Bayesian Analysis of Hot-Jupiter Radius Anomalies: Evidence for Ohmic Dissipation?

Daniel P. Thorngren and Jonathan J. Fortney

1 Department of Physics, University of California, Santa Cruz, CA, USA
2 Department of Astronomy and Astrophysics, University of California, Santa Cruz, CA, USA

Received 2017 September 13; revised 2018 March 15; accepted 2018 March 23; published 2018 April 27

Abstract

The cause of hot-Jupiter radius inflation, where giant planets with \( T_{\text{eq}} > 1000 \) K are significantly larger than expected, is an open question and the subject of many proposed explanations. Many of these hypotheses postulate an additional anomalous power that heats planets’ convective interiors, leading to larger radii. Rather than examine these proposed models individually, we determine what anomalous powers are needed to explain the observed population’s radii, and consider which models are most consistent with this. We examine 281 giant planets with well-determined masses and radii and apply thermal evolution and Bayesian statistical models to infer the anomalous power as a fraction of (and varying with) incident flux \( \epsilon(F) \) that best reproduces the observed radii. First, we observe that the inflation of planets below about \( M = 0.5 \, M_J \) appears very different than their higher-mass counterparts, perhaps as the result of mass loss or an inefficient heating mechanism. As such, we exclude planets below this threshold. Next, we show with strong significance that \( \epsilon(F) \) increases with \( T_{\text{eq}} \) toward a maximum of \( \sim 2.5\% \) at \( T_{\text{eq}} \approx 1500 \) K, and then decreases as temperatures increase further, falling to \( \sim 0.2\% \) at \( T_{\text{eff}} = 2500 \) K. This high-flux decrease in inflation efficiency was predicted by the Ohmic dissipation model of giant planet inflation but not other models. We also show that the thermal tides model predicts far more variance in radii than is observed. Thus, our results provide evidence for the Ohmic dissipation model and a functional form for \( \epsilon(F) \) that any future theories of hot-Jupiter radii can be tested against.

Key words: planets and satellites: gaseous planets – planets and satellites: interiors – planets and satellites: physical evolution

Supporting material: tar.gz file

1. Introduction

The longest-standing open question in exoplanetary physics is what causes the inflated radii of “hot Jupiters,” gas giant planets on short-period orbits heated to equilibrium temperatures \( T_{\text{eq}} > 1000 \) K (Miller & Fortney 2011). Since the first detection of planet HD 209458 b (Charbonneau et al. 2000; Henry et al. 2000), the radii of the vast majority of these transiting gas giants have exceeded the expected radius of \( \sim 1.1 \) times that of Jupiter, sometimes approaching 2 Jupiter radii. This excess radius appears to correlate with the level of incident stellar irradiation (Guillot & Showman 2002; Laughlin et al. 2011), rather than, e.g., semimajor axis (Weiss et al. 2013). A wide range of theories have been proposed to explain this, most of which postulate an additional “anomalous” power that heats the convective interior of the planet, leading to larger radii. Typically, these theories are tested by directly modeling the physics to determine if they can produce large enough radii to explain the observations (e.g., Ginzburg & Sari 2016; Tremblin et al. 2017). We shall take a more complete approach by determining what anomalous powers are needed to explain the radii of the whole observed population, and then considering what models are most consistent with this.

This approach is feasible thanks to the work of surveys such as WASP, HAT, and Kepler, which have identified a large number of transiting giant planets. Follow-up radial-velocity measurements have yielded mass measurements for many of these. Merging data from the NASA Exoplanet Archive (Akeson et al. 2013) and exoplanet.eu (Schneider et al. 2011), we examine the set of transiting planets with measured masses and radii with relative uncertainties of less than 50%, in the mass range \( 20 \, M_{\oplus} < M < 13 \, M_J \). The resulting flux-radius-mass data are shown in Figure 1. Several patterns are apparent. First, many planets with high incident flux are anomalously large—these are the hot Jupiters. The flux at which these excess radii become apparent has been estimated to occur at 0.2 Gerg \( \text{s}^{-1} \text{ cm}^{-2} \) (Miller & Fortney 2011), equivalent to an equilibrium temperature \( T_{\text{eq}} \approx 1000 \) K. Second, the degree of radius inflation increases steadily with flux. Finally, the degree of radius inflation is greater at lower masses. This is more visible in Figure 2, which plots radius against planetary mass.

In modeling the interior structure of a transiting giant planet with a measured mass, there are two key variables that are not directly observable: the bulk heavy-element abundance and the anomalous power. Planets at fluxes below the inflation threshold, including Jupiter and Saturn, are well described by evolution models with zero anomalous power. In this cool giant regime, we can directly infer the heavy-element mass from the observables. Our previous work, Thorngren et al. (2016), did this for the \( \sim 50 \) known cool transiting giant planets (those with \( T_{\text{eq}} < 1000 \) K), and observed a correlation between the planetary heavy-element mass and the total planet mass of \( (M_{\text{He}}/M_J) \approx 58(M/M_J)^{0.1} \). That cool giant sample and this hot giant sample do not differ much in semimajor axis (typically \( \sim 0.1 \) versus \( \sim 0.03 \) au), so we do not expect their formation mechanisms or composition trends to differ. Thus, for this work, we apply this relation with its predictive uncertainty as a population-level prior on the heavy-element masses of the hot Jupiters. By doing this, we constrain one of the two unobserved variables, allowing us to infer planetary anomalous power. Individually, planets may vary in composition so by themselves our predictions are not particularly precise. However, since planets as a population will follow the trend line, a hierarchical Bayesian model based on this prior allows us to combine
anomalous power. Since it seems plausible that a mass-loss process affects this planets in that region. This upturn is a feature of any plausible model of the radii increasing dramatically at lower masses, coinciding with the absence of excess radius correlated with Fortney 2011 effect, an approximate upper limit on the non-independent model of in...radiation power using our Gaussian... 155:214 (10pp), 2018 May...information from our whole sample to infer the shape of the function. These features do not eliminate systematic error, but they do allow for more confidence in our results.

