Precision Measurements of $\mathcal{B}[\psi(3686) \to \pi^+\pi^- J/\psi]$ and $\mathcal{B}[J/\psi \to l^+l^-]$
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I. INTRODUCTION

Since the discovery almost four decades ago of the first charmonium state, the $J/\psi$, the states that have been studied the most among the various conventional charmonium states found have been the $J/\psi$ and $\psi(3686)$. However, the largest branching fraction in $\psi(3686)$ decays, $\mathcal{B}[\psi(3686) \to \pi^+\pi^-J/\psi] (\mathcal{B}_{\pi\pi\psi})$ still remains interesting both experimentally and theoretically. On the experimental side, the mass recoiling against the dipion system ($M_{\pi^+\pi^-}^{\text{rec.}}$) of this common decay mode can be used to identify $J/\psi$ decays. This makes $\mathcal{B}_{\pi\pi\psi}$ crucial for the relevant measurements in charmonium decays and searching for new particles, such as invisible particles in $J/\psi$ decays, as well as the measurements of charmonium production rates in higher energy collisions. Because of its large size, the branching fraction, $\mathcal{B}_{\pi\pi\psi}$, also imposes a limit on the rest of the decay channels of $\psi(3686)$. On the theoretical side, the transition $\psi(3686) \to \pi^+\pi^-J/\psi$ relates

Abstract

Based on $(106.41 \pm 0.86) \times 10^6 \psi(3686)$ events collected with the BESIII detector at the BEPCII collider, the branching fractions of $\psi(3686) \to \pi^+\pi^-J/\psi$, $J/\psi \to e^+e^-$, and $J/\psi \to \mu^+\mu^-$ are measured. We obtain $\mathcal{B}[\psi(3686) \to \pi^+\pi^-J/\psi] = (34.98 \pm 0.02 \pm 0.45)\%$, $\mathcal{B}[J/\psi \to e^+e^-] = (5.983 \pm 0.007 \pm 0.037)\%$ and $\mathcal{B}[J/\psi \to \mu^+\mu^-] = (5.973 \pm 0.007 \pm 0.038)\%$. The measurement of $\mathcal{B}[\psi(3686) \to \pi^+\pi^-J/\psi]$ confirms the CLEO-c measurement, and is apparently larger than the others. The measured $J/\psi$ leptonic decay branching fractions agree with previous experiments within one standard deviation. These results lead to $\mathcal{B}[J/\psi \to l^+l^-] = (5.978 \pm 0.005 \pm 0.040)\%$ by averaging over the $e^+e^-$ and $\mu^+\mu^-$ channels and a ratio of $\mathcal{B}[J/\psi \to e^+e^-]/\mathcal{B}[J/\psi \to \mu^+\mu^-] = 1.0017 \pm 0.0017 \pm 0.0033$, which tests $e-\mu$ universality at the four tenths of a percent level. All the measurements presented in this paper are the most precise in the world to date.

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to the interaction between heavy quarks and gluons as well as hadronization, providing an excellent testing ground for some theoretical predictions such as the QCD multipole expansion [2] and chiral symmetry [3].

$B_{\pi\pi\psi}$, however, has changed dramatically in the last decades [3][8]. For example, the most recent result from CLEO-c, $B_{\pi\pi\psi} = (35.04 \pm 0.8)\%$ [5], is apparently larger than the former most precise result $(32.3 \pm 1.4)\%$ from BESII [8]. The situation, thus, demands additional, high precision measurements of $B_{\pi\pi\psi}$. The data sample of $\psi(3686)$ collected with the BESIII detector, which is the world’s largest such sample, makes it possible to remeasure $B_{\pi\pi\psi}$ and clarify the discrepancy.

