THE RELATIONSHIP BETWEEN THE TURBULENCE-DRIVING-LENGTH AND THE LENGTH-SCALE OF DENSITY STRUCTURES IN MAGNETOHYDRODYNAMIC TURBULENCE

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ABSTRACT

Density fluctuations produced by supersonic turbulence are of great importance to astrophysical chemical models. A property of these density fluctuations is that the two point correlation function decreases with increasing scale separation. The relation between the density decorrelation length scale ($L_{\text{dec}}$) and the turbulence driving scale ($L_{\text{drive}}$) determines how turbulence affects the density and chemical structures in the interstellar medium (ISM), and is a key component for using observations of atomic and molecular tracers to constrain turbulence properties. We run a set of numerical simulations of supersonic magnetohydrodynamic turbulence, driven on varying scales from 1/2.5 to 1/7 the box length, and derive the $L_{\text{dec}} - L_{\text{drive}}$ relation as a function of driving-scale and the orientation of the line-of-sight (LOS) in respect to the mean magnetic field. We find that $L_{\text{drive}}$, $L_{\text{dec}}/L_{\text{drive}} = 0.231$ when averaging over all LOS. For LOS parallel to the magnetic field the density structures are statistically smaller and the $L_{\text{dec}} - L_{\text{drive}}$ relation is tighter, with $L_{\text{dec}}/L_{\text{drive}} = 0.129 \pm 0.011$. We discuss our results in the context of using observations of chemical tracers in the ISM to constrain the dominant turbulence driving scale.

1. INTRODUCTION

Understanding turbulence in galaxies is of central importance to a number of areas of astrophysical interest including star formation (e.g., Krumholz et al. 2009; Ostriker et al. 2010; Burkhart et al. 2015c), cosmic ray acceleration and diffusion (Schlickeiser 2002; Lazarian & Yan 2014; Xu et al. 2016), and accretion disks around planets, stars and black holes (Balbus & Hawley 1991; Hughes et al. 2010; Ross et al. 2017). Compressible turbulence is ubiquitous throughout the interstellar medium (ISM) of galaxies from scales of at least tens of parsecs down to the sub-parsec scales (Armstrong et al. 1995; McKee & Ostriker 2007; Lazarian 2007; Chepurnov et al. 2010; Krumholz 2014; Burkhart et al. 2015a). Turbulence in the ISM may be driven by multiple energy injection sources on different scales (Elmegreen & Scalo 2004; Chepurnov et al. 2015; Pingel et al. 2018), from disk instabilities and supernova acting on the largest scales (Krumholz & Burkhal 2016) to stellar winds and jets on sub-cloud scales (Offner et al. 2014). Considering the wide range of scales, galaxy turbulence affects there has been significant effort to connect observed levels of turbulence with theoretical predictions and simulations (Goldreich & Sridhar 1995; Cho & Lazarian 2003; Federrath et al. 2008; Burkhart et al. 2010; Correia et al. 2016; Herron et al. 2017). However, it is still unclear which driving mechanism dominates the turbulent energy budget in the ISM (Krumholz 2014) and on what scales the turbulence is dissipated (Burkhart et al. 2015b).

An important feature of a compressible turbulent cascade is that density fluctuations exhibit statistical correlations in relation to the driving scale (Burkhart et al. 2009; Portillo et al. 2017). Numerical and analytic studies found that the correlation of density fluctuations decreases with increasing spatial separation (Kowal et al. 2007). The characteristic scale over which the correlation decreases is the density decorrelation scale, $L_{\text{dec}}$, and it is found to be of order of the driving scale, $L_{\text{drive}}$, with $L_{\text{dec}} = \phi L_{\text{drive}}$ (1)

with $\phi \approx 0.1 - 0.3$ (Vazquez-Semadeni & Garcia 2001, hereafter VG, Fischera & Dopita 2004, Kowal et al. 2007, Bialy et al. 2017, hereafter BBS). The exact value of $\phi$ depends on the method used to measure the decorrelation scale, for example, VG define $L_{\text{dec}}$ as the point at which the autocorrelation function (ACF) falls to a fraction 0.1 of its initial value, whereas BBS derive $L_{\text{dec}}$ using an analytic model that describes the correlation of the smoothed-density field as a function of the smoothing-length (see §2 below; cf. Squire & Hopkins 2017).

