Multifractal analysis of spatial–temporal characteristics in the Alpine–Himalayan seismic zone

Jiemin Chen¹, Zelin Yan¹, Linfeng Xu¹, Zhixin Liu¹*, Yan Liu² and Jiawei Tian²

¹School of Life Science, Shaoxing University, Shaoxing 312000, Zhejiang, P. R. China
²School of Automation, University of Electronic Science and Technology of China, Chengdu 610054, Sichuan, P. R. China

*Corresponding author e-mail: liuzhixin@usx.edu.cn

Abstract. The occurrence regularity and propagation mechanism of seismic activity are complicated and variable to be described by existing models. This paper presents a multifractal analysis of the Alpine–Himalayan seismic zone in time–space series. The multifractal spectrum of energy released from the seismic zone is obtained from different angles. Results show that earthquakes in the seismic zone possess complex multifractal characteristics in time–space series. Moreover, the clustering feature of the seismic activity intensity in the seismic zone is similar among the time–space series.

1. Introduction

Multifractal method is applied to study the time series, spatial distribution sequence, release and intensity of earthquake activity as well as determine its corresponding characteristics [1, 2, 3]. Seismic activity exhibits distinct stages or periodicity; in periodicity, complex fluctuation alternation occurs at the centre of the intense earthquake activity. In this paper, the spatial and temporal characteristics of the seismic activity in the Alpine–Himalayan seismic zone are analysed and described. The earthquake time series relationship of $r(q) - q$, $D_q - q$ and $f(\alpha) - \alpha$ is obtained through multifractal analysis of 1900–2015 seismic data. The multifractal characteristics in the Alpine–Himalayan seismic zone is also analysed from the aspects of time and space. The multifractal spectrum of the spatial–temporal characteristics is recorded to explore the space–time characteristics in the seismic zone [4,5].

2. Literature Review

Earthquake is a natural phenomenon with scale-invariant features [6] and is a critical process of random self-organisation [7, 8]. Earthquakes and other natural hazards have caused huge losses to human life. Scientists are committed to the prediction and impact analysis of natural disasters [9-12] to minimize the damage [13]. Earthquake research needs to combine complex disciplines such as dynamics [14,15], physics [16,17], geology [18,19], and earth science [20,21], which made the early Research on it has little effect. The successful prediction of the Haicheng earthquake in China in 1975 has gained significant research attention worldwide [22]. However, the 1976 Tangshan earthquake in China [23] and the earthquake in the west coast of the United States in 1978 [24] were timely predicted, indicating the randomness and complexity of seismic activities. To date, many of the earthquake prediction models are not ideal [25]. The occurrence and propagation mechanism of earthquakes are considerably complicated and variable to be described by previous models [26].
with the beginning of a new generation of industrial revolution [27,28], the emergence of machine learning [29-33], and later deep learning research [34-38], applied to various fields such as image processing [39,40], The further application of statistics [41,42] to seismic research has led to further development of seismic research.

Through continuous application of seismic fractal theory, researchers found that seismic characteristics are not a single fractal process but a multifractal process both in time series and space sequence [43]. S.M.Potirakis and G.Minadakis [44] used multifractal theory to study changes in the variation sequence of electromagnetic flow in the seismic area. Results showed that the electromagnetic flow dimension of the study area presents an abnormal value before the earthquake occurs. David Harte and Chen Shijun [45] investigated the earthquakes of seismically active regions in New Zealand; multifractal analysis on the epicentre, source and time interval showed that earthquakes with different epicentres exhibit obvious differences. Md.Nurujjaman and Ramesh Narayanan et al. [46] calculated the multifractal spectrum of the earthquake time series of Salvatore in Central America in 2001; the result showed that the fractal dimension fluctuates around 1. Kawamura M. et al. [47] studied the spatial distribution sequence and release of wide stress in an earthquake phenomenon in China through multifractal analysis; the phenomenon was characterised as multifractal.

3. Data Description, Data Processing and Analysis Method

3.1 Multifractal method

The object of the study is a series of attribute values with time series or a series of things arranged in space according to a certain rule. The research object is divided into N scale of different units \( s_i \) \( (i=1, 2, \ldots, N) \); the earthquake time series is divided into different time periods. Each \( s_i \) is not a single unit but occupies a certain proportion and satisfies \( \sum_{i} p_i = 1 \). The regional units also differ, that is, \( s_i, p_i \); this difference can be characterised by different scaling exponents \( \alpha_i \), and its relationship to \( p_i \) and linear degree size \( r_i \) can be expressed by Eq. (1):

\[
p_i = r_i^{\alpha_i} (i=1, 2, \ldots, N)
\]

where \( r_i \) indicates the size of the area \( i \); \( p_i \) represents the generation probability of \( s_i \) in the area (physical quantity); and \( \alpha_i \), a Lipschitz–Holder index (referred to the Holder index), is the fractal dimension of a small region and also known as the local fractal dimension. This parameter reflects the size of the generated probability of area \( i \).

