Proposal for the Use of a New Parameter in the Composite Road Traffic Model

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Abstract. Comparative analysis of the models used in the literature for the fundamental traffic diagram reveals that the improvement of these models can only be achieved by taking into account the vehicle tracking models. Given that it is of great interest to model the relation between density and speed, to identify the conditions in which maximum flow can be achieved with a high operating speed and safely. For the non-homogeneous traffic, met in our country due to the lack of highways, the model describing the minimum range of vehicles was developed based on the relation of the minimum distance between vehicles taking into account the difference between the vehicle braking modes. Finally it is explained the maximum volume of traffic according to a parameter that includes the decelerations of the two vehicles, which determined the proposed use of this new concept in road traffic theory. This has made it possible to conclude that in the case of non-homogeneous traffic the volume increases only to a certain speed value, then decreases, while in the case of homogeneous traffic the volume continues to increase to a limit value. The paper argues the necessity of using this new parameter for road traffic models.

1. Papers objectives
The comparative analysis of the various models used in specialised literature for the fundamental traffic diagram (single-regime models: Greenshields, Greenberg, Underwood, Northwestern, multi-regime models: Edie, linear-two regimes, modified Greenberg, linear-three regimes...) leads to the conclusion that the improvement of these models can only be achieved by taking into account the vehicle tracking models, which involves the microscopic modelling of road traffic flows [2].

Given that what is of great practical interest is the modelling of the relation between density and speed in order to identify the conditions in which maximum flow can be achieved (this is the interest of the road infrastructure designer), but also with an operating speed as high as possible (this is the interest of the user of the infrastructure, who wishes to reach his/her destination as soon as possible), and under safe circumstances (this is true for all parties involved), a detailed analysis of the most interesting cases has been carried out to obtain maximum traffic (to ensure high efficiency for road infrastructure) at high operating speeds (to ensure the economical use of vehicles) and under safe circumstances [5].

The analysis of traffic flows as a whole (macroscopically) also uses the parameters of the microscopic traffic flow - which describe the flow behaviour of the individual vehicle (singular vehicle speed) or of the pairs of vehicles inside the flow (the intervals between successive vehicles) [9].
Therefore, it is important for the vehicle range inside the flow to be able to be evaluated, a range that can be measured as time interval or space interval. The intervals between vehicles are primarily dependent on the flow velocity (they increase by the flow rate, for security reasons), but depend largely on the characteristics and condition of the track, these factors being the same for all road users. But, for the same conditions of speed and road, the intervals between vehicles are not constant since they depend on the other two factors in the traffic system - the driver (the human factor) and the vehicle (the technical factor), which causes a pronounced variability of the intervals between vehicles [7]. If, in what the technical factor is concerned, in the case of traffic on upgraded arteries, with many lanes in each traffic direction, the flows get homogenised (because vehicles with similar technical characteristics and dynamic performance will drive on the same lane), in the case of the vehicle drivers, their very different driving style is the main reason for the variability of the intervals between vehicles of the same type on the same lane.

As a result, since, within the actual flows, a constant interval between the vehicles cannot be maintained, the most suitable modelling should be based on a statistical approach, in view of the diversity of temper types, so it is necessary to evaluate the average of intervals [6]. Thus, the objective of this paper is to identify ways to improve the performance of traffic by analysing the intervals between successive vehicles.

2. Research undertaken

Empirical data are used to calculate the statistical indicators of vehicle intervals distribution within the flow, starting from the definition of the intervals depending on the measurement method. The interval between vehicles is an important quantitative parameter of traffic flows, by means of which performance of the road system or the level of road service can be measured, as well as the efficiency of traffic signals.

The time interval between vehicles is a measure of the time needed for the passage of two successive vehicles in the flow, through a designated benchmark on the roadside. The measure of the level of service of a road artery is closely linked to the time intervals between vehicles and can be expressed in two ways:

- the weight of the time interval in which a vehicle is forced to follow another vehicle on a road with two lanes;
- frequency of speed of travel adjustment manoeuvres a driver performs in order to keep a minimum time interval to the vehicle in front.

The movement of the two vehicles is shown in figure 1, where the following notations are used:

- \( l_1 \) – length of the vehicle in front (the first vehicle, the vehicle followed);
- \( d_{\text{min}} \) – space of the vehicle in the back (the second vehicle, the vehicle that is following) and the vehicle followed;
- \( s_0 \) – safety space;
- \( t_r \) – response time of the driver of the vehicle that is following;
- \( s_1 \) – the space covered by the vehicle in front in the time interval \( t_1 \), braking with the deceleration \( a_1 \) to a halt;
- \( s_2 \) – the space covered by the vehicle in the back, made up of two components; the space covered at the speed \( v \) in the response time \( t_r \) until the driver operates the braking system; the space covered in the time interval \( t_2 \) braking with the deceleration \( a_2 \) to a halt.