Importantly, as discussed by BBS and Bialy et al. (2019), $L_{\text{dec}}$ may be constrained from observations of the column density PDF of various atomic and molecular tracers (H, H$_2$, OH$^+$, OH$^-$, H$_3^+$, Ar$^+$). This is because the chemical reactions in the ISM are sensitive to the gas density and its structure. In particular, the absorption of ultraviolet (UV) radiation by H$_2$ lines (i.e., H$_2$ self-shielding) is very sensitive to the length-scales of density fluctuations. In turn, other molecular species depend on the H$_2$ abundance, and are therefore also sensitive to $L_{\text{dec}}$. Given a robust relation between $L_{\text{dec}}$ and $L_{\text{drive}}$, observations of chemical tracers may be used to constrain the turbulence driving scale (see Fig. 4 and §5.1 below).

However, previous numerical studies have obtained $L_{\text{dec}}$ considering only large driving scales, of order of the simulation box-size. In this paper, we use a large set of MHD simulations, driven on different scales from large-scale $k_{\text{drive}} = 2.5$ driving (we denote the wavenumber $k = 1/L_{\text{box}}$), intermediate scale $k_{\text{drive}} = 5$, and down to small scale $k_{\text{drive}} = 7$ driving, and derive the $L_{\text{dec}} - L_{\text{drive}}$ relation as a function of $k_{\text{drive}}$. For each driving-scale we further investigate the dependence of

\[ \frac{L_{\text{dec}}}{L_{\text{drive}}} = \phi \]
the density structures on the line-of-sight (LOS) orientation, parallel and perpendicular to the large scale magnetic field.

This paper is organized as follows: in §2 we provide a theoretical overview for methods for deriving the decorrelation scale. In §3 we describe our numerical set up. In §4 we present results for the density structures and the decorrelation scale, and their dependence on LOS orientation and driving scale. We discuss our results in §5, and conclude in §6.

2. THEORETICAL BACKGROUND

The decorrelation scale, $L_{\text{dec}}$, is the characteristic scale over which density correlations decrease. We consider two definitions for $L_{\text{dec}}$: (1) via the smoothed-density method (BBS), and (2) using the ACF (i.e., VG). The smoothed-density method was developed for modeling the distribution of integrated column densities of chemical species and inferring properties of the 3D density field (BBS). The idea is quite intuitive: if we smooth (average) the density over a scale $\ell$, and let $\ell$ vary from small to large, we expect the dispersion of the smoothed density to decrease with increasing $\ell$, as more density fluctuations are smoothed-out within the smoothing length.

More quantitatively, given the field $x \equiv n/(n)$ (i.e., normalized density), we define the smoothed-density

$$x_\ell(\ell) \equiv \frac{\int_{x_\ell}^{x_{\ell}+\ell} x \, dz}{\ell},$$

where $\ell$ is the smoothing length and $z$ the line-of-sight (LOS) direction along which the density is smoothed. If $x$ is a 3D field, then $x_\ell$ is also a 3D field but unlike $x$, $x_\ell$ also depends on $\ell$. The distribution of $x_\ell$ is tightly related to that of the column density of slab of size $\ell$: $N = x_\ell(n)\ell$.

