Fuzzy Hungarian Method for Solving Intuitionistic Fuzzy Travelling Salesman Problem

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ABSTRACT
The travelling salesman problem is to identify the shortest route that the salesman journey all the places and return the starting place with minimum cost. We develop a fuzzy version of Hungarian algorithm for the solution of intuitionistic fuzzy travelling salesman problem using triangular intuitionistic fuzzy numbers without changing them to classical travelling salesman problem. The purposed method is easy to empathize and to implement for finding solution of intuitionistic travelling salesman problem happening in real life situations. To illustrate the proposed method numerical example are provided.

1. INTRODUCTION
The Traveling Salesman Problem (TSP) was introduced by Irish mathematician W.R Hamilton in the 19th century. A huge number of methods were formulated to solve the problem. The aim or goal of the problem is to identify the shortest route of the salesman starting from a given city, journey all other cities only once and finally return to the same city where he started. Traveling salesman problems are classical and widely studied in Combinatorial Optimization. It is an important problem in Artificial Intelligence and Operations Research domain and has been studied intensively, in both Operations Research and Computer Science. Sometimes information available in real life system is of vague, imprecise and uncertain nature. Under such situations it is highly impossible to formulate the mathematical model through the classical traditional methods. Therefore the use of fuzzy set theory proposed by Zadeh[25] in 1965 is more appropriate to model and solve the real world problems involving imprecise parameters.

In 1970s, Bellman and Zadeh [5] introduced the concepts and problem of building under uncertain environment. Tanaka et al. [24] proposed the theory of fuzzy mathematical programming. Zimmerman [27] proposed the initial conceptualization of Fuzzy linear programming to deal with the impreciseness and inexactness of the parameters in LPP with fuzzy constraints and objective functions. Sometimes the idea of fuzzy set theory is not good enough to deal with the vagueness in linear programming problems. In 1986, the idea of intuitionistic fuzzy sets was proposed by Atanassov [4] as an extension of Lotfi Zadeh’s [25] idea of fuzzy sets. In intuitionistic fuzzy sets, degree of membership (acceptance) and degree of non-membership (elimination) are defined simultaneously so that addition of these values is at most one. Since intuitionistic fuzzy set theory involves both the degree of acceptance and degree of rejection of each element, So Intuitionistic fuzzy set theory more suitable to deal with ambiguity and vagueness comparing with fuzzy sets.
In recent years, fuzzy travelling salesman problem has got great tending and the problems in fuzzy travelling salesman problem have been approached using several methods. Jain [13] introduced a new algorithm for fractional transshipment problem. Mukerjee and Basu [21] proposed a new method to solve fuzzy travelling salesman problem. Majumdar J. et al. [18] used genetic algorithm to solve asymmetric travelling salesman problem with fuzzy costs. Kumar and Gupta [2] has been solved the fuzzy travelling salesman problem for LR-fuzzy parameters. Fischer and Richter [8] proposed a method for solving a multi objective travelling salesman problem by dynamic programming. Zimmermann [27] used fuzzy programming and linear programming with several objective functions. Jain and Lachhwani [14] proposed a new method for solving fuzzy bi-level linear programming problem. Here we investigate a more realistic problem, namely intuitionistic fuzzy travelling salesman problem using new arithmetic operations and new ranking method.

This paper is organized as follows: In section 2, we recall fundamental concept and outcome of Triangular intuitionistic fuzzy numbers, a new type of arithmetic operations, a new ranking method on Intuitionistic fuzzy numbers and some related results. In section 3, we define Intuitionistic fuzzy travelling salesman problem as an extension of the classical travelling salesman problem and propose fuzzy Hungarian algorithm. In section 4, numerical examples are furnished and the obtained results are discussed.

2. PRELIMINARIES

This section provides fundamental concept and outcome of Triangular intuitionistic fuzzy numbers which are useful for our future discussions.

Definition 2.1
An Intuitionistic Fuzzy set (IFS) in the universe of discourse X is defined as
\[ \tilde{a}^I = \left\{ (x, \mu_a(x), \gamma_a(x)) / x \in X \right\} \]
where the functions \( \mu_a : X \rightarrow [0,1] \) and \( \gamma_a : X \rightarrow [0,1] \) determine the degree of membership and degree of non membership of the element \( x \in X \), respectively, and for every \( x \in X \), \( 0 \leq \mu_a(x) + \gamma_a(x) \leq 1 \).

