Numerical investigations of single bubble oscillations generated by a dual frequency excitation

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Abstract. The oscillations of a single bubble excited with a dual frequency acoustic field are numerically investigated. Computations are made for an air bubble in water exposed to an acoustic field with a linearly varying amplitude. The bubble response to an excitation containing two frequencies \( f_1 = 500\text{kHz} \) and \( f_2 = 400\text{kHz} \) at the same amplitude is compared to the monofrequency case where only \( f_1 \) is present. Time-frequency representations show a sharp transition in the bifrequency case, for which the low frequency component \( f_2 \) becomes resonant while the high frequency component \( f_1 \) is strongly attenuated. The temporal evolution of the power spectra reveals that the resonance of the low frequency component is correlated with the time varying mean radius of the bubble. It is also observed that the total power of the bubble response in the bifrequency case can reach almost twice the power obtained in the monofrequency case, which indicates a strong enhancement of the cavitating behavior of the bubble for this specific frequency combination.

1. Introduction

Controlling the cavitation activity is of primary interest for specific clinical uses of ultrasound whose efficiency can be enhanced with the generation of implosive bubbles. For example, when used for thrombolysis, focused ultrasound-induced cavitation plays a major role in the destruction of blood clots. For this type of application, reducing the cavitation threshold is challenging as it promotes the mechanical effects of cavitation while limiting the heat deposition on tissues. A number of studies have already shown that this threshold can be significantly lowered by using a multifrequency excitation. Several frequency combinations have been tested such as fundamental and second harmonic superimposition [1, 2], low frequency component addition [3–6] or dual neighboring frequencies emission [7, 8]. While all these experiments demonstrated the beneficial effect of dual frequency emission on cavitation activity, quite few works have been devoted to the theoretical investigation of this phenomenon. In the present study, this effect is quantified through the analysis of the response of a single bubble to a dual frequency excitation based on the numerical solving of the Keller-Miksis equation [9]. Computations are made for an air bubble in water exposed to a mono- or bi-frequency acoustic field with a linearly time-varying amplitude, and the temporal evolutions of the power spectra are determined for both mono- and bi-frequency cases.
2. Single bubble response to a dual frequency excitation

The present study deals with the nonlinear oscillations of a gas bubble in a slightly compressible liquid. Assuming that the size of the bubble is much smaller than the acoustic wavelength allows to keep spherical symmetry of the problem, so that the bubble radius $R$ is governed by the Keller-Miksis equation [9]:

$$R\ddot{R} \left(1 - \frac{\dot{R}}{c}\right) + \frac{3}{2} \dot{R}^2 \left(1 - \frac{\dot{R}}{3c}\right) = \left(1 + \frac{\dot{R}}{c}\right) \frac{p_i - p(t)}{\rho} + \frac{R \dot{p}_i - \dot{p}(t)}{\rho} - \frac{2\sigma}{\rho R} - \frac{4\mu}{\rho R},$$

(1)

where $c$ is speed of sound in the liquid, $\rho$ and $\mu$ are the liquid density and viscosity, and $\sigma$ denotes the gas-liquid surface tension. The motion of the bubble is driven by the pressure difference existing between its internal pressure $p_i$, and the liquid pressure $p(t)$. Assuming that the bubble is only filled with non-condensable gas, thus neglecting the partial vapor pressure, $p_i$ is given by the following equation:

$$p_i = \left(p_{\infty} + \frac{2\sigma}{R_0}\right) \left(\frac{R_0}{R}\right)^{3\gamma},$$

(2)

where $\gamma$ is gas polytropic exponent, $R_0$ is the static radius of the bubble and $p_{\infty}$ is the ambient pressure. In the specific case of a dual frequency acoustic excitation, the liquid pressure $p(t)$ is written as:

$$p(t) = p_{\infty} - \frac{p_a}{2} \left(\sin(2\pi f_1 t) + \sin(2\pi f_2 t)\right).$$

(3)

The minus sign ensures that the motion begins at $t = 0$ with a rarefaction phase and thus an expansion of the bubble. Note that in order to simplify further analysis, we state here that both frequency components are generated with the same amplitude $p_a/2$, corresponding to half the amplitude of the monofrequency case $p(t) = p_{\infty} - p_a \sin(2\pi f_1 t)$ (same peak amplitude for both mono- and bi-frequency cases).

