State permutations from manipulation of near level-crossings.

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We discuss some systematic methods for implementing state manipulations in systems formally similar to chains of a few spins with nearest-neighbor interactions, arranged such that there are strong and weak scales of coupling links. States are permuted by means of bias potentials applied to a few selected sites. This generic structure is then related to an atoms-in-a-cavity model that has been proposed in the literature as a way of achieving a decoherence free subspace. A new method using adiabatically varying laser detuning to implement a CNOT gate in this model is proposed.

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I. INTRODUCTION

Two central concerns of quantum information theory are representation of data through occupancy of quantum states and the manipulation of these data. Roughly speaking, for these purposes, we desire systems that have the two properties:

1) When the system is left alone it should have some eigenstates that are quite localized, in the sense that the occupancy of such a state identifies a property of a localized physical element (e.g., a particular spin being up or down, or a state of a single atom being in one energy level or the other.

2) There is a simple mechanics for applying a signal from the outside that will move the system from one of these “localized” states to another.

In this paper we discuss two models that show some interesting potentialities for this program. The first is a novel and rather idealized “spin chain” type of model; the second is an “atoms-in-a-cavity” model, still quite idealized, that has been discussed at some length in several previous works by other authors.

All of the models that we shall consider (and some others in the literature) have a unifying feature that can be stated as a nearly-general property of a class of Hamiltonians. We define this class as follows:

a) We picture the states as points distributed in a space, and connect each pair of points that correspond to a non-vanishing matrix element of $H$. We consider only cases in which each point is connected to a few neighbors. A natural extension that will be required for the atoms-in-a-cavity discussion is to include the “environment” as one of the points in the space, with dissipative links to it from some of the other states.

b) Now let there be two scales of strength in these connections, strong (S) and weak (W). If there are small neighborhoods, or blocs, in this space in which all connections, as well as the diagonal elements, are weak, then there will generally be energy eigenfunctions that are nearly confined to these blocs. This can be seen by treating the weak links to the outside perturbatively after diagonalizing the weak bloc in the absence of connections to the outside. Since the denominators in this perturbation are generally of strong order and the numerators of weak order, the perturbation will make little difference to the state. We can characterize the perturbation of the state as being of order W/S. Likewise, we note that although other weak blocs somewhere else in the space of states will indeed have energies of weak scale, we are in general protected from a small denominator by the fact that we must go through a weak link to get from the first bloc into an intervening strong bloc, and then another weak link to get to the second weak bloc, where we would find energies of the weak order characterizing the first bloc. Thus the largest term with a dangerous denominator would generally be of order W^2/SW. (The dissipative connections to the environment in the atoms-in-a-cavity model to be considered later will come to be classified as, effectively, a set of additional strong links.)

The above statement is, to be sure, neither a theorem nor an unfamiliar kind of conclusion. In much more sophisticated work on infinite systems much is known about the relations between disorder and localization. But we will show that the result above, tempered as it is with “generally” (meaning “most of the time”) is a very useful design criterion for systems that meet the objectives enunciated at the beginning of this paper.

We will begin by considering cases where the strong bloc consists of two states connected by a single strong link, such that the Hamiltonian is of the form

$$
\begin{pmatrix}
0 & S \\
S & 0
\end{pmatrix},
$$

(1)

and the eigenvalues of the two energy eigensates are $\pm S$. Adding weak scale connections to other states, will make
only a small perturbation to these eigenstates. We may
generalise this to chains of an arbitrary even number of
states connected by strong links of similar size – all the
eigenvalues will be of order strong. However, if we have
an odd number of states, there will always be a zero eigen-
value, for example, three states connected via a Hamilton-
ian of the form
\[
\begin{pmatrix}
0 & S_{12} & 0 \\
S_{12} & 0 & S_{23} \\
0 & S_{23} & 0
\end{pmatrix},
\]
will have eigenvalues \(\{0, \pm \sqrt{S_{12}^2 + S_{23}^2}\}\). Due to the pre-
sence of a zero eigenvalue, adding weak scale links to
states outside of this bloc will greatly perturb the eigen-
states. We regain of a set of eigenvalues, all of strong
order, however, if we introduce either a coupling \(S_{13}\) be-
tween the 1st and the 3rd states, or diagonal elements of
strong scale. Dissipative connections, for example, would
enter as imaginary diagonal terms.

