Chiral perturbation theory in the meson sector

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The present status of Chiral Perturbation Theory in the meson sector is discussed concentrating on recent developments. This write-up contains short discussions on a listing a few historical papers, the principles behind ChPT, two-flavour ChPT including some comments about the pion polarizability, three-flavour ChPT with a discussion of the recently found relations as tests of ChPT and preliminary results of a new fit of the NLO low-energy-constants (LECs). It discusses somewhat deeper $\eta \rightarrow 3\pi$ and the arguments for the existence of a “hard pion ChPT” and its application to $K \rightarrow 2\pi$.

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1. Introduction

Chiral Perturbation Theory (ChPT) and effective field theory (EFT) play a major role at this now 15 year old conference series. In this talk I review some aspects of mesonic ChPT. This talk has a large overlap with earlier talks given at Lattice 2007 [1] and EFT09 [2] as well as with my review on ChPT at two-loop order [3]. I concentrate on some of the newer results in this written version, consult the earlier references for the topics discussed here in less detail.

In the talk I discussed several topics not included in this write-up. The discussion and references for the partially quenched calculations and recent progress in renormalization group and effective field theory can be found in [2] and for the results for ChPT for the weak interaction a recent review is [4].

This write-up contains short discussions on a listing a few historical papers, the principles behind ChPT, two-flavour ChPT including some comments about the pion polarizability, three-flavour ChPT with a discussion of the recently found relations and preliminary results of a new fit of the NLO low-energy-constants (LECs). I devote quite some space to $\eta \to 3\pi$ and the arguments for the existence of a “hard pion ChPT” and its application to $K \to 2\pi$.

2. Some History: 50, 40, 35, 30, 25, 20 and 15 years ago

In this “capital city” of Chiral Perturbation Theory it is appropriate to look back at some of its history. ChPT has a 50 year history by now as was reviewed by S. Weinberg in his talk [5]. It should also be remarked that this conference series is now 15 years old.

I picked some papers which fell at or close to jubilee years. About 50 years ago our subject started with the Goldberger-Treiman relation [6] and the advent of PCAC, the partially conserved axial-current [7], and how this reproduced the Goldberger-Treiman relation. About 40 years ago a lot of work had been done within the framework of PCAC but 1968 and 1969 saw some very important papers: the Gell-Mann–Oakes–Renner relation [8] and the proper way how to implement chiral symmetry in all generality in phenomenological Lagrangians [9] after Weinberg’s derivation for the two flavour case [10]. Shortly afterwards loop calculations started with e.g. loop results for $\pi\pi$ scattering [11] and $\eta \to 3\pi$ [12]. 30 years ago the start with the modern way of including higher order Lagrangians and performing a consistent renormalization came with [13]. At the same time there was also the beautiful paper by Gasser and Zepeda about the types of non-analytical corrections that can appear [14]. The seminal papers by Gasser and Leutwyler of 25 years ago then put the entire subject on a modern firm footing [15, 16]. The same period also had my own entry into the subject [17]. Many one-loop calculations were done and the understanding that the coefficients in the higher-order Lagrangians could be understood from the contributions of resonances was put on a firm footing 20 years ago [15, 18]. Let me close this historical part with two 15 year old papers, a very clear discussion of the basics of ChPT [19] and the first full two-loop calculation [20].

3. Chiral Perturbation Theory: ChPT, CHPT or $\chi$PT

ChPT is best described as “Exploring the consequences of the chiral symmetry of QCD and its spontaneous breaking using effective field theory techniques” and a clear discussion about its
derivation and underlying assumptions is in [19]. Some reviews are [21, 3]. More reviews and references to introductory lectures can be found on the webpage [22].

For effective field theories, there are three principles that are needed and for ChPT they are

- **Degrees of freedom:** Goldstone Bosons from the spontaneous chiral symmetry breakdown.
- **Power counting:** This is what allows a systematic ordering of terms and is here essentially dimensional counting in momenta and masses.
- **Expected breakdown scale:** The scale of the not explicitly included physics, here resonances, so the scale is of order $M_p$, but this is channel dependent.