3. Enhancement of cavitation activity: spectral analysis

The motion equation Eq. (1) is solved using a standard numerical method to obtain the temporal evolution of the bubble radius. In the following, the computations are made for an air bubble
Figure 2. Time-averaged power of the spectral components of the bubble response signal in the monofrequency (a) and bifrequency (b) cases. On graph (a) are given the power amplitudes of the fundamental frequency $f_1$ and its first harmonic $2f_1$, as well as the power amplitude of the static component (mean radius). On graph (b) are given the power amplitudes of the primary frequencies $f_1$ and $f_2$ and low frequency modulation $f_1 - f_2$, as well as the power amplitude of the static component.

in water at ambient temperature $T = 300$K and atmospheric pressure $p_\infty = 1.013$bar. The initial bubble radius is fixed to $R_0 = 7.058\mu$m, corresponding to the linear damped resonant radius at frequency $f_1 = 500$kHz. The acoustic pressure amplitude $p_a$ is linearly varying from 0 to 100kPa (thus from 0 to 50kPa in bifrequency case) over 4ms, corresponding to 2000 acoustic cycles at frequency $f_1$. Figure 1 gives the time-frequency representations of the bubble response in the monofrequency case for $f_1 = 500$kHz (a) and in the bifrequency case when a frequency component $f_2 = 400$kHz is added in the excitation term (b). The typical nonlinear harmonics generation is recovered in the monofrequency case with the rising of $2f_1$, $3f_1$ and $4f_1$ components (Note that because of the proper choice of the bubble radius and pressure amplitude, no subharmonics are generated in the bubble response). When a second frequency $f_2$ is added to the excitation, a spectral broadening occurs with the generation of harmonics and combinations of both frequencies. The most important observation concerns the transition towards strongly nonlinear oscillations at $t \simeq 1.8$ms associated with the generation of a broadband spectrum, which is clearly absent in the monofrequency case. A second transition is observed at $t \simeq 2$ms when the bubble recovers a periodic behavior. After this transition, one can observe a prevailing of the $f_2$-component over the $f_1$-component which is considerably attenuated.

To better understand the role of the additional $f_2$ component on the bubble response, the time evolution of the power spectra is given in Fig. 2 in both monofrequency (a) and bifrequency (b) cases. First analyzing the monofrequency case, what is clearly not seen on the time-frequency representations in Fig. 1 is the emergence of the static component of the bubble motion (see Fig. 2) which carries an important part of the total energy, with the same order of magnitude than the fundamental frequency and much larger than the first harmonic. This static component arises due to the asymmetrical oscillations of $R(t)$ around its mean value $R_m$ and corresponds to a change of the bubble mean radius with time, to which one can associate a variation of the linear resonant frequency of the bubble [10]:

$$f_r = \frac{1}{2\pi R_m \sqrt{\rho}} \sqrt{3\gamma \left( \frac{p_\infty R_0}{R_m} + \frac{2\sigma}{R_m} \right) - \frac{2\sigma}{R_m} - \frac{4\mu^2}{\rho R_m^2}}. \quad (4)$$
As the mean radius increases with time, the linear resonant frequency decreases, as shown by circles (●) in Fig. 1, being also responsible for the saturation of the fundamental and harmonics power amplitudes (Fig. 2) as \( f_1 \) deviates from the resonance. In the bifrequency case, the transition happens when the lower frequency \( f_2 \) becomes resonant. An important part of energy is then transferred to this component, which gives rise to the \( o \)-curve in Fig. 2(b). One can also observe that the static component is prevailing and that the total power is clearly increased, being almost twice the total power obtained in the monofrequency case at \( t = 4 \text{ms} \). Finally, Fig. 2(b) also shows that the \( f_1 \)-component completely vanishes at the transition and all the energy supplied at this frequency is transferred into others such as the low-frequency modulation \( f_1 - f_2 \) (×) and the higher order combination \( 2f_2 - f_1 \) (not shown for readability). The latter becomes resonant as soon as it matches with the shifted linear frequency given by Eq. (4) (see Fig. 1).

4. Conclusion

Numerical computations of the single bubble response to an acoustic field have been conducted for a monofrequency excitation (\( f_1 = 500 \text{kHz} \)) and for a bifrequency excitation (\( f_1 = 500 \text{kHz} \) and \( f_2 = 400 \text{kHz} \)). In the bifrequency case, the time-frequency representation showed the emergence of multiple combinations of both frequencies, followed by a sharp transition towards strongly nonlinear oscillations and the generation of a broadband spectrum. After this transition, the recovered periodic behavior of the bubble was characterized by the rising of the lower frequency component (\( f_2 \)) and the strong attenuation of the higher one (\( f_1 \)). The emergence of the \( f_2 \)-component was correlated with the increase of the mean bubble radius with time, associated with a decrease of its linear resonant frequency. Due to the resonant character of the \( f_2 \)-component or of higher-order combinations, the total power reached twice the power obtained in the monofrequency case after the transition, meaning that the cavitating behavior of the bubble was largely enhanced by the application of this dual frequency combination. In conclusion, this study highlights some fundamental aspects of the bifrequency driven single bubble oscillations and thus constitutes a step towards a better understanding of the role of dual frequency excitations in the enhancement and control of cavitation activity.

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