Cosmology with extended Chaplygin gas media and $w = -1$ crossing

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In this paper the accelerating expansion of our universe at the late cosmic evolution time in a generally modified (extended) Chaplygin gas (Dark Fluid) model is detailed, which is characterized by two parameters $(m, \alpha)$. Different choices to the parameters $m$ and $\alpha$ divide this model into two main kinds of situation by different properties with cosmological interests. With proper choices of parameters, we find that this extended model can realize the phantom divide $w = -1$ (Equation Of State parameter) crossing phenomenon with interesting ranges of the scale factor value $a(t)$ corresponding to $w = -1$ and present value of state parameter $w$.

Additionally, through Taylor series expansion of the function $A(a)$ in the extended Chaplygin gas model, a specific Equation of State is gained. Under the framework of Friedman-Robertson-Walker cosmological model, it can successfully explain the accelerating expansion of our universe. However, the value of $w$ in this case is larger than $-1$, that is, indicating it like a quintessence fluid and no $w = -1$ crossing occurs.

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I. INTRODUCTION

Dark side of the Universe has been puzzling us across the century [1], especially the recent years discovery as coined Dark Energy. Observations of type Ia supernova (SNe Ia) directly suggest that the expansion of the universe is accelerating, and the measurement of the cosmic microwave background (CMB) [2] and the galaxy power spectrum for large scale structure [3] indicate that in spatially flat isotropic universe, about two-thirds of the critical energy density seems to be stored in a so called dark energy component with enough negative pressure responsible for the currently cosmic accelerating expansion [4]. It is clear from observations that most of the matter in the Universe is in a dark (non-baryonic) form (see, for instance, [5]). To understand the mysterious dark components is even great challenge for temporary physicists from this new century crossings.

The simplest candidate for the dark energy is a cosmological constant $\Lambda$, which has a specially simple pressure expression $p_\Lambda = -\rho_\Lambda$. However, the $\Lambda$-term requires that the vacuum energy density be fine tuned to have the observed very tiny value, the famous "old" cosmological constant problem. So, many other different forms of dynamically changing dark energy models have been proposed instead of the only cosmological constant incorporated model. Usually, the equation of state (EOS) for describing dark energy can be assumedly factorized into the form of $p_{DE} = w \rho_{DE}$, where $w$ may depend on cosmological redshift $z$, or scale factor $a(t)$ and so on. By the way, the case for $w = -1$ corresponding to the cosmological constant, which has involved singularity in perturbation calculations, was thought as border-case called the phantom divide [6].

To further explore the properties of the so called dark energy or its concrete form of EOS $w$, some authors propose kinds of models such as $p_X = \frac{\omega}{\rho_X}$ (Chaplygin gas model [7]), which is not consistent with astrophysics observations as it can produce oscillations and exponential blow-up to the matter power spectrum, its generalizations like $p_X = \frac{\omega}{\rho_X}$ (generalized Chaplygin gas model [8], or scale factor $a$)

$I_\alpha$ like $p_X = \frac{\omega}{\rho_X}$ (generalized Chaplygin gas model [9], or generalizing the constant $A$ to variable forms [10], $p_X = \alpha (\rho_X - \rho_0)$ (linearized EOS model [11], compared with the conventionally perfect fluid EOS $p_X = \gamma (\rho_X - \rho_0)$, and so on (subscript $X$ represents dark energy fluid except for the cosmological constant). Among these models, the fluid of GCGM has a dual behavior: it mimics matter ($p \approx 0$) domination at very early stage in the history of the universe evolution and a cosmological constant much later (with a smooth transition in between), which is highly suggestive of a unified description of dark matter and dark energy (see reference [12]) that we may address it as Dark Fluid, as reason above. Moreover, dark energy may be made up of different energy fluids with different EOS, and these different energy fluids interact with each other through $\rho_s = \frac{\rho_1 + \rho_2}{\rho_1 + \rho_2}$, where subscript $s$ indicates a component, which is called the self-interacting fluids (see reference [13]).

With the merits of GCGM, we further discuss a modified general Chaplygin gas model by extending its EOS as an explicitly scale factor related form, which is based on the original relation $p_X = \frac{\omega}{\rho_X}$. Moreover, through the mathematically Taylor series expansion of the relatively general EOS we elaborately demonstrate concrete values of two characteristic parameters $(m, \alpha)$ for this ECG model, and we find the cosmic effective compositions (matter, radiation and vacuum energy) are from the extended Chaplygin gas component.

