Uniformly convergent Fourier series and multiplication of functions

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Let $U(T)$ be the space of all continuous functions on the circle $T$ whose Fourier series converges uniformly. Salem’s well-known example shows that a product of two functions in $U(T)$ does not always belong to $U(T)$ even if one of the factors belongs to the Wiener algebra $A(T)$. In this paper we consider pointwise multipliers of the space $U(T)$, i.e., the functions $m$ such that $mf \in U(T)$ whenever $f \in U(T)$. We present certain sufficient conditions for a function to be a multiplier and also obtain some results of Salem type.

**Key words:** uniformly convergent Fourier series, function spaces, multiplication operators.

1. Introduction

We consider functions $f$ on the circle $T = \mathbb{R}/(2\pi \mathbb{Z})$ and their Fourier expansions

$$f(t) \sim \sum_{k \in \mathbb{Z}} \hat{f}(k)e^{ikt}. \quad (1)$$

Here $\mathbb{R}$ is the real line and $\mathbb{Z}$ is the additive group of integers (we naturally identify the functions on $T$ with the $2\pi$-periodic functions on $\mathbb{R}$).

Let $C(T)$ be the space of all continuous functions $f$ on $T$ with the usual norm $\|f\|_{C(T)} = \sup_{t \in T} |f(t)|$. Consider the space $U(T)$ of all functions $f \in C(T)$ which have uniformly convergent Fourier series, i.e., satisfy the condition $\|f - S_N(f)\|_{C(T)} \to 0$ as $N \to \infty$, where $S_N(f)$ is the $N$th partial sum of the Fourier series of $f$:

$$S_N(f)(t) = \sum_{|k| \leq N} \hat{f}(k)e^{ikt}.$$
The norm on $U(\mathbb{T})$ is defined by

$$\|f\|_{U(\mathbb{T})} = \sup_N \|S_N(f)\|_{C(\mathbb{T})}.$$  

We note that the space $U(\mathbb{T})$ is a Banach space. Clearly, the inclusion $U(\mathbb{T}) \subseteq C(\mathbb{T})$ holds with the corresponding relation for the norms: $\|\cdot\|_{C(\mathbb{T})} \leq \|\cdot\|_{U(\mathbb{T})}$.

Another well-known space related to expansion (1) is the space $A(\mathbb{T})$ of all functions $f \in C(\mathbb{T})$ whose Fourier series converges absolutely. Endowed with the natural norm

$$\|f\|_{A(\mathbb{T})} = \sum_{k \in \mathbb{Z}} |\hat{f}(k)|,$$

the space $A(\mathbb{T})$ is a Banach space. Moreover, under the usual multiplication of functions $A(\mathbb{T})$ is a Banach algebra (known as the Wiener algebra). Obviously, $A(\mathbb{T}) \subseteq U(\mathbb{T})$ and $\|\cdot\|_{U(\mathbb{T})} \leq \|\cdot\|_{A(\mathbb{T})}$.

It is known that the space $U(\mathbb{T})$, unlike $A(\mathbb{T})$ and $C(\mathbb{T})$, is not an algebra; the product of two functions in $U(\mathbb{T})$ does not necessarily belong to $U(\mathbb{T})$. Moreover, there exist $f \in A(\mathbb{T})$ and $g \in U(\mathbb{T})$ such that $fg \notin U(\mathbb{T})$. This result is due to Salem; see [1, Ch. 1, Sec. 6]. In this paper we present certain sufficient conditions for a function $m$ to have the property that $mf \in U(\mathbb{T})$ whenever $f \in U(\mathbb{T})$. We also obtain some results of Salem type.

2. Pointwise multipliers: Sufficient conditions

Let $m \in C(\mathbb{T})$. We say that $m$ is a pointwise multiplier of the space $U(\mathbb{T})$ if for every function $f$ in $U(\mathbb{T})$ the product $mf$ is in $U(\mathbb{T})$ as well. We denote the space of all these multipliers by $MU(\mathbb{T})$. Clearly, if $m \in MU(\mathbb{T})$, then the operator $f \rightarrow mf$ of multiplication by $m$ is a bounded operator on $U(\mathbb{T})$. The space $MU(\mathbb{T})$ endowed with the natural norm

$$\|m\|_{MU(\mathbb{T})} = \sup_{\|f\|_{U(\mathbb{T})} \leq 1} \|mf\|_{U(\mathbb{T})}$$

is a Banach algebra. Obviously, $MU(\mathbb{T}) \subseteq U(\mathbb{T})$.

