IMPORTANCE OF CLUSTER STRUCTURAL EVOLUTION IN USING X-RAY AND SUNYAEV-ZELDOVICH EFFECT GALAXY CLUSTER SURVEYS TO STUDY DARK ENERGY

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Received 2002 June 13; accepted 2002 November 18

ABSTRACT

We examine the prospects for measuring the dark energy equation-of-state parameter \(w\) within the context of the still uncertain redshift evolution of galaxy cluster structure. We show that for a particular X-ray survey (Sunyaev-Zeldovich effect [SZE] survey), the constraints on \(w\) degrade by roughly a factor of 3 (factor of 2) when one accounts for the possibility of nonstandard cluster evolution. With follow-up measurements of a cosmology-independent, masslike quantity it is possible to measure cluster evolution, improving constraints on cosmological parameters (such as \(w\) and \(\Omega_M\)). We examine scenarios in which 1%, 10%, and 100% of detected clusters are followed up, showing that even a modest follow-up program can enhance the final cosmological constraints. For the case of follow-up measurements on 1% of the cluster sample with an uncertainty of 30% on individual cluster masslike quantities, constraints on \(w\) are improved by a factor of 2–3. For the best-case scenario of a zero-curvature universe, these particular X-ray and SZE surveys can deliver uncertainties on \(w\) of \(\approx 4%–6\%\).

Subject headings: cosmic microwave background — cosmological parameters — cosmology: theory — X-rays: galaxies: clusters

On-line material: color figures

1 INTRODUCTION

Galaxy clusters have been used extensively to determine the cosmological matter density parameter and the amplitude of density fluctuations. Cluster surveys in the local universe are particularly useful for constraining a combination of the matter density parameter \(\Omega_M\) and the normalization of the power spectrum of density fluctuations (we describe the normalization using \(\sigma_8\), the rms fluctuations of overdensity within spheres of \(8 h^{-1}\) Mpc radius; see, e.g., Henry 1997; Viana & Liddle 1999; Reiprich \\& Böhringer 2002); surveys that probe the cluster population at higher redshift are sensitive to the growth of density fluctuations, allowing one to break the \(\Omega_M-\sigma_8\) degeneracy that arises from local cluster abundance constraints (Eke, Cole, \\& Frenk 1996; Bahcall \\& Fan 1998). Wang \\& Steinhardt (1998) argued that a measurement of the changes of cluster abundance with redshift would provide constraints on the dark energy equation-of-state parameter \(w \equiv p/\rho\).

Describing the problem in terms of cluster abundance only makes sense in the local universe, because, of course, one cannot measure the cluster abundance without knowing the survey volume; the survey volume beyond \(z \sim 0.1\) is sensitive to cosmological parameters that affect the expansion history of the universe—namely, the matter density \(\Omega_M\), the dark energy density \(\Omega_E\), and the dark energy equation-of-state parameter \(w\). A cluster survey of a particular piece of the sky with appropriate follow-up actually delivers a list of clusters with mass estimates and redshifts—that is, the redshift distribution of galaxy clusters above some detection limit.

Recently, it has been recognized that with current instrumentation it is possible to use such surveys of galaxy clusters extending to redshifts \(z > 1\) to precisely study the amount and nature of the dark energy (Haiman, Mohr, \\& Holder 2001). Clusters are promising tools for precision cosmological measurements, because they exhibit striking regularity and they exist throughout the epoch of dark energy domination. Moreover, their use is complementary to studies of cosmic microwave background (CMB) anisotropy and Type Ia supernova (SN Ia) distance measurements (Haiman et al. 2001; Levine, Schulz, \\& White 2002; Hu \\& Kravtsov 2003). Following Haiman et al. (2001), a series of analyses appeared that explore the theoretical and observational obstacles to precise cosmological measurements with cluster surveys. Holder, Haiman, \\& Mohr (2001) applied the Fisher matrix formalism to the cluster survey problem and showed that high-yield Sunyaev-Zeldovich effect (SZE) cluster surveys can provide precise constraints on the geometry of the universe through simultaneous measurements of \(\Omega_E\) and \(\Omega_M\). Weller, Battye, \\& Kneissl (2002) demonstrated that future SZE surveys might constrain the variation of the dark energy equation-of-state parameter \(w(z)\). Hu \\& Kravtsov (2003) examined the effects of cosmic variance on cluster surveys, including as well the effects of imprecise knowledge of a more complete list of cosmological parameters. Levine et al. (2002) examined an X-ray cluster survey, showing that a sufficiently large survey allows one to measure cosmological parameters and constrain the all-important cluster mass-observable relation simultaneously.

An important caveat to these works is that the authors assumed that the evolution of cluster structure with redshift was perfectly known. In this paper, we examine the effects of uncertainties about cluster structural evolution on cosmological constraints from cluster surveys, finding that current survey projections that ignore this evolution uncertainty overstate the cosmological sensitivity of the survey.

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Furthermore, we examine the effects of survey follow-up to measure a cluster mass–like quantity \( M_f \), demonstrating that an appropriately designed survey can overcome this evolution uncertainty.

