Students Reasoning Capabilities to Solve HOTS Geometry Materials

Esti Setyahastuti¹, Riyadi¹, Triyanto¹

¹ Mathematics Education Department, Sebelas Maret University, Indonesia

Abstract. This study aims to describe students’ reasoning abilities to solve HOTS questions on geometry material. Type of this research is descriptive qualitative. Subjects of the research were 10 students of 12th grade at Senior High School which were taken using a purposive sampling technique. Data collection was obtained from the result of reasoning ability tests and interviews. The validity of the data in this study used a triangulation method. Based on the analysis of data obtained: 40% of students can submit allegations, students that can do mathematical manipulation and give a reason with proof through the solution both of them are 30%, and only 10% of students can draw conclusions from statements and check the validity of an argument. The results of this study indicate that students’ reasoning abilities are still low. Many students are not able to draw conclusions from statements and check the validity of an argument. This is caused students having difficulty determining the distance in space, besides that it has not been able to connect statements that correspond to the problem of question.

Keyword: Reasoning Ability, HOTS Questions, Geometry

1. Introduction

Nowadays, era development is known as disruption and industrial eras can’t be separated from the education rule to build the character and to develop the self-ability. The process of developing self-ability can be obtained by students at school, one of the ways through mathematics learning. According to NCTM (2000) states that there is five learning process standard, they are problem-solving, reasoning, communicating, connecting, and representing. The five standard processes refer to certain mathematical processes that the students are expected to be able to use and acquire mathematical knowledge [1]. Reasoning ability is one of several basic competencies of students in learning, besides having the same vision with mathematics to meet future needs. Based on [2] that students have the ability to reason.

Reasoning ability is the student’s ability to think logically based on mathematical knowledge through the process of identifying, guessing, establishing, connecting, and confirming all information used in solving problems [3]. Reasoning has a role in bloom’s cognitive taxonomy in high-level thinking called HOTS (Higher Order Thinking Skills). Anderson and Krathwohl classify the dimensions of the cognitive process to measure the realm of analysis, evaluation, and creation [4]. HOTS questions can be used to measure metacognitive dimensions, one of them to an argument (reasoning) and being able to make the right decision [5].
The Organization for Economic Cooperation and Development (OECD) announced the results of the Programme for International Student Assessment (PISA) score for Indonesia’s mathematical abilities in 2018 ranked 72 out of 78 countries. From these results, it can be seen that student’s mathematical abilities in thinking at a high level are still low. Prepare 21\textsuperscript{st} century demands by providing high-level thinking skills so that competitive skills can develop and focus on developing HOTS [6]. Students find it difficult to solve mathematical problems with geometric material HOTS forms because students don’t have mathematical skills [7].

The results of the National Examination in Senior High School data for the 2019/2020 academic year show that absorption in the mathematical lesson is low. Geometry material has a relatively low percentage of absorption. In the National Examination results, the absorption of geometry material was 34.59 \%, it shows that the students have problems with geometry material. Based on the indicators tested on material geometry shows that the absorption of students is still low. In the geometry material to determine the distance of points to lines and points to the planes, the percentage of absorption is 30.85 and 49.40 [8]. The students experience misconceptions about knowledge and geometrical concepts so that it impacts on subsequent learning that experiences obstacles [9]. The National Mathematics Examination questions contain three levels, consists of understanding, application, and reasoning. These three levels are very required in mathematics because each other is interrelated to build students' mathematical abilities. National Examination results show that students' mathematical abilities are still low.

Based on observations, students are not able to use their reasoning power optimally. When learning mathematics students tend to take notes and listen to the material presented, not found activities that can stimulate students' reasoning ability to solve problems. Students only refer to the method and solutions that have been obtained, if students get problems that are different than usual that students will find it difficult to solve the questions. Many students assume that the question has never been discussed and given or taught how to solve it. Based on the results of students’ daily tests on the material determines the distance between points, points to lines, and points to planes are still low because there are not many students get a score above the standard. Seen the students have not good mathematical abilities, especially the ability to reason. The ability of reasoning in patterns and properties, mathematical manipulation, generalization as well as explaining ideas and statements can be done by students to train and apply reasoning in learning [10].

