A NEW EVOLUTIONARY PATH TO TYPE Ia SUPERNOVAE: A HELIUM-RICH SUPERSOFT X-RAY SOURCE CHANNEL

IZUMI HACHISU
Department of Earth Science and Astronomy, College of Arts and Sciences, University of Tokyo, Komaba, Meguro-ku, Tokyo 153-8902, Japan; hachisu@chianti.c.u-tokyo.ac.jp

MARIKO KATO
Department of Astronomy, Keio University, Hiyoshi, Kouhoku-ku, Yokohama 223-8521, Japan; mariko@educ.cc.keio.ac.jp

AND
KEN’ICHI NOMOTO AND HIDEYUKI UMEDA
Department of Astronomy and Research Center for the Early Universe, University of Tokyo, Bunkyo-ku, Tokyo 113-0033, Japan; nomoto@astron.s.u-tokyo.ac.jp, umeda@astron.s.u-tokyo.ac.jp

Received 1998 July 6; accepted 1999 February 4

ABSTRACT

We have found a new evolutionary path to Type Ia supernovae (SNe Ia) that has been overlooked in previous work. In this scenario, a carbon-oxygen white dwarf (C+O WD) is originated not from an asymptotic giant branch star with a C+O core but from a red giant star with a helium core of \( \sim 0.8-2.0 \, M_\odot \). The helium star, which is formed after the first common envelope evolution, evolves to form a C+O WD of \( \sim 0.8-1.1 \, M_\odot \), transferring a part of the helium envelope onto the secondary main-sequence star. This new evolutionary path, together with the optically thick wind from mass-accreting white dwarf, provides a much wider channel to SNe Ia than previous scenarios. A part of the progenitor systems are identified as luminous supersoft X-ray sources or recurrent novae such as U Sco, which are characterized by the accretion of helium-rich matter. The white dwarf accretes hydrogen-rich, helium-enhanced matter from a lobe-filling, slightly evolved companion at a critical rate and blows excess matter into the wind. The white dwarf grows in mass to the Chandrasekhar mass limit and explodes as an SN Ia. A theoretical estimate indicates that this channel contributes a considerable part of the inferred rate of SNe Ia in our Galaxy, i.e., the rate is about 10 times larger than the previous theoretical estimates for white dwarfs with slightly evolved companions.

Subject headings: binaries: close — stars: evolution — stars: interiors — supernovae: general — white dwarfs

1. INTRODUCTION

Type Ia supernovae (SNe Ia) have been widely believed to be thermonuclear explosions of mass-accreting white dwarfs (WDs) (e.g., Nomoto, Iwamoto, & Kishimoto 1997 for a recent review). However, the immediate progenitor binary systems have not been identified yet (Branch et al. 1995). There exist two models discussed frequently as progenitors of SNe Ia: (1) the Chandrasekhar (Ch) mass model, in which a mass-accreting carbon-oxygen white dwarf (C+O WD) grows in mass up to the Ch mass and explodes as an SN Ia; and (2) the sub-Ch model, in which an accreted layer of helium on a C+O WD ignites off center for a WD mass well below the Ch mass. The early time spectra of the majority of SNe Ia are in excellent agreement with the synthetic spectra of the Ch mass models, while the spectra of the sub-Ch mass models are too blue to be comparable with the observations (Höflich & Khokhlov 1996; Nugent et al. 1997).

For the evolution of accreting WDs toward the Ch mass, two scenarios have been proposed: (1) a double degenerate (DD) scenario, i.e., merging of double C+O WDs with a combined mass surpassing the Ch mass limit (Iben & Tutukov 1984; Webbink 1984); and (2) a single degenerate (SD) scenario, i.e., accretion of hydrogen-rich matter via mass transfer from a binary companion (e.g., Nomoto 1982a; Nomoto et al. 1994). The issue of DD versus SD is still debated (e.g., Branch et al. 1995), although theoretical modeling has indicated that the merging of WDs leads to accretion-induced collapse rather than to SN Ia explosion (Saio & Nomoto 1985, 1998; Segretain, Chabrier, & Mochkovitch 1997).

For the Ch/SD scenario, a new evolutionary model has been proposed by Hachisu, Kato, & Nomoto (1996; hereafter HKKN). HKKN have shown that if accretion exceeds a critical rate, the WD blows a strong wind, burns hydrogen steadily at this critical rate, and expels excess matter in the wind. Since the wind avoids formation of a common envelope, the WD increases its mass up to the Ch mass. Li & van den Heuvel (1997) have extended the HKKN model to a system consisting of a mass-accreting WD and a lobe-filling, more massive, main-sequence (MS) or subgiant star (hereafter called a WD+MS system), identified with luminous supersoft X-ray sources. They found that such systems are one of the main progenitors of SNe Ia as well as the WD+RG systems proposed by HKKN, which consist of a lobe-filling, less massive, red giant.

Recently Yungelson & Livio (1998) reanalyzed the models by HKKN and Li & van den Heuvel (1997) on the basis of their population synthesis code and concluded that both HKKN’s WD+RG and Li & van den Heuvel’s WD+MS systems can account (at most) for only 10% of the inferred rate of SNe Ia in our Galaxy. However, Yungelson & Livio (1998) overlooked important evolutionary processes in both the WD+MS and WD+RG systems. In this paper, we first describe an important evolutionary process in the formation of the WD+MS system that has been overlooked in previous works (e.g., Di Stefano & Rappaport 1994; Yungelson et al. 1996; Yungelson & Livio...
Another evolutionary process leading to the WD + RG system is discussed elsewhere (Hachisu et al. 1999, hereafter HKN99). In § 2 we describe the new evolutionary process for the formation of WD + MS systems. Including this new evolutionary path, we will show that the secondary (slightly evolved MS) star becomes a helium-rich star such as U Sco (e.g., Williams et al. 1981), which is transferring helium-rich matter onto the primary (WD) star. We have reanalyzed such helium-rich matter accretion onto the WD on the basis of the optically thick wind theory developed by Kato & Hachisu (1994). If the secondary MS star has a mass of \( \sim 2-3.5 \, M_\odot \), a WD with an initial mass of 0.8–1.1 \( M_\odot \) grows in mass to the Ch mass and explodes as an SN Ia. We describe the evolution of such a WD + MS system in § 3. The new parameter region thus obtained is much wider than those obtained by Li & van den Heuvel (1997) and Yungelson & Livio (1998). Discussion follows in § 4, in which we have estimated a realization frequency of our WD + MS systems that accounts for about one-third of the inferred rate of SNe Ia in our Galaxy.

