How to Complete the Quantum-Mechanical Description?

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If the statement by Einstein, Podolsky and Rosen on incompleteness of Quantum-Mechanical description of nature is correct, then we can regard Quantum Mechanics as a Method of Indirect Computation. The problem is, whether the theory is incomplete or the nature itself does not allow complete description? And if the first option is correct, how is it possible to complete the Quantum-Mechanical description? Here we try to complement de-Broglie’s idea on wave-pilot the stochastic gravitation gives origin to. We assume that de-Broglie’s wave-pilots are gravitational stochastic ones, and we shall regard micro-objects as test classical particles being subject to the influence of de-Broglie’s waves stochastic gravitation.

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1. Introduction.

The Quantum Theory exists for many decades. But is everything OK with it completeness[1]? To our opinion, it is not just so. The incompleteness of Quantum-Mechanical description gives rise to various paradoxes, such as Einstein-Podolsky-Rosen (EPR) one, the paradox of the Schroedinger’s cat, the Paradox of Quantum Nonlocality and Paradox of the Quantum Teleportation. In this study we shall call the phenomena of quantum nonlocal behavior and teleportation of the quantum states as paradoxes because they follow from Stochastic Gravitation Model of Quantum Mechanic. It can be
easily seen that these are paradoxes, and indeed they are brought about by the drawbacks in the Quantum Theory rather than being actual properties of nature. This is due to the fact that time in Quantum Theory plays the role not conforming to the physical reality. In particular, the Quantum Theory employs the concept of Hilbert Space, in which time acts as a parameter. Henceforth, this parameter (i.e. the time) may be the same in different points of the Hilbert Space. This property of time in the Hilbert Space brings about the effects of simultaneous quantum states of microobjects at different space points (or transfer of the state from one Hilbert Space point to another with velocities exceeding the velocity of light). These effects of the Quantum Theory that are apparently real we call here the Quantum Nonlocality Paradox. The Paradox of Quantum Teleportation is a sort of Quantum Nonlocality Paradox. These paradoxes do not exist in the Classical Physics and in the Stochastic Gravitation Model of the Quantum Mechanic, and General Relativity Theory (employing the 4-dimensional space), in which different points of time-space correspond to different values of the time.

And another question is whether the quantum-mechanical wave-function interpretation of micro-objects is complete?

Let us consider the electron diffraction experiment on a set of the slits. We shall consider this electron interference for the case when electrons pass through the slits one-by-one with a small time gap between them. To describe the observable pattern, we must solve the following dilemma: Either each electron passes simultaneously through several slits, which seems impossible from the classical physics viewpoint, or each of the wave-electrons is coherent to others, which seems more correct and natural. These wave-electrons must be coherent if the difference in their amplitudes and phases is rather small and almost constant in time. Otherwise, the interference pattern would be smeared due to varying amplitude and phase difference.

Then, the question arises, why the wave functions within the Quantum-Mechanical Description of different electrons are coherent. It is perhaps more strange than the Quantum-Mechanical electron wave/particle dualism. Postulating Quantum-Mechanical wave properties to be possessed by each electron would not suffice, and to explain the interference pattern we must complement the description with coherence of electron waves. This is the additional requirement to account for electron interference. We can call it the Phenomena of Quantum Coherence.

There exists a simple way to tell which of the slits has the electron passed through. It is to leave open only a single slit. We can open any slit, either
the first or the second one, but we must leave open only one slit. Maybe, it is possible to choose the time of opening and closing the slits so that we see the interference pattern.

Now, let us consider the history of the Quantum Mechanical Description incompleteness. In the EPR effect [1] two particles, $P$ and $Q$, interact at the initial moment and then scatter in opposite directions. Let the first of them be described by the wave function $\psi_P$, the other by $\psi_Q$. The system of the two particles $P$ and $Q$ is described by the wave function $\psi_{PQ}$. For each of particles there are two distinct descriptions depending on whether we take into account presence of the second particle or not.