Once we have reason to take a weak bloc as isolated, we
then wish to address the movements of probability
through the action of time dependent external interven-
tions. In the first chain-of-spins model that we shall
present, with five sites, this will take the form of a bias
signal to be applied to the middle site, which is changed
slowly enough for adiabaticity of a kind to be realized.
The actual state transformations that we discuss can be
regarded as fairly elaborate “avoided-crossing” phenom-
ena. In the atoms-in-a-cavity models of Refs. 2, 3, and
4 that we consider, after noting how the creation therein of
decoherence-free subspace can be qualitatively un-
derstood using the strong-weak picture, we demonstrate
an adiabatic method of manipulation by tuning and detun-
ing that is different from the pulsed, and exactly-
timed, laser techniques that are suggested in these refer-
ences.

The remainder of this paper is structured thus: The
next section analyses spin-chain models with a W and
S hierarchy of nearest-neighbor couplings and time-
dependent biases on selected sites. Section III then re-
examines atoms-in-a-cavity models, while Sec. IV pro-
vides further discussion and a conclusion.

II. SPIN-CHAIN MODELS

Our first example uses a chain of five spins. We take
the spins to be numbered consecutively from left to right,
with pure exchange interactions built of the operators
between adjacent spins \(i, j\),
\[
h^{(i,j)} = \sigma_+^{(i)} \sigma_-^{(j)} + \sigma_-^{(i)} \sigma_+^{(j)}.
\]
We choose the Hamiltonian
\[
H = g_1 h^{(1,2)} + \lambda h^{(2,3)} + \lambda h^{(3,4)} + g_2 h^{(4,5)} + f(t) [\sigma_z^{(3)} - 1],
\]
where we have added a single time-dependent term in-
volving the operator of the middle spin \(\sigma_z^{(3)}\). We refer
to the function \(f(t)\) as the bias on site \#3. Since this
Hamiltonian commutes with the operator \(\sum_i \sigma_z^{(i)}\), we
can operate within the set of five states with four of the spins
up and one spin down. The S-W structure comes from
taking \(\lambda \ll g_1, g_2\), and \(g_1 \neq g_2\). When \(f(t) = 0\), this
choice has the effect of creating eigenstates of \(H\) that are
very nearly the following,
\[
\begin{align*}
\psi_1 &= (\downarrow \downarrow \downarrow) (\uparrow \uparrow \uparrow)/\sqrt{2}, \\
\psi_2 &= (\uparrow \uparrow \downarrow) (\uparrow \uparrow \uparrow)/\sqrt{2}, \\
\psi_3 &= \langle \uparrow \uparrow \downarrow \downarrow \downarrow \downarrow \downarrow \rangle, \\
\psi_4 &= \langle \downarrow \uparrow \downarrow \uparrow \rangle (\downarrow \downarrow \uparrow)/\sqrt{2}, \\
\psi_5 &= \langle \uparrow \uparrow \downarrow \downarrow \downarrow \rangle (\downarrow \downarrow \downarrow)/\sqrt{2}.
\end{align*}
\]
We understand these combinations qualitatively by not-
ing that they would be the exact eigenstates if the small
coupling \(\lambda\) were set to zero, and that, treating \(\lambda\) as a perturbation, the link between the pairs of states
\((\#1,\#2)\) and \((\#4,\#5)\) is of order \(\lambda^2\).

Now we ask the question: Of the 120 permutations on
this set of five states, how many can we implement by
applying a small series of simple pulses in \(f(t)\)? In form-
ing these pulses we shall tailor \(f(t)\) to the permutation
that is sought, but we shall insist that \(f(t)\) begins at and
ends at the value zero. By a permutation we mean just
the reshuffling of the states, modulo phase. This demand
effectively rules out setting relative phases, which would
require fine-tuning in any system that is to be considered
over a period of time. The answer to the question is “all
120”.

