Abstract—We propose a novel scheme for surveillance of a dynamic ground convoy moving along a non-linear trajectory, by aerial agents that maintain a uniformly spaced formation on a time-varying elliptical orbit encompassing the convoy. Elliptical orbits are used as they are more economical than circular orbits for circumnavigating the group of targets in the moving convoy. The proposed scheme includes an algorithm for computing feasible elliptical orbits, a vector guidance law for agent motion along the desired orbit, and a cooperative strategy to control the speeds of the aerial agents in order to quickly achieve and maintain the desired formation. It achieves mission objectives while accounting for linear and angular speed constraints on the aerial agents. The scheme is validated through simulations and actual experiments with a convoy of ground robots and a team of quadrotors as the aerial agents, in a motion capture environment.

I. INTRODUCTION

We consider the problem of autonomously monitoring a convoy with a group of aerial agents. A convoy is a group of ground vehicles trailing one behind the other on a common path. The problem of tracking ground targets using aerial agents has been well studied for both single and multiple Unmanned Aerial Vehicles (UAVs). Most solutions proposed earlier focus on circumnavigating the targets using circular orbits. However, for circumnavigating a moving convoy, elliptical orbits are often more economical to traverse as compared to circular orbits. The convoy monitoring problem has generally been studied for straight line paths or for paths whose curvature is known a priori. In this work, we propose a cooperative strategy for multiple aerial agents to traverse a time varying elliptical orbit around a moving convoy while maintaining an equi-distributed formation, without any knowledge of the convoy path.

In [1] and [2], the authors proposed an algorithm to compute a time varying elliptical orbit encompassing the convoy and a vector-field guidance law to drive a single aerial agent, modeled using unicycle kinematics, to track this orbit. For a multi-UAV mission, the objective is to achieve and maintain a multi-agent formation on an elliptical orbit that equally divides the elliptical orbit in terms of the parametric length of the ellipse. The original contributions of this work are:

1. Improvements to the vector field guidance law for better tracking of the desired elliptical orbit.
2. An updated algorithm for computing feasible elliptical orbits to encompass the target convoy, while accounting for the motion constraints of the aerial agents.
3. A strategy to control the linear speeds of the aerial agents to achieve uniform spacing on the elliptical orbit.
4. A cooperation scheme for fast convergence to the formation having equal parametric spacing on the elliptical orbit.

Our work is applicable to a large class of UAVs such as autonomous fixed wing aircraft, helicopters and quadrotors. This is because we approximate the aerial agents as unicycle kinematic agents with constraints on minimum and maximum linear speed and maximum angular speed. We validate our results with MATLAB simulations for unicycle agents, and for a quadrotor model using a ROS-Gazebo based software in the loop (SITL) simulator. We have also implemented the proposed strategy with a team of quadrotors and a convoy of ground robots in a motion capture environment.

The paper is organized as follows: Section II reviews some related work. Section III describes the problem setting, assumptions and notation. Section IV discusses some preliminary concepts that are used for developing the proposed strategy. Section V discusses the vector field guidance law used to drive the aerial agents to the elliptical orbit. In Section VI, the linear speed profile of the aerial agents is designed to maintain the desired formation. Section VII discusses the scheme for calculating the time varying elliptical orbit around the convoy. In Section VIII, the main cooperation algorithm for attaining the desired aerial formation is discussed. Section IX validates the proposed strategy by simulations and actual experiments.

II. RELATED WORK

This section is a short overview of related literature, see [3] for an extended survey. Our proposed scheme focuses on monitoring a ground convoy by aerially circumnavigating it with UAVs using a vector field based guidance law to track elliptical orbits encompassing the ground convoy. In literature such strategies are generally used for tracking circular orbits for UAV path-planning [4] and atmospheric sensing [5] applications. They have also been developed in [6] for tracking circular orbits around stationary and moving targets while respecting curvature demands of the underlying vector field. For multiple UAVs monitoring a single target [7] or a closely moving group of targets [8] while flying on a circular orbit, the UAVs are treated as unicycle kinematic agents and the phase separation is maintained by controlling the linear speed of the UAVs. This is close in spirit to our linear speed control strategy for maintaining a formation on the time varying elliptical orbit.
The vector field guidance law in [7] is extended to track a racetrack like orbit in [9], where the UAV alternates between tracking a circular segment and a straight line. Frew [10] extends it to tracking an elliptical orbit whose parameters depend on estimation uncertainties of the target states. In both, the UAVs track a target moving in a straight line. In contrast, our strategy guides the aerial agents to track a flexible ellipse that adapts its shape and orientation to the convoy trajectory.

Another approach for aerial monitoring of ground targets is the cooperative cyclic pursuit, where multiple aerial agents that follow unicycle kinematics cooperatively converge to a circular orbit with an equispaced formation around a ground target, both stationary [11] and moving [12]. In [13], such a cooperative scheme is proposed for achieving a circular formation on orbits having varying radii around the target. Our scheme generalizes to ellipses and circular orbits of varying sizes and gives a more robust orbit tracking as a result of its vector field guidance law.

