Bouncing Braneworld with Born-Infeld and Gauss-Bonnet

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ABSTRACT

We show the existence of some bouncing cosmological solutions in the braneworld scenario. More specifically, we consider a dynamical three-brane in the background of Born-Infeld and electrically charged Gauss-Bonnet black hole. We find that, in certain range of parameter space, the brane universe, at least classically, never shrinks to a zero size, resulting in a singularity-free cosmology within the classical domain.

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Singularity-free cosmological solutions are very rare within the framework of the conventional Einstein theory of gravity. It is however believed that at a short distance scale, Einstein action would require modifications. These modifications can, in principle, have cosmological consequences and may induce nonsingular cosmologies by evading the singularity theorem [1]. Indeed, in string theory, one expects drastic modifications of Einstein action at high energies. These modifications appear due to worldsheet corrections and due to contributions from the quantum loops. These corrections may violate strong energy condition, making it possible to construct nonsingular time dependent solutions [2–4].

Physics at a small scale receives modifications in the braneworld scenario as well. In the cosmological sector, this is substantiated via a modified version of the Friedmann equations resulting from the embedding geometry. Indeed in many braneworld models, it has been found that the universe can have a smooth transition from a contracting to an expanding phase without reaching a singularity [5–8]. In particular, in [5], a non-singular cosmological solution was presented by considering a dynamical three-brane in a five dimensional charged AdS black hole background. This happens because the charge of the bulk black hole induces a negative energy density on the brane world volume. This, in turn, prevents the brane to fall in to the black hole singularity. However, this bounce, unfortunately, occurs inside the Cauchy horizon of the black hole. Consequently, regardless of the initial energy of the brane, the bulk space-time collapses due to the back-reaction at the Cauchy horizon [9].

Within the framework of braneworld scenario, in this short note, we search for other probable models, where, at least classically, one can construct non-singular cosmological universe. In the following, we show the existence of such solutions by considering an empty three-brane in the background of Born-Infeld black hole [10] and in the background of charged Gauss-Bonnet (GB) black hole [11–13]. In both the cases, by considering the effective potentials, we show that, within a certain range of parameter space, there do exist bouncing solutions in these backgrounds as well.

An important conclusion of this analysis is that the charge of the bulk black hole plays the key role in avoiding cosmological singularity on the brane. It is however important to study the stability of such solution. We have not done this in this note and expect to come back to it in the future.

**Bounce with Born-Infeld**

The Einstein-Born-Infeld action in five dimensions in the presence of a negative
bulk cosmological constant $\Lambda$ is given by
\[
S = \int d^5x \sqrt{-g} \left[ \frac{1}{\kappa} (R - 2\Lambda) + L(F) \right],
\]
where $L(F)$ is the contribution from the Born-Infeld gravity
\[
L(F) = 4\beta^2 \left( 1 - \sqrt{1 + \frac{F_{\mu\nu} F^{\mu\nu}}{2\beta^2}} \right).
\]

Here the constant $\beta$ is the Born-Infeld parameter, having the dimension of mass. In the limit $\beta \to \infty$, $L(F)$ reduces to the standard Maxwell form
\[
L(F) = -F_{\mu\nu} F^{\mu\nu} + O(F^4).
\]

