J/ψ and ηc in the Deconfined Plasma from Lattice QCD

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Analyzing correlation functions of charmonia at finite temperature (T) on 32³×(32–96) anisotropic lattices by the maximum entropy method (MEM), we find that J/ψ and ηc survive as distinct resonances in the plasma even up to T ≃ 1.6Tc and that they eventually dissociate between 1.6Tc and 1.9Tc (Tc is the critical temperature of deconfinement). This suggests that the deconfined plasma is non-perturbative enough to hold heavy-quark bound states. The importance of having sufficient number of temporal data points in MEM analyses is also emphasized.

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Whether hadrons survive even in the deconfined quarkgluon plasma is one of the key questions in quantum chromodynamics (QCD). This problem was first examined in [1] and [2] in different contexts. In the former, it was shown that collective q̄q excitations with a low mass and a narrow width in the πσ channels exist even above Tc (the critical temperature) from analyses of the spectral functions in the Nambu-Jona-Lasinio model. The fate of heavy mesons such as J/ψ in the deconfined plasma was also investigated in a phenomenological potential picture taking into account the Debye screening [3]. In general, there is no a priori reason to believe that the dissociation of bound states should take place exactly at the phase transition point Tc.

Experimentally, measurements of dileptons (diphotons) in heavy ion collisions may provide a clue to the properties of vector (pseudo-scalar) mesons in hot/dense matter. Indeed, data from CERN SPS indicate an anomaly in the dilepton spectra relevant to ρ and J/ψ. Also, RHIC is going to produce ample data of dileptons in a few years [4].

From the theoretical point of view, the spectral function (SPF) at finite temperature T, which has all the information of in-medium hadron properties, is a key quantity to be studied. Recently, the present authors have shown [5] that the first-principle lattice QCD simulation of SPFs is possible by utilizing the maximum entropy method (MEM). We have also formulated the basic concepts and applications of MEM on the lattice at T = 0 and T ≠ 0 in [6]. The purpose of this Letter is to report our latest results of SPFs of low lying charmonia (J/ψ and ηc) above Tc extracted from quenched QCD simulations on anisotropic lattices. We find that J/ψ and ηc survive even up to T ≃ 1.6Tc and disappear between 1.6Tc and 1.9Tc. This together with our previous results for s̅s mesons on the same lattice [3] suggests that the system is still strongly interacting above Tc.

To draw the above conclusion with a firm ground, we put special emphases on [1] the MEM error analysis of the resultant SPFs and [II] the sensitivity of the SPFs to Ndata (the number of the temporal data points adopted in MEM). These tests are crucial to prevent fake generation and/or smearing of the peaks and must be always carried out as emphasized in [6, 7].

Let us first summarize the basic formulation of MEM applied to lattice data at finite T. We consider the Euclidean correlation function of the local interpolating operator Jl(τ, x) = ϵlγμ with l = 1, 2, 3 for J/ψ and l = 5 for ηc. The spectral decomposition of the correlator in the imaginary time 0 < τ < 1/T reads

\[ D(τ) = \int \langle J_l(τ, x) J_l^\dagger(0) \rangle d^3x = \int_{0}^{∞} K(τ, ω) A(ω) dω, \tag{1} \]

where ω is a real frequency and A(ω) is the spectral function. The sum over l = 1, 2, 3 is taken for J/ψ. K(τ, ω) is the integral kernel, K(τ, ω) = (e^−τω + e^−(1/T−τ)ω)/(1 − e^−ωT). For simplicity, we take the three momentum of the correlation function to be zero.

