Anti-De Sitter BPS Black Holes in $N = 2$ Gauged Supergravity

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ABSTRACT

Electrically charged solutions breaking half of the supersymmetry in Anti-De Sitter four dimensional $N = 2$ supergravity coupled to vector supermultiplets are constructed. These static black holes live in an asymptotic $AdS_4$ space time. The Killing spinor, i.e., the spinor for supersymmetry variation is explicitly constructed for these solutions.


1 Introduction

Supersymmetric solutions of $N = 2$ ungauged supergravity theory in four and five dimensions have received lots of attention in recent years [1, 2, 3]. Such solutions play a fundamental role in probing the (non)-perturbative phase transitions taking place among $N = 2$ string vacua and M-theory [4]. These solutions admit perturbative and non-perturbative corrections and exhibit a rich structure due to less supersymmetry and the special and very special geometries underlying $D = 4$ and $D = 5$ $N = 2$ supergravity theories. BPS solutions provide a link between the structure of the internal space, e.g. a Calabi-Yau threefold, and the physical properties of the four-dimensional space-time. Singularities in space-time could be related to special points in the internal space. Using non-trivial space-time dependent solutions of $N = 2$ supergravity, some information can be obtained about the dynamical nature of the non-perturbative topological transitions [5].

Recently, there has been renewed interest in gauged supergravity theories in various dimensions. It is motivated by the fact that the ground state of these theories is anti-de Sitter (AdS) space-time and thus they may have implications for the recently proposed AdS/conformal field theory (CFT) correspondence [6]. This implies an equivalence of Type IIB string theory (or M-theory) on anti-de Sitter (AdS) space-time and the conformal field theory (CFT) on the boundary of this space. It is of special interest to address cases with lower or no supersymmetry, in order to shed light on the nature of the correspondence there. Supergravity vacua with less supersymmetry may have an interpretation on the CFT side as an expansion of the theory around non-zero vacuum expectation value of certain operators. Solutions with no supersymmetry could also be viewed as excitations above the ground state of the theory. This makes the study of black holes in gauged supergravity a subject of current research interest. Moreover, it has been realised recently that AdS spaces admit the so-called topological black holes which have some unusual geometric and physical features [7].

The purpose of this paper is to describe static electric BPS black hole solutions of $D=4$ gauged supergravity theories with vector supermultiplets. These gauged supergravity theories admit the AdS$_4$ space-time as a ground state. The analysis is general as we formulate our solutions in terms of the holomorphic sections of $D=4$, $N=2$ supergravity. $N = 4, 8$ black holes appear as a special subclass for special choices of the prepotential of the $N = 2$ theory. For the pure supergravity case (i.e., no vector multiplets), Reissner-Nordström solutions have been discussed in [8]. Supersymmetric solutions have been obtained recently for the theory of $N = 2$ supergravity in five dimensions [9] as well as for the $N = 8$, $D = 4$ theory [10]. Also BPS topological black hole solutions for the pure $N = 2$ supergravity case have been constructed in [11]. The spherically symmetric BPS electric solutions can be obtained by solving for the vanishing of the gravitino and gaugino supersymmetry variations for a particular choice for the supersymmetry parameter. We will present here only static non-rotating spherically symmetric electric solutions.
which break half of supersymmetry and leave the rotating, magnetic and more general solutions for a separate publication. The construction of our solutions relies very much on special geometry of the $N = 2$ supergravity theories. For this reason, we review this subject and collect some formulae and expressions of $N = 2$ supergravity which will be important for the following discussion. Also the general BPS solutions of $N = 2$ black holes \[2\] in the theory of ungauged $N = 2$ supergravity with vector supermultiplets are briefly discussed. Our conventions are collected in the Appendix.

