Advanced Speed-and-current control approach for dynamic electric car modelling

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Abstract
Considering environmental conditions and reduced fuel availability, electric cars (ECs) play a vital role in many applications such as consumer cars and short-distance transportation. This paper proposes a detailed dynamic modelling of battery, motor, and inverter developed for the design of an EC. In addition, an improved controller is developed with a different geometrical method using the sensitivity gain of the current sensor and tachometer to assure the optimal performance of the EC. For achieving linear vehicle operation and improved stability, a system transfer function model is designed by considering various uncertainties such as force acting on the car, wheel, road, and wind speed conditions. To offer better regulation and excellent tracking operation of the EC, a combined proportional–integral–derivative controller-based outer-speed and inner-current control approach is suggested to regulate the nonlinear parameters for different driving profile applications. The proposed designed control approach and system model are tested using two input conditions such as step and driving profile inputs through MATLAB/Simulink software, and performance is analysed through various open-loop and closed-loop test scenarios.

1 | INTRODUCTION

Considering environmental conditions and greater awareness about energy conservation, researchers have been paying more attention to the design of zero-polluting electric cars (ECs) of late. Recently, improvements in EC/hybrid-EC modelling have attracted greater interest at an augmented pace [1]. Particularly, lesser-weight ECs are becoming popular for many applications such as patrol and short-distance transportation cars. Many EC modelling techniques have been suggested to offer a longer driving range and linear operation [2]. Generally, EC modelling is designed by considering two subsystems, such as electric motors (EMs) for the drive system with a car platform, as shown in Figure 1. The main components of ECs are battery energy storage (BES) devices, central control structures, a tachometer, and a voltage source converter to convert DC–AC power. A single EM is used to drive each wheel [3]. However, with increasing costs and complex modelling, [4] the EC can lose its attraction for real-time applications. To achieve greater simplicity and easier control action, the DC EM is popularly selected for the traction of ECs [5]. In addition, DC motors also supply high starting torque. Therefore, to develop a robust/light, high-efficiency, reduced-cost EC, it is necessary to derive an appropriate mathematical model of ECs and EMs for different driving profile operations.

Simple EC design leads to a simple control strategy that decreases the overall cost of the vehicle. However, the development of the simple EC model is difficult because of uncertainty and non-linearity in the environment and wheel and road conditions [6,7]. Mostly, disturbances are categorised into two types: (1) parametric uncertainty and (2) inner/outer disturbances. The first type of disturbance is caused by a lack of appropriate information regarding EC modelling, friction modelling, and parameter fault conditions, and the second type of disturbance is generated by unidentified effects of existing physical constraints in the environment [8,9]. Therefore, there is a necessity to design improved mathematical modelling of EC by considering possible real-time disturbances.

Generally, ECs are known as ‘power management’ machines [10,11]. Therefore, there is a requirement to design a coordinated control strategy to provide satisfactory driving performance and linear operation by optimally consuming...
power [12]. Because of the significant growth of microprocessors like digital signal processor boards, it is easier to develop a complex control algorithm for optimal EC operation [13]. The just-mentioned capabilities of the microprocessor-based controller enhance EC performance by improving the safety conditions for specific applications [14]. Many power engineers have proposed various adaptive methods to overcome the first type of problem [15]. Because of the incapability of adaptive filters to solve the second type of problem, a robust adaptive controller is proposed to tackle real-time problems [16–18]. To overcome non-linearity and disturbance conditions, adaptive feedback linearisation techniques are used to approximate a disturbance component [19]. However, offline disturbance identification control techniques are not well suited because of changes in disturbances over time. As a solution, various online adaptation techniques such as fuzzy logic controller (FLC)- and artificial neural network (ANN)-based control techniques deal with the unknown disturbances. Because they use various linguistic variables, FLC techniques are widely accepted as offering better solutions than other soft techniques [20]. In reviewing the related literature such as [21–24], it can be seen that adaptive FLC is divided into two specific classes, direct and indirect, depending on the system condition and applications. In a direct adaptive FLC, fuzzy systems are used as a controller using IF–THEN fuzzy rules [21,22]. In an indirect FLC, fuzzy systems are used to describe the system model through the IF–THEN rule [23,24]. However, FLC-based systems reduce performance because of the requirement for an increasing number of fuzzy rules during real-time and multi-input–multi-output systems [22–25]. Moreover, owing to the increasing number of rules, the structure of an FLC becomes more complex in design and provides an additional computational burden on the design model. Similarly, the ANN-based control approach is used as an alternative to deal with the uncertainty because of its excellent tracking capability [24–26]. In adaptive ANN-based approaches, the direct and indirect classes are used to track the unknown disturbances to save time and reduce the effort of complex system modelling [27]. For real-time applications, ANN-based approaches have decreased importance because of their requirements for larger nodes, excess training time, slower training speeds, and appropriate input and output data as well as problems with tracking of local minima and filtering. Therefore, there is a necessity to design an improved controller for optimal performance of EC operation in real-time applications. The major objectives of the proposed approach are as follows:

1. For designing a lighter-weight and reduced-cost EC system, the detailed mathematical modelling of EC and EM is presented.
2. By viewing the inner and outer disturbances, two load models are designed.
3. Different subsystem transfer functions of EC components such as the battery, inverter, and motor model are developed.
4. Improved control models are designed using various geometrical methods using the sensitivity gain of both current sensors and tachometer.
5. Using only the speed sensor and combined current-and-speed sensor, two EC models such as model-1 and model-2 are designed.
6. To offer linear output and linear EC operation, a combined proportional–integral–derivative (PID) control-based outer-speed and inner-current control approach is suggested.

7. The performance of the individual designed models is studied during open-loop, speed, and combined speed and current closed-loop control applications.

