Studies of $CP$ Violation and Mixing in the $D$ Mesons
decays from $BABAR$

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Abstract. After an introduction on the mixing and $CP$ violation ($CPV$) in the neutral charmed mesons system, we present the current status of the measurements. We discuss some recent analyses performed on the data sample collected by the $BABAR$ detector at the PEPII $e^+e^−$ asymmetric collider.

1. Introduction
Mixing occurs in neutral mesons systems in which the flavour eigenstates differ from the mass eigenstates. The $D^0 - \bar{D}^0$ system is one of the four systems in which this phenomenon is predicted by the Standard Model (SM) and observed experimentally, the other systems are $B^0 - \bar{B}^0$, $K^0 - \bar{K}^0$ and $B_s - \bar{B}_s$. The oscillation rate is different in each system, the $D$ system is the one that shows the smallest mixing, this is the reason why it’s the last one in which evidence of mixing has been found experimentally.

2. Theory and Formalism
The following is a brief overview on mixing and $CP$ violation, for more details see PDG [1].

The time evolution in the $D^0 - \bar{D}^0$ system is described by a Schrödinger equation:

$$i \frac{\partial}{\partial t} \begin{pmatrix} D^0(t) \\ \bar{D}^0(t) \end{pmatrix} = \begin{pmatrix} M - i \frac{\Gamma}{2} \\ \bar{M} + i \frac{\Gamma}{2} \end{pmatrix} \begin{pmatrix} D^0(t) \\ \bar{D}^0(t) \end{pmatrix}$$

where $M$ and $\Gamma$ are two Hermitian matrices. $C$, $P$ and $T$ conservations impose constraints on the elements of $M$ and $\Gamma$. In the following we will assume $CPT$ conservation, which implies $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$.

The two $D^0$ mass eigenstates are obtained diagonalizing the equation above and can be represented as:

$$|D_1\rangle = p|D^0\rangle + q|\bar{D}^0\rangle \quad |D_2\rangle = p|D^0\rangle - q|\bar{D}^0\rangle$$

where

$$\begin{pmatrix} p \\ q \end{pmatrix}^2 = \frac{M_{12}^* + \frac{\Gamma}{2} \Gamma_{12}}{M_{12} - \frac{\Gamma}{2} \Gamma_{12}} \quad (3)$$

The normalization condition is $|p|^2 + |q|^2 = 1$. The mass eigenstate $|D_i\rangle$ has a mass $M_i$ and width $\Gamma_i$. Assuming $CP|D^0\rangle = +|\bar{D}^0\rangle$, in case of $CP$ conservation the $|D_1\rangle$ is the $CP$-even state.
2.1. CP Violation

CP Violation is an important ingredient when neutral meson mixing is involved. The only source of CP Violation (CPV) in the Standard Model is the complex phase in the CKM matrix.

There are three possible manifestations of CPV:

(i) in the decay (direct CPV);
(ii) in mixing (indirect CPV);
(iii) in the interference between mixing and decay (indirect CPV).

There is direct CPV when the probability of a meson $D$ to decay into a final state $f$ differs from the probability of the anti-meson to decay into the CP conjugate state $\bar{f}$. In other words when:

$$\frac{|A_f|}{|\bar{A}_{\bar{f}}|} \neq 1$$

where $A_f = \langle f | H_d | D \rangle$ and $\bar{A}_{\bar{f}} = \langle \bar{f} | H_d | \bar{D} \rangle$, $H_d$ is the Hamiltonian responsible of the decay. It’s important to underline that the moduli $|A_f|$ and $|\bar{A}_{\bar{f}}|$ can be different only if the decay proceeds through the sum of two different contributions, with different phases. If the phase is the same for the two contributions, then the asymmetry will be zero.

This kind of CPV is independent of mixing and it is expected to be negligible in the decays of $D$ mesons [2].

When CPV in mixing occurs then the probability of the transition $D^0 \rightarrow \bar{D}^0$ is different from the probability of the inverted process $\bar{D}^0 \rightarrow D^0$. The signature for CPV in mixing is:

$$R_M = \left| \frac{q}{p} \right| \neq 1.$$  

The CPV in the interference between mixing and decay can occur when both the $D^0$ and the $\bar{D}^0$ have access to a common final state $f$. In this case $f$ can be reached in two possible ways:

(i) with a direct decay: $D^0 \rightarrow f$;
(ii) with a decay after the mixing: $D^0 \rightarrow D^0 \rightarrow f$.

