CP-safe gravity mediation and muon $g - 2$

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We propose a CP-safe minimal supersymmetric (SUSY) standard model in gravity mediation, where the phases of the Higgs $B$ parameter, scalar trilinear couplings, and gaugino mass parameters are all aligned. Since all dangerous CP-violating phases are suppressed, we are now safe to consider low-energy SUSY scenarios under the assumption that the SUSY flavor-changing neutral current problem is solved. As an application, we consider a gravity mediation model explaining the observed muon $g - 2$ anomaly. The CP-safe property originates in two simple assumptions: SUSY breaking in the Kähler potential and the shift symmetry of a SUSY-breaking field $Z$. As a result of the shift symmetry, the imaginary part of $Z$ behaves as a QCD (quantum chromodynamics) axion, leading to an intriguing possibility: the strong CP problem in QCD and the SUSY CP problem are solved simultaneously.

1. Introduction

Large CP violation is a generic problem in the supersymmetric (SUSY) standard model (SSM). In particular, this problem becomes extremely serious when sleptons, bino, and wino are as light as $\mathcal{O}(100)$ GeV. In such cases, SUSY contributions to the electric dipole moment (EDM) of the electron usually exceed the experimental bound [1] by many orders of magnitude, due to CP violation in the SUSY-breaking sector [2–6]. On the other hand, the observed muon $g - 2$ anomaly [7,8] suggests such light sleptons, bino, and wino; the experimental value of the muon $g - 2$ can be explained by the contributions from the bino, wino, and sleptons of masses $\mathcal{O}(100)$ GeV [9–11]. Moreover, models with such light SUSY particles are fascinating possibilities for the ILC. Therefore, it is important to construct a mechanism to suppress CP-violating phases of the SSM. The SSM has many sources of CP-violating phases: soft SUSY-breaking masses of sfermions, gaugino mass parameters, scalar trilinear couplings, and the Higgs $B$-term. The first source, linked to the flavor structure, can be eliminated by assuming that the soft SUSY-breaking masses are universal or zero at the high-energy scale. Thus, we restrict our discussion to the other sources.

In this paper, we propose a CP-safe framework of gravity mediation in the SSM, where all of the relevant phases are aligned. Consequently, the dangerous CP-violating phases are sufficiently suppressed, and hence low-energy SUSY scenarios become attractive. Encouraged by this theoretical

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1 In this paper, we concentrate on frameworks that are free from the SUSY flavor-changing neutral current problem. In these frameworks, it is expected that the dangerous flavor-violating masses for sfermions are suppressed.
proposal, we construct a model consistent with the observed anomaly of the muon $g - 2$. We also show how to test this model at the LHC and linear colliders.

2. CP problem in the SSM

CP violation in soft SUSY-breaking terms is a serious obstacle for low-energy SUSY. Apart from CP violation in mass matrices of sfermions, there are dangerous CP-violating phases in the Higgs sector, gaugino masses, and trilinear couplings of sfermions. In fact, these phases are severely constrained from the EDM experiments if the masses of the SUSY particles are $O(100)$ GeV.\footnote{Masses of non-colored SUSY particles are still allowed to be $O(100)$ GeV.}

The Lagrangian of the relevant part is given by

$$
\mathcal{L} = \int d^2 \theta \left[ (\mu - B_\mu \theta^2) H_u H_d - \frac{1}{2} M_i \theta^2 \lambda_i \lambda_i + (1 - (A_e)_{ij} \theta^2) (Y_e)_{ij} L_l H_d E_j^c + (1 - (A_d)_{ij} \theta^2) (Y_d)_{ij} Q_i H_d D_j^c + (1 - (A_u)_{ij} \theta^2) (Y_u)_{ij} Q_i H_u U_j^c + \text{h.c.} \right],
$$

where $\lambda_1$, $\lambda_2$, and $\lambda_3$ denote bino, wino, and gluino, respectively. CP violation arises from the gaugino mass parameters $M_i$, the trilinear couplings $A_{e,d,u}$, the Higgsino mass parameter $\mu$, and the Higgs $B$-term $B_\mu$. Physical CP-violating phases are given by combinations of the phases of $B_\mu$, $\mu$, $M_i$, $A_{e,d,u}$ as\footnote{This fact can be understood by taking the convention in which $\mu$ and $B_\mu$ are real, where the parameters are rotated as $\mu, B_\mu \rightarrow |\mu|, |B_\mu|$, $M_i \rightarrow M_i \exp(i \arg(\mu B^*_\mu)) = |M_i| e^{i\theta_0}$, $A_f \rightarrow A_f \exp(i \arg(\mu B^*_\mu)) = |A_f| e^{i\theta_A}$.}

$$
\theta_i = \arg(M_i(\mu B^*_\mu)), \quad \theta_{A_{e,d,u}} = \arg(A_{e,d,u}(\mu B^*_\mu)),
$$

where we neglect small CP-violating phases of the Higgs vacuum expectation values (VEVs).

In the following, we focus on the electron EDM. It is generated by the phases $\theta_1$, $\theta_2$, and $\theta_{A_e}$ and constrained as

$$
|d_e| < 8.7 \times 10^{-29} \text{ e cm}
$$

by the ACME experiment [1]. It generally gives the most stringent constraint for scenarios with light sleptons, and is especially severe for models with $|\mu| \tan \beta \gg M_1, M_2$, which we will utilize in the rest of this paper. The large $|\mu| \tan \beta$ enhances the SUSY contribution to the muon $g - 2$ and, simultaneously, to the electron EDM. Taking the common mass scale $m_{\text{SUSY}}$ for $|M_1|, |M_2|$, and selectron masses, the contribution due to $\theta_1$ is approximated as [12]

$$
(|d_e|/e)_{\theta_1} \approx \frac{m_{\text{SUSY}}}{48\pi \cos^2 \theta_w} \frac{|\mu| \tan \beta}{m_{\text{SUSY}}} (3 \text{ TeV} \times 10^3) ^3.
$$

Assuming no cancellation with contributions from the other phases, the experimental bound gives a constraint

$$
\theta_1 \lesssim 1.2 \times 10^{-4} \left( \frac{m_{\text{SUSY}}}{300 \text{ GeV}} \right) ^3 \left( \frac{3 \text{ TeV} \times 10^3}{|\mu| \tan \beta} \right).
$$

Thus the phases of $B_\mu B^*_\mu$ and $M_1$ must be very accurately aligned at the scale of electroweak symmetry breaking (EWSB), or very small (see Eq. (2)).
It should be noted that the phase of $B_\mu \mu^*$ depends on the renormalization scale and is changed by those of the trilinear couplings $A_{u,d,e}$ and the gluino mass $M_3$. The relevant renormalization group equations for $B_\mu$ are [13]

\[
(16\pi^2) \frac{d(B_\mu/\mu)}{dt} \simeq 6(A_u)_{33}|(Y_u)_{33}|^2 + 6(A_d)_{33}|(Y_d)_{33}|^2 + 2(A_e)_{33}|(Y_e)_{33}|^2 + 6g_2^2M_2 + \frac{6}{5}g_1^2M_1,
\]

\[
(16\pi^2) \frac{d(A_{u,d})_{33}}{dt} \simeq \frac{32}{3}g_3^2M_3 + \cdots,
\]

(6)

where we neglect small Yukawa couplings. Here, $g_i$ are the gauge coupling constants for the SU(3), SU(2), and U(1) gauge groups. As seen from Eq. (6), even if the phases of $B_\mu \mu^*$ and $M_1$ are aligned at high energy such as $\simeq 10^{16}$ GeV, the resultant CP phase $\theta_1$ at low energy becomes $O(1)$ due to the phases of $A_{u,d,e}$ and $M_3$. Consequently, the constraints Eq. (5) are satisfied only when all the relevant phases, i.e., those of $B_\mu \mu^*$, $M_i$, and $A_{u,d,e}$, are precisely aligned or very small at the high-energy scale as long as there are no miraculous cancellations.

