A New Evaluation of Polarized Parton Densities in the Nucleon

Elliot Leader  
*Imperial College, University of London*  
*London WC1E 7HX, England*  
*E-mail: e.leader@ic.ac.uk*

Aleksander V. Sidorov  
*Bogoliubov Theoretical Laboratory*  
*Joint Institute for Nuclear Research*  
*141980 Dubna, Russia*  
*E-mail: sidorov@thsun1.jinr.ru*

Dimiter B. Stamenov  
*Institute for Nuclear Research and Nuclear Energy*  
*Bulgarian Academy of Sciences*  
*bld. Tsarigradsko Chaussee 72, Sofia 1784, Bulgaria*  
*E-mail: stamenov@inrne.bas.bg*

Abstract

We present a new leading and next-to-leading QCD analysis of the world data on inclusive polarized deep inelastic scattering. A new set of polarized parton densities is extracted from the data and the sensitivity of the results to the newly incorporated SLAC/E155 proton data is discussed.

PACS numbers:13.60.Hb; 13.88.+e; 14.20.Dh; 12.38.-t
1 Introduction

Deep inelastic scattering (DIS) of leptons on nucleons has remained the prime source of our understanding of the internal partonic structure of the nucleon and one of the key areas for the testing of perturbative QCD. Decades of experiments on unpolarized targets have led to a rather precise determination of the unpolarized parton densities. Spurred on by the famous EMC experiment [1] at CERN in 1988, there has been a huge growth of interest in polarized DIS experiments which yield more refined information about the partonic structure of the nucleon, i.e., how the nucleon spin is divided up among its constituents, quarks and gluons. Many experiments have been carried out at SLAC, CERN and DESY.

In this paper we present an updated version of our NLO polarized parton densities in both the $\overline{\text{MS}}$ and the JET (or so-called chirally invariant) [2] factorization schemes as well as the LO ones determined from the world data [1, 3, 4] on inclusive polarized DIS. Comparing to our previous analysis [5]:

i) For the axial charges $a_3$ and $a_8$ their updated values are used [3, 7]:

$$a_3 = g_A = F + D = 1.2670 \pm 0.0035, \quad a_8 = 3F - D = 0.585 \pm 0.025.$$  (1)

ii) In our ansatz for the input polarized parton densities

$$\Delta f_i(x, Q_0^2) = \rho_i(x) f_i^{\text{MRST}}(x, Q_0^2)$$  (2)

we now utilize the MRST’99 set [8] of unpolarized parton densities $f_i(x, Q_0^2)$ instead of the MRST’98 one.

iii) The recent SLAC/E155 proton data [4] are incorporated in the analysis.

iv) The positivity constraints on the polarized parton densities are discussed.

2 Method of Analysis

The nucleon spin-dependent structure function of interest, $g_1^N(x, Q^2)$, is a linear combination of the asymmetries $A_1^N$ and $A_1^N$ (or the related virtual photon-nucleon asymmetries $A_{1,2}^N$) measured with the target polarized longitudinally or perpendicularly to the lepton beam, respectively. Neglecting as usual the subdominant contributions, $A_1^N(x, Q^2)$ can be expressed via the polarized structure function $g_1^N(x, Q^2)$ as

$$A_1^N(x, Q^2) \approx (1 + \gamma^2) \frac{g_1^N(x, Q^2)}{F_1^N(x, Q^2)}$$  (3)
or
\[ A_1^N(x, Q^2) \cong \frac{g_1^N(x, Q^2)}{F_2^N(x, Q^2)} 2x[1 + R^N(x, Q^2)] \]  
(4)

using the relation between the unpolarized structure function \( F_1(x, Q^2) \) and the usually extracted from unpolarized DIS experiments \( F_2(x, Q^2) \) and \( R(x, Q^2) \)
\[ 2x F_1^N = F_2^N(1 + \gamma^2)/(1 + R^N) \quad (N = p, n, d). \]  
(5)

In Eq. (3) the kinematic factor \( \gamma^2 \) is given by
\[ \gamma^2 = \frac{4M_N^2x^2}{Q^2}. \]  
(6)

In (3) \( M_N \) is the nucleon mass. It should be noted that in the SLAC and HERMES kinematic region \( \gamma \) cannot be neglected.

