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Impact of Technological Shock on the Sierra Leone Economy: A Dynamic Stochastic General Equilibrium (DSGE) Approach

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Abstract

The neoclassical growth model has emphasised the importance of technology shocks, which supposedly affect macroeconomic variables’ heterogeneously in a small open economy like Sierra Leone. Using a Bayesian DSGE methodology for a non-linear model, we found that investment-specific technological shock partly explains business cycle fluctuations in Sierra Leone. Moreover, the analysis indicates that technology shock on output, capital, and consumption is more persistent than that of interest rate. The key implication is that technological innovation is crucial for long-term steady-state growth in Sierra Leone. The results also partly confirm the neoclassical growth model prediction – that is, in the long run, productivity growth is driven only by technological progress. The model specified for this research is largely inward-looking, with a minimal role for the Bank of Sierra Leone to influence investment in technology-related investment directly. Despite this limitation and more so given the fact that the DSGE modelling concept is quite a new venture at the BSL, thoughts have been given to enhance the model’s future capabilities to incorporate both the monetary bloc and external blocs to fully assess the impact of technological shock’s transmission in the entire economy in future research.

Keywords: Neoclassical growth model, Bayesian DSGE, Technological Shock, Impulse Response, Sierra Leone.

JEL Classification: C11, E30, E32, E37

1.0 Introduction

The basic DSGE model introduces several (specific) assumptions about the capital accumulation process. For instance, it is assumed that savings transform directly into physical capital through the investment process at no cost. Additionally, the capital accumulation equation assumes that physical capital remains homogenous over time and just new capital assets are added to the existing capital stock through investment. However, in practice, technological progress changes the characteristics of physical capital as technology is embodied in capital assets. When new capital assets are incorporated into the economy through the investment process, these assets have different characteristics to those already in existence – an indication that they are not homogenous over time as different vintages of capital can exist. For example, a computer produced in 1990 can be perceived as obsolete in terms of its features when compared to a computer produced in 2020. Thus, the cost of incorporating an additional computer into the production process may be the same over time, but its productivity is much higher - the implication being, that it would be equivalent to having more capital units because computer production in the year 2020

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incorporates modern technological progress. This is the so-called Investment-Specific Technological Change (ISTC). Recall that the neoclassical growth model predicts in the long run, productivity growth is driven only by technological growth. Traditionally, the concept used in economics for technological progress is associated with the increase in Total Factor Productivity, which affects all of the factors of production. Because of that, it is called neutral technological or total factor productivity. However, there is also specific technological progress associated with capital inputs, which depends on the investment process, and occurs when new vintages of capital assets are incorporated into the capital stock.

In this context, it is appropriate to posit that Sierra Leone is still a pre-industrial economy, with a need to absorb a lot of modern technologies. Essentially, the adoption of modern technologies has occurred in all facets of the economy – notable highlights include agriculture, fishing, services sector, construction, banking, etc. Therefore, there is a need to determine the impacts of technological shock on the Sierra Leonean economy. This means adopting and also extending the analysis of the theoretical foundation of the DSGE model developed by Schmitt-Grohe and Uribe (2004). This is to enable us to capture the impact of the technological shock on output, capital stock, consumption, and interest rate in Sierra Leone. Our primary motivation therefore for this study, has stemmed from the need to assess the effect of the technological shock on key macroeconomic variables stated above, using a Bayesian non-linear DSGE model approach. We believe that this will inform policymakers on the appropriate response to the ongoing technological transformation. This is particularly important in a context wherein the government has an ambitious target of transforming Sierra Leone into a middle-income country, where technology will need to play a significant role. Therefore, it will be quite instructive and informative for the fiscal and monetary policy authorities to understand the impact of these technologies and the response of selected macroeconomic variables with a model that accounts for international trade and agencies intervention (notably the BSL and Ministry of Finance) to be utilised for future studies.

To achieve this objective, we have employed non-linear Bayesian DSGE due to its ability to incorporate initial values that account for the peculiarities of the Sierra Leone economy, and for which data alone may seem inadequate. Relative to the Maximum Likelihood DSGE approach, the Bayesian DSGE model can provide more efficient estimations of the model parameters, as well as consistent estimates of the observed technological shock that drives economic development, which is imperative for policymaking (Smet and Wouters, 2007).

Following Kydland and Prescott’s (1982) study, many researchers have resorted to calibrating dynamic stochastic general equilibrium (DSGE) models, with the incorporation of the formal use of econometric methods to parameterise model outcomes in a bid to study their quantitative implications. To build our case, we have decided to measure the contribution of both neutral and investment-specific technology shock to movements in gross national income and consumption as a percentage of Gross Domestic Product. We also compared our answers obtained with the calibration and likelihood-based estimation of a neoclassical stochastic growth model. The key parameter in the analysis is the technological shock and its pass-through effects on gross capital formation. To make our empirical analysis as transparent as possible, we have deliberately chosen a fairly closed-economy, but quite simple DSGE model, rather than a more sophisticated specification. Our model allows us to focus on a single parameter – the application of technological shock, which seems quite challenging to measure, while at the same time crucial for the quantitative result. The difficulty in determining the technological shock is in part caused by the stylised nature of our choice of the theoretical model.

Our paper differs from others in distinct ways. To the best of our knowledge, no study on the Sierra Leone economy has employed the DSGE Bayesian DSGE framework to provide empirical results on the impacts of technological shock and the response of output, capital stock, consumption, and interest rate. The study will undoubtedly contribute significantly to a niche
body of literature, particularly in the area of technological shock and its impact on the macro-economy of Sierra Leone. The use of a non-linear Bayesian DSGE model is a step forward in supporting effective policy formulation and implementation by authorities at the BSL and Ministries of Finance and Development. Therefore, the remaining sections of the paper are planned as follows: Section 2 presents a generalized literature review (historically profiling emerging development from non-structured to complex structured models), while Section 3 describes the basic structure of the DSGE model. Section 4 explains the methodology and data used for the estimation of the DSGE model. Section 5 analyses the empirical results and conclusion, with some thoughtful insights for future expansion considered necessary in supporting effective policy formulation at the BSL.