A $(n + 1)$ dimensional black hole solution of this theory has been obtained in [10] by considering the general form of the metric as
\[
ds^2 = -V(r) dt^2 + \frac{dr^2}{V(r)} + r^2 d\Omega_3^2,
\]
which gives, after some rigorous calculations, a solution for the metric function $V(r)$ in $(n + 1)$ dimensions [10]. In $(4 + 1)$ dimensions, the metric function $V(r)$ can be written in a compact form in terms of Hypergeometric and algebraic functions as
\[
V(r) = 1 - \frac{m}{r^2} + \left[ \frac{\beta^2}{3} + \frac{1}{r^2} \right] r^2 - \frac{\sqrt{2} \beta}{6r} \sqrt{2\beta^2 r^6 + 6q^2} + \frac{3}{2r} q^2 \, _2F_1\left[ \frac{1}{3}, \frac{1}{2}, \frac{4}{3}, -\frac{3q^2}{\beta^2 r^6} \right].
\]
The solution of the corresponding gauge field equations of motion is
\[
F^{rt} = \frac{\sqrt{3} \beta q}{\sqrt{\beta^2 r^6 + 3q^2}},
\]
with all the other components of $F^{\mu\nu}$ being zero. In (5), $m$ and $q$ are related to the ADM mass and charge of the black hole respectively. The constant $l$, appearing in various equations, parametrises the cosmological constant $\Lambda$ as $\Lambda = -6/l^2$. The asymptotic behaviour (large $r$ limit) of the metric function can be obtained by using the properties of the Hypergeometric function in (5), which gives
\[
V(r)|_{r \to \infty} = 1 - \frac{m}{r^2} + \frac{q^2}{r^4} + \frac{r^2}{l^2} - \frac{3q^4}{16\beta^2 r^{10}}.
\]
Note that in the limit of large $\beta$, it has the form of Reissner-Nordstrom AdS black hole, which is obvious from the fact that, in the large $\beta$ limit, the non-zero gauge field component reduces to
\[
F^{rt}|_{\beta \to \infty} \sim \frac{q}{r^3}.
\]
It is instructive to consider the nature of the horizon(s) associated with this black hole. This is perhaps best described by a plot of the mass of the hole as a function of the horizon radius. The horizons correspond to the zeros of $V(r)$. Denoting the radius by $r_h$, from (5), we get

$$m = r_h^2 + \left[ \beta^2 + \frac{1}{l^2} \right] r_h^4 - \frac{\sqrt{2} \beta r_h}{6} \sqrt{2 \beta^2 r_h^6 + 6q^2} + \frac{3}{2r_h^2} q^2 _2 F_1 \left[ \frac{1}{3}, \frac{1}{2}, \frac{4}{3}, -\frac{3q^2}{\beta^2 r_h^6} \right].$$  \hspace{1cm} (9)

In figure (1), we have plotted $m(r_h)$ for fixed $q$ and for different values of $\beta$. For a given mass, we clearly see that the number of horizons depend on the value of $\beta$. For fixed $q$, there is a single horizon unless $\beta$ is above a critical value, which we call $\beta_c$. This critical value $\beta_c$ can be found out as follows. From the figure, we see that if we decrease the mass, at some value of $m$, two horizons meet. We call this an extremal hole\(^3\). This minimum can be found by extremising (9). At the extremal point, $r_h$ then satisfies

$$2r_h + 4 \left( \frac{\beta^2}{3} + \frac{1}{l^2} \right) r_h^3 - \frac{2 \sqrt{2}}{3} \beta \sqrt{2 \beta^2 r_h^6 + 6q^2} = 0.$$  \hspace{1cm} (10)

\(^3\)Black hole then has zero temperature.
It can be checked that, for a given $q$, there exists a real solution for $r_h$ only when $\beta$ is larger than a critical value. We previously called this $\beta_c$. It can also be checked that $\beta_c$ decreases as we increase $q$, which reveals the important role of the charge parameter in the bouncing cosmology in the braneworld context.

We would now like to study cosmology of an empty three-brane moving in this background. In what follows, we will show the existence of a bouncing cosmology on the brane for $\beta > \beta_c$.

