Search for Dormant Black Holes in Ellipsoidal Variables II. A Binary Modified Minimum Mass Ratio

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ABSTRACT
This is the second of a series of papers that focuses on searching large sets of photometric light curves for evidence of close binaries with a dormant black hole, and, in some cases, a dormant neutron star. The detection of such a binary is based on identifying a star that displays a large ellipsoidal periodic modulation, induced by tidal interaction with its companion. Based on the observed ellipsoidal amplitude and the primary mass and radius, one can derive a minimum mass ratio of the binary. A binary with a minimum mass ratio significantly larger than unity might be a candidate for having a dormant compact-object companion. Unfortunately, the photometric search is hampered by the fact that in many cases the primary mass and radius are not well known. In this paper we present a simple approach that circumvents this problem by suggesting a robust modified minimum mass ratio, assuming the primary fills its Roche lobe. The newly defined modified minimum mass ratio is always smaller than the minimum mass ratio, which is, in its turn, smaller than the actual mass ratio. Therefore, binaries with a modified minimum mass ratio larger than unity are candidates for having a compact-object secondary.

Key words: methods: data analysis – techniques: photometric – binaries: close – stars: black holes – X-rays: binaries

1 INTRODUCTION
This is the second paper of a series (Paper I: Gomel, Faigler & Mazeh 2021) that focuses on searching large sets of stellar photometric light curves for evidence of close binaries with a dormant black-hole (BH), and, in some cases, a dormant neutron-star (NS) secondary. Such systems were not discovered yet as they do not emit X-rays, either because there is no mass transfer between the primary star and the compact object, or because the accretion disc is in a quiescent state. The detection of such a dormant binary is based on identifying a star that displays a large ellipsoidal periodic modulation, induced by tidal interaction with its companion.

The tidal interaction distorts the stellar surface, resulting in an observed modulation with half the orbital period. The amplitude of the ellipsoidal modulation depends, for a circular orbit, on the stellar radius, the semi-major axis and the inclination of the orbit, the binary mass ratio, and, to some extent, on the stellar temperature (e.g., Kopal 1959; Morris 1985; Bochkarev et al. 1979; Morris & Naftilan 1993).

Based on the observed ellipsoidal amplitude and the mass and radius of the more luminous primary star, one can derive a minimum mass ratio (MMR) of the binary, defined as the mass ratio obtained for an inclination of 90°, provided most of the light is coming from the primary star only (e.g., Faigler & Mazeh 2011; Faigler et al. 2015). This is similar to the MMR derived for a single-line spectroscopic binary (e.g., Mazeh & Goldberg 1992; Boffin et al. 1993; Shahaf et al. 2017).

A binary with an MMR significantly larger than unity may be a candidate for having a dormant compact-object companion. This is so because for a binary with two main-sequence (MS) stars we expect the more massive star to overshine its companion. The fact that we observe the less massive star in the system might indicate that the other star is a compact object.

Note, however, that this is not always the case. Algol-type binaries, which probably went through a mass-transfer phase during their evolution (e.g., Erdem & Öztürk 2014; Dervişoglu et al. 2018; Chen et al. 2020), are famous counterexamples. In these binaries, a giant or a sub-giant primary is the less massive component, but nevertheless the brighter star of the system (e.g., Nelson & Eggleton 2001; Budding et al. 2004;
Mennekens & Vanbeveren 2017; Negu & Tessema 2018). Indeed, the recently suggested systems consisting of an evolved primary and a dormant compact-object secondary (e.g., Thompson et al. 2019; Liu et al. 2019; Rivinius et al. 2020; Jayasinghe et al. 2021), could be Algol-type binaries (e.g., van den Heuvel & Tauris 2020; Irrgang et al. 2020; Shenar et al. 2020; Bodensteiner et al. 2020; Mazeh & Faigler 2020; El-Badry & Quataert 2020).

Therefore, a more restricted list of candidates should include only stars that lie on or near the main sequence, with MMRs larger than unity. This applies to spectroscopic and photometric candidates alike.

Unfortunately, the photometric search for dormant BHs is hampered by the fact that in many cases the primary mass and radius are not well known, even in the Gaia (Gaia Collaboration et al. 2016) EDR3 era (Gaia Collaboration et al. 2020), when the stellar parallaxes are better known (e.g., Stassun & Torres 2021). In many cases, the Gaia parallaxes are still not accurate enough (Fabricius et al. 2020), and the constraints on the extinction are also not known to high precision (Green et al. 2018). This is especially true towards the Galactic bulge, where most of the known Galactic binaries that contain BHs reside (e.g., Corral-Santana et al. 2016; Tetarenko et al. 2016). In such cases, the MMR cannot be reliably obtained, especially because the ellipsoidal modulation depends on the stellar radius to the third power.