2.0 Literature Review (including Evolutionary Discourse)

While taking cognisance of the scantiness of literature on DSGE study to address the peculiarities of the Sierra Leone economy, we believe that the few studies that have been pursued will certainly lend support to the effort in expanding knowledge exploration for future work. Most importantly, the relevance of (macro)economic modelling to support effective policy formulation at the Bank of Sierra Leone (BSL) is now taking centre stage with the provision of the revised BSL of 2019, which so far has incorporated both price and financial stability as the core mandates (BSL, 2019). While it is a well-known fact that models, in general, are not full proof of problem solvers, their development is considered very vital in approximating reality within an economic system. As emphasised by Mordi, Adebiyi, Adenuga, Abeng, Adeboye, Adamgbe, and Evbuomwan (2013), models can be construed as a form of art, which sits within a scientific platform in explaining the inter-linkages (as presented by existing data outputs) that exists within an economic system. The thrust of such a process is to support existing fundamentals of economic theory that explain concepts around causation or shock emanating from data manipulation or some form of exogenous abnormalities.

Therefore, the importance of this section to the study is to help the authors set out the platform on which the empirical output is to be guided, which is built on the application of the existing body of literature on DSGE modelling to assess the impact of the technological shock on the Sierra Leone economy. On this note, we are very much guided in providing a brief narrative of the categorization of models (typically non-structural and structural) and their (historical) evolution to the present vogue of DSGE models. Generally speaking, models are usually constructed to enable predictions that guide effective policy outcomes - to say the least, the use of non-structural or typically referred to as simple linear (difference) equations like ARMA/ARIMA, B(VAR), and ARDL makes use of time-series data to support short term predictions of a given phenomenon (see works produced by Warburton and Jackson, 2020; Barrie, 2020; Jackson, Tamuke and Jabbie, 2019; Tamuke, Jackson and Sillah, 2018; Bangura, Caulker and Pessima, 2012). Given the inherent downside of serial correlation and the lack of structural determinants that typically exist with non-structural models, it is difficult for non-structural to be used to support long-term predictions (Mordi et al, 2013). Such limitations inherent in the use of non-structural models have however resulted in the development of a more structural form of macroeconomic models as explored in the 1970s (Diebold, 1998) - historically, these types of models could be traced back to 1936, with the construction of the Dutch model by Tinbergen (Mordi et al, 2013).

Progression into the construction of macro-econometrics models has given way to establishing inter-linkages between variables to assess outcomes that are relevant to support policy decision-making in institutions like central banks across the world. As technically explained, by Jayawickrama (2007) the concept is seen as a set of stochastic equations that explained relationships in behaviours of economic agents or variables. The construction of macro-
econometric models varies in complexity depending on their purpose or use - in the case of central banks, they are mostly utilized to support effective (monetary) policy formulation, which is also generally linked along the line of economic theory and intuition. Although macro-econometric models have brought in the advantage of utilising multiple equations, decisions relating to (ir)rational choices of variables that are included in an equation can render them not-so-suitable as explained by Lucas (1976).

The shortfall inherent in the use of macro-econometric models has made it more explorative for pursued efforts to be devoted to developing the Real Business Cycle (RBC) model (Kydland and Prescott, 1982). This is seen as the emergence of structural model development given its inherent characteristics of economic agents’ ability to make decisions that are built on expectation patterns. As resonated on the background of this study, the RBC model is intuitively factored with the inclusion of randomised technological shock, which is the backbone on which DSGE theoretically emerged. On a more critical note, such types of models are also limited in their flexible use of prices - implying that changes afforded to the interest rate can also be affected by an almost proportional level of inflationary pressures, thereby making it almost impossible for the interest rate to be changed (Mordi et al, 2013: 7). Contrary to the thinking of the Keynesian thought about (economic) recession and its link with the under-utilisation of resources, the RBC model had carved its impetus on the assumption that fluctuations in business cycles could be associated with optimal responses to shock, while an effort to use policies to stabilise situations can also end up being counterproductive.

The stride to construct structural models perceived as suitable for addressing the vagaries of macroeconomic problems inherent with economic agents’ behavioural patterns has also given rise to the development of the New Keynesian Model (NKM). This is considered to be a cross-road between the RBC and which then emerged into becoming the DSGE model. In this situation, we assume imperfection and rigidity in the economic system. This, therefore, makes it possible for NKM to factor in monopolistic completion that is backed by real and nominal rigidities, and together with varied forms of shocks akin to sticky prices, wage, and price indexation (Calvin, 1983; Christiano, Eichenbaum and Evans, 2005) – this brings the importance of monetary stabilisation policies to the fore, and mostly championed through the interest rate channel.

After the emergence of NKM, both the Computable General Equilibrium (CGE) and Dynamic Stochastic General Equilibrium (DSGE) models were developed to address limitations inherent in earlier models constructed. Despite the relevance/usefulness of the CGE, which mostly incorporates multi-sectoral components and features for input-output analysis (Valadkhanai, 2004), the DSGE model was seen as an advancement in assessing shortfall in New Keynesian thinking by incorporating supply-side components in the structure (Mordi et al, 2013). The model has made it possible to address structural changes, which can be accounted for by the structural linkages of equations amongst agents - typically including households, firms, government, central bank, and other entities within an economic system. DSGE model has become very useful in central banks for the deliberation of policy outcomes, particularly in predicting economic fluctuations (Coletti and Murchison, 2002). The application of DSGE modelling has become very widespread, particularly with its inherent complexity of incorporating monetary policy transmission channels, with the added feature of linking empirical outcomes from IS/LM models as illustrated by Adebiyi and Mordi (2011). The usefulness and application of DSGE models have been very well captured by authors like Torres (2015), and thereafter summarised by Jackson (2018).

The highlight of DSGE applications, and particularly that which is concerned with the implications of technological shock is highly reflected in empirical outputs around the world. A notable case of this is cited by Christofzik et al (2021) in their study as applied to the German economy, which has experienced a slowdown in economic performance despite the high level of
digitisation efforts to transform the economy. The paper utilised three empirical reasons for the said development – in the first place, the use of quarterly adjusted Total Factor Productivity (TFP) for the German economy proved that a slowdown in the United States of America (USA) productivity growth level from the middle of the 2000s seems to have made very little impact on Germany’s productivity. Secondly, the decision to shift into the services industry could also be cited for a share weak level of aggregate productivity advances. This transformation process is associated with a strong labour market performance. Thirdly, it was proven that technological progress in the area of Information and Communication Technology (ICT) in Germany seems to stimulate some level of slow growth rate in employment.

