Minimal Quark-Lepton Symmetry Model
and Possible Limits on Z'-mass from TRISTAN and LEP200

A.D. Smirnov*
Division of Theoretical Physics, Department of Physics,
Yaroslavl State University, Sovietskaya 14,
150000 Yaroslavl, Russia.

ABSTRACT

A minimal extension of the Standard Model containing the four-color quark-lepton symmetry is discussed. Some features of an additional Z'-boson originated from this symmetry are investigated and the limits on \( m_{Z'} \) from the current TRISTAN data and the possible limits on \( m_{Z'} \) from the future LEP200 collider are obtained.

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*E-mail: phystheo@univ.yars.free.net
The search for a new physics beyond the Standard Model (SM) is now one of the main problems of the high energy physics. It seems that the $SU_L(2) \times U(1) \times SU_c(3)$-symmetry of the SM is only the first stage in the hierarchy of possible symmetries of fundamental interactions. To search for the next such stage it seems reasonable to investigate various minimal extensions of the SM by adding to it some additional symmetries such as $SU_R(2)$-symmetry, supersymmetry, etc. One of such symmetries possibly existing in nature and being worthy of the detailed investigation now is the four-color quark-lepton symmetry regarding the lepton number as the fourth color. In this work we discuss the minimal quark-lepton symmetry model of the unification of the strong and electroweak interactions (MQLS-model) which is the minimal extension of the SM containing the four-color quark-lepton symmetry and investigate the limits on the mass of $Z'$-boson originated from this symmetry using the current TRISTAN data and the possible limits on $m_{Z'}$ which can be achieved at LEP200.

The model to be discussed here is based on the $SU_V(4) \times SU_L(2) \times U_R(1)$-group as the minimal group containing the four-color quark-lepton symmetry and investigate the limits on the mass of $Z'$-boson originated from this symmetry using the current TRISTAN data and the possible limits on $m_{Z'}$ which can be achieved at LEP200.

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\[ Q^{L,R} = \sqrt{\frac{2}{3}} L_{15}^R + \frac{\tau_3^L}{2} + \frac{Y^R}{2}, \]

where \( t_{15}, \tau_3/2 \) are the corresponding generators, \( \tau_3 \) is the Pauli matrix.

According to the structure of the group the gauge sector consists of 19 fields \( A^i_\mu, i = 1, 2, \ldots, 15; W^k_\mu, k = 1, 2, 3 \) and \( B_\mu \). The first eight of them are the gluons \( G^j_\mu = A^j_\mu, j = 1, 2, \ldots, 8; \) the next six fields form the triplets of the leptoquarks \( V^\pm_\alpha \), \( \alpha = 1, 2, 3 \) with the electric charge \( Q_V = \pm 2/3; \) \( W^1_\mu, W^2_\mu \) form the \( W^\pm \)-bosons in a usual way and the remained fields \( A^1_\mu, A^3_\mu, B_\mu \) form the photon, the \( Z \)-boson and an extra \( Z' \)-boson.

The electromagnetic field \( A_\mu \) is related to \( A^{15}_\mu, A^3_\mu, B_\mu \) by

\[ A_\mu = s_S A^{15}_\mu + \sqrt{1 - s_W^2 - s_S^2 B_\mu} + s_W W^3_\mu, \]

and two orthogonal to \( A_\mu \) fields \( Z_{1\mu} \) and \( Z_{2\mu} \) can be written as

\[ Z_{1\mu} = -t_W (s_S A^{15}_\mu + \sqrt{1 - s_W^2 - s_S^2 B_\mu}) + c_W W^3_\mu, \]
\[ Z_{2\mu} = (\sqrt{1 - s_W^2 - s_S^2 A^{15}_\mu} - s_S B_\mu) / c_W, \]

where \( s_{W,S} = \sin \theta_{W,S}, c_W = \cos \theta_W, t_W = \tan \theta_W \). The angles \( \theta_W \) and \( \theta_S \) of the weak and strong mixings are defined as

\[ s_W^2 = \frac{\alpha(m)}{\alpha_W(m)}; \]
\[ s_S^2 = \frac{2}{3} \frac{\alpha(m)}{\alpha_{15}(m)} = \frac{2}{3} \frac{\alpha(m)}{\alpha_S(m)} \left( 1 + \frac{\alpha_S(m)}{2\pi} b \ln \frac{M_C}{m} \right), \]

where \( \alpha(m), \alpha_W(m), \alpha_S(m) \) are the electromagnetic, weak and strong coupling constants at the scale \( m, M_C \) is the mass scale of the \( SU_V(4) \)-symmetry breaking, \( b = b_S - b_{15} = 11, b_S = 11 - \frac{2}{3} n_f, b_{15} = -\frac{2}{3} n_f, n_f \) is the number of fermions with masses below \( M_C \). The last equality in (2) is obtained by the elimination of the \( SU_V(4) \) unified gauge coupling constant \( \alpha_4(M_C) = g^2_4/4\pi \) from the one-loop approximation relations

\[ \alpha_{S,15}(m) = \alpha_4(M_C) / (1 + \frac{\alpha_4(M_C)}{2\pi} b_{S,15} \ln \frac{M_C}{m}) \]

between \( \alpha_4(M_C) \) and the \( A^{15} \)-interaction constant \( \alpha_{15}(m) \) and \( \alpha_S(m) \).