I will not go into more details here, links to lectures can be found on the website [22], short introductions can be found in [2] and [3].

4. Two-flavour ChPT at NNLO

References to order $p^2$ and $p^4$ work can be found in [3]. The first work at NNLO used dispersive methods to obtain the nonanalytic dependence on kinematical quantities, $q^2, s, t, u$ at NNLO. This was done for the vector (electromagnetic) and scalar form-factor of the pion in [23] (numerically) and [24] (analytically) and for $\pi\pi$-scattering analytically in [25].

Basically all processes of interest are calculated to NNLO in ChPT: $\gamma\gamma \to \pi^0\pi^0$ [20, 26], $\gamma\gamma \to \pi^+\pi^-$ [27, 28], $F_\pi$ and $m_\pi$ [27, 29, 30], $\pi\pi$-scattering [29], the pion scalar and vector form-factors [30] and pion radiative decay $\pi \to \ell\nu\gamma$ [31]. The pion mass is known at order $p^6$ in finite volume [32]. Recently $\pi^0 \to \gamma\gamma$ has been done to this order as discussed in the talk by Moussallam [33].

The LECs have been fitted in several processes. $\tilde{l}_4$ from fitting to the pion scalar radius [31, 34], $\tilde{l}_3$ from an estimate of the pion mass dependence on the quark masses [15, 34] and $\tilde{l}_1, \tilde{l}_2, \tilde{l}_5, \tilde{l}_6$ from the agreement with $\pi\pi$-scattering [34], $\tilde{l}_6$ from the pion charge radius [30] and $\tilde{l}_6 - \tilde{l}_5$ from the axial form-factor in $\pi \to \ell\nu\gamma$. There is also a recent determination of $\tilde{l}_5$ from hadronic tau decays [35]. The final best values are [30, 31, 34, 35]

$$\begin{align*}
\tilde{l}_1 &= -0.4 \pm 0.6, & \tilde{l}_2 &= 4.3 \pm 0.1, & \tilde{l}_3 &= 2.9 \pm 2.4, & \tilde{l}_4 &= 4.4 \pm 0.2, \\
\tilde{l}_6 - \tilde{l}_5 &= 3.0 \pm 0.3, & \tilde{l}_6 &= 16.0 \pm 0.5 \pm 0.7, & \tilde{l}_5 &= 12.24 \pm 0.21.
\end{align*}$$

Values of $\tilde{l}_3$ and $\tilde{l}_4$ have also been obtained by the lattice as discussed in several talks at this conference. There is information on some combinations of $p^6$ LECs. These are basically via the curvature in the vector and scalar form-factor of the pion [30] and two combinations from $\pi\pi$-scattering [34] from the knowledge of $b_5$ and $b_6$ in that reference. The order $p^6$ LECs $c_i^f$ are estimated to have a small effect for $m_\pi, f_\pi$ and $\pi\pi$-scattering.

A possible problem for ChPT are the pion polarizabilities. These are cleanly predicted in ChPT, the latest numbers from ChPT [28] and experiment [56] are:

$$\text{ChPT : } (\alpha_1 - \beta_1)_{\pi^\pm} = (5.7 \pm 1.0) \cdot 10^{-4} \text{ fm}^3$$

$$\text{Exp : } (\alpha_1 - \beta_1)_{\pi^\pm} = (11.6 \pm 1.5_{\text{stat}} \pm 3.0_{\text{syst}} \pm 0.5_{\text{mod}}) \cdot 10^{-4} \text{ fm}^3$$

A possible problem in this experiment is the background from direct $\gamma N \to \gamma N\pi$ production. Large values also follow from the older Primakoff experiments and a dispersive analysis [37] from $\gamma\gamma \to \gamma\gamma \to \ldots$
\[ \pi \pi: \]

Primakoff: \( (\alpha_1 - \beta_1)_{\pi^\pm} = (13.6 \pm 2.8_{\text{stat}} \pm 2.4_{\text{syst}}) \cdot 10^{-4} \text{ fm}^3 \) \hfill (4.4)

dispersive: \( (\alpha_1 - \beta_1) = (13.0 + 3.6 - 1.9) \cdot 10^{-4} \text{ fm}^3 \). \hfill (4.5)

The latter value has been criticized in [38] who argue that [37] has a large uncontrolled model dependence and conclude “Our calculations... are in reasonable agreement with ChPT for charged pions”. See their presentations in these proceedings for more details. ChPT in our present understanding cannot produce a value as large as in (4.3).

