CRITICAL AND CHAOTIC BEHAVIORS OF QUARKS

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The critical behavior of quarks undergoing phase transition to hadrons is considered in the framework of the Ising model. It is found that spatial fluctuations do not alter the $F$-scaling result obtained earlier in the Ginzberg-Landau formalism. For the study of chaos a new measure must be found for the quark-gluon system, for which the usual description appropriate for classical nonlinear systems is inapplicable. It is shown that the entropy indices $\mu_q$ which characterize the fluctuations of spatial patterns, are as effective as the Lyapunov exponent in measuring chaos. When applied to the QCD parton showers, it is found that the quark jets are more chaotic than gluon jets. The analysis is highly appropriate for a quantitative treatment of the erratic fluctuations of multiparticle production in leptonic, hadronic and nuclear collision processes. It represents a step beyond the traditional intermittency analysis.

1 Introduction

The topics to be discussed here concern the nature of fluctuations of hadronic observables in high-energy collisions. The first is on quark-hadron phase phase transition, a subject that has been investigated from many approaches. Our emphasis is on the scaling behavior of what can be observed experimentally. Our earlier study of the subject has been on the basis of a mean-field theory. We now improve upon that by incorporating spatial fluctuations.

The second topic is on the possibility of chaotic behaviors of a quark-gluon system. Since the number of degrees of freedom of such a system is not conserved, the traditional method of following the distance between two nearby trajectories of a classical system cannot be applied. Thus a major part of our effort in that problem is to identify an appropriate measure of chaos and to show that it is as good as the usual one when applied to the classical problems. With the new measure it is then possible to investigate the question of chaos for QCD parton showers. The significance of this line of investigation is in uncovering the fluctuating properties of multiparticle production that have hitherto been averaged over. The experimental determination of the key measures that quantify those properties will then present a challenge to theoretical models, which may not have been sufficiently accurate (at least in soft interaction) to reproduce them.

The two topics will be discussed separately below, with more space given
to the latter, since it involves newer and more unfamiliar concepts.

2 Critical Behavior

In a series of papers \[ \text{1,2} \] the consequences of quark-hadron phase transition in terms of hadronic observables have been studied in the framework of the Ginzburg-Landau formalism. It is found that the normalized factorial moments \( F_q \) do not exhibit intermittency (i.e., the power-law behavior \( M^q_F \)), but they do satisfy \( F \)-scaling

\[
F_q \propto F_2^{\beta_q},
\]

where

\[
\beta_q = (q - 1)^\nu, \quad \nu = 1.304.
\]

The value of the scaling exponent \( \nu \), which bears no relationship to any of the usual critical exponents, is a definitive characteristic of second-order phase transition, in which the temperature is not a measurable quantity, and the observables are multiplicities of the quanta produced. Heavy-ion collision experiments have shown the validity of (1), but currently the value of \( \nu \) is not 1.304, signifying the absence of quark-hadron phase transition. But in a laser experiment at the threshold of lasing, where the physics is known to be that of a second-order phase transition, it has been shown that (2) is valid to a high degree of accuracy.

The Ginzburg-Landau theory is a mean-field theory, so the spatial properties of the system are assumed to be smooth. To improve on that description, it is necessary to take into account the fluctuating character of hadronization. That has been done recently in two different directions. One is done by simulation on a 2D lattice using the Ising model for hadronization. The other is also done on the lattice, but surface fluctuations (perpendicular to the lattice) are introduced with the thermal effects being constrained by the color confinement potential. We discuss them in turn.

