Application of the theory of stochastic processes and Monte-Carlo simulations for the analysis of the operation of charging stations for electric vehicles

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Abstract. Considering the development of the Electric Vehicle industry, and the growing number of such vehicles in public transport, the charging problem of these vehicles is essential. The service process of a charging station seems a fundamental question, that must be analyzed.

Using the theory of stochastic processes, especially the Poisson process and the queuing theory; the characterization of the process can be studied. Some fundamental system parameters like the average waiting time, the average time in the system, the average time in the queue, and the average length of the queue, etc., can be modeled by the queuing theory using the service rate, the charging rate, and the number of servers, etc., as parameters.

Using the conclusions of this model, and considering the demands of Electric Vehicle users, the necessary/minimal/optimal number of servers, parking lots, etc. can be estimated. Using the theory of the Markov process, a stochastic model can be proposed for studying the long-term characterization of Electric Vehicle owners’ charging behavior; that which’s problem is strongly affected by the properties of charging stations. In this article, the queuing model and the Monte-Carlo method are presented for characterizing a charging station. For every calculation, Matlab has been used.

1. Introduction

Due to the growing number of electric vehicles, the charging of these vehicles is a fundamental problem, that efficiently must be solved. For example, in Hungary, the number of electric vehicles (pure electric and plug-in hybrid) in 2021 was around 30 thousand [1]. Comparing this quantity and the total number of vehicles in Hungary, which is around 4 million [2], the ratio of electric vehicles, which is around 0,75% seems negligible. According to worldwide statistics [3], recently the global ratio of electric car sales is 3%, which will reach 10% by 2025 and predicted to rise up to 58% by 2040. Due to trends for recent years, the profile change of car factories and the increasing number of such vehicles, the charging of these cars will be an essential problem of public transport soon [4]. This is a new challenge for existing, already working and, potential service providers. New charging stations must be constructed. But there are some expectations for these charging stations. On the one hand, which is fundamental for the provider, it must be economical and profitable. On the other hand, from the customers' point of view, it must be efficient, as fast as possible, there must be enough parking slots, and the waiting time must be as short as possible. In this article, the characterization of charging stations are given. During this investigation, various system parameters are considered. System parameters are properties of the station that essentially affect the operation of the station. The following parameters have been selected: the number of servers, in other words, the number of charging plugs, the number of
parking lots and the charging time. The charging time is partially determined by the type of charging device (several fast-charging types exist), therefore it can be considered as a property of the station.

The paper is organized as follows: Section 2 presents the application of the basic mathematical model, the part of the mass service theory, the queuing theory [5, 6, 7], which is a specific stochastic process [8, 9], a continuous-time Markov process. In section 3, using the theory described in section 2, the effect of system parameters for the key features of the operation of a charging station will be studied. These key features, which are basic for customers, are as follows: the waiting time, the length of the queue, and the occurrence of the event when the station is full, therefore the arriving vehicle must be rejected. In section 4 the Monte-Carlo simulation method is applied [10], which provides more information about a charging station, it characterizes the charging station from another point of view. Section 5 summarizes conclusions and sketches the possible future work.

2. The $M/M/s/K$ finite queue model

For modeling a charging station, an extremely suitable mathematical tool is the queuing theory [5, 6]. Customers (vehicle owners) sit in a queue and wait for the service. This process is depicted in Figure 1., assuming that the number of servers (charging plugs) is $s$, and the number of parking slots is $K$ including places where cars are being charged.

![Figure 1. The illustration of the charging station as a multiple server queuing system](image)

In Figure 1., the structure of a queuing system can be seen, including the queue itself at the station, servers, parking slots, etc. A gray rectangle refers to, that the parking slot is busy. White means that the parking slot is free. For a customer, only the number of gray slots matter, because it is the length of the queue, and the waiting time which basically depends on it. Clearly, if there are free servers, there is no queue, no waiting time, so only the illustrated situation is interesting in practice.