CPV occurs in the interference between these two paths and it is possible also when CP is conserved in the decay and in the mixing separately. In this case we define $\lambda_f$:

$$\lambda_f = \frac{q A_f}{p A_{\bar{f}}} = R_M \left| \frac{A_f}{A_{\bar{f}}} \right| e^{i(\delta_f + \phi_f)}$$

where $\delta_f$ and $\phi_f$ are, respectively, the strong and the weak phase differences between the two paths. There is CPV when the weak phase difference is different from zero:

$$\phi_f \neq 0$$

3. Mixing and CPV in the charm sector

The mixing in the $D$ system is described by two real parameters, the mixing parameters that are proportional to the difference of masses and widths of the mass eigenstates:

$$x_D = \frac{M_1 - M_2}{\Gamma}, \quad y_D = \frac{\Gamma_1 - \Gamma_2}{2\Gamma}.$$  

with $\Gamma \equiv (\Gamma_1 + \Gamma_2)/2$. If either $x_D$ or $y_D$ is non-zero, mixing will occur. If $D_1$ and $D_2$ have different lifetimes then $y_D \neq 0$. 


The mixing parameters defined above are the ones directly related to the physical quantities that drive mixing. Other parameters not directly sensitive to these quantities may be defined. The parameter $y_D^{CP}$ [3] measures how much the lifetime of the $CP$-even eigenstate ($\tau_{CP+}$) differs from the $D^0$ lifetime:

$$y_D^{CP} = \frac{\tau_D^0}{\tau_{CP+}} - 1.$$  \hspace{1cm} (9)

In case of no $CPV$ then $y_D^{CP} = y_D$.

The $CPV$ is also quantified in the parameter $\Delta Y$:

$$\Delta Y = \frac{\tau_D^0}{\tau_{CP+}} A_r \quad \text{with} \quad A_r = \frac{\Gamma(D^0 \to f_+)}{\Gamma(D^0 \to f_-)} + \frac{\Gamma(\bar{D}^0 \to f_-)}{\Gamma(\bar{D}^0 \to f_+)}$$ \hspace{1cm} (10)

In case of no mixing $\Delta Y = 0$. In the limit of $CP$ conservation $y_D^{CP} = y_D$ and $\Delta Y = 0$. $\Delta Y \neq 0$ is the signature for indirect $CPV$.

3.1. Standard Model Predictions and Measurements

Standard Model predictions for $x_D$ and $y_D$ are of the order of $10^{-2}$ or smaller for both, as showed in Fig. 1.

![Figure 1](image1.png)  
**Figure 1.** left: SM predictions for $|x_D|$ (triangles) and $|y_D|$ (squares) published by different authors; right: predictions for $x_D$ coming from theories beyond the SM. See [4] for the reference index.

The suppression of mixing in the $D$ system with respect to the other systems has been understood within the Standard Model. Since the mixing process changes the flavour of two units, the transition must proceed through an intermediate state that can be either real or virtual. One possible transition proceeding through a virtual intermediate state is represented in Fig. 2 (left). The contribution of these diagrams is small [5] due to the CKM factors associated to the vertices: the contribution of the $b$ quark is doubly Cabibbo suppressed (DCS) while the contributions of the $d$ and $s$ quark are GIM suppressed. The biggest uncertainty related to the computation of these diagrams is due to the poor knowledge of the hadronization processes. It’s important to notice that the $D$ system is the only one in which the virtual quarks are down-type since the top quark is too heavy to form long-living bound states and the $\pi^0$ coincides with its anti-particle. From this point of view the $D$ system is unique.

Some examples of real diagrams are reported in Fig. 2 (right), the intermediate states are all the possible final states common to the $D^0$ and the $\bar{D}^0$ decays, such as $K^+K^-$ and $\pi^+\pi^-$. This second class of diagrams is expected to be dominant in the mixing process though the uncertainty associated to their computation is even larger [5].
Figure 2. left: Feynman diagrams of $D^0$ mixing through virtual intermediate states; right: mixing processes involving real intermediate states.

The first evidence of $D^0$ mixing was found by the Belle experiment [6] and the BABAR experiment [7] in 2007, and it was confirmed a year later by the CDF experiment [8]. The measurements are all in agreement with the SM predictions and show no evidence of $CPV$. Fig. 3 shows the mean world values for $x_D$ e $y_D$, extracted from all measurements, computed by the HFAG group [9].

Figure 3. World mean values computed by the HFAG group under the hypothesis of $CP$ conservation (left) and allowing for $CPV$ (right).