2.1 Spatial fluctuations simulated on the Ising lattice

In the Ginzburg-Landau (GL) approach to the problem of phase transition we relate the order parameter \( \phi(z) \) to the hadron density by \( \rho(z) = |\phi(z)|^2 \). For a bin of area \( \delta^2 \) we assume that \( \rho(z) \) is constant in the bin so the GL free energy is simplified to the form \( F[\phi] = \delta^2 \left( a |\phi|^2 + b |\phi|^4 \right) \). The multiplicity
distribution $P_n$ of hadrons for $T < T_c$ is then given by
\[
P_n = Z^{-1} \int D\phi \frac{1}{n!} (\delta^2 |\phi|^2)^n e^{-\delta^2 |\phi|^2 - F[\phi]}
\]
where $Z = \int D\phi e^{-F[\phi]}$. To improve on this, we allow $\phi(z)$ to vary within a bin by adapting the GL theory of ferromagnetism, using the Ising model for $\phi(z)$
\[
\phi(z) = A^{-1/2} \sum_{j \in A} s_j,
\]
where $s_j$ is the spin $\pm 1$ at site $j$, and $z$ is now the location of the center of a cell of size $A$, which is $< \delta^2$, but large enough to contain several sites. The hadron multiplicity at the $i$th cell is then
\[
n_i = \lambda \left| \sum_{j \in A_i(i)} s_j \right|^2 \theta \left( \sum_j s_j \right)
\]
where $\lambda$ is a scale factor that relates the quark density of the plasma at $T_c$ to the lattice site density in the Ising model. The bin multiplicity is $n_\delta = \sum_{i=1}^{N_\delta} n_i$, where $N_\delta = (\delta/\varepsilon)^2$.

By using the Wolff algorithm to simulate the bin configurations in the Ising model, we can calculate $F_q(\delta)$ and therefore $F_q(M)$, where $M = (L/\delta)^2$, $L$ being the lattice size. We have found that the strict scaling behavior, $F_q \propto M^{2\nu}$, is valid only at $T = T_c$. In fact, $T_c$ can be determined by varying $T$ until that behavior is manifested. However, for $T \leq T_c$ the more general $F$-scaling behavior (1) is valid for a range of $T$. Furthermore, (2) is also valid, but the value of $\nu$ depends on $T$.

Our result is that $\nu \simeq 1.0$ at $T_c$ but $\nu$ becomes bigger at $T < T_c$. For a range of $T < T_c$, the values of $\nu$ are between 1.0 and 1.6 so that the average value of $\nu$ is about 1.3. This is a very satisfying solution to a dilemma posed by an earlier analytical result, where Satz found $\varphi_q = (q - 1)/8$ at $T = T_c$ in the 2D Ising model. It means that $\beta_q = \varphi_q/\varphi_2 = q - 1$, so that $\nu = 1.0$. Now, we see that the GL mean-field result of $\nu = 1.304$ is an average of $\nu(T)$ over a range of $T < T_c$, without invalidating the analytical result at precisely $T = T_c$.

2.2 Surface fluctuations

There is another way to consider the spatial fluctuations of hadronization sites inside a bin. Imagining the 2D surface to be a membrane that can have displacements normal to a flat reference surface, then an outward protrusion from
the plasma interior can be identified as a bump where hadronization is more likely to occur than a dent, an inward indentation. The two competing mechanisms that control the nature of the surface fluctuations are (a) the thermal fluctuations, and (b) the confining potential on the partons that prefers no deformation of the flat membrane. For the latter we can consider an additional term to the free energy, $F_s$, representing an increase due to spatial displacement. Specifically, we take it to be $F_s = C \sum_{<ij>} |z_i - z_j|$, where $C$ is a parameter proportional to the confinement strength, and $z_i$ is the vertical displacement from a flat surface at site $i$. $C$ contains the thermal effect represented by $(k_B T)^{-1}$. The competition between the two mechanisms is then introduced by the Boltzmann factor $\exp(-F_s)$.

In our calculation, we use the GL formalism to describe the mean result, which is to be convoluted with the fluctuation result that is computed on a lattice by Monte Carlo simulation using the $\exp(-F_s)$ factor to determine the probability of nonzero $z_i$ displacements. Consideration of cells in bins is again necessary, as in the Ising problem described above. Omitting the details we summarize here that $F$-scaling as in (1) is still valid; moreover, the slopes $\beta_q$ are nearly independent of $C$ in the range $0 < C < 2$, which includes the values that has the maximum effect due to surface fluctuations. The formula in (2) is also still valid, and the value of $\nu$ is found to be $1.306 \pm 0.035$.

Our conclusion is then that the GL result for the scaling exponent $\nu$ for quark-hadron phase transition is unaffected, when spatial fluctuations due to surface displacements are taken into account.

3 Chaotic Behavior

We now come to fluctuations of a different nature. For phase transitions, because of the competition between the ordered (collective) and disordered (thermal) motions of the constituents at the critical point, there can be large fluctuations over extended domains. But for jets of hadrons produced in hard processes or for multiparticle production at low $p_T$ in hadronic collisions, we do not have dense thermal systems of partons (unless it is a heavy-ion collision at high energy) and phase transition is not the sort of physics that one is concerned with. There are, nevertheless, fluctuations in the event multiplicity and in the phase-space distribution of the particles produced. Are those fluctuations unpredictable? In what way can they be recognized as chaotic behavior?

