Research Article

Design of an Anti-Windup PID Algorithm for Differential Torque Steering Systems

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The EV (electric vehicle) with a wheel hub motor has the advantage of independent driving torque distribution for each wheel, which allows the vehicle performance to be improved. Therefore, a lot of work has been done to investigate the torque allocation algorithm for the mechanical and differential torque integration steering system. To investigate the differential steering process, the 2 DOF (degree of freedom) dual-track reference models with linear and nonlinear tire models are established, and based on the steering process analysis and yaw rate gain calculation, a BP-NN (backpropagation neural network) model is initiated to maintain the accuracy of the calculated yaw rate gain. The limitation of DTSS (differential torque steering system) and the difference of reference models with linear and nonlinear tires are drawn. In addition, an anti-windup variable PID (proportion integration differentiation) controller is designed for the torque distribution. Based on the built 8 DOF model, the vehicle performance indicators are calculated and compared, the gap between the models with linear and nonlinear tires is non-negligible, and a reference model with a nonlinear tire model is recommended for the relevant research. The anti-windup variable PID controller has a better performance than the normal PID controller except for the stability phase plane indicator.

1. Introduction

The EV (electric vehicle) has attracted the researcher’s attention for some time due to its advantage in energy conservation and environment protection. Among the different kinds of EVs, the EV with wheel-motor shows superior performance in advanced chassis control algorithms because of its simplified chassis and independent driving torque.

The independent driving torque on the vehicle wheels allows the vehicle to have a better ability on torque vectoring (TV) control. The TV control can improve the vehicle’s dynamic performance according to the driver’s attention without significantly damaging the vehicle’s speed. In addition, it provides another steering method, differential torque steering system (DTSS) [1].

Most of the relevant research takes the DTSS as a supplementary method of the Ackerman or articulated steering system to improve the vehicle performance [2, 3]. Based on the coordination of active front steering (AFS), direct yaw control (DYC), and TV control, an integrated control algorithm is provided to maintain the vehicle stability during extreme work conditions [4]. In the work of Zheng Hongyu et al., the TV control is combined with the AFS to enhance the yaw and roll stability of a coach [5]. Normally, the torque control distribution algorithm is made up of 2 or 3 layers [6]. The number of layers depends on the preference of the researchers, but the function of the distribution algorithms is similar, including the stability judgment, slip rate calculation, torque allocation, and torque distribution. The phase plane method can represent the nonlinear characteristics; therefore, it is widely used in vehicle stability judgment. The phase plane method can be classified according to the state variable used in the phase plane, including sideslip angle-sideslip angular velocity [7], yaw rate-sideslip angle [8], and front-rear tire sideslip angle [9]. Different torque control algorithms, such as PID (proportion integration differentiation) [10], LQR (linear quadratic regulator control) [11], MPC (model predictive
2. Vehicle Dynamic Model

To investigate the differential torque steering control algorithms, the vehicle dynamic model is established based on the vehicle dynamic theory. For an operating vehicle, lots of freedom exists in the whole system, and the simplification must be done based on the researchers’ focus. Normally, a 2 DOF model is established as the reference model and the 8 DOF model is established to verify the torque distribution algorithms in the relevant research [16, 17]. Since the 8 DOF model is much more complicated than the 2 DOF model, the 8 DOF model is introduced first. The 8 DOF model is made up of the vehicle body, wheel, motor, and tire model.

2.1. Vehicle Body Model. The 8 DOF model has the motion of longitudinal, lateral, yaw, roll, and rotation of the four wheels, as shown in Figure 1. The four motions can be identified as the following equations:

\[
m(\dot{v}_x - v_y) + m_x h_x \dot{\phi} = \sum_{i=1}^{4} F_{x_i} - \frac{1}{2} C_d A \rho v_x^2 - mgf,
\]

\[
m(\dot{v}_y + v_x \dot{y}) - m_y h_y \dot{\phi} = \sum_{i=1}^{4} F_{y_i},
\]

\[
I_x \ddot{\phi} - m_x h_x (\dot{v}_y + y \dot{v}_x) - I_{xz} \dot{y} = m_x g h_x \phi - (K_f + K_r) \phi - (C_f + C_r) \dot{\phi},
\]

\[
I_x \ddot{y} - I_{xz} \ddot{\phi} = M + (F_{y_1} + F_{y_2}) L_f - (F_{y_3} + F_{y_4}) L_r,
\]

\[
M = \frac{L_w}{2} (F_{x_2} + F_{x_4} - F_{x_1} - F_{x_3}),
\]

