Neutral SU(2) Gauge Extension of the Standard Model
and a Vector-Boson Dark-Matter Candidate

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Abstract

If the standard model of particle interactions is extended to include a neutral
$SU(2)_N$ gauge factor, with $SU(3)_C \times SU(2)_L \times U(1)_Y \times SU(2)_N$ embedded in $E_6$
or $[SU(3)]^3$, a conserved generalized $R$ parity may appear. As a result, apart from the
recent postulate of a separate non-Abelian gauge factor in the hidden sector, we have
the first example of a possible dark-matter candidate $X_1$ which is a non-Abelian vector
boson coming from a known unified model. Using current data, its mass is predicted to
be less than about 1 TeV. The associated $Z'$ of this model, as well as some signatures
of the Higgs sector, should then be observable at the LHC (Large Hadron Collider).
Introduction: Whereas dark matter [1] is generally accepted as being an important component of the Universe, its nature remains unclear. Myriad hypotheses exist, but so far, almost all particles which have been considered as dark-matter candidates are spin-zero scalars, or spin-one-half fermions, or a combination of both [2]. Spin-one Abelian vector bosons are also possible, but only in the context of more exotic scenarios, such as those of universal extra dimensions [3] and little Higgs models [4]. Spin-one non-Abelian vector bosons from a hidden sector have also been considered [5, 6, 7]. In this paper, we will show for the first time that a spin-one non-Abelian vector boson which interacts directly with known quarks and leptons may also be a dark-matter candidate, motivated by an extension of the standard SU(3)C × SU(2)L × U(1)Y gauge model of particle interactions with an extra neutral SU(2)N gauge factor, which is derivable from a decomposition of E6 or [SU(3)]3.

We will show how a conserved generalized lepton number may be defined, in analogy with the previously proposed dark left-right gauge models [8, 9, 10, 11, 12]. The difference is that the vector bosons corresponding to W±R are now electrically neutral and may become dark-matter candidates. We will also show how the decomposition of E6 or [SU(3)]3 leads to three different models of the form SU(3)C × SU(2)L × SU(2)′ × U(1)′. The first is the conventional left-right model where SU(2)′ = SU(2)R and U(1)′ = U(1)B−L, the second is the alternative left-right model [13], and the third is the case [14] where U(1)′ = U(1)Y and SU(2)′ = SU(2)N, with some of its Z′ phenomenology already discussed [15]. We do not use the original notation of SU(2)I, where the subscript I stands for “inert”, because this new gauge group certainly has interactions linking the known quarks and leptons with the exotic fermions.

We will discuss the phenomenology of this model, assuming that the real vector gauge boson X1 of SU(2)N is the lightest particle of odd R parity, where \( R = (-1)^{3B+L+2j} \), to account for the dark-matter relic abundance of the Universe. This is a new and important
possibility not discussed previously in the applications of this model. Combining it with the recent CDMS data \[16\], we find \( m_X \) to be less than about 1 TeV. This means that the associated \( Z' (= X_3) \) boson (with even \( R \) parity) should not be much heavier, and be observable at the Large Hadron Collider (LHC). The Higgs sector of this model also has some salient characteristics, with good signatures at the LHC. Note that our proposal is very different from the hidden-sector case, where all three gauge bosons, i.e. \( X_{1,2,3} \), would all be dark-matter candidates having the same mass.

\textbf{Model} : Under \( SU(3)_C \times SU(2)_L \times U(1)_Y \times SU(2)_N \), where \( Q = T_{3L} + Y \), the fermion content of this nonsupersymmetric model is given by

\[
\begin{pmatrix}
u \\ E \\ e \\
\end{pmatrix} \sim (1, 2, 1; 1), \quad (\nu^c, n^c) \sim (1, 1, 0; 2),
\]

where all fields are left-handed. The \( SU(2)_L \) doublet assignments are vertical with \( T_{3L} = \pm 1/2 \) for the upper (lower) entries. The \( SU(2)_N \) doublet assignments are horizontal with \( T_{3N} = \pm 1/2 \) for the right (left) entries. There are three copies of the above to accommodate the known three generations of quarks and leptons, together with their exotic counterparts. It is easy to check that all anomalies are canceled.

