Detecting the neutral top-pion at $e^+e^-$ colliders

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Abstract

We investigate some processes of the associated production of a neutral top-pion $\Pi_0^\pm$ with a pair of fermions($e^+e^- \to f\bar{f}\Pi_0^\pm$) in the context of top-color-assisted technicolor (TC2) theory at future $e^+e^-$ colliders. The studies show that the largest cross sections of the processes $e^+e^- \to f\bar{f}\Pi_0^\pm$($f = u, d, c, s, \mu, \tau$) could only reach the level of 0.01fb, we can hardly detect a neutral top-pion through these processes. For the processes $e^+e^- \to e^+e^-\Pi_0^\pm, e^+e^- \to \bar{u}\bar{d}\Pi_0^\pm$ and $e^+e^- \to \bar{d}\bar{u}\Pi_0^\pm$, the cross sections of these processes are at the level of a few fb for the favorable parameters and a few tens, even hundreds, of neutral top-pion events can be produced at future $e^+e^-$ colliders each year through these processes. With the clean background of the flavor-changing $t\bar{c}$ channel, the top-pion events can possibly be detected at the planned high luminosity $e^+e^-$ colliders. Therefore, such neutral top-pion production processes provide a useful way to detect a neutral top-pion and test the TC2 model directly.

I. INTRODUCTION

Essential elements of fundamental constituents of matter and their interactions have been discovered in the past three decades by operating $e^+e^-$ colliders. A coherent picture of the structure of matter has emerged, that is adequately described by the standard model (SM), in many of its facets at a level of very high accuracy. However, the SM does not provide a comprehensive theory of matter. Neither the fundamental parameters, masses and couplings, nor the symmetry pattern can be explained. Therefore, some new theories beyond the SM (new physics models) have been studied to solve the problems of the SM. On the other hand, there exist new particles in the new physics models, probing these particles at future high-energy colliders can provide a direct way to test these models.

A. The top-color-assisted technicolor model

The top quark is the heaviest particle yet experimentally discovered and its mass of 174 GeV [1] is close to the electroweak symmetry breaking (EWSB) scale. Much theoretical work has been carried out in connection to the top quark and EWSB. The top-color-assisted technicolor (TC2) [2] model, the top seesaw model [3] and the flavor universal TC2 model [4] are three such examples. The TC2 model is a more realistic one, which generates the large top quark mass through a dynamical $< t\bar{t} >$ condensation and provides a possible dynamical mechanism for breaking electroweak symmetry. In the TC2 model, the new strong dynamics is assumed to be chirally critical but spontaneously broken by TC at the scale $\sim 1$ TeV and the EWSB is driven mainly by TC interaction. The extended technicolor (ETC) interaction gives the contribution to all ordinary quark and lepton masses including a very small portion of the top quark mass: $m_t = \varepsilon m_t$ ($0.03 \leq \varepsilon \leq 0.1$) [5]. The top-color interaction also makes a small contribution to the EWSB and gives rise to the main part of the top quark mass: $(1-\varepsilon)m_t$. The TC2 model also predicts the existence of a CP-even scalar ($h_0^0$) called the top-Higgs and three CP-odd pseudo Goldstone bosons (PGB’s) called top-pions ($\Pi_0^\pm, \Pi_0^0$) in a few hundreds GeV region. The physical particle top-pions can be regarded as a typical feature of the TC2 model. Thus, the study of the possible signatures of top-pions and top-pion contribution to some processes at high-energy colliders can be regarded as a good method to test the TC2 model and further to probe the EWSB mechanism.

At the energy scale $\Lambda \sim 1$ TeV, the new strong dynamics is coupled preferentially to the third generation. The dynamics of a general TC2 model involves the following structure [2,5]:

$$SU(3)_1 \otimes SU(3)_2 \otimes U(1)_{Y_1} \otimes U(1)_{Y_2} \otimes SU(2)_L \rightarrow SU(3)_{QCD} \otimes U(1)_{EM}$$

where $SU(3)_1 \otimes U(1)_{Y_1} (SU(3)_2 \otimes U(1)_{Y_2})$ couples preferentially to the third generation (the first and the second generations). The $U(1)_{Y_i}$ is just strongly rescaled versions of electroweak $U(1)_Y$. $SU(3)_1 \otimes U(1)_{Y_1}$ is assumed strong enough to produce a large top condensate which is responsible for the main part of the top quark mass. The b-quark mass is an interesting issue, involving a combination of ETC effects and instanton effects in $SU(3)_1$. The instanton induced b-quark mass can then be estimated as [6]:

