Non-linear characterisation of the physical model of an ancient masonry bridge

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Abstract. This paper presents the non-linear investigations carried out on a scaled model of a two-span masonry arch bridge. The model has been built in order to study the effect of the central pile settlement due to riverbank erosion. Progressive damage was induced in several steps by applying increasing settlements at the central pier. For each settlement step, harmonic shaker tests were conducted under different excitation levels, this allowing for the non-linear identification of the progressively damaged system. The shaker tests have been performed at resonance with the modal frequency of the structure, which were determined from a previous linear identification. Estimated non-linearity parameters, which result from the systematic application of restoring force based identification algorithms, can corroborate models to be used in the reassessment of existing structures. The method used for non-linear identification allows monitoring the evolution of non-linear parameters or indicators which can be used in damage and safety assessment.

1. Introduction

Non-linear identification is still scarcely applied to the identification of real structures, due to the large quantity of data required and to the difficulty of the problem. However, information about the non-linear behaviour of a structure can be essential in the assessment of the seismic behaviour of a real structure. Moreover, changes in the structure’s dynamic properties, including deviations from linearity, may provide quantitative evidence that damaging phenomena are underway.

The Restoring Force Surface (RFS) method was first developed by Masri & Caughey (1979) in order to achieve a non-parametric identification of non-linearity in structural systems. The basic idea behind the method is that the restoring force of a non-linear system can be expressed in terms of the state variables of the system itself. Once the restoring force surface is defined, it is possible to set the parameters of an analytical model in terms of displacements and velocities using a least square approximation. Masri & Caughey [1] at first used an orthogonal Chebyshev polynomial representation to construct a model of the restoring force.

This paper deals with the non-linear characterisation of an experimental masonry bridge. The structure underwent various damage steps, allowing for the collection of dynamic response data at each step. In particular, an electro-mechanical shaker has been used to apply different excitation levels in order to investigate the behaviour of the bridge at resonant frequencies, also in order to perform a non-linear identification of the structure based on a simple Single Degree of Freedom (SDoF) equation form. Thus, the shaker was placed so as to apply harmonic excitations at sensible points of the structure.

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2. The experimental masonry bridge model

The 1:2 scaled model of the masonry arch bridge shown in figure 1 was built in the laboratory of the Department of Structural, Building and Geotechnical Engineering at the Politecnico di Torino [2,3] consisting of a twin-arch bridge with a length of 5.90m, a width of 1.60m and it is 1.75m high. The two arches are segmental arches with a radius of 2.00m and an angular opening of 30°. Each span is 2.00m long between the supports and the thickness of the arch is equal to 0.20m. The model was built with handmade clay bricks also scaled to 130x65x30mm to respect the adopted modelling scale law. Low compressive strength elements were chosen and a mortar with poor mechanical properties was used to bound them in order to reproduce the typical materials of historical constructions.

![Figure 1. The scaled masonry bridge: notice the settlement application device under the central pier.](image)

The mid-span masonry pier, which was cut at a hypothetical middle-height section to allow the insertion of a settlement application system, is imagined to be placed inside the streambed and subjected to the scour of its foundation.

Foundation settlements and rotations were applied on the bridge model by means of the four independent screws installed at the extremities of the settlement application system. The spherical plain bearings placed at the head of the screws allow the rotations of the plate which supports the central pier about axes parallel to the longitudinal and transversal directions of the bridge.

2.1. Dynamic experimental test

The experimental study involved three different testing campaigns. The first campaign regarded the undamaged structure: an extensive set of dynamic tests was carried out on the bridge model in order to characterise its “healthy” state. The second campaign started after the application of additional masses on the central pier, in order to take in account the weight of the missing part of the pier. In the same campaign the first four settlement steps were applied on the upstream side of the pier. Dynamic tests were conducted in correspondence of each settlement step. During the third campaign five further settlement steps were applied (11 mm total settlement).
Figure 2. Shaker application in positions 9A and 6C (right), experimental setups for vibration tests (left).

Measuring points were selected in order to achieve a sufficient mode shapes resolution. The arch barrels were subdivided in 11 segments whose ends were assumed as measuring points for both the edge and the middle lines. Other 6 positions at the springing sections of the pier were materialised to capture the longitudinal displacements. The 4 mid-span sections of the arch barrels lateral faces and the 2 pier frontal faces were considered for the lateral and torsional modes. Finally, the 2 positions on the longitudinal spandrel walls at the middle section of the deck were added to identify the vertical modes.

Forced vibration tests were performed by using a shaker TIRA TV 51220, capable of supplying a rated peak force of 200 N. The force applied was acquired by using a mechanical impedance sensor PCB Piezotronics 288D01 (measurement range ±222.4 N pk). Figure 2 shows both the location where the shaker has been applied and the accelerometers’ location on the structure.

Through a first linear identification, at each damage step, the modal frequencies were identified, in order to plan the forced vibration tests. In fact, at each modal frequency a harmonic excitation has been applied by using different force levels, usually 33 N, 66 N and 100 N for one minute. Both the force and the acceleration input to the system were acquired. Using the accelerometers, dynamic measurements were collected at 18 different locations, as shown in figure 2.