However, it is challenging in general to align all of the phases at the high-energy scale. To see this, we consider the following simple Kähler potential and the superpotential, the so-called Polonyi model (J. Polonyi, Hungary Central Research Institute report KFKI–77–93, unpublished):

\[
K = Z^*Z + (Q^*_{SM})_{ij} (Q_{SM})_{ij}, \quad W = \mu H_u H_d + W_{Yukawa} + \mu_2^2Z + \mathcal{C},
\]

(7)

where $Z$ is the SUSY-breaking field and $\mathcal{C}$ is a complex constant. Gaugino masses are generated from couplings between $Z$ and field strength superfields,

\[
\mathcal{L} \supset \int d^2\theta \left( \frac{1}{4g_i^2} + k_i \frac{Z}{M_P} \right) W^{i\alpha} W^{i\alpha}_\alpha + \text{h.c.},
\]

(8)

where $M_P$ is the reduced Planck mass. To focus on CP violation in the SUSY-breaking sector, we assume that the phases of $k_i$ are universal as $k_i = |k_i|e^{i\theta}$ in this section. For instance, the SU(5) grand unified theory (GUT) model satisfies this condition. Then, by a rotation of $Z$, the coefficients $k_i$ can be taken to be real.\(^4\) In this basis, the scalar potential is given by\(^5\)

\[
V = e^{K/M_P^2} \left[ \mu_Z^2 + Z^* \frac{W}{M_P^2} \right]^2 + \frac{\partial W}{\partial (Q_{SM})_{ij}} \left( Q^*_{SM} \right)_{ij} \frac{W}{M_P^2} \right] - \frac{3}{2} \frac{|W|^2}{M_P^2},
\]

(9)

and the gaugino mass term of Eq. (8) is now written as

\[
\mathcal{L} \supset k_i g_i^2 \frac{(F_Z)}{M_P} \lambda^i \lambda^i + \text{h.c.};
\]

(10)

the gaugino masses are proportional to $(F_Z)$:

\[
M_i = 2k_i g_i^2 \frac{(F_Z)}{M_P}.
\]

\(^4\) By using $R$-rotation, the constant $\mathcal{C}$ can be taken as real. However, to clarify the point, we leave $\mathcal{C}$ as a complex parameter.

\(^5\) The replacement of the coupling, $1/(4g_i^2) + \text{Re}(k_i) \rightarrow 1/(4g_i^2)$, has been done. The gauge kinetic functions are taken to be canonical.
SUSY breaking occurs by the nonzero $F$-term of $Z$:

$$\langle F_Z \rangle = -e^{K/(2M_p^2)} \left[ \frac{\partial W^*}{\partial Z^*} + \frac{\partial K}{\partial Z^*} \frac{W^*}{M_p^2} \right],$$

$$= -e^{i|z|^2/(2M_p^2)} \left[ \frac{\mu_Z^*}{M_p^2} + \frac{(\mu_Z^2 z^2 + C^*)}{M_p^2} \right].$$

(12)

Here, $z$ is the VEV of $Z$, which is found to be

$$z \equiv \langle Z \rangle = (\sqrt{3} - 1) e^{i\theta_Z} M_p, \quad \theta_Z = \arg(C) - \arg(\mu_Z^2)$$

for the stable SUSY-breaking vacuum with vanishing cosmological constant $V = 0$. As a result, the phase of $\langle F_Z \rangle$ turns out to be $\arg(\langle F_Z \rangle) = \pi - \arg(\mu_Z^2)$. The constant term is found to be

$$|C| = (2 - \sqrt{3}) |\mu_Z^2| M_p.$$  

(13)

The Higgs $B$-term and the $A$-terms are generated from

$$V \ni -e^{K/(2M_p^2)} \langle F_Z \rangle \left\{ \frac{\partial K}{\partial Z} \frac{W_{vis}}{M_p^2} + e^{K/M_p^2} \left[ \frac{\partial W_{vis}}{\partial (Q_{SM})} (Q_{SM}) - 3 W_{vis} \right] \frac{\langle W^* \rangle}{M_p^2} \right\} + B_\mu = \left( 2/\sqrt{3} - 1 \right) e^{-i \arg(C)} \frac{\langle F_Z \rangle}{M_p} \mu, \quad A_{ijk} = \left( 3 - 1 \right) e^{-i \arg(C)} \frac{\langle F_Z \rangle}{M_p}.$$  

(14)

In this basis, $B_\mu \mu^*$ and $A_{ijk}$ have the same phases as $C^*$, while the phases of the gaugino masses are given by $\arg(M_i) = \arg(F_Z) = \pi - \arg(\mu_Z^2)$. As $\arg(C) \neq \arg(\mu_Z^2)$ in general, the phases of $B_\mu \mu^*$ ($A_{ijk}$) and $M_i$ are different at high energy. Thus the resultant CP-violating phases become too large at the electroweak scale, and the EDM exceeds the constraints of Eq. (5) unless the sleptons are as heavy as $O(10)$ TeV.

### 3. CP-safe gravity mediation

The Polonyi model has a large CP violation unless $\arg(\mu_Z^2) = \arg(C)$. As one can see from Eqs. (11) and (15), the phases of the $B$-terms and $A$-terms are determined by those of $\langle F_Z \rangle (\partial K)/(\partial Z)$ and $\langle W^* \rangle$, while those of the gaugino masses are determined by the phase of $\langle F_Z \rangle$. When the phases of $\mu_Z^2$ and $C$ are aligned, $\langle Z \rangle$ becomes real (see Eq. (13)). Then, $(\partial K)/(\partial Z)$ is real and $\arg((F_Z)) = \arg((W^*))$ is satisfied (see Eq. (12)). As a result, the phases of the $B$-term, $A$-terms, and gaugino masses are all aligned. However, there is no reason why the phases of $\mu_Z^2$ and $C$ are aligned.

We now propose a generic gravity mediation model where the CP-safe conditions, (a) $(\partial K)/(\partial Z) \in \mathbb{R}$ and (b) $\arg((F_Z)) = \arg((W^*))$, are automatically satisfied. We adopt the gravitational SUSY-breaking scenario [14] (see also Appendix B for details), where SUSY is broken in the Kähler potential rather than in the superpotential: SUSY is broken with a constant superpotential. Furthermore, we assume a shift symmetry of $Z$, which guarantees $(\partial K)/(\partial Z)$ to be real, as shown below.

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6 Of course, the superpotential can be more generic; $\arg(B_\mu \mu^*) = \arg(A_{ijk})$ is not satisfied in the general case (see Appendix A).

7 Considering that the super-Weyl–Kähler transformation is anomalous, we adapt the constant superpotential from the beginning.
Consequently, conditions (a) and (b) are automatically satisfied (see Eq. (12)). In this paper, we concentrate our discussion only on the minimal supersymmetric standard model and do not consider GUTs. However, it may be interesting that we can construct a CP-safe model consistent with GUTs.

To demonstrate the above CP-safe gravity mediation, we consider the following Lagrangian:

\[ K = s(x) + (Q^c_{SM})_i (Q^{}_{SM})_i, \quad W = \mu H_u H_d + W_{Yukawa} + C, \]

(17)

where \( s(x) \) is a real function of \( x \), and \( x = (Z + Z^*) \). The Lagrangian has a shift symmetry

\[ Z \rightarrow Z + i \, R, \]

(18)

where \( R \) is a real constant. Notice that the \( R \)-charge of \( (H_u H_d) \) is 2, which forbids regeneration of \( \mu \) and \( B_{\mu} \) from \( K \equiv c_n (Z + Z^*)^n (H_u H_d) + h.c.\). This is very important, since otherwise we may have a CP-violating phase in the Higgs B-term. Note that \( K \equiv d_n C^* (Z + Z^*)^n (H_u H_d) + h.c. \) allowed by the symmetry is not dangerous, since the generated \( \mu \)-term and B-term are suppressed by \( O(m_{3/2}/M_P) \). Here, we consider that the \( R \)-symmetry breaking is \( O(m_{3/2}) \), which is much smaller than the Planck scale and explains the smallness of the constant term \( C \) in the superpotential.