$$m_b^\gamma \approx \frac{3k m_t}{8\pi^2} \sim 6.6k \ GeV$$

where we generally expect $k \sim 1 \times 10^{-1}$ as in QCD. In the TC2 model, the top-color gauge bosons include
the color-octet colorons $B_A^\pm$ and color-singlet extra $U(1)$
gauge boson $Z'$. These gauge bosons have very large masses
which can be up to several TeV. Such large masses
will depress the contribution to the cross sections. So, in
our calculation, we can neglect the contributions of the
gauge bosons.

B. Search for the new particles at planned $e^+e^-$
colliders

The planned linear $e^+e^-$ colliders(LC) with energy in
the range from a few hundred GeV up to several TeV are
under intense studies around the world. These studies are
being done at the Next Linear Collider(NLC)(USA)
[7], the Japan Linear Collider(JLC)(Japan) [8] and the
DESY TeV Energy Superconducting Linear Accelerator(TESLA)(Europe) [9]. One task of these high-energy
$e^+e^-$ colliders is to search for Higgs particle in the SM or
some new particles predicted in the models beyond the
SM[such as Higgs bosons $A^0, H^0, h^0$, $H^\pm$ in the minimal
supersymmetric standard model(MSSM) and PGB's in
the TC2 model]. So, the study of some new particle
production processes can provide a theoretical instruction to
search for these particles experimentally.

As it is known, top-pions are the typical particles in
the TC2 model. Some neutral top-pion production pro-
cesses have been studied in Ref. [10]. On the other hand,
Ref. [11] has studied a top-charm associated production
process at LHC to probe the top-pion. The above stud-
ies provide us with some useful information to detect
top-pion events and test the TC2 model. In this pa-
per, we study the neutral top-pion production processes
$e^+e^- \rightarrow f f \Pi_0^\pm$ in the framework of the TC2 model,
where $f$ represents $u, d, c, s, t, b$ quarks and $e, \mu, \tau$
leptons. Our results show that the cross sections of the processes
$e^+e^- \rightarrow f f' \Pi_0^\pm(f' = u, d, c, s, \mu, \tau)$ are very small.
The largest cross section could only reach an order of magni-
tude $0.01 fb$. With such small cross sections, the neu-
tral top-pion can be hardly detected via these processes.
So, we pay attention to the processes $e^+e^- \rightarrow e^+e^- \Pi_0^\pm$,
$e^+e^- \rightarrow \tau t \Pi_0^\pm$ and $e^+e^- \rightarrow b b \Pi_0^\pm$. These cross sections are
about two orders or even three orders of magnitude larger
than that of processes $e^+e^- \rightarrow f f' \Pi_0^\pm$. The study in this
paper is a useful addition to the previous studies. Some
similar processes in the context of the SM and MSSM
have also been studied quite extensively in the LC [12],
Tevatron and LHC [13]. Reference [14] has investigated
the production of the neutral scalar with a pair of top
quarks at the hadron collider. They find that the neu-
tral scalar may be observed at the LHC via the process
$e^+e^- \rightarrow t\bar{t}f$. The $qqH$ production mode is extremely
interesting for physicists because this production mode
provides a direct way to measure the Yukawa couplings
of the quarks with scalar particles, on the other hand, we
can detect these scalar particles at LC, LHC and Teva-
tron II with high luminosity through these processes.

The calculation of the production cross sections of the
processes $e^+e^- \rightarrow f f \Pi_0^\pm$ is presented in Sec II.

II. CROSS SECTIONS OF THESE PROCESSES

It is noticeable that the TC2 model may have rich top-
quark phenomenology since it treats the top quark differ-
cently from other quarks. The couplings of top-pions to
three family fermions are nonuniversal and the top-pions
have large Yukawa couplings to the third generation. The
Yukawa interactions of top-pions to
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Yukawa interactions of top-pions to
quark mass induced by instanton.

