The Emergence of the Planck Scale

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Abstract

In this paper we first observe some interesting parallels between Planck scale considerations and elementary particle Compton wavelength scale considerations, particularly in the context of Wheeler’s space time foam and a space time arising out of a stochastic random heap of elementary particles discussed in previous papers. These parallels lead to a semi qualitative picture which shows how the short lived Planck scale arises from the Compton wavelength considerations. Finally all this is quantified.

1 Introduction

About a century ago Max Planck had pointed out that the quantity \( \left( \frac{\hbar G}{c^4} \right)^{\frac{1}{2}} \sim 10^{-33}cms \) is a fundamental length. This so called Planck length ties up Quantum Mechanics, Gravitation and Special Relativity and leads to the Planck mass \( \sim 10^{-5}gms \). It is but natural that the Planck length has played a crucial role in Quantum Gravity as also in String Theory which includes a description of Gravitation, unlike Quantum Theory or Quantum Field Theory.

It turns out to be the scale at which we have no longer the smooth space time of Classical Theory and Quantum Theory, but rather we have the space time foam of Wheeler\(^1, 2\). This is inextricably linked with gravitational collapse.
which has been described by Wheeler as "The greatest crisis of Physics". As he puts it, "These are small scale fluctuations telling one that something like gravitational collapse is taking place everywhere in space and all the time; that gravitational collapse is in effect perpetually being done and undone .... at the Planck scale of distances." In this space time foam, worm holes and non local effects abound.

On the other hand there is also a stochastic fluctuational picture of space time that deals with phenomena at the Compton wavelength scale and leads to meaningful physics and cosmology including a unified description of gravitation and electromagnetism consistent with observation[3, 4, 5, 6, 7]. In this picture, space time has been considered to be a random heap of elementary particles. If we consider a typical elementary particle to be a pion with Compton wavelength \( l \), then the above picture leads to a dispersion length in the Gaussian distribution \( \sim \sqrt{Nl} \), \( N \sim 10^{80} \) being the number of elementary particles in the universe, this being the correct dimension of the universe itself.

We will now show a parallel between the Planck length considerations and the Compton wavelength considerations referred to above, which will then show us how the Planck length considerations emerge.

2 The Emergence of the Planck Scale

We first show the parallels between the Compton wavelength picture and the Planck length picture. We note that in the former scenario, particles are fluctuationally created at the Compton wavelength from a background pre space time Zero Point Field (ZPF) of the kind considered in stochastic electrodynamics[9, 10]. The energy content in terms of the magnetic field of such a particle is given by (Cf.ref.[3])

\[
\Delta B \sim \frac{(\hbar c)^{1/2}}{L^2}
\]  

(1)

where \( L \) is the dimension under consideration, which in this case is of the order of the particle’s Compton wavelength. We note that in (1) if \( \hbar c \) or equivalently \( 137e^2 \) is replaced by its gravitational counterpart, namely \( 137Gm^2 \).
then we get, as in the fluctuation of the metric[1],

$$\Delta g \sim \frac{L_p}{L}$$

where $L_p$ is the Planck length and $L$ as in (1) is of the order of the dimension under consideration.

The space time foam referred to above arises at the Planck scale because the right hand side in (2) becomes unity, indicating perpetual collapse and creation.

From this point of view, as Wheeler points out our space time is an approximation, an average swathe at the Planck scale of several probable spaces and topologies which form the super space (Cf.ref.[2]). There is an immediately parallel in terms of the Compton wavelength considerations also: As pointed out by Nottale, Abbot-wise, El Naschie, the author and others[11, 12, 13, 14] the Quantum behaviour below a critical length is fractal and as pointed out by the author[8], our space time is the thick brush stroke of thickness of the order of the Compton wavelength of a jagged, fractal coastline like underpinning.

In the light of the above considerations the fluctuational creation of particles considered by Hayakawa[15] and the author[5] have a parallel in the non local worm hole related appearance of particles and fields at the Planck scale[2].

We will now quantify the above parallels and show the actual emergence of the Planck scale particles from the Compton wavelength considerations.

