Limits on Cosmological Variation of Strong Interaction and Quark Masses from Big Bang Nucleosynthesis, Cosmic, Laboratory and Oklo Data

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Recent data on cosmological variation of the electromagnetic fine structure constant from distant quasar (QSO) absorption spectra have inspired a more general discussion of possible variation of other constants. We discuss variation of strong scale and quark masses. We derive the limits on their relative change from (i) primordial Big-Bang Nucleosynthesis (BBN); (ii) Oklo natural nuclear reactor, (iii) quasar absorption spectra, and (iv) laboratory measurements of hyperfine intervals.

I. INTRODUCTION

Time variation of major constants of physics is an old and fascinating topic, its discussion by many great physicists – Dirac as the most famous example – had surfaced many times in the past. Recent attention to this issue was caused by astronomical data which seem to suggest a variation of electromagnetic \(\alpha\) at the \(10^{-5}\) level for the time scale 10 bn years, see [1]. The issue discussed in this work is related to it, although indirectly. Instead of looking into atomic spectra and testing a stability of electric charge, we will discuss possible variations of nuclear properties induced by a change in strong and weak scales. We will not go into theoretical discussion of why such changes may occur and how they can be related to modification of electromagnetic \(\alpha\). Our aim is to identify the most stringent phenomenological limitations on such a change, at (i) a time of the order of few minutes, when the Big Bang Nucleosynthesis (BBN) took place, as well as (ii) at the time of Oklo natural nuclear reactor (1.8 bn years ago), (iii) when quasar radiation has been absorbed in the most distant gas clouds (3-10 bn years ago) and (iv) at the present time.

Mentioning relevant literature we start with the BBN limits on electromagnetic \(\alpha\), obtained in [2]. The main results come from variation of late-time nuclear reactions. Because of low temperatures and velocities involved at this stage, those reactions have quite significant suppression due to Coulomb barriers, in spite of the fact that only \(Z=1-3\) is involved. These limits are in the following range

\[ |\delta \alpha|_{BBN} / \alpha < 0.02 \] (1)

In general, all the models for time variations of electromagnetic/weak/strong interactions can be divided into two distinct classes, depending on whether it originates in (i) infrared or (ii) ultraviolet. The former approach ascribes variations to some hypothetical interaction of the corresponding gauge bosons with some matter in Universe, such as vacuum expectation values (VEVs) or “condensates” of some scalar fields. Those typically have zero momentum but can have cosmological time dependence. We would not discuss it: for recent example and references see [3].

We would however mention few details from two recent examples of the latter approach, by Calmet and H. Fritzsch \(^*\) and Langacker, Segre and Strassler \(^\ddagger\). Their main assumption is that Grand Unification \(^\ddagger\) of electromagnetic, weak and strong forces holds at any time. Therefore, a relation between all three coupling constants exists: truly modified 2 parameters are in this approach the unification scale \(\Lambda_{GUT}\) and the value of the unified coupling \(\alpha_{GUT}\) at this scale. Their time variation is assumed to propagate down the scales by the usual (unmodified) renormalization group.

If this assumption is correct, any variation of electromagnetic \(\alpha\) should be accompanied by a variation of strong and weak couplings as well. Specific predictions need a model, we will mention the one discussed in \(^\ddagger\). In it the QCD scale \(\Lambda_{QCD}\) (determined as usual by a continuation of the running coupling constant into its – unphysical – Landau pole) is modified as follows

\[ \frac{\delta \Lambda_{QCD}}{\Lambda_{QCD}} \approx 34 \frac{\delta \alpha}{\alpha} \] (2)

Another focus of our work is possible limits on cosmological modifications of quark masses. According to Standard Model, they are related to electroweak symmetry breaking scale, as well as to some Yukawa couplings \(h_i\). In \(^\ddagger\) running of those has been considered, with a (model-dependent) conclusion that quark mass indeed may have a different (and stronger) change

\[ \frac{\delta m_q}{m_q} \approx 70 \frac{\delta \alpha}{\alpha} \] (3)

Large coefficients in these expressions are generic for GUT and other approaches, in which modifications come from high scales: they appear because weak and strong couplings run more.

* One might think that if the GUT scale be used to set units, its variation would be impossible to detect without explicit measurements related to gravity. But it is not so, since the cosmological expansion itself (which is quite important for BBN) contains the Newton’s constant (or the Plank mass) in the Hubble constant.
If a coefficients of such a magnitude are indeed there, at the BBN time of few minutes the QCD scale and quark masses would be modified quite a bit, if the upper limit [9] be used in the r.h.s. The type of questions we are trying to answer in this work are: Do we know whether it might or might not actually happened? Which simultaneous change of strong and weak interaction scales is or is not observable? What observables are the most useful ones, for that purpose? What are the actual limits on their variation which can be determined from BBN and other cosmological and laboratory data?

Let us repeat, that although we use the above mentioned papers as a motivation, we do not rely on any particular model. Nevertheless, we will at the end of the paper return to these predictions in order to see whether our limits on time variation imply stronger or weaker effects than the electromagnetic ones.

II. THE ROLE OF HEAVY AND STRANGE QUARK MASSES IN HADRONIC/NUCLEAR OBSERVABLES

Both papers just mentioned [6,7] argue for what we would call the zeroth approximation to QCD modification. It assumes that the QCD scale \( \Lambda_{QCD} \) is so dominant in all hadronic and nuclear phenomena that all dimensional parameters – hadronic masses, magnetic moments, energies of nuclear levels, etc – are to a good approximation simply proportional to its respective powers.

If so, any time variation of the overall strong interaction scale would not change dimensionless quantities (such as mass ratios or g-factors) which we can observe. The absolute scale of hadronic and nuclear spectroscopy may change, but even if we would be able to observe it from a cosmological distances, such a modification would easily be confused with the overall redshift.

