Annihilation of vortex dipoles in an oblate Bose–Einstein condensate

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Abstract
We theoretically explore the annihilation of vortex dipoles, generated when an obstacle moves through an oblate Bose–Einstein condensate, and examine the energetics of the annihilation event. We show that the grey soliton, which results from the vortex dipole annihilation, is lower in energy than the vortex dipole. We also investigate the annihilation events numerically and observe that annihilation occurs only when the vortex dipole overtakes the obstacle and comes closer than the coherence length. Furthermore, we find that noise reduces the probability of annihilation events. This may explain the lack of annihilation events in experimental realizations.

(Some figures may appear in colour only in the online journal)

1. Introduction

One of the important developments in recent experiments on atomic Bose–Einstein condensates (BECs) is the creation of vortices and the study of their dynamics [1, 2]. Equally important is the recent experimental observation of a vortex dipole, which consists of a vortex–antivortex pair, when an obstacle moves through a BEC [3], and the observation of vortex dipoles produced through phase imprinting [4, 5]. In superfluids, the vortices carry quantized angular momenta and are the topological defects, which often serve as the conclusive evidence of superfluidity. In a vortex dipole, vortices of opposite circulation cancel each other's angular momentum and thus carry only linear momentum. This is the cause of several exotic phenomena such as leapfrogging, snake instability [6], orbital motion [7], trapping [8] and others. The effects of vortices are widespread in classical fluid flow [9] and optical manipulation [10]. A good description of vortices in superfluids is given in [11] and the review articles [12, 13]. A detailed discussion of vortices is given in [14].

Among the important phenomena associated with the BEC, the creation, dynamics and annihilation of vortex dipoles carry useful information associated with the system. Several methods have been suggested to nucleate vortices and recently, nucleation of the vortices has been observed experimentally by passing a Gaussian obstacle through the BEC with a speed greater than some critical speed [3]. The trajectories of these vortex dipoles are ring-structured as described in [15, 16]. The annihilation of vortices or the vortex dipole in the BEC has been mentioned in a number of theoretical studies [17–19]. However, there is the lack of extensive study on this topic and more importantly, no definite signatures of vortex dipole annihilation were observed in the experiment [3]. The study of vortex dipole annihilation will shed light on the process which influences the separation between the vortex and antivortex, as well as the conditions for annihilation along with other phenomena arising from the dynamics of vortex dipoles.

In this work, we present analytical as well as numerical results related to vortex dipole annihilation for an oblate BEC at zero temperature. The results are obtained using the Gross–Pitaevskii (GP) equation. In section 2 of this paper, we provide a brief description of the two-dimensional (2D) GP equation and vortex dipole solutions. Condensates with a diametric vortex dipole and a grey soliton are studied, and this is described in section 3. Section 3 contains studies done in the strong as well as weak interacting systems. The annihilation of vortex dipoles is analysed from the energies obtained from the analytical calculations. The numerical results, confirming the analytic results, are discussed in section 4, and we then conclude.
2. Superfluid vortex dipole and its generation

In the mean-field approximation, the dynamics of a dilute BEC is very well described by the GP equation:

\[ i\hbar \partial_t \Psi(r, t) = [\hat{H} + U] \Psi(r, t) \],

where \( \hat{H} \) and \( U \) are the single-particle Hamiltonian, interaction strength and order parameter of the condensate, respectively. The order parameter, \( \Psi \), is normalized to \( N \), the total number of atoms in the condensate. In the present case, the single-particle Hamiltonian \( \hat{H} \) consists of the kinetic energy operator, an axisymmetric harmonic trapping potential and a Gaussian obstacle potential, that is,

\[ \hat{H} = -\frac{\hbar^2}{2m} \nabla^2 + \frac{m\omega^2}{2}(x^2 + \alpha^2 + z^2) + V_{\text{obs}}(x, y, t), \]

where \( \alpha \) and \( \beta \) are the anisotropies along the \( y \)- and \( z \)-axes, respectively, \( m \) is the mass of particles used in the condensate, \( \omega \) is the trapping potential frequency along the \( x \)-axis and \( V_{\text{obs}}(x, y, t) \) is the repulsive Gaussian obstacle potential. Experimentally, a blue-detuned laser beam is used to generate the \( V_{\text{obs}}(x, y, t) \) and it can be written as