In addition, our calculations underscore the importance of incorporating information from multiple observables into future cluster surveys. Clusters of galaxies are dark matter–dominated objects with baryon reservoirs in the form of an intracluster medium (ICM) and a galaxy population. Clusters can be found through the light the galaxies emit, the gravitational lensing distortions the cluster mass introduces into the morphologies of background galaxies, the X-rays emitted by the energetic ICM, the distortion that the hot ICM introduces into the CMB spectrum (SZE), and the effects that the ICM has on jet structures associated with active galaxies in the cluster. These methods are largely complementary, each having different strengths. It appears that X-ray and SZE signatures of clusters are higher contrast observables than are weak lensing or galaxy light. That is, massive galaxy clusters are more prominent relative to the far more abundant lower mass halos and the large-scale filaments when viewed with the SZE or X-rays; projection effects are a far more serious concern when using galaxy light or weak-lensing signatures. Studies of the highest redshift galaxy clusters will likely be done with the SZE, because of the redshift independence of the spectral distortion in the CMB. Any effort to carry out a precise cosmological study using galaxy clusters will undoubtedly be most effective through some combination of these complementary cluster observables.

The paper is arranged in the following way. In § 2 we describe two representative surveys and survey follow-up. Section 3 contains a description of our estimates of the survey sensitivity when follow-up is included, as well as a description of our fiducial model. Results are presented in § 4 and discussed further in § 5.

2. FUTURE GALAXY CLUSTER SURVEYS

A study of using cluster surveys to probe dark energy begins with the redshift distribution of detectable clusters within a survey solid angle \( \Delta \Omega \),

\[
\frac{dN}{dz} = \Delta \Omega \frac{dV}{dz d\Omega}(z) \int_0^\infty f(M) \frac{dn(M,z)}{dM} dM,
\]

(1)

where \( dV/dzd\Omega \) is the comoving volume element, \( (dn/dM)dM \) is the comoving density of clusters of mass \( M \), and \( f(M) \) is the cluster selection function for the survey. In this analysis we take \( f(M) \) to be a step function at some limiting mass \( M_{\text{lim}} \), which corresponds to the mass of a cluster that lies at the survey detection threshold. We use the cluster mass function \( dn/dM \) determined from structure formation simulations (Jenkins et al. 2001).

In practice, surveys select clusters using observables like the X-ray flux, SZE flux, galaxy light, or weak-lensing shear. Thus, in addition to the ingredients above, one requires a virial mass–observable relation (such as \( M_{\text{X}} - L_X \), \( M_{\text{SZE}} - L_{\text{SZE}} \), or \( M - r_{200} \)). Low-redshift clusters do exhibit regularity (see, e.g., David et al. 1993; Mohr & Evrard 1997), suggesting that observables like the ICM X-ray luminosity and temperature are good mass estimators (Finoguenov, Reiprich, & Böhringer 2001; Reiprich & Böhringer 2002). Cluster mass-to-light ratios have been studied for decades, and it may be that this body of work, together with modern data sets, will allow more conclusive statements about how well galaxy light traces cluster halo mass. Hydrodynamical simulations lead us to expect that the SZE luminosity (related to the total thermal energy within the virial region) should be the best ICM observable for predicting mass, but we await new observations with next-generation SZE instruments to demonstrate this.

A central feature of these mass-observable relations is that they evolve with redshift because of the increasing density of the universe at earlier times (and the changing ratio of distance to lens and source, in the case of weak lensing). Within standard structure formation models, galaxy clusters form self-similarly, and so there are standard evolution models for each mass-observable relation (see, e.g., Bryan & Norman 1998; Mohr et al. 2000; Evrard et al. 2002). Results to date suggest that the degree of cluster regularity locally and at intermediate redshift is comparable (Mushotzky & Scharf 1997; Mohr et al. 2000; Vikhlinin et al. 2002). However, given the central importance of cluster mass estimates in using surveys to study dark energy, we can only regard these standard structure formation models as a guide; ultimately, one needs to determine the evolution of cluster structure observationally. In this paper we examine the effects that nonstandard redshift evolution of cluster structure would have on our ability to use cluster surveys to study the dark energy. As detailed below, we explore nonstandard redshift evolution by allowing an additional dependence of \((1+z)^\gamma\) in the evolution of the relevant mass-observable relations.

2.1. An X-Ray and an SZE Survey

To examine these new effects, we adopt two representative surveys that are being promoted as ways of measuring the dark energy equation of state. Namely, we examine the following two high-yield surveys: (1) a \( 10^4 \) deg\(^2\) flux-limited X-ray survey proposed as part of the DUET mission to the NASA Medium-class Explorer Program, and (2) a 4000 deg\(^2\) SZE survey to be carried out with an NSF-funded 8 m South Pole Telescope (SPT). Figure 1 contains a plot of the redshift distribution and limiting mass for both surveys.