The interaction of the gauge fields with the fermions can be written as
\[ \mathcal{L}_{\text{int}}^\psi = \frac{g_4}{\sqrt{2}} [V_\mu^A (Q'_\alpha \gamma^\mu Q'_\ell) + h.c.] + \frac{g_2}{\sqrt{2}} [W_\mu^+ (\bar{\psi}_a^L \gamma^\mu (\tau^+)_{ab} \psi_{bA}) + h.c.] \]
\[ + g_{\text{st}} G_\mu^J (Q \gamma^\mu t J Q) + e A_\mu (\bar{\psi} \gamma^\mu \psi) + \mathcal{L}_{\text{NC}}. \] (4)

Here the first term describes the interaction of leptoquarks with quarks and leptons by the constant \( g_4 \) related to \( \alpha_S(m) \) by (3). This interaction contains, in general, the new generation mixing due to the matrices \( K_{aL}^{L,R} = (A_{Qa}^{L,R}) + A_{\ell a}^{L,R} \). The second term in (4) describes the weak charged current interaction of \( W^\pm \)-bosons with quarks or leptons by the constant \( g_2 \) related to the Fermi constant \( G_F \) and \( m_W \) in a usual way. This interaction contains the well known Cabibbo-Kobayashi-Maskawa mixing of the quarks due to the matrix \( C_Q = (A_{QL})^+ A_{Qa} \), and analogous mixing in the lepton sector due to the lepton mixing matrix \( C_\ell = (A_{\ell L})^+ A_{\ell a} \). The next two terms are the QCD- and QED-interactions. The neutral current interaction \( \mathcal{L}_{\text{NC}} \) can be written as

\[ \mathcal{L}_{\text{NC}} = Z_\mu J_\mu^Z + Z'_\mu J'_\mu^Z, \] (5)

where

\[
\begin{align*}
Z_\mu & = Z_{1\mu} \cos \theta_m + Z_{2\mu} \sin \theta_m, \\
Z'_\mu & = -Z_{1\mu} \sin \theta_m + Z_{2\mu} \cos \theta_m,
\end{align*}
\]
are the mass eigenstate fields and

\[
\begin{align*}
J_\mu^Z & = J_{\mu_1}^Z \cos \theta_m + J_{\mu_2}^Z \sin \theta_m, \\
J'_\mu^Z & = -J_{\mu_1}^Z \sin \theta_m + J_{\mu_2}^Z \cos \theta_m, \\
J_{\mu_1}^Z & = \frac{e}{s_{\text{CW}} c_{\text{CW}}} (J_{\mu_2}^{3L} - s_{\text{W}} J_{\mu}^Q), \\
J_{\mu_2}^Z & = \frac{e}{s_{\text{SCW}} \sqrt{1 - s_{\text{W}}^2 - s_{\text{S}}^2}} \left[ c_{\text{W}}^2 \sqrt{\frac{2}{3}} J_{\mu_2}^{15} - s_{\text{S}}^2 (J_{\mu_2}^Q - J_{\mu_2}^{3L}) \right].
\end{align*}
\] (9)

with the currents \( J_{\mu}^Q = (\bar{\psi}_{paA} \gamma_\mu (Q_{aA}) \psi_{paA}) \), \( J_{\mu}^{3L} = \frac{1}{2} (\bar{\psi}_{paA} \gamma_\mu (1 + \gamma_5) (\tau_3/2)_{aa} \psi_{paA}) \), \( J_{\mu_2}^{15} = (\bar{\psi}_{paA} \gamma_\mu (t_{15})_{AA} \psi_{paA}) \). The \( Z_1 \)-current (3) is the usual neutral current of the Standard Model, but the structure of the \( Z_2 \)-current (4) is specified by the model under consideration. The \( Z-Z' \)-mixing angle \( \theta_m \) is defined by the symmetry breaking mechanism of the model and is found to be small.