2. FORMATION OF A NAKED HELIUM CORE AND ITS EVOLUTION INTO A C + O WHITE DWARF

In this section, we describe the evolutionary path of the formation of a WD + MS system in which the secondary (MS) star has a helium-rich envelope. This important evolutionary path, which is shown in Figure 1, has been overlooked in previous work. Yungelson & Livio (1998) applied their population synthesis code to the WD + MS systems and found that the realization frequency of the WD + MS systems is at most 1/10 of the inferred rate of SNe Ia in our Galaxy. In their population synthesis code, they consider only initial systems consisting of a more massive asymptotic giant branch (AGB) star with a C + O core and a less massive main-sequence star. This system undergoes a common envelope evolution and finally yields a binary system of a mass-accreting C + O WD and a lobe-filling MS or subgiant star. Yungelson & Livio’s code does not include another important evolutionary path in which a more massive component fills up its inner critical Roche lobe and

![Diagram](image-url)
when it develops a helium core of \( \sim 0.8 \text{–} 2.0 M_\text{\odot} \) in its red giant phase.

2.1. Common Envelope Evolution at the Red Giant Phase with a Helium Core

This evolutionary path from stage A to stage F in Figure 1 was first introduced by Hachisu & Kato (1999) to explain the helium-rich companion of the recurrent nova U Sco. We consider, for example, a close binary at a separation, \( a \), with a primary of mass \( M_1 = 7 M_\odot \) and a secondary of \( M_2 = 2 M_\odot \) (stage A). When the primary has evolved into a red giant with the radius \( R_1 \), forming a helium core of mass \( M_{1,\text{He}} \) (stage B), it fills up its inner critical Roche lobe, i.e., \( R_1 = R_1^\text{f} \). Here \( R_1^\text{f} \) is the effective radius of the inner critical Roche lobe of the primary, which is approximated by Eggleton’s (1983) formula,

\[
R_1^\text{f} = f(q) = \frac{0.49q^{2/3}}{0.6q^{2/3} + \ln(1 + q^{1/3})}, \tag{1}
\]

for the mass ratio \( q = M_1/M_2 \). The long-dashed line in Figure 2 shows the primary radius \( R_1 \) plotted against the helium core mass \( M_{1,\text{He}} \) (Bressan et al. 1993). The radius of the primary increases with the helium core mass from \( M_{1,\text{He}} \sim 0.2 M_\odot \) to \( \sim 1.4 M_\odot \) until helium burning ignites at the center of the helium star. If the primary fills its inner critical Roche lobe at a certain \( M_{1,\text{He}} \), i.e., \( R_1(M_{1,\text{He}}) = R_1^\text{f} \), the separation of the binary, \( a \), is given by

\[
a = \frac{R_1(M_{1,\text{He}})}{f(q)}. \tag{2}
\]

Using this relation, we can plot the initial separation \( \alpha_1 \) against \( M_{1,\text{He}} \) as shown by the thick solid line in Figure 2.

Then the mass transfer begins. This mass transfer is dynamically unstable because the primary red giant star has a convective envelope. The binary undergoes common envelope (CE) evolution (stage C), which yields a much more compact close binary consisting of a naked helium star of \( M_{1,\text{He}} \) and a main-sequence star of \( M_2 = 2 M_\odot \) (stage D). Figure 2 shows the separation \( a_{f,\text{CE}} \) and the inner critical Roche lobe radius of the secondary \( R_2^* \) after the common envelope evolution. Here we assume the relation

\[
a_{f,\text{CE}} \approx \alpha_{\text{CE}} \left( \frac{M_{1,\text{He}}}{M_1(i)} \right) \left( \frac{M_2}{M_1(i) - M_{1,\text{He}}} \right), \tag{3}
\]

with the efficiency \( \alpha_{\text{CE}} = 1.0 \) for the common envelope evolution (e.g., Iben & Tutukov 1984; Iben & Livio 1993; Yungelson & Livio 1998).

After the common envelope evolution, the radii of the inner critical Roche lobes of the primary and the secondary become \( R_1^* \sim 0.36a_{f,\text{CE}} \) and \( R_2^* \sim 0.4a_{f,\text{CE}} \), respectively. Since the secondary radius (shown by the dashed line in Fig. 2 for the \( 2 M_\odot \) zero-age main sequence (ZAMS)) should be smaller than its inner critical Roche lobe, i.e., \( R_2 < R_2^* \), the initial separation \( \alpha_1 \) should exceed \( \sim 80 R_\odot \), as shown in Figure 2. The upper bound of the initial separation is obtained from the maximum radius of a \( 7 M_\odot \) star that has formed a helium core, i.e., \( \alpha_1 \approx 2R_{1,\text{max}} \leq 2 \times 300 R_\odot \). Thus, the allowable range of the initial separations for our model is \( 80 R_\odot \leq \alpha_1 \leq 600 R_\odot \), so that \( R_1^* \sim 1.4 \text{–} 18 R_\odot \) and \( R_2^* \sim 1.6 \text{–} 20 R_\odot \) for the case shown in Figure 2.