Recall a classical fact (see, e.g., [2, Ch. VIII, Sec. 1, Theorem 3]): if a function $g \in C(\mathbb{T})$ satisfies the uniform Dini condition

$$\sup_t \int_{|t-\theta| \leq \varepsilon} \frac{|g(t) - g(\theta)|}{|t - \theta|} d\theta = o(1), \quad \varepsilon \rightarrow +0, \quad (2)$$

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then $g \in U(T)$.

We shall prove the following theorem.

**Theorem 1.** If a function $m \in C(T)$ satisfies the uniform Dini condition, then $m \in MU(T)$. Furthermore, the following estimate holds

$$\|m\|_{MU(T)} \leq \|m\|_{C(T)} + \sup_t \int_{|t-\theta|\leq \pi} \frac{|m(t) - m(\theta)|}{|t - \theta|} d\theta.$$

Theorem 1 immediately implies a sufficient condition for a function to belong to $MU(T)$ in terms of the modulus of continuity; namely, we obtain the following theorem.

**Theorem 2.** If $m \in C(T)$ and

$$\int_0^\pi \frac{\omega(m, \delta)}{\delta} d\delta < \infty,$$

where $\omega(m, \delta) = \sup_{|t_1 - t_2| \leq \delta} |m(t_1) - m(t_2)|$, then $m \in MU(T)$. Furthermore, one has

$$\|m\|_{MU(T)} \leq \|m\|_{C(T)} + 2 \int_0^\pi \frac{\omega(m, \delta)}{\delta} d\delta.$$

The following theorem, which gives a sufficient condition in terms of the Fourier transform (proved by the author as a lemma in [3]) is an immediate consequence of Theorem 2.

**Theorem 3.** If $m \in C(T)$ and

$$\sum_{k \in \mathbb{Z}} |\hat{m}(k)| \log(|k| + 2) < \infty,$$

then $m \in MU(T)$. Furthermore,

$$\|m\|_{MU} \leq c \sum_{k \in \mathbb{Z}} |\hat{m}(k)| \log(|k| + 2),$$

where $c > 0$ does not depend on $m$.  

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To derive Theorem 3 from Theorem 2, it suffices to note that
\[
\omega(m, \delta) \leq \sum_{k \neq 0} |\hat{m}(k)||2 \sin(k\delta/2)|
\]
and use the obvious relation\footnote{We write \(a(n) \lesssim b(n)\) or \(b(n) \gtrsim a(n)\) in the case when \(a(n) \leq cb(n)\) for all sufficiently large \(n\), where \(c > 0\) does not depend on \(n\). The relation \(a(n) \simeq b(n)\) means that \(a(n) \lesssim b(n)\) and \(b(n) \lesssim a(n)\) simultaneously.}
\[
\int_0^\pi \frac{|\sin(k\delta/2)|}{\delta} d\delta \simeq \log |k|.
\]

**Proof of Theorem 1.** For each \(N = 0, 1, 2, \ldots\) let \(Q_N\) be the commutator of the operator of multiplication by \(m\) and the partial sum operator \(S_N\), i.e.,
\[
Q_N : f \rightarrow mS_N(f) - S_N(mf).
\]

Considering these commutators as operators on \(C(T)\), let us show that (under the assumptions of the theorem on \(m\)) the following two conditions hold:

(i) the sequence of the norms \(\|Q_N\|_{C(T) \rightarrow C(T)}\), \(N = 0, 1, 2, \ldots\), is bounded;

(ii) for each \(n \in \mathbb{Z}\), the sequence of the norms \(\|Q_N e_n\|_{C(T)}\), \(N = 0, 1, 2, \ldots\), of the images of the exponential function \(e_n(t) = e^{int}\) tends to zero as \(N \to \infty\).

