Plasmons in Two-Dimensional Topological Insulators

Henning Schlömer,1 Zhihao Jiang,2 and Stephan Haas2

1Institute for Theoretical Solid State Physics, RWTH Aachen University, 52056 Aachen, Germany
2Department of Physics and Astronomy, University of Southern California, Los Angeles, CA 90089-0484

(Dated: December 1, 2020)

We analyze collective excitations in models of two-dimensional topological insulators using the random phase approximation. In the two-dimensional extension of the Su-Schrieffer-Heeger model, we reveal regimes of enhanced inter-band wave function correlations, leading to characteristic hardening and softening of inter- and intra-band plasmonic branches, respectively. In real space, plasmonic modes governed by electronic transitions involving the mid-gap topological edge states are found, resulting in induced charge-density distributions localized at the boundaries of the system. In the Haldane model, single-particle band structure details are instead identified as the dominant contribution influencing bulk plasmonic responses in different topological phases.

Introduction.— Non-interacting models of topological insulators have been studied to a great extent in recent years, and have become one of the most active and rapidly growing research areas in condensed matter physics [1–28]. However, effects of electron-electron correlations in topological insulators have not yet received equal attention. Nonetheless, measurements of collective electronic excitations arising from long-range Coulomb interaction are experimentally more accessible than probing directly single particle states, as it has been demonstrated e.g. for thin micro-ribbon arrays of single layer graphene [29] and three-dimensional topological insulator thin films (Bi2Se3) [30, 31] using infrared spectroscopy. Recently, plasmons in the one-dimensional (1D) Su-Schrieffer-Heeger (SSH) model were analyzed in real space, where it was found that single particle edge states appearing in the non-trivial phase lead to strongly localized plasmon charge distributions [32]. Furthermore, bilayer architectures composed of two unhybridized but Coulomb coupled massless Dirac electron systems, realized on surfaces of three-dimensional (3D) topological insulators, have been studied theoretically [33]. Here, the typically large bulk dielectric screening was observed to lock the low-frequency plasmon modes at energies above the particle-hole (p-h) continuum. In magnetically doped thin films of 3D topological insulators, band-inversions were shown to enhance inter-band correlations, which in turn lead to the appearance of inter-band plasmonic responses in certain topological phases [34].

In this letter, we report results for collective plasmonic excitations in two-dimensional (2D) topological insulators. Specifically, we investigate plasmonic responses in reciprocal and real space, capturing their bulk and surface properties, respectively. For the 2D extension of the SSH model, we identify two distinct regimes, namely dimerized (D) and anti-dimerized (AD), which depend on the choice of hopping parameters and on the real space lattice modulation and significantly influence the plasmonic response. We observe strong dispersion hardening (i.e., blue shift) of the high-frequency plasmonic mode in anti-dimerized regimes due to enhanced inter-band wave function correlations. This results in high-energy plasmons well above the p-h continua, which are hence intrinsically undamped. For low-energy plasmonic branches, on the other hand, a transition between the two regimes leads to a dispersion softening (i.e., red shift) controlled by a suppression of intra-band overlap functions. We envision an experimental setup on a momentum-space lattice [35, 36], allowing for manipulation of the hopping parameters and real space lattice structure, thus enabling access to the different plasmonic excitation spectra, which can be observed via electronic energy loss spectroscopy [37] or electromagnetic radiation combined with sub-wavelength grated surface probes [29, 30]. For the detection of high-energy, intrinsically undamped plasmons, we propose a method analogous to the concepts introduced in [38], where it was argued that typical speckle patterns produced by elastic scattering processes can be observed via spatial near-field imaging in regimes where Landau damping is quenched.

Methods.— We account for long-range Coulomb interactions via the random-phase approximation (RPA). The complex dielectric function, whose nodes yield a diverging dynamical response to an external electric perturbation, is given by

$$\varepsilon(\omega, \mathbf{q}) = 1 - V q \Pi(\omega, \mathbf{q}),$$

where \(V_q = 2\pi e^2/\kappa q\) is the Fourier transformed Coulomb potential in two spatial dimensions with background dielectric screening \(\kappa\), and \(\Pi(\omega, \mathbf{q})\) denotes the bare polarization function. For a system with \(n\) sites per unit cell and sublattice index \(\sigma = 1...n\), the bare polarization bubble can be evaluated [39],

$$\Pi(\omega, \mathbf{q}) = \frac{g_s}{V} \sum_{\mathbf{k}, \sigma, \sigma'} \frac{n_F(E_{\mathbf{k}, \sigma}) - n_F(E_{\mathbf{k} + \mathbf{q}, \sigma'})}{\omega + i\eta + E_{\mathbf{k}, \sigma} - E_{\mathbf{k} + \mathbf{q}, \sigma'}} F_{\mathbf{k}, \sigma, \sigma'}^{\prime}. \tag{2}$$

