Contactless non-destructive thermal control of materials

A G Divin¹, A S Egorov¹, S V Ponomarev¹, S S Al-Busaidi¹, G V Shishkina¹ and A I Tiurin²

¹ Tambov State Technical University, Tambov, Russia
² Derzhavin Tambov State University, Tambov, Russia

E-mail: egorov.andrey@list.ru

Abstract. Physical and mathematical models of the heat transfer process in the reviewed control object are considered. On the basis of the formulated boundary value problem of thermal conductivity (using the Laplace and Hankel transformations) the calculated dependences for calculating the desired values of thermal conductivity and thermal diffusivity are obtained. An algorithm for the functioning of the information-measuring and control system for implementing the developed method for measuring the thermophysical properties of materials is presented.

1. Introduction

Modern trends in the development of aerospace technology, power equipment, powerful optical systems, chemical and petrochemical complex are characterized by the desire to increase the density of energy fluxes, the temperature of the working fluid, and the duration of reliable operation. An increase in energy intensity leads to an increase in the efficiency of engines, energy conversion efficiency, a decrease in the mass and dimensional characteristics of products, an increase in fuel efficiency, etc. As a consequence, the role of thermophysical characteristics (TPC) of the materials used, their stability during operation, the importance of controlling these characteristics during production and operation are continuously increasing. Existing methods of thermal control [1] often do not meet the increasing requirements for them. In their overwhelming majority, they have limited functionality, high labor intensity (mainly associated with the need to prepare samples of a certain shape and size), low productivity, and require expensive stationary equipment and qualified personnel. Therefore, the development of scientific and technical foundations for new rapid methods of thermal diagnostics and determination of TPC are urgent and very relevant for many branches of technology. The creation of such means will make it possible to move from selective laboratory control, carried out on special samples, to continuous non-destructive testing in production and operation conditions, which will lead to an increase in the reliability of equipment while simultaneously increasing the quality and yield of suitable products.

As a rule, methods for measuring thermophysical characteristics are active, that is, they involve exposure of the test sample from an external energy source. To eliminate thermal active resistances, it is advisable to use the energy of laser radiation, as implemented in the Parker method, known as the laser flash method [2]. This method is currently used in the commercially available Laser Flash Apparatus LFA 427 from NETZSCH [3].

These devices make it possible to measure the thermal diffusivity coefficient $a$, the volumetric heat capacity $c_p$, as well as the thermal conductivity $\lambda$ of specially prepared samples of the materials under...
study at a given temperature. In this case, the error in measuring TPC can reach several percent, but the cost of such devices for researchers is extremely high. Therefore, in this article, an approach is considered, the implementation of which will allow you to create a device with the same functionality, acceptable errors, but much more affordable.

2. Physical and mathematical models of unsteady heat transfer for the method of measuring thermophysical properties

The physical model of the device for implementing the measurement method is shown in figure 1.

We assume that the following conditions are fulfilled in the experiment:

- the investigated object in relation to the thermal effect is semi-limited $0 \leq z < \infty$, $0 \leq r < \infty$;
- at the initial moment of time $t = 0$ the body has a constant temperature $U(r, z, 0)$. For convenience, we consider it equal to zero;
- heat flux constant along the coordinate $r$ density $q_e(r, t) = q_e(t)$ supplied (in the form of laser radiation) to the body through a round section $0 \leq r < R$ surface at $z = 0$ object;
- heat flow is limited in time: $q_e(t) = \begin{cases} q_e & \text{if } 0 < t \leq t_i, \\ 0 & \text{if } t > t_i; \end{cases}$
- heat flux density $q_e(r, t)$ such that the maximum excess temperature $U(r, z, t)$ during the experiment it was on the order of several degrees, but not more than 30 K. This condition gives us the right to describe the heating process with a high degree of reliability by a linear mathematical model, i.e. thermal conductivity and thermal diffusivity of the material of the investigated object can be considered constant;
- above the plane $z = 0$ a thermal imager is located near the source of thermal effect, which allows obtaining information on the spatial integral characteristic (SIC) of the temperature of the entire heated area of the body $0 \leq r < R$.