$K_{DL,R}^{t}$, $K_{UL,R}^{t}$, $K_{UR}^{tt}$ the elements of the rotation matrices
$K_{UL,R}$ and $K_{DL,R}$. The rotation matrices $K_{UL,R}$, and $K_{DL,R}$
are needed for diagonalizing the up- and down-quark
mass matrices $M_U$ and $M_D$, i.e., $K_{UL}^t M_U K_{UR} = M_U^\text{diag}$
and $K_{DL}^t M_D K_{DR} = M_D^\text{diag}$, from which the Cabibbo-
Kobayashi-Maskawa(CKM) matrix is defined as $V = K_{UL}^t K_{DL}$.
The matrix elements are given as

\[
K_{UL}^{tt} \approx K_{DL}^{bb} \approx 1 \quad K_{UR}^{t} = 1 - \varepsilon
\]

Here, we take $\varepsilon$ as a free parameter changing from 0.03
to 0.1.

With $tt\Pi_0^\pm$ coupling, the neutral top-pion $\Pi_0^\pm$ can
couple to a pair of gauge bosons through the top quark triangle
loops in an isospin violating way. Calculating the top
quark triangle loops, we can explicitly obtain the couplings
of $\Pi_0^\pm - \gamma - \gamma$, $\Pi_0^\pm - \gamma - Z$ and $\Pi_0^\pm - Z - Z$

\[
\Pi_0^\pm - \gamma - \gamma : \\
i_N \cos \beta \left( \frac{8}{9} \frac{\tan \beta}{v_w} m_t^2 (1 - \varepsilon) \alpha_c \varepsilon_{\mu \nu \rho \delta} p_{in}^\mu \not{p}_{out}^\nu C_0 \right.
\]

\[
\Pi_0^\pm - \gamma - Z : \\
i_N \cos \beta \left( \frac{4}{3} \frac{\tan \beta}{v_w} m_t^2 (1 - \varepsilon) \varepsilon_{\mu \nu \rho \delta} p_{in}^\mu \not{p}_{out}^\nu \right.
\]

\[
\left( 1 - \frac{8}{3} s_w^2 \right) C_0
\]
\[ \Pi^0_e = \gamma - \gamma : \]
\[ iN_c \frac{\alpha_e}{8\pi^2 c^2 s_w^2} \frac{\tan \beta}{v_w} m_t^2 (1 - \varepsilon) \varepsilon_{\mu
u\rho\delta} p_{\mu} p_{\nu} p_{\rho} p_{\delta} \]
\[ \{[(1 - \frac{8}{3}s_w^2)^2 - 1]C_0 - 2C_{11} \} \tag{4} \]

where \( N_c \) is the color index with \( N_c = 3 \), \( s_w = \sin \theta_w \), \( c_w = \cos \theta_w \) (\( \theta_w \) is the Weinberg angle), \( C_0 = C_0(-p_{\text{in}}, p_{\text{out}}, m_t, m_t, m_t) \) and \( C_{11} = C_{11}(-p_{\text{in}}, p_{\text{out}}, m_t, m_t, m_t) \) are standard three-point scalar integrals with \( p_{\text{in}} \) and \( p_{\text{out}} \) denoting the momenta of the incoming gauge boson and the outgoing top-pion, respectively.

With the couplings of \( \Pi^0_e \gamma \gamma, \Pi^0_e Z \gamma, \Pi^0_e ZZ \), the neutral top-pion can be produced via the processes \( e^+ e^- \to f f \Pi^0_e \). The Feynman diagrams of these processes are shown in Fig. 1.

From the diagrams, we can see that the process \( e^+ e^- \to e^+ e^- \Pi^0_e \) can take place through t-channel and s-channel as shown in Fig. 1(a),(b). The light quark pairs (\( u \bar{u}, d \bar{d}, c \bar{c}, s \bar{s} \)) or lepton pairs (\( \mu \bar{\mu}, \tau \bar{\tau} \)) can only be produced via Fig.1(b). The heavy quark pair production processes \( e^+ e^- \to tt \Pi^0_e \) and \( e^+ e^- \to bb \Pi^0_e \) are shown in Fig.1(b),(c),(d),(e). \( h^0_f \) shown in Fig.1(c) is a CP even particle in TC2 model, the couplings \( h^0_f q \bar{q} \) and \( Z h^0_f \) are given in Ref. [16].

