An explicit calculation of pseudo-goldstino mass at the leading three-loop level

Jianpeng Dai\textsuperscript{a,b}, Tao Liu\textsuperscript{c,b} and Jin Min Yang\textsuperscript{a,b}

\textsuperscript{a}CAS Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China
\textsuperscript{b}School of Physics, University of Chinese Academy of Sciences, Beijing 100049, China
\textsuperscript{c}Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China

E-mail: daijianpeng@mail.itp.ac.cn, liutao86@ihep.ac.cn, jmyang@itp.ac.cn

ABSTRACT: Pseudo-goldstinos appear in the scenario of multi-sector SUSY breaking. Unlike the true goldstino which is massless and absorbed by the gravitino, pseudo-goldstinos could obtain mass from radiative effects. In this note, working in the scenario of two-sector SUSY breaking with gauge mediation, we explicitly calculate the pseudo-goldstino mass at the leading three-loop level and provide the analytical results after performing Taylor expansions in the loop integrals. In our calculation we consider the general case of messenger masses (not necessarily equal) and include the higher order terms of SUSY breaking scales. Our results can reproduce the numerical value estimated previously at the leading order of SUSY breaking scales with the assumption of equal messenger masses. It turns out that the results are very sensitive to the ratio of messenger masses, while the higher order terms of SUSY breaking scales are rather small in magnitude. Depending on the ratio of messenger masses, the pseudo-goldstino mass can be as low as $\mathcal{O}(0.1)$ GeV.

KEYWORDS: NLO Computations, Supersymmetry Phenomenology

ArXiv ePrint: 2104.12656
1 Introduction

Albeit lack of direct evidence at the LHC, supersymmetry (SUSY) as a generalization of space-time symmetry in quantum field theory remains one of the most popular extensions of the Standard Model of particle physics. The SUSY breaking and mediation mechanism plays a key role in the phenomenology of SUSY. If the spontaneous SUSY breaking is generated independently in different hidden sectors, then each sector provides a massless goldstino $\eta_i$ with SUSY breaking scale $F_i$. One linear combination of $\eta_i$ is identified as the true massless goldstino that is absorbed by gravitino, other orthogonal combinations named pseudo-goldstinos could acquire masses from radiative corrections. However, unconstrained by the supercurrent, the interactions between pseudo-goldstinos and ordinary superparticles could be sizable enough to induce unconventional signatures in collider physics and cosmology [1, 3–18]. For example, these physical pseudo-goldstinos with loop induced masses could be lighter than the lightest neutralino so that the neutralino could decay to a pseudo-goldstino plus a gauge boson or Higgs boson, leading to intriguing phenomenology at the LHC and future lepton colliders [15–17]. On the other hand, since the thermally produced bino-like neutralinos may decay readily to pseudo-goldstinos (the pseudo-goldstinos finally decay to gravitino dark matter), the stringent constraints on SUSY from dark matter detection and relic density can be relaxed (note that most GUT-constrained SUSY models like CMSSM/mSUGRA suffer from such a tension between dark matter constraints and muon g-2 explanation if the dark matter is the lightest bino-like neutralino [19]).

Obviously, the mass of pseudo-goldstino is a crucial parameter for all the phenomenological analysis in the multi-sector SUSY breaking scenario. Due to the intrinsic property of supergravity, there is a universal mass which is twice the gravitino mass $m_{3/2}$ at tree level. Another tree-level contribution coming from electroweak symmetry breaking in the low energy part is negligible. The possible loop corrections to the mass of pseudo-goldstino have been discussed in ref. [1]. If SUSY breaking sectors only communicate via gauge interactions which are common from the viewpoint of model building, it is argued that the leading order contribution to the pseudo-goldstino mass arises at the three-loop level and it could be described through the two-point correlation functions defined in General Gauge
Mediation [2]. Also a numerical value of the pseudo-goldstino mass was estimated from the evaluation of these functions assuming the SUSY breaking scales $M_i$ in the two sectors are equal [1]. Note that the analysis in ref. [1] is based on arguments and approximation, while an independent cross-check or an explicit calculation of the three-loop contributions with different messenger masses is still missing to date. Thus in this note we perform an explicit three-loop calculation for the mass corrections to the pseudo-goldstino, which is also applicable to the minimal gauge mediation. Although the realistic SUSY breaking and gauge mediation models might not be as simple as the minimal one mentioned above, the physics behind them for the mass corrections at the leading three-loop level is almost the same. So our result could be easily generalized to other cases or used to make model-dependent estimations.

This work is organized as follows. In section 2 we will make a brief review on the framework with pseudo-goldstinos and mention some technical details used for our calculation. Section 3 contains the analytical results and related discussions. Finally, we conclude in section 4.

