Modelling the behavior of the positron plasma temperature in antihydrogen experimentation

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Abstract Antihydrogen is now routinely produced at CERN by overlapping clouds of positrons and antiprotons. The mechanisms responsible for antihydrogen formation (radiative capture and the three-body reaction) are both dependent on the temperature of the positrons ($T_e$), though with a different weight. Here we present a simple model of the behavior of the positron temperature based on the main processes involved during antihydrogen synthesis, namely: antiproton–positron collisions, positron heating due to plasma expansion and cooling via the emission of synchrotron radiation. The time evolution of $T_e$ has been simulated by changing the relevant parameters of the mechanisms involved in order to highlight the importance of the different (competing) effects.

Keywords Antiproton · Positron · Antihydrogen · Antimatter · Temperature

1 Introduction

The main motivation of the studies of antihydrogen ($\bar{H}$), the bound state of an antiproton ($\bar{p}$) and a positron ($e^+$), is the possibility to test CPT symmetry through comparison between $\bar{H}$ properties, a pure antimatter system, and those of its counterpart of matter, the hydrogen atom. Since up to now there is no experimental evidence of any violation of CPT, the study of antimatter can also shed light on the properties of the constituents of matter and on the
features of the physical processes between them, even though antimatter study is usually more difficult than that of ordinary matter. For example, the measurement of the $\bar{p}$ mass achieved from high precision spectroscopy of the antiprotonic system ($\bar{p}$He$^+$) [1] can be used to increase the precision of the knowledge of the mass of the proton [2].

From the first detections of cold $\bar{H}$ in 2002 [3, 4], several achievements have been made even if many questions remain with no clear answer. One topic to be clarified is the role of the different mechanisms in $\bar{H}$ production when $e^+$ and $\bar{p}$ clouds are merged together in dedicated experiments.

The expected mechanisms are spontaneous radiative recombination:

$$\bar{p} + e^+ \rightarrow \bar{H} + h\nu,$$

and a three-body recombination (see e.g. [5] for a review):

$$\bar{p} + e^+ + e^+ \rightarrow \bar{H} + e^+.$$

The outcomes of both mechanisms depend on the density ($n_e$) and the temperature ($T_e$) of the $e^+$ plasma and produce markedly different $\bar{H}$ binding energies. For spontaneous radiative recombination $\bar{H}$ is produced in low-lying quantum states and the rate varies as $n_e \times T_e^{-0.63}$. In the case of three-body recombination the favored $\bar{H}$ states are weakly bound and the production rate depends strongly on the $e^+$ plasma parameters, since in equilibrium it is proportional to $n_e^2 \times T_e^{-4.5}$. The observation of reactions 1 and 2 is complicated by the multi-step nature of antihydrogen formation [6] and by the difference between produced and detected $\bar{H}$s [7]. The experimental results seem to indicate that three-body recombination plays a role in $\bar{H}$ production in some cases [4, 8–14], while in other instances [15–19] this is less clear.

Due to the influence of the $e^+$ plasma temperature in $\bar{H}$ production, we have developed a simple model to determine the behavior of $T_e$ during antihydrogen synthesis.

2 The model

The physical processes considered in the model to be responsible for the modification of the $e^+$ temperature are: cooling due to the radiation emitted by the cyclotron motion of the $e^+$s (synchrotron radiation); heating due to the energy lost by the $\bar{p}$s injected into the $e^+$ plasma; heating due to the decrease of the electrostatic energy of the $e^+$ plasma following its expansion.

Actually, at high magnetic fields and low plasma temperatures it is necessary to consider two different temperatures: one for the component of the motion along the magnetic field ($T_{e\parallel}$) and one for the perpendicular direction ($T_{e\perp}$). This happens when the positron-positron collision rate is reduced so much that the energy transfer from the motion along the magnetic field ($B$) to those in the perpendicular direction, where it is dissipated via synchrotron radiation, is ineffective.

