Application of tangent modulus method in nonlinear settlement calculation of foundation

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Abstract. Foundation settlement calculation has been a difficult and hot problem in geotechnical engineering study. The problems lies in the large differences between the geotechnical parameters of field and indoor test. The using of the tangent modulus method based on the field loading test to determine the calculation parameters can overcome this shortcoming, and reflect the non-linear of foundation settlement, which is a new progress of foundation settlement calculation. However, compared with other tests, the in situ plate loading test has higher difficulty, and it is also difficult to reflect the characteristics of deep soil. In this paper, by making use of systematic geotechnical test data by Riverside Campus, Texas A & M University, United States, it first examines the feasibility of the tangent modulus method used in sand foundation nonlinear settlement calculation, and then studies on the tangent modulus calculation methods of adopting lateral pressure test, The CTP test and other field tests to determine the different soil layers and lastly tests the results of loading tests for different plate sizes. The results show that the calculated parameters needed in the tangent modulus method determined by the lateral pressure test, The CTP test and other field tests are feasible. In addition, when the parameters thus determined are used in the tangent modulus method to calculate the load experimental curves of different plate sizes, the calculated nonlinear curve has good consistency with and experimental curves.

1. Introduction

Due to the complexity of geotechnical materials, the foundation settlement is difficult to be inaccurate, which can be specifically attributed to less accurate value of geotechnical parameters and the unreasonable calculation method and. The compression modulus usually used in engineering adopts the layer-wise summation method to calculate the foundation settlement, while the compression modulus is accessed from the confined compression test at side of the interior room. As the stress state of the majority of the foundation soil does not match with the confined compression, theoretically the compression modulus is larger than modulus of deformation and the settlement value calculated by the compression modulus should be small. However, according to the engineering practice on in situ plate loading tests of the residual soil and hard soil base in the Guangdong region, the obtained deformation modulus can be 6-10 times of indoor compression modulus, while the foundation settlement calculated by the indoor compression modulus will be much larger than the actual settlement. Due to the disturbance of undisturbed soil in sampling and making preparation process, the foundation settlement parameters accessed by the indoor experiment will be distorted. Therefore, the development of settlement deformation calculation method based on the in situ tests has important practical significance on the improvement of accuracy of the ground deformation calculation.

Based on plate loading test, Yang Guanghua\(^1\)\(^-\)\(^8\) proposes that the foundation nonlinear settlement...
analyzed tangent modulus method can better solve the problem of the accuracy of the foundation settlement. However, as parameters are derived from the plate loading test which has a high cost, the deep soil test is difficult. Therefore, to explore other simple test instead of in situ plate loading test to determine the required parameters is very important. This paper\cite{9} discusses how to use the lateral pressure test to determine the required parameters.

American scholar Briaud JL, Gibbens R M., et, al\cite{10} have carried out the plate loading test with five different sizes' plates, static cone penetration test, lateral pressure test and other systematic foundation soil mechanics tests on the silty sand soil in Riverside Campus, Texas A & M University, of which the purpose is to test the effectiveness of a variety of foundation settlement calculation methods, including the finite element method of the constitutive model. By making use of these valuable test data, this paper further tests the applicability of the foundation nonlinear settlement calculated tangent modulus method\cite{4} and studies how to use the static cone penetration test and lateral pressure test and other simple in situ tests to determine the parameters of the foundation parameters required in the tangent modulus method.

2. Calculation Principle of Tangent Modulus Method [4]

Assuming that the basis load settlement curve can be fitted by using the hyperbolic curve, and let:

\[
p = \frac{s}{a + bs}
\]

(1)

\(a\) and \(b\) are constants, with a clear physical meaning: when the load reaches the ultimate load \(p_u\), \(s \rightarrow \infty\), take the limit of the above formula:

\[
b = \frac{1}{p_u}
\]

(2)

Therefore, the \(b\) is a parameter related to the foundation strength. Based on the Boussinesq solution of homogeneous elastic half infinite space, the settlement of foundation is:

\[
s = \frac{pB(1-\nu^2)}{E} \omega
\]

(3)

In the above formula, \(B\) is the foundation width or diameter, \(\omega\) is the shape factor (0.79 for round and 0.88 for square), \(\nu\) is the Poisson ratio and \(E\) is Young modulus. Formula (1) can be expressed as \(s/p = a + bs\). When loading at start, \(s \rightarrow 0\), \(s/p = a\), at this time the elastic modulus is the initial tangent modulus, represented by \(E_{t0}\). Compared with Formula (3):

\[
a = \frac{B(1-\nu^2)}{E_{t0}} \omega
\]

(4)