Note 1: Throughout this paper \( \mu_a(x) \) represents membership value and \( \gamma_a(x) \) represents non membership value of \( x \) in \( \tilde{a}^I \).

Definition 2.2
The Intuitionistic Fuzzy Index of \( x \) in \( \tilde{a}^I \) is defined as \( \pi_a(x) = 1 - \mu_a(x) - \gamma_a(x) \). It is also known as degree of hesitancy or degree of uncertainty of the element \( x \) in \( \tilde{a}^I \). Obviously, for every \( x \in X \), \( 0 \leq \pi_a(x) \leq 1 \).

Definition 2.3
An Intuitionistic Fuzzy Number (IFN) \( \tilde{a}^I \) is

(i) an intuitionistic fuzzy subset of the real line,
(ii). normal, that is there is any \( x_0 \in \mathbb{R} \), such that \( \mu_d(x_0) = 1, \gamma_d(x_0) = 0 \).

(iii). Convex for the membership function \( \mu_d(x) \), that is,

\[
\mu_d(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_d(x_1),\mu_d(x_2)) \text{ for } x_1, x_2 \in \mathbb{R}, \lambda \in [0,1].
\]

(iv). Concave for the non-membership function \( \gamma_d(x) \), that is,

\[
\gamma_d(\lambda x_1 + (1-\lambda)x_2) \leq \max(\gamma_d(x_1),\gamma_d(x_2)) \text{ for } x_1, x_2 \in \mathbb{R}, \lambda \in [0,1].
\]

**Definition 2.4**

A triangular intuitionistic fuzzy number (TIFN) with parameters \( a_1 \leq a_i \leq a_2 \leq a_j \leq a_3 \) is an intuitionistic fuzzy number having the membership function and non-membership function as follows:

\[
\mu_d(x) = \begin{cases} 
0 & \text{for } x < a_1 \\
\frac{x-a_1}{a_2-a_1} & \text{for } a_1 \leq x \leq a_2 \\
1 & \text{for } x = a_2 \\
\frac{a_3-x}{a_3-a_2} & \text{for } a_2 \leq x \leq a_3 \\
0 & \text{for } x > a_3
\end{cases}
\]

and

\[
\gamma_d(x) = \begin{cases} 
1 & \text{for } x < a_1 \\
\frac{a_3-x}{a_3-a_2} & \text{for } a_1 \leq x \leq a_2 \\
0 & \text{for } x = a_2 \\
\frac{x-a_3}{a_3-a_2} & \text{for } a_2 \leq x \leq a_3 \\
1 & \text{for } x > a_3
\end{cases}
\]

and is denoted by \( \tilde{a}^l = (a_1, a_2, a_3; a_1, a_2, a_3) \).
Let $F(R)$ denote the set of all triangular intuitionistic fuzzy numbers.

**Note 2:** Here $\mu_{a_{i}}(x)$ growing with constant rate for $x \in [a_{1}, a_{2}]$ and diminishes with constant rate for $x \in [a_{2}, a_{3}]$, but $\gamma_{a_{i}}(x)$ diminishes with constant rate for $x \in [a_{1}, a_{2}]$ and growing with constant rate for $x \in [a_{2}, a_{3}]$.

**Particular Case:**

Let $\tilde{a}^{1} = (a_{1}, a_{2}, a_{3}; a_{1}^{*}, a_{2}^{*}, a_{3}^{*})$ be a triangular intuitionistic fuzzy number then the following cases arises.

**Case 1:** If $a_{1}^{*} = a_{1}$, $a_{3}^{*} = a_{3}$ then $\tilde{a}^{1}$ represent Triangular Fuzzy number (TFN).

**Case 2:** If $a_{1}^{*} = a_{1}$, $a_{2}^{*} = a_{2}$, $a_{3}^{*} = a_{3}$, $m$ then $\tilde{a}^{1}$ represent a real number $m$.

Also if $m = a_{2}$ represents the modal value (or) midpoint, $\alpha_{1} = (a_{2} - a_{1})$ represents the left spread and $\beta_{1} = (a_{1} - a_{2})$ right spread of membership function and $\alpha_{2} = (a_{2} - a_{1})$ represents the left spread and $\beta_{2} = (a_{3} - a_{2})$ right spread of non-membership function.

