A modified estimation method of GLONASS inter-frequency bias for RTK based on short baseline

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Abstract. Different from other satellite systems, GLONASS adopts frequency-division multiple access (FDMA) technology to distinguish satellites, which directly leads to frequency deviation during signal receiving. Additionally, carrier phase inter-frequency bias (IFB) between different types of receivers are usually not zero, hindering the ambiguity resolution and thus limits the positioning accuracy. To solve this problem, a new estimating method of GLONASS ambiguity is proposed in this paper. According to the variation range of frequency deviation rate, the influence of phase IFB on ambiguity was removed by using searching method and quadratic linear model, thus reducing phase IFB estimating problem to an optimizing one. Feasibility of the method was verified using zero baseline data. Results show that the method can guarantee the reliability of RTK solution effectively.

1. Introduction

Due to the lack of an effective integer ambiguity fixing method, there is still a gap between GLONASS and GPS in practical efficiency of fast and real-time accurate positioning. One of the reasons is that the FDMA technology adopted by GLONASS leads to carrier phase inter-frequency bias(IFB)[1-3]. Previous results have shown that GLONASS receivers of the same brand have similar phase IFB values, but for receivers with different brands, discrepancies exist[4].

Based on the linear relationship between IFB and signal frequency number, many methods for estimating IFB rate have been proposed[5]. Although in many cases the relative frequencies of deviation are linear, this assumption is not given with 100% certainty. In engineering practice, we found that phase IFB of some receivers is not purely linear, and it is for this reason that the estimation accuracy of ordinary linear method in some cases needs to be improved.

In this paper, based on the receiver channel nonlinear effect and the unknown IFB rate value, the linear relation between phase IFB and frequency number is upgraded to a quadratic square according to the practical problems encountered in engineering. The fixing efficiency of ambiguity and RTK solution result are analyzed in the end.

2. Construction of GLONASS Double-Difference observation model

2.1. Double-Difference observation error elimination

The phase and pseudorange observation equations of GLONASS can be expressed as[6] :

\[
P_i = \rho_i^l - c(dt^i - dt_i) - \Delta_i^l + \delta_i + e_i
\]  

(1)
\[ \lambda_a^i \phi^i_a = \rho^i_a + \lambda_a^i B^i_a + \gamma^i_a - c(d t^i_a - d t_a) - I^i_a + T^i_a + \varepsilon^i_a \]  \hspace{1cm} (2)

In the above equation, \( \rho, \phi \) represent pseudorange (unit: m) and carrier-phase (unit: cycle) measurement, \( i, a \) represent the satellite and receiver number, respectively; \( n = 1, 2 \) are the L1, L2 frequency number; for other satellite system, all satellites carry the same frequency, but for GLONASS with FDMA technology, the carrier frequencies of different satellites have subtle differences; \( \lambda, B \) represent the wavelength and integer ambiguity; \( R, l, T \) the geometric distance between satellite and receiver antenna, ionospheric and tropospheric delays, respectively; \( c \) the speed of light in vacuum; \( d t_a, d t^i_a \) the satellite and receiver clock offset (unit: s); \( \delta, \gamma \) the pseudorange and carrier-phase multipath error; \( \xi, \varepsilon \) the measurement noise for range and phase.

In GPS data processing, errors that affect ambiguity fixing can be eliminated by Double-Difference (DD) method, but GLONASS DD observation is different from other satellite systems. This is due to the fact that GLONASS send signals at different frequencies. Therefore, hardware errors between receiver channels cannot be eliminated even though under zero baseline condition.

Satellite clock error could be eliminated by Single Difference (SD) method between observation values, and for a short baseline within 15 km, the ionosphere and troposphere delay could also be decreased to a negligible level. However, the phase IFB still exist, for they depend on receiver type. Even receivers from the same manufacturer or with the same type also have a chance to be different. Thus, we can obtain the Single Difference observation equation of GLONASS as:

\[ P_{ab}^i = \rho_{ab}^i + \delta_{ab}^i + \xi_{ab}^i \]  \hspace{1cm} (3)

\[ \lambda_{ab}^i \phi_{ab}^i = \rho_{ab}^i + \lambda_{ab}^i B_{ab}^i + \gamma_{ab}^i - c d t_{ab} + \varepsilon_{ab}^i \]  \hspace{1cm} (4)

Where \( b \) stands for the other receiver on the baseline.