The plot of this paper is arranged as: In Sect 11 first we introduce the framework by mathematically integrable expression of the extended Chaplygin gas model with an-
alytical investigations. From the mathematical point of view, we divide the model into two classes described by two characters \( m \) and \( \alpha \). And then in Subsects.IIA and IIB, we investigate the two classes with general theoretical discussions, respectively. Secondly, through the Taylor series expansion method, a set of parameters \((m, \alpha)\) is determined and its physical indications are presented in Sect.III. At the last part, we summarize with discussions.

II. EXTENDED CHAPLYGIN GAS MODEL

The metric of a homogeneous and isotropic universe is usually written as follows

\[
ds^2 = dt^2 - a^2(t)dl^2,
\]

where \( dl^2 \) is the metric of a 3-manifold of constant curvature \((k = 0, \pm 1)\), and \( a(t) \) is the expansion factor.

Under the framework of Friedman-Robertson-Walker cosmology, the dynamic evolution of cosmology is completely determined by the Friedman equation

\[
\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \rho
\]

and energy conservation equation

\[
d(\rho a^3) = -pd(a^3)
\]

with state equation \( p = f(\rho) \) if we have. With the consideration that our Universe is filled mainly with both matter (mostly non-relative dark matter) and dark energy (the radiation component negligible at the later phase for the Universe evolution), so in the above Eq. (4) \( \rho = \rho_m + \rho_{DE} \), \( p = p_m + p_{DE} \) and we have used the unit convention \( 8\pi G/3 = c = 1 \) throughout.

As for EOS, taking \( p = f(\rho) \) into Eq. (4), we get generally:

\[
\int \frac{d\rho}{\rho + f(\rho)} = -3 \int \frac{da}{a}
\]

If only we know the concrete form of the function \( f(\rho) \), the evolution of scale factor \( a \) with the energy density \( \rho \), may be deduced out.

In order to describing the behavior of dark energy with more possibilities, an extension of the Chaplygin gas model is considered, and its relatively general EOS takes the following form:

\[
p_{X} = -\frac{A(a)}{\rho_{X}}\rho_{X}
\]

where \( p_{X} \) is pressure, \( \rho_{X}(\text{positive}) \) with \( \alpha > 0 \) is energy density of the dynamically changing part for the dark energy component if we also include the cosmological constant contribution, and \( A(a) \) is a positive function depending on scale factor \( a \) or its inverse. In some generalized Chaplygin gas model the cosmological constant can not be obtained at later cosmic times. The total EOS of dark energy can be, therefore, divided into two parts—the cosmological constant \( \rho_{\Lambda} \) and dynamically changing part \( \rho_{X} \). That is formerly:

\[
p_{DE} = \rho_{\Lambda} + \rho_{X}, \quad p_{DE} = p_{\Lambda} + p_{X}
\]

where the interrelations are,

\[
p_{\Lambda} = -\rho_{\Lambda}, \quad p_{X} = -\frac{A(a)}{\rho_{X}^2}\rho_{X}
\]

Also Eq. (3) is divided into, with the assumptions they decouple each other:

\[
d(\rho_{m}a^3) = -p_{m}d(a^3), \quad d(\rho_{\Lambda}a^3) = -p_{\Lambda}d(a^3),
\]

\[
d(p_{X}a^3) = -p_{X}d(a^3)
\]

\( i.e. \) by assuming that there is no interaction between different components. In this paper, our main attention is put on the ‘\( X \)-component’.

When \( A(a) \) takes a constant value \( A \) and \( \alpha = 2 \), it results in the following relation, the CG case (see reference [10]):

\[
\rho_{X} = \sqrt{A + \frac{B}{a^6}}
\]

where \( B \) is an integrated constant. We can get a more general relationship between density \( \rho_{X} \) and scale factor \( a \) by considering [5] and [8], that is

\[
\int_{a^{3\alpha}p_{X,0}^\alpha}^{a^{3\alpha}p_{X}^\alpha} d(a^3 \rho^\alpha) = \int_{a}^{a_0} 3\alpha a^{3\alpha-1} A(a) da
\]

where \( a_0 \) is the present value of the scale factor and \( \rho_{X,0} \) is the present X-term energy density.

