Properties of the Ground-State $q\bar{q}$ Mesons and Possible Classification of Observed Mesons in the $\tilde{U}(12)_{SF} \times O(3,1)_L$ Scheme

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We examine possible assignments for known mesons to the predicted ground-state $q\bar{q}$ multiplets in the $\tilde{U}(12)_{SF}$-classification scheme of hadrons, based on their observed properties. The masses of missing members of the ground-state multiplets are estimated, using a phenomenological mixing scheme for the normal- and extra-spin wave functions specific to this classification scheme. We also examine experimental candidates for the excited $1^P$, $1^D$ and $2^S$ multiplets in a framework of the $\tilde{U}(12)_{SF} \times O(3,1)_L$-classification scheme, based on the analysis of the ground-state masses. Then we see that the two exotic mesons $\pi_1(1400)$ and $\pi_1(1600)$ with $J^{PC} = 1^{--}$ have suitable masses to be assigned to the $P$-wave excitation and also there are a number of observed mesonic states which could be classified in terms of the $\tilde{U}(12)_{SF} \times O(3,1)_L$ scheme.

§1. Introduction

Recently, Ishida et al. have proposed the covariant $\tilde{U}(12)_{SF}$-classification scheme of hadrons with $\tilde{U}(12)_{SF} \times O(3,1)_L$ which gives covariant quark representations for composite hadrons with definite Lorentz and chiral transformation properties. The $\tilde{U}(12)_{SF}$-classification scheme has a “static” unitary $U(12)_{SF}$ spin-flavor symmetry in the rest frame of hadrons embedded in the covariant $\tilde{U}(12)_{SF}$-representation space, which includes subgroups as $\tilde{U}(12)_{SF} \supset U(4)_D \times U(3)_F$ ($U(4)_D$ being the pseudounitary homogeneous Lorentz group for Dirac spinors). Since

$$U(12)_{SF} \supset U(4)_D \times U(3)_F$$

with

$$U(4)_D \supset SU(2)_\rho \times SU(2)_\sigma,$$

(1.1a)

the static $U(12)_{SF}$ symmetry includes as its subgroup both the nonrelativistic spin-flavor $SU(6)_{SF}$ and the chiral $U(3)_L \times U(3)_R$ symmetry as

$$U(12)_{SF} \supset SU(6)_{SF} \times SU(2)_\rho$$

(1.1b)

and

$$U(12)_{SF} \supset U(3)_L \times U(3)_R \times SU(2)_\sigma,$$

(1.2a)

where $SU(2)_\rho$ and $SU(2)_\sigma$ are the Pauli-spin groups concerning the boosting and intrinsic spin rotation, respectively, of constituent quarks (being connected with decomposition of Dirac $\gamma$-matrices, $\gamma = \rho \otimes \sigma$). This implies that the $\tilde{U}(12)_{SF}$-classification scheme is able to incorporate effectively the effects of chiral symmetry and its spontaneous breaking, essential for understanding of properties of the low-lying hadrons, into what is called a constituent quark model.
§2. Experimental candidates for the ground-state $q\bar{q}$ mesons

2.1. Essential features of the $\tilde{U}(12)_{SF}$-classification scheme

An essential feature of the $\tilde{U}(12)_{SF}$-classification scheme is to have the static $U(4)_{D}$-spin symmetry in Eq. (1.1b) for light $u,d,s$ quarks confined inside hadrons. The degree of freedom on the $\rho$-spin, being indispensable for covariant description of spin 1/2 particles, offers a basis to define the rule of chiral transformation for quark-composite hadrons.\[\[\]

Since we have the $\rho$-spin degree of freedom, which is discriminated by the eigenvalues $r = \pm$ of $\rho_{3}$, in addition to the ordinary Pauli-spin, the ground states of light-quark $q\bar{q}$ mesons are composed of eight $SU(3)_{F}$ multiplets with respective $J^{PC}$ quantum numbers, two pseudoscalars $\{P^{(N)}(0^{-}), P^{(E)}(0^{-})\}$, two scalars $\{S^{(N)}(0^{++}), S^{(E)}(0^{+-})\}$, two vectors $\{V^{(N)}(1^{--}), V^{(E)}(1^{--})\}$, and two axial-vectors $\{A^{(N)}(1^{++}), B^{(E)}(1^{+-})\}$ ($N$ and $E$ denoting “normal” and “extra”), where each $N(E)$ even-parity multiplet is the chiral partner of the corresponding $N(E)$ odd-parity multiplet and they form linear representations of the chiral $U(3)_{L} \times U(3)_{R}$ symmetry.

For heavy-light mesons we have two heavy-spin multiplets $\{P(0^{-}), V(1^{-})\}$ and $\{S(0^{+}), A(1^{+})\}$, which are the chiral partner of each other, since the eigenstates only with the $\rho_{3}$-eigenvalue of $r = +$ are taken for heavy quarks. For heavy-heavy mesons we have the same $\{P(0^{-}), V(1^{-})\}$-spin multiplets as in the conventional quark model. For both the heavy-light and heavy-heavy systems, their spin wave functions are the perfect mixtures, equally weighted sum, of the $N$ and $E$ states concerning the respective $J^{P}$ states.