Other approaches to formation control include [14] and [15], where the agents switch between revolutions on a circular orbit around slow moving targets and fixed formation paths to move with faster moving targets, e.g., to track a point moving on a piece-wise linear path. In [16], linear and circular paths are considered for the target and the emphasis is on analyzing the error caused by wind. In [17], the target is assumed to follow a smooth trajectory specified a priori as the level curve of a known function. In [18], a backstepping-based controller is designed to follow a vector field to achieve a multi-UAV formation on a circular orbit around a target while respecting the input constraints of the UAVs. Most of these approaches are designed towards tracking circular orbits. For a convoy of ground targets, an elliptical orbit can be more economical in terms of path length, see [11, 2] for the single agent case. A consensus based strategy for elliptical formation around a ground target is proposed in [19]. Here the agent models considered are single and double integrator. Our strategy, for a unicycle type aerial agent with input constraints, is applicable for a large class of UAV platforms.

III. PROBLEM DESCRIPTION AND NOTATION

We assume a target ground convoy of $N_T$ vehicles trailing one behind the other on some unknown common path. Each vehicle moves with a maximum speed of $V_{T_{max}}$. We number the vehicles as $1, 2, \ldots, N_T$ along the direction of travel of the convoy with vehicle $N_T$ being the lead vehicle. Our objective is to design a cooperation strategy for a team of $N_A$ aerial agents for monitoring the moving convoy. The aerial agents are modeled using unicycle kinematics flying at a commanded altitude as follows:

$$\dot{x}_{Ai}(t) = V_{Ai} \cos(\psi_{Ai}(t)), \quad \dot{y}_{Ai}(t) = V_{Ai} \sin(\psi_{Ai}(t)), \quad \dot{\psi}_{Ai}(t) = \omega_{Ai}(t), \quad (1)$$

where for agent $i \in \{1, 2, \ldots, N_A\}$, $(x_{Ai}(t), y_{Ai}(t))$ are the position coordinates, $\psi_{Ai}(t)$ is the heading angle, $V_{Ai} \in [V_{A_{min}}, V_{A_{max}}]$ is the commanded linear speed, and $\omega_{Ai}(t)$ is the commanded angular velocity of the monitoring agent satisfying $|\omega_{Ai}(t)| \leq \omega_{max}$. We make the following assumptions:

1. The upper bound on convoy speed $V_{T_{max}}$ and the convoy vehicle positions $(x_{Ti}(t), y_{Ti}(t))$ for all $i \in \{1, \ldots, N_T\}$ are known to the aerial agents, e.g., through communication for a cooperating convoy or through on board sensors.

2. The maximum speed of the ground convoy and the linear speed bounds of the aerial agents are related as follows:

$$0 \leq V_{T_{max}} < V_{A_{min}} < V_{A_{max}} - 2V_{T_{max}}. \quad (2)$$

That is, the aerial agents are assumed to have a sufficient speed advantage over the ground convoy, which is reasonable if they have to circumnavigate the moving convoy.

3. The aerial agents have a sufficiently large communication range for effective cooperation.

4. The aerial agent altitudes are high enough to ignore terrain.

We denote the rotation matrix from the right handed global reference frame to a tilted frame with tilt angle $\theta(t)$ as

$$R_\theta(t) = \begin{bmatrix} \cos(\theta(t)) & \sin(\theta(t)) \\ -\sin(\theta(t)) & \cos(\theta(t)) \end{bmatrix}. \quad (3)$$

IV. PRELIMINARIES

We now discuss some useful properties of elliptical orbits, used later for developing the guidance law and the cooperation strategy between aerial agents for achieving the desired formation. The parametric equation of the ellipse is:

$$x(s) = a \cos(s), \quad y(s) = b \sin(s). \quad (4)$$

Here $s$ is a parameter taking values in $[0, 2\pi)$, $a$ is the semi-major axis length and $b$ is the semi-minor axis length, with $a \geq b > 0$. The equation of the ellipse then is $x(s)^2/a^2 + y(s)^2/b^2 = 1$.

The first and second time-derivatives of $\dot{x}(s) = -a \sin(s) \dot{s}$, $\ddot{x}(s) = -a \sin(s) \ddot{s} - a \cos(s) \dot{s}^2$, $\dddot{x}(s) = -b \sin(s) \dddot{s}^2 + b \cos(s) \dot{s}$, $\dddot{y}(s) = -b \sin(s) \dddot{s}^2 + b \cos(s) \dot{s}$.

