A composite model of quarks and leptons is proposed. The quark s and leptons are
given by three body states which are composed of constituents \((w_1, w_2, c_1, c_2, c_3)\) of
\(SU(5)_{\text{flavor}}\) and \((f_1, f_2, f_3)\) of \(SU(3)_{\text{family}}\)

1. Introduction

Nowadays, we know that hadrons (baryons and mesons) are composite states composed of quarks[1], so that "elementary particles" are quarks and leptons.

We know that quarks and leptons are described as \((\bar{10} + 5)\) of \(SU(5)_{\text{GUT}}\) [2]:

\[
\begin{align*}
5_L &= \begin{pmatrix} \nu_L \\ e_L \\ d_R \end{pmatrix}, \\
\bar{10}_L &= \begin{pmatrix} \bar{e_R} \\ u_L \\ d_L \\ u_R \end{pmatrix}.
\end{align*}
\] (1.1)

If we introduce a family symmetry \(SU(3)\), as we discuss in Sec.2, it is suggested that quarks
and leptons are described by a symmetry \(SU(5)_{\text{flavor}} \times SU(3)_{\text{family}}\) as 
\((5, \bar{3})\) and \((10, 3)\).

A model with such a symmetry \(SU(5)_{\text{flavor}} \times SU(3)_{\text{family}}\) has been discussed by the author already [3]. However, since the model has been discussed only on the base of the symmetry, some unwelcome states appeared in addition to the desirable quarks and leptons.

In this paper, we discuss a possibility that the quarks and leptons are not elementary
particles, but they are composed of more fundamental constituents ("preons")[4]. By introducing
some dynamical constraint for the composite states, we will remove unwelcome states from all
the combinations of preons except for the desirable quark and lepton states.

2. Preons and composite states

We know that quarks and leptons are described as \((\bar{10} + 5)\) of \(SU(5)_{\text{GUT}}\):

\[
\begin{align*}
5 &= \{(\nu_L, e_L, \bar{e_R})\}, \\
10 &= \{(e_R, (u_L, d_L), u_R)\}.
\end{align*}
\] (2.1)

If we denote the \(SU(5)\) symmetric states by the Young diagrams, the state \(10\) can be given by
a two box state which is arranged longitudinally

\[
10 = \begin{array}{c} \hline \hline 5 \\ \hline 5 \end{array}.
\] (2.2)
Table 1: **Quantum numbers of preons** Here, \( a = 1, 2, 3 \) (color numbers) and \( i = 1, 2, 3 \) (family numbers).

| preon | \( I_3 \) | \( Q \) |
|-------|---------|--------|
| \( w_1 \) | \( +\frac{1}{2} \) | 0     |
| \( w_2 \) | \( -\frac{1}{2} \) | -1    |
| \( c_a \) | 0       | \( +\frac{1}{3} \) |
| \( f_i \) | 0       | 0     |

We define more fundamental constituents (preons)

\[
\mathbf{5} = (w_1, w_2, c_1, c_2, c_3),
\]

where \((w_1, w_2)\) is a doublet of SU(2)\(_L\) and \((c_1, c_2, c_3)\) is a triplet of SU(3)\(_{\text{color}}\). Then, we can describe quarks and leptons as

\[
\begin{pmatrix} \mathbf{5}_L \\ \mathbf{5}_R \end{pmatrix} = \begin{pmatrix} 5 \\ 5 \\ 3 \end{pmatrix}_L + \begin{pmatrix} 5 \\ 5 \\ 3 \end{pmatrix}_R,
\]

i.e. the quarks and leptons can be described by the composite states \((2.4)\) of preons \((2.3)\). Here, the assignment of the chirality \((L\ and\ R)\) was chosen from a phenomenological point of view. The rule of the chirality will be discussed in the next section.

The relation \((2.4)\) suggests that if we introduce additional preons

\[
\mathbf{3} = (f_1, f_2, f_3),
\]

of SU(3) family, quarks and leptons can be regarded as three-body composite states. That is, we can regard quarks and leptons with three families as the three-body composite states of the preons

\[
\begin{pmatrix} 5 \\ 5 \\ 3 \end{pmatrix}_R + \begin{pmatrix} 5 \\ 3 \\ 3 \end{pmatrix}_L = (10, \mathbf{3})_R + (\mathbf{5}, \mathbf{3})_L.
\]

Here, we assume that the three preon states are bounded by the SU(3) hyper-color force.