\[ V_{\text{obs}}(x, y, t) = V_0(t) \exp \left[ -\frac{2}{w_o^2} (x - vt)^2 + y^2 \right], \]

where \( V_0(t) \) is the potential at the centre of the Gaussian obstacle at time \( t \), \( v \) is the velocity of the obstacle along the \( x \)-axis, and \( w_0 \) is the radius of the repulsive obstacle potential. In this work, we consider the motion of the obstacle along the \( x \)-axis only. Defining the oscillator length of the trapping potential as \( a_{\text{osc}} = \sqrt{\hbar/(m\omega)} \) and considering \( \hbar \) as the unit of energy, we can then rewrite the equations in a dimensionless form with the transformations \( \tilde{r} = r/a_{\text{osc}}, \tilde{t} = t\omega \), and the transformed order parameter assumes the form

\[ \phi(\tilde{r}, \tilde{t}) = \sqrt{\frac{a_{\text{osc}}}{N}} \Psi(r, t). \]

For the sake of notational simplicity, hereafter, we denote the scaled quantities without the tilde in the rest of the paper. In a pancake-shaped trap, \( \alpha = 1 \) and \( \beta \gg 1 \), and the order parameter can then be written as

\[ \phi(\tilde{r}, \tilde{t}) = \psi(x, y, t) \xi(z) \exp(-i\tilde{\beta}t/2), \]

where \( \xi(z) = (\beta/(2\pi))^{1/4} \exp(-\beta z^2/4) \). Equation (1) is then reduced to the 2D form

\[ \left[ -\frac{1}{2} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) + \frac{x^2 + y^2}{2} + \frac{V_{\text{obs}}(x, y, t)}{\hbar\omega} \right] \psi(x, y, t) = 0, \]

where \( u = 2a\sqrt{2\pi\beta}/a_{\text{osc}}, \) with \( a \) being the s-wave scattering length, is the modified interaction strength. In this work, we consider a condensate consisting of \(^{87}\text{Rb}\) atoms in the \( F = 1, m_F = -1 \) state, with \( a = 99a_0 \) [20]. We have neglected a constant term corresponding to the energy along the axial direction as it only shifts the energies and chemical potentials by a constant without affecting the dynamics. We solve this equation numerically using the Crank–Nicolson method [21].

There are several theoretical and experimental proposals to generate vortices in non-rotating traps. These include stirring of the condensate using a blue-detuned laser or several laser beams [3, 15], an adiabatic passage [22], Raman transitions in binary condensate systems [23], laser beam vortex guiding [24] and phase imprinting [5]. Among these methods, the easiest way to nucleate vortex dipoles is by stirring a BEC with a blue-detuned laser beam. When the velocity of the laser beam exceeds a critical velocity, vortex–antivortex pairs are released from the localized dip in the number density created due to the laser beam. These vortex dipoles then move through the BEC and exhibit various interesting dynamics [4, 15, 25]. The critical velocity depends on the number density, width and intensity of the laser beam and the frequency of the trapping potential. This nucleation process exhibits a high degree of coherence and stability, allowing us to map out the annihilation of the dipoles. In an axis-symmetric trap, a vortex dipole is a metastable state of superfluid flow with a long lifetime.

3. Condensates with a vortex dipole or grey soliton

To analyse the vortex dipole annihilation, we consider a model system where the vortex–antivortex dipole pair and grey soliton, which may be generated when the annihilation of vortex dipole occurs, are static. However, we vary the distance of separation and examine the energy of the total system. The present system can be studied under two regimes: a strongly interacting system and a weakly interacting system. The strongly interacting system is studied considering \( \phi \) with the Thomas–Fermi (TF) approximation, and the weakly interacting system is studied considering the Gaussian form of \( \phi \).

3.1. Strongly interacting system with TF approximation

For the \( Na/a_{\text{osc}} \gg 1 \) case, we use the TF approximation to determine the steady-state density profile and energy of the condensate. To begin with, we consider a condensate with a vortex dipole and later with a grey soliton.

