Radio resource allocation in OFDMA multi-cell networks

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Abstract. In this paper, the problem of allocating users to radio resources (i.e., subcarriers) in the downlink of an OFDMA cellular network is addressed. We consider a multi-cellular environment with a realistic interference model and a margin adaptive approach, i.e., we aim at minimizing total transmission power while maintaining a certain given rate for each user. The computational complexity issues of the resulting model is discussed and proving that the problem is NP-hard in the strong sense. Heuristic approaches, based on network flow models, that finds optima under suitable conditions, or “reasonably good” solutions in the general case are presented. Computational experiences show that, in a comparison with a commercial state-of-the-art optimization solver, the proposed algorithms are effective in terms of solution quality and CPU times.

Keywords: radio resource allocation, network flow models, heuristic algorithms.

1 Introduction

Resource allocation (RA) is one of the most efficient techniques to increase the performance of multicarrier systems. In an orthogonal frequency-division multiple access (OFDMA) scheme, each user of a communication system is allocated a different subset of orthogonal subcarriers of a radio frame. If the transmitter possesses full knowledge of channel state information, the subcarriers can be assigned to the various users following a certain optimality criterion to increase the overall spectral efficiency. In fact, propagation channels are independent for each user and thus the subcarriers that are in a deep fade for one user may be good ones for another and the goal is to assign only high quality channels to each different user to exploit the so-called multiuser diversity. One of the major drawback of efficient RA schemes is that their complexity is in general high and tends to grow larger with the number of users and subcarriers. Many resource allocation algorithms have been designed for taking advantage of both the frequency selective nature of the channel and the multi-user diversity [10]-[9] but all of them exhibit a trade-off between complexity and performance: low complexity [5, 8] algorithms tend to be outperformed by those requiring high computational loads [10, 6].
In this paper, we consider the downlink of a multi-cellular OFDMA system, and we deal with the assignment both of radio resources (i.e., subcarriers) of a radio frame and of transmission formats to users minimizing the overall transmission power, while providing a given transmission rate for each user. The allocation is performed on a radio frame basis, and we assume that the propagation channel is quasi static, i.e., it does not vary within a radio frame. As a consequence, even if the problem is intrinsically dynamic, for the allocation decisions within a radio frame, data may be regarded as static.

The paper is organized as follows. Section 2 describes the system model and defines the RA problem. Section 3 addresses the computational complexity of the problem. In particular, it is proved that the addressed problem is NP-hard in the strong sense. Heuristic approaches, based on network flow models, that finds optima under suitable conditions, or “reasonably good” solutions in the general case are presented in Section 4. In Section 5, experimental results are presented and discussed. Finally, Section 6 provides conclusive remarks.

2 System model and problem definition

The problem we address, that we call RRAP, is a constrained minimization problem in which subcarriers and transmission formats must be assigned to the users, in such a way that a given bit-rate is provided to each user and that the total transmission power is minimized. In particular, we consider a multi-cell scenario with users belonging to different cells and a frequency bandwidth divided into orthogonal subcarriers.

Let $N = \{1, \ldots, n\}$ and $C = \{1, \ldots, c\}$ be the sets of the users and cells, respectively, and let $N_k$ be the set of users in cell $k$. (Hence, $N = \{N_1 \cup \ldots \cup N_c\}$.) Each user belongs to exactly one cell, and, given a user $i \in N$, we denote by $b(i)$ the cell of user $i$. Let $M = \{1, \ldots, m\}$ be the set of the available subcarriers and $Q = \{q_1, \ldots, q_p\}$ be the set of possible transmission formats.

Let $R_i$ be the required transmission rate of user $i$, $i \in N$. A given transmission format $q$ corresponds to the usage of a certain error correction code and symbol modulation that leads to a spectral efficiency $\eta_q$: a user employing format $q$ on a certain subcarrier transmits with rate $R = B\eta_q$, $B$ being the bandwidth of each subcarrier. The target Signal-to-interference ratio (SIR) to achieve the spectral efficiency $\eta_q$ is $SIR(q) = 2^{\eta_q} - 1$.