Here, \(g_s = 2\) is the spin degeneracy factor, \(V\) is the volume of the unit cell, \(\sum_{\mathbf{k}}\) runs over the first Brillouin zone (BZ), \(E_{\mathbf{k}, \sigma}\) are the single particle states, \(n_F(x) = (1 + \exp[-(x - \mu)/k_B T])^{-1}\) is the Fermi-Dirac distribution function at chemical potential \(\mu\), and \(F_{\mathbf{k}, \sigma, \sigma'}^{\prime}\) is the
overlap function of the corresponding pseudo-spinors,

$$F_{k,k+q} = |\langle \Psi_{k,\sigma} | \Psi_{k+q,\sigma'} \rangle|^2.$$  \hspace{1cm} (3)

The plasmon dispersion can then be extracted from the electronic energy loss spectrum (EELS) given by EELS(ω, q) = −Im1/ε(ω, q). In real space, assuming the system consists of M unit cells and thus N = nM total sites, we calculate the N × N response matrix ε(ω) = 1 − VIT^0, where V is the real space Coulomb interaction matrix,

$$\mathbf{V}_{ab} = \begin{cases} e^2/k|r_a - r_b| & a \neq b \\ U_0/k & a = b \end{cases}$$  \hspace{1cm} (4)

with $U_0 = 17.38$ eV [32] and

$$[\mathbf{\Pi}^0]_{ab} = g_s \sum_{i,j} \frac{n_F(E_i) - n_F(E_j)}{E_i - E_j - \omega - i\eta} \psi_{ia}^* \psi_{jb} \psi_{ja} \psi_{ia},$$  \hspace{1cm} (5)

the bare polarization function in real space. Here, $E_i$, $n_F(E_i)$ and $\psi_{ia}$ are the i-th electronic eigenenergy, the corresponding Fermi function at chemical potential $\mu$ and the wave function coefficient of tight binding orbital $a$, respectively. The real space non-interacting density response is efficiently calculated using fast Fourier transforms [40]. We then extract the electronic energy loss function by choosing the eigenvalue $\epsilon_a(\omega)$ and eigenvector $\mathbf{v}_{\text{max}}$ of $\epsilon(\omega)$ such that EELS(ω) = −Im1/ε_a(ω) is maximized [41, 42]. For a qualitative picture of the induced charge distribution $\mathbf{\rho}(\omega) \propto \mathbf{\Pi}^{0} \mathbf{v}_{\text{max}}$ of the plasmon modes, we approximate the tight binding orbital around site $a$ as a two-dimensional Gaussian distribution $\phi_a(\mathbf{r})$ with variance $\sigma$ and transform the charge density distribution into the $\mathbf{r}$-space representation via $\rho(\omega, \mathbf{r}) = \sum_a \mathbf{\rho}_a(\omega) \phi_a(\mathbf{r})$. Throughout our computations, we set $\hbar = 1, T = 0$ K, and use numerical broadenings $\eta = 0.01$ eV and $\eta = 0.08$ eV in momentum and real space, respectively.

**2D Su-Schrieffer-Heeger Model.**— We start by analyzing a two-dimensional extension of the SSH model [43, 44], i.e., a square lattice with 2 × 2 sites per unit cell and intra-cell (inter-cell) hopping $w$ ($v$), Fig. 1 (a). The 4 × 4 Hamiltonian in reciprocal space has the entries $H_{12} = H_{24} = w \exp\{ik_x\delta\} + v \exp\{-ik_x(L - \delta)\}$, $H_{13} = H_{34} = w \exp\{ik_y\delta\} + v \exp\{-ik_y(L - \delta)\}$, with their corresponding complex conjugate partners at transposed matrix elements, $L^2$ the surface of the unit cell, and $\delta$ ($L - \delta$) the intra-cell (inter-cell) nearest neighbor distance. Having time reversal (TR) and inversion symmetry, the Berry curvature vanishes throughout the entire BZ, except at $C_4$, invariant points $|k_x| = |k_y|$, where oscillating divergences appear due to the degeneracy of energy bands. These, however, integrate to zero and thus result in a vanishing Chern number. Nevertheless, a non-trivial topological classification arises through a finite 2D Zak-phase, resulting in a fractional wave polarization and topological edge states for $w < v$ [43], hence resembling its analogue in one spatial dimension [45, 46].

