Testing Supersymmetric Grand Unified Models of Inflation

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Abstract

We reconsider a class of well motivated supersymmetric models in which inflation is associated with the breaking of a gauge symmetry $G$ to $H$, with the symmetry breaking scale $M \sim 10^{16}$ GeV. Starting with a renormalizable superpotential, we include both radiative and supergravity corrections to derive the inflationary potential. The scalar spectral index $n_s$ can exceed unity in some cases, and it cannot be smaller than 0.98 if the number of e-foldings corresponding to the present horizon scale is around 60. Two distinct variations of this scenario are discussed in which non-renormalizable terms allowed by the symmetries are included in the superpotential, and one finds $n_s \geq 0.97$. The models discussed feature a tensor to scalar ratio $r \lesssim 10^{-4}$, while $dn_s/d\ln k \lesssim 10^{-3}$. If $G$ corresponds to SO(10) or one of its rank five subgroups, the observed baryon asymmetry is naturally explained via leptogenesis.

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1 Introduction

Supersymmetric grand unified theories in four and higher dimensions continue to play a prominent role in high energy physics, and it is therefore tempting to speculate that they may also play a key role in realizing an inflationary epoch in the very early universe. Indeed, in a class of realistic supersymmetric models, inflation is associated with the breaking either of a grand unified symmetry or one of its subgroups.

In the simplest models, inflation is driven by quantum corrections generated by supersymmetry breaking in the early universe, and the temperature fluctuations \( \delta T/T \) are proportional to \( (M/M_P)^2 \), where \( M \) denotes the symmetry breaking scale of \( G \), and \( M_P = 1.2 \times 10^{19} \) GeV denotes the Planck mass \( \frac{1}{2} \). It turns out that for \( M \sim 10^{16} \) GeV, one predicts an essentially scale invariant spectrum which is consistent with a variety of CMB measurements including the recent WMAP results \( \frac{3}{2} \). With inflation ‘driven’ solely by radiative corrections the scalar spectral index \( n_s \) is very close to 0.98, if the number of e-foldings \( N_\Omega \) after the present horizon scale crossed outside the inflationary horizon is close to 60.

As an example, if \( G = SO(10) \), one could associate inflation with the breaking of \( SO(10) \) to \( SU(5) \). A realistic model along these lines is most easily realized in a five dimensional setting \( \frac{1}{2} \), in which compactification on an orbifold can be exploited to break \( SO(10) \) down to the MSSM. Interesting examples for \( G \) in four dimensions include the gauge symmetry \( SU(4)_c \times SU(2)_L \times SU(2)_R \) \( \frac{5}{2} \frac{3}{2} \frac{1}{2} \) as well as \( SU(2)_L \times SU(2)_R \times U(1)_{B-L} \) \( \frac{8}{2} \frac{9}{2} \). If the unified gauge group \( G \) is identified with \( SO(10) \) or one of its subgroups listed above, the inflaton naturally decays into massive right-handed neutrinos whose out of equilibrium decay lead to the observed baryon asymmetry via leptogenesis.

Motivated by the prospects that much more precise information about the scalar index \( n_s \) and other related quantities will become available in the not too distant future, especially from a successful launch of the Planck satellite, we reconsider a realistic set of inflationary models in which not only radiative but also the canonical supergravity (SUGRA) contribution to the inflationary potential is taken into account. As noted in \( \frac{10}{2} \frac{11}{2} \frac{12}{2} \), the presence of SUGRA corrections can give rise to \( n_s \) values that exceed unity. We find that \( n_s \) does indeed exceed unity for some parameter choices, and its value cannot fall below 0.97 for e-foldings close to 60.

There are no tiny dimensionless parameters in the class of models that we consider.
Indeed, in the tree level superpotential there appears a dimensionless coupling $\kappa$ whose value, it turns out, is restricted to be $\lesssim 0.1$. Otherwise the scalar spectral index exceeds unity due to SUGRA corrections by an amount that is not favored by the data on smaller scales. With $dn_s/d\ln k \lesssim 10^{-3}$, this requires that the vacuum energy density that drives inflation is somewhat below $M^4_{\text{GUT}}$, and the tensor to scalar ratio $r$ turns out to be $\lesssim 10^{-4}$.

