Fermion-Higgs model with Reduced Staggered Fermions

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Abstract

We introduce a lattice fermion-Higgs model with one component ‘reduced staggered’ fermions. In order to use the fermion field as efficiently as possible we couple the two staggered flavors to the O(4) Higgs field leading to a model with only one SU(2) doublet in the scaling region. The number of fermions is doubled in a numerical investigation of the model with the hybrid Monte Carlo algorithm. We present results for the phase diagram, particle masses and renormalized couplings on lattices ranging in size from $6^324$ to $16^324$.

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1. Introduction

Recent years have witnessed a lot of interest in the non-perturbative understanding of the symmetry breaking sector of the standard model. An important question is whether the scalar field self-coupling and the Yukawa coupling are trivial as suggested by the signs of the perturbative $\beta$ functions. If the model is influenced by the Gaussian fixed point for all allowed values of the bare couplings, one can derive non-perturbative upper bounds on the renormalized Yukawa coupling $y_R$ and quartic self-coupling $\lambda_R$ from the constraint that the cut-off must be sufficiently larger than the masses of the particles. Using the relations $m_H = \sqrt{2\lambda_R v_R}$ and $m_F = y_R v_R$ this is equivalent to finding upper bounds on the Higgs mass $m_H$ and the heavy fermion mass $m_F$ ($v_R \approx 246$ GeV is the scalar field expectation value).

It is reasonable to address these questions first in simplified lattice fermion-Higgs models without gauge fields. For recent reviews on such models we refer the reader to ref. [1].

In a model with naive fermions the number of mass degenerate doublets is as large as 16. With the usual staggered fermions one can reduce this number of doublets to four and using the mirror fermion model with Wilson fermions of ref. [2] it appears to be possible to reduce the number of doublets to one, while the 30 doublers acquire masses of order of the cut-off [3]. For numerical investigations of these models with the hybrid Monte Carlo algorithm (HMCA) it is necessary to double these numbers of fermion doublets.

In this letter we shall investigate a new proposal [4] for a fermion-Higgs model which is based on the reduced staggered fermion formalism and which describes one SU(2) doublet in the scaling region (two doublets when investigating the model with the HMCA). In this paper we shall introduce the model and present some preliminary results. A detailed account of our investigation will appear elsewhere [5].

2. The model and its symmetries

The usual euclidean staggered fermions on a four-dimensional hypercubic lattice describe four flavors in the scaling region. By using the ‘reduced’ staggered formalism [6, 7] this number can be halved to two. If two of these staggered fields are placed in an SU(2) doublet one would get a model with two degenerate isospin doublets in the scaling region. In this letter, however, we follow another strategy. In order to use the fermion field as efficiently as possible we want to couple the two staggered flavors to the Higgs field, leading to a model with only one doublet in the scaling region.

To show how the Higgs field can be coupled to the staggered fermion flavors while preserving as much symmetry as possible, we first write down the target action that we want to reproduce with the lattice model,

$$S_T = -\int d^4x \left[ \bar{\psi} \gamma^\mu \partial_\mu \psi + y (\bar{\psi}_R \phi \psi_L + \bar{\psi}_L \phi \psi_R) \right]$$

(1)

The Dirac doublet $\psi$ interacts with the scalar field $\phi \in SU(2)$ which contains the O(4) components $\varphi_\mu$ of the Higgs field in the familiar way, $\phi = i \sum_{j=1}^3 \varphi_j \tau_j + \varphi_4 \mathds{1}$ where $\tau_j$ are the usual Pauli matrices.

In order to reproduce this action with staggered fermions, we need to exhibit their spin-
flavor structure and it is convenient to introduce the $4 \times 4$ matrix fields 

$$
\Psi_x = \frac{1}{8} \sum_b \gamma^{x+b} \chi_{x+b}, \quad \overline{\Psi}_x = \frac{1}{8} \sum_b (\gamma^{x+b})^\dagger \chi_{x+b}. 
$$

(2)

