Deducing the Lowest Rest Mass Quarks and Baryons of all Kinds with an Expanded Form of Planck-Bohr’s Quantization Method

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Abstract

Using an expanded form of Planck-Bohr’s quantization method and phenomenological formulae, we deduce the rest masses and intrinsic quantum numbers (I, S, C, b and Q) of the lowest energy quarks and baryons of all kinds, from only one elementary quark family \( \epsilon \) with \( S=C=b=0 \). The deduced quantum numbers match those found in experiments, and the deduced rest masses are consistent with experimental results. This paper predicts some quarks \( u_c(6073) \), \( d_s(9613) \) and \( d_b(9333) \) and baryons \( \Lambda_c^+(6699) \), \( \Lambda_b^0(9959) \) and \( \Lambda(10239) \).

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I Introduction

One hundred years ago, most physicists thought that the study of physics had been “completed.” They believed that it could explain “all” physical phenomena. Black body spectrum, however, could not be explained by the physics of that time, leading Planck to propose a quantization postulate to solve this problem [1]. This quantization eventually led to quantum mechanics.

Today we face a similar situation. The standard model [2] “is in excellent accord with almost all current data.... It has been enormously successful in predicting a wide
range of phenomena,” but it cannot deduce the mass spectra of quarks, baryons and mesons. So far, no theory can successfully do so. This case hints that physics need a “more fundamental theory” \[2\] than the standard model. What is it? Nobody knows! Thus we should encourage attempts of all kinds. We try to deduce the spectra using the expanded form of Planck-Bohr’s \[3\] quantization method and phenomenological formula. We work with systems (quarks and hadrons) that are a level deeper than the system (atoms and nuclei) faced by Planck and Bohr. Therefore, if Planck and Bohr got correct quantizations for atoms and nuclei using only one simple quantization, we must use two steps and more complex quantizations. It is worth emphasizing that we must expand upon Planck-Bohr’s quantization, performing it twice rather than once, in order to obtain the short-lived and scarce quarks (a deeper quantization than atoms and nuclei). Hopefully, this method will help physicists to discover a more fundamental theory which underlies the standard model \[2\], just as Planck-Bohr’ method has done.

II The Elementary Quarks in the Vacuum and Their
Free Excited Quarks

1. We assume that there is only one elementary quark family \( \epsilon \) with two isospin states (\( \epsilon_u \) has \( I_Z = \frac{1}{2} \) and \( Q = +\frac{2}{3} \), \( \epsilon_d \) has \( I_Z = -\frac{1}{2} \) and \( Q = -\frac{1}{3} \)). For \( \epsilon_u \) (or \( \epsilon_d \)), there are three colored quarks. Thus, there are six Fermi \( (s = \frac{1}{2}) \) elementary quarks in the \( \epsilon \) family with \( S = C = b = 0 \) in the vacuum.

2. As a colored elementary quark \( \epsilon_u \) (or \( \epsilon_d \)) is excited from the vacuum, its color, electric charge and spin do not change, but it will get energy \( V \) (the minimum excited energy from the vacuum). The excited state of the elementary quark \( \epsilon_u \) is the u-quark with \( Q = \frac{2}{3} \) and \( s = \frac{1}{2} \). The excited state of the elementary quark \( \epsilon_d \) is the d-quark with \( Q = -\frac{1}{3} \) and \( s = \frac{1}{2} \). For the excited quark free motion, in general case, we shall use the
Dirac equation. Our purpose is to find the rest masses of the excited quarks. The rest masses are the low energy limits of the excited quark masses. If we omit the spin of the quark, the low energy limit of the Dirac equation is the Schrödinger equation. When we use the Schrödinger equation to approach the Dirac equation, we cannot forget the static energy of the excited quark. We will deal with this energy as a potential energy (V). The approximate Schrödinger equation is:

\[
\frac{\hbar^2}{2m_\epsilon} \nabla^2 \psi + (\varepsilon - V) \psi = 0 \tag{1}
\]

where \(m_\epsilon\) is the unknown mass of the elementary quark \(\epsilon\). Omitting electromagnetic mass, we assume \(m_\epsilon \gg M_p = 938\) Mev; this is one of the reasons we use the Schrödinger equation instead of the Dirac equation. Our results will show that this approach is a very good approximation. V is the static energy, and it is the minimum excited energy of an elementary quark from the vacuum. The solution of (1) is the eigen wave function and the eigen energy of the u-quark or the d-quark:

\[
\psi_{-\rightarrow \vec{k}}(\vec{r}) \sim \exp(i \vec{k} \cdot \vec{r}), \\
\varepsilon = V + \frac{\hbar^2}{2m_\epsilon} [(k_1^2 + k_2^2 + k_3^2)].
\tag{2}
\]

According to the Quark Model [4] a proton \(p = uud\) and a neutron \(n = udd\), omitting electromagnetic mass of quarks, from (2), at \(\vec{k} = \vec{0}\), we have

\[
M_p = m_u + m_u + m_d - |E_{\text{bind}}| \approx M_n = m_u + m_d + m_d - |E_{\text{bind}}| = 939\ \text{Mev}
\tag{3}
\]

\[
\rightarrow m_u = m_d = V = \frac{1}{3}(939 + |E_{\text{bind}}|) = 313 + \Delta \ (\text{Mev})
\tag{4}
\]

where \(E_{\text{bind}}\) is the total binding energy of the three quarks in a baryon. \(\Delta\) represents \(\frac{1}{3} |E_{\text{bind}}|\), and is an unknown large positive constant for all baryons.

\[
\Delta = \frac{1}{3} |E_{\text{bind}}| \gg M_p.
\tag{5}
\]
Now that we have free excited the $u(313+\Delta)$-quark and the $d(313+\Delta)$-quark, they are long-lived and common quarks; how, then, do we deduce the short-lived and scarce quarks?

