Differentiable Artificial Reverberation

Sungho Lee, Hyeong-Seok Choi, and Kyogu Lee

Abstract—Artificial reverberation (AR) models play a central role in various audio applications. Therefore, estimating the AR model parameters (ARPs) of a reference reverberation is a crucial task. Although a few recent deep-learning-based approaches have shown promising performance, their non-end-to-end training scheme prevents them from fully exploiting the potential of deep neural networks. This motivates the introduction of differentiable artificial reverberation (DAR) models, allowing loss gradients to be back-propagated end-to-end. However, implementing the AR models with their difference equations “as is” in the deep learning framework severely bottlenecks the training speed when executed with a parallel processor like GPU due to their infinite impulse response (IIR) components. We tackle this problem by replacing the IIR filters with finite impulse response (FIR) approximations with the frequency-sampling method. Using this technique, we implement three DAR models—differentiable Filtered Velvet Noise (FVN), Advanced Filtered Velvet Noise (AFVN), and Delay Network (DN). For each AR model, we train its ARP estimation networks for analysis-synthesis (RIR-to-ARP) and blind estimation (reverberant-speech-to-ARP) task in an end-to-end manner with its DAR model counterpart. Experiment results show that the proposed method achieves consistent performance improvement over the non-end-to-end approaches in both objective metrics and subjective listening test results. Audio samples are available at https://sh-lee97.github.io/DAR-samples/.

Index Terms—Digital Signal Processing, Acoustics, Reverberation, Artificial Reverberation, Deep Learning.

I. INTRODUCTION

Reverberation is ubiquitous in a real acoustic environment. It provides the listeners psychoacoustic cues for spatial characteristics. Therefore, adding an appropriate reverberation to a dry audio is desirable for plausible listening [1]–[3]. Artificial reverberation (AR), efficient digital filters that may be used to mimic real-world reverberation, have been developed to achieve such auditory effects [4]–[6] and applied to room acoustic enhancement [7], auditory scene generation [8], [9], post-production [10], and many more.

Nevertheless, estimating the AR model parameters (ARPs) that match the reference reverberation remains challenging. This is because the mapping from the ARPs to the reverberation is highly nontrivial. We refer to this task as analysis-synthesis when the reference is a room impulse response (room IR, RIR). When the reference is indirectly provided with a reverberant signal, i.e., an RIR convolved with a dry signal, we refer to this task as blind estimation. We limit the scope to reverberant speech signals in this paper, yet the framework can easily be extended to other types of signals, such as musical ones.

ARP estimation is a classical problem, and various attempts have been made [11]–[14]. However, they are only applicable to a specific AR model (AR-model-dependent) and the analysis-synthesis task (task-dependent). This inflexibility is problematic since every application has its own suitable AR models and reference forms (e.g., RIR and reverberant speech).

We can overcome such limitations with differentiable artificial reverberation (DAR) models. Each DAR model takes ARPs as input and generates its corresponding AR model’s IR (see Figure 1a). After computing a loss between the generated IR and the reference RIR, they allow the loss gradients to be back-propagated through all of their components. As a result, we can train a deep neural network (DNN) to match the reference RIR in an end-to-end manner. By leveraging the DNN’s flexibility and expressiveness, we can train the network to perform any task with any AR model with minimal architecture change.

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Since the AR models are exactly or close to linear time-invariant (LTI), we can implement them differentiably “as is” with their difference equations [15]–[18]. However, this approach has been impractical because the training speed is limited by the recurrent sample generation of their IIR filter components when executed on a parallel processor, such as a GPU. In this context, we aim to modify the vanilla DAR models to sidestep the aforementioned problem. Inspired by recent works [19], [20], we replace the IIR filters with finite impulse response (FIR) approximations using the frequency-sampling method [21]. This way, each DAR model becomes an FIR filter whose IR can be generated without any recurrent step. Using this technique, we present differentiable Filtered Velvet Noise (FVN) [13], Advanced Filtered Velvet Noise (AFVN), and Delay Network (DN) [22], [23]. Note that the AR-to-DAR conversion can be applied systematically to any other AR model with IIR components. While the DAR models are not identical to their originals, the differences are little in most cases, allowing us to train the ARP estimation networks reliably with them. Furthermore, even when differences exist, the benefits of the end-to-end learning enabled by the DAR models outweigh them; we show that the proposed approach brings consistent performance improvement over non-end-to-end baselines where the difference is only in the training scheme. After the training, we can revert to the original AR models for efficient computation (see Figure [16]).

II. RELATED WORKS

A. Differentiable Digital Signal Processing

Digital signal processing (DSP) and deep learning are closely related. For example, a temporal convolutional network [24] is a stack of dilated one-dimensional convolutional layers with nonlinear activation layers and it has been used for modeling dynamic range compression [25] and reverberation [26]. From the DSP viewpoint, it is FIR filter banks serially connected with memoryless nonlinearities, i.e., an extension of the classical Wiener-Hammerstein model [27].

If we look for more explicit DSP-deep learning relationships, so-called “differentiable digital signal processing (DDSP)” approaches exist which aim to import DSP components into deep learning frameworks as they provide strong structural priors and interpretable representations. For example, a harmonic-plus-noise model [28] was implemented in such a manner; then, a DNN was trained to estimate its parameters to synthesize a monophonic signal and used for various applications, including controllable synthesis and timbre transfer [29].

In the case of LTI filters, it is well-known that both an FIR and IIR filter are differentiable and can be directly imported into the deep learning framework as a convolutional layer and recurrent layer, respectively [15]–[18]. Since the IIR filter, e.g., parametric equalizer (PEQ), bottleneck the training speed, the frequency-sampling method [21] was utilized [19], [20]. Upon these works, we investigate the reliability of this technique, present a general differentiable IIR filter based on state-variable filter (SVF) [30], [31], and use it to implement the DAR models.

B. ARP Estimation

1) Analysis-synthesis: Various methods have been proposed to systematically and automatically estimate ARPs that mimic certain characteristics of a given RIR. Some works first extract perceptually relevant features from the RIR, then estimate the ARPs from them. For example, FDN parameters were derived from the RIR’s energy decay relief (EDR) representation which encodes the frequency-dependent reverberation decay [11]. Along with the FVN and AFVN, their parameter estimation algorithm based on linear prediction were proposed together [13]. Another thread of work utilizes optimization. A genetic algorithm was proposed to find the FDN parameters that achieve the desired “fitness” to the reference RIR [12]. Since all these methods assume availability of the RIR, they cannot be applied to the blind estimation task.

2) Blind Estimation: End-users (e.g., audio engineers) often interact with the AR models provided as audio plug-ins. As a result, several attempts have been made to implement blind estimation algorithms for the plug-ins. For example, a preset recommendation method based on the Gaussian mixture model was proposed [14]. More recently, DNNs were trained for the plug-in parameter estimation and preset recommendation [10].

While these methods have shown promise, there is still room for improvement. First, they generated the training data using the plug-ins to train their models. However, this procedure may be time-consuming, and relying on data generated with a single plug-in may increase the generalization error. Second, their training objectives could be suboptimal. When the task is defined as parameter regression [10], one must weigh each parameter’s perceptual importance and apply it to the loss function design, which is a highly challenging task. When the task is defined as classification [10], [14], it could suffer from scalability problems. Finally, most plug-ins provide only a few parameters, and each of them controls multiple internal ARPs simultaneously. This might limit the performance due to the reduced degree of freedom.

We tackle these problems with the DAR models as follows. First, we can use a training objective that directly compares the estimated IR with ground-truth RIR. Therefore, we can use any RIR for training and bypass the need to obtain the ground-truth ARPs. Second, we replace the parameter-matching loss with the end-to-end IR-matching loss and show that it improves the estimation performance by a large margin in both objective metrics and subjective listening test scores. Finally, we estimate the ARPs directly instead of relying on the plug-in parameters.

3) Two-Stage Approaches: Note that both analysis-synthesis and blind estimation tasks can be divided into two subtasks. First, we can obtain reverberation parameters, e.g., reverberation time, from a given reference (reference-to-reverberation parameters). We can directly compute the parameters from the RIR [32] for the analysis-synthesis case. For the blind estimation, various methods have been proposed [33]–[36]. Then, we can tune the ARPs to match the desired reverberation parameter values (reverberation parameters-to-ARPs) [37]–[39]. This two-stage approach can be a possible alternative to our single-stage end-to-end estimation method. However, perceptually different reverberations can have the same reverberation parameters, resulting in information loss.
C. Modeling Reverberation in the Deep Learning Framework

An alternative to the proposed DAR approach could be using existing deep learning modules to build another reverberation model. The most simplest approach could be using a high-order FIR directly as an RIR model [29]. While it is viable when the objective is to learn a single RIR, training a DNN to generate various raw RIRs is highly challenging and inefficient.

