Symmetries and Unification

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Abstract

Symmetries concerning the ordinary coordinate spacetime and internal spacetime are discussed. A possible unification model of electroweak, strong and gravitational interactions is briefly described.

Symmetry has played a crucial role in physics. I first learned the importance of symmetries in physics was from the lecture talks given by professor C.S. Wu when I was a student in the Department of Physics at Nanjing University. The $\beta$ decay experiment by Wu et al [1] and the $\pi$-$\mu$ decay experiment by Garwin, Lederman and Weinrich [2] and by Friedman and Telegdi [3] were the most excellent experiments that first decisively established parity nonconservation discovered first by T.D. Lee and C.N. Yang [4]. In fact, these experiments proved not only parity $P$ violation, but also charge asymmetry under particle-antiparticle conjugation $C$. Late on, the experiment by Christenson, J. Cronin, V.L. Fitch and R. Turlay [5] in 1964 established $CP$ violation in kaon decays. From CPT symmetry, $CP$ violation implies time reversal $T$ asymmetry. All physical laws are related to these discrete symmetries as they are the basic symmetries of spacetime. The present status of these discrete symmetries has been known that: weak interaction violates the parity $P$, charge conjugation $C$ and $CP$ symmetries, while strong and electromagnetic interactions appear to be invariant under these discrete symmetries.
The papers by Glashow, Weinberg and Salam unified the weak interaction with electromagnetic force\cite{6}. Such a unified interaction is called electroweak interaction characterized by the local gauge symmetry SU(2) × U(1). The strong interaction is described by QCD with gauge symmetry group SU(3). The model with gauge symmetry group U(1) × SU(2) × SU(3) is usually called as the standard model which has successfully described the electroweak and strong interactions. The gravitational force is characterized by Einstein’s general relativity which is invariant under general coordinate transformations.

However, in the standard model, one has to introduce eighteen parameters to describe real world. Three charged lepton masses, six quark masses, three quark mixing angles and one CP-violating phase, three gauge coupling constants and two weak gauge boson masses, all of unknown origins. Therefore, the standard model cannot be considered as a complete model. Thus the outstanding puzzles that confront us today are:

1) Origin of CP violation and fermion masses.
2) Basic symmetries of nature and symmetry breaking mechanism.

Numerous efforts have been made to understand these puzzles. In this talk, I briefly outline the progresses in our recent studies.

I. Mechanisms of CP Violation in General 2HDM. In order to understand origin and mechanism of CP violation, we consider a simple extension of the standard model by just adding a Higgs doublet. Since the discovery of CP violation in 1964, two of the interesting ideas about CP-violating scheme have been known as the superweak interaction proposed by Wolfenstein\cite{7} and spontaneous breaking of CP symmetry suggested by T.D. Lee\cite{8}. These two ideas were found to be simultaneously realized in two-Higgs doublet models\cite{9, 10, 11}. In the most general two-Higgs doublet model proposed recently by Wolfenstein and myself\cite{11}, we observed that there exist rich sources of CP violation. In this general 2HDM, we have assumed that CP violation arises solely through the Higgs potential and there is no discrete symmetry that distinguishes the two Higgs bosons. It was found that an approximate global family symmetry is sufficient to suppress flavor-changing neutral scalar interactions. The model have four major sources of CP violation induced from a single CP-violating phase, i.e., the relative phase of the two vacuum expectation values:

(A) The CKM CP-violating source. Its effects are related to the magnitudes of the flavor-changing scalar interactions. This is easily understood
because in the case of natural flavor conservation ensured by imposing discrete symmetry, the model is CP-conserved. In addition to the usual CP violation in $W^{\pm}$ exchange, there is also in all two-Higgs doublet models a similar CP violation in the charged-Higgs-boson sector.

(B) New CP-violating sources that are independent of the CKM phase. Their effects are related to the diagonal couplings of the two Higgs doublets to the fermions. Therefore, this type of CP violation occurs not only in the charged-Higgs-boson exchange processes but also in flavor-conserving scalar interactions.

