Universal dielectric response across a continuous metal-insulator transition

Prosenjit Haldar,1,2,* M. S. Laad,2,† and S. R. Hassan2,†

1Centre for Condensed Matter Theory, Department of Physics,
Indian Institute of Sciences, Bangalore 560012, India
2Institute of Mathematical Sciences, Taramani, Chennai 600113, India and
Homi Bhabha National Institute Training School Complex, Anushakti Nagar, Mumbai 400085, India

A wide range of disordered materials, including disordered correlated systems, show “Universal Dielectric Response” (UDR), followed by a superlinear power-law increase in their optical responses over exceptionally broad frequency regimes. While extensively used in various contexts over the years, the microscopics underpinning UDR remains controversial. Here, we investigate the optical response of the simplest model of correlated fermions, Falicov-Kimball model (FKM), across the continuous metal-insulator transition (MIT) and analyze the associated quantum criticality in detail using cluster expansion of dynamical mean field theory (CDMFT). Surprisingly, we find that UDR naturally emerges in the quantum critical region associated with the continuous MIT. We tie the emergence of these novel features to a many-body orthogonality catastrophe accompanying the onset of strongly correlated electronic glassy dynamics close to the MIT, providing a microscopic realization of Jonscher’s time-honored proposal as well as a rationale for similarities in optical responses between correlated electronic matter and canonical glass formers.

I. INTRODUCTION

Optical conductivity has long been used to characterize elementary excitations in condensed matter. Response of matter to ac electromagnetic fields is usually encoded in the complex conductivity [1], \( \sigma_{xx}(\omega) = \sigma'(\omega) + i\sigma''(\omega) \) or the complex dielectric constant, \( \epsilon(\omega) \), related to each other by \( \sigma'(\omega) = \omega \epsilon(\omega) \epsilon''(\omega) \), where \( \epsilon'(\omega) \) quantifies the dielectric loss, and \( \epsilon_0 \) is the permittivity of free space. Optical studies have been especially valuable in strongly correlated electronic matter [2] and, as a particular example, have lead to insights into breakdown of traditional concepts in cuprates [3].

Such studies have also led to much progress in understanding of complex charge dynamics in disordered matter. In the seventies, pioneering work of Jonscher [4, 5] showed a universal dielectric response (UDR) of disordered quantum matter to ac electromagnetic fields, wherein \( \sigma_{xx}(\omega) \sim \omega^\alpha \) with \( \alpha \leq 1 \) in the sub-GHz regime. More recently, Lunkenheimer et al. [6] report astonishingly similar responses in a wide class of disordered matter over a more extended energy window: among others, doped, weakly and strongly correlated semiconductors exhibit UDR, followed by a superlinear power-law increase in \( \sigma(\omega) \), bridging the gap between classical dielectric and infra-red regions. That this behavior is also common to dipolar and ionic liquids as well as to canonical glass formers. Very recently, materials which belong to the elusive class of spin liquids [7] have also been interpreted in terms of UDR: in this case, it is possible that intrinsic disorder, arising from geometric frustration, is implicated in emergence of UDR. This suggests involvement of a deeper, more fundamental and common element, related to onset of a possibly intrinsic, glassy dynamics in emergence of UDR. In the context of correlated quantum matter (such as the Mott insulator \( \text{LaTiO}_3 \) and \( \text{Pr}_{0.65}(\text{Ca}_{0.8}\text{Sr}_{0.2})_{0.35}\text{MnO}_{0.35} \) (PC-SMO) [6]), such unconventional “glassy” dynamics can emerge near the doping-induced MIT as a consequence of substitutional and/or intrinsic disorder due to inhomogeneous electronic phase(s) near the MIT. On the other hand, early on, Jonscher himself suggested the relevance of many-body processes akin to the seminal Anderson orthogonality catastrophe (AOC) for UDR. Thus, the link between AOC and an emergent, slow glassy dynamics underlying the electronic processes leading to UDR in disordered, interacting electronic systems remains a challenging and largely unaddressed issue for theory, to our best knowledge.

