Static behavior of fine particle clouds in fine particle (dusty) plasmas under gravity

Hiroo Totsuji

Graduate School of Natural Science and Technology, Okayama University, Tsushimanaka 3-1-1, Kitaku, Okayama 700-8530, Japan totsuji-090@okadai.jp

Keywords: fine particle (dusty) plasma, gravity, enhanced charge neutrality, drift-diffusion equation, simple model

Abstract

Static behavior of fine particle cloud in fine particle (dusty) plasmas under gravity is analyzed numerically on the basis of the drift-diffusion equations. The system is assumed to have one-dimensional structure and the effect of the ion drag force is taken into account. The results confirm the analytically obtained prediction that we have an enhanced charge neutrality in fine particle clouds under gravity as well as under microgravity. They are also compared with a simple model based on the enhanced charge neutrality in clouds to show its applicability.

1. Introduction

Fine particle (dusty) plasmas are weakly ionized plasmas containing macroscopic fine particles typically of micrometer size. Fine particles included there are negatively charged to large magnitudes and often form strongly coupled classical quasi-Coulombic system. Due to relatively easy observability of fine particles, fine particle plasmas have been the target of intense investigations on the behavior of fine particles and also of surrounding plasmas [1–5].

Since the gravitation on the ground has a large effect on macroscopic fine particles and tends to mask their intrinsic properties as strongly coupled system, experiments under microgravity have been and are being performed, for example, on the International Space Station (ISS) [6, 7]. With the sacrifice of the uniformity of the system to some extent, the gravity can also be compensated by the thermophoretic effect by making necessary vertical temperature gradient [8, 9]. Though the gravity has little direct effect on microscopic plasma particles (electrons and ions), it also influences them through the interaction between fine particles and the ambient plasma. For usual experiments on the ground, it is therefore helpful to have theoretical guidelines to understand the behavior of fine particle clouds in fine particle plasmas under gravity. The purpose of this paper is to contribute to the static aspect of such guidelines.

Based on the drift–diffusion equations, we have shown that, including numerical solutions, the existence of fine particles leads to the enhanced charge neutrality in fine particle clouds under microgravity [10, 11] and, on the basis of this observation, have given simple models of fine particle clouds [12]. The analytical result is recently extended to the case of one-dimensional structures under gravity and a simple model of fine particle cloud under gravity is also constructed [13].

Experimental data of the electrostatic potential or densities of electron and ions in fine particle plasma, especially in fine particle clouds, are not easy to access directly and are usually obtained by combinations of some experimental observations and simulations. The tendency of enhanced charge neutrality has been shown in [14–17] and the parallel behavior of the ion charge density and particle charge density shown in [10] is observable in figure 4 in [18].

Under the condition of microgravity, clouds of fine particles are formed around the center of the plasma where we have relatively weak electric field and ion flow. In the case under gravity, on the other hand, they are formed near the periphery of the plasma where we have rather strong electric field which can support fine particles against gravity. The effect of the ion flow is also different from the case of microgravity. Fine particle clouds under microgravity and gravity are thus in very different conditions and, even with analytical result under
gravity [13], we need further analysis similar to those under microgravity [10]. In this article, we numerically analyze the drift-diffusion equations under gravity and clarify the behavior of fine particle clouds, confirming the prediction on the behavior of fine particle cloud under gravity and the applicability of our simple model. Set of parameters we consider is a typical one in experiments [6, 7, 14–18]: the partially ionized plasma of inert gases such as Ar and Ne at 10–100 Pa with the plasma (electron and ion) and fine particle densities around 10^8–10^9 cm^-3 and 10^–3–10^5 cm^-3, respectively, and the electron temperature of a few eV. A preliminary result in limited cases has been reported in [19].

In section 2, basic equations are described with boundary conditions and numerical procedures of solution. Results of numerical analyses showing the enhanced charge neutrality are given in section 3 with discussions. Conclusions are given in section 4.

2. Basic equations

2.1. Drift-diffusion equations in one dimension under gravity

Since fine particles of micron size have the mass of the order of 10^-15 kg or more, they are strongly influenced by the gravity on the ground. For example, in plasmas generated between horizontal parallel plates (electrodes), the cloud of fine particles are often formed just above the lower electrode. In cylindrical apparatus with the horizontal symmetry axis, the fine particle cloud formed near the axis under microgravity moves downward with the increase of the gravity [10, 20, 21]. We consider fine particle plasmas under gravity on the ground and assume that the system has the one-dimensional structure or, at least, some part of the system can be regarded as one-dimensional.