where \( m \) means the vehicle mass; \( m_x \) is the sprung mass; \( v_x \) and \( v_y \) present the longitudinal and lateral speed; \( \phi \) is the yaw rate; \( \phi \) is the roll angle; \( F_{x_i} \) and \( F_{y_i} \) are the longitudinal and lateral force of the four wheels, \( i = 1, 2, 3, 4; C_d \) presents the air drag coefficient; \( A \) is the front area of the vehicle; \( \rho \) means the air density; \( f \) is the rolling resistance coefficient; \( I_x \) and \( I_z \) present the roll and yaw inertial of the vehicle; \( I_{xz} \) means the product of inertia of \( x, z \), and axes; \( h_x \) is the height of the sprung mass; \( L_f, L_r, \) and \( L_w \) mean the distance between the vehicle gravity center to the front axle, rear axle, and the wheel distance of axle; \( K_f \) and \( K_r \) are the rolling stiffness of the front and rear axle; and \( C_f \) and \( C_r \) are the rolling damper of the front and rear axle.

2.2. Wheel Model. The rotation of the four wheels is taken into consideration in the wheel dynamic model. Each wheel can be demonstrated as follows:

\[
I_w \dot{\omega}_i = T_i - F_{xi} R_w,
\]

where \( I_w \) is inertial of the wheel, \( T_i \) means the driving torque on the wheel, and \( R_w \) is the rolling radius of the wheel. The slip rate of the wheel, \( s_i \), can be acquired by

\[
s_i = \frac{\omega_i R_w - v_{wi}}{\omega_i R_w},
\]

where \( v_{wi} \) is the speed of the wheel. For the pure DTSS, there is no steering angle for the front and rear axle. For the wheels on the left side, the wheel speeds, \( v_{w1} \) and \( v_{w3} \), can be expressed as

\[
v_{w1,3} = \left( v_x - \frac{L_w}{2} \gamma \right).
\]

For the wheels on the right side, the wheel speeds, \( v_{w2} \) and \( v_{w4} \), can be expressed as

\[
v_{w2,4} = \left( v_x + \frac{L_w}{2} \gamma \right).
\]
2.3. Motor Model. For the electrical vehicle, as the driving unit, the motor is quite critical for the vehicle dynamic model. There are kinds of methods to establish the motor model [18]. In this case, the torque distribution algorithm is the concern of our work. Therefore, the model is built with a simple method that can represent the torque characteristic of the motor.

The motor model calculates the maximum torque in the current rotation speed, based on the dynamic response characteristic simulated by the first-order inertial response unit, and the output torque of the motor, $T$, can be acquired.

$$T = \frac{1}{1 + \tau_t} \max(T_{m_{\text{max}}}, T_d), \quad (10)$$

where $\tau_t$ is the constant time in a first-order system, $T_{m_{\text{max}}}$ means the maximum output of the motor in the current rotation speed, and $T_d$ represents the demand torque. In this case, the $\tau_t$ is 0.002 s. The external characteristic curve of the motor is shown in Figure 2.

2.4. Tire Model. The tire system contacts the vehicle to the road surface, and all of the vibration and forces are translated by it. The accuracy of the tire model has a great influence on vehicle performance. A lot of work has been done to establish the tire model, the Fila tire model [19], UA tire model [20], Gim tire model [21], Dugoff tire model [22], HRSI tire model [23], UniTire model [24], and Magic Formula tire model (MF tire) [25] are widely used in the vehicle dynamic model. In this case, two tire models are used: linear tire and MF tire. The linear tire can be expressed as follows:

$$\begin{align*}
F_{y1,2} &= -k_f \alpha_f, \\
F_{y3,4} &= -k_r \alpha_r, \\
\alpha_f &= \frac{v_y + L_f \gamma}{v_x}, \\
\alpha_r &= \frac{v_y - L_r \gamma}{v_x},
\end{align*} \quad (11)$$

where $k_f$ and $k_r$ are the cornering stiffness of the front and rear tire and $\alpha_f$ and $\alpha_r$ are the sideslip angle of the front and rear tires, respectively.

For the MF tire model, its parameters are fit from the analysis of the test. It can be expressed as follows:

$$Y(X) = D \sin[C \arctan[BX - E(BX - \arctan(BX))]]. \quad (13)$$

When $Y$ is the longitudinal force, $F_x$, the variable, $X$, is the slip rate of the wheel, $s$. When $Y$ is the lateral force, $F_y$, the corresponding variable, $X$, is the sideslip angle of the wheel, $\alpha$. The $B$, $C$, $D$, and $E$ are parameters fitted based on the test on different road types, wheel load, camber angle, temperature, inflation, and tread wear.

