The application of vector concepts on two skew lines

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Abstract. The purpose of this study is knowing how to apply vector concepts on two skew lines in three-dimensional (3D) coordinate and its utilization. Several mathematical concepts have a related function for the other, but the related between the concept of vector and 3D have not applied in learning classroom. In fact, there are studies show that female students have difficulties in learning of 3D than male. It is because of personal spatial intelligence. The relevance of vector concepts creates both learning achievement and mathematical ability of male and female students enables to be balanced. The distance like on a cube, cuboid, or pyramid whose are drawn on the rectangular coordinates of a point in space. Two coordinate points of the lines can be created a vector. The vector of two skew lines has the shortest distance and the angle. Calculating of the shortest distance is started to create two vectors as a representation of line by vector position concept, next to determining a norm-vector of two vector which was obtained by cross-product, and then to create a vector from two combination of pair-points which was passed by two skew line, the shortest distance is scalar orthogonal projection of norm-vector on a vector which is a combination of pair-points. While calculating the angle are used two vectors as a representation of line to dot-product, and the inverse of cosine is yield. The utilization of its application on mathematics learning and orthographic projection method.

1. Introduction

An issue about STEM (Science, Technology, Engineering, and Mathematics) still to be discussed until now and continues to be consistently a trending topic, especially in education since the beginning emergence of the last few decades. However, the focus of the discussion is on gender gaps which are the percentage of career opportunities for women in STEM areas are lower than men [8,11]. The reason isn’t a historical heritage as a reflection where are women not to be encouraged to be a worker [7], but it is a spatial intelligence problem [2,10]. The occurrence of gender gaps has been caused more than the expectation of men careers related to physics, mathematics, or engineering, wherever more than the expectation of women careers in science or health-related professions [7,8,11], So that the professional areas of occupied bring out many gender similarities within a scope of work [7].

However, there is a development of the condition in Indonesia that the student who comes from a vocational high school still free to choose a faculty in the university program, which isn’t rare to be found that the faculty of the students chosen was not relevant to the program or the vocational practiced at their senior high school (SMA) or vocational education. This condition is due to several
factors including the maturity, self-regulated, and thinking skills are matured [3], so there is a tendency that student was realized about their talent and interest whichever they did, and the students will be brave to take a decision and choose a department on university program with their intelligence. This becomes interesting because still there a chance for a female student to get study at STEM areas which will become a focus of their career in the future. However, this chance will opponent with the founded at some case studies result, that is gender gap happened when an education system gives a freedom to students to choose majors [8,11].

Efforts to reduce the gender gap in STEM areas is recommended to get involved with parents for exploring their career [8] because parents who work in the STEM areas will reflect his talent even to his daughters. Nevertheless, the involvement of parents also should not exceed the limits of children abilities, because educational selected often as a selection of parents and not compatible with their child and then their child can’t be responsible [3]. The gender gap in STEM areas is started on many options of high schools with science program as a possible selected so there are pursuing career gaps generated like in engineering area [7,8,11,12]. The problem of the gender gap marked by career expectation to an achievement of educational and working in STEM areas [11,12], this is shown by a little number of female representations [7]. The possible impact on gender gap because the option and working placement occur in old ages and then an ability and interest in STEM area to be down [12]. In addition, senior high school structural system can create to be bad of gender gap [11], so the initiative promote gender equality in STEM must be focused on senior high school structural system [8].

In addition to a focus on the gender gaps, the STEM is also influenced by dynamic of science development, so that should be accompanied by a demand on human resources with competence in particular areas of the STEM [5,12]. Mathematics has contributed ideas in the development of STEM. Application of mathematics in STEM areas give a view that mathematics is one of the scaffolding its development because science is closely related to mathematics and technology [1]. The advantages of mathematics learning are very well in daily life [9,10] because its integration involves particularly STEM subjects [9], so mathematics is one of the important subjects in the development of STEM [5,9,12]. Not only beneficial to the development of science and technology but also mathematics is beneficial to self-development. In this case, exemplified like the application vector concepts on 3D learning then mathematics has much beneficial and utilization. Vector is the findings by Josiah Willard Gibbs which has a great contribution to mathematics area [2,4]. Nevertheless, learning of 3D in senior high school by utilizing the geometric concept of the plane and involves of technology for learning assisted that is based on spatial intelligence. The effort to give a facility for students as a mean [2], whereas the problem on geometry can be solved easily by vector concepts [4].

