Dark Matter in the Private Higgs Model

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Abstract

The extremely large hierarchy observed in the fermion mass spectrum remains as one of the most puzzling and unresolved issues in particle physics. In a recent proposal, however, it was demonstrated that by introducing one Higgs doublet (or Private Higgs) per fermion this hierarchy could be made natural by making the Yukawa couplings between each fermion and its respective Higgs boson of order unity. Among the interesting predictions of the Private Higgs scenario is a variety of scalars which could be probed at future collider experiments and a possible dark matter candidate. In this paper, we study in some detail the dark matter sector of the Private Higgs model. We first calculate the annihilation cross sections of dark matter in this model and find that one can easily account for the observed density of dark matter in the Universe with relatively natural values of the model’s parameters. Finally, we investigate the possibility of detecting Private Higgs dark matter indirectly via the observation of anomalous gamma rays originating from the galactic halo. We show that a substantial flux of photons can be produced from the annihilation of Private Higgs dark matter such that, if there is considerable clumping of dark matter in the galactic halo, the flux of these gamma rays could be observed by ground-based telescope arrays such as VERITAS and HESS.

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I. INTRODUCTION

One of the most puzzling issues in the Standard Model is the large hierarchy observed in the masses of fermions. For example, in the quark sector alone, the masses of the heaviest (top) and lightest (up) quarks are separated by nearly five orders of magnitude. Conversely, if one assumes that all fermions receive their mass via interactions with the 

*same* Higgs doublet (as in the Standard Model (SM)), the large hierarchy of masses observed in the fermion sector translates into a large hierarchy in the Yukawa couplings of the fermions.

Recently, it has been proposed that the hierarchy of fermion masses can be made natural by extending the scalar sector of the SM to include one Higgs doublet (or *Private Higgs* (PH)) per fermion [1]. In this scenario, all of the Yukawa couplings can be made of $O(1)$ by tuning parameters of the model. In other words, the vacuum expectation values (vev’s) of each respective PH field can be made to satisfy $v_f \sim m_f$ such that the hierarchy in the fermion mass spectrum becomes natural.

The approach to electroweak symmetry breaking (EWSB) in the PH model is quite different than those of other multi-Higgs models. First, one introduces one gauge singlet scalar $S_q$ per quark flavor $q$ and uses the vev’s of these fields along with certain interactions between these fields and the various PH fields to induce “negative-mass-squared” instabilities. By using different terms in the Lagrangian for the top PH and non-top PH fields, one can easily explain the hierarchy in vev’s by tuning certain parameters of the model. As a consequence of this approach, the lighter the fermion is, the heavier its associated PH particle must be in order to explain the smallness of the respective vev. In particular, the mass of the PH particle associated with the up quark can be shown to lie in the $10^2 - 10^3$ TeV range which is definitely beyond the reach of the Large Hadron Collider (LHC). However, there is interesting phenomenology originating from the sector of the top and bottom PH fields along with the singlet scalars. In this work, we study a scenario where the physical spectrum of this sector contains a light SM-like Higgs boson, a heavy scalar Higgs boson, a pair of charged Higgs bosons and a pseudoscalar Higgs boson. The last three of these arise mainly from the bottom PH field and all have masses in the $\sim$ TeV range. In addition to these, there are also two light scalars which are admixtures of the singlet states associated with the top and bottom quarks ($S_t$ and $S_b$). By construction, $S_t$ and $S_b$ are *dark* to interactions with SM gauge fields and fermions. While we will focus mainly on the light scalars in this work,
the heavier Higgs bosons could be probed at the LHC via production with bottom quarks (since the Yukawa coupling between the bottom quarks and the bottom PH field is of order unity)\textsuperscript{1}.