It is possible to speak about only one temperature ($T_e$) when $T_{e\parallel} = T_{e\perp}$ and this occurs when the condition $\lambda_{\text{coll}} \tau_e \gg 1$ is satisfied, where $\lambda_{\text{coll}}$ is the positron–positron collision frequency and $\tau_e$ is the synchrotron radiation time. To check the above condition we use the following formulae [20]:

$$\lambda_{\text{coll}} = n_e \bar{v} \bar{b}^2 I(\bar{k}), \quad \tau_e = 4\pi \varepsilon_0 m_e^3 e^2 / e^4 B^2.$$

Here $\bar{v} = \sqrt{2k_B T_{e\parallel} / m_e}$ is the mean velocity parallel to the magnetic field ($k_B$ is Boltzmann’s constant, $m_e$ is the positron mass), $\bar{b} = 2e^2 / 4\pi \varepsilon_0 k_B T_{e\parallel}$ is twice the distance
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of closest approach ($e$ is the positron charge, $\varepsilon_0$ is the vacuum permittivity with $c$ the speed of light). The quantity $I(\bar{k})$ is a function of $\bar{k} = \frac{eB}{m_e v}$ and in the region where $\bar{k} \gg 1$ (i.e. in the magnetised plasma regime) the following analytical expression is valid [21]:

$$I(\bar{k}) \approx \left( \frac{1.83}{k^{7/15}} + \frac{20.9}{k^{11/15}} + \frac{0.347}{k^{13/15}} + \frac{87.8}{k^{15/15}} + \frac{6.68}{k^{17/15}} \right) \times \exp \left( -\frac{5(3\pi \bar{k})^{2/5}}{6} \right).$$

In Fig. 1 the quantity $\lambda_{coll} \tau_e$ is plotted for the small $T_{e\parallel}$ where the parameterization of (4) can be used (i.e., when $\bar{k} \gg 1$). The condition $\lambda_{coll} \tau_e \gg 1$ (equivalent to considering only one temperature) is satisfied with the exception of very low temperatures $T_{e\parallel}$ and densities with relatively high magnetic fields. In the present paper we will only discuss the case where a single value for the $e^\pm$ temperature can be used.

In a standard antihydrogen experiment using a Penning trap, a cloud of $N_i$ antiprotons is injected into a plasma of $N_e$ positrons. Due to Coulomb collisions the two particles species are thermalized and their rate equations can be written as:

$$\frac{dT_i}{dt} = - \frac{L}{\tau_i} (T_i - T_e),$$

$$\frac{dT_e}{dt} = K \frac{L}{\tau_i} \frac{N_i}{N_e} (T_i - T_e).$$

Here $T_i$ and $T_e$ represent the temperatures of antiprotons (with mass $m_i$) and positrons, respectively. The rate $\tau_i$ is the Spitzer relaxation time given by:

$$\tau_i = \frac{3m_em_i c^3}{8\sqrt{2\pi n_e \ln(\Lambda)}} \left( \frac{4\pi \varepsilon_0}{e^2} \right)^2 \left( \frac{k_BT_i}{m_ic^2} + \frac{k_BT_e}{m_ec^2} \right)^\frac{1}{2}.$$
where the so-called Coulomb logarithm is:

$$
\ln(\Lambda) = \ln\left(4\pi \left(\frac{\varepsilon_0 k_B}{e^2}\right)^{\frac{3}{2}} \sqrt{\frac{T_e}{n_e}} \left(T_e + \frac{m_e T_i}{m_i} + 2 \sqrt{\frac{m_e}{m_i} T_e T_i}\right)\right).
$$

In (5) and (6) the factors $L$ and $K$ (with $0 < L \leq 1$ and $0 < K \leq 1$) are introduced to consider the partial overlap between antiprotons and positrons along the axial and radial directions, respectively. In particular the parameter $K$ is introduced to describe the effect occurring when the antiproton cloud has a larger radius than the positron plasma [22]. The antiprotons which do not radially overlap the positron plasma are assumed, in this model, to not interact with those that do.