Similar to the transition $\psi(3686) \to \pi^+\pi^- J/\psi$, $J/\psi \to e^+e^-$ and $\mu^+\mu^-$ are often used to identify the $J/\psi$ experimentally for they are the two largest and cleanest decay modes of $J/\psi$. The branching fractions for the leptonic decays $J/\psi \to e^+e^-$ ($B_{ee}$) and $J/\psi \to \mu^+\mu^-$ ($B_{\mu\mu}$) are fundamental parameters of the $J/\psi$ resonance, and hence of general interest. The process of a vector charmonium decaying into a lepton pair is thought to occur through the annihilation of the $c\bar{c}$ pair into a virtual photon, and thereby is related to the $c\bar{c}$ wave function overlap at the origin, which plays a direct role in potential models [3]. Furthermore, the ratio $B_{ee}/B_{\mu\mu}$ provides a test of lepton universality. The standard model predicts exact lepton universality for $ee$ and $\mu\mu$, and any deviation from unity will indicate possible new physics effects or new decay mechanisms for $J/\psi$ to $l^+l^-$, where $l$ may be either $e$ or $\mu$. Also, as the branching fraction of $J/\psi \to l^+l^-$ ($B_l$) is important in the determination of the $J/\psi$ leptonic and total widths, ($\Gamma_{ee}$ and $\Gamma_{tot}$) [10], its precision is important for their uncertainties.

$B_{ee}$ and $B_{\mu\mu}$ have been measured to be approximately equal, as expected from lepton universality combined with a negligible phase space correction. A relative precision of 1% on both $B_{ee}$ and $B_{\mu\mu}$ has been achieved through an average [11] over measurements, which are dominated by the results from CLEO-c [12] and BESI [13].

This paper describes the measurement of the branching fraction $B_{\pi\pi\psi}$, as well as $B_l$ via the decay $\psi(3686) \to \pi^+\pi^- J/\psi$. Measuring $B_l$ via $\psi(3686) \to \pi^+\pi^- J/\psi$ has the advantage that there is no interference with Bhabha or dimuon production, that would need to be considered in measurements via direct $J/\psi$ production and decay in an electron-positron collider.

Our overall analysis procedure is as follows. The observed number of events, $N_{\pi\pi J/\psi}$ and $N_{ll}$ ($ll$ represents $\pi^+\pi^- l^+l^-$ final states), are extracted by fitting to data distributions or counting the signal candidate events directly. The corresponding acceptances, $\epsilon_{\pi\pi J/\psi}$ and $\epsilon_{ll}$, are calculated based on Monte Carlo (MC) samples. Then $B_{\pi\pi\psi}$ is calculated with the equation

$$B_{\pi\pi\psi} = \frac{N_{\pi\pi\psi}}{\epsilon_{\pi\pi\psi} N_{tot}}$$  \hspace{1cm} (1)

where $N_{tot}$ is the number of $\psi(3686)$ events. $B_{ll}$ is calculated with

$$B_{ll} = \frac{B_{\pi\pi\psi} \times B_l}{B_{\pi\pi\psi}} = \frac{N_{ll}/(\epsilon_{ll} \times N_{tot})}{N_{\pi\pi\psi}/(\epsilon_{\pi\pi\psi} \times N_{tot})} = \frac{N_{ll}/\epsilon_{ll}}{N_{\pi\pi\psi}/\epsilon_{\pi\pi\psi}}$$  \hspace{1cm} (2)

Here it should be noted that Eq. 2 is independent of the number of $\psi(3686)$ events, which is one of the major sources of systematic uncertainties in the determination of $B_{\pi\pi\psi}$.

II. BEPCII AND BESIII

BESIII/BEPCII, described in detail in Ref. [14], is a major upgrade of the BESII detector and the BEPC accelerator [15] for studies of hadron spectroscopy and $\tau$-charm physics [16]. The design peak luminosity of the double-ring $e^+e^-$ collider, BEPCII, is $10^{33}\text{ cm}^{-2}\text{s}^{-1}$ at a beam current of 0.93 A.
The BESIII detector with a geometrical acceptance of 93% of 4π, consists of the following main components: 1) a main drift chamber (MDC) equipped with 6796 signal wires and 21884 field wires arranged in a small cell configuration with 43 layers working in a gas mixture of He (40%) and C₃H₈ (60%). The single wire resolution on average is 135 μm, and the momentum resolution for charged particles in a 1 T magnetic field is 0.5% at 1 GeV/c; 2) an electromagnetic calorimeter (EMC) made of 6240 CsI (Tl) crystals arranged in a cylindrical shape plus two end-caps. The energy resolution is 2.5% in the barrel and 5% in the end-caps at 1.0 GeV; the position resolution is 6 mm in the barrel and 9 mm in the end-caps at 1.0 GeV; 3) a Time-Of-Flight system (TOF) for particle identification with a cylindrically shaped barrel portion, made with two layers for particle identification with a cylindrically shaped barrel portion, made with two layers with 176 pieces of 5 cm thick, 2.4 m long plastic scintillators in each layer, and end-caps each with 96 fan-shaped, 5 cm thick, plastic scintillators. The time resolution is 80 ps in the barrel, and 110 ps in the end-caps, corresponding to a K/π separation at the 2σ level up to about 1.0 GeV/c; 4) a muon chamber system (MUC) made of 1000 m² of Resistive Plate Chambers (RPC) arranged in 9 layers in the barrel and 8 layers in the end-caps. The position resolution is about 2 cm.