Now entering the essence of this paper, we consider the function \( A(a) \) with the form \( A(a) = A_0 a^{-m} (A_0 \text{ is constant, and } m > 0) \), and then the characteristics of our model are determined by three undetermined parameters \((m, \alpha, A_0)\). A special case with \( \alpha = 2 \) results in the following relation [8]:

\[
\rho_{X} = \sqrt{\frac{6}{6 - m} \frac{A_0}{a^m} + \frac{B}{a^6}}
\]

Obviously it can easily return to the original CG model result by taking \( m = 0 \) directly.

Otherwise, in the point of view for integrability to [9], two cases are obvious as following. We will discuss \( m = 3\alpha \) in the section II.A and \( m \neq 3\alpha \) in the section II.B

We have known that present observation data constrains the range of the equation of state parameter for dark energy as \(-1.38 < w < -0.82\). Hao and Li [13] have demonstrated that \( w = -1 \) state is an attractor for the Chaplygin gas model and the equation of state of this gas could approach this attractor from either \( w < -1 \) or \( w > -1 \) sides. So in following sections, we would also consider this attractor approaches on the background of the extended Chaplygin gas model.
FIG. 1: On the $\rho_i - a$ plane, $\rho_i$ changes depending on $a$. $i$ represents $M, X, \Lambda$. (assuming $A_0 = 1$). And The total density $\rho = \sum_i \rho_i$ is also showed up.

A. The $m = 3\alpha$ case

In this section, we take $m = 3\alpha$ so Eq. (9) gives the following result:

$$\rho_X^0 = 3\alpha A_0 \frac{\ln a/a_0}{a^{3\alpha}} + a_0^{3\alpha} \rho_{X,0}^{\alpha}$$

(10)

We can see that,

$$\rho_X \propto a^{-3}, \quad a \to a_0$$

$$\rho_X \propto a^{-3}(\ln a)^{\frac{\alpha}{2}}, \quad a \to \infty$$

It means that around $a = a_0$ the dynamically changing part dark energy density evolves similar to dust-like matter with $\rho \sim a^{-3}$. In contrast, it evolves slower as $a \to \infty$ than that of $a \sim a_0$.

Figure 1 shows

from $a = 0.5134a_0$ the zero ECG density point, and with the increasing of scale factor $a$, the dynamically changing part of dark energy density has reached a biggest value (for example: corresponding to $\alpha = 2$ case, $(\rho_X)_{max} = \rho_X(0.6065a_0)$ seeing Figure 2), and then it approaches to a constant near zero.

As to $\alpha = 0$, we gain $p = -A_0 \rho$, which can be described directly by conventional EOS $p = w\rho$ ($w$ is constant). More discussions for this situation can be found in references [13, 14, 15], for example.

Taking Eq. (10) into Friedman Eq. (2), we get:

$$\int_{a_0}^{a} \frac{a^{1/2} da}{(3\alpha A_0 \ln a/a_0 + a_0^{3\alpha} \rho_{X,0}^{\alpha})^{1/2\alpha}} = \int_{t_0}^{t} dt$$

Because the uncertainty of $\alpha$, the above integration is not easy to solve. But for a general picture about the scale factor, we can simply put $\alpha = 1/2$. Thus we get

the integration result:

$$\ln Y + Y + \frac{Y^2}{2 \cdot 2!} + \frac{Y^3}{3 \cdot 3!} + \cdots = \frac{3A_0}{2a_0^{3/2}} \sqrt{\rho_{X,0} A_0} t + C$$

(11)

where $Y = \frac{3}{2} \ln \frac{a}{a_0} + \sqrt{\rho_{X,0} A_0}$. Eq. (11) can be directly figured as in Figure 3, which explicitly shows scale factor in a accelerating state with $a > 0$ and there is no singularity during the process.