2 Special Geometry and Black Holes in $N = 2$ Supergravity

Special geometry comes about when one couples vector supermultiplets to $N = 2$ supergravity in four space-time dimensions (for a recent discussion and references therein see \[12\]). The complex scalars $z^A$ of the vector supermultiplets of the $N = 2$ supergravity theory are coordinates which parametrise a special Kähler manifold. Roughly, this is a Kähler-Hodge manifold with a constraint on the curvature,

$$R_{ABCD} = g_{AB} g_{CD} + g_{AD} g_{CB} - C_{ACE} C_{BDL} g^{EL},$$

where $g_{AB} = \partial_A \partial_B K$, is the Kähler metric with $K$ the Kähler potential and $C_{ABC}$ is a completely symmetric covariantly holomorphic tensor. Kähler-Hodge manifolds are characterised by a $U(1)$ bundle whose first Chern class is equal to the Kähler class, thus, locally, the $U(1)$ connection can be represented by

$$Q = -i \frac{1}{2} \left( \partial_A K d z^A - \overline{\partial_A} K d \overline{z^A} \right).$$

A definition of special Kähler manifold is given in terms of a flat $(2n + 2)$ dimensional symplectic bundle over the Kähler-Hodge manifold, with the covariantly holomorphic sections

$$V = \begin{pmatrix} L^I \\ M_I \end{pmatrix}, \quad I = 0, \ldots, n \quad D_A V = (\partial_A - \frac{1}{2} \partial_A K) V = 0,$$

obeying the symplectic constraint

$$i \langle V | \tilde{V} \rangle = i (\overline{L^I} M_I - L^I \overline{M_I}) = 1.$$ 

One also defines

$$U_A = D_A V = (\partial_A + \frac{1}{2} \partial_A K) V = \left( f_A^I \right) \frac{f_A^I}{h_{AI}},$$

where $D_A$ and $\overline{D_A}$ are the covariant derivatives\[4\]. In general,

$$M_I = N_{IJ} L^J, \quad h_{AI} = N_{IJ} f_A^J.$$

\[2\] for a generic field $\phi^A$ which transforms under the Kähler transformation, $K \rightarrow K + f + \bar{f}$, by the $U(1)$ transformation $\phi^A \rightarrow e^{-\frac{1}{2} (f + \bar{f})} \phi^A$, we have $D_A \phi^B = \partial_A \phi^B + \Gamma_{AC}^B \phi^C + \frac{1}{2} \partial_A K \phi^B$, $D_A$ is defined in the same way.
Special geometry can be defined in terms of the following differential constraints

\[ D_A V = U_A, \]
\[ D_A U_B = iC_{ABC}g^{CL}U_L, \]
\[ D_A U_B = g_{AB}V, \]
\[ D_A \bar{V} = 0 \]  

(6)

and \[ \langle V, U_A \rangle = 0. \]

(7)

The Kähler potential can be constructed in a symplectic invariant manner by defining the holomorphic sections \( \Omega = e^{-\frac{i}{4} V} \) by

\[ K = -\log \left( i\langle \Omega|\bar{\Omega} \rangle \right) = -\log i\left( \bar{X}^IF_I - X^I\bar{F}_I \right). \]

(8)

Moreover, special geometry implies the following relations

\[ g_{AB} = -i\langle U_A|\bar{U}_B \rangle = -2f_A^I\text{Im}N_{IJ}\bar{f}_B^I, \]
\[ g^{AB}f_A^I\bar{f}_B^I = -\frac{1}{2}(\text{Im}N)^{IJ} - \bar{L}^IL^J, \]
\[ F_I\partial_\mu X^I - X^I\partial_\mu F_I = 0. \]

(9)

The \( N = 2 \) supergravity action includes in addition to the gravitational supermultiplets, a number of vector and hypermultiplets. Throughout this work, the hypermultiplets are assumed to be constants. In this case, the bosonic \( N = 2 \) action is given by

\[ S_{N=2} = \int \sqrt{-g}d^4x \left( -\frac{1}{2}R + g_{AB}\partial^\mu z^A\partial_\mu z^B + i\left( \mathcal{N}_{IJ}F_{\mu\nu}^{-I}F_{\mu\nu}^{-J} - \mathcal{N}_{IJ}F_{\mu\nu}^{+I}F_{\mu\nu}^{+J} \right) \right) \]

where \( F^{\pm I}_{\mu\nu} = \frac{1}{2}\left( F_I^{\mu\nu} \pm \frac{i}{2} \varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}^I \right) \).

