A Method of Transforming Plain Text into Cipher Text using Gear Graphs and Product Cordial Labeling

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Abstract. This research article includes the development of encoding technique of ordinary text with the help of Gear structure $G_n$ by using vertex multiplications, the graph $G_n$ is vertex product cordial for n not even and there is no vertex product cordial labeling for n even. By using three graphs $G_5$, $G_7$ and $G_{11}$, illustrations are given for converting Text in to Matrix and developed flowchart for coding technique. At present the encoding method includes computer algorithm techniques, but here the researcher develop a new method of coding without computer knowledge.

1. Introduction for graph theory and coding theory

The awesome mind learns, receives and observes. As a result, the journey in the pursuit of knowledge turns out to fruitful. A gear in a vehicle man made have provided coding techniques to the researcher which is presented in this paper. Cryptography is the methods of transforming original text into unpredictable but to one who known how to decode it. Whether it is hiding messages under the stamps on letter or writing invisible ink, people
have always found ingenious ways of using whatever technology they have available to write secret messages. Computer scientists have now invented a way to hide secret messages in ordinary text by imperceptibly changing shapes of letters. In the digital world, we won’t quite as far with a jar of juice, but the premise of secret messages hiding plain sight. To make a secret encrypted letter from a normal one, we need to have a secret key. Getting text back to normal is called decryption.

1.1. Literature Review:
Perhaps of all the creations of man, language is the most astonishing. The first business of language is simply to be transparent. The mind of man, then struck at the idea of coding languages ([2], [3]. The concept of cordial labeling was introduced by Cahit [1]. The researchers were founding the split graphs of star graphs are prime cordial. Narayanan and R. Kala introduced difference cordial labeling [5] and worked on some special graphs [6], [16] introduced the concept of the edge product cordial labeling as an edge analogue of the product cordial labeling, [15]. A deep discussion of $H_n$ and graphs based on $G_n$ was given by Neelam [4]. Graph theory and the Amateur cryptographer computer math application was given by Read [7]. Alphabets shifting by three was implemented by Rizwan [8], [12], [15] and [16]. After inspiring these work, we worked on Gear graphs $G_5$, $G_7$ and $G_{11}$ by applying product cordial labeling and GMJ (Graph Message Jumbled) coding method and hence this paper.

2. Definitions

Definition 2.1. A binary point label $\mu : P(G) \rightarrow \{\text{zero, one}\}$ of a diagram $G$ with induced line label $\mu^* : L(G) \rightarrow \{\text{zero, one}\}$ classified by $\mu^*(xy) = \mu(x)\mu(y)$ is said to be PCL if $|p_\mu(\text{zero}) - p_\mu(\text{one})| \leq 1$ and $|l_\mu(\text{zero}) - l_\mu(\text{one})| \leq 1$, where $p_\mu(\text{zero}), p_\mu(\text{one})$ defines the number of points of $G$ having numbers zero and one under $\mu$ and $l_\mu(\text{zero}), l_\mu(\text{one})$ denotes the number of lines of $G$ having numbers zero, one respectively under $\mu$. A graph $G$ is Product Cordial if it admits PCL.

Definition 2.2. Gear graph $G_n$, that obtained from wheel $W_n$ by subdividing each of its rim edge.

Definition 2.3. The original intelligible message is known as Plain text.

Definition 2.4. The transformed message is known as Cipher text.
3. Method of coding:
Caesar used a key three for his technique to communicate and it is called as Caesar Cipher. GMJ coding method, procedure for encoding a message referred by [12]. Figure 1 shows the coding process.

![Figure 1. Encryption and Decryption](image)

Here we establish a method of coding a message using graph labeling and split the alphabet into 13 parts of two each. To code a text message we would begin by finding the letter in numbering of alphabets and identify the letter which belongs to the pair $x$ or $y$ co-ordinate if it is $x$ co-ordinate which gives the code $I_{k,1}$, similarly if it is $y$ co-ordinate gives the code $I_{k,2}$ keep going through the whole message like that. The flow chart for the method of coding is given in Figure 4.

3.1. Coding of Numbers:
By arranging the 16 Greek letters according to the alphabetical order (in English), the first ten letters are used to denote the numbers $0, 1, 2, \cdots, 9$.