III. EVENT SELECTION

The data sample used for this analysis consists of (106.41 ± 0.86) × 10⁶ ψ(3686) decays produced at the resonance peak [17] and an additional 44 pb⁻¹ of data collected at √s = 3.65 GeV to determine the non-resonant background contributions. A MC sample of 106 × 10⁶ ψ(3686) inclusive decay events is used to obtain the detection efficiencies as well as to estimate the backgrounds. This sample is generated with KKMC [18] and EvtGen [19] for decays with known branching fractions [20], or by Lund-Charm [21] for unmeasured decays. The signal process of ψ(3686) → π⁺π⁻J/ψ is generated according to the formulas and measured results in Ref. [22], which takes the small D-wave contribution into account. The J/ψ → ℓ⁺ℓ⁻ processes are generated with an angular distribution of (1 + cos²θₗ), where θₗ is the lepton angle relative to the beam line in the J/ψ rest frame, and PHOTOS [23] is used for the final state radiation. These MC events are then processed with the detector simulation package based on GEANT4 [24].

In order to suppress tracks due to cosmic rays and beam associated events, charged tracks are required to pass within ±10 cm of the run-by-run determined interaction point along the beam direction and within 1 cm of the beam line in the plane perpendicular to the beam. To guarantee good agreement between data and MC simulation, all the charged tracks must lie in the barrel region, i.e., |cos θ| < 0.8, where θ is the polar angle with respect to the positron beam direction.

To identify π⁺π⁻J/ψ candidates, M_{rec} is determined for all pairs of charged tracks of opposite charge with momentum less than 450 MeV/c, that are assumed to be pions, and all the combinations with M_{rec} near the J/ψ peak are kept ([3.04, 3.16] GeV/c²). The (n)γJ/ψ backgrounds with an electron-positron pair converted from a photon are removed by requiring the cosine of the angle between the two charged tracks be less than 0.95. N_{ππJ/ψ} is determined from a fit to the distribution of M_{rec}. The left plot in Fig. 1 shows the distribution of M_{rec} for data, non-π⁺π⁻J/ψ decays, the scaled continuum events, and the sum of the signal from MC simulation and all backgrounds. Note that the mass resolutions of data (black dots) and MC simulation (red histogram) are different, which is considered in the following sections.

For the selection of π⁺π⁻ℓ⁺ℓ⁻ candidates, the pion pair is identified in the same way as for π⁺π⁻J/ψ. When multiple entries occur, the one with the minimum |M_{rec} − m_{J/ψ}|
E/p with both \( \pi^+ \pi^- \) energy deposition in the EMC of each track with their ratios, where \( E/p \) is the measured momentum. The events are shown with blue dash-dotted lines. The arrows shown in each plot indicate nominal selection criteria, which are applied for the other plots in the figure.

FIG. 1. (left) Distributions of \( M_{\pi^+\pi^-}^{\text{rec.}} \), where candidate events are represented by black dots, the non-\( \pi^+ \pi^- \) \( J/\psi \) decays of \( \psi(3686) \) background by the purple long dashed line, the scaled continuum by the blue dashed dotted line, and the \( \psi(3686) \) inclusive MC plus the scaled continuum and non-\( \pi^+ \pi^- \) \( J/\psi \) background by the red histogram. Distributions of \( M_{\pi^+\pi^-}^{\text{rec.}} \) (top right) \( J/\psi \rightarrow e^+e^- \) and (bottom right) \( J/\psi \rightarrow \mu^+\mu^- \) candidate events, where only total backgrounds are shown with blue dash-dotted lines. The arrows indicate the mass windows to count the number of signal candidates.

