Abstract

We study the role of fermionic resonances in realistic composite Higgs models. We consider the low energy effective description of a model in which the Higgs arises as the pseudo-Goldstone boson of an $SO(5)/SO(4)$ global symmetry breaking pattern. Assuming that only fermionic resonances are present below the cut-off of our effective theory, we perform a detailed analysis of the electroweak constraints on such a model. This includes the exact one-loop calculation of the $T$ parameter and the anomalous $Zb_L\bar{b}_L$ coupling for arbitrary new fermions and couplings. Other relevant observables, like $b \to s\gamma$ and $\Delta B = 2$ processes have also been examined. We find that, while minimal models are difficult to make compatible with electroweak precision tests, models with several fermionic resonances, such as the ones that appear in the spectrum of viable composite Higgs models from warped extra dimensions, are fully realistic in a large region of parameter space. These fermionic resonances could be the first observable signature of the model at the LHC.
I. INTRODUCTION

One main objective of the Large Hadron Collider (LHC) is to discover the precise mechanism of electroweak symmetry breaking (EWSB). A well motivated hypothesis is that there exists a Higgs boson which is not a fundamental scalar. Instead it could be a composite state of a strongly coupled theory, the pseudo-Goldstone boson of a spontaneously broken global symmetry [1]. Compositeness can explain the insensitivity of EWSB to ultraviolet physics, while the pseudo-Goldstone nature of the Higgs boson may explain the little hierarchy between the scale of new physics and the scale of EWSB.

This mechanism has recently received increased attention, due to the realization that calculable composite Higgs models can be constructed in five dimensions [2]. The main idea is an old one [3], but only recently realistic models in warped extra dimensions [2, 4, 5, 6, 7] have been constructed. The experience with five-dimensional models indicates that custodial symmetry [8] and a custodial protection of the $Zb_L\bar{b}_L$ coupling [9] are likely ingredients of realistic constructions. Little Higgs models [10] also use the idea of a Higgs boson which is the pseudo-Goldstone boson of a spontaneously broken global symmetry. In order to solve the little hierarchy problem, they employ the mechanism of collective symmetry breaking, which ensures that the Higgs mass remains insensitive to ultraviolet physics at one loop.

The main phenomenological implications of a Higgs boson which is the pseudo-Goldstone boson of an extended broken symmetry are largely independent of the particular details of how the global symmetry is broken. They can therefore be conveniently described using an effective Lagrangian approach [11, 12]. A reasonable starting point is a symmetry breaking pattern that includes custodial symmetry. A minimal example of such a pattern is given by a global $SO(5)$ symmetry broken at a scale $f$ to its custodially symmetric subgroup $SO(4)$ [4, 5].

Using this effective Lagrangian approach, it was argued in [12] that composite Higgs models with an $SO(5)/SO(4)$ symmetry breaking pattern are difficult to make compatible with electroweak and flavor precision data without introducing a substantial fine-tuning. The argument was based on very minimal models, in which fermionic resonances did not span full representations of the $SO(5)$ group. In addition, the estimation of electroweak observables was made neglecting contributions which are formally subleading, but can be relevant in specific situations. Two recent works extended the analysis of [12] to models in
which the fermionic composites span full representations of $SO(5)$ \cite{13,14}. Although these analyses differ in several aspects, like the degree of explicit $SO(5)$ symmetry breaking and the symmetry breaking patterns, their outcome is somewhat similar. Only a small region of parameter space is allowed by electroweak precision data in models with no significant fine-tuning and one set of fermionic composites spanning a vector representation of $SO(5)$.

In this paper, we investigate thoroughly the viability of models with an $SO(5)/SO(4)$ symmetry breaking pattern. Our analysis extends previous works in two ways, by making a careful computation of the effect of the new fermionic states on electroweak observables and by considering the effect of an extended fermionic sector.

In our study of electroweak precision constraints we use an exact one-loop calculation of the relevant electroweak observables. We do this in complete generality, and our analytic formulae can be used in other models. In particular, our result for the anomalous $Zb_L\bar{b}_L$ coupling is, to the best of our knowledge, the first complete calculation for an arbitrary number of new quarks with generic couplings. We find that such an exact computation can be important when formally subleading effects in commonly employed approximations are enhanced.

We also examine the possibility of multiple sets of fermionic composites, departing from minimal constructions. Our motivation is the model presented in \cite{7}, in which a five-dimensional realization of a composite Higgs model with $SO(5)/SO(4)$ symmetry breaking pattern was shown to be fully compatible with electroweak precision tests, flavor observables, electroweak symmetry breaking and the observed dark matter relic abundance. The non-minimal fermion sector of the model in \cite{7} was indispensable in order to render its predictions compatible with experimental data. The goal of our article is to present an effective four-dimensional description of composite Higgs models with a non-minimal fermionic content and discuss their electroweak and flavor constraints. We show that there are large regions of parameter space in which composite Higgs models can provide a fully realistic description of EWSB without fine tuning.

The paper is organized as follows. In section II we briefly review the effective description of composite Higgs models with an $SO(5)/SO(4)$ symmetry breaking pattern, including the experimental constraints of minimal models. Section III is devoted to a description of the relevant fermionic sector of the theory and its effects on electroweak precision observables and flavor physics. In section IV we present exact one-loop expressions for electroweak
and flavor precision observables which are valid in general extensions of the SM. The main phenomenological implications of our model are discussed in section \text{V} where we describe two options for realistic composite Higgs models, a very simple one and a slightly more involved one which is closer to the realistic examples we know from extra dimensions. Finally, we conclude in section \text{VI}.

II. EFFECTIVE DESCRIPTION OF COMPOSITE HIGGS MODELS

The low energy effective description of a composite Higgs model with $SO(5)/SO(4)$ symmetry breaking pattern can be described by a scalar $\phi$ in the fundamental representation of $SO(5)$, subject to the constraint

$$\phi^2 = f^2, \quad (1)$$

where $f$ is the scale of the global symmetry breaking, assumed to be somewhat larger than the EWSB scale $v \approx 174$ GeV. The first four components of $\phi$, which transform as the fundamental representation of SO(4), are denoted by $\vec{\phi}$. The $SU(2)_L \times U(1)_Y$ subgroup of $SO(4) = SU(2)_L \times SU(2)_R$, where the hypercharge corresponds to the $T^{3R}$ generator, is weakly gauged.\(^1\) The vacuum expectation value of $\vec{\phi}$ breaks the EW symmetry,

$$m_W^2 = \frac{g^2 v^2}{2}, \quad v^2 = \frac{1}{2} \langle \vec{\phi}^2 \rangle. \quad (2)$$

From Eq. (1) we see that the ratio

$$s_\alpha \equiv \sin \alpha \equiv \sqrt{2} \frac{v}{f}, \quad (3)$$

measures the Higgs compositeness, \textit{i.e.} how the vev of $\phi$ is split between $\vec{\phi}$ and $\phi_5$. Canonical normalization of the different components in $\phi$, expanded around its vev, requires a rescaling of the physical Higgs

$$h \rightarrow \cos \alpha \ h \equiv c_\alpha \ h, \quad (4)$$

whereas the would be Goldstone bosons are not modified. This redefinition implies an important feature of Higgs compositeness, namely that Higgs couplings to gauge bosons are suppressed with respect to the couplings in the SM by the factor $c_\alpha = \sqrt{1 - 2v^2/f^2}$.

\(^1\) An extra $U(1)$ group is required to generate the correct Weinberg angle, but it is irrelevant for the present discussion and will be disregarded.
In the case of fermions, the suppression factor depends also on the embedding of the SM fermions in $SO(5)$ representations \cite{6, 15}. This suppression of the Higgs couplings affects the quantum corrections to electroweak precision observables, leading to some sensitivity to the ultraviolet cut-off \cite{12}. The leading effect can be taken into account by replacing the Higgs mass with an effective (heavier) Higgs in the SM expressions of the one-loop corrections to the electroweak precision observables,

$$m_{\text{EWPT,eff}} = m_h (\Lambda/m_h)^{s_\alpha^2},$$

where $\Lambda = 4\pi f/\sqrt{N_G} = 2\pi f$ is the ultraviolet cut-off of our effective theory, $N_G$ being the number of Goldstone bosons. This modification gives rise to an additional contribution to the Peskin-Takeuchi \cite{16} $S$ and $T$ parameters

$$\Delta S = \frac{1}{12\pi} \ln \left( \frac{m_{\text{EWPT,eff}}^2}{m_{h,\text{ref}}^2} \right), \quad \Delta T = -\frac{3}{16\pi c_W^2} \ln \left( \frac{m_{\text{EWPT,eff}}^2}{m_{h,\text{ref}}^2} \right),$$

where $m_{h,\text{ref}}$ is the reference Higgs mass used in the electroweak fit and $c_W$ is the cosine of the Weinberg angle.

