Belief Evolution Network: Probability Transformation of Basic Belief Assignment and Fusion Conflict Probability

Qianli Zhou, Yusheng Huang, Yong Deng*

Abstract—We give a new interpretation of basic belief assignment (BBA) transformation into probability distribution, and use directed acyclic network called belief evolution network to describe the causality between the focal elements of a BBA. On this basis, a new probability transformations method called full causality probability transformation is proposed, and this method is superior to all previous method after verification from the process and the result. In addition, using this method combined with disjunctive combination rule, we propose a new probabilistic combination rule called disjunctive transformation combination rule. It has an excellent ability to merge conflicts and an interesting pseudo-Matthew effect, which offer a new idea to information fusion besides the combination rule of Dempster.

Index Terms—Belief evolution network, Full causality function, Full causality probability transformation, Probability fusion, Disjunctive combination rule, Disjunctive transform combination rule

I. INTRODUCTION

As a generalization of probability theory (PT), Dempster-Shafer evidence theory (DSET) was proposed by Dempster [1] based on the mapping of upper and lower probabilities, and then summarized by Shafer [2] into a handling uncertainty theory. DSET expresses the uncertainty through the distribution of mass functions on frame’s power set, so it can be compatible with many methods of modeling uncertainty such as fuzzy sets [3], [4], rough sets [5] and possibility theory [6]. In addition, in terms of information fusion, the combination rule proposed by Dempster can be backward compatible with PT and has the Matthew effect when combining the same evidence, which makes DSET widely used in pattern recognition [7], reliability analysis [8], decision-making [9], clustering [10] and classification [11]. However, DSET still has some drawbacks, in order to make it closer to the reality to handle uncertainty, some DSET-based models and theories are proposed. Smets [12] proposed the transfer belief model (TBM) to reason utilizing belief functions, Smarandache and Dezert [13] proposed the Dezert-Smarandache Theory (DSmT) to handle conflict evidence, Deng [14] removed the assumption that the elements are completely mutually exclusive and proposed the D number theory and Xiao [15], [16] extended DSET to complex numbers to predict the interference effect in decision making.

The goal of all theories expressing uncertainty is to find the required objects under the frame, so transforming the final reasoning result to probability distribution is significant in process of decision-making. In DSET, the uncertain information is expressed by the distribution of mass functions called basic belief assignment (BBA). There are many methods to transform BBA to probability distribution has been proposed [17]–[25], and different transformation methods can be applied to different fields. The most well-known method is the pignistic probability transformation (PPT) proposed by Smets [17], which is applied in the decision-making layer of TBM. Cobb and Shenoy [18] utilized the normalized plausibility function of elements to propose the probability transformation (PTM), which is the only transformation method that satisfies the consistency of combination rule of Dempster (CRD). Sudano [19] redistributed the mass functions of multi-element focal element to generate the probability distribution (PraPl). Dezert and Smarandache [20] proposed the most optimistic probability transformation based on DSmT (DSmP). Cuzzing was committed to study the visual DSET [21], and proposed probability transformation from the perspective of graphs (CuzzP) [21]. Facing these methods, Dezert et al. [27] proposed 3 requirements of probability transformation and Han et al. [28] extended them to establish evaluation methods.
to fit different applications. However, the previous evaluation methods are result-oriented, which may cause uneven using of known information in order to achieve certain results. Probability transformation of BBA is reduction the dimension of the data, which definitely causes information loss. From the perspective of decision-making, smaller Shannon entropy of transformed probability means more easier in decision, but from the perspective of discord and non-specificity, transformation should loss the information of non-specificity reasonably.

Information fusion always be a significant research theme in DSET. CRD as the most widely applied method of information fusion satisfies the commutativity and associativity, but when it combines the conflict evidence, it will distribute the belief to object which has no evidence support. Aiming this counter-intuitive result, the improvements of CRD can be divided 3 types.

1) **Keeping conflicts in frame:** Yager [30] pointed out that the normalization in CRD leading to the counter-intuitive result, so he distributed the part of conflict to the total ignorance set. Dubois and Prade [31] thought that Yager’s method was too conservative, and they assigned conflicting information to their union, which is called disjunctive combination rule (DCR). Smets [32] assigned conflicting information to the empty set, because he thought the conflicts caused by ignorance in open world. The combination rule proposed by Smets is unnormalized CRD called conjunctive combination rule (CCR). These methods just give an explanation to conflicts, but how to deal with conflict information fusion is still an open issue when decision-making.

2) **Change the frame:** Smarandache and Dezert [13] change the frame of evidence according to DSmT and proposed corresponding combination rule.

3) **Fusion after weighted evidence:** Murphy [33] did not change CRD but chooses to preprocess the conflict information, and use CRD to combine the averaged evidence, which is the most common method to deal with information fusion now. Based on this method, scholars use different methods to obtain the weight coefficients of evidence and combine the weighted evidence by CRD to get satisfactory results [34], [35].

The CRD-based information fusion algorithms also have the characteristic of the Matthew effect. When combining the same information, the information will gradually tend to polarize until the probability of a certain target is 1. So even for a target with a small support, as long as its support is greater than other targets, after a finite number of fusions, its probability will reach 1. In addition combination, decomposition also is an essential topic, Fan et al. [36] proposes an optimized evidence decomposition method.

In this paper, we build a directed acyclic network for a BBA called belief evolution network (BEN), which can describe the causal relationship of different focal elements. According to the structure of BEN, we propose the full causality (FC) function to express uncertainty of BBA in a new view. In addition, we give a new explanation of probability transformation and propose full causality probability transformation (FCPT) based on BEN and FC function. Finally, we combine the new transformation method and DCR to propose a new probability fusion method called disjunctive transform combination rule (DTCR). The contributions can be summarized as follows: (1) A new method to express BBA by causality is proposed, and a directed acyclic network is used to represent the evolution process of belief. (2) According to the proposed network, a new belief function called FC function is defined and a new probability transformation method is given. (3) According to the DCR and new transformation method, a new combination rule of probability distribution is proposed, which can not only manage the conflicts reasonably, but have more intuitive pseudo Matthew effect in reality.

The rest of paper is organized as follows: Section II introduces the basic concepts of DSET and display the common combination rules and probability transformations. Section III proposes the belief evolution network and defines the full confidence function. In Section IV we give a new interpretation of probability transformation and propose the full causality probability transformation. The new probability fusion method is proposed in Section V which utilizes the DSET’s stronger ability to handle uncertainty to manage conflicts rationally. In Section VI we summarize the whole paper and discuss the future research directions of belief evolution network.