Motivated hereby, we investigate these issues by a careful study of the optical response of the FKM. The FKM is the simplest representative model of correlated electrons on a lattice, and possesses an exact solution within both DMFT [8] and its cluster extensions [9, 10]. Remarkably, it can be solved almost analytically, even in CDMFT [10], leading to enormous computational simplifications in transport studies [11–13]. Across a critical \( U \), the FKM is known to undergo a \( T = 0 \) continuous MIT of the Hubbard band-splitting type [8].

As found earlier for transport properties, it turns out that precise computation of the optical response for the FKM within two-site cellular-DMFT [14] is facilitated by the fact that: (i) explicit closed-form expressions for the cluster propagators, \( G(\mathbf{K}, \omega) \) with \( \mathbf{K} = (0, 0), (\pi, \pi) \) greatly reduces computational cost, even in CDMFT, and (ii) the cluster-resolved irreducible particle-hole vertex functions are negligibly small and we ignore them in the Bethe-Salpeter equations (BSE) for all conductivities, thanks to an almost rigorous symmetry argument [15]: upon a cluster-to-orbital mapping (as is implicit in our mapping of the two-site CDMFT to two, “\( S, P \)” channels [14]). In this “multi-orbital” scenario,
the irreducible vertex corrections entering the Bethe-Salpeter equation for the conductivity are still negligibly small \([16]\). Thus, the optical conductivity acquires a form similar to DMFT, but with an additional sum over cluster momenta (or the two “S” and “P” channels on the cluster).

The rest of this paper is organized as follows: In Sec. II we describe the model we study in this work and the calculation of optical conductivity using Cluster DMFT formalism. In Sec. III we present our numerical results and analyze (i) Mott-like quantum criticality in optical response using CDMFT and (ii) universal dielectric response across the MIT. We then tie the UDR to an emergent many-body orthogonality catastrophe in the FKM within our CDMFT approach. In Sec. IV, we discuss our findings in the context of real materials exhibiting UDR.

II. GENERAL FORMULATION OF OPTICAL CONDUCTIVITY WITHIN CLUSTER DMFT

The Hamiltonian of the spinless FKM model is:

\[
H_{FKM} = -t \sum_{<i,j>} (c_i^\dagger c_j + h.c) - \mu \sum_i n_{i,c} + U \sum_i n_{i,d} n_{i,c}
\]  

(1)

The Hamiltonian describes a band of dispersive fermions \((c, c^\dagger)\) interacting locally via a “Hubbard” type interaction with dispersionless \(d\)-fermions. Since \(v_i = U n_{i,d}\) is a random (binary) potential in the symmetry-unbroken phases of the FKM, Eq. 1 can also be viewed (as is long known \([17]\)) as a model of fermions in a random binary alloy potential.

In recent work \([14]\), we used a cluster extension of DMFT to solve FKM using the Dyson-Schwinger equation of motion technique. Remarkably, the cluster-local Green’s function in two-site cluster DMFT is obtained analytically, and reads

\[
\hat{G} = \begin{bmatrix} G_{00}(\omega) & G_{0\alpha}(\omega) \\ G_{\alpha 0}(\omega) & G_{\alpha \alpha}(\omega) \end{bmatrix}
\]

where, the matrix element \(G_{ij}(\omega)\), with \(i, j = 0, \alpha\),

\[
G_{ij}(\omega) = \left[ \frac{1 - \langle n_{\alpha d} \rangle - \langle n_{\alpha d} \rangle + \langle n_{0 d} n_{\alpha d} \rangle}{\xi_2(\omega)} + \frac{\langle n_{\alpha d} \rangle - \langle n_{0 d} n_{\alpha d} \rangle}{\xi_2(\omega) - U} \right] \left[ \frac{\delta_{ij} - \frac{F_2(\omega)}{(t - \Delta_0(\omega))(1 - \delta_{ij})}}{\xi_1(\omega) - U} \right] \left[ \frac{\delta_{ij} - \frac{F_1(\omega)}{(t - \Delta_0(\omega))(1 - \delta_{ij})}}{\xi_1(\omega) - U} \right]
\]

(2)