Furthermore, custodial symmetry can naturally account for a suppressed contribution of ultraviolet physics to the $T$ parameter, but there is no reason not to expect a contribution to the $S$ parameter from higher dimensional operators. A reasonable estimate, assuming that new physics couples linearly to the SM (otherwise this estimate should have an extra loop suppression), is

$$\Delta S_{\lambda} \approx \frac{4s_W^2 g^2 v^2}{\alpha_{\text{em}}^2 \Lambda^2} \approx 0.16 \left( \frac{3 \text{ TeV}}{\Lambda} \right)^2.$$  \hfill (7)

The combination of the two corrections, \cite{6} and \cite{7}, results in a positive shift to the $S$ parameter and a negative shift to the $T$ parameter. We show in Fig. \ref{fig:constraints} the current constraints on the $S$ and $T$ parameters, assuming a Higgs mass $m_h = 120$ GeV. We also show the contributions from Higgs compositeness and UV physics for different values of $s_\alpha$. These constraints are obtained as follows. For reasons that will become apparent in the next section, we have performed a fit to all the relevant electroweak observables, allowing the $S$ and $T$ parameters and an anomalous coupling of the $b_L$ quark to the $Z$, that we denote by $\delta g_{b_L}$, to vary (note that we have set $U = 0$ in our fit, as it is expected to be vanishingly small in our model). \footnote{We use Ref. \cite{17} for the fit updated to the most recent experimental data \cite{18}. $\alpha_s(m_Z) = 0.1183$ and $\alpha_s(m_Z) = 0.1183$} In Fig. \ref{fig:constraints} we have fixed the optimal value of $\delta g_{b_L} = -2.5 \times 10^{-4},$
FIG. 1: 68%, 95% and 99% C.L. limits on $S$ and $T$ for a fit to electroweak observables with three independent parameters ($S$, $T$ and $\delta g_{bL}$), fixing the optimal value of $\delta g_{bL} = -2.5 \times 10^{-4}$ and a reference Higgs mass $m_{h,\text{ref}} = 120$ GeV. The dots at the start and the tip of the arrows show the effects due to Higgs compositeness, Eq. (6), and to UV effects on $S$, Eq. (7), respectively. The effect is shown (from top to bottom) for the values of $s_\alpha = 0.25, 0.33$ and 0.5.

although the result of a proper projection over the $S-T$ plane does not differ significantly. As we see in the figure, the above corrections put the model in gross contradiction with experimental observations for any sizable Higgs compositeness. As emphasized in [12], this is a quite generic implication of Higgs compositeness and custodial symmetry, that seems to disfavor composite Higgs models as a realistic description of electroweak symmetry breaking. However, we have not included yet in our discussion the effect of other composite states that might be lighter than the cut-off of our effective description. In the next section we discuss the effect that new fermionic states can have on electroweak precision tests.  

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$m_t = 172.4$ GeV have been fixed to the best fit values from a 5 parameter ($\alpha_s(m_Z)$, $m_t$, $S$, $T$ and $\delta g_{bL}$).

We would like to thank E. Pontón for help with the fit and with Fig. 1.

New bosonic resonances have been recently shown to be able to make even Higgs-less models compatible with experimental data [19].
III. THE FERMIONIC SECTOR

If the Higgs is a composite state of a new strongly interacting theory, it is natural to assume that the large top mass is due to partial top compositeness. The partners under $SO(5)$ of the composite states with which the top mixes can then play an important role in electroweak precision tests. In particular, if the top sector is partly composite, large corrections to the $Zb_L\bar{b}_L$ coupling are typically expected, unless some symmetry forbids them [9]. Fermions, in the fundamental or adjoint representation of the $SO(5)$ group, are natural building blocks that incorporate the left-right symmetry guaranteeing the absence of large tree-level corrections to the $Zb_L\bar{b}_L$ coupling. In this article we will consider the former possibility and include new (composite) vector-like fermions that transform in the five-dimensional vector representation of $SO(5)$, which decomposes under $SO(4) = SU(2)_L \times SU(2)_R$ as
\[
\Psi = (Q,X,T) \Rightarrow (5) = (2,2) \oplus (1).
\]

$Q$ and $X$ form a bidoublet under $SU(2)_L \times SU(2)_R$, they are $SU(2)_L$ doublets with hypercharges $1/6$ and $7/6$, respectively (and $T^R_3 = -1/2$ and $1/2$, respectively). $T$ is an $SU(2)_L \times SU(2)_R$ singlet with hypercharge $2/3$. The SM quarks $q_L$ and $t_R$ have the same quantum numbers under the SM gauge group as $Q$ and $T$, respectively.

The question is, given the large corrections to the $S$ and $T$ parameters from the Higgs sector, can the one-loop contribution of this new fermionic sector to $T$ be such that the model is compatible with experimental data? As shown in Fig. 1, a large positive contribution to $T$ is required, but this has to be obtained without spoiling the good agreement with other experimental observables. In particular, $\delta g_{b_L}$ is extremely constrained experimentally. The situation is exemplified in Fig. 2 in which we show the constraints on the contribution to the $T$ parameter and $\delta g_{b_L}$ from the new quarks, for a fixed $s_\alpha \approx 0.5$ ($f = 500$ GeV). In the figure, we have shown both the actual value of the allowed $T$ and $\delta g_{b_L}$ (bottom and left axes, respectively) together with the values relative to the one-loop SM corrections (top and right axes). We see that, for this value of $s_\alpha$, even at the 99% C.L. the $T$ parameter must be positive and it is allowed to vary at most by about 20% in units of the SM one-loop correction. We also observe that $\delta g_{b_L}$ is constrained in the range $-0.5 \lesssim \delta g_{b_L}/\delta g_{b_L}^{SM} \lesssim 0.4$.

The effects on the $T$ parameter and the $Zb_L\bar{b}_L$ coupling of new vector-like quarks with the above quantum numbers through their mixing with the top were discussed in detail in
FIG. 2: 95% and 99% C.L. limits on $T$ and $\delta g_{bL}$ for a fit to electroweak observables with three independent parameters ($S, T$ and $\delta g_{bL}$) assuming $s_\alpha \approx 0.5$ ($f = 500$ GeV) and a reference Higgs mass $m_{h,\text{ref}} = 120$ GeV (no projection). The top and right axes show the value of the $T$ parameter and $\delta g_{bL}$ in terms of the SM one-loop contribution. For this value of $s_\alpha$ there is no allowed region at 68% C.L..

Ref. [20]. The $SU(2)_L$ quantum numbers of the different multiplets determine, to a great degree, the sign of the contribution to the $T$ parameter. Through their mixing with the top, the different $SU(2)_L$ multiplets typically contribute (assuming only one type of mixing at a time) as follows:

- **Singlets** ($T$) contribute positively to the $T$ parameter and the $Zb_L\bar{b}_L$ coupling.

- **Hypercharge $7/6$ doublets** ($X$) contribute negatively to the $T$ parameter.

- **Hypercharge $1/6$ doublets** ($Q$) contribute positively to the $T$ parameter.

Furthermore, there is a strong correlation between the contribution of vector-like singlets to the $T$ parameter and the $Zb_L\bar{b}_L$ coupling (they are both positive and governed by the same parameters) which implies a large positive correction to the anomalous $Zb_L\bar{b}_L$ coupling (making it less compatible with experimental observations) in the case that the singlet contributes a large positive amount to $T$. Due to the particular chirality of the final state, doublets ($Q, X$) do not contribute significantly to the $Zb_L\bar{b}_L$ coupling (they can contribute
to the $Zb_R\bar{b}_R$ coupling but that is a more model dependent issue that is not correlated with the contribution to the $T$ parameter).

Given the constraints shown in Fig. 2, it is clear that a large contribution from the singlet is bound to give problems, as it will also give a large contribution to the $Zb_L\bar{b}_L$ coupling, which is extremely constrained experimentally. Ideally, one would want a large contribution from $Q$, but this is also difficult, due to the constraints imposed by the global $SO(5)$ symmetry which usually mean a larger (negative) contribution from $X$. The general situation is more complicated, and several modes can give large contributions to make the model compatible with EWPT. The difficulties we have outlined, however, mean that the model can be quite predictive, and only a few patterns can be realized. Our goal is to describe these patterns and their implications at the LHC.

