A supersymmetric model based on a left-right symmetric gauge group is proposed where hybrid inflation, baryogenesis and neutrino oscillations are linked. This scheme, supplemented by a familiar ansatz for the neutrino Dirac masses and mixing of the two heaviest families and with the MSW resolution of the solar neutrino puzzle, implies that $1 \text{ eV} < m_{\nu_{\tau}} < 9 \text{ eV}$. The mixing angle $\theta_{\mu\tau}$ is predicted to lie in a narrow range which will be partially tested by the Chorus/Nomad experiment.

After the end of inflation, the system falls towards the supersymmetric vacua, oscillates about them and finally decays ‘reheating’ the universe. The ‘inflaton’ (oscillating system) consists of two complex scalar fields, $S$ and $\theta = (\delta \phi + \delta \bar{\phi})/\sqrt{2}$, with $\delta \phi = \phi - M$, $\delta \bar{\phi} = \bar{\phi} - M$ and mass $m_{\text{infl}} = \sqrt{2} \kappa M S$. $S$ can decay first through the superpotential coupling $S h^{(1)} h^{(2)}$, where $h^{(1)}$, $h^{(2)}$ are the higgs superfields coupled to up and down type quarks respectively. So we concentrate on the decay of $\theta$. The relevant coupling is given $^3$ by the non-renormalizable effective superpotential term $\delta W = (M_{\nu_{e}}/2 M^2) \delta \phi \nu_{e} \bar{\nu}_{e}$, where $M_{\nu_{e}}$ is the Majorana mass of the relevant right handed neutrino $\nu_{e}$. The field $\theta$ decays predominantly to the heaviest $\nu_{c}$ with $M_{\nu_{c}} \leq m_{\text{infl}}/2$. The ‘reheating’
temperature, assuming the MSSM spectrum, is found from the decay width of $\theta$, $\Gamma_\theta \approx (1/16\pi)(\sqrt{2}m_{\nu^c}/M)^2 m_{infl}$, to be $T_R \approx M_{\nu^c}/9.23$. We will assume that $T_R$ is restricted by the gravitino constraint, $T_R \lesssim 10^9$ GeV. Note that $T_R$ is closely linked to the mass of the heaviest $\nu^c$ satisfying $M_{\nu^c} \leq m_{infl}/2$.

In order to obtain information about the light neutrino masses and mixing of the two heaviest families, we ignore the first family assuming it has small mixings. The relevant ‘asymptotic’ (at $M_{GUT}$) $2 \times 2$ mass matrices are $M^L$, the mass matrix of charged leptons ($L^c$, $L$), $M^D$, the Dirac mass matrix of neutrinos ($\nu^c$, $\nu$), and $M^R$, the Majorana mass matrix of $\nu^c$'s. We shall first diagonalize $M^L$, $M^D$:

$$M^L \rightarrow M^L' = \tilde{U}^L M^L U^L = \begin{pmatrix} m_{\mu} & m_{\tau} \\ m_{\tau} & m_{\mu} \end{pmatrix},$$

$$M^D \rightarrow M^D' = \tilde{U}^\nu M^D U^\nu = \begin{pmatrix} m^D_2 & m^D_3 \\ m^D_3 & m^D_2 \end{pmatrix},$$

where the diagonal entries are positive. This gives rise to the ‘Dirac’ mixing matrix $U^\nu U^L$ in the leptonic charged currents. Using the remaining phase freedom, we can bring this matrix to the form

$$U^\nu U^L \rightarrow \begin{pmatrix} \cos \theta^D & \sin \theta^D \\ -\sin \theta^D & \cos \theta^D \end{pmatrix},$$

where $\theta^D (0 \leq \theta^D \leq \pi/2)$ is the ‘Dirac’ (not the physical) mixing angle in the 2-3 leptonic sector. In this basis, the Majorana mass matrix can be written as $M^R = U^{-1} M_0 U^{-1}$, where $M_0 = \text{diag}(M_2, M_3)$, with $M_2, M_3$ (both positive) being the two Majorana masses, and $U$ is a unitary matrix which can be parametrized as

