PETSCHEK-LIKE RECONNECTION WITH CURRENT-DRIVEN ANOMALOUS RESISTIVITY AND ITS APPLICATION TO SOLAR FLARES

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ABSTRACT

Recent simulations of magnetic reconnection with localized resistivity demonstrated the development of a Petschek-like configuration with the width of the inner diffusion region of the order of the resistivity localization scale. In this paper, we combine this fact with a realistic model for locally enhanced current-driven anomalous resistivity. In the qualitative model that results, the size of the diffusion region and hence the reconnection rate are determined self-consistently by the functional dependence of anomalous resistivity on the current density. For the specific case of anomalous resistivity due to ion-acoustic turbulence, we express the main reconnection parameters directly in terms of the basic plasma parameters. Finally, we apply our model to solar flares and obtain typical reconnection times that are consistent with observations.

Subject headings: MHD — Sun: flares — Sun: magnetic fields

1. INTRODUCTION

Magnetic reconnection is a basic plasma physics phenomenon of great importance in many astrophysical systems (Tsuneta 1996; Kulsrud 1998) and in laboratory plasma devices (Yamada et al. 1997), including tokamaks (Kadomtsev 1975; Yamada et al. 1994). Historically, the earliest theoretical models of magnetic reconnection have aimed at explaining the solar flare phenomenon. In the very first model, developed by Sweet (1958) and Parker (1957, 1963), magnetic field is frozen into the plasma everywhere except in a very thin layer where the current density is so high that resistive effects become important no matter how small the resistivity is. It is inside this thin current layer that the actual breaking (and reconnecting) of the magnetic lines of force takes place, accompanied by a violent release of enormous amounts of magnetically stored energy, thus leading to the observed flare. The Sweet-Parker theory predicts that the current layer thickness, $\delta_{\text{SP}}$, scales as $\delta_{\text{SP}} \sim L/\sqrt{S}$, where $L$ is the global system size (typically of order $10^9$ cm in the solar corona) and $S = LV_A/\eta$ is the global Lundquist number (here $V_A$ is the Alfvén velocity and $\eta$ is the magnetic diffusivity; in the rest of this paper we refer to $\eta$ as the resistivity). Correspondingly, the typical reconnection timescale is found to be $\tau_{\text{rec}} \sim \tau_A(L)/\sqrt{S}$, where $\tau_A(L) \equiv L/V_A$ is the global Alfvén transit time. It was immediately realized that the resulting reconnection time turns out to be too long; in the solar corona one typically has $S \sim 10^{12} - 10^{14}$ and $\tau_A(L) \sim 1$ s, which leads to $\tau_{\text{rec}}$ of order a few months. This is in sharp contrast with the typical observed solar flare duration of order $10^5 - 10^7$ s. Thus, since its early years, the main thrust of magnetic reconnection research has been to explain reconnection rates that are much faster than the Sweet-Parker theory predicts.

Starting from the 1960s, two major routes toward faster reconnection were proposed. One of them was to use the so-called anomalous resistivity instead of the classical Spitzer resistivity used in the original Sweet-Parker model (Coppi & Friedland 1971; Smith & Priest 1972; Coroniti & Evitar 1977; Kulsrud 1998). The idea was that, as a current layer forms, its thickness becomes so small, and hence the current density becomes so high, that the drift velocity of the current-carrying electrons exceeds a certain threshold, such as the ion-sound speed or the electron thermal speed. This leads to the excitation of current-driven kinetic microturbulence, which, in turn, provides a more efficient (compared to particle-particle collisions) mechanism for the scattering of electrons (via wave-particle interactions). As a result, one ends up with a greatly enhanced effective resistivity and hence a greatly reduced effective Lundquist number. When substituted into the Sweet-Parker scaling, this results in a greatly enhanced reconnection rate. In fact, controlled laboratory studies of magnetic reconnection have shown a good agreement with a simple Sweet-Parker model augmented with some (experimentally measured) anomalously enhanced resistivity (Ji et al. 1998, 1999). On the other hand, in the solar flare context, the Sweet-Parker model with anomalous resistivity gives typical reconnection times of order several hours (e.g., Kulsrud 1998), a great improvement over Spitzer resistivity. Among various anomalous resistivity mechanisms, the one most frequently quoted has been the ion-acoustic turbulence (IAT). Rapid magnetic energy dissipation due to the IAT-driven anomalous resistivity has in fact also been invoked to explain coronal heating (Rosner et al. 1978).