In order to introduce a quantitative measure of unpredictability we shall proceed in stages. First, we shall discuss the notion of erraticity, which concerns the fluctuations of the spatial patterns of the final states in momentum...
space. Then we shall discuss entropy indices, followed by their applications to both the classical nonlinear problems and the QCD parton showers. The consideration involves an area of phenomenology that has hitherto been unexplored.

3.1 Erraticity

Intermittency refers to the scaling behavior of the multiplicity fluctuations in bins of size \( \delta \). In determining the normalized factorial moments by averaging over all events at a fixed bin and then averaging over all bins, information on the spatial patterns from event to event is lost. Erraticity analysis is an attempt to capture that information.

Let \( F_q \) denote a specific quantification of the spatial pattern in the phase space of a final state. It can be the horizontal moments, the correlation integrals, or the result of wavelet analysis. After many events one obtains a distribution \( P(F_q) \) of \( F_q \). Now, let us define the normalized (vertical) moments

\[
C_{p,q} = \frac{\langle F_q^p \rangle}{\langle F_q \rangle^p},
\]

where the averages are done with \( P(F_q) \) as the probability distribution. The fluctuations of \( F_q \) become more simple to categorize and more interesting to study, if \( C_{p,q} \) exhibits scaling behavior, i.e.,

\[
C_{p,q} \propto M^{\psi_{q}(p)},
\]

where \( M \) is the number of bins in a fixed space. For convenience, we refer to this behavior as erraticity. It is a natural extension of the notion of intermittency. The value of \( p \) can be any positive real number. It should not be negative because \( F_q \) may vanish for some events, so \( P(F_q = 0) \neq 0 \) for some \( q \). For \( 0 < p < 1 \), it is the \( F_q < 1 \) region that is probed by \( \psi_q(p) \), while for \( p > 1 \) it is the spiky events with high \( F_q \) that \( \psi_q(p) \) describes. In practice it is not necessary to consider \( p \) greater than 3.

Quantities similar to \( C_{p,q} \) have been considered before. In statistical physics the random energy model for spin glass has been investigated with the consideration of the quantity \( \langle Z^p(\beta) \rangle \), where \( Z(\beta) \) is the partition function \( \sum \exp [-\beta E(\omega)] \), \( \beta \) being the inverse temperature. It led Brax and Peschanski to study in the \( \alpha \) model the quantity \( \langle Z_q^p \rangle \) where \( Z_q = \sum_m (\rho_m / \sum_m \rho_m)^q \), \( \rho_m \) being the density of particles in the \( m \)th bin. Since \( q \) plays the role of \( \beta \) in \( Z(\beta) \), the possibility of “non-thermal” phase transition is considered in Ref. 12. The same quantity \( \langle Z_q^p \rangle \) has similarly been studied in Ref. 13 also
in the framework of the $\alpha$ model. The emphasis of erraticity analysis is not on the theoretical possibility of a phase transition, but on determining the scaling behavior of $C_{p,q}$ from data (experimental and model simulation) and on extracting the erraticity indices $\psi_q(p)$. It is reasonable to suggest that $\psi_q(p)$ provides a stringent test of the reality of any model on multiparticle production.

The study of erraticity can be applied to many problems in physics, chemistry and beyond. Fluctuation in spatial patterns is a ubiquitous phenomenon. The Ising model, for instance, has many types of patterns in the lattice space depending on the temperature. One can even determine the critical temperature for a finite lattice in any dimension by studying the behavior of $\psi_q(p)$

It is also possible to determine the erraticity spectrum $e(\alpha)$ analogous to the multifractal spectrum $f(\alpha)$. The definition for $e(\alpha)$, for a fixed $q$, is

$$e(\alpha) = p\alpha - \psi(p), \quad \alpha = \frac{d\psi(p)}{dp}.$$  

(8)

At $p = 1$, $e(\alpha) = \alpha$, since $\psi(1) = 0$. That value of the spectrum has a particular significance, as we shall discuss next. The function $e(\alpha)$ exhibits that value explicitly, unlike $\psi(p)$.