We model the DUET X-ray survey as having a bolometric flux limit of \( f_X > 1.25 \times 10^{-13} \) ergs s\(^{-1}\) cm\(^{-2}\) (corresponding to \( f_X > 5 \times 10^{-14} \) ergs s\(^{-1}\) cm\(^{-2}\) in the 0.5–2 keV band). For our fiducial cosmological model (see § 3.2 below), this survey yields \( \sim 21,600 \) detected clusters, consistent with the known X-ray log \( N \)–log \( S \) relation for clusters (see, e.g., Gioia et al. 2001). For our mass-observable relation, we adopt a bolometric X-ray luminosity–mass relation,

\[
4\pi d_L^2 = A X M_{200}^{0.8} E(z)(1+z)^\gamma,
\]

(2)

where \( f_X \) is the observed flux in units of ergs s\(^{-1}\) cm\(^{-2}\), \( d_L \) is in units of megaparsecs, \( M_{200} \), in units of \( 10^{15} M_\odot \), is the mass enclosed within a radius \( r_{200} \) having an overdensity of \( 200 \) with respect to critical density, and \( E(z) = H_0 E(z) \), where \( E(z) = \Omega_M(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda^{1/2} \). The \( E(z) \) factors follow the evolution of the critical density of the universe, \( \rho_{\text{crit}} = 3H^2/8\pi G \). We convert \( M_{200} \) to \( M \), the halo mass appropriate for our mass function at redshift \( z \), using a halo model (Navarro, Frenk, & White 1997, hereafter NFW; see discussion below regarding the effects of uncertainties in this conversion). Our standard evolution model ignores the \( T^{1/2} \)-dependence of the bolometric bremsstrahlung
radiation, because X-ray surveys detect clusters using detected photons rather than detected energy. We introduce the possibility of nonstandard evolution of the mass-observable relation with the parameter $\gamma_X$. We take $\gamma_X = 0$ to be consistent with the observed weak evolution in the luminosity-temperature relation (Vikhlinin et al. 2002), and we choose $\beta_X = 1.807$ and $\log \Delta X = -3.926$, consistent with observations (Reiprich & B"ohringer 2002). The overall $h$-scaling of the limiting mass is $h^{-1.11}$.

We model the SPT SZE survey as a flux-limited survey, with $f_{SZ} > 5 \text{ mJy}$ at 150 GHz. Within our fiducial cosmological model this survey would yield $\sim 13,500$ clusters with measured fluxes. The mass-observable relation is

$$f_{SZ}(z, \nu) dA = 3.781 f(\nu) f_{ICM} T M_{200} (1+z)^{\gamma_{SZ}},$$

$$M_{200} = A_{SZ} \frac{k_B T}{E(z)},$$

where $f(\nu)$ is the frequency dependence of the SZE distortion, $f_{SZ}$ is the observed flux in millijansky, $T$ is in kelvins, $M_{200}$ is in units of $10^{15} M_\odot$, $f_{ICM} = 0.12$ (see, e.g., Mohr, Mathiesen, & Evrard 1999), and $d_A$ is in units of megaparsecs (see Diego et al. 2002). We use $\log A_{SZ} = 13.466$, $\gamma_{SZ} = 1.48$ (Finoguenov et al. 2001), and $\gamma_{SZ} = 0$ to model standard structure evolution. In this form, the overall $h$-scaling of the limiting mass is $h^{-1.11}$. Note that in determining the estimated uncertainties on cosmological parameters, we allow the normalization of these mass-observable relations to be free to vary. The survey contains enough information to solve for the best normalization and the cosmological parameters simultaneously; therefore, shifts in model inputs like $f_{ICM}$ within the observational uncertainties have minimal effect on our conclusions.

A generic problem with flux-limited surveys is that at low redshift the implied mass limit drops well below those masses corresponding to galaxy clusters. The flux from a nearby object is spread over a much larger portion of the sky, and surface brightness selection effects become important. We model these complications by imposing a minimum cluster mass of $10^{14} h^{-1} M_\odot$. This lower limit on the survey mass limit is readily apparent below $z \sim 0.25$ in Figure 1.

2.2. Follow-up of Large Solid Angle Surveys

The redshift distribution of clusters contains far more cosmological information than the surface density of clusters (Haiman et al. 2001) or the angular correlation function (see, e.g., Komatsu & Seljak 2002). Thus, in both these surveys each detected cluster will be followed up with multi-band optical and near-IR photometry to provide photometric redshift estimates. These same data can be used to estimate cluster masses through their weak-lensing effects on background galaxies (see, e.g., Bartelmann & Schneider 2001) and the total detected light from cluster galaxies.

In addition, some of these clusters can be followed up with detailed X-ray, SZE, or galaxy spectroscopic observations that allow one to measure the masslike quantity $M_f(\theta) = M(\theta)/d_A$, which we refer to as the follow-up mass. As an example, in the case of follow-up X-ray observations that deliver the projected ICM temperature profile and surface brightness profile, it is straightforward to extract the underlying ICM density $\rho(\theta)$ and temperature profile $T(\theta)$, to then estimate the follow-up mass $M_f$ as

$$M_f(\theta) = -\theta k_B T(\theta) \left( \frac{d \ln \rho}{d \ln \theta} \frac{d \ln T}{d \ln \theta} \right),$$

where $m_p$ is the proton mass, $k_B$ is Boltzmann’s constant, $G$ is Newton’s constant, and the ICM number density $n \equiv \rho/\mu m_p$. Note that only the shape of the ICM density and temperature profiles is required (i.e., knowledge of the actual distance to the system is not required). The follow-up mass $M_f$ can then be examined within some angle $\theta$, along with the cluster X-ray or SZE flux. At fixed redshift, this follow-up would produce an $M_f-f_X$ or $M_f-f_{SZ}$ relation, which would provide direct constraints on the structural evolution of the clusters. As we see in the next section, the parameter sensitivity of these scaling relation observations can exhibit quite different degeneracies from those of the cluster redshift distribution, making the scaling relations and the cluster redshift distribution complementary. In § 3.1, we describe how these survey follow-up observations are included in our estimates of the cosmological sensitivity of the survey.