2.2. Experimental candidates for the ground-state $q\bar{q}$ mesons in the $\tilde{U}(12)_{SF}$-classification scheme

We try to assign some of the known mesons to the predicted ground-state $q\bar{q}$ multiplets, resorting to their $J^{PC}$ quantum numbers and masses. The experimental data are taken from the Particle Data Group 2004 edition. The resulting assignments, though some of them are ambiguous, are shown in the Table II

Here we make some comments on these assignments as follows:

(i) The light scalar mesons $\{a_{0}(980), \sigma, f_{0}(980), \kappa\}$ are assigned to the $S^{(N)}_{A}(0^{++})$ nonet as a chiral partner of the $\pi$-meson $P^{(N)}(0^{-})$ nonet. Recently the existence of the $\kappa$ meson has been confirmed in two independent partial-wave analyses of the decay $J/\psi \to K^{*}(892)^{0}K^{+}\pi^{-}$ by the BES Collaboration.

(ii) The $B^{(E)}(1^{+-})$ nonet is composed of the $b_{1}(1235)$, $h_{1}(1170)$, $h_{1}(1380)$ and $K_{1}(1400)$ mesons. The low-mass vector meson $K^{*}(1410)$ is assigned as a member of the $V^{(E)}(1^{--})$ nonet which is a chiral partner of the $B^{(E)}(1^{+-})$ nonet.

(iii) The axial-vector mesons $\{a_{1}(1260), f_{1}(1285), f_{1}(1420), K_{1}(1270)\}$ are assigned to the $A^{(N)}(1^{++})$ nonet which is a chiral partner of the $\rho(770)$-meson $V^{(N)}(1^{--})$ nonet. The $a_{1}(1260)$ and $f_{1}(1285)$ mesons are tentatively assigned to this nonet, while their masses seem to be higher than could be expected and also their observed properties of radiative transitions does not seem to be consistent with
Table I. Experimental candidates for the ground-state $q\bar{q}$ mesons in the $\tilde{U}_{SF}(12)$-classification scheme. The data are taken from Ref. 3.

| $q\bar{q}$ | $p(N)$ | $S(N)$ | $s(N)$ | $v(N)$ | $A(N)$ | $V(E)$ | $B(E)$ |
|------------|--------|--------|--------|--------|--------|--------|--------|
|            | $P_0$  | $S_0$  | $P_1$  | $S_1$  | $A_1$  | $V_1$  | $B_1$  |
| $n\bar{n}$ | $\pi$  | $\pi(980)$ | $\pi(1300)$ | $\rho(770)$ | $a_1(1260)$ | $b_1(1235)$ |
| $\eta$     | $\eta$ | $\eta(1295)$ | $\omega(782)$ | $f_1(1285)$ | $h_1(1170)$ |
| $s\bar{s}$ | $\eta'(958)$ | $f_0(980)$ | $\eta(1475)$ | $\phi(1020)$ | $f_1(1420)$ | $h_1(1380)$ |
| $s\bar{n}$ | $K$    | $\kappa$ | $K(1460)$ | $K^*(892)$ | $K_1(1270)$ | $K^*(1410)$ | $K_1(1400)$ |
| $c\bar{n}$ | $D$    | $D^*$  |        |        |        |        |        |
| $c\bar{s}$ | $D_s$  | $D^*_{sJ}(2317)$ | $D_s^*$ | $D_{sJ}(2460)$ |        |        |        |
| $b\bar{n}$ | $B$    | $B^*$  |        |        |        |        |        |
| $b\bar{s}$ | $B_s$  |        |        |        |        |        |        |
| $c\bar{c}$ | $\eta_c(1S)$ |        | $J/\psi(1S)$ |        |        |        |        |
| $b\bar{b}$ | $\eta_b(1S)$ |        | $\Upsilon(1S)$ |        |        |        |        |

(ⅳ) The recent discovered mesons $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$ are just assigned to the $\{S(0^+), A(1^+)\}$-spin multiplet as a chiral partner of the $\{P(0^-), V(1^-)\}$-spin multiplet, $\{D_s, D^*_s\}$. These newly observed states, together with the $\sigma$-meson nonet, are the best candidates for the hadronic states with the $\rho_3$-eigenvalue of $r = -$ whose existence is expected in the $\tilde{U}_{SF}(12)$-classification scheme.

