Analysis of Assembly Error Effect on Stability Accuracy of Unmanned Aerial Vehicle Photoelectric Detection System

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Received: 4 March 2020; Accepted: 20 March 2020; Published: 28 March 2020

Featured Application: For photoelectric detection devices based on unmanned aerial vehicles, the analysis results are used as standards of manufacturing process improvements.

Abstract: A photoelectric detection system is a typical type of device widely used for detecting purposes based on unmanned aerial vehicles (UAV). Stability accuracy is the key performance index. Compared to traditional analysis methods aimed at unpredictable error-causing sources, assembly errors can be easily controlled during the manufacturing processes. In this research, an analysis method of assembly error effect on stability accuracy is proposed. First, by using kinematics analysis of homogeneous coordinate transformation, stability accuracy is comprehensively modeled and simulated. Then, by analyzing the manufacturing process, assembly errors of axis perpendicularities, run-outs and gyroscopes are defined and modeled. By simulating different carrier movements, the effects caused by assembly errors under various environments are studied. Finally, error sensitivity is proposed by using standard deviation analysis. Results show that the most sensitive assembly errors are identified, and ranked in order of sensitivity as follows: x-component of pitch axis perpendicularity, y-component of the azimuth gyroscope assembly, and z-component of the pitch gyroscope assembly. In conclusion, the results can be used as standards of manufacturing process improvements, and the proposed methods can be used to provide valuable references for real application scenarios.

Keywords: photoelectric detection system; unmanned aerial vehicle; stability accuracy; assembly errors; error sensitivity analysis

1. Introduction

The unmanned aerial vehicle (UAV) photoelectric detection system (hereinafter, the system) is a typical type of device widely used in both military and civilian fields. Specifically, the system can be used to transfer target information and images among the fields, client–server stations (CS), and the satellite situation center (SSC). The typical application scenario is shown in Figure 1. This kind of system is especially used for detecting purposes such as target search, identification, localization, tracking, damage effect feedback, etc. Furthermore, they have been playing the roles of information transfer centers, and they are widely considered to be the most critical device for battlefield scouting based on UAV. The field of sight (FOS) can be considered to be the vision plane chosen by the UAV operator. FOS is chosen to observe the target properly, and it is considered temporally stationary relative to the ground (or to the inertia space). The line of sight (LOS) is defined as the central axis of the system camera.
Ideally, to obtain clear images, LOS should always be kept vertical to FOS. However, there are various disturbances in the working process, and those disturbances will result in angular deviations of LOS in real time. Therefore, the stability of LOS is a fundamental basis for proper working. Over a certain period time of the working process, the intention of stabilizing the LOS or the system is to minimize the angular deviations of LOS relative to FOS. The index to evaluate the stability performance of the system is defined as stability accuracy. It is treated as the focus of this research, and will be defined and analyzed later in detail. On the other hand, those disturbances are considered to be casual elements, and cannot be easily predicted, because they are often caused by movements of UAV and environmental factors such as wind and noises, etc.

From the perspective of product mechanism design, the system is often divided into four major parts: photoelectric and infrared detectors, which are used to capture vision images; multi-axis frame (or gimbal), for supporting physical frames and compensation movements of axial rotations; drive, control panel, and related system interfaces, for controlling the motions of motors on axes and transferring data; and passive vibration damper, for dissolving some of the disturbances. The typical composite of the system is shown in Figure 2.

As mentioned, the stability can be affected by various unpredictable factors. Therefore, the servo-control of the system is the core for realizing stability. The overall structure of the servo-control functions shown in Figure 3 mainly consist of loops of stabilizing, tracking, and positioning. The control loops can correspond to the system’s working modes of stabilizing, tracking, and positioning, respectively. The working modes can be selected and switched by the operator.
Based on the commonly used mechanism structure shown in Figure 2 and the servo-control structure shown in Figure 3, the working modes can be explained as follows: while the system is working on the stabilizing mode, the gyroscopes are used to measure angular velocities coupled to the system in real time. The velocity data will be transferred by the system interfaces. Then, the velocities will be compensated by the motor motions of the stabilizing loop, ensuring the stability of LOS to obtain clear images. In addition, the interface performance of the system is very important, both for data transmission speed and even overall system performance. The reasons for interface failure include damages or wrong assembly of hardware, incompatibilities of data formats, and inferiority of control algorithms. The failures of interfaces will directly lead to losing real-time states, and therefore the overall failures of servo-control functions.

While working on the tracking mode, based on the stabilizing loop, the miss distances of the target are obtained by analyzing the vision images. Then, motor rotations will be controlled to keep the LOS always aligned with the target. Specifically, the target is always kept in the center of FOS. While working on the positioning mode, the real-time positions of the frames are obtained by position sensors, and then the motors will be controlled to rotate to the specified positions.

Obviously, the stabilizing function is the core of the whole system, and the basis of further applications such as tracking and positioning. Therefore, stability accuracy of the stabilizing function is the key performance index, and it is treated as the focus of this research. Compared to unpredictable factors, the assembly errors of system components will also affect that index greatly. Those errors can be controlled relatively easily. The relative analysis will provide valuable guidance for the design, development, and manufacturing process.

In this research, a method to analyze assembly error effect on stability accuracy is proposed. Stability accuracy is comprehensively modeled and simulated. The system’s general kinematics, compensation principle, and major components are studied. Assembly errors of axis perpendicularities, run-outs, and gyroscopes are defined, modeled, and introduced into the overall model. The effects caused by assembly errors under different disturbance frequencies are studied. Error sensitivity is proposed by using standard deviation analysis. The most sensitive assembly errors are identified, and the results can be used to determine standards and priorities of improvements in the future. The modeling and simulations are all carried out in MATLAB R2019b Simulink. The MATLAB codes of the simulation works are available online at Supplementary Materials.

2. Literature Review

Recently, many researchers have highlighted the stabilization accuracy control of photoelectric devices. For instance, Song, et al. [1] studied the stabilization precision control methods of a photoelectric aim-stabilized system. Lin, et al. [2] developed an inertial-stabilized platform for airborne remote sensing using magnetic bearings. Lei, et al. [3] proposed a composite control method for two-axis inertial-stabilized platforms. Liu, et al. [4] compared ANFIS and fuzzy PID control model for performance in a two-axis inertial-stabilized platform. Fei, et al. [5] proposed a dynamic model and control method. Kwak, et al. [6] proposed a dual-stage and digital image-based method for sight stabilization. It can be concluded that the stability control of LOS is the key factor to
keep the relevant systems working properly. Stability accuracy is always treated as the key performance index to evaluate the whole system.

On the other hand, the overall stability accuracy of the system can be severely affected by various kinds of disturbances, including carrier movements, winds, noises, etc. Therefore, many researchers have been focusing on the effect caused by different disturbances: Wu, et al. [7] studied the video stabilization with total warping variation model. Spampinato, et al. [8] studied a low-cost rot-translational video stabilization algorithm. Mao, et al. [9] proposed the continuous second-order sliding mode control. Chen, et al. [10] studied the disturbance observer-based control and related methods. Dasgupta, et al. [11] designed the disturbance observer by using a Hirschhorn inverse approach. Those unpredictable disturbances contribute to the stability performance as casual elements, and they can only be compensated for by using real-time mechanism motions.

