Existence of Fermion Zero Modes and Deconfinement of Spinons in Quantum Antiferromagnetism resulting from Algebraic Spin Liquid

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We investigate the quantum antiferromagnetism arising from algebraic spin liquid via spontaneous chiral symmetry breaking. We claim that in the antiferromagnetic massive Dirac spinons can appear to make broad continuum spectrum at high energies in inelastic neutron scattering. The mechanism of spinon deconfinement results from the existence of fermion zero modes in single monopole potentials. Neel vectors can make a skyrmion configuration around a magnetic monopole of compact U(1) gauge fields. Remarkably, in the monopole-skyrmion composite potential the Dirac fermion is shown to have a zero mode. The emergence of the fermion zero mode forbids the condensation of monopoles, resulting in deconfinement of Dirac spinons in the quantum antiferromagnet.

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I. INTRODUCTION

High $T_c$ superconductivity is believed to result from hole doping to an antiferromagnetic Mott insulator. Hole doping to an antiferromagnetic Mott insulator destroys antiferromagnetic long range order, resulting in one quantum disordered paramagnetic Mott insulator that is considered to be the pseudogap phase in high $T_c$ cuprates. High $T_c$ superconductivity is expected to arise from further hole doping to the paramagnetic Mott insulator [1]. In this respect it should be a starting point for the theory of high $T_c$ superconductivity to understand the nature of the antiferromagnetic Mott insulator.

Recently, the paramagnetic Mott insulator was proposed to be algebraic spin liquid, where spin 1 antiferromagnetic fluctuations break up into more elementary spin 1/2 fractionalized excitations called spinons [8]. If the algebraic spin liquid correctly describes spin degrees of freedom in the pseudogap phase, its parent antiferromagnet may include the trace of fractionalized spinon excitations. In the present paper we find a trace of deconfined spinons in the quantum antiferromagnet.

There are some experimental reports that the spin spectrum in the antiferromagnetic Mott insulator is difficult to understand only by antiferromagnons [2]. Although the dispersing peaks observed in inelastic neutron scattering measurements can be interpreted as antiferromagnons, an analysis of the spectral weight shows that long range order and antiferromagnons can account for only about 1/2 of the observed spectrum [2]. This difficulty originates from the presence of finite spectral weight at high energies [2]. The unidentified spectral weight indicates the presence of excitations beyond one magnon mode [2].

In the present paper we claim that the unidentified spectral weight at high energies in the spin spectrum can be identified with deconfined gapped spinons. In a theoretical point of view, if antiferromagnetism is supposed to originate from algebraic spin liquid via spontaneous chiral symmetry breaking, the emergence of deconfined spinons seems to be possible. The mechanism of spinon deconfinement results from the existence of fermion zero modes in single monopole potentials. It should be noted that the mechanism of monopole suppression in the present antiferromagnet completely differs from that in the algebraic spin liquid, where existence of quantum criticality (critical fluctuations of matter fields) is the origin of spinon deconfinement [3]. In high energy physics the mechanism of monopole suppression by fermion zero modes is well known [4]. In the context of quantum antiferromagnetism there was a try to find fermion zero modes. Marston has tried to find fermion zero modes in the algebraic spin liquid [5]. Unfortunately, the fermion zero mode was not found.

In the present communication we find a fermion zero mode in the antiferromagnetism arising from the algebraic spin liquid via spontaneous chiral symmetry breaking. In the present antiferromagnet the effective action is pretty much similar to a well studied action in high energy physics, where a fermion zero mode exists. In the antiferromagnet Neel vectors can make a hedgehog configuration around a magnetic monopole of compact U(1) gauge fields. Remarkably, in the monopole-skyrmion composite potential the Dirac fermion is shown to have a zero mode. The emergence of the fermion zero mode forbids condensation of magnetic monopoles, resulting in deconfinement of Dirac spinons. Notice that Dirac spinons are massive owing to spontaneous chiral symmetry breaking. These gapped spinons would appear to make broad continuum spectrum above their mass gap in inelastic neutron scattering [2, 6, 10].

The main body of the present paper consists of three major parts. The first is to introduce a fermionic nonlinear $\sigma$ model [Eq. (1)] as a low energy effective field theory for the quantum antiferromagnetism. The second is to prove the existence of fermion zero modes based on the proposed fermionic nonlinear $\sigma$ model [Eq. (3)]. The last is to investigate the effect of the fermion zero mode [Eq. (13)].
II. EXISTENCE OF FERMIAN ZERO MODES AND DECONFINEMENT OF SPINONS

A. Fermionic Nonlinear $\sigma$ Model for Quantum Antiferromagnetism

We consider the following effective action called fermionic nonlinear $\sigma$ model

$$Z = \int D\bar{\psi}D\psi_a e^{-S_{\text{ASL}} - S_M - S_{\text{NL}\sigma M}},$$

$$S_{\text{ASL}} = \int d^3x \left[ \bar{\psi}_n \gamma_\mu (\partial_\mu - ia_\mu) \psi_n + \frac{1}{2c^2} |\vec{a}|^2 \right],$$

$$S_M = \int d^3x m_\psi \bar{\psi}_n (\vec{n} \cdot \vec{\tau}_{nm}) \psi_m,$$

$$S_{\text{NL}\sigma M} = \int d^3x \left[ \frac{Nm_\psi}{4\pi} |\partial_\mu \vec{n}|^2 - i\lambda (|\vec{n}|^2 - 1) \right].$$  \hspace{0.5cm} (1)

In $S_{\text{ASL}}$, $\psi_n$ is a massless Dirac spinon with a flavor index $n = 1, ..., N$ associated with SU(N) spin symmetry. $a_\mu$ is a compact U(1) gauge field mediating long range interactions between Dirac spinons. $e$ is an internal electric charge of the Dirac spinon. $S_{\text{ASL}}$ in Eq. (1) was proposed to be an effective field theory for one possible quantum paramagnetism, called algebraic spin liquid, of SU(N) quantum antiferromagnets on two dimensional square lattices [1]. However, the stability of the algebraic spin liquid has been suspected owing to instanton excitations of compact U(1) gauge fields [1]. Condensation of instantons (magnetic monopoles) [12] is well known to cause confinement of charged particles [13, 14, 15], here Dirac spinons. Recently, Hermele et al. showed that the algebraic spin liquid can be stable against magnetic monopole excitations [2]. Ignoring the compactness of U(1) gauge fields $a_\mu$, one can show that the $S_{\text{ASL}}$ has a nontrivial charged fixed point in two space and one time dimensions $[(2 + 1)D]$ in the limit of large flavors [3, 10, 17], identified with the algebraic spin liquid. Hermele et al. showed that the charged critical point in the case of noncompact U(1) gauge fields can be stable against magnetic monopole excitations of compact U(1) gauge fields in the limit of large flavors [2]. Condensation of monopoles can be forbidden at the stable charged fixed point owing to critical fluctuations of Dirac fermions. The $S_{\text{ASL}}$ in Eq. (1) is a critical field theory at the charged critical point, where correlation functions exhibit power law behaviors with anomalous critical exponents resulting from long range gauge interactions [14, 18]. This is the reason why the state described by the $S_{\text{ASL}}$ is called the algebraic spin liquid. In appendix A we briefly discuss how the effective quantum electrodynamics in $(2 + 1)D$ (QED$_3$), the $S_{\text{ASL}}$ in Eq. (1) can be derived from the antiferromagnetic Heisenberg model on square lattices.

