Charmed and Charmed-Strange Mesons
in Kaluza-Klein Picture

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Abstract

In the present paper, we continue our study the structure of hadronic spectra, started in Refs. [1, 2, 3, 4, 5], from the view point of existence the extra dimensions in the spirit of Kaluza-Klein approach. We show that all charmed and charmed-strange mesons, including the recently observed new states [7, 8], are excellently incorporated in the systematics provided by Kaluza-Klein approach.

1 Introduction

In the present paper, we continue our study the structure of hadronic spectra, started in Refs. [1, 2, 3, 4, 5], from the view point of existence the extra dimensions in the spirit of Kaluza-Klein approach. Here we shall concern the spectra of charmed and charmed-strange mesons. The physics of charmed and charmed-strange mesons is really charmed and charmed-strange one. On the one hand the fundamental QCD Lagrangian predicted the charmed and charmed-strange states containing c and s quarks, while on the other hand there are serious problems to describe that sort of experimentally observed states in the framework of known QCD-inspired potential models with the quark and gluon degrees of freedom. The best currently performed lattice computations in QCD [9] can not help us to understand the exact nature of the recently observed new states in the spectrum of charmed-strange mesons too. However, we show below that all charmed and charmed-strange mesons, including the recently observed new states, are excellently incorporated in the systematics provided by Kaluza-Klein approach.

2 Charmed mesons

As it was established in previous paper [5], the charmed $D^0(1864)$-meson may occupy $M^{2K}_{19}(1860 – 1864)$-Storey in Kaluza-Klein tower of KK excitations in two-kaon system and $M^{2K}_{21}(2003 – 2006)$-Storey in the same tower is acceptable for the $D^{*0}(2006)$-meson. However, $D^0(2006)$-meson has been observed as a resonance in $D^0\pi^0$-system. That is why, first of all, let us build the Kaluza-Klein tower of KK excitations for $D\pi$-system by the formula

$$M^{D\pi}_n = \sqrt{m^2_D + \frac{n^2}{R^2}} + \sqrt{m^2_\pi + \frac{n^2}{R^2}}, \quad (n = 1, 2, 3, \ldots).$$  (1)
where $R$ is the fundamental scale characterizing the size of extra dimensions calculated early from the analysis of nucleon-nucleon dynamics at low energies [1, 2]

$$\frac{1}{R} = 41.481 \text{ MeV} \quad \text{or} \quad R = 24.1 \text{ GeV}^{-1} = 4.75 \times 10^{-13} \text{ cm}.$$ (2)

The Kaluza-Klein tower such built is shown in Table 1 where the comparison with experimentally observed mass spectrum of $D^*$-mesons is also presented.

Throughout we have used Review of Particle Physics [6] where the experimental data on mass spectrum of the resonance states have been extracted from. In particular, we have used $m_{D^0} = 1864.1 \pm 1.0 \text{ Mev}$ and $m_{D^\pm} = 1869.4 \pm 0.5 \text{ Mev}$ for the masses of $D$-mesons calculating the Kaluza-Klein tower for $D\pi$-system. In Table 2–Table 3 we collected some known experimental information. The Tables 1–3 show a remarkable correspondence of the calculated KK excitations for $D\pi$ system with the experimentally measured masses of the $D^*$-mesons. In fact, there are many empty cells in Table 1 where we have not found the corresponding experimental data.

At the next step we built the Kaluza-Klein tower of KK excitations for the $D^*\pi$-system by the formula

$$M_{D^*\pi}^{n} = \sqrt{m_{D^*}^2 + \frac{n^2}{R^2}} + \sqrt{m_{\pi}^2 + \frac{n^2}{R^2}}, \quad (n = 1, 2, 3, \ldots),$$ (3)

and this is shown in Table 4 where the comparison with experimentally observed mass spectrum is also presented. Some known experimental information in that case is collected in separate tables: Table 5–Table 6. We have used $m_{D^0} = 2006.71 \pm 0.5 \text{ Mev}$ and $m_{D^\pm} = 2010.0 \pm 0.5 \text{ Mev}$ for the masses of $D^*$-mesons calculating the Kaluza-Klein tower for $D^*\pi$-system. Again we see from Tables 4–6 that there is a remarkable correspondence of the calculated KK excitations for $D^*\pi$-system with the experimentally measured masses of the resonance states. Here, there are many empty cells in Table 4 as well, where we have not found the corresponding experimental data. However, it should be noted that the decay modes $D_1^0(2420) \rightarrow D^+(2010)\pi^-$ and $D_1^+(2420) \rightarrow D^{*0}(2007)\pi^+$ have been seen but the decay modes $D_0^0(2420) \rightarrow D^+\pi^-$ and $D_1^+(2420) \rightarrow D^0\pi^+$ have not been observed [6]. Table 1 and Table 4, as it were, confirm that observation.