(10)

The supersymmetry transformations for the chiral gravitino \( \psi_{\alpha\mu} \) and gauginos \( \lambda^{A\alpha} \) in a bosonic background of \( N = 2 \) supergravity are given by

\[ \delta \psi_{\alpha\mu} = \nabla_\mu \epsilon_\alpha - \frac{1}{4}T_{\rho\sigma}^\gamma \epsilon_\alpha \gamma^\rho \gamma^\sigma \varepsilon_{\alpha\beta} \epsilon_\beta, \]
\[ \delta \lambda^{A\alpha} = i\gamma^\mu \partial_\mu z^A \epsilon_\alpha + G_{\rho\sigma}^{-A} \gamma^\rho \gamma^\sigma \varepsilon_{\alpha\beta} \epsilon_\beta \]

(11)

where

\[ T_{\mu\nu} = M_I F^{I}_{\mu\nu} - L^I G_{I\mu\nu} = 2i(\text{Im}N_{IJ})L^I F^{-J}_{\mu\nu}, \]
\[ G_{\mu\nu}^{-A} = -g^{AB}f_B^I(\text{Im}N)_{IJ}F_{I\mu\nu}^J, \]
\[ G_{I\mu\nu} = \text{Re}N_{IJ}F_{I\mu\nu}^J - \text{Im}N_{IJ}^*F_{I\mu\nu}^J, \]
\[ \nabla_\mu \epsilon_\alpha = (\partial_\mu - \frac{1}{4}w_{\mu}^{ab}\gamma_a\gamma_b + i\frac{1}{2}Q_{\mu})\epsilon_\alpha \]

(12)
where $\epsilon_\beta$ is the chiral supersymmetry parameter, $w_{\mu}^{ab}$ is the spin connection and $Q_\mu$ is the Kähler connection.

We now review the construction of the stationary solutions to the theory of ungauged $N = 2$ supergravity [2]. It is well known from the work of Tod [13] that such metrics can be brought to the following form

$$ds^2 = -e^{2U}(dt + \omega_m dx^m)^2 + e^{-2U} dx^m dx^m. \tag{13}$$

For this metric, the components of the spin connection are given as follows

$$w_{10i} = -e^{2U} \partial_i U, \quad w_{ij} = \frac{1}{2} e^{2U}(\partial_i w_j - \partial_j w_i),$$

$$w_{m0i} = \frac{1}{2} e^{2U}(\partial_m w_i - \partial_i w_m) - e^{2U} w_m \partial_i U,$$

$$w_{mij} = \partial_i U \delta_{mj} - \partial_j U \delta_{mi} + \frac{1}{2} e^{4U} w_m (\partial_i w_j - \partial_j w_i). \tag{14}$$

The equations of motion and the Bianchi identities for the gauge fields can be solved by

$$F_{ij} = \frac{1}{2} e^{2U} \varepsilon_{ijm} \partial_m \tilde{H}^I, \quad G_{ij} = \frac{1}{2} e^{2U} \varepsilon_{ijm} \partial_m H_I,$$

where $(\tilde{H}^I(x), H_I(x))$ are harmonic functions.

The BPS solution can be expressed in the following form [2]

$$e^{-2U} \equiv Z \bar{Z} \equiv i(\bar{Y}^I F_I(Y) - Y^I \bar{F}_I(\bar{Y}))$$

$$\frac{1}{2} e^{2U} \varepsilon_{mnp} \partial_n w_p \equiv Q_m = e^{2U} \text{Re}(\bar{F}_I(\bar{Y}) \partial^I \bar{F}_I(Y) - \bar{Y}^I \partial^I F_I(Y),$$

$$i(Y^I - \bar{Y}^I) = i(\bar{Z} \bar{L}^I - Z \bar{L}^I) = \tilde{H}^I$$

$$i(F_I(Y) - \bar{F}_I(\bar{Y})) = i(\bar{Z} M_I - Z \bar{M}_I) = H_I \tag{15}$$

where $Q_\mu = Q_\mu - i \partial_\mu \log(\frac{\bar{Z}}{Z}).$

For this particular choice of the metric, we obtain

$$T_{ij}^- = \frac{1}{2} e^{2U} \varepsilon_{ijm} (F_I(Y) \partial_m \tilde{H}^I - Y^I \partial_m H_I),$$

$$T_{0k}^- = \frac{i}{2} e^{2U} \varepsilon_{ijm} (F_I(Y) \partial_k \tilde{H}^I - Y^I \partial_k H_I). \tag{16}$$