The method uses adiabatic avoided-level-crossing dy-
namics, with an additional feature that can be embodied
in the above Hamiltonian, namely, that the bias on the
weakly coupled site can be changed suddenly, without
affecting the state of the system appreciably, at all times
when the system is far from any (near) level-crossing. As
an example, we choose the parameters \(g_1 = 30, g_2 = 60,
\lambda = 1\), and begin by defining two bias operations \(f(t)\),
\[
\begin{align*}
f_a(t) &= (t - t_0) \theta(t - t_0) \theta(t_o + \tau - t) \rightarrow U_a(t_0), \\
f_b(t) &= -f_a(t) \rightarrow U_b(t_0).
\end{align*}
\]
The signal begins at \(t = t_0\), grows linearly until \(t = t_0 + \tau\)
when it is switched off abruptly. We take \(\tau = 20\) in
the applications that follow. Any bias \(f_a(t)\) defines a trans-
formation,
\[
U_a(t_0) = T[\exp(-i \int_{t_0}^{t_0 + \tau} dt' H(t'))].
\]
We will show [5] that the operation \(U_a\) effects the per-
mutation \((13)\), while \(U_b\) effects \((23)\). The turn-off of the
bias at \(t = t_0 + \tau\) leaves the state at time \(t = t_0 + \tau\)
evolving with the Hamiltonian of Eq. 3 with \(f = 0\), so
that we are prepared to apply another signal to get a
remains there during the sudden bias jump at state #1, then it first adiabatically transforms to state #3, one of these states, follows the continuous path connected thereto. For example, if the system is initially prepared in state #1, then it first adiabatically transforms to state #3, remains there during the sudden bias jump at \( t = 40 \), then adiabatically transforms to state #2. States #4 and #5 remain virtually unmixed and at their original energies. The energy is in units of \( \lambda \) and the time in units of \( \lambda^{-1} \).

The Hamiltonian of Eq. (4) with the bias function given by, Figure 1 shows plots of the energy levels for the first three states as they evolve under the above successive changes. The plots show clearly the effects of each of the constituent transitions in turn, giving rise to the factorization indicated in Eq. (8).

To show that the states really follow these paths, we construct the matrix \( U(t) = T[\exp(-i \int_0^t dt' H(t'))] \) by directly solving the Schrödinger equation with the time-dependent bias of Eq. (4). In Figs. 2 and 3, we plot \(|(1, 2, 3|U(t)|1)|^2 \) and \(|(1, 2, 3|U(t)|2)|^2 \) against time in this solution, giving the expected behavior, \( |U(3\tau)|^2 = |U_{1\to 2}|^2 \) with negligible contamination either from non-adiabaticity in the region of small level separation, or from transitions induced in any of the three sudden changes. The plots clearly show the effects of each of the constituent transitions in turn, giving rise to the factorization indicated in Eq. (8).

To implement the transformations involving states #4 and #5, we introduce four more bias functions,

\[
f_d(t) = -f_c(t) \to U_d \to \langle 341 \rangle,
\]

\[
f_c(t) = -2(\tau - t/2)\theta(\tau - t)\theta(t) \to U_c \to \langle 34 \rangle,
\]

\[
f_f(t) = -f_e(t) \to U_f \to \langle 35 \rangle,
\]

where, for simplicity, we have chosen \( t_0 = 0 \). By straightforward multiplication of the operations in Eqs. (7) and (10), we obtain \( U_d U_a \to \langle 14 \rangle, U_c U_b \to \langle 25 \rangle, U_a U_f U_a \to \langle 15 \rangle, U_b U_c U_b \to \langle 24 \rangle, U_c U_f U_c \to \langle 45 \rangle \). These five composite operations, together with \( U_d U_b U_a \to \langle 12 \rangle \) and the four single pulse operations \( U_a, U_b, U_c, U_f \), give all of the simple interchanges. All permutations can be built from these interchanges, although in most cases it would be more efficient to draw on the cycle-of-three permutation opera-
operators $U_c$ and $U_d$ as well, or on further primary pulse
shape variants that directly embody other operators.

Now we consider longer chains with nearest neighbor
couplings,

$$H_0 = \sum_{i=j+1} g_{ij} h^{(ij)} - E_0,$$

(11)

where $g_{ij}$ are coupling coefficients, and $E_0$ is the
energy that gives $\langle H_0 \rangle = 0$ for the case of all spins up.
We again adopt two scales of coupling strength: $S$ and
$W$. As an example, we consider the pattern of couplings
$\{ g_{1,2}, g_{2,3}, g_{3,4}, \ldots \}$ to be $\{ S, W, W, S, W, W, W, \ldots \}$. Under this scheme, the states #3,#6,#9... are
only weakly coupled to their neighbors.