The scalar potential is then given by

\[ V = e^{K/M_P} \left[ \frac{1}{M_P^2} \left( \frac{\partial s}{\partial x} \right)^2 \left( \frac{\partial^2 s}{\partial x^2} \right)^{-1} - 3 \right] \left| \frac{W}{M_P^2} \right|^2 e^{K/M_P} \left[ \frac{1}{\partial (Q^{}_{SM})_i} \frac{\partial K}{\partial (Q^{}_{SM})_i} \right] \frac{W}{M_P^2} + e^{K/M_P} \left[ \frac{1}{\partial (Q^{}_{SM})_i} \frac{\partial K}{\partial (Q^{}_{SM})_i} \right] \frac{W}{M_P^2} \right]. \]

(19)

The SUSY-breaking \( F \)-term of the hidden-sector field \( Z \) is proportional to the constant term

\[ \langle F_Z \rangle = -e^{s/(2M_P^2)} \left( \frac{\partial s}{\partial x} \right) \left( \frac{\partial^2 s}{\partial x^2} \right)^{-1} \left| \frac{C^*}{M_P^2} \right|. \]

(20)

Note that the shift symmetry of \( Z \) guarantees \( (\partial s)/(\partial x) \in \mathbb{R} \), and the phase of \( \langle F_Z \rangle \) is the same as that of \( \langle W^* \rangle = C^* \). Now, it is clear that the CP-safe conditions (a) and (b) are satisfied.

The condition for the vanishing cosmological constant, \( V = 0 \), is satisfied with an appropriate choice of Kähler potential rather than tuning of the constant \( C \):

\[ \left. \frac{1}{M_P^2} \left( \frac{\partial s}{\partial x} \right)^2 \left( \frac{\partial^2 s}{\partial x^2} \right)^{-1} \right|_{x=\langle x \rangle} = 3, \]

(21)

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8 In Refs. [15,16], CP violation in SUSY breaking is discussed with a Lagrangian motivated by string theory. In their setup, parameters in a superpotential of SUSY-breaking moduli can be taken to be real by approximate shift symmetries of moduli, resulting in no CP violation in the gaugino mass parameters and trilinear couplings. However, it is not clear if the CP-violating phase of the Higgs B parameter is sufficiently aligned with the other phases in their approaches; for our purpose of explaining the muon \( g - 2 \) anomaly, we need the CP-violating phase suppressed as tightly as \( O(10^{-4}) \) level, because the SUSY contribution to the electron EDM is enhanced as well as that to the muon \( g - 2 \).

9 As shown in Appendix D, the sequestered form of the Kähler potential is also consistent with a CP-safe gravity mediation.

10 In this paper, we have introduced the bare \( \mu \) parameter in the superpotential. One can also consider the case that this \( \mu \) parameter is generated dynamically. For instance, the \( \mu \)-term can arise in \( Z_2 \)-invariant NMSSM (next-to-minimal supersymmetric standard model) without generating a new CP-violating phase.
where the VEV $\langle x \rangle$ is determined by a stationary condition:

$$\frac{\partial}{\partial x} \left[ \frac{\partial s}{\partial x} \right]^2 \left( \frac{\partial^2 s}{\partial x^2} \right)^{-1} = 0.$$  \hspace{1cm} (22)

A concrete example model that satisfies Eqs. (21) and (22) with a stable minimum is shown in Appendix C. The constant $C$ can be written using the gravitino mass $m_{3/2}$ as

$$|C|^2 = e^{-s/(2M_P)} m_{3/2}^2 M_P^4. \hspace{1cm} (23)$$

The Higgs $B$-term and $A$-term arise from the second term of Eq. (19) and they are proportional to the constant $C^*$ as

$$B_\mu = e^{s/(2M_P)} \frac{2C^*}{M_P^2} \mu, \quad A_{ijk} = e^{s/(2M_P)} \frac{3C^*}{M_P^2}.$$

(24)

Notice that the phases of $B_\mu \mu^*$ and $A_{ijk}$ are aligned with that of $\langle F_Z \rangle$. By using Eq. (20), they are rewritten as

$$B_\mu = -\frac{2}{3} \left( \frac{\partial s}{\partial x} \right) \frac{\langle F_Z \rangle}{M_P^2} \mu, \quad A_{ijk} = -\left( \frac{\partial s}{\partial x} \right) \frac{\langle F_Z \rangle}{M_P^2}.$$

(25)

The scalar masses are the same as the gravitino mass:

$$m_0^2 = |\langle F_Z \rangle|^2 \left( \frac{\partial s}{\partial x} \right)^2 = m_{3/2}^2. \hspace{1cm} (26)$$

Finally, let us consider the gaugino masses, which arise from the couplings between $Z$ and field strength superfields $W_i$:

$$\int d^2 \theta \left( \frac{1}{4g_i^2} + k_i \frac{Z}{M_P} \right) W^i W^{\alpha i}.$$

However, the required $ZW^i_\alpha W^{\alpha i}$ terms violate the shift symmetry, and hence the terms must vanish; $k_i = 0$. However, we consider that the $ZW^i_\alpha W^{\alpha i}$ terms are generated by gauge anomalies of the shift symmetry, resulting in $k_i \in \mathbb{R}$. The constants $k_i$ depend on unknown high-energy physics, and hence we take them as free parameters in this paper.

Now, we see that all relevant phases are aligned as $\arg(B_\mu \mu^*) = \arg(M_i) = \arg(A_{ijk})$ and that the SUSY contributions to the EDM are successfully suppressed. We call this CP-safe gravity mediation. Note that this feature is not affected by the anomaly-mediation effect [17,18]. This is because the contributions from the anomaly mediation to $A_{ijk}$, $B_\mu / \mu$, and gaugino masses are also aligned with $C^* (= \arg(F_Z))$ [19,20], and any additional CP-violating phases are not introduced by the anomaly mediation.

4. Application: SUSY solving the muon $g-2$ anomaly

We have seen that the model that we have proposed, CP-safe gravity mediation, provides SUSY breaking without CP violation. This feature is very helpful for SUSY scenarios with light, i.e., $\mathcal{O}(100)$ GeV, SUSY particles. In this section, focusing on this advantage, we will consider an application of the CP-safe gravity mediation model.
The anomalous magnetic moment of muons, or the muon $g - 2$, has a $3\sigma$-level discrepancy between the experimental value measured in the Brookhaven E821 experiment [7,8] and theoretical predictions based on the Standard Model [21–26]. SUSY is capable of solving the anomaly with contributions of loop diagrams in which smuons, sneutrinos, neutralinos, and charginos are involved [9–11]. The SUSY contribution, which we call $\Delta \alpha^\text{SUSY}_{\mu}$, can be large enough to solve the discrepancy if the masses of the bino and/or winos are $\lesssim 1$ TeV and those of the smuons and/or muon sneutrinos are of the same order (see, e.g., Ref. [27] for a recent study). However, as we have discussed, this scenario generally confronts too large a CP violation because of the small SUSY particle masses. This is the main reason why we apply the CP-safe gravity mediation model to explain the anomaly of the muon $g - 2$.

In this section, we consider a model with a slightly extended Kähler potential,

$$K = s(x) + (Q^b_{\text{SM}})_i (Q_{\text{SM}})^i \left[ 1 + \alpha_1 x + \alpha_2 x^2 \right],$$

$$W = \mu H_u H_d + W_{\text{Yukawa}} + \mathcal{C},$$

where $\alpha_1$ and $\alpha_2$ are real constants and $x = Z + Z^*$. A normalization factor $r = 1 + \alpha_1 \langle x \rangle + \alpha_2 \langle x \rangle^2$ is introduced for canonical kinetic terms. Note that $\alpha_1 = \alpha_2 = 0$ corresponds to the model discussed in the previous section (Eq. (17)), and that this extension introduces no additional CP phases.\(^{12}\) The Higgs $B$-term and $A$-terms are given by

$$B_\mu = -\frac{2}{3} \left[ \left( \frac{\partial s}{\partial x} \right) r - 3 \alpha_1 M^2 P \right] \langle F_Z \rangle / M^2 P, \quad A_{ijk} = -\left( \frac{\partial s}{\partial x} \right) r - 3 \alpha_1 M^2 P \langle F_Z \rangle / M^2 P,$$

and the universal scalar mass by

$$m^2_0 = \frac{|\langle F_Z \rangle|^2}{M^4 P} \left[ \frac{1}{9} \left( \frac{\partial s}{\partial x} \right)^2 + \left( \alpha_1^2 - \alpha_2^2 \right) M^4 P \right],$$

where $\alpha_1' = (\alpha_1 + 2 \alpha_2 \langle x \rangle) / r$ and $\alpha_2' = 2 \alpha_2 / r$.