Let $\sigma_x$ and $\sigma_x(\ell)$ be the standard deviations (SDs) of $x$ and $x_\ell$. To obtain an analytic description for $\sigma_x(\ell)$, we assume that the correlation may be described with a single parameter, $L_{\text{dec}}$, such that when $\ell < L_{\text{dec}}$ the density is correlated, while when $\ell > L_{\text{dec}}$ the density is uncorrelated. The number of independent density cells along a LOS of length $\ell$ is

$$N(\ell) \approx \ell/L_{\text{dec}} + 1,$$

and the $x_\ell$ distribution may be viewed as the sampling distribution of the mean (encountered in the error estimation of repeated measurements; Barlow 1989). The $x_\ell$ SD obeys

$$\frac{\sigma_{x_\ell}}{\sigma_x} = \frac{1}{\sqrt{N(\ell)}} = \frac{1}{\sqrt{1+\ell/L_{\text{dec}}}}.$$  

In the limit $\ell/L_{\text{dec}} \ll 1$, the smoothing length is smaller than a single density fluctuation, $N \approx 1$, and $\sigma_{x_\ell} \rightarrow \sigma_x$. In the other extreme, when $\ell/L_{\text{dec}} \gg 1$, $N \gg 1$, many turbulent fluctuations are smoothed-over within $\ell$, and $\sigma_{x_\ell}/\sigma_x$ vanishes. For more details and examples, see §4 in BBS. See also Squire & Hopkins (2017) for an alternative derivation of the density PDF as a function of scale.

Another way to define $L_{\text{dec}}$ is from the ACF of the density field. The ACF generally decreases with increasing lag, and we may define $L_{\text{dec}}$ as the point at which the ACF falls below some fraction $\varepsilon$ of its (initial) maximum value. This method depends on the somewhat arbitrary choice of $\varepsilon$. Interestingly, as we show below, the suggestion of VG to use $\varepsilon = 0.1$ yields $L_{\text{dec}}$ values that are in very good agreement with those obtained via our smoothed-density method (§4.3).

3. NUMERICAL METHOD

3.1. MHD simulations

We run 3D numerical simulations of isothermal compressible MHD turbulence. The code and setup is similar to that of a number of past works (Kowal et al. 2007, Burkhart et al. 2009, BBS). We refer to these works for the details of the numerical set-up and here provide a short overview. The code is a third-order accurate ENO scheme which solves the ideal MHD equations in a periodic box with purely solenoidally driving (Cho & Lazarian 2003). The magnetic field consists of the uniform background field and a turbulent field, i.e: $B = B_0 + b$ with the magnetic field initialized along a single preferred direction. Previous studies used driving on large scales, with $k_{\text{drive}} = 2 - 2.5$. Here we run simulations with different driving scales: $k_{\text{drive}} = 2.5, 5, 7$. We also ran simulations of $k_{\text{drive}} = 10$ but for this high $k_{\text{drive}}$ the results do not robustly converge and thus we do not discuss this simulation further. The sonic and Alfvenic Mach numbers in all the simulations are $\mathcal{M}_s = 4.5$, and $\mathcal{M}_A = 0.7$.

To test numerical convergence, we run simulations of various resolutions: $N_{\text{res}} = 256^3, 512^3, 1024^3$ resolution elements. As we discuss in §3.2, in our analysis of $L_{\text{dec}}$, we consider 5 time snapshots for each simulations, to evaluate statistical errors. In total, we analyze $3 \times 3 \times 5 = 45$ density fields. For each density field we compute the function $\sigma_{x_\ell}(\ell)$ and $L_{\text{dec}}$ along three LOS orientations, as discussed in §3.2 below.

3.2. Calculating $L_{\text{dec}}$ for the MHD boxes

For each simulation, characterized by $(k_{\text{drive}}, N_{\text{res}})$ we calculate $L_{\text{dec}}$ as follows:

1. calculate $\sigma_x$ for that simulation.

2. calculate $\sigma_{x_\ell}(\ell)$: We choose $\ell$, and integration orientation (hereafter denoted by line-of-sight, LOS). We pick $5 \times 10^4$ random locations (cells) in the simulation and for each location we compute the smoothed density $x_\ell$ using Eq. (2) (we use periodic boundaries). This gives the $x_\ell$ distribution at scale $\ell$. We repeat this for $\ell$ values ranging from 0 to 1 (we adopt units normalized to the box length) and calculate $\sigma_{x_\ell}$ as a function of $\ell$.