5. Three-flavour ChPT

5.1 Calculations

In this section I discuss several results at NNLO in mesonic three-flavour ChPT. The formulas here are much larger than in two-flavour ChPT and while the expressions have been reduced to a series of well-defined two-loop integrals, the latter are evaluated numerically. Both are the consequence of the different masses present. The vector two-point functions [39, 40] and the isospin breaking in the \( \rho \omega \) channel [41] were among the first calculated. The flavour disconnected scalar two-point function relevant for bounds on \( L_9' \) and \( L_6' \) was worked out in [42]. The remaining scalar two-point functions are known, available from the speaker but unpublished. Masses and decay constants as well as axial-vector two-point functions were the first calculations requiring full two-loop integrals, done in the \( \pi \) and \( \eta \) [39, 43] and the \( K \) channel [39]. Including isospin breaking contributions to masses and decay constants was done in [44]. After \( K_{\ell 4} \) had also been evaluated to NNLO [45] a fit to the LECs was done as described below. The vacuum expectation values in the isospin limit were done in [45], with isospin breaking in [44] and at finite volume in [46].

Vector (electromagnetic) form-factors for pions and kaons were calculated in [47, 48] and in [48] a NNLO fit for \( L_9' \) was performed. \( L_{10}' \) can be had from hadronic tau decays [55] or the axial form-factor in \( \pi, K \to \ell \nu \gamma \). The NNLO calculation is done, but no data fitting was performed [49]. A rather important calculation is the \( K_{\ell 3} \) form-factor. This calculation was done in [50, 51] and a rather interesting relation between the value at zero, the slope and the curvature for the scalar form-factor obtained [50]. Isospin-breaking has been included as well [52].

Scalar form-factors including sigma terms and scalar radii [53] and \( \pi \pi \) [54] and \( \pi K \)-scattering [55] are known and used to place limits on \( L_4' \) and \( L_6' \). Finally, the relations between the \( l_i', c_i' \) and \( L_i', C_i' \) have been extended to the accuracy needed to compare order \( p^6 \) results in two and three-flavour calculations [56] and there has been some progress towards fully analytical results for \( m_\pi^2 \) [57] and \( \pi K \)-scattering lengths [58]. The most recent results are \( \eta \to 3\pi \) [59], isospin breaking in \( K_{\ell 3} \) [52].

5.2 Testing ChPT and estimates of the order \( p^6 \) LECs

Most numerical analysis at order \( p^6 \) use a (single) resonance approximation to the order \( p^6 \) LECs. The main underlying motivation is the large \( N_c \) limit and phenomenological success at order \( p^4 \) [18]. There is a large volume of work on this, some references are [60]. The numerical work I will report has used a rather simple resonance Lagrangian [18, 29, 44, 45]. The estimates of
the $C_i^r$ is the weakest point in the numerical fitting at present, however, many results are not very sensitive to this. The main problem is that the $C_i^r$ which contribute to the masses, are estimated to be zero, except for $\eta'$ effects, and how these might affect the determination of the others. The estimate is $\mu$-independent while the $C_i^r$ are not.

The fits done in [44, 45, 53] tried to check this by varying the total resonance contribution by a factor of two, varying the scale $\mu$ from 550 to 1000 MeV and compare estimated $C_i^r$ to experimentally determined ones. The latter works well, but the experimentally well determined ones are those with dependence on kinematic variables only, not ones relevant for quark-mass dependence.

A new fit is in progress but in order to check whether ChPT with three flavours works, one would like a test that is as much as possible independent of the estimated values of the $C_i$. In [61] we studied 76 observables leading to 35 combinations that are independent of the $C_i$, or 35 relations. For these we found 13 with good “data,” $\pi\pi$ [54] and $\pi K$ [62] threshold parameters from dispersion theory and $K_{\ell 4}$ from experiment [53, 54].