Now, the parameters of this model are summarized as $(m_0, \tan \beta, \text{sgn} \mu, M_1)$; $m_0$ and $M_1$ are the scalar and gaugino mass parameters at the high-energy scale, respectively, and $\tan \beta$ is the ratio of the VEVs of the Higgs fields: $\tan \beta = v_\mu / v_\lambda$. Note that trilinear couplings are universal and fixed as $A_{ijk} / (B_\mu / \mu) = 3 / 2$ at the high-energy scale even in this extended model.

A series of benchmark points are shown in Fig. 1. The Higgs boson mass is plotted with black lines, while the muon $g - 2$ discrepancy is relaxed to less than the $1\sigma$ ($2\sigma$) level in the orange (yellow) region. Green lines show the size of the $\mu$ parameter at the SUSY scale. The lightest neutralino is the lightest SUSY particle (LSP), and it is bino-like, in the displayed parameter space. The gray-shaded region is invalid or excluded because of the stau LSP, the existence of tachyonic SUSY particles, and/or EWSB vacuum instability [29,30]. The lighter stau is degenerate with the lightest neutralino on the blue line in the figure, where the dark-matter relic density can be explained by the coannihilation mechanism [31,32]. The values of the fixed parameters are $(m_0, \tan \beta, \text{sgn} \mu, M_1) = (300 \text{ GeV}, 10, +, 400 \text{ GeV})$. As a reference, the parameter space of the

\(^{12}\) The original model with $\alpha_1 = \alpha_2 = 0$ has a strict constraint that the universal scalar mass parameter at the high-energy scale is related to the Higgs $B$-term as $(B_\mu / \mu)^2 = 4m^2_0$ (cf. Eqs. (25) and (26)).
**Fig. 1.** Higgs boson mass and the SUSY contribution to the muon $g - 2$ in CP-safe gravity mediation. The muon $g - 2$ is compatible with the experimental value at the 1σ level in the orange region and at the 2σ level in the yellow. The black contours show the Higgs boson mass, while the green contours describe the value of the $\mu$ parameter at the SUSY scale. The gray-shaded region is invalid since the vacuum is unstable or the LSP is the lighter stau; the lighter stau is degenerate with the lightest neutralino on the blue line. The plotted points correspond to the model point that we analyzed. Red circles are the model points excluded by ATLAS slepton searches [28], while blue stars are those that we have checked are not excluded in any collider searches. The collider statuses of the other points (gray dots) are not determined in this study. The parameter space with $\alpha_1 = \alpha_2 = 0$ is described with the red-dashed line as a reference.

When we fix $M_2$ and increase $M_3$, $\Delta a_{\mu}^{\text{SUSY}}$ becomes larger and the discrepancy of the muon $g - 2$ is relaxed, but with too large a value of $M_3$ we face vacuum instability or the stau LSP. This feature can be understood with the following discussion on the renormalization group evolution from a high-energy scale of $\sim 10^{16}$ GeV to the SUSY scale. Firstly, the large $M_3$ increases squark masses during the evolution. Then, the large scalar-top mass, which on the one hand raises the Higgs boson mass, affects the soft mass of the up-type Higgs. For successful EWSB, the $\mu$ parameter is forced to be large, as shown in Fig. 1. This results in a large mixing between $\tilde{\mu}_L$ and $\tilde{\mu}_R$, which enhances the bino–smuon contribution (a loop diagram of $\tilde{B} - \tilde{\mu}_L - \tilde{\mu}_R$). Note that the other contributions, e.g., the sneutrino–chargino contribution, are insignificant because Higgsinos are decoupled. However, too much mixing is not allowed because it makes the lighter stau too light or induces vacuum instability [29,30], and this is why $M_3$ is bounded from above.

Figure 1 also contains information on the status of collider searches. Because squarks and gluinos are much heavier than the current collider bounds [41,42], LHC searches for electroweakinos (sleptons, staus, neutralinos, and charginos) give the most severe constraint on our benchmark points.

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13 See Refs. [38,39] for detailed discussion on two-loop level contributions in scenarios with hierarchical SUSY mass spectrum.

14 In this section we use the term “lepton” (“l”) for electrons and muons, not taus.
Table 1. Masses of SUSY particles, the Higgs boson mass, and the SUSY contribution to the muon $g - 2$ at two benchmark points. Masses are given in units of GeV. $M_2$ and $M_3$ are values at the high-energy scale in units of GeV. Values in parentheses for $\Delta a_\mu^{\text{SUSY}}$ are the deviation from the experimental results, where we use the value $(26.1 \pm 8.0) \times 10^{-10}$ as the discrepancy of the muon $g - 2$ [24–26]. Note that $\tilde{\chi}_1^0$ and $\tilde{\chi}_1^\pm$ are degenerate because they are mostly pure-wino.

| $(M_2, M_3)$  | $\tilde{\chi}_2^0$, $\tilde{\chi}_1^\pm$ | $\tilde{\ell}_L$, $\tilde{\ell}_R$ | $\tilde{\tau}_1$, $\tilde{\tau}_2$ | $\tilde{\chi}_1^0$ | $m_h$ | $\Delta a_\mu^{\text{SUSY}} \times 10^{10}$ |
|---------------|-----------------------------------|----------------|----------------|----------------|--------|---------------------------------|
| (460, 3300)   | 333                               | 329            | 348            | 182            | 400    | 150                             | 25  |
| (1000, 5000)  | 774                               | 578            | 365            | 247            | 589    | 142                             | 25  |

Fig. 2. Masses of electroweakinos in the parameter space of Fig. 1. Blue solid contours show the masses of winos (the lightest chargino $\tilde{\chi}_1^\pm$ and the second-lightest neutralino $\tilde{\chi}_2^0$), and blue-dashed contours those of the bino-like LSP $\tilde{\chi}_1^0$. Red-solid (dashed) lines describe the masses of left- (right-) handed selectrons and smuons, and green solid lines the masses of the lighter staus.

However, in order to discuss the collider search status on all the model points, we would have to perform a dedicated Monte Carlo simulation, which is far beyond the scope of this paper. We thus leave the status of collider searches unascertained for most of the points, and show the statuses of clearly excluded and clearly surviving points in Fig. 1. The model points depicted with red circles are excluded by the dilepton plus large missing energy ($2l + E_T$) signature from $pp \to \ell\ell^*\ell\ell^*$ [28], while those with blue stars are not excluded by any searches. The other points, i.e., those with gray dots, are unascertained points. Here, because there is no significant difference between the results from the CMS Collaboration [43,44] and those from ATLAS [28,45,46], the status is determined using the results from the ATLAS Collaboration.