The analytical equation that indicates the minimum distance to be maintained between the two vehicles during movement at speed \( v \) is rendered in what follows:

\[
l_v + d_{\text{min}} + s_1 = s_2 + l_v + s_0 \tag{1}
\]

There thus ensues the value for the minimum distance \( d_{\text{min}} \) function of the parameters that define the movements of the two vehicles (speed \( v \), driver’s response time \( t_r \), the decelerations of the two
Figure 1. Graphical representation of the distance between two vehicles that follow each other if the first vehicle brakes to a halt [3].

vehicles, \( a_1 \) și \( a_2 \), and the safety space \( s_0 \):

\[
d_{\text{min}} = s_2 + s_0 - s_1 = \left( v \cdot t_\tau + \frac{v^2}{2a_2} \right) + s_0 - \frac{v^2}{2a_1} = v \cdot t_\tau + \frac{v^2}{2} \left( \frac{1}{a_2} - \frac{1}{a_1} \right) + s_0
\]

(2)

There are two cases to be discussed for the minimum distance:

1 – homogeneous traffic: \( a_1 = a_2 \) - the optimum case because the minimum distance between vehicles will have the lowest value:

\[
d_{\text{min}} = v \cdot t_\tau + s_0
\]

(3)

In this case the variability range of vehicles is given by the different options for drivers, depending on their physical and mental characteristics.

2 – non-homogeneous traffic: \( a_1 \neq a_2 \)

In this case, the vehicles have different braking capacities, and the succession of the various vehicles in the flow is random, so that the minimum interval between vehicles can be described through the equation that uses the absolute value for the difference \( \frac{1}{a_1} - \frac{1}{a_2} \):

\[
d_{\text{min}} = v \cdot t_\tau + \frac{v^2}{2} \left| \frac{1}{a_2} - \frac{1}{a_1} \right| + s_0
\]

(4)

Thus, for two cars that follow each other and that are moving at high speed, there are three possible cases [8]:

a) both the vehicle followed and the vehicle that is following brake under emergency conditions – given the fact that the vehicle that is following brakes under emergency conditions, this is the case of maximum safety (take into account the fact that the vehicle followed will hit the brakes vigorously) and discomfort for the vehicle that is following (because it has to hit the brakes vigorously);

b) the vehicle followed brakes under emergency conditions and the vehicle that is following brakes under normal conditions – this is the case of maximum safety (take into account the fact that the vehicle followed will hit the brakes vigorously) and comfort for the vehicle that is following (which does not have to hit the brakes vigorously, it will brake under normal conditions);

c) both the vehicle followed and the vehicle that is following brake under normal conditions – this is the case of reduced safety (one does not take into account the fact that the vehicle that is following
could brake vigorously, one takes into account only the variant in which it brakes under normal conditions) and comfort for the vehicle that is following (which does not have to hit the brakes vigorously, it will brake under normal conditions).

It can be noticed that, when the vehicles brake in the same way (with the same decelerations), the minimum required distance between them is the same regardless of the speed of travel. But to ensure safe travel in maximum comfort, the case that is considered to be appropriate is case b) the vehicle followed brakes in emergency, and the vehicle that is following brakes under normal conditions.

Of great practical interest, however, is the modelling of the relation between density and speed to identify conditions in which a maximum flow rate can be obtained. As a result, further on there will be examined the most interesting cases to obtain a high volume of traffic at high operating speed (to ensure economic use of the vehicle), under safe conditions. Thus, the most interesting case, not only from a theoretical point of view, is that providing the highest traffic volume possible, which is the one that allows that a minimum distance \( d_{\text{min}} \) be kept between vehicles (this is the case of homogeneous traffic, when the vehicles belong to the same category), a case in which the distance between the vehicles that can brake with similar decelerations is rendered through the equation (3), presented before. The analytical equation for the interval between two vehicles \( I_v \) that succeed one another during the movement at speed \( v \) is:

\[
I_v = l_v + d_{\text{min}} = l_v + v \cdot t_r + s_0
\]

where \( l_v \) is the length of the vehicle. As a result, traffic density will be:

\[
k \left[ \frac{\text{veh}}{\text{km}} \right] = \frac{1000}{l_v[m] + \frac{1}{5.8} \left[ \frac{\text{km}}{\text{h}} \right] t_r[s] + s_0[m]} = \frac{1000}{l_v[m] + \frac{1}{5.8} \left[ \frac{\text{km}}{\text{h}} \right] t_r[s] + s_0[m]}
\]

