Research on signal processing technology in $\varphi$ - OTDR system based on non-uniform sampling

Jin Chen$^1$, Xiao Rui Qiao$^1$ * and Zhi Gao$^2$

$^1$ School of Electronics Engineering, Navy University of Engineering, Wuhan 430033, China  
$^2$ Unit 92458 of the People's Liberation Army, Qingdao, 266100, China

Abstract. In the optical cable communication system, because the optical cable often has a long span, the sampling frequency is limited when $\varphi$ - OTDR is used as the monitoring method. When the system adopts the traditional uniform sampling method, the sampling frequency must be greater than 2 times of the signal frequency, otherwise spectrum aliasing will occur. How to get the frequency information of high frequency signal with lower sampling frequency, and how to solve the problem that the highest response frequency of the system decreases when the sensing distance of $\varphi$ - OTDR system is extended, is a major technical problem for the monitoring of optical cable communication system. In this paper, through the analysis of the common methods of broadening the response frequency band of $\Phi$ - OTDR system, a non-uniform sampling method is selected to analyze the optimal value of some parameters of $\Phi$ - OTDR system based on non-uniform sampling, and a method of using the mean square change value to locate the vibration source is proposed. The simulation results show that this method is suitable for monitoring vibration signals with sparse characteristics in frequency domain, and the location SNR is better than the traditional method, and has good anti noise performance.

1. Introduction

The transmitting frequency of the traditional $\Phi$ - OTDR system is fixed. At this time, the highest response frequency of the system is limited by the sensing distance. The longer the distance is, the lower the highest response frequency is. Because the communication distance of optical cable is often more than 100 kilometers, when $\Phi$ - OTDR is used for optical cable monitoring, the highest detectable frequency of the system is only a few hundred hertz. In reference [1], the detection distance of $\Phi$ - OTDR system is 175km, but the highest response frequency of the system is only 285hz. In order to solve this problem, researchers put forward various technologies to broaden the response frequency band of $\Phi$ - OTDR system, including frequency division multiplexing technology, interferometer multiplexing technology and non-uniform sampling technology. Among the above three technologies, frequency division multiplexing technology needs to prepare multiple ultra narrow linewidth lasers, which has high cost. Moreover, the system using frequency division multiplexing to realize fiber-optic sensing is more complex. If the interferometer multiplexing technology is used for optical cable, it needs to occupy an additional optical fiber, and the receiver needs to realize the integration of two sensor technologies, with high system complexity[2]. Because of the long span of optical cable, when $\Phi$ - OTDR is used as the monitoring means of optical cable, the sampling frequency will be limited, which is very consistent with the characteristics of non-uniform sampling technology to obtain high frequency signal through low sampling frequency. Moreover, the non-uniform sampling technology does not need to change the structure of $\Phi$ - OTDR system, only by controlling the AWG to modulate the AOM, the response band can be broadened [3]. Therefore, the application prospect of nonuniform sampling technology in the field of
optical cable monitoring is the best. Firstly, the principle of nonuniform sampling is analyzed. Secondly, the optimal value of some parameters of \( \Phi \)-OTDR system based on non-uniform sampling is analyzed, on this basis, a method of using the change value of standard deviation of frequency signal to locate the position of vibration source is proposed, and the anti noise performance of this method and the frequency-domain sparsity of vibration signal are analyzed.

2. Technical principle of nonuniform sampling

2.1 Basic structure

The non-uniform sampling technology is to obtain the signal with constantly changing sampling frequency by constantly changing the transmission interval of detection pulse of \( \Phi \)-OTDR system. The bandwidth of the frequency domain signal is not limited by the sampling frequency, so the response bandwidth of the system can be extended. The non-uniform sampling technology does not need to change the structure of the \( \Phi \)-OTDR system. It only needs to control the AWG to modulate the AOM, which can widen the response frequency band. The basic structure of the specific non-uniform sampling system is shown in Figure 1.

![Non-uniform sampling system structure](image)

2.2 Principle of nonuniform sampling technology

The traditional sampling method is uniform sampling, and the sampling time is equal interval. According to Nyquist sampling theorem, the sampling frequency should not be less than twice of the signal frequency, otherwise spectrum aliasing will occur.