Since no singularity (see reference [16] for the classification of singularity) occurs during the evolution of $\rho_X(a)$, we can discuss the $w = -1$ crossing with it. So far as we know, $w = \frac{p\rho_X}{\rho_D}$ results in the following equations with subscript denoting today’s value:

$$w = -\frac{\rho_X}{\rho + \rho_X} - \frac{1}{\eta} \eta \rho_X + \rho + \rho_X$$

$$w_0 = -\frac{\eta \rho_{X,0} + \rho + \rho_{X,0}}{\rho_{X,0} + \rho + \rho_{X,0}}$$

(12)

where $\eta$ is defined by $\eta = A_0/(a^m \rho_{X,0}^\alpha)$ called Judge for
Taking (10) into above equation, we can get:

$$\ln \frac{a}{a_0} \begin{cases} < & \frac{\eta - 1}{3\alpha \eta} \text{ provided } (w < -1) \\ > & \frac{\eta - 1}{3\alpha \eta} \text{ provided } (w > -1) \\ = & \frac{\eta - 1}{3\alpha \eta} \text{ provided } (w = -1) \end{cases}$$

Then we can see that through the evolution of scale factor $a(t)$, the value of state parameter $w$ changes continuously. So that crossing state barrier $(w = -1)$ would occur during the procedure of the evolution. It has also told that Judge $\eta$ here played an important role of relating to the different ranges of the present state parameter $w_0$ and the scale factor (denoted by $a^w$) in accordance with Phantom Divide($w = -1$) crossing as tabulated in [II A] below.

**Table IIA** Parameter ranges and $w = -1$

| $\eta$ | $w_0$ | $a^w(at) = w = -1$ |
|--------|-------|-------------------|
| $1$    | $-1$  | $a_0$             |
| $< 1$  | $> -1$| $< a_0$           |
| $> 1$  | $< -1$| $> a_0$           |

Why can the Judge $\eta$ have such important property? In the next part, we will work out its physical meaning in the more complete background.

Now, as for eq. (2), we can get its equivalent expression as:

$$h^2 = \frac{\rho_m}{H_0^2} + \frac{\rho_X}{H_0^2} + W_{\Lambda,0} - \frac{k}{H_0^2 a^2} = W_{m,0}(1 + z)^3 + W_{X,0}(1 + z)^3 + [3\alpha \eta \ln(1 + z) + 1]^\frac{1}{\alpha} + W_{\Lambda,0} + W_{k,0}(1 + z)^2$$

(13)

where $h(z)$ with $h(z) = \frac{H(z)}{H_0}$ is reduced Hubble parameter and $z = a_0/a - 1$ is red-shift, $W_{m,0} = \frac{\rho_m}{\rho_c,0}$, $W_{X,0} = \frac{\rho_X}{\rho_c,0}$, $W_{\Lambda,0} = \frac{\Lambda}{\rho_c,0}$, and $W_{k,0} = \frac{k}{2\pi \rho_c,0} (\rho_m,0, \rho_X,0, H_0, a_0$ are, respectively, present non-relative matter density, dynamically changing dark energy density, Hubble parameter, and scale factor); $\rho_c,0 = H_0^2$ is the current critical density. These density parameters should satisfy the following relation:

$$W_{m,0} + W_{X,0} + W_{\Lambda,0} + W_{k,0} = 1$$

(14)

Previously, we have pointed out that at the stage of $a \to a_0$ the density $\rho_X$ evolves like dust-like matter and now we can also see this point from following. Around $a = a_0$, we can expand the term $[3\alpha \eta \ln(1 + z) + 1]^1/\alpha$ around $z = 0$, and preserve the first order of $z$ and plus other terms. Thus, we get:

$$h^2 = \overline{W}_{m,0}(1 + z)^3 + 3W_{X,0,0}\eta(1 + z)^4 + W_{\Lambda,0} + W_{k,0}(1 + z)^2$$

(15)

The equivalent density parameter $\overline{W}_{m,0}$ now is defined by:

$$\overline{W}_{m,0} = W_{m,0} + W_{X,0}(1 - 3\eta)$$

(16)

which has included the dark energy density contribution part. The $(z + 1)^4$ term corresponds to the radiation component, which can be neglected in the beginning.

