Magnetic Field Induced Gap and Kink Behavior of Thermal Conductivity

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The thermal conductivity of a quasiparticle (QP) system described by a relativistic four-fermion interaction model in the presence of an external magnetic field is calculated. It is shown that, for narrow width of quasiparticles, the thermal conductivity, as a function of the applied magnetic field, exhibits a kink behavior at a critical field $B_c \sim T^2$. The kink is due to the opening of a gap in the QP spectrum at a critical magnetic field $B_c$ and to the enhancement of the transitions between the zeroth and first Landau levels. Possible applications of the results are discussed.

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It is now well known from the study of field theoretical models that an external magnetic field can be a strong catalyst for a symmetry breaking leading to the generation of a fermion dynamical mass even at the weakest attractive interactions \cite{1}. This phenomenon of magnetic catalysis (MC) of symmetry breaking has been shown to be essentially model independent and to have wide applications in several physical systems \cite{5,6}, in particular, in condensed matter physics \cite{5,6}. The essence of the MC phenomenon is that an external magnetic field can be a strong catalyst (MC) of symmetry breaking has been shown to be essentially model independent and to have wide applications in several physical systems \cite{1–4}, in particular, in condensed matter physics \cite{5,6}. The essence of the MC effect lies in the dimensional reduction of the fermion pairing dynamics when the pairing energy is much less than the Landau gap $\sqrt{eB}$ ($B$ is the magnitude of the magnetic field induction). In this case, the fermions are mostly confined to the lowest Landau level (LL) and their attractive interaction, regardless how small it may be, leads to mass generation.

In recent years, several authors \cite{1,7} suggested that the magnetic catalysis may play a relevant role in the physics of high-$T_c$ superconductors in external magnetic fields. This idea was inspired in part by the outcome of various experiments \cite{7,8,9} initiated by Krishana’s et al. on the thermal conductivity of high-$T_c$ compounds in the presence of an external magnetic field perpendicularly applied to the cuprate plane. According to these experiments, at temperatures significantly lower than the critical temperature of superconductivity, the profile of the thermal conductivity $\kappa$ versus the magnetic field displays a sharp break in its slope at a transition field $B_c$, followed then by a plateau region in which it ceases to change with increasing field up to the highest attainable fields $\sim 14T$.

To understand how the magnetic catalysis was related to these experimental observations, let us recall that high-$T_c$ superconducting cuprates are d-wave superconductors which implies that their Fermi surface is characterized by nodal points where the gap function vanishes. Then, the low-energy quasiparticle (QP) dynamics is concentrated mainly around these nodal points because these states can be occupied even at very low temperatures. The QP excitations in the vicinity of each node can be described by a massless Dirac Lagrangian \cite{12,13}. Hence, a natural candidate to modelling the QP interactions in a d-wave superconductor is a 3-dimensional relativistic four-fermion interaction (Nambu-Jona-Lasinio (NJL) type) model. It is well known \cite{12,13} that the gap equations of such a kind of models in the presence of an external magnetic field lead to the generation of a dynamical fermion mass (QP gap), which scales (at zero temperature) with the field as $m \sim \sqrt{eB}$. The critical temperature at which the dynamical mass vanishes is determined by the dynamical mass at zero temperature, and therefore, it scales with the magnetic field with the same law as the zero-temperature mass. One of the outcomes of the above mentioned experiments \cite{1} was that the critical temperature for the appearance of the kink-like behavior of the thermal conductivity scales with the magnetic field as $T_c \sim \sqrt{B}$. Then, the square root field dependence of the critical temperature in $(2 + 1)$-dimensional NJL models was considered as a hint that the MC phenomenon could be important to explain the experimental results of Krishana et al. \cite{8,9}.