The plan of the paper is as follows. In section 2 we discuss the simplest supersymmetric GUT inflation model realized with a renormalizable superpotential whose form is fixed by a $U(1)$ R-symmetry. The termination of inflation in this case is abrupt, leading to a monopole problem for models such as $SU(4)_c \times SU(2)_L \times SU(2)_R$. (For a resolution in $SU(5)$ see [13].) In sections 3 and 4, we consider two extensions of this scenario in which non-renormalizable terms allowed by the symmetries are included in the superpotential, and the monopole problem is circumvented. In one extension, called ‘shifted’ GUT inflation [14], the gauge symmetry is already broken along the inflationary trajectory. In the other extension, called ‘smooth’ GUT inflation [14], the inflationary path possesses a classical inclination and the termination of inflation is smooth. Finally, in section 5 we note that consideration of leptogenesis in an $SO(10)$ model results in a somewhat more stringent upper bound on $\kappa$ ($\lesssim 10^{-2}$).

## 2 Supersymmetric GUT Inflation

An elegant inflationary scenario is most readily realized starting with the renormalizable superpotential [15]

$$W_1 = \kappa S (\phi \phibar - M^2)$$

where $\phi(\phibar)$ denote a conjugate pair of superfields transforming as nontrivial representations of some gauge group $G$, $S$ is a gauge singlet superfield, and $\kappa (> 0)$ is a dimensionless coupling. A suitable $U(1)$ R-symmetry, under which $W_1$ and $S$ transform the same way, ensures the uniqueness of this superpotential at the renormalizable level [11]. In the absence of supersymmetry breaking, the potential energy minimum corresponds to non-zero (and equal in magnitude) vevs ($= M$) for the scalar components in $\phi$ and $\phibar$, while the vev of $S$ is zero. (We use the same notation for superfields and their scalar components.) Thus, $G$ is broken to some subgroup $H$. In the presence of $N = 1$ supergravity, $S$ acquires a vev comparable to the gravitino
mass $m_{3/2} \sim \text{TeV}$.

In order to realize inflation, the scalar fields $\phi$, $\bar{\phi}$, $S$ must be displayed from their present minima. Thus for $|S| > M$, the $\phi$, $\bar{\phi}$ vevs both vanish so that the gauge symmetry is restored, and the tree level potential energy density $\kappa^2 M^4$ dominates the universe. With supersymmetry thus broken, there are radiative corrections from the $\phi - \bar{\phi}$ supermultiplets that provide logarithmic corrections to the potential which drives inflation. In one loop approximation the inflationary effective potential is given by $[1]$

$$V_{\text{LOOP}} = \kappa^2 M^4 \left[ 1 + \frac{\kappa^2 N}{32 \pi^2} \left( 2 \ln \frac{\kappa^2 |S|^2}{\Lambda^2} + (z + 1)^2 \ln(1 + z^{-1}) + (z - 1)^2 \ln(1 - z^{-1}) \right) \right], \tag{2}$$

where $z = x^2 = |S|^2/M^2$, $N$ is the dimensionality of the representations to which $\phi$, $\bar{\phi}$ belong, and $\Lambda$ is a renormalization mass scale. From Eq. (2) the quadrupole anisotropy is found to be $[1, 2, 8]$:

$$\left( \frac{\delta T}{T} \right)_Q \approx \frac{8 \pi}{\sqrt{N}} \left( \frac{N_Q}{45} \right)^{\frac{1}{2}} \left( \frac{M}{M_P} \right)^2 x_Q^{-1} y_Q^{-1} f(x_Q^{-1}), \tag{3}$$

with

$$f(z) = (z + 1) \ln (1 + z^{-1}) + (z - 1) \ln (1 - z^{-1}), \tag{4}$$

$$y_Q^2 = \int_1^{x_Q^2} \frac{dz}{z f(z)} \quad y_Q \geq 0. \tag{5}$$

Here, the subscript $Q$ denotes the epoch when the present horizon scale crossed outside the inflationary horizon and $N_Q$ is the number of e-foldings it underwent during inflation. From Eq. (2), one also obtains

$$\kappa \approx \frac{8 \pi^{3/2}}{\sqrt{N N_Q}} y_Q \frac{M}{M_P}. \tag{6}$$

For relevant values of the parameters ($\kappa \ll 1$), the slow roll conditions are violated only ‘infinitesimally’ close to the critical point at $x = 1$ ($|S| = M$) $[2]$. So inflation continues practically until this point is reached, where the ‘waterfall’ occurs.