We first observe that in an actual random heap of particles, the smaller particles (in our case those having smaller Compton wavelengths and therefore higher mass) tend to settle down together due to gravity. In a fluctuationally created random heap of particles, there is no gravity, but as this space time heap is not only non differentiable, but is also not required to be even a continuum the random motion would have a similar effect: Of the $N' = \sqrt{N}$ particles which are less dispersed, $\sqrt{N'}$ particles would similarly fluctuationally, that is non locally be together. This fluctuationally bound group would have a mass $\sqrt{N'm} \sim 10^{-5}gms$ or the Planck mass, since $m$ is the mass of the pion. (Cf.ref."Ramification" for another interesting perspective).

One way of looking at this is that in the above scenario, space time no longer has the rigid features of Classical and Quantum Physics - on the average it is a measure of dispersion of a random distribution of particles which themselves have a stochastic underpinning. So the length scale or dispersion would
be less, the less dispersed the random collection of particles is - this leads to the Planck scale from the Compton scale. However it must be borne in mind that a Planck mass has a life time $\sim 10^{-42}$ seconds, and can hardly be detected.

The Planck scale corresponds to the extreme classical limit of Quantum Mechanics, as can be immediately seen from the fact that the Planck mass $m_P \sim 10^{-5} gms$ corresponds to a Schwarzschild Black Hole of radius $L_P \sim 10^{-33} cms$, the Planck length. At this stage the spinorial Quantum Mechanical feature as brought out by the Kerr-Newman type Black Hole and the Compton wavelength (Cf.detailed discussion in refs.[3, 4]) disappears. In fact at the Planck scale we have

$$Gm_P c^2 = \hbar / m_P c$$

(3)

In (3), the left side gives the Schwarzschild radius while the right side gives the Quantum Mechanical Compton wavelength. Another way of writing (3) is,

$$Gm_P^2 / e^2 \approx 1,$$

(4)

Equation (4) expresses the well known fact that at this scale the entire energy is gravitational, rather than electromagnetic, in contrast to equation (1) for a typical elementary particle mass, vi.

$$Gm^2 \approx \frac{1}{\sqrt{N}} e^2 \sim 10^{-40} e^2$$

Interestingly from the background ZPF, Planck particles can be produced at the Planck scales given by (3), exactly as in the case of pions, as seen earlier. They have been considered to be what may be called a Zero Point Scale[17, 18, 19]. But these shortlived Planck particles can at best describe a space time foam.

We will now throw further light on the fact that at the Planck scale it is gravitation alone that manifests itself. Indeed Rosen[20] has pointed out that one could use a Schrodinger equation with a gravitational interaction to deduce a mini universe, namely the Planck particle. The Schrodinger equation for a self gravitating particle has also been considered[21], from a different point of view. We merely quote the main results.
The energy of such a particle is given by

\[ \frac{Gm^2}{L} \sim \frac{2m^5G^2}{\hbar^2} \quad (5) \]

where

\[ L = \frac{\hbar^2}{2m^3G} \quad (6) \]

(5) and (6) bring out the characteristic of the Planck particles and also the difference with elementary particles, as we will now see. We first observe that for a Planck mass, (5) gives, self consistently,

\[ \text{Energy} = m_Pc^2, \]

while (6) gives,

\[ L = 10^{-33} \text{cms}, \]

as required.

However, the situation for pions is different: They are parts of the universe and do not constitute a mini universe. Indeed, if, as above there are \( N \) pions in the universe, then the total gravitational energy is given by, from (5),

\[ \frac{NGm^2}{L} \]

where now \( L \) stands for the radius of the universe \( \sim 10^{28} \text{cm} \). As this equals \( mc^2 \), we get back as can easily be verified, the pion mass!

Indeed given the pion mass, one can verify from (6) that \( L = 10^{28} \text{cms} \) which is the radius of the universe, \( R \). Remembering that \( R \approx \frac{c}{H} \), (6) infact gives back the supposedly mysterious and adhoc Weinberg formula, relating the Hubble constant to the pion mass\[22\].

This provides a justification for taking a pion as a typical particle of the universe, and not a Planck particle, besides re-emphasizing the basic unified picture of gravitation and electromagnetism. It must be mentioned that just as the Planck particle constitutes a mini universe or Black Hole, so also the \( N \sim 10^{80} \) pion filled universe can itself considered to be a Black hole\[23\]!