However, we should remember that the criticality of algebraic spin liquid holds only for large flavors of critical Dirac spinons. If the flavor number is not sufficiently large, the internal charge $e$ is not screened out satisfactorily by critical Dirac fermions. Then, gauge interactions can make bound states of Dirac fermions, resulting in massive Dirac spinons. This is known to be spontaneous chiral symmetry breaking ($S_{\chi SB}$) [14, 20, 21, 22, 23]. In the case of physical SU(2) antiferromagnets it is not clear if the algebraic spin liquid criticality remains owing to the $S_{\chi SB}$ causing antiferromagnetism. It is believed that there exists the critical flavor number $N_c$ associated with $S_{\chi SB}$ in the $QED_3$ [12, 20]. But, the precise value of the critical number is far from consensus [24]. If the critical value is larger than 2, the $S_{\chi SB}$ is expected to occur for the physical $N = 2$ case. Then, the Dirac fermions become massive. On the other hand, in the case of $N_c < 2$ the algebraic spin liquid criticality remains stable against the $S_{\chi SB}$. Experimentally, antiferromagnetic long range order is clearly observed in the SU(2) antiferromagnet. This leads us to consider the $S_{\chi SB}$ in the algebraic spin liquid. In Eq. (1) $S_M$ represents the contribution of a fermion mass due to the $S_{\chi SB}$. $m_\psi$ is a mass parameter corresponding to staggered magnetization in the context of antiferromagnetism [21]. $\vec{n}$ represents fluctuations of Neel order parameter fields regarded as Goldstone bosons in the $S_{\chi SB}$. $\vec{\tau}$ is Pauli matrix acting on the spin (flavor) space of Dirac spinons. The mass parameter $m_\psi$ can be determined by a self-consistent gap equation, given by $m_\psi \approx e^2 \exp[-2\pi/\sqrt{N_c/N - 1}]$ in the $1/N$ approximation [14]. For completeness of this paper we briefly sketch the derivation of dynamical mass generation in appendix B. There are additional fermion bilinears connected with the Neel state by “chiral” transformations because the $S_{\text{ASL}}$ in Eq. (1) has more symmetries than those of the Heisenberg model [22, 24, 25]. These order parameters are associated with valence bond orders [22, 24, 27]. But, in the present paper we consider only the Neel order parameter in order to obtain the O(3) nonlinear $\sigma$ model.

Contributions of high energy spinons in $S_{\text{ASL}} + S_M$ lead to the $S_{\text{NL}\sigma M}$ in the gradient expansion [23, 24]. This effective action is nothing but the O(3) nonlinear $\sigma$ model describing quantum antiferromagnetism. In the $S_{\text{NL}\sigma M}$ we introduced a Lagrange multiplier field $\lambda$ to impose the rigid rotor constraint $|\vec{n}|^2 = 1$. In appendix C we give a detailed derivation of the nonlinear $\sigma$ model. The total effective action $S_{\text{ASL}} + S_M + S_{\text{NL}\sigma M}$ called fermionic nonlinear $\sigma$ model in Eq. (1) naturally describes the quantum antiferromagnetism resulting from the algebraic spin liquid via $S_{\chi SB}$ at half filling. Based on this fermionic O(3) nonlinear $\sigma$ model we discuss physics of the quantum antiferromagnetism.

In Eq. (1) we focus our attention on the Kondo-like spin coupling term $\vec{n} \cdot \bar{\psi} \psi \vec{\tau}$ between antiferromagnetic spin fluctuations $\vec{n}$ and Dirac fermions $\psi_n$. This term shows that an antiferromagnetic excitation of spin 1 can fractionalize into two fermionic spinons of spin 1/2. But, long range gauge interactions prohibit antiferromagnetic spin fluctuations from decaying into spinons. More quantitatively, massive Dirac spinons should be confined to form spin 1 antiferromagnetic fluctuations owing to the effect of magnetic monopoles of compact U(1) gauge
fields $a_\mu$. $S\chi_{SB}$ makes the criticality of algebraic spin liquid disappear, causing massive spinons. These spinons can generate only the Maxwell kinetic energy for the gauge field $a_\mu$ via particle-hole polarizations. It is well known that this Maxwell gauge theory shows confinement of charged matter fields in $(2 + 1)D$ owing to the condensation of magnetic monopoles\[3, 14, 15\]. Spin $1/2$ massive Dirac spinons are confined to make spin $1$ antiferromagnetic fluctuations, i.e., $\pi = (\bar{\psi}\tau\psi)$, where $\langle\ldots\rangle$ denotes a vacuum expectation value. A resulting effective field theory for this antiferromagnet is obtained to be

$$Z_{AF} = \int D\pi e^{-S_\pi},$$

$$S_\pi = \int d^3x N m_\psi \frac{1}{4\pi} \left( |\partial_\mu \pi|^2 + \frac{(\bar{\pi} \cdot \pi)^2}{1 - |\pi|^2} \right),$$

where the Neel vector is given by $\bar{\pi} = (\bar{\pi}, \bar{n}^3)$ with $n^3 = \sqrt{1 - |\pi|^2}$. In the last line we obtained an effective field theory for small fluctuations of $\pi$ fields around the Neel axis $n^3$ in the antiferromagnet. The $\pi$ fields are nothing but aniferromagnons, considered to be spinon-antispinon composites $\pi^\pm = \langle \bar{\psi}\tau^\pm\psi \rangle = n_1 \pm i n_2$ with relativistic spectrum, $\omega = k$ in the low energy limit. The last term in the last line represents interactions between antiferromagnons. Low energy physics in this conventional quantum antiferromagnet is well described by the interacting antiferromagnons\[24\]. This is the result of conventional antiferromagnetism when we do not consider fermion zero modes. In the present paper we show that the presence of fermion zero modes can alter this effective field theory Eq. (2) at high energies. We will see that the fermion zero mode can appear from the Kondo-like coupling term, resulting in magnon decaying into spinons.

