Dynamics of quantized rings and loops after reconnection in superfluid helium

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Abstract. The presented work is devoted to the study of the motion of vortex loops after reconnection in superfluid helium. The helium temperature as well as the initial arrangement of the vortex loops varied. The data on the change in the length, average radius of curvature and velocity of vortex loops from the moment of reconnection to the moment of their collapse were obtained. A comparative analysis of the motion of a single vortex ring and a deformed vortex loop was carried out. All calculations were performed within the framework of the vortex filament method using the full Biot-Savart equation.

1. Introduction

Quantum or superfluid turbulence attracts considerable attention from both theoretical and applied points of view [1]. As a rule, superfluid turbulence is understood as a tangle of quantized vortex filaments or loops that occurs in the superfluid component of helium. With an increase in the volume of liquid helium, the proportion of vortex loops (closed vortex filaments) in a vortex tangle consisting of vortex loops and vortex filaments, which ends at the boundaries of the volume of liquid superfluid helium, usually increases. Therefore, the study of quantized vortex loops dynamics is especially important. The reconnections of quantum vortex loops have a significant effect on the structure and properties of vortex tangles, also. At present, there is no consistent theory describing both the process of reconnection itself and the motion of substantially deformed vortex loops arising after reconnection. To develop approximate models of these processes, it would be useful to analyze the differences and similarities in the motion of the vortex ring and the vortex loop that originated after the reconnection. The vortex ring in this case is a relatively well theoretically studied object; see, for example, [2-4]. Thus, the main goal of the present work is to compare the motion of the vortex loop after reconnection and the vortex ring under different initial conditions and temperatures, as well as analyzing the influence of disturbances that occur after reconnection on the motion of the vortex loop as a whole. The main parameters for comparison are the change in the length of the vortex configuration, average curvature and velocity in time. For the vortex ring, there are analytical estimates of these quantities [2-4]. The motion of a vortex loop is studied using the vortex filament method and the Biot-Savart equation.

2. Equations of motion and numerical simulation

The motion of quantized vortex loops was simulated by the vortex filament method. Details related to this method may be found in articles [5]. According to this method, the velocity of the point of vortex
in superfluid component of helium is determined by the velocity induced by the all vortices in system, which is calculated from the Biot-Savart equation:

\[ \mathbf{V}_B = \frac{\kappa}{4\pi} \int \frac{\mathbf{s}_i \times d\mathbf{s}_i}{|\mathbf{s}_i - \mathbf{s}|^3}, \]

here \( \mathbf{s}, \mathbf{s}_i \) are coordinates of vortex points, \( \kappa \) is the quantum of circulation; integration is performed along the whole vortex loop configuration. For the resting helium, i.e. for the helium in which velocity of normal and superfluid components are equal to zero, the equation of motion for the point of the vortex is as follows:

\[ \mathbf{V}_l = \mathbf{V}_B - \alpha \mathbf{s}' \times \mathbf{V}_B + \alpha' \mathbf{s}' \times [\mathbf{s}' \times \mathbf{V}_B]. \]

here \( \alpha \) and \( \alpha' \) are friction coefficients, and \( \mathbf{s}' \) is a tangent vector. The motion of loops was modeled at temperatures equal to zero: \( \alpha = 0, \alpha' = 0 \) and \( 1.9 \, K; \alpha = 0.21, \alpha' = 0.009 \).

Initially, two identical smooth vortex rings were generated. The initial radii of the rings \( R_0 \) were the same and equal to \( 10^{-3} \) cm. The angles between vortex planes were varied. Further, the calculation continued until the implementation of reconnection. The Runge–Kutta method with the precision factor of the fourth order was used for integration of the equations of motion. More details on the initial conditions, mathematical modeling and specific numerical procedures implemented in this study can be found in works [6,7]. Here the mentioned calculations continued from the moment of reconnection.

As noted in the introduction, there are theoretical estimates for the properties of quantized vortex ring. Generally, the Biot-Savart equation in the local approximation is used. The local approximation takes into account the effect on the movement of only closest points of the vortex filament, it gives [5]:

\[ \mathbf{V}_B = \beta \mathbf{s}' \times \mathbf{s}' + \frac{\kappa}{4\pi} \ln \frac{R}{a_0}, \]

here \( a_0 \) is the radius of vortex filament core, and \( R \) is the vortex ring radius. In the polar, coordinate system with the \( z \)-axis is chosen along the direction motion of the vortex ring. The projection of the last equation on the coordinate axes, taking into account the friction forces, will be:

\[ \mathbf{V}_B = \frac{\beta}{R(t)} \mathbf{e}_z - \frac{\alpha\beta}{R(t)} \mathbf{e}_z \frac{\partial z}{\partial t} = \frac{\beta}{R(t)} \frac{\partial R}{\partial t} = -\frac{\alpha\beta}{R(t)}. \]

By integrating this equation, the dependence of the collapse time and dependence of vortex diameter on time can be obtained for the different initial rings sizes and the friction coefficients:

\[ R(t) = \sqrt{R_0 - 2\alpha\beta}. \]

A more accurate estimate of the velocity for the ring [5], gives:

\[ \mathbf{V}_\text{ring} = \frac{\kappa}{4\pi R} \left( \ln \frac{8R - 1}{a_0} - \frac{1}{4} \right). \]

The dependences of the simulated quantities for the vortex ring, obtained by simulation, were compared with the given analytical estimates. Good agreement has been received. The largest deviation was observed in the velocity values immediately before the vortex collapse, but it did not exceed a few percent, which is quite enough to solve the tasks set in the current work.