III Deducing Energy Bands With an Expanded Form of Planck-Bohr’s Quantization Method

1. In order to deduce the short-lived and scarce quarks with the expanded quantization method, we recall the works of Planck and Bohr.

Planck’s [1] energy quantization postulate states that “any physical entity whose single ‘coordinate’ execute simple harmonic oscillations (i.e., is a sinusoidal function of time) can possess only total energy $\varepsilon$ which satisfy the relation $\varepsilon = nh\nu$, $n = 0, 1, 2, 3, \ldots$ where $\nu$ is the frequency of the oscillation and $h$ is a universal constant.” Planck selects reasonable energy from a continuous energy spectrum.

Bohr’s [3] orbit quantization tell us that “an electron in an atom moves in a circular orbit about the nucleus...obeying the laws of classical mechanics, But instead of the infinity of orbits which would be possible in classical mechanics, it is only possible for an electron to move in an orbit for which its orbital angular momentum $L$ is an integral multiple of Planck’s constant $h$, divided by $2\pi$.” Using the quantization condition ($L = \frac{nh}{2\pi}$, $n = 1, 2, 3, \ldots$ ), Bohr selects reasonable orbits from the infinite orbits.

Drawing from these great physicists’ works, we find the most important law is to use quantized conditions and symmetries (as circular orbit) to select reasonable energy levels from a continuous energy spectrum.

2. In order to get the short-lived scarce quarks, we quantize the free motion of an excited quark [2] to select energy bands from the continuous energy. The energy bands correspond to short-lived and scarce quarks.
a. For free motion of an excited quark with continuous energy (2), we assume the wave vector $\vec{k}$ has the symmetries of the regular rhombic dodecahedron in $\vec{k}$-space (see Fig. 1). We assume that the axis $\Gamma$-$H$ in Fig.1 has length $\frac{2\pi}{a}$, with an unknown constant $a$.

The expanded quantizing conditions are:

$$k_1 = \frac{2\pi}{a}(n_1 - \xi),$$
$$k_2 = \frac{2\pi}{a}(n_2 - \eta),$$
$$k_3 = \frac{2\pi}{a}(n_3 - \zeta).$$

Putting (6) into (2), we get (7) and (8):

$$\psi(\vec{k}) \sim \exp\frac{2\pi}{a}[(n_1 - \xi)x + (n_2 - \eta)y + (n_3 - \zeta)z]$$

$$E(\vec{k}, \vec{n}) = 313 + \Delta + \alpha[(n_1 - \xi)^2 + (n_2 - \eta)^2 + (n_3 - \zeta)^2].$$

Where $\alpha = \frac{\hbar^2}{2ma^2}$. For $\vec{n} = (n_1, n_2, n_3)$, $n_1$, $n_2$ and $n_3$ are ± integers and zero. The $\vec{n}$ = ($\xi$, $\eta$, $\zeta$) has the symmetries of a regular rhombic dodecahedron. In order to deduce the deeper short-lived and scarce quarks, we must further quantize the $\vec{n}$ as follows.

b. Where $n_1$, $n_2$ and $n_3$ are integers to satisfy the expanded quantizing condition (9): If we assume $n_1 = l_2 + l_3$, $n_2 = l_3 + l_1$ and $n_3 = l_1 + l_2$, so that

$$l_1 = \frac{1}{2}(-n_1 + n_2 + n_3),$$
$$l_2 = \frac{1}{2}(+n_1 - n_2 + n_3),$$
$$l_3 = \frac{1}{2}(+n_1 + n_2 - n_3),$$

the condition is that only those values of $\vec{l} = (n_1, n_2, n_3)$ are allowed that make $\vec{l}$ = $(l_1, l_2, l_3)$ an integer vector. This is the expanded quantization for $\vec{n}$ values. For example, $\vec{n}$ cannot take the values (1, 0, 0) or (1, 1, 1), but can take (0, 0, 2) and (1, 1, 2). The low level allowed $n = (n_1, n_2, n_3)$ values are shown in the following table:
c. The vector $\vec{\kappa} = (\xi, \eta, \zeta)$ in (7) and (8) has the symmetries of the regular rhombic dodecahedron in k-space (see Fig. 1). From Fig. 1, we can see that there are four kinds of symmetry points ($\Gamma$, H, P and N) and six kinds of symmetry axes ($\Delta$, $\Lambda$, $\Sigma$, $\Delta$, $\Gamma$ and G) in the regular rhombic dodecahedron. The coordinates $(\xi, \eta, \zeta)$ of the symmetry axes are:

\[
\Delta = (0, 0, \zeta), \quad 0 < \zeta < 1; \quad \Lambda = (\xi, \zeta, \xi), \quad 0 < \xi < \frac{1}{2};
\]

\[
\Sigma = (\xi, \xi, 0), \quad 0 < \xi < \frac{1}{2}; \quad D = (\frac{1}{2}, \frac{1}{2}, \xi), \quad 0 < \xi < \frac{1}{2};
\]

\[
\Gamma = (\xi, 1-\xi, 0), \quad \frac{1}{2} < \xi < 1; \quad F = (\xi, \xi, 1-\xi), \quad 0 < \xi < \frac{1}{2}.
\]

