**XFEM analysis of concrete arch dam to assess deformations and propagation of artificial crack due to the combination of earthquake and uplift pressure**

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**Abstract**— In this study, the extended finite element method (XFEM) is further generalized to study the crack propagation in the concrete arch dam. Static and dynamic analysis performed on scaled-down prototype plane concrete arch dam to assess the deformation occurs due to water pressure and moderate earthquake magnitude of 5.8M. The most obvious finding to emerge from this study is that XFEM allows the entire crack to represented independently of the mesh, and so re-meshing is not necessary to model crack growth. The study has also shown that emergence of new cracks in different locations on the dam's body.

1. **Introduction**  
Concrete is one of the basic materials used very widely in various parts of the world for construction purposes. As a result of mixing the cement with the aggregate and water, it will produce concrete, which is characterized by providing high compressive strength in addition to durability. One of the defects that occur in concrete is the appearance of cracks. Cracks are one of the most important defects that must be studied and monitored their growth in concrete. The use of the finite element method for modeling discontinuities is very complex due to the need to change the mesh in proportion to the type of discontinuities. New methods of finite elements have been found for modeling cracks and monitoring their growth without changing the mesh. These incorporate the joining of a discontinuous mode on an element level [1], a moving mesh procedure [2], and an enhancement method for finite elements based on a partition-of-unity which involves minimal re-meshing [3]. curved cracks were treated by mapping the straight crack enriched field. Usually not promptly pertinent to long cracks or three dimensions. The extended finite element method (XFEM) will be used in this investigation through which to combine the discontinuous field between crack tip and crack face. The discontinuous displacement field near cracks can be approx by using the (XFEM) without the need for the use of interpolation functions of FE mesh. Therefore, the extended finite element method will be easier to use for cracking modeling and analysis than the classic finite element method. Since information concerning the crack geometry is in demand to determine the interpolation features in the extended finite element, the degree set method, which voices the geometry implicitly as the zero contour of the phase set function, able to be utilized to simplify the computation system in the extended finite element analysis. the extended finite element method can decrease the time needed to exhaustion mesh and division processes and that’s because of its ability to model cracks independently of the finite factor models. Thus, the extended finite element method able to be used to perform crack propagation analyses, which is not applicable in pursuit by using the classic finite element method, which regularly demands re-meshing procedures [3-8].

Therefore, the utilize of (XFEM) technique to simulate fracture behaviors of structures can decrease the time to evaluate the safety of engineering structures and minimize test costs. A lot of researchers study the (XFEM) field to simulate fracture behavior. Modelling quasi-static crack rise in 2-D problems for isotropic and biomaterial media utilizing the extended finite element is described in Sukumar and Prevost in which the implementation of the crack growth the usage of the extent within an ordinary object FE code is also described. The numerical functions are outrighted by Sukumar et al [9]. A 2-D numerical model of microstructural outcome and quasi-static crack propagation in brittle materials the
utilizing of extended finite element is in Sukumar et al [10]. The modelling of cracks with more than one sections, more than one holes and cracks emanating from holes is described in Daux et al [11]. The implementation is depending on the utilize of the equal enrichment functions for the cracks (discontinuous and tip functions) and the enrichment scheme is advanced based on the interaction of the discontinuous geometric aspects with the mesh. Whilst for holes, new enrichment characteristic is introduced. Modeling 3-d planar using the extended finite element was first introduced in Sukumar et al [12], who solved countless planar crack mode-I issues and present that the method in contrast totally with analytical solutions.

2. Numerical Modelling of the Dam

2.1 Arch dam modeling

Arched dams are massive and solid concrete dams that are constructed for the purpose of storing water and producing energy in narrow canyons with steep abutments. Arched dams are completely different from Gravity dams in terms of their resistance to loads. Gravity dams only resist loads by their weight, while the arched geometrical shape of arched dams gives the ability to transfer loads the canyon walls and dam foundation in addition to resisting part of the loads by its weight [13,14]. The dam in this investigation is depending on optional information, the site, water reservoir information, weather condition and other important information for dam plan and design were presumed. Typical values were chosen to know the properties of the material. The main object in this study is to assess stress distribution over the dam displacement of the concrete arch dam and follow-up to the spread of crack underwater pressure applied on the back of the dam and intensity of an earthquake.