Thus, the maximum volume of homogenous traffic will be:

\[
q \left[ \frac{\text{veh}}{\text{h}} \right] = k \left[ \frac{\text{veh}}{\text{km}} \right] \cdot v \left[ \frac{\text{km}}{\text{h}} \right] = \frac{1000}{l_v[m] + \frac{1}{5.8} \left[ \frac{\text{km}}{\text{h}} \right] t_r[s] + s_0[m]} \cdot v \left[ \frac{\text{km}}{\text{h}} \right] = \frac{3600}{\frac{l_v[m]}{v} + \frac{s_0[m]}{v} + t_r[s]}
\]

Analyzing the last equation, it is to be noted that the first order derivative of the function \( q(v) \) is always positive and continuously decreasing:

\[
\frac{dq}{dv} = \frac{l_v + s_0}{v^2}
\]

so it is not cancelled for any value of speed \( v \), which means that, in this case (which is not just theoretical, but is rather frequently met with on highways, where it can be considered that each lane is populated by vehicles that are equipped with similar braking capacities) the maximum volume of traffic \( q \) continuously increases along with increasing the operation speed (from value zero, for \( v \to 0 \), but increasingly less, aiming asymptotically at (for \( v \to \infty \)) the value \( 3600/t_r[s] \) – figure 2. As, in the present, vehicles of the same type are comparable in braking performance, it appears that the only factor that makes a difference in terms of the required distance between vehicles is the drivers’ behaviour, in terms of response time. The ideal situation is that where highways have three lanes, because trucks will drive on the first lane (which have a maximum speed limit), and cars will drive on the second and third lanes (on the second lane at an average speed, and on the third lane at a higher speed). This is how the „layering” of cars is performed on the second and third lanes, depending on the speed adopted and therefore the response time characterizing the drivers of the respective vehicles.

A second case is that of a non-homogeneous traffic (with various categories of vehicles - cars, trucks, buses ...), when the vehicle decelerations values are different, so there are situations when the vehicle followed brakes by a deceleration greater than the vehicle that is following [3]. The model for
This second case is based on the relation that describes the minimum interval between vehicles, in view of the difference of braking modes of vehicles that are part of the road flow (different decelerations) - equation (4), presented above. The analytical equation for the interval between two vehicles that succeed one another during the movement at speed $v$ is in this case:

$$I_v = I_v + d_{\text{min}} = I_v + v \cdot t_r + \frac{v^2}{2 \cdot \frac{1}{a_{f_2}} - \frac{1}{a_{f_1}}} + s_0$$

(9)

The traffic density will be:

$$k_{\text{[veh/km]}} = \frac{1}{I_v + \frac{1}{\frac{1}{a_{f_2}} - \frac{1}{a_{f_1}}} + \frac{v^2}{2}} \cdot \frac{1}{a_{f_2}} + \frac{1}{a_{f_1}} + s_0$$

(10)

where speed $v$ is expressed in [km/h]. As a result, the maximum volume of non-homogeneous traffic will be:

$$q_{\text{[veh/h]}} = \frac{1000 \cdot \frac{1}{a_{f_2}} + \frac{1}{a_{f_1}} + s_0}{I_v + \frac{1}{\frac{1}{a_{f_2}} - \frac{1}{a_{f_1}}} + \frac{v^2}{2}} \cdot v$$

(11)

It is to be noted that, in this case, the first order derivative of the function $q(v)$ is cancelled for a certain value of the speed, at which traffic volume records a maximum [3], a value that corresponds to a maximum traffic volume and is noted as $v_M$:

$$\frac{dq}{dv} = 0 \Rightarrow (I_v + s_0) + \frac{1}{2 \cdot \frac{1}{a_{f_2}} - \frac{1}{a_{f_1}}} = 0$$

(12)

It results:

$$v_M = 3.6 \cdot \sqrt{\frac{2 \cdot (I_v + s_0)}{a_{f_2} - a_{f_1}}} = 3.6 \cdot \sqrt{\frac{2 \cdot (I_v + s_0)}{a_{f_2} - a_{f_1}}} = 3.6 \cdot \sqrt{\frac{2 \cdot (I_v + s_0)}{C}}$$

(13)

where, for simplicity reasons, the following notation was introduced:

$$C = \frac{a_{f_1} - a_{f_2}}{a_{f_1} a_{f_2}}$$

(14)