$$U = \begin{pmatrix} \cos \theta & \sin \theta e^{-i\delta} \\ -\sin \theta e^{i\delta} & \cos \theta \end{pmatrix} \begin{pmatrix} e^{i\alpha_2} & \\ e^{i\alpha_3} \end{pmatrix},$$

with $0 \leq \theta \leq \pi/2$ and $0 \leq \delta < \pi$. The light neutrino mass matrix is

$$m = -\tilde{M}^D' \frac{1}{M^R} M^D = \begin{pmatrix} e^{i\alpha_2} \\ e^{i\alpha_3} \end{pmatrix} \Psi(\theta, \delta) \begin{pmatrix} e^{i\alpha_2} & \\ e^{i\alpha_3} \end{pmatrix},$$

where $\Psi(\theta, \delta)$ depends also on $M_2$, $M_3$, $m^D_2$, $m^D_3$.

We will denote the two positive eigenvalues of the light neutrino mass matrix by $m_2$ (or $m_{\nu_\mu}$), $m_3$ (or $m_{\nu_\tau}$). Recall that all the quantities here
masses, mixings) are ‘asymptotic’. The determinant and the trace invariance of $m^\dagger m$ provide us with two constraints on the (asymptotic) parameters:

$$m_2m_3 = \frac{(m_2 D 3 m_3 D 3)^2}{M_2 M_3}, \quad (6)$$

$$m_2^2 + m_3^2 = \frac{(m_2 D 2 c^2 + m_3 D 2 s^2)^2}{M_2^2} + \frac{(m_3 D 2 c^2 + m_2 D 2 s^2)^2}{M_3^2} + \frac{2(m_3 D 2 m_2 D 2)^2 c^2 s^2 \cos 2\delta}{M_2 M_3}, \quad (7)$$

where $\theta, \delta$ are defined in Eq. 4, $c = \cos \theta$, $s = \sin \theta$. Note that the phases $\alpha_2, \alpha_3$ in Eq. 5 cancel out in these constraints and, thus, remain undetermined.

The mass matrix $m$ is diagonalized by a unitary rotation $V$ on $\nu$'s:

$$V = \begin{pmatrix} e^{i\beta_2} & e^{i\beta_3} \\ e^{i\delta_3} & -e^{i\delta_3} \end{pmatrix} \begin{pmatrix} \cos \varphi & i\sin \varphi e^{-i\epsilon} \\ -i\sin \varphi e^{i\epsilon} & \cos \varphi \end{pmatrix}, \quad (8)$$

where $0 \leq \varphi \leq \pi/2$, $0 \leq \epsilon < \pi$. The ‘Dirac’ mixing matrix in Eq. 3 is now multiplied by $V^\dagger$ on the left and, after phase absorptions, takes the form

$$\begin{pmatrix} \cos \theta_{23} & \sin \theta_{23} e^{-i\epsilon_{23}} \\ -\sin \theta_{23} e^{i\epsilon_{23}} & \cos \theta_{23} \end{pmatrix}, \quad (9)$$

where $0 \leq \theta_{23} \leq \pi/2$, $0 \leq \delta_{23} < \pi$. Here, $\theta_{23}$ (or $\theta_{\mu\tau}$) is the physical mixing angle in the 2-3 leptonic sector and its cosine equals the modulus of the complex number $\cos \varphi \cos \theta^D + \sin \varphi \sin \theta^D e^{i(\xi - \epsilon)}$, where $-\pi \leq \xi - \epsilon = \beta_2 - \beta_3 - \epsilon \leq \pi$. The phases $\beta_2, \beta_3$ and $\xi$ remain undetermined due to the arbitrariness of $\alpha_2, \alpha_3$. Thus, the precise value of $\theta_{23}$ cannot be found. However, we can determine the range in which $\theta_{23}$ lies: $|\varphi - \theta^D| \leq \theta_{23} \leq \varphi + \theta^D$, for $\varphi + \theta^D \leq \pi/2$.

