Stochastic Fluctuations in the Spectrophotometric Properties of Star Clusters

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Abstract. Integrated spectrophotometric properties of stellar systems are intrinsically dispersed due to the stochastic nature of the small numbers of bright stars they contain. Among clusters, only the most massive ones can be used individually for comparison with the mean properties predicted by population synthesis calculations. The appropriate minimal masses depend, among others, on the waveband or colour index studied and on age. Selected indices (near-IR CO and H$_2$O, EW(H$_{\alpha}$)) are used to illustrate the asymmetric nature of the probability distribution of observable properties and their dependence on cluster mass.

1. Introduction

Most population synthesis codes (those based on Monte Carlo simulations excepted) predict mean properties of stellar populations as a function of fundamental model parameters such as the stellar initial mass function (IMF) and the star formation history (SFH). However, for a given model the number of stars in each area of the HR diagram is a statistical variable obeying Poisson statistics. The resulting intrinsic dispersion of integrated spectrophotometric properties is observed, both as pixel to pixel fluctuations in otherwise uniform objects (“surface brightness fluctuations”, Tonry & Schneider 1988) and as cluster to cluster variations among cluster samples restricted to similar SFHs (Ferraro et al. 1995, Girardi et al. 1995). Clusters are tempting targets for the tests and calibrations of population synthesis prediction because of the coeval nature of their stars. In this context, it is important to remember that the properties of individual clusters are representative of the mean properties only in the limit of large star numbers, i.e. large cluster masses (assuming a universal IMF). Discussing how large these masses need to be in practice is the purpose of this paper.

All results presented here are based on the population synthesis code Pégase (Fioc & Rocca-Volmerange 2000) and extensions thereof. They assume solar metallicity and a Salpeter IMF extending from 0.1 to 120 M$_{\odot}$.

2. Luminosity fluctuations

The variance of the luminosity $L$ of a population (or of $L_\lambda$ to allow for wavelength dependence) is proportional to $\sum n_i L_i^2$, where the sum extends over all luminosities $L_i$ of the HR diagram, and where $n_i$ is the corresponding expec-
tation number of stars. Intrinsically luminous stars, which already contribute significantly to the integrated luminosity despite their relatively small numbers, contribute even more exclusively to the variance (Figs. 1a, b). Figs. 1c, d and e illustrate how the strongest contributors to the flux density depend on wavelength, using a 1 Gyr old stellar population as an example. Fig. 1f shows the resulting mean spectral distribution of the flux and of the relative rms flux deviations from this mean, $\sigma_L/L$, for a population of $10^6 M_\odot$ of stars.

At all but the youngest ages, the most luminous of the red stars determine the bolometric as well as the near-IR luminosity fluctuations. Turn-off stars also contribute to $\text{var}(L_{\text{bol}})$ during the first few $10^7$ yrs; at all times they are responsible for the optical fluctuations together with horizontal branch and red clump stars.

The relative fluctuations around the mean flux, $\sigma_L/L$, are large when the subpopulation that contributes most of the variance consists of stars of large intrinsic luminosity and when the total number of these stars is small (i.e. one is not in the large $n_i$ limit considered by Tonry & Schneider 1988). Consider the simplified case of a population of $N$ stars of which a mean number $\alpha N$ belongs to the subpopulation of interest (at the wavelength of interest). Let $L_s$ be the intrinsic individual luminosity of each subpopulation stars, and $L_o$ that of each other star: $L = \alpha N L_s + (1 - \alpha) N L_o = N L$. Assuming Poisson statistics for the number of stars in the subpopulation ($\alpha N$), one gets

$$\frac{\sigma_L}{L} = \frac{\sqrt{\alpha N (L_s - L_o)}}{\alpha N (L_s - L_o) + N L_o} = \frac{1}{\sqrt{N}} \frac{\sqrt{\alpha (L_s - L_o)}}{l}$$