To see details of the collider search statuses, we pick two benchmark points among the surviving points and show the mass spectrum as well as the Higgs boson mass and $\Delta a_\mu^{\text{SUSY}}$ in Table 1. The first point, $(M_2, M_3) = (460 \text{ GeV}, 3.3 \text{ TeV})$, explains the muon $g - 2$ at the 0.9$\sigma$ level. For this point, the sleptons are slightly heavier than the current bound [28] ($m_{\tilde{\ell}} > 324 \text{ GeV}$ for $m_{\tilde{\chi}_2^0} = 150 \text{ GeV}$ if $m_{\tilde{\ell}_L} = m_{\tilde{\ell}_R}$). Wino pair-production $pp \to \tilde{\chi}_2^0\tilde{\chi}_1^\pm$ is generally the most important for SUSY scenarios compatible with large $\Delta a_\mu^{\text{SUSY}}$ [47] for its cross section of $\mathcal{O}(10) \text{ fb}$. At this point, however, the winos mainly decay into staus and tau-sneutrinos to result in a multi-tau signature, which is less constrained than the signature with winos decaying into leptons ($e$ or $\mu$). For the second point, $(M_2, M_3) = (1 \text{ TeV}, 5 \text{ TeV})$, the masses of electroweakinos are clearly above the collider bounds in Refs. [28,45,46]. Further information on the electroweakino masses can be found in Fig. 2.
Let us discuss future prospects of collider searches for this scenario. From the viewpoint of future prospects, the CP-safe gravity mediation model as a solution to the muon $g - 2$ anomaly has three important features. The first one is the large $M_3$, which is indicated by the collider bound and the Higgs boson mass, and results in the large $\mu$ parameter and decoupled Higgsinos. It diminishes the chargino contribution to the muon $g - 2$. Consequently, as the second point, both $\tilde{\mu}_L$ and $\tilde{\mu}_R$ are as light as $O(100)$ GeV to yield sufficient contribution to the muon $g - 2$. Thirdly, the universal scalar soft masses (26) ensure that the lighter stau is lighter than the smuons.

The most important signature is $2l + \slashed{E}_T$ from $pp \to \tilde{\ell}\ell$. It is generally important for any SUSY scenarios explaining the muon $g - 2$ anomaly but because of light $\tilde{\mu}_L$ and $\tilde{\mu}_R$ it is more effective for this scenario. Because the cross section $\sigma(pp \to \tilde{\mu}_L\tilde{\mu}_L^\ast)$ at the 14 TeV LHC with $m_{\tilde{\mu}_L} = 400$ GeV is approximately equal to that at 8 TeV with $m_{\tilde{\mu}_L} = 300$ GeV, the whole of the $1\sigma$ region in Fig. 1 will be examined at the 14 TeV LHC.

Another promising production channel is wino production $pp \to \tilde{\chi}_2^0\tilde{\chi}_1^\pm$, which has the largest production cross section. The future prospects of this channel have been widely discussed [48–51] in terms of the $WZ + \slashed{E}_T$ signature, i.e., assuming that the winos produced exclusively decay as $\tilde{\chi}_2^0\tilde{\chi}_1^\pm \to (Z\tilde{\chi}_1^0)(W^+\tilde{\chi}_1^0)$. Reportedly, winos with a mass of 700–800 GeV are searched for at 14 TeV LHC with an integrated luminosity of 300 fb$^{-1}$ under this assumption. However, the branching ratio is negligible in our benchmark points; the winos mostly decay via sleptons, sneut inos, or staus, and produce a multi-$l + \slashed{E}_T$ signature. As the capability of searches for these signatures is seriously dependent on mass gaps among SUSY particles as well as the resultant lepton species [47], we will here just comment that the multi-$l + \slashed{E}_T$ signature generally provides a tighter constraint than $WZ + \slashed{E}_T$, while multi-$\tau + \slashed{E}_T$ gives a looser bound [44,45].

For experiments at linear colliders, e.g., ILC or CLIC, the stau is a particularly interesting target because of its small mass and large mixing. After staus are discovered, the stau mixing angle as well as stau mass should be measured, because it will be a test of this scenario. Furthermore, if all the sleptons are within the reach of linear colliders, we can estimate the size of $\Delta a^\mu_{\text{SUSY}}$ through measurements of the stau mixing angle, slepton production cross section, and masses [53].

5. Discussion and conclusions

We have discussed CP-safe gravity mediation models, which are free from the SUSY CP problem. The SUSY CP problem is a serious obstacle for low-energy SUSY. The CP-safe gravity mediation models assume that SUSY is broken in the Kähler potential with a non-vanishing constant term in the superpotential. Together with a shift symmetry of the SUSY-breaking field, the Higgs $B$-term, trilinear couplings as well as gaugino masses have a common phase determined by the constant term in the superpotential, and hence their phases are all aligned.

Note that CP-safe gravity mediation is also consistent with the sequestered form of the Kähler potential. (A concrete model is shown in Appendix D.) In this case, the Higgs $B$-term, $A$-terms, and scalar masses vanish at the tree level; then, gaugino mediation with a large gravitino mass is possible [54,55], and the gravitino problem may be relaxed. It is very important that the stable EWSB vacuum is easily realized, since the Higgs $B$-term ($B_\mu/\mu$) becomes of the order of gaugino masses.

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15 In Ref. [52], the future prospects of this channel are discussed in terms of the $WH + \slashed{E}_T$ signature.

16 For a large gravitino mass of $O(100)$ TeV, the coupling constants $k_i(i = 1–3)$ in Eq. (27) may be taken as small as $k_i = O(0.01)$ to produce $O(1)$ TeV gaugino masses. Such small $k_i$ seem more likely if the terms in Eq. (27) are generated from gauge anomalies of the shift symmetry.
rather than $O(m^3_{3/2})$. This is a clear advantage over the framework with a $Z$-dependent superpotential, where the Higgs $B$-term of $O(m^3_{3/2})$ arises (see the discussion in Appendix D).

In CP-safe gravity mediation, the Lagrangian has the shift symmetry of the SUSY-breaking field $Z$: $Z \rightarrow Z + iR$ ($R$ is a real constant). Therefore, the imaginary part of $Z$ is massless at the perturbative level. This imaginary part gets a mass via the QCD (quantum chromodynamics) non-perturbative effect, since it couples to the gluon field strength superfield, as shown in Eq. (27);\(^{17}\) the imaginary part of $Z$ behaves as a QCD axion [56,57] with the decay constant

$$f_a = \frac{M_P}{k_{316}^{\sqrt{2/\pi}}} \left( \frac{\partial^2 K}{\partial \phi^i \partial \phi^j} \right)^{1/2} \sim 10^{16} \text{GeV}/k_3.$$ 

\(^{(31)}\)

It is very intriguing that the strong CP problem and the SUSY CP problem are solved simultaneously in the present framework. If the inflation scale is high, as in chaotic inflation, we have an axion iso-curvature problem. It may be solved in more complicated frameworks [58], which are, however, beyond the scope of the present paper.

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Appendix A. General superpotential

The scalar potential is given by

$$V = F_i^* K^{ij} F_j - 3 e^K |W|^2,$$

where

$$K^{ij} = \frac{\partial^2 K}{\partial \phi^i \partial \phi^j}, \quad F_i^* = -e^{K/2} \left( K^{-1} \right)_{il} \left( \frac{\partial W}{\partial \phi^l} + \frac{\partial K}{\partial \phi^l} W \right).$$

We take units of $M_P = 1$. The gravitino mass is $m_{3/2} = (F_i^* K^{ij} F_j)/3 \simeq e^K |W|^2$. Assuming the minimal Kähler potential, $K = Z^* Z + (Q_{SM})^* (Q_{SM})$, the scalar potential is written as

$$e^{-|z|^2/2} V_{vis} \equiv \frac{\partial W_{vis}}{\partial \phi^l}_{SM} \left( (W_{hid})^* + C^* \right) + \left[ \frac{\partial W_{hid}^*}{\partial Z^*} z^* + (|z|^2 - 3) \left( (W_{hid})^* + C^* \right) \right] W_{vis} + \text{h.c.}$$

\(^{(A3)}\)

for the superpotential $W = W_{vis} + W_{hid} + C$, where $z = \langle Z \rangle$ and $W_{vis} = \mu H_u H_d + W_{Yukawa}$. The Higgs $B$-term and the $A$ parameter are given by

$$B_{\mu}/\mu = \left( z + \frac{1}{W_{hid}^* + C^*} \frac{\partial W_{hid}^*}{\partial Z^*} \right)^{-1} - z^* \langle F_Z \rangle,$$

$$A_{ijk} = -z^* \langle F_Z \rangle,$$

\(^{(A4)}\)

One might introduce a shift-symmetry-breaking term, e.g., $K \ni \epsilon_2 |Z|^2$ ($K \ni \epsilon_4 |Z|^4$). With this shift-breaking term, the axionic part of $Z$ can get a larger mass of $\sim \epsilon_2 m_{3/2} \sim \sqrt{|\epsilon_4| m_{3/2}}$ without inducing a CP-violating phase. However, such explicit breaking terms must be extremely small, otherwise the Peccei–Quinn mechanism does not work for solving the strong CP problem in QCD.
where

\[ \langle F_Z \rangle = -e^{\frac{|z|^2}{2}} \left[ \frac{\partial W^*_{\text{hid}}}{\partial Z^*} + \frac{\partial K}{\partial Z^*} (W^*_{\text{hid}} + C^*) \right]. \]

The gaugino mass arises from the coupling between \( Z \) and the gauge kinetic function:

\[ \int d^2 \theta \left( \frac{1}{4g_i^2} + k_i Z \right) W^{i*} W_i + \text{h.c.} \quad (A5) \]

The resultant gaugino masses are \( M_i = 2k_i g_i^2 F_Z \).