In our momentum space calculations, we fix the intra-cell nearest neighbor distance, $\delta = 0.1L$. For clarity, we refrain here from using the terms topologically trivial and non-trivial, and instead use the following terminology: if the tunneling amplitude $t_s$ associated with bond of length $\min\{\delta, L - \delta\}$ is larger [resp. smaller] than the hopping $t_l$ corresponding to two atomic sites separated by $\max\{\delta, L - \delta\}$, the phase is referred to as dimerized (D) [resp. anti-dimerized (AD)]. For our choice of $\delta$, phase D (AD) corresponds to the topologically trivial (non-trivial) phase of the 2D SSH model. A change between the D↔AD regimes can then either be induced by a topological phase transition $t_s \leftrightarrow t_l$ or by a change $\delta \leftrightarrow L - \delta$. Figs. 2 (a) and (b) show the electronic loss functions of the model in the dimerized and anti-dimerized phase in momentum space, respectively. The inset of Fig. 2 (a) illustrates the four single-particle energy bands through high-symmetry points of the BZ, referred to as $s, p_x, p_y$ and $d_{xy}$ from lowest to highest energy. The chemical potential is chosen to lie inside the $p_y$ band. Focusing first on the dimerized regime, Fig. 2 (a), where $w = t_s = 3$ eV, $v = t_l = 1$ eV, we see that the energetically lowest plasmon branch governed by $p_y$ intra-band transitions is characterized by a steep dispersion, entering the p-h continua for small momentum transfers close to the $\Gamma$ point when moving towards $M$. Inter-band plasmonic excitations, on the other hand, feature flat energy dispersions, hybridizing with the p-h continua and hence being Landau damped throughout the BZ.

When tuning the bulk into the AD phase, the resulting collective excitation spectra are in stark contrast to the D regime. The gapless plasmonic intra-band mode is

![FIG. 1. Topological insulator models analyzed in this letter. (a) Two-dimensional extension of the SSH model. Each unit cell (light blue solid line) includes four sites, with intra-cell nearest neighbor hopping $w$ and inter-cell tunneling $v$. Nearest neighbor distances within each unit cell plaquette are given by $\delta$, whereas inter-cell neighbors measure a distance of $L - \delta$. (b) Haldane model, a Chern insulator on the honeycomb lattice consisting of nearest neighbor (black solid line) and complex next-nearest neighbor hoppings (light and dark blue solid lines) sharing the same chirality (indicated by arrows). Furthermore, a staggered sublattice on-site potential $\Delta (-\Delta)$ on A (B) sites is present.](image-url)
FIG. 2. (a) & (b) Momentum space: Electron energy loss spectra (log[EELS(ω, q)]) of the 2D SSH model for (a) the dimerized (ω = t_2 = 3 eV, v = t_1 = 1 eV), and (b) the anti-dimerized regime (ω = t_2 = 1 eV, v = t_1 = 3 eV). Inset (a) shows the single-particle energy dispersion (solid lines) as well as the choice of the chemical potential μ = 1 eV (dashed line). Lower plots show zooms of the low energy plasmon modes, where q is chosen to reach from Γ to 1/5 of the distance to M. The unit cell size is set to L = 2 Å, and δ = 0.1L. (c) Real space: EELS(ω) and real space plasmonic charge distributions for a system of size 30 × 30 sites with uniform atomic distance δ = L − δ = 1 Å. The center plot shows EELS(ω) for the topologically trivial (red solid line) and non-trivial (blue solid line) phase. The second leading eigenvalue is also shown for the non-trivial phase (blue dashed line). The inset illustrates the mid-gap edge states (black dots) in the upper band gap appearing in addition to bulk states (grey dots) in the non-trivial phase, as well as the chemical potential μ = 3 eV (black dashed line). Charge distributions ρ(ω, r) of chosen peaks of the loss spectra are illustrated in the top and bottom row for the topologically non-trivial and trivial regime, respectively. A background dielectric screening κ = 2.5 is introduced in the real space calculations and σ = 1 Å.

softened, reaching up to only about half of the maximum energy compared to its analogue in the D phase, and then hybridizing with the p-h continuum. For high energy, gapped plasmons we observe the opposite, namely a strong hardening of the inter-band modes. This sharp increase of collective excitation energies leads to intrinsically undamped plasmonic modes already for small momentum transfers q, whereby Landau damping via p-h excitations is almost entirely quenched.