Let us illustrate the origin of these sources. For this purpose, one can simply neglect the off-diagonal elements of the couplings. For each fermions, we have

$$m_{f_i} e^{i\delta_{f_i}} = (g_{1f_i} e^{i\delta} \cos \beta + g_{2f_i} \sin \beta) v$$

where $g_{1f_i}$ and $g_{2f_i}$ are Yukawa couplings corresponding to the two Higgs doublets $\phi_1$ and $\phi_2$. $\delta$ is the relative phase of the two vacuum expectation values $v_1 e^{i\delta}$ and $v_2$. The angle $\beta$ is given by $\tan \beta = v_1 / v_2$ with $\sqrt{v_1^2 + v_2^2} = v = 246$GeV. $m_i$ are the fermion masses and $\delta_i$ are phases associated with the masses. One gets rid of $\delta_i$ by redefining the corresponding right-handed fermions. These phases then enter into the scalar boson interactions with effective couplings

$$ (g_{1f_i} e^{i\delta} \sin \beta - g_{2f_i} \cos \beta) e^{-i\delta_{f_i}} \equiv \xi_{f_i} m_{f_i} / v$$

(C) Superweak type CP violation. The effective CP-violating phases arise from the small off-diagonal couplings of two Higgs doublets. These yield CP violation in flavor-changing processes mediated by the exchange of neutral scalar bosons (FCNE).

(D) CP violation due to neutral scalar boson mixings. This source arises from the matrix $O^H$ that diagonalizes the Higgs boson mass matrix. Even in the absence of fermions this $O^H$ may violate CP invariance.

As a consequence, we observed the following interesting features arising from the new sources:

(1) The new sources through charged Higgs boson exchange can make a contribution to direct CP-violating parameter $\epsilon'/\epsilon$ which has the order of magnitude

$$\frac{\epsilon'}{\epsilon} \approx 10^{-4} \sim 10^{-5}, \quad \tan \beta \sim 1,$$
\[ \frac{\epsilon'}{\epsilon} \simeq 10^{-3}, \quad \tan \beta \sim 10 \left[ \frac{m_{H^\pm}}{(200 \text{GeV})} \right] \] (3)

without conflicting with other constraints. These predictions are comparable with those from the standard CKM CP source.

(2) The indirect CP-violating parameter \( \epsilon \) can be fitted by the new sources from box diagrams containing \( H^\pm \). It may also receive significant contribution from superweak FCNE.

(3) CP asymmetry in the decay \( b \to s\gamma \) may arise from the new sources from the charged Higgs boson interactions with fermions. The asymmetry may be larger than in the standard model and can lie between 0.01 and 0.1 [12].

(4) The new sources may also seriously change the expectations for CP violation in the \( B^0 \) system. In general, if there are large contributions to \( B^0 - \bar{B}^0 \) mixing from superweak and new sources, the measured three angles corresponding to the unitarity triangle of the CKM matrix may not be closed.

(5) The new sources could provide large CP violation in hyperon decays. The resulting values can reach the present experimental sensitivity [14].

(6) The new sources may also lead to a significant time reversal \( T \) violation. Such as the electric dipole moment \( D_e \) of the electron and the electric dipole moment \( D_n \) of the neutron. From both charged and neutral Higgs boson contributions to \( D_n \) and \( D_e \) via the two-loop Barr-Zee mechanism, resulting values of \( D_n \) of the order \( 10^{-25} \) to \( 10^{-26} \) e cm and of \( D_e \) of the order \( 10^{-26} \) to \( 10^{-27} \) e cm close to the present limits are allowed without conflicting with other constraints.

In a word, this general 2HDM, as one of the most simplest extenstions of the standard model, does contain rich physical phenomena. Though more unkown parameters have been introduced, it does show us from where one may look for the possible new physics.

II. Predictive SUSY GUTs. Let us extend the standard model along the direction of supersymmetric grand unification theories (SUSY GUTs) for the purposes of reducing the parameters. As the eighteen parameters in the standard model have been improved to be more and more accuracy. It reminds us that we are in a stage similar to that of atomic spectroscopy before Balmer. Much effort has been made along this direction [13]. Here I briefly describe an SUSY GUT model proposed recently by Chou and myself [13, 16, 17]. Our SUSY GUT model was based on the symmetry group SUSY \( \text{SO}(10) \times \Delta(48) \times U(1) \). Where \( \text{SO}(10) \) [18] unifies all leptons.
and quarks of a single generation into a single 16-dimensional spinor representation of SO(10). The dihedral group \( \Delta(48) \), a subgroup of SU(3), is taken as the family group. U(1) is family-independent and is introduced to distinguish various fields which belong to the same representations of SO(10) \( \times \Delta(48) \). The irreducible representations of \( \Delta(48) \) consisting of five triplets and three singlets have been found to be sufficient to build an interesting texture structure for fermion mass matrices. The symmetry \( \Delta(48) \times U(1) \) naturally ensures the texture structure with zeros for Yukawa coupling matrices. To reduce the possible free parameters, the universality of coupling constants in the superpotential is assumed, i.e., all the coupling coefficients are assumed to be equal and have the same origins from perhaps a more fundamental theory. With these considerations, Yukawa coupling matrices which determine the masses and mixings of all quarks and leptons can be obtained by carefully choosing the structure of the physical vacuum and integrating out the heavy fermions at the GUT scale

