THERMAL STABILITY OF COLD CLOUDS IN GALAXY HALOS

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ABSTRACT

We consider the thermal properties of cold, dense clouds of molecular hydrogen and atomic helium. For cloud masses below $10^{-17} M_\odot$, the internal pressure is sufficient to permit the existence of particles of solid or liquid hydrogen at temperatures above the cosmic microwave background temperature. Optically thin thermal continuum emission by these particles can balance cosmic-ray heating of the cloud, leading to equilibria that are thermally stable even though the heating rate is independent of cloud temperature. For the Galaxy, the known heating rate in the disk sets a minimum mass of order $10^{-6} M_\odot$ necessary for survival. Clouds of this type may in principle comprise most of the dark matter in the Galactic halo. However, we caution that the equilibria do not exist at redshifts $z \geq 1$ when the temperature of the microwave background was substantially larger than its current value; therefore, the formation and the survival of such clouds to the present epoch remain open questions.

Subject headings: dark matter — galaxies: halos — ISM: clouds

1. INTRODUCTION

Walker & Wardle (1998) show that a population of neutral, AU-sized clouds in the Galactic halo could be responsible for the “extreme scattering events” (ESEs) observed in the radio flux toward several quasars (Fiedler et al. 1987, 1994). In this model, the cloud surfaces are exposed to UV radiation from hot stars in the Galactic disk, producing a photoionized wind. When one of these clouds crosses the line of sight to a compact radio source, the flux varies as a result of refraction by the ionized gas (cf. Henriksen & Widrow 1995). This model explains the observed flux variations quite naturally, but if the clouds are self-gravitating, then the ESE rate implies that the cloud population comprises a significant fraction of the Galaxy’s mass.

This halo cloud population cannot contain much dust mixed with the gas since this would lead to optical extinction events of distant stars: either the clouds have extremely low metallicity or any dust grains have sedimented to the cloud center. Given this, several factors make the clouds difficult to detect (Pfenniger, Combes, & Martinet 1994): cold molecular hydrogen is, by and large, invisible; the clouds are small; they are transparent in most regions of the electromagnetic spectrum; and they cover by and large, invisible; the clouds are small; they are transparent in most regions of the electromagnetic spectrum; and they cover a small fraction of the sky. The clouds are not sufficiently compact to cause gravitational lensing toward the LMC, although Draine (1998) has shown that there is substantial optical refraction by the neutral gas, so that microlensing experiments (Paczyński 1996) already place useful constraints on the properties of low-mass halo clouds.

Given that this hypothesized cloud population does not violate observational constraints, the primary issues that need to be addressed are theoretical: (1) How and when did these clouds form? (2) How do they resist gravitational collapse? The second of these questions is addressed in this Letter.

We begin by writing down equations describing a simple “one-zone” model of a cloud, characterized by a single temperature and pressure (§ 2), and show that particles of solid H$_2$ may exist in the clouds (cf. Pfenniger & Combes 1994). At temperatures above the microwave background temperature, these particles cool the cloud by thermal continuum radiation, admitting equilibria in which this cooling balances the heating by cosmic rays. In § 3, we show that (for optically thin emission) these equilibria are thermally stable: if the cloud contracts, the coolant is destroyed by the increase in temperature, and the power deposited by cosmic rays causes the cloud to expand and the temperature to return to its original value. We conclude that, within the context of our one-zone model, the viable mass range for Galactic clouds is $10^{-6}$ to $10^{-17} M_\odot$.

2. CLOUD MODEL

Virial equilibrium implies that for a self-gravitating cloud characterized by mass $M$, temperature $T$, and radius $R$,

$$R \approx \frac{G M \mu k T}{v^2},$$

where $\mu$ is the mean molecular weight. Hydrostatic equilibrium implies that the pressure $P$ in the cloud is $\sim GM^2/R^4$; thus we write

$$P = \frac{q}{G M^2} \left(\frac{k T}{\mu}\right)^4,$$

where $q$ depends on the cloud’s structure. For polytropes, $q$ rises monotonically from 9.2 to 40 as the polytropic index runs from 3/2 to 9/2, so we adopt $q = 20$.