Sharing identical single-particle energy dispersions in both phases due to the sublattice symmetry, the only factor leading to different plasmonic excitation spectra lies in the overlap of the wave functions \( \rho_{k,k+q} \) which store the real space and topological properties of the system and hence influence the collective modes accordingly. Indeed, we find that inter-band (intra-band) correlations are greatly enhanced (suppressed) in the AD regime. All 16 overlap functions for paths along high-symmetry points of the BZ in both phases are shown in the Supplementary Material [47]. The effect of the real space lattice configuration on bulk plasmons in topological insulator models as well as their experimental realization using trapped atoms will be extensively discussed in an upcoming work, where it is explicitly shown how the lattice modulation and winding of the Hamiltonian enter the bulk plasmon dispersion for the 1D SSH atomic chain. Further note that the enhancement of plasmonic energies into intrinsically undamped regimes is reminiscent of what was found in flat-band Hamiltonians such as twisted-bilayer graphene due to large fine structure constants [38], which is, however, of a different physical nature. The overlap functions, entering via the Coulomb interaction matrix elements, also affect the p-h continua given by peaks of Im\( \Pi^\sigma(\omega,q) \). This results in single-particle transitions to be suppressed along certain momentum transfer paths in the BZ. Prominent suppression occurs especially along the \( \Gamma - X \) and \( \Gamma - M \) directions, the latter being less noticeable in the AD regime due to the previously discussed larger inter-band overlaps.

Fig. 2 (c) depicts the real space results in a finite system of 30 × 30 sites on a uniform square lattice, i.e., \( \delta = L/2 \), in which case we can again return to the terminology trivial ↔ non-trivial. Examining the electronic energy loss spectrum using the leading eigenvalue method, we observe that for plasmonic energies \( \omega \lesssim 2.5 \text{ eV} \), a collective excitation continuum arises in the topologically non-trivial phase, which is absent in the trivial system. The real space plasmonic induced charge modulations reveal that charge distributions corresponding to the emerging peaks are strongly localized at the boundaries of the slab, with increasing charge modulation frequency for rising plasmonic energies (upper row in Fig. 2 (c)). Bulk plasmons (\( \omega \gtrsim 4.5 \text{ eV} \)), on the other hand, have expected similar structures in both phases and are characterized by induced charge distributions delocalized throughout the bulk of the system, as explicitly shown for four ex-
citation peaks in the trivial phase in the lower row of Fig. 2 (c). The appearance of excitations in the non-trivial phase, along with the observed localization, reflect that the single-particle transitions governing these confined plasmons are the edge states emerging in the gap, akin to results presented in [32]. The localized plasmonic peak at the lower border of the bulk continuum (ω ∼ 4.2 eV), additionally appearing in the non-trivial phase, shows a charge modulation perpendicular instead of parallel to the surface, which leads to a larger generated Coulomb energy and hence separates it from the edge continuum.

Analyzing the eigenvalue of the dielectric matrix that produces the second largest EELS(ω), we find that most of the local maxima are degenerate with the leading eigenvalue. Indeed, the resulting induced charge-density distributions differ from the leading eigenvalues shown in Fig. 2 only by a charge sign flip or a C4v symmetry operation. Note that for the real space calculation, a background dielectric constant κ = 2.5 was chosen, which enables an energetic decoupling of the surface and bulk plasmons and thus makes it easier to discern them.

Chern Insulator on the Honeycomb Lattice. — We now analyze plasmonic excitation spectra in the Haldane model [48], the most prominent example of a quantum anomalous Hall (QAH) insulator [49] featuring the quantum Hall effect with vanishing net-magnetic flux. It is a tight binding model on a honeycomb lattice, allowing for real nearest neighbor hoppings, complex (TR symmetry breaking) next-nearest neighbor tunneling terms, as well as a staggered (sub-lattice symmetry breaking) on-site potential opening a gap. Fig. 1 (b) illustrates the Haldane model. The Hamiltonian in momentum space takes the form

$$\mathcal{H}(k) = \mathcal{H}_G + \left( \Delta + 2t' \sum_i \sin(k \cdot b_i) \right) \sigma_z,$$  \hspace{1cm} (6)

where $\mathcal{H}_G$ is the nearest-neighbor tight binding Hamiltonian for graphene [50], $\Delta$ is the gap parameter, $t'$ is the hopping strength for next-nearest neighbors and $b_i$, $i = 1, 2, 3$ denote the three different types of vectors connecting a site with its next-nearest neighbors. At $t' = \pm t_{\text{crit}} = \pm \Delta/3\sqrt{3}$, the gap closes at one of the Dirac points (i.e., they become massless and sources of Berry curvature) and the system undergoes a topological phase transition. Different from the 2D SSH model, the topological invariant is given by the Chern number, which can be calculated as $C = -1, 0, 1$ for $t' < -t_{\text{crit}}, -t_{\text{crit}} < t' < t_{\text{crit}}, t' > t_{\text{crit}}$, respectively. Using ultracold fermionic atoms, the Haldane model has been successfully realized and explored experimentally [51].