is kept, where \( m_{J/\psi} \) is the nominal \( J/\psi \) mass \([11]\). The fastest positive and negative tracks are taken as the lepton candidates. The lepton species are identified with their \( E/p \) ratios, where \( E \) is the measured energy deposition in the EMC of each track and \( p \) is its measured momentum. The events with both \( [E/p]^+ < 0.26 \) and \( [E/p]^- < 0.26 \) are taken as \( \mu^+\mu^- \) events, and those with \( [E/p]^+ > 0.80 \), \( [E/p]^- > 0.80 \), or \( \sqrt{([E/p]^+ - 1)^2 + ([E/p]^− - 1)^2} < 0.4 \) are taken as \( e^+e^- \) events. The backgrounds, such as \( J/\psi \rightarrow \pi^+\pi^−\pi^0 \), are removed by requiring the cosine of the angle between two lepton candidates be less than −0.95. The invariant mass of the lepton pair must be consistent with that of a \( J/\psi \), i.e., \( M_{e^+e^-} \in [2.7, 3.2] \) GeV/\( c^2 \) or \( M_{\mu^+\mu^-} \in [3.0, 3.2] \) GeV/\( c^2 \), where different mass windows are used since the \( e^+e^- \) final state has more final state radiation than \( \mu^+\mu^- \) does. Fig. 2 shows the invariant masses of the dipion pair (top) and the dilepton (bottom) pairs for \( \pi^+\pi^-e^+e^- \) (left) and \( \pi^+\pi^-\mu^+\mu^- \) (right) final states. To extract \( N_{\ell\ell} \), we count the number of events directly in a narrower mass window of \( M_{\ell\ell}^{\text{rec.}} \). Fig. 1 shows the distributions of the invariant mass recoiling against the dipion for the \( e^+e^- \) (top right) and \( \mu^+\mu^- \) (bottom right) channels for the \( \pi^+\pi^-l^+l^- \) candidates.

FIG. 2. Distributions of \( \psi(3686) \rightarrow \pi^+\pi^-J/\psi, J/\psi \rightarrow e^+e^- \) (left) and \( J/\psi \rightarrow \mu^+\mu^- \) (right) candidate events in the \( \psi(3686) \) data (black dots with error bars), MC simulation of signal plus background (red solid histogram), and backgrounds (blue dashed dotted line). The top panel shows distributions of the dipion invariant mass, and the bottom panel shows the dilepton invariant mass. The arrows shown in each plot indicate nominal selection criteria, which are applied for the other plots in the figure.
IV. BACKGROUND STUDY

For the $\pi^+\pi^- J/\psi$ final state, the backgrounds are studied with the $\psi(3686)$ inclusive MC and the continuum data sample. The backgrounds can be classified into three categories: (1) the non-$\pi^+\pi^- J/\psi$ decays of $\psi(3686)$, such as $\psi(3686) \rightarrow$ light hadrons or $\psi(3686) \rightarrow \eta J/\psi$; (2) the $\psi(3686) \rightarrow \pi^+\pi^- J/\psi$ decays, but one or both soft pions are from $J/\psi$ decays; and (3) other backgrounds, including the continuum process in $e^+e^-$ annihilation, beam-related, and cosmic ray backgrounds. As shown in the left plot of Fig. 1, the backgrounds from the non-$\pi^+\pi^- J/\psi$ and non-$\psi(3686)$ events are smooth and produce no peak at the $J/\psi$ mass. The second kind of background is studied with toy MC simulation in which the contributions with one or two charged tracks from $J/\psi$ decays are studied. The background shape is also found to be smooth with no peak at $J/\psi$ mass.

After all the requirements described above, the $\pi^+\pi^- l^+l^-$ event samples are rather clean. In the window of the invariant mass recoiling against the dipion $[m_{J/\psi} - 15, m_{J/\psi} + 15]$ MeV/$c^2$, for the $\pi^+\pi^- e^+e^-$ final state, the background level is estimated to be less than 0.10%. The largest background is $\psi(3686) \rightarrow \eta J/\psi, \eta \rightarrow \gamma \pi^+\pi^-, J/\psi \rightarrow e^+e^- (\sim 0.04\%)$, and the second largest background is $\psi(3686) \rightarrow \pi^+\pi^- J/\psi, J/\psi \rightarrow \pi^+\pi^-\pi^0 (\sim 0.03\%)$. For the $\pi^+\pi^-\mu^+\mu^-$ final state, the total background level is found to be 0.15%. The largest background is from $\psi(3686) \rightarrow \pi^+\pi^- J/\psi, J/\psi \rightarrow \pi^+\pi^- (\sim 0.09\%)$, and the second largest background is $\psi(3686) \rightarrow \eta J/\psi, \eta \rightarrow \gamma \pi^+\pi^-, J/\psi \rightarrow \mu^+\mu^- (\sim 0.02\%)$. Since the dominant backgrounds are exclusively simulated and subtracted from the signal region according to the known branching fractions and the scaled continuum data is subtracted, the remaining background is only 0.03 (0.04\%) for the $e^+e^- (\mu^+\mu^-)$ channel.