At sufficiently high pressures, a fraction $x$ of the molecular hydrogen assumes solid (or liquid) form. Then $\mu = (1-x + 2y)m/(1-x+y)$, where $m$ is the mass of an H$_2$ molecule, and $y \approx 1/6$ is the abundance ratio He : H$_2$ by number. Neglecting the temperature difference between the phases, in equilibrium, the partial pressure of H$_2$ equals the saturated vapor pressure, i.e.,

$$\frac{1-x}{1-x+y} P = \left(\frac{2\pi m}{h^2}\right)^{3/2} (k T)^{5/2} e^{-\gamma T/k T},$$

(valid for $0 < x < 1$), where $k T_c$ is the heat of vaporization for H$_2$ (Phinney 1985). With $T_c = 91.5 K$, the vapor pressure given by the right-hand side of equation (3) is within 20% of the available experimental data (Souers 1986).

Hydrogen grains can cool the gas in a manner similar to dust grains in molecular clouds: the gas cools via collisions with...
the model, this assumption represents one of our main areas of uncertainty. To calculate the cooling by solid \( \text{H}_2 \), let us first consider the net power radiated by a single particle (we employ an "escape probability" formulation of radiative transfer):

\[
L_p = 4 \sigma \left[ \frac{C(T)T^2_p}{1 + \epsilon} - \frac{C(T)T^2_p}{1 + \epsilon} \right],
\]

where \( C(T) \) is the Planck mean absorption cross section, \( T_p \) is the cosmic microwave background temperature, and \( \epsilon = \tau C(T)/C(T) \) are the Planck mean optical depths of the cloud for thermal radiation characterized by \( T \) and \( T_p \), respectively. Assuming that the particle size is \( \ll \lambda \) (\( \sim 0.1 \) cm at the temperatures of relevance here), we may write \( C(T) = C_n(T)m_p \), with \( m_p \) being the particle mass, and for spherical grains, we have (Draine & Lee 1984)

\[
C_n(T) \approx \frac{15(4\pi)^3}{28\rho_t} \frac{\lambda_2}{(\epsilon_2 + 2)^2} \left( \frac{kT}{\hbar c} \right)^2,
\]

where \( \rho_t = 0.087 \) g cm\(^{-3} \) is the density of solid \( \text{H}_2 \) (Souers 1986), the complex dielectric function of the solid is \( \epsilon_2 + i\epsilon_2 \), and we have assumed that \( \epsilon_2 = \lambda_2/\lambda \) as expected at low frequencies. The net cooling rate per unit mass of cloud material (gas and solid) is then

\[
\Lambda = \frac{4\pi R^2 \sigma}{M} \left( \frac{\tau^4 \epsilon_2 T^4_p}{1 + \tau} \right) - \tau \frac{C_n}{1 + 2y} \frac{M}{\pi R^2}.
\]

where \( \tau = \tau(T_p/T)^2 \), and

\[
\tau = \frac{C_n x}{1 + 2y} \frac{M}{\pi R^2}.
\]

To evaluate \( \Lambda \), we require optical constants for solid \( \text{H}_2 \) in the microwave. The particles are expected to be almost pure para-hydrogen as an ortho-para mixture of the solid relaxes to para \( (J = 0) \) form in a few days (Souers 1986). The low-frequency value of \( \epsilon_2 \) for para-hydrogen has been measured (Souers 1986) as \( \epsilon_2 \approx 1.25 \); the low-frequency limit of \( \epsilon_2 \) is less certain. Jochemsen et al. (1978) measured the extinction coefficient of a single crystal of solid para-hydrogen in the region of interest \( (\lambda \sim 0.1 \) cm \) but could not determine whether this continuum extinction was due to absorption or to scattering within the crystal. Because these measurements do not conform to the anticipated low-frequency behavior (proportional to \( 1/\lambda \)) and because absorption bands are not expected below the \( \lambda(0) \) line, it is likely that the absorption of pure crystalline \( \text{H}_2 \) is much smaller than the measured extinction, and we can only infer a limit: \( \epsilon_2 \leq 1.8 \times 10^{-3} \). However, the low-frequency absorption of solid \( \text{H}_2 \) grains could be strongly enhanced by impurity species and lattice defects. For the purposes of this Letter, we adopt \( \epsilon_2 = \lambda_2/\lambda \) and \( \lambda_2 = 10^{-5} \) cm. Within the confines of the model, this assumption represents one of our main areas of uncertainty.

