Formal Verification of Safety Properties Using Interpolation and $k$-induction

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Abstract. This technical report presents implementation of two symbolic model checking algorithms that use SAT/SMT Solvers, namely interpolation based model checking and $k$-induction based model checking. We also do a comparative analysis of these two model checking algorithms.

1 Introduction

In any software development process, testing and validation remains an important step before releasing the final product. Formal verification methods are designed to find bugs and check performance with specifications in the system. With a given finite state machine description and a temporal logic property, we can automatically check whether a given property holds or not using model checking. Model checking is a method for formally verifying the correctness of finite state systems. It provides counter examples if the system does not satisfy the given specification.

Bounded model checking (BMC) is a formal method based approach that may be used to find bugs in a system given as a finite state machine. In BMC, we unfold the transition relation of the finite state machine for $k$ steps and check if there is any property violation. After every iteration, it increases the bound to search for bugs in longer traces. For each $k$, a Boolean formula is constructed which return sat, if there is a counterexample of length $k$. We can check the satisfiability of Boolean formulas using SAT solvers.

The main problem in BMC is that the property validation can only be checked up to $k$ steps, to check for longer traces we have to increment the $k$. A complete bound can be calculated based on the longest loop-free path between two states. But the longest loop-free path can be exponentially larger than the reachable diameter of the state space. The calculation of such an upper bound can be computationally expensive.

Interpolation is an efficient methodology to enhance BMC to the unbounded case. The concept of interpolation in model checking was first introduced by K.C. Mcmillan in 2003[1]. In contrast to BMC, here we do not have to compute an upper bound. In BMC, the longest loop free path can become exponentially larger than the reachable diameter but in interpolation-based model checking FSM can unfold upto the diameter of state space. Generally, it terminates before it reaches the diameter. The algorithm and its implementation are discussed in detail in the next sections.
2 Logic

Logic allows us to reason about mathematical models for which it has to follow a set of guidelines. Based on the differences in guidelines (syntax and semantics), there are different types of logic. One such type is Modal Logic. It is a specific type of Zero Order Logic. It is the logic of possibility and necessity, past and future, knowledge and belief, and dynamic change. It is developed to study different modes of truth. For different types of modals there exist different types of modal logic[2]. One of the most studied modal logic is tense logic (temporal logic) or logic of time. Temporal logic allows us to make deductive statements about what was, what has always been and what always will be.

Propositional logic and first-order logic do not have a time component, therefore in some cases, we cannot apply this logic directly. For example, if you want to specify the system functionality it requires a notion of time. Temporal logic is a special kind of modal logic where truth values of assertion vary with time[3]. It is used in the verification of sequential circuits. An automation system is also a sequential circuit because it depends on the previous and next states. So, we need temporal logic for specifying behavior. Potential models of temporal logic are sequences if states (paths) and property can hold in some states and does not hold in other states. One such model is the Kripke structure which is very similar to a finite state machine.

In Linear-time Temporal Logic (LTL), we use two fundamental temporal operators to construct more complex formulas from simpler formulas. These operators are as follows:

- Next operator \( \neg X \alpha \) - property holds in the next state.
- Until operator \( \alpha U \beta \) - \( \alpha \) holds in all states until \( \beta \) holds.

Also, the LTL language has two derived temporal operators which are quite useful.

- Future operator \( F \alpha \) - \( \alpha \) holds in some future states.
- Always operator \( G \alpha \) - \( \alpha \) holds in all states.

The set of LTL formulas over atomic propositions (AP) is defined as follows:

- If \( p \in AP \), then \( p \) is an LTL formula.
- If \( \alpha \) and \( \beta \) are LTL formulas then \( \neg \alpha \ | \ \alpha \lor \beta \ | \ \alpha \land \beta \ | \ X \alpha \ | \ \alpha U \beta \) are LTL formulas.

Temporal logics include interval temporal logic, linear-time temporal logic, computation tree logic, and many other types of logic. The most convenient and common formalism used for specifying properties is an extension of \( G, F \), and \( X \) expressions which are called linear-time temporal logic (LTL). This system only contains operators that deal with the future namely \( G, X \), and \( U \). It is a subset of more complex CTL*, which allows branching. One application of LTL is that it can express safety and liveness properties. For more information about automata-based LTL model checking, visit [4]. Safety and liveness properties are discussed in the next sections.