The optical conductivity within Cluster DMFT then reads

\[
\sigma'(\omega) = \sigma_0 \sum_{K \in [S,P]} \int_{-\infty}^{\infty} d\epsilon v^2(\epsilon) \rho_0^K(\epsilon) \times \int_{-\infty}^{\infty} d\tilde{\omega} A_K(\epsilon, \tilde{\omega}) A_K(\epsilon, \tilde{\omega} + \omega) \frac{f(\tilde{\omega}) - f(\tilde{\omega} + \omega)}{\omega}
\]

(4)

with

\[
A_K(\epsilon, \omega) = \text{Im} \left[ \frac{1}{\omega + \mu - \epsilon - \Sigma_K(\omega)} \right]
\]

(5)

Here, \(\rho_0^K(\epsilon)\) is non-interacting spectral function of the even and odd orbitals and \(f(\omega)\) is the Fermi distribution. This simplification allows comprehensive study of the optical response of the FKM within CDMFT, which we now describe.
III. RESULTS AND DISCUSSION

We consider the Bethe lattice with the halfbandwidth of the conduction electron (c-fermions) as unity ($2t = 1$). We define the short-range order parameter $f_{0a}$ as, $f_{0a} = \langle n_{0a} n_{ad} \rangle - \langle n_{0a} \rangle \langle n_{ad} \rangle$.

A. Quantum Criticality near MIT

We exhibit the real part of the optical conductivity near and across the MIT ($1.6 \leq U \leq 2.0$), computed from Eq. 4 as a function of $U$ for (a) the completely disordered case (short-range order parameter $f_{0a} = 0$ in our earlier work [14]) in the upper panel of Fig. 1 and (b) the short-range ordered case ($f_{0a} \neq 0$) in Fig. 2. Several features stand out clearly: In Case (a), (1) $\sigma'(\omega)$ shows an incoherent low-energy bump centered at $\omega = 0$, whose weight decreases continuously as the MIT is approached (at $U = 1.8$). It is important to note that (i) there is no low-energy Drude component in $\sigma'(\omega)$, since the CDMFT propagators have no pole structure [14], and (ii) as expected, low-energy spectral weight is continuously transferred from the bad-metallic and mid-infra-red (MIR) regions to high energies $O(U)$ across the MIT. This is characteristic of a correlation-driven MIT and the continuous depletion of low-energy weight is a consequence of the continuous MIT in the FKM driven by increasing $U$. In Fig. 2, we exhibit the effect of “anti-ferro alloy” (AF-A) SRO. Apart from the fact that the MIT now occurs at ($U \approx 1.35$ [14], the above features seem to be reproduced in this case as well. Looking more closely, however, we see marked changes in the low- and mid-infra-red energy range: the “bad metallic” bump centered at $\omega = 0$ is suppressed by SRO, and $\sigma'(\omega)$ rises faster with $\omega$ in the MIR, showing up the emergence of a low-energy pseudogap. These changes are to be expected, since AF-A SRO reduces the effective kinetic energy, increases the effective $U$, leading to reduction of low-energy spectral weight and a low-energy pseudogap in optics.

