Large Effects of CP- and T-Violation in $K^0$ Decay

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The $\epsilon$-impurity of the $K_L$ wave-function gives rise to a huge $CP$-violating effect in the decay $K_L \rightarrow \pi^+\pi^-\gamma$ which is hidden in the polarization state (Stokes vector) of the photon. One component of the Stokes vector is $CP$-odd and $T$-even, and may be identified with circular polarization. Another component is $CP$-odd and $T$-odd ("oblique polarization") and reveals itself as a large asymmetry (14%) in the decay $K_L \rightarrow \pi^+\pi^-e^+e^-$. Striking time-dependent effects are predicted in the angular distribution of the $\pi^+\pi^-e^+e^-$ system emanating from an initial $K^0$ or $\bar{K}^0$ state.

It is not customary to use the word "large" in association with $CP$-violating effects in $K^0$ decays. Nevertheless, a large effect has been observed in the decay $K_L \rightarrow \pi^+\pi^-e^+e^-$, in agreement with theoretical predictions. In this talk, I explain the origin of this effect, and some of its ramifications.

1. $CP$- and $T$-Violation in $K_L \rightarrow \pi^+\pi^-\gamma$

The decay $K_S \rightarrow \pi^+\pi^-\gamma$ is known to be well-described by pure bremsstrahlung. By contrast, the branching ratio and the photon energy spectrum of $K_L \rightarrow \pi^+\pi^-\gamma$ require a mixture of bremsstrahlung and direct $M_1$ emission. A simple ansatz for the matrix elements is

$$M(K_{SL} \rightarrow \pi^+\pi^-\gamma) =$$

$$\frac{|s|}{M_k} E_{S,L}(\omega, \cos \theta) \left[ \epsilon \cdot p_+ k \cdot p_- - \epsilon \cdot p_- k \cdot p_+ \right] + M_{S,L}(\omega, \cos \theta) \epsilon_{\mu\nu\rho\sigma} \epsilon^\mu k^\nu p_+^\rho p_-^\sigma,$$

(1)

where

$$E_S = \left( \frac{2M_k}{\omega} \right)^2 \epsilon^{i\delta_0(s=M_k)} \frac{1}{1 - \beta^2 \cos^2 \theta}, \quad M_S = 0,$$

$$E_L = \left( \frac{2M_k}{\omega} \right)^2 \eta_{\epsilon+} \epsilon^{i\delta_0(M_k)} \frac{1}{1 - \beta^2 \cos^2 \theta},$$

$$M_L = i (0.76)e^{i\delta(s)}.$$  \hspace{1cm} (2)

Here $\omega$ is the photon energy in the $K_L$ rest frame, $\theta$ the angle between $\pi^+$ and $\gamma$ in the $\pi^+\pi^-$ c.m. frame, and $\beta = \sqrt{1 - \frac{4m^2}{s}}$, $s$ being the $\pi^+\pi^-$ invariant mass. The coefficient 0.76 in $M_L$ is determined from the empirical strength of direct emission in $K_L \rightarrow \pi^+\pi^-\gamma$. The phase factor $e^{i\delta_0(M_k)}$ in the bremsstrahlung amplitudes $E_{S,L}$ is dictated by the Low theorem, while the factor $i e^{i\delta(s)}$ in $M_L$ is determined by CPT invariance and the Watson theorem. The important feature of the $K_L$ amplitude is that the bremsstrahlung component $E_L$, proportional to $\eta_{\epsilon+}$, is enhanced by the factor $(2M_K/\omega)^2$, making it comparable to the direct emission amplitude $M_L$. The interference of the electric multipoles ($CP = +1$) with the magnetic multipole ($CP = -1$) opens the way to large $CP$-violating observables. Such interference effects vanish, however, if one sums over the photon polarization. Thus $CP$ violation is encrypted in the polarization state of the photon.