3.2. Impact of the measurement

Historically mixing measurements have played an important role as SM tests and in building New Physics (NP) theories. The $D$ system is the last one in which mixing has been observed, still not yet with a precision that can bring information on a possible or impossible NP structure and on $CPV$.

Measuring mixing in the $D$ system completes the picture of neutral meson mixing within the SM bringing new information since this is the only system in which the virtual quark are down-type. Moreover, since the mixing parameters values are quite small, NP could manifest itself with a higher relative value (v. Fig. 1). Unfortunately, except for big values of $x_D$ and $y_D$ (hypothesis currently not favored by the experimental results), the interpretation of the results is not straightforward, it is limited by the uncertainties in the SM predictions.

The search for $CPV$ is probably the most effective probe for NP: an evidence of $CPV$ with the current experimental sensitivity would be a clear sign of NP.
4. The BABAR Experiment

The BABAR detector [10] has been running at the B-Factory PEP II, the $e^+e^-$ asymmetric collider located at the SLAC National Accelerator Laboratory. The data taking started in 1999 and ended in April 2008, resulting in a total integrated luminosity of 530 fb$^{-1}$. The collider operated mostly at a center of mass energy of 10.58 GeV, corresponding to the mass of the $\Upsilon(4S)$ resonance (Run1 to Run6). During the last period of data taking it also operated at lower energies, corresponding to the masses of the $\Upsilon(3S)$ and $\Upsilon(2S)$ (Run7). The asymmetry in the beam energies led to a boost $\beta\gamma = 0.56$. The internal part of the detector consists of the Silicon Vertex Tracker (SVT), the drift chamber (DCH), the Cherenkov light detector (DIRC) and the electromagnetic calorimeter (EMC). Outside there is the instrumented flux return (IFR) and a superconductive solenoid which produces a uniform magnetic field of 1.5 T parallel to the direction of the electron beam. The B Factory produced around 690 millions of $e^+e^- \rightarrow e\bar{e}$ events, working also as a Charm Factory.

4.1. The $D^0$ mesons tagging

In CPV measurements it is of crucial importance to identify the flavour of the neutral $D$ meson at production. In order to do so the decay chain $D^{++} \rightarrow D^0\pi^+$ is reconstructed and the charge of the pion tag the flavour of the $D$ meson. Samples of $D$ mesons reconstructed in this way are referred to as tagged samples. In mixing analyses the knowledge of the flavour of the $D$ is not always needed and in this cases we talk of untagged samples.

The tagged samples contain a much lower level of background but are around four times less populated than the correspondent untagged samples.

5. A time-dependent analysis on the Dalitz Plot: $D^0 \rightarrow K^0_S\pi^+\pi^-$, $K^0_SK^+K^-$

A direct measurement of the mixing parameters is possible through a full time-dependent analysis of the Dalitz plot (DP) for the three-body final states $K^0_S h^+h^-$ ($h = \pi, K$) with the $K^0_S$ reconstructed in the $\pi^+\pi^-$ final state.

The three-body $D^0$ decay is assumed to proceed through two-body intermediate states where one is a resonance, and it is described by two independent Dalitz variables, $m_{2\pi}$ and $m^2$ which are the reconstructed invariant mass squared of $K^0_S h^+$ and $K^0_S h^-$. In the following we assume CP conservation in the decay, i.e. $A(m^2, m_{2\pi}^2) = \bar{A}(m^2, m_{2\pi}^2)$. The total amplitude $A(m^2, m_{2\pi}^2)$ can be written as the superimposition of the amplitudes of each resonance ($r$) plus a non-resonant (NR) term:

$$A(m^2, m_{2\pi}^2) = \sum_r a_r e^{i\phi_r} A_r(m_{2\pi}^2, m^2) + a_{NR} e^{i\phi_{NR}}$$

(11)

where $a_r$ and $\phi_r$ are the modulus and the phase of the amplitudes and $A_r(m_{2\pi}^2, m^2)$ reproduces the dependence on the Dalitz variables. According to [11] the modulus and the phases are extracted from data while a phenomenological model for each $A_r(m_{2\pi}^2, m^2)$ is assumed. After selecting a sample of $D^0$ of purity greater than 98% for both the $D^0$ decay modes, we discriminate signal from background events fitting the distribution in the $(\Delta m = m_{D^\pi} - m_{D^\pi}, m_{D^\pi})$ plane for the signal and background categories. An extended maximum likelihood fit is then performed on the combined samples of $D^0 \rightarrow K^0_S\pi^+\pi^-$ and $D^0 \rightarrow K^0_SK^+K^-$ decays in order to extract the Dalitz model and the resolution function parameters, yielding [12]:

$$x_D = [1.6 \pm 2.3(\text{stat}) \pm 1.2(\text{syst}) \pm 0.8(\text{model})] \times 10^{-3}$$

$$y_D = [5.7 \pm 2.0(\text{stat}) \pm 1.3(\text{syst}) \pm 0.7(\text{model})] \times 10^{-3}$$

(12)

This measurement, done on 485 fb$^{-1}$, is the most precise direct measurement of $x_D$ and $y_D$. The contour plot for the 1 to 3 standard deviations regions is reported in Fig. 4.