2 | OVERALL STRUCTURE AND DETAILED MODELLING OF ELECTRIC CAR

Figure 1 shows the complete system architecture of the EC model. The basic model of the EC is designed by focussing on two subsystems, dynamic modelling of an electric motor (DMEM) and dynamic modelling of an electric car (DMEC). The modelled EC is coupled with wheel rotational speed through an EM to achieve the desired speed. In addition, the actual performance of the EC depends on the force acting on it. Therefore, considering all the above factors, there is a necessity to develop an appropriate EC model. An appropriate torque and power model is computed to provide the detailed dynamic modelling of the respective EM and EC presented below.

2.1 | Dynamic modelling of electric motor

The main role of an EM is to provide necessary force for EC speed regulation as indicated in Figure 2. Therefore, appropriate mathematical modelling of the EM is much more important for EC operation. To assure a suitable speed-up time, the driving EM necessitates excess torque output at slower speeds and reduced torque output at higher speeds. In addition, to achieve sustained high speeds, a driving EM is necessary to attain a certain power output during high-speed operation [28]. The appropriate dynamic equation of EM is obtained by combining Newton’s and Kirchhoff’s laws.

The basic mathematical equations of any EM are presented as follows:

\[
\begin{align*}
V_s &= R_f i_f + \frac{d\psi_f}{dt} \pm a_f \psi_f \\
\psi_f &= L_f i_f + L_m (i_f + i_a) \\
\psi_{fm} &= L_m (i_f + i_a) \\
V_r &= R_a i_a + \frac{d\psi_a}{dt} \pm (a_f - a_e) \psi_a \\
\psi_a &= L_a i_a + L_m (i_f + i_a) \\
\psi_{am} &= L_m (i_f + i_a)
\end{align*}
\]

where \(V_s\) and \(V_r\) are the field and armature voltages of EM, \(R_f\) and \(L_f\) are the field resistance and inductance of EM, \(R_a\)

and \(L_a\) are the armature resistance and inductance of EM, \(L_m\) is the mutual inductance of EM, \(\psi_f\) and \(\psi_a\) are the field and armature fluxes of EM, \(\psi_{fm}\) and \(\psi_{am}\) are the mutual field and armature flux components, and \(I_f\) and \(I_a\) are the field and armature currents of EM.

A simplified equivalent circuit of an EM is illustrated in Figure 2. In Figure 2, both electrical and mechanical components of the EM are illustrated. A detailed explanation of the electrical and mechanical modelling of the motor is provided below.

2.1.1 | Electrical modelling of motor

As shown in Figure 2, by providing an input voltage \((V_{in})\) to the EM, the EM coil generates an electrical torque \((T_e)\) in the armature winding. The generated \(T_e\) is computed by multiplying the armature current \((I_a)\) with the torque constant \((K_t)\) and represented as

\[T_e = K_t \times I_a\] (3)

During the armature action between the stator field, the EM produces an electromotive force \((E_b)\) reverse to the direction of \(I_a\). \(E_b\) is computed by multiplying the \(E_b\) constant \((K_b)\) with an angular speed of the motor \((\omega_e)\) and represented as

\[E_b(t) = K_b \times \frac{d\theta_e(t)}{dt} = K_b \omega_e\] (4)

Applying Kirchhoff’s law to the electrical side of the motor, the total voltage \((V_T)\) is computed as

\[V_T = \sum V = V_{in} - V_{R_a} - V_{L_a} - E_b = 0\] (5)
where $V_{Ra}$ and $V_{La}$ are denoted as the voltage drop across armature resistance and inductance, respectively. The armature current of the motor can be computed as

$$V_{in} = R_a I_a + L_a \frac{dI_a}{dt} + K_i \frac{d\theta}{dt} \quad (6)$$

The Laplace transform of Equation 6 becomes

$$V_{in}(s) = R_a I_a(s) + L_a s I_a(s) + K_i \frac{d\theta}{dt}(s)$$

$$I_a(s) = \frac{V_{in}(s) - K_i \frac{d\theta}{dt}(s)}{(R_a + L_a s)} \quad (7)$$

### 2.1.2 Mechanical modelling of motor

Because of the moment of inertia of motor ($J_M$), damping motor friction constant ($B_M$), and load, the torque produced by the motor generates an angular speed ($\omega_M = d\theta_M/dt$). By balancing the energy of the motor, the mathematical modelling describing the mechanical characteristics of the motor can be presented as follows:

$$T_{\theta_M} = J_M \times \frac{d^2 \theta_M}{dt^2} \quad (9)$$

$$T_M = K_t \times I_a \quad (10)$$

$$T_{\omega_M} = B_M \frac{d\theta_M}{dt} \quad (11)$$

The total torque ($T$) equation becomes

$$\sum T_i = T_M - T_{\theta_M} - T_{\omega_M} = 0 \quad (12)$$

$$T = K_t \times I_a - J_M \times \frac{d^2 \theta_M}{dt^2} - B_M \frac{d\theta_M}{dt} = 0 \quad (13)$$

Taking the Laplace transform of Equation 13,

$$T(s) = K_t \times I_a(s) - J_M \times s^2 \theta_M(s) - B_M \theta_M(s) = 0 \quad (14)$$

$$I_a(s) = \frac{(sJ_M + B_M) \theta_M(s)}{K_t} \quad (15)$$

### 2.1.3 Developing the motor open-loop transfer function

From Equations 8 and 15, the transfer function values are presented as follows:

$$\frac{I_a(s)}{V_{in}(s) - K_i \frac{d\theta}{dt}(s)} = \frac{1}{(R_a + L_a s)} \quad (16)$$

$$\frac{\omega_M(s)}{I_a(s)K_t} = \frac{1}{(sJ_M + B_M)} \quad (17)$$

Substituting Equation (16) into Equation (14), the equation becomes

$$K_t \frac{V_{in}(s) - K_i \frac{d\theta}{dt}(s)}{(R_a + L_a s)} = (J_M \times s + B_M) \theta_M(s) \quad (18)$$