2. Lack of Inflated Sub-Saturns

An interesting feature is apparent in the mass–radius relationship. Figure 2 shows the masses and radii of our sample of planets, along with prediction lines of constant temperature and inflation power. The relationship between the temperature (color) and inflation power is posterior to our model (discussed in Section 4), but the general shape of the lines themselves is generic, and appears for any mass-independent model of inflation power. It is apparent that with decreasing mass and constant inflation power, the radius anomaly becomes larger exponentially. This is not seen in the observed planet radii. In fact, giant planets are not observed with surface gravity less than about 3 m s\(^{-2}\), even though our models allow it and the transits of such large planets would be readily detectable. This might be the result of an inflation mechanism that is inefficient at low masses, but this possibility is weakened by examining the frequency of planets in mass–flux space (see Figure 3).

Consider the population of high-mass Jupiters compared to lower-mass Saturns, separating the groups at 0.5 \( M_J \). Among Jupiters, many high-flux planets are observed: 58% (164/281) have more than 1 Gerg \( s^{-1} cm^{-2}\). Among Saturns, we find only 22% (21/97) that experience this level of insolation. This discrepancy does not appear to result from any observational biases. It is possible that significant mass loss could occur if planets inflate too much. Because radii increase with decreasing mass, any mass loss that occurs might experience positive feedback. This is similar to what was seen in Baraffe et al. (2004), though their mass-loss rate appears to have been too high (Hubbard et al. 2007). The best alternative hypothesis appears to be that Saturns preferentially stop migration further from the parent star and that planets at these masses also experience a significantly less efficient inflation effect. Further study will require more advanced models, which we leave to future work. To avoid this issue, we restrict our attention to planets with \( M > 0.5 \, M_J \).

3. Planet Models

Our interior structure models are broadly the same as those in Thorngren et al. (2016), with only two changes for this work on inflated giant planets. We solve the equations of hydrostatic equilibrium, conservation of mass, and an equation of state (EOS) based on the SCvH (Saumon et al. 1995) solar H/He EOS and the EOS of a 50/50 ice/rock mixture (Thompson 1990):

\[
\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^2},
\]

\[
\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho},
\]

\[
\rho = \rho(P, T).
\]
Metals were fully mixed into the convective envelope using the additive volumes approximation. No core was included because for planets of this mass the radius difference would be minor (see Thorngren et al. 2016). Heat flow out of the planet (and therefore thermal and structural evolution) was regulated using the atmospheric models of Fortney et al. (2007). Additional details and analysis of the effect of our modeling choices can be found in Thorngren et al. (2016). Sample evolution calculations are shown in Figure 4.

The most important modeling addition is the inclusion of an additional heating power $\epsilon F \pi R^2$. The resulting power balance of the interior of the planet is

$$\frac{\partial E}{\partial t} = \pi R^2 (\epsilon F - 4 F_{int}).$$

(4)

Here, $F_{int}$ is the intrinsic flux of energy radiated out of the planet as computed by the atmosphere model. Note that our definition of $\epsilon$ differs slightly from other authors, such as Komacek & Youdin (2017), who deposit the energy at a particular depth within the planet. Using their results, our definitions agree for their models where the power is deposited at the radiative-convective boundary or deeper. Otherwise, our $\epsilon$ is smaller than theirs by a factor of $<1$ depending on depth and stage of evolution.

The other change was an improvement to the thermal evolution integration system. The new system uses the SciPy (Van Der Walt et al. 2011) function Odeint to adaptively integrate the changes in planet internal entropy. We have also added a system to detect when the planet is near thermal equilibrium (when $\epsilon F \pi R^2 \approx L_{int}$), and quickly completes the evolution accordingly. This serves to handle the stiffness of the ODE near an equilibrium of high specific entropy.

4. Bayesian Statistical Analysis

Our statistical analysis is based on a hierarchical Bayesian approach, with two levels in the hierarchy. The lower level consists of our beliefs about the properties of individual planets given the observations and our planetary mass–metallicity relation from Thorngren et al. (2016) as a prior on bulk metallicity. The upper level combines information about the individual planets to infer population-level patterns in anomalous power. The variables we will use are listed and described in Table 1.