Where could the dependence of the object $P$ on the object $Q$ and vice versa originate from, these objects $P$ and $Q$ being considered as distant and non-interacting? The authors EPR came to the conclusion on incompleteness of the quantum-mechanical description. To solve this contradiction, an idea has been put forward in [1] on existence of hidden variables that would make it possible to consistently interpret the results of the experiments without altering the mathematical apparatus of quantum mechanics.

Later, it has been proved by von Neumann [2] that quantum-mechanical axiomatic does not allow introduction of hidden variables. It is, however, important that the argument presented in [2] would not hold valid in certain cases, e.g., for pairwise observable microobjects (for Hilbert space with pairwise commutable operators) [4]. In 1964, J. S. Bell [5] has formulated the experimental criterion enabling to decide, within the framework of the problem statement [1], on the existence of the local hidden variables. The essence of the experiment proposed by Bell is as follows. Let us consider the following experimental scheme. Let there be two photons that can have orthogonal polarizations $A$ and $B$ or $A'$ and $B'$, respectively. Let us denote the probability of observation of the pair of photons with polarizations $A$ and $B$ as $\psi_{AB}^2$. Bell has introduced the quantity $|\langle S \rangle| = \frac{1}{2} \left| \psi_{AB}^2 + \psi_{A'B'}^2 - \psi_{A'B'}^2 - \psi_{AB}^2 \right|$, called the Bell’s observable; it has been shown that if the local hidden variables do exist, then $|\langle S \rangle| \leq 1$. The possibility of experimental verification of actual existence of local hidden variables has been demonstrated in [5]. The above inequalities are called Bell’s inequalities. The series of experiments has shown that there does not exist any experimental evidence of existence of local hidden variables as yet, and the existing theories comprising hidden variables are indistinguishable experimentally. Because very interesting the contextualist viewpoint to the probabilistic foundation of the quantum mechanics [6].
Further, considering the wave-pilot concept of de Broglie, we have to complement it with the statement of these wave-pilots having to possess the stochastic character in the space. These waves must generate mechanical fluctuations of classical test particles; then, these particles can be considered “smeared” in space and should be described by the quantum-mechanical wave functions.

Einstein [2] has noted that it is impossible to extend geometrical interpretation to the submolecular (sizes smaller than a molecule) scale, as this would be as erroneous as to speak of a particle temperature for an individual molecular-scale particle.

2. Microobjects in the Curved Pro-Hilbert Space.

Recent years a very fascinating idea to put QM into geometric language attracts the attention of many physicists. The starting point for such an approach is the projective interpretation of the Hilbert space \( \mathcal{H} \) as the space of rays. To illustrate the main idea it is convenient to decompose the Hermitian inner product \( \langle \cdot | \cdot \rangle \) in \( \mathcal{H} \) into real and imaginary parts by putting for the two \( L_2 \)-vectors \( |\psi_1\rangle = u_1 + iv_1 \) and \( |\psi_2\rangle = u_2 + iv_2 \):

\[
\langle \psi_1 | \psi_2 \rangle = G(\psi_1, \psi_2) - i\Omega(\psi_1, \psi_2),
\]

where \( G \) is a Riemannian inner product on Projective Hilbert space or in further description Pro-Hilbert space \( \mathcal{H} \) and \( \Omega \) is a symplectic form, that is

\[
G(\psi_1, \psi_2) = (u_1, u_2) + (v_1, v_2); \quad \Omega(\psi_1, \psi_2) = (v_1, u_2) - (u_1, v_2),
\]

with \( \langle \cdot, \cdot \rangle \) denoting standard \( L_2 \) inner product. The symplectic form \( \Omega \) can acquire its dynamical content if one uses the special stochastic representation of QM.

