Fundamental tradeoff between packetized model predictive control and forward error correction in wireless feedback control systems

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Abstract: In wireless feedback control systems, loss of control packets due to unreliable wireless channels is one of the most important problems. To cope with this problem, we propose a novel concept that combines a packetized model predictive control (MPC), which is control layer’s redundancy, and a forward error correction (FEC), which is communication layer’s redundancy. Simulation results show that under the same length of redundancy, there is a tradeoff performance between the packetized MPC and the FEC, and the proposed method can provide well-balanced tolerance to channel error.

Keywords: Wireless control, feedback control, model predictive control, forward error correction, error correction coding

Classification: Wireless communication technologies

References

[1] D. E. Quevedo, and J. Østergaard, and D. Nešić, “Input-to-state stability of packetized predictive control over unreliable networks affected by packet-dropouts,” IEEE Trans. Autom. Control, vol. 56, no. 2, pp. 370 – 375, Feb. 2011.
[2] M. Nagahara, D. E. Quevedo, and J. Østergaard, “Sparse Packetized Predictive Control for Networked Control Over Erasure Channels,” IEEE Trans. Autom. Control, vol. 59, no. 7, pp. 1899 – 1905, July 2014.
[3] S. Hattori, K. Kobayashi, H. Okada, and M. Katayama, “ON-OFF error control coding scheme for minimizing tracking error in wireless feedback control systems,” IEEE Trans. Ind. Informat., vol. 11, no. 6, pp. 1411 – 1421, Oct. 2015.
[4] Y. Miwa, K. Kobayashi, H. Okada, and M. Katayama, “A study on variable length channel coding for state feedback in wireless control systems,” Proc. IEEE Wireless Commun. Netw. Conf. (WCNC), Mar. 2017.
[5] M. Cannon, “C21 Model Predictive Control: Lecture notes,” Oxford University, 2018. [Online]. Available: https://markcannon.github.io/assets/downloads/teaching/C21_Model_Predictive_Control/mpc_notes.pdf
1 Introduction

In factory automation, process monitoring, smart grids, and so on, there is a growing interest in wireless control of machines. Wireless control systems have several advantages, such as reconfigurability, mobility, and no maintenance cost associated with cables. However, because of unreliability of wireless channels, there is a disadvantage that the control performance deteriorates due to loss of control packets; therefore, it is very important how to suppress such deterioration of the control performance. To cope with this problem, packetized MPC methods [1,2] have been proposed to cope with successive packet loss by the approach from the control layer; by the approach from the communication layer, adaptive FEC methods [3,4] have been proposed to cope with wireless channel errors. These methods have a common view point of adding redundancy to control information. The packetized MPC method adds predicted control information as redundancy to compensate the loss of control packets and the FEC method adds error-correction bits as redundancy to reduce the probability of packet loss.

In this paper, we propose a cross-layer combination of the redundancy of MPC in the control layer and that of FEC in the communication layer, and we show that under the same total length of redundancy, there is a tradeoff between the redundancy lengths of MPC and FEC, and the proposed method can provide better tolerance to channel error.

2 System model

Figure 1 shows the model of wireless feedback control with the proposed method. It consists of a MPC controller, an encoder and decoder of FEC, and a buffer of received MPC control information followed by a controlled plant. State information of the plant is observed by sensors, and the state information is estimated by a state estimator at the controller. The forward channel from the controller side to the plant side is assumed to be a noisy channel, but the feedback channel is assumed to be ideal with no error in order to focus on the effect of channel error in control information.