Then the triangular intuitionistic fuzzy number $\tilde{a}^{1}$ can be represented by the triplet $\tilde{a}^{1} = (\alpha_{1}, m, \beta_{1}; \alpha_{2}, m, \beta_{2})$ i.e., $\tilde{a}^{1} = (a_{1}, a_{2}, a_{3}; a_{1}^{*}, a_{2}^{*}, a_{3}^{*}) = (\alpha_{1}, m, \beta_{1}; \alpha_{2}, m, \beta_{2})$

**Definition 2.5**

Triangular Intuitionistic fuzzy number $\tilde{a}^{1} \in F(R)$ can also be represented as a pair $\tilde{a}^{1} = (a, \overline{a}; a^{*}, \overline{a}^{*})$ of functions $a(r), \overline{a}(r), a^{*}(r)$ and $\overline{a}^{*}(r)$ for $0 \leq r \leq 1$ which satisfies the following requirements:
(i). \( \underline{a}(r) \) is a bounded monotonic increasing left continuous function for membership function

(ii). \( \overline{a}(r) \) is a bounded monotonic decreasing left continuous function for membership function.

(iii). \( \underline{a}(r) \leq \overline{a}(r), 0 \leq r \leq 1 \)

(iv). \( \overline{a}'(r) \) is a bounded monotonic decreasing left continuous function for non-membership function.

(v). \( \overline{a}(r) \) is a bounded monotonic increasing left continuous function for non-membership function.

(vi). \( \underline{a}'(r) \leq \overline{a}'(r), 0 \leq r \leq 1 \).

Definition 2.6

For an arbitrary triangular Intuitionistic fuzzy number \( \tilde{a}^l = \left( \underline{a}, \overline{a}; \underline{a}', \overline{a}' \right) \), the number

\[
a_0 = \frac{\left( \underline{a}(1) + \overline{a}(1) \right)}{2} \quad \text{or} \quad \overline{a}_0 = \frac{\left( \underline{a}'(1) + \overline{a}'(1) \right)}{2}
\]

are said to be a location index number of membership and non-membership functions. The non-decreasing left continuous functions

\[a_0 = (a_0 - \underline{a}), \quad a_0 = (\overline{a} - a_0)\]

are called the left fuzziness index function and right fuzziness index function for the membership function respectively.

The non-decreasing left continuous functions

\[a_0 = (a_0 - \underline{a}), \quad a_0 = (\overline{a} - a_0)\]

are called the left fuzziness index function and right fuzziness index function for the non-membership function respectively for the triangular intuitionistic fuzzy number \( \tilde{a}^l \). Hence every triangular Intuitionistic fuzzy number \( \tilde{a}^l = (a_1, a_2; a_1', a_2', a_3) \) can also be represented by \( \tilde{a}^l = (a_0, a_0, a_0; a_0, a_0, a_0) \).

2.1 Arithmetic operations on triangular intuitionistic fuzzy numbers

We extend the fuzzy arithmetic operations on the set of triangular fuzzy numbers of Ming Ma et al.\[19\] based upon both location index and fuzziness index functions. The location index number is taken in the ordinary arithmetic, whereas the fuzziness index functions are considered to follow the lattice rule which is least upper bound in the lattice \( L \). That is for \( a, b \in L \) we define \( a \lor b = \max\{a, b\} \) and \( a \land b = \min\{a, b\} \).

We extend this fuzzy arithmetic operations for the set of triangular intuitionistic fuzzy numbers. That is for arbitrary triangular intuitionistic fuzzy numbers \( \tilde{a}^l, \tilde{b}^l \in F(\mathbb{R}) \) and \( * = \{+, -, \times, \div\} \), the arithmetic operations on them are defined by

\[\tilde{a}^l \ast \tilde{b}^l = (a_0 \ast b_0, \ a_0 \lor b_0, \ a_0 \lor b_0, \ a_0 \lor b_0, \ a_0 \lor b_0, \ a_0 \lor b_0)\].
In particular for any two triangular intuitionistic fuzzy numbers \( \tilde{a} = (a_0, a^*, a^+; a_0, a^+, a^+) \) and \( \tilde{b} = (b_0, b^*, b^+; b_0, b^+, b^+) \), we define