4.1. Planetary Statistical Models

We wish to understand the observed radii of giant planets, which have normally distributed errors, in terms of our interior structure models $R(t, M_i, M_z, \epsilon, F)$. As such, we construct the following normal likelihood for observing the $i$th planet’s radius to be $R_{obs}$ given the structure models parameters:

$$p(R_{obs} | t^i, M_i^s, M_z, \epsilon^i)$$

$$= \mathcal{N}(R_{obs} | R(t^i, M_i^s, M_z, \epsilon^i, F), \sigma_{\text{obs}}^i).$$

(5)

Here, $\mathcal{N}$ refers to the normal distribution, and $\mathcal{N}(x | \mu, \sigma)$ is the PDF of $\mathcal{N}(\mu, \sigma)$ evaluated at $x$ (similarly for the uniform $\mathcal{U}$ and log-normal $\mathcal{L}\mathcal{N}$ distributions). The observed flux $F_i$ is known precisely enough (compared to the other observations) that we will neglect the effect its uncertainty has on the model radius uncertainty. Previous studies provide us with observational constraints on $M_i$ and $t^i$, which we will use as priors. Combined with the motivated prior on $M_z^s$ from our mass–metallicity relationship, we have

$$t^i \sim \mathcal{U}(t_{10}, t_1),$$

(6)

$$M_i^s \sim \mathcal{L}\mathcal{N}(\alpha + \beta \log(M_z^s), \sigma),$$

(7)

$$M_z^s \sim \mathcal{N}(\mu_{\text{obs}}, \sigma_{\text{fit}}^2).$$

(8)

$\mathcal{L}\mathcal{N}(\mu, \sigma)$ is the base-10 log-normal distribution with location $\mu$ and scale $\sigma$ (i.e., the log$_{10}$ of the variable is distributed as $\mathcal{N}(\mu, \sigma)$). Using these priors, we can write a posterior distribution for the structure model parameters $(t^i, M_z^s, M_i^s, \epsilon^i)$ as follows:

$$p(t^i, M_z^s, M_i^s, \epsilon^i | R_{obs})$$

$$= p(R_{obs} | t^i, M_z^s, M_i^s, \epsilon^i) p(t^i, M_z^s, M_i^s, \epsilon^i) / p(R_{obs})$$

(9)

$$\propto p(R_{obs} | t^i, M_z^s, M_i^s, \epsilon^i) p(t^i) p(M_z^s | M_i^s) p(M_i^s) p(\epsilon^i)$$

(10)
The true age of the \( t_i \), based observational studies retrieved from exoplanets.org.

We include the model parameters as explicit arguments, and let the constants be indicated by the index \( i \). This function reduces the right-hand side of Equation (11) to \( Q^i(t_i, M_i^e, M_i^l, \epsilon^i) p(\epsilon) \).

### 4.2. Models of Anomalous Power

For convenience, we define the function \( Q^i \) as follows:

\[
Q^i(t_i, M_i^e, M_i^l, \epsilon^i) = N(R_{\text{obs}}^i|R(t_i, M_i^e, M_i^l, \epsilon^i, F_i), \sigma_{R}^i) \\
\times \mathcal{U}(t_i|\mu_{t_i}, \sigma_{t}^i) \\
\times \mathcal{N}(M_i^l|\alpha + \beta \log(M_i^e), \sigma_{M}^l) \\
\times p(\epsilon).
\]

(11)

The purpose of this model is to infer \( \epsilon^i \). If we apply a simple uniform prior \( \epsilon^i \sim \mathcal{U}(0\%, 5\%) \), we can infer the interior structure parameters for the \( i \)th planet. Figure 5 shows the results of this approach for HD 209458 b. Unfortunately, as seen in the figure, data from a single planet do not provide enough information to infer much about \( \epsilon^i \). In the next section, we describe a hierarchical model that combines the information from many planets to draw conclusions about the anomalous power as a function of flux \( \epsilon(F) \).

### 4.2. Models of Anomalous Power

For convenience, we define the function \( Q^i \) as follows:

\[
Q^i(t_i, M_i^e, M_i^l, \epsilon^i) = \mathcal{N}(R_{\text{obs}}^i|R(t_i, M_i^e, M_i^l, \epsilon^i, F_i), \sigma_{R}^i) \\
\times \mathcal{U}(t_i|\mu_{t_i}, \sigma_{t}^i) \\
\times \mathcal{N}(M_i^l|\alpha + \beta \log(M_i^e), \sigma_{M}^l) \\
\times p(\epsilon).
\]

(12)

We can now focus on constructing models of \( \epsilon^i \). First, we consider the models in which the heating efficiency \( \epsilon \) is given by a deterministic function of several hyperparameters \( \phi \). We will refer to this function generally as \( \epsilon(F^i, \phi) \), and consider several specific functions (power-law, logistic, and Gaussian), differentiated by their subscripts. These models were chosen because they all allow for low heating efficiencies at low fluxes, but exhibit differing behavior at high fluxes. The power-law model is a classic and simple model for many astronomical phenomena, the logistic model captures the possibility that the inflation effect “turns on” at some flux, and the Gaussian model covers the case that heating efficiency declines at high flux.

\[
\epsilon_p(F, \phi_p) = \epsilon_0 F^k,
\]

(14)

\[
\epsilon_l(F, \phi_l) = \frac{\epsilon_0}{1 + (F/F_0)^k},
\]

(15)

\[
\epsilon_g(F, \phi_g) = \epsilon_0 \exp \left( -\frac{\log_{10}(F/F_0)^2}{2\sigma^2} \right).
\]

(16)

For each of these models, we choose the following weakly informative proper priors for the hyperparameters:

\[
p(\phi_p) \propto \mathcal{U}(\epsilon|0\%, 5\%) \times \mathcal{N}(k|0, 2),
\]