3. Fit Eq. (4) to the numerical data, $\sigma_{x_\ell}(\ell)/\sigma_x$, as a function of $\ell$, with $L_{\text{dec}}$ being the best-fitting parameter that minimizes $\chi^2$.

We follow the procedure above for three LOS orientations, 1 parallel and 2 perpendicular to $B_0$. For each simulation and LOS orientation, we repeat the steps above 5 times for 5 time snapshots and adopt the average $L_{\text{dec}}$ as the value of the decorrelation scale. For the error we sum in quadrature the error from the $\chi^2$ fitting (step 3), and the SD $L_{\text{dec}}$ over the 5 time snapshots. In conclusion we obtain $L_{\text{dec}} = \Delta L_{\text{dec}}$ as a function of $k_{\text{drive}}, N_{\text{res}}$, and LOS orientation.

4. RESULTS

In this section we present results for the density structures in the turbulent boxes, and particularly the dependence of the density decorrelation scale, $L_{\text{dec}}$, on driving scale, LOS orientation, and resolution.
4.1. Density Slices

We start with some visual examples of the data. In Fig. 1 we show density slices, parallel to \( B_0 \) (upper panels) and perpendicular to \( B_0 \) (lower panels), for the \( k_{\text{drive}} = 2.5 \) (left), 5 (middle), and 7 (right) simulations. Comparing the panels left-to-right, it is evident that density structures are typically smaller as the driving scale decreases. This is expected as the density fluctuations develop as a result of the driving process. In the upper panels, we see that density structures are relatively isotropic (compared to the lower panels). This is because in these panels \( B_0 \) is directed into the plane and thus there is no preferred direction. Thus the structure is more reminiscent of pure hydro turbulence. On the other hand in the lower panel, where \( B_0 \) is in the plane, the density structures are not isotropic and tend to have their shorter dimension along \( B_0 \). This behavior makes sense physically as the gas may stream more freely in directions along the magnetic field and thus gas compressions are more efficient. As we show in §4.3, our calculated decorrelation scale as a function of \( k_{\text{drive}} \) and LOS orientation captures these trends in a quantitative manner.

4.2. The SDs of the smoothed and non-smoothed density

In Fig. 2 we show an example of the calculated \( \sigma_x \ell \sigma_x \) as a function of the smoothing length \( \ell \), for the \( (k_{\text{drive}}, N_{\text{res}}) = (2.5, 1024^3) \) simulation. The red points correspond to the LOS \( \parallel B_0 \) and the blue points to a LOS \( \perp B_0 \). The black curves are the best \( \chi^2 \) fits to Eq. (4) which yield \( L_{\text{dec}} \) for these LOS orientations. As expected, \( \sigma_x \ell \sigma_x \) decreases as \( \ell \) increases. More of the density structures are averaged-out and \( \sigma_x \ell \) falls faster for the LOS \( \parallel B_0 \) than that of the LOS \( \perp B_0 \) as the density structures are non-isotropic and are typically shorter along the direction of \( B_0 \) (see §4.1 and Fig. 1). The corresponding \( L_{\text{dec}} \) is thus smaller for LOS \( \parallel B_0 \), with \( L_{\text{dec}} = 4.9 \times 10^{-2} \) and \( 8.3 \times 10^{-2} \) for LOS \( B_0 \) and LOS \( B_0 \), respectively. As we show in §4.3, this difference remains also after time averaging and is seen in all simulations from small to large \( k_{\text{drive}} \).

