Abstract

Old fashioned duality used to derive the closed string field theory for magnetic vortex from the gauge theory with Higgs scalar, is applied to the string theories. The bosonic string theory coupled to the Kalb-Ramond 2-form field is dually transformed to the 6-brane theory coupled to the 7-form field. The old fashioned dual transformation is also examined for the Type IIA and IIB superstrings. For this study, the string field theoretical treatment of the bosonic and fermionic strings is developed based on the reparametrization invariant formulation of strings by Marshall and Ramond. In order for the self-duality of the Type IIB superstring to appear, the following dual correspondence may happen: the dual transformation of the NS-NS field functional is the bosonization of the R-R one.

1 Introduction

In the late 70’s, we were in the first time excitement on the duality, in which the quark confinement mechanism was searched based on the Mandelstam-’t Hooft duality [1]. Many works were carried out in this direction, using the so-called “dual(-ity) transformation” [2], [3], [4]. For example, the Abelian Higgs model is “dually transformed” into the relativistic hydrodynamics of Kalb-Ramond and of Nambu [5] coupled to the vorticity source. The latter model is further transformed to a closed string field theory of the magnetic vortex coupled to the anti-symmetric tensor fields called Kalb-Ramond fields, or the gauge field of strings [6]. In the course of the dual transformation the coupling constant \(g\) is naturally replaced by its inverse \(1/g\).

Recently, D-brane has been introduced as a source of the closed strings, and is understood as a soliton in the superstring theories, like a monopole or a magnetic string in the gauge
theories. Since then, the D-brane dynamics has begun to reveal the strong coupling regime of the superstring theories, generating the second time excitement of the string duality.

The first and the second time dualities are quite analogous: Starting from a gauge theory having the minimal coupling for an electric charge, another theory is derived by the dual transformation, which has the minimal coupling for the magnetically charged soliton of the former theory. Therefore, it is interesting to study the D-brane, a key element of the second time duality, based on the old fashioned duality of the first time. The relation between the electrically charged scalar field theory and the string field theory of magnetic vortices is analogous to the relation between the fundamental string theory and the D-brane theory.

More recently, a duality is conjectured by Mardacena between the gauge theories on the 4 dimensional boundary and the string theory (or the supergravity theory as a low energy effective theory of the string theory) on the 10 dimensional bulk. The example mentioned above, giving the old-fashioned duality between the Abelian Higgs model and the closed string field theory, is quite interesting, since the dual relationship between the gauge theory and the closed string theory is quite analogous to the Mardacena’s one. In the Mardacena’s duality existence of the extra 5th dimension is essential, which represents the energy scale of the theory. In the old-fashioned duality, the role of the 5th dimension is played in a sense by the vacuum expectation value of the Higgs scalar. We do not have yet the constructive way of finding the dual partner of a given realistic theory, not having conformal symmetry nor supersymmetry. In this respect, the old fashioned duality is worth re-examining, from the present insights.

Preliminary stage of this work can be found in the lecture given at Kashikojima Summer Institute by one of the authors (A.S.) and a part of this work has been studied in the PhD thesis of another one of the authors (R.E).

In this paper, the fundamentals such as the deformation and the sum of curves, etc. are refined more rigorously, and the duality between the fundamental string and the D-brane is clarified. Furthermore, the old fashioned dual transformation is examined for the superstrings of Neveu-Schwarz and of Ramond, based on the field theoretical formulation of the theories by Marshall and Ramond.
2 Old Fashioned Dual Transformation

Dual transformation is a kind of Fourier transformation performed in the integrand of the partition function. It is similar to the picture changing transformation from x-representation to p-representation in quantum mechanics, where commutation relations

\[ [\hat{x}, \hat{p}] = i, \quad \text{and} \quad [\hat{x}, \hat{x}] = [\hat{p}, \hat{p}] = 0. \]  

hold and its physical manifestation is the uncertainty relation \((\Delta p)^2 \cdot (\Delta x)^2 \geq 1\). Similarly, dual transformation from \(E\)-representation to \(B\)-representation gives 't Hooft algebra which is given for the SU(N) gauge theory by

\[ \hat{A}[C] \hat{B}[C'] = \hat{B}[C'] \hat{A}[C] \times e^{2\pi i L(C,C')/N}, \quad \text{and} \quad [\hat{A}[C], \hat{A}[C']] = [\hat{B}[C], \hat{B}[C']] = 0, \]

where \(L(C, C')\) is the linking number of two closed curves \(C\) and \(C'\). Its uncertainty relation \((\Delta E)^2 \cdot (\Delta B)^2 \geq 1\) gives a physical picture of the duality, that is, the squeezing of the electric flux and that of the magnetic flux cannot be observed at the same time.

In order to change \(E\)-representation to \(B\)-representation, we simply perform the following dual transformation, a Fourier transformation with respect to the field strengths performed in the integrand of the partition function [3][4]:

\[ \exp \left\{ i \int d^4x - \frac{\varepsilon}{4} F_{\mu\nu} F^{\alpha\mu\nu} \right\} \]

\[ \propto \int D W^\alpha_{\mu\nu}(x) \exp \left\{ i \int d^4x \left[ \frac{1}{4\varepsilon} W^\alpha_{\mu\nu} W^{\alpha\mu\nu} - \frac{1}{2} \tilde{W}^\alpha_{\mu\nu} F^{\alpha\mu\nu} \right] \right\}, \]

where \(\varepsilon\) is a dielectric constant, \(F_{\mu\nu}^\alpha\) is the field strength of the gauge field \(A_{\mu}^\alpha\), and \(W^\alpha_{\mu\nu}(x)\) is an anti-symmetric tensor field (Kalb-Ramond field), which becomes a velocity potential of the hydrodynamics after the dual transformation is performed.