If a point moves on the ellipse with a parametric speed $\dot{s} > 0$, then, the actual speed of the point is given by

$$V(s) = \sqrt{\dot{x}(s)^2 + \dot{y}(s)^2} = \sqrt{a^2 \sin^2(s) + b^2 \cos^2(s)} \dot{s}. \quad (5)$$

Similarly the curvature and radius of curvature of the ellipse at parameter value $s$ are given by

$$\kappa(s) = \frac{[\ddot{x}(s)\dot{y}(s) - \ddot{y}(s)\dot{x}(s)]}{(\dot{x}(s)^2 + \dot{y}(s)^2)^{3/2}} = \frac{ab}{(a^2 \sin^2(s) + b^2 \cos^2(s))^{3/2}}, \quad (6)$$

$$R(s) = \kappa^{-1}(s) = \frac{(a^2 \sin^2(s) + b^2 \cos^2(s))^{3/2}}{ab}. \quad (7)$$

Define $G(s) = \sqrt{a^2 \sin^2(s) + b^2 \cos^2(s)}$. Successively differentiating $G(s)$ with respect to $s$, we have

$$G^{(1)}(s) = \left( (a^2 - b^2)^2 \sin(2s) / 2\sqrt{a^2 \sin^2(s) + b^2 \cos^2(s)} \right), \quad (8)$$

$$G^{(2)}(s) = \left( (a^2 - b^2)^3 \cos(2s) / 2G(s) - (a^2 - b^2)^2 \sin^2(2s) / (aG(s))^3 \right). \quad (9)$$

By the first order necessary conditions for maxima or minima of $G$, setting $G^{(1)}(s) = 0$ gives the following cases:

**Case 1:** If $a = b$, then the ellipse is a circle with $G^{(1)}(s) = 0$, for all $s \in [0, 2\pi)$, i.e., $G(s) = K$ where $K$ is a constant.
Case 2: If $a > b$, then $\sin(2s) = 0$, so $s = (2n+1)\pi/2$, $n\pi$ for $n \in \{0, 1\}$, as $s \in [0, 2\pi)$. By the second order sufficiency condition for maxima or minima of $G$, from (9),
\[ G'(n\pi) = \frac{a^2 - b^2}{b} > 0, \quad G'(\frac{(2n+1)\pi}{2}) = \frac{b^2 - a^2}{a} < 0. \]
Thus $s = n\pi$ minimizes and $s = (2n+1)\pi/2$ maximizes $G(s)$.

![Figure 1](image_url) Geometric representation of $(x_{AE}, y_{AE})$, $(r_A, \theta_A)$ and $(s_A, \gamma_A)$. The $s$ values listed in Table I are shown in magenta. The desired orbit is plotted in solid black. The commanded aerial agent heading and local tangential heading shown in red and green respectively.

Consider a point moving with constant parametric speed $\dot{s} = s_u$ on the ellipse. From (5), (6) and (7), $V(s) = G(s)s_u$, $\kappa(s) = \frac{ab\sqrt{G(s)}}{G(s)}$ and $R(s) = G(s)^{3/2}$, respectively, which implies that $s = n\pi$ minimizes $V(s)$ and $R(s)$ and maximizes $\kappa(s)$, and vice versa for $(2n+1)\pi/2$. Table I summarizes the maximum and minimum values of $V(s)$, $R(s)$ and $\kappa(s)$.

| $s$          | $V(s)$  | $R(s)$  | $\kappa(s)$ |
|--------------|---------|---------|-------------|
| $n\pi$       | $V_{min} = bs_u$ | $R_{min} = b^2/a$ | $\kappa_{min} = a/s^2$ |
| $(2n+1)\pi/2$| $V_{max} = as_u$ | $R_{max} = a^2/b$ | $\kappa_{max} = b/a^2$ |

Parameter computation for the aerial agents

If the aerial agents start with equi-parametric separations on the ellipse, then they can maintain it by moving at a constant parametric speed $\dot{s} = s_u$. But it is not necessary that they start on the desired elliptical orbit. For an agent at position $(x_{AE}, y_{AE})$ expressed in the orbit-centered coordinate frame, we need to calculate the parameter value $s_A$ for the agent. Given a desired orbit centered at origin $O$ with axis constants $a$ and $b$, every point in space lies on an ellipse of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \gamma$ concentric with the desired orbit, where $\gamma < 1$ for points inside the desired orbit, $\gamma = 1$ for points on it, and $\gamma > 1$ for points outside, as shown in Fig. 1. Then the parametric equation for these concentric ellipses is given by
\[ x(\gamma, s) = a\sqrt{\gamma} \cos(s), \quad y(\gamma, s) = b\sqrt{\gamma} \sin(s). \] (10)