Moving into more emerging economies in the European bloc like Croatia, Palic´ (2018) empirically tests the amenability of monetary policy shock with a calibrated DSGE model that supposedly incorporates financial frictions with that of monetary policy disturbances in Croatia using the Vector Autoregression (VAR) model. Upon calibration of the DSGE model, the VAR model was then used to estimate outcomes for the Croatian economy – this provided a comparative analysis of impulse response functions of the identified DSGE and that of the VAR model. The outcome shows that for both models, monetary policy shock has an earlier (positive) impact on the interest rate, but a negative early impact on house prices and the output gap. The conclusion from the study revealed that monetary policy shock imitates the impact of monetary shock with the use of the DSGE model incorporating financial frictions (highly influenced by technological change).

In the African continent, for example, Alimi and Chakroun (2022) analysed the effects of nominal wage rigidity on inflation persistence and unemployment using a Bayesian DSGE model for a small open economy like Tunisia. The findings show that interest rates seem to be highly influenced by wage policy, which is linked with technological change. There is also an indication to unearth the rigidity of nominal wage on unemployment. The study also proved the power of monetary policy tightening, which has the power of slowing down economic activities and its ultimate negative outcome on the level of unemployment. In conclusion, the study proved that policy action is needed to deal with nominal wage rigidities as a way of allowing a better means of executing monetary policy transmission.

Notwithstanding the positive developments that DSGE modelling has brought to mainstream economics with its inherent theoretical foundations, it has also come under great criticism by authors like Sims (2006) for being atheoretical and lacking other features that make it inapplicable in a variety of contexts. However, the novelty to address its usefulness with technology shock and its applicability to a small open economy like Sierra Leone has warranted the need to explore literature that lends support to the continued applicability of the model.

3.0 Methodology

This section derives from a neoclassical growth model the long-run identifying assumptions exploited in the empirical analysis. The model is deliberately stripped down to make the discussion as transparent as possible. Some short-run implications of the model are also discussed to motivate the analyses and to assess the plausibility of the empirical findings.

The usual way to consider a technological change in DSGE models is to assume the existence of a shock that affects the aggregate production function of the economy. This is the so-called Total Factor Productivity (TFP)\(^2\) shock or neutral technology change. However, another source of technological change is derived from the technical and performance characteristics of assets that

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\(^2\) See highlight on Jackson, 2020 for an extended literature on the Economics of Technology Innovation.
do not remain constant over time. In general, capital assets have embodied better technical and performance characteristics over time. This is especially true in the case of equipment (transport, telecommunications, machinery, etc). This implies the existence of different vintages of capital assets, with different productivity.

The neoclassical model generally predicts that long-run productivity growth can only be driven by technological progress. Technology in turn can be differentiated into neutral progress and investment-specific progress. Whilst the first is associated with multi-factor productivity, the second is the amount of technology that can be acquired by using one unit of a particular asset. In reality, the amount of technology that can be transferred to productivity widely differs among the different capital assets.

Greenwood, Hercowitz and Huffman (2000) were the first to develop a DSGE model with specific technological progress in the capital accumulation function as an exogenous stochastic process associated with the investment. One way to introduce Investment-specific Technological Change in a DSGE model is to define the capital accumulation process as follows:

$$K_t = (1 - \delta)K_t + Z_t I_t$$ (1)

Where $\delta$ in Eq. 1 is the physical capital depreciation rate and $Z_t$ represents technological progress specific to the investment. Following Greenwood et al. (1997), $Z_t$ determines the amount of capital that can be purchased with a production unit, which also represents the current state of technology needed to produce capital. The standard neoclassical model would invariably imply that $Z_t = 1$ for all $t$ – this simply means the amount of capital that can be purchased with a final production unit is constant over time. However, in reality, the relative price of capital falls broadly implying that over time, we can buy a larger amount of capital with the same amount of final production. Thus, the higher the value of $Z_t$, the greater the amount of capital that can be incorporated into the economy with an investment unit, thereby reflecting the fact that the quality of capital has increased. An increase in $Z_t$ can be associated with a positive technology shock, which reduces the slope of transforming the investment into a consumer good (i.e., a reduction in the average cost of producing consumption goods). To obtain a measure of technological progress specific to investment, it is necessary to have prices of capital assets adjusted for quality. This is what is called hedonic price, i.e., the price of a particular capital asset whose quality remains constant over time (see Gordon, 1990; Cummins and Violante, 2002).

3.1 Model Specification

Here, we present a very simple version of the DSGE model with specific technological change investment. The model blocs (Household and Firm) incorporate two shocks: aggregate productivity, which measures the neutral technological change, and specific productivity, which measures technological change associated with the new capital assets. Two changes are introduced in the (basic) model – firstly, the capital accumulation (equation) that accounts for changes in the quality of new vintages of capital through the investment process, and secondly, a defined stochastic process for investment-specific shock.

3.2 Household

The economy is inhibited by an infinitely lived-representative household that has time-separable preferences in terms of final goods consumed $\{C_t\}_{t=0}^{\infty}$, and leisure $\{1 - L_t\}$ max $\sum_{t=0}^{\infty} \beta t [\gamma \log C_t + (1 - \gamma) \log (1 - L_t)]$ (2)

Where $\beta$ in equation (2) is a discount factor and $\gamma \epsilon (0,1)$ is the elasticity of substitution between consumption and leisure. Budget constraints faced by the consumer imply that consumption and saving $S_t$ cannot exceed the sum of labour and (rental) capital income:

$$C_t + S_t = W_t L_t + R_t K_t$$ (3)
Where $W_t$ in equation (3) is the wage and $R_t$ is the rental price of capital. To keep things simple, we assume that savings are transformed into an investment at no cost; $I_t = S_t$.

