Semiclassical interpretation of microscopic processes

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Abstract

There are three upper limits (2, 2√2, 2√3) of the Bell operator corresponding to different physical concepts: classical, hidden-variable and quantum-mechanical. Only the classical concept corresponding to the lowest limit has been excluded by experimental data, while the other two should be regarded as acceptable for the interpretation of EPR experiments and all microscopic processes. A corresponding hidden-variable or semiclassical model (based on the extended Hilbert space) will be proposed and shortly described.

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1. Introduction

The interpretation of EPR experiments represents one of the key problems of contemporary physics. It seemed that the question would be fully answered on the basis of experimental data when J. Bell derived his famous inequalities. However, the basic questions remained practically open, even if Aspect et. al. showed that the given inequalities have been violated in the experiment and the data have been practically in agreement with quantum-mechanical predictions.

We should like to show that the given situation has been burdened from the very beginning by two following facts:

- It has been believed that Bell’s inequalities have been derived without any important assumption; however, a far-reaching assumption has been involved (see Sec. 2).
- It was stated by Belinfante that the prediction of any hidden-variable theory for photon transmission through a polarizer pair should differ strongly from the quantum-mechanical one, which has been based on an assumption contradicting the reality (see Sec. 3).

We will show in Sec. 2 that there are different limits of expectation values of Bell operator that correspond to divers physical concepts. Only the classical limit (being regarded mistakenly until now as the hidden-variable one) is clearly excluded by experimental data, while the other two (hidden-variable and quantum-mechanical) must be denoted as acceptable. In Sec. 3 the correct hidden-variable formula for photon transmission through two polarizers will be derived. And finally, a corresponding mathematical hidden-variable model will be proposed and described in Sec. 4.

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2. Different limits of Bell operator

Bell (and also his followers) has derived the given inequalities for a combination of different coincidence probabilities for two spin particles (photons), having included an assumption concerning the individual transition probabilities. We will show now its consequence specifying various conditions that lead to divers limit values and correspond to different physical concepts. It will be done in the language of the so called Bell operator obtained by substituting individual probabilities by basic operators that correspond to individual measurement acts (see, e.g., [5]).

The Bell operator $B$ is defined in the Hilbert space

$$\mathcal{H} = \mathcal{H}_a \otimes \mathcal{H}_b$$

where the subspaces $\mathcal{H}_a$ and $\mathcal{H}_b$ represent individual measuring devices (polarizers) in the coincidence arrangement. It is then possible to write

$$B = a_1 b_1 + a_1 b_2 + a_2 b_1 - a_2 b_2$$

where $a_j$ and $b_k$ are operators acting in subspaces $\mathcal{H}_a$ and $\mathcal{H}_b$ and corresponding to measurements in individual polarizers. It holds for the expectation values of these operators

$$0 \leq |\langle a_j \rangle|, |\langle b_k \rangle| \leq 1.$$

The expectation values $|\langle B \rangle|$ of the Bell operator may then possess different upper limits according to the mutual commutation relations of the operators $a_j$ and $b_k$.

If it holds

$$[a_1, a_2] \neq 0, [b_1, b_2] \neq 0,$$

and also $[a_j, b_k] \neq 0$,

one can obtain by a rough estimate [6]

$$\langle BB^+ \rangle \leq 16 \text{ or } \langle B \rangle \leq 4.$$

However, after more detailed calculation one obtains [7]

$$\langle BB^+ \rangle \leq 12, \quad |\langle B \rangle| \leq 2\sqrt{3}. \quad (3)$$

If

$$[a_1, a_2] \neq 0, [b_1, b_2] \neq 0,$$

but $[a_j, b_k] = 0$,

it holds

$$\langle BB^+ \rangle \leq 8, \quad |\langle B \rangle| \leq 2\sqrt{2}. \quad (4)$$

And finally, if all operators $a_j$ and $b_k$ commute mutually one obtains

$$\langle BB^+ \rangle \leq 4, \quad |\langle B \rangle| \leq 2; \quad (5)$$

the same limit being obtained also if all operators at least in one of the subspaces $\mathcal{H}_a$ and $\mathcal{H}_b$ commute mutually [7].