For the above Ansatz, one can demonstrate that the time-component for the gravitino supersymmetry transformation as well as those of the gaugino vanish for the following choice of the spinor supersymmetry parameter

$$\sqrt{\frac{Z}{\bar{Z}}} \epsilon_\alpha = i \gamma_0 \epsilon_\alpha \beta \epsilon^\beta. \tag{17}$$
Also the equation for the supersymmetry spinor is given by

\[ \left( \partial_m + iQ_m + \frac{1}{2} \partial_m \log \bar{Z} \right) \epsilon_\alpha = 0. \]  

(18)

The integrability condition enforces the condition that the field strength of \( Q_m \) has to vanish. For static non-singular solutions, i.e., \( w_m = 0 \), one imposes the vanishing of \( Q_m \).

### 3 Electric BPS solutions In 4D N=2 Anti-De Sitter Supergravity

In this section we derive the solution for electric BPS states in the theory of Abelian gauged \( N = 2 \) supergravity coupled to vector supermultiplets \[14\]. The theory of gauged \( N = 2 \) supergravity without vector multiplets was first discussed in \[3\] and BPS solutions of this theory were discussed in \[3, 14\]. More recently, supersymmetric topological black holes were obtained in the theory of \( N = 2 \) anti-de Sitter supergravity in \[11\]. Here we will concentrate on electrically charged spherically symmetric BPS solutions of the theory of gauged \( N = 2 \) supergravity coupled to vector supermultiplets. The Abelian gauging is achieved by introducing a linear combination of the abelian vector fields \( A^I_\mu \) already present in the theory, \( A_\mu = \kappa_I A^I_\mu \) with a coupling constant \( g \), where \( \kappa_I \) are constants. The coupling of the fermi-fields to this vector field breaks supersymmetry which in order to preserve one has to introduce gauge-invariant \( g \)-dependent terms. In a bosonic background, these additional terms result in a scalar potential \[14\]

\[ V = g^2 \left( g^{AB} \kappa_I \kappa_J f^I_A f^J_B - 3 \kappa_I \kappa_J \bar{L}^I L^J \right). \]  

(19)

Moreover, the supersymmetry transformations for the gauginos and the gravitino in a bosonic background become (in terms of complex spinors),

\[ \delta \psi_\mu = \left( \mathcal{D}_\mu + \frac{i}{4} T_{\rho\sigma} \gamma^\rho \gamma^\sigma \gamma_\mu - ig \kappa_I A^I_\mu + \frac{i}{2} g \kappa_I \bar{L}^I_\gamma_\mu \right) \epsilon, \]  

(20)

\[ \delta \lambda^A = \left( i \gamma^\mu \partial_\mu z^A + i G^{-A} \gamma^\rho \gamma^\sigma - gg^{AB} \kappa_I f^I_A \right) \epsilon. \]  

(21)

In what follows we are mainly concerned with bosonic backgrounds which break half of supersymmetry. Consider the following general form for the metric

\[ ds^2 = -e^{2A} dt^2 + e^{2B} dr^2 + e^{2C} r^2 (d\theta^2 + \sin^2 \theta d\phi^2). \]  

(22)

The vielbeins of this metric are

\[ e^0_\tau = e^A, \quad e^1_r = e^B, \quad e^2_\theta = r e^C, \quad e^3_\phi = r e^C \sin \theta, \]

\[ e^0_t = -e^A, \quad e^1_r = e^{-B}, \quad e^2_\theta = \frac{1}{r e^C}, \quad e^3_\phi = \frac{1}{r e^C \sin \theta}. \]  

(23)
and the spin connections
\[
\begin{align*}
\omega^{01}_t &= A'e^{A-B}, \\
\omega^{12}_\theta &= -(1 + r C')e^{C-B}, \\
\omega^{13}_\phi &= -(1 + r C')e^{C-B} \sin \theta, \\
\omega^{23}_\phi &= -\cos \theta.
\end{align*}
\]
(24)