Under these assumptions, the temperature field [3] in the investigated body will be described by the solution [5] by the following axisymmetric boundary value problem of heat conduction [6]:

$$\frac{1}{a} \frac{\partial U(r, z, t)}{\partial t} = \frac{\partial^2 U(r, z, t)}{\partial r^2} + \frac{1}{r} \frac{\partial U(r, z, t)}{\partial r} + \frac{\partial^2 U(r, z, t)}{\partial z^2},$$  \hspace{1cm} (1)
\[(0 \leq r < +\infty, 0 \leq z < +\infty, t > 0);\]
\[U(r, z, 0) = 0;\]  
\[
\frac{\partial U(r, z, t)}{\partial r} \bigg|_{r=0} = 0; \]  
\[
\frac{\lambda}{r} \frac{\partial U(r, 0,t)}{\partial z} \bigg|_{r\in R} = -q_e(r, t), \quad q_e(r, t) = \begin{cases} q_e(r) & \text{if } t \leq t_e, \\ 0 & \text{if } t > t_e; \end{cases} \]
\[U(r, z, t) = 0 \text{ if } r, z \to +\infty. \]  

We apply to problem (1) - (5) the integral Laplace transform with respect to time \(t\) [7], assuming that the function \(U(t, r, z)\) measured in the course of the experiment is continuous and has continuous derivatives with respect to coordinates \(r\) and \(z\) in the domains \(t > 0, 0 \leq r < +\infty, 0 \leq z < +\infty\). As a result, we obtain the Laplace-transformed differential equation:
\[
\frac{p}{a} U^*(r, z, p) = \frac{\partial^2 U^*(r, z, p)}{\partial r^2} + \frac{1}{r} \frac{\partial U^*(r, z, p)}{\partial r} + \frac{\partial^2 U^*(r, z, p)}{\partial z^2},
\]  
\[(0 \leq r < +\infty, 0 \leq z < +\infty, p > 0);\]
with boundary conditions:
\[
\frac{\partial U^*(r, z, p)}{\partial r} \bigg|_{r=0} = 0; \]  
\[
\frac{\lambda}{r} \frac{\partial U^*(r, 0, p)}{\partial z} \bigg|_{r\in R} = -(1 - \exp(-pt_e))q_e^*(r, p); \]
\[U^*(r, z, p) = 0 \text{ if } r, z \to \infty. \]  

We apply to (6) - (9) the Hankel integral transform with an infinite limit along the coordinate \(r\) [7] and, taking into account its properties [7], we obtain a second-order differential equation in total derivatives [8]:
\[
\frac{d^2 \bar{U}^*(\xi, z, p)}{dz^2} - \xi^2 \bar{U}^*(\xi, z, p) - \frac{p}{a} \bar{U}^*(\xi, z, p) = 0
\]  
\[(0 \leq \xi < +\infty, 0 \leq z < +\infty, p > 0);\]
with boundary conditions:
\[
\frac{\lambda}{d} \frac{d \bar{U}^*(\xi, 0, p)}{dz} = -(1 - \exp(-pt_e))\bar{q}_e^*(\xi, p),
\]  
\[\bar{U}^*(\xi, z, p) = 0 \text{ if } z \to \infty, \]
where: \(\bar{U}^*(\xi, z, p) = \int_0^\infty U^*(r, z, p)J_0(\xi r)dr; \quad \bar{q}_e^*(\xi, p) = \int_0^\infty q_e^*(r, p)J_0(\xi r)dr; \quad \xi\) -parameter of the Hankel integral transform, \(J_0\) - zero-order Bessel function of the first kind.
Taking into account the fact that information is collected only from the surface of the investigated body at \( z = 0 \), solution of differential equation (10) taking into account conditions (11) and (12) for \( z = 0 \) looks like:
\[
\tilde{U}^{*}(\xi, 0, p) = \frac{(1 - \exp(-pt_c))\tilde{q}^{*}(\xi, p)}{\lambda \sqrt{\xi^2 + p/a}}.
\] (13)