The energy (8) of the excited quarks has six kinds of symmetry axes (10) (see Table A2 and Table A3 of [5]).

d. The energy (8) with a $\vec{n} = (n_1, n_2, n_3)$ along a symmetry axis (coordinates $(\xi, \eta, \zeta)$ of (10)) forms an energy band.

e. Each energy band corresponds to a short-lived and scarce quark. Any excited elementary quark that is not along a symmetry axis is the long-lived u-quark (or the d-quark).

3. After getting (8), (9) and (10), using (25) we can deduce low energy bands of the six symmetry axes (see Table B1-B3 and Table B4-B7 of [5]). As an example, we can show the single energy bands of the $\Delta$-axis and the $\Sigma$-axis in Table 1:
Table 1. The Single Energy Bands

| The \( \Delta \)-Axis | The \( \Sigma \)-Axis |
|----------------------|----------------------|
| \( E_{\text{Start}} \) | \( E_{\text{Point}} \) |
| \( (n_1, n_2, n_3) \) | \( n_1 n_2 n_3 \) |
| \( E(\vec{k}, \vec{n}) \) | \( E(\vec{k}, \vec{n}) \) |
| \( E_{\text{end}} \) | \( E_{\text{end}} \) |
| \( E_\Gamma=0 \) | \( E_\Gamma=0 \) |
| \( (0, 0, 0) \) | \( (0, 0, 0) \) |
| 313 | 313 |
| \( E_\Gamma=4 \) | \( E_\Gamma=4 \) |
| \( (0, 0, \frac{\pi}{2}) \) | \( (1, 1, 0) \) |
| 1753 | 493 |
| \( E_\Gamma=9 \) | \( E_\Gamma=9 \) |
| \( (0, 0, \frac{\pi}{4}) \) | \( (-1, -1, 0) \) |
| 3553 | 1033 |
| \( E_\Gamma=16 \) | \( E_\Gamma=16 \) |
| \( (0, 0, \frac{3\pi}{4}) \) | \( (-2, -2, 0) \) |
| 6073 | 3193 |
| \( E_\Gamma=25 \) | \( E_\Gamma=25 \) |
| \( (0, 0, \frac{5\pi}{4}) \) | \( (-3, -3, 0) \) |
| 9313 | 6793 |

IV Deducing the Lowest Energy Quarks of all Kinds with Phenomenological Formulae

A The Phenomenological Formulae for Intrinsic Quantum Numbers of Quarks

In order to deduce the short-lived and scarce quarks with phenomenological formulae we assume the following phenomenological formulae:

1). For a group of degenerate energy bands (number=\( \text{deg} \)) with the same energy and equivalent \( \vec{n} \) values (66) of \( [5] \), the isospin is

\[
I = \frac{\text{deg} - 1}{2}
\]  

(11)

2). The strange number \( S \) of an excited quark that lies on an axis with a rotary fold \( R \) of the regular rhombic dodecahedron is

\[
S = R - 4.
\]  

(12)
3). For the single energy bands on the Γ-H and the Γ-N axis, the strange number is

\[ S = S_{axis} + \Delta S, \quad \Delta S = \delta(\tilde{n}) + [1-2\delta(S_{axis})]\text{Sign}(\tilde{n}) \]  

(13)

where \( \delta(\tilde{n}) \) and \( \delta(S_{axis}) \) are Dirac functions, and \( S_{axis} \) is the strange number (12) of the axis (see Table A3 of [5]). For an energy band with \( \vec{n} = (n_1, n_2, n_3) \), \( \tilde{n} \) is defined as

\[ \tilde{n} \equiv \frac{n_1+n_2+n_3}{|n_1| + |n_2| + |n_3|}. \]

\[ \text{Sign}(\tilde{n}) = \begin{cases} 
+1 & \text{for } \tilde{n} > 0 \\
0 & \text{for } \tilde{n} = 0 \\
-1 & \text{for } \tilde{n} < 0 
\end{cases} \]  

(14)

If \( \tilde{n} = 0 \quad \Delta S = \delta(0) = +1 \) from (13) and (14).

(15)

If \( \tilde{n} = \frac{0}{0} \), \( \Delta S = -S_{Axis} \).