In order to avoid cross-talk between different quarks, the PH model contains a set of six discrete symmetries (one for each quark flavor). Under these symmetries, the right-handed quarks, their respective PH fields and the gauge singlet scalars $S_q$ are all odd, while all other SM fields are even. The existence of these discrete symmetries provides one of the most interesting features of the PH scenario which is the possibility of a dark matter (DM) candidate. Scalar DM was originally proposed over twenty years ago in Ref. \cite{2} and has been studied more recently in several different scenarios including singlet scalar DM \cite{3, 4, 5, 6, 7, 8} and in the so-called Inert Doublet Model \cite{9, 10, 11, 12, 13, 14, 15}. However, as we will demonstrate, the features of PHDM can be quite different from previously studied scenarios.

The remainder of the paper is structured as follows. First, in Section \textbf{II} we review the structure of the PH model and demonstrate how EWSB is realized in this model. In section Section \textbf{III} utilizing the observations from WMAP \cite{16}, we show that the PH model is able to account for all of the observed dark matter in the Universe for relatively natural values of the model’s parameters. In addition, in Section \textbf{IV} we consider the possibility of detecting PHDM via its annihilation into anomalous gamma rays in the galactic halo. We show that, with a favorable distribution of DM in the halo, PHDM could be detected by ground-based telescopes, but is probably beyond the reach of the space-based GLAST telescope \cite{17}. Finally, in Section \textbf{V} we conclude.

\section{The Model}

The main goal of the Private Higgs model is to account for the extremely large hierarchy observed in the fermion mass spectrum \cite{1}. For purposes of this paper, we will focus on the quark sector. In contrast to the SM, where one introduces a single scalar doublet which couples to all quarks, the PH scenario democratically introduces one Higgs doublet $\phi_q$ ($q = u, d, s, c, t, b$) per quark. All of the PH fields are assumed to have identical $SU(2) \times U(1)$

\textsuperscript{1} Presumably, the PH partner of the $\tau$ could also provide interesting phenomenology; however, we will focus on the quark sector here.
quantum numbers as the SM Higgs. In addition to the PH fields, the scalar sector of the PH model also contains a set of gauge singlet scalars $S_q$. In order to avoid cross talk between quarks of different flavors, a set of six discrete symmetries $K_q$ is imposed on the model. Under the $K_q$ symmetries, the right-handed quarks $(U_q, D_q)$ along with the PH fields and $S_q$ are all odd, i.e.:

$$U_q \rightarrow -U_q (D_q \rightarrow -D_q), \quad \phi_q \rightarrow -\phi_q, \quad S_q \rightarrow -S_q,$$

(1)

while all other fields are considered even. The Lagrangian which is symmetric under the $K_q$ symmetries is then given by:

$$\mathcal{L} = \mathcal{L}_{SM-H} - \sum_q (Y_{PH}^D \overline{q} \phi_D D_q + Y_{PH}^U \overline{q} \tilde{\phi}_U U_q)$$

$$+ \sum_q \left[ \partial_\mu S_q \partial^\mu S_q + (D_\mu \phi_q)^\dagger D^\mu \phi_q \right] - V(S_q, \phi_q),$$

(2)

where $\tilde{\phi}_U = i \sigma_2 \phi_U$, $\mathcal{L}_{SM-H}$ is the SM Lagrangian without the Higgs terms and $Y_{PH}^D, Y_{PH}^U$ are Yukawa matrices. The scalar potential $V(S_q, \phi_q)$ takes the form:

$$V(S_q, \phi_q) = \sum_q \left\{ \frac{1}{2} M_{\phi_q}^2 S_q^2 + \lambda_S^q \phi_q^4 + \frac{1}{2} M_{\phi_q}^2 \phi_q^4 + \lambda_q (\phi_q^\dagger \phi_q)^2 - g_{sq} S_q^2 \phi_q \phi_q^\dagger \phi_q \right\}$$

$$+ \sum_{q \neq q'} \left\{ a_{qq'} S_q^2 S_{q'}^2 + b_{qq'} S_q \phi_q \phi_{q'}^\dagger \phi_{q'} + c_{qq'} S_q^2 \phi_q \phi_{q'} \phi_{q'}^\dagger \phi_{q'} \right\} + h.c.,$$

where, for stability of the potential, $a_{qq'}, b_{qq'}, c_{qq'} < 0$. In our analysis, we will assume these terms are small and neglect them in the following.