The explicit expressions of the amplitudes for different diagrams can be directly written as:

\[ M^a_{\gamma\gamma \Pi^0_e} = iN_c \alpha_e^2 \frac{32}{9} \frac{\tan \beta}{v_w} m_t^2 (1 - \varepsilon) \varepsilon_{\mu\nu\rho\delta} (p_2 - p_4)_{\rho} p_{5\delta} C_0 \]
\[ G(p_2 - p_4, 0)G(p_2 - p_4 - p_5, 0) \]
\[ \bar{\nu}_{e^-} (p_4) \gamma_{\mu} u_{e^-} (p_2) \bar{\nu}_{e^+} (p_1) \gamma_{\nu} v_{e^+} (p_3) \]
\[ \bar{M}^a_{\gamma\gamma \Pi^0_e} = -iN_c \frac{2\pi}{3} \frac{\alpha_e^2}{c_w^2 s_w^2} \frac{\tan \beta}{v_w} m_t^2 (1 - \varepsilon) \varepsilon_{\mu\nu\rho\delta} (p_2 - p_4)_{\rho} p_{5\delta} \]
\[ (1 - \frac{8}{3}s_w^2)C_0 G(p_2 - p_4, 0)G(p_2 - p_4 - p_5, M_Z) \]
\[ \bar{\nu}_{e^-} (p_4) \gamma_{\mu} u_{e^-} (p_2) \bar{\nu}_{e^+} (p_1) \gamma_{\nu} v_{e^+} (p_3) \]
\[ M^a_{Z\gamma \Pi^0_e} = -iN_c \frac{2\pi}{3} \frac{\alpha_e^2}{c_w^2 s_w^2} \frac{\tan \beta}{v_w} m_t^2 (1 - \varepsilon) \varepsilon_{\mu\nu\rho\delta} (p_2 - p_4)_{\rho} p_{5\delta} \]
\[ (1 - \frac{8}{3}s_w^2)C_0 G(p_2 - p_4, M_Z)G(p_2 - p_4 - p_5, 0) \]
\[ \bar{\nu}_{e^-} (p_4) \gamma_{\mu} (\frac{1}{2}L + s_w^2) u_{e^-} (p_2) \bar{\nu}_{e^+} (p_1) \gamma_{\nu} v_{e^+} (p_3) \]
\[ M^a_{Z\gamma \Pi^0_e} = iN_c \frac{2\pi}{3} \frac{\alpha_e^2}{c_w^2 s_w^2} \frac{\tan \beta}{v_w} m_t^2 (1 - \varepsilon) \varepsilon_{\mu\nu\rho\delta} (p_2 - p_4)_{\rho} p_{5\delta} \]
\[ [(1 - \frac{8}{3}s_w^2)^2 - 1]C_0 - 2C_{11} \}G(p_2 - p_4, M_Z) \]
\[ G(p_2 - p_4 - p_5, M_Z) \bar{\nu}_{e^-} (p_4) \gamma_{\mu} (\frac{1}{2}L + s_w^2) u_{e^-} (p_2) \bar{\nu}_{e^+} (p_1) \gamma_{\nu} v_{e^+} (p_3) \]
\[ M^b_{\gamma\gamma \Pi^0_e} = -iQ_f N_c \frac{2\pi}{3} \frac{\alpha_e^2}{c_w^2 s_w^2} \frac{\tan \beta}{v_w} m_t^2 (1 - \varepsilon) \varepsilon_{\mu\nu\rho\delta} (p_1 + p_2)_{\rho} \]
\[ p_{5\delta} C_0^f G(p_1 + p_2, 0)G(p_1 + p_2 - p_5, 0) \]
\[ \bar{\nu}_{e^+} (p_1) \gamma_{\mu} u_{e^-} (p_2) \bar{\nu}_{e^+} (p_1) \gamma_{\nu} v_{e^+} (p_3) \]
\[ M^b_{\gamma\gamma \Pi^0_e} = -iN_c \frac{2\pi}{3} \frac{\alpha_e^2}{c_w^2 s_w^2} \frac{\tan \beta}{v_w} m_t^2 (1 - \varepsilon) \varepsilon_{\mu\nu\rho\delta} (p_1 + p_2)_{\rho} \]
\[ p_{5\delta} (1 - \frac{8}{3}s_w^2)C_0^f G(p_1 + p_2 - p_5, M_Z) \]
\[ G(p_1 + p_2, 0) \bar{\nu}_{e^+} (p_1) \gamma_{\mu} u_{e^-} (p_2) \]
