Investigation on the Precision Loss of Ball Screw Considering the Full Ball Load Distribution

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Abstract: Ball screws are the driving functional components most frequently used for the machining equipments. However, the precision positioning error of ball screws decide directly machining accuracy of machine tools. This paper aims to investigate the precision loss of ball screw considering full ball load distribution. The raceway wear model is built to predict the precision loss of ball screw in different structural parameters and operating parameters. Meanwhile, the precision loss rate is obtained and analyzed during the whole running life.

1. Introduction
With the development of high speed cutting, the dynamic performance of ball screw is getting higher and higher. As in many mechanisms, the wear in ball screws complicates the control design progress and reduces the positioning accuracy significantly. Lin \cite{1} investigated the quasi-static contact characteristics of ball screw. Yoshida T \cite{2} established a new finite element model to study the uneven distribution of stress. Mei \cite{3} studied the influence of geometrical random error on the load distribution of ball screw. C. Braccesi \cite{4} presented a general elastic plastic approach to impact analysis for stress state limit evaluation in ball screw bearings return system. Song \cite{5} established a new online measuring system of ball screw movement precision. With the wear of ball screw, the accuracy of ball screw is reduced rapidly, which directly affects the life and the reliability of machine tool. A new model is built to predict the precision loss rate in different structural parameters and operating parameters.

2. Theoretical analysis

2.1 The load distribution of the full ball

The contact deformation $\delta$ between the ball and the nut or the screw are expressed as follows:

$$\delta_{ji} = C_i \frac{Q_{j,ii}^2}{L_i^3}.$$  \hspace{1cm} (1)
\( \delta_j (j=s \text{ or } n) \) are the contact deformation between the \( i \)th ball and the screw or the nut respectively, \( C_j (j=s \text{ or } n) \) are the contact stiffness of the screw or the nut respectively, \( Q_j \) is the normal load between the \( i \)th ball and the raceway. The axial deformation of the screw or the nut can be obtained respectively.

\[
\varepsilon_j = \frac{F_{i-1,i} \cdot \Delta L}{E \cdot A_j}, \quad \Delta L = l_p / N'.
\] (2)

\( e_j (j=s \text{ or } n) \) are the axial deformation of the screw segment or the nut segment between the \( i-1 \)th ball and the \( i \)th ball, \( E \) is the elastic modulus of the screw and the nut, \( A_j (j=s \text{ or } n) \) are the cross-sectional area of the screw or the nut in the contact point, \( \Delta L \) is the axial distance between two adjacent ball, \( F_{i-1,i} \) is the axial load of the screw segment between the \( i-1 \)th ball and the \( i \)th ball. \( l_p \) is the lead of the screw, \( N' \) is the number of balls in a single lead. Where the axial force relation of the screw along the axis is shown in Fig.1.

**Figure 1.** The axial force diagram of the screw axis.

Where \( F_a \) is the axial load, \( F_i \) is the axial force between the \( i \)th ball and the raceway. The relation of the axial force at each contact point can be obtained as follows:

\[
F_{i-1,i} = F_a - \sum_{j=1}^{N} F_j = \sum_{j=1}^{N} F_j - \sum_{j=i}^{N} F_j = \sum_{j=i}^{N} F_j \cdot F_j = Q_i \cdot \sin \alpha \cdot \cos \lambda.
\] (3)

Where \( N \) is the number of ball, \( \alpha \) is the contact angle between the ball and the raceway, \( \lambda \) is the nominal helix angle of the screw. The deformation relation of ball screw between the \( i-1 \)th ball and the \( i \)th ball is shown in Fig.2. \( \delta_{sai-1} \) and \( \delta_{mai} \) are the axial component of the Hertz contact deformation of the \( i-1 \)th ball and the \( i \)th ball with the nut raceway or the screw raceway respectively \((j=s \text{ or } n)\). The deformation coordination relationship can be obtained as follows:

\[
\varepsilon_{ai} + e_{mi} = (\delta_{sai-1} - \delta_{mai}) + (\delta_{msi-1} - \delta_{mai}).
\] (4)

**Figure 2.** The deformation diagram of ball screw.

The axial component of the Hertz contact deformation between the ball and the nut raceway or the screw raceway can be obtained as follows [6]

\[
\delta_a = \delta_s i n \alpha \cdot c o s \lambda
\] (5)

By substituting (1), (2), (3) and (5) into (4), the iterative formula of full ball load distribution can be obtained as follows:
\[
Q_i^2 - Q_{j-i}^2 + \frac{\Delta L}{E} \left( \frac{1}{A_i} + \frac{1}{A_n} \right) f \left( C_s + C_n \right) \sum_{j=1}^{N} Q_j = 0
\]
\[
F_n = \sum_{i=1}^{N} Q_i \cdot \sin \alpha \cdot \cos \lambda
\]