2.1 Safety Properties

In this section, we formally define the notion of a property and, also, the notions of safety and liveness properties. In this report, we focus on safety properties and their model checking. It
there has been argued elsewhere that model checking safety properties is computationally cheaper than model checking liveness properties.

**Definition 1.** Given a set of atomic propositions \((AP)\), a property over \(AP\) is a subset of infinite words over \(2^AP\).

Here is an example of a property:

**Example 1.** \(p_1\) is true at least once and \(p_2\) is always true.

\[
\{A_0A_1A_2\ldots \mid \forall i, A_i \subseteq AP \text{ and } \exists i, p_1 \in A_i \text{ and } \forall j, p_2 \in A_j\}. 
\]

Correctness of a property is always expressed in terms of safety and liveness properties. Any property can be expressed as a conjunction of safety and liveness property \([5]\). A characteristic of safety property is that something bad is absent which means it is used to verify that something bad never happens. \(P\) is a safety property if there exists a set of bad prefixes such that \(P\) is the set of all words that do not start with a bad prefix.

A Property \(P\) is a safety property if:

- Given any execution \(Ex\), \(P(Ex)\) is false.
- There exists a prefix of \(F\), such that every extension that prefix gives an execution \(F\) such that \(P(F)\) is false.

**Example 2.** Always: if \(p_1\) is true, then in the next step \(p_2\) is true. this property can be written as \(G(p_1 \implies Xp_2)\)

### 2.2 Liveness

Liveness property asserts that something good eventually happens whereas safety property asserts that something bad is absent. Liveness property can not be violated in a finite time because something good can happen later whereas safety property can only be violated in a finite time. But liveness property can be satisfied in a finite time contrary to that safety property can be satisfied in infinite time. Liveness is concerned with a program eventually reaching a good state. We can describe the liveness property by combining the \(G\) and \(F\) operator:-

\[GFp : \text{infinitely often } p\]

### 3 Bounded Model Checking

The main problem in model checking is the capacity issue that arises mainly due to the size of the state space. BDDs \([6]\) can represent the state transition relation in canonical form and we can efficiently traverse the state space to check whether a given property holds or not. Typically, for a large number of variables, it is not feasible for us to construct a BDD because that may require
a sufficiently large amount of memory. For linear increases in variables, BDDs grow exponentially \cite{7}. In this section, we look at another approach to tackle this problem, namely Bounded model checking (BMC).

The key idea of bounded model checking is that it looks for a counter-example path of increasing length $k$. We have to increase the bound to search for longer traces\cite{8}. For each $k$, it builds a Boolean formula that is satisfiable if there is a counterexample of length $k$. So, as you can see BMC is more oriented toward finding bugs. The satisfiability of Boolean formulas is checked using SAT procedure and according to that, it returns a satisfying assignment. As we have to increase the bound successively, the correctness of the property can be verified in that limited bound. We can calculate a complete bound to check safety properties based on the longest loop-free path between the states. Unfortunately, the longest loop-free path can be exponentially larger than the reachable diameter of the state space.

For BMC we need the symbolic representation of a system $M = (I, T)$, where Boolean formulas $I$ and $T$ describe the initial states and transition relation respectively. For a given safety property $P$, we can check whether $P$ holds for all states at a distance of $k$ by constructing a Boolean formula $F_k$ with following three clauses:

1. $I(S_0)$
2. $\bigwedge_{i=0}^{k-1} P(S_i, S_{i+1})$
3. $\neg P(S_k)$

If $F_k$ is satisfiable then it means that some state in $S_k$ (at a distance of $k$ from some initial state in $S_0$) is unsafe (does not satisfy safety property $P$) and we may terminate the procedure. If $F_k$ is unsatisfiable we can increment $k$ and search for longer traces. If $P$ holds for all reachable states then we say that $M$ is $P$-safe.

