Gauss-Bonnet holographic superconductors with magnetic field

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Abstract – We study the Gauss-Bonnet (GB) holographic superconductors in the presence of an external magnetic field. We describe the phenomena away from the probe limit. We derive the critical magnetic field of the GB holographic superconductors with backreaction. Our analytical approach matches the numerical calculations. We calculate the backreaction corrections up to first order of \(\kappa^2\) \(O(\kappa^2 = 8\pi G)\) at the critical temperature \(T_C\) and the critical magnetic field \(B_C\) for a GB superconductor. We show that the GB coupling \(\alpha\) makes the condensation weaker but the backreaction corrections \(O(\kappa^2)\) make the critical magnetic field stronger.

Introduction. – The anti de Sitter/conformal field theory (AdS/CFT) correspondence [1] provides a powerful theoretical method to investigate the strongly coupled field theories. It may have useful applications in condensed matter physics, especially for studying scale-invariant strongly coupled systems, for example, low-temperature systems near quantum criticality (see for example [2] and references therein). Recently, it has been proposed that the AdS/CFT correspondence also can be used to describe superconductor phase transition [3]. Since the high-\(T_C\) superconductors are shown to be in the strong-coupling regime, one expects that the holographic method could give some insights into the pairing mechanism in the high-\(T_C\) superconductors. Inspired by the idea of spontaneous symmetry breaking in the presence of horizon [4] various holographic superconductors have been studied in Einstein theory [5] or extended versions as the Gauss-Bonnet (GB) theory [6,7], the Horava-Lifshitz theory [8,9] and the Weyl corrected theories [10]. AdS/CFT can also describe superfluid states in which the condensing operator is a vector and hence rotational symmetry is broken, that is, \(p\)-wave superfluid states [11]. All these works are based on a numerical analysis of the equations of motion (EOM) near the horizon and the asymptotic limit by a suitable shooting method. But as we know that the analytical methods are better and easy to invoke in different problems, recently some attempts have been done on analytical methods in superconductors (see for example [12] and references therein). In [12] the authors have shown that one can obtain the critical exponent and the critical temperature by applying a variational method to the EOM. Their method and terminology is simple and very sound. Instead of involving in numerical problems, we can obtain the critical temperature \(T_C\) and the exponent of the criticality very easily by computing a simple variational approach. They studied different modes of supercriticality s-wave, p-wave and even d-wave. Their good and efficient method can be applied to other condensers with higher-order Lagrangian black hole. Recently, we applied this method to superconductors in the presence of the magnetic field [13], and our results are in good agreement with numerical results produced previously [14]. Another semi-analytical method is based on the matching method, in which we match the asymptotic solutions at a midpoint. After matching, we can easily obtain the expectation values of the dual operators \((O_\pm)\) and the critical temperature \(T_C\). Indeed, this method has been used by several authors [6,9,15]. In these topics the effect of the external magnetic field is very important. A holographic model of superconductor with external magnetic field has been previously studied numerically [14]. Here as we can observe, the higher-order terms make the condensation harder. We can study the backreaction effects in the presence of magnetic field. Our main goal in this paper is to investigate the effect of the backreaction on the magnitude of the critical magnetic field \(B_C\). We calculate the backreaction corrections up to first order of \(\kappa^2 = 8\pi G\) at the

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critical temperature $T_C$ and the critical magnetic field $B_C$ for a GB superconductor (to see more about $B_C$ in superconductors refer to [16]). We show that the GB coupling $\alpha$ makes the condensation weaker but the backreaction corrections make the critical field stronger.

**Gauss-Bonnet holographic superconductors away from the probe limit.** – We write the action for a Maxwell field and a charged complex scalar field coupled to the Einstein-Gauss-Bonnet (EGB) as

$$S = \frac{1}{2\kappa^2} \int \sqrt{-g} d^5x \times \left[ R + 12 + \frac{\alpha}{2} (R_{\mu
u\rho\sigma}R^{\mu
u\rho\sigma} - 4R_{\mu
u}R_{\mu\nu} + R^2) \right] + \int \sqrt{-g} d^5x \left[ -\frac{1}{4} F_{\mu\nu}F_{\mu\nu} - |D_\mu \psi|^2 - m^2 |\psi|^2 \right],$$

(1)

where in it the GB coupling is $\alpha$, the AdS radius $l = 1$, the field strength tensor is defined through $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and $D_\mu = \partial_\mu - iq A_\mu$, where the charge of the scalar field is $q$. The mass of the scalar field is chosen such that it remains below to the Breitenlohner-Freedman (BF) bound [17]. For the 5-dimension case, the GB coupling $\alpha$ is bounded within the range $-\frac{7}{36} < \alpha < 0.09$ (see for example [18]). The hairy black hole (BH) solution in the EGB and with plane symmetry is bounded within the range $\alpha < 0.09$. The hairy black hole (BH) solution in the EGB and with plane symmetry is bounded within the range $\alpha < 0.09$.