To simplify the resource allocation algorithm, we impose that all user requests are expressed as an integer multiple of a certain fixed rate $R_0 = B\eta_0$, i.e., the rate requested by user $i$ is $R_i = r_iR_0$ with $r_i$ an integer number. In the same way, we assume that if user $i$ transmits on subcarrier $j$ transmission format $q \in Q$, she gets a transmission rate of $B\eta_q = Bq\eta_0$. Hence, bigger is the transmission format bigger is the transmission rate. Moreover, given a certain $\eta_0$, the target SIR to achieve the spectral efficiency $\eta_q$ is $SIR(q) = 2^{\eta_q} - 1 = 2^{q\eta_0} - 1$.

In general, users belonging to different cells can share the same subcarrier, while interference phenomena do not allow two users in the same cell to transmit
on the same subcarrier. However, the power required for the transmission on a given subcarrier increases both with the set of users transmitting on that subcarrier and with the used transmission formats. More precisely, let $S(j)$ be the set of users (belonging to different cells) which are assigned to (i.e., that are transmitting on) the same subcarrier $j$, and let $q_i \in Q$ be the transmission format of user $i \in S(j)$ on subcarrier $j$. Let $p_{ijq}$ be the transmission power needed to user $i$ for transmitting with format $q$ on subcarrier $j$. The transmission powers $p_{ijq}$ have to satisfy the following system.

\[
p_{ijq} = SIR(q_i) \frac{\sum_{h \in S(j), h \neq i} G_i^{b(h)}(j) p_{hjq} + B_0}{G_i(j)} \quad \forall i \in S(j) \quad (1)
\]

\[
p_{ijq} \geq 0 \quad \forall i \in S(j) \quad (2)
\]

where $G_i(j)$ is the channel gain of user $i$ on subcarrier $j$, $G_i^{b(h)}(j)$ is the channel gain between user $i$ and the base station of cell $k \neq b(i)$ on subcarrier $j$. Values $G_i^{b(h)}(j)$ are a measure of the interference between user $i$ and users of other cells transmitting on the same subcarrier $j$. In Equation (1), we refer to the term $\sum_{h \in S(j), h \neq i} G_i^{b(h)}(j) p_{hjq}$ as to interference term.

Note that, power $p_{ijq}$ increases as the interference term increases, moreover, the interference term depends on the set of users, other than $i$, which are assigned to the same subcarrier, and by their transmission formats. Note also that, system (1)–(2) may not have a feasible solution. On the other hand, if only user $i$ is assigned subcarrier $j$ (i.e., if the interference term is 0), by (1), $p_{ijq} = \frac{SIR(q_i) B_0}{G_i(j)}$.

A feasible radio resource allocation for RRAP consists in assigning subcarriers to users and, for each subcarrier-user pair, in choosing a transmission format, in such a way that (a) for each user $i$, a bit-rate $R_i$ is achieved, (b) the users in the same cell are not assigned to the same radio resource (i.e., subcarrier), (c) given the set $S(j)$ of users assigned to radio resource $j$, System (1)–(2) has a feasible solution, for any subcarrier $j \in M$.

RRAP consists in finding a feasible radio resource allocation minimizing the overall transmission power.

### 2.1 A MILP formulation for RRAP

In this section, a Mixed Integer Linear Programming (MILP) formulation of RRAP is presented. Let $x_{ijq}$ be a binary variable equal to 1 if user $i$ is assigned to subcarrier $j$ with format $q$ (and 0 otherwise), and let $p_{ijq}$ be a positive real variable denoting the transmission power allocated for user $i$ on subcarrier $j$ with format $q$. RRAP can be formulated as follows.
\[
\min \sum_{i \in N, j \in M, q \in Q} p_{ijq} \quad (3)
\]
\[
p_{ijq} \leq L x_{ijq} \quad \forall i \in N, j \in M, q \in Q \quad (4)
\]
\[
G_i(j)p_{ijq} - \sum_{h: b(h) \neq b(i), v \in Q} SIR(q)G^{b(h)}_i(j)p_{hjv} \geq SIR(q)BN_0(1 - L(1 - x_{ijq})) \quad (5)
\]
\[
\forall i \in N, j \in M, q \in Q
\]
\[
\sum_{i \in N, q \in Q} x_{ijq} \leq 1 \quad \forall j \in M, k \in C \quad (6)
\]
\[
p_{ijq} \geq \frac{SIR(q)BN_0}{G_i(j)} x_{ijq} \quad \forall i \in N, j \in M, q \in Q \quad (7)
\]
\[
p_{ijq} \geq 0 \quad \forall i \in N, j \in M, q \in Q \quad (8)
\]
\[
x_{ijq} \in \{0, 1\} \quad \forall i \in N, j \in M, q \in Q \quad (10)
\]