V. DATA ANALYSIS

Since the dipion emission occurs independently of the subsequent $J/\psi$ decay, the dipion recoil mass shape can be taken from any cleanly determined $J/\psi$ decay. We use $J/\psi \rightarrow e^+e^-$, which is almost background-free and has less background than $J/\psi \rightarrow \mu^+\mu^-$, for the signal shape of the dipion recoil mass distribution, and use a second-order polynomial to model the background shape. Increasing the order of the polynomial does not substantially improve the fit. However, a study shows that the resolution of the $\pi^+\pi^-$ recoil mass depends on the charged track multiplicity of $J/\psi$ decays. As a result, the mass resolution from leptonic exclusive decays of $J/\psi$ is slightly better than that of $J/\psi$ inclusive decays, and the difference produces a bad fit quality ($\chi^2/ndof \sim 50$, where $ndof$ is the number of degrees of freedom). To improve the fit quality, the signal shapes are smeared by convoluting them with two Gaussian functions, whose parameters are determined by directly fitting to data. While this procedure obviously improves the quality ($\chi^2/ndof \sim 4$), it changes the resultant $B_{\pi\pi\psi}$ by only 0.37\%, which is taken as one of the sources of systematic uncertainty. Fig. 3 shows the fit to the dipion recoil mass spectrum for $\psi(3686) \rightarrow \pi^+\pi^- J/\psi, J/\psi \rightarrow \text{anything}$.

For the $\pi^+\pi^- l^+l^-$ final states, the number of signal candidates in the distribution of $M^\text{rec}_{\pi^+\pi^-}$ are counted directly, since they are almost background free. However, as shown in the right column of Fig. 1, the resolutions of data (black dots) and MC simulation (red histogram) are different. Thus, the MC distributions are smeared according to data in determining their reconstruction efficiencies. A mass window of $[m_{J/\psi} - 15, m_{J/\psi} + 15]$ MeV/$c^2$ ($\sim 5\sigma$) is used in counting the signal candidates. Fig. 4 shows the comparison between data and the smeared MC simulation, in which the data points, as well as the regions, are the same as those in the right
FIG. 3. The dipion recoil mass spectrum for \( \psi(3686) \rightarrow \pi^+\pi^- J/\psi, J/\psi \rightarrow \text{anything} \). Top: data points (black) overlaid with the fit result (solid blue curve) obtained using the signal shape from \( \psi(3686) \rightarrow \pi^+\pi^- J/\psi \), \( J/\psi \rightarrow e^+e^- \) (blue dashed curve) and a second-order polynomial background shape (red dashed curve). Middle: the same plot as the top but with a log scale. Bottom: the fractional difference between the fit and the data.

panel of Fig. 1.

To validate the analysis method, MC input/output checks are performed based on the \( 10^6 \times 10^6 \) \( \psi(3686) \) inclusive MC sample, which has input values \( B_{\pi\pi\psi}, B_{ee}, \) and \( B_{\mu\mu} \) of 32.6\%, 5.93\%, and 5.94\%, respectively. Since this sample cannot be used at the same time to determine the efficiencies, an alternative \( 10^7 \) \( \psi(3686) \) inclusive MC sample is used for their determination. In order to make these two samples look more like real data, we also add in the scaled continuum data. As shown in Table I, all the extracted branching fractions are consistent with the input branching fractions within their uncertainties.

