If Gauss-Bonnet interaction plays the role of dark energy

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Abstract

Gauss-Bonnet-scalar interaction has been found to play a crucial role from the beginning till the late time of cosmological evolution. A cosmological model has been constructed where the Universe starts with exponential expansion but with infinite deceleration, $q \to \infty$ and infinite equation of state parameter, $w \to \infty$. During evolution it passes through the stiff fluid era, $q = 2, w = 1$, the radiation dominated era, $q = 1, w = 1/3$ and the matter dominated era, $q = 1/2, w = 0$. Finally, deceleration halts, $q = 0, w = -1/3$, and it then encounters a transition to the accelerating phase. Asymptotically the Universe reaches yet another inflationary phase $q \to -1, w \to -1$. Such evolution is independent of the form of the potential and the sign of the kinetic energy term i.e., even a noncanonical kinetic energy is unable to phantomize ($w < -1$) the model.

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1 Introduction

In the recent years lot of observations have been carried out that lead to a precise knowledge of the cosmological evolution. Important cosmological observations like abundance of galaxy clusters [1], statistics of large scale redshift surveys [1], angular power spectrum of cosmic microwave background radiation (CMBR) [2] and baryon oscillations [3] suggest that the Universe is nearly flat and almost 73% of the matter density is in the form of dark energy [4]. Further, the magnitude-redshift relation from standard candles such as type Ia supernovae (SNIa) [5] indicates that the Universe has recently entered a phase of accelerating expansion. Reconciling all these astronomical observations it is now quite clear that the so called dark energy is slowly varying with negative pressure $p$ having repulsive properties that can encounter the attractive gravitational force and the corresponding equation of state parameter ($w = p/\rho$) is presently pretty close to minus one ($-1$) [7]. So we can now brief the knowledge of the cosmological evolution that we have gathered so far from all the astronomical observations. Soon after the beginning, the Universe passed through a phase of exponential expansion (inflation) and during evolution it went through the stiff fluid era - when pressure balanced the energy density ($p = \rho$, i.e., $w = 1$). Thereafter, it encountered the radiation dominated era ($p = \rho/3$, i.e., $w = 1/3$) and finally the Universe entered the pressureless dust era ($p = 0$, i.e., $w = 0$). Presently, as already mentioned, it is accelerating with the equation state parameter $w$ pretty close to minus one ($-1$) [7]. So far all the attempts made to construct different dark energy models of the Universe encompassing all the observable phenomena went in vain [6, 8].

An alternative approach to accommodate dark energy is to modify the General Theory of Relativity by considering additional curvature invariant terms such as Gauss-Bonnet (GB) term. GB term arises naturally as the leading order of the $\alpha'$ expansion of heterotic superstring theory, where, $\alpha'$ is the inverse string tension [9]. Some interesting results appear in the literature with GB interaction. Avoidance of naked singularities in dilatonic brane world scenarios and the problem of fine tuning with scalar fields and GB interaction have been discussed in [10]. Further in string induced gravity near initial singularity GB coupling with scalar field has been found [11] important for the occurrence of nonsingular cosmology. In addition there are also some recent investigations [12, 13] in the context of dark energy models. In these works issues (scalar-GB gravity is equivalent classically to so-called modified GB gravity) like phantom cosmology with GB correction, interplay between GB term and quintessence scalar, experimental constraints on astronomical and cosmological observations and cosmological models with scalar dependent GB interaction have been addressed.
Gauss-Bonnet term is a topological invariant one and so to get some contribution in the four dimensional space-time it requires dynamic dilatonic scalar coupling. In this work we shall consider such coupling and investigate cosmological consequence.

The paper has been organized as follows. In the following section we write the action and the field equations. In section 3 we explore a set of solutions with negative GB interaction. It is found that such solutions require noncanonical kinetic energy. In section 4 we present another set of solutions for both positive and negative GB interacting terms. It has been observed that a noncanonical kinetic energy evolves through to a canonical one, without affecting the cosmic evolution.

