Anomalous Couplings and Chiral Lagrangians: 
An Update†

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ABSTRACT

We present an update of the limits expected for anomalous gauge boson couplings in the language of chiral Lagrangian operators at the LHC and the Linear Collider. Both, the $e^+e^-$ and the $\gamma\gamma$ mode of the Linear Collider are analyzed. With a 500 GeV $e^+e^-$ collider, and an integrated luminosity of $50 - 80 \text{ fb}^{-1}$, one reaches the domain of precision measurements.

†To appear in the Proceedings of the LC2000 Workshop, Annecy, Gran Sasso and Hamburg, February – September 1995
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1 Introduction

In case a Higgs boson is not found by the time the linear collider is running, one will have to rely on studying the dynamics of the Goldstone Bosons, \textit{i.e.} how the $W$ and $Z$ interact, in order to probe the mechanism of symmetry breaking. In this scenario one expects the weak interactions to enter a new regime where the weak bosons become strongly interacting at effective energies of the order of a few TeV. At this scale new resonances might appear. However, the new dynamics could already be felt at lower energies through more subtle and indirect effects on the properties of the weak bosons. These effects may be revealed by precision measurements with the most prominent effects appearing in the self-couplings of the $W$ and $Z$ bosons. Detailed investigations of these self-couplings therefore would provide a window to the mechanism of symmetry breaking.

Present precision data leave no doubt about the local gauge symmetry \cite{1,2} while the proximity of the $\rho$ parameter to one can be considered as strong evidence for a residual global (custodial) $SU(2)$ symmetry. Whatever the new dynamics may be, it should respect these “low energy” constraints. In turn, this makes it possible to easily parameterize possible effects of new physics in the $W$ sector by the introduction of a restricted set of higher order operators. Such constructions have been discussed at length \cite{3,4} and therefore, in this short note, we will only recall the minimal set of operators that describe the residual effect of the new dynamics in the absence of Higgs boson field:

\begin{equation}
\begin{aligned}
\mathcal{L}_{9R} &= -ig\frac{L_{9R}}{16\pi^2} \text{Tr}(B^{\mu\nu}D_\mu \Sigma^\dagger D_\nu \Sigma) \\
\mathcal{L}_{9L} &= -ig\frac{L_{9L}}{16\pi^2} \text{Tr}(W^{\mu\nu}D_\mu \Sigma^\dagger D_\nu \Sigma^\dagger) \\
\mathcal{L}_1 &= \frac{L_1}{16\pi^2} \left(\text{Tr}(D^\mu \Sigma^\dagger D_\mu \Sigma)\right)^2 \\
\mathcal{L}_2 &= \frac{L_2}{16\pi^2} \left(\text{Tr}(D^\mu \Sigma^\dagger D_\nu \Sigma)\right)^2
\end{aligned}
\end{equation}

(1)

The first two operators contribute to the tri-linear couplings $\Delta\kappa_\gamma$ and $\Delta\kappa_Z$. Only $L_{9L}$ affects $\Delta g_\gamma^Z$. $L_{1,2}$ contributes solely to the quartic couplings (see for instance \cite{3}). There is, in fact, yet another operator in conformity with the above symmetries:

\begin{equation}
\mathcal{L}_{10} = gg'\frac{L_{10}}{16\pi^2} \text{Tr}(B^{\mu\nu} \Sigma^\dagger W^{\mu\nu} \Sigma)
\end{equation}

(2)

However, because it contributes to the two-point function, it is strongly constrained.
through the $S$ parameter \cite{5} by LEP data: $L_{10} = -\pi S$. Current limits \cite{2} lead to

$$-0.7 < L_{10} < 2.4$$

(3)