As before, we take only states with one spin down and
the remainder up. With the $W$ coupling constants turned
toff, the eigenvalues of the states in which the down spin
occupies one of the large blocs, i.e., $\{ (\#1,\#2), (\#4,\#5),
(\#7,\#8), \ldots \}$, come in pairs of $\pm g_{i,i+1}$. We assume that
the $S$ couplings are sufficiently irregular so that the
differences in energies between nearest strong blocs, e.g.,
g$_{1,2}$–$g_{4,5}$, are still of the strong ($S$) scale. When we add
the $W$ couplings, the eigenvectors will remain most al-
local, as can be seen from a perturbation expansion.
In general, as argued previously, small energy differences
between two non-adjacent blocs are not a concern, since
as we move across $n$ additional weak links we can toler-
ate energy denominators that are smaller by a factor of
$(\omega/W)^{2n} \ll 1$.

Thus for most cases the eigenvectors can be ar-
 ranged in a list $\{ (\xi_1,\uparrow,\uparrow,\ldots), (\eta_1,\uparrow,\uparrow,\ldots),
(\uparrow,\uparrow,\downarrow,\uparrow,\ldots), (\uparrow,\downarrow,\uparrow,\uparrow,\ldots),
(\uparrow,\uparrow,\uparrow,\xi_2,\uparrow,\ldots), (\uparrow,\uparrow,\uparrow,\eta_2,\uparrow,\ldots),
(\uparrow,\uparrow,\uparrow,\uparrow,\uparrow,\uparrow,\ldots), \ldots \}$. Here, the $\xi$’s and $\eta$’s stand for the sym-
metric and anti-symmetric two-component eigenstates of
the strong blocs with one spin up and the other down. If
the system begins with all of its amplitude in one bloc, the
amplitude will stay almost within that bloc under the
evolution governed by $H_0$.

Now, generalizing the earlier five-site example, we can
ask whether biases placed at every third site, #3, #6,
#9, ... , which are weakly coupled both to the right and
to the left, have the capability of moving information
around the whole system. We have looked at an eight-
site example, with independently manipulable biases at
sites #3 and #6. In complete analogy with the five-
site case, we find it is easy, for example, to move one of
the $(\#1,\#2)$ eigenstates successively to the blocs $(\#3),
(\#4,\#5), (\#6), (\#7,\#8)$, again by numerically solving
the Schrödinger equation for the system.

As a qualitative summary of these outcomes, we restate
and sharpen some of the remarks in the introduction:

(1) If we have a chain of blocs of states connected, one
to another, through an intermediary state that is weakly
coupled to both blocs, the dynamics within the blocs will
be nearly self-contained. That is, if the probability is lo-
calized within a particular bloc at a particular time, in
the form of any superposition of the eigenstates belong-
ing to that bloc, then the probability will remain in that
bloc. Likewise, a bloc of several states in which the mu-
tual couplings and the diagonal elements are weak will,
in general, not admix appreciably with strongly coupled
states to its left or right.

(2) By putting controllable biases on a weak connection
site to bring the energy of an associated level to (nearly)
coincide with a level in an adjacent strong bloc, shifts of
probability from bloc to bloc can be implemented in an
orderly and complete fashion.

III. ATOMS-IN-A-CAVITY MODELS

An exact analogue to the above mechanics for moving
a system from one state to another can be applied to
some models of atoms-in-a-cavity type that have been the
subject of a series of recent papers. As an example, we
consider the model of Ref.[2], with two identical atoms
in a cavity, and a dynamics that is effectively confined
to three states, which form a decoherence free subspace
(DFS).

