Research on the Kinematic Compatibility of Position Solution of a Four-DOFs Mechanical Structure Mechanism

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Abstract. Upper extremity rehabilitation exoskeletons are the key devices for stroke patients with movement function disorders. Based on the configuration synthesis and optimum principles of 4-DOFs upper limb exoskeleton mechanisms, an exoskeleton mechanism compatible with upper limb were presented. When the exoskeleton was worn to the upper extremity, a kinematic chain with two closed loops was formed. Moreover, the kinematic equations of the chain were established and a position solution method was addressed. Based on the presented position solution, the kinematic characteristics during eating movement were investigate. The conclusion suggested that the exoskeleton was applied to the upper extremity rehabilitation and can be applied as main structural design parameters for the presented exoskeleton.

1. Introduction

The movement dysfunction of upper limbs caused by stroke will seriously affect the daily life and quality of life of patients. Traditional rehabilitation therapy involved one-to-one rehabilitation training between physical therapists and patients, which has many visible disadvantages, such as high labour intensity and low working efficiency [1]. Accordingly, the development of rehabilitation exoskeletons has practical significance for the treatment of upper extremity dyskinesia [2].

For the design of exoskeletons, how to achieve the kinematic compatibility between the exoskeleton and upper extremity has become a critical issue to be considered [3]. If the human-exoskeleton kinematics is incompatible, the undesired interaction forces will be generated at the physical connection interfaces during rehabilitation. These forces will increase the constraint between the exoskeleton and upper extremity, leading to poor comfort or safety issues. Studies have shown that the main cause of kinematic incompatibility was the dislocation of the upper extremity joints and the exoskeleton joints [4]. The addition of additional passive joints into the main exoskeletons have become one of the effective methods to resolve dislocation of joints axes [5]. For example, Intelli-ARM system at Northwestern University introduced three passive joints in the exoskeleton to track the center of the shoulder joint, weakening the undesired interaction forces [6]. MEDARM [7], designed by Queen's University of Canada, also adopted a similar method to achieve kinematic compatibility of upper limb and exoskeleton. In addition, by establishing a generalized degrees of freedom (DOFs) model of the upper extremity and based on the DOFs analysis theory of the spatial multiloop chain, literature 8 presented a configuration synthesis method of the four-DOFs exoskeleton mechanism [8]. The results showed that all four-DOFs exoskeleton mechanisms were compatible with the upper...
extremity. In addition to meeting kinematic compatibility, the design of the exoskeleton should refer to the mature exoskeletons to determine the distribution characteristics of the joints, so as to select the appropriate rehabilitation exoskeleton mechanism [9].

Generally speaking, it was very difficult to derive the inverse position solution of the series exoskeleton mechanism, of which the exoskeletons are one. Previous studies have shown that the inverse position solution was of extreme importance in the analysis of exoskeleton mechanisms, especially for the human-exoskeleton closed chain [10]. In our work, a novel four-DOFs upper extremity exoskeleton was presented. Furthermore, the kinematic equations of human-exoskeleton closed chain were established and the position solution was addressed. Meanwhile, the kinematic properties of the mechanism were analyzed with the joints angle trajectories related to the eating movement as input.

2. The upper extremity exoskeletal mechanism

2.1. The Model of the Upper Extremity

The flexibility of the shoulder complex was mainly reflected in the large range of movement (ROM) of the upper extremity. As shown in Figure 1, the shoulder complex consists of bones and joints, the bones including humerus, clavicle, sternum, and scapula, and the joints including glenohumeral joint (GH), scapularthoracic joint (ST), sternoclavicular joint (SC), acromioclavicular joint (AC). Because of the complexity of the shoulder complex kinematic properties, it was a difficult problem to develop the kinematics model which can reflect its biological characteristics of shoulder complex.

Investigations shown [10] that motion of shoulder complex was comprised of 3-DOFs rotation movement of humerus and the movement of shoulder girdle. The rotational movement of the humerus around its own axis was independent of the movement of shoulder girdle. Meanwhile, the rotational motion of the other two axes for humerus was coupled with the movement of shoulder girdle, and the coupling relationship resulted in the change of the rotational center of shoulder joint. The elbow joint comprises of the humerus, ulna, and radius, and the main movement was flexion/extension movement [11]. Previous studies have studied the kinematics model of the upper extremity, but mainly focused on quantifying the range of joint movement. NEWKIRK [12] et al. proposed a complete characterization of shoulder girdle movement relative to humeral pointing and addressed the coupling relationship between the movement of the shoulder girdle and the shoulder joint. As shown in Fig. 2, based on the research findings of NEWKIRK et al., the position coordinates of the shoulder joint in sternum coordinate system \((O_s-X_sY_sZ_s)\) can be expressed as follows:

\[
T_s = T(x_0, x_0) \cdot T(y_0, y_0) \cdot T(z_0, z_0) \tag{1}
\]

Among them, detailed expressions of \(x_0, y_0\) and \(z_0\) are presented in reference 12. The Eq. (1) represent the position transformation from the GH joint frame \((O_c-X_cY_cZ_c)\) to the sternum coordinate.
system \((O_s-X_sY_sZ_s)\), and variables \(\rho\) and \(\phi\) determine the positional angles of the humerus movement in \(O_c-X_cY_cZ_c\).