B. Existence of Fermion Zero Modes

Now we show that a fermion zero mode can arise in a monopole-skyrmion composite potential. In order to introduce monopole potentials we separate the compact U(1) gauge field $a_\mu$ into $a_\mu = a_\mu^{el} + a_\mu^{qu}$, where $a_\mu^{el}$ represent magnetic monopole (instanton) potentials and $a_\mu^{qu}$, gaussian quantum fluctuations. In addition, we consider skyrmion configurations $\bar{r}^{sky}$. The single monopole and skyrmion potentials are given by $a_\mu^{el} = a(r)\epsilon_{\lambda\alpha\mu}x_\lambda$ and $n_\mu^{qu} = \Phi(r)x_\mu$, respectively, where $a(r)$ and $\Phi(r)$ are proportional to $r^{-2}$ in $r \to \infty$ with $r = \sqrt{\chi^2 + x^2 + y^2}$\[1, 27\]. Integrating over the Dirac spinon fields in the fermionic nonlinear $\sigma$ model Eq. (1), we obtain the following fermion determinant in the monopole-skyrmion composite potential, $S_\psi = -N\text{ndet} \left[ \gamma_\mu (\partial_\mu - ia_\mu^{el} - ia_\mu^{qu}) + m_\psi \bar{r}^{sky} \cdot \bar{r} \right]$. For the time being, we ignore gaussian quantum fluctuations $a_\mu^{qu}$ because our objective is to find a fermion zero mode in the single monopole potential $a_\mu^{el}$. In order to calculate the determinant we solve an equation of motion in the monopole-skyrmion potential

$$\langle \gamma_\mu \partial_\mu \delta_{nm} + ia(r)(\gamma \times x)_3 + m_\psi \Phi(r)x_\mu \tau_3^{\mu} \rangle \psi_m = E \psi_n.$$  

(3)

Remember that in the absence of the skyrmion potential ($m_\psi = 0$), i.e., in the algebraic spin liquid the fermion zero mode was not found\[8\]. As mentioned in the introduction, the presence of the Neel vector makes the Dirac equation (3) have the essentially same structure as that utilized in the context of high energy physics\[1, 2, 3, 4]. SU(2) gauge theory in terms of massless Dirac fermions and adjoint Higgs fields interacting via SU(2) gauge fields has been intensively studied in the presence of Kondo-like isospin couplings between these matter fields\[1, 2, 3, 4\]. When the Higgs fields are condensed, topologically nontrivial stable excitations called ‘t Hooft-Polyakov monopoles can appear\[27, 28\]. The ‘t Hooft-Polyakov monopoles consist of gauge-Higgs composite potentials\[27, 28\]. Jackiw and Rebbi showed that the Dirac equation in the presence of the Kondo-like isospin couplings has a fermion zero mode in a ‘t Hooft-Polyakov monopole potential\[2\]. Here, the Neel order parameter fields play the same role as the adjoint Higgs fields.

Following Jackiw and Rebbi\[1\], we explicitly demonstrate that Eq. (3) has a zero mode. We rewrite Eq. (3) in terms of the two component spinors $\psi_{\pm}$ with $E = 0$

$$\langle \sigma_3 \partial_\tau \rangle_{ij} \chi_{j_n}^\pm + (\sigma_2 \partial_\tau \chi_{j_n}^\pm + (\sigma_1 \partial_\rho \chi_{j_n}^\pm
$$

$$+ i a_3 \sigma_2 \chi_{j_n}^\pm - i a_3 \sigma_3 \chi_{j_n}^\pm + m_\psi \Phi \chi_{j_n}^\pm \tau_3^{\mu} \rangle_{mn} = 0.$$  

(4)

Inserting $\psi_{\pm} = M_{\pm}^{\mu \nu} \tau^2_{mn}$ with a two-by-two matrix $M_{\pm}$ into the above, we obtain

$$\langle \sigma_3 \partial_\tau M_{\pm}^{\mu} + \sigma_2 \partial_\rho M_{\pm}^{\mu} + \sigma_1 \partial_\mu M_{\pm}^{\mu} + i a_3 \sigma_2 M_{\pm}^{\mu} - i a_3 \sigma_3 M_{\pm}^{\mu} + m_\psi \Phi M_{\pm}^{\mu} \sigma^\mu = 0.$$  

(5)

Now the spin matrices $\tau^k$ and the Dirac matrices $\sigma^k$ are indistinguishable\[1\]. Finally, representing the matrix $M_{\pm}$ in $M_{\pm}^{\mu \nu} = g_{\mu}^{\nu} \sigma_{\mu}^{\nu}$, we obtain the coupled equations of motion for the numbers $g_{\mu}^{\nu}$ and $g_{\mu}^{\nu}$

$$\langle \partial_\tau + m_\psi \Phi \rangle g_3^{\mu} - i (\partial_\rho + i a_3 \mp m_\psi \Phi) g_3^{\mu} = 0,$$

$$\langle \partial_\mu + m_\psi \Phi \rangle g_2^{\mu} + i (\partial_\tau + m_\psi \Phi) g_2^{\mu} = 0,$$

$$\langle \partial_\mu + m_\psi \Phi \rangle g_1^{\mu} + i (\partial_\nu + m_\psi \Phi) g_1^{\mu} = 0,$$

$$\langle \partial_\nu + m_\psi \Phi \rangle g_2^{\mu} + (\partial_\mu + i a_3 \mp m_\psi \Phi) g_2^{\mu} = 0,$$

$$\langle \partial_\nu + m_\psi \Phi \rangle g_3^{\mu} + (\partial_\tau + i a_3 \mp m_\psi \Phi) g_3^{\mu} = 0.$$  

(6)
These equations yield the following zero mode equations

\[
(\partial_x + m_\psi \Phi \tilde{\tau}) g^- = 0,
(\partial_y + i a x + m_\psi \Phi \tilde{\tau}) g^- = 0,
(\partial_y - i a x + m_\psi \Phi \tilde{\tau}) g^- = 0. \tag{7}
\]

A normalizable zero mode solution \( g^- \) is given by \( g^- = \exp[-i \int dx(r)y + i \int dy(r)x] \ exp[-\int dm_\psi \Phi \tilde{\tau} \ - \ \int dm_\psi \Phi(r)y - \int dm_\psi \Phi(r)x] \). Existence of the zero mode makes the fermion determinant zero in the single monopole excitation. As a result single monopole excitations are suppressed by the zero mode and thus, the condensation of magnetic monopoles is forbidden. The suppression of monopole condensation results in deconfinement of internally charged particles, here the Dirac spinons. This deconfined antiferromagnetism differs from the usual confined one described by Eq. (2). Antiferromagnetic spin fluctuations fractionalize into spinons because the U(1) gauge interactions are not confining any more. Below we discuss an effective field theory to describe this unusual antiferromagnetism.