In the NLO QCD approximation the quark-parton decomposition of the proton structure function \( g_1^p(x, Q^2) \) has the following form (a similar formula holds for \( g_1^n \)):
\[ g_1^p(x, Q^2) = \frac{1}{2} \sum_q e_q^2 \left[ (\Delta q + \Delta \bar{q}) \otimes (1 + \frac{\alpha_s(Q^2)}{2\pi} \delta C_q) + \frac{\alpha_s(Q^2)}{2\pi} \Delta G \otimes \frac{\delta C_G}{N_f} \right], \]  
(7)

where \( \Delta q(x, Q^2), \Delta \bar{q}(x, Q^2) \) and \( \Delta G(x, Q^2) \) are quark, anti-quark and gluon polarized densities in the proton which evolve in \( Q^2 \) according to the spin-dependent NLO DGLAP equations. \( \delta C_{q,G} \) are the NLO terms in the spin-dependent Wilson coefficient functions and the symbol \( \otimes \) denotes the usual convolution in Bjorken \( x \) space. \( N_f \) is the number of flavours.

It is well known that at NLO and beyond, the parton densities as well as the Wilson coefficient functions become dependent on the renormalization (or factorization) scheme employed. Both the NLO polarized coefficient functions and the NLO polarized splitting functions (anomalous dimensions) needed for the calculation of \( g_1(x, Q^2) \) in the \( \overline{\text{MS}} \) scheme are well known at present. The corresponding expressions for these quantities in the JET scheme can be found in [11].

All details of our approach to the fit of the data are given in [12]. Here we would like to emphasize only that according to this approach first used in [13], the NLO QCD predictions have been confronted to the data on the spin asymmetry \( A_1^N(x, Q^2) \), rather than on the \( g_1^N(x, Q^2) \). The choice of \( A_1^N \) appears to minimize the higher twist (HT) contributions to \( g_1^N \) which are expected to partly cancel with those of \( F_1^N \) in the ratio (3), allowing use of data at lower \( Q^2 \) (in polarized DIS most of the small 

\[ \text{†} \] Of course, physical quantities such as the virtual photon-nucleon asymmetry \( A_1(x, Q^2) \) and the polarized structure function \( g_1(x, Q^2) \) are independent of choice of the factorization convention.
experimental data points are at low $Q^2$). Indeed, we have found \[14\] that if for $g_1$ and $F_1$ leading-twist (LT) QCD expressions are used in \[3\], the HT corrections $h(x)$ to $A_1(x, Q^2) = A_1(x, Q^2)_{LT} + h(x)/Q^2$, extracted from the data, are negligible and consistent with zero within the errors (see Fig. 1). (Note that the polarized parton densities in QCD are related to the leading-twist expression of $g_1$.) On the other hand, it was shown \[15\] that if $F_2$ and $R$ in Eq. (4) are taken from experiment (as has been done in some of the analyses) the higher twist corrections to $A_1(x, Q^2)$ are sizeable and important. So, in order to extract the polarized parton densities from $g_1$ data the HT contribution to $g_1$ (unknown at present) has to be included into data fit. Note that a QCD fit to the $g_1$ data keeping in $g_1(x, Q^2)_{QCD}$ only the leading-twist expression leads to some ”effective” parton densities which involve in themselves the HT effects and therefore, are not quite correct. These results suggest that in order to determine polarized parton densities less sensitive to higher twist effects, it is preferable at present to analyze $A_1$ data directly using for $g_1$ and $F_1$ their leading twist expressions. Bearing in mind this discussion one must be careful when comparing the polarized parton densities determined from the inclusive DIS data with those extracted from semi-inclusive DIS data \[16\]. The results will depend, especially in a LO QCD analysis, upon whether the higher twist terms are or are not taken into account.

