Tool path optimization of globoidal cam flank milling based on spatial linear-regression analysis

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Abstract
In order to tackle the incongruous error of the normal vector of globoidal cam in non-equal diameter machining, the tool axis vector with minimum machining error is preliminarily fitted through the principle of ruled surface generation by using spatial linear-regression algorithm. The vector error of the initial tool axis is gradually reduced by linear regression iteration. The improved tool axis is used as the generatrix of NURBS ruled surface to reconstruct the theoretical tool axis surface. Furthermore, the least square method is carried out to optimize the tool path, and the optimization model of flank milling error of the tool axis path is constructed. Subsequently, a real-coded artificial immune algorithm for solving the optimization model is proposed. The validity of the algorithm is verified by the results of simulation and calculation of tool axis vector of globoidal cam machining.

Keywords: Globoidal cam, Linear-regression analysis, Non-equal diameter machining, Non-uniform rational b-Splines (NURBS) ruled surface

1. Introduction

As an efficient and simple transmission indexing mechanism, globoidal cam has broad application prospects. The globoidal cam mechanism not only has the advantages of high precision, compact structure, light weight and high yield, but also has the characteristics of strong carrying capacity, good dynamic performance and high reliability. Therefore, it is increasingly used in textile, automobile manufacturing, packing service and aerospace(Kong et al.2008), and so on. Meanwhile, the requirements for the manufacturing process of globoidal cam are also more rigorous. Globoidal cam is a precision mechanical element, and cam production is a complicated task (J.N. Lee, and R.S Lee. 2006). Most recently, the machining methods of globoidal cam are divided into two categories: equal-diameter machining and non-equal diameter machining. The tool with the same geometrical parameters of the roller surface is used in equal diameter machining, and then the tool and the workpiece are moved in a conjugate manner by the mechanism meshing transmission to realize the processing of the globoidal cam. Although the equal-diameter machining is performed easily, its accuracy is poor. The non-equal diameter machining is the processing that globoidal cams are machined with a tool smaller than the radius of the roller, which is superior to equal diameter machining. With the development of NC machining technology, the non-equal diameter machining method of the cam is widely adopted.

For several decades, the focus of research has been on the tool path optimization of globoidal cam in non-equal diameter flank milling. Machining methods are mainly divided two types, i.e., double-envelope method and tool position compensation method. The former is a point-machining method and each part of its profile has different tool location points, and the theoretical errors in the reconstruction process will inevitably lead to poor machining accuracy and surface quality. Although the direction of tool position compensation is relatively fixed, the latter is regarded as a free-form surface when dealing with the cam profile, which results in a larger theoretical error in the reconstructing cam profile. Based on existing machining methods of globoidal cam profile, an optimization algorithm for calculating the tool location data of globoidal cam in one-side NC machining was proposed by Kong (Kong et al.2008), which aimed at minimizing the extremum error between the trajectory surface of the cutter axis and the equidistant surface of the cam profile surface. The calculation method of searching the optimal tool axis vector through a moving point slip
was conducted by Lin Xiaojun et al (Lin et al. 2014). A normal error calculation method of tool position compensation was discovered by Ge Rongyu et al (Ge et al. 2010), which effectively optimized tool position and significantly reduced the machining error. The tool path was optimized according to a definition for evaluation of the design surface function through the algorithm approach (Zhu et al. 2013). In order to refine the approximation error for the data points discretization and great fluctuation of ruled surface of the tool path, HU (HU et al. 2018) devised an optimization algorithm of the tool path of spatial cam flank milling based on NURBS surface. A reliable method was presented by Bu Fanhua to define the cam profile error based on the minimum distance from the point on the actual profile to the desired (Bu et al. 2012). Based on the differential geometry and conjugate theory, the deviation between the machined surface and the desired one of globoidal cam was derived (Zhang et al. 2016). In terms of the principle of NURBS surface reconstruction, the least square method was adopted to improve the theoretical machining accuracy as a whole (Ge et al. 2007). In the light of the theory of the ruled surface reconstruction, the tool position of tool path surface of the theoretical non equal-diameter machining was optimized by genetic algorithm (Ge et al. 2007).