Thus the coordinates of the point $(x_{AE}, y_{AE})$ can be represented by (10) for some value of $s_A$ and $\gamma_A$. If the corresponding polar coordinates of $(x_{AE}, y_{AE})$ in the orbit-centric reference frame are $(r_A, \theta_A)$, then
\[ \theta_A = \arctan 2(y_{AE}, x_{AE}), \] (11)

and $(x_A, y_A) = (r_A \cos(\theta_A), r_A \sin(\theta_A))$. Equating these with $(x(\gamma_A, s_A), y(\gamma_A, s_A))$ in (10), we have $a\sqrt{\gamma_A} \cos(s_A) = r_A \cos(\theta_A)$ and $b\sqrt{\gamma_A} \sin(s_A) = r_A \sin(\theta_A)$. Hence $s_A$ is computed as
\[ s_A = d_c \arctan 2(a \sin(\theta_A), b \cos(\theta_A)), \] (12)

where $d_c$ is a direction selection input chosen as
\[ d_c = \begin{cases} -1, & \text{for clockwise motion on orbit} \\ 1, & \text{for counter-clockwise motion on orbit}. \end{cases} \] (13)

Hence given agent’s position $(x_{AE}, y_{AE})$ and desired orbit constants $a, b$, we can compute the parameter $s_A$ on the concentric ellipse given by (10), using (11) and (12). The value of $\gamma_A$ is given by
\[ \gamma_A = \frac{x_{AE}^2}{a^2} + \frac{y_{AE}^2}{b^2}. \] (14)

From (12), it can be shown that $s_A = d_c \theta_A$, when $\theta_A = n\pi$ and $(2n+1)\pi/2$ for $n \in \{0, 1\}$, as illustrated in Fig. 1.

V. VECTOR-FIELD GUIDANCE LAW

The vector field guidance law computes a desired heading command that guides the aerial agent towards the desired elliptical orbit described by (4). Our guidance law is an update of that in [2], with a modification for improved tracking performance. If $(x_{AE}, y_{AE})$ are the agent position coordinates in the coordinate frame centered and aligned with the desired elliptical orbit (see Fig. 1), then the commanded heading $\psi_D$ of the aerial agent has two components remove space: $\psi_T$ is the tangential direction to the concentric ellipse described by (10) at position $(x_{AE}, y_{AE})$ as in Fig. 1 and $\psi_O$ is an offset term responsible for generating the heading command $\psi_D$ to reach the desired orbit. These are computed as
\[ \psi_T = \arctan 2 \left( d_c b^2 x_{AE}, -d_c a^2 y_{AE} \right), \]
\[ \psi_O = d_c \arctan (k_\gamma \kappa(s_A)(\gamma_A - 1)), \]
\[ \psi_D = \psi_T + \psi_O. \] (15)

d$_c$ is chosen corresponding to the direction of motion along the elliptical orbit, according to (13). $k_\gamma > 0$ is a gain used as tuning parameter. $s_A, \gamma_A$ and $\kappa(s_A)$ are computed using (12), (14) and (6) respectively. As shown in Fig. 1 $\psi_O$ adds an appropriate offset to $\psi_T$, such that the resulting heading command $\psi_D$ drives the agent towards the desired elliptical orbit. The aerial agents track $\psi_D$ by commanding the angular velocity of the aerial agent as follows:
\[ \omega_A(t) = k_\psi (\psi_D(t) - \psi_{AE}), \] (16)

where $k_\psi$ is a control gain, $\psi_{AE} = \psi_A - \theta_E$ is the heading angle of the aerial agent expressed relative to the orbit-centric reference frame, and $\theta_E$ is the tilt angle of the ellipse, relative to the global reference frame. The angular velocity of the aerial agent given by (16) is constrained as $|\omega_A(t)| \leq \omega_{max}$. For proof of convergence of the guidance law to the desired elliptical orbit, see [2], as the same proof applies here.

In (15), we improve the guidance law in [2] by introducing the curvature term $\kappa(s_A)$ in the $\psi_O$ to increase the sensitivity
of the guidance law in the high curvature regions of the elliptical law in [2]. We demonstrate the improvement in the tracking performance of this updated guidance law simulating two unicycle agents moving with a constant speed $V_A = 0.4$ m/sec, $|\omega_A| \leq 1.5$ rad/sec, $k_v = 1$ and $d_c = 1$. Both track an elliptical orbit centered around the origin with $a = 2.5$ m, $b = 1$ m. From Table I the minimum curvature is $\kappa_{\text{min}} = 0.16$ m$^{-1}$. For a fair comparison, the agent 1 uses the guidance law in [2] with $k_1 = 2$, and the agent 2 uses the guidance law given by (15), with the gain $k_2 = 2/\kappa_{\text{min}}$. From the plots in Fig. 2, using $\gamma_A$ was commanded to fly with a constant linear speed, and the angular speed was controlled so as to guide the agent to the desired elliptical orbit. Switching to the multi-agent framework, the objective now is to obtain a parametrically equispaced formation of $N_A$ aerial agents on the ellipse. That is, the parametric separation between any pair of consecutive aerial agents on the ellipse is $\delta_e = 2\pi/N_A$. This is done by controlling the linear speeds of the aerial agents.

