2.5D relativistic electromagnetic PIC code for simulation of the beam interaction with plasma in axial-symmetric geometry

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Abstract. 2.5D relativistic electromagnetic PIC code for simulation of the beam interaction with plasma in axial-symmetric geometry was developed. Accurate charge weighting scheme and difference schemes near the system axis were introduced. Simulation tests of electromagnetic wave interaction with inhomogeneous plasma were carried out.

1. Introduction
A lot of problems in plasma electronics (the nonlinear stage of beam-plasma interaction, dynamics of electron bunches in wake fields excited by them in plasma, the virtual cathode formation during the injection of strong electron beams, etc.) can be solved only by means of computer simulation.

The method of particles in cells is used intensively for numerical simulation of plasma [1]. This method is used for various phenomena in space, ionosphere, various discharges, plasma display cells etc.

Electrostatic PIC codes were used widely for the simulation of above mentioned phenomena [2-3]. But using such codes we cannot observe effects of electromagnetic waves propagation or radiation in plasmas. Equations of the particles’ motion are not relativistic, but it is necessary to solve the relativistic equations of motion in many cases.

Relativistic PIC codes are also used for plasma simulation (see, e.g., [4]). But usually every type of code is convenient only for some specific problems.

The aim of this work is to present the relativistic electromagnetic code for axial-symmetric beam-plasma systems simulation with a longitudinal magnetic field using the method of particles in cells.

2. Description of the difference scheme
If you don’t wish to use the Word template provided, please set the margins of your Word document Two-dimensional cell in cylindrical coordinates (r-z) for the presented axial-symmetric code is shown on Fig. 1. Points are marked where each component of field, potential, charge and current are calculated. Large particle have a shape of the ring which can move along z axis, vary its radius moving in radial direction and rotate azimuthally. It is possible to simulate propagation of electromagnetic waves of E type in two dimensional systems without z component of magnetic field H accordingly.
But in some cases the presence of this component is important (e.g., dynamics of an electron bunch injected along the magnetic field).

In the proposed code the space grid remains two-dimensional, but large particles have three components of velocity which results in three components of electric and magnetic field. Therefore electromagnetic waves of $H$ type arise in the system.

For solution of Maxwell equations’ set the method of finite differences in time domain (FDTD) was used [5-6]. This method is based on Yee algorithm, which allows finding both electrical and magnetic field in time and space using first pair of the Maxwell equations (the law of Ampere’s circuital law with Maxwell’s correction and the electromagnetic induction law).

3D elementary cell in cylindrical geometry is represented on Fig. 2. Each point where electric field component is calculated is surrounded in a plane perpendicular to its direction by four points, where magnetic field components are calculated. Oppositely, each point where the magnetic field component is calculated is surrounded by four points where electric field components are calculated.

Thus difference schemes for $E$ and $H$ components of the field take a form:

\[
E_{r, i+1/2, k} = E_{r, i, k} + \frac{1}{\Delta z} \left[ H_{\varphi, i+1/2, k+1/2} + H_{\varphi, i+1/2, k-1/2} - \frac{\Delta t}{\varepsilon_0 \varepsilon_{r, i, k}} \left( E_{r, i+1/2, k+1/2} - E_{r, i+1/2, k-1/2} \right) \right],
\]

\[
E_{\varphi, i, k+1/2} = E_{\varphi, i, k} - \frac{1}{\Delta z} \left[ H_{r, i+1/2, k+1/2} - H_{r, i+1/2, k-1/2} \right] + \frac{1}{\Delta r} \left( H_{z, i+1/2, k+1/2} - H_{z, i+1/2, k-1/2} \right) \frac{\Delta t}{\varepsilon_0 \varepsilon_{r, i, k}},
\]

\[
E_z = E_{z, i, k+1/2} - E_{z, i+1/2, k}.
\]

\[
H_{r, i+1/2, k+1/2} = H_{r, i, k+1/2} - \frac{1}{2} \frac{\Delta t}{\Delta z \mu_0} \left( E_{\varphi, i, k+1/2} - E_{\varphi, i, k-1/2} \right),
\]

\[
H_{z, i, k+1/2} = H_{z, i, k+1/2} + \frac{1}{2} \frac{\Delta t}{\Delta z \mu_0} \left( E_{\varphi, i, k+1/2} + E_{\varphi, i, k-1/2} \right).
\]
3. Field calculation near the axis

Solution of equations (1)-(2) has some specific features in the cylindrical coordinate system connected with the calculation of field’s components near the system axis. One can see from Fig. 2 that electromagnetic field components \( E_\phi, E_z \) and \( H_r \) are found on the axis, and \( E_r, H_\phi \) and \( H_z \) components are found at the distance \( \Delta r/2 \) from the axis.