3.1.1. Diametric vortex dipole. We consider a condensate consisting of \( N \) atoms in a purely harmonic potential

\[ V(x, y) = \frac{x^2 + y^2}{2}. \]

Consider the condensate has a vortex dipole, consisting of a vortex and an antivortex located at \((0, v_2)\) and \((0, -v_2)\), respectively. The cores of the vortex and antivortex can be approximated as circular regions centred around \((0, v_2)\) and \((0, -v_2)\) with radii equal to the coherence length \(\xi\). At the cores, we consider the density to be equal to zero. Hence, we use the TF approximation and adopt the following piecewise ansatz for the density of the condensate:

\[ n(x, y) = \begin{cases} 0 & \text{for } x^2 + y^2 > R^2 \\ 0 & \text{for } [x^2 + (y \pm v_2)^2] \leq \xi^2 \\ \left[\frac{\mu - V(x, y)}{\hbar \omega} \right] & \text{for } [x^2 + (y \pm v_2)^2] > \xi^2 \\ \end{cases}. \]
where $R = \sqrt{2\mu}$ is the spatial extent of the condensate in the TF approximation, and $\xi = 1/R$ is the coherence length at the centre of the trap. Normalizing this ansatz yields

$$\frac{\pi (2 - 4R^2 + R^8 + 4R^2v_2^2)}{4R^4u} = 1.$$  \hspace{1cm} (9)

This equation defines the radius of the condensate. The TF ansatz can be used to calculate the total potential energy arising from the regions outside the cores of the vortices and is given as

$$E_0 = \frac{\pi}{12R^8u} \left[1 - 3R^8 + R^{12} + 3R^2v_2^2(2 + R^4v_2^2)\right].$$  \hspace{1cm} (10)

The main energy contribution from the vortex dipole is the kinetic energy due to the velocity field associated with it. This energy can be approximated as [26]

$$E_{KE} = \frac{R^2}{u} \log \left(\frac{2v_2}{\xi}\right).$$  \hspace{1cm} (11)

This relation is valid when $\xi \ll v_2 \ll R$ and in this work, $\xi \sim 0.06$ and $R = 15.5 a_{osc}$. In order to estimate the energy contributions from the cores of the vortices, we approximate the density within the cores as

$$n(x, y) = \begin{cases} 
2n_0[x^2 + (y - v_2)^2] / x^2 + (y - v_2)^2 + \xi^2 & \text{for } [x^2 + (y - v_2)^2] < \xi^2 \\
2n_0[x^2 + (y + v_2)^2] / x^2 + (y + v_2)^2 + \xi^2 & \text{for } [x^2 + (y + v_2)^2] < \xi^2,
\end{cases}$$  \hspace{1cm} (12)

where $n_0$ is the average TF density on the circle $x^2 + (y \pm v_2)^2 = \xi^2$. Assuming that the normalization is still defined by equation (9), equation (12) can be used to calculate energy contribution from the core region. The energy within the core consists of

$$E_0 = \frac{6\pi n_0}{8},$$  \hspace{1cm} (13)

$$E'' = \pi \xi^4 (\log[4] - 1)n_0,$$  \hspace{1cm} (14)

$$E'''' = 2\pi u^2(3 - \log[16])n_0^2,$$  \hspace{1cm} (15)

where $E_0$, $E''$, and $E''''$ are the energies arising from the quantum pressure, trapping potential and interaction within the core region, respectively. Thus, the total energy of the condensate with a vortex dipole is

$$E_{vd} = E_0 + E_{KE} + E_0'' + E''' + E''''.$$  \hspace{1cm} (16)

The variation of $E_{vd}$ as a function of $v_2$ is shown in figure 1.