The results for the 5 relations found in $\pi K$ scattering are shown in Tab. I. The equality of the remainder of LHS and RHS gives a test of ChPT. The results are encouraging but not 100% conclusive. More details and more results can be found in [61, 65].

### 5.3 The fitting and results

The inputs used for the standard fit, as discussed more extensively in [44, 45], are

- $K_{\ell 4}$: $F(0)$, $G(0)$, $\lambda$ from E865 at BNL [53].
- $m_{\pi^+}^2$, $m_{K^+}^2$, $m_{K^0}^2$, electromagnetic corrections include the violation of Dashen’s theorem.
- $F_{\pi^+}$ and $F_{K^+}/F_{\pi^+}$.
- $m_i/\bar{m} = 24$. Variations with $m_i/\bar{m}$ were studied in [44, 45].
- $L_{4}^0, L_{6}^0$ the main fit, 10, has them equal to zero, but see below and the arguments in [42].

| Roy-Steiner | NLO 1-loop | NLO LECs | NNLO 2-loop | NNLO 1-loop | remainder |
|-------------|------------|----------|-------------|-------------|-----------|
| LHS (I)     | 5.4 ± 0.3  | 0.16     | 0.97        | 0.77        | −0.11     |
| RHS (I)     | 6.9 ± 0.6  | 0.42     | 0.97        | 0.77        | −0.03     |
| 10 LHS (II) | 0.32 ± 0.01| 0.03     | 0.12        | 0.11        | 0.00      |
| 10 RHS (II) | 0.37 ± 0.01| 0.02     | 0.12        | 0.10        | 0.01      |
| 100 LHS (III)| −0.49 ± 0.02| 0.08   | −0.25      | −0.17       | 0.05      |
| 100 RHS (III)| −0.85 ± 0.60| 0.03  | −0.25      | 0.11        | −0.03     |
| 100 LHS (IV)| 0.13 ± 0.01| 0.04     | 0.00        | 0.01        | 0.03      |
| 100 RHS (IV)| 0.01 ± 0.01| 0.01     | 0.00        | 0.00        | −0.01     |
| 10³ LHS (V) | 0.29 ± 0.05| 0.09     | 0.00        | 0.06        | 0.01      |
| 10³ RHS (V) | 0.31 ± 0.07| 0.03     | 0.00        | 0.06        | 0.05      |

**Table 1:** $\pi K$-scattering: The numerical results for relations I-V, both left- (LHS) and right-hand side for the dispersive result from [62] and the NLO, NNLO 2-loop and NNLO $\ell$-dependent part (1-loop). The tree level for LHS and RHS of (I) is 3.01 and vanishes for the others. The equality of the remainder of LHS and RHS gives a test of ChPT.
Some results of this fit are given in Tab. 3. The errors are very correlated, see Fig. 6 in [45] for an example. Varying the values of \( L'_4, L'_6 \) as input can be done with a reasonable fitting chi-squared when varying \( \sqrt{\mu}^{3} L'_4 \) from \(-0.4\) to \(0.6\) and \( L'_6 \) from \(-0.3\) to \(0.6\) [53]. The variation of many quantities with \( L'_4, L'_6 \) (including the changes via the changed values of the other \( L'_i \)) are shown in [53, 54, 55]. Fit B was one of the fits with a good fit to the pion scalar radius and fairly small quantities with squared when varying \( \sqrt{\mu}^{10} \) including isospin breaking. This column is included as reference. The estimates for the fit 10 in [44] but without isospin breaking. The results are essentially identical to those of the fit first fitting results are given in Tab. 3. The column labeled fit10 iso uses the input as used for uniform treatment and an easier handling of the LECs. This program is only partly completed but most. First we add the better information on the have some more constraints included. The small boxes indicate the LECs which have changed and NA48 has measured the \( \ell_{K}^{p} \) for an example. Varying the values of \( \pi_{K} \) and \( \eta \): 