Another possibility leading to shorter reconnection times was proposed by Petschek (1964). His very elegant model makes use of a somewhat more complicated reconnection layer geometry while still relying on simple resistive magnetohydrodynamics (MHD) without invoking any new physics at the microscopic level. The Petschek model actually does not predict a unique configuration of the reconnection layer and a unique reconnection rate. Instead, this model encompasses an entire one-parametric family of solutions. Each of these configurations has at its center a small Sweet-Parker–like layer, called the diffusion region, and four standing slow-mode shocks emanating from the ends of this central layer. The members of this family of solutions can be labeled by the width (or length) $\Delta$ of the inner diffusion region. As Petschek noticed, in the Sweet-Parker model the reconnection process had been slowed down by the very large aspect ratio $L/\delta$ of the reconnection layer. He
suggested that the width $\Delta$ of the layer’s diffusion region does not have to be as large as the global size $L$; if $\Delta$ can be made sufficiently short, the reconnection process will go much faster than in the Sweet-Parker model. The maximum value of $\Delta$ corresponds to the Sweet-Parker solution, which is therefore just one of the family members. The reconnection rate ranges from the slowest (Sweet-Parker) rate to the so-called maximum Petschek rate, which scales as $1/\log S$. We thus see that the relatively strong square-root dependence on the resistivity, characteristic for the Sweet-Parker model, is replaced here by a much weaker logarithmic dependence; even for a very large $S \approx 10^{14}$, the resulting reconnection timescale turns out to be reasonably short, of order $10^{2} \tau_{A}(L)$.

This model had remained the favorite model of reconnection until the 1980s, when two-dimensional resistive-MHD numerical simulations by Biskamp (1986) showed that, in the case of a spatially uniform resistivity, a Petschek-like configuration fails to form and that a long (of order $L$) current layer tends to form instead, consistent with the Sweet-Parker picture. This finding has been confirmed in a number of numerical simulations performed by several other groups (Scholer 1989; Ugai 1992, 1999; Yokoyama & Shibata 1994; Uzdensky & Kulsrud 2000; Erkaev, Semenov, & Jamitzky 2000; Erkaev et al. 2001). A theoretical explanation has been put forward by Kulsrud (2001). He noticed that, when the resistivity is uniform, the relatively large transverse magnetic field that is needed to support the standing shocks in the Petschek model is rapidly swept out of the diffusion region by the downstream flow, while its regeneration due to the nonuniform merging is not fast enough. As a result, the diffusion region’s size $\Delta$ (which Kulsrud calls $L'$) increases until it reaches the global scale $L$, and the reconnection rate correspondingly slows down to the usual Sweet-Parker rate. This explanation has been confirmed numerically by Uzdensky & Kulsrud (2000; see also Kulsrud 1998).

It has been noticed, however, that the key assumption leading to the above conclusion was the assumption of uniform resistivity (which is a very common assumption in numerical simulations in general). Simulations featuring a nonuniform resistivity have shown that a Petschek-like structure does form and can be stable whenever the resistivity is locally enhanced in some small region near the X-point at the center of the reconnection region (Ugai & Tsuda 1977; Sato & Hayashi 1979; Ugai 1986, 1992, 1999; Scholer 1989; Yokoyama & Shibata 1994; Erkaev et al. 2000, 2001; Ugai & Kondoh 2001; Biskamp & Schwarz 2001). A nice plausible theoretical explanation of this phenomenon has again been provided by Kulsrud (2001), who analyzed how a locally enhanced resistivity may lead to a more efficient regeneration of the transverse magnetic field through nonuniform merging and thus to the sustainment of Petschek’s shocks. In addition, recent numerical work by Erkaev et al. (2000, 2001) and by Biskamp & Schwarz (2001) has shown that the particular Petschek-like configuration that forms in the localized-resistivity situation is characterized by the width $\Delta$ of the inner diffusion region being of the order of the resistivity localization scale, which in this paper we call $l_0$ (Erkaev et al. 2000, 2001; Biskamp & Schwarz 2001). We thus see that the resistivity-localization mechanism plays a crucial role in determining the final reconnection rate.

From the point of view of physical reality, the main motivating force behind these localized-resistivity studies has been the idea that anomalous resistivity, being such a sensitive function of the local current density, may in fact be triggered only in a small neighborhood of the X-point, where the current density is highest. In other words, anomalous resistivity can enhance the reconnection rate not only directly (by simply being higher than the collisional resistivity) but also indirectly, via enabling the Petschek mechanism (by being strongly localized). This is one of the key ideas behind the so-called spontaneous fast reconnection model suggested by Ugai (1986, 1992, 1999; also see Ugai & Tsuda 1977; Yokoyama & Shibata 1994; Ugai & Kondoh 2001) on the basis of numerical simulations.

Up until now, however, with the notable exception of the work by Kulsrud (2001), there have been no analytical attempts to combine the Petschek reconnection model with any physically realistic model of anomalous resistivity, which would provide unique theoretical predictions for the main parameters of the reconnection region, such as the width of the diffusion region and the reconnection rate, in terms of the basic parameters of the plasma. The goal of this paper is to remedy this situation by attempting to build a simple theoretical framework incorporating a particular anomalous resistivity mechanism (ion-acoustic turbulence).

In §2 we present our model of the Petschek reconnection layer with a generic form of anomalous resistivity. In particular, in §2.1 we describe the basic elements of the model, e.g., the Sweet-Parker relationships for the inner diffusion region and the functional dependence of anomalous resistivity, $\eta$, on the current density $j$. In §2.2 we discuss possible solutions and their stability. In §2.3 we calculate the reconnection rate in terms of the model parameters. Section 3 is devoted to the specific case of anomalous resistivity due to IAT; in this section we use the theory of IAT to express all major reconnection layer parameters, including the reconnection rate, in terms of the basic plasma parameters (such as the magnetic field strength, plasma density, and the electron and ion temperatures). We apply the obtained results to a solar flare environment in §4 and find a very reasonable agreement in terms of general timescales. We conclude in §5.