These three different limits correspond to divers physical alternatives:
(i) In contradiction to common opinion the last limit (5) corresponds to the conditions of classical physics. The assumption (interchange of transmission probabilities between different pairs of photons) introduced by Bell [2] (see also, e.g., [8]) has been equivalent in its consequences to that used by von Neumann [9] in 1932, which has been mentioned recently also by Malley [10].

(ii) The limit (4) represents the properties of an hidden-variable alternative; it is not the limit (5) as commonly assumed until now. The corresponding hidden parameters involve the variables characterizing the properties of measuring devices (such as exact coordinates of individual atoms and similarly), the values of which may be hardly exactly specified, even if they influence measurement results.

(iii) As to the limit (3) it represents the case when the results of both the measuring devices are being mutually influenced; i.e., the case of the orthodox quantum mechanics. In the past it has been stated that only the quantum-mechanical alternative has been allowed by experimental data.

However, it is only the classical limit that has been excluded by experimental data. As to the hidden-variable alternative it does not contradict the experiment and may be brought to agreement (see Sec. 3) with experimentally established coincidence polarization data (obtained, e.g., by Aspect et al. [3]).

3. Photon transmission through a polarizer pair and hidden-variable prediction

The last statement seems, however, to be in disagreement with that of Belinfante (see [4], p. 284) that the hidden-variable prediction must differ significantly from the experimental data obtained for two polarizers and represented in principle by Malus law

\[ M(\alpha) \cong \cos^2 \alpha \]

where \( \alpha \) is the mutual deviation of polarizer axes. However, Belinfante started from a non-physical assumption. In fact, already a very simple hidden-variable alternative may give approximately required results, as will be shown in the following.

The transmission probability of unpolarized beam through a polarizer pair in the hidden-variable description may be expressed by

\[ P(\alpha) = \int_{-\pi/2}^{\pi/2} d\lambda \ p_1(\lambda) \ p_1(\alpha - \lambda) \]  

(6)

where \( \lambda \) are the spin (polarization) deviations of individual incoming photons from the axis of the first polarizer and \( p_1(\lambda) \) is the corresponding transition probability through the polarizer; parameter \( \lambda \) being introduced by Bell. Formula (6) is valid for both one-side and coincidence arrangements of two polarizers.

We can then easily obtain

\[ P(\alpha)/P(0) \cong M(\alpha) \]

if, e.g.,

\[ p_1(\lambda) = 1 - \frac{1 - \exp(-a|\lambda|e)}{1 + c \exp(-a|\lambda|e)}, \quad a = 1.95, \ e = 3.56, \ c = 500; \]

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(a λ), e, c being dimensionless numbers. Function \( p_1(\lambda) = p_1(-\lambda) \) is represented by full line and \( P(\alpha) \) by dashed line in Fig. 1; \( \lambda \) or \( \alpha \) being shown on abscissa. It holds also for the intensity transmitted through the first polarizer

\[
I_1/I_0 = \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} d\lambda \, p_1(\lambda) \cong 0.45.
\]

Belinfante came to his conclusion when he put quite arbitrarily: \( p_1(\lambda) = \cos^2 \lambda \); more detailed explanation of the given problem being found in [11].

Thus, both the previous arguments against the hidden-variable theory have been removed and nothing seems to prevent the polarization EPR experiments from being described with the help of the given theoretical alternative. And the last question should be put: Is it possible to propose a corresponding mathematical model that would fulfill all required properties? Such a model has been already proposed [12, 13]; the goal having been reached by adapting the basic assumptions of the standard quantum-mechanical model, which will be mentioned shortly in the next section.