## 2 Action and the field equations

We start with the following action containing Gauss-Bonnet interaction

\[
S = \int d^4x \sqrt{-g} \left(\frac{R^2}{2\kappa^2} + \frac{\Lambda(\phi)}{8} G(R) - g(\phi) \phi_{,\mu} \phi^{,\mu} - V(\phi)\right),
\]  

(1)

where, \(G(R) = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\) is the Gauss-Bonnet term which is appearing in the action with a coupling parameter \(\Lambda(\phi)\). In the action there is yet another coupling parameter viz., \(g(\phi)\). For the spatially flat Robertson-Walker space-time

\[
ds^2 = -dt^2 + a(t)^2 [dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2]
\]

The field equations are

\[
2 \ddot{a} + \frac{\dot{a}^2}{a} = -\kappa^2 [g \dot{\phi}^2 - V(\phi)] + 2\Lambda' \frac{\dot{\phi} \ddot{a}}{a^2} + (\Lambda' \dot{\phi} + \Lambda'' \phi^2) \frac{\dot{a}^2}{a^2} = -8\pi G p,
\]

(2)

\[
3 \frac{\dot{a}^2}{a^2} = \kappa^2 [g \dot{\phi}^2 + V(\phi)] - 3\Lambda' \frac{\dot{\phi}^2}{a^2} = 8\pi G \rho,
\]

(3)

where, \(p\) and \(\rho\) are the effective pressure and density generated by the scalar field and the Gauss-Bonnet interaction. In addition we have got the \(\phi\) variation equation

\[
2g(\ddot{\phi} + \frac{3}{a} \dot{\phi} + \frac{1}{2} \frac{g'}{g} \dot{\phi}^2 + \frac{V'}{2g}) = 3\Lambda \frac{\dot{a}^2 \ddot{a}}{a^3}
\]

(4)

which may not be considered to be an independent equation, since it is derivable from the above two equations (2) and (3). In the above, over-dot and dash (') stand for differentiations with respect to the proper time \(t\) and \(\phi\) respectively. Now, we are to solve for \(a, \phi, g(\phi), V(\phi)\) and \(\Lambda(\phi)\) in view of the above two field equations (2) and (3), which requires three assumptions. The first assumption that we make is

\[
\Lambda' \phi = \lambda
\]

(5)

where \(\lambda\) is a constant, which is physically reasonable, since it implies that the Gauss-Bonnet coupling parameter \(\Lambda(\phi(t)) = \lambda t\) grows in time, and as a result it might contribute at the later epoch of cosmological evolution. Such an assumption also mathematically simplifies the field equations (2) and (3) considerably, which are,

\[
2\dot{H} + 3H^2 = -\kappa^2 [g \dot{\phi}^2 - V(\phi)] + 2\lambda H \dot{H} + 2\lambda H^3 = -\kappa^2 p,
\]

(6)

\[
3H^2 = \kappa^2 [g \dot{\phi}^2 + V(\phi)] - 3\lambda H^3 = \kappa^2 \rho,
\]

(7)

where, \(H = \dot{a}/a\). Now eliminating \(V(\phi)\) between equations (6) and (7) we find,

\[
\dot{H} + \kappa^2 [g \dot{\phi}^2 + \lambda H \dot{H} - \frac{\lambda}{2} H^3] = 0.
\]

(8)

One can also eliminate \(g \dot{\phi}^2\) between the same pair of equations to express the potential as,

\[
V = \frac{3}{n^2} H^2 + \frac{5\lambda - n^2}{2} H^3 - \frac{\kappa^2 n^2 \lambda}{2} H^4.
\]

(9)

