Shock Focusing upon Interaction with Heavy Gas Bubble of Different Density

Pavel Georgievskiy, Vladimir Levin, Oleg Sutyrin
Lomonosov Moscow State University, Institute of Mechanics, Russian Federation
E-mail: sutyrin@imec.msu.ru

Abstract. Interaction of a shock with round area of heavy gas is simulated using two-dimensional plane and axisymmetrical Euler’s equations. Plane formulation describes interaction of a shock with heavy gas cylinder and axisymmetrical models interaction with heavy gas bubble. Incident shock refraction and focusing processes are described including two different interaction patterns – external and internal. Three consecutive pressure peaks are found to occur at symmetry axis (or plane) inside and outside of the shocked inhomogeneity. A dependence of peak pressure values on initial gas density in the inhomogeneity is determined for three different shock Mach numbers. The highest pressure for plane and axisymmetric cases is shown to be reached in different interaction patterns.

1. Introduction
Interaction of shock waves with local inhomogeneities (high or low density areas, dust clouds) takes place in wide range of problems, including large-scale interstellar flows and technologies of inertial confinement fusion and shock-driven powder coating. Well-known key features of these processes are uneven inhomogeneity acceleration and turbulent mixing [1], generation of multiscale vortices [2] and shock refraction and focusing, especially pronounced for heavy gas bubbles [3]. Most studies are focused on experimental and numerical investigation of vorticity deposition and evolution with subsequent gas mixing [4].

New area of interest is the initiation of combustion and detonation using shock focusing upon interaction with reactive gas bubble [5, 6] and thus the focusing effect is studied more closely. Two distinct focusing patterns [7] are known to take place depending on shock intensity, bubble shape and gas density. In addition, the shape of the bubble influences shock focusing intensity [8] and total flow vorticity [9]. Bubble gas density is equally important: insufficient density leads to weaker focusing and inferior combustion initiation effectiveness [5].

The present work deals with the study of qualitative and quantitative features of shock focusing process depending on density of local gas inhomogeneity in both two-dimensional plane and axisymmetrical formulations.

2. Problem formulation, mathematical model and numerical method
Plane shock wave of Mach number $M$ propagates from left to right through motionless air containing round cylindrical or spherical area filled with gas of elevated density. Ahead of the shock, gas pressure $p = p_0 = 1$, density $\rho = \rho_0 = 1$ outside of the inhomogeneity and $\rho = \omega > 1$ inside it.
Figure 1. Early (a) and intermediate (b) flow stages for $M=2,\omega=3$, axisymmetric case: density flood, pressure contours for $\Delta p = 0.2$. Incident shock $is$, reflected and transmitted shocks $rs, ts$; initial (dashed line) and shocked bubble edges $ic, sc$; transversal shock $tts$, triple point $tp$, secondary shock $iss$, main vortex $mv$.

inside of it. At the initial time moment $t = 0$ the shock touches the leftmost point of the inhomogeneity.

Two-dimensional Euler’s equation are used:

$$
\frac{\partial}{\partial t} \begin{pmatrix} 
\rho r^\alpha \\
\rho u r^\alpha \\
\rho v r^\alpha \\
e r^\alpha 
\end{pmatrix} + \frac{\partial}{\partial x} \begin{pmatrix} 
\frac{\rho u}{r^\alpha} (p + \rho u^2) r^\alpha \\
\frac{\rho v}{r^\alpha} (p + \rho u^2) r^\alpha \\
(\rho v + p) vr^\alpha \\
(\rho v + p) vr^\alpha 
\end{pmatrix} + \frac{\partial}{\partial r} \begin{pmatrix} 
\frac{\rho u}{r^\alpha} (p + \rho u^2) r^\alpha \\
\frac{\rho v}{r^\alpha} (p + \rho u^2) r^\alpha \\
(\rho v + p) vr^\alpha \\
(\rho v + p) vr^\alpha 
\end{pmatrix} = \begin{pmatrix} 0 \\
0 \\
0 \\
0 
\end{pmatrix},
$$

$$
e = \frac{p}{\gamma - 1} + \frac{\sqrt{\gamma}}{\gamma - 1} p_0
$$

here $p, \rho, u, v$ are pressure, density and velocity components along axes $x, r$; $e$ – total gas energy per volume unit and $\gamma$ is heat capacity ratio. $\alpha$ governs the coordinate system type: $\alpha = 0$ is for plane flows and $\alpha = 1$ corresponds to axisymmetrical ones.

Only one length scale – inhomogeneity diameter $l_0$ – is present in the problem formulation. The time scale thus may be given by $t_0 = l_0/a_0$ where $a_0$ is the ambient speed of sound: $a_0 = \sqrt{\gamma \rho_0 / \rho_0} = \sqrt{\gamma}$. Therefore, the problem is governed by three parameters: heat ratio $\gamma$, Mach number $M$ and density ratio $\omega$ (or Atwood number $At = (\omega - 1) / (\omega + 1)$). For the present simulations, the parameters were taken as follows: $\gamma = 1.4, M = 2, 2.5, 3$ and $\omega = 1.5, 2, 2.5, 3, 3.5, 4, 4.5$ or $5$ (which correspond to $At = 0.2, 0.33, 0.43, 0.5, 0.56, 0.6, 0.64$ or $0.67$).