\[
\Gamma^G_u = \frac{2}{3} \lambda_H \begin{pmatrix} 0 & \frac{3}{2}z_u \epsilon_P^2 e^{i \phi} & 0 \\ \frac{3}{2} z_u \epsilon_P^2 e^{-i \phi} & -3 y_u \epsilon_G^2 e^{i \phi} & -\frac{\sqrt{3}}{2} x_u \epsilon_G^2 \\ 0 & -\frac{\sqrt{3}}{2} x_u \epsilon_G^2 & w_u \end{pmatrix}
\]

(4)

and

\[
\Gamma^G_f = \frac{2}{3} \lambda_H \frac{(-1)^{n+1}}{3^n} \begin{pmatrix} 0 & \frac{3}{2} z_f \epsilon_P^2 e^{i \phi} & 0 \\ \frac{3}{2} z_f \epsilon_P^2 e^{-i \phi} & 3 y_f \epsilon_G^2 e^{i \phi} & -\frac{1}{2} x_f \epsilon_G^2 \\ 0 & -\frac{1}{2} x_f \epsilon_G^2 & w_f \end{pmatrix}
\]

(5)

for \( f = d, e \), and

\[
\Gamma^G_\nu = \frac{2}{3} \lambda_H \frac{(-1)^{n+1}}{3^n} \frac{1}{5^{n+1}} \begin{pmatrix} 0 & \frac{15}{2} z_\nu \epsilon_P^2 e^{i \phi} & 0 \\ \frac{15}{2} z_\nu \epsilon_P^2 e^{-i \phi} & 15 y_\nu \epsilon_G^2 e^{i \phi} & -\frac{1}{2} x_\nu \epsilon_G^2 \\ 0 & -\frac{1}{2} x_\nu \epsilon_G^2 & w_\nu \end{pmatrix}
\]

(6)

for Dirac-type neutrino coupling. The Majorana neutrino mass matrix is chosen to be

\[
M_N^G = M_R \begin{pmatrix} 0 & 0 & \frac{1}{2} z_N \epsilon_P^2 e^{i (\delta_\nu + \phi_3)} \\ 0 & y_N e^{2i \phi_2} & 0 \\ \frac{1}{2} z_N \epsilon_P^2 e^{i (\delta_\nu + \phi_3)} & 0 & w_N \epsilon_P^4 e^{2i \phi_3} \end{pmatrix}
\]

(7)

We choose \( n = 4 \) for a realistic case. \( \lambda_H = \lambda_H^0 r_3 = 2 \lambda_4^G / 3, \epsilon_G \equiv \left( \frac{v_{10}}{v_{10}^r} \right) \sqrt{\frac{v_3}{r_3}} \) and \( \epsilon_P \equiv \left( \frac{v_{10}^P}{M_P} \right) \sqrt{\frac{v_3}{r_3}} \) are three parameters. Where \( \lambda_H^0 \) is a universal coupling constant expected to be of order one, \( r_1, r_2 \) and \( r_3 \) denote the ratios
of the coupling constants of the superpotential at the GUT scale for the
textures ‘12’, ‘22’ (‘32’) and ‘33’ respectively. They represent the possible
renormalization group (RG) effects running from the scale \( M_P \) to the GUT
scale. \( M_P, v_{10} \) and \( v_5 \) are the VEVs for \( U(1) \times \Delta(48), SO(10) \) and \( SU(5) \)
symmetry breaking respectively. \( \phi \) is the physical CP phase\(^1\) arising from
the VEVs. The assumption of maximum CP violation implies that
\( \phi = \pi/2. \)