We choose the gauge $A_\mu = (\phi(r), 0, B_C x, 0)$, where $B_C$ is the critical magnetic field in direction $z$. In the absence of the magnetic field, the superconductor phase has been described previously [6,7]. If we ignore the backreaction, and setting $\psi = 0$ (normal phase), the gravity sector decoupled from the matter part and the EGB field equations give a charged black hole in AdS background

$$f_0(r) = \frac{r^2}{2\alpha} \left[ 1 - \sqrt{1 - 4\alpha \left( 1 - \frac{b^2}{r^2} \right) + \frac{8\kappa^2 \alpha^2}{3h^2 r^4} \left( 1 - \frac{b^2}{r^2} \right)^2} \right],$$

$$\chi_0(r) = 0,$$

$$\phi_0(r) = \mu - \frac{\rho}{r^2}.$$  

(3)

We choose the minus sign of the solutions so that we have a solution in the Einstein limit of GB theory $\alpha \to 0$. The horizon locates at $r = h$ and the BH temperature reads as

$$T_{BH} = \frac{1}{4\pi h}.$$  

The effective asymptotic AdS scale is given by

$$l_{ef} = \sqrt{\frac{2\alpha}{1 - \sqrt{1 - 4\alpha}}}.$$  

(4)

Figure 1 shows the behavior of the $l_{ef}$ for $-\frac{7}{36} < \alpha < 0.09$.

In next section we will use from the zeroth order solutions given in (3) in a perturbation method based on the work of [15]. The field equations can be written as the following forms:

$$\nabla_\mu F^{\mu\nu} = iq(\psi^\dagger D^\nu \psi - \psi D^\nu \psi^*),$$

$$D_\mu D^\mu \psi - m^2 \psi = 0,$$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} - 6g_{\mu\nu} + 4\alpha [R_{\mu\sigma\kappa\tau} R^{\sigma\kappa\tau} - 2R_{\mu\rho\sigma\tau} R^{\rho\sigma\tau} + RR_{\mu\nu} - \frac{1}{2} g_{\mu\nu} (R_{\rho\sigma\kappa\tau} R^{\rho\sigma\kappa\tau}) - 4R_{\mu\nu} R_{\rho\sigma} + R^2] = \kappa^2 T_{\mu\nu},$$

(7)

here the $T_{\mu\nu}$ denotes the total energy-momentum tensor of matter fields.

In the limit of the zero magnetic field $B_C = 0$, the field equations are given by

$$f' = \frac{2r^2 - f}{r^2 - 4\alpha f} - \frac{\kappa^2 r^3 \chi}{f},$$

$$\frac{2\varphi^2 \psi^2 + f(2m^2 \psi^2 e^{-\chi} + \bar{\phi}^2) + f^2 e^{-\chi} \psi'^2}{r^2 - 4\alpha f},$$

$$\chi' e^{-\chi} = -2\kappa^2 r^3 (\varphi^2 \psi^2 + e^{-\chi} f^2 \psi'^2),$$

$$\varphi'' = -\left( \frac{3}{r} + \frac{\chi'}{2} \right) \phi' + \frac{2\varphi^2 \psi^2 \phi}{f},$$

$$\psi'' = -\left( \frac{3}{r} + \frac{f'}{f} + \frac{\chi'}{2} \right) \psi' - \left( \varphi^2 \psi^2 e^{-\chi} - \frac{m^2}{f} \right) \psi.$$  

(11)
This system has been studied completely using numerical schemas [7]. It has been shown that the critical temperature in the absence of any magnetic field is \( T_C \approx \rho^{1/3} \). For \( T < T_C \) these solutions will be unstable to form scalar hair, i.e. develop a non-vanishing value of \( \psi \) on the horizon. In the gauge-theory–gravity duality, \( T_C \) is the temperature below which superconductivity appears.