Numerical results of the electronic energy loss spectra are shown in Figs. 3 (a) and (b) for $C = 0$ and $C = 1$ respectively. Examining the low energy collective modes, we find that for $C = 1$, the plasmon dispersion is softened when comparing it to the trivial phase. For the energetically higher plasmonic branch, although the total energy of the non-trivial phase plasmon is slightly enhanced, the bandwidth remains indistinguishable from the $C = 0$ high-energy plasmon. Changing the chemical potential into the gap and/or increasing the next-nearest neighbor hopping strength to move the system further into the $C = 1$ regime does not fundamentally change these observations. Analyzing the intra- and inter-band overlap functions, Figs. 3 (c)-(f), we see that they barely vary when switching from $C = 0$ to $C = 1$. The term dominating a variation of the plasmon dispersion is hence identified as the change of single particle band structure details, illustrated in the insets of Figs. 3. Indeed, we find that the low-$q$ expansion of the gapless plasmon mode is given by [47]

$$\omega^2(q) = \frac{2e^2}{\kappa} \left[ \mu - \frac{(\Delta^2 + (3\sqrt{3}t')^2)}{\mu} \right] q,$$  \hspace{1cm} (7)

which is included in the lower parts of Figs. 3 (a) and (b). The squared energy dispersion in the gapped system is...
hence quadratically softened by the gap parameter and the next-nearest neighbor hopping, in contrast to a linear decrease when simply lowering the doping level in non-gapped graphene, whose low energy plasmon mode is given by $\omega^2(q) = 2e^2\mu q/\kappa$ [47, 52, 53].

Conclusions. — We have examined the plasmonic excitations arising from long-range Coulomb interactions in two-dimensional models of topological insulators. Our analysis of the 2D SSH model predicts strong dispersion hardening and softening in bulk plasmonic excitation spectra when tuning the system between the dimerized and the anti-dimerized regimes, controlled by the intra- and inter-band wave function correlations. The strong enhancement of inter-band plasmonic energies can be used to access regimes where Landau damping is entirely quenched, thus enabling applications based on dissipationless light-matter coupling. Bulk plasmons in the Haldane model, in contrast, were shown to almost entirely be controlled by single-particle band structure details when changing the system’s topological phase, which we investigated analytically for small momentum transfers. The observed characteristics of the plasmon dispersion in both the 2D SSH as well as the Haldane model can, however, not be considered a result originating from non-trivial topology and should rather be interpreted as a tunable feature in topological insulator models.

In real space, on the other hand, we could clearly establish that topology directly influences surface properties of the plasmonic response. In non-trivial phases, characteristic charge-density distributions localized on the boundaries were found, identified to be governed by single-particle mid-gap edge states appearing in finite systems with open boundaries. For future work, it will be interesting to investigate the dispersion of surface plasmon modes as well as their stability against disorder.

Acknowledgements. — We would like to thank Stefan Wessel, Hubert Saleur, Manfred Sigrist, Masao Ogata, and Amnon Fischer for useful discussions.