Fortunately, this pessimistic situation [9] is in fact rather far from reality. Quark masses do play significant role in hadronic/ nuclear physics, and, if they have time modification different from that of \( \Lambda_{QCD} \), those can be detected.

Logistically it is convenient to start with masses of heavy – c,b,t – quarks. They do play a role in running of the strong charge, from high scales down, changing beta function each time a corresponding mass scale is passed. However, as it is well known, those effects can be readily absorbed in re-definition of \( \Lambda_{QCD} \).

If strong coupling is the same at some high normalization point \( M \), between \( m_b \) and \( m_t \), the corresponding relations between \( \Lambda_{QCD} \) with all experimental quarks and without heavy c,b,t quarks is as follows:

\[
\Lambda_{without-c,b,t} = \Lambda_{QCD}(\frac{M^2}{m_cm_b})^{2/27}
\]

(4)

Note that rather small powers of the masses are involved in this relation. In effect, we indeed may pretend that c,b,t quarks do not exist at all, as far as basic hadronic/nuclear physics is concerned.

The situation is completely different with the next quark flavor we have to discuss, the strange quark. It is still true, that if one fixes strong coupling \( \alpha_s(k) \) at some sufficiently high scale [4] and then considers the role of non-zero \( m_s \) in perturbative beta function, the effect is negligible. (Since its scale is too low for QCD to be passed by.)

But hadronic/nuclear masses and properties are not determined by perturbative diagrams, leading to beta function: they are of course determined by much more complicated non – perturbative dynamics. Although it is far from being completely understood, it is clear that it does indeed depend strongly on quark masses. In particular, the “strange part” of the vacuum energy density can be estimated, because the derivative of the vacuum energy

\[
\frac{\partial \epsilon_{vac}}{\partial m_s} = <0|\bar{s}s|0> \approx -1.4 \text{fm}^{-3}
\]

is known [7]. Thus the linear term in the strange part of the vacuum energy

\[
\epsilon_s = m_s <\bar{s}s> \approx -0.2 \text{GeV/fm}^3
\]

is not negligible compared to gluonic vacuum energy [3]

\[
\epsilon_g = -\frac{(11/3)N_c - (2/3)N_f}{128\pi^2} <0|gG_{\mu\nu}^a|0> \approx -(0.5 - 1) \text{GeV/fm}^3
\]

making 20-40 percent of it. (The numerator in this expression is the familiar coefficient of the QCD beta function, with \( N_c = 3, N_f = 3 \) being the number of colors and relevant flavors. It appears because this expression, known as the scale anomaly to lowest order, has the same origin as the beta function itself.)

\footnote{Motivated historically by large number of colors limit or quenched lattice QCD. Both may be reasonable starting approximations, which are not expected to be really accurate.}

\footnote{It is in fact done on the lattice, where k is the inverse lattice spacing, typically 2-3 GeV.}

\footnote{For definiteness, this number had come from the QCD sum rules, which typically correspond to operator normalization at \( \mu = 1 \text{GeV} \). The lattice numbers are similar, but normalized at inverse lattice spacing, typically \( \mu = 2 \text{GeV} \). Anomalous dimension of this operator lead to small difference between two normalizations which we ignore here.}
Furthermore, for the nucleon one finds that similar “strange fractions” of their masses are of the same magnitude, e.g. \(\frac{\partial m_N}{\partial m_s} = \langle N|\bar{s}s|N \rangle \approx 1.5\) (9)

Putting into linear expansion the strange quark mass \(m_s = 120 - 140 \text{ MeV}\) one finds that about 1/5 of the nucleon mass comes from the “strange term”.

(Although in this paper we cannot go into discussion of why it is the case, let us make a small digression. First of all, the reader should not be confused with the fact that only a very small fraction of the energy of a fast moving nucleon is due to the strange sea, as experimentally measured partonic densities tell us. Such drastic difference between vector-like (or chiral-even) and scalar-like (or chiral-odd) operators is very common feature of the non-perturbative QCD. Its origin is related with dominance of the instanton-induced t Hooft interaction, see [10]. In short, it happens because topological tunneling events should necessarily involve all light fermion flavors.)

Returning to cosmology, we conclude that due to strange terms, any variation of quark masses would imply significant modification of hadronic masses and other properties. It remains a challenging task for model-builders and lattice practitioners to establish to what extent the \(O(m_s)\) part of hadronic masses is or is not universal.

In principle, what we would call the most pessimistic scenario is possible, in which \(\Lambda_{QCD}\) and \(m_s\) enter into all hadronic observables in one combination

\[ \Lambda_{\text{eff}} = \Lambda_{QCD} + K m_s \]

where \(K\) is some universal constant. If such a scenario happens to be true, its time modification can indeed be neutralized by a change of units, since experiments can only measure dimensionless ratios.

In fact, when lattice practitioners express the obtained results in terms of the so called "physical units" the dependence on \(m_s\) indeed tends to become weaker.

However at the moment this is just a hint, with accuracy not better than say 10 percent, and there is no reason to expect this scenario to be the case. We mentioned such a pessimistic case provocatively, emphasizing that at the moment we lack solid theory which would explain how any particular hadronic observable depends on \(m_s\). In general, variation of \(m_s\) alone can noticeably influence strong interaction parameters, since different quantities in general depends differently on it. Let us give an example.