- Alpha–α–0
- Beta–β–1
- Delta–δ–2
- Epsilon–ε–3
- Eta–η–4
- Gamma–γ–5
- Kappa–κ–6
- Lambda–λ–7
- Mu–μ–8
- Nu–ν–9

3.2. Numbering of alphabets:
OTOT (One three one two)
The 26 alphabets are divided into 13 parts of two each, the first and the second are allotted the numbers 1 and 2 respectively. $I_{k,1}$ stands for the first letter of the $k^{th}$ set and $I_{k,2}$ for the second letter of the $k^{th}$ set. The odd positioned pairs are allotted $I_1, I_2, \ldots, I_7$ in order and then the even positioned pairs. $I_1 = \{A, B\}$, $I_{1,1} = A$, $I_2 = \{E, F\}$, $I_{2,2} = F, \ldots, I_7 = \{Y, Z\}$, $I_{7,2} = Z$, $I_8 = \{C, D\}$, $I_{8,1} = C$ and so on. Thirteen is hidden in OT(one three) and one and two are understood by (one two) OT again. This method of numbering the alphabets is called OTOT.

\[
\begin{array}{cccccccccc}
I_1 & I_8 & I_2 & I_9 & I_3 & I_{10} & I_4 & I_{11} & I_5 \\
A & B & C & D & E & F & G & H & I & J & K & L & M & N & O & P & Q & R \\
I_{12} & I_6 & I_{13} & I_7 \\
S & T & U & V & W & X & Y & Z
\end{array}
\]

The function $g$ on the set of alphabets is given by
\[
g(a_{(4\ell+1)}) = I_{\ell+1,i}, \quad \ell = 0, 1, 2, \ldots, 6, \quad i = 1, 2
\]
\[
g(a_{(2+4\ell+i)}) = I_{\ell+8,i}, \quad \ell = 0, 1, 2, \ldots, 5, \quad i = 1, 2.
\]

3.3. **Coding a letter:**

\[
\begin{align*}
I_{k,1} &= f(v_{k-1}) = 1, \quad k = 1, 2, 3 \cdots, 6 \\
I_{k,1} &= f(u_s) = 1, \quad k \equiv s \pmod{6}, k = 7 \text{ to } 10 \\
I_{k,1} &= f(v_s) = 1, \quad k = 11, 12, 13, \quad k \equiv s \pmod{6} \\
I_{k,2} &= f(v_{5+k}) = 0, \quad k = 1, 2, 3, 4 \\
I_{k,2} &= f(u_k) = 0, \quad k = 5, 6, 7, 8, 9 \\
I_{k,2} &= f(u_{s+1}) = 0, \quad k \equiv s \pmod{6}, \quad k = 10, 11 \\
I_{k,2} &= f(u_{\frac{s+1}{2}+1}) = 0, \quad k = 12 \\
I_{k,2} &= f(u_{\frac{s}{2}+1}) = 0, \quad k = 13.
\end{align*}
\]

After development of encoding we continuously use two terms original message and coded message. The plain text that the person wants to secure and given to the next person. Coded message is the message that is transformed over the insecure channel. In this method in place of key there is a clue for gussing the graph and numbering of alphabets. There are three messages using three graphs $G_5, G_7$ and $G_{11}$ by using EPCL for converting ordinary message in to matrix form. $G_n$ is EPCL if $n$ is odd and no EPCL if $n$ is even.
Illustration 1:

(i) **Message:** Meet me in the forest at zero dark night.

(ii) **clue:** He put the car in to fifth.

(iii) **Graph:** The Gear graph $G_5$ is considered.

(iv) **Verification:**

   (a) The vertices $v_1$ to $v_3$ and $u_1$ and $u_2$ are allotted number 0. A total of 5 vertices take the value 0.

   (1)

   (b) The rim vertices $v_4$ and $v_5$ and $u_3$ to $u_5$ and the apex vertex $v_0$ are allotted number 1. A total of 6 vertices take the value 1.

   (2)

From (1) and (2), $|e_f(0) - e_f(1)| = 1 \leq 1$. The definition of vertex product cordial labeling is satisfied here. Figure 2 shows the gear graph with five vertices.