3. COSMOLOGICAL SENSITIVITY OF A SURVEY

3.1. Fisher Matrix Technique

We employ the Fisher matrix technique to probe the relative sensitivities of two cluster surveys to different cosmological and cluster structural parameters. The Fisher matrix information for a data set (see Tegmark, Taylor, & Heavens 1997; Eisenstein, Hu, & Tegmark 1998) is defined as $F_{ij} \equiv -(\partial^2 \ln L / \partial p_i \partial p_j)$, where $L$ is the likelihood for an observable $(dN/dz$ for the survey and $M_f$ for the follow-up) and $p_i$ describes our parameter set. The inverse $F^{-1}$ describes the best attainable covariance matrix $[C_{ij}]$ for measurement of the parameters considered. The diagonal terms in $[C_{ij}]$ then give the uncertainties on each of our parameters. In calculating these uncertainties, we have added the Fisher matrix for the follow-up observations ($F_{f}^{\ell}$), the Fisher matrix for the cluster redshift distribution ($F_f^R$), and several external priors that are discussed below.
We construct the survey Fisher matrix $F^s_{ij}$, following Holder et al. (2001), as

$$F^s_{ij} = \sum_n \frac{\partial N_n}{\partial p_i} \frac{\partial N_n}{\partial p_j} \frac{1}{N_n},$$

(5)

where we sum over redshift bins of size $\Delta z = 0.01$ to $z_{\text{max}} = 3.0$ and $N_n$ represents the number of surveyed clusters in each redshift bin $n$. The Fisher matrix for the follow-up is constructed as

$$F^f_{ij} = \sum_n \frac{dV}{dz} \int dM f \frac{dn}{dM} \left( \frac{\partial M_f}{\partial p_i} \frac{\partial M_f}{\partial p_j} \frac{1}{\sigma_{M_f}} \right),$$

(6)

where $M_f$ is a function of halo mass $M$ and angular radius $\theta$ and $f(dn/dM)$ represents the number of clusters of mass $M$ for which follow-up mass measurements are available in a particular redshift bin. We examine cases in which the follow-up fraction is 1%, 10%, and 100%.

To generate the follow-up Fisher matrix, we calculate the cluster binding mass within radius $r = d_M \theta$ for a halo with virial mass $M$. To do this calculation, we assume that cluster mass profiles are well represented, on average, by NFW models with concentration index $c = 5$. In practice clusters undergo merging quite frequently, and there is a range of halo shapes. This introduces a "theoretical" uncertainty to the follow-up mass. In this analysis we take $\sigma_M = 0.3 M_f$ to be the characteristic uncertainty in the follow-up mass measurements. This uncertainty reflects the observational uncertainty on individual cluster follow-up mass measurements, as well as the uncertainties inherent in predicting the follow-up mass from the halo virial mass. As is clear from equation (6), the redshift and mass distributions of the follow-up clusters match those of the full cluster survey sample; that is, we do not choose to follow up only high-redshift clusters, which would presumably provide the tightest constraints on our evolution parameter. We also choose $\theta$ to be a dynamically varying quantity, fixed to be 95% of the virial radius corresponding to the cluster limiting mass at each redshift. Thus, follow-up at all redshifts corresponds to masslike measurements at radii within the virial radius $r(\theta(M) < r_{200}(M))$.

3.2. Fiducial Cosmology and External Constraints

The fiducial cosmological parameters of our model are $h = 0.65$ (see, e.g., Hendry et al. 2001; Ajhar et al. 2001; Reese et al. 2002), $\Omega_M = 0.3$ (see, e.g., Mohr et al. 1999; Grego et al. 2001), $\Omega_{\text{tot}} = \Omega_M + \Omega_L = 1$ (see, e.g., Netterfield et al. 2002; Pryke et al. 2002), $w = -1$, $n = 0.96$ (Netterfield et al. 2002), $\Omega_B = 0.047$ (Burles & Tytler 1998), and a COBE-normalized $\sigma_8 = 0.72$ (Bunn & White 1997). Note that we use a rather low value of $\sigma_8$, which is consistent with the recent 2dF analysis (Lahav et al. 2002). Because the expected number density of clusters is very sensitive to the value of $\sigma_8$, our fiducial SZE survey has fewer clusters when compared to some previous studies (see, e.g., Holder et al. 2001).

Cosmological constraints from cluster surveys are complementary to constraints from SN Ia distance measurements and observations of the anisotropy of cosmic microwave background. This is particularly true when it comes to using cluster surveys to measure the dark energy equation-of-state parameter $w$ (Haiman et al. 2001). In combination with precise CMB constraints on the curvature ($\Omega_k = 0$), cluster surveys enable precise measurements of the dark energy equation of state; however, when curvature is allowed to depart from zero—even slightly—the cluster constraints on $w$ weaken considerably. For a flat universe, the constraints on $w$ ($\Omega_M$) from our SZE survey, assuming standard evolution and a 100% follow-up, are 0.0406 (0.0139), whereas for $\sigma_k = 0.01$ the constraints are 0.1207 (0.0139).