In addition, for the other satellite, the DD observation equation between two satellites and receivers can be derived as:

\[ P_{ab}^j = \rho_{ab}^j + \delta_{ab}^j + \xi_{ab}^j \]  \hspace{1cm} (5)

\[ \lambda_{ab}^j \phi_{ab}^j - \lambda_{ab}^i \phi_{ab}^i = \rho_{ab}^j + \lambda_{ab}^j B_{ab}^j - \lambda_{ab}^i B_{ab}^i + \gamma_{ab}^j + \epsilon_{ab}^j \]  \hspace{1cm} (6)

2.2. General estimation method of carrier phase frequency deviation

Modern receivers assign different signals to each satellite, resulting in hardware deviation between channels[7]. Previous results have shown that receiver terminal phase IFB can be expressed as a linear function of frequency number, which contains a constant term and a primary term proportional to frequency[8].

In the same frequency band, constant term and primary term coefficient are consistent for all GLONASS satellites, so the constant term can be eliminated in DD process. While primary term coefficient changes with the change of satellite number, so it cannot be eliminated. According to the linear relationship between phase IFB and frequency number, the phase IFB of each DD observation can be simplified by frequency number and IFB rate as follows:

\[ \gamma_{ab}^i = (k^i - k^j) \Delta \gamma_{ab} \]  \hspace{1cm} (7)

Where \( \gamma_{ab}^i \) is the phase IFB of DD observed value; \( \Delta \gamma_{ab} \) refers to the differential phase IFB rate between two receivers; \( k \) represents the satellite frequency number.

Insert Equation (7) into Equation (5)(6) to get:

\[ P_{ab}^i = \rho_{ab}^i + (k^i - k^j) \Delta \gamma_{ab} + \xi_{ab}^{ij} \]  \hspace{1cm} (8)

\[ \lambda_{ab}^i \phi_{ab}^i - \lambda_{ab}^j \phi_{ab}^j = \rho_{ab}^i + \lambda_{ab}^i B_{ab}^i - \lambda_{ab}^j B_{ab}^j + (k^i - k^j) \Delta \gamma_{ab} + \varepsilon_{ab}^{ij} \]  \hspace{1cm} (9)
According to the observation result (9), the station coordinates hidden in the geometric distance, ambiguity parameter and IFB rate can be estimated simultaneously.

3. Improved nonlinear IFB rate estimation method

Medium and short baselines are usually used in studies to estimate the carrier phase deviation of GLONASS[9]. In practical engineering applications, it is found that due to the inconsistent hardware conditions of receiver, the inter-frequency deviation sometimes does not have a complete linear relationship with frequency number, and the continuous use of the linear estimation method will result in the reduction of ambiguity fixation. In this paper, we adopt the following strategy to solve this contradicting problem:

The inter-frequency bias of receiver \( a, b \) equipped in zero-base line test at each frequency point of satellite \( i, j \) are recorded as \( \gamma'_i, \gamma'_j, \gamma'_i, \gamma'_j \). In view of the fact that the frequency offset of some receivers at various frequency points is found to be nonlinear, the above frequency offset can be written as follow(in meters):

\[
\gamma'_i = \text{ifb}_a \cdot k_i^2 + \text{ifb}_b \cdot k_i + \gamma'_i = \text{ifb}_a \cdot k_j^2 + \text{ifb}_b \cdot k_j
\]

\[
\gamma'_b = \text{ifb}_a \cdot k_i^2 + \text{ifb}_b \cdot k_i + \gamma'_b = \text{ifb}_a \cdot k_j^2 + \text{ifb}_b \cdot k_j
\]

Where \( k \) is the satellite frequency value, and the value range is (-7~6).

The frequency bias between receivers can be expressed as follows:

\[
\gamma'_{ab} = (\text{ifb}_a - \text{ifb}_b) \cdot k_i + (\text{ifb}_a - \text{ifb}_b) \cdot k_j
\]

\[
\gamma'_{ab} = (\text{ifb}_a - \text{ifb}_b) \cdot k_i + (\text{ifb}_a - \text{ifb}_b) \cdot k_j
\]

The following is an example of how to estimate the difference in frequency based on the carrier phase output of two receivers.