In the aforementioned literatures, only the real-time local error of the tool and workpiece was taken into account, and the requirements of NC machining tool path generation are not considered. Besides, the theoretical tool-path surface needs to be discretized and reconstructed, which requires a large amount of initial data, the collection process is fussy and low efficiency. Consequently, in this paper, the application of the NURBS fitting algorithm is studied through the principle of ruled surface approximation. An optimization algorithm of the tool path of globoidal cam flank milling based on linear regression reconstruction of the NURBS ruled surface is proposed, which reduces the machining errors and simplifies the calculation process simultaneously.

2. The mathematical model of globoidal cam
2.1 Theoretical expression of globoidal cam

A 3-dimensional coordinate system of globoidal cam mechanism is established by differential geometry. In view of the characteristic of the conjugate surface of the roller and globoidal cam in the actual meshing process of globoidal cam mechanism, thus geometric model of globoidal cam mechanism is constructed, as shown in Figure 1.
In terms of conjugate principle of the space envelope surface, then profile equation of globoidal indexing cam is derived.

\[
\mathbf{C}_n = \mathbf{C}_i \mathbf{C}_3 \mathbf{C}_n = \begin{bmatrix}
\cos \gamma_2 & \sin \gamma_2 & 0 & 0 \\
-\sin \gamma_2 & \cos \gamma_2 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\cos \gamma_3 & \sin \gamma_3 & 0 & L \\
-\sin \gamma_3 & \cos \gamma_3 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\cos \Gamma_i & 0 & -\cos \Gamma_i & p \cos \Gamma_i \\
-\sin \Gamma_i & 0 & -\sin \Gamma_i & p \cos \Gamma_i \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

(1)

\[
\Gamma_i = (i-1) \times 60^\circ
\]

(2)

Where \( o_1 - x_1 y_1 z_1 \) is the absolute coordinate system, also known as the fixed coordinate system; \( o_2 - x_2 y_2 z_2 \) stands for the globoidal cam coordinate system; \( o_3 - x_3 y_3 z_3 \) refers to the coordinate system of the indexing plate; \( o_n - x_n y_n z_n \) means the coordinate system of the roller; \( \mathbf{C}_n \) denotes the transformation matrix from coordinate system \( o_j - x_j y_j z_j \) to coordinate system \( o_i - x_i y_i z_i \); \( \gamma_2 \) is the cam angle; \( \gamma_3 \) is the rotary angle of the indexing plate; \( L \) means the center distance between the indexing plate and the cam, i.e., the distance between \( z_2 \)-axis and \( z_3 \)-axis; \( \xi \) is the distance from the top of the roller to \( z_3 \)-axis; \( l \) is the length of the roller; \( \Gamma_i \) stands for the angle relationship of the rollers on the indexing plate.

As shown in local enlarged view in Figure 1(b), \( n \mathbf{F} \) is the surface equation formed by the conjugate movement of the roller and the cam in the roller coordinate system \( o_n - x_n y_n z_n \).

\[
n \mathbf{F} = \begin{bmatrix}
 r \cos \alpha \\
 r \sin \alpha \\
 1
\end{bmatrix}
\]

(3)

\( r \) means the radius of the roller; \( v \) denotes the roller surface parameter, i.e., the distance from the inner end of the roller to the contact point of roller and cam, also known as the meshing depth; \( \alpha \) stands for the contact angle of the roller in meshing with the cam profile surface.

\[
\mathbf{F} = n \mathbf{F} = \begin{bmatrix}
 L \cos \gamma_2 - (\xi + v) \cos \gamma_2 \cos (\gamma_3 + \Gamma_i) + r \cos \alpha \cos \gamma_2 \sin (\gamma_3 + \Gamma_i) + r \sin \alpha \sin \gamma_2 \\
 -L \sin \gamma_2 + (\xi + v) \sin \gamma_2 \cos (\gamma_3 + \Gamma_i) - r \cos \alpha \sin \gamma_2 \sin (\gamma_3 + \Gamma_i) + r \sin \alpha \sin \gamma_2 \\
 -\xi \sin (\gamma_3 + \Gamma_i) - r \cos \alpha \cos (\gamma_3 + \Gamma_i) \\
\end{bmatrix}
\]

(4)

According to the properties of equidistant surface of the ruled surface, then the working profile surface of globoidal cam is obtained as a non-ruled and anisotropy-shape surface. In terms of equation (4), the normal vector \( \mathbf{n} \) of the roller axis trajectory surface is derived.
Based on the principle of the space meshing, consequently, it is drawn that the relative velocity of the normal vector direction at the meshing point is zero.