We set
\[
c = \sup_t \int_{|t-\theta|\leq \pi} \frac{|m(t) - m(\theta)|}{|t-\theta|} d\theta.
\]

For \(0 < |x| \leq \pi\), the Dirichlet kernel \(D_N(x) = \sum_{|k| \leq N} e^{ikx}\) is estimated as
\[
|D_N(x)| = \left| \frac{\sin(N + 1/2)x}{\sin(x/2)} \right| \leq \frac{1}{|\sin(x/2)|} \leq \frac{\pi}{|x|}.
\]

So, if \(\|f\|_{C(T)} \leq 1\), then for every \(t \in T\) we obtain
\[
|Q_N f(t)| = \left| \frac{1}{2\pi} \int_T D_N(t - \theta) f(\theta) d\theta - \frac{1}{2\pi} \int_T D_N(t - \theta) m(\theta) f(\theta) d\theta \right| =
\]
\[ \frac{1}{2\pi} \left| \int_{\mathbb{T}} D_N(t - \theta)(m(t) - m(\theta))f(\theta)d\theta \right| \leq \frac{1}{2\pi} \int_{|t-\theta|\leq\pi} |D_N(t - \theta)||m(t) - m(\theta)|d\theta \leq c. \]

Hence,
\[ \|Q_N\|_{C(\mathbb{T}) \to C(\mathbb{T})} \leq c. \tag{3} \]

Thus condition (i) holds.

It is obvious that the product of two continuous functions satisfying the uniform Dini condition (see (2)) satisfies the uniform Dini condition as well and therefore belongs to \( U(\mathbb{T}) \). So, for all \( n \in \mathbb{Z} \) we have \( me_n \in U(\mathbb{T}) \), whence
\[ \|Q_N e_n\|_{C(\mathbb{T})} = \|m S_N(e_n) - S_N(me_n)\|_{C(\mathbb{T})} \to 0 \quad \text{as} \quad N \to \infty. \]

Thus, (ii) holds.

It remains to note that from (i) and (ii) it follows that \( \|Q_N f\|_{C(\mathbb{T})} \to 0 \) for every function \( f \in C(\mathbb{T}) \), whence for every \( f \in U(\mathbb{T}) \) we obtain
\[ \|m f - S_N(m f)\|_{C(\mathbb{T})} = \|m (f - S_N(f)) + Q_N(f)\|_{C(\mathbb{T})} \leq \|m\|_{C(\mathbb{T})}\|f - S_N(f)\|_{C(\mathbb{T})} + \|Q_N(f)\|_{C(\mathbb{T})} \to 0. \]

The bound of the norm \( \|m\|_{MU(\mathbb{T})} \) is obvious from (3) and the identity \( S_N(m f) = m S_N(f) - Q_N(f) \). This completes the proof of Theorem 1 and, thereby, Theorems 2 and 3.

### 3. Two results of Salem type and their corollaries

Recall Salem’s result mentioned in the introduction: there exist functions \( f \in A(\mathbb{T}) \) and \( g \in U(\mathbb{T}) \) such that \( fg \notin U(\mathbb{T}) \). We note that the proof of this result given in [1, Ch. 1, Sec. 6] yields in addition that \( \hat{g}(k) = o(1/|k|) \). Modifying this proof, we obtain two theorems of a similar type. As a consequence we shall see that Theorem 3 is sharp.

Given a positive sequence \( \gamma = \{\gamma(n), n = 0, 1, 2, \ldots\} \) consider the space \( A_\gamma(\mathbb{T}) \) of all functions \( f \in C(\mathbb{T}) \) satisfying
\[ \|f\|_{A_\gamma(\mathbb{T})} = \sum_{k \in \mathbb{Z}} |\hat{f}(k)|\gamma(|k|) < \infty. \]

We shall deal with sequences \( \gamma \) bounded away from zero, i.e., such that \( \inf_n \gamma(n) > 0 \); this is why we assume the continuity of functions in \( A_\gamma(\mathbb{T}) \). If
\[ \gamma(n) \equiv 1, \] we obtain the Wiener algebra \( A(\mathbb{T}) \). Clearly, \( A_\gamma(\mathbb{T}) \) is a Banach space (provided that \( \gamma \) is bounded away from zero).