Instead, recent approaches either use a DNN as a reverberator and control them via conditioning methods [26, 40] or design a differentiable RIR generator and use a DNN as a parameter estimator [41]. Our method is closely related to the latter since the DAR model and the proposed ARP estimation network act in that way. However, the key difference is that our DAR models are direct imports of the AR models, which bring the following benefits. First, they are more directly interpretable and controllable, allowing end-users to further tune the parameters. Second, their structure can prevent unexpected artifacts. Finally, they use sparse IIR and FIR filters, enabling efficient real-time computation in CPU.

III. FREQUENCY-SAMPLING METHOD FOR DIFFERENTIABLE IIR FILTERS

For any real IIR filter $H$, its frequency response $H(e^{j\omega})$ is continuous, 2π-periodic, and conjugate symmetric. To obtain its FIR approximation, we frequency-sample $H(e^{j\omega})$ at angular frequencies $\omega_k = 2\pi k/N$ where $k = 0, \ldots, \lfloor N/2 \rfloor$ [21]. We denote the frequency-sampled filter with subscript $N$, i.e., $H_N$.

$$H_N[k] = H(e^{j\omega_k}).$$  \hspace{1cm} (1)

Note that the frequency-sampling can also be defined on transfer functions, e.g., $H(z)$ with $z = e^{j\omega_k}$.

Since each sampling is independent, $H_N$ can be generated simultaneously with a parallel processor like GPU. Furthermore, the order of frequency-sampling ($\cdot N$) and other basic arithmetic operations does not matter. Therefore, we can frequency-sample an IIR filter by combining frequency-sampled $m$-sample delays $(z^{-m})_N \in \mathbb{C}^{(N/2)+1}$ as follows,

$$H_N[k] = \left( \sum_{m=0}^{M} \beta_m z^{-m} \right)_N [k] = \frac{\sum_{m=0}^{M} \beta_m(z^{-m})_N[k]}{\sum_{m=0}^{M} \alpha_m(z^{-m})_N[k]}.$$ \hspace{1cm} (2)

Since $(z^{-m})_N[k] = e^{-j2\pi km/N}$, using the widely-used deep learning libraries, we obtain $(z^{-m})_N$ as a tensor $z_{-m}$ by

$$\text{angle} = 2\pi \text{angle}(N/2+1)/N, \hspace{1cm} \text{angle} \equiv (-1) \text{angle}.$$ \hspace{1cm} (3a)

$$z_{-m} = e^{-(1) \text{angle} \times m}.$$ \hspace{1cm} (3b)

Then, we apply equation (2) to $z_{0}, \ldots, z_{M}$ with basic tensor operations and obtain every sample of $H_N$ in parallel. A time-domain representation of $H_N$ is a length-$N$ FIR $h_N[n]$, inverse discrete Fourier transform (inverse DFT) of $H_N[k]$, i.e.,

$$h_N[n] = \frac{1}{N} \sum_{k=0}^{N-1} H_N[k] e^{j\omega_n / N} (u[n] - u[n - N])$$ \hspace{1cm} (4)

where $u[n]$ is a unit step function.

Therefore, using the frequency-sampling method, we can replace any IIR filter $H$ that causes the bottleneck with an FIR $h_N[n]$ whose convolution can be performed efficiently via Fast Fourier Transform (FFT). In this paper, the “differentiable IIR” filter refers to $H_N$, which is in fact an FIR filter.

A. Reliability of the Frequency-Sampling Method

Every IIR filter’s frequency response has “infinite bandwidth” (in the time domain) so that the frequency-sampling causes time-aliasing [42]. Specifically, the frequency-sampled IR $h_N[n]$ is sum of length-$N$ segments of the original IR $h[n]$.

$$h_N[n] = \sum_{m=0}^{\infty} h[mN + n](u[n] - u[n - N]).$$ \hspace{1cm} (5)

Since the energy of every stable IIR filter’s IR decays over time, more frequency-sampling points $N$ give less time-aliasing, i.e., a closer approximation (see Figure 2). Therefore, we can use the differentiable filter $H_N$ in place of the original $H$ to match a reference response $H_{Ref}$. We elaborate this with following analytical results.

1) Time-aliasing Error: If $H$ has $M$ distinct poles where each of them $\nu_i \in \mathbb{C}$ has multiplicity $r_i \in \mathbb{N}$, the time-aliasing error asymptotically decreases as $N$ increases as follows,

$$\|H - H_N\|_2 = \sum_{i=1}^{M} O(N^{\nu_i - 1}|\nu_i|^N).$$ \hspace{1cm} (6)

2) Loss Error: The triangle inequality indicates that the loss error is bounded to the time-aliasing error.

$$\|H_{Ref} - H\|_2^2 - \|H_{Ref} - H_N\|_2^2 \leq \|H - H_N\|_2^2.$$  \hspace{1cm} (7)

3) Loss Gradient Error: The loss gradient error has similar asymptotic behavior to the time-aliasing error as follows,

$$\left| \frac{\partial}{\partial p} \|H_{Ref} - H\|_2^2 - \frac{\partial}{\partial p} \|H_{Ref} - H_N\|_2^2 \right| = \sum_{i=1}^{M} O(N^{3r_i - 1}|\nu_i|^N)$$ \hspace{1cm} (8)

where $p$ is a parameter of $H$. See Appendix C for the proofs.

B. Differentiable Dense IIR Filter with State-variable Filters

Since arbitrary IIR filter can be expressed as serially cascaded biquads (second-order IIR filters), we can obtain a differentiable

| Magnitude (dB) | Original Filter | Frequency-Sampled |
|---------------|----------------|-------------------|
| $N = 10$      | ![Graph](image1) | ![Graph](image2) |
| $N = 20$      | ![Graph](image3) | ![Graph](image4) |
| $N = 40$      | ![Graph](image5) | ![Graph](image6) |
| $N = 80$      | ![Graph](image7) | ![Graph](image8) |

Fig. 2. The frequency-sampling method with a various number of sampling points $N$. Blue curves represent its magnitude response $|H(e^{j\omega})|$ (top row) and IR $h[n]$ (bottom row). Red curves are the magnitude response and IR of the frequency-sampled filter $H_N$.
IIR filter $\hat{H}$ as a product of frequency-sampled biquads $(H_i)_N$.

$$\hat{H}[k] = \prod_i (H_i)_N[k] = \prod_i \left( \sum_{m=0}^{2} \beta_{i,m} z^{-m} \right) \left( \sum_{m=0}^{2} \alpha_{i,m} z^{-m} \right)_N[k]. \quad (9)$$

Moreover, we use the state variable filter (SVF) parameters $f_i$, $R_i$, $m_i^{LP}$, $m_i^{BP}$, and $m_i^{HP}$ to express each biquad as follows,

$$\beta_{i,0} = f_i^2 m_i^{LP} + f_i m_i^{BP} + m_i^{HP}, \quad (10a)$$
$$\beta_{i,1} = 2 f_i^2 m_i^{LP} - 2 m_i^{HP}, \quad (10b)$$
$$\beta_{i,2} = f_i^2 m_i^{LP} - f_i m_i^{BP} + m_i^{HP}, \quad (10c)$$
$$\alpha_{i,0} = f_i^2 + 2 R_i f_i + 1, \quad (10d)$$
$$\alpha_{i,1} = 2 f_i^2 - 2, \quad (10e)$$
$$\alpha_{i,2} = f_i^2 - 2 R_i f_i + 1. \quad (10f)$$

We prefer to use the SVF parameters rather than the biquad coefficients due to the following reasons.

- **Interpretability.** Each SVF has lowpass $H_i^{LP}$, bandpass $H_i^{BP}$, and highpass filter $H_i^{HP}$. They share a resonance $R_i$ and cutoff parameter $f_i = \tan(\pi \omega_c / \omega_s)$ where $\omega_c$ is their cutoff frequency and $\omega_s$ is the sampling rate. The filter outputs are multiplied with gains $m_i^{LP}$, $m_i^{BP}$, and $m_i^{HP}$ and then summed. This interpretability comes without any generality loss since an SVF can express any biquad [31].