Thus, for \( \vec{n} = (0, 0, 0) \), from (16), we have

\[ S = S_{Axis} + \Delta S = S_{Axis} - S_{Axis} = 0. \]  

(17)

4). If \( S = +1 \), we call it the charmed number \( C (= 1) \):

if \( \Delta S = +1 \rightarrow S = S_{Ax} + \Delta S = +1, \quad C \equiv +1. \)  

(18)

If \( S = -1 \), which originates from \( \Delta S = +1 \) on a single energy band \( (S_{Ax} = -2) \), and there is an energy fluctuation, we call it the bottom number \( b \):

for single bands, if \( \Delta S = +1 \rightarrow S = -1 \) and \( \Delta E \neq 0, \quad b \equiv -1. \)  

(19)

5). The sixfold energy bands of the F-axis \( (R = 3 \) and \( S = -1) \) need two divisions. In the first division, the sixfold band divides into two \( (\frac{6}{R} = 2) \) threefold bands; the energy and the strange number do not change \( (S = -1 \text{ steel}) \). In the second division \( (K = 1), \)
the threefold bands with non-equivalent $n$ values divide into a twofold band and a single band. For the twofold energy bands with two equivalent $\pi$ values (see (66) of [5])

\[
[(\text{for sixfold bands}) \; \Delta S = +1 \text{ and } E > m_{uc}(1753 \text{ Mev})] \rightarrow q_{\Xi C},
\]

the twofold band represents a twofold family $q_{\Xi C}$-quark with $I = \frac{1}{2}$, $S = -1$ and $C = +1$.

The sixfold energy bands of the G-axis ($R = 2$ and $S = -2$) also need two divisions. In the first division, the sixfold band divides into three ($\frac{6}{R} = 3$) twofold bands. The energy and the strange number do not change ($S = -2$). In the second division ($K = 1$), the twofold band with non-equivalent $\pi$ values divides into two single bands. For a single energy band,

\[
[(\text{for sixfold bands}) \; \text{if } \Delta S = +1 \text{ and } E > m_{uc}(1753 \text{ Mev})] \rightarrow d_{\Omega C},
\]

the single energy band represents a $d_{\Omega C}$-quark with $I = 0$, $S = -2$ and $C = +1$.

6). The elementary quark $\epsilon_u$ (or $\epsilon_d$) determines the electric charge $Q$ of an excited quark. For an excited quark of $\epsilon_u$ (or $\epsilon_d$), $Q = +\frac{2}{3}$ (or $-\frac{1}{3}$). For an excited quark with isospin $I$, there are $2I + 1$ members. $I_z > 0$, $Q = +\frac{2}{3}$; $I_z < 0$, $Q = -\frac{2}{3}$;

\[
I_z = 0, \text{ if } S + C + b > 0, \quad Q = Q_{\epsilon_u(0)} = \frac{2}{3}; \quad (22)
\]

\[
I_z = 0, \text{ if } S + C + b < 0, \quad Q = Q_{\epsilon_d(0)} = -\frac{1}{3}. \quad (23)
\]

There is no quark with $I_z = 0$ and $S + C + b = 0$.

7). We assume a fluctuation $\Delta E$ of a quark energy is

\[
\Delta E = 100 \; S[(1+S_{Ax})(J_{Sz}+S_{Ax})] \Delta S \quad J_{S} = |S_{Ax}| + 1, 2, 3, \ldots \quad (24)
\]

Fitting experimental results, we can get

\[
\alpha = 360 \text{ Mev}. \quad (25)
\]
The rest mass \( m^* \) of a quark, from (8), (4), (25) and (24) is

\[
m^* = \{ 313 + 360 \text{ minimum}[(n_1-\xi)^2 + (n_2-\eta)^2 + (n_3-\zeta)^2] + \Delta E + \Delta \} \text{ (Mev)}
\]

\[
= m + \Delta \text{ (Mev)},
\]

This formula (26) is the united quark mass formula.

## B  Deducing Quarks with the Phenomenological Formulae from Energy Bands

Using the above formulae (11)-(26) and the deduced energy bands shown in Table B1-B3 and Table B5-B7 of [5], we can deduce all low energy quarks that are sufficient to cover all experimental data (see Table 11 of [5]). Since the five ground quarks are all born on the single energy bands of the \( \Delta \)-axis and the \( \Sigma \)-axis, we deduce the quarks of these single energy bands as examples using Table 2 and Table 3.

### Table 2. The \( u_C(m^*) \)-quarks and the \( d_S(m^*) \)-quarks on the \( \Delta \)-axis (\( S_\Delta = 0 \))

| \( E_{\text{Point}} \) | \( E_{(\vec{x}, \vec{n})} \) | \( n_1, n_2, n_3 \) | \( \Delta S \) | \( J \) | \( I \) | \( S \) | \( C \) | \( Q \) | \( \Delta E \) | \( q_{\text{Name}}(m^*) \) |
|-----------------|-----------------|-----------------|---------|------|------|------|------|------|--------|-----------------|
| \( E_T = 0 \)   | 313             | 0, 0, 0         | 0       | J = 0 | 0    | 0    | 0    | 0    | \( \frac{1}{2} \) | \( u_C(313+\Delta) \) |
| \( E_H = 1 \)   | 673             | 0, 0, 2         | -1      | \( J_{S,H} = 1 \) | 0    | -1   | 0    | -\( \frac{1}{3} \) | 100    | \( d_S(773+\Delta) \) |
| \( E_T = 4 \)   | 1753            | 0, 0, -2        | +1      | \( J_{C,\Gamma} = 1 \) | 0    | 0    | 1    | \( \frac{2}{3} \) | 0      | \( u_C(1753+\Delta) \) |
| \( E_H = 9 \)   | 3553            | 0, 0, 4         | -1      | \( J_{S,H} = 2 \) | 0    | -1   | 0    | -\( \frac{1}{3} \) | 200    | \( d_S(3753+\Delta) \) |
| \( E_T = 16 \)  | 6073            | 0, 0, -4        | +1      | \( J_{C,\Gamma} = 2 \) | 0    | 0    | 1    | \( \frac{2}{3} \) | 0      | \( u_C(6073+\Delta) \) |
| \( E_H = 25 \)  | 9313            | 0, 0, 6         | -1      | \( J_{S,H} = 3 \) | 0    | -1   | 0    | -\( \frac{1}{3} \) | 300    | \( d_S(9613+\Delta) \) |