### 3 Charmed-strange mesons

Now, we go to the charmed-strange mesons. In the first, we calculate the Kaluza-Klein tower of KK excitations for the $K^{*0}K^{*\pm}$-system by the formula

$$M_{K^{*0}K^{*\pm}}^{n} = \sqrt{m_{K^{*0}}^2 + \frac{n^2}{R^2}} + \sqrt{m_{K^{*\pm}}^2 + \frac{n^2}{R^2}}, \quad (n = 1, 2, 3, \ldots),$$ (4)

which is shown in Table 7. We see that $D_s^\pm(1969)$-meson lives in $M_{10}^{K^{*0}K^{*\pm}}$-Storey of that Kaluza-Klein tower. We have used $m_{K^{*0}} = 896.1 \pm 0.27 \text{ Mev}$ and $m_{K^{*\pm}} = 891.66 \pm 0.26 \text{ Mev}$ for the masses of $K^{*}(892)$-mesons calculating the Kaluza-Klein tower. It should be emphasized that $M_{9}^{K^{*0}K^{*\pm}}$-Storey in this Kaluza-Klein tower is quite acceptable for the $f_2(0^+2^{++})(1950)$-meson (LASS 91)[6]. In Table 8 we presented some known experimental information concerning $D_s^\pm$-meson.
We further present the results calculating the Kaluza-Klein tower of KK excitations for the $D^K$-system by the formula

$$M_{n}^{D^K} = \sqrt{m_{D}^{2} + \frac{n^{2}}{R^{2}}} + \sqrt{m_{K}^{2} + \frac{n^{2}}{R^{2}}}, \quad (n = 1, 2, 3, \ldots). \quad (5)$$

These results are shown in Table 9. We have used $m_{D^{0}} = 1864.1 \pm 1.0$ Mev and $m_{D^{\pm}} = 1869.4 \pm 0.5$ Mev for the masses of $D$-mesons calculating the Kaluza-Klein tower. Table 10 concerns the experimental data of resonance states in $D^K$-system extracted from [6]. Recently CLEO Collaboration reported the observation of a narrow resonance $D_{sJ}^{+}(2463)$ in $D_{s}^{+} \pi^{0}$-system [7]. From Table 9 we see that $M_{7}^{DK}(2459 - 2468)$-Storey is acceptable for the resonance in $D^{+}K^{0}$-system with a similar mass.

We have also calculated the Kaluza-Klein tower of KK excitations for the $D^*K$-system by the formula

$$M_{n}^{D^*K} = \sqrt{m_{D^*}^{2} + \frac{n^{2}}{R^{2}}} + \sqrt{m_{K}^{2} + \frac{n^{2}}{R^{2}}}, \quad (n = 1, 2, 3, \ldots). \quad (6)$$

This is shown in Table 11. Here, we have used $m_{D^{*0}} = 2006.7 \pm 0.5$ Mev and $m_{D^{*\pm}} = 2010.0 \pm 0.5$ Mev for the masses of $D^*$-mesons calculating the Kaluza-Klein tower. As it is seen, the $D_{s1}^{+}(2536)$-meson is excellently incorporated in Table 11. We have also found that $D_{sJ}^{\pm}(2573)$-meson may occupy the $M_{6}^{D^*K}$-Storey even though the decay $D_{sJ}^{+}(2573) \to D^{0(+)0}(2007)K^{+(0)}$ has not been seen so far. Table 12 contains the experimental data in respect of $D_{s1}^{+}(2536)$-meson.

In Table 13 we present the results calculating the Kaluza-Klein tower of KK excitations for the $D_{s}^{(+)}(1969)\pi$-system by the formula

$$M_{n}^{D_{s}^{+}\pi} = \sqrt{m_{D_{s}^{+}}^{2} + \frac{n^{2}}{R^{2}}} + \sqrt{m_{\pi}^{2} + \frac{n^{2}}{R^{2}}}, \quad (n = 1, 2, 3, \ldots), \quad (7)$$

where it follows that the $D_{sJ}^{+}(2112)$ lives in the first Storey of this Kaluza-Klein tower from. The seventh Storey of this Kaluza-Klein tower is acceptable for the recently discovered in [8] and confirmed in [7] $D_{sJ}^{+}(2317)$-meson as well.