Rearranging Equation (18) without load angle, the open-loop transfer function ($G_a(s)$) related to the input voltage ($V_{in}$) and output angle ($\theta_M(s)$) of the motor can be computed as

$$G_a(s) = \frac{\theta_M(s)}{V_{in}(s)} = \frac{K_t}{s \{sL_a + R_a\}(sJ_M + B_M) + K_t K_b} \quad (19)$$

Rearranging Equation (18) without load, the speed open-loop transfer function ($G_s(s)$) related to the input voltage ($V_{in}$), and output angular velocity ($\omega_M(s)$) of the motor can be computed as

$$G_s(s) = \frac{\omega_M(s)}{V_{in}(s)} = \frac{K_t}{s \{sL_a + R_a\}(sJ_M + B_M) + K_t K_b} \quad (20)$$

To design an appropriate open-loop transfer function for EC operation, it is necessary to compute all moments of inertia for better results. Generally, the EC platform can be a shape of cuboid or cubic shape. Therefore, the total moment of inertia ($J_f$) and total damping factor ($B_f$) at the armature of EM with gear ratio ($n$) is computed using the conservation principle:

$$B_f = B_M + B_L \left(\frac{N_t}{N_m}\right) \quad (21)$$

$$J_f = J_M + J_L \left(\frac{N_t}{N_m}\right) \quad (22)$$

$$J_L = M_f V^2 \omega_M^2 \quad (23)$$

where $J_L$ is the load inertia, $M_f$ is the total mass of the system, and $N_t$ and $N_m$ are defined as the number of teeth presented in the load and motor gears, respectively. By considering the linear velocity of EC ($V$), the angular speed of the motor ($\omega_M$), tyre radius ($r$), and gear ratio ($n$), the moment of inertia of load ($J_L$) is computed as

$$\omega_M = \omega_r \times n = \frac{V \times n}{r} \quad (24)$$
\[ V = \frac{\omega_M \times r}{n} \]  

(25)

Applying Equation (25) to Equation (23), \( J_L \) becomes,

\[ J_L = \frac{M_T r^2}{n^2} \]  

(26)

By considering the above-discussed equations, the equivalent EC open-loop transfer function \( G_y(s) \) can be presented as

\[ G_y(s) = \frac{\omega_M(s)}{V_{in}(s)} = \frac{K_i / n}{(sL_a + R_a)(sT + B_T) + K_i K_h} \]  

(27)

By considering the armature voltage input \( (V_{in}) \) and the output voltage of the tachometer \( (V_{tach}) \) with the corresponding load torque \( (T_L) \), the EC open-loop transfer function \( G_y(s) \) can be presented as

\[ G_y(s) = \frac{V_o(s)}{V_{in}(s)} = \frac{K_{tach} \times \omega_M(s)}{V_{in}(s)} = \frac{K_i \times K_{tach}}{(sL_a + R_a)(sT + B_T) + (sL_a + R_a) \times T + K_i K_h} \]  

(28)

where \( T \) is denoted as the disturbance torque including the Coulomb friction \( (T_F) \). To track the actual speed of the EC and feed it back to the control system, a tachometer is used in the EC. The tachometer dynamics and corresponding transfer function are illustrated in Equation (29). To achieve a linear speed for the EC of 23 m/s, the tachometer constant \( (K_{tach}) \) selected is 0.4696 [28]:

\[ V_o(t) = K_{tach} \times \frac{d\omega_M(t)}{dt} \Rightarrow V_o(s) = K_{tach} \times \omega_M(s) \]  

(29)

where \( V_o \) is denoted as the system output voltage.

2.2 Dynamic modelling of electric car

Figure 3 illustrates the overall motion diagram of ECs by showing various forces. By balancing the magneto and electromotive forces of the electric motor and operating resistive forces [28], the speed of the EC is decided. To derive an accurate DMEC, it is much more important to track the dynamics among road, wheel condition and acting forces such as the wind force \( (F_W) \), inertia force \( (F_I) \), rolling forces \( (F_R) \), traction force \( (F_T) \), and normal force \( (F_N) \) on the EC. EC torque disturbance is the resultant torque produced by all the resistive forces acting on the EC as presented below:

\[ F_{x,acc} = I_s \left[ \frac{G}{r} \right]^2 \]  

(31)

where \( M_C \) is the mass of the car \((\text{kg})\), \( V_C \) is the velocity of EC \((\text{m/s})\), \( V_C \) is the acceleration of EC \((\text{m/s}^2)\), \( g \) is the gravitational constant \((\text{m/s}^2)\), \( \theta \) is the driving angle of EC \((\text{rad})\), \( C_V \) is the rolling coefficient of EC, \( \rho \) is the density of air at \( 20^\circ \), \( C_d \) is the drag coefficient of EC, \( A_f \) is the front area of EC, \( V_w \) is the wind velocity \((\text{m/s})\), \( I_w \) is the wheel moment of inertia of EC, \( r \) is the radius of the wheel, \( G \) is the gear ratio, \( I_s \) is the armature current of the motor, and \( F_{x,acc} \) is the linear acceleration force.