One of the most important quantity for astronomical and laboratory experiments is the magnetic moments of nuclei. For example, the ratio of hyperfine splitting to molecular rotational intervals is proportional to \(a^2 g\) where \(g\) is defined by the magnetic moment

\[ \mu = \frac{g e \hbar}{2 m_p c} \]

In zeroth approximation as well as in the most pessimistic scenario discussed above, only one dimensional parameter (\(\Lambda_{QCD}\) and \(\Lambda_{QCD} + K m_s\), respectively) exist, so a dimensionless g factor cannot have any time variation.

But, we repeat, there is no general argument for such approximations to be accurate. The magnetic moment and the nucleon mass are not directly related to each other at the QCD level\(^{[1]}\) and the dimensionless derivative

\[ \mu^N_s = \frac{\partial (1/\mu_N)}{\partial m_s} \]

remains unknown even for the proton and neutron, to say nothing about the composite nuclei. If it is not the same as derivative\(^{[2]}\) for the nucleon mass, any time variation of \(m_s/\Lambda\) would induce a variation of the nuclear g factors.

The role of light quark masses is another issue, and a part of magnetic moments related to the contribution of the so called "pion cloud" will be briefly discussed in section\(^{[2]}\).

In summary, any dimensionless ratios should be viewed as a function of the ratio

\[ g(t) = g\left(\frac{m_s(t)}{\Lambda_{QCD}(t)}\right) \]

which for small variations is reduced to partial derivatives such as mentioned above.

### III. THE ROLE OF LIGHT QUARK MASSES

Unlike strange quark mass, we have more solid theory explaining what the effect of a change of light quark

\(^{**}\) The reader may find discussion of phenomenological situation in the first paper from\(^{[1]}\), while the second contains lattice calculations of this quantity. For reference, their conclusion the r.h.s. of\(^{[1]}\) is 1.53 ± 0.07, with the errors being statistical only.

\(^{††}\) Those units are defined by some non-perturbative observable such as (i) \(\rho\) meson mass or (ii) the string tension, or (iii) the force between two point-like charges at fixed distance. All of those are measured on the lattice, with whatever quark masses one wants to have, and then put equal to its observable value in real world by a decree.

\(^{†††}\) For example, in non-relativistic quark model the nucleon mass is approximately 3 times constituent quark mass, and quark magnetic moment is given by “quark magneton”. However even in this model there is also a binding energy and other corrections. A constituent quark itself is a complicated composite object, so one should not expect its magnetic moment to be exactly equal to the Dirac value related to its mass.
masses $m_u, m_d$ relative to that of the strong scale can be. The main focus of this work is this particular variation, which at the end we will be able to constrain rather well.

Since the pion is a Goldstone boson, its mass scales as a geometric mean between weak and strong scales:

$$m_\pi^2 \sim (m_u + m_d)\Lambda_{QCD}$$

Therefore the appropriate parameter characterizing the relative change of the pion mass ratio to the strong scale can be defined as

$$\delta_\pi = \frac{\delta(m_\pi/\Lambda_{QCD})}{(m_\pi/\Lambda_{QCD})}$$

Because the pion mass determines the range of nuclear forces, its modification leads directly to changes in nuclear properties. The main question we would like to study is how such a change in the pion mass relative to that of other hadrons, described by a non-zero $\delta_\pi$, is limited at the cosmological time when primordial nucleosynthesis took place.

The first limit on relative change of quark masses have been put in [12],

$$-0.1 \leq \delta(m_u/m_p) < 0.02$$

where both numerical values come from the observational uncertainty of $He^4$ production, and $T_\nu$ is the freezeout temperature for neutrino-induced weak processes. This effect is however not very restrictive, for the following reasons:

(i) The sensitivity of $He^4$ production to any variation is itself not very impressive. (Below we will discuss $d$ or $Li^7$ yields which may vary by orders of magnitude and is much more sensitive.);

(ii) As correctly explained in [12], $T_\nu$ scales as

$$T_\nu \sim \sqrt[3]{v^4/\Lambda_{QCD}}$$

as a function of weak scale (Higgs VEV) $v$ and Plank mass, while $m_u - m_p$ variation is expected to come primarily from $\delta(m_d - m_u)$, with smaller electromagnetic correction. Therefore, both numerator and denominator in (15) mostly reflect the same physics and thus a large portion of the modifications, if exist, would tend to cancel in this combination.

IV. CRUDE LIMITS ON SCALE VARIATIONS AT THE BBN TIME

Before we come to specifics, let us explain our basic philosophy in selection of the observables. To get the maximal sensitivity, one has to focus on phenomena which may vary by very large factors. Basically, there are two sources for that, inside the dynamics which drives the BBN. One is the Boltzmann factor, which may at late times reach 10 orders of magnitude or more; the other is Gamow factor due to Coulomb barriers, which may reach 3-4 orders of magnitude. So naturally, production of the heaviest primordial nuclides – especially $Li^7$, which is sensitive to both of those – would be most promising place to look.

We start however with a preliminary discussion of possible drastic changes when $\delta_\pi$ is of the order of several percents, and then return to more delicate limits on scale variation.

The nucleosynthesis starts with the two-nucleon states. Standard nuclear forces produce one bound state – the deuteron, $pn$ with $T=0,J=1$ – and a virtual triplet of states – $pp, pn, nn$ with $T=1,S=L=J=0$.

If pion was lighter at the BBN time relative to its present value, it leads to better binding. Rather dramatic effects should have occured if the virtual states unbound by standard nuclear forces were bound.

Specifically, a binding of $(pp)$ state, with its subsequent beta decay into the deuteron, could eliminate free protons, in obvious contradiction with observations [21]. Conditions for a binding of $pn$ and $nn$ states have been studied recently in [22]. These states may add new paths to nucleosynthesis.

Note also, that if the pion was so much heavier than now that the deuteron gets unbound, no primordial nucleosynthesis could possible proceed at all. We will not discuss those in this paper, as they are superseded by the modification of the deuteron discussed below.