Table I summarizes the resultant signal yields, efficiencies, and branching fractions based on data, along with their statistical uncertainties.

| modes                  | \( B_{\text{in}} \)   | \( N_{\text{obs}}(10^4) \) | \( B \)          |
|------------------------|------------------------|-----------------------------|------------------|
| \( \pi^+\pi^- J/\psi \)| 32.6 \( 18783.4 \pm 5.1 \) | 32.64 \( \pm 0.03 \)       |                  |
| \( \pi^+\pi^- e^+e^- \)| 5.93 \( 660.6 \pm 0.8 \)    | 5.912 \( \pm 0.024 \)      |                  |
| \( \pi^+\pi^- \mu^+\mu^- \)| 5.94 \( 707.5 \pm 0.8 \)    | 5.930 \( \pm 0.024 \)      |                  |
TABLE II. Summary of $\psi(3686) \rightarrow \pi^+\pi^-J/\psi$ and $J/\psi \rightarrow l^+l^-$ results, showing numbers of the three decays, $N_{\pi\pi\psi}$, $N_{ee}$ and $N_{\mu\mu}$; efficiencies for observing those decays, $\epsilon_{\pi\pi\psi}$, $\epsilon_{ee}$ and $\epsilon_{\mu\mu}$; and the calculated branching fractions of the three channels, along with the statistical uncertainties on all quantities.

| $N(10^3)$ | $\pi^+\pi^-J/\psi$ | $\pi^+\pi^-e^+e^-$ | $\pi^+\pi^-\mu^+\mu^-$ |
|------------|-----------------------|----------------------|----------------------|
| 20235 ± 6  | 718.8 ± 0.9           | 771.1 ± 0.9          |
| $\epsilon(\%)$ | 54.37 ± 0.02          | 32.19 ± 0.04         | 34.54 ± 0.04         |
| $B(\%)$    | 34.98 ± 0.02          | 5.983 ± 0.007        | 5.973 ± 0.007        |

VI. STUDY OF SYSTEMATIC UNCERTAINTIES

We consider systematic uncertainties from many different sources. The uncertainty of the number of $\psi(3686)$ decays, 0.81% [17], which is measured by counting the hadronic events from $\psi(3686)$ decay directly, is the dominant uncertainty of $B_{\pi\pi\psi}$, while $B_{ee}$ and $B_{\mu\mu}$ are independent of it. The difference of tracking efficiency between data and MC simulation is measured from a comparison of yields of partially and fully reconstructed $\psi(3686) \rightarrow \pi^+\pi^-J/\psi$ and $J/\psi \rightarrow l^+l^-$ decays in real and simulated data. The differences depending on the polar angle and the transverse momentum of the track are used to re-weight the MC samples. And the uncertainty of the re-weighting factor is estimated to be 0.1% per lepton and 0.4% per pion. The systematic effects related to the soft pion tracking cancel in the calculation of $B_{ee}$ and $B_{\mu\mu}$. The tracking uncertainties of $\pi^+$ and $\pi^-$, or $l^+$ and $l^-$ are considered as fully correlated and are added linearly.

In the inclusive analysis, even though we only reconstruct two soft charged pions, the reconstruction efficiency depends on the track multiplicity of the subsequent $J/\psi$ decays. However, since the sum of known exclusive $J/\psi$ partial widths is small compared to the total width, a MC sample must be used to represent all $J/\psi$ decays and to obtain $\epsilon_{\pi\pi\psi}$. The global efficiency found is about 54% but varies about 15% (relative) from low to high charged track multiplicities of $J/\psi$ decays, similar to that reported in BESIII [13], but the variation is much larger than that in CLEO-c [12]. We attribute the difference to the finer segmentation in the CLEO-c tracking system, which was designed for physics at higher energy [26] relative to that of BESI [27] and BESIII [14], as well as the consequent robustness of track reconstruction in the presence of many charged particles.

To study the dependence of the detection efficiency $\epsilon_{\pi\pi\psi}$ on the generated charged track multiplicity distribution for $J/\psi$ decays in $\pi^+\pi^-J/\psi$ events, we first use the inclusive MC sample to determine the detection efficiency ($\epsilon_k$) as a function of generated track multiplicity ($k$), as shown in Table III, and then determine $\epsilon_{\pi\pi\psi}$ considering alternative generated multiplicity distributions. Two methods are used to determine the fraction $w_k$ of each multiplicity from data directly and $\epsilon_{\pi\pi\psi}$. The first is the method used in Ref. [13], which fits the observed multiplicities in data using the efficiency matrix, $\epsilon_{ij}$, which describes the efficiency of a MC event generated with $j$ charged tracks to be reconstructed with $i$ charged tracks, to determine the true generated charged track multiplicity distribution. The second method fits the observed multiplicity distribution with exclusive MC based templates as in Ref. [12]. Fig. 5 shows the multiplicity distribution fitted by the generated multiplicity distribution of the inclusive MC. Table III summarizes the multiplicity distribution obtained from the $\psi(3686)$ inclusive MC and the two methods mentioned above, as well as the overall $\epsilon_{\pi\pi\psi}$ for each case. Consistent results are obtained, which indicates that $\epsilon_{\pi\pi\psi}$ is not very sensitive to the generated multiplicity distribution of $J/\psi$ decays. We assign the largest difference as the systematic uncertainty due to our imperfect simulation of the charged track multiplicity, $N_{trk}$, in $J/\psi$ decays.
FIG. 5. Fit (histogram) to the multiplicity distribution of data (points) with that of the MC sample.