For a given cloud mass, the fraction of \( \text{H}_2 \) in the solid phase can be determined from \( T \) using equations (2) and (3). This allows the cooling by solid particles to be calculated as a function of \( T \). The top panel in Figure 1 illustrates this for a cloud of mass \( 10^{-3} M_\odot \). For comparison, the cooling contributed by rotational lines of gas-phase \( \text{H}_2 \), HD, and LiH is also plotted; energies and \( A \)-values are from Turner, Kirby-Docken, & Dal-garno (1977), Abgrall, Roueff, & Viala (1982), and Gianturco et al. (1996). The adopted deuterium and LiH abundances are \( 3 \times 10^{-4} \) and \( 1.2 \times 10^{-10} \), respectively (Schramm & Turner 1998). We employ an escape-probability formulation of radiative transfer, in which the optically thin cooling rates are divided by \( (1 + \tau) \), where \( \tau \) is the optical depth at line center (rotational transitions) or the Planck mean (continuum emission).

There is a critical temperature \( T_c \) at which \( \text{H}_2 \) in the cloud lies on the border between the solid and gaseous phases; i.e., the partial pressure of \( \text{H}_2 \) is equal to its saturated vapor pressure. From this point, \( x \) and \( \Lambda \) increase precipitously as \( T \) is reduced, until \( \tau \approx 1 \), and the cooling rate approximates that of a blackbody. \( \Lambda \) then increases more slowly, peaking when \( x \approx 1/2 \) and subsequently dropping to zero as the temperature approaches that of the microwave background. At lower temperatures, the cloud is heated by the background radiation.

The solid \( \text{H}_2 \) cooling curves for cloud masses of \( 10^{-3} - 10^{-5}, \) and \( 10^{-7} M_\odot \) are compared in the bottom panel of Figure 1. Decreasing the cloud mass has two consequences: the critical temperature \( T_c \) increases, whereas the maximum value of \( \Lambda \) decreases (because, in this circumstance, the emission is optically thick and the cloud surface area is proportional to \( M^2 \) at a given temperature). For cloud masses \( \leq 10^{-7} M_\odot \), \( T_c \) is above the \( \text{H}_2 \) triple point (13.8 K), and liquid droplets of \( \text{H}_2 \)
form instead. This does not affect the calculations qualitatively since the density and saturated vapor pressure of the liquid are within 50% of those of the solid for \( T \approx 20 \) K (Souers 1986), and the optical properties are similar (in the sense that \( \varepsilon_g \) is small and uncertain); thus, in Figure 1, we continue the cooling curve for a \( 10^{-3} M_\odot \) cloud above the triple point as a dotted curve. Thermally stable solutions do not exist for masses \( \lesssim 10^{-5} M_\odot \) since the partial pressure of \( \text{H}_2 \) exceeds the saturated vapor pressure unless \( x > 0.5 \) (see § 3). On the other hand, for \( M \approx 10^{-17} M_\odot \), \( T \) is below the cosmic microwave background temperature, and the solid phase warms the cloud rather than cooling it.

