Polarized Hyperons from $pA$ Scattering in the Gluon Saturation Regime

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We study the production of transversely polarized \(\Lambda\) hyperons in high-energy collisions of protons with large nuclei. The large gluon density of the target at saturation provides an intrinsic semi-hard scale which should naturally allow for a weak-coupling QCD description of the process in terms of a convolution of the quark distribution of the proton with the elementary quark-nucleus scattering cross section (resummed to all twists) and a fragmentation function. In this case of transversely polarized \(\Lambda\) production we employ a so-called polarizing fragmentation function, which is an odd function of the transverse momentum of the \(\Lambda\) relative to the fragmenting quark. Due to this \(k_t\)-odd nature, the resulting \(\Lambda\) polarization is essentially proportional to the derivative of the quark-nucleus cross section with respect to transverse momentum, which peaks near the saturation momentum scale. Such processes might therefore provide generic signatures for high parton density effects and for the approach to the "black-body" (unitarity) limit of hadronic scattering.

It has been known for over 25 years that \(\Lambda\)'s produced in collisions of unpolarized hadrons exhibit polarization perpendicular to the production plane. As of yet, such data are not available for very high energies where one expects that hadronic cross sections are close to their geometrical values (the "black body limit"). However, the BNL-RHIC collider will soon collide protons and deuterons on gold nuclei at energies of \(\sim 200\) GeV in the nucleon-nucleon center of mass frame; later on, much higher energies will be accessible at the CERN-LHC. In this letter, we demonstrate that the polarization of \(\Lambda\) hyperons produced in the forward region in high-energy collisions of protons and heavy nuclei may generically be a sensitive probe of high-density effects and gluon saturation in the target.

The wave function of a hadron (or nucleus) boosted to large rapidity exhibits a large number of gluons at small \(x\), which is the fraction of the light-cone momentum carried by the gluon. The density of gluons is expected to saturate when it becomes, parametrically, of the order of the inverse QCD coupling constant \(\alpha_s\) [1]. The parton density at saturation is denoted by \(Q_s^2\), the so-called saturation momentum. This provides an intrinsic momentum scale [2] which grows with atomic number and with rapidity because more gluons can be radiated in the initial state when phase space is big. For sufficiently high energies and/or large nuclei, the saturation momentum \(Q_s\) can become much larger than \(\Lambda_{QCD}\), such that weak coupling methods are applicable.

Forward \(\Lambda\) production in \(pA\) collisions is dominated by high-\(x\) quarks from the proton traversing the high gluon density region of the heavy nucleus. The quarks typically experience interactions with momentum transfers of the order of the saturation momentum. Thus, for large gluon densities in the target, such that the saturation momentum is in the perturbative regime, \(Q_s \gtrsim 1\) GeV, the coherence of the projectile is lost, and the scattered quarks (having an average transverse momentum proportional to \(Q_s\)) fragment independently [3]. While nonperturbative constituent-quark and diquark scattering and hadronization models [4] have been employed to understand hyperon polarization in collisions of protons with dilute targets, we expect that in the high-energy limit the presence of the intrinsic semi-hard scale \(Q_s\) should naturally allow for a weak-coupling QCD description of the process. One can thus calculate the cross section for \(qA\) scattering in this kinematical domain within pQCD [5], and the deflected, outgoing quark will subsequently fragment into hadrons, which is described by a fragmentation function.

In order to explain the transverse \(\Lambda\) polarization in unpolarized hadron collisions within such a factorized pQCD description, it has been suggested that unpolarized quarks can fragment into transversely polarized hadrons, for instance \(\Lambda\) hyperons. The associated probability [6,7] is described by a so-called polarizing fragmentation function, sometimes also called Sivers (effect) fragmentation function. Its main properties are that it is an odd function of the transverse momentum relative to the quark, \(\vec{k}_t\), and that the \(\Lambda\) polarization is orthogonal to \(\vec{k}_t\), because of parity invariance. The polarizing fragmentation function is defined as [7]:

