New experimental tests of nuclear supersymmetry are suggested. They involve the measurement of one- and two-nucleon transfer reactions between nuclei that belong to the same supermultiplet. These reactions provide a direct test of the 'fermionic' sector, i.e. of the operators that change a boson into a fermion or vice versa. We present some theoretical predictions for the supersymmetric quartet of nuclei: $^{194}$Pt, $^{195}$Pt, $^{195}$Au and $^{196}$Au.

1 Introduction

Dynamical supersymmetries were introduced in nuclear physics in 1980 by Franco Iachello in the context of the Interacting Boson Model (IBM) and its extensions. The spectroscopy of atomic nuclei is characterized by the interplay between collective (bosonic) and single-particle (fermionic) degrees of freedom. The IBM describes collective excitations in even-even nuclei in terms of a system of interacting monopole and quadrupole bosons with angular momentum $l = 0, 2$. The bosons are associated with the number of correlated proton and neutron pairs, and hence the number of bosons $N$ is half the number of valence nucleons.

For odd-mass nuclei the IBM has been extended to include single-particle degrees of freedom. The Interacting Boson-Fermion Model (IBFM) has as its building blocks $N$ bosons with $l = 0, 2$ and $M = 1$ fermion with $j = j_1, j_2, \ldots$. The IBM and IBFM can be unified into a superalgebra $U(n/m)$, where $n = \sum_l (2l+1) = 6$ is the dimension of the boson space and $m = \sum_j (2j+1)$ of the fermion space. In this framework, even-even and odd-mass nuclei form the members of a supermultiplet. The inclusion of the neutron-proton degree of freedom leads to supersymmetric quartets of nuclei consisting of an even-even, an odd-even, an even-odd and an odd-odd nucleus.

Supersymmetry (SUSY) distinguishes itself from other symmetries in that it includes, in addition to transformations among fermions or among bosons, also transformations between bosons and fermions. The spectroscopic properties of the nuclei that belong to the same supermultiplet, are linked and correlated by SUSY, i.e. they are described by the same form of the operators. Most tests of supersymmetry that have been discussed in the literature involve energies and transitions. These observables are described by the bosonic generators that transform bosons into bosons and fermions into fermions. Whereas the bosonic generators describe observables within a given nucleus, the fermionic generators that change a boson into a fermion or vice versa, describe the transitions between different nuclei of the same supermultiplet, such as observed in single-particle transfer reactions. Unlike for the bosonic sector, there are relatively few direct tests of the fermionic generators.
It is the purpose of this contribution to investigate one-nucleon transfer reactions in the context of nuclear supersymmetry, and to establish possible correlations between different transfer reactions. As an example, we consider the supersymmetric quartet of nuclei: $^{194}$Pt, $^{195}$Pt, $^{195}$Au and $^{196}$Au, whose energy spectra have been classified and described successfully in terms of the $U(6/12)\nu \otimes U(6/4)\pi$ supersymmetry $^5$. 

2 The $U(6/12)\nu \otimes U(6/4)\pi$ supersymmetry

The mass region $A \sim 190$ has been a rich source of possible empirical evidence for the existence of (super)symmetries in nuclei. The even-even nucleus $^{196}$Pt is the standard example of the $SO(6)$ limit of the IBM $^9$. The odd-proton nuclei $^{191,193}$Ir and $^{193,195}$Au were suggested as examples of the $Spin(6)$ limit $^1$, in which the odd-proton is allowed to occupy the $\pi d_{3/2}$ orbit, whereas the pairs of nuclei $^{192}$Os - $^{191}$Ir, $^{194}$Os - $^{193}$Ir, $^{192}$Pt - $^{193}$Au and $^{194}$Pt - $^{195}$Au have been analyzed as examples of a $U(6/4)$ supersymmetry $^4$. The odd-neutron nucleus $^{195}$Pt, together with $^{194}$Pt, were studied in terms of a $U(6/12)\nu$ supersymmetry, in which the odd neutron occupies the $\nu p_{1/2}$, $\nu p_{3/2}$ and $\nu f_{5/2}$ orbits $^10$. These ideas were later extended to the case where neutron and proton bosons are distinguished $^5$, predicting in this way a correlation among quartets of nuclei, consisting of an even-even, an odd-proton, an odd-neutron and an odd-odd nucleus. The best experimental example of such a quartet with $U(6/12)\nu \otimes U(6/4)\pi$ supersymmetry is provided by the nuclei $^{194}$Pt, $^{195}$Au, $^{195}$Pt and $^{196}$Au. The number of bosons $N$ is taken to be the number of bosons in the odd-odd nucleus $^{196}$Au: $N = N_\nu + N_\pi = 4 + 1 = 5$.