(ⅴ) The $N$ and $E$ states with the same $J^{PC}$, that is, the vector $V^{(N,E)}$ and pseudoscalar $P^{(N,E)}$ states generally mix together, due to the spontaneous as well as explicit breaking of chiral symmetry, and so do the strange scalar $\kappa^{(N,E)}$ and axial-vector $K^{(N,E)}_1$ states. However, as far as the pseudoscalar $P^{(N,E)}$ octet is concerned, it is assumed that no mixing occurs so as to preserve the property of both the $\pi$-meson octet, the Nambu-Goldstone bosons associated with the spontaneous breaking of the axial $SU(3)_{A}$ symmetry, and the $\sigma$-meson nonet belonging to the same chiral multiplet. As for the flavor-singlet $P^{(N)}$ and $P^{(E)}$ states, the $\eta'$-like mesons, there would be mixing between them according to a common understanding that the axial $U(1)_A$ is not a true symmetry in the strong interactions.

§3. Masses and $N-E$ mixings of the ground-state $q\bar{q}$ mesons

In the following we examine the respective $N$- and $E$-state mixings of $V^{(N,E)}$, $K^{(N,E)}_1$ and $\kappa^{(N,E)}$ in a simple phenomenological model. Through this analyses the flavor mixing between $n\bar{n}$ and $s\bar{s}$ states is neglected for the isoscalar channels and the masses of $n\bar{n}(I = 0)$ states are taken to be equal to those of $n\bar{n}(I = 1)$ states. It is also assumed that the $N$- and $E$-state masses of different flavor states are related
to each other as
\[ M_{(N,E)}(s\bar{s}) - M_{(N,E)}(s\bar{n}) = M_{(N,E)}(s\bar{n}) - M_{(N,E)}(n\bar{n}) \equiv \Delta m_s, \tag{3.1} \]
where \( \Delta m_s \) is taken to be 120 MeV.

3.1. Phenomenological mixing scheme of the N and E states

In the normal and extra basis (\(|E\rangle, |N\rangle\)), the mass-squared matrix describing the N-E mixing can be written as
\[ M^2 = \begin{pmatrix} M^2_E & \Delta \\ \Delta & M^2_N \end{pmatrix}, \tag{3.2} \]
where \( M_N \) and \( M_E \) are the masses of the N and E states, respectively, before mixing and \( \Delta \) is a phenomenological parameter corresponding to the mixing strength. Assuming that the physical-state basis (\(|L\rangle, |H\rangle\) (\(L\) and \(H\) denoting “low” and “high”) is an eigenvector of the mass-squared matrix \( M^2 \) with the eigenvalues \( M^2_L \) and \( M^2_H \), we can diagonalize the matrix \( M^2 \) as
\[ U^{-1}M^2U = \begin{pmatrix} M^2_L & 0 \\ 0 & M^2_H \end{pmatrix}, \tag{3.3} \]
by the unitary transformation
\[ \begin{pmatrix} |L\rangle \\ |H\rangle \end{pmatrix} = U^{-1} \begin{pmatrix} |E\rangle \\ |N\rangle \end{pmatrix} \tag{3.4a} \]
with
\[ U^{-1} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}, \tag{3.4b} \]
in which the N-E mixing angle is defined. It is noted that the low- and high-mass states have dominantly \(|E\rangle\) and \(|N\rangle\) components, respectively, if the mixing angle is \(|\theta| < 45^\circ\).

3.2. Masses and mixing properties of the vector \( V^{(N)} \) and \( V^{(E)} \) nonets

In the analysis of the vector meson nonets \( V^{(N)}(1^{--}) \) and \( V^{(E)}(1^{--}) \) we assume that the \( V^{(N)} \) and \( V^{(E)} \) states are mixed maximally, that is, the N-E mixing angle is \(|\theta| = 45^\circ\), in accord with the fact that the lowest-lying vector mesons \( \rho(770), \omega(782), \phi(1020), K^*(892) \) are well described as nonrelativistic \( q\bar{q} \) states and the analysis of their radiative transitions in the \( \tilde{U}(12)_{SF} \)-classification scheme. These maximally mixed low- and high-mass vector states, \( V^{(NR)} \) and \( V^{(ER)} \), mean to be nonrelativistic (NR) and extremely relativistic (ER) states which have the \( r_3\)-eigenvalues of \((r_q, r_{\bar{q}}) = (+, +) \) and \((-,-)\), respectively.

First, we consider the \( K^* \) system, whose mass-squared matrix relation is given by
\[ U^{-1} \begin{pmatrix} M^2_{K^*^{(E)}} & \Delta_{K^*} \\ \Delta_{K^*} & M^2_{K^*^{(N)}} \end{pmatrix} U = \begin{pmatrix} M^2_{K^*^{(892)}} & 0 \\ 0 & M^2_{K^*^{(1410)}} \end{pmatrix}, \tag{3.5} \]
where $K^*(892)$ and $K^*(1410)$ are members of the $V^\mathcal(N)(1^{--})$ and $V^\mathcal(E)(1^{--})$ nonets as mentioned in \cite{22}. Using the mixing angle $|\theta| = 45^\circ$, which means $M_{K^*(N)} = M_{K^*(E)}$, and $M_{K^*(892)} = 894 \text{ MeV}$ and $M_{K^*(1410)} = 1414 \text{ MeV}$, we obtain

$$M_{K^*(N,E)} = 1183 \text{ MeV}, \quad |\Delta_{K^*}| = 0.6001 \text{ GeV}^2.$$  \hspace{1cm} (3.6)