\[ \bar{\nu}_{e^+} (p_1) \gamma_{\nu} (aL + b) v_{e^+} (p_3) \]
\[ M^b_{Z\gamma \Pi^0_e} = iQ_f N_c \frac{2\pi}{3} \frac{\alpha_e^2}{c_w^2 s_w^2} \frac{\tan \beta}{v_w} m_t^2 (1 - \varepsilon) \varepsilon_{\mu\nu\rho\delta} (p_1 + p_2)_{\rho} \]
\[ p_{5\delta} (1 - \frac{8}{3}s_w^2)C_0^f G(p_1 + p_2 - p_5, 0)G(p_1 + p_2, M_Z) \]
\[ \bar{\nu}_{e^+} (p_1) \gamma_{\mu} (\frac{1}{2}L + s_w^2) u_{e^-} (p_2) \]
\[ \bar{\nu}_{e^+} (p_1) \gamma_{\nu} (aL + b) v_{e^+} (p_3) \]
\[ M^b_{Z\gamma \Pi^0_e} = iQ_f N_c \frac{2\pi}{3} \frac{\alpha_e^2}{c_w^2 s_w^2} \frac{\tan \beta}{v_w} m_t^2 (1 - \varepsilon) \varepsilon_{\mu\nu\rho\delta} (p_1 + p_2)_{\rho} p_{5\delta} \]
\[ [(1 - \frac{8}{3}s_w^2)^2 - 1]C_0 - 2C_{11} \}G(p_1 + p_2, M_Z) \]
\[ G(p_1 + p_2 - p_5, M_Z) \bar{\nu}_{e^+} (p_1) \gamma_{\mu} (\frac{1}{2}L + s_w^2) u_{e^-} (p_2) \]
\[ \bar{\nu}_{e^-} (p_2) \bar{\nu}_{e^+} (p_1) \gamma_{\nu} v_{e^+} (p_3) \]
\[ M^b_{Z\gamma \Pi^0_e} = iQ_f N_c \frac{2\pi}{3} \frac{\alpha_e^2}{c_w^2 s_w^2} \frac{\tan \beta}{v_w} m_t^2 (1 - \varepsilon) \varepsilon_{\mu\nu\rho\delta} (p_1 + p_2)_{\rho} \]
\[ q_3 \bar{q}_3 \] represents the third generation quarks \( t, b \).
\[ M_Z^2 = -4\pi \frac{\alpha_e}{s_h^2 s_w^2} \tan \beta \frac{1}{v_w} m_{q_3}^* G(p_4 + p_5 + m_{q_3}) G(p_1 + p_2), \]
\[ M_Z^d \overline{p}_{q_3}(p_4) \gamma_5 (p_4 + p_5 + m_{q_3}) \gamma^\mu (aL + b) \]
\[ v_{q_3}(p_3) \overline{p}_e^+(p_1) \gamma_\mu \left( -\frac{1}{2} L + s_h^2 \right) u_e^-(p_2) \]
\[ M_Z^d = -4Qf \frac{\alpha_e}{v_w} m_{q_3}^* G(p_3 + p_5 + m_{q_3}) \]
\[ G(p_1 + p_2, 0) \overline{p}_{q_3}(p_4) \gamma_5 (p_4 + p_5 + m_{q_3}) \gamma^\mu u_{q_3}(p_3) \overline{p}_e^+(p_1) \gamma_\mu \left( -\frac{1}{2} L + s_h^2 \right) u_e^-(p_2) \]
\[ M_Z^d = 4Qf \frac{\alpha_e}{v_w} m_{q_3}^* G(p_3 + p_5 + m_{q_3}) G(p_1 + p_2), \]
\[ M_Z^d \overline{p}_{q_3}(p_4) \gamma^\mu (aL + b)(p_4 + p_5 + m_{q_3}) \gamma_5 u_{q_3}(p_3) \overline{p}_e^+(p_1) \gamma_\mu \left( -\frac{1}{2} L + s_h^2 \right) u_e^-(p_2) \]
\[ M_Z^d = -2\pi \frac{\alpha_e}{s_h^2 s_w^2} \tan \beta \frac{1}{v_w} m_{q_3}^* G(p_3 + p_4 + m_h) G(p_1 + p_2), \]
\[ M_Z^d \overline{p}_e^+(p_1) (p_4 - p_3 - p_4) u_e^-(p_2) \]
\[ \overline{p}_{q_3}(p_4) \overline{p}_{q_3}(p_5) \]

