Fast tunable coupling scheme of Kerr-nonlinear parametric oscillators based on shortcuts to adiabaticity

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Kerr-nonlinear parametric oscillators (KPOs), which can be implemented with superconducting parametrons possessing large Kerr nonlinearity, have been attracting much attention in terms of their applications to quantum annealing, universal quantum computation and studies of quantum many-body systems. It is of practical importance for these studies to realize fast and accurate tunable coupling between KPOs in a simple manner. We develop a simple scheme of fast tunable coupling of KPOs with high tunability in speed and amplitude using the fast transitionless rotation of a KPO in the phase space based on the shortcuts to adiabaticity. Our scheme enables rapid switching of the effective coupling between KPOs, and can be implemented with always-on linear coupling between KPOs, simply by controlling the phase and frequency of the pump field without controlling the amplitude of the pump field nor using additional drive fields. We apply the coupling scheme to a two-qubit gate, and show that our scheme realizes high gate fidelity compared to a purely adiabatic one, by mitigating undesired nonadiabatic transitions.

In the mid-twentieth century, classical parametric phase-locked oscillators, called parametrons were utilized as classical bits of digital computers. Recently, a Kerr-nonlinear parametric oscillators (KPOs), which are parametrons in the single-photon Kerr regime where the nonlinearity is larger than the decay rate, attracted increasing attention in terms of their applications to quantum information processing and studies of quantum many-body systems.

In the circuit-QED architecture, which is a promising platform of quantum information processing, KPOs can be implemented by a superconducting resonator with Kerr-nonlinearity realized by the Josephson junctions, driven by an oscillating pump field. Two coherent states with opposite phases can exist stably in a KPO, and are used as qubit states. Bit-flip error of a KPO is suppressed because of the stability of the coherent states against photon loss, and thus phase-flip error dominates bit-flip error in a KPO. It is expected that quantum error correction for KPOs can be performed with less overhead owing to such biased errors compared to conventional qubits with unbiased errors.

Quantum annealing and universal quantum computation using KPOs have been studied theoretically. Single-qubit operations were experimentally demonstrated. Two-qubit gate operations, which preserve the biased feature of errors and allow one to use its advantage, were studied theoretically, and high error-correction performance by concatenating the XZZX surface code with KPOs were numerically demonstrated. Fast and accurate controls, spectroscopy, and dynamics not confined in qubit space, Boltzmann sampling, effect of strong pump field, quantum phase transitions and quantum chaos have been the subject of investigations of KPO systems. Many of the above studies use multi-KPO systems, where the inter-KPO coupling plays a major role determining the property of the system. Simple coupling scheme of KPOs with high tunability in terms of speed and amplitude will extend the degrees of freedom of controls, and is highly desirable for significant advances in the fields.

Many of the relevant control schemes of KPOs resort to quantum adiabatic dynamics. However, in practice, there are unwanted excitations due to the violation of the quantum adiabatic theorem in the controls, when performed in a short time. Fast and accurate manipulations of KPOs have been studied using the shortcuts to adiabaticity (STA), a group of protocols which mitigate or eliminate completely such unwanted excitations realizing the desired final state. Fast creation of a cat state (a superposition of two coherent states with opposite phases) and traveling cat states and geometric quantum computation with cat qubits were proposed based on the STA.

In this paper, we develop a scheme of fast tunable ZZ coupling of KPOs using the counter-diabatic (CD) protocol, which is categorized to the STA. The coupling scheme is based on a fast transitionless rotation of a KPO in the phase space, and importantly can be implemented with the fixed amplitude of the pump field and with always-on coupling between resonators.
constituting the KPOs in contrast to other schemes [28, 30], and moreover does not require additional driving fields. In our scheme, the coupled KPOs can be identical because the controlled relative phase of the pump fields can eliminate undesired energy transfers between KPOs. Thus, the scheme will mitigate hardware requirements, complexity of sample design and frequency crowding, which are critical and ubiquitous problem of current quantum computing technologies. We apply this novel method of change of the control using the STA which can be implemented with fixed amplitude of the oscillating external field. Furthermore, this is the first proposal of transitionless rotation of a quantum system in phase space, although rotation of quantum systems in real space have been studied in different systems [42, 48, 49].