The key point of the model is that capital holdings evolve according to:

$$k_{t+1} + (1 - \delta)K_t + Z_t I_t$$

(4)

Where $\delta$ in Eq. 4 is the depreciation rate of physical capital, while $Z_t$ determines the amount of capital that can be purchased by one unit of output – signifying a representation of the current state of technology for producing capital. Therefore, investment can be defined as illustrated in equation (5):

$$I_t = \frac{K_t(1 - \delta)}{Z_t}$$

(5)

The budget constraint as shown in equation (6) can be written as:

$$C_t + \frac{K_{t+1}}{Z_t} = W_t L_t + R_t K_t + \frac{(1 - \delta)K_t}{Z_t}$$

(6)

The Langrangian problem to be solved is based on $C_t, L_t$ and $I_t$ to maximise budget constraints as shown represented in equation (7) below:

$$\max_{C_t, L_t, I_t} l = \sum_{t=0}^{\infty} \gamma \log C_t + (1 - \gamma) \log (1 - L_t) \beta^t \left[-\lambda_t \left(C_t + \frac{K_{t+1}}{Z_t} - R_t K_t - \frac{(1 - \delta)K_t}{K_t}\right)\right]$$

Eq. 7

The first-order conditions for the households are represented in equations (8), (9) and (10) as follows:

$$\frac{dl}{dC_t} \gamma C_t^{-1} - \lambda_t = 0$$

(8)

$$\frac{dl}{dL_t} = -\frac{1 - \gamma}{1 - L_t} + \lambda_t W_t = 0$$

(9)

$$\frac{dl}{dK_t} \beta \lambda_t^t \left(\frac{Z_t^{-1}}{Z_t} + \frac{1 - \delta}{Z_t} - \beta^{t-1} \lambda_{t-1}\right) = 0$$

(10)

By combining equations (8) and (9), we obtain equation (11), which represents a condition that equates the marginal rate of substitution between consumption and leisure to the opportunity cost of one additional unit of leisure.

$$\frac{1 - \gamma}{\gamma} \frac{C_t}{1 - L_t} = W_t$$

(11)

On the other hand, combining Eq. 8 with Eq. 10 eventually results in the derivation of equation (12).

$$\frac{1}{\beta} \frac{C_t}{C_{t-1}} = \frac{Z_{t-1}^{-1}}{Z_t} \left[Z_t R_t + 1 - \delta\right]$$

(12)

This implies an equilibrium condition that equates the marginal consumption rates to the rate of return on investment, which now depends on the investment-specific technological change.

### 3.3 The Firm

The problem of the firm is to find optimal values for the utilisation of labour and capital. The production of the final output ($Y$) requires the services of $L$ and $K$. The firm rent capital and employ labour to maximise profit at period $t$, taking factor prices as given. The technology is given by a constant return to scale Cobb-Douglas production function as given herein equation (13)

$$Y_t = A_t K^\alpha L^{1-\alpha}$$

(13)

Where $A_t$ is a measure of total factor or sector-neutral productivity, and $0 \leq \alpha \leq 1$. The static maximisation for the firm is represented in equation (14):
\[
\max_{K_t, L_t} \pi_t = A_t K_t^{\alpha} L_t^{1-\alpha} - R_t K_t - W_t L_t
\]

The first-order conditions for the firm’s profit maximization are represented in equations (15) and (16).

\[
d\pi_t \frac{d\pi_t}{dK_t} = R_t - \alpha_t A_t K_t^{\alpha-1} L_t^{1-\alpha} = 0
\]

\[
d\pi_t \frac{d\pi_t}{dL_t} = W_t - (1 - \alpha) A_t K_t^{\alpha-1} L_t^{1-\alpha} = 0
\]

From the above, the first-order conditions for profit maximization equilibrium prices for production inputs are specified in equations (17) and (18):

\[
R_t = \alpha_t A_t K_t^{\alpha-1} L_t^{1-\alpha}
\]

\[
W_t = (1 - \alpha) A_t K_t^{\alpha-1} L_t^{1-\alpha}
\]

3.4 Equilibrium Of The Model

The equilibrium of our model economy is obtained by combining the first-order conditions for the “average representative household with that of the representative firm”, thereby resulting in the output of equations (19) and (20) as indicated below:

\[
\frac{1-\phi}{\phi} \frac{C_t}{1-L_t} = (1-\alpha) \frac{C_t}{L_t}
\]

\[
\frac{1}{\beta} (C_t - C_{t-1}) = Z_{t-1} \left[ \beta \frac{Y_t}{K_t} + 1 - \delta \right]
\]

To close the model, the feasibility constraint of the economy must be defined as shown in equation

\[
C_t + L_t = Y_t
\]

A more formal definition of the equilibrium condition can be defined as a sequence of consumption, leisure, and private investment for the consumers – typically represented as \{C_t, 1 - L_t, 0, \} for consumption, \{K_t, L_t, 0\} for labour utilisation of the firm, and \{Z_t, 0\} for the state of technology in the production of each capital asset. This also takes cognisance of a given sequence of prices\{W_t, R_t\} that involve: (i) the optimised condition of the consumer; (ii) the first-order condition of the firm hold; and (iii) the feasibility constraint of the economy holds.

3.5 The Bayesian DSGE Model

We now turn to the search for exploring the importance of technological shock for movements in the specified macroeconomic variables using formal econometric methods. We use ‘Bayesian estimation techniques’, partly because of their efficient outcome in modern econometric estimation and also based on personal taste and the researchers’ expert knowledge. We commenced by selecting the observables and the model specifications to be estimated and then followed by a description of how we calibrated the priors for typical DSGE model parameters.