**Addition**

\[
\tilde{a} + \tilde{b} = (a_0, a^*, a^+; a_0, a^+, a^+) + (b_0, b^*, b^+; b_0, b^+, b^+)
\]

\[
= (a_0 + b_0, \max\{a^+, b^+\}; a_0 + b_0, \max\{a^+, b^+\}; a_0 + b_0, \max\{a^+, b^+\})
\]

**Subtraction**

\[
\tilde{a} - \tilde{b} = (a_0, a^*, a^+; a_0, a^+, a^+) - (b_0, b^*, b^+; b_0, b^+, b^+)
\]

\[
= (a_0 - b_0, \max\{a^+, b^+\}; a_0 - b_0, \max\{a^+, b^+\}; a_0 - b_0, \max\{a^+, b^+\})
\]

**Multiplication**

\[
\tilde{a} \times \tilde{b} = (a_0, a^*, a^+; a_0, a^+, a^+) \times (b_0, b^*, b^+; b_0, b^+, b^+)
\]

\[
= (a_0 \times b_0, \max\{a^+, b^+\}; a_0 \times b_0, \max\{a^+, b^+\}; a_0 \times b_0, \max\{a^+, b^+\})
\]

**Division**

\[
\tilde{a} \div \tilde{b} = (a_0, a^*, a^+; a_0, a^+, a^+) \div (b_0, b^*, b^+; b_0, b^+, b^+)
\]

\[
= (a_0 \div b_0, \max\{a^+, b^+\}; a_0 \div b_0, \max\{a^+, b^+\}; a_0 \div b_0, \max\{a^+, b^+\})
\]

Provided \( b_0 \neq 0 \)

**Scalar Multiplication**

\[
k\tilde{a} = \begin{cases} 
  (ka_0, a^*, a^+; ka_0, a^+, a^+) & \text{for } k \geq 0 \\
  (-ka_0, a^*, a^+; -ka_0, a^+, a^+) & \text{for } k < 0
\end{cases}
\]

### 2.2 Ranking of triangular intuitionistic fuzzy number

Ranking of fuzzy numbers is one of the key issues of choosing the best choices in fuzzy decision making. Fuzzy numbers must be compared ahead an action is chosen by a decision maker. Different researches approaches for the ranking of intuitionistic fuzzy numbers in the literature. Abbasbandy and Hajjari [1] proposed a new ranking method for triangular fuzzy numbers based on the left and the right spreads at some \( \alpha \)-levels of fuzzy numbers.

We extend this fuzzy ranking method to triangular intuitionistic fuzzy numbers.

For an arbitrary triangular intuitionistic fuzzy number \( \tilde{a} = (a_1, a_2, a_3; a^1, a^2, a^3) = (a_0, a^+, a^+; a_0, a^+, a^+) \), the magnitude of the triangular intuitionistic fuzzy number \( \tilde{a} \) is by
where the function \( f(r) \) is a non-negative and increasing function on \([0,1]\) with \( f(0) = 0 \) and \( f(1) = 1 \). The function \( f(r) \) can be considered as a weighting function. In real life applications, \( f(r) \) can be chosen by the decision maker according to the situation. In this paper, for convenience we use \( 2f(r) = r^2 \).

Hence \[ \text{Mag}(\bar{a}^l) = \frac{1}{2} \left( \frac{a^* + a^- + 6a_o - a_s - a_i}{6} \right) \]

The magnitude of a triangular intuitionistic fuzzy number \( \bar{a}^l \) synthetically reflects the information on every membership degree, and meaning of this magnitude is visual and natural. \( \text{Mag}(\bar{a}^l) \) is used to rank fuzzy numbers. The larger \( \text{Mag}(\bar{a}^l) \) is larger intuitionistic fuzzy number.