In contrast, the assembly errors of components can be easily controlled during the manufacturing processes. For instance, Xiang, et al. [12] carried out analysis and evaluation of installation uncertainty in rotary pair measurement. Liu, et al. [13] carried out a machining error analysis of a freeform surface off-axis three-mirror system based on optical performance evaluation. Ren and Wang [14] studied the impacts of installation errors on the calibration accuracy of a gyro accelerometer tested on centrifuge. Wang, et al. [15] studied SINS installation error calibration based on multi-position combinations, including installation errors of gyroscopes and accelerometers. Chen, et al. [16] analyzed a novel method based on a magnetically suspended gyroscope in the presence of installation error. However, it is common that only one kind of assembly error is studied, such as that of axis, gyroscope, or accelerometer, without comprehensive modeling and analysis. Furthermore, these researchers barely considered a two-axis stabilization mechanism, which is commonly adopted by UAV photoelectric detection. In summary, the analysis of assembly error effect on the stability accuracy of a UAV photoelectric detection system deserves further study.

3. Modeling and Simulations of the Assembly Error Effect on the Stability Accuracy

To study the dynamic control performance of the system stabilizing function, each component must be modeled and simulated separately. These will be integrated together in the final step for the simulation experiments.

3.1. Definition and Modeling of Stability Accuracy Based on Pointing Error

Based on the kinematics of space mechanisms, those disturbances can be decomposed into angular motions around three vertical axes of inertial space, and they will all be coupled to LOS. To stabilize LOS, at least two of those motions must be compensated for, and typically they are the rotations around the y- and z-axes of inertial space [17]. The compensations will be realized by the two motor-driven frames, namely the frames of azimuth and pitch, respectively. In this research, they are abbreviated as the azimuth and the pitch hereafter.

UAV photoelectric detection commonly adopts this two-axis stabilization mechanism because of advantages of simplicity and low weight [18]. Based on the above background, an index of stability accuracy $\Delta \theta$ is proposed for evaluation of the angular deviations. $\Delta \theta$ is defined as the root mean square (RMS) of the deviations between ideal angles and real angles. In a statistical sense, the value of $\Delta \theta$ can be estimated by sampling within a certain total sampling time $t_s$ (with sampling period $T_s$ and sampling number $n_s$). In that case, $\Delta \theta$ can be obtained by using Equation (1), where $\Delta \theta$ is the mean of the sampling values, $\Delta \theta_{si}$ is the sample value of the $i$th sample, and $\sigma_{\Delta \theta}$ is the standard error.

$$
\begin{align*}
\Delta \theta &= \frac{\sum_{i=1}^{n_s} \Delta \theta_{si}}{n_s}, \\
\sigma_{\Delta \theta}^2 &= \frac{1}{n_s-1} \sum_{i=1}^{n_s} (\Delta \theta_{si} - \Delta \theta)^2.
\end{align*}
$$

Equation (1)
As shown in Figure 4, in practice, the value of $\Delta \theta_{si}$ is calculated by using sampling values, which include the angular deviations along the y-axis $\Delta \theta_{syi}$ and that of z-axis $\Delta \theta_{szi}$. The calculation method is expressed as Equation (2). $\Delta \theta_{syi}$ and $\Delta \theta_{szi}$ will be obtained by using the signals of the two gyroscopes assembled on the frames.

\begin{equation}
\Delta \theta_{si} = \sqrt{\Delta \theta_{syi}^2 + \Delta \theta_{szi}^2},
\end{equation}

3.2. Definitions of Coordinates and Kinematics Analysis of the System

To stabilize LOS, the angular deviations must be compensated for by the two motor-driven frames. To realize that, establishment of coordinates and relevant kinematics analysis is essential. To describe the motion relationships among the components in the system, their coordinates and motions should be defined separately. Those definitions of the commonly used azimuth-pitch two-axis mechanism are as follows, and shown in Figure 5.

- **The inertial coordinate**
  This is denoted as $\{i\}$ with axes of $(o_i x_i y_i z_i)$. The center point of the Earth is taken as the coordinate origin $o_i$; $o_i x_i$ is along with the Earth’s rotation axis; $o_i y_i$ and $o_i z_i$ are orthogonal to each other in the equatorial plane of the Earth, pointing to two fixed stars. $o_i x_i$ and $o_i y_i$ form a right-hand coordinate with $o_i z_i$ perpendicularly.

- **The coordinate of the system base**
  This is denoted as $\{b\}$ with $(o_b x_b y_b z_b)$. To simplify the modeling, the system is treated as fixed to the UAV, neglecting effects of the vibration damper, which is not the focus of this research. Hence, the coordinates of the UAV and the system base are unified as $\{b\}$. $o_b$ is the geometric center point of the location where the azimuth frame is assembled on the base; $o_b x_b$, $o_b y_b$ and $o_b z_b$ are along the heading, left-moving, and rising directions of the UAV. Then, the linear and angular velocity
vectors of the system base in the inertial coordinate \{i\} are denoted as \( \vec{v}_{ib} = [v_{ibx}, v_{iby}, v_{ibz}]^T \) and \( \vec{\omega}_{ib} = [\omega_{ibx}, \omega_{iby}, \omega_{ibz}]^T \).

- The coordinate of the azimuth frame and axis

  This is denoted as \{a\} with \((o_a x_a y_a z_a)\). The coordinates of the azimuth frame and axis are unified as \{a\} for simplification. \( o_a \) is the geometric center where the pitch frame is connected to the azimuth frame. \( o_a x_a, o_a y_a \) and \( o_a z_a \) form a coordinate, parallel to \{b\} when the system has no azimuth rotation; the compensation azimuth rotation is around \( o_a z_a \). The angular velocity vector of the azimuth in \{i\} is denoted as \( \vec{\omega}_{ia} = [\omega_{iax}, \omega_{iay}, \omega_{iaz}]^T \); the rotation angle and angular velocity of azimuth compensation driven by the azimuth motor are denoted as \( \theta_a \) and \( \dot{\theta}_a \), which are around \( o_a z_a \) in \{a\}.

- The coordinate of the pitch frame and axis

  This is denoted as \{p\} with \((o_p x_p y_p z_p)\). \( o_p \) is treated as the start point of the LOS, and \( o_p x_p \) is treated as the LOS for simplification; \{p\} is parallel to \{a\} when the system has no pitch rotation; the compensation pitch rotation is around \( o_p y_p \), the angular velocity vector of the pitch in \{i\} is denoted as \( \vec{\omega}_{ip} = [\omega_{ipx}, \omega_{ipy}, \omega_{ipz}]^T \); the rotation angle and angular velocity of pitch compensation around \( o_p y_p \) in \{p\} are denoted as \( \theta_p \) and \( \dot{\theta}_p \).

- The coordinates of the gyroscopes

  These include \{ga\} with \((o_{ga} x_{ga} y_{ga} z_{ga})\) for the azimuth gyroscope, and \{gp\} with \((o_{gp} x_{gp} y_{gp} z_{gp})\) for the pitch one. \{ga\} and \{gp\} are ideally parallel to \{p\}. \( o_{ga} \) and \( o_{gp} \) are the original points where the gyroscopes are assembled. The sensitive axis for LOS angular velocity of \( \omega_{ipz} \) is \( o_{ga} z_{ga} \) of \{ga\}, and that for \( \omega_{ipy} \) is \( o_{gp} y_{gp} \) of \{gp\}.