![Figure 2. The rate diagram of death and birth process for the $M/M/s/K$ model](image)
The basis of this theory is the "birth and death process". The meaning of "birth" is a new vehicle in the system (arrival), and the "death" means a served vehicle (departure). This is a stochastic process, that can be illustrated by the "rate diagram" (Figure 2.). In this article the $M/M/1/K$ queuing model is applied, as it is illustrated in Figure 1. and Figure 2. The "rate diagram" illustrates the possible states of the system, and "transition rates". The meaning of the letters and concepts are the following:

The first $M$ refers to that the arrival process is "Markovian", which means that the arrival process is modeled by a Poisson distribution [11, 12], with parameter $\lambda$. The parameter $\lambda$ is the "arrival rate," the average number of arrived vehicles per unit time. Clearly, the arrival rate $\lambda$ is independent of the state of the system, therefore it is constant. The Poisson distribution is essential in this study, the distribution can be given by the following formula:

$$ p_k = \frac{\lambda^k}{k!} e^{-\lambda}; \quad k = 0, 1, 2, ... $$  

(1)

A basic property of a Poisson distribution is that the modeled events are "rare," which means that by selecting a short time interval, the occurrence of more than one event in the given short time interval is negligible, its probability is zero. Therefore, at any time, at most one arrival must be considered.

The basic law in probability theory [11, 12], is that if the arriving process follows a Poisson distribution, then the interarrival time - the time between two consecutive arrivals - follows an exponential distribution. The exponential distribution is also fundamental in this study, so the basic formulas are recalled. The probability density function (pdf) and the cumulative distribution function (cdf) are respectively:

$$ f(x) = \begin{cases} 
0; & \text{if } x \leq 0 \\
\lambda e^{-\lambda x}; & \text{if } x > 0 
\end{cases} $$

$$ F(x) = \begin{cases} 
0; & \text{if } x \leq 0 \\
1 - e^{-\lambda x}; & \text{if } x > 0 
\end{cases} $$  

(2)

The second $M$ also refers to that the serving process is also "Markovian", which means that the service time is modeled by Exponential distribution, with parameter $\mu$. The parameter $\mu$ is the "service rate", the average number of served vehicles per unit of time. Assuming that every charging device is equipped with the same type of charging plug, $\mu$ is naturally the same constant for every server. This is an essential assumption of the theory. Over the rate diagram, the number of free and busy servers must be considered, therefore the "actual" service rate depends on the current state of the system and on the current number of free servers.

The "Markovian property" is equivalent to the application of exponential distribution for the arrival and service process. A fundamental feature of the exponential distribution is the "memoryless" property. Using the language of the theory-of-probability-theory, a stochastic process is said to be Markovian = memoryless, if the following holds:

$$ P(W > t + t' | W > t) = P(W > t) . $$  

(3)

This means that the probability is independent of time delay. More precisely, if $W$ is the waiting time, the probability of the event, that someone has to wait longer for the service than time $t$, is the same $t'$ time later. In other words, assuming that someone has already waited $t'$ time for the service, the probability of the event that he/she has to wait more $t$ time, is the same, independent to time $t'$. It doesn't matter when someone stands in the queue, in the morning, afternoon or tomorrow, the probability for this characteristic of the station, is the same.
This is a common property of exponential distribution and the continuous-time Markov process. This assumption is clearly valid for a queuing system. The queuing system is a continuous-time Markov process, because a new arrival or a new departure may occur at any time. A continuous-time Markov process can be defined by its "infinitesimal generator" matrix $Q$, which contains transition rates [5, 7, 9]. In the studied case, considering Figure 2., it is the following $(K+1)\times(K+1)$ quadratic matrix:

$$Q = \begin{pmatrix}
-\lambda & \lambda & 0 & 0 & \ldots \\
\mu & -(\lambda+\mu) & \lambda & 0 & \ldots \\
0 & 2\mu & -(\lambda+2\mu) & \lambda & \ldots \\
\vdots & \vdots & \vdots & \vdots & \ddots \\
(\lambda+(s-1)\mu) & -\lambda & \lambda & 0 & 0 \\
0 & s\mu & -(\lambda+s\mu) & \lambda & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots \\
0 & s\mu & -(\lambda+s\mu) & \lambda \\
\end{pmatrix} \tag{4}$$