(17)

\[
p(\phi_l) \propto \mathcal{U}(\epsilon|0\%, 5\%) \times \mathcal{N}(F_0|1, 2) \times \mathcal{N}(k|3, 1),
\]

(18)

\[
p(\phi_g) \propto \mathcal{U}(\epsilon|0\%, 5\%) \times \mathcal{L}(F_0|1, 2) \times \mathcal{L}(s|0, 2).
\]

(19)

In the power-law case, the uniform distribution demands \( \epsilon_0 \) and \( k \) be such that that no planet’s \( \epsilon \) leave the [0%, 5%] bounds. In the logistic case, the prior on \( k \) is fairly informative, demanding that the transition be somewhat similar to the scale of the data; this parameter would be poorly constrained otherwise. Now we substitute \( \epsilon(F^i, \phi) \) into Equation (13), which together with the hyperpriors gives us the following posterior:

\[
p(t, M_e, M_l, \phi|R_{\text{obs}}) \propto p(\phi) \prod_{i=1}^{N} Q^i(t_i, M_i^e, M_i^l, \epsilon(F^i, \phi)).
\]

(20)

The Gaussian process (GP) model takes a slightly different form. In it, we model \( \log_{10}(\epsilon) \) as a GP with mean 0 and covariance matrix \( K \). We use the squared exponential kernel with a small white noise component \( \sigma^2 = 10^{-3} \) for numerical convenience, which amounts to a relative spread of about 7% in linear space. Thus, the covariance matrix for the process is
given by

\[
K_{jk}(\phi_{gp}) = \sigma_j^2 \exp\left(-\frac{\log^2(F_j/F_k)}{2\ell^2}\right) + \sigma^2_{\delta,jk}. \tag{21}
\]

We define some weakly informative priors for \( \phi_{gp} \) as follows:

\[
p(\phi_{gp}) \propto \mathcal{L}\mathcal{N}(\sigma_j^2|0, 1) \times \mathcal{L}\mathcal{N}(\epsilon|0, 1). \tag{22}
\]

Because we do not have simple normal distributions for them, we cannot marginalize out \( \epsilon \), and instead must keep them as parameters hierarchically connected through the GP prior. To provide an appropriate lower boundary condition on the function, we include an independent portion of the prior on \( \epsilon' \) in combination with the GP such that the model is

\[
p(\epsilon_{gp}) \propto \mathcal{L}\mathcal{N}(\epsilon|0, K(F, \phi_{gp})) \prod_{i=1}^{N} \mathcal{U}(\epsilon'|0, 5\%) G^i(\epsilon'), \tag{23}
\]

\[
G^i(\epsilon') \equiv \begin{cases} 
1 & \text{for } F_i \geq 10^8 \\
\mathcal{L}\mathcal{N}(\epsilon'|-2, 1) & \text{for } F_i < 10^8.
\end{cases} \tag{24}
\]

The log-normal portion is the Gaussian process. The \( G^i \) component is useful because it sets an appropriate lower boundary condition for \( \epsilon(F) \). Experimentation reveals that this boundary condition has little effect when \( F > 2 \text{ Gerg s}^{-1} \text{ cm}^{-2} \) (the region of interest); we merely include it to best represent our belief about the function for the full range of fluxes. With these priors and likelihood, Bayes’s theorem yields the

**Figure 5.** Inferred parameters for HD 209458 b, using Equation (11). The parameters are mass in Jupiter masses, planetary metal mass fraction, inflation efficiency, and age in gigayears. The planet is old enough that its age uncertainty has little effect on the other parameters. As expected, the main driver of \( \epsilon \) uncertainty is \( Z_{pl} \). For this planet, we disfavor an inflation efficiency below \( \sim 1\% \). Together with other planets, some of which disfavor high \( \epsilon \), this forms the basis for our inference of \( \epsilon(F) \).
posterior for the GP model:

\[
p(t, M_1, M_2, \epsilon, \phi_{\text{GP}} | R_{\text{obs}}) \\
\propto p(\phi_{\text{GP}}) f_{\text{GP}}(\epsilon | \theta) \\
\times \sum_{i=1}^{N} Q^i(t^i, M^i_1, M^i_2, \epsilon^i) L(\epsilon^i | 0, 5\%) G^i(\epsilon^i). \quad (25)
\]

Finally, we constructed a simple model for the thermal tides model of hot-Jupiter inflation (Arras & Socrates 2009). We adapt the scaling relations of Socrates (2013), \( L \propto T_0^2 R^3 P^{-2} \), where \( L \) is the total anomalous power, \( P \) is the period, \( T_0 \) is the equilibrium temperature, and \( R \) is the planet radius. We model this as follows, where \( c_0 \) is a model parameter, using the present-day radius and flux for simplicity:

\[
\epsilon_i(F) = c_0 R_i^2 p^{-2} F^{-25}. \quad (26)
\]