The Higgs sector of the model is taken in the simplest way and consists of the four multiplets \((4,1,1), (1,2,1), (15,2,1), (15,1,0)\) of \( SU_V(4) \times SU_L(2) \times U_R(1) \)-group with the vacuum expectation values (VEV’s) \( \langle \phi_A^{(1)} \rangle = \delta_{AA} \eta_1 / \sqrt{2} \), \( \langle \phi_A^{(2)} \rangle = \delta_{aa} \eta_2 / \sqrt{2} \), \( \langle \phi_{aB}^{(3)} \rangle = \delta_{a2} (t_{15})_{AB} \eta_3 \) and \( \langle \phi_{AB}^{(4)} \rangle = (t_{15})_{AB} \eta_4 \) respectively. After breaking the symmetry
Table 1. The strong mixing angle $\sin^2 \theta_S$ and the parameter $\sigma$ depending on the mass scale $M_C$ in the MQLS-model.

| $M_C$, GeV | $\sin^2 \theta_S$ | $\sigma$ |
|------------|-------------------|---------|
| $10^6$     | 0.130             | 0.216   |
| $10^{10}$  | 0.213             | 0.297   |
| $10^{14}$  | 0.297             | 0.380   |

in such a way the masses of quarks and leptons are defined by VEV’s $\eta_2$, $\eta_3$ and by Yukawa coupling constants and may be arbitrary just as they are in the Standard Model; the photon and the gluons are still massless but all the other gauge fields acquire the masses. The VEV $\eta_4$ contributes only to the masses of leptoquarks, and hence the leptoquarks may be heavy enough irrespective of the other gauge particles.

For the masses of the $W^-$, $Z^-$ and $Z'^-$-bosons the model predicts the mass relation

$$(\mu^2 - \rho_0)(\rho_0 - 1) = \rho_0^2 \sigma^2,$$  \hspace{1cm} (10)

where $\mu \equiv m_{Z'}/m_Z$, $\rho_0 \equiv m_W^2/m_Z^2$ and

$$\sigma = \frac{s_W s_S}{\sqrt{1 - s_W^2 - s_S^2}}.$$  \hspace{1cm} (11)

Simultaneously the model gives for the $Z^-$ - $Z'$ mixing angle $\theta_m$ the expression

$$\sin \theta_m = \left[1 + \left(\frac{\rho_0 \sigma}{\rho_0 - 1}\right)^2\right]^{1/2}.$$  \hspace{1cm} (12)

For $\theta_m \ll 1$ and $\rho_0 \simeq 1$ we also obtain from (10), (12) that $\theta_m \simeq \sigma m_{Z'}^2/m_Z^2$.

It should be noted that all the coupling constants in (4), (5) may be expressed by means of $s_W$, $s_S$ in terms of the known coupling constants $\alpha(m)$, $\alpha_W(m)$, $\alpha_S(m)$ at the scale $m = m_Z$ and the $SU_V(4)$ unification mass scale $M_C$ which enters $s_S$ and $\alpha_4(M_C)$ according to (2), (3) and is found to be the only unknown parameter of the Lagrangian (4), (5). Of course, to calculate the neutral current processes described by (5) it is necessary to know $m_{Z'}$, which actually results in two unknown parameters $M_C$ and $m_{Z'}$.

The mass relation (10) gives the limit on the $Z'$-mass. Using the experimental values of $G_F$, $m_W$ and $\alpha(m_Z)$ we have $s_W^2 = 0.2298 \pm 0.0014$. Then taking the most stringent limit $M_C \geq 10^5 \div 10^6 GeV$ resulting from $\text{Br}(K^0_L \rightarrow \mu e) < 0.94 \cdot 10^{-10}$ into account and using the experimental values $\alpha_s(m_Z) = 0.1134 \pm 0.0035$ we evaluate $s_S^2$ and $\sigma$ from (3) and (11) for $M_C = 10^6 \div 10^{14} GeV$ (see Table 1). For these values of the $\sigma$ the relation $m_{Z'}/m_Z$ and $\sin \theta_m$ as functions of the $\Delta \rho_0 \equiv \rho_0 - 1$ are presented on Fig.1. Extracting the allowed values of $\Delta \rho_0$ from the experimental value.
we obtain $\Delta \rho_0 = (4 \pm 2) \cdot 10^{-3}, (2 \pm 2) \cdot 10^{-3}, (0 \pm 2) \cdot 10^{-3}$ for $m_t = 100 \text{GeV}, 125 \text{GeV}$ and $150 \text{GeV}$ respectively. Then we see from Fig.1 that the $Z'$-boson may be rather light. Thus for $M_C = 10^6 \text{GeV}$ and $m_t = 125 \text{GeV}$ we get the limits $m_{Z'} > 4 m_Z$ on $Z'$-mass and $\theta_m < 0.01$ on the $Z-Z'$-mixing angle. This upper limit on $\theta_m$ is compatible with those obtained in the extended gauge models.\cite{3–5}