Here $\chi$ and $\overline{\chi}$ are the usual (one-component) staggered fields, the sums in eq. (2) are over the 16 corners of the unit hypercube, $b_\mu = 0, 1$ and $\gamma^{x+b}$ is a short hand notation for the product $\gamma_1^{x_1+b_1} \gamma_2^{x_2+b_2} \gamma_3^{x_3+b_3} \gamma_4^{x_4+b_4}$. Since $\Psi$ contains 16 times as many degrees of freedom as $\chi$, not all components $\Psi^{\alpha \kappa}$ are independent. However, the components of the low momentum modes $\tilde{\Psi}(p)$ with $-\pi/2 < p_\mu \leq \pi/2$ are independent. Spin-flavor transformations on $\chi$ correspond to discrete transformations on $\Psi$ from the left or right. A shift transformation $\chi_x \to \zeta_{\mu x} \chi_{x+\hat{\mu}}$, to be interpreted as a discrete flavor transformation, translates into $\Psi_x \to \Psi_{x+\hat{\mu}} \gamma_\mu$ and a ‘spin’ transformation $\chi_x \to \eta_{\mu x} \chi_{x+\hat{\mu}}$ translates into $\Psi_x \to \gamma_\mu \Psi_{x+\hat{\mu}}$. The sign factors $\eta_{\mu x}$ and $\zeta_{\mu x}$ are defined as $\eta_{\mu x} = (-1)^{x_1+\cdots+x_{\mu-1}}$ and $\zeta_{\mu x} = (-1)^{x_{\mu+1}+\cdots+x_4}$.

The kinetic part of the free staggered fermion action

$$
S_K = -\frac{1}{2} \sum_{x \mu} \eta_{\mu x} (\overline{\chi}_x \chi_{x+\hat{\mu}} - \overline{\chi}_{x+\hat{\mu}} \chi_x)
$$

(3)

can now be written as

$$
S_K = -\sum_{x \mu} \frac{1}{2} \text{Tr}(\overline{\Psi}_x \gamma_\mu \Psi_{x+\hat{\mu}} - \overline{\Psi}_{x+\hat{\mu}} \gamma_\mu \Psi_x),
$$

(4)

which reduces in the classical continuum limit to the gradient term in eq. (1), however with two doublets.

In the reduced staggered formalism the field $\chi$ is restricted to the odd sites $\chi_x \to \frac{1}{2} (1 - \epsilon_x) \chi_x$ and $\overline{\chi}_x$ to the even sites $\overline{\chi}_x \to \frac{1}{2} (1 + \epsilon_x) \overline{\chi}_x$, with $\epsilon_x = (-1)^{x_1+x_2+x_3+x_4}$. Inserting these restricted fields into (2) and dropping the bar on $\overline{\chi}$ gives

$$
\Psi_x = \frac{1}{8} \sum_b \gamma^{x+b} \frac{1}{2} (1 - \epsilon_{x+b}) \chi_{x+b} , \quad \overline{\Psi}_x = \frac{1}{8} \sum_b (\gamma^{x+b})^\dagger \frac{1}{2} (1 + \epsilon_{x+b}) \chi_{x+b}.
$$

(5)

The action (3) with $\Psi$ and $\overline{\Psi}$ defined as in (4), reproduces the kinetic term of the action for reduced (‘real’ or ‘Majorana-like’) staggered fermions,

$$
S_K = -\frac{1}{2} \sum_{x \mu} \eta_{\mu x} \chi_x \chi_{x+\hat{\mu}}.
$$

(6)

The restriction of $\chi$ and $\overline{\chi}$ to odd and even sites corresponds to the projections $\Psi \to \frac{1}{2} (\Psi - \gamma_5 \Psi \gamma_5)$ and $\overline{\Psi} \to \frac{1}{2} (\overline{\Psi} + \gamma_5 \overline{\Psi} \gamma_5)$. More explicitly this implies that the matrix fields have the structure

$$
\overline{\Psi} = \left( \begin{array}{cc} \overline{\psi}_L & 0 \\ 0 & \overline{\psi}_R \end{array} \right), \quad \Psi = \left( \begin{array}{cc} 0 & \psi_R \\ \psi_L & 0 \end{array} \right).
$$

(7)

The relation of the $2 \times 2$ matrix fields $\psi_{L,R}$ and $\overline{\psi}_{L,R}$ to the fields $\psi$ in the target action becomes clear when writing the Yukawa interaction in (4) in the form $y(\overline{\psi}_{L,\alpha} \psi_{R,\alpha i} \phi_{ij} + \overline{\psi}_{R,\alpha i} \psi_{L,\alpha j} \phi_{ij})$, where $\alpha = 1, 2$ and $i, j = 1, 2$ are the Weyl spinor and flavor indices, respectively.