Consider a Higgs sector of one bidoublet and two doublets:

\[
\begin{pmatrix}
\phi_1^0 & \phi_2^0 \\
\phi_1^- & \phi_2^- \\
\end{pmatrix} \sim (1, 2, -1/2; 2), \quad \begin{pmatrix}
\eta^+ \\
\eta^0 \\
\end{pmatrix} \sim (1, 2, 1/2; 1), \quad (\chi^0_1, \chi^0_2) \sim (1, 1, 0; 2).
\]

The allowed Yukawa couplings are thus

\[
(d\phi_1^0 - u\phi_2^-)d^c - (d\phi_2^0 - u\phi_2^-)h^c, \quad (u\eta^0 - d\eta^+)u^c, \quad (h^c\chi_2^0 - d^c\chi_1^0)h,
\]

\[
(N\phi_2^- - \nu\phi_1^- - E\phi_2^0 + e\phi_1^0)e^c, \quad (E\eta^+ - N\eta^0)n^c - (e\eta^+ - \nu\eta^0)\nu^c,
\]

\[
(EE^c - NN^c)\chi_2^0 - (eE^c - \nu N^c)\chi_1^0,
\]
as well as

\[(EE^c - NN^c)\bar{\chi}_1^0 + (eE^c - \nu N^c)\bar{\chi}_2^0.\]  

(9)

If Eq. (9) is disallowed, then a generalized lepton number may be defined, with the assignments

\[L = 0 : u, d, N, E, \phi_1, \eta, \chi_2^0, n^c, \quad L = 1 : \nu, e, h, \phi_2, \quad L = -1 : \chi_1^0,\]  

so that the neutral vector gauge boson X linking E to e has \(L = 1\). In this scenario, \(\phi_2^0\) and \(\chi_1^0\) cannot have vacuum expectation values. Fermion masses are obtained from the other neutral scalar fields as follows: \(m_d, m_e\) from \(\langle \phi_1^0 \rangle = v_1\); \(m_u, m_\nu\) from \(\langle \eta^0 \rangle = v_3\); \(m_h, m_E, m_N\) from \(\langle \chi_2^0 \rangle = u_2\). Actually, because of the \(Nn^c\) mass term from \(v_3\), \(N\) pairs up with a linear combination of \(N^c\) and \(n^c\) to form a Dirac fermion, leaving the orthogonal combination massless. We will return to the resolution of this problem in a later section.

To forbid Eq. (9), an additional global U(1) symmetry \(S\) is imposed, as discussed in the two original dark left-right models \([8, 11]\), where \(S = L \pm T_{3R}\). Here we have \(S = L - T_{3N}\) instead. An alternative solution is to make the model supersymmetric, in which case Eq. (9) is also forbidden. We note that the structure of this model guarantees the absence of flavor-changing neutral currents, allowing thus \(SU(2)_N\) to be broken at the relatively low scale of 1 TeV.

**\(E_6\) origin**: As listed in Eqs. (1) to (4), there are 27 chiral fermion fields per generation in this model. This number is not an accident, because it comes from the fundamental representation of \(E_6\) or \([SU(3)]^3 = SU(3)_C \times SU(3)_L \times SU(3)_R\). Under the latter which is the maximal subgroup of the former, these fields transform as \((3, 3^*, 1) + (1, 3, 3^*) + (3^*, 1, 3)\), i.e.

\[
\begin{pmatrix}
  d & u & h \\
  d & u & h \\
  d & u & h \\
\end{pmatrix}
+ \begin{pmatrix}
  N & E^c & \nu \\
  E & N^c & e \\
  \nu^c & e^c & n^c \\
\end{pmatrix}
+ \begin{pmatrix}
  d^c & d^c & d^c \\
  u^c & u^c & u^c \\
  h^c & h^c & h^c \\
\end{pmatrix}
\]

(11)

The decomposition of \(SU(3)_L \rightarrow SU(2)_L \times U(1)_{Y_L}\) is completely fixed because of the standard
model. However, the decomposition of \( SU(3)_R \to SU(2)' \times U(1)' \) is not. If we choose the conventional path, then we see from the above that \((\nu^c, e^c)\) and \((u^c, d^c)\) are \( SU(2)_R \) doublets. However, another choice is to switch the first and third columns of \((1, 3, 3^*)\) and the first and third rows of \((3^*, 1, 3)\), i.e.

\[
\begin{pmatrix}
d & u & h \\
d & u & h \\
d & u & h \\
\end{pmatrix} + \begin{pmatrix}
\nu & E^c & N \\
e & N^c & E \\
n^c & e^c & \nu^c \\
\end{pmatrix} + \begin{pmatrix}
h^c & h^c & h^c \\
u^c & u^c & u^c \\
d^c & d^c & d^c \\
\end{pmatrix}.
\]

This is the alternative left-right model \([13]\), where \((n^c, e^c)\) and \((u^c, h^c)\) are \( SU(2)_R \) doublets.