By now, several possible mechanisms for the arising plateau have been proposed \cite{1,14,15}. All of them are based on the adoption of the QP picture as the low-energy excitations at the nodes of the $d-$wave symmetric order parameter. In the approach of Refs. \cite{1,14}, following the ideas of Ref. \cite{15}, it was assumed that the QP spectrum in the magnetic field was characterized by Landau levels. Furthermore, in Ref. \cite{1,14,15} it was supposed that a QP gap opens at the critical field $B_c$ leading to the exponential vanishing of the quasiparticle contribution in the thermal conductivity. Since the total conductivity is assumed to be the sum of the QP term $\kappa_{QP}$ and the phonon term $\kappa_{ph}$ with the phononic part independent of the field, this would lead to a plateau for high fields. Laughlin \cite{16} relates the QP gap to a weakly first order phase transition leading to the development of an additional small parity-violating superconducting order parameter. In another line of reasoning \cite{1,14}, the drop in the QP’s conductivity, hence the plateau in the total conductivity, is ascribed to the opening of a field-induced energy gap in d-wave superconductors due to a
The two fluxes, due to the external magnetic field and the phononic part of the thermal conductivity is also considered to be field-independent, in Franz' scenario the QPs conductivity itself becomes field-independent above a crossover field $B_c$. The effect is primarily due to the compensation of the enhancement of the QP density of states, arising from the Doppler shift of QP’s spectrum in the superfluid velocity field around vortices (Volovik effect [19]), and the growth of QP’s width caused by the scattering on vortices. The Volovik effect certainly plays a decisive role at weak magnetic fields and low temperatures (less than 1 K), where the increase in the thermal conductivity was predicted theoretically [20], and later confirmed in the experiment [10], to follow the $\sqrt{B}$ dependence of the density of states. However, Volovik’s arguments are based on semiclassical calculations valid in case one has well isolated vortices with supercurrents circulating around them. At higher fields, when vortices begin to overlap, this semiclassical picture should be replaced by a more adequate quantum mechanical treatment.

It should be stressed that none of the above works address the question of the kink itself in the thermal conductivity. Moreover, we should mention that the applicability of the Landau level quantization scheme used in the QP picture of the d-wave superconductor in a constant magnetic field has been recently subjected to intense criticism by several authors [21–23]. The Landau level-like spectrum proposed by Gorkov and Schiffer [17], and Anderson [18] was based on the assumption that the spatially dependent superfluid velocity can be neglected, but according to Melnikov [21], the superfluidity strongly mixes individual Landau levels. In order to take into account the effects of spatially varying supercurrents, Franz and Tesanovich [22] introduced a singular gauge transformation to trade the phase of the order parameter for a fictitious $U(1)$ gauge field. As a result, the two fluxes, due to the external magnetic field and the $U(1)$ field, cancel each other on average, washing out the whole picture of Landau level quantization. In this framework the low-energy quasiparticles are in fact Bloch waves described by massless Dirac fermions moving in a vector potential of physical supercurrents but zero average magnetic field. While the arising scenario might seem promising, the approach of Ref. [22] itself is not free of problems: there is no good foundation from a physical point of view to use singular gauge transformations and to introduce fictitious gauge fields, both of which are needed in this scheme. Moreover, because of the singular transformations, the physical picture itself becomes strongly dependent on the way such transformations are implemented (i.e., different singular gauge transformations can give rise to different spectra, depending on the regularization scheme [23]). Finally, as far as we know, it has not been proved up to now that such a formulation can be applicable in the regime of weak-to-moderate magnetic fields, where the individual vortices overlap and the magnetic field can be considered to be fairly uniform.

The aim of the present paper is to discuss a mechanism that generates the kink effect within the framework of the MC phenomenon. We emphasize that although, as shown in the derivations that follow, our numerical curves are in close qualitative agreement with the experimental curves of Ref. [3], we do not pretend to claim that our results explain the experimental findings of Krishana et. al. [9], as our calculations are based on the Landau level quantization whose applicability to d-wave superconductivity, as mentioned above, is not well established. Our main result in this paper is then of a more general and theoretical character. That is, we show that the MC phenomenon can be responsible for a kink effect in the thermal conductivity of (2+1)-dimensional relativistic fermion systems, and we point out the very important finding that the kink-like effect close to the critical field is essentially model independent, since it is determined by the critical behavior of the dynamically generated mass near the phase transition point. This fact makes our result relevant beyond the particular model under consideration, linking it to the universality class of theories with such a critical behavior.

In connection with this, we would like to call the attention of the reader to a new class of recently discovered materials [24], which could be a playground for applications of the phenomenon of magnetic catalysis. The so-called highly oriented pyrolitic graphites (HOPG) have layered structure with two isolated points in the Brillouin zone where the dispersion is linear, so that the electronic states can be described here too in terms of relativistic Dirac particles [23]. It was already suggested [24] that the magnetic catalysis in the system of Coulomb-interacting Dirac fermions can provide an explanation for the semimetal-insulator phase transition observed in HOPG in the presence of a magnetic field perpendicular to the layers. Since the quasiparticles in a graphite do not suffer from the Bloch waves versus Landau levels dilemma, our present results should have full strength there and we anticipate that the thermal conductivity of these systems should exhibit a behavior similar to the one we are reporting here.