It can be known that \(a\) is a parameter related to foundation medium deformation parameters. Considering that the p-s curve of foundation is nonlinear. In the foundation loading process, the deformation modulus gradually decreases, represented by \(E_t\), and then evaluates the tangent countdown to Formula (1) and (3):

\[
\frac{dp}{ds} = \frac{(1-bp)^2}{a}
\]

(5)

\[
\frac{dp}{ds} = \frac{E_t}{B(1-\nu^2)\omega} = \frac{E_t}{aE_{t0}}
\]

(6)

Compare the above two formulas, it can be obtained:

\[
E_t = (1 - \frac{p}{p_u})^2 \cdot E_{t0}
\]

(7)

Considering that strength indicator \(p_u\) measured by the test of may be smaller, plus a damage ratio
coefficient $R_f$ in the above formula, then:

$$E_i = (1 - R_f \frac{P}{P_u})^2 \cdot E_{i0}$$  \hspace{1cm} (8)

As $E_{i0}$ is a material constant, Formula (8) demonstrates that the tangent modulus of certain depth in the foundation can be determined by the ultimate bearing capacity of foundation soil $P_u$, the initial tangent modulus $E_{i0}$ and the additional stress $P$, which will decrease in non-linear with the increase of the additional stress. When considering layered foundation settlement calculation, the corresponding ultimate bearing capacity of soil $P_u$ will increase with the increase of the depth. Thus, $E_i$ will non-linearly increase with the increase of depth.

When the tangent modulus $E_i$ is calculated at different soil depths according to Formula (8), it can be used in the layer-wise summation method for settlement calculation. Assuming the added load of layered-soil $\Delta h_i$ under load $P$ is expressed by $\Delta P_i$, and the compression of the soil layer is:

$$\Delta s_i = \frac{\Delta P_i \cdot \alpha \cdot \Delta h_i}{E_i}$$  \hspace{1cm} (9)

$E_i$ is the tangent modulus on the soil layer corresponding to the load $P$. $\alpha$ is the additional stress distribution coefficient. At this time, $P_u$ is equivalent to the ultimate bearing capacity of the foundation deeply buried in the soil by assumption. After the compression of each layer is calculated, according to layer-wise summation method, the total settlement under added load is:

$$\Delta s = \sum_{j=1}^{n} \Delta s_j$$  \hspace{1cm} (10)

When using this method to calculate nonlinear settlement of foundation, the key is to determine three parameters, including the tangent modulus value $E_i$ of each foundation layer of soil, the strength index $c$ and $\phi$ of soil parameter required in Formula (7) or Formula (8) calculation as well as the initial tangent modulus $E_{i0}$, which are simple. The soil strength index $c$ and $\phi$ and basis size are used to calculate the ultimate bearing capacity $P_u$ of foundation at different depths. Considering the disturbance of sampling test on the undisturbed soil, and to improve the accuracy of the calculation, these three different soil parameters should be determined by in situ test.

3. Foundation settlement calculation parameters determined by loading test

As to the $p$-$s$ curve measured by the loading test, if the measured $(p, s)$ data can be fitted with aforementioned Formula (1) and the parameters $a, b$ can be obtained, then it can inversely calculate the internal friction angle $\phi$ of foundation settlement parameters, cohesion $c$ and the initial tangent modulus $E_{i0}$, resulting in a more reasonable calculation parameters.

Loading test site is located in the Riverside Campus, Texas A & M University, with silty sand foundations at 0 to 10.5m deep and black hard clay at the lower layer. The groundwater is at depth of 4.9m, and the main physical and mechanical parameters of the indoor tested silty sand are shown in Table 1.

A total of five plate loading tests are conducted in the site, and the plates are square at width of 1 ~ 3m. The test is carried out at depth of 0.76m. Fit the $p$-$s$ curves of 5 plates by Formula (1), and let $s/p = a + bs$. By using test data of five plates, it can linearly fit the $(s/p) \sim s$ data points, and calculate the hyperbolic parameters $a$ and $b$.

Table 1. physical mechanic index of ground soil
By using of parameters a and b obtained by fitting, it can calculate initial tangent modulus $E_t$ and bearing capacity limit $P_u$ of each plate. Now, we use two methods to forecast the p-s curve, e.g. the first method uses the aforementioned fitted hyperbolic parameters a and b to directly draw p-s curve based on the assumption\cite{8,9} that p-s curve complies with the hyperbolic model; the second method is the aforementioned tangent modulus method: first calculate the tangent modulus $E_t$ under different loads, then calculate the amount of compression of each layer, and lastly make summary of the foundation settlement under different load by layer-wise summation, and draw p-s curve\cite{4}.