Based on the vehicle body, wheel, motor, and tire model, the 8 DOF model is established. The 2 DOF model can be built based on the simplification of equations (2) and (4) and the tire model. Equations (2) and (4) can be simplified as

$$m(\ddot{v}_y + \dot{v}_x \gamma) = \sum_{i=1}^{4} F_{y_i}, \quad (14)$$

$$I_{z} \ddot{\gamma} = M + (F_{y1} + F_{y2})L_f - (F_{y3} + F_{y4})L_r. \quad (15)$$

3. Differential Torque Steering Process Analysis

To simplify the analysis, the following assumptions are made in this case. There is no side slip during the differential steering process. The sideslip angle of the tires is the same. The linear tire is used in this section.
3.1 Differential Torque Steering Process. Based on the above-mentioned assumption, the state function of the differential torque system can be expressed as follows:

\[ \dot{x} = Ax + Bu, \]

\[ x = \begin{bmatrix} \nu_y \\ \nu_x \end{bmatrix}, \]

\[ A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \]

\[ B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}^T, \]

\[ u = M. \]

To access the stability of this system, equation (17) is established as follows:

\[ \text{det}(sI - A) = \begin{vmatrix} s - a_{11} \\ -a_{21} & s - a_{22} \end{vmatrix} = s^2 - (a_{11} + a_{22})s + a_{11}a_{22} - a_{12}a_{21}, \]

\[ \Delta = (a_{11} + a_{22})^2 - 4(a_{11}a_{22} - a_{12}a_{21}), \]

where \( s \) can be gained as follows:

\[ s = \frac{a_{11} + a_{22}}{2} \pm \frac{\sqrt{(a_{11} + a_{22})^2 - 4(a_{11}a_{22} - a_{12}a_{21})}}{2}, \]

\[ \Delta \geq 0, \quad \Delta < 0. \]

The condition for a steady system is that all of the eigenvalues of the state matrix, \( A \), have a negative real part. Since the values of \( k_f \) and \( k_r \) in \( a_{11} \) and \( a_{22} \) are negative, \( a_{11} \) and \( a_{22} \) always have a negative value. Therefore, if \( a_{11}a_{22} > a_{12}a_{21} \), this system is stable.

\[ a_{11}a_{22} - a_{12}a_{21} = \frac{4(L_f + L_r)^2k_fk_r}{I_zmv_x^2} + \frac{2(L_fk_f - L_rk_r)}{I_z} > 0, \]

in which \( k_f, k_r > 0 \); therefore, \( 4(L_f + L_r)^2k_fk_r/I_zmv_x^2 > 0 \). Thus, when \( L_fk_f - L_rk_r > 0 \), the system is always stable. If it is lower than zero, the system will be unstable when the velocity equals the critical speed, \( v_{crr} \). Otherwise, the system is stable.

\[ v_{crr} = \sqrt{\frac{2(L_fk_f - L_rk_r)}{m}}. \]

3.1.1 Yaw Rate Gain during the Steady Steering Process. During the steady steering process, the gradient of yaw rate and lateral speed is zero. The equations (2) and (3) can be expressed as

\[ 2L_fk_f\alpha_f - 2L_rk_r\alpha_r + M = 0, \]

\[ 2k_f\alpha_f + 2k_r\alpha_r = mv_y. \]

Based on equations (11) and (12), and (15), the yaw rate gain during the steady steering process can be acquired.

\[ \frac{\nu_y}{M} = \frac{-(k_f + k_r)v_x}{(L_fk_f - L_rk_r)mv_x^2 + 2k_fk_r(L_f + L_r)^2}. \]

3.2 Torque Distribution Algorithm. According to equation (23), the torque needed to steer in the differential torque steering can be expressed as follows:

\[ M = \frac{(L_fk_f - L_rk_r)mv_x^2 + 2k_fk_r(L_f + L_r)^2}{(k_f + k_r)v_x} \gamma, \]

where \( L_f, L_r, \) and \( m \) are constant values. The relevant parameters of the vehicle are shown in Table 1 [26].

Based on the parameters in Table 1, the impact factor of vehicle speed, tire cornering stiffness on differential torque can be analyzed.

Equation (24) is established based on the assumption of the linear tire. However, the relationship between the lateral force and tire sideslip angle is nonlinear. When the tire sideslip angle exceeds the range of ±5°, the gap of the lateral force of the tire gets non-negligible. The comparison of the lateral force acquired from the linear tire model and MF tire model of the front and the rear tire is shown in Figures 3 and 4.

According to equation (24) and the variation of sideslip angle and tire cornering stiffness during the steering process, the needed torque can be calculated. In Figure 5, the dash lines represent the torque based on the linear tire, and the solid lines mean the tire cornering stiffness is nonlinear during the process.

According to Figure 5, the needed torque increases with the yaw rate and decreases with the vehicle velocity. The needed torque of taking the tire cornering stiffness as constant is higher than taking it as the variable. When the yaw rate is lower than 0.2 rad/s, the velocity is higher than 4 m/s, and the calculated torque in the two methods is similar.

