Urbanization and population contraction

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Received: 22 April 2021 / Accepted: 25 May 2022 / Published online: 9 July 2022
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Abstract
Urbanization is treated in the literature as a process that occurs along with economic development. We consider an overlapping generations model with two regions, designated as ‘urban’ and ‘rural’. Concentration of population to urban areas involves population inflows from rural areas, thereby exacerbating urban congestion. Inverse agglomeration economies in rural areas exert negative effects on rural income, consequently increasing the attractiveness of urban areas. Because of lower urban fertility rates, urbanization involves population decreases. Therefore, population dynamics might be explained as simultaneous urbanization and population contraction. However, depopulation mitigates congestion, which increases the fertility rate and the worker’s lifetime utility level. Eventually, it can lead to a stationary population size.

Keywords  Fertility · Migration · Population contraction · Reversely directed agglomeration · Urbanization

JEL Classification  J11 · J13 · R13 · R23

1 Introduction

In most reports of the literature describing aspects of economic geography, urbanization is treated as a monotonic process that occurs along with economic development. That has been true since a seminal paper published by Krugman (1991), who presents an analysis of how an economy can, endogenously, become differentiated into industrialized urban areas and peripheral areas. For instance, Arcalean et al. (2019) describe that Asia and Africa lag behind in urbanization.

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The UN World Urbanization Prospects (2018) report describes that urbanization is proceeding steadily worldwide. In fact, rates of urbanization are higher in more economically developed regions than in less-developed regions (Fig. 1). According to the UN World Population Prospects (2019) report, total fertility rates have been lower than the replacement rate in most economically developed regions for decades, although the rates are still higher in less-developed economies. Figure 2 portrays changes in the population aged 15–64 since 1990. Reflecting fertility differences, population sizes in more economically developed regions which are highly urbanized have recently declined, although those in least-developed regions with the lowest degree of urbanization have rather increased during the period.¹

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It has been recognized as a stylized fact that concentrations of population increase income through agglomeration economies, whereas congestion diseconomies lower fertility rates through a negative income effect in urban areas (e.g., de la

¹ The population growth of young generations has also slowed in less-developed countries during the period, although the total population size continues to increase. Changes in population growth rates of countries might be influenced by immigrations, as they have been in Germany and Italy.
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The relevant literature reports determination of the dynamics of overall population in an economy through interaction between the regional distribution and the population size (Sato and Yamamoto 2005; Sato 2007; Yakita 2011; Muroishi and Yakita 2021). Particularly, Sato and Yamamoto (2005) and Yakita (2011) assume that the concentration of population causes agglomeration effects only in urban areas, but not in rural areas. Sato (2007) and Muroishi and Yakita (2021) assume that the degrees of congestion diseconomy are equal among regions, although all regions are urban.

In contrast, to analyze the population dynamics of a two-region economy in this paper, we assume that concentrations of population also exert agglomeration effects in rural areas. Moreover, because of agglomeration economies in rural areas, the rural wage rate increases with the number of workers. Although the degree of agglomeration economies is higher in urban than in rural areas, concentration of populations in urban areas causes depopulation in rural areas. Population declines in rural areas decrease agglomeration economies in those regions, thereby lowering regional wages. The consequently lowered income might induce more population outflows. Nevertheless, we also assume that congestion diseconomies exist only in urban areas.

Baldwin and Forslid (2000) find that agglomeration economies also enhance real wage growth in the periphery through interregional transactions. Gruber and Soci (2010) describe that the immobile factor “land” provides housing space for people and firms to set up their plants and offices, especially in peripheral (rural) zones. As they describe, the land-intensive sector, such as tourism and recreation, already has large employment shares in rural areas of economically developed countries. Therefore, rural areas might also have agglomeration effects. We incorporate this consideration into our model.

The main result is that, for highly urbanized economies such as those of economically developed countries, the economy will undergo both urbanization and population contraction simultaneously. Depopulation in rural areas accelerates through reversely directed agglomeration economies. As time passes, the populations become fully urbanized. However, they converge to a stationary size because population contraction mitigates congestion in urban areas.

The next section introduces a simple overlapping-generations model of a two-region economy of urban and rural regions. Section 3 presents analysis of the population dynamics. Section 4 concludes the paper.

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2 Fujita and Thisse (2013) emphasize (i) increasing returns to scale economies in firm-level production and (ii) the spread of information among close-knit agents (workers and firms) as major causes of agglomeration economies and workers’ commuting time to business districts as a primary reason for congestion diseconomies.