To proceed, let us now use the fact that our minimum space time intervals are \( (l_P, \tau_P) \), the Planck scale, instead of \( (l, \tau) \) of the pion, as above.
With this new limit, it can be easily verified that the total mass in the volume \( \sim l^3 \) is given by
\[
\rho_P \times l^3 = M
\] (7)
where \( \rho_P \) is the Planck density and \( M \) is the mass of the universe.

Moreover the number of Planck masses in the above volume \( \sim l^3 \) can easily be seen to be \( \sim 10^{60} \). However, it must be remembered that in the physical time period \( \tau \), there are \( 10^{20} \) (that is \( \frac{\tau}{\tau_P} \)) Planck life times. In other words the number of Planck particles in the physical interval \((l, \tau)\) is \( N \sim 10^{80} \), the total particle number, as if all these were the seeds of the fixed number of \( N \) particles in the universe. This is symptomatic of the fact that instead of the elementary particle Compton wavelength scale of the physical universe we are using the Planck scale (cf. also considerations before equation (3)). That is from the typical physical interval \((l, \tau)\) we recover the entire mass and also the entire number of particles in the universe, as in the Big Bang theory. This also provides the explanation for the above puzzling relations like (7).

That is the Big Bang theory is a characterization of the new Compton wavelength model in the classical limit at Planck scales, but then, in this latter case we cannot deduce from theory the relations like the Dirac coincidences or the Weinberg formula.

In the spirit of [7], one can now see the semi-classical and Quantum Mechanical divide between Planck particles and elementary particles in the following way. We will see that Planck particles have a life time given by the Hawking Radiation Law of Black Hole Thermodynamics, whereas elementary particles are characterised by Quantum Mechanical life times.

It is well known that [24] the life time due to the Hawking Radiation Law is given by
\[
t = \frac{G^2 m^3}{\hbar c^4}
\] (8)
which for the Planck particles gives the usual Planck time.

However this formulation is not valid for elementary particles. In this case, we consider the gravitational energy \( \Delta E \) of a pion as given by an equation like (3) and use instead the Quantum Mechanical relation
\[
\Delta E \Delta t \sim \hbar
\] (9)
to get
\[
Gm_\pi^2 (\hbar/m_\pi c) \Delta t \sim \hbar
\] (10)
which is correct if in (9) $\Delta t \sim \frac{1}{T}$, the age of the universe! (cf. also ref. [24]).

In this case equation (10) gives the well known and supposedly mysterious and empirical formula of Weinberg referred to earlier, viz.,

$$m_\pi^3 \sim \frac{H^2 \hbar^2}{Gc}$$  \hspace{1cm} (11)

One way of looking at this is that it is the emergence of Quantum Mechanical effects and electromagnetism at the Compton wavelength scales from classical gravitational considerations at the Planck scale as seen above, which gives stability to the universe as expressed by (9) and (10).

All this has been justified from stochastic considerations [7].

Another way of looking at all this is the following: The gravitational constant $G$ is taken to be a universal constant in most conventional theories. However in the above formulation it turns out that,

$$G = \frac{G_0}{\sqrt{N}} \propto \frac{1}{T}$$  \hspace{1cm} (12)

where $N$ is the number of elementary particles in the universe and $T$ is the age of the universe. This time varying gravitational constant can be shown to lead to consistent results including an explanation for the all important precision of the perihelion of the Planet Mercury [8, 25]. The equation (12) also shows a Machian or holistic character. In any case for a single particle universe, $N = 1$ the $G$ above leads to the Planck length or Planck mass, while for $N \sim 10^{80}$ the same equation leads to the pion Compton wavelength and the usual Physics and Cosmology. In fact if the pion Compton time scales $(l, \tau)$ tends to zero or the Planck scale we recover the big bang scenario and the usual space time of Classical and Quantum Physics or the Prigogine Cosmology [26]. In these cases we cannot explain the large number “coincidences” and Weinberg’s mysterious formula (11), whereas at the elementary particle Compton scale these features can be deduced as consequences of the theory.

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