For any two triangular intuitionistic fuzzy numbers \( \bar{a}^l = (a_o, a^*, a^-; a_o, a^*, a^-) \) and \( \bar{b}^l = (b_o, b^*, b^-; b_o, b^*, b^-) \) in \( F(\mathbb{R}) \), we define the ranking of \( \bar{a}^l \) and \( \bar{b}^l \) by comparing the \( \text{Mag}(\bar{a}^l) \) and \( \text{Mag}(\bar{b}^l) \) on \( \mathbb{R} \) as follows:

(i). \( \bar{a}^l \succeq \bar{b}^l \) if and only if \( \text{Mag}(\bar{a}^l) \geq \text{Mag}(\bar{b}^l) \)

(ii). \( \bar{a}^l \preceq \bar{b}^l \) if and only if \( \text{Mag}(\bar{a}^l) \leq \text{Mag}(\bar{b}^l) \)

(iii). \( \bar{a}^l \approx \bar{b}^l \) if and only if \( \text{Mag}(\bar{a}^l) = \text{Mag}(\bar{b}^l) \)

**Definition 2.7** A triangular intuitionistic fuzzy number \( \bar{a}^l = (a_o, a^*, a^-; a_o, a^*, a^-) \) is said to be non-negative if and only if \( \text{Mag}(\bar{a}^l) \geq 0 \) and denoted by \( \bar{a}^l \succeq 0^l \). Further if \( \text{Mag}(\bar{a}^l) > 0 \), then \( \bar{a}^l = (a_o, a^*, a^-; a_o, a^*, a^-) \) is said to be a positive intuitionistic fuzzy number and is denoted by \( \bar{a}^l \succ 0^l \).

**Definition 2.8** Two triangular intuitionistic fuzzy numbers \( \bar{a}^l = (a_o, a^*, a^-; a_o, a^*, a^-) \) and \( \bar{b}^l = (b_o, b^*, b^-; b_o, b^*, b^-) \) in \( F(\mathbb{R}) \) are said to be equivalent if and only if \( \text{Mag}(\bar{a}^l) = \text{Mag}(\bar{b}^l) \) (i.e.) \( \bar{a}^l \equiv \bar{b}^l \) if and only if \( \text{Mag}(\bar{a}^l) = \text{Mag}(\bar{b}^l) \). Two triangular intuitionistic fuzzy numbers \( \bar{a}^l = (a_o, a^*, a^-; a_o, a^*, a^-) \) and \( \bar{b}^l = (b_o, b^*, b^-; b_o, b^*, b^-) \) in \( F(\mathbb{R}) \) are said to be equal if and only if \( a_o = b_o, a^* = b^*, a^- = b^- \) and \( a_o = b_o, a^* = b^*, a^- = b^- \) (i.e.) \( \bar{a}^l = \bar{b}^l \) if and only if \( a_o = b_o, a^* = b^*, a^- = b^- \) and \( a_o = b_o, a^* = b^*, a^- = b^- \).
3. INTUITIONISTIC FUZZY TRAVELLING SALESMAN PROBLEM

The Intuitionistic fuzzy travelling salesman problem is like the fuzzy assignment problem expects that in the former, there is an additional constraint. Suppose a fuzzy salesman has to visit n cities. He wishes to start from a particular city, visit each city once, and then return to his starting point. The objective is to select the sequence in which the cities are visited in such a way that his total fuzzy travelling time is minimized. Since the salesman has to visit all n cities, the fuzzy optimal solution remains independent of selection of starting point.

The mathematical form of the fuzzy travelling salesman is given below

\[
\text{Minimize } z^I = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}^I \bar{x}_{ij}^I
\]

subject to \( \sum_{i=1}^{n} \bar{x}_{ij}^I = \bar{1}, \ j = 1, 2, ..., n \)

\( \sum_{j=1}^{n} \bar{x}_{ij}^I = \bar{1}, \ i = 1, 2, ..., n \)

where \( \bar{x}_{ij}^I = \begin{cases} 
\bar{1}, & \text{if the salesman travel from city } i \text{ to city } j \\
\bar{0}, & \text{otherwise}
\end{cases} \)

This Intuitionistic fuzzy travelling salesman can be stated in the form of \((n \times n)\) fuzzy cost matrix \(c_{ij}^I\) be the corresponding Intuitionistic fuzzy cost of going from city \(i\) to city \(j\).