The objective function accounts for the overall transmission power. In Constraints (4) and (5), \( L \) is a suitable large positive number. Constraints (4) are logic constraints, forcing power \( p_{ijq} \) to be 0 if subcarrier \( j \) is not assigned to user \( i \) with format \( q \). In accordance with Equations (1), Constraints (5) state that if user \( i \) is assigned subcarrier \( j \) with format \( q \) (i.e., if \( x_{ijq} = 1 \)) power \( p_{ijq} \) cannot be smaller than \( SIR(q)BN_0(1 - L(1 - x_{ijq})) \). On the other hand, if \( x_{ijq} = 0 \), the right term of (5) is a large negative number, and Constraints (5) are always satisfied. Constraints (6) state that at most one user per cell (using only one transmission format) can be assigned to a given subcarrier. Constraints (7) require that a rate of at least \( R_i = B \eta_0 r_i \) is assigned to each user \( i \). Recall that, if subcarrier \( j \) is assigned to user \( i \) with format \( q \), a rate of \( B \eta_0 q \) is assigned to user \( i \). Constraints (7) can be divided by the term \( B \eta_0 \). Finally, Constraints (8) are redundant, but improve the solution of the linear relaxation used by the algorithm introduced in Section 4. They state that if user \( i \) is assigned to subcarrier \( j \) with format \( q \), the transmission power \( p_{ijq} \) cannot be smaller than \( \frac{SIR(q)BN_0}{G_i(j)} \) (corresponding to the case in which \( i \) is the unique user assigned to subcarrier \( j \)). In Section 5, we solve the above MILP formulation on randomly generated RRAP problems.

3 Problem complexity

In this section, we show that RRAP is strongly \( NP \)-hard even if a single cell exists (i.e., \( c = 1 \)) and only two transmission formats can be used. In particular, the following theorem holds.

**Theorem 1.** RRAP is strongly \( NP \)-hard even when \( c = 1 \) and the set of the available transmission formats is \( Q = \{1, \bar{q}\} \).
**Proof.** Consider the case in which only one single cell exists. Note that, in this case, by Constraints (6), at most one user can be assigned to each subcarrier. Problem RRAP in its decision form can be stated as follows. We are given a set of \( n \) users, each requiring a given transmission rate, a set of subcarriers, and a set of transmission formats \( Q \) and a real value \( \alpha \). All the users belong to the same cell.

**Question:** Is there an assignment of users to subcarriers and of transmission formats to each user-subcarrier pair such that (a) at most one user can be assigned to each subcarrier, (b) the transmission requirements of the users are fulfilled, and (c) the total transmission power does not exceed \( \alpha \)?

The proof is by reduction from the strongly NP-hard scheduling problem \( P|M_j|C_{\text{max}} \leq 2 \) [3]. According to the notation for machine scheduling problems [4], problem \( P|M_j|C_{\text{max}} \leq 2 \) can be described as follows. Given a set of \( n \) identical parallel machines of capacity \( C_{\text{max}} = 2 \), a set of jobs with processing times 1 or 2 (in the following called lengths), and for each job \( j \) a set of machines \( M_j \) able to process it, the problem is of finding, if it is possible, an assignment of the jobs to the machines in such a way that (a) the sum of the lengths of the jobs assigned to a given machine does not exceed the machine capacity \( C_{\text{max}} \), and (b) each job \( j \) is assigned to exactly one machine in \( M_j \).