Since we assume that in the experiment the heat flux supplied to the circle of the surface \( z = 0 \), has constant density along the coordinate \( r \), then (taking into account the properties of the Bessel function under the Hankel transform for a constant value [10]) we obtain the following expression for the heat flux density in the region of double space-time Laplace-Hankel transformations:
\[
\tilde{q}^{*}(\xi, p) = \frac{q_e(p)R}{\xi} J_1(R\xi),
\] (14)

where: \( J_1 \) - first-order Bessel function of the first kind.

Substituting (14) into (13) and applying the inverse Hankel transform [11] to the obtained dependence, we find the relationship of the time integral characteristic (TIC) of temperature \( U^*(r,0, p) \) surface at \( z = 0 \) test body with TIC heat flux \( q_e^*(p) \) on the same surface at \( z = 0 \):
\[
U^*(r,0, p) = \frac{(1 - \exp(-pt_c))q_e^*(p)R}{\lambda} \int_{0}^{\infty} \frac{1}{\sqrt{\xi^2 + p/a}} J_1(R\xi) J_0(r\xi) d\xi.
\] (15)

In our method and device for non-destructive testing (NDT) of thermophysical properties (TPP), the main information about the surface temperature \( z = 0 \) is taken in the form (13), taking into account that the PTIC of temperature (15) of the entire surface of a circle with a radius \( R \), i.e. the space-time integral characteristic (STIC) of the temperature of the investigated body will have the form [12]:
\[
S^*(p) = \frac{2(1 - \exp(-pt_c))q_e^*(p)}{\lambda} \int_{0}^{\infty} \frac{1}{\sqrt{\xi^2 + p/a}} J_1^2(R\xi) d\xi.
\] (16)

3. Calculated dependencies in determining TPC by the developed method
To obtain calculation formulas in the domain of the integral Laplace transform, we use the similarity of the Fourier number in the form of a dimensionless parameter [11]:
\[
g = \frac{pR^2}{a},
\] (17)
and also introduce dimensionless variables:
\[
\tau = pt_c \quad \text{and} \quad \mu = \xi R
\] (18)
and denote the functions:
\[
V(g) = \int_{0}^{\infty} \frac{J_1^2(\mu)}{\mu^2 + g} d\mu, \quad W(g, \tau) = (1 - \exp(-\tau))V(g).
\] (19)

We assume that in the experiment the heat flux supplied to the circle of the surface \( z = 0 \) over time \( t \in [0; t_c] \) has constant density. Then (taking into account the properties of the Laplace transform for a
constant value), taking into account (17) - (19), we obtain the expression for the PTIC of body temperature (16):

\[ S'(p) = \frac{2Rq^*}{p\lambda} W(g, \tau). \]  

(20)

For two values of the parameter of the integral Laplace transform \( p_1 = p \) and \( p_2 = kp \), \( (k > 1) \), according to the well-known method [11] from expression (20) we obtain the equation for determining the parameter \( g \):

\[ \Theta \equiv \frac{S'(p)}{kS'(kp)} = \frac{W(g, \tau)}{W(kg, k\tau)} \equiv \Phi(g, k, \tau). \]  

(21)

The left side of equation (21) - the value \( \Theta \) is determined by calculation based on data obtained from the results of experimental measurements \( (S(t), q_e) \), and known quantities \( (p, k) \). The function \( \Phi(g, k, \tau) \) is calculated in advance for certain \( k \) and \( \tau \), and from the dependence \( \Phi(g, k, \tau) = \Theta \) at fixed \( k \) and \( \tau \) it is \( g \) determined by the numerical value of which the thermal diffusivity of the material under study is found:

\[ a = \frac{pR^2}{g}. \]  

(22)

The value of thermal conductivity, expressed from (20), is calculated by the formula:

\[ \lambda = \frac{2Rq^*}{pS'(p)} W(g, \tau). \]  

(23)

