Gauge Unification and Flavor Hierarchy from Extra Dimensions*

Kiwoon Choi†

Department of Physics,
Korea Advanced Institute of Science and Technology
Daejeon 305-701, Korea

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Extra dimension provides an attractive way to break symmetry by boundary conditions which can be useful to construct a natural grand unified theory avoiding the doublet-triplet splitting problem and the proton decay problem. It can provide also an elegant mechanism to generate hierarchical 4-dimensional Yukawa couplings, involving the quasi-localization of matter fields in extra dimension. We discuss the Kaluza-Klein threshold corrections to low energy gauge couplings in generic 5-dimensional orbifold grand unified theories with quasi-localized matter fields, and apply the results to some class of SU(5) models on $S^1/Z_2 \times Z_2$.

I. INTRODUCTION

Grand unification of the strong and electroweak forces is a highly persuasive idea for physics at high energy scales. However conventional 4-dimensional (4D) grand unified theories (GUTs) have suffered from the doublet-triplet splitting problem and also the problem of too rapid proton decay. GUTs in (orbifolded) higher dimensional spacetime can avoid these problems by employing the symmetry breaking by boundary conditions [1]. Extra dimension can provide also an elegant mechanism to generate hierarchical Yukawa couplings [2]. The quark and lepton fields can be quasi-localized in extra dimension in a natural manner, and then their 4D Yukawa couplings are determined by the wavefunction factor $e^{-M \pi R}$ where $M$ is a combination of mass parameters in higher dimensional theory and $R$ is the radius of extra dimension. This allows that hierarchical Yukawa couplings are obtained from fundamental mass parameters having the same order of magnitudes.

In any GUT, heavy particle threshold effects at GUT symmetry breaking scale should be taken into account for a precision analysis of low energy gauge couplings $g^2_a$. In conventional 4D GUT, those threshold corrections to $1/g^2_a$ are generically of the order of $1/8 \pi^2$ and thus not so important. However higher dimensional field theory contain (infinitely) many Kaluza-Klein (KK) modes, so can have a sizable threshold correction [3]. It is then essential to include the KK threshold corrections in the analysis of low energy couplings in higher dimensional gauge theories. In this talk, we discuss the KK threshold corrections in generic 5D orbifold field theories with quasi-localized matter fields [4], and apply the results to some class of SU(5) models on $S^1/Z_2 \times Z_2$.

II. YUKAWA COUPLINGS OF QUASI-LOCALIZED MATTER FIELDS

Hierarchical Yukawa couplings can be naturally obtained if the matter fermions are quasi-localized in extra dimension [2]. To see this, consider a 5D theory on $S^1/Z_2$ with coordinate $y \equiv y + 2\pi R$, containing generic bulk fermions and also a brane Higgs field confined at $y = \pi R$:

$$S = -\int d^5x \left[ i \bar{\Psi}_I (\gamma^M D_M + M_I \epsilon(y)) \Psi_I + \delta(y - \pi R) \left( D_\mu HD_\mu H^* + \frac{\lambda_{IJ}}{\Lambda} H \psi_I \psi_J \right) \right].$$

(1)

Here $\epsilon(y) = y/|y|$, $\Lambda$ denotes the cutoff scale, and $\lambda_{IJ}$ are dimensionless brane Yukawa couplings. The 5D fermion $\Psi_I$ has the boundary condition

$$\Psi_I(-y) = z_I \gamma_5 \Psi_I(y), \quad \Psi_I(-y') = z_I \gamma_5 \Psi_I(y'),$$

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† kchoi@hep.kaist.ac.kr
where \( y' = y - \pi R \), \( z_I = \pm 1 \), and \( \Psi_I = \frac{1}{z} (1 + \gamma_5) \Psi_I \) (if \( z_I = 1 \)) or \( \frac{1}{z} (1 + \gamma_5) \Psi_I^\dagger \) (if \( z_I = -1 \)). For any value of the kink mass \( M_I \epsilon(y) \), \( \Psi_I \) has a chiral zero mode

\[
\psi_{0I} = \exp(-z_I M_I R y),
\]

which is quasi-localized at \( y = 0 \) or \( \pi \) depending upon the sign of \( M_I \). The 4D Yukawa couplings of these quasi-localized zero modes are given by

\[
y_{IJ} = \sqrt{Z(z_I M_I) Z(z_J M_J)} \lambda_{IJ}
\]

where

\[
Z(M) = \frac{M}{\Lambda} \frac{1}{e^{2 M \pi R} - 1}.
\]