We can also rewrite the Yukawa term in terms of the matrix fields $\Psi$ and $\overline{\Psi}$ if we introduce the $4 \times 4$ matrix

$$
\Phi = \left( \begin{array}{cc} 0 & \phi \\ \phi^\dagger & 0 \end{array} \right) = -\sum_{\mu} \varphi_\mu \gamma_\mu.
$$

(8)
The total fermionic action
\[ S_F = -\sum_x \left[ \frac{1}{2} \text{Tr}(\overline{\Psi}_x \gamma_\mu \Psi_{x+\hat{\mu}} - \overline{\Psi}_{x+\hat{\mu}} \gamma_\mu \Psi_x) + y \text{Tr}(\overline{\Psi}_x \Psi_x \Phi_x^T) \right] \] (9)
reduces in the classical continuum limit to eq. (1). Using (5) we finally obtain the action in terms of the independent \( \chi \) fields
\[ S_F = -\frac{1}{2} \sum_{x,\mu} \chi_x \chi_{x+\hat{\mu}} (\eta_{\mu x} + y \xi_{\mu x} \overline{\varphi}_{\mu x}) = -\frac{1}{2} \sum_{x,y} \chi_x M_{xy} \chi_y, \] (10)
where \( \overline{\varphi}_{\mu x} = \frac{1}{16} \sum_b \varphi_{\mu x-b} \) is the average of the scalar field over a lattice hypercube. The fermion matrix \( M \) in eq. (10) is antisymmetric and real.

For completeness we also show the action of the Higgs fields,
\[ S_H = \kappa \sum_{x,\mu} \frac{1}{2} \text{Tr}(\phi_{\mu x}^\dagger \phi_{\mu x} + \phi_{\mu x+\hat{\mu}}^\dagger \phi_{\mu x+\hat{\mu}} - \lambda (\phi_{\mu x}^\dagger \phi_{\mu x} - 1)^2). \] (11)
The total action \( S = S_H + S_F \) depends on three coupling constants, the Yukawa coupling \( y \), the scalar field hopping parameter \( \kappa \) and the quartic self-coupling \( \lambda \).

We emphasize that this action is invariant under the staggered fermion symmetry group: One can check the invariance under shifts, \( \chi_x \rightarrow \zeta_{\mu x} \chi_{x+\hat{\mu}} \), \( \varphi_{\mu x} \rightarrow \varphi_{\mu x+\hat{\mu}}(1 - 2\delta_{\mu \rho}) \), 90° rotations, axis reversal and global U(1) transformations of the form \( \chi_x \rightarrow \exp(i \alpha \xi_{\mu x}) \chi_x \) (cf. refs. [7, 9]).

The action is not invariant, however, under the full O(4) flavor group, and one expects to need counterterms to recover this invariance in the scaling region. In the scaling region operators with dimension larger than four become irrelevant. There are two operators with dimension four which respect the discrete symmetries but break O(4):
\[ O^{(1)} = \sum_{x,\mu} \varphi_{\mu x}^4, \quad O^{(2)} = \sum_{x,\mu} (\varphi_{\mu x+\hat{\mu}} - \varphi_{\mu x})^2. \] (12)
In general one has to add these operators as counterterms to the action, \( S \rightarrow S + c_1 O^{(1)} + c_2 O^{(2)} \) and tune \( c_{1,2} \) as a function of \( \kappa \) and \( y \) in order to recover the O(4) invariance in the scaling region.

Here and also in the numerical work we will restrict ourselves to the case of radially frozen Higgs fields corresponding to \( \phi \in \text{SU(2)} \) or equivalently \( \lambda = +\infty \).

3. Phase diagram and fermion mass

To investigate the phase diagram we carry out a simple mean field calculation in the saddle point formulation with the replacement \( \varphi_{\mu x} \rightarrow f_{\mu} + \varepsilon_x f_{\mu}^{\text{st}} \). The constant fields \( f_{\mu}, f_{\mu}^{\text{st}} \), account for a possible ferromagnetic and antiferromagnetic ordering of the scalar field. In this approximation we find \( \varphi_{\mu x} \rightarrow f_{\mu} \) which implies that the fermion fields in eq. (10) do not couple to the staggered mode \( f_{\mu}^{\text{st}} \).