Following the procedure of our previous analyses we have extracted the NLO (as well as LO) polarized parton densities from the fit to the world data on $A_N^V(x, Q^2)$ using for the flavour non-singlet combinations of their first moments

$$a_3 = (\Delta u + \Delta \bar{u})(Q^2) - (\Delta d + \Delta \bar{d})(Q^2), \quad (8)$$
$$a_8 = (\Delta u + \Delta \bar{u})(Q^2) + (\Delta d + \Delta \bar{d})(Q^2) - 2(\Delta s + \Delta \bar{s})(Q^2), \quad (9)$$

the sum rule values \[1\]. The sensitivity of the polarized parton densities to the deviation of $a_8$ from its SU(3) flavour symmetric value (0.58) has been studied and the results are given in \[17\]. Here we will present only the polarized parton densities corresponding to the SU(3) symmetric value of $a_8$ in \[14\].

What we can deduce from inclusive DIS in the absence of charged current neutrino data is the sum of the polarized quark and anti-quark densities

$$(\Delta u + \Delta \bar{u})(x, Q^2), \quad (\Delta d + \Delta \bar{d})(x, Q^2), \quad (\Delta s + \Delta \bar{s})(x, Q^2) \quad (10)$$

and the polarized gluon density $\Delta G(x, Q^2)$. The non-strange polarized sea-quark densities $\Delta \bar{u}(x, Q^2)$ and $\Delta \bar{d}(x, Q^2)$, as well as the valence quark densities $\Delta u_v(x, Q^2)$ and
\[ \Delta d_v(x, Q^2) : \]
\[ \Delta u_v \equiv \Delta u - \Delta \bar{u}, \quad \Delta d_v \equiv \Delta d - \Delta \bar{d} \]  \hspace{1cm} (11)

cannot be determined without additional assumptions about the flavour decomposition of the sea. Nonetheless (because of the universality of the parton densities) they are of interest for predicting the behaviour of other processes, like polarized pp reactions, etc. That is why, we extract from the data not only the quark densities (10) \[ \Delta G(x, Q^2) , \]

but also the valence parts \[ \Delta u_v(x, Q^2), \Delta d_v(x, Q^2) \]

and anti-quark densities using the assumption on the flavour symmetric sea

\[ \Delta u_{sea} = \Delta \bar{u} = \Delta d_{sea} = \Delta \bar{d} = \Delta s = \Delta \bar{s} . \]  \hspace{1cm} (12)

For the input LO and NLO polarized parton densities at \( Q_0^2 = 1 \text{ GeV}^2 \) we have adopted a very simple parametrization

\[ x \Delta u_v(x, Q_0^2) = \eta_u A_u x^{a_u} x u_v(x, Q_0^2) , \]
\[ x \Delta d_v(x, Q_0^2) = \eta_d A_d x^{a_d} x d_v(x, Q_0^2) , \]
\[ x \Delta s(x, Q_0^2) = \eta_s A_s x^{a_s} x s(x, Q_0^2) , \]
\[ x \Delta G(x, Q_0^2) = \eta_g A_g x^{a_g} x G(x, Q_0^2) , \]  \hspace{1cm} (13)

where on RHS of (13) we have used the MRST98 (central gluon) \[ 18 \] and MRST99 (central gluon) \[ 8 \] parametrizations for the LO and NLO(\overline{\text{MS}}) unpolarized densities, respectively. The normalization factors \( A_f \) in (13) are fixed such that \( \eta_f \) are the first moments of the polarized densities. The first moments of the valence quark densities \( \eta_u \) and \( \eta_d \) are constrained by the sum rules (1). The rest of the free parameters in (13),

\[ \{ a_u, a_d, \eta_s, a_s, \eta_g, a_g \} , \]  \hspace{1cm} (14)

have been determined from the best fit to the data.