Therefore, the reasoning is very important for the mathematical problem-solving students, especially in the HOTS questions form. The use of reasoning in mathematics learning is expected to train students’ reasoning abilities to be better so that they have intelligent and insightful thinking [11]. Student’s reason abilities can be trained by often giving diverse mathematical problems, so the students become accustomed to solving the problems [12]. The students of High School are not used to giving mathematical reasons because based on the prior mathematical knowledge students’ abilities are related to the reasoning and proof are not formed and accustomed to learning [13].

The mathematic lesson is not only on practicing procedural activities and memorizing the formula but it is also about reasoned student’s skill or critical thinking and creativity [14]. On reasoned activities, students who have not able to use the relation of pattern to analyze the condition, making analogical or doing generalization. By giving the mathematical problems routinely and which is discussed by the teacher intensively giving a positive impact on the students reasoning skill achievement [15]. The school curriculum development also takes an important role to train the student reason skill, because in curriculum consists of reasoning skill material [16]. The presentation of geometry concept is expected to train the reasoning student’s skill, so geometrical knowledge, visual perception, and logical arguments can increase [17]. Therefore, the intention of this research is to identify the student’s reasoning ability to solve HOTS questions on geometry material at Senior High School.
2. Method
The methodological of this research is qualitative descriptive. Qualitative descriptive research places more emphasis on checking in depth by conducting understanding in students such as their behavior, experiences, and explaining a situation that occurs [18]. Validity is done by collecting data from various sources to find out what will be studied, referring to the research design to avoid obtaining data that is distorted. This study uses a triangulation method, which is collecting data from several sources as well as various data collection methods such as observation, interviews, and documentation [19]. The data collecting technique is using test method and interview method. The test method use geometry material essay test, then interviewing in semi-structured type. Geometry material essay test based on HOTS. HOTS essay test on geometry material have been validated by mathematics lecturers adjusting indicators of reasoning ability and higher-order thinking skills level. The HOTS question compilation module is also used as a reference for preparing the instrument. The intention of this research is to identify the student’s reasoning ability through solving HOTS questions on geometry material at Senior High School. The subject of this research are ten students of grade 12 through subject election using a purposive sampling technique. Here is the indicator to analyze the reasoning students ability, there are: 1) submitting the assumption, 2) doing the mathematic manipulation, 3) giving the reason and proof through the solution, 4) drawing the conclusion from the statement, and 5) checking the validity of an argument [20].

3. Results and Discussion
The results were obtained based on students' reasoning ability test data and interviews. The test is given in the form of description questions according to HOTS level, namely analyzing, evaluating, and creating. The material used is geometry. Geometry material about calculating the distance between points, the distance of points to lines, and the distance of points to planes. Students' reasoning abilities are needed in solving given geometry problems, so students must analyze the problems properly by giving reasons, besides being able to provide evidence accompanied by students' opinions. The following is an achievement of the reasoning ability indicator shown in Table 1.

| Indicator                                      | Percentage (%) |
|-----------------------------------------------|----------------|
| Submitting the Assumption                     | 40             |
| Doing the Mathematic Manipulation             | 30             |
| Giving the Reason and Proof Through the Solution | 30             |
| Drawing the Conclusion from the Statement     | 10             |
| Checking the Validity of an Argument          | 10             |

3.1. Analysis problem number 1
In question number 1, only several students can solve and give reasons that match the problem. The student can write down all possible distances between the points of the LED lighting in a pyramid frame and be able to give a good reason. In addition, there are students who only find a few possible distances between points but are able to give appropriate reasons. There are still many students who are not able to finish in accordance with the basic concept of point to point distance, so students make drawings that are not suitable for calculating the length of a particular line with the Pythagoras theorem.
Results of written work and interviews S1

Figure 1. Answers of S1

Dialog 1 from S1 Interview

1. P: What do you understand from problem number 1?
2. S1: Looking for possible distance, miss.
3. P: How many possibilities do you have?
4. S1: Five.
5. P: Try to explain these five possibilities.
6. S1: First find a path that can be passed from point A to point C.
   There are A to C, A-B-C, A-D-C, A-D-B-C, A-B-D-C, then the length is calculated.
7. P: Why did you choose 20 cm as the right distance from point A to C?
8. S1: Because that's the closest distance, the more distance added, the more results.