The objective of the linear speed control is to achieve and maintain an aerial agent formation that divides the parametric length of the ellipse $(2\pi)$ into $N_A$ equal parts. To do this the commanded linear speed for each agent comprises of three components:

1) Nominal speed profile $V_{Ei}$
2) Correction component $V_{Ci}$
3) Motion compensation component $\tilde{V}_T$.

Agents moving with equal constant parametric speed $\dot{s}_A$ on an ellipse maintain their initial parametric separation on the ellipse. Thus we first compute a nominal linear speed profile for the agent corresponding to the constant parametric speed $\dot{s}_A = s_v$ for motion on the stationary elliptical orbit described by (10). From Table I the maximum and minimum speeds on the ellipse for such a motion are $V_{\text{max}} = a\sqrt{\gamma_A}s_v$ and $V_{\text{min}} = b\sqrt{\gamma_A}s_v$. Suppose we want to constrain $V_{\text{min}}, V_{\text{max}}$ to a given interval $[V_{E\text{min}}, V_{E\text{max}}]$, then in order to symmetrically utilize the interval, we select $s_v$ such that the average speed $(V_{\text{min}} + V_{\text{max}})/2$ resulting from constant parametric rate $\dot{s} = s_v$ equals $(V_{E\text{min}} + V_{E\text{max}})/2$, leading to

$$s_v = \frac{V_{E\text{min}} + V_{E\text{max}}}{(a + b)\sqrt{\gamma_A}}. \quad (17)$$

To ensure that the motion due to this parametric rate does not violate the given constraint, we need $[V_{\text{min}}, V_{\text{max}}] \subseteq [V_{E\text{min}}, V_{E\text{max}}]$. This leads to:

$$\frac{V_{E\text{min}}}{b\sqrt{\gamma_A}} \leq s_v \quad \text{and} \quad s_v \leq \frac{V_{E\text{max}}}{a\sqrt{\gamma_A}} \Rightarrow \frac{V_{E\text{min}}}{V_{E\text{max}}} \leq \frac{b}{a}. \quad (18)$$

We select $a, b$ in Section VII such that (18) is satisfied. The nominal linear speed corresponding to the constant parametric rate $s_v$ for an aerial agent $i$ at position coordinates $(x_{AEi}, y_{AEi})$, moving on a stationary ellipse described by [10], is now computed using (5) as

$$V_{Ei}(s_{Ai}, \gamma_{Ai}) = \sqrt{\gamma_{Ai}(a^2 \sin^2(s_{Ai}) + b^2 \cos^2(s_{Ai}))}s_v, \quad (19)$$

where $s_{Ai}$ is calculated using (11) and (12), and $\gamma_{Ai}$ and $\dot{s}_{Ai}$ computed using (13) and (17) respectively. Then from (17) and assuming (18) is satisfied, $V_{Ei} \in [V_{E\text{min}}, V_{E\text{max}}]$. To drive the parametric spacing between neighboring agents to the desired separation of $2\pi/N_A$ on the ellipse, we add a control component $V_{Ci}$ to the nominal speed $V_{Ei}$ of the agents. $V_{Ci}$ is computed based on cooperation with a neighboring agent as explained in the Section VIII. The resulting linear speed magnitude command then is

$$\dot{V}_{Ai} = V_{Ei}(s_{Ai}, \gamma_{Ai}) + V_{Ci}. \quad (20)$$

For monitoring a moving convoy, the elliptical orbit is not stationary. We have designed the speed profile $\dot{V}_{Ai}$ in the orbit centric frame, and to compensate for orbit motion we add the
the orbit center velocity to \( \vec{V}_i \), to compute the linear speed command relative to the global reference frame. We assume that the center of the ellipse moves with a velocity vector \((\vec{V}_{T\bar{x}}, \vec{V}_{T\bar{y}})\), with a magnitude bounded above by \( \vec{V}_{T\text{max}} \). In Section VII we propose orbit computation strategy to guarantee this. Since the aerial agents follow unicycle kinematics, their linear velocity components along the axes of the global reference frame cannot be commanded separately due to the non-holonomic constraint on the unicycle model. Thus an approximate compensation is used where the magnitude of the resultant vector is commanded as the linear speed of the agent \( i \) given by

\[
V_{Ai} = \sqrt{(\vec{V}_{Ai}\cos(\psi_{Ai}) + \vec{V}_{T\bar{x}})^2 + (\vec{V}_{Ai}\sin(\psi_{Ai}) + \vec{V}_{T\bar{y}})^2}.
\]

(21)

Since the motion compensation component can at most increase or decrease the agent speed by \( \vec{V}_{T\text{max}} \), the range of speeds available to design \( V_{Ai} \) is \([V_{R_{\text{min}}}, V_{R_{\text{max}}}]\), where \( V_{R_{\text{max}}} = V_{A_{\text{max}}} - \vec{V}_{T\text{max}} \) and \( V_{R_{\text{min}}} = V_{A_{\text{min}}} + \vec{V}_{T\text{max}} \). This interval is not empty due to assumption (2).