The guiding arguments to choose for the input polarized parton densities the ansatz (13) are simplicity (not too many free parameters) and the expectation that polarized and unpolarized densities have similar behaviour at large \( x \). Also, such an ansatz allows easy control of the positivity condition, which in LO QCD implies:

\[ |\Delta f_i(x, Q_0^2)| \leq f_i(x, Q_0^2) , \quad |\Delta \bar{f}_i(x, Q_0^2)| \leq \bar{f}_i(x, Q_0^2) . \]  \hspace{1cm} (15)

The constraints (13) are consequence of a probabilistic interpretation of the parton densities in the naive parton model, which is still correct in LO QCD. Beyond LO the parton densities are not physical quantities and the positivity constraints on
the polarized parton densities are more complicated. They follow from the positivity condition for the polarized lepton-hadron cross-sections $\Delta\sigma_i$ with the unpolarized ones ($|\Delta\sigma_i| \leq \sigma_i$) and include also the Wilson coefficient functions. It was shown [20], however, that for all practical purposes it is enough at the present stage to consider LO positivity bounds, since NLO corrections are only relevant at the level of accuracy of a few percent.

3 Results

In this section we present the numerical results of our fits to the world data on $A_1(x, Q^2)$. The data used (185 experimental points) cover the following kinematic region:

$$0.005 \leq x \leq 0.75, \quad 1 < Q^2 \leq 58 \text{ GeV}^2.$$  \hspace{1cm} (16)

The total (statistical and systematic) errors are taken into account. The systematic errors are added quadratically.

We prefer to discuss the NLO results of analysis in the JET scheme. To compare our NLO polarized parton densities with those extracted by other groups, we present them also in the usually used $\overline{\text{MS}}$ scheme using the renormalization group transformation rules for the parton densities.

It is useful to recall the transformation rules relating the first moments of the singlet quark density, $\Delta\Sigma(Q^2)$, and the strange sea, $(\Delta s + \Delta \bar{s})(Q^2)$, in the JET and $\overline{\text{MS}}$ schemes:

$$\Delta\Sigma_{\text{JET}} = \Delta\Sigma_{\overline{\text{MS}}}(Q^2) + N_f \frac{\alpha_s(Q^2)}{2\pi} \Delta G(Q^2),$$  \hspace{1cm} (17)

$$(\Delta s + \Delta \bar{s})_{\text{JET}} = (\Delta s + \Delta \bar{s})_{\overline{\text{MS}}}(Q^2) + \frac{\alpha_s(Q^2)}{2\pi} \Delta G(Q^2),$$  \hspace{1cm} (18)

where $\Delta G(Q^2)$ is the first moment of the polarized gluon density $\Delta G(x, Q^2)$ (note that $\Delta G$ is the same in the factorization schemes under consideration).

A remarkable property of the JET (and so-called Adler-Bardeen (AB) [19]) schemes is that the singlet $\Delta\Sigma(Q^2)$, as well as the strange sea polarization $(\Delta s + \Delta \bar{s})(Q^2)$, are $Q^2$ independent quantities. Then, in these schemes it is meaningful to directly interpret $\Delta\Sigma$ as the contribution of the quark spins to the nucleon spin and to compare its value obtained from DIS region with the predictions of the different (constituent, chiral, etc.) quark models at low $Q^2$.

It is important to mention that the difference between the values of the strange sea polarization, obtained in the $\overline{\text{MS}}$ and JET schemes could be large if $\Delta G$ in (18) is
positive and large. To illustrate how large it can be, we present the values of $(\Delta s + \Delta \bar{s})$ at $Q^2 = 1 \text{ GeV}^2$ obtained in our analysis of the world DIS data in the $\overline{\text{MS}}$ and JET schemes ($\Delta G = 0.68$):

\[
\begin{align*}
(\Delta s + \Delta \bar{s})_{\overline{\text{MS}}} &= -0.13 \pm 0.04 \\
(\Delta s + \Delta \bar{s})_{\text{JET}} &= -0.07 \pm 0.02.
\end{align*}
\]  

(19)

Note that if $\Delta G$ is larger than 0.68, $(\Delta s + \Delta \bar{s})_{\text{JET}}$ could vanish in agreement with what is intuitively expected in quark models at low-$Q^2$ region ($Q^2 \approx 0$).