![Gear graph $G_5$](image)

**Figure 2.** Gear graph $G_5$
(v) Coding (word wise)

Meet  $- f(v_3)f(v_1)f(v_1)f(w_7)$
me   $- f(v_3)f(v_1)$
in   $- f(v_2)f(v_0)$
the  $- f(u_7)f(u_0)f(v_1)$
forest $- f(v_7)f(v_5)f(u_5)f(v_1)f(v_0)f(u_7)$
at  $- f(v_0)f(u_7)$
zero  $- f(u_7)f(v_1)f(u_5)f(v_0)$
dark  $- f(u_8)f(v_0)f(u_5)f(u_4)$
night $- f(v_9)f(v_2)f(u_3)f(u_9)f(u_7)$

(vi) Presenting the letter codes:

By using the rule for coding a letter, the 32 letters of the given message are written along a horizontal string in the following manner. Any alphabet belongs to some $I_{k,i}$ which is associated to some $f(v_7)$. By omitting ‘f’ and using the Greek letters, the coding is written along the horizontal string. For example, the letter M is understood as $f(v_3)$. Here $f(v_3)$ is written as $v_\epsilon$.

The coded message is made starting with the first letter, the 32nd letter, second letter, the 31st letter and so on, by jumbling the letters, making the recognition of the message a bit difficult.

The long coding as a string is shown:

(vii) String Structure:

$v_\epsilon u_\lambda v_\beta u_\nu v_\beta u_\kappa u_\lambda v_\delta v_\gamma u_\eta u_\rho v_\nu v_\alpha u_\lambda u_\mu u_\nu v_\beta u_\gamma v_\lambda v_\alpha v_\gamma u_\lambda u_\mu u_\nu v_\beta v_\alpha v_\alpha u_\lambda$

$$
\begin{pmatrix}
v_\epsilon & u_\lambda & v_\beta & u_\nu \\
v_\beta & u_\epsilon & u_\lambda & v_\delta \\
v_\delta & u_\nu & v_\nu & v_\alpha \\
u_\lambda & u_\mu & u_\nu & v_\gamma \\
v_\beta & u_\gamma & v_\lambda & v_\alpha \\
v_\gamma & u_\lambda & u_\gamma & u_\lambda \\
v_\beta & v_\alpha & v_\alpha & u_\lambda
\end{pmatrix}
$$

(viii) Matrix Representation:

Illustration 2:

(i) Message: Conduct meeting between new moon and full moon.
(ii) **clue:** Dinesh drove cautiously along in seventh.

(iii) **Graph:** The Gear graph $G_7$ is considered.

Figure 3 shows the gear graph with seven vertices.

(iv) **Verification:**

(a) The vertices $v_1, v_4, u_1$ to $u_3$, and apex vertex $v_0$ are allotted number 1. A total of 8 vertices take the value 1.

(3)

(b) The rim vertices $v_4$ to $v_7$ and $u_4$ to $u_7$ are allotted number 0. A total of 7 vertices take the value 0.

(4)

From (3) and (4), $|e_f(0) - e_f(1)| = 1 \leq 1$. The definition of vertex product cordial labeling is satisfied here.

**Figure 3.** Gear graph $G_7$
(v) Coding: (word wise)

Conduct \(- f(u_2)f(v_5)f(v_9)f(u_8)f(u_2)f(u_7)\)
meeting \(- f(v_3)f(v_1)f(u_7)f(v_2)f(v_9)f(u_3)\)
between \(- f(v_6)f(v_1)f(u_7)f(v_1)f(v_1)f(v_9)\)
new \(- f(v_9)f(v_1)f(v_1)\)
moon \(- f(v_3)f(v_5)f(v_9)\)
and \(- f(v_0)f(v_9)f(u_8)\)
full \(- f(v_7)f(v_5)f(u_5)f(u_5)\)
moon \(- f(v_3)f(v_5)f(v_5)\)

(vi) Presenting the letter codes:
As followed by previous illustration the presentation of letter codes are by using the rule
for coding a letter, the 39 letters of the given message are written along a horizontal
string in the following manner. The horizontal string is written starting with the first
letter, the 39th letter, second letter, the 38th letter and so on.

