HOW TO PUT A HEAVIER HIGGS ON THE LATTICE *

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The cutoff dependence of the Scalar Sector of the Minimal Standard Model can result in an increase of the existing triviality bound estimates of the Higgs mass. We present a large \(N\) calculation and some preliminary \(N = 4\) results that suggest that the increase can be as large as 30%, resulting to a bound of about 850 GeV.

Investigation of the triviality of the \(\lambda(\Phi^2)^2\) theory on the lattice has resulted in an upper bound for the Higgs mass of about 590 – 640 GeV. A Higgs with mass of about 640 GeV will have a width of about 130 GeV and it should be possible to observe experimentally. On the other hand if the Higgs is heavier than about 820 GeV its width will be larger than about 275 GeV (larger than a third of its mass) and it will be hard to observe experimentally. Recall that in a technicolor scenario the Higgs is a very wide enhancement, analogous to the "\(\sigma\)-particle" in QCD, centered somewhere in the range 1 TeV–2 TeV. It is therefore very important to ask if a lattice action can be found that will produce a heavier Higgs.

The Scalar Sector of the Minimal Standard Model is an effective theory that describes the physics for energies less than some cutoff \(\Lambda\). The leading cutoff effects (order \(\Lambda^{-2}\)) can be parametrized by adding to the \(\lambda(\Phi^2)^2\) theory two bare dimension six operators with freely adjustable coefficients. Recall that in a technicolor scenario the Higgs is a very wide enhancement, analogous to the "\(\sigma\)-particle" in QCD, centered somewhere in the range 1 TeV–2 TeV. It is therefore very important to ask if a lattice action can be found that will produce a heavier Higgs.

The action is:

\[
S = \int_x \left[ \frac{1}{2}\Phi_E g(-\partial^2)\Phi_E - \frac{b_1}{2N}(\partial_{\mu}\Phi_E \cdot \partial_{\mu}\Phi_E)^2 - \frac{b_2}{2N}(\partial_{\mu}\Phi_E \cdot \partial_{\nu}\Phi_E - \delta_{\mu\nu}\partial_{\sigma}\Phi_E \cdot \partial_{\sigma}\Phi_E)^2 \right]
\]

where \(\Phi_E g(-\partial^2)\Phi_E\) is a regularized kinetic en-
ergy term, $\Phi_c^2 = N \beta$, and the partition function is $Z = \int d\Phi e^{-S}$. We have investigated this action to leading order at large $N$ with Pauli Villars regularization and have found that the phase diagram does not depend on $b_2$ (for classical fields the term is not Lorentz invariant), and also the ratio of the Higgs mass to the weak scale $f_\pi$ ($f_\pi = 246$ GeV) does not depend on $b_2$ to leading order in $m^2_R$, where $m_R$ is the renormalized mass. Under these approximations we can set $b_2 = 0$ and we are left with a two parameter space to search. This we have done, but since these are the proceedings of a lattice conference we now turn our attention to the lattice regularization.

The lattice best suited for the study of leading cutoff effects ($\Lambda^{-2}$) is the $F_4$ lattice since it does not introduce Euclidean $O(4)$ violations by lattice artifacts to that order [3] (as the more commonly used hypercubic lattice does). Denoting sites on $F_4$ by $x, x', x''$, and links by $<x,x'>$, $l, l'$, the action is:

$$S = -2N \beta_0 \sum_{<x,x'>} \Phi(x) \cdot \Phi(x') - N \beta_1 \sum_{<x,x'>} [\Phi(x) \cdot \Phi(x')]^2 - N \frac{\beta_2}{48} \sqrt{6} N \sum_x \left( \sum_{l \cap x \neq \emptyset} \Phi(x) \cdot \Phi(x') \right)^2.$$  \hspace{1cm} (2)

Here the field is constrained by $\Phi^2(x) = 1$. To obtain the action in eq. (1) above, the field has to be rescaled $\Phi_c = \sqrt{6} N (\beta_0 + \beta_1 + \beta_2) \Phi$ (we only consider the region $\beta_0 + \beta_1 + \beta_2 > 0$). From the relation of the parameters with those of action (1) we find that the region of interest $b_2 = 0$ corresponds to $\beta_1 = 0$. We then are left to search the two parameter space $\beta_0, \beta_2$. We do that using the large $N$ saddle point approximation and what follows from now on is done in that approximation unless otherwise noted.