Following Ref. [3], we start with the action of the Abelian Higgs model,

\[ S = \int d^4x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + |(\partial_\mu + ie A_\mu) \phi|^2 - V(|\phi|^2) \right]. \]

If the above dual transformation in Eq.(3) is carried out in the partition function of the Abelian Higgs model and the subsequent integration over \(A_\mu\) is performed, the following dual
action is obtained:

\[ S^* = \int d^4x \left[ -\frac{1}{2\epsilon^2|\phi|^2} \frac{1}{2} (V_\mu)^2 - \frac{1}{4} (W_{\mu\nu})^2 + (\partial_\mu |\phi|)^2 - V(|\phi|) \right. \\
\left. + \frac{12\pi}{2} \frac{1}{e} W_{\mu\nu} \times \frac{1}{4\pi} \epsilon_{\mu\nu\lambda\rho} (\partial_\lambda \partial_\rho - \partial_\rho \partial_\lambda) \chi(x) \right] . \]  

Here, \( V_\mu = \partial^\nu \tilde{W}_{\nu\mu} \) is the velocity field of the fluid, satisfying the continuity equation, \( \partial^\mu V_\mu = 0 \), and \( \phi = |\phi| \exp(i\chi) \). This model gives a kind of “relativistic hydrodynamics” which has been originally formulated by Kalb and Ramond and by Nambu \[5\]. The singularity of the Higgs’ phase \( \chi \) (that is the point around which the phase \( \chi \) changes by \( 2\pi \) times integer \( n \)) gives a vorticity source,

\[ \omega^{\mu,\nu} = \frac{1}{4\pi} \epsilon^{\mu\nu\lambda\rho} (\partial_\lambda \partial_\rho - \partial_\rho \partial_\lambda) \chi(x) = n \int d\tau d\sigma \frac{\partial(y^\mu, y^{\nu})}{\partial(\tau, \sigma)} \delta^{(4)}(x - y(\tau, \sigma)). \]  

Here \( x^\mu = y^\mu(\tau, \sigma) \) determines the world sheet swept by a magnetic vortex, on which the \( \chi \) field becomes singular, and the integer \( n \) represents the quantized vortex number. Recently this kind of correspondence is rediscovered, and is applied to the study of the motion of vortices in the superfluidity \[12\].

In this manner, the gauge coupling of the world sheet of the vortex \( y^\mu(\tau, \sigma) \) with the Kalb-Ramond field \( W_{\mu\nu} \) is derived, and the contribution of this interaction in the partition function reads

\[ \exp \left( i \int_S d\sigma^{\mu\nu} \frac{1}{2} \frac{12\pi}{e} W_{\mu\nu} \right) . \]  

The above method is a general one of introducing a vortex (a “bare soliton”) and of obtaining its coupling to the “gauge field” (Kalb-Ramond field in this case) with a strength \( 2\pi/e \), so that the dual model obtained is suitable for the strong coupling expansion. Therefore, this method may be useful to introduce D-branes and to obtain their coupling to the Kalb-Ramond fields, since D-branes are the “bare solitons” of the string theory. We will discuss this issue in the next section.

The contribution to the partition function of the classically charged particle interacting with the electromagnetic field is \( \exp(i e \int_C dx^\mu A_\mu(x)) \). Furthermore, the field theory of charged particles, that is the charged Higgs model, is known to be derived, by summing up all the possible classical configurations having any number of closed circles depicted by the charged
particles. Similarly, summing up all the possible classical shapes of the world sheets of the vortices, the field theory of string is obtained, since the coupling of vortex and Kalb-Ramond field given in Eq. (7) is the same minimal coupling as the electromagnetic one. Namely, we can arrive at the following action of the closed string field theory, which is dually related to the Higgs model [3]:

\[ S^\star = \sum_C \left\{ -\frac{1}{f_C} \oint_C dt \left( \frac{\delta}{\delta \sigma^{\mu t}} - i \frac{2\pi}{e} W^{\mu t} \right) \Phi[C] \right\}^2 - M_0 |\Phi[C]|^2, \]

where \( C \) is a closed curve on which string field \( \Phi[C] \) is defined, and \( M_0 \) is a constant including the entropy effect. Once we have the quantum field theory of the string, the probability for a closed string to take the shape \( C \) is given by \(|\Phi[C]|^2\) as usual.

Furthermore, if we add the fermionic action \( S_f \) to the Higgs model:

\[ S_f = \int d^4x \left[ \bar{\psi}(i\gamma^\mu - m_f)\psi + e\bar{\psi}\gamma_\mu A^\mu \right], \]

then the corresponding dual action has the following additional term:

\[ S_f^\star = \int d^4x \left[ \bar{\psi}(i\gamma^\mu - m_f)\psi - n\bar{\psi}\gamma_\mu \psi \bar{\phi}^\mu \theta + \frac{1}{2} W^{\mu \nu} \omega_{\mu \nu}^F - \frac{1}{4} \frac{1}{|\phi|^2} (\bar{\psi}\gamma_\mu \psi)(\bar{\psi}\gamma_\mu \psi) \right], \]

where the additional vorticity appears from the fermion,

\[ \omega_{\mu \nu}^F = \frac{1}{4\pi} \epsilon_{\mu \nu \lambda \rho} \partial^\lambda \left( \frac{1}{|\phi|^2} \bar{\psi}\gamma^\rho \psi \right). \]

3 Application of Old Fashioned Duality for D-branes

In this section, we start with the closed string field theory coupled to the Kalb-Ramond field (2-form field) which is a kind of the hydrodynamics obtained by the dual transformation from the Abelian Higgs model in the previous section. Here, we take the superstringy space-time dimension of \( D = 10 \).