**Appendix B. Gravitational SUSY breaking**

Here, we consider a model where SUSY is broken with a constant superpotential. The SUGRA Lagrangian is written as

\[ \mathcal{L} = \int d^4 \theta \phi \phi^* f + \int d^2 \theta \phi^3 (C + W_{\text{vis}}) + \text{h.c.,} \quad (B1) \]

where \( \phi \) is the chiral compensator and \( \phi = 1 + F \theta^2 \). Here, \( W_{\text{vis}} \) is a function of the SSM fields.

Then, the scalar potential is given by

\[ -V = |F|^2 f + F^* F_i \frac{\partial f}{\partial Q_i} + F^*_i F \frac{\partial f}{\partial Q_i} + \frac{\partial^2 f}{\partial Q_i \partial Q_j} F^*_i F_j + 3(C + W_{\text{vis}}) F + 3(C^* + W_{\text{vis}}^*) F^* + \frac{\partial W_{\text{vis}}}{\partial Q_i} F_i + \frac{\partial W_{\text{vis}}^*}{\partial Q_i^*} F_i^*, \quad (B2) \]

where \( Q_i \) contains both the SUSY-breaking field \( Z \) and the SSM fields. The equations of motion are

\[ F f + F_i \frac{\partial f}{\partial Q_i} + 3(C^* + W_{\text{vis}}^*) = 0, \quad F \frac{\partial f}{\partial Q_i} + \frac{\partial^2 f}{\partial Q_i \partial Q_j} F_j + \frac{\partial W_{\text{vis}}^*}{\partial Q_i^*} = 0. \quad (B3) \]

Solving the equations of motion, \( F_i \) is written as

\[ F_i = -\left( \tilde{f}^{-1} \right)_{ik} \left[ \frac{\partial W_{\text{vis}}^*}{\partial Q_k^*} \frac{1}{f} \frac{\partial f}{\partial Q_i} + \frac{1}{f} \frac{\partial f}{\partial Q_i} \frac{3(C^* + W_{\text{vis}}^*)}{f} \right], \quad (B4) \]

where \( \tilde{f}^{-1} \) is the inverse of the matrix

\[ \tilde{f}^{ij} = \frac{\partial^2 f}{\partial Q_i^* Q_j} - \frac{1}{f} \frac{\partial f}{\partial Q_i} \frac{\partial f}{\partial Q_j}. \quad (B5) \]

The scalar potential is

\[ V = -3(C + W_{\text{vis}}) F - \frac{\partial W_{\text{vis}}}{\partial Q_i} F_i \]

\[ = \frac{9|C + W_{\text{vis}}|^2}{f} + \left[ 3(C + W_{\text{vis}}) \frac{1}{f} \frac{\partial f}{\partial Q_i} - \frac{\partial W_{\text{vis}}}{\partial Q_i} \right] \left( \tilde{f}^{-1} \right)_{ij} \left[ 3(C^* + W_{\text{vis}}^*) \frac{1}{f} \frac{\partial f}{\partial Q_j^*} - \frac{\partial W_{\text{vis}}^*}{\partial Q_j^*} \right]. \quad (B6) \]
The scalar potential in the Einstein frame is obtained by the rescaling, \( V \rightarrow (-3/f)^2 V \). The vanishing cosmological constant is obtained by requiring that \( V = 0 \), resulting in

\[
- \langle F \rangle = \frac{1}{f} \left( F_Z \frac{\partial f}{\partial Z} + 3C^* \right)
\]

\[
= \frac{3C^*}{f} \left[ \frac{1}{f} \left| \frac{\partial f}{\partial Z} \right|^2 \left( \frac{\partial^2 f}{\partial Z \partial Z^*} - \frac{1}{f} \left| \frac{\partial f}{\partial Z} \right|^2 \right)^{-1} + 1 \right]
\]

\[
= 0.
\]

Then, the Kähler potential \( K = -3 \log(-f/3) \) must satisfy the condition

\[
\left| \frac{\partial K}{\partial Z} \right|^2 \left( \frac{\partial^2 K}{\partial Z \partial Z^*} \right)^{-1} = 3.
\]

To satisfy the condition for the vanishing cosmological constant, SUSY must be broken when \( C \neq 0 \) [14]. This is because \( \langle F_i (\partial f)/(\partial Q_i) \rangle \) cannot be zero for \( \langle F \rangle = 0 \) (see Eq. (B3)).

Since \( \langle F \rangle = 0 \), the Higgs B-term and the A-terms arise from \( V \equiv -(\partial W_{\text{vis}})/(\partial Q_i) F_i \). The scalar masses arise from \( V \equiv -3C F \equiv 3C(1/f)(\partial f/\partial Q_i) F_i \).

**Appendix C. A model of CP-safe gravity mediation**

We consider the following Kähler potential of the hidden sector:

\[
K = s(x) = -3 \log(-f(x)/3),
\]

where \( f \) is a function of \( x = Z + Z^4 \) and is invariant under the shift \( Z \rightarrow Z + iR \) with a real constant \( R \). The superpotential is taken as \( W = C \); SUSY is broken with a constant superpotential.

By using the equations of motion, the scalar potential is written as \( V = -3CF \), where

\[
- \langle F \rangle = \frac{3C^*}{f} \left[ \frac{1}{f} \left| \frac{\partial f}{\partial x} \right|^2 \left( \frac{\partial^2 f}{\partial x \partial x^2} - \frac{1}{f} \left| \frac{\partial f}{\partial x} \right|^2 \right)^{-1} + 1 \right] = 0.
\]

The vanishing cosmological constant is given by \( \langle F \rangle = 0 \), which leads to

\[
\frac{\partial^2 f}{\partial x^2} \bigg|_{x = \langle x \rangle} = 0.
\]

The minimum of \( x \) is determined by \( (\partial V)/(\partial x) = 0 \):

\[
\frac{\partial^3 f}{\partial x^3} \bigg|_{x = \langle x \rangle} = 0.
\]

The \( F \)-term of \( Z \) is

\[
\langle F_Z \rangle = \left( f^{-1} \right)^{ZZ} \left( \frac{1}{f} \frac{\partial f}{\partial x} 3C^* \right) = -3 \left( \frac{\partial f}{\partial x} \right)^{-1} C^*.
\]

Now, let us consider the following \( f \): \( f = -3 + c_1x + c_2x^2 + c_3x^3 + c_4x^4 \). The conditions \( \frac{\partial f}{\partial x} \neq 0, \frac{\partial^2 f}{\partial x^2} = 0, \) and \( \frac{\partial^3 f}{\partial x^3} = 0 \) give

\[
\langle x \rangle = -c_3/(4c_4), \quad c_2 + 3 \langle x \rangle (c_3 + 2c_4 \langle x \rangle) = 0 \rightarrow c_2 = (3/8)c_3^2/c_4.
\]

The SUSY-breaking vacuum is stable for \( 768c_4^3 + 64c_1c_3c_4^2 - 3c_3^4 > 0 \).
Appendix D. A model with a sequestered Kähler potential

Here, we consider the sequestered form of the Kähler potential and superpotential:

\[ f = -3 + f_{\text{vis}} + f_{\text{hid}}, \quad K = -3 \log(-f/3), \quad W = C + W_{\text{vis}}. \]  

(D1)

where \( f_{\text{hid}} \) is a function of \( x = Z + Z^† \); \( f \) is invariant under the shift \( Z \rightarrow Z + i \mathcal{R} \). The superpotential in the visible sector is \( W_{\text{vis}} = (\mu_{ij}/2) Q_i Q_j + (y_{ijk}/6) Q_i Q_j Q_k \).