In this paper we present for the first time a consistent calculation of the thermal conductivity in the presence of an external magnetic field in a model with the simplest four-fermion interaction for quasiparticles. We use the same constant magnetic field approximation that was already explored in Ref. [34] to calculate the thermal conductivity. However, our calculation deviates considerably from what was done in [34]. Not only we take into account the contribution of all Landau levels, but the definition of the heat current itself is different. In the limit of narrow width of quasiparticles ($\Gamma \ll T, \sqrt{B}$), after a gap is opened, the thermal conductivity exhibits a new term proportional to $T^2 / \alpha^2$ ($\alpha$ is the gap). This term originates from the compensation of the matrix elements...
of transitions between the zeroth and the first LLs (be- 
thaving as 1/B) and the LL density of states which in
turn is proportional to B. This is one more manifesta-
tion of the MC phenomenon: not only a gap is induced 
by the magnetic field, but the transitions between the 
zeroth and the first LLs are enhanced. In mean-field ap-
proximation and near the phase transition point the gap 
behaves like σ ≈ 0.523√eB − eBc, what yields a positive 
contribution into the slope of the thermal conductivity 
leading to a jump (kink effect) of κ(B) at B = Bc.

We start from the following NJL model in an external 
magnetic field in (2 + 1) dimensions,
\[ \mathcal{L} = \bar{\psi}_a [i\gamma^0 \hbar \frac{\partial}{\partial t} + v_D \gamma^1 (i\hbar \frac{\partial}{\partial x^1} - \frac{e}{c} A_1)] \psi_a + \frac{g}{2N} (\bar{\psi}_a \psi_a)^2, \]

(1)

In the vector potential is taken in the symmetric 
gauge \[ A_1 = (0, -\frac{B}{2} x_2, \frac{B}{2} x_1) \], and \[ v_D \] is a characteristic 
velocity \[ \frac{\hbar}{2N} \]. In what follows we take \[ h = v_D = k_B = 1 \] 
and absorb \[ c \] into the charge \[ e \]. We will restore these 
constants when needed. We assume also that the fermions 
carry an additional flavor index \[ a = 1, \ldots, N \] \((N = 2 \text{ for} \) realistic \textit{d}-wave superconductors). The Dirac \[ \gamma \] matrices 
are taken in a reducible four-component representation.

The Lagrangian density \( \mathcal{L} \) is invariant under the dis-
crete (chiral) symmetry \[ \psi \rightarrow \gamma_5 \psi, \bar{\psi} \rightarrow -\bar{\psi} \gamma_5 \], which 
forbids the fermion mass generation in perturbation theory. 
The mass generation can be studied introducing an aux-
iliary field \[ \sigma = -(g/N) \bar{\psi} \psi \] by means of the Hubbard-
Stratanovich trick which permits one to integrate over 
fermion fields in the path integral representation of the 
partition function. The field \[ \sigma \] has no dynamics at the 
tree level but it acquires the kinetic term due to fermion 
loops. Likewise, a nontrivial minimum of the effective po-
tential (the expectation value of \[ \sigma \]) gives mass to fermions 
and spontaneously breaks the discrete symmetry leading 
to a neutral condensate of fermion-antifermion pairs.

Studying the minimum of the effective potential we find 
that, at a fixed temperature \( T \), there is a critical value of 
the magnetic field \( \sqrt{eB}/T \approx 4.118 \) such that for subcrit-
cial fields \[ eB \leq eB_c \], the gap is zero, while for \[ eB > eB_c \] 
a nontrivial gap appears. The critical curve equation has the 
form \( (v_D/c)^2 10^{10} B = 21.5 \cdot T^2 \) for \( B \) measured 
Tesla. Using the approximated value of the characteris-
tic velocity \( v_D \approx 10^7 \text{cm/s} \) \[ \frac{2\pi}{2\pi} \], we obtain the critical 
curve \( B = 0.014 \cdot T^2 \). Notice that the obtained critical 
curve fits well the experimental curve of Ref. [3].