Our starting action is tentatively

\[ S^{(0)} = \sum_C \sum_{x \in C} \left| \left( \frac{\delta}{\delta \sigma^{\mu t}(x)} - ig A_\mu(x) \right) \Phi[C] \right|^2 + \sum_x \frac{\varepsilon}{2 \cdot 3!} F_{\mu \nu \lambda} F^{\mu \nu \lambda} - V(|\Phi[C]|^2), \]
where $F_{\mu \nu \lambda} = \partial_{\mu} A_{\nu \lambda} + \partial_{\nu} A_{\lambda \mu} + \partial_{\lambda} A_{\mu \nu}$. We consider the massless Kalb-Ramond field, so that the model has the gauge symmetry of the string theory (i.e. Kalb-Ramond symmetry):

$$
\Phi[C] \rightarrow \Phi'[C] = e^{i \oint_C dx^i A_t(x) \Phi[C]},
$$

$$
A_{\mu \nu}(x) \rightarrow A'_{\mu \nu}(x) = A_{\mu \nu}(x) + \partial_{\mu} \Lambda_{\nu}(x) - \partial_{\nu} \Lambda_{\mu}(x).
$$

(13)

Then, we perform the old-fashioned dual transformation:

$$
\exp \left\{ i \int d^{10} x - \frac{\varepsilon}{2 \cdot 3!} F_{\mu \nu \lambda} F^{\mu \nu \lambda} \right\}
\propto \int D W_{\mu_1 \cdots \mu_7}(x)
\exp \left\{ i \int d^{10} x \left[ \frac{1}{2 \cdot 7! \varepsilon} W_{\mu_1 \cdots \mu_7} W^{\mu_1 \cdots \mu_7} - \frac{1}{7! 3!} \varepsilon^{\mu_1 \cdots \mu_{10}} W_{\mu_1 \cdots \mu_7} F_{\mu_8 \mu_9 \mu_{10}} \right] \right\},
$$

(14)

and integrate out the original variables $A_{\mu \nu}(x)$ as in the previous section. The lecture note [9] giving the preliminary stage of this work starts with the action $S^{(0)}$ and proceeds in this way.

Then, we arrive at the dual action $S^*$:

$$
S^* = \sum_x - \frac{1}{8 \cdot 8!(7!)^2} \sum_{C(\exists x)} \frac{1}{\Psi[C]} F_{\mu_1 \cdots \mu_7} F^{\mu_1 \cdots \mu_7}
+ \sum_{S_6} \sum_x \left| \frac{\delta}{\delta \sigma^{\mu_1 \cdots \mu_6}(x)} - W_{\mu_1 \cdots \mu_6}(x) \right| \Psi[S_6]^2
+ \cdots.
$$

(15)

This is the Kalb-Ramond field theory in which the “6-brane” ($S_6$) interacts with the 7-form field, $W_{\mu_1 \cdots \mu_7}$. Similarly, we obtain the following correspondences between different brane theories and their dual brane theories:

| Original Brane | Dual Brane |
|---------------|------------|
| 0-brane       | 7-brane    (= “vortex” connecting 6-branes “monopole”) |
| 1-brane       | 6-brane    (= “vortex” connecting 5-branes “monopole”) |
| 2-brane       | 5-brane    (= “vortex” connecting 4-branes “monopole”) |
| 3-brane       | 4-brane    (= “vortex” connecting 3-branes “monopole”) |

(16)

A similar study of duality for D-branes is recently carried out in [18].

In the above, the present terminology of “p-brane” is used to represent the p-dimensionally extended object in the old days. The membrane and the more extended objects (generally p-branes) are now inevitable ingredients, but have not been popular until 5 years ago. Curiously, they have the longer history than the string, since the membranic model of muon (the first
excited state of the elastic ball on which the electric charge is distributed) by P. A. M. Dirac in 1962 \[13\]. Subsequently, the bosonic membrane (p=2) was studied in \[14\], and the bosonic membrane as well as the general p-brane was studied in \[15\]. As for the spinning membrane, it was first formulated in \[16\]. Hereafter, the supermembrane theory is formulated as a matrix model \[17\] and it becomes very popular now as a candidate of the M-theory.

So far we have given the rather naive study of the old fashioned duality for D-branes. It is, however, better to examine more carefully each step of the dual transformation, when we treat the non-local objects such as the string field functional \(\Phi[C]\) and the path integration over the functionals. This erases the complexity existing in the preliminary version of this work, that is the admixture of the local and the non-local objects, and may give a clear demonstration of the dual transformation in string theory. For this purpose, useful reference is the paper by Marshall and Ramond \[11\] even now.

Let a closed curve \(C\) be parametrized by \(X^\mu(\lambda)\) with \(0 \leq \lambda < 2\pi\). It has the following normal mode expansion,

\[
X^\mu(\lambda) = \sum_n x_n^\mu f_n(\lambda),
\]

(17)

where the normal modes \(\{f_n(\lambda)\}\) satisfy the orthonormality and the completeness conditions:

\[
\int d\lambda f_m(\lambda)f_n(\lambda) = \delta_{mn}, \quad \text{and} \quad \sum_n f_n(\lambda)f_n(\lambda') = \delta(\lambda - \lambda').
\]

(18)

Therefore, the string field functional \(\Phi[C]\) is a function of infinite number of variables \(\{x_n^\mu\}\). Then, the functional derivative giving the deformation of the curve \(C\),

\[
\frac{\delta}{\delta\sigma^\mu(x)} \Phi[C]
\]

(19)

can be defined reparametrization invariant way, by using the orthonormal expansion:

\[
\frac{1}{\sqrt{-\left(dX^\mu/d\lambda\right)^2}} \frac{\delta}{\delta X^\mu(\lambda)} \Phi[X^\mu(\lambda)],
\]

(20)

where

\[
\frac{\delta\Phi}{\delta X^\mu(\lambda)} = \sum_n f_n(\lambda) \frac{\partial\Phi}{\partial x_n^\mu}.
\]

(21)

If the parametrization \(X^\mu(\lambda)\) of the curve \(C\) in terms of \(\lambda\) is changed to \(Y^\mu(\rho)\) in terms of \(\rho\), defined by \(\lambda = g(\rho)\), the coefficients of the normal mode expansion is changed accordingly from
\{x'_\mu\} \text{ to } \{y'_\mu\}. \text{ Then, we have }
\begin{equation}
y'_n = \sum_m x'_m \int d\rho f_n(\rho) f_m(g(\rho)). \tag{22}\end{equation}

From this we can understand that the functional derivative given in Eq.(20) is reparametrization invariant. Similarly, the reparametrization invariant line integral along the curve \(C\) is given by \(\int d\lambda \sqrt{-(dX^\mu/d\lambda)^2}\). The unit vector \(t^\mu\) tangential to the curve \(C\) reads \(t_\mu(\lambda) \equiv X'_\mu(\lambda)/\sqrt{-X'^2(\lambda)}\), satisfying \(t^2 = -1\), where \(X'^\mu(\lambda)\) means \(dX^\mu/d\lambda\) as usual.