Hence, we shall now deal with equations (5), (7), (8) and/or (9) to solve the field variables \(a, \phi\) the potential \(V(\phi)\) and the parameters \(g, \lambda\) of the theory. Two important parameters that we deal with, in the cosmological context, are the equation of state \((w = \frac{p}{\rho})\) and the deceleration \((q = -\frac{\ddot{a}}{a^2})\) parameters, which are expressed as,

\[
w = -1 - \frac{2\dot{H}}{3H^2} \quad \text{and} \quad q = -1 - \frac{\dot{H}}{H^2} = \frac{1 + 3w}{2}.
\]

(10)
3 Solution with negative Gauss-Bonnet interaction

The Gauss-Bonnet term interacts with the dilatonic scalar field through $\Lambda(\phi)$ which may appear with both the sign positive and negative. Here we consider the negative Gauss-Bonnet interaction. Let us assume,

$$g\dot{\phi}^2 + \lambda H \dot{H} = 0$$  \hspace{1cm} (11)

Under the above choice, equation (8) gets solved immediately to yield

$$H = \frac{1}{\kappa n \sqrt{t}} \ & \ & a = a_0 \ e^{\frac{2n^2}{\kappa}};$$  \hspace{1cm} (12)

where, $\lambda = -n^2$ and we have considered positive sign only for both $\kappa$ and $n$ to ensure expanding model. Now in view of equation (10) we can find the state and the deceleration parameters as

$$w = -1 + \frac{\kappa n}{3\sqrt{t}} \ & \ & q = -1 + \frac{\kappa n}{2\sqrt{t}}.$$  \hspace{1cm} (13)

It is to be noted that the above set of solutions (12) and (13) depends on the constant $n$, which determines the GB coupling parameter $\Lambda(\phi)$. Thus at the very early stage of the evolution of the Universe, i.e., at $t \to 0$, $q \to \infty$ and $w \to \infty$. Thereafter at $t = \frac{2n^2}{\kappa}$, the Universe enters stiff fluid era with $q = 2$ and $w = 1$. Radiation dominated era starts at $t = \frac{4n^2}{\kappa}$, when $q = 1$ and $w = \frac{3}{2}$. At $t = \frac{\kappa^2}{2n^2}$ the Universe becomes dust filled, with $q = \frac{1}{2}$ and $w = 0$. There is a transition from the decelerating to the accelerating Universe at $t = \frac{4n^2}{\kappa}$, when $q = 0$ and $w = -\frac{3}{2}$. Finally with the increase of the proper time $q \to -1$ and $w \to -1$ asymptotically. It is most important to notice that such evolution from the very early to the late time of the Universe encompassing all the experimental observations is independent of the form of the potential $V(\phi)$.

Now in view of equation (11) we get

$$g\dot{\phi}^2 = -\frac{\kappa^2 n^2}{2} H^4 = -\frac{1}{2\kappa^2 t^2}.$$  \hspace{1cm} (14)

Thus the solutions obtained demands $g(\phi)$ has to be negative, i.e the kinetic term appears with a wrong sign. It is rather interesting to note that even a wrong sign of kinetic energy does not phantomize the cosmological model under consideration, i.e., the state parameter $w$ never goes beyond $-1$. However, to find explicit solutions of the model under consideration, we have to fix up either $g(\phi)$, $\Lambda(\phi)$, $\phi$ or the potential $V(\phi)$. As an example we consider the most natural choice, viz., $g = -\frac{1}{2}$. Thus equation (14) gets solve for $\phi$ as

$$\phi = \frac{\ln t}{\kappa}.$$  \hspace{1cm} (15)

Equation (5) now solves $\Lambda(\phi)$ as

$$\Lambda = -n^2 e^{\kappa\phi},$$  \hspace{1cm} (16)

and equation (7) solves the potential as

$$V = \frac{1}{2\kappa^2 n^2\lambda^2}[e^{-\kappa\phi} + \kappa^2 n^2 e^{-2\kappa\phi} - 6\kappa ne^{-\frac{3\kappa\phi}{4}}].$$  \hspace{1cm} (17)