(vii) Horizontal string:

\[
\begin{pmatrix}
  u_\delta & v_\gamma & v_\eta \\
  v_\gamma & v_\nu & u_\mu \\
  v_\epsilon & u_\gamma & v_\eta \\
  u_\gamma & u_\delta & u_\eta \\
  u_\lambda & v_\gamma & v_\epsilon \\
  v_\lambda & v_\beta & u_\mu \\
  v_\beta & v_\nu & u_\lambda \\
  v_\alpha & v_\delta & v_\nu \\
  v_\nu & v_\gamma & u_\epsilon \\
  v_\gamma & v_\chi & v_\epsilon \\
  v_\beta & v_\alpha & u_\lambda \\
  v_\alpha & v_\beta & v_\nu \\
  v_\beta & v_\nu & v_\beta \\
\end{pmatrix}
\]
Illustration 3:

(i) **Message:** DONT change the place.

(ii) **clue:** His mountain bike had eleventh.

(iii) **Graph:** The Gear graph $G_{11}$ is considered.

Figure 4 shows the gear graph eleven vertices.

(iv) **Verification:**

(a) The vertices $v_1$ to $v_6$ and $u_1$ to $u_6$ and the apex vertex are alloted number 1. A total of 13 vertices take the value 1.

(b) The rim vertices $v_7$ to $v_{11}$ and $u_6$ to $u_{11}$ are alloted number 0. A total of 12 vertices take the value 0.

From (5) and (6), $|e_f(0) - e_f(1)| = 1 \leq 1$.

The definition of vertex product cordial labeling is satisfied here.

![Figure 4. Gear graph $G_{11}$](image)
(v) **Coding:** (word wise)
- Dont: $f(u_2)f(v_5)f(v_9)f(u_8)f(v_5)f(u_7)$
- change: $f(v_3)f(v_1)f(u_7)f(v_2)f(v_9)f(u_3)$
- the: $f(v_6)f(v_1)f(u_7)f(v_1)f(v_1)f(v_9)$
- place: $f(v_9)f(v_1)f(v_1)$

(vi) **Presenting the letter codes:**
As followed by previous illustrations the presentation of letter codes are by using the rule for coding a letter, the 18 letters of the given message are written along a horizontal string in the following manner. The horizontal string is written starting with the first letter, the 18th letter, second letter, the 17th letter and so on.

(vii) **Horizontal string:**
$u_\mu v_\beta v_\gamma u_\delta v_\alpha u_\lambda u_\gamma u_\delta u_\kappa u_\rho v_\beta v_\alpha v_\nu u_\lambda v_\epsilon v_\beta$

(viii) **Matrix Representation:**
\[
\begin{pmatrix}
 u_\mu & v_\beta & v_\gamma \\
 u_\delta & v_\nu & v_\alpha \\
 u_\lambda & u_\gamma & u_\delta \\
 u_\kappa & u_\nu & v_\beta \\
 v_\alpha & u_\nu & v_\nu \\
 u_\lambda & u_\epsilon & v_\beta \\
\end{pmatrix}
\]

4. **Coding Technology Algorithm**
- **case 1:** Get the graph with given hint.
- **case 2:** Label the graph $G_n$ using the given instruction.
- **case 3:** Rearrange the 26 letters, use OTOT.
- **case 4:** Write the coding method.
- **case 5:** Write the text message to be code.
- **case 6**: Apply method of coding for each letter and each word.

- **case 7**: String Structure.

- **case 8**: Matrix Structure.

The sender has to forward to the receiver the following:

- Hint to know the diagram
- Hint to know the alphabets splitting.
- Coded Message

Figure 5 shows flow chart for the Coding process.

![Flow Chart](image-url)

**Figure 5.** Flow Chart
5. Concluded with Suggestion for the next work:

This research article is settled with the gear graph is an vertex product cordial labeling for n odd and not product cordial labeling for n even. Secrecy is the practice of hiding informations from certain individuals or groups who do not have the need to know, perhaps while sharing it with other individuals that which is kept hidden is known as secret. Here the researcher have described the process for transforming secret messages into coded messages. we propose novel approaches to guess the graph mathematical or non-mathematical such that graph could not be easily found. We planned to work EPCL for other graphs and construct coding technology to them.

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