\[
\begin{vmatrix}
\frac{\partial^2 F}{\partial v^2} & \frac{\partial^2 F}{\partial \alpha^2} \\
\frac{\partial^2 F}{\partial v^2} & \frac{\partial^2 F}{\partial \alpha^2}
\end{vmatrix}
\begin{bmatrix}
\cos \alpha \cos \gamma_2 \sin(\gamma_3 + \Gamma_i) + \sin \alpha \sin \gamma_2 \\
-\cos \alpha \cos \gamma_2 \sin(\gamma_3 + \Gamma_i) + \sin \alpha \sin \gamma_2 \\
-\cos \alpha \cos(\gamma_3 + \Gamma_i) \\
0
\end{bmatrix}
\]

\text{(5)}

Based on the principle of the space meshing, consequently, it is drawn that the relative velocity of the normal vector direction at the meshing point is zero.

\[
(d^2 F/dt^2)_{n} = 0
\]

\text{(6)}

The equation (6) is substituted into equations (4) and (5), thus the equation (7) is derived.

\[
\theta = \arctan \left( \frac{\frac{d\alpha_3}{d\alpha_2}(\tilde{\xi} + \alpha)}{L - (\tilde{\xi} + \alpha) \cos(\gamma_3 + \Gamma_i)} \right)
\]

\text{(7)}

Based on the equations of ruled surface and equidistant surface, the parametric expression of theoretical tool path surface is drawn.

\[
S(\tilde{\xi}, \gamma) = rF(\tilde{\xi}, \gamma) + r\tilde{n}(\tilde{\xi}, \gamma)
\]

Where $\gamma$ denotes the rotary angle of globoidal cam.

\text{(8)}

### 2.2 Machinability analysis of non-equal diameter flank milling

The essence of tool position generation in flank milling is that the machined surface generated through the tool axis is regarded as the ruled surface generatrix. While the desired tool path surface is formed by the contacts between the side blade of actual rotary cutter and workpiece. As for flank milling of globoidal cams, the actual machine contact line is a spatial spiral. For the traditional error control method, the machining error is analyzed by measuring the distance between the tool contact line at the beginning and end of the tool axis and the contact line of the small size tool. The optimal compensation amount $\Delta = R - r$, namely the deviation between the roller radius and the tool. In recent years, the tool-path approach was mainly adopted into error control of the non-equal diameter milling. Its basic idea is that the reconstructed surface approaches theoretical tool path surface of the non-equal diameter milling. For instance, according to the characters that the actual tool path surface is a ruled surface, and the theoretical tool path surface was reconstructed by NURBS ruled surface, then the tool position of flank milling was optimized by least square method, consequently, the theoretical machining error model was established (Ge et al.2007). With regard to the deviation reduction between the theoretical profile and practical one, then an optimized calculation algorithm of tool position was devised, its objective was to minimize the deviation between the tool axis trajectory surface and the offset surface of theoretical cam profile (Kong et al.2008). YIN proposed the principle of tool position control of the single-side machining of the globoidal cam was investigated and the tool position control methods were studied by YIN, which effectively reduced the machining error of the cam surface (Yin et al.2008). In the light of the equation (7), the contact angle $\theta$ is a nonlinear function with regard to the center distance $L$. Assuming a given cam angle $\gamma_2$, the corresponding theoretical contact line of the cam and the roller will change, such as Figure 2. $L_D$ stands for the theoretical tool path for equal diameter machining; $L_N$ denotes the theoretical tool path for non-equal diameter machining; $L_C$ refers to the theoretical tool path of critical position in non-equal diameter machining; $R_o$ is the radius of tool in equal diameter machining; $r_i$ is the radius of tool in non-equal diameter machining.
With the change of $\Delta \gamma$, the tool axis surface will be generated through tool axis trajectory of the non-equal diameter machining, thus the surface is a non-developable ruled surface. The $L_D$, $L_N$ and $L_C$ are curves formed by all the point sets in the normal vector direction of the vertical roller axis at each point on the theoretical contact line between the cam and the roller.