C. Effect of Fermion Zero Modes: ’t Hooft Effective Interaction and Deconfinement of Massive Spinons

In high energy physics it is well known that the contribution of instantons (monopoles)\(^{12}\) in the presence of a fermion zero mode gives rise to an effective interaction to fermions\(^2\) \& \(^3\). This interaction is usually called ’t Hooft effective interaction. In order to obtain the effective fermion interaction it is necessary to average the partition function in Eq. (1) over various instanton and anti-instanton configurations. Following Ref. \(^{22}\), we first consider a partition function in a single instanton potential

\[
Z_\psi = \int D\psi_n e^{-\int d^3x \bar{\psi}_n \gamma_\mu \partial_\mu \psi_n \left( m - V^I[\psi_n] \right)},
V^I[\psi_n] = \int d^3x \left( \bar{\psi}_n(x) \gamma_\mu \partial_\mu \Phi_n^I(x) \right)
\times \int d^3y \left( \Phi_n^I(y) \gamma_\mu \partial_\mu \psi_n(y) \right). \tag{8}
\]

Here \( \Phi_n^I \) is the zero mode obtained from Eq. (7). A fermion mass \( m \) is introduced. Later the chiral limit \( m \to 0 \) will be taken. The effective action including the effective potential \( V^I[\psi_n] \) in Eq. (8) gives a correct green function in a single instanton potential\(^{22}\),

\[
S^I(x, y) = \int d^3x \bar{\psi}_n(x) \gamma_\mu \partial_\mu \psi_n(y) + S_0(x, y)
\]

with the bare propagator \( S_0(x, y) = (\gamma_\mu \partial_\mu)^{-1} \delta(x - y) \). Thus the partition function in Eq. (8) can be used for average of instantons\(^{22}\). The partition function in the presence of \( N_+ \) instantons and \( N_- \) anti-instantons can be easily built up\(^{23}\)

\[
Z_\psi = \int D\psi_{n_+} D\psi_{n_-} e^{-\int d^3x \bar{\psi}_{n_+} \gamma_\mu \partial_\mu \psi_{n_+} \left( m - V^I[\psi_{n_+}] \right) \times \int d^3y \bar{\psi}_{n_-} \gamma_\mu \partial_\mu \psi_{n_-}}, \tag{9}
\]

Here we admit the noncompact U(1) gauge field \( a_\mu^u \) representing gaussian quantum fluctuations. In the following the index \( qu \) is omitted. (\dots) means averaging over individual instantons. Introducing instanton averaged nonlocal fermion vertices \( Y_{\pm} = -V < V^I[\psi_n] >= -\int d^3z \eta(i) V^I[\psi_n] \) with volume \( V \), where \( \eta(i) \) represent instanton (anti-instanton) positions\(^{22}\), we obtain the following partition function in the chiral limit \( m \to 0 \)

\[
\int d\lambda_\pm \frac{2}{2\pi} \int d\Gamma_\pm e^{i\lambda_+(Y_+-\Gamma_+)+N_+} \ln \frac{\lambda_+}{\lambda_+-(+---)} \tag{10}
\]

Integration over \( \lambda_\pm \) and \( \Gamma_\pm \) recovers Eq. (9) in the chiral limit. In the thermodynamic limit \( N_+/V \to \infty \) and \( N_+/V \) fixed, integration over \( \Gamma_\pm \) and \( \lambda_\pm \) can be performed by the saddle point method\(^{22}\). Integrating over \( \Gamma_\pm \) first, we obtain

\[
Z_\psi = \int d\lambda_+ \frac{2}{2\pi} e^{N_+ \left( \ln \frac{\lambda_+}{\lambda_+-(+---)} \right)} \int D\psi_{n_+} D\psi_{n_-} e^{-\int d^3x \bar{\psi}_{n_+} \gamma_\mu \partial_\mu \psi_{n_+} \int d^3y \bar{\psi}_{n_-} \gamma_\mu \partial_\mu \psi_{n_-}}. \tag{11}
\]

An explicit calculation for the instanton average shows that the vertex \( Y_{\pm} \) corresponds to a mass\(^{22}\),

\[
Y_{\pm} = \int \frac{d^3k}{(2\pi)^3} \frac{1}{(2\pi)^3} = \psi_n \frac{1}{\psi_n} \psi_n \gamma_5 \psi_n \gamma_5 = \psi_n \left( \begin{array}{c} 0 \\ I \\ -I \\ 0 \end{array} \right).
\]

Here \( F(k) \) is associated with the fermion zero mode in the effective potential \( V^I[\psi_n] \) in Eq. (8). In the present paper we do not perform an explicit calculation for the instanton average in \( Y_{\pm} \) and thus, we do not know the exact form of \( F(k) \). Our objective is to see how the ’t Hooft interaction appears as an instanton effect. Here \( \rho \) is the size of an instanton. Although the instanton (magnetic monopole) can be considered to be a point particle, the instanton-skyrmion composite would have its characteristic size \( \rho \). In order to determine the size \( \rho \) we should solve an equation of motion for the Neel vector \( \vec{n} \) in the presence of an instanton configuration. In the present paper we do not examine this issue. We assume its existence. Owing to the neutrality condition of magnetic charges, \( N_+ = N_- = N_I/2 \) is obtained in Eq. (11), where \( N_I \) is the total number of instantons and anti-instantons. The saddle point solution of \( \lambda_+ = \lambda_- = \lambda \) in Eq. (11) gives rise to cancellation of the \( \gamma_5 \) term in the mass, causing a momentum dependent mass

\[
m(k) = m_1 F^2(k) \text{ with } m_1 = \lambda \left( 2\pi \rho \right)^2 \tag{12}
\]