3.3. Modeling and Simulations of the General Kinematics and Compensation Principle

According to kinematics analysis of homogeneous coordinate transformation [19], the rotation transformation matrices from \{b\} to \{a\} and from \{a\} to \{p\} can be expressed as Equation (3), where \( c\theta_a \) means \( \cos\theta_a \), \( s\theta_a \) means \( \sin\theta_a \), and likewise hereinafter for simplicity.

\[
R_a^b = \begin{bmatrix}
    c\theta_a & s\theta_a & 0 \\
    -s\theta_a & c\theta_a & 0 \\
    0 & 0 & 1
\end{bmatrix},
R_p^a = \begin{bmatrix}
    c\theta_p & 0 & -s\theta_p \\
    0 & 1 & 0 \\
    s\theta_p & 0 & c\theta_p
\end{bmatrix}.
\]  

(3)

The rotation of the azimuth in \{i\} can be decomposed into that of the azimuth in \{b\} and that of the system base in \{i\}, which is expressed as Equation (4), where \( \vec{\omega}_{ba} = [0, 0, \theta_a] \), representing the angular velocity vector of the azimuth in \{b\}.

\[
\vec{\omega}_{ia} = R_a^b \vec{\omega}_{ib} + \vec{\omega}_{ba},
\]  

(4)

Similarly, the rotation of the pitch in \{i\} can be decomposed into the pitch rotation in \{a\} and the azimuth rotation in \{i\}, which is expressed as Equation (5), where \( \vec{\omega}_{ap} = [0, \theta_p, 0] \), representing the angular velocity vector of the pitch in \{a\}.

\[
\vec{\omega}_{ip} = R_a^p \vec{\omega}_{ia} + \vec{\omega}_{ap},
\]  

(5)

By introducing Equation (3) and Equation (4) into Equation (5) and decomposing as needed, Equation (6) is obtained, where \( \vec{\omega}_{ib}^p = [\omega_{ibx}^p, \omega_{iby}^p, \omega_{ibz}^p]^T \), representing the projection of the system base angular velocity relative to the inertia coordinate in \{p\}, and \( \vec{\omega}_{ib} = [\omega_{ibx}, \omega_{iby}, \omega_{ibz}]^T \), representing the projection of the pitch angular velocity relative to the system base in \{p\}.

\[
\vec{\omega}_{ip} = \begin{bmatrix}
    \omega_{ipx} \\
    \omega_{ipy} \\
    \omega_{ipz}
\end{bmatrix} = \begin{bmatrix}
    \omega_{ibx}^p \\
    \omega_{iby}^p \\
    \omega_{ibz}^p
\end{bmatrix} + \frac{\omega_{ibx} c\theta_a c\theta_p + \omega_{iby} s\theta_a c\theta_p - \omega_{ibz} s\theta_p}{\dot{\theta}_a s\theta_p} \begin{bmatrix}
    \dot{\theta}_a s\theta_p \\
    \dot{\theta}_p \\
    \dot{\theta}_a c\theta_p
\end{bmatrix}.
\]  

(6)

As mentioned, axial rotation movements of y- and z-axes of the inertial coordinate coupled to the LOS must be compensated. Those are realized by using the motor-driven azimuth and pitch
frames to make \( \omega_{ipy} = 0 \) and \( \omega_{ipz} = 0 \). In that case, the compensation angular velocities \( \hat{\theta}_a \) and \( \hat{\theta}_p \) can be inferred from Equation (6), which is expressed as Equation (7), where \( tg\theta_p \) represents \( \tan\theta_p \).

\[
\begin{align*}
\hat{\theta}_a &= -\frac{\omega_{ibz}}{eq} = -\left(\omega_{ibx}c\theta_a tg\theta_p + \omega_{iby} s\theta_a tg\theta_p + \omega_{ibz}\right) \\
\hat{\theta}_p &= -\omega_{iby} = -\omega_{iby} c\theta_a + \omega_{ibz} s\theta_a
\end{align*}
\] (7)

Practically, the azimuth and pitch gyroscopes are assembled on the pitch frame to detect the real-time value of \( \omega_{ibz} \) and \( \omega_{iby} \). The sensitive velocities of the two gyroscopes are denoted as \( \omega_{gyro,x} = \omega_{ibz} \) and \( \omega_{gyro,y} = \omega_{iby} \). Therefore, the relationship between the gyroscope sensitive velocities and the compensation velocities are expressed as Equation (8):

\[
\begin{align*}
\hat{\theta}_a &= -\omega_{ibz} / c\theta_a = -\omega_{gyro,x} c\theta_p s\theta_a \\
\hat{\theta}_p &= -\omega_{iby} = -\omega_{gyro,y}
\end{align*}
\] (8)

Based on the analysis above, the general kinematics and compensation principle of the system are simulated in Simulink as shown in Figure 6. In the simulation module of the azimuth kinematics from \( b \) to \( a \), e.g., the angular velocity vector of the system base in \( i \), namely \( \omega_{ib} \), is set as one of the inputs. The other input is the z-axial component of the azimuth velocity vector \( \omega_{iaz} \), namely \( \omega_{iaz} \) can be obtained by using signals of the motor modules, which will be discussed later. The velocity vector of the azimuth in \( i \), namely \( \omega_{iaz} \), is set as the output. Based on Equation (4), the transformation matrices of \( R_a^b \) are imbedded in the module. The azimuth motor velocity and the rotation angle, namely \( \hat{\theta}_a \) and \( \theta_a \), can also be calculated by using inverse operation of Equation (4). Similarly, the simulation module for the pitch kinematics from \( a \) to \( b \) can also be interpreted by using Equation (5). Those velocity vectors and parameters will be used for later analysis and modeling.

3.4. Modeling and Simulation of the Motor Kinematics

All the compensation movements are driven by the motors including the azimuth motor and the pitch motor. The equivalent circuit of the motor is shown in Figure 7, where \( U_m \) is the motor armature voltage; \( I_m \) is the armature current; \( R_m \) is the total resistor of the armature circuit; \( L_m \) is the total inductance; \( M_m \) is the motor output torque; \( J_m \) and \( J_L \) are the rotary inertia of the motor and the load; and \( D_L \) and \( K_L \) are the damping and elastic coefficients between the motor and the load. \( \theta_m, d\theta_m/dt \), and \( d^2\theta_m/dt^2 \) are the outputs of angle, angular velocity, and angular acceleration of the motor side. \( \theta_L, d\theta_L/dt \), and \( d^2\theta_L/dt^2 \) are those of the load side, respectively.

![Figure 6. Simulation modules of the System’s General Kinematics and Compensations.](image)

![Figure 7. Equivalent armature circuit of the motor.](image)
The equations of voltage balance, back electro-dynamic force (EMF), and moment for DC torque motor are shown in Equation (9), where $E_m$ is the EMF, $C_e$ is the coefficient of EMF, $C_m$ is the coefficient of the motor torque.

$$\begin{align*}
U_m &= E_m + I_m R_m + L_m (d I_m / dt) \\
E_m &= C_e (d \theta_m / dt) \\
M_m &= C_m I_m
\end{align*}$$ (9)

By combining the three equations of Equation (9), Equation (10) is obtained:

$$U_m = C_e \frac{\partial \theta_m}{\partial t} + \frac{R_m}{C_m} I_m + \frac{L_m}{C_m} \frac{d M_m}{d t}$$ (10)

The kinematics of the motor is analyzed and expressed as Equation (11):

$$\begin{align*}
I_m \frac{d^2 \theta_m}{d t^2} + D_l \left( \frac{d \theta_m}{d t} - \frac{d \theta_L}{d t} \right) + K_L (\theta_m - \theta_L) &= M_m \\
J_L \frac{d^2 \theta_L}{d t^2} + D_l \left( \frac{d \theta_L}{d t} - \frac{d \theta_m}{d t} \right) + K_L (\theta_L - \theta_m) &= 0
\end{align*}$$ (11)