As for the tool axis $L_D$ of the equal-diameter machining, the reconstruction of globoidal cam profile is realized, i.e., the roller radius is equal to the tool radius, therefore the theoretical expression of the cam axis can be drawn:

$$
R(\Delta r, \theta) = p(\theta) + \Delta rv(\theta) = \begin{bmatrix}
\cos \gamma_2 l(\theta) - L \cos \gamma_2 \\
\sin \gamma_2 l(\theta) \\
\sin \gamma_2 l(\theta) - L \sin \gamma_2
\end{bmatrix}
+ \Delta
\begin{bmatrix}
\sin \gamma_2 \sin \theta - \sin \gamma_3 \cos \gamma_2 \cos \theta \\
\cos \gamma_2 \cos \theta \\
\cos \gamma_2 \sin \theta - \sin \gamma_3 \sin \gamma_2 \cos \theta
\end{bmatrix}
$$

The theoretical surface of the non-equal machining tool axis is the curved surface generated by the $L_N$ and $L_C$ sweeping. In essence, the reconstruction of the tool path surface approach method is a constraint algorithm based on
adaptive multi-point bias. The tool position and compensation amount of a certain point on the contact line can be reasonably optimized through adaptive optimization algorithm. In order to effectively shorten the optimization compensation time, as for selection of compensation direction, a small section of the middle position of the effective length of the roller is regarded as the domain of the compensation parameter, i.e., \( \nu \in (l / 2, \varepsilon) \). Then adaptive optimization algorithm of the tool position compensation is transformed into the optimization problem of two-dimension nonlinear constraint interval. Accordingly, the \( \text{fmincon} \)-function in the optimization toolbox of MATLAB is employed to find the optimal solution. In the process of optimizing the flank milling algorithm, the data points are collected by meshing from the directions \( u \) and \( v \), then the surface reconstruction is performed based on the data points.

### 2.3 Discrete analysis of trajectory surface of the theoretical tool axis

The sampling intensity of discrete data points is determined by whether the final calculation results of the theory error can satisfy the machining accuracy. One of the methods to improve the machining accuracy is to add the sampling density of data points. In the past, the data points are mostly sampled through an equidistant and topological rectangular array (LIU et al.2007). If the topological rectangular array is applied to divide the theoretical profile, the data points reflecting the curvature distribution of the surface cannot be collected reasonably according to the actual curvature distribution. Therefore, an algorithm is needed to collect data points based on the geometric properties of the globoidal cam surface. The process of data points acquisition is considered as a sampling process in probability, the principle should be based on the fact that the distribution of data points collected should reflect the characteristic of curves and surfaces as much as possible. The simplest method is to use a dense sampling in areas with large curvature changes and sparse sampling in relatively flat areas.

### 3. The vector fitting of tool axis of globoidal cam flank milling

#### 3.1 The ruled surface of NURBS generation based on linear regression

**Pre-conditions of the regression problem:**

1) Effective data are collected firstly;
2) Then a hypothetical model is established, i.e., a function that contains unknown parameters, which can be estimated through learning. Then this model is served as predict new cases.

1. Preliminary determination of nodal points

In the process of machining, the regression function \( u(x) \) is unknown, and it needs to be speculated through effective experimental data. In the non-equal diameter flank milling, it is necessary to ensure the minimum machining error in order to obtain the optimal tool position. It can be seen from equation (9) that the theoretical axis of the non-equal diameter machining is a spatial curve, and the climax value of the curvature \( k \) and deflection \( \tau \) of the spatial curve occur in the parts where the shape of curves changes drastically. Therefore, the data points can be obtained through equations (9), (10), and (11).

\[
k = \frac{|R'(\Delta, \theta) \times R''(\Delta, \theta)|}{|R'(\Delta, \theta)|^3} \quad (10)
\]

\[
\tau = \frac{(R'(\Delta, \theta), R''(\Delta, \theta), R'''(\Delta, \theta))}{|R'(\Delta, \theta) \times R''(\Delta, \theta)|^2} \quad (11)
\]

Besides, the shape of the curve is also manipulated by the end point of \( L_0 \), therefore end point can be served as a valid data point whose value can be found in \( l \in [l_0, l_1] \) according to equation (9). Where the inner end of the roller along the roller axis is \( l_0 \), the top end is \( l_1 \).

2. The fitting model of spatial nodal points based on linear regression

The equation of the space line can be transformed into:
\[
\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z}{1}
\] (12)

The required parameters are: \(x_0, \ y_0, \ a, \ b\).

