Estimation of crack initiation and crack damage stress by means of mechanical, physical and dynamical properties in brecciated marbles

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Abstract. The objective of this study is the determination of the various stages of deformation of brecciated marbles, under uniaxial compression, and the application of regression techniques, to investigate the relations of these stages, with physical, dynamical, and mechanical properties. Therefore, a total number of fifteen specimens, prepared from samples obtained from various locations from the southern part of the East Attica Prefecture, Greece. The laboratory program involved the determination of the uniaxial compressive strength (UCS), the crack initiation (Cci) and crack damage stress (Ccd) levels, the static elastic modulus (Es), as well as the basic physical and dynamical properties, such as the dry density (γd), the effective porosity (n_{eff}) and the ultrasonic velocities of both primary (V_P) and secondary (V_S) waves. Results obtained from the mechanical tests reveal that the onset of stable (C_{si}) and unstable (C_{cd}) crack growth varies between 0.19–0.38 and 0.66–0.92 of the UCS values, respectively. Applying simple nonlinear regression models, it was found that crack initiation and crack damage stresses, decrease exponentially with the increase of effective porosity, while increase exponentially with the increase of static elastic modulus, dynamic elastic modulus (E_d), and the primary wave velocity.

1. Introduction

It is generally accepted that the experimental stress-strain curves of uniaxial compression tests up to the ultimate strength (UCS) can be distinguished into four sections corresponding to different stages of rock deformation [1, 2, 3]. The initial section (stage I) is related to pre-existing microcracks closure. When the majority of these microcracks are closed the rock can be roughly considered to behave as an elastic material (stage II). The onset of dilatancy delimits the beginning of the stable cataclastic phase (stage III). The tensile microcracks, which initiate at this stage, propagate mainly parallel to the applied axial stress direction. The stress level associated with the beginning of this stage is defined as the crack initiation stress (C_{ci}). With the increase of the applied load, new microcracks initiate and propagate while at the same time new and old microcracks coalesce and interact at oblique angles to the applied loading direction (stage IV). The stress level where unstable crack growth begins is defined as crack damage stress (C_{cd}).

The study of the brittle fracture of rocks concludes that the ultimate strength determined by a uniaxial compressive test is not an intrinsic property of the material but depends on the boundary conditions of the experiment [4]. On the other hand, C_{ci} and C_{cd} are characteristic properties that do...
not depend on testing conditions [3]. As the onset of stable and unstable crack growth is considered to be the lower and upper bound of in situ long-term strength [5, 6], it becomes clear that these stress levels are key parameters for the proper design and maintenance of civil and mining engineering works.

Over the years various methods have been proposed for the direct determination of the aforementioned cataclastic stress levels, which primarily depend on the induced axial and lateral strains and the calculated volumetric ones [7]. These methods can also incorporate non--destructive techniques that are capable of monitoring phenomena accompanying the brittle fracture of rocks, such as, the acoustic emission (AE) energy release [8].

Regardless of the methodology that can be used, it is clear that these are time-consuming and require expensive experimental configurations. An alternative and practical approach to problems that can arise due to experimental restrictions is the usage of empirical correlations that relates $C_{ci}$ and $C_{cd}$ with parameters that can more easily be measured. So, in the context of this research, an effort has been made to derive statistical correlations, in the case of brecciated marbles, for these stress levels with characteristic material properties that have been proven to affect the values of UCS, such as, the effective porosity ($n_{eff}$), the static elastic modulus ($E_s$), the dynamic elastic modulus ($E_d$), and the compressional wave velocity ($V_P$) [9].

2. Material characterization

The marbles studied herein were sampled from various locations from the southern part of the East Attica Prefecture, Greece. In the field, they are intensively brecciated with mm to dm–sized clasts and strongly fractured by a chaotic and dense network of discontinuities, either closed or open.

Mineralogical and petrographical properties of these marbles were studied through powder X-ray diffraction (PXRD) analyses and the microscopic examination under polarized light of 30 mm thin section of the samples. These studies are in agreement with previously published data [10].