Given an instance of \( P|M_j|C_{\text{max}} \leq 2 \), let \( F_1 \) and \( F_2 \) be the number of jobs in the instance of length 1 and 2, respectively. Without loss of generality, we can suppose that \( F_1 + 2F_2 = 2n \), i.e., we can restrict to consider instances in which all machine capacity is used in any feasible assignment. In fact, a feasible instance in which \( F_1 + 2F_2 < 2n \) can be transformed into a feasible instance where \( F_1 + 2F_2 = 2n \), by adding \( 2N - F_1 - 2F_2 \) jobs of length 1 that can be processed by all the \( n \) machines (hence, for such jobs we have that \( M_j \) is equal to the set of machines).

Given an instance of \( P|M_j|C_{\text{max}} \leq 2 \), we build an instance \( I \) of RRAP as follows. In \( I \), two transmission formats exists, namely, \( Q = \{1, \bar{q}\} \), where \( \bar{q} > 1 \) is a suitable value. In \( I \), the number of subcarriers is equal to the number of jobs. Subcarriers are of two types. In particular, for each job \( j \) of length 1 or 2, we introduce a subcarrier \( \bar{q} \) of type 1 or 2, respectively. Users correspond to machines. Hence, \( n \) users are considered in \( I \). Each user requires at least a rate \( R_i = 2B\eta_0 \) (i.e. \( r_i = 2 \)), for \( i = 1, \ldots, n \). For each subcarrier \( j \), let \( A_j \) be the set containing the users that corresponds to the machines in \( M_j \) (containing the machines able to process job \( j \)).

Given a subcarrier \( j \) of type 1, for each user \( i \), \( G_i(j) \) is set in such a way that \( P_{ij1} = \frac{2F_i}{2F_1 + F_2} \) if \( i \in A_j \) and \( P_{ij1} > \alpha \) if \( i \notin A_j \). Format \( \bar{q} \) is chosen in such a way that \( P_{ij\bar{q}} > \alpha \) for all users \( i \) and subcarriers \( j \) of type 1. Hence, the subcarriers of type 1 can be used only with format 1 (providing a total transmission rate of \( F_1 \)), in any solution with total power not greater than \( \alpha \). For each subcarrier \( j \) of type 2, \( G_i(j) \) is chosen in such a way that \( P_{ij\bar{q}} = \frac{2F_i}{2F_1 + F_2} \) (\( P_{ij1} \) is obviously smaller) if \( i \in A_j \) and \( P_{ij\bar{q}} > \alpha \) if \( i \notin A_j \).

Note that, to get a solution with total power at most \( \alpha \), the \( F_1 \) subcarriers of type-1 can be only used with format 1. As a consequence and since \( F_1 + 2F_2 = 2n \),
the subcarriers of type 2 must be necessarily used with format $\bar{q}$ to satisfy the transmission requirements of all the users (i.e., constraints (7) with $r_i = 2$). Hence, an assignment of subcarriers to users of total power $\alpha$ in $I$, if any, assigns each subcarrier $j$ of type 1 (of type 2) to a user of $A_j$ with format 1 (format $\bar{q}$). Since $r_i = 2$, either two subcarriers of type 1 or one subcarrier of type 2 (getting a rate $B_{\eta_0\bar{q}} \geq 2B_{\eta_0}$) are assigned to each user $i$. Such an assignment corresponds to a feasible solution for the instance of problem $P|\sum_{j} C_j|\text{max} \leq 2$. On the other hand, if no feasible solution exists for the $P|\sum_{j} C_j|\text{max} \leq 2$ instance, by construction, no assignment of subcarriers to users exists in $I$ with total power equal to $\alpha$ or smaller. Hence, a feasible solution exists for the $P|\sum_{j} C_j|\text{max} \leq 2$ instance if and only if an assignment of subcarriers to users exists in $I$ with total power equal to $\alpha$, and the thesis follows.

\[\square\]

4 Heuristic algorithms for RRAP

Usually, small computational times (about few tens of milliseconds) are required for solving RRAP, so that an exact approaches can not used in practice for solving the problem. In this section, two heuristic algorithms are proposed. The two heuristics, called H-LP and H-LAGR, are an extension and an improvement of two algorithms from the literature\cite{1,7}, designed for solving RRAP when only one single transmission format is considered (i.e. $|Q| = 1$).

