Efficacious Extra U(1) Factor for the Supersymmetric Standard Model

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Abstract

The totality of neutrino-oscillation phenomena appears to require the existence of a light singlet neutrino. As pointed out recently, this can be naturally accommodated with a specific extra U(1) factor contained in the superstring-inspired E$_6$ model and its implied particle spectrum. We analyze this model for other possible consequences. We discuss specifically the oblique corrections from Z-Z' mixing, the phenomenology of the two-Higgs-doublet sector and the associated neutralino sector, as well as possible scenarios of gauge-coupling unification.
1 Introduction

There are experimental indications at present for three types of neutrino oscillations: solar[1], atmospheric[2], and laboratory[3]. Each may be explained in terms of two neutrinos differing in the square of their masses by roughly $10^{-5}\ eV^2$, $10^{-2}\ eV^2$, and $1\ eV^2$ respectively. To accommodate all three possibilities, it is clear that three neutrinos are not enough. On the other hand, the invisible width of the $Z$ boson is saturated already with the three known neutrinos, each transforming as part of a left-handed doublet under the standard electroweak $SU(2)\times U(1)$ gauge group. There is thus no alternative but to assume a light singlet neutrino which also mixes with the known three doublet neutrinos. As pointed out recently[4], this can be realized naturally with a specific extra $U(1)$ factor contained in the superstring-inspired $E_6$ model and its implied particle spectrum.

In Section 2 we map out the essential features of this supersymmetric $SU(3)_C\times SU(2)_L\times U(1)_Y \times U(1)_N$ model. In Section 3 we study the mixing of the standard $Z$ boson with the $Z'$ boson required by the extra $U(1)_N$. We derive the effective contributions of this mixing to the electroweak oblique parameters $\epsilon_{1,2,3}$ or $S, T, U$, and show that the $U(1)_N$ mass scale could be a few TeV. In Section 4 we discuss the reduced Higgs potential at the electroweak scale and show how the two-Higgs-doublet structure of this model differs from that of the minimal supersymmetric standard model (MSSM). In Section 5 we consider the neutralino sector and show how the lightest supersymmetric particle (LSP) of this model is constrained by the Higgs sector. In Section 6 we venture into the realm of gauge-coupling unification and propose two possible scenarios, each with some additional particles. Finally in Section 7 there are some concluding remarks.
2 Description of the Model

The supersymmetric particle content of this model is given by the fundamental 27 representation of $E_6$. Under $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_N$, the individual left-handed fermion components transform as follows[4].

\begin{align*}
(u, d) & \sim (3; 2; \frac{1}{6}; 1), \quad u^c \sim (3^*; 1; -\frac{2}{3}; 1), \quad d^c \sim (3^*; 1; \frac{1}{3}; 2), \quad (1) \\
(\nu_e, e) & \sim (1; 2; -\frac{1}{2}; 2), \quad e^c \sim (1; 1; 1; 1), \quad N \sim (1; 1; 0; 0), \quad (2) \\
(\nu_E, E) & \sim (1; 2; -\frac{1}{2}; -3), \quad (E^c, N_E^c) \sim (1; 2; \frac{1}{2}; -2), \quad (3) \\
h & \sim (3; 1; -\frac{1}{3}; -2), \quad h^c \sim (3^*; 1; \frac{1}{3}; -3), \quad S \sim (1; 1; 0; 5). \quad (4)
\end{align*}

As it stands, the allowed cubic terms of the superpotential are $u^c (uN^c_E - dE^c)$, $d^c (uE - d\nu_E)$, $e^c (\nu_e E - e \nu_E)$, $Shh^c$, $S (EE^c - \nu_E N_E^c)$, and $N (\nu_e N_E^c - e E^c)$, as well as $hu^c e^c$, $hd^c N$, and $N^3$.

We now impose a $Z_2$ discrete symmetry where all superfields are odd, except one copy each of $(\nu_E, E)$, $(E^c, N_E^c)$, and $S$, which are even. This gets rid of the cubic terms $hu^c e^c$, $hd^c N$, and $N^3$, but allows the quadratic terms $hd^c$, $\nu_e N_E^c - e E^c$, and $N^2$.