The equation of the line can be simplified:

\[
\begin{align*}
  x &= x_0 + az \\
  y &= y_0 + bz
\end{align*}
\] (13)

The matrix form is as follows:

\[
\begin{bmatrix}
  a & x_0 \\
  b & y_0
\end{bmatrix}
\begin{bmatrix}
  z
\end{bmatrix}
= \begin{bmatrix}
  x \\
  y
\end{bmatrix}
\] (14)

When the number of points is \(b\), the equation of the \(i\)-th point is as follows:

\[
\begin{bmatrix}
  a & x_0 \\
  b & y_0
\end{bmatrix}
\begin{bmatrix}
  z_i
\end{bmatrix}
= \begin{bmatrix}
  x_i \\
  y_i
\end{bmatrix}
\] (15)

Equation (16) is derived by parallel \(b\) equations:

\[
\begin{bmatrix}
  m & x_0 \\
  n & y_0
\end{bmatrix}
\begin{bmatrix}
  z_1, \ldots, z_n \\
  1, \ldots, 1
\end{bmatrix}
= \begin{bmatrix}
  x_1, \ldots, x_n \\
  y_1, \ldots, y_n
\end{bmatrix}
\] (16)

Then line fitting equation of the tool path is obtained:

\[
\begin{bmatrix}
  a & x_0 \\
  b & y_0
\end{bmatrix}
\begin{bmatrix}
  z_1, \ldots, z_n \\
  1, \ldots, 1
\end{bmatrix}
= \begin{bmatrix}
  x_1, \ldots, x_n \\
  y_1, \ldots, y_n
\end{bmatrix}
\] (17)

The simplification of equation (17) is as follows:

\[
\begin{bmatrix}
  a & x_0 \\
  b & y_0
\end{bmatrix}
= \begin{bmatrix}
  \sum x_i z_i & \sum z_i \\
  \sum y_i z_i & \sum z_i
\end{bmatrix}^{-1}
\] (18)

Consequently, the spatial position of the instantaneous tool point is solved by the regression model can be obtained from equation (18).

3. Iterative optimization of linear regression

Through the characters of linear regression, the results of the parameters \(m\) and \(n\) are solved through the method of searching for the extremum of the orthogonal function square. In the solving process of probability-problem, the robustness of the algorithm needs to be performed. The outlier points can occur in the solution of the regression function, that is, where the individual point change is dramatically (FENG et al. 2008). While the method of finding the extremum by the orthogonal function square is more sensitive to outlier points. The optimal solution process of the tool axis is to find a certain line segment in the space so that the maximum distance of the theoretical tool axis of non-equal
diameter machining to the line segment is the smallest. Since the so-called "outlier points" are all applicable points on
the tool axis in practice, and the sensitivity of regression algorithm is used to increase the number of "outlier points" by
inserting new nodal points. Therefore the spatial position of the instantaneous point of the tool axis is remarkably
optimized by iteration.

The optimization method can be carried out by the following processes:
1) The line segment of the tool axis is gained by preliminary fitting from equation (18);
2) The distance \( d'_0 \) from the point \( (x_i, y_i, z_i) \) of the theoretical tool axis to \( I_0 \) can be calculated through the
searching stepwisely, then \( \max[d'_0] \) is found;
3) Whether \( d'_{0\text{max}} \) satisfies \( d'_{0\text{max}} \leq d \) is judged;
4) If 3) is satisfied, the line segment of the tool axis will be output. If not, the new nodal points are added by the
isoparametric method, and then the line segment of the tool axis is obtained by the formula (18) again, i.e., the process
returns to step 1.

4. The ruled surface generation

Since the weight of data points of the theoretical tool path are equal, therefore the ownership factor \( w = 1 \) of the
NURBS surface. The curves and surfaces is widely expressed by 3-order NURBS ruled surface, and the 3-order
NURBS surface is also served for the reconstruction of the non-developable ruled surface of the tool path, then let
\( k = 3 \). Based on the differential geometry, the two alignments of the NURBS ruled surface can be expressed as 3-order
B-Spline forms (Shi et al.2013):

\[
S^{1,2}(u) = \sum_{i=0}^{n} d_{i}^{1,2} N_{i,3}(u)
\]  

(19)

Where \( d_{i}^{1,2} \) denote the control vertexs of the alignment \( S^{1}(u) \) and \( S^{2}(u) \), and \( N_{i,3}(u) \) is the basis function on
the alignment. The ruled surface is a curved surface formed by a line segment continuous motion between two
directrixes. The equation can be defined by two directrixes:

\[
S(u, v) = (1-v)S^{1}(u) + vS^{2}(u)
\]