$\alpha, L_s, L_o$ are given by population synthesis calculations. Other assumptions (e.g. fixing other quantities than the total number of stars $N$) lead to slightly different formulae with similar behaviour.

| cluster age | $\lambda = 200 \text{ nm}$ | 550 nm | 2.2 $\mu$m |
|-------------|----------------|--------|---------|
| 5 Myr       | $5 \times 10^4$  | $10^5$ | $8 \times 10^5$ |
| 10 Myr      | $5 \times 10^4$  | $10^5$ | $6 \times 10^5$ |
| 50 Myr      | $2 \times 10^4$  | $2 \times 10^4$ | $3 \times 10^5$ |
| 200 Myr     | $10^4$           | $10^4$ | $10^6$   |
| 1 Gyr       | $5 \times 10^5$  | $6 \times 10^3$ | $6 \times 10^5$ |
| 10 Gyr      | > $10^6$         | $2 \times 10^4$ | $5 \times 10^5$ |

Table 1 lists the masses required to ensure the relative luminosity fluctuations are below 10%, with the IMF of Fig. 1. The luminosity fluctuations directly translate into stochastic fluctuations of the mass-to-light ratio ($M/L$); note that differences in the lower IMF could add significantly to the spread in $M/L$ in cluster samples.

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1In practice, the luminosities are binned and the rms luminosity of each bin must be used; the formula assumes statistical independence of the bins, which is justified when many bins of relatively large $n_i$ but low $L_i$ contribute a negligible amount to the total luminosity or variance.
Figure 1. Plots a, b, c, d, e show the contribution of bins of the HR diagram (labelled in Log($T_{\text{eff}}$) and Log($L/L_\odot$)) to $L_{\text{bol}}$ and to the variance of $L_{\text{bol}}$, $L_{200\text{ nm}}$, $L_V$, $L_K$ for a solar metallicity stellar population at the age of 1 Gyr (Salpeter IMF, 0.1-120 $M_\odot$). Post-AGB evolutionary phases that lie outside the plotted areas are responsible for most of the UV fluctuations. The greyscales are normalised separately in each diagram; the comparison of figures c, d and e with b shows that UV-V fluctuations contribute little to the variance of $L_{\text{bol}}$. Plot f shows the corresponding integrated spectrum (Log($F_\lambda$) in arbitrary units versus $\lambda$ in Å), as well as the relative fluctuations Log($\sigma_{F_\lambda}/F_\lambda$) for a population of $10^6 M_\odot$ of stars.
3. Fluctuations in colours and spectrophotometric indices

The statistics of flux ratios are non trivial. The 2 fluxes that define colours or other spectrophotometric indices are in general not independent and not Gaussian (the Gaussian approximation can be used for the large number limit, in which case however the fluctuations are too small to require consideration).

As a consequence of Poisson statistics, the most likely HR diagrams for a given stellar population model underpopulate areas where the expectation number of stars is smaller than or of the order of 1. 80% probability intervals, defined so that the statistical variable (e.g. the number of stars of interest) has probabilities of 10% to lie outside the interval on either side, are centered on a number smaller than the expectation value; their size is not equal to $2 \times 1.28 \sigma$, as it would be for Gaussians. In practice, the stars with high $L$ and small expectation numbers are red, and the most likely colours will thus be bluer than the mean. This explains the behaviour of the Monte-Carlo simulations of Santos & Frogel (1997), and in particular their Fig. 5: in a population of $10^3$ stars, the number of post main sequence stars evolves with time from about 10 at 50 Myr to about 100 at 1 Gyr, and the expected number of luminous cool giants (red supergiants of AGB stars) is of the order of one, resulting in most common J-K colours significantly below the large cluster limit.