It is to be mentioned that the GB coupling parameter $\Lambda$ is expressed (eg., N.E.Mavromatos and J.Rizos in [10]) in the form,

$$\Lambda = -\lambda_0 e^{\phi},$$  \hspace{1cm} (18)

where, $l = -4/\sqrt{6}$ in four dimension. So, instead of fixing $g$ if one chooses $\lambda$ as in (18), then the solutions are found as

$$\phi = \frac{1}{l} \ln \frac{\lambda_0}{n^2}; \ & \ & g = -\frac{l^2}{2\kappa^2},$$  \hspace{1cm} (19)

and

$$V = \frac{1}{2\kappa^2 \lambda_0^2}[2\lambda_0 e^{\phi} + n^4 e^{2\phi} - 3\kappa n^2 \sqrt{\lambda_0} e^{\frac{3\phi}{4}}].$$  \hspace{1cm} (20)

It is to be noted that the GB coupling parameter $\Lambda(\phi)$ and the potential $V(\phi)$ carry exponents with opposite signs automatically, and asymptotically the potential becomes a constant. As mentioned, one can also choose different forms of the potential to find explicit solutions which we shall not consider in this section. What we have observed is that the negative interaction with Gauss-Bonnet term leads to a noncanonical kinetic energy, which has got some interest in the context of phantom cosmology [13], but not in general. In the following section we study both positive and negative interactions and transition from noncanonical to canonical kinetic energy.
4 Solution with both signs of Gauss-Bonnet interaction

In the previous section we have observed that the condition (11) led to negative GB interaction which ultimately made the kinetic energy noncanonical. In this section we make a different assumption so that the GB interaction \( \Lambda(\phi) \) may be positive as well. Let us consider,

\[
g\dot{\phi}^2 + \lambda H \dot{H} - \frac{\lambda}{2} H^3 = \frac{n_1^2}{2} H^3, \tag{21}
\]

where, \( n_1 \) is a constant. As a result equation (8) is solved to yield

\[
H = \frac{1}{\kappa n_1 \sqrt{t}}, \quad a = a_0 e^{\kappa n_1 t}. \tag{22}
\]

Further, equation (10) can be expressed as,

\[
w = -1 + \frac{\kappa n_1}{3 \sqrt{t}} \quad \text{and} \quad q = -1 + \frac{\kappa n_1}{2 \sqrt{t}}. \tag{23}
\]

Thus we obtain the same set of solutions as in the previous section with the only difference that the constant parameter \( n \) that determines the solutions (12) and (13) in the previous section 3, determines the GB coupling parameter \( \Lambda(\phi) \) too, while in the above solutions (22) and (23) \( n_1 \) has nothing to do with \( \Lambda(\phi) \). Thus \( \lambda \) here may be positive as well as negative and so is the GB interaction parameter \( \Lambda(\phi) \). The Universe evolves in the same manner as discussed in the previous section, starting from an exponential expansion with infinite deceleration and passing through the phases of stiff fluid, radiation dominated and the matter dominated era. It then finally encounters a transition to the accelerating phase when \( w = -1/3 \) and asymptotically reaches \( q = -1, w = -1 \).

We would like to mention once again that such cosmological evolution remains independent of the form of potential \( V(\phi) \). Equation (21) now reduces to

\[
g\dot{\phi}^2 = \frac{1}{\kappa^2 n_1^2} \left[ \frac{\lambda}{t^2} + \frac{\lambda + n_1^2}{\kappa n_1 t^{3/2}} \right]. \tag{24}
\]

To find the complete set of solutions we can explore different situations fixing up either \( g(\phi), \Lambda(\phi) \) or the potential \( V(\phi) \). In the following we cite a few examples.