4.3. Statistical Computation

We wish to use a Metropolis–Hastings MCMC (Hastings 1970) sampler to draw samples from the posteriors given above. However, if we do this with no further simplifications, we will end up exploring the parameters very slowly. This is because the models listed above have a very large number of parameters (~1100) thanks to the many nuisance parameters \( (M^i_1, M^i_2, \epsilon^i, F, \theta) \), each of which have one parameter per planet. The complexity of our Metropolis–Hastings sampler scales with dimension at roughly \( O(d^2) : O(d) \) posterior PDF evaluations (see Roberts & Rosenthal 2004) that cost \( O(d) \). However, we are really only interested in \( \phi \) for the various models, plus \( \epsilon \) in the GP case. We can save a great deal of computational effort by directly sampling marginal distribution and rewriting the posteriors as follows:

\[
p(\phi | R_{\text{obs}}) \\
= \int p(t, M_1, M_2, \phi | R_{\text{obs}}) dt dM_1 dM_2 d\phi \quad (27)
\]

\[
= \int p(\phi) \sum_{i=1}^{N} Q^i(t^i, M^i_1, M^i_2, \epsilon(F^i, \phi)) dt dM^i_1 dM^i_2 d\phi \quad (28)
\]

\[
= p(\phi) \sum_{i=1}^{N} \int Q^i(t^i, M^i_1, M^i_2, \epsilon(F^i, \phi)) dt dM^i_1 dM^i_2 \\
= p(\phi) \sum_{i=1}^{N} Q^i_0(\epsilon(F^i, \phi)). \quad (30)
\]

We use \( dt \) and the like as shorthand for integration over every component of \( t \) in sequence over their full domain; Equation (28) has 843 nested integrals. \( Q^i_0 \) is defined as \( Q^i \) integrated over \( t^i, M^i_1, M^i_2 \):

\[
Q^i_0(\epsilon) \equiv \int Q^i(t^i, M^i_1, M^i_2, \epsilon) dt^i, dM^i_1, dM^i_2. \quad (31)
\]

in a similar fashion:

\[
p(\epsilon, \phi | R_{\text{obs}}) \propto p(\phi) f_{\text{GP}}(\epsilon | \theta) \\
\times \sum_{i=1}^{N} Q^i_0(\epsilon(F^i, \phi)) L(\epsilon | 0, 5\%) G(F^i, \epsilon^i). \quad (32)
\]

Figure 6. Histogram and kernel density estimate (the black line) of the posterior inflation power \( \epsilon \) (proportional to \( Q^i(\epsilon) \)) for WASP-43 b. In this case, smaller values of \( \epsilon \) are more likely, but larger values are not ruled out. Note that the KDE matches the histogram, as is required for us to be able to use it as a likelihood for the upper level of the hierarchical model.
(DIC), which is similar to the AIC in interpretation, but which makes use of an estimate of the effective number of parameters (Spiegelhalter et al. 2002), derived from the variance of the log posterior likelihood. The empirical DIC from a set of samples is

\[
\text{DIC} = -2 \log \left( \mathbb{E}(\log \pi(x|y)) \right) + 4 \text{Var}(\log \pi(x|y|\epsilon)).
\]

As described in Equation (21), the empirical DIC from a set of samples is

\[
\text{DIC} = -2 \log \left( \mathbb{E}(\log \pi(x|y)) \right) + 4 \text{Var}(\log \pi(x|y|\epsilon)).
\]

Here, \( \hat{\epsilon} \) is the posterior mean of \( \epsilon \) and \( \text{Var}_\epsilon \) is the variance of the log likelihood across samples. Note that while the samples in question are taken using the posterior, this computation is

Figure 7. Scatterplot matrix of the GP hyperparameter posterior (see Equation (21)). It is fairly well behaved, but has a long right tail. This is a common feature for Gaussian processes.

Figure 8. Scatterplot matrix of the Gaussian function hyperparameter posterior (see Equation (16)). Two modes were observed, differing primarily in height \( \epsilon_{\text{max}} \); the model with a peak of \( \epsilon \approx 2\% \) is favored over the model with peak \( \epsilon \approx 3.5\% \) by a probability ratio of about 3:1. The discovery of more giant planets around the \( \approx 1500 \text{ K} \) peak will help to resolve this further.

Figure 9. Scatterplot matrix of the power-law hyperparameter posterior (see Equation (14)). A strong correlation between the coefficient \( F_0 \) and the power \( k \) is seen. This likely reflects the constraint that the function achieves adequate power for the many planets at around \( T_{\text{eq}} \approx 1300 \text{ K} \), yet avoids exceeding 5% for the hottest planets, which would exceed the bounds of our grid. Such constraints are difficult for the power law to achieve. Regardless, as a result of its overestimate of high \( T_{\text{eq}} \) radii, this model had a comparatively disfavorable DIC.

Figure 10. Scatterplot matrix of the logistic function hyperparameter posterior (see Equation (15)). Thanks to our prior on \( k \), which demanded the transition be similar to the scale of the data, the resulting posterior is well behaved and easy to sample from. The model is not bad, but its DIC indicates that it is still inferior to a model that decreases at high equilibrium temperatures.

Here, \( \epsilon \) is the posterior mean of \( \epsilon \) and \( \text{Var}_\epsilon \) is the variance of the log likelihood across samples. Note that while the samples in question are taken using the posterior, this computation is
done using the likelihood. In the results, the model with the more negative DIC is favored. The interpretation of $\Delta$DIC is similar to that of the AIC and BIC, in which differences of $\Delta$DIC > 6.5 are strong evidence in favor of the model with the lesser DIC (e.g., Kass & Raftery 1995 for BIC).