\[
M_R = \lambda_H \epsilon_P^2 \epsilon_G^2 v_{10}^2 / M_P, \quad \lambda_1^N = \epsilon_P^2 M_R, \quad \lambda_2^N = M_R/\epsilon_G^2 \quad \text{and} \quad \lambda_3^N = \epsilon_P M_R. \quad \text{x}_f, \quad \text{y}_f, \quad \text{z}_f, \quad \text{and} \quad w_f \quad (f = u, d, e, \nu) \quad \text{and} \quad y_N, z_N \quad \text{and} \quad w_N \quad \text{are the Clebsch factors of} \quad SO(10) \quad \text{determined by the directions of symmetry breaking of the ad-
}
\]

\[
\begin{align*}
x_u &= 5/9, & x_d &= 7/27, & x_e &= -1/3, & x_\nu &= 1/5, & y_u &= 0, & y_d &= y_e/3 = 2/27, & y_\nu &= 4/225, & z_u &= 1, & z_d &= z_e = -27, & z_\nu &= -15^3 = -3375, & z'_u &= 1 - 5/9 = 4/9, & z'_d &= z_d + 7/729 \simeq z_d, & z'_e &= z_e - 1/81 \simeq z_e, & z'_\nu &= z_\nu + 1/15^3 \simeq z_\nu. \\
y_N &= 9/25, & z_N &= 4, & w_N &= 256/27
\end{align*}
\]

By diagonalizing the mass matrices and taking into account the renormal-
ization group effects from GUT scale down to low energies, we can obtain
twenty three predictions with four input parameters. Four predictions for
\( |V_{us}|, |V_{ub}/V_{cb}|, |V_{ub}/V_{cb}| \) and \( m_s/m_d \) are RG scaling-independent. All the
predictions are presented in Table 1.

\footnote{We have rotated away other possible phases by a phase redefinition of the fermion fields.}
Table 1. Output observables and model parameters and their predicted values with input parameters $m_e = 0.511$ MeV, $m_\mu = 105.66$ MeV, $m_\tau = 1.777$ GeV and $m_b(m_b) = 4.32$ GeV for $\alpha_s(M_Z) = 0.113$

| Output para. | Values | Data | Output para. Values |
|-------------|--------|------|----------------------|
| $M_1$ [GeV] | 179 | $175 \pm 6$ | $J_{CP} = A^2 \lambda^b \eta$ | $2.62 \cdot 10^{-5}$ |
| $m_e(m_e)$ [GeV] | 1.21 | $1.27 \pm 0.05$ | $\alpha$ | $86.28^\circ$ |
| $m_u$(1GeV) [MeV] | 4.11 | $4.75 \pm 1.65$ | $\beta$ | $22.11^\circ$ |
| $m_s$(1GeV) [MeV] | 156.5 | $165 \pm 65$ | $\gamma$ | $71.61^\circ$ |
| $m_d$(1GeV) [MeV] | 6.26 | $8.5 \pm 3.0$ | $m_{\nu_e}$ [eV] | 2.4515 |
| $\left| V_{us} \right| = \lambda$ | 0.22 | $0.221 \pm 0.003$ | $m_{\nu_\mu}$ [eV] | 2.4485 |
| $\left| V_{ub} \right|$ | 0.083 | $0.08 \pm 0.03$ | $m_{\nu_e}$ [eV] | $1.27 \cdot 10^{-3}$ |
| $\left| V_{ts} \right|$ | 0.209 | $0.24 \pm 0.11$ | $m_{\nu_\mu}$ [eV] | $2.8 \cdot 10^{-3}$ |
| $\left| V_{cb} \right|$ | 0.0389 | $0.039 \pm 0.005$ | $\left| V_{\nu_e \nu_e} \right|$ | -0.049 |
| $\lambda^G_e$ | 1.20 | - | $\left| V_{\nu_e \nu_e} \right|$ | 0.000 |
| $\tan \beta = v_2/v_1$ | 2.12 | - | $\left| V_{\nu_\mu \nu_e} \right|$ | -0.049 |
| $\epsilon_G$ | 0.2987 | - | $\left| V_{\nu_\mu \nu_\mu} \right|$ | -0.707 |
| $\epsilon_P$ | 0.0101 | - | $\left| V_{\nu_e \nu_e} \right|$ | 0.038 |
| $B_K$ | 0.96 | $0.82 \pm 0.10$ | $M_{N_1}$ [GeV] | $\sim 361$ |
| $f_B \sqrt{B}$ [MeV] | 212 | $200 \pm 70$ | $M_{N_2}$ [GeV] | $1.77 \cdot 10^6$ |
| $\text{Re}(\epsilon'/\epsilon)/10^{-3}$ | 1.4 $\pm$ 1.0 | 1.5 $\pm$ 0.8 | $M_{N_3}$ [GeV] | 361 |