The parameters of different test points inversely calculated by the tangent modulus method are shown in Table 2.

| Plate loading test Number | $E_t$ /MPa | $\varphi$ /° | $c$ /kPa | $P_u$ /kPa |
|---------------------------|------------|-------------|----------|------------|
| 5#(1.0m×1.0m)             | 83.4       | 37.2        | 0        | 1399       |
| 2#(1.5m×1.5m)             | 84.4       | 35.5        | 0        | 1202       |
| 4#(2.5m×2.5m)             | 84.7       | 34.8        | 0        | 1340       |
| 1#(3.0m×3.0m)             | 90.9       | 37          | 0        | 1405       |
| 3#(3.0m×3.0m)             | 86.4       | 35.6        | 0        | 1128       |

The average of soil parameters in Table 2 parameters is shown as follows: $E_t = 86$MPa, $\varphi = 36^\circ$, and the settlement load curve of each plate calculated by taking these soil parameters and using tangent modulus method is shown in Figure 1 to 5 below:
4. The initial tangent modulus of soil layer determined by lateral pressure test

It should be pointed out that the above calculated tangent modulus is the level indicator. If the sand foundation is recognized as isotropic media, then the above calculation results can be used as the vertical indicators in settlement calculation.

According to the typical lateral pressure test curve (PMT-2, the standard method of test execution literature[17]) on loading test sites in Riverside Campus, Texas A & M University, $p_i = 400kP_a$, $p_f = 280kP_a$, $p_0 = 20kP_a$. After substituting into the above formula, it can get:

$$E_{10} = \frac{(400 - 20)}{(400 - 280)} \cdot E_m = 3.2E_m$$

(11)

5. Foundation strength parameters determined by static cone penetration test

In accordance with the foregoing numerical analysis method, it selects different soil strength indicators $\phi$ to calculate the cone tip resistance and cone lateral resistance, and by using of the size data of the CPT probe, it can calculate the penetration resistance of CPT test. As to the comparison between $\phi \sim p_s$ data determined by the numerical analysis and the empirical relationship proposed by the railway specifications shown in Figure 19, $\phi$ determined by the numerical calculation is slightly larger than the value recommended by railway specifications. After fitting the $\phi \sim p_s$ data by power function, they have good correlation and the fitted empirical relationship is shown as follows:

$$\phi = 29.352 \times p_s^{0.0915}$$

(12)

Through the initial tangent modulus and shear strength indicator determined by situ lateral pressure test and static cone penetration test, it can calculate the parameters required by the layered foundation settlement by using the tangent modulus method.

Firstly by using the testing data of PMT-2 typical lateral pressure test on the test site, it can calculate the initial tangent modulus $E_{10}$ of each layer by Formula (11) as shown in Figure 6. Then by using the CPT testing data of at the location of each plate on the test site, it can calculate the shear strength indicator $\phi$ by Formula (12). The Figure 7 shows the distribution of $\phi$ calculated by CPT-1 data with the depth change.

By using of initial tangent modulus $E_{10}$ and shear strength indicator $\phi$ of each layer determined by above lateral pressure test and the static cone penetration test, it can re-calculate the settlement of each plate by tangent modulus method, and compare with the measured results and the calculated results of each plate by the tangent modulus method when average foundation inversely calculated value $E_{10}$=86MPa, $\phi$=36°in the loading test.
6. Conclusions
By analyzing the nonlinear settlement calculation results of five-plate load test by tangent modulus method on the sand foundation in Riverside Campus, Texas A & M University, this paper shows that the tangent modulus method can calculate the nonlinear settlement of foundation in a more effective approach. In the meantime, this paper also seeks more simple in situ tests such as lateral pressure test and the static cone penetration test to replace the field plate loading test to determine the foundation parameters required by the tangent modulus method calculation and nonlinear calculations of foundation settlement, which achieves a better effect. Therefore, this paper provides a more convenient way to promote the use of the tangent modulus method to calculate the nonlinear settlement of foundation.

The tangent modulus method is equally applicable to the nonlinear settlement analysis of sand foundation; when the foundation soil properties are unevenly distributed with the change of depth, it can adopt the in situ tests, such as static cone penetration test and lateral pressure test to determine the calculation parameters of each layer in the deep soil; the nonlinear settlement of calculated plate are in good consistency with experimental results.

Sand foundation strength parameter $\phi$ can use numerical method to simulate the static cone penetration test process, and establish the relationship with penetration resistance $ps$; according to the numerical simulation results in the static cone penetration test process, the sand foundation strength parameters $\phi$ and penetration resistance $ps$ are in a power-function relation.

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