We treat the stationary state of the one-dimensional system with the space coordinate y under the gravitational acceleration in the (−y)-direction, \( g = (0, -g, 0) \) (g > 0). The drift-diffusion equations [11] for densities of electrons(e), ions(i), and fine particles(p), \( n_e, n_i, n_p \), with the charge \( -e, e \), and \( -Q \) (\( Q > 0 \)) are now written respectively as

\[
\frac{d}{dy} D_e \left[ -\frac{d}{dy} n_e + \frac{\varepsilon_0}{e} \frac{e}{\lambda_e^2} (eE_y) \right] = \frac{\delta n_e}{\delta t} = (\varepsilon_g n_e - \varepsilon_p n_p),
\]

(1)

\[
\frac{d}{dy} D_i \left[ -\frac{d}{dy} n_i + \frac{\varepsilon_0}{e} \frac{e}{\lambda_i^2} (eE_y) \right] = \frac{\delta n_i}{\delta t} = (\varepsilon_g n_e - \varepsilon_p n_p),
\]

(2)

and

\[
D_p \left[ -\frac{d}{dy} n_p + \frac{\varepsilon_0}{Q^2 e} \frac{e}{\lambda_p^2} (-QeE_y + E^{id} - mg) \right] = 0,
\]

(3)

and the Poisson’s equation is given by

\[
\frac{d}{dy} \varepsilon_0 E_y = e (n_i - n_e - Q n_p).
\]

(4)

Here \( D_{e,i,p} \) are the diffusion coefficient of each component (Einstein relations between diffusion coefficients and mobilities being assumed) and \( \lambda_{e,i,p} \) are Debye lengths defined respectively by

\[
\frac{1}{\lambda_{e,i}^2} = \frac{n_{e,i} e^2}{\varepsilon_0 k_B T_{e,i}} \quad \text{and} \quad \frac{1}{\lambda_p^2} = \frac{n_p (Qe)^2}{\varepsilon_0 k_B T_p}
\]

(5)

with temperatures \( T_{e,i,p} \) respectively. The \( y \)-component of the electric field is denoted by \( E_y \) and the gravitational and ion drag forces on a fine particle are denoted respectively by \( mg \) (\( m \) being the fine particle mass) and \( F^{id} \).

In order to close the system of particles, electrons, and ions with respect to momentum conservation, the reaction of \( F^{id} = -(n_p/n_i) F^{id} \), is included in (2) for ions. (Since \( F^{id} \) is at most of the order of \( QeE_y \), from the balance of forces on particles appearing in (3), this term is small when \( Qn_p < n_i \).) On the right-hand sides of (1) and (2), \( \delta n_e/\delta t = \delta n_i/\delta t \) are the electron and ion sources per unit time and volume which are expressed by the generation rate \( \varepsilon_g \) and the electron/ion fluxes onto the surface of a particle \( \varepsilon_p \) [11]. The electrostatic potential \( \Psi \) is related to \( E_y \) via

\[
-\frac{d}{dy} \Psi = E_y.
\]

(6)
As is shown in Appendix, we rewrite (1)–(4) introducing \( N_{\text{i,ip}} \), defined by
\[
N_{\text{i,ip}} \equiv \int_0^y dy \ n_{\text{i,ip}}
\] (7)
in numerical solutions.

We estimate the ion drag force \( F_{\text{id}} \) as [2, 22]
\[
\sim \frac{1}{4 \pi \varepsilon_0} \frac{n_i e^2}{e_0 k_B T_i} \frac{u_i}{v_i} \quad (u_i < 1),
\sim \frac{1}{4 \pi \varepsilon_0} \frac{n_i e^2}{e_0 k_B T_i} \frac{v_i}{u_i} \quad (u_i > 1).
\] (8)
Here \( u_i \) is the average ion drift velocity, \( v_i \equiv (k_B T_i/m_i)^{1/2} \) is the ion thermal velocity (\( m_i \) being the ion mass) and the coefficient \( c_i \) is given in Appendix. The value of \( u_i \) is determined by
\[
n_i u_i = -D_i \left[ \frac{d}{dy} n_i - \frac{e_0}{e \lambda_i^2} \left( E_y - \frac{F_{\text{id}} n_p}{e n_i} \right) \right],
\] (9)
or in terms of \( N_{\text{i,ip}} \),
\[
n_i u_i = c_g \left( N_e - c_g N_p \right) - D_i \left[ \frac{d}{dy} n_i - \frac{e_0}{e \lambda_i^2} \left( E_y - \frac{F_{\text{id}} n_p}{e n_i} \right) \right]_{y=0}.
\] (10)