The bosonic components of the even superfields serve as Higgs bosons which break the gauge symmetry spontaneously. Specifically, $\langle S \rangle$ breaks $U(1)_N$ and generates $m_h$ and $m_E$; the electroweak $SU(2)_L \times U(1)_Y$ is then broken by two Higgs doublets as in the MSSM, with $\langle N_E^c \rangle$ responsible for $m_u$, $m_D$, and $m_1$, and $\langle \nu_E \rangle$ for $m_d$, $m_e$, and $m_2$. The mass matrix spanning the fermionic components of $\nu_e$, $N$, and the odd $\nu_E$, $N_E^c$, and $S$ is then given by

\[
\mathcal{M} = \begin{bmatrix}
0 & m_D & 0 & m_3 & 0 \\
m_D & m_N & 0 & 0 & 0 \\
0 & 0 & 0 & m_E & m_1 \\
m_3 & 0 & m_E & 0 & m_2 \\
0 & 0 & m_1 & m_2 & 0
\end{bmatrix},
\]

where the mass term $m_N$ is expected to be large because $N$ is trivial under $U(1)_N$ and may thus acquire a large Majorana mass through gravitationally induced nonrenormalizable
interactions\cite{5}, and \(m_3\) comes from the allowed quadratic term \(\nu_eN_E^c - eE^c\). This means that the usual seesaw mechanism holds for the three doublet neutrinos: \(m_\nu \sim m_D^2/m_N\), whereas the two singlet neutrinos have masses \(m_S \sim 2m_1m_2/m_E\) and mix with the former through \(m_3\). Note that \(\mathcal{M}\) is really a 12 \(\times\) 12 matrix because there are 3 copies of \((\nu_e, N)\) and 2 copies of \((\nu_E, N_E^c, S)\).

\section{Z-Z' Mixing}

Let the bosonic components of the even superfields \((\nu_E, E), (E^c, N_E^c),\) and \(S\) be denoted as follows:

\[
\Phi_1 \equiv \begin{pmatrix} \bar{\nu}_1 \\ \bar{E} \end{pmatrix}, \quad \Phi_2 \equiv \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix}, \quad \chi \equiv \tilde{S}. \quad (6)
\]

The part of the Lagrangian containing the interaction of the above Higgs bosons with the vector gauge bosons \(A_i\) \((i = 1, 2, 3)\), \(B\), and \(Z'\) belonging to the gauge factors \(SU(2)_L, U(1)_Y,\) and \(U(1)_N\) respectively is given by

\[
\mathcal{L} = |(\partial^\mu - \frac{ig_2}{2} \tau_i A_i^\mu + \frac{ig_1}{2} B^\mu + \frac{3ig_N}{2\sqrt{10}} Z'^\mu)\bar{\Phi}_1|^2 + |(\partial^\mu - \frac{ig_2}{2} \tau_i A_i^\mu - \frac{ig_1}{2} B^\mu + \frac{ig_N}{\sqrt{10}} Z'^\mu)\Phi_2|^2 + |(\partial^\mu - \frac{5ig_N}{2\sqrt{10}} Z'\mu)\chi|^2, \quad (7)
\]

where \(\tau_i\) are the usual 2 \(\times\) 2 Pauli matrices and the gauge coupling \(g_N\) has been normalized to equal \(g_2\) in the \(E_6\) symmetry limit. Let \(\langle \phi_{1,2}^0 \rangle = v_{1,2}\) and \(\langle \chi \rangle = u\), then for

\[
W^\pm = \frac{1}{\sqrt{2}}(A_1 \mp iA_2), \quad Z = \frac{g_2A_3 - g_1B}{\sqrt{g_1^2 + g_2^2}}, \quad (8)
\]

we have \(M_W^2 = (1/2)g_2^2(v_1^2 + v_2^2)\), and the mass-squared matrix spanning \(Z\) and \(Z'\) is given by

\[
\mathcal{M}_{Z,Z'}^2 = \begin{bmatrix}
(1/2)g_2^2(v_1^2 + v_2^2) & (g_N g_Z/2\sqrt{10})(-3v_1^2 + 2v_2^2) \\
(g_N g_Z/2\sqrt{10})(-3v_1^2 + 2v_2^2) & (g_N^2/20)(25u^2 + 9v_1^2 + 4v_2^2)
\end{bmatrix}, \quad (9)
\]
where \( g_Z \equiv \sqrt{g_1^2 + g_2^2} \).