A closer look at Fig. 1 reveals very interesting features. We uncover a crossover scale ($\Omega_0(U)$), separating “metallic” and “insulator-like” behaviours in $\sigma'(\omega)$. As expected, it collapses at the MIT: interestingly, we find $\Omega_0(\delta U) \simeq (\delta U)^{1.29}$, quite close to $\nu = 4/3$ found in earlier work [11]. This also motivates us to investigate underlying quantum-criticality in optical response. In Fig. 3, we show that $\log(\sigma_c/\sigma'(\omega))$ plotted versus $\omega/\Omega_0(U)$ (the latter taken from Fig. 1) indeed reveals clean quantum-critical scaling: the insulating (I) and metallic (M) data fall on two master curves, and the beautiful mirror symmetry relating the two testifies to the unambiguous manifestation of the “Mott” QCP in optical response. Further, we also find that $\Omega_0(\delta U) \simeq c|\delta U|^{\eta}$ with $\eta = 1.3 \approx 4/3$,
in excellent accord with both Fig. 1 and our previous study. Using our earlier result \( \xi \approx (U - U_c)^{-\nu} \) with \( \nu = 4/3 \) and \( z = 1 \), we thus expect that \( \sigma'(\omega)/\sigma_c \) should scale as \( y = (U - U_c)/U_c \omega = 1/\omega \xi^{1/3} \), i.e. that \( \sigma'(\omega)/\sigma_c = F(\omega \xi^{1/3}) \). This is indeed adequately borne out in Fig. 4, for both \( M \) and \( I \) phases. This is a manifestation of the "Time-Temperature Superposition Principle" [5], expressible as a scaling law, \( \sigma(\omega) = F(\omega \xi^{1/3}) \) with \( F \) a \( T \)-independent scaling function and \( \Omega_0(U) \) a scaling parameter corresponding to the onset of conductivity dispersion, precisely as found here. The variation of \( \Omega_0 \) with \( U \) reflects the non-trivial interplay between itinerance (hopping) and Mott-like localization in the FKM. In analogy with the parameter \( T_0(\delta U) \approx c(\delta U)^z \) for the \( dc \) transport criticality [11], \( \Omega_0 \) also scales like \( (\delta U)^z \). Finally, the fact that \( \Omega_0(\delta U) \approx (\delta U)^{1/2} \) in both \( M \)- and \( I \)-phases reflects the fact, alluded to in earlier work [11, 18], that the basic electronic processes governing the \( I \)-phase are also relevant deep into the \( M \)-phase.

### B. Ununiversal Dielectric Response

Having shown a novel Mott-like quasi-local quantum criticality, we now turn to the UDR near the MIT. Since the FKM is isomorphic to the binary-alloy Anderson disorder problem, we inquire how CDMFT performs in the context of the remarkable universality in dielectric response in disordered quantum matter alluded to before [6]. In Fig. 5, Fig. 6, we show \( \log \sigma'(\omega) \) and the dielectric loss, \( \log(\sigma'(\omega)/\omega) \) versus \( \log(\omega) \) as functions of \( U \) to facilitate meaningful comparison with data of Lunkenheimer et al. It is indeed quite remarkable that all the basic features reported for disordered quantum matter are faithfully reproduced by our CDMFT calculation. Specifically, (i) for \( 1.5 < (U) < 1.8 \), a "\( dc \)" conductivity regime at lowest energy (up to \( 10^{-4} - 10^{-3} \)) smoothly goes over to a sublinear-in-\( \omega \) regime (UDR, in the region \( 10^{-2} - 10^{-1} \)) followed by a superlinear-in-\( \omega \) regime (around \( 10^{-1} \)), connecting up smoothly into the "boson" peak. These regimes are especially visible around \( (U) = 1.8 \), precisely where the MIT occurs. (ii) Moreover, corroborating behavior is also clearly seen in Fig. 6, where we exhibit the dielectric loss function vs \( \omega \) on a log-log scale. It is clearly seen that a shallow minimum separates the UDR and superlinear regimes at approximately \( \ln(\omega) = -0.8 \) (in the meV region) in the very bad-metallic state close to the MIT. This is in excellent accord with results for both LaTiO\(_3\) and PCCSW [6]. Moreover, the energy dependence of the optical conductivity also seems to be in good qualitative accord with data when we compare our results with Figures(1),(2)

---

**Figure 3.** (Color online) Clean quantum criticality as revealed in the scaling behaviour of the CDMFT optical conductivity, \( \sigma'(\omega)/\sigma_c \) versus \( y = (U - U_c)/U_c \omega \). Since the localization length, \( \xi \approx |U - U_c|^{-4/3} \) (see text), this implies that \( \sigma'(\omega)/\sigma_c = F(\omega \xi^{1/3}) \). This is a manifestation of the "Time Temperature Superposition Principle" [5] following from Jonscher’s UDR.