Then, from the assumption in Eq. \[(3\text{-}1)\] we have $M_{\rho(N,E)} = M_{\omega(N,E)} = 1063 \text{ MeV}$ for the $\rho$ and $\omega$ systems. For the $\rho$ system the mass-squared matrix relation is given by

$$U^{-1} \begin{pmatrix} M_{\rho}^2(N(1063)) & \Delta_{\rho} \\ \Delta_{\rho} & M_{\rho}^2(N(1063)) \end{pmatrix} U = \begin{pmatrix} M_{\rho}^2(770) & 0 \\ 0 & M_{\rho'}^2 \end{pmatrix},$$  \hspace{1cm} (3.7)

which gives, taking the mass value of $M_{\rho(770)} = M_{\omega(782)} = 776 \text{ MeV}$,

$$M_{\rho'} = M_{\omega'} = 1290 \text{ MeV}, \quad |\Delta_{\rho}| = |\Delta_{\omega}| = 0.5284 \text{ GeV}^2.$$  \hspace{1cm} (3.8)

In a similar way, we obtain

$$M_{\phi(N,E)} = 1303 \text{ MeV}, \quad M_{\phi'} = 1535 \text{ MeV},$$  \hspace{1cm} (3.9a)

$$|\Delta_{\phi}| = 0.6585 \text{ GeV}^2$$  \hspace{1cm} (3.9b)

for the $\phi$ system. These results are tabulated in Table III.

Here we point out that there exist observed candidates suitable for the predicted low-mass vector mesons $\rho'(1290)$ and $\omega'(1290)$. In fact, there is some experimental evidence for the $\rho(1250)$ reported by the LASS\cite{22} and OBELIX\cite{23} Collaborations and the existence of $\omega(1250)$ is claimed in the analysis of the $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ cross section by the SND\cite{9} and BABAR\cite{10} Collaborations. Furthermore, a recent reanalysis of the BABAR, SND\cite{9} and CMD-2\cite{12} data on the $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ and $e^+e^- \rightarrow \omega\pi^0$ cross sections indicates the existence of these two vector states\cite{13}.

It is worthwhile to mention that only the $V^\mathcal(N)(1^{--})$ states and not the $V^\mathcal(E)(1^{--})$ can be produced in the $e^+e^-$ annihilation process $e^+e^- \rightarrow \gamma^* \rightarrow V$ due to the chirality conservation of quarks\cite{1} since both the constituent quark and antiquark of $V^\mathcal(N)(1^{--})$ have the same chirality, while those of $V^\mathcal(E)(1^{--})$ have opposite one in the $\overline{U}(12)_{SF}$-classification scheme. Therefore we can expect that the predicted $\rho'(1290)$ and $\omega'(1290)$ mesons, which have the same amount of $V^\mathcal(N)(1^{--})$ component as the $\rho(770)$ and $\omega(782)$, are seen in the above $e^+e^-$ annihilation process.

3.3. Masses and mixing properties of the axial-vector $A^\mathcal(N)$ and $B^\mathcal(E)$ nonets

In the axial-vector meson nonets, $A^\mathcal(N)(1^{++})$ and $B^\mathcal(E)(1^{++})$, their mixing could occur only for the strange $K_1$ system and therefore the $N$ and $E$ states are physical ones for the isoscalar and isovector channels. In this analysis we select, as input data, the $b_1(1235)$, $K_1(1270)$ and $K_1(1400)$ mesons from among the assigned mesons in \cite{22,22}.

\textsuperscript{1} A vertex operator for the quark-photon interaction is assumed to be proportional to $\gamma_{\mu}$.
Table II. Masses and mixing properties of the vector meson $V^{(N)}(1^{--})$ and $V^{(E)}(1^{--})$ nonets. Fitted values are underlined.

| $qar{q}$ | State  | Predicted mass | Observed mass | Mixing angle $|\theta|$ | Mixing strength $|\Delta|$ |
|-----------|--------|----------------|---------------|--------------------------|--------------------------|
| $nar{n}$ | $\rho(770)$ | 776 | 775.8 ± 0.5 | 45° | 0.5284 |
|           | $\rho'$       | 1290 | 0 | 0 | 0 |
| $\omega(782)$ | 776 | 782.59 ± 0.11 | 45° | 0.6585 |
|           | $\omega'$     | 1290 | 0 | 0 | 0 |
| $s\bar{s}$ | $\phi(1020)$ | 1019 | 1019.456 ± 0.020 | 45° | 0.6001 |
|           | $\phi'$       | 1535 | 0 | 0 | 0 |
| $sar{n}$ | $K^*(892)$ | 894 | 893.9 ± 2.5 | 45° | 0.1700 GeV² |
|           | $K^*(1410)$ | 1414 | 1414 ± 15 | 0 | 0 |

a) The data are taken from Ref. [3].