Monte Carlo simulations provide D(τi) with statistical error on a discrete set of temporal points τi. Although there exist infinitely many A(ω) which give the same D(τi) through the integral Eq. (1), MEM provides a way to select a unique A(ω) without introducing parametrization of SPF and to give its statistical significance on the basis of the Bayes’ theorem. The most probable A(ω) given lattice data D is obtained by maximizing the conditional probability \( P[A|D] \propto e^αS−L \), where L is the standard likelihood function and S is the Shannon-Jaynes entropy:

\[ S = \int_{0}^{∞} \left[ A(ω) − m(ω) − A(ω) \log \left( \frac{A(ω)}{m(ω)} \right) \right] dω. \tag{2} \]

Here α is a parameter dictating the relative weight of S and L. The statistical significance (error) of the resultant A(ω) is estimated by the second variation, \( (δ/δA)^2P[A|D] \). The default model m in Eq. (2), the first estimate of which is obtained from the perturbative QCD calculation of A(ω ≫ 1 GeV), may be chosen so that the MEM errors become minimum. The final result A(ω) is given by the weighted average over
γ Wilson quark action. The corresponding bare anisotropy take the naive plaquette gauge action and the standard a

anisotropic lattice. The renormalized anisotropy is ξ it becomes more difficult to keep enough N_{\text{data}} to obtain reliable SPFs as T increases. In other words, simulations up to a few times T_c with N_{\text{data}} as large as 30 require fine lattices at least in the temporal direction (anisotropic lattices).

On the basis of the above observation, we have carried out quenched simulations with $\beta = 7.0$ on $32^4 \times N_T$ anisotropic lattice. The renormalized anisotropy is $\xi = a_\sigma/a_\tau = 4.0$ with $a_\sigma$ being the spatial lattice spacing. We take the naive plaquette gauge action and the standard Wilson quark action. The corresponding bare anisotropy $\xi_0 = 3.5$ is determined from the data given in [11]. The fermion anisotropy $\gamma_F \equiv \kappa_\tau/\kappa_\sigma$ with $\kappa_\tau$ ($\kappa_\sigma$) being the spatial (temporal) hopping parameter is determined by comparing the temporal and spatial effective masses of the pseudoscalar and vector mesons on a $32^2 \times 48 \times 128$ lattice. $a_\tau = a_\sigma/4 = 9.75 \times 10^{-3}$ fm is determined from the ρ meson mass in the chiral limit [21]. The physical lattice size in the spatial direction $L_\sigma = 1.25$ fm can well accommodate $J/\psi$ or $\eta_c$, whose root mean square radius is about 0.5 fm. $N_T$, the corresponding T/T_c, and the number of gauge configurations $N_{\text{gauge}}$ are summarized in TABLE I. Calculations close to T_c (T/T_c = 1.04 and 0.93) are also currently undertaken. Gauge configurations are generated by the pseudo heatbath and over-

relaxation algorithms with a ratio 1:4. Initially, the gauge field is thermalized with 10000 sweeps and, then each configuration is separated by 1000 sweeps.

TABLE I: Temporal lattice size, the corresponding temperature, and the number of gauge configurations in our simulation.

| $N_T$ | 32 | 40 | 46 | 54 | 96 |
|-------|----|----|----|----|----|
| T/T_c | 2.33 | 1.87 | 1.62 | 1.38 | 0.78 |
| $N_{\text{gauge}}$ | 141 | 181 | 182 | 150 | 194 |

We have measured the point-point $q\bar{q}$ correlation functions in the scalar (S), pseudo-scalar (PS), vector (V), and axial-vector (AV) channels for the spatial hopping parameters $\kappa_\sigma = 0.08285, 0.0850, 0.0853,$ and 0.08545. In this Letter, we focus our attention on the heaviest quark mass ($\kappa_\sigma = 0.08285$ with $\gamma_F = 3.476$), and study the V ($J/\psi$) and PS ($\eta_c$) channels. The masses determined on the $T = 0$ lattice ($32^2 \times 48 \times 128$) are $m_{J/\psi} = 3.10$ GeV and $m_{\eta_c} = 3.03$ GeV.