### Table 3. The \( d_b \)-quarks, \( d_S \)-quarks and \( d_\Omega \)-quarks on the \( \Sigma \)-axis (\( S_\Sigma = -2 \))

| \( E_{\text{Point}} \) | \( E_{(\vec{x}, \vec{n})} \) | \( n_1, n_2, n_3 \) | \( \Delta S \) | \( S \) | \( b \) | \( Q \) | \( J \) | \( I \) | \( \Delta E \) | \( q_{\text{Name}}(m^*) \) |
|-----------------|-----------------|-----------------|---------|------|------|------|------|------|--------|-----------------|
| \( E_T = 0 \)   | 313             | (0, 0, 0)       | +2      | 0    | 0    | \( \frac{1}{3} \) | \( J_{S,\Gamma} = 0 \) | \( \frac{1}{2} \) | 0      | \( d(313+\Delta) \) |
| \( E_N = \frac{1}{2} \) | 493         | (1, 1, 0)       | +1      | -1   | 0    | \( \frac{1}{3} \) | \( J_{S,N} = 1 \) | 0      | 0      | \( d_S(493+\Delta) \) |
| \( E_T = 2 \)   | 1033            | (-1, -1, 0)     | -1      | -3   | 0    | \( \frac{1}{3} \) | \( J_{S,\Gamma} = 1 \) | 0      | 0      | \( d_\Omega(1033+\Delta) \) |
| \( E_N = \frac{9}{2} \) | 1933     | (2, 2, 0)       | +1      | -1   | 0    | \( \frac{1}{3} \) | \( J_{S,N} = 2 \) | 0      | 0      | \( d_S(1933+\Delta) \) |
| \( E_T = 8 \)   | 3193            | (-2, -2, 0)     | -1      | -3   | 0    | \( \frac{1}{3} \) | \( J_{S,\Gamma} = 2 \) | 0      | 0      | \( d_\Omega(3193+\Delta) \) |
| \( E_N = \frac{25}{2} \) | 4813   | (3, 3, 0)       | +1      | 0    | -1   | \( \frac{1}{3} \) | \( J_{S,N} = 3 \) | 0      | 100    | \( d_b(4913+\Delta) \) |
| \( E_T = 18 \)  | 6793            | (-3, -3, 0)     | -1      | -3   | 0    | \( \frac{1}{3} \) | \( J_{S,\Gamma} = 3 \) | 0      | -300   | \( d_\Omega(6493+\Delta) \) |
| \( E_N = \frac{49}{2} \) | 9133    | (4, 4, 0)       | +1      | 0    | -1   | \( \frac{1}{3} \) | \( J_{S,N} = 4 \) | 0      | 200    | \( d_b(9333+\Delta) \) |
As with Table 2 and Table 3, we can deduce the lowest energy quarks of all kinds shown in Table 4A and Table 4B.

Table 4A The Lowest Energy (Ground) Quarks of all Kinds

| $I^Z_q$ | $\frac{1}{2}^+_{N}$ | $\frac{1}{2}^+_{\Lambda}$ | $\frac{1}{2}^+_{\Sigma}$ | $\frac{1}{2}^+_{\Xi}$ | $\frac{1}{2}^+$ | $\frac{1}{2}^-$ | $\frac{1}{2}^-$ | $\frac{1}{2}^-$ | $\frac{1}{2}^-$ | $\frac{1}{2}^-$ | $\frac{1}{2}^-$ |
|--------|------------------|------------------|------------------|------------------|----------|----------|----------|----------|----------|----------|----------|
| S      | 0                | 0                | 0                | 0                | 0        | -1       | -1       | -1       | -1       | -1       | -1       |
| C      | 0                | 0                | 0                | 0                | 0        | 0        | 0        | 0        | 0        | 0        | 0        |
| b      | 0                | 0                | 0                | 0                | 0        | 0        | 0        | 0        | 0        | 0        | 0        |
| I      | $\frac{1}{2}$    | $\frac{1}{2}$    | $\frac{1}{2}$    | $\frac{1}{2}$    | $\frac{1}{2}$| $\frac{1}{2}$| $\frac{1}{2}$| $\frac{1}{2}$| $\frac{1}{2}$| $\frac{1}{2}$| $\frac{1}{2}$|
| $I^Z_q[\frac{1}{2}(B+S+C)]$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
| $Q_q$  | $\frac{2}{3}$    | $-\frac{1}{3}$   | $\frac{2}{3}$    | $-\frac{1}{3}$   | $\frac{2}{3}$ | $-\frac{1}{3}$| $\frac{2}{3}$ | $-\frac{1}{3}$| $\frac{2}{3}$ | $-\frac{1}{3}$| $\frac{2}{3}$|
| m (Mev)$^*$ | 313            | 313            | 673            | 673            | 673       | 583       | 583       | 583       | 583       | 583       | 583       |
|
| $\epsilon_{I^Z}$ | $u^+_{\Xi}$ | $d^+_{\Xi}$ | $u^+_{\Omega}$ | $u^+_{\Sigma}$ | $d^+_{\Sigma}$ | $u^+_{\Delta}$ | $u^+_{\Xi}$ | $d^+_{\Xi}$ | $u^+_{\Omega}$ | $u^+_{\Sigma}$ | $d^+_{\Sigma}$ |
|--------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|

$I^Z_q$ = I, I-1, I-2, ... -I. $#I^Z_q = Q_q\frac{1}{2}(B+S+C+b)$. $^*$m$^*$ = m + $\Delta$ → m = m$^*$ - $\Delta$.