- **Simple Activation Functions.** The SVF parameters have simpler stability conditions, $R_i > 0$ and $f_i > 0$, than the biquad coefficients [18], which leads to simpler activation function design. Refer to Section V-C for details.

- **Better Performance.** We found that our ARP estimation networks performed better when they estimate the SVF parameters rather than the biquad coefficients. Refer to Appendix A-B for the comparison results.

From now on, we denote each SVF-parameterized biquad with a superscript $H_{i\text{SVF}}$ and call it simply “SVF”.

**C. Differentiable Parametric Equalizer**

A low-shelving $H_i^{LS}$, peaking $H_i^{Peak}$ and high-shelving filter $H_i^{HS}$ are widely used audio filters. Each of them can be obtained using the SVF $H_{i\text{SVF}}(f, R, m_i^{LP}, m_i^{BP}, m_i^{HP})$ as follows [30],

$$H_i^{LS}(f, R, G) = H_{i\text{SVF}}(f, R, G, 2R\sqrt{G}, 1), \quad (11a)$$
$$H_i^{Peak}(f, R, G) = H_{i\text{SVF}}(f, R, 1, 2RG, 1), \quad (11b)$$
$$H_i^{HS}(f, R, G) = H_{i\text{SVF}}(f, R, 1, 2\sqrt{G}, G). \quad (11c)$$

Here, $G$ is a new parameter that gives $20 \log_{10}(G)$ dB gain. A parametric equalizer (PEQ) is serial composition of such filters. Following the recent works [19], [20], we use one low-shelving, one high-shelving, and $K - 2$ peaking filters,

$$H_{i\text{PEQ}}(z) = H_i^{LS}(z) H_i^{HS}(z) \prod_{i=1}^{K-2} H_i^{Peak}(z). \quad (12)$$

Same as the IIR filter case, frequency-sampling the components and multiplying them results in a differentiable PEQ $H_{i\text{PEQ}}^N$.

**IV. DIFFERENTIABLE ARTIFICIAL REVERBERATION**

In this section, we derive differentiable FVN, AFVN, and DN. We first briefly review the original AR models. Then, we obtain their DAR counterparts by frequency-sampling the IIR components, replacing them with FIRs. Also, we slightly modify the original models for efficient training, plausible reverberation, and better overall estimation performance.

**A. Differentiable Filtered Velvet Noise**

1) Filtered Velvet Noise: One can divide an RIR $h[n]$ into $S$ segments $h_i[n]$ and approximate each segment with a velvet noise $v_i$ filtered with an allpass filter $U_i$ and a coloration filter $C_i$. We obtain the differentiable FVN and AFVN by converting the IIR filters $U_i$ and $C_i$ into FIR filters $u_i$ and $(c_i)_N$. Each IIR filter is expressed with the double border box.

![Fig. 3. Modified FVN, AFVN’s RIR approximation, and their differentiable implementation strategy.](image)

In this section, we derive differentiable FVN, AFVN, and DN. We first briefly review the original AR models. Then, we obtain their DAR counterparts by frequency-sampling the IIR components, replacing them with FIRs. Also, we slightly modify the original models for efficient training, plausible reverberation, and better overall estimation performance.

**A. Differentiable Filtered Velvet Noise**

1) Filtered Velvet Noise: One can divide an RIR $h[n]$ into $S$ segments (see Figure 3). As reverberation can be regarded as mostly stochastic, we can model each length-$L_i$ segment $h_i[n]$ with a source noise signal $s_i[n]$ “colored with” a filter $C_i$. When we use velvet noise for each source, this source-filter model becomes Filtered Velvet Noise (FVN) [13], [43], [44]. The velvet noise $v_i[n]$ is sparse; in every length-$T_i$ interval, it contains a single nonzero sample $\pm 1$ with random sign/position such that its time-domain convolution is highly efficient. Moreover, an allpass filter $U_i$ is introduced to smooth each source and make plausible sound. Finally, an deterministic (bypass) FIR $h_0[n]$ models the direct arrival and early reflection. Then, an IR of FVN $h[n]$ becomes

$$h[n] = h_0[n] + \sum_{i=1}^{S} v_i[n - d_i] \ast u_i[n] \ast c_i[n] \quad (13)$$

where each $d_i$ is a delay for the segment alignment ($d_i = 0$). In practice, each allpass filter $U_i$ is composed with Schroeder allpass filters (SAPs), $U_i^{SAP}(z) = (1 + \gamma_{i,j} z^{-\tau_{i,j}})/(\gamma_{i,j} + z^{-\tau_{i,j}})$, where $\tau_{i,j}$ is a delay-line length and $\gamma_{i,j}$ is a feed-forward/back gain) for efficiency [45]. A low-order dense IIR filter is used as each coloration filter $C_i$. 

![Fig. 3. Modified FVN, AFVN’s RIR approximation, and their differentiable implementation strategy. FVN and AFVN divide a target RIR $h[n]$ into segments $h_i[n]$ and approximate each segment with a velvet noise $v_i$ filtered with an allpass filter $U_i$ and a coloration filter $C_i$. We obtain the differentiable FVN and AFVN by converting the IIR filters $U_i$ and $C_i$ into FIR filters $u_i$ and $(c_i)_N$. Each IIR filter is expressed with the double border box.](image)
2) Differentiable Implementation: We modify each FVN component to obtain differentiable FVN.

- Velvet filters. To batch the IR segments, we use the same segment lengths \( L_1, \ldots, L_S \) and average pulse distance values \( T_1, \ldots, T_S \) for every reference RIR.
- Allpass filters. We tune and fix the allpass filter parameters jointly with the velvet filter parameters to avoid perceptual artifacts, e.g., discontinuous and rough sound, while being computationally efficient. Since we fixed the filter, instead of the frequency-sampling method, we simply crop its IR to obtain an FIR \( \hat{h}_i[n] \).
- Coloration filters. We model \( C_i \) with \( K \) serial SVFs, i.e.,
\[
C_i(z) = \prod_{k=1}^{K} C_{i,k}^{SVF}(z).
\]
We frequency-sample each filter \( C_{i,k}^{SVF}(z) \) and convert it into a length-\( N \) FIR \( (c_i)_N[n] \) so that we can train our network to estimate its parameters.

Therefore, IR of the differentiable FVN \( \hat{h}[n] \) becomes
\[
\hat{h}[n] = \hat{h}_0[n] + \text{overlap-add} \left( \sum_{i=1}^{M} (c_i)_N[n] \right).
\]

Since every component \( v_i[n], \hat{u}_i[n], \) and \( (c_i)_N[n] \) is an FIR, we can compute each segment \( \hat{h}_i[n] = v_i[n] * \hat{u}_i[n] * (c_i)_N[n] \) efficiently by convolving the FIRs in the frequency domain, i.e., zero-padding followed by FFT, multiplication, then IFFT. Finally, we overlap-add all the segments and add the deterministic FIR \( \hat{h}_0[n] \) to obtain the full IR \( \hat{h}[n] \). The overlap-add can be implemented with a transposed convolution layer.

While the modifications for the differentiable FVN introduce approximation error, it is negligible in practice. We explain this with the EDR \( E^{EDR} \), a reverse cumulative sum of an IR’s short-time Fourier transform (STFT) energy \( |H[n]|^2 \).
\[
E^{EDR} = 10 \log_{10} H^{EDR} = 10 \log_{10} \hat{H}^{EDR}.
\]

Figure 4a shows the EDRs and the EDR error. It reveals that the error is small (at most 3dB) and mostly in the low-energy region, (below \(-40\)dB). Also, there is little EDR error at the first time index, i.e., \( |E^{EDR}[k,0]| \ll 1 \), indicating that the overall magnitude response remains the same during the conversion process. In conclusion, the error is hardly noticeable, and the differentiable FVN can replace the original FVN for the training (we use a loss function related to the EDR; see Section V-D).

3) Modifications to the Original Model: FVN was initially designed for the analysis-synthesis of late reverberation; it used the direct arrival and early reflections cropped from the RIR and modeled only the late part. However, we also aim to solve the blind estimation problem where we must estimate the entire RIR. To this end, we modify the original model as follows.
- Finer segment gain. We divide each velvet segment \( v_i[n] \) into \( M \) smaller segments \( v_{i,1}[n], \ldots, v_{i,M}[n] \) multiplied with gains \( g_{i,1}, \ldots, g_{i,M} \). Figure 5c shows the resulting
velvet filter \( V_r \) in the time domain.

