The Kelvin-Helmholtz instability in weakly ionised plasmas: Ambipolar dominated and Hall dominated flows.

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Accepted –. Received –; in original form –

\textbf{ABSTRACT}

The Kelvin-Helmholtz (KH) instability is well known to be capable of converting well-ordered flows into more disordered, even turbulent, flows. As such it could represent a path by which the energy in, for example, bowshocks from stellar jets could be converted into turbulent energy thereby driving molecular cloud turbulence. We present the results of a suite of fully multifluid magnetohydrodynamic simulations of this instability using the HYDRA code. We investigate the behaviour of the instability in a Hall dominated and an ambipolar diffusion dominated plasma as might be expected in certain regions of accretion disks and molecular clouds respectively.

We find that, while the linear growth rates of the instability are unaffected by multifluid effects, the non-linear behaviour is remarkably different with ambipolar diffusion removing large quantities of magnetic energy while the Hall effect, if strong enough, introduces a dynamo effect which leads to continuing strong growth of the magnetic field well into the non-linear regime and a lack of true saturation of the instability.

\textbf{Key words:} mhd – instabilities – ISM:clouds – ISM:kinematics and dynamics

\section{INTRODUCTION}

The Kelvin-Helmholtz (KH) instability is an important instability in almost any system involving fluids: it can occur anywhere that has a velocity shear. In astrophysical plasmas the KH instability can provide the means of producing turbulence in a medium or the mixing of material between two boundary layers.

The KH instability has been studied in a variety of astrophysical systems, from solar winds (Amerstorfer et al. 2007; Bettarini et al. 2006; Hasegawa et al. 2004) and pulsar winds (Bucciantini & Del Zanna 2006) to thermal flares (Venter & Meintjes 2006). Due to its ability to drive mixing and turbulence, the KH instability has been considered relevant in protoplanetary disks (Johansen et al. 2006; Gómez & Ostriker 2007), accretion disks and magnetospheres (Li & Narayana 2004), and other jets and outflows (Baty & Keppens 2006). Generally speaking, the assumptions of ideal magnetohydrodynamics (MHD) have been used in order to simplify the system of equations to be solved. These assumptions are, however, not always valid. Weakly ionised plasmas, for example, contain a large fraction of neutral particles as well as a number of charged particle fluids with differing physical characteristics. Interactions between the various species can introduce non-ideal effects. Ambipolar dissipation and the Hall effect are two non-ideal effects that can greatly influence the development of the KH instability in a system by altering the dynamics of the plasma and the evolution of the magnetic field. Astrophysical examples of such weakly ionised systems include dense molecular clouds (e.g. Ciolek & Roberge 2002) and accretion disks around young stellar objects (e.g. Wardle 1999).

In these systems, the relevant length scales are such that non-ideal effects can play an important role (Wardle 2004a; Downes & O’Sullivan 2003, 2011).

Many authors have investigated the role of the KH instability in both magnetised and unmagnetised astrophysical flows (e.g. Frank et al. 1996; Malagoli et al. 1996; Hardee et al. 1997; Downes & Ray 1998; Keppens et al. 1999). Most of these studies have investigated the KH instability in the context of either hydrodynamics or ideal MHD. We know that non-ideal effects are important in molecular clouds at length scales below about 0.2 pc (e.g. Oishi & Mac Low 2004; Downes & O’Sullivan 2009) and hence it is of interest to explore the KH instability in the context of either non-ideal MHD or, preferably, fully multifluid MHD. In more recent years the emphasis of KH studies has been on including non-ideal effects. Keppens et al.
scales and for super-Alfvénic flows, the fastest growing mode at short length scales, the growth rate is well approximated. It is found that species are not participating in the turbulent interface with detectably narrower line profile in ionised species tracing the stellar outflow compared with neutral species, since ionised flows. A detailed study of this instability in the spectrum of equations, given below, incorporates finite parallel, axial, and parallel resistivity in these simulations and we analyse the role each of the former two effects on the development of the instability.

In a linear study of stellar outflows, Watson et al. (2004) studied both the linear growth and subsequent non-linear saturation of the KH instability using resistive MHD numerical simulations. The inclusion of diffusion allowed for magnetic reconnection and non-ideal effects were observed through tearing instabilities and the formation of magnetic islands.

Palotti et al. (2008) also carried out a series of simulations using resistive MHD. They found that, following its initial growth, the KH instability decays at a rate that decreases with decreasing plasma resistivity, at least within the range of resistivities accessible to their simulations. They also found that magnetisation increases the efficiency of momentum transport, and that the transport increases with decreasing resistivity. Birk & Wiechen (2002) examined the case of a partially ionised dusty plasma, using a multifluid approach in which collisions could be included or ignored. They found that collisions between the neutral fluid and dust particles could lead to the stabilisation of KH modes of particular wavelengths. The unstable modes led to a significant local amplification of the magnetic field strength through the formation of vortices and current sheets. In the nonlinear regime they observed the magnetic flux being redistributed by magnetic reconnection. It was suggested that this could be applicable to dense molecular clouds and have important implications for the magnetic flux loss problem (Umebayashi & Nakano 1991).

A comprehensive study was carried out by Wiechen (2006) which demonstrated the effect of dealing with the plasma using a multifluid scheme. This study focused on the effect of varying the properties of the dust grains. The results of the simulations led to the conclusions that more massive dust grains have a stabilising effect on the system while higher charged numbers have a destabilising effect. It was found that there is no significant dependence on the charge polarity of the dust.