**Case I**

The most natural choice, \( g = 1/2 \) leads to a well behaved solution, with

\[
\phi = 4 \left( \frac{\lambda}{(\kappa n_1)^3/2} \right) \left[ \lambda \kappa n_1 + (\lambda + n_1^2) \sqrt{t} \right]^{1/2} + \frac{\sqrt{\lambda \kappa n_1}}{2} \ln \left\{ \frac{\sqrt{\lambda \kappa n_1} + (\lambda + n_1^2) \sqrt{t} - \sqrt{\lambda \kappa n_1}}{\sqrt{\lambda \kappa n_1} + (\lambda + n_1^2) \sqrt{t} + \lambda \kappa n_1} \right\}, \tag{25}
\]

while the potential can be expressed as a function of time as,

\[
V = \frac{1}{\kappa^2 n_1^2} \left[ \frac{3}{\kappa^2 t} + \frac{5 \lambda - n_1^2}{2 \kappa n_1 t^{3/2}} - \frac{\lambda}{2 t^2} \right], \tag{26}
\]

but it is extremely difficult to express the potential and the GB interaction parameter \( \Lambda \) as a function of \( \phi \).

**Case II**

Next let us choose \( \Lambda \) as,

\[
\Lambda = \frac{\beta}{\phi}. \tag{27}
\]

Thus \( \phi \) can be solved in view of equation (5) as,

\[
\phi = \frac{\beta}{\lambda t^2}, \tag{28}
\]

which decreases while \( \Lambda \) increases linearly with time. \( g(\phi) \) can be found in view of equation (24) as,

\[
g = \sqrt{\beta} \left( \frac{n_1^2 + \lambda}{\kappa^3 n_1^2} \right) \phi^{-5/2} + \frac{\lambda}{2 \kappa^2 n_1^2} \phi^{-2}. \tag{29}
\]
Thus $g(\phi) \rightarrow \infty$ asymptotically. Now the potential takes the following form,

$$V = \frac{\lambda}{\kappa^2 n_1^2 \beta} \phi^2 + \sqrt{\frac{\lambda}{\beta}} \left( \frac{5\lambda - n_1^2}{2\kappa n_1} \right) \phi^{5/2} - \frac{\lambda^2}{2\beta} \phi^2.$$ \hspace{1cm} (30)

It is interesting to observe that if one sets $\lambda = -m^2$, then $\beta$ has to be negative making the Gauss-Bonnet interaction parameter negative. For such a situation, with $\beta = -c^2$, we have

$$g = \frac{c}{m} \left( \frac{n_1^2 - m^2}{2\kappa^2 n_1^4} \right) \phi^{-5/2} - \frac{m^2}{2\kappa^2 n_1^2} \phi^{-2}.$$ \hspace{1cm} (31)

Hence, for $n_1^2 > m^2$, $g(\phi) < 0$ at the beginning but becomes positive at a later stage of the cosmic evolution. So a noncanonical kinetic energy turns canonical at

$$\phi < \frac{c^2(n_1^2 - m^2)^2}{\kappa^2 n_1^2 m^6}, \text{ i.e., at } t > \left( \frac{\kappa^2 n_1^2 m^4}{(n_1^2 - m^2)^2} \right).$$ \hspace{1cm} (32)

This proper time has nothing to do with the transitions of the state parameter or the deceleration parameter in general. So even though a noncanonical kinetic energy evolves through to a canonical one, it does not play any role in the evolution of the Universe and the state parameter $w$ always remains over the phantom divide line. However, by properly choosing $m$ in terms of $n_1$, eg., choosing $m = n_1/\sqrt{3}$, one can relate the time of flipping of the sign of the kinetic energy term to the time of transition of a decelerating Universe to an accelerating one.

**Case III**

Let us now choose $\Lambda$ in the form

$$\Lambda = \beta e^{\phi} \hspace{1cm} (33)$$

where, $\beta$ is a constant. So, in view of equations (5) and (24)

$$\phi = \frac{1}{l} \ln \left( \frac{\lambda}{\beta} t \right), \text{ & } g = \frac{l^2}{2\kappa^2 n_1^2} \left[ \lambda + \sqrt{\frac{\beta}{\lambda}} \left( \frac{\lambda + n_1^2}{2\kappa n_1} \right) e^{\frac{\lambda}{\beta} \phi} \right],$$ \hspace{1cm} (34)

and the potential can be found from equation (9) as,

$$V = \frac{\lambda}{2\kappa^2 n_1^2 \beta^2} \left[ 6\beta n_1^2 e^{-\phi} - \kappa^2 n_1^2 \lambda^2 e^{-2\phi} + \kappa n_1 \sqrt{\beta} \lambda (5\lambda - n_1^2) e^{-\frac{5\lambda}{\beta} \phi} \right].$$ \hspace{1cm} (35)