The plant is assumed to be a linear time-invariant and its state space representation is given as $\mathbf{x}[k+1] = A\mathbf{x}[k] + B\mathbf{v}[k] + \mathbf{w}[k]$ and $y[k] = C\mathbf{x}[k] + z[k]$, where $\mathbf{v}[k], \mathbf{x}[k], y[k]$ are a control input vector, a state vector, and an observed state vector at time $kT_s$ ($k = 0, 1, 2, \ldots, T_s$: control period), respectively; $A$, $B$, and $C$ are coefficient matrices that represent dynamics and observation and are controllable and observable; $\mathbf{w}[k]$ represents a system disturbance and assumed to be a white Gaussian random vector with zero mean and covariance matrix $\mathbf{W}$; $\mathbf{z}[k]$ represents an observation noise and assumed to be with zero mean and covariance matrix $\mathbf{Z}$.

At the controller side, the MPC controller calculates control information up to $N_p$ cycles ahead as

$$\mathbf{u}[k] = (\mathbf{u}[k|k] \quad \mathbf{u}[k+1|k] \quad \cdots \quad \mathbf{u}[k+N_p|k])^T = \text{MPC}(\mathbf{x}_{\text{est}}[k], \mathbf{x}_{\text{ref}}[k]), \quad (1)$$

where $\mathbf{u}[k+n|k]$ ($n = 0, 1, \ldots, N_p$) is a predictive control input vector to
be input at \( k + n \). \( x_{\text{ref}}[k] \) is a reference vector, and the function \( \text{MPC}() \) minimizes \( J[k] \) of (2) by performing conditional optimization of estimation

\[
x[k+n+1|k] = Ax[k+n|k] + Bu[k+n|k]
\]

under \( u[k+n|k] \leq u_{\text{max}} \) [5].

\[
J[k] = \sum_{n=0}^{N_p} \left( ||x[k+n|k] - x_{\text{ref}}[k]||_Q^2 + ||u[k+n|k]||_R^2 \right),
\]

(2)

where \( x[k|k] = x_{\text{est}}[k] \) and \( ||x||_X^2 = x^T X x \); \( Q \) and \( R \) are positive definite weight matrices. Before calculating the control information, a Kalman filter-based state estimator is performed to obtain \( x_{\text{est}}[k] \) from the previously received \( y[k-1] \). Due to space limitations, the formulas of the estimator and the controller are omitted here.

## 2.1 Proposed method

The proposed method combines the redundancy of MPC and FEC to be added to the control information \( u[k|k] \). By adding the redundancy of MPC and storing it in the buffer on the plant, it is possible to control the plant using the stored predictive control information even if packet loss occurs. Also, by adding the redundancy of FEC, it is possible to correct channel error and reduce the probability of packet loss. Under a fixed total redundancy, longer FEC redundancy can decrease the probability of packet loss but cannot cope with successive packet loss; on the other hand, longer MPC redundancy can cope with successive packet loss, but the probability of packet loss becomes higher. Therefore, there will be a tradeoff between the lengths of the MPC redundancy and the FEC redundancy.

The control input vector \( u[k|k] \) to be input at \( k \) and the \( N \) redundancy of MPC, i.e., \( u[k+1|k], \ldots, u[k+N|k] \), are packetized as a control packet to be sent to the plant. Thereafter, the packet including a header is encoded and the redundancy of FEC is added to the packet. The encoded packet passes through the noisy channel. The received packet will be discarded (i.e., packet loss) if a bit error remains after error correction by the decoder. If correctly received, the control information is stored in the buffer, i.e., the buffer \( b[k] = (b_1[k], b_2[k], \ldots, b_N[k])^T = (u[k|k], u[k+1|k], \ldots, u[k+N|k])^T \), and the top of the buffer is input to the plant, i.e., \( v[k] = u[k|k] \); otherwise, the predictive control information stored in the buffer is shifted as \( b[k-1] = \ldots, b_N[k-1] \).
\[ (b_2[k-1] b_3[k-1] \cdots b_N[k-1] 0)^\top, \] and the top of the buffer is input to the plant as \( v[k] = b_1[k-1] \).