After computing the possible force acting on the EC model, it is necessary to design the battery model. The battery is used only to provide the supply voltage for EC operation. Before computing the battery capacity of the EC, it is necessary to estimate the total required electrical energy for EC operation. The power demand is measured in kW, and the power is used to regulate the speed of the EC. The electric power \( (P_e) \) is computed by multiplying the total traction force \( (F_T) \) and \( V_C \), and is represented as follows:

\[ P_e = \sum F \times V_C = F_T \times V_C \]  

(32)

\[
F_T = M_C V_C + M_C g \cdot \sin(\theta) + \text{sign}(V_C) \cdot M_C g \cdot \cos(\theta) \cdot C_V + \text{sign}(V_C + V_W)^{\frac{1}{2}} \cdot \rho C_d A_f (V_C + V_W)^2 + \left[ \frac{M_C + I_w}{r} \right] \times V_C
\]  

(30)
The battery is the key element for EC applications. In recent times, many different types of batteries, such as lead-acid, nickel hydride, and lithium-ion, have been used for various purposes [28]. However, from a real-time application point of view, a lithium-ion-based battery storage device is selected due to the relative increase in specific energy and power [2–4].

2.2.1 Battery electric model

The equivalent battery model is illustrated in Figure 4. As shown, the equivalent battery model is designed using the internal voltage source ($V_{ib}$), battery voltage ($V_b$), charging and discharging diode ($D_{bc}$ and $D_{bd}$), and charging and discharging resistance ($R_{bc}$, and $R_{bd}$), respectively. $D_b$ is known as the forward diode of the battery, and $I_b$ is known as the obtained battery current. The two diodes are generally ideal and used only to facilitate charging and discharging operations. Charging currents are denoted with a ‘+’ sign, and discharging currents are denoted with a ‘-’ sign. The ratings of $V_{ib}$, $R_{bd}$, and $R_{bc}$ depend on the depth of battery discharge capability. As indicated in Figure 2, equivalent circuit $V_b$ is computed as follows:

$$V_b = \begin{cases} V_{ib} - R_{bd}I_b & \geq 0 \\ V_{ib} - R_{bc}I_b & < 0 \end{cases}$$

After generating the necessary electric power from the battery ($P_e = V_C^2I$) and power available in the wheel of the EC ($P_w$), the driving angle ($\theta$) of the EC is computed as follows:

$$\theta = \frac{P_w - P_e}{MC \times V_C}$$

After the successful modelling of the battery and DMEC, to obtain more accurate precision for the value of the disturbance force ($F_D$) acting on the EC, additional factors are considered. By viewing the accuracy demand of the EC, various constraints such as total driving resistance force ($F_{dr}$) and EC dynamics can be considered. For smooth acceleration of the EC, the EM of the EC must be able to overcome the $F_{dr}$. The modelling of the EC dynamics is simplified in [26–28], and the corresponding equations are presented below. A detailed explanation of the following equations is presented in [29]:

$$F_D(t) = R_C M CG \sin(\theta) + M_C g r C_r$$

$$T_D(t) = \frac{1}{2} \rho A_f C_d r (V_w + V_C)^2$$

$$F_D(t) = M_C V_C \left( t \right) + \frac{1}{2} \rho V_C^2 \left( \frac{d\omega}{dt} \right) + \frac{1}{2} M_C g r \sin(\theta) + \omega C_r$$

$$F_D(t) = M_C K_{ma} + \frac{1}{2} M_C \sin(\theta) \left( V_w \left( t \right) \right)^2$$

where $R_w$ is the wheel resistance, $K_{ma}$ is the equilibrium constant and ‘a’ is the acceleration constant of the EC. From the derived dynamic equations presented as Equations (30, 35–38), two load models are derived and presented in Figure 5a,b. To meet the accuracy level, all the related parameters stated above are considered for the design of the accurate load model. The combined load model to provide an accurate idea of disturbance torque ($T_D$) is illustrated in Figure 6.

$$G_F(s) = \frac{2K_t K_{tach} \omega}{2B_s s(L_a + R_a) + r^2 M_C s(L_a + R_a) + C_v(sL_a + R_a) + 2J_e(sL_a + R_a) + 2K_b K_t}$$

Simplifying Equation (39), $G_F(s)$ becomes,

$$G_F(s) = \frac{2K_t K_{tach} \omega}{(sL_a + R_a)(2J_e + 2B_s s + C_v) + s(L_a + R_a)r^2 M_C + 2K_b K_t}$$

Depending on all the derived force/torque equations, armature input voltage ($V_{im}$ ($s$)), and output voltage of the tachometer ($V_{tach}$), and by considering all the combined load parameters, a simplified open-loop transfer ($G_F(s)$) function for the EC model is presented in Equation (39). The
fundamental closed-loop transfer function Simulink model is illustrated in Figure 9a by considering DMEC, wheel rotational velocity, tachometer voltage, EM modelling, and disturbance forces acting on the system.

3 | RESULTS

After considering uncertainty and assessing costs and complexity, two types of EC model (model-1 and model-2) are suggested. Model-1 is designed by considering only the speed sensor. Similarly, model-2 is designed by combining both the speed and the current sensor. In this section, individual model results are tested through both a single-loop and a two-loop control system. The individual test results are analysed in the following sections.

3.1 | Comparison and validation of the proposed controller

In this section, the performance of the open-loop test model EC presented in Equation 40 is compared with the proportional–integral (PI) control-based EC model through
different responses such as frequency response, impulse response, Hankel singular values and relative error between the two systems as shown in Figure 7. As illustrated in the Figure 7a bode diagram, the closed-loop PI control-based EC model captures a resonance of less than 50 rad/s. Although it looks like a substantially impressive result, the tracking of the lower frequency region (<5 rad/s) is poor. Because of the different load torque, the conventional PI control-based closed-loop model does not fully track the dynamics of the proposed EC model within 30–50 rad/s. As a result, the possibility of large errors and lower gain in the EC model arises at a lower frequency range. Therefore, large errors at low frequency contribute little to increasing the overall error.