The polarization state of the photon can be described by the Stokes vector $\vec{S} = (S_1, S_2, S_3)$ whose components are (dropping the subscript $L$ in $E_L, M_L$)

$$S_1 = \frac{2 \Re E^* M}{|E|^2 + |M|^2},$$

$$S_2 = \frac{2 \Im E^* M}{|E|^2 + |M|^2},$$

and

$$S_3 = \frac{2 \Im E^* M}{|E|^2 + |M|^2}.$$
\[
S_3 = \frac{|E|^2 - |M|^2}{|E|^2 + |M|^2}.
\] (3)

These are plotted in Fig. 1 as a function of the photon energy. The components \(S_1\) and \(S_2\) are \(CP\)-violating observables, and are remarkably large, considering that they originate in the small parameter \(\epsilon \approx \eta_{+-}\). To see the physical meaning of these parameters, we choose a frame in which \(\vec{k} = (0, 0, \omega)\) and \(\vec{n}_\pi = (\vec{p}_+ \times \vec{p}_-)/|\vec{p}_+ \times \vec{p}_-| = (1, 0, 0)\). The parameter \(S_2\) is then recognized as the circular polarization, i.e.

\[
S_2 = \frac{d\Gamma(L) - d\Gamma(R)}{d\Gamma(L) + d\Gamma(R)} \quad (4)
\]

where \(L\) and \(R\) refer to the polarization vectors \(\vec{e}_{R,L} = (1, \pm i, 0)/\sqrt{2}\). This is a \(CP\)-odd but \(T\)-even observable, and vanishes in the hermitian limit \(\delta_0 = \delta_1 = 0\), \(\text{arg} \eta_{+-} = \pi/2\ [7]\). To understand the significance of \(S_1\), we consider the linear polarization vector \(\vec{e} = (\cos \phi, \sin \phi, 0)\), where \(\phi\) is the polarization direction relative to \(\vec{n}_\pi\), the normal to the decay plane. One then discovers that \(S_1\) is the “oblique polarization”, defined as the difference between the decay rates for \(\phi = 45^\circ\) and \(\phi = 135^\circ\):

\[
S_1 = \frac{d\Gamma(45^\circ) - d\Gamma(135^\circ)}{d\Gamma(45^\circ) + d\Gamma(135^\circ)} \quad (5)
\]

More generally, the decay rate as a function of \(\phi\) is

\[
\frac{d\Gamma}{d\phi} \sim 1 - [S_3 \cos 2\phi + S_1 \sin 2\phi]. \quad (6)
\]

Fig. 2 illustrates how this pattern changes from

![Figure 1. Stokes parameters of the photon in \(K_L \to \pi^+\pi^-\gamma\)](image)

![Figure 2. Distribution of \(K_L \to \pi^+\pi^-\gamma\) in angle \(\phi\) between polarization vector \(\vec{e}\) and normal to decay plane. (a) \(\omega \to 0\), (b) \(\omega \to 170\ MeV\), (c) average over \(20\ MeV < \omega < 170\ MeV\).](image)
an electric distribution \( d\Gamma/d\phi \sim \sin^2 \phi \) at low \( \omega \) to a magnetic distribution \( d\Gamma/d\phi \sim \cos^2 \phi \) as the photon energy increases. The presence of the parameter \( S_1 \) produces a tilted pattern in which there is an asymmetry between the quadrants I+III compared to II+IV. It is this tilt that signals a violation of \( CP \). By examining the behaviour of \( k, \vec{z} \) and \( \vec{n}_\pi \) under \( CP \) and \( T \), we find that \( \sin 2\phi \) transforms as \( CP = -, T = - \). Thus the oblique polarization \( S_2 \) is a \( CP \)-odd, \( T \)-odd observable \( \vec{n}_\pi \). Unlike the parameter \( S_2 \), the parameter \( S_1 \) survives in the hermitian limit. In this sense, the \( T \)-odd property of \( S_1 \) is not an artifact of dynamical phases, but rather an example of a magnetic distribution accompanying \( CP \)-violation in a \( CP \)-odd, \( T \)-odd parameter. The study of the Dalitz pair reaction \( K_L \rightarrow \pi^+\pi^-e^+e^- \) may be viewed as an attempt to detect the oblique polarization of the photon in \( K_L \rightarrow \pi^+\pi^- \gamma \), using the plane of the \( e^+e^- \) pair as an analyser.