The utilization of science and technology is one of the keys as a preparation to face times today [9]. Science development is important in factor variety and easiness especially in helping students overcome their learning obstacle. Students will not face an obstacle and difficulties on learning of 3D when they have a good spatial intelligence [10]. A part of all students may feel difficulties, it will impact on geometric idea development and learning achievement when was seen on gender views, so it needs more than understanding and strategy, one of all by applying the vector concept [2]. Writing this article aims to know how to apply vector concepts on two skew lines and its utilization. Two skew lines to be known the shortest distance between of them is one of the discussions in learning of 3D in senior high school. Mathematics curricula in Indonesia set vector as a chapter which is studied in basic mathematics, while the 3D chapter in additional mathematics and both have interrelated. Application of vector on 3D allows for a balance between students’ mathematical abilities of male and female [2] so that the gender gap in STEM areas do not continue to be the focus of the discussions.

2. Methods
This study was focused on several articles which published at the end of the year from 2010, related to examples of the application of vector on STEM areas utilization, and vector concepts application. Next, reading and relating its article content. This study is describing how to applying vector concepts on two skew lines and the utilization of STEM area. This study is analyzing the calculation of the
distance between two skew lines for three examples of the different geometrical object, that is a cube, cuboid, and pyramid, to find a common step calculation of the distance and how many degrees of the angle. Each a geometrical object is drawn on the rectangular coordinates of a point in space (3D coordinate) with the length of an edge is the one, so the vertex of a geometrical object can be named on the 3D coordinate, and two points of an edge can be represented in vector form. Next, vector concept will find both how long of the shortest distance and how many degrees of the angle on two skew lines.

3. Results and Discussion

Basically, the conventional 3D learning in senior high school by drawing 3D objects manually on the board in front of the class by a teacher and without helping of technology [2], is a part of practices as drawing, spatial thinking, spatial transformation, create a representation of a model, and arithmetic [10]. Its practices are directed as self-development of students in STEM areas. Because mathematics learning based on STEM education is a learning which the integrated design and expected a contribution to improving the competitiveness of Indonesian human resources in all areas [5,9]. Conventionally, mathematics learning is a study about drawing techniques that require 3D effects for adding magnificence of image and then getting the best outcome. Some technique of drawing like compatibility of orthogonal comparison and horizontal visual angle on the drawing orthogonal lines on 3D objects. The orthogonal comparison refers to the comparison of the length of an orthogonal line on the image and the actual. The orthogonal line is the line which perpendicular to the plane of the image. The angle of inclination is an angle between horizontal lines and orthogonal lines.

3D Object defined the object which has a spatial dimension on the 3D coordinate like on solid, the examples are a cube, cuboid, and pyramid [2]. On the 3D object, can be created two skew lines where have both how long of the shortest distance and how many degrees of the angle on two skew lines. Both of them can be calculated by applying vector concept. Application of vector is one of all alternative ways for determining how long distance and how many degrees of angle on two skew lines, for students who have a spatial disability problem on 3D learning conventionally [2].

3.1. Calculate Distance and Angle Between Two Skew Lines on 3D Objects

The means of two skew lines refers two lines which can be created on the 3D coordinate and it does not intersect [2]. Two skew lines are the example of the 3D object. On a cube, cuboid, or pyramid, can be created two skew lines within a distance between of them. The distance of between two skew lines is calculated by applying vector analysis concepts where the 3D object like a cube, cuboid, or pyramid is the first step was made on 3D coordinate with the length of an edge which is according to need [2].