To derive an expression for the static thermal conduc-
tivity in an isotropic system, we follow the familiar linear 
response method and apply Kubo’s formula \[ \frac{28}{28} \]
\[ \kappa = -\frac{1}{TV} \text{Im} \int_0^\infty dt \int d^2 x_1 d^2 x_2 \langle u_i(x_1, 0) u_i(x_2, t) \rangle, \] (2)

where \( V \) is the volume of the system and \( u_i(x, t) \) is the 
heat-current density operator. The brackets denote aver-
gaging in the canonical ensemble. Physically, the thermal 
conductivity \( \kappa \) appears as a coefficient in the equation re-
lying the heat current to the temperature gradient \( \bar{u} = 
-\kappa \nabla T \) under the condition of absence of particle flow. If 
we neglect the chemical potential, the heat density coin-
dides with the energy density \( \epsilon \), hence the quantity \( \bar{u} \) that 
satisfies the continuity equation \[ \epsilon(x) + \nabla \cdot \bar{u}(x) = 0 \] can 
be interpreted as the heat current. From the Lagrangian 
density [4] we find \( u_i = \frac{1}{2} (\bar{\psi} \gamma_i \partial_t \psi - \partial_t \bar{\psi} \gamma_i \psi) \).

Neglecting vertex corrections [29] the calculation of 
the thermal conductivity reduces to the evaluation of 
the bubble diagram [30]. In this approximation, making use 
of the spectral representation for the fermion Green’s 
function, we arrive at the following expression for the 
thermal conductivity in the Matsubara formalism

\[ \kappa = \frac{1}{32 \pi T^2} \int_{-\infty}^\infty d\omega \omega^2 \int d^2 k \text{ tr} \left[ \gamma^i A(\omega, \vec{k}) \gamma^j A(\omega, \vec{k}) \right]. \]

The spectral function \( A(\omega, \vec{k}) = -\pi^{-1} \text{Im} S(\omega + i\epsilon, \vec{k}) \) is 
derived from the fermion propagator in a magnetic field 
decomposed over Landau level poles \[ [31, \text{31}] \].

Here \( P_\pm = (1 \pm i \gamma_1 \gamma_2)/2 \) are projectors operators, \( L_n, L^1_n \) 
are Laguerre’s polynomials \((L^1_n = 0)\), \( s = 2\vec{k}^2/eB \), and \[ \sigma \] is the dynamical fermion mass obtained from the finite 
temperature gap equation in the magnetic field. We 
introduced also the quasiparticles width replacing \( \epsilon \) in 
the definition of \( A(\omega, \vec{k}) \) by finite \( \Gamma \), which is due to inter-
action processes, in particular, scatterings on impurities. 
Straightforward calculation of the trace and further in-
tegration over momenta in Eq. \( [3] \) produces Kronecker’s 
δ symbols \( \delta_{n, n-1} + \delta_{n, n-1} \) due to orthogonality of La-
guerre’s polynomials. This implies that only those trans-
sitions between neighbor Landau levels contribute to the 
heat transfer. After performing one of the sums in [3] 
we obtain

\[ \kappa = \frac{e B \Gamma^2 N}{\pi^2 T^2} \sum_{n=0}^\infty \int_0^\infty \frac{d\omega \omega^2}{\cosh^2 \frac{\omega}{2T}} \frac{1}{(\omega^2 + M_n^2 + \Gamma^2)^2 - 4\omega^2 M_n^2} \]
\[ \times \left[ (\omega^2 + M_n^2 + \Gamma^2)(\omega^2 + M_{n+1}^2 + \Gamma^2)^2 - 4\omega^2 \sigma^2 \right]. \]

Further summation over \( n \) in Eq. [3] can be carried out expanding the integrand in terms of partial fractions. 
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tions and the final expression for \( \kappa \) is written as follows
\[ \kappa = \frac{N \Gamma^2}{2 \pi^2 T^2} \int_0^\infty \frac{d\omega^2}{\cosh^2 \frac{\omega}{2T}} \left( \frac{1}{(eB)^2 + (2\omega)^2} \right) \left( 2\omega^2 + \frac{(\omega^2 + \sigma^2 + \Gamma^2)(eB)^2 - 2\omega^2 (\omega^2 - \sigma^2 + \Gamma^2) eB}{(\omega^2 - \sigma^2 - \Gamma^2)^2 + 4\omega^4 \Gamma^2} \right) \frac{\omega(\omega^2 - \sigma^2 + \Gamma^2)}{\Gamma} \Im \psi \left( \frac{\sigma^2 + \Gamma^2 - \omega^2 - 2i\omega \Gamma}{2eB} \right) \right). \] (6)

This formula is the main result of our paper and we can use it now to study the different asymptotic regimes. It is easy to show that in the limit of zero field \( \kappa \) reduces to

\[ \kappa_0 = \frac{N}{4\pi^2 T^2} \int_0^\infty \frac{d\omega^2}{\cosh^2 \frac{\omega}{2T}} \left[ 1 + \frac{\omega^2 + \Gamma^2}{\omega \Gamma} \arctan \frac{\omega}{\Gamma} \right], \] (7)

where we put \( \sigma = 0 \), since the mass is not generated in the zero-field weak coupling NJL model. Eq. (6), up to an overall factor \( k_F^2(v_F^2 + v_A^2)/\hbar v_F \), coincides with the corresponding expression obtained in Ref. [1].