\[
\Delta^N D_{h+/q}(z,\vec{k}_t) \equiv \hat{D}_{h+/q}(z,\vec{k}_t) - \hat{D}_{h+/q}(z,\vec{k}_t) - \hat{D}_{h+/q}(z,-\vec{k}_t),
\]

and denotes the difference between the densities \(\hat{D}_{h+/q}(z,\vec{k}_t)\) and \(\hat{D}_{h+/q}(z,\vec{k}_t)\) of spin-1/2 hadrons \(h\), with longitudinal

\[1\] Another commonly used notation for the polarizing fragmentation function is \(D_{h+/q}^{+1}\), but with a slightly different definition [6].
momentum fraction $z$, transverse momentum $\vec{k}_t$ and transverse polarization $\uparrow$ or $\downarrow$, in a jet originating from the fragmentation of an unpolarized parton $q$. Clearly, this $k_t$-odd function vanishes when integrated over transverse momentum and also when the transverse momentum and the transverse spin are parallel. In order to set the sign convention for the $\Lambda$ polarization we define

$$
\Delta^N D_{h\uparrow/q}(z, \vec{k}_t) \equiv \Delta^N D_{h\uparrow/q}(z, |\vec{k}_t|) \frac{\vec{P}_h \cdot (\vec{q} \times \vec{k}_t)}{|\vec{q} \times \vec{k}_t|},
$$

where $\vec{q}$ is the momentum of the unpolarized quark that fragments and $\vec{P}_h$ is the direction of the polarization vector of the hadron $h$ (the $\uparrow$ direction). Fig. 1 shows the kinematics of the process under consideration and indicates the direction of positive $\Lambda$ polarization for each quadrant in the $\Lambda$ production plane.

It should be emphasized that such a nonzero probability difference $\Delta^N D_{h\uparrow/q}(z, \vec{k}_t)$ is allowed by both parity and time reversal invariance. Generally it is expected to occur due to final state interactions in the fragmentation process, where the direction of the transverse momentum yields an oriented orbital angular momentum compensated by the transverse spin of the final observed hadron. This polarizing fragmentation function is the analogue of the so-called Sivers effect for parton distribution functions [8], which yields different probabilities of finding an unpolarized quark in a transversely polarized hadron, depending on the directions of the transverse spin of the hadron and the transverse momentum of the quark. The Sivers effect can lead to single spin asymmetries, for instance in $p^\uparrow p \rightarrow \pi X$, a process for which such (large) asymmetries have been observed in several experiments.

Recently, such a single spin asymmetry in $e^+ p^\uparrow \rightarrow e^' \pi X$ has been calculated in a one-gluon exchange model [9]. Shortly afterwards it was understood [10] as providing a model for the Sivers effect distribution function. A similar calculation has recently been performed by Metz [11] for the production of polarized spin-1/2 hadrons in unpolarized scattering, which can be viewed as providing a model for the polarizing fragmentation function. Here we will not employ such a model calculation, but rather use a parametrization for the polarizing fragmentation functions obtained from a fit to data [7]. However, these model calculations do demonstrate that nonzero Sivers effect functions can arise in principle.

Due to the $k_t$-odd nature of the polarizing fragmentation function it is accompanied by a different part of the partonic cross section (essentially the first derivative w.r.t. $k_t$) compared to the ordinary, unpolarized $\Lambda$ fragmentation function, which is $k_t$-even. The characteristics of the resulting $\Lambda$ polarization will turn out to be rather different from presently available data for hadronic collisions at moderately high energies and with “dilute” targets. These data show a $\Lambda$ polarization that increases approximately linearly as a function of the transverse momentum $l_t$ of the $\Lambda$, up to $l_t \sim 1$ GeV/c, after which it becomes flat, up to the highest measured $l_t$ values: $l_t \sim 4$ GeV/c. No indication of a decrease at these high $l_t$ values has been observed. Furthermore, the polarization increases with the longitudinal momentum fraction $\xi$ and is to a large extent $\sqrt{s}$ independent. These features do not change with increasing $A$ [12–14]. The only $A$ dependence observed is a slight overall suppression of the $\Lambda$ polarization for large $A$ and higher energies. For Cu and Pb fixed targets, probed with a 400 GeV/c proton beam [13,14], the magnitude of the polarization is about 30% lower than for light nuclei. This effect is usually attributed to secondary $\Lambda$ production through $\pi^- N$ interactions [14]. The slight suppression shows no evidence for a dependence on $l_t$ in the investigated range $0.9 < l_t < 2.6$ GeV/c, albeit with rather low statistical accuracy. It is clear that this data on heavy nuclei is not in the kinematic region where
saturation is expected to play a dominant role and the main differences to the results presented below are that in the saturation regime the transverse $\Lambda$ polarization will depend on the collision energy and no plateau region is expected.