### 3.1.2. Grey soliton

For the grey soliton extending from $(0, -v_2)$ to $(0, v_2)$ along the $y$-axis, we use the following piecewise ansatz in the TF approximation:

$$n(x, y) = \begin{cases} 
0 & \text{for } x^2 + y^2 > R^2, \\
\frac{\mu - V(x, y)}{u} & \text{for } \begin{cases} 
|y| > v_2, \\
|x| > \xi, \\
|x| \leq \xi \text{ and } |y| \leq v_2.
\end{cases}
\end{cases}$$  \hspace{1cm} (17)

And the normalization condition leads to the following constraint on the radius of the condensate:

$$\frac{1}{12R^8u} \left[3\pi R^7 + 4v_2(10 + 6R^4 - 3(1 + R^4)(-2 + \pi R^2 v_2^2)\right] = 1.$$  \hspace{1cm} (18)

For the grey soliton, other than the quantum pressure, there is no need to separate out the energy associated with the trapping and interaction potential within the soliton. So, the total energy of the system is

$$E_s = E_0 + E_0^3,$$  \hspace{1cm} (19)

where $E_0$ is the potential energy associated with the system and $E_0^3$ is the energy arising from the quantum pressure. These are given as

$$E_0 = \frac{\int \int [V(x, y)n(x, y) + \frac{u}{2} n(x, y)^2] dx dy,}{\int \int [V(x, y)n(x, y)] dx dy}.$$  \hspace{1cm} (20)

From the expression $n(x, y)$ in equation (17), we obtained

$$E_0 = \frac{1}{180R^8u} \left[15\pi R^1 + 3|236 - 75\pi + 20(19 - 6\pi)R^4 + 15(8 - 3\pi)R^6 v_2 + 10R^2[-28 + 9\pi + 6(-3 + \pi)R^4 v_2^2 - 9(-4 + \pi)R^6 v_2^2]\right]$$  \hspace{1cm} (21)

$$E_0^3 = \frac{1}{48u^2}.$$  \hspace{1cm} (22)

Interestingly, $E_0^3$ has a $1/\xi$ dependence, which is to be expected, as smaller $\xi$ implies a larger density variation and translates to higher quantum pressure.

For illustration, the vortex dipole and grey soliton inside the condensate are shown in figure 2. The vortex dipole is...
located at (1, 0) and (−1, 0), while the grey soliton extends from (−1, 0) to (1, 0) along the x-axis. In the case of the vortex dipole, the phase varies from 0 to 2π, if one goes around the point of singularity, whereas in the case of the grey soliton, there is a phase discontinuity of π along the line forming the soliton. The number density at the point of singularity is zero. In figure 1, the soliton. The number density at the point of singularity is shown in figure 3. So, there is a discontinuity across the system. However, when v2 > 0, the vortex dipole state is the energetically favourable state. This analytical result provides a compelling reason to study the annihilation of vortex dipoles and formation of grey solitons.

3.2. Weakly interacting system with Gaussian approximation

In the Na/\textit{a}_osc \ll 1 regime, a simplistic model of a vortex dipole in the BEC of trapped dilute atomic gases can be considered as the superposition of harmonic oscillator eigenstates. The minimalistic wavefunction which supports a vortex and an antivortex at the coordinates \((-a/c, -\sqrt{b/d})\) and \((-a/c, \sqrt{b/d})\) is

\[ \psi(x, y, t) = e^{-i\mu t} (a - b + ic x + dy^2) e^{-(x^2+y^2)/f}, \]  

(23)

where \(a, b, c, d\) and \(f\) are positive variational parameters and \(\mu\) is the chemical potential of the system. The wavefunction is a superposition of the scaled ground state and the first and second excited states of a harmonic oscillator along the \(x\) and \(y\)-axes, respectively. The wavefunction is ideal for weakly interacting condensates.

We have considered that the vortex and antivortex are located on the diameter of the condensate. Without loss of generality, we consider the diameter as coinciding with the \(y\)-axis, which is equivalent to \(a = 0\) in equation (23). Such an assumption does not modify qualitative descriptions, but expressions are far less complicated. The wavefunction is then

\[ \psi(x, y, t) = e^{-i\mu t} [-b + ic x + dy^2] e^{-(x^2+y^2)/f}. \]  

(24)

The nontrivial phase of the wavefunction \(\theta\) is discontinuous along the \(x = 0\) line for \(-\sqrt{b/d} \leq y \leq \sqrt{b/d}\). Across the discontinuity, there is a phase change from \(-\pi\) to \(\pi\) as we traverse along the \(x\)-axis from \(0^-\) to \(0^+\), and this phase variation is shown in figure 3. So, there is a discontinuity across the \(y\)-axis and this is the typical phase pattern associated with vortex dipoles. For the present case, the ground state wavefunction is

\[ \psi_g(x, y, t) = e^{-i\mu t} e^{-(x^2+y^2)/f}, \]  