Table 4B. The Lowest Energy (Ground) Quarks of all Kinds

| $I^Z_q$ | $\frac{1}{2}^+_{\Xi}$ | $\frac{1}{2}^-_{\Xi}$ | $\frac{1}{2}^-_{\Omega}$ | $\frac{1}{2}^-_{\Sigma}$ | $\frac{1}{2}^-_{\Xi}$ | $\frac{1}{2}^-_{\Xi}$ |
|--------|------------------|------------------|------------------|------------------|------------------|------------------|
| S      | -2               | -2               | -1               | -3               | 0                | 0                |
| C      | 0                | 0                | 0                | 0                | 1                | 1                |
| b      | 0                | 0                | 0                | 0                | -1               | 0                |
| I      | $\frac{1}{2}$   | $\frac{1}{2}$   | 0                | 0                | 0                | 0                |
| $I^Z_q[\frac{1}{2}(B+S+C)]$ | $\frac{1}{2}$ | $\frac{1}{2}$ | 0 | 1 | 0 | 0 | $\frac{1}{2}$ | $-\frac{1}{2}$ |
| $Q_q$  | $\frac{2}{3}$   | $-\frac{1}{3}$  | $\frac{2}{3}$   | $-\frac{1}{3}$  | $\frac{2}{3}$   | $-\frac{1}{3}$  |
| m (Mev)$^*$ | 673            | 673            | 493            | 1033            | 1753            | 4913 |
| $\epsilon_{I^Z}$ | $u^+_{\Xi}$ | $d^+_{\Xi}$ | $u^+_{\Omega}$ | $u^+_{\Sigma}$ | $d^+_{\Sigma}$ | $u^+_{\Delta}$ | $u^+_{\Xi}$ | $d^+_{\Xi}$ | $u^+_{\Omega}$ | $u^+_{\Sigma}$ | $d^+_{\Sigma}$ |
|--------|----------------|----------------|----------------|----------------|----------------|----------------|

$I^Z_q$ = I, I-1, I-2, ... -I. $#I^Z_q = Q_q\frac{1}{2}(B+S+C+b)$. $^*$m$^*$ = m + $\Delta$ → m = m$^*$ - $\Delta$.

V Deducing the Ground Baryons of all Kinds

According to the Quark Model [11], a baryon is composed of three quarks with different colors. For each flavor, the three different colored quarks have the same I, S, C, b, Q and m. Thus we can omit the color when we deduce the rest masses and intrinsic quantum numbers of the baryons. We must remember, however, that three colored
quarks compose a colorless baryon. For the lowest energy baryons, the sum laws are:

\[
\begin{align*}
S_B &= S_{q_1} + S_{q_N(313)} + S_{q_N(313)} = S_{q_1(m)}, \\
C_B &= C_{q_1} + C_{q_N(313)} + C_{q_N(313)} = C_{q_1(m)}, \\
b_B &= b_{q_1} + b_{q_N(313)} + b_{q_N(313)} = b_{q_1(m)}, \\
Q_B &= Q_{q_1} + Q_{q_N(313)} + Q_{q_N(313)}, \\
M_B &= m_{q_1} + m_{q_N(313)} + m_{q_N(313)}.
\end{align*}
\] (27)

where \(S_B\) is the baryon’s strange number, \(C_B\) is the baryon’s charmed number, \(b_B\) is the baryon’s bottom number, \(Q_B\) is the baryon’s electric charge and \(M_B\) is the baryon’s rest mass. There are strong interactions among the three quarks (colors), but we do not know how strong. Since the rest masses of the quarks in a baryon are large (from \(\Delta\)) and the rest mass of the baryon composed by three quarks is not, we infer that there will be a strong binding energy \(E_{\text{Bind}} = -3\Delta\) to cancel \(3\Delta\) from the three quarks:

\[
M_B = m_{q_1} + m_{q_N(313)} + m_{q_N(313)} - |E_{\text{Bind}}| = m_{q_1} + m_{q_N(313)} + m_{q_N(313)} + 3\Delta - 3\Delta = m_{q_1} + m_{q_N(313)} + m_{q_N(313)}.
\]