The charge on a fine particle \( -Q_e \) is determined by the balance of the ion and the electron currents onto its surface [11]. Based on previous analyses [2], we take into account the effect of ion-neutral collisions to the ion current [23] in the domain of our parameters (expressions of these currents and the equation for \( Q \) are given in (16)–(22) in [9]).

2.2. Solution without fine particles
Without fine particles, our plasma is approximately described by the well-known ambipolar diffusion equation for the density of electrons and ions \( n(n_e n_i) \) [24, 25]
\[
\frac{d^2}{dy^2} n + \frac{1}{R_a^2} n = 0.
\] (11)
Here the characteristic length \( R_a \) is determined by \( c_g \) and the ambipolar diffusion coefficient \( D_a \),
\[
D_a = \left( 1 + \frac{T_e}{T_i} \right) D_i,
\] (12)
as
\[
R_a^2 = \frac{D_a}{c_g}.
\] (13)
From (11), we have the density distribution
\[
n \sim n(y = 0) \cos(y/R_a),
\] (14)
the electric field
\[
E_y \sim -\frac{k_B T_e}{e} \frac{d}{dy} \ln n = \frac{k_B T_e}{e R_a} \tan(y/R_a),
\] (15)
the potential
\[
\Psi \sim \frac{k_B T_e}{e} \ln \cos(y/R_a),
\] (16)
and the net charge density
\[
e(n_i - n_e) \sim n(y = 0) \frac{\lambda_i^2(y = 0)}{R_a^2} \frac{1}{\cos^2(y/R_a)}.
\] (17)
When \( n_e \sim n_i \) and \( F_{\text{id}} \) is neglected, our equations (1) and (2) reduce to (11) giving (14)–(17).

2.3. Boundary conditions and numerical procedures
We take \( y = 0 \) at the position where the forces on a fine particle mutually balance or
\[
-Q(0)eE_y(0) + F_{\text{id}}(0) = mg = 0
\] (18)
Table 1. Values of parameters adopted for numerical solutions.

| Neutral gas | Ar, 40 Pa, 300 K |
|-------------|------------------|
| $k_B T_e$   | 1 eV             | 3 eV           |
| $T_e$       | 300 K            |                |
| $n_i(0)$    | $10^6$ cm$^{-3}$ |                |
| $2R_a$      | 3.0 cm           |                |
| $R_a = R_n / (\pi/2)$ | 0.95 cm | |
| $T_p$       | 300 K            |                |
| $\tau_p$    | 1 $\mu$m         |                |
| $\rho$      | 1 g/cm$^3$        |                |
| $n_p(0)$    | $1 \times 10^4$ cm$^{-3}$ | $2.5 \times 10^4$ cm$^{-3}$ | $0.5 \times 10^4$ cm$^{-3}$ | $1 \times 10^4$ cm$^{-3}$ |
| $n_p/R_a$   | 0.030             | 0.022          | 0.038          | 0.030          |

and therefore

$$\frac{dn_p(0)}{dy} = 0 \quad (19)$$

by (3). Since the ion drag force $F_{id}(0)$ is proportional to the ion drift velocity $u_i(0)$ which, in turn, depends on $dn_i/dy(0)$, $E_i(0)$, and $F_{id}(0)$, $F_{id}(0)$ is consistently expressed by $dn_i(0)/dy$ and $E_i(0)$ in the form

$$\frac{F_{id}(0)}{k_B T_e / R_a} = - \frac{\alpha_i Q_e A_i A_2}{\left(1 + \alpha_i Q_e A_i A_2 (\xi_i / n_i(0))\right)} \frac{\xi_i}{n_i(0)} \left[ \frac{d}{dy} n_i - \frac{n_i E_i}{k_B T_e}\right]_{y=0}, \quad (20)$$