Considering the deceleration parameter $q = -a\dot{a}/a^2$, it can be expressed by the use of reduced Hubble parameter as:

$$q(z) = -1 + \frac{1 + z}{2h(z)^2} \frac{d}{dz} h(z)^2$$

(17)

Therefore, $(dh(z)^2/dz)_{z=0} = 2(1 + q_0)(q_0$ is the present deceleration parameter). Evaluating $(dh(z)^2/dz)_{z=0}$ from Eq. (13) and solving with respect to $\eta$, we get:

$$\eta = \frac{2q_0 + 2W_{\Lambda,0} - W_{m,0} - W_{X,0}}{3W_{X,0}}$$

(18)

So far, the physical meaning of parameter term $\eta$ has clearly stood before us. It reflects the combination of density parameters and deceleration parameter at present. In other words, if we know the values of density parameters and deceleration parameter, we would know the ranges of $w_0$ and $a^w$. In addition, from Eq. (18), we also know: $2(q_0 + W_{\Lambda,0}) > W_{m,0} + W_{X,0}$.

**B. The $m \neq 3\alpha$ case**

In this section, we have $m \neq 3\alpha$ and Eq. (9) has the following result:

$$\rho_X^\alpha = \rho_{X,0}^\alpha [(1 - \frac{3\alpha}{3\alpha - m})x^{3\alpha} + \frac{3\alpha}{3\alpha - m} \eta x^m]$$

(19)

where $x$ is defined by $x = a_0/a$ called reduced scale factor. We can see that dynamically changing part dark energy density evolves depending on two terms of $x^{3\alpha}$ and $x^m$, and the values of pure parameters term $x^{3\alpha}$ determine the effect. Additionally, the equation (19) also tells us that $\rho_X^\alpha$ decreases while the scale factor $a$ increases.

Combined with the EOS $p_{DE} = w\rho_{DE}$, the state parameter $w$ is expressed as:

$$w = -\frac{A_0}{\rho_X^\alpha(\rho_X^\alpha + \rho_\Lambda)}$$

(20)

. Taking (19) into consideration, we can get the following relation:

$$x^{3\alpha - m} < \frac{m}{(3\alpha - m)/\eta - 3\alpha}$$

provided $w < -1$

$$w = -1$$

$$w > -1$$

(21)

Through the evolution of scale factor $a$, the value of $x^{3\alpha - m}$ would satisfy the above relation, that is, the
$w = -1$ crossing phenomenon would occur. The different ranges of the present state parameter $w_0$ and the scale factor $a^w$ corresponding to the state barrier ($w = -1$) can be shown from the following table no matter whether $3\alpha > m$ or $3\alpha < m$.

| $\eta$ | $w_0$ | $a^w(w = -1)$ |
|--------|-------|---------------|
| $= 1$  | $= -1$| $= a_0$       |
| $< 1$  | $> -1$| $< a_0$       |
| $> 1$  | $< -1$| $> a_0$       |

Note: it is similar to Table II A but the Judge $\eta$ has changed.

Corresponding to Eq. (13), we can write out the expression of the $m \neq 3\alpha$ case as:

\[
h^2 = W_{m,0}(1 + z)^3 + W_{X,0}(1 + z)^3[1 + \frac{3\alpha}{3\alpha - m} \eta((1 + z)^{m-3\alpha} - 1)]^{\frac{1}{2}} + W_{A,0} + W_{k,0}(1 + z)^2
\]

(21)

It would be interesting to discuss the behavior around $a = a_0$. Thus, we can expand the $[1 + \frac{3\alpha}{3\alpha - m} \eta((1 + z)^{m-3\alpha} - 1)]^{\frac{1}{2}}$ around $z = 0$, preserve the first order of $z$, and get:

\[
h^2 = \widetilde{W}_{m,0}(1 + z)^3 - 3W_{X,0}\eta(1 + z)^4 + W_{A,0} + W_{k,0}(1 + z)^2
\]

(22)

where $\widetilde{W}_{m,0}$ is defined by

\[
\widetilde{W}_{m,0} = W_{m,0} + W_{X,0}(1 + 3\eta)
\]

(23)

The effective matter component also gets the contribution from the dark energy component.

With the use of Eq. (17), we can get the Judge as:

\[
\eta = \frac{-2q_0 + 2W_{A,0} - W_{m,0} - W_{X,0}}{3W_{X,0}}
\]

(24)

It results in: $2(q_0 + W_{A,0}) < W_{m,0} + W_{X,0}$ which is quite different from $m = 3\alpha$ case, and this difference can be used as to distinguishing the two cases.