Based on the results of work and interviews, S1 is able to analyze the problem sentences well, so that they can submit allegations in accordance with the problem. S1 writes down a number of possible paths that can be crossed with a starting point at A and ends at C, the answer can be seen in Figure 1. Then S1 calculates the distance of the possible path that was passed from point A to C by adding up the length of the internodes between the points. In addition, S1 is able to provide reasons with appropriate evidence, because it chooses the distance from point A to C, which is 20 cm. S1 chooses the distance because it is the closest line segment from one point to another or the shortest distance (look at Dialog 1).

Results of written work and interviews S2

Figure 2. Answers of S2

The right distance to install LED decorative lights from Point A to Point C is the ADC line, because it has the closest distance from Point A to Point C.
Dialog 2 from S2 Interview
(1) P : Try explaining about question number 1?
(2) S2 : Asked the distance from A to C.
         Can be from A-D to C and A-B to C.
         Then look for the middle first.
(3) P : How to?
(4) S2 : Using the Pythagorean formula.
         Because in the middle half BC.
(5) P : Why did you get these two pictures?
(6) S2 : From the triangle ADC and ABC image, miss.
(7) P : How can be like that?
(8) S1 : I don't know, miss.

According to the result of work and interviews, S2 did not comprehend the problem yet, so S2 did the error by making the new sketch which is different from the problem. Finally, S2 made two triangles then it was completed by counting the line from point A to point O and point A to point P. S2 hadn’t able to submit the problem assumption well, so making the two sketches are false. Other than that S2 could not give the reason regarding the prior sketch. S2 used the Pythagoras rule that doesn’t have a relation with the problems because it was not able to comprehend the problem of the sketch and count the distance between the points. From the result, S2 still had difficulties in counting mathematical operation well (regard in Figure 2).

The results of S1 and S2 appear to have differences from the completion process. In question 1st number the indicator of reasoning ability is about submitting the assumption and giving the reasons with proof through the solution. S2 answers differ from the purpose of the question command. S2 makes ABCD pyramid images into ABC triangles and ADC triangles (flat shape). S2 encountered a visual error, where the shape of the space presented in the two dimensions. Understanding the wrong problem resulted in the process and method of resolution used. Submitting the assumption to the problem is very important to do the preliminary analysis as the main step before working. On indicators giving the reasons and proof through the solution, S1 can write answers with clear reasons. While S2 writes the reasons but it is not right and different from the concept of distance between points.

3.2. Analysis problem number 2

Through solving question number 2, many students who were not able to do well, even some students only draw the pyramid or did not draw it at all. Many students had difficulties to determine the distance of a point to the plane, so they were not able to find the projection line from point O to the TBC plane yet.

Results of written work and interviews S3

![The distance of point O to the TBC Plane](image)

Figure 3. Answers of S3

Dialog 3 from S3 Interview
(1) P : What are the steps for your work in solving problem number 2?
(2) S3 : Confused miss, because there are no numbers
(3) P : Does the TP line you get represents the distance of the point O to the TBC plane?
(4) S3 : I think not yet, miss.
(5) P : Is TP line needed to calculate the distance of the point O to the TBC plane?
(6) S3 : Maybe Miss, because TP line in the TBC plane.
I’m confused about the next method miss, to determine the distance of the point O to the TBC plane.
(7) P : Are the square root of the results that you get is correct?
(8) S3 : It seems already

According to the result of work and interviews, S3 was not able to solve the problem well. S3 only counted the line from point T to point P, then they did false mathematic manipulation. It seemed from the wrong TP lines calculation (Figure 3), S3 made a mistake in calculating the square root. Based on the interview, S3 is still confused to determine the distance of a point O to the TBC plane and is not able to continue the next solving way. S3 had difficulties solving number 2 because it doesn’t have a number as information that can be count as the usual problem has been done (look at Dialog 3). So S3 was not able to do mathematical manipulation on 2nd number yet.

The indicators of reasoning ability in question number 2 about mathematical manipulation. Many students have trouble solving this problem because they do not have numbers. Besides that, the students have trouble doing calculations with symbols. Mathematical manipulation is used as a process to find the results. The process of mathematical manipulation by students on this problem is still low, because of a lack to do practice various questions. Some students are still having trouble doing mathematical operations properly and correctly. The problem in question number 3 requires careful thinking and reasoning.

3.3. Analysis problem number 3
On the 3rd number, many students felt the difficulties solving the problems. It was seen at the student work-sheet. Some students are confused to determine true statements. Specially adjusted for the determining steps of distance point F to the BEG plane. Many students only draw cubs form and they didn’t write the right statements option and the reasons.