To produce posterior predictive mass–flux–radius relations, we assume the planets are old (5 Gyr), and for given $M$ and $F$, we draw $M_s$ from Equation (8) and $\epsilon$ from $\epsilon(F, \phi)$ marginalized over the posterior $p(\phi|R_{\text{obs}})$. These sampled values are then plugged into the structure models $R(t, M_s, M, \epsilon, F)$. The result is a probability distribution in $R$ for the given parameters.

5. Results

The results for $\epsilon(F)$ in Figure 11. All functional forms yield similar results below about 0.5 Gerg s$^{-1}$ cm$^{-2}$, but differ significantly above this. The GP model reaches a peak at around 1600 K and decreases toward zero with high statistical confidence, as shown by the uncertainty bounds. At high fluxes, the uncertainty in heating power is roughly constant, and so declines as a fraction of flux. Figure 2 shows the predicted radius for a given mass of 5 Gyr old planets of average (posterior mean) composition and inflation power using the GP model. The predictions align well with planets of similar mass and temperature. The shape of $\epsilon(F)$ presented by the GP is corroborated by comparison of the DIC values. Of the parametric models, the Gaussian model is most favored, with a DIC of $-1723$. The logistic model was next, at $-1648$, followed by the power-law model at $-1641$. We interpret this to mean that $\epsilon$ decreases toward zero at high fluxes with high statistical significance, in agreement with our conclusions from the GP approach. The DIC of the GP model is $-1723$, so there is no significant preference between it and the Gaussian model.

We present the Gaussian model since it takes a simple analytic form, as a percent of flux and with flux in units of Gerg s$^{-1}$ cm$^{-2}$:

$$\epsilon = (2.37^{+1.3}_{-2.6}) \text{Exp} \left[ -\frac{(\log(F) - (1.43^{+0.60}_{-0.69}))^2}{2(3.738^{+0.261}_{-0.260})^2} \right].$$  \hspace{1cm} (34)

Note that for planets whose interiors are in thermal equilibrium where $E_{\text{in}} = E_{\text{out}}$ and therefore $dR/dt = 0$ (which may happen quite early—see Figure 4), the intrinsic temperature is directly related to $\epsilon$ as

$$T_{\text{int}} = \left( \frac{\epsilon F}{4\sigma} \right)^{1/4} = \epsilon^{1/4} T_{\text{eq}},$$

where $\sigma$ is the Stefan–Boltzmann constant, and the conversion from flux to equilibrium temperature assumes an ideal black-body with full heat redistribution.

To visualize why the Gaussian model is preferred, we compute the posterior predictive radius distributions, and compare them to the radii of our observed planets. Figure 12 compares these predictions for the favored GP model and the next-best logistic model to the observed radii as a function of incident flux, divided into six mass bins. The models only diverge at high fluxes, about 2 Gerg s$^{-1}$ cm$^{-2}$. Beyond this, the logistic model systematically overestimates the radii, and the GP does not. To make this clear, Figure 13 shows the residual to the expected radius (the radius anomaly) for high fluxes under a no-inflation model, the logistic model, and the Gaussian model. Here, the increasing bias of the logistic model for the 30 planets at such high fluxes is apparent. Even a flat $\epsilon$ at high flux predicts overly large planets, hence our conclusion that $\epsilon(F)$ must decline.

For our model of thermal tides (Arras & Socrates 2009), we examined the scaling relations for thermal tides from Socrates (2013) (Equation (26)), and found this potential power source to much too strongly increase with flux to reproduce the observed radii. The variance also appears overly high; for example, the scaling relations force $\epsilon$ to vary by more than an order of magnitude in planets with fluxes between 0.8 and 1.2 Gerg s$^{-1}$ cm$^{-2}$. As a result, we encountered considerable difficulty getting the model (see Section 4.1) to fit. We were only able to fit a model by imposing the regularizing constraint that $\epsilon$ for any individual planet cannot exceed 4.5%, a level far above what is otherwise needed to explain the observed radii. Under this requirement, we measure $\log_{10}(\epsilon_0) = -1.61 \pm 0.065$; Figure 14 shows the the inferred heating efficiencies for the sample planets as a function of flux. The
MCMC was able to fit the bulk of the data by placing them in the 5%–3% range, but the scaling is far too extreme. In explaining the bulk of the planets, a huge 43% (122/281) of the data exceeded the upper bound. Without the constraint, very few of the planets actually end up inflated; the range of coefficients to $\epsilon_0$ given by the scaling relation from Socrates (2013) is simply too large. As such, we conclude that the dominant source of inflation power in the observed population does not follow the thermal tides scaling relation.

6. Discussion

The Gaussian shape is significant because it exclusively matches predictions of hot-Jupiter inflation from the Ohmic dissipation mechanism. Under this model, magnetic interactions transfer energy from the atmosphere of a planet into its interior (Batygin & Stevenson 2010). The effect is initially increasing with greater atmospheric temperatures and therefore ionization, but at very high temperatures the magnetic drag on atmospheric winds (Perna et al. 2010) inhibit the process (Batygin et al. 2011; Menou 2012). Batygin et al. (2011) predict a scaling with equilibrium temperature as $\epsilon \propto (1500 \, K / T_{eq})^2$. Menou also derives scaling laws for this effect, estimating the peak $\epsilon$ to occur at 1600 K, depending on the planetary magnetic field strength (Menou 2012). Ginzburg & Sari (2016) support this conclusion, estimating a peak $\epsilon$ to occur at 1500 K, with power-law tails on either side. Finally, MHD simulations in Rogers & Komacek (2014) find a peak at 1500–1600 K. Figure 11 shows that the posteriors of our favored models match these predictions well. If Ohmic dissipation is responsible for our observation, then our measured $\epsilon(F)$ is presumably the average over various planetary magnetic field strengths.