### 3. THERMAL EQUILIBRIUM

In thermal equilibrium, the cloud temperature is set by the balance between cooling and heating. We assume that clouds in the Galactic halo are heated primarily by cosmic rays. The local interstellar cosmic-ray ionization rate in the Galactic disk, \( \approx 3 \times 10^{-7} \text{ s}^{-1} \text{ H}^{-1} \) (Webber 1998), implies a heating rate of \( \Gamma \approx 3 \times 10^{-4} \text{ ergs g}^{-1} \text{ s}^{-1} \) (Cravens & Dalgarno 1978). The cosmic-ray heating in the halo is uncertain but should be somewhat lower, say, \( \approx 10^{-5} \text{ ergs g}^{-1} \text{ s}^{-1} \). In Figure 2, we show the cooling rates for clouds of masses from \( 10^{-3} \) to \( 10^{-17} M_\odot \). The solid curves are contours of constant optical depth; the dashed curve shows the optically thick limit and represents the maximum cooling rate for each cloud mass. It appears that solid hydrogen can provide the necessary cooling for planetary-mass gas clouds at the cosmic-ray heating rates expected in the Galactic disk and halo.

The top panel of Figure 1 shows that there are typically three equilibrium temperatures available for cloud masses between \( 10^{-3} \) and \( 10^{-17} M_\odot \) at the expected heating rates: solid \( \text{H}_2 \) provides one barely above \( T_\text{s} \) and one a few degrees higher; the gas-phase coolants provide an equilibrium above 30 K. We now show that thermal stability requires \( \Lambda \) to be a decreasing function of \( T \), and therefore only the second of these three equilibria is stable.

In virial equilibrium, the total energy per unit mass is approximately \(-3(3/2)kT/\mu\) (the internal excitation of the gas is negligible at the low temperatures of interest here), so the thermal evolution of the cloud is determined by

\[
\frac{3k}{2\mu} \frac{dT}{dt} = \Lambda - \Gamma.
\]  

(8)

In the absence of heating, the cloud contracts on the Kelvin-Helmholtz timescale \( t_{\text{KH}} = (3/2)kT/(\mu\Lambda) \). Note that this timescale can be a substantial fraction of the Hubble time for temperatures of a few kelvins: the dashed curves in Figure 1 show the cooling rate that yields \( t_{\text{KH}} \approx 10 \) Gyr in thermal equilibrium, cosmic-ray heating replaces the energy radiated away by the cloud, implying, for example, \( t_{\text{KH}} \approx 2 \times 10^6 \) yr for \( \Gamma \approx 10^{-5} \text{ ergs g}^{-1} \text{ s}^{-1} \) at \( T \approx 10 \) K. This is much greater than the sound-crossing time (~10^2 yr), so the response of a cloud to dynamical perturbations is adiabatic, to a good approximation, and dynamical stability is assured. However, equation (8) shows that perturbations to the cloud temperature grow or decay as \( e^{\alpha t} \), where

\[
\alpha = t_{\text{KH}}^{-1} \frac{T}{\Lambda} \frac{d}{dT} (\Lambda - \Gamma),
\]  

(9)

and the right-hand side of this equation is evaluated at the equilibrium temperature. Thus, a cloud is thermally stable only if a decrease (increase) in cloud temperature leads to cooling outstripping (lagging) heating. For cosmic-ray heating, \( \Gamma \) is independent of \( T \) if the column through the cloud is insufficient to cause significant attenuation of cosmic rays (changes in temperature affect the cloud’s column density through the virial relationship \( R \propto 1/T \)). Thermal stability then requires that \( \Lambda \) be a decreasing function of \( T \), and we conclude that only the equilibrium on the high-temperature shoulder of the solid hydrogen cooling curve is stable. In fact, the column density of each cloud (~10^2 g cm^-2; Walker 1999) is sufficient to stop sub-GeV cosmic-ray protons (and all electrons), leading to a dependence of \( \Gamma \) on \( T \); this dependence is too weak to affect our conclusions concerning stability.