Several comments are in order:

• For $x_Q \gg 1$ (but $|S_Q| \ll M_P$), $y_Q \to x_Q$ and $x_Q y_Q f(x_Q^2) \to 1^{-}$. 

3
• Comparison of Eq. (3) with the COBE result \((\delta T/T)_Q \approx 6.3 \times 10^{-6}\) shows that the gauge symmetry breaking scale \(M\) is naturally of order \(10^{16}\) GeV.

• Suppose we take \(G = SO(10)\), with \(\phi(\bar{\phi})\) belonging to the \(16(\bar{16})\) representation, so that \(G\) is spontaneously broken to \(SU(5)\) at scale \(M\). Taking \(N = 16\), and \(x_Q y_Q f(x_Q) \to 1^-\), \(M\) is determined to be \(10^{16}\) GeV, which essentially coincides with the SUSY GUT scale. The dependence of \(M\) on \(\kappa\) is displayed in Fig. 1. Note that a five dimensional supersymmetric \(SO(10)\) model in which inflation is associated with this symmetry breaking was presented in [17].

• Another realistic example is given by \(G = SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}\), corresponding to \(N = 2\), and the scale \(M\) is then associated with the breaking of \(SU(2)_R \times U(1)_{B-L} \to U(1)_Y\) [8, 9].

• The scalar spectral index \(n_s\) is given by [18]

\[
n_s \simeq 1 - 6\epsilon + 2\eta, \quad \epsilon \equiv \frac{m_P^2}{2} \left(\frac{V'}{V}\right)^2, \quad \eta \equiv \frac{m_P^2V''}{V}, \quad (7)
\]

where \(m_P\) is the reduced Planck mass \(M_P/\sqrt{8\pi}\); hereafter we take \(m_P = 1\). The primes denote derivatives with respect to the normalized real scalar field \(\sigma \equiv \sqrt{2}|S|\). For \(x_Q \gg 1\) (but \(\sigma_Q \ll 1\)), \(n_s\) approaches [11]

\[
n_s \simeq 1 + 2\eta \simeq 1 - \frac{1}{N_Q} \simeq 0.98 \quad (8)
\]

where \(N_Q \approx 60\) denotes the number of e-foldings.\(^3\) The dependence of \(n_s\) on \(\kappa\) is displayed in Fig. 2 (the behavior of \(n_s\) for large \(\kappa\) is influenced by the SUGRA correction, as discussed below).

• The minimum number of e-foldings (\(\approx 60\)) required to solve the horizon and flatness problems can be achieved even for \(x_Q\) very close to unity, provided that \(\kappa\) is taken to be sufficiently small. This follows from Eq. (6). An important constraint on \(\kappa\) can arise from considerations of the reheat temperature \(T_r\) after inflation, taking into account the gravitino problem. The latter requires that

\[^3N_Q \simeq 56.5 + (1/3)\ln(T_r/10^9\text{GeV}) + (2/3)\ln(\mu/10^{15}\text{GeV})\] [2], where \(T_r\) is the reheating temperature and \(\mu\) is the false vacuum energy density.
$T_r \lesssim 10^{10}$ GeV [19], unless some mechanism is available to subsequently dilute the gravitinos.

The inflaton mass is $\sqrt{2} \kappa M$ (recall that both $S$, and $\phi$, $\bar{\phi}$ oscillate about their minima after inflation is over, and they have the same mass), and so to prevent inflaton decay via gauge interactions which would cause $T_r$ to be too high ($\sim M \gg 10^{10}$ GeV), the coupling $\kappa$ should not exceed unity. A more stringent constraint on $\kappa$ ($\leq 0.1$) appears when SUGRA corrections are included.

- For $G = SO(10)$ or $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, the inflaton produces right handed neutrinos [20, 8, 21, 22] whose subsequent out of equilibrium decay leads to the observed baryon asymmetry via leptogenesis [26, 20].

- For sufficiently large values of $\kappa$, SUGRA corrections become important, and more often than not, these tend to derail an otherwise successful inflationary scenario by giving rise to scalar mass $^2$ terms of order $H^2$, where $H$ denotes the Hubble constant. Remarkably, it turns out that for a canonical SUGRA potential (with minimal Kähler potential $|S|^2 + |\phi|^2 + |\bar{\phi}|^2$), the problematic mass $^2$ term cancels out for the superpotential $W_1$ in Eq. (11) [15]. This may be considered an attractive feature of the inflationary scenario. Note that this property persists even when non-renormalizable terms that are permitted by the $U(1)_R$ symmetry are included in the superpotential.