Let \( V(\mathbb{T}) \) be the space of all functions of bounded variation on \( \mathbb{T} \). We denote the variation of \( f \) on \( \mathbb{T} \) by \( \|f\|_{V(\mathbb{T})} \).

Consider also the space \( W^{1/2}_2(\mathbb{T}) \) (the Sobolev space) of all integrable functions \( f \) on \( \mathbb{T} \) with

\[ \|f\|_{W^{1/2}_2(\mathbb{T})} = \left( \sum_{k \in \mathbb{Z}} |\hat{f}(k)|^2 |k| \right)^{1/2} < \infty. \]

Recall that \( V \cap C(\mathbb{T}) \subseteq U(\mathbb{T}) \) and \( W^{1/2}_2 \cap C(\mathbb{T}) \subseteq U(\mathbb{T}) \). These inclusions are well known. It is also clear that if \( \gamma \) is bounded away from zero, then \( A_\gamma(\mathbb{T}) \subseteq A(\mathbb{T}) \subseteq U(\mathbb{T}) \).

**Theorem 4.** If

\[ \lim_{n \to \infty} \frac{\gamma(n)}{\log n} = 0, \]

then there exist two (real) functions \( f \) and \( g \) such that \( f \in A_\gamma(\mathbb{T}) \) and \( g \in V \cap C(\mathbb{T}) \) but \( fg \notin U(\mathbb{T}) \).

**Theorem 5.** If

\[ \lim_{n \to \infty} \frac{\gamma(n)}{(\log n)^{1/2}} = 0, \]

then there exist two (real) functions \( f \) and \( g \) such that \( f \in A_\gamma(\mathbb{T}) \) and \( g \in W^{1/2}_2 \cap V \cap C(\mathbb{T}) \) but \( fg \notin U(\mathbb{T}) \).

Theorem 4 shows, in particular, that the logarithmic weight in Theorem 3 cannot be replaced by a weight of slower growth. Combining Theorems 3 and 4 we obtain the following corollary.

**Corollary 1.** The inclusion \( A_\gamma(\mathbb{T}) \subseteq MU(\mathbb{T}) \) holds if and only if

\[ \lim_{n \to \infty} \frac{\gamma(n)}{\log n} > 0. \]

\(^2\)The former was obtained by Jordan, see [4, Ch. I, Sec. 39]. The latter can be proved as follows. Assuming that \( g \in W^{1/2}_2(\mathbb{T}) \), we have \( \sum_{|k| \leq N} |k||\hat{g}(k)| = o(N) \), which for \( g \in C(\mathbb{T}) \) implies \( g \in U(\mathbb{T}) \), see [4, Ch. I, Sec. 64].
The next corollary is an obvious consequence of Theorem 5.

**Corollary 2.** $W^{1/2}_2 \cap V \cap C(\mathbb{T}) \not\subseteq MU(\mathbb{T})$.

To prove Theorems 4 and 5 we need some lemmas.

It is convenient to consider the general case of linear normed spaces $S$ embedded in $C(\mathbb{T})$, i.e., such that $S$ is contained in $C(\mathbb{T})$ as a linear subspace and $\|f\|_{C(\mathbb{T})} \leq \text{const} \|f\|_S$ for all $f \in S$.

If $X$ and $Y$ are two sets in $C(\mathbb{T})$ we let

$$XY = \{xy : x \in X, y \in Y\}.$$  

The following general lemma on multiplication is possibly known in one form or another. We provide a short proof.

**Lemma 1.** Let $X$, $Y$, and $Z$ be Banach spaces embedded in $C(\mathbb{T})$. Suppose that $XY \subseteq Z$. Then there exists a $c > 0$ such that $\|xy\|_Z \leq c \|x\|_X \|y\|_Y$ for all $x \in X$ and $y \in Y$.

