Primordial Black Holes and Scotogenic dark matter

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Abstract. We study the correlations between the initial density of primordial black holes and the lepton flavor violating \( \mu \rightarrow e\gamma \) process via scotogenic dark matter. The initial density of PBHs may be constrained with the future MEG II experiment.

1. Introduction
The primordial black holes (PBHs) are the one of the type of black hole that produced in the early Universe \([1, 2]\). A PBH emits particles via the Hawking radiation. The particle dark matter is also produced by Hawking radiation of the PBHs \([3]\).

On the other hand, the lepton flavor violating phenomena, such as \( \mu \rightarrow e\gamma \) process, are directly related to the new physics beyond the standard model of particle physics. In addition, since the physics run of the MEG II experiment to search \( \mu \rightarrow e\gamma \) process with 10 times better sensitivity than the MEG experiment will be started very near future \([4, 5]\), a study which is related on the \( \mu \rightarrow e\gamma \) process is interesting and timely.

In this study, we show the correlations between the initial density of PBHs and the branching ratio of \( \mu \rightarrow e\gamma \) with scotogenic dark matter \([6]\). Since the scotogenic model can account for dark matter candidates and predict the lepton flavor violating processes simultaneously, and the scotogenic dark matter can be also produced by Hawking radiation of PBH \([7]\), the initial density of the PBHs and the branching ratio of \( \mu \rightarrow e\gamma \) is related via scotogenic dark matter.

2. Scotogenic model
The scotogenic model \([6]\) is an extension of the standard model in the particle physics. In this model, three new Majorana \( SU(2)_L \) singlets \( N_k \) \((k = 1, 2, 3)\) with mass \( M_k \) and one new scalar \( SU(2)_L \) doublet \((\eta^+, \eta^0)\) are introduced. These new particles are odd under exact \( Z_2 \) symmetry. Owing to the \( Z_2 \) symmetry, the tree level neutrino mass should vanish but they acquire masses via one-loop interactions. Since the lightest \( Z_2 \) odd particle is stable, it becomes a dark matter candidate. We assume that the lightest Majorana singlet fermion, \( N_1 \), is the dark matter particle. Hereafter we use a notation of \( m_{\text{DM}} = M_1 \). The relic abundance of scotogenic dark matter can be produced by freeze-out mechanism. In this model, flavor violating processes such as \( \mu \rightarrow e\gamma \) are induced at the one-loop level \([8]\).

We use the best-fit values of neutrino parameters in Ref. \([9]\). For simplicity, we assume the normal mass ordering for the neutrinos. Since the relic density of the scotogenic dark matter depends only weakly on CP-violating phases, the Majorana CP phases \([10]\) are neglected.
We introduce the dimensionless parameter

$$\beta = \frac{\rho_{\text{PBH}}(T_{\text{in}})}{\rho_{\text{rad}}(T_{\text{in}})},$$

(1)

to represent the initial energy density of PBHs at the time of its formation, $\rho_{\text{PBH}}(T_{\text{in}})$, where $\rho_{\text{rad}}(T_{\text{in}})$ is the energy density of radiation.

Gondolo et al. [3] show that if PBH evaporate after the freeze-out of the dark matter, $T_{\text{FO}} > T_{\text{evap}}$, where $T_{\text{FO}}$ denotes the freeze-out temperature and $T_{\text{evap}}$ denotes the temperature of the Universe right after PBH evaporation, then the dark matter particles produced from PBHs may contribute to the final relic abundance of the dark matter.

Since $\rho_{\text{PBH}} \propto a^{-3}$ and $\rho_{\text{rad}} \propto a^{-4}$, where $a$ denotes the scale factor, $\rho_{\text{PBH}}(t_{\text{early-eq}}) \approx \rho_{\text{rad}}(t_{\text{early-eq}})$ may be happen at the early equality time $t_{\text{early-eq}}$. In order for PBH evaporation to occur before the early equality time (radiation dominated era), $t_{\text{evap}} < t_{\text{early-eq}}$, the initial density of PBHs should be less than the following critical density ($\beta < \beta_c$) [13, 14, 15]:

$$\beta_c = \frac{T_{\text{evap}}}{T_{\text{in}}}$$

(2)

In this case the final relic abundance of the scotogenic dark matter is to be

$$\Omega_{\text{DM}} h^2 = \Omega_{\text{FO}} h^2 + \Omega_{\text{PBH}} h^2,$$

(3)

where $\Omega_{\text{FO}} h^2$ and $\Omega_{\text{PBH}} h^2$ are the relic abundance via freeze-out and PBH evaporation, respectively, and $h$ denotes the dimension less Hubble parameter.

Figure 1 shows the correlations between the branching ratio $\text{BR}(\mu \rightarrow e\gamma)$ and initial density of PBHs $\beta$ for observed relic abundance of dark matter $\Omega_{\text{DM}} h^2 = 0.12 \pm 0.0009$ [16] in the case...
of $\beta < \beta_c$. We see that if the lepton flavor violating $\mu \rightarrow e\gamma$ processes is observed in the MEG II experiment, the initial density of PBH should be
\[ \beta \lesssim 3 \times 10^{-16}, \]
for observed relic abundance of dark matter $\Omega_{\text{DM}}h^2 = 0.12 \pm 0.0009$ with scotogenic dark matter in the case of $\beta < \beta_c$.