At the same time we have calculated Kaluza-Klein tower of KK excitations for the $D_{s}^{*\pm}(2112)\pi$-system by the formula

$$M_{n}^{D_{s}^{*\pm}\pi} = \sqrt{m_{D_{s}^{*\pm}}^{2} + \frac{n^{2}}{R^{2}}} + \sqrt{m_{\pi}^{2} + \frac{n^{2}}{R^{2}}}, \quad (n = 1, 2, 3, \ldots), \quad (8)$$

and found that the recently discovered narrow resonance of mass 2.46 GeV decaying to $D_{s}^{*\pm} \pi^{0}$ [7] lives in the seventh Storey of this Kaluza-Klein tower: see Table 14.

At last, we present in Table 15 the results calculating the Kaluza-Klein tower of KK excitations for the $K^{*}(892)K$-system by the formula

$$M_{n}^{K^{*}K} = \sqrt{m_{K^{*}}^{2} + \frac{n^{2}}{R^{2}}} + \sqrt{m_{K}^{2} + \frac{n^{2}}{R^{2}}}, \quad (n = 1, 2, 3, \ldots). \quad (9)$$

Here we have found that the recently discovered in [8] and confirmed in [7] $D_{sJ}^{+}(2317)$-meson [8] may excellently be incorporated in $M_{22}^{K^{*}K}(2315 - 2317)$-Storey of that Kaluza-Klein tower. Moreover, the first Storey in this tower is acceptable for the $h_{1}(?^{-1^{+}+})(1380)$-meson with experimentally measured mass 1386\pm 19 MeV ($\Gamma = 91 \pm 30$) MeV(AVERAGE
and $M_4^{K^*K}(1432)$-Storey is very acceptable for the $f_1(0^{++})(1420)$-meson with experimentally measured mass $1433.4 \pm 0.8$ MeV ($\Gamma = 58.8 \pm 3.3$) MeV(SPEC 99) as well, $M_6^{K^*K}(1482)$-Storey is a good place for the $\eta(0^{0++})(1440)$-meson with experimentally measured mass $1475 \pm 5$ MeV ($\Gamma = 81 \pm 11$) MeV(AVERAGE PDG), the $f_1(0^{++})(1510)$-meson with experimentally measured mass $1512 \pm 4$ MeV ($\Gamma = 35 \pm 15$) MeV(MPS 88) may all rights to occupy $M_7^{K^*K}(1514)$-Storey, and finally $M_7^{K^*K}(1632)$-Storey is very acceptable for the $\eta_2(0^{+2-})(1645)$-meson with experimentally measured mass $1632 \pm 14$ MeV ($\Gamma = 180^{+22}_{-20}$) MeV(AVERAGE PDG) too.

4 Summary

We calculated the Kaluza-Klein towers of KK excitations for the different experimentally observed charmed and charmed-strange hadronic systems and found that all known charmed and charmed-strange mesons, including the recently observed new states, are excellently incorporated in the systematics provided by Kaluza-Klein picture. This is a very non-trivial fact, even though there are many empty cells in Tables 1,4,7,9,11, 13,14,15 where we have no the corresponding experimental information.

Of course, it would be very desirable to state new experiments to search new states, and we believe that the Tables presented here may serve as a guide for the physicists–experimenters.

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Table 1: Kaluza-Klein tower of KK excitations for $D\pi$-system and experimental data.