### 4. DISCUSSION

The suggestion that cold gas could comprise a significant fraction of the Galaxy’s dark matter is not new. Previous proposals include a fractal medium in the outer reaches of the Galactic disk (Pfenniger et al. 1994), isolated halo clouds (Gerhard & Silk 1996), and miniclusters of clouds in the halo (De Paolis et al. 1995; Gerhard & Silk 1996). However, to date, there has been no compelling reason to believe that isolated, cold gas clouds—as inferred by Walker & Wardle (1998)—could support themselves for long periods against gravitational collapse. We have shown that such clouds can be stabilized by the precipitation/sublimation of particles of solid hydrogen (or by the condensation/evaporation of droplets of liquid hydrogen) if these particles dominate the radiative cooling of the cloud. The key feature that confers thermal stability is that these particles are destroyed; hence, cooling becomes less efficient as the cloud temperature increases. This feature will be present in any model where condensed hydrogen is the principal coolant, and consequently we expect that more sophisticated structural treatments will also admit stable solutions.

The masses of thermally stable clouds lie in the approximate range from \( 10^{-3} \) to \( 10^{-17} M_\odot \). The lower limit is increased to \( 10^{-6} M_\odot \) if subject to cosmic-ray heating similar to that in the Galactic disk for an interval \( \approx kT/\Gamma \approx 10^3 \) yr. Since halo clouds take much longer than this to pass through the cosmic-ray disk, this limit is appropriate even for a halo cloud population. For cloud masses in the range from \( 10^{-6} \) to \( 10^{-17} M_\odot \), the radiative cooling simply readjusts, on the timescale \( t_{\text{KH}} \).
maintain equilibrium as \( \Gamma \) varies through the orbit (∼10⁶ yr) of a cloud around the Galaxy.

The typical particle radius \( a \) is constrained by the requirement that the clouds not produce significant extinction at optical wavelengths: the geometrical optical depth of the particles, \( \tau \approx \pi C_{\text{m}} a n / \lambda \), should be less than 1. Adopting \( T = 5 \) K and \( \tau = 0.01 \), this translates to \( a (\text{mm}) \geq (10^{-4} \text{ cm}) / \lambda_{\gamma} \). Millimeter-size particles settle at the center of the cloud in \( \sim 10^6 \) yr; this time is shortened if \( \lambda_{\gamma} \) is significantly less than \( 10^{-4} \text{ cm} \), and the particles are required to be larger. Settling may be counteracted by convective motions or sublimation resulting from the higher temperatures deeper within the cloud—issues that must await a more sophisticated treatment of the cloud structure.

If the hypothesized population of clouds exists, their thermal microwave emission may be detectable as a Galactic continuum background at temperatures just above the cosmic microwave background; a Galactic component of this kind has in fact been isolated in the COBE Far-Infrared Absolute Spectrometer (FIRAS) data (Reach et al. 1985). One would like to compare these data with the theory presented here, but it is difficult to predict the total microwave intensity for our model because the distribution of cosmic-ray density away from the Galactic plane is only loosely constrained (see Webber et al. 1994). A similar uncertainty afflicts the modeling of \( \gamma \)-ray production from barionic material in the Galactic halo (cf. De Paolis et al. 1995, 1999; Salati et al. 1999; Kalberla, Shchekinov, & Dettmar 1999). Nevertheless, microwave and \( \gamma \)-ray emissivities are each proportional to the local cosmic-ray flux (assuming the cosmic-ray spectrum does not vary greatly), so we can write \( I_\gamma = I_j \gamma / I_j \), for emissivities \( j \) and intensities \( I_j \). At high latitudes, the Galactic \( \gamma \)-ray background is \( I_\gamma \sim 10^{-6} \text{ photons cm}^{-2} \text{ sr}^{-1} \), above 1 GeV (Dixon et al. 1998); local to the Sun, the corresponding (optically thin) emissivity is \( 1.1 \times 10^{-3} \text{ photons s}^{-1} \text{ g}^{-1} \text{ sr}^{-1} \) (Bertsch et al. 1993). Thus, for a cosmic-ray heating rate (again, local to the Sun) of \( \Gamma = 4\pi j_{\gamma} \sim 3 \times 10^{-4} \text{ ergs s}^{-1} \text{ g}^{-1} \) (Cravens & Dalgarno 1978), we expect \( I_\gamma \sim 2 \times 10^{-8} \text{ ergs cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \). This is roughly 1% of that observed in the FIRAS cold component at high latitude (Reach et al. 1985), so the microwave data do not exclude the possibility of a cold cloud population heated by cosmic rays. Sciana (1999) proposed that all of the cold excess may be accounted for by cosmic-ray heating of cold clouds, but this appears to be based on an overestimate of the \( \gamma \)-ray flux and an underestimate of the high-latitude FIRAS flux.