Let us start with a three-brane with tension $\Lambda_{br}$ moving in the geometry given by (4) with $V(r)$ given in (5). The brane metric, consistent with the orthonormality conditions and junction conditions for embedding, is compatible to FRW geometry

$$ds^2 = -d\tau^2 + a(\tau)^2 d\Omega^2_3,$$

(11)

$\tau$ being the proper time on the brane. Using the junction conditions relating the extrinsic curvature with the bulk quantities, one can obtain an FRW metric on the brane of the above form if one identifies the scale factor $a(\tau)$ on the brane with the radial trajectory $r(\tau)$ of the bulk black hole. From now on we shall use $a$ for $r$, wherever it appears. The embedding geometry thus gives [14]

$$\frac{\bar{\Lambda}_{br}}{3} = \bar{H}^2 + \frac{V(\bar{a})}{\bar{a}^2}. \tag{12}$$

In the above, the quantities with bar are all dimensionless and are defined as

$$\bar{\Lambda}_{br} = l^2 \Lambda_{br}, \quad \bar{a} = \frac{a}{l}, \quad \bar{H} = \frac{d\bar{a}}{d\bar{\tau}}. \tag{13}$$

In the limit $\beta \to \infty$, (12) reduces to

$$\frac{\bar{\Lambda}_{br}}{3} = 1 + \bar{H}^2 + \frac{1}{\bar{a}^2} - \frac{\bar{m}}{\bar{a}^4} + \frac{\bar{q}^2}{\bar{a}^6}. \tag{14}$$

Here $\bar{m} = m/l^2, \bar{q} = q/l^2$. This equation has a simple solution when we tune the brane tension such that $\bar{\Lambda}_{br} = 3$. This corresponds to setting the brane tension same as that of the curvature radius of the asymptotic AdS space. In conformal gauge, the solution of (14) is

$$\bar{a}(\bar{\eta}) = \sqrt{\frac{\bar{m}}{2}} \left[1 - \sqrt{1 - \frac{4\bar{q}^2}{\bar{m}^2} \cos(2\bar{\eta})}\right], \tag{15}$$

where $\eta$ is the conformal time defined as $d\tau = a(\eta)d\eta$ and $\bar{\eta} = \eta/l$. For $\bar{m} > 2\bar{q}$, this generates a non-singular cyclic cosmology with brane universe reaching a minimum radius at $\eta = n\pi$ for $n$ being integer, while the maximum radius is reached at $\eta = (n + 1/2)\pi$. 

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For finite $\beta$, it is difficult to find an exact closed form solution of (12), which now includes terms involving $\beta$ in the Friedmann equation and, in turn, make the results more complicated. It is obvious from the inclusion of the Hypergeometric function which does not have a simple algebraic expression for a general value of $\beta$ as such. However, taking the asymptotic behaviour (7) and rearranging terms, we can arrive at the following expression for the modified Friedmann equations on the brane for large $\bar{a}$ limit:

$$\frac{\bar{\Lambda}_{\text{br}}}{3} = 1 + \bar{H}^2 + \frac{1}{\bar{a}^2} - \frac{\bar{m}}{\bar{a}^3} + \frac{\bar{q}^2}{\bar{a}^6} - \frac{3\bar{q}^4}{16\beta^2\bar{a}^{12}},$$

(16)

where the term involving $\bar{m}$ is the usual dark radiation, the one with $\bar{q}^2$ gives a geometric stiff matter-like contribution (obtained in the case of infinite $\beta$ as well) and the one with $\bar{q}^4$ and $\beta$ gives another geometric contribution (stiffer matter), which reflects the precise role of a finite Born-Infeld parameter in this context. We have been able to obtain an analytical solution for the scale factor at least in this asymptotic limit but the expressions are too lengthy to produce here. As apparent, complicated equations sometime defer us from obtaining a simple analytical result but it is more important to overcome the hindrance and to extract the underlying physics than to obtain a simple analytical solution.

So, irrespective of whether or not there is a simple analytical solution for a general Born-Infeld parameter, it is rather more relevant to ask whether we can have a bouncing cosmology in this Born-Infeld background with a finite $\beta$ as well. In the following, we incorporate a mechanism by analysing the effective potential to find that the bouncing solution still survives if $\beta > \beta_c$. In order to see this, we rewrite (12) as

$$\left(\frac{d\bar{a}}{d\bar{t}}\right)^2 + U(\bar{a}) = 0,$$

(17)

Here $U(\bar{a})$ is the effective potential given by

$$U(\bar{a}) = V(\bar{a}) - \bar{a}^2.$$

(18)

where we have set, as before, $\bar{\Lambda}_{\text{be}} = 3$.