If one chooses \( f_{\text{vis}} = (Q_{SM})^i (Q_{SM})_i \), the Higgs \( B \)-term, \( A \)-terms, and scalar masses vanish. This can be seen clearly on the basis that the SM fields are rescaled as \( (Q_{SM})_i \rightarrow (Q_{SM})_i \varphi \). Then, the scalar potential is written as

\[
-V = f'_{\text{hid}} |F|^2 + \left( \frac{\partial f'_{\text{hid}}}{\partial x} F Z F^* + \text{h.c.} \right) \\
+ \frac{\partial^2 f'_{\text{hid}}}{\partial x^2} |FZ|^2 + F_i^* F_i + \left( 3C F + \frac{\mu_{ij}}{2} Q_i Q_j F + \text{h.c.} \right) + \left( \frac{\partial W_{\text{vis}}}{\partial Q_i} F_i + \text{h.c.} \right),
\]

(D2)

where \( f'_{\text{hid}} = -3 + f_{\text{hid}} \). The equations of motion are given by

\[
f'_{\text{hid}} F + \frac{\partial f'_{\text{hid}}}{\partial x} F Z + \left( 3C^* + \frac{\mu_{ij}}{2} Q_i^* Q_j^* \right) = 0, \\
\frac{\partial f'_{\text{hid}}}{\partial x} F + \frac{\partial^2 f'_{\text{hid}}}{\partial x^2} F_Z = 0, \\
F_i + \frac{\partial W^*}{\partial Q_i} = 0.
\]

Using the equations of motion, we have

\[
V = - \left( 3C + \frac{\mu_{ij}}{2} Q_i Q_j \right) F - \frac{\partial W_{\text{vis}}}{\partial Q_i} F_i
\]

(D4)

where \( F_i = - (\partial W^* / (\partial Q_i^*) \). Since the vanishing cosmological constant is satisfied with \( \langle F \rangle = 0 \) [14], the \( B \)-terms and \( A \)-terms, as well as the scalar masses, vanish at tree level.

This vanishing \( B \)-term is an important advantage of our setup, because it allows us to easily realize EWSB even if the gravitino is as heavy as \( O(10) \) TeV. This may be very useful for gaugino mediation models [54,55]. We can see this advantage by observing the EWSB conditions

\[
m_Z^2 = -|\mu|^2 - \frac{m_{H_u}^2 \tan^2 \beta - m_{H_d}^2}{\tan^2 \beta - 1}.
\]

(D5)

\[
B_{\mu}(\tan \beta + \cot \beta) = 2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2.
\]

(D6)

If the typical mass scale of SUSY particles is \( m_{\text{soft}} = O(1) \) TeV, \( B_{\mu}/\mu \) must be as small as \( \lesssim O(1) \) TeV. In our setup, this condition is fulfilled easily.

By contrast, if the superpotential depends on \( Z \), i.e., \( \partial W / (\partial Z) \neq 0 \), the Higgs \( B \)-term \( B_{\mu}/\mu = O(m_{3/2}) \) arises from the VEV of the compensator field, and it is difficult to satisfy the EWSB conditions with \( m_{3/2} = O(10) \) TeV and \( m_{\text{soft}} = O(1) \) TeV. To see this, we consider a \( Z \)-dependent
superpotential

\[ W = C + W_{\text{hid}}(Z) + W_{\text{vis}}. \]  

(D7)

The Lagrangian is, with rescaling of \( Q_i \varphi \to Q_i \),

\[ \mathcal{L} = \int d^4 \theta (f^{i*}_{\text{hid}}|\varphi|^2 + Q_i^\dagger Q_i) + \left\{ \int d^2 \theta \left[ (C + W_{\text{hid}})\varphi^3 + \frac{\mu_{ij}}{2} \varphi Q_i Q_j + \frac{\lambda_{ijk}}{6} Q_i Q_j Q_k \right] + \text{h.c.} \right\}. \]

(D8)

The equations of motion are given by

\[ f^{i*}_{\text{hid}} F + \frac{\partial f^{i*}_{\text{hid}}}{\partial x} F_Z + \left( 3C^* + 3W_{\text{hid}}^* + \frac{\mu_{ij}^*}{2} Q_i^* Q_j^* \right) = 0, \]

\[ \frac{\partial f^{i*}_{\text{hid}}}{\partial x} F + \frac{\partial^2 f^{i*}_{\text{hid}}}{\partial x^2} F_Z + \frac{\partial W_{\text{hid}}^*}{\partial Z^*} F_i = 0, \]

\[ F_i + \frac{\partial W_{\text{vis}}^*}{\partial Q_i^*} = 0. \]

(D9)

Using the above equations, the scalar potential is now written in a simple form:

\[ -V = \left( 3C + 3W_{\text{hid}} + \frac{\mu_{ij}}{2} Q_i Q_j \right) F + \frac{\partial W_{\text{hid}}}{\partial Z} F_Z + \frac{\partial W_{\text{vis}}}{\partial Q_i} F_i. \]

The condition for the vanishing cosmological constant, \( V = 0 \), is read as

\[ 3(C + W_{\text{hid}})F + \frac{\partial W_{\text{hid}}}{\partial Z} F_Z = 0 \]

at the vacuum. Because of nonzero \( W_{\text{hid}} \), \( \langle F \rangle \) is not zero but \( \mathcal{O}(m_{3/2}) \), and it induces the Higgs \( B \)-term as \( (B_\mu/\mu) \sim m_{3/2} \).\(^{18}\) This \( B \)-term is problematic for the condition (D5) when we consider scenarios with \( m_{3/2} = \mathcal{O}(10) \text{ TeV} \) and \( (\mu, m_{H_u}, m_{H_d}) = \mathcal{O}(0.1–1) \text{ TeV} \).

References

[1] ACME Collaboration, Science 343, 269 (2014) [arXiv:1310.7534] [Search iNSPIRE].

[2] J. R. Ellis, S. Ferrara, and D. V. Nanopoulos, Phys. Lett. B 114, 231 (1982).

[3] W. Buchmuller and D. Wyler, Phys. Lett. B 121, 321 (1983).

[4] F. del Aguila, M. B. Gavela, J. A. Grifols, and A. Mendez, Phys. Lett. B 126, 71 (1983); 129, 473 (1983) [erratum].

[5] J. Polchinski and M. B. Wise, Phys. Lett. B 125, 393 (1983).

[6] E. Franco and M. L. Mangano, Phys. Lett. B 135, 445 (1984).

[7] Muon G-2 Collaboration, Phys. Rev. D 73, 072003 (2006) [arXiv:hep-ex/0602035] [Search iNSPIRE].

[8] B. L. Roberts, Chin. Phys. C 34, 741 (2010) [arXiv:1001.2898] [Search iNSPIRE].

[9] J. L. Lopez, D. V. Nanopoulos, and X. Wang, Phys. Rev. D 49, 366 (1994) [arXiv:9308336 [hep-ph]].

[10] U. Chattopadhyay and P. Nath, Phys. Rev. D 53, 1648 (1996) [arXiv:9507386 [hep-ph]] [Search iNSPIRE].

[11] T. Moroi, Phys. Rev. D 53, 6565 (1996) [arXiv:9512396 [hep-ph]] [Search iNSPIRE]; 56, 4424 (1997) [erratum].

[12] S. Pokorski, J. Rosiek, and C. A. Savoy, Nucl. Phys. B 570, 81 (2000) [arXiv:9906206 [hep-ph]] [Search iNSPIRE].