The numerical results of our fits to the data are summarized in Table 1. The best LO and NLO(JET) fits correspond to $\chi^2$ per degree of freedom of $\chi^2_{\text{DF,LO}} = 0.921$ and to $\chi^2_{\text{DF,NLO}} = 0.871$. In LO QCD $\Delta G(x, Q^2)$ does not contribute directly in $g_1$ and the gluons cannot be determined from DIS data alone. For that reason the LO fit to the data was performed using for the input polarized gluon density $\Delta G(x, Q_{0}^2)$ that one extracted by the NLO fit to the data:

\[
\Delta G(x, Q_{0}^2)_{\text{LO}} = \Delta G(x, Q_{0}^2)_{\text{NLO(JET)}}.
\]  

(20)

We consider that such a procedure leads to non-realistic errors of the rest of parameters and therefore, we present only their central values in Table 1. It is important to note that in the polarized case the LO approximation has some peculiarities compared to the unpolarized one. As has been already mentioned above, as a consequence of the axial anomaly, the difference between NLO anti-quark polarizations $\Delta \bar{q}_i$ in different factorization schemes could be quite large, in order of magnitude of $\Delta \bar{q}_i$ themselves. In this case the leading order will be a bad approximation, at least for the polarized sea-quark densities extracted. Also, bearing in mind that in polarized DIS most of the data points are at low $Q^2$, lower than the usual cuts in the analyses of unpolarized data ($Q^2 \geq 4 - 5 \text{ GeV}^2$), the NLO corrections to all polarized parton densities are large in this region and it is better to take them into account. Nevertheless, the LO polarized parton densities may be useful for many practical purposes: For preliminary estimations of the cross sections in future polarized experiments, etc. For that reason we present them in this paper, at $Q^2 = 1 \text{ GeV}^2$ (see Fig. 2 and Appendix) and for any $Q^2$ in the region: $1 \leq Q^2 \leq 6.10^5$ (the FORTRAN code is available\[^1\]). Further, we will discuss mainly the NLO QCD results.

As in our previous analysis [5] a very good description of the world data on $A_1^N$ and $g_1^N$ is achieved. The new NLO theoretical curves for $A_1$ corresponding to the

\[^1\]http://durpdg.dur.ac.uk/HEPDATA/PDF
best fit practically coincide with the old ones (see Fig. 3). The agreement with the SLAC/E155p data involved in this analysis is also very good. This is illustrated in Fig. 4. The NLO QCD theoretical predictions for $A_1$ corresponding to our previous analysis of the data are also shown. They are in a good agreement with the SLAC/E155 data not available at the time our previous analysis was performed. In Fig. 5 we compare the new NLO $g_1$ curves with the SMC and SLAC/E143 proton data. The extrapolations for $g_1^p$ in the yet unmeasured small $x$ region at different $Q^2$ are also shown. As seen from Fig. 5, the proton spin dependent structure function $g_1^p$ changes sign at $x$ smaller than $10^{-3}$ and becomes negative if the gluon polarization is positive. One of the challenges to future polarized DIS experiments is to confirm or reject this behaviour, quite different from the usual Regge type behaviour.

The extracted NLO(JET) polarized parton densities at $Q^2 = 1 \text{ GeV}^2$ are shown in Figs. 6(a) and 6(b). (The explicit expressions are given in the Appendix.) The new parton densities are found to be within the error bands of the old ones. The positivity constraints have not been imposed during the fit. Except for the strange sea density $\Delta s(x)$, the polarized parton densities determined from the data are compatible with the LO positivity bounds (15) imposed by the MRST99 unpolarized parton densities. However, if one uses the more accurate LO positivity bounds on $\Delta s(x)$ obtained by using the unpolarized strange sea density $s(x)_{BPZ}$ (Barone et al. [21]), $\Delta s(x)$ also lies in the allowed region. It is important to mention that $s(x)_{BPZ}$ is determined with a higher accuracy compared to other global fits.

In Fig. 7 we illustrate the difference between the singlet quark density $\Delta \Sigma(x, Q^2)$ and the strange sea density $(\Delta s+\Delta \bar{s})(x, Q^2)$ determined in the JET and $\overline{\text{MS}}$ schemes at $Q^2 = 1 \text{ GeV}^2$. They differ essentially in the small $x$ region and this difference increases with $\Delta G$ increasing. In Figs. 8(a) and 8(b) we compare our NLO($\overline{\text{MS}}$) polarized parton densities with those obtained by AAC [7] and GRSV [15] using almost the same set of data. This comparison is a good illustration of the present situation in polarized DIS. While the quality and the kinematic range of the data are sufficient to determine the valence polarized densities $\Delta u_v(x, Q^2)$ and $\Delta d_v(x, Q^2)$ with a good accuracy (if an SU(3) symmetry of the flavour decomposition of the sea is assumed), the polarized strange quark density $\Delta s(x, Q^2)$ as well as the polarized gluon density $\Delta G(x, Q^2)$ are still weakly constrained, especially $\Delta G$.