The servo-control of the motor generally adheres to principles of pulse width modulation (PWM) or automatic drive (AD). The input is the compensation velocity as shown in Equation (7), and denoted as $\theta_L$ here. The output is the control voltage $U_c$. In this research, the driving circuit of the motor is taken as a lagging amplification model. Therefore, the Laplace transform function can be expressed as Equation (12), where $K_p$ is the amplification coefficient; $T_p$ is the lagging time. Because the working frequency of the circuit is much larger than the cut-off frequency of the motor, the effect of lagging time can be ignored.

$$U_c(s) = K_p e^{-T_p s} \theta_L(s) \approx K_p \theta_L(s)$$ (12)

Using Laplace inverse transformation, Equation (12) can be transformed as Equation (13):

$$U_c(t) = K_p \dot{\theta}_L(t)$$ (13)

The voltage balance of the servo-control circuit is expressed as (14), where $E_s$ is the EMF of the voltage source, and $R_s$ is the resistor.

$$\begin{align*}
\{ U_c = E_s + U_m \\
E_s = R_s I_m
\end{align*}$$ (14)

Based on the above analyses of Equation (10), Equation (11), Equation (13), and Equation (14), the $y$-axial component of the pitch velocity vector $\dot{\omega}_{'y}$ is equal to $\theta_L$ by definition. So $\dot{\omega}_{'y}$ is set as the final output of this module and it will be used in other modules. The simulation

**Figure 8.** Simulation module of the pitch motor.

Based on the above analyses of Equation (10), Equation (11), Equation (13), and Equation (14), the $y$-axial component of the pitch velocity vector $\dot{\omega}_{'y}$, namely $\omega_{gyro,y}$, is the other input, and it can be obtained by using signals of the gyroscope module, which will be discussed later.
module for the azimuth motor is similar, and the output will be $\omega_{iax}$, which is discussed and used in Section 3.3.

3.5. Modeling and Simulation of the Assembly Error Effects on Stability Accuracy

From Equation (6) and Equation (7), it can be seen how the UAV movements and the system attitude generally affect the stability accuracy. However, considerations of other error-causing sources, including environment disturbances, component noises, or assembly errors, are neglected. Those will deteriorate the overall stability accuracy index once they are coupled to the system movements, which cannot be ignored in practice. In this research, major error sources including assembly errors of axis perpendicularities, run-outs and gyroscope locations are modeled. Those analysis results will provide guidance for future possible improvements.

3.5.1. Modeling and Simulation of Assembly Errors of Axis Perpendicularities

Ideally, the coordinate of the azimuth $\{a\}$ is parallel to $\{b\}$ when the system has no azimuth rotation. However, due to the effect of assembly error, the real axis location is not perfectly perpendicular to the base, and the perpendicularity deviation can be denoted as a vector of $\delta_{ba} = [\delta_{bax}, \delta_{bay}, 0]^T$.

As a result, the coordinate $\{a\}$ and especially for the rotation axis of the azimuth compensation $o_{az}$ will also deviate from the ideal direction by $\delta_{ba}$. (For simplification, $o_{az}$ is taken as the instantaneous rotation axis for error analysis of perpendicularity, rather than the average one.) For the pitch compensation, the perpendicularity deviation vector is denoted as $\delta_{ap} = [\delta_{apx}, 0, \delta_{apz}]^T$ respectively. The meaning of axis perpendicularity deviations is shown in Figure 9. In this case, the rotation transformation matrix affected by errors of axis perpendicularity can be expressed as Equation (15):

$$
R_a = \begin{bmatrix}
   c\delta_{bay} & 0 & -s\delta_{bay} \\
   0 & 1 & 0 \\
   s\delta_{bay} & 0 & c\delta_{bay}
\end{bmatrix},
R_p = \begin{bmatrix}
   c\delta_{apx} & s\delta_{apx} & 0 \\
   -s\delta_{apx} & c\delta_{apx} & 0 \\
   0 & 0 & 1
\end{bmatrix},
$$

The angular velocity vectors of the azimuth and the pitch in $\{i\}$ affected by errors of axis perpendicularity are denoted as $\tilde{\omega}_{ia}^i$ and $\tilde{\omega}_{ip}^i$, and they can be calculated by using Equation (16):

$$
\tilde{\omega}_{ia}^i = R_a^p R_a^i \tilde{\omega}_{ib} + \tilde{\omega}_{ba},
\tilde{\omega}_{ip}^i = R_p^p R_p^i \tilde{\omega}_{ip} + \tilde{\omega}_{ap}.
$$

Based on Equation (16) and analysis above, the signals of the gyroscopes affected by $\delta_{ba}$ and $\delta_{ap}$ are simulated in Simulink as shown in Figure 10. Taking the azimuth, for example, based on Equation (16), $\tilde{\omega}_{ib}$ is set as the input; the perpendicularity deviation vector $\delta_{ba}$ and the
transformation matrix $R'_a$ are imbedded in the middle; the output is $R'_a\bar{o}_{lp}$, which will be used in other modules. The value of $\delta_{pa}$ can be set as needed in the future simulation experiments. The simulation module for the pitch frame is similar. Additionally, the value of $\delta_{pa}$ and $\delta_{ap}$ are determined by the assembly accuracies of the axes and frames. They can be measured by using high-precision instruments [20].

**Figure 10.** The signals affected by errors of axis perpendicularities.

### 3.5.2. Modeling and Simulation of Assembly Errors of Run-Outs

As mentioned, the instantaneous rotation axes are taken as the rotation center lines for error analysis of perpendicularity, rather than the average one. However, in practice, due to bad assembly of the bearings, the rotation center lines can be dynamically unsteady under working state. The maximum deviations between the instantaneous rotation axes and the average ones are defined as the assembly errors of run-outs.

Specifically, run-outs include axial shifts $\Delta s$, radial run-outs $\Delta R_{ro}$, and tilting oscillations $\Delta \tilde{y}$ ($\Delta R_o$ and $\Delta \tilde{y}$ can be decomposed along two axes of the coordinate [21]). Taking the azimuth for example, the assembly errors of run-outs are shown in Figure 11. Apparently, displacement errors such as $\Delta s$ and $\Delta R_{ro}$ had no effect on the stability accuracy, and angular tilting oscillations $\Delta \tilde{y}$ are what this research focuses on.

**Figure 11.** The assembly errors of azimuth run-outs.

The oscillation vector of the azimuth is expressed as $\Delta \tilde{y}_a = [\Delta \alpha_{am}, \Delta \beta_{am}, 0]^T$, and that of the pitch is expressed as $\Delta \tilde{y}_p = [\Delta \alpha_{pm}, \Delta \beta_{pm}, 0]^T$. $\Delta \alpha_{am}, \Delta \beta_{am}, \Delta \alpha_{pm}, \Delta \beta_{pm}$ are the decomposed components of the maximum tilting oscillations along the coordinates $\{b\}$ and $\{a\}$, and they can also be measured [22].