| n | $M_n^{D_n^0\pi^+}$ MeV | $M_n^{D^*_n\pi^0}$ MeV | $M_n^{D_n^0\pi^0}$ MeV | $M_n^{D^*_n\pi^0}$ MeV | $M_{exp}^{D^0}$ MeV |
|---|------------------------|------------------------|------------------------|------------------------|---------------------|
| 1 | 2005.77                | 2010.17                | 2011.07                | 2015.46                | $D^{*0,\pm}(\frac{1}{2}1^-)$ |
| 2 | 2024.38                | 2028.31                | 2029.67                | 2033.61                |                     |
| 3 | 2051.84                | 2055.24                | 2057.13                | 2060.53                |                     |
| 4 | 2085.36                | 2088.29                | 2090.64                | 2093.57                |                     |
| 5 | 2123.06                | 2125.60                | 2128.33                | 2130.86                |                     |
| 6 | 2163.77                | 2165.99                | 2169.03                | 2171.25                |                     |
| 7 | 2206.79                | 2208.75                | 2212.02                | 2213.99                |                     |
| 8 | 2251.66                | 2253.41                | 2256.88                | 2258.63                |                     |
| 9 | 2298.10                | 2299.68                | 2303.30                | 2304.88                |                     |
| 10 | 2345.92               | 2347.36               | 2351.09               | 2352.53               |                     |
| 11 | 2394.97              | 2396.30              | 2400.12              | 2401.44              |                     |
| 12 | 2445.17              | 2446.39              | 2450.29              | 2451.51              |                     |
| 13 | 2496.42              | 2497.56              | 2501.52              | 2502.65              |                     |
| 14 | 2548.68              | 2549.74              | 2553.74              | 2554.80              |                     |
| 15 | 2601.89              | 2602.88              | 2606.92              | 2607.91              |                     |
| 16 | 2656.01              | 2656.94              | 2661.01              | 2661.94              |                     |
| 17 | 2711.01              | 2711.89              | 2715.97              | 2716.84              |                     |
| 18 | 2766.84              | 2767.67              | 2771.76              | 2772.59              |                     |
| 19 | 2823.49              | 2824.27              | 2828.37              | 2829.16              |                     |
| 20 | 2880.91              | 2881.66              | 2885.76              | 2886.50              |                     |
| 21 | 2939.10              | 2939.81              | 2943.90              | 2944.61              |                     |
| 22 | 2998.01              | 2998.69              | 3002.77              | 3003.46              |                     |
| 23 | 3057.64              | 3058.29              | 3062.35              | 3063.01              |                     |
| 24 | 3117.95              | 3118.57              | 3122.62              | 3123.25              |                     |
| 25 | 3178.92              | 3179.52              | 3183.55              | 3184.16              |                     |
| 26 | 3240.54              | 3241.12              | 3245.13              | 3245.71              |                     |
| 27 | 3302.78              | 3303.34              | 3307.33              | 3307.89              |                     |
| 28 | 3365.63              | 3366.17              | 3370.13              | 3370.67              |                     |
| 29 | 3429.06              | 3429.58              | 3433.51              | 3434.03              |                     |
| 30 | 3493.05              | 3493.55              | 3497.46              | 3497.96              |                     |
Table 2: $M_1^{D\pi}(2006 - 2015)$ - Storey.

| $R(IJ^P)$ | $M_R$ MeV | $\Gamma_R$ MeV | Reaction | Collab. |
|------------|------------|----------------|----------|---------|
| $D^{*0}(\frac{1}{2}^-_1)$ | 2006.7 ± 0.5 | < 2.1 | AVERAGE | PDG 00 |
| $D^{*+}(\frac{3}{2}^-_1)$ | 2010.0 ± 0.5 | < 0.131 | AVERAGE | PDG 00 |

Table 3: $M_{12}^{D\pi}(2445 - 2452)$ - Storey.

| $R(IJ^P)$ | $M_R$ MeV | $\Gamma_R$ MeV | Reaction | Collab. |
|------------|------------|----------------|----------|---------|
| $D^{*0}_{2}(\frac{1}{2}^+_{2})$ | 2453±3±2 | 25±10±5 | $\gamma Be \rightarrow D^+ \pi^- X$ | E687 94 |
| $D^{*+}_{2}(\frac{1}{2}^+_{2})$ | 2459 ± 4 | 25 ± 7 | AVERAGE | PDG 00 |
| $D^{*+}_{2}(\frac{1}{2}^+_{2})$ | 2458.9 ± 2.0 | 23 ± 5 | $\gamma Be \rightarrow D^0 \pi^+ X$ | E687 94 |
| $D^{*+}_{2}(\frac{1}{2}^+_{2})$ | 2459 ± 4 | 23±9±5 | AVERAGE | PDG 00 |
Table 4: Kaluza-Klein tower of KK excitations in $D^*\pi$-system and experimental data.