Finally, let us turn to the quark and gluon polarizations (the first moments of the polarized parton densities). The results of the new as well as of the old analysis are presented in Table 2 ($Q^2 = 1 \text{ GeV}^2$). The corresponding values in the $\overline{\text{MS}}$ scheme are also given. The changes of the central values of the parton polarizations (JET scheme)
are negligible and within the errors of the quantities. Note that for the central value of the axial charge $a_0(Q^2)$ (equal to $\Delta \Sigma(Q^2)_{\overline{\text{MS}}}$ in $\overline{\text{MS}}$ scheme) we obtain now somewhat smaller value: $a_0(1 \text{ GeV}^2) = 0.21 \pm 0.10$ than the old one: $a_0 = 0.26 \pm 0.10$. The values of the LO parton polarizations are between those of the JET and $\overline{\text{MS}}$ schemes.

For the gluon polarization $\Delta G$ corresponding to the best NLO(JET) fit we have found $\Delta G = 0.68 \pm 0.32$ at $Q^2 = 1 \text{ GeV}^2$. However, if one takes into account the sensitivity of $\Delta G$ to variation of the non-singlet axial charge $a_8$ from its SU(3) symmetric value of 3F-D, the positive values of $\Delta G$ could lie in the wider range $[0, 1.5]$ [17]. A negative $\Delta G$ is still not excluded from the present DIS inclusive data.

4 Conclusion

We have re-analyzed the world data on inclusive polarized deep inelastic lepton-nucleon scattering in leading and next-to-leading order of QCD adding to the old set of data the SLAC/E155 proton data. Compared to our previous analysis: i) the updated values for the non-singlet axial charges $g_A$ and $a_8$ and ii) in the ansatz for the input polarized parton densities the MRST99 set of unpolarized parton densities instead of the MRST98 one have been used. It was demonstrated that the polarized DIS data are in an excellent agreement with the pQCD predictions for $A_1^N(x, Q^2)$ and $g_1^N(x, Q^2)$ and that the new theoretical curves practically coincide with the old ones. A new set of NLO polarized parton densities in the JET and $\overline{\text{MS}}$ factorization schemes as well as LO polarized patron densities have been extracted from the data. We note that one must be careful when using LO polarized densities. The LO QCD approximation could be a bad one, at least for the polarized sea-quark densities, if it will turn out that the gluon polarization is positive and large (the first direct measurement of $\Delta G$ [22] as well as the QCD analyses support such a possibility). We have found that the new NLO(JET) polarized parton densities do not change essentially and lie within the error bands of the old parton densities.

What follows from our analysis is that the limited kinematic range and the precision of the present generation of inclusive DIS experiments are enough to determine with a good accuracy only the polarized parton densities $(\Delta u + \Delta \bar{u})(x, Q^2)$ and $(\Delta d + \Delta \bar{d})(x, Q^2)$. The polarized strange sea density $(\Delta s + \Delta \bar{s})(x, Q^2)$ as well as the polarized gluon density $\Delta G(x, Q^2)$ are still weakly constrained, especially $\Delta G$. The non-strange polarized sea-quark densities $\Delta \bar{u}$ and $\Delta \bar{d}$ cannot be determined, in principle, from the inclusive DIS experiments alone without additional assumptions. The further study of flavour decomposition of the sea as well as a more accurate determini-
nation of the gluon polarization are important next steps in our understanding of the partonic structure of the nucleon and this will be done in the forthcoming and future polarized lepton-hadron and hadron-hadron experiments.