Let us consider two classic particles in random gravitational fields or waves (in the relict gravitational background for example) with number \( j = 1, 2, 3, \ldots N \). In General Relativity Theory the interval in this fields is

\[
ds^2 = \sum_{j=1}^{N} g_{\mu\nu}(j)dx^\mu dx^\nu = g^0_{\mu\nu}dx^\mu dx^\nu,
\]

where the stochastic metric in the linear approach is

\[
g_{\mu\nu}(j) = \eta_{\mu\nu}(j) + h_{\mu\nu}(j),
\]

\[4\]
being $\eta_{\mu\nu}$ the Minkowsky metric, constituting the unity diagonal matrix and $h_{\mu\nu}$ is perturbation of the metric. Here $g_{\mu\nu}^0 = \sum_{j=1}^{N} g_{j\mu\nu}$ we call a metric of the Stochastic Curved Space. Hereinafter, the indices $\mu, \nu, \gamma, m, n$ acquire values $0, 1, 2, 3$. Indices encountered twice imply summation thereupon.

Hilbert Space is non-curved space. But in Projective Hilbert space which we are call the Pro-Hilbert Space the wave function $\psi^\mu$ play the role of coordinates.

Accepting Riemann’s definition of the interval,
\[
ds^2 = g_{\mu\nu}^0 dx^\mu dx^\nu = g_{\mu\nu}^0 \frac{dx^\mu}{d\psi^i} \frac{dx^\nu}{d\psi^k} d\psi^i d\psi^k = G_{ik} d\psi^i d\psi^k = G_{ik} d\psi_i d\psi_k
\]

where $x^\mu$ being coordinates in the Riemann’s space, $\mu, \nu = 0, 1, 2, 3$ and $i, k = 1, 2, 3, ..., N$, where in common case $N \to \infty$,

we are denote
\[
g_{\mu\nu}^0 \frac{dx^\mu}{d\psi^i} \frac{dx^\nu}{d\psi^k} = G_{ik},
\]

where $\psi^\mu$ is wave function of microobject and $G_{\mu\nu}$ is metric in Pro-Hilbert Space.

We will have the definition of the probabilities in Pro-Hilbert Space, if we shall normalize the equation by condition
\[
\int G_{\mu\nu} d\psi^\mu d\psi^\nu = P \leq 1,
\]

where $P$ means the probability’s function with maximum $P = 1$. The probability $P$ in the Pro-Hilbert Space is the scalar product of two vectors $\psi_i$ and $\psi_k$ with metric $G^{ik}$

\[
P = G^{ik} \psi_i \psi_k.
\]

This space in common case is the curved. Really, if $l = 1, 2, 3, ..., N$, where $l$ is the number of a point, the volume of such figure is determined by the formula
\[
V_N = \frac{1}{N!} \begin{vmatrix}
\psi_1^1 & \psi_1^2 & \cdots & \psi_1^N \\
\psi_2^1 & \psi_2^2 & \cdots & \psi_2^N \\
\vdots & \vdots & \ddots & \vdots \\
\psi_N^1 & \psi_N^2 & \cdots & \psi_N^N
\end{vmatrix}
\]
In common case this space is curved, because $V_n \neq 0$ means that this space is non-Euclidian.

The Hilbert Space with Euclidian metric $H_{ik}$ is particular case of the Pro-Gilbert Space with non-Euclidian metric $G_{ik}$. If we denote $\Pi_{ik}$ as weak perturbation of the Euclidian metric than

$$G_{ik} = H_{ik} + \Pi_{ik}, \quad \Pi_{ik} << H_{ik}.$$  

Let us consider now the experiment with interference of two electrons on two slits. The electron interference experiment is the one in which it is impossible to determine the electron trajectory. Any attempt to determine the electron trajectory fails due to any infinitesimal affecting of an electron with the purpose of determination of its trajectory would alter the interference pattern. This is the first aspect. On the other hand, interaction of classical and stochastic fields and waves in such experiments is usually neglected. Such interactions must exist in compliance with the existing provisions of classical physics, and in particular, of the General Relativity Theory. Moreover, this is experimentally confirmed by the pre-quantum classical physics, hence, they require verification of their effect onto quantum micro-objects.