4.3. The decorrelation-scale driving-scale relation

In Fig. 3 we show the ratio \( \phi = L_{\text{dec}} / L_{\text{drive}} \) as a function of \( k_{\text{drive}} \) for various resolutions (different symbols), and LOS ori-
entation: the LOS∥B₀ (left panel), the average over the two LOS⊥B₀ (middle) and an average over all three LOSs (right). As discussed in §3.2 each L_{dec} is also an average over 5 time snapshot and the error bars correspond to the quadrature sum of the fitting process error and the statistical error (over the 5 time snapshots and, for left and right panels, also over the averaged LOS).

If correlations in the density field are imposed by the driving scale, we expect \( \phi \equiv L_{dec}/L_{drive} \) to be constant in respect to \( k_{drive} \). Starting from the left panel of Fig. 3 we see that in the case of the LOS∥B₀, \( \phi \) indeed remains nearly constant (albeit a weak increasing trend). The average \( \phi \) (at the highest resolution, \( N_{res} = 1024^3 \)) is

\[
\langle \phi \rangle_{B_0} = 0.129 \pm 0.011 ,
\]

where the error corresponds to half the SD (over \( k_{drive} \)), which is 0.021. The mean and the SD range are shown by the red horizontal line and strip.

For the LOS⊥B₀ (middle panel) the \( \phi \) values are higher, with

\[
\langle \phi \rangle_{\perp B_0} = 0.282 \pm 0.058 ,
\]

and have a larger SD of 0.12. The larger \( L_{dec,\perp B_0} \), compared to \( L_{dec,\parallel B_0} \), may be seen by-eye in the density slices presented in Fig. 1, and may be explained by the fact that gas compression is limited in the direction perpendicular to the magnetic field (see §4.1). The SD deviation is also much larger in the \( \perp B_0 \) case. Furthermore, \( \phi \) shows an increasing trend with increasing \( k_{drive} \). However, numerical convergence is not optimal for these LOS (as evident by comparing the various resolution markers), and the number of points across \( k_{drive} \) is limited. A \( \phi \) that increases with \( k_{drive} \) may be expected if the large scale \( B_0 \) field induces correlations on large scales proportional to the simulation box length rather than the driving scale.

Finally, in the right panel we show the average \( L_{dec} \) over all LOS (with appropriate weights: 2/3 for the LOS∥B₀ and 1/3 for the LOS⊥B₀). We obtain

\[
\langle \phi \rangle_{all \ LOS} = 0.231 \pm 0.057 ,
\]

We also calculated the decorrelation lengths from the ACF (crosses), by finding the point at which the ACF falls to a fraction \( \epsilon = 0.1 \) of its maximal value. This measure was suggested by VG in their study of column density PDFs (which are tightly related to the \( x\ell \) distribution). Interestingly, the decorrelation length obtained from the ACF agrees well with our method of fitting the decline of the \( \sigma_{\phi}/\sigma_{\phi} \) ratio. However, while the ACF method depends on the arbitrary choice of \( \epsilon \), our method does not require any tuning as it relies on an analytic model that describes the dependence of \( \sigma_{\phi}(\ell)/\sigma_{\phi} \) on \( L_{dec} \) (§2).

5. DISCUSSION

In this paper we have explored the density structures that arise in a supersonic magnetized driven turbulence box simulations. In particular, we focused on quantifying the relation between the decorrelation scale (\( L_{dec} \)) and the turbulence driving scale (\( L_{drive} \)) as well as on the orientation relative to the large scale magnetic field. We find that the \( L_{dec} - L_{drive} \) relation may be approximated by a constant ratio \( \phi \equiv L_{dec}/L_{drive} \approx 0.2 - 0.3 \). When only the LOS parallel to \( B₀ \) is considered, the decorrelation scale is smaller, with \( \phi \approx 0.13 \), and the \( L_{dec} - L_{drive} \) relation is tighter.

5.1 Implications to Observations

In a broader context, the \( L_{dec} - L_{drive} \) relation is a key component in the quest to constrain turbulence properties from observations. This is depicted in Fig. 4. The diagram shows that the turbulence, the density structure, and the chemical structure of interstellar gas are connected:

(A) Turbulence (when supersonic) produces strong density fluctuations in the gas such that the properties of the density field depend on the turbulence properties

(B) The density structure, in turn, controls the abundances of various chemical species since the rates of chemical reactions are sensitive to gas density.