Figure 1. Cube ABCD EFGH on 3D coordinate axis with a length of the edge in 4 units.
Based on the coordinate points of the cube in Figure 1 above, the length of an edge of a 3D object is 4 units. On a cube can be created a variety of pair of two skew lines from a combination of four vertices. Not only on the vertices of figure 1, but also another vertex whose new can be made for one need of calculating the shortest distance between two skew lines, and a note that its vertices on a cube and the 3D coordinate can be determined. Wherever the position of coordinate points of two skew lines on a cube, vector concepts application still can be used in calculating the shortest distance on between of them. The application can apply also to the cuboid and pyramid [2]. For example of two skew lines on the cube is a line of $CE$ and $AF$. The way of calculating the distance between its line with vector concept application is starting by making two skew lines in a representation of vector respectively [2].

$$CE = AE - AC = (-4, -4, 4) \text{ and } AF = (4, 0, 4).$$

The shortest distance between two skew lines is the length of a line whose are perpendicular to its lines. On two skew line, can be determined one vector whose are perpendicular to its lines. The vector is commonly known as a norm-vector. Norm-vector is cross-product of two representation of vector respectively as a line of $CE$ and $AF$. The following is the calculation of the norm-vector.

$$CE \times AF = n = (-4i - 4j + 4k) \times (4i + 4k)$$
$$= -16(i \times k) - 16(j \times i) - 16(j \times k) + 16(k \times i)$$
$$= -16(-j - k) - 16i + 16j$$
$$= -16i + 32j + 16k$$
$$= \lambda(-1, 2, 1).$$

On figure 2, made the illustration of norm-vector ($\nu$), the line of $CE$ and $AF$ as the line whose skew, points of $P$ and $Q$ as the point of intersection between the norm-vector with two skew lines, and then the line of $PQ$ whose is perpendicular to two skew lines and collinear on norm-vector. Based on the figure, the shortest distance between two skew lines is the length of $PQ$ [2].

![Figure 2](image)

**Figure 2.** The intersection of two skew lines and norm-vector at points of $P$ and $Q$.

Some of the vector concepts related to perpendicularity are dot-product, cross-product, norm-vector, scalar orthogonal projection, and vector orthogonal projection [2,4]. Because of the one which is calculated to be a scalar, then the shortest distance between two skew lines is a scalar orthogonal projection of vector $FC$ on norm-vector, or vector $FE$ on norm-vector, or vector $AE$ on norm-vector, or vector $AC$ on norm-vector. The scalar which is obtained by some calculating options is the same value [2]. So that one of all is chosen, it is a scalar orthogonal projection of vector $FC$ on norm-vector, where

$$\vec{FC} = (0, 4, -4), \text{ then } |\vec{FQ}| = \vec{FC} \cdot n = \lambda(0 + 8 - 4) = \frac{4}{\sqrt{6}} = \frac{2}{3} \sqrt{6} \text{ units.}$$

In addition, to calculating the shortest distance, the application of vector concepts on 3D objects also can be used to calculating of how many degrees of the acute angle between two skew lines. The
degree of the angle is calculated with dot-product of vector concept [2,4], where are two vectors is meant refers to a representation of a line of $CE$ and $AF$. So the angle is,

$$\theta = \cos^{-1} \left( \frac{CE \cdot AF}{||CE|| \cdot ||AF||} \right) = \cos^{-1} \left( \frac{-16 + 0 + 16}{\sqrt{48} \sqrt{32}} \right) = 90^\circ.$$

The way of calculation by vector concept to determine the shortest distance of two skew line isn’t only on a cube but also applies to a cuboid and pyramid [2]. Here is a figure of cuboid $ABCD EFGH$ with the length, width, and height respectively is 8, 4, and 3 units.