Specific volumetric \( c\rho \) heat capacity is determined from the ratio:

\[ c\rho = \lambda/a. \]  

(24)

Thus, to determine the sought-for TPC values by the developed method from the obtained calculated dependencies, it is necessary:

- know the value of the coefficient \( k, t = t_n \) and heater radius \( R \) before starting the experiment;
- measure SIC temperatures throughout the experiment \( S(t) \), and up to the point in time \( t = t_e \) - heat flux density \( q_e(t) \);
- determine the optimal value of the Laplace \( p \) transform parameter according to experimental data;
- calculate the temperature \( S'(p), S'(kp) \) TIC, and, using formula (21), determine the value \( \Theta \);
- find the value of the parameter \( g \) from the equation \( \Phi(g, k, \tau) = \Theta \);
- calculate the TPC of the material under study by dependences (22) and (23) by the found \( g \).

To calculate the values of thermal conductivity and thermal diffusivity when using this method, an experimental determination of the heat flux directed to the investigated body is required. This is due to a difficult technical problem with the use of calibration measurements on standard optically opaque (at a wavelength of 404 nm) samples. Development of an algorithm for solving coefficient inverse problems of thermal conductivity using a non-contact method for measuring temperature in the IR range using a
thermal imager. The non-contact method for measuring TPC is based on the measuring setup shown in figure 2.

The laser has a maximum output power of 0.5 W and a wavelength of 405 nm. Focusing the laser makes it possible to obtain a light spot with a diameter of a few tenths of a millimeter to 10 mm at a distance of 10 mm to 200 mm to the object surface. The laser power control unit allows you to control the output power using pulse width modulation.

To obtain primary information about the body surface temperature, a FLIR model A35 thermal imager is used. The thermal imager is designed for technical vision systems and can form a video stream at a speed of up to 60 frames per second.

4. Results and discussion
Using the presented method and a working model, calibration measurements of the thermophysical properties of polymethyl methacrylate, which has well-known thermophysical properties, were carried out. The sample was 25 mm thick and over 100 mm in diameter. Since the test material is optically
transparent for laser radiation at a wavelength of 405 nm, its surface was coated with a thin layer of acrylic matte black enamel. The diameter of the defocused laser spot on the sample was about 10 mm and was determined visually using a ruler with an upper measurement limit of 150 mm (GOST 427-75). The excess of the surface temperature of the sample during the active stage of the experiment was no more than 30 °C, which makes it possible to ignore the change in the thermophysical characteristics of the material during the measurements.

During the calibration, such a value of the heat flux density \( q_0 \) was found, at which the calculated value of the thermal conductivity of the sample material was 0.195 W/(m·K).

Next, we studied the thermophysical characteristics of a single crystal of barium fluoride (BaF\(_2\)), which is actively used in thermal imaging optics. A sample of this material had the shape of a disk, 25 mm thick, 50 mm in diameter, and its surface was also covered with a thin layer of black acrylic enamel. For this sample, 10 measurements were carried out, in each of which the excess temperature (with respect to the initial temperature) of the surface heated by the laser was no more than 2 °C (at the same supplied heat flux density as in the experiment with polymethyl methacrylate).

The results of determining the thermophysical characteristics of a single crystal of barium fluoride in accordance with the algorithm presented in figure 3 are shown in table 1.

### Table 1. Results of an experimental study of the thermophysical characteristics of a single crystal of barium fluoride at room temperature.

| Material | Measured and values \( a \times 10^{-6} \), m\(^2\)/s | Test number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|----------|-----------------------------------------------|-------------|---|---|---|---|---|---|---|---|---|---|
| Sample BaF\(_2\) | \( \sigma_\alpha \times 10^{-6} \), m\(^2\)/s | 3.365 | 3.251 | 3.563 | 3.161 | 3.648 | 3.546 | 3.252 | 3.244 | 3.672 | 3.280 |
| \( \bar{a} \times 10^{-6} \), m\(^2\)/s | 0.2 | 3.40 |
| \( \bar{\lambda} \), W/(m·K) | 6.81 | 7.01 | 6.6 | 6.25 | 6.32 | 6.54 | 6.42 | 6.718 | 6.83 | 7.03 |
| \( \sigma_\lambda \), W/(m·K) | 0.3 |
| \( \bar{\lambda} \), W/(m·K) | 6.65 |