We shall now present our calculation of $\Lambda$ polarization in the gluon saturation regime, following ref. [7] regarding the treatment of the polarizing fragmentation functions.

As mentioned above, in the calculation of the $qA$ cross section one is dealing with small coupling if the target nucleus is very dense; however, the well known leading-twist pQCD can not be used when the density of gluons is large. Rather, scattering amplitudes have to be resummed to all orders in $\alpha_s^2$ times the density. When the target is probed at a scale $\sim Q_s$, scattering cross sections approach the geometrical “black body” limit, while for momentum transfer far above $Q_s$ the target appears dilute and cross sections are approximately determined by the known leading-twist pQCD expressions.

At high energies, and in the eikonal approximation, the transverse momentum distribution of quarks is essentially given by the correlation function of two Wilson lines $V$ running along the light-cone at transverse separation $r_\perp$ (in the amplitude and its complex conjugate),

$$
\sigma^{qA} = \int \frac{d^2q_1 dq_1^+}{(2\pi)^2} \delta(q_1^+ - p^+) \left( \frac{1}{N_c} \text{tr} \left( \int d^2z_1 e^{i\vec{q}_1 \cdot \vec{r}_1} [V(z_1) - 1] \right)^2 \right).$$

Here, $p^+$ is the large light-cone component of the momentum of the incident proton, and that of the incoming quark is $p_1^+ = xP^+ + q_1^+$ (for the outgoing quark). The correlator of Wilson lines has to be evaluated in the background field of the target nucleus. A relatively simple closed expression can be obtained [5] in the “Color Glass Condensate” model of the small-$x$ gluon distribution of the dense target [2]. In that model, the small-$x$ gluons are described as a classical non-abelian Yang-Mills field arising from a stochastic source of color charge on the light-cone which is averaged over with a Gaussian distribution. The quark $q_1$ distribution is then given by [5]

$$
\frac{d\sigma^{qA}}{dq_1^+ dq_1^+ d^2b} = \frac{q_1^+}{P^+} \delta\left(\frac{p^+ - q_1^+}{P^+}\right) \frac{1}{(2\pi)^2} C(q_1),
$$

$$
C(q_1) = \int d^2r_\perp e^{i\vec{q}_1 \cdot \vec{r}_1} \left\{ \exp\left[ -2Q_s^2 \int \frac{dp_{t}}{(2\pi)^2} \frac{1}{p_{t}^2} (1 - \exp(i\vec{p}_t \cdot \vec{r}_1)) \right] - 2 \exp\left[ -Q_s^2 \int \frac{dp_{t}}{(2\pi)^2} \frac{1}{p_{t}^2} + 1 \right] \right\}.
$$

This expression is valid to leading order in $\alpha_s$ (tree level), but to all orders in $Q_s$ since it resums any number of scatterings of the impinging quark in the strong field of the nucleus. The saturation momentum $Q_s$, as introduced in eq. (4), is related to $\chi$, the total color charge density squared (per unit area) from the nucleus integrated up to the rapidity $y$ of the probe (i.e. the projectile quark), by

