Anomalous Radiative Decay of Heavy Higgs Boson

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The radiative decay width of a heavy Higgs boson $H \rightarrow W^+W^-\gamma$ for a hard photon is calculated in the Standard Model and its extension with anomalous $\gamma WW$ couplings. Its dependence on the Higgs mass, the two unknown anomalous couplings, and the photon energy cutoff are studied in detail. We show that this radiative decay of a heavy Higgs is not very sensitive to a wide range of the anomalous couplings compared to the Standard Model result.

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I. INTRODUCTION

If the Standard Model (SM) scalar Higgs boson is heavier than twice the W- or Z-boson masses, it will decay predominantly into the two gauge bosons. Hunting for such a heavy Higgs is one of the primary goals at the future hadron colliders like SSC or LHC [1]. The decay rate for \( H \to W^+W^- \) is given by

\[
\Gamma(H \to W^+W^-) = \frac{\alpha h}{16 \sin^2 \theta_w} \left(1 - \frac{r^2}{2}\right)^{3/2} (3r^4 - 4r^2 + 4) ,
\]

where \( r = 2w/h \) (\( w \) and \( h \) denote the W-boson and Higgs masses respectively). This partial width increases monotonically with the Higgs mass and eventually violates the unitarity bound – this indicates the heavy Higgs boson couples strongly with the longitudinal component of the gauge boson. Similar feature holds for \( H \to ZZ \). The radiative decay \( H \to W^+W^-\gamma \) for a hard photon with energy \( E_\gamma \geq 5 \) GeV has also been considered in the SM [2]. Despite the branching ratio

\[
R_{\text{hard}} = \frac{\Gamma(H \to W^+W^-\gamma)}{\Gamma(H \to W^+W^-)}
\]

is only about several percent, it grows with the Higgs mass. (We note that the multi-soft photons contribution of this process as well as the full SM one-loop electroweak corrections to \( H \to W^+W^- \) have been thoroughly studied recently in [3].) In this paper we extend this previous work of [2] by including the anomalous \( \gamma WW \) couplings which are allowed by the discrete T, C, and P invariance and consistent with electromagnetism. These new contributions to the \( \gamma WW \) vertex can be induced at the one-loop level in SM or its various extension (for example, the two-Higgs-doublets model [4]). Thus the radiative decay mode can be used to probe either the SM at the quantum level or new physics (for example compositeness, supersymmetry, extended Higgs sector, ...) or both! In the next section, we present the matrix element of the process \( H \to W^+W^-\gamma \). We then study the Higgs mass dependence of the branching ratio \( R_{\text{hard}} \) for a hard photon in a wide class of model parametrized by the anomalous couplings in section 3. Analytic formulas for the decay rate are relegated to an Appendix.
II. DECAY RATE OF $\Gamma(H \rightarrow W^+W^-\gamma)$

The general $\gamma W W$ couplings that are allowed by electromagnetism and the discrete T, C, and P invariance have been written down in [5],

$$\mathcal{L}_{\gamma W W} = -ie \left[ (W_{\mu
u}^\dagger W_{\mu}^\nu A_\nu - W_{\mu}^\dagger A_\mu W_{\nu\nu}^\mu) + \kappa W_{\mu}^\dagger W_{\mu}^\nu F^\mu\nu + \frac{\lambda}{w^2} W_{\lambda\mu}^\dagger W_{\mu}^{\nu\nu} F_{\nu\lambda} \right].$$  (3)

The anomalous couplings $\kappa$ and $\lambda$ are related to the magnetic moment $\mu_W$ and the electric quadrupole moment $Q_W$ of the W-boson defined by

$$\mu_W = \frac{e}{2w}(1 + \kappa + \lambda), \quad Q_W = -\frac{e}{w^2}(\kappa - \lambda).$$  (4)

In SM, $\delta\kappa \equiv \kappa - 1 = 0$ and $\lambda = 0$ at tree level. There are two Feynman diagrams contribute to the process $H \rightarrow W^+(k_1)W^-(k_2)\gamma(q)$, since the photon can couple either to $W^+$ or $W^-$. The decay rate can be calculated readily and is given by (we follow some of the notations of Ref. [2])

$$\Gamma(H \rightarrow W^+W^-\gamma) = \frac{\alpha^2 h}{16\pi \sin^2 \theta_W} \int_y^{1-r^2} dx \int_{x_{min}^+}^{x_{max}^+} dx_+ W,$$  (5)

where $x = \frac{2E_\gamma}{h}$ and $x_+ = \frac{2E_{W^+}}{h}$ are the rescaled energies of the photon and the $W^+$-boson respectively. The integration range of $x_+$ is

$$x_{max,min}^+ = 1 - \frac{x}{2} \pm \frac{x}{2(1-x)} R(x, r), \quad R(x, r) = \sqrt{(1-x)(1-x-r^2)}.$$  (6)