In this paper we present a simple approach that circumvents this problem by deriving a robust modified minimum mass ratio (mMMR), assuming the primary fills its Roche lobe. This is done with the correction to the Morris & Naftilan (1993, MN93) formula, developed in paper I, where we extended the MN93 formalism to include binaries with a high fillout factor, based on the PHOEBE software package (Prša & Zwitter 2005; Prša et al. 2016; Horvat et al. 2018; Jones et al. 2019).

The newly defined mMMR is always smaller than the MMR, which is, in its turn, smaller than the actual mass ratio. Therefore, binaries with mMMR larger than unity are good candidates for having a compact object secondary, even though we cannot reliably constrain their primary mass and radius.

Interestingly, one can reverse the process and use the observed ellipsoidal amplitude to impose a mass-radius relation on the primary, according to which the radius goes like the cube root of the mass. The coefficient of this relation depends on the fillout factor and binary inclination, which are not known. However, the coefficient values of these relations are quite limited in most cases. We can use the range of allowed mass-radius relations, together with some additional constraints, to obtain some boundaries of the mass and radius of the primary. One can use, for example, an MS mass-radius relation, if we know the primary star is on or not too far from the MS. These boundaries can help us better understand the system under study.

The modified minimum mass ratio can be an efficient tool for searching large sets of stellar light curves for binaries with dormant BHs, and sometimes NSs. Short-period ellipsoidal binaries with dormant compact objects can substantially enlarge the population of known systems with BH and NS companions.

Section 2 presents our modified minimum mass ratio, Section 3 explains the newly derived mass-radius relation for the primary, Section 4 analyses two low-mass x-ray binaries (LMXB) to illustrate the potential of our technique and Section 5 discusses our results.

2 MODIFIED MINIMUM MASS RATIO

Consider a binary system of two stars with masses $M_1$ and $M_2$, for which we observe the light coming from the primary star, $M_1$, only. Note that according to the terminology used here the secondary, $M_2$, is the assumed unseen companion. We are interested in the primary’s relative ellipsoidal modulation at a given optical band, caused by the tidal interaction with the secondary.

An approximation for the first four harmonics of the primary-star ellipsoidal modulation is given by MN93, assuming a tidally-locked primary in a circular orbit, adopting linear limb- and gravity-darkening laws. The leading term of the approximation is

$$\frac{\Delta L}{L} \approx \frac{1}{L/L_0} a^2 \left( \frac{R_1}{a} \right)^3 q \sin^2 i \cos 2\phi \equiv A_2 \cos 2\phi \quad (1)$$

where $q$ is the secondary-to-primary mass ratio $q \equiv M_2 / M_1$, $E(q)$ is the Eggleton (1983) approximation for the volume-averaged Roche-lobe radius in binary semi-major axis units, $i$ is the orbital inclination, the $a_2$ coefficient is a function of the linear limb- and gravity-darkening coefficients of the primary of order unity, $\bar{T}$ is the average luminosity of the star, and $L_0$ being the stellar brightness with no secondary. The angle $\phi$ represents the orbital phase, with $\phi=0$ defined to be at superior conjunction, and $A_2$ is the amplitude of the ellipsoidal at half the orbital period.

In paper I we express the MN93 approximation with the fillout factor $f \equiv R_i/R_{Roche,i}$, where $R_i$ is the primary radius and $R_{Roche,i}$ is the volume-averaged radius of the primary Roche lobe. The leading MN93 term, proportional to $f^3$, can be written, with a correction term derived by paper I, as

$$A_2 \approx \frac{1}{L_0} a_2^3 E^3(q) \frac{q}{\sin i} C(q, f), \quad (2)$$

where the correction coefficient $C(q, f)$ starts at 1 for $f = 0$ (no correction), as expected, and rises monotonically as $f \to 1$, obtaining a value of $\sim 1.5$ at $f \gtrapprox 0.9$.

Equation (2) expresses the ellipsoidal amplitude with three unknowns — the fillout factor, the mass ratio and the inclination. Consider a binary that its ellipsoidal amplitude is determined by the observations and assume for a minute that we can estimate its fillout factor too. The fillout factor can be estimated by using the orbital period, and mass and radius of the primary, as is shown below. In such a case, one can obtain a minimum mass ratio, $q_{\text{min}}$, using Equation (2) for an inclination of 90°.