The PXRD measurements were carried out on a Bruker D8 Focus diffractometer in a θ–θ geometry with CuKα radiation. The diffraction diagrams were collected between 2 and 70° 2θ, in 0.02° steps and 1 s per step. According to PXRD analysis, these rocks are mostly characterized as pure calcite marbles; only at places, dolomite is also present.

The intense cataclastic texture of these rocks was characterized by a pervasive network of fractures and marble clasts embedded in an equidimensional fine-grained matrix (figure 1).

![Clasts](image1.png)

![Filled fractures with calcite](image2.png)

Figure 1. Photomicrograph of optical microscope showing marble clasts and fractures filled with calcite (double polarized transmitted light).
3. Testing program and results

The results presented in this study refer to right circular cylinders of NX diameter (54.4 mm) with a slenderness ratio between 2.5 and 3. For the mechanical tests, a servo-hydraulic-driven loading frame of 5000 kN capacity was used. The axial load and the induced strains were measured and recorded throughout the experiments. The axial strain was taken as the average value of the measurements of three linear variable differential transducers (LVDTs) at an angle of 120° apart, to eliminate any bending moment effects from the measured strains [11]. The LVDTs were mounted between two aluminium rings and this system was attached to the middle third of the specimen. The lateral strain was obtained by using a circumferential extensometer mounted around the specimen at mid-height. All tests were conducted under lateral displacement control with a constant rate of 15 μm/min.

The static elastic modulus (E_d) was taken as a least-square fit along the near-linear portion of the axial stiffness–stress curve. From the measured strains the volumetric strains are calculated, and the onset of crack initiation (C_in) and crack damage (C_do) stress level determined by characteristic inflection points on the volumetric stiffness–axial stress curve [8].

Before mechanical testing, the basic physical and dynamical properties of the specimens are also measured. For the determination of the effective porosity (n_eff) and dry density (γ_d), the specimens were immersed in a vacuum chamber filled with deionized water for several hours. Then the samples were oven-dried at the temperature of 45°C for several days until a constant mass was reached. It is pointed out that this temperature level was deemed appropriate to avoid any thermal damage to the properties of the specimens [12]. Using the ultrasonic pulse method, the wave velocities (V_P, V_S) were determined by dividing the distance traversed by the waves with travel time. The operating frequency of the transducers was 1 MHz. The dynamic elastic modulus (E_d) is calculated from the ultrasonic wave velocities and the dry density [13].

The results of the testing program are summarized in Table 1. The latter also presents the mineralogical composition of each sample.

| Specimen code | γ_d (gr/cm³) | n_eff (%) | V_p (km/s) | V_s (km/s) | E_d (GPa) | UCS (MPa) | E_s (GPa) | C_in (MPa) | C_do (MPa) | mineralogy          |
|---------------|--------------|-----------|------------|------------|-----------|-----------|-----------|------------|------------|---------------------|
| BM1           | 2.627        | 1.98      | 6.200      | 3.299      | 74.5      | 84.3      | 74.0      | 30.0       | 71.3       | calcite             |
| BM2           | 2.595        | 3.51      | 6.092      | 3.260      | 71.7      | 74.2      | 69.4      | 22.3       | 62.7       | calcite             |
| BM3           | 2.646        | 0.74      | 6.151      | 3.298      | 74.7      | 126.9     | 67.4      | 42.0       | 105.5      | calcite             |
| BM4           | 2.595        | 3.63      | 5.978      | 3.193      | 68.8      | 81.9      | 62.8      | 30.0       | 66.0       | calcite             |
| BM5           | 2.608        | 3.22      | 5.820      | 3.097      | 65.1      | 72.9      | 59.8      | 28.0       | 58.8       | calcite             |
| BM6           | 2.609        | 3.19      | 5.941      | 3.173      | 68.3      | 77.7      | 60.1      | 25.0       | 70.0       | calcite             |
| BM7           | 2.607        | 2.94      | 5.450      | 2.961      | 59.0      | 72.8      | 49.4      | 25.2       | 50.0       | calcite             |
| BM8           | 2.637        | 2.16      | 5.589      | 3.015      | 62.1      | 72.8      | 60.1      | 19.0       | 63.0       | calcite             |
| BM9           | 2.617        | 2.50      | 6.271      | 3.254      | 72.9      | 75.6      | 59.5      | 28.0       | 56.6       | calcite             |
| BM10          | 2.642        | 1.86      | 6.257      | 3.170      | 70.5      | 84.6      | 70.2      | 24.0       | 72.0       | calcite             |
| BM11          | 2.622        | 2.15      | 6.264      | 3.223      | 71.9      | 100.6     | 71.4      | 28.0       | 92.1       | calcite             |
| BM12          | 2.628        | 2.02      | 6.273      | 3.273      | 73.9      | 102.3     | 73.1      | 36.0       | 91.4       | calcite             |
| BM13          | 2.637        | 2.15      | 6.251      | 3.300      | 75.1      | 89.5      | 70.2      | 28.0       | 78.5       | calcite             |
| BM14          | 2.435        | 10.03     | 4.702      | 2.397      | 37.1      | 29.5      | 15.3      | 5.5        | 19.4       | Calcite and dolomite |
| BM15          | 2.630        | 6.12      | 5.443      | 2.884      | 57.1      | 53.6      | 38.3      | 10.0       | 39.5       | Calcite and dolomite |