Since formulation of probability distributions is very complicated in general, the steady-state of the system is studied instead. The steady-state of the continuous Markov process can be given by "balance equations". These equations can be combined in an equation system that is simply the following:

$$\pi Q = 0; \tag{5}$$

In this system $0$ is the constant zero row vector, and

$$\pi = [P_0, P_1, P_2, P_3, \ldots, P_K]; \tag{6}$$

is the probability distribution of the states of the queuing system. In this distribution, $P_n$ is the probability of the event, that there are exactly $n$ vehicles in the system ($n = 0, 1, 2, \ldots, K$). For characterizing the charging station, the probability distribution $P_n$ must be given. Considering (5), balance equations can be given as follows [4, 5, 6]:

$$\begin{align*}
\mu P_0 &= \lambda P_0 \\
\lambda P_0 + 2\mu P_2 &= (\lambda+\mu) P_1 \\
\lambda P_1 + 3\mu P_3 &= (\lambda+2\mu) P_2 \\
&\quad \ldots \\
\lambda P_{K-2} + s\mu P_K &= (\lambda+s\mu) P_{K-1} \\
s\mu P_K &= \lambda P_{K-1}
\end{align*} \tag{7}$$

This equation system provides the following results: Every probability can be given by the product

$$P_n = C_n \cdot P_0; \tag{8}$$

where on the one hand.
The letter $s$, as it is already defined, refers to the number of servers of the system. One "server" means one charging plug in the charging system, therefore $s$ is equal to the number of charging plugs at one charging station. It can be assumed in practice, that the number of servers is greater than one, therefore here a "multiple server system" is being studied and applied.

Finally, $K$ means that the proposed model is the "finite queue" model. $K$ is equal to the number of parking slots in the charging system, including places where vehicles are being charged. In other words, $K$ is the maximum number of vehicles, the capacity of the charging system. Clearly, every charging system has a finite number of places for vehicles, therefore if the system is full, the next arriving vehicle can't stand into the queue, it must be rejected. Rejection of a vehicle is a key problem for the customer and the provider too. This problem will also be studied in section 3.

The finite queue model has a great advantage over the infinite queue model. For describing this advantage, the "utilization factor" must be defined. If the average interarrival time is $ta$, and the average service time is $ts$, then clearly

$$
\lambda = \frac{1}{ta}; \quad \mu = \frac{1}{ts};
$$

The utilization factor of a charging system is defined by the ratio

$$
\rho = \frac{\lambda}{s \mu} = \frac{ts}{s \cdot ta};
$$

The utilization factor [5] is a fundamental quantity for management because it is a measure of the efficiency and effectiveness of the charging station, therefore it is desirable to be as big as possible. So, in practice, this factor should be greater than or equal to one, $\rho \geq 1$ can be considered as an everyday situation. (For example, if the average interarrival time is 10 minutes, the average service time is 60 minutes, and there are 4 charging plugs in the system, then $\rho = 1.5$; etc.)

The finite queue model has a great advantage against the infinite model. The infinite model makes sense only if $\rho < 1$, because if $\rho \geq 1$, according to the behavior of the geometric series, the "system blows up," the number of customers in the queuing system approaches infinity as time goes on and tends towards infinity. But the finite queue model is applicable also for the case when $\rho \geq 1$ [5, 6, 7]. This advantage will be strongly exploited in this study.

Naturally, the following questions arise: What are the key features of a queuing system, and what are the most important parameters of the charging station, which affect key features?
On the one hand, as it has already been mentioned in the introduction; from the point of view of the owner of a vehicle, clearly the most essential characteristics are the waiting time and the length of the queue. Furthermore, the case when a system is full, therefore the vehicle must be rejected is also important.

On the other hand, for the service provider, fundamental parameters of the system are the number of servers (number of charging plugs = s), the number of parking slots (capacity = K), and in some sense the charging time, because if the system was equipped with fast charging devices, it would be much more efficient.

The time, spent in the system (W, including service time) and the number of vehicles in the system (L, including vehicles that are being served) are also random variables. The formulation of these random variables by their density or distribution function can be very complicated in general. This is the reason why the expected value, in other words, the mean of these random variables is studied instead of their distribution function. These mean values come from balance equations, using the classic definition of expected value in the probability theory.

It is self-evident, that for a customer (owner of a vehicle) the number of vehicles being served is not important, only the length of the "actual" queue matters. Similarly, the time spent with charging his/her vehicle is personal, depending on the owner’s intention, the type of the vehicle, the charge level of the accumulator (battery), etc., so it is not a feature of the system. Therefore, only the “actual” waiting time must be investigated. The following notations and definitions will be used [5]:

- \( L_q \): The average (mean) length of the queue in the system (excluding vehicles that are being charged).
- \( W_q \): The average (mean) waiting time in the system (excluding service time).

