Structure of the magnetic fields in A stars and white dwarfs

J. Braithwaite, H.C. Spruit

Max-Planck-Institut für Astrophysik, Postfach 1317, 85741 Garching, Germany.

Some main-sequence stars of spectral type A are observed to have a strong (0.03 - 3 tesla), static, large-scale magnetic field, of a chiefly dipolar shape – the ‘Ap stars’\(^1\)\(^−\)\(^4\) (for example Alioth, the fifth star in the Big Dipper). Following their discovery fifty years ago, it was speculated that these fields are remnants of the star’s formation, a ‘fossil’ field\(^6\)\(^,\)\(^7\). Alternatively, they could be generated by a dynamo process in the star’s convective core\(^5\). The dynamo hypothesis has difficulty explaining high field strengths and the lack of a correlation with rotation. The weakness of the fossil-field theory has been the absence of known field configurations stable enough to survive in a star over its lifetime. We demonstrate here the formation of stable magnetic field configurations, with properties agreeing with those observed, by evolution from arbitrary, unstable initial fields. The results are applicable equally to Ap stars, magnetic white dwarfs and some highly magnetised neutron stars known as magnetars. This establishes fossil fields as the natural, unifying explanation for the magnetism of all these stars.

For a fossil field to survive over millions of years it has to be stable on an Alfvén
time-scale (the time for an Alfvén (magnetic) wave to cross the star, of the order of years for the observed field strengths). In addition there is the unavoidable decay of the field by the finite electrical resistivity of the plasma, but for an A star this is a slow process.

Although there have been educated guesses as to what shape a field in a stable equilibrium might have\textsuperscript{8,9}, all configurations studied with the analytic methods available so far were found to be unstable on the Alfvén time-scale; it has been impossible to prove stability for even a single field configuration\textsuperscript{10,11}.

Where analytic methods fail, numerical simulations can sometimes be useful. The field configurations sought here, if they exist, are global, with length scales of the order of the size of the star. This makes them amenable to full time-dependent 3-D numerical simulations. This is in contrast to dynamo-generated magnetic fields like that of the Sun, which have global length scales as well as very small length scales that cannot be simultaneously resolved with current technology.

With a grid-based magnetohydrodynamics code\textsuperscript{12}, we model a non-rotating, self-gravitating body of electrically conducting plasma with pressure and temperature profiles approximating those of a real star (a polytrope of index $n = 3$). The initial magnetic field is random, with the smallest length scales dominating. The initial ratio of magnetic to thermal energy density is roughly constant through the star.

From this state, the field decays quickly at the beginning of the simulation. However, after a few Alfvén time-scales, the decay slows down as a stable equilibrium is approached. Remarkably, this equilibrium was found to be always of the same general shape, regardless of the particular realisation of the random initial field. It is roughly
axisymmetric, consisting of both toroidal (azimuthal) and poloidal (meridional) components, in the shape of a roughly circular torus. Its structure is consistent with guesses based on previous analytical work\textsuperscript{8,10,13} which suggested that any stable field configuration must be twisted, with a mixture of poloidal and toroidal fields of similar strengths.

The orientation of the torus and the handedness of its twist (right or left-handed) depend on the particular realisation of the initial random field. Usually the resulting torus is also slightly distorted and somewhat offset from the centre of the star. An example of such a field configuration is shown in Fig. 1. The surface field strength also depends on the initial conditions, but stable fields as high as 1 T are easily reached.

In the atmosphere, the field is of the poloidal, ‘offset dipole’ type seen in most observations, but the structure below the surface is quite different. This must be so, since any untwisted, purely poloidal field in the star is known to be highly unstable\textsuperscript{10,14,15}. This can be seen by considering a configuration consisting of a pure dipolar field outside the star, matched onto a uniform field inside\textsuperscript{16}. With respect to a plane, parallel to the field lines, which divides the star into two halves, this configuration is like two bar magnets parallel to each other with their north poles adjacent. The magnets will tend to rotate until the north pole of one is next to the south pole of the other. The same happens to a star with a purely poloidal field: one half of the star can rotate by 180° with respect to the other half. We have numerically followed the evolution of this example, and found the field to decay completely in a few Alfvén time-scales. In the configurations starting with random fields, on the other hand, the decay is halted by the azimuthal field component of the twisted torus. It provides stability, as long as it
carries enough magnetic flux.

Once this stable field has formed it continues to evolve, albeit on a much longer time-scale, as a result of electrical resistivity. Following this evolution numerically, we find that the field strength on the surface of the star now increases again, as a result of gradual outwards diffusion of the field lines, even though the magnetic energy as a whole is going down. Scaling the results to realistic values of the resistivity, the time-scale of this increase corresponds to around $2 \times 10^9$ yrs. This agrees with recent observations\textsuperscript{17} which suggest that the A stars with detectable magnetic fields are about 30% older than normal A stars.

In the course of this outward diffusion phase, the shape of the field gradually changes. The potential field in the atmosphere does not support twisted field lines, as this would require the atmosphere to carry an electric current. Field lines of the torus therefore ‘unwind’ when entering the atmosphere. The toroidal component, which initially accounts for most of the magnetic energy, thereby decreases until the poloidal component starts dominating. When this point is reached, the field becomes distorted in just the way expected from the aforementioned bar-magnet analogy (Fig. 1, lower left). After this the evolution speeds up, and the field disappears within a short time.