4.1 Algorithm H-LAGR

H-LAGR is an iterative algorithm based on a network flow approach. It consists in iteratively solving a relaxation of the MILP formulation (3)–(10), as described in the following. When only one single transmission format is considered, it is easy to note that (see for example \cite{1}), the MILP formulation obtained by relaxing the interference constraints (5) in the Lagrangian way (see, for example, \cite{2} for the description of the Lagrangian relaxation technique) of the formulation of Section 2.1 can be solved in polynomial time as a minimum cost network flow problem. In fact, when constraints (5) are relaxed and only one single transmission format is used, the formulation can be decomposed into $c$, i.e., one per cell, minimum cost flows problems, where the problem formulation related to cell $k \in C$ is reported in the following. Note that, since a single transmission format exists, we use variables $x_{ij}$ and $p_{ij}$, where $x_{ij}$ is 1 if user $i$ is assigned to subcarrier $j$ (with the unique transmission format) and 0 otherwise and variable
$p_{ij}$ is the related transmission power.

\[
\begin{align*}
\min \sum_{i \in N_k, j \in M} p_{ij} \\
p_{ij} &\leq L x_{ij} \quad \forall i \in N_k, j \in M \\
\sum_{i \in U_k} x_{ij} &\leq 1 \quad \forall j \in M \\
\sum_{j} x_{ij} &= r_i \quad \forall i \in N_k \\
p_{ij} &\geq \frac{SIRBN_0}{G_i(j)} x_{ij} \quad i \in N_k, j \in M \\
p_{ij} &\geq 0 \quad \forall i \in N_k, j \in M \\
x_{ij} &\in \{0, 1\} \quad \forall i \in N_k, j \in M
\end{align*}
\]

Observe that, by constraints (15), variables $p_{ij}$ can be replaced by $\frac{SIRBN_0}{G_i(j)} x_{ij}$ throughout the formulation, and it can be rewritten by only using variables $x_{ij}$. Hence, the term $\frac{SIRBN_0}{G_i(j)} x_{ij}$ can be viewed as the "cost" of assigning user $i$ to subcarrier $j$. It is easy to see that formulation (11)–(17) can be solved as a minimum cost network flow problem [1].

Each iteration of H-LAGR is composed of two phases. In the first phase, the $c$ minimum cost network flow problems are simultaneously solved to get a user-to-subcarrier assignment (using a single transmission format). In the second phase, the solution found in the first phase (that may be not feasible for the original problem, since the interference constraints (5) are ignored) is "adjusted". In particular, the following two types of adjustments are considered in the second phase.

- Users’ removal: If under the current assignment, users transmitting on a given subcarrier $j$ requires high transmission power levels, or if system (1)–(2) for subcarrier $j$ is not feasible, a peeling procedure is used to remove some users from that subcarrier (the removed users are those that make smaller the transmission power of the subcarrier). At the same time, the "cost" of the removed user, say $i$, (for subcarrier $j$) is increased to

$$\lambda_{ij} \frac{SIRBN_0}{G_i(j)}$$

where $\lambda_{ij}$ is a suitable value greater than 1. Such cost updating is performed to make less profitable the assignment of the user $i$ to the subcarrier $j$, at the next iteration of H-LAGR.

- Transmission format adjustment: We illustrate this procedure through an example. Suppose that, during the users’ removal procedure, a user $i$ is removed from subcarrier $j$, while it still transmits on some subcarriers, say for example $j'$ and $j''$. (Hence, after the removal procedure, user $i$ does not satisfy her transmission requirements.) In this second procedure, the algorithm increases, if it possible, the transmission format of user $i$ on the subcarriers $j'$ and $j''$, until the transmission rate requirements of the user are fulfilled.
H-LAGR stops when a prefixed number of iterations is reached providing the best solution found so far (i.e. that maximizing the overall transmission rate and minimizing the total transmission power).