$C$ is a coefficient that expresses the variability of braking decelerations for the vehicles in the continuous flow [4], a value inverse to deceleration: C [1/(m/s²)]. Considering further on speed $v$ expressed in [km/h] and the newly-adopted coefficient $C$ (that can be referred to as a “deceleration variation coefficient”), the highest value for the maximum traffic volume will be:

$$q_M = \frac{1000}{I_v + \frac{1}{\frac{1}{a_{f_2}} - \frac{1}{a_{f_1}}} + \frac{v^2}{2}} \cdot \frac{a_{f_1} a_{f_2}}{a_{f_2} - a_{f_1}} = \frac{3600}{t_r + \frac{v_M}{2 \cdot \frac{1}{a_{f_2}} - \frac{1}{a_{f_1}}} \cdot \frac{a_{f_1} a_{f_2}}{a_{f_2} - a_{f_1}}} = \frac{3600}{t_r + \frac{v_M}{2 \cdot \frac{1}{a_{f_2}} - \frac{1}{a_{f_1}}} \cdot \frac{a_{f_1} a_{f_2}}{a_{f_2} - a_{f_1}}} = \frac{3600}{t_r + \sqrt{2 \cdot (I_v + s_0)} \cdot \sqrt{\frac{C}{a_{f_2} - a_{f_1}}}$$

(15)

The graph presenting the maximum traffic volume for the two cases is shown in figure 2 [1]. It is to be noticed that, in fact, the case of homogeneous traffic is a particular case of the model developed for non-homogeneous traffic: the case $a_{f_1} = a_{f_2}$, i.e. when there is no deceleration variability, so the deceleration variability coefficient is null: $C = 0$. For this model it is equally interesting to determine the traffic density that corresponds to the maximum traffic volume:

$$k_M = \frac{q_M}{v_M} = \frac{1000}{3.6 \sqrt{2 \cdot (I_v + s_0)} \cdot \sqrt{\frac{C}{a_{f_2} - a_{f_1}}}} = \frac{3600}{t_r + \sqrt{2 \cdot (I_v + s_0)}} \cdot \sqrt{\frac{C}{a_{f_2} - a_{f_1}}} = \frac{1000}{2 \cdot (I_v + s_0) + t_r \sqrt{2 \cdot (I_v + s_0)}}$$

(16)
Figure 2. Relation maximum output - speed for homogeneous and non-homogeneous traffic.

One notes that for C = 0 (the case of homogeneous traffic), the density of maximum traffic tends to reach the value of zero, which seems surprising, but can be explained through the fact that for the finite value of maximum traffic (equal to 3600/t_r) the speed tends to reach the infinite, and this can be justified mathematically only through the case of non-determination for the product q_M·k_M.

As far as the deceleration variation coefficient C is concerned, it may be considered to have the following extreme values:

• for a_0 = a_2, the minimum value: C_{min} = 0;
• for a_0 ≠ a_2, one may take into account a range of values starting from 3 m/s^2 to 9 m/s^2, resulting in the maximum value for C:

\[ C_{max} = \frac{|a_{f_1} - a_{f_2}|}{a_{f_1} a_{f_2}} = \frac{3 - 0}{6} = 0.2 \]

Thus, using the model for the maximum flows of traffic obtained (equation 16) and following the highlighting of size C as a parameter, the variation of the maximum volume of traffic for the following values of deceleration variation coefficient was represented: C = 0; C = 0.05; C = 0.10; C = 0.20 and for the fixed values of the other parameters: \( l_v = 4.5 \text{ m}; s_0 = 1.5 \text{ m}; t_r = 1 \text{ s} \) [3].

There were determined the highest values of maximum traffic output, q_M, and the speeds v_M at which these maximum values of traffic output can be obtained.

Thus, the maximum volume of traffic has been highlighted as a function of speed (in km/h), where the variation deceleration coefficient C is a parameter, through the equation:

\[ q \left[ \frac{\text{veh}}{h} \right] = \frac{1000}{t_r + \frac{1}{\nu} + \frac{1}{3.6} \cdot t_r + \frac{1}{v^2 + 2.25 \cdot \nu}} \cdot \frac{|a_{f_1} - a_{f_2}|}{a_{f_1} a_{f_2}} = \frac{1000}{\frac{3.6 + 1.5}{3.6} + \frac{1}{v^2 + 2.25 \cdot \nu} C} = \frac{1000}{\frac{1}{v^2 + 2.25 \cdot \nu} C} \]  

(17)

In order to represent the variation of the maximum volume of traffic function of the speed, for the five values of the parameter C an analytical calculation was carried out according to the equation above, using the Excel program for speed values from 0 to 150 km/h, at a pace of 10 km/h, there emerging the values from table 1.