**Figure 4.** (Color online) Clean quantum criticality as revealed in the scaling behaviour of the CDMFT optical conductivity, \( \sigma'(\omega)/\sigma_c \) versus \( y = (U - U_c)/U_c \omega \). Since the localization length, \( \xi \approx |U - U_c|^{-4/3} \) (see text), this implies that \( \sigma'(\omega)/\sigma_c = F(\omega \xi^{1/3}) \). This is a manifestation of the "Time Temperature Superposition Principle" [5] following from Jonscher’s UDR.
and (3) of Lunkenheimer et al. In Fig. 6(b), we also show that short-range spatial correlations do not qualitatively modify these conclusions, attesting to their robustness against finite short-range order. Finally, precisely at the MIT (red curves in Fig. 6, we unearth a very interesting feature: \( \text{Im}(0, \omega) \simeq \omega^{-\eta} \) with \( \eta = 0.8, 0.75 \) for \( f_{\text{trunc}} = 0.0, -0.15 \). Hence, apart from a “dielectric phase angle”, \( \cot(\pi \eta/2) \), the real part of the dielectric constant also varies like \( \omega^{-\eta} \) as the MIT is approached from the metallic side. Remarkably, this is a concrete manifestation of the dielectric (or polarization) catastrophe that is expected to occur at a QCP associated with a continuous MIT.

This wide-ranging accord with data is quite remarkable, and begs a microscopic clarification in terms of basic electronic processes at work near the MIT. A phenomenological way is to posit that the universality is linked to glassiness [6] as follows: (i) first, our finding [11] of \( \nu = 4/3 \) (and \( z = 1 \)) is characteristic of percolative transport that is naturally expected to arise in glassy systems. (ii) it has also been shown [19] (for the disorder problem) that electronic glassy behavior precedes an insulating phase. Thanks to the mapping between the FKM and a binary-alloy ”disorder” problem, we also expect an intrinsic electronic glassy phase near the continuous MIT in the FKM. This suggests a close link between UDR and onset of electronic glassy dynamics near the continuous MIT in the FKM.

On a more microscopic level, it is very interesting that Jonscher himself [5] hinted at relevance of many-body processes at the heart of UDR. In particular, an explicit parallel with the seminal X-ray edge like physics was already (phenomenologically) invoked to account for such features. It is indeed remarkable that such X-ray-edge physics naturally falls out in the DMFT and CDMFT solution of the FKM in high-\( D \) [20]: in DMFT, the mobile (\( c \)-fermion) propagator exhibits a pseudogapped metal going over to an insulator across a critical \( U = U_c \), while the \( d \)-fermion propagator exhibits a “X-ray-edge” singular behavior linked to the seminal Anderson orthogonality catastrophe (AOC). In our CDMFT [14], we find similar behaviour: (i) a clear correlation-induced pseudogapped metal without Landau quasiparticles going over to an insulator at \( U_c = 1.8 \), with an anomalous self-energy, \( \text{Im}\Sigma^{(c)}(\omega) \simeq |\omega|^{1/3} \) and density-of-states \( \rho_c(\omega) \simeq |\omega|^{1/3} \) at the Mott QCP, and (ii) unusual power-law behavior of the dynamical charge susceptibility near \( U_c \), as well as, most importantly, anomalous energy dependence of the \( d \)-fermion propagator at long times. In CDMFT, these features arise precisely from the fact that \( a \) the dispersive \( c \)-fermions interact with the \( d_{\pm} = (d_1 \pm d_2)/\sqrt{2} \).
dispersionless fermion modes at the intra-cluster level via $U$, while (b) the $c$-fermions do not hybridize at all with the $d_{\pm}$ localized mode at single-fermion level. Physically, the origin of the AOC is that the dynamical screening arising from strong intra-cluster interactions in the FKM is non-trivially affected by hopping motion of carriers: since the $c_{\pm}$ fermions do not hybridize with the $d_{\pm}$ fermion modes at the one-fermion level (there is no term of the form $V(c_{+}^\dagger d_{\pm} + h.c)$ on the cluster), this screening is non-trivial, and arises from the "slow" reaction of the $d_{\pm}$ modes to the "sudden" jumps of the $c_{\pm}$-fermions (the latter occurs on a much shorter time scale of $\hbar/t$ in the FKM). Due to the local symmetry implied by $[n_{i,d}, H_{FKM}] = 0$ at each $i$, a hopping carrier experiences a sudden change in the local potential on the cluster (from 0 to $U$ and vice versa while hopping), now in the sense of a "sudden local quench". Rigorous absence of $c-d$-fermion one-particle mixing in the FKM implies lack of heavy-particle recoil in the "two-impurity" cluster problem, leading to "Kondo destruction" and generation of an AOC in our two-site CDMFT as above, in precise accord with Jonscher's original suggestion. We present the origin of the many-body AOC using an analytical approach in the following subsection.