From the perspective of dynamic signal processing, the tilting oscillations are periodic signals, with the angular frequencies of $\theta_a$ and $\dot{\theta}_p$. Taking $\Delta \alpha_{am}$ for example, its real-time value expression $\Delta \alpha_{am}(t)$ is a periodic signal with angular frequency of $\dot{\theta}_a$ and period of $T_a = 2\pi/\dot{\theta}_a$, then $\Delta \alpha_{am}$ can be expressed as Equation (17) by using Fourier series:

\[ \Delta \alpha_{am} = \frac{1}{T_a} \sum_{n=-\infty}^{\infty} A_n \sin(n \dot{\theta}_a t + \phi_n) \]
\[ \Delta a(t) = \frac{\Delta \theta_{a0}}{2} + \sum_{n=1}^{\infty} \left( a_n c(n\theta_a t) + a_n s(n\theta_a t) \right) \]

\[ a_n = \frac{2}{T_a} \int_{T_a}^{T_0} \Delta a(t) c(n\theta_a t) \, dt, \quad (n = 0, 1, 2, \ldots), \]
\[ b_n = \frac{2}{T_a} \int_{T_a}^{T_0} \Delta a(t) s(n\theta_a t) \, dt \]

For simplicity, \( \Delta a(t) \) is taken as an odd function, and high-frequency signals with \( n > 1 \) are ignored. Then \( \Delta a(t) \) can be expressed as Equation (12), and \( \Delta a_{am} \) is considered to be the amplitude of this signal. Similarly, other tilting oscillations are also shown in Equation (18).

\[ \Delta a(t) \approx \frac{2}{T_a} \int_{T_a}^{T_0} \Delta a(t) s(\theta_a t) \, dt \cdot s(\dot{\theta}_a t) = \Delta a_{am} s(\theta_a) \]

Based on the kinematic analysis of axis perpendicularity errors earlier, the rotation transformation matrices caused by run-outs \( \Delta R_a \) and \( \Delta R_p \) can be expressed as Equation (19):

\[
\begin{bmatrix}
  c \Delta \beta_a(t) & 0 & -s \Delta \beta_a(t) \\
  s \Delta \beta_a(t) & 0 & c \Delta \beta_a(t) \\
  0 & 1 & 0
\end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c \Delta \alpha_a(t) & s \Delta \alpha_a(t) \\ 0 & -s \Delta \alpha_a(t) & c \Delta \alpha_a(t) \end{bmatrix},
\]

\[
\begin{bmatrix}
  c \Delta \beta_p(t) & 0 & -s \Delta \beta_p(t) \\
  s \Delta \beta_p(t) & 0 & c \Delta \beta_p(t) \\
  0 & 0 & 1
\end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c \Delta \alpha_p(t) & s \Delta \alpha_p(t) \\ 0 & -s \Delta \alpha_p(t) & c \Delta \alpha_p(t) \end{bmatrix},
\]

The angular velocity vectors of the azimuth and the pitch in \( \{i\} \) affected by errors of run-outs are denoted as \( \Delta \omega_{ia} \) and \( \Delta \omega_{ip} \), and can be calculated by using Equation (20):

\[
\Delta \omega_{ia} = R_a^b \Delta R_a \omega_{ib} + \omega_{ba},
\]
\[
\Delta \omega_{ip} = R_p^b \Delta R_p \Delta \omega_{ia} + \omega_{ap},
\]

Based on analysis above, the signals of the gyroscopes affected by \( \Delta \gamma_{ia} \) and \( \Delta \gamma_{ip} \) are simulated in Simulink as shown in Figure 12. Taking the azimuth, for example, based on Equation (18) and Equation (20), \( \omega_{ib} \) and the motor rotation angle \( \theta_a \) are set as the inputs; the maximum tilting oscillations including \( \Delta a_{am} \) and \( \Delta \beta_{am} \) and the transformation matrix \( \Delta R_a \) are imbedded in the middle; the output is \( \Delta R_a \omega_{ib} \), which will be used in other modules. The value of \( \Delta a_{am} \) and \( \Delta \beta_{am} \) can be set as needed in the future simulation experiments. The value of \( \theta_a \) can be obtained from the motor module in Section 3.4. The simulation module for the pitch frame is similar.

\[\text{Figure 12. The signals affected by errors of run-outs.}\]
3.5.3. Modeling and Simulation of Assembly Errors of Gyroscope Locations

The gyroscopes are the core components for realizing the stabilizing function, and the assembly of them will directly affect the stability accuracy. The coordinates of the gyroscopes \( \{ ga \} \) and \( \{ gp \} \) are ideally parallel to \( \{ p \} \). Due to bad assembly, there are angular deviations between the real and the ideal locations. The deviations are denoted as \( \vec{\epsilon}_a = [\epsilon_{ax}, \epsilon_{ay}, 0]^T \) for the azimuth gyroscope and \( \vec{\epsilon}_p = [\epsilon_{px}, 0, \epsilon_{pz}]^T \) for the pitch one. Based on the earlier analysis, it is easy to infer the rotation transformation matrixes caused by assembly errors of gyroscope locations \( R_{ag} \) and \( R_{pg} \), which are expressed as Equation (21).

\[
R_{ag} = \begin{bmatrix}
\cos \epsilon_{ay} & 0 & -\sin \epsilon_{ay} \\
0 & 1 & 0 \\
\sin \epsilon_{ay} & 0 & \cos \epsilon_{ay}
\end{bmatrix} \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \epsilon_{ax} & -\sin \epsilon_{ax} \\
0 & \sin \epsilon_{ax} & \cos \epsilon_{ax}
\end{bmatrix},
\]

\[
R_{pg} = \begin{bmatrix}
\cos \epsilon_{pz} & \sin \epsilon_{pz} & 0 \\
-\sin \epsilon_{pz} & \cos \epsilon_{pz} & 0 \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \epsilon_{px} & -\sin \epsilon_{px} \\
0 & \sin \epsilon_{px} & \cos \epsilon_{px}
\end{bmatrix},\]

(21)

Based on Equation (8) and Equation (21), the sensitive velocities of the two gyroscopes in \( \{ i \} \) affected by \( R_{ag} \) and \( R_{pg} \) are denoted as \( \omega'_{gyro, x} \) and \( \omega'_{gyro, y} \), and they could be calculated by using Equation (22).

\[
\omega'_{gyro, x} = [0, 0, 1]R_{ag}\vec{\omega}_p,
\]

\[
\omega'_{gyro, y} = [0, 1, 0]R_{pg}\vec{\omega}_p,
\]

(22)

By using a typical second-order model, the transformation function between the input and output signals of gyroscopes \( Trans_g(s) \) could be expressed as Equation (23), where \( K_{gy}, \omega_g = 2\pi f_g \) and \( \zeta_g \) represent the amplification coefficient, natural frequency, and damping coefficient of the gyroscopes.

\[
Trans_g(s) = \frac{K_{gy}\omega_0^2}{s^2 + 2\zeta_g\omega_0 s + \omega_0^2},
\]

(23)

Based on Equation (23) and analysis above, the signals and assembly errors of the gyroscopes are simulated in Simulink as shown in Figure 13. Based on Equation (22), \( \vec{\omega}_p \) is set as the input. Furthermore, the value of \( \vec{\omega}_p \) can be considered equal to the real-time value of \( \vec{\omega}_p \), which can be obtained from the kinematics module in Section 3.3. The assembly error vectors of \( \vec{\epsilon}_a \) and \( \vec{\epsilon}_p \), the transformation matrixes \( R_{ag} \) and \( R_{pg} \), and \( Trans_g(s) \) are imbedded in the middle. The final sensitive velocities of the two gyroscopes \( \omega'_{gyro, x} \) and \( \omega'_{gyro, y} \) are set as the output, and they will be used in other modules. The value of \( \vec{\epsilon}_a \) and \( \vec{\epsilon}_p \) can be set as needed in the future simulation experiments, and also can be measured by using high-precision instruments and relevant methods [23,24].