2 Technical details

The superpotential with multi-sector SUSY breaking in the minimal gauge mediation is given by

$$W = X_i \Phi_i \Phi_i^\dagger + M_i \Phi_i \Phi_i^\dagger,$$

(2.1)

with goldstino $\eta_i$ contained in the $\theta$ component of chiral spurion superfield $X_i$, whose auxiliary component acquires F-term vacuum expectation value $F_i$. To get more general results, here the messenger fields $\Phi_i$ and $\Phi_i^\dagger$ are assumed to fill the $N + \bar{N}$ representation of a general SU($N$) gauge group. One could get similar corrections for U(1) gauge transformations through replacing the overall SU($N$) color factor $C_N$ by 1 after exchanging the gauge couplings (the rules might not be so simple when going to higher loops). The fermionic part of the messengers obtain Dirac masses $M_i$ which are not affected by SUSY breaking. And the bosonic parts have the mass matrix

$$\begin{pmatrix} |M_i|^2 & F_i \\ F_i^* & |M_i|^2 \end{pmatrix},$$

(2.2)

whose two eigenvalues are $|M_i|^2 \pm F_i$. In order to avoid tachyonic scalar masses and gauge symmetry breaking, usually it is assumed that $F_i \ll |M_i|^2$.

Next, let us turn to the goldstino part in eq. (2.1). In the two hidden sector scenario we define $F = \sqrt{F_1^2 + F_2^2}$ and $\tan \theta = F_2 / F_1$, then the combination $G = \eta_1 \cos \theta + \eta_2 \sin \theta$ is eaten by the super-Higgs mechanism, while one pseudo-goldstino $G' = -\eta_1 \sin \theta + \eta_2 \cos \theta$ is left. Due to the fact that $G$ has to be massless before the gravitational effects are taken into account, the radiatively generated mass matrix for goldstinos $\eta_{1,2}$ has to be of the following form

$$\begin{pmatrix} -\frac{F_2}{F_1} M_{12} & M_{12} \\ M_{12} & -\frac{F_1}{F_2} M_{12} \end{pmatrix},$$

(2.3)
where $\mathcal{M}_{12}$ denotes the loop induced mass mixing between the two goldstinos $\eta_1, \eta_2$, i.e., $-\frac{1}{2} \eta_1 \mathcal{M}_{12} \eta_2$. Now it is easy to get the Majorana mass of the pseudo-goldstino

$$m_{G'} = \left( \frac{F_1}{F_2} + \frac{F_2}{F_1} \right) \mathcal{M}_{12}. \tag{2.4}$$

The typical non-vanishing Feynman diagrams contributing to $\mathcal{M}_{12}$ which start at three loops are shown in figure 1, where the sectors are connected by supersymmetric gauge interactions. Physically only the zero external momentum could generate the mass correction, in other words we have to calculate the three-loop vacuum Feynman integrals. On the technical side, the analytical formulas written in terms of polylogarithms for general vacuum integrals at two-loop level have been known for some time. But the general results at three-loop level are still not available \[20–29\]. In the case of hierarchical momenta and masses, asymptotic expansions (see, e.g., refs. \[30, 31\]) could be applied to evaluate Feynman integrals. As mentioned above, the scalar mass of the messengers will be split to $|M_i|^2 \pm F_i$ after SUSY breaking. So there are four different mass scales in the first four diagrams and six scales in the last one of figure 1. Although these multi-scale integrals can not be evaluated directly, we can perform Taylor expansion on the scalar propagators to reduce the number of scales appearing in the integral

$$\frac{1}{\ell^2 - (M_i^2 \pm F_i)} = \frac{1}{\ell^2 - M_i^2} \left[ 1 + \frac{\pm F_i}{\ell^2 - M_i^2} + \ldots \right]. \tag{2.5}$$

This trick has been used by one of us in calculating the next-to-next-to-leading order hadronic correction to the muon anomalous magnetic momentum \[32\]. We also checked that in this way the same expanded results for the one-loop gaugino mass corrections would be reproduced as in ref. \[33\]. Thus, at the cost of increasing the number of integrals, we are left with two-scale vacuum integrals which could be solved analytically.
In our calculation the algebraic part is done with the help of FORM [34]. FIRE [35] which implements the Laporta algorithm [36] is used to reduce all the scalar integrals into a basis of master integrals (MI). The analytical results for these MIs which require the introduction of harmonic polylogarithms (HPLs) $H(a_1, \ldots, a_k; x)$ are already known [23]. For simplicity, we show the simplest HPLs and more details on their properties could be found in refs. [37–39]:

$$H(1; x) = - \log(1 - x),$$
$$H(0; x) = \log(x),$$
$$H(-1; x) = \log(1 + x).$$

(2.6)