The most important systematic errors are related to the Dalitz model assumption (referred to as “model” in the error), the fit bias and the detector misalignment.
6. Lifetime Ratio analyses: $D^0 \rightarrow K^-\pi^+, K^+K^-, \pi^+\pi^+$

The BABAR Collaboration has presented two analyses on the measurement of the ratio between the lifetimes of $D^0$ reconstructed in CP eigenstates and in CP mixed states, $y_D^{CP}$. One was performed on an untagged sample, the other on a statistically independent sample of tagged events. In both analyses $\tau_{D^0}$ is extracted from the proper time distribution of $D^0$ reconstructed in the final state $K^-\pi^+$ assuming an exponential decay. The same hypothesis is under the extraction of the value of $\tau_{CP^+}$ from $K^+K^-$ final state and, in the tagged analysis only, the $\pi^+\pi^-$ final state. The value of $y_D^{CP}$ extracted from the untagged sample of $D^0$ is [13]:

$$y_D^{CP} = (1.12 \pm 0.26 \pm 0.22)\%.$$

(13)

In the case of the tagged sample a measurement of CPV is also possible through the parameter $\Delta Y$. The extracted values for mixing and CPV are [7]:

$$y_D^{CP} = (1.24 \pm 0.39 \pm 0.13)\%$$

$$\Delta Y = (-0.26 \pm 0.36 \pm 0.08)\%$$

(14)

By combining the tagged and the untagged samples, BABAR obtained a value of

$$y_D^{CP} = (1.16 \pm 0.22 \pm 0.18)\%,$$

(15)

with a significance for mixing of 4.1 standard deviations, resulting the most significant mixing measurement.

7. T violation with $D^0 \rightarrow K^+K^-\pi^+\pi^+$

Considering the four-body decay $D^0 \rightarrow K^+K^-\pi^+\pi^+$, the T odd observable $C_T \equiv \vec{p}_{K^+} \cdot (\vec{p}_{\pi^+} \times \vec{p}_{\pi^-})$ built with the momenta of the tracks, is used to measure a T asymmetry:

$$A_T = \frac{\Gamma(D^0, C_T > 0) - \Gamma(D^0, C_T < 0)}{\Gamma(D^0, C_T > 0) + \Gamma(D^0, C_T < 0)}$$

(16)

Since final state interactions can produce a non zero value of $A_T$, the relevant parameter is $\mathcal{A}_T = \frac{A_T - \bar{A}_T}{\sqrt{2}}$, where $\bar{A}_T$ is extracted from a $D^0$ sample, and therefore a tagged sample of $D^0$ is

1 Since the mixing effect is smaller than 1%, the assumption is experimentally acceptable.
needed. In order to extract the value of $A_T$ from a sample of 470 fb$^{-1}$ a two-steps analysis is performed. First, the resolution function parameters for signal and background components are extracted from data in the $(\Delta m, m_D)$ plane. Then the sample is divided in four sub-samples (depending on the flavour of the $D$ and the sign of $C_T$) and, after fixing the resolution function, the number of events in each sub-sample is extracted and $A_T$ is computed.

The measurement is still statistically limited [15]:

$$A_T = [1.0 \pm 5.1\text{(stat)} \pm 4.4\text{(syst)}] \times 10^{-3}$$  \hspace{1cm} (17)

but is represents an improvement of one order of magnitude with respect to the previous FOCUS measurement. The most important systematic error is associated to the particle identification, other less important contributions come from the cut on the $D^0$ momentum in the center of mass, the detector misalignment and the fit bias.

8. $CP$ violation with $D^+ \rightarrow K^0\pi^+$

A measurement of the direct $CP$ asymmetry is possible studying the decay $D^+ \rightarrow K^0\pi^+$ and computing:

$$A_{CP} = \frac{\Gamma(D^+ \rightarrow K^0\pi^+) - \Gamma(D^- \rightarrow K^0\pi^-)}{\Gamma(D^+ \rightarrow K^0\pi^+) + \Gamma(D^- \rightarrow K^0\pi^-)}.$$  \hspace{1cm} (18)

A $CP$ violation contribution from $K^0 - \bar{K}^0$ mixing must also be taken into account. The expected value of $A_{CP}$ due to the $CP$ violation in the $K^0$ system is $A_{CP}^{K^0} = [-0.332 \pm 0.006]\%$ [14], any deviation from it would be evidence of $CP$ violation.