As the only exception to this rule, we briefly discuss another important bottleneck on the way toward heavy nuclides, the absence of the $A=5$ bound states. We have shown below that with $\delta_\pi = -0.052$ it can be bridged in a modified world.

V. DEFINING TWO MOST IMPORTANT TEMPERATURES OF THE BBN

As we will argue throughout this work that the most sensitive parameter in BBN remains the binding energy of the deuteron $|E_d|$ ($E_d$ is negative), we provide a (very brief) account of BBN, with emphasis on the role of deuterons. For details of the evolution, the role of all processes involved the reader should consult appropriate reviews, e.g. [32].

When the basic reaction producing deuterons $p + n \rightarrow d + \gamma$ is in equilibrium at high enough $T$, the density of
deuterons relative to photons is of the order of
\[ \frac{n_d}{n_\gamma} \sim \eta^2 \exp(|E_d|/T) \] (18)

Here \( \eta \approx 3 \times 10^{-10} \) is the famous primordial baryon-to-photon ratio. Although it remains unknown what have created it and even at which stage of the Bing Bang it happened, we will assume it happens early and do not consider its variations.

So, the density of deuterium remains negligible small till the Boltzmann factor (needed for the photons to split d) helps. In particular, when this factor is so large than it can compensate one of the \( \eta \), the deuteron fraction may be comparable to that of p,n. This condition determine the first crucial temperature value, to be denoted by \( T_d \)
\[ \exp(|E_d|/T_d) \sim 1 \] (19)

Note that \( T_d \) will be modified below together with the value of \( E_d \). At standard BBN parameters it is about 70 keV, at which \( n_d \) reaches its maximum, see fig.1(a).

What happens after d reaches this maximum, is significant reduction of the density of d (and other species) due to several reactions leading into the best bound light nuclei, \( He^4 \). At such low T the equilibrium configuration would wipe out all of other nuclides. Indeed, an advantage in binding of \( He^4 \) by more than 20 MeV the Boltzmann factor is enormous.

However, the universe expansion does not allow to reach such equilibrium. Its rate leads to the final freeze-out of all production reactions, the stage when reaction rates and the Hubble expansion rates are comparable. The Hubble rate, according to one of the Friedman eqn. for flat Universe and zero cosmological constant (not important so early anyway) is
\[ H^2 \equiv (\ddot{R}/R)^2 = (8\pi/3)G_N\epsilon \] (20)

where \( G_N, \epsilon \) are Newton’s constant and mater energy density. Ignoring numerical constants, \( H \sim T^2/M_P \), with \( M_P \) being the Plank mass, and ignoring the T-dependence of the reaction rate itself \( <\sigma v>_T \), one can obtain the freeze-out temperature
\[ T_f \sim 1/(\eta M_P <\sigma v>_T) \] (21)

Note how small parameter \( \eta \) fight with large \( M_P \). For standard BBN it is \( T_f \approx 35 \text{ keV} \).

VI. HOW VARIATION OF THE DEUTERON BINDING AFFECT THE BBN

Now we can proceed to discussion of how modification of the fundamental interactions would affect the BBN yields.

Let us start with the most pessimistic case when both the strong scale and quark masses are modified identically, so that this change can be eliminated from the discussion by simply using modified units. In such units all reactions rates, bindings and \( T_d \) would be the same as in the standard BBN. The only difference appears in the freeze-out time and temperature \( T_f \). The reason is, as we changed the units to adjust to time varying scales, the value of the Plank mass entering relation gets modified instead.

We can estimate crudely the effect of that modification as follows. At the second stage of the BBN, \( T_d < T < T_f \), a decrease of d and \( Li^7 \) is roughly power-like
\[ f_{d, Li^7} \sim T^{-a} \quad a = 6 - 7 \] (22)

The fall is by about 2 orders of magnitude. Using this trend we conclude that the modification of their yields within a factor 2 (the magnitude of current error bars) corresponds to certain change in \( T_f \), which implies the following limits on variation of the strong scale (relative to gravity)
\[ (\delta(\Lambda_{QCD}/M_P))/(\Lambda_{QCD}/M_P) < 0.1 \] (23)

We now switch to discussion of the relative change, between quark masses and hadronic scale, leading to modification of the pion-induced forces and consequently, of the deuteron binding energy. We will look at modification of \( T_d \), which is so important for BBN final observable yields.

![FIG. 1. (a) Schematic dependence of the deuterium mass fraction \( f_d \) on temperature T (MeV). (b) Schematic dependence of the deuterium and \( He^4 \) mass fractions on the binding energy of the deuteron \( E_d \) (MeV). The vertical dotted line indicate its experimental value, -2.2 MeV.](image-url)
A qualitative dependence of the yields of different species on the magnitude of the deuteron binding is schematically shown in Fig.1(b). As \( T_d \) is allowed to vary, the first thing to note is that there exist an optimum for \( d \) production corresponding to the case when two crucial temperatures are close to each other,

\[
T_f \approx T_d
\]

(24)

It is easy to see from expressions given above, that it happens if the deuteron binding is around \( E_d \approx -1 \text{ MeV} \), and its fraction reaches at this time about a percent level.

When the value of \( T_d \) reduces further, so that the relation between the two temperatures is inverted and \( T_f \) becomes smaller than \( T_d \), the stage at which \( d \) and other light nuclei are “eaten” by \( He^4 \) is no longer present. All heavier nuclei, \( t, He^3, He^4 \) etc. reach their final yields from below, basically tracing the deuteron production. There is no time for \( He^4 \) to grab most of the available neutrons any more, and its final yield start declining (see Fig.1(b)). Finally, when deuterons become nearly unbound, all of the primordial nucleosynthesis is wiped out altogether.