BMC in its original form can be used for falsification. For completeness, it requires an upper bound. Unfortunately, the calculation of such an upper bound can be computationally expensive. It is important to know if the system is $P$-safe and when to terminate the program or stop incrementing $k$. This property is called completeness. There are several techniques for achieving completeness. Here we discuss the properties such as $F_p$ and $G_p$. In the next section, we introduce how completeness can be achieved with $k$-induction.

The bound where we can stop our BMC algorithm and conclude that the system is $P$-safe is called the completeness threshold (CT)\cite{9}. So for $G_p$ formulas, the completeness threshold is the minimum number of steps required to reach all states which is also called reachability diameter. We can also use the largest distance of any reachable state from an initial state like CT. Unfortunately, computation of diameter can be expensive. Instead of computing the diameter, we can compute the over-approximation of reachable diameter which is called recurrence diameter, which is just the longest loop-free path from an initial state. As every shortest path is a loop-free path, we can say recurrence diameter is an over-approximation of the reachability diameter.

3.1 $k$-induction

We can also use $k$-induction for completeness of BMC, with induction we can check safety properties. In this algorithm, first, the base case is checked if it is unsatisfiable then only we move forward towards the induction step. The base case for $k$-induction is: $I(S_0) \land \neg P(S_0)$. 

If the result is sat then we can simply abort the program and conclude that input FSM is unsafe at the initial state. Otherwise, if the result is unsat, we can proceed further toward the induction step. Also, we can conclude that all initial states are P-safe for \( k = 0 \).

The induction formula which is to be checked with every increment of \( k \) is:

\[
\bigwedge_{i=0}^{k-1} T(S_i, S_{i+1}) \land \neg P(S_k)
\]

For base case, \( k = 0 \) if there is no counter example, it increments the \( k \) and checks for the next step. For \( k = 1 \) it checks for \( T(S_0, S_1) \land \neg P(S_1) \) and if it returns satisfiable then input FSM is unsafe for \( k = 1 \) and we abort the program. Otherwise, we keep on incrementing \( k \) and check for every iteration. For example, for \( k = 2 \), it checks for the formula \( T(S_0, S_1) \land T(S_1, S_2) \land \neg P(S_2) \) and do the same checks.

How to know when it is safe to stop incrementing and conclude that the system is P-safe? We can not wait for \( I(S_0) \land T(S_0,S_1) \cdots \land T(S_{i-1}, S_i) \) to become a contradiction because given that there is an initial state, this will never happen because we assume that every state has successor through \( T \), so there will always be loops in state space. Instead, we can check the following:

\[
I(S_0) \land \text{loop-free}(S_0, S_1) \cdots \land \text{loop-free}(S_{i-1}, S_i)
\]

and if there is a contradiction then we can safely conclude that the system is P-safe.

**Algorithm 1 BMC with \( k \)-induction**

\[
V = [] \quad \triangleright \quad \text{List} \ V \text{ holds the variables used in the runs}
\]

\[
\text{Run} = [] \quad \triangleright \quad \text{List} \ \text{run}\text{ holds the expressions run}_0, \text{run}_1, \text{etc.}
\]

\[
\text{Path} = [] \quad \triangleright \quad \text{List} \ \text{path}\text{ holds the expressions path}_0, \text{path}_1, \text{etc}
\]

\[
\text{Run}_0 = I, \ \text{subp}_0 = P
\]

if \( (\text{Run}_0 \land \neg \text{subp}_0) = \text{sat} \) then

input FSM is unsafe ... abort

end if

while \( k < 2^n \) do

\[
\text{run}_k = T
\]

for \( i \) in \( n \) do

\[
\text{run}_k = \text{substitute}(\text{run}_k, (X[i],V[n*(k-1)+i]),(Y[i],V[n*k+i]))
\]

end for

\[
\text{run}_k = \text{Run}[k-1] \land \text{run}_k
\]

\[
\text{subp}_k = P
\]

for \( i \) in \( n \) do

\[
\text{subp}_k = \text{substitute}(\text{subp}_k, (X[i],V[n*k+i]))
\]

end for

if \( (\text{run}_k \land \neg \text{subp}_k) = \text{sat} \) then

FSM is unsafe ... abort

else

all states at distance \( i \) are P-states

\[
k = k + 1
\]

end if

end while
4 Interpolation Technique

This technique was first introduced by Kenneth L McMillan [1]. In this section we first discuss the flaws in BMC and how interpolation can be used to iron out these problems. Thereafter interpolation-based model checking algorithm is described in detail and later we compare these techniques on various examples.