Based on the torque characteristic of the motor, the output torque decreases with the rotation speed. For the vehicle equipped with a wheel-hub motor, the output also decreases with the vehicle speed. According to equation (1), the driving torque needs to overcome the resistance force, including the wind, slope, and rolling resistance force. In this case, the relationship of the force in the longitudinal direction is shown in Figure 6.

According to the research of Antanaitis and Brent [27], there is a different maximum speed for the vehicle on slopes
with a different angle. The vehicle has the greatest resistance force when driving on a slope of 4 degrees; therefore, they are set as the boundary condition in Figure 5. This vehicle will have to decrease the speed during the steering process when the speed reaches 35 m/s. The maximum driving force drops sharply at the velocity of 20 m/s, the force that can be used for differential steering also drops dramatically. Thus, the steering process at the speed of 20 m/s is chosen as the work condition for further analysis.

Different yaw rate and tire models are set in the 8 DOF model to investigate the impact factor of the tire model, and the result is shown in Figure 4. The gap between the result of the linear tire and MF tire gets smaller with the increase of the vehicle speed; therefore, only the minimum gap condition, velocity is 20 m/s, is compared.

As shown in Figure 7, the yaw rate can reach the ideal value in all the conditions. The response of the yaw rate with the speed reaches 35 m/s. The maximum driving force drops sharply at the velocity of 20 m/s, the force that can be used for differential steering also drops dramatically. Thus, the steering process at the speed of 20 m/s is chosen as the work condition for further analysis.

Different yaw rate and tire models are set in the 8 DOF model to investigate the impact factor of the tire model, and the result is shown in Figure 4. The gap between the result of the linear tire and MF tire gets smaller with the increase of the vehicle speed; therefore, only the minimum gap condition, velocity is 20 m/s, is compared.

As shown in Figure 7, the yaw rate can reach the ideal value in all the conditions. The response of the yaw rate with

| Parameter                        | Value   | Unit   |
|----------------------------------|---------|--------|
| Vehicle mass                     | 1704.7  | kg     |
| Yawing moment inertia            | 3048.1  | kg·m²  |
| Front tire cornering stiffness   | -49412  | N/rad  |
| Rear tire cornering stiffness    | -47642  | N/rad  |
| Distance between the front axle and CG | 1.035  | m      |
| Distance between the rear axle and CG | 1.675  | m      |
| Rotation moment inertia of the wheel | 2.5    | kg·m²  |
| Wheel radius                     | 0.313   | m      |

![Figure 3: Comparison of lateral force of the front tire.](image)

![Figure 4: Comparison of lateral force of the rear tire.](image)

![Figure 5: Comparison of need torque during the steering process.](image)

![Figure 6: Variation of the force in the longitudinal direction.](image)
the linear tire is worse than the MF tire, and the gap between them gets larger with the increase of the wanted yaw rate. In conclusion, the gap gets larger when the vehicle speed is lower. Therefore, to maintain the accuracy of the torque acquired by equation (24), the cornering stiffness of the front and rear tire, \( k_f \) and \( k_r \), should be adjusted with the vehicle’s working condition. The sideslip angle of the tire is hard to measure when the vehicle is operating. Thus, a BP-NN (backpropagation neural network) model is established to acquire the tire cornering stiffness based on the vehicle velocity and yaw rate.

3.3. BP-NN Model. The BP-NN is the improved feedforward algorithm of ANN (artificial neural network), which was developed by Rumelhart et al. [28]. BP-NN is widely used in parameter fitting and optimization in many fields, such as manufacturing, aerospace, and vehicle design [29–32]. Figure 8 illustrates a typical configuration of the BP-NN.

In Figure 8, there is one input layer, one output layer, and one or more hidden layers between them. In the input layer, there are \( n \) inputs; for the output layer, \( m \) outputs are included. In addition, \( s \) neurons lie in the hidden layer. Suppose the output of the hidden layer and output layer are \( b_j \) and \( y_k \), the threshold value of the hidden layer and output layer is \( \theta_j \) and \( \theta_k \), the transfer functions of the hidden layer and output layer are \( f_1 \) and \( f_2 \), and the weight factor between the input layer and hidden layer is \( w_{ij} \), while the factor between the hidden layer and output layer is \( w_{jk} \). For the hidden layer, the output of \( j \)th neuro can be gained by

\[
b_j = f_1 \sum_{i=1}^{n} (w_{ij}x_i - \theta_j).
\]  

(25)

The result for the output layer is as follows:

\[
y_k = f_2 \sum_{j=1}^{s} (w_{jk}b_j - \theta_k). \tag{26}
\]

In equations (25) and (26), \( i = 1, 2, \ldots, n; \quad j = 1, 2, \ldots, s; \quad k = 1, 2, \ldots, m. \)

In this case, there are two inputs, the yaw rate and vehicle speed, and two outputs, the cornering stiffness of the front and rear tire. The samples of the inputs and outputs used for the training process are gained by a 2 DOF reference model. The training function of the established BP-NN is set as “Levenberg–Marquardt,” and the adaption learning function is set as “Gradient descent with momentum weight and bias learning function.” The detailed setting of the layers is shown in Table 2.