Transitionless rotation of a KPO. — Before introducing the coupling scheme, we first develop the method of the fast transitionless rotation of a KPO used for the coupling scheme. We consider a KPO of which Hamiltonian is written in a rotating frame as [4, 5, 27]

$$\frac{H(\theta)}{\hbar} = \frac{K}{2} a^4 + \frac{p}{2} a^2 e^{2i\theta} + \frac{a^2 e^{-2i\theta}}{2},$$  

where $K$ is the nonlinearity parameter, $p$ and $2\theta$ are the amplitude and phase of the pump field (see also Supplemental Material section S1). Hereafter, we assume that $K$ and $p$ are positive for simplicity, although they are not for realized KPO reported e.g. in Ref. [17], because the overall sign of the Hamiltonian is not of physical importance. This system has two degenerate ground states represented as $(|\alpha e^{i\theta}| ± |\alpha e^{-i\theta}|)/\sqrt{2}$ with $\alpha = \sqrt{p/K}$ when $p \gg K$, which are called the even and odd cat states, respectively.

The phase of the pump field determines the orientation of the Wigner function of energy eigenstates of the KPO [31] (see Supplemental Material section S2 for definition of the Wigner function). Figure 1 shows the Wigner function of the even cat state for $\theta = 0$ (a) and $\pi/4$ (b). We set $p/K = 7$. Illustration of effective potential $V(\theta)$ for $\theta = 0$ (c) and $\pi/4$ (d).

The Wigner function can be rotated by changing $\theta$ gradually. When the rate of change of $\theta$ is sufficiently small, an adiabatic dynamics leads to a simple rotation of the Wigner function. On the other hand, when the rate of change of $\theta$ is large, the Wigner function is disturbed due to unwanted nonadiabatic transitions as shown in Supplemental Material section S2.

Now we derive a modified Hamiltonian, which realizes an ideal rotation, using the CD protocol [47]. We consider a dynamics in the system with $\theta = 0$ as a reference. Suppose that $|\Psi(t)\rangle$ is a solution of the Schrödinger equation

$$i\hbar \frac{d}{dt} |\Psi(t)\rangle = H(0)|\Psi(t)\rangle,$$

where $H(0)$ denotes $H(\theta = 0)$. The state rotated by $\theta(t)$ is represented as $U(\theta(t))|\Psi(t)\rangle$, where $U(\theta)$ is defined by

$$U(\theta) = e^{i\theta a^\dagger a}$$

(see Supplemental Material section S3). We also have the relation

$$H(\theta) = U(\theta) H(0) U^\dagger(\theta).$$

We can straightforwardly obtain the relation

$$i\hbar \frac{d}{dt} \{U(\theta(t))|\Psi(t)\rangle\} = H' U(\theta(t))|\Psi(t)\rangle,$$

FIG. 1. The Wigner function of the even cat state for $\theta = 0$ (a) and $\pi/4$ (b). We set $p/K = 7$. Illustration of effective potential $V(\alpha)$ for $\theta = 0$ (c) and $\pi/4$ (d).
with
\[ H' = H(\theta(t)) - \hbar \dot{\theta}(t) a^\dagger a, \]
where dot denotes time derivative, and we have used Eqs. (2), (3), (4) and \( U^\dagger(\theta(t)) U(\theta(t)) = 1 \). The rotated state \( U(\theta(t)) \ket{\Psi(t)} \) is a solution of the Schrödinger equation corresponding to Hamiltonian \( H' \) composed of \( H(\theta) \) in Eq. (1) and \(-\hbar \dot{\theta}(t) a^\dagger a\), which we call CD term. The CD term can be implemented by the detuning \( \Delta \) in KPOs, which is the difference between the resonance frequency of the KPO and half of the pump frequency \[ \frac{\omega_p}{2} \] and appears as a term, \( \hbar \Delta a^\dagger a \), in the KPO Hamiltonian [7]. The detuning can be tuned either by controlling the resonance frequency of the KPO via the magnetic flux [17] or by changing the frequency of the pump field (see Supplemental Material section S1). Therefore, controlling the phase and frequency of the pump field can rotate a KPO without any disturbance. The performances of the controls with and without the CD term are compared by numerical simulations in Supplemental Material section S2.