In the PH model, instead of inducing EWSB through the usual “negative-mass-squared” approach where $M_{\phi_q}^2 < 0$, one utilizes the vev’s of the singlet fields $S_q$ and the interactions between the $S_q$’s and the PH fields. In particular, for the top PH one assumes $M_{\phi_t}^2 > 0$ and induces EWSB through the $g_{st}$ and $\chi_{qt}$ couplings as well as the vev’s of the $S_q$ fields. Thus, taking $g_{st}, \chi_{qt} > 0$ and $\frac{1}{2} M_{\phi_t}^2 - g_{st} \langle S_q \rangle^2 - \sum_{q \neq t} \chi_{qt} \langle S_q \rangle^2 \equiv \mu_t^2 < 0$, the top PH is forced to develop a negative-mass-squared instability which, in turn, spontaneously breaks the $SU(2)_L \times U(1)_Y$ gauge symmetry. Therefore, in a sense, the top PH plays the role of the SM Higgs.
In general, the PH scenario can contain many new free parameters in addition to those of the SM. In order to simplify our analysis in the following sections, we will make a succession of approximations. Thus, our results will not probe the full parameter space of the PH scenario, but should be viewed as a first step in this direction. To begin, we follow Ref. [1] and assume that $M_{\phi_t}^2 \ll g_{st} v_s^2$ which is in accordance with the symmetry breaking pattern discussed above. We also consider the case where $g_{st} \sim \chi_{qt}$ and $a_{qq'}^S \ll 1$. To give the $S_q$ fields a vev, one introduces an instability $M_{S_q}^2 < 0$ such that, under our assumptions, the potential in the $S_q - \phi_t$ sector reduces to:

$$V(S_q, \phi_t) = \frac{\lambda_q^2}{4} \left( S_q^2 - \frac{(v_q^q)^2}{2} \right)^2 + \lambda_t (\phi_t^\dagger \phi_t)^2 - g_{st} S_q^2 \phi_t^\dagger \phi_t, \quad (3)$$

where summation over quark flavor $q$ is implicit and, in principle, the quantity $v_q^q$ is a bare parameter. Minimizing this potential, we find the conditions:

$$\frac{\partial V(S_q, \phi_t)}{\partial S_q} \bigg|_{\langle S_q \rangle, \langle \phi_t \rangle} = \lambda_q^2 \left( \langle S_q \rangle^2 - \frac{(v_q^q)^2}{2} \right) - 2 g_{st} \langle \phi_t \rangle^2 = 0, \quad (4)$$

and:

$$\frac{\partial V(S_q, \phi_t)}{\partial \phi_t} \bigg|_{\langle S_q \rangle, \langle \phi_t \rangle} = 2 \lambda_t \langle \phi_t \rangle^2 - g_{st} \langle S_q \rangle^2 = 0. \quad (5)$$

Solving these equations for the individual vev’s we find:

$$\langle S_q \rangle^2 = \frac{(v_q^q)^2}{2} \left( \frac{\lambda_q^2 \lambda_t}{\lambda_q^2 \lambda_t - g_{st}^2} \right) \equiv \frac{v_s^2}{2}, \quad (6)$$

for the vev of $S_q$ and for the top PH vev:

$$\langle \phi_t \rangle^2 = \frac{(v_q^q)^2}{4} \left( \frac{g_{st} \lambda_q^q}{\lambda_q^2 \lambda_t - g_{st}^2} \right) \equiv \frac{v_h^2}{2}. \quad (7)$$