Due to the infrared divergencies associated with emission of soft photons, we cutoff the lower end of the $x$-integral at $y = \frac{2E_{min}^\gamma}{h}$. In terms of these rescaled variables, we have

$$q \cdot k_1 = -\frac{h^2}{2}(1-x-x_+), \quad q \cdot k_2 = \frac{h^2}{2}(1-x_+), \quad k_1 \cdot k_2 = \frac{h^2}{2}(1-x - \frac{r^2}{2}).$$  (7)

The matrix element squared is given by

$$W = W_{SM} + \lambda W_\lambda + \delta\kappa W_{\delta\kappa} + \lambda^2 W_{\lambda^2} + (\delta\kappa)^2 W_{\delta\kappa^2} + \lambda\delta\kappa W_{\lambda\delta\kappa},$$  (8)

where
The above results give us the branching ratio

\[ W_{SM} = +2 + 4 \frac{k_1 \cdot k_2}{w^2} + \left( 2 + \frac{(k_1 \cdot k_2)^2}{w^4} \right) \left[ \frac{2 w^2 k_1 \cdot k_2}{q \cdot k_1 \cdot k_2} - \frac{w^4}{(q \cdot k_1)^2} - \frac{w^4}{(q \cdot k_2)^2} \right] \\
+ \frac{2 q k_2}{q \cdot k_2} \left[ q \cdot k_1 - k_1 \cdot k_2 + \frac{1}{w^2} q \cdot k_2 k_1 \cdot k_2 + \frac{2}{w^2} (k_1 \cdot k_2)^2 \right] \\
+ \frac{2 q k_1}{q \cdot k_2} \left[ q \cdot k_1 - k_1 \cdot k_2 + \frac{1}{w^2} q \cdot k_1 k_2 \cdot k_2 + \frac{2}{w^2} (k_1 \cdot k_2)^2 \right] \\
+ \frac{1}{(q \cdot k_1)^2} \left[ (q \cdot k_2)^2 - 2 q \cdot k_2 k_1 \cdot k_2 \right] \\
+ \frac{1}{(q \cdot k_2)^2} \left[ (q \cdot k_1)^2 - 2 q \cdot k_1 k_1 \cdot k_2 \right] , \tag{9} \]

\[ W_\lambda = +4 + \frac{2 q k_1}{q \cdot k_2} \left[ q \cdot k_1 + q \cdot k_2 \right] + 2 \frac{q k_1}{q \cdot k_2} \left[ 2 + \frac{1}{w^2} (q \cdot k_2 + k_1 \cdot k_2) \right] \\
+ \frac{2 q k_1}{q \cdot k_2} \left[ 2 + \frac{1}{w^2} (q \cdot k_1 + k_1 \cdot k_2) \right] , \tag{10} \]

\[ W_{\delta \kappa} = +8 - \frac{2}{w^2} \left[ q \cdot k_1 + q \cdot k_2 \right] + 2 \left[ \frac{(q \cdot k_1)^2}{(q \cdot k_2)^2} + \frac{(q \cdot k_2)^2}{(q \cdot k_1)^2} \right] \\
+ 2 \frac{q k_1}{q \cdot k_2} \left[ 2 - \frac{1}{w^2} (q \cdot k_2 + k_1 \cdot k_2) \right] + 2 \frac{q k_1}{q \cdot k_2} \left[ 2 - \frac{1}{w^2} (q \cdot k_1 + k_1 \cdot k_2) \right] , \tag{11} \]