where

$$A_1 \equiv \frac{R_i^2}{3 \tau_i(y = 0)}, \quad (21)$$

$$A_2 \equiv \frac{e^2}{4\pi \varepsilon_0 k_B T_e R_a}, \quad (22)$$

and $\xi_i$ is the ion mean free path related to the diffusion coefficient as $D_i = \xi_i v_i$. From the neutral gas density $n_n$ and the ion-neutral collision cross section $\sigma_{in}$ (for example, $\sigma_{in} \sim 10^{-14}$ cm$^2$ for Ar), we evaluate $\xi_i$ by $\xi_i = 1/n_i \sigma_{in}$. The balance (18) gives the value of the electric field at $y = 0$, $E_i(0)$, in the form

$$Q(0) \frac{e E_i(0) R_a}{k_B T_e} = - \frac{\alpha_i Q_e A_i A_2 (\xi_i / n_i(0))dn_i(0) / dy}{\left(1 + \alpha_i Q_e A_i A_2 (\xi_i / R_a) (n_i(n_i(0))\tau_i(T_e / T_i))\right)^2}, \quad (23)$$

where the values of variables on the right-hand side (including $Q$) are those at $y = 0$.

We assume $n_i(0) \sim 10^6$ cm$^{-3}$ at $y = 0$ in accordance with typical experiments. We then take some value for $n_p(0)/n_i(0)$ and integrate the drift-diffusion equations from $y = 0$ in both directions with some assumed values for $n_i(0)$ and $dn_i(0)/dy$. The values of $n_i(0)$ and $dn_i(0)/dy$ are adjusted so as to satisfy the conditions (a) and (b): (a) $n_a$, $n_b$, nor $n_p$ diverges with the increase of $|y|$ and, (b) $n_p(0+)=n_p(0-)$. In principle, $dn_i(0)/dy$ is also a parameter to be adjusted. We assume, however,

$$\frac{dn_i}{dy} + \frac{n_i E_i}{k_B T_e} \approx 0 \quad (24)$$

at least around $y = 0$, noting that $D_i / D_e \ll 1$ and electrons approximately follow the Boltzmann distribution.

3. Result and discussions

In numerical solutions, we assume the discharge in the Ar gas of 40 Pa and adopt the values of parameters shown in table 1, $r_p$ and $\rho$ being the radius and specific mass of fine particles, respectively. The values with the argument (0) are those at the position $y = 0$ in the cloud used as the starting point of numerical integration. The particle mean distance $a_i$ is defined by
The height (width) of plasma (without fine particles), the distance between the upper and lower boundaries of plasma, is denoted by $2R_w$; since our drift-diffusion equations are not accurate enough on the peripheries of the plasma, $2R_w$ is to be regarded as the plasma height extrapolated from the behavior around the center (where those equations are applicable) and the actual height of plasma (and of apparatus) may naturally be larger than $2R_w$ [12, 13]. We identify the value of $R_w$ with the first zero of the solution of the ambipolar diffusion equation (14) and determine the value of $R_w$. We have checked that the resultant value is appropriate in the light of known estimations of $D_i$ and $c_e$.

The existence of the fine particle cloud naturally makes the height of the plasma larger than $2R_w$. The increase which can be roughly estimated by the width of the cloud, however, is sufficiently small in our examples and we may still regard $2R_w$ as the approximate plasma width: the width of the cloud $D$ is at most 5% of $2R_w$ as shown below.

In the case without fine particles, the solution of our drift-diffusion equations is compared with those of the ambipolar equations (11), (14)–(17), in figures 1(a), (b), and (c). We observe that our solution gives the values consistent with those obtained by the ambipolar diffusion equation. Also in the case of $k_B T_e = 3\text{ eV}$, we have the consistency of our solution with those of ambipolar diffusion equation, while the net charge density is somewhat larger than the case of $k_B T_e = 1\text{ eV}$ [11].

The fine particle clouds under microgravity are shown in figure 2; Left and right figures are for $k_B T_e = 1\text{ eV}$ and $3\text{ eV}$, respectively. As shown in top figures, clouds are centered around the center of the plasma corresponding, in this case, to $y = 0$. In the middle figures, we observe that the net charge density outside of the cloud is larger for $k_B T_e = 3\text{ eV}$ but the charge neutrality is largely enhanced in clouds in both cases. In the bottom figures, it is also shown that the ion drag force is in the direction from the center with increasing magnitude in accordance with the average ion flow. We confirm that the condition

$$Q^2 n_p \gg n_e, n_i$$

is satisfied. We have also solved the case where the central density of fine particles is reduced to half and observed that the width of clouds is approximately proportional to the density of fine particles at the center.