In BMC, the output is only in the Boolean form, whether it is satisfiable or unsatisfiable but with interpolation, we get a proof of unsatisfiability and from that, we can create an interpolant which is used to prove that all reachable states are safe. As we have seen in the BMC algorithm we can only increment $k$ by 1, but in interpolation, we are able to increment $k$ by any number, as mentioned in next sections. Interpolation is an efficient methodology to enhance BMC towards the model checking of infinite-state systems and verification of safety and liveness properties. We give a brief introduction to Craig’s interpolation theorem in the next section.

4.1 Craig’s Interpolant

Craig’s interpolation theorem states that for a contradiction $A \land B$, a Craig interpolant is a formula $P[10]$, such that

- $A$ implies $P$
- $P \land B$ is a contradiction
- All non-logical symbols of $P$ occur in $A$ as well as $B$.

An interpolant can be computed from the proof of unsatisfiability of $A \land B$. For more information about efficient interpolation computation, visit[11]. In this report, we have chosen a basic algorithm for interpolant computation.

Algorithm 2 Computing Interpolant

Ensure: $Var = Var(A) - Var(B)$
for $x$ in $Var$ do
    $P = A[True|x] \lor A[False|x]$
end for
$P = simplify(P)$

We consider the tuple $(VarA, VarB)$ which store the list of states contained in clauses $A$ and $B$ respectively. Then, we take the difference between the two lists and store them in $Var$. After that, for every item $x$ in list $Var$, first, we replace True with $x$ in the $A$ clause and then replace False with $x$ and then do the OR operation. We repeat the same procedure in every iteration. After that to remove the redundant variables, we simplify our interpolant using simplify() which is an inbuilt function in the Z3 theorem prover[12].
4.2 Algorithm

Given the original BMC formula for some $k$, we divide it into two clauses $A$ and $B$. 'A' clause stores the initial state and first transition state whereas $B$ consists of the remaining transition state and property condition. Then an interpolant is constructed which is an over approximated image operator as the interpolant depends only on the $A$ clause.

Let us consider the complete BMC formula [1]:

$$I(S_0) \land \bigwedge_{i=0}^{k-1} T(S_i, S_{i+1}) \land \bigvee_{i=j}^{k} \neg P(S_k)$$

The $A$ and $B$ formulas in this case are:

$$A = I(S_0) \land T(S_0, S_1)$$  \hspace{1cm} (1)

$$B = \bigwedge_{i=0}^{k-1} T(S_i, S_{i+1}) \land \bigvee_{i=j}^{k} \neg P(S_k)$$  \hspace{1cm} (2)

We have to introduce new indexed variables for each run which means we have to replace our state variables with path variables. Here $S_i = s_{i1}, s_{i2}, \ldots, s_{in}$.

We now consider a finite state machine Figure. 1 to better understand interpolant computation.

![Finite State Machine with 4 states](image_url)

**Fig. 1.** Finite State Machine with 4 states

For representing four states we need two lists which contains two Boolean variables (let’s say $X$ and $Y$).

- $S_0$ is represented by $\neg X_1 \land \neg X_0$
- $S_1$ is represented by $\neg X_1 \land X_0$
- $S_2$ is represented by $X_1 \land \neg X_0$
- $S_3$ is represented by $X_1 \land X_0$
Suppose the property to be checked is $P = X_1 \land X_0$. Initially, we consider $k = 1$. First we construct $A$ and $B$ formulas:

- $A = S_{00} \land (S_{00} \implies S_{11}) \land (S_{10} \implies S_{21}) \land (S_{20} \implies S_{31}) \land (S_{30} \implies (S_{31} \lor S_{11}))$
- $B = \neg P$

where $S_{00} = \neg X_{01} \land \neg X_{00}$, $S_{11} = \neg X_{11} \land X_{10}$ etc. Using the above algorithm, we compute interpolant $ITP = \neg X_{11} \land X_{10}$ which, essentially, represents state $S_{11}$.