The important issue is how we perform path integration over the string field functional \(\Phi[C]\). To solve this problem, we have to know the way to sum up all the possible shapes of curves \(C\). The integration measure can be read from the metric (or the distance) of the integration variables. The distance \(ds\) between two closed curves \(C\) and \(C + \delta C\) can be defined as the minimum area of the membrane connecting \(C\) and \(C + \delta C\) as its boundaries. Since
\begin{equation}
ds^2 = \int d\lambda \sqrt{X'^2} \left[-(\delta X^\mu)^2 - (t^\mu \delta X^\mu)^2\right] = \sum_{m,n} \sum_{\mu,\nu} g^{mn}_{\mu\nu} \delta x'_m \delta x'_n, \tag{23}\end{equation}

the metric in the space of closed curves is given by
\begin{equation}
g^{mn}_{\mu\nu} \equiv \int d\lambda \sqrt{X'^2} \left[-g_{\mu\nu} - t^\mu t^\nu\right] f_m(\lambda) f_n(\lambda). \tag{24}\end{equation}

Therefore, the sum of curves can be done by
\begin{equation}
\sum_C \equiv \int \mathcal{D}X \equiv \int \prod_{\mu,n} dx'_n \sqrt{g} \quad \text{with} \quad g \equiv \det g^{mn}_{\mu\nu}. \tag{25}\end{equation}

After these preparations, we can write down definitely the action \(S\) which we start with, as follows
\begin{equation}
S = \int \mathcal{D}X(\lambda) \int d\lambda \sqrt{-X'^2} \left[ \left\| \left( \frac{1}{\sqrt{-X'^2}} \delta X^\mu(\lambda) \right) - it^\mu(\lambda) A_{\mu\nu}[C, X(\lambda)] \right\| \Phi[C] \right]^2 \\
+ \frac{1}{12} F_{\mu\nu\rho}[C, X(\lambda)] F^{\mu\nu\rho}[C, X(\lambda)] - V(\|\Phi[C]\|^2) \right], \tag{26}\end{equation}

where \(F_{\mu\nu\rho}[C, X(\lambda)]\) is the field strength of the Kalb-Ramond field \(A_{\mu\nu}[C, X(\lambda)]\), defined by
\begin{equation}
F_{\mu\nu\rho}[C, X(\lambda)] \equiv \frac{1}{\sqrt{-X'^2}} \frac{\delta A_{\nu\rho}[C, X(\lambda)]}{\delta X^\mu(\lambda)} + \text{(cyclic permutations)}. \tag{27}\end{equation}

It should be noted that the Kalb-Ramond field, or the gauge field of the string, is taken to be a non-local one defined on the curve \(C\) and the point \(X^\mu(\lambda)\). This non-locality is necessary
to carry out the dual transformation consistently. The action is shown to be invariant under the Kalb-Ramond transformation, or the gauge transformation of the 2-form field, namely,

$$\Phi[C] \rightarrow \exp(i\Lambda[C])\Phi[C],$$

$$A_{\mu\nu}[C, X(\lambda)] \rightarrow A_{\mu\nu}[C, X(\lambda)] + \frac{t_\mu(\lambda)}{\sqrt{-X'^2}} \frac{\delta}{\delta X^\mu(\lambda)} - \frac{t_\nu(\lambda)}{\sqrt{-X'^2}} \frac{\delta}{\delta X^\nu(\lambda)}.$$ (28)

It is also possible to choose the simpler action $S'$ rather than $S$,

$$S' = \int DX^\mu(\lambda) \int d\lambda \sqrt{-X'^2} \left[ \left( \frac{1}{\sqrt{-X'^2}} \frac{\delta}{\delta X^\mu(\lambda)} - iA_{\mu}[C, X(\lambda)] \right) \Phi[C] \right]^2$$

$$- \frac{1}{4} F_{\mu\nu}[C, X(\lambda)] F^{\mu\nu}[C, X(\lambda)] - V(|\Phi[C]|^2),$$ (29)

where the field strength of the non-local gauge field is given by

$$F_{\mu\nu}[C, X(\lambda)] = \frac{1}{\sqrt{-X'^2}} \frac{\delta A_{\nu}[C, X(\lambda)]}{\delta X^\nu(\lambda)} - (\mu \leftrightarrow \nu),$$ (30)

and the action is invariant under the following gauge transformation

$$A_{\mu}[C, X(\lambda)] \rightarrow A_{\mu}[C, X(\lambda)] + \frac{1}{\sqrt{-X'^2}} \frac{\delta \Lambda[C]}{\delta X^\mu(\lambda)}.$$ (31)

We examine in detail the dual transformation of the former action $S$ in the following. For the kinetic term of the Kalb-Ramond fields in $S$, we apply the following dual transformation:

$$\exp\left[ \frac{i}{12} \int DX \int d\lambda \sqrt{-X'^2} F_{\mu\nu\rho} F^{\mu\nu\rho}[C, X(\lambda)] \right]$$

$$= \int DW^{\mu_1\mu_2...\mu_7} \exp\left[ \frac{i}{12} \int DX^\mu(\lambda) \int d\lambda \sqrt{-X'^2} \left\{ - \left( \tilde{W}^{\mu\nu\rho} \right)^2 + 2\tilde{W}^{\mu\nu\rho} F_{\mu\nu\rho} \right\} \right].$$ (32)

Here, the path integration over $X^\mu(\lambda)$ is defined in the above in terms of the metric

$$\int DX^\mu(\lambda) \equiv \int d\xi^\mu_n \sqrt{g(\xi)}.$$ (33)