In order to express intuitively, it is assumed that there is a vibration signal \( s(t) \) whose vibration frequency \( f_0 = 240 \text{Hz} \), the expression of \( s(t) \) is:

\[
s(t) = \cos(2\pi f_0 t)
\]  

(1)

The signal can be discretized by sampling, and the spectrum of discrete signal can be obtained by discrete Fourier transform (DFT). The expression of DFT is as follows:

\[
F(j\omega) = \sum_{n=1}^{N-1} f(t_n) e^{j\omega t_n}
\]  

(2)

Different from uniform sampling, the spectrum of non-uniform sampling signal needs to be obtained by non-uniform discrete Fourier transform (NDFT). The expression of NDFT is [3]:

\[
F'(j\omega) = \sum_{n=1}^{N-1} [f(t_n) e^{j\omega (t_{n+1} - t_n)}]
\]  

(3)

Compared with DFT formula (3), when the sampling interval is equal, the spectrum shape obtained by the above formula is the same as that of uniform sampling. The non-uniform sampling can be regarded as the result of the accumulation of many uniform sampling spectrum. Because the real frequency of the signal is the common part of the spectrum obtained by the uniform sampling of different frequencies, the maximum value always appears at the frequency position of the real value of the signal, which is the principle of the non-uniform sampling technology to expand the response frequency band of \( \Phi \)-OTDR system.
3. Parameter optimization of non-uniform sampling φ - OTDR system

3.1 Maximum response frequency of the system

According to the principle of non-uniform sampling, if the sampling frequency can be completely random, the bandwidth of the non-uniform sampling signal can be any value when the frequency spectrum is calculated by NDFT, but in the actual non-uniform sampling Φ - OTDR system, there is no completely random non-uniform sampling. Since the transmission frequency of the detection pulse is controlled by the AWG modulation frequency[4], there is a minimum unit interval time $T_0$ in the non-uniform sampling Φ - OTDR system, and the interval time of all the detection pulses is a multiple of $T_0$.

Because of the existence of $T_0$, the highest response frequency of the system can only be obtained to 1/2, which is still limited by the sampling theorem. However, because the frequency of AWG is only related to the performance of electronic devices, it is not limited by the detection distance, therefore, the highest response frequency of the system is still much higher than the Φ - OTDR system with uniform sampling method.

For intuitive expression, set the frequency of AWG ($f_{\text{AWG}}=100$KHz), so $T_0=0.01$ms. 1000 sampling points are obtained by nonuniform sampling of cosine signal with frequency of 40KHz, the signal spectrum obtained by NDFT is shown in Figure 2, the red dotted line in the figure is the position $f_{\text{AWG}}/2$, it can be seen from the figure that the highest response frequency can only be taken as 50KHz, and the frequency beyond this range starts to repeat.

![Fig. 2. Spectrogram of non-uniformly sampled signal at $T_0=0.01$ms](image)

Theoretically, the AWG modulation frequency can reach the order of MHz. Limited by the performance of computer, this paper takes 100 kHz, at this time, the highest response frequency of the system is 50KHz, that is, the response frequency band width of the system is $B = 50$KHz.

3.2 Sampling interval

In the actual Φ-OTDR system, the maximum transmission frequency of the detection pulse is limited by the sensing distance L, so the detection pulse interval time has a minimum value $T_{p,\text{min}}$:

$$T_{p,\text{min}} = \frac{1}{f_{p,\text{max}}} = 2n_0 \times L/c$$  \hspace{1cm} (4)

When the non-uniform sampling technology is applied to the Φ - OTDR system, the minimum interval time of detection pulse shall not be less than $T_{p,\text{min}}$. However, the system has no rigid requirement for the maximum interval time $T_{p,\text{max}}$ of the detection pulse. In order to find a suitable one $T_{p,\text{max}}$, the cosine signal with the frequency of 10kHz is non-uniform sampled, and the interval time of sampling is uniformly distributed in the interval. The number of sampling points is $n = 1000$. NDFT is used to get the spectrum of the sampled discrete signal. The ratio of the amplitude value of the real frequency of the signal in the spectrum to the maximum value of other aliasing values is defined as the frequency signal-
to-noise ratio (FSNR). Obviously, the larger the FSNR, the better the detection performance of the system. The relationship between FSNR and ΔT is shown in Figure 3 by changing ΔT continuously.