2.2. Transfer of the Helium Envelope to the Secondary

After the hydrogen-rich envelope is stripped away and hydrogen shell burning vanishes, the naked helium core contracts to ignite central helium burning and becomes a helium main-sequence star of mass \( M_{1,\text{He}} \) (stage D). For \( M_{1,\text{He}} \leq 2 M_\odot \), its C + O core mass is less than 1.07 \( M_\odot \), which is the lower mass limit to the nondegenerate carbon ignition (e.g., Umeda et al. 1999; see Nomoto & Hashimoto 1988 for a review). Then the helium star forms a degenerate C + O core, whose mass \( M_{\text{C+O}} \) grows by helium shell burning. When \( M_{\text{C+O}} \) becomes 0.9 \text{–} 1.0 \( M_\odot \) and the core becomes strongly degenerate, its helium envelope expands to \( R_1 \sim 1.4 \text{–} 18 R_\odot \) (e.g., Paczyński 1971a; Nomoto 1982b) to fill its inner critical Roche lobe again (stage E). The helium transferred to the secondary stably on an evolutionary timescale of \( \tau_{\text{EV}} \sim 10^5 \) yr because the mass ratio is smaller than 0.79 (\( q = M_1/M_2 < 0.79 \)). The resultant mass transfer (MT) rate is \( \dot{M}_1 \sim 10^{-5} M_\odot \) yr\(^{-1} \), which is too low to form a common envelope (e.g., Neo et al. 1977; Kippenhahn & Meyer-Hofmeister 1977). After the helium envelope is lost, the primary becomes a C + O WD of \( M_{\text{WD}} \sim 0.9 \text{–} 1.1 M_\odot \) (stage F). In Figure 3 \( M_{1,\text{f,MT}} = M_{\text{WD}} \) is plotted as a function of \( M_{1,\text{He}} \). Here we assume the relation (in solar mass units of \( M_\odot \))

\[
M_{\text{WD}} = \begin{cases} 
0.2(M_{1,\text{He}} - 0.85) + 0.85, & \text{for } 0.85 < M_{1,\text{He}} < 2, \\
M_{1,\text{He}}, & \text{for } 0.46 \lesssim M_{1,\text{He}} \lesssim 0.85, \\
0, & \text{for } M_{1,\text{He}} \leq 0.46, \\
\end{cases} \tag{4}
\]

for the final degenerate C + O WD mass versus the initial helium star mass relation (reduced from the evolutionary paths of Paczyński 1971a and Nomoto 1982b). Helium stars with \( M_{1,\text{He}} \leq 0.85 M_\odot \) do not expand to \( \sim 2 \text{–} 10 R_\odot \), thus burning most of helium to C + O without transferring.
helium to the secondary. For $M_{1,\text{He}} \gtrsim 2.0 \, M_\odot$, the C+O core has a mass larger than 1.07 $M_\odot$ before becoming degenerate, thus igniting carbon to form an O+Ne+Mg core (e.g., Nomoto 1984), and thus we do not include their mass range. For $M_{1,\text{He}} \lesssim 0.46 \, M_\odot$, helium is not ignited to form a C+O core.

The secondary increases its mass $M_2$ by receiving almost pure helium matter of $\Delta M_{1,\text{He}} \sim 0.1$–0.6 $M_\odot$, which is a function of $M_{1,\text{He}}$, as plotted in Figure 3. Then a helium-enriched envelope is formed as illustrated in Figure 1 (stage F). After the helium mass transfer (MT), the separation increases by 10%–40%, i.e., $a_{f,\text{MT}} \sim (1.1$–1.4)$a_{f,\text{CE}} \sim (4$–70) $R_\odot$. Here we assume the conservation of the total mass and angular momentum during the helium mass transfer, which leads to the relation

$$a_{f,\text{MT}} = \frac{M_{1,\text{He}}}{M_{1,\text{He}} - \Delta M_{1,\text{He}}} \left(\frac{M_{2,i}}{M_{2,i} + \Delta M_{1,\text{He}}}ight)^2,$$

and we use this to obtain $a_{f,\text{MT}}$ and then the orbital period $P_0 \equiv P_{f,\text{MT}}$ in Figure 3.

Since the secondary receives $\sim 0.1$–0.6 $M_\odot$ helium, its hydrogen content in the envelope decreases to $X \sim 0.6$ if helium is completely mixed into the central part of the star. However, the envelope of the mass-receiving star is not convective but radiative so that the helium content may be higher in the outer part of the star.

2.3. Helium-enriched Main-Sequence Companion

We have examined 25 cases, all the combinations of $M_{1,i} = 4, 5, 6, 7$, and $9 \, M_\odot$ and $M_{2,i} = 1, 1.5, 2.0, 2.5$, and $3.0 \, M_\odot$, and have found the possible progenitors to be in the ranges of $M_{1,\text{C+O-WD}} \sim 0.8$–1.1 $M_\odot$ and $M_{2,\text{MS}} \sim 1.7$–3.5 $M_\odot$, with the separation of $a_{f,\text{MT}} \sim 4$–80 $R_\odot$. In these cases, the secondary forms a helium-enriched envelope for the primary mass of $M_{1,\text{WD}} \sim 0.9$–1.1 $M_\odot$ (but not for $M_{1,\text{WD}} \sim 0.8$–0.85 $M_\odot$). We assume in this paper that the average mass fractions of hydrogen and helium in the envelope are $X = 0.50$ and $Y = 0.48$, respectively (assuming the solar abundance of heavy elements, $Z = 0.02$). For the 9 $M_\odot + 2.5 \, M_\odot$ case, much more helium is transferred, while much less helium is transferred for the $6 \, M_\odot + 2 \, M_\odot$ case (see Table 1).