Table 3.1: Intuitionistic Fuzzy cost matrix of Intuitionistic fuzzy travelling salesman problem.

| Cities | 1 | 2 | 3 | ... | j | ... | n |
|--------|---|---|---|-----|---|-----|---|
| 1      | $c_{11}^I$ | $\infty$ | $c_{13}^I$ | ... | $c_{1j}^I$ | ... | $c_{1n}^I$ |
| 2      | $c_{21}^I$ | $c_{21}^I$ | $\infty$ | $c_{23}^I$ | ... | $c_{2j}^I$ | ... | $c_{2n}^I$ |
| ...    | ... | ... | ... | ... | ... | ... | ... |
| i      | $c_{i1}^I$ | $c_{i2}^I$ | $c_{i3}^I$ | ... | $\bar{c}_{ij}^I$ | ... | $c_{in}^I$ |
| ...    | ... | ... | ... | ... | ... | ... | ... |
| n      | $c_{n1}^I$ | $c_{n2}^I$ | $c_{n3}^I$ | ... | $c_{nj}^I$ | ... | $\infty$ |

3.1 Solution Procedure for Intuitionistic Fuzzy travelling salesman problem.

We, now introduce a algorithm called the Intuitionistic fuzzy Hungarian method for identifying a Intuitionistic fuzzy travelling salesman problem.

**Step 1**: Express all the triangular intuitionistic fuzzy numbers in the given problem based upon both location index and fuzziness index functions.

**Step 2**: Deduct the least amount element in every row from all the elements of that row. See that every row having at least one Intuitionistic fuzzy zero.

**Step 3**: Deduct the least amount element in every column of resulting Intuitionistic Fuzzy Assignment table after using step 3. Now every column and row has minimum a single fuzzy zero.
Step 4: Within resulting Intuitionistic fuzzy assignment table obtained by step 4, now we try to investigate for Intuitionistic fuzzy optimal assignment.
(a) Look at all the rows continuously until a row with accurately one unmarked Intuitionistic fuzzy zero was identified. Allocate this Intuitionistic fuzzy zero in addition to cancel every other Intuitionistic fuzzy zeros in that column, as these will not be considered for further assignment. Keep on this process used for all other rows.
(b) After examining all the rows completely, now examine the same process for every column of resultant Intuitionistic fuzzy assignment table.
(c) If a row and / or column have two or more than two Intuitionistic fuzzy zeros assign randomly any one of these Intuitionistic fuzzy zeros and cancel all other Intuitionistic fuzzy zeros of that row/column. Continue the above steps consecutively throughout the sequence of assigning or cross ends.

Step 5: If the number of allocations is equal to size of the matrix the order of the Intuitionistic fuzzy cost matrix, Intuitionistic fuzzy optimal solution is reached. If the number of assignments is less than n, (i.e) the order of the Intuitionistic fuzzy zeros of the Intuitionistic fuzzy cost matrix is less than n, go to the step 6.

Step 6: Sketch the least number of horizontal and / or vertical lines to cover up all the Intuitionistic fuzzy zeros in the resultant Intuitionistic fuzzy assignment matrix. That can be complete by means of the next steps:
(i) note the rows so as to do not contain any marked Intuitionistic fuzzy zero.
(ii) note the columns(not already marked) so as to have Intuitionistic fuzzy zeros in the noted rows.
(iii) note the rows(not already marked) so as to do have Intuitionistic allocated fuzzy zeros in the noted columns.
(iv) Continue the above said procedure till the sequence of mark is finished.
Sketch lines from end to end all the unnoted rows and noted columns.
This will ensure the preferred least amount of lines.

Step 7: Expand the fresh revised Intuitionistic fuzzy cost matrix as given below: Find the smallest entry of the reduced fuzzy Intuitionistic cost matrix not covered by any of the lines. Subtract this entry from all the uncovered entries and add the same to all the entries lying at the intersection of any two lines and do not change the remaining entries which lying on the lines.

Step 8: Do again 6th step to 8th step until Intuitionistic fuzzy optimal solution to the specified Intuitionistic fuzzy assignment problem is achieved.

Step 9: Finally check whether the route conditions are satisfied or not.