- **Cumulative allpass.** The original model smoothes every velvet noise with the same allpass filter. This is undesirable in our context since it could smear the direct arrival and early reflections. Instead, we gradually cascade the SAPs to obtain each allpass filter \( U_j(z) = \prod_{j=1}^{K_j} U_{j,SAP}(z) \) so the earlier segments are less smeared than the later ones. As shown in Figure 5a, this modification can be implemented in the time domain by inserting each SAP \( U_{j,SAP} \) before the corresponding velvet filter \( V_r \).

- **Shorter deterministic FIR.** With the above changes, the stochastic segments can fit the early RIR to some extent. Therefore, we shorten the early deterministic FIR \( h_0[i] \).

Refer to Appendix A-A for the evaluation of the modifications.

4) **Model Configuration Details.** Exact configurations of our FVN used for the experiments are as follows.

We generated 2.5 seconds of IR (48kHz sampling rate, total \( L = 120k \) samples) with FVN. We used \( S = 20 \) non-uniform velvet segments whose lengths \( L_i \) are \( L/40, L/20, \) and \( L/10 \) for 10, 5, and 5 segments, respectively, and their average pulse distances were set to \( T = [10, 20, 35, 50, 65, 90, 120, 135, 180, 220, 270, 320, 370, 420, 480, 540, 610, 680, 750, 820] \). The SAPs \( U_{j,SAP} \) were forced to have gains \( g_j = 0.75 + 0.01j \) and delay lengths \( \tau_j = [23, 48, 79, 109, 113, 127, 163, 191, 229, 251, 293, 337, 379, 421, 449, 509, 541, 601, 641, 691] \). Each velvet segment had \( M = 4 \) sub-segment gains and filtered with \( K = 8 \) SVFs. We frequency-sampled the coloration filters with \( N = 4k \) points. The length of the deterministic FIR \( h_0[i] \) was set to 50. We set the gain \( g \in \mathbb{R}^{S \times M} \), SVF parameters \( f, \mathbf{m}, \mathbf{M} \), and the bypass FIR \( h_0[i] \in \mathbb{R}^Z \) to estimation targets (total 930 ARPs). The time-domain FVN \( \hat{H} \) requires 2166 floating point operations per sample (FLOPs).

B. **Differentiable Advanced Filtered Velvet Noise**

Since frequency-dependent decay of reverberation is gradual, one can model each coloration filter \( C_1 \) as an initial coloration filter \( C_1 \) cascaded with delta-coloration filters \( C_{\Delta_2}, \ldots, C_{\Delta_i} \):

\[
C_i(z) = C_1(z) \prod_{j=2}^{i} C_{\Delta_j}(z). \tag{17}
\]

FVN with this modification is called Advanced Filtered Velvet Noise (AFVN) [13]. The delta filters’ orders \( \Delta_{K_2}, \ldots, \Delta_{K_S} \) are set lower than the initial filter’s \( K_1 \) for the efficiency. See Figure 5b for its time-domain implementation. We used \( K_1 = 8 \) and \( K_{\Delta} = K_{\Delta_2} = \cdots = K_{\Delta_S} = 2 \) in the experiments. This results in a total of \( K = K_1 + (S-1)K_{\Delta} = 46 \) SVFs. The other settings are the same as the FVN. This AFVN has 360 ARPs to estimate and its time-domain model \( \hat{H} \) requires 1511 FLOPs. While order of the coloration filters are higher than the FVN’s counterparts, the introduced frequency-sampling error remains approximately the same in practice (see Figure 5b).

C. **Differentiable Delay Network**

1) **Delay Network:** By interconnecting multiple delay lines in a recursive manner, one can obtain a Delay Network (DN) [22, 23] structure. A difference equation of the DN can be written in a general form as follows (we omit the bracket notation for the filtering).

\[
y[n] = C^T \hat{y}[n] + \hat{H}_0 x[n], \tag{18a}
\]

\[
y[n] + d = A y[n] + B x[n]. \tag{18b}
\]

That is, an input \( x[n] \) is distributed and filtered (or simply scaled) with \( B \), then go through \( d \)-sample parallel delay lines which are recursively interconnected to themselves through a mixing filter matrix \( A \). The delay line outputs \( \hat{y}[n] \) are filtered and summed with \( C \). We add a bypass signal filtered with \( \hat{H}_0 \), resulting in an output \( y[n] \). Transfer function of the DN is

\[
\hat{H}(z) = C(z)^T (D^{-1}(z) - A(z))^{-1} B(z) + \hat{H}_0(z). \tag{19}
\]

Here, \( D(z) = \text{diag}(z^{-d}) \) is an \( M \times M \) transfer function matrix for the delay lines, i.e., \( D_{ii}(z) = z^{-d_i} \). \( B(z), C(z), \) and \( A(z) \) are input, output, and feedback transfer function matrices of shape \( M \times 1, 1 \times M, \) and \( M \times M, \) respectively.

2) **Differentiable Implementation:** The DN components are recursively interconnected such that one cannot divide its IR into independent segments and generate them in parallel like we did with the FVN and AFVN. Instead, we frequency-sample its entire transfer function to obtain differentiable DN, which is equivalent to a composition of individually frequency-sampled transfer function matrices.

\[
\hat{h}[n] = \text{IFFT} \left\{ C_y^T (D_N^{-1} - A_N)^{-1} B_N(n) + \hat{H}_0(n) \right\}. \tag{20}
\]

Here, \( D_N = \text{diag}(z^{-d}) \in \mathbb{C}^{M \times M \times [N/2+1]} \) is a frequency-sampled version of the delay line transfer function matrix \( D \), i.e., \( (D_N)_{ii} = (z^{-d_i})_N \in \mathbb{C}^{[N/2+1]} \). Likewise, we can frequency-sample the transfer function matrices \( B, C, \) and \( A \) to obtain their approximations \( B_N, C_N, \) and \( A_N \), respectively.

Each frequency-sampled transfer function matrix is a batch of \( [N/2+1] \) matrices, the matrix multiplications and inversions of equation (20) can be performed in parallel.
3) Restrictions to the General Model: The presented DN structure is fully general and able to express many variants, including the Feedback Delay Network (FDN) [46]. Therefore, we can also derive their differentiable versions following the proposed method. However, when following the general practice that uses many delay lines (e.g., $M = 16$) for a high-quality reverberation, its differentiable model’s frequency-sampled transfer function matrices consume too much memory, and their multiplications and inversions bottleneck the training speed. To tackle this, we reduce $M$ and modify the components as follows to make DN plausible with low $M$ (see Figure 6).

- **Pre and Post Filter Matrix.** Following most previous works [11], [47], [48], the pre filter matrix $B$ is set to a (constant) gain vector $b$. The post filter matrix $C$ is a combination of a gain vector $c$ and a common filter $C_1$, i.e., $C = C_1 c$. The common filter $C_1$ is composed of serial $K_{C_1}$ SVFs.

- **Feedback Filter Matrix.** While most DN model combines channel-wise parallel delta-coloration (absorption) filters $C_\Delta$ and an inter-channel mixing matrix $Q$ to composite the feedback filter matrix $A = QC_\Delta$. Additionally, we insert channel-wise allpass filters $U$ and introduce time-variability by modulating the mixing matrix $Q$. This results in the time-varying feedback filter matrix $A_n = Q_n U C_\Delta$.

- **Allpass Filters.** To achieve faster echo density build-up without adding more delay lines, we insert serial $K_U$ SAPs in each feedback path. Unlike the FVN and AFVN cases, we estimate the SAP gains $\gamma \in \mathbb{R}^{M \times K_U}$ with the estimation network and frequency-sample the allpass filter matrix $U$ for the differentiable model.

- **Time-varying Mixing Matrix.** We further reduce the audible “ringing” artifacts by modulating the stationary poles with the time-varying mixing matrix, which is set to $Q_n = Q_0 R^n$ where $Q_0$ is a Householder matrix and $R$ is a tiny rotational matrix constructed from a random matrix [48], [49]. Since the frequency-sampling method is only applicable to LTI filters, we fix the mixing matrix to $Q_0$ when obtaining the differentiable model.

- **Absorption Filters.** Unlike the conventional FDN [46], we use the same absorption filter $C_\Delta$ for every channel. Since the mixing matrix $Q_n$ is always unitary, our DN becomes stable when $C_\Delta$ has magnitude response less than 1 [48]. We achieve this by using a PEQ with additional constraints (see Section V.C) as $C_\Delta$.