On the axis \( H_r=0 \) and \( H_z=0 \), equation (5) can be rewritten in the form:

\[
\frac{
\left[ j_\phi \left|_{i-1/2,k+1/2}^{n+1/2} \right] - \frac{2}{\Delta r} \left[ H_z \left|^{n+1/2}_{i-1/2,k-1/2} \right| - \left[ H_z \left|^{n+1/2}_{i-1/2,k+1/2} \right| \right] \right] \Delta t}{\varepsilon_0 \varepsilon_{0,k}}.
\]  

(6)

Figure 3. Calculation of electric field components \( E_z \) (a) and \( E_\phi \) (b) near the axis of the system.
4. Weighting procedure in cylindrical coordinates

Procedure of current density weighting also has the specific features in cylindrical geometry. The current density in each node

\[ j = \frac{\Delta q}{\Delta t \Delta \xi} \]  

should be defined so that the continuity equation is fulfilled.

Weighting of the first order was used in the program, where large particles’ cross-section has the square shape in the plane \((r, z)\). Therefore, they can distribute the charge to four cells simultaneously. In order to find the current density caused by each large particle it is necessary to trace how the particle passes through each edge of the elementary cell (Fig. 4) during each time step of simulation [7].

Maximum distance that particle can pass during one time step should not exceed the cell size (so called Courant condition):

\[ v_{\text{max}} \Delta t \leq \Delta \xi \]  

Let's consider the possible variants of particle current distribution to the neighboring nodes. To reduce the quantity of the possible variants we use Courant condition in more strict form in comparison to (8): limit maximum distance the particle passes per time step to \(\Delta \xi / 2\).

During its move per time step the particle can give the contribution to 4 sides (Fig. 4 a), to 7 sides (Fig. 4 b), or to 10 sides (Fig. 4 c). It is also necessary to consider that large particle charge density is a function of its radius, in contrast to be constant in the rectangular geometry.
5. Test simulations of electromagnetic wave interacting with inhomogeneous plasma
A simulation volume has a form of cylindrical resonator with radius of 0.4 m and length of 12.8 m. A cylindrical waveguide mode E01 with frequency $f = 400$ MHz is excited by the rod with alternating current. It is located on the axis of the resonator and has a length of 0.37 m. Electromagnetic absorbing layers is imposed on the both sides of resonator.

The system is partly filled with inhomogeneous plasma with linear density profile. The plasma consists of electrons and ions of hydrogen with near zero temperature.

The spatial distribution of $z$ component of electric field for the case of wave interaction with subcritical inhomogeneous plasma is showed on Fig.6. The plasma density changes from $n_p(z = 4.5 \text{ m}) = 0.2 \cdot 10^{15} \text{ m}^{-3}$ to $n_p(z = 12.8 \text{ m}) = 0.6 \cdot 10^{15} \text{ m}^{-3}$. One can see that in this case the wave remains periodical but changes its wavelength while propagating through the plasma. This happens because subcritical plasma has lower dielectric permeability than vacuum that leads to the decrease of phase velocity of the wave.

The spatial distribution of $z$ component of electric field for the case of interaction with subcritical inhomogeneous plasma with the reflection point on the density profile is showed on Fig.5. The plasma density changes from $n_p(z = 4.5 \text{ m}) = 0.6 \cdot 10^{15} \text{ m}^{-3}$ to $n_p(z = 12.8 \text{ m}) = 1.5 \cdot 10^{15} \text{ m}^{-3}$. One can see that now the electromagnetic wave reflects from the inhomogeneous plasma. There are also low intensity oscillations of electric field beyond the reflection point which could be explained by generation of second harmonic.

Figure 5. Spatial distribution of $z$ component of electric field for the case of electromagnetic wave interaction with subcritical plasma.

Figure 6. Spatial distribution of $z$ component of electric field for the case of reflection of electromagnetic wave from the inhomogeneous plasma.

Figure 7. Spatial distribution of intensity of electromagnetic field for the case of reflection of electromagnetic wave from the inhomogeneous plasma.

The distribution of the intensity of the electric field is showed on the Fig.7. One can see that the standing wave is formed in plasma by the incident and the reflected wave. The reflection point is defined by the transversal wave number $k_\perp$. It’s well known from the theory [8] that in the case of non-zero angle $\theta$ between
wave number and the density gradient the reflection density is

\[ n_{\text{ref}} = n_{cr} \sin^2 \theta = n_{cr} \frac{k^2 - 2\pi \nu_{01}/R}{k^2}, \]  

(9)

where \( n_{cr} = 1.9 \cdot 10^{15} \text{ m}^{-3} \) is critical density, \( R \) – radius of the resonator, \( \nu_{01} = 2.4 \)– root of the Bessel’s function. For parameters of simulation \( n_{\text{ref}} = 0.97 \cdot 10^{15} \text{ m}^{-3} \). This value is close to the observed value of \( n_{\text{ref}, \text{exp}} = 0.8 \cdot 10^{15} \text{ m}^{-3} \).

6. Conclusions
The 2.5D electromagnetic relativistic PIC code for simulation of beam- and wave-plasma interaction in cylindrical geometry is presented in the paper. The accurate charge weighting scheme is introduced which allows the charge conservation law to be automatically fulfilled. The correctness of the program is confirmed by the comparison of simulation results with theoretical predictions.

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