We restrict our discussion to the top sector $^4q_L,t_R$, which couples to a set of new vector-like quarks, transforming as the $(5)$ representation of $SO(5)$, $\Psi_{L,R} = (Q,X,T)_{L,R}$ as

$$- \mathcal{L}_{\text{int}} = m_i^L \bar{q}_L Q^i_R + m_i^R \bar{T}^i_L t_R + \text{h.c..} \tag{9}$$

The new sector is $SO(5)$ invariant, with a mass Lagrangian given by

$$- \mathcal{L}_{SO(5)} = m^i_i \bar{\Psi}^i \Psi^i + \frac{\mu_{ij}}{f} (\bar{\Psi}^i \phi)(\phi^T \Psi^j), \tag{10}$$

where the indices $i,j$ allow for the possibility of more than one set of fermionic composites and the brackets in the second term indicate the contraction of the $SO(5)$ indices. $\mu_{ij}$ is a hermitian matrix. $\phi$ is our scalar 5-plet and the terms involving it can be written, in an $SU(2)_L \times U(1)_Y$ invariant way, as

$$\bar{\Psi} \phi = \bar{Q} \bar{\varphi} + \bar{X} \varphi + \bar{T} \phi_5, \tag{11}$$

where $\varphi$ and $\bar{\varphi} = i\sigma^2 \varphi^*$ are the SM Higgs doublets with hypercharge $1/2$ and $-1/2$, respectively. Note that these have to incorporate the rescaling of the physical Higgs. For instance we have

$$\varphi = \begin{pmatrix} \varphi^+ \\ v + c_\alpha \frac{\varphi_0 + \varphi_0^*}{2} + \frac{\varphi_0 - \varphi_0^*}{2} \end{pmatrix}, \tag{12}$$

$^4$ For simplicity we will assume the bottom mass to come from a direct Yukawa coupling, $\bar{q}_L \phi b_R$. This small explicit breaking of the $SO(5)/SO(4)$ pattern will not have appreciable effects.
with \( \langle \varphi_0 \rangle = \langle \varphi^+ \rangle = 0 \). We also have \( \phi_5 = f c_\alpha \frac{s^2}{f} (\varphi_0 + \varphi^*_0) + \ldots \), where the dots denote terms with two or more scalars. From these expressions and the Lagrangians in Eqs. (9) and (10) we can compute the mass matrix and the Yukawa couplings for the quarks in the model (including the couplings to the would be Goldstone bosons that will be required for the calculation of the anomalous \( Z b_L \bar{b}_L \) coupling). The mass terms can be written in matrix form as

\[
- \mathcal{L} = \begin{pmatrix} t_L \\ Q^u_L \\ X^u_L \\ T_L \end{pmatrix} \begin{pmatrix} 0 & m^T_L & 0 & 0 \\ 0 & m_\Psi + \frac{s^2}{2} f_\mu & \frac{s^2}{2} f_\mu & c_\alpha v_\mu \\ 0 & \frac{s^2}{2} f_\mu & m_\Psi + \frac{s^2}{2} f_\mu & c_\alpha v_\mu \\ m_R & c_\alpha v_\mu & c_\alpha v_\mu & m_\Psi + \frac{c^2}{2} f_\mu \end{pmatrix} \begin{pmatrix} t_R \\ Q^u_R \\ X^u_R \\ T_R \end{pmatrix} + \text{h.c.,} 
\]  

where we have implicitly written the mass matrix in block form and \( Q^u_{L,R}, X^u_{L,R} \) are the charge 2/3 components of \( Q \) and \( X \), respectively. The interaction Lagrangian, Eq. (9), gives a mixing between the fundamental fields \( q_L, t_R \) and the composite states \( Q \) and \( T \), which makes the Lagrangian non-diagonal before EWSB. If we diagonalize the mass matrix before EWSB (with \( v = 0 \)), we will end up with a massless \( SU(2)_L \) doublet and a massless singlet that are now an admixture of fundamental and composite states. These new massless states have Yukawa couplings, thanks to their composite components (since, assuming that the only explicit breaking of \( SO(5) \) is through \( m^L_{L,R} \), the Higgs only couples to composites).

In order to better understand this mechanism, we consider the case that there is only one set of composite fermionic states below the cut-off of our theory. We can then diagonalize the mass matrix, for \( v = 0 \), by means of the following rotations

\[
q_L \rightarrow \cos \theta_L q_L + \sin \theta_L Q_L, \quad Q_L \rightarrow -\sin \theta_L q_L + \cos \theta_L Q_L, 
\]  

and

\[
t_R \rightarrow \cos \theta_R t_R + \sin \theta_R T_R, \quad T_R \rightarrow -\sin \theta_R t_R + \cos \theta_R T_R. 
\]  

Note that in the case of \( q_L \) and \( Q_L \) we are rotating entire doublets; these have the same quantum numbers and therefore no traceable physical footprint of the rotation is left. The mixing angles determining the degree of compositeness of \( q_L \) and \( t_R \) are, respectively, \( \tan \theta_L = \frac{m_L}{m_\Psi} \) and \( \tan \theta_R = \frac{m_T}{m_\Psi} \), where we have defined \( m_T \equiv m_\Psi + f_\mu \) (note that now \( \mu \) is not a matrix but a number and that for \( v = 0 \) we have \( c_\alpha = 1 \)).
With these field rotations, the mass Lagrangian for the charge 2/3 quarks reads

$$-\mathcal{L}_m = \begin{pmatrix} t_L \\ Q_L^c \\ X_L^c \\ T_L \end{pmatrix} \begin{pmatrix} s_L s_R c_\alpha v_\mu & -s_L \frac{3}{2} f_\mu & -s_L \frac{3}{2} f_\mu & -s_L c_R c_\alpha v_\mu \\ -s_R c_L c_\alpha v_\mu & \frac{m_\tau}{c_L} + c_L \frac{3}{2} f_\mu & c_L \frac{3}{2} f_\mu & c_L c_R c_\alpha v_\mu \\ -s_R c_\alpha v_\mu & \frac{3}{2} f_\mu & m_\nu + \frac{3}{2} f_\mu & c_R c_\alpha v_\mu \\ s_R s_\alpha^2 f_\mu & c_\alpha v_\mu & c_\alpha v_\mu & \frac{m_\tau}{c_R} - c_R s_\alpha^2 f_\mu \end{pmatrix} \begin{pmatrix} t_R \\ Q_R^c \\ X_R^c \\ T_R \end{pmatrix} + \text{h.c.}$$

where we have denoted $s_{L,R} \equiv \sin \theta_{L,R}$, $c_{L,R} \equiv \cos \theta_{L,R}$.

We already see in this mass Lagrangian some of the constraints imposed by the global symmetry. First, the top quark acquires mass through its mixing with the composite states. In order to have a large enough top mass, $t_L$ and $t_R$ cannot be simultaneously mostly fundamental (small $s_L$ and $s_R$). Second, the mixing of the hypercharge 1/6 doublet with the top (one possible source of positive contribution to the $T$ parameter) is always smaller, by a factor $c_L$, than the mixing of the hypercharge 7/6 multiplet. If the two are degenerate or $X$ is lighter than $Q$ (as happens in minimal five-dimensional models), then the system $Q, X$ usually contributes negatively to the $T$ parameter. This effect together with the additional negative contribution from the fact that the Higgs boson is composite lead to the problem discussed previously. Either the fermion contribution to the $T$ parameter is not large enough and therefore incompatible with electroweak precision data (given the positive contribution to the $S$ parameter from UV physics) or large corrections to flavor preserving and violating $b$ couplings are introduced, again in conflict with experimental data.

A precise assessment of the model viability requires a study after a complete diagonalization of the matrix in Eq. (16), since several modes can simultaneously give relevant contributions which are difficult to disentangle qualitatively. A detailed analysis of electroweak constraints in our model is therefore required to see if there are regions of parameter space compatible with current data. This detailed analysis includes a precise calculation of the main electroweak observables, which in our case can be encoded in the $T$ parameter and $\delta g_{bl}$, including formally subleading contributions not proportional to large Yukawa couplings and a careful scan over parameter space. This is the subject of the next two sections.
IV. EVALUATION OF PRECISION ELECTROWEAK OBSERVABLES

In this section, we compute the one-loop contribution of the new fermionic sector to the most relevant electroweak observables, which receive large corrections due to the large value of the top mass. The most important observables are the $T$ parameter and the anomalous $Zb_L\bar{b}_L$ coupling. Other observables, $B_{d,s} - \bar{B}_{d,s}$ mixing, $B_{d,s} \rightarrow \mu^+\mu^-$, and $b \rightarrow s\gamma$, may also receive large one-loop corrections which are however less generic, depending for example on the details of how the bottom quark gets a mass. These observables provide typically weaker constraints than the $T$ parameter and the anomalous $Zb_L\bar{b}_L$ coupling (see for instance [21] for a discussion in the context of MFV scenarios). Nevertheless, we have explicitly checked that the constraints from $B - \bar{B}$ mixing and $b \rightarrow s\gamma$ are indeed typically weaker in our model.

Given the stringent constraints on the new fermionic contributions to the $T$ parameter and $\delta g_{bL}$ we have found it important to calculate these observables as precisely as possible. In our results we do not discard any one-loop diagram. For $\delta g_{bL}$, in particular, we compute the full dependence of the corresponding amplitude on all mass parameters (including the $Z$, $W$ and Goldstone boson masses) except for the bottom or lighter quark masses. Our calculation of $\delta g_{bL}$ is general and the result can be readily used in other models.