Let the mass eigenstates of the \( Z - Z' \) system be

\[
Z_1 = Z \cos \theta + Z' \sin \theta, \quad Z_2 = -Z \sin \theta + Z' \cos \theta,
\]

then the experimentally observed neutral gauge boson is identified in this model as \( Z_1 \), with mass given by

\[
M_{Z_1}^2 \equiv M_Z^2 \simeq \frac{1}{2} g_Z^2 v^2 \left[ 1 - \left( \sin^2 \beta - \frac{3}{5} \right) \frac{v^2}{u^2} \right],
\]

where

\[
v^2 \equiv v_1^2 + v_2^2, \quad \tan \beta \equiv \frac{v_2}{v_1},
\]

and

\[
\theta \simeq -\sqrt{\frac{2 g_Z}{g_N}} \left( \sin^2 \beta - \frac{3}{5} \right) \frac{v^2}{u^2}.
\]

The interaction Lagrangian of \( Z_1 \) with the leptons is now given by

\[
\mathcal{L} = \left( \frac{1}{2} g_Z \cos \theta + \frac{g_N}{\sqrt{10}} \sin \theta \right) \bar{\nu}_L \gamma_\mu \nu_L Z_1^\mu \\
+ \left( -\frac{1}{2} + \sin^2 \theta_W \right) g_Z \cos \theta + \frac{g_N}{\sqrt{10}} \sin \theta \right) \bar{e}_L \gamma_\mu e_L Z_1^\mu \\
+ \left( \sin^2 \theta_W \right) g_Z \cos \theta - \frac{g_N}{2\sqrt{10}} \sin \theta \right) \bar{e}_R \gamma_\mu e_R Z_1^\mu,
\]

where the subscripts \( L(R) \) refer to left(right)-handed projections and \( \sin^2 \theta_W = \frac{g_1^2}{g_Z^2} \) is the usual electroweak mixing parameter of the standard model. Using the leptonic widths and the forward-backward asymmetries, the deviations from the standard model are conveniently parametrized \( \epsilon_1, \epsilon_2, \epsilon_3 \):

\[
\epsilon_1 = \left( \sin^4 \beta - \frac{9}{25} \right) \frac{v^2}{u^2} = \alpha T,
\]

\[
\epsilon_2 = \left( \sin^2 \beta - \frac{3}{5} \right) \frac{v^2}{u^2} = -\frac{\alpha U}{4 \sin^2 \theta_W},
\]

\[
\epsilon_3 = \frac{2}{5} \left( 1 + \frac{1}{4 \sin^2 \theta_W} \right) \left( \sin^2 \beta - \frac{3}{5} \right) \frac{v^2}{u^2} = \frac{\alpha S}{4 \sin^2 \theta_W},
\]
where $\alpha$ is the electromagnetic fine-structure constant. In the above we have also indicated how $Z - Z'$ mixing as measured in the lepton sector would affect the oblique $S, T, U$ parameters defined originally for the gauge-boson self energies only\cite{7}. The present precision data from LEP at CERN are consistent with the standard model but the experimental error bars are of order a few $\times 10^{-3}$\cite{8}. This means that $u \sim \text{TeV}$ is allowed. Note also that the relative sign of $\epsilon_{1,2,3}$ is necessarily the same in this model.

4 Two-Higgs-Doublet Sector

The Higgs superfields of this model $(\nu_E, E)$, $(\nu_E^c, N^c_E)$, and $S$ are such that the term $f(\nu_E N_E^c - E E^c) S$ is the only allowed one in the superpotential. This means that a supersymmetric mass term for $S$ is not possible and for $U(1)_N$ to be spontaneously broken, the supersymmetry must also be broken. Consider now the Higgs potential. The quartic terms are given by the sum of

$$V_F = |f|^2[(\Phi_1^\dagger \Phi_2)(\Phi_1^\dagger \Phi_2) + (\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1)(\bar{\chi}\chi)],$$

and

$$V_D = \frac{1}{8} g_2^2[(\Phi_1^\dagger \Phi_2)^2 + (\Phi_2^\dagger \Phi_2)^2 + 2(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) - 4(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1)]$$

$$+ \frac{1}{8} g_1^2[(\Phi_1^\dagger \Phi_1)^2 + (\Phi_2^\dagger \Phi_2)^2 - 2(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2)]$$

$$+ \frac{1}{80} g_N^2[9(\Phi_1^\dagger \Phi_1)^2 + 4(\Phi_2^\dagger \Phi_2)^2 + 12(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) - 30(\Phi_1^\dagger \Phi_1)(\bar{\chi}\chi)]$$

$$- 20(\Phi_2^\dagger \Phi_2)(\bar{\chi}\chi) + 25(\bar{\chi}\chi)^2].$$

The soft terms which also break the supersymmetry are given by

$$V_{soft} = \mu_1^2(\Phi_1^\dagger \Phi_1 + \mu_2^2(\Phi_2^\dagger \Phi_2 + m^2\bar{\chi}\chi + f A\Phi_1^\dagger \Phi_2\chi + (f A)^*\bar{\chi}\Phi_2^\dagger \Phi_1).$$