For $\beta < \beta_c$ and $\beta > \beta_c$, the plots are shown in figure (2). Physical domain of the solution corresponds to the region $U(\bar{a}) \leq 0$ with $U(\bar{a}) = 0$ representing the turning points. From the plots we see that for very small $\beta$, the effective potential does not show any signature for bounce. On the contrary, above a specific value for the BI parameter $\beta = \beta_c$, there are, indeed, bouncing cosmological solutions as indicated by the nature of the effective potential. We have previously termed this $\beta_c$ as the
critical value for the BI parameter. It readily follows from the above analysis that, so far as the BI parameter is above a critical value, we have non-singular cosmology on the brane. This is true for infinite value of $\beta$ as well, as supported by the analytical solution (15).

In the rest of this note, we show that even in the Gauss-Bonnet theory, in the presence of electric charge, it is possible to construct a bouncing cosmological solution.

**Bounce with Gauss-Bonnet**

The Gauss-Bonnet action with a Maxwell term is given by

$$S = \int d^5 x \sqrt{-g} \left[ \frac{R}{\kappa} - 2\Lambda + \alpha (R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd}) - \frac{F^2}{\kappa} \right].$$  \hspace{1cm} (19)
Here, $\alpha$ is the GB coupling. This action possesses black hole solutions carrying electric charge [12, 13]. These solutions have the form

$$ds^2 = -V(r)dt^2 + \frac{dr^2}{V(r)} + r^2d\Omega_3^2,$$  

(20)

where $V(r)$ is given by

$$V(r) = 1 + \frac{r^2}{2\hat{\alpha}} - \frac{r^2}{2\hat{\alpha}}[1 - \frac{4\hat{\alpha}}{l^2} + \frac{4\hat{\alpha}m}{r^4} - \frac{4\hat{\alpha}q^2}{r^6}]^{\frac{1}{2}}.$$  

(21)

In the above expression, $\hat{\alpha} = 2\alpha\kappa$, while $l$ is related to the cosmological constant as $l^2 = -6/(\kappa\Lambda)$. As before, the parameters $m$ and $q$ are related to the ADM mass and the charge respectively. The gauge potential is given by

$$A_t = -\frac{\sqrt{3}}{2} \frac{q}{r^2} + \Phi,$$  

(22)

where $\Phi$ is a constant which we will fix below. Denoting $r_+$ as the largest real positive root of $V(r)$, we find that the metric (20) describes a black hole with a non-singular horizon if

$$2r_+^6 + l^2r_+^4 \geq q^2l^2.$$  

(23)

In the following we would choose the gauge potential $A_t$ to vanish at the horizon, which, in turn, fixes $\Phi$ as

$$\Phi = -\frac{\sqrt{3}}{2} \frac{q}{r_+^2}.$$  

(24)

Asymptotically, the metric (20) is an AdS space with

$$V(r)|_{r\to\infty} = 1 + \left[\frac{1}{2\hat{\alpha}} - \frac{1}{2\hat{\alpha}}(1 - \frac{4\hat{\alpha}}{l^2})^{\frac{1}{2}}\right]r^2.$$  

(25)

Therefore, the effective AdS length associated with the asymptotic metric is given by

$$L^2 = \left[\frac{1}{2\hat{\alpha}} - \frac{1}{2\hat{\alpha}}(1 - \frac{4\hat{\alpha}}{l^2})^{\frac{1}{2}}\right]^{-1}.$$  

(26)

We now note that the metric is real if,

$$\hat{\alpha} \leq \frac{l^2}{4}.$$  

(27)

In the rest of our discussion in this section, we will always work with $\hat{\alpha}$ satisfying this bound.
As in Born-Infeld, it would be convenient for us to define a few dimensionless quantities

\[ \bar{r} = \frac{r}{l}, \quad \bar{\alpha} = \frac{\alpha}{l^2}, \quad \bar{q} = \frac{q}{l^2}, \quad \text{and} \quad \bar{m} = \frac{m}{l^2}. \]  