We perform all our calculations in the 't Hooft-Feynman gauge, in which the Goldstone bosons and the corresponding gauge bosons have the same mass. We consider an arbitrary number of new quarks $\psi^i_Q$, with electric charge $Q$ ($Q = -1/3, 2/3$ or $5/3$ in our model) and mass $m^i_Q$. We parametrize their couplings to the $Z$ and $W$ bosons in the mass eigenstate basis as (an implicit sum over the particle charges $Q$ is always understood)

\begin{align}
\mathcal{L}^Z &= \frac{g}{2c_W} \bar{\psi}_Q^i \gamma^\mu [X^Q_{ij} P_L + X^Q_{ij} P_R - 2s^2_W Q \delta_{ij}] \psi^j Q Z^\mu, \\
\mathcal{L}^W &= \frac{g}{\sqrt{2}} \bar{\psi}_Q^i \gamma^\mu [V^Q_{ij} P_L + V^Q_{ij} P_R] \psi^j_{(Q-1)} W_\mu^+ + \text{h.c.},
\end{align}

where $V^Q = 0$ for the minimum $Q$ in the model and $P_{L,R} = (1 \mp \gamma_5)/2$ are chirality projectors.

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5 The one-loop contribution from the fermionic sector to $S$, which was computed in [20], is negligible compared with the tree level contribution from UV physics.

6 Our calculations are done assuming point particles. The observables computed here are however finite and dominated by scales of the order of the masses of the particles involved. In the composite Higgs models we are interested in, the relevant masses are much smaller than the strong coupling scale and therefore our approximation is valid.
In the SM, \(X_{ij}^L = -X_{ij}^{-L} = \delta_{ij}\), \(V_{ij}^L = V_{ij}^{CKM}\) and all the other couplings vanish at tree level. The couplings to the Goldstone bosons are denoted by

\[
L^{G^0} = \frac{g}{2c_W} \bar{\psi}_Q^i \left[ Y_{ij}^{QL} P_L + Y_{ij}^{QR} P_R \right] \psi_Q^j G^0, \tag{19}
\]

\[
L^{G^\pm} = \frac{g}{\sqrt{2}} \bar{\psi}_Q^i \left[ W_{ij}^{QL} P_L + W_{ij}^{QR} P_R \right] \psi_{(Q-1)}^j G^+ + \text{h.c.}, \tag{20}
\]

Note that we have extracted a factor of \(g/2c_W\) and \(g/\sqrt{2}\) in the couplings of the Goldstone bosons to simplify the equations of the observables. Finally, the trilinear gauge boson and the gauge-Goldstone boson interactions are those of the SM [22],

\[
L^{\text{int}}_{g-g-g} = -g \left\{ c_W \left[ g^{\mu\nu}(k_1 - k_2)^\rho + g^{\rho\nu}(k_2 - k_3)^\mu + g^{\rho\mu}(k_3 - k_1)^\nu \right] Z_\mu(k_1) W^+_{\nu}(k_2) W^-_{\rho}(k_3) \right. \\
+ \frac{1 - 2s_W^2}{2c_W} \left( k^- - k^+ \right)^\mu Z_\mu G^+(k^+)G^-(k^-) \\
+ \left. \left( m_W s_W^2 c_W g^{\mu\nu} Z_\mu G^+ W^-_{\nu} + \text{h.c.} \right) \right\}, \tag{21}
\]

where all momenta are taken to flow into the vertices.

A. Result for the \(T\) parameter

The \(T\) parameter measures the amount of custodial symmetry breaking and can be defined in terms of the vacuum polarization amplitudes of the \(SU(2)_L W^i\) gauge bosons as [16]

\[
T = \frac{4\pi}{m_Z^2 s_W^2 c_W^2} \left[ \Pi_{+-}(0) - \Pi_{33}(0) \right], \tag{22}
\]

where \(\Pi_{ij}(0)\) denotes the transverse part of the vacuum polarization amplitude evaluated at zero momentum,

\[
ig^{\mu\nu} \Pi_{ij}(p^2) + (p^\mu p^\nu \text{ terms}) \equiv \int \! d^4x \, e^{-ipx} \langle J_\mu^i(x) J_\nu^j(0) \rangle. \tag{23}
\]

The \(T\) parameter was computed in [23] for an arbitrary number of vector-like singlets and doublets. We have extended their calculation to arbitrary couplings. The final result, which we reproduce here for completeness, is almost unchanged,

\[
\Delta T = \frac{N_c}{16\pi s_W^2 c_W^2} \left\{ \sum_{\alpha,\beta} \left[ \left| V^{QL}_{\alpha\beta} \right|^2 + \left| V^{QR}_{\alpha\beta} \right|^2 \right] \theta_+ (y_\alpha, y_\beta) + 2 \text{Re} \left( V^{QL}_{\alpha\beta} V^{QR*}_{\alpha\beta} \right) \theta_-(y_\alpha, y_\beta) \right. \\
\left. - \frac{1}{2} \sum_{\alpha,\beta} \left[ \left| X^{QL}_{\alpha\beta} \right|^2 + \left| X^{QR}_{\alpha\beta} \right|^2 \right] \theta_+ (y_\alpha, y_\beta) + 2 \text{Re} \left( X^{QL}_{\alpha\beta} X^{QR*}_{\alpha\beta} \right) \theta_-(y_\alpha, y_\beta) \right\}. \tag{24}
\]
Here again the sum over $Q$ is left implicit; we have defined $y \equiv m^2/m_Z^2$, with $m$ the mass of the corresponding quark, and the functions $\theta_\pm$ read

$$\theta_+(y_1, y_2) \equiv y_1 + y_2 - \frac{2y_1y_2}{y_1 - y_2} \log \frac{y_1}{y_2} - 2(y_1 \log y_1 + y_2 \log y_2) + \frac{y_1 + y_2}{2} \Delta, \quad (25)$$

$$\theta_-(y_1, y_2) \equiv 2\sqrt{y_1y_2} \left( \frac{y_1 + y_2}{y_1 - y_2} \log \frac{y_1}{y_2} - 2 + \log(y_1y_2) - \frac{\Delta}{2} \right), \quad (26)$$

where $\Delta$ is a divergent term that comes from dimensional regularization. We have left it explicit so that it is possible to check the cancellation of poles in the $T$ parameter. Taking the SM limit (only $t$ and $b$ quarks) we obtain

$$\Delta T_{SM} = \frac{N_c}{16\pi^2 s_W^2 c_W^2} \left\{ \theta_+(y_t, y_b) - \frac{1}{2} (\theta_+(y_t, y_t) + \theta_+(y_b, y_b)) \right\}$$

$$= \frac{N_c}{16\pi^2 s_W^2 c_W^2 m_Z^2} \left[ m_t^2 + m_b^2 - 2 \frac{m_t^2 m_b^2}{m_t^2 - m_b^2} \log \frac{m_t^2}{m_b^2} \right] \approx 1.19. \quad (27)$$

### B. Anomalous $Zb_L\bar{b}_L$ coupling at one loop

The amplitude for the decay of a $Z$ boson to massless left-handed b-quarks,

$$Z(q) \to \bar{b}_L(p_2)b_L(p_1), \quad (28)$$

can be written

$$\mathcal{M}_{Z\to \bar{b}_Lb_L} = -\frac{e g_L}{s_W c_W} \bar{b}(p_2) \gamma_5(q) \frac{1 - \gamma_5}{2} b(p_1), \quad (29)$$

where the $g_L$ coupling can be modified from its value in the SM

$$g_L = g_L^{SM} + \delta g_L \quad (30)$$

due to new heavy quarks with $W$ and $Z$ boson interactions described by the Lagrangian of Eqs. \[17\]-\[20\]. There are severe constraints on the value of $g_L$ from the LEP experiments. In our model, a symmetry protection of this coupling has been implemented to forbid tree level corrections. However, modifications can occur via radiative corrections. Taking into account one-loop effects we have

$$\delta g_L = \frac{\alpha}{2\pi} \left( F_{heavy}^{top} - F_{SM}^{top} \right), \quad (31)$$

where we denote with $F_{heavy}^{top}$ one-loop contributions from all the heavy quarks in the general BSM model, including the top quark with couplings as in the BSM model. The prediction
FIG. 3: Diagrams with heavy quarks contributing to the $Zb\bar{b}$ amplitude.

for $g^\text{SM}_L$ includes already contributions from the top quark with SM couplings, $F^\text{top}_\text{SM}$; these have been removed explicitly in the above equation.