3.6 Model Estimation Procedure

To solve the model, we assume the following: K to be an indeterministic (endogenous) state variable (rooted in theory). Thus, the equation for K is specified without a shock. We also note that only exogenous state variables are subject to shocks. Y is observed, while both C and R are unobserved. This assumption is required to satisfy the condition that the number of exogenous state variables is equal to the number of observed control variables. In other words, we must have exactly as many exogenous state variables (and thus, as many shocks) as the number of observed control variables. The competitive equilibrium of the model is given by a set of 5 equations that

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3 A detailed review of the Bayesian estimation of DSGE models can be found in Schorfheide (2007).
derives the dynamics of the seven endogenous macroeconomic variables \( \{Y_t, C_t, K_t, R_t, N_t, I_t, P_t\} \) plus the two technology variables \( \{A_t \text{ and } Z_t\} \), which is assumed to follow an AR (1) process. This set of equations is defined as follows:

\[
1 = \beta E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-1} (1 + R_{t+1} - \delta) \right\} \quad \text{(Consumption)} \tag{22}
\]

Equation (22) defines a relationship between consumption growth \( \left( \frac{C_{t+1}}{C_t} \right) \) and the interest rate \( R_{t+1} \).

\[
Y_t = Z_t K_t^\alpha \quad \text{(Production function)} \tag{23}
\]

Equation (23) is a production function for output \( Y_t \), productivity \( Z_t \) and capital \( K_t \).

\[
R_t = \alpha \frac{Y_t}{K_t} \quad \text{(Interest rate/Capital demand)} \tag{24}
\]

Equation (24) is a model for the interest rate.

\[
K_{t+1} = Y_t - C_t + (1 - \delta)K_t \quad \text{(Capital Accumulation)} \tag{25}
\]

Equation (25) is the equation for capital accumulation - capital in the next period \( K_{t+1} \) is equal to under-appreciated capital in this period \( (1 - \delta)K_t \) plus unconsumed output \( Y_t - C_t \).

\[
\ln(Z_{t+1}) = \rho \ln(Z_t) + \epsilon_{t+1} \quad \text{(State Variable)} \tag{26}
\]

Equation (26) is a law of motion for productivity \( Z_t \).

The state variables are the current-period capital stock and the level of productivity, \( K_t, Z_t \). The control variables are consumption, interest rate, and output \( C_t, R_t, Y_t \).

### 3.7 Prior and Posterior Calibration

To perform the Bayesian estimation of the model, we need to specify priors for the parameters. Parameters in a DSGE model typically have economic interpretation. We use those interpretations to specify informative priors. The model has four structural parameters \( \{\beta, \alpha, \delta, \rho\} \) and one standard deviation parameter \( \sigma \). The parameters of the beta distributions were chosen to put the weight of prior mass on theoretically appropriate values. The parameter \( \beta \) is a discount factor in the consumption equation. It must lie between 0 and 1, and is probably in the higher end of the range. We use a prior beta distribution with parameters \( (0.90, 0.99) \). These parameters are consistent with a prior mean of 0.95. The parameter \( \alpha \) is a production parameter in the output equation – based on literature, it usually lies between 0 and 1, and is generally in the lower end of the range for small pre-industrial economies, where national productivity is very low. Based on this presupposed theoretical grounding, we use an Alpha density distribution with parameters \( (0.3, 0.4) \). The parameters are consistent with the prior mean of 0.35. The parameter \( \delta \) is a depreciation parameter in the capital accumulation equation. It commonly lies between 0 and 1, mostly in the lower end of this distribution. Economic theory indicates that this will be negative. This is so because capital depreciation is high in small developing economies since maintenance appears to be minimal. Therefore, we use a Beta distribution with parameters \( (0.03, 0.05) \). The parameters here are also consistent with the prior mean of 0.04.

Finally, for the autoregressive parameter, the parameter \( \rho \) is perceived as persistent in the productivity equation. It normally lies between 0 and 1 and it is at the higher end of this density distribution. Therefore, we use a Rho distribution, with parameters \( (0.66, 0.99) \). This is also consistent with the prior mean of 0.8. The model has two technology shocks, namely neutral or total factor productivity technological change and investment-specific technological change. However, since both perturbations represent technological change, alternatively it can be assumed that there may be some level of relationship existing between them. Note that these are parameters defining the stochastic process for investment-specific technological shock and are exactly equal to the process for a neutral shock - see Pakko, 2005, Rodriguez and Torres, 2010. Our prior choices for all parameters are driven by the aforementioned theoretical considerations. Given that all four parameters are plausibly restricted to the unit interval, a beta distribution is chosen for all four priors. The parameters of the beta distribution were chosen to confirm the weight of prior mass on theoretically appropriate values.
3.8 Data: Description & Sources

A crucial step in the estimation process is the choice of observables $Y$ that enter the likelihood function ($Y | \Theta$). Since the goal is to determine the contribution of a technological shock to the variation in the key macroeconomic variables, we have decided to follow the tradition in econometrics. For the estimation processes of our DSGE model, we have employed quarterly data spanning 2006Q1 to 2020Q4. Variables used for the study include the observed growth rate of real GDP ($y$), Interest rate spread [lending rate minus deposit rate, expressed in percentage term] ($r$) unobserved, the growth rate of consumption ($c$) unobserved, the growth rate of hours worked/GNI Growth (Annual %) ($n$). Data utilized for this study were sourced from the World Bank - World Development Indicators (WDI) for Sierra Leone.

3.9 Convergence Diagnostic

We checked for the convergence diagnostic of the model to determine whether there is convergence in the Markov Chain Monte Carlo (MCMC) simulation. MCMC simulation convergence indicates reliable parameter estimates (Fernandez-Villaverde and Guerron-Quintana, 2021: 237-238). This then resulted in us graphing the behaviour of individual parameters, as well as generating the effective sample size (ESS) summary statistics. To ensure there is convergence efficiency, the trace plot should not exhibit a time trend and must be mean-reverting, with an exhibit of constant variance and decaying autocorrelation. The density of the chain should not vary throughout the MCMC sample. Also, the constant of the density distribution can be assessed by examining both the 1-half and 2-half density plots and must be symmetrical or worse, differentials should be minimal. Significant differences in the density may invariably imply no convergence in the chain.

Attainment of low-level efficiency for the estimated non-linear DSGE model could also indicate convergence problems in the iteration processes. If such is the case, the recommendation is to estimate the model with block options - implying an imposition of restrictions/blocks on selected parameters to adjust for the observed high autocorrelation, which invariably may enhance the efficiency of the MCMC sampling. However, to properly identify the parameters (both structural and blocked state), an algorithm for the density functions command will need to be written for all the parameters and immediately followed by a graphing of the diagnostic outputs to display comparison. The structural parameters with the best performance should be restricted/blocked and followed by a re-estimation of the model. Simply put, identified parameters for the restricted / block options must be informed by the behaviour of the density functions.