4.2 Algorithm H-LP

H-LP is based on a decentralized approach [7]. Also in H-LP, an iterative scheme is followed in which the single format allocation problem (11)-(17) is separately solved on each cell. At each iteration, all cells in the system change their allocations simultaneously. The power costs $p_{ij}$ are updated on the base of the interference measured at the end of the previous iteration. The algorithm stops when a steady-state is reached (i.e., in which no cell has interest in changing its allocation). As described in [7], the convergence of this decentralized algorithm is not guaranteed and thus after a certain number of iterations the rate requirements of the users are progressively reduced determining a certain rate loss with respect to the initial targets.

5 Experimental results

In this section, preliminary experimental results on random generated instances are presented. Instances have been generated as in [1]. In all the instances the number of cells is $c = 7$, containing the same number of users, the number of subcarriers is $m = 16$, the overall signal bandwidth is $B_{tot} = 5$ MHz and the channel is frequency selective Rayleigh fading with an exponential power delay profile. The rms delay spread is $\sigma_\tau = 0.5 \mu s$, typical of a urban environment. We also assume a fixed throughput per cell evenly shared among the $|N_k| = n_k$ users, which are uniformly distributed in hexagonal cells of radius $R = 500$ meters. Hence, for each user $i \in C_k$, $r_i = m/n_k$ is set. Three classes of instances have been generated varying $n_k$. In particular, we set $n_k = 2, 4, 8$, so that $r_i = 8, 4, 2$, respectively, for each user in $N$, in the different classes. For each value of $n_k$, 10 instances have been generated through simulation of realistic scenarios.

The performances of the heuristics H-LAGR and H-LP have been compared with a truncated branch and cut algorithm that uses the Integer Linear Programming formulation of Section 2.1 solved with CPLEX 9.1. All the experiments have been performed on a 1.6GHz Pentium M laptop equipped with 1GB RAM.

In Table 1, a performance comparison of H-LP, H-LAGR and CPLEX on the MILP formulation of Section 2.1 is given. In CPLEX, a limit of 100,000 branch and bound nodes has been set. In the table, the first column reports on the value of $n_k$. For each value of $n_k$, the results are an average on the 10 instances. For each algorithm, $\text{pow.}$ is the total transmission power (in Watt), $\%\text{rate loss}$ is the percentage of not assigned required transmission rate (respect to the total number of required sub-carriers in the 10 instances), and $\text{time}$ is the average computational time in seconds.

In all the instances, the three algorithms find solutions satisfying all the requirements on the transmission rate. The transmission powers in the solutions
found of H-LP and H-LAGR are very similar (H-LP finds slightly better solutions) and are bigger than those found by CPLEX. However, the solutions values found by the two heuristics are quite close to the values found by CPLEX, especially in the instances with 4 and 8 users per cell.

From the computational time point of view, we have that H-LP and H-LAGR are very fast (about tens of milliseconds on average) while CPLEX requires more than 2500 seconds, on average. Observe that, H-LAGR requires higher computational times on instances with smaller values of $n_k$. This is due to the "Transmission format adjustment" procedure that requires more calls when $n_k$ is smaller.

\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline
$n_k$ & pow. & \%rate & time & pow. & \%rate & time & pow. & \%rate & time \\
\hline
2 & 64.5 & 0 & 0.020 & 60.9 & 0 & 0.089 & 40.0 & 4 & 514.8 \\
4 & 37.1 & 0 & 0.022 & 38.5 & 0 & 0.020 & 32.1 & 0 & 1562 \\
8 & 26.5 & 0 & 0.024 & 27.8 & 0 & 0.012 & 25.2 & 0 & 5631.7 \\
\hline
\end{tabular}

Table 1. Comparison results on random instances.

6 Conclusions

In this paper, we addressed a radio resource allocation problem arising in wireless cellular networks. We study the computational complexity of the problem and proposed an efficient heuristics algorithms for its solution. A preliminary computational study shows that the algorithms are suitable for real world applications both for the solution quality and for the short computational times.

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