In order to allow for sufficient band of speeds above and below the nominal linear speed profile given by (19) for the interval is not empty due to assumption (2).

such that:

\[
\begin{align*}
(V_{E_{\text{min}}} + V_{E_{\text{max}}})/2 &= (V_{A_{\text{max}}} + V_{A_{\text{min}}})/2, \\
V_{E_{\text{max}}} - V_{E_{\text{min}}} &= \delta(V_{R_{\text{max}}} - V_{R_{\text{min}}}).
\end{align*}
\]

Solving this system of linear equations yields

\[
\begin{align*}
V_{E_{\text{min}}} &= (1 - \delta)V_{A_{\text{max}}} + (1 + \delta)V_{A_{\text{min}}}/2, \\
V_{E_{\text{max}}} &= (1 + \delta)V_{R_{\text{max}}} + (1 - \delta)V_{R_{\text{min}}}/2.
\end{align*}
\]

(22)

(23)

The intervals \([V_{E_{\text{max}}}, V_{R_{\text{max}}}]\) and \([V_{R_{\text{min}}}, V_{E_{\text{min}}}]\) allow room for the action of \( V_{E_{\text{min}}} \) and \( V_{E_{\text{max}}} \) on the nominal speed profile (19) such that:

\[
\begin{align*}
&\text{if } V_{E_{\text{min}}} > V_{E_{\text{max}}} + \delta V_{R_{\text{max}}} + \delta V_{R_{\text{min}}}/2 & \text{then } l_1(k) = 2p_{\text{max}}, l_2(k) = 0, \text{Algorithm 1 takes current convex position values as inputs and computes the values of } l_1(k), l_2(k). \text{ To select the ellipse that contains this bounding box, we consider the minimum area ellipse circumscribing it according to the following Lemma.}
\end{align*}
\]

Lemma 1. (2) The area of an ellipse \( E : x^2/a^2 + y^2/b^2 = 1 \) circumscribing a rectangle of dimensions \( l_1 \times l_2 \) with \( l_1 > l_2 > 0 \) is minimized by \( a = l_1/\sqrt{2} \) and \( b = l_2/\sqrt{2} \). We now have the following constraints on \( a \) and \( b \):

1. To ensure that the elliptical orbit is feasible, its minimum radius of curvature given in Table I should exceed the worst case minimum turn radius of the aerial agents, i.e.,
\[
R_{\text{min}} > V_{A_{\text{max}}} \omega_{\text{max}}^2 b \Rightarrow b > \sqrt{a/V_{A_{\text{max}}} \omega_{\text{max}}^2}.
\]

2. Since \( a \geq b \), \( a \geq \sqrt{a/V_{A_{\text{max}}} \omega_{\text{max}}^2} \Rightarrow a \geq V_{A_{\text{max}}} \omega_{\text{max}}^2 \).

3. The constraint on \( a \) and \( b \) given by (18).

Imposing these constraints along with Lemma I we select the

![Figure 4: Bounding box for a target convoy with \( N_T = 4 \)](image-url)
major and minor axes of the elliptical orbit as:

\[
a(k) = \max \left( \frac{l_1(k)}{\sqrt{2}}, \frac{l_2(k)}{\sqrt{2}}, \frac{V_{a_{max}}}{\omega_{max}} \right),
\]

\[
b(k) = \max \left( \frac{l_1(k)}{\sqrt{2}}, \frac{a(k)V_{E_{min}}}{V_{E_{max}}}, \frac{a(k)V_{a_{max}}}{\omega_{max}} \right).
\]

The target centroid velocity is computed by filtering the target centroid position using an exponential smoothing filter, and then computing the discrete derivative \(V_T = \frac{\Delta x}{\Delta t}, \frac{\Delta y}{\Delta t}\). This is then used as compensation in (21).

VIII. COOPERATION FOR EQUI-PARAMETRIC FORMATION

In cooperative approaches for target detection such as cyclic pursuit, there is a possibility that the final multi-agent formation attained at equilibrium has more than one agent at the same position on the orbit around the target (equi-parametric formations having separations \(\Delta_s\) which are integer multiples of \(\frac{2\pi}{N_A}\)). Such formations are generally initial condition dependent (as shown in Fig. 9 and 12 in [20]) or can be caused by saturation of inputs. Our cooperation scheme avoids such scenarios by first driving the agents to the desired equi-parametric equilibrium formation and then maintains this formation for the remaining mission. Also, the neighbors on the ellipse are not predefined and get assigned based on the order in which the agents reach the ellipse. This allows the agents to settle to the desired equi-parametric formation faster.