Motivated by the form of the BPS solution in $D = 5, N = 2$ gauged supergravity \cite{9}, we choose the following Ansatz for the electrically charged BPS solutions in four-dimensional AdS supergravity
\[
\begin{align*}
e^{2A} &= e^{-2B} - e^{2U} f^2, & e^{2C} &= e^{-2U}, \\
e^{-2U} &= Z \bar{Z} \equiv i(Y^I \bar{F}_I(Y) - Y^J \bar{F}_J(\bar{Y})) = Y^I H_I, \\
i(Y^I - \bar{Y}^I) &= 0, \\
i(\bar{F}_I(Y) - \bar{F}_I(\bar{Y})) &= H_I, \\
A^I_t &= e^{2U} Y^I.
\end{align*}
\]
(25)

where $U$ and $f$ are functions depending only on the radial distance and are to be determined.

In terms of the coordinates $(Y^I, F_I(Y))$, the vanishing of the time component for the gravitino supersymmetry transformation gives the following equation
\[
\left( \partial_t - \frac{1}{2}(e^{2U} f \partial_r f + f^2 e^U \partial_r (e^U)) \right) \gamma_0 \gamma_1 + \frac{f}{4Z} e^{3U} \partial_r e^{-2U} \gamma_1 - i g e^{2U} \kappa_I Y^I + \frac{i}{2} g \kappa_I L^I e^U \gamma_0 \right) = 0
\]
(26)

In what follows we set $Z = \bar{Z}$. Assuming that the solution breaks supersymmetry in such a way that the spinor $\epsilon$ satisfies
\[
\epsilon = (a \gamma_0 + b \gamma_1) \epsilon
\]
(27)

where $a$ and $b$ are functions satisfying $a^2 - b^2 = 1$ and to be determined. The condition (27) breaks half of supersymmetry. From Eq. (26) one obtains the following equations
\[
\begin{align*}
\left( \partial_t + \frac{a}{2b} e^{2U} f \partial_r f - i g e^{2U} \kappa_I Y^I \right) \epsilon &= 0, \\
a f \partial_r e^U + \frac{1}{2} e^{3U} \partial_r (e^{-2U}) &= 0, \\
- \frac{1}{b} e^U \partial_r f + i g \kappa_I L^I + f b \partial_r e^U &= 0.
\end{align*}
\]
(28)

The gaugino supersymmetry transformation is given by
\[
\delta \lambda^A = \left( i \gamma^\mu \partial_\mu z^A + i \bar{G}^{-A}_{\rho \sigma} \gamma^\rho \gamma^\sigma - g g^{A\bar{B}} \kappa_I f^I_{\bar{B}} \right) \epsilon
\]
(29)

where $G^{-A}_{\rho \sigma} = -g^{A\bar{B}} \bar{f}^I_{\bar{B}} (\text{Im} \mathcal{N}_{IJ}) F^J_{\rho \sigma}$, $g^{A\bar{B}}$ is the inverse Kähler metric and $\bar{f}^I_{\bar{B}} = (\partial_{\bar{B}} + \frac{1}{2} \partial_{\bar{B}} K) \bar{L}^I$. To show the vanishing of the gaugino supersymmetry variations for the choice
of $\epsilon$ given in (27), it is more convenient to multiply Eq. (29) with $f^I_A$. This gives using the second relation in (9) following from special geometry

$$f^I_A \delta \lambda^A = (i \gamma^\mu \partial_\mu z^A (\partial_A + \frac{1}{2} \partial_A K) L^I + \frac{i}{2} (F^I_{\mu \nu} - i \bar{L}^I T^-_{\mu \nu}) \gamma^\mu \gamma^\nu + \frac{g}{2} \text{Im} N^{IJ} \kappa_J + g \kappa_J L^J \bar{L}^I) \epsilon$$

We now try to simplify the above rather complicated form by evaluating each term separately. One has

$$\partial_\mu z^A (\partial_A + \frac{1}{2} \partial_A K) L^I = e^{\frac{2}{e^2}} (\partial_\mu X^I - \partial_\mu z^A (\partial_A e^{-K}) e^{\frac{2K}{e^2}} X^I)$$

which for our Ansatz becomes

$$\partial_\mu z^A (\partial_A + \frac{1}{2} \partial_A K) L^I = -e^{3U} Y^I (Y^J \partial_r H_J) + e^{U} \partial_r Y^I.$$  (33)