We now need to know the asymptotic values of $m_{2,3}^D$, $\theta^D$. Approximate $SU(4)_c$ invariance in the up quark and neutrino sectors gives $m_2^D = m_c, m_3^D = m_t$, $\sin \theta^D = |V_{tb}|$ ‘asymptotically’. Renormalization of light neutrino masses and mixing between $M_{GUT}$ and $M_2$ is also included assuming the MSSM spectrum and large $\tan \beta \approx m_t/m_b$. In the framework of ‘hierarchical’ light neutrino masses ($m_3 \gg m_2 \gg m_1$), the small angle MSW resolution of the solar neutrino puzzle implies $1.7 \times 10^{-3} \text{ eV} \lesssim m_2 \lesssim 3.5 \times 10^{-3} \text{ eV}$. Finally, $m_3$ is restricted by the cosmological bound $m_3 \lesssim 23 \text{ eV}$ (for $h \approx 0.5$).

We are now ready to derive useful restrictions on $M_{2,3}$. Assume that both $M_{2,3} \leq \frac{1}{2} m_{infl}$. Then the inflaton predominantly decays to the heaviest of the two. The determinant condition implies that the lowest possible value
of the heaviest $M_{2,3}$ is about $10^{11}$ GeV giving $T_R \gtrsim 10^{10}$ GeV, in conflict with the gravitino constraint. So we must take $1.72 \times 10^{13}$ GeV $\approx \frac{1}{2} m_{\text{infl}} \leq M_3 \lesssim 2.5 \times 10^{13}$ GeV, where the upper bound comes from the requirement that the coupling constant of the non-renormalizable term responsible for the mass of the heaviest $\nu^c$ does not exceed unity. (This requirement also implies the upper bound on $x_Q$.) In summary, we see that i) $M_3$ is constraint in a narrow range, and ii) the inflaton decays to the second heaviest $\nu^c$ with mass $M_2$.

Baryons can be produced, in the present scheme, only via a primordial leptogenesis from the decay of $\nu^c$’s emerging as decay products of the inflaton. The lepton asymmetry is then partially converted into the observed baryon asymmetry of the universe by ‘sphaleron’ effects. The lepton asymmetry is

$$n_L = \frac{9 T_R}{8 \pi m_{\text{infl}} M_3} \frac{c^2 s^2}{\varepsilon^2} \sin 2 \delta \left( m_{\nu}^2 - m_{\nu}^2 \right)^2 - m_{\nu}^2 c^2 \right) .$$

(10)

Here $\nu$ is the electroweak vev at $M_{\text{GUT}}$. Renormalization effects should also be included. Assuming the MSSM spectrum between 1 TeV and $M_{\text{GUT}}$, the observed baryon asymmetry $n_B/s$ is related to $n_L/s$ by $n_B/s = -\frac{3}{2} (n_L/s)$.