In the above equations only the effect of $\bar{p}$ energy loss has been considered. In order to take into account the heating due to the $e^+$ plasma expansion and the cooling due to the synchrotron radiation, (6) has to be modified. The former effect is a consequence of the torque exerted on the $e^+$ plasma by collisions with the residual gas or by the lack of perfect cylindrical symmetry of the electric and magnetic fields of the trap [23, 24]. When the $e^+$ plasma expands, the electrostatic energy of its charge decreases and is converted into internal energy thereby heating the plasma. By modeling the plasma as an infinite cylinder with an increasing radius $r_e$, the term $\frac{e^2}{6\varepsilon_0 k_B} (n_e r_e^2) \frac{1}{0} \frac{dr_e}{dt}$ must be added to the right side of (6). The subscript 0 indicates that the quantity $n_e r_e^2$ is assumed time-independent and $\frac{dr_e}{dt}$ is the plasma expansion rate ($v_e$).

When we consider also the effect of the synchrotron radiation which depends on the parameter $T_{res}$ (that is the ambient temperature surrounding the $e^+$ plasma) the new rate equation can be written as:

$$
\frac{dT_e}{dt} = K \frac{L}{\tau_i} \frac{N_i}{N_e} (T_i - T_e) + \frac{e^2}{6\varepsilon_0 k_B} (n_e r_e^2) \frac{1}{0} \frac{dr_e}{dt} - \frac{(T_e - T_{res})}{\tau_e}.
$$

The $e^+$ plasma and $\bar{p}$ temperatures, $T_e$ and $T_i$, can be determined by numerically solving the system of differential equations (5) and (9). The solutions will depend also on the initial temperature of the positrons ($T_{0e}$), the initial kinetic energy of antiprotons ($E_0$) and the initial $e^+$ plasma radius ($r_0$) which accounts for the effective value at the antiproton injection time.

3 Simulations

Since a general solution of (5) and (9) cannot be determined due to the large number of parameters involved, particular cases have to be considered individually. Here we give some examples by varying the parameters that influence the behavior of $T_e$. Since (5) and (9) are linear in respect of $T_{res}$ and of $T_e$, we will consider the quantity $\Delta T = T_e - T_{res}$ whose behavior is the same as $T_e$. We want to point out that some of the simulated conditions may be far from real experimental situations since the aim of these evaluations is to highlight the relative importance of the mechanisms responsible for the time evolution of $T_e$. In the following we consider two broad examples, namely, (i) when the number of positrons is much greater than the number of antiprotons and (ii) when the relative numbers are much closer.
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Fig. 2 The values of $T_e$ (dashed lines) and $T_i$ (full lines) versus time over 50 s for the examples for which $N_e \gg N_i$ at different positron densities ($0.7 \times 10^{14}$ m$^{-3}$ in black, $7 \times 10^{14}$ m$^{-3}$ in red and $70 \times 10^{14}$ m$^{-3}$ in green). The upper plates are for $v_e = 0$. The lower plates are for $v_e = 0.2, 2, 20 \mu$m s$^{-1}$ (from left to right).

3.1 EXAMPLE 1: $N_e \gg N_i$

In Fig. 2 the simulated $\Delta T$ behaviors are plotted versus time for different values of the initial $e^+$ plasma density ($0.7 \times 10^{14}$, $7 \times 10^{14}$ and $70 \times 10^{14}$ m$^{-3}$) and expansion velocity ($0.2, 2$ and $20 \mu$m s$^{-1}$). The common parameters used in these simulations are: $N_e = 7.5 \times 10^7$, $N_i = 10^4$, $E_0 = 16$ eV, $r_0 = 1$ mm, $L = 0.3$ and $K = 1$. The initial value of $\Delta T$ is set equal zero (i.e. $T_e^0 = T_{res}$).

In the upper panel the ideal case with $v_e = 0$ is shown: on the left the plot is in logarithmic scale and includes also the $T_i - T_{res}$ behavior. It is clear that $T_i$ and $T_e$ equalise rapidly (0.1-10 ms) in a time whose value depends linearly on the positron density as a consequence of the different rate of the energy released by the antiprotons. The maximum increase is around 25 K and in less than 1 s the $T_e$ value reaches the ambient temperature $T_{res}$, as clearly shown in the plot (in linear scale) on the right side.