**Proof.** Given Banach spaces $E$ and $F$, let $B(E, F)$ denote the space of bounded operators from $E$ to $F$. For $y \in Y$, let $M_y : X \to Z$ be the operator that takes each element $x \in X$ to the product $xy$. Applying the closed graph theorem, we obtain $M_y \in B(X, Z)$. Consider then the operator $Q : Y \to B(X, Z)$ that takes each element $y$ of $Y$ to the operator $M_y$. Applying the closed graph theorem, we obtain $Q \in B(Y, B(X, Z))$. Setting $c = \|Q\|_{B(Y, B(X, Z))}$, we see that $\|xy\|_Z = \|M_yx\|_Z \leq \|M_y\|_{B(X, Z)} \|x\|_X = \|Qy\|_{B(X, Z)} \|x\|_X \leq c \|y\|_Y \|x\|_X$. The lemma is proved.

For $n = 1, 2, \ldots$, let $g_n$ be the functions defined by

$$g_n(t) = \sum_{k=1}^{n} \left(1 - \frac{k}{n}\right) \frac{1}{k} \sin kt. \quad (4)$$

As above, by $e_n$ we denote the function $e_n(t) = e^{int}$.

**Lemma 2.** The estimate $\|e_n g_n\|_{U(\mathbb{T})} \gtrsim \log n$ holds.
Proof. We have
\[ g_n(t) = \sum_{1 \leq |k| \leq n} \left( 1 - \frac{|k|}{n} \right) \frac{1}{2ik} e^{ikt}, \]
whence
\[ e_n(t)g_n(t) = \sum_{1 \leq |k| \leq n} \left( 1 - \frac{|k|}{n} \right) \frac{1}{2ik} e^{i(k+n)t}. \]
Therefore,
\[ S_n(e_n g_n)(t) = \sum_{1 \leq |k| \leq n, |k+n| \leq n} \left( 1 - \frac{|k|}{n} \right) \frac{1}{2ik} e^{i(k+n)t}. \]
This implies
\[ S_n(e_n g_n)(0) = \sum_{-n \leq k \leq -1} \left( 1 - \frac{k}{n} \right) \frac{1}{2k} = \sum_{k=1}^{n} \left( 1 - \frac{k}{n} \right) \frac{1}{2k}. \]
So,
\[ \|e_n g_n\|_{U(T)} \geq |S_n(e_n g_n)(0)| = \sum_{k=1}^{n} \left( 1 - \frac{k}{n} \right) \frac{1}{2k} = \sum_{k=1}^{n} \frac{1}{2k} + O(1) \simeq \log n. \]
The lemma is proved.

Lemma 3. The estimates \( \|g_n\|_{V(T)} \leq 2\pi \) and \( \|g_n\|_{C(T)} \leq 2\pi \) hold.

Proof. For the derivative \( g'_n \) we have (see (4))
\[ g'_n(t) = \sum_{k=1}^{n} \left( 1 - \frac{k}{n} \right) \cos kt = \frac{1}{2} F_n(t) - \frac{1}{2}, \]
where \( F_n \) is the Fejér kernel. As is known, \( F_n(t) \geq 0 \) for all \( t \), so,
\[ |g'_n(t)| \leq \left| g'_n(t) + \frac{1}{2} \right| + \frac{1}{2} = g'_n(t) + 1, \]
whence
\[ \|g_n\|_{V(T)} = \int_0^{2\pi} |g'_n(t)| dt \leq 2\pi, \]
which, in turn, implies \( \|g_n\|_{C(T)} \leq 2\pi \), because \( g_n(0) = 0 \). The lemma is proved.

**Proof of Theorems 4 and 5.** We set
\[ \|f\|_{V \cap C(T)} = \|f\|_V(T) + \|f\|_{C(T)} \]
\[ \|f\|_{W^{1/2} \cap V \cap C(T)} = \|f\|_{W^{1/2} \cap V(T)} + \|f\|_V(T) + \|f\|_{C(T)}. \]
The spaces \( V \cap C(T) \) and \( W^{1/2} \cap V \cap C(T) \) endowed with these norms are Banach spaces (we leave a simple verification to the reader).