A solid 3D curved plane concrete Dam model used in this study shown in figure 1, fixed from the bottom with dimensions is as follows: 1m inner length, 1.5m outer length, 0.15 thickness, and 0.75m height as. There is a crack (red line) as cleared in the Figure with 10 cm length, 2cm height, and 2cm depth at the mid surface.

2.2 Material properties

In the current study, only one material, concrete, was used. The concrete is plane from the steel reinforcement, considered homogeneous, isotropic and linear. For analysis, there are several properties of the concrete which effect on the analysis of arch dams These values are calculated according to the following equation and summarized in Table (1).

![Figure 1: Concrete Arch Dam Model with Artificial Crack.](image-url)
in simulia Abaqus/Cae depends upon the nodal displacements of the elements nearby the crack tip. The approximation for the vector function for the displacement is [17]:

\[ u = \sum_{i=1}^{N} N_i(x) [u_i + H(x)a_i + \sum_{a=1}^{4} f_a (x) b_{a}^i] \] ...

where; (u: displacement vector, N: shape function, H: jump function, a_i: nodal enriched degree of freedom vector, f_a: asymptotic crack-tip functions and b_a^i: nodal enriched degree of freedom vector).

In the extended finite element, the unique function is the division of unit, that approves the existence of discontinuities in an aspect by using enriching degrees of freedom with one of a kind displacements function (Equation 3). The technique is beneficial to solve an approximate fraction of the computational non-smooth domain.

### Table 1: Material Properties

| Properties                      | Value         |
|---------------------------------|---------------|
| Compressive strength, \( f_{cc} \) | 50 (Mpa)      |
| Density of Concrete             | 2400 (Kg/m³)  |
| Poisson’s ratio                 | 0.2           |
| Modulus of elasticity, \( E_c \) | 33234 (Mpa)   |
| Fracture Energy, \( G_f \)     | 1008.6 (Nm/m) |

All nodes in the model should be applied by shape function as well as all nodes should be applied by jump function, whose shape function backing is cut by the crack interior; Asymptotic crack-tip function effects to the node whose shape function support is cut by the crack tip (Figure 2). The jump function can be written as Equation 4, which is support cut by the crack interior.

![Figure 2: Functions applied on the crack part.](image)

\[ H (X) = \begin{cases} \frac{1}{n!} f (x-x_1), & n \geq 0 \\ -1, & \text{otherwise} \end{cases} \] ...

Where: [x= Sample (Gauss) point, x1= The nearest point to x on the crack and n= Unit outward normal to the crack at x1]. The tip function is given by

\[ F_s = [ \sqrt{r} \sin \frac{\theta}{2}, \sqrt{r} \cos \frac{\theta}{2}, \sqrt{r} \sin \theta \cdot \sin \frac{\theta}{2}, \sqrt{r} \sin \theta \cdot \cos \frac{\theta}{2} ] \] ...

Where: r, \( \theta \) = Polar coordinate system whit its origin at \( \theta=0 \) and the crack tip [17].
The prediction of the initiation of a crack is based totally on the max principal stress; crack initiation takes place if

\[ f = \frac{\langle \sigma_{\text{max}} \rangle}{\sigma_{\text{max}}} \quad (6) \]

Where: The symbol \( \langle \cdot \rangle \) represents the Macaulay bracket with the usual interpretation.

\( \sigma_{\text{max}}^{*} \): Maximum allowable principal stress.

\( \sigma_{\text{max}} \): Maximum normal principle stress.

\( \langle \sigma_{\text{max}} \rangle = \begin{cases} 2, & \sigma_{\text{max}} < 2 \\ \sigma_{\text{max}}, & \sigma_{\text{max}} \geq 2 \end{cases} \)

Equation 4 implies that the purely compressive stress state does not initiate damage [17].