Writing the complex string field functional as $\Phi[C] = |\Phi[C]| e^{i\chi[C]}$, and using the integration by parts, the relevant terms to $A_{\mu\nu}[C, X(\lambda)]$ read

$$\int DX^\mu(\lambda) \int d\lambda \sqrt{-X'^2} \left[ (t^\mu(\lambda) A_{\nu\mu})(t^\nu(\lambda) A_{\mu\nu})|\Phi[C]|^2 - \frac{1}{2\sqrt{g}} \frac{1}{\sqrt{-X'^2}} \frac{\delta}{\delta X^\mu(\lambda)} \frac{\delta}{\delta X^\nu(\lambda)} A_{\nu\mu} \right]$$

$$- 2|\Phi[C]|^2 \frac{1}{\sqrt{-X'^2}} \frac{\delta}{\delta X^\mu(\lambda)} A_{\nu\mu}.$$ (34)
Here, we decompose the vector index into the tangential direction $t^\mu$ along $C$, and the transverse directions to it. Defining $B_\nu \equiv t^\mu A_{\mu \nu}$ and $V_{\nu \rho} \equiv t^\mu \tilde{W}_{\mu \nu \rho}$, the decomposition of $A_{\mu \nu}$, and $\tilde{W}_{\mu \nu \lambda}$ is given by

\[
A_{\mu \nu} = -(t_\mu B_\nu - t_\nu B_\mu) + A^T_{\mu \nu},
\]

\[
\tilde{W}_{\mu \nu \lambda} = -(t_\mu V_{\nu \lambda} + \text{cyclic perm.}) + \tilde{W}^T_{\mu \nu \lambda},
\]

where we have $t_\mu B_\nu = 0$, $t^\mu A^T_{\mu \nu} = 0$, $t^\mu \tilde{V}_{\nu \mu} = 0$, and $t^\mu \tilde{W}^T_{\mu \nu \lambda} = 0$. Then, the path integration over $A^T_{\mu \nu}$ gives a constraint

\[
\frac{\delta}{\delta X^\rho(\lambda)} \left( \sqrt{g} \tilde{W}^T_{\mu \nu \rho} \right) = 0,
\]

while the path integration over $B_\mu$ is the Gaussian integration.

Accordingly, the following dual action $S^*$ is derived, starting with the original string action $S$,

\[
S^* = \int D^X(\lambda) \int d\lambda \sqrt{-X^\prime} \left[ -\frac{1}{4g |\Phi[C]|^2} \left\{ \frac{1}{\sqrt{-X^\prime}} \frac{\delta}{\delta X^\mu} \left( \sqrt{g} \tilde{V}^\mu \right) \right\}^2 + \frac{1}{4} \tilde{V}^2_{\mu \nu} 
- \frac{1}{12} (\tilde{W}^T_{\mu \nu \rho})^2 \right]
- \frac{1}{\sqrt{-X^\prime}} \frac{\delta}{\delta X^\mu} \left( \frac{1}{\sqrt{-X^\prime}} \frac{\delta \chi[C]}{\delta X^\nu} \right) \tilde{V}^\mu \nu + \left( \frac{1}{\sqrt{-X^\prime}} \frac{\delta |\Phi[C]|}{\delta X^\mu} \right)^2 \right].
\]

Similarly, from the second action of $S'$, we can derive the following dual action:

\[
S'^* = \int D^X(\lambda) \int d\lambda \sqrt{-X^\prime} \left[ -\frac{1}{4g |\Phi[C]|^2} \left\{ \frac{1}{\sqrt{-X^\prime}} \frac{\delta}{\delta X^\mu} \left( \sqrt{g} \tilde{W}^\mu \right) \right\}^2 + \frac{1}{4} \tilde{W}^2_{\mu \nu} 
- \frac{1}{\sqrt{-X^\prime}} \frac{\delta}{\delta X^\mu} \left( \frac{1}{\sqrt{-X^\prime}} \frac{\delta \chi[C]}{\delta X^\nu} \right) \tilde{W}^\mu \nu + \left( \frac{1}{\sqrt{-X^\prime}} \frac{\delta |\Phi[C]|}{\delta X^\mu} \right)^2 \right].
\]

These two dual actions are equivalent, since they are related to each other:

\[
S'^* = S^* + \int D^X(\lambda) \int d\lambda \sqrt{-X^\prime} \frac{1}{4g |\Phi[C]|^2} \left\{ \frac{1}{\sqrt{-X^\prime}} \frac{\delta}{\delta X^\mu} \left( \sqrt{g} C^\mu \right) \right\}^2 - \frac{1}{2} C^2_{\mu},
\]

where $C_\mu \equiv t^\mu \tilde{W}_{\mu \nu}$, and $\tilde{W}_{\mu \nu} \equiv -(t_\mu C_\nu - t_\nu C_\mu) + \tilde{V}_{\mu \nu}$ with $t^\mu C_\mu = 0$ and $t^\mu \tilde{V}_{\mu \nu} = 0$.

In the dual actions in Eqs. \[37\] \[38\], the “vorticity source” $\omega_7(S_7)$ is given by

\[
\omega_{\mu_1 \mu_2 \ldots \mu_7}(S_7) \equiv \frac{1}{4\pi} \epsilon_{\mu_1 \mu_2 \ldots \mu_7 \nu \nu \rho} t^\rho 
\times \left( \frac{1}{\sqrt{-X^\prime}} \frac{\delta}{\delta X^{\nu_1}} \frac{1}{\sqrt{-X^\prime}} \frac{\delta}{\delta X^{\nu_2}} - \frac{1}{\sqrt{-X^\prime}} \frac{\delta}{\delta X^{\nu_2}} \frac{1}{\sqrt{-X^\prime}} \frac{\delta}{\delta X^{\nu_1}} \right) \chi[C].
\]
To extract the “vorticity source” from the singularity of the phase $\chi[C]$ of the wave functional, is not a simple task. The task means to understand how the various topological quantum numbers are defined using the non-local field, such as the string field $\Phi[C]$ or $\chi[C]$. More generally, the problem is how are the topological quantum numbers defined with the p-branes’ wave functional? This is an interesting mathematical problem probably to be solved in the non-commutative geometry [19]. It is because, the string and the p-brane theories in general give the non-commutativity in space-time, so that the geometry behind these topological quantum numbers should be the non-commutative geometry.