2. MODEL OF THE RECONNECTION LAYER

2.1. Three Main Ingredients of the Model

We now describe our semiempirical model$^1$ of a Petschek-like reconnection configuration that is formed in the presence of anomalous resistivity due to a current-driven microturbulence. The model represents a synthesis of three external ingredients:

1. The numerically observed fact that, whenever the resistivity is strongly localized, a Petschek-like structure tends to develop, with the width of the diffusion region being of the order of the resistivity localization scale (Erkaev et al. 2000, 2001; Biskamp & Schwarz 2001).

2. The Sweet-Parker model for the central diffusion region of the Petschek configuration.

3. A physically motivated model for anomalous resistivity.

Figure 1 shows schematically the central part of a Petschek-like configuration. Here, $y$ is the direction along

$^1$ See Uzdensky (2002) for a more elaborate presentation and discussion.
the layer and \( x \) across the layer. The shaded rectangular area (of characteristic thickness \( \delta \) and width \( \Lambda \)) in the center of the figure is the inner diffusion region where the current density is concentrated and where the actual breaking of magnetic field lines occurs. For definiteness, we use Ampere’s law to define the thickness \( \delta \) of this central current layer in terms of the central current density \( j_0 \equiv j(0, 0) \) and the outside magnetic field \( B_0 \) as

\[
\delta \equiv \frac{c B_0}{4 \pi j_0}. \tag{1}
\]

We also define the characteristic width \( \Lambda \) of the diffusion region in terms of the function \( j(y) \equiv j(x = 0, y) \) as the distance from the center \( y = 0 \) to the point where the current density drops by a factor of \( e \), i.e., \( j(y = \Lambda) = j_0/e \). Similarly, we define the characteristic scale \( l_y \) for the variation of the plasma resistivity \( \eta(y) \) along the layer as \( \eta(y = l_y) = \eta(y = 0)/e \).

In general, \( \Lambda \) and \( l_y \) may differ. However, as discussed in §1, if the resistivity is strongly localized \( (l_y \ll L) \), a Petschek-like configuration that develops has \( \Lambda \sim l_y \). This condition enables one to select a unique solution out of the entire family of Petschek configurations. We call this unique Petschek configuration an equilibrium configuration. In general, \( \Lambda \) and \( l_y \) in this equilibrium Petschek configuration may differ by a finite factor, so we define a dimensionless parameter \( K \) such that the equilibrium configuration has \( \Lambda = \Lambda_{\text{eq}} \equiv KL_y \). In our analysis it is actually more convenient to use another dimensionless parameter, \( \xi \), which we expect to be of order 1, to describe the equilibrium Petschek configuration; we define it in terms of \( j(y) \) as \( j(l_y) = \xi j_0/e \). Since \( j(y) \) is a monotonically decreasing function of \( y \), we have \( \xi > 1 \) whenever \( \Lambda_{\text{eq}} > l_y \) \((K > 1)\) and \( \xi < 1 \) whenever \( \Lambda_{\text{eq}} < l_y \) \((K < 1)\). The analysis in this paper is restricted to the situation where \( \Lambda_{\text{eq}} > l_y \), i.e., \( \xi > 1 \); this choice is made purely for reasons of convenience, as is elucidated below.

The second ingredient of our model is the model for the inner diffusion region. In Petschek’s theory, this region is a Sweet-Parker–like current layer with the thickness \( \delta \) related to its width \( \Delta \) via the relationship \( \delta/\Delta \simeq 1/\sqrt{S_A} \), where \( S_A \equiv V_A \Delta/\eta_{\text{eff}} \) is the Lundquist number for the scale \( \Delta \). Here we take \( \eta_{\text{eff}} = \eta(j_0) \), the resistivity at the center. Then we can write

\[
\delta = \sqrt{\frac{\Delta}{V_A \eta(j_0)}}. \tag{2}
\]

We invoke this very important relationship many times throughout the paper.

The third ingredient of our model is a physically plausible model for anomalous resistivity \( \eta(j) \); it is needed to determine the actual values of \( l_y \) and hence \( \Lambda \). Our choice of the function \( \eta(j) \) is motivated by anomalous resistivity due to a current-driven microturbulence. Thus, here we adopt a very simple, minimal model for \( \eta(j) \), which, however, has to exhibit the following general properties: first, there exists a current-density threshold, \( j_c \), for triggering anomalous resistivity, and second, the rapid rise of \( \eta \) after the threshold is exceeded stops at some large but finite value \( \eta_1 \), after which \( \eta(j) \) continues to rise with increased \( j \) more slowly. Thus, we take \( \eta(j) \) to be a prescribed function with the following behavior (see Fig. 2):