A noteworthy difficulty with identifying our results with the Ohmic dissipation model is the depth at which the anomalous heat is deposited. Our model assumes that anomalous heating is efficiently conducted into the interior adiabat. Ohmic heating, however, is generally believed to be deposited at pressures low enough that only a portion of the deposited energy is inducted into the adiabat and a delayed cooling effect is produced (Spiegel & Burrows 2013; Wu & Lithwick 2013; Komacek & Youdin 2017). Indeed, Rogers & Komacek (2014) do not see sufficient heating to explain the observed radii. As well as differing from our modeling assumptions, this appears inconsistent with the results of Hartman et al. (2016), who observe reinflation of giants as their parent stars age and brighten over their main-sequence lifetime. This effect would be prohibitively slow in the shallow deposition case (Ginzburg & Sari 2016). Thus, if Ohmic heating is to explain our results, it must either violate these predictions or be modified by an additional effect that ushers the heat further into the planet. The advection effects proposed by Tremblin et al. (2017) show that such effects are plausible and that there is still a great deal left to understand about atmospheric flows in hot Jupiters.

As the results of Tremblin et al. (2017) stand, our observations do not seem to support them as the sole cause of inflation. They predict observable inflation occurring well below or at the observed 0.2 Gerg s$^{-1}$ cm$^{-2}$ threshold, and do not appear to support a decrease in efficiency at high flux. However, our results might align better if temperature-dependent wind speeds are considered within their model, which could slow flows both at especially low and high $T_{eq}$. Slower winds at high $T_{eq}$ would be a natural consequence of magnetic drag (Perna et al. 2010). We view our results here as support for the idea that magnetic drag is quite important in the hottest atmospheres.

Other candidate inflation models do not match our results very well. Tidal heating may introduce non-negligible energy into planet interiors, but cannot fully explain the anomalous radii (Miller et al. 2009; Leconte et al. 2010), and would not reproduce our relationship with flux. The thermal tides mechanism (Socrates 2013) appears to predict more variation in $\epsilon$ than can plausibly exist (see Figure 14). Delayed cooling models propose that no anomalous heating occurs and that radii anomalies instead result from phenomena that prevent the escape of formation energy, such as enhanced atmospheric opacities (Burrows et al. 2007) or inefficient heat transport in the interior (Chabrier & Baraffe 2007). This energy would
otherwise rapidly radiate away. The issue with these proposals is that they do not inherently depend on flux and cannot explain the results of Hartman et al. (2016). Furthermore, in the case of layered convection (see Leconte & Chabrier 2012) resulting in delayed cooling (Chabrier & Baraffe 2007), structure evolution simulations in Kurokawa & Inutsuka (2015) show that layered convection would not occur in young giants, and that even if layers are imposed, they would need to be implausibly thin (1−1000 cm) to achieve the observed radii.

The situation for Saturn-mass planets (those excluded from our model) remains puzzling. As described in Section 2, these exhibit a different relationship with flux than Jupiter-mass planets (Figure 2) and have been found less frequently in high-flux orbits than their higher-mass analogs (Figure 3). Inefficiency in the heating mechanism, perhaps by lower magnetic field strengths, could explain the former observation, but not the latter. Furthermore, Pu & Valencia (2017) recently showed that Ohmic dissipation should occur in Neptunes, so we can reasonably expect that it would work on Satsum as well. Some observational biases are doubtless present, but would likely not produce the effects seen. Thus, it seems possible that mass loss is occurring. However, the exact mechanism would be unclear; for example, neither XUV-driven mass loss (Yelle 2004; Lopez et al. 2012) nor boil-off (Owen & Wu 2016) appear to significantly affect planets in this mass range. As such, the cause of these observations is an open question.

There is still much work to be done in understanding hot-Jupiter radius inflation. A promising avenue are the case of “reinflated” hot Jupiters, which are planets whose radii may be increasing over time as their stars evolve off the main sequence and brighten (Lopez & Fortney 2016). Grunblatt et al. (2017) have conducted promising observations of two potentially reinflated planets around subgiant stars. Our posterior radius predictions are closer to their observations under the reinflated case, but more planets will be needed to establish strong statistical significance. Comparing the main-sequence re-inflation results of Hartman et al. (2016) with structure models could reveal the timescale of re-inflation, which is closely related to the depth of energy deposition (Ginzburg & Sari 2016; Komacek & Youdin 2017). If re-inflation does indeed occur, delayed cooling models are ruled out. Follow-up work of Tremblin et al. (2017) to determine how their results would be affected by temperature-dependant wind speeds would also be helpful. Finally, further magnetohydrodynamic simulations are needed to properly understand heat flow in the outer layers of these planets. Our results add to this picture by providing strong evidence of a heating efficiency drop at high temperatures and thereby pointing us toward the Ohmic dissipation model; they also suggest that 3D atmospheric circulation models need to take magnetic fields into account.