If the line size \( r_i \) tends to become zero, then the following relation is obtained:

\[
\alpha_i = \lim_{r_i \to 0} \frac{\ln p_i}{\ln r_i}
\]

(2)

The fractal dimension of the fractal region is expressed by \( \alpha \). When the value \( \alpha \) of each small unit divided from each subset is the same, then the following relationships are obtained:

\[
N_\alpha(e) \propto e^{-f(\alpha)}
\]

(3)

\[
f(\alpha) = -\lim_{e \to 0} \frac{\ln N_\alpha(e)}{\ln e}
\]

(4)

In addition to the representation \( \alpha \sim f(\alpha) \), multifractal can also use another parameter, that is, \( q \sim D_q \), which is obtained from the perspective of information theory. Firstly, the order \( q \) is introduced on both sides of Eq. (1) by \( q \) times, and the two sides of the equation are added as follows:

\[
\Sigma_i^N p_i^q = \Sigma_i^N (r_i)^{\alpha_i q} = X(q)
\]

(5)

We define partition function \( M \) as follows:

\[
M(e, q) = \Sigma_i^N p_i^q (e)
\]

(6)

where \( q \) is the order of the partition function \( (q \in (-\infty, +\infty)) \), which can characterise the multifractal uneven degree. When a given \( q \) satisfies the condition \( M(e, q) \propto e^{\tau(q)} \), the \( \tau(q) \), known
as the quality index function, is the characteristic function of the fractal behaviour. When \( \tau(q) \) is a straight line with a constant slope, the object of the study is a single fractal. When the slope of \( \tau(q) \) is not distinct but a convex function of \( q \), the research object presents multifractal characteristics. In the study of multifractal characteristics of the earthquake, we use \( \tau(q) \) to determine whether the earthquake exhibits multifractal properties. Information dimension \( D_q \) is represented by Eq. (7):

\[
D_q = \lim_{L \to 0} \frac{1}{q-1} \frac{\ln x(q)}{\ln L} = D(q)
\]  

(7)

\( q \sim D_q \) and \( \alpha \sim f(\alpha) \) possess a certain link and can be connected using the Legendre transform,

\[
D_q = \frac{1}{q-1} [q \alpha - f(\alpha)]
\]  

(8)

or \( f(\alpha) = q \alpha - \tau(q) \); then, \( \tau(q) = (q - 1)D_q \)

\[
\alpha(q) = \frac{d\tau(q)}{dq} = \frac{d(q-1)D_q}{dq}
\]  

(9)

The formula for calculating the multifractal dimension \( D_q \) [Eq. (8)] is widely used in the study of multifractal. The generalised dimension \( D_q \) shows different fractal dimensions under different values of \( q \). When \( q=0 \), the box dimension or capacity dimension \( D_0 \) is obtained; hence,

\[
D_0 = f(\alpha(0)) = -\tau(0)
\]

When \( q = 0 \), \( D_1 = f(\alpha(1)) = \alpha(1) \), that is the information dimension.

3.2 Multifractal analysis of seismic time characteristics in the Alpine–Himalayan seismic zone

In this paper, the spatial–temporal characteristics of seismic activity are explored according to space and time variation characteristics of the energy release by seismic activity. According to Eq. (10), the energy released by the earthquake can be estimated by the following:

\[
E = \Omega = 10^{1.5M+c}
\]  

(10)

where \( M \) is the magnitude of the earthquake magnitude, and \( c \) is a constant (\( c=4.8 \)).

In the present study, a total of 182360 seismic data satisfying the condition \( M \geq 2 \) are selected for multifractal analysis from the preprocessed seismic activity (1900–2015) data of the Alpine–Himalayan seismic zone [48].

On the basis of the principle above, the time series of the Alpine–Himalayan seismic zone is calculated. According to Eq. (1), the quality function \( \tau(q) \) is calculated (Fig. 1-a). The relationship between the mass function of \( \tau(q) \) and \( q \) is not an obvious linear correlation, which indicates that the earthquake exhibits a strong multifractal characteristic in the time series of the earthquake zone. In general, the entire curve is a strong nonlinear one, which shows that multifractal degree of the research object is strong.
Figure 1. Diagram of time series in the Alpine–Himalayan seismic zone (a) $\tau(q) - q$ relation (b) $D_q - q$ relation (c) $f(\alpha) - \alpha$ relation.

According to Eqs. (7) and (8), the relationship of $D_q - q$ of the seismic activity in the time series can be obtained (Fig. 1-b). Figure 1-b shows that the curve is a nonlinear minus function; different $D_q$ values represent the characteristics of dense and sparse parts of the observation. With the variation of $D_q$ to time and the change of energy in the study area, an obvious decrease is observed before the earthquake, which is a good observation before a strong earthquake. However, the spatial location of the earthquake zone is extensive; thus, the singularity of the local small range is likely weakened or ignored. This characteristic causes this phenomenon in the diagram of $D_q - q$ not particularly obvious.