(25)

and from the normalization condition \(\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left| \psi_g \right|^2 \, dx \, dy = 1\), we obtain the constraint equation

\[ b^2 = \frac{2}{f\pi}. \]  

(26)

For general considerations, rewrite the additional term as

\[ \delta \psi(x, y, t) = e^{-i\mu t} (ic x + dy^2) e^{-(x^2+y^2)/f}, \]  

(27)

so that the total wavefunction \(\psi = \psi_g + \delta \psi\), where \(\delta \psi\) represents an elementary excitation of the condensate. We can calculate the total energy of the system, without the obstacle potential, as

\[ E_{vd} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ \frac{1}{2} \left| \nabla _\perp \psi(x, y) \right|^2 + \frac{x^2 + y^2}{2} |\psi(x, y)|^2 + u |\psi(x, y)|^4 \right] \, dx \, dy. \]  

(28)

This is the energy of the condensate with a vortex dipole with the assumption that it is a weakly interacting system. Energy without the vortex may be calculated trivially [11]. In general, the energy added to the system due to the vortex dipole is not large compared to the total, and for obvious reasons, the angular momentum of the condensate is still zero.

A slight modification to the wavefunction can describe a solitonic solution along the \(y\)-axis. The form of the modified wavefunction is

\[ \psi(x, y) = [b + ic x + dy^2] e^{-(x^2+y^2)/f}. \]  

(29)

where except for the change in the sign of \(b\), all the terms remain unaltered as in equation (23). It is a grey soliton as the density \(n \propto (b + dy^2)^2 + (cx)^2\) has a dip but is different from zero. The phase varies smoothly from \(-\pi/2\) to \(\pi/2\) along the normal to the line which connects \((0, -\sqrt{b/d})\) and \((0, \sqrt{b/d})\). This phase variation is shown in figure 3(b).

Using the wavefunction in equation (29), we can then evaluate the total energy of the system \(E_{gs}\) and calculate the energy difference between two possible states of the system:

\[ \Delta E = E_{vd} - E_{gs}, \]  

(30)
which after evaluation is
\[ \Delta E = \frac{\hbar d f^2 \pi}{256} \left[ 64d^2u + 15d^2 f^2u + 8f(8 + c^2u) \right]. \] (31)

The most general solution is that when all the constants are positive, then \(\Delta E > 0\) and the grey soliton is lower in energy. This shows that when the vortex and antivortex collide, it is energetically favourable for them to decay into the grey soliton. 

As discussed in the results section, this is confirmed in the numerical calculations.

The analysis so far is for an ideal system at zero temperature, where we have neglected the quantum and thermal fluctuations and perturbations from imperfections. In addition, there is dissipation from three-body collision losses in the condensates of dilute atomic gases.

4. Numerical results

For the numerical computation, we choose \(^{87}\text{Rb}\) with \(N = 2 \times 10^6\) atoms. The trapping potential and obstacle laser potential parameters are similar to those considered in [3], i.e. \(\omega/(2\pi) = 8 \text{ Hz}, \beta = 11.25, V_0(0) = 93.0 \text{ h} \omega\) and \(u_0 = 10 \mu\text{m}\). To nucleate the vortices on the edges of the condensate, the obstacle potential \(V_{\text{obs}}\) is initially located at \((-12.5a_{\text{osc}}, 0)\) and moves along the \(x\) direction at a constant velocity with decreasing intensity, until \(V_{\text{obs}}\) vanishes at \((5.18a_{\text{osc}}, 0)\).

