour Violation and Beyond,” JHEP 0702 (2007) 067 [arXiv:hep-ph/0611095].

[71] S Davidson, S Descotes-Genon, G Isidori, work in progress.

[72] N. Arkani-Hamed and M. Schmaltz, Phys. Rev. D 61, 033005 (2000) [arXiv:hep-ph/9903417]; Y. Grossman and M. Neubert, Phys. Lett. B 474 (2000) 361 [arXiv:hep-ph/9912408]. T. Gherghetta and A. Pomarol, Nucl. Phys. B 586 (2000) 141 [arXiv:hep-ph/0003129].

[73] S. Davidson, G. Isidori and S. Uhlig, “Solving the flavour problem with hierarchical fermion wave functions,” Phys. Lett. B 663 (2008) 73 [arXiv:0711.3370 [hep-ph]].