| \( 10^{3} L'_4 \) | 0.43 ± 0.12 | 0.38 | 0.44 | 0.44 |
| \( 10^{3} L'_2 \) | 0.73 ± 0.12 | 1.59 | 0.60 | 0.69 |
| \( 10^{3} L'_3 \) | \(-2.53 \pm 0.37\) | \(-2.91\) | \(-2.31\) | \(-2.33\) |
| \( 10^{3} L'_4 \) | \(\equiv 0\) | \(\equiv 0\) | \(\equiv 0.5\) | \(\equiv 0.2\) |
| \( 10^{3} L'_5 \) | 0.97 ± 0.11 | 1.46 | 0.82 | 0.88 |
| \( 10^{3} L'_6 \) | \(\equiv 0\) | \(\equiv 0\) | \(\equiv 0.1\) | \(\equiv 0\) |
| \( 10^{3} L'_8 \) | \(-0.31 \pm 0.14\) | \(-0.49\) | \(-0.26\) | \(-0.28\) |
| \( 10^{3} L'_9 \) | 0.60 ± 0.18 | 1.00 | 0.50 | 0.54 |
| \( 10^{3} L'_{10} \) | 5.93 ± 0.43 | 7.0 | – | – |

\[
2B_{0}m_{\pi}/m_{\pi}^{3} = 0.736 \quad \text{same} \quad \text{p}^4 \quad \text{fit B} \quad \text{fit D}
\]

\[
2B_{0}m_{\pi}/m_{\pi}^{3} = 0.006,0.258 \quad 0.009,\equiv 0 \quad -0.138,0.009 \quad -0.091,0.133
\]

\[
m^{2}_{\pi} : p^4, p^6 \quad 0.007,0.306 \quad 0.075,\equiv 0 \quad -0.149,0.094 \quad -0.096,0.201
\]

\[
m^{2}_{K} : p^4, p^6 \quad -0.052,0.318 \quad 0.013,\equiv 0 \quad -0.197,0.073 \quad -0.151,0.197
\]

\[
m_{u}/m_{d} = 0.45\pm0.05 \quad 0.52 \quad 0.52 \quad 0.50
\]

\[
F_{0} \text{[MeV]} = 87.7 \quad 81.1 \quad 70.4 \quad 80.4
\]

\[
F_{K}^{\pi} = 0.169,0.051 \quad 0.22,\equiv 0 \quad 0.153,0.067 \quad 0.159,0.061
\]

Table 2: The fits of the \( L'_{i} \) and some results, see text for a detailed description. They are all quoted at \( \mu = 0.77 \text{ GeV} \). Table with values from [44, 53, 54, 55].

Note that \( m_{u}/m_{d} = 0 \) is never even close to the best fit and this remains true for the entire variation with \( L'_4, L'_6 \). The value of \( F_{0} \), the pion decay constant in the three-flavour chiral limit, can vary significantly, even though I believe that fit B is an extreme case.

We, JB and I. Jemos, are working on a new general fit. The preferred value of \( F_{K}/F_{\pi} \) is changed and NA48 has measured the \( K_{44} \) formfactors more accurately. In addition, we want to include more constraints directly. Some preliminary results are also discussed in I. Jemos talk [55]. For this work, all the calculated processes are being programmed in C++ to allow for a more uniform treatment and an easier handling of the LECs. This program is only partly completed but first fitting results are given in Tab. 3. The column labeled fit 10 iso uses the input as used for fit 10 in [44] but without isospin breaking. The results are essentially identical to those of the fit including isospin breaking. This column is included as reference. The estimates for the \( C_{i}^{p} \) used are the same as used in [44] for all the fits shown in the table except the last. The other columns always have some more constraints included. The small boxes indicate the LECs which have changed most. First we add the better information on the \( K_{44} \) form-factors from NA48 [64]. This produces sizable changes in \( L'_4 \) and \( L'_5 \). The newer value of the PDG for \( F_{K}/F_{\pi} = 1.193 \) then changes the fitted value of \( L'_4 \) which influences \( L'_{8} \) via the fitted masses. \( L'_5 \) gets lowered somewhat more when
we include the scattering lengths $a_0^0$, $a_0^2$, $a_0^{1/2}$ and $a_0^{3/2}$. Adding the pion scalar radius requires a nonzero for one of $L_4^c$ and $L_5^c$. This is what is shown in the column labeled “All.” In the last column we have set the estimated value of the $C_i^f = 0$. This is work in progress but some puzzles appear at this stage, the large $N_c$ relation $2L_1^c = L_2^c$ is now badly broken and the central value of $L_4^c$ is not small compared to $L_5^c$. We are working on including more scattering lengths and trying to include also some lattice results on the meson masses and decay constants. This is especially important since the simple estimate of the $C_i^f$ used has none contributing to masses and decay constants.