As data tell us that \( d \) fraction is at the level \( 10^{-4} - 10^{-5} \), the BBN happened clearly away from the maximum. Furthermore, as \( He^4 \) fraction is large, we are definitely at the left branch of the curve in Fig.1(b). This shows exponential growth, and thus sensitivity of the results to the variation of \( E_d \) and \( T_d \) can be estimated from its slope, which we obtained from \( \frac{\delta E_d}{E_d} \). We conclude from this estimate that a change of the \( d \) yield by the factor 2 corresponds to a constrain on relative variation of deuteron binding by

\[
\left( \frac{\delta E_d}{E_d} \right)_d < 0.075
\]

(25)

VII. FURTHER LIMITATIONS AND THE COULOMB BARRIERS

Now we can discuss the refinement of qualitative picture described above, in which the absolute change of strong scale has been eliminated by a change of units, and the only weak effect included has been a quark mass. Now we let the electromagnetic effects come into the game, in their simplest and most important (exponential) form. We mean the Coulomb barriers, described be well known Gamow factors

\[
f_{\text{Gamow}} = \frac{2\pi e^2 Z_1 Z_2}{\nu} \exp\left( -\frac{2\pi e^2 Z_1 Z_2}{\nu} \right)
\]

(26)

Here \( \nu \) is the relative velocity, which scales as \( (T/m)^{1/2} \). As explained above, the relevant \( T \) varies in between \( T_d \) and \( T_f \). As the former one is modified, together with \( E_d \), the Gamow factors change accordingly.

For standard BBN we estimated that the product of Gamow factors of reactions leading to \( Li^7 \) is about \( \exp(14 \times v_0/\nu) \), where \( v_0 \) is the unmodified value. So, assuming that experimental data on \( Li^7 \) are restricted within a factor 2, we see that only a variation of

\[
\left( \frac{\delta E_d}{E_d} \right)_{Li^7} < 0.1
\]

(27)

can be tolerated. It leads to a limit on \( \delta \pi \) comparable to the one obtained above from deuteron yield.

VIII. THE MODIFIED DEUTERON

In this section we try to relate the change of deuteron binding \( \delta E_d \) to the modification of the fundamental parameter \( \delta \pi \) introduced above.

In general, it is a very non-trivial dynamical issue, which is far from being really understood. We have discussed above to which extent the QCD vacuum energy \( \delta \pi \) and the nucleon mass \( \delta \) depends on quark masses. May be one day such information will be available from lattice QCD for nuclear forces as well, but right now we do not have it and have to rely on model-dependent potentials.

Of course, one can identify single pion exchange forces. Especially for the deuteron channel, those lead to well known tensor forces producing deuteron quadrupole moment. A textbook 1-pion exchange corresponds to the following potential

\[
V_{1\pi} = \frac{f^2}{m_{\pi}^2} \left( \vec{r}_1 \cdot \vec{r}_2 \right) \left( \vec{\sigma}_1 \cdot \vec{\sigma}_2 \right) \frac{\exp(-m_{\pi}r)}{r}
\]

(28)

where \( f^2 = 0.08 \). The pion mass in denominator has a tilde; it indicates that this mass has been put there by hand for normalization purposes, and, unlike masses in other places, it will not have any variations. (It has been put there in order to cancel the pion mass squared coming from differentiation of the exponent, which we will evaluate first.)

When the variation of the pion mass is sufficiently small, the variation is simply given in the first order of the perturbation theory \( \delta E_d =< 0/\delta V | 0 > \). For any weakly bound state the wave function outside the potential range can be approximated by simple expression

\[
\psi_0(r) = \sqrt{\frac{\kappa}{2\pi}} \exp(-\kappa r)/r
\]

(29)

where parameter \( \kappa \) is related to binding energy by \( E_d = \kappa^2/m_N \). If we use this expression till the core size, we can easily evaluate it

***We have also checked that this estimate agrees with the results by \( \frac{\delta}{\delta \alpha} \) of the dependence of its yield on \( \alpha \), which come from running a compete BBN code.
\[
\frac{\partial E_d}{\partial m_\pi} = -2f^2 \frac{\kappa}{m_\pi} \left( 2 \ln \frac{1}{(2\kappa + m_\pi)r_0} - \frac{m_\pi}{2\kappa + m_\pi} \right) \approx -0.05 \]  

We therefore get from these terms
\[
\frac{\delta |E_d|}{|E_d|} \approx 3\delta_\pi \]  

The sign tells us that reducing the pion mass we reduce binding: this counterintuitive result comes from the \(m_\pi^2\) in pre-exponent.

One more one-pion-exchange term we have ignored is the one in which two derivatives act on \(1/r\): this leads to delta function. Naively it does not contribute since at \(r=0\) the wave function is vanishingly small due to repulsive core. This can be cured by the account for a finite size of the nucleon.

However the main problem with perturbative calculation outlined above is that the 1-pion-exchange terms are actually several times smaller compared to phenomenological potentials needed to reproduce NN scattering and the deuteron binding. Furthermore, the phenomenological potentials which fit scattering and d data actually ascribe most of the potential to 2 and even 3-pion exchanges. As an example, we used well known work by Hamada and Johnston \[14\], which has the central potential in the deuteron channel in the form of subsequent pion exchanges. We have only modified the pion mass in the exponents:
\[
V(r) = V_c Y(x)[1 + a_\pi Y(x) + b_\pi Y(x)^2] \]  

\[
Y(x) = \frac{e^{-x(1+\delta_\pi)}}{x} \]  

where \(x = r \ast m_\pi\) is distance in units of unmodified pion mass, and \(a_\pi = 6, b_\pi = -1\) \[14\]. The potential also has infinite repulsive core with the radius 0.4 fm.