The first stage of symmetry breaking occurs with $\langle \chi \rangle = u$. From $V_{soft}$ and $V_D$, we see that $u^2 = -8m^2/5g_N^2$. Consequently, $\sqrt{2}Im\chi$ combines with $Z'$ to form a massive vector
gauge boson and $\sqrt{2}Re\chi$ is a massive scalar boson. Both have the same mass:

$$M_{Z'}^2 = m_\chi^2 = \frac{5}{4}g_N^2u^2.$$  \hfill (21)

The reduced Higgs potential involving only the two doublets is then of the standard form:

$$V = m_1^2\Phi_1^\dagger\Phi_1 + m_2^2\Phi_2^\dagger\Phi_2 + m_{12}^2(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1)$$

$$+ \frac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1)^2 + \frac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1),$$  \hfill (22)

where

$$m_1^2 = \mu_1^2 - \frac{3}{8}g_N^2u^2, \quad m_2^2 = \mu_2^2 - \frac{1}{4}g_N^2u^2, \quad m_{12}^2 = fAu,$$  \hfill (23)

assuming that $f$ and $A$ are real for simplicity. In the above, we have of course also assumed implicitly that $m_1^2$, $m_2^2$, and $m_{12}^2$ are all small in magnitude relative to $u^2$. The quartic scalar couplings $\lambda_{1,2,3,4}$ receive contributions not only from the coefficients of the corresponding terms in $V_D$ and $V_F$, but also from the cubic couplings of $\sqrt{2}Re\chi$ to the doublets which are proportional to $u$, as shown in Fig. 1. As a result,

$$\lambda_1 = \frac{1}{4}(g_1^2 + g_2^2) + \frac{9}{40}g_N^2 - \frac{8(f^2 - 3g_N^2/8)^2}{5g_N^2}$$

$$= \frac{1}{4}(g_1^2 + g_2^2) + \frac{6}{5}f^2 - \frac{8f^4}{5g_N^2},$$  \hfill (24)

$$\lambda_2 = \frac{1}{4}(g_1^2 + g_2^2) + \frac{1}{10}g_N^2 - \frac{8(f^2 - g_N^2/4)^2}{5g_N^2}$$

$$= \frac{1}{4}(g_1^2 + g_2^2) + \frac{4}{5}f^2 - \frac{8f^4}{5g_N^2},$$  \hfill (25)

$$\lambda_3 = -\frac{1}{4}g_1^2 + \frac{1}{4}g_2^2 + \frac{3}{20}g_N^2 - \frac{8(f^2 - 3g_N^2/8)(f^2 - g_N^2/4)}{5g_N^2}$$

$$= -\frac{1}{4}g_1^2 + \frac{1}{4}g_2^2 + f^2 - \frac{8f^4}{5g_N^2},$$  \hfill (26)

$$\lambda_4 = -\frac{1}{2}g_1^2 + f^2.$$  \hfill (27)

It is obvious from the above that the two-Higgs-doublet sector of this model differs from that of the minimal supersymmetric standard model (MSSM) and reduces to the latter only in the limit $f = 0$. Note that if $m_{12}^2$ is of order $m_\chi^2$, then it is not consistent to assume that both $\Phi_1$ and $\Phi_2$ are light. In that case, only a linear combination of $\Phi_1$ and $\Phi_2$ may be light and
the electroweak Higgs sector reduces to that of just one doublet, as in the minimal standard model.

Since $V$ of Eq. (22) should be bounded from below, we must have

$$
\lambda_1 > 0, \quad \lambda_2 > 0, \quad \lambda_1 \lambda_2 - (\lambda_3 + \lambda_4)^2 > 0 \quad \text{if} \quad \lambda_3 + \lambda_4 < 0. \quad (28)
$$

Hence $f^2$ has an upper bound. For $g_N^2 = (5/3)g_1^2$ which is a very good approximation if $U(1)_Y$ and $U(1)_N$ are unified only at a very high energy scale, we find that the ratio $f^2/g_Z^2$ has to be less than about 0.35. After electroweak symmetry breaking, the upper bound on the lighter of the two neutral scalar Higgs bosons is given in general by

$$
(m_h^2)_{\text{max}} = 2v^2[\lambda_1 \cos^4 \beta + \lambda_2 \sin^4 \beta + 2(\lambda_3 + \lambda_4) \sin^2 \beta \cos^2 \beta] + \epsilon, \quad (29)
$$

where $\epsilon$ comes from radiative corrections, the largest contribution being that of the top quark:

$$
\epsilon \simeq \frac{3g_2^2m_t^4}{8\pi^2M_W^2} \ln \left(1 + \frac{\tilde{m}^2}{m_t^2}\right), \quad (30)
$$

with $\tilde{m}$ coming from soft supersymmetry breaking. In the present model, this becomes

$$
(m_h^2)_{\text{max}} = 2v^2 \left[ \frac{1}{4}g_Z^2 \cos^2 2\beta + f^2 \left( \frac{3}{2} + \frac{1}{5} \cos 2\beta - \frac{1}{2} \cos^2 2\beta \right) - \frac{8f^4}{5g_N^2} \right] + \epsilon. \quad (31)
$$