From the above results, we observe the following features for the neutrinos

1. A $\nu_\mu(\bar{\nu}_\mu) \rightarrow \nu_e(\bar{\nu}_e)$ short wave-length oscillation with

$$\Delta m_{\nu_e}^2 = m_{\nu_\mu}^2 - m_{\nu_e}^2 \approx 6 \, eV^2, \quad \sin^2 2\theta_{\nu_e \nu_\mu} \approx 1.0 \times 10^{-2}, \quad (8)$$

which is consistent with the LSND experiment[19]

$$\Delta m_{\nu_e}^2 = m_{\nu_\mu}^2 - m_{\nu_e}^2 \approx (4-6) eV^2, \quad \sin^2 2\theta_{\nu_e \nu_\mu} \approx 1.8 \times 10^{-2} \sim 3 \times 10^{-3}; \quad (9)$$

2. A $\nu_\mu(\bar{\nu}_\mu) \rightarrow \nu_\tau(\bar{\nu}_\tau)$ long-wave length oscillation with

$$\Delta m_{\nu_\tau}^2 = m_{\nu_\mu}^2 - m_{\nu_\tau}^2 \approx 1.5 \times 10^{-2} eV^2, \quad \sin^2 2\theta_{\nu_{\tau} \nu_\mu} \approx 0.987, \quad (10)$$

which could explain the atmospheric neutrino deficit[20]:

$$\Delta m_{\nu_\tau}^2 = m_{\nu_\tau}^2 - m_{\nu_\mu}^2 \approx (0.5-2.4) \times 10^{-2} eV^2, \quad \sin^2 2\theta_{\nu_{\tau} \nu_\mu} \approx 0.6-1.0, \quad (11)$$

with the best fit[20]

$$\Delta m_{\nu_\tau}^2 = m_{\nu_\tau}^2 - m_{\nu_\mu}^2 \approx 1.6 \times 10^{-2} eV^2, \quad \sin^2 2\theta_{\nu_{\tau} \nu_\mu} \approx 1.0; \quad (12)$$
3. Two massive neutrinos $\nu_\mu$ and $\nu_\tau$ with

$$m_{\nu_\mu} \simeq m_{\nu_\tau} \simeq 2.45 \text{ eV}$$

fall in the range required by possible hot dark matter\textsuperscript{21}.

4. $(\nu_\mu - \nu_\tau)$ oscillation will be beyond the reach of CHORUS/NOMAD and E803. However, $(\nu_e - \nu_\tau)$ oscillation may become interesting as a short wave-length oscillation with

$$\Delta m^2_{e\tau} = m^2_{\nu_\tau} - m^2_{\nu_e} \simeq 6 \text{ eV}^2, \quad \sin^2 2\theta_{e\tau} \simeq 1.0 \times 10^{-2}, \quad (14)$$

which should provide an independent test on the pattern of the present Majorana neutrino mass matrix.

5. Majorana neutrino allows neutrinoless double beta decay $(\beta\beta_0\nu)$\textsuperscript{22}. Its decay amplitude is known to depend on the masses of Majorana neutrinos $m_{\nu_i}$ and the lepton mixing matrix elements $V_{ei}$. The present model is compatible with the present experimental upper bound on neutrinoless double beta decay

$$\bar{m}_{\nu_e} = \sum_{i=1}^{3} |V_{e_i}^2 m_{\nu_i} \zeta_i| \simeq 1.18 \times 10^{-2} \text{ eV} < \bar{m}_{\nu}^{\text{upper}} \simeq 0.7 \text{ eV} \quad (15)$$

The decay rate is found to be

$$\Gamma_{\beta\beta} \simeq \frac{Q^5 G_F^4 \bar{m}_{\nu_e}^2 p_F^2}{60 \pi^3} \simeq 1.0 \times 10^{-61} \text{GeV} \quad (16)$$

with the two electron energy $Q \simeq 2 \text{ MeV}$ and $p_F \simeq 50 \text{ MeV}$.