| n  | $M^{D^*[n]}_{n}\pi^0$ MeV | $M^{D^*[n]}_{n}\pi^\pm$ MeV | $M^{D^*[n]}_{n}\pi^0$ MeV | $M^{D^*[n]}_{n}\pi^\pm$ MeV | $M^{D}{_{exp}}^\pi$ MeV |
|----|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|
| 1  | 2148.34                 | 2152.73                 | 2151.63                 | 2156.03                 |                         |
| 2  | 2166.85                 | 2170.78                 | 2170.15                 | 2174.08                 |                         |
| 3  | 2194.14                 | 2197.55                 | 2197.44                 | 2200.84                 |                         |
| 4  | 2227.44                 | 2230.37                 | 2230.73                 | 2233.66                 |                         |
| 5  | 2264.85                 | 2267.38                 | 2268.13                 | 2270.67                 |                         |
| 6  | 2305.21                 | 2307.43                 | 2308.48                 | 2310.70                 |                         |
| 7  | 2347.81                 | 2349.77                 | 2351.07                 | 2353.04                 |                         |
| 8  | 2392.20                 | 2393.96                 | 2395.46                 | 2397.22                 |                         |
| 9  | 2438.11                 | 2439.70                 | 2441.36                 | 2442.94                 | D1(2420)                |
| 10 | 2485.35                 | 2486.79                 | 2488.58                 | 2490.02                 |                         |
| 11 | 2533.76                 | 2535.09                 | 2536.98                 | 2538.30                 |                         |
| 12 | 2583.27                 | 2584.49                 | 2586.47                 | 2587.69                 |                         |
| 13 | 2633.79                 | 2634.92                 | 2636.97                 | 2638.11                 | D*±(2640)               |
| 14 | 2685.26                 | 2686.32                 | 2688.43                 | 2689.49                 |                         |
| 15 | 2737.64                 | 2738.63                 | 2740.79                 | 2741.78                 |                         |
| 16 | 2790.89                 | 2791.82                 | 2794.03                 | 2794.96                 |                         |
| 17 | 2844.98                 | 2845.86                 | 2848.10                 | 2848.97                 |                         |
| 18 | 2899.87                 | 2900.70                 | 2902.97                 | 2903.80                 |                         |
| 19 | 2955.54                 | 2956.33                 | 2958.62                 | 2959.40                 |                         |
| 20 | 3011.97                 | 3012.72                 | 3015.02                 | 3015.77                 |                         |
| 21 | 3069.12                 | 3069.83                 | 3072.15                 | 3072.86                 |                         |
| 22 | 3126.98                 | 3127.66                 | 3129.98                 | 3130.67                 |                         |
| 23 | 3185.53                 | 3186.18                 | 3188.51                 | 3189.16                 |                         |
| 24 | 3244.74                 | 3245.37                 | 3247.70                 | 3248.32                 |                         |
| 25 | 3304.60                 | 3305.20                 | 3307.53                 | 3308.14                 |                         |
| 26 | 3365.09                 | 3365.67                 | 3367.10                 | 3368.58                 |                         |
| 27 | 3426.19                 | 3426.75                 | 3429.07                 | 3429.63                 |                         |
| 28 | 3487.88                 | 3488.42                 | 3490.74                 | 3491.28                 |                         |
| 29 | 3550.15                 | 3550.67                 | 3552.98                 | 3553.50                 |                         |
| 30 | 3612.98                 | 3613.48                 | 3615.78                 | 3616.29                 |                         |
Table 5: $M_{9}^{D}π(2438 - 2443)$–Storey.

| $R(IJ^{P})$ | $M_R$ MeV | $Γ_R$ MeV | Reaction | Collab. |
|-------------|-----------|-----------|----------|---------|
| $D_0^0(1^{++})$ | 2428±3±2 | 23_{-6}^{+8}±10 | $e^+e^- → D^{++}π^−X$ | CLEO 90 |
| | 2428±8±5 | 58 ± 14 ± 10 | $γN → D^{++}π^−X$ | TPS 89 |
| | 2422.2 ± 1.8 | 18.9_{-3.5}^{+4.6} | AVERAGE | PDG 00 |
| $D_1^+(1^{++})$ | 2425 ± 2 ± 2 | 26_{-4}^{+8} | $e^+e^- → D^{*0}π^+X$ | CLE2 94 |
| | 2443 ± 7 ± 5 | 41 ± 19 ± 8 | $γN → D^{*0}π^+X$ | TPS 89 |
| | 2427 ± 5 | 28 ± 8 | AVERAGE | PDG 00 |

Table 6: $M_{13}^{D}π(2634 - 2638)$–Storey.

| $R(IJ^{P})$ | $M_R$ MeV | $Γ_R$ MeV | Reaction | Collab. |
|-------------|-----------|-----------|----------|---------|
| $D^{*±}(1^{−+})$ | 2637 ± 2 ± 6 | < 15 | $e^+e^- → D^{*±}π^±X$ | DLPH 98 |
Table 7: Kaluza-Klein tower of KK excitations in $K^{*0}K^{*\pm}$-system and $D_s^{\pm}$-meson.