In a linear study of stellar outflows, Watson et al. (2004) described how the charged and neutral fluids are affected differently by the presence of a magnetic field. This study is carried out using parameters chosen to reflect those of molecular clouds, and so is particularly relevant to our own study. The principal result of this paper is that for much of the relevant parameter space, neutrals and ions are sufficiently decoupled that the neutrals are unstable while the ions are held in place by the magnetic field. Since the magnetic field is frozen to the ionised plasma, it is not tangled by the turbulence in the boundary layer. The authors predict that with well-resolved observations, there should be a detectably narrower line profile in ionised species tracing the stellar outflow compared with neutral species, since ionised species are not participating in the turbulent interface with the ambient interstellar medium. The paper also includes a study of the growth rate of the instability. It is found that at short length scales, the growth rate is well approximated by the growth rate of the hydrodynamic system. At larger scales and for super-Alfvénic flows, the fastest growing mode is equal to that of the ideal MHD case.

Shadmehri & Downes (2008) carried out an analytical study of the Kelvin-Helmholtz instability in dusty and partially ionised outflows. They investigated primarily the effect of the presence of dust particles by varying their mass, charge and charge polarity. It was found that as the charge of the grain increased, the growth timescales also increased, implying a stabilising effect on the system. The stability of the system was also examined for dependence on the mass of the dust particles. It was found that for stronger magnetic fields, this did not affect the stability of the system. However, for weaker magnetic fields, the larger dust particles had a stabilising effect on the growing modes. This was in agreement with previous laboratory experiments (Luo et al. 2001) and numerical simulations (Wiechen 2006). Finally, as the magnetic field strength increased, the growth timescale of the unstable modes at a particular perturbation wavelength decreased. By examining the combinations of the wavelength of the perturbation used, and the resultant growth timescales of the instability, Shadmehri & Downes (2008) concluded that the Kelvin-Helmholtz instability is a possible candidate for causing the formation of some of the physical structures observed in molecular outflows from young stars.

In this paper we perform numerical simulations of the complete evolution of the Kelvin-Helmholtz instability in a weakly ionised, multifluid plasma including both its linear development, saturation and its subsequent behaviour. We include the physics of the Hall effect, ambipolar diffusion and parallel resistivity in these simulations and we analyse the role each of the former two effects on the development of the instability.

The aim of this work is to investigate the influence of multifluid effects on the growth and saturation of the KH instability. In order to develop a full understanding of the roles of the various non-ideal effects, in particular the Hall effect and ambipolar diffusion, we run simulations with parameters chosen to simulate very high, medium and very low magnetic Reynolds number systems and with parameters chosen to ensure ambipolar-dominated flows and Hall-dominated flows. We focus on gaining an understanding of the general characteristics of the KH instability in weakly ionised flows. A detailed study of this instability in the specific context of molecular clouds, and which is of interest from the point of view of turbulence generation by stellar outflows, is the subject of a future work.

In section 2 we outline the numerical and physical model employed, in section 3 we discuss how we analyse the results of our simulations while in section 4 we detail the results in both the linear and non-linear regimes, separating out the effects of ambipolar diffusion and the Hall effect in order to more fully understand the influence of each.

2 NUMERICAL APPROACH

The simulations described in this work are performed using the HYDRA code (O’Sullivan & Downes 2006, 2007) for multifluid magnetohydrodynamics in the weakly ionised regime. We further assume the flow is isothermal. The assumption of weak ionisation allows us to ignore the inertia of the charged species and allows us to derive a (relatively) straightforward generalised Ohm’s law. The resulting system of equations, given below, incorporates finite parallel, Hall and Pederson conductivity and is valid in, for example, molecular clouds. In such regions the viscous length scales are much smaller than those over which nonide al effects are important. This leads to high Prandtl numbers and plasma flows in these regions can be considered to be effectively inviscid. In this work we examine low Mach number flows...
which, taken in concert with the isothermal assumption, means that features in the flow such as shocks are unlikely to create regions of high ionisation.

2.1 Multifluid equations

The HYDRA code solves the following equations for a system of $N$ fluids. The simulations described in this paper consist of three fluids, indexed by $i = 0$ for the neutral fluid and $i = 1$ and $i = 2$ for the electron and ion fluids respectively. The equations to be solved are

$$\frac{\partial \rho_i}{\partial t} + \nabla \cdot (\rho_i \mathbf{v}_i) = 0 \quad (0 \leq i \leq N - 1), \quad (1)$$

$$\frac{\partial \rho_i \mathbf{v}_i}{\partial t} + \nabla \cdot (\rho_i \mathbf{v}_i \mathbf{v}_i + a^2 \rho_i \mathbf{I}) = \mathbf{J} \times \mathbf{B}, \quad (2)$$

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \cdot (\mathbf{v}_i \mathbf{B} - \mathbf{B} \mathbf{v}_i) = -\nabla \times \mathbf{E}', \quad (3)$$

$$\alpha_i \rho_i (\mathbf{E} + \mathbf{v}_i \mathbf{B}) + \rho_i \rho_0 K_{i0} (\mathbf{q}_0 - \mathbf{q}_i) = 0 \quad (1 \leq i \leq N - 1), \quad (4)$$

$$\nabla \times \mathbf{B} = 0, \quad (5)$$

$$\nabla \times \mathbf{J} = 0, \quad (6)$$

$$\sum_{i=1}^{N-1} \alpha_i \rho_i = 0, \quad (7)$$

where $\rho_i$, $\mathbf{v}_i$, $\mathbf{B}$, and $\mathbf{J}$ are the mass densities, velocities, magnetic field and current density, respectively. $a$ denotes the sound speed, and $\alpha_i$ and $K_{i0}$ are the charge-to-mass ratios and the collision coefficients between the charged species and the neutral fluid, respectively.