However, in many cases, the stellar or radius of the primary cannot be reliably estimated, and therefore we do not know the fillout factor. Nevertheless, we can still obtain a stringent lower limit for $q$ — defined as the modified minimum mass ratio, mMMR or $q_{\text{min}}$, by assuming the maximum values for $f$ and $\sin i$, namely that the primary star fills its Roche lobe, $f \sim 1$, and $\sin i = 1$. One can show that the resulting $q_{\text{min}}$ is always smaller than $q_{\text{min}}$, attained when one
assumes only one factor of Equation (2) — \( \sin i \), to have its maximal value.

In Fig. 1 we plot \( \hat{q}_{\text{min}} \) as a function of \( A_2 \), using Equation (2) with \( f \approx 1 \), and three typical \( \alpha_2 \) values for the V-band (Claret & Bloemen 2011). We set \( f \) to be 0.98, in order to be able to use the analytical approximation of paper I. The resulting \( \hat{q}_{\text{min}} \) depends only on \( A_2 \), and gets larger than unity as \( A_2 \gtrsim 0.1 \), independent from the orbital period and mass and radius of the primary.

To display the effect of the maximal value we attributed for \( f \), the figure also shows the MMR derived from Equation (2) for \( f = 0.85 \), with the same \( \alpha_2 \) values. The diagram assumes \( \sin i = 1 \) for all functions. One can notice that for, say, \( A_3 = 0.08 \), \( \hat{q}_{\text{min}} \approx 0.4 \) while \( \hat{q}_{\text{min}}(f = 0.85) \approx 5 \). This emphasizes the fact that the nMMR can substantially underestimate the mass ratio, even if the primary is close to filling its Roche lobe.

The monotonic relation between \( \hat{q}_{\text{min}} \) and \( A_2 \) imposed by Equation (2) suggests that a binary with ellipsoidal amplitude \( A_2 \gtrsim 0.1 \) probably contains an unseen companion more massive than the primary. Furthermore, Fig. 1 suggests that such systems are expected to have a primary fillout factor not far from unity — say, \( f \gtrsim 0.85 \).

### 3 MASS-RADIUS RELATION FOR THE PRIMARY

To obtain another constraint on the mass ratio that does use the binary period and the mass and radius of the primary one can use the Eggleton (1983) approximation for the volume-averaged Roche-lobe radius and Kepler’s third law

\[
\left( \frac{a}{R_\odot} \right)^3 = 4.23 \left( \frac{M_1}{M_\odot} \right) \left( \frac{P}{\text{1 day}} \right)^2 (1 + q),
\]

where \( M_\odot \) and \( R_\odot \) are the solar mass and radius, \( a \) is the semi-major axis and \( P \) is the orbital period, to obtain another relation between \( q \) and \( f \):

\[
f (1 + q)^{1/3} E(q) = 0.238 \left( \frac{R_1}{R_\odot} \right) \left( \frac{M_1}{M_\odot} \right)^{-1/3} \left( \frac{P}{\text{1 day}} \right)^{-2/3}.
\]

For a given set of \( A_2 \), \( P \), \( \alpha_2 \), \( M_i \), and \( R_i \), Equations (2) and (4) can be solved to obtain the MMR, \( q_{\text{min}} \), and the corresponding \( f \) of the system, assuming \( \sin i = 1 \).

In fact, Equation (4) enables us to impose an interesting constraint on the mass-radius relation of the primary star:

\[
\frac{R_i}{R_\odot} = 4.2 \left[ f (1 + q)^{1/3} E(q) \right]^{2/3} \left( \frac{M_i}{M_\odot} \right)^{1/3}.
\]

For a given ellipsoidal amplitude, which allows us to obtain \( q \) for assumed values of \( f \) and \( i \), one can derive \( g'(f,i) = f (1 + q)^{1/3} E(q) \). Interestingly, the permitted \( g' \) range is quite limited, depending on \( A_2 \). This is demonstrated in Fig. 2, where we present two examples of numerically derived 2-D maps of \( g' \) for two values of \( A_2 = 0.05 \) and 0.1.

The figure shows that \( g' \) attains its maximum when \( f \) and \( \sin i \) are at unity, with \( q \sim 0.1 \) (left panel) or \( \sim 0.9 \) (right panel). Its minimum is obtained at \( f \sim 0.7 \) (left panel) or \( \sim 0.85 \) (right panel), but still at \( \sin i = 1 \) and \( q \sim 20 \). In any given binary, we can use the maximum and minimum of \( g' \), which depend only on \( A_2 \), to impose two limiting constraints on the mass-radius relation of the primary. These are used in Fig. 3 to plot the resulting limiting mass-radius relations for two different values of \( P \) and \( A_2 \). In the same plots we present mass-radius relations for zero-age main-sequence stars, \( R = R_{\text{ZAMS}} \), and for \( R = 2R_{\text{ZAMS}} \). We expect the primary star of the four virtual binaries to be within the region bounded by the four graphs.
A combined analysis of the spectroscopic and photometric data of the soft X-ray transient A0620-00 was performed by Cantrell et al. 2010, taking into account the non-trivial BH-disk contribution to the light curve. They assumed that the primary star fills its Roche lobe. The V-band ellipsoidal component presented a semi-amplitude of about $A_2 \simeq 0.1$ mag, as illustrated in Cantrell et al. (2010, Fig. 5).