As shown above $ITP$ is the over-approximation of the forward image of the initial state (I). Here, initially, we take $k = 1$, as we increase $k$, in each iteration, we will be able to compute the over-approximation of all reachable states. In BMC, we have to calculate the exact image operator but in the interpolation technique, we are using the over-approximation technique. With an increase in $k$, we will, at last, find a true counterexample or we can conclude that system is $P$-safe. If we increment $k$ by a too-small number, it will take more time to reach the conclusion or if we increment $k$ by a large number, it is possible that it will give a wrong result.

In algorithm 3, the $create\_A$ and $B(Q, T, Not(P))$ function takes transition relation and property and returns $A$ and $B$ clauses which have been discussed earlier. $create\_Interpolant(A, B)$ takes $A$ and $B$ clauses and returns the interpolant which has been created from Algorithm 2.

**Algorithm 3** Property Checking Procedure

```plaintext
procedure PropertyChecking(prevK, K)
    if (I \land Not(P)) = sat then
        Property not satisfied
        abort
    end if
    Q = I
    while True do
        A, B = create\_A and B(Q, T, Not(P))
        if A \land B = sat then
            if Q == I then
                Property not satisfied
                abort
            else
                PropertyChecking(k, k + n)
            end if
        else
            ITP = create\_Interpolant(A, B)
            if Not(Implies(ITP, Q)) then
                Property holds
                abort
            end if
            Q = Q \lor ITP
        end if
    end while
end procedure
```
5 Comparison between $k$-induction and interpolation

In this section, we consider two finite state machines and perform model checking on them using the techniques described above: $k$-induction and interpolation, to illustrate the advantages and disadvantages of these algorithms.

Example 1

Now, we will show how BMC with $k$-induction works with an example from Figure 2.

![Finite State Machine with 4 states](image)

**Fig. 2.** Finite State Machine with 4 states

The initial state is $S_3$, and the property that we are checking is $P = X[1] \oplus X[0]$ which means it should be satisfied in states $S_1$ and $S_2$ and the FSM is $P$-safe for states $S_0$ and $S_3$. For each $k$ we have a $run_k$ and $subp_k$. These variables will then be passed into a solver and the result is shown after solving SAT queries.

| $k$ | $run_k$ | $subp_k$ | result |
|-----|---------|----------|--------|
| 0   | $run_0 : S3_0$ | $P = X0_0 \oplus X0_1$ | UNSAT |
| 1   | $run_1 : run_0 \wedge S3_0 \implies S0_1 \wedge S0_0 \implies (S1_1 \lor S2_1) \wedge S1_0 \implies S0_1 \wedge S2_0 \implies S0_1$ | $P = X1_1 \oplus X1_0$ | UNSAT |
| 2   | $run_2 : run_0 \wedge run_1 \wedge S3_1 \implies S0_2 \wedge S0_1 \implies (S1_2 \lor S2_2) \wedge S1_1 \implies S0_2 \wedge S2_1 \implies S0_2$ | $P = X2_1 \oplus X2_0$ | SAT |

**Table 1.** BMC with $k$-induction

With $k = 1$, all states reachable in one transition is $S_0$, and the result is unsatisfiable. Hence, we increment to $k = 2$, where all states reachable in two transitions are $S_0$, $S_1$, $S_2$ and property is satisfiable for $S_1$ and $S_2$ so it returns satisfiable and we can conclude that FSM is not $P$-safe and then we terminate the program.
We discuss the same example using interpolation-based model checking. Here we have to keep track of previous and current \( k \) variables. we can increase \( k \) by any amount but for this example, we increment it by \( n \), where \( n \) is the number of Boolean variables required to represent all states in FSM. So, initially, we pass our previous and current \( k \) as 1 and 1 respectively.

| \( k \) | Queries | Interpolant | \( Q \) | Result |
|-------|---------|-------------|--------|--------|
| \( k = 1 \) | \( A \land B \) | \( S_0 \) | SAT |
| \( k = 1 \) | \( \neg (P \implies Q) \) | \( S_3 \lor S_1 \) | SAT |
| \( k = 3 \) | \( A \land B \) | SAT |

Table 2. BMC with Interpolation

Here, all the iterations are explained in detail.