The accuracy of the established BP-NN model should be verified before further analysis. Normally, the coefficient of determination, \( R^2 \); root mean square error, RMSE; and maximum absolute percentage error, MAPE, are used to assess the accuracy of the fitted model [33, 34]. These indicators can be gained according to the equations as follows:
According to the above equations and the original and fitted model and real value of the samples, num means the number of samples, and $\bar{y}$ is the mean value of the test result. According to the above equations and the original and fitted values, the $R^2$, RMSE, and MAPE of the two tire cornering stiffness fitted models can be acquired and listed in Table 3.

For $R^2$, the value closer to 1 means better accuracy. For RMSE and MAPE, the value closer to 0 means better accuracy. The error analysis in Table 3 shows that the established BP-NN model meets the requirement for further analysis.

3.4. Anti-Windup Variable PID Controller. To keep the output torque of the motor from saturation, an anti-windup variable PID controller is established based on the traditional PID controller. The output torque of the motors, $T_i$, can be expressed as follows:

$$T_i = T'_i \pm M \frac{R_{eff}}{2L_w} \Delta T'_i$$

where the footprint, $i = 1, 2, 3, 4$, means the four motors of the vehicle, $T'_i$ is the drive torque in the normal work condition, $\Delta T'_i$ represents the torque needed to initiate the differential torque steering process, and $T_{max}$ and $T_{min}$ are the maximum and minimum torque of the motor in the current state.

For a typical negative feedback PID control system, the controller can be expressed as

$$u = K_p \left( e + \frac{K_i}{K_p} \int e dt + \frac{K_d}{K_p} \frac{de}{dt} \right),$$

where the control parameters $K_p$, $K_i$, and $K_d$ are tuned according to the object model or the output curve. $u$ is the output of the controller, and $e$ is the error between the input and output. For the vehicle system, it is a time-varying system with nonlinear characteristics, and a PID controller with constant parameters is not effective enough [35]. To deal with the situation, the Gaussian PID method is proposed, in which the Gaussian function is used to define the control parameter of the PID controller. The Gaussian function can be given by [36]

$$f(\delta) = k_1 - (k_1 - k_0)\exp(-q\delta^2),$$

where $\delta$ is the input signal; $k_0$ and $k_1$ are the bonding limit for the zero and infinity input, respectively; and the parameter $q$ defines the concavity openness of the output curve, which can be gained based on the reference input $\delta_r$ and the other variable in equation (28).

$$q = \frac{-\ln(f(\delta) - k_1/k_0 - k_1)}{\delta_r^2}$$

where $-\delta_r$ and $\delta_r$ are the startup and ending point of the upward and downward transient in the upward Gaussian function (as Figure 9 shows); at these two points, $f(\delta)$ reaches to the $\lambda$ percent of the $k_0$-to-$k_1$ range.

To determine the Gaussian function type of the PID controller parameters, the yaw rate with different $K_p$, $K_i$, and $K_d$ is gained and compared in Figures 10, 11, and 12. In Figure 10, the yaw rate response oscillation amplitude gets smaller with the increase of $K_p$; the fluctuation disappears when the value is 27000, and it keeps increasing, which will lead to a higher accommodation time. Therefore, a higher $K_p$ when the response error is relatively big and a lower $K_p$ when the error gets smaller are preferred, and the upward Gaussian function is chosen for $K_p$.

In Figure 11, a higher $K_i$ means bigger oscillation amplitude and accommodation time, and the ideal $K_i$ should increase with the response error; thus, the downward Gaussian function is set for $K_i$.

In Figure 12, a higher $K_d$ means longer accommodation time and lower oscillation amplitude when the error is bigger, and the ideal $K_d$ should decrease with the response error; thus, the upward Gaussian function is set for $K_d$. Based on the (30) and analysis of Figures 10, 11, and 12, the three control parameters are determined based on the following equations [37]:

$$K_p = K_{p0}\left[1 + K_{p1}\left(1 - \exp(-q\delta^2)\right)\right],$$

$$K_i = K_{i0}\left[1 + K_{i1}\exp(-q\delta^2)\right],$$

$$K_d = K_{d0}\left[1 + K_{d1}\left(1 - \exp(-q\delta^2)\right)\right].$$

4. Simulation and Discussion

The torque distributed by reference model with a linear tire with PID controller, a nonlinear tire with constant PID, and Gaussian PID controller during the steady-state steering performance of the vehicle system. Table 2 presents the layer properties for the BP-NN model and Table 3 shows the error analysis of the approximation model.