Furthermore, according to the dynamic change characteristics of curve $D_q - q$ of the time series, complexity can be observed. $D_q$ firstly shows a sharp decrease and a subsequent slow decline, and $D_q = 1.55$. This phenomenon is probably explained by that earthquake zone subjected to some stress effects leads to an uneven distribution of energy release to the various regions. Hence, the complexity of multifractal is high, which indicates that the time series of seismic activity is high. This result is consistent with the complexity of its geographic structure.

The $f(\alpha) - \alpha$ curve of the time series of the earthquake zone is obtained by using Eq. (5) (Fig. 1-c); the curve shows sparse multifractal characteristic: its vertex is directed to the left, and the curve is an asymmetric and smooth continuous one. The front and back parts of the curve are different, which shows that the characteristics of earthquake intensity in time series are similar in the two periods of time clustering. With the peak as the cut-off point, the left in the boundary represents the sparse region of the seismic zone; the right is the representative of the cluster area of the seismic zone. No exact value $\alpha$ is obtained for the left half of the vertex, which shows that the intensity distribution of the Alpine–Himalayan seismic zone in time sequence only display an infinite cluster. Additionally, when a difference exists between the two stages, the seismic activity of small earthquake cluster dominates in the previous period, and the latter is more uniform. This result can be significantly explained by the geological structure of the seismic zone.

3.3 Spatial multifractal characteristics of the Alpine–Himalayan seismic zone

Different resolution discretization of the Alpine–Himalayan seismic zone is conducted. From the operational perspective, such discretization is a process of establishing regular grids, which results in that the seismic data of the study area are divided into different grids according to certain standards (Fig. 2). After obtaining the earthquake information of each grid unit, the total energy released by the earthquake in each grid cell is calculated according to Eq. 10. The result of energy release in the seismic zone after gridding statistics is expressed as the following formula:

$$ E = \begin{bmatrix} E_{11} & E_{12} & \cdots & E_{1n} \\ E_{21} & E_{22} & \cdots & E_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ E_{m1} & E_{m2} & \cdots & E_{mn} \end{bmatrix} $$

(11)

where $E$ represents the energy matrix, that is, the total energy released by the earthquake zone. $E_{ij}$
represents the total energy in gridding row i and column j units.

Figure 2. Grid map of the Alpine–Himalayan seismic zone.

Similar to the time series analysis, $\tau(q) - q$, the contrast relationship diagram of the earthquake zone in the spatial distribution sequence is obtained (Fig. 3-a). The slope of the curve shows a continuous change; it becomes large firstly and subsequently small, which shows a multifractal characteristic in the spatial sequence of earthquake energy. In general, the nonlinearity of the whole curve is strong, which indicates that the multifractal degree of the seismic activity in space is strong. This result is consistent with the result of multiple analyses.

Figure 3. Diagrams of spatial distribution sequence of seismic activity in the Alpine–Himalayan seismic zone (a) $\tau(q) - q$ relation (b) $D_q - q$ relation (c) $f(\alpha) - \alpha$ relation.

Furthermore, the change of the two stages in the space sequence diagram of $\tau(q) - q$, as well as the nonlinearity, is obvious. Compared with the time series set, the complexity of the seismic activity sequence set is relatively high, and its multifractal characteristics are considerably obvious.

The relationships of $D_q - q$ and $f(\alpha) - \alpha$ in the spatial distribution of the earthquake zone are shown in Figs.3-b and 3-c, respectively. The results obtained from the diagram of $D_q - q$ in the spatial distribution and time series show consistency. The energy released by seismic activity is not uniform, which leads to a high degree of complexity; moreover, the energy release of the seismic zone is a complex process, which is consistent with the geological structure of the background.

4. Discussion and Conclusion

This study conducts the research according to the original seismic data from 1900 to 2015 of the Alpine–Himalayan seismic zone. Multifractal method is used to analyse time spatial distribution characteristics of seismic activities, and the following conclusions are obtained:

(1) The $\tau(q) - q$ relation diagram of time series in the Alpine–Himalayan seismic zone shows that the energy released by the earthquake presents obvious multifractal characteristics in time and space.

(2) The $f(\alpha)$, the singular spectra of the time series in the earthquake zone, is a nonsymmetrical smooth continuous curve. Obvious differences are observed in the curve $f(\alpha) - \alpha$, which shows that the seismic activity intensities of the earthquake sequence in time and space are similar.

(3) The left multifractal spectrum of the time series is not convergent, which indicates that the
intensity of activities in the spatial and temporal distribution almost only displays infinite clusters.

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