Using (27) and Table 4(A and B), we can find the rest masses and the intrinsic quantum numbers (I, S, C, b and Q) of the lowest energy (ground) baryons of all kinds shown in Table 5. The experimental results are from [6].
Table 5. The Ground Baryons (Lowest Energy) of all Kinds

| \( q_i \) | \( q_j \) | \( q_k \) | I | S | C | b | Q | M | Baryon                  | Exper.       | \( \Delta M \)/% |
|--------|--------|--------|----|----|----|----|----|----|-------------------------|--------------|------------------|
| \( u_1^2 \) (313) | u     | d     | \( 1/2 \) | 0 | 0 | 0 | 1  | 939 | p(939)                  | p(938)       | 0.11             |
| \( d_1^2 \) (313) | u     | d     | \( 1/2 \) | 0 | 0 | 0 | 0  | 939 | n(939)                  | n(940)       | 0.11             |
| \( d_0^2 \) (493) | u     | d     | 0    | -1| 0 | 0 | 0  | 1119| \( \Lambda(1119) \)    | \( \Lambda^0(1116) \) | 0.27             |
| \( u_0^2 \) (1753) | u     | d     | 0    | 1 | 0 | 1 | 0  | 2379| \( \Lambda_c(2379) \)  | \( \Lambda_c^+(2285) \) | 4.1              |
| \( u_0^2 \) (1753) | u     | u     | 1    | 0 | 1 | 0 | 2  | 2379| \( \Sigma_c^+(2379) \)  | \( \Sigma_c^+(2455) \) | 3.1              |
| \( u_0^2 \) (1753) | u     | d     | 1    | 0 | 1 | 0 | 1  | 2379| \( \Sigma_c^+(2379) \)  | \( \Sigma_c^+(2455) \) | 3.1              |
| \( u_0^2 \) (1753) | d     | d     | 1    | 0 | 1 | 0 | 0  | 2379| \( \Sigma_c^+(2379) \)  | \( \Sigma_c^+(2455) \) | 3.1              |
| \( d_0^2 \) (4913) | u     | d     | 0    | 0 | 0 | 1 | -1 | 5539| \( \Lambda_b(2379) \)  | \( \Lambda_b^0(5624) \) | 1.5              |
| \( u_2 \) (583)   | u     | d     | 1    | -1| 0 | 0 | 1  | 1209| \( \Sigma^+(1209) \)    | \( \Sigma^+(1189) \) | 1.7              |
| \( d_2 \) (583)   | u     | d     | 1    | -1| 0 | 0 | 0  | 1209| \( \Sigma^0(1209) \)    | \( \Sigma^0(1193) \) | 1.4              |
| \( d_2 \) (583)   | d     | d     | 1    | -1| 0 | 0 | -1 | 1209| \( \Sigma^{-1}(1209) \) | \( \Sigma^{-}(1197) \) | 1.0              |
| \( u_2 \) (673)   | d     | d     | \( \frac{1}{2} \) | -2| 0 | 0 | 0  | 1299| \( \Xi^0(1299) \)      | \( \Xi^0(1315) \) | 1.2              |
| \( d_2 \) (673)   | d     | d     | \( \frac{1}{2} \) | -2| 0 | 0 | -1 | 1299| \( \Xi^-(1299) \)      | \( \Xi^-(1321) \) | 1.7              |
| \( d_0 \) (1033)  | d     | d     | 0    | -3| 0 | 0 | -1 | 1659| \( \Omega^-(1659) \)   | \( \Omega^-(1672) \) | 0.78             |
| \( d_2 \) (1213)  | u     | d     | 0    | -2| 1 | 0 | 0  | 2759| \( \Omega_C(2759) \)   | \( \Omega_C(2698) \) | 2.0              |
| \( u_2 \) (1873)  | u     | d     | \( \frac{1}{2} \) | -1| 1 | 0 | 1  | 2499| \( \Xi_C(2499) \)     | \( \Xi_c^{-}(2466) \) | 1.4              |
| \( d_0 \) (1873)  | u     | d     | \( \frac{1}{2} \) | -1| 1 | 0 | 0  | 2499| \( \Xi_C(2499) \)     | \( \Xi^0(2472) \) | 1.2.             |
| \( u_2 \) (673)   | u     | u     | \( \frac{3}{2} \) | 0 | 0 | 0 | 2  | 1299| \( \Delta^{++}(1299) \)| \( \Delta^{++}(1232) \) | \( \Gamma=120 \) |
| \( u_2 \) (673)   | u     | d     | \( \frac{3}{2} \) | 0 | 0 | 0 | 1  | 1299| \( \Delta^{+}(1299) \) | \( \Delta^{+}(1232) \) | \( \Gamma=120 \) |
| \( d_0 \) (673)   | d     | d     | \( \frac{3}{2} \) | 0 | 0 | 0 | -1 | 1299| \( \Delta^0(1299) \)   | \( \Delta^0(1232) \) | \( \Gamma=120 \) |
| \( d_0 \) (673)   | d     | d     | \( \frac{3}{2} \) | 0 | 0 | 0 | 0  | 1299| \( \Delta^{-}(1299) \) | \( \Delta^{-}(1232) \) | \( \Gamma=120 \) |

In the Table, \( u \equiv u_1^2(313) \) and \( d \equiv d_1^2(313) \).

Table 5 shows that the intrinsic quantum numbers (I, S, C, b and Q) of the deduced baryons are the same as the experimental results and the deduced masses are consistent with the experimental results.
VI  Predictions

From Table 2, Table 3, Table 4A and Table 4B, we can predict some quarks and baryons as shown in the following list:

| Quark# | d_s(773) | d_s(3753) | d_s(9613) | u_c(6073) | d_s(9333) | d_Ω(3193) |
|-------|---------|---------|---------|--------|---------|--------|
| q_i   | u(313)  | u(313)  | u(313)  | u(313)  | d(313)  | d(313)  |
| q_j   | d(313)  | d(313)  | d(313)  | d(313)  | d(313)  | d(313)  |
| Prediction | Λ^0(1399) | Λ^0(4379) | Λ^0(10239) | Λ^+_c (6699) | Λ^0_b (9959) | Ω^-(3819) |
| S, C, b | Λ^0(1406)$^*$ | S = -1 | S = -1 | C = +1 | b = 1 | S = -3 |

* Predicted quarks  $^*$Λ^0(1399) has discovered Λ^0(1406) with ($\frac{\Delta M}{M}$%) = 0.5%.