C. Cluster Orthogonality Catastrophe in Two-site CDMFT for the FKM

Here, we present an analytic argument that exposes the venerated orthogonality catastrophe in our CDMFT approach. It turns out that this is most conveniently done by taking recourse to the underlying two-impurity problem of our CDMFT, which we now describe.

The "two-impurity" FKM is:

$$H_{FKM} = t(c_{+}^\dagger c_{-} + h.c.) + \sum_{k} \epsilon_k c_{+}^\dagger c_{-} + t \sum_{k,i=0,a} (e^{ikR} c_{+}^\dagger c_{+} + h.c.) + U \sum_{i=0,a} n_{i,c} n_{i,d}$$

(6)

Introduce the bonding-antibonding fermions, $c_{\pm} = (c_{0} \pm c_{a})/\sqrt{2}, d_{\pm} = (d_{0} \pm d_{a})/\sqrt{2}$. Then, $H_{FKM} = H_{12} + H_{coup} + H_{band}$ with

$$H_{12} = t(n_{c,+} - n_{c,-}) + \frac{U}{2} \sum_{a,a',=\pm} (n_{c,a} n_{d,a'} + c_{a}^\dagger c_{a'}^\dagger d_{a'}^\dagger d_{a'})$$

(7)

$$H_{coup} = \sqrt{2}t \sum_{k} [\cos(ka/2)c_{+}^\dagger c_{k} + \sin(ka/2)c_{-}^\dagger c_{k} + h.c.]$$

(8)

and,

$$H_{band} = \sum_{k} \epsilon_k c_{+}^\dagger c_{-}$$

(9)

This "cluster-to-orbital" mapping exposes the novel structure of the cluster-local correlations at the Mott QCP. Specifically, we observe that while the $d_{\pm}$-fermions interact with the $c_{\pm}$-fermions via $U$, they do not hybridize with each other at the one-fermion level (i.e., there is no term of the form $V(c_{+}^\dagger d_{\pm} + h.c)$ in the FKM). Thus, the two-impurity FKM maps rigorously onto a cluster version of the classic problem of recoilless, “infinite-mass” $d_{\pm}$ scatterers in a "Fermi sea" of the $c_{\pm}$; i.e., to the cluster version of the venerated X-ray edge problem. One now directly reads off that the spectral function of the $d_{\pm}$ fermions is infra-red singular with an interaction-dependent power-law behavior [21]:

$$\rho_{d_{\pm}}(\omega) \simeq \frac{\theta(\omega)}{|\omega|^{1-\eta_{\pm}}}$$

(10)

where $\eta_{\pm} = (\delta_{\pm}/\pi)^2$ and $\pi \delta_{\pm} = \tan^{-1}(U\rho_{c_{\pm}}(0)\pi)$ is the scattering phase shift. There will be an additional contribution to the scattering phase shift arising from the term $\frac{t}{2} \sum_{a,a'} c_{a}^\dagger c_{a'}^\dagger d_{a'}^\dagger d_{a'}$, but this will not qualitatively change the power-law behavior above. This many-body orthogonality catastrophe will carry over into the self-consistently embedded two-site CDMFT solution of the FKM.

Interestingly, we thus find that the orthogonality catastrophe and the accompanying breakdown of adiabatic continuity also holds for the case of spatially separated recoilless scatterers on the length scale of the cluster. Using a different approach, this aspect has also been studied earlier [22]. Thus, incorporation of inter-site correlations between spatially separated scattering centers does not qualitatively modify the orthogonality catastrophe, an interesting result in itself. In modern parlance, this means that the vanishing fidelity as well as the anomalous long-time behavior of the Loschmidt echo, both manifestations of the orthogonality catastrophe [23] also hold for spatially separated, sudden local quenches, a result that may have more widespread applications.