We have ignored tensor and spin-orbit potentials, as having minor effect on the deuteron binding, adjusted the overall coefficient to have the experimental binding energy \(-2.2\) MeV. After that, we have modified the pion mass in \[12\], and calculated binding energy by solving the s-wave Schroedinger eqn. For small pion modifications we need, we found good linear dependence with the following coefficient
\[
\frac{\delta |E_d|}{|E_d|} \approx -18\delta_\pi \]  

We have presented these two estimates as reasonable minimal and maximal bounds on this derivative. Presumably the true value is somewhere in between 3 and -18.

The issue has been discussed in literature. A general discussion of how all nuclear physics would change if quark masses, the number of flavors or even the number of colors be modified, can be found in \[24\]. Probably the latest paper addressing deuteron binding with modern methods is ref. \[23\]. Like us, these authors have shown examples producing opposite sign of the derivative and conclude that, strictly speaking, neither the magnitude nor even the sign can be definitely obtained at this time. Further extensive lattice and chiral perturbation theory studies are needed.

If the derivative is small near zero, our arguments lose its weight. However, it would be very unnatural to get a small value of this derivative. The attractive and repulsive parts of the potential have already conspired to get deuteron at the binding edge. One more fine tuning, exactly at the physical quark mass values (which does not have any special meaning from the QCD point of view) is very unlikely to happen.

**IX. BINDING OF \(He^5\)**

We now return to \(He^4 + n\) channel mentioned above, and evaluate which BBN limits on modification of the pion mass it will produce.

In this channel there is a \(p_{3/2}\) resonance at energy 0.77 MeV. If reduction of the pion mass can make it bound, it will drastically enhance production of \(Li\). Without bound \(He^5\) BBN has to jump over \(A=5\), e.g. by a reaction \(He^4 + t\), which is impaired by the Coulomb barrier as well as by very low concentration of t.

We now study the sensitivity of \(He^5\) binding to modifications of the nuclear potential. Before we discuss the calculations, let us make a couple of qualitative points. First of all, this case is more complicated compared to deuteron, because we now discussed very weakly bound states, and so no simple linear dependence on \(\delta_\pi\) is expected. Furthermore, let us point out the interesting quantum-mechanical distinction between low-lying levels with zero and non-zero angular momentum l. The wave function at large r, outside of the interaction range, for zero energy level can easily be found from Schroedinger equation
\[
\psi(r, E = 0) \sim r^{-(1+l)} \]  

In the s-wave, \(l=0\), it is unacceptable because such wave function is non-normalizable solution. In other words, keeping \(exp(-kr)\) in \[29\] is crucial, no matter how small \(\kappa\) is. It is no longer so for \(l=1\): the tail of the wave function in question is now normalizable. In short, for \(l > 0\) the centrifugal barrier keeps particles inside the attractive potential, contrary to the \(l = 0\) states. This effect makes \(l > 0\) states more sensitive to the change of the potential.

We have modeled the potential of the interaction between a neutron and \(He^4\) in the following form
\[
V(r) = Q \frac{1 + w(r/c)^2}{exp[(r-c)/\Delta] + 1} \]  

where \(w=0.445, \Delta = 0.327\) fm. If \(c=1.01\) fm it is a good fit to experimental density distribution in \(He^4\) \[15\]. In the
potential we simply changed it to \( c = 1.01 fm + 1/\mpi \), with variable pion mass. The depth of the potential \( Q \) has been tuned to reproduce the position of the above mentioned resonance, it gave \( Q = -32.6 \) MeV. After that we start changing the pion mass till this level becomes bound, \( E = 0 \). This happens at the magnitude of the pion mass modification
\[
\delta_{\pi}^{He^5} > -0.052 \tag{37}
\]
If that would happen at the BBN time, the yields of Li would be dramatically enhanced, by orders of magnitude, contrary to observations. (This is why it is written as the inequality.)

This value can be compared to the natural small parameter of the virtual \( He^5 \) level \( E_{\text{res}}/V \sim 0.02 \). The needed shift is somewhat larger because, in spite of general argument given above, its wave function is spread to large \( r \), see fig. 2.

Note that this limit (37) would become an order of magnitude weaker if we base our consideration on the direct effect of the 1-pion exchange eq. (28) - see discussion in the previous section. For \( He^5 \) there is an additional suppression because the average value of the potential (28) over closed 1s shell is zero (the effect, however, appears due to the exchange interaction and correlation corrections).

As a result, we conclude that the limit from the \( He^5 \) binding cannot compete with that from the deuteron modification in its importance in BBN.

The next bottleneck, effectively blocking synthesis of heavy elements, is the gap at \( A=8 \) nuclei, which too readily decay into two alpha-particles. Although we have not investigated this case, we do not expect it to beat the limits related to deuteron modification as well.

**X. LIMITATIONS FROM OKLO DATA**

Finally, let us deviate from our discussion of BBN to a related subject, namely similar limits following from data on natural nuclear reactor in Oklo active about 2 bn years ago. The most sensitive phenomenon (used previously for limits on the variations of the electromagnetic \( \alpha \) is disappearance of certain isotopes (especially \( Sm^{149} \)) possessing a neutron resonance close to zero (K). Today the lowest resonance energy is only \( E_0 = 0.0973 \pm 0.0002 \) eV is larger compared to its width, so the neutron capture cross section \( \sigma \sim 1/E_0^2 \). The data constrain the ratio of this cross section to the non-resonance one. It therefore implies that these data constrain the variation of the following ratio \( \delta(E_0/E_1) \) where \( E_1 \sim 1 \) MeV is a typical single-particle energy scale, which may be viewed as the energy of some 1-body “doorway” state.