The physical properties show a significant dispersion in their values and this had a direct effect on the values of the dynamic parameters [13], wherein general, a higher effective porosity corresponds to lower ultrasonic wave velocities and dynamic elastic modulus (E_d). There is also a clear tendency...
where stiffer samples exhibit higher values of ultimate strength and ultrasonic wave velocities and lower values of effective porosity.

The crack initiation ($C_{ci}$) and the crack damage stress level ($C_{cd}$) vary between 0.19%–0.38% and 0.66%–0.92% of the ultimate strength (UCS), respectively. Applying linear regression with zero constant to the results, suggests that the average stress levels for the onset of stable and unstable crack growth are 0.32% and 0.84% of the ultimate strength (UCS), respectively (figure 2).

**Figure 2.** Empirical relation of crack initiation stress ($C_{ci}$) and crack damage stress ($C_{cd}$) with ultimate strength (UCS).

### 4. Statistical analysis

Least-squared regression techniques were applied to establish empirical correlations for crack initiation ($C_{ci}$) and crack damage stress ($C_{cd}$) level with characteristic properties of the tested marbles. The equation describing the fitted regression line is calculated along with the respective coefficient of determination ($R^2$) for each equation. Because the independent variables correlate well with each other, no effort has been made for conducting multiple regression analyses.

#### 4.1. Relations with effective porosity ($n_{eff}$)

Figure 3 illustrates the dependence of crack initiation stress ($C_{ci}$) and crack damage ($C_{cd}$) stress level on the variation of the effective porosity ($n_{eff}$). These stress levels are inversely related to effective porosity. The derived relations show good coefficients of determination ($R^2 = 0.86–0.88$) and are described by equations (1) and (2), respectively.

\[
C_{ci} = 45.8e^{-0.212n_{eff}} \quad (1)
\]

\[
C_{cd} = 107.7e^{-0.170n_{eff}} \quad (2)
\]

These results are following the general assumption that open defects act as stress concentrators and can cause crack initiation and growth under applied load [14, 15, 16]. For limestones and dolomites, negative exponential, and polynomial equations of second order have been formulated for the relation of $C_{ci}$ and $C_{cd}$ with total porosity [14, 15]. For granitic rocks, noticeable effects of effective and total porosity on the values of $C_{ci}$ have been expressed via linear relations [17].
Figure 3. Empirical relation of crack initiation stress ($C_{ci}$) and crack damage stress ($C_{cd}$) with effective porosity ($n_{\text{eff}}$).

4.2. Relations with the static elastic modulus ($E_s$)
Static elastic modulus ($E_s$) is a measure of the overall stiffness of the rock [18], depending on factors such as porosity, stiffness of the individual grains, and their contacts. It has been extensively correlated with the ultimate strength values of rock and the proposed relations are either linear or nonlinear [19].