4.1. Vortex dipole nucleation

We study the nucleation of vortices by \(V_{\text{obs}}\) with the translation speed \(v\) ranging from 80 to 200 \(\mu\text{m s}^{-1}\). Vortices are not nucleated when the speed is 80 \(\mu\text{m s}^{-1}\). However, a vortex–antivortex pair or a vortex dipole is nucleated when the speed is in the range of 90 \(\mu\text{m s}^{-1} < v < 140 \mu\text{m s}^{-1}\). Increasing the speed of the obstacle generates two pairs of vortex dipoles when 140 \(\mu\text{m s}^{-1} \leq v < 160 \mu\text{m s}^{-1}\), and more than two pairs when \(v \geq 160 \mu\text{m s}^{-1}\). In other words, the number of vortex dipoles created can be controlled with the speed of the obstacle. The creation of vortex dipoles above a critical speed \(v_c\) is natural as the vortex nucleation must satisfy the Landau criterion [27]. The density and phase of the condensate after the nucleation of the vortex dipole for \(v = 120 \mu\text{m s}^{-1}\) are shown in figure 4. The figure clearly shows the nucleation dynamics of the vortex dipoles.

From numerical calculations, we have determined \(v_c \approx 90 \mu\text{m s}^{-1}\). This is, however, less than the local acoustic velocity of the medium \(s = \sqrt{\hbar U/m}\), which depends on the local condensate density. This also explains the reason for the predominant vortex dipole nucleation around the edge of the condensate where \(n\) is lower and \(s\) is accordingly lower.

4.2. Vortex dipole annihilation

To determine the energetically preferred state of the system, we examine the energy of the condensate with the vortex dipole and grey soliton as a function of the separation \(v_2\). The result is shown as the inset plot in figure 1. As in the TF calculations, the vortex dipole is the stable solution for larger \(v_2\), but for \(v_2 < 0.5a_{\text{osc}}\), a grey soliton is the stable solution. However, in the numerical results, the critical value of \(v_2\) at which the vortex soliton overcomes the grey soliton as the stable solution is higher than the TF values. This may be an account of the piecewise nature of the TF ansatz.

It is observed that the vortex dipole annihilation is critically dependent on the initial conditions of the nucleation, in particular, the vortex–antivortex separation, \(v_2\). The annihilation occurs when the vortex dipole is generated with \(v_2 < 0.5a_{\text{osc}}\), which is consistent with the analytical results and solutions of the time-independent GP equation. The initial \(v_2\) is, however, dependent on the velocity \(v\) of the obstacle potential. For this reason, the annihilation events are observed only for a specific range of \(v\). As an example, the annihilation event when \(v = 120 \mu\text{m s}^{-1}\) is shown in figure 4. In figure 4, we can notice the density minima arising from the annihilation and propagating away from the obstacle potential.

A reliable and qualitative way to describe the occurrence of annihilation could be achieved by observing the density at the cores of the vortex and antivortex which form the dipole. For the vortex, the matter density at the core when \(v = 120 \mu\text{m s}^{-1}\) is shown in figure 5. In the plot, at time \(\approx 3.19\) (scaled units), the core density starts increasing. This is because the core starts to fill in with the atoms from around the vortex after the annihilation. This filling process may not complete till it reaches the edge of the condensate and gets reflected inside the condensate.

After the annihilation of the vortex–antivortex dipole pair, a grey soliton gets generated. We can clearly observe the propagation of this soliton in figure 6. The speed of propagation is 2000 \(\mu\text{m s}^{-1}\), which is similar to the speed of sound in a condensate. During the propagation, the number density on the location of the soliton increases, which is clearly visible from figure 6 as well as from figure 5. To estimate the energy...
of the grey soliton, we have obtained the stationary state with the same position of vortex dipoles and the obstacle potential. The energy difference between the stationary and the dynamic state will provide us with the energy of the grey soliton, as discussed in [28]. The energy released due to the annihilation is  0.004 $\hbar\omega$, and is similar to the energy difference observed in figure 1, obtained from the TF approximation. We have also observed that this soliton gets reflected back and forth from the edge of the condensate. This reflection is similar to the reflection of any pulse from the circular edges.