When the baseline between the two passbands defining a spectrophotometric index is small, the corresponding fluxes in most cases originate from the same population and can be considered 100% correlated. Fig. 2 shows the 80% probability limits obtained with this assumption for 3 commonly used indices (gas recombination is radiation not included). As expected, the behaviour of the probability intervals is complex. The stochastic fluctuations of the CO index for instance are largest when red supergiants exist, because the latter have particularly strong CO bands; at later stages, fluctuations in the numbers of the current most luminous red stars matters less, since red giants of various ages and luminosities have relatively similar CO bands.

The mass required for a meaningful direct comparison between predicted mean properties and those of a real stellar population depends on the scientific application. H$_2$O bands are strongest in the stars of the upper AGB (TP-AGB), which are most important between $10^8$ and a few $10^9$ yrs. These stars are still poorly understood. If the purpose of a cluster observation is to test the effective temperature scale of TP-AGB star spectra (e.g. Lançon et al. 1999), Fig. 2 shows that $10^4$ M$_\odot$ of stars (i.e. typical LMC cluster masses) are only marginally sufficient. More appropriate clusters should have $10^5$ M$_\odot$ or more.

4. Conclusions

Stochastic fluctuations due to small numbers of bright stars need to be considered when stellar populations are compared to population synthesis models, be it using star counts or using integrated properties. The most probable spectropho-

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2Exceptions are indices such as the strength of optical or near-IR emission lines, for which the line flux is dominated, and the continuum contaminated, by recombination radiation due to the presence of hot stars.
Figure 2. In $a$, $b$ and $c$, the solid line describes the mean evolution of selected spectrophotometric indices. The dashed and dotted lines show the upper and lower boundaries of 80% probability intervals for these indices, for stellar populations of respectively $10^3$ and $10^4 \, M_\odot$ of stars. Plot $d$ compares the predicted mean evolutions of the H$_2$O index of plot $c$, with three different assumptions for the effective temperature scale of variable stars of the TP-AGB: the dotted lines are considered as extremes still compatible with the available literature, the solid line is a currently adopted intermediate scale (note that slightly different timesteps and stellar libraries were used for plots $c$ and $d$). Here, all TP-AGB stars are assumed to be oxygen rich.
tometric properties of small clusters are usually different from their expectation value (i.e. the properties in the large cluster limit), leading to systematic effects in the determination of age, metallicity or other fundamental parameters. The adequate definition of a massive cluster, for which these effects would be negligible, depends strongly on the spectrophotometric property studied and on the star formation history. The cluster populations formed in galaxy mergers, thoroughly discussed during this workshop and known to contain objects of more than $10^6 M_\odot$, are becoming accessible to spectrographs on large telescopes and clearly represent important targets for population synthesis studies of the near future (e.g. Mouhcine & Lançon, this volume).

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**Discussion**

_**J. Gallagher:**_ WR stars being intrinsically rare objects, how can there be so many WR clusters?

_**A. Lançon:**_ Actually, WR stars aren’t that rare, at least at solar metallicity. According to Schaerer & Conti (1998, ApJ, 497, 618), their numbers are of the same order as those of O stars over significant starburst age ranges and, with a solar neighbourhood initial mass function, one finds about one O star for every $10^3 M_\odot$ of newly formed stars (e.g. Leitherer & Heckman 1995, ApJS, 96, 9). Stochastic fluctuations must be considered in individual young clusters of less than $\sim 10^5 M_\odot$, in particular when measuring ratios of different types of WR stars. They will average out over the many clusters of a WR galaxy.

_**S. Portegies Zwart:**_ Current dynamical simulations of clusters usually don’t exceed $10^4$ stars. Are their predictions useless?

_**A. Lançon:**_ The good thing about numerical simulations is that they keep you aware of the number of stars you are dealing with, a point that one easily forgets when looking at the integrated photometric properties of a population from a distance! Maybe one should focus on the properties that don’t depend so much on small numbers of bright stars first; by the time we will have tested these predictions thoroughly, it is likely that computers will have improved enough to increase the sizes of simulations...