\[ W_{\lambda^2, \delta \kappa} = +\frac{3}{2} + \frac{(k_1 \cdot k_2)^2}{w^4} + \frac{2}{w^2} \left( 1 - \frac{k_1 \cdot k_2}{2w^2} \right) \left[ q \cdot k_1 + q \cdot k_2 \right] \\
+ \left( 1 - \frac{k_1 \cdot k_2}{2w^2} \right) \left[ \frac{q k_2}{q \cdot k_2} + \frac{q k_1}{q \cdot k_1} \right] + \frac{1}{4} \left[ \frac{(q \cdot k_1)^2}{(q \cdot k_2)^2} + \frac{(q \cdot k_2)^2}{(q \cdot k_1)^2} \right] \\
+ \frac{1}{w^2} \left[ (q \cdot k_2)^2 \frac{q \cdot k_1}{q \cdot k_1} + (q \cdot k_1)^2 \frac{q \cdot k_2}{q \cdot k_2} \right] + \frac{1}{2w^2} \left[ (q \cdot k_1)^2 + (q \cdot k_2)^2 \right] \tag{12} , \]

and

\[ W_{\lambda \delta \kappa} = W_{\lambda^2} + W_{\delta \kappa^2} - \frac{2}{w^4} \left[ (q \cdot k_1)^2 + (q \cdot k_2)^2 \right] . \tag{13} \]

The SM result of \( W_{SM} \) agrees with Ref. [2]. Our calculation was performed in the unitary gauge. Noteworthy, for the process that we are interested in, the \( \lambda \)-term of the anomalous couplings in Eq. (3) do not contribute to the longitudinal piece of the W-boson propagator.

The above results give us the branching ratio

\[ R_{\text{hard}} = \frac{\alpha}{\pi} r^2 (1 - r^2)^{-\frac{1}{2}} (3r^4 - 4r^2 + 4)^{-1} \int_y^{1-r^2} dx \int_{x_{\text{min}}}^{x_{\text{max}}} dx_{\pm} W . \tag{14} \]

All the double integrals in Eq. (14) can be done analytically. The final formulas are tedious and not illuminative, we therefore relegate them to the Appendix.
III. DISCUSSIONS

Previously, a very weak experimental limit on $\kappa$ ($-73.5 \leq \kappa \leq 37$ with 90 % CL) has been derived from PEP and PETRA by studying the process $e^+e^- \rightarrow \gamma\nu\bar{\nu}$. Recently, more stringent limits of $-3.5 < \kappa < 5.9$ (for $\lambda = 0$) and $-3.6 < \lambda < 3.5$ (for $\kappa = 0$) with 95 % CL were obtained from the study of the process $\bar{p}p \rightarrow e\nu\gamma + X$ by the UA2 Collaboration. These limits are of course agree well with the SM tree level prediction. Nevertheless, they are expected to be improved considerably at the TEVATRON in the near future. More accurate measurements on the anomalous couplings $|\delta\kappa|$ and $|\lambda|$ at the level of $\sim 0.1 - 0.2$ are expected at LEP II by studying the process $e^+e^- \rightarrow W^+W^-$ with $95\%$ CL. Also, several recent studies of the process $e^\pm p \rightarrow \nu\gamma + X$ conclude a somewhat less sensitivity of the anomalous couplings at HERA.

Without referring to any particular values for the anomalous couplings in any specific models, we are free to vary their magnitudes that are consistent with the present UA2 experimental constraints. In Figures (1a) and (1b), we plot the ratio $R_{hard}$ as function of the Higgs mass with a photon energy cutoff $E_{\gamma}^{min} = 10$ GeV for $(\delta\kappa, \lambda) = (\pm 0.5, \pm 0.5)$ and $(\pm 1, \pm 1)$ respectively. The SM contribution is also presented for comparison. One can see that in SM the branching ratio is less than 6 percent for the entire range of the Higgs mass that we are interested in (from 200 GeV to 1 TeV). The anomalous contributions are not significant unless the magnitude of the anomalous couplings $\delta\kappa$ and $\lambda$ are significantly larger than 1. $R_{hard}$ is always less than 10 % for the values of the anomalous couplings chosen in Figure 1. For somewhat larger anomalous couplings, say $(\delta\kappa, \lambda) = (2.5, 2.5)$, we find that $R_{hard}$ can be as large as 7 and 24 % for a 500 GeV and 1 TeV Higgs respectively using the same photon energy cutoff. As is evident in Figure 1, destructive effects occur mainly for a positive $\delta\kappa$ and a negative $\lambda$. In other cases, the anomalous contributions tend to have constructive interference with the SM result as the Higgs mass grows heavier. Increasing (decreasing) the photon energy cutoff tends to decrease (increase) the branching ratio. We also see that $R_{hard}$ increases monotonically with the Higgs mass when all the other
parameters are held fixed. For \((\delta \kappa, \lambda) = (0.5, 0.5)\) and \((1,1)\), \(R_{\text{hard}}\) approaches to 100 % as the Higgs mass becomes 10 and 5 TeV respectively. On the other hand, \(R_{\text{hard}}\) climbs up to about 22 % for a 10 TeV Higgs in the SM with \((\delta \kappa, \lambda) = (0, 0)\). At any rate, perturbative calculation is no longer trustworthy for such a heavy Higgs.