Using the structure \( (5) \) - \( (9) \) of the neutral current interaction we have calculated in the tree approximation the cross sections $\sigma_{\bar{f}f} = \sigma(e^+e^- \rightarrow \gamma, Z, Z' \rightarrow \bar{f}f)$. The leptonic cross section $\sigma_{\bar{f}\ell}$ is found to be less than the one predicted by the SM. This effect is due to the destructive $\gamma-Z'$-interference.\cite{6} The magnitude of this deviation depends on the MQLS-model parameters $m_{Z'}$ and $M_C$ and can be used for the derivations of the limits on $m_{Z'}$ and $M_C$ from the experimental data. Calculating the relative deviations $\delta_{\bar{f}\ell} = (\sigma_{\bar{f}\ell} - \sigma_{\bar{f}\ell}^{SM})/\sigma_{\bar{f}\ell}^{SM}$ at $\sqrt{s} = 60 \text{GeV}$ and using the current values of the leptonic cross section measured by AMY-, VENUS- and TOPAZ-groups at TRISTAN\cite{7} we have obtained the limits on $Z'$-mass in dependence on $M_C$ (or $\sin^2 \theta_S$). As an example, such limits on $m_{Z'}$ extracted from $\sigma_{\mu^+\mu^-}$ and corresponding to the experimental error of the one standard deviation are presented on Fig. 2. We see from Fig. 2 that within the experimental error of one standard deviation the
current $\sigma_{\mu^+\mu^-}$ data give the lower limit on $m_{Z'}$ about a few hundreds $GeV$ depending on $M_C$. As concerns the upper limits on $m_{Z'}$ extracted from $\sigma_{\mu^+\mu^-}$ VENUS- and TOPAZ- data, they should be regarded as preliminary because these limits depend crucially on the experimental errors and can be taken into account more seriously only at high experimental accuracy.

The hadronic cross section $\sigma_h = \sum_q \sigma_{q\bar{q}}$ is found to be somewhat more than that predicted by the SM but this deviation is smaller than that in the leptonic case. Hence, the measurements of the leptonic cross sections are more favourable for the search for the possible manifestations of the extra $Z'$-boson than the measurements of the hadronic cross sections.

We have calculated also the forward-backward asymmetry $A_{FB}$ of $e^+e^- \rightarrow \gamma, Z, Z' \rightarrow \ell\ell$ reactions and have analysed the limits on $Z'$- mass from $A_{FB}$ asymmetry measured at TRISTAN. These limits turned out to be weaker than those
extracted from cross sections \(\sigma_{\mu^+\mu^-}\).

Fig. 3. The possible lower limits on \(Z'\)-mass in MQLS-model from future measurements of the deviations \(\delta_{\ell\ell}\) of the leptonic cross sections \(\sigma_{\ell\ell}\) from the SM predictions at LEP200.

The evaluation of leptonic cross sections \(\sigma_{\ell\ell}\) at \(\sqrt{s} = 200\, GeV\) shows that their deviations \(\delta_{\ell\ell}\) from the SM predictions are significantly larger at such energies and, hence, the measurements of these deviations at LEP200 will allow either to observe the manifestation of \(Z'\)-boson originated from the four-color quark-lepton symmetry or to obtain the most stringent limits on the MQLS-model parameters \(m_{Z'}\) and \(M_C\). On Fig. 3 we show the lower limits on \(m_{Z'}\) in dependence on \(M_C\) (or \(\sin^2\theta_S\)) which can be achieved at LEP200 by measurements of the deviations \(\delta_{\ell\ell}\) of leptonic cross sections from the SM predictions to an accuracy of 20\%, 10\% and 5\% if the results of these measurements will be negative. It can be seen from Fig. 3 that, in particular, the lower limit on \(m_{Z'}\) for \(M_C \sim 10^6\, GeV\) and at 5\% accuracy of \(\delta_{\ell\ell}\) measurements can be raised up to about 1.2\, TeV.

In summary we can conclude that the minimal extension of the Standard Model
containing the four-color quark-lepton symmetry discussed here leads, in particular, to the existence of an additional $Z'$- boson originated from this symmetry. The limits on its mass $m_{Z'}$ obtained here from the mass relation and from the current TRISTAN data on $\sigma_{\bar{u}u}$, $\sigma_{h}$ and $A_{FB}$ show that such $Z'$- boson may be rather light ($m_{Z'}$ is larger a few hundred GeV) and may be interesting for experimental search at LEP200 and future colliders.

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