**Calculating the corrections to \( B_C \).** — The critical temperature with backreaction has been obtained analytically in [15] and we briefly summarize the main results first. In order to solve the field equations (8)–(11), firstly we define a new coordinate \( z = \frac{r}{r_+} \). This map converts the interval \((h, \infty)\) of the coordinate \( r \) to the inside of the strip \((0, 1)\) of the new coordinate \( z \). It is useful in numerical analysis of the field equations. We examine the near critical point behavior of the system. It is convenient to introduce a small parameter for our perturbation analysis

\[
\epsilon \equiv \langle O_{\Delta_+} \rangle. \tag{12}
\]

Near the critical point, the value of the scalar field \( \psi \) is small. Thus we can expand the fields, metric functions and the chemical potential \( \mu \) as the following series:

\[
\phi = \sum_0^\infty \epsilon^{2n} \phi_{2n}, \tag{13}
\]

\[
\psi = \sum_0^\infty \epsilon^{2n+1} \psi_{2n+1}, \tag{14}
\]

\[
f = \sum_0^\infty \epsilon^{2n} f_{2n}, \tag{15}
\]

\[
\nu = \sum_0^\infty \epsilon^{2n} \nu_{2n}, \tag{16}
\]

\[
\mu = \mu_0 + \epsilon^2 \delta \mu_2, \tag{17}
\]

Here \( \epsilon \ll 1 \). From (17), we obtain the familiar exponent \( \frac{1}{2} \) for phase transition. The critical value of the chemical potential is \( \mu_C = \mu_0 \). Including the backreaction effects on the critical temperature when \( \kappa^2 \ll 1 \) has been discussed in [15]. The corrected critical temperature in first order of \( \kappa^2 \) and with the matching point \( z_m = \frac{1}{2} \), mass \( m^2 = -3 \) and the conformal dimension \( \Delta_+ = 3 \) is

\[
T_C = T_0(1 - 2\kappa^2 \delta T), \tag{18}
\]

where in it

\[
T_0 = \frac{\sqrt{f_0} \sqrt{2/\pi \pi}}{2} (0.778 + 1.5\alpha)^{-1/6}, \tag{19}
\]

\[
\delta T = 1.641 + 2.667\alpha. \tag{20}
\]

Figure 2 shows the behavior of the \( T_C/T_0 \) as a function of the GB coupling \( \alpha \) for different values of \( \kappa^2 \). As we can observe that, when we fix the backreaction coupling, then in fixed coupling \( \kappa^2 \), increasing the GB coupling \( \alpha \) decreases the rate of \( T_C/T_0 \). It means that the condensate becomes harder when we increase the GB coupling in a fixed backreaction’s correction.

Now, we consider an external magnetic field, so \( \psi_1 \) satisfy following differential equation:

\[
\psi_1'' + \left( \frac{f_0}{f_0} \frac{1}{z} \right) \psi_1' + \frac{h^2}{z^2} \left( \frac{\phi_0^2 e^{\chi_0} - m^2}{f_0} \right) \psi_1 = \frac{\left( f_0 \rho^3 \right) (\partial_x^2 + (\partial_y - i B_C x)^2)}{m^2} \psi_1. \tag{21}
\]

By separation of variables we have

\[
\psi_1 = e^{i k_x y} X_n(x) Z_n(z), \tag{22}
\]

where \( X(x) \) satisfies following equation for a two-dimensional harmonic oscillator:

\[
-\left( \partial_x^2 - (k_y - B_C x)^2 \right) X_n(x) = u_n B_C X_n(x). \tag{23}
\]

Simply one can find \( X(x) \) in terms of the Hermite function \( H_n \) as

\[
X(x) = e^{-a(x-x_0)^2} H_n(x), \tag{24}
\]

where \( u_n = 2n + 1, a = (B_C / 2) x_0 = C \). By considering the lowest mode \( n = 0 \), one can obtain the following equation for \( Z_0(z) \):

\[
Z_0'' + \left( \frac{f_0}{f_0} \frac{1}{z} \right) Z_0' + \frac{h^2}{z^2} \left( \frac{\phi_0^2 e^{\chi_0} - m^2}{f_0} \right) Z_0 = \frac{B_C t^2}{z^2 f_0} Z_0. \tag{25}
\]

Due to the regularity at the horizon we have

\[
Z_0' = \frac{m^2 h^2 + B_C t^2}{f_0 (1)} Z_0(1). \tag{26}
\]

In details, we must remove the singularity from the term \( \frac{\phi_0^2 e^{\chi_0}}{f_0} \) near the horizon, located at \( z = \frac{1}{2} \). From (3), it is obvious that when \( \frac{z}{\rho} = 1 \), then \( f_0 \rightarrow 0, \phi_0 \rightarrow 0 \). Using Hopital’s rule for removing the singularity, from the term \( \lim_{z \rightarrow 0} \left( \frac{h^2}{z^2} \left( \frac{\phi_0^2 e^{\chi_0}}{f_0} - m^2 \right) / \frac{B_C t^2}{z^2 f_0} \right) \) we obtain:

\[
\frac{\phi_0^2 e^{\chi_0}}{f_0} \rightarrow 0. \tag{27}
\]