3 Results and discussions

From eq. (2.4) we learn that the mass of pseudo-goldstino could be easily derived from the radiative corrections to the matrix element $\mathcal{M}_{12}$. In the following we choose to present $\mathcal{M}_{12}$ by grouping terms according to the power of two independent SUSY breaking scales $F_{1,2}$:

$$\mathcal{M}_{12} = \frac{1}{4} \left( \frac{g^4}{(16\pi^2)^3} \right) C_N \left[ A_1 \frac{F_1}{M_1} + A_2 \frac{F_2}{M_1} + B_1 \frac{F_3^3}{M_1^3} + B_2 \frac{F_2^2}{M_1^2} + B_3 \frac{F_1 F_2^2}{M_1^2} + B_4 \frac{F_3^3}{M_1^3} ight. + C_1 \frac{F_1^3}{M_1^3} + C_2 \frac{F_1^3 F_2}{M_1^3} + C_3 \frac{F_2^3}{M_1^3} + C_4 \frac{F_1^3 F_2^2}{M_1^3} + C_5 \frac{F_1^3 F_2}{M_1^3} + C_6^{13} \frac{F_3^3}{M_1^3} + \ldots \right],$$

(3.1)

where $C_N = (N^2 - 1)/4$ is the color factor for SU($N$) gauge group and $g$ is the corresponding gauge coupling. One new dimensionless variable $x = M_2/M_1$ is introduced to eliminate the second messenger mass scale $M_2$, so the coefficients $A_i$, $B_i$ and $C_i$ in the above equation are just functions of $x$. Although $\mathcal{O}(F_1^3)$ terms with coefficients $D_i$ are too long to be shown, their contributions are considered in the following analysis. The expressions of coefficients $A_i$, $B_i$ and $C_i$ are given by

$$A_1 = \frac{8}{(-1 + x^2)} \left[ 2x H(0, 0; x) + (-1 + x^2) \left( H(-1, 0, 0; x) + H(1, 0, 0; x) \right) \right],$$

(3.2)

$$A_2 = \frac{8}{x(-1 + x^2)} \left[ -2x H(0, 0; x) + (-1 + x^2) \left( H(-1, 0, 0; x) + H(1, 0, 0; x) \right) \right],$$

(3.3)

$$B_1 = \frac{1}{48x^5((-1 + x^2)^3)} \left[ -6x^4 + 22x^6 - 26x^8 + 10x^{10} + 6x^4 H(0; x) ight. \\ \\ - 2x^6 H(0; x) + 2x^8 H(0; x) - 6x^{10} H(0; x) - 6x^4 H(0; x) \\ + 22x^6 H(0, 0; x) + 22x^8 H(0, 0; x) - 6x^{10} H(0, 0; x) + 3x^5 H(-1, 0, 0; x) \\ - 12x^5 H(-1, 0, 0; x) + 18x^7 H(-1, 0, 0; x) - 12x^9 H(-1, 0, 0; x) \\ + 3x^{11} H(-1, 0, 0; x) + 3x^3 H(1, 0, 0; x) - 12x^5 H(1, 0, 0; x) \\ + 18x^7 H(1, 0, 0; x) - 12x^9 H(1, 0, 0; x) + 3x^{11} H(1, 0, 0; x) \right],$$

(3.4)
\[
B_2 = \frac{1}{48x^5(-1 + x^2)^3} \left[ -12x^3 + 28x^5 - 20x^7 + 4x^9 + 12x^3H(0; x)
- 116x^5H(0; x) + 164x^7H(0; x) - 60x^9H(0; x) - 12x^3H(0, 0; x)
- 164x^5H(0, 0; x) + 172x^7H(0, 0; x) - 60x^9H(0, 0; x) + 6x^2H(1, 0; 0; x)
- 48x^4H(-1, 0, 0; x) + 108x^6H(-1, 0; x) - 96x^8H(-1, 0; x)
+ 30x^{10}H(-1, 0, 0; x) + 6x^2H(1, 0, 0; x) - 48x^4H(1, 0, 0; x)
+ 108x^6H(1, 0, 0; x) - 96x^8H(1, 0, 0; x) + 30x^{10}H(1, 0, 0; x) \right],
\]

\[
B_3 = \frac{1}{48x^5(-1 + x^2)^3} \left[ -4x^2 + 20x^4 - 28x^6 + 12x^8 - 60x^2H(0; x) + 164x^4H(0; x)
- 116x^6H(0; x) + 12x^8H(0; x) + 60x^2H(0, 0; x) - 172x^4H(0, 0; x)
+ 164x^6H(0, 0; x) + 12x^8H(0, 0; x) - 30xH(1, 0, 0; x) + 96x^3H(-1, 0, 0; x)
- 108x^5H(-1, 0, 0; x) + 48x^7H(-1, 0, 0; x) - 6x^9H(-1, 0, 0; x)
- 30xH(1, 0, 0; x) + 96x^3H(1, 0, 0; x) - 108x^5H(1, 0, 0; x)
+ 48x^7H(1, 0, 0; x) - 6x^9H(1, 0, 0; x) \right],
\]