Had the system been a short-period binary with a dormant BH component, we probably would be able to identify it as a BH candidate based on the large amplitude of the ellipsoidal modulation, with $q_{\text{min}} \sim 1$.

Using $P$ and $A_2$ and Equations (2) & (4), we derive two mass-radius relations for this system, shown in Fig. 4. The actual location of the primary is presented on the mass-radius diagram using the values of Cantrell et al. (2010), within the permitted region of our analysis. The diagram suggests that the star is very close to filling its Roche lobe, and its radius is about 1.5 larger than the corresponding ZAMS value, consistent with the results of Cantrell et al. (2010).

4.1 A0620-00

A0620-00 is a low-mass X-ray binary consisting of a K-star primary orbiting around a compact companion, presumably a BH, with an orbital period of $P = 0.323$ day (Johannsen et al. 2009). A combined analysis of the spectroscopic and photometric data of the soft X-ray transient A0620-00 was performed by Cantrell et al. (2010), taking into account the non-trivial BH-disk contribution to the light curve. They assumed that the primary star fills its Roche lobe and derived an orbital inclination of $i = 59 \pm 0.9$, a stellar mass of $M_1 = 0.40 \pm 0.045 M_\odot$, and an estimated BH mass of $M_2 = 6.61 \pm 0.25 M_\odot$. According to their analysis, the V-band ellipsoidal component presented a semi-amplitude of about $A_2 \simeq 0.07$ mag, as illustrated in Wu et al. (2016, Fig. 6).

Had the system been a short-period binary with a dormant BH component, we probably would identify it as a

4.2 GRS 1124-683

Another example is GRS 1124-683, consisting of a K-star primary orbiting a compact companion, presumably a BH, with an orbital period of $P = 0.433$ day (Wu et al. 2015). A similar analysis of Wu et al. (2016) determined an orbital inclination of $i = 43.2 \pm 2.1$, a stellar mass of $M_1 = 0.89^{+0.08}_{-0.11} M_\odot$ and a BH mass of $M_2 = 11.0^{+1.4}_{-1.1} M_\odot$, assuming that the primary star fills its Roche lobe. The I-band ellipsoidal component presented a semi-amplitude of about $A_2 \simeq 0.07$ mag, as illustrated in Wu et al. (2016, Fig. 6).

Had the system been a short-period binary with a dormant BH component, we probably would identify it as a
Figure 3. Mass-radius relations of ellipsoidal variables for different sets of \((A_2, P)\). Blue and red lines correspond to the highest and lowest slopes of the mass-radius relations respectively, obtained for mass ratio and fillout factor indicated in the legend, and inclination of 90°. Both relations are plotted with \(\alpha \approx 1.4\). Dashed (dotted) line presents \(R = R_{\text{ZAMS}} \) \((R = 2R_{\text{ZAMS}})\) of Eker et al. (2018), smoothed by a fifth-degree polynomial for convenience. We expect the primary star of these virtual systems to reside within the shaded region in most cases.

Figure 4. Mass-radius relations of A0620-00, derived for \(P = 0.323\) d, \(A_2 = 0.1\) mag, and a typical \(\alpha\) value of 1.4 for the V band. The origin of each plotted line is described in the caption of Fig. 3. The black circle marks A0620-00 position using the values of Cantrell et al. (2010) and adopting a stellar-radius uncertainty of 0.05 \(R_\odot\).

Figure 5. Mass-radius relations for GRS 1124-683, derived by using \(P = 0.433\) d, \(A_2 = 0.07\) mag and \(\alpha = 1.2\), typical for the I band. The origin of each plotted line is described in the caption of Fig. 3. The black circle marks GRS 1124-683 position by adopting the values of Wu et al. (2016).

Our mass-radius relations are presented in Fig. 5, with the system position according to the values of Wu et al. (2016), which is close to the line of fillout factor of unity, and close to the MS border of our permitted region.
5 Discussion

As shown above, one can derive the mMMR of a system, based on the ellipsoidal amplitude alone, assuming an inclination of 90° and a fillout factor close to unity. The mMMR is substantially smaller than the actual mass ratio in most cases, and therefore an mMMR larger than unity can be used to identify ellipsoidal variables that might have a compact secondary.