\( u, v \in [0,1] \)

(20)

The equation of NURBS ruled surface is derived through substituting (19) into (20).
\[
S(u, v) = \sum_{i=0}^{n} [(1-v)N_{i,0}(u) + vN_{i,1}(u)d_i^2] \tag{21}
\]

5. The least square optimization

Since the tool axis vector of the aforementioned algorithm is obtained through linear regression, the optimality of the tool position of the single point is only guaranteed. Therefore, the ruled surface generated by the skinning interpolation method still needs to be optimized in order to the minimum overall error. Due to the trajectory surface of the roller axis is a ruled surface (GONG et al.2005), the objective function of the least square optimization can be established by approaching the square sum of the distance between the ruled surface and the trajectory surface of roller axis.

\[
\min \phi = \sum_{i=1}^{n} [C_i(\theta_i, w_j) - P_i(\tilde{u}_i, \tilde{v}_i)] = \|C_i - \Phi X_i\|^2 \tag{22}
\]

Where \((\tilde{u}_i, \tilde{v}_i)\) stands for the surface coordinate of the theoretical tool axis trajectory, and \((\theta_i, w_j)\) denotes the coordinate of the ruled surface of tool axis, \(X_i\) is the discrete data points on the theoretical tool axis, \(C_i\) is the discrete data points on tool axis of non-equal diameter machining, \(\Phi\) is the node vector.

3.2 The solution of artificial immune algorithm

Artificial immune method is a global probability search algorithm based on biological genetic organisms such as natural selection and genetic variation (LI et al.2004). Its selection probability includes fitness and concentration information, which simulates the function of the biological immune system in nature. The mathematical solution process is vividly simulated through the process of immunization of human or other superior animals, and then the genetic manipulations carried out through the operators of replication, crossover and mutation, subsequently the progeny antibodies superior to the parents are produced. Ultimately the optimal solution is gained through successive iteration.

1. Antibody gene coding

Antibodies in artificial immune algorithms are produced through the genes that are recognized by antigens, then the specific binding of antibodies to antigens (i.e., the degree of gene binding) is called affinity (Deng et al.2015). The basic calculation unit is similar to a binary string of chromosomes, thus the optimized data is discretely collected. Since the solution of the tool position optimization of the non-equal diameter flank milling is a problem of solving the multivariable function, the each variable position is arranged in binary code, consequently the affinity between the antibody and antigen is calculated based on the information entropy.

Suppose the capacity of the antibody is \(N\), the length of the chromosome is \(M\), and the number of alleles is \(S\). The gene of the antibody is encoded in binary code, \(U_i = \{\mu_1, \mu_2, \ldots, \mu_n\}\), \(U_i \in M, \{m_1, m_2, \ldots, m_n\}\), \(M\) is the total antibody population, \(\mu_i \in [0, 1]\) is the allele, then the information entropy of the i-th position is obtained:

\[
S_i(\zeta) = \sum_{j=1}^{S} q_{ij} \log_2 q_{ij} \tag{23}
\]

\(\zeta\) is the basic parameter, \(\zeta \in [2, N]\); \(q_{ij}\) is the probability that the allele will appear at the i-th position in the antibody.

\[
q_{ij} = \frac{\text{the sum of alleles on gene } i}{\zeta} \tag{24}
\]

The average information entropy of antibodies is:
\[
S(\zeta) = \frac{1}{M} \sum_{j=1}^{M} S_j(\zeta)
\]  

(25)

2. The affinity calculation of antibody and antigen

According to the binary code of the antigen gene, the affinity between antigen \( \sigma \) and antibody \( \omega \) is defined as \( a_{\sigma,\omega} \).

\[
a_{\sigma,\omega} = \frac{1}{1 + S(\sigma, \omega)}
\]

(26)

\( a_{\sigma,\omega} \in (0, 1) \), \( S(\sigma, \omega) \) is the binding strength of the antigen \( \delta \) and the antibody \( \beta \).