3 More recently, Grafeneder-Weissteiner and Prettner (2013) and Jedwab et al. (2017) consider the effects of lower mortality on urbanization and urban increases in recent decades. However, the study reported herein is concerned only with fertility dynamics and urbanization.
2 Model

The regions are designated, respectively, as ‘urban ($u$)’ and ‘rural ($r$)’. Each generation consists of individuals who have identical preferences and who live for two periods. In adulthood, individuals choose a region in which to reside and work. They rear children during adulthood. The adult population in the economy in period $t$ is denoted as $N_t = N_{ut} + N_{rt}$, where $N_{ii}$ stands for the number of adults in region $i$ ($i = r, u$). The fertility rate of the economy in period $t$ is $n_t = (n_{i1}N_{ut} + n_{i2}N_{rt})/N_t$, where $n_{ii}$ is the regional fertility rate ($i = r, u$). The change of the population size can be expressed as $N_{t+1} = n_tN_t$.

Assuming that the congestion cost in urban areas is measured in time costs, the net time endowment available for urban adults can be written as

$$l_{ut} = \exp(-GN_{ut}) < 1.$$  

This functional form is also assumed for their work by Sato and Yamamoto (2005). Parameter $G > 0$ reflects the degree of congestion diseconomies. The gross time endowment is normalized to one. Workers in the urban area commute to the central business district (CBD). Residing far from the CBD entails greater commuting costs and a lower land rent. Commuting time, which is defined as the ratio of the congestion cost divided by the wage rate, increases with population concentration, which reduces the available time for working, parenting, and other activities. In an urban spatial equilibrium, land rents are determined to compensate for differences in commuting costs. The sum of these costs is the same value for all urban workers. Therefore, workers no longer have an incentive to move to another location within the region. In sum, the urban congestion cost is $1 - l_{ut} > 0$. By contrast, without congestion, the time endowment for adults in rural regions is therefore equal to one, i.e., $l_{rt} = 1$.

Denoting the rearing time per child as $z$, which is assumed to be constant, the budget constraint of adults in region $i$ at period $t$ can be written as

$$(l_{it} - zn_{it})w_{it} = c_{it},$$

where $w_{it}$ and $c_{it}$ respectively stand for the wage rate and consumption in region $i$ during period $t$. Assuming the lifetime utility of an individual in region $i$ at period $t$ to be represented as $U_{it} = c_{it}^{\alpha}n_{it}^{1-\alpha}$, where $0 < \alpha < 1$ ($i = r, u$), we obtain the optimal plans as

$$c_{it} = \alpha l_{it}w_{it}$$

4 Workers in the urban area commute to the central business district (CBD). Land is owned by absentee landowners. Each worker consumes one unit of land in each region, receiving the same wage rate. When the sum of commuting costs to the CBD and land rent is the same value for all workers in a region, they have no incentive to move to another location within the region, i.e., intra-region spatial (or residential) equilibrium.
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The individual chooses the consumption level \( c_{it} \) (or working time \( l_{it} - zn_{it} \)) and the number of children \( n_{it} \) (or child-rearing time \( zn_{it} \)) to maximize the utility, given the wage rate \( w_{it} \).

Moreover, we assume for this study that the degree of agglomeration effects and therefore the wage rate is higher in the urban region than in the rural region, i.e., the urban wage rate of

\[
  w_{ut} = B \exp(\beta N_{ut})
\]

is higher than the rural wage rate of

\[
  w_{rt} = R \exp(\gamma N_{rt}).
\]

The urban wage function (5) is common in the literature in the sense that concentration of workers generates positive externalities on the wage rate through agglomeration effects in urban areas (Sato and Yamamoto 2005). By contrast, the rural wage function (6) is a salient feature of the present analyses, as described in the Introduction. The usual assumption in the literature of New Economic Geography (NEG) is that the population in rural regions should be engaged primarily in agriculture (e.g., Fujita et al. 1999) and that the employment share in the land-intensive sector is very low and decreasing (e.g., Tabuchi and Thisse, 2006). However, as described by Gruber and Soci (2010), the immobile factor “land” provides housing space for people and firms to set up their plants and offices, especially in peripheral (rural) zones. Therefore, there might also exist agglomeration effects attributable to the concentration of workers even in rural areas, although the rural area has no definite business center. Concentration of agricultural tourism and recreation sites, for instance, might exert external economies among them, i.e., agglomeration economies.

For the analyses presented in this paper, we assume that \( B > R(> 0) \) and \( \beta \geq \gamma(> 0) \) hold. The former inequality guarantees the possibility that adults choose to reside in an urban area, although they face congestion costs. The last inequality indicates that the degree of agglomeration effect is greater in the urban area.