Typical examples of the solution with fine particles under gravity are shown in figures 3(a)–(f) for $k_B T_e = 1\text{ eV}$ and in figures 4(a)–(f) for $k_B T_e = 3\text{ eV}$. The gravitational force is in the direction of $-y$. The magnitude of the particle charge number $Q(0)$ obtained in solving drift-diffusion equations are listed in table 2 and we confirm that the condition (26) is satisfied in all cases.

As shown in the top figures in figures 3 and 4, we have fine particle clouds located in the lower part of the plasma under the influence of the gravity in the direction of $-y$. In the middle figures, we observe the enhancement of the charge neutrality in the domain of fine particles’ existence similarly to the case under microgravity: while the electron density increases with the increase of $y$ apparently with little effect by particles’ distribution, the ion distribution in the domain of fine particle cloud changes so as to to cancel the negative charge density of particles and recover the charge neutrality. Correspondingly, the electric field is almost constant (flat) in the domain of particle cloud as shown in the bottom figures. The sum of the forces by the electric field and the gravitation (plotted by dotted lines) is positive and balances the downward ion drag force as shown by solid lines which include the latter.

For these solutions, we estimate approximate values of the positions of the center, $y_c$, and the thickness, $D$, of fine particle clouds as shown in table 2. From the cloud thickness $D$ and the particle mean distance $a_p$, we may estimate the number of layers in the cloud by $D/a_p$ as shown in thick letters. As shown there, we have the cases from one layer to five layers in these examples. We also see that $D$ increases with $n_p(0)$ almost linearly with the proportionality constant dependent on $T_e$ (exactly, the ratio $D/n_p(0)$ weakly increases with $n_p(0)$).

Outside of the cloud in the direction of increasing plasma density, we assume that the electron and ion distributions are described by the ambipolar diffusion equation (11) and connect the electron distribution to its solution to have the position of the peak of the electron (or plasma) distribution at $y_p$. Assuming also that, in the direction of decreasing density, the electron and ion distributions outside of the cloud are described by the ambipolar diffusion equation, we have the center of the whole plasma (which includes the cloud) at $y = y_p - D/2$. We then evaluate the distance between the center of cloud $y_c$ and the center of the plasma by $(y_p - D/2) - y_c$ as listed in table 2 in thick letters. In our simple model [13], this distance is denoted by $y_0$ and determined by the mass and the magnitude of charge of fine particle as

\[
a_p = \left(\frac{3}{4\pi n_p(0)}\right)^{1/3}.
\]
Figure 1. Electron and ion distributions normalized by $n_e(0)$ (top), potential and net charge density (middle), and electric field (bottom) without fine particles. They are compared with those of the ambipolar diffusion equations (14)–(17), plotted by broken lines.
where

$$\frac{mgR_a}{Q(0)/k_B T_e} = 2.56 \left( \rho [\text{g/cm}^3] (\tau_p [\mu s]^3 (g [\text{m/s}^2]/9.80) (R_a [\text{cm}]) / (Q(0) / 10^5) T_e [\text{eV}] \right).$$

(28)

By the value of the charge magnitude at \( y = 0 \), \( Q(0) \), we estimate \( y_0 \) as shown in Table 2 in thick letters. We observe that \( y_0 \) serves as an approximation for \( (y_p - D/2) - y_c \), the former being somewhat smaller than the
latter by at most 20%. We may attribute this difference to the effect of the downward ion drag force which has been neglected in the model [13].

Figure 3. Ion, electron, and fine particle charge densities normalized by $n_e(0)$ (top), the net charge density normalized by $n_e(0)$ (middle), and force on a fine particle normalized by $k_B T_e/R_e$ (bottom) in the case of $T_e = 1$ eV; $n_e(0) = 10^4$ cm$^{-3}$ and $n_p(0) = 2.5 \times 10^4$ cm$^{-3}$ in the left and right columns, respectively.
4. Conclusion

In conclusion, we have confirmed that, when $Q_{p}^{2}n_{p} \gg n_{c} \sim n_{e}$, the charge neutrality is largely enhanced in fine particle clouds under gravity as well as under microgravity by numerically solving the drift–diffusion equations. This extends the observation of the enhancement of charge neutrality from the case under microgravity to general cases on the ground where the particle mass plays an important role and the structure is one-

Figure 4. Ion, electron, and fine particle charge densities normalized by $n_{e}(0)$ (top), the net charge density normalized by $n_{e}(0)$ (middle), and force on a fine particle normalized by $k_{b}T_{e}/R_{e}$ (bottom) in the case of $T_{e} = 3$ eV; $n_{e}(0) = 5 \times 10^{3}$ cm$^{-3}$ and $n_{e}(0) = 10^{4}$ cm$^{-3}$ in the left and right columns, respectively.

