Cosmo MSW effect for mass varying neutrinos

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Abstract

We consider neutrinos with varying masses which arise in scenarios relating neutrino masses to the dark energy density in the universe. We point out that the neutrino mass variation can lead to level crossing and thus a cosmo MSW effect, having dramatic consequences for the flavor ratio of astrophysical neutrinos.

Two of the most mysterious puzzles in particle physics and cosmology concern the mechanism generating small neutrino masses and the origin of the dark energy in the universe. There exists, however, an amazing coincidence between the order of magnitude of neutrino masses, $m_\nu < 1 \text{ eV}$ and the vacuum density being responsible for the dark energy, $\rho_V \approx (10^{-3} \text{ eV})^4$. In ref. \cite{1} (compare also \cite{2}) this coincidence was explained by generating sterile neutrino masses of order $10^{-3} \text{ eV}$ via interactions with a scalar field, the “acceleron”. Recently a more elaborated scenario \cite{3} was published, where the sterile neutrino mass variation is transmitted to the active sector in a seesaw framework, when integrating out the singlets under the Standard Model gauge group. This scenario conserves the ratio of the neutrino and dark energy contributions to the total energy density of the universe and implies the sterile neutrino mass term $M_s$ to decrease with the cosmic evolution, while the effective active neutrino masses $m_\nu$ vary like the inverse neutrino density,

\begin{align}
    m_\nu(z) &= (1 + z)^{3w} m_{\nu0} \rightarrow (1 + z)^{-3} m_{\nu0}, \\
    M_s(z) &\propto m_\nu^{-1}(z) \rightarrow (1 + z)^3 M_{s0}.
\end{align}

Here $z$ is the cosmological redshift, $w \approx -1$ defines the equation of state of the dark energy, and the neutrinos are assumed to propagate in a non-relativistic background.

Several effects to test this hypothesis have been discussed in \cite{3}, including observation of sterile neutrino states being in conflict with big bang nucleosynthesis, conflicts of terrestrial...
and astronomical neutrino mass measurements, as well as the predicted relation of the dark energy equation of state and the cosmological neutrino mass. Implications for baryogenesis via leptogenesis in mass varying neutrino scenarios have been discussed in [4]. For more recent work see also [5]. Here we focus on a spectacular effect which seems to be overlooked so far, namely the possibility of an MSW effect for cosmological neutrinos in vacuo, which is possible if the variation of neutrino masses leads to level crossing of the associated mass eigenstates (for a similar discussion in a different context, see [6]). Such level-crossings appear naturally in a large class of models. An obvious possibility is to assume active states with constant masses being only weakly coupled to mass varying sterile states, as would be the case in [1]. Another possibility is a seesaw framework as in [3], with some of the singlet states being light and only weakly mixed with the active states, leading to a mass matrix of the kind

\[
M \sim \begin{pmatrix} 0 & m_D & \epsilon \\ m_D & M_s & 0 \\ \epsilon & 0 & m_s \end{pmatrix},
\]

with \( \epsilon \ll m_D \lesssim m_s \ll M_s \), in the early universe. Here \( m_D \) denotes the Dirac mass and \( m_s \), \( M_s \) are Majorana masses of the light and heavy singlets, respectively. Integrating out \( M_s \) yields a mass matrix

\[
M \sim \begin{pmatrix} -m_D^2/M_s & \epsilon \\ \epsilon & m_s \end{pmatrix}.
\]

Assuming now the masses of the sterile states to decrease, i.e. \( m_s, M_s \to 0 \) while keeping \( m_s/M_s \) constant, the mass eigenstates experience active-sterile level crossing at a resonance point with \( -m_D^2/M_s = m_s \). This scheme is motivated by assuming the Majorana masses to be generated by couplings to the same singlet acceleron field, while the Dirac masses originate from couplings to the Standard Model Higgs. The evolution of all three mass eigenstates \( \tilde{m}_i \) with \( M_s \) in such a scenario is illustrated in Fig. 1. Further possibilities to generate level crossing involve more complicated flavor structures, allowing the individual flavors to evolve with different time-dependence.

The two illustrative examples discussed above can both be described with one sterile neutrino flavor state \( \nu_s \) possessing a mass term varying according to (2) and an active state \( \nu_a \) with a constant mass 1. Generalization to different scenarios is straightforward, since a flavor-blind potential doesn’t enter the effective mixing angles and oscillation probabilities, and thus each logical possibility can be reduced to the chosen option.

1In the seesaw case the varying sterile mass dominates the mass squared difference to the active state.
Figure 1: Level crossing in the seesaw scheme for mass varying neutrinos – schematically. Shown is the evolution of the three mass eigenstates $\tilde{m}_i$ in a scenario described by the mass matrix (3).