The concentration of any antibody \( \omega \) in a given population of antibodies is:

\[
y_{\omega} = \frac{1}{M} \sum_{\omega} a_{\sigma,\omega}
\]

(27)

\[
a_{\sigma,\omega} = \begin{cases} 
1 & a_{\sigma,\omega} \geq T_a, \text{ and } |a_{\sigma,\omega} - a_{\sigma,\omega'}| < m, \\
0 & \text{others}
\end{cases}
\]

where \( a_{\sigma,\omega} \) is the affinity of the antibody \( \sigma, \omega \), standing for the degree of similarity. \( T_a \) and \( m \) are preset threshold and constant respectively.

3.3 The theoretical error model of globoidal cam

The theoretical error model of the flank milling process is measured by the absolute value of the difference among the corresponding points on the tool path surface and the roller ruled surface. Since the theoretical trajectory surface of the tool axis is \( S(u, v) \), the generatrix of ruled surface is corresponding tool axis. The theoretical machining error of the approaching method is:

\[
\delta_{\text{max}} = \max_{1 \leq i \leq 4} \left\| C(\theta, w) - S(u, v) \right\| 
\]

(28)

4. The simulation and analysis of tool path

Now a left-hand globoidal cam is machined with non-equal diameter tool, as shown in Figure 4. The radius of the follower plate is \( R = 20 \text{mm} \), the center distance is \( L = 180 \text{mm} \), the movement cycle of the follower index adhere to sinusoidal law, and the indexing cycle of globoidal cam is \( 120^\circ, 72 \text{mm} \leq \zeta \leq 102 \text{mm} \).
According to formula (8), 36 generatrices of the initial ruled surface are obtained. The $36 \times 2$ control vertices of the 3-order NURBS ruled surface are optimized by linear regression algorithm through collecting data points. In the light of formula (21) and formula (22), the ruled approach surface of the theoretical tool path of the optimized cam is obtained, and the tool-path surface is generated simultaneously, as shown in Figure 5a, 5b.

![Fig. 4 3D model of globoidal cam mechanism](image)

The non-equal diameter flank milling is carried out by cylindrical cutters with different radius, and the maximum machining error is calculated through equation (19). Compared with the tool position compensation and the double-envelope methods, the results are shown in Table 1.
The maximum error of the machining in different tool radius

| \( \Delta r / \text{mm} \) | \( \delta_{\text{max}} / \text{mm} \) this research | \( \delta_{\text{max}} / \text{mm} \) tool-position compensation method | \( \delta_{\text{max}} / \text{mm} \) double-envelope method |
|---|---|---|---|
| 1 | 0.01084 | 0.01203 | 0.01392 |
| 2 | 0.02817 | 0.02844 | 0.02851 |
| 3 | 0.04113 | 0.04568 | 0.04732 |
| 5 | 0.06692 | 0.07183 | 0.07413 |

The error fluctuation of the fitted tool path surface of the tool position compensation method, the double-envelope method, and this research by the spatial linear regression algorithm are shown in Figure 6.

![Graph showing error distribution](image)

The conclusion is drawn from Table 1 and Figure 6 that the fitting error of the linear regression algorithm is smaller than the tool position compensation method and the double-envelope method when the different radius of tool are used. Compared with the tool position compensation method and the double envelope method, the spatial linear regression algorithm is not prominent in the interval with a small radius. In the interval with large tool radius, the tool position control ability of the spatial linear regression algorithm is significantly better than the tool position compensation method and the double envelope method. The reason is that the more control points in the interval with larger tool radius of curvature are attained though the spatial linear regression algorithm, which effectively improves the control ability of the algorithm, ultimately the machining accuracy is improved by 9%~13%.

5. Conclusion

1. There is an incongruous error of the normal vector on the globoidal cam profile in the non-equal diameter machining, and the normal error distribution at the instantaneous tool point along the normal direction is a non-developable ruled surface.

2. On the basis of the defects of NURBS, the trend of error variation is analyzed, and it is proved that the proposed algorithm is better than the multi-point offset algorithm based upon the ruled surface fitting of NURBS when the difference between the radius of cam roller and tool is large.

3. The trivial algorithm process is effectively avoided through tool position calculation method of multi-point free ruled surface algorithm by using linear regression for non-quantitative points, and the theoretical machining accuracy is improved concurrently. The defects of the previous NURBS algorithm are refined to large extent, then the continuity and smoothness of the tool path are guaranteed simultaneously.
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Conflict of Interests

The authors have declared that there is no conflict of interests regarding the publication of this paper.

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