At \( y = 0 \) the model reduces to the O(4) model which has three phases: a broken (or ferromagnetic (FM)) phase for \( \kappa > \kappa_c \) (\( \kappa_c = 0.30411(1) \)), a symmetric (or paramagnetic (PM)) phase for \( -\kappa_c < \kappa < \kappa_c \) and an antiferromagnetic (AM) phase for \( \kappa < -\kappa_c \). Appropriate order parameters for a distinction of the various phases are the magnetization \( v_{\mu} = \langle \frac{1}{V} \sum_x \varphi_{\mu x} \rangle \)
Figure 1: The phase diagram at $\lambda = \infty$. The squares represent the transition points determined on an $8^4$ lattice, the dashed lines are the results of the mean field calculation.

and the staggered magnetization $v_{\mu}^{st} = \langle \frac{1}{V} \sum_x \varepsilon_x \varphi_{\mu x} \rangle$ with magnitudes $v = (\sum_\mu v_{\mu}^2)^{1/2}$ and $v^{st} = (\sum_\mu v_{\mu}^{st 2})^{1/2}$. In the mean field approximation $v_\mu = f_\mu$, $v_{\mu}^{st} = f_{\mu}^{st}$.

Since the fermions do not couple to $f_{\mu}^{st}$ the phase transition between the PM and AM phases comes out independent of $y$, $\kappa_{cAM - PM}^c(y) = -1/4$. For the position of the FM-PM phase transition we find

$$\kappa_{cFM - PM}^c(y) = \frac{1}{4} - \frac{N_D}{128} y^2 \sum_\mu \frac{\cos^2 p_\mu}{\sin^2 p_\mu} \approx \frac{1}{4} - 0.012 N_D y^2,$$

(13)

where the $\sum_p$ is a normalized sum over lattice momenta $p_\mu \in (\frac{-\pi}{2}, \frac{\pi}{2}]$. We have inserted in (13) the number of SU(2) doublets as the variable $N_D$. In the numerical simulation with the HMCA, $N_D$ has to be chosen as a multiple of two in order to guarantee a positive Boltzmann weight, $[\pm (\text{Det} M)^{1/2}]^{N_D} e^{S_H} > 0$. The mean field results for $\kappa_{cFM - PM}^c(y)$ and $\kappa_{cPM - AM}^c(y)$ ($N_D = 2$) are represented by the two dashed curves in fig. 1. The lines intersect at the point $(\kappa, y) \approx (-0.25, 4.67)$. Thus there is a ferrimagnetic (FI) phase where both $v_\mu$ and $v_{\mu}^{st}$ are nonzero.

With the mean field ansatz for $\varphi_{\mu x}$ the model describes free fermions and it is straightforward to compute the fermion propagator. After Fourier transforming the $\chi$ field in the action (10) and following the steps outlined in refs. [7, 9] we find (using $v_\mu = f_\mu$)

$$S_{AB}(p) = \frac{\langle \sum_{x,y} e^{i(p+\pi_A)x} M_{xy}^{-1} e^{-i(p+\pi_B)y} \rangle}{V},$$

(14)

$$\rightarrow -i \sum_\mu \Gamma_{AB} \sin p_\mu - \sum_\mu y v_\mu (\Xi_5 \Gamma_5 \Xi_5)_{AB} \cos p_\mu \right) \sum_\mu \sin^2 p_\mu + \sum_\mu y^2 v^2_\mu \cos^2 p_\mu,$$

(15)

with $p_\mu \in (\frac{-\pi}{2}, \frac{\pi}{2}]$ and $\pi_A$ denoting the 16 momentum four-vectors with components equal to 0 or $\pi$. The 16 dimensional gamma and flavor matrices $\Gamma_\mu$ and $\Xi_\mu$ are defined in refs. [7, 9]. For $p \rightarrow 0$ we can read off the fermion mass from eq. (15): $m_F = y v$. This reproduces the usual tree level relation between $m_F$ and $v$. 

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\[ v_{\mu}^{st} = \langle \frac{1}{V} \sum_x \varepsilon_x \varphi_{\mu x} \rangle \text{ with magnitudes } v = (\sum_\mu v_{\mu}^2)^{1/2} \text{ and } v^{st} = (\sum_\mu v_{\mu}^{st 2})^{1/2}. \]

\[ \text{In the mean field approximation } v_\mu = f_\mu, \; v_{\mu}^{st} = f_{\mu}^{st}. \]

\[ \text{Since the fermions do not couple to } f_{\mu}^{st} \text{ the phase transition between the PM and AM phases comes out independent of } y, \; \kappa_{cAM - PM}^c(y) = -\frac{1}{4}. \]

\[ \text{For the position of the FM-PM phase transition we find } \kappa_{cFM - PM}^c(y) = \frac{1}{4} - \frac{N_D}{128} y^2, \]
4. Symmetry breaking terms