Summing over all the diagrams in Fig. 3 and carrying out the renormalization procedure, we obtain

\footnote{In order to make our result more compact, here we already substituted for the trilinear gauge boson and for the gauge-Goldstone boson couplings. Therefore, when applying this formula one should be careful in writing all the fermion-(gauge/Goldstone) boson couplings appearing in it consistently with the conventions of Eq. (21).}
\[
F^{\text{heavy}} = -\frac{1}{8s_W^2} \sum_i \left\{ \sum_j \left[ V_{i,j}^{QL}V_{i,b}^{QL*} \left( 2\tilde{X}_{ij}^{QR}E_1^{ij} + \tilde{X}_{ij}^{QL}E_2^{ij} \right) \\
+ W_{i,j}^{QL}W_{i,b}^{QL*} \left( \tilde{X}_{ij}^{QL}E_3^{ij} + \tilde{X}_{ij}^{QR}E_4^{ij} \right) \right] \\
+ \tilde{X}_{bb}^{L} |V_{i,b}^{QL}|^2(2E_5^i - 1) + |W_{i,b}^{QL}|^2E_6^i \\
+ (2s_W^2 - 1)|W_{i,b}^{QL}|^2E_5^i - 2c_W^2|V_{i,b}^{QL}|^2E_6^i + 4s_W^2 \text{Re}(V_{i,b}^{QL*}W_{i,b}^{QL})E_7^i \right\}.
\]

(32)

We have defined

\[
\tilde{X}_{ij}^{Q(L,R)} = X_{ij}^{Q(L,R)} - 2s_W^2 Q\delta_{ij},
\]

and

\[
E_1^{ij} = \sqrt{y_iy_j}I_3(y_i, y_W, y_j),
\]

(34)

\[
E_2^{ij} = \Delta - 2 + y_i + y_j - 2y_W - I_2(y_i, y_j)(y_i + y_j - 2y_W - 3) \\
+ 2I_3(y_i, y_W, y_j)(y_i - y_W - 1)(y_j - y_W - 1) + \log(y_i) \left( \frac{2y_i}{y_i - y_W} - y_i \right) \\
+ \log(y_j) \left( \frac{2y_j}{y_j - y_W} - y_j \right) + 2y_W \log(y_W) \left( 1 - \frac{y_i + y_j - 2y_W}{(y_i - y_W)(y_j - y_W)} \right),
\]

(35)

\[
E_3^{ij} = \frac{1}{2} \left[ \Delta + 1 + y_i + y_j - 2y_W - I_2(y_i, y_j)(y_i + y_j - 2y_W + 1) \\
+ 2I_3(y_i, y_W, y_j)(y_i - y_W)(y_j - y_W) - y_i \log(y_i) - y_j \log(y_j) \\
+ 2y_W \log(y_W) \right],
\]

(36)

\[
E_4^i = \frac{1}{2} \left[ -\Delta + 1 + \frac{y_i}{y_i - y_W} - \log(y_i) \frac{y_i^2}{(y_i - y_W)^2} + y_W \log(y_W) \frac{2y_i - y_W}{(y_i - y_W)^2} \right],
\]

(37)

\[
E_5^i = \frac{\Delta}{2} - 1 + y_i - y_W - I_2(y_W, y_W) \left( y_i - y_W + \frac{1}{2} \right) \\
- I_3(y_W, y_i, y_W) \left( (y_i - y_W)^2 + y_i \right) - y_i \log(y_i) + y_W \log(y_W),
\]

(38)

\[
E_6^i = 3\Delta - 4 + 2(y_i - y_W) - I_2(y_W, y_W)(2y_i - 2y_W - 1) \\
- 2I_3(y_W, y_i, y_W)((y_i - y_W)^2 + 2y_W) \\
+ 2\log(y_i) \left( \frac{2y_i}{y_i - y_W} - y_i \right) + 2\log(y_W) \left( -\frac{2y_W}{y_i - y_W} + y_W \right),
\]

(39)

and

\[
E_7^i = \sqrt{y_Wy_i}I_3(y_W, y_i, y_W).
\]

(40)
The finite parts of the two-point and three-point master integrals can be easily evaluated numerically from their integral representations:

\[ \mathcal{I}_2(y_1, y_2) = -\int_0^1 dx \log[xy_1 + (1 - x)y_2 - x(1 - x)] , \]  
\[ \mathcal{I}_3(y_1, y_2, y_3) = -\int_0^1 dx \frac{1}{x + y_2 - y_3} \log \left[ \frac{xy_1 + (1 - x)y_2}{xy_1 + (1 - x)y_3 - x(1 - x)} \right] . \]  

The calculation of the anomalous \( Zb_L\bar{b}_L \) coupling required tensor one-loop Feynman integrals with up to three propagators with different masses, and the external \( Z \) boson invariant mass, which we computed using the scalar form factors decomposition in [24, 25] and the program AIR [26]. We note that our exact expressions above agree with the limits for the \( Zb_L\bar{b}_L \) amplitude that have been presented in Ref. [27].

The result for \( F_{\text{SM}}^{\text{top}} \) can be obtained from the above results in the special case of \( i = j = t \) by substituting appropriately the SM couplings of the top quark with gauge and Goldstone bosons.

In the model presented here the only relevant contribution to the anomalous \( Zb_L\bar{b}_L \) coupling is given from charge 2/3 quarks in the loop. Yet, the result we give can be extended to models with a different quark content. For example, if heavy charge \(-1/3\) quarks were relevant, their contribution to the anomalous \( Zb_L\bar{b}_L \) coupling can be obtained from the first three lines of Eq. (32) with the substitutions

\[ y \rightarrow y(m_W^2/m_Z^2) , \quad V_{ij}^{Q(L,R)} \rightarrow \frac{1}{\sqrt{2}c_W} \tilde{X}_{ij}^{Q(L,R)} \quad \text{and} \quad W_{ij}^{Q(L,R)} \rightarrow \frac{1}{\sqrt{2}c_W} Y_{ij}^{Q(L,R)} . \]  

\[ (43) \]

\[ (44) \]

C. Result for \( B_q - \bar{B}_q \)

\( \Delta B = 2 \) processes can be conveniently parametrized in terms of the following dimension 6 effective Lagrangian,

\[ \mathcal{L}^{\Delta B=2} = \sum_{i=1}^5 C_i Q_i^{bq} + \sum_{i=1}^3 \tilde{C}_i \tilde{Q}_i^{bq} , \]  

\[ (44) \]
where we have used standard notation

\[ Q_1^{bq} = q_L^\alpha \gamma_\mu b_R^\alpha q_L^\beta b_L^\beta, \quad Q_1^{bq} = q_R^\alpha \gamma_\mu b_R^\alpha d_R^\beta b_R^\beta, \]
\[ Q_2^{bq} = q_R^\alpha b_L^\beta q_L^\mu b_L^\beta, \quad \tilde{Q}_2^{bq} = q_L^\alpha b_R^\beta q_L^\beta b_R^\alpha, \]
\[ Q_3^{bq} = q_R^\beta b_L^\alpha q_L^\mu b_L^\alpha, \quad \tilde{Q}_3^{bq} = q_L^\beta b_R^\alpha q_L^\alpha b_R^\alpha, \]
\[ Q_4^{bq} = q_R^\alpha b_L^\beta q_L^\beta b_R^\alpha, \]
\[ Q_5^{bq} = q_R^\beta b_L^\alpha q_L^\beta b_R^\alpha. \]

We have computed the Wilson coefficients for the different operators \(C_i, \tilde{C}_i\), due to the exchange, in box diagrams, of charge \(2/3\) quarks with arbitrary couplings as parametrized in Eqs. (18) and (20). The procedure requires Fierz rearrangement but is otherwise standard.

We define the mass ratios

\[ x_i = \frac{m_i^2}{m_W^2}. \]

The final result reads:

\[ C_1 = \frac{G_F^2 m_W^2}{8\pi^2} \sum_{i,j} g_1(x_i, x_j) \left( \frac{1}{2} W_{is}^L W_{ib}^L W_{js}^L W_{jb}^L + 2 V_{is}^L V_{ib}^L V_{js}^L V_{jb}^L \right) \]
\[ - 4 \sqrt{x_i x_j} g_0(x_i, x_j) W_{is}^L V_{ib}^L V_{js}^L W_{jb}^L, \]
\[ C_2 = \frac{G_F^2 m_W^2}{8\pi^2} \sum_{i,j} 2 \sqrt{x_i x_j} g_0(x_i, x_j) W_{is}^R W_{ib}^L W_{js}^L W_{jb}^L, \]
\[ C_3 = \frac{G_F^2 m_W^2}{8\pi^2} \sum_{i,j} \left[ 8 g_1(x_i, x_j) W_{is}^R V_{ib}^L V_{js}^R W_{jb}^L - 16 \sqrt{x_i x_j} g_0(x_i, x_j) V_{is}^R V_{ib}^L V_{js}^R V_{jb}^L \right], \]
\[ C_4 = \frac{G_F^2 m_W^2}{8\pi^2} \sum_{i,j} \left[ 4 \sqrt{x_i x_j} g_0(x_i, x_j) (W_{is}^R W_{ib}^L W_{js}^L W_{jb}^R + 4 V_{is}^R V_{ib}^L V_{js}^L V_{jb}^R) \right. \]
\[ - 4 g_1(x_i, x_j) (W_{is}^L V_{ib}^L V_{js}^R W_{jb}^R + W_{is}^L V_{ib}^R V_{js}^L W_{jb}^R), \]
\[ C_5 = \frac{G_F^2 m_W^2}{8\pi^2} \sum_{i,j} \left[ g_1(x_i, x_j) \left( -2 W_{is}^L W_{ib}^L W_{js}^R W_{jb}^R - 32 V_{is}^L V_{ib}^L V_{js}^R V_{jb}^R \right) \right. \]
\[ + 8 \sqrt{x_i x_j} g_0(x_i, x_j) (W_{is}^L V_{ib}^L V_{js}^R W_{jb}^R + W_{is}^R V_{ib}^L V_{js}^L W_{jb}^L) \right]. \]

Also \( \tilde{C}_i = C_i(L \leftrightarrow R) \).