**II. Preliminaries**

A. Dempster-Shafer evidence theory

For a finite set $\Theta$ called frame of discernment (FoD) with $n$ elements, DSET utilizes $2^n$ mass functions to form a distribution to express its uncertainty. Compared with the probability distribution, the focal elements in power set $2^\Theta$ are not completely mutually exclusive, which leads to some interesting properties. We introduce DSET from modeling uncertainty and combination rules.
1) Modeling uncertainty:

Definition 2.1 (BBA): The power set of $\Theta$ is $2^\Theta = \{\emptyset, \{\theta_1\}, \ldots, \{\theta_n\}, \{\theta_1, \ldots, \theta_n\}\}$, and the basic belief mass functions (BBM) $m(F_i)$ is mapping from $2^n \rightarrow [0, 1]$. The $m(F_i)$ satisfies

$$\sum_{\{F_i\} \in 2^\Theta} m(F_i) = 1; \quad m(F_i) \in [0, 1]. \quad (1)$$

The distribution composed with BMMs is called basic belief assignment (BBA), and When $m(\emptyset) = 0$, the BBA is normalized.

BBA is the basic form of DSET to express information, but it cannot intuitively reflect the information of the entire focal element, so belief functions are also a common forms to express information.

Definition 2.2 (Belief functions): For a BBA $m(2^\Theta)$ under FoD $\Theta$, the belief (Bel) function, plausibility (Pl) function and commonality ($q$) function are defined as

$$Bel(F_i) = \sum_{G_i \subseteq F_i} m(G_i) = 1 - PL(\overline{F_i}) = \{G_i \neq \emptyset\}, \quad (2)$$

$$Pl(F_i) = \sum_{G_i \cap F_i = \emptyset} m(G_i) = 1 - Bel(\overline{F_i}), \quad (3)$$

$$q(F_i) = \sum_{G_i \supseteq F_i} m(G_i), \quad (4)$$

$$m(F_i) = \sum_{B \supseteq A} (-1)^{|B|-|A|} q(B) \forall F_i \in 2^\Theta, \quad (5)$$

where the $\overline{F_i}$ is the complement of $F_i$ in $\Theta$.

They express the uncertainty of $m(2^\Theta)$ from different perspectives. Bel function $Bel(F_i)$ and Pl function $Pl(F_i)$ express the total support degree and non-negativity degree for elements $\theta \in \{F_i\}$ respectively, and they compose the belief interval $[Bel(F_i), Pl(F_i)]$, whose length can express the non-specificity of BBA in certain extent. Commonality function is the dual of Bel function which express the support degree for whole focal element in BBA. Smets [38] given a method to encode BBA elements into the binary system. For a FoD $\Theta = \{\theta_1, \ldots, \theta_n\}$, the focal element $\{F_i\}$ can be written as an $n$-bit binary number. If $\theta_i \in F_i$, the $i$th binary number equals 1, otherwise, the binary number equals 0. For example, the binary representation of focal element $\{AB\}$ under FoD $X = \{A, B, C\}$ is 011. In addition, Smets generated the vector of BBA based on its order and utilized invertible matrix to transform between above belief functions.

2) Combination rules:

Definition 2.3 (CCR): Under a FoD $\Theta$, there are two BBAs $m_1$ and $m_2$ from different sensors at the same time. The conjunctive combination rule (CCR) $m_1 \otimes m_2$ is defined as

$$m_{1 \otimes 2}(F_i) = \sum_{G_i \cap H_i = F_i} m_1(G_i)m(H_i). \quad (5)$$

Transforming the BBMs to commonality ($q$) functions, the CCR also can be written as:

$$q_{1 \otimes 2}(F_i) = q_1(F_i)q_2(F_i). \quad (6)$$

Normalizing the results of CCR ($m(\emptyset = 0)$) can give the definition of combination rule of Dempster (CRD). Definition 2.4 (CRD): For a FoD $\Theta$, suppose $m_{1\otimes 2}$ is the result of CCR, the combination rule of Dempster (CRD) is defined as

$$m_{1 \oplus 2}(F_i) = \begin{cases} K^{-1} \cdot m_{1 \otimes 2}(F_i) & F_i \neq \emptyset \\ 0 & F_i = \emptyset \end{cases}, \quad (7)$$

where $K = \sum_{G_i \cap H_i = \emptyset} m_1(G_i)m_2(H_i)$ is conflict coefficient, which can express the conflict degree between two BBAs.

Definition 2.5 (DCR): For a FoD $\Theta$, there are two BBAs $m_1$ and $m_2$ from different sensors at the same time. The disjunctive combination rule (DCR) $m_1 \oplus m_2$ is defined as

$$m_{1 \oplus 2}(F_i) = \sum_{G_i \cup H_i = F_i} m_1(G_i)m_2(H_i), \quad (8)$$

which also can be written as the form of belief function,

$$Bel_{1 \oplus 2}(F_i) = Bel_1(F_i)Bel_2(F_i). \quad (9)$$

When appearing conflicts, DCR chooses to assign belief to their union and expresses the meaning of $A$ or $B$. Smets [37] applied it to the conditional belief function and proposed the generalized Bayesian theorem.

B. Probability transformation

1) Common probability transformation method:

Definition 2.6 (PT): Give a BBA $m(2^\Theta)$ under FoD $\Theta = \{\theta_1, \ldots, \theta_n\}$, some probability transformation methods are shown in TABLE 1.

Existing probability transformation methods have their own merits by analyzing from different perspectives. In expressions of TABLE 1, $C_x = \sum_{F_i \subseteq \Theta, |F_i| = 2} m(F_i) \sum_{G_i \in \Theta \setminus F_i} m(G_i)$ indicates the normalization support degree of $\theta_i$ in $F_i$, please refer to [25] for specific calculation method. The expression of common probability transformation methods (except PMT) can be written as
Suppose the entropy and similarity was evaluate. The similarity between BBA and probability indicates that only entropy is not sufficient to evaluate. For a probability distribution, the larger PIC, the more conducive to the content (PIC). For a probability distribution, the result of probability transformation is itself. For two BBAs, that PPT is the transformation method that maintains the largest correlation coefficient. For two BBAs $m_1$ and $m_2$ under FoD $\Theta$, the correlation coefficient is defined as

$$r_{BBA}(m_1, m_2) = \frac{c(m_1, m_2)}{\sqrt{c(m_1, m_1) \cdot c(m_2, m_2)}},$$

where $c(m_1, m_2)$ is

$$c(m_1, m_2) = \sum_{F_i \subseteq \Theta, G_i \subseteq \Theta} m_1(F_i) m_2(G_i) \frac{|F_i \cap G_i|}{|F_i \cup G_i|}.$$  

So the correlation coefficient also can be the similarity index in Bi-Criteria evaluation.