In all the figures below, we adopt the continuum kernel, $K(\tau, \omega)$, defined after Eq. (1). Use of the lattice kernel does not lead to any appreciable difference. This is because our lattice spacing is quite small, $a_\tau \sim 0.01$ fm. Following [7], we define dimensionless SPFs: $A(\omega) = \omega^2 \rho(\omega)$ for $\eta_c$ and $A(\omega) = 3\omega^2 \rho(\omega)$ for $J/\psi$. If the temporal distance between the source and the sink is closer than $\xi a_\tau$, lattice artifact due to anisotropy would appear in the SPFs for $\omega \geq \pi/a_\tau$. To avoid this, we exclude the six points near the edge ($\tau_c = 1, 2, 3, N_\tau - 3, N_\tau - 2, N_\tau - 1$) and adopt the points $\tau_c = 4, 5, \ldots, N_\tau - 4, N_\tau - 5, \ldots$ until we reach the total number of points $N_{\text{data}}(< N_\tau - 7)$.

The current operator on anisotropic lattice $J^{\text{LAT}}(x)$ and that in the continuum $J^{\text{CON}}(x)$ are related by $J^{\text{LAT}}_i(\omega) = \sqrt{a_\tau a_\sigma} J^{\text{CON}}_i/(2Z_1 \sqrt{\kappa_\tau \kappa_\sigma})$, where $Z_1$ is a nonperturbative renormalization constant. The default model $m(\omega)$ extracted from the perturbative evaluation of $A(\omega) \gg 1$ GeV also depends on $Z_1$. We vary $Z_1$ from 1 (the value in the weak coupling limit) and the known value at $\beta = 6.0$ on the isotropic lattice [11]. The central values of $m$ thus obtained are $m(\omega)/(3\omega^2) = 0.40$ and $m(\omega)/\omega^2 = 1.15$ for the V and PS channels, respectively. We use these values throughout the following figures. We have checked that our conclusions are not modified within the variation of $Z_1$ mentioned above.

Shown in Fig. I are $\rho(\omega)$s for $J/\psi$ at $T/T_c = 0.78, 1.38,$ and 1.62 (Fig. I(a)) and those at $T/T_c = 1.87$ and 2.33 (Fig. I(b)). The corresponding $N_{\text{data}}$ used in these figures are $N_{\text{data}} = 89, 40, 34, 33,$ and 25 from low T to high T). If the deconfined plasma were composed of almost free quarks and gluons, SPFs would show a smooth structure with no pronounced peaks above the $q\bar{q}$ threshold. To the contrary, we find a sharp peak near the zero temperature mass even up to $T \simeq 1.6 T_c$ as shown in Fig. I(a), while the peak disappears at $T \simeq 1.9 T_c$ as shown in Fig. I(b). The width of the first peak in Fig. I(a) partly reflects the unphysical broadening due to the statistics of the lattice data and partly reflects possible physical broadening at finite T. At the moment, the former width of a few hundred MeV seems to dominate and we are not able to draw definite conclusions on the thermal mass shift and broadening.

The second and third peaks in Fig. I(a) may be related to the fermion doublers as first pointed out for light mesons at $T = 0$. This must be checked by studying whether the peak position scales as $1/a_\tau$ by varying $a_\tau$, which remains to be a future problem. Our error analysis shows that the peak around $\omega = 0$ in Fig. I(b) is not significant in the present statistics. The qualitative
change of the spectral structure between $T/T_c = 1.62$ and 1.87, and other features seen in the $J/\psi$ channel are also observed in the $\eta_c$ channel as shown in Fig.2.

Let us now evaluate the reliability of the existence (absence) of the sharp peak at $T/T_c = 1.62(1.87)$ by the two tests [I] and [II] mentioned before. The first test is the error analysis of the peak. Shown in Fig 3 are the SPFs for $J/\psi$ at $T/T_c = 1.62$ and 1.87 with MEM error bars. The frequency interval over which the SPF is averaged is characterized by the horizontal position and extension of the bars, while the mean value and the 1$\sigma$ uncertainty of the integrated strength within the interval are characterized by the heights of the bars). The sharp peak at $T = 1.62T_c$ is statistically significant, and the absence of the peak at the same position at $T = 1.87T_c$ is also statistically significant. The same features are also observed for $\eta_c$.