\(^{18}\)Even with a \( Z \)-dependent superpotential, it is possible to suppress the CP-violating phase. An example is the case that the arguments of \( C, (W_{\text{hid}}), \) and \( (\partial W_{\text{hid}}/\partial Z) \) can be taken as the same by shifting \( Z \), i.e., \( Z \to Z + i\theta \). (For instance, \( W_{\text{hid}} = Ae^{-bZ} \) with \( b \in \mathbb{R} \).) Then we observe \( \arg(\langle F \rangle) = \arg(\langle F_Z \rangle) \) from the equations of motion (D8), and the \( B \)-term is free from CP-violating phases.
[13] S. P. Martin and M. T. Vaughn, Phys. Rev. D 50, 2282 (1994) [arXiv:9311340 [hep-ph]] [Search inSPIRE].

[14] K. I. Izawa, T. Kugo, and T. T. Yanagida, Prog. Theor. Phys. 125, 261 (2011) [arXiv:1008.4641 [Search inSPIRE]].

[15] K. Choi, Phys. Rev. Lett. 72, 1592 (1994) [arXiv:9311352 [hep-ph]] [Search inSPIRE].

[16] K. Choi and K. S. Jeong, J. High Energy Phys. 0701, 103 (2007) [arXiv:0611279 [hep-th]] [Search inSPIRE].

[17] L. Randall and R. Sundrum, Nucl. Phys. B 557, 79 (1999) [arXiv:9810155 [hep-th]] [Search inSPIRE].

[18] G. F. Giudice, M. A. Luty, H. Murayama, and R. Rattazzi, J. High Energy Phys. 9812, 027 (1998) [arXiv:hep-ph/9810442 [Search inSPIRE]].

[19] J. A. Bagger, T. Moroi, and E. Poppitz, J. High Energy Phys. 0004, 009 (2000) [arXiv:hep-th/9911029 [Search inSPIRE]].

[20] M. K. Gaillard and B. D. Nelson, Nucl. Phys. B 588, 197 (2000) [arXiv:0004170 [hep-th]] [Search inSPIRE].

[21] M. Davier et al., Eur. Phys. J. C 66, 127 (2010) [arXiv:0906.5443 [Search inSPIRE]].

[22] M. Davier, A. Hoecker, B. Malaescu, C. Z. Yuan, and Z. Zhang, Eur. Phys. J. C 66, 1 (2010) [arXiv:0908.4300 [Search inSPIRE]].

[23] M. Davier, A. Hoecker, B. Malaescu, and Z. Zhang, Eur. Phys. J. C 71, 1515 (2011) [arXiv:1010.4180 [Search inSPIRE]]; 72, 1874 (2012) [Search inSPIRE].

[24] K. Hagiwara, A. D. Martin, D. Nomura, and T. Teubner, Phys. Lett. B 649, 173 (2007) [arXiv:0611102 [hep-ph]] [Search inSPIRE].

[25] T. Teubner, K. Hagiwara, R. Liao, A. D. Martin, and D. Nomura, Chin. Phys. C 34, 728 (2010) [arXiv:1001.5401 [Search inSPIRE]].

[26] K. Hagiwara, R. Liao, A. D. Martin, D. Nomura, and T. Teubner, J. Phys. G 38, 085003 (2011) [arXiv:1105.3149 [Search inSPIRE]].

[27] M. Chakraborti, U. Chattopadhyay, A. Choudhury, A. Datta, and S. Poddar, J. High Energy Phys. 07, 019 (2014) [arXiv:1404.4841 [Search inSPIRE]].

[28] ATLAS Collaboration, J. High Energy Phys. 1405, 071 (2014) [arXiv:1403.5294 [Search inSPIRE]].

[29] J. Hisano and S. Sugiyama, Phys. Lett. B 696, 92 (2011) [arXiv:1011.0260 [Search inSPIRE]].

[30] M. Endo, K. Hamaguchi, T. Kitahara, and T. Yoshinaga, J. High Energy Phys. 1311, 013 (2013) [arXiv:1309.3065 [Search inSPIRE]].

[31] J. R. Ellis, T. Falk, and K. A. Olive, Phys. Lett. B 444, 367 (1998) [arXiv:9810360 [hep-ph]] [Search inSPIRE].

[32] J. R. Ellis, T. Falk, K. A. Olive, and M. Srednicki, Astropart. Phys. 13, 181 (2000) [arXiv:9905481 [hep-ph]] [Search inSPIRE].

[33] B. C. Allanach, Comput. Phys. Commun. 143, 305 (2002) [arXiv:0104145 [hep-ph]] [Search inSPIRE].

[34] S. Heinemeyer, W. Hollik, and G. Weiglein, Comput. Phys. Commun. 124, 76 (2000) [arXiv:9812320 [hep-ph]] [Search inSPIRE].

[35] S. Heinemeyer, W. Hollik, and G. Weiglein, Eur. Phys. J. C 9, 343 (1999) [arXiv:9812472 [hep-ph]] [Search inSPIRE].

[36] G. Degrassi, S. Heinemeyer, W. Hollik, P. Slavich, and G. Weiglein, Eur. Phys. J. C 28, 133 (2003) [arXiv:0212020 [hep-ph]] [Search inSPIRE].

[37] M. Frank et al., J. High Energy Phys. 0702, 047 (2007) [arXiv:0611326 [hep-ph]] [Search inSPIRE].

[38] H. G. Fargnoli, C. Gnendiger, S. Paßehr, D. Stöckinger, and H. Stöckinger-Kim, Phys. Lett. B 726, 717 (2013) [arXiv:1309.0980 [Search inSPIRE]].

[39] H. Fargnoli, C. Gnendiger, S. Paßehr, D. Stöckinger, and H. Stöckinger-Kim, J. High Energy Phys. 1402, 070 (2014) [arXiv:1311.1775 [Search inSPIRE]].

[40] M. Muhlleitner, A. Djouadi, and Y. Mambrini, Comput. Phys. Commun. 168, 46 (2005) [arXiv:0311167 [hep-ph]] [Search inSPIRE].

[41] ATLAS Collaboration, J. High Energy Phys. 1409, 176 (2014) [arXiv:1405.7875 [Search inSPIRE]].

[42] CMS Collaboration, CMS PAS SUS–13–019 (2014).

[43] CMS Collaboration, Eur. Phys. J. C 74, 3036 (2014) [arXiv:1405.7570 [Search inSPIRE]].

[44] CMS Collaboration, Phys. Rev. D 90, 032006 (2014) [arXiv:1404.5801 [Search inSPIRE]].

[45] ATLAS Collaboration, J. High Energy Phys. 1404, 169 (2014) [arXiv:1402.7029 [Search inSPIRE]].

[46] ATLAS Collaboration, Phys. Rev. D 90, 052001 (2014) [arXiv:1405.5086 [Search inSPIRE]].
[47] M. Endo, K. Hamaguchi, S. Iwamoto, and T. Yoshinaga, J. High Energy Phys. 1401, 123 (2014) [arXiv:1303.4256] [Search INSPIRE].

[48] ATLAS Collaboration, ATL–PHYS–PUB–2013–002 (2013).

[49] ATLAS Collaboration, ATL–PHYS–PUB–2013–011 (2013).

[50] CMS Collaboration, [arXiv:1307.7135] [Search INSPIRE].

[51] CMS Collaboration, CMS PAS FTR–13–014 (2013).

[52] H. Baer, V. Barger, A. Lessa, and X. Tata, Phys. Rev. D 86, 117701 (2012) [arXiv:1207.4846] [Search INSPIRE].

[53] M. Endo, K. Hamaguchi, S. Iwamoto, T. Kitahara, and T. Moroi, Phys. Lett. B 728, 274 (2014) [arXiv:1310.4496] [Search INSPIRE].

[54] K. Inoue, M. Kawasaki, M. Yamaguchi, and T. Yanagida, Phys. Rev. D 45, 328 (1992).

[55] T. T. Yanagida and N. Yokozaki, Phys. Lett. B 722, 355 (2013) [arXiv:1301.1137] [Search INSPIRE].

[56] R. D. Peccei and H. R. Quinn, Phys. Rev. Lett. 38, 1440 (1977).

[57] R. D. Peccei and H. R. Quinn, Phys. Rev. D 16, 1791 (1977).

[58] J. L. Evans, M. Ibe, K. A. Olive, and T. T. Yanagida, Eur. Phys. J. C 74, 2931 (2014) [arXiv:1402.5989] [Search INSPIRE].