- **Bypass FIR.** We use a length-Z FIR $h_0[n]$ for the bypass.

We summarize the modifications. The difference equation of our time-varying DN $H_n^N$ is

$$y[n] = C_1 e^{T} \hat{y}[n] + H_0 x[n],$$  
$$\hat{y}[n + d] = Q_n U C_\Delta \hat{y}[n] + b x[n].$$

The transfer function of its LTI approximation $\hat{H}(z)$ is given as equation (19) with the restrictions $B(z) = b$, $C(z) = C_1 c$, and $A(z) = Q_0 \text{diag}(U_N(z) \cap C_\Delta(z))$. Hence, the IFFT of

$$(C_1)_N e^{T}(D_N^{-1} - Q_0 \text{diag}(U_N \cap (C_\Delta)_N))^{-1} b$$

summed with the bypass FIR $\hat{h}_0[n]$ results in our differentiable DN’s IR $\hat{h}[n]$.

Figure 7 shows the EDR errors between the three different DN modes, i.e., differentiable DN, original LTI, and the time-varying one. The only difference between the first two is the frequency-sampling; unless the DN decays very slowly, there is little time-aliasing and EDR error. However, due to the pole modulation, the EDR error increases significantly when the time-varying mixing matrix is applied. Despite this, we find that other perceptually important factors, such as reverberation time, remain largely unchanged (see Section VI-A).

4) Model Configuration Details: We used $M = 6$ delay lines with delay lengths $d = [233, 311, 421, 461, 587, 613]$ in samples. We used $K_U = 4$ SAPs for each delay line. We fixed the SAP delay lengths to $\tau = [131, 151, 337, 353, 103, 173, 331, 373, [89, 181, 307, 401], [79, 197, 281, 419], [61, 211, 257, 431], [47, 229, 251, 443]$]. Note that the SAPs introduce additional delays so that the effective delay lengths are [1205, 1291, 1399, 1437, 1547, 1583] or about [25.1, 26.9, 29.1, 29.9, 32.2, 33.0] in milliseconds. Our rotational matrix satisfies $R^{M_k} = I$ so that the mixing matrix $Q_n$ has a period of 0.625 second. Both post $C_1$ and absorption filter $C_\Delta$ have $K_{C_1} = K_{C_\Delta} = 8$ components. We used $N = 120k$ frequency-sampling points. The order of the bypass FIR is $Z = 100$. The post filter parameters $f_{C_1}, R_{C_1}, m_{BP}, m_{CF}, m_{BP} \in \mathbb{R}^{1 \times K_{C_1}}$, feedback filter parameters $f_{C_\Delta}, R_{C_\Delta}, G_{C_{\Delta}} \in \mathbb{R}^{K_{C_\Delta}}$, pre and post gain vectors $b, c \in \mathbb{R}^{M \times 1}$, SAP gain vector $\gamma \in \mathbb{R}^{M \times K_U}$, and the bypass FIR $h_0 \in \mathbb{R}^Z$ are the estimation targets (total 200 ARFs). The resulting time-domain model $\hat{H}$ and the time-varying model $\hat{H}_n^N$ consume 889 and 1285 FLOPs, respectively.

V. Artificial Reverberation Parameter Estimation with a Deep Neural Network

The details of our ARP estimation network are as follows. As shown in Figure 8, it transforms single channel audio input (either IR or reverberant speech) into a shared latent $\mathbf{z}$ with the model/task-agnostic encoder. Then, each ARP-groupwise layer projects the latent $\mathbf{z}$ into an ARP tensor $\mathbf{P}_i$. 
A. AR-Model/Task-Agnostic Encoder

We first transform the reference RIR or reverberant speech into a log-frequency log-magnitude spectrogram. Then, the spectrogram goes through five two-dimensional convolutional layers, each followed by a rectified linear unit (ReLU) activation. We apply gated recurrent unit (GRU) layers [50] along with the frequency axis regarding the channels as features and the time axis regarding the channels and frequencies as features, widening the receptive field. Finally, we apply two linear layers followed by layer normalization [51] and ReLU, resulting in the two-dimensional shared latent $z$. The detailed network configurations are shown in Figure 8, and the encoder has about 7.3M parameters.

B. ARP-Groupwise Projection Layers

Now we aim to transform the two-dimensional shared latent $z$ into each ARP tensor $P_i$, which is at most two-dimensional. To achieve this, we use two axis-by-axis linear layers. We first transpose and apply a linear layer to the latent to match the first axis shape. Then, another transposition and a linear layer transpose and apply a linear layer to the latent to match the second axis. This approach factorizes a single large layer into two smaller ones, reducing the number of parameters.

C. Nonlinear Activation Functions and Bias Initialization

Each ARP group has different stability conditions and desired distribution. To satisfy these, we attach a nonlinear activation to each projection layer and initialize the last linear layer’s distribution. To satisfy these, we attach a nonlinear activation $C$. Nonlinear Activation Functions and Bias Initialization match the other axis. This approach factorizes a single large first axis shape. Then, another transposition and a linear layer transpose and apply a linear layer to the latent to match the into each ARP tensor $z$ has about

$\bar{\omega}$ having higher initial cutoff frequencies. For the resonance $R$, we use a scaled softplus $\zeta(x) = \log(1 + e^x) / \log(2)$ to center the initial distribution at 1 and satisfy the stability condition $R > 0$.

For cutoff $f$ we also have the same stability condition $f > 0$. However, instead of the softplus, we use $\tan^{-1}(\pi f(x)/2)$ where $\sigma(x) = 1/(1 + e^{-x})$ is a logistic sigmoid. With this activation, $\sigma(x) = 0$ and 1 represent cutoff frequency of 0Hz and half of the sampling rate, respectively.

The delta-coloration PEQ $C_{\Delta}$ inside the DN must satisfy the stability condition $|C_{\Delta}(e^{j\omega})| < 1$. To achieve this, we use $10^{-\zeta(x)}$ for the component gain $G$. Also, the shelving filters’ resonances should satisfy $R > \sqrt{2}/2$ to prevent their magnitude responses to “spike” over 1. Therefore, we add $\sqrt{2}/2$ after the softplus activation.

For the feed-forward/back gains $\gamma$ of channel-wise allpass filters $U$ inside the DN, we use sigmoid activation $\sigma(x)$.

2) Bias Initialization:

- We initialize bias of the $f$ projection layer to $\sigma^{-1}(2\omega_k/\omega_s)$ where each $\omega_s$ is desired initial cutoff frequency of $k$th SVF. We set $\omega_k = \omega_{\min}(\omega_{\max}/\omega_{\min})^{(K-1)/(k-1)}$ such that the frequencies are equally spaced in the logarithmic scale from $\omega_{\min} = 40Hz$ to $\omega_{\max} = 12kHz$. To the SVFs inside the AFVN’s initial and delta-coloration filters, we perform a random permutation to the frequency index $k$ and obtain a new index $k$. This prevents SVFs for the latter segments having higher initial cutoff frequencies.

- Decoder biases for the SVF mixing coefficients $m^{\text{LP}}$, $m^{\text{BP}}$, and $m^{\text{HP}}$ are initialized to 1, 2, and 1, respectively, so that each SVF’s initial magnitude response slightly deviates from 1. This prevents the coloration filters’ responses and the estimation network loss gradients to vanish or explode.

- For the PEQ gains $G$ of the DN’s delta-coloration filters, we set each of their biases to $-10$ such that the initially generated IRs have long enough reverberation time.

D. Loss Function

1) Match Loss: We utilize a multi-scale spectral loss [29] and define a match loss as follows,

$$L_{\text{MATCH}}(h, \hat{h}) = \sum_i \left\| H^\text{STFT}_i - \hat{H}^\text{STFT}_i \right\|_1.$$ (23)

$i$ denotes STFT with different FFT sizes. We use the FFT sizes of 256, 512, 1024, 2048, and 4096, hop sizes of 25% of the respective FFT sizes, and Hann windows. Each spectrogram’s frequency axis is log-scaled like the encoder’s spectrogram.