4. Mesh
The mesh properties and details for the dam model are listed in Table 2 and Figure 3.

| Element Type | Element Shape | Geometric Order | Elements |
|--------------|---------------|-----------------|----------|
| C3D8R        | hexahedral    | linear          | 2889     |

![Figure 3: Model Mesh](image)

5. Input Loads and Seismic Motion

Dams are subjected to a variety of loads, both static and dynamic such as (gravity loads, uplift pressure, seismic loads etc.), the gravity load was applied at this study and uplift pressure of water Equivalent to real reservoir full to a height of 100 m. To simulate the response of, the input seismic motion used in the dynamic crack propagation analysis was designed by assuming the occurrence of a moderate magnitude M5.8 decided by "Northwest Calif-03" earthquake of 1951 which cased strike-slip mechanism. The simulated seismic motion used is shown in Figure (4). The ground motion effects on the arch dam are considered with X-direction, Y-direction and vertical components of the "Northwest Calif-03" earthquake records with 0.05 damping ratio was used in calculations. The numerical analyses are realized during 20 sec. Besides, 0.01 second was selected as the time step.

6. Analysis Results

Before applying the dynamic crack propagation analysis, the static analysis was performed to get initial stress taking into account the dead weight of the dam structure, the static pressure of the reservoir water, and the uplift pressure on the surface of the dam body. The results of the dynamic crack
propagation analysis are shown in Figure (5), which depicts relative displacement between the top and bottom at the center of the dam body.

![Figure 5: Relative Displacement](image)

Figures 6 show the crack propagation distribution with time. It can be seen that a crack initially progressed a long way from the center of the dam to the bottom of the dam with angle approximately equal to 45 degrees, then it settled at a certain limit and stopped spreading until the earthquake was applied, and it began to spread again. In addition, cracks began to appear on the inner surface of the dam’s body and the lower base as well until the cracks on the inner surface of the dam met with those generated on the lower base of the dam in the time 15 seconds of the earthquake. At second 17 the concrete dam was divided into two parts thus reaching the final stage of failure.

![Figure 4: Magnitude](image)
Figure 6: Crack propagation distribution.

Figures (7) and (8) show the crack propagation stress distribution in (N/m²) along with the dam body and the maximum at representative times. In about a third of the time of the earthquake, we notice that there is a high concentration of stresses at the bottom of the concrete arch dam, while the value of these stresses decreases in other parts of the dam body, and this is because the bottom is tightly fixed, which creates strong reactions to resist the stresses of water pressure and earthquake. As we note from Figure (8), before the earthquake reaches its end and before the final failure of the concrete arch dam occurs, the value of stresses in the bottom of the dam and the rest parts will decrease as a result of entering the unloading stage, and the maximum value of the stresses will be noticed at the failure area which is shown in red color. It should also be noted that the cracks that were visible near the support at the bottom are the result of the maximum shear forces in this area, as well as there are no reinforcements in the dam's concrete.

Figure 7: Stress distribution at t = 7 sec.
Figure 8: Stress distribution at $t = 17$ sec.

7. Conclusion
Static and dynamic analysis to check crack propagation during water pressure and earthquake has been presented. The XFEM has shown a good numerical technique based on the generalized finite element method and the partition of unity method. The powerful for discontinuous crack growth, including the Independent of the internal geometry and physical interfaces, such that meshing and re-meshing difficulties in discontinuous problems can be overcome.

When the static analysis applied the crack start it to propagate and no deformations noticed. After earthquake occurs, the deformations appeared very quick in different locations at dam's body, and the main crack continues to spread in an angle of about 45 degrees, as well as cracks appear in the bottom of the dam near the support. The main reason of these deformities and cracks and their rapid spread because no reinforcement resists the tensile stresses generated in the concrete; in addition to the absence of reinforcement that resists the shear stresses generated at the bottom of the dam. It has noticed that if the earthquake lasts for extra than 10 seconds, the entire model fails.

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