3 Numerical examples

3.1 Simulation setup

A rotary inverted pendulum, which is a typical under-actuated object, is employed as an example of the plant. Its control objective is to enable the arm angle to follow a reference while maintaining upright of the pendulum attached at the tip of the rotating arm. \( v[k] \) is an one-dimensional vector consisting of the input voltage to the DC motor that drives the arm. \( x[k] \) is a four-dimensional vector consisting of the pendulum angle, pendulum velocity, arm angle, and angular velocity. The physical parameters are based on REAL TECH RTC05; \( A, B, \) and \( C \) at \( T_s = 0.01 \) s are given as

\[
A = \begin{bmatrix}
1.006 & 0 & 1.002 \times 10^{-2} & 7.457 \times 10^{-5} \\
-3.265 \times 10^{-4} & 1 & -1.089 \times 10^{-6} & 9.963 \times 10^{-3} \\
1.113 & 0 & 1.006 & 1.491 \times 10^{-2} \\
-6.528 \times 10^{-2} & 0 & -3.265 \times 10^{-4} & 9.926 \times 10^{-1}
\end{bmatrix}, \\
B = \begin{bmatrix}
-4.332 \times 10^{-4} \\
2.160 \times 10^{-4} \\
-8.643 \times 10^{-2} \\
4.315 \times 10^{-2}
\end{bmatrix}, \quad C = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0
\end{bmatrix}.
\]

The reference is set to a rectangular signal switching \( \pm \pi/4 \) rad every 10 s. For simplicity, \( W \) is set to be diagonal with diagonal elements are \( 10^{-8} \). \( Z \) is diagonal with diagonal elements are \( \delta/12 \), where \( \delta = 2\pi/2^{16} \) rad for 2-byte quantization. The controller is performed with \( N_p = 10, u_{\text{max}} = [8.0], Q = C^\top C \) and \( R = [0.24] \). The simulation time is 100 s, and once the pendulum angle > \( \pi/2 \), it is assumed that the pendulum has fallen. The stability is evaluated by the probability of pendulum fall and the trackability is evaluated by the RMSE of the arm angle against the ideal control case.

The size of each control packet is 16 bytes and the header is 6 bytes. The size of each control input vector is 2 bytes and the redundancy of MPC is \( 2 \times N \) bytes. A CRC-aided Polar code under successive cancellation list decoding with 16 list size and 2-byte CRC is adopted as the FEC, and its code rate is \( R_c = (6 + 2 + 2N + 2)/16 \) except for the CRC. The control performance for \( N = 0, 1, 2, \) and 3 is compared in the below subsection. Note that \( N = 0 \) is the case with only the FEC redundancy and \( N = 3 \) is the case with only the MPC redundancy. To see the effect of both redundancy of MPC and FEC, the noisy channel is assumed to be a burst error channel given as a Gilbert model with average bit error rate \( P \) and average burst length \( L \), where the good state causes no error and the bad state causes random bit error, and the states change only every control period with the same probability.

3.2 Simulation results

The control performance for various burst length and bit error rate are shown in Figure 2. Figure 2(a) shows the case with smaller bit error rate. In this case, the longer the redundancy of MPC, the worse the control performance because it leads to larger packet loss rate. The larger redundancy of FEC
Fig. 2. Control performance for various burst length and bit error rate

provides better performance because it leads to smaller packet loss rate. In contrast, figure 2(c) shows the case with larger bit error rate. In this case, the longer the redundancy of MPC, the better the control performance because it compensates successive packet loss. As also shown in Figure 2(b), we can see that there is a tradeoff between the lengths of the MPC redundancy and the FEC redundancy. From these results, we can also see that overall, $N = 1$ of the proposed method provides well-balanced control performance.

4 Conclusion

We have proposed a method of combining MPC redundancy in the control layer and FEC redundancy in communication layer for reliable wireless feedback control. It has been shown that under a fixed total redundancy, there is a tradeoff between the redundancy lengths of MPC and FEC, and the proposed method provides well-balanced control performance. Future work combining it with adaptive coding may provide further improvement.
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