Note that, for simplicity, we have identified the individual vev’s $v_q^q$ with one common parameter $v_s$. Finally, we also note the relationship between $v_s$ and $v_h$:

$$v_h^2 = \frac{g_{st}}{2 \lambda_t} v_s^2. \quad (8)$$

Next, we consider the non-top PH fields which acquire their vev’s in a slightly different manner. First, as in the case of the top PH, the mass parameter $M_{\phi_q}^2$ is assumed to be positive. However, for the $\phi_q$ fields (where $q \neq t$), one imposes the condition $M_{\phi_q}^2 > g_{sq} v_s^2$ in contrast to the case of the top PH. Then, vev’s for the non-top PH fields are induced
through the cubic term $\gamma_{qq}$ and the vev’s $v_s$ and $v_h$. Again, to simplify our analysis, we will make some assumptions. Specifically, we will assume that:

$$M_{\phi_q}^2 \gg g_{sq} v_s^2, \lambda_q$$

which is consistent with the symmetry breaking pattern discussed above. Then, after $S_q$ and $\phi_t$ pick up vev’s, the relevant part of the $\phi_q$ potential is:

$$\frac{1}{2} M_{\phi_q}^2 \phi_q^\dagger \phi_q - \frac{\gamma_{qt}}{\sqrt{2}} \frac{v_h v_s^2}{2} \phi_q.$$  

(10)

Minimizing this potential, the vev’s for the non-top Higgs fields are found to be:

$$\langle \phi_q \rangle = \frac{\gamma_{qt} v_h v_s^2}{\sqrt{2} M_{\phi_q}^2} \equiv v_q.$$  

(11)

Eq. (11) summarizes the main result of the PH scenario. By having the parameter $\gamma_{qt}$ to be small while keeping $M_{\phi_q}^2$ large, one is able to make all Yukawa couplings (which are given by $m_q/v_q$) of $O(1)$ without fine-tuning. As a consequence of this relation, one can show from Eq. (11) that the lighter quarks have associated PH particles in the $10^2 - 10^3$ TeV range which are definitely beyond the reach of current or future experiments [1]. However, the masses from the $\phi_t - S_q$ sector can be naturally light (100’s GeV), while the bottom PH particle can have masses in the TeV range.

Finally, inserting Eqs. (7) and (11) into the Lagrangian of Eq. (2), it is easy to show that the $W^\pm$ mass is given in the PH model by:

$$m_W^2 = \frac{1}{2} g v_h^2 \left[ 1 + \sum_{q \neq t} \left( \frac{v_q^2}{v_h^2} \right) \right].$$

(12)

Obviously, the leading term in the sum comes from the bottom PH; however, even in this case, the contribution is of order $m_b^2/m_t^2 \sim 0.001$. Thus, the contributions to EWSB from quarks lighter than the top are negligible and our statement above that the role of the SM Higgs boson is being played by the top PH is verified.

A. Mass Eigenstates and Their Interactions

In this section, we study the top-bottom sector of the PH model in some detail. In particular, we will consider the case where $\lambda^t_S = \lambda^b_S \equiv \lambda_S$ and $\lambda_S \ll \lambda^{q}_{S}$ for $q \neq t, b$. Thus, the gauge singlet scalars associated with the lighter quarks become heavy and effectively
decouple from our analysis. Then, under our assumptions, the scalar potential in the top-bottom sector reduces to:

\[
V(S, \phi_t, \phi_b) = \frac{\lambda_S}{4} \left[ (S_t^2 - \frac{v_s^2}{2})^2 + (S_b^2 - \frac{v_s^2}{2})^2 \right] + \lambda_t (\phi_t^+ \phi_t)^2 + \frac{1}{2} M_{\phi_b}^2 \phi_b^+ \phi_b
- a_{tb} S_t^2 S_b^2 \right] - \gamma_{tb} S_t S_b (\phi_t^+ \phi_t + \phi_b^+ \phi_b) - g_{st} \right] \left[ S_t^2 \phi_t^+ \phi_t + S_b^2 \phi_b^+ \phi_b + S_t^2 \phi_t^+ \phi_t \right].
\] (13)

To begin, we expand the PH Higgs fields in the usual way:

\[
\phi_t = \begin{pmatrix} \omega^+ \\ \frac{1}{\sqrt{2}} (v_h + h_t + i\chi^0) \end{pmatrix},
\] (14)

\[
\phi_b = \begin{pmatrix} H^+ \\ v_b + H_b + iA_b \end{pmatrix},
\] (15)

while the singlet fields are expanded as:

\[
S_{t,b} = \frac{1}{\sqrt{2}} (v_s + \sigma_{t,b}).
\] (16)