If the condition $\beta > \beta_c$ is satisfied, PBH evaporation occurs after the early equality time (PBH dominated era). In the PBH dominated era, the entropy production via the evaporation of PBHs leads to a dilution of the freeze-out-origin scotogenic dark matter [13, 14, 15]. The final relic abundance of scotogenic dark matter is obtained as
\[ \Omega_{\text{DM}}h^2 = \alpha^{-1}\Omega_{\text{FO}}h^2 + \Omega_{\text{PBH}}h^2, \]
where $\alpha$ denotes the entropy boost factor. The factor $\alpha$ is the ratio of the entropy prior $S_{\text{before}}$ and after $S_{\text{after}}$ the PBH evaporation, $\alpha(s_{\text{before}}\alpha_3^{\text{before}}) = s_{\text{after}}\alpha_3^{\text{after}}$, and is given by
\[ \alpha = \frac{S_{\text{after}}}{S_{\text{before}}} = \frac{Y_{\text{in}}}{Y_{\text{evap}}}, \]
where
\[ Y_{\text{in}} = \frac{n_{\text{BH}}(t_{\text{in}})}{s(t_{\text{in}})} = \beta \frac{\rho_{\text{rad}}(t_{\text{in}})}{M_{\text{in}}s(t_{\text{in}})} , \quad Y_{\text{evap}} = \frac{n_{\text{BH}}(t_{\text{evap}})}{s(t_{\text{evap}})} = \frac{\rho_{\text{rad}}(t_{\text{evap}})}{M_{\text{evap}}s(t_{\text{evap}})} , \]
with
\[ \rho_{\text{rad}}(T) = \frac{\pi^2}{30} g_s(T) T^4 , \quad s(T) = \frac{2\pi^2}{45} g_s(T) T^3 . \]
where $g_{s*}$ is the relativistic effective degrees of freedom for entropy density. By combining Eqs. (6), (7) and (8), we obtain
\[ \alpha = \beta \frac{g_s(t_{\text{in}})}{g_s(t_{\text{evap}})} \frac{g_{s*}(T_{\text{in}})}{g_{s*}(T_{\text{evap}})} \frac{T_{\text{in}}}{T_{\text{evap}}} . \]
According to the relation $g_s(T) \simeq g_{s*}(T)$ for high temperature, the entropy boost factor becomes the ratio of the initial density of PBH $\beta$ and critical density $\beta_c$ [17]:
\[ \alpha = \frac{\beta}{\beta_c} . \]

Figure 2 shows the dependence of relic abundance of PBH-origin scotogenic dark matter $\Omega_{\text{PBH}}h^2$ on dark matter mass $m_{\text{DM}}$. The horizontal line shows the observed relic abundance of dark matter. Since the relation $\Omega_{\text{PBH}}h^2 < 0.12 \pm 0.0009$ should be satisfied, the following regions of scotogenic dark matter mass and initial PBH mass
\[ 1\text{GeV} \lesssim m_{\text{DM}} \lesssim 3\text{GeV}, \quad 2 \times 10^{12} \lesssim M_{\text{in}}/M_{\text{Pl}} \lesssim 2 \times 10^{13} , \]
are relevant for $\beta > \beta_c$.

Figure 3 shows the correlations between the branching ratio $\text{BR}(\mu \rightarrow e\gamma)$ and initial density of PBHs $\beta$ for $\Omega_{\text{DM}}h^2 = 0.12 \pm 0.0009$ and $1\text{GeV} \lesssim m_{\text{DM}} \lesssim 3\text{GeV}$ in the case of $\beta > \beta_c$ (PBH dominated era). We see that if the lepton flavor violating $\mu \rightarrow e\gamma$ processes is observed in the MEG II experiment, the initial density of PBH should be
\[ 1 \times 10^{-8} \lesssim \beta \lesssim 9 \times 10^{-8} , \]
for $\Omega_{\text{DM}}h^2 = 0.12 \pm 0.0009$ and $M_{\text{in}}/M_{\text{Pl}} \sim 2 \times 10^{13}$ and $1\text{GeV} \lesssim m_{\text{DM}} \lesssim 3\text{GeV}$ in the case of $\beta > \beta_c$. 


4. Summary

In this study, we have shown the correlations between the initial density of PBHs and the branching ratio of $\mu \to e\gamma$ with scotogenic dark matter. It turned out that if the lepton flavor violating $\mu \to e\gamma$ processes is observed in the MEG II experiment, the initial density of PBH may be $\beta \lesssim 3 \times 10^{-16}$ for $m_{DM} \approx 1500$ GeV in the case of $\beta < \beta_c$, or $1 \times 10^{-8} \lesssim \beta \lesssim 9 \times 10^{-8}$ for $M_{\text{in}}/M_{Pl} \sim 2 \times 10^{13}$ and $1\text{GeV} \lesssim m_{DM} \lesssim 3\text{GeV}$ in the case of $\beta > \beta_c$. Since the physics run in the MEG II experiment is about to be started, the predictions in this study may be tested in less than 5 years.

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