Supersymmetric Q-balls: theory and cosmology

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Abstract

MSSM predicts the existence of Q-balls, some of which can be entirely stable. Both stable and unstable Q-balls can play an important role in cosmology. In particular, Affleck–Dine baryogenesis can result in a copious production of stable baryonic Q-balls, which can presently exist as a form of dark matter.
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In a class of theories with interacting scalar fields \( \phi \) that carry some conserved global charge, the ground state is a Q-ball, a lump of coherent scalar condensate that can be described semiclassically as a non-topological soliton and has a form

\[
\phi(x, t) = e^{i\omega t}\bar{\phi}(x).
\]

Q-balls exist whenever the scalar potential satisfies certain conditions that were first derived for a single charged degree of freedom and were later generalized to a theory of many scalar fields with different charges.

It was recently pointed out that all phenomenologically viable supersymmetric extensions of the Standard Model predict the existence of non-topological solitons associated with the conservation of baryon and lepton number. The MSSM admits a large number of different Q-balls, characterized by (i) the quantum numbers of the fields that form a spatially-inhomogeneous ground state and (ii) the net global charge of this state.

First, there is a class of Q-balls associated with the tri-linear interactions that are inevitably present in the MSSM. The masses of such Q-balls grow linearly with their global charge, which can be an arbitrary integer number. Baryonic and leptonic Q-balls of this variety are, in general, unstable with respect to their decay into fermions. However, they could form in the early universe through the accretion of global charge. In the false vacuum such a process could precipitate an otherwise impossible or slow phase transition. In a metastable vacuum, a Q-ball tends to have a negative energy density in its interior. When the charge reaches some critical value, a Q-ball expands and converts space into a true-vacuum phase. In the case of tunneling, the critical bubble is formed through coincidental coalescence of random quanta into an extended coherent object. This is a small-probability event. If, however, a Q-ball grows through charge accretion, it reaches the critical size with probability one, as long as the conditions for growth are satisfied. Therefore, the phase transition can proceed at a much faster rate than it would by tunneling.
The second class of solitons comprises the Q-balls whose VEV is aligned with some flat direction of the MSSM and is a gauge-singlet combination of squarks and sleptons with a non-zero baryon or lepton number. The potential along a flat direction is lifted by some soft supersymmetry-breaking terms that originate in a “hidden sector” of the theory some scale \( \Lambda_s \) and are communicated to the observable sector by some interaction with a coupling \( g \), so that \( g\Lambda \sim 100 \text{ GeV} \). Depending on the strength of the mediating interaction, the scale \( \Lambda_s \) can be as low as a few TeV (as in the case of gauge-mediate SUSY breaking), or it can be some intermediate scale if the mediating interaction is weaker (for instance, \( g \sim \Lambda_s/m_{\text{Planck}} \) and \( \Lambda_s \sim 10^{10} \text{ GeV} \) in the case of gravity-mediated SUSY breaking). For the lack of a definitive scenario, we take \( \Lambda_s \) to be a free parameter. Below \( \Lambda_s \) the mass terms are generated for all the scalar degrees of freedom, including those that parameterize the flat direction. At the energy scales larger than \( \Lambda_s \), the mass terms turn off and the potential is “flat” up to some logarithmic corrections. If the Q-ball VEV extends beyond \( \Lambda_s \), the mass of a soliton, is no longer proportional to its global charge \( Q \), but rather to \( Q^{3/4} \). This allows for the existence of some entirely stable Q-balls with a large baryon number \( B \) (B-balls). Indeed, if the mass of a B-ball is \( M_B \sim (1 \text{ TeV}) \times B^{3/4} \), then the energy per baryon number \( (M_B/B) \sim (1 \text{ TeV}) \times B^{-1/4} \) is less than 1 GeV for \( B > 10^{12} \). Such large B-balls cannot dissociate into protons and neutrons and are entirely stable thanks to the conservation of energy and the baryon number. If they were produced in the early universe, they would exist at present as a form of dark matter. 

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At the end of inflation, the scalar fields of the MSSM develop some large expectation values along the flat directions, some of which have a non-zero baryon number. Initially, the scalar condensate has the form given in eq. (1) with \( \tilde{\phi}(x) = \text{const} \) over the length scales greater than a horizon size. One can think of it as a universe filled with Q-matter. The relaxation of this condensate to the potential minimum is the cornerstone of the Affleck–Dine (AD) scenario for baryogenesis.

It was often assumed that the condensate remains spatially homogeneous from the time of formation until its decay into the matter baryons. This assumption is, in general, incorrect. In fact, the initially homogeneous condensate can become unstable and break up into Q-balls whose size is determined by the potential and the rate of expansion of the Universe. B-balls with \( 12 < \log_{10} B < 30 \) can form naturally from the breakdown of the AD condensate. These are entirely stable if the flat direction is “sufficiently flat”, that is if the potential grows slower than \( \phi^2 \) on the scales or the order of \( \phi(0) \).

The evolution of the primordial condensate can be summarized as follows:
Conceivably, the cold dark matter in the Universe can be made up entirely of SUSY Q-balls. Since the baryonic matter and the dark matter share the same origin in this scenario, their contributions to the mass density of the Universe are related. Therefore, it is easy to understand why the observations find $\Omega_{DARK} \sim \Omega_B$ within an order of magnitude. This fact is extremely difficult to explain in models that invoke a dark-matter candidate whose present-day abundance is determined by the process of freeze-out, independent of baryogenesis. If this is the case, one could expect $\Omega_{DARK}$ and $\Omega_B$ to be different by many orders of magnitude. If one doesn’t want to accept this equality as fortuitous, one is forced to hypothesize some \textit{ad hoc} symmetries that could relate the two quantities. In the MSSM with AD baryogenesis, the amounts of dark-matter Q-balls and the ordinary matter baryons are naturally related. One predicts $\Omega_{DARK} = \Omega_B$ for B-balls with $B \sim 10^{26}$.

This size is well above the present experimental lower limit on the baryon number of an average relic B-ball, under the assumption that all or most of cold dark matter is made up of Q-balls. On their passage through matter, the electrically neutral baryonic SUSY Q-balls can cause a proton decay, while the electrically charged B-balls produce massive ionization. Although the condensate inside a Q-ball is electrically neutral, it may pick up some electric charge through its interaction with matter. Regardless of its ability to retain electric charge, the Q-ball would produce a straight track in a detector and would release the energy of, roughly, 10 GeV/mm. The present limits constrain the baryon number of a relic dark-matter B-ball to be greater than $10^{22}$. Future experiments are expected to improve this limit. It would take a detector with the area of several square kilometers to cover the entire interesting range $B \sim 10^{22}...10^{30}$.

The relic Q-balls can accumulate in neutron stars and can lead to their ultimate destruction over a time period from one billion years to longer than the age of the Universe. If the lifetime of a neutron star is in a few Gyr range, the predicted mini-supernova explosions may be observable and may be related to the gamma-ray bursts.
A different scenario that relates the amounts of baryonic and dark matter in the Universe, and in which the dark-matter particles are produced from the decay of unstable B-balls was proposed by Enqvist and McDonald.\(^\text{13}\)

The following diagram illustrates our conclusions and the assumptions on which they rely:

- **Q-balls exist**
- **MSSM (SM+SUSY)**
- **stable baryonic Q-balls exist**
- **"flat" directions grow slower than $\phi^2$ after SUSY breaking**
- **stable Q-balls copiously produced in the early Universe; now: dark matter**
- **inflation**

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