Table 3: The changes in the $L_i$ compared to the isospin symmetric fit to the same input as the old fit 10 of [44]. The other columns include one new effect at a time, see text.

| $10^3 L_1^c$ | $10^3 L_2^c$ | $10^3 L_3^c$ | $10^3 L_4^c$ | $10^3 L_5^c$ | $10^3 L_6^c$ | $10^3 L_7^c$ | $10^3 L_8^c$ |
|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| $0.40 \pm 0.12$ | $0.76 \pm 0.12$ | $-2.40 \pm 0.37$ | $\equiv 0$ | $0.97 \pm 0.11$ | $-0.30 \pm 0.15$ | $0.61 \pm 0.20$ |
| $0.98$ | $0.78$ | $-3.14$ | $0.93$ | $0.72$ | $0.59$ |
| $0.97$ | $0.79$ | $-3.12$ | $0.72$ | $0.56$ |
| $0.98 \pm 0.11$ | $0.59 \pm 0.18$ |
| $0.75$ | $0.09$ | $-1.49$ | $0.67$ | $0.18$ |
| $5.38 (5)$ | $1.44 (4)$ |
| $\chi^2$ (dof) | $0.25 (1)$ | $0.17 (1)$ | $0.19 (1)$ | $1.51 (4)$ |

6. $\eta \rightarrow \pi \pi \pi$

In the limit of conserved isospin, no electromagnetism and $m_u = m_d$, the $\eta$ is stable. Direct electromagnetic effects are small [57]. The decay thus proceeds mainly through the quark-mass difference $m_u - m_d$. The lowest order was done in [58], order $p^4$ in [59] and recently the full order $p^6$ has been evaluated [59]. The momenta for the decay $\eta \rightarrow \pi^+ \pi^- \pi^0$ we label as $p_\eta$, $p_+$, $p_-$ and $p_0$ respectively and we introduce the kinematical Mandelstam variables $s = (p_+ + p^-)^2$, $t = (p_+ + p_0)^2$, $u = (p_- + p_0)^2$. These are linearly dependent, $s + t + u = m_{\pi}^2 + m_{\pi}^2 + m_{\eta}^2 = 3s_0$. The amplitudes for the charged, $A(s,t,u)$, and neutral, $\bar{A}(s,t,u)$ are related

$$\bar{A}(s_1,s_2,s_3) = A(s_1,s_2,s_3) + A(s_2,s_3,s_1) + A(s_3,s_1,s_2).$$

The relation in (6.1) is only valid to first order in $m_u - m_d$. The overall factor of $m_u - m_d$ can be put in different quantities, two common choices are

$$A(s,t,u) = \frac{\sqrt{3}}{4R} M(s,t,u) \quad \text{or} \quad A(s,t,u) = \frac{1}{Q^2} \frac{m_{\pi}^2}{m_{\pi}^2 - m_{K}^2} \frac{s + t + u}{3\sqrt{3}F_{\pi}^2},$$

with $R = (m_s - \hat{m})/(m_d - m_u)$ or $Q^2 = R(m_s + m_d)/(2\hat{m})$ pulled out. The lowest order result is