Here, we study a simple case in which the mean value $<\chi[C]>$ of $\chi[C]$ changes $2\pi n$ while we deform the curve $C$ transversally and return to the original shape of $C$. The homotopy of this deformation can be given as

$$X_\mu(\lambda,s_1) = \sum_m f_m(\lambda)x_m(\lambda)x_\mu(x_m, x_n)$$

(41)

This means

$$\frac{\partial^2 \chi}{\partial x_m \partial x_n} = 4\pi n \left( \int d\lambda \sqrt{-X'^2} \right) \frac{1}{\sqrt{-X'^2}} \frac{\partial(s_1, s_2)}{(x_m, x_n)} \delta(s_1 - s_1^*) \delta(s_2 - s_2^*) \delta(\lambda - \lambda^*)$$

(42)

If we add 7 parameters ($\sigma_1, \cdots, \sigma_7$) in addition to ($\lambda, s_1, s_2$), then the whole 10 dimensional space can be parametrized by $x^\mu = Y^\mu(\lambda, s_1, s_2, \sigma_1, \cdots, \sigma_7)$. The singularity existing at ($\lambda, s_1, s_2$) = ($\lambda^*, s_1^*, s_2^*$) determines $S_7$ (the world volume of the 6-brane) by \{\{Y^\mu(\sigma_1, \cdots, \sigma_7)\}, on which the vorticity takes the non-vanishing value. Now, the vorticity source can be written as

$$\omega_{\mu_1 \mu_2 \cdots \mu_7}(S_7) = -n \left( \int d\lambda \sqrt{-X'^2} \right) \frac{\partial(Y_{\mu_1}, Y_{\mu_2}, \cdots, Y_{\mu_7})}{\partial(\sigma_1, \sigma_2, \cdots, \sigma_7)} \delta^{(10)}(x^\mu - Y^\mu(\sigma_1, \sigma_2, \cdots, \sigma_7))$$

(43)

Then, the world volume $S_7$ of the 6-brane couples minimally to the Kalb-Ramond 7-form field, or $\tilde{W}_{\mu \nu \rho}$, where $\tilde{W}_{\mu \rho}$ and $A_{\mu \nu}$ are dually related.
Our formulation is adequate for extracting the vortex rather than the monopole, so that the derived 6-brane is considered to be the bare “magnetic vortex”, connecting the pair of “magnetic monopoles” on the both ends. Therefore, the “magnetic monopole” is the 5-brane which is the dual object of the “fundamental string” of the 1 brane. The other correspondences are given in Eq. (16).

4 Old-Fashioned Duality for Superstrings

We know that the place where the duality plays the powerful role is the superstring theories, so that our next step should be the application of the old-fashioned duality for superstrings and see how it works. However, we have to restrict ourselves only to a small step towards this project, by examining the old fashioned dual transformation in the “field theories of the spinning strings”.

In the spinning string of Neveu-Schwarz and Ramond, ten spins are attached on the curve $C$, which are represented by the two sets $(i = 1, 2)$ of ten 2-dimensional spinors $\psi_{(i)}^\mu(\lambda)(\mu = 0 - 9, \text{ and } i = 1, 2)$. They satisfy the commutation relations,

$$\{\psi_{(i)}^\mu(\lambda), \psi_{(j)}^\nu(\lambda')\} = G^{\mu\nu}\delta_{ij}\delta(\lambda - \lambda').$$

(44)

Following [11], the mode expansion of $\psi_{(i)}^\mu$ for the Neveu-Schwarz (NS) sector is given by

$$\psi_{(i)}^\mu(\lambda)_{(NS)} \equiv b_{(i)}^\mu(\lambda) = \sum_{k:\text{half-integer}} b_{(i)k}^\mu f_k(\lambda),$$

(45)

while for the Ramond (R) sector,

$$\psi_{(1)}^\mu(\lambda)_{(R)} \equiv \Gamma_{(1)}^\mu(\lambda)/\sqrt{2} = \gamma^\mu/\sqrt{2} + \gamma_{11} \sum_{n:\text{integer} \neq 0} d_{(1)n}^\mu f_n(\lambda),$$

$$\psi_{(2)}^\mu(\lambda)_{(R)} \equiv \Gamma_{(2)}^\mu(\lambda)/\sqrt{2} = \gamma_{11} \sum_{n:\text{integer} \text{ including } 0} d_{(2)n}^\mu f_n(\lambda).$$

(46)

Here, $\psi_{(i)}^\mu(\lambda)$ is related to the right-moving mode $\psi(\tau - \sigma)$ and the left-moving mode $\tilde{\psi}(\tau + \sigma)$ as follows [21]:

$$\psi_{(1)}^\mu = \tilde{\psi}^\mu + \psi^\mu,$$

$$\psi_{(2)}^\mu = \tilde{\psi}^\mu - \psi^\mu.$$  

(47)
The 10 dimensional spinor structure is naturally build in with the help of the $\gamma^\mu$ matrices which are lifted to the position dependent fields $\Gamma^\mu_{(1)}(\lambda)$ on the curve $C$ (See (44)). From the commutation relation in Eq. (44), we understand the product $[-X'^2]^{-1/4}\psi^\mu(\lambda) \equiv \hat{\psi}^\mu(\lambda)$ is reparametrization invariant. Similarly, the reparametrization invariant fields $\hat{\Gamma}^\mu(\lambda)$ and $\hat{b}^\mu(\lambda)$ are introduced by multiplying $[-X'^2]^{-1/4}$. The string field functional of the spinning string depends on the shape of the curve $C$, as well as the spins’ configuration. Therefore, the string field functional in the NS sector is bosonic and is given by $\Phi[X^\mu(\lambda), b^\mu_{1,2}(\lambda)]$, while the string field functional in the R sector is a 10 dimensional spinor $\Psi[X^\mu(\lambda), d^\mu_{1,2}(\lambda)]$.