3. Growth of C+O White Dwarfs

Starting from a close binary of $M_{\text{WD,0}} \equiv M_{1,\text{C+O-WD}} \sim 0.8$–1.1 $M_\odot$ and $M_{\text{MS,0}} \equiv M_{2,\text{MS}} \sim 1.7$–3.5 $M_\odot$, with the separation of $a_0 \equiv a_{f,\text{MT}} \sim 4$–80 $R_\odot$, we have followed a growth of the WD component to examine whether the WD reaches 1.38 $M_\odot$ and explodes as an SN Ia (from stage F to stage J in Fig. 4).

The initial secondary now has a helium-rich envelope (stage F). It evolves to expand and fill its inner critical Roche lobe near the end of main-sequence (MS) phase. Then it starts mass transfer (stage G). This is a cataclysm-like mass transfer after Iben & Tutukov (1984). Since the donor is more massive than the accretor (WD component), the separation decreases and the inner critical Roche lobe decreases and the inner critical Roche lobe decreases with the envelope of the donor star being scraped off. Thus, the mass transfer proceeds on a thermal timescale rather than an evolutionary timescale (stage H). The transferred matter is helium-rich, as observed in the recurrent nova U Sco.

3.1. Optically Thick Winds from Mass-accreting White Dwarfs

Hachisu et al. (1996) have shown that optically thick winds blow from the white dwarf when the mass accretion rate exceeds a critical value (stage H). In the present case, the matter accreted to form the white dwarf envelope is helium-rich, which is different from the solar abundance in HKN96. Assuming $X = 0.50$, $Y = 0.48$, and $Z = 0.02$, we

| $M_{2,i}$ | $M_{1,i} = 5 \, M_\odot$ | $M_{1,i} = 6 \, M_\odot$ | $M_{1,i} = 7 \, M_\odot$ | $M_{1,i} = 9 \, M_\odot$ |
|----------|----------------|----------------|----------------|----------------|
| $a_{f,\text{CE}} = 1$ | 0.3 | 0.3 | 0.3 | 0.3 |
| $\Delta M_{1,\text{He}}$ | 0.0, 0.0 | 0.0, 0.2 | 0.1, 0.2 | 0.37, 0.37 |
| 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| 1.5 | 1.9, 2.4 | 2.4, 2.6 | 1.8, 2.3 | 2.3, 2.8 |
| 2.0 | 1.8, 2.4 | 2.3, 2.7 | 1.8, 2.3 | 2.4, 2.8 |
| 2.5 | 2.3, 2.5 | 2.1, 2.4 | 2.4, 2.7 | 2.0, 2.3 |
| 3.0 | 2.3, 2.5 | 2.1, 2.4 | 2.4, 2.7 | 2.0, 2.3 |

* Helium mass transferred to the secondary (minimum, maximum).
* Initial separation of log ($a/\rho R_\odot$) (minimum, maximum).
have calculated the envelope models of accreting white dwarfs for various accretion rates and white dwarf masses, i.e., $M_{\text{WD}} = 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.35,$ and $1.377 M_\odot$. Optically thick winds occur for all 10 cases of $M_{\text{WD}}$, as shown for the five cases of $M_{\text{WD}} = 0.8, 1.0, 1.2, 1.3,$ and $1.377 M_\odot$ in Figures 5 and 6. Our numerical methods have been described in Kato & Hachisu (1994). The envelope solution is uniquely determined if the envelope mass $M_{\text{env}}$ is given, where $M_{\text{env}}$ is the mass above the base of the hydrogen-burning shell. Therefore, the wind mass-loss rate $M_{\text{wind}}$ and the nuclear burning rate $M_{\text{nuc}}$ are obtained as a function of the envelope mass $M_{\text{env}}$, i.e., $M_{\text{wind}}(M_{\text{env}})$ and $M_{\text{nuc}}(M_{\text{env}})$, as shown in Figure 5.

The envelope mass of the white dwarf is determined by

$$M_{\text{env}} = M_2 - (M_{\text{wind}} + M_{\text{nuc}}).$$

(6)

If the mass transfer rate does not change much in a thermal timescale of the WD envelope, the steady state $M_{\text{env}} = 0$, i.e.,

$$M_2 = M_{\text{wind}} + M_{\text{nuc}},$$

(7)

is a good approximation. In such a steady state approximation, the ordinates in Figures 5 and 6, $M_{\text{wind}} + M_{\text{nuc}}$, are regarded as the mass transfer rate from the secondary $M_2$. Thus, the envelope solution is determined from the relation in Figure 5 for the given mass transfer rate $M_2$. The photospheric radius, temperature, and velocity are also obtained from the relations in Figure 6.

Optically thick winds blow when the mass transfer rate exceeds the critical rate, which corresponds to the break of each solid line in Figure 5. There exists only a static (no wind) envelope solution for the mass transfer rate below this break. Its critical accretion rate is approximated as

$$M_\text{cr} = 1.2 \times 10^{-6} \left( \frac{M_{\text{WD}}}{M_\odot} - 0.40 \right) M_\odot \text{ yr}^{-1}$$

(8)

for $X = 0.50$, $Y = 0.48$, and $Z = 0.02$. If the mass accretion rate exceeds the critical rate, i.e., $M_2 > M_\text{cr}$, the strong wind blows from the white dwarf. The white dwarf accretes helium almost at the critical rate, i.e., $M_{\text{nuc}} \approx M_\text{cr}$, and expels the excess matter in the wind at a rate of $M_{\text{wind}} \approx M_2 - M_\text{cr}$. Here we assume that the white dwarf accretes helium-rich matter from the equator via the accretion disk and blows winds to the pole or off the equator.