In the above expansions, $\omega^\pm$ and $\chi^0$ are assumed to play the roles of the usual Goldstone bosons which are eaten by the $W^\pm$ and $Z$, while $H^\pm$ and $A_b$ are charged and pseudoscalar Higgs bosons, respectively. Both the $H^\pm$ and $A_b$ will have masses on the order of $M_{\phi_b} \sim \text{TeV}$ and could provide interesting phenomenology at the LHC. Note that we are neglecting mixing between the “pure Goldstones” ($\omega^\pm, \chi^0$) and the “physical Higgs bosons” ($H^\pm, A_b$). These mixings are typically of order $\gamma_{tb} v_b^2 / M_{\phi_b}^2$ and, thus, are extremely small.

Inserting the expansions of the Higgs fields from Eqs. (14) - (16) into (13), we first extract the mass terms of the Goldstone bosons which we require to vanish:

\[
m_{\omega^\pm}^2 = m_{\chi^0}^2 = \lambda_t v_h^2 - \frac{1}{2} g_{st} v_s^2 = 0.
\] (17)

Note that this equation is in agreement with Eq. (8).

Next, we could attempt to diagonalize the full $4 \times 4$ mass matrix in the $(h_t, H_b, \sigma_t, \sigma_b)$ basis. However, as shown in Ref. [1], for values of the model parameters that we consider in our analysis most of the mixings between the various scalars are negligible. In particular, the mixing between $h_t$ and $\sigma_q$ is negligible provided:

\[
8 g_{st}^3 \ll (2 g_{st} - \lambda_S)^2 \lambda_t \quad (\text{for } q = t),
\] (18)
\[ \gamma_{tb} \left( \frac{m_b}{m_t} \right) \ll g_{st} \quad \text{(for } q = b) \].

Similarly, the mixing between \( \sigma_q \) and \( H_b \) can be neglected provided:

\[ v_h \left( \frac{m_b}{m_t} \right) \ll v_s. \]

All of these conditions are satisfied for the parameter choices in our analysis, hence we choose to neglect the above mixings. However, it should be noted that the mixing between \( h_t \) and \( H_b \) serves to reproduce the small SM coupling between bottom quarks and the SM-like Higgs boson. In the following, we identify \( h_t \) and \( H_b \) with approximate mass eigenstates \( h^0 \) and \( H^0 \) respectively and assume the coupling between \( h^0 \) and a pair of bottom quarks takes its SM value.

Finally, there can be substantial mixing between the two singlet scalars \( \sigma_t \) and \( \sigma_b \) via the \( a_{tb}^s \) and \( \gamma_{tb} \) terms. Diagonalizing the 2 \( \times \) 2 mass matrix in this sector, we find two mass eigenstates (\( \Sigma_1 \) and \( \Sigma_2 \)) with mass eigenvalues:

\[ m_{\Sigma_1}^2 = \frac{1}{2} \left( \lambda_S^2 - g_{st} v_h^2 - a_{tb}^s v_s^2 \right) - \left( a_{tb}^s v_s^2 + \frac{\gamma_{tb}}{\sqrt{2}} v_h v_b \right) \sin 2\alpha, \quad (21) \]

\[ m_{\Sigma_2}^2 = \frac{1}{2} \left( \lambda_S^2 - g_{st} v_h^2 - a_{tb}^s v_s^2 \right) + \left( a_{tb}^s v_s^2 + \frac{\gamma_{tb}}{\sqrt{2}} v_h v_b \right) \sin 2\alpha. \]  \quad (22)

Note that for \( a_{tb}^s, \gamma_{tb} > 0 \) and \( 0 < \alpha < \pi/2 \), \( \Sigma_1 \) plays the role of the lightest PH particle (LPHP) which is stable against decay and, thus, provides a candidate for DM. Using Eqs. (21) and (22), we can exchange two of the free parameters (e.g., \( \lambda_S \) and \( a_{tb}^s \)) for the masses of the two singlet scalars. This is the approach we will take. Therefore, in the analysis to follow, we will take as our free parameters the masses \( m_{\Sigma_1}, m_{\Sigma_2} \) as well as the couplings \( g_{st} \) and \( \gamma_{tb} \) and the mixing angle \( \alpha \). Note that the conditions for small mixings between the \( \sigma_q \)'s and the PH fields forces \( g_{st} \) to take small values.