Now, we can write down the field theoretical superstring actions. Bosonic part of the action $S_b$ is common for Type IIA and Type IIB, namely

$$S_b = \int \mathcal{D}X^\mu(\lambda) \int \mathcal{D}b^\mu_{(1)}(\lambda) \mathcal{D}b^\mu_{(2)}(\lambda) \int d\lambda \sqrt{-X'^2} \left[ \Phi[C, b^\mu_{(1,2)}(\lambda)]*\hat{b}^0_{(1)}(\lambda) \right.$$  
\[\times \left\{ e^{D[C,X(\lambda)]}E^\mu_{\mu}(C, X(\lambda)) \right\} \left\{ \frac{1}{\sqrt{-X'^2}} \delta_\lambda + it^\nu(\lambda)B_\rho\nu[C, X(\lambda)] \
+ (1/8) \left[ \hat{b}^\mu_{(1)}(\lambda)\hat{b}^\mu_{(1)\nu}(\lambda) \right] \Omega^\nu\rho_{\mu}(C, X(\lambda)) \right\} + (1/\alpha')\hat{b}^\mu_{(2)}(\lambda)t_\nu(\lambda) \right]\right], \tag{48}
\]
where the dilaton, vierbein, spin connection, and the 2-form field (Kalb-Ramond field) are denoted by $D[C, X(\lambda)], E^\mu_{\mu}(C, X(\lambda)), \Omega^\nu\rho_{\mu}(C, X(\lambda))$, and $B_\rho\nu[C, X(\lambda)]$, respectively. We use the indices without and with bars for the flat and curved ones, respectively.

The fermionic action depends on the type of the closed string theories, namely Type IIA or IIB. For the Type IIB superstring, we have the coupling of the chiral fermion with the even form fields $A^{(p)}$ ($p =$even) in the R-R sector, so that the action $S_f$(IIB) is given by

$$S_f(IIB) = \int \mathcal{D}X^\mu(\lambda) \int \mathcal{D}d^\mu_{(1)}(\lambda) \int \mathcal{D}d^\mu_{(2)}(\lambda) \int d\lambda \sqrt{-X'^2} \left[ \Psi_L[C, d^\mu_{(1,2)}(\lambda)]^T \hat{\Gamma}^\mu_{(1)}(\lambda) \right.$$  
\[\times \left\{ i\hat{\Gamma}_{(1)}^\mu(\lambda) \frac{1}{\sqrt{-X'^2}} \delta_\lambda + t^\tau(\lambda)\hat{\Gamma}_{(1)}^\mu(\lambda)A^{(0)}[C, X(\lambda)] \
+ t^\tau(\lambda)\hat{\Gamma}_{(1)}^\mu(\lambda)A^{(2)}[C, X(\lambda)] \
+ \left[ t^\tau(\lambda)\hat{\Gamma}_{(2)}^\mu(\lambda)\hat{\Gamma}_{(1)}^\nu(\lambda)\hat{\Gamma}_{(1)}^\rho(\lambda)A^{(4)}_{\tau\mu\nu\rho}[C, X(\lambda)] \right] \Psi_L[C, d^\mu_{(1,2)}(\lambda)] \right]\right]. \tag{49}
\]
Here, the product of even number of $\hat{\Gamma}_{(1)}^\mu$’s has appeared between $[\Psi_L]^T$ and $\Psi_L$, so that the theory becomes chiral, without having the R-handed field functional. Notice that the $\hat{\Gamma}_{(2)}^\mu$ does not change the chirality of the fermionic field functional.
As for the Type IIA superstrings, the fermionic field functionals of both chiralities are introduced, which couple to the odd form fields in the R-R sector, giving the following fermionic action $S_f$ (IIA):

$$S_f(\text{IIA}) = \int \mathcal{D}X^\mu(\lambda) \int \mathcal{D}d^{(1)}_{\lambda}(\lambda) \int \mathcal{D}d^{(2)}_{\lambda}(\lambda) \int d\lambda \sqrt{-X'^2} \left[ \Psi[C, d^{(1)}_{\lambda}(\lambda)] \Gamma^0(1)_{\lambda} \right]$$

In this case, the coupling of the odd form fields $A^{(p)} (p = \text{odd})$ in the R-R sector changes the chirality of the field functional.

The transformation property of the R-R p-form field $A^{(p)}$ for $p \geq 2$ are summarized as follows

$$\Psi[C, d^{(\mu)}(\lambda)] \rightarrow \exp \left( i \hat{\Gamma}^{[\mu_1, \cdots, \mu_{p-2}, C]} \Lambda_{\mu_1, \cdots, \mu_{p-2}} \right) \Psi[C, d^{(\mu)}(\lambda)], \quad (51)$$

$$A^{(p)}_{\mu_1 \mu_2, \cdots, \mu_p} \rightarrow A^{(p)}_{\mu_1 \mu_2, \cdots, \mu_p} + \frac{t_{\mu_1}}{\sqrt{-X'^2}} \frac{\delta A_{\mu_3, \cdots, \mu_p}}{\delta X^\mu(\lambda)} + \text{(cyclic perm.)}. \quad (52)$$

The transformation property of the 0- and 1-form fields of the R-R sector is not clear in this formulation, but the reason why even and odd forms of the R-R sector couple to the fermionic field functional of Type IIB and IIA superstrings, respectively, is well understood. Transformation property of the NS-NS sector is not special. The local Lorentz transformation is generated by

$$M^{\hat{\mu}}(\lambda) = \frac{1}{4} \left[ \Gamma^{\hat{\mu}}(\lambda), \Gamma^{\nu}(\lambda) \right], \quad (53)$$

while the general coordinate transformation is generated by

$$T^{\mu \nu \cdots}[C, X](\lambda) = \left( \frac{\sqrt{-X'^2(\lambda)} \delta X'^\mu(\lambda)}{\sqrt{-X^2(\lambda)} \delta X^\mu(\lambda)} \right) \left( \frac{\sqrt{-X'^2(\lambda)} \delta X'^\nu(\lambda)}{\sqrt{-X^2(\lambda)} \delta X^\nu(\lambda)} \right) \cdots T^{\mu \nu \cdots}[C, X](\lambda), \quad (54)$$

where the general coordinate transformation is naturally modified $\lambda$-dependently:

$$\frac{\delta X^\nu(\lambda)}{\delta X^\mu(\lambda)} = \sum_{n,m} \frac{\partial x_n^{\nu}}{\partial x_m^{\mu}} f_n(\lambda) f_m(\lambda). \quad (55)$$
Now, starting with these superstring actions, we examine the old-fashioned dual transformation for them. In addition to the bosonic and fermionic actions, we have the kinetic terms of the NS-NS fields and of the R-R p-from fields \( A_p \) \((p = 0, 2, 4)\). The kinetic action of the R-R fields reads

\[
S_0(\text{IIB}) = \int \mathcal{D}X^\mu(\lambda) \int d\lambda \sqrt{-X'^2} \left[ (1/2) F_{(0)}^{\mu\nu}[C, X(\lambda)] F^\mu_{(0)}[C, X(\lambda)] \\
+ (1/12) F_{(2)\mu
u\rho}[C, X(\lambda)] F^{\mu\nu\rho}_{(2)}[C, X(\lambda)] \\
+ (1/240) F_{(4)\mu
u\rho\sigma}[C, X(\lambda)] F^{\mu
u\rho\sigma}_{(4)}[C, X(\lambda)] \right].
\]