2.2. Mechanism Configuration of Exoskeleton

When the users wear an exoskeleton and is tightly connected to it, a spatially closed chain is formed between the upper extremity and exoskeleton. Moreover, the transmission of power and coordination of movement in the rehabilitation task are implemented through the physical interaction on the connective interfaces. To guarantee the smoothness of human-exoskeleton interaction and avoid the secondary injury of affected limbs, strong constraint forces and larger relative movement offsets cannot be generated in the closed chain. However, because of the dislocation of the axes between the human-exoskeleton joints, the exoskeleton rehabilitation training occurred some problems. The axes misalignments caused hyperstatic forces at the human-exoskeleton physical connective interfaces. Furthermore, these forces will change as the closed chain moves, and the training may become uncomfortable or unsafe in extreme cases.

The introduction of the passive DOFs is one of the most effective approaches to solve the axes misalignments. These introduced passive joints can achieve the kinematic compatibility, thus reducing the hyperstatic forces at the connection interfaces. Based on this design concept, LI [13] et al. proposed a number synthesis approach of exoskeleton configurations that was compatible with upper limb. Based on the reason of the incompatibility between the upper extremity and exoskeleton was explained from the perspective of mechanism theory, which was caused by the kinematic over-actuation characteristics of the closed chain. According to this method, the configurations of an exoskeleton were presented, as shown in Fig.3. In this article, five passive joints were introduced to decrease the detrimental effects of hyperstaticity in exoskeleton on upper extremity training [14].

![Figure 3. Schematic diagram of exoskeleton.](image1)

![Figure 4. Schematic diagram of coordinate system.](image2)

3. The position solution for exoskeleton mechanism

3.1. The Kinematics Constraint Equations

To facilitate the inverse position solution of exoskeleton, several dimensions and reference coordinate frames are defined. As shown in Fig. 4, the additional parameters are described as below: Let \(O_1O_2=l_1\), \(O_2O_3=l_2\), \(O_3O_4=l_3\), \(O_4O_5=l_4\), \(O_4O_5=l_5\), \(O_5O_6=l_6\), \(O_6O_7=l_7\), \(O_8O_7=l_8\), \(O_6O_7=l_9\), \(O_7O_8=l_{10}\), \(O_8O_9=l_{11}\),
\( O_0O_\varphi = l_{12}, \quad O_0O_{\theta_0} = l_{13}, \quad O_0O_{\theta_0} = l_{14}, \quad O_0O_{\theta_0} = l_{15}, \) and \( O_0O_{\theta_0} = l_{16} \). An inertial coordinate system \( O_sX_sY_sZ_s \) is established on the sternum with the \( Z_s \)-axis perpendicular to the horizontal. To represent the movement characteristics of the GH joint, two frames \( O_\varphi X_\varphi Y_\varphi Z_\varphi \) and \( O_\theta X_\theta Y_\theta Z_\theta \) are established at the GH joint. Frames \( O_sX_sY_sZ_s \) and \( O_\varphi X_\varphi Y_\varphi Z_\varphi \) denote the position and posture of the GH joint in the frame \( O_\varphi X_\varphi Y_\varphi Z_\varphi \), respectively, and they are coincident under the initial conditions. Additionally, frame \( O_\varphi X_\varphi Y_\varphi Z_\varphi \) is defined at the elbow joint to describe the flexion/extension motion of the elbow joint. The fixed frame on the exoskeleton is represented by frame \( O_0X_0Y_0Z_0 \). The \( O_\varphi X_\varphi Y_\varphi Z_\varphi \) \( (i=1,2, \ldots,12) \) denote the local coordinate systems of each joint, which are established at the center of each joint.

The transformation matrix \( T_0^e \) of a fixed frame \( O_0X_0Y_0Z_0 \) relative to the inertial coordinate system \( O_sX_sY_sZ_s \) can be expressed as:

\[
T_0^e = T(x_0, A_0 + a_0)T(y_0, b_0)T(z_0, C_0 + c_0)R(x_\alpha, \alpha_0)R(y_\beta, \beta_0)R(z_\gamma, \gamma_0)
\]

\[(2)\]

Where \([a_0, b_0, c_0]^\top \) and \([\alpha_0, \beta_0, \gamma_0]^\top \) represent the position vector and the pose vector of the human-exoskeleton initial wear deviation. \( A_{ij} \) \( (i, j=1,2,3,4) \) is a polynomial with respect to the deviation parameters \( a_0, b_0, c_0, a_\alpha, b_\alpha, c_\alpha \).