Next we want to estimate the magnitude of the O(4) symmetry breaking terms which are induced by the fermions, using renormalized perturbation theory. To that end we expand $(N_D/2) \text{Tr} \ln M$ in powers of the scalar field. From the two-point contribution we find a contact term $(\delta_R/2) \sum \mu \varphi_{\mu,R}^2$, corresponding to the counterterm $O^{(2)}$. For the coefficient we obtain

$$\delta_R = \frac{N_D}{32} y_R^2 \left\{ 1 - \sum \frac{\mu \cos^2 p_{\mu}}{p_{\mu}^2 + y_R^2 \sum \mu \varphi_{\mu,R}^2 \cos^2 p_{\mu}} \right\} \equiv \delta_N y_R^2,$$

where the subscript $R$ indicates a renormalized quantity. We remark that keeping the term $y_R^2 \sum \mu \varphi_{\mu,R}^2 \cos^2 p_{\mu}$ in the fermion loop, gives $O(a^2)$ corrections to $f_\delta$ which are negligible deep enough in the scaling region but which we want to include if $m_F = y_R v_R$ is of order one.

The second term $\varepsilon_R \sum \mu \varphi_{\mu,R}^4$ appears in the four-point contribution to the effective action. We find,

$$\varepsilon_R = \frac{N_D}{32} y_R^4 \sum \frac{\mu \cos^4 p_{\mu} - \frac{1}{2} \sum_{\mu \neq \nu} \cos^2 p_{\mu} \cos^2 p_{\nu}}{(\sum \mu \sin^2 p_{\mu} + y_R^2 \sum \mu \varphi_{\mu,R}^2 \sin^2 p_{\mu})^2} \equiv \varepsilon_N y_R^4.$$

This leads us to consider the following tree-level effective action,

$$S_{\text{eff}} = - \int d^4x \frac{1}{2} \sum_{\mu \nu} \partial_\mu \varphi_{\nu,R} \partial_\mu \varphi_{\nu,R} (1 + \delta_R \delta_{\mu \nu}) + \frac{m_R^2}{2} \sum \varphi_{\mu,R}^2 + \frac{\lambda_R}{4} (\sum \varphi_{\mu,R}^2)^2 + \varepsilon_R \sum \varphi_{\mu,R}^4.$$

As a consequence of the $\varepsilon_R$-term the shape of the effective potential differs from the usual Mexican hat form. The potential now has 16 discrete minima at $\varphi_R = (\pm \frac{1}{2}, \pm \frac{1}{2}, \pm \frac{1}{2}, \pm \frac{1}{2}) v_R$ with

$$v_R = (-m_R^2/\lambda_R + \varepsilon_R)^{1/2}.$$

In the infinite volume limit the scalar field will be frozen to one of these minima. Then we can replace $y_R^2 \sum \mu \varphi_{\mu,R}^2 \cos^2 p_{\mu}$ in eqs. (16) and (17) by $(m_R^2/4) \sum \mu \cos^2 p_{\mu}$ and compute the prefactors $f_\delta$ and $f_\varepsilon$ as a function of $m_F$. We find $f_\delta \approx -0.0150, -0.0110, -0.0070$ and $f_\varepsilon \approx 0.0054, 0.0050, 0.0043$ for $m_F = 0, 0.3, 0.5$.

In order to estimate the effect of the symmetry breaking terms on masses of the longitudinal ($\sigma$) and transversal ($\pi$) modes we compute now the scalar field propagator for the effective action in eq. (18). For this purpose we decompose $\varphi$ into its longitudinal and transversal components, $\varphi_{\mu,R} = (v_R + \sigma_R) e^{\mu}_l + \pi_R e^{\mu}_t$, where $e^{l}_l$ is chosen to point in the direction of one of the 16 minima and $e^{l}_t$ are three orthogonal vectors. We shall make the convenient choice $e^1 = \frac{1}{2}(1, -1, -1, 1), e^2 = \frac{1}{2}(-1, 1, -1, 1), e^3 = \frac{1}{2}(-1, -1, 1, 1), e^4 = \frac{1}{2}(1, 1, 1, 1)$. In this basis the inverse scalar propagator has the form

$$G^{-1}(p)^{\alpha \beta} = (1 + \frac{\delta_R}{4}) p^2 \delta^{\alpha \beta} + 2(\lambda_R + \varepsilon_R) v_R^2 \delta^{\alpha 4} \delta^{\beta 4} + 2\varepsilon_R v_R^2 \delta^{\alpha \beta} + \delta_R \sum \mu \mu \epsilon^{\alpha \mu} \epsilon^{\beta \mu} - \frac{p^2}{4} \delta^{\alpha \beta}.$$