In the above equations the functions \( g_0(x, y) \) and \( g_1(x, y) \) are

\[ g_0(x, y) = - \frac{J_0(x) - J_0(y)}{x - y}, \]
\[ g_1(x, y) = - \frac{J_1(x) - J_1(y)}{x - y}, \]
with
\[ J_0(x) = \frac{x \log(x)}{(1-x)^2} + \frac{1}{1-x}, \]
\[ J_1(x) = \frac{x^2 \log(x)}{(1-x)^2} + \frac{1}{1-x}. \]  
\( (57) \)

D. Results for \( b \to s\gamma \)

\( B \to X_s\gamma \) is an interesting observable, as it can be very sensitive to new physics. It probes different combinations of top couplings than the other observables that we have considered so far, and in principle it restricts further the allowed parameter space in our model. However, because of the same reason, deviations in \( b \to s\gamma \) are not necessarily correlated to those of \( T \) and \( \delta g_{bl} \). This means that while arbitrary points in parameter space can be constrained by \( b \to s\gamma \), small modifications of other sectors in the model outside the top (like for instance details of how the \( b \) quark gets a mass) can easily render this observable compatible with experimental measurements without modifying the values of \( T \) or \( \delta g_{bl} \).

We use the results of Ref. [28], in which the relevant matching conditions are computed including the NLO QCD corrections in an arbitrary extension of the SM (however, we use only LO QCD correction in our estimation). The relevant operators are
\[ Q_7 = \frac{e m_b}{16\pi^2} (\bar{s}_L \sigma^{\mu\nu} b_R) F_{\mu\nu}, \quad Q_8 = \frac{g_s m_b}{16\pi^2} (\bar{s}_L \sigma^{\mu\nu} T^a b_R) G_{\mu\nu}^a, \]
\( (58) \)
where \( e \) and \( g_s \) are the electromagnetic and strong coupling constants, respectively, \( F_{\mu\nu} \) and \( G_{\mu\nu}^a \) the electromagnetic and gluonic field strength tensors and \( T^a \) are the color matrices normalized to \( \text{Tr} T^a T^b = \delta_{ab}/2 \). Splitting the corresponding Wilson coefficients into a SM part and a new physics part, at the matching scale,
\[ C_{7,8} = C_{7,8}^{\text{SM}} + \Delta C_{7,8}, \]
\( (59) \)
we can express the constraint on the new physics contribution as:
\[ B(\bar{B} \to X_s\gamma) = \left[ 3.15 \pm 0.23 - 8.03 \Delta C_7 - 1.92 \Delta C_8 \\
+ 4.96(\Delta C_7)^2 + 0.36(\Delta C_8)^2 + 2.33 \Delta C_7 \Delta C_8 \right] \times 10^{-4}, \]
\( (60) \)
where the SM contribution includes NNLO results [29]. This result is to be compared with the experimental average [30]
\[ B(B \to X_s\gamma)_{\text{exp}} = (3.52 \pm 0.23 \pm 0.09) \times 10^{-4}. \]
\( (61) \)
V. CONSTRAINTS ON THE FERMIONIC SECTOR AND COLLIDER IMPLICATIONS

In this section we analyze the main electroweak and flavor experimental constraints on our class of composite Higgs models. The fact that we require a very constrained but non-negligible contribution from the fermionic sector to the $T$-parameter while the other observables must not be disturbed significantly, renders the $\chi^2$ quite sensitive to the model parameters. This sensitivity could be reduced by choosing large values of $f$, since in the infinite $f$ limit we recover the SM. However, this possibility is not attractive because large values of $f$ do not help with addressing the usual SM fine-tuning problem. For small values, EWSB is less fine-tuned but a larger contribution to $T$ from the fermionic sector is required to make the model compatible with precision data. From now on we will consider as our benchmark scenario $f = 500$ GeV. This corresponds to the lowest point in Fig. 1 and the model is subject to non-trivial constraints (see Fig. 2) while at the same time its naive fine-tuning measure is better than $\sim 10\%$.

From Figs. 1 and 2 we see that the region of parameter space allowed by precision observables is presumably not only very sensitive to the input parameters but also small.\footnote{Whether this constitutes further fine-tuning is a debatable issue. The fermionic sector will also contribute, in a full composite Higgs model, to the Higgs potential and a sensible measure of fine-tuning might be the overlap between the regions with good EWSB and good compatibility with precision data. Ref. \cite{7} presented a 5D composite Higgs model in which the two regions overlap nicely.}

We have found that in such a situation, computing the electroweak observables exactly or estimating them in the large Yukawa approximation can lead to important phenomenological differences. In order to estimate this effect and to ensure probing all relevant regions in parameter space, we have performed several scans, based on adaptive Monte-Carlo methods \cite{31}, which we require to search for phase-space regions with a small value of $\chi^2$. We have implemented two types of scans: in the first type the $\chi^2$ is obtained using the complete one-loop calculation of the electroweak observables and in the second type the $\chi^2$ is obtained from the calculation of $\delta g_{bL}$ in the large Yukawa approximation.

In our scans we have restricted $|\mu_{ij}| \lesssim 4\pi$. We have also checked that the mass parameters $m_{iL,R}^j$ and $m_\psi^i$ are typically below $\Lambda$ in the case of two multiplets (but either $m_\psi$ or $m_L$ are close or above $\Lambda$ in most of the parameter space for one multiplet) and the masses of the
fermions that affect the relevant observables are also typically below the cut-off of our effective theory.

A. One multiplet

We now discuss the constraints imposed by electroweak and flavor precision data in the case that there is only one fermionic (5) of $SO(5)$ below the cut-off of our effective theory. This case has been previously considered in the literature, in the same or similar models (see [13, 14, 32]). The new result of our paper is an exact treatment at one loop of all the important precision observables; we also elaborate further on the implications of this minimal model for collider phenomenology (Ref. [32] also emphasized the collider implications of a composite top). We view the case of one multiplet in this section as a preparation for the more interesting case of two multiplets in the following section, where we will discuss in detail the role of a non-minimal sector of fermionic composites below the cut-off.

We first show that it is often important to use an exact one-loop calculation for $\delta g_{b_L}$. We start by performing a scan of the parameter space using the large Yukawa approximation. As expected for the case of one multiplet, the allowed parameter space is quite small. For $f = 500$ GeV, using the large Yukawa coupling approximation, we find two generic regions in parameter space compatible with electroweak precision data at the 99% C.L.: one for $0.1 \lesssim s_L \lesssim 0.2$ and one for $s_L \sim 1$. These regions are shown with empty (green) squares in the plots of Fig. 4. The region $0.1 \lesssim s_L \lesssim 0.2$ (left plot in Fig. 4) is, however, a misleading artifact of the large Yukawa coupling approximation. When we repeat the computation of $\delta g_{b_L}$ exactly at one loop, corresponding to the full (red) squares in Fig. 4, this region is excluded at the 99% C.L. (the region survives only at the 99.9% C.L.). On the other hand, the large $s_L$ region (right plot in Fig. 4) survives when the full one-loop calculation is used, but it turns out to be significantly smaller than what the analysis using the large Yukawa coupling approximation indicates.

In Fig. 5 we show the fermionic spectrum below the cut-off of the theory $\Lambda \sim 3$ TeV, in the region of parameter space which is compatible with electroweak precision data. Above the top, there is always a very light ($m_{\frac{5}{3}} \lesssim 500$ GeV) charge $\frac{5}{3}$ quark, then a charge $\frac{2}{3}$ quark very close in mass to it ($0 \leq m_{\frac{2}{3}}^{(1)} - m_{\frac{5}{3}} \lesssim 100$ GeV for $m_{\frac{5}{3}} \gtrsim 300$ GeV) and finally a heavier charge $\frac{2}{3}$ quark ($800$ GeV $\lesssim m_{\frac{2}{3}}^{(2)} \lesssim 2$ TeV). The other two quarks, with electric
FIG. 4: Contribution to $\delta g_{bL}$ as a function of $s_L$. The other input parameters are left free (keeping the total $\chi^2$ using the estimation for $\delta g_{bL}$ within 99% C.L.). The full red (empty green) squares correspond to the full contribution (large Yukawa estimation) of $\delta g_{bL}$. The horizontal lines correspond to the maximum allowed contribution to $\delta g_{bL}$ (assuming an optimal contribution to $T$).