3.2 Entropy indices

The unpredictability of the outcome of a dynamical process should be related to some quantity like the entropy that measures the ignorance about the system. We do not want to follow the time evolution of the system, which is not observable. Our entropy must refer to an ensemble of final states. To that end we use $F_q$ as in Sec. 3.1, to describe the spatial pattern of an event, and $P(F_q)$ to describe the distribution of $F_q$ after $N$ events. The average of any function $f(F_q)$ can also be written as

$$\int dF_q P(F_q) f(F_q) = N^{-1} \sum_{e=1}^{N} f(F_{q}^e),$$  

(9)

where $F_{q}^e$ is the $F_q$ for the $e$th event. Introduce now

$$P_e = F_{q}^e / \sum_{e=1}^{N} F_{q}^e,$$  

(10)

in terms of which we can define an entropy in the “event space”

$$S = - \sum_{e} P_e \ln P_e,$$  

(11)
(which should probably be called “eventropy”, since it is not the usual entropy.)
To calculate $S$ it is more convenient to introduce the moments $H_p$ such that

$$S = - \frac{d}{dp} \ln H_p \bigg|_{p=1}, \quad H_p = \sum_e (P_e)^p .$$

(12)

From (9) we have

$$H_p = \mathcal{N} \int dF_q P(F_q) \left[ \frac{F_q}{\mathcal{N} \int dF_q P(F_q) F_q} \right]^p = \mathcal{N} \left( \frac{\langle F_q \rangle}{\langle F_q \rangle} \right)^p = \mathcal{N}^{1-p} C_{p,q} .$$

(13)

If the system exhibits erraticity, it then follows from (7) that

$$S = \ln \left( \mathcal{N} M^{-\mu_q} \right) ,$$

(14)

where

$$\mu_q = \left. \frac{d}{dp} \psi_q(p) \right|_{p=1} .$$

(15)

We refer to $\mu_q$ as the entropy indices.

If $\mu_q = 0$, then $S = \ln \mathcal{N}$, which is large. One may think that it describes a system that is chaotic. Actually, the opposite is true. It corresponds to $P_e = 1/\mathcal{N}$ in (11), i.e., every event has the same value of $F_q$, according to (11). That is hardly what one would expect of a chaotic system, since the spatial patterns are the same for every event. $S$ is large because in the event space $F_q$ is evenly distributed over the entire space. $S$ would be small if $F_q^e \neq 0$ is restricted to only a few events; that would mean large fluctuations in $F_q^e$. Thus finite, nonvanishing positive values of $\mu_q$ corresponds to wide $P(F_q)$, which in turn means unpredictable spatial pattern from event to event. In short, $\mu_q$ is a measure of unpredictability.

An alternative way of calculating $\mu_q$ is to circumvent $S$ and work directly with $C_{p,q}$ by expressing it as

$$C_{p,q} = \langle \Phi_q^p \rangle , \quad \Phi_q = \frac{F_q}{\langle F_q \rangle} .$$

(16)

Then we have

$$\left. \frac{d}{dp} C_{p,q} \right|_{p=1} = \langle \Phi_q \ln \Phi_q \rangle .$$

(17)
On the other hand, if $C_{p,q}$ has the scaling behavior \( [p] \), then we also have

\[
\frac{d}{dp} C_{p,q} \bigg|_{p=1} = \mu_q \ln M
\]

in the scaling range of $M$. Consequently, we obtain

\[
\mu_q = \frac{\partial}{\partial \ln M} \langle \Phi_q \ln \Phi_q \rangle, \tag{19}
\]

which is to be determined in the region where $\sigma_q \equiv \langle \Phi_q \ln \Phi_q \rangle$ exhibits a linear dependence on $\ln M$.

### 3.3 Chaoticity

We now consider the question whether multiparticle production processes are chaotic. Let us first recall the properties of chaotic behavior in classical nonlinear dynamics. Since in such systems trajectories in space-time exist, the distance function $d(t)$ between two trajectories can be defined. A system is chaotic if $d(t) \sim e^{\lambda t}$, $\lambda > 0$, no matter how small $d(0) = \epsilon$ may be, for $\epsilon > 0$.

For classical Yang-Mills dynamics the Lyapunov exponent $\lambda$ has been shown to be positive by lattice calculation. For quantized Yang-Mills fields the problem is vastly more complicated. In addition to the ambiguity associated with quantum chaos in the realm of first quantization, we now have also the problems of nonconservation of the number of degrees of freedom and of the lack of a meaningful definition of trajectory. The absence of an unambiguous notion of time in production processes further results in the nonexistence of the Lyapunov exponent for the problem.