Note that this approach cannot find all binaries with compact objects. Systems with dormant BHs (of NSs) for which the primary did not evolve yet to fill its Roche lobe do not display large enough amplitudes to yield mMMR larger than unity. In those cases, only additional information on the mass and radius of the primary can lead to a derivation of the MMR, which is always larger than the mMMR and is closer to the actual mass ratio.

On the other hand, many BH and NS binaries with primaries that fill their Roche lobes are dormant, as they happen to be in an X-ray quiescent phase, in between outbursts (e.g., Cackett et al. 2005; Kneivit et al. 2014). For these systems, the mMMR could be significantly larger than unity. Ellipsoidal variables of this kind, favored by our searching approach, are more likely to be detected.

The photometric search faces another problem — the assumption that the inclination is close to 90°. Even systems with a fillout factor close to unity can yield small mMMR, if their inclination is low, as in the two cases discussed above, with actual mass ratios of ~10. This would cause many systems to be missed by a search based on the ellipsoidal amplitude alone.

This problem is inherent to spectroscopic and photometric searches alike. Fortunately, the random orientations of systems tend to favor inclinations of 90°, as the expected value for a sample of binaries is $\sin i = \pi/4 = 0.78$. In a practical search, one might consider lowering the threshold of $\tilde{q}_{\text{min}}$ that identifies BH candidates, allowing for binaries with $\sin i < 1$ and $f < 1$. This is also true for the MMR, if the mass and radius of the primary are known. To determine the best threshold for a given project one can apply the False Discovery Rate (FDR) approach of Benjamini & Hochberg (1995) that controls the purity of the resulting sample of discoveries.

Our method relies on identifying large sets of stellar light curves as ellipsoidal variables. For that we can use ellipsoidal-variables catalogs that are available for a few photometric surveys, such as CATALINA (Drake et al. 2014), ASAS-SN (Pojmanski 2002; Pigulski et al. 2009), OGLE (Soszyński et al. 2016; Pawlak et al. 2016), and Kepler (Kirk et al. 2016). For a detailed review of photometric-variables catalogs see the introduction of Drake et al. (2014). In the future, the space mission Gaia, which follows the photometry of more than a billion stars (Gaia Collaboration et al. 2016), is expected to publish a large catalog of variable stars, as done for DR2 (Gaia Collaboration et al. 2019). This will probably be the most extensive catalog of ellipsoidal variables.

For future photometric surveys, if a variable-star catalog is not available, one can identify the ellipsoidal variables by using, for example, classification methods similar to the ones described in Gaia Collaboration et al. (2019). Such methods can use a combination of the stellar position in the color-magnitude diagram, if available, together with Fourier-harmonics amplitude ratios (Soszyński et al. 2009), to distinguish between ellipsoids and other variables.

Obviously, any catalog may include misclassified ellipsoidal variables, especially contact binaries and stellar rotational variables with stable periodicity. Those might result in false-positive compact-object candidates. Our false-positive ratio depends on the contamination of the ellipsoidal catalogs.

In the immediate next stage of this project, we are applying our technique to the large samples of short-period ellipsoidal variables (Soszyński et al. 2016; Pawlak et al. 2016) identified by the OGLE project (Udalski et al. 2015).

The photometric search we envision is focused on short-period binaries. The ellipsoidal amplitude falls as the period squared, so the search is sensitive up to at most ~5-day binaries for MS stars. Note that for late-type MS stars the search is sensitive to NS companions as well, as the mass ratio can be around 3 for a K-star of ~0.5$M_\odot$ and an NS with a mass of ~1.4$M_\odot$ (see a review by Enoto et al. (2019), and Haniewicz et al. (2021), for a detailed discussion of one specific system). A sample of BH/NS candidates should be followed by radial-velocity observations, to confirm they have massive companions. The availability of follow-up resources might, among others, determine the FDR threshold of a project.

Ellipsoidal searches can substantially enlarge the number of known short-period binaries with compact objects. This is especially true because almost all known systems were discovered by their X-ray emission. We do not know yet of any short-period system with a compact object for which the optical star does not fill its Roche lobe. The proposed project can reveal the systems for which the optical star is on its way to fill its Roche lobe, and therefore was not yet discovered. With the newly discovered systems, we will be able to better study the statistical characteristics of close systems with BHs or NSs companions.

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Data Availability

No new data were generated or analysed in support of this research.

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