TABLE III. The fractions of each charged track multiplicity of $J/\psi$ decays from the $\psi(3686)$ inclusive MC (column 2), from the method of Ref. [13] (column 3), and that of Ref. [12] (column 4). The MC efficiency for $k$-charged tracks is shown as $\epsilon_k$. The overall efficiency $\epsilon_{\pi\pi\psi}$ for each of the three cases is also shown.

| $N_{\text{trk}}^{J/\psi}$ | $w_k$ (incl.) (BES) (CLEO-c) | $\epsilon_k(\%)$ | $\epsilon_{\pi\pi\psi}(\%)$ |
|---|---|---|---|
| 0 | 0.0175 | 0.0225 | 0.0231 | 56.56 |
| 2 | 0.3440 | 0.3881 | 0.3945 | 55.82 |
| 4 | 0.4310 | 0.4015 | 0.4012 | 53.97 |
| 6 | 0.1871 | 0.1644 | 0.1627 | 52.03 |
| 8 | 0.0199 | 0.0200 | 0.0185 | 49.49 |

From the above analysis, the uncertainty from the charged track multiplicity distribution was found to be less than 0.2%, including all the contributions from the fit, the sideband selection, and the backgrounds. The efficiency does exhibit a weak dependence not only on the charged multiplicity, but also slightly on the neutral track multiplicity. More neutral particles in the $J/\psi$ decay soften the momentum spectrum of the charged tracks, which makes the tracks harder to detect, and produces more photon conversions in the material in the inner detectors, which also changes the charged track multiplicity. But a MC study suggests that such effects are very small and can be neglected.

The dipion invariant mass distribution is simulated with the measurement of Ref. [22], in which a small amount of $D$-wave contribution is included. However, there is still a slight difference between the data and MC simulation, so the MC simulation is re-weighted by the distribution in data, and the difference before and after the re-weighting, which is 0.35% for $\epsilon_{\pi\pi\psi}$, is taken as a systematic uncertainty. The difference is much smaller for $\epsilon_{l^+l^-}$, $\sim 0.01\%$, since the effect cancels in a relative measurement.

The fit to the huge statistics of the distribution of mass recoiling against the dipion gave a poor $\chi^2$/$ndof$, since the resolutions in the exclusive and inclusive decays are a bit different. The signal shapes of the exclusive channel are smeared by convoluting with double Gaussian functions to improve the fit quality. And as a result, $B_{\pi\pi\psi}$ is changed by 0.37% before and after the smearing, which is taken as one of the systematic uncertainties.

The shapes of the invariant mass distributions of lepton pairs are affected by the simulation of final state radiation (FSR), which is simulated with the PHOTOS package [28]. Differences between data and MC simulation are still observed. The invariant mass requirement on lepton pairs is studied by an alternative control sample, in which the lepton pairs are identified by the information of the EMC, MUC, and specific ionization ($dE/dx$) measured in MDC, while demanding $M_{\pi^+\pi^-}$ to be consistent with the $J/\psi$ mass, but without any requirement on the invariant mass of $e^+e^-$ or $\mu^+\mu^-$. The differences are determined to be 0.29% ($e^+e^-$) and 0.45% ($\mu^+\mu^-$). To reduce this type of uncertainty, corrections are made based on this study, and the final contributions to the total systematic un-
uncertainty are 0.10% and 0.23%, respectively.