![Figure 3. Relationship between FSNR and ΔT](image)

(a) \(T_{P,\text{min}}=1\text{ms}\)  
(b) \(T_{P,\text{min}}=2\text{ms}\)

The above figure (a) and (b) show the relationship between FSNR and ΔT at 1ms and 2ms respectively, it can be seen from the figure that, if \(ΔT < T_{P,\text{min}}\), with the increase of ΔT, the detection performance of the system will gradually improve, and then if \(ΔT = T_{P,\text{min}}\), the detection performance of the system tends to be stable, the larger ΔT, the greater the detection delay of the system. The optimal value of ΔT is \(ΔT = T_{P,\text{min}}\).

Set fiber refractive index \(n_0 = 1.467\), \(c = 3 \times 10^8\text{m/s}\), according to formula (1), the length \(L\) of sensing optical fiber corresponding to figure 3 (b) is 200km, therefore, when \(L=200\text{km}\), the launch interval of detection pulse should be between \([2\text{ms}, 4\text{ms}]\).

3.3 Number of sampling points required for a single operation

When \(\Phi\) - OTDR is used in the submarine cable monitoring system, due to the continuous emission probe pulse, so the sampling points has been increasing. However, the number of sampling points needed for a single operation is limited, so it is necessary to find the most suitable sampling points for a single operation of the system. Set the vibration signal as the cosine signal of 20kHz, the sampling interval time is evenly distributed between \([2\text{ms}, 4\text{ms}]\), and the detection bandwidth of the system is 50KHz, when the number of sampling points is 200, the spectrum image obtained by NDFT is as shown in Figure 4 (a). When \(n = 1000\), the spectrum image can be obtained as shown in Figure 4 (b), it can be found that when the number of sampling points increases, the interference value caused by the spectrum aliasing in the figure decreases obviously.

However, if there are too many sampling points, the system delay will increase. In addition, if the duration of ordinary commercial ultra narrow linewidth laser exceeds 2S, there will be obvious frequency drift [64], so the duration of sampling should not exceed 2S. Take different sampling points and draw the relationship between sampling point \(n\) and FSNR as shown in Figure 5.
It can be seen from the figure that the FSNR will increase gradually with the increase of $N$. Make the sixth order fitting curve as shown by the black line in the figure, and derive the fitting curve, the results are shown in Figure 6.

It can be seen from the above figure that the curve after derivation tends to be gentle when $n$ is greater than 600. Combined with the limitation that the sampling duration shall not be greater than $2S$, when $L=200\text{km}$, $n=600$ is taken as the number of single operation sampling points of the system.
4. Research and Simulation of mean square change value location method

For the non-uniform sampling signal, the sampling frequency is often less than the signal frequency. Because the sampling points cover multiple periods of the signal, the mean value obtained by moving average will only fluctuate in a very small range and will not smooth the signal curve. Due to the extreme value of the non-uniform sampling signal at the vibration position in the frequency domain, and the noise only exists at other non vibration positions in the frequency domain, based on this feature, this section proposes that the location can be carried out by filtering out the extreme value of the frequency domain and use the mean square change value of the discrete frequency domain signal at each position before and after filtering.

4.1 Analysis of positioning principle

Let X be a column vector of N × 1 dimension, and the mean square value A of X can be expressed as:

\[ A = \frac{1}{N} \sum_{i=1}^{N} X_i^2 \]  

(5)

If there is a large component in X, the above formula can be changed to:

\[ A = \frac{X_a^2}{N} + \frac{1}{N} \sum_{i=1, i \neq a}^{N} X_i^2 \]  

(6)

Among them, the larger the proportion of this item in \( X_a \) is, and the \( \frac{X_a^2}{N} \) has an amplification effect on the proportion. Set X to divide \( X_a \) in the largest component outside a is \( X_b \). If \( X_a \) is M times of \( X_b \), then after the square, \( X_a^2 \) is \( m^2 \) times of \( X_b^2 \). So \( X_a \) relative to the larger the \( X_b \), the more obvious the amplification effect of the relative proportion in the mean square value is. If the larger component in X is filtered out, the change of the mean square value will be more obvious.

If x is a noise signal, the root-mean-square value of the noise signal is generally stable, so when there are enough sampling points, filtering out some of them, the root-mean-square value will not change significantly.