Results of written work and interviews S4
b. Connected F with BD, obtained BF ⊥ BD
   > true BF ⊥ BD in point B

c. Connected F with BE, obtained the intersection in point P,
   FP represents F
   > False FP does not represents F → BEG because FP is not the closest distance to PF, but F emphasis with BEG (FG)

Figure 4. Answers of S4

Dialog 4 from S4 Interview
(1) P : In question number 3, which statements is correct?
(2) S4 : The statement (a) and (b) are true, then (c) is false, true, true
Just statement (c) is wrong
(3) P : Why the statement (c) is wrong?
(4) S4 : Because FP is not representative of F and FP is not the closest distance, where is the point P?
Because point P has nothing to do
(5) P : Why the statement (b) is true?
(6) S4 : That’s right, this angle is 90, FB perpendicular BD
(7) P : Is the statement (b) a step that fits the problem?
(8) S4 : Yes, the distance to Q is not to P
Results of written work and interviews S5

Figure 5. Answers of S5

Dialog 5 from S5 Interview
(1) P : In question number 3, which statement is correct?
(2) S5 : Statement (c) is true
(3) P : Why only statement (c) is true?
(4) S5 : Because the point P in the middle of the BEG is the one near the FP point. FP represents the distance from point F to BEG
(5) P : Why the other statement untrue?
(6) S5 : Because the statement (c) already represents distance.

According to the result of work and interviews, S4 was able to check the problem statement validity well, but they were not able to give a good conclusion. It has one false statement justified by S4, there was statement (b) which did not any correlation with statements (a), (d), and (e). S4 also wrote the false steps included their reasons (the answer can be seen in Figure 4). On the other hand, S5 was not able to solve the problem well, S5 only wrote one statement which was true, c statement. It is seen that S5 was not able to check each statement well (Figure 5), so it couldn’t find what are the right steps, so it could determine the distance of point F to the BEG plane.

The indicator of reasoning ability in question number 3 is about drawing conclusions from statements and checking the validity of an argument. S4 and S5 have not been able to check the question statement. S4 writes the answers with incorrect reasons, while S5 only writes one statement of questions. It seems that students have not able to examine the statement of questions about a problem. Based on students' explanations, there is no precise and appropriate explanation yet. Test results and interviews show students do not conduct checks on every statement, but only choose statements that they think are true without any conclusions (look at Dialog 4 and Dialog 5).

Mathematical problems about the three dimensions become a challenge for students, because the ability of students in reasoning the shape, location, and position of both points, lines, and planes are very influential to obtain solutions. Especially the third dimension presented in the second dimension becomes an obstacle for students because not a few people make mistakes. Errors experienced by students include the use of settlement methods, formulas and determine a line that represents the distance, both the distance between points, points to lines, and points to planes. In this material, some students only apply the Pythagorean theorem formula and trigonometric formulas, because they have a misunderstanding about the purpose and objectives of the problem image.

4. Conclusion
Based on the analysis of 10 students, it shows that students who can submit allegations of 40%, students who are able to do mathematical manipulation and give a reason with proof through the solution by 30%, and only 10% of students are able to draw conclusions from statements and check the validity of an argument. From the results of students' work and interviews, it can be seen that students' reasoning abilities are still low. Some students were unable to use reasoning skills in solving problems that are more varied or different from the problems they often encounter. Only a few students can do mathematical manipulation of the problems given, especially student’s have difficult to work the
questions that do not have numbers or sizes. Students assume that the problem is difficult to do, even some of them do calculations that are not by the concept of the material. Students were unable to check and choose which statements are true and appropriate to the problem given problem. Students have not able to understand abstract mathematical problems, which require good analysis and logic. Besides that, systematic in the process of doing mathematical problems is also a supporting factor in obtaining problems and solutions.

The development of science nowadays is very much in need of students' mathematical abilities, one of which is reasoning. Based on the current assessment students are still unable to express their opinions and reasons for the results of their work. Reasoning is inseparable from an analytical thought process of a problem faced especially new problems that have never been encountered or experienced. It is better to improve reasoning skills by training to solve a more varied problem and mathematical problems. At least with these efforts, students are able to gradually be trained to use the ability of reasoning because the process of reasoning students is needed to deal with diverse problems both in knowledge and for the daily lives of students.