By inserting the optimal plans into the utility function, we obtain the indirect function as \( V_{it} = \alpha^a (1 - a) l_{it}^{(1-a)} (w_{it})^a \). Because of the free residence choices of adults between regions, the utility levels are equalized in the migration equilibrium, i.e., \( V_{ut} = V_{rt} \). By inserting \( l_{ut}, w_{ut}, \) and \( w_{rt} \) from (1), (5), and (6) into the equality, the arbitrary condition for migration equilibrium can be rewritten as

\[
  \exp(-GN_{ut}) [B \exp(\beta N_{ut})]^{\alpha} = [R \exp(\gamma N_{rt})]^{\alpha}.
\]

In the inter-regional migration equilibrium, each region is also in intra-regional spatial equilibrium. From (7), the equilibrium distribution of adults between regions in period \( t \) is obtainable for a given total population in period \( t, N_t \). From (7), the urban population is given as
\[ N_{ut} = \left( \lambda + \gamma N_i \right) / \left( \beta - \frac{G}{\alpha} + \gamma \right), \]  

where \( \lambda = \ln R - \ln B < 0 \) from assumption. The migration-stability condition is \( \frac{d}{dN_{ut}} (V_{ut} - V_{rt}) < 0 \), i.e.,

\[ \alpha \left\{ B \exp \left[ \left( \beta - \frac{G}{\alpha} \right) N_{ut} \right] \right\}^a \left( \beta - \frac{G}{\alpha} + \gamma \right) < 0. \]  

The condition might be interpreted as follows. An adult is worse off if the adult migrates to the other region. The utility gain from moving to the urban area \( \beta - (G/\alpha) \) is less than the utility loss from moving out of the rural area \( \gamma \), i.e., \( \beta - (G/\alpha) < -\gamma \). We assume that condition \( \beta - (G/\alpha) + \gamma < 0 \) is satisfied. Nevertheless, because we must have condition \( (\lambda < 0) \) \( \lambda + \gamma N_i < 0 \) for \( N_{ut} \) to be positive in (8), \( B \) is sufficiently greater than \( R \), i.e., the urban wage rate is sufficiently high to attract workers to the urban area.

Given the migration-stability condition, we analyze the population dynamics of the economy for a given population size \( N_i \). First, from (8), we have positive urban population \( N_{ut} > 0 \) only for \( N_i < -\lambda / \gamma \). When \( N_i \geq -\lambda / \gamma \), we have a corner solution of zero urban population, i.e., \( N_{ut} = 0 \). In this case, the time change in the population of the economy is given as \( N_{t+1} = n_r N_t \), where \( n_r = (1 - \alpha / z) \) is the rural fertility rate. All adults reside in the rural region. We assume here that \( n_r > 1 \).

Second, from (8), we can obtain \( N_i = N_{ut} \) when the overall population size is \( N_i = \lambda / [\beta - (G/\alpha)] \). The entire population resides in the urban area. If otherwise the population is less than the critical size, i.e., \( \lambda / (\beta - G/\alpha) > N_i \), then we must have \( N_{ut} > (\lambda + \gamma \beta - G/\alpha) / (\beta - G/\alpha + \gamma) = \lambda / (\beta - G/\alpha) > N_i \), which is a contradiction. Therefore, the case of \( \lambda / (\beta - G/\alpha) > N_i \) is impossible. We also have a corner solution \( N_i = N_{ut} \) for \( \lambda / (\beta - (G/\alpha)) > N_i \). In this case, all adults choose to reside and work in the urban area. The evolution of the overall population is given as \( N_{t+1} = n_{ut} N_i = n_r \exp(-GN_i)N_i \) from (1).

Finally, because of the migration-stability condition (9), we have \( 0 < N_{ut} < N_i \) for \( -\lambda / \gamma > N_i > \lambda / [\beta - (G/\alpha)] \). Adults are distributed both in urban and in rural areas. In this case, the evolution of population is given as

\[ N_{t+1} = n_{ut} N_{ut} + n_r N_{rt} \]

\[ = n_r \left\{ -\frac{\lambda}{\Omega} + \left( 1 - \frac{\gamma}{\Omega} \right) N_i + \frac{\lambda + \gamma N_i}{\Omega} \exp \left( -G \frac{\lambda + \gamma N_i}{\Omega} \right) \right\}, \]

where \( \Omega = \beta - (G/\alpha) + \gamma < 0 \). From (8) we obtain

\[ \frac{d}{dN_i} \left( \frac{N_{ut}}{N_i} \right) = \frac{-\lambda}{(N_i)^2 \Omega} < 0. \]