The existence of the stable equilibria found here can be interpreted in terms of magnetic helicity, a concept which has also proven useful in other contexts, for instance in the magnetohydrodynamics of the solar corona\textsuperscript{18} and in laboratory plasma experiments\textsuperscript{19}. Defined as the volume integral of the dot product of magnetic field with its vector potential, magnetic helicity is a measure of the net ‘twist’ of a field configuration. It is strictly conserved in the absence of reconnection of field lines, i.e.
when the resistivity vanishes. A magnetic field of non-zero magnetic helicity residing in a perfectly conducting fluid cannot, therefore, decay to nothing. Instead, it relaxes to a stable equilibrium (a local minimum of magnetic energy) with the same value of magnetic helicity. In the laboratory a tendency towards helicity conservation is seen even when resistivity cannot be ignored. In our simulations strict conservation does not hold either (mostly because helicity can leave the star through the atmosphere), but the tendency is still strong enough to allow stable equilibria to form in the interior.

The stars modelled here are non-rotating, guided by the observations which do not show systematic differences between rapidly and slowly rotating stars. Rotation also introduces an additional time-scale, making the calculations an order of magnitude more expensive. It is conceivable, however, that rotation actually has a significant influence on the initial magnetic helicity present, and on observables like the orientation of the magnetic axis in the final state. The effect of the convective core, not modelled here, is expected to be small, as its radius is only around a tenth of the radius of the star, and contains only a negligible fraction of the magnetic energy of the configuration.

The smallness of the convective core is a problem for the dynamo theory of A-star fields, since it would require an implausibly large field strength in this core to produce surface fields as large as a tesla. A second difficulty for this theory is that the observations show no correlation between rotation (an essential ingredient for a dynamo) and field strength.

The field configurations resulting from our quantitative numerical calculations reproduce the key observations: the static nature of the field on historical time-scales, their ‘offset dipole’ shape, the observation that the presence of a field does not depend
on the star’s rotation, the large strengths attainable\(^1\text{−}\(4\), and the observational indication that Ap stars may be somewhat older on average\(^17\).

The deviations from a simple dipole are still significant at the end of the fast initial evolution of the field, before magnetic diffusion simplifies the configuration further on a slower time-scale. From this we predict that the strongest deviations from a dipole should occur in the youngest stars. The results also predict that the field will disappear in a final phase of rapid evolution, though it is not quite certain that this will happen within the star’s lifetime.

The results presented here also apply to objects other than Ap stars. A subgroup of white dwarfs possess a strong magnetic field of similar geometry and total flux to the Ap stars\(^21,22\). Since, unlike the Ap stars, white dwarfs contain no convective core, dynamo processes cannot account for this magnetic field. In addition, there is a small class of neutron stars, the magnetars\(^23\), known for their X-ray outbursts\(^24\), which possess a very strong magnetic field, of the order of \(10^{11}\) tesla\(^25,26\). In other neutron stars, for instance the classical pulsars, the field is weak enough to be held in place by the solid crust on the surface, whereas the strong field of a magnetar would be able to break the crust. A slowly evolving stable configuration in the fluid interior, of the type found here, would explain both the long-term stability of a magnetar’s field and the periodic cracking of the crust observed as X-ray outbursts.

The structure and stability of the magnetic fields in these three types of object can thus be understood by a single process: the spontaneous evolution of a magnetic field under the action of its own dynamics in a self-gravitating fluid.

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Structure of the stable magnetic fields of A-stars and magnetic White Dwarfs, as found with 3-D numerical simulations. (upper left panels) Stereographic view of the long-lived magnetic field configuration as it evolves from a random initial condition. The stable core of the configuration is formed by a torus of twisted field lines inside the star (blue, with axis of torus shown in grey). Field lines which pass through the stellar surface (red) are stabilised by the torus. The configuration slowly evolves outwards by magnetic diffusion. When the torus reaches the surface it becomes unstable (lower panels) and distorts into a shape like the seam on a tennis ball. The initial field configuration for the simulations is constructed from a randomly generated vector potential. The spectrum of spatial wavenumbers $k$ of the variations in the resulting magnetic field is flat (energy spectrum $E(k)$ is a constant). This initial field is tapered such that the ratio of magnetic to thermal energy density in the stellar interior is about 1%, roughly constant as a function of radius. Twenty different random realisations were used. Vacuum conditions outside the star are modelled by embedding the model star in a tenuous, poorly conducting atmosphere. This guarantees that the magnetic field outside the star’s surface stays close to its lowest energy state, a potential field. The numerical code used\textsuperscript{12} is a grid based, explicit MHD code, 6\textsuperscript{th} order in spatial dimensions and 3\textsuperscript{rd} order time. It was run at a resolution of $144^3$ during the initial evolution of the field on the dynamical (Alfvén) time-scale, and $96^3$ to model the slower, diffusive evolution of the stable field and its eventual decay. The value of the electrical resistivity was set such that the magnetic diffusion time was 100 times the initial
Alfvén time-scale. (upper right panel) An azimuthal average of the toroidal (shaded in blue) and poloidal (red lines) field components. The surface of the star, as in the left panels, is shown in yellow. (lower right panel) Schematic showing the axis of the torus field (grey), the torus field lines which are closed within the star (blue) and the untwisted, poloidal field which emerges into the atmosphere (red).