FIG. 5: Spectrum of light (below $\Lambda$) fermionic states (including the top quark) for the region of parameter space compatible with EWPT. The (green) crosses, (blue) dots and (red) empty squares correspond to charge $5/3$, $2/3$ and $-1/3$ quarks, respectively. The latter, together with one charge $2/3$ quark, are typically above $\Lambda$.

charges $2/3$ and $-1/3$ respectively, are quite degenerate and typically heavier than $\Lambda$.

All these fields mix very strongly with the top and among themselves to provide the required positive contribution to the $T$ parameter without violating the bounds on $\delta g_{bL}$. 
This gives rise to large corrections to the top gauge couplings, $V_{tb}^L$ and $X_{tt}^{L,R}$ (the correction to $V_{tb}^R$ depends on the details of how the bottom quark is embedded in the theory but they are expected to be suppressed by Yukawas of the order of the bottom Yukawa). We show in Fig. 6 the values of these couplings in the allowed region of our model. It is interesting

\begin{figure}
\centering
\includegraphics[width=0.8\textwidth]{figure6}
\caption{$X_{tt}^L$ (empty green squares) and $X_{tt}^R$ (full red squares) as a function of $s_L$ (left panel) and $V_{tb}^R$ vs $V_{tb}^L$ (right panel). In both cases all the points are compatible with precision data at the 99\% C.L. The SM values for these couplings are $X_{tt}^L = V_{tb}^L = 1$ and $X_{tt}^R = V_{tb}^R = 0$.}
\end{figure}

that while these couplings receive large corrections, the indirect constraints on them from radiative corrections to electroweak and flavor precision data are still satisfied thanks to the extra contribution of the new quarks in the theory. The charged current top-bottom coupling is only now starting to be constrained by single top production measurements at the Tevatron [33] (see also [34])

$$|V_{tb}^L| \gtrsim 0.66,$$

(62)

where we have assumed (as it happens in our model) that the only sizable correction to the $Wtb$ vertex is in $V_{tb}^L$. From Fig. 6 we see that this precision is not yet sufficient in order to constrain our model. The situation will improve at the LHC, where this coupling can be determined with an accuracy of about $\sim 10\%$ [35] and therefore the model could be probed not only through direct production of the new quarks but also in single top production. The measurement of the $Zt\bar{t}$ coupling is much more difficult. It is currently unconstrained and although measurements of $Zt\bar{t}$ production at the LHC can be in principle used to measure it, we have checked that the achievable precision is likely insufficient to constrain our model [36, 37].
Let us now turn to the collider implications of the new fermionic sector. Pair production of new vector-like quarks with quantum numbers similar to ours has been discussed in detail in the literature \[6, 7, 20, 32, 38, 39, 40\]. In particular, it was shown in \[39, 40\] that pair production of charge $5/3$ quarks as light as the ones in our model can be discovered in the very early runs of the LHC. The lightest of the charge $2/3$ quarks has sizable couplings to the top, the bottom and the charge $5/3$ quark. Thus, pair production of this quark will result in many $Z Z t\bar{t}$ and $W^+ W^- b\bar{b}$, which should allow for a relatively easy search. However, the mass difference with the charge $5/3$ quark will typically be too small to allow for a significant fraction of cascade decays involving both quarks.

Similar properties are shared by the second charge $2/3$ quark (order one coupling to all the lighter modes) with the notable distinction that now the mass difference with the charge $5/3$ quark is large enough to allow for a large branching ratio. Thus, we have a sizable set of events with the spectacular signature

$$pp \rightarrow q_{2/3}^{(2)} \rightarrow q_{5/3} \bar{q}_{5/3} W^- W^+ \rightarrow W^+ W^+ W^- W^- b\bar{b}. \quad (63)$$

Note however that the larger mass of this quark will significantly reduce the pair production cross section. A detailed analysis of the signal and background is necessary to decide the reach of the LHC on this channel.

The large couplings of all these fields to the top and bottom indicates that they can be further tested through single production (for a discussion of single production of vector-like quarks with these quantum numbers mixing with valence quarks at the Tevatron see \[41\]).

**B. Two multiplets**

We have seen in the previous subsection that the allowed region of parameter space compatible with experimental data is quite small if only one set of fermionic composites is below the cut-off of our effective theory. Due to the explicit breaking of the $SO(5)$ symmetry induced by the mixing with the fundamental fields of the SM top sector, these composite fermions also contribute to the Higgs potential. Naturalness suggests that, ideally, they should provide the leading contribution to the effective potential, cutting-off the Higgs mass not at $\Lambda$ but rather at the mass of these fermionic resonances. However, given the severe constraints that precision data impose on the model, it would be rather coincidental if the
permitted small region in parameter space generated also a viable pattern of electroweak symmetry breaking with the correct vacuum expectation value. This problem has already been observed in 5D models of composite Higgs, in which a non trivial fermionic spectrum was required in order to obtain a successful realization of EWSB \[6, 7\].

As a first step in preparation of realistic composite Higgs models which fully incorporate a satisfactory pattern of EWSB, we find it useful to study the effect on electroweak and flavor precision data of a second set of light composite fermions. Note that here we do not mean to simply include the effect of *more Kaluza-Klein modes* (in an analogous 5D picture) or heavier resonances in a purely 4D picture. We are rather considering the possibility that the spectrum of fermionic resonances at the scale of our strongly coupled theory is richer than what we have considered so far. \(^9\)

We go back to our original mass term, Eq. (13), in which now \(m_L\) and \(m_R\) are two-dimensional vectors and \(m_\Psi\) and \(\mu\) are \(2 \times 2\) matrices (the former diagonal, the latter hermitian). The generalization of \(s_{L,R}\) to measure the degree of top compositeness in the presence of more than one composite is

\[
\begin{align*}
  s_{L}^{\text{eff}} & \equiv \sqrt{1 - (U_{q}^{(0)})^{2}}, \\
  s_{R}^{\text{eff}} & \equiv \sqrt{1 - (U_{t}^{(0)})^{2}},
\end{align*}
\]

where \(U_{q}^{(0)}\) \((U_{t}^{(0)})\) is the \(3 \times 3\) unitary matrix that mixes \(q_L, Q_L^1\) and \(Q_L^2\) \((t_R, T_R^1\) and \(T_R^2\)) in order to make the mass matrix diagonal before EWSB. This definition generalizes to an arbitrary number of extra composites.

The result of our scans in this case shows that the region of parameter space compatible with electroweak and flavor precision data expands dramatically. This was to be expected, due to the increase in the number of degrees of freedom. Nevertheless, it should be emphasized that the constraints from the pattern of \(SO(5)/SO(4)\) global symmetry breaking are still imposed on our extended sector. Even more interesting is the fact that the allowed parameter space not only is it larger, but we also find a plethora of patterns of phenomena, some of which we discuss below.

The most important phenomenological feature that is allowed by experimental data when two sets of composite fermions are below \(\Lambda\) is a much richer spectrum. The number and couplings of light quarks are no longer fixed. We can have from just one single charge \(\frac{2}{3}\)

\(^9\) This is precisely what happens in realistic 5D composite Higgs models. For a 4D interpretation, along the lines of the models discussed in this paper, see \[42\] and \[43\].

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quark to a full, almost degenerate, set of quarks arranged in a (4) of $SO(4)$. They can be very strongly coupled to the top and bottom or almost not coupled at all. This has a number of interesting consequences, that can be subjects for exploration at colliders:

**-Complex collider signatures.** The presence of a relatively large, sometimes quite degenerate, set of new particles will require sophisticated analysis to disentangle the contribution of different modes. Also, the possibility of long decay chains ending in a large number of leptons, jets and missing energy can make it more difficult to distinguish composite Higgs models from other new physics models in the early data.

**-Importance of single heavy quark production.** Single production, which tests directly the couplings of the new quarks to the top and bottom, becomes an important tool, complementary to pair production and to indirect tests through electroweak precision observables, to fully reconstruct the fermionic sector of the model.

**-Strongly composite $t_R$.** A strongly composite $t_R$ (as opposed to a strongly composite $t_L$ as in the case of one multiplet) is now a quite common occurrence. This is a welcome feature, as there are some UV completions for which extra sources of flavor violation can be enhanced for a strongly composite $t_L$.

We now discuss in a bit more detail some indicative spectrum patterns of new quarks that we have found in our scans. One interesting possibility is the presence of a rather light ($m_{(4)} \sim 500$ GeV) full, almost degenerate, (4) of $SO(4)$ as the only set of particles below 1 TeV. This set of new quarks does not contribute significantly to the $T$ parameter or $\delta g_{bL}$ (as it barely breaks the custodial symmetry), a role that is mainly played by heavier modes which are more difficult to produce at the LHC. The light quarks (two charge 2/3, one 5/3 and one $-1/3$) all decay mainly to the top and therefore a large number of $VVt\bar{t}$ events should be produced at the LHC (Refs. [39, 40] showed that the charge 5/3 quarks should be easily discovered with very early data at the LHC). Once their masses are known and some information on their couplings from single production is gathered, a reanalysis of electroweak precision data should give a clear picture of the fermionic content of the model.