In this model, the two atoms have three levels, $\{0,1,2\}$,
and are placed in a cavity tuned exactly to the 1–2 level
spacing, with the atoms separately addressable by weak
laser fields that drive Rabi oscillations. The cavity mode,
if excited, is also allowed to escape through conversion
to photons at a partially transmitting wall. This is the
only decohering process. The connection scheme for the
relevant states is depicted in Fig. 4:

The excitation of the cavity mode from the states in
the chain that are coupled thereto are the S links, denoted
by fat double arrows. The thin double arrows are W links,
and the double arrows leading to D represent the cavity
mode leakage to the outside. All W links come from the
laser interactions, which are three-fold: on atom #2, an

\[
\begin{align*}
D & \leftrightarrow |1,1\rangle_1 \leftrightarrow |s\rangle_0 \\
D & \leftrightarrow |1,0\rangle_1 \leftrightarrow |2,0\rangle_0 \leftrightarrow |1,0\rangle_0 \\
D & \leftrightarrow |s\rangle_1 \leftrightarrow |2,2\rangle_0 \leftrightarrow |d\rangle_0 \\
D & \leftrightarrow |1,1\rangle_2 \leftrightarrow |1,1\rangle_0
\end{align*}
\]

FIG. 4: In a notation close to that of Ref. [2], the states are
labeled as $|\text{atom } #1, \text{atom } #2\rangle_{\text{cav}}$ where “cav” is the excita-
tion of the cavity mode. The states denoted “s” and “a” (for
the atomic part) are $|s\rangle = (|1,2\rangle \pm |2,1\rangle)/\sqrt{2}$. The connec-
tions are those variously induced by the three laser couplings
and the cavity mode, as explained in the text.
with the five states reduced to three by taking only the
small amount, but allowing the laser frequencies to be detuned by a
rather than turned on and off. The applied laser fields are slowly changed in frequency,
in the first example in this paper. In this mechanism
states, following an exact analogue of the protocol used
exchanged through imaginary diagonal terms in the Hamil-
has a zero eigenvalue. With the dissipative links, $D$, in-
cluded through imaginary diagonal terms in the Hamiltonian matrix, all three eigenvalues are of “strong” level,
and are thus approximately decoupled from the three
states $\{|1, 1\rangle_0, |a\rangle_0, |1, 0\rangle_0\}$ which form a DFS, as empha-
sized in Ref. [2].

Ref. [2] proceeds from this observation to the construc-
tion of a CNOT gate in which the transformation of states is effected by means of accurately timed laser pulses. We now demonstrate the action of an alterna-
tive mechanism for the controlled transformation of the states, following an exact analogue of the protocol used
in the first example in this paper. In this mechanism the applied laser fields are slowly changed in frequency,
rather than turned on and off.

To show this, we first write an effective Hamiltonian operating within the bloc of Eq. (12), as given in Ref. [2],
but allowing the laser frequencies to be detuned by a small amount,

$$H_{\text{eff}} = \frac{\Omega}{2} \left[ e^{\Delta_A t} |1, 0\rangle_0 \langle a| - e^{\Delta_B t} |a\rangle_0 \langle 1, 1| \right] + \text{h.c.}$$ \hspace{1cm} (14)

Here $\Omega$ is a Rabi frequency, and the detunings of the
$0 \leftrightarrow 2$ and $1 \leftrightarrow 2$ lasers are given by $\Delta_A$ and $\Delta_B$ respect-
tively. Transition probabilities in the indicated basis will be
affected by the transformation (acting in the atomic
space only), $\Psi(t) = \exp(-i\Delta t)\Psi(0)$, where

$$2 \Lambda = (\Delta_A + \Delta_B) |1, 0\rangle_0 \langle 1, 0| + (\Delta_B - \Delta_A) |a\rangle_0 \langle a|$$
$$- (\Delta_A + \Delta_B) |1, 1\rangle_1 \langle 1, 1|,$$ \hspace{1cm} (15)

giving the new effective Hamiltonian,

$$H'_{\text{eff}} = \Lambda + \frac{\Omega}{2} [ |1, 0\rangle_0 \langle a| + |a\rangle_0 \langle 1, 1| ] + \text{h.c.}$$ \hspace{1cm} (16)

This is exactly the first example of the present paper,
with the five states reduced to three by taking only the
symmetric states in place of the (#1,#2) and (#4,#5)
complexes. We may thus interchange the states, ex-
actly as before, by adiabatically varying the detunings.
For example, by adiabatically changing the combination
$\Delta_A + \Delta_B$, while keeping $\Delta_A - \Delta_B$ constant, we can ef-
fect the interchange of the states $|1, 1\rangle_0$ and $|1, 0\rangle_0$, thus
performing a CNOT operation.

