Logarithmic rate dependence in deforming granular materials

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Rate-independence for stresses within a granular material is a basic tenet of many models for slow dense granular flows[1, 2, 3, 4, 5]. By contrast, logarithmic rate dependence of stresses is found in solid-on-solid friction[6, 7, 8, 9], in geological settings[10, 11, 12], and elsewhere[13, 14, 15]. In this work, we show that logarithmic rate-dependence occurs in granular materials for plastic (irreversible) deformations that occur during shearing but not for elastic (reversible) deformations, such as those that occur under moderate repetitive compression. Increasing the shearing rate, \( \Omega \), leads to an increase in the stress and the stress fluctuations that at least qualitatively resemble what occurs due to an increase in the density. Increases in \( \Omega \) also lead to qualitative changes in the distributions of stress build-up and relaxation events. If shearing is stopped at \( t = 0 \), stress relaxations occur with \( \sigma(t)/\sigma(t = 0) \approx A \log(t/t_0) \). This collective relaxation of the stress network over logarithmically long times provides a mechanism for rate-dependent strengthening.

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Slow granular flows, the subject of this letter, are typically described in the context of Mohr-Coulomb friction models[2] that resemble those used for describing friction between two solid bodies[17]. In the well-known solid friction scenario[15, 16], an object on a frictional surface will resist a force and remain at rest provided the magnitude of the tangential force is less than the product of a static friction coefficient and the normal force. This picture was translated into the granular context (for dense granular systems characterized by networks of force chains[18]) by Coulomb[19] and more recent authors[1, 2], where the normal and tangential forces are replaced by corresponding normal and shear stresses, and the surface of interaction is replaced by a plane within the material. For large enough tangential force (shear stress) relative to the normal force (normal stress) sliding friction (failure in a granular material) occurs. In these pictures, sliding friction (deformation following failure) is independent of the speed of sliding (the shear rate).

In reality, experiments in diverse contexts have shown that solid friction exhibits a logarithmic dependence on rate and that static frictional contacts strengthen logarithmically with age. These experiments span a vast range of lengths, and include studies at the atomic[14], lab[1, 2, 3, 4, 10, 11], and geological scale[7, 8]. Recent experiments by Ovarlez et al.[9] using granular materials sliding against the interior wall of a piston showed clear rate dependence that was associated with aging effects of individual solid-friction contacts and with the force network. Additionally, experiments by Nasuno et al.[13, 14] in which a solid surface was pushed across a granular bed showed a slow strengthening (or aging) with time of the (quasi-static) force. In the present experiments, which zoom in uniquely on the grains, we show that there is a logarithmic rate-dependence in slowly sheared granular materials that is associated with irreversible rearrangements of the grain contacts within the material itself. This is manifest as a strengthening with rate, and it is unique to granular systems and possibly other jammed systems[20], i.e., it does not depend per se on the logarithmic strengthening of frictional contacts.

The experiments described here were carried out with a 2D realisation of a granular system. The particles were made of a photoelastic material and were either relatively thin disks or flat particles with a pentagonal cross section. By using a photoelastic material, we could determine forces at the grain scale. More detailed descriptions of the experimental apparatus and methods[21] have been given elsewhere for related experiments[18, 22, 23], and here we provide only essential information for two qualitatively different experiments. For each experiment, the humidity varied by no more than \( \pm 5\% \). In the first of these two experiments, the photoelastic particles were sheared in an annular geometry (inset, Fig. 2) that was bounded on the inside by a rough wheel, the source of shearing, and on the outside by an equally rough static ring. The particles sat on a horizontal, flat powder-lubricated Perspex sheet, and were contained from above by a similar sheet. The whole was placed in a polariscope consisting of a pair of right and left circular polarizers, a light source and a video camera. Photoelastic images were obtained at rates up to 15 Hz, captured by a framegrabber, and processed on-the-fly to determine the forces on the particles within the field of view of the camera (~100 to ~200 particles). Specifically, we computed the force on each particle in each frame and summed over all particles in a frame. We stored this integrated force for each image to produce a time series over very long sampling times, between \( 2^{14} \) and \( 2^{16} \) points depending on the rate of shearing. To a reasonable approximation (~10\%), this quantity is proportional to the pressure, and for simplicity, we will refer to it as the stress.

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Key control parameters are the rate of shearing, \( \Omega = 2\pi/T \), where \( T \) is the period for one rotation of the shearing wheel, and the density of the system, which we give in terms of the area fraction, \( \gamma \), occupied by the particles. As shown previously [18, 22], there is a critical value of \( \gamma_c \), below which the shearing ceases, and we typically reference \( \gamma \) to this value. \( \gamma_c \) depends on properties such as the particle shape and possibly the geometry of the container. It is also useful to give the rotation rate, \( f = 1/T = \Omega/2\pi \). In these experiments, 0.03 mHz \( \leq f \leq 60 \) mHz.