For \( k = 1 \),
First, we check whether \( I \land \neg (P) \) is unsatisfiable, if it is satisfiable then FSM does not satisfy the property and we terminate the program. In the above example, if the output is unsatisfiable, now we create \( A \) and \( B \) variables and check \( A \land B \). The output is unsatisfiable. So it satisfies the property at \( k = 1 \). We then created the interpolant, \( P \), as you can see from the table 2, which represents \( (S_0) \) which is the next state from the initial state. Then, after replacing path variables with state variables, and storing them in \( R_1 \), the solver checks \( \neg (R_1 \implies Q) \). There are two options if it is unsatisfiable then property holds for all reachable states, else we will update our \( R \) and do the next iteration.

So, it again creates \( A \) and \( B \) clauses and checks \( A \land B \), now the output is satisfiable. Now we have another condition, if \( Q = I \), we terminate the program and concludes that the system does not satisfy the property. But this is not the case here, so \( k \) is incremented by \( n \) which in this case is 2 because the number of states is 4.

For \( k = 3 \),
After checking \( A \land B \), which returns satisfiable, we check if \( Q = I \), which is true here. So we terminate the program and conclude that the FSM does not satisfy the property for \( 1 < k \leq 3 \).

Example 2

Now we look into another example as given in Figure. 3 where property holds for all reachable states and check how these two algorithms perform.

Here the initial state is \( S_5 \) and the property that we are checking is \( \text{Prop} = X_2 \land X_1 \land X_0 \) which does not satisfy any of the reachable states. So the system will be property safe. Let’s run our example with \( BMC \) with the \( k \)-induction program.

After every result which is unsatisfiable, it reiterates till \( k < 2^n \), and does the same checks. After \( k = 7 \) our program will end and it will conclude that the system is P-safe. Here, \( T_i = (\ T_0, T_1, T_2, \cdots ) \) represents the transition relation for each run, \( k \).

At the end if result for \( \neg (P \implies B) \) becomes unsat we can conclude that property holds for all reachable states.
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**Fig. 3.** Finite State Machine with 6 states

```
| $k$ | $run_k$ | $subp_k$ | Result  |
|-----|---------|----------|---------|
| 0   | $S0_0$  | $P = X0 \land X1 \land X2$ | UNSAT   |
| 1   | $run_0 \land T_0$ | $P = X10 \land X11 \land X12$ | UNSAT   |
| 2   | $run_1 \land T_1$ | $P = X20 \land X21 \land X22$ | UNSAT   |
| 3   | $run_2 \land T_2$ | $P = X30 \land X31 \land X32$ | UNSAT   |
| 4   | $run_3 \land T_3$ | $P = X40 \land X41 \land X42$ | UNSAT   |
| 5   | $run_4 \land T_4$ | $P = X50 \land X51 \land X52$ | UNSAT   |
| 6   | $run_5 \land T_5$ | $P = X60 \land X61 \land X62$ | UNSAT   |
| 7   | $run_6 \land T_6$ | $P = X70 \land X71 \land X72$ | UNSAT   |
```

**Table 3.** BMC with $k$-induction

```
| $k$ | $i$ | Queries | Interpolant($P_i$) | $Q_i$ | Result    |
|-----|-----|---------|--------------------|------|-----------|
| 1   | 0   | $A \land B$ | $S0_1$ | $S0 \lor S5$ | UNSAT |
| 1   | 1   | $A \land B$ | $P_0 \lor S1_1$ | $Q_0 \lor S1$ | SAT    |
| 1   | 2   | $A \land B$ | $P_1 \lor S2_1 \lor S3_1$ | $Q_1 \lor S2 \lor S3$ | SAT    |
| 1   | 3   | $A \land B$ | $P_2 \lor S4_1 \lor S5_1$ | $Q_2 \lor S4$ | SAT    |
| 1   | 4   | $A \land B$ | $P_3$ | $Q_3$ | UNSAT |
|     | 1   | $A \land B$ | $P_1$ | $Q_1$ | UNSAT |
```

**Table 4.** Interpolation
6 Implementation

We need a SAT/SMT solver in the model checking algorithms presented in this report. We have used Z3 theorem prover (constraint solver) which is an efficient and open source SAT/SMT solver. We use ANTLR (Another Tool for Language Recognition) for input preprocessing as with ANTLR we can define our own input format and process it easily. In the next sections, we have introduced these tools and their usage in our implementation.