Of course, we must discuss further three-body bound states \\{\mathbf{5},\mathbf{5},\mathbf{5}\\} and \\{\mathbf{3},\mathbf{3},\mathbf{3}\\}. This problem will be discussed in the next section Sec.3. The problem on the choice \(L\) or \(R\) will be also discussed in the next section.

Lastly, let us summarize the quantum numbers of preons in Table 1 and their composite states in Table 2. In both Tables 1 and 2, we show the electric charge \(Q = I_3 + Y/2\) instead of the hypercharge \(Y\).
3. Chirality selection rule

Now, we must discuss “chirality” in the composite states. However, rigorously speaking, there is no theory on the chirality for composite states.

In this paper, we put an empirical rule of the chirality on the composite states as follows:

(i) Chirality is defined for the two states $L$ and $R$, so that the state $L$ or $R$ will be given by two quantum numbers $\chi = -\frac{1}{2}$ and $\chi = +\frac{1}{2}$ analogously to two components in spin $J = \frac{1}{2}$ particle. We define the chiral quantum number as follows:

\[
\chi(5_R) = +\frac{1}{2}, \quad \chi(3_L) = -\frac{1}{2}.
\]  

(ii) The chirality of a composite state is given by

\[
\chi_{\text{comp}} = \sum \chi_{\text{preon}}.
\]

Then, the chirality of composite state is given by

\[
\chi(5_R5_R3_L) = +\frac{1}{2}, \quad \chi(5_R3_L3_L) = -\frac{1}{2},
\]  

so that we obtain

\[
(5_R5_R3_L)_R, \quad (5_R3_L3_L)_L.
\]

On the other hand, for the composite states

\[
(5_R5_R5_R), \quad (3_L3_L3_L),
\]  

we obtain $\chi = +\frac{3}{2}$ and $\chi = -\frac{3}{2}$, respectively. Since we consider that the value of $\chi$ must be $\pm \frac{1}{2}$, we consider that those states (3.5) are unphysical. Therefore, we can remove such unwelcome states (exotic states) (3.5).

This is not only to remove unwelcome states, but also our model makes anomaly free for the SU(5) flavor. (See Table 3.) However, this choice does not make anomaly free for SU(3)
family. We consider that SU(3) family is not gauge symmetry, but "apparent" symmetry. In fact, we know the family symmetry is badly broken even if it is "symmetry".

4. Concluding remarks

In conclusion, if we take that number of families is three, we can easily accept the picture that the quarks and leptons are members of \((\mathbf{10} + \mathbf{5}, \mathbf{3})_L\) of SU(5)\_GUT×SU(3)\_family, i.e. \((\mathbf{10}, \mathbf{3})_R + (\mathbf{5}, \mathbf{3})_L\).

In this paper, we have proposed a composite model for quarks and leptons where those are given by three body states of preons with 5 of SU(5)-flavor and 3 of SU(3)-family. In this model, we would like to emphasis that family number is three as seen in Eq.(2.6). Of course, in this composite model, it is important that those preons are confined by the SU(3) hyper-color.

For flavor symmetry, we have taken a picture of SU(5)-GUT. Regrettably, at present, we do not have any evidence for the proton decay. However, considering our success of the unified description of our quarks and leptons based on the composite model, we have recognized the SU(5) GUT picture and we consider that SU(5)-GUT phenomena will be confirmed in future experiments.

The author previously have discussed a similar model under the SU(5)\_GUT×SU(3)\_family\[3\]. However, because the author took a conventional field theoretical approach, we obtained unwelcome states \((\mathbf{1}, \mathbf{1})\) and \((\mathbf{1}, \mathbf{1})\) which correspond to the states \((\mathbf{5} 5 5)\) and \((\mathbf{3} 3 3)\) in Table 3 in addition to the states given in Table 2. In this paper, we adopt a composite picture, and we put a selection rule on the chirality in the composite system (Sec.3). This rule is somewhat forcible. However, with the help of this selection rule, we can obtain the desired quarks and lepton states only. In future, the selection rule given in Sec.3 will be replaced by more reasonable theory.

It is likely picture that quarks and leptons are composite states which are consisted of further fundamental constituents "preons".

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