The remaining sources of systematic uncertainty not addressed above are the requirements on $E/p$, the angles between the two leptons and the two pions, the background contamination for $\pi^+\pi^- l^+ l^-$ final states, and the uncertainty related to the fitting (counting) procedure. The first two items are determined with independent samples selected with alternative selection criteria, and the uncertainties of the $E/p$ requirement are found to be 0.18% and 0.09% for muon and electron pairs, respectively; the uncertainties of the two angle requirements are found to be less than 0.1%. The uncertainties of the backgrounds of the $\pi^+\pi^- l^+ l^-$ exclusive final states are only 0.03~0.04%, after subtracting the background using known branching ratios. The uncertainties of the fitting (except the uncertainty of the resolution in $B_{\pi\pi\psi}$), which are all at the part per thousand level, are estimated by changing the signal shape, background shape, fitting ranges (mass windows), and bin size. The uncertainties of the trigger efficiency in the three measurements are taken as 0.10% for $B_{\pi\pi\psi}$ and 0.30% for $B_{ll}$ according to the study in [29].

The systematic uncertainties in the branching fractions are summarized in Table IV. The systematic uncertainty in $B[\psi(3686) \to \pi^+\pi^- J/\psi]$ is dominated by the number of $\psi(3686)$ events and the tracking efficiency of the two soft pions, and the total contribution of the other sources is less than 0.5%. The systematic uncertainty in $B[J/\psi \to l^+ l^-]$ is dominated by the uncertainty of the determination of $N_{\pi\pi\psi}$.

**TABLE IV. Summary of the systematic uncertainties (%) in the branching fractions.**

| Sources                        | $\pi^+\pi^- J/\psi$ | $e^+e^-$ | $\mu^+\mu^-$ |
|--------------------------------|----------------------|----------|--------------|
| Tracking                       | 0.80                 | 0.20     | 0.20         |
| Multiplicity of $J/\psi$       | 0.20                 | 0.20     | 0.20         |
| $M_{\pi^+\pi^-}$ distribution  | 0.35                 | 0.01     | 0.01         |
| Background shape               | 0.03                 | 0.03     | 0.04         |
| Fit/Count range                | 0.06                 | 0.14     | 0.14         |
| Bin size                       | 0.06                 | 0.06     | 0.06         |
| $E/p$                          | —                    | 0.18     | 0.09         |
| $\cos \theta_{\pi^+\pi^-}$    | 0.13                 | 0.07     | 0.07         |
| $\cos \theta_{l^+l^-}$         | —                    | 0.04     | 0.05         |
| FSR effect of $l^+l^-$         | —                    | 0.10     | 0.23         |
| Fit method                     | 0.37                 | 0.37     | 0.37         |
| Trigger                        | 0.10                 | 0.30     | 0.30         |
| Number of $\psi(3686)$         | 0.81                 | —        | —            |
| Sum in quadrature              | 1.28                 | 0.62     | 0.63         |

The branching fractions of three processes $\psi(3686) \to \pi^+\pi^- J/\psi$, $J/\psi \to e^+e^-$, and $J/\psi \to \mu^+\mu^-$, are measured with $(106.41 \pm 0.86) \times 10^6 \psi(3686)$ decays. The results are $B_{\pi\pi\psi} = (34.98 \pm 0.02 (0.45))\%$, $B_{ee} = (5.983 \pm 0.007 (0.037))\%$, and $B_{\mu\mu} = (5.973 \pm 0.007 (0.038))\%$, where the first uncertainties are statistical and the second are systematic. We also measure $B_{ee}/B_{\mu\mu} = 1.0017 \pm 0.0017 \pm 0.0033$, where the common systematic uncertainties have been canceled out. This tests $e-\mu$ universality at the four tenths of a percent level. The precision is significantly improved with respect to the PDG average $B_{ee}/B_{\mu\mu} = 0.998 \pm 0.012$ [11]. Assuming leptonic universality, the average of $B_{ee}$ and $B_{\mu\mu}$ is $B[J/\psi \to l^+l^-] = (5.978 \pm 0.005 \pm 0.040)\%$, in which the correlations among the uncertainties are accounted for. The measured branching fractions of $J/\psi \to e^+e^-/\mu^+\mu^-$ are consistent with previous measurements, and will allow improvements in potential models [9] and the determinations of $\Gamma_{ee}$ and $\Gamma_{\text{tot}}$ of $J/\psi$ [10].

Figure 3 shows a comparison of $B_{\pi\pi\psi}$ among various experiments. Our measured $B_{\pi\pi\psi}$ is the most precise to date and is consistent with the latest CLEO-c [3] measurement, but higher than most of the previous measurements.
FIG. 6. Comparison of $B(\psi(3686) \to \pi^+\pi^- J/\psi)$ among different experiments.

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