These equations lead to an expression for the electric field in the frame of the fluid, $\mathbf{E}'$, given by the generalised Ohm’s Law

$$\mathbf{E}' = \mathbf{E}_o + \mathbf{E}_H + \mathbf{E}_\Lambda, \quad (8)$$

where the components of the field are given by

$$\mathbf{E}_o = (\mathbf{J} \cdot \mathbf{a}_o) \mathbf{a}_o, \quad (9)$$

$$\mathbf{E}_H = \mathbf{J} \times \mathbf{a}_H, \quad (10)$$

$$\mathbf{E}_\Lambda = - (\mathbf{J} \times \mathbf{a}_H) \times \mathbf{a}_H, \quad (11)$$

using the definitions $\mathbf{a}_o \equiv f_o \mathbf{B}$, $\mathbf{a}_H \equiv f_H \mathbf{B}$, $\mathbf{a}_\Lambda \equiv f_\Lambda \mathbf{B}$, where $f_o \equiv \sqrt{\rho_o / B}$, $f_H \equiv r_H / B$ and $f_\Lambda \equiv \sqrt{\sigma_\Lambda / B}$. The resistivities given here are the Ohmic, Hall and ambipolar resistivities, respectively, and are defined by

$$r_o \equiv \frac{1}{\sigma_o}, \quad (12)$$

$$r_H \equiv \frac{\sigma_H}{\sigma_{H}^2 + \sigma_\Lambda^2}, \quad (13)$$

$$r_\Lambda \equiv \frac{\sigma_\Lambda}{\sigma_{H}^2 + \sigma_\Lambda^2}, \quad (14)$$

where the conductivities are given by

$$\sigma_o = \frac{1}{B} \sum_{i=1}^{N-1} \alpha_i \rho_i \beta_i, \quad (15)$$

$$\sigma_H = \frac{1}{B} \sum_{i=1}^{N-1} \alpha_i \rho_i \beta_i, \quad (16)$$

$$\sigma_\Lambda = \frac{1}{B} \sum_{i=1}^{N-1} \frac{\alpha_i \rho_i}{1 + \beta_i^2}. \quad (17)$$

where the Hall parameter $\beta_i$ for a charged species is given by

$$\beta_i = \frac{\alpha_i B}{K_{i0} \rho_0}. \quad (18)$$

To solve these equations numerically we use three different operators:

(i) solve equations (1), (2), (3), including the restriction of equation (5) and for $i = 0$, using a standard second order, finite volume shock-capturing scheme. Note that for this operator the resistivity terms in equation (3) are not incorporated. Equation (5) is incorporated using the method of Dedner (Dedner et al. 2002).

(ii) Incorporate the resistive effects in equation (3) using super-time-stepping to accelerate the ambipolar diffusion term and the Hall Diffusion Scheme to deal with the Hall term.

(iii) Solve equations (4) for the charged species velocities and use these to update equation (1) (with $i = 1, \ldots, N - 1$)

These operators are applied using Strang operator splitting in order to maintain the second order accuracy of the overall scheme. We refer the reader to O’Sullivan & Downes (2002, 2007) for a more detailed description.

2.2 Initial conditions

The simulations are carried out on a 2.5D slab grid in the $xy$-plane. The grid consists of $6400 \times 200 \times 1$ cells, in the $x$, $y$, and $z$ directions respectively. This resolution was chosen on the basis that it reproduces the initial linear growth of the ideal MHD system in Keppens et al. (1999). Resolution studies were performed to confirm the resolution as being appropriate (see Sect. 2.3.1). The initial set-up used was that of two plasmas flowing anti-parallel side-by-side on a grid of size $x = [0, 32L]$ and $y = [0, L]$. The plasma velocities are given by $+\frac{x}{a}$ and $-\frac{x}{a}$ in the $x$-direction, with a tangential shear layer of width $2a$ at the interface at $x = 16L$. This velocity profile is described by

$$v_0 = \frac{V_0}{2} \tanh \left( \frac{x - 16L}{a} \right) y. \quad (19)$$

The width of the shear layer is chosen to be $\frac{a}{L} = 0.05$, or approximately 20 grid zones. The magnetic field is initially set to be uniform and aligned with the plasma flow.

The initial background for all three fluids in the system is now an exact equilibrium. The initial neutral velocity field, $V_0$ is then augmented with a perturbation given by

$$\delta v_y = \delta V_0 \sin (-k_y y) \exp \left( -\frac{(x - 16L)^2}{\sigma^2} \right). \quad (20)$$

where $\delta V_0$ is set to $10^{-4} V_0$. The wavelength of the perturbation is set equal to the characteristic length scale, $\lambda \equiv \frac{a}{L} = L$, so that a single wavelength fits exactly into the computational domain. This maximises the possibility of resolving structures that are small relative to the initial perturbed wavelength. (Frank et al. 1998). The perturbation attenuation scale is chosen so that it is larger than the
Table 1. The density, collision coefficient ($K_{i0}$) and charge-to-mass ratios ($\alpha_i$) for each of the charged fluids in simulation full-low-hr (see table 2). These parameters are modified to vary the resistivities as necessary for the other simulations in this work. See text.

| Fluid     | Density   | $K_{i0}$     | $\alpha_i$ |
|-----------|-----------|--------------|------------|
| 1         | $2.84 \times 10^{-13}$ | $2 \times 10^4$ | $-1 \times 10^{17}$ |
| 2         | $2 \times 10^{-7}$      | $1.42 \times 10^{10}$ | $1.42 \times 10^{11}$ |

shear layer, but small enough so that the instability can be assumed to interact only minimally with the $z$-boundaries (see Palotti et al. 2008), and is set using $\frac{L}{c} = 0.2$ (see Keppens et al. 1999, Palotti et al. 2008).