$$
\begin{align*}
\text{odd-odd} & \quad N_\nu, N_\pi, j_\nu, j_\pi \\
^{196}_{79}\text{Au}_{117} & \leftrightarrow ^{195}_{79}\text{Au}_{116} \\
\uparrow & \uparrow \\
^{195}_{78}\text{Pt}_{117} & \leftrightarrow ^{194}_{78}\text{Pt}_{116} \\
\text{odd-even} & \quad N_\nu, N_\pi + 1, j_\nu \\
\text{even-odd} & \quad N_\nu + 1, N_\pi, j_\pi \\
N_\nu + 1, N_\pi + 1
\end{align*}
(1)
$$

The supersymmetric classification of nuclear levels in the Pt and Au isotopes has been re-examined by taking advantage of the significant improvements in experimental capabilities developed in the last decade. High resolution transfer experiments with protons and polarized deuterons have led to strong evidence for the existence of supersymmetry (SUSY) in atomic nuclei. The experiments include high resolution transfer experiments to $^{196}$Au at TU/LMU München $^5$. 

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and in-beam gamma ray and conversion electron spectroscopy following the reactions $^{196}\text{Pt}(d, 2n)$ and $^{196}\text{Pt}(p, n)$ at the cyclotrons of the PSI and Bonn. These studies have achieved an improved classification of states in $^{195}\text{Pt}$ and $^{196}\text{Au}$ which give further support to the original ideas and extend and refine previous experimental work in this research area.

In a dynamical (super)symmetry, the Hamiltonian is expressed in terms of the Casimir invariants of the subgroups in a group chain. The relevant subgroup chain of $U(6/12)_\nu \otimes U(6/4)_\pi$ for the Pt and Au nuclei is given by

$$U(6/12)_\nu \otimes U(6/4)_\pi \supset U^{BF}(6) \otimes U^{F}(12) \otimes U^{B}(6) \otimes U^{F}(4)$$

$$\supset U^{B}(6) \otimes U^{F}(6) \otimes U^{F}(2) \otimes U^{F}(4)$$

$$\supset U^{BF}(6) \otimes U^{F}(2) \otimes U^{F}(4)$$

$$\supset SO^{BF}(6) \otimes U^{F}(2) \otimes SU^{F}(4)$$

$$\supset Spin(6) \otimes U^{F}(2)$$

$$\supset Spin(5) \otimes U^{F}(2)$$

$$\supset Spin(3) \otimes SU^{F}(2)$$

$$\supset SU(2) .$$

In this case, the Hamiltonian

$$H = \alpha C_{2U^{BF}(6)} + \beta C_{2SO^{BF}(6)} + \gamma C_{2Spin(6)}$$

$$+ \delta C_{2Spin(5)} + \epsilon C_{2Spin(3)} + \eta C_{2SU(2)} ,$$

describes simultaneously the excitation spectra of the quartet of nuclei. Here we have neglected terms that only contribute to binding energies. The coefficients $\alpha$, $\beta$, $\gamma$, $\delta$, $\epsilon$ and $\eta$ have been determined in a simultaneous fit of the excitation energies of the four nuclei of Eq.