For the $K_1$ system we obtain $M_{K^{(E)}_{1B}} = 1350$ MeV from the assumption in Eq. (3.1) with the measured mass of $M_{b_1(1235)} = 1230$ MeV and the mass-squared matrix relation is given by

$$U^{-1} \begin{pmatrix} M_{K^{(E)}_{1B}}^2(1350) \\ \Delta K_1 \\ M_{K^{(N)}_{1A}}^2 \\ \Delta K_1 \\ M_{K^{(E)}_{1B}}^2(1350) \\ M_{K^{(N)}_{1A}}^2 \\ M_{K^{(E)}_{1B}}^2(1350) \\ M_{K^{(N)}_{1A}}^2 \end{pmatrix} U = \begin{pmatrix} M_{K(1270)}^2 \\ 0 \\ 0 \\ M_{K(1400)}^2 \end{pmatrix}.$$ (3.10)

Taking the mass values of $M_{K(1270)} = 1273$ MeV and $M_{K(1400)} = 1402$ MeV, we derive

$$M_{K^{(N)}_{1A}} = 1328 \text{ MeV},$$ (3.11a)

$$|\Delta K_1| = 0.1700 \text{ GeV}^2, \quad |\theta_{K_1}| = 49.9°$$ (3.11b)

and then

$$M_{a_1} = M_{f_1(n\bar{n})} = 1210 \text{ MeV}, \quad M_{f_1'(s\bar{s})} = 1450 \text{ MeV},$$ (3.12a)

$$M_{b_1} = M_{h_1(n\bar{n})} = 1230 \text{ MeV}, \quad M_{h_1'(s\bar{s})} = 1470 \text{ MeV}$$ (3.12b)

for the isoscalar and isovector channels. These results are given in Table III.

It should be noted here that $K^{(N)}_{1A}$ and $K^{(E)}_{1B}$ are the ${}^1D_1$ and ${}^1D_1$ states respectively in the conventional quark model, while they are relativistic $S$-wave states in which each $q$ and $\bar{q}$ has the opposite $\rho_3$-eigenvalue in the $\tilde{U}(12)_{SF}$-classification scheme. The resulting mixing angle $|\theta_{K_1}| = 49.9°$ means that the dominant components of $K_1(1270)$ and $K_1(1400)$ are the $K^{(N)}_{1A}$ and $K^{(E)}_{1B}$ states, respectively.

3.4. Masses and mixing properties of the scalar $S^{(N)}_A$ and $S^{(E)}_B$ nonets

The scalar $S^{(N)}_A(0^{++})$ and $S^{(E)}_B(0^{+-})$ states could mix only for the strange $\kappa$ system, as in the $K_1$ system. For lack of experimental information and theoretical
Table III. Masses and mixing properties of the axial-vector meson $A^{(N)(1^{++})}$ and $B^{(E)(1^{+-})}$ nonets. Fitted values are underlined.

| $qar{q}$ | State | Predicted mass | Observed mass | Mixing angle $|\theta|$ | Mixing strength $|\Delta|$ |
|-----------|-------|---------------|---------------|------------------|------------------|
| $n\bar{n}$ | $a_1(1260)$ | 1210 | 1230 $\pm$ 40 | - | - |
| | $f_1(1285)$ | 1210 | 1281.8 $\pm$ 0.6 | - | - |
| | $b_1(1235)$ | 1230 | 1229.5 $\pm$ 3.2 | - | - |
| | $h_1(1170)$ | 1230 | 1170 $\pm$ 20 | - | - |
| $s\bar{s}$ | $f_1(1420)$ | 1450 | 1426.3 $\pm$ 0.9 | - | - |
| | $h_1(1380)$ | 1470 | 1386 $\pm$ 19 | - | - |
| $s\bar{n}$ | $K_1(1270)$ | 1273 | 1273 $\pm$ 7 | 49.9$^\circ$ | 0.1700 |
| | $K_1(1400)$ | 1402 | 1402 $\pm$ 7 | - | - |

a) The data are taken from Ref. [3].

understanding on the light scalar mesons, we assume here that the mixing strength between $\kappa_A^{(N)}$ and $\kappa_B^{(E)}$ is equal to that of the $K_1$ system, that is,

$$\Delta_{\kappa} = \Delta_{K_1},$$

which might be realized, provided that the $\kappa_A^{(N)}$-$\kappa_B^{(E)}$ mixing originates from the spontaneous breaking of chiral symmetry. As input data in this analysis we take the mass values of the $a_0(980)$ and $\kappa$ mesons to be 985 MeV and 875 MeV, respectively.