3.2. Efficiency of Mass Accretion in Hydrogen Shell Burning

Steady hydrogen shell burning converts hydrogen into helium atop the C+O core, which can be regarded as a helium matter accretion onto the C+O WD. To estimate
whether the white dwarf mass grows to 1.38 $M_\odot$, we must calculate the mass accumulation efficiency, that is, the ratio between the mass accumulated in the WD after H/He burning and the mass transferred from the companion. We denote the efficiency by $\eta_H$ and $\eta_{He}$ for hydrogen shell burning and helium shell burning, respectively. As shown in the previous subsection, the excess matter is blown in the wind when the mass transfer rate exceeds the critical rate, which leads to $\eta_H = (M_2 - M_{\text{wind}})/M_2 < 1$ for the wind phase.

During the evolution of mass-accreting white dwarfs, the accretion rate becomes lower than $M_{\text{crit}}$ in some cases. Then the wind stops (stage I in Fig. 4). The hydrogen burns steadily for $M_2 > M_{\text{crit}} \approx 0.5 M_{\text{crit}}$. Then we have $\eta_H = 1$. For $M_2 < M_{\text{crit}}$, hydrogen shell burning becomes unstable to trigger weak shell flashes. Once a weak hydrogen shell flash occurs, a part of the envelope mass of the white dwarf may be lost from the system (e.g., Kovetz & Prialnik 1994). In the present study, no mass loss is assumed during the weak hydrogen shell flashes until the mass transfer rate becomes lower than $M_{\text{low}} = 1 \times 10^{-7} M_\odot$ yr$^{-1}$, i.e., $\eta_H = 1$ for $M_{\text{low}} < M_2 < M_{\text{crit}}$. When the mass transfer rate becomes lower than $M_{\text{low}}$, however, no mass accumulation is expected from such relatively strong hydrogen shell flashes (e.g., Kovetz & Prialnik 1994), i.e., $\eta_H = 0$ for $M_2 \leq M_{\text{low}}$. Therefore, we stop calculating the binary evolution either when the primary reaches 1.38 $M_\odot$ (stage I in Fig. 4), i.e., $M_{1,\text{WD}} = 1.38 M_\odot$, or when the mass transfer rate becomes lower than $M_{\text{low}}$.

To summarize, we have used the following simplified relation

$$\eta_H = \begin{cases} 1, & (M_2 - M_{\text{crit}}) < 1, \\ 0, & (M_{\text{low}} \leq M_2 \leq M_{\text{crit}}), \\ (M_2 < M_{\text{low}}), \end{cases}$$

for the mass accumulation efficiency of hydrogen shell burning.

3.3. Efficiency of Mass Accretion in Helium Shell Burning

For $M_2 \geq M_{\text{crit}}$, steady hydrogen burning is equivalent to the helium accretion at the critical rate of $M_{\text{crit}}$ given by equation (8). In this case, weak helium shell flashes are triggered (e.g., Kato, Saio, & Hachisu 1989) and almost all processed matter is accumulated on the C + O WD. Recently, Kato & Hachisu (1999) have recalculated the helium wind model after the helium shell flashes and estimated the mass accumulation efficiency with the updated OPAL opacity (Iglesias & Rogers 1996). Here we adopt their new results in a simple analytic form, i.e.,

$$\eta_{He} = \begin{cases} 1, & (-5.9 \leq \log M_{He} \leq -5), \\ -0.175(\log M_{He} + 5.35)^2 + 1.05, & (-7.8 < \log M < -5.9), \end{cases}$$

where the mass accretion rate, $M_{He}$, is in units of $M_\odot$ yr$^{-1}$. We use this formula for various white dwarf masses and accretion rates, although their results are given for only the 1.3 $M_\odot$ white dwarf (Kato & Hachisu 1999).

The wind velocity in helium shell flashes reaches as high as ~1000 km s$^{-1}$ (Kato & Hachisu 1999), which is much faster than the orbital velocities of our WD+MS binary systems, $a\Omega_{\text{orb}} \sim 300$ km s$^{-1}$. It should be noted that neither a Roche lobe overflow nor a common envelope plays a role as a mass ejection mechanism because the envelope matter leaves the system quickly without interacting with the orbital motion (see Kato & Hachisu 1999 for more details).

3.4. Mass Transfer Rate of the Secondary

We have followed binary evolutions from the initial state of $(M_{WD,0}, M_{MS,0}, a_0)$ or $(M_{WD,0}, M_{MS,0}, P_0)$, where $P_0$ is...
the initial orbital period. Here the subscript zero (0) denotes stage F in Figure 4, that is, the stage before the mass transfer from the secondary starts. The radius and luminosity of slightly evolved main-sequence stars are calculated from the analytic form given by Tout et al. (1997). The mass transfer proceeds on a thermal timescale for the mass ratio of $M_2/M_1 > 0.79$. We approximate the mass transfer rate as

$$M_2 = \frac{M_2^*}{\tau_{KH}} \max \left( \frac{\zeta_{RL} - \zeta_{MS}}{\zeta_{MS}}, 0 \right),$$  \hspace{1cm} (11)

where $\tau_{KH}$ is the Kelvin-Helmholtz timescale (e.g., Paczyński 1971b), and $\zeta_{RL} = d \log R^*/d \log M$ and $\zeta_{MS} = d \log R_{MS}/d \log M$ are the mass-radius exponents of the inner critical Roche lobe and the main-sequence component, respectively (e.g., Hjellming & Webbink 1987). The effective radius of the inner critical Roche lobe, $R^*$, is calculated from equation (1).