Each aerial agent indexed by \(i \in \{1, \ldots, N_A\}\) knows its own pose, i.e., \((x_{Ai}, y_{Ai}, \psi_{Ai})\), using a localisation source (GPS, motion capture, etc.), and computes its parameters \(s_{Ai}\) and \(\gamma_{Ai}\), using (11), (12) and (14). In order to achieve an equi-parametric formation, each aerial agent repeatedly broadcasts a communication packet (using the BROADCAST_PACKET function in Algorithm 2) containing the following:

1) Its identification index \(i \in \{1, \ldots, N_A\}\).
2) Agent parameter \(s_{Ai}\).
3) Ready flag \(f_{RI}\).
4) Formation height flag \(f_{HI}\).
5) Orbit flag \(f_{OI}\).

For each such packet received, the RECEIVE_PACKET function in Algorithm 2 updates the array entries in \(s_{A_{arr}}, f_{R_{arr}}, f_{H_{arr}}\) and \(f_{O_{arr}}\) at the index locations corresponding to the agent index \(i\) received in the packet.

Before attaining the desired formation, in order to ensure collision free motion, all aerial agents are initialized to fly at different altitudes to ensure sufficient vertical separation. The guidance law of Section VII drives all aerial agents to reach and latch onto the desired orbit, thus for this discussion we assume \(\gamma_{Ai} \approx 1\) for all agents once they reach the desired orbit. The agents start cooperation only after reaching the desired orbit. This is indicated by setting the flag \(f_{O}\), when error \(\gamma_{Th} = 1\) is within some threshold \(\gamma_{Th}\) (say, \(\gamma_{Th} = 0.1\)). Also, each agent \(i\) identifies the next agent \(i_N\) on the desired elliptical orbit along the direction of motion on ellipse, based on the parameter values received in \(s_{A_{arr}}\). This is done in algorithm 2 by the FIND_NEIGHBOUR function. If two agents \(i\) and \(j\) with \(j > i\) have the same parameter value, the tie is broken by assigning the \(i_N = j\) for agent \(i\).

To ensure that the agents first attain the equispaced formation, agent 1 acts as a reference leader and moves with a linear speed computed using (20) and (21) with \(V_{CI} = 0\). Thus from (20), in the orbit centered reference frame, agent 1 moves with linear speed \(V_{A1} = \dot{V}_{E1}(s_{A1}) \in [V_{Emin}, V_{Emax}]\) corresponding to \(s_{A1} = s_{C}\). The remaining agents compute the parametric separation error \(\Delta x\) (error between the actual separation of agent \(i\) from \(i_N\), and the desired separation \(\Delta_s = 2\pi/N_A\)) and the resulting correction component \(V_{CI}\) as

\[
V_{CI} = \sqrt{\gamma_{AI}(a^2 \sin^2(s_{Ai}) + b^2 \cos^2(s_{Ai}))} k_s D_{si}.
\]

The function NORMALIZE in Algorithm 2 is used to normalize the parameter differences in the interval \([-\pi, \pi]\). Suppose \(i_N = 1\) for an agent \(i'\), and the separation \(s_{A1} - s_{A_{i'}} > \Delta_s\), then the commanded \(V_{CV'} > 0\), which implies that \(V_{Av'} = \dot{V}_{E1}(s_{A_{i'}}) + V_{CV'}\) corresponds to \(s_{A_{i'}} > s_{C}\) and vice-versa. Because of the choice of \(\delta < 1\) in Section VI, \(V_{Av'}\) can take values less than \(V_{Emin}\) or above \(V_{Emax}\). This guarantees that agent \(i'\) achieves and maintains desired separation relative to agent 1. Upon achieving the desired separation \(V_{CV'} \approx 0\), and a similar argument follows for the agent with \(i_N = i'\) and so on. Thus convergence of all agents to the desired separation relative to agent 1, is guaranteed.

Algorithm 2 AERIAL_AGENT_COOPERATION

Inputs: \(i, s_{Ai}, \gamma_{Ai}, D_{Th}\)

Functions: FIND_NEIGHBOUR, BROADCAST_PACKET, RECEIVE_PACKET, NORMALIZE

Initialization: \(\Delta_s \approx \frac{2\pi}{N_A}\), \(f_{RI} \leftarrow False, f_{HI} \leftarrow False, f_{O1} \leftarrow False\) for all \(i \in \{1, \ldots, N_A\}\)
1: \(s_{A_{arr}}, f_{R_{arr}}, f_{H_{arr}}, f_{O_{arr}} \leftarrow \) RECEIVE_PACKET()
2: \(i_N \leftarrow \) FIND_NEIGHBOUR\((s_{A_{arr}})\)
3: \(s_{A_{arr}}, f_{R_{arr}}, f_{H_{arr}}, f_{O_{arr}} \leftarrow \) RECEIVE_PACKET()
4: \(D_{s} \leftarrow NORMALIZE(NORMALIZE(s_{A_{arr}}/s_{Ai}) - \Delta_s)\)
5: \(V_O \leftarrow 0\)
6: if \(|\gamma_{Ai} - 1| < \gamma_{Th}\) then
7: \(f_{O1} \leftarrow True\)
8: if \(f_{O1} = True\) and \(f_{O_{arr}} = True\) then
9: if \(i_N = 1\) and \(|D_s| < D_{Th}\) then
10: \(f_{RI} \leftarrow True\)
11: else if \(f_{RI} = True\) then \(|D_s| < D_{Th}\) then
12: \(f_{HI} \leftarrow True\)
13: if \(f_{HI} = True\) then
14: \(f_{HI} \leftarrow True\)
15: if \(i = 1\) then
16: \(V_{CI} \leftarrow 0\)
17: if \(f_{RI} = True\) then
18: \(V_{CI} \leftarrow \sqrt{\gamma_{AI}(a^2 \sin^2(s_{Ai}) + b^2 \cos^2(s_{Ai}))} k_s D_{s}\)
19: \(f_{HI} \leftarrow True\)
20: else
21: \(V_{CI} \leftarrow \sqrt{\gamma_{AI} \sin^2(s_{Ai}) + b^2 \cos^2(s_{Ai})} k_s D_{s}\)

