1 Photonic reservoir model

The numerical model follows the Lang-Kobayashi rate equations of a SL with time-delayed feedback and with an additional optical injection dynamical term. The slowly varying electrical field amplitude \( E_r(t) \) that corresponds to the optical emission of the response laser is calculated using the following equations:

\[
\frac{dE_r(t)}{dt} = \frac{1}{2} (1 + j\alpha) \left[ G_r(t) - \frac{1}{t_{ph}} \right] \cdot E_r(t) + \frac{r_c}{t_{in}} \cdot E_r(t - \tau) e^{j(\omega_0 \tau + \phi)} + \frac{r_{inj}}{t_{in}} \cdot E_{inj}(t) e^{-j\Delta \omega t} \tag{1}
\]

\[
\frac{dN_r(t)}{dt} = \frac{I_r}{q} - \frac{N_r(t)}{t_s} - G_r \cdot |E_r|^2 \tag{2}
\]

\[
G_r(t) = g_n \cdot [1 + s |E_r(t)|^2]^{-1} \cdot [N_r(t) - N_0] \tag{3}
\]

\[
E_{inj}(t) = E_{inj,0} \cdot h(t) \cdot J(t) \tag{4}
\]

The information input to be processed by the photonic reservoir \( J(t) \) is in our case the detected signal that emerges from the optical communication task. It is normalized in the \([0, 1]\) range, pre-processed (masked) by a \( h(t) \) random temporal sequence with repetition time \( \tau \) and is inserted into the reservoir as a linear modulation of the drive SL emission. The injected electrical field to the reservoir is of the form of (4), with an arbitrary CW amplitude of \( E_{inj,0} \). The parameter set used for simulating the physical system of the photonic reservoir appears in the Supplementary Table 1.

**Supplementary Table 1:** Numerical simulation parameters for the Lang-Kobayashi rate equations model

| Parameter                      | Value |
|--------------------------------|-------|
| Frequency detuning \( \Delta f \) | -60 to 60 GHz |
| Angular frequency detuning \( \Delta \omega \) | \( 2\pi \cdot \Delta f \) |
| Response laser threshold current \( I_{th} \) | \( 15.37 \cdot 10^{-3} \) A |
| Response laser bias current \( I_r \) | \( 0.995 \cdot I_{th} \) |
| Linewidth enhancement factor \( \alpha \) | 3 |
| Gain coefficient \( g_n \) | \( 1.2 \cdot 10^{-5} \) ns\(^{-1} \) |
| Gain saturation coefficient \( s \) | \( 5 \cdot 10^{-7} \) |
| Carrier number at transparency \( N_0 \) | \( 1.5 \cdot 10^8 \) |
| Carrier lifetime \( t_s \) | \( 2 \) ns |
| Reservoir SL roundtrip time \( t_{in} \) | \( 10^{-2} \) ns |
2 Optical communications signal recovery task

We have applied the TDRC technique to recover binary streams that are encoded in a 180km-long coherent transmission system at 1550 nm at 28 GBd/s. The parameters used for the numerical simulation of the transmission system are provided in Supplementary Table 2. For each binary encoding value in the optical communication task, we obtain a pattern of 8 samples (blue lines, Suppl. Fig. 1) that represent a binary value of “0” or “1” (red lines, Suppl. Fig. 1). However, the nonlinear distortion after 180 km of transmission results in patterns from which it is not easy to identify the initially encoded bits. A simple decision threshold detection applied in these patterns results in a decoding log(BER) as high as -0.7.

### Supplementary Figure 1:
Segment of the input stream obtained from the transmission task (A) before, and (B) after the masking pre-processing. Each binary pattern (red) has 8 samples resolution in the fiber transmission task (blue). Each sample value is assigned to 10 virtual nodes with different mask weights. Thus, the pattern of each bit time-frame is masked into $N$ different values that represent the $N = 80$ virtual nodes of the delay-based reservoir.