Figure 13. Simulation module of the signals and assembly errors of the gyroscopes.
3.6. Integrating Simulation of the Overall System Stability Accuracy

By integrating all the above modules, the overall system stability accuracy is modeled in Simulink, which is shown in Figure 14. The original system base angular velocity \( \tilde{\omega}_{ib} = [\omega_{ibx}, \omega_{iby}, \omega_{ibz}]^T \) is set as the input. All the modules of the kinematics, components, and assembly errors are imbedded in the middle. The final index of stability accuracy \( \Delta \theta \) is set as the output by referring to Equation (1) and Equation (2).

![Figure 14. Overall simulation modeling of the system stability accuracy.](image)

4. Simulation Experiments and Analysis Based on Error Sensitivity

In the real application process of the system, those disturbances caused by UAV, environment factors such as wind, noises, etc., will all couple to the system pedestal base in the end. They are all integrated and defined as the base disturbances (BD) for simplicity. Apparently, in the normal working process of the system, BD is the strongest factor affecting the stability accuracy. That effect will be deteriorated by those modeled assembly errors. According to previous definitions in Section 3.2, the angular velocities of BD disturbances are denoted as \( \tilde{\omega}_{bd} = [\omega_{bdx}, \omega_{bdy}, \omega_{bdz}] \).

By carrying out dynamic simulations, the effects caused by BD and the modeled assembly errors can be analyzed. The analysis results will provide valuable guidance for future possible improvements. For simulation study, according to Equation (1), the sampling period \( T_s \) is equal to the Simulink step size, and set as 1ms. For other parameter settings: Total sampling time: \( t_o = 30s \); Sampling number: \( n_o = 30000 \). By referring to real measurement data of a newly developing system, the necessary parameters are predefined before the simulation experiments. Those parameters are listed in Table 1, and the prototype of the product is shown in Figure 15.

![Figure 15. The assembly prototype of the newly developing system.](image)
To study the sensitivity of the assembly error effect on the stability accuracy, the simulation experiments affected by each kind error source was carried out separately. Specifically, the simulations of each type of error source are carried out with other errors are equal to zero.

The variables of each group of simulations will be the levels of each error source, and the disturbance frequencies of BD. The two variables are determined by referring to principles as follows: according to engineering experiences and the real conditions of manufacturing accuracies, the error source levels are set to range from 1 m° to 50 m°. By referring to [25,26], the angular velocities of the three axes of BD disturbances, specifically \( \dot{\omega}_{ib} = [\omega_{ibx}, \omega_{iby}, \omega_{ibz}] \), are all simulated as sinusoidal signals of low frequencies, and they range from 1.5°/0.5Hz to 1.5°/4Hz. The simulation results of stability accuracy obtained \( \Delta \theta \) are presented with three decimal places, and listed in the central area of Tables 2–6. The results of each kind of BD disturbance are listed in a separate table.

**Table 1.** Necessary measured parameters before simulation experiments.

| Parameters | Descriptions | Values | Units |
|------------|--------------|--------|-------|
| \( J_{ia} \) | Rotary inertia of the azimuth motor | 0.02 kg \( \cdot \) m² |
| \( J_{ia} \) | Rotary inertia of the azimuth load | 0.04 kg \( \cdot \) m² |
| \( J_{ip} \) | Rotary inertia of the pitch motor | 0.01 kg \( \cdot \) m² |
| \( J_{ip} \) | Rotary inertia of the pitch load | 0.02 kg \( \cdot \) m² |
| \( R_m \) | Total resistor of the two armature circuits | 8.60 Ω |
| \( L_m \) | Total inductance of the armature circuits | 0.01 H |
| \( D_k \) | Damping coefficient of the two motors | 10 / |
| \( K_e \) | Elastic coefficient of the two motors | \( \approx 100000 \) / |
| \( C_m \) | EMF coefficient of the two motors | 0.33 / |
| \( K_m \) | Motor torque coefficient of the two motors | 0.33 / |
| \( K_p \) | Amplification coefficient of the driving circuit | \( \approx 20 \) / |
| \( R_s \) | Resistor of the driving circuit | 0.4 Ω |
| \( K_g \) | Amplification coefficient of the gyroscopes | \( 15/\pi \) / |
| \( \omega_g \) | Natural frequency of the gyroscopes | \( \approx 200\pi \) rad/s |
| \( \zeta_g \) | Damping coefficient of the gyroscopes | 0.7 / |

**Table 2.** Simulation results under BD Dis of 1.5°/0.5Hz

| Stability Accuracy \( \Delta \theta \) [mrad] (Data in the central area with three decimal places) | Error Levels [m] (For each column, the first data is the error level of this column, and for the following each line of this column, each relevant error type on the left is equal to the error level, with all of the other errors are equal to 0 simultaneously.) |
|-------------------------------------------------|-------------------------------------------------|
| \( \delta_{ix} \)                           | 112.117 112.143 112.056 112.127 112.261 |
| \( \delta_{iy} \)                           | 112.074 112.024 112.257 112.103 112.108 |
| \( \delta_{iz} \)                           | 112.359 113.085 113.733 115.457 120.446 |
| \( \delta_{ax} \)                           | 112.222 112.148 112.153 112.138 112.104 |
| \( \triangle \theta_{x} \)                  | 112.155 112.205 112.065 112.123 112.111 |
| \( \triangle \theta_{y} \)                  | 112.191 112.085 112.096 112.145 112.069 |
| \( \triangle \theta_{z} \)                  | 112.101 112.178 112.235 112.044 112.066 |
| \( \delta_{x} \)                           | 112.146 112.155 112.097 112.150 112.109 |
| \( \delta_{y} \)                           | 112.028 112.183 112.094 112.123 112.144 |
| \( \delta_{z} \)                           | 111.922 111.302 110.480 108.808 103.845 |
| \( \varepsilon_{x} \)                      | 112.057 112.212 112.121 112.100 112.077 |
| \( \varepsilon_{y} \)                      | 112.216 112.331 112.689 113.233 115.184 |

**Table 3.** Simulation results under BD Dis of 1.5°/1Hz

| Stability Accuracy \( \Delta \theta \) [mrad] | Error Levels [m] |
|---------------------------------------------|-----------------|
| \( \delta_{ix} \)                           | 39.544 39.552 39.527 39.558 39.526 |
| \( \delta_{iy} \)                           | 39.535 39.492 39.499 39.534 39.524 |
| \( \delta_{iz} \)                           | 39.614 39.929 40.316 41.128 43.521 |
| \( \delta_{ax} \)                           | 39.546 39.535 39.539 39.531 39.520 |
| \( \delta_{ay} \)                           | 39.540 39.516 39.515 39.518 39.505 |
| \( \delta_{az} \)                           | 39.496 39.528 39.825 39.571 39.564 |
| \( \delta_{x} \)                            | 39.511 39.548 39.526 39.530 39.508 |
| \( \delta_{y} \)                            | 39.536 39.538 39.548 39.523 39.555 |
| \( \delta_{z} \)                            | 39.519 39.531 39.530 39.532 39.542 |
| \( \varepsilon_{x} \)                       | 39.434 39.093 38.755 37.982 35.669 |
error sensitivity, which are denoted as errors equal to 0 simultaneously. For each line, the first grid is the error type of this line, and for the rest of the grid, the error increases corresponding to the error level on the top of the column, and index which is used to evaluate the fluctuations changing along with the error levels. Therefore, $S(x)$ calculation results of changing along with the BD disturbance frequencies are shown in Figure 16.