3.1.1 | Solution

To overcome the above problem, this proposed approach uses a multiplicative error method such as ‘bstmr’. This technique emphasises relative error rather than absolute error because this technique does not work under near-zero gain. Therefore, in this approach, a minimum gain threshold is added to the original open-loop EC model. After adding the gain, the open-loop model is converted to a closed-loop EC model using a PI controller. The PI controller-based model is not worried about errors below −100 dB gain. In addition, a minimum value nearer to $10^{-5}$ is added to the gain to reduce the error. To validate system performance, a comparative impulse response is presented using the above approach as illustrated in Figure 7b.
The impulse response is plotted between the open-loop EC and proposed PI control-based EC models. Figure 7b shows that the settling time of the open-loop system is 10.7 s, and the settling time of the PI control-based closed-loop system is 6.8 s. The illustrated impulse response gives a clear idea about the improvement of the PI control-based EC model \( G_{F_{\text{closed}}} \) over the open-loop EC model \( G_{F_{\text{open}}} \) through the settling time.

### 3.1.2 Validation of results

Generally, all techniques offer bounds on the approximation error. In this approach, using additive-error methods like ‘balancmr’, the approximation error is measured through the maximum peak gain \( P_{g_{\text{max}}} \) of the error model \( G_{E_{\text{closed}}} \) across all frequencies. This \( P_{g_{\text{max}}} \) is also identified as the \( H_\infty \) norm of the \( G_{E_{\text{closed}}} \). The error bound for the additive-error technique is expressed as follows:

\[
G_{F_{\text{open}}} - G_{F_{\text{closed}}} \leq 2 \sum_{i=9}^{45} \sigma_i = \text{error bound} \quad (41)
\]

where the sum is over all discarded Hankel singular values of \( G_{F_{\text{open}}} \) (entries 9 through 45 of Hankel singular values of \( G_{F_{\text{open}}} \) indicated in Figure 7c. The Hankel singular value response illustrates that there are four dominant modes in \( G_{F_{\text{open}}} \). However, the contribution of the remaining modes is still significant. In this approach, a line is drawn at eight states, and the remaining ones are discarded, to find the eighth-order reduced \( G_{F_{\text{closed}}} \) that best approximates the original system response \( G_{F_{\text{open}}} \). From the relative error plot indicated in Figure 7d, there is up to 65% relative error at 25.5 rad/s frequency and 8.54 dB \( P_{g_{\text{max}}} \) which may be represent a better choice that offers a better response than that of \( G_{F_{\text{open}}} \) alone. Therefore, in this proposed approach, the combined load-based EC model is operated through both PI-control-based speed and current loop.

### 3.2 Scenario-1: open-loop testing model of electric car

By considering the developed system models DMEM and DMEC, the overall mathematical model of the EC is designed. Step input signals \( (V_{in} = 36) \) and driving cycle-based reference input signals are used to test the performance of the designed EC model. The driving cycle-based input signal is modelled by considering the acceleration of EC at rated EC speed and braking of the EC until null velocity is achieved. This scenario is tested to show the accuracy of the control system and the overall performance of the designed EC.

Figure 8 shows the results of linear speed, armature current, motor torque, the angular speed of the wheel, linear position, and nearly linear acceleration of the EC. All the characteristic responses show that modelling of the EC is correctly designed. The designed model is tested by considering a disturbance load.
torque model and without any controlling devices such as a P, PI, or PID controller. Due to open-loop EC model testing, the speed of the EC is kept lower at 8 m/s (around 28.8 km/h). It is shown that the linear speed of the EC is achieved within 4–5 s time intervals. During that period, the armature current and motor torque responses reach a higher value during the starting of the vehicle and settle to a constant value after a certain time interval. After a certain time, the angular speed of the motor is also achieved at a constant value. However, because of the absence of any controller, the speed of the EC is set to a lower value. It is clearly shown that because of the absence of any controller, the acceleration of the EC is non-linear. Therefore, after noting the necessity for high-speed driving, road conditions, the weight of the EC, forces acting on the EC, and wheel condition, it is necessary to design appropriate, suitable controller-based closed-loop EC control action to offer suitable and efficient operation of the EC.

3.3 | Scenario-2: closed-loop testing model of electric car (single-loop control approach for model-1)

The control system design of the EC is not an easy task because of the design constraints of the EC as well as road conditions (varying in nature). Therefore, the design of a robust/adaptive controller is essential to offer better driving operation, easy riding, and a null steady-state error and to increase the tolerance capability of the EC during both steady and dynamic state conditions.

Generally, the EC speed regulator takes a constant input voltage from the BES device and provides a variable output voltage for regulating the motor at variable-speed operation. The output voltage of EC is regulated by the control signal provided through the accelerator according to the user requirement. Due to voltage regulation, the motor and EC speeds are regulated [30]. During operation of the EC accelerator, the battery discharges a specific amount of current to the EM to achieve the required EC speed. In addition, the car sensors sense the actual speed of the EC and send that information back to the controller to offer closed-loop control action. As the battery supplies DC voltage and current, an inverter plays an important role in DC–AC voltage and current conversion for EM action. For appropriate voltage regulation, the inverter switches must operate through the pulse width modulation (PWM) technique. Using the PWM technique, the controller sends the required AC power pulses to the EM at a ratio of a thousand times per second. The shorter pulses slow the motor speed, and longer pulses increase the motor speed. Different control strategies are suggested for the specific operation of the EC with specific merits and demerits. In this proposed approach, the most important controllers, such as PI, PID, and PI, with a deadbeat controller and prefilter are selected for specific control structure design action.