It is to be mentioned that for the parameters considered in this work, the speed of sound is $2190 \mu$m s$^{-1}$ and the coherence length of the system is $\sim 0.229 \mu$m. These are in agreement with the minimum separation between the vortex and antivortex observed in the analytical work. The energy gap for the vortex dipole and grey soliton of the same size matches with the estimates from the ansatz based on the TF approximation. The vortex dipole annihilation is not only observed for $V_{\text{obs}} = 120 \mu$m s$^{-1}$, it also occurs for other obstacle velocities as well. Once such case, for $V_{\text{obs}} = 160 \mu$m s$^{-1}$, is shown in figure 7. In this case, the difference in energy of the vortex dipole and grey soliton is $0.0025 \hbar\omega$.

One observation which is common to all the vortex dipoles getting annihilated is the nature of their trajectories. All of them traverse through $V_{\text{obs}}$, whereas the ones which do not get annihilated avoid $V_{\text{obs}}$. The vortex dipoles are generally nucleated at the aft region of the $V_{\text{obs}}$ where there is a trailing superflow. When nucleated very close to each other ($v_2 < 0.5$) and with high velocity, the mutual force further increases the velocity of the vortex dipoles. At the same time, it decreases the distance separating the vortex and antivortex. So, the kinetic energy is high enough to surpass $V_{\text{obs}}$. Later, at some point, the vortex and antivortex separation is less than $\xi$, and they annihilate.

4.3. Effect of noise and dissipation

In the numerical studies, the annihilation events are not rare. But this is in contradiction with the experimental results of Neely and collaborators [3]—they observed no signatures of annihilation events. One possible reason is that our numerical calculations are too ideal, and an immediate remedy is to include quantum and thermal fluctuations. The rigorous way to study these fluctuations is to use methods like the truncated Wigner approximation [29]; however, in this work, we use the simple but widely accepted method of adding white noise [30, 31], as white noise constitutes random fluctuations and is
the speed of the obstacle is 180 \mu m s^{-1}.

hence able to change the number density of the condensate. It is added numerically using a random number generator. We have used the Mersenne Twister pseudo-random number generator. The strength of random noise used in our numerical calculation is 0.01% of the maximum density of the condensate. This noise is added/subtracted at every time-step of the real-time evolution of the condensate. One immediate outcome is that the symmetry in the position of the vortex and antivortex is lost. The superflow around the vortex is no longer a mirror reflection of the antivortex, which was nearly the case without white noise. The deviations are shown for an example case in figure 8, where \( V_{\text{obs}} = 180 \mu m s^{-1} \). This change in path leads to the suppression of annihilation events of the vortex dipoles. We have also studied the effect of large white noise (10%) added at the beginning, and avoided adding any noise in the subsequent time-steps. In such cases, the noise gets damped throughout the condensate and there are no observable influences on the annihilation event.

The other important effect is the loss of atoms from the trap. We have examined the effect of loss terms, which arise from inelastic collisions in the condensate. There are two types of inelastic collisions that lead to the loss of atoms from the trap: the two-body inelastic collision loss and the three-body loss. To model the effect of loss of atoms from the trap, we add the loss terms

\[
-\frac{i\hbar}{2}[K_2|\Psi(r, t)|^2 + K_3|\Psi(r, t)|^3],
\]

(32)
to the Hamiltonian \( \mathcal{H} \). Based on the previous work [32], for \(^{87}\text{Rb}\), the inelastic two-body loss rate coefficient \( K_2 = 4.5 \times 10^{-17} \text{cm}^3 \text{s}^{-1} \) and the inelastic three-body loss rate coefficient \( K_3 = 3.8 \times 10^{-29} \text{cm}^6 \text{s}^{-1} \). With trap loss, the annihilation events continue to occur. However, during the destructive time of flight observations in the experiments, the decreased atom numbers may lower the contrast and reduce the possibility of observing an annihilation event.

5. Conclusions

When an obstacle moves through a condensate above a critical speed, it nucleates the vortex dipoles, and the number of dipoles seeded depends on the obstacle velocity. Depending on the initial condition of nucleation, vortex and antivortex annihilation events occur under ideal conditions: at zero temperature, at no loss and without noise. These events are found to be energetically favourable theoretically and observed numerically. In the case of weakly interacting condensates, the energy of the grey soliton is always less than that of the vortex dipole, and provides higher possibility for annihilation events. Similarly, in the case of strongly interacting condensates, we use TF approximation to study the system and find that if the separation between the vortex–antivortex pair is less than the coherence length, the energy of the vortex dipole is more than that of the grey soliton, and this leads to annihilation. The generated grey soliton propagates through the condensate and shows the phenomena of reflection from the circular edge of the condensate. The speed of propagation is found to be similar to the speed of sound in a BEC. However, noise, thermal fluctuations and dissipation destroy the superflow reflection symmetry around the vortex and antivortex. Breaking the symmetry reduces the possibility of annihilation events and may explain the lack of annihilation events in experimental observations.