2) Regularization: We additionally apply a regularization to reduce time-aliasing of the estimation. As shown in equation [5] and [6], pole radii of each SVF affects the reliability of the frequency-sampled one (hence DAR model) and its parameter...
estimator. Since each SVF’s IR \((e_{SVF}^{SVF})_N[i]\) can be computed, we encourage reducing its pole radii by penalizing the IR’s decreasing speed. Each decreasing speed \(\gamma_{i,k}\) is calculated by the ratio of the average amplitude of first and last \(n_0\) samples of \((e_{SVF}^{SVF})_N[i]\). We fix \(n_0\) to \(N/8\). Then, each \(\gamma_{i,k}\) is weighted with a softmax function along the SVF-axis to penalize higher \(\gamma_{i,k}\) more. Sum of the weighted decreasing speed values results in the regularization loss \(L_{\text{REG}}\) as follows,

\[
\gamma_{i,k} = \frac{\sum_{n=N-n_0}^{N} \left| (e_{SVF}^{SVF})_N[i] \right|}{\sum_{n=0}^{N-n_0} \left| (e_{SVF}^{SVF})_N[i] \right|}, \tag{24a}
\]

\[
L_{\text{REG}} = \beta \sum_{i=1}^{N} \sum_{k=1}^{K} \gamma_{i,k} e^{\gamma_{i,k}}. \tag{24b}
\]

We omit the regularization loss for the DN networks since with DN the time-aliasing is inevitable to match the reference with reverberation parameter statistics.

Therefore, our full loss function \(L\) is

\[
L(h, \hat{h}) = L_{\text{MATCH}}(h, \hat{h}) + \beta L_{\text{REG}}(e_{SVF}^N) \tag{25}
\]

where \(\beta = 1\) for the FVN and AFVN and \(\beta = 0\) for the DN.

VI. EXPERIMENTAL SETUP

A. Data

1) Room Impulse Response: We collected 1835 real-world RIR measurements from various datasets including OpenAIR \([52]\) and ACE Challenge \([53]\). The amount of the RIRs was insufficient for the training purpose, so we used them only for the validation and test (836 and 999 RIRs, respectively), ensuring that each set consists of RIRs from different rooms.

For the training, we synthesized 200k RIRs with shoebox room simulations using the image-source method \([54, 55]\). To reflect various acoustic environments, we randomized simulation parameters for every RIR synthesis, which include room size, each wall’s frequency-dependent absorption coefficient, and source/microphone positions. We tuned the randomization scheme to match the training dataset to the validation set in terms of reverberation parameter statistics.

In addition, we pre-processed each RIR as follows:

- We removed the pre-onset part of the RIR. We detected the onset by extracting a local energy envelope of the RIR then finding its maximum point \([56]\).
- Following the recent RIR augmentation method \([57]\), we multiplied random gain sampled from \(U(-12\text{dB}, 3\text{dB})\) to the first 5\(\text{ms}\) of each train RIR. This slightly reduces the average direct-to-reverberant ratio (DRR) of the training set and matches the validation set. At the evaluation/test, we omitted this procedure.
- Finally, we normalized the RIR to have the energy of 1.

2) Reverberant Speech: We used VCTK \([58]\) for dry speech. We split it into the train (61808), validation (21608), and test (4912) set so that each set is composed of the speech recordings from different speakers. We sampled an RIR and a dry speech sample from their respective datasets and convolved them to generate reverberant speech. We random-cropped 2.5-second segment from it for the input.

B. Training

We trained each network with every proposed DAR model for the three tasks: analysis-synthesis, blind estimation, and both tasks. For the both-performing ones, we fed an RIR or reverberant speech in a 50-50% probability.

We set the initial learning rate to \(10^{-4}\) for FVN and AFVN and \(10^{-5}\) for DN. We used Adam optimizer \([59]\). For DN, gradients were clipped to \(±10\) for stable training. After 250k steps, we performed the learning rate decay. We multiplied the learning rate by \(10^{-0.2}\) and \(10^{-0.1}\) every 50k step for the analysis-synthesis networks and the others, respectively. We finished the training if the validation loss did not improve after 50k steps, which took no more than 500k steps for the analysis-synthesis and 1M steps for the others.

C. Evaluation Metrics

We evaluated our networks with the match loss \(L_{\text{MATCH}}\) and EDR distance, defined as an average of the absolute EDR error. Furthermore, we evaluated reverberation parameter differences, i.e., reverberation time \(T_{30}\), DRR, and clarity \(C_{50}\) difference, denoted as \(\Delta T_{30}\), \(\Delta DRR\), and \(\Delta C_{50}\), respectively \([32, 60]\). We calculated both full-band differences and average of differences measured at octave bands of center frequencies of 125, 250, 500, 1k, 2k, 4k and 8kHz. Then, we obtained their respective median values.

We can interpret the reverberation parameter differences by comparing them with their respective just noticeable differences \([9]\). Reported just noticeable differences of \(T_{30}\) from previous works vary from 5\% \([61]\) up to about 25\% \([62]\). For \(C_{50}\), 1dB \([63]\) and 1.1dB \([64]\) were reported. For DRR it differs for various range, e.g., 6dB at \(-10\text{dB}, 2\text{dB}\) at 0dB, 3dB at 10dB, and 8dB at 20dB \([65]\).

D. Subjective Listening Test

Following the previous researches \([10, 41]\), we conducted a modified MUltiple Stimuli with Hidden Reference and Anchor (MUSHRA) test \([66]\). We asked subjects to score similarities of reverberation between a given reference reverberant speech \(h \ast x\) and followings \((h \ast x)\) (and dry speech signal, respectively).

- A hidden reference \(h \ast x\) (exactly the same audio).
- A lower anchor \(h' \ast x\) obtained by averaging the test set RIRs \([41]\).
- Another anchor \(h'' \ast x\) obtained by random-sampling an RIR from the test set \([10]\).
- Estimations to evaluate \(\hat{H}_1(x), \cdots, \hat{H}_n(x)\). We used the AR models \(\hat{H}_1, \cdots, \hat{H}_n\) to obtain the reverberant speech.

The similarity score ranged from 0 to 100, where we provided text descriptions for 5 equally-divided ranges (0-20: totally different, 20-40: considerably different, 40-60: slightly different, 60-80: noticeable, but not much, and 80-100: imperceptible). While we asked the participants to score both analysis-synthesis and blind estimation results, we divided them into two separate pages. As a result, a total of 10 stimulus (3 proposed models, 4 baselines which we will explain in Section VI-E, 2 anchors, and 1 hidden reference) were scored for each task. We conducted
the test using the webMUSHRA API [67]. A total of 24 sets were scored for each task, where the first 3 sets were used for training. A total of 12 people participated where 7 people were audio engineers and the others were instrument players. We discarded 1 subject’s results since he scored the hidden references lower than 90 for more than 15% of the entire sets.

E. Baselines

We compared our framework with following baselines. Refer to Appendix B for further details.

1) Parameter-matching Networks: Assuming that the DAR models are not available, similar to the previous non-end-to-end approach [10], we generated the training data (reference-and-ARPs pair) with the DAR models and trained the same networks with a ARP-matching loss (see Figure 9a). Note that we used the DAR models simply for the convenience of on-the-fly data generation in GPU, but the same procedure can be performed with the AR models.

2) CNN Decoder: We trained the same estimation network but instead of the DAR model we attached a decoder composed of one-dimensional transposed convolutional layers and TCNs. With this decoder, the entire network resembles an autoencoder (see Figure 9b). From now, we denote this model as CNN. We trained two CNNs, one for each of the two tasks.

Note that we did not compare the our methods with possible two-stage solutions [32]–[39] (see Section II-B3) since each DAR model’s architecture became different from its original.

VII. EVALUATION RESULTS

A. Comparison with the Baseline Models

1) Advantage of the End-to-end Learning: Figure 10 compares the proposed end-to-end approach with the baselines on objective metrics and subjective scores. By a large margin, the end-to-end model outperformed the non-end-to-end ARP-matching baselines. The baselines struggled to match the reverberation decay, reporting noticeably large reverberation time differences $\Delta T_{30} \geq 20\%$. A two-sided t-test for each AR/DAR model and task showed that the performance difference was statistically significant (all $p < 10^{-5}$). Subjective listening test results also agreed with the objective metrics; median scores of the end-to-end approaches were from 70 to 80, showing close matches to the reference, while the parameter matching models scored less than 60. Also, refer to Figure 13 that visualizes each estimation result with an EDR and EDR error plots. This shows that the end-to-end learning enabled by the DAR models gives a huge performance gain relative to the non-end-to-end parameter-matching scheme.