### 3.5. Late Binary Evolution toward an SN Ia

The separation is determined by

$$\dot{a} = \frac{M_1 + M_2}{a} - 2 \frac{\dot{M}_1}{M_1} - 2 \frac{\dot{M}_2}{M_2} + 2 \frac{J}{J}. \hspace{1cm} (12)$$

We estimate the total mass and angular momentum losses by the winds as

$$\dot{M} \equiv \dot{M}_1 + \dot{M}_2 = \dot{M}_{\text{wind}}, \hspace{1cm} (13)$$

and

$$\frac{J}{J} = l \left( \frac{M_1 + M_2}{M_1 M_2} \right)^2 \frac{\dot{M}}{M}, \hspace{1cm} (14)$$

where $l$ is a numeric factor expressing the specific angular momentum of the wind, i.e.,

$$\frac{J}{M} = l a^2 \Omega_{\text{orb}}. \hspace{1cm} (15)$$

For the very fast winds such as $v_{\text{wind}} \gtrsim 2a \Omega_{\text{orb}}$, the wind has, on average, the same specific angular momentum as that of the WD component, i.e.,

$$l = \left( \frac{M_2}{M_1 + M_2} \right)^2, \hspace{1cm} (16)$$

because the wind is too fast to interact with the orbital motion (see also HKN99). In equations (12)-(15), we must take into account the sign of the mass-loss rates, i.e., $\dot{M} = \dot{M}_{\text{wind}} \leq 0$, $\dot{M}_2 \leq 0$, $J \leq 0$, and so on.

Figure 7 shows an example of such a close binary evolution that leads to the SN Ia explosion. Here we calculate the formation of the white dwarf in the main critical Roche lobe. The initial parameters are $M_{WD,0} = 1.0 \, M_\odot$, $M_{\text{MS},0} = 2.0 \, M_\odot$, and $a_0 = 9.6 \, R_\odot$ ($P_0 = 2.0$ d). The mass transfer begins at a rate as high as $\dot{M}_2 = 2.2 \times 10^{-6} \, M_\odot \, yr^{-1}$. The WD burns hydrogen to form a helium layer at the critical rate of $\dot{M}_{\text{wind}} = 0.7 \times 10^{-6} \, M_\odot \, yr^{-1}$, and the wind mass-loss rate is $\dot{M}_{\text{wind}} = 1.5 \times 10^{-6} \, M_\odot \, yr^{-1}$. Thus a large part of the transferred matter is blown off in the wind.

Since the mass ratio $M_2/M_1$ decreases, the mass transfer rate determined by equation (11) gradually decreases below $M_{\text{crit}}$. The wind stops at $t = 1.9 \times 10^5 \, yr$. The mass transfer rate becomes lower than $\dot{M}_{\text{crit}} = 5 \times 10^{-7} \, M_\odot \, yr^{-1}$ for $(M_{WD,0} = 1.2 \, M_\odot)$ at $t = 3.8 \times 10^5 \, yr$, and very weak shell flashes may occur. The WD mass gradually grows to reach $1.38 \, M_\odot$ at $t = 6.7 \times 10^5 \, yr$. At this time, the mass transfer rate is still as high as $\dot{M}_2 = 3.6 \times 10^{-7} \, M_\odot \, yr^{-1}$ because the mass ratio $M_2/M_1$ is still larger than 0.79, which implies thermally unstable mass transfer.

This WD + MS system may not be observed in X-rays during the strong wind phase because of the self-absorption of X-rays. However, it is certainly identified as a luminous supersoft X-ray source from $t = 1.9 \times 10^5$ to $3.8 \times 10^5 \, yr$ because it is in a steady hydrogen shell burning phase without a strong wind. Just before the explosion, it may be observed as a recurrent nova like U Sco, which indicates a helium-rich accretion in quiescence (Hanes 1985).

### 3.6. Outcome of Late Binary Evolution

Thus we have obtained the final outcome of close binary evolutions for various sets of $(M_{WD,0}, M_{\text{MS},0}, a_0)$ or $(M_{WD,0}, M_{\text{MS},0}, P_0)$. Figure 8 depicts the final outcomes in the $M_{\text{MS},0} - \log P_0$ plane for $M_{WD,0} = 1.1 \, M_\odot$. The final outcome is one of the following:

1. Forming a common envelope (crosses) because the mass transfer rate at the beginning is large enough to form a common envelope, i.e., $R_{1,ph} > a \sim 10 \, R_\odot$ for $M_2 \gtrsim 1 \times 10^{-4} \, M_\odot \, yr^{-1}$, as seen in Figure 6.
2. Triggering an SN Ia explosion (circled plus signs, open circles, or circled dots) when $M_{1, WD} = 1.38 \, M_\odot$.
3. Triggering repeated nova cycles (open triangles), i.e., $M_2 < \dot{M}_{\text{low}}$ when $M_{1, WD} < 1.38 \, M_\odot$

Among the SN Ia cases, the wind status at the explosion depends on $M_2$ as follows:

2a. Wind continues at the SN Ia explosion for $\dot{M}_{\text{crit}} < M_2 \lesssim 1 \times 10^{-4} \, M_\odot \, yr^{-1}$ (circled plus signs).
2b. Wind stops before the SN Ia explosion, but the mass transfer rate is still high enough to keep steady hydrogen shell burning for $\dot{M}_{\text{crit}} < M_2 < \dot{M}_{\text{crit}}$ (open circles).
The final outcome of the evolution is also plotted in Figures 9–11 for other initial white dwarf masses of $M_{\text{WD,0}} = 1.0 M_\odot$, 0.9 $M_\odot$, and 1.1 $M_\odot$ (thick solid lines) together with $M_{\text{WD,0}} = 0.7 M_\odot$ (thin solid lines). The region for $M_{\text{WD,0}} = 0.7 M_\odot$ vanishes. The shrinking of the upper bound for smaller $M_{\text{WD,0}}$ is due to larger initial
The white dwarf with smaller \( M_{\text{WD},0} \) needs to accrete more mass than the companion to supply hydrogen-rich matter to the white dwarf. The thermal timescale of the companion is longer for smaller masses, thereby decreasing the mass transfer rate down to the nova region.