To conclude, we have studied in detail the radiative decay mode of a heavy Higgs \(H \rightarrow W^+W^-\gamma\) for a hard photon in a wide class of model (including the SM) parametrized by the anomalous couplings \(\kappa\) and \(\lambda\). The SM prediction for the branching ratio \(R_{\text{hard}}\) is only a few percent and the anomalous contributions tend to increase its value somewhat but never exceeds 10 percent unless the magnitudes of the anomalous couplings turn out to be much larger than unity or the Higgs boson becomes ultra-heavy.
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In this Appendix, we collect the analytic results for all the integrals defined in the branching ratio $R_{\text{hard}}$. One can split the total contribution into the SM piece and an extra piece arise from the anomalous couplings,

$$R_{\text{hard}} = \frac{2\alpha}{\pi} (1 - r^2)^{-\frac{1}{2}} [C_{SM} + \delta C].$$  

(15)

The SM contribution is given by \cite{10}

$$C_{SM} = (1 - \frac{r^2}{2})A_1 - A_2 - B_1 + 8(3r^4 - 4r^2 + 4)^{-1}B_5,$$  

(16)

and the anomalous piece is

$$\delta C = (3r^4 - 4r^2 + 4)^{-1} \left\{ 2\lambda [2(A_3 - B_2 + B_4) + r^2A_3] - 2\delta \kappa [2(A_3 - B_2 + B_4 - 2B_5) - r^2A_3] \\
+ \frac{\lambda^2}{6r^2} [4(B_2 - B_4 + 2B_5 - 7B_6) + r^2(B_2 - B_3 - 2B_4 - B_5 - 6A_3 + 18A_4) + 6r^4A_3] \\
+ \frac{(\delta \kappa)^2}{6r^2} [4(B_2 - B_4 + 2B_5 - 6B_6) + r^2(B_2 - B_3 - 2B_4 - B_5 - 6A_3 - 6A_4) + 6r^4A_3] \\
- \frac{\delta \kappa}{3r^2} [4(B_2 - B_4 - 4B_5 + 2B_6) + r^2(B_2 - B_3 - 2B_4 - B_5 + 6A_3 - 6A_4) - 6r^4A_3]\right\}. 

(17)

$A_i(i = 1 \text{ to } 4)$ and $B_i(i = 1 \text{ to } 6)$ are the integrals defined by

$$A_{1,2,3,4} = \int_y^{1-r^2} dx \left( \frac{1}{x}, \frac{1}{1-x}, x, x^2 \right) \ln \left[ \frac{1 - x + R(x, r)}{1 - x - R(x, r)} \right],$$  

(18)

$$B_{1,2,3,4,5,6} = \int_y^{1-r^2} dx \left( \frac{1}{x}, \frac{1}{1-x}, \frac{1}{(1-x)^2}, 1, x, x^2 \right) R(x, r),$$  

(19)

where $R(x, r)$ was defined in Eq.(6). Evaluating these integrals are laborious. The final results are
\[A_1 = -\ln^2 2 + \ln y \ln r^2 + \ln^2 \left(\frac{1-y-R(y,r)}{r^2} \right) - 2 \ln 2 \ln \left(\frac{1-y+R(y,r)}{r^2} \right) + 2 \ln (1 + \sqrt{1 - r^2}) \ln \left(\frac{(1-r^2)(1-y) + \sqrt{1-r^2} R(y,r)}{r^2 y} \right) + 2 \ln (1 - \sqrt{1 - r^2}) \ln \left(\frac{(1-r^2)(1-y) - \sqrt{1-r^2} R(y,r)}{r^2 y} \right) + 2 \text{Li}_2 \left(\frac{r^2(1-y) - (1-r^2)(1+y+R(y,r))}{r^2(1-y)} \right) + 2 \text{Li}_2 \left(\frac{r^2(1-y) - (1-r^2)(1-y+R(y,r))}{r^2(1-y)} \right) - 2 \text{Li}_2 \left(\frac{-1 + r^2 \sqrt{1-r^2}}{r^2} \right) - 2 \text{Li}_2 \left(\frac{-1 + r^2 \sqrt{1-r^2}}{r^2} \right) + \text{Li}_2(y) + \text{Li}_2(r^2) - 2 \text{Li}_2 \left(\frac{1-y-R(y,r)}{2(1-y)} \right),
\]