### III. PRIVATE HIGGS DARK MATTER

As mentioned earlier, one of the most interesting aspects of the PH scenario is the prospect of a Weakly Interacting Massive Particle (WIMP) with masses in the expected natural range for DM. In this context, the PH model is similar to other scalar DM models such as the gauge singlet models of Refs. [2, 3, 4, 5, 6, 7, 8] and the Inert Doublet Model (IDM) [9, 10].
FIG. 1: Leading s-channel processes which maintain the singlet scalar $\Sigma_1$ in equilibrium with the rest of the cosmic fluid.

In this section, we calculate the annihilation cross sections of PHDM into SM particles and show that, for relatively natural values of the model parameters, one can account for all of the observed dark matter in the Universe. In the next section, we investigate the possibility of indirectly detecting PHDM via its annihilation into anomalous gamma rays in the galactic halo.

First, let us consider the present relic abundance of PHDM in the Universe. In the following, we will assume that the mass splitting $m_{\Sigma_2} - m_{\Sigma_1}$ is large enough that coannihilation reactions between $\Sigma_1$ and $\Sigma_2$ do not significantly affect the relic abundance. These effects will be considered in future work. In the early Universe, the singlet scalar $\Sigma_1$ would have been in equilibrium with the rest of the cosmic fluid. This equilibrium is maintained via $\Sigma_1$ pair-annihilation and pair-creation reactions which proceed through the s-channel exchange of the SM-like Higgs $h^0$. The leading $2 \to 2$ s-channel reactions which contribute to these processes are shown in Fig. 1.

The present relic abundance of PHDM is determined by the pair-annihilation rates in the non-relativistic limit. The rates for each allowed channel are given in the non-relativistic limit as:

$$a(X) \equiv \lim_{u \to 0} \sigma(\Sigma_1 \Sigma_1 \to X) u$$

where $u$ is the relative velocity of the annihilating particles. The total annihilation cross section is then given by summing over each of the allowed channels. Computing the cross
sections for the diagrams in Fig. 1 we find:

\[ a(W^+W^-) = \frac{g_{\Sigma_1 h^0}^2}{2\pi v_h^2} \sqrt{1 - \mu_w} \left( 1 - \mu_w + \frac{3}{4} \mu_w^2 \right), \]  

(24)

\[ a(ZZ) = \frac{g_{\Sigma_1 h^0}^2}{4\pi v_h^2} \sqrt{1 - \mu_z} \left( 1 - \mu_z + \frac{3}{4} \mu_z^2 \right), \]  

(25)

\[ a(f\bar{f}) = \frac{g_{\Sigma_1 h^0}^2}{4\pi v_h^2} \left( 1 - \mu_f \right)^2 \left( 1 - \mu_f + \frac{3}{4} \mu_f^2 \right), \]  

(26)

where \( \mu_i = m_i^2/m_{\Sigma_1}^2 \), \( \Gamma_{h^0} \) is the width of the \( h^0 \) for which we use SM values and the expression for the coupling \( g_{\Sigma_1 h^0} \) is given by:

\[ g_{\Sigma_1 h^0} = -g_{st} v_h + \frac{2\gamma_{tb} v_b}{\sqrt{2}} c_\alpha s_\alpha. \]  

(27)

The WMAP collaboration [16] provides a very precise determination of the present DM abundance which, at the two-sigma level, is given by:

\[ \Omega_{DM} h^2 = 0.111 \pm 0.018. \]  

(28)