These quantities can be derived from formulas (8)-(10). Using the classic definition of expected value [11, 12] of a discrete random variable. For the average (mean) length of the queue, the following is obtained:

\[
L_q = \sum_{n=0}^{K} (n-s) P_n = P_0 \frac{\bar{\lambda}}{s(1-\rho)} \left( \frac{\lambda}{\mu} \right)^s \left( 1 - \rho^{K-s} - (K-s) \rho^{K-s} (1-\rho) \right); \tag{13}
\]

For giving the average waiting time, Little's formula [4, 5, 6] is necessary:

\[
W_q = \frac{L_q}{\bar{\lambda}}; \tag{14}
\]

Where \( \bar{\lambda} \) is the average arrival rate in the \( M/M/s/K \) system:

\[
\bar{\lambda} = \sum_{n=0}^{\infty} \lambda P_n = \sum_{n=0}^{K-1} \lambda P_n = \lambda (1 - P_K) \tag{15}
\]

Summarizing these results the average waiting time in the queue can be given by the following expression:

\[
W_q = \frac{L_q}{\lambda (1 - P_K)}; \tag{16}
\]

In this formula probability \( P_K \) - the probability of the event that the system is full, there are \( K \) vehicles in the system - also comes from expressions (8)-(10).
3. Characterization of a charging station

In this section, the \( M/M/s/K \) queuing system will be applied as a model for characterizing the charging station. The basis for every further calculation is presented in the previous section by formulas (1)-(16). Calculations are performed by Matlab. The mean values \( L_q, W_q \), and the probability \( P_K \) will be studied, because these are the key features of a charging station, in other words, the most important questions for a customer.

The following questions will be investigated: How these key features are affected by the essential parameters of a charging system, such as the number of servers \( (s) \), the charging (service) time \( (t_s) \), and the number of parking slots \( (K) \). In every subsection, the connection between key features and one of these essential parameters will be studied and illustrated, using Matlab.

3.1. The effect of the number of servers

The number of servers (charging plugs) seems to be the most essential parameter of a charging station. In this section, the effect of the number of servers is studied. First, the effect on the average waiting time.

![Figure 3](image1.png)

**Figure 3.** The average waiting time \( (W_q) \) as a function of the number of servers \( (s) \)

![Figure 4](image2.png)

**Figure 4.** The average length of the queue \( (L_q) \) as a function of the number of servers \( (s) \)
The following key feature is the length of the queue. Figure 4. illustrates the average length of the queue ($L_q$) as a function of servers ($s$), for several numbers of charging time ($t_s$) but also for fixed capacity ($K$) and for fixed interarrival time ($t_a$).

Finally, the probability of the event, that one vehicle must be rejected ($P_K$) because the charging station is full, is studied. Figure 5. depicts the probability $P_K$ as a function of $s$ for different numbers of service time ($t_s$) and also for a fixed capacity ($K$) and for fixed interarrival time ($t_a$).

![Figure 5. The probability of rejection of a vehicle ($P_K$) as a function of the number of servers ($s$)](image)

It can be seen on these figures, that either the average waiting time or the average length of the queue and also the probability of the rejection of a vehicle is a decreasing function of the number of servers. These features are significantly affected by the number of servers, especially if $s$ is a small value. But clearly what is "small" depends on the average interarrival time. The situation is worse if the arrival rate is greater. These relationships can be used for estimating the necessary/desirable/"optimal" number of servers at a station.

3.2. The effect of the charging (service) time ($t_s$)

The length of the charging time is also a fundamental parameter of a charging station, but obviously depends not only on the type of chargers, but also on the intentions of customers, types of vehicles, and the charge level of the accumulators (batteries), which are random. Here the role of the first aspect is emphasized. In this section, the effect of the charging time is studied, first of all, the effect on the average waiting time.

![Figure 6. The average waiting time ($W_q$) as a function of the service time ($t_s$)](image)
Figure 6. depicts the average waiting time ($W_q$) as a function of the service time ($t_s$) for different numbers of servers ($s$), if the capacity of the charging station ($K$) and the interarrival time ($t_a$) is fixed.

The following key feature is the length of the queue. Figure 7. illustrates the average length of the queue ($L_q$) as a function of the service time ($t_s$), for several numbers of servers ($s$) but also for fixed capacity ($K$) and for fixed interarrival time ($t_a$).