Our first task must be the search for a measure of chaos appropriate to our problem. Since nonperturbative QCD is too difficult to implement, we focus on parton showers in pQCD, i.e., quark and gluon jets in a tree-diagram approximation of QCD branching processes. In place of classical trajectories whose initial points are all in a small neighborhood of one another, we consider branching processes all starting from exactly the same virtuality $Q^2$ and let quantum fluctuations take them to different final states. Since we have no experimental access to the process of branching, and theoretically the degradation of virtuality does not have an unique association with the temporal evolution, our measure of chaos must be focused on what we can observe, viz, the characteristics of the final states. It is in that respect that the subject of erraticity and entropy indices becomes relevant.

There are two issues here. One is to find a measure of chaos, and the other concerns QCD branching processes. We discuss them separately.
3.3.1 A new measure of chaos

Our proposal is to use $\mu_q$ as an alternative measure of chaos in problems where only the spatial patterns can be observed. Whether that approach agrees with the conventional approach can be determined only by applying the measure to known chaotic systems, of which there are many with simple nonlinear dynamics. They are classical systems with well-defined trajectories, and the positivity of the maximum Lyapunov exponent $\lambda$ is a sufficient criterion for chaos. Since our measure is concerned with spatial patterns rather than temporal evolution, it is necessary to construct a spatial pattern for every trajectory. That is achieved by considering a set of points corresponding to the positions of a trajectory at a discrete collection of times, separated by finite intervals apart. Clearly, two nearby trajectories for $\lambda < 0$ would generate two very similar patterns, while two chaotic trajectories for $\lambda > 0$ would have very dissimilar patterns. We can generate $N$ events by considering $N$ trajectories all starting from a small neighborhood of an initial point. Studying the $P(F_q)$ distribution of those events and then comparing the resultant $\mu_q$ to $\lambda$ constitute our procedure to check the effectiveness of $\mu_q$ as a measure of chaos.

We have applied this procedure to the logistic map and the Lorenz attractor, the details of which are omitted here. They are nonlinear systems that become chaotic when their control parameters $r$ exceed certain critical values. We present here only the result for the logistic map. In Fig. 1 the dashed line shows the value of $\lambda$ as a function of $r$. For $r > 3.57$, $\lambda$ becomes positive, although there are short intervals where $\lambda$ drops below zero. The solid line indicates the value of $\mu_2$ (normalized by a specific factor), calculated at discrete points in the same range of $r$. Evidently $\mu_2$ coincides very well with $\lambda$ except that it cannot be negative but vanishes when $\lambda \leq 0$. The multiplicative factor used for $\mu_2$ in the plot depends on the number of spatial points taken to determine the $P(F_q)$ distribution, and has no particular significance. What is of great significance is that we now have an alternative measure of chaos. The positivity of $\lambda$ is coordinated with the positivity of $\mu_2$ (and all other $\mu_q$), so that the fluctuations of spatial patterns can equally serve as a means for revealing chaotic behaviors.

There are many complex systems possessing complex patterns. The determination of their entropy indices may reveal certain features that are not otherwise recognizable. For some nonlinear systems that are known theoretically to be chaotic, the experimental verification of $\lambda$ may be difficult, since the precise adjustment of the initial condition may not always be possible. Our method of studying the spatial patterns may therefore offer a feasible alternative.
3.3.2 Chaos in QCD parton showers

Since perturbative QCD is well developed theoretically, and the final states of quark and gluon jets are well measured experimentally, we propose that the erraticity of hard processes be used as a new arena for comprehensive tests and that the entropy indices of parton showers be determined to reveal the chaoticity of QCD.

Since hadronization is irrelevant to the question of chaos in pQCD, we have developed an efficient Monte Carlo generator of quark and gluon jets with the usual Sudakov form factor and splitting functions incorporated in the code.\[1\]

The initial virtuality of a quark (Q jet) or a gluon (G jet) is fixed at a \(Q^2\) value for all events, and all partons evolve and branch until the virtuality of a parton reaches \(\leq Q^2_0\), where branching is terminated for that parton. From the spatial pattern of each event in the cumulative variable \(\tau\) (in terms of which the inclusive distribution is flat), we calculate the horizontal normalized factorial moments \(F_q\), then the distribution \(P(F_q)\), the vertical moments \(C_{p,q}\), the erraticity indices \(\psi_q(p)\), and finally the entropy indices \(\mu_q\). The results of the calculation for \(Q/Q_0 = 10^3\) are shown in Fig. 2. The positivity of \(\mu_q\) for both Q and G jets indicates that the QCD branching processes are chaotic. Moreover, the Q jets are more chaotic than the G jets. The reason is that quark jets have fewer partons produced, so the multiplicity fluctuations relative to the mean is larger.