6. In this case, solar neutrino deficit has to be explained by oscillation between $\nu_e$ and a sterile neutrino $\nu_s$\textsuperscript{23}. Since strong bounds on the number of neutrino species both from the invisible $Z^0$-width and from primordial nucleosynthesis\textsuperscript{24} require the additional neutrino to be sterile (singlet of SU(2)$\times$ U(1), or singlet of SO(10) in the GUT SO(10) model). Masses and mixings of the triplet sterile neutrinos can be chosen by introducing an additional singlet scalar with VEV $v_s \simeq 336 \text{ GeV}$. We find

$$m_{\nu_s} = \lambda_H v_s^2 / v_{10} \simeq 2.8 \times 10^{-3} \text{eV}$$

$$\sin \theta_{es} \simeq \frac{m_{\nu_s} v_{s}}{m_{\nu_s}} = \frac{v_2}{2 v_s} \frac{\epsilon_F}{\epsilon_G} \simeq 3.8 \times 10^{-2} \quad (17)$$
with the mixing angle consistent with the requirement necessary for primordial nucleosynthesis [25] given in [24]. The resulting parameters

$$\Delta m_{es}^2 = m_{\nu_s}^2 - m_{\nu_e}^2 \simeq 6.2 \times 10^{-6} \text{eV}^2, \quad \sin^2 2\theta_{es} \simeq 5.8 \times 10^{-3}$$

(18)

are consistent with the values [23] obtained from fitting the experimental data:

$$\Delta m_{es}^2 = m_{\nu_s}^2 - m_{\nu_e}^2 \simeq (4-9) \times 10^{-6} \text{eV}^2, \quad \sin^2 2\theta_{es} \simeq (1.6-14) \times 10^{-3}$$

(19)

This scenario can be tested by the next generation solar neutrino experiments in Sudhuray Neutrino Observatory (SNO) and Super-kamiokanda (Super-K), both planning to start operation in 1996. From measuring neutral current events, one could identify $\nu_e \rightarrow \nu_s$ or $\nu_e \rightarrow \nu_\mu$ ($\nu_\tau$) since the sterile neutrinos have no weak gauge interactions. From measuring seasonal variation, one can further distinguish the small-angle MSW [26] oscillation from vacuum mixing oscillation.

III. Unification of All Basic Forces. We finally consider a possible unification of the standard model with gravity. This is one of the great theoretical endeavours in this century. One of the difficulties arises from the no-go theorem. Most of the attempts to unify all basic forces involve higher dimensional spacetime, such as Kaluza-Klein Yang-Mills theories, supergravity theories and superstring theories, etc. The Kaluza-Klein approach is not rich enough to support the fermionic representations of the standard model. The maximum supergravity has SO(8) symmetry which is too small to include the standard model. In superstring theories, all the known particle interactions can be reproduced, but millions of vacua have been found. The outstanding problem is to find which one is the true vacuum of the theory. We then presented an alternative scheme [27]. Firstly, we observe that quarks and leptons in the standard model can be unified into a single 16-dimensional representation of complex chiral spinors in SO(10) [18]. Each complex chiral spinor belong to a single 4-dimensional representation of SO(1,3). In an unified theory, it is an attractive idea to treat these 64 real spinor components on the same footing, i.e., they have to be a single representation of a larger group. It is therefore natural to consider SO(1,13) as our unified group and the gauge potential of SO(1,13) as the fundamental interaction that unifies the four basic forces (strong, electromagnetic, weak and gravitational) of nature. Secondly, to avoid the restriction given by no-go theorem and other
problems mentioned above, we consider the ordinary coordinate spacetime remains to be a 4-dimensional manifold $S_4$ with metric $g_{\mu\nu}(x)$, $\mu, \nu=0,1,2,3$.

At each point $P$: $x^\mu$, there is an $d$-dimensional flat space $M_d$ with $d > 4$ and signature $(1, -1, \cdots, -1)$. We assume that the tangent space $T_4$ of $S_4$ at point $P$ to be an 4-dimensional submanifold of $M_d$ spanned by four vectors $e^A_\mu(x)$ $\mu=0,1,2,3$; $A \equiv (\alpha, a)$ with $\alpha = 0, 1, 2, 3$ and $a = 1, \cdots, d - 4$ such that

$$g_{\mu\nu}(x) = e^A_\mu(x)e^A_\nu(x)\eta_{AB} \quad (20)$$

where $\eta_{AB} = \text{diag.}(1, -1, \cdots, -1)$ can be considered as the metric of the flat space $M_d$. We shall call $e^A_\mu(x)$ to be the generalized vierbein fields or simply the frame fields.