\[
B_4 = \frac{1}{48x^5(-1 + x^2)^3} \left[ -10x + 26x^3 - 22x^5 + 6x^7 - 6xH(0; x) + 2x^3H(0; x)
- 2x^5H(0; x) + 6x^7H(0; x) + 6xH(0, 0; x) - 22x^3H(0, 0; x) - 22x^5H(0, 0; x)
+ 6x^7H(0, 0; x) - 3H(-1, 0, 0; x) + 12x^2H(-1, 0, 0; x) - 18x^4H(-1, 0, 0; x)
+ 12x^6H(-1, 0, 0; x) - 3x^8H(-1, 0, 0; x) - 3H(1, 0, 0; x) + 12x^2H(1, 0, 0; x)
- 18x^4H(1, 0, 0; x) + 12x^6H(1, 0, 0; x) - 3x^8H(1, 0, 0; x) \right],
\]

\[
C_1 = \frac{1}{92160x^5(-1 + x^2)^3} \left[ 2430x^6 - 16560x^8 + 42062x^{10} - 51536x^{12} + 32154x^{14}
- 9728x^{16} + 1178x^{18} - 2430x^6H(0; x) + 16380x^8H(0; x) - 66020x^{10}H(0; x)
+ 133224x^{12}H(0; x) - 127026x^{14}H(0; x) + 56540x^{16}H(0; x) - 10086x^{18}H(0; x)
+ 2430x^6H(0, 0; x) - 15840x^8H(0, 0; x) + 27246x^{10}H(0, 0; x) - 59280x^{12}H(0, 0; x)
+ 100218x^{14}H(0, 0; x) - 63120x^{16}H(0, 0; x) + 14490x^{18}H(0, 0; x)
- 1215x^5H(-1, 0, 0; x) + 8325x^7H(-1, 0, 0; x) - 16155x^9H(-1, 0, 0; x)
- 1575x^{11}H(-1, 0, 0; x) + 43875x^{13}H(-1, 0, 0; x) - 59985x^{15}H(-1, 0, 0; x)
+ 33975x^{17}H(-1, 0, 0; x) - 7245x^{19}H(-1, 0, 0; x) - 1215x^5H(1, 0, 0; x)
+ 8325x^7H(1, 0, 0; x) - 16155x^9H(1, 0, 0; x) - 1575x^{11}H(1, 0, 0; x)
+ 43875x^{13}H(1, 0, 0; x) - 59985x^{15}H(1, 0, 0; x) + 33975x^{17}H(1, 0, 0; x)
- 7245x^{19}H(1, 0, 0; x) \right],
\]
\begin{align}
C_2 &= \frac{1}{92160x^9(-1 + x^2)^5} \left[ -2430x^5 + 5760x^7 - 9342x^9 + 18288x^{11} - 22266x^{13} + 12816x^{15} - 282x^{17} + 2430x^5H(0; x) - 5580x^7H(0; x) - 93318x^9H(0; x) + 293112x^{11}H(0; x) - 345006x^{13}H(0; x) + 188052x^{15}H(0; x) - 39690x^{17}H(0; x) - 2430x^5H(0; x) + 5040x^7H(0; x) - 148254x^9H(0; x) + 341040x^{11}H(0; x) - 365658x^{13}H(0; x) + 191520x^{15}H(0; x) - 39690x^{17}H(0; x) + 1215x^4H(-1, 0, 0; x) - 2925x^6H(-1, 0, 0; x) - 23445x^8H(-1, 0, 0; x) + 118575x^{10}H(-1, 0, 0; x) - 23875x^{12}H(-1, 0, 0; x) + 212985x^{14}H(-1, 0, 0; x) - 102375x^{16}H(-1, 0, 0; x) + 19845x^{18}H(-1, 0, 0; x) + 1215x^4H(1, 0, 0; x) - 2925x^6H(1, 0, 0; x) - 23445x^8H(1, 0, 0; x) + 118575x^{10}H(1, 0, 0; x) - 23875x^{12}H(1, 0, 0; x) + 212985x^{14}H(1, 0, 0; x) - 102375x^{16}H(1, 0, 0; x) + 19845x^{18}H(1, 0, 0; x) \right], \quad (3.9) \\