4.0 Results and Discussion

4.1 Model Estimation without Block Options

The output header below repeats the prior specification and reminds us that we are fitting a DSGE model. The model summary as referenced in Table 1 reports the prior and likelihood specifications, including the default inverse-gamma before the standard deviation of the shock. The output header reports the burn-in length and MCMC sample size and information about the efficiency of the Metropolis-Hastings sampler.
Table 1. Bayesian first-order DSGE model without block options

| Bayesian first-order DSGE | Model stings sampling 4 | MCMC iterations = 12,500 |
|---------------------------|-------------------------|---------------------------|
| Random-walk Metropolis–Ha|                         | Burn-in = 2,500 MCMC sample size = 10,000 |
| Sample: 2006q1 thru 2020q |                         | Number of obs = 60 |
| Log marginal-likelihood = |                         | Acceptance rate = .1515 |
|                           | -191.77231              | Efficiency: min = .02826 |

| Parameter | Mean   | Std. dev. | MCSE  | Median | Equal-tailed 95% cred. interval |
|-----------|--------|-----------|-------|--------|-------------------------------|
| beta      | .9494578 | .0221166 | .001316 | .952777 | .8932913 - .9819287 |
| delta     | .0397577 | .0062944 | .000362 | .0397073 | .0291956 - .0528338 |
| alpha     | .3403352 | .0475408 | .0024  | .3394153 | .2528225 - .4430147 |
| rhoz      | .7879269 | .0365272 | .00179 | .7890018 | .7093168 - .855767 |
| sd(e.z)   | 5.324392 | .4913897 | .022624 | 5.269096 | 4.51084 - 6.433976 |

Source: STATA Output

The overall acceptance rate is 0.15% - acceptance rates that are too low indicate that a large portion of the proposed MCMC iterations was rejected so that regions of high posterior probability were not sufficiently explored. Whilst sampling efficiencies are between 0.028 and 0.047, efficiency is linked to the autocorrelation of the MCMC draws, with higher efficiency indicating lower autocorrelation. The posterior mean for {beta} is 0.95, seemingly identical to its prior mean of 0.95. The posterior mean for {delta} is 0.039, almost identical to its prior mean of 0.04. The posterior mean for {alpha} is 0.34, near to its prior mean of 0.35.

The posterior mean for {rhoz} is 0.78 and proved different from its prior mean of 0.80. Overall, most of the parameters show little updating, indicating that the likelihood is uninformative along several dimensions of the model’s parameter space. The posterior results for {beta}, {delta}, and {alpha} are mainly driven by the prior. However, the posterior for {rhoz} shows significant updating, as such, we check the posterior diagnostics to determine which parameters need to be restricted/blocked.

4.2 Posterior Diagnostics and Plots

We begin by investigating effective sample sizes for each parameter in the estimated Bayesian model. Concerning Table 2, the effective sample size for the discount factor {beta} is somewhat low relative to the other parameters, which indicates that blocking/restriction may improve sampling efficiency. Because {rhoz} was the only internal parameter to receive substantial updating, we look at its full set of posterior diagnostic plots as shown below. All the parameters have more than 1% sampling efficiency, indicating good mixing. However, standard deviation has an efficiency of less than 1% - low efficiency implies that it takes longer for the MCMC chain to explore the posterior distribution.

Table 2. Efficiency Summary Statistics without block options

| Efficiency summaries | MC sample size = 10,000 |           |            |
|----------------------|-------------------------|-----------|------------|
| Efficiency: min =    | .02826                  |           |            |
| Avg =                | .03733                  |           |            |
| Max =                | .04717                  |           |            |
| ESS                  | 282.63                  | 35.38     | .0283      |
| Corr. time           | 302.97                  | 33.01     | .0303      |
| Efficiency           | 392.50                  | 25.48     | .0392      |
| 416.51               | 24.01                   | .0417     |
| 471.74               | 21.20                   | .0472     |
From Figure 1, all four parameters show that Autocorrelations tail off or decay at a moderate pace. The trace plot shows reasonable mixing, while the density plot shows that the first and second-half densities do not substantially differ, but are not symmetrical from the full-sample density.

Figure 1. Post Estimation Diagnostics plots for the parameters in the without block options model

Source: STATA Output

Next, we generate prior–posterior plots for all four parameters as shown below: The posterior distribution of \( \beta \) as shown in Figure 2 above is almost identical to its prior, indicating that the data provide little information along this dimension of the model. The posterior density of \( \alpha \) differs from its prior density. This situation indicates a flat likelihood along the \( \alpha \) dimension. The posterior distribution of \( \delta \) is identical to its prior, indicating that the data provide little information along this dimension of the model. By contrast, the posterior density of \( \rho_z \) differs from its prior density. The posterior mean has fallen to a value of less than 0.8.
The above posterior-prior plots in Figure 2 indicate the need for blocking/restricting the informative priors. Moreover, the low acceptance rate from the estimated model is also an indication of a convergence problem. Finally, the visual plots of autocorrelations of the parameters show that the pace of decay is slow and there is a need for increasing the Markov Chain Monte Carlo (MCMC) sample size or blocking the parameters of the informative priors and re-estimating the model with the so-called blocked options.

4.3 Estimating Model with Block Options

The re-estimated model as shown in Table 3 placed three parameters into their own blocks: \{delta\}, \{rhoz\}, and \{sd(e.z)\}. Blocking/restriction improves efficiency at the cost of longer run time. The significant variation between the prior and posterior plots of some of the estimated parameters and the relatively low estimated Efficiency Summary Statistics (ESS) necessitated the estimation of the model with block parameters.