The full set of equations describing the evolution of all
the states depicted in Fig. 4 is given below, where we have used
the Hamiltonian from Ref. [2] and added detuning as in Eq. (16):

$$\dot{c}_{[a]}(t) = -i\frac{\Omega}{2} \left[ c_{[1]}(t) - c_{[1]}(t) + c_{[2]}(t) \right]$$
$$-i\frac{\Delta_A - \Delta_B}{2} c_{[a]}(t),$$
$$\dot{c}_{[1]}(t) = -i\frac{\Omega}{2} \left[ c_{[1]}(t) + \frac{1}{\sqrt{2}} c_{[2]}(t) + c_{[a]}(t) \right]$$
$$-i\frac{\Delta_A + \Delta_B}{2} c_{[1]}(t),$$
$$\dot{c}_{[1]}(t) = -i\frac{\Omega}{2} c_{[1]}(t) + \frac{i}{2} (\Delta_A + \Delta_B) c_{[1]}(t),$$
$$\dot{c}_{[a]}(t) = -i\frac{\Omega}{2} c_{[a]}(t) + \frac{i}{\sqrt{2}} g c_{[1]}(t),$$
$$\dot{c}_{[1]}(t) = \sqrt{2} g c_{[a]}(t) - \kappa c_{[1]}(t),$$
$$\dot{c}_{[a]}(t) = g \left( c_{[1]}(t) - \sqrt{2} c_{[1]}(t) \right) - g c_{[1]}(t),$$
$$\dot{c}_{[2]}(t) = -i\frac{\Omega}{2} \left[ c_{[2]}(t) - \sqrt{2} c_{[2]}(t) \right] - g c_{[1]}(t),$$
$$\dot{c}_{[1]}(t) = g c_{[2]}(t) - \kappa c_{[1]}(t),$$
$$\dot{c}_{[1]}(t) = \sqrt{2} g c_{[a]}(t) - 2 \kappa c_{[1]}(t).$$ \hspace{1cm} (17)

In these equations, $g$ is the parameter setting the strong
scale depicted by the fat double-arrows in Fig. 4 while $\kappa$ sets the (strong) scale of the dissipative links $D$ to the
environment (decay of cavity modes).

We have solved this set of equation numerically, to
demonstrate the interchange of states $|10\rangle_0 \leftrightarrow |11\rangle_0$. We
begin with a steady situation in which detuning parame-
ters $\Delta_A$ and $\Delta_B$ are substantially greater than $\Omega$, and
the initial state is (very stably) either
$|1, 1\rangle_0$ or $|1, 0\rangle_0$. Then the detuning is manipulated, using simultaneous
slow changes of both laser frequencies in order to inter-
change these two states, in a process similar to those
described in section 11. The evolution is shown graphically
in Fig. 4. By contrast, Ref. [2] uses the perfectly tuned
case $\Delta_A, \Delta_B = 0$, and takes $\Omega = 0$ until a pulse turn-on
time. The pulse is then turned off at exactly the time for the
interchange $|1, 0\rangle_0 \leftrightarrow |1, 1\rangle_0$, to have occurred under
the influence of precession alone. In either method, the
transformation represents a CNOT gate, the states $|0, 1\rangle_0$
and $|0, 0\rangle_0$ being frozen due to the S-W effect. Note that
our implementation does not require accurate timing of
applied fields.
IV. DISCUSSION AND CONCLUSION

We have shown that in systems with appropriate arrangements of strong (S) and weak (W) couplings, variable potentials applied to a relatively small number of sites can efficiently effect state permutations for spin chains with pure-exchange coupling. Using the same approach, we found a new way to implement a CNOT gate in an atoms-in-a-cavity model discussed by previous authors [2].

A number of models similar to that of Ref. [2] can be found in the recent literature [3, 4, 6, 7, 9, 10]. For many of these cases, the generic S-W paradigm developed in the present paper provides a unified basis for understanding how isolated subspaces are generated.

As our final comment we note that the S-W non-dissipative links, and decay-induced dissipation – the two cases specifically studied here – are not the only means by which subspaces can become mutually isolated. For instance, Ref. [8] and many subsequent works have discussed cases in which rapid incoherent scattering can freeze a system in a single state or subspace of states.

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