The SUGRA scalar potential is given by

$$V = e^{K/m_P^2} \left[ \left| \frac{\partial W}{\partial z_i} + \frac{z^*_i W}{m_P^2} \right|^2 - 3 \frac{|W|^2}{m_P^2} \right], \quad (9)$$

where the sum extends over all fields $z_i$, and $K = \sum_i |z_i|^2$ is the minimal Kähler potential. (In general, $K$ is expanded as $K = |S|^2 + |\phi|^2 + |\bar{\phi}|^2 + \alpha |S|^4/m_P^2 + \ldots$, and only the $|S|^4$ term in $K$ generates a mass $^2$ for $S$, which would ruin inflation for $\alpha \sim 1$ [23, 24].)

From the requirement $\sigma < m_P$, one obtains an upper bound on $\alpha$ ($\lesssim 10^{-2}$) [25].) From Eq. (9), the SUGRA correction to the potential is [15, 10, 11, 12]

$$V_{SUGRA} = \kappa^2 M^4 \left[ \frac{1}{8} \sigma^4 + \ldots \right], \quad (10)$$

where $\sigma = \sqrt{2}|S|$ is a normalized real scalar field, and we have set the reduced Planck mass $m_P = 1$. The effective inflationary potential $V_1$ can be written to
a good approximation as the sum of the radiative and SUGRA corrections. For $1 \gg \sigma \gg \sqrt{2}M$,

$$V_1 \approx \kappa^2 M^4 \left[ 1 + \frac{\kappa^2 N}{32\pi^2} 2 \ln \frac{\kappa^2\sigma^2}{2\Lambda^2} + \frac{1}{8} \sigma^4 \right], \quad \text{(11)}$$

and comparing the derivatives of the radiative and SUGRA corrections one sees that the radiative term dominates for $\sigma^2 \lesssim \kappa \sqrt{N}/2\pi$. From $3H\dot{\sigma} = -V'$, $\sigma^2_Q \approx \kappa^2 N_N Q/4\pi^2$ for the one-loop effective potential, so that SUGRA effects are negligible only for $\kappa \ll 2\pi/\sqrt{N}_N Q \approx 0.1/\sqrt{N}$. (For $N = 1$, this essentially agrees with $\text{[11]}$).

From Eq. (11), the scalar spectral index is given by

$$n_s \simeq 1 + 2\eta \simeq 1 + 2 \left( 3\sigma^2 - \frac{\kappa^2 N}{8\pi^2 \sigma^2} \right), \quad \text{(12)}$$

and it exceeds unity for $\sigma^2 \gtrsim \kappa \sqrt{N}/2\sqrt{3}\pi$. For $x_Q \gg 1$,

$$N_Q = \int_{\sigma_{end}}^{\sigma_Q} \frac{V}{V'} d\sigma \approx \frac{\pi}{2\sigma^2_Q \kappa_c} \tan \left( \frac{\pi \kappa}{2 \kappa_c} \right), \quad \text{(13)}$$

where $\kappa_c = \pi^2/\sqrt{N}_N Q \approx 0.16/\sqrt{N}$. Using Eq. (13), one finds that the spectral index exceeds unity for $\kappa \approx 2\pi/\sqrt{3}\pi N_Q \approx 0.06/\sqrt{N}$.

The quadrupole anisotropy is found from Eq. (11) to be

$$\left( \frac{\delta T}{T} \right)_Q \approx \frac{1}{4\pi \sqrt{45}} \frac{V_1^{3/2}}{V'} \approx \frac{1}{2\pi \sqrt{45}} \frac{\kappa M^2}{\sigma^2_Q}. \quad \text{(14)}$$

In the absence of the SUGRA correction, the gauge symmetry breaking scale $M$ calculated from the observed quadrupole anisotropy approaches the value $N^{1/4} \cdot 6 \times 10^{15}$ GeV for $x_Q \gg 1$ (from Eq. (13), with $x_Q y_Q f(x_Q^2) \to 1^-$). The presence of the SUGRA term leads to larger values of $\sigma_Q$ and hence larger values of $M$ for $\kappa \gtrsim 0.06/\sqrt{N}$. The dependence of $M$ on $\kappa$ including the full one-loop potential (Eq. (2)) and the leading SUGRA correction is presented in Fig. 1.