3. Nonlinear identification

3.1. Non-linear identification in principal coordinates

Classical linear theory allows the decoupling of motion equations in terms of modal or principal coordinates. On the other hand, several authors investigated the non-linear vibration of mechanical systems also in the principal coordinates - for instance see Bellizzi & Bouc [4] or Spottswood & Alleman [5] - though in such a case the equations of motion are coupled, as it is shown by the equation referred to the k-th coordinate:

\[ \dddot{p}_k + 2 \zeta_k \omega_k \ddot{p}_k + \omega_k^2 p_k = \theta_k \left( p_1(t), p_2(t), \ldots, p_n(t) \right) = U_{ik} F_i(t) \]  

(1)

where \( p_i \) represents the principal coordinate, \( \zeta \) the modal damping, \( \omega_k \) the k-th natural frequency, \( F_i(t) \) the force applied at position \( i \) in physical coordinates, \( U_{ik} \) the modal or basis vector and \( \theta_k \) the non-linear component that couples the equations of motion.
In light of what has been shown, whenever the excitation is high enough to induce a non-linear response in the system, modal interaction and internal resonances between different modes are expected.

In some circumstances, especially in the case of resonant excitation applied to a weakly non-linear system, most of the energy is carried by a single resonant component, \( p_k \), the other components being negligible. This means that the coupling in Equation (1) vanishes and the system approximately behaves like a SIMO (Single Input Multiple Output) system. In the specific case of the resonance tests conducted on the masonry bridge, the latter circumstance has been confirmed by experimental evidence [6]. In fact, while at higher excitation levels the presence of internal resonances was evident (peculiarly in the 100 N excitation case), all resonance peaks were still related to the super-harmonics of the excited frequency, the coupling components being excluded [6].

For a non-linear characterisation purpose, one can neglect the interactions among the principal responses and non-linearity may be ultimately studied through the following equation:

\[
\ddot{p}_k(t) + 2\xi_k\omega_k \dot{p}_k(t) + \omega_k^2 p_k(t) + f_k(p(t)_k) = U_k F_i(t)
\]  

(2)

where the non-linear modal component \( f_k \) is decoupled.

4. Non-linear polynomial identification from shaker tests

As previously stated, the bridge dynamic response has been treated as a series of SISO systems, using resonant excitation. In particular, the model proposed in equation (2) may be expressed also in physical coordinates. For an excitation applied at position \( i \) and a response measured at position \( j \) one has:

\[
\begin{align*}
\ddot{x}_{ki} + f_{st}(x_{ki}, \dot{x}_{ki}) + f_{hyd}(\dot{x}_{ki}, f) &= U_{kj} U_k F_i(t) \\
f_{st}(x_{ki}, \dot{x}_{ki}) &= \sum_{m=0}^{3} \sum_{n=0}^{3} \alpha_{mn} x_{ki}^m \dot{x}_{ki}^n
\end{align*}
\]

(3)

The lower excitation levels are expected to show a predominance of the linear behaviour. In the optimisation process, linear modal parameters, including a viscous equivalent damping, have been initialised to those available from a previous identification from ambient response signals (SSI algorithm) [2]. An initial estimate of the factor \( U_{kj} U_k \) has been conveniently obtained from the classical formula of viscous damping at resonant excitation.

4.1. Non-linear identification results

The results of the fittings are reported in figure 3 for one single response point that is representative of the dynamic behaviour of the structure relative to the first mode at 17.4 Hz (point 6C – see figure 2). Several nonlinear terms assume relevant values with the increase of the excitation level. The higher excitation level (100 N) reported in figure 3, shows clearly a nonlinear behaviour. Red lines in figure 3 correspond to the nonlinear fitting carried out by minimising the \( \alpha \) coefficients in equation (3).
Figure 3. Comparison between experimental data and identified results for three excitation levels in terms of acceleration. The identified restoring force vs displacement plot is reported on the right side.

Figure 4 quantifies the polynomial coefficients as estimated for the various excitation levels. When the system is excited by a 50 N resonant force, only the viscous damping term is activated by the minimisation process, while at higher excitation levels the whole set of nonlinear terms is activated. Estimates for various coefficients are normalised to their maximum value. The value estimated for viscous damping is similar at different excitation levels (in Figure 4 this term seems to vanish, but this is due to normalization with respect to nonlinear coefficients). In fact, at 100 N the most significant nonlinear terms resulted to be the $x^2$, $x\dot{x}$ and $x^2\ddot{x}$. 
Figure 4. Nonlinear coefficients results for three different excitation levels (sixth damage step applied to the bridge). In the bottom part the corresponding nonlinear terms laws is highlighted.

5. Conclusions

In the last years, non-linear identification has been an area of increasing research interest. In the specific context of cultural heritage structures, this paper has presented a polynomial identification of the non-linear restoring force associated to the resonant response of a scaled model of a masonry arch bridge subjected to shaker dynamic tests at three different excitation levels. Research on the dynamics of masonry offers some perspectives for increasing the knowledge in the fields of structural reassessment of ancient structures, as well as damage detection procedures.

6. References

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