The measured asymmetry $A = \frac{N_{D^+} - N_{D^-}}{N_{D^+} + N_{D^-}}$ will have two contributions besides the $CP$ asymmetry: the forward-backward asymmetry and the detector-induced asymmetry in the reconstruction of the charged particles. The former is extracted directly from data exploiting its oddness in the center of mass polar angle. In order to remove the latter a new data-driven technique has been developed.

8.1. The track reconstruction efficiency

If the detector is not completely symmetric in charge reconstruction, the measurement of $CP$ violation will be affected by a detector-induced asymmetry. In the signal channel the only track affected by this asymmetry is the charged pion from the $D$ decay. In order to estimate this effect and subtract it from the final result a high populated control sample with no asymmetry from physics is selected: if no detector asymmetry is present then the number of reconstructed $\pi^+$ ($N_{\pi^+}^{rec}$) should be equal to the number of reconstructed $\pi^-$ ($N_{\pi^-}^{rec}$). Using the $\Upsilon(4S)$ decays\footnote{these events are produced at rest and decay isotropically, continuum subtraction was applied.} the ratio of the charged pion detection efficiency as a function of the $D$ momentum was computed as:

$$R(\vec{p}) = \frac{\epsilon^+(\vec{p})}{\epsilon^-(\vec{p})} = \frac{N_{\pi^+}^{rec}(\vec{p})}{N_{\pi^-}^{rec}(\vec{p})}$$  \hspace{1cm} (19)

and this correction is applied to the negative $D$ candidates. In Fig. 5 the value of the ratio of the detection efficiency and its error are reported: the plots shows that the correction is small and dominated by statistical fluctuations.

8.2. Results

After the subtraction of the forward-backward asymmetry (v. fig 6), the measured $CP$ asymmetry is:

$$A_{CP} = [-4.4 \pm 1.3\text{(stat)} \pm 1.0\text{(syst)}] \times 10^{-3}.$$  \hspace{1cm} (20)
it is consistent with $A_{\text{CP}}^K$ and therefore with no $CP$ violation in the charm sector, as predicted by the Standard Model at these levels of error. This measurement is the most precise $CP$ asymmetry measurement at B-Factories. The most important systematic errors are relative to the charge asymmetry correction and the $K^0 - \bar{K}^0$ regeneration in the material.

References
[1] Nakamura, K. and others (Particle Data Group Collaboration), J. Phys., G37 (2010) 075021;
[2] Buccella, F. and Lusignoli, M. and Miele, G. and Pugliese, A. and Santorelli, P., Phys. Rev. D 51 (1995) 3478;
[3] Bergmann, S. and Grossman, Y. and Ligeti, Z. and Nir, Y. and A.A. Petrov, Phys. Lett. B 486 (2000) 418;
[4] Petrov, Alexey A, Int. J. Mod. Phys. A21 (2006) 5686-5693;
[5] Golowich, E. and Hewett, J. and Pakvasa, S. and Petrov, A. A., Phys. Rev. D 76 (2007) 095009;
[6] Staric, M. and others (Belle collaboration), Phys. Rev. Lett. 98 (2007) 211803;
[7] Aubert, B. and others (BABAR Collaboration), Phys. Rev. D 78 (2008) 011105;
[8] Aaltonen, T. and others (CDF Collaboration), Phys. Rev. Lett. 100 (2008) 121802;
[9] Asner, D. and others (Heavy Flavor Averaging Group), http://www.slac.stanford.edu/xorg/hfag/;
[10] Aubert, B. and others (BABAR Collaboration) Nucl. Instrum. Meth. A479 (2002) 1-116;
[11] Aubert B. and others (BABAR collaboration), Phys. Rev. D 78 (2008) 034023
[12] del Amo Sanchez P. and others (BABAR collaboration), Phys. Rev. Lett. 105 (2010) 081803
[13] Aubert, B. and others (BABAR Collaboration), Phys. Rev. D 80(7), (2009) 071103;
[14] H. J. Lipkin and Z. z. Xing, Phys. Lett. B 450, 405,(1999)
[15] del Amo Sanchez P. and others (BABAR collaboration), Phys. Rev. D 81, (2010) 111103