To summarize, the scalar spectral index in this class of models is close to unity for small $\kappa$, has a minimum at $\simeq 0.98$ for $\kappa \simeq 0.02/\sqrt{N}$, and exceeds unity for $\kappa \gtrsim 0.06/\sqrt{N}$ (Fig. 2). The experimental data seems not to favor $n_s$ values in excess of unity on smaller scales (say $k \sim 0.05$ Mpc$^{-1}$), which leads us to restrict ourselves to $\kappa \lesssim 0.06/\sqrt{N}$. Thus, even though the symmetry breaking scale $M$ is of order $10^{16}$ GeV (Fig. 1), the vacuum energy density during inflation is smaller than $M_{GUT}^4$. Indeed, the tensor to scalar ratio $r \lesssim 10^{-4}$ (Fig. 4). Finally, the quantity $dn_s/d\ln k$
is negligible for small $\kappa$ and $\sim 10^{-3}$ as the spectral index crosses unity \[12\] (Fig. 3). The WMAP team has reported a value for $dn_s/d\ln k = -0.042_{-0.020}^{+0.021}$ \[3\], but the statistical significance of this conclusion has been questioned by the authors of \[27\]. Clearly, more data is necessary to resolve this important issue. Modifications of the models discussed here has been proposed in \[12\] to generate a much more significant variation of $n_s$ with $k$.

### 3 Shifted GUT Inflation

The inflationary scenario based on the superpotential $W_1$ in Eq. (1) has the characteristic feature that the end of inflation essentially coincides with the gauge symmetry breaking $G$ (e.g. $SO(10)$) to $H$ ($SU(5)$). Thus, modifications should be made to $W_1$ if the breaking of $G$ to $H$ leads to the appearance of topological defects such as monopoles, strings or domain walls. For instance, the breaking of $SU(4)_c \times SU(2)_L \times SU(2)_R$ \[5\] to the MSSM by fields belonging to $\phi(4,1,2), \bar{\phi}(4,1,2)$ produces magnetic monopoles that carry two quanta of Dirac magnetic charge \[28\].

As shown in \[7\], one simple resolution of the monopole problem is achieved by supplementing $W_1$ with a non-renormalizable term consistent with $U(1)$ R-symmetry, whose presence enables an inflationary trajectory along which the gauge symmetry is broken. Thus, the magnetic monopoles are inflated away. The part of the superpotential relevant for inflation is given by

$$W_2 = \kappa S(\phi\phi - M^2) - \beta \frac{S(\phi\phi)^2}{M_S^2},$$

(15)

where $M$ is comparable to the GUT scale, $M_S \sim 5 \times 10^{17}$ GeV is a superheavy cutoff scale, and the dimensionless coefficient $\beta$ is of order unity.

Remarkably, the inflationary potential including one loop radiative corrections and the leading SUGRA correction is obtained from Eq. (2) by substituting $N = 2, m^2 = M^2(1/4\xi - 1), \sigma = \sqrt{2}|S|$, with $\xi = \beta M^2/\kappa M_S^2$ and $z = x^2 = \sigma^2/m^2$ \[7\]:

$$V_2 \approx \kappa^2 m^4 \left[ 1 + \frac{\kappa^2}{16\pi^2} \left( 2 \ln \frac{2\kappa^2\sigma^2}{\Lambda^2} + (z + 1)^2 \ln(1 + z^{-1}) + (z - 1)^2 \ln(1 - z^{-1}) \right) + \frac{1}{8}\sigma^4 \right].$$

(16)

For $\kappa \lesssim 0.01$ the SUGRA correction is negligible, and one obtains Eq. (3) with $N$ replaced by 2, Eq. (6) with $N$ replaced by 4, and $M$ replaced by $m$ in both equations.
The dependence of the spectral index on $\kappa$ is depicted in Fig. 5. The tensor to scalar ratio $r \lesssim 10^{-4}$, while $dn_s/d\ln k \lesssim 10^{-3}$.