![Figure 3. Cuboid $ABCD EFGH$, length, width and height are 8, 4 and 3 units respectively.](image)

Suppose a sample taken of two skew lines are the same as on the cube in above, it is the line of $CE$ and $AF$. In units of length, the shortest distance between the line of $CE$ and $AF$ on a cube is different on a cuboid, because of the length of an edge are also different, so a representation of vector on the lines is different exactly, as shown below.

$$\vec{CE} = (-8,-4,3) \text{ and } \vec{AF} = (8,0,3).$$

Calculating steps is same like on a cube, it is for finding a norm-vector in the cross-product way of two representation of vectors in above.

$$\vec{CE} \times \vec{AF} = n = (-8i - 4j + 3k) \times (8i + 3k)$$

$$= -24(i \times k) - 32(j \times i) - 12(j \times k) + 24(k \times i)$$

$$= -24(-j) - 32(-k) - 12i + 24j$$

$$= -12i + 48j + 32k$$

$$= \lambda (-3,12,8).$$

After that, the shortest distance between the line of $CE$ and $AF$ on a cuboid, and let $PQ$ is it, it can be directly calculated by scalar orthogonal projection, with

$$\vec{FC} = (0,4,-3) \text{ and then } \vec{PQ} = \frac{\vec{FC} \cdot n}{||n||} = \frac{\lambda(0 + 48 - 24)}{\sqrt{(-3)^2 + 12^2 + 8^2}} = \frac{24}{\sqrt{217}} = \frac{24}{\sqrt{217}} \text{ units.}$$

Next, calculate the angle between two skew lines were also in the same way like in a cube, using a dot-product, so obtained,

$$\theta = \cos^{-1} \left( \frac{\vec{CE} \cdot \vec{AF}}{||\vec{CE}|| \cdot ||\vec{AF}||} \right) = \cos^{-1} \left( \frac{-64 + 0 + 9}{\sqrt{89} \sqrt{73}} \right) = \cos^{-1} \left( \frac{-55}{\sqrt{6497}} \right) \approx 39.4^\circ.$$
On 3D learning conventionally, in abstraction process, there is a little of difficult to find a 2D representation of the results a spatial transformation form on two skew lines. Perpendicularity concept and the projection ways must be thoroughly understood well because it is the key to find a spatial transformation form and shortest distance. The similarity to another of a 3D geometric object like a pyramid, thus applying vector concepts could be one solution of all. For example, calculating the distance between the line of \( AD \) and \( TB \) at pyramid \( T.ABCD \) where is a square as a base, the length of an edge on base and the height is 4 units, is drawn on below.

**Figure 4.** Pyramid \( T.ABCD \), base form square, length of an edge on base and altitude are 4 units.

Based on figure 3 in above, the altitude of a pyramid is the line which is pull up from at intersection of base diagonals as an original point to top. According to 3D coordinate, the point of that intersection position at \((2, 2, 2)\). Because of the altitude is 4 units, so the distance from the vertex to the plane of \( ABCD \) is 4 units, on position of 3D coordinate at \((4, 2, 2)\), and then a representation of vector of line \( AD \) and \( TB \) are,

\[
\vec{AD} = (0, 4, 0) \text{ and } \vec{TB} = (2, -2, -4).
\]

Then, in the same way, like on cube and cuboid, calculating of the norm-vector of two vectors in above like at before, and obtained,

\[
\vec{AD} \times \vec{TB} = n = (4j) \times (2i - 2j - 4k)
\]

\[
= 8(j \times i) - 16(j \times k)
\]

\[
= 8(-k) - 16(i)
\]

\[
= -16i - 8k = \lambda(2, 0, 1).
\]

Therefore, the shortest distance between lines of \( AD \) and \( TB \) on pyramid \( T.ABCD \) if the length of \( PQ \) is supposed that, is

\[
\vec{AB} = (4, 0, 0) \text{ and then } |PQ| = \frac{\vec{AB} \cdot n}{|n|} = \frac{\lambda(8 + 0 + 8)}{\lambda \sqrt{2^2 + 0^2 + 1^2}} = \frac{8}{\sqrt{5}} = \frac{8}{5}\sqrt{5} \text{ units.}
\]