We will now extract restrictions on light neutrino masses and mixing. Take a specific allowed value of $M_3$ (in practice, we take its two extreme values $\frac{1}{2} m_{\text{infl}}$ or $2.5 \times 10^{13}$ GeV). For any pair $m_2$, $m_3$, we use the determinant condition to evaluate $M_2$ and, subsequently, $T_R$. The gravitino constraint ($T_R \lesssim 10^9$ GeV) then gives a lower bound in the $m_2, m_3$ plane. This bound together with the MSW restriction on $m_3$ yields a lower bound on $m_3$, namely $m_3 \gtrsim 0.9$ eV (for $M_3 \approx 2.5 \times 10^{13}$ GeV) or 1.3 eV (for $M_3 \approx \frac{1}{2} m_{\text{infl}}$). The trace condition is solved with respect to $\delta = \delta(\theta)$, $0 \leq \theta \leq \pi/2$, which is then substituted in Eq. (10) to yield $n_L/s = n_L/s(\theta)$. Imposing the ‘low’ deuterium bound on $n_B/s$ (0.02 $\lesssim \Omega_B h^2 \lesssim 0.03$), we find the range of $\theta$ where this bound is satisfied. If such a range exists, we keep $m_2$, $m_3$ as satisfying the baryogenesis constraint. This gives an upper bound in the $m_2, m_3$ plane which together with the MSW restriction on $m_3$ yields an upper bound on $m_3$, namely $m_3 \lesssim 5.1$ eV (for $M_3 \approx 2.5 \times 10^{13}$ GeV) or 8.8 eV (for $M_3 \approx m_{\text{infl}}/2$). The allowed area in the $m_2, m_3$ plane is depicted in Fig. 1, where the thick solid (dashed) line corresponds to $M_3 = m_{\text{infl}}/2$ ($M_3 = 2.5 \times 10^{13}$ GeV). The overall allowed range for $m_{\nu_e}$ is $1$ eV $\lesssim m_{\nu_e} \lesssim 9$ eV, which is interesting for the cold plus hot dark matter scenario for large scale structure formation in the universe.

The mixing angle $\theta_{\mu\tau}$ can also be restricted. For every allowed $m_2$, $m_3$ pair and every $\theta$ satisfying the baryogenesis constraint we construct $V$ in Eq. (8) and, consequently, $\varphi, \epsilon$ and the allowed range of $\theta_{\mu\tau}$, $|\varphi - \theta^D| \leq \theta_{\mu\tau} \leq \varphi + \theta^D$. The union of all these ranges for all allowed $\theta$’s and $m_2$’s for a given $m_3$ gives the range of $\theta_{\mu\tau}$ which is allowed for this value of $m_3$. All these ranges for
all allowed $m_3$'s constitute the allowed area on the oscillation diagram, which is depicted in Fig. 2 (notation as in Fig. 1) in confrontation to past, ongoing and planned experiments. The central part of this allowed area will be tested by the ongoing SBLE at CERN, NOMAD/CHORUS. Possibly negative result from NOMAD/CHORUS will exclude a significant part of the allowed domains in Figs. 1,2 reducing the upper bound on $M_{\nu_{\mu}}$ to 3.7 eV. The new CERN-SBLE(TOSCA) together with the new CERN-MBLE (ICARUS-JURA (600t,DIS)) will cover all our predicted area on the oscillation diagram.

In summary, hybrid inflation, baryogenesis and neutrino oscillations have been linked in the context of a supersymmetric model based on a left-right symmetric gauge group. Our scheme leads to stringent restriction on $m_{\nu_{\tau}}$ and $\theta_{\mu\tau}$ to be tested by ongoing and planned experiments. These restrictions are derived by mainly ‘physical’ arguments (gravitino and baryogenesis constraints) supplemented by a ‘minimal’ input from fermion mass matrix ansaetze (only 3 input parameters) and experiments (MSW resolution of the solar neutrino problem). The choice of the gauge group is crucial since, in this case, $\phi$ has the quantum numbers of $\nu^{c}$ and, thus, decays to $\nu^{c}$'s producing an initial lepton asymmetry. As a consequence, the gravitino and baryogenesis constraints restrict the neutrino parameters. Supersymmetry is also crucial since together with an $R$ symmetry provides a ‘natural’ frame for hybrid inflation.

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Figure Captions

Fig.1. The allowed regions in the $m_{\nu_{\mu}}, m_{\nu_{\tau}}$ plane.

Fig.2. The allowed regions in the $\nu_{\mu}$-$\nu_{\tau}$ oscillation plot.
Fig. 1

\[ m_{\nu_\tau} \quad (\text{eV}) \]

\[ m_{\nu_\mu} \quad (10^{-3} \text{ eV}) \]
$\Delta m_{\mu\tau}^2$ (eV$^2$)

$\sin^2(2\theta_{\mu\tau})$

Fig. 2