Quark-meson coupling model for finite nuclei

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Abstract

A Quark-Meson Coupling (QMC) model is extended to finite nuclei in the relativistic mean-field or Hartree approximation. The ultra-relativistic quarks are assumed to be bound in non-overlapping nucleon bags, and the interaction between nucleons arises from a coupling of vector and scalar meson fields to the quarks. We develop a perturbative scheme for treating the spatial nonuniformity of the meson fields over the volume of the nucleon as well as the nucleus. Results of calculations for spherical nuclei are given, based on a fit to the equilibrium properties of nuclear matter. Several possible extensions of the model are also considered.

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I. INTRODUCTION

Quantum Chromodynamics (QCD), a theory of quarks and gluons, is the current paradigm for the strong interaction. Nuclei are bound by the strong interaction, but hadronic degrees of freedom account for the most of the physical properties. This contrast suggests both questions and opportunities. One might ask how and why this standard picture of nuclear physics as a system of clustered color-singlet objects emerges from the fundamental theory. The opportunities arise from the challenge of discovering small (at normal nuclear densities) but interesting corrections to the standard picture which arise from the underlying constituents. One example was the discovery of the EMC effect [1], in which the structure function of a nucleon was shown to be modified by the nuclear medium.

Thus the need for a theory of nuclei that incorporates quark-gluon degrees of freedom, but also respects the vast body of information substantiating the standard picture, is apparent. There are many possibilities, and here we wish to focus on the Quark-Meson Coupling (QMC) model.

In a recent series of papers Saito and Thomas [2] have considered a model for nuclear matter involving color-singlet clusters of quarks and also mesons. This model is a variation on one originally proposed by Guichon [3], with corrections by Fleck et al. [4]. The quarks are assumed to be bound in non-overlapping nucleon bags, and the interaction between nucleons arises from a coupling of meson fields to the quarks. The ultra-relativistic quarks are described by a mean-field Dirac equation together with the MIT bag model boundary conditions [5]. The nucleons are assumed to be also described by the Dirac equation in the effective mean fields arising from meson fields coupling to the quarks in the nucleon. The model is conceptually similar to the QHD model of Walecka and collaborators [6], in which the meson fields couple to point-like nucleons. Indeed, in the limit that the quark mass becomes very large the present model reduces to the Walecka model. But the presence of the quark degrees of freedom provides important physical content absent from QHD.

The purpose of the present paper is to outline the formalism for applying this model to
finite nuclei. Justifications for the adiabatic approximation we use, in which the quarks do not respond to the motion of the nucleon within the nucleus, have already been obtained by Guichon et al. [7]. These authors also reported preliminary results for $^{16}$O within a local density approximation to the nuclear matter results. Here we provide a more detailed formalism which avoids the need for some of the approximations they use. We also discuss extensions of the model to account for other effects that may depend on the nuclear density [8], and the use of a relativistic oscillator description of the nucleon [9] as an alternative to the bag model.

An outline of the paper is as follows. The next section is concerned with the formalism for both finite nuclei and nuclear matter. Results are then given for spherical nuclei, based on a fit of the model parameters to the properties of nuclear matter. Extensions of the model are then considered, followed by a summary and outlook in the conclusions.

II. THEORY OF FINITE NUCLEI

A. Nuclear Energy and General Field Equations

The nucleus is treated as a set of color singlet objects carrying nucleonic quantum numbers. These objects (which we shall denote as bound nucleons) are not the same as free nucleons because the forces that bind the nucleus also influence the baryonic wave function.

In the present model the forces between the bound nucleons are generated by the exchange of vector ($\omega$) and scalar ($\sigma$) mesons coupling to quarks. The static Hartree approximation is used to describe the meson mean-fields and the bound nucleon single particle states.

As in the QHD model [6], we will assume the nucleons obey the Dirac equation

$$\left[ -i\gamma \cdot \nabla + M^*(\mathbf{X}) + \gamma^0 V_{\nu}(\mathbf{X}) \right] \psi_i(\mathbf{X}) = E_i \gamma^0 \psi_i(\mathbf{X}),$$

with nucleon states labeled by quantum numbers $i$. The quantity $M^*(\mathbf{X})$ is the mass of the bound nucleon, which differs from the mass of the free nucleon due to the influence of the
scalar potential generated by the sigma field on the quarks inside the nucleon. As will be shown, \( M^*(X) \) is a nonlinear function of the sigma field in the QMC models. \( V_v(X) \) is the time component of the vector potential, and depends linearly on the time component of the omega field. The vector component of the omega field averages to zero in the nuclear rest frame, so we drop it here.