C_3 &= \frac{1}{92160x^9(-1 + x^2)^5} \left[ -3150x^4 + 13120x^6 - 18190x^8 + 9840x^{10} - 490x^{12} - 2480x^{14} + 1350x^{16} + 3150x^4H(0; x) - 16300x^6H(0; x) + 31530x^8H(0; x) - 16200x^{10}H(0; x) + 3970x^{12}H(0; x) - 7500x^{14}H(0; x) + 1350x^{16}H(0; x) - 3150x^4H(0; x) + 15600x^6H(0; x) - 32430x^8H(0; x) + 37680x^{10}H(0; x) + 18870x^{12}H(0; x) - 7200x^{14}H(0; x) + 1350x^{16}H(0; x) + 1575x^3H(-1, 0, 0; x) - 8325x^5H(-1, 0, 0; x) + 18675x^7H(-1, 0, 0; x) - 23625x^9H(-1, 0, 0; x) + 19125x^{11}H(-1, 0, 0; x) - 10575x^{13}H(-1, 0, 0; x) + 3825x^{15}H(-1, 0, 0; x) - 675x^{17}H(-1, 0, 0; x) + 1575x^3H(1, 0, 0; x) + 18675x^7H(1, 0, 0; x) - 23625x^9H(1, 0, 0; x) + 19125x^{11}H(1, 0, 0; x) - 10575x^{13}H(1, 0, 0; x) + 3825x^{15}H(1, 0, 0; x) - 675x^{17}H(1, 0, 0; x) \right], \quad (3.10) \\
C_4 &= \frac{1}{92160x^9(-1 + x^2)^5} \left[ -1350x^3 + 2480x^5 + 490x^7 - 9840x^9 + 18190x^{11} - 13120x^{13} + 3150x^{15} + 1350x^3H(0; x) - 7500x^5H(0; x) + 3970x^7H(0; x) - 16200x^{10}H(0; x) + 31530x^{11}H(0; x) - 16300x^{13}H(0; x) + 3150x^{15}H(0; x) - 1350x^3H(0; x) + 7200x^5H(0; x) - 18870x^7H(0; x) + 37680x^9H(0; x) + 32430x^{11}H(0; x) - 15600x^{13}H(0; x) + 3150x^{15}H(0; x) + 675x^2H(-1, 0, 0; x) - 3825x^4H(-1, 0, 0; x) + 10575x^6H(-1, 0, 0; x) - 19125x^8H(-1, 0, 0; x) + 23625x^{10}H(-1, 0, 0; x) - 18675x^{12}H(-1, 0, 0; x) + 8325x^{14}H(-1, 0, 0; x) - 1575x^{16}H(-1, 0, 0; x) + 675x^2H(1, 0, 0; x) - 3825x^4H(1, 0, 0; x) + 10575x^6H(1, 0, 0; x) - 19125x^8H(1, 0, 0; x) + 23625x^{10}H(1, 0, 0; x) - 18675x^{12}H(1, 0, 0; x) + 8325x^{14}H(1, 0, 0; x) - 1575x^{16}H(1, 0, 0; x) \right], \quad (3.11)
\end{align}
\[ C_5 = -\frac{1}{92160x^9(-1 + x^2)^5} \left[ -2826x^2 + 12816x^4 - 22266x^6 + 18288x^8 - 9342x^{10} \\
+ 5760x^{12} - 2430x^{14} + 39690x^2H(0; x) - 188052x^4H(0; x) + 345006x^6H(0; x) \\
- 293112x^8H(0; x) + 93318x^{10}H(0; x) + 5580x^{12}H(0; x) - 2430x^{14}H(0; x) \\
- 39690x^2H(0, 0; x) + 191520x^4H(0, 0; x) - 365658x^6H(0, 0; x) \\
+ 341040x^8H(0, 0; x) - 148254x^{10}H(0, 0; x) + 5040x^{12}H(0, 0; x) \\
- 2430x^{14}H(0, 0; x) + 19845xH(-1, 0, 0; x) - 102375x^3H(-1, 0, 0; x) \\
+ 212985x^5H(-1, 0, 0; x) - 223875x^7H(-1, 0, 0; x) + 118575x^9H(-1, 0, 0; x) \\
- 23445x^11H(-1, 0, 0; x) - 2925x^{13}H(-1, 0, 0; x) + 1215x^{15}H(-1, 0, 0; x) \\
+ 19845xH(1, 0, 0; x) - 102375x^3H(1, 0, 0; x) + 212985x^5H(1, 0, 0; x) \\
- 223875x^7H(1, 0, 0; x) + 118575x^9H(1, 0, 0; x) - 23445x^{11}H(1, 0, 0; x) \\
- 2925x^{13}H(1, 0, 0; x) + 1215x^{15}H(1, 0, 0; x) \right], \quad (3.12) \]