The mass \( m_1 \) is determined by the saddle point equation for \( \lambda \) usually called a self-consistent gap equation\(^{22}\).
Ignoring the momentum dependence by setting $F(k) = 1$ for simplicity, we obtain the ’t Hooft mass $m_I = \frac{\pi}{2 a m_{\text{inst}}} \left(\frac{\Lambda}{y_{m}}\right)^{1/2}$ with a momentum cut-off $\Lambda$ in the small mass limit. Since the mean density of instantons is proportional to the instanton fugacity, $N_I/V \sim y_m = e^{-S_{\text{inst}}} \sim 1/\epsilon^2$ with an instanton action $S_{\text{inst}} \sim 1/\epsilon^2$, the fermion mass is roughly given by $m_I \sim y_m^{1/2}$. We can obtain the following effective Lagrangian in terms of Dirac spinons $\psi_n$ with the ’t Hooft effective mass $m_I$ interacting via noncompact $U(1)$ gauge fields $a_\mu$, $\mathcal{L}_\psi = \bar{\psi}_n \gamma_\mu (\partial_\mu - ia_\mu) \psi_n + m_I \bar{\psi}_n \psi_n$. The formal appearance of the fermion mass does not necessarily mean that the fermions are massive. This is because the fermion mass is determined by density of instantons. In order to determine the fermion mass, we should find the instanton fugacity, i.e., reveal the state of instantons.

It is one of the most difficult problems to determine the state of monopoles (instantons) in the presence of matter fields. In this paper we do not pursue to determine the state of monopoles precisely. Instead, we consider all possibilities. Generally speaking, there would be three possible monopole states. These are monopole plasma, monopole dipolar and monopole "liquid" phases. These phases are characterized by $y_m \to \infty$, $y_m \to 0$ and $y_m \neq 0$, respectively, where $y_m$ is the monopole fugacity. We exclude the first, monopole plasma phase because the presence of fermion zero modes does not allow the condensation of monopoles.

At first glance a liquid phase of magnetic monopoles sounds quite strange. It is known that the liquid state does not appear in the Abelian Higgs model without fermions. But, in the present case we do not have any evidence to exclude this monopole state. In (2+1)D the basic trend is confinement, i.e., $y_m \to \infty$ away from quantum criticality. Owing to the confinement tendency many instantons would be excited although the presence of fermion zero modes does not allow instanton condensation. Thus, we should treat dense uncondensed monopoles. We claim that in order to solve this problem a new methodology beyond the dilute approximation of monopoles is required. As far as we know, this methodology is not found yet. Note that the usual renormalization group equation for the monopole fugacity is obtained from the dilute approximation of monopoles. It is not clear if this standard renormalization group equation is applicable to high density limit. In this respect we have no clear evidence to exclude the monopole liquid phase in the presence of fermion zero modes. We may view the emergence of the liquid state as the proximate effect of the Higgs-confinement phase in the presence of fermion zero modes. There exist some reports about a new phase instead of plasma and dipolar phases in two dimensional Coulomb gas when the density of particles is high. Furthermore, a new fixed point with nonzero monopole fugacity was recently reported even in the $QED_3$ only with massless Dirac fermions.

In the liquid phase of monopoles Dirac spinons obtain the ’t Hooft effective mass ($m_I$) while in the dipolar phase of monopoles the ’t Hooft mass vanishes. But, it should be noted that we are considering antiferromagnetism. In antiferromagnetism Dirac spinons are massive ($m_\psi$) via $S\chi SB$. This mass completely differs from the ’t Hooft mass. Although it is not easy to evaluate the ’t Hooft mass, we expect that the chiral mass $m_\psi$ would be larger than the ’t Hooft mass $m_I$. This assumption is based on the fact that the chiral mass $m_\psi$ would be identified with the staggered magnetization observed in quantum antiferromagnets. In both phases of magnetic monopoles the deconfined Dirac spinons would be massive owing to the antiferromagnetism. At present we do not have any idea how to distinguish these two monopole states.

Combining the existence of fermion zero modes in section II-B and their physical effects in section II-C, we can reach the following effective Lagrangian from Eq. (1) in the deconfined antiferromagnet

$$ Z_{AF} = \int D\bar{\psi}_n D\bar{a}_\mu D\bar{\pi} e^{-\int d^dx \mathcal{L}_{AF}}, $$

$$ \mathcal{L}_{AF} = \bar{\psi}_n \gamma_\mu (\partial_\mu - ia_\mu) \psi_n + m_\psi \bar{\psi}_n \chi_3^3 \psi_n \psi_n + m_\psi \bar{\psi}_n \chi_3^3 \psi_n \cdot \bar{\psi}_n + \frac{N m_\psi}{4\pi} \left((\partial_\mu \bar{\pi}^2 + \bar{\pi} \cdot \partial_\mu \bar{\pi}^2)^2 (13) \right), $$

where we used $n^3 = \sqrt{1 - |\bar{\pi}|^2} \approx 1 - \frac{1}{2} |\bar{\pi}|^2$ as Eq. (2). In this effective Lagrangian we should understand that the gauge field $a_\mu$ is noncompact in Eq. (13) owing to the effect of fermion zero modes while the gauge field $a_\mu$ in Eq. (1) is compact, causing confinement of Dirac spinons and thus, resulting in Eq. (2). This antiferromagnet low lying excitations are antiferromagnetic spin waves and (noncompact) $U(1)$ gauge fluctuations. Both gapless excitations cause $C_0 \sim T^2$, where $C_0$ is specific heat and $T$, temperature. Furthermore, the gapped Dirac spinons would be deconfined to emerge above their mass gap. These gapped spinons interact with the antiferromagnons as shown by the fourth and fifth terms. In Fig. 1 we show schematic spin spectrum in inelastic neutron scattering. Here $\chi_3^3(\omega)$ is the imaginary part of the transverse dynamic spin susceptibility at momentum $(\pi, \pi)$, given by $\chi_3(\omega) \approx \chi_3(\omega + \chi_3^3(\omega) + \chi_3(\omega))$ where $\chi_3^3(\omega) \approx \langle \pi \tau \pi \rangle$ is a magnon susceptibility, $\chi_3^3(\omega) \sim \langle \pi \tau \pi \rangle$, a spinon susceptibility, and $\chi_3^3(\omega) \sim \langle \psi \pi \psi \psi \rangle + \langle \pi \psi \pi \pi \rangle$, a "coupling" susceptibility in a highly schematic form. At low energies antiferromagnons give dominant spectral weight $\chi_3^3(\omega)$ while at high energies Dirac spinons exhibit broad continuum spectrum $\chi_3^3(\omega)$.