2) Advantage of the DSP Prior: Since the CNN baselines have more trainable parameters and complex architecture, they reach lower losses than the DAR-equipped networks. However, they produce audible artifacts. While being more restrictive, the AR models avoid such artifacts and achieve higher subjective listening scores than the CNN baselines. Again, these subjective score differences were statistically significant (all $p < 10^{-5}$).


### B. Reliability of the DAR Models

We demonstrated in previous sections that each DAR model is close to the original AR model, making them reliable to use as an alternative for the training. Here, we empirically validate this argument again. Figure 11 summarizes the objective metrics calculated with the DAR and AR models. We used the trained analysis-synthesis network for this evaluation. First, metrics calculated between the AR model IR and the DAR model IR were orders of magnitude smaller than the metrics calculated with the ground truth. DN was the only exception, showing more deviation than the other AR models due to the time variance. Nevertheless, the reverberation parameter differences between the DN IR and differentiable model IR were less than the just noticeable difference values (for example, $\Delta T_{30} < 3\%$). Second, as expected with equation (7), each metric difference was smaller than the corresponding AR-to-DAR conversion error. As a result, overall evaluation results were very similar regardless of the used model. In short, the DAR models are reliable for training in practice.

### C. Performance Difference Between Target Tasks

Without surprise, the analysis-synthesis networks performed better in most metrics than the blind estimation networks since the reference reverberation is directly given, i.e., the former ones have fewer burdens than the latter. Likewise, the both-performing networks (shown as dots in Figure 10) show slightly degraded results than their single-task counterparts. Yet, the reverberation parameter difference only slightly increased (less than the just noticeable differences), which hint at a possibility of a universal reference-form-agnostic ARP estimation network.

### D. AR Model Efficiency and Estimation Performance

Each AR model approximates reference reverberation differently, and consequently, its computational efficiency and its estimation networks’ performance varies.

1) **Filtered Velvet Noise:** FVN filters each source segment independently (see Figure [12]). Such flexibility enables its parameter estimators to perform better than the other AR models’ counterparts experimented in this paper. However, at the same time, it also has the largest number of ARPs (930), which makes it cumbersome to control manually. Also, it could be computationally expensive to use multiple FVN instances in real-time (2546 FLOPs for each).

2) **Advanced Filtered Velvet Noise:** Instead, AFVN optimizes each coloration filter by decomposing it into the initial coloration and delta-coloration filters, making the model more controllable and efficient (360 ARPs and 1511 FLOPs) than the FVN. Such simplification costs estimation performance since the underlying assumption of the optimization that coloration of real-world RIRs changes gradually does not necessarily hold. In addition, the AFVN networks could be more challenging to optimize than the FVN’s since each segment’s coloration depends on the previous ones’. Indeed, we empirically observed that their training losses decreased slower than the FVN’s.

3) **Delay Network:** In DN, we simplified the filter structure even further by using a single absorption filter for each feedback loop. Similar to the AVFN case, DN trade-offs its estimators’ performance with its controllability and efficiency (200 ARPs and 1285 FLOPs). Again, the estimation performance degradation comes from the DN’s restricted expressibility (forced exponential decay) and training difficulty.

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**Fig. 11.** Reliability of the DAR models. Diff, LTI, and LTV denote the differentiable, original LTI, and the time-varying model, respectively. The hatched results are the same values of Figure 10 (analysis-synthesis).

**Fig. 12.** Magnitude responses of the coloration filters of the AR models. We used the proposed analysis-synthesis networks to obtain the filter parameters for this plot. Each AR model has a different approach to model the frequency-dependent characteristic of given reverberation.
Fig. 13. EDR plots of the reference RIRs and their estimations with the trained networks. For each reference, a total of twelve networks’ estimations and their errors are visualized where each network is trained for one of the three AR models (FVN, AFVN, and DN), one of the two tasks (analysis-synthesis and blind estimation), and one of the two training approaches (end-to-end learning with the proposed DAR models and the baseline non-end-to-end parameter-matching learning). The number provided above each plot is the EDR distance of the estimation.
VIII. Conclusion
We proposed differentiable artificial reverberation (DAR) models that can be integrated with deep neural networks (DNNs). Among numerous pre-existing artificial reverberation (AR) models, we selectively implemented Filtered Velocity Noise (FVN), Advanced Filtered Velvet Noise (AFVN), and Delay Network (DN) differentially. Nonetheless, the proposed method, which replaces the infinite impulse response (IIR) components with finite impulse response (FIR) approximations via frequency-sampling, is applicable to any other AR model. The evaluation results showed that our networks captured the target reverberation accurately in both analysis-synthesis and blind estimation tasks. We showed, in particular, that the end-to-end training significantly improves the estimation performance at the tolerable cost of the approximation errors caused by the frequency-sampling. Additionally, we demonstrated that the structural priors of the AR models avoid perceptual artifacts that a DNN as a room impulse response (RIR) generator might produce. Thus, our framework successfully combined powerful and fully general deep learning techniques with well-established domain knowledge of reverberation.

Here, we outline the remaining challenges and future work. First, we used the shoe-box simulation to obtain a large amount of training data, which is slightly different from the real-world RIRs. Interestingly, energy decay relief (EDR) errors of the end-to-end models showed similar patterns regardless of the equipped AR/DAR model (see Figure [1]). This suggests that the data characteristics discrepancy might be the cause of the performance degradation. Therefore, collecting more real-world RIRs and using powerful augmentation methods could replace the simulation-based data and improve the performance. Second, we only used the RIRs and the reverberant speech signals as references. However, other possible applications with different reference types exist (for example, automatic mixing of musical signals). Finally, we investigated how each AR model can be compared in terms of its expressive power and difficulty of parameter optimization under the deep learning environment. The evaluation results revealed a trade-off relationship; when a more compact AR model is used, the estimation performance was upper-bounded by its limited expressive power. Also, the proposed modifications to the original AR models improved estimation performance and training speed. All of these indicate that other than the presented AR models and estimation networks, there may be more “deep-learning-friendly” models and “AR-model-friendly” DNNs that yield better performance-efficiency trade-offs, and seeking those is left as future work.

APPENDIX A
Ablations and Comparisons of the AR/DAR Models
A. AR/DAR Model Ablations
We conducted ablation studies to verify that the modifications to the original AR models improve the estimation performance. We evaluated the modified models with the analysis-synthesis task since their performance differences remained the same regardless of the task in our initial experiments.

Table I summarizes the results. FVN, AFVN, and DN denote the proposed models. $T_i = 10$, $\Delta l_i = L/S$, $-h_0[n]$, and $M = 1$ denote the proposed FVN model without non-uniform pulse distance and allpass filters, non-uniform segmentation, a deterministic FIR, and finer segment gains. Their results show that the proposed modifications contribute to better performance. We omitted the same ablations for the AFVN. Instead, we discarded the initial filter $C_1$, delta filter $C_j$, or both. We also evaluated the DN without the initial filter $C_1$ or delta filter $C_{\Delta}$. The results reveal the importance of modelling the frequency-dependent nature of reverberation.

For DN, $-Q_n$, $-U$, and $-U, Q_n$ denote DN without the time-varying mixing matrix $Q_n$, the allpass filters $U$, and both, respectively. While $-U$ and $Q_n$ improve plausibility of reverberation with an acceptable performance degradation. Refer to the provided audio samples.

B. Comparisons on Different Coloration Filters
We compared different IIR/FIR filters and parameterization approaches. We evaluated them with the analysis-synthesis task using FVN. Table II summarizes the results. SSVF, PSVF, PEQ, SBIQ, PBIQ, FIR, and LFIR denote serial SVF, parallel SVF, PEQ, serial biquad, parallel biquad, FIR, and linear-phase FIR
TABLE III

| Model | Frequency-Sampling Resolution |
|-------|-----------------------------|
|       | ×1 (FULL) | ×0.5 | ×0.25 | ×0.125 |
| DAR AR | DAR AR | DAR AR | DAR AR | DAR AR |
| FVNβ=1 | 1.236 | 1.236 | 1.259 | 1.259 | 1.252 | 1.249 | 1.574 | 1.570 |
| FVNβ=0 | 1.231 | 1.237 | 1.283 | 1.282 | 1.264 | 1.263 | 1.280 | 1.773 |
| DN     | 1.277 | 1.307 | - | 1.326 | - | 1.361 | - | 1.393 |

from [29]. The order of every filter was set to 40. The activation functions and bias initialization for each filter’s decoder are as follows. PSVF has the same setting as SSVF. For SBIQ and PBIQ decoders, we used activations from [29], and their biases are initialized to match that of SSVF and PSVF. For PEQ, unlike DN, we used $10^{-2}$ for the activation of the gain G. FIR/LFIR have no activation and custom bias initialization.