$$
Q_s^2 = 4\pi^2 \alpha_s^2 N_c^2 - 1 \chi.
$$

In the low-density limit, $\chi$ is related to the ordinary leading-twist gluon distribution function of the nucleus, see for example [15]. From BFKL evolution, $Q_s^2$ evolves as $\sim \exp(\lambda y) \sim x^\lambda$, with the intercept $\lambda \simeq 0.3$ [16]. Thus, if $Q_s^2 \simeq 10$ GeV$^2$ at the proton beam rapidity (i.e. $x = 1$) and for $A \simeq 200$ targets [5], then $Q_s \simeq 3$ GeV at $x = 0.6$, decreasing to $Q_s \simeq 2$ GeV at $x = 0.05$; furthermore, assuming $Q_s^2 \sim A^{1/3}$ scaling, then at $x = 0.6$, $Q_s$ drops from 3 GeV to 2 GeV when the atomic number $A$ of the target decreases from 200 to 20. It is clear therefore that in order to be sensitive to high-density effects, experimentally one should study high-energy $pA$ collisions in the forward region (where the polarization is largest anyway, see below) and with large target nuclei, and then compare to $pp$ collisions. Below, we shall focus on polarized $\Lambda$ production in a relatively small rapidity interval in the forward region, and so take $Q_s$ as a constant of order $2 - 3$ GeV.

The integrals over $p_t$ in eq. (4) are cut off in the infrared by some cutoff $\Lambda$, which we assume is of order $\Lambda_{QCD}$. We denote the momentum of the produced $\Lambda$ by $\vec{t} = z\vec{q} + \vec{k}$, with $\vec{k}$ the transverse momentum relative to the fragmenting quark. Assuming parity conservation in the hadronization process, only the component of $\vec{k}$ in the production plane contributes to the polarization $P_\Lambda$, therefore in order to simplify the kinematics we choose $k_y = 0$ as was done in Ref. [7]. For forward kinematics, $q_1^+ \gg q_t$, one then finds $zq_t \simeq l_t - k_t$. The polarized cross section is given by

$$
P_\Lambda(l_t, \xi) \frac{d\sigma}{d\xi d^2l_t d^2b} = \int d\left(\frac{q^+}{P^+}\right) \int \frac{dz}{z^2} f_{q/p}(x, Q^2) \int \frac{d^2k_t}{(2\pi)^2} \Delta^N D_{\Lambda/q}(z, Q^2, \vec{k}_t) q^+ \frac{\delta\left(\frac{q^+}{P^+} - x\right)}{P^+} C(q_t)
$$

$$
= \int \frac{dz}{z^2} f_{q/p}(x, Q^2) \int \frac{d^2k_t}{(2\pi)^2} \Delta^N D_{\Lambda/q}(z, Q^2, \vec{k}_t) \times C(q_t)
$$

$$
= \int \frac{dx}{\xi} f_{q/p}(x, Q^2) \int \frac{d^2k_t}{(2\pi)^2} \Delta^N D_{\Lambda/q}(x, Q^2, \vec{k}_t) C(q_t),
$$

(6)
where $\xi = l_z/P_z \simeq x z$ is the longitudinal momentum fraction carried by the $\Lambda$. We assume that $\Delta^N D_{\Lambda^+/q}(z, \vec{k}_t)$ is strongly peaked around an average $\vec{k}_t^0$ lying in the production plane, such that [7]

$$
\int d^2k_t \Delta^N D_{\Lambda^+/q}(z, Q^2) F(\vec{k}_t) \simeq \Delta^N D_{\Lambda^+/q}(z, Q^2) \left[ F(k_t^0) - F(-k_t^0) \right].
$$

Note that $k_t^0$ is a function of $z$, see below. Alternatively, one could consider Gaussian distributions over $k_t$ [17], though the above simplified treatment is sufficient for our purposes.

Considering the unpolarized cross section, we can safely neglect $k_t$-smearing from hadronization, which is of order $\Lambda_{QCD}$, while the quarks are typically scattered to much larger transverse momenta, namely of order $Q_s$. In our numerical results shown below we also include the contribution from anti-quarks and gluons to the unpolarized cross section, although this is a small correction for $\xi \gtrsim 0.1$. For the polarizing fragmentation functions, only contributions from $u, d$, and $s$ quarks (the valence quarks of the $\Lambda$) are considered [7]. Thus, we obtain

$$
P_{\Lambda}(u, t, \xi) = \int_{\xi}^{1} dx x f_{q/p}(x, Q^2) \Delta^N D_{\Lambda^+/q}(z, Q^2) \left[ C \left( \frac{\xi (l_t - k_t^0)}{z} \right) - C \left( \frac{\xi (l_t + k_t^0)}{z} \right) \right].
$$