**Fast tunable coupling scheme.** — We introduce a scheme of fast tunable coupling for KPOs based on the above transitionless rotation of a single KPO. We consider two linearly coupled KPOs with the same resonance frequencies with Hamiltonian
\[
\frac{H_{\text{tot}}(t)}{\hbar} = \sum_{i=1}^{2} \left[ \begin{array}{cc}
K_i \frac{\partial}{\partial t} a_i^\dagger a_i^\dagger & -\frac{p_i}{2} a_i^\dagger e^{2i\theta_i(t)} + a_i^\dagger e^{-2i\theta_i(t)} \\
\frac{p_i}{2} a_i e^{2i\theta_i(t)} & -\frac{p_i}{2} a_i e^{-2i\theta_i(t)}
\end{array} \right] + J(a_1 a_2^\dagger + a_2 a_1^\dagger) - \dot{\theta}_1 a_1^\dagger a_1 + \dot{\theta}_2 a_2^\dagger a_2,
\]
where \( K_i \) is the nonlinearity parameter, \( p_i \) and \( 2\theta_i \) are the amplitude and phase of the pump field of KPO \( i \). Here, \( J \) is the fixed coupling coefficient between the KPOs. We emphasize that the effective coupling between KPOs can be turned off even with fixed \( J \) as shown below. The last term in Eq. (7) is the CD term for transitionless rotation of KPO 1. We, hereafter, assume that \( p_i = p, K_i = K \) and \( \theta_1(t) = \theta(t) \) and \( \theta_2 = 0 \) for simplicity. Note that only the phase of KPO 1 is modulated, while that of KPO 2 is fixed.

In the parameter regime where \( p \gg J \) and \( K \), four states represented by \( |\alpha e^{i\theta}, \alpha \rangle, |\alpha e^{i\theta}, -\alpha \rangle, |-\alpha e^{i\theta}, \alpha \rangle, |-\alpha e^{i\theta}, -\alpha \rangle \) with \( \alpha = \sqrt{p/K} \) are stable due to the exponential suppression of bit-flip rate caused with the increase of \( \alpha \) [18]. Hereafter, these states are denoted by \( |0, 0 \rangle, |0, 1 \rangle, |1, 0 \rangle \) and \( |1, 1 \rangle \), respectively. The interaction terms in the Hamiltonian shift the energy of the states because
\[
\langle 0, 0 |(a_2^\dagger a_1^\dagger + a_1 a_2) |0, 0 \rangle = 2|\alpha|^2 \cos\theta,
\langle 0, 1 |(a_2^\dagger a_1^\dagger + a_1 a_2) |0, 1 \rangle = -2|\alpha|^2 \cos\theta
\]
while off-diagonal elements, such as \( \langle 0, 0 |a_1 a_2^\dagger + a_2 a_1 |0, 1 \rangle \), are negligible (note that \( \langle -\alpha |\alpha \rangle \simeq 0 \)). Importantly, the shift of the energies can be controlled via \( \theta \) as seen in Eq. (8), and the shift of the energies becomes zero when \( \theta = \pi/2 + \pi n \), where \( n \) is an integer. Thus, the effective coupling between KPOs can be tuned and even turned off. A pulsed effective coupling can be used to perform a \( R_{z\pi} \) gate as shown below.