1. For \( j < j_c \), \( \eta(j < j_c) = \eta_0 = \text{const.} \)
2. At \( j = j_c \), \( \eta(j) \) rises rapidly to a large value \( \eta_1 \gg \eta_0 \) over some very small interval of width \( \Delta j \ll j_c \); the exact value of \( \Delta j \) is unimportant in our analysis. We assume that \( \eta(j) \) is linear in the interval \( j \in [j_c j_c + \Delta j] \).
3. When \( j \) is increased even further, \( j > j_c + \Delta j \), the resistivity continues to rise monotonically with increased \( j \) but in a much slower manner. In particular, we take \( \eta \sim j \) in this region, which is the case for IAT (e.g., Bychenkov, Silin, & Uryupin 1988). Thus, we take

\[
\eta(j > j_c + \Delta j) \simeq \eta_1 \frac{j}{j_c}. \tag{3}
\]

In addition to the three main parameters \((j_c, \eta_0, \eta_1)\) describing the dependence \( \eta(j) \), we introduce the critical
layer thickness $\delta_c$:

$$
\delta_c = \frac{cB_0}{4\pi j_c}.
$$

For the analysis in this section this generic level of description will suffice. In §3, however, we consider the case of anomalous resistivity due to IAT and give specific expressions for these parameters.

Finally, let us make a remark concerning the above prescription for $\eta(j)$. In our model, $\eta$ is determined solely by the local value of $j$. In reality the coefficients $j_c$, $\eta_0$, and $\eta_1$ will depend on the plasma density and temperature and so will vary in space inside the reconnection layer. In our simple model we ignore this aspect and assume these parameters to be constant in space and time and hence the same profile $\eta(j)$ to apply everywhere.

2.2. Two Possible Solutions and Their Stability

We now use the $\eta(j)$ dependence shown in Figure 2 to find the equilibrium Petschek solution. At $y = 0$, let us have some value $j_0$ and the corresponding value $\eta(j_0)$. The requirement $j_s = j(l_0) = \xi_0/e$ means that

$$
\eta(y = l_0) = \eta(\xi_0/e), \\
\eta(y = l_0) = \eta(j_0)/e.
$$

Thus, $j_0$ is determined in terms of the function $\eta(j)$ as

$$
\eta(j_0) = \eta(j_s) = \eta\left(\frac{\xi_0}{e}\right).
$$

The solution of this equation can be found by drawing a set of parabolas $\eta \propto j^\alpha$ with $\alpha = 1/(1 - \ln \xi)$ and selecting out of this set those parabolae for which the points of their intersection with the curve $\eta(j)$ are separated by a factor of $e$ in their values of $\eta$, as shown in Figure 3. It is clearly seen from this figure that, for any $\xi > 1$ ($\alpha > 1$), one finds two such solutions. The first solution (curve I) corresponds to $j^I_0 \in [j_c, j_c + \Delta j]$, while $j^I_s = j^I(l_0) = \xi_0/e < j_c$. Then, $\eta^I(y = l_0) = \eta_0$, and hence $\eta^I(j_0) = e\eta_0$. We thus see that the corresponding resistivity enhancement over $\eta_0$ is not very large in this case, just by a factor $e$. In the second solution (curve II), the resistivity enhancement is much stronger.

Here it is $j^I_0$ that falls within the narrow rapidly rising part of the $\eta(j)$ curve, and we have

$$
\eta^I(j_0) = \eta^I(j_0^I) = \eta_0 e \frac{\xi_0}{\xi} \gg \eta_0.
$$

Any Petschek-like configuration with $j_0$ between $j^I_0$ and $j^II_0$, and hence with $e\eta_0 < \eta < e\eta_0/\xi$, will not be in an equilibrium and will evolve so as to increase $j_0$. Indeed, if $\eta(j_0) > e\eta_0$, then $\eta(j_s) > \eta_0$ and hence $j_s > j_c$, but then we have $j_s/j_0 > e/\xi$ and so $\Delta > \Delta_{eq}$, which will lead to shrinkage of $\Delta$. As is shown below, this will result in a decrease in $\delta$ and an increase in $j_0$. The evolution will then presumably reach a stationary state when (the stable) solution II is reached.

Now, which one of the two solutions I and II will be realized in a real physical system? We suggest that, in general, the answer to this question is determined by the stability of the solutions with respect to a small change in $j_0$. In particular, we demonstrate that the first solution is unstable while the second one is stable. This enables us to conclude that the system will evolve toward the second solution corresponding to higher $\eta_0$ and hence to a higher reconnection rate.

Let us consider an equilibrium Petschek-like configuration with $\Delta = K l_{eq}$. Let this configuration be characterized by unperturbed values $\delta$, $j_0$, $j_s$, $\Delta$, $l_{eq}$, etc. Now imagine that at $t = 0$ this equilibrium Petschek configuration is suddenly perturbed, namely, changed into a neighboring Petschek configuration. This new configuration is generally not in equilibrium; i.e., it does not satisfy condition (5). It will then evolve through a sequence of Petschek states. For simplicity, here we envision the process as occurring in two parts with different timescales: the adjustment between $\Delta$ and $\delta$ to conform to the Sweet-Parker structure of the diffusion region is instantaneous, whereas the adjustment of $\Delta$ to $l_{eq}$ occurs on a longer timescale.