Suppose that the closed chain is opened from the passive connective joint \( P_6 \) at the upper arm, so as to obtain the kinematic chain \( O_0O_\theta O_\varphi O_2O_3O_4O_5O_6 \) and kinematic chain \( O_\varphi GHO_6 \). The pose matrix \( T_6^e \) of the kinematic chain \( O_0O_\theta O_\varphi O_2O_3O_4O_5O_6 \) can be calculated. The \( T_6^e \) represents the transformation matrix from frame \( O_0X_0Y_0Z_0 \) to inertial coordinate system \( O_sX_sY_sZ_s \).

\[
T_6^e = T_5^e \cdot T_1^0 \cdot T_2^1 \cdot T_3^2 \cdot T_4^3 \cdot T_5^4 \cdot T_6^5 \quad (3)
\]

Where \( T_{j+1}^e \) \( (j=0,1, \ldots,5) \) are the transformation matrices from coordinate system \( O_sX_sY_sZ_s \) to inertial coordinate system \( O_\varphi X_\varphi Y_\varphi Z_\varphi \), which are presented in the appendix A. The \( i \) represents the \( i \)-th category configuration of the exoskeleton. The \([n_{\varphi_0}, n_{\theta_0}, n_{\zeta_0}]^\top \), \([\alpha_{\varphi_0}, \alpha_{\theta_0}, \alpha_{\zeta_0}]^\top \) and \([n_{\varphi_0}, \alpha_{\varphi_0}, \alpha_{\theta_0}, \alpha_{\zeta_0}]^\top \) denote the direction vectors of the passive connective joints. The \([X_{\varphi_0}, Y_{\varphi_0}, Z_{\varphi_0}]^\top \) is the position vectors of the passive connective joints. With the similar theory and method, the pose transformation matrix \( T_6^e \) of the kinematic chain \( O_\varphi GHO_6 \) can be calculated, as described below.

\[
T_6^e = T_5^e \cdot T_6^e \cdot T_7^e \quad (4)
\]
For the movement description method of upper limbs, this paper adopted the movement description approach proposed by the literature 14, and $T_y$ can be expressed as:

$$T_y = R(z_y, \alpha) \cdot R(y_y, \beta) \cdot R(z_y, \gamma)$$  \hspace{1cm} (5)

Where $\alpha$, $\beta$ and $\gamma$ are based on ISB to describe the three angles of upper extremity movement. The $[n_x, o_x, a_x]^T$, $[n_y, o_y, a_y]^T$ and $[n_z, o_z, a_z]^T$ denote the direction vectors of the passive connective joints. The $[X_{y71}, Y_{y71}, Z_{y71}]^T$ is the position vector of the passive connective joints.

For the of exoskeleton configuration, passive connective joint is prismatic joint $P_6$, and the position coordinates and direction coordinates of the two kinematic sub-chains in the center $O_6$ is always equal. Therefore, the established kinematic equations can be expressed as:

$$\begin{cases}
X_{y61(1)} = X_{y71(1)} \\
Y_{y61(1)} = Y_{y71(1)} \\
Z_{y61(1)} = Z_{y71(1)} \\
n_{y6(1)} \cdot n_{y7(1)} + o_{y6(1)} \cdot o_{y7(1)} + a_{y6(1)} \cdot a_{y7(1)} = 0 \\
n_{y6(2)} \cdot n_{y7(2)} + o_{y6(2)} \cdot o_{y7(2)} + a_{y6(2)} \cdot a_{y7(2)} = 0 \\
n_{y6(3)} \cdot n_{y7(3)} + o_{y6(3)} \cdot o_{y7(3)} + a_{y6(3)} \cdot a_{y7(3)} = 0
\end{cases}$$  \hspace{1cm} (6)

3.2. Position Solution for the Exoskeleton

By eliminating the intermediate variables from the constraint equations, and $x = \tan(\theta_5/2)$ and $y = \tan(\theta_4/2)$, the following expressions can be achieved.

$$\sum_{i=1}^{5} f_i (x) y^{i-1} = 0, \sum_{i=1}^{5} g_i (x) y^{i-1} = 0$$  \hspace{1cm} (7)

To eliminate parameter $y$, the construction of matrix must be performed on Eq. (7), and the constructed expression is shown below.