For the analysis presented here, we adopt relatively conservative priors from future CMB anisotropy studies and distance measurements. We assume that the power spectrum index $n$ will be known to 5%, i.e., $\sigma_n = 0.05$, that the Hubble parameter will also be known to 5%, i.e., $\sigma_h = 0.0325$, and that the total density parameter $\Omega_{\text{tot}}$ will be known to 1%, i.e., $\sigma_{\Omega_{\text{tot}}} = 0.01$. In addition, we take the prior on the baryon density parameter ($\Omega_B = 0.047$) to be $\sigma_{\Omega_B} = 0.004$. For reasonable values of $\Omega_B$, surveys are affected by $\Omega_B$ variations only through minor effects on the transfer function for density perturbations (see also Levine et al. 2002). Finally, we neglect the possibility of a variation with redshift in the equation-of-state parameter $w$ (Weller et al. 2002).

4. COSMOLOGICAL PARAMETER CONSTRAINTS

Our results are listed in Table 1 and highlighted in Figures 2 and 3. Table 1 contains a listing of 1 $\sigma$ uncertainties on all seven cosmological and three mass-observable relation parameters (see eqs. [2] and [3] for definitions). The first row contains a listing of the priors adopted for the runs. Following that are the results for the SZE and X-ray surveys. For each survey we show the constraints in the case of standard evolution (i.e., $\gamma = 0$), followed by those for nonstandard evolution ($\gamma$ is free parameter). The rows that follow highlight the effect of 1%, 10%, and 100% follow-up. Following those scenarios is what we consider to be the ideal case of a flat universe with 100% survey follow-up. Finally, we show the case for follow-up only (i.e., the Fisher matrix derived from the redshift distribution is not used). The constraints in this line provide some insights into the parameter leverage that is afforded by cluster follow-up observations. In all cases, the uncertainties are absolute (i.e., $\sigma_M = 0.0177$ means $\Omega_M = 0.3 \pm 0.0177$).

4.1. Importance of Nonstandard Evolution

In the standard evolution case, the SZE and X-ray surveys compare favorably, yielding 1 $\sigma$ absolute uncertainties on $w$ ($\Omega_M$) of 0.1629 and 0.1659 (0.0177 and 0.0147), respectively. However, when one takes into account the possibility of nonstandard evolution, the constraints on $w$ weaken by almost a factor of 2, to $\sim 0.25$, for the SZE and a factor of 3, to $\sim 0.45$, for the X-ray survey; $\Omega_M$ constraints weaken by close to a factor of 2, to $\sim 0.026$, for the X-ray but are only slightly affected in the SZE survey. The constraints from the cluster redshift distribution $dN/dz$ on $\gamma_{\text{SZ,X}}$ are very weak, at 0.46 and 1.29, respectively; this large uncertainty in the evolution of the mass-observable relation contributes to the weakened sensitivity to other cosmological parameters.

The importance of evolution in interpreting the cluster redshift distribution contrasts somewhat with the results of the Levine et al. (2002) study, which showed that prior knowledge of the normalization of the mass-observable relation has only a weak effect on the cosmological sensitivity
TABLE 1
Estimated Parameter Constraints

| Description                      | $\Omega_M$ | $\Omega_{\text{tot}}$ | $\sigma_8$ | $w$  | $h$  | $\Omega_B$ | $\log A$ | $\beta$ | $\gamma$ |
|----------------------------------|------------|------------------------|------------|------|------|------------|----------|--------|--------|
| Priors                           | ...        | 0.0100                 | ...        | 0.0325 | 0.0500 | 0.0040     | ...      | ...    | ...    |

**SPT SZE Survey**

| Description                      | $\Omega_M$ | $\Omega_{\text{tot}}$ | $\sigma_8$ | $w$  | $h$  | $\Omega_B$ | $\log A$ | $\beta$ | $\gamma$ |
|----------------------------------|------------|------------------------|------------|------|------|------------|----------|--------|--------|
| Standard evolution               | 0.0177     | 0.0077                 | 0.0134     | 0.1629 | 0.0323 | 0.0478     | 0.0040   | 0.1392 | 0.0064 |
| Nonstandard evolution            | 0.0184     | 0.0079                 | 0.0195     | 0.2487 | 0.0323 | 0.0479     | 0.0040   | 0.1505 | 0.0064 |
| +1% follow-up                    | 0.0167     | 0.0074                 | 0.0157     | 0.1404 | 0.0285 | 0.0476     | 0.0040   | 0.1423 | 0.0064 |
| +10% follow-up                   | 0.0147     | 0.0070                 | 0.0122     | 0.1237 | 0.0259 | 0.0472     | 0.0040   | 0.1320 | 0.0061 |
| +100% follow-up                  | 0.0119     | 0.0068                 | 0.0109     | 0.1207 | 0.0248 | 0.0471     | 0.0040   | 0.1004 | 0.0046 |
| +flat +100% follow-up            | 0.0139     | ...                    | 0.0109     | 0.0406 | 0.0235 | 0.0439     | 0.0039   | 0.1001 | 0.0046 |
| 100% follow-up only              | 0.2360     | 0.0099                 | ...        | 0.6569 | ...    | ...        | ...      | 0.2204 | 0.0067 |