**Table 2: Parameter of the BP-NN model.**

| Properties for layer | Layer 1 | Layer 2 | Layer 3 |
|---------------------|---------|---------|---------|
| Number of neurons   | 20      | 15      | 15      |
| Transfer functions  | Hyperbolic tangent sigmoid transfer function |

**Table 3: Error analysis of the approximation model.**

| Fitted model | $R^2$ | RMSE | MAPE (%) |
|--------------|-------|------|----------|
| Front tire   | 0.98  | 0.91 | 1.01     |
| Rear tire    | 0.99  | 1.15 | 2.12     |
process is initiated to the 8 DOF model. The yaw rate (Figure 13), roll angle (Figure 14), sideslip angle and sideslip angular velocity (Figure 15), routine (Figure 16) of the vehicle, and tire sideslip angle (Figure 17), and output torque of the motors (Figure 18) are simulated and discussed in this part.

According to the comparison in Figure 13, the linear tire with PID controller has the maximum peak value, longest accommodation time, but minimum delay time. The MF tire model with constant PID and Gaussian PID has a relatively close response, and the Gaussian PID controller has a lower peak value and accommodation time; in conclusion, it has the best response.

In Figure 14, the linear tire model has the maximum and minimum peak value, the MF tire model with the Gaussian PID algorithm has a higher peak value than the MF tire model with the constant PID algorithm. When the vehicle maintaining in a stable state (0~2.5 s and after 15 s), the roll angle gap between these models can be ignored. During the variation period (2.5~15 s), the roll angle of the Gaussian PID algorithm has the lowest fluctuation range.

Figure 15 shows the sideslip angle and sideslip angular velocity phase plane and the variation of these two indicators of the three models. The area between the purplish red dash lines is the stable area. It is obvious that the track of the linear tire model with PID exceeds the stable boundary, the track of nonlinear tires stays in the stable region, and the MF tire with constant PID has the smallest region; thus, it is more stable.
Figure 13: Comparison of yaw rate response.

Figure 14: Comparison of the vehicle roll angle.

Figure 15: Comparison of sideslip angle and side slip angular velocity phase plane.
**Figure 16:** Comparison of vehicle routine.

**Figure 17:** Continued.
Figure 17: Comparison of tire sideslip angle. (a) Front left tire. (b) Front right tire. (c) Rear left tire. (d) Rear right tire.

Figure 18: Continued.
Figure 16 illustrates the vehicle routine of these three models. The routine of the two MF tires is close to each other, and the gap between the MF tire and linear tire is obvious.

The sideslip angle of the tires is compared in Figure 17. The trend of the tires is the same, and the sideslip angle decreases with the increase of yaw rate, starts to increase (about 7 s) after the yaw rate reached 0.4 rad/s (5 s), and becomes stable after some fluctuation. The linear tire has the minimum valley value, while the maximum peak value is calculated by the MF tire with a constant PID algorithm. The sideslip angle of the MF tire model with Gaussian PID control has the minimum fluctuation range.

According to Figure 18, the output torque of the left tires and right tires has the same trend. For the left tires, two valley values occur around 4 and 9 seconds, and a peak value emerges around 7 s. The torque of the linear tire model has the maximum peak value, while the MF tire with Gaussian PID has the minimum valley value. The torque variation range of the two models with MF tires is similar, which is lower than the linear tire model. Compared to the left tire, the output torque of the right tires has the opposite trend before the torque reaches a stable value, the torque increases with the yaw rate at 2.5 s, reaches a peak value around 5 s, and then decreases to 7 s and climb to a peak value at 9 s; at last, it decreases to a stable value. The linear tire has the minimum valley value, the first peak value of the three models is close to each other, and the linear tire model and MF tire model with constant PID have a higher 2nd peak value than the MF tire model with Gaussian PID algorithm.

5. Conclusion

The EV equipped with a wheel-hub motor allows each wheel to have different driving torque, which is a benefit to the vehicle’s dynamic control and performance improvement. Most of the current research focuses on the integrated control of DTSS and mechanic steering system. In this case, the DTSS itself is analyzed and an anti-windup PID control algorithm to distribute the motor torque is provided to improve the vehicle performance.

Based on the vehicle dynamic theory, the 2 DOF reference models with linear and nonlinear tire models are established, the differential torque steering process is analyzed, and the yaw rate gain during the steady steering process is calculated. The DTSS has to kill the speed or acceleration ability to maintain the steering performance during critical conditions, such as running on the upward slope with a relatively high speed. The difference between the two 2 DOF models is compared. To maintain the accuracy of the torque distribution method, a BP-NN model is established, and an anti-windup PID controller is provided.

According to the simulation of the distributed torque by linear tire model with PID, nonlinear tire model with PID, and nonlinear tire model with Gaussian PID, the vehicle performance indicators, such as the yaw rate, roll angle, sideslip angle, sideslip angular velocity, sideslip angle of tires, and motor torques are compared. The gap between the model with linear tire and nonlinear tire is non-negligible, and a reference model with a nonlinear tire model is recommended for the relevant research. Except for the sideslip angle and sideslip angular velocity phase plane track, the anti-windup variable PID controller has a better result than the normal PID control.