$$\begin{bmatrix}
f_1 & f_2 & f_3 & f_4 & f_5 & 0 & 0 & 0 & 0 \\
g_1 & g_2 & g_3 & g_4 & g_5 & 0 & 0 & 0 & 0 \\
0 & f_1 & f_2 & f_3 & f_4 & f_5 & 0 & 0 & 0 \\
0 & g_1 & g_2 & g_3 & g_4 & g_5 & 0 & 0 & 0 \\
0 & 0 & f_1 & f_2 & f_3 & f_4 & f_5 & 0 & 0 \\
0 & 0 & g_1 & g_2 & g_3 & g_4 & g_5 & 0 & 0 \\
0 & 0 & 0 & f_1 & f_2 & f_3 & f_4 & f_5 & y^3 \\
0 & 0 & 0 & g_1 & g_2 & g_3 & g_4 & g_5 & y^1
\end{bmatrix}\begin{bmatrix}
y^7 \\
y^6 \\
y^5 \\
y^4 \\
y^3 \\
y^2 \\
y^1 \\
y^0
\end{bmatrix} = 0$$  \hspace{1cm} (8)

Where $f_i$ and $g_i$ are the eighth order functions of the variable $x$. In Eq. (8), the square matrix is represented by $A$. And it is pretty obvious that the necessary and sufficient condition for Eq. (8) to have a nonzero solution is $|A| = 0$. So, if we rearrange the $|A| = 0$, we find that it is a 24-degree equation with respect to $x$, and its solution is the analytical solutions to the third category of exoskeleton. If the variable $x$ is known, all other joint displacements of the exoskeleton mechanism can be calculated. According to the above derivation, the inverse position solution of exoskeleton can be solved.
4. Example position solution for the exoskeleton

In this section, an example for position solution was presented. The joint angle trajectories of upper extremity during eating movement were regarded as the inputs of the position solution. The curve trajectories of the upper extremity joints were shown in Fig. 5, which were obtained through Vicon capturing system [8]. Moreover, the dimensional parameters of the human-exoskeleton closed chain were described in Table 1.

Table 1. Parameters associated with the human-exoskeleton closed chain

|   |   |   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 40 | 166 | 280 | 280 | 200 | 280 | 70 | 30 | 125 | 120 | 148 |
| 70 | 30 | 180 | 120 | 160 |

When the upper limb performed the actions corresponding to the movements of eating, the angle trajectories of joints were inserted into the position solution derived in Chapter 3.2. Based on this, the joint angular displacements of the exoskeleton mechanisms were calculated, and the trajectories were shown in Fig. 6 to 9. Furthermore, the angular trajectory curves of exoskeleton joints were analyzed.

When the human-exoskeleton closed chain was moving during eating movement, the ROMs of the active joints were shown in Fig. 6, and the trajectory curves of the active joints can be regards as a reference for the trajectory planning of the exoskeleton mechanism.

Figure 5. Human joints angle trajectories.

Figure 6. Angle trajectories of the active joints.

Figure 7. Linear trajectories of prismatic joints.

Figure 8. Angle trajectories of connection joints.

The ROMs of passive joints associated with the eating movement were shown in Figs. 7-8. It can be seen from Fig. 7 to Fig. 8 that in the presented 4-DOFs exoskeleton mechanism, the ROMs of the passive joints were relatively great. The following conclusions can be presented. Firstly, when all the passive joints were locked, the resulting displacement was borne by the upper extremity. Therefore, large hyperstatic forces will be generated in the closed chain, and these forces will change with the motion of the closed chain. If these forces were generated sufficiently, the rehabilitation training can be uncomfortable. Secondly, it can be concluded from the Fig. 7 to Fig. 8 that the initial wearing
deviation may lead to a larger variation ROMs of the passive joints. Thus, when the exoskeleton was worn on the upper extremity, the axis dislocation of the human-exoskeleton joints was minimized. Finally, the ROMs of the passive joints presented a reasonable distribution and the curve trajectories were smooth, which is convenient for the smooth movement control of upper extremity exoskeleton mechanism.

5. Conclusion
In this work, a 4-DOFs upper extremity exoskeleton mechanism was addressed. The inverser position solution of the kinematic chain was presented, and kinematic properties of upper extremity exoskeleton were investigated by the upper extremity’s joint trajectories corresponding to eating. The findings suggested that the addition of the passive joints made the upper extremity exoskeleton to be kinematically compatible with upper extremity, thus decreasing the hyperstatic forces. The movement range of all joints in the human-exoskeleton closed chain were reasonable and the curve trajectories were smooth. Consequently, the presented upper extremity exoskeleton was suitable for rehabilitation training, on which the results can be applied as the main parameters of the structure designed.

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