The physical parameters are then chosen using normalised, dimensionless quantities. The wavenumber $k_y$ is chosen to be $2\pi$ in order to maximise the growth rate of the instability (Keppens et al. 1999). This normalises the length scale of the simulation so that $L = 1$. The timescale is then normalised by the sound speed, so that $c_s = \frac{c}{L} = 1$. The mass scale is chosen such that the initial mass density is set to unity, $\rho_0 = 1$. In the isothermal case the adiabatic index $\gamma = 1$, and this gives us a sound speed $c_s = \sqrt{\frac{\gamma}{\gamma - 1}} = 1$, and the initial pressure is therefore also equal to unity, $p_0 = 1$. A transonic flow is chosen with sonic Mach number $M_s = \frac{c_s}{c} = 1$, so that the plasma has velocity $\pm \frac{c_s}{2} = \pm 0.5$. In order for the KH instability to be driven, it is required that the flow be super-Alfvénic (Chandrasekhar 1961), and so for this study, an Alfvén Mach number $M_A = \frac{c_A}{c} = 10$ is chosen. This sets the Alfvén velocity, $v_A = \sqrt{\frac{\rho_0 c_s}{\mu_0}} = 0.1$, and the initial magnetic field strength to $B_0 = 0.1$.

We use periodic boundary conditions at the high and low $y$ boundaries. Since we wish to study not only the initial growth phase of the instability, but also its subsequent nonlinear behaviour we must ensure that waves interacting with the high and low $x$ boundaries do not reflect back into the domain to influence the dynamics. Several test simulations for various parameters have shown that a large width of 32 is necessary to ensure this. We use gradient zero boundary conditions at the high and low $x$ and $z$ boundaries.

Finally we must choose the parameters describing the properties of our charged fluids in our multifluid system. Our basic parameter set is contained in table 1. See Jones (2011) for more details.

Table 2 contains the nomenclature we will use for the rest of this paper when referring to the simulations. Each simulation is denoted by yyyy-yyyy-zz where the first set of characters denote the dominant resistivity (Hall or ambipolar or “full” if both resistivities have the same magnitudes), the second set denote the level of the resistivity (low, medium or high) and the final two characters denote the resolution (low, medium or high resolution denoted by lr, mr and hr respectively). The high resolution simulations ($6400 \times 200$) took of the order of 1500 – 3000 core hours on a quad-core Xeon E5540 based system.

Note that simulation full-low-hr is effectively an ideal MHD simulation and produced results virtually identical to mhd-zero-hr which is a true MHD simulation (see Sect. 4.1) and which was used for comparison with previous literature.

| Simulation         | Resolution | $r_H$ | $r_A$ |
|--------------------|------------|-------|-------|
| mhd-zero-hr        | 400 $\times$ 200 | 0     | 0     |
| hd-zero-hr         | 400 $\times$ 200 | 0     | 0     |
| ambi-high-lr       | 1600 $\times$ 50 | $3.52 \times 10^{-6}$ | $3.5 \times 10^{-2}$ |
| ambi-high-mr       | 3200 $\times$ 100 | $3.52 \times 10^{-6}$ | $3.5 \times 10^{-2}$ |
| ambi-high-hr       | 6400 $\times$ 200 | $3.52 \times 10^{-6}$ | $3.5 \times 10^{-6}$ |
| full-low-hr        | 6400 $\times$ 200 | $3.52 \times 10^{-6}$ | $3.5 \times 10^{-6}$ |
| full-med-hr        | 6400 $\times$ 200 | $3.52 \times 10^{-6}$ | $3.5 \times 10^{-6}$ |
| full-high-hr       | 6400 $\times$ 200 | $3.52 \times 10^{-6}$ | $3.5 \times 10^{-3}$ |
| hall-high-lr       | 1600 $\times$ 50 | $3.52 \times 10^{-2}$ | $3.5 \times 10^{-6}$ |
| hall-high-mr       | 3200 $\times$ 100 | $3.52 \times 10^{-3}$ | $3.5 \times 10^{-6}$ |
| hall-high-hr       | 6400 $\times$ 200 | $3.52 \times 10^{-2}$ | $3.5 \times 10^{-6}$ |
| hall-med-hr        | 6400 $\times$ 200 | $3.52 \times 10^{-3}$ | $3.5 \times 10^{-6}$ |
| hall-high-hr       | 6400 $\times$ 200 | $3.52 \times 10^{-3}$ | $3.5 \times 10^{-6}$ |