4. Conclusion

In conclusion, we have confirmed that, when $Q_{p}^{2}n_{p} \gg n_{c} \sim n_{e}$, the charge neutrality is largely enhanced in fine particle clouds under gravity as well as under microgravity by numerically solving the drift–diffusion equations. This extends the observation of the enhancement of charge neutrality from the case under microgravity to general cases on the ground where the particle mass plays an important role and the structure is one-

4. Conclusion

In conclusion, we have confirmed that, when $Q_{p}^{2}n_{p} \gg n_{c} \sim n_{e}$, the charge neutrality is largely enhanced in fine particle clouds under gravity as well as under microgravity by numerically solving the drift–diffusion equations. This extends the observation of the enhancement of charge neutrality from the case under microgravity to general cases on the ground where the particle mass plays an important role and the structure is one-
dimensional. This extension has been predicted on the basis of the analytic treatment of the drift-diffusion equations and enabled to construct a simple model of fine particle cloud under gravity [13]. We have shown that the model works as an approximation by comparing its prediction with numerically obtained results. Though there seem to be no experimental data of electron, ion and particle densities or the potential under gravity for more or less direct comparison, we expect they may become available, for example, similarly to those in [18].

Appendix

We integrate both sides of (1)–(4) and (6) and have

\[
\begin{align*}
\left[ \frac{d}{dy} n_e + \frac{\varepsilon_0}{e \lambda_p^2} E_y \right]_0^y &= - \frac{c_p}{D_p} \left( N_e - \frac{c_p}{c_e} N_p \right), \\
\left[ \frac{d}{dy} n_i - \frac{\varepsilon_0}{e \lambda_i^2} \left( E_y - \frac{F_{id} n_p}{e} \right) \right]_0^y &= - \frac{c_p}{D_i} \left( N_i - \frac{c_p}{c_e} N_p \right), \\
\frac{d}{dy} n_p &= - \frac{\varepsilon_0}{Q e \lambda_p^2} \left( E_y + \frac{mg}{Q e} - \frac{F_{id}}{Q e} \right), \\
E_y - E_y(0) &= \frac{e}{\varepsilon_0} \left( N_i - N_e - \int_0^y dy \left( Q n_p \right) \right),
\end{align*}
\]

and

\[
\Psi = - \int_0^y dy E_y,
\]

where variables without arguments denote those at \( y = 0 \). We note that, when the system is symmetric with respect to \( y = 0 \) as the one under microgravity, the terms evaluated at \( y = 0 \) reduce to 0. When \( g \) is finite, however, those terms are also finite.

In the ion drag force (8), the coefficient \( c_1 \) is given by [22]

\[
c_1 = \frac{1}{3} \left( \frac{2}{\pi} \right)^{1/2} \Lambda,
\]

(A.1)

Here \( \Lambda \) is the generalized Coulomb logarithm

\[
\Lambda(\beta_T, r_p/\lambda) = -e^{\beta_T/2} \text{Ei}(-\beta_T/2) + e^{(\beta_T/2)(\lambda/r_p)} \text{Ei}(-\beta_T/2)(\lambda/r_p),
\]

\( r_p \) and \( \text{Ei}(x) \) being the particle radius and the exponential integral, respectively (\( \beta_T \sim [Q e^2/4\pi\varepsilon_0 k_B T_p]/(\varepsilon_0 k_B T/\varepsilon^2 m)^{1/2} \) as defined in appendix A in [5]).