A generic expression for the level energy in terms of fundamental parameters of QCD can be written as follows
\[
E_i = A_i \Lambda_{QCD} + B_i m_q + C_i \alpha \Lambda_{QCD} + D_i (m_q \Lambda_{QCD})^{1/2} + ... \tag{38}
\]
where \( A_i, B_i, C_i, D_i \) are some coefficients. The first term is the basic QCD term, while others are corrections due to quark mass, pions and electromagnetism. Without \( B_i, D_i \) terms one can see that in the \( E_0/E_1 \) ratio the QCD scale drops out, confirming our general statement above that in such kind of approximations the variation of \( \Lambda_{QCD} \) itself cannot be seen.

The sum is very small for \( E_0 \), just because we deliberately picked up the lowest resonance, but (and this is our main point) there is no reason to expect each term to be especially small. For example the electromagnetic term is about 1 MeV.

Let us estimate what variation of the resonance energy would result from a modification of the pion mass. As we did above for the deuteron, we assume that the main effect comes from increase of the radius \( R \) of the nuclear potential well. The energy of the resonances \( E_i = E_{\text{excitation}} - S_n \) consists of excitation energy of a compound nucleus, minus the neutron separation energy \( S_n \). This, in turn, is a depth of the potential well \( V \) minus

\[^{\dagger\dagger}\text{Of course, under assumption that the same resonance was the lowest one at the time of Oklo reactor.}\]
the neutron Fermi energy $\epsilon_F$, $S_n = V - \epsilon_F$. The latter scales like $1/R^2$ if the radius of the well is changed. The kinetic part of the excitation energy $E_{\text{excitation}}$ scales in the same way. Adding both, one gets shift of the resonance

$$\delta E_i = - (\epsilon_F + E_{\text{excitation}}) \frac{2\delta R}{R}$$  (39)

For resonance near zero the combination in brackets is approximately $V \sim 50$ MeV. If $R = 5 fm + 1/m_\pi$ then

$$\frac{\delta^2 E_i}{E_i} = 3 \times 10^8 |\delta_\pi| < 0.2$$  (40)

The r.h.s. above comes from the observational limits claimed in [13]. The resulting limitation on pion modification at time $\approx 1.8$ bn years ago is

$$\delta_{\pi} \approx 7 \times 10^{-10}$$  (41)

Note that the authors of the last work in [13] found also the non-zero solution $\delta E_i = -1 \pm 0.1$. This solution corresponds to the same resonance moved below thermal neutron energy. In this case $\delta_\pi \approx -4 \times 10^{-9}$. In principle, the total number of the solutions can be very large since $Sm^{149}$ nucleus has millions of resonances and each of them can provide two new solutions (thermal neutron energy on the right tale or left tale of the resonance). However, these extra solutions are probably excluded by the measurements of the neutron capture cross-sections for other nuclei since no significant changes have been observed there also, see [13].

As in the cases of the deuteron and $He^5$ binding one may argue that the limits on $\delta_\pi$ presented above should be weaker by an order of magnitude since the direct contribution of the 1-pion exchange eq. (28) to the energy is smaller. To clarify this point it may be useful to perform a numerical calculation of this contribution to the neutron separation energy in $Sm^{150}$ and $He^5$.

XI. LIMITS FROM ASTROPHYSICAL AND LABORATORY MEASUREMENTS

Comparison of atomic H 21 cm (hyperfine) transition with molecular rotational transitions gave the following limits on $Y = \alpha^2 g_p$ [4]

$$\delta Y/Y = (-20 \pm 0.4) \times 10^{-5}$$  for redshift $z=0.2467$

$$\delta Y/Y = (-16 \pm 0.54) \times 10^{-5}$$  for $z=0.6847$. The second limit corresponds to roughly $t=6$ bn years ago.

As we have already discussed above, only in the most pessimistic scenario all strong interaction phenomena depends on only one parameter, e.g. $\Lambda_{\text{eff}} = \Lambda_{\text{QCD}} + K\times m_\pi$, its time variation cannot change dimensionless quantities like proton magnetic $g$-factor $g_p$. If so, the limits given above are just limits on variation of $\alpha$. However, in general there is no reason to think this to be the case, and one may wander which limits can be put from these data on a cosmological variation of $m_\pi/\Lambda_{\text{QCD}}$.

Another issue here is a contribution proportional to light quark masses, $m_u, m_d$. Let us first make a qualitative point, suggesting that their role in magnetic moments is expected to be larger than in hadronic masses. Hadrons are surrounded by the so called “pion cloud”, which have small virtual momenta $p$. Masses depends on it in the form $\sqrt{p^2 + m_\pi^2}$ while magnetic moments have $p \times r$, $r$ being

the distance form the center. Small masses are partly compensated by large $r$ in the latter but not former case.

Model-dependent estimates support this idea. The magnitude of the effect varies a lot between models, and as an example we use rather conservative treatment by T.Sato and S.Sawada [23]. It can be seen as a minimal estimate: they identified the contribution of small virtual momenta $p < \Lambda$ by using form-factors $\sim \Lambda^2/(\Lambda^2 + p^2)$, and have shown that a consistent picture for p,n,d and hyperon magnetic moment emerges if the cutoff is $\Lambda \sim m_\pi$. Furthermore, this contribution is shown to be basically proportional to $\Lambda^2 \sim m_\pi^2 \sim m_\pi$. They found that the cloud contribution is about 1 percent of the proton magnetic moment, but 7.3 percent for the neutron (to be compared to the 1 percent level for the masses).