Table 3. Bayesian First-Order DSGE Model with block options

| Parameter | Mean     | Std. dev. | MCSE  | Median  | Equal-tailed 95% cred | 95% interval |
|-----------|----------|-----------|-------|---------|------------------------|--------------|
| beta      | 0.9502194| 0.0214217 | 0.00629| 0.9538787| 0.9000438 - 0.982186  |              |
| delta     | 0.0401786| 0.006096  | 0.0012 | 0.0399732| 0.0286841 - 0.052781  |              |
| alpha     | 0.3444854| 0.0455994 | 0.00153| 0.3429863| 0.2572353 - 0.436150  |              |
| rhoz      | 0.7900874| 0.0378697 | 0.00077| 0.7920865| 0.7128953 - 0.85940    |              |
| sd(e.z)   | 5.342249 | 0.5069394 | 0.01169| 5.295409 | 4.462074 - 6.46708    |              |

Source: STATA OUTPUT

The overall acceptance rate has increased to 0.39%, which shows significant portions of the proposed MCMC iterations were accepted – implying that regions of high posterior probability were sufficiently explored. Moreover, the sampling efficiencies have improved and now lie between 0.09 and 0.26, which also indicates higher efficiency indicating lower autocorrelation of the MCMC draws. The posterior mean for \{beta\} is now exactly 0.95, identical to its prior mean of 0.95. The posterior mean for \{delta\} is now 0.04, identical to its prior mean of 0.04. The posterior mean for \{alpha\} remains at 0.34, which is near its prior mean of 0.35. The posterior mean for \{rhoz\} has 0.79, significantly closer to its prior mean of 0.80. In the blocked option
model, almost all the parameters show little updating, indicating that the likelihood is uninformative along several dimensions of the model’s parameter space.

4.4 Posterior Diagnostics and Plots

In the re-estimated model with block options (a reference to Table 4), all the parameters have sampling efficiencies of more than 1%, indicating good mixing. This also implies that it is quicker for the MCMC chain to explore the posterior distribution.

### Table 4. Efficiency Summary Statistics with block options

| Efficiency summaries | MC | MC sample size = 10000 |
|----------------------|----|------------------------|
| Efficiency:         |    | 0.000                  |
|                      | min = 0.0826               |
|                      | avg = 0.1776               |
|                      | Max = 0.2558               |

|        | ESS | Corr. time | Efficiency |
|--------|-----|------------|------------|
| beta   | 1161.03 | 8.61       | 0.1161     |
| delta  | 2557.85 | 3.91       | 0.2558     |
| alpha  | 682.57  | 11.33      | 0.0883     |
| rhoz   | 2397.15 | 11.33      | 0.2397     |
| sd(e.z)| 1879.51 | 5.32       | 0.1880     |

Source: STATA Output

The re-estimated model with block options for all four parameters in Figure 3 shows that Autocorrelations decays more quickly, and the trace plot shows good mixing and mean reversion, whilst the density plots show that the first- and second-half densities do not substantially differ from the full-sample density.

![Figure 3. Post Estimation Diagnostics plots for the parameters in the block option model](source: STATA Output)

After adjusting for autocorrelations and remedying the identified convergence problems, we thereafter proceeded to account for impulse response functions, which shows the impact of the state variable technological shock and the response of model variables – specifically output, capital stock, consumption, and interest rate.
4.5 Impulse Response Functions

Each panel in Figure 4 displays the response of one model variable to the impulse of technological shock. Each step is one quarter, implying four steps equating to one year after the shock. In the top-left panel, consumption crises follow a mostly flat trajectory for the first eight periods after the shock and then fall to return to their steady-state. In the top-left panel, the capital stock (k) does not move in the first period but rises afterwards in a hump-shaped pattern. In the top-right panel, the interest rate (r) rises on impact, remains elevated for the first four periods, and then dips below its steady-state value in the fifth period - it then returns to its steady-state from below. In the bottom-left panel, output y rises on impact and then declines monotonically back to its steady-state. For a numerical impulse response function, see the result output in the Appendix.

Figure 4 above also indicates the individual and combined impulse response of different variables to a positive investment-specific technological shock. As observed, this type of technological shock generates dynamic responses of the relevant variables differently relative to an aggregate productivity shock.

![Figure 4. Impulse response function output](image)

The response of consumption (c) is positive, but its initial values are lower than at subsequent peak periods. This model indicates that in Sierra Leone, the technological shock has a positive relationship with consumption since surprise advances in technology imply higher employment, and by extension higher income and consumption.

Capital stock (k) does not move in the first period but rises afterwards in a hump-shaped pattern. While k is also positively related to the shock, it does not move in the initial period due to the rigidities involved in acquiring new capital. The intuition is that in Sierra Leone, positive technological shock makes it profitable to invest in new capital, as its productivity is higher than the productivity of the installed capital stock. This is what causes a rise in investment, which is accumulated into capital stock. The investment-specific technological shock causes investment units to be cheaper than consumer units. This provokes an inter-temporal substitution effect between consumption and saving and an intra-temporal effect between consumption and leisure.

The relationship between interest rate and the shock is mixed. The interest rate (r) rises on impact, remains elevated for the first four periods, and then dips below its steady-state value - it then returns to its steady-state from below. Increases in income and consumption are inflationary (at least in the short run) and therefore, raising the level of interest rate above its steady-state is not
unexpected. However, this policy action is usually short-lived as a prolonged high-interest rate may exert a drag on long-run growth. Interest rates, which reflect the dynamics of marginal productivity of production factors, show different behaviour. The interest rate or cost of acquiring new capital rises on the impact of technological shock, but the response is negative afterwards. This indicates that in Sierra Leone, interest rates spread grows with the impact of a positive technological shock. The practical intuition behind this is that firms in Sierra Leone save less and borrow more to acquire new technologies. The increased borrowing drags the lending rates upwards and the savings rate in the reverse direction. However, (with the inclusion of a monetary institution bloc in the model), Sierra Leone may intervene in the long run coupled with firms’ investment returns arising from the acquired funds that were used to finance technological upgrades - and as such, this may reduce the interest spread in long-run steady-state from below optimal.

Output (y) shows a positive relationship with the shock as expected. Surprise advances in technology stimulate economic activities and thus raise the level of output (y). In summary, Investment-specific technological shock generates an inter-temporal substitution between “investment and consumption”, as well as “consumption and leisure”, and when put together may result in an upward response in the level of output.

5.0 Conclusion

The result shows that investment-specific technological shock partly explains business cycle fluctuations in Sierra Leone. Since these results are based on a procedure that abstracts from orthogonal transitory technology shock, the findings may be viewed as representing a lower bound on the overall contribution of technology shock to business cycles. Therefore, the results strongly suggest that technology shock, or more generally, shocks to the efficiency of producing goods are important for understanding business cycles in Sierra Leone. We, therefore, identify the following as implications of the findings:

- The impact of technology shock on output, capital stock, and consumption is more persistent than that of interest rate on the Sierra Leone economy.
- Advances in technology can be used to stimulate long-term growth - in other words, technological innovation is crucial for long-term growth in Sierra Leone.
- Monetary policy authority (the BSL to speak) and the entire financial system have a role to play in stabilising the aftermath of high income and consumption catalysed through means of technology shock – with an expansion of the model blocs.
- The research partly confirms the prediction of neoclassical growth theory for Sierra Leone – typically proving that long-run productivity growth is driven (mainly) by technological progress.