The evolution equation in flavor space reads [7] [8]

$$i \frac{d}{dt} \begin{pmatrix} \nu_a(t) \\ \nu_s(t) \end{pmatrix} = \tilde{H} \begin{pmatrix} \nu_a(t) \\ \nu_s(t) \end{pmatrix},$$

(5)

where

$$\tilde{H} = E + \frac{m_1^2 + m_2^2}{4E} + \begin{pmatrix} \frac{\delta m^2}{4E} \cos 2\theta + \frac{\delta m_{\text{cosm}}^2}{4E} & \frac{\delta m_{\text{cosm}}^2}{4E} \\ \frac{\delta m_{\text{cosm}}^2}{4E} & \frac{\delta m^2}{4E} \sin 2\theta \end{pmatrix}.$$

(6)

Here $m_i$ and $E$ denote the neutrino mass eigenstates and energy, respectively, and the mixing angle

$$\theta = \frac{1}{2} \arcsin \left( \frac{\epsilon}{m_1 - m_2} \right)$$

(7)

parametrizes the $z = 0$ neutrino mixing matrix.

The mass squared differences as measured in present experiments and due to the cosmological mass variation [2] are $\delta m^2 = |m_2^2 - m_1^2|$ and

$$\delta m_{\text{cosm}}^2 = m_{\alpha 0}^2 \left[ (1 + z)^6 - 1 \right],$$

(8)

respectively, assuming $w = -1$. The Hamiltonian $\tilde{H}$ is diagonalized by the effective mixing angle

$$\tan 2\tilde{\theta} = \frac{\delta m^2 \sin 2\theta}{\delta m^2 \cos 2\theta - \delta m_{\text{cosm}}^2}.$$  

(9)
The mass eigenvalues are given by

\[ E_\alpha = E + \frac{\tilde{m}_\alpha^2}{2E}, \]

(10)

where

\[ \tilde{m}_{1,2} = \frac{1}{2} \left[ (m_1^2 + m_2^2 + \delta m_\text{cosm}^2) \mp \sqrt{(\delta m^2 \cos 2\theta - \delta m_\text{cosm}^2)^2 + \delta m^2 \sin^2 2\theta} \right], \]

(11)

and the resonance occurs for

\[ \delta m_\text{cosm}^2 = \delta m^2 \cos 2\theta. \]

(12)

For an adiabatic transition, the adiabaticity parameter, evaluated at the resonance, has to be large,

\[ \gamma = \frac{\delta m^2 \sin^2 2\theta}{E \cos 2\theta} \left| \frac{H_0}{\delta m_\text{cosm}^2} \frac{d(f(z))}{dz} \right|^{-1} \gg 1, \]

(13)

where the Hubble relation, \( z = H_0 f(z) d \) with

\[ f(z) = \sqrt{\Omega_M(1 + z)^3 + \Omega_\Lambda} \]

(14)

for a flat universe has been assumed \cite{9}. The cosmological parameters are given by \( H_0 = 70 \text{ km/(s Mpc)}, \Omega_\Lambda = 0.73 \) and \( \Omega_M = 0.27 \) (see, e.g. \cite{10}), so that \( f(z) \approx 1 \) for small \( z \ll 1 \) and \( f(z) \approx 0.3 z^{3/2} \) for large \( z \gg 1 \). Since \( H_0 \approx 10^{-33} \text{ eV} \), adiabaticity is easily fulfilled, even for PeV neutrinos. In this case, the Landau-Zener-Stückelberg probability \( P^{\text{LZS}} = \exp \left( -\frac{\pi}{4} \gamma \right) \) vanishes, and the oscillation probability is given by

\[ P(\nu_a \rightarrow \nu_s) = \frac{1}{2} (1 - \cos 2\theta \cos 2\hat{\theta}). \]

(15)

It is interesting to note, that, contrary to the common MSW effect, the cosmo MSW effect depends via (8) on both \( z \) and \( m_{\nu_0} \), and thus exhibits information on absolute neutrino masses.

In the following we calculate the oscillation probabilities for neutrinos from distant astrophysical sources. For this purpose we generalize the two-neutrino framework to a 3+1 generation framework, by assuming \( m_{D}^2/M_s \) to be a 3 \times 3 matrix. The mixing \( \sin \theta \sim 0.1 \) and mass squared difference \( \delta m^2 \sim 0.1 \text{ eV}^2 \) are chosen, as can be assumed in 3+2 models \cite{11} in order to accomodate the LSND result \cite{12}, and \( m_{\nu_0} \approx 0.1 \text{ eV} \) is assumed. In this case the active states are degenerated with masses at a scale \( \sim \sqrt{m_{\nu_0}^2 + \delta m^2} \sim 0.45 \text{ eV}^2 \), in accordance with the recently claimed evidence for neutrinoless double beta decay \cite{13}, and the heavy mass \( M_{s0} \approx 20 \text{ eV} \) could play the role of the fifth state \cite{11}.