$$M(s,t,u)_{LO} = ((4/3)m_{\pi}^2 - s)/F_{\pi}^2.$$  

The tree level determination of $R$ in terms of meson masses gives with (6.3) a decay rate of 66 eV which should be compared with the experimental results of $295 \pm 17$ eV [70]. In principle, since the
The Dalitz plot in $\eta \to 3\pi$ is parameterized in terms of $x$ and $y$ defined in terms of the kinetic energies of the pions $T_i$ and $Q_{\eta} = m_\eta - 2m_{\pi^0} - m_{\pi^0}$ for the charged decay and $z$ defined in terms of the pion energies $E_i$. The amplitudes are expanded in $x = \sqrt{3}(T_+ - T_-)/Q_\eta$, $y = 3T_0/Q_\eta - 1$, $z = (2/3)\sum_{i=1,3}(3E_i - m_{\eta})^2/(m_\eta - 3m_{\pi^0})^2$, via

$$|M(s,t,u)|^2 = A_0^2 (1 + ay + by^2 + dx^2 + fy^3 + \cdots), \quad \overline{|M(s,t,u)|^2} = A_0^2 (1 + 2az + \cdots). \quad (6.4)$$

Recent experimental results for these parameters are shown in Tabs. 4 and 5. There are discrepancies among the experiments but the latest precision measurements of $\alpha$ agree. The predictions from ChPT to order $p^6$ with the input parameters as described earlier are given in Tabs. 4 and 5. The different lines correspond to variations on the input and the order of ChPT. The lines labeled NNLO are the central results. The agreement with experiment is not too good and clearly needs

| Exp.            | a      | b      | c      | d                      |
|-----------------|--------|--------|--------|------------------------|
| KLOE            | −1.090±0.005±0.008  | 0.124±0.006±0.010  | 0.057±0.006±0.007  |
| Crystal Barrel  | −1.22±0.07  | 0.22±0.11  | 0.06±0.04 (input)    |
| Layter et al.   | −1.08±0.014  | 0.034±0.027  | 0.046±0.031          |
| Gormley et al.  | −1.17±0.02  | 0.21±0.03  | 0.06±0.04            |

Table 4: Measurements of the Dalitz plot distributions in $\eta \to \pi^+\pi^-\pi^0$. Quoted in the order cited in [74]. The KLOE result $f$ is $f = 0.14±0.01±0.02$.

| Order          | $A_0^2$ | a      | b      | c      | d      | f      |
|----------------|---------|--------|--------|--------|--------|--------|
| LO             | 120     | −1.039 | 0.270  | 0.000  | 0.000  |        |
| NLO            | 314     | −1.371 | 0.452  | 0.053  | 0.027  |        |
| NLO ($L^\pi = 0$) | 235 | −1.263 | 0.407  | 0.050  | 0.015  |        |
| NNLO           | 538     | −1.271 | 0.394  | 0.055  | 0.025  |        |
| NNLO ($\mu = 0.6$ GeV) | 543 | −1.300 | 0.415  | 0.055  | 0.024  |        |
| NNLO ($\mu = 0.9$ GeV) | 548 | −1.241 | 0.374  | 0.054  | 0.025  |        |
| NNLO ($C^\eta = 0$) | 465 | −1.297 | 0.404  | 0.058  | 0.032  |        |
| NNLO ($L^\eta = C^\eta = 0$) | 251 | −1.241 | 0.424  | 0.050  | 0.007  |        |

Table 5: Theoretical estimate of the Dalitz plot distributions in $\eta \to \pi^+\pi^-\pi^0$. decay rate is proportional to $1/R^2$ or $1/Q^4$, this should allow for a precise determination of $R$ and $Q$. However, the change required seems large. The order $p^4$ calculation [69] increased the predicted decay rate to 150 eV albeit with a large error. About half of the enhancement in the amplitude came from $\pi\pi$ rescattering and the other half from other effects like the chiral logarithms [69]. The rescattering effects have been studied at higher orders using dispersive methods in [71] and [72]. Both calculations found an enhancement in the decay rate to about 220 eV but differ in the way the Dalitz plot distributions look. That difference and the facts that in $K_{44}$ the dispersive estimate [73] was about half the full ChPT calculation [45] and at order $p^4$ the dispersive effect was about half of the correction for $\eta \to 3\pi$ makes it clear that a full order $p^6$ calculation was desirable. The calculation [59] generalized the methods of [44] to deal with $\pi^0,\eta$ mixing. The correction found in [59] at order $p^6$ is 20-30% in amplitude, larger in magnitude than the dispersive estimates [71, 72] but with a shape similar to [72].
In this section I discuss some recent work that argues that chiral effects should also be calculable for processes with hard pions. This type of arguments was given by Flynn and Sachrajda for $K\ell_3$ decays away from $q_2^{\text{max}}$ [78]. It was argued that those arguments apply much more generally in [79] and there they were also applied to $K \rightarrow \pi \pi$.