In Fig. 1, we show several examples of stress time series for \( \Omega \)'s ranging over about three orders of magnitude in the shearing rate. The three images of this figure give an indication of the effect of shearing rate: the time series for the slowest rate is clearly of lower amplitude and more intermittent than that for the highest rate.

The rate dependence is seen clearly in the mean (time-averaged) stress vs. \( \Omega \), Fig. 2. These data are consistent with a logarithmic variation of the stress with \( \Omega \). Inset: sketch of Couette shear apparatus.

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\Omega_1 > \Omega_o.
\]

If a sample is sheared under steady state conditions and the shearing wheel is abruptly stopped, the stress relaxes over very long time scales. In Fig. 3 we show the relaxation of stress vs. time on a semi-log plot. Note that there are two time scales: a quick initial decay lasting \( \approx 20 \) s and a much longer process such that the stress network may still be relaxing after 20 hrs. Fig. 3 shows fits to the stress decrease with time, of the form \( \sigma(t = 0)/\sigma(t) \approx A\log(t/t_o) \). Although the relaxation is not completely uniform in time, a logarithmic functional form is not unreasonable for many cases (after an initial rapid relaxation). This was not always the case, and an interesting example of a major relaxation event is seen in one of these runs about 350 s after the stopping of shearing; thereafter, continued slow relaxation occurred. A key insight is that, while the micro-contacts between particles may be strengthening in time [8, 11, 13], the stress network as a whole is relaxing over very long time scales.

We gain additional insight into the effect of rate changes by determining the distribution of build-up events (monotonic increases in stress) and release events ("avalanches" – monotonic decreases in stress). Here, there are two aspects that are of interest: the size of the build ups and avalanches, \( \Delta \sigma \), and the angle, \( \theta \), through which the shearing wheel turns during the course of a build-up or an avalanche. In the top part of Fig. 4 we show an example of an "avalanche" event, and in the bottom, we show distributions for \( \Delta \sigma \) and \( \theta \) for avalanches. Note that for near-critical \( \gamma \)'s and for the slowest \( \Omega \)'s, the avalanche distributions are consistent with power laws. At higher \( \gamma \) and \( \Omega \) the distributions for both build-up (not shown due to space constraints) and avalanche events are roughly exponentials. Interestingly, both increases in \( \Omega \) and \( \gamma \) have a logarithmic strengthening with waiting time and/or humidity. If we fit these data to the form \( \bar{\sigma} = A\log(\Omega/\Omega_o) \), then the amplitude, \( A \), is sensitive to the density, but the characteristic frequency, \( \Omega_o \), depends only weakly on \( \gamma \), and corresponds to a typical time of \( 2\pi/\Omega_o \approx 30 \) days. However, there is no compelling reason to believe that this relation holds in the limit \( \Omega \to 0 \). A more realistic description may be \( \bar{\sigma} = A\log([\Omega + \Omega_1]/\Omega_o) \), where...
and in $\gamma$ tend to have the same qualitative effect on the statistics of avalanches and build-ups.

In order to further explore the origin of this rate dependence, we have carried out a second set of experiments, using the same photoelastic techniques, in which the particles were confined to a box with three fixed sides. The sample was then compressed and released repeatedly on the fourth side by means of an oscillating piston, as in the sketch of the inset of Fig. 5. The driving velocity of the piston was chosen to match the corresponding velocities of the shear wheel in the first experiment. The key difference between the two sets of experiments was that in the first set there was continuous plastic deformation while in the latter set there was none; i.e., the particles maintained their relative position. As shown in the body of the figure, which gives $\bar{\sigma}$ vs. $\bar{\Omega}$, there was no rate dependence of this process, within experimental error.

To conclude, we have demonstrated that sheared granular materials exhibit logarithmic rate dependence that is tied in an essential way to particle rearrangements (plastic deformation). The nature of this rate dependence is further illuminated by the slow relaxation that occurs when shearing is halted. The latter experiments in particular suggest that there are slow collective rearrangements of the particles that can occur over large time scales. This slow relaxation of stresses over long times is consistent with increases in with rate: force chains generated by shearing cannot completely relax on the time scales over which new chains are formed, an effect that is exacerbated by increasing $\Omega$. It seems likely that this effect is intimately related to the long time scales reported for compaction in granular materials[24]. For compaction, collective rearrangements become progressively more difficult over time because each new rearrangement requires the involvement of larger collections of particles. Similarly, in the present experiments, relaxation events involving collections of particles become less probable over time due to geometric effects and to the fact that the available elastic energy is gradually reduced with each rearrangement. An interesting question is whether similar properties are seen in other jammed systems[20] such as colloids or foams.
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