**DUET X-Ray Survey**

| Description                      | $\Omega_M$ | $\Omega_{\text{tot}}$ | $\sigma_8$ | $w$  | $h$  | $\Omega_B$ | $\log A$ | $\beta$ | $\gamma$ |
|----------------------------------|------------|------------------------|------------|------|------|------------|----------|--------|--------|
| Standard evolution               | 0.0147     | 0.0087                 | 0.0113     | 0.1659 | 0.0323 | 0.0476     | 0.0040   | 0.0928 | 0.0060 |
| Nonstandard evolution            | 0.0255     | 0.0092                 | 0.0441     | 0.4505 | 0.0324 | 0.0476     | 0.0040   | 0.2216 | 0.0096 |
| +1% follow-up                    | 0.0157     | 0.0086                 | 0.0167     | 0.1655 | 0.0233 | 0.0475     | 0.0040   | 0.1060 | 0.0062 |
| +10% follow-up                   | 0.0139     | 0.0085                 | 0.0116     | 0.1454 | 0.0201 | 0.0475     | 0.0040   | 0.0814 | 0.0055 |
| +100% follow-up                  | 0.0134     | 0.0084                 | 0.0105     | 0.1414 | 0.0191 | 0.0473     | 0.0040   | 0.0586 | 0.0040 |
| +flat +100% follow-up            | 0.0133     | ...                    | 0.0100     | 0.0593 | 0.0187 | 0.0467     | 0.0039   | 0.0585 | 0.0040 |
| 100% follow-up only              | 0.2018     | 0.0100                 | ...        | 0.5500 | ...    | ...        | ...      | 0.1408 | 0.0056 |

Fig. 2.—Constraints on $w$ and $\Omega_M$ for an SZE (top) and an X-ray survey (bottom). The star marks the fiducial model. Contours denote joint 1 $\sigma$ constraints in five scenarios: (1) constraints from $dN/dz$ when the cluster evolution is unknown (long-dashed line); constraints from $dN/dz$ and follow-up mass measurements for (2) 1% of sample (short-dashed line), (3) 10% of sample (dotted line), and (4) 100% of sample (solid line); and (5) the case of 100% follow-up plus added prior of a flat universe (dot-dashed line). The follow-up mass measurements are estimated to have fractional uncertainties of 30%. [See the electronic edition of the Journal for a color version of this figure.]

Fig. 3.—Constraints on the mass-observable relation normalization $A$ and redshift evolution $(1+z)^{\gamma_x}$ for an SZE (top) and an X-ray survey (bottom). The star marks the fiducial model. Contours denote joint 1 $\sigma$ constraints in four scenarios: (1) constraints from $dN/dz$ where the non-standard cluster evolution is unknown (long-dashed line) and constraints from $dN/dz$ and follow-up mass measurements for (2) 1% of sample (short-dashed line), (3) 10% of sample (dotted line), and (4) 100% of sample (solid line). The follow-up mass measurements are estimated to have fractional uncertainties of 30%. [See the electronic edition of the Journal for a color version of this figure.]
of cluster surveys (see also Diego et al. 2001). In their study, they only considered the standard evolution model. Within the context of uncertain evolution of the mass-observable relation one needs observations, in addition to $dN/dz$, to determine the evolution parameter $\gamma$ and regain high sensitivity to the equation-of-state parameter $w$. Next we examine the effects of including follow-up mass measurements.

### 4.2. Effects of Follow-up Mass Measurements

Figure 2 contains joint constraints on $\Omega_m$ and $w$ for the two surveys that highlight the effect of survey follow-up. For each survey we show constraints with nonstandard evolution and no follow-up (long-dashed line), 1% (short-dashed line), 10% (dotted line), and 100% (solid line) follow-up, along with 100% follow-up in a flat universe (dot-dashed line). The figure makes clear that even a limited follow-up program can greatly improve cosmological constraints.

Table 1 shows that as the follow-up fraction increases, the constraints on $w$ in the SZE survey tighten from 0.25 (no follow-up) to 0.14 (1% follow-up) to 0.12 (10% and 100% follow-up). The difference between 10% and 100% follow-up is minimal, suggesting that the 10 times higher cost of full survey follow-up is not a worthwhile investment when viewed solely from the perspective of obtaining constraints on the equation of state of the dark energy.

Notice that the constraints on $w$ in the cases of even limited follow-up are somewhat better than the constraints in the cases in which we assume complete knowledge of the evolution of the mass-observable relation. In the X-ray survey, a program to follow up as few as 1% of the clusters can offset the increase in uncertainties that we see in going from standard evolution to nonstandard evolution. In the SZE survey, 1% follow-up produces constraints that are somewhat better, reducing the uncertainty on $w$ from 0.1629 in the standard evolution scenario to 0.1404 in the nonstandard evolution case. This can be traced to our assumption in these calculations that the redshift distribution of follow-up mass measurements matches the redshift distribution of the full survey. The higher redshift follow-up measurements contain more information about evolution, and the SZE survey probes to higher redshift than does the X-ray survey (see Fig. 1).