A FORTRAN package containing our NLO polarized parton densities in both the JET and $\overline{\text{MS}}$ factorization schemes as well as LO parton densities can be found at [http://durpdg.dur.ac.uk/HEPDATA/PDF](http://durpdg.dur.ac.uk/HEPDATA/PDF) or obtained by electronic mail: sidorov@thsun1.jinr.ru or stamenov@inrne.bas.bg.

Acknowledgments

One of us (D.S.) is grateful for the hospitality of the High Energy Section of the Abdus Salam International Centre for Theoretical Physics, Trieste, where this work has been completed. This research was supported by the UK Royal Society and the JINR-Bulgaria Collaborative Grants, by the RFBR (No 00-02-16696), INTAS 2000 (No 587) and by the Bulgarian National Science Foundation under Contract Ph-1010.

Appendix

For practical purposes we present here explicitly our LO and NLO(JET) polarized parton densities at $Q^2 = 1$ GeV$^2$. The polarized valence quark densities correspond to SU(3) flavour symmetric sea.

LSS - LO:

$$
\begin{align*}
  x\Delta u_v(x) &= 0.3112 \ x^{0.4222} \ (1 - x)^{3.177} \ (1 - 0.4085 \ x^{1/2} + 17.60 \ x) , \\
  x\Delta d_v(x) &= -0.01563 \ x^{0.2560} \ (1 - x)^{3.398} \ (1 + 37.25 \ x^{1/2} + 31.14 \ x) , \\
  x\Delta s(x) &= -0.08548 \ x^{0.5645} \ (1 - x)^{8.653} \ (1 - 0.9052 \ x^{1/2} + 11.53 \ x) , \\
  x\Delta G(x) &= 19.14 \ x^{1.118} \ (1 - x)^{6.879} \ (1 - 3.147 \ x^{1/2} + 3.148 \ x) . \quad (A.1)
\end{align*}
$$

LSS - NLO(JET):

$$
\begin{align*}
  x\Delta u_v(x) &= 0.5052 \ x^{0.6700} \ (1 - x)^{3.428} \ (1 + 2.179 \ x^{1/2} + 14.57 \ x) , \\
  x\Delta d_v(x) &= -0.01852 \ x^{0.2704} \ (1 - x)^{3.864} \ (1 + 35.47 \ x^{1/2} + 28.97 \ x) , \\
  x\Delta s(x) &= -0.1525 \ x^{1.332} \ (1 - x)^{7.649} \ (1 + 3.656 \ x^{1/2} + 19.50 \ x) , \\
  x\Delta G(x) &= 19.14 \ x^{1.118} \ (1 - x)^{6.879} \ (1 - 3.147 \ x^{1/2} + 3.148 \ x) . \quad (A.2)
\end{align*}
$$
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Table 1. Results of the LO and NLO fits to the world $A_i^N$ data ($Q_0^2 = 1 \text{ GeV}^2$). The errors shown are total (statistical and systematic). The parameters marked by (*) are fixed.

| Fit       | LO       | NLO(JET) |
|-----------|----------|----------|
| DF        | 185 - 4  | 185 - 6  |
| $\chi^2$ | 166.7    | 155.9    |
| $\chi^2$/DF | 0.921    | 0.871    |
| $\eta_u$  | 0.926*   | 0.926*   |
| $a_u$     | 0.121    | 0.253 ± 0.027 |
| $\eta_d$  | -0.341*  | -0.341*  |
| $a_d$     | 0.102    | 0.000 ± 0.054 |
| $\eta_s$  | -0.055   | -0.036 ± 0.007 |
| $a_s$     | 0.754    | 1.613 ± 0.429 |
| $\eta_g$  | 0.681*   | 0.681 ± 0.141 |
| $a_g$     | 0.149*   | 0.149 ± 0.741 |

Table 2. First moments (polarizations) of polarized parton densities at $Q^2 = 1 \text{ GeV}^2$.