### III. Belief Evolution Network and Full Causality Function

In this Section, we express BBA in a directed acyclic network called belief evolution network (BEN), and use causality to describe the relationship between different focal elements, which provides a new idea for the expression of BBA. In addition, according to BEN, we propose a new belief function, called full causality (CF) function, which can describe the degree of causality of a certain focal element in the entire BBA.

#### A. Belief evolution network

1) The relationship of BBA and probability distribution: In [43], we used a new model to describe the relationship of BBA and probability distribution. For a event described by $n$-element FoD $\Theta$, a timeline can be used to describe the evolution of our understanding. The initial point of timeline represents $m(\Theta) = 1$, which means that we know nothing about FoD. As time goes by, our understanding gradually becomes clear. The end point of timeline is a probability distribution, which means that the development of the event is over, and the distribution of elements is certain. The probability at the end point is not the same as the empirical probability, which is an established fact that

\[
    \text{PIC}(P) = 1 - E_H = 1 + \frac{\sum_{i=1}^{n} p(\theta_i) \log p(\theta_i)}{\log n},
\]

where $E_H$ is the ratio of Shannon entropy. In evaluation process, the larger PIC, the more conducive to the decision-making for transformation result. Han et al. [28] indicates that only entropy is not sufficient to evaluate. The similarity between BBA and probability distribution after transformation should also be an evaluation index, and the more similar transformation, the more reasonable. Based on above, the evaluation method comprehensive the entropy and similarity was proposed. Suppose $n$ probability distributions $P = \{P_1, \ldots, P_n\}$ after transformation, the Bi-Criteria evaluation is defined as

\[
    C_{joint}(P_i) = \alpha \cdot \text{Ent}^t(P_i) + (1 - \alpha) \cdot d'(P_i),
\]

where the $\text{Ent}^t(P_i)$ and $d'(P_i)$ are entropy index and similarity index respectively.
will not change with time under this framework, so it cannot be used as prior knowledge under this FoD. The points on the timeline are BBAs, which represents the development process of the event. Although the points on the timeline exist in form of BBAs, they are observed in the form of probability distributions, which are usually used for fusion, prediction and reasoning. For example, using the FoD $Θ = \{\{\text{Head}\}, \{\text{Tail}\}\}$ to represent the coin tossing, the probability we observe is $p(\{\text{Head}\}) = 0.5, p(\{\text{Tail}\}) = 0.5$, but because the coin tossing is still happening, its actual form should be $m(\{\text{Head}\}) = 0.5(1 − ε), m(\{\text{Tail}\}) = 0.5(1 − ε), m(\{\text{Tail, Head}\}) = ε$, which is consistent with Denœux’s view in [44]. According to the above, BBA is the actual form of existence, and the probability distribution is the form observed by people. Fig. 1 shows their relationship at different moments.

![Fig. 1: The relationship between BBA and probability distribution at same time.](image)

2) Belief evolution network: The combination rules (CRD and CCR) and probability transformation methods (PPT and PMT etc.) both can transform BBAs into probability distributions. But the transformation process is an open issue. In [45], we simulated a possibly PPT transformation process and proposed a probability transformation process model: In the process of probability transformation, a focal element’s belief comes from the focal elements which contain it, and BBMs are allocated to their subsets continuously in a certain proportion. But this model lacks explanation for other transformation methods, so we add Bayesian network idea to this process: This process can be regarded as causal, so we utilize focal elements as nodes to form a new directed acyclic network called belief evolution network (BEN). We stratify the focal elements according to their cardinality, and belief of $n$-element focal elements can only evolve to the $(n − 1)$-element focal element with each step. The belief evolution network under FoD $X = \{A, B, C\}$ is shown in Fig. 2.

Fig. 2: Belief evolution network under FoD $X = \{A, B, C\}$

For a BBA, the BBMs can be marked on the corresponding nodes of BEN, which can describe the current state of the event. So from the perspective of BEN, Model of Fig. 1 can be described as a process in which the belief starts from the $Θ$ and reaches the elements $θ_i$ through different paths. According to the binary encoding of focal elements, above evolution process can also be simulated on a reversible Boolean circuit. The left and right parts in Fig. 3 are the reversible Boolean circuits that evolve the belief of $\{ABC\}$ to $\{A\}$ through two paths, $\{ABC\} \rightarrow \{AB\} \rightarrow \{A\}$ and $\{ABC\} \rightarrow \{AC\} \rightarrow \{A\}$.

Fig. 3: Boolean circuits simulation of BEN.

B. Full causality function

View belief functions based on the structure of BEN. For focal element $\{F_i\}$, $Bel(F_i)$ represents the belief of all potential focal elements which can be evolved from $\{F_i\}$, $Pl(F_i)$ represents the belief of related focal elements with $\{F_i\}$, and $q(F_i)$ represents the belief of focal elements which can evolve to $\{F_i\}$. Based on above, both Bel function and q function can partially express causality, but Pl function only can express correlation. Hence, we define a new belief function, called full causality (FC) function, to express the belief of focal elements which have causal relationship with $\{F_i\}$.

**Definition 3.1 (FC function):** For a BBA $m(2^Θ)$ under FoD $Θ$, the full causality (FC) function are defined as

$$FC(F_i) = \sum_{G_i \subseteq F_i; F_i \subseteq G_i; G_i \neq ∅} m(G_i)$$

When $\{F_i\}$ is equal to $Θ$ and $θ_i$, $FC(F_i)$ is equal to $Bel(F_i)$ and $q(F_i)$ respectively.

FC function can be regarded as the complete set of Bel function and q function. When all focal
IV. FULL CAUSALITY PROBABILITY TRANSFORMATION

A. A new interpretation of probability transformation

According to the above description, people’s events cognition can be simulated on BEN as process of completely unknown (top layer) evolution to precise description (bottom layer). In the process of evolution, BBA can be represented on BEN, which means the cognitive state of people at this moment, but in actual observations, what people get at this moment is a probability distribution. This process from BBA to probability distribution can be interpreted as probability transformation, which utilizes the known information to predict the remaining evolution process on BEN, and transform the belief to bottom layer. Based on above, the generalized probability transformation model (GPTM) is summarized in Fig. 4. Because the belief evolves from multi-element focal elements to few-element focal elements, and the end of the evolution is the probability distribution composed of elements, the GPTM satisfies the requirements of p Consistency and ULB Consistency in [27].