For the $\kappa$ system we have $M_{\kappa_A^{(N)}} = 1105$ MeV from the assumption in Eq. (3.13) with the $a_0(980)$ mass and the mass-squared matrix relation is given by

$$U^{-1} \begin{pmatrix} M^2_{\kappa_A^{(E)}} & \Delta_{\kappa} \\ \Delta_{\kappa} & M^2_{\kappa_A^{(N)}} \end{pmatrix} U = \begin{pmatrix} M^2_{\kappa(875)} & 0 \\ 0 & M^2_{\kappa'} \end{pmatrix}$$

with $|\Delta_{\kappa}| = 0.1700$ GeV$^2$. This relation gives

$$M_{\kappa_B^{(E)}} = 911 \text{ MeV}, \quad M_{\kappa'} = 1135 \text{ MeV},$$

$$|\theta_{\kappa}| = 20.5^\circ$$

and then

$$M_{a_0} = M_{f_0(n\bar{n})} = 985 \text{ MeV}, \quad M_{f_0(s\bar{s})} = 1225 \text{ MeV},$$

$$M_{b_0} = M_{h_0(n\bar{n})} = 790 \text{ MeV}, \quad M_{h_0(s\bar{s})} = 1030 \text{ MeV}$$

for the isoscalar and isovector channels. These results are given in Table IV.

Here it is noticeable that a dominant component of the $\kappa(875)$ is $\kappa_B^{(E)}$ while that of the $\kappa'(1135)$ is $\kappa_A^{(N)}$, due to the mixing angle of $|\theta_{\kappa}| = 20.5^\circ$. This implies that the unknown $\kappa'(1135)$ rather than the $\kappa(875)$ is a member of the light scalar $\sigma$-meson
nonet. In this connection, it is of great interest that a recent lattice-QCD study of light scalar mesons with the interpolating field $\bar{\psi}\psi$ gives the results of the lightest $a_0$ meson having a mass of $1.01 \pm 0.04$ GeV and the $K_0^*$ meson $100-130$ MeV heavier than the $a_0$ meson. These results are in conformity with our prediction, since the $\kappa_A^{(N)}$ and $\kappa_B^{(E)}$ are considered to be states corresponding to the interpolating fields $\bar{\psi}\psi$ and $\bar{\psi}\gamma_\mu \partial_\mu \psi$, respectively.

Table IV. Masses and mixing properties of the scalar meson $S_A^{(N)}(0^{++})$ and $S_B^{(E)}(0^{+-})$ nonets. Fitted values are underlined.

| $q\bar{q}$ | State | Predicted mass (MeV) | Observed mass$^a)$ (MeV) | Mixing angle $|\theta|$ | Mixing strength $|\Delta|$ (GeV$^2$) |
|------------|-------|----------------------|--------------------------|------------------------|-----------------|
| $n\bar{n}$ | $a_0$(980) | 985 | 984.7 ± 1.2 | -- | -- |
|            | $\sigma$ | 985 | $\sim$ 400-600 | -- | -- |
| $b_0$      | 790 | -- | -- | -- |
| $h_0$      | 790 | -- | -- | -- |
| $s\bar{s}$ | $f_0$(980) | 1225 | 980 ± 10 | -- | -- |
|            | $h_0'$ | 1030 | -- | -- | -- |
| $s\bar{n}$ | $\kappa'$ | 1135 | 20.5$^\circ$ | 0.1700 |
| $\kappa$   | 875 | 878 ± 23$^{+64}_{-55}$ |

$^a)$ The data are taken from Ref. [3], unless the mass of the $\kappa$ from Ref. [4].

§4. Mass spectra and possible assignments for observed mesons in the $\bar{U}(12)_{SF} \times O(3,1)_L$-classification scheme

4.1. Mass spectra of $n\bar{n}$ mesons

We examine excited states of the respective ground-state sectors in the extended $\bar{U}(12)_{SF}$-classification scheme with $\bar{U}(12)_{SF} \times O(3,1)_L$, in which the degree of freedom concerning the orbital motion of quarks is incorporated. In this classification scheme it is predicted that there exist some $q\bar{q}$ exotic states with $J^{PC}$ quantum numbers, such as $0^{++}$ in the ground states, $0^{--}$ and $1^{--}$ in the excited $P$-wave states, $2^{++}$ in the excited $D$-wave states, which never appear in any nonrelativistic quark model. There is presently the experimental observation of two exotic mesons with $J^{PC} = 1^{--}$, the $\pi_1(1400)$ and $\pi_1(1600)$. Since, as far as the $1^{--}$ exotic state is concerned, we have just two states in the $P$-wave excitation in this scheme, the $\pi_1(1400)$ and $\pi_1(1600)$ mesons are expected to be promising experimental candidates for these predicted exotics.