where \( L = \frac{1}{2}(1 - \gamma_5) \), \( G(p,m) = \frac{1}{m - m^*} \) denotes the propagator of the particle and \( C_0 = C_0(-p_2 + p_1, p_2, m_t, m_b, m_{q_3}), C_0' = C_0(-p_1 - p_2, p_1, m_t, m_b, m_{q_3}). m_{q_3} \) represents the masses of the third generation quarks and \( m_{q_3}^* \) denotes \( m_t^* \) and \( m_b^* \). \( m_0^* \) is induced by topcolor interaction and \( m_b^* \) is the b quark mass produced by instanton. The values of \( Qf, a, b \) are taken as following:

| f | Qf | a | b |
|---|---|---|---|
| up-quarks(u,c,t) | 1/2 | 1/2 | -1/2 |
| down-quarks(d,s,b) | -1/3 | -1/3 | -1/2 |
| leptons(e,µ,τ) | -1 | -1 | -1 |

The production amplitudes for different processes are:

\[ M_{e^+ e^- \to \Pi^0} = M^a + M^b \quad (e^+ e^- \to e^+ e^- \Pi^0) \]
\[ M_{q_3 \bar{q}_3 \Pi^0} = M^b + M^c + M^d + M^e \quad (e^+ e^- \to q_3 \bar{q}_3 \Pi^0) \]
\[ M_{f \bar{f} \Pi^0} = M^b + M^c \quad (f' = u, d, c, s, µ, τ) \quad (e^+ e^- \to f' \bar{f'} \Pi^0) \]

With the above production amplitudes, we can obtain the production cross sections directly.

### III. NUMERICAL RESULTS AND CONCLUSIONS

In our calculation, we take \( m_e = 0, m_\mu = 0.105 \) GeV, \( m_\tau = 1.784 \) GeV, \( m_\nu = 0.005 \) GeV, \( m_d = 0.009 \) GeV, \( m_c = 4.14 \) GeV, \( m_s = 0.15 \) GeV, \( m_b = 174 \) GeV, \( m_{q_3} = 4.9 \) GeV, \( M_Z = 91.187 \) GeV, \( v_1 = 60 \) GeV and \( s_h^2 = 0.15 \). The electromagnetic fine-structure constant \( \alpha_e \) at a certain energy scale is calculated from the simple QED one-loop evolution with the boundary value \( \alpha_e = \frac{1}{137} \). There are three free parameters \( \epsilon, M_{H_1}, s \) in the cross sections. To see the influence of these parameters on the cross sections, we take the mass of the top-pion \( M_{H_1} \) to vary in the range of \( 150 \text{ GeV} \leq M_{H_1} \leq 450 \text{ GeV} \) and \( \epsilon = 0.03, 0.06, 0.1 \), respectively. Considering the center-of-mass energy \( \sqrt{s} \) in the planned \( e^+ e^- \) linear colliders (for example, TESLA), we take \( \sqrt{s} = 800 \) GeV and 1600 GeV, respectively.