The third choice \([14]\) is to switch the second and third columns of \((1, 3, 3^*)\) and the second and third rows of \((3^*, 1, 3)\), i.e.

\[
\begin{pmatrix}
d & u & h \\
d & u & h \\
d & u & h \\
\end{pmatrix} + \begin{pmatrix}
N & \nu & E^c \\
E & e & N^c \\
\nu^c & n^c & e^c \\
\end{pmatrix} + \begin{pmatrix}
h^c & h^c & h^c \\
u^c & u^c & u^c \\
d^c & d^c & d^c \\
\end{pmatrix}.
\]

This then results in Eqs. (1) to (4).

In analyzing \( Z' \) models from \( E_6 \), the usual convention is to define the two possible extra \( U(1) \) gauge factors as coming from \( E_6 \to SO(10) \times U(1)_\psi \) and \( SO(10) \to SU(5) \times U(1)_\chi \). The special case \( U(1)_{\eta} = \sqrt{3/8} \ U(1)_\chi - \sqrt{5/8} \ U(1)_\psi \) is often also considered. Here the \( Z' \) of \( SU(2)_N \) couples to the orthogonal combination, i.e. \( \sqrt{5/8} \ U(1)_\chi + \sqrt{3/8} \ U(1)_\psi \). Under the conventional \( SU(3)_R \) assignments, this is equivalent to \((1/2)T_{3R} - (3/2)Y_R\), hence \( n^c \) is +1/2 and \( \nu^c \) is −1/2 as expected.

**Gauge boson masses**: The extra gauge symmetry \( SU(2)_N \) is completely broken by \( \langle \chi^0_2 \rangle = u_2 \), so that each of the three gauge bosons \( X_{1,2,3} \) has the same mass, i.e. \( m_X^2 = (1/2)g_N^2u_2^2 \). Whereas \( X_3 \) should be identified with the extra \( Z' \) of this model, coupling to fermions according to \( T_{3N}, (X_1 \mp iX_2)/\sqrt{2} \) are the neutral analogs of \( W^\pm_R \) with \( L = \pm 1 \).

With the Higgs content of Eq. (5), there is a massless fermion per generation, corresponding to a linear combination of \( n^c \) and \( N^c \). At the same time, the neutrino has only a Dirac
mass, from the pairing of $\nu$ with $\nu^c$. Consider then the addition of the scalar triplet

$$(\xi_3^0, \xi_4^0, \xi_5^0) \sim (1, 1, 0; 3),$$

with $S = 1$, so that $\xi_3^0$ couples to $n^c n^c$ and $\xi_5^0$ couples to $\nu^c \nu^c$. Let these have nonzero vacuum expectation values $u_3$ and $u_5$ respectively, then $L$ is broken by the latter to $(-1)^L$ so that $\nu$ gets a seesaw Majorana mass in the usual way. There is also a large Majorana mass for $n^c$, so that no massless particle remains. At the same time, $R$ parity, i.e. $R = (-1)^{3B+L+2j}$, remains valid. All standard-model particles have even $R$. New particles of even $R$ are $\phi_1, \eta, \chi_2^0, Z'$; those of odd $R$ are $N, E, n^c, h, \phi_2, \chi_1^0, X_{1,2}$, the lightest of which is stable and a good dark-matter candidate if it is also neutral. However, $N$ and $\phi_2^0$ are ruled out by direct-search experiments because they have $Z$ interactions; $n^c$ and $\chi_1^0$ are also ruled out because they are mass partners of $N$ and $\phi_2^0$. That leaves only $X_{1,2}$.

The masses of the gauge bosons are now given by

$$m_W^2 = \frac{1}{2} g_2^2 (v_1^2 + v_3^2), \quad m_{X_{1,2}}^2 = \frac{1}{2} g_N^2 [u_2^2 + 2(u_3 + u_5)^2],$$

$$m_{Z,Z'}^2 = \frac{1}{2} \begin{pmatrix}
(g_1^2 + g_2^2)(v_1^2 + v_3^2) & -g_N \sqrt{g_1^4 + g_2^4 v_1^2} \\
-g_N \sqrt{g_1^4 + g_2^4 v_1^2} & g_N^2 [u_2^2 + v_1^2 + 4(u_3^2 + u_5^2)]
\end{pmatrix}. \quad (15)$$

Note that $X_{1,2}$ are split in mass, and the splitting is typically large. There is also $Z - Z'$ mixing, which is approximately given by $-\{(g_1^2 + g_2^2)/g_N\} (v_1^2/[u_2^2 + 4(u_3^2 + u_5^2)])$. Experimentally, $m_{Z'}$ is constrained [15] to be greater than about 900 GeV, and the $Z - Z'$ mixing less than a few times $10^{-4}$. Here, since $v_1$ couples to the $d$ and $e$ sectors, we may set it at around 10 GeV, which is then consistent with $SU(2)_N$ breaking to be at the TeV scale. Note that this scale is not motivated by supersymmetry, but rather by dark-matter phenomenology as detailed below.