Obviously, the 4D Yukawa couplings \( y_{IJ} \) can have hierarchical values even when the values of \( \lambda_{IJ} \) are similar to each other. For instance, if \( z_I M_I \) and \( z_J M_J \lesssim -1/R \), we have

\[
y_{IJ} \approx \lambda_{IJ} \frac{\sqrt{|M_I M_J|/\Lambda^2|}}{R},
\]

while for \( z_I M_I \) and \( z_J M_J \gtrsim 1/R \),

\[
y_{IJ} \approx \lambda_{IJ} \frac{\sqrt{|M_I M_J|/\Lambda^2|}}{R} e^{-(z_I M_I + z_J M_J)\pi R}.
\]

The physical interpretation of this result is simple. If \( z_I M_I, z_J M_J \gtrsim 1/R \), the corresponding zero modes are quasi-localized at \( y = 0 \), so the Yukawa couplings are exponentially suppressed as they originate from \( y = \pi \). On the other hand, for \( z_I M_I, z_J M_J \lesssim -1/R \), the zero modes are localized at \( y = \pi \), so there is no suppression of Yukawa couplings.

### III. KALUZA-KLEIN THRESHOLD CORRECTIONS

The model we study here is a generic 5D gauge theory on \( S^1/Z_2 \times Z_2 \) which is described by

\[
S = - \int d^4 x d y \left[ \frac{1}{\sqrt{g_{5a}}} F_{\mu \nu}^a \left( F_{\mu \nu}^a + D_{\mu} \phi \phi^* + m^2 \phi \phi^* \right.ight.
\]

\[
+ i \bar{\Psi} \left( \gamma^\mu D_{\mu} + M \epsilon(y) \right) \Psi \Big]\]

where

\[
\frac{1}{g_{5a}} = \frac{1}{g_{5a}} + \delta(y) \frac{1}{g_{5a}} + \delta(y - \pi R) \frac{1}{g_{5a}},
\]

\[
m^2 = m^2 + 2 \mu \delta(y) - 2 \mu \delta(y - \pi R).
\]

The 5D fields in the model can have arbitrary \( Z_2 \times Z_2 \) boundary condition,

\[
\phi(-y) = z_{\phi} \phi(y), \quad \phi(-y') = z'_{\phi} \phi(y'),
\]

\[
\Psi(-y) = z_{\Psi} \gamma_5 \Psi(y), \quad \Psi(-y') = z_{\Psi} \gamma_5 \Psi(y'),
\]

\[
A_{\mu}^a(-y) = z_{A} A_{\mu}^a(y), \quad A_{\mu}^a(-y') = z_{A} A_{\mu}^a(y'),
\]

with \( z_{\phi}, z'_{\phi} = \pm 1 \) (\( \Phi = \phi, \Psi, A_{\mu}^a \)). A 5D complex scalar field \( \phi \) has a zero mode for any value of \( R \) if \( z_{\phi} = z'_{\phi} = 1 \) and \( m = \mu = \mu' \), while \( \Psi \) has a chiral zero mode for arbitrary values of \( M \) and \( R \) if \( z_{\Psi} = z'_{\Psi} = 1 \).

The one-loop gauge couplings at low momentum scale \( p \) can be written as

\[
\frac{1}{g_{\mathbf{a}}^2(p)} = \left( \frac{1}{g_{\mathbf{a}}^2} \right)_{\text{bare}} + \frac{1}{8 \pi^2} \left[ \Delta_{a}(\ln \Lambda, m, \mu, \mu', M, R) + b_{a} \ln \left( \frac{\Lambda R}{p} \right) \right],
\]

where

\[
\left( \frac{1}{g_{\mathbf{a}}^2} \right)_{\text{bare}} = \frac{\pi R}{g_{5a}} + \frac{1}{g_{5a}} + \frac{1}{g_{\mathbf{a}}^2} + \frac{\gamma_a}{24 \pi^3} \Lambda \pi R,
\]

\[
\Delta_{a}(\ln \Lambda, m, \mu, \mu', M, R) = \left[ \frac{1}{g_{\mathbf{a}}^2} \right]_{\text{bare}} - \frac{2 g_{5a}}{g_{\mathbf{a}}^2} - \frac{2 g_{\mathbf{a}}^2}{g_{5a}^2} - \frac{1}{g_{5a}^2} - \frac{\gamma_a}{g_{\mathbf{a}}^2} - \frac{\gamma_a}{24 \pi^3} \Lambda \pi R,
\]

\[
b_{a} = \frac{1}{g_{5a}^2} + \frac{1}{g_{\mathbf{a}}^2} + \frac{\gamma_a}{24 \pi^3} \Lambda \pi R.
\]
\( \Delta_a \) stand for KK threshold corrections, and \( b_a \) are the 4D one-loop beta function coefficients. Note that \( \Delta_a \) are not sensitive to the unknown physics at \( \Lambda \), so are calculable within orbifold field theory, while the linearly divergent one-loop corrections in \( (1/g_s^2)_{\text{bare}} \) are UV-sensitive and thus not calculable. The computation of \( \Delta_a \) involves the summation over all massive KK modes [5], yielding [4]