For, $l = -4/\sqrt{6}$, $\phi$ becomes negative but that does not create any problem whatsoever, and the potential carries positive exponents. As before, here again we can choose $\lambda = -m^2, \beta = -c^2$ and $l = -s^2$ and find

$$g = \frac{s^4}{2\kappa^2 n_1^2} \left[ -m^2 + \frac{n_1^2 - m^2}{2\kappa n_1} \left( \frac{c}{m} e^{-\frac{2s^2}{m}} \right) \right].$$ \hspace{1cm} (36)

Now since $\phi = \frac{1}{s^2} \ln \left( \frac{6}{m^4 t} \right)$, so at $t \rightarrow 0, \phi \rightarrow \infty$ and $g < 0$. It has a turning point at a later epoch $t = \frac{4\kappa^2 n_1^2 m^4}{(n_1^2 - m^2)^2}$ after which $g$ becomes positive provided $n_1 > m$. So, here again we encounter the same situation as discussed in case II, that the solutions remain unaffected even though a noncanonical kinetic energy evolves through to a canonical one.

**Case IV**

Here we choose a simple quadratic form of the potential, viz.,

$$V(\phi) = \lambda_0 \phi^2 = \frac{V_0^2}{2\kappa^2} \phi^2.$$ \hspace{1cm} (37)

As a result,

$$\phi = \frac{1}{\kappa n_1 V_0} \left[ 6 - \frac{\kappa(n_1^2 - 5\lambda)}{t} - \frac{\kappa^2 \lambda}{t \sqrt{4}} \right]^{1/2}.$$ \hspace{1cm} (38)

Here, for a positive GB interaction $\lambda > 0, \Lambda > 0$ even an imaginary scalar field evolves to a real one, without affecting the nature of cosmic evolution. In view of equations (5) and (24) it is now in principle possible to find the forms of $\Lambda(\phi)$ and $g(\phi)$. 

5 Concluding remarks

A viable cosmological model has been presented in four dimensions considering Gauss-Bonnet-scalar coupling. To explain recent accelerating expansion of the Universe the Gauss-Bonnet term should dominate at the later epoch of cosmological evolution. Therefore under such physically reasonable assumption that the coupling parameter ($\Lambda$) grows linearly in time a set of solutions have been presented which are independent of the signature of the coupling parameter ($\Lambda$). The solutions demonstrate that the Universe starts with an exponential expansion, but with infinite deceleration ($q \to \infty$) and the equation of state ($w \to \infty$) parameters. In the process of evolution it then passes through the stiff fluid era ($w = 1$), the radiation dominated era ($w = 1/3$) and the matter dominated (pressureless dust) era ($w = 0$). It then encounters a turning point when ($w = -1/3$), after which the Universe starts accelerating. Asymptotically, both the deceleration and the equation of state parameters go over to $-1$. Hence the observations suggest that we are now living at the final stage of cosmological evolution and the dark energy is presently evolving rapidly from $w = -1/3$ to $w = -1$. For a negative coupling parameter ($\Lambda < 0$), the kinetic energy is noncanonical. It has been observed that a noncanonical kinetic energy might evolve to a canonical one without influencing the nature of the solutions in any way, and the equation of state parameter ($w$) never goes beyond the phantom divide line. Even an imaginary scalar field might evolve to a real one without affecting the cosmological evolution. Thus dark energy in the form of Gauss-Bonnet interaction has been found to play a crucial role in the cosmological evolution.

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