4. NUMERICAL EXAMPLE
Example 4.1 Consider an intuitionistic fuzzy travelling salesman problem with 5 cities namely A, B, C, D and E. The cost matrix \( \tilde{C}_{ij} \) is given whose elements are triangular intuitionistic fuzzy numbers.
The problem is to find a route that starts from his home city (A), passes through each city exactly once and returns to his home city at lowest possible cost.
\[
\begin{bmatrix}
  \infty & (23, 25, 27; 21, 25, 29) & (38, 40, 42; 36, 40, 44) & (8, 10, 12; 6, 10, 14) & (10, 12, 14; 8, 12, 16) \\
(23, 25, 27; 21, 25, 29) & \infty & (18, 20, 22; 16, 20, 24) & (21, 23, 25; 19, 23, 27) & (9, 11, 13; 7, 11, 15) \\
(38, 40, 42; 36, 40, 44) & (18, 20, 22; 16, 20, 24) & \infty & (19, 23, 27; 17, 23, 29) & (31, 33, 35; 29, 33, 37) \\
(6, 10, 14; 8, 10, 12) & (19, 23, 27; 21, 23, 25) & (17, 23, 29; 19, 23, 27) & \infty & (16, 20, 24; 18, 20, 22) \\
(10, 12, 14; 8, 12, 16) & (9, 11, 13; 7, 11, 15) & (31, 33, 35; 29, 33, 37) & (18, 20, 22; 16, 20, 24) & \infty 
\end{bmatrix}
\]
To implement the proposed algorithm and the fuzzy arithmetic, let us rewrite all the triangular intuitionistic fuzzy numbers in the given problem based upon both location index and fuzziness index functions. That is in the form of we have

\[
\begin{align*}
&\Rightarrow (25,2+r,2-r;25,4-r,4-r) \quad (40,2-r,2-r;40,4-r,4-r) \quad (10,2-r,2-r;10,4-r,4-r) \quad (12,2-r,2-r;12,4-r,4-r) \\
&\quad (25,2-r,2-r;25,4-r,4-r) \quad \Rightarrow (20,2-r,2-r;20,4-r,4-r) \quad (23,2-r,2-r;23,4-r,4-r) \quad (11,2-r,2-r;11,4-r,4-r) \\
&\quad (40,2-r,2-r;40,4-r,4-r) \quad (20,2-r,2-r;20,4-r,4-r) \quad \Rightarrow (23,4-r,4-r;23,6-r,6-r) \quad (33,2-r,2-r;33,4-r,4-r) \\
&\quad (10,4-r,4-r;10,2-r,2-r) \quad (23,4-r,4-r;23,2-r,2-r) \quad (23,6-r,6-r;23,4-r,4-r) \quad \Rightarrow (20,4-r,4-r;20,2-r,2-r) \\
&\quad (12,2-r,2-r;12,4-r,4-r) \quad (11,2-r,2-r;11,4-r,4-r) \quad (33,2-r,2-r;33,4-r,4-r) \quad \Rightarrow (20,2-r,2-r;20,4-r,4-r) \\
\end{align*}
\]
Therefore, the Intuitionistic fuzzy optimal assignment for the given Intuitionistic fuzzy travelling salesman problem is

\[ A \rightarrow D, B \rightarrow E, C \rightarrow B, D \rightarrow C, E \rightarrow A. \]

and the route conditions \( A \rightarrow D \rightarrow C \rightarrow B \rightarrow E \rightarrow A. \)

The Intuitionistic fuzzy optimal total cost is calculated as

\[
\begin{align*}
&= (10, 2 - 2r, 2 - 2r; 20, 4 - 4r, 4 - 4r) + \\
&(23, 6 - 6r, 6 - 6r; 23, 4 - 4r, 4 - 4r) + \\
&(20, 2 - 2r, 2 - 2r; 20, 4 - 4r, 4 - 4r) + \\
&(11, 2 - 2r, 2 - 2r; 11, 4 - 4r, 4 - 4r) + \\
&(12, 2 - 2r, 2 - 2r; 12, 4 - 4r, 4 - 4r)
\end{align*}
\]

\[= (76, 6 - 6r, 6 - 6r; 76, 4 - 4r, 4 - 4r)\]

Intuitionistic fuzzy optimal cost is (if \( r = 0 \)) = (70, 76, 82; 72, 76, 80) units.

5. CONCLUSION

We have obtained an optimal solution for travelling salesman problem involving triangular Intuitionistic fuzzy number using the proposed algorithm and proposed arithmetic operations. The examples shown in this paper guarantees the correctness and effectiveness of the working procedure of the algorithm.

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