Algorithm 1 Full causality probability transformation

\begin{algorithm}
\caption{Full causality probability transformation}
\begin{algorithmic}[1]
\State \textbf{Input:} $n$-element FoD with BBA $m(2^\Theta)$;
\State \textbf{Output:} The probability distribution $\text{FCP}(\Theta)$;
\State 1: Establish $n$-element BEN;
\State 2: Represent BBMs on BEN nodes;
\State 3: Calculation FC function of $m(2^\Theta)$ $\text{FC}(F_i)$;
\State 4: Order layer $k = 1$;
\State 5: for $k = 1 : n-1$ do
\State 6: for Traverse $k$-layer nodes $\{F_i\}$ do
\State 7: for Traverse $k-1$-layer nodes $\{G_i\}$ do
\State 8: $\text{FC}_{F_i}^{G_i} = \sum_{G_i \subseteq F_i} \text{FC}(G_i)$; \% sum of FC functions of $\{G_i\}$ which can be evolved from $\{F_i\}$ in layer $k-1$.
\State 9: $p_{G_i}^F = \frac{\text{FC}(G_i)}{\text{FC}_{F_i}^{G_i}}$; \% the proportion of $\{F_i\}$ evolution belief to $\{G_i\}$.
\State 10: $m(G_i) = m(G_i) + p_{G_i}^F m(F_i)$; \% renew the belief of $\{G_i\}$.
\State 11: \textbf{end for}
\State 12: \textbf{end for}
\State 13: \textbf{end for}
\State 14: $m(\theta_i) = \text{FCP}(\theta_i)$;
\State 15: \textbf{return} $\text{FCP}(\Theta)$;
\end{algorithmic}
\end{algorithm}

We realize the transformation from BBA to probability distribution by simulating the evolution of belief.

Fig. 4: Generalized probability transformation model

The new interpretation and GPTM are extensions of original proposed probability transformation. In TBM [17], Smets puts pignistic probability transformation at the decision-making layer, which means that when a decision has to be made, BBA is transformed into probability for decision-making. In our model, probability transformation is not only a forced choice when decision-making, but a characterization of BBA under observation. Therefore, our model can be backward compatible with PPT. When all probability distributions $\text{P}^{k-k+1}$ in Fig. 4 are uniform probability distribution, the evolution result is PPT, i.e. we know nothing for future evolution, so uniform distribution representing ignorance is used for prediction. However, ignorance is also a kind of information, which is equivalent to fusion other information to BBA, and the probability distribution after transformation is the most conservative case.
So we use evolution to describe the process, and the result is called transformation. According to Algorithm 1 for n-element FoD, it needs n−1 times updating to transform BBA to probability distribution. Example 4.1 intuitively shows the evolution process in the form of BEN.

**Example 4.1:**
For a 4-element FoD $X = \{A, B, C, D\}$, the corresponding BBA is as follows:

1) 1-element focal element:
   \[
   m(A) = 0.16, m(B) = 0.14, m(C) = 0.01, m(D) = 0.005
   \]
2) 2-element focal element:
   \[
   m(AB) = 0.20, m(AC) = 0.09, m(AD) = 0.04, \\
   m(BC) = 0.04, m(BD) = 0.02, m(CD) = 0.01;
   \]
3) 3-element focal element:
   \[
   m(ABC) = 0.10, m(ABD) = 0.03, \\
   m(ACD) = 0.03, m(BCD) = 0.03;
   \]
4) 4-element focal element:
   \[
   m(ABCD) = 0.08.
   \]

Based on Algorithm 1 the FCP is
\[
\{FCP(A) = 0.4787, FCP(B) = 0.3702, \\
FCP(C) = 0.0985, FCP(D) = 0.0526\},
\]
and the specific evolution process on BEN is shown in Fig. 5.

![Evolution Process](image5)

Fig. 5: The evolution process in Example 4.1

FCPT is the only way to describe probability transformation as a process. We combine the FC function and BEN to simulate the belief evolution, which is superior to all previous methods in this aspect. According to Figure 4, GPTM can also simulate the PPT process. In view of causality, PPT neither uses prior knowledge, nor updates prior knowledge in evolution process, so PPT is not applicable in decision-making, which also be verified in [28] by comparing its PIC value with other methods.

### C. Evaluate the FCPT

In previous part, we interpret the superiority of CFPT from perspective of transformation process, and now we compare CFPT with other methods through result-oriented evaluation methods.

1) Recognition result and PIC value: According to Equ. (17) when decision-making, on the premise that the recognition result is correct, a larger PIC value is more favorable for decision-making.

**Example 4.2:** For a 3-element FoD $X = \{A, B, C\}$ with BBA $m(2^X) = \{m(A) = 0.1, m(AB) = 0.2, m(BC) = 0.3, m(ABC) = 0.4\}$, the results and PIC of transformation methods in Table I BetP, PnPl, PraPl, CuzzP, DSmP, ITP and FCP are shown in Table II

| Methods | $P(A)$ | $P(B)$ | $P(C)$ | Result | PIC($\{P\}$) |
|---------|--------|--------|--------|--------|--------------|
| CuzzP   | 0.3455 | 0.3681 | 0.2864 | $\{B\}$ | 0.0050       |
| PnPl    | 0.3043 | 0.3913 | 0.3043 | $\{B\}$ | 0.0067       |
| BetP    | 0.3333 | 0.3833 | 0.2834 | $\{B\}$ | 0.0068       |
| DSmP$_{1}$ | 0.3591 | 0.3659 | 0.2750 | $\{B\}$ | 0.0073       |
| PraPl   | 0.3739 | 0.3522 | 0.2739 | $\{A\}$ | 0.0077       |
| FCP     | 0.2951 | 0.4688 | 0.2361 | $\{B\}$ | 0.0387       |
| ITP     | 0.4140 | 0.3885 | 0.1975 | $\{A\}$ | 0.0418       |

**TABLE II:** The transformation results of Example 4.2

In Example 4.2 although the PIC value of ITP is the largest, its recognition result is different from most classical methods. This is because it over-utilizes the belief of single-element focal element. In the methods of the recognition result is $\{B\}$, the PIC value of FCP is significantly greater than other methods, which proves its effectiveness in decision-making.

2) Bi-Criteria evaluation: In [28], Han et al. pointed out that it is not comprehensive to evaluate probability transformation methods only by entropy-like methods (PIC), because some methods neglect the correlation between the transformation result and the original BBA because of excessive decision-making tendency. So the bi-criteria evaluation is proposed. According to Equ. (17) and (18) the entropy index can be expressed by PIC values:

\[
PIC'((\{P_i\})) = \frac{\max(PIC) - PIC((\{P_i\}))}{\max(PIC) - \min(PIC)},
\]  

so the bi-criteria evaluation is

\[
C_{\text{joint}}((\{P_i\})) = \alpha \cdot PIC'((\{P_i\})) + (1 - \alpha) \cdot PIC((\{P_i\}))
\]
We expect that the probability after transformation has a larger PIC value and a smaller evidence distance, so a smaller \( C_{\text{joint}} \) value proves the more rational probability transformation method. The BetP, PnPl, CuzzP, DSmP, PraPl and FCP of \( m(2^X) \) in Example 4.1 and their evaluation indexes are shown in Table III.

