We study the correlation between instantons and QCD-monopoles both in the lattice gauge theory and in the multi-instanton system using the maximally abelian gauge. First, we find the existence of an almost linear correlation between the total length of monopole trajectories and the total number of pseudoparticles (instantons and anti-instantons) in the $16^3 \times 4$ SU(2) lattice. Second, we study the features of QCD-monopole in the SU(2) multi-instanton vacuum on the $16^4$ lattice as a random ensemble of pseudoparticles. A signal of monopole condensation is found as the clustering of monopole trajectories, when the topological pseudoparticles is sufficiently dense.

1 Topological Objects in QCD

The appearance of QCD-monopole was pointed out by 't Hooft using abelian gauge fixing. Lattice QCD calculations show that this topological object plays an essential role on color confinement through its condensation, which is characterized by the appearance of the large monopole clusters and can be interpreted as a Kosterlitz-Thouless-type phase transition. In QCD, there is also another non-trivial topological object, an instanton. Instantons and QCD-monopoles are thought to be hardly related to each other since these topological objects appear from different non-trivial homotopy groups. Recently, however, the existence of a relation between these two objects has been shown by analytic studies and lattice QCD simulations.

In our previous analytical works, we conjectured that the existence of instantons promotes the clustering of monopole trajectories as a signal of monopole condensation. First, we examine an evidence for our conjecture by measuring the monopole-loop length and the number of instantons simultaneously in the lattice QCD simulation. For this discussion, we take the SU(2) gauge theory.

2 Lattice Study for Monopole Trajectory and Instantons

We study the correlation between the total monopole-loop length $L_{\text{total}}$
and the integral of the absolute value of the topological density $I_Q$, which corresponds to the total number of instantons and anti-instantons, by use of the Monte Carlo simulation in the lattice gauge theory of the SU(2) Wilson action. We can measure $L_{\text{total}}$ and $I_Q$ using the following procedure.

In the maximally abelian (MA) gauge, the abelian gauge fixing is done by maximizing $R \equiv \sum_{s,\mu} \text{Tr}\{U_\mu(s)\tau^3 U_\mu^{-1}(s)\tau^3\}$. The SU(2) link variable $U_\mu(s)$ is then factorized into the abelian link variable; $u_\mu(s) = \exp\{i\tau_3 \theta(s)\}$ and off-diagonal part; $M_\mu(s) \equiv \exp\{i\tau_1 C_1^\mu(s) + i\tau_2 C_2^\mu(s)\}$ as $U_\mu(s) = M_\mu(s) u_\mu(s)$. The Dirac string is extracted from the abelian field strength $\theta_{\mu\nu} \equiv \partial_\mu \theta_{\nu} - \partial_\nu \theta_{\mu}$ by decomposition as $\theta_{\mu\nu} = \bar{\theta}_{\mu\nu} + 2\pi M_{\mu\nu}$ with $-\pi \leq \bar{\theta}_{\mu\nu} < \pi$ and $M_{\mu\nu} \in \mathbb{Z}$. Here, $\bar{\theta}_{\mu\nu}$ and $M_{\mu\nu}$ correspond to the regular part and the Dirac string part, respectively. The monopole current is derived from the Dirac string part as

$$k_\mu(s) = \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} \partial_\nu M_{\rho\sigma}(s + \hat{\mu}) ,$$

which forms a closed loop, since the monopole current is topologically conserved. The total monopole-loop length is measured as $L_{\text{total}} = \sum_{\mu,s} |k_\mu(s)|$. In order to examine the total number of topological pseudoparticles, the integral of the absolute value of the topological density is defined as

$$I_Q = \frac{1}{32\pi^2} \sum_s \varepsilon_{\mu\nu\rho\sigma} |\text{Tr}\{U_{\mu\nu}(s)U_{\rho\sigma}(s)\}|$$

where $U_{\mu\nu}(s)$ is the plaquette variable. This value is measured with the Cabibbo-Marinari cooling method in the same way as the topological charge.

We make lattice calculations for various $\beta = 2.2 \sim 2.35$ on the $16^3 \times 4$ lattice, where the deconfinement transition occurs at $\beta_c \simeq 2.3$. As shown in Fig.1, we find the almost linear correlation between $I_Q$ after 3 cooling sweeps and $L_{\text{total}}$. Hence, the monopole-loop length would be largely enhanced in the dense instanton system.

### 3 Clustering of Monopoles in the Multi-Instanton System

We study the multi-instanton system by measuring the monopole clustering in the abelian gauge in order to understand the role of instantons on confinement. The field configuration for single instanton with the size $\rho$ and the center $z_\mu$ in the singular gauge is

$$A_\mu(x; z_k, \rho_k, O_k) = \frac{i\rho^2 \tau_\mu O^a \bar{\eta}_\mu^a(x - z)\nu}{(x - z)^2[(x - z)^2 + \rho^2]} ,$$

where $O_a$ is the color charge. The field configuration for $N$ instantons is

$$A_\mu(x; z_k, \rho_k, O_k) = \sum_{n=1}^N \frac{i\rho^2 \tau_\mu O^a \bar{\eta}_\mu^a(x - z)\nu}{(x - z)^2[(x - z)^2 + \rho^2]}$$