In our scenario the unexplained 1/2 spectral weight in the inelastic neutron scattering would be identified with the gapped Dirac spinons. Although scattering between spinons and magnons in Eq. (13) does not alter the spin spectrum in Fig. 1 qualitatively, elaborate calculations are required in order to
understand the spin spectrum more quantitatively and precisely.

III. SUMMARY

In summary, we investigated the two dimensional quantum antiferromagnet resulting from the algebraic spin liquid via spontaneous chiral symmetry breaking. This antiferromagnet is described by the fermionic nonlinear σ model, Eq. (1) in terms of Dirac spinons $\psi_n$ interacting via not only U(1) gauge fluctuations $a_\mu$, but also antiferromagnetic spin fluctuations $\bar{a}$. We showed that the Kondo-like spin couplings between the Dirac spinons and Neel vectors give rise to the fermion zero mode in the single monopole potential. The existence of fermion zero modes suppresses the condensation of monopoles, thus causing the deconfinement of spinons. As a result we obtained the effective field theory Eq. (13) in the deconfined antiferromagnet, which differs from the conventional confined antiferromagnet described by the O(3) nonlinear σ model Eq. (2). From the effective field theory Eq. (13) we argued that the deconfined massive spinons would be observed as particle-hole continuum above their mass gap in the dynamic spin susceptibility [2, 3, 10].

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APPENDIX A

In appendix A we briefly sketch how we can obtain the $S_{ASL}$ in Eq. (1) from the antiferromagnetic Heisenberg model on two dimensional square lattices, $H = J \sum_{<i,j>} \vec{S}_i \cdot \vec{S}_j$ with $J > 0$. Inserting the following spinon representation for spin, $\vec{S}_i = \frac{1}{2} f_{1a}^\dagger \vec{\tau}_\alpha f_{1\alpha}$ into the above Heisenberg model, and performing the standard Hubbard-Stratonovich transformation for an exchange interaction channel, we obtain an effective one body Hamiltonian for fermions coupled to an order parameter, $H_{eff} = -J \sum_{<i,j>} f_{1a}^\dagger \chi_{ij} f_{j\alpha} - h.c$. Here $f_{1\alpha}$ is a fermionic spinon with spin $\alpha = \uparrow, \downarrow$, and $\chi_{ij}$ is an auxiliary field called a hopping order parameter. Notice that the hopping order parameter $\chi_{ij}$ is a complex number defined on links $ij$. Thus, it can be decomposed into $\chi_{ij} = |\chi_{ij}| e^{i\theta_{ij}}$, where $|\chi_{ij}|$ and $\theta_{ij}$ are the amplitude and phase of the hopping order parameter, respectively. Inserting this representation for the $\chi_{ij}$ into the effective Hamiltonian, we obtain $H_{eff} = -J \sum_{<i,j>} |\chi_{ij}| f_{1a}^\dagger e^{i\theta_{ij}} f_{j\alpha} - h.c$. We can easily see that this effective Hamiltonian has an internal U(1) gauge symmetry, $H_{eff} = H_{eff}[f_{1\alpha}, \theta_{ij}] = H_{eff}[f_{1\alpha}, \theta']_{ij}$ under the following U(1) phase transformations, $f_{1a}^\dagger = e^{i\phi} f_{1a}$ and $\theta'_{ij} = \theta_{ij} + \phi_i - \phi_j$. This implies that the phase field $\theta'_{ij}$ of the hopping order parameter plays the same role as a U(1) gauge field $a_{ij}$. When a spinon hops on lattices, it obtains an Aharonov-Bohm phase owing to the U(1) gauge field $a_{ij}$. It is known that a stable mean field phase is a $\pi$ flux state at half filling [4, 21]. This means that a spinon gains the phase of $\pi$ when it turns around one plaquette. In the $\pi$ flux phase low energy elementary excitations are massless Dirac spinons near nodal points showing gapless Dirac spectrum and U(1) gauge fluctuations [11, 21]. In the low energy limit the amplitude $|\chi_{ij}|$ is frozen to $|\chi_{ij}| = |J| < f_{1a}^\dagger f_{j\alpha} > |$. A resulting effective field theory for one possible quantum disordered paramagnetism of the antiferromagnetic Heisenberg model is QED$\_3$ in terms of massless Dirac spinons interacting via compact U(1) gauge fields, $S_{ASL}$ in Eq. (1).