We consider the phase of the pump field controlled for \( 0 \leq t \leq T \) as \( \theta(t) = \frac{\pi}{2} - \theta_{\text{amp}} \pi [1 - \cos(2\pi t/T)] \), where \( \theta_{\text{amp}} \) is a constant parameter, which determines maximum strength of the effective coupling during the control. \( \theta \) is chosen to be \( \pi/2 \) at \( t = 0 \) and \( T \) so that the effective coupling is off at the initial and final times of the control. We assume that the initial state is one of the states, \( |0, 0 \rangle, |0, 1 \rangle, |1, 0 \rangle \) and \( |1, 1 \rangle \). For sufficiently large \( T \), the state of the system evolves adiabatically from \( |i, j \rangle \) to \( e^{i\varphi_{ij}} |i, j \rangle \), where \( i, j = 0, 1 \). Here, \( \varphi_{ij} \) is the dynamical phase at \( t = T \) due to the energy shift, and is written as
\[
\varphi_{ij} = \left\{ \begin{array}{ll}
2J|\alpha|^2 \int_0^T \cos\theta(t) dt & \text{for } i = j, \\
-2J|\alpha|^2 \int_0^T \cos\theta(t) dt & \text{for } i \neq j.
\end{array} \right.
\]
Thus, we can perform \( R_{z\pi} \) gates simply by controlling the phase of the pump field. When \( T \) is sufficiently large, the CD term is not necessary because it is proportional to \( \theta \), and therefore much smaller than the other parameters. However, for the small \( T \) regime where \( \theta \) is comparable to or greater than the other parameters, the final state considerably deviates from \( e^{i\varphi_{ij}} |i, j \rangle \) due to nonadiabatic transitions unless the CD term is used.

In order to compare the performance of the controls with and without the CD term, we numerically simulate the dynamics with the initial state of \( |\Psi(0)\rangle = |i, j \rangle \) and \( |\Psi_s\rangle \equiv \sum_i |i, j \rangle / \sqrt{4} \), and obtain the fidelity defined by \( |\langle \Psi_s | \Psi(T) \rangle|^2 \), where \( |\Psi_{\text{ideal}}\rangle = U_{\text{ideal}} |\Psi(0)\rangle \), and \( U_{\text{ideal}} \) denotes the operator representing the ideal gate operation, e.g., \( U_{\text{ideal}} |i, j \rangle = e^{i\varphi_{ij}} |i, j \rangle \). Figure 2 shows the dependence of the fidelity on \( T \) for the controls with and without the CD term for \( \theta_{\text{amp}} = 0.1 \). The fidelity of the control without the CD term (purely adiabatic scheme) is degraded by the nonadiabatic transitions as \( T \) decreases. On the other hand, the fidelity of the control with the CD term is approximately unity in such small \( T \) regime. For example, the fidelity of the control with the CD term averaged over the initial states is approximately 0.999, while the averaged fidelity of the control without the CD term is less than 0.89 for \( T = K^{-1} \). The fidelity of the control with the CD term slightly decreases from unity as \( T \) decreases. We attribute this to the fact that the CD term is designed for transitionless rotation of an individual KPO (\( J = 0 \)), and therefore there is finite nonadiabatic transitions for \( J \neq 0 \). However, it is noteworthy that the CD term can work well also for the case with \( J \neq 0 \).

Figure 3(a) shows the dependence of the fidelity on \( \theta_{\text{amp}} \) for the controls with \( T = K^{-1} \). It is seen that the fidelity of the control with the CD term is much
higher than that of the control without the CD term for large $\theta_{\text{amp}}$. We numerically obtain phase $\varphi_{ij}$, which the system acquires during the control with the CD term, and compare it with the analytic one in Eq. 9. We define the phase at $t = T$ as $\varphi_{ij} = \arg\langle ij|\Psi(T)\rangle$ for the simulation with the initial state $|ij\rangle$ and $|\Psi_s\rangle$, respectively. The purple squares are for the control with CD term, and the data points for all initial states are almost overlapping. The used parameters are $p/K = 7$, $J/K$ and $\theta_{\text{amp}} = 0.1$.

Conclusions. — A fast tunable ZZ coupling scheme of KPOs has been developed using the transitionless rotation of a KPO in the phase space based on the CD protocol. The effective coupling between KPOs can be turned off even with always-on linear coupling between the resonators constituting the KPOs. We have examined the performance of our scheme applying it to $R_{zz}$ gate, and compared with the results of a purely adiabatic scheme, which utilizes only a controlled phase of the pump field. It has been shown that our scheme greatly enhances the fidelity of $R_{zz}$ gate compared to the adiabatic scheme by eliminating undesired nonadiabatic transitions, when applied in a short time. We emphasize that our scheme can be implemented simply using the time dependent phase and frequency of the pump field, which can be accurately tailored with hardwares out of the cryostat such as arbitrary waveform generator, and does not require additional microwave drives to KPOs nor controlling the amplitudes of the linear coupling between KPOs. This makes our scheme experimentally feasible.