[1] Y. Ando, Journal of the Physical Society of Japan 82, 102001 (2013).
[2] D. Hsieh, D. Qian, L. Wray, Y. Xia, Y. S. Hor, R. J. Cava, and M. Z. Hasan, Nature 452, 970 (2008).
[3] C.-Z. Chang, J. Zhang, X. Feng, J. Shen, Z. Zhang, M. Guo, K. Li, Y. Ou, P. Wei, L.-L. Wang, Z.-Q. Ji, Y. Feng, S. Ji, X. Chen, J. Jia, X. Dai, Z. Fang, S.-C. Zhang, K. He, Y. Wang, L. Lu, X.-C. Ma, and Q.-K. Xue, Science 340, 167 (2013).
[4] M. Sato and S. Fujimoto, Journal of the Physical Society of Japan 85, 072001 (2016).
[5] M. Z. Hasan and C. L. Kane, Rev. Mod. Phys. 82, 3045 (2010).
[6] X.-L. Qi and S.-C. Zhang, Rev. Mod. Phys. 83, 1057 (2011).
[7] L. Fu and C. L. Kane, Phys. Rev. Lett. 100, 096407 (2008).
[8] R. B. Laughlin, Phys. Rev. B 23, 5632 (1981).
[9] T. Ando and Y. Uemura, Journal of the Physical Society of Japan 36, 959 (1974).
[10] D. J. Thouless, M. Kohmoto, M. P. Nightingale, and M. den Nijs, Phys. Rev. Lett. 49, 405 (1982).
[11] R. B. Laughlin, Phys. Rev. Lett. 50, 1395 (1983).
[12] J. E. Hirsch, Phys. Rev. Lett. 83, 1834 (1999).
[13] Y. L. Chen, J. G. Analytis, J.-H. Chu, Z. K. Liu, S.-K. Mo, X. L. Qi, H. J. Zhang, D. H. Lu, X. Dai, Z. Fang, S. C. Zhang, I. R. Fisher, Z. Hussain, and Z.-X. Shen, Science 325, 178 (2009).
[14] S. Murakami, N. Nagaosa, and S.-C. Zhang, Phys. Rev. Lett. 93, 156804 (2004).
[15] C. L. Kane and E. J. Mele, Phys. Rev. Lett. 95, 226801 (2005).
[16] C. L. Kane and E. J. Mele, Phys. Rev. Lett. 95, 146802 (2005).
[17] B. A. Bernevig and S.-C. Zhang, Phys. Rev. Lett. 96, 106802 (2006).
[18] B. A. Bernevig, T. L. Hughes, and S.-C. Zhang, Science 314, 1757 (2006).
[19] M. König, S. Wiedmann, C. Brüne, A. Roth, H. Buhmann, L. W. Molenkamp, X.-L. Qi, and S.-C. Zhang, Science 318, 766 (2007).
[20] J. E. Moore and L. Balents, Phys. Rev. B 75, 121306 (2007).
[21] L. Fu, C. L. Kane, and E. J. Mele, Phys. Rev. Lett. 98, 106803 (2007).
[22] L. Fu and C. L. Kane, Phys. Rev. B 76, 045302 (2007).
[23] S.-C. Zhang and J. Hu, Science 294, 823 (2001).
[24] X.-L. Qi, T. L. Hughes, and S.-C. Zhang, Phys. Rev. B 78, 195424 (2008).
[25] K. Nomura and N. Nagaosa, Phys. Rev. Lett. 106, 166802 (2011).
[26] D. Kim, S. Cho, N. P. Butch, P. Syers, K. Kirshenbaum, S. Adam, J. Paglione, and M. S. Fuhrer, Nature Physics 8, 459 (2012).
[27] Y. Tanaka, Z. Ren, T. Sato, K. Nakayama, S. Souma, T. Takahashi, K. Segawa, and Y. Ando, Nature Physics 8, 800 (2012).
[28] F. D. M. Haldane, Phys. Rev. Lett. 93, 206602 (2004).
[29] L. Ju, B. Geng, J. Horng, C. Girit, M. Martin, Z. Hao, H. A. Bechtel, X. Liang, A. Zettl, Y. R. Shen, and F. Wang, Nature Nanotechnology 6, 630 (2011).
[30] P. Di Pietro, M. Orlotani, O. Limaj, A. Di Gaspare, V. Giliberti, F. Giorgianni, M. Brahlek, N. Bansal, N. Koirala, S. Oh, P. Calvani, and S. Lupi, Nature Nanotechnology 8, 556 (2013).
[31] P. Di Pietro, N. Adhlakha, F. Piccirilli, A. Di Gaspare, J. Moon, S. Oh, S. Di Mitri, S. Spampinati, A. Perucchi, and S. Lupi, Phys. Rev. Lett. 124, 226403 (2020).
[32] Z. Jiang, M. Rösmers, R. E. Groeneveld, and S. Haas, Phys. Rev. B 101, 045106 (2020).
[33] R. E. V. Profumo, R. Asgari, M. Polini, and A. H. MacDonald, Phys. Rev. B 85, 085443 (2012).
[34] F. Zhang, J. Zhou, D. Xiao, and Y. Yao, Phys. Rev. Lett. 106, 226801 (2011).
[35] E. J. Meier, F. A. An, and B. Gadway, Phys. Rev. A 93, 051602 (2016).
[36] E. J. Meier, F. A. An, and B. Gadway, Nature Communications 7, 13986 (2016).
[37] T. Eberlein, U. Bangert, R. R. Nair, R. Jones, M. Gass, A. L. Bleloch, K. S. Novoselov, A. Geim, and P. R.
[38] C. Lewandowski and L. Levitov, Proceedings of the National Academy of Sciences 116, 20869 (2019).
[39] T. Ando, Journal of the Physical Society of Japan 75, 074716 (2006).
[40] S. Thongrattanasiri, A. Manjavacas, and F. J. Garcia de Abajo, ACS Nano 6, 1766 (2012).
[41] T. Westerhout, E. van Veen, M. I. Katsnelson, and S. Yuan, Phys. Rev. B 97, 205434 (2018).
[42] W. Wang, T. Christensen, A.-P. Jauho, K. S. Thygesen, M. Wubs, and N. A. Mortensen, Scientific Reports 5, 9535 (2015).
[43] F. Liu and K. Wakabayashi, Phys. Rev. Lett. 118, 076803 (2017).
[44] D. Obana, F. Liu, and K. Wakabayashi, Physical Review B 100 (2019).
[45] W. P. Su, J. R. Schrieffer, and A. J. Heeger, Phys. Rev. B 22, 2099 (1980).
[46] A. J. Heeger, S. Kivelson, J. R. Schrieffer, and W. P. Su, Rev. Mod. Phys. 60, 781 (1988).
[47] See Supplemental Material for a full illustration of the 2D-SSH overlap functions and analytical analysis of the Haldane model.
[48] F. D. M. Haldane, Phys. Rev. Lett. 61, 2015 (1988).
[49] C.-X. Liu, S.-C. Zhang, and X.-L. Qi, Annual Review of Condensed Matter Physics 7, 301 (2016).
[50] The nearest neighbor tight-binding Hamiltonian for graphene is given by