\[
\Delta_a = -\frac{1}{6} T_a(\phi^{(0)++}) \ln \left( \frac{\Lambda(e^{m_{++}R} - e^{-m_{++}R})}{2m_{++}} \right) \\
-\frac{1}{6} T_a(\phi_{++}) \ln \left( \frac{(m_{++} + \mu_{++})(m_{++} + \mu'_{++})e^{m_{++}R}}{2m_{++} \Lambda} \right) \\
- \frac{1}{6} T_a(\phi_{+-}) \ln \left( \frac{(m_{+-} + \mu_{+-})(m_{+-} + \mu'_{+-})e^{m_{+-}R}}{2m_{+-}} \right) \\
- \frac{1}{6} T_a(\phi_{-+}) \ln \left( \frac{(m_{-+} + \mu_{-+})(m_{-+} + \mu'_{-+})e^{m_{-+}R}}{2m_{-+}} \right) \\
- \frac{1}{6} T_a(\phi_{- -}) \ln \left( \frac{(m_{--} + \mu_{--})(m_{--} + \mu'_{--})e^{m_{--}R}}{2m_{--}} \right) \\
- \frac{2}{3} T_a(\Psi_{++}) \ln \left( \frac{\Lambda(e^{M_{++}R} - e^{-M_{++}R})}{2M_{++}} \right) \\
- \frac{2}{3} T_a(\Psi_{+-}) \ln \left( e^{-M_{+-}R} \right) \\
- \frac{2}{3} T_a(\Psi_{-+}) \ln \left( e^{M_{-+}R} \right) \\
- \frac{2}{3} T_a(\Psi_{- -}) \ln \left( \frac{\Lambda(e^{M_{--}R} - e^{-M_{--}R})}{2M_{--}} \right) \\
+ \frac{21}{12} \left[ T_a(A_{++}^M) + T_a(A_{- -}^M) \right] \ln(\Lambda \pi R)
\]

(9)

where the subscripts represent the \( Z_2 \times Z_2 \) boundary conditions. Here \( \phi^{(0)++} \) is a 5D scalar field having zero mode, i.e. scalar field with \( \mu = \mu' = m \), and \( \phi_{\pm \pm} \) stand for other scalar fields without zero mode. The one-loop beta function coefficients \( b_a \) are given by

\[
b_a = -\frac{11}{3} T_a(A_{++}^M) + \frac{1}{6} T_a(A_{- -}^M) + \frac{1}{3} T_a(\phi^{(0)++}) + \frac{2}{7} T_a(\Psi_{++}) + \frac{2}{3} T_a(\Psi_{- -})
\]

(10)

since \( A_{++}^M \) gives a massless 4D vector, \( A_{- -}^M \) a massless real 4D scalar, and \( \Psi_{\pm \pm} \) a massless 4D chiral fermion. In supersymmetric limit, we have

\[
(\Delta_a)_{\text{SUSY}} = [T_a(V_{++}) + T_a(V_{- -})] \ln(\Lambda \pi R) \\
- T_a(H_{++}) \ln \left( \frac{\Lambda(e^{M_{++}R} - e^{-M_{++}R})}{2M_{++}} \right) \\
- T_a(H_{-+}) \ln \left( e^{-M_{-+}R} \right) \\
- T_a(H_{+-}) \ln \left( e^{M_{+-}R} \right) \\
- T_a(H_{- -}) \ln \left( \frac{\Lambda(e^{M_{--}R} - e^{-M_{--}R})}{2M_{--}} \right)
\]

(11)

where \( V_{zz'} \) denotes a vector multiplet, and \( H_{zz'} \) is a hypermultiplet with kink mass \( M_{zz'} \). In fact, \((\Delta_a)_{\text{SUSY}}\) can be computed by a different method based on 4D effective supergravity [6], which gives the same result as the direct summation over the KK modes. A similar calculation of KK threshold effects can be done for 5D theories with warped extra dimension [6].
IV. APPLICATION TO ORBIFOLD GUTS

To see the importance of KK threshold corrections, let us consider a class of 5D $SU(5)$ orbifold GUTs whose effective 4D theory is the MSSM. To break $SU(5)$ by orbifolding, $Z_2 \times Z'_2$ is embedded into $SU(5)$ as

\[
    Z_2 = \text{diag}(+1, +1, +1, +1, +1), \\
    Z'_2 = \text{diag}(+1, +1, +1, -1, -1).
\]  