### Supplementary Table 2: Numerical simulation parameters for the transmission link

| Parameter                                      | Value                        |
|-----------------------------------------------|------------------------------|
| SSMF transmission loss coefficient            | 0.2 dB/km                    |
| SSMF chromatic dispersion coefficient         | 17 ps/(nm-km)                |
| DCF transmission loss coefficient             | 0.6 dB/km                    |
| DCF chromatic dispersion coefficient          | -100 ps/(nm-km)              |
| Nonlinear refractive index                    | $2.6 \cdot 10^{-20}$ m$^2$/W |
## At the output layer of the TDRC, the ridge regression algorithm is used to train a classifier that recovers the binary sequence which was encoded in the communication task. As we discussed in the manuscript, nonlinear effects transfer information from one pattern to its neighboring patterns. Thus, an optimized training of the system occurs when considering the virtual nodes’ responses from neighboring patterns. For example, if information from 4 neighboring patterns is affecting the properties of the current pattern, then the total number of the virtual nodes’ responses that will be considered in the linear classifier will be 5, i.e. $5 \cdot N = 400$ responses, when $N = 80$. For example, training on the 3rd bit in Suppl. Fig. 1 (A) (samples 17-24), the number of reservoir responses considered in training will be the samples 1 to 400 of Suppl. Fig. 1 (B) after going through the reservoir. The linear classifier is constructed via ridge regression algorithm, as defined in the function "ridge" in Matlab programming libraries. The ridge regression parameter $\lambda$ has been optimized for the task and fixed to a value of $\lambda = 0.01$.

3 Benchmark test on Santa Fe time-series prediction

Here we apply our consideration in a second benchmark test, which has been commonly used in reservoir computing community. The Santa Fe time-series is one of the six data sets that were used in the "Time Series Prediction Competition" in 1994 [1] and was obtained from measuring the output of a pulsing laser. We aim at this time-series prediction task to predict the future value of the time-series, $y(t+1)$, by considering the previous values up to time $t$. We use this task to generalize the findings of the bandwidth enhanced operation of the TDRC under strong optical injection conditions. Specifically, each time-step input takes one sample value from the Santa Fe time-series, its is masked with $N$ random weight values and then inserted into the reservoir. Thus, each sample point of the time-series is expanded through interpolation over $N = 80$ virtual nodes defined within the delay loop $\tau$. Suppl. Fig. 2 illustrates the masking procedure.

The Santa Fe data set contains in total 9000 sample points, from which we use the first 2000 points as a training set, then we leave a gap of 1000 points, and finally, we use the following 4000 points to test the system. The linear classifier (output layer) is constructed via the same ridge regression algorithm that was used in the optical communications task. The performance of the prediction task is evaluated by calculating normalized mean square error (NMSE) between the target and the reservoir output prediction:

$$\text{NMSE} = \frac{1}{L} \sum_{n=1}^{L} (y(n) - \bar{y}(n))^2$$

(5)

3
where the predicted value $y$, and the expected value $\bar{y}$ are normalized to zero mean and unit variance. $L = 4000$ is the number of data points used in the test set. By considering the same photonic reservoir model explained in Sec. 1, we evaluate the prediction performance for two different sizes of the time delay $\tau$ ($\theta = 12$ ps and $\theta = 100$ ps) and two different values of the injection parameter ($r_{inj} = 0.4$ and $r_{inj} = 2$).

### 3.1 Results and discussion for time-series prediction task

Initially we consider a photonic reservoir with feedback time delay of $\tau = 8$ ns ($\theta = 100$ ps) and evaluate its efficiency to predict the next point of the Santa Fe time-series. For the two cases of optical injection ($r_{inj} = 0.4$ and $r_{inj} = 2$), we make a physical parameters $\{r_c, \Delta f\}$ scan of the photonic reservoir. The results are shown in Suppl. Fig. 3 (A) and (B), respectively.

For a photonic reservoir with the smaller virtual node separation ($\theta = 12$ ps) and thus short feedback cavity ($\tau = 960$ ps), we repeat the above prediction evaluation, by taking into account the optical feedback phase dependence. Thus, the physical parameters scan is now in a three-dimensional $\{r_c, \Delta f, \varphi\}$ parameter space. The NMSE comparison is shown in Suppl. Fig. 3 (C) and (D) for $r_{inj} = 0.4$ and $r_{inj} = 2$, respectively. In these two-dimensional $\{r_c, \Delta f\}$ graphs, each value is obtained by scanning the optical feedback phase value in the range $[0, 2\pi]$. Then, the NMSE with the smallest value is recorded.