Since $\delta_R$ is small compared to 1 we can treat the off-diagonal part (i.e. the last term in (20)) as a perturbation. Then we find for the propagators $G_\sigma(p) = G^{44}(p)$ and $G_\pi = \frac{1}{3} \sum_j G^{jj}(p)$ the forms

$$G_{\sigma,\pi}(p) = (1 + \frac{\delta_R}{4})^{-1} \frac{1}{p^2 + m^2_{\sigma,\pi}} (1 + O(\delta_R^2)),$$
where the $\delta^2_R$ correction is bounded by $\frac{2}{3}R^2$ for all momenta. The particle masses are given by

$$m^2_\sigma = \frac{2(\lambda_R + \varepsilon_R)v^2_R}{1 + \delta_R/4}, \quad m^2_\pi = \frac{2\varepsilon_R v^2_R}{1 + \delta_R/4}.$$  (22)

This shows that the three transversal modes in the scalar spectrum acquire a mass as a consequence of the symmetry breaking term in the potential.

We can get an impression of the size of the symmetry breaking corrections for $y_R < 3$, $N_D = 2$ and $m_F = 0$. Then $|\delta_R/4| < 0.068$, $\frac{2}{3}R^2 < 0.055$ and $\varepsilon_R < 0.88$ where the latter one has to be compared with typical values of $\lambda_R = 5 - 10$. The values which we find for these quantities in the physically relevant region are actually smaller than these bounds. It should be kept in mind that we have neglected in the relations (22) the usual loop corrections not involving the symmetry breaking terms.

Note that to leading order in $\delta_R$ the propagators $G_\sigma(p)$ and $G_\pi(p)$ in (21) are covariant functions of $p_\mu$ and $\epsilon^4_{\mu\nu}$, such that we can rotate our frame of reference and choose for example $\epsilon^4 = 4$. In the following we shall use the approximation (21) for our analysis of the numerical propagator results.

5. Results of the numerical simulations

In this section we present our results for the phase diagram and give some preliminary results for the renormalized couplings $y_R$ and $\lambda_R$. The results were obtained at $\lambda = \infty$ and without the two counterterms. The numerical simulations were performed on $L^3 T$ lattices with periodic boundary conditions in all directions except for the fermion fields which had antiperiodic boundary conditions in the time direction. For the calculation of the fermion and scalar propagators we carried out the simulations on $L^3 24$ lattices with $L = 6, 8, 10, 12$ and 16 and at $\kappa = 0$.

The phase diagram is shown in fig. 1. The squares represent the transition points which were obtained by scanning systematically the $(\kappa, y)$ coupling parameter space in vertical and horizontal directions. For each point in the raster we have typically accumulated a statistics of 500 scalar field configurations on an $8^4$ lattice. We used the rotation technique for the calculation of the order parameters $v$ and $v_{st}$ [11, 12]. The numerical results are qualitatively well described by the mean field results. The phase diagram in fig. 1 is very similar to the phase diagrams of other models with a hypercubic Yukawa coupling [3].

For the determination of the fermion mass $m_F$ we have measured the fermion propagator $S_{AB}(p)$ given in (24) for momenta $p = (0, p_4)$ and computed from it the projections $S_0(p_4) = i\text{Tr}[\Gamma_4 S(p_4)]/16$, $S_\mu(p_4) = \text{Tr}[\xi_\mu \xi_5 \Gamma_5 S(p_4)]/16$. The components $S_\mu$ $(\mu \neq 0)$ are non-invariant under cubic rotations of the scalar field and would vanish when averaging over many configurations. To avoid finding zero for all of them we have rotated the scalar fields such that $v_\mu = v \delta_{\mu 4}$. Then, the numerical values for $S_i$, $i = 1, \ldots, 3$ are found to be zero within errors and we have fitted the non-vanishing components $S_0$ and $S_4$ to a free fermion propagator form (cf. (13))

$$S_0(p_4) \to \frac{Z_{F,0}}{(1 - m^2_{F,0}) \sin^2 p_4 + m^2_{F,0}}, \quad S_4(p_4) \to \frac{Z_{F,4} m_{F,4} \cos p_4}{(1 - m^2_{F,4}) \sin^2 p_4 + m^2_{F,4}},$$  (23)

where we allow for two different masses and wave-function renormalization constants in $S_0$ and $S_4$. In fig. 2 we have plotted $S_0^{-1}(p_4) \sin p_4$ and $S_4^{-1}(p_4) \cos p_4$ for various values of $y$ as
Figure 2: $S_0(p_4) \sin p_4$ and $S_4(p_4) \cos p_4$ as a function of $\sin^2 p_4$ for several values of $y$ ($\kappa = 0, V = 12^3 24$). The error bars are in all cases much smaller than the symbols. The straight lines were obtained by fitting $S_0(p_4)$ and $S_4(p_4)$ to the forms (23).