### Table III: The results of probability transformation methods and their evaluation indexes.

| Methods         | \( P(A) \)  | \( P(B) \)  | \( P(C) \)  | \( P(D) \)  |
|-----------------|--------------|--------------|--------------|--------------|
| \( PnPl \)     | 0.3614       | 0.3168       | 0.1931       | 0.1287       |
| \( CuzzP \)    | 0.3860       | 0.3382       | 0.1607       | 0.1151       |
| \( BetP \)     | 0.3983       | 0.3433       | 0.1533       | 0.1050       |
| \( PraPl \)    | 0.4021       | 0.3523       | 0.1394       | 0.1062       |
| \( DSmP_{0.01} \) | 0.5176       | 0.4051       | 0.0303       | 0.0470       |
| \( DSmP_{0.001} \) | 0.5162       | 0.4043       | 0.0319       | 0.0477       |
| FCP            | 0.4787       | 0.3702       | 0.0985       | 0.0526       |

### Table IV: The correlation coefficients in Example 4.3.

| \( |A| \) | 1   | 2   | 3   | 4   | 5   |
|-------|-----|-----|-----|-----|-----|
| \( FCP \) | 0.9714 | 0.6967 | 0.5853 | 0.5127 | 0.4639 |
| \( BetP \) | 0.9723 | 0.6976 | 0.5903 | 0.5256 | 0.4808 |
| \( CuzzP \) | 0.9712 | 0.6807 | 0.5864 | 0.5239 | 0.4793 |
| \( PnPl \) | 0.8920 | 0.6831 | 0.5870 | 0.5234 | 0.4783 |

### Table V: The PIC values in Example 4.3.

| \( |A| \) | 1   | 2   | 3   | 4   | 5   |
|-------|-----|-----|-----|-----|-----|
| \( FCP \) | 0.4460 | 0.4103 | 0.3768 | 0.3456 | 0.3172 |
| \( BetP \) | 0.4625 | 0.4355 | 0.4129 | 0.3938 | 0.3772 |
| \( CuzzP \) | 0.4608 | 0.4333 | 0.4102 | 0.3903 | 0.3751 |
| \( PnPl \) | 0.4602 | 0.4326 | 0.4092 | 0.3891 | 0.3717 |

Fig. 6: The \( C_{\text{joint}} \) of methods under different \( \alpha \).

After sampling 11 values of \( \alpha \) from 0 to 1, the \( C_{\text{joint}} \) of them are shown in the Fig. 6. The circle represents the corresponding \( C_{\text{joint}} \) under different \( \alpha \), and the triangle represents the average of all the sampled values. When \( \alpha = 0.5 \), \( C_{\text{joint}}(FCP) \) is smallest, and it is obvious that the oscillation amplitude of \( C_{\text{joint}}(FCP) \) is more stable than other methods, which shows that FCP has a good balance between correlation and decision-making advantages. In addition, the average value of \( C_{\text{joint}}(FCP) \) is also the smallest. Therefore, for the BBA in Example 4.1, using bi-criteria evaluation can conclude that FCP is better than other methods.

In Example 4.3, we utilize bi-criteria to evaluate the transformation methods of dynamic BBA qualitatively, which further demonstrates the superiority of our proposed method. The correlation coefficient \( \rho \) proposed by Jiang has been proven to express similarity more reasonable than distance. So we choose correlation coefficient and PIC as bi-criteria to evaluate transformation methods respectively.

Example 4.3: For a 10-element FoD \( \Theta = \{\theta_1, \cdots, \theta_{10}\} \) with BBA

\[
m(2^\Theta) = \{m(\theta_3\theta_4\theta_5) = 0.15, \]

\[
m(\theta_6) = 0.05, m(\Theta) = 0.1, m(A) = 0.7, \]

when \( \{A\} \) change from \( \{\theta_1\}, \{\theta_2\} \) to \( \Theta \), the PIC values and correlation coefficients of CuzzP, PnPl, BetP and FCP are shown in Table IV and V, and the change trends are shown in Fig. 7 and 8.

In Example 4.3, we can find that BetP has the greatest correlation with original BBA, and for other methods, their correlation is very close to BetP. But in terms of PIC values, the results of FCP is obviously greater than other methods. Although these methods are relatively similar, FCP is still the better choice.
close under the similarity index, FCP has absolute advantages in decision-making, especially when the multi-element focal elements’ belief is high.

In this section, we give a probability transformation physical model from a new perspective based on belief evolution network (BEN), and propose a new probability transformation method called full causality probability transformation (FCPT). For the new method, we evaluate it comprehensively through 2 dimensions (transformation process and transformation result). It is proved that the result after transformation by FCPT can not only maintain a high degree of similarity but be conducive to decision making, which is superior to all previous methods. Based on the above advantages, we combine the new method with DCR and propose a new probability fusion method to deal with conflicts in decision-making.

V. APPLICATION OF BENPT: DISJUNCTIVE TRANSFORMATION COMBINATION RULE

Handling conflict has always been an open issue in information fusion. The main way of previous resolve conflicts is to first transform the conflicts into the same information by weight, and then use Matthew effect of CRD to fuse same information. In this section, we utilize the ability of BBA to store more information and the properties of FCPT in probability transformation, combining DCR and FCPT to propose a new probability fusion method called disjunctive transformation combination rule (DTCR).

A. Disjunctive transformation combination rule

According to Equation 2.5, DCR can store conflict information in their union to generate a BBA with higher uncertainty. For two probabilities \( p_1 \) and \( p_2 \), we propose disjunctive transformation combination rule (DTCR) to fuse probability distributions. As shown in Fig. 9, firstly, we generate a BBA to store conflict through DCR, and then transform it into probability distribution through FCPT.

![Diagram](Image)

Fig. 9: The process of disjunctive transformation combination rule

**Definition 5.1 (DTCR):** For an \( n \)-dimensional random variable with probability distributions \( P_1 \) and \( P_2 \), the disjunctive transformation combination rule (DTCR) \( P_1 \oplus P_2 \) can be divided as 2 steps.