Using this result, we can remove the singularity from the term near the horizon, located at \( z = \frac{1}{2} \). From (26), it is obvious that when \( \frac{z}{\rho} = 1 \), then \( f_0 \rightarrow 0, \phi_0 \rightarrow 0 \). Using Hopital’s rule for removing the singularity, from the term \( \lim_{z \rightarrow 0} \left( \frac{h^2}{z^2} \left( \frac{\phi_0^2 e^{\chi_0}}{f_0} - m^2 \right) / \frac{B_C t^2}{z^2 f_0} \right) \), by simple
algebra we obtain the non-singular part $\frac{m^2h^2 + Bc e^{l/2}}{f_0(1)} Z_0(1)$. Collecting it with the same limit process we obtain (26). At the AdS boundary $z = 0$ we can write

$$Z_0 = D_+ z^{\Delta_+},$$  

(27)

where $\Delta_+ = 2 + \sqrt{4 + m^2 2e / f_0'}$. Now we would like to obtain the solution of $Z_0$ using the matching method [15] (see also [19]). For this purpose we can expand $Z_0$ in a Taylor series near the horizon as

$$Z_0 = \sum_{n=0}^{\infty} Z_0^{(n)}(1)(1-z)^n.$$  

(28)

From (25), we obtain $Z_0''(1)$ as

$$Z_0''(1) = -\frac{1}{2} \left(3 + \frac{f_0''(1)}{f_0(1)} - \frac{m^2 h^2 + Bc e^{l/2}}{f_0(1)}\right) Z_0(1)$$

$$- \frac{h^2 \phi_0(1)^2}{2 f_0'^2(1)} Z_0(1).$$  

(29)

So an approximate solution near the horizon is given by

$$Z_0(z) = Z_0(1) - \frac{m^2 h^2 + Bc e^{l/2}}{f_0(1)} Z_0(1)(1-z)$$

$$- \left[\frac{m^2 h^2 + Bc e^{l/2}}{f_0(1)} - \frac{m^2 h^2 + Bc e^{l/2}}{f_0(1)}\right] Z_0(1)(1-z)^2 + O((1-z)^3).$$  

(30)

Now we connect the solutions eqs. (27) and (30) at the matching point $z_m = \frac{T}{T_c}$ smoothly and by defining a new parameter $\xi = m^2 h^2 + Bc e^{l/2}$ we have

$$D_+ = Z_0(1) - \frac{\xi}{2 f_0'(1)} Z_0(1)$$

$$- \left[\frac{\xi}{4 f_0'(1)} + \frac{f_0''(1)}{f_0(1)} - \frac{\xi}{f_0'(1)}\right] Z_0(1)/4,$$  

(31)

$$\Delta_+ \left(\frac{1}{2}\right)^{\Delta_+ - 1} D_+ = \frac{\xi}{f_0'(1)} Z_0(1)$$

$$+ \left[\frac{\xi}{4 f_0'(1)} + \frac{f_0''(1)}{f_0(1)} - \frac{\xi}{f_0'(1)}\right] Z_0(1)/2.$$  

(32)

From these equations, we obtain the relation between $D_+$, and $Z_0(1)$

$$D_+ = \frac{2 \Delta_+}{1 + \Delta_+} \left(1 - \frac{\xi}{4 f_0'(1)}\right) Z_0(1).$$  

(33)

Substituting $D_+$ from above equation into eq. (30), we find following relation in case of $Z_0(1) \neq 0$:

$$\frac{2 \Delta_+}{1 + \Delta_+} = \left(\frac{\Delta_+}{1 + \Delta_+} + \frac{7}{4}\right) \frac{\xi}{f_0'(1)} - \frac{\xi}{4 f_0'(1)} Z_0(1)$$

$$+ \frac{1}{4} \frac{f_0''(1)}{f_0'(1)^2} \frac{h^2 \phi_0(1)^2}{4 f_0'(1)^2} = 0.$$  

(34)

By substituting the values of $f_0''(1)$, $f_0''(1)$ and $\phi_0(1)$ into (35), we obtain an equation for $\mu_0$

$$\kappa^4 \frac{4 \pi^2 T^2}{9 \hbar^4} \left[\frac{2 \Delta_+}{1 + \Delta_+} - \frac{\xi \Delta_+}{2 \hbar^2}\right] \mu_0^2$$

$$- \frac{4 \pi^2 T^2}{16 \hbar^4} \left[\frac{1}{1 + \Delta_+} + \frac{\xi \Delta_+}{2 \hbar^2}\right] \mu_0^2$$

$$+ \frac{2 \xi}{\hbar^2} \frac{8 \xi}{\hbar^2} \frac{\Delta_+}{2} \left(\frac{4 + \xi \Delta_+}{4 h^2}\right) \mu_0^2$$

$$+ \frac{3 \xi^2 \Delta_+}{8} \frac{1}{\hbar^2} + \frac{\xi \Delta_+}{8 \hbar^2} \alpha = 0.$$  