While we are preparing our manuscript, we came to know that another group independently studied the tunability of the effective coupling solely by the phase of the pump field [52]. However, this method has recourse to an adiabatic evolution of the system and, therefore, is not suitable for fast tuning of the coupling. Our scheme resolves the shortcoming of the adiabatic scheme.

Before closing, we point out that our coupling scheme will find wider applications in quantum technologies, although we particularly applied to $R_{zz}$ gate in this paper to demonstrate the effectiveness of the scheme. For example, the coupling scheme can be useful for quantum annealing and quantum simulation in which time dependent qubit-qubit couplings are utilized. Our scheme is used to decrease undesired population transfers out of the qubit space caused by the rotation of a KPO (not nonadiabatic transitions of the whole system which may be caused by time dependent effective coupling), and therefore has a different motivation from other studies based on the STA which consider ideal spin models and aim decreasing nonadiabatic transitions of the model.

FIG. 2. Dependence of the fidelity on $T$ for the controls with and without the CD term. The labels, $ij$ and $\Psi_s$, on the figure indicate the results of the control without the CD term with initial state $|ij\rangle$ and $|\Psi_s\rangle$, respectively. The purple squares are for the control with CD term, and the data points for all initial states are almost overlapping. The used parameters are $p/K = 7$, $J/K$ and $\theta_{\text{amp}} = 0.1$.

FIG. 3. (a) Dependence of the fidelity on $\theta_{\text{amp}}$ for the controls with and without the CD term. The labels, $ij$ and $\Psi_s$, on the figure indicate the results of the control without the CD term with initial state $|ij\rangle$ and $|\Psi_s\rangle$, respectively. The purple squares are for the control with CD term, and the data points for all initial states are almost overlapping. The dashed lines are guides to eyes. (b) Dependence of $\varphi_{ij}$ on $\theta_{\text{amp}}$ of the control with the CD term. The solid curve represents $\varphi_{ij}$ in Eq. 9. The used parameters are $p/K = 7$, $J/K = 0.2$ and $T = K^{-1}$. 

systems. Our scheme can be implemented by the simple manner and even independent of energy-level structure of the system. Performance of our scheme in quantum annealing and quantum simulation deserves further quantitative investigation.

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Supplemental Material:
Fast tunable coupling scheme of Kerr-nonlinear parametric oscillators based on shortcuts to adiabaticity

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S1 HAMILTONIAN AND IMPLEMENTATION OF TIME DEPENDENT DETUNING

We consider the case that the frequency of the pump field is time dependent. Hamiltonian of a KPO in a lab frame is represented as (see e.g., [1] for the case with fixed frequency of the pump field)

$$H = \frac{\hbar}{\omega_0}a^\dagger a + \frac{K}{12}(a^\dagger + a)^4 - p(a^\dagger + a)^2 \cos \left( \int_0^t \omega_p(t') dt' - 2\theta(t) \right),$$  \hspace{1cm} (S1)

where $p$ and $2\theta$ are the amplitude and phase of the pump field, $\omega_0$ and $K$ are the resonance angular frequency and nonlinear parameter of the resonator constituting the KPO. We move on a rotating frame defined by an unitary operator $U(t) = \exp \left[ i \frac{\hbar}{\omega_p} \omega_p(t')dt' a^\dagger a \right]$, and omit non-resonant rapidly oscillating terms (the rotating wave approximation) to obtain

$$H = \Delta(t) a^\dagger a + \frac{K}{2}a^\dagger a^2 - \frac{p}{2}(a^2 e^{2i\theta(t)} + a^2 e^{-2i\theta(t)}),$$  \hspace{1cm} (S2)

where we assumed that $\dot{\theta}(t)$ is much smaller than $\omega_p(t)$ for any $t$. The detuning $\Delta$ is written as

$$\Delta(t) = \omega_0 - K - \omega_p(t)/2.$$  \hspace{1cm} (S3)

Hamiltonian in Eq. (S2) is the same as Eq. (1) when $\Delta(t) = 0$. As seen in Eq. (S3), the time dependent detuning can be implemented by the time dependent frequency of the pump field.