| n | $M_n^{K^{*0}K^{*\pm}}$ MeV | $M_{exp}^{K^{*0}K^{*\pm}}$ MeV |
|---|-----------------------------|-------------------------------|
| 1 | 1789.68                     |                               |
| 2 | 1795.44                     |                               |
| 3 | 1805.00                     |                               |
| 4 | 1818.30                     |                               |
| 5 | 1835.25                     |                               |
| 6 | 1855.77                     |                               |
| 7 | 1879.72                     |                               |
| 8 | 1906.98                     |                               |
| 9 | 1937.42                     | $f_2(0^{+}2^{++})$            |
| 10| 1970.88                     | $D_s^{\pm}(00^-)$             |
| 11| 2007.21                     |                               |
| 12| 2046.27                     |                               |
| 13| 2087.89                     |                               |
| 14| 2131.93                     |                               |
| 15| 2178.24                     |                               |
| 16| 2226.67                     |                               |
| 17| 2277.12                     |                               |
| 18| 2329.40                     |                               |
| 19| 2383.44                     |                               |
| 20| 2439.10                     |                               |
| 21| 2496.28                     |                               |
| 22| 2554.87                     |                               |
| 23| 2614.78                     |                               |
| 24| 2675.92                     |                               |
| 25| 2738.22                     |                               |
| 26| 2801.58                     |                               |
| 27| 2865.94                     |                               |
| 28| 2931.24                     |                               |
| 29| 2997.42                     |                               |
| 30| 3064.41                     |                               |

Table 8: $M_{10}^{K^{*0}K^{*\pm}}(1971)$–Storey.

| $R(J^{P})$ | $M_R$ MeV | $\Gamma_R$ MeV | Reaction | Collab.   |
|------------|-----------|----------------|----------|-----------|
| $D_s^{\pm}(00^-)$ | $1970 \pm 5 \pm 5$ | | $e^+e^-$ | CLEO 83 |
|            | $1969.0 \pm 1.4$ | | AVERAGE | PDG 00 |
Table 9: Kaluza-Klein tower of KK excitations in $DK$-system and experimental data.

| n  | $M^{D+K^0}_n$ MeV | $M^{D^0K^+}_n$ MeV | $M^{DK}_{exp}$ MeV |
|----|------------------|------------------|------------------|
| 1  | 2369.26          | 2359.98          |                  |
| 2  | 2375.78          | 2366.54          |                  |
| 3  | 2386.53          | 2377.37          |                  |
| 4  | 2401.35          | 2392.28          |                  |
| 5  | 2420.03          | 2411.08          |                  |
| 6  | 2442.33          | 2433.51          |                  |
| 7  | 2468.00          | 2459.32          | $D_{sJ}(2463)$? |
| 8  | 2496.79          | 2488.25          |                  |
| 9  | 2528.45          | 2520.06          |                  |
| 10 | 2562.75          | 2554.51          | $D^\pm_{sJ}(2573)$ |
| 11 | 2599.47          | 2591.38          |                  |
| 12 | 2638.42          | 2630.48          |                  |
| 13 | 2679.43          | 2671.64          |                  |
| 14 | 2722.34          | 2714.68          |                  |
| 15 | 2767.00          | 2759.48          |                  |
| 16 | 2813.29          | 2805.90          |                  |
| 17 | 2861.09          | 2853.84          |                  |
| 18 | 2910.32          | 2903.19          |                  |
| 19 | 2960.87          | 2953.86          |                  |
| 20 | 3012.67          | 3005.78          |                  |
| 21 | 3065.64          | 3058.86          |                  |
| 22 | 3119.73          | 3113.06          |                  |
| 23 | 3174.85          | 3168.29          |                  |
| 24 | 3230.98          | 3224.52          |                  |
| 25 | 3288.04          | 3281.69          |                  |
| 26 | 3346.00          | 3339.75          |                  |
| 27 | 3404.82          | 3398.65          |                  |
| 28 | 3464.44          | 3458.37          |                  |
| 29 | 3524.84          | 3518.87          |                  |
| 30 | 3585.99          | 3580.10          |                  |

Table 10: $M^{DK}_{40}(2555−2563)$–Storey.

| $R(IJP^\prime)$ | $M_R$ MeV | $\Gamma_R$ MeV | Reaction | Collab.     |
|-----------------|-----------|----------------|----------|-------------|
| $D_{sJ}^\pm(0??)$ | 2573.5 ± 1.7 | 15 ± 0.4 | AVERAGE | PDG 00      |
Table 11: Kaluza-Klein tower of KK excitations in $D^*K$-system and experimental data.