Considered as a function of $f^2$, the above quantity is maximized at

$$
f_0^2 = \frac{5g_N^2}{16} \left( \frac{3}{2} + \frac{1}{5} \cos 2\beta - \frac{1}{2} \cos^2 2\beta \right). \quad (32)
$$

Assuming that $g_N^2 = (5/3)g_1^2$ as before, we find $f_0^2/g_Z^2$ to be always smaller than the upper bound we obtained earlier from requiring $V > 0$. Hence we plot $(m_h)_{\text{max}}$ in Fig. 2 for $f = f_0$ and $f = 0$ as functions of $\cos^2 \beta$, as the maximum allowed values of $m_h$ in this model and in the MSSM respectively. It is seen that for $m_t = 175$ GeV and $\tilde{m} = 1$ TeV, $m_h$ may be as high as 140 GeV in this model, as compared to 128 GeV in the MSSM.
For the charged Higgs boson $H^\pm = \sin \beta \phi^\pm_1 - \cos \beta \phi^\pm_2$ and the pseudoscalar Higgs boson $A = \sqrt{2}(\sin \beta \text{Im} \phi^0_1 - \cos \beta \text{Im} \phi^0_2)$, we now have the sum rule

$$m^2_{H^\pm} = m^2_A + M^2_W - f^2 v^2,$$

(33)

where $m^2_A = -m^2_{12}/\sin \beta \cos \beta$. Note that the above equation is common to all extensions\[9\] of the MSSM with the term $f \Phi^1_1 \Phi_2 \chi$ in the superpotential and would serve as an unambiguous signal of physics beyond the MSSM at the supersymmetry breaking scale.

### 5 The Neutralino Sector

In the MSSM, there are four neutralinos (two gauge fermions and two Higgs fermions) which mix in a well-known $4 \times 4$ mass matrix\[10\]. Here we have six neutralinos: the gauginos of $U(1)_Y$ and the third component of $SU(2)_L$, the Higgsinos of $\phi^0_1$ and $\phi^0_2$, the $U(1)_N$ gaugino and the $\chi$ Higgsino. The corresponding mass matrix is then given by

$$M_N = \begin{bmatrix}
M_1 & 0 & -g_1 v_1/\sqrt{2} & g_1 v_2/\sqrt{2} & 0 & 0 \\
0 & M_2 & g_2 v_1/\sqrt{2} & -g_2 v_2/\sqrt{2} & 0 & 0 \\
-g_1 v_1/\sqrt{2} & g_2 v_1/\sqrt{2} & 0 & fu & -3g_N v_1/2\sqrt{5} & fv_2 \\
g_1 v_2/\sqrt{2} & -g_2 v_2/\sqrt{2} & fu & 0 & -g_N v_2/\sqrt{5} & fv_1 \\
0 & 0 & -3g_N v_1/2\sqrt{5} & -g_N v_2/\sqrt{5} & M_1 & \sqrt{5}g_N u/2 \\
0 & 0 & fv_2 & fv_1 & \sqrt{5}g_N u/2 & 0
\end{bmatrix},
$$

(34)

where $M_{1,2}$ are allowed $U(1)$ and $SU(2)$ gauge-invariant Majorana mass terms which break the supersymmetry softly. Note that without the last two rows and columns, the above matrix does reduce to that of the MSSM if $fu$ is identified with $-\mu$. Recall that if $f$ is very small, then the two-Higgs-doublet sector of this model is essentially indistinguishable from that of the MSSM, but now a difference will show up in the neutralino sector unless the $\mu$ parameter of the MSSM accidentally also happens to be very small. In other words, there is an important correlation between the Higgs sector and the neutralino sector of this model.
which is not required in the MSSM.