5 If otherwise \( n_r \leq 1 \), then the overall population never increases.
The share of the urban population decreases with the overall population size. Increases (decreases) in the population size lower (raise) the urban share of population. That is, urbanization is associated with population contraction because of lower urban fertility as a result of congestion. In contrast, rural areas can enjoy agglomeration economies without congestion costs in this paper. Therefore, when the overall population becomes large, individuals are likely to reside in rural areas to take the agglomeration benefits while avoiding congestion costs. When the population becomes smaller, the opposite occurs. The negative effect of the population size, i.e., (11), is greater when the measure of agglomeration effects in rural areas ($\gamma$) is greater. It is noteworthy that this result contrasts to those presented in earlier reports of the conventional literature without agglomeration in rural areas.

3 Dynamics

Given the fact of progressive urbanization illustrated in the Introduction, we can consider that the initial condition satisfies $N_0 < -\lambda/\gamma$, where $N_0$ is the initial overall population in the economy. For analyses of the population dynamics, two possibilities can be considered at the critical overall population size of $N_t = \lambda/(\beta - G/\alpha)$: (i) $n_{ut} = n_r \exp[-G\lambda/(\beta - G/\alpha)] \geq 1$ and (ii) $n_{ut} = n_r \exp[-G\lambda/(\beta - G/\alpha)] < 1$.

In case (i), the urban fertility rate is greater than or equal to unity at the critical population size. In this case, from arguments presented in the preceding section, we have $N_{t+1} = n_{ut}N_t \geq N_t$ for $N_t \leq \lambda/(\beta - G/\alpha)$. Therefore, the overall population size increases under the critical population size. When $N_t$ becomes greater than $\lambda/(\beta - G/\alpha)$, the share of urban population $N_{ut}/N_t$ decreases from (11). Both the overall population size and the rural population share increase as time passes. The population dynamics runs against fact in economically developed countries, as reported in the Introduction. Therefore, this case is implausible.

Next, in case (ii), the urban fertility rate is lower than unity at the critical population size. In this case, a stationary equilibrium prevails because $N_{t+1} = n_rN_t > N_t$, at $N_t = -\lambda/\gamma$ and because $N_{t+1} = n_{ut}N_t \geq N_t$ for $N_t \leq \lambda/(\beta - G/\alpha)$, where $-\lambda/\gamma > N_t > \lambda/(\beta - G/\alpha)$. In other words, a stationary equilibrium population size exists if condition $1 < n_r < \exp[\lambda G/(\beta - G/\alpha)]$ is satisfied. This case is presented in Fig. 3, in which curve $N_{t+1} = n_rN_t \equiv \Lambda(N_t)$ crosses the 45-degree line from below. Therefore, the stationary population equilibrium is unstable.

Urbanization might therefore involve population contraction. In economically developed regions, the population size has not increased for decades. Therefore, this is the case for economically developed regions. When the economy is below the 45-degree line, it has $n_{ut} \leq n_t < 1$. After the overall population size shrinks

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6 Because all adults reside in rural areas when $N_t = -\lambda/\gamma$, we have $N_{r+1} = n_rN_r > N_r$.

7 Curve $N_{t+1} = n_rN_t$ is presented as a line for expositional simplicity. However, it might be nonlinear, i.e., the sign of $dN_{t+1}/dN_t$ is ambiguous a priori. Therefore, although the stationary equilibrium might be multiple, the equilibrium with the smallest population size between $\lambda/(\beta - G/\alpha)$ and $-\lambda/\gamma$ is unstable. At a stable stationary equilibrium between $\lambda/(\beta - G/\alpha)$ and $-\lambda/\gamma$, if it exists, then there are regions of two types: rural and urban. The multiple equilibrium case is shown in Fig. 3b.

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to become smaller than \( \frac{\lambda}{\beta - G/\alpha} \), the population becomes fully urbanized. Moreover, it declines further as time passes because \( n_t = n_{ut} < 1 \) still holds below the 45-degree line.

However, we eventually have \( N_{t+1} = N_t \) at \( N_t = \ln n_r / G \), where \( n_r \exp(-GN_t) = 1 \). Fully urbanized economies might have a stable stationary population. In the
stationary equilibrium, a constant size of population resides in urban areas. No population resides in the rural areas.\(^8\)

Therefore, from summation of the arguments presented above, we have the following propositions.

**Proposition 1** Insofar as the rural fertility rate satisfies condition \(1 < n_r < \exp[\frac{\lambda G}{(\beta - G/\alpha)}]\), a stationary equilibrium of population size residing in both urban and rural regions exists. The equilibrium might be multiple. However, the stationary equilibrium with the smallest population size between \(\frac{n_u}{\beta - G/\alpha}\) and \(-\frac{\lambda}{\gamma}\) is unstable.