1. **Step 1:** Fuse \( P_1 \) and \( P_2 \) by disjunctive combination rule (DCR) to get \( m_{1\oplus 2} = P_1 \ominus P_2 \).
2. **Step 2:** Transform \( m_{1\oplus 2} \) to \( P_{1\oplus 2} \) utilizing full causality probability transformation (FCPT).

DTCR is the same as CRD in terms of mathematical algorithm. It satisfies the commutative law but not the associative law. In terms of Matthew effect and conflict handling, we use Example 5.1 and Example 5.2 to show their differences.

**Example 5.1:** For probability distributions \( P_1 = \{0.9, 0.09, 0.01\} \) and \( P_2 = \{0.01, 0.14, 0.85\} \) received from 2 sensors of objects frame \( X = \{A, B, C\} \), using CRD and DTCR to fuse them respectively.

\[
P_{1\oplus 2} = \{0.2990, 0.4186, 0.2824\}; \quad P_{1\oplus 2} = \{0.5046, 0.0531, 0.4423\}.
\]

**Example 5.2:** Suppose 2 sensors received same probability distributions \( P_1 = P_2 = \{0.5, 0.25, 0.25\} \) under frame \( X = \{A, B, C\} \), using CRD and DTCR to fuse them respectively.

\[
P_{1\oplus 2} = \{0.6667, 0.1667, 0, 1667\}; \quad P_{1\oplus 2} = \{0.5658, 0.2171, 0.2171\}.
\]

When the number of sensors is 15, we suppose that all received information is the same probability distribution \( P = \{p(A), 1-p(A), 1-p(A)\} \). When the support to \( \{A\} \) of the original distribution increases from 0.34 to 1, the \( p(A) \) after each step fusion by CRD and DTCR are shown in Figures 10 and 11.

According to Equation 21, the DTCR is similar to CRD in that it amplifies tendency to object when fusing the same information, which is more convenient to make decisions. But according to Figure 10 and 11, we can find that any support degree to object with tendency will reach 1 after a limited number of CRDs, which is the unique Matthew effect of CRD. For DTCR, fusion the same information having object with tendency also can increase the support to the object with tendency, but it will approach a limit. We
call this property the pseudo-Matthew effect. For the probability distribution in Example 5.2, after limited times fusion by CRD and DTCR respectively, the $p(A)$ are 1 and 0.7016. In actual expert decision-making, for the three outcomes \{A, B, C\}, all experts believe that there is a half probability of outcome \{A\}. This does not allow us to determine that \{A\} is the final outcome, but can magnify the probability of \{A\} to more than half probability. From this perspective, the pseudo-Matthew effect of DTCR is more reasonable than the Matthew effect of CRD. In summary, DTCR produces more reasonable results than CRD in terms of conflict information fusion and the same information fusion.

B. Ablation experiment

According to Fig. 9, the second step of DTCR is probability transformation. In Example 5.3, we replaced FCP with the most classic four methods: BetP, PnPI, DSmP\(_0\) and CuzzP. The results show that only FCP can realize the performance of DTCR.

Example 5.3: We use BetP, PnPl, DSmP\(_0\) and CuzzP to replace FCP in Definition 5.1 and fusion the probability distributions in Example 5.1 and 5.2, the results $P^1$ and $P^2$ are shown in Table VI.

| Methods (A) | FCP | DSmP\(_0\) | BetP | PnPl | CuzzP |
|-------------|-----|------------|------|------|-------|
| $P^1(A)$    | 0.5046 | 0.2990 | 0.4550 | 0.4574 | 0.4550 |
| $P^1(B)$    | 0.0531 | 0.4186 | 0.1550 | 0.1104 | 0.1550 |
| $P^1(C)$    | 0.4423 | 0.2824 | 0.4300 | 0.4323 | 0.4300 |

| $P^2(A)$    | 0.5658 | 0.6667 | 0.5000 | 0.4574 | 0.4615 |
| $P^2(B)$    | 0.2171 | 0.1667 | 0.2500 | 0.1104 | 0.2692 |
| $P^2(C)$    | 0.2171 | 0.1667 | 0.2500 | 0.4323 | 0.2692 |

TABLE VI: The combination results in Example 5.3

In terms of conflict handling, DSmP\(_0\) produces the similar counter-intuitive results as CRD, because it only considers the belief of elements, so the results obtained are the same as CRD. Although the results of the other 3 methods assign the degree of support into \{A\} and \{C\}, in terms of the degree of support to \{B\}, their results are the intermediate value of the two pieces of information ($P^1(B) \in [P_1(B), P_2(B)]$), which shows that they does not reduce the possibility of \{B\} based on the information. So all of them is unreasonable in conflicts fusion. In terms of fusion the same information, DSmP\(_0\) and CRD have the same Matthew effect. The results of BetP and CuzzP both are same as the results before fusion. After PnPl fusion, the degree of support of \{A\} is reduced. Based on above, Example 5.3 proves sufficiently that only FCP is adapt to compose the combination rule in Fig. 9.

C. Fusion multi-source probability

Since DTCR does not satisfy the associative law, we choose to use Murphy’s idea \cite{33} for multi-source information fusion. For $n$ pieces of probability information, we average the distributions and use DTCR to fuse $n$ times, and the specific process is shown in Algorithm 2.

Example 5.4: For $n$ pieces of probability information under 4-dimensional frame, we use multi-source probability disjunctive transformation combination rule (MSPDTCR) and Murphy’s method to fuse them respectively. The probability distributions and their fusion results are shown in Table VII.

Example 5.4 shows the difference between Murphy’s CRD-based method and our DTCR-based method when fusing multi-source conflicting information. Since our method and Murphy’s method both use the first averaging and then the fusion method, the weight of each piece of information is equal, so we only need to compare with Murphy method to prove our methods’ advantages. According to 8 pieces of probability information,
Algorithm 2 Multi-source probability disjunctive transformation combination rule

Input: $n$-dimension probability distributions $P_1 \cdots P_m$;  
Output: Fused probability distribution $\mathbb{P}_{1 \cap \cdots \cap 2}$;  
1: for $i = 1 : m - 1$ do  
2: $P_{i+1} = P_i + P_{i+1}$;  
3: end for  
4: $\mathbb{P} = P_m / m$; %Get probability average.  
5: $P_0 = \mathbb{P}$;  
6: for $i=1:m$ do \%Fusion average probability $m$ times.  
7: $\mathbb{P} = \mathbb{P} P_0$;  
8: end for  
9: $\mathbb{P}_{1 \cap \cdots \cap 2} = \mathbb{P}$;  
10: return $\mathbb{P}_{1 \cap \cdots \cap 2}$; 

| Probability | $A$ | $B$ | $C$ | $D$ | Result |
|-------------|-----|-----|-----|-----|--------|
| $P_1$       | 0.30| 0.60| 0.09| 0.01| $\{B\}$ |
| $P_2$       | 0.30| 0.01| 0.01| 0.68| $(D)$ |
| $P_3$       | 0.02| 0.02| 0.30| 0.66| $(D)$ |
| $P_4$       | 0.20| 0.10| 0.70| 0.00| $(C)$ |
| $P_5$       | 0.02| 0.80| 0.08| 0.10| $(B)$ |
| $P_6$       | 0.60| 0.30| 0.05| 0.05| $(A)$ |
| $P_7$       | 0.90| 0.50| 0.50| 0.35| $(A)$ |
| $P_8$       | 0.30| 0.30| 0.40| 0.00| $(C)$ |