The factorization scale is chosen to be the saturation momentum of the dense nucleus, $Q^2 = Q_s^2$. A parametrization for $\Delta^N D_{\Lambda^+/q}(z)$ in terms of the unpolarized fragmentation function $D_{\Lambda/q}(z)$ was given in ref. [7]. It was obtained by performing a fit to available $pA \rightarrow \Lambda^+X$ data (for light nuclei only), where the transverse momentum $l_t$ was required to be larger than 1 GeV/c, in order to justify the application of a factorized expression and of pQCD for the partonic cross section. Although doubts have arisen about the applicability of pQCD in the kinematic region covered by the available data [17], the resulting functions do exhibit reasonable features. Here, we shall employ those functions as an Ansatz to investigate the dependence of the $\Lambda$ polarization on the saturation momentum $Q_s$, which turns out not to depend on the detailed parameterization of the polarizing fragmentation functions. Rather, it is the $k_t$-odd structure (and the fact that it is peaked around an average nonzero transverse momentum) that is responsible for the dependence. Of course, future parameterizations can be easily implemented.

To be explicit, we use

$$
\Delta^N D_{\Lambda^+/q}(z, Q^2) \equiv N_q \left( 1 - z \right) d_q \frac{D_{\Lambda/q}(z, Q^2)}{2},
$$

where

$$
N_u = N_d = -28.13, \quad N_s = 57.53, \quad c_q = 11.64, \quad d_q = 1.23.
$$

The average transverse momentum $k_t^0$ acquired in the fragmentation is parameterized as

$$
k_t^0 = 0.66 z^{0.37} (1 - z)^{0.50} \text{ GeV/c}.
$$

For the unpolarized fragmentation function $D_{\Lambda/q}$ in eq. (11) the parameterization of ref. [18] is to be used; strictly speaking, that parameterization holds for the fragmentation into $\Lambda + \Lambda$. However, in the forward region ($\xi \gtrsim 0.1$), one expects $P_{\Lambda+\Lambda} \approx P_{\Lambda}$. Furthermore, the parameterization of [18] assumes $SU(3)$ symmetry: $D_{\Lambda/u} = D_{\Lambda/d} = D_{\Lambda/s}$. However, the polarizing fragmentation functions $\Delta^N D_{\Lambda^+/q}$ reduce the flavor symmetry to $SU(2)$, since $N_{u,d} \neq N_{s}$ ($N_u = N_d$ was imposed in [7] to reduce the number of fit parameters). But even though $\Delta^N D_{\Lambda^+/s} > |\Delta^N D_{\Lambda^+/u,d}|$, the overall $\Lambda$ polarization in the process under consideration is in fact dominated by the valence-like quarks of the proton, not by the strange quark.

The polarizing fragmentation function describes the probability of an unpolarized quark to fragment into a transversely polarized $\Lambda$. Here, no difference is made as to whether the $\Lambda$ is produced directly or as a secondary particle, for instance as a decay product of heavier hyperons like the $\Sigma^0$ or $\Sigma^{++}$. This second type is usually expected to have a depolarizing effect, which means that the degree of polarization is higher for the directly produced $\Lambda$’s. The number of directly produced $\Lambda$’s is estimated to be roughly 75% of the total, such that the depolarizing effect could be on the order of 30%. The polarizing fragmentation functions of ref. [7] thus effectively account for the depolarizing effect from decays, since they were obtained by a fit to data that does not discriminate between direct and decay contributions either.
A numerical evaluation of eq. (10) is shown in Fig. 2, using the CTEQ5L LO parton distribution functions for the proton [19]. Generically, one observes that $P_\Lambda$ is negative (due to the fact that $u$ and $d$ quarks dominate); it first increases with transverse momentum, then peaks at $l_t \sim Q_s$, and asymptotically approaches zero again. The fact that $P_\Lambda$ peaks at $l_t \sim Q_s$ has its origin in the $k_t$-odd nature of the polarizing fragmentation function: from eq. (10), $P_\Lambda$ corresponds to the difference of the cross sections taken with “intrinsic” transverse momentum $k^0_t$ parallel and antiparallel to the quark transverse momentum $q_t$. Since $k^0_t$ is small, $P_\Lambda$ is essentially proportional to the derivative of $d\sigma^{qA}/d^2q_t$, the differential quark-nucleus cross section, which varies most rapidly at $q_t \sim Q_s$ (see also eq. (17) below). Consequently, $|P_\Lambda|$ exhibits a maximum at such transverse momentum. This conclusion is independent of the details of the polarizing fragmentation functions; only the $k_t$-odd nature and the fact that they are strongly peaked about an average $k^0_t$ matters. In contrast, $k_t$-even distribution and fragmentation functions only probe the $qA$ cross section itself but not its derivative with respect to $q_t$.