In this case, only the steady steering scenario is taken as the input, and the vehicle performance in the other typical condition, such as double lane-change, fishhook, and snaking, should be taken into consideration. The PID parameter determination method is based on experience, and some adaptive or optimization algorithms, such as the genetic algorithm (GA) and particle swarm optimization (PSO), can be initiated in the parameter determination of the controller.
Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare no conflicts of interest.

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References

[1] N. Borchardt, R. Kasper, and W. Heinemann, "Design of a wheel-hub motor with air gap winding and simultaneous utilization of all magnetic poles," in Proceedings of the IEEE International Electric Vehicle Conference, April 2012.
[2] Z. Gao, D. Li, and L. Xie, "Study on the differential braking of an eddy current retarder axle used for articulated vehicles," in Proceedings of the 2020 IEEE 5th International Conference on Intelligent Transportation Engineering (ICITE), September 2020.
[3] J. Wang, T. Yan, Y. Bai, Z. Luo, and X. L. B. Yang, "Assistance quality analysis and robust control of electric vehicle with differential drive assisted steering system," IEEE Access, vol. 8, Article ID 136339, 2020.
[4] N. Aouadj, K. Hartani, and M. Fatihia, "New integrated vehicle dynamics control system based on the coordination of active front steering, direct yaw control, and electric differential for improvements in vehicle handling and stability," SAE International Journal of Vehicle Dynamics, Stability, and NVH, vol. 4, no. 2, pp. 119–133, 2020.
[5] H. Zheng, M. Yangyang, L. Wang, and J. Zhang, "Comparison of active front wheel steering and differential braking for yaw/roll stability enhancement of a coach," SAE International Journal of Vehicle Dynamics, Stability, and NVH, vol. 2, no. 4, pp. 267–283, 2018.
[6] L. Guo, P. Ge, and D. Sun, "Torque distribution algorithm for stability control of electric vehicle driven by four in-wheel motors under emergency conditions," IEEE Access, vol. 7, pp. 104737–104748, 2019.
[7] S. Inagaki, I. Kushiro, and M. Yamamoto, "Analysis on vehicle stability in critical cornering using phase-plane method," ISAE Review, vol. 2, no. 16, p. 216, 1995.
[8] E. Ono, S. Hosoe, and D. Hoang, "Bifurcation in vehicle dynamics and robust front wheel steering control," IEEE Transactions on Control Systems Technology, vol. 6, no. 3, pp. 412–420, 1998.
[9] C. G. Bobier and J. C. Gerdes, "Staying within the nullcline boundary for vehicle envelope control using a sliding surface," Vehicle System Dynamics, vol. 51, no. 2, pp. 199–217, 2013.
[10] Z. Liu, X. Pei, Z. Chen, and B. Yang, Differential Speed Steering Control for Four-Wheel Distributed Electric Vehicle. No. 2019-01-1235, SAE Technical Paper, PA, USA, 2019.
[11] Y. Dai, L. Yu, J. Song, and W. Zhao, The Differential Braking Steering Control of Special Purpose Flat-Bed Electric Vehicle. No. 2019-01-0440, SAE Technical Paper, PA, United States, 2019.
[12] L. Zhang and G. Wu, Combination of Front Steering and Differential Braking Control for the Path Tracking of Autonomous Vehicle. No. 2016-01-1627, SAE Technical Paper, PA, United States, 2016.
[13] Y. Chen, S. Chen, Y. Zhao, Z. Gao, and C. Li, "Optimized handling stability control strategy for a four in-wheel motor independent-drive electric vehicle," Ieee Access, vol. 7, Article ID 17032, 2019.
[14] K. Xu, Y. Luo, Y. Yang, and G. Xu, "Review on state perception and control for distributed drive electric vehicles," Journal of Mechanical Engineering, vol. 55, no. 22, pp. 60–79, 2019.
[15] K. Hartani, Y. Miloud, and A. Miloudi, "Improved direct torque control of permanent magnet synchronous electrical vehicle motor with proportional-integral resistance estimator," Journal of Electrical Engineering and Technology, vol. 5, no. 3, pp. 451–461, 2010.
[16] S. M. M. Jafarani and K. H. Shirazi, "A comparison on optimal torque vectoring strategies in overall performance enhancement of a passenger car," Proceedings of the Institution of Mechanical Engineers - Part K: Journal of Multi-Body Dynamics, vol. 230, no. 4, pp. 469–488, 2016.
[17] A. Goodarzi, S. Amir, and E. Esmailzadeh, "Active variable wheelbase as an innovative approach in vehicle dynamic control," International Design Engineering Technical Conferences and Computers and Information in Engineering Conference, vol. 54853, 2011.
[18] A. A. Adam, "Accurate modeling of PMSM for differential mode current and differential torque calculation," in Proceedings of the International Conference On Computing, Electrical And Electronic Engineering (Iccsee), August 2013.
[19] M. V. Blundell, "The modelling and simulation of vehicle handling Part 3: tyre modelling," Proceedings of the Institution of Mechanical Engineers - Part K: Journal of Multi-Body Dynamics, vol. 214, no. 1, pp. 1–32, 2000.
[20] G. Gim, Y. Choi, and S. Kim, "A semi-physical tyre model for vehicle dynamics analysis of handling and braking," Vehicle System Dynamics, vol. 43, no. sup1, pp. 267–280, 2005.
[21] G. Gim and P. E. Nikravesh, "An analytical model of pneumatic tyres for vehicle dynamic simulations. Part 1: pure slips," International Journal of Vehicle Design, vol. 11, no. 6, pp. 589–618, 1990.
[22] Z. Qi, S. Taheri, B. Wang, and H. Yu, "Estimation of the tyre-road maximum friction coefficient and slip slope based on a novel tyre model," Vehicle System Dynamics, vol. 53, no. 4, pp. 506–525, 2015.
[23] J. T. Tielking, HSRI Digital Computer Programs for Semi-emipirical Tire Models, University of Michigan, Ann Arbor, Michigan, USA, 1974.
[24] K. Guo, D. Lu, S. K. Chen, C. William, and X. P. Lu, "The UniTire model: a nonlinear and non-steady-state tyre model for vehicle dynamics simulation," Vehicle System Dynamics, vol. 43, no. sup1, pp. 341–358, 2005.
[25] H. B. Pacejka and I. J. M. Besselink, "Magic formula tyre model with transient properties," Vehicle System Dynamics, vol. 27, no. S1, pp. 234–249, 1997.
[26] X. Yuan, Integrated Control of Active Steering and Hybrid Braking in Distributed Electric Drive Vehicles, Graduate School of Hunan University, Changsha, China, 2015.
[27] D. B. Antanaitis and L. Brelle, "Braking with a trailer and mountain pass descent," SAE International Journal of Advances and Current Practices in Mobility, vol. 2, pp. 851–869, 2019.
[28] D. E. Rumelhart, G. E. Hinton, and R. J. Williams, Learning Internal Representations by Error Propagation, California
[29] R. Cespi, R. Galluzzi, R. A. Ramirez-Mendoza, and S. D. Gennaro, “Artificial intelligence for stability control of actuated in-wheel electric vehicles with CarSim validation,” Mathematics, vol. 9, 2021.