A function of $\sin^2 p_4$. The data points fall nicely on straight lines which were obtained by fitting $S_0$ and $S_4$ to the forms (23). Within the statistical errors, we find that $m_{F,0} \approx m_{F,4}$. We therefore have taken the average, which will be denoted by $m_F$ in the following. The results for $m_F$ are summarized in table 1 for several $y$ values and several lattice sizes. We find $Z_{F,0} \approx 0.8$ and $Z_{F,4}$ approximately 5% smaller than $Z_{F,0}$. When rotating $v_\mu$ in a different direction we find that $m_{F,0}$, $Z_{F,0}$ and $m_{F,4}$, $Z_{F,4}$ may differ by about 7% for $y < 4.2$. Such systematic effects are presumably due to scaling violations.

In order to monitor finite size effects and extrapolate to infinite volume, we have computed $m_F$ and $v$ on lattices of different spatial extent $L$. In a previous work with naive fermions both observables were found to obey the relations $v_L = v_\infty + a_1/L^2$ and $m_{F,L} = m_{F,\infty} + a_2/L^2$ with $a_1, a_2 > 0$ [12]. This finite size dependence of $v$ and $m_F$ is due to the massless Goldstone bosons. In our model the three transversal modes acquire a mass $m_\pi > 0$ as a consequence of the O(4) symmetry breaking and we expect deviations from the $1/L^2$ behavior if $L$ becomes larger than $O(1/m_\pi)$. In fig. 3 we have displayed the $1/L^2$ dependence of $v$ and $m_F$. The plot shows that the $1/L^2$ dependence is well fulfilled for $y = 3.8$ though small deviations are visible for $m_F$ on the largest lattice with $L = 16$. The deviations become more pronounced at $y = 4.0$, which is consistent with the observed increase of $m_\pi$ (see fig. 4). The effect of the symmetry breaking on $v$ and $m_F$, as measured by the deviations from the straight line behavior, appears to be smaller than 15%. The effect on the ratio $m_F/v$ is even smaller.

The masses $m_\sigma$ and $m_\pi$ of the longitudinal and transversal modes were obtained by fitting the scalar momentum space propagators $G_\sigma(p) = \langle \sum_{x,y} \sigma_x \sigma_y e^{ip(x-y)} \rangle / V$ and $G_\pi(p) = \langle \sum_{x,y} \pi_x \pi_y e^{ip(x-y)} \rangle / V$.
The quantity $\hat{p}^2 = 2 \sum_\mu (1 - \cos p_\mu)$ is a lattice equivalent of the momentum squared in the continuum. Here we have chosen the magnetization to point in the four-direction as for the computation of the fermion mass (see also the remarks at the end of sect. 4). When plotting the inverse propagator $G_{\sigma,\pi}^{-1}(p)$ as a function of $\hat{p}^2$ we expect to find a linear behavior if $\hat{p}^2 \ll m^2_F$, as is found in the O(4) model. We measured the $\sigma$ and $\pi$ propagators on a 12^324 lattice where $G_{\sigma,\pi}^{-1}(p)$ for the three smallest momenta has an approximately linear behavior. We note that because of curvature in $G_{\sigma,\pi}^{-1}(p)$, the results of a linear fit tend to overestimate $m_{\sigma,\pi}$ and $Z_{\sigma,\pi}$. In a forthcoming publication [5] we will extend the analysis of the scalar propagators by taking into account the one-fermion-loop contribution to the self-energy which allows for a good description of the curvature in $G_{\sigma,\pi}^{-1}(p)$.

In fig. 4 we have plotted the fit results for $m_\pi$ (squares) as a function of $y$. The mass has a minimum close to $y = 3.8$ which is due to two different effects. The increase towards small $y$ is a consequence of the finite lattice size as in the pure O(4) theory. The mass has to approach the scalar mass $m_S$ in the PM phase continuously when crossing the FM-PM phase boundary region. On a finite lattice $m_S$ does not vanish at the FM-PM phase transition and consequently $m_\pi$ has to grow when approaching the FM-PM phase transition. The increase at large $y$ is due to the O(4) symmetry breaking. The numerical data for $m_\pi$ may be compared with the one-loop results given in eqs. (17) and (22). The values obtained for $m_\pi$ are represented in fig. 4 by the diamonds. For $y \approx 4.0$ the agreement with the numerical results is reasonable.
Figure 4: Numerical results for \( m_\pi \) (squares) as a function of \( y \) \( (\kappa = 0, V = 12^324) \). The diamonds represent the values for \( m_\pi \) which were obtained by inserting the numerical results for \( m_F, y_R \) and \( v_R \) into eqs. (17) and (22).