Under general coordinate transformations and the rotations in $M_d$, $e^A_\mu(x)$ transform as a covariant vector in ordinary coordinate spacetime and a vector in the $M_d$ rotation, $e^A_m(x)$ transform as a covariant vector in the $C_{d-4}$ rotation and a vector in the $M_d$ rotation. For a theory to be invariant under both general coordinate transformations and local rotations in the flat space $M_d$, it is necessary to introduce affine connection $\Gamma^\rho_\mu\nu(x)$ for general coordinate transformations and gauge potential $\Omega^A_\mu^B(x) = -\Omega^B_\mu^A(x)$ for $d$-dimensional rotation SO$(1,d-1)$ in $M_d$. These transformations are connected by the requirement that $T_4$ has to be the submanifold of $M_d$ spanned by four vectors $e^A_\mu(x)$ at point $P$ and $e^A_\mu(x)$ should be a covariantly constant frame and satisfy the condition

$$D_\mu e^A_\rho = \partial_\mu e^A_\rho - \Gamma^\sigma_\mu\rho e^A_\sigma + g_U\Omega^A_\mu^B e^B_\rho = 0 \quad (21)$$

It is then easily verified that

$$D_\mu g_{\rho\sigma} = \partial_\mu g_{\rho\sigma} - \Gamma^\lambda_\mu\rho g_{\lambda\sigma} - \Gamma^\lambda_\mu\sigma g_{\rho\lambda} = 0 \quad (22)$$

$$D_\mu e^\rho_A = \partial_\mu e^\rho_A + \Gamma^\rho_\mu e_A^\sigma - g_U\Omega^R_\mu_A e^\rho_B = 0 \quad (23)$$

With the above considerations, we can now construct an invariant action under general coordinate transformations in the ordinary coordinate space-time and the local SO$(1,d-1)$ group symmetry in $M_d$ with $D_\mu e^A_\rho = 0$ as a constraint. In addition, the action is required to have no dimensional parameters and to be renormalizable in the sense of the power counting. The general form of the action which satisfies these requirements is

$$S_B = \int d^4x \sqrt{-g}\left\{-\frac{1}{4}F^{AB}_\mu F^{CD}_\rho \eta_{\mu\rho} g^{\nu\sigma} \eta_{AC} \eta_{BD}\right\}$$

10
\[ -\frac{1}{2} \xi \phi^2 F_{\mu \nu}^{AB} \epsilon_{A}^{\mu} \epsilon_{B}^{\nu} + \frac{1}{2} g^{\mu \nu} \partial_{\mu} \phi \partial_{\nu} \phi + \frac{1}{4} \lambda \phi^4 \]

\[ + \zeta F_{\mu \rho}^{AB} F_{\rho \sigma}^{CD} g^{\mu \sigma} \eta_{AB} e_{C}^{\nu} e_{D}^{\sigma} + a_1 F_{\mu \nu}^{AB} F_{\rho \sigma}^{CD} e_{C}^{\mu} e_{D}^{\nu} e_{A}^{\rho} e_{B}^{\sigma} + a_2 F_{\mu \nu}^{AB} F_{\rho \sigma}^{CD} e_{C}^{\mu} e_{D}^{\nu} e_{A}^{\rho} e_{B}^{\sigma} + a_3 F_{\mu \nu}^{AB} F_{\rho \sigma}^{CD} e_{A}^{\mu} e_{B}^{\nu} e_{C}^{\rho} e_{D}^{\sigma} \}

where \( \phi(x) \) is a scalar field introduced to avoid the dimensional coupling constants. \( a_i \) (i=1,2,3), \( \zeta, \xi \) and \( \lambda \) are dimensionless parameters. \( F_{\mu \nu}^{AB} \) is the field strength defined in a standard way

\[ F_{\mu \nu}^{AB} = \partial_{\mu} \Omega_{\nu}^{AB} - \partial_{\nu} \Omega_{\mu}^{AB} + g_{U} (\Omega_{\mu}^{A} \Omega_{\nu}^{CB} - \Omega_{\nu}^{A} \Omega_{\mu}^{CB}) \]