The findings obtained from the NMSE performance of the Santa Fe time-series prediction are aligned completely with the overall performance obtained for the optical communications task. For the slow transient operation ($\theta = 100$ ps and $\tau = 8$ ns), we find the lowest value of the NMSE (0.0064) for a certain combination of $\{r_c, \Delta f\}$ when $r_{inj} = 0.4$. When considering $r_{inj} = 2$, the minimum NMSE is slightly higher (0.0126) but we obtain more tolerance to the operating parameter space variations. For the fast transient operation ($\theta = 12$ ps and $\tau = 960$ ps), with moderate injection ($r_{inj} = 0.4$), we find a minimum NMSE = 0.0114. The best results are obtained for the combination of fast transient operation ($\theta = 12$ ps) and strong optical injection ($r_{inj} = 2$), where we find a minimum NMSE of 0.0045, along with a large tolerance to the operating parameter space variations.
Supplementary Figure 3: NMSE performance for the Santa-Fe time-series prediction task. Figures (A) and (B) correspond to NMSE after RC post-processing in the parameter space \( \{r_c, \Delta f\} \) for the slow transient regime (\( \theta = 100 \) ps) with \( r_{\text{inj}} = 0.4 \) and \( r_{\text{inj}} = 2 \), respectively. For the fast transient regime (\( \theta = 12 \) ps), the NMSE is calculated after the RC post-processing in the parameter space \( \{r_c, \Delta f, \phi\} \) for (C) moderate optical injection (\( r_{\text{inj}} = 0.4 \)) and (D) for strong optical injection (\( r_{\text{inj}} = 2 \)).

4 Experimental configuration and parameters

Our experimental photonic reservoir is shown in Suppl. Figure 4. It has one input port that receives the information to be processed (data in) and one output port that provides the responses for the reservoir computation (data out). The masked data from the assigned task are uploaded in an 9.6 GHz arbitrary waveform generator (AWG) and transformed into an electrical signal with a temporal resolution of 100 ps (10 GSamples/s). The AWG’s electrical output is amplified with a 30 GHz broadband RF amplifier (RFA) and is uploaded onto the optical carrier of the driver laser, via a 20 GHz Mach-Zehnder modulator (MZM). The wavelength of the drive DFB SL is tuned around 1542 nm, by controlling the operating temperature of the SL. The optical injection power that is sent to the response laser is tuned by an electrically controlled variable optical attenuator (ATT-1). The polarization state of the optical carrier is tuned via a polarization controller (POL-1) and is aligned with the polarization state of the response laser’s emission. The coupling of the light into the response laser is done through a 75/25 optical splitter (SPL-1) and a 3-port optical circulator (CIR). The response laser is biased at 11.8 mA (or at 0.995 its current threshold), while the wavelength of its optical emission is also at 1542 nm and is always fixed. The frequency detuning between the drive and the response laser is tuned with 2 GHz resolution. The optical feedback time delay of the photonic reservoir is \( \tau = 16.8 \) ns and includes a polarization controller (POL-2) to tune the cavity polarization state, an electrically controlled variable optical attenuator (ATT-2) to tune the optical feedback strength and a 50/50 splitter (SPL-2) to close the feedback loop. The other splitter port is used as the output port of the photonic reservoir. The signal from this port is amplified by a semiconductor optical amplifier (SOA) and a tunable optical filter (OF) that minimizes spontaneous optical emission noise effects. An optical isolator (OI) minimizes any back reflections to the reservoir cavity. Finally, the optical signal is detected by a 20GHz photoreceiver and the electrical
signal is recorded by a 16GHz (80 Gsamples/s) real-time oscilloscope. Averaging mode (16 repetitions) is used in the signal recording, in order to improve its signal to noise ratio. The frequency detuning $\Delta f$ is determined experimentally, by measuring the wavelength difference between the response and the drive laser’s optical emission with a 10MHz resolution optical spectrum analyzer.

**Supplementary Figure 4:** Experimental configuration of the photonic reservoir. AWG: Arbitrary waveform generator, RFA: RF broadband amplifier, MZM: Optical Mach-Zehnder modulator, ATT: Optical attenuator, POL: Optical polarization controller, SPL: Optical splitter, CIR: 3-port optical circulator, OI: Optical isolator, SOA: Semiconductor optical amplifier, OF: Tunable optical filter, OSC: Real time oscilloscope.

**References**

[1] Gershenfeld NA, Weigend AS. The future of time series. Proceedings of the NATO advanced Research Workshop on Comparative Time Series Analysis, 1992.