3 ANALYSIS

In order to study the growth of the instability, the evolution of a number of parameters can be measured with time. In particular, we measure the transverse kinetic energy

$$E_{k,x} = \int \int \frac{1}{2} \rho v_x^2 \, dx \, dy$$

and the magnetic energy

$$E_b = \frac{1}{2} \int \int \{ B_x^2 + B_y^2 + B_z^2 \} \, dx \, dy$$

in the system where $B_0$ is the magnitude of the magnetic field at $t = 0$. As the entire plasma flow is initially in the $y$-direction, with only a very small perturbation in the $x$-direction, any growth of $E_{k,x}$ is due to the growth of the instability. It is possible to determine the growth rate of the instability directly from the growth of the transverse kinetic energy (Keppens et al. 1999): the transverse kinetic energy can be expressed as

$$E_{k,x} = (\rho_0 + \delta \rho)(|v_x|_0 + \delta v_x)^2$$

$$= (\rho_0 + \delta \rho)(\delta v_x)^2$$

$$\approx \rho_0 \delta v_x^2$$

$$\propto \exp[2i(k \cdot r - \omega t)]$$

presuming the initial perturbation is proportional to $\exp[i(k \cdot r - \omega t)]$. Hence the kinetic energy grows at a rate of $2\omega$, where $\omega$ is the growth rate of the instability.

4 RESULTS

We start by describing the validation of the set-up used by comparing an ideal MHD simulation run using HYDRA with previously published literature. We then go on to discuss the behaviour of the KH instability in ambipolar-dominated and Hall-dominated flows respectively.
The Kelvin-Helmholtz instability in weakly ionised plasmas

4.1 Validation

In order first to validate our set-up we examine our mhd-zero-hr simulation with HYDRA and determine the growth rate using the kinetic energy of motions in the x direction as described in Sect. 3. Figure 1 contains a plot of the log of the transverse kinetic energy as a function of time. At early times this growth is clearly exponential and can be fitted with a line of slope 2.63 implying a growth rate, normalised by the width of the shear layer and the initial relative velocity, for the dominant mode of the KH instability of 0.1315. We can compare this with the value of the growth rate calculated analytically by Miura & Pritchett (1982) (their Figure 4) at this wavenumber of 0.13. While comparisons between linear studies of incompressible flows, and numerical studies of compressible flows are bound to differ to some extent, these results are seen to agree exceptionally well.

We wish to examine not only the linear regime but also the non-linear regime. We compare our results for the growth of magnetic energy with those of Malagoli et al. (1996) (the upper panel of their Figure 5). Figure 2 contains a plot of the magnetic energy, calculated as \(\int \int \frac{1}{2} (B_x^2 + B_y^2 + B_z^2) \, dx \, dy\), as a function of time. The maximum magnetic energy reached in our simulations matches that of Malagoli et al. (1996) to within 10%. Our simulation reaches saturation at a later time but the exact time of saturation depends on the initial amplitude of the perturbation and so this is not a concern.

We are therefore confident of the behaviour of HYDRA in simulating the KH instability. We now move on to investigating the influence of multifluid MHD effects on the growth, saturation and non-linear behaviour of this instability.

4.2 Ambipolar dominated flows

We begin our study of the multifluid KH instability by choosing our fluid parameters to ensure our ambipolar resistivity is dynamically significant while minimising the Hall resistivity. This allows us to isolate the influence of the ambipolar resistivity on the instability. In order to increase the ambipolar resistivity we change the value of the collision coefficient for species 2 so that \(K_{2,0}\) is decreased by 3 orders of magnitude from that given in Table 1 for simulation ambi-med-hr and by 4 orders of magnitude for ambi-high-hr. These alterations of \(K_{2,0}\) give values of \(\tau_A\) of \(3.5 \times 10^{-3}\) and \(3.5 \times 10^{-2}\) respectively, and (ambipolar) magnetic Reynold’s numbers, \(R_{m}\), of \(2.84 \times 10^2\) and \(2.84 \times 10^3\) respectively. These simulations are examined in comparison to the full-low-hr simulation. With a formal magnetic Reynolds number \(2.84 \times 10^5\), the diffusion in this set-up is predominantly numerical, and as such, it is effectively an ideal MHD simulation.

4.2.1 Resolution study

In non-ideal MHD we must ensure that the length scales over which the diffusion of the magnetic field (or the whistler waves in the case of Hall dominated flows) must be resolved in order to properly track the dynamics of the system. To this end we perform a resolution study using simulations ambi-high-lr, ambi-high-mr and ambi-high-hr (see Table 2).

Figures 3 and 4 contain plots of the evolution of \(E_{k,x}\) and \(E_{b}\) for each of the simulations in our resolution study. It can be seen that the linear growth in ambi-high-lr is significantly lower than the two other simulations. However, the linear behaviour is almost identical for ambi-high-mr and ambi-high-hr. The subsequent non-linear behaviour is similar with only relatively small variations after \(t \sim 11\).

The results of this study indicate that a resolution of 6400 \times 200 is sufficient to capture the initial growth and saturation of the instability. Subsequently, the dynamics is captured at least qualitatively.

4.2.2 The linear regime

Generally speaking, the evolution of the KH instability in ideal MHD leads to a wind-up of both the plasma and the magnetic field at the interface between the two fluids, resulting in the “Kelvin’s cat’s eye” vortex. Multifluid effects
alter the nature of this significantly. Figure 5 contains plots of the magnetic field at $t = 8t_*$, which is the time at which the instability saturates for both full-low-hr (effectively ideal MHD) and ambi-high-hr. The difference in the morphology is clear.