(28)

In terms of these quantities the temperature of the black hole \( \bar{T} \) is given by

\[ \bar{T} = Tl = \frac{\bar{r}^2 + 2\bar{r}^4 - \bar{q}^2}{2\pi \bar{r}(\bar{r}^2 + \bar{2}\bar{\alpha})}. \]  

(29)

Again, it would be instructive to study the structure of the horizon(s) associated with the black hole. Firstly notice that, from (21), we can write a relation between mass and horizon radius as

\[ \bar{m} = \bar{r}_h^4 + \bar{r}_h^2 + \bar{\alpha} + \frac{\bar{q}^2}{\bar{r}_h^2}, \]  

(30)

where \( \bar{r}_h = r_h/l \) and \( r_h \) represents the roots of the equation \( V(r) = 0 \). The largest root was defined as \( r_\text{+} \) earlier. For generic values of \( \bar{q} \) and \( \bar{\alpha} \), at small \( \bar{r}_h \), \( \bar{m} \) decreases with \( \bar{r}_h \) while for large \( \bar{r}_h \), it increases as \( \bar{r}_h^4 \). In between, there is a single minimum. At the minimum, \( \bar{r}_h \) satisfies

\[ 2\bar{r}_h^6 + \bar{r}_h^4 - \bar{q}^2 = 0. \]  

(31)

The behaviour of \( \bar{m} \) as a function of \( \bar{r}_h \) is shown in Figure (3). It is clear from the figure that in general for a given \( \bar{q} \), as long as \( \bar{m} \) is more than a critical value,
there are two horizons. They will be called inner and outer horizons subsequently. At the critical value of $\bar{m}$, when (31) is satisfied, two horizons coincide. From (29), we see that the temperature of the hole is then zero. We call this hole as an extremal black hole. When we further decrease the mass, there are no horizons. One then has a naked singularity.

Let us now consider a three-brane with tension $\Lambda_{br}$ moving in this background. Choosing the brane metric as in equation (11), we get the Hubble equation of the form [15]

$$\frac{\Lambda_{br}}{3} = \left(\frac{V(\bar{a})}{\bar{a}^2} + \bar{H}^2 \right)[1 + \frac{2\bar{a}}{3}\left(\frac{3 - V(\bar{a})}{\bar{a}^2} + 2\bar{H}^2\right)]^2.$$  \(32\)

In the above, we have used $\bar{a} = a/l$. $\bar{H}$, the Hubble parameter, is given by $\bar{H} = d\bar{a}/(\bar{a}d\bar{\tau})$ where $\bar{\tau} = \tau/l$. Furthermore, we have defined $\bar{\Lambda}_{br} = l^2\Lambda_{br}$.

In the limit $\bar{\alpha} \to 0$, (32) simplifies and goes over to the form given in equation (14). The solution for the scale factor for $\bar{\alpha} \to 0$ is then given by (15), resulting a cyclic universe. Thus, for a vanishing $\bar{\alpha}$, we have non-singular cosmological solutions on the brane.

Though for finite $\bar{\alpha}$, it is hard to find an exact closed form solution of (32), however, it is easy to see that the bouncing solutions still continue to survive. Note that the effective AdS scale for non-zero $\bar{\alpha}$ and large $r$ is given by $L$ as in (26). As before, and for the sake of comparison, we make effective brane tension to zero at large $r$ by setting

$$\Lambda_{br} = 3L^{-2}$$  \(33\)

Hence the equation (32) reduces to

$$\left[\frac{1}{2\bar{\alpha}} - \frac{1}{2\bar{\alpha}}(1 - 4\bar{\alpha})^\frac{1}{2}\right] = \left(\frac{V(\bar{a})}{\bar{a}^2} + \bar{H}^2 \right)[1 + \frac{2\bar{a}}{3}\left(\frac{3 - V(\bar{a})}{\bar{a}^2} + 2\bar{H}^2\right)]^2.$$  \(34\)