We have also considered the fixed coupling problem, since by varying \(\alpha_s\) as a control parameter one can study the behavior of the system at the onset of chaos. It turns out that there is no threshold of chaos. When \(\alpha_s\) is small,
the multiplicity is so small that $\mu_q$ is large for both Q and G jets. Thus chaoticity is an inherent property of QCD dynamics and cannot be tuned out by decreasing $\alpha_s$, the only tunable parameter that we can associate directly with the strength of the nonlinear term.

Using the positivity of $\mu_q$ as a criterion for chaos, we have not only found that the non-Abelian, nonlinear dynamics of QCD is chaotic, but also come to realize that any quantum system in which the number of quanta of interest is not conserved is likely to have positive $\mu_q$. Thus while chaotic behaviors are spectacular and remarkable phenomena in classical dynamics, their occurrences seem to be common and generic in quantum systems. Perhaps the notion of chaos should not be more emphasized in multiparticle production than the realization that the unpredictability of the final states can be quantified. To calculate the entropy indices for all collision processes, soft as well as hard, that can agree with the experimental data would be a theoretical challenge. Our results obtained should not be taken quantitatively for comparison with experiments, since the MC code has not been tuned to check the other features of parton showers. However, they have been effective in demonstrating the procedure of determining $\mu_q$ and in elucidating the question of chaos in QCD.

3.4 Hadronic and nuclear collisions

For hadronic collisions we expect more fluctuations than in $e^+e^-$ annihilation, since there is impact-parameter variation from event to event. Even at fixed impact parameter there can be various cut-Pomerons, each of which can have large variations in the final states corresponding to many possible ways that the branching processes can proceed, at least according to the geometrical branching model (GBM) for soft processes. That does not even include the consideration of hard subprocesses at high energies that would further increase the degree of fluctuations. Gianini has found in his preliminary analysis of the old Fermilab data that $\mu_2$ is in excess of 0.4. Cao is currently upgrading ECCO, an event generator that is based on GBM, by taking into account resonance production. The preliminary result on the entropy indices is that they are also very large.

In heavy-ion collisions the fluctuations in impact parameter are usually controlled by making cuts in $E_T$. Because of the high multiplicity per event not much has been observed in the intermittency behavior. More may be revealed in the erraticity analysis. I venture to speculate here what the results on $\mu_q$ would be in the four possible scenarios: with and without a hadron-gas phase, and with and without a quark-to-hadron phase transition. If there is a hadron gas phase before hadrons are emitted so that there is thermalization
in the final state, then the emitted hadrons are randomized. Random spatial patterns of many particles can differ significantly from event to event, so I would not expect $\mu_q$ to be small whether or not there is a phase transition. If there is no hadron gas phase that randomizes the hadronic final states, then one should expect to see the dynamical effects of the production processes in the erraticity analysis. If there is a phase transition, the produced hadrons should exhibit long-range correlations that result in clusters of all sizes (assuming second-order phase transition and Kadanoff scaling). In that case we should expect larger values of $\mu_q$ than in the case of no phase transition.

4 Conclusion

Significant progress has been made recently on the subject of fluctuations in multiparticle production. On critical behavior considerations of fluctuations beyond the result of mean-field theory have not altered the scaling exponent $\nu$. For noncritical phenomena new measures of fluctuations from event to event are proposed. Experimental determination of $\nu$, $\psi_q(p)$ and $\mu_q$ is urgently needed. For $\nu$ it is necessary to consider high $q$ moments. For $\psi_q(p)$ and $\mu_q$ low $q$ values like 2 and 3 are enough and $p$ in the range 0.5 to 2 is sufficient. It is highly likely that many existing event simulators may not be able to reproduce the experimental data on those quantities. Significant change of $\mu_q$ due to quark-gluon plasma formation would be very exciting.

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