Given these striking differences at saturation it is interesting to investigate whether the linear growth rate is influenced by the addition of ambipolar diffusion. Figure 6 contains plots of $E_{k,x}$ with time for the various simulations. The linear growth rate remains almost unchanged with the addition of ambipolar diffusion. On the other hand, figure 7 contains plots of $E_k$ as a function of time. It is clear that the perturbed magnetic energy is strongly influenced, even well before saturation, by the presence of ambipolar diffusion. We will discuss this in more detail in section 4.2.3.

4.2.3 The nonlinear regime

It is clear from figure 7 that the magnetic energy is strongly influenced by the presence of ambipolar diffusion. This is not too surprising as ambipolar diffusion, being a genuinely diffusive process (unlike the Hall effect) allows the magnetic field to diffuse relative to the bulk flow.

As the collision rate between the ion and neutral fluids is decreased, the ion fluid decouples from the bulk fluid, and thus the magnetic field becomes decoupled from the bulk flow: the frozen-in approximation of ideal MHD is broken. As a result, the magnetic field is able to diffuse through the bulk fluid rather than being tied to it. In figure 7 diffusion can be identified as the cause of the decrease of the amplification of the magnetic energy with time for increasing ambipolar resistivity. A cursory examination of the topology of the magnetic field for the low resistivity case (full-low-hr in figure 5) demonstrates clearly that there are regions in the flow which will be susceptible to diffusion: regions in which the magnetic field lines have been compressed and amplified.

It is clear, again from figure 7, that there is a significant decrease in the growth of magnetic energy as a result of diffusion for even a moderate amount of ambipolar resistivity. The diffusion has an influence on the dynamics of the neutral fluid: when the magnetic energy has not been
as strongly amplified, the field can no longer exert the same effect on the neutral and ion fluid. Examination of figure 8 reveals that the peak reached by $E_{b,x}$ increases with increasing ambipolar resistivity. The value reached tends toward the hydrodynamic limit for two reasons: increasing resistivity implies increasing diffusion and hence a decreasing field strength, and it also implies less coupling between the magnetic field and the neutral fluid with increasing resistivity. As expected, the weaker magnetic field strength allows the vortex to become more rolled up (Faganello et al. 2009).

It can be shown that for the ambimed-hr simulation, the increase in the wind-up of the bulk fluid is due solely to the first source: the magnetic diffusion. The slight decoupling of the ion and neutral fluid is sufficiently high to allow magnetic diffusion, while still being sufficiently low to force the charged fluids to behave in a manner similar to the bulk fluid. This can be seen in a plot of the transverse kinetic energy of the ion fluid, as shown in figure 8. With moderate amounts of ambipolar diffusion, the transverse kinetic energy of the ion fluid (and electron fluid) reaches a higher maximum, meaning that the fluid experiences more wind-up.

However, as the ambipolar resistivity is increased further, the ion fluid becomes more decoupled from the neutral fluid, and instead is influenced primarily by the dynamics of the magnetic field. This significant decoupling occurs only for magnetic Reynolds number lower than $Re_m \approx 100$. As the magnetic field therefore undergoes less wind-up than before, so does the ion (and electron) fluid. This decrease in the wind-up of the charged fluid velocity field is the cause of the decrease in the transverse kinetic energy of the ion fluid, as is seen in ambi-high-hr in figure 8.

### 4.3 Hall dominated flows

We now turn our attention to the likely influence of the Hall effect on the KH instability in multifluid MHD flows. In order to attain the resistivities we want (see table 2), we reduce the charge-to-mass ratio of the ion fluid, causing its Hall parameter to become smaller, causing the ion fluid dynamics to more closely resemble the bulk (neutral) fluid dynamics. The electrons are, however, still well-tied to the field lines and so a relative drift emerges between the ion and electron fluids leading to a current perpendicular to the magnetic field and hence the Hall effect.

#### 4.3.1 Resolution study

As in the ambipolar resistivity study, a resolution study is carried out to ensure that the small-scale non-ideal dynamics are captured. For this purpose, simulations are again run at three different resolutions (see Table 2). These simulations are run with the highest level of Hall resistivity to ensure that the smallest-scale dispersive effects are in place when examining whether they are sufficiently resolved. The inclusion of the Hall term in the dispersion relation in principle allows for the introduction of waves with a signal speed which tends towards infinity as their wavelength tends towards zero. While the Hall term is handled in the equations by the HYDRA code using the explicit Hall Diffusion Scheme (HDS) (O’Sullivan & Downes 2006, 2007), the code naturally does not resolve these waves of vanishing wavelength.

As has been seen, the introduction of non-ideal effects does not tend to greatly influence the linear growth rate of the instability. As a result, the growth rate does not provide a good means of measuring convergence with increasing resolution. The nonlinear evolution of the transverse kinetic energy, $E_{b,x}$ is, however, strongly influenced by the non-ideal effects. We do not examine the evolution of the perturbed magnetic energy as, in the case of high Hall resistivity, it no longer demonstrates a simple growth to an initial maximum. The evolution of the transverse kinetic energy for each simulation is plotted in figure 9. It can clearly be seen that the simulations have started to converge at higher resolutions. While there is a notable difference between the two simulations of lower resolution, the gap closes significantly in the comparison between the two simulations of higher resolution. In particular, the initial maxima of $E_{b,x}$ are almost identical in simulations hall-high-mr and hall-high-hr.

We are, therefore, confident that the dynamics in the hall-hr are well resolved and that our conclusions as to the physical processes occurring are well-founded.