**Proposition 2** If the population residing in both urban and rural regions is decreasing, then the population eventually urbanizes fully through reversely directed agglomeration effects in rural areas: in this case, simultaneously, its size decreases over time. However, when it becomes \(N_t = \ln n_r/G\), the fully urbanized population becomes stationary.

Results of simultaneous depopulation and urbanization accelerated by (reversely directed) agglomeration effects in rural areas, have not been described in reports of the literature, such as reports by Sato and Yamamoto (2005) and Yakita (2011). In our model, when the population is highly urbanized, the urban fertility rate becomes less than the replacement rate, thereby representing a decrease in the population size. This theoretical prediction in Proposition 2 is apparently consistent with the facts presented in the Introduction (Figs. 1, 2).\(^9\) However, the depopulation consequently leads to a stationary urban population.

The intuition underlying these results is the following. The initial population is smaller than the unstable stationary level: The population size decreases and converges toward a stable fully urbanized population. In fact, concentration of the population in urban regions enhances agglomeration economies in the urban region, consequently raising the urban wage rate, but higher concentrations also decrease agglomeration economies in rural areas, thereby lowering the rural wage rate. Higher urban wages attract workers to urban areas, consequently leading to full urbanization of the overall population. Increases in the number of urban workers accompany a lower overall fertility rate because the urban fertility rate is less than unity. However, depopulation mitigates urban congestion in full urbanization, thereby affecting the urban fertility rate positively. Consequently, the economy converges to a long-term equilibrium with a stationary urban population size. The convergence also increases the utility level of urbanized workers during the transition.\(^10\)

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\(^8\) Condition \(\Lambda(N_t) = N_t\) is also satisfied at the origin in Fig. 3. The origin is an unstable equilibrium.

\(^9\) The growth rates of population aged 15–64 became negative in France and Italy during 2015–2020, although the rate has been negative in Japan since 1995. Japan and France have received few immigrants. By contrast, because of immigration, the population growth rate became positive in Germany during 2010–2020.

\(^10\) If the initial population is less than \(\ln n_r/G\), then the size increases to a stationary size with both agglomeration and congestion effects, thereby decreasing the fertility rate. We do not examine this trivial case.
Remark: The greater the degree of congestion diseconomy \((G)\) becomes in urban areas, the wider the range of instability \((\ln n_r/G < N_t < \lambda/(\beta - G/\alpha))\) becomes for the overall population, as depicted in Fig. 3. The stationary population size of the fully urbanized economy \((\ln n_r/G)\) is greater if the degree of congestion diseconomy is smaller and if the rural fertility rate without congestion is higher.

4 Concluding remarks

Results of this study have demonstrated that population urbanization and contraction occur simultaneously in a simple model with rural agglomeration economies. For economically developed countries, this is apparently the case (Figs. 1, 2). Especially, Proposition 2 indicates that urbanization leads the fertility rate below the replacement rate at earlier stages of urbanization. However, after full urbanization, decreased population size mitigates congestion diseconomies, thereby exerting positive effects on fertility, and eventually bringing the economy to a stable, fully urbanized stationary population. Such a process of urbanization has not been reported in the literature. It is noteworthy that the utility level of workers increases during the depopulation process, although the effects on wage income are ambiguous.

The simplicity of the model leads to some important deficits. If the initial population size is sufficiently great, then the population might continue to increase. However, this prediction apparently runs counter to factual outcomes reported for economically developed countries. Second, we have not considered agricultural goods production explicitly. The agricultural sector might become a business centralized at the central business district (CBD) in the long-term urbanized equilibrium.11 As described in Gruber and Soci (2010), the agricultural population does not necessarily coincide with the rural population. Finally, we have not considered economic growth engines such as human capital accumulation and policy intervention. These are interesting subjects which shall be reserved for future research.

Acknowledgements The authors thank two anonymous reviewers and Editor of this journal for their heartwarming and insightful comments and suggestions. An earlier version of this paper was read at Nagoya City University. The authors thank the seminar participants for their comments and suggestions. The second author acknowledges financial support from the Japanese Society for the Promotion of Science KAKENHI [Grant Nos. 19H01503 and 20K01621].

Data availability Data sharing is not applicable to the study described in this paper: no new data were analyzed.

Declarations

Conflict of interest The authors have no conflict of interest, financial or otherwise, related to this study.

11 The NEG literature conventionally includes the presumption that peripheral (rural) regions act as a supplier of agricultural products for all workers.