Finally, the probability of the event, that a vehicle must be rejected ($P_K$) because the charging station is full, is studied. Figure 8. depicts the probability $P_K$ as a function of $t_s$ for different numbers of servers ($s$) and also for a fixed capacity ($K$) and for fixed interarrival time ($t_a$).

It can be seen on these plots, that either the average waiting time or the average length of the queue and the probability of the rejection of a vehicle is an increasing function of the service time. These features are significantly affected by the length of the service time, especially if $t_s$ exceeds a specific value that depends on the number of servers. The situation is again worse if the arrival rate is greater. These relationships can be used for selecting the appropriate/suitable type of charger.
3.3. The effect of the number of parking slots

The effect of the number of parking slots, the capacity of the charging station is the last studied quantity that is also considered as a fundamental system parameter. In practice at an ordinary charging station, there are more parking slots than the number of servers, providing parking possibilities for those who wait for the service. In this section, the effect of this parameter is being considered.

Figure 9. The average waiting time \((Wq)\) as a function of the capacity \((K)\)

\[
\text{a) } t_a = 20 \text{ min, } t_s = 90 \text{ min; } \quad \text{b) } t_a = 10 \text{ min, } t_s = 90 \text{ min; }
\]

Figure 9. depicts the average waiting time \((Wq)\) as a function of the capacity \((K)\) for different numbers of servers \((s)\) if the service time \((ts)\) and the interarrival time \((ta)\) is fixed.

The following key feature is the length of the queue. Figure 10. reflects the average length of the queue \((Lq)\) as a function of the capacity \((K)\), for several numbers of servers \((s)\) but also for fixed service time \((ts)\) and for fixed interarrival time \((ta)\).

Figure 10. The average length of the queue \((Lq)\) as a function of the capacity \((K)\)

\[
\text{a) } t_a = 20 \text{ min, } t_s = 90 \text{ min; } \quad \text{b) } t_a = 10 \text{ min, } t_s = 90 \text{ min; }
\]

It can be seen on these plots, that either the average waiting time or the average length of the queue is an increasing function of the capacity, if the arrival rates and the service times are fixed. These features are also affected by the number of parking slots. But, comparing this case and the previous subsections, the nature of the effect is different. In this case, the function is close to linear. The situation is again worse if the arrival rate is greater. These relationships can be used for planning the appropriate/necessary number of parking slots.
Finally, the probability of the event, that a vehicle must be rejected ($P_K$) because the charging station is full, is studied. Figure 11. depicts the probability $P_K$ as a function of $K$ for different numbers of service time ($t_s$) and a fixed number of servers ($s$) and for a fixed interarrival time ($t_a$).

![Image](image1.png)

**Figure 11.** The probability of rejection of a vehicle ($P_K$) as a function of the capacity ($K$)

Considering Figure 11. it can be visualized that the behavior of the probability-capacity function strongly depends on the arrival rate. The probability of rejection via capacity is more complicated than previous connections. Its monotonicity depends on the arrival rate, as it can be seen in Figure 11. for a fixed number of servers. So, if someone plans a charging station, the necessary number of parking slots seem to be a critical problem, which depends on the arrival rate.

3.4. *The effect of the utilization factor*

The utilization factor ($\rho$) measures the efficiency of the charging station, ("to what extent it is utilized"), which is a basic economic property for the providers. The greater the utilization factor the more efficient the station is; therefore, it must be as big as possible. According to (12) $\rho$ depends on the average interarrival time, the average service time, and the number of servers.

![Image](image2.png)

**Figure 12.** The connection between interarrival time ($t_a$) and the service time ($t_s$) for fixed utilization factor ($\rho$) and various numbers of servers ($s$)
Figure 12. depicts the corresponding $ta$ and $ts$ values for several numbers of servers and some fixed $\rho$ values. These plots facilitate visualizing some cases. The point is that one key feature, the average length of the queue ($Lq$), can be considered as a function of the utilization factor ($\rho$). In other words, this feature depends explicitly on $\rho$ [5]. This is not true for the average waiting time! Figure 13. visualizes the average length of the queue as a function of the utilization factor.