In a dynamical supersymmetry, closed expressions can be derived for energies, and selection rules and intensities for electromagnetic transitions and single-particle transfer reactions. While a simultaneous description and classification of these observables in terms of the $U(6/12)_\nu \otimes U(6/4)_\pi$ supersymmetry has been shown to be fulfilled to a good approximation for the quartet of nuclei $^{194}\text{Pt}$, $^{195}\text{Au}$, $^{195}\text{Pt}$ and $^{196}\text{Au}$, there are important predictions still not fully verified by experiments. These tests involve the transfer reaction intensities among the supersymmetric partners. In the next section we concentrate on the latter and, in particular, on the one-proton transfer reactions $^{194}\text{Pt} \rightarrow ^{195}\text{Au}$ and $^{195}\text{Pt} \rightarrow ^{196}\text{Au}$. 

3 One-nucleon transfer reactions

The single-particle transfer operator that is commonly used in the Interacting Boson-Fermion Model (IBFM), has been derived in the seniority scheme. Although strictly speaking this derivation is only valid in the vibrational regime, it has been used for deformed nuclei as well. An alternative method is based on symmetry considerations. It consists in expressing the single-particle transfer operator in terms of tensor operators under the subgroups that appear in the group chain of a dynamical (super)symmetry. The single-particle transfer between different
Figure 1. Allowed one-proton transfer reactions for $^{194}\text{Pt} \rightarrow ^{195}\text{Au}$. The spectroscopic factors are normalized to 100 for the ground state to ground state transition for the operators $T_1/T_2$.

members of the same supermultiplet provides an important test of supersymmetries, since it involves the transformation of a boson into a fermion or vice versa, but it conserves the total number of bosons plus fermions.

The operators that describe one-proton transfer reactions in the $U(6/12)_\nu \otimes U(6/4)_\pi$ supersymmetry are given by

$$T_1^{(\frac{1}{2}, \frac{1}{2}, -\frac{1}{2})}; (\frac{1}{2}, \frac{1}{2}, \frac{3}{2}) = -\sqrt{\frac{1}{6}} \left( \hat{s}_\nu \times a^\dagger_{\pi, \frac{3}{2}} \right)_m \left( \frac{3}{2} \right) + \sqrt{\frac{5}{6}} \left( \hat{d}_\pi \times a^\dagger_{\pi, \frac{3}{2}} \right)_m \left( \frac{3}{2} \right) \right),$$

$$T_2^{(\frac{1}{2}, \frac{1}{2}, \frac{3}{2})}; (\frac{1}{2}, \frac{1}{2}, \frac{3}{2}) = \sqrt{\frac{5}{6}} \left( \hat{s}_\nu \times a^\dagger_{\pi, \frac{3}{2}} \right)_m \left( \frac{3}{2} \right) + \frac{1}{6} \left( \hat{d}_\pi \times a^\dagger_{\pi, \frac{3}{2}} \right)_m \left( \frac{3}{2} \right) \right). \quad (4)$$

The operators $T_1$ and $T_2$ are, by construction, tensor operators under $Spin(6)$, $Spin(5)$ and $Spin(3)$ \cite{13}. The upper indices $(\sigma_1, \sigma_2, \sigma_3), (\tau_1, \tau_2), J$ specify the tensorial properties under $Spin(6)$, $Spin(5)$ and $Spin(3)$. The use of tensor operators to describe single-particle transfer reactions in the supersymmetry scheme has the advantage of giving rise to selection rules and closed expressions for the spectroscopic factors.

Fig. 1 shows the allowed transitions for the transfer operators of Eq. (4) that describe the one-proton transfer from the ground state $|(N + 2, 0, 0), (0, 0, 0)\rangle$ of the even-even nucleus $^{194}\text{Pt}$ to the even-odd nucleus $^{195}\text{Au}$ belonging to the same supermultiplet $[N_\nu + 1]_\nu \otimes [N_\pi + 1]_\pi$. The operators $T_1$ and $T_2$ have the same transformation character under $Spin(5)$ and $Spin(3)$, and therefore can only excite states with $(\tau_1, \tau_2) = (\frac{1}{2}, \frac{1}{2})$ and $J = \frac{3}{2}$. However, they differ in their $Spin(6)$ selection rules. Whereas $T_1$ can only excite the ground state of the even-odd nucleus with $(\sigma_1, \sigma_2, \sigma_3) = (N + \frac{3}{2}, \frac{3}{2}, \frac{3}{2})$, the operator $T_2$ also allows the transfer to an
Figure 2. As Fig. 1, but for $^{195}\text{Pt} \rightarrow ^{196}\text{Au}$.