| Fit       | $\Delta u + \Delta \bar{u}$ | $\Delta d + \Delta \bar{d}$ | $\Delta s + \Delta \bar{s}$ | $\Delta G$ | $\Delta \Sigma$ | $a_0$   |
|-----------|-----------------------------|-----------------------------|-----------------------------|------------|-----------------|---------|
| old/JET   | 0.86±0.03                   | -0.40±0.05                  | -0.06±0.02                  | 0.57±0.31  | 0.40±0.06       | 0.26±0.10|
| new/JET   | 0.85±0.03                   | -0.41±0.05                  | -0.07±0.02                  | 0.68±0.32  | 0.37±0.07       | 0.21±0.10|
| new/MS    | 0.80                        | -0.47                       | -0.13                       | 0.68       | 0.21            | 0.21    |
| LO        | 0.82                        | -0.45                       | -0.11                       | 0.68       | 0.25            | 0.25    |
Figure Captions

**Fig. 1.** Higher twist contribution $h^N(x)$ to the spin asymmetry $A_1^N(x, Q^2)$ extracted from the data.

**Fig. 2.** Leading order polarized parton densities at $Q^2 = 1 \text{ GeV}^2$.

**Fig. 3.** Comparison of our NLO results in JET scheme for $A_1^N(x, Q^2)$ with the experimental data at the measured $x$ and $Q^2$ values. Errors bars represent the total (statistical and systematic) error. Our old NLO results are shown for comparison.

**Fig. 4.** Comparison of our NLO(JET) result for $A_1^p$ with SLAC/E155p experimental data. Error bars represent the total errors. The predictions for $A_1^p$ (dot curves) from our old analysis are also shown.

**Fig. 5.** Comparison of our NLO(JET) results for $g_1^p$ with SMC and SLAC/E143 proton data. The extrapolations at small $x$ are also shown.

**Fig. 6(a).** NLO(JET) polarized parton densities $x(\Delta u + \Delta \bar{u})$ and $x(\Delta d + \Delta \bar{d})$ at $Q^2 = 1 \text{ GeV}^2$. The old parton densities together with their error bands are presented for comparison.

**Fig. 6(b).** NLO(JET) polarized strange sea $x(\Delta s + \Delta \bar{s})$ and gluon polarized parton densities at $Q^2 = 1 \text{ GeV}^2$. The old parton densities together with their error bands are presented for comparison.

**Fig. 7.** Comparison between NLO polarized singlet and strange sea parton densities at $Q^2 = 1 \text{ GeV}^2$ in the JET and \(\overline{\text{MS}}\) schemes.

**Fig. 8(a).** Comparison between our NLO(\(\overline{\text{MS}}\)) polarized valence quark densities at $Q^2 = 1 \text{ GeV}^2$ with those obtained by AAC (NLO-2 set) and GRSV (‘standard’ scenario).

**Fig. 8(b).** Comparison between our NLO(\(\overline{\text{MS}}\)) polarized strange quark and gluon densities at $Q^2 = 1 \text{ GeV}^2$ with those obtained by AAC (NLO-2 set) and GRSV (‘standard’ scenario).
Fig. 1
Fig. 2

\[ Q^2 = 1 \text{ GeV}^2 \]

\[ x(\Delta u + \Delta \bar{u}) \]

\[ x\Delta G \]

\[ x(\Delta s + \Delta \bar{s}) \]

\[ x(\Delta d + \Delta \bar{d}) \]

LO
Fig. 3
Fig. 4

\[ \frac{g_1}{F_1} \]

(2.75° spec.)

(5.5° spec.)

(10.5° spec.)

- **new**
- **old**

E155p
Fig. 5
Fig. 6 (a)

JET \quad Q^2 = 1 \text{GeV}^2

x\Delta u + x\Delta\bar{u}

x\Delta d + x\Delta\bar{d}

-0.4
-0.3
-0.2
-0.1
0.0

-0.4
-0.3
-0.2
-0.1
0.0

old

new
Fig. 6 (b)
Fig. 7

$Q^2 = 1 \text{GeV}^2$

$\Delta\Sigma$

$\Delta s + \Delta \bar{s}$
$x \Delta u_v$

$Q^2 = 1 \text{ GeV}^2$

$LSS$

$GRSV$

$AAC$

Fig. 8 (a)
Fig. 8(b)

$x\Delta G$

$Q^2 = 1 \text{ GeV}^2$

$x\Delta s$

- LSS
- GRSV
- AAC