The total nuclear energy \( E \) is obtained as a sum of the nucleon and meson field energies:

\[
E = \sum_{i=\text{occ}} E_i + E_v + E_s. \tag{2}
\]

The first term is a sum over the occupied positive energy nucleon states. The contribution of the vector mesons to the total energy is given by

\[
E_v = -\frac{1}{2} \int d^3r \omega(r) \left( m_v^2 - \nabla^2 \right) \omega(r), \tag{3}
\]

and the sigma meson contribution is

\[
E_s = \frac{1}{2} \int d^3r \sigma(r) \left( m_s^2 - \nabla^2 \right) \sigma(r). \tag{4}
\]

The mass of the vector meson is taken here to be the same as the omega (\( m_v=783 \text{ MeV} \)), whereas the mass of the scalar meson is a parameter of the model, typically in the range \( m_s \sim 400 - 550 \text{ MeV} \).

The fields \( \omega(r) \) and \( \sigma(r) \) provide vector

\[
V^q_v(r) = g^q_v \omega(r), \tag{5}
\]

and scalar quark potentials

\[
V^q_s(r) = g^q_s \sigma(r), \tag{6}
\]

that act on confined quarks located at \( r \). Here \( g^q_v \) and \( g^q_s \) are the relevant quark-meson coupling constants. The quark potentials determine the quark wave functions and densities, which in turn are needed to obtain the functions \( \omega(r) \) and \( \sigma(r) \). Thus there is an additional self-consistency requirement in the present model — the quark and nuclear wave functions must be determined together.
The meson field equations are derived by minimizing $\mathcal{E}$ with respect to variations in the meson fields $\omega(r)$ and $\sigma(r)$, yielding

\[
\left(-\nabla^2 + m_v^2\right) \omega(r) = \int d^3X \, \rho_v(X) \frac{\delta V_v(X)}{\delta \omega(r)}, \quad (7)
\]
\[
\left(-\nabla^2 + m_s^2\right) \sigma(r) = -\int d^3X \, \rho_s(X) \frac{\delta M^s(X)}{\delta \sigma(r)}. \quad (8)
\]

Here $\rho_{v,s}(X)$ represents the vector ($v$) and scalar ($s$) nucleon densities at a position $X$, with

\[
\rho_v(X) = \sum_{i=\text{occ}} \psi_i^\dagger(X) \psi_i(X), \quad (9)
\]
\[
\rho_s(X) = \sum_{i=\text{occ}} \bar{\psi}_i(X) \psi_i(X). \quad (10)
\]

The relation between the nuclear potentials and the meson fields will be derived in the next sub-section. In anticipation of these results, we set

\[
\frac{\delta V_v(X)}{\delta \omega(r)} = 3g_v^q \rho_v^q(r - X; X), \quad (11)
\]

and, for the simplest of the QMC models,

\[
\frac{\delta M^s(X)}{\delta \sigma(r)} = -3g_s^q \rho_s^q(r - X; X). \quad (12)
\]

The quantities $\rho_v^q(r - X; X)$ and $\rho_s^q(r - X; X)$ represent the nucleonic expectation values of the vector and scalar quark field operators for a nucleon with center-of-mass located at $X$. The variable $r - X$ is the displacement between a quark and the center of its nucleon. For simplicity, we have assumed that the three quarks inside the nucleon have identical masses, and therefore identical density distributions. The calculation of these quark densities is described in the next sub-section.

With these results, the meson field equations become:

\[
\left(-\nabla^2 + m_v^2\right) \omega(r) = 3g_v^q \int d^3X \, \rho_v(X) \rho_v^q(r - X; X), \quad (13)
\]
\[
\left(-\nabla^2 + m_s^2\right) \sigma(r) = 3g_s^q \int d^3X \, \rho_s(X) \rho_s^q(r - X; X). \quad (14)
\]

Thus the source term for $\omega(r)$ in Eq. (13) is a convolution of the vector density of the nucleon with the vector density of the quarks in the nucleon. Similarly, the source term for
\( \sigma(r) \) in Eq. (14) is a convolution of the scalar density of the nucleon with the scalar density of the quarks in the nucleon. This distinction, which was not present in the original work of Guichon [3], was first pointed out by Fleck et al. [4], who derived it from an analysis of a boost of the composite system. In Ref. [7] the convolution of densities has been replaced by an evaluation of the nuclear densities at the point \( r \). This is equivalent to a local density approximation to the nuclear matter result, where the nucleon densities are constant over the volume of the nucleon.