excited state with $(N + \frac{1}{2}, \frac{1}{2}, -\frac{1}{2})$. The ratio of the intensities is given by

$$R_1 = \frac{I_{gs \rightarrow exc}}{I_{gs \rightarrow gs}} = 0,$$

$$R_2 = \frac{I_{gs \rightarrow exc}}{I_{gs \rightarrow gs}} = \frac{9(N + 1)(N + 5)}{4(N + 6)^2}, \quad (5)$$

for $T_1$ and $T_2$, respectively. In the case of the one-proton transfer $^{194}\text{Pt} \rightarrow ^{195}\text{Au}$, the second ratio is given by $R_2 = 1.12 \ (N = 5)$.

The available experimental data from the proton stripping reactions $^{194}\text{Pt}(\alpha, t)^{195}\text{Au}$ and $^{194}\text{Pt}(^{3}\text{He}, d)^{195}\text{Au}$ shows that the $J = 3/2$ ground state of $^{195}\text{Au}$ is excited strongly with $C^2S = 0.175$, whereas the first excited $J = 3/2$ state is excited weakly with $C^2S = 0.019$. In the SUSY scheme, the latter state is assigned as a member of the ground state band with $(\tau_1, \tau_2) = (5/2, 1/2)$. Therefore the one proton transfer to this state is forbidden by the $Spin(5)$ selection rule of the tensor operators of Eq. (4). The relatively small strength to excited $J = 3/2$ states suggests that the operator $T_1$ of Eq. (1) can be used to describe the data.

In Fig. 2 we show the allowed transitions for the one-proton transfer from the ground state $|(N + 2, 0, 0), (0, 0), 0, \frac{1}{2} \rangle$ of the odd-even nucleus $^{195}\text{Pt}$ to the odd-odd nucleus $^{196}\text{Au}$. Also in this case, the operator $T_1$ only excites the ground state doublet of $^{196}\text{Au}$ with $(\sigma_1, \sigma_2, \sigma_3) = (N + \frac{3}{2}, \frac{1}{2}, \frac{1}{2})$, $(\tau_1, \tau_2) = (\frac{1}{2}, \frac{1}{2})$, $J = \frac{3}{2}$ and $L = J \pm \frac{1}{2}$, whereas $T_2$ also populates the excited state with $(N + \frac{3}{2}, \frac{1}{2}, -\frac{1}{2})$. The ratio of the intensities is the same as for the $^{194}\text{Pt} \rightarrow ^{195}\text{Au}$ transfer reaction

$$R_1(^{195}\text{Pt} \rightarrow ^{196}\text{Au}) = R_1(^{194}\text{Pt} \rightarrow ^{195}\text{Au}) = 0,$$

$$R_2(^{195}\text{Pt} \rightarrow ^{196}\text{Au}) = R_2(^{194}\text{Pt} \rightarrow ^{195}\text{Au}) = \frac{9(N + 1)(N + 5)}{4(N + 6)^2}. \quad (6)$$
This is direct consequence of the supersymmetry. Just as the energies and the electromagnetic transition rates of the supersymmetric quartet of nuclei were calculated with the same form of the Hamiltonian and the transition operator, here we have extended this idea to the one-proton transfer reactions. We find definite predictions for the spectroscopic factors of the $^{195}$Pt → $^{196}$Au transfer reactions, which can be tested experimentally. To the best of our knowledge, there are no data available for this reaction.