Murphy's method  
MSPDTCR

|               | 0.7685| 0.1661| 0.0207| 0.0447| $\{A\}$ |
|---------------|-------|-------|-------|-------|--------|
|               | 0.4110| 0.2659| 0.1430| 0.1802| $\{A\}$ |

TABLE VII: The probability distributions and fusion results in Example 5.4

In this paper, we give a new interpretation to probability transformation based on the relationship between BBA and probability distribution, which is the extension of Smets proposed in transfer belief model. On the basis of the interpretation, we propose a belief evolution network (BEN) and a full causal (FC) belief function, which establish a causal relationship for all focal elements in a BBA. According to the BEN, we established a generalized probability transformation model (GPTM) to make the transformation process clearer. In addition, we use BEN and FC function to propose a new probability transformation method, called the full causal probability transformation method (FCPTM), which completes the probability transformation by evolving belief on BEN. Through process-oriented and result-oriented verification, we prove that FCPT is superior to all previous methods under multi-criteria. In terms of application, we combine the FCPT with the disjunctive combination rule (DRC) and propose a new probabilistic combination rule called disjunctive transformation combination rule (DTCR). After verification, it has better performance than combination rule of Dempster (CRD) in dealing with handling conflicts and fusing same information. In summary, the contributions of this paper can be summarized as follows: (1) For the first time, the causal relationship between focal elements is established, and BBA is expressed on a directed acyclic network. (2) The GPTM can evaluate the rationality of a transformation method in its process. (3) FC function as a new belief function can not only express the support degree to focal elements with causality, but be regarded as the complete set of the Bel function and the q function as well. (4) The FCPT is proved to be the only method that takes into account both the correlation and the PIC value. (5) DTCR is a new idea for information fusion, its pseudo-Matthew effect and method to handle conflicts are very novel.

In future research, we can extend this work from the following aspects. (1) Regarding the relationship between BBA and probability distribution, we can find more ways to improve this model. (2) For the BEN and FC functions, we can explore more applications for it. (3) For FCPT, the physical meaning of intermediate quantities in the evolution process can be further explored. (4) For DTCR, we can extend it to the information fusion of Dempster-Shafer evidence theory and other uncertainty theories.

VI. CONCLUSION

In this paper, we give a new interpretation to probability transformation based on the relationship between BBA and probability distribution, which is the extension of Smets proposed in transfer belief model. On the basis of the interpretation, we propose a belief evolution network (BEN) and a full causal (FC) belief function, which establish a causal relationship for all focal elements in a BBA. According to the BEN, we established a generalized probability transformation model (GPTM) to make the transformation process clearer. In addition, we use BEN and FC function to propose a new probability transformation method, called the full causal probability transformation method (FCPTM), which completes the probability transformation by evolving belief on BEN. Through process-oriented and result-oriented verification, we prove that FCPT is superior to all previous methods under multi-criteria. In terms of application, we combine the FCPT with the disjunctive combination rule (DRC) and propose a new probabilistic combination rule called disjunctive transformation combination rule (DTCR). After verification, it has better performance than combination rule of Dempster (CRD) in dealing with handling conflicts and fusing same information. In summary, the contributions of this paper can be summarized as follows: (1) For the first time, the causal relationship between focal elements is established, and BBA is expressed on a directed acyclic network. (2) The GPTM can evaluate the rationality of a transformation method in its process. (3) FC function as a new belief function can not only express the support degree to focal elements with causality, but be regarded as the complete set of the Bel function and the q function as well. (4) The FCPT is proved to be the only method that takes into account both the correlation and the PIC value. (5) DTCR is a new idea for information fusion, its pseudo-Matthew effect and method to handle conflicts are very novel.

In future research, we can extend this work from the following aspects. (1) Regarding the relationship between BBA and probability distribution, we can find more ways to improve this model. (2) For the BEN and FC functions, we can explore more applications for it. (3) For FCPT, the physical meaning of intermediate quantities in the evolution process can be further explored. (4) For DTCR, we can extend it to the information fusion of Dempster-Shafer evidence theory and other uncertainty theories.

ACKNOWLEDGMENT

The work is partially supported by National Natural Science Foundation of China (Grant No. 61973332), JSPS Invitational Fellowships for Research in Japan (Short-term).

REFERENCES

[1] A. P. Dempster, “Upper and lower probabilities induced by a multivalued mapping,” in Classic works of the Dempster-Shafer theory of belief functions. Springer, 2008, pp. 57–72.
[2] G. Shafer, A mathematical theory of evidence. Princeton university press, 1976, vol. 42.
[3] Q. Zhou, H. Mo, and Y. Deng, “A new divergence measure of pythagorean fuzzy sets based on belief function and its application in medical diagnosis,” Mathematics, vol. 8, no. 1, p. 142, 2020.
[4] J. Deng and Y. Deng, “Information volume of fuzzy membership function.” International Journal of Computers, Communications & Control, vol. 16, no. 1, 2021.
[5] W.-Z. Wu, Y. Leung, and W.-X. Zhang, “Connections between rough set theory and dempster-shafer theory of evidence,” International Journal of General Systems, vol. 31, no. 4, pp. 405–430, 2002.

[6] D. Dubois, M. Grabisch, H. Prade, and P. Smets, “Assessing the value of a candidate. comparing belief function and possibility theories,” arXiv preprint arXiv:1301.6692, 2013.

[7] C. Zhu and F. Xiao, “A belief hellingfer distance for d-s evidence theory and its application in pattern recognition,” Engineering Applications of Artificial Intelligence, vol. 106, p. 104452, 2021.

[8] X. Gan, X. Su, H. Qian, and X. Pan, “Dependence assessment in Human Reliability Analysis under uncertain and dynamic situations,” Nuclear Engineering and Technology, 2021.

[9] J.-B. Yang and D.-L. Xu, “On the evidential reasoning algorithm for multiple attribute decision analysis under uncertainty,” IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans, vol. 32, no. 3, pp. 289–304, 2002.