2. \( CP \)- and \( T \)-Violation in \( K_L \rightarrow \pi^+\pi^-e^+e^- \)

The matrix element of the reaction \( K_L \rightarrow \pi^+\pi^-e^+e^- \) can be written as

\[
\mathcal{M} = \mathcal{M}_{br} + \mathcal{M}_{mag} + \mathcal{M}_{CR} + \mathcal{M}_{SD} \tag{7}
\]

where the \( \mathcal{M}_{br} \) and \( \mathcal{M}_{mag} \) are associated with the bremsstrahlung and magnetic components of the radiative amplitude. The term \( \mathcal{M}_{CR} \) denotes a “charge radius” contribution, corresponding to \( \pi^+\pi^- \) emission in an s-wave, not possible for the real radiative process \( K_L \rightarrow \pi^+\pi^- \gamma \). The term \( \mathcal{M}_{SD} \) represents the contribution of the short-distance interaction \( \vec{s}_{\pi} \rightarrow e^+e^- \). Estimates in \( \mathcal{M}_{SD} \) showed that the amplitude is dominated by the first two terms in Eq. \( \mathcal{M}_{SD} \), as is now borne out by the data \( \mathcal{M}_{SD} \). The differential decay rate may be calculated in the form

\[
d\Gamma = I(s_\pi, s_1, \cos \theta_1, \cos \theta_\pi, \phi) ds_\pi ds_1 d\cos \theta_1 d\cos \theta_\pi d\phi \tag{8}
\]

where \( s_\pi(s_1) \) is the invariant mass of the pion (lepton) pair and \( \theta_\pi(\theta_1) \) is the angle of the \( \pi^+(l^+) \) in the \( \pi^+\pi^- (l^+l^-) \) rest frame, relative to the dilepton (dipion) direction. For the purpose of detecting the oblique polarization in \( K_L \rightarrow \pi^+\pi^- \gamma \), the relevant variable is \( \phi \), the angle between the normals to the \( \pi^+\pi^- \) and \( l^+l^- \) planes. Defining the unit vectors

\[
\vec{n}_\pi = \frac{\vec{p}_+ \times \vec{p}_-}{|\vec{p}_+ \times \vec{p}_-|}, \\
\vec{n}_l = \frac{\vec{k}_+ \times \vec{k}_-}{|\vec{k}_+ \times \vec{k}_-|}, \\
\vec{z} = \frac{\vec{p}_+ + \vec{p}_-}{|\vec{p}_+ + \vec{p}_-|},
\]

we have

\[
\sin \phi = \vec{n}_\pi \cdot \vec{n}_l \cdot \vec{z} \quad (CP = -, T = -) \\
\cos \phi = \vec{n}_l \cdot \vec{n}_\pi \quad (CP = +, T = +). \tag{10}
\]

Figure 3. Distribution of \( K_L \rightarrow \pi^+\pi^-e^+e^- \) in angle \( \phi \) between \( \pi^+\pi^- \) and \( e^+e^- \) planes.

Integrating over all variables other than \( \phi \), one obtains

\[
\frac{d\Gamma}{d\phi} \sim 1 - (\Sigma_3 \cos 2\phi + \Sigma_1 \sin 2\phi) \tag{11}
\]

with \( \Sigma_3 = -0.133 \) and \( \Sigma_1 = 0.23 \). This distribution is plotted in Fig. 3, and shows clearly the \( CP \)- and \( T \)-violating “tilt” similar to the oblique polarisation in Fig. 2. The KTeV and NA48 experiments confirm the predicted \( \phi \)-distribution \( \mathcal{A}_\phi \), and also the integrated asymmetry

\[
\mathcal{A}_\phi = \frac{\left( f_0^{\pi/2} - f_{\pi/2}^{\pi} + f_{3\pi/2}^{3\pi/2} - f_{3\pi/2}^{2\pi} \right) \frac{d\Gamma}{d\phi}}{\left( f_0^{\pi/2} + f_{\pi/2}^{\pi} + f_{3\pi/2}^{3\pi/2} + f_{3\pi/2}^{2\pi} \right) \frac{d\Gamma}{d\phi}}
\]
3. Time-Evolution of the Decay Spectrum in $K^0(K^0)\to \pi^+\pi^-e^+e^-$ \[8\]

