Proportionality between powder cohesion and unconfined yield strength from shear cell testing

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Abstract

From an analysis of the geometry of the yield locus and the Mohr’s circle for determining unconfined yield strength ($f_c$) in shear cell testing, it has been shown that powder cohesion is proportional to $f_c$, where the proportionality constant is a function of angle of linearized yield locus, $(1-\sin \theta)/(2\cos \theta)$. While both parameters are routinely included in shear cell data, only one parameter is needed to characterize flow properties of a new powder.

Keywords: Chemical engineering, Pharmaceutical science

1. Introduction

Powder flowability is one of the critical material attributes that affect powder handling in several industries, including pharmaceutical manufacturing [1, 2]. For example, high speed tableting requires the powder to have “acceptable flow properties” for consistent die filling, tablet weight, and content uniformity [3]. Powder flow properties can be measured using different techniques [4, 5, 6], but shear cells are the most fundamental, versatile, and reliable technique [7]. Consequently, it has been
widely applied to characterize pharmaceutical powders to guide formulation development and optimization [8, 9, 10, 11].

A shear cell measurement can produce a number of useful parameters to describe powder flow properties [7, 12, 13, 14]. The unconfined yield strength ($f_c$) is the most commonly used parameter to quantify powder flowability. A flowability index, $ff$, defined as the ratio of major principal stress ($\sigma_1$) to $f_c$ [15, 16], has been used to quantify powder flowability. In general, a powder with a higher $f_c$ at the same $\sigma_1$ tends to exhibit poorer powder flow. Recently, it was observed that $f_c$ and powder cohesion, $C$, exhibit a peculiarly similar relationship with $\sigma_1$ (Fig. 1) [15], indicating a possible intrinsic proportional relationship between these two parameters. A proportionality between the two parameters is also suggested by the observation of nearly perfect linear relationship between experimental $f_c$ and $C$ for 25 fine powders [17]. Here, we derive a mathematical relationship between the two parameters to explain this phenomenon.

![Image](https://doi.org/10.1016/j.heliyon.2019.e01171)

**2. Theory**

In the classical approach of analyzing powder flowability using a shear cell, a yield locus is experimentally determined by measuring the maximum shear stress the powder can sustain under a given normal stress [18]. Repeating the measurement at different normal stresses delineates the yield locus. According to the shear cell theory, $C$ is the intercept of a yield locus with the shear stress ($\tau$) axis. A series of semicircles that are below and tangent to the yield locus can be drawn to represent the stress distribution inside the powder under different external stress conditions. The semicircle passing the origin crosses the normal stress ($\sigma_n$) axis at $f_c$ (Fig. 2) [14]. An inspection of these basic elements of the figure indicates these two parameters can be related by taking a trigonometric approach.

To solve the problem, we can connect the tangent point, D, and the center of the circle, B. The radius BD forms a right angle with the yield locus line. If the yield locus
is extrapolated to intersect with the $\sigma_n$ axis at point A, angle $\theta$ is identified. In cases where the yield locus is not strictly linear, this angle corresponds to the angle of linearized yield locus [14]. The two right triangles, ADB and AOC, are similar because they share the same angle, $\theta$. Therefore, Eq. (1) is written by applying the rule of proportionality between similar triangles.

$$\frac{AB}{AC} = \frac{BD}{CO}$$  

Expressing the lengths of each side in terms of $f_c$ and C yields Eq. (2).

$$\left(\frac{C + \frac{f_c}{2}}{\sin \theta}\right) = \left(\frac{\frac{f_c}{2}}{C}\right)$$  

We can substitute $\tan(\theta)$ in Eq. (2) with $\sin(\theta)/\cos(\theta)$ and rearrange terms to get Eq. (3).

$$\left(C \frac{\cos \theta}{\sin \theta} + \frac{f_c}{2}\right) \left(\frac{\sin \theta}{C}\right) = \frac{f_c}{2C}$$  

Solving for $C$ by rearranging terms in Eq. (3) leads to Eq. (4),

$$C = f_c \frac{(1 - \sin \theta)}{2\cos \theta}$$  

Eq. (4) explicitly indicates a proportionality between $f_c$ and C, with the proportionality constant, $(1-\sin \theta)/(2\cos \theta)$. This equation was recently suggested but derivation was not given [19]. The proportionality between C and $f_c$ explains the similar shape of $f_c$ and C plots shown in Fig. 1. This relation is consistent with the observed proportionality between $f_c$ and C for the same material under different initial consolidation stresses. The proportionality constants differed for different materials, which may be attributed to their different $\theta$ values [20].
3. Results & discussion

Proportionality in Eq. (4) is a function of $q$, which is different among powders. To validate this relationship, we used shear cell data of powders exhibiting a wide range of flow properties tested under different normal stresses, i.e., limestone reference powder for shear cell (pre-shear normal stresses of 3, 6, 9, 15 kPa), microcrystalline cellulose (Avicel Ph105 and Avicel PH102, pre-shear normal stresses of 1, 3, 6, 9 kPa). Data were collected using a rotational shear cell (RST-XS, Dietmar Schulze, Wolfsenbüttel, Germany). The angle of linearized yield locus was used to calculate the proportionality constant. Subsequently, for each experimentally determined $f_c$ value, a corresponding $C$ value was predicted from Eq. (4), which was then plotted against $C$ values obtained by extrapolating the yield locus (Fig. 3). The linear regression equation has a slope of essentially unity (1.0003) and an intercept of essentially zero (0.2981 Pa). Thus, the regression line closely resembles the ideal line, which suggests the validity of the derived relationship between $f_c$ and $C$. Eq. (4) is valid as long as the yield locus between the y-intercept and the tangency point, i.e., CD in Fig. 2, is linear. However, when the yield locus is curved, deviations from Eq. (4) are expected. Therefore, a gross deviation from the proportionality is a clear indication of curvature of the yield locus in the pressure range below the tangent point, D, in Fig. 2. This occurs usually when the powder is very cohesive and, hence, flows poorly.

4. Conclusion

The similarity between cohesion and flow function plots derived from shear cell data is explained by the proportionality between $f_c$ and $C$ values derived from a given

![Graph showing the relationship between experimental and calculated $C$ values and the linear regression equation with $y = 1.0003x - 0.2981$, $R^2 = 1$.](https://doi.org/10.1016/j.heliyon.2019.e01171)
yield locus, where the proportionality constant is equal to \((1-\sin \theta)/(2\cos \theta)\). Given the proportionality, only one of the two flow parameters needs to be analyzed to compare flowability among different powders or to predict flow behavior of a given powder.

**Declarations**

**Author contribution statement**

David J. Sun: Performed the experiments; Analyzed and interpreted the data; Wrote the paper.

Changquan Calvin Sun: Conceived and designed the experiments; Analyzed and interpreted the data; Wrote the paper.

**Funding statement**

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

**Competing interest statement**

The authors declare no conflict of interest.

**Additional information**

No additional information is available for this paper.

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