References

[1] NCTM, “Principles and Standards for School Mathematics”, Reston: VA NCTM, 2000.
[2] BSNP, “Standar Kompetensi dan Kompetensi Dasar Pendidikan Dasar dan Menengah”, Jakarta: BSNP, 2006.
[3] E. S. W. Astuti, et al, “Mathematical Reasoning Ability Based on Self Regulated Learning by Using The Learning of Reciprocal Teaching With RME Approach”, Unnes Journal of Mathematics Education Research, vol. 8, no. 1, pp. 49-56, 2019.
[4] L. W. Anderson and D. R. Krathwohl, “A Taxonomy for Learning, Teaching, and Assessing Revision of Bloom’s Taxonomy of Educational Objectives”, Allyn & Bacon Boston: MA Pearson Education Group, 2001.
[5] I. W. Widana, “Modul Penyusunan Soal HOTS”, Jakarta: Direktorat Pembinaan SMA Ditjen Pendidikan Dasar dan Menengah, 2017.
[6] W. Conklin, “Higher-order thinking skills to develop 21 st century learners”, Huntington Beach: Shell Education Publishing, Inc, 2012.
[7] R. K. N. Karimah, “Analysis of difficulties in mathematics learning on students with guardian personality type in problem-solving HOTS geometry test”, International Conference on Combinatorics, Graph Theory and Network Topology 1008, 2018.
[8] BNSP, “Laporan Hasil Ujian Nasional Tahun Pelajaran 2018/2019”, 2019.
[9] A. Ozerem, “Misconceptions in Geometry and Suggested Solutions for Seventh Grade Students” International Journal of New Trends in Arts, Sports & Science Education, vol. 1, no. 4, pp. 23-35, 2012.
[10] S. I. Hasanah, “Mathematical Reasoning: The Characteristics of Students’ Mathematical Abilities In Problem Solving” J. Phys.: Conf. Ser. vol. 1188, no. 012057, 2019.
[11] Shadiq, “Pemecahan Masalah Penalaran dan Komunikasi”, Yogyakarta: Makalah Penataran Guru PPPG, 2004.
[12] E. Surya, “An Analysis of Students’ Mathematical Reasoning Ability In VIII Grade of Sabilina Tembung Junior High School” International Journal Of Advance Research And Innovative Ideas In Education, vol. 3, pp. 2395-4396, 2017.
[13] D. Zhang and C. Qi, “Reasoning and Proof in Eighth-grade Mathematics Textbooks in China”, International Journal of Educational Research, vol. 98, pp. 77-90, 2019.
[14] A. Minarni and E. E. Napitupulu, “Developing Instruction Materials Based on Joyful PBL to Improve Students Mathematical Representation Ability”, International Education Studies, vol. 10, no. 9, pp. 23-38, 2017.
[15] E. E. Napitupulu, “Analyzing the Teaching and Learning of Mathematical Reasoning Skills in Secondary School”, Asian Social Science, vol. 13, no. 12, 2017.
[16] A. Zaman, et al, “Predictive Validity of Scores in Mathematics for Reasoning Ability in Mathematics for Grade 9 Students in Khyber Pakhtunkhwa Based on Curriculum of Mathematics”, *Procedia-Social and Behavioral Sciences*, vol. 12, pp. 588-594, 2011.

[17] B. C. Gunhan, “A case study on Investigation of Reasoning Skills in Geometry”, *South African Journal Of Education*, vol. 34, no. 2, 2014.

[18] H. Nassaji, “Qualitative and Descriptive Research: Data Type Versus Data Analysis”, *SAGE Journals*, vol. 19, no. 2, pp. 129-132, 2015.

[19] P. Fusch, et al, “Denzin’s Paradigm Shift: Revisiting Triangulation in Qualitative Research”, *Journal of Social Change*, vol. 10, issue 1, pp. 19-32, 2018.

[20] N. R. Rizqi and E. Surya, “An Analysis of Students Mathematical Reasoning Ability In VIII Grade of Sabilina Tembung Junior High School” *International Journal Of Advance Research And Innovative Ideas In Education*, pp. 2395-4396, 2017.

**Acknowledgements**

Thanks a lot to Sebelas Maret University for supporting this research. I’m very grateful to Senior High School 1 Wuryantoro for its availability as research data.