As shown in Ref. [18], for a generic model of DM, the present abundance of DM is mainly determined by \( J_0 \) (the angular momentum of the dominant partial wave contributing to DM annihilation) and the annihilation cross section. In contrast, the relic abundance depends only weakly on the mass or spin of the DM particle. Thus, the very precise constraints from WMAP on \( \Omega_{DM} h^2 \) translate into very precise constraints on the quantity \( a \equiv \sum_X a(X) \) depending on the value of \( J_0 \). In particular, for an “s-wave annihilator” \( (J_0 = 0) \) such as the case considered here, the WMAP measurement translates into the bounds:

\[ a = 0.8 \pm 0.1 \text{ pb}, \]  

(29)

nearly independent of the mass or spin of the DM particle (see Fig. 1 of Ref. [18]).

In Fig. 2 we plot the values of \( a(X) \) for two different values of the coupling \( g_{st} \) and several different values of the mixing angle \( \alpha \). In these plots, we have set the bottom PH vev equal to the bottom quark mass and \( \gamma_{tb} = 1 \). The horizontal dashed lines indicate the limits on \( a(X) \) from Eq. (29). Clearly, from Eqs. (24) - (27), we see that the annihilation cross sections depend quadratically on \( g_{st} \). This is evident in the plot of Fig. 2 where we see a small shift in \( g_{st} \) results in a large shift in the ranges of \( m_{\Sigma_1} \) allowed by the WMAP data.
Finally, we note that these plots are only meant to show that for relatively natural values of the model parameters it is indeed possible to account for the observed density of DM in the Universe. A full scan of the PH parameter space would probably find other choices of parameters which could fulfill the constraints from Eq. (29).

IV. INDIRECT DETECTION OF PHDM

Next, we would like to investigate the possibility of detecting PHDM. We will focus here on indirect detection and save an analysis of direct detection for future work.

As we have seen, the annihilation rates for PHDM are approximately velocity-independent in the non-relativistic regime. In general, this implies that DM collected in galactic halos has a substantial probability to pair-annihilate resulting in anomalous high-energy cosmic rays which can be distinguished from astrophysical backgrounds. In particular, gamma rays from these annihilations provide a chance to extract information about DM, since they can travel over galactic scales without scattering.

The production of gamma rays from $\Sigma_1 \Sigma_1$ annihilation can originate from several different processes (including hadronization, factorization and radiation from final-state particles). However, for simplicity, we will assume the dominate source is from direct annihilation into a two-body final state as shown in Fig. 3. Note that, under our assumptions, only SM particles circulate the loop. In the full parameter space of the PH model, it would be possible to have charged Higgs circulating the loop. However, their couplings to $h^0$ are always of order $v_b/v_h$ or $v_b/v_s$ and, thus, can be safely ignored in comparison to the SM loops. The cross section for photon pair-production in the PH scenario can be written as:

$$\sigma_{\gamma\gamma} = \frac{2 g_{\Sigma_1 \Sigma_1 h^0}^2}{(s - m_{h^0}^2)^2 + \Gamma_{h^0} m_{h^0}^2} \frac{\hat{\Gamma}(h^0 \to \gamma\gamma)}{\sqrt{s}},$$  

where the expression for $g_{\Sigma_1 \Sigma_1 h^0}$ is given above and, in the non-relativistic regime, $s \approx 4 m_{\Sigma_1}^2$. The hat on $\hat{\Gamma}$ indicates that one should replace $m_{h^0} \to \sqrt{s}$ in the standard expressions for on-shell Higgs decays. The expressions needed to construct $\hat{\Gamma}(h^0 \to \gamma\gamma)$ can be found in several reviews (e.g., see Ref. [19]) and, hence, we will not repeat them here.

Here, we concentrate on the dominant $\gamma\gamma$ signal and save a discussion of the $Z\gamma$ and/or $h^0\gamma$ channels for future work.
FIG. 2: The annihilation cross section as a function of the $\Sigma_1$ mass and the mixing parameter $\alpha$. The dashed horizontal lines indicate the WMAP constraints on the annihilation cross sections given by Eq. [29].
FIG. 3: Diagrams which dominate photon pair-production in the $\Sigma_1$ annihilation in the galactic halo.