\[ H_G = \begin{pmatrix} 0 & h(k) \\ h^*(k) & 0 \end{pmatrix} = t \sum_i \left( \sigma_x \cos(k \cdot a_i) - \sigma_y \sin(k \cdot a_i) \right), \]

where \( h(k) = t \sum_i \exp(ik \cdot a_i) \) with \( a_i \), \( i = 1, 2, 3 \) the connecting vectors from a given site to its three nearest neighbors and \( \sigma_x, \sigma_y \) Pauli matrices.
[51] G. Jotzu, M. Messer, R. Desbuquois, M. Lebrat, T. Uehlinger, D. Greif, and T. Esslinger, Nature 515, 237 (2014).
[52] B. Wunsch, T. Stauber, F. Sols, and F. Guinea, New Journal of Physics 8, 318 (2006).
[53] E. H. Hwang and S. Das Sarma, Phys. Rev. B 75, 205418 (2007).
SUPPLEMENTARY MATERIAL

Overlap Functions in the 2D SSH Model

The overlap functions, often also called coherence or form factors,
\[ F_{\sigma\sigma'}^{k,k+q} = \left| \langle \Psi_{k,\sigma} | \Psi_{k+q,\sigma'} \rangle \right|^2, \] (S1)
are identified in the main text as the only actors influencing the plasmonic dispersion in the 2D SSH model when switching between the relevant regimes D\leftrightarrow AD. All 16 overlap functions for chosen values of momentum \( k \) and transferred momentum \( q \) along high-symmetry points of the BZ (for \( \delta = 0.1 L \) as in the main text) are shown in Fig. S1 (a) and (b) for the dimerized and anti-dimerized regime, respectively. Here, intra-band (inter-band) overlap functions are shown as diagonal (off-diagonal) elements. As a sanity check, note that for \( q = 0 \), i.e., at the \( \Gamma \) point on the \( q \)-axis, the intra-band (inter-band) overlap is one (zero) for all \( k \), as the hermiticity of the Hamiltonian demands. Generally, one can observe that the overlap of inter-band (intra-band) wave functions is greatly enhanced (suppressed) in the AD phase, Fig. S1 (b), compared to its analogue in the D regime, Fig. S1 (a). For the inter-band contributions, we hence observe that single particle transitions of high energy difference are enhanced (or, strictly speaking, less suppressed) in the AD phase and thus lead to significantly larger contributions to the polarization function when summing over \( k \) for a given \( q \). On the other hand, low energy intra-band transitions are suppressed in AD regimes, resulting in smaller contributions to \( \Pi^0(\omega,q) \). This leads to the observed hardening (softening) of the high energy (low energy) plasmon mode, which is governed by inter-band (intra-band) transitions. In the dimerized phase, we further see that only inter-band transitions between \( p \) bands play a significant role, such that one would expect the upper energy plasmon mode to be further softened when tuning the chemical potential to lie inside the gap between the \( p_y \) and \( d_{xy} \) band, hence prohibiting single-particle transitions between the two \( p \) bands. The high-energy plasmon modes in AD phases, on the other hand, are not expected to alter by a considerable amount due to strong contributions coming from all other inter-band transitions. We confirmed this numerically.