We will present now some preliminary results for the renormalized couplings \( y_R \) and \( \lambda_R \), using the tree level motivated definitions \( y_R = m_F / v_R \) and \( \lambda_R = \frac{1}{2}(1 + \delta_R / 4)(m_\sigma / v_R)^2 - \varepsilon_R \). The definition of \( \lambda_R \) is based on eq. (22) and we shall use the one-loop values given in eqs. (16) and (17) for \( \delta_R \) and \( \varepsilon_R \). For the normalization of the scalar fields we use the wave-function renormalization constant \( Z_\pi \) (not \( Z_\sigma \) because the \( \sigma \) particle is unstable). The renormalized field expectation value is then defined by \( v_R = v / \sqrt{Z_\pi} \). We find that \( Z_\pi \) is smaller than in the pure O(4) model by roughly a factor four. In table 1 we give the results for \( y_R, \sqrt{2\lambda_R} \) and the ratio \( m_\sigma / v_R \). The table shows that the numerical values for \( \sqrt{2\lambda_R} \) and the ratio \( m_\sigma / v_R \), which differ by the \( \varepsilon_R \) and \( \delta_R \) corrections, agree within 5%. This indicates that the effect of the symmetry breaking is relatively small.

The bare Yukawa couplings are relatively large and it is therefore interesting to compare the values for \( y_R \) listed in table 1 with the tree level unitarity bound which for this model is given by \( y_R^{u.b.} = 2\sqrt{\pi / N_D} \approx 2.51 \). At the edge of the scaling region with \( m_\sigma \approx 0.75 \) the numerical results for \( y_R \) are quite close to this value. This indicates that the renormalized couplings are relatively weak, but they appear to be stronger than in the model with naive fermions where the numerical results for \( y_R \) are significantly smaller than \( y_R^{u.b.} \).

The values for the quantity \( \sqrt{2\lambda_R} \) with \( L = 12 \) may be compared with the infinite volume results obtained previously in the O(4) model \( [14] \). There the ratio \( \sqrt{2\lambda_{R,\infty}} = m_{\sigma,\infty} / v_{R,\infty} \) ranges from 2.5 to 3.1 for \( m_{\sigma,\infty} = 0.4 - 1.0 \). The finite volume results in table 1 with \( m_{\sigma,L} < 1 \) lie clearly above this range which indicates that also the upper bound of the Higgs mass grows when the Yukawa coupling is turned on.

6. Conclusion

We have introduced a fermion-Higgs model with reduced staggered fermions in which the staggered flavors are coupled to the Higgs field. In a numerical simulation with the hybrid
Monte Carlo algorithm the model contains two isospin doublets in the scaling region. To recover the full O(4) symmetry we have to add two counterterms to the scalar part of the action. The addition of counterterms can perhaps be avoided using an $F_4$ lattice.

In this paper we have studied the model without these counterterms. We find that a one-loop computation of the effect of the symmetry breaking can account reasonably well for the measured values of $m_\pi$ and predicts a 5% correction in the relation between $m_\sigma/v_R$ and $\sqrt{2\lambda_R}$. Also the finite volume dependence of $v$ and $m_F$ indicates that the effect of the symmetry breaking is relatively small.

As a preliminary result we find that at $\kappa = 0$ and at relatively large bare Yukawa coupling $y$ the renormalized Yukawa coupling cannot be significantly larger than the tree level unitarity bound in the region with $m_\sigma < 0.7$, $m_F < 0.5$. The quantity $\sqrt{2\lambda_R}$ comes out to be larger than the infinite volume results in the pure O(4) model.

The model has proven to be very efficient in a numerical simulation and it is possible to perform calculations on relatively large lattices. The time for an update of a scalar field configuration is smaller by roughly a factor 10 than in a model with naive fermions [11, 12].

We have investigated here a simple example in a class of models in which the staggered flavors are coupled to scalar or gauge degrees of freedom [4, 8]. We will investigate in future more complicated models which might provide a non-perturbative formulation of a chiral gauge theory on the lattice.

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