\[ C_6 = -\frac{1}{92160x^9(-1 + x^2)^5} \left[ -1178x + 9728x^3 - 32154x^5 + 51536x^7 - 42062x^9 \\
+ 16560x^{11} - 2430x^{13} - 10086xH(0; x) + 56540x^3H(0; x) - 127026x^5H(0; x) \\
+ 133224x^7H(0; x) - 66602x^9H(0; x) + 16380x^{11}H(0; x) - 2430x^{13}H(0; x) \\
- 14490xH(0, 0; x) + 63120x^3H(0, 0; x) - 100218x^5H(0, 0; x) + 59280x^7H(0, 0; x) \\
- 27246x^9H(0, 0; x) + 15840x^{11}H(0, 0; x) - 2430x^{13}H(0, 0; x) + 7245H(-1, 0, 0; x) \\
- 33975x^2H(-1, 0, 0; x) + 59985x^4H(-1, 0, 0; x) - 43875x^6H(-1, 0, 0; x) \\
+ 1575x^8H(-1, 0, 0; x) + 16155x^{10}H(-1, 0, 0; x) - 8325x^{12}H(-1, 0, 0; x) \\
+ 1215x^{14}H(-1, 0, 0; x) + 7245H(1, 0, 0; x) - 33975x^2H(1, 0, 0; x) \\
+ 59985x^4H(1, 0, 0; x) - 43875x^6H(1, 0, 0; x) + 1575x^8H(1, 0, 0; x) \\
+ 16155x^{10}H(1, 0, 0; x) - 8325x^{12}H(1, 0, 0; x) + 1215x^{14}H(1, 0, 0; x) \right]. \quad (3.13) \]

From above equations, it is easy to see that there are no contributions with even powers of \( F_i \). The reason is rather simple: perturbative expansion of \( F_i \) in the Feynman diagrams plays the role of converting the scalar components of the superfields from one to another, and even times of these transformations lead to SUSY-preserving interactions which should vanish due to the symmetry constraint. Same behaviors have also been found for the one-loop gaugino mass corrections in the model of minimal gauge mediation \cite{33}, where even powers of \( F_i \) also do not contribute.

Next, another important check besides UV-finiteness\(^{1}\) on the correctness of our result will be explained. When \( x \) is fixed to be 1, the result for the related functions expressed in terms of complicated HPLs could be greatly simplified. In this case, one will get the

\(^{1}\)Here no renormalization is needed since the first non-vanishing correction starts at three loops. All the calculations are performed in four dimensions before obtaining the finite intermediate results which are expressed in terms of scalar integrals. So, there should be no violations of SUSY relations as discussed, e.g., in ref.\cite{40}.
Table 1. Values of $A_i$, $B_i$, $C_i$ and $D_i$ defined in eq. (3.1) when the two messenger masses are equal. Symmetric parts are not displayed. $\zeta_n$ denotes the Riemann’s zeta function.

Figure 2. Plots of the functions $A$, $B$, $C$ and $D$.

expressions listed in table 1. It means that when the two messenger masses are the same, i.e., we can set $M_1 = M_2$ at the beginning of the calculation, then we do the reduction for single-scale integrals and obtain single-scale MIs. The second approach has been done independently and thus provides a good cross-check.

With the assumption $F_A = F_B$, at the leading power of $F_i$ one could define a new function $A = \sum_i A_i$ to parameterize the radiative corrections. From the arguments based on internal symmetry $M_1 \leftrightarrow M_2$, we know that $A(x)$ should equal to $A(1/x)/x$ which has been verified. Similar self-consistent checks have also been performed for new functions $B$, $C$ and $D$. In order to get an estimation of the numerical effects of the higher order terms expanded in $F_i$, two plots are shown in figure 2. Obviously, the functions $B$, $C$, $D$ are much smaller than $A$ for moderate values of $x$. The conclusion does not change for large values of $x$, e.g., we get $A \simeq 4.30$, $B \simeq 0.003$, $C \simeq -0.01$, $D \simeq -0.01$ when $x = 100$. So, the higher order terms could be safely neglected. From table 2 and figure 2(a) one could see that the value of $A$ decreases with the increase of $x$. It means we can get a much lighter pseudo-goldstino when the messenger scales are different.

Note that comparing to the usual low energy soft SUSY breaking parameter $m_{\text{soft}}$, the pseudo-goldstino mass is suppressed by a factor $g^2/(16\pi^2)^2$ when both the SUSY breaking and messenger scales are comparable. Considering the summation effects over different gauge sectors, the authors in ref. [1] concluded that the mass of pseudo-goldstino is around
the GeV scale in this case. Thus, it is rather simple to see from table 2 that the pseudo-goldstino mass can be further suppressed by the ratio of messenger masses, e.g., as low as $\mathcal{O}(0.1) \text{GeV}$ when $x = M_2/M_1 = 100$.

Before the end of this section, let us make a direct comparison with the result obtained in [1]. The authors considered the scenario of equal messenger scales and then performed numerical evaluation. The approximated value 4.21 in appendix B of ref. [1] agrees well with our analytical result at $\mathcal{O}(F_i)$ which corresponding to $\frac{7}{2}\zeta_3$, when the same convention as theirs is used.