We conclude that at least due to the pion cloud effect one should expect that the g-magnetic ratios $g$ for nuclei have a term proportional to $m_{\text{light}}/\Lambda_{\text{QCD}}$ contributing of the order of several percents. Combining it with the contribution proportional to $m_q/\Lambda_{\text{QCD}}$ (which we hope does not exactly conspire with the effect on masses to cancel completely), we expect overall effect at the level of 1/10 of $g$ is of this origin.

Assuming that it has such magnitude, that a variation of $\alpha$ and $m_q/\Lambda_{\text{QCD}}$ do not conspire to produce observed zero, and a simplest linear dependence

$$g_p = g_p(m_q = 0)(1 + q \frac{m_q}{\Lambda_{\text{QCD}}})$$  (42)

we may interpret the above mentioned limits as the following on variation of this ratio

$$|\delta(\frac{m_q}{\Lambda_{\text{QCD}}})/(\frac{m_q}{\Lambda_{\text{QCD}}})| < 10^{-4}$$  (43)

This should be compared with the limit on $X = \alpha^2 g_p m_e/m_p$ [4] $\delta X/X = (0.7 \pm 1.1) \times 10^{-5}$ for $z=1.8$. This limit was interpreted as a limit on variation of $\alpha$ or $m_e/m_p$. It also can be viewed as a limit on variation of $m_q/\Lambda_{\text{QCD}}$. Although few times weaker ($\sim 2 \times 10^{-4}$) than the limit (43), it corresponds to higher redshift.

The limits on variation of $m_q/\Lambda_{\text{QCD}}$ can also be obtained from laboratory measurements of ratios of hyperfine splittings. By comparison the rates of two clocks based on different atoms, say H and $Hg^+$ [8], we com-
pare \( \delta \)-factors of quite different nuclei \[19\]. In terms of the standard shell model description, this gives the ratio of proton and neutron spin \( \delta \)-factor. Other examples, such as using hyperfine transition in \( Cs \) as a frequency standard, would also involve the orbital \( \delta \)-factor, with \( g_1 = 1 \). In principle, corrections to shell model include “exchange currents” which also contribute to magnetic moments. All of the above may have different dependence on \( m_q/\Lambda_{QCD} \), and we conclude that

\[
\frac{d}{dt} \ln A_1 = K \frac{d}{dt} \ln m_q / \Lambda_{QCD}
\]

where \( A_1, A_2 \) are hyperfine structure constants of different atoms, where \( K \), a combination of derivatives \[12\], remains unknown but can be as big as \( 1/10 \) or even larger. Using such tentative value and \( H, Cs \) and \( Hg^+ \) measurements \[15,21\], we obtain the limit on variation of \( m_q/\Lambda_{QCD} \) about \( 5 \times 10^{-13} \) per year.

**XII. SUMMARY**

Combining our strongest limits on the deuteron binding, from deuteron \[23\] and \( Li^7 \) \[27\], corresponding to variation of their production by a factor 2, with a relation between modification of the deuteron binding and modification of the pion mass \[34\]. Both effects suggest about the same BBN limit on the modification of the pion mass relative to strong interaction scale \( \Lambda_{QCD} \).

Using eq. \[24\] we obtain:

\[
|\delta_\pi|_{BBN} < 0.005
\]

Eq. \[31\] provides a more conservative limit

\[
|\delta_\pi|_{BBN} < 0.03
\]

and we think the true limit is somewhere in between.

We have investigated other effects, such as binding of \( He^5 \) or \( pp, nm, np \) (\( S=0 \)) states, but found that in these cases the needed pion modification about an order of magnitude larger. (It is expected, since all these states are more loosely bound than the deuteron.) If \( m_s \) modification relative to strong scale are as large as our limit on light quark modification just mentioned, it means the nucleon mass can be modified within \( \pm 2 \) MeV due to strange term. Note also, that our limit on quark mass modification is stronger than the limit \[7\] coming from proton-neutron mass difference \[10\].

We also pointed out significantly weaker limits on a simultaneous modification of strong scale and \( m_q \) scale at the same rate, relative to the gravity scale.

\[ \frac{\delta (\Lambda_{QCD}/M_P)}{(\Lambda_{QCD}/M_P)} < 0.1 \]  (47)

Limits on a variation of the quark mass relative to strong scale at the \( 10^{-4} \) level 3-10 bn years ago follows from observations of distant objects \[43\], while at the time 1.8 bn years ago the Oklo data lead to even better limits, at the \( 10^{-8} - 10^{-9} \) level.

Although there is no general relation between variation of weak, strong and electromagnetic constants, as we mention in the Introduction it is implied by Grand Unification \[4\]. If one uses those \[23\], one finds that all our limits on relative weak/strong modification are much more restrictive than the corresponding limits on the modification of the electromagnetic \( \alpha \). In the case of astronomical observations, in which variation of \( \alpha \) seems to be seen, one may either soon find the variation of \( \delta_\pi \) or rule out relations between couplings based on Grand Unification idea.

Finally, let us emphasize that our discussion is semi- qualitative in many aspects, and a lot of quantitative work remains to be done. Theory-wise, the most straightforward thing to do is to add modifications directly into the BBN code, and get more quantitative limits. Although there seem to be no particular problem with the standard BBN at the moment, it is still true that the calculated yields and observations typically differ by 1-2 standard deviations \[6\]. Therefore it seem to be worth while to make a global fit to data with unrestricted modification parameters (like our \( \delta_\pi \)) and see whether zero value would or would not be the best one.

Another challenge to the theory, probably mostly lattice simulations, is to clarify the issue of the dependence of various hadronic parameters on the strange quark mass \( m_s \), especially how universal are the derivatives like \[7\] for all hadrons.

Experimental laboratory work and astronomical observations of distant objects can significantly enhance the limits available today, hopefully with a non-zero effect eventually observed.

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