3.3.1 | Control structure design for EC (model-1)

In this proposed approach, a single PID regulator-based closed-loop control approach is used to regulate the total EC system. This is a fundamental closed-loop transfer function Simulink model.

The total control approach for the EC as illustrated in Figure 9a is known as a speed regulator. The load torque model illustrated in Figure 6 is used for EC model design and testing purposes.

3.3.2 | Testing

During testing of the model, two reference input signals such as step input ($V_{in} = 36$) and driving profile input are
considered. The reference input signals are operated through a manual switch as illustrated in Figure 9a. To test EC performance, first the driving profile is taken as input for the closed-loop EC model. In this test condition, the most preferred proportional integral and derivative controllers are selected for controlling the errors generated during the transient condition. Different strategies like optimisation, self-tuning operations, and Zeigler Nicolas are commonly used to compute the constant parameters. However, during the transient condition, computations are not well performed [15]. Therefore, in this proposed approach, selection of the PID controller parameter is achieved using the simple mathematical closed and open-loop time-domain analysis. The detailed explanations of constant parameters such as proportional ($K_p$), integral ($K_i$), and derivative ($K_d$) are presented in [15] by showing the stability criteria of the system. Similar to [15], in this approach, the $K_p$, $K_i$, and $K_d$ values are computed as 6.734528, 9.652537, and 1.248723, respectively.

In this condition using the PID controller, the system responses are shown in Figure 9b. Figure 9b shows the linear responses of current, speed, torque, and power by considering the driving profile input. Figure 9b shows that the EC system achieves a linear speed of 23 m/s (approximately 82.8 km/h) within 5–6 s time intervals. The linear control output using the PID controller is shown in Figure 9b. According to the input condition profile, the angular speed of the motor is computed. The angular speed of EM is computed at around 80 rad/s. Using the computed angular speed, the linear position of the motor speed is achieved at its desired value. The derivative of the linear speed generates linear acceleration of the EC.

As indicated in Figure 9b, during linear speed operation, acceleration of the EC reaches zero, and the responses change linearly at a certain time interval. The linear system responses indicate that using the controller, system error is significantly minimised. However, the armature current and load torque of the EM are increased to a higher value of around 250 A. Due to the single-loop control approach, the motor draws a high current and consumes more power. Therefore, high-rating motors of around 35 kW are required for EC operation. Similar to the driving profile input, the single-loop control approach is also tested for step input responses. The corresponding linear response results are shown in Figure 9c. Compared with the open-loop system, the proposed controller provides faster responses as indicated in Figure 9c. After evaluating the above two output responses, it is suggested that a single-loop control approach is used for smaller-rating ECs, robotic system operation, go-karts and moveable power chairs for the disabled persons.

### 3.4 | Scenario-3: closed-loop testing model of electric car (single-loop control approach for model-2)

Similar to model-1, in this scenario a single-loop speed control approach is used to test the EC model.

#### 3.4.1 | Control structure design for EC model-2

In this proposed approach, a step input ($V_{in} = 36$ V) is considered as an input reference signal for the test model. In this case, a similar combined load condition is used to test the EC model. To make the system faster than that of the model-1 response results, in model-2, the current sensors are used. The complete system model diagram with the current sensor is illustrated in Figure 10a.

#### 3.4.2 | Testing

In EC design, current sensors play an important role by sensing the accurate armature current. The main aim of the current sensor is to relate the load torque ($T_L$) to the motor torque ($T_M$) by generating appropriate armature current ($I_a$) signals. $T_M$ is computed by multiplying the torque constant $K_t$ with $I_a$. After getting the torque constant, the load torque is divided by $K_t$ to generate appropriate current, and it is sensed using the current sensor. The current sensor has a sensitivity gain ($K_{se} = 0.00238$). Using the sensitivity gain, the current responses become the voltage signals. Using the load fluctuation constraints based on Equations 30 and 38, the angular speed output of the motor is considered to generate an appropriate voltage error component by multiplying the sensitivity gain of the current sensor ($K_{sc}$) and tachometer sensitivity gain ($K_{tach}$). The generated load voltage and tachometer voltage error are now added to generate the optimum voltage error. After generating the optimum voltage error, it is compared with the reference step input voltage profile ($V_{in} = 36$) and fed to the PID controller to generate an appropriate control signal for the EC model. In this testing model, two test conditions, such as error voltage computed with the current sensor and tachometer sensitivity constraints as indicated in Figure 10a, and error voltage computed through tachometer sensitivity constraints as indicated in Figure 10b, are used to test the performance of the design model. During the use of only tachometer sensitivity to compute the voltage error, the load torque is used for comparison with the motor torque responses. Evaluating both of the test responses as indicated in Figures 10b and 10c, it is shown that using the combined voltage error approach, the control signal is settled in minimum time. Because of the faster control responses, linear output responses such as the speed of EC, angular speed, linear position, and linear acceleration are achieved with a minimum time interval. Therefore, it is suggested that the EC model is operated through the single-loop speed control model with both the current sensor and tachometer sensitivity gain.

### 3.5 | Scenario-4: closed-loop testing model of electric car (two-loop control approach for model-1)

In accord with the previously discussed scenarios, all the above methods draw high armature current. However, due
to the high current drawn, the electrical motor rating of the EC must be increased to a higher value. The increase in motor rating also increases the weight and cost of the EC. As a solution to the above high current-draw problems, two control loop approaches, the inner-current regulator and outer-speed regulator approaches, are

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**FIGURE 9** (a) Speed control model with tachometer sensitivity gain and proportional-integral (PI) controller, (b) Output responses of speed control model with tachometer sensitivity gain and PI controller using driving profile input condition, (c) Output responses of speed control model with tachometer sensitivity gain and PI controller using step input condition.
suggested for model-1 of the EC. The two control loops require two PI controllers for speed and current regulation. According to EC current demand, the inner loop controls the current and EC speed demand, and the outer loop controls the speed of the EM. In this proposed approach, the current and speed regulator are separately modelled because two different subsystems are used to regulate ECs with different characteristics.