The carrier phase observations of receiver \( a \) and \( b \) after clock difference correcting at different satellite frequency points are respectively represented as \( \phi'_i, \phi'_j \) (\( i \) is satellite number, and the unit is converted into meters), the corresponding value \( k \) of different satellite numbers are denoted as \( k_i, k_j \), and the SD carrier phase of different satellites is calculated, as shown in the following two equations:

\[
\Delta \phi = \phi' - \phi'_b, \quad \Delta \rho = \rho' - \rho'_b
\]

Select a SD carrier phase data without cycle slip for each satellite, and calculate the mean value:

\[
\Delta \phi_i = \Delta \phi_m \left( i \neq m \right), \quad \Delta \rho_i = \Delta \rho_m \left( i \neq m \right)
\]

Take any satellite as a reference to calculate the DD pseudorange and carrier phase:

\[
\hat{\Delta} \phi_{i,m} = \Delta \phi_i - \Delta \phi_m, \quad \hat{\Delta} \rho_{i,m} = \Delta \rho_i - \Delta \rho_m
\]

The DD results of pseudorange and carrier phase after group delay modifying are shown in the following two equations:

\[
\Delta \phi_{i,m} = \Delta \phi_i - \Delta \phi_m \left( i \neq m \right) \quad \Delta \rho_{i,m} = \Delta \rho_i - \Delta \rho_m \left( i \neq m \right)
\]

The \( m \) satellite SD ambiguity of carrier phase is denoted as \( B_m \), and the SD pseudorange is used to estimate it, as shown in the following formula:

\[
B = \text{round}[\left( \Delta \rho_m / \lambda_m \right)]
\]
Where $\lambda_m$ is the signal carrier wavelength of satellite $m$. Due to the influence of pseudorange measurement noise, there will be errors. The pseudorange SD error of the monitoring receiver is within 2 meters, and the value range of $B_m$ is $(\hat{B}_m - 10, \hat{B}_m + 10)$.

The SD ambiguity of other satellites is obtained by direct rounding:

$$B_i = \text{round}((\Delta^2 \phi_{r,m} + B_m \lambda_m) / \lambda_i)$$

(20)

The DD carrier phase after deducting the SD ambiguity of each satellite is shown as below:

$$\Delta^2 \phi_{r,m}(\text{ifb\_rate}, \text{ifb\_rate'}, B_m) = \Delta^2 \phi_{r,m}(\text{ifb\_rate}, \text{ifb\_rate'}) - (B_i \lambda_i - B_m \lambda_m)$$

(21)

The ergodic search method is used to find a combination of $\text{ifb\_rate}$ and $B_m$ to get to a minimum value of the following expression:

$$\epsilon = \sum_{i=1}^{M} |\Delta^2 \phi_{r,m}(\text{ifb\_rate}, \text{ifb\_rate'}, B_m)|$$

(22)

Where the searching step of $\text{ifb\_rate}$ is 0.0005 meters and searching range is (-0.025,0.025), the searching range of $B_m$ is (-10,10) with a 1 searching step.

In order to minimize $\epsilon$, record the estimation of $\text{ifb\_rate}$, $B_m$ as $\hat{\text{ifb\_rate}}$ and $\hat{B}_m$, then the carrier phase and pseudorange DD after the group delay and SD ambiguity are corrected is shown in the following two equations:

$$\Delta^2 \phi_{r,m} = \Delta^2 \phi_{r,m} - \text{ifb\_rate}[k_i - k_m] - \text{ifb\_rate'}[k_i - k_m] - (\hat{B}_i \lambda_i - \hat{B}_m \lambda_m)$$

(23)

$$\Delta^2 \rho_{r,m} = \Delta^2 \rho_{r,m} - \text{ifb\_rate}[k_i - k_m] - \text{ifb\_rate'}[k_i - k_m]$$

(24)

Where $\hat{B}_i = \text{round}((\Delta^2 \phi_{r,m}(\text{ifb\_rate}, \text{ifb\_rate'}) + \hat{B}_m \lambda_m) / \lambda_i)$.