The possibility we have just mentioned, a light full (4) representation of $SO(4)$, would clearly point to the underlying symmetry structure of the theory. There are however other regions of parameter space in which this structure is not so obvious. One example is the case
in which the only light mode, easily accessible at the LHC, is a charge $2/3$ quark. Of course, we know that in the context of composite Higgs models with little fine-tuning, a single charge $2/3$ singlet is not compatible with electroweak data. However, without additional information, it will not be easy to disentangle the contribution of Higgs compositeness and the new quark to electroweak precision data. Our study of electroweak constraints shows however that the heavier modes must significantly contribute to the $T$-parameter and $\delta g_{bl}$ and therefore should have large couplings to the top and/or bottom that would make them accessible through single production.

A variety of viable spectra allows for the possibility in which we have several non-degenerate modes below 1 TeV. For instance, there are regions in which the lightest new quark is a charge $5/3$ one, then a heavier charge $2/3$ and then a heavier (but still relatively light, $m_{-1/3} \sim 800$ GeV) charge $-1/3$. The mass difference is commonly large enough to allow for cascade decays that end in the top and up to three gauge bosons. This means we can easily have up to eight gauge bosons and two b’s in the final state,

\[
pp \to q_{-1/3} \bar{q}_{-1/3} \to W^- q_{2/3} \bar{q}_{2/3} \to W^- q_{2/3} W^+ W^+ q_{2/3} \\
\to W^- W^- W^+ W^- t W^+ W^- b \to W^- W^- W^+ W^- b W^+ W^- b.
\] (65)

A fraction of the time $q_{2/3}$ will decay into $Zt$ so that the final state can also be $6W + Z + b\bar{b}$ or $4W + 2Z + b\bar{b}$. Thus, we have processes with a relatively large production cross section (for quarks which are not too heavy) and a very complex final state with many jets, leptons and missing energy. Although a detailed analysis is required to assess our capability of understanding these complex processes, it is likely that they will be easy to discover but very difficult to fully reconstruct and the detailed information on the quark masses may not be extracted.

We note that the same features as we find here, a rich spectrum of light modes, some of which do not contribute very strongly to electroweak observables, and a number of heavier modes (but still lighter than the bosonic resonances) which contribute to render electroweak observables compatible with experimental data has been recently observed in composite Higgs models in five dimensions. In fact, this kind of spectrum proved crucial to make the model in \[7\] simultaneously compatible with electroweak precision tests and with a realistic pattern of electroweak symmetry breaking.
VI. CONCLUSIONS

The realization of electroweak symmetry breaking is a long-standing mystery that will be soon tested in detail at the LHC. Composite Higgs models, in which the Higgs boson arises as the (composite) pseudo-Goldstone boson of a spontaneously broken global symmetry in a strongly coupled gauge theory, is an appealing candidate. It naturally protects the electroweak scale from short distance physics and can even explain the suppression of the electroweak scale with respect to the scale of new physics. Compatibility with current experimental data seem to point to a relatively large scale of new bosonic resonances $\Lambda \gtrsim 3$ TeV and to a custodially preserving symmetry breaking pattern to protect the $T$ parameter. Similarly, a left-right symmetry within the custodial symmetry naturally protects the $Zb_L\bar{b}_L$ coupling from large corrections. A minimal symmetry breaking pattern that contains the Higgs as a pseudo-Goldstone boson, is custodially symmetric and can protect the $Zb_L\bar{b}_L$ coupling is a global $SO(5)$ symmetry spontaneously broken to $SO(4)$ and fermions in the fundamental representation of $SO(5)$ \cite{1,2,8}.

The low energy implications of this set-up can be simply analyzed with the aid of an effective description based on an $SO(5)/SO(4)$ non-linear sigma model. If the Higgs is quite composite, i.e. if the scale of the global symmetry breaking is not far from the electroweak scale, as one would expect in a natural (non fine-tuned) model, its couplings to the Standard Model fields are significantly reduced. This results in a sensitivity to the cut-off of the theory whose effect can be taken into account by defining an effective (heavier) Higgs mass. This effective Higgs mass is the one that should be included in the calculation of electroweak precision observables, giving a positive contribution to the $S$ parameter and a negative contribution to the $T$ parameter. These contributions simply come from the fact that the Higgs is composite and are therefore quite generic. UV physics, being custodially symmetric, is not expected to contribute to the $T$ parameter but it can give a tree level contribution to the $S$ parameter that, together with the contributions to $S$ and $T$ from Higgs compositeness, put the model in gross contradiction with current experimental tests.

The large mass of the top quark, however, makes it natural to assume that it is partially composite. In that case it will strongly couple to the fermionic resonances of the strongly coupled theory which, if lighter than the cut-off of the theory, can have an important impact on electroweak observables (due to their mixing with the top, which breaks explicitly the
custodial symmetry). We have considered the presence of one or more sets of fermionic resonances, spanning full fundamental representations of $SO(5)$. The $SO(5)$ symmetry is explicitly broken by the SM quarks, $q_L$ and $t_R$ (the former also breaks the custodial $SO(4)$ symmetry), which couple linearly to the strongly coupled theory. This coupling makes $q_L$ and $t_R$ partly composite and, through their composite components, couple to the Higgs and get a mass. We have computed the exact contribution of this new sector to the relevant electroweak precision observables, $T$ and $\delta g_{b_L}$, and also to some of the flavor observables that are strongly correlated to this new fermionic sector, $B_q - \bar{B}_q$ mixing and $b \to s\gamma$. We have performed an exact one-loop calculation which does not rely on any approximation and presented the results in a general enough way that can be easily extended to models beyond the one we have considered. Our exact calculation, that goes beyond the large Yukawa approximation which is commonly used to estimate the contribution to the anomalous $Zb_L\bar{b}_L$ coupling, can have an important impact if the model is strongly constrained, as happens in our case in some regions of parameter space, or if one wants to do precision analysis of new physics, as it will become necessary if new quarks are discovered at the LHC.

Armed with these detailed calculations of the most relevant electroweak observables, we have used adaptive Monte-Carlo methods to scan the parameter space in search of the regions allowed by experimental data. The result depends dramatically on whether there exist just one or more fermionic multiplets below the cut-off of our theory.

The case of only one multiplet below $\Lambda$ is quite constrained. When all the experimental constraints are taken into account, only a small region with a very composite LH top survives. In this region the spectrum of new quarks and their couplings are almost univocally determined by electroweak and flavor precision data. Above the top, there are typically two quite light ($\lesssim 500$ GeV) quarks of charge $5/3$ and $2/3$, respectively (the former typically slightly lighter) followed by another charge $2/3$ quark with mass $800$ GeV $\lesssim m_{(2)}^{(2)} \lesssim 2$ TeV. The lighter two quarks should be easily produced at the LHC and have large enough couplings to the top to make single production an interesting channel to study. Furthermore, they typically induce large enough corrections to the $V_{tb}$ coupling to be detectable at the LHC. The strong constraints on this possibility and the fact that we have simply assumed a realistic pattern of electroweak symmetry breaking - which is fully calculable and therefore imposes further constraints in a UV completion of the model - have motivated us to consider the possibility that more than one set of fermionic resonances contribute to electroweak ob-
servables. This motivation is reinforced by the experience with 5D UV completions of the model, in which electroweak symmetry breaking imposes non-trivial constraints on the low energy spectrum, usually requiring a more complex spectrum of light fermionic resonances than the one we have found in the case of one multiplet.

The situation dramatically changes when two fermionic multiplets are allowed to contribute to electroweak precision observables. The spectrum of light resonances is no longer constrained; we have found from one single quark of charge $2/3$ to four quarks in a full degenerate multiplet (4) of $SO(4)$ below 1 TeV. Their couplings to the top can also vastly change, which makes the study of single production even more interesting. When pair-produced, these new quarks can produce long decay chains that contain a large number (up to eight) of SM electroweak vector bosons and a $b\bar{b}$ pair. Thus, final states with many jets, leptons and missing energy would be a common signature in these models.

We conclude that composite Higgs models with no bosonic resonances (apart from the Higgs itself) below the cut-off of the low energy effective theory can be fully compatible with current experimental constraints provided a quite rich spectrum of light fermionic resonances is present in the model. These new fermionic resonances should be easily produced at the LHC and would most likely be the first signal of new physics beyond the SM. Establishing the symmetry pattern from the fermionic spectrum can however prove more difficult and will require a detailed analysis of the full experimental information, including pair production, single production and a detailed analysis of electroweak and flavor observables. This is really important as other signatures that would definitely pin down the model as a composite Higgs model, like the measurement of the Higgs couplings, longitudinal gauge boson scattering and the production of new bosonic resonances of the strongly coupled theory can take much longer at the LHC.

Acknowledgements

We would like to thank J.A. Aguilar-Saavedra, R. Contino, M. Gillioz, U. Haisch, A. Pomarol, E. Pontón and Z. Kunszt for useful discussions. This work was supported by the
Swiss National Science Foundation under contract 200021-117873.

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