(36)

In the above equation we assume that $\kappa^4 \ll 1$, then by considering the relation $\mu_0 = \frac{T}{T_c}$, (which is obtained from $\phi_0(1) = 0$ from eq. (2)), we can write following expression for $B_C$:

$$B_C = B_1 + \kappa^2 \delta B.$$  

(37)

The resulting critical magnetic field is the upper critical magnetic field, not the lower one:

$$B_1 = \frac{\pi^2 T^2}{(2 + \Delta_+)} \left[16(3 - 4\alpha)^2$$

$$+ 16(7 - 32\alpha + 16\alpha^2) \Delta_+ + 4(16\alpha^2 - 40\alpha + 9) \Delta_+$$

$$+ (2 + \Delta_+)(96\alpha \Delta_+ + 192\alpha + 145\Delta_+ - 126) T_c^6\right]^{1/2}$$

$$\times \left[3 + 2\Delta_+ - 2(2 + \Delta_+) - 2 \left[18 + 19\Delta_+ + 6\Delta_+^2$$

$$+ 8\alpha^2(2 + \Delta_+) - 6\alpha(8 + 10\Delta_+ + 3\Delta_+^2)\right]\left[16(3 - 4\alpha)^2$$

$$+ 16(7 - 32\alpha + 16\alpha^2) \Delta_+ + 4(16\alpha^2 - 40\alpha + 9) \Delta_+$$

$$+ (2 + \Delta_+)(96\alpha \Delta_+ + 192\alpha + 145\Delta_+ - 126) T_c^6\right]^{1/2}\right].$$  

(39)

In figs. 3, 4, we plot the variation of the log $B_C$ as a function of the temperature $T/T_c$ for different values of the backreaction $\kappa$, GB coupling $\alpha$. Figure 3 shows the variation of log $B_C$ for $\alpha = 0.01$. As we observe that when we fix the GB coupling at $\alpha = 0.01$, varying the backreaction effects term $\kappa^2$, the critical magnetic field, with backreaction corrections becomes larger. Thus the backreaction terms $\kappa^2$ in fixed GB coupling $\alpha$ makes the condensation harder. It is observed from fig. 4, in which we fixed the backreaction effects term $\kappa = 0.01$ and varied with respect to the GB coupling $\alpha = -0.19, 0, 0.07$, that the effect of changes in $\alpha$ makes the critical magnetic
are interested in the Gauss-Bonnet corrections [6,7]. In the present paper we have investigated the implication of the Gauss-Bonnet correction to the holographic superconductor in the presence of an external magnetic field, and away from the probe limit. Here we have done our calculation analytically, where the results match with the numerical calculations. We have obtained the critical magnetic filed $\log B_C$ up to order $k^2$. The resulting critical magnetic field is the upper critical magnetic field, not the lower one. Our results show that the GB coupling $\alpha$ makes the critical magnetic field weaker, because it is always smaller than the critical magnetic field up to $\alpha = 0$. But the backreaction corrections make the critical magnetic field stronger.

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Additional remark: When we were busy with the calculations for this paper, a paper [19] appeared in the arXiv where a similar problem has been discussed.

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Fig. 3: The plot of the critical magnetic field $\log B_C$ as a function of the temperature $T$ for different values of the backreaction $\kappa$ for GB coupling $\alpha = 0.01$.

Fig. 4: The plot of the Critical magnetic field $\log B_C$ as a function of the temperature $T$ for different values of the GB coupling $\alpha$ for the value of $\kappa = 0.01$.

field weaker, because it is always smaller than the critical magnetic field up to $\alpha = 0$. As one can see $\delta B$ is positive for possible values of the Hawking temperature below the critical temperature and the Gauss-Bonnet coupling. Due to this, the backreaction makes the critical magnetic field stronger. Previously such investigations in the case $\kappa^2 = 0$ have been done by Ge et al. in [20]. Our result in this special case matches with the result of [20]. One can see the behavior of backreaction term $\delta B$ with respect to temperature, it shows that higher temperature $T$ leads to a lower $\delta B$.

Conclusions. – There are many interesting features for critical phenomena and superconductivity when we are working on higher-orders corrections, especially when we

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