The second term in the gaugino transformation gives

$$\frac{i}{2} (F^I_{\mu \nu} - i \bar{L}^I T^-_{\mu \nu}) \gamma^\mu \gamma^\nu \epsilon = -2i (F^I_{01} - i \bar{L}^I T^-_{01}) \gamma_0 \gamma_1 \epsilon.$$  (34)

Using the supersymmetry breaking condition (27)

$$\gamma_0 \gamma_1 \epsilon = -(\frac{\gamma_1}{a} + \frac{b}{a}) \epsilon$$

we get

$$\frac{i}{2} (F^I_{\mu \nu} - i \bar{L}^I T^-_{\mu \nu}) \gamma^\mu \gamma^\nu \epsilon = \frac{2}{a} (i F^I_{01} \gamma_1 + i b F^I_{01} + \bar{L}^I T^-_{01} \gamma_1 + b \bar{L}^I T^-_{01}) \epsilon.$$  (36)

Collecting terms and imposing the vanishing of the gaugino supersymmetry transformation gives the following equations

$$\left( -ie^{4U} f y^I (Y^J \partial_r H_J) + ie^{2U} f \partial_r Y^I \right) + \frac{2}{a} \bar{L}^I T^-_{01} + \frac{2i}{a} F^I_{01} = 0,$$

$$\frac{2b}{a} \bar{L}^I T^-_{01} + \frac{2ib}{a} F^I_{01} + \frac{g}{2} \text{Im} N^{IJ} \kappa_J + g \kappa_J L^J \bar{L}^I = 0.$$  (37)

It can be shown that all the conditions imposed by unbroken supersymmetry which are given by (37) and (28) are satisfied if we set

$$a = \frac{1}{f},$$

$$b = -\frac{g r e^{-2U}}{f},$$

$$f^2 = 1 + g^2 r^2 e^{-4U}.$$

(38)
and take the harmonic functions to be $H_I = \kappa_I + \frac{q_I}{r}$, where $q_I$ are electric charges.

Here we summarize our solution

$$ds^2 = -(e^{2U} + g^2 r^2 e^{-2U}) dt^2 + \frac{1}{(e^{2U} + g^2 r^2 e^{-2U})} dr^2 + e^{2U} r^2 (d\theta^2 + \sin^2 \theta d\phi^2),$$

$$e^{-2U} = Y^I H_I,$$

$$i(\mathcal{F}_I(Y) - \bar{\mathcal{F}}_I(\bar{Y}) = H_I, \quad Y^I = \bar{Y}^I,$$

$$A_I^r = e^{2U} Y^r.$$  (39)

For the above solution, the vanishing of the time-component of the gravitino supersymmetry variation simply gives

$$\left( \partial_t - i\frac{g}{2} \right) = 0.$$  (40)

From the vanishing of the space-component of the gravitino supersymmetry transformation we obtain the following equations

$$\left( \partial_r + \frac{1}{2} \frac{1}{fr} (1 - 2r\partial_r U) \gamma_0 - \frac{1}{2} \frac{1}{fr} (1 - r\partial_r U) \right) \epsilon = 0,$$

$$\left( \partial_\theta + \frac{1}{2} \gamma_0 \gamma_1 \gamma_2 \right) \epsilon = 0,$$

$$\left( \partial_\phi + \frac{1}{2} \cos \theta \gamma_2 \gamma_3 + \frac{1}{2} \gamma_0 \gamma_1 \gamma_3 \sin \theta \right) \epsilon = 0.$$  (41)