The phase diagram is given in fig. 1. The second order line terminates at a tricritical point (TCP) where a first order line begins. The phase diagram obtained numerically for the physical $N = 4$ looks very similar to the one in fig. 1 and it has an approximately straight second order line [3].

The renormalized mass (defined from the smallest positive zero of the real part of the determinand of the matrix that appears in the quadratic term at the end of the calculation) is a good approximation to the Higgs mass in the perturbative regime ($f_\pi^2 \sim p^2 \sim m^2_R << 1$). We find to leading order in $m^2_R$:

$$m_R = C(\beta_2) \exp \left[ -16\pi^2 \frac{f_\pi^2}{Nm^2_R} \right]$$  \hspace{1cm} (3)

with

$$C(\beta_2) = \exp \left[ 8\pi^2 c_1 - \frac{16\pi^2 r_0^2}{1 - \frac{4\pi^2}{3r_0^2}} \right]$$  \hspace{1cm} (4)

where $m_R$ is in lattice units, $r_0$ is a constant equal to the momentum integral of the inverse ki-
netic energy term and $c_1$ is a constant that comes from the “bubble” integral. Both constants have been calculated in \[3\]. By demanding $\beta_2$ to be able to stay arbitrarily close to the critical surface and at the same time keep $\beta_0 + \beta_2$ positive we find that $\beta_2$ has to be larger than $-\frac{4}{3}r_0^2$ which is the point where the second order line cuts the $x$ axis in fig.1. As a result we find that

$$\frac{C(0)}{C(-\frac{4}{3}r_0^2)} = \exp\left[8\pi^2 r_0^2\right] = 4.521 \quad (5)$$

To check the sensitivity of keeping only leading order in $m_R^2$ we included the next order (still neglecting $\beta_1$). The effect was less than a 1% -- 2% correction on $m_R/f_\pi$ even for $m_R$ close to 1. Of course one does not expect the large $N$ calculation to give good results for $N = 4$. We are however only interested in relative changes. To that end, as a check, we compare $m_R/f_\pi$ from the $N = 4$ numerical work of \[3\] with our large $N$ result for the case $\beta_1 = \beta_2 = 0$. Their relative difference is about 25% but it remains basically constant (within errors) for $m_R$ as large as 1. If we restrict the cutoff effects to the differential cross section of $\pi^-\pi^+$ scattering at right angles with energies up to $3m_R$ to be less than 10%, then we have calculated that $m_R$ must stay less than about $\sqrt{2}/2$ for the range of interest $-\frac{4}{3}r_0^2 < \beta_2 < 0$. In fig. 2 we plot $m_R/f_\pi$ vs. $m_R$ (in lattice units). At $m_R = \sqrt{2}/2$ the percent increase of $m_R/f_\pi$ from $\beta_2 = 0$ to $\beta_2 = -\frac{4}{3}r_0^2$ is about 30% which brings the Higgs mass bound to $\approx 850$ GeV.

Preliminary numerical work for the physical case, $N = 4$, \[3\] shows that at $m_R$ around 0.55 the corresponding increase is about 23% within errors (from fig. 2 the large $N$ calculation gives an increase of about 27%).

In conclusion, large $N$ calculations suggest that the cutoff dependence of the effective theory (Scalar Sector of Minimal Standard Model) can increase the triviality bound by $\approx 30\%$. A Higgs of that mass will have a width of $\approx 300$ GeV which is more than a third of its mass. If it turns out that this conclusion holds also for $N = 4$, as our preliminary results indicate, it will be hard to observe the Higgs experimentally, should nature choose to saturate the bound.

References

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