Results obtained in this study suggest that crack initiation stress ($C_{ci}$) and crack damage ($C_{cd}$) stress levels increase exponentially with the increase of static elastic modulus ($E_s$), as shown in figure 4. These relationships are characterized by good coefficients of determination ($R^2=0.80–0.88$) and expressed by equations (3) and (4), respectively.

$$C_{ci}=4.0e^{0.029E_s}$$ (3)

$$C_{cd}=14.4e^{0.024E_s}$$ (4)

Figure 4. Empirical relation of crack initiation stress ($C_{ci}$) and crack damage stress ($C_{cd}$) with the static elastic modulus ($E_s$).
For carbonate rocks, similar results have been reported [14, 15], where the cataclastic stress levels ($C_{ci}$ and $C_{cd}$), increased with the increase of elastic modulus, either by negative exponential or second-order polynomial laws.

4.3. Relations with dynamical properties

Ultrasonic wave velocities depend primarily on rock type, texture, density, and porosity [13], factors that contribute to ultimate strength. Compressional wave velocity ($V_P$) has been used by many researchers for the estimation of UCS, as summarized in [9]. The general tendency of the established empirical laws is the linear or non-linear increase of UCS with the increase of $V_P$.

For the studied rocks it was found that the onset of stable and unstable crack growth increases exponentially with the increase of $V_P$, as shown in figure 5. These empirical relations showed fairly good coefficients of determination ($R^2=0.73-0.79$) and were defined by equations (5) and (6), respectively.

$$C_{ci} = 0.1e^{-0.977V_P}$$  \( \text{(5)} \)

$$C_{cd} = 0.5e^{-0.810V_P}$$  \( \text{(6)} \)

![Figure 5. Empirical relation of crack initiation stress ($C_{ci}$) and crack damage stress ($C_{cd}$) with primary wave velocity ($V_P$).](image)

In contrast with the static elastic modulus ($E_s$), dynamic elastic modulus ($E_d$) is not often used for the estimation of the ultimate strength of rocks, in practical geotechnical applications. This fact can be attributed to the difficulty of measuring transverse waves ($V_S$) at low pressures, especially for high porosity rocks. For limestones, power-law relations have been reported between UCS and dynamic elastic modulus [20]. Through this research, it was found that crack initiation stress ($C_{ci}$) and crack damage stress ($C_{cd}$) increase exponentially with the increase of $E_d$, as is presented in figure 6 and equations (7) and (8), respectively.

$$C_{ci} = 1.1e^{-0.046E_d}$$  \( \text{(7)} \)

$$C_{cd} = 5.1e^{-0.038E_d}$$  \( \text{(8)} \)
The coefficients of determination for equations (7) and (8) are increased compared to the corresponding ones for equations (5) and (6) due to that the determination of the dynamic elastic modulus ($E_d$) requires the determination of two more parameters, namely the dry density ($\gamma_d$) and the shear wave velocity ($V_S$). Similar remarks have been reported elsewhere, in terms of UCS values [20].

![Image of Figure 6](image)

**Figure 6.** Empirical relation of crack initiation stress ($C_{ci}$) and crack damage stress ($C_{cd}$) with the dynamic elastic modulus ($E_d$).

5. **Conclusions**

In this study, fifteen samples of brecciated marbles were prepared from blocks collected from natural outcrops in the southern part of the East Attica Prefecture, Greece. The experimental program involved the determination of the basic physical and dynamical properties, as well as the basic mechanical properties under uniaxial compression loading conditions. Through the results of this research, it was found that effective porosity ($n_{eff}$), elastic constants ($E_s$ and $E_d$), and primary wave velocity ($V_P$) have pronounced effects on the variation of crack initiation stress ($C_{ci}$) and crack damage stress ($C_{cd}$). In all cases, reasonably good relations were obtained via single regression techniques, in the form of exponential equations between these stress levels and the aforementioned parameters. Our results are in general agreement with the limited published data on this subject and can be used for preliminary investigations in land planning and development structures, at least in the study region.

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