It will be very difficult to improve this limit in experiments at future colliders. This poses a naturalness problem, since one would expect the other operators to be of the same order \cite{3}. If this were the case the improvement that might result from high energy colliders would, at best, be marginal. For instance, such a situation is encountered with a naive scaled-up-QCD technicolour. One way out is to associate the smallness of $L_{10}$ to a symmetry that forbids its appearance, in the same way that the custodial $SU(2)$ symmetry prevents large deviations of the $\rho$ parameter from one. $L_{10}$ represents the breaking of the axial global $SU(2)$ symmetry \cite{7}. Models that naturally incorporate the $L_{10}$ constraint include dynamical vector models that deviate from the usual scaled up versions of QCD by having (heavy) degenerate vectors and axial-vectors like the extended BESS model \cite{8}. The latter implements an $(SU(2)_L \times SU(2)_R)^3$ symmetry. In the following we will follow a model independent approach, assuming that the couplings of Eq. (1) are independent parameters. Taking $L_{10} \sim 0$ we investigate whether future machines could do as well as LEP1 in constraining the remaining operators.

2 Tri-linear Couplings: $L_{9L}, L_{9R}$

In the past, extensive studies on the extraction on the anomalous tri-linear couplings have been performed. Our aim here is to update some of those results for the particular case of the chiral lagrangian approach (Eq. (1)).

2.1 Strategy and Analysis at the NLC

At the NLC, the best channels to look for $L_{9L,R}$ are $e^+e^- \rightarrow W^+W^-$ and $\gamma\gamma \rightarrow W^+W^-$. In the $e^+e^-$ mode one could also use $e^+e^- \rightarrow \nu_\tau\bar{\nu}_\tau$ \cite{2} (which singles out the photonic part) or $e^+e^- \rightarrow \nu\bar{\nu}Z$ \cite{10} (which isolates the $WWZ$ part). Beside the fact that within the chiral Lagrangian approach both non-standard $WW\gamma$ and $WWZ$ couplings appear, the latter channels are found not to be competitive with $W$ pair production. One reason is that $W$ pair production has a much richer helicity structure that can directly access the longitudinal modes of the $W$. $WW\gamma$ and $WWZ$ \cite{11} production in $e^+e^-$ collisions are quite interesting but, as far as tri-linear couplings are concerned, they can compete with the $WW$ channel only for TeV energies since they suffer from much lower rates \cite{11}. However they are well suited to study possible quartic couplings. In this respect $WWZ$ production can probe the all important $L_{1,2}$ operator.

In the $e^-e^-$ mode one can investigate the tri-linear couplings through $e^-e^- \rightarrow e^-W^+\nu$ \cite{12}, which unfortunately exhibits the same shortcomings as $e^+e^- \rightarrow \bar{\nu}_\tau\nu_\tau Z$. Cuypers has studied the potential of this reaction in probing the tri-linear couplings by taking into account the possibility of polarized beams, fits have only been done to the scattering
angle of the final electron. The limits do not compare well with those obtained from \( e^+e^- \rightarrow W^+W^- \). The \( \gamma\gamma \) mode is ideal in probing the photonic couplings due to the very large cross section for \( WW \) production \[13\]. In the chiral approach this reaction will only constrain the combination \( L_{9L} + L_{9R} \) but, as we shall see, in conjunction with the \( e^+e^- \) mode this is very helpful.

\( W \) pair production, both in \( \gamma\gamma \) and \( e^+e^- \) collisions, provides a large data sample and involves different helicity states. To fully exploit these reactions it is then important to reconstruct all the elements of the density matrix for these reactions (both in the \( \gamma\gamma \) and \( e^+e^- \) mode). This can be achieved by analysing all the information provided by the complete set of the kinematical variables related to the decay products of the \( W \)’s, rather than restricting the analysis to the angular distribution at the level of the \( W \). Since binning in the 5 variables characterizing a \( WW \rightarrow 4 \) fermion event requires high statistics, a \( \chi^2 \) fit is not very efficient and a maximum likelihood technique is used. Initial state polarization can also be easily implemented. The results presented here are based on using the semi-leptonic final states only. The impact of the the non-resonant diagrams (which could introduce a bias) is also quantified. The issue of luminosity and the improvement in the limits by going to higher energy will be discussed, and we shall compare our results with those of Barklow \[14\] which include initial state radiation (ISR) effects.