In spite of having identical expressive power, PSVF/PSVF performed better than SBIQ/PBIQ by large margins. PEQ performed slightly worse than SSVF due to its restricted degree of freedom. FIR/LFIR also performs worse than SSVF/PSVF and PEQ because they cannot change their frequency responses radically as other IIR filters could.

C. Effects of the Frequency-Sampling Resolution

Table III demonstrates the effects of the frequency-sampling resolution and the regularization loss $\mathcal{L}_{\text{REG}}$. From the table, $\times x$ denotes the relative sampling resolution compared to the default configuration, e.g., $\times 0.5$ denotes $N = 2k$ for the FVN and $N = 6k$ for the DN. For each resolution and regularization setup, we report match loss with the DAR and AR models. Results of the differentiable DNs with lower resolutions are omitted since they are calculated with shorter IRs.

As expected, higher sampling resolution led to smaller loss difference. Also, introducing the regularization term ($\beta = 1$) reduced the difference further. As a result, the final differentiable FVN model (with full resolution and regularization) showed little loss difference. Moreover, higher resolution led to better performance, which might be because increased resolution helped each network to find better ARPs.

APPENDIX B

DETAILS ON THE BASELINE METHODS

A. ARP Match Models

1) Training: We trained the ARP match baseline models as follows. First, we randomized ARPs $P_1, \ldots, P_n$ and generated an IR with using the DAR model. Then, each baseline network estimated ARPs $P_1, \ldots, P_n$ from the given IR. We evaluated the estimation and trained the baseline using an ARP-match loss $\mathcal{L}_{\text{ARP}}$ defined as follows,

$$\mathcal{L}_{\text{ARP}}(P_i, \hat{P}_i) = \sum_i \alpha_i \| f_i(P_i) - f_i(\hat{P}_i) \|_1. \quad (26)$$

Here, the index $i$ denotes a different ARP tensor, e.g., $P_0 = g$ and $P_1 = h_0$. $f_i(\cdot)$ and $\alpha_i$ are an elementwise function and a constant, respectively. We set $\alpha_i = 10$ for the all DAR models’ bypass FIRs and the DN’s absorption filter parameters. We used $f_i(x) = \log_{10}(x)$ for the FVN and AFVN segment gains. We chose $f_i(x) = x$ and $\alpha_i = 1$ otherwise.

2) Network Architecture: We trained an almost identical network to the proposed network for each model/task. The only difference is that we added an activation $10^{-\frac{(x)}{20}}$ to the g decoders of FVN and AFVN to improve their performance.

3) Data Generation: We tuned each DAR model’s ARP randomization scheme to match the synthesized IRs’ reverberation parameter statistics to the validation set. For the FVN baselines, we first sampled a reverberation time $T_{30} \sim U_{log}[50\text{ms}, 88]$ then generated the segment gains $g$ that match the $T_{30}$. Here, $U_{log}[\cdot]$ denotes a log uniform distribution in log scale. In this procedure, we also compensated the average pulse distance by dividing each gain $g_i$ by $\sqrt{T_i}$. For the initial coloration filter $C_1$, we first generated PEQ parameters as $\omega_i \sim U_{log}[40\text{Hz}, 16\text{kHz}]$, $R_i \sim U_{log}[0.2, 5]$, and $G_i \sim U[-18\text{dB}, 18\text{dB}]$ (we sorted the cutoff frequencies after the sampling) and derived their SVF parameters. After that, we perturbed each parameter’s value slightly. This ARP generation method was motivated by the observation that the differentiable SVFs act like a relaxed PEQ (see Figure 12).

B. DNN Decoder Models

Regarding the first and second axis of the shared latent z as a time and channel axis, respectively, our decoder upsamples z with seven one-dimensional transposed convolution layers. Configurations of these layers are as follows. Channels: $128, 128, 64, 64, 64, 64, 64$, then $64$. Kernel sizes: $5, 5, 5, 5, 5, 5, 5$, then 3. Strides: $3, 3, 3, 3, 3, 3, 3$, then 2. Furthermore, after each upsampling, we inserted a small temporal convolutional network with configurations as follows. Channels: $128, 128, 64, 64, 64, 64, 64$, and 64. The number of layers: $3, 3, 3, 2, 2, 2, 2$, and 2. Kernel sizes: all 7. Finally, we added $1 \times 1$ convolution as the last layer to mix all channels. We used the ReLU activation between the layers. This decoder has about 2.5M parameters.

APPENDIX C

DETAILS ON THE FREQUENCY-SAMPLING METHOD

A. Proof of Equation 5

With equation [5], the time-aliasing error can be written as

$$\|H - H_N\|_2^2 = \sum_{n=0}^{N-1} \sum_{m=0}^{\infty} |h[mN+n]|^2 + \sum_{n=N}^{\infty} |h[n]|^2. \quad (27)$$

With the triangle inequality and $\|X\|_2 \leq \|X\|_2$, we can upper-bound $\|H - H_N\|_2^2$ with $2S_N^2$, where $S_N = \sum_{n=N}^{\infty} |h[n]|$ is an absolute sum of the tail of the original IR $h[n]$ ($n \geq N$). Next, we expand $h[n]$ with partial fraction expansion [68, 69]. For
\[ H(z) \] with \( M \) distinct poles where each of \( \nu_i \in \mathbb{C} \) has multiplicity \( r_i \in \mathbb{N}_0 \), we can express \( b[n] \) as a sum of an FIR \( h_{\text{FIR}}[n] \) and IIRs \( h_{\text{IIR}}[n] \) where following holds,

\[
H(z) = H_{\text{FIR}}(z) + \sum_{i=1}^{M} \sum_{k=1}^{r_i} \left( 1 - \nu_i z^{-1} \right) \hat{g}_{i,k} \cdot \left( \hat{g}_{i,k} \in \mathbb{C} \right) \tag{28}
\]

Then, applying the triangle inequality to the summation gives an upper bound \( U_N \), an absolute sum of the IIRs' tails.

\[
S_N \leq \sum_{i=1}^{M} \sum_{k=1}^{r_i} \sum_{n=1}^{\infty} |h_{\text{IIR}}[n]| = U_N. \tag{29}
\]

From above equation, each IIR, first sum, and second sum show \( O(n^{k-1}|\nu_i|^n) \), \( O(N^{k-1}|\nu_i|^N) \), and \( O(N^{r_i-1}|\nu_i|^N) \) asymptotic behavior, respectively \( \left[59\right] \).

We prove the conclusion:

\[
\|H - H_N\|_2 \leq \sqrt{2S_N} \leq \sqrt{2U_N} = \sum_{i=1}^{M} O(N^{r_i-1}|\nu_i|^N). \tag{30}
\]

### B. Proof of Equation 8

We denote \( H(e^{j\omega}) \) and \( \partial H / \partial p \) with \( H \) and \( H' \), \( X_N' \) denotes \( \langle X_N'y \rangle_N \). We upper-bound the gradient difference \( \Delta G \) using integral inequalities as

\[
\Delta G \leq \int_{0}^{2\pi} |H_{\text{Ref}}(H' - H_N')| \frac{d\omega}{\pi} \leq 2\|H_{\text{Ref}}\|_2 \|H' - H_N'\|_2 \leq 2\|H' - H_N'\|_2 \|H_N'\|_2 \tag{31}
\]

where the under-braced ones are the Cauchy-Schwarz inequalities. Considering the first integral’s upper bound, \( |H_{\text{Ref}}| \) is a finite constant. In \( \|H' - H_N'\|_2 \), \( H' \) is another stable LTI filter if \( p \) is well-defined. If \( \nu_i \) is a function of \( p \), its multiplicity \( r_i \) is doubled in \( H' \). In the worst-case, \( p \) controls every pole of \( H \) and \( \|H' - H_N'\|_2 = \sum_{i=1}^{M} O(N^{2r_i-1}|\nu_i|^N) \) holds. Similarly, the upper bound of the second integral consists an \( l_2 \) distance between \( H' \) and \( H_N' \). Since \( H' \) has \( 2r_i \) or \( 3r_i \) multiplicity for each pole \( \nu_i \), \( \|H' - H_N'\|_2 = \sum_{i=1}^{M} O(N^{3r_i-1}|\nu_i|^N) \) holds at the worst case, which concludes the proof.

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