Let us review some provisions of the General Relativity Theory. We consider the motion of electrons from the source $S$ to the screen through slits 1 and 2.

$$\langle x \mid s \rangle_1 = G_{ik}(1) \langle x \mid s \rangle_{ik} = H_{ik} \langle x \mid s \rangle_{ik} + \Pi_{ik}(1) \langle x \mid s \rangle_{ik},$$

$$\langle x \mid s \rangle_2 = G_{ik}(2) \langle x \mid s \rangle_{ik} = H_{ik} \langle x \mid s \rangle_{ik} + \Pi_{ik}(2) \langle x \mid s \rangle_{ik},$$

where $G_{ik}(1) \neq G_{ik}(2)$. Due to the propagation difference between the two trajectories in space and time, the interference pattern is generated. In the stochastic curved space one needs not to know the electron trajectory. The interference pattern emerges due to the difference in metrics $G_{ik}(1)$ and $G_{ik}(2)$. Thus, we have separated the wave function of the space from the particle, because it is the property of the space but not particle in our model. In Stochastic Gravitation Model the microobjects is the test classical particle in the stochastic gravitation fields and waves.

Let us select harmonic coordinates (the condition of harmonicity of coordinates mean selection of concomitant frame $\frac{\partial h_{\mu\nu}}{\partial x_{\mu}} = \frac{1}{2} \frac{\partial h_{\mu\nu}}{\partial x_{\nu}}$) and let us take into consideration that $h_{\mu\nu}$ satisfies the gravitational field equations

$$\Box h_{\mu\nu}(j) = -16\pi GS_{\mu\nu}(j),$$
which follow from the General Theory of Relativity; here $S_{mn}$ is energy-
momentum tensor of gravitational field sources with d’Alemberian $\Box$ and
gravity constant $G$. Then, the solution shall acquire the form

$$h_{\mu\nu}(j) = e_{\mu\nu}(j) \exp(ik_\gamma(j)x^\gamma) + e^*_{\mu\nu}(j) \exp(ik_\gamma(j)x^\gamma),$$

where the value $h_{\mu\nu}(j)$ is called metric perturbation, $e_{\mu\nu}(j)$ polarization,
and $k_\gamma(j)$ is 4-dimensional wave vector.

We shall assume that this metric perturbation $h_{\mu\nu}(j)$ is distributed in
space with an unknown distribution function $\rho = \rho(h_{\mu\nu})$. Relative oscillations $\ell$ of two particles in classic gravitational fields are described in the General
Theory of Relativity by deviation equations, which we can write for the
stochastic case as

$$\frac{D^2}{D\tau^2}\ell^\mu(j) + R^\mu_{\nu mn}(j)\ell^m \frac{dx^\nu}{d\tau} \frac{dx^n}{d\tau} = F(j),$$

being $R^\mu_{\nu mn}(j)$ the gravitational field Riemann’s tensor with gravitational
field number $j$ of the stohastic gravitational fields and $F(j)$ is the stochastic
constant (for the non-stochastic case this constant is zero $F(j) = 0$).

Specifically, the deviation equations give the equations for two particles
oscillations

$$\ell^{-1} + c^2 R^1_{010}\ell^1 = 0, \quad \omega = c\sqrt{R^1_{010}}.$$

The solution of this equation has the form

$$\ell^1(j) = \ell_0 \exp(k_a x^a + i\omega(j)t),$$

being $a = 1, 2, 3$. Each gravitational field or wave with index $j$ and
Riemann’s tensor $R^a_{\nu mn}(j)$ shall be corresponding to the value $\ell^\mu(j)$ with
stochastically modulated phase $\Phi(j) = \omega(j)t$. If we to sum the all fields, we
can write $\Phi(t) = \omega(t)t$, where $t$ is the time coordinate.