#### 4.3.2 The linear regime

Figures 10 and 11 contain plots of $E_{b,x}$ and $E_b$ as a function of time for simulations hall-med-hr and hall-low-hr. In the linear regime, the influence of the Hall effect on the growth...
rate of the instability, and on the magnetic and kinetic energy in the system at saturation can be seen to be negligible. Hence we can conclude that neither ambipolar diffusion nor the Hall effect influence the energetics of the system in the linear regime.

We expect the Hall effect to re-orient the magnetic field out of the $xy$-plane in which it resides at $t = 0$. Figure 12 contains plots of $E_b$, the perturbed magnetic energy in the $xy$-plane and the perturbed magnetic energy in the $z$ direction as functions of time. It is clear that there is some growth of the magnetic field in the $z$ direction. Interestingly, at saturation the magnetic energy in the $xy$-plane is noticeably less than $E_b$ and yet the overall value of $E_b$ is the same as that derived from the full-low-hr (i.e. quasi-ideal MHD) simulation. An interesting point to note about this is that, whereas it has been shown (Jones et al. 1999) that the strength of the field in the direction of the initial flow is what is important in determining the effect of the magnetic field on the KH instability, here we can see that the strength of the field perpendicular to this plane appears to play a role also.

\[ \rho v^2 - \frac{1}{2} B_z^2 \]

### 4.3.3 The nonlinear regime

Following the initial linear growth of the instability, the system experiences a period of transferring energy back and forth between the magnetic field and velocity field, as in full-low-hr (see figure 13). Interestingly, the amplitude of the oscillations - i.e. the amount of energy being transferred between motion and the magnetic field - is larger in hall-med-hr than in full-low-hr. This may be due to re-orientation of the magnetic field out of the plane of the instability and hence a reduction in (numerical) reconnection. It is worth recalling here that the Hall effect, although it appears similar to a diffusion term in the induction equation, is a dispersive effect which conserves magnetic energy.

Perhaps the most important difference between full-low-hr and hall-med-hr is that while the kinetic energy in the $y$-direction gradually tends towards a constant value in the low resistivity case, it continues to steadily decay in hall-med-hr. The conclusion is that in full-low-hr, the instability has completed its growth and is returning to a quasi-steady state. In hall-med-hr however, the instability is continuing to consume the parallel kinetic energy available to it and the instability undergoes a further stage of development as a result of the inclusion of the Hall effect.

Since the magnetic field gains a component in the $z$ direction in hall-med-hr due to the Hall effect it is interesting to examine the behaviour of the kinetic energy in the $z$ direction also. Figure 14 contains plots of the transverse kinetic energy and the kinetic energy in the $z$ direction (i.e. $E_{k,z} = \frac{1}{2} \rho v_z^2$). Clearly the transverse kinetic energy grows...
The Kelvin-Helmholtz instability in weakly ionised plasmas

rapidly during the linear development of the instability but \( E_{k,z} \) also grows and, eventually, even becomes larger than \( E_{k,x} \). Figure 15 contains plots of the total energy (kinetic and magnetic) in full-low-hr, hall-med-hr and hall-high-hr. Somewhat surprisingly we find that the hall-med-hr simulation loses energy somewhat faster than full-low-hr. This is, on the face of it, a little puzzling since the Hall effect does not itself dissipate magnetic energy. If we consider the hall-high-hr simulation we find that the total energy is roughly constant - it behaves roughly the same as hd-zero-hr. In fact, the magnetic energy in the hall-high-hr simulation behaves qualitatively differently to the hall-med-hr. Figure 16 contains plots of the perturbed magnetic energy as a function of time for hall-high-hr, and full-low-hr. It is clear that something dramatic is happening in the hall-high-hr simulation. To gain insight into this, in figure 17 we plot the total perturbed magnetic energy and the magnetic energy in the \( xy \)-plane and in the \( z \)-direction for simulation hall-high-hr. The growth of the magnetic field in the \( z \) direction is significant, as we expect from a system with the Hall effect. In fact, the growth of the magnetic field in the \( xy \)-plane is faster in the hall-med-hr case. This isn’t too surprising as, in order to increase the Hall effect, the coupling between the electrons and the neutrals is much weaker than that between the ions and neutrals. Hence, while the neutrals and ions generate the usual cat’s eye vortex, the electrons (and the magnetic field) do not. Figure 16 demonstrates these morphological differences between the various fluids.

Furthermore, in the case of full-low-hr, the magnetic field in the \( xy \)-plane grows to such an extent that it opposes further wind-up of the fluid in the vortex. In the hall-high-hr case this does not happen as magnetic energy is redistributed to the \( z \) component of the field which does not oppose this wind-up. Hence the vortex is not destroyed in the non-linear regime in the hall-high-hr case. This is the main difference between the hall-med-hr and hall-high-hr simulations. In the non-linear regime, then, the ion fluid remains spinning in the KH vortex while the electron fluid remains tied to the magnetic field. This maintains a velocity difference between the electrons and ions which, in turn, causes the magnetic energy to be further re-distributed to the \( z \) component. In this way the Hall effect, if strong enough, introduces strong dynamo action into the KH instability. This dynamo behaviour, which is not possible in a 2.5D, ideal MHD system is made possible by the Hall term introducing a handedness into the flow (e.g. Wardle 1994).