The second test is the $N_{\text{data}}$ dependence of the SPFs. Shown in Fig 4(a) is a comparison of the SPF obtained with $N_{\text{data}} = 34$ and that with $N_{\text{data}} = 39$ for the same temperature $T = 1.62T_c$ ($N_{\tau} = 46$). The two curves are almost identical with each other (Note that the maximum number of temporal data available in this case is 46 − 7 = 39). Fig 4(b) shows SPFs obtained with $N_{\text{data}} = 26$ and 33 for a higher temperature $T = 1.87T_c$ ($N_{\tau} = 40$). The maximum number of data points available is 40 − 7 = 33 in this case. Again the two curves are almost identical. Therefore, the qualitative change of the SPF between $T = 1.62T_c$ and $1.87T_c$ is the real thermal effect and not caused by the artifact of the insufficient number of data points.

Here we make a brief comment on related works aiming at studying the charmonia above $T_c$ using MEM. In contrast to our large temporal grids ($N_{\tau} = 46$ for $T = 1.62T_c$), $N_{\tau}$ in these papers are 3-4 times smaller ($N_{\tau} = 17$ in [17] and $N_{\tau} = 12$ in [16] at $T = 1.62T_c$). For such small $N_{\tau}$ with point-point correlation, the reliability tests [I] and [II] need to be done before drawing physics conclusions. There is indeed evidence that $N_{\tau} = 17$ fails the test [II] as shown in [15].

In summary, we have extracted the spectral functions of $J/\psi$ and $\eta_c$ in the deconfined plasma using lattice Monte Carlo data and the maximum entropy method. In the quenched approximation, the number of temporal sites $N_c$ is taken as large as 46 and 40 for $T/T_c = 1.62$ and 1.87, respectively. Careful analyses of the MEM errors and the $N_{\text{data}}$ dependence of the results are carried out. It is found that there are distinct resonances up to $T \approx 1.6T_c$ and they disappear between 1.6$T_c$ and 1.9$T_c$. This together with our previous results on $s\bar{s}$ mesons on the same lattice indicates that the quark-gluon plasma is still strongly interacting above $T_c$ so that it can develop hadronic resonances. Whether these resonances found in quenched simulation survive in full simulation with dynamical quarks is an interesting future problem. It is also important to unravel the nature of such resonances by studying their spatial structure on the lattice or by using non-relativistic potential approaches.
FIG. 3: Spectral functions for $J/\psi$ with MEM errors (a) for $T/T_c=1.62$ (b) $T/T_c=1.87$.

FIG. 4: Comparison of the SPF for $J/\psi$ (a) for $N_{\text{data}}=34$ and 39 with $N_T=46$ ($T/T_c=1.62$) (b) for $N_{\text{data}}=26$ and 33 with $N_T=40$ ($T/T_c=1.87$).

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[1] T. Hatsuda and T. Kunihiro, Phys. Rev. Lett. 55 (1985) 158.
[2] C. E. DeTar, Phys. Rev. D 32 (1985) 276.
[3] T. Matsui and H. Satz, Phys. Lett. B178 (1986) 416.
[4] An example is the superconductivity in the strong coupling. See, P. Nozières and S. Schmitt-Rink, J. Low Temp. Phys. 59 (1985) 195; E. Babaev, Int. J. Mod. Phys. A16 (2001) 1175.
[5] See e.g. the Proceedings of QM2002, Nucl. Phys. A715 (2003).
[6] Y. Nakahara, M. Asakawa, and T. Hatsuda, Phys. Rev. D 60 (1999) 091503.
[7] M. Asakawa, T. Hatsuda, and Y. Nakahara, Prog. Part. Nucl. Phys. 46 (2001) 459. See also, M. Jarrell and J. E. Gubernatis, Phys. Rep. 269 (1996) 133.
[8] M. Asakawa, T. Hatsuda, and Y. Nakahara, Nucl. Phys.