The contribution of monopole clustering to the total monopole length is measured as $L_{\text{mono}} = \sum_{\mu,s} |k_\mu(s)|$. The density of monopoles is derived from $L_{\text{mono}}$ as $\rho_{\text{mono}} = L_{\text{mono}} / V$, where $V$ is the lattice volume. The clustering of monopoles is measured as $I_{\text{mono}} = \int_{-\pi}^{\pi} \rho_{\text{mono}}(\theta) d\theta$, which is compared with the result of the Cabibbo-Marinari cooling method. The clustering factor is defined as $C = I_{\text{mono}} / I_Q$. As shown in Fig.2, we find the almost linear correlation between $C$ and $I_Q$ after 3 cooling sweeps and $L_{\text{total}}$. Hence, the monopole-loop length would be largely enhanced in the dense instanton system.
where $O^{ai}$ is the color orientation matrix and $\bar{\eta}^{i}_{\mu\nu}$ the 't Hooft symbol. For anti-instantons $A_{I}^{\mu}(x; z_{k}, \rho_{k}, O_{k})$, one has to replace the $\bar{\eta}^{i}_{\mu\nu}$ symbol by $\eta^{i}_{\mu\nu}$.

The multi-instanton configurations are assumed as the sum of instanton ($I$) and anti-instanton ($\bar{I}$) solutions,

$$A_{\mu}(x) = \sum_{k} A^{I}_{\mu}(x; z_{k}, \rho_{k}, O_{k}) + \sum_{k} A^{\bar{I}}_{\mu}(x; z_{k}, \rho_{k}, O_{k}) .$$

(4)

We generate ensembles of $N$ pseudoparticles with random orientations and random positions. The size of pseudoparticles are taken according to the size distribution $f(\rho)$. Here, we adopt the following instanton size distribution as

$$f(\rho) = \frac{1}{\left(\frac{\rho}{\rho_{IR}}\right)^{\nu} + \left(\frac{\rho}{\rho_{UV}}\right)^{b-5}}$$

(5)

with $b = \frac{11}{3} N_{c}$. Here, $\rho_{IR}$ and $\rho_{UV}$ are certain parameters such that the distribution is normalized to unity, while the maximum of the distribution is fixed to a give value $\rho_{0}$, which is the most probable size of the pseudoparticles in the ensemble. It is noted that $f(\rho)$ is reduced to the one obtained in the dilute instanton case: $f(\rho) \sim \rho^{b-5}$ in the limit of $\rho \to 0$. For large size instantons, $f(\rho)$ falls off with negative power of $\nu$, since large size instantons are suppressed by the instanton interaction. Here, we adopt $\nu = 3$, which is proposed by Diakonov and Petrov. We then introduce a lattice and express the gauge field in terms of the unitary matrices $U_{\mu} = \exp(i\alpha A_{\mu})$ living on the links. Then, we can do exactly the same procedure as done in lattice simulations in order to extract the monopole trajectories in the MA gauge.

We take a $16^{4}$ lattice with the lattice spacing of $a = 0.15$fm and the most probable instanton size, $\rho_{0} = 0.4$fm. The volume is thus fixed and equal to $V = (2.4$fm$)^{4}$. In this calculation, we take equal numbers of instantons and anti-instantons; $N_{I} = N_{\bar{I}} = N/2$. In Fig.2, we show the histograms of monopole loop length for two typical cases with the total pseudoparticle number $N = 20$ and 60, which correspond to the density $(N/V)^{2} = 174$ and 228MeV, respectively. At low instanton density, only relatively short monopole loops are found. At high density, there appears one very long monopole loop in each gauge configuration.

4 Summary

From above results, instantons would be the source of large size monopole clustering, which indicates occurring of monopole condensation in the similar argument as the Kosterlitz-Thouless-type phase transition. Thus, instantons seem to play a relevant role on color confinement.
Fig. 1 Correlation between $L_{\text{total}}$ and $I_Q$ in MA gauge on $16^3 \times 4$ lattice with various $\beta$. We use 3 cooling sweeps for the calculation of $I_Q$.

Fig. 2 Histogram of monopole loop length in the multi-instanton system; (a) dilute case ($N = 20$) and (b) dense case ($N = 60$).

References

1. G. ’t Hooft, *Nucl. Phys.* B **190**, 1981 (455).
2. A. S. Kronfeld et al., *Nucl. Phys.* B **293**, 461 (1987).
3. M. I. Polikarpov, in *Lattice 96*, hep-lat/9609020 and references therein.
4. Z. F. Ezawa et al., *Phys. Rev.* D **25**, 2681 (1982); D **26**, 631 (1982).
5. H. Suganuma, A. Tanaka, S. Sasaki and O. Miyamura, *Nucl. Phys.* B (Proc. Suppl.) **47**, 302 (1996).
6. H. Markum et al., *Nucl. Phys.* B (Proc. Suppl.) **47**, 254 (1996).
7. M. Fukushima, S. Sasaki, H. Suganuma, A. Tanaka, H. Toki and D. Diakonov, hep-lat/9608084.