[30] G. Wang, X. Xu, Y. Yao, and J. Tong, “A novel BPNN-based method to overcome the GPS outages for INS/GPS system,” IEEE access, vol. 7, pp. 82134–82143, 2019.

[31] Y. Rong, Z. Zhang, G. Zhang et al., “Parameters optimization of laser brazing in crimping butt using Taguchi and BPNN-GA,” Optics and Lasers in Engineering, vol. 67, pp. 94–104, 2015.

[32] Y. Li, X. Xu, and W. Wang, “GA-BPNN based hybrid steering control approach for unmanned driving electric vehicle with in-wheel motors,” Complexity, vol. 2018, pp. 1–15, 2018.

[33] R. Jiang, Z. Jin, D. Liu, and D. Wang, “Multi-objective lightweight optimization of parameterized suspension components based on NSGA-II algorithm coupling with surrogate model,” Machines, vol. 9, no. 6, p. 107, 2021.

[34] D. Peng, G. Tan, J. Tang, and X. Guo, “Design and optimization of forced-air cooling system for commercial vehicle brake system,” SAE International Journal of Commercial Vehicles, vol. 15, no. 1, pp. 15–25, 2021.

[35] B. Leng, L. Xiong, C. Jin, J. Liu, and Z. Yu, “Differential drive assisted steering control for an in-wheel motor electric vehicle,” SAE International Journal of Passenger Cars - Electronic and Electrical Systems, vol. 8, no. 2, pp. 433–441, 2015.

[36] E. D. P. V. Puchta, H. V. Siqueira, and M. D. S. Kaster, “Optimization tools based on metaheuristics for performance enhancement in a Gaussian adaptive PID controller,” IEEE Transactions on Cybernetics, vol. 50, no. 3, pp. 1185–1194, 2019.

[37] D. P. E. Puchta, L. Ricardo, R. V. F. Ferreira, and H. V. Siqueira, “Gaussian adaptive PID control optimized via genetic algorithm applied to a step-down DC-DC converter,” in Proceedings of the 2016 12th IEEE International Conference on Industry Applications (INDUSCON), November 2016.