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References

[1] Anglin J R and Ketterle W 2002 Nature 416 211
[2] Matthews M R, Anderson B P, Haldajn C P, Hall D S, Wieman C E and Cornell E A 1999 Phys. Rev. Lett. 83 2948
[3] Neely T W, Samson E C, Bradley A S, Davis M J and Anderson B P 2010 Phys. Rev. Lett. 104 160401
[4] Freilich D V, Bianchi D M, Kaufman A M, Langin T K and Hall D S 2010 Science 329 1182
[5] Andrejczyk G, Brenczyk M, Dobrev L, Gajda M and Lewenstein M 2001 Phys. Rev. A 64 043601
[6] Brand J and Reinhardt W P 2002 Phys. Rev. A 65 043612
[7] Middelkamp S, Torres P J, Kevrekidis P G, Frantzeskakis D J, Carretero-González R, Schmelcher P, Freilich D V and Hall D S 2011 Phys. Rev. A 84 011605
[8] Aioi T, Kadokura T, Kishimoto T and Saito H 2011 Phys. Rev. X 1 012003
[9] Couder Y and Basdevant C 1986 J. Fluid. Mech. 173 225
[10] Grier D G 2003 Nature 424 810
[11] Pethick C J and Smith H 2008 Bose–Einstein Condensation in Dilute Gases (Cambridge: Cambridge University Press)
[12] Fetter A L 2009 Rev. Mod. Phys. 81 647
[13] Sonin E B 1987 Rev. Mod. Phys. 59 87
[14] Aleskeenko S V, Kuibin P A and Okulov V L 2007 Theory of Concentrated Vortices (New York: Springer)
[15] Jackson B, McCann J F and Adams C S 1999 Phys. Rev. A 61 013604
[16] Li W, Haque M and Komineas S 2008 Phys. Rev. A 77 055610
[17] Nazarenko S and Onorato M 2007 J. Low Temp. Phys. 146 146
[18] Rooney S J, Blakie P B, Anderson B P and Bradley A S 2011 Phys. Rev. A 84 023637
[19] Nowak B, Sexty D and Gasenzer T 2011 Phys. Rev. B 84 020506
[20] van Kempen E G M, Kokkelmans S J M F, Heinzen D J and Verhaar B J 2002 Phys. Rev. Lett. 88 093201
[21] Muruganandam P and Adhikari S K 2009 Comput. Phys. Commun. 180 1888
[22] Dum R, Cirac J I, Lewenstein M and Zoller P 1998 Phys. Rev. Lett. 80 2972
[23] Marzlin K P, Zhang W and Wright E M 1997 Phys. Rev. Lett. 79 4728
[24] Staliunas K 2000 Appl. Phys. B 71 555
[25] Kuopanportti P, Huhtamäki J A M and Möttönen M 2011 Phys. Rev. A 83 011603
[26] Zhou Q and Zhai H 2004 Phys. Rev. A 70 043619
[27] Lifshitz E M and Pitaevskii L P 1980 Statistical Physics, Part 2 (Oxford: Pergamon)
[28] Parker N G and Adams C S 2005 Phys. Rev. Lett. 95 145301
[29] Martin A D and Ruostekoski J 2010 Phys. Rev. Lett. 107 194102
[30] Sasaki K, Suzuki N, Akamatsu D and Saito H 2009 Phys. Rev. A 80 063611
[31] Kadokura T, Aioi T, Sasaki K, Kishimoto T and Saito H 2012 Phys. Rev. A 85 013602
[32] Burke J P, Bohn J L, Esry E D and Greene C H 1998 Phys. Rev. Lett. 80 2097