### 2.2 Analysis at the LHC

Before discussing the limits on the parameters of the chiral Lagrangian that one hopes to achieve at the different modes of the linear colliders it is essential to compare with the situation at the LHC. For this we assume the high luminosity option with 100 fb\(^{-1}\). The LHC limits are based on a very careful study \[15\] that includes the very important effects of the QCD NLO corrections as well as implementing the full spin correlations for the most interesting channel \( pp \rightarrow WZ \). \( WW \) production with \( W \rightarrow jets \) production is fraught with a huge QCD background, while the leptonic mode is extremely difficult to reconstruct due to the two missing neutrinos. Even so, a thorough investigation (including NLO QCD corrections) for this channel has been done \[16\], which confirms the superiority of the \( WZ \) channel. The NLO corrections for \( WZ \) and \( WW \) production at the LHC are huge, especially at large \( W \) and \( Z \) boson transverse momenta where effects of the anomalous couplings are expected to show up. In the inclusive cross section this is mainly due to, first, the importance of the subprocess \( q_1g \rightarrow Z/Wq_1 \) (large gluon density at the LHC) followed by the “splitting” of the quark \( q_1 \) into \( W/Z \). The probability for this splitting increases with the \( p_T \) of the quark (or \( Z/W \)): \( \text{Prob}(q_1 \rightarrow q_2W) \sim \frac{\alpha_s}{4\pi} \ln^2\left(\frac{p_T^2}{M_W^2}\right) \). To reduce this effect one has to define an exclusive cross section that should be as close to the LO \( WZ \) cross section as possible by cutting on the extra high \( p_T \) quark, rejecting any jet with \( p_T^{\text{jet}} > 50 \text{ GeV}, |\eta_{\text{jet}}| < 3 \). This defines a NLO \( WZ/WW + \text{“0 jet”} \) cross section which is stable against variations in the choice of the \( Q^2 \) but which nonetheless can be off by as much as 20% from the prediction of the Born \( SM \) result.
2.3 Comparison and Discussion

Figure 1: Limits on \( (L_{9L}, L_{9R}) \) from \( e^+e^- \) collisions including ISR and beam polarization effects with only the resonant diagrams. The effect of keeping all resonant diagrams for the semi-leptonic final state is also shown. Limits from \( \gamma\gamma \rightarrow W^+W^- \), and \( WZ \) and \( WW \) production at the LHC, are also shown for comparison.

Fig. 1 compares the limits one expects from the NLC and LHC while quantifying the various approximations that can affect the \( e^+e^- \rightarrow W^+W^- \) analysis. First, we address the issue of non-resonant contributions. Gintner, Godfrey and Couture [17] considered all diagrams that contribute to the semi-leptonic \( WW \) final state. Requiring that the \( jj \) and \( \ell\nu \) invariant masses are both within 10 GeV of \( M_W \), one essentially selects the doubly-resonant, \( WW \) mediated, process. Taking into account all contributions including the background to \( WW \) production only marginally degrades the limits. Changing the luminosity can to a very good approximation be accounted for by a scaling factor, \( \sim \sqrt{\mathcal{L}} \) (compare the analyses performed with an integrated luminosity of 10 fb\(^{-1}\) and 50 fb\(^{-1}\)). This result is confirmed also by the analysis conducted by Barklow [14] which assumes higher luminosities but incorporates ISR effects as well as beam polarization.
When polarization is assumed, the total luminosity shown on the plot is shared equally between a left-handed and a right-handed electron (assuming 90% longitudinal electron polarization).