### III. REMARKS ON $A(x)$

In the above treatment $A(a) = A_0a^{-m}$ as in the last section is just one of the possible forms for $A(a)$. Generally, we are not able to obtain $A(a)$ directly from theoretical analysis. However, as to the far future universe in which $x$(here we also define $x = a_0/a$) will approach to zero with redshift negative if we define today’s redshift value as zero, the behaviour of $A(a)$ is then determined by the small quantity $x$. It is natural to think of Taylor series expansion. We substitute $A(a)$ by $A(x)$ for simplification, and then expand $A(x)$.

\[
A(x) = \sum_{m} \frac{A^{(m)}(0)}{m!} x^m
\]

\[
= A(0) + A'(0)x + \frac{1}{2} A''(0)x^2 + 
\]

\[
\cdots + \frac{1}{m!} A^{(m)}(0)x^m + \cdots
\]

Taking it into the expression of EOS, the pressure becomes

\[
p_X = \sum_{m} \frac{A^{(m)}(0)}{m!\rho_X^m} \rho_X
\]

(25)

Compared with $A(a) = A_0a^{-m}$, the above expression is a combination of Eq. (13) with different values of $m$. This property may also imply that at the different cosmic stages there is an accordant effective value of $m$ in the EOS (ref eq state equation) to describe the dark energy fluid. As for far future stage, $m$ takes zero after neglecting small quantities.

Calculating density from Eq. (9), we get

\[
\rho_X^\alpha = \sum_{m} \frac{1}{m!} \frac{3\alpha}{3\alpha - m} A^{(m)}(0) x^m
\]

(26)

Often, $\rho_X$ and $\alpha$ depend on each other and we can deduce out the interrelationship of them from integration formula (9). Thus, pressure $p_X$ can be at least expressed as a function with one variable, for example, $p_X(\rho_X)$, but not the two variables function $p_X(\rho_X, \alpha)$. As for EOS $p_X = \frac{A(x)}{\rho_X}$, $A(x)$ and $\alpha$ are both used to determine $p_X(\rho_X)$ and at last $p_X(\rho_X)$ has no relation with $\alpha$. According to this assumption, we would try to choose $A(x)$ to satisfy this requirement. Through a series of analysis, we finally get

\[
A(x) = B^\alpha (1 + x)^{3\alpha - 1}
\]

where $B$ is a lower limit for density $\rho_X^\alpha$ as following. Thus, $\rho_x$ becomes

\[
\rho_x = B(1 + C_3^{1\alpha} + \cdots + C_{3\alpha}^{m} x^m + \cdots + x^{3\alpha})^{\alpha/3} = B(1 + x)^3
\]

where $C_{3\alpha}^{m} = \frac{(3\alpha)!}{m!(3\alpha-m)!}$ and $B$ is the density lower limit defined by

\[
\lim_{x\to\infty} \rho_X = B
\]

As a result, the EOS

\[
p_X = -B^\frac{1}{2} \rho_X^\frac{3}{2}
\]

(26)

is irrelevant to $\alpha$. Then, we will discuss what information about universe this EOS gives.
can get: 

As for \(X\)-term, we combined them with the other terms, we have

\[
h(z)^2 = W_{m,0}(1 + z)^3 + W_{k,0}(1 + z)^2 + \frac{B}{\rho_{X,0}}(2 + z)^3 + W_{\Lambda,0}
\]

(28)

where \(a_0\) and \(t_0\) is the present scale factor and age of universe, respectively. This function can easily be shown in figure 4 on which the reduced scale factor increases accompanying the elapse of cosmic time. And also \(\ddot{a} > 0\) implies our universe at the acceleration phase of state.

B. contributions from dynamical dark energy

Under the help of Eq.(2), reduced Hubble parameter \(h(z)^2\) can take the following form:

\[
h(z)^2 = \tilde{W}_{m,0}(1 + z)^3 + \tilde{W}_{k,0}(1 + z)^2 + \frac{3B}{\rho_{X,0}}(1 + z) + \tilde{W}_{\Lambda,0}
\]

(29)

where

\[
\tilde{W}_{m,0} = W_{m,0} + W_{X,0} \frac{B}{\rho_{X,0}}, \quad \tilde{W}_{k,0} = W_{k,0} + W_{X,0} \frac{3B}{\rho_{X,0}}
\]

\[
\tilde{W}_{\Lambda,0} = W_{\Lambda,0} + W_{X,0} \frac{B}{\rho_{X,0}}
\]