### 4 Conclusion

We calculated the three-loop contribution to the mass of pseudo-goldstino explicitly. Since the exact evaluation of a basis of MIs for general vacuum integrals at the three-loop level is still unavailable, the implicit conditions $F_i \ll M_i^2$ are used to reduce the number of scales we have to deal with. After performing expansions in $F_i$, we provided analytical results which are in good agreement with the numerical value in the literature. It has been proved that the mass correction is dominated by the leading order contribution at $\mathcal{O}(F_i)$. We also found that the mass correction could be greatly reduced if there is a hierarchy between different messenger scales. Note that although the whole calculation is based on the two-sector SUSY-breaking scenario, our results could also be generalized to multi-sector cases as long as these sectors communicate with each other via gauge interactions. So, our result presented in this note could be useful for phenomenological analysis or model-buildings on pseudo-goldstino in the future.

### Acknowledgments

This work was supported in part by IHEP under Grant No. Y9515570U1, by the National Natural Science Foundation of China (NNSFC) under grant Nos. 11821505 and 12075300, by Peng-Huan-Wu Theoretical Physics Innovation Center (12047503), by the CAS Center for Excellence in Particle Physics (CCEPP), by the CAS Key Research Program of Frontier Sciences, and by a Key R&D Program of Ministry of Science and Technology of China under number 2017YFA0402204.

**Open Access.** This article is distributed under the terms of the Creative Commons Attribution License (CC-BY 4.0), which permits any use, distribution and reproduction in any medium, provided the original author(s) and source are credited.
References

[1] R. Argurio, Z. Komargodski and A. Mariotti, Pseudo-Goldstini in Field Theory, Phys. Rev. Lett. 107 (2011) 061601 [arXiv:1102.2386] [inSPIRE].

[2] P. Meade, N. Seiberg and D. Shih, General Gauge Mediation, Prog. Theor. Phys. Suppl. 177 (2009) 143 [arXiv:0811.3278] [inSPIRE].

[3] C. Cheung, Y. Nomura and J. Thaler, Goldstini, JHEP 03 (2010) 073 [arXiv:1002.1967] [inSPIRE].

[4] C. Cheung, J. Mardon, Y. Nomura and J. Thaler, A Definitive Signal of Multiple Supersymmetry Breaking, JHEP 07 (2010) 035 [arXiv:1004.4637] [inSPIRE].

[5] N. Craig, J. March-Russell and M. McCullough, The Goldstini Variations, JHEP 10 (2010) 095 [arXiv:1007.1239] [inSPIRE].

[6] M. McCullough, Stimulated Supersymmetry Breaking, Phys. Rev. D 82 (2010) 115016 [arXiv:1010.3203] [inSPIRE].

[7] H.-C. Cheng, W.-C. Huang, I. Low and A. Menon, Goldstini as the decaying dark matter, JHEP 03 (2011) 019 [arXiv:1012.5300] [inSPIRE].

[8] K.I. Izawa, Y. Nakai and T. Shimomura, Higgs Portal to Visible Supersymmetry Breaking, JHEP 03 (2011) 007 [arXiv:1101.4633] [inSPIRE].

[9] J. Thaler and Z. Thomas, Goldstini Can Give the Higgs a Boost, JHEP 07 (2011) 060 [arXiv:1103.1631] [inSPIRE].

[10] C. Cheung, F. D’Eramo and J. Thaler, The Spectrum of Goldstini and Modulini, JHEP 08 (2011) 115 [arXiv:1104.2600] [inSPIRE].

[11] E. Dudas, G. von Gersdorff, D.M. Ghilencea, S. Lavignac and J. Parmentier, On non-universal Goldstino couplings to matter, Nucl. Phys. B 855 (2012) 570 [arXiv:1106.5792] [inSPIRE].

[12] R. Argurio, K. De Causmaecker, G. Ferretti, A. Mariotti, K. Mawatari and Y. Takaesu, Collider signatures of goldstini in gauge mediation, JHEP 06 (2012) 096 [arXiv:1112.5058] [inSPIRE].

[13] T. Liu, L. Wang and J.M. Yang, Higgs decay to goldstini and its observability at the LHC, Phys. Lett. B 726 (2013) 228 [arXiv:1301.5479] [inSPIRE].

[14] G. Ferretti, A. Mariotti, K. Mawatari and C. Petersson, Multiphoton signatures of goldstini at the LHC, JHEP 04 (2014) 126 [arXiv:1312.1698] [inSPIRE].

[15] K.-i. Hikasa, T. Liu, L. Wang and J.M. Yang, Pseudo-goldstino and electroweak gauginos at the LHC, JHEP 07 (2014) 065 [arXiv:1403.5731] [inSPIRE].

[16] T. Liu, L. Wang and J.M. Yang, Pseudo-goldstino and electroweakinos via VBF processes at LHC, JHEP 02 (2015) 177 [arXiv:1411.6105] [inSPIRE].

[17] J. Chen, C. Han, J.M. Yang and M. Zhang, Probing bino NLSP at lepton colliders, arXiv:2101.12131 [inSPIRE].

[18] J. Cao, X. Du, Z. Li, F. Wang and Y. Zhang, Explaining The XENON1T Excess With Light Goldstini Dark Matter, arXiv:2007.09981 [inSPIRE].