The authors thank Eric Lopez, Vivien Parmentier, Thad Komacek, and Ruth Murray-Clay for helpful discussions. Funding for this work was provided by NASA XRP grant NNX16AB49G.

References

Akeson, R. L., Chen, X., Ciardi, D., et al. 2013, PASP, 125, 989
Arras, P., & Socrates, A. 2009, arXiv:0901.0735
Baraffe, I., Selsis, F., Chabrier, G., et al. 2004, A&A, 419, L13
Batygin, K., & Stevenson, D. J. 2010, ApJ, 714, L238
Batygin, K., Stevenson, D. J., & Bodenheimer, P. H. 2011, ApJ, 738, 1
Burrows, A., Hubeny, I., Budaj, J., & Hubbard, W. B. 2007, ApJ, 661, 502
Chabrier, G., & Baraffe, I. 2007, ApJ, 661, L81
Charbonneau, D., Brown, T. M., Latham, D. W., & Mayor, M. 2000, ApJL, 529, L45
Demory, B.-O., & Seager, S. 2011, ApJS, 197, 12
Foreman-Mackey, D. 2016, The Journal of Open Source Software, 1, 24
Fortney, J. J., Marley, M. S., & Barnes, J. W. 2007, ApJ, 659, 1661
Gelman, A., Carlin, J. B., Stern, H., et al. 2013, Bayesian Data Analysis (3rd ed.; Taylor and Francis: London)
Gelman, A., Hwang, J., & Vehari, A. 2014, Statistics and Computing, 24, 997
Ginzburg, S., & Sari, R. 2016, ApJ, 819, 116
Grunblatt, S. K., Huber, D., Gaidos, E., et al. 2017, arXiv:1706.05865
Guillot, T., & Showman, A. P. 2002, A&A, 385, 156
Hartman, J. D., Bakos, G. Á., Bhatti, W., et al. 2016, AJ, 152, 182
Hastings, W., & K. 1970, Biometrika, 57, 97
Henry, G. W., Marcy, G. W., Butler, R. P., & Vogt, S. S. 2000, ApJL, 529, L41
Hubbard, W. B., Hattori, M. F., Burrows, A., Hubeny, I., & Sudarsky, D. 2007, Icar, 187, 358
Kass, R. E., & Raftery, A. E. 1995, J. Am. Stat. Assoc., 90, 773
Komacek, T. D., & Youdin, A. N. 2017, arXiv:1706.07605
Kurokawa, H., & Inutsuka, S.-i. 2015, ApJ, 815, 78
Laughlin, G., Crismani, M., & Adams, F. C. 2011, ApJL, 729, L7
Leconte, J., & Chabrier, G. 2012, A&A, 540, A20
Leconte, J., Chabrier, G., Baraffe, I., & Levrard, B. 2010, A&A, 516, A64
Lopez, E. D., & Fortney, J. J. 2016, ApJ, 818, 4
Lopez, E. D., Fortney, J. J., & Miller, N. 2012, ApJ, 761, 59
Menou, K. 2012, ApJ, 745, 138
Miller, N., & Fortney, J. J. 2011, ApJL, 736, L29
Miller, N., Fortney, J. J., & Jackson, B. 2009, ApJ, 702, 1413
Owen, J. E., & Wu, Y. 2016, ApJ, 817, 107
Perna, R., Menou, K., & Rauscher, E. 2010, ApJ, 719, 1421
Pu, B., & Valencia, D. 2017, ApJ, 846, 47
Roberts, G. O., & Rosenthal, J. S. 2004, Probab. Surveys, 1, 20
Rogers, T. M., & Komacek, T. D. 2014, ApJ, 794, 132
Saumon, D., Chabrier, G., & van Horn, H. M. 1995, ApJS, 99, 713
Schneider, J., Dedieu, C., Le Sidaner, P., Savalle, R., & Zolotukhin, I. 2011, A&A, 532, A79
Silverman, B. W. 1986, Density estimation for statistics and data analysis (Boca Raton, FL: CRC Press)
Socrates, A. 2013, arXiv:1304.4121
Spiegel, D. S., & Burrows, A. 2013, ApJ, 772, 76
Spiegelhalter, D. J., Best, N. G., Carlin, B. P., & Van Der Linde, A. 2002, Journal of the Royal Statistical Society: Series B, Statistical Methodology, 64, 583
Thompson, S. L. 1990, ANEOS—Analytic Equations of State for Shock Physics Codes, Sandia Natl. Lab. Doc. SAND89-2051, http://prod.sandia.gov/techlib/access-control.cgi/1989/829251.pdf
Thornren, D. P., Fortney, J. J., Murray-Clay, R. A., & Lopez, E. D. 2016, ApJ, 831, 64
Tremblin, P., Chabrier, G., Mayne, N. J., et al. 2017, ApJ, 841, 30
Van Der Walt, S., Colbert, S. C., & Varoquaux, G. 2011, arXiv:1102.1523
Weiss, L. M., Marcy, G. W., Rowe, J. F., et al. 2013, ApJ, 768, 14
Wu, Y., & Lithwick, Y. 2013, ApJ, 763, 13
Yelle, R. V. 2004, Icar, 170, 167

ORCID iDs

Daniel P. Thornren @ https://orcid.org/0000-0002-5113-8558
Jonathan J. Fortney @ https://orcid.org/0000-0002-9843-4354