Specifically, in the non-uniform sampling Φ-OTDR system, if the continuous sampling gets n periods of BRS signals, then every point on the optical fiber can get an n-dimensional array in the time domain. The array is regarded as a non-uniform discrete time domain signal x (z), Z represents the fiber link position, and the discrete frequency domain signal s (z) corresponding to X (z) is obtained by NDFT. If the vibration position \( z=a \), S (a) will have a maximum value at the vibration frequency. But at other positions, the position point \( z=B (B \neq a) \), because there is no vibration, the signal x (b) is a noise signal, the energy distribution of noise signal in frequency domain is relatively uniform, and there is no particularly prominent extreme point. Use this feature, we can locate the vibration position by finding out the large frequency component at each position of the optical fiber, filtering it out, and comparing the change degree of the mean square value of the signal in the frequency domain before and after filtering.

4.2 Threshold function

It can be seen from the analysis in the previous section that the key of vibration positioning is to find out the relatively large frequency component at each position. This process can be realized by setting the threshold T. Because the frequency spectrum at different locations is not the same, and the number of large frequency components will be slightly different, so threshold T must have the ability to adaptively adjust the size according to the spectrum signal. The expression of threshold T is deduced below.

In addition to the additive white Gaussian noise introduced in the transmission process, the spectrum of the non-uniform sampling signal also including the interference noise generated by the spectrum superposition, and the spectrum noise distribution conforms to the Rayleigh distribution. If the probability that the amplitude of a single spectral line in the spectrum noise is less than the threshold T is Q (T), then:

\[ Q(T) = 1 - \int_{T}^{\infty} \frac{1}{\sigma^2} e^{-\frac{z^2}{2\sigma^2}} d\xi = 1 - e^{-T^2/\sigma^2} \]  

(7)
σ is the standard deviation of Rayleigh distribution. If the total length of the frequency domain signal is n, and the number of spectral lines greater than the threshold value is k, then:

\[ P(T) = \left(1 - e^{-t^2/\sigma^2}\right)^{N-K} \]  

(8)

\(P(T)\) is the ratio of the number of spectral lines less than the threshold value to the total number of spectral lines in the spectrum. From equation (8), we can get the formula to calculate t:

\[ T = \sqrt{-\sigma^2 \ln \left(1 - P(T)^{\frac{1}{N-K}}\right)} \]  

(9)

Because the frequency band of the non-uniform sampling signal is wide, the frequency domain signal at the vibration position can be regarded as a sparse signal, \(K < n\), so formula (9) can be simplified as:

\[ T = \sqrt{-\sigma^2 \ln \left(1 - P(T)^{\frac{1}{N}}\right)} \]  

(10)

4.3 Positioning steps

The steps to determine the vibration position by the mean square change value positioning method can be described as follows:

1. During the vibration duration, N strip BRS curves were obtained.
2. The mean square value of N strip BRS curves in the discrete frequency domain at each position of the optical fiber is calculated, and the mean square value array a along the optical fiber is obtained.
3. Use equation (10) to calculate the threshold value T at each position, filter out the components larger than T in the discrete frequency domain signal at each position of the optical fiber, and calculate the mean square value of the filtered discrete frequency domain signal again, and get the mean square value array B.
4. The corresponding elements in array A and B are subtracted to obtain the mean square change value array D, and the point corresponding to the maximum value in Array D is the vibration position.

4.4 Simulation verification

It is assumed that there is a vibration signal \(s(t)\) with a strain size of 200με at the position of 190km with a length of 200km:

\[ s(t) = \sum_{i=1}^{4} \cos(2\pi f_i t) \]  

(11)

\(s(t)\) is the superposition of four cosine signals with different vibration frequencies. Let the four vibration frequencies be \(f_1=6\text{KHz}\), \(f_2=18\text{KHz}\), \(f_3=30\text{KHz}\), \(f_4=42\text{KHz}\), the vibration signal is substituted into the simulation model of Φ-OTDR system, regardless of the attenuation and gain of optical signal (\(g_R = 0, \alpha_r = 0\)), the launch interval time of the detection pulse is between [2ms, 4ms] and the random value meets the uniform distribution, and other simulation parameters are the same as table 1. The BRS curves of 600 cycles are obtained by simulation.