Thus, the classic orthogonality catastrophe, arising from the “sudden local” but spatially correlated quenches in our two-impurity model, is a genuine feature in our CDMFT used in the main text. This also provides a concrete microscopic rationalization linking the Jonscher UDR to this novel many-body effect, as conjectured early on by Jonscher himself.

IV. DISCUSSION AND CONCLUSION

Our findings can be profitably utilized to interpret wider range of data on dielectric responses of a wide range
of disordered electronic matter, e.g., disordered semiconductors, doped Mott insulators, PN junction devices [5], etc. In reality, the optical response at low energy will now be \( \sigma_{xx}(\omega) = \sigma_{dc} + \sigma_0 \omega^\alpha \), with \( 0 < \alpha < 1 \). This directly implies that \( \Im \sigma_{xx}(\omega) = \tan(\pi \alpha/2) \sigma_0 \omega^\alpha + \omega \epsilon_0 \epsilon\infty \), with \( \sigma_{dc} \) the dc conductivity and \( \epsilon_\infty \) the bare dielectric constant. The corresponding (dynamic) capacitance and impedance read \( C(\omega) = C_\infty + \left( \sigma_0 / 2 \pi \right) \tan(\pi \alpha/2) \omega^{\alpha-1} \) and \( Z(\omega) = (i \omega \epsilon^*(\omega))^{-1} \). Along with causality (Kramers-Kröning relations), UDR implies that the real and imaginary parts of the dielectric function (thus of the susceptibility) are related to each other by a “dielectric phase angle”, \( \chi'(\omega)/\chi''(\omega) = \cot(\pi \alpha/2) \) [5], independent of \( \omega \), in stark contrast to Debye-like relaxation, where this ratio equals \( \omega \tau \). Such forms have been widely used to analyze data in detail for a wide range of disordered matter [4, 5, 24] for a long time. Within CDMFT, our findings provide a microscopic rationale for use of these relations.

Theoretically, it is very interesting that such features appear near a correlation-driven MIT in the FKM, since this is a band-splitting type Mott (rather than pure Anderson localization in a disorder model, or a first-order Mott transition in the pure Hubbard model) transition. It supports views [11, 25] that the “disorder” problem at strong coupling, where \( kp \approx O(1) \), is characterized by a different Mott-like quantum criticality, a view nicely supported by our earlier finding [11] of \( \beta(\omega) \approx \ln(g) \) instead of \( \beta(\omega) \approx (d-2) - 1/g \) even deep in the (bad) metallic phase. This is not unreasonable, as it is long known [26] that the coherent potential approximation (CPA), the best mean-field theory for the Anderson disorder problem, is equivalent to the Hubbard III band-splitting view of the Mott transition (the latter becomes exact for the FKM in \( d = \infty \) [27]).

As concrete examples on the materials front, we note that (1) various aspects of manganite physics have also been successfully modeled by an effective FKM, where the c-fermions represent effectively spinless fermions (due to strong Hund coupling) strongly scattered by a disordered “liquid” of effectively localized Jahn-Teller polarons [28]. In this context, it is also interesting to notice that a field-induced percolative MIT is also long known to occur in manganites [29]. Thus, our model can serve as a simple effective model for PCSMO [6]. Application to LaTiO\(_3\) would require using a full Hubbard model very close to half-filling, where intrinsic disorder due to inhomogeneous phases near the MIT would generally be expected to be relevant. (2) It is also very interesting that related features, namely \( i \) non-Landau quasiparticle (Drude) “strange”, but \( \text{infra-red singular} \) power-law optical response and \( ii \) anomalous optical phase angle characterize strange metallicity in near-optimally doped cuprates [3]. One scenario, based on the hidden Fermi-liquid idea, posits that an \textit{inverse} orthogonality catastrophe also underlies [30] such non-Landau Fermi liquid ob-
servations in cuprates. However, the microscopics in this case involves momentum-selective Mott physics within CDMFT [31] and such novel responses could be linked to co-existing nodal (“itinerant”) and Mott-localized anti-nodal states. While more work is certainly needed to cleanly exhibit such features in the cuprate context, it is out of scope of our present study.