It is worth noting that this study is built on the neoclassical growth theory, which postulates technological change as a driving force for long-term growth. The model in its current state is somehow endogenously focused, with minimal scope for external influence and the monetary authority’s influence to utilise its mandate of influencing changes in policy rates needed to induce investments through savings or technology-induced means. Despite this limitation and the emerging focus of DSGE modelling as a tool to support effective policy formulation at the BSL, thoughts have been given to enhancing the model’s future capabilities to incorporate both the monetary and external blocs to fully assess transmission impact of technological shock on the entire economy.

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Appendix. Impulse Response Function Numerical Results

| Step | (1) irf | (1) Lower | (2) irf | (2) Lower | (3) irf | (3) Lower | (4) irf | (4) Lower | (4) Upper |
|------|---------|-----------|---------|-----------|---------|-----------|---------|-----------|-----------|
| 0    | 1.61123 | 1.0034    | 2.38143 | 0         | 0       | 5.34225   | 4.46207 | 6.46708   | 5.42899   |
| 1    | 1.90351 | 1.20025   | 2.53327 | 1.55438   | 0.94414 | 1.65555   | 1.35158 | 2.00261   | 1.48511   | 2.56046   | 1.71185   | 5.42899   |
| 2    | 2.10113 | 1.32234   | 3.08386 | 1.8118    | 1.44008 | 2.70848   | 2.14739 | 1.2020    | 1.37469   | 1.95919   | 3.06884   | 5.02997   |
| 3    | 2.19453 | 1.39675   | 3.19756 | 2.31018   | 1.49223 | 3.35999   | 1.12465 | 1.22099   | 2.19461   | 1.44027   | 2.50208   | 4.57523   |
| 4    | 2.22179 | 1.42974   | 3.23553 | 2.52924   | 1.59258 | 3.72273   | 3.96231 | 2.85044   | 1.82351   | 4.00969   | 4.80166   | 4.10999   |
| 5    | 2.20189 | 1.42052   | 3.18972 | 2.08458   | 1.68731 | 3.90263   | -0.65269 | 1.09737   | 2.84751   | 2.64019   | 1.71996   | 3.78814   |
| 6    | 2.14712 | 1.39903   | 3.00957 | 2.84371   | 1.03736 | 3.97519   | -0.54052 | 1.37571   | -0.59498  | 2.32965   | 1.45532   | 4.37635   |
| 7    | 3.00687 | 1.35549   | 3.18615 | 2.88858   | 1.90662 | 3.93036   | -0.24184 | 1.55548   | -0.19301  | 0.60436   | 1.24501   | 3.11911   |
| 8    | 1.97515 | 1.20351   | 2.84042 | 2.84096   | 1.94583 | 3.96694   | -1.00438 | 1.61349   | -2.29673  | 1.83629   | 1.07787   | 2.90645   |
| 9    | 1.83802 | 1.22781   | 2.78452 | 2.756     | 1.89287 | 3.79243   | -1.11804 | 1.85344   | -0.22121  | 1.61996   | 0.94019   | 2.50606   |
| 10   | 1.79084 | 1.18839   | 2.52755 | 2.56647   | 1.82002 | 3.67912   | -1.10932 | 1.84605   | -0.66188  | 0.46976   | 0.81951   | 2.34585   |
| 11   | 1.66222 | 1.00691   | 2.4976   | 2.52319   | 1.72058 | 3.53147   | -1.20341 | 1.65255   | -0.76967  | 1.31768   | 0.72169   | 2.42384   |
| 12   | 1.55551 | 0.96985   | 2.3672   | 2.38669   | 1.61734 | 3.50114   | -1.2005   | 1.60908   | -0.95641  | 1.18689   | 0.63554   | 2.08026   |
| 13   | 1.45114 | 0.90354   | 2.2869   | 2.28759   | 1.58056 | 3.25051   | -1.17789 | 1.56088   | -0.82807  | 0.79005   | 0.55958   | 1.85035   |
| 14   | 1.35102 | 0.87127   | 2.1987   | 2.09831   | 1.57872 | 3.03812   | -1.14148 | 1.51974   | -0.80928  | 0.96735   | 0.49337   | 1.74011   |
| 15   | 1.25313 | 0.79956   | 2.00318 | 1.92314   | 1.52174 | 2.84616   | -1.09912 | 2.65173   | -0.76425  | 1.46040   | 0.44266   | 1.60554   |
| 16   | 1.16419 | 0.69269   | 1.88554 | 1.85379   | 1.39224 | 2.70224   | -1.04402 | 1.42941   | -0.71118  | 0.74172   | 0.38085   | 1.48136   |
| 17   | 1.07843 | 0.59854   | 1.78013 | 1.71101   | 1.10505 | 1.65718   | -0.90897 | 1.37413   | -0.65135  | 0.52979   | 0.40778   | 1.36742   |
| 18   | 0.99943 | 0.52509   | 1.68701 | 1.50325   | 0.88715 | 1.50714   | -0.32286 | 1.32116   | -0.58335  | 0.55063   | 0.30196   | 0.28275   |
| 19   | 0.92267 | 0.45515   | 1.56234 | 1.47481   | 0.81393 | 1.37255   | -0.35546 | 1.27277   | -0.50630  | 0.58567   | 0.26538   | 0.18462   |
| 20   | 0.85250 | 0.40938   | 1.48092 | 1.36549   | 0.72002 | 2.24341   | -0.19696 | 1.22126   | -0.46807  | 0.54793   | 0.21988   | 0.107829  |

Footnote means reported.

| irfname | tdlgdsb, irf, impulse = z, and response = c. | irfname | tdlgdsb, irf, impulse = z, and response = k. | irfname | tdlgdsb, irf, impulse = z, and response = r. | irfname | tdlgdsb, irf, impulse = z, and response = y. |

Source: STATA Output