The vev $v_0 = |\langle \phi \rangle| = |\langle \bar{\phi} \rangle|$ at the SUSY minimum is given by \[ (\frac{v_0}{M^2})^2 = \frac{1}{2\xi} \left(1 - \sqrt{1 - 4\xi}\right), \] (17) and is $\sim 10^{16} - 10^{17}$ depending on $\kappa$ (Fig. 6). Requiring $v_0$ to be $\lesssim M_{\text{GUT}} \simeq 2.86 \times 10^{16}$ GeV restricts $\kappa$ to $\lesssim 0.01$ for $\beta = 1$. However, if one allows a smaller value for the effective cutoff scale which controls the non-renormalizable terms in the theory, say $M_S/\sqrt{\beta} \approx 2 \times 10^{17}$ for $\beta = 6$, then $v_0$ remains below $M_{\text{GUT}}$ even for $\kappa$ close to 0.1 (Fig. 6).

4 Smooth GUT Inflation

A variation on the inflationary scenarios in sections 2 and 3 is obtained by imposing a $Z_2$ symmetry on the superpotential, so that only even powers of the combination $\phi \bar{\phi}$ are allowed [14]. To leading order,

$$W_3 = S \left(-\mu^2 + \frac{(\phi \bar{\phi})^2}{M_S^2}\right),$$
(18)

where the dimensionless parameters $\kappa$ and $\beta$ (see Eq. (15)) are absorbed in $\mu$ and $M_S$ [14]. This idea has also been implemented in a SUSY GUT model [29] based on the gauge group $SU(4)_c \times SU(2)_L \times SU(2)_R$ [5]. The inflationary superpotential coincides with the ‘shifted’ model of the previous section, except that the trilinear term is now absent. The resulting scalar potential possesses two (symmetric) valleys of local minima which are suitable for inflation and along which the GUT symmetry is broken. The inclination of these valleys is already non-zero at the classical level and the end of inflation is smooth, in contrast to inflation based on the superpotential $W_1$ (Eq. (1)). An important consequence is that, as in the case of shifted GUT inflation, potential problems associated with primordial topological effects are avoided.

The common vev at the SUSY minimum $M = |\langle \phi \rangle| = |\langle \bar{\phi} \rangle| = (\mu M_S)^{1/2}$. For $\sigma^2 \gg M^2$, the inflationary potential is given by

$$V_3 \approx \mu^4 \left[1 - \frac{2}{27} \frac{M^4}{\sigma^4} + \frac{1}{8}\sigma^4\right].$$
(19)
where the last term arises from the canonical SUGRA correction. If we set $M$ equal to the SUSY GUT scale $M_G$, we get $\mu \simeq 1.8 \times 10^{15}$ GeV and $M_S \simeq 4.6 \times 10^{17}$ GeV. (Note that, if we express Eq. (18) in terms of the coupling parameters $\kappa$ and $\beta$, these values correspond to $\kappa \sim O(\mu^2/M^2_{GUT}) \sim 10^{-2}$ and $\beta \simeq 1$.) The value of the field $\sigma$ is $1.3 \times 10^{17}$ GeV at the end of inflation (corresponding to $\eta = -1$) and is $\sigma_Q = 2.7 \times 10^{17}$ GeV at horizon exit. In the absence of the SUGRA correction (which is small for $M \lesssim 10^{16}$ GeV), $\sigma \propto M^{2/3} M_p^{1/3}$, $(\delta T/T)_Q \propto M^{10/3}/(M_S^2 M_p^{4/3})$ and the spectral index is given by [14]

$$n_s \simeq 1 - \frac{5}{3N_Q} \simeq 0.97,$$

(20)
a value which coincides with the prediction of some D-brane inflation models [30]. This may not be surprising since, in the absence of SUGRA correction, the potential $V_3$ (Eq. (19)) has a form familiar from D-brane inflation. The SUGRA correction raises $n_s$ from 0.97 to 1.0 for $M \sim 10^{16}$ GeV, and above unity for $M \gtrsim 2 \times 10^{16}$ (Fig. 7).

5 Gravitino Constraint on $\kappa$

We have seen that the dimensionless superpotential coupling $\kappa$ in Eq. (1) satisfies the constraint $\kappa \lesssim 0.1$, so that the scalar spectral index does not exceed unity by much on smaller scales ($k \gtrsim 0.05$ Mpc$^{-1}$) from SUGRA corrections. A somewhat more stringent upper bound on $\kappa$ can be realized in an $SO(10)$ model from consideration of the inflaton decay into right-handed neutrinos, taking into account the gravitino constraint $T_r \lesssim 10^{10}$ GeV on the reheat temperature. Consider the $SO(10)$ superpotential couplings

$$\frac{1}{m_p\gamma_{ij}\phi\bar{\phi}}16_i16_j$$

(21)

which provide large masses for the right-handed neutrinos after symmetry breaking (say of $SO(10)$ to $SU(5)$). Here $16_i, j$ ($i, j = 1, 2, 3$) denote the three chiral families, and $\phi, \bar{\phi}$ vevs break the gauge symmetry.