S2 PERFORMANCE OF ROTATION SCHEMES FOR SINGLE KPO

We compare the performance of the rotation schemes with and without the CD term. As an example, we consider the case that $\theta$ is increased from 0 to $\pi/2$ for $0 \leq t \leq T$ as

$$\theta(t) = \frac{\pi}{4} \left[ 1 - \cos \left( \frac{\pi t}{T} \right) \right].$$  \hspace{1cm} (S4)

The initial state is a ground state well approximated by $|\alpha\rangle + |\alpha\rangle)/\sqrt{2}$, where $\alpha = \sqrt{p/K}$. The Wigner function of the initial state is presented in Fig. 1(a). The Wigner function is defined by $W(\xi) = \frac{2}{\pi} \text{Tr}[D(-\xi)\rho D(\xi)P]$, with $\xi = x + iy$, density operator $\rho$, displacement operator $D(\xi) = \exp(\xi a^\dagger - \xi^\dagger a)$ and parity operator $P = \exp(i\pi a^\dagger a)$. We fix $p$ and $K$, while $\theta$ is changed during the control.

Figures 1(a) and 1(b) show the Wigner function at $t = T$ for $T = 0.6K^{-1}$ and $1.5K^{-1}$, respectively, for the control without the CD term. The Wigner function at $t = T$ is disturbed due to nonadiabatic transitions for $T = 0.6K^{-1}$, while the Wigner function is almost ideally rotated for $1.5K^{-1}$ because the system evolves almost adiabatically.

We define the fidelity of the control as $|\langle \Psi_{\theta(T)} |\Psi(T) \rangle|^2$, where $|\Psi(T)\rangle$ and $|\Psi_{\theta(T)}\rangle$ are the final state of the control.
FIG. S1. The Wigner function of the final state of the control without the CD term for $T = 0.6K^{-1}$ (b) and $T = 1.5K^{-1}$. The used parameters are the same as Fig. 1.

and the state ideally rotated by angle $\theta(T)$, respectively. Figure S2 shows the $T$-dependence of the fidelity for both the controls. In the control without the CD term, the fidelity is degraded due to nonadiabatic transitions for small $T$, while the fidelity becomes close to unity for sufficiently large $T$. On the other hand, the fidelity of the control with the CD term is unity. The inset of Fig. S2 shows the time dependence of the detuning $\Delta(t) = -\dot{\theta}(t)$ for $T = 0.6K^{-1}$.

FIG. S2. $T$-dependence of the fidelity of a rotation of a KPO. The black circles and red crosses correspond to the controls with and without the CD term. The inset shows the time dependence of the detuning $\Delta(t) = -\dot{\theta}(t)$ for $T = 0.6K^{-1}$ in the control with the CD term.

S3 THEORY OF ROTATION

The rotation of a KPO is characterized by operator $U$ defined by

$$U(\theta) = e^{i\theta a^{\dagger}a}.$$  \hfill (S5)

$U(\theta)$ rotates a state of a KPO in the $\alpha$ space. This fact is easily confirmed by letting $U$ act on coherent state $|\alpha\rangle$ to obtain

$$aU(\theta)|\alpha\rangle = \alpha e^{i\theta}U(\theta)|\alpha\rangle,$$  \hfill (S6)

where we used $U^{\dagger}(\theta)U(\theta) = ae^{i\theta}$.

Suppose that $|\phi_m\rangle$ is $m$th eigenstate of $H(0)$ with eigenenergy $E_m$. The time independent Schrödinger equation is written as

$$H(0)|\phi_m\rangle = E_m|\phi_m\rangle.$$  \hfill (S7)
Then, we can obtain

\[ H(\theta)U(\theta)\ket{\phi_m} = E_m U(\theta)\ket{\phi_m}, \]  

(S8)

where we used Eq. (4). The above discussion shows that if \( \ket{\phi_m} \) is an eigenstate of \( H(0) \), \( U(\theta)\ket{\phi_m} \), which is a rotated state by \( \theta \), is an eigenstate of \( H(\theta) \). This fact is independent of energy eigenstates. Therefore, we can rotate an arbitrary state of a KPO by adiabatically changing \( \theta \).

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