In the $\bar{U}(12)_{SF} \times O(3,1)_L$-classification scheme the mass of excited states is given by

$$M_N^2 = M_0^2 + N\Omega, \quad N = L + 2N', \quad (4.1)$$

$^a)$ Chiral transformation properties of various interpolating fields of mesons have been examined in Ref. [15].
where $M_0$ is the ground-state mass, $\Omega^{-1}$ the slope parameter of linear Regge trajectories, and $L$ ($N'$) the orbital-angular-momentum (radial) quantum number. We take here a value of $\Omega$ to be 1.136 GeV$^2$ for the $n\bar{n}$ meson system, determined from the mass-squared distance between the $\rho(770)$ (775.8 MeV) and $a_2(1320)$ (1318.3 MeV) mesons and also take the respective ground-state masses obtained in \[3\] as values of the $M_0(n\bar{n})$. For the $P(E)$ sector we use a mass, 1300 MeV, of the $\pi(1300)$ meson, which was assigned to the isovector $P(E)$ state in \[2,\] \[2\] Since we have no appropriate ground state as input for the $P(N)$ sector, due to its Nambu-Goldstone nature, we use a mass, 1880 MeV, of the $\pi_2(1880)$ meson which is considered as an experimental candidate for the excited $1^1D_2$ isovector state. Then we estimate masses of the excited $1P, 1D$ and $2S$ states for the respective sectors of $n\bar{n}$ mesons. The results are presented in Table \[V\].

Table V. Estimated masses (in units of GeV) of the $1S, 1P, 2S$ and $1D$ states for the respective sectors of states in the $n\bar{n}$ system. Input values are underlined.

| $N$ | $L$ | $P(N)$ | $S_A^{(N)}$ | $P(E)$ | $S_B^{(E)}$ | $V^{(NR)}$ | $A^{(N)}$ | $V^{(ER)}$ | $B^{(E)}$ |
|-----|-----|--------|-------------|--------|-------------|------------|----------|----------|----------|
| 0   | 0   | 0$^+$  | 0$^+$       | 1$^-$  | 1$^+$       | 1$^-$      | 1$^+$    | 1$^+$    | 1$^+$    |
|     |     | 0.985  | 1.30        | 0.790  | 0.776       | 1.21       | 1.29     | 1.23     |          |
| 1   | 1   | 1$^-$  | 1$^-$       | 1$^+$  | 1$^+$       | 1$^+$      | 1$^+$    | 1$^+$    |          |
| 2   | 0   | 0$^+$  | 0$^+$       | 0$^+$  | 0$^-$       | 1$^-$      | 1$^+$    | 1$^+$    | 1$^+$    |
|     |     | 1.88   | 1.80        | 1.99   | 1.70        | 1.70       | 1.93     | 1.98     | 1.95     |
| 2   | 2   | 2$^+$  | 2$^+$       | 2$^-$  | 2$^+$       | 2$^-$      | 2$^+$    | 2$^+$    |          |
|     |     | 1.88   | 1.80        | 1.99   | 1.70        | 1.70       | 1.93     | 1.98     | 1.95     |

4.2. The $\tilde{U}(12)_{SF} \times O(3,1)_L$ classification of observed mesons below $\sim 2$ GeV

We seek experimental candidates for the $1S, 1P, 2S$ and $1D$ states out of the known mesons\[7\] listed in the Particle Data Group 2004,\[3\] in addition to the states

\[\)\] They include meson states which are listed under the section ‘Further States’ in Ref. \[3\].
assigned in the previous sections, based on their $J^{PC}$ quantum numbers, measured masses and decay modes. The resulting possible assignments are given in Table VI for the mesons selected in this way, though very tentative. From this table it is found that we have a number of observed states which could be classified well in terms of the $\tilde{U}(12)_{SF} \times O(3,1)_L$ scheme.

Since the $1^{-+}$ exotic states are in the $P$-wave excitation of the $S_B^{(E)}(0^{+-}; 1^1S_0)$ and $B^{(E)}(1^{-+}; 1^3S_1)$ ground states, their masses are given as

$$M(1^{-+}; 1^1P_1 S_B^{(E)}) = 1.33 \text{ GeV}, \quad M(1^{-+}; 1^3P_1 B^{(E)}) = 1.63 \text{ GeV}, \quad (4.2)$$

corresponding to the respective ground-state masses, 0.790 GeV and 1.230 GeV. We see that these masses are quite consistent with their measured values as

$$M_{\pi_1(1400)} = 1376 \pm 17 \text{ MeV}, \quad M_{\pi_1(1600)} = 1653^{+18}_{-15} \text{ MeV}. \quad (4.3)$$

Recently, the BES collaboration has reported the observation of a broad $K^+K^-$ resonance $X(1576)$ with $J^{PC} = 1^{--}$ and the pole position of $1576^{+49}_{-55}(\text{stat})^{+98}_{-91}(\text{syst}) - i409^{+11}_{-12}(\text{stat})^{+32}_{-26}(\text{syst})$ MeV in the decay $J/\psi \rightarrow K^+K^-\pi^0$. Subsequently, it is shown that this resonance should be an isovector state. In the $\tilde{U}(12)_{SF} \times O(3,1)_L$ classification scheme, taking into account its measured mass of $\sim 1.5$-1.6 GeV, we have a proper place to assign the $X(1576)$, that is, $(1^{--}; 1^3P_1 A^{(N)})$ in the $P$-wave excitation of the $A^{(N)}(1^{++}; 1^3S_1)$. Since both the quark and antiquark of this state are the same chirality like $V^{(N)}(1^{--}; 1^3S_1)$, the $X(1576)$ and its isoscalar partners might be expected to be seen in the $e^+e^-$ annihilation process, as was mentioned in §3.2, though in this case the production rates may be considerably suppressed as compared with the $S$-wave states due to their $P$-wave spatial wave functions.