3. Results and discussion

For a comparative analysis of the dynamics of the vortex ring and loop, hereinafter a ring, whose length is equal to the length of the vortex loop after reconnection, is considered. This choice is because the energy of the vortex loop is mainly associated with its length and the comparison of energetically
close objects looks reasonable. Further, in all the figures, the Theta symbol designates the angles between the initial positions of the planes of the vortex rings, for more details see the previous section and works [6,7]. In addition, the dependences presented below in Figures 1–4, 6, 7 are normalized to the values of the corresponding quantities just at the time of reconnection.

Let us now consider how the length of the vortex ring and loop changes after reconnection. At zero temperature, the length of the vortex ring does not change, see Figure 1.

![Figure 1](image)

**Figure 1.** The dependences of the length of the vortex ring and loop on time at zero temperature.

The length of the vortex loops after reconnection periodically changes around a certain constant value, which is associated with the propagation of disturbances or Kelvin waves that occurred during reconnection, the corresponding geometric configurations of loops are described in detail in [8]. It is worth noting that these deviations do not exceed one percent.

At a temperature of 1.9 K, the length of the vortices decreases, and the rate of decrease in length increases with increasing angle between the initial planes of the vortex rings, see Figure 2.

![Figure 2](image)

**Figure 2.** The dependences of the length of the vortex ring and loop on time at temperature of 1.9 K.

This fact is due to the fact that at large theta angles at the time of reconnection, Kelvin waves of greater amplitude appear, as well as propagating along the vortex loop with greater velocity. An increase in this velocity leads to the fact that a large friction force acts on the corresponding parts of the vortex loop and, as a result, the length of the vortex ring decreases faster (region of 0.001-0.002 second in Figure 2). Further, after the damping of the main perturbations, the length of the vortex loops decreases according to a quadratic dependence, as for the vortex ring.
At zero temperature, the average radius of curvature of the vortex ring does not change, see Figure 3.

When reconnecting two identical vortex loops and in the absence of additional small loops arising during reconnection (for details, see [8]), the length of the vortex loop formed should be approximately equal to the sum of the lengths of the reconnected loops. When saving the length, a logical assumption will be that the resulting loop will be twice as long as each of the initial ones and the corresponding radius of curvature will double. However, the opposite is observed, i.e. the average radius of curvature is significantly reduced, see Fig. 3. Such behavior is due to the formation of Kelvin waves on the resulting loop. The curvature of these waves is significant [8], which considerably reduces the average radius of curvature. At zero temperature, the Kelvin waves do not decay; therefore, upon further movement, the average radius of curvature remains almost unchanged.

At elevated temperatures, the vortex ring shrinks and collapses, i.e. turns into thermal excitation (the radius of curvature decreases to zero), Figure 4.

The radius of curvature of the vortex loop after reconnection under the action of friction begins to increase, i.e. the loop begins to turn into a vortex ring due to the smoothing and attenuation of the
disturbances that have arisen. In parallel, there is a process of general compression of the ring under the action of friction between the vortex loop and the normal component of superfluid helium.

Let us construct the “effective length” of the vortex loop on the base of the average radius of curvature, multiplying the latter by $2\pi$. Figure 5 shows in a dimensional form the dependences of such an “effective length” and the actual length of the vortex loop on time.

![Figure 5](image)

**Figure 5.** The dependences of the length of the vortex loops on the time and the effective length of the vortex loops, determined by the average radius of curvature on time at temperature of 1.9 K.

The coincidence of dependences means the fulfillment of the ratio $L = 2\pi R$ valid for the vortex ring, in other words, the attenuation of disturbances arising during the reconnection. Such dependence is observed immediately before the collapse.

The velocity of the vortex ring at zero temperature does not change over time (Figure 6).

![Figure 6](image)

**Figure 6.** The dependences of the velocity of the vortex ring and loop on time at zero temperature.

In contrast, the average velocity of the vortex loop also does not change, but its values periodically significantly change, which is connected with the movement of Kelvin waves along the vortex loop, the corresponding illustrations can be found in [8]. It is worth noting that the amplitude of the Kelvin waves and the corresponding amplitude of oscillations of the average velocity of the vortex loop significantly depends on the initial conditions.
At a nonzero temperature, the average radius of curvature of the vortex ring and loop decreases, which leads to an increase in their average velocity, Figure 7.

Moreover, for the vortex rings, when moving directly after reconnection, some fluctuations of the average velocity are observed due to the propagation of damped Kelvin waves.

**Conclusion**
The main difference in the dynamics of the vortex ring and loop that occurred during reconnection can be attributed to the presence of Kelvin waves propagating through the latter. At zero temperature, these perturbations do not decay, which leads to fluctuations in the quantities characterizing the vortex loop (especially the average velocity) at some average values. However, as with the vortex ring, the average values do not change over time. At elevated temperatures, the vortex ring and loop collapse under the action of friction force. The nature of averages, as for zero temperature, is also similar here. However, the presence of Kelvin waves arising during reconnection leads to a local increase in the velocity of the vortex loops and, as a consequence, an increase in the friction force. This leads to the fact that the lifetime of the vortex loop (collapse time) decreases compared to the lifetime of the vortex ring of the same length.

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