Thus, each of the four spaces \( U(T), A_\gamma(T), V \cap C(T), \) and \( W^{1/2} \cap V \cap C(T) \) is a Banach space embedded in \( C(T) \).

We apply Lemma 1. Assuming that the conclusion of Theorem 4 is false, we obtain
\[ \|fg\|_U \leq c \|f\|_{A_\gamma} \|g\|_{V \cap C}. \]
In particular,
\[ \|e_n g_n\|_U \lesssim \|e_n\|_{A_\gamma} \|g_n\|_{V \cap C}. \] \hspace{1cm} (5)
Clearly, \( \|e_n\|_{A_\gamma} = \gamma(n) \). Taking Lemmas 2 and 3 into account and using (5), we obtain \( \log n \lesssim \gamma(n) \), which contradicts the condition of Theorem 4.

Assuming that the conclusion of Theorem 5 is false, we have
\[ \|e_n g_n\|_U \lesssim \|e_n\|_{A_\gamma} \|g_n\|_{W^{1/2} \cap V \cap C}. \] \hspace{1cm} (6)
Obviously, \( \|g_n\|_{W^{1/2}} \simeq (\log n)^{1/2} \), whence, taking Lemma 3 into account, we obtain
\[ \|g_n\|_{W^{1/2} \cap V \cap C} \simeq (\log n)^{1/2}. \]
Thus, relation (6) and Lemma 2 yield \( \log n \lesssim \gamma(n) (\log n)^{1/2} \), which contradicts the condition of Theorem 5.

**Remarks.** 1. The following result, directly related to the topic of this paper, was obtained by Olevskii [5]: The algebra generated by \( U(T) \) coincides with \( C'(T) \); moreover, each function \( f \in C(T) \) can be represented in the form \( f = \varphi_1 \varphi_2 + \varphi_3 \), where \( \varphi_j \in U(T), \ j = 1, 2, 3 \). Apparently, it is unknown whether \( \varphi_3 \) can be set to zero. It would be interesting to find out if one can chose the functions \( \varphi_j \) so that they belong to more narrow classes rather then \( U(T) \); for instance, can we chose \( \varphi_1 \in A(T) \)?
2. Theorem 3 immediately implies $\|e_n\|_{MU} \lesssim \log |n|$. At the same time it is easy to see that the functions $g_n$ defined by (4) satisfy $\|g_n\|_U = O(1)$. So, using Lemma 2, we see that $\log |n| \lesssim \|e_n g_n\|_U \leq \|e_n\|_{MU} \|g_n\|_U$. Thus, $\|e_n\|_{MU(T)} \simeq \log |n|$.

3. It is natural to consider the nonsymmetric analogue of the space $U(T)$, namely, the space $U^\text{asym}(T)$ defined in the similar way as $U(T)$ with the only difference that instead of the partial sums $S_N(f)(t)$ one uses the partial sums $S_{N,M}(f)(t) = \sum_{N \leq k \leq M} \hat{f}(k)e^{ikt}$. The space $MU^\text{asym}(T)$ of multipliers is defined in a natural way. Note that the spaces of multipliers in the symmetric and nonsymmetric cases are different. It is easy to verify that $\|e_n\|_{MU^\text{asym}(T)} = O(1)$ and, therefore, $A(T) \subseteq MU^\text{asym}(T)$. It is worth mentioning that this embedding has a counterpart for functions analytic in the disk $D = \{z \in \mathbb{C} : |z| < 1\}$ of the complex plane $\mathbb{C}$. Let $U^+(D)$ be the class of functions analytic in $D$ whose Taylor series converges uniformly in $D$, and let $A^+(D)$ be the class of functions analytic in $D$ whose sequence of Taylor coefficients belongs to $l^1$. If $f \in A^+(D)$ and $g \in U^+(D)$, then $fg \in U^+(D)$.

4. The conditions in Theorems 1–3 of this paper are resemblant to the conditions that appear in the work by Vinogradov, Goluzina, and Khavin [6, Theorem 3] on multipliers of the space $K$ of Cauchy–Stieltjes-type integrals. The resemblance of these conditions is a reflection of a certain likeness between the character of singularity of the Dirichlet and Cauchy kernels.

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