4. DISCUSSION

We have estimated the rate of SNe Ia originating from our WD + MS systems in our Galaxy by using equation (1) of Iben & Tutukov (1984), i.e.,

\[
v = 0.2 \Delta a \int_{M_{4}}^{M_{B}} \frac{dM}{M_{2/3}} \Delta \log A \text{ yr}^{-1},
\]

where \( \Delta a \) and \( \Delta \log A \) denote the appropriate ranges of the initial mass ratio and the initial separation, respectively, and \( M_{4} \) and \( M_{B} \) are the lower and the upper limits, respectively, of the primary mass that leads to SN Ia explosions. For the WD + MS progenitors, we assume that \( a_{i} \lesssim 1500 \ R_{\odot} \) in order to obtain a relatively compact condition of \( a_{f,CE} \) after the common envelope evolution.

If the \( \sim 1 \ M_{\odot} \) C + O WD is a descendant of an AGB star, its zero-age main-sequence mass is \( \sim 7 \ M_{\odot} \) (see, e.g., eq. [11] of Yungelson et al. 1995), and the binary separation is larger than \( a_{i} \approx 1300 \ R_{\odot} \) (e.g., Iben & Tutukov 1984). Its separation shrinks to \( a_{f,CE} \approx 60 \ R_{\odot} \) after the common envelope evolution for the case of \( \Delta a_{\text{CE}} = 1 \) and an \( \sim 2 \ M_{\odot} \) secondary. Then the orbital period becomes \( P_{o} \approx 30 \) days, which is too long to become an SN Ia (e.g., Li & van den Heuvel 1997; see also Fig. 9). Therefore, the WD + MS systems descending from an AGB star may be rare, as pointed out by Yungelson & Livio (1998), and may not be a main channel to SNe Ia.

To obtain the realization frequency of our WD + MS system descending from a red giant with a helium core, we have followed a total of \( \sim 500 \) evolutions with the different initial set of \((M_{1,1}, M_{2,1}, a_{i})\) and estimated the appropriate range for the initial separation of \( \Delta \log A = \log a_{i,\max} - \log a_{i,\min} \) for a total of \( 5 \times 5 = 25 \) cases of \((M_{1,1}, M_{2,1})\), both for the primary mass of \( M_{1,0} = 4, 5, 6, 7, \) and \( 9 \ M_{\odot} \) and the secondary mass of \( M_{2,0} = 1, 1.5, 2, 2.5, \) and \( 3 \ M_{\odot} \) (see Table 1). In Table 1, we omit the case of \( M_{1,0} = 4 \ M_{\odot} \) because it never leads to SN Ia explosions. We find that SN Ia explosions occur for the ranges of \( M_{1,1} = 5.5 \)–\( 5.8 \ M_{\odot} \), \( M_{2,1} = 1.8 \)–\( 3.4 \ M_{\odot} \), and \( \Delta \log A = 0.5 \). We thus obtain the realization frequency of SNe Ia from the WD + MS systems \( v_{\text{MS}} = 0.0010 \text{ yr}^{-1} \) for \( x_{\text{CE}} = 1 \) by substituting \( \Delta a = (3.4/5.5)(1.8/8.5) = 0.41, M_{A} = 5.5 \ M_{\odot}, M_{B} = 8.5 \ M_{\odot}, \) and \( \Delta \log A = 0.5 \) into equation (17). For comparison, we have obtained a realization frequency of SNe Ia for \( x_{\text{CE}} = 0.3 \). It is still as high as \( v_{\text{MS}} \approx 0.0007 \text{ yr}^{-1} \), which is about one-fourth of the inferred rate. Our new rate of \( v_{\text{MS}} = 0.0010 \text{ yr}^{-1} \) is about one-third of the inferred rate of SNe Ia in our Galaxy and much higher than \( v_{\text{MS}} = 0.0002 \text{ yr}^{-1} \) (as an upper limit), which was obtained by Yungelson & Livio (1998). The reason for their low frequency is probably the absence of a path through the primary’s helium star phase in their scenario. However, it should be noted here that Yungelson & Livio (1998) have also obtained a realization frequency of \( v_{\text{MS}} \sim 0.001 \text{ yr}^{-1} \) under the assumption of no restrictions in their binary evolutions, the conditions of which are unlikely.

Part of our WD + MS systems are identified as luminous supersoft X-ray sources (SSSs) (van den Heuvel et al. 1992). SSSs are characterized by a luminosity of \( \sim 10^{38} \text{ ergs s}^{-1} \) and a temperature of \( T \sim 4 \times 10^{6} \text{ K (kT} \approx 35 \text{ eV)} \), which have been established as a new class of X-ray sources through ROSAT observations (e.g., Kahabka & van den Heuvel 1997 for a review). A population synthesis for SSSs was first done by Rappaport, Di Stefano, & Smith (1994), followed by a more recent one by Yungelson et al. (1996). These calculations predicted a total number of the Galactic SSSs of \( \sim 1000 \) and led to the conclusion that the SSS birthrate is roughly consistent with the observations (see, e.g., Kahabka & van den Heuvel 1997 for a review). Our SN Ia progenitors should be observed as an SSS during the steady hydrogen shell burning phase without winds, which is about a few times \( 10^{3} \text{ yr} \), as shown in Figure 7. Then the number of SSSs from our scenario is estimated roughly to be at least \( \sim 3 \times 10^{5} \text{ yr} \times v_{\text{MS}} \approx 300 \), which should be added to the 1000 by Yungelson et al. (1996). The total number is still consistent with observations.

Our WD + MS progenitor model predicts helium-enriched matter accretion onto a WD. Strong He ii \( \lambda 4686 \) lines are prominent in the luminous supersoft X-ray sources (see, e.g., Kahabka & van den Heuvel 1997 for a recent review) as well as in the recurrent novae like U Sco (Hanes 1985; Johnston & Kulkarni 1992) and V394 CrA (Sekiguchi et al. 1989). Thus the weakness of the hydrogen emission lines relative to the He ii and CNO lines is very consistent with the requirement that the accreted matter and hence the envelope of the secondary have a hydrogen-poor (helium-rich) composition.