To solve the equations, a 5th order finite-difference WENO scheme [10] with 3rd order Runge-Kutta time discretization and H-correction [11] was implemented. Parallel computations were performed using uniform square grid with 1000 cells per inhomogeneity diameter. For axisymmetrical case, the symmetry condition $\partial/\partial r = 0$ was imposed at the axis.

3. Shock refraction and focusing

The early and intermediate stages of the flow for the axisymmetrical case is shown in Fig. 1. The bottom edge is the symmetry axis. The main features of the flow are the same for both the plane and axisymmetrical cases. Incident shock $is$ propagates from left to right, convex shock $rs$ is reflected off the bubble edge $ic$ while slower concave transmitted shock $ts$ travels inside the bubble. The concaveness of $ts$ increases with time and transversal shock $tts$ and triple point $tp$ that move towards the symmetry axis are formed. Bent part $bis$ of the shock $is$ skirts the bubble on the outside and the main vortex $mv$ stands out in the vortex lane.

Depending on the bubble gas density (and, generally, its shape [7, 8, 9]) two different flow types follow, shown in Fig. 2. For lesser $\omega$ values the shock $ts$ and triple point $tp$ reach the
Figure 2. External (a, $\omega = 2$) and internal (b, $\omega = 5$) flow patterns, time moments just before the focusing. Retransmitted shock and triple point $rts$, $rtp$; reflected expansion wave $rew$, parts of focusing transversal shock $fs1$, $fs2$; incident shock reflected off the axis $ris$.

opposite bubble edge and retransmit into the ambient gas. Expansion wave $rew$ is reflected off the edge and transversal focusing shock $fs$ propagate towards the axis inside the shocked bubble. This flow regime (known as “Type-II” [7]) may also be called “external” since $ts$ and $tp$ focus – reach the symmetry axis – outside of the shocked bubble shortly after.

For the higher density ratios the speed of $ts$ is even lower and incident shock is has enough time to completely circumflex the bubble and start to reflect off the axis (Fig. 2b). Shocks $ts$, $fts$ and triple point $tp$ confine local area of undisturbed gas and collapse on it inside the bubble – an internal shock focusing takes place (Type-I [7]).

4. Local pressure peaks
Three local pressure peaks are observed while transversal shocks reflect off the symmetry axis (or plane). First momentary peak $P_1$ is reached when triple point $tp$ hits the axis outside or inside the bubble, depending on focusing regime. Longer-living second ($P_2$) and third ($P_3$) peaks occur upon collapse of parts $fs1$, $fs2$ of transversal focusing shock on the axis to the right and left of the expansion wave $rew$. Both $rew$ and $P_2$ exist only in external focusing regime.

Peak pressure values are sensitive to grid resolution; in plane formulation, they increase for 5-10% for each resolution doubling (from 250 to 500 or 500 to 1000 cells per $l_0$). In axisymmetrical case peak values does not converge at all, due to absence of physical viscosity: for the inviscid cylindrical shock focusing, peak pressure is infinite. Such limitation of the adopted mathematical model does not allow determination of exact shock focusing pressure values. Nevertheless, based on series of simulations using the same grid, it is possible to obtain some qualitative conclusions of peak pressure relations between themselves and their dependence on problem parameters.

Dependence of $P_1$, $P_2$ and $P_3$ on initial gas density for plane flows is shown in Fig. 3. For external focusing regimes, all three peak values are close to each other; upon transition to internal pattern, $P_3$ sharply increases while $P_1$ is nearly stable or decreases. External shock focusing regimes are relatively weaker and yet peak pressures are several time higher than the original shock pressure (which equals 4.5, 7.1, 10.3 for $M = 2, 2.5, 3$). This effect may probably be used for combustion initiation in the case of reactive mixture surrounding heavy gas cylinder.

Similar charts for axisymmetrical case are given in Fig. 4; due to cylindrical symmetry, shock focusing effect is much more pronounced. In all cases except $\omega \geq 4$ for $M = 2.5, 3$ the first peak, reached upon triple point hitting the axis, is dominant. The highest pressure is reached upon external focusing close to bubble edge; $P_3$ stabilizes in the internal regime or shortly before. In axisymmetrical case, transition from external to internal pattern happens at lower $\omega$ values due
Figure 3. Dependence of peak pressure values on $\omega$ for plane flows: (a) $M = 2$, (b) $M = 2.5$, (c) $M = 3$. Dashed line separates external and internal shock focusing pattern areas.

Figure 4. Dependence of peak pressure values on $\omega$ for axisymmetrical flows.

to shock strengthening and acceleration during convergence near rightmost bubble point.

Conclusion
During interaction of a shock with local area of heavier gas, transversal shocks form and later focus on the symmetry axis or plane. Two distinct interaction patterns – external and internal shock focusing – occur depending on density ratio. Three consecutive pressure peaks on symmetry axis (plane) occur in external flow regime – one outside and two inside the shocked inhomogeneity. For internal pattern, two peaks are observed inside the inhomogeneity.

With density ratio increase, peak pressure values significantly rise. For plane flows, the highest pressure is reached in internal focusing regime upon transversal shock reflection off the symmetry plane inside the shocked gas cylinder and for axisymmetrical case – when triple point hits the symmetry axis outside shocked bubble close to its edge.

The effect of multifold pressure – and thus temperature – rise inside and outside the inhomogeneity allows the development of new methods for combustion and detonation initiation using shock focusing phenomenon upon propagation through inhomogeneous gas mixtures; although, implementation of gas mixture combustion models is necessary for further conclusions.

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