Thus, the central message of our work is that non-perturbative \textit{dynamical} effects of strong intrinsic scattering in the FKM lead to onset of a many-body AOC. We find that it is this specific aspect that is at the heart of the “universal” dielectric response.
Phys. Rev. B 95, 125116 (2017).

[15] K. Haule and G. Kotliar, “Optical conductivity and kinetic energy of the superconducting state: A cluster dynamical mean field study,” EPL (Europhysics Letters) 77, 27007 (2007).

[16] Jan M. Tomczak and Silke Biermann, “Optical properties of correlated materials: Generalized peierls approach and its application to vo2,” Phys. Rev. B 80, 085117 (2009).

[17] Romuald L/yżwa and Zbigniew Domański, “Falicov-kimball model and its relation to the hubbard model: Studies on clusters,” Phys. Rev. B 50, 11381–11385 (1994).

[18] H. Terletska, J. Vučičević, D. Tanasković, and V. Dobrosavljević, “Quantum critical transport near the mott transition,” Phys. Rev. Lett. 107, 026401 (2011).

[19] A. A. Pastor and V. Dobrosavljević, “Melting of the electron glass,” Phys. Rev. Lett. 83, 4642–4645 (1999).

[20] V Janis, “A lattice model for x-ray emission and absorption: the edge singularity,” Journal of Physics: Condensed Matter 5, L425 (1993).

[21] P. NOZIÈRES and C. T. DE DOMINICIS, “Singularities in the x-ray absorption and emission of metals. iii. one-body theory exact solution,” Phys. Rev. 178, 1097–1107 (1969).

[22] Yuval Gefen, Richard Berkovits, Igor V. Lerner, and Boris L. Altshuler, “Anderson orthogonality catastrophe in disordered systems,” Phys. Rev. B 65, 081106 (2002).

[23] Alessandro Silva, “Statistics of the work done on a quantum critical system by quenching a control parameter,” Phys. Rev. Lett. 101, 120603 (2008).

[24] P Lunkenheimer, S Krohns, F Gemander, WW Schmahl, and A Loidl, “Dielectric characterization of a nonlinear optical material,” Scientific reports 4 (2014).

[25] V. Dobrosavljević, Elihu Abrahams, E. Miranda, and Sudip Chakravarty, “Scaling theory of two-dimensional metal-insulator transitions,” Phys. Rev. Lett. 79, 455–458 (1997).

[26] B. Velický, “Theory of electronic transport in disordered binary alloys: Coherent-potential approximation,” Phys. Rev. 184, 614–627 (1969).

[27] P. G. J. van Dongen and D. Vollhardt, “Exact mean-field hamiltonian for fermionic lattice models in high dimensions,” Phys. Rev. Lett. 65, 1663–1666 (1990).

[28] T. V. Ramakrishnan, H. R. Krishnamurthy, S. R. Hassan, and G. Venketeswara Pai, “Theory of insulator metal transition and colossal magnetoresistance in doped manganites,” Phys. Rev. Lett. 92, 157203 (2004).

[29] M. Fäth, S. Freisem, A. A. Menovsky, Y. Tomioka, J. Aarts, and J. A. Mydosh, “Spatially inhomogeneous metal-insulator transition in doped manganites,” Science 285, 1540–1542 (1999), http://science.sciencemag.org/content/285/5433/1540.full.pdf.

[30] Philip W Anderson, “The strange metal is a projected fermi liquid with edge singularities,” Nature Physics 2, 626–630 (2006).

[31] Michel Ferrero, Pablo S. Cornaglia, Lorenzo De Leo, Olivier Parcollet, Gabriel Kotliar, and Antoine Georges, “Pseudogap opening and formation of fermi arcs as an orbital-selective mott transition in momentum space,” Phys. Rev. B 80, 064501 (2009).