The new particles $h, h^c, \nu^c, n^c$ are singlets with respect to the standard-model gauge group, whereas the doublets $(N, E)$ and $(E^c, N^c)$ obtain masses through $\langle \chi_2^0 \rangle$, which is a standard-model singlet. This means that their contributions to the oblique electroweak parameters
are negligible and will not upset the precision tests of the standard model. The $SU(2)_N$ interactions of $d^c$ are crucial for dark-matter search and dark-matter relic abundance. They, as well as those of $e^c$, are not in conflict with any known experimental constraint. There is also no mixing between the standard-model fermions and the new ones of this model, because the former have even $R$ and the latter odd.

**$X_1$ as dark-matter candidate**: Assuming that $X_1$ is the lightest particle of odd $R$, its relic abundance is easily estimated. The nonrelativistic cross section of $X_1X_1$ annihilation to $d\bar{d}$, $\nu\bar{\nu}$, $e^-e^+$, and $\phi_1\phi_1^\dagger$ through $h$, $N$, $E$, and $\phi_2$ exchange respectively, multiplied by their relative velocity, is given by

$$\sigma v_{rel} = \frac{g_N^4 m_X^2}{72\pi} \left[ \sum_h \left( \frac{3}{m_h^2 + m_X^2} \right)^2 + \sum_E \left( \frac{2}{m_E^2 + m_X^2} \right)^2 + \frac{2}{(m_{\phi_2}^2 + m_X^2)} \right],$$

where the sum over $h$, $E$ is for 3 generations. The factor of 3 for $h$ is the number of colors, the factor of 2 for $E$ and $\phi_2$ is to include $N$ which has the same mass of $E$ and the two $SU(2)_L$ components of $\phi_2$. Note that there is no $X_1X_1Z'$ interaction; only $X_1X_2Z'$ is allowed. We take the usual ansatz that $\sigma v_{rel}$ is about 1 pb to account for the dark-matter relic abundance. Assuming that $g_N^2 = g_2^2 = e^2 / \sin^2 \theta_W \simeq 0.4$, and setting all exotic particle masses equal, i.e. $m_h = m_E = m_{\phi_2}$, we find $(m_h^2 + m_X^2)/m_X \simeq 2.16$ TeV. Since $m_X < m_h$ must hold in this scenario, an upper bound of 1.08 TeV on $m_X$ is obtained.

The interaction of $X_1$ with nuclei is only through the $d$ quark, i.e. $X_1d \rightarrow h \rightarrow X_1d$. The coherent spin-independent elastic cross section is given by

$$\sigma_0 = \frac{3g_N^4 m_r^2}{512\pi(m_h^2 - m_X^2)^2} \frac{[Z + 2(A - Z)]^2}{A^2},$$

where $m_r$ is the reduced mass which is just the nucleon mass for heavy $m_X$, and $(Z, A)$ are the atomic and mass numbers of the target nucleus, which we take to be $^{73}$Ge, i.e. $Z = 32$ and $A - Z = 41$. The recent experimental bound [16] on $\sigma_0$ is very well approximated in the
range $0.3 < m_X < 1.0$ TeV by the expression [11]

$$\sigma_0 < 2.2 \times 10^{-7} \text{ pb } (m_X/1 \text{ TeV})^{0.86}.$$  \hspace{1cm} (19)

Using this, we obtain

$$m_h^2 > m_X^2 + (1.03 \text{ TeV})^2 \left(1 \text{ TeV}/m_X\right)^{0.43}.$$  \hspace{1cm} (20)

Here $m_h$ refers only to the mass of the $h$ leptoquark which couples to $d$ of the nucleon. It is easy to see that this requires $m_h > 1.29$ TeV. Combining Eq. (20) with $\sigma v_{rel} = 1$ pb from Eq. (17), the prediction of our model is then

$$m_X < 1.03 \text{ TeV}.$$  \hspace{1cm} (21)

To obtain a lower bound on $m_X$, we assume that no exotic particle has a mass greater than $\sqrt{5}$ times $m_X$ (basically from requiring that no Yukawa coupling exceeds 1). In that case, Eq. (20) implies $m_X > 0.58$ TeV. It is also interesting to note that for a real-vector-boson dark-matter candidate, a large spin-dependent cross section is allowed [18].