The underlying argument is that the main predictions of ChPT, namely chiral logarithms come from soft pion lines. In pure ChPT as discussed above the powercounting works since all lines are considered soft. In the baryon sector, power counting has also been developed. There the meson lines are soft and the baryons are always close to their mass-shell. The heavy momentum always follows a baryon line. Here two momentum regions are important, those close to the baryon momentum $p = M_B v + k$ and the soft ones $p = k$, $k$ soft for both. As argued in [80], the $M_B$-dependence of loop-diagrams is analytic and can be absorbed in the LECs. This is a broad area of research as can be judged from many of the talks in working group 2. Similarly, ChPT with mesons with a heavy-quark relies on having two momentum regimes with one line always carrying the large momentum, [81, 82]. Thereafter it has been argued that ChPT could also be constructed for unstable particles near their mass-shell [83], see especially the discussion in [84]. In all these cases, the underlying argument is always the same. The heavy-mass dependence is analytic and can be absorbed in the LECs.

Ref. [79] argued that the same type of reasoning works for processes with hard pions. The reasoning is depicted in Fig. [1]: Take a process with a given external momentum configuration, identify the soft lines and cut them. The resulting part is analytic in the soft stuff and should thus be describable by an effective Lagrangian with coupling constants depending on the given...
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Figure 1: An example of the argument for a “hard pion ChPT.” The thick lines contain a large momentum, the thin lines a soft momentum. Left: a general Feynman diagram with hard and soft lines. Middle-left: we cut the soft lines to remove the soft singularity. Middle-right: The contracted version where the hard part is assumed to be correctly described by a “vertex” of an effective Lagrangian. Right: the contracted version as a loop diagram. This is expected to reproduce the chiral logarithm of the left diagram.

external momenta. The Lagrangian should be complete in the neighbourhood, in both momenta and processes, and should respect the symmetries present in the problem. Loop diagrams with this effective Lagrangian should reproduce the nonanalyticities in the light masses.

In [78] and [79] it was proven that respectively for $K_{\ell 3}$-formfactors and $K \rightarrow \pi \pi$-decays the lowest order Lagrangian is sufficiently complete to be able to calculate uniquely the pionic chiral logarithm. [79] explicitly kept some higher order terms to illustrate the argument and found that

$$A_{0}^{NLO} = A_{0}^{LO} \left( 1 + \frac{3}{8F^2} \bar{\Lambda}(M^2) \right) + \lambda_0 M^2 + \mathcal{O}(M^4),$$

$$A_{2}^{NLO} = A_{2}^{LO} \left( 1 + \frac{15}{8F^2} \bar{\Lambda}(M^2) \right) + \lambda_2 M^2 + \mathcal{O}(M^4),$$

(7.1)

with $\bar{\Lambda}(M^2) = -1/(16\pi^2)M^2 \log(M^2/\mu^2)$ and $M^2$ the lowest order pion mass. $\lambda_0$ and $\lambda_2$ depend on higher order terms in the Lagrangian and are not calculable. Another check was that the three-flavour ChPT did have the same pionic chiral logarithms. Notice that the logarithms in (7.1) are not due to the final state interaction, that effect goes into the couplings in this approach, and actually go against the $\Delta I = 1/2$-rule.

8. Conclusions

ChPT in the meson sector is progressing and finds new application areas. I have shortly reviewed two- and three-flavour ChPT for mesons and discussed or provided a references to several areas where there has been recent progress.

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