Thus, we may potentially use observations of chemical abundances to constrain the density field and turbulence properties. For this we need to quantify the connections of (A) and
is a self-gravitating medium, and the inclusion of feedback in a non-isothermal medium. Other important generalizations are focused on how \( \Sigma_{\text{B}} \) in Figure 4, and established the link between the decorrelation-scale, \( L_{\text{dec}} \), and the driving-scale, \( L_{\text{drive}} \), via a set of MHD simulations driven on varying scales. More generally, turbulence driving is also described by other parameters, such as the velocity dispersion at the driving scale, and the ratio of solenoidal versus compressional modes, which also affect the density field.

5.2. Limitations and Future Work

In this study we have analyzed 3D MHD driven box simulations with an isothermal equation of state (see §3.1). In the realistic ISM, the density field is affected by active cooling and heating processes, which render the equation of state non-isothermal, and leading to the formation of a multiphase medium composed of cold-dense and warm-diffuse gas (Field et al. 1969; Wolfire et al. 2003; Bialy & Sternberg 2019), although the phase separation vanishes when turbulence is sufficiently strong (Gazol & Kim 2013; Kritsuk et al. 2017).

In a future study, it would be interesting to investigate the structure of the density field (i.e., the \( L_{\text{dec}} - L_{\text{drive}} \) relation) in a non-isothermal medium. Other important generalizations is a self-gravitating medium, and the inclusion of feedback (i.e., supernova feedback), which can drive turbulence and provide gas heating.

6. CONCLUSION

We found that the decorrelation-scale of the density field, \( L_{\text{dec}} \), is related to the turbulence driving scale, following an approximately constant ratio, \( L_{\text{dec}} / L_{\text{drive}} \approx \phi \) where \( \phi = 0.129 \pm 0.011 \) for density fluctuations along the large scale \( B_0 \) field, and with \( \phi = 0.282 \pm 0.058 \) for fluctuations perpendicular to \( B_0 \). On average over all directions, \( \phi = 0.251 \pm 0.057 \). The decorrelation scale calculated with our smoothed density method is in good agreement with that obtained from the autocorrelation function. The \( L_{\text{dec}} - L_{\text{drive}} \) relation is a key step for constraining the turbulence driving scale from observations of column density PDFs. This may shed light on the relative importance of various turbulence stirring mechanisms in the Galaxy.

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Fig. 4.— Schematic diagram demonstrating how turbulence determines the density field which in turns affect the PDF of chemical abundances (solid arrows). Thus observations of column density PDFs may be used to constrain the density field and turbulence properties (dashed arrows). Therefore it is important to quantify the connections between properties of turbulence driving, the density field, and chemical structure.

setup of isothermal, non-gravitational, Fourier-driven simulations, as they constitute a clean numerical experiment that are useful for deriving and understanding the basic form of the \( L_{\text{dec}} - L_{\text{drive}} \) relation.

6. CONCLUSION

We found that the decorrelation-scale of the density field, \( L_{\text{dec}} \), is related to the turbulence driving scale, following an approximately constant ratio, \( L_{\text{dec}} / L_{\text{drive}} \approx \phi \) where \( \phi = 0.129 \pm 0.011 \) for density fluctuations along the large scale \( B_0 \) field, and with \( \phi = 0.282 \pm 0.058 \) for fluctuations perpendicular to \( B_0 \). On average over all directions, \( \phi = 0.251 \pm 0.057 \). The decorrelation scale calculated with our smoothed density method is in good agreement with that obtained from the autocorrelation function. The \( L_{\text{dec}} - L_{\text{drive}} \) relation is a key step for constraining the turbulence driving scale from observations of column density PDFs. This may shed light on the relative importance of various turbulence stirring mechanisms in the Galaxy.

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