| n | $M_n^{D^*K}$ MeV | $M_n^{D^*K^*}$ MeV | $M_{exp}^{D^*K}$ MeV |
|---|----------------|-----------------|----------------|
| 1 | 2509.83       | 2502.55         |                |
| 2 | 2516.25       | 2509.01         |                |
| 3 | 2526.84       | 2519.67         |                |
| 4 | 2541.44       | 2534.36         | $D^\pm_s(2536)$|
| 5 | 2559.83       | 2552.87         |                |
| 6 | 2581.79       | 2574.94         | $D^\pm_sJ(2573)$? |
| 7 | 2607.05       | 2600.34         |                |
| 8 | 2635.38       | 2628.80         |                |
| 9 | 2666.51       | 2660.08         |                |
| 10| 2700.24       | 2693.94         |                |
| 11| 2736.33       | 2730.17         |                |
| 12| 2774.61       | 2768.58         |                |
| 13| 2814.89       | 2809.00         |                |
| 14| 2857.02       | 2851.26         |                |
| 15| 2900.87       | 2895.23         |                |
| 16| 2946.31       | 2940.78         |                |
| 17| 2993.22       | 2987.81         |                |
| 18| 3041.52       | 3036.22         |                |
| 19| 3091.12       | 3085.92         |                |
| 20| 3141.93       | 3136.83         |                |
| 21| 3193.89       | 3188.89         |                |
| 22| 3246.94       | 3242.02         |                |
| 23| 3301.01       | 3296.18         |                |
| 24| 3356.05       | 3351.31         |                |
| 25| 3412.02       | 3407.36         |                |
| 26| 3468.87       | 3464.29         |                |
| 27| 3526.56       | 3522.06         |                |
| 28| 3585.05       | 3580.63         |                |
| 29| 3644.31       | 3639.96         |                |
| 30| 3704.31       | 3700.03         |                |
Table 12: $M^D_{4K}(2534 - 2541)$–Storey.

| $R(IJ^P)$ | $M_R$ MeV | $\Gamma_R$ MeV | Reaction | Collab. |
|-----------|-----------|----------------|----------|---------|
| $D_{s1}^{*+}(01^+)$ | 2536.6 ± 0.7 ± 0.4 | < 5.44 | $e^+e^- \rightarrow D^{*+}K^0X$ | CLEO 90 |
| 2535.2 ± 0.5 ± 1.5 | < 3.9 | $e^+e^- \rightarrow D^{*0}K^+X$ | ARG 92 |
| 2534.8 ± 0.6 ± 0.6 | < 2.3 | $e^+e^- \rightarrow D^{*+}K^0X$ | CLEO 93 |
| 2535.3 ± 0.2 ± 0.5 | < 2.3 | $e^+e^- \rightarrow D^{*0}K^+X$ | CLEO 93 |
| 2535 ± 0.6 ± 1 | < 3.2 | $\gamma Be \rightarrow D^{*0}K^+X$ | E687 94 |
| 2535.35 ± 0.34 | < 2.3 | AVERAGE | PDG 00 |
Table 13: Kaluza-Klein tower of KK excitations in $D_s^\pm\pi$-system and experimental data.

| n  | $M_{n}^{D_s^\pm\pi^n}$ MeV | $M_{n}^{D_{s}^{\pm\pi}}$ MeV | $M_{exp}^{D_s^\pm\pi}$ MeV |
|----|----------------------------|----------------------------|-----------------------------|
| 1  | 2110.64                    | 2115.04                    | $D_s^{*\pm}(2112)$         |
| 2  | 2129.18                    | 2133.11                    |                             |
| 3  | 2156.52                    | 2159.92                    |                             |
| 4  | 2189.87                    | 2192.80                    |                             |
| 5  | 2227.35                    | 2229.89                    |                             |
| 6  | 2267.80                    | 2270.02                    |                             |
| 7  | 2310.50                    | 2312.47                    | $D_{sJ}(2317)$              |
| 8  | 2355.02                    | 2356.77                    |                             |
| 9  | 2401.06                    | 2402.65                    |                             |
| 10 | 2448.44                    | 2449.88                    |                             |
| 11 | 2497.02                    | 2498.34                    |                             |
| 12 | 2546.70                    | 2547.92                    |                             |
| 13 | 2597.40                    | 2598.53                    |                             |
| 14 | 2649.07                    | 2650.13                    |                             |
| 15 | 2701.66                    | 2702.65                    |                             |
| 16 | 2755.14                    | 2756.07                    |                             |
| 17 | 2809.45                    | 2810.33                    |                             |
| 18 | 2864.58                    | 2865.41                    |                             |
| 19 | 2920.50                    | 2921.29                    |                             |
| 20 | 2977.17                    | 2977.92                    |                             |
| 21 | 3034.59                    | 3035.30                    |                             |
| 22 | 3092.72                    | 3093.40                    |                             |
| 23 | 3151.54                    | 3152.19                    |                             |
| 24 | 3211.03                    | 3211.66                    |                             |
| 25 | 3271.18                    | 3271.78                    |                             |
| 26 | 3331.95                    | 3332.53                    |                             |
| 27 | 3393.34                    | 3393.90                    |                             |
| 28 | 3455.33                    | 3455.87                    |                             |
| 29 | 3517.90                    | 3518.42                    |                             |
| 30 | 3581.02                    | 3581.53                    |                             |
Table 14: Kaluza-Klein tower of KK excitations in $D_{s}^{*\pm}\pi$-system and $D_{sJ}^{+}(2463)$-meson.