The model contains matter hypermultiplets $F_p(\bar{5})$, $F'_p(\bar{5})$, $T_p(10)$ and $T'_p(10)$ $(p = 1, 2, 3)$ with kink masses $M_{F_p}$, $M_{F'_p}$, $M_{T_p}$ and $M_{T'_p}$, and also the Higgs hypermultiplets $H(5)$ and $H'(\bar{5})$ with kink masses $M_H$ and $M_{H'}$, where the numbers in brackets represent the $SU(5)$ representation. We assign the $Z_2 \times Z'_2$ parities of these hypermultiplets as

\[
    Z_2(F_p) = Z_2(F'_p) = Z_2(T_p) = Z_2(T'_p) = Z_2(H) = Z_2(H') = 1, \\
    Z'_2(F_p) = -Z'_2(F'_p) = Z'_2(T_p) = -Z'_2(T'_p) = Z'_2(H) = Z'_2(H') = -1.
\]

Then using (9), we find that the QCD coupling constant at the weak scale is predicted to be [4]

\[
    \frac{1}{\alpha_3(M_Z)} = 7.8 + \frac{1}{2\pi} \left[ \Delta_{\text{gauge}} + \Delta_{\text{higgs}} + \Delta_{\text{matter}} \right] + O \left( \frac{1}{\pi} \right)
\]

where

\[
    \frac{1}{2\pi} \Delta_{\text{gauge}} = \frac{3}{4\pi} \ln(\pi R \Lambda) \approx 0.8
\]

corresponds to the KK threshold correction from the 5D vector multiplet,

\[
    \Delta_{\text{higgs}} = \frac{9}{14} \left[ \ln \left( \frac{\sinh \pi R M_H \sinh \pi R M_{H'}}{\pi R M_H M_{H'}} \right) + \pi R (M_H + M_{H'}) \right]
\]

is the correction from Higgs hypermultiplets, and

\[
    \Delta_{\text{matter}} = \frac{9}{14} \sum_p \left[ \ln \left( \frac{\sinh \pi R M_{a,p} \sinh \pi R M_{a,p}'}{\pi R M_{F_p} M_{F'_p}} \right) + \pi R (M_{F_p} - M_{F'_p}) \right] \\
    + \frac{3}{2} \sum_p \left[ \ln \left( \frac{\sinh \pi R M_{T_p} \sinh \pi R M_{T'_p}}{\pi R M_{T_p} M_{T'_p}} \right) + \pi R (M_{T_p} - M_{T'_p}) \right]
\]

is the correction from matter hypermultiplets. For the prediction (13), we made the usual assumption [7] that the theory is strongly coupled at $\Lambda$, so the brane gauge couplings at $\Lambda$ are estimated as $1/g^2_{\text{br.},\pi\pi} = O(1/\pi^2)$.

In order to be consistent with $(1/\alpha_3(M_Z))_{\text{exp}} = 8.55 \pm 0.15$, $\Delta_{\text{higgs}} + \Delta_{\text{matter}}$ are required to be not so large, which is a nontrivial condition for the model. A simple way to avoid a too large $\Delta_{\text{higgs}}$ is that both Higgs hypermultiplets have $M_{\pi R} \ll -1$. Then the Higgs zero modes are localized at $y = \pi R$ and

\[
    \frac{1}{2\pi} \Delta_{\text{higgs}} \approx -\frac{9}{14\pi} \ln \left( \sqrt{M_H M_{H'} \pi R} \right) \approx -0.45,
\]

where the final number is obtained for $M_{H} \pi R \approx M_{H'} \pi R \approx -10$. For matter hypermultiplets, to generate hierarchical 4D Yukawa couplings, some kink masses should be positive, while some others are negative. If the Higgs zero modes are localized at $y = \pi R$, the matter hypermultiplets with $M_{\pi R} \gg 1$ have small Yukawa couplings $Y$ suppressed by $e^{-M_{\pi R}}$. The above form of $\Delta_{\text{matter}}$ shows that such hypermultiplets give a contribution of $O(M_{\pi R}) = O(\ln Y)$ to $\Delta_{\text{matter}}$. Then, in order for $\Delta_{\text{matter}}$ to be small enough, one needs a nontrivial cancellation between the contributions from different matter hypermultiplets. A simple example realizing such cancellation is

\[
    M_{F_p} = M_{F'_p}, \quad M_{T_p} = M_{T'_p},
\]

for which $\Delta_{\text{matter}} = 0$. 

V. CONCLUSION

In this talk, we discussed the KK threshold corrections to low energy gauge couplings $g^2_a$ from bulk matter fields whose zero modes are quasi-localized to generate hierarchical 4D Yukawa couplings. We presented the explicit form of threshold corrections in generic 5D orbifold field theory on $S^1/Z_a \times Z'_b$. The typical size of KK threshold corrections to $1/g^2_a$ is of the order of $\ln(Y)/8\pi^2$ where $Y$ denotes small 4D Yukawa couplings generated by quasi-localization. So KK threshold corrections can significantly affect the gauge coupling unification. We applied these results to some class of 5D $SU(5)$ models to find the condition to get hierarchical Yukawa couplings without spoiling the successful gauge coupling unification.

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