Next, we would like to compute the flux of photons observed on Earth from $\Sigma_1$ annihilation in the galactic halo. The monochromatic flux due to the $\gamma\gamma$ final state, observed by a telescope with a line of sight parameterized by $\Psi = (\theta, \phi)$ and a field of view $\Delta \Omega$ can be written as [20]:

$$\Phi = (1.1 \times 10^{-9} \text{s}^{-1} \text{cm}^{-2}) \left(\frac{\sigma_{\gamma\gamma} u}{1 \text{ pb}}\right) \left(\frac{100 \text{ GeV}}{m_{\Sigma_1}}\right) \bar{J}(\Psi, \Delta \Omega) \Delta \Omega,$$

(31)

where the dependence of the flux on the halo dark matter density distribution is contained in $\bar{J}$. Many models predict a large spike in the DM density in the neighborhood of the galactic center, making the line of sight towards the center of the galaxy the preferred one. However, the features of the peak are highly model-dependent resulting in values of $\bar{J}$ ranging from $10^3$ to $10^7$ for $\Delta \Omega = 10^{-3}$ sr (typical for ground-based atmospheric Cherenkov telescopes) [21, 22, 23, 24].

The monochromatic photon flux predicted from PHDM annihilation for the two values of $g_{st}$ studied previously are shown in Fig. 4. For these plots, we have assumed there is no substantial spiking in the galactic center (i.e., $\bar{J}\Delta \Omega = 1$). In the energy range considered in these plots, ground-based atmospheric Cherenkov telescopes (such as VERITAS [25] and HESS [26]) typically have a flux sensitivity down to the $10^{-12}$ s$^{-1}$ cm$^{-2}$ level. On the other hand, the upcoming space-based telescope GLAST [17] is limited by statistics to the $10^{-10}$ s$^{-1}$ cm$^{-2}$ level over the energy range considered. From these plots, it is clear that without a substantial spike in the galactic center, PHDM will be difficult to observe in either ground- or space-based observatories. However, if the halo does exhibit a substantial spike or strong clumping (e.g., if $\bar{J} \geq 10^5$ at $\Delta \Omega \simeq 10^{-3}$), PHDM could be observed at ground-based telescopes assuming small values of $g_{st}$ and relatively light masses ($m_{\Sigma_1} \simeq 100 - 120 \text{ GeV}$).
FIG. 4: The flux of monochromatic photons from the reaction $\Sigma_1 \Sigma_1 \to \gamma \gamma$ for $\bar{J} \Delta \Omega = 1$ for two different values of the coupling $g_{st}$. 

$g_{st} = 0.04$

$g_{st} = 0.06$
V. CONCLUSIONS

The Private Higgs model attempts to address the large hierarchy observed in the fermion mass spectrum by introducing one Higgs doublet for each fermion. EWSB is achieved not by the usual “negative-mass-squared” approach, but by introducing a set of gauge singlet scalars and using the vev’s of these fields and their interactions with the PH fields to induce instabilities. In order to avoid cross-talk between quarks of different flavors, one also introduces a set of discrete symmetries. This provides one of the most interesting features of the Private Higgs model: a possible dark matter candidate.

In this paper, we have begun an investigation of the PH dark matter sector. We found that for relatively natural values of the model’s parameters the PH model provides a candidate which can account for the relic density of dark matter observed in the present Universe. To show this, we calculated the annihilation cross section for PHDM into SM particles and compared to limits on the cross section which can be obtained from the WMAP observations.

Finally, we investigated the possibility of detecting PHDM via anomalous gamma rays originating from the annihilation of PHDM in the galactic halo. While the observation of these gamma rays may be difficult for the space-based GLAST observatory, we showed that evidence of PHDM could be observed at ground-based atmospheric Cerenkov telescopes such as VERITAS and HESS if there is substantial clustering of dark matter in the galactic halo.

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