| n  | $M_{n}^{D_{s}^{*\pm}\pi}$ MeV | $M_{n}^{D_{s}^{*\pm}\pi}$ MeV | $M_{exp}^{D_{s}^{*\pm}\pi}$ MeV |
|----|-------------------------------|-------------------------------|-------------------------------|
| 1  | 2254.01                       | 2258.41                       |                               |
| 2  | 2272.46                       | 2276.39                       |                               |
| 3  | 2299.65                       | 2303.05                       |                               |
| 4  | 2332.80                       | 2335.73                       |                               |
| 5  | 2370.02                       | 2372.55                       |                               |
| 6  | 2410.14                       | 2412.36                       |                               |
| 7  | 2452.47                       | 2454.43                       | $D_{sJ}^{+}(2463)$            |
| 8  | 2496.56                       | 2498.31                       |                               |
| 9  | 2542.12                       | 2543.70                       |                               |
| 10 | 2588.96                       | 2590.41                       |                               |
| 11 | 2636.96                       | 2638.28                       |                               |
| 12 | 2686.01                       | 2687.23                       |                               |
| 13 | 2736.04                       | 2737.17                       |                               |
| 14 | 2786.99                       | 2788.05                       |                               |
| 15 | 2838.82                       | 2839.81                       |                               |
| 16 | 2891.50                       | 2892.43                       |                               |
| 17 | 2944.98                       | 2945.86                       |                               |
| 18 | 2999.24                       | 3000.07                       |                               |
| 19 | 3054.26                       | 3055.05                       |                               |
| 20 | 3110.01                       | 3110.76                       |                               |
| 21 | 3166.46                       | 3167.18                       |                               |
| 22 | 3223.61                       | 3224.30                       |                               |
| 23 | 3281.43                       | 3282.08                       |                               |
| 24 | 3339.90                       | 3340.53                       |                               |
| 25 | 3399.00                       | 3399.61                       |                               |
| 26 | 3458.72                       | 3459.30                       |                               |
| 27 | 3519.04                       | 3519.60                       |                               |
| 28 | 3579.95                       | 3580.49                       |                               |
| 29 | 3641.42                       | 3641.94                       |                               |
| 30 | 3703.44                       | 3703.94                       |                               |
Table 15: Kaluza-Klein tower of KK excitations in $K^*K$-system and $D_{sJ}(2317)$-meson.

| n  | $M_{n}^{K^+K^0}$ MeV | $M_{n}^{K^-K^+}$ MeV | $M_{exp}^{K^*K}$ MeV |
|----|----------------------|----------------------|-----------------------|
| 1  | 1392.02              | 1392.48              | $h_1(1^{+-})$         |
| 2  | 1400.05              | 1400.53              |                       |
| 3  | 1413.30              | 1413.82              |                       |
| 4  | 1431.57              | 1432.15              | $f_1(0^{1++})$        |
| 5  | 1454.63              | 1455.27              |                       |
| 6  | 1482.18              | 1482.89              | $\eta(0^{0-+})$      |
| 7  | 1513.94              | 1514.71              | $f_1(0^{1++})$        |
| 8  | 1549.58              | 1550.42              |                       |
| 9  | 1588.80              | 1589.70              |                       |
| 10 | 1631.30              | 1632.27              | $\eta_2(0^{2-+})$    |
| 11 | 1676.82              | 1677.83              |                       |
| 12 | 1725.08              | 1726.14              |                       |
| 13 | 1775.85              | 1776.95              |                       |
| 14 | 1828.91              | 1830.04              |                       |
| 15 | 1884.06              | 1885.22              |                       |
| 16 | 1941.12              | 1942.29              |                       |
| 17 | 1999.92              | 2001.11              |                       |
| 18 | 2060.32              | 2061.51              |                       |
| 19 | 2122.17              | 2123.38              |                       |
| 20 | 2185.37              | 2186.57              |                       |
| 21 | 2249.79              | 2251.00              | $D_{sJ}(2317)$        |
| 22 | 2315.35              | 2316.55              |                       |
| 23 | 2381.94              | 2383.14              |                       |
| 24 | 2449.49              | 2450.68              |                       |
| 25 | 2517.92              | 2519.10              |                       |
| 26 | 2587.17              | 2588.34              |                       |
| 27 | 2657.17              | 2658.33              |                       |
| 28 | 2727.87              | 2729.01              |                       |
| 29 | 2799.22              | 2800.35              |                       |
| 30 | 2871.17              | 2872.28              |                       |