In the table 1 there were identified and marked with bold characters the peaks of output for each case (corresponding to some of the values adopted at a pace of 10 km/h), but the exact values for the highest outputs, as well as for the corresponding appropriate speeds, were calculated analytically according to the analytical equations (13) and (15).
Table 1. The volumes of traffic function of speed for various values of C.

| C [s²/m] | 0.00 | 0.05 | 0.10 | 0.15 | 0.20 |
|----------|------|------|------|------|------|
| v[km/h]  |      |      |      |      |      |
| 10       | 1139 | 1115 | 1091 | 1069 | 1047 |
| 20       | 1731 | 1622 | 1527 | 1442 | 1366 |
| 30       | 2093 | 1867 | 1685 | 1535 | 1410 |
| 40       | 2338 | 1980 | 1718 | 1517 | 1358 |
| 50       | 2514 | 2023 | 1693 | 1455 | 1276 |
| 60       | 2647 | 2076 | 1641 | 1379 | 1189 |
| 70       | 2751 | 2006 | 1578 | 1301 | 1107 |
| 80       | 2835 | 1972 | 1512 | 1226 | 1031 |
| 90       | 2903 | 1930 | 1446 | 1156 |  963 |
| 100      | 2961 | 1884 | 1382 | 1091 |  901 |
| 110      | 3009 | 1836 | 1322 | 1032 |  847 |
| 120      | 3051 | 1788 | 1265 |  978 |  798 |
| 130      | 3087 | 1740 | 1211 |  929 |  754 |
| 140      | 3119 | 1693 | 1162 |  884 |  714 |
| 150      | 3147 | 1647 | 1115 |  843 |  678 |

The exact calculation - achieved analytically according to these equations using Excel, for the 5 values of parameter C - led to the values recorded in table 2.

Table 2. The maximum volume of traffic and the corresponding speed, for different values of C.

| C [s²/m] | 0.00 | 0.05 | 0.10 | 0.15 | 0.20 |
|----------|------|------|------|------|------|
| vM [km/h]| ∞    | 55.8 | 39.4 | 32.2 | 27.9 |
| qM [veh/h]| 3600 | 2029 | 1718 | 1537 | 1412 |

Figure 3. Variation of traffic volume with speed, for various values of C.

The graphical representation (figure 3) shows that, in the case of non-homogeneous traffic (when this parameter has non-zero values, for example = 0.05; 0.10; 0.15 or 0.20) the volume of traffic
increases only up to a certain speed value (the smaller it is, the more non-homogeneous the traffic is),
then decreases. On the other hand, in the case of homogeneous traffic (C = 0.00), the volume of traffic
continues to increase at a constant speed, increasing to a high value and tending asymptotically to a
limit value.

3. Conclusions
It has been demonstrated that, in order to achieve a high volume of traffic, it is necessary to ensure
conditions for maintaining a minimum distance between vehicles, and this can only be achieved in the
case of homogeneous traffic (when the vehicles belong to the same category, therefore they have
similar dynamic and braking performance). For this case (which can be met with on roads with
restricted access for larger vehicles or on highways with several lanes in each direction, where traffic
is naturally layered on lanes), the conclusion can be drawn that the only factor that makes the
difference in what the distance between vehicles is concerned is the drivers’ response time. For the
case of non-homogeneous traffic (with a composite structure of vehicles, which have different braking
and acceleration possibilities), frequently met with in our country due to the lack of highways, the
mathematical model describing the minimum range of vehicles was developed based on the relation
describing the minimum distance between vehicles taking into account the difference between the
vehicle braking modes (different decelerations). Thus, the value for the minimum distance between
two successive vehicles was defined according to the parameters defining the movements of the two
vehicles (speed, driver response time, deceleration of the two vehicles and safety space) and, finally,
the point is reached when the maximum volume of traffic is explained according to a parameter that
includes the deceleration of the two vehicles, which determined the proposed use of this value concept
as a new concept in road traffic theory: \( C = \text{coefficient expressing the variability of braking}
deceleration for vehicles in the composite continuous flows } \) [3], this coefficient having the equivalent
of deceleration inverse: \( C \left[1/(\text{m/s}^2)\right] \). This has made it possible to conclude that, in the case of non-
homogeneous traffic (when \( C \) has non-zero values) the volume of traffic increases only to a certain
speed value (the smaller it is, the more non-homogeneous the traffic is), then decreases, while in the
case of homogeneous traffic (\( C=0 \)), the volume of traffic continues to increase at a constant speed,
increasing to a high value and tending asymptotically to a limit value that depends on the value of
the drivers’ response time.

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