6.1 Z3 Theorem Prover

Z3 is used in hardware and software verification, constraint solving, and testing. Let us see a few examples to better understand its working. Here, we show how it is used for constraint solving and simplification of a Boolean expression.

```python
a = Int('a')
b = Int('b')
solve(a > 7, b >= 10, a + 2*b == 28)
OUTPUT
 [b = 10, a = 8]
```

In case a solution does not exist it returns “No Solution”. We can also simplify the complex Boolean expression.

```python
a = Int('a')
b = Int('b')
simplify(a**2 + 2*a**2 + b**2 - b**2 + 5)
OUTPUT
  5 + 3·a²
```

We have used `simplify()` function in our implementation for simplification of interpolants generated in the course of execution of the algorithm because the interpolant produces redundant expressions.

Now we look at `prove()` function which prove a given statement by checking whether the statement negation is unsatisfiable or not.

```python
a = Bool("a")
b = Bool("b")
statement = Xor(a,b)
prove(statement)
OUTPUT
counterexample
 [b = False, a = False]
```

It gives a counterexample along with the values when it is not able to prove the given statement. Next, an example using `Solver()` is demonstrated.
a = Bool("a")  
b = Bool("b")  
s = Solver()  
s.add(Implies(a,b))  
s.check()

It returns sat because for \( a = \text{True} \) and \( b = \text{True} \) a solution exists. Solver() creates a general purpose solver and we can add constraints in the Solver() using add(). For solving the constraints we use check() which can return any of the three possible outputs. sat if there exists a solution, unsat if no solution exists and unknown if Solver() is not able to solve given constraints.

### 6.2 ANTLR

We have used ANTLR (Another Tool for Language Recognition) for input preprocessing of finite state machines. ANTLR is a parser generator. For getting a parse tree, first, we have to write a lexer and parser grammar then after running ANTLR it generates a lexer and parser in a targeted programming language. We then pass our finite state machine input in propositional formulas to obtain a parse tree. The interested reader may refer to the book [13] for a detailed study of ANTLR.

![Finite State Machine with 4 states](image)

**Fig. 4.** Finite State Machine with 4 states

For a finite state machine with 4 states given in Figure. 4, the following formulas have to be provided as input for model checking. As one can see, it is not possible to manually write such formulas for more complex systems.

\[
I = \text{And}(\text{Not}(X[1]), \text{Not}(X[0]))
\]

\[
T = \text{And}(\text{Implies}(\text{And}(\text{Not}(X[1]), \text{Not}(X[0])), \text{And}(\text{Not}(Y[1]), Y[0])),
\text{Implies}(\text{And}(\text{Not}(X[1]), X[0]), \text{And}(Y[1], \text{Not}(Y[0]))),
\text{Implies}(\text{And}(X[1], \text{Not}(X[0])), \text{And}(Y[1], Y[0])),
\text{Implies}(\text{And}(X[1], X[0]), \text{Or}(\text{And}(Y[1], Y[0]),\text{And}(\text{Not}(Y[1]), Y[0]))))
\]

# define some properties.
Prop = \text{And}(X[1], X[0])
But with the help of ANTLR, we define our own input format, which is recognized using ANTLR, and feed it to our program. In this case, we can pass the input in this format:

```
SIZE : 2
s0 : 00
s1 : 01
s2 : 10
s3 : 11

INIT(s0)
NEXT(s0) := s1
NEXT(s1) := s2
NEXT(s2) := s3
NEXT(s3) := s3
NEXT(s3) := s1
```

The above input represents the FSM in Figure 4. Here SIZE refers to the number of Boolean variables required to create all states. INIT means the initial state of FSM and NEXT represents the transition state. NEXT(s0) := s1, means there is a transition from s0 to s1.