[10] T. Denoeux and O. Kanjanatarakul, “Evidential clustering: a review,” in International symposium on integrated uncertainty in knowledge modelling and decision making. Springer, 2016, pp. 24–35.

[11] Z. Liu, Q. Pan, J. Dezert, J.-W. Han, and Y. He, “Classifier fusion with contextual reliability evaluation,” IEEE transactions on cybernetics, vol. 48, no. 5, pp. 1665–1618, 2017.

[12] P. Smets and R. Kennes, “The transferable belief model,” Artificial intelligence, vol. 62, no. 2, pp. 191–234, 1994.

[13] F. Smarandache and J. Dezert, Advances and Applications of DS and I information fusion (Collected works), second volume: Collected Works. Infinite Study, 2006, pp. 94–106.

[14] X. Deng, Y. Hu, Y. Dong, and S. Mahadevan, “Supplier selection using ahp methodology extended by d numbers,” Expert Systems with Applications, vol. 41, no. 1, pp. 156–167, 2014.

[15] F. Xiao, “CEQD: A complex mass function to predict interference effects,” IEEE Transactions on Cybernetics, p. DOI: 10.1109/TCYB.2020.3040770, 2021.

[16] ——, “CaFR: A fuzzy complex event processing method,” International Journal of Fuzzy Systems, pp. DOI: 10.1007/s40815-021-01118-6, 2021.

[17] P. Smets, “Decision making in the tbm: the necessity of the pignistic transformation,” International journal of approximate reasoning, vol. 38, no. 2, pp. 133–147, 2005.

[18] B. K. Coe and P. P. Shenoy, “On the plausibility transformation method for translating belief function models to probability models,” International journal of approximate reasoning, vol. 41, no. 3, pp. 314–330, 2006.

[19] L. Martin and J. Sudano, “Yet another paradigm illustrating evidence fusion (yapief),” in 2006 9th International Conference on Information Fusion, IEEE, 2006, pp. 69–76.

[20] J. Dezert and F. Smarandache, “A new probabilistic transformation of belief mass assignment,” in 2008 11th International Conference on Information Fusion. IEEE, 2008, pp. 1–8.

[21] F. Cuzzolin, “On the relative belief transform,” International Journal of Approximate Reasoning, vol. 53, no. 5, pp. 786–804, 2012.

[22] Y. Dong, X. Li, and J. Dezert, “A new probabilistic transformation based on evolutionary algorithm for decision making,” in 2017 20th International Conference on Information Fusion (Fusion). IEEE, 2017, pp. 1–8.

[23] L. Chen, Y. Deng, and K. H. Cheong, “Probability transformation of mass function: A weighted network method based on the ordered visibility graph,” Engineering Applications of Artificial Intelligence, vol. 105, p. 104438, 2021.

[24] C. Huang, X. Mi, and B. Kang, “Basic probability assignment to probability distribution function based on the shapley value approach,” International Journal of Intelligent Systems, 2021.

[25] Z. Deng and J. Wang, “A novel decision probability transformation method based on belief interval,” Knowledge-Based Systems, vol. 208, p. 106427, 2020.

[26] F. Cuzzolin, The geometry of uncertainty. Springer, Cham, 2021.

[27] J. Dezert, D. Han, Z. Liu, and J.-M. Tancret, “Hierarchical proportional redistribution for bba approximation,” in Belief functions: theory and applications. Springer, 2012, pp. 275–283.

[28] D. Han, J. Dezert, and Z. Duan, “Evaluation of probability transformations of belief functions for decision making,” IEEE Transactions on Systems, Man, and Cybernetics: Systems, vol. 46, no. 1, pp. 93–108, 2015.

[29] R. R. Yager, “Entropy and specificity in a mathematical theory of evidence,” in Classic Works of the Dempster-Shafer Theory of Belief Functions. Springer, 2008, pp. 291–310.

[30] ——, “On the dempster-shafer framework and new combination rules,” Information sciences, vol. 41, no. 2, pp. 93–137, 1987.

[31] D. Dubois and H. Prade, “A set-theoretic view of belief functions,” in Classic Works of the Dempster-Shafer Theory of Belief Functions. Springer, 2008, pp. 375–410.

[32] P. Smets, “Data fusion in the transferable belief model,” in Proceedings of the third international conference on information fusion, vol. 1. IEEE, 2000, pp. P521–P585.

[33] C. K. Murphy, “Combining belief functions when evidence conflicts,” Decision support systems, vol. 29, no. 1, pp. 1–9, 2000.

[34] J. Deng, Y. Deng, and K. H. Cheong, “Combining conflicting evidence based on pearson correlation coefficient and weighted graph,” International Journal of Intelligent Systems, p. DOI: 10.1002/int.22593, 2021.

[35] L. Xiong, X. Su, and H. Qian, “Conflicting evidence combination from the perspective of networks,” Information Sciences, vol. 580, pp. 408–418, 2021.

[36] X. Fan, D. Han, Y. Yang, and J. Dezert, “De-combination of belief function based on optimization,” Chinese Journal of Aeronautics, 2021.

[37] P. Smets, “Belief functions: the disjunctive rule of combination and the generalized bayesian theorem,” International Journal of approximate reasoning, vol. 9, no. 1, pp. 1–35, 1993.

[38] ——, “The application of the matrix calculus to belief functions,” International Journal of Approximate Reasoning, vol. 31, no. 1–2, pp. 1–30, 2002.

[39] J. J. Sudano, “Pignistic probability transforms for mixes of low and high-probability events,” arXiv preprint arXiv:1505.07751, 2015.

[40] A.-L. Jousselme, D. Grenier, and É. Bossé, “A new distance between two bodies of evidence,” Information fusion, vol. 2, no. 2, pp. 91–101, 2001.

[41] W. Jiang, “A correlation coefficient for belief functions,” International Journal of Approximate Reasoning, vol. 103, pp. 94–106, 2018.

[42] W. Jiang, C. Huang, and X. Deng, “A new probability transformation method based on a correlation coefficient of belief functions,” International Journal of Intelligent Systems, vol. 34, no. 6, pp. 1337–1347, 2019.

[43] Q. Zhou and Y. Deng, “Higher order information volume of mass function,” arXiv preprint arXiv:2012.07697, 2020.

[44] T. Denoeux, “Conjunctive and disjunctive combination of belief functions induced by nondistinct bodies of evidence,” Artificial Intelligence, vol. 172, no. 2–3, pp. 234–247, 2008.

[45] Q. Zhou and Y. Deng, “Fractal-based belief entropy,” arXiv preprint arXiv:2012.00235, 2020.