The tensor \( F_{\mu}^{A} \) is defined as \( F_{\mu}^{A} = F_{\mu \nu}^{AB} \epsilon_{B}^{\nu} \). Note that not all the gauge fields \( \Omega_{\mu}^{AB}(x) \) are simply new propagating fields due to the constraints \( D_{\mu} e_{\rho}^{A} = 0 \). By counting the constraint equations (4 x 4 x d), unknowns \( \Omega_{\mu}^{AB}(x) \) (with 4d(d-1)/2 degrees of freedom) and \( e_{\mu}^{A}(x) \) (with 4 x d degrees of freedom) as well as \( \Gamma_{\mu}^{\rho} \) (with 40 degrees of freedom for the symmetric parts \( \Gamma_{(\mu \rho)}^{\alpha} = \Gamma_{(\rho \mu)}^{\alpha} \)) and 24 degrees of freedom for antisymmetric parts \( \Gamma_{[\mu \rho]}^{\alpha} = -\Gamma_{[\rho \mu]}^{\alpha} \)), one sees that besides the antisymmetric parts \( \Gamma_{[\mu \rho]}^{\alpha} \), the independent degrees of freedom are \( (4d + 4(d-4)(d-5)/2) \). These independent degrees of freedom coincide with the degrees of freedom of the frame fields \( e_{\mu}^{A}(x) \) and the gauge fields \( A_{\mu}^{ab}(x) \) \( (a, b = 1, \cdots, d - 4) \) of the subgroup SO(d-4). In addition, the gauge conditions in the coset SO(1,d-1)/SO(d-4) lead to additional constraints (4d-10). Thus the independent degrees of freedom are reduced to \( (10 + 4(d-4)(d-5)/2) \) which exactly match with the degrees of freedom of the metric tensor \( g_{\mu \nu}(x) \) and the gauge fields \( A_{\mu}^{ab}(x) \) of the group SO(d-4). For \( d=14 \), the resulting independent degrees of freedom of the fields are sufficient to describe the four basic forces. Where the general relativity of the Einstein theory is described by the metric tensor. Photon, W-bosons and gluons, that mediate the electromagnetic, weak and strong interactions respectively, are different manifestations of the gauge potential \( A_{\mu}^{ab}(x) \) of the symmetry group SO(10). The curvature tensor \( R_{\mu \nu \sigma}^{\rho} \) and the Ricci tensor \( R_{\nu \sigma}^{\rho} = R_{\mu \nu \sigma}^{\rho} g_{\mu}^{\rho} \) as well as the scalar curvature \( R = R_{\nu \sigma} g^{\rho \sigma} \) via \( R_{\mu \nu \sigma}^{\rho} = g_{\nu} F_{\mu \rho}^{A B} e_{A}^{\nu} e_{B}^{\sigma}, R_{\nu \sigma}^{\rho} = g_{\nu} F_{\mu \rho}^{A B} e_{A}^{\nu} e_{B}^{\sigma} \) and \( R = g_{\nu} F_{\mu \rho}^{A B} e_{A}^{\nu} e_{B}^{\nu} \).
Weyl fermion $\Psi_+(x)$ belonging to the fundamental spinor representation of $SO(1,13)$. The action for fermions is given by

$$S_F = \int d^4x \sqrt{-g} \left\{ \frac{1}{2} \bar{\Psi}_+ \gamma^A \left( i \partial_\mu + g_\nu \Omega_{\nu}^{BC} \frac{1}{2} \Sigma_{BC} \right) \Psi_+ + h.c. \right\}$$

(26)

where $\Sigma_{AB}$ are the generators of the $SO(1, d-1)$ in the spinor representations and given by $\Sigma_{AB} = \frac{i}{4} [\Gamma_A, \Gamma_B]$. $\Gamma^A$ are the gamma matrices that obey $\{\Gamma^A, \Gamma^B\} = 2\eta^{AB}$.

It is not difficult to check that the action can be decomposed into two parts. One of the parts has the same form as the action of a multiplicatively renormalized unified gauge theory including so-called $R^2$-gravity and a renormalizable scalar matter field as well a nonminimal gravitational-scalar coupling. Another part represents the direct interactions between the gauge fields and the gravitational fields. It is expected that such a model has provided us a new insight for unifying all the basic forces within the framework of quantum field theory. Though the ideas and the resulting model are both simple, there remains more theoretical work and experimental efforts needed to test whether they are the true choice of nature.

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