Analytical Expansion of the Polarization Function of the Haldane Model

Here, we present an analytical approximation of the low-energy \( \sqrt{\alpha} \) plasmonic mode in the Haldane model. Let us start with the standard tight binding model for graphene [50], for which
\[ F_{\sigma\sigma'}^{k,k+q} = \frac{1}{2} \left( 1 + \sigma\sigma' \cos(\phi_k - \phi_{k+q}) \right), \] (S2)
where \( \phi_k = \arg\{h(k)\} \). For finite electronic doping \( \mu > 0 \), the low energy plasmon mode is formed by intra-band transitions in the conduction band, for which the overlap function can be approximated by \( F_{\sigma\sigma'=+}^{k,k+q} = 1 + O(q^2) \). Denoting the conduction band energies by \( E_k \) and expanding the Lindhard term in Eq. (2) in...
\[ \omega \gg |E_k - E_{k+q}|, \text{ one finds} \]

\[
\Pi^0_{\text{intra}} \approx \frac{g_s}{\omega^2} \sum_k n_F(E_k)\{E_{k+q} - E_{k-q} - 2E_k\} \\
\approx \frac{g_s}{\omega^2} \sum_k n_F(E_k)(q \cdot \nabla_k)^2 E_k. \tag{S3}
\]

Approximating the conduction band as \( E_k = v_F k \) throughout the whole BZ and accounting for the additional valley degeneracy factor \( g_v = 2 \), one finds

\[
(q \cdot \nabla_k)^2 E_k = \frac{v_F q^2}{2E_k} \sin^2(\alpha) q^2 / k, \text{ with } \alpha \text{ being the angle between } q \text{ and } k. \] 

Hence,

\[
\Pi^0_{\text{intra}} \approx \frac{4v_F q^2}{(2\pi)^2 \omega^2} \int_0^{2\pi} \sin^2(\alpha) d\alpha \int_0^{k_F} dk = \frac{\mu q^2}{\pi \omega^2}, \tag{S4}
\]

which within RPA (Eq. 1 in the main text) results in the well known low-\( q \) plasmon square-root dispersion for graphene, \([52, 53]\)

\[
\omega^2(q) = \frac{2e^2 \mu}{\kappa} q. \tag{S5}
\]

When introducing a staggered sublattice potential and complex next-nearest neighbor hoppings to the Hamiltonian, the valley degeneracy is broken and the (now gapped) system can be approximated by two Dirac cones with a corresponding mass term

\[
E_{k,\pm} = \pm \sqrt{\Delta_P^2 + (v_F k)^2}, \tag{S6}
\]

where \( \Delta_P = \Delta \pm 3\sqrt{3}t' \) at \( P = K' (+) \) and \( P = K (-) \). Again focusing on the intra-band transitions \( \rightarrow F_\sigma \rightarrow \sigma' = \pm 1 + \mathcal{O}(q^2) \) and keeping the chemical potential inside the conduction band, we find that

\[
(q \cdot \nabla_k)^2 E_{k,\pm}^P = \frac{(v_F q)^2}{E_{k,\pm}^P} - \frac{(v_F q)^2(v_F k)^2}{(E_{k,\pm}^P)^3} \cos^2(\alpha). \tag{S7}
\]

The polarization function hence reads

\[
\Pi^0_{\text{intra}} \approx \frac{2q^2}{(2\pi)^2 \omega^2} \sum_{P=K,K'} \int_0^{2\pi} \frac{d\alpha}{k_F} \int_0^{k_F} dk \left\{ \frac{k}{\sqrt{\Delta_P^2 + k^2}} = \frac{k^3}{\sqrt{\Delta_P^2 + k^2}} \cos^2(\alpha) \right\} \tag{S8}
\]

\[
= \frac{\mu - [\Delta^2 + (3\sqrt{3}t')^2]/\mu}{\pi \omega^2} q^2,
\]

where \( k_F^P \) is defined such that \( \mu = \sqrt{\Delta_P^2 + (v_F k_F^P)^2} \). Within RPA, this results in the plasmonic energy dispersion

\[
\omega^2(q) = 2e^2 \left[ \frac{\mu - (\Delta^2 + (3\sqrt{3}t')^2)/\mu}{\kappa} \right] q. \tag{S9}
\]