After the parse tree is generated, we can walk the tree and there are two different ways, listeners and visitors. ANTLR generates a parse tree listener interface if we do not provide any specification. For trigger calls to listeners, ANTLR has `ParseTreeWalker`. It also generates a subclass that is unique for each grammar, `ExprListener` for `Expr.g4` grammar file, for example. For each rule, we have to provide enter and exit functions. So, the moment walker finds a node it encounters the node enter method and we can store or transform the data present at that node. The advantage of listener mechanisms is that you do not have to worry about the flow you just have to override the needed functions. If you want more control over the nodes of your parse tree you can use an alternative mechanism called `visitor`. In this implementation, we have used the listener mechanism. We stored the data on each node and transform it into required formulas with the help of given information.

## 7 Conclusion

In this report we have discussed the interpolation and $k$-induction algorithms for model checking and their implementation in detail. First, a brief introduction on foundational topics viz. model checking, logic, safety, and liveness properties was given. Then, we discussed the bounded model checking algorithm, its pros and cons, and its implementation in detail. Later, an interpolation-based model checking algorithm was discussed. Next, we considered two examples to elucidate the internal working of these algorithms. Finally, we discussed the implementation, in particular the use of Z3 theorem prover and ANTLR in the project.

We can not just conclude that the interpolation algorithm is better than BMC with $k$-induction. In example 5, where we selected a safe model, in the worst case, the upper bound can be the same as that of BMC with $k$-induction. But generally, the number of iterations is very small. In example 5, as we have shown, interpolant computation on an unsafe model is slower when compared to BMC with $k$-induction on the same model. Arguably, the importance of computing interpolants should not be ignored as there are other algorithms which have been developed using interpolants, like interpolation-sequence-based model checking[14], which is not in the scope of the report.
References

[1] Kenneth L McMillan. “Interpolation and SAT-based model checking”. In: CAV. Springer. 2003, pp. 1–13.
[2] Alexander V. Chagrov and Michael Zakharyaschev. Modal Logic. Vol. 35. Oxford logic guides. Oxford University Press, 1997.
[3] Nicholas Rescher and Alasdair Urquhart. Temporal logic. Vol. 3. Springer Science & Business Media, 2012.
[4] Moshe Y Vardi. “An automata-theoretic approach to linear temporal logic”. In: Logics for concurrency (1996), pp. 238–266.
[5] Bowen Alpern and Fred B Schneider. “Recognizing safety and liveness”. In: Distributed computing 2.3 (1987), pp. 117–126.
[6] Henrik Reif Andersen. “An introduction to binary decision diagrams”. In: Lecture notes, available online, IT University of Copenhagen (1997), p. 5.
[7] Stefan Edelkamp and Peter Kissmann. “Limits and possibilities of BDDs in state space search”. In: Annual Conference on Artificial Intelligence. Springer. 2008, pp. 46–53.
[8] Armin Biere et al. “Bounded model checking.” In: Handbook of satisfiability 185.99 (2009), pp. 457–481.
[9] Edmund Clarke et al. “Completeness and complexity of bounded model checking”. In: International Workshop on Verification, Model Checking, and Abstract Interpretation. Springer. 2004, pp. 85–96.
[10] Kenneth L McMillan. “Applications of Craig interpolants in model checking”. In: International Conference on Tools and Algorithms for the Construction and Analysis of Systems. Springer. 2005, pp. 1–12.
[11] Pavel Pudlák. “Lower bounds for resolution and cutting plane proofs and monotone computations”. In: The Journal of Symbolic Logic 62.3 (1997), pp. 981–998.
[12] Leonardo de Moura and Nikolaj Bjørner. “Z3: An efficient SMT solver”. In: International conference on Tools and Algorithms for the Construction and Analysis of Systems. Springer. 2008, pp. 337–340.
[13] Terence Parr. The definitive ANTLR 4 reference. 2013.
[14] Yakir Vizel and Orna Grumberg. “Interpolation-sequence based model checking”. In: 2009 Formal Methods in Computer-Aided Design. IEEE. 2009, pp. 1–8.