\footnote{Note, that cosmological neutrino mass bounds do not apply in the mass varying neutrino scenario.}
While local neutrino sources such as a supernova in the Large Magellanic Cloud (LMC) are not significantly affected by cosmological neutrino mass variation, neutrino telescopes may be sensitive to neutrinos from active galactic nuclei (AGN’s) at distances of 1000 Mpc \((z = 0.3)\) and energies of a PeV \([14]\). The corresponding effect on the absolute neutrino mass is

\[
\left( \frac{m_s(z)}{m_{s0}} \right)^2 = 4.8 \quad \text{(AGN)} \tag{16}
\]

and we obtain

\[
P(\nu_a \rightarrow \nu_s) = 0.96 \quad \text{(AGN)}. \tag{17}
\]

It is obvious that the flavor ratios of astrophysical neutrino fluxes obtained may significantly deviate from the expected \([15]\) \(\nu_e : \nu_\mu : \nu_\tau : \nu_s \) ratio of 1:1:1:0 after decoherence of flavor into mass eigenstates. For normal hierarchical neutrinos after the first level crossing the information about the \(\tau\) neutrino flux is lost and the initial flavor spectrum 1 : 2 : 0 : 0 is transformed resonantly into the characteristic 1 : \(\frac{1}{2} : \frac{1}{2} : 1\). Such a flavor ratio corresponds to a muon to shower ratio of about 5 in next generation neutrino telescopes such as IceCube \([17]\). For inverse hierarchical neutrinos, the electron neutrinos become sterile and disappear, and the resulting flavor ratio is 0.3:0.85:0.85:1, corresponding to a muon to shower ratio of 20 \([17]\). These results do not depend on whether the resonance was reached before or after decoherence. While the standard MSW resonance depends on the beam energy via the adiabaticity condition \([13]\), the effect here is essentially energy independent due to the enhancement of the adiabaticity parameter \(\gamma\) by \(H_0^{-1}\).

As has been mentioned above, local astrophysical sources such as SN87A in the LMC would not exhibit this effect. Such a distance dependent characteristic flavor composition would provide a strong evidence for neutrino mass variation. It also could be a unique possibility to study the parameters triggering the neutrino mass evolution, such as the acceleron potential and the relic neutrino density. It should be stressed, though, that a clear signature of this effect can be spoiled by neutrino overdensities at the source and, in non-standard neutrino scenarios, also in the galactic neighborhood, which fake the cosmological level crossing. However, since typical neutrino source candidates such as AGN’s bear large neutrino densities and the neutrino propagation in these backgrounds occurs on small time scales, while the neutrinos are extremely high energetic, the adiabaticity condition,

\[
\gamma = \frac{\delta m^2 \sin^2 2\theta}{E \cos 2\theta} \left| \frac{d(\ln \delta m^2_{\text{cosm}})}{dt} \right|^{-1}_{t_{\text{res}}} \gg 1, \tag{18}
\]

\(^3\)These flavor ratios have also been derived in \([10]\).
is not necessarily fulfilled. A conservative estimate assuming $O(\Delta \ln \delta m^2_{\text{cosm}}) \simeq 1$ and $\Delta t$ to be of the size of the Schwarzschild radius of a $10^9$ solar mass black hole, $r_s = 3 \cdot 10^{12}$ m, results in $1 < \gamma \simeq 40$ for $\delta m^2_{\text{atm}} = 2.6 \cdot 10^{-3}$ eV$^2 < \delta m^2 < 0.1$ eV$^2$ and PeV neutrino energies. Thus at least for large neutrino energies in the multi-PeV region the process is not adiabatic so that the resulting neutrino spectra are clearly distinguishable from level crossing due to the cosmological mass shift implied by the relic neutrino density. The non-adiabaticity is even stronger for a Gamma Ray Burst source, where the time variability gives an estimate of the source size of about 1 light-second $\simeq 10^9$ m. Since these subtleties depend strongly on the astrophysical source, on assumptions about neutrino densities and on the scenario for mass variation chosen, a detailed discussion is beyond the scope of this work. It should be kept in mind though, as a possible caveat.

In conclusion we discussed the effect of level-crossing in a mass-varying neutrino scenario. The resulting MSW effect in vacuo is the same for neutrinos and antineutrinos, unlike the common MSW effect in matter. It can significantly distort the flavor ratios of neutrino fluxes emitted in active galactic nuclei, predicting characteristic muon to shower ratios in next generation neutrino telescopes, which depend on the distance to the source.

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