Table 1 contains a listing of the effects of follow-up on all parameters. It is clear that follow-up mass measurements dramatically reduce the projected uncertainties on cosmological and scaling relation parameters. As is evident from the last column in the table, even a modest follow-up of 10% of the clusters reduces the uncertainty on $\gamma$ from 0.46 to 0.10 for the X-ray case and 1.29 to 0.12 for the SZE case. With full follow-up, the constraint on $w$ shrinks from 0.25 to 0.12 in the SZE and from 0.45 to 0.14 in the X-ray case. Even with follow-up of only 1% of the clusters in the SZE survey, one reduces the uncertainty on $w$ by half. Comparison of the uncertainties on $\gamma$ for nonstandard evolution with no follow-up with the follow-up--only case shows clearly that follow-up is very effective in constraining $\gamma$. For example, for the X-ray case the follow-up itself can constrain $\gamma$ to $\sim 0.19$, compared to 1.29 that one can achieve from the survey only. This underscores the advantage of having a follow-up program for cluster surveys.

In Figure 3, we show the constraints on the mass-observable relation normalization $A$ and the evolution parameter $\gamma$ for the four cases: no follow-up (long-dashed line) and a follow-up of 1% (short-dashed line), 10% (dotted line), and 100% (solid line) of the clusters. Follow-up has strikingly different effects in the SZE and X-ray surveys. Follow-up in the SZE survey is more effective at constraining the evolution parameter $\gamma_{SZ}$, because of the greater redshift depth of this survey. The differences in the constraints on $\log_{10} A$ generally reflect the different definitions of the normalization and its relationship to halo mass (see eqs. [2] and [3]). In contrast to the previous figure, which shows the effects of follow-up on $w$ constraints, it is clear from this figure that if one really wants to understand the normalization and evolution of the mass-observable relations, more follow-up is better.

### 4.3. Redshift Variation of Parameter Sensitivity

In Figure 4, we display an estimate of the redshift variation of the survey and follow-up sensitivity to $w$. We do this by examining the derivatives with respect to redshift of the $w$--$w$ components of the survey and follow-up Fisher matrices. These are shown as heavy lines for both the X-ray (dashed line) and SZE (solid line) surveys. We also show an estimate of the per-cluster sensitivity as lighter lines for the X-ray (dot-dashed line) and SZE (dotted line) surveys. The axes for the heavier curves are included on the left of

![Fig. 4.—Sensitivity of the follow-up (top) and survey (bottom) to change in $w$ for both X-ray and SZE surveys.](See the electronic edition of the Journal for a color version of this figure.)
the figure, and the axes corresponding to the per unit cluster values are located on the right of the figure. In both the X-ray and SZE cases and for both follow-up and the survey itself, the heavy curves show sensitivity that peaks at lower redshift than in the lighter curves that show the per-unit cluster sensitivity. This is simply a reflection of the redshift distributions of the clusters detected in the surveys, which peak at $z < 0.5$ in both surveys.

Consider now the lower panel. The higher redshift nature of the SPT SZE survey relative to the DUET X-ray is clear in this panel, where we see that the $w$-sensitivity of the SZE survey peaks near $z \sim 1$ (solid line). For the X-ray survey, the $w$-sensitivity peaks at $z \sim 0.7$ (dashed line). However, it is clear that the sensitivity per unit cluster $[(dP_{wN}/dz)/(dN/dz)]$, denoted by the dotted line for SZE and the dot-dashed line for X-ray) increases as we go to higher redshift. This emphasizes the importance of high-redshift cluster surveys for probing $w$. High-redshift clusters in the DUET survey are even more sensitive to $w$, because they are more massive and lie well beyond the exponential cutoff in the mass function; however, these clusters are so rare that none are detected in the DUET survey.

The sharp cutoff and first minipeak, at redshift $z \sim 0.3$, of $dP_{wN}/dz$ for the survey is a direct result of our requirement that clusters have masses above $10^{14} h^{-1} M_\odot$. In Figure 1, the plot of the limiting mass becomes flat at redshifts below $z \sim 0.3$; we include this mass cutoff for several reasons: (1) the mass-observable scaling relations we have adopted are for clusters, and they are inappropriate for group scale systems; and (2) a flux-limited survey becomes sensitive to surface brightness limitations at low redshift, at which the flux from these nearby objects is spread over a larger and larger solid angle. Introducing a minimum mass in our survey causes this interesting artifact in the $w$-sensitivity at low redshift, which can be understood by considering the competing $w$-dependences of the limiting mass, the survey volume element, and the growth factor of density perturbations. As long as the limiting mass is constant, the opposite sensitivities of the volume element and the growth factor to $w$ determine the net $w$-sensitivity of the survey. At low redshift, the $w$-sensitivity of the volume element dominates over that of the growth factor. Beyond redshift $z \sim 0.3$, the limiting mass suddenly rises above the minimum, allowing the net $w$-sensitivity to include that of the limiting mass. The limiting mass is sensitive to $w$ primarily through the angular diameter or luminosity distance to the redshift of interest, and this sensitivity combines with that of the growth factor to offset to a larger degree the $w$-sensitivity of the volume element. Note that $dP_{wN}/dz$ is positive definite, which is the reason for the visual appearance of a break in the sensitivity, which is actually reflecting an underlying change in sign of the $w$-sensitivity.