We have further another $1^{--}$ vector multiplet in the $P$-wave excitation of the $S_A^{(N)}(0^{++}; 1^1S_0)$ and vector mesons belonging to this multiplet is composed of a quark and antiquark which have the opposite chirality each other the same as $S_A^{(N)}$. Therefore, it is expected that isoscalar and isovector members of this multiplet would be doubly hard to be produced in the $e^+e^-$ annihilation process.

§5. Concluding remarks

We have proposed the possible assignments for a number of observed mesons below $\sim 2$ GeV in the $\tilde{U}(12)_{SF} \times O(3,1)_L$-classification scheme. Considering the phenomenological mixing scheme of the normal and extra states, we estimated the masses of missing members of the ground-state multiplets and also the masses of the excited $1P$, $1D$ and $2S$ states. We see that enigmatic states, such as the light scalar $\sigma$, $\kappa$, $a_0(980)$ and $f_0(980)$, the exotic $1^{--}$ states $\pi_1(1400)$ and $\pi_1(1600)$, the low-mass vector $p(1250)$ and $\omega(1250)$, and the unexpectedly low-mass states $D^{*}(2317)$ and $D_{sJ}(2460)$ could be classified naturally as conventional $q\bar{q}$ states in the $\tilde{U}(12)_{SF} \times O(3,1)_L$-classification scheme, without resort to more exotic or far-fetched interpretations like a multiquark, molecule or low-mass hybrid. Since it goes without saying that only mass spectra are insufficient to establish their assignments,
Table VI. Possible assignments for the known mesons to the 1S, 1P, 2S and 1D states in the \( \bar{U}(12)_{SF} \times O(3,1)_L \)-classification scheme. Mesons in brackets are the unknown states whose masses are estimated in the present work.

| \( P(N) \) | \( S_A^{(N)} \) | \( P(E) \) | \( S_B^{(E)} \) | \( V(N,R) \) | \( A(N) \) | \( V(E,R) \) | \( B(E) \) |
|-----------|-----------------|------------|----------------|-------------|-------------|-------------|-------------|
| 0\(^-\)   | 0\(^+\)         | 0\(^-\)    | 0\(^+\)        | 1\(^-\)     | 1\(^+\)     | 1\(^-\)     | 1\(^+\)     |
| \( \pi \) | \( a_0(980) \)  | \( \pi(1300) \) | \( \eta(1295) \) | \( \eta(1475) \) | \( \pi(1450) \) | \( \eta(1405) \) | \( \pi(1400) \) |
| \( \eta \) | \( \sigma \)    | \( \eta(1295) \) | \( \omega(782) \) | \( \phi(1020) \) | \( X(1570) \) | \( a_1(1640) \) | \( \pi_1(1600) \) |
| \( \eta'(958) \) | \( f_0(980) \) | \( \eta(1475) \) | \( \phi(1020) \) | \( f_1(1420) \) | \( a_2(1320) \) | \( f_2(1520) \) | \( K_2(1430) \) |
| \( K \) | \( [\kappa(1135)] \) | \( K(1460) \) | \( \kappa \) | \( K^*(892) \) | \( K_1(1270) \) | \( K^*(1410) \) | \( K_1(1400) \) |

\begin{align*}
&1^1P_0 & 1^3P_1 & 1^3P_2 & 1^3P_3 & 1^3D_1 & 1^3D_2 \\
0^+ & 0^- & 0^+ & 0^- & 1^- & 1^+ & 1^- & 1^+ & 2^- & 2^- & 2^- & 2^- & 2^- & 2^- & 2^- & 2^- & 2^- & 2^- & 2^- & 2^- & 2^- \end{align*}

\begin{align*}
&1^1S_0 & 1^3S_1 & 1^3S_2 & 1^3S_3 & 1^3D_1 & 1^3D_2 \\
0^+ & 0^{-} & 0^+ & 0^- & 1^- & 1^+ & 1^- & 1^+ & 2^- & 2^- & 2^- & 2^- & 2^- & 2^- & 2^- & 2^- & 2^- & 2^- & 2^- & 2^- & 2^- \end{align*}

\begin{align*}
&1^1D_0 & 1^3D_1 & 1^3D_2 \\
1^- & 1^+ & 1^- & 1^+ & 1^- & 1^+ & 1^- & 1^+ \end{align*}

\begin{align*}
&1^1D_0 & 1^3D_1 & 1^3D_2 \\
1^- & 1^+ & 1^- & 1^+ & 1^- & 1^+ & 1^- & 1^+ \end{align*}
it is important to examine the production and decay properties, such as pionic and radiative transitions, of the assigned states in the $\tilde{U}(12)_{SF} \times O(3,1)_L$-classification scheme.

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