![Figure 12](image-url)

**Figure 12.** The corresponding $ta$ and $ts$ values for several numbers of servers and some fixed $\rho$ values.

Figure 13. The average length of the queue ($Lq$) as a function of the utilization factor ($\rho$)

Considering Figure 13. it is obvious, that the average length of the queue is significantly affected by the utilization factor, and surprisingly, the sharply increasing section of the function is in the neighborhood of $\rho = 1$, independently to the number of servers. Beyond $\rho = 1.5$ the function becomes nearly constant for every $s$ value.

4. Monte-Carlo simulations

In section 3. the average/mean value of several probability distributions has been studied because the formulation of the distribution functions can be very complicated. The expected value is essential information for a probability distribution, but naturally, the distribution itself provides more information about the random variable than the mean, which is only one single real number. In this section, instead of giving complicated formulas for distributions, the Monte-Carlo simulation method is applied for investigating some previously applied distributions. The basis of the Monte-Carlo method is the simulation of random numbers [10]. Obviously, an accurate distribution is not equivalent to a random simulation. But, comparing the Monte-Carlo method [12, 13] to the "mean value approach," the Monte-Carlo method provides more information about random variables for practice from another perspective, which can be interesting for the service provider and for customers too.

4.1. Simulation of the waiting time

In section 3. the average waiting time has been investigated. According to the theory, the conditional waiting time in the queue, assuming that the waiting time is positive, is a random variable that follows exponential distribution [5]. If the parameter of the exponential distribution is known, which is the reciprocal of the expected value, in other words, the average waiting time; then the distribution can be simulated using the Monte-Carlo method by the following way.
The cumulative distribution function is given by (2). If a uniformly distributed random number \( r \) is generated in the interval \([0, 1]\[, then the inverse of the equation (2), which is

\[
r = 1 - e^{-\lambda W} \quad \Rightarrow \quad W = \frac{\ln(1-r)}{-\lambda}
\]  

(17)

Which provides an exponentially distributed random variable, where \( W \) is the random waiting time. So, if \( r \in [0, 1] \[, then \( W \) provided by (17) follows an exponential distribution. It must be emphasized that in this formula \( \lambda \) is a general notation for the parameter of exponential distribution and not the arrival rate!!! If the expected value as the mean waiting time is given, then \( \lambda \) is it's reciprocal.

![Figure 14. Exponentially distributed random waiting time, if \( \lambda = 1/10 \ [1/\text{min}] \)](image)

Figure 14. depicts two sequences of experiments. Both plots illustrate 100 random trials on the basis of (17), assuming that, for example, the average waiting time is 10 minutes. The only difference between these plots is that they are two independent sequences of experiments, but the parameter is the same. The horizontal dashed line illustrates the average waiting time. The point is, which is not self-evident for those who don't deal with probability theory, is that the actual waiting time can be greater, even much greater than the average value; considering these simulations, the number of events when the actual waiting time is longer than the average is 33 and 34, and the longest waiting time is 45 and 70 minutes respectively.

The first result is "natural", because the accurate probability of the event, that the waiting time is longer than the average is the following:

\[
P\left( W > \frac{1}{\lambda} \right) = 1 - F\left( \frac{1}{\lambda} \right) = 1 - \left( 1 - e^{-\frac{1}{\lambda}} \right) = e^{-1} \approx 0.3678
\]  

(18)

Multiplying this probability by the number of trials, the result is absolutely consistent with the theory. But the second result, that the real waiting time can be up to 5 or 6 times the average waiting time, or even more, is also a remarkable property of the exponential distribution. Consequently, it must be underlined, that the average waiting time as an expected value, provides a fundamental characterization of the random variable. But, for customers in a queuing system it should be clear, that the actual waiting time can be much greater sometimes, and the factor can be up to 6 or even higher.
4.2. Simulation of the total number of vehicles in the charging system

In subsection 4.1. a simple exponential distribution has been simulated by random numbers, using the formula (17). This simple method can be used as a basis for studying a more complex system, the charging station itself by random numbers. Assuming that the number of arrivals follow a Poisson distribution, the interarrival time follows an exponential distribution, with parameter $\lambda$ (arrival rate), and that the service time also follows an exponential distribution, with parameter $\mu$ (service rate), for every server; the evolution of the number of vehicles can be simulated by the Monte-Carlo method.