Thus, the measurement uncertainties for type A presented in the table, in the form of standard deviations \( \sigma_\alpha \) and \( 2\sigma_\lambda \), indicate relative errors in measuring the coefficient of thermal conductivity and thermal diffusivity, respectively, no more than 5 and 7%. However, the real error may be somewhat larger, since the measurements have a number of methodological errors caused by the fact that the experiment does not take into account important factors, such as the non-uniformity of the heat flux density in the laser-heated sample surface, the presence of radiation heat exchange between the sample and the environment, the final dimensions sample. Nevertheless, the measurement results agree well with the data of other authors [13], and the low excess temperature of the sample during the active stage of the measurement makes it possible to control materials whose structure is sensitive to overheating.

### 5. Conclusion

Application of the proposed method of integral characteristics allows to reduce measurement errors caused by noise in thermal imaging cameras by integrating and averaging temperature; reduce overheating of the sample to 20-30 °C, thereby reducing methodological errors.

When carrying out the research carried out in the course of preparing this article, the experience of developing methods and devices for measuring the TPC of materials was used, accumulated at the
Federal State Budgetary Educational Institution of Higher Education "Tambov State Technical University" at the Department of "Mechatronics and Technological Measurements".

Acknowledgments
This work was supported by the Russian Science Foundation, grant no. 20-19-00602.

References
[1] Vavilov V P 2014 Thermal NDT: historical milestones, state-of-the-art and trends *Quantitative InfraRed Thermography Journal* 11 66-83
[2] Parker W J, Jenkins R J, Butler C P and Abbot G L 1961 Flash method of determining thermal diffusivity, heat capacity and thermal conductivity *J. Appl. Phys.* 32 1679-84
[3] Laser Flash Analysis – LFA 2019 *Method Technique Applications. NETZSCH-Geratebau GmbH* Retrieved from: http://www.netzsch.com
[4] Antonova L L and Churikov AA 2005 Method of non-destructive thermophysical control of ceramic electrical insulating products *Proceedings of TSTU: collection of scientific articles of young scientists and students* (Tambov) 65-8
[5] Lykov AV 1967 *Theory of Thermal Conductivity* (Moscow: Higher School) p 700
[6] Carslow G and Jaeger D 1964 *Thermal Conductivity of Solids* (Moscow: Nauka) p 487
[7] Kartashov E M 2001 *Analytical Methods in the Theory of Thermal Conductivity of Solids* (Moscow: Vysshaya Shkola) p 550
[8] Koshlyakov N S, Gliner E B and Smirnov M M 1970 *Partial Differential Equations of Mathematical Physics* (Moscow) p 712
[9] Watson GN 1949 *Theory of Bessel functions CH I* (Moscow: foreign literature publishing house) p 799
[10] Vlasov V V, Shatalov Yu S, Zotov E N, Labovskaya A S, Mishchenko S V, Pankov A K, Ponomarev S V, Puchkov N P, Seregina V G and Churikov A A 1975 Thermophysical measurements. Handbook on methods of calculating fields, characteristics of heat and mass transfer and automation of measurements (Tambov Publishing House of VNIIRTMASH) p 256
[11] Churikov A A 1980 *Development and research of methods and devices for automatic non-destructive testing of temperature-dependent thermophysical properties of solid heat-shielding materials* (Moscow) p 149
[12] Ponomarev S V, Mishchenko S V, Divin A G, Vertogradskiy V A and Churikov A A 2008 *Theoretical and practical foundations of thermophysical measurements: monograph* (Moscow: FIZMATLIT) p 408
[13] Thermal conductivity of single crystals of Ba1 - X Yb x F2 + x solid solution. *Article in Doklady Physics* · July 2008 DOI: 10.1134/S1028335808070045