In Fig. 1 we show the lower bound of $m_h$ (for the $h$ which couples to the $d$ quark of the nucleon) versus $m_X$ from Eq. (20), as well as the upper bound on the mass of at least one other exotic particle (call it $m_E$), corresponding to 1 pb and 0.5 pb for $\sigma v_{rel}$ of Eq. (17). We impose the constraint that $m_X$ is lighter than all other exotic particles and assume that no mass is greater than $\sqrt{5}m_X$. We note that $\sigma v_{rel} = 0.5$ pb allows $m_X$ to extend to about 1.5 TeV.

**LHC phenomenology:** As mentioned already, the $Z'$ of our model is a particular linear combination of the $Z_\psi$ and $Z_X$ of $E_6$ models, which have been studied widely in the literature. Its mass is constrained by present data to greater than about 1 TeV. In our model, from Eqs. (15) and (16), $m_{Z'}$ exceeds $m_X$ by the contributions of $u_3$ and $u_5$, which should be of order 1 TeV. It will decay into all three generations of $\bar{d}d$, $\nu\bar{\nu}$, and $e^-e^+$, and into $\phi_1\phi_1^\dagger$. 

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Figure 1: Lower bound on $m_h$ versus $m_X$ from CDMS and upper bound on the mass of at least one other exotic particle for $\sigma v_{rel} = 1$ pb and 0.5 pb.

Its decay into $\mu^-\mu^+$ (with branching fraction 1/16) should be a very good signature of its observation [8, 11]. Once $Z'$ is discovered, the ratios $B(Z' \to t\bar{t})/B(Z' \to \mu^-\mu^+) = 0$ and $B(Z' \to b\bar{b})/B(Z' \to \mu^-\mu^+) = 3$ should discriminate [17] it from other possible $Z'$ models.

The Higgs sector of this model also has some interesting features that could be tested at the LHC. For instance, since $m_{d,s,b}$ and $m_{e,\mu,\tau}$ come from $v_1$ which is constrained by $Z - Z'$ mixing to be small, $v_1$ itself could be of order 10 GeV or less. This means that a physical neutral Higgs boson with a significant component of $\phi_1^0$ will have large Yukawa couplings to $b\bar{b}$. This can induce a large enhancement on the cross section for the associated production...
of some neutral Higgs bosons with $b$ quarks, which may be detectable at the LHC. Writing the Yukawa interaction as $y_b \bar{b} b e(\phi_0^0)$, with $y_b = \kappa m_b/v$ ($v = 174$ GeV), we can estimate the values of $\kappa$ that can be tested at the LHC. In particular, a Higgs mass of $(200, 400, 800)$ GeV can be tested with $\kappa \sim (2.7, 4, 8)$, which is well within the enhancement that can be achieved in this model for $v_1 \sim 10$ GeV. A detailed study of the Higgs sector of this model, including these signals, will be given elsewhere.

**Conclusion**: A neutral $SU(2)_N$ gauge extension of the standard model is studied and a non-Abelian vector boson $X_1$ is identified as a possible dark-matter candidate. This is the first example of such a particle coming from a well-motivated unified model, namely $E_6$ or its maximal subgroup $[SU(3)]^3 = SU(3)_C \times SU(3)_L \times SU(3)_R$. We show that the annihilation of $X_1$ to standard-model particles through their $SU(2)_N$ interactions may account for the dark-matter relic abundance of the Universe. Together with the constraint from the recent CDMS dark-matter direct-search experiment, we find that $m_{X'}$ is less than about 1 TeV. The associated $Z'$ of this model is predicted to be not too much heavier and should be observable at the LHC, along with some associated Higgs signatures.

**Addendum**: Variants of our model are easily contemplated. For example, the $SU(2)_N$ Higgs triplet $\xi$ may be eliminated, in which case the massless state in the $(N, N^c, n^c)$ sector can become massive by adding a singlet $n$ with $S = 0$. If a similar singlet $\nu'$ with $S = 1$ is also added, neutrinos can get a Majorana mass through the inverse seesaw mechanism in the $(\nu, \nu^c, \nu')$ sector with a small Majorana mass term for $\nu'$ which breaks $L$ to $(-1)^L$ softly. In this scenario, the splitting of $X_{1,2}$ is radiative and finite, but very small. This implies a very different phenomenology for dark matter, because $XX^\dagger$ annihilation as well as $Xd$ scattering through $Z'$ must be taken into account. These and other issues will be discussed elsewhere.

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