2.2 The wear model of the ball-screw contacts

Based on the Archard theory, the precision loss of the ball screw in the wear can be expressed as follows [7]

\[
W_{ji} = K \frac{Q_{L}}{H}.
\]

Where \( W_{ji} (j=s \text{ or } n) \) is the amount of wear volume, \( Q_i \) is the normal pressure of \( i \)th ball contact point, \( L_j \) is the relative sliding distance between ball and raceway at contact points, \( K \) is the coefficient of wear, \( H \) is the hardness of raceway. The contact area of ball with the screw or the nut can be written as follows:

\[
A_{ji} = \pi a_{ji} \cdot b_{ji}. \tag{8}
\]

Where \( a_{ji} \) and \( b_{ji} \) are the semi-major axis and semi-minor axis on the contact ellipse respectively.

Considering to the period of a ball passing through the contact point on the raceways of the screw or the nut, the coefficient \( f_s \) and \( f_n \) is introduced respectively. Thus, the total sliding distance of a contact point on the raceways of the screw or the nut can be obtained as follows:

\[
L_s = f_s \cdot |V_s^\perp| t = \frac{b_{ji} \sin \lambda}{l_s} |V_s^\perp| t, \quad L_n = f_n \cdot |V_n^\perp| t = \frac{b_{ji} \sin \lambda}{m_l} |V_n| t. \tag{9}
\]

Where \( t \) is the running time, \( l_s \) is the effective travel of the screw, \( m \) is the number of turns in one chain and \( V_s (j=s \text{ or } n) \) is the ball’s slipping velocity relative to the raceway at the contact point between a ball and the screw or the nut respectively.

By combining (7), (8), (9) and (10), the wear depth of the raceway can be obtained as follows:

\[
\Delta \delta = \sum_{i=1}^{N} W_{ji} \cdot \left( \sum_{j=1}^{N} \frac{K_s Q_s \sin \lambda |V_s^\perp|}{\pi H a_{ji}} + \frac{K_n Q_n \sin \lambda |V_n^\perp|}{\pi H a_{ji} m_{l_j}} \right) t = \delta_{\text{init}} \cdot t. \tag{10}
\]

Where \( \delta_{\text{init}} \) is the precision loss rate of ball screw.

**Table 1.** Parameters used in the simulation test

| Parameters               | Value | Unit |
|--------------------------|-------|------|
| Nominal diameter         | 40    | mm   |
| Ball diameter            | 4.763 | mm   |
| Lead angle               | 3.64  | °    |
| Lead                     | 8     | mm   |
| Rows \( \times \)Turns   | 3x1   |      |
| Contact angle            | 45    | °    |
| Poisson’s ratio          | 0.3   |      |
| Young’s modulus          | 205   | GPa  |

3. Results and discussion

It is worth mentioning that the running-in process is ignored in this paper to study the precision loss of ball screw because of the wear. The precision loss rate of ball screw versus axial load in different rotational speeds are shown in Fig.3. The relation of precision loss rate versus the axial load and rotational speed can be described as syntagmatic spatial plane. The precision loss rate is increased with an increasing axial load and rotational speed. When the axial load is increased, the normal force and the contact ellipse area between the ball and the raceway increase with the increasing axial load. However, the increasing rate of contact normal force is larger than the increasing rate of contact.
deformation area. It is obvious that the precision loss rate is proportional to the normal force and inversely proportional to the contact area. Thus, for a given rotational speed, the precision loss rate increases with increasing axial load. For a given axial load, the precision loss rate increases with increasing rotational speed.

**Figure 3.** Precision loss rate of the raceway versus axial load at different rotational speeds

Fig. 4 shows the precision loss rate of ball screw in different structure parameters. The precision loss rate of ball screw decreases with the increase of the contact angle. Thus, the greater contact angle can reduce the precision loss of ball screw. In addition, the precision loss rate of ball screw increases with the increase of the helix angle. Thus, the precision loss of large lead ball screw is serious, and properly reducing helix angle can decrease the precision loss of ball screw. The precision loss rate of ball screw increases with the increase of the raceway radius. While, the precision loss rate of ball screw decreases with the increase of ball diameter. The turning point is the situation that the ball radius is equal to the raceway radius. Therefore, it’s obvious that reasonable structure parameters can improve the precision preservation of ball screw.
Figure 4. Precision loss rate of the raceway in different structure parameters, (a) contact angle, (b) helix angle, (c) raceway radius and (d) ball diameter.

4. Conclusion
High-precision machine tools can not be separated from high-precision functional components. As a key functional component, the accuracy of ball screw directly affects the positioning precision of machine tools. The wear model is built to predict the precision loss of the ball screw in different structural parameters and operating parameters. Meanwhile, the precision loss rate is obtained and analyzed during the whole running life.

5. Acknowledgement
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6. References
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