Since $g_N u$ cannot be small, the neutralino mass matrix $M_N$ reduces to either a $4 \times 4$ or $2 \times 2$ matrix, depending on whether $fu$ is small or not. In the former case, it reduces to that of the MSSM but with the stipulation that the $\mu$ parameter must be small, i.e. of order 100 GeV. This means that the two gauginos mix significantly with the two Higgsinos and the lightest supersymmetric particle (LSP) is likely to have nonnegligible components from all four states. In the latter case, the effective $2 \times 2$ mass matrix becomes

$$M_N' = \begin{pmatrix} M_1 + g_1^2 v_1 v_2 / fu & -g_1 g_2 v_1 v_2 / fu \\ -g_1 g_2 v_1 v_2 / fu & M_2 + g_2^2 v_1 v_2 / fu \end{pmatrix}.$$  (35)

Since $v_1 v_2 / u$ is small, the mass eigenstates of $M_N'$ are approximately the gauginos $\tilde{B}$ and $\tilde{A}_3$, with masses $M_1$ and $M_2$ respectively. In supergravity models,

$$M_1 = \frac{5 g_1^2}{3 g_2^2} M_2 \simeq 0.5 M_2,$$  (36)

hence $\tilde{B}$ would be the LSP.

In the chargino sector, the corresponding mass matrix is

$$M_\chi = \begin{pmatrix} M_2 & g_2 v_2 \\ g_2 v_1 & -fu \end{pmatrix}.$$  (37)

If $fu$ is small, then both charginos can be of order 100 GeV, but if $fu$ is large (say of order 1 TeV), then only one may be light and its mass would be $M_2$. In the MSSM, the superpotential has the allowed term $\mu \Phi_1^+ \Phi_2$. Hence there is no understanding as to why $\mu$ should be of order of the supersymmetry breaking scale, and not in fact very much greater. Here $fu$ is naturally of order of the $U(1)_N$ breaking scale, and since the latter cannot be broken without also breaking supersymmetry, the two scales are necessarily equivalent. This solves the so-called $\mu$ problem of the MSSM.
6  Gauge-Coupling Unification

In the MSSM, the three gauge couplings $g_3$, $g_2$, and $g_Y = (5/3)^{1/4} g_1$ have been shown to converge to a single value at around $10^{16}$ GeV\[11\]. In the present model, with particle content belonging to complete 27 representations of $E_6$ and nothing else, this unification simply does not occur. This is a general phenomenon of all grand unified models: the experimental values of the three known gauge couplings at the electroweak energy scale are not compatible with a single value at some higher scale unless the particle content (excluding the gauge bosons) has different total contributions to the evolution of each coupling as a function of energy scale. The evolution equations of $\alpha_i \equiv g_i^2/4\pi$ are generically given to two-loop order by

$$\mu \frac{\partial \alpha_i}{\partial \mu} = \frac{1}{2\pi} \left[ b_i + \frac{b_{ij}}{4\pi} \alpha_j(\mu) \right] \alpha_i^2(\mu),$$

where $\mu$ is the running energy scale and the coefficients $b_i$ and $b_{ij}$ are determined by the particle content of the model. To one loop, the above equation is easily solved:

$$\alpha_i^{-1}(M_1) = \alpha_i^{-1}(M_2) - \frac{b_i}{2\pi} \ln \frac{M_1}{M_2}.$$  \hspace{1cm} (39)

Below $M_{\text{SUSY}}$, assume the standard model with two Higgs doublets, then

$$b_Y = \frac{21}{5}, \quad b_2 = -3, \quad b_3 = -7.$$  \hspace{1cm} (40)

Above $M_{\text{SUSY}}$ in the MSSM,

$$b_Y = 3(2) + \frac{3}{5}(4) \left( \frac{1}{4} \right), \quad b_2 = -6 + 3(2) + 2 \left( \frac{1}{2} \right), \quad b_3 = -9 + 3(2).$$  \hspace{1cm} (41)

Note that in the above, the three supersymmetric families of quarks and leptons contribute equally to each coupling, whereas the two supersymmetric Higgs doublets do not. The reason is that the former belong to complete representations of $SU(5)$ but not the latter. For $M_{\text{SUSY}} \sim 10^4$ GeV, the gauge couplings would then unify at $M_U \sim 10^{16}$ GeV in the MSSM.
In the present model as it is, the one-loop coefficients of Eq. (38) above $M_{\text{SUSY}}(\sim u)$ are

$$
\begin{align*}
    b_Y &= 3(3), \\
    b_2 &= -6 + 3(3), \\
    b_3 &= -9 + 3(3), \\
    b_N &= 3(3),
\end{align*}
$$

(42)

because there are three complete $27$ supermultiplets of $E_6$. [Actually $N$ is superheavy but it transforms trivially under $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_N$.] To achieve gauge-coupling unification, we must add new particles in a judicious manner. One possibility is to mimic the MSSM by adding one extra copy of the anomaly-free combination $(\nu_e, e)$ and $(E^c, N^c_E)$. Then

$$
\Delta b_Y = \frac{3}{5}, \quad \Delta b_2 = 1, \quad \Delta b_3 = 0, \quad \Delta b_N = \frac{2}{5}.
$$

(43)

Since the relative differences of $b_Y, b_2, \text{and } b_3$ are now the same as in the MSSM, we have again unification at $M_U \sim 10^{16}$ GeV, from which we can predict the value of $g_N$ at $M_{\text{SUSY}}$.