4. Experimental analysis

In order to analyze the effectiveness of the method proposed in this paper, a set of zero baseline data is selected. Two different types of receivers are used for data acquisition, namely IGMAS and the self-innovative one by nudt navigation center. The baseline data sampling interval was 1s, and the observation duration was 24 h.

Figure 1. Antenna location and receivers for zero-baseline experiment

4.1. Ambiguity fixing effect

According to the variation range of IFB rate, the error curve of each traversal value is obtained by searching. The success rate of ambiguity fixing will fluctuate with the change of IFB rate. When the error reaches the minimum value, that is, when a clear error trough is found, the IFB rate value of this group is locked and the ambiguity is considered to be unaffected by the inter-frequency bias at this time, as shown in figure 2.
The correction value of inter-frequency bias is obtained by solving the zero baseline, as shown in figure 3. Only a limited part of searching is fixed and thus we could find a minimum error value point, because the hardware deviation has already been proved to exist within a certain scope. The residual error of carrier phase DD after correction for simulating is shown figure 4(a) and in order to verify the feasibility of this method on actual data, we select one of the satellite pairs, and the residual error after correction is shown in figure 4(b). It can be seen that the corrected residuals of the DD observations are all white noise, thus the systematic deviation caused by the inter-frequency bias can be treated as already been eliminated.

4.2. RTK results analysis
LAMBDA algorithm is used to calculate the DD ambiguity float to fixed solution, and the threshold value of RATIO is set as 3.0. The single epoch ambiguity fixing mode is adopted in the process. The obtained correction values are applied to the short baseline ambiguity resolution. In order to further study the influence of carrier phase bias on ambiguity resolution, the two modes before and after correction were respectively analyzed and compared experimentally. The results of the ambiguity resolution were counted, details are showed in Table 1.
It can be seen that the success rate of ambiguity resolutions before data correction is obviously low. After correcting the inter-phase bias of carrier phase observation, the success rate is greatly improved, reaching 98.54% and 82.32%, which proves the necessity of bias correcting and the correctness of the optimizing method proposed in this paper. Using the relative after event positioning result as the true value for comparison, the positioning accuracy statistics were carried out for all epochs with fixed solutions, figure 5, 6 and Table 2 are obtained.

| Mode               | Success rate          |
|--------------------|-----------------------|
|                    | float | fixed   |
| With correction    | 98.5412% | 82.3203% |
| Without correction | 90.6106% | 67.8291% |

Table 2. Standard deviation of zero baseline with and without bias corrections applied to GLONASS observations

| Standard deviation(mm) | IGMAS - Self-developed receiver | E     | N     | U     |
|------------------------|---------------------------------|-------|-------|-------|
| With correction        | 0.003                           | 0.005 | 0.020 |
| Without correction     | 0.071                           | 0.084 | 0.091 |

Figures 5 and 6 show the mixed receiver baseline solution before and after applying deviation correction. The experimental pot was on the roof of Navigation Center of National University of Defense Technology. IGMAS and self-developed receivers were connected to the same antenna through a splitter to form a zero baseline condition. Dual-frequency GLONASS static RTK solutions are obtained by GLONASS correction and non-correction observations.

Data analysis shows that dual-frequency positioning between receivers is not possible without calibration of GLONASS inter-channel deviation. However, the ambiguity fixing success rate reached 98.5% after the GLONASS carrier phase deviation nonlinear estimation.

As with the previous mixed receiver baseline processing, the number of GLONASS ambiguities that were fixed after applying the GLONASS inter-channel bias corrections is significantly larger than if the corrections were not applied. Consequently, a higher standard deviation was assured after applying the corrections. Therefore, after the application of the correction, a smaller standard deviation is ensured (Table 2), and the correction method proposed in this paper is feasible.
5. Summary
GLONASS is not widely used compared with GPS. The main reason is that GLONASS adopts FDMA technology, which leads to the phase IFB of different satellites at the receiver end. In this paper, GLONASS DD observation equation considering the influence of IFB is derived. Through the phase IFB nonlinear relationship analysis, we consider that the phase IFB estimation problem can be reduced to an optimization problem. The experimental results show that under the condition of zero base line, by using method described in this paper, the success rate of ambiguity fixing and the accuracy of RTK positioning are effectively improved.

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