This is a cubic equation of $\bar{H}^2$ and can be re-written as

$$\frac{16\bar{\alpha}^2}{9}\bar{H}^6 + \frac{8\bar{\alpha}}{3}\left(1 + \frac{2\bar{\alpha}}{\bar{a}^2}\right)\bar{H}^4 + \frac{1}{3\bar{\alpha}^4}\left(3\bar{a}^2 + 6\bar{\alpha} - 2\bar{a}V\right)(\bar{a}^2 + 2\bar{\alpha} + 2\bar{\alpha}V)\bar{H}^2$$
$$+ \frac{1}{18\bar{\alpha}^6\bar{a}}\left(9\bar{a}^6(\sqrt{1 - 4\bar{\alpha}} - 1) + 18\bar{a}^4\bar{a}V\right)$$
$$- 24\bar{\alpha}^2\bar{a}^2(V - 3)V + 8\bar{\alpha}^3(V - 3)^2V = 0.$$  \(35\)

This equation can, in principle, be explicitly solved to get $\bar{H}^2$. The physical domain, however, corresponds to the region $\bar{H}^2 \geq 0$. The points where the equality holds represent the turning points. Unlike the case of $\bar{\alpha} = 0$, we have not been able to
Figure 4: A plot of the potential $U(\bar{a})$ with $\bar{a}$. Three curves correspond to $\bar{\alpha} = 10^{-7}$ (dotted), $\bar{\alpha} = 10^{-6}$ (dashed) and the other is for $\bar{\alpha} = 0$. We have also set $\bar{m} = 10$ and $\bar{q} = .1$. From (36), allowed physical region is $U(\bar{a}) \leq 0$. Since $U(\bar{a}) = 0$ occurs only at non-zero finite values of $\bar{a}$, we get non-singular cyclic brane universe.

Integrate the above equation to get the scale factor $\bar{a}$ as a function of time in a closed form. However, as in the Born-Infeld case, it is possible to re-write (35) as

$$\left(\frac{d\bar{a}}{d\tau}\right)^2 + U(\bar{a}) = 0,$$

with $U(\bar{a})$, the effective potential. By studying $U(\bar{a})$ for different values of parameters, we would be able to infer the nature of the brane universe. This is what we do in the following.

Figure (4) shows a plot of the effective potential $U(\bar{a})$ for different values of $\bar{\alpha}$ including the Reissner-Nordstrom black hole ($\bar{\alpha} = 0$). Notice that the potential does not allow the brane to hit the black hole singularity. It bounces before reaching the singularity and thus generates a bouncing universe. It can be checked that for large $\bar{a}$, $U(\bar{a})$ changes sign at some value of $\bar{a}$, becomes positive and asymptotically reaches a constant. We can therefore conclude that the brane again bounces before reaching an infinite size. This is typical of a bouncing cyclic universe. However, we must mention here that, as in the case of $\bar{\alpha} = 0$ [5], it can however be checked that for a bounce at small $\bar{a}$ to occur, the brane has to cross the inner horizon. As in $\bar{\alpha} = 0$, this may
induce instabilities in the bulk causing an appearance of a singularity on the inner horizon [9]. For that, one needs to study perturbations on the bulk due to the brane. We leave this for a future study.

To conclude, in this note, we have shown the existence of bouncing cosmological solutions by studying the dynamics of three-brane in certain black hole backgrounds. These black holes are the classical solutions of Born-Infeld and Gauss-Bonnet theories in the presence of negative cosmological constant. The analysis guides us to infer that the charge parameter of the bulk black holes play crucial role, inducing bouncing cosmology on the brane. More precisely, the charge induces a negative energy density on the brane thereby preventing the brane to fall in the black hole singularity, and in turn, succeeding to give singularity-free cosmology on the four-dimensional world. Finally, it is important to study the stability of these solutions. We plan to return to this issue in the future.

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