In particular, let us imagine that at $t = 0$ the thickness $\delta$ of the diffusion region is reduced slightly and hence the central current density is correspondingly slightly increased:

$$
\delta \rightarrow \delta' = \delta(1 - \epsilon), \quad j_0 \rightarrow j_0' = j_0(1 + \epsilon), \quad \epsilon \ll 1.
$$

This increase in $j_0$ leads to an increase in the resistivity at the
Since $j_0$ in solution I lies on the steeply rising part of the
$\eta(j)$ dependence, this increase in central resistivity is large.
On this part of the curve, we have $\eta'(j_0) \simeq \eta_1/\Delta j$, and so
\begin{equation}
\eta(j_0) = \eta(j_0) + \eta'(j_0)j_0 \varepsilon .
\end{equation}

Then, $\eta(L_0) = \eta(j_0)/\varepsilon = \eta_0 + (\varepsilon/\eta_1)\eta_0(j_0/\Delta j)$, and therefore,
$j_0 \approx j_0$. If the perturbed current density $j'(y)$
has a characteristic scale $\Delta$, then $j_0 \approx j_0$ implies that $L_0 \ll \Delta$.
Thus, we see that, as the principal effect of the initial perturbation,
the size $L_0$ of the resistivity enhancement region drops very sharply,
whereas all other quantities change rather smoothly. Then we can again invoke our empirical
fact that a configuration like this will not stay stationary but
will evolve so that the current layer width $\Delta$ will tend to decrease
to become comparable with $L_0$.

However, as $\Delta$ changes, there will be a feedback on $\delta$.
Indeed, since the diffusion region is a Sweet-Parker layer,
$\delta$ and $\Delta$ are always related via equation (2). Since $\eta(j_0)$
is a monotonically increasing function of $j_0 \propto 1/\delta$, we see that $\delta$
is a monotonically increasing function of $\Delta$. For solution I, we can express the resistivity as
$\eta(j_0) \simeq \eta(j_0/\Delta j)(\delta - \delta_0)/\delta_0$ and thus obtain from equation (2)
\begin{equation}
\left( \frac{\delta - \delta_0}{\delta_0} \right) = \frac{\delta_0 V_0 L_0}{\Delta} \frac{\eta_1}{\eta_0}.
\end{equation}

We thus see that the decrease in $\Delta(r)$ leads to a proportional
increase in $\delta(r)$, and so $\delta$ changes in the same direction as $\Delta$.

From the above arguments it is clear that, in the case of
solution I, the initial decrease in $\delta$ and $\Delta$ tends to amplify
further. This means that this solution is unstable: a slight increase in $j_0$ will lead to further increase,
and thus the system will move away from the initial equilibrium, toward
higher values of central current density and resistivity, until
it approaches solution II. A similar chain of arguments can be
used to show that solution II is stable: after their initial decrease,
$\Delta$ and hence $\delta$ will tend to increase (negative feedback),
thereby reducing the initial perturbation. (See Uzdensky 2002 for a more rigorous and detailed stability
analysis than the one presented here.)

Let us notice that, in practice, solution I is going to be
irrelevant for one more reason. Real astrophysical systems
always have a finite (albeit very large) size $L$. For the
Petschek model to work, one must have $\Delta \leq L$, which
imposes an upper limit on $\Delta$ and hence a lower limit on
$\eta(j_0)$. For $j_0 \in [j_0, j_0 + \Delta j]$, this condition can be cast as
\begin{equation}
\eta(j_0) > \eta_{\min}(L) \equiv \frac{\eta_0^2 V_0}{L}.
\end{equation}

If $\eta_{\min} > \eta_0$, then solution I does not even exist for a given
global size $L$. For typical solar corona conditions, collisional resistivity $\eta_0$ is so small that inequality (12)
is not satisfied; i.e., one gets a value for $\Delta$ much larger than
$L \approx 10^9$ cm.

Here is how the requirement $\eta(j_0) > \eta_{\min}(L)$ comes into
play from the evolutionary point of view. As a reconnection layer starts to form, its thickness decreases rapidly until it
reaches $\delta_0$; after that, $\delta$ and the central current density $j_0$
both stay approximately constant while $\eta(j_0)$ increases rap-

dily. When $\eta(j_0)$ reaches $\eta_{\min}$, the corresponding width $\Delta$ of the
diffusion region, determined by equation (2), becomes
equal to $L$. From this moment on, the reconnection system
can be described by the Petschek model. The particular
Petschek configuration found exactly at this moment (i.e.,
when $\eta = \eta_{\min}$ and hence $\Delta = L$) is that of Sweet-Parker. If
$\eta_{\min} > \eta_0$, then $j(j_0) > j_0$ and hence is very close to $j_0$; this
means that resistivity is very strongly localized, $L_0 \ll \Delta$.
Therefore, this configuration will not be at equilibrium, and the
system will evolve through a sequence of Petschek config-

urations with ever-increasing $j_0$ and $\eta(j_0)$. Each of these
configurations with $j_0$ on the rapidly rising part of the $\eta(j)$
curve will have $\Delta > \Delta_{\text{eq}} = \Delta L_0$ until the configuration corre-

csponding to the stable equilibrium solution II is reached.