For the follow-up (Fig. 4, top panel), the sensitivity per unit cluster peaks at $z \sim 0.8$ for both the surveys. However, survey-weighted sensitivities of the follow-up to variation in $w$ are peaked at much lower redshifts, where the survey yields are much higher. This is due to the fact that the $w$-sensitivity of the follow-up is a balance between the $w$-sensitivities of the mass observables and the number of clusters one has at that redshift to make the measurement. The fact that the redshift dependence of the cluster mass observable is similar for both X-ray and SZE just reflects the weak mass dependence of this sensitivity. Survey strategists should consider a follow-up program that targets predominantly redshift $z \sim 1$ clusters if their goal is to constrain the evolution parameter and improve constraints on $w$; however, having evolution information over the entire redshift range of the survey is critical to testing the form of the nonstandard evolution model, which we parametrize here simply as $(1+z)^{\gamma}$.

5. DISCUSSION AND CONCLUSIONS

Any attempt to precisely measure the dark energy equation-of-state parameter $w$ with cluster surveys will require (1) a strong external prior on the curvature (presumably from CMB anisotropy studies) and (2) an understanding of the evolution of the relation between cluster halo mass and observable properties such as the X-ray luminosity, SZE luminosity, galaxy light, or weak-lensing shear. We have examined the effects of current uncertainties about cluster structural evolution; for two recently proposed cluster surveys the estimated constraints on $w$ are $\sim 2-3$ times weaker than if one assumes full knowledge of cluster evolution. Constraints on other interesting cosmological parameters are also weakened (see Table 1).

Follow-up observations to measure cluster masses directly will enable one to solve for cluster structure evolution and to enhance cosmological constraints. We have examined the effects of follow-up mass observations from hydrostatic or dynamical methods, and we find that even modest follow-up of 1% of the cluster sample can improve survey constraints. Full follow-up with mass measurements that are 30% uncertain, on average, provides cosmological constraints that match or surpass those possible through $dN/dz$ alone with full knowledge of cluster evolution. Full follow-up with weak-lensing mass measurements is currently being planned for the SPT SZE survey.

The implications are quite interesting. Essentially, to do precision cosmology with cluster surveys and follow-up, we need only know that clusters conform to mass-observable scaling relations and that these relations evolve in some well-behaved manner. Then, together with our well-established theoretical framework for structure formation, the observed cluster redshift distribution and follow-up masses of as few as 1% of the sample provide enough information to deliver precise constraints on cosmological parameters and the character and evolution of the mass-observable relation simultaneously. In a sense, cluster surveys with limited follow-up are self-calibrating: one gains detailed knowledge of the structure of the tracers (i.e., galaxy clusters) and detailed knowledge of the evolution of the universe from the same data set.

We have focused here on the mean equation-of-state parameter $w$, and for the two surveys considered we examine the redshift variation of the sensitivity to $w$. In the SPT SZE survey, the sensitivity to the dark energy equation of state peaks at $z \sim 1$, with sensitivity at or above half the peak for $0.65 \leq z \leq 1.5$. In the DUET X-ray survey, the sensitivity peaks at $z \sim 0.7$, with sensitivity at or above half the peak for $0.45 \leq z \leq 1.0$. In both surveys, the $w$-sensitivity of follow-up mass measurements peaks near redshift $z \sim 0.35$. In the case of the cluster redshift distribution, the most information about $w$ is provided by the highest redshift clusters, and so deeper, more sensitive surveys will in general be better for studies of the dark energy equation of state.
One interesting feature of our analysis is the orientation of the elliptical constraints on $w$ and $\Omega_M$ (see Fig. 2). In general, the rotation of the parameter degeneracy can be understood as the result of competing effects of changes in the volume element and the growth factor as parameters vary. Variations in $w$ and $\Omega_M$ affect the survey yield in different ways at different redshifts, and so the $w$-$\Omega_M$ degeneracy depends on the redshift distribution of a particular survey. Rotations of parameter degeneracies occur as the maximum redshift of the survey is varied (Levine et al. 2002; Hu & Kravtsov 2003). We have further found that changing the prior on $\Omega_{\text{tot}}$ and changing the degree of mass follow-up on a survey also result in rotations of the parameter degeneracy. This behavior has interesting implications for the design of cluster surveys that are optimally complementary to CMB anisotropy and SN Ia distance measurements, and it deserves further study.

In addition, we emphasize that the final constraint on the determination of cosmological parameters depends sensitively on the survey strategy and also on the details of the follow-up. For example, a different definition of $M_f(\theta)$ would lead to slightly different uncertainties. Changing $\theta$ from a quantity that varies with redshift to some fixed value leads to modest variations of the constraints. In general, the best results can be obtained by optimizing $M_f(\theta)$ so that one probes as much of the virial mass as possible.

S. M. thanks Ben Wandelt and Shiv Sethi for helpful conversations. J. J. M. thanks Zoltán Haiman for many fun discussions of cluster surveys. This work has been supported by NASA Long-Term Space Astrophysics grant NAG 5-11415 and Chandra X-Ray Observatory archival grant AR1-2002X, awarded through the Smithsonian Astrophysical Observatory.

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