We show in Fig. 3 the evolution of $\alpha_i^{-1}$ using also the two-loop coefficients

$$
\begin{bmatrix}
    \frac{234}{25} & \frac{54}{5} & \frac{84}{5} & \frac{339}{100} \\
    \frac{18}{5} & 39 & 24 & \frac{73}{20} \\
    3 & 9 & 48 & 3 \\
    \frac{339}{100} & \frac{219}{20} & 24 & \frac{1897}{200}
\end{bmatrix}
$$

(44)

We work in the $\overline{\text{MS}}$ scheme, and take the two-loop matching conditions accordingly[12]. As an example, we use $\alpha = 1/127.9, \sin^2 \theta_W = 0.2317$, and $\alpha_s = 0.116$ at the scale $M_Z = 91.187$ GeV. We also choose $M_{\text{SUSY}} = 1$ TeV and use the top quark mass $m_t = 175$ GeV. Note that the value of $\alpha_N$ is always close to that of $\alpha_Y$ since their one-loop beta functions are close in value to each other and they are required to be unified at the scale $M_U$.

Another possibility is to exploit the allowed variation of particle masses near the superstring scale of $M_U \approx 7g_U \cdot 10^{17}$ GeV [13] in the $\overline{\text{MS}}$ scheme. Just as Yukawa couplings are assumed to be subject only to the constraints of the unbroken gauge symmetry, the masses of the superheavy $27$ and $27^*$ multiplet components may also be allowed to vary accordingly. For example, take three copies of $(u, d) + (u^*, d^*)$ and $(\nu_e, e) + (\nu^*_e, e^*)$ with $M'$ much below

12
Then between $M'$ and $M_U$, 

$$\Delta b_Y = 3 \times \left( \frac{1}{5} + \frac{3}{5} \right) = \frac{12}{5}, \quad \Delta b_2 = 3 \times (3 + 1) = 12, \quad (45)$$

$$\Delta b_3 = 3 \times (2 + 0) = 6, \quad \Delta b_N = 3 \times \left( \frac{3}{10} + \frac{2}{5} \right) = \frac{21}{10}. \quad (46)$$

For $M' \sim 10^{16}$ GeV, gauge-coupling unification at $M_U \sim 7 \times 10^{17}$ GeV is again achieved. We show in Fig. 4 the evolution of $\alpha_i^{-1}$ using also the two-loop coefficients

$$b_{ij} = \begin{bmatrix} 9 & 9 & \frac{84}{5} & 3 \\ 3 & 39 & 24 & 3 \\ 3 & 9 & 48 & 3 \\ 3 & 9 & 24 & 9 \end{bmatrix} \quad (47)$$

between $M_{\text{SUSY}}$ and $M'$, and

$$b_{ij} = \begin{bmatrix} \frac{253}{25} & \frac{81}{5} & 20 & \frac{102}{25} \\ \frac{26}{5} & 123 & 72 & \frac{51}{100} \\ 3 & 27 & 116 & \frac{18}{5} \\ \frac{102}{25} & \frac{153}{10} & 24 & \frac{957}{100} \end{bmatrix} \quad (48)$$

between $M'$ and $M_U$. As an example, we use $\alpha = 1/127.9$, $\sin^2 \theta_W = 0.2317$, and $\alpha_s = 0.123 \pm 0.006$ at the scale $M_Z = 91.187$ GeV, $M_{\text{SUSY}} = 1$ TeV and the top quark mass $m_t = 175$ GeV. For $\alpha_s(M_Z) = 0.123$, we find $M' = 5.9 \times 10^{16}$ GeV. It should be emphasized that the sharp turn at $M'$ should not be taken too literally but only as an indication that gauge couplings may in fact evolve drastically near the unification energy scale. This possibility allows us to have unification without having split multiplets containing both superheavy and light components, as in most grand unified models. The size of $\alpha_N$ is always very close to that of $\alpha_Y$ since they have the same one-loop beta functions for scales beneath $M'$ and are required to be unified at $M_U$. We note that the two-loop corrections are larger here than in the MSSM due to the much larger particle content. We also observe that for $\alpha_s(M_Z) = 0.123$ we obtain an $M_U$ which is about 1.5 times the superstring scale of $7g_U \cdot 10^{17}$ GeV, whereas the $M_U$ in most supersymmetric grand unified models is about 0.04 times that number.
7 Concluding Remarks