Finally, let us remark that our model works only for
$\xi > 1$. The case $\xi = 1$ ($\Delta_{\text{eq}} = L_0$) is degenerate: in addition to
an unstable solution I, one gets a continuum of neutrally
stable solutions II that correspond to $j_0 \approx j_c$. By considering
a sequence of stable solutions II with $\xi > 1$ and taking
the limit $\xi \to 1$, one can break this degeneracy and arrive at
a single limiting solution (with $j_0 = j_c$). In the case $\xi < 1$,
however, our analysis does not work: no solutions of type II
exist, which suggests that $j_0$ and $\eta(j_0)$ will continuously
increase, whereas the reconnection process will continuously
accelerate until some new resistivity saturation mechanism sets in.

2.3. Reconnection Rate

Thus, a physical system with anomalous resistivity of the

type shown in Figure 1 will evolve toward a stable solution
II, i.e., a Petschek-like configuration with $\delta = \delta_c/\zeta$, central
current density $j_0 = \zeta j_c$, and central resistivity $\eta = \zeta \eta_0$.

According to our analysis, $\zeta = \varepsilon/\xi$, but in general, it is just a
finite constant, $\zeta = O(1)$, which we will leave unspecified
here.

What is the reconnection rate associated with this config-

uration? Using expression (2), we find the aspect ratio of the
central diffusion region:
\begin{equation}
\frac{\Delta}{\delta} = \frac{V_\Lambda \delta}{\eta(j_0)} = \frac{S_e}{\zeta^2},
\end{equation}
where we define
\begin{equation}
S_e = \frac{V_\Lambda \delta_c}{\eta_0}.
\end{equation}

The reconnection velocity is (assuming uniform density)
\begin{equation}
\frac{V_{\text{rec}}}{V_\Lambda} = \frac{\delta}{\Delta} = \zeta^2 S_e^{-1},
\end{equation}
and the typical reconnection timescale is
\begin{equation}
\tau_{\text{rec}} = \frac{L}{V_{\text{rec}}} = \tau_\Lambda(L) \frac{S_e}{\zeta^2},
\end{equation}
where $\tau_\Lambda(L) \equiv L/V_\Lambda$ is the global Alfvén crossing time.

Note that both the aspect ratio (eq. [13]) and the reconnec-
tion velocity (eq. [15]) turn out to be independent of the
global size $L$. Also note that our expression (15) for the
reconnection velocity differs from the result $V_{\text{rec}}/V_\Lambda \sim (\delta_c \eta_0^2/V_\Lambda L^2)^{1/3}$ obtained previously by Kulsrud (see eq. [26]
of Kulsrud 2001). This discrepancy can presumably be attributed to a somewhat different form of the $\eta(j)$
dependence.
3. PETSCHEK RECONNECTION IN THE PRESENCE OF ANOMALOUS RESISTIVITY DUE TO ION-ACOUSTIC TURBULENCE

In this section we assume that the anomalous resistivity is caused by the scattering of electrons by ion-acoustic waves that are excited when the current density exceeds the threshold for ion-acoustic instability. The theory of anomalous resistivity due to IAT has been developed during the past 40 years (e.g., Kadomtsev 1965; Rudakov & Korabiev 1966; Sagdeev 1967; Kaplan & Tsytovich 1973; Biskamp & Chodura 1973; Coroniti & Eviatar 1977; Bychenkov et al. 1988). While in the case of a homogeneous unmagnetized plasma the theory seems to be very mature and capable of producing reliable quantitative results regarding anomalous resistivity, the situation is not as clear in the presence of a magnetic field. A stationary anomalous resistivity may not be useful expression, along with equation (20), into our equation (13) for \( \Lambda \), we find

\[
\Lambda \simeq \frac{a_4}{\zeta^3} \frac{j}{m_i \gamma} \frac{T_i}{Z_{me} \gamma} \frac{V_{A}}{m_i \gamma} ,
\]

where \( a_4 \equiv 2/a_1^2a_2 \simeq 7.3 \).

We also get an expression for the Alfvén crossing time for the diffusion region—one of the most important timescales in the problem:

\[
\tau_A(\Lambda) \simeq \frac{\Lambda}{V_A} \simeq \frac{a_4}{\zeta^3} \frac{1}{\omega_p^2 \beta_e^2} \frac{T_i}{m_i} ,
\]

In addition, substituting equation (20) into equation (16) for the reconnection time \( \tau_{rec} \), we find a very simple relationship expressing \( \tau_{rec} \) in terms of the light crossing time \( L/c \) and \( \beta_e \):

\[
\tau_{rec} \simeq \frac{a_3}{\zeta^3} \frac{L}{T_i} \frac{m_i}{Z_{me} T_e} .
\]

For a pure hydrogen plasma (\( Z = 1, m_i = m_p = 1836m_e \)), we get

\[
S_* \simeq 2 \times 10^4 \frac{V_A}{c \sqrt{\beta_e}} \frac{T_i}{T_e} ,
\]

\[
\Lambda \simeq 1.3 \times 10^4 \frac{a_2}{\zeta^3} \frac{T_i}{T_e} \frac{V_A}{c \sqrt{\beta_e}} ,
\]

\[
\tau_A(\Lambda) \simeq 1.3 \times 10^4 \frac{a_2}{\zeta^3} \frac{1}{\omega_p^2 \beta_e^2} \frac{T_i}{T_e} ,
\]

\[
\tau_{rec} \simeq 2 \times 10^4 \frac{L}{T_i} \frac{m_i}{Z_{me} T_e} .
\]

Note that in all these expressions \( n_e \) and \( T_e \) are to be taken at the center of the reconnection layer (\( x = y = 0 \)), while the magnetic field \( B_0 \) is the reconnecting magnetic field outside the layer, at \( x > \delta, y = 0 \).