In those tables, for each column, the first piece of data is the error level of this column, and for the rest of the grid, all of the other errors equal to 0 simultaneously. For each line, the first grid is the error type of this line, and for the rest of the grid, the error increases corresponding to the error level on the top of the column, and other error types are still all equal to 0 simultaneously.

The standard deviations of the experimental samples under different error levels are defined as error sensitivity, which are denoted as $S(x)$. $x$ represents the objective error source. $S(x)$ is an index which is used to evaluate the fluctuations changing along with the error levels. Therefore, $S(x)$ can be used to evaluate the sensitivity of those error sources.

$S(x)$ is calculated by using Equation (24), where $n_i$ is the number of error levels; $\Delta \theta_i(x)$ is the accuracy sample value of the $ith$ error level of $x$; $\overline{\Delta \theta(x)}$ is the average value of $\Delta \theta_i(x)$. The calculation results of $S(x)$ are listed in Table 7. The error sensitivities $S(x)$ of different error sources changing along with the BD disturbance frequencies are shown in Figure 16.
Table 7. Error sensitivity of different error sources.

| Error Types (Each line contains the S(\(x\)) of the error type under different BD frequency on the top) | BD Frequency [Hz] (For each column, the first data is the value of the BD frequency, and the following are the S(\(x\)) of each error type on the left side) |
|---|---|
| | \(0.5\) | \(1\) | \(2\) | \(3\) | \(4\) |
| \(\delta_{ax}\) | 7.483 | 1.448 | 0.517 | 0.487 | 0.332 |
| \(\delta_{ay}\) | 8.704 | 2.007 | 0.673 | 0.261 | 0.249 |
| \(\delta_{apx}\) | 324.476 | 156.998 | 74.231 | 47.222 | 34.866 |
| \(\delta_{apz}\) | 4.305 | 0.968 | 0.303 | 0.277 | 0.277 |
| \(\Delta \alpha_{am}\) | 5.213 | 1.287 | 0.502 | 0.261 | 0.339 |
| \(\Delta \beta_{am}\) | 5.007 | 3.078 | 0.704 | 0.217 | 0.187 |
| \(\Delta \alpha_{pm}\) | 7.989 | 1.612 | 0.612 | 0.507 | 0.297 |
| \(\Delta \beta_{pm}\) | 2.646 | 1.223 | 0.838 | 0.421 | 0.152 |
| \(\varepsilon_{ax}\) | 5.817 | 0.817 | 0.853 | 0.477 | 0.158 |
| \(\varepsilon_{ay}\) | 325.078 | 150.672 | 70.202 | 44.542 | 31.868 |
| \(\varepsilon_{px}\) | 6.014 | 1.408 | 0.912 | 0.265 | 0.308 |
| \(\varepsilon_{pz}\) | 121.427 | 86.072 | 53.342 | 38.140 | 30.272 |

Figure 16. Error sensitivity of different error sources.

From the above tables and Figure 16, the most sensitive error sources are identified as x-axial component of pitch axis perpendicularity deviation \(\delta_{apx}\), y-axial component of the azimuth gyroscopes assembly deviation \(\varepsilon_{ax}\), and z-axial component of the pitch gyroscopes assembly deviation \(\varepsilon_{pz}\). Error sensitivities of the identified three errors are ranked as the order of \(S(\delta_{apx}) > S(\varepsilon_{ax}) > S(\varepsilon_{pz})\). Additionally, with the rising of disturbance frequency, the stability accuracies \(\Delta \theta\) can be improved, and the error sensitivities \(S(\chi)\) decrease.

5. Discussions, Conclusions, and Future Work

The major contributions of this research are listed as follows: (1) Stability accuracy of UAV detection system is comprehensively modeled and simulated. The system’s general kinematics, compensation principle, and major components including servo-motors and gyroscopes are all covered. (2) Assembly errors of axis perpendicularities, run-outs, and gyroscopes are defined, modeled, and introduced into the overall model. The effects caused by various assembly error sources can be comprehensively analyzed. (3) The effects caused by those assembly errors under different disturbance frequencies are studied. The simulation experiment results provide valuable data for real application scenarios. (4) Error sensitivity is proposed by carrying out standard deviation analysis. Based on the analysis, the most sensitive assembly errors are identified, and ranked in order of sensitivity as follows: x-axial component of pitch axis perpendicularity deviation,
y-axial component of the azimuth gyroscope assembly deviation, and z-axial component of the pitch gyroscope assembly deviation. In real processes of manufacturing and assembly, the analysis results can be used to determine standards and priorities for possible improvements.

The limits of this research are listed as follows: (1) So far, only assembly errors and UAV angular disturbances are studied. Other error sources and situations to further verify the proposed modeling and analysis method should be considered. (2) Simulation results provide only limited information, and further validations based on real experiments are needed. (3) The limitations, constraints used in this research, and the possibility of uncertainty will affect the results, such as the changes of error distributions, UAV disturbances, and target locations. (4) The effects of series versus parallel (or hybrid) component arrangement are not considered. (5) The effects of interface failures are not covered.

As the next step, validations based on real experiments will be carried out. Other error sources and situations will be investigated, such as frictions, structural eccentricity, and inertia coupling. The linear disturbances of UAV and target movements may have great influence on the results. The effects of series versus parallel (or hybrid) component arrangement will be further studied. The interface performance of the system is very important. The reasons for interface failure include damage or wrong assembly of hardware, inconsistencies of data formats, and inferiority of control algorithms, and will be studied in detail. A similar control mechanism of a more complex system with more dimensions or new materials will be studied, such as the structures of two-axis–four-frame and three-axis–four-frame used on larger carriers. How to incorporate the proposed method to other working models of the system will be studied, such as target identification, tracking, positioning, etc.

**Supplementary Materials:** The following are available online at www.mdpi.com/xxx/s1, MATLAB codes of the simulation works.

**Author Contributions:** Conceptualization, K.H.; Data curation, K.H. and G.J.; Formal analysis, K.H.; Funding acquisition, K.H. and H.H.; Investigation, K.H., H.H. and G.J.; Methodology, K.H. and H.H.; Project administration, K.H. and H.H.; Resources, G.J.; Software, H.G.; Supervision, H.H.; Validation, G.J.; Visualization, G.J.; Writing–original draft, K.H.; Writing–review & editing, H.G. All authors have read and agreed to the published version of the manuscript.

**Funding:** This research was funded by China Postdoctoral Science Foundation (Grant No.47662, the 66th general grant batch of the military system).

**Acknowledgments:** The authors would like to convey their sincere thanks to Mr Hui Guo and Mr Zenan Xu from School of Intelligence and Technology, National University of Defense Technology, and all the anonymous reviewers for their helpful suggestions on the quality improvement of our paper.