The behavior of $P_\Lambda(l_t)$ is qualitatively rather different when the quark cross section is taken at leading twist. In that case, not only is the magnitude of the polarization larger, but moreover $P_\Lambda$ in the forward region peaks about small transverse momentum $\lesssim 1$ GeV. This can be understood by noting that the derivative of the $qA$ cross section at leading twist peaks in the infrared, contrary to eqs. (4, 17). For a more quantitative evaluation of polarized $\Lambda$ production in the “dilute regime” (hadronic collisions far below the unitarity limit, e.g. $pp$ collisions at RHIC) we refer to Ref. [7].

To understand the behavior of eq. (10) in more detail, consider first large transverse momentum, $q_t \gg Q_s$. Here, the last two terms of eq. (4) can be dropped, since they contribute only via a $\delta(q_t)$ term. At large transverse momentum the phase factor $\exp(i\vec{q}_t \cdot \vec{r}_l)$ in eq. (4) effectively restricts the integral over $d^2r_l$ to the region $r_l \lesssim 1/q_t \ll 1/Q_s$; the first exponential can then be expanded order by order to generate the usual power series in $1/q^2_t$. The leading and subleading twists are (see also [20])

$$C(q_t) = \frac{2Q^2_t}{q_t^2} \left[ 1 + \frac{4Q^2_t}{\pi q_t^2} \log \frac{q_t}{\Lambda} + O\left(\frac{Q^2_t}{q_t^2}\right) \right].$$

This expression is valid to leading logarithmic accuracy. The first term corresponds to the perturbative one-gluon $t$-channel exchange contribution to $qg \to qg$ scattering [20]. To leading order in $k^0_t/l_t$, the polarization given in eq. (10) thus becomes

$$P_\Lambda(l_t, \xi) = \frac{\int_x dxf_{q/p}(x, Q^2)\Delta N D_{\Lambda/q}(\xi, Q^2) x^{-4} \left[ 1 + \frac{4Q^2_t}{\pi x q_t^2} \log \frac{l_t}{\Lambda} \right] k^0_t/l_t}{\int_x dxf_{q/p}(x, Q^2)D_{\Lambda/q}(\xi, Q^2) x^{-4} \left[ 1 + \frac{4Q^2_t}{\pi x^2 q_t^2} \log \frac{l_t}{\Lambda} \right]}.$$

It is known [7] that the polarization (for large $l_t$) is a higher-twist effect, i.e. it is suppressed by powers of the “intrinsic” transverse momentum at hadronization, $k^0_t$, over the external momentum scale $l_t$. Eq. (15) shows that despite a partial cancellation the first power-suppressed correction to the quark-nucleus cross section (the subleading terms in the square brackets) enhance $P_\Lambda$ at large $l_t$, in agreement with the behavior at $l_t \gtrsim 5$ GeV in Fig. 2.