4. APPLICATION TO SOLAR FLARES

Let us now try to apply the above results to solar flare conditions. First, we discuss the relevant plasma parameters (see also Uzdensky 2002). Table 1 lists the values of the key parameters for two sets of conditions (both are for fully ionized hydrogen plasma). The first set (third column) illustrates the fiducial solar coronal conditions: \( B_0 = 100 \) G, \( n_e = 10^8 \) cm\(^{-3} \), and \( T_e = T_i = 2 \times 10^6 \) K \( \simeq 200 \) eV. We see that the characteristic reconnection time turns out to be of order \( 10^4 \) s, somewhat longer than the typical flare duration. However, the parameters used in the third column might not be appropriate for the center of the flare reconnection layer. Indeed, one can expect that the turbulence will lead to rapid heating of the plasma, resulting in a substantial rise of the electron temperature (Sagdeev 1967; Biskamp & Chodura 1973; Bychenkov et al. 1988; Kingsep 1991). (The fact that electrons are heated more efficiently than ions leads to \( T_e \gg T_i \) and thus creates conditions for the conversion of an initial Buneman [1959] instability into the ion-acoustic regime [Roussev, Galgaard, & Judge 2002] and the development of the IAT.)

In general, the central electron temperature is going to be determined by the balance between the turbulent ohmic heating and the cooling due to electron thermal
TABLE 1
VALUES OF SOME BASIC PLASMA AND RECONNECTION REGION PARAMETERS IN THE TYPICAL CORONAL ENVIRONMENT AND IN THE FLARE ENVIRONMENT

| Parameter | Expression | Typical Coronal Value | Typical Flare Value |
|-----------|------------|-----------------------|---------------------|
| $T_e$ (K) | $T_e$       | $2 \times 10^6$        | $3.3 \times 10^6$    |
| $T_i$ (K) | $T_i$       | $2 \times 10^6$        | $3.3 \times 10^6$    |
| $n_e$ (cm$^{-3}$) | $n_e$       | $1 \times 10^9$        | $1 \times 10^{10}$   |
| $B_0$ (G) | $B_0$       | 100                   | 100                 |
| $v_i$ (cm s$^{-1}$) | $\sqrt{T_e/m_e}$ | $1.3 \times 10^7$      | $5 \times 10^7$      |
| $V_A$ (cm s$^{-1}$) | $B_0/\sqrt{4\pi n_e m_e}$ | $6.9 \times 10^8$     | $2.2 \times 10^8$     |
| $\omega_{pe}$ (s$^{-1}$) | $\sqrt{4\pi n_e e^2/m_e}$ | $1.8 \times 10^9$     | $5.6 \times 10^9$    |
| $\omega_{pi}$ (s$^{-1}$) | $\sqrt{4\pi n_e m_e/e^2}$ | $4.2 \times 10^7$     | $1.3 \times 10^8$    |
| $\Omega_\parallel$ (s$^{-1}$) | $eB_0/m_e c$ | $1.8 \times 10^9$     | $1.8 \times 10^9$    |
| $\Omega_\perp$ (s$^{-1}$) | $zeB_0/m_e c$ | $9.6 \times 10^5$     | $9.6 \times 10^5$    |
| $\lambda_{B0}$ (cm) | $T_e/4\pi n_e e^2$ | 0.31                  | 0.38                 |
| $\lambda_{B0}$ (cm) | $\sqrt{T_i/4\pi n_e m_e}$ | 0.31                  | 0.12                 |
| $d_e$ (cm) | $c/\omega_{pe}$ | 17                   | 5.3                 |
| $d_i$ (cm) | $c/\omega_{pi}$ | 720                  | 230                 |
| $\lambda_e$ (cm) | $2.14 n_e e v_s$ | $1.3 \times 10^7$    | $5.1 \times 10^9$    |
| $\lambda_i$ (cm) | $eB_0/4\pi e^2 n_e$ | $1.8 \times 10^4$   | 470                 |
| $\eta_e$ (cm$^2$ s$^{-1}$) | $1.4 \times 10^{-3} (T_e/T_i) e^2/\omega_{pe}$ | $7.1 \times 10^8$ | $2.2 \times 10^9$    |
| $\beta_e$ | $8\pi n_e T_e/B_0^2$ | $7. \times 10^{-4}$ | 0.10               |
| $S_e$ | $2 \times 10^3 (T_e/T_i) V_A/e\sqrt{\beta_e}$ | $1.7 \times 10^4$ | 45                 |
| $\Delta$ (cm) | $\sqrt{eS_e \lambda_e}$ | $4 \times 10^7$       | $2.6 \times 10^3$    |
| $\tau_e$ (s) | $\Delta/V_A$ | 0.057                 | $1.2 \times 10^{-5}$ |
| $\tau_{rec}$ (s) | $V_{rec}/c$ | $1.6 \times 10^7$     | $1.9 \times 10^7$    |
| $E$ (e) | $4\eta_e^2 n_e e^4/\eta_e c^2$ | $5.2 \times 10^{-4}$ | $6.4 \times 10^{-2}$ |
| $\tau_{rec}$ (s) | $L_i/V_{rec}$ | $6.3 \times 10^3$     | 52                  |