To accommodate a naturally light singlet neutrino, an extra $U(1)$ factor is called for. It has been shown\cite{4} that the superstring-inspired $E_6$ model is tailor-made for this purpose as it contains $U(1)_N$ which has exactly the required properties. To obtain $U(1)_N$ as an unbroken gauge group, we need to break $E_6$ spontaneously along the $N$ and $N^*$ directions with superheavy $27$’s and $27^*$’s while preserving supersymmetry. This is impossible if the superpotential is allowed only terms up to cubic order so that the theory is renormalizable. On the other hand, the requirement of renormalizability may not be applicable at the superstring unification scale, in which case the quartic term $M^{-1} 27 27^* 27 27^*$ in conjunction with the quadratic term $m 27 27^*$ in the superpotential would result in $\langle 27 \rangle = \langle 27^* \rangle = (-2mM)^{1/2}$ without breaking supersymmetry.

The addition of $U(1)_N$ has several other interesting phenomenological consequences. (1) The $U(1)_N$ neutral gauge boson $Z'$ mixes with the standard-model $Z$ and affects the precision data at LEP. From the present experimental error bars on the $\epsilon_{1,2,3}$ parameters, we find that the $U(1)_N$ breaking scale could be as low as a few TeV. (2) The spontaneous breaking of $U(1)_N$ is accomplished only with the presence of a mass term in the Higgs potential which breaks the supersymmetry softly. Hence the reduced two-doublet Higgs potential at the electroweak energy scale is not guaranteed to be that of the MSSM. In fact, the scalar quartic couplings now depend also on a new Yukawa coupling $f$ as well as the gauge coupling $g_N$. Assuming that $g_N = (5/3)^{1/2} g_1$, one result is that the upper bound on the lighter of the two neutral scalar Higgs bosons is now 140 GeV instead of 128 GeV in the MSSM. (3) The neutralino mass matrix also depends on $f$, hence there is a correlation here with the Higgs sector. Such a connection is not present in the MSSM. (4) This model may also be compatible with gauge-coupling unification. We identify two possible scenarios. One is just like the MSSM with two light doublets presumably belonging to complete multiplets (of the
grand unified group) whose other members are superheavy; the other requires no light-heavy splitting but assumes a large variation of superheavy masses near the unification scale.

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Figure Captions

Fig. 1: The tree-level Feynman diagrams due to the cubic couplings of the scalar $\sqrt{2}\text{Re}\chi$ to the scalar Higgs doublets $\Phi_{1,2}$ which contribute to the quartic scalar couplings $\lambda_{1,2,3,4}$ of the reduced Higgs potential given in Eq. (22).

Fig. 2: The upper bound on the mass of the lighter of the two neutral scalar Higgs bosons $(m_h)_{\text{max}}$ for $f = f_0$ and $f = 0$ as functions of $\cos^2\beta$, as the maximum allowed values of $m_h$ in the model discussed here and in the MSSM respectively. We have used $\alpha = 1/127.9$ and $\sin^2\theta_W = 0.2317$ at the $M_Z$ scale, $m_t = 175$ GeV, and $\tilde{m} = 1$ TeV.

Fig. 3: The two-loop evolution of the gauge couplings of the unification scenario involving three complete $27$ supermultiplets and one extra copy of $(\nu_e, e)$ and $(E^c, N^c_E)$ with mass of order $M_{\text{SUSY}}$ as explained in the text. We have used $\alpha = 1/127.9$, $\sin^2\theta_W = 0.2317$, and $\alpha_s = 0.116$ at the scale $M_Z = 91.187$ GeV, $M_{\text{SUSY}} = 1$ TeV, and $m_t = 175$ GeV.

Fig. 4: The two-loop evolution of the gauge couplings of the unification scenario explained in the text which involves three complete $27$ supermultiplets for scales above $M_{\text{SUSY}}$ and with three additional copies of $(u, d) + (u^*, d^*)$ and $(\nu_e, e) + (\nu_e^*, e^*)$ with mass of order the intermediate scale $M'$. We have used $\alpha = 1/127.9$, $\sin^2\theta_W = 0.2317$, and $\alpha_s = 0.123 \pm 0.006$ at the scale $M_Z = 91.187$ GeV, $M_{\text{SUSY}} = 1$ TeV, and $m_t = 175$ GeV. The dashed lines correspond to $\alpha_s(M_Z) = 0.117$ and 0.129.
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