![FIG. 2. Transverse momentum distribution of the transverse $\Lambda$ polarization. Left: at fixed longitudinal momentum fraction $\xi = 0.5$ and varying target saturation scale, $Q_s = 2$, $3$ GeV, respectively. Right: For $Q_s = 2$ GeV and various $\xi$.](attachment:image.png)
Regarding the scaling of \( \mathcal{P}_\lambda \) at the peak, consider the quark-nucleus cross section for \( q_t \sim Q_s \gg \Lambda \). Again, the last two terms of eq. (4) can be dropped, while in the leading logarithmic approximation the argument of the first exponential reads

\[
\frac{-Q_s^2 r^2}{4\pi} \log \frac{1}{r_\Lambda} + O(Q_s^2 r_\Lambda^2) .
\]

The phase factor effectively cuts off the integral at \( r_t \sim 1/q_t \sim 1/Q_s \), and so \( 1/r_t \Lambda \) is large. We therefore replace \( 1/r_t \rightarrow Q_s \) in the argument of the above logarithm, since it is slowly varying and formally makes the expression well-behaved at large \( r_t \). The remaining integral leads to

\[
C(q_t) \simeq \frac{4\pi^2}{Q_s^2 \log Q_s/\Lambda} \exp \left( -\frac{\pi q_t^2}{Q_s^2 \log Q_s/\Lambda} \right) .
\]

This approximation reproduces the behavior of the full expression (4) about \( q_t \sim Q_s \) reasonably well. Expressions (16,17) are useful only when the cutoff \( \Lambda \ll Q_s \), that is, when color neutrality is enforced on distance scales of order \( 1/\Lambda \gg 1/Q_s \). If, however, color neutrality in the target nucleus were to occur on distances of order \( 1/Q_s \) [21] then \( \Lambda \sim Q_s \) and one would have to go beyond the leading-logarithmic approximation.

From eq. (17), \( \mathcal{P}_\lambda \) is given by (to leading order in \( k_0^2/L_t \))

\[
\mathcal{P}_\lambda(l_t, \xi) = \frac{4\pi}{Q_s^2 \log Q_s/\Lambda} \frac{l_t^2}{L_t^2} \int_{l_t}^{1} dx x f_{q/p}(x, Q^2) \Delta N_{q/p} (x, Q^2, \xi_0) \exp \left( -\frac{-\pi l_t^2}{Q_s^2 \log Q_s/\Lambda} \right) \frac{\xi^2}{\xi_t^2} k_0^0 / L_t .
\]

Thus, at the peak \( \mathcal{P}_\lambda \) scales approximately with \( 1/(Q_s \sqrt{\log Q_s/\Lambda}) \), as indeed seen in Fig. 2. The strong dependence on the target gluon density, as parametrized by \( Q_s \), is rather different from leading-twist perturbation theory.

As mentioned above, there is a related \( k_t \)-odd effect in processes with one transversely polarized hadron in the initial state (the Sivers effect), which can lead to asymmetries in \( p^\uparrow p \rightarrow \pi X \) [22], for example. Since at RHIC polarized proton beams are also available (for recent preliminary \( p^\uparrow p \rightarrow \pi X \) data from STAR, see Ref. [23]), one could investigate the process \( p^\uparrow A \rightarrow \pi X \) in the saturation regime. Similar signatures should arise in that process as for \( p A \rightarrow \Lambda^\uparrow X \) pointed out here.

In summary, we have studied transverse \( \Lambda \) polarization in \( p A \) collisions at high energies and with dense targets. The resulting \( \Lambda \) polarization is quite different from that observed in \( p A \) and \( pp \) collisions to date, which presumably did not probe the saturation regime yet. To study the high-density limit, we have performed a weak coupling analysis of the hard \( q \Lambda \) scattering, determined by the saturation momentum \( Q_s \), and described the unpolarized quark fragmentation into a transversely polarized \( \Lambda \) hyperon by the so-called polarizing fragmentation functions. We observe that the \( \Lambda \) polarization peaks at transverse momentum \( \sim Q_s \), where it also scales approximately as \( 1/(Q_s \sqrt{\log Q_s/\Lambda}) \) and hence is collision energy dependent. Moreover, no plateau region for larger transverse momenta is present. These features are independent of the details of the polarizing fragmentation functions, but rather occur due to their \( k_t \)-odd nature. Similar effects are expected in the process of \( p^\uparrow A \rightarrow \pi X \) in the saturation region. Both processes can be studied, in principle, at the BNL-RHIC collider, and perhaps in the future at the CERN-LHC.

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