**Conflicts of Interest:** The authors declare no conflict of interest.

**Nomenclature**

| Symbol       | Description                                                                 |
|--------------|-----------------------------------------------------------------------------|
| $\Delta \theta$          | System accuracy; Root mean square (RMS) of the angular deviation between the ideal and the real pointing positions |
| $t_s, T_s, n_s$               | Total sampling time in [s], sampling period in [s], sampling number of system accuracy |
| $\bar{\theta}$              | Mean of sampling values                                                     |
| $\Delta \theta_{yi}$, $\Delta \theta_{yi}$ | Sample value of the $i$th sample and its components along the $y$ and $z$-axes of the inertial coordinate |
| $\sigma_{\Delta \theta}$      | Standard error of sample values                                             |
| $\vec{v}_{ib}, \vec{v}_{ibx}, \vec{v}_{iby}, \vec{v}_{ibz}$ | Linear velocity vector of the system base in the inertial coordinated and its components along the three axes of the coordinate in [m/s] |
| $\vec{\omega}_{ib}, \vec{\omega}_{ibx}, \vec{\omega}_{iby}, \vec{\omega}_{ibz}$ | Angular velocity vector of the system base in the inertial coordinated and its components around the three axes of the coordinate in [rad/s] |
| $\vec{\omega}_{ia}, \vec{\omega}_{iax}, \vec{\omega}_{iay}, \vec{\omega}_{iaz}$ | Angular velocity vector of the azimuth in the inertial coordinate and its components around the three axes of the coordinate in [rad/s] |
| $\theta_s, \theta_a$        | Rotation angle in [rad] and angular velocity in [rad/s] of azimuth compensation |
Angular velocity vector of the pitch in the inertial coordinate and its components around the three axes of the coordinate in [rad/s]

$\omega_{\text{ip}}$, $\omega_{\text{ix}}$, $\omega_{\text{iy}}$, $\omega_{\text{iz}}$

Rotation angle in [rad] and angular velocity in [rad/s] of pitch compensation

$\theta_{p}$, $\theta_{p}$

Rotation transformation matrix from the system base coordinate to that of the azimuth

$R_{b}$

Rotation transformation matrix from the azimuth coordinate to that of the pitch

$R_{a}$

Angular velocity vector of the azimuth in the system base coordinate in [rad/s]

$\omega_{\text{az}}$

Angular velocity vector of the pitch in the azimuth coordinate in [rad/s]

$\omega_{\text{az}}$

Projection of the system base angular velocity and its components, relative to the inertia coordinate in the pitch coordinate in [rad/s]

$\omega_{\text{bp}}$, $\omega_{\text{bpx}}$, $\omega_{\text{bpy}}$, $\omega_{\text{bpz}}$

Projection of the pitch angular velocity relative to the system base in the pitch coordinate in [rad/s]

$\omega_{\text{gyro}_x}$, $\omega_{\text{gyro}_y}$

Sensitive velocities of the azimuth and pitch gyroscopes in [rad/s]

$U_{m}$

Motor armature voltage in [v]

$I_{m}$

Armature current in [A]

$R_{m}$

Total resistor of the armature circuit in [Ω]

$L_{m}$

Total inductance of the armature circuit in [H]

$J_{ma}$

Rotary inertia of the azimuth motor in [kg · m²]

$J_{la}$

Rotary inertia of the azimuth load in [kg · m²]

$J_{mp}$

Rotary inertia of the pitch motor in [kg · m²]

$J_{lp}$

Rotary inertia of the pitch load in [kg · m²]

$D_{L}$

Damping coefficient of the two motors

$K_{L}$

Elastic coefficient of the two motors

$\theta_{m}$

Output angle of the motor side in [rad]

$\theta_{L}$

Output angle of the load side in [rad]

$E_{m}$

Back electro-dynamic force (EMF) of the armature circuit in [v]

$C_{e}$

EMF coefficient of the two motors

$C_{m}$

Motor torque coefficient of the two motors

$U_{c}$

Control voltage of the motor armature in [v]

$K_{p}$

Amplification coefficient of the driving circuit

$T_{p}$

Lagging time of the motor armature in [s]

$E_{s}$

EMF of the driving circuit in [v]

$R_{s}$

Resistor of the driving circuit in [Ω]

$\delta_{\text{baz}}$, $\delta_{\text{bax}}$, $\delta_{\text{bay}}$

Perpendicularity deviation vector of azimuth axis assembly in the system base coordinate, and its components around the x and y axes of the coordinate in [rad]

$\delta_{\text{apz}}$, $\delta_{\text{apx}}$, $\delta_{\text{apz}}$

Perpendicularity deviation vector of pitch axis assembly in the azimuth coordinate, and its components around the x and z-axes of the coordinate in [rad]

$R'_{a}$, $R'_{p}$

Rotation transformation matrixes affected by errors of axis perpendicularity of the azimuth and the pitch

$\omega_{\text{az}}$, $\omega_{\text{ip}}$

Angular velocity vectors of the azimuth and the pitch in the inertial coordinate affected by errors of axis perpendicularity in [rad/s]

$\Delta s$

Axial shifts

$\Delta \bar{R}_{o}$

Vector of radial run-outs

$\Delta \bar{Y}$

Vector of tilting oscillations

$\Delta \bar{Y}_a$, $\Delta \alpha_{am}$, $\beta_{am}$

Oscillation vector of the azimuth, and its decomposed components of the maximum tilting oscillations around the x and y axes of the system base coordinate in [rad]

$\Delta \bar{Y}_p$, $\Delta \alpha_{pm}$, $\beta_{pm}$

Oscillation vector of the pitch, and its decomposed components of the maximum tilting oscillations around the x and z-axes of the azimuth coordinate in [rad]

$\Delta \alpha_{a}(t)$, $\Delta \beta_{a}(t)$

Real-time value expressions of the tilting oscillations of the azimuth along the x and y axes of the system base coordinate in [rad]

$\Delta \alpha_{p}(t)$, $\Delta \beta_{p}(t)$

Real-time value expressions of the tilting oscillations of the pitch along the x and z-axes of the azimuth coordinate in [rad]

$\Delta R_{a}$, $\Delta R_{p}$

Rotation transformation matrixes affected by errors of run-outs of the azimuth and the pitch

$\Delta \omega_{\text{az}}$, $\Delta \omega_{\text{ip}}$

Angular velocity vectors of the azimuth and the pitch in the inertial coordinate affected by errors of run-outs in [rad/s]
\[ \vec{e}_{\theta}, \vec{e}_{\phi}, \vec{e}_{\psi} \] Vector of the azimuth gyroscope assembly deviation in the pitch coordinate, and its components around the x and y axes of the coordinate in [rad]

\[ \vec{e}_{\phi}, \vec{e}_{\psi}, \vec{e}_{\theta} \] Vector of the pitch gyroscope assembly deviation in the pitch coordinate, and its components around the x and z-axes of the coordinate in [rad]

\[ R_{\alpha \phi}, R_{\phi \psi}, R_{\psi \theta} \] Rotation transformation matrixes affected by assembly errors of the gyroscope locations

\[ \omega'_{gyro,x}, \omega'_{gyro,y} \] Angular velocity vectors of the gyroscopes affected by errors of assembly errors of the gyroscope locations in [rad/s]

\[ K_g \] Amplification coefficient of the gyroscopes

\[ \omega_0 \] Natural frequency of the gyroscopes in [rad/s]

\[ \xi_g \] Damping coefficient of the gyroscopes

\[ S(x) \] Error sensitivity; the standard deviations of the experiment samples under different error levels

\[ n_1 \] Number of error levels

\[ \Delta \theta_i(x) \] Accuracy sample value of the ith error level of x

\[ \overline{\Delta \theta}(x) \] Average value of accuracy sample values

UAV Unmanned aerial vehicle

FOS Field of sight

LOS Line of sight

BD Disturbances of the system base

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