In the $S_{ASL}$ $\psi_n = \begin{pmatrix} \chi^n_+ \\ \chi^n_- \\ \chi^n_\uparrow \\ \chi^n_\downarrow \end{pmatrix}$ is a four component massless Dirac fermion, where $n = 1, 2$ represents its SU(2) spin $|\uparrow, \downarrow|$ and $\pm$ denote the nodal points of $(\pi/2, \pm \pi/2)$ in momentum space. Usually, SU(N) quantum antiferromagnets are considered by generalizing the spin components $n = 1, 2$ into $n = 1, 2, ..., N$. The two component spinors $\chi^n_{\alpha}$ are given by $\chi^n_+ = \begin{pmatrix} f_{11e}^\dagger \\ f_{11o}^\dagger \end{pmatrix}$, $\chi^n_- = \begin{pmatrix} f_{22e}^\dagger \\ f_{22o}^\dagger \end{pmatrix}$, $\chi^n_\uparrow = \begin{pmatrix} f_{11e}^\dagger \\ f_{11o}^\dagger \end{pmatrix}$, and $\chi^n_\downarrow = \begin{pmatrix} f_{22e}^\dagger \\ f_{22o}^\dagger \end{pmatrix}$, respectively. In the spinon field $f_{abc}$, $a = \uparrow, \downarrow$ represents its SU(2) spin, $b = 1, 2$, the nodal points $\{\pm, \mp\}$, and $c = e, o$, even and odd sites, respectively [21]. Dirac matrices $\gamma_\mu$ are given by $\gamma_0 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix}$, $\gamma_1 = \begin{pmatrix} \sigma_2 & 0 \\ 0 & -\sigma_2 \end{pmatrix}$, and $\gamma_2 = \begin{pmatrix} \sigma_1 & 0 \\ 0 & -\sigma_1 \end{pmatrix}$, respectively, where they satisfy the Clifford algebra $[\gamma_\mu, \gamma_\nu]_+ = 2\delta_{\mu\nu}$ [21].
In appendix B, for completeness of this paper we briefly sketch how we obtain the dynamically generated spinon mass $m_\psi$ in $S_{M}$ in Eq. (1). A single spinon propagator is given by $G^{-1}(k) = G_0^{-1}(k) - \Sigma(k)$, where $G_0^{-1}(k) = i\gamma_\mu k_\mu$ is the inverse of a bare spinon propagator, and $\Sigma(k)$, a spinon self-energy resulting from long range gauge interactions. The spinon self-energy is determined by the self-consistent gap equation, $\Sigma(k) = \frac{N\hbar}{4\pi} d\mathbf{q} Tr[\gamma_\mu G(k-q)\gamma_\nu D_\mu(q)]$, where $D_\mu(q)$ is a renormalized propagator of the U(1) gauge field $a_\mu$ due to particle-hole excitations of massless Dirac fermions. The self-energy can be written as $\Sigma(k) = -m_\psi(k)^3$ for staggered magnetization.[21] Inserting this representation into the above self-consistent gap equation, we obtain the following expression for the spinon mass, $m_\psi(p) = \int \frac{d^3k}{(2\pi)^3} k^2\psi_m(k) D_\mu(p-k)$. The renormalized gauge propagator $D_\mu(q)$ is obtained to be $D_\mu(q) = \Pi_\mu^{-1}(q) = -N \int \frac{d^3k}{(2\pi)^3} Tr[\gamma_\mu G_0(k)\gamma_\nu G_0(k-q)]$ is the polarization function of massless Dirac fermions in the 1/N approximation. Inserting this gauge propagator into the gap equation and performing an angular integration, one can find $m_\psi(p) = \frac{s}{N\pi^2} k_0^3 \int d^3k \psi_m(k) (k^2 + p^2 - |k-p|^2)$, where $\Lambda$ is a momentum cutoff.[21] This integral expression is equivalent to the differential equation, $\frac{d}{dp}\left[p^2\frac{dm_\psi}{dp}\right] = \frac{8}{\pi^2 N p^2 + m_\psi^2}$ with boundary conditions, $\Lambda \frac{dm_\psi}{dp} = m_\psi(\Lambda) = 0$ and $0 \leq m_\psi(0) \leq \infty$.[10] In Ref. [10] this equation is well analyzed in detail. Its solution is given by $m_\psi \approx e^2 \text{exp}[-2\pi/\sqrt{N_c/N-1}]$ in the case of $N < N_c$. [10 22].

\[ \frac{N}{2} \ln \text{det} \left[ 1 - \frac{m_\psi \gamma_\mu \partial_\mu (\vec{n} \cdot \vec{\tau})}{-\partial^2 + m_\psi^2} \right] \]

\[ = -\frac{N}{2} Tr \int d^3x (x\ln \left[ 1 - \frac{m_\psi \gamma_\mu \partial_\mu (\vec{n} \cdot \vec{\tau})}{-\partial^2 + m_\psi^2} \right] |x\rangle \]

\[ = -\frac{N}{2} \int d^3x \int \frac{d^3k}{(2\pi)^3} e^{-ikx} \text{Tr} \ln \left[ 1 - \frac{m_\psi \gamma_\mu \partial_\mu (\vec{n} \cdot \vec{\tau})}{-\partial^2 + m_\psi^2} \right] e^{ikx} \]

\[ = \frac{N}{2} \int d^3x \int \frac{d^3k}{(2\pi)^3} \text{Tr} \ln \left[ 1 - \frac{m_\psi \gamma_\mu \partial_\mu (\vec{n} \cdot \vec{\tau})}{-\partial^2 + m_\psi^2} \right] e^{ikx} \]

\[ \approx \int \frac{d^3x}{4\pi} \int \frac{d^3k}{(2\pi)^3} Tr \left[ \frac{m_\psi \gamma_\mu \partial_\mu (\vec{n} \cdot \vec{\tau})}{k^2 + m_\psi^2} \right]^2 \]

\[ = \int \frac{d^3x}{4\pi} \frac{N m_\psi}{4\pi} |\partial_\mu \vec{n}|^2. \] (C2)

In the above $Tr$ stands for not a functional but a usual matrix trace for both flavor (spin) and spinor indices. In going from the third to the fourth line we have dragged the factor $e^{ikx}$ through the operator, thus shifting all differential operators $\partial_\mu \rightarrow \partial_\mu + ik_\mu$. Expanding the argument of the logarithmic term in powers of $\partial_\mu \vec{n}$ and of $2ik_\mu \partial_\mu - \partial^2$, one can easily obtain the expression in the fifth line. Performing the momentum integration, we obtain an effective spin stiffness proportional to the mass parameter $m_\psi$. This implies that the rigidity of fluctuations in the Neel field is controlled by the mass parameter $m_\psi$ of Dirac spinons. Eq. (C2) is nothing but the O(3) nonlinear $\sigma$ model describing quantum antiferromagnetism. Note that higher order derivative terms in the gradient expansion are irrelevant in $(2+1)D$ in the renormalization group sense.

Next, we sketch the derivation of the Maxwell gauge action. Expanding the argument of the logarithmic term in Eq. (C1) to the second order of the gauge field $a_\mu$, we obtain $S_{gauge} = \int \frac{d^4x}{(2\pi)^4} \frac{1}{2} \rho_\mu (q) \Pi_\mu(q) a_\nu(q) - q^2 \delta_{\mu\nu} - q_\mu q_\nu$, where the fermion polarization function $\Pi_\mu(q)$ is given by $\Pi_\mu(q) = -N \int \frac{d^4k}{(2\pi)^4} Tr \left[ \gamma_\mu G(k+q) \gamma_\nu G(k) \right]$ with the single spinon propagator $G(k) = \left[ \gamma_\mu k_\mu + m_\psi \vec{n} \cdot \vec{\tau} \right]^{-1}$. Utilizing the Feynman identity and trace identity of Dirac gamma matrices, one can obtain the following expression for the polarization function, $\Pi_\mu(q) = 2N (Tr |1/2\tilde{D}/D|) (q^2 - q_\mu q_\nu) \int dx (1-x)x m_\psi^2 + q^2 x(1-x)D_{\mu\nu}^{-2} + (\tau r) N_\psi (q^2 - q_\mu q_\nu) \left( \frac{m_\psi^2}{2\pi^2} + \frac{\delta^2 - 4m_\psi^2}{4\pi^2} \sin^2 \left( \frac{q}{2\sqrt{4m_\psi^2 + \delta^2}} \right) \right)$. This leads to the Maxwell gauge action in Eq. (C1).