Notes.—All values are calculated for pure hydrogen plasma ($Z = 1$, $m_i = m_e$) and for $\zeta = 2.0$ ($\zeta = e/\zeta \approx 1.36$); $\tau_{rec}$ is calculated for $L = 10^6$ cm.

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2 Radiative cooling of the diffusion region appears to be ineffective in the context of solar flares, since the radiative cooling time greatly exceeds the time $\tau_A(\Delta)$ that a fluid element spends inside the diffusion region.

Then, equating $\tau_{min}^cool$ and $\tau_{heat}$, we get a lower bound on $\beta_e$:

$$\beta_{e,\text{min}} \sim \left( \frac{m_i}{m_e} \right)^{1/3} \ll 1 .$$ (31)

This regime is illustrated in the fourth column of Table 1. Here we adopt the following “fiducial solar flare conditions”: $B_0 = 100$ G, $n_e = 10^{10}$ cm$^{-3}$, $T_e = 3 \times 10^7$ K $\approx 3000$ eV, and $T_i = 3 \times 10^6$ K $\approx 300$ eV, which correspond to $\beta_e \approx 0.1$. The resulting reconnection timescale is $\tau_{rec} \approx 10^2$ s, which is fast enough to explain the observed very short duration of the impulsive phase of solar flares.

Thus, we can constrain $\beta_e$ to lie between $\beta_{e,\text{min}} \sim 10^{-1}$ and $\beta_{e,\text{max}} = O(1)$. We then see from equation (21) that, even in the case of the lowest possible $\beta_e$, the thickness $\delta$ of the diffusion region is close to $d_e$. This suggests that the Hall-MHD regime of reconnection (see, e.g., Drake, Kleva, & Mandt 1994; Biskamp 1997; Bhattacharjee, Ma, & Wang 2001) may be very relevant in solar flare physics.

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3 In this scenario, thermal pressure cannot contribute significantly to the pressure balance across the reconnection layer, and so a relatively strong axial magnetic field $B_z$ needs to be present to provide the necessary pressure support: $B_{z,\text{inside}}^2 - B_{z,\text{outside}}^2 = B_0^2$. 
5. CONCLUSIONS

In this paper we have presented a simple model of magnetic reconnection in the presence of a current-driven anomalous resistivity and obtained specific predictions for the scaling of the reconnection rate with various plasma parameters. In this model, we have combined the following three ingredients: (1) a simulation-based observation that a reconnecting system with a strongly localized resistivity develops a Petschek-like configuration with the width of the inner diffusion region of the order of the resistivity localization scale; (2) the Sweet-Parker model for the Petschek diffusion region; and (3) a physically motivated (by the theory of ion-acoustic turbulence) model for a current-driven anomalous resistivity $\eta(j)$ exhibiting a sudden increase of $\eta$ up to a very large value $\eta_1$ at a certain threshold $j = j_1$, followed by a subsequent linear growth $\eta \propto j$ for $j > j_1$. This very strong sensitivity of $\eta(j)$ near the threshold makes a highly localized enhancement of the resistivity possible and thus leads to a Petschek-like configuration. Thus, reconnection is accelerated by a combined action of anomalous resistivity and of the Petschek mechanism. The role of anomalous resistivity in this process is twofold: in addition to its direct action (lowering the global Lundquist number), it accelerates reconnection by turning on the Petschek mechanism.

Out of two possible Petschek-like solutions in our model, only one is stable. The central current density $j_0$ and the central resistivity $\eta(j_0)$ of this stable Petschek-like configuration exceed $j_1$ and $\eta_1$, respectively, by a finite factor of order 1; the reconnection velocity scales as $V_{\text{rec}} \sim \eta_1/\delta_c$, where $\delta_c = cB_0/4\pi j_0$ is the critical thickness of the layer.

In § 3 we considered a specific example of anomalous resistivity due to IAT and derived some very simple expressions for the reconnecting system parameters in terms of the basic plasma parameters $\eta_c$, $T_e$, $T_i$, and $B_0$. Finally, in § 4 we applied our model to the solar flare environment, taking into account the significant electron heating that is expected at the center of the reconnection layer. We obtained typical reconnection times of order $10^5$ s, short enough to explain the very fast timescale of impulsive flares. We note, however, that the heating inside the diffusion region may quickly shrink its thickness down to the ion skin-depth, $d_i \equiv c/\omega_{pi}$; then, new physical processes (described by Hall MHD) may become important.

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