## 5. Conclusions

We have demonstrated that, by virtue of the solid/gas-phase transition of hydrogen, cold planetary-mass Galactic gas clouds can be thermally stable even when they are heated at a temperature-independent rate. Our analysis applies to the present epoch, with the microwave background temperature at \( T_b < 3 \) K; for background temperatures \( T_b > 6 \) K, our model admits no stable mass range. Consequently, the longevity of the clouds at redshifts \( z \approx 1 \) is problematic. However, we cannot hope to address this issue until a firm theoretical basis for the formation of such clouds has been established.

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## References

Abgrall, H., Roueff, E., & Viala, Y. 1982, A&AS, 50, 505

Bertsch, D. L., Dame, T. M., Fichtel, C. E., Hunter, S. D., Sreekumar, P., Stacy, J. G., & Thaddeus, P. 1993, ApJ, 416, 587

Cravens, T. E., & Dalgarno, A. 1978, ApJ, 219, 750

De Paolis, F., Ingrisso, G., Jetzer, Ph., & Roncadelli, M. 1995, A&A, 295, 567

De Paolis, F., Ingrisso, G., Jetzer, Ph., & Roncadelli, M. 1995, A&A, 295, 567

Draine, B. T. 1998, ApJ, 509, L41

Draine, B. T., & Lee, M. H. 1984, ApJ, 285, 89

Fiedler, R. L., Dennis, B., Johnston, K. J., & Hewish, A. 1987, Nature, 326, 675

Fiedler, R., Dennison, B., Johnston, K. J., Waltham, E. B., & Simon, R. S. 1994, ApJ, 430, 581

Gerhard, O., & Silk, J. 1996, ApJ, 472, 34

Gianturco, F. A., Giorgi, P. G., Berriche, H., & Gadea, F. X. 1996, A&AS, 117, 377

Henriksen, R. N., & Widrow, L. M. 1995, ApJ, 441, 70

Jochens, R., Berlisnky, A. J., Verspaandonk, F., & Silvera, I. F. 1978, J. Low Temp. Phys., 32, 185

Kalberla, P. M. W., Shchekinov, Yu. A., & Dettmar, R. J. 1999, A&A, 350, L9

Paczynski, B. 1996, ARA&A, 34, 419

Pfenniger, D., & Combes, F. 1994, A&A, 285, 94

Pfenniger, D., Combes, F., & Martinet, L. 1994, A&A, 285, 79

Phinney, E. S. 1985, preprint

Reach, W. T., et al. 1995, ApJ, 451, 188

Salati, P., Chardonnet, P., Luo, X. C., Silk, J., & Taillet, R. 1996, A&A, 313, 1

Schramm, D. N., & Turner, M. S. 1998, Rev. Mod. Phys., 70, 303

Sciana, D. W. 1999, MNRAS, submitted (astro-ph/9906159)

Sousers, P. C. 1986, Hydrogen Properties for Fusion Energy (Berkeley: Univ. California Press)

Turner, J., Kirby-Docken, K., & Dalgarno, A. 1977, ApJS, 35, 281

Walker, M. 1999, MNRAS, 308, 551

Walker, M., & Wardle, M. 1998, ApJ, 498, L125

Webber, W. R. 1998, ApJ, 506, 329

Webber, W. R., Binns, W. R., Crary, D., & Westphall, M. 1994, ApJ, 429, 764