Similarly to calculating how many degrees of the angle between lines of \( AD \) and \( TB \) on pyramid \( T.ABCD \), isn’t different with the way of calculating how many degrees between two skew line on a cube or cuboid, like below.
\[
\theta = \cos^{-1} \frac{\vec{AD} \cdot \vec{TB}}{\|\vec{AD}\| \|\vec{TB}\|} = \cos^{-1} \frac{0 - 8 + 0}{\sqrt{16} \sqrt{24}} = \cos^{-1} \frac{-8}{8\sqrt{6}} \approx 65.9^\circ.
\]

Thus, calculating how long of the shortest distance and how many degree of the angle between two skew lines on solid like a cube, cuboid, or pyramid, by applying vector concepts, it will help to be easy for student who have difficulties on spatial problem, because the calculation for three 3D objects is using the same of steps and ways, so this learning technique can be used on 3D learning at senior high school then learning achievement in this chapter especially for female student don’t have any problem.

All of the 3D objects in the example, one vertex for each solid figure is always in the position of \((0,0,0)\). However, it shouldn’t always be like that, because wherever a position coordinate of a 3D object and it is different of points, but the length of the edge is according to all, and then the vector of representation two skew lines will always be same, so the position of solid on 3D coordinates might be wherever. Anyhow, the application of vector on the calculation of the shortest distance and the angle isn’t applicable to parallel lines, because two parallel lines have the same vector [2,4]. However, knowing the distance between two parallel lines may be not difficult because the level of abstraction and spatial thinking is not like the skew lines, and the angle of its projection shape 0 or 180 degrees certainly [4].

### 3.2. Utilization in STEM Area

In mathematics, arithmetic techniques utilization of shortest distance two skew lines can be applying on learning at senior high school in the 3D chapter for the additional mathematics subject group. Importance is an alternative learning to help students who tend to have problems in spatial thinking in the space object of 3D. Calculating for the third space object of 3D with the step and the same way, so it will facilitate learning achievement, especially for female students, and mathematics performance of male and female students will be possible to be balanced. By vector concepts, students neither need to imagine the position of two skew lines on 3D objects nor think to create a spatial transformation form that is generally a two-dimension figure as a spatial orientation two skew lines on a solid. Students only need 3D coordinate points that formed two skew lines and understanding of vector analysis concepts. However, this calculation technique isn’t a priority on spatial intelligence training [2].

Beginning to applying this calculation technique, students who have a problem to imagine of two skew lines position on the 3D object, they will be led to find coordinate points of a 3D object at a figure in the classroom. If it does often repeatedly then students will be familiar with the position where the points on 3D coordinate so it will form a student imagination. This habit is a training method because a training can improve spatial thinking ability [2].

On technology area, vector application to calculating the distance between two skew lines is used for estimating a position of intersection point accurately of two virtual lines. This estimation is called by orthographic projection method where is more excellent than conventional perspective geometric method. Orthographic projection method used a mathematical vector calculation on a remote eye tracking system. This is an indication of mathematics development to continue showing its existence and facilitate of level achievement accurately better than conventional ways. It is important because human vision orientation on 3D way show the focus and attention of observing to objects that used a device with the perspective of the geometric conventional algorithm [6].

### 4. Conclusion

Based on results and discussion, application of vector on skew line is using to calculate the shortest distance and the angle between its lines. Three examples are in applying on solid, it is the way of applying vector concept in calculating the distance between two skew lines on a 3D object like a cube, cuboid, or pyramid, so that can be created a common steps of calculating: (1) two skew lines that each pass through at least two points on 3D coordinate was known, with those points, the line is
created in vector representation respectively; (2) determining norm-vector by cross-product from two vectors was formed; (3) determining a vector from two points, the vector is formed by two points which was a combination of pair-points that was passed by two skew line; (4) the shortest distance between two skew lines is scalar orthogonal projection of norm-vector on a vector which is a combination of pair-points. Utilization application of vector on two skew lines as an alternative 3D learning in senior high school to help students who have spatial disabilities in solving a problem on determining the distance of a 3D geometrical object. On technology area, is used in orthographic projection method to estimate of intersection point position accurately of two virtual lines in 3D coordinates.

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