The important conclusion drawn from Fig. 1 is that through $e^+e^- \rightarrow W^+W^-$ alone one indeed reaches the domain of precision measurements with an integrated luminosity of $50 - 80 \text{ fb}^{-1}$, matching the accuracy of LEP1 for $L_{10}$. It is quite fascinating that we can achieve this level of precision already with $\sqrt{s} = 500 \text{ GeV}$. Moving to the TeV range, the limits improve by an additional order of magnitude (see Fig. 1).

$\gamma \gamma \rightarrow W^+W^-$ is quite important. The results [18] shown in Fig. 1 consider a luminosity of 20 fb$^{-1}$ with a peaked fixed spectrum corresponding to a center of mass $\sqrt{s}$ energy of 400 GeV (80% of $\sqrt{s_{ee}} = 500 \text{ GeV}$). Convolution with a photon spectrum has not been done, since recent studies show that there is still much uncertainty concerning the form of the spectrum due to multiple rescattering effects [19]. Clearly the $\gamma \gamma$ mode helps to improve the limits extracted for $e^+e^-$ collisions at $\sqrt{s} = 500 \text{ GeV}$ and integrated luminosities of 50 fb$^{-1}$ or less.

Fig. 1 also compares the situation with the LHC. One observes that the limits obtained from $W^\pm Z$ production are considerably better than those derived from the $WW$ channel. This is mostly due to the absence of serious background contributions in the $WZ$ case. In the $WW$ case, $tt$ production is the main background which is difficult to suppress [17]. However, since $pp \rightarrow WZ$ effectively only constrains $L_{9L}$ (through $\Delta g_1^Z$), the LHC is not very sensitive to $L_{9R}$. As a result, with 50 fb$^{-1}$ and 500 GeV, the NLC constrains the two-parameter space much better than the LHC.

Finally, we briefly comment on the genuine quartic couplings, which are parameterized through $L_{1,2}$. These are extremely important as they are the only couplings which involve the longitudinal modes and hence are of crucial relevance when probing the Goldstone interaction. They are best probed through $VV_L \rightarrow V_LV_L$ scattering. However, for $\sqrt{s} = 500 \text{ GeV}$ the $V_L$ luminosity inside an electron is unfortunately rather small, and one has to revert to $e^+e^- \rightarrow W^+W^-Z$ production, as suggested in [11]. This channel has been re-investigated by A. Miyamoto [20] who conducted a detailed simulation including $b$-tagging to reduce the very large background from top pair production. With a luminosity of 50 fb$^{-1}$ at 500 GeV, the limits are not very promising and do not pass the benchmark criterion $L_i < 10$. It is found that $-95 < L_1 < 71$, $-103 < L_2 < 100$ (one parameter fits). These limits agree very well with the results of a previous analysis [11]. To seriously probe these special operators one needs energies in excess of 1 TeV. At 1 TeV the bounds improve to $L_{1,2} \sim 6$ [3]. However, it is difficult to beat the LHC here, where limits of $\mathcal{O}(1)$ are possible [3] through $pp \rightarrow W_L^+W_L^-$.

In conclusion, it is clear that already with a 500 GeV $e^+e^-$ collider and an integrated luminosity of about 50 – 80 fb$^{-1}$ one can reach a precision on the parameters that probe $SB$ in the genuine tri-linear $WWV$ couplings which is similar to that which can be achieved with LEP1 from oblique corrections to the $Z$ boson parameters. The sensitivity of the NLC is further enhanced if $\gamma \gamma \rightarrow W^+W^-$ can be studied.
Results from this channel would provide invaluable information on the mechanisms of symmetry breaking, if no new particle is observed at the LHC or NLC (Light Higgs and SUSY). The NLC is unique in probing the vector models that contribute to $L_9$ (with $L_{1,2} \sim 0$) and hence is complementary to the LHC. The latter is extremely efficient at constraining the “scalar” models. To probe deeper into the structure of symmetry breaking, a linear collider with an energy range $\sqrt{s} \geq 1.5$ TeV would be most welcome.

Acknowledgements

We would like to thank Tim Barklow for providing us with the results of his analysis.

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