The combined two-control approach of a single-machine electric car test case is illustrated in Figure 11. The regulator models are presented in the following sections.

3.5.1 | Current regulator

As shown in Figure 11, the current regulator is the inner loop attached to the stator (field winding) of EM in an SMEC system. The proposed current regulator is used to regulate the current within a certain limit through an inductor during variations in load. In this proposed approach, to design an inner current control loop, the PID/PI regulator is chosen to offer a small peak overshoot and better tracking performance of current control in the type-1 system. The transfer function of the PI regulator \(G_c(s)\) is presented below:
\[ G_c(s) = K_{pc} \times \frac{sT_c + 1}{sT_c} \]  

(42)

where \( K_{pc} \) is known as the proportional gain (nearer to 1.68), \( K_{ic} \) is known as the integral gain, and \( T_c \) is denoted as the time constant of the PI regulator (near to 0.08 for faster action). During controller design, it is assumed that the proposed inner loop operates faster than the outer loop. During controller action, the \( P_{z_{cross}} \) (\( Z_0 = K_{ic}/K_{pc} \)) factor inversely affects system performance. Therefore, to eliminate the \( Z_0 \) factor, a prefilter is used in the SMEC system. The transfer function of the prefilter \( (G_p(s)) \) is presented as follows:

\[ G_p(s) = \frac{z_0}{s} = \frac{1}{sT_c + 1} \]  

(43)

3.5.2 | Speed regulator

As shown in Figure 11, the speed regulator is the outer loop of the SMEC model. The proposed speed regulator loop offers smooth and comfortable riding, zero steady-state error, and reduced disturbance during transient conditions. In this proposed approach to achieving a comfortable condition, a PID/PI control-based speed regulator is suggested. The transfer function of the PI regulator for the speed regulator \( (G_a(s)) \) is presented as follows:

\[ G_a(s) = \frac{sK_{ps} + K_{ii}}{s} = \frac{K_{ps}}{s} \left( \frac{s + K_{ii}}{K_{ps}} \right) = K_{ps} \left( \frac{s + 1}{K_{ps} s} \right) \]  

(44)

where \( K_{ps} \) is known as the proportional gain, \( K_{ii} \) is known as the integral gain, and \( T_s \) is denoted as the time constant of the PI regulator. The parameters of the PI regulator computed from the open-loop transfer function are presented as follows:

\[ K_{ps} = \frac{J}{2TD}, \quad K_{ii} = \frac{J}{4TD} \]  

(45)

where \( T_D \) is denoted as the combination of time delay due to the outer loop. In outer loop condition, the prefilter approach is also used to cancel the \( Z_0 \) factor.

3.5.3 | Inverter model

In this approach, the input voltage \( (V_{in}) \) is equal to 36, which is fed to the inverter. The role of the inverter is to convert the

**FIGURE 12** (a) Output response of electric car (EC) control model-1 using both current sensor and tachometer sensitivity gain with driving profile input (b) Output response of electric control model-1 using both current and speed control loop with driving profile input (c) Output response of EC control model-1 using both current and speed control loop with step input profile
FIGURE 13  (a) Electric car (EC) control model-2 using both current and speed control loop with step input profile without prefiltre (b) EC control model-2 using both current and speed control loop with driving profile input with prefiltre (c) EC control model-2 using both current and speed control loop with step input profile and prefiltre
DC voltage to AC voltage. The conversion depends on the PWM strategy. The output voltage of the system is regulated through the duty ratio \( D \) of the PWM signal. The transfer function of the inverter model \( G_{i}(s) \) and related detailed explanation are presented in [2–4] and reflected in Equation 46. The inner loop PI regulator regulates the inverter switching frequency to decrease the ripples in the motor torque and current in an SMEC system:

\[
G_{i}(s) = \frac{K_{pum}}{sT_{sw} + 1}
\]  

(46)

where \( K_{pum} \) is known as the inverter gain (near or equal to 5), and \( T_{sw} \) is the switching time constant of the PWM controller (near 0.25 ms).

### 3.5.4 | Testing

In this scenario, the testing of the SMEC occurs by considering both reference inputs and two combined load torque systems. The reference inputs are regulated through a manual switch. In this scenario, the proposed approach is used to test the SMEC without using the current sensors. As shown in Figure 11, using both outer and inner control loops and applying the driving profile input condition results in linear responses such as speed, angular speed, armature current, motor torque, acceleration, and angular position, as illustrated in Figure 12a. The same model-1-based SMEC system is tested using the proposed design structures such as load torque with current sensor sensitivity \( K_{a} \), tachometer sensitivity \( K_{tech} \), and corresponding voltages. The proposed system results in linear responses such as speed, angular speed, armature current, motor torque, acceleration, and angular position as illustrated in Figure 12b. By comparing Figure 12a,b, it is concluded that using the proposed approach with both sensitivity gains, the system tracks the required speed by drawing nearly 60% less current (around 100 A) than the single-loop-based SMEC armature current (240 A). A similar test is also applicable for the step input reference signal. The corresponding results are illustrated in Figure 12c. Therefore, by analysing the above results, it is suggested that the systems can be operated and designed by combining both the current and the speed loops with the appropriate sensitivity gain of the current sensor and tachometer.