If we follow the dynamics further we find that the KH instability, in the presence of high Hall resistivity does not saturate to a quasi-steady state as it does in, for example, the full-low-hr case. As the \( z \) components of the current and magnetic field continue to grow, the Hall effect now acts on the non-parallel currents and magnetic fields that have arisen between the \( z \)-directions and the \( xy \)-plane. This has the result of re-orienting some of the magnetic field, and electron fluid flow, back onto the \( xy \)-plane. During this process the electron fluid obtains a velocity away from the KH vortex, which results in a broader volume of plasma being disturbed. This feeds the continuous growth of the magnetic energy in the \( xy \)-plane, and thus causes continuous growth.
The perturbed magnetic energy in the system is plotted against time for hall-high-hr (solid line). Also plotted for comparison are the growth of the magnetic energies in the $xy$-plane (dashed line) and in the $z$-direction (dot-dashed line). It can be seen that there is significant growth in the $z$-direction.

The perturbed magnetic energy in the system is plotted against time for hall-high-hr (solid line). Also plotted for comparison are the growth of the magnetic energies in the $xy$-plane (dashed line) and in the $z$-direction (dot-dashed line). It can be seen that there is significant growth in the $z$-direction.

Figure 18. Plot of the magnitude and vector field of the magnetic field in the $xy$-plane in hall-high-hr at time $8t_s$. It can be seen that the magnetic field does not undergo as much wind-up as is seen in the bulk velocity field (upper panel of figure 19).

To summarise, through their strong decoupling from the magnetic field, the dynamics of the bulk fluid and ion fluid demonstrate behaviour very similar to that of hd-zero-hr in which the KH vortex remains intact. The high Hall case can in fact be thought of as two separate systems occurring simultaneously; the bulk fluid demonstrating hydrodynamic behaviour and the continuously widening volume of perturbed fluid perpetually feeding the growth of the magnetic field through the Hall effect. These two systems are intrinsically entwined through the requirement of charge neutrality, by which the electron fluid causes a widened area of perturbed ion fluid, and thus the bulk fluid. Both systems are relatively energy efficient and, following the initial growth of the KH instability, the overall system experiences little further loss of energy. The supply of energy to feed the magnetic field is limited only to the physical size of the computational domain over which the simulation is run.

This dynamo action occurs only under certain conditions. As the initial Hall resistivity is increased from moder-
ate to high, the electron fluid, and thus the magnetic field, becomes increasingly decoupled from the neutrals. As a result, the neutral fluid tends toward hydrodynamic behavior. This dynamo action is observed only when the Hall resistivity is increased to a sufficiently high value that the KH vortex formed by the neutral fluid is no longer constrained by the magnetic field. Unlike the pure hydrodynamic case, the presence of charged fluids undergoing different dynamics leads to the dynamo action observed. If the Hall resistivity is not sufficiently large, the magnetic field eventually leads to the destruction of the KH vortex in the neutral fluid through reconnection as in the ideal MHD case.

Even a moderate amount of Hall resistivity results in a wider volume of fluid being disturbed by the instability. This agrees with previous studies of the Hall effect on the KH instability (e.g. Haba 1994). The re-orientation of the magnetic field lines within the KH instability has also been observed in studies of MRI in accretion disks (e.g. Wardle 1999, Kunz 2008). Kunz (2008) investigated a simple model of accretion disks without rotation and demonstrated that the combined actions of the shear instability and the Hall effect leads to increased stretching of the magnetic field lines. This was shown to result in continued growth of the instability, which corresponds well to the dynamo action observed in our simulations with high Hall resistivity. It is important to note, though, that studies by both Kunz (2008) and Wardle (1999) are linear studies and, as such, do not extend to the nonlinear regime of the instability. A more recent numerical study by Nykyri & Otto (2004) demonstrates the twisting of magnetic field lines, but due to the inclusion of magnetic reconnection, doesn’t produce the magnetic dynamo observed here.

5 CONCLUSIONS

We have presented the results of a suite of fully multifluid MHD simulations of the KH instability in weakly ionised fluids such as, for example, molecular cloud material. Through varying the collision coefficients between the various charged species and the neutrals we were able to investigate systems in which ambipolar diffusion dominates the multifluid effects and ones in which the Hall effect dominates. We validated our KH simulations through comparison of an ideal MHD simulation with previously published results and performed resolution studies for each of these cases to ensure that our conclusions are not unduly effected by our numerical resolution.

Our findings can be summarised as follows:

- The multifluid effects do not significantly influence the linear growth rate of the instability.
- Ambipolar diffusion dramatically reduces the energy associated with the perturbed magnetic field. This happens through diffusion for moderate ambipolar resistivity, but through both diffusion and decoupling of the magnetic field from the bulk flow for higher resistivity.
- The Hall effect, as expected, rotates the magnetic field out of the initial xy-plane.
- In contrast to both ambipolar dominated and ideal MHD flows, the Hall effect causes the system to fail to settle to a quasi-steady state after saturation of the instability.
- For moderate Hall resistivity the perturbed magnetic field contains higher energy than in the ideal MHD case, presumably due to a field topology which impedes (numerical) reconnection.
- For high Hall resistivity strong dynamo action is seen as energy associated with the magnetic field grows without any apparent signs of saturation.

ACKNOWLEDGEMENTS

The research of A.C.J. has been part supported by the CosmoGrid project funded under the Programme for Research in Third Level Institutions (PRTLI) administered by the Irish Higher Education Authority under the National Development Plan and with partial support from the European Regional Development Fund.

The authors wish to acknowledge the SFI/HEA Irish Centre for High-End Computing (ICHEC) for the provision of computational facilities and support.

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