\(\tilde{W}_{m,0}, \tilde{W}_{k,0}, \text{and} \tilde{W}_{\Lambda,0}\) are the effective density parameters and show the \(X\)-term behaviors. Noting our prerequisite at cosmic far future evolution stage, the reduced Hubble parameter can be approximately written as

\[
h(z)^2 \approx \tilde{W}_{\Lambda,0}
\]

which means the de sitter phase. If our prerequisite is not so strict, we can see that for the bigger values of \(z\),

\[
h(z)^2 \approx \tilde{W}_{m,0}(1 + z)^3
\]

that means a dust-like phase. On the whole, our universe has experienced a phase transition from dust-like stage to a de sitter stage during the process of cosmic evolution.

C. The \(w = -1\) crossing problem

In section two we have discussed the crossing problem in general cases, and now we reconsider it with the specific situation. Taking EOS into the state parameter formula \(w = \frac{\rho_{DE}}{\rho_{X}}\), we get:

\[
w = \frac{(\frac{B}{\rho_{X}})^{\frac{3}{2}} \rho_{X} + \rho_{\Lambda}}{\rho_{X} + \rho_{\Lambda}}
\]

(30)

It is obvious that \(w > -1\), that is to say, there is no crossing phenomenon in this specific model and this particular case belongs to a quintessence-like dark energy.

IV. CONCLUSION AND DISCUSSION

In this paper, in order to discuss the properties of dark energy with more possibilities, which is used to explain currently cosmic accelerating expansion, we have extended the Chaplygin gas model by which the Dark Fluid concept can be realized with its contributions to effective universe compositions. Through a series of mathematical treatment, it shows that there are indeed some cases that can realize state parameter \(w = -1\) crossing. Moreover, the physical meanings behind the parameter terms like \(3A_0/a_0^6 \rho_{X,0}^{\alpha} \) in Sec.1 is interpreted.

Firstly, as for the \(m = 3\alpha\) case, we find that \(\rho_{X}\) first increases a maximum, and then decreases approaching to a constant near 0. During the procedure, the pressure \(p\) decreases to \(-\infty\). At the neighborhood of \(a = a_0\), the behavior of the density \(\rho_{X}\) is similar to non-relative matter with \(\rho \propto a^3\). With further analysis, we find that the state parameter \(w\) can cross its traditional barrier \(w = -1\) smoothly without singularity. By the calculations of the reduced Hubble parameter \(h(z)\), we find that the dynamically changing part of dark energy density is effectively equivalent to the sum of the matter-like part and radiation-like part. After considering the deceleration parameter \(q(z)\), the parameter term \(\frac{3A_0}{a_0^6 \rho_{X,0}^{\alpha}}\) is explicitly expressed as combination of the present density parameters \(W_{i,0}(\text{subscript } i = m, X, \Lambda, k)\) and the present deceleration parameter \(q_0\).
Secondly, as to the \( m \neq 3\alpha \) case, mostly it is similar to the \( m = 3\alpha \) case. However, different from the former case, the density \( \rho_X^m = (a_0/a)^m \) is made up of \( (a_0/a)^{3\alpha} \)-term and \( (a_0/a)^m \)-term, and it is equivalent to total effects of both a non-relative matter and some kinds of dark energy fluid with density \( \rho_X = (a_0/a)^m \). Obviously, from Eq. (19) \( \rho_X \) always decreases to approach zero with the scale factor \( a(t) \) increasing.

Thirdly, after describing the general situations, we also discuss a set of specific values of \( m = 3\alpha - 1 = 0 \). Then, with parameter independent assumption, concrete calculations show that the universe is in the state of accelerating expansion with \( \ddot{a} > 0 \). At last, we compare our model with the typical state parameter \( w = p_{DE}/\rho_{DE} \) obtaining \( w > -1 \), which means that there is never \( w = -1 \) crossing phenomenon in this case. To sum up, we have discussed the extended Chaplygin gas model in detail, with the hope to show more possible properties for dark energy and to realize the concept of Dark Fluid for a suggestive describing the cosmic dark components. To constrain the EOS characterizing parameters, work will be done to maximize the following likelihood function (see reference [23]):

\[
L \propto \exp\left[ -\chi^2(P)/2 \right].
\]

And also in this paper, interactions between the different components of cosmology are not considered. We will publish the contents elsewhere soon.

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