The stochastic phase $\Phi = \Phi(t)$ accounts for the Phenomena of Quantum
Coherence and we can use the stochastic phase $\Phi = \Phi(t)$ to understand the
nature of quantum interference.

3. Bell’s Inequalities as Experimental Criteria.
We shall consider the physical model with the Stochastic Gravitational Background [i.e. with the background of gravitational fields and waves]. This means that we assume existence of fluctuations in gravitational waves and fields expressed mathematically by metric fluctuations.

Considering quantum micro-objects in the stochastic curved space, we shall take into consideration the fact that the scalar product of two 4-vectors \( A^\mu \) and \( B^\nu \) equals \( g^0_{\mu\nu} A^\mu B^\nu \), where for weak gravitational fields one can use the value \( h^\mu_{\mu} \), which is the solution of Einstein’s equations for the case of weak gravitational field in harmonic coordinates.

Correlation factor \( M \) of random variables \( \lambda^i \) are projections onto directions \( A^\nu \) and \( B^m \) defined by polarizers (all these vectors being unit) is

\[
|M_{AB}| = |\langle AB \rangle| = |\langle \lambda^i A^k g_{ik} \lambda^m B^m g_{mn} \rangle|
\]

The differential geometry gives

\[
\cos \Phi = \frac{g_{\lambda^i A^k}}{\sqrt{A^k A^k}},
\]

\[
\cos(\Phi + \theta) = \frac{g_{\lambda^m B^m}}{\sqrt{B^m B^m}}.
\]

Here \( i, k, m, n \) possess 0,1,2,3; \( \theta \) is angle between polarizers, then

\[
|M_{AB}| = \left| \frac{1}{2} \int_0^{2\pi} \rho(\Phi) \cos \Phi \cos(\Phi + \theta) d\Phi \right| = \frac{1}{2} \rho \cos \theta,
\]

if \( \rho(\Phi) = \rho = \text{const} \) is the distribution function of \( \Phi \). Finally, the real part of the correlation factor for \( \rho = 2 \) is

\[
|M_{AB}| = |\cos \theta|.
\]

Then, we obtain the maximum value the Bell’s observable \( S \) in Riemann’s space for \( \theta = \frac{\pi}{4} \)

\[
|\langle S \rangle| = \left| \frac{1}{2} [\langle M_{AB} \rangle + \langle M_{A'B'} \rangle + \langle M_{AB} \rangle - \langle M_{A'B'} \rangle] \right| = \left| \frac{1}{2} [\cos(-\frac{\pi}{4}) + \cos(\frac{\pi}{4}) + \cos(\frac{\pi}{4}) - \cos(\frac{3\pi}{4})] \right| = \left| \sqrt{2} \right|,
\]

which agrees fairly with the experimental data. The Bell in equality in Riemann’s space shall take on the form \( |\langle S \rangle| \leq \sqrt{2} \).

Therefore, we have shown that the Classical Physics with the Stochastic Gravitational Background gives the value of the Bell’s observable matching both the experimental data and the quantum mechanical value of the Bell’s
observable. To sum it up, the description of microobjects by the classical physics accounting for the effects brought about by the Gravitational Background is equivalent to the Quantum-Mechanical descriptions, both agreeing with the experimental data.

4. Conclusion.

We are describe here the Stochastic Gravitation Model of Quantum Theory with stochastic gravitation de-Broigle waves which have the classical gravitational fields and waves (i.e. Gravity Background with random nature) origin.

Complementing the wave functions with the requirement of the Stochastic Geometrical Fluctuation (in other words, with the stochastic nature of gravitational fields and waves) enables us to get a new interpretation of Quantum Mechanics. To put it otherwise, complementing the Quantum-Mechanical description with stochastic gravitational fields and waves yields another approach to Quantum-Mechanical microobject description

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