Consider the decay $K^0(K^0)\to \pi^+\pi^-\gamma$ of a state that is prepared as an eigenstate of strangeness +1(-1). The decay amplitude at a subsequent time $t$ can be expressed in terms of the amplitudes $E_{L,S}$, $M_{L,S}$ as follows:

$$
E(t, \omega, \cos\theta) = e^{-i\lambda_{L,S} t} E_{L,S}(\omega, \cos\theta),
$$

$$
M(t, \omega, \cos\theta) = e^{-i\lambda_{L,S} t} M_{L,S}(\omega, \cos\theta),
$$

where

$$
E = e^{-i\lambda_{L,S} t} E_S(\omega, \cos\theta) + e^{-i\lambda_{L,S} t} E_L(\omega, \cos\theta),
$$

$$
M = e^{-i\lambda_{L,S} t} M_S(\omega, \cos\theta) - e^{-i\lambda_{L,S} t} M_L(\omega, \cos\theta)
$$

with $\lambda_{L,S} = m_{L,S} - \frac{s}{2}\Gamma_{L,S}$.

Figure 4. Components $S_1$ and $S_2$ of the Stokes vector of the photon as a function of photon energy and time for the decay $K^0 \to \pi^+\pi^-\gamma$.

Figure 5. Components $S_1$ and $S_2$ of the Stokes vector of the photon as a function of photon energy and time for the decay $K^0 \to \pi^+\pi^-\gamma$. 

$$
\mathcal{M}(K^0(t) \to \pi^+\pi^-\gamma) \sim \{E(t, \omega, \cos\theta)[\epsilon \cdot p_+ k \cdot p_- - \epsilon \cdot p_- k \cdot p_+] + M(t, \omega, \cos\theta)\epsilon\mu\nu\rho\sigma M_{\mu\nu}^0 p_{\rho\sigma}^0\},
$$
The amplitudes in Eq.(13) allow us to determine the Stokes vector of the photon at any time \( t \). As an example, Figs. 4, 5 show the components \( S_1(t) \) and \( S_2(t) \) as function of energy for an initial \( K^0 \) or \( \bar{K}^0 \). It is interesting to ask how this time dependence would be reflected in the decay spectrum of \( K^0(\bar{K}^0) \to \pi^+\pi^-e^+e^- \).

This question has been analysed in [8]. In particular, we have calculated the time-dependent correlation of the \( \pi^+\pi^- \) and \( e^+e^- \) planes,

\[
\frac{d\Gamma}{d\phi} \sim 1 - (\Sigma_3(t) \cos 2\phi + \Sigma_1(t) \sin 2\phi),
\]

and the associated asymmetry \( A_\phi(t) \) defined as in Eq.(12).

The result is displayed in Fig. 6, and shows a remarkable time variation, that differs between \( K^0 \) and \( \bar{K}^0 \). As expected (and as measured by NA48 [3]), the asymmetry vanishes at short times, where the \( K \) meson decays essentially as \( K_S \) (and the amplitude of \( K_S \to \pi^+\pi^-\gamma \) is purely electric). At large times the asymmetry approaches the asymptotic value \( A_\phi = -14\% \) expected for \( K_L \to \pi^+\pi^-e^+e^- \). Also shown in Fig. 6 is the result to be expected if the source of \( K \)-mesons is an untagged equal mixture of \( K^0 \) and \( \bar{K}^0 \) (such as derived from \( \phi \to K^0\bar{K}^0 \) at DAΦNE). The non-zero value of \( A_\phi(t) \) for such an untagged beam represents a \( CP^- \) and \( T \)-violating effect at any decay time.

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