VII  Discussion

1. The fact that physicists have not found any free quark shows that the binding energies are strong. The binding energy (-3Δ) is a phenomenological approximation of the color’s strong interaction energy. The binding energy (-3Δ) is always cancelled by the corresponding parts (3Δ) of the rest masses of the three quarks. Thus we can omit the binding energy and the corresponding rest mass parts of the three quarks. This effect makes it appear as if there is no binding energy in baryons.

2. The energy band excited quarks of the elementary quark $\epsilon_u$ (or $\epsilon_d$) are the short-lived and scarce quarks, such as $d_s(493)$, $u_c(1753)$, $d_b(4913)$, $q_\Xi(673)$, $q_\Sigma(583)$ and $q_\Delta(673)$. The $q_N(313)$ corresponding to the energy band with $\vec{n} = (0, 0, 0)$ will be a short-lived and scarce quark. It is, however, a lowest energy quark with strong binding energy. Since there is no lower energy position that they can decay to, they are not short-lived quarks. Because they have the same rest mass and intrinsic quantum numbers as the free excited quarks $u(313)$ and $d(313)$, they cannot be distinguished from $u(313)$ and $d(313)$ by experiments; therefore, they are not scarce quarks. The $u(313)$ and $d(313)$ with $\vec{n} = (0, 0, 0)$ will be covered up by free excited $u(313)$ and $d(313)$ in experiments. Thus, we can omit $u(313)$ and $d(313)$ with $\vec{n} = (0, 0, 0)$. There are only long-lived
free excited the u(313)-quark and the d(313)-quark in both theory and experiments.

3. The five quarks of the current Quark Model correspond to the five deduced ground quarks [u↔u(313), d↔d(313), s↔d_s(493), c↔u_c(1753) and b↔d_b(4913)] (see Table 4B and 4B as well as Table 11 of [5]). The current Quark Model uses only these five quarks to explain baryons and mesons. Using quantized energy bands and the phenomenological formulae, the new quark model can deduce the rest masses and the intrinsic quantum numbers of excited quarks, baryons and mesons [5] from one elementary quark family. Thus, the current Quark Model is the five ground quark approximation of the new quark model.

VIII Conclusions

1. There is only one elementary quark family \( \epsilon \) with three colors and two isospin states (\( \epsilon_u \) with \( I_Z = \frac{1}{2} \) and \( Q = +\frac{2}{3} \), \( \epsilon_d \) with \( I_Z = \frac{1}{2} \) and \( Q = -\frac{1}{3} \)) for each color. Thus there are six Fermi (\( s = \frac{1}{2} \)) elementary quarks with \( S = C = b = 0 \) in the vacuum.

2. All quarks in hadrons are the excited state of the elementary quark \( \epsilon \). There are two types of excited states: free excited states and energy band excited states. The free excited states are only the u(313)-quark and the d(313)-quark; the energy band excited states are the short-lived and scarce quarks, such as \( d_s(493) \), \( u_c(1753) \), \( d_b(4913) \), \( q_N(583) \), \( q_{\Sigma}(583) \) and \( q_{\Delta}(673) \).

3. The short-lived and scarce quarks mainly originate from the expanded quantization. They are the energy band excited states of the elementary quarks.

4. There is a strong binding energy (-3\( \Delta \)) among the three quarks (colors) inside a baryon. It may be a possible foundation of the quark confinement.

5. This paper predicts new baryons \( \Lambda_c^+(6699) \), \( \Lambda_b^0(9959) \), \( \Lambda^0(4379) \) and \( \Lambda^0(10239) \).
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The axis $\Delta$ (the axis $\Gamma$-$H$) is a four fold rotation axis.

The axis $\Lambda$ (the axis $\Gamma$-$P$) is a three fold rotation axis.

The axis $\Sigma$ (the axis $\Gamma$-$N$) is a two fold rotation axis.

Figure 1: The regular rhombic dodecahedron. The symmetry points and axes are indicated. The $\Delta$-axis is a fourfold rotation axis, the strange number $S = 0$ and the fourfold baryon family $\Delta$ ($\Delta^{++}$, $\Delta^{+}$, $\Delta^{0}$, $\Delta^{-}$) will appear on the axis. The axes $\Lambda$ and $F$ are threefold rotation axes, the strange number $S = -1$ and the threefold baryon family $\Sigma$ ($\Sigma^{+}$, $\Sigma^{0}$, $\Sigma^{-}$) will appear on the axes. The axes $\Sigma$ and $G$ are twofold rotation axes, the strange number $S = -2$ and the twofold baryon family $\Xi$ ($\Xi^{0}$, $\Xi^{-}$) will appear on the axes. The axis $D$ is parallel to the axis $\Delta$, $S = 0$. And it is a twofold rotation axis, the twofold baryon family $N$ ($N^{+}$, $N^{0}$) will be on the axis.