For SNe Ia, several attempts have been made to detect signatures of circumstellar matter. There have been no radio detections so far. Radio observations of SN 1986G have provided the most stringent upper limit to the circum-

---

**Fig. 12.**—Regions that lead to SN Ia explosions are plotted in the \( \log p_{o} - M_{WD,0} \) plane for five cases of the initial white dwarf mass, i.e., \( M_{WD,0} = 0.75, 0.8, 0.9, \) and 1.1 \( M_{\odot} \) (thin solid lines) together with \( M_{WD,0} = 1.0 \) (thick solid line). However, the region for \( M_{WD,0} = 0.7 \ M_{\odot} \) vanishes.
stellar density as $\dot{M}/v_{10} = 1 \times 10^{-7} M_\odot \text{yr}^{-1}$ (Eck et al. 1995), where $v_{10}$ means $v_{10} = v/10 \text{ km s}^{-1}$. This is still 10–100 times higher than the density predicted for the white dwarf winds because the WD wind velocity is as fast as $\sim 1000 \text{ km s}^{-1}$. Further attempts to detect high-velocity hydrogen signatures are encouraged.

We thank the anonymous referee for helpful comments, which improved the manuscript. This research has been supported in part by the Grants-in-Aid for Scientific Research (05242102, 06233101, 08640321, and 09640325) and COE Research (07CE2002) of the Japanese Ministry of Education, Science, Culture, and Sports.

REFERENCES

Branch, D., Livio, M., Yungelson, L. R., Boffi, F. R., & Baron, E. 1995, PASP, 107, 717
Bressan, A., Fagotto, F., Bertelli, G., & Chiosi, C. 1993, A&AS, 100, 647
Di Stefano, R., & Rappaport, S. 1994, ApJ, 437, 733
Eck, C. R., Cowan, J. J., Roberts, D. A., Boffi, F. R., & Branch, D. 1995, ApJ, 451, L53
Eggleton, P. P. 1983, ApJ, 268, 368
Hachisu, I., & Kato, M. 1999, ApJ, 521, in press
Hachisu, I., Kato, M., & Nomoto, K. 1996, ApJ, 470, L97 (HKN96)
Hanes, D. A. 1985, MNRAS, 213, 443
Hjellming, M. S., & Webbink, R. F. 1987, ApJ, 318, 794
Höflich, P., & Khokhlov, A. 1996, ApJ, 457, 500
Iben, I. Jr., & Livio, M. 1993, PASP, 105, 1373
Iben, I. Jr., & Tutukov, A. V. 1984, ApJS, 54, 335
Iglesias, C. A., & Rogers, F. 1996, ApJ, 464, 943
Johnston, H. M., & Kulkarni, S. R. 1992, ApJ, 396, 267
Kahabka, P., & van den Heuvel, E. P. J. 1997, ARA&A, 35, 69
Kato, M., & Hachisu, I. 1994, ApJ, 437, 802
Kato, M., Saio, H., & Hachisu, I. 1989, ApJ, 340, 509
Kippenhahn, R., & Meyer-Hofmeister, E. 1977, A&A, 54, 539
Kippenhahn, R., & Weigert, A. 1967, Z. Ap., 65, 251
Kovetz, A., & Prialnik, D. 1994, ApJ, 429, 319
Li, X.-D., & van den Heuvel, E. P. J. 1997, A&A, 322, L9
Neo, S., Miyaji, S., Nomoto, K., & Sugimoto, D. 1977, PASJ, 29, 249
Nomoto, K. 1982a, ApJ, 253, 798
Nomoto, K. 1982b, in Supernovae: A Survey of Current Research, ed. M. Rees & R. J. Stoneham (Dordrecht: Reidel), 205
Nomoto, K. 1984, ApJ, 277, 791
Nomoto, K., & Hashimoto, M. 1988, Phys. Rep., 163, 13
Nomoto, K., Iwamoto, K., & Kishimoto, N. 1997, Science, 276, 1378
Nomoto, K., Yamaoka, H., Shigeyama, T., Kumagai, S., & Tsujimato, T. 1994, in Les Houches, Session LIV, Supernovae, ed. S. A. Bludman, R. Mochovitch, & J. Zinn-Justin (Amsterdam: Elsevier), 199
Nugent, P., Baron, E., Branch, D., Fisher, A., & Hauschildt, P. H. 1997, ApJ, 485, 812
Paczyński, B. 1971a, Acta Astron., 21, 1
Paczyński, B. 1971b, ARA&A, 9, 183
Rappaport, S., Di Stefano, R., & Smith, J. D. 1994, ApJ, 426, 692
Saio, H., & Nomoto, K. 1985, A&A, 150, L21
Saio, H., & Nomoto, K. 1998, ApJ, 500, 388
Sekiguchi, K., et al. 1998, MNRAS, 296, 611
Segretain, L., Chabrier, G., & Mochkovitch, R. 1997, ApJ, 481, 355
Tout, C. A., Aarseth, S. J., Pols, O. R., & Eggleton, P. P. 1997, MNRAS, 291, 732
Umeda, H., Nomoto, K., Yamaoka, H., & Wanajo, S. 1999, ApJ, 513, 861
van den Heuvel, E. P. J., Bhattacharya, D., Nomoto, K., & Rappaport, S. 1992, A&A, 262, 97
Webbink, R. F. 1984, ApJ, 277, 355
Williams, R. E., Sparks, W. M., Gallagher, J. S., Ney, E. P., Starrfield, S. G., & Truran, J. W. 1981, ApJ, 251, 221
Yungelson, L., & Livio, M. 1999, ApJ, 497, 168
Yungelson, L., Livio, M., Truran, J. W., Tutukov, A., & Fedorova, A. 1996, ApJ, 466, 890
Yungelson, L., Livio, M., Tutukov, A., & Kenyon, S. 1995, ApJ, 447, 656