On the lower panel the $v_e > 0$ cases are plotted. The main differences in respect to the $v_e = 0$ case are: a) $T_e$ values have larger maxima; b) $T_e$ takes more time to reach equilibrium; c) the new equilibrium values are higher than $T_{res}$. These effects are stronger for the high positron densities and $v_e$ values.

For $v_e = 0.2 \mu$m s$^{-1}$ the maximum increase of $T_e$ (25 K) is the same as for $v_e = 0$ while the $T_e$ values at equilibrium are some kelvin over $T_{res}$. It must be noted that the effects on $\bar{H}$ production are negligible as far as the $\Delta T$ values are low in respect to $T_{res}$.

For $v_e = 2 \mu$m s$^{-1}$ and $v_e = 20 \mu$m s$^{-1}$ the maximum $\Delta T$ ranges from some tens to hundreds of kelvin. In addition the equilibrium values are achieved very late for the extreme $e^+$ plasma density ($70 \times 10^{14}$ m$^{-3}$) with $\Delta T$ remaining above several tens of kelvin for
most of the time of the mixing cycle. Clearly in these conditions if $T_{\text{res}}$ is around several tens of kelvin, $\bar{H}$ formation is expected to be deeply influenced, with a strong reduction of the $\bar{H}$ yields.

3.2 EXAMPLE 2: $N_e \geq N_i$

Here we consider the case in which the numbers of antiprotons and positrons are much closer. For the $T_e$ simulations we use the following parameters: $N_i = 3 \times 10^5$, $E_0 = 13$ eV, $r_0 = 0.5$ mm, $L = 0.3$ and $K = 1$ and $B = 2.7$ T. The considered values of the number of positrons and their initial density ($n_0^e$) are: $N_e = 3 \times 10^6$ and $n_0^e = 1 \times 10^{13}$ m$^{-3}$; red for $N_e = 3 \times 10^6$ and $n_0^e = 1 \times 10^{13}$ m$^{-3}$; green for $N_e = 30 \times 10^6$ and $n_0^e = 10 \times 10^{13}$ m$^{-3}$. The upper plates are for $v_e = 0$, while the lower plates are for $v_e = 1, 10, 100 \mu$m s$^{-1}$ (from left to right).

Fig. 3 The temporal behaviors of $T_e$ (dashed lines) and $T_i$ (full lines) over 50 s for EXAMPLE 2 (with $N_e \geq N_i$) for different positron numbers ($N_e$) and initial densities ($n_0^e$); black for $N_e = 0.3 \times 10^6$ and $n_0^e = 0.1 \times 10^{13}$ m$^{-3}$; red for $N_e = 3 \times 10^6$ and $n_0^e = 1 \times 10^{13}$ m$^{-3}$; green for $N_e = 30 \times 10^6$ and $n_0^e = 10 \times 10^{13}$ m$^{-3}$). The upper plates are for $v_e = 0$, while the lower plates are for $v_e = 1, 10, 100 \mu$m s$^{-1}$.
the behavior at late times, especially for the lowest selected number of positrons \(N_e = 3 \times 10^5\) where, for \(v_e = 100 \mu m s^{-1}\), \(\Delta T\) is around 500 K even 30 s after the start of mixing.

In addition it is worth pointing out that the naive expectation that with more positrons the heating is lower is not always correct: at the equilibrium the \(T_e\) values tend to be higher for larger numbers of positrons. This is a consequence of the \(e^+\) plasma expansion which converts the electrostatic energy into thermal energy, and this is increased when \((n_e e^2)_{0}\) is large (see (9)).

4 Conclusions

We have developed a simple model to assess the behavior of the positron plasma temperature in antihydrogen experiments. It is based on the heating from antiproton-positron collisions, from the positron plasma expansion and on the cooling from the positron synchrotron radiation emission. It is the first time that these mechanisms have been included together in a model.

The solutions depend upon several parameters and not all of them are usually known from experiment. The results indicate when the heating of the positron plasma could be relevant in reducing the antihydrogen production rates. The proposed model can also be used to help the interpretation of the experimental data not only about antihydrogen production \([25]\) but also for the recent analysis on protonium formation \([26, 27]\).

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