**ORCID iDs**

Hiroo Totsuji © https://orcid.org/0000-0003-3915-603X

---

**Table 2. Resultant values of parameters compared with model.**

| \( k_0 T_e \) | \( n_p(0) \) | 1 eV | 2.5 \( \times 10^4 \) cm\(^{-3} \) | 5 \( \times 10^4 \) cm\(^{-3} \) | 2.5 \( \times 10^4 \) cm\(^{-3} \) | 5 \( \times 10^4 \) cm\(^{-3} \) |
|---|---|---|---|---|---|---|
| \( Q(0) \) / \( \Omega_{l0} \) \( n_p(0) \) | 9.7 \( \times 10^2 \) / 9.4 \( \times 10 \) | 9.5 \( \times 10^2 \) / 2.3 \( \times 10^2 \) | 1.75 \( \times 10^3 \) / 1.5 \( \times 10^2 \) | 1.73 \( \times 10^3 \) / 3.0 \( \times 10^2 \) |
| \( \gamma_p / R_e \) | 0.0055 / 0.025 | 0.025 / 0.075 | 0.0098 / 0.064 | 0.030 / 0.140 |
| \( D_i / R_d \) | 0.83 / 3.4 | 1.7 / 4.7 |
| \( D / \mu_p \) | 1.272 / 1.315 | 0.51 / 0.62 |

| \( \gamma_p / R_e \) | 1.254 / 1.253 | 0.47 / 0.52 |
| \( (\gamma_p - D / 2 - \gamma) / R_i \) | 2.6 / 0.46 | 0.47 / 0.47 |

| \( \frac{m}{m_p} \) | 2.5 / 2.6 | 0.46 / 0.47 |
| \( \frac{\lambda}{\lambda_p} = \tan^{-1} \left( \frac{m_{\lambda_p}}{m_{\lambda}} \right) \) | 1.19 / 1.20 | 0.43 / 0.44 |
References

[1] For an example Shukla P K and Mamun A A 2002 Introduction to Dusty Plasma Physics (London: Institute of Physics Publishing)
[2] Fortov V E, Ivlev A V, Khrapak S A, Khrapak A G and Morfill G E 2005 Phys. Rep. 421 1
[3] Tsytovich V N, Morfill G E, Vladimirov S V and Thomas H 2008 Elementary Physics of Complex Plasmas (Berlin: Springer)
[4] Morfill G E and Ivlev A V 2009 Rev. Mod. Phys. 81 1355
[5] Fortov V E and Morfill G E (ed) 2010 Complex and Dusty Plasmas—From Laboratory to Space (Boca Raton, London, New York: CRC Press, Taylor and Francis Group)
[6] Fortov V et al 2005 Plasma Phys. Control. Fusion 47 B537
[7] Thomas H M et al 2008 New J. Phys. 10 033036
[8] Rothermel H, Hagl T, Morfill G E, Thoma M H and Thomas H M 2002 Phys. Rev. Lett. 89 175001
[9] Arp O, Block D and Piel A 2004 Phys. Rev. Lett. 93 165004
[10] Totsuji H 2014 J. Plasma Phys. 80 843
[11] Totsuji H 2016 Plasma Phys. Control. Fusion 58 045010
[12] Totsuji H 2016 Phys. Letters A 380 1445
[13] Totsuji H 2017 Phys. Letters A 381 903
[14] Polyakov D N, Shumova V V and Vasilyak L M 2017 Plasma Phys. Rep. 43 397
[15] Vasilyak L M, Polyakov D N, Fortov V E and Shumova V V 2011 High Temp. 49 623
[16] Polyakov D N, Shumova V V and Vasilyak L M 2013 Surf. Eng. Appl. Electrochem. 49 114
[17] Sukhinin G I, Fedoseev A V, Antipov S N, Petrov O F and Fortov V E 2013 Phys. Rev. E 87 013101
[18] Pustylnik M Y, Semenov I L, Zähringer E and Thomas H M 2017 Phys. Rev. E 96 033203
[19] Totsuji H 2017 Contrib. Plasma Phys. 57 463
[20] Totsuji H, Totsuji C, Takahashi K and Adachi S 2014 Int. J. Microgravity Sci. Appl. 31 55
[21] Totsuji H, Takahashi K and Adachi S 2016 Int. J. Microgravity Sci. Appl. 33 330209
[22] Khrapak S A, Ivlev A V, Morfill G E and Thomas H M 2002 Phys. Rev. E 66 046414
[23] Khrapak S A et al 2005 Phys. Rev. E 72 016406
[24] Chen F F 1974 Introduction to Plasma Physics (New York: Plenum Press) ch 5
[25] Franklin R N 1976 Plasma Phenomena in Gas Discharges (Oxford: Clarendon Press) ch 2