The applied procedure is the following: roughly speaking, $s + 1$ exponential distributions must be simulated and must be combined. More precisely, considering $\lambda$ (arrival rate) for arrivals, and taking into account every server, considering $\mu$ (service rate) for the departures, using formulas

$$ ta = \frac{\ln(1-r)}{-\lambda}; \quad ts_k = \frac{\ln(1-r)}{-\mu}; \quad k = 1, 2, 3, ..., s; $$

arrivals and departures are simulated for every server at the same time, simultaneously. If one vehicle arrives, a new interarrival time is simulated when the following vehicle arrives, and so on. Similarly, if one vehicle is served and departs, a new service time is simulated, when a new vehicle departs, etc., for every server independently and simultaneously. If a new vehicle arrives or a vehicle is served, the total number of cars are calculated, and if there are free servers, the simulation of service times for those servers is suspended, etc. This is how the evolution of the total number of vehicles at the charging station can be simulated by the Monte-Carlo method.

For illustration three cases have been selected. The evolution of the number of the vehicles is illustrated if $\rho < 1$, $\rho = 1$ and if $\rho > 1$ respectively, where $\rho$ is the utilization factor (12). As typical parameters, $s = 10$ and $ts = 120$ min have been chosen, and several interarrival times ($ta$). For possible further study, it is assumed that the capacity of the station is twice the number of servers.

Figure 15. illustrates the evolution of vehicles at a charging station if $ta = 15$ min, therefore if $\rho = 0.8 < 1$. The initial number of vehicles is $N_0 = s$. The evolution is clear, one step up means one arrival, and one step down means one served vehicle. The growth can be at most one, due to the property of the Poisson distribution [10, 11], but the decrease can be greater than one because there are $s$ servers, as is presented in Figures 15-17.

![Figure 15. Evolution of the number of vehicles if $\rho = 0.8$ and $N_0 = s$](image-url)
Figure 16. reflects the evolution of vehicles at a charging station if $ta = 12$ min, therefore if $\rho = 1$. The initial number of vehicles is also $N_0 = s$.

Finally Figure 17. illustrates the evolution of vehicles at a charging station if $ta = 6$ min, therefore if $\rho = 2 > 1$. In this case the initial number of vehicles is $N_0 = 0$.

These figures demonstrate typical trends and clearly describe the importance and meaning of the utilization factor. The advantage of the Monte-Carlo method is that the random simulation provides more information about the behavior of the system, the evolution of the number of vehicles at the charging station, than a simple expected value. For example, considering the assumption, that the capacity $K/s = 2$, or any other ratio, the obtained plots help study the problem of the necessary/"optimal"/desirable number of parking slots, and help reduce the probability of rejection.

5. Conclusions

In the light of what has been presented in previous sections, it is demonstrated that the queuing theory, especially the finite queue model and the Monte-Carlo simulation method are suitable and efficient tools for describing the characteristics of a charging station. Results, presented in sections 3.
and 4. help investigate the operation of existing charging stations and help plan and construct a new station, that meets the expectations and needs of customers and providers.

Considering system parameters such as the number of servers, the charging time, and the capacity of the station; it has been presented that these parameters affect the operation and efficiency of the charging station. The quantitative relationship can be estimated by the obtained graphs. The key features of the station, such as the length of the queue, the waiting time, and the probability of rejection of a vehicle are essentially affected by system parameters, which are especially true for the number of servers and the charging time. The nature of the effect of capacity is different because as it was presented, the monotonicity of the probability-capacity function is not unique, it depends on more parameters.

The obtained functions are only depicted by plots, but for users, these curves are enough for studying properties in question, and for numerical estimations.

For example, summarizing some results from section 3, it is obvious, that for improving the efficiency of a charging station an increasing the number of chargers are required, while decreasing the charging time. But, depending on the actual/general/average interarrival time, that comes from experience, there is a reasonable upper limit for the number of chargers, and a reasonable lower limit for the length of charging time, because beyond specific limits, key features don't change significantly. The decision depends on the user and the provider.

The Monte-Carlo simulation extends this picture with some more data. This procedure provides information not only about the expected values but about the behavior of random variables, such as the exponential and Poisson distribution, and what is more important; the overall behavior of the charging station, such as the evolution of the numbers of vehicles in the system. This study also helps estimate the necessary capacity of a station and reduce the probability of rejection. Furthermore, it points out the role and importance of the utilization factor.

The Author’s proposed prospective future research direction is the study of applicability of discrete time Markov chains, for investigating the long-term behavior of the distribution of electric vehicles between home and public charging stations.

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