Firstly, the separation effect of threshold T on the large signal frequency component is tested, and the ratio of the RMS value of BRS curve and the RMS value of noise is set as the simulated signal-to-noise ratio (SSNR), add the noise with SSNR = 10dB to the curve, make \(P(T) = 0.99\), extract the signals at 180km and 190km and substitute them into equation (10) to get their respective thresholds. The results of threshold separation of large frequency components are shown in Figure 7.

| Symbol | Value | Explain |
|--------|-------|---------|
| \(\lambda_p\) | 1550nm | Detection pulse wavelength |
| \(\tau\) | 200ns | Detection pulse time width |
| \(\varphi_p(0)\) | 0 | Initial phase of detection pulse |
| \(f_p\) | 490Hz | Detection pulse transmission frequency |
| \(L\) | 206km | Sensing fiber length (sensing distance) |
Effective group index of single mode fiber at 1550nm

Transmission loss coefficient of single-mode fiber at 1550nm

Standard deviation of refractive index distribution on optical fiber

Length of optical fiber micro element

Vacuum speed of light

(a) Vibration signal spectrum and threshold  (b) Noise signal spectrum and threshold

Fig. 7. Signal spectra and threshold

Threshold T has the ability to adaptively adjust the size and separate large signal frequency components. In Figure 7 (a), there are 7 frequency components greater than the threshold, and in Figure 7 (b), there are 6 frequency components greater than the threshold. It can be seen that the number of frequency components filtered by the threshold is almost the same regardless of whether the frequency spectrum contains vibration frequency or not. To further verify this, set the number of frequency components greater than t in the discrete frequency domain signal to K_T. Calculate the K_T value of each position of the optical fiber as shown in Figure 8.

Fig. 8. Number of frequency components greater than the threshold at each position of the fiber

Remove the frequency component greater than the threshold value at each position, and calculate the mean square change value of the discrete frequency domain signal at the position before and after removal. Normalize the change value and draw a positioning curve, as shown in Figure 9.
According to figure 9, there is a peak value at 190km. The ratio of the peak value at 190km location to the peak value at other locations is defined as the positioning signal-to-noise ratio (LSNR). SSNR takes different values and repeats the above positioning process to get the relationship between LSNR and SSNR, as shown in Figure 10.

The red scatter in the figure above is the LSNR value obtained by the mean square change value positioning method, and the curve connecting the scatter is a 5th order fitting curve. It can be seen from the above figure that when SSNR = 0dB, LSNR = 2.84db is still greater than 0, indicating that this method has good noise resistance performance.

Use the new positioning method, repeat the simulation, and know from formula (1) that the shortest interval time of the detection pulse is 2.014ms. Therefore, according to the content of Section 2.2, the transmission interval time of the detection pulse is set between [2.02ms, 4.04ms] and meets the uniform distribution. Add 240Hz and 24kHz vibration signals at 156km from the transmitter, and the other simulation parameters are the same as table 1. The BRS curve of 600 cycles is obtained by simulation, and the noise of -83dbm is added. Use the new positioning method, the mean square change value of the discrete frequency domain signal at each point of the optical fiber is calculated, and the positioning image is drawn as shown in Figure 11.

Figure 11 (a) is a positioning image with a vibration frequency of 240Hz, which can clearly distinguish the peak value at the vibration location. At this time, LSNR = 7.2db, compared with the original positioning results, the new method has better positioning effect. Figure 11 (b) is a positioning image with a vibration frequency of 24kHz.
with a vibration frequency of 24khz. It can be seen that there is a peak value at 156km. At this time, LSNR = 5.9db. Although it is slightly lower than that at 240Hz, it has little impact on the positioning effect. It can be seen that this method is still effective for high-frequency vibration signals.

5. Conclusion
The φ - OTDR system based on non-uniform sampling method can break through the limitation of Nyquist sampling theorem on the maximum response frequency of the system. Because of the traditional signal processing methods, such as moving average method, the effect is not ideal. In order to solve this problem, this paper proposes a method of using the change value of the standard deviation of signal frequency domain to locate the vibration source, compares it with the traditional method, and makes simulation analysis on its anti noise and signal frequency domain sparsity. From the simulation results, the positioning effect and anti noise performance of this method are better than the traditional moving average signal processing method for the high frequency vibration signal with frequency domain sparse characteristics. This is of great significance for the further optimization and application of φ - OTDR system based on non-uniform sampling method.

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