[19] F. Wang, L. Wu, Y. Xiao, J.M. Yang and Y. Zhang, GUT-scale constrained SUSY in light of E989 muon g-2 measurement, arXiv:2104.03262 [inSPIRE].
[20] J. Fleischer and M.Y. Kalmykov, *Single mass scale diagrams: Construction of a basis for the $\epsilon$-expansion*, Phys. Lett. B **470** (1999) 168 [hep-ph/9910223] [inSPIRE].

[21] K.G. Chetyrkin and M. Steinhauser, *The Relation between the MS-bar and the on-shell quark mass at order $\alpha^3_3$*, Nucl. Phys. B **573** (2000) 617 [hep-ph/9911434] [inSPIRE].

[22] Y. Schröder and A. Vuorinen, *High-precision $\epsilon$-expansions of single-mass-scale four-loop vacuum bubbles*, JHEP **06** (2005) 051 [hep-ph/0503209] [inSPIRE].

[23] J. Grigo, J. Hoff, P. Marquard and M. Steinhauser, *Moments of heavy quark correlators with two masses: exact mass dependence to three loops*, Nucl. Phys. B **864** (2012) 580 [arXiv:1206.3418] [inSPIRE].

[24] S. Bekavac, A.G. Grozin, D. Seidel and V.A. Smirnov, *Three-loop on-shell Feynman integrals with two masses*, Nucl. Phys. B **819** (2009) 183 [arXiv:0903.4760] [inSPIRE].

[25] A. Freitas, *Three-loop vacuum integrals with arbitrary masses*, JHEP **11** (2016) 145 [arXiv:1609.09159] [inSPIRE].

[26] S.P. Martin and D.G. Robertson, *Evaluation of the general 3-loop vacuum Feynman integral*, Phys. Rev. D **95** (2017) 016008 [arXiv:1610.07720] [inSPIRE].

[27] J.J. van der Bij, K.G. Chetyrkin, M. Faisst, G. Jikia and T. Seidensticker, *Three loop leading top mass contributions to the rho parameter*, Phys. Lett. B **498** (2001) 156 [hep-ph/0011373] [inSPIRE].

[28] M. Faisst, J.H. Kühn, T. Seidensticker and O. Veretin, *Three loop top quark contributions to the rho parameter*, Nucl. Phys. B **665** (2003) 649 [hep-ph/0302275] [inSPIRE].

[29] V.A. Smirnov, *Applied asymptotic expansions in momenta and masses*, Springer Tracts Mod. Phys. **177** (2002) 1 [inSPIRE].

[30] A. Kurz, T. Liu, P. Marquard and M. Steinhauser, *Hadronic contribution to the muon anomalous magnetic moment to next-to-next-to-leading order*, Phys. Lett. B **734** (2014) 144 [arXiv:1403.6400] [inSPIRE].

[31] T. Seidensticker, *Automatic application of successive asymptotic expansions of Feynman diagrams*, in 6th International Workshop on New Computing Techniques in Physics Research: Software Engineering, Artificial Intelligence Neural Nets, Genetic Algorithms, Symbolic Algebra, Automatic Calculation, (1999) [hep-ph/9905298] [inSPIRE].

[32] S.P. Martin, *Generalized messengers of supersymmetry breaking and the sparticle mass spectrum*, Phys. Rev. D **55** (1997) 3177 [hep-ph/9608224] [inSPIRE].

[33] J. Kuipers, T. Ueda, J.A.M. Vermaseren and J. Vollinga, *FORM version 4.0*, Comput. Phys. Commun. **184** (2013) 1453 [arXiv:1203.6543] [inSPIRE].

[34] A.V. Smirnov, *FIRE5: a C++ implementation of Feynman Integral REduction*, Comput. Phys. Commun. **189** (2015) 182 [arXiv:1408.2372] [inSPIRE].

[35] S. Laporta, *High precision calculation of multiloop Feynman integrals by difference equations*, Int. J. Mod. Phys. A **15** (2000) 5087 [hep-ph/0102033] [inSPIRE].

[36] E. Remiddi and J.A.M. Vermaseren, *Harmonic polylogarithms*, Int. J. Mod. Phys. A **15** (2000) 725 [hep-ph/9905237] [inSPIRE].
[38] D. Maître, *HPL, a mathematica implementation of the harmonic polylogarithms*, *Comput. Phys. Commun.* **174** (2006) 222 [hep-ph/0507152] [inSPIRE].

[39] D. Maître, *Extension of HPL to complex arguments*, *Comput. Phys. Commun.* **183** (2012) 846 [hep-ph/0703052] [inSPIRE].

[40] D. Stöckinger and J. Unger, *Three-loop MSSM Higgs-boson mass predictions and regularization by dimensional reduction*, *Nucl. Phys. B* **935** (2018) 1 [arXiv:1804.05619] [inSPIRE].