With the overall factor replacing \( N = 2 \) in real \( d \)-wave superconductor, Eq. (6) reproduces the universal (or residual) thermal conductivity at low \( T \) in the so-called “dirty” limit, \( T \ll \Gamma \) [2]. The residual conductivity was recently observed in experiments [3] giving explicit confirmation of the existence of gapless quasiparticles in \( d \)-wave cuprates at \( T < T_c \).

Let us consider the case \( T \ll \Gamma \) with \( B \neq 0 \). In this approximation we obtain

\[ \kappa \simeq \frac{N \Gamma}{4\pi^2 T^2} \left\{ \frac{\sigma^2}{\cosh^2 \frac{\sigma}{2T}} + 4 \sum_{n=1}^\infty \frac{n(\sigma^2 + 2eBn)}{\cosh^2 \sqrt{\sigma^2 + 2eBn}} \right\}. \] (8)

Asymptotically, at \( \sqrt{eB} \gg \sqrt{eB_c} \simeq T \), the dynamical mass behaves as \( \sigma \sim \sqrt{eB} \) what leads to an exponential decrease as in the case of gapless fermions. However, the term \( \cosh^{-2} \sigma/2T \) in Eq. (8) is of order one when \( \sigma \lesssim 2T \), thus, there is no suppression of this term for a certain range of fields where it is almost field independent (plateau region).

Near the phase transition point \( \sigma \simeq a \sqrt{eB - eB_c} \) and the first term in Eq. (8) gives positive contribution to the slope of the thermal conductivity at \( eB \geq eB_c \) leading to the jump in the slope of \( \kappa \) (kink effect) at \( eB = eB_c \). The parameter \( a \) is model-dependent, but the scaling of \( \sigma \) is determined by the universality class of the theory. For the NJL model [4] we find, in mean-field approximation, \( a \simeq 0.523 \).

The explicit appearance of the kink has been corroborated by numerical calculations as shown in Fig.1. Notice the break in the slope of \( \kappa \) (kink effect) at the critical value \( B_c \) in the presence of \( \sigma \). For \( B > B_c \) the kink is followed by a region where \( \kappa \) is only weakly dependent on the field. While decreasing the temperature, the position of the kink moves to the left in accordance with the critical line \( B_c = 0.014T^2 \). Notice the similarity of our results (Fig. 1) with the experimental behavior reported in [5] for high-Tc superconductors. Moreover, Fig. 1 also reproduces the experimentally observed crossing of the curves that occurs with decreasing \( T \) in such a way that the lower \( T \) curve reaches the higher value at large fields.

Because of the important role played by the first term in (6) to get the kink effect, it is instructive to clarify its origin. From Eq. (6) one can see that while the contribution of transitions between Landau levels with \( n \geq 1 \) in the integrand behaves as \( 1/(eB)^2 \) for large fields, the contribution of transitions between the zeroth and the first LLs decreases only as the first power of the field \( \sim 1/eB \). Since the density of LLs is proportional to \( eB \), this implies that the transitions between the zeroth and the first LLs are not suppressed, even though the gap between the levels grows with the field. Note that for gapless QPs the transitions with \( n = 0 \) are completely suppressed in the regime \( \Gamma \to 0 \); indeed, their contribution is proportional to \( \delta(\omega) \) and the integrand in (6) contains an \( \omega^2 \) factor. For gapped QPs, \( \delta(\omega) \) is replaced by \( \delta(\omega \pm \sigma) \) and the transitions with \( n = 0 \) survive.

In summary, in the present paper the thermal conductivity at \( B \neq 0 \) in the framework of a relativistic four-fermion model was calculated. Assuming a uniform magnetic field approximation, we showed that the thermal conductivity exhibits a kink behavior when the field reaches the critical value \( B_c = 0.014 \cdot T^2 \). Two main features determine this effect: the generation of a QA gap in a magnetic field (MC phenomenon) and the lack of suppression of zeroth-first LLs transitions. We stress that the appearance of a kink in the thermal conductivity is model independent, being only determined by the critical behavior of the induced dynamical mass near the phase transition. The universality character of this result leaves an open window for possible future applications.

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