For the process \( e^+ e^- \to e^+ e^- \Pi^0 \), we can see that there exists a t-channel resonance effect induced by the photon propagator in fig.1(a) and an s-channel resonance effect induced by the Z boson propagator in fig.1(b). So we should take into account the effect of the width of the Z boson in the calculations, i.e. we should take the complex mass term \( M_Z^2 - iM_Z \Gamma_Z \) instead of the simple Z boson mass term \( M_Z^2 \) in the Z boson propagator. Here, we take \( \Gamma_Z = 2.49 \) GeV. All the resonance effects will enhance the cross section significantly. We find that the contribution to the cross section of \( e^+ e^- \to e^+ e^- \Pi^0 \) arises mainly from Fig.1(a). The final numerical results of the cross section are summarized in Figs 2. The figure is the plots of the cross section as a function of \( M_{H_1} \). One can see that there is a peak in the plot when \( M_{H_1} \) is close to 350 GeV, which arises from the top quark triangle loop. The largest cross sections are 1.78 fb and 4.15 fb, when we take \( \epsilon = 0.03 \) and \( \sqrt{s} = 800 \) and 1600 GeV, respectively. The cross section increases with \( \sqrt{s} \). With a luminosity of 100 fb\(^{-1}\)/yr, there are several tens or even hundreds of \( \Pi^0 \) events to be produced with the rate \( e^+ e^- \to e^+ e^- \Pi^0 \). For \( M_{H_1} \leq 350 \) GeV, the dominate decay channel of \( \Pi^0 \) is \( \Pi^0 \to \ell \ell \). As has been investigated in Ref. [17], the branching ratio Br(\( \Pi^0 \to \ell \ell \)) can reach about 60%. Because there is no tree level flavor-changing neutral current in the SM, the background of \( e^+ e^- \to e^+ e^- \Pi^0 \to e^+ e^- \ell \ell \) is very clean. Therefore, \( e^+ e^- \to e^+ e^- \Pi^0 \to e^+ e^- \ell \ell \) is an ideal channel to detect the neutral top-pion with small top-pion mass. For \( M_{H_1} \geq 350 \) GeV, \( \Pi^0 \to \ell \ell \) is permitted and the total width of \( \Pi^0 \) increases significantly. The branching ratio of the decay channel \( \Pi^0 \to \ell \ell \) is close to 100%, all other decay modes may be ignored. In this case, we should detect \( \Pi^0 \) through the \( \ell \ell \) channel.
The numerical results of the process $e^+e^- \rightarrow \bar{t}\bar{t}\Pi^0_0$ are shown in Fig.3. The contributions arise from

The results show that although the coupling of $\Pi^0_0\bar{t}\bar{t}$ in Fig.1(c)(d) can enhance the cross section to the level of a few fb. When $\sqrt{s}$ = 800 GeV, the cross section will be much smaller than that for $\sqrt{s}$ = 800 GeV and it hardly varies with $M_{\Pi}$. So we only draw up the plots for the case of $\sqrt{s}$ = 800 GeV. The plots show that the cross section is not sensitive to $M_{\Pi}$. The neutral top-pion can be more easily detected via the process $e^+e^- \rightarrow b\bar{b}\Pi^0_0$ than via the process $e^+e^- \rightarrow t\bar{t}\Pi^0_0$ with b-tagging. Therefore, the process $e^+e^- \rightarrow b\bar{b}\Pi^0_0$ provides us with another useful way to search for a neutral top-pion.

For the other processes $e^+e^- \rightarrow f'\bar{f}'\Pi^0_0$ ($f' = u, d, c, s, \mu, \tau$), the largest cross section could only be up to 0.01 fb. Therefore, we could hardly detect the neutral top-pion through these processes. So we do not discuss these processes in detail.

In conclusion, we have studied some neutral top-pion production processes $e^+e^- \rightarrow f\bar{f}\Pi^0_0$ ($f = u, d, c, s, t, b, e, \mu, \tau$) at the future $e^+e^-$ colliders in the framework of the TC2 model. We find that there are the
following features for these processes: (i) The cross sections of the processes \( e^+e^- \rightarrow f\bar{f}\Pi_0^0 \) \((f' = u, d, c, s, \mu, \tau)\) are too small to detect a neutral top-pion. (ii) Due to the effect of the top quark triangle loops, there exists a narrow peak in the cross section plots of the process \( e^+e^- \rightarrow e^+e^-\Pi_0^0 \). Because of the resonance effect, the cross section of \( e^+e^- \rightarrow e^+e^-\Pi_0^0 \) could be up to a few fb and a few tens or even hundreds of \( \Pi_0^0 \) events can be produced, which causes the neutral top-pion to become experimentally detectable through the flavor-changing \( t\bar{t}e \) channel with the clean background. (iii) The cross sections of both \( e^+e^- \rightarrow b\bar{b}\Pi_0^0 \) and \( e^+e^- \rightarrow t\bar{t}\Pi_0^0 \) can reach the level of a few fb, the strong coupling \( t\bar{t}\Pi_0^0 \) in the process \( e^+e^- \rightarrow t\bar{t}\Pi_0^0 \) and the resonance effect of the \( h_t \) propagator in the process \( e^+e^- \rightarrow b\bar{b}\Pi_0^0 \) could enhance the cross sections significantly. These two processes provide us with another way to search for a neutral top-pion. Therefore, our studies could provide a direct way to test the TC2 model by detecting top-pion signals.

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