4. Hidden-variable (semiclassical) model of EPR experiments

It is possible to say that the standard quantum-mechanical mathematical model is based on the following three assumptions:

(i) The description of a physical system is given by the complex function \( \Psi(\tilde{x}, t) \) obtained as a solution of the time-dependent Schrödinger function

\[
i \frac{\partial}{\partial t} \Psi(\tilde{x}, t) = H \Psi(\tilde{x}, t)
\]  
(7)

where the Hamiltonian

\[
H = \sum_{k=1}^{N} \sum_{j=1}^{3} \frac{p_{jk}^2}{2m_k} + V(\tilde{x})
\]  
(8)

represents its total energy;

(ii) Individual states \( \Psi(\tilde{x}, t) \) are represented by vectors in the Hilbert space \( \mathcal{H} \) spanned on the eigenfunctions of the Hamiltonian:

\[
H u_E(\tilde{x}) = E u_E(\tilde{x})
\]

The expectation physical values are then established with the help of standard rules.

(iii) A physical meaning is attributed to the mathematical superposition principle holding in such a space. Only in some more complex physical systems the so-called superselection rules have been applied to, even if no theoretical reason for different handling has been given in principle.

It is possible to state that the measurement postulate proposed by von Neumann and involving the so-called wave collapse has been the direct consequence of these three mathematical conclusions.

A corresponding mathematical hidden-variable model has been then obtained when the first assumption has conserved, the last assumption has been abandoned (superselection rules being extended inside \( \mathcal{H} \)), and the second assumption substituted by:
(ii) The Hilbert space has been extended (twice doubled in the general case) as proposed and described to a greater detail in Refs. [12, 13]; see also [14]. There is also one-to-one correspondence between a vector in the Hilbert space and an actual experimental state in the framework of such a model.

However, as to the EPR problem (and all microscopic collision processes) a much simpler Hilbert space structure is sufficient that corresponds to the scattering theory proposed by Lax and Phillips [15, 16] many years ago. The Hilbert subspace \( \mathcal{H} \) consists in such a case of two subspaces

\[
\mathcal{H} = \mathcal{D}_- \oplus \mathcal{D}_+
\]

where each subspace \( \mathcal{D}_- \) and \( \mathcal{D}_+ \) is spanned on one set of Hamiltonian eigenfunctions; both the subspaces being related mutually with the help of evolution operator \( U(t) = \exp(-iHt) \). The evolution goes from incoming states (\( \mathcal{D}_- \)) to outgoing states (\( \mathcal{D}_+ \)) in an irreversible way; it holds, e.g., \( \langle U(t)\mathcal{D}_+|\mathcal{D}_- \rangle = 0 \).

In the description of coincidence EPR system the time evolution in individual subspaces may be, of course, neglected. The incoming and outgoing states may be represented by simple vectors in agreement with Eq. \( \text{(1)} \). Only the transition of an incoming state to an outgoing one is important, which may be described by the products of operators \( a_j b_k \) representing the transition probabilities of corresponding photon-pair states from \( \mathcal{D}_- \) to \( \mathcal{D}_+ \). The hidden-variable model of polarization EPR experiments may be, therefore, brought to harmony with the general mathematical structure proposed independently already earlier [12, 13].

5. Conclusion

To conclude we should like to stress that the correlation of diverse limit values of the Bell operator to different physical concepts and theoretical approaches has opened quite new views how to interpret EPR experiments (and other microscopic phenomena) and to contribute to looking for new ways in physical thinking. It is evident that the hidden-variable (semiclassical) theory cannot be excluded from further considerations concerning the EPR problem, while the standard quantum-mechanical description seems to be unnecessarily broad. A definite decision between these two alternatives must be done, however, on the basis of other experiments.

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Figure 1: Malus law and hidden variables; $p_1(\lambda)$ - full line, $P_2(\alpha)$ - dashed line; Malus law - individual points.