It was shown in [8] that the reheat temperature for the model discussed in section 2 can be approximated as
\[ T_r \sim \frac{1}{12} \sqrt{y_Q} M_i \tag{22} \]

where \( y_Q \) is defined in Eq. (5), and \( M_i \) denotes the heaviest right handed neutrino that satisfies \( 2M_i < m_{\text{infl}} \), with

\[ m_{\text{infl}} = \sqrt{2\kappa M} . \tag{23} \]

From Eq. (21) we expect the heaviest neutrino to have a mass of around \( 10^{14} \) GeV or so, which is in the right ball park to provide via the seesaw a mass scale of about .05 eV to explain the atmospheric neutrino anomaly through oscillations [17]. From Eq. (22), assuming \( y_Q \) of order unity, we conclude that \( M_i \) should not be identified with the heaviest right handed neutrino, otherwise \( T_r \) would be too high. Thus, we require that [31]

\[ \frac{m_{\text{infl}}}{2} \lesssim M_3 = \frac{2M^2}{m_P} \quad (\text{with } \gamma_3 = 1) . \tag{24} \]

Using Eq. (23) and Eq. (6), this implies \( y_Q \lesssim \sqrt{\mathcal{N} N_Q}/\pi \), which corresponds to \( \kappa \lesssim 0.015 \) (0.008) for \( \mathcal{N} = 16 \) (\( \mathcal{N} = 2 \)).

### 6 Conclusion

Motivated by the prospects of much more precise data becoming available in the not too distant future, we have explored the predictions of the scalar spectral index \( n_s \), \( \text{d}n_s/\text{d}\ln k \) and the tensor to scalar ratio \( r \) for three classes of closely related supersymmetric models. A characteristic feature shared by all these models is that inflation becomes an integral part of realistic supersymmetric grand unified theories, with the grand unification scale playing an essential role. If the grand unified theory is identified with \( SO(10) \) or one of its rank five subgroups, then leptogenesis also becomes an integral part of the setup. We find that the inclusion of SUGRA corrections can give rise to a blue spectrum [10] [11], with the upper bound on \( n_s \) (say \( \leq 1.0 \) for \( k = 0.05 \) Mpc\(^{-1} \)) providing an upper bound on \( \kappa \) that is of order \( 10^{-1} \)–\( 10^{-2} \) depending on the model. The quantity \( \text{d}n_s/\text{d}\ln k \) is always small, of order \( 10^{-3} \) or less [1] [12].

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Figure 1: The gauge symmetry breaking scale $M$ as a function of the coupling constant $\kappa$. $\mathcal{N} = 16$ (2) corresponds to the breaking $SO(10) \rightarrow SU(5)$ and $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \rightarrow SU(2) \times U(1)$ respectively.

Figure 2: The spectral index $n_s$ at $k = 0.05 \text{ Mpc}^{-1}$ as a function of the coupling constant $\kappa$ (dashed line–without SUGRA correction, solid line–with SUGRA correction).
Figure 3: The spectral index $n_s$ as a function of the wavenumber $k$ ($N = 16$):
$\kappa = 0.004$ (dot-dashed), $1 \times 10^{-4}$ (dotted), 0.015 (dashed), 0.02 (solid).

Figure 4: The tensor to scalar ratio $r$ as a function of the coupling constant $\kappa$. 
$N = 16$, $N = 2$.
Figure 5: The spectral index $n_s$ at $k = 0.05 \text{ Mpc}^{-1}$ as a function of the coupling constant $\kappa$ for shifted GUT inflation (dashed line–without SUGRA correction, solid line–with SUGRA correction).

Figure 6: The gauge symmetry breaking scale $v_0$ in shifted GUT inflation as a function of the coupling constant $\kappa$ for $\beta = 1$ (solid line) and $\beta = 6$ (dashed line).
Figure 7: The spectral index $n_s$ at $k = 0.05 \text{ Mpc}^{-1}$ as a function of the gauge symmetry breaking scale $M$ for smooth GUT inflation (dashed line–without SUGRA correction, solid line–with SUGRA correction).