### 3.6 | Scenario-5: closed-loop testing model of electric car (two-loop control approach for model-2)

#### 3.6.1 | Control structure design for EC model-2

In this proposed approach, EC control model-2 is designed by tracking the armature current through current sensors. The tracking current is given as a feedback signal to the current control loop. The speed control loop is similar to Scenario-4.
As discussed previously, in model-2 a prefilter is connected, and the performance is studied by comparing it with the without-prefilter model-2. Figure 11 shows the EC model-2 diagram without prefilter, and Figure 14 shows the EC model-2 diagram of the with-prefilter design. The performance of both control approaches is tested using both input conditions —step input and driving input profile.

3.6.2 Testing

By simulating the simulation model shown in Figure 13a with a step input signal, the controller takes 4 s to obtain linear output responses such as linear speed, angular speed, and load torque. However, using the proposed control approach, system performance lags in achieving linear acceleration. To solve the above problem, a prefilter is connected before the speed and current control loop as indicated in Figure 14. The modelled system is now simulated with both driving input and step input profile responses, and the output responses are compared with the responses shown in Figure 11. The output responses of model-2 during the driving input profile and the step input profile are illustrated in Figure 13b,c, respectively. The results indicate that using prefilters, the output responses are more linear, have minimal overshoot, and offer excellent tracking operation compared with the without-prefilter-based EC model-2. Figure 13c shows that the settling time of the output responses is much less at around 2.5 s. Therefore, it is suggested that both the prefilter-based current and speed-control loop are designed for the EC to achieve best results.

4 CONCLUSION

The control dynamics of the EC are complex and require an extraordinary number of interconnected electrical systems to perform the desired operation. To achieve the desired operation, it is first necessary to compute all external disturbances acting as forces on the EC, such as wind velocity and wheel position, and internal disturbances such as battery and motor conditions. By considering all possible disturbances, the load model is designed. In considering the combined load model, EC performance is tested using various control strategies. The mathematical modelling of the EC and related control strategies is designed by focussing on three major factors:

- properly identifying all possible operating modes such as starting and stopping of the EC;
- appropriately computing all probable transitions between starting and stopping conditions; and
- arbitration of the urgencies between the simultaneous transitions.

By properly evaluating all of the above conditions in the proposed approach, a combined outer-speed and inner-current control loop is suggested and tested for different inputs, control models, and disturbance conditions. In addition, a prefilter, current sensor, and tachometer sensitivity gain are used to generate improved linear output response with robustness and adaptive performance during both steady-state and dynamic conditions of the system. To provide a clear understanding of the system and control approaches, detailed mathematical modelling is presented.

Using the proposed EC model, the system offers enhanced performance in many areas such as

- an increase in torque at low speed for starting and uphill as well as high power at high speed for travelling operation;
- a wide choice of speeds at persistent torque and power operation;
- fast torque response;
- increase in efficiency during high speed and torque ranges;
- increase in efficiency and reliability during regenerative braking; and
- affordable cost and simpler design.

In the future, because they are environmentally friendly, simple and advanced, renewable energy—based hybrid ECs will gain in interest compared with internal combustion engine vehicles. To provide better efficiency and renewable operation, a focus on battery life and capacity is required. Therefore, during design, battery size, storage capacity and cost aspects must be focussed on by power engineers. To make the environment pollution free, plug-in hybrid vehicles offer excellent performance compared with traditional vehicle operation. From a vehicle-improvement point of view, control and coordination between generating and charging stations are also very important. To improve stability and performance, synchronisation between the electrical and mechanical parts will be a necessary focus in the future. By circumventing the above factors, the future of the electric vehicle becomes brighter and the first choice of consumers. In this proposed model, the obtained and analysed simulated outcomes serve as the basis for an appropriately robust and adaptive control approach for the EC model in a real-time application.

ACKNOWLEDGEMENT

Assistance provided by Council of Scientific and Industrial Research (CSIR) and Siksha O Anusandhan University

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APPENDIX

Nomenclature

| Load Parameter | Symbol | Values |
|----------------|--------|--------|
| Mass of the car | $M_c$ | 1000 kg |
| Velocity of EC | $V_C$ | 82.8 km/h |
| Driving angle | $\theta$ | 60 deg |
| Rolling coefficient | $C_r$ | 0.01 |
| Drag coefficient | $C_d$ | 0.8 |
| Density of air at 20°C | $\rho$ | 1.2041 kg/m$^3$ |
| Front area | $A_f$ | 1.5 m$^2$ |
| Moment of inertia | $I_W$ | 0.02 kgm$^2$ |
| Radius of the wheel | $r$ | 0.33 |

| Machine Parameter | Symbol | Values |
|-------------------|--------|--------|
| Input voltage | $V_{in}$ | 36 V |
| Moment of inertia | $J_I$ | 0.02 kgm$^2$ |
| Damping friction | $B_d$ | 0.03 Nm/rad.s |
| Armature resistance and inductance | $R_a$, $L_a$ | 1 $\Omega$, 0.23 H |
| Torque constant | $K_t$ | 0.023 N-m/A |
| Electromotive constant | $K_b$ | 0.023 V-s/rad |
| Gear ratio | $n$ | 3:1 |

| Modelling parameter | Symbol | Values |
|---------------------|--------|--------|
| Current sensor gain | $K_i$ | 0.00238 |
| Tachometer gain | $K_{stab}$ | 0.4696 |
| PWM constant | $K_{PWM}$ | 5 |
| Switching time instant | $T_{sw}$ | 0.0025 |

How to cite this article: Sahoo B, Routray SK, Rout PK. Advanced Speed-and-current control approach for dynamic electric car modelling. IET Electr. Syst. Transp. 2021;1–18. https://doi.org/10.1049/els2.12015