When the desired formation is reached, \(V_{CI} \approx 0\) and \(\dot{V}_{A1} \approx \dot{V}_{E1}(s_{A1})\) for all agents, corresponding \(s_{Ai} \approx s_{C}\). Thus the inter-agent parametric separations are maintained. Each agent raises its ready flag \(f_{RI}\) when \(D_s\) is small (within a threshold of, say, \(D_{Th} = 0.1\)), starting from the agent \(i'\). Since the ellipse is a closed curve, once the remaining agents achieve
the desired relative separations with respect to agent 1, the parametric separation between agent 1 and its neighbor \( i_N \) is approximately \( \Delta_s = \frac{2\pi}{N_A} \). Thus in algorithm 2 when all the remaining agents have reached their formation positions relative to agent 1 and have set their ready flag \( f_{Ri} \), agent 1 also sets its \( f_{Hi} \) flag and applies the control command given by (27). The \( f_{Hi} \) flag is set throughout the formation to indicate that all agents should attain the same mission altitude for monitoring the convoy. As a result there is no longer a leader in the formation and all agents maintain the formation about the stable equilibrium of \( \tilde{V}_{Ai} = V_{Ei}(s_{Ai}) \) using control \( V_{Ci} \).

Algorithm 2 gives the pseudo-code to implement the proposed cooperation strategy when a communication packet is received.

IX. SIMULATIONS AND EXPERIMENTS

We validated the proposed multi-agent convoy monitoring strategy in simulation and with real hardware experiments. We modelled the agents with unicycle kinematics and considered for different convoy paths with smooth turns of different curvatures, and non-smooth paths with sharp turns in the MATLAB simulations. Here we do not consider the z-axis separation as it is a simulation to validate that the cooperative scheme achieves the desired formation. Since most aerial platforms can be approximated as unicycle agents, our strategy can be used with fixed-winged UAVs, helicopters and quadcopters. To demonstrate this, the proposed strategy has been simulated with a ROS-Gazebo based Software-In-The-Loop (SITL) simulator, where the aerial agents follow the 6-DOF dynamical model of a quadrotor. The presented strategy has also been implemented using the Bitcraze Crazyflie 2.0 quadrotors (equipped with the flowdeck sensor) in a motion capture arena for two different cases. Table II summarises the cases considered for the simulations and the experiments.

For the SITL simulation, the ground convoy follows waypoints on a road network like path with sharp turns to simulate motion in an urban environment. The SITL setup is the same as in [21], wherein the 3DR Iris drone with the Pixhawk autopilot board is simulated. For the ground convoy we use a generic differential drive robot model. For both the SITL simulation and the experiments, the quadrotors have internal PID control loops which track linear velocity commands along the global coordinate frame axes, i.e., \( (V_x, V_y, V_z) \) and the angular velocity command \( \omega_z \) about the vertical \( z \) axes. We approximate the quadrotor as a unicycle agent, by mapping the linear speed \( V_A \) and angular velocity \( \omega_A \) given by (21) and (16) as follows:

\[
V_x = V_A \cos(\psi_A), \quad V_y = V_A \sin(\psi_A), \\
V_z = k_z(z_{cmd} - z_A), \quad \omega_z = \omega_A,
\]

where \( z_{cmd} \) is the commanded height of the quadrotor and \( z_A \) is the height of the quadrotor in the global reference frame. The PID loops in the flight controller firmware, for SITL simulation and the Crazyflie quadrotor, track the commanded linear speed and angular velocity satisfactorily as shown in Fig 5.

For the experiments we used small differential drive robots (designed in-house, refer Remark 1) as the convoy. The positions of the convoy robots and the quadrotors are available from the motion capture system comprising of eight Vicon MX.
The authors thank Sri Theja Vuppala and John M. Hart for their assistance with logistics for the experiments, which were implemented at the Intelligent Robotics Laboratory, University of Illinois at Urbana Champaign. The work in this paper was funded in part by AFOSR#FA9550-15-1-0146.
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