Lastly, we should comment the reason why imaginary terms do not arise in the present nonlinear $\sigma$ model because some previous studies have shown the emergence of an imaginary term in the effective potential or effective action.

\[ \left\{ \psi^n \gamma_\mu (\partial_\mu - i a_\mu) \psi_n \right\} \]

\[ = -N \ln \left[ \left\{ \gamma_\mu (\partial_\mu - i a_\mu) \right\} \psi_m \right] \]

\[ \approx -\frac{N}{2} \ln \text{det}(-\partial^2 + m_\psi^2) \]

\[ + \int d^3x \left( \frac{N m_\psi}{4\pi} |\partial_\mu \vec{n}|^2 + \frac{N}{12\pi m_\psi} |\partial \times a|^2 \right). \] (C1)

\[ \text{The second term in the last line is well derived in Ref. [22].} \]
of imaginary terms [36, 37]. Following the evaluations in Ref. [36], one can obtain two imaginary terms in the irreducible representation of gamma matrices; one is a coupling term \( i a_\mu J_\mu \) between topologically nontrivial fermionic currents \( J_\mu = \frac{1}{8\pi} \epsilon_{\mu\nu\lambda} \chi_\alpha \gamma^\nu \partial_\nu \psi \gamma^\lambda \partial_\lambda \psi \) and U(1) gauge fields \( a_\mu \), and the other, a geometrical phase term \( i N \pi \Gamma[\bar{n}] [36, 37] \). The key point is the representation of Dirac gamma matrices [38]. Here, we utilized four-by-four Dirac matrices by combining the two nodal points. Note that the signs of Pauli matrices in the Dirac gamma matrices are opposite for the nodal points ±. This fact results in cancellation of the imaginary terms. The first imaginary term can be considered from a variation of the logarithmic term in Eq. (C1) with respect to the gauge field \( a_\mu \)

\[
i NTr \left[ \frac{\gamma_\mu \delta a_\mu}{\gamma_\mu (\partial_\mu - ia_\mu) + m_\psi (\bar{n} \cdot \bar{\tau})} \right] = i NTr \left[ \frac{\gamma_\mu \delta a_\mu}{\gamma_\mu (\partial_\mu - ia_\mu) + m_\psi (\bar{n} \cdot \bar{\tau})} \right] \approx i NTr \left[ \frac{m_\psi (\bar{n} \cdot \bar{\tau})}{-\partial^2 + m_\psi^2 \left( \frac{m_\psi \gamma_\mu \partial_\mu (\bar{n} \cdot \bar{\tau})}{-\partial^2 + m_\psi^2} \right)^2} \right]. \tag{C3}
\]

In this expression the key point is the triple product of Dirac matrices, \( \gamma_\mu \gamma_\nu \gamma_\lambda \) in the last line. In the irreducible representation of gamma matrices, i.e., Pauli matrices, this contribution is nonzero, leading to \( \epsilon_{\mu\nu\lambda} \). As a result the imaginary term of \( i a_\mu \epsilon_{\mu\nu\lambda} \chi_\alpha \gamma^\nu \partial_\nu \psi \gamma^\lambda \partial_\lambda \psi \) can be obtained [37]. Another \( \epsilon_{\alpha\beta\gamma} \) associated with the Neel vectors appears from the triple product of Pauli matrices, \( \tau_\alpha \tau_\beta \tau_\gamma \). On the other hand, in the present representation of Dirac matrices the contribution of the + nodal point leads to \( + \epsilon_{\mu\nu\lambda} \) while that of the − nodal point, \( - \epsilon_{\mu\nu\lambda} \). Thus, these two contributions are exactly cancelled. The geometrical phase term, considered from a variation of the logarithmic term in Eq. (C1) with respect to the Neel field \( \bar{n} \) [36, 37],

\[
-NTr \left[ \frac{m_\psi (\bar{n} \cdot \bar{\tau})}{\gamma_\mu \partial_\mu + m_\psi (\bar{n} \cdot \bar{\tau})} \right] = -NImTr \left[ \frac{m_\psi (\bar{n} \cdot \bar{\tau})}{\gamma_\mu \partial_\mu + m_\psi (\bar{n} \cdot \bar{\tau})} \right] \approx NTr \left[ \frac{m_\psi (\bar{n} \cdot \bar{\tau})}{\gamma_\mu \partial_\mu + m_\psi (\bar{n} \cdot \bar{\tau})} \right] \tag{C4}
\]

is also exactly zero owing to the same reason. Another way to say this is that the signs of mass terms for the Dirac fermions \( (\chi^+_n \text{ and } \chi^-_n) \) at the two Dirac nodes (+ and −) are opposite, resulting in cancellation of the parity anomaly [36]. If we fix the Neel vector in the z direction \( (\bar{n} = \bar{z}) \), we can see the opposite signs explicitly from \( m_\psi \bar{n}_\mu \tau_\mu \bar{\tau}_n \psi_m = m_\psi \bar{n}_\mu \tau_\mu \psi_m = m_\psi \chi^+_n \gamma^\mu \tau_\mu \chi^-_n - m_\psi \chi^-_n \gamma^\mu \tau_\mu \chi^+_n \). Both massive Dirac fermions \( (\chi^+_n \text{ and } \chi^-_n) \) contribute to the imaginary terms, respectively. However, the signs of the imaginary terms are opposite and thus, the cancellation occurs. As a result the imaginary terms do not appear in the present nonlinear σ model. This was already discussed in Ref. [39, 40].

It is possible that the mass terms have the same signs. Considering the two gamma matrices of \( \gamma_4 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \) and \( \gamma_5 = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix} \) [21], we can obtain the following mass terms with the same signs, \( \tilde{\Lambda}_M = m_\psi \bar{n}_\mu \gamma_4 \gamma_5 \tau^\mu \psi_m = -m_\psi \chi^+_n \tau^\mu \lambda^+_n - m_\psi \chi^-_n \tau^\mu \lambda^-_n \). These mass terms can arise from the algebraic spin liquid, \( S_{\text{ASL}} \) in Eq. (1) via \( S_{\chi \Sigma B} \) because the algebraic spin liquid has the enlarged symmetry [21, 24], as discussed earlier. In this case the cancellation does not occur and thus, the imaginary terms necessarily arise. This antiferromagnetism would not be conventional since it breaks not only time reversal symmetry but also parity symmetry. When this antiferromagnetism disappears via strong quantum fluctuations, its corresponding quantum disordered paramagnet is expected to be the chiral spin liquid [21, 34, 40]. In the present paper we did not discuss the chiral spin liquid.

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