For the one-neutron transfer reactions there exists a similar situation. The available experimental data from the neutron stripping reactions $^{194}$Pt($d,p$)$^{195}$Pt can be used to determine the appropriate form of the one-neutron transfer operator, which then can be used to predict the spectroscopic factors for the transfer reaction $^{195}$Au → $^{196}$Au. We believe that, as a consequence of the supersymmetry classification, a number of additional correlations exist for transfer reactions between different pairs of nuclei. This would be the first time that such relations are predicted for nuclear reactions which may provide a challenge and motivation for future experiments.

4 Summary and outlook

The recent measurements of the spectroscopic properties of the odd-odd nucleus $^{196}$Au have rekindled the interest in nuclear supersymmetry. The available data on the spectroscopy of the quartet of nuclei $^{194}$Pt, $^{195}$Au, $^{195}$Pt and $^{196}$Au can, to a good approximation, be described in terms of the $U(6/4)\otimes U(6/12)$ supersymmetry. However, there is a still another important set of experiments which can further test the predictions of the supersymmetry scheme. These involve transfer reactions between nuclei belonging to the same supermultiplet, in particular between the even-odd (odd-even) and odd-odd members of the supersymmetric quartet. Theoretically, these transfers are described by the supersymmetric generators which change a boson into a fermion, or vice versa.

We investigated in some detail the example of proton transfer between the SUSY partners: $^{194}$Pt → $^{195}$Au and $^{195}$Pt → $^{196}$Au. The supersymmetry implies strong correlations for the spectroscopic factors of these two reactions which can be tested experimentally. A similar set of relations can be derived for the one-neutron transfer reactions $^{194}$Pt ↔ $^{195}$Pt and $^{195}$Au ↔ $^{196}$Au. Another interesting extension of supersymmetry concerns the recently measured two-nucleon transfer reaction $^{194}$Pt($\alpha,d$)$^{196}$Au, in which a neutron-proton pair is transferred to the target nucleus. This reaction presents a very sensitive test of the wave functions, since it provides a measure of the correlation within the transferred neutron-proton pair. Whether it is possible to describe this process by a transfer operator that is correlated by SUSY to that of the one-proton and one-neutron transfer reactions is an open question.

In conclusion, we emphasize the need for new experiments taking advantage of the new experimental capabilities and suggest that particular attention be paid to one- and two-nucleon transfer reactions between the SUSY partners $^{194}$Pt, $^{195}$Au, $^{195}$Pt and $^{196}$Au, since such experiments provide the most stringent tests of nuclear supersymmetry. It remains to be seen whether the correlations predicted
by SUSY are indeed verified by experiments.

Dedication

It is a great pleasure to dedicate this contribution to Franco Iachello on the occasion of the conference ‘Symmetries in Nuclear Structure’, held in his honor. Unfortunately it was not possible to attend the meeting, but I am grateful to the organizers for the invitation to write a contribution for the proceedings.

In the fall of 1977, I took a course on nuclear structure presented by a young Italian professor at the University of Groningen. The lectures were characterized by their clarity of presentation, a contagious enthusiasm, a link between the material presented in the course and ongoing research and, last but not least, the connection between theory and experiment. These ingredients have formed the basis and provided the motivation and inspiration for my own scientific career, first as a graduate student at the KVI in Groningen, and later as a research scientist. Over the years I have had the pleasure to collaborate with Franco on different subjects, such as supersymmetry, baryon spectroscopy and nuclear clusters which has resulted in 12 joint publications between 1983 and 2002.

I will not mention his other career as a racecar driver, nor comment on his uncanny likeness to Woody Allen (with the exception of the title of this contribution). Finally, I wish Franco an equally productive and creative second half of his career. Congratulations, Franco!

Acknowledgments

We are grateful to Gerhard Graw for numerous stimulating discussions and for sharing the new data on the two-nucleon transfer reactions $^{198}\text{Hg}(d,\alpha)^{196}\text{Au}$ and $^{194}\text{Pt}(\alpha,d)^{196}\text{Au}$ prior to publication. The work presented in this contribution is motivated by the renewed experimental interest in this mass region. This work was supported in part by CONACyT.

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