(56)

Using the similar equation to Eq. (32), we can replace the R-R fields to their dual fields. For example for the R-R 2-form field, the path integration over the field \( A_{(2)} \) gives the following constraint:

\[
\begin{align*}
\frac{1}{\sqrt{g}} \frac{1}{\sqrt{-X'^2}} \frac{\delta}{\delta X^\mu} (\sqrt{g} \tilde{W}_{(2)\mu\nu}) &= 2 \int \mathcal{D}d^\mu_{(1)}(\lambda) \int \mathcal{D}d^\mu_{(2)}(\lambda) \Psi_L[C, d_{(1,2)}] T \hat{\Gamma}^\mu_{(1)} T \hat{\Gamma}^\nu_{(1)} \Psi_L[C, d_{(1,2)}] \equiv J^{\mu\nu}[C, X(\lambda)]. \\
\end{align*}
\]

(57)

This constraint can be solved as

\[
\sqrt{g} \tilde{W}_{(2)\mu\nu} = \frac{1}{6 \iota^\mu_{\nu\rho\sigma_1,\ldots,\sigma_6}} \frac{1}{\sqrt{-X'^2}} \frac{\delta}{\delta X^\mu} B_{(6)}^{\sigma_1,\ldots,\sigma_6} \\
+ \int \mathcal{D}Y^\nu(\rho) \frac{1}{\sqrt{-X'^2}} \frac{\delta}{\delta X^\nu} D[C\{X^\mu(\lambda)\}, C'\{Y^\nu(\rho)\}] \sqrt{g} J^{\nu\mu}[C'].
\]

(58)

Here, the first term in the r.h.s. is the general solution in the case of \( J^{\nu\mu} = 0 \), and the second term is given, using the propagator \( D[C, C'] \) of the bosonic string field theory. Namely, \( D[C, C'] \) is defined by

\[
\left( \frac{1}{\sqrt{-X'^2}} \frac{\delta}{\delta X^\mu} \right)^2 D[C\{X^\mu(\lambda)\}, C'\{Y^\nu(\rho)\}] = \delta[C\{X^\mu(\lambda)\}, C'\{Y^\nu(\rho)\}].
\]

(59)

Now, the relevant terms of the dual action become

\[
S^* = \int \mathcal{D}X^\mu(\lambda) \int d\lambda \sqrt{-X'^2} \left[ \mathcal{D}d^\mu_{(1)}(\lambda) \int \mathcal{D}d^\mu_{(2)}(\lambda) \\
\times \Psi_L[C, d^\mu_{(1,2)}(\lambda)] T \hat{\Gamma}^\mu_{(1)}(\lambda) i \hat{\Gamma}^\nu_{(1)}(\lambda) \frac{1}{\sqrt{-X'^2}} \frac{\delta}{\delta X^\mu(\lambda)} \Psi_L[C, d^\mu_{(1,2)}(\lambda)] \\
+ \int \mathcal{D}X^\mu(\lambda) \int d\lambda \sqrt{-X'^2} \left[ -\frac{1}{12} \left( \frac{1}{\sqrt{-X'^2}} \frac{\delta}{\delta X^\mu} \Phi^{\nu\mu}[C] + \tilde{H}^{\mu\nu} \right)^2 + \ldots \right]
\]

(60)
Here, the $\tilde{H}^{\mu\nu\rho}$ is the dual tensor of the field strength of $B_{(6)}$,
\[\tilde{H}_{\mu\nu\rho} = \frac{1}{6!} \epsilon_{\mu\nu\rho\sigma_1\sigma_2\cdots\sigma_6} \frac{1}{\sqrt{-\Delta X}} \frac{1}{\delta X_{\sigma_1}} \delta \sigma_2 \cdots \delta \sigma_6,\] (61)
and
\[\Phi^{\mu\nu}[C] = \int DY^\mu(\rho) D[C\{X^\mu(\lambda)\}, C'\{Y^\mu(\rho)\}] \sqrt{g} J^{\mu\nu}[C].\] (62)

After the dual transformation of the Type IIB superstring is performed, the obtained action in Eq. (60) includes the free fermionic and 4-fermionic terms as well as the coupling of the fermionic current to the field $H_{(3)}$ which is dually related to the RR 2-form field $A_{(2)}$.

From the above consideration, a kind of “bosonization” should work for the NS-NS and R-R field functionals, $\Phi[C]$ and $\Psi_L[C]$. Namely the $\Phi[C]^* \in$ the dual action is the “bosonization” of the $\Psi_L[C] \in$ the original action, and the $\Psi_L[C]^* \in$ the dual action is the “fermionization” of the $\Phi[C] \in$ the original action. This conjecture is by no means unrealistic, since the field functionals are defined on the closed curve $C$ and are essentially the 2 dimensional objects. If this kind of “bosonization” happens, then the free fermionic term and the 4-fermionic current-current interaction term for the R-R field functional appearing in the dual action (60) becomes the free bosonic action of the NS-NS field functional, so that the dual action approaches the NS-NS action of the Type IIB superstring. The self-duality of the Type IIB superstring may be proved in this way within the framework of the old fashioned duality.

The old fashioned dual transformation transforms the gauge theory to the hydrodynamics of Kalb-Ramond and Nambu [5] coupled to the extended objects of p-branes. Therefore, the same technique is applicable to the problem of swimming of microorganisms in the fluid. Since the envelope of the microorganisms can be considered as the closed string or membrane which couples naturally to the outside viscous fluid [21]. The study in this direction is another interesting application of the duality [22].

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