\[A_2 = -R(y,r) + (1 - y - \frac{r^2}{2}) L(y, r),
\]

\[A_3 = \frac{1}{8} \left[3r^2 - 8 + 2(1 - y)\right] R(y, r) + \frac{1}{16} \left[r^2(3r^2 - 8) + 8(1 - y^2)\right] L(y, r),
\]

\[A_4 = \frac{1}{72} \left[r^2(44 + 10y - 15r^2) - 4(11 + 5y + 2y^2)\right] R(y, r) - \frac{1}{48} \left[r^2(5r^4 - 18r^2 + 24) - 16(1 - y^2)\right] L(y, r),
\]

\[B_1 = -R(y,r) + \frac{1}{2}(r^2 - 2) L(y, r) + (1 - r^2)^{\frac{1}{2}} L'(y, r),
\]

\[B_2 = R(y,r) - \frac{r^2}{2} L(y, r),
\]

\[B_3 = -\frac{2}{1-y} R(y,r) + L(y, r),
\]

\[B_4 = -\frac{1}{4} \left[r^2 - 2(1-y)\right] R(y, r) - \frac{r^4}{8} L(y, r),
\]

\[B_5 = -\frac{1}{24} \left[r^2(2(2+y) - 3r^2) - 4(1+y(1-2y))\right] R(y, r) + \frac{r^4(r^2-2)}{16} L(y, r),
\]

\[B_6 = -\frac{1}{192} [15r^6 - 2r^4(19 + 5y) + 8r^2(3 + 2y + y^2) - 16(1 + y + y^2 - 3y^3)] R(y, r) - \frac{r^4}{128}(5r^4 - 16r^2 + 16) L(y, r).
\]

In the above equations, we have defined

\[L(y, r) = \ln \left(\frac{2(1-y)-r^2+2R(y,r)}{r^2} \right),
\]

\[L'(y, r) = \ln \left(\frac{2(1-r^2)+(r^2-2)y+2(1-r^2)^{\frac{1}{2}} R(y,r)}{r^2 y} \right).
\]
REFERENCES

[1] See for example, J. Gunion, H. Haber, G. Kane, and S. Dawson, *The Higgs Hunter’s Guide*, Addison-Wesley Publishing Company (1990) and references therein.

[2] D. A. Dicus, S. Willenbrock, T. Imbo, W.–Y. Keung, and T. Rizzo, Phys. Rev. **D34**, 2157 (1986); T. Rizzo, Phys. Rev. **D31**, 2366 (1985).

[3] B. A. Kniehl, Nucl. Phys. **B357**, 439 (1991).

[4] T. Rizzo, Argonne preprint, May (1992).

[5] K. J. F. Gaemers and G. J. Gounaris, Z. Physik C, **1**, 259 (1979); K. Hagiwara, R. D. Peccei, and D. Zeppenfeld, Nucl. Phys. **B282**, 253 (1987).

[6] H. Grotch and R. W. Robinett, Phys. Rev. **D36**, 2153 (1987).

[7] UA2 Collaboration, Phys. Lett. **B277**, 194 (1992).

[8] U. Baur and E. L. Berger, Phys. Rev. **D41**, 1476 (1990).

[9] T. Helbig and H. Spiesberger, Nucl. Phys. **B373**, 73 (1992); U. Baur and M. A. Doncheski, Madison preprint, MAD/PH/692, February (1992).

[10] The result of $C_{SM}$ in Eq.(A2) agrees with Ref.[1]. However, the formula of $C_{SM}$ given in Ref.[1] is only semi-analytic since the integral $A_1$ was evaluated using the mean valued theorem in calculus. We have checked their semi-analytic result agrees well with our analytic result using Eq.(A6) of $A_1$. 
FIGURES

FIG. 1. $R_{\text{hard}}$ as function of Higgs mass with $E_{\gamma}^{\text{min}} = 10$ GeV. (a) $(\delta\kappa, \lambda) = (\pm 0.5, \pm 0.5)$ and (b) $(\delta\kappa, \lambda) = (\pm 1, \pm 1)$. The Standard Model prediction $(\delta\kappa = 0, \lambda = 0)$ is also presented for comparison. We take $w = 80$ GeV and $\alpha = 1/128$. 