The radial differential equation can be solved using the techniques described in the Appendix of [8]. If one has for the spinor $\Psi(r)$ the following differential equation

$$\partial_r \Psi(r) = \left( A(r) + B(r) \Gamma_1 \right) \Psi(r),$$  (42)

where $\Gamma_1$ is an operator satisfying $\Gamma_1^2 = 1$. Also suppose that $\Psi(r)$ is subject to the constraint

$$\Psi(r) = -\left( X(r) \Gamma_1 + Y(r) \Gamma_2 \right) \Psi(r), \quad \Gamma_2 = 1, \quad \{\Gamma_1, \Gamma_2\} = 0,$$  (43)

then integrability implies that

$$\frac{dX}{dr} + 2BY^2 = 0,$$  (44)

and the solution for $\Psi(r)$ is given by

$$\Psi(r) = \frac{1}{2} \left( V(r) + W(r) \Gamma_2 \right) (1 - \Gamma_1) \Psi_0,$$  (45)
where

\[ V(r) = \sqrt{1 + \frac{X}{Y}} e^T, \]
\[ W(r) = -\sqrt{1 - \frac{X}{Y}} e^T, \]
\[ T = \int^r A(r')dr' \quad (46) \]

and \( \Psi_0 \) is a constant arbitrary spinor. For the case at hand, we have the following identifications:

\[ \Gamma_1 = \gamma_0, \quad \Gamma_2 = i\gamma_1, \]
\[ A(r) = \frac{1}{2r}(1 - r\partial_r U), \]
\[ B(r) = -\frac{1}{2rf}(1 - 2r\partial_r U), \]
\[ X(r) = -\frac{1}{f}, \]
\[ Y(r) = \frac{gre^{-2U}}{f} \quad (47) \]

By combining with the solution for the differential equations for time and the angular variables, we obtain the following solution

\[ \epsilon(r) = \frac{1}{2\sqrt{gr}} e^{\frac{\text{int}}{2}} e^{-\frac{1}{2}\gamma_0\gamma_1\gamma_2\theta} e^{-\frac{1}{2}\gamma_2\gamma_3\phi} e^{U + T} \left( \sqrt{f - 1 - i\gamma_1}\sqrt{f + 1}(1 - \gamma_0)\epsilon_0 \right) \quad (48) \]

where \( \epsilon_0 \) is an arbitrary constant spinor and \( T = \int^r \frac{1}{2r}(1 - r'\partial_{r'} U(r'))dr' \).

In summary, we have obtained spherically symmetric electric BPS solutions which break half of supersymmetry in the theory of \( N = 2 \) anti-De Sitter supergravity with vector supermultiplets. The solution is expressed in terms of the holomorphic sections and therefore independent of the existence of a holomorphic prepotential. Note that a subclass of solutions of \( N = 2 \) supergravity (for a particular choice of prepotential) are actually also solutions of supergravity theories with more, i.e. \( N = 4 \) or \( N = 8 \) supersymmetries. The D=4 static BPS-saturated electric black holes of gauged supergravity have naked singularities. The generalisation of the results of this paper to the non-extreme static black hole solutions is straightforward and can be done along the same lines of [17].

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\( ^3 \)notice that the integrability condition (44) is satisfied.
Appendix: conventions and notation We took the space–time metric to have signature \((- + + +)\). Curved indices were denoted by \(\mu = t, m\). Flat indices were denoted by \(a = 0, i\) where \(i = 1, 2, 3\). Antisymmetrized indices were defined as follows: 
\[
[ab] = \frac{1}{2}(ab - ba).
\]
We defined (anti) selfdual components as follows:
\[
F_{ab}^\pm = \frac{1}{2}(F_{ab} \pm i \star F_{ab})
\]
with
\[
\star F_{ab} = \frac{1}{2} \epsilon_{abcd} F_{cd}
\]
and \(\epsilon^{0123} = 1 = -\epsilon_{0123}\) (**\(F = -F\)). For the \(\gamma\) matrices we used the relation
\[
\gamma^a \gamma^b = -\eta^{ab} + \frac{i}{2} \gamma_5 \epsilon^{abcd} \gamma^c \gamma^d
\]
with \(\gamma_5 = -i \gamma_0 \gamma_1 \gamma_2 \gamma_3\) (\(\gamma_5^2 = 1\)). Using these definitions we find for any antiselfdual tensor the identity
\[
T^-_{ab} \gamma_a \gamma_b = 2(1 - \gamma_5) T^-_{0m} \gamma_0 \gamma_m
\]
and
\[
T^-_{mn} = -i \epsilon_{mpq} T^-_{0p}.
\]
References

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