Strictly speaking, Eq. (13) is incomplete. The \( \omega \)-meson couples to a quark baryon current of the simple form \( j^\mu_q = \bar{\psi}_q \gamma^\mu \psi_q \). However, the nucleonic matrix element of \( j^\mu_q \), which is the effective baryon current of the composite nucleon (\( \psi \)), will have both a \( \bar{\psi} \gamma^\mu \psi \) term and an “anomalous” term \( \partial_\nu \bar{\psi} \sigma^{\mu\nu} \psi \). This is the well-known “tensor” coupling for vector mesons and photons. It is proportional to the anomalous magnetic moment of the nucleon, and emerges naturally in the present model. In the mean-field approximation there is a non-vanishing contribution from this term for the time-like part of the current (\( \mu = 0 \)). We give a more detailed discussion of this point in Appendix A. Because the tensor coupling of the \( \omega \)-meson is known to be small, we choose to ignore its influence in the computations performed here.

The essential feature of the present sub-section is that the self-consistent solution of Eqs. (1), (13) and (14) give the energy and wave function of the nucleus. These equations are similar to the ones of QHD [6], but with the essential difference being the dependence on the quark vector and scalar densities.

**B. Quark Wave Functions and Densities**

Computing the quark wave functions and the resulting nucleonic vector potential \( V_v(X) \) and effective mass \( M^*(X) \) starts with using a fairly general representation of the field equation of a confined quark,

\[
\left[ -i\gamma^0 \frac{\partial}{\partial t} - i\gamma \cdot \nabla_r + m_q + V_{con}(r - X) \right] \psi^f_q(t, r - X) = 0. \tag{15}
\]
Here $X$ is the position of the center of the free nucleon, $m_q$ is the bare quark mass, and the confining potential is defined as $V_{con}(r - X)$. This potential depends on the distance between the quark and the center of its nucleon. This central confining potential can be understood to represent the MIT bag, when complemented by the bag boundary conditions. In relativistic potential models one can take $V_{con}(r) = C r^n$, as did Tegen et al. [9], who consider $n = 2$ and $n = 3$. $V_{con}$ can also be obtained as a Hartree approximation to a more realistic treatment of confinement based on two or three quark interactions. The superscript $f$ is meant to denote that the nucleon is free.

Next we consider how Eq. (15) is modified when the nucleon is bound. Each quark feels a vector $V^q_v(r)$ and scalar $V^q_s(r)$ potential. If the nucleon is centered at a position $X$, then one expects that the quark wave functions will depend on both $r$ and $X$. Thus we obtain

$$\left[ -i\gamma^0 \frac{\partial}{\partial t} - i\gamma \cdot \nabla_r + m_q - V^q_s(r) + V_{con}(r - X) + \gamma^0 V^q_v(r) \right] \psi_q(t, r - X, r) = 0. \quad (16)$$

This dependence can be fairly complicated. The $\omega$ and $\sigma$ fields and resulting quark potentials are functions of $r$, so that such fields are not functions of $r - X$, and are not central in the frame of the nucleon. Hence the ground state of a bound nucleon will not be spherically symmetric in general.

To handle the difficulties that this entails, we develop a new approximation scheme based on the notion that

$$V^q_v(s) \psi_q(t, r - X, r) \approx U_{v,s}(X) \phi_q(t, r - X; X), \quad (17)$$

where $U_{v,s}(X)$ is some suitable averaging of $V^q_v(s)$ over the volume of the nucleon. The notation $\phi_q(t, r - X; X)$ is meant to denote that the quark wave function depends on the coordinate variable $r - X$, and is only an implicit function of $X$ through $V^q_v(s)$. This notion is plausible because $\psi_q(t, r - X, r)$ vanishes if $r - X$ is much bigger than the mean radius of a nucleon, and $V^q_s(r)$ and $V^q_v(r)$ are not expected to vary much over this region. The variation of the nuclear fields is governed by two scales — the nuclear radius, and the nuclear skin thickness (distance at the surface over which the density decreases from 90% to
10% of its maximum value) of about 2.5 fm. Either distance is much larger than the radius of the nucleon, so that replacing $V_q^s$ and $V_q^v$ by some suitable average over the volume of the nucleon seems reasonable, at least at the start.

The procedure (but not the theory) of Ref. [7] is to evaluate the external fields at the nucleon center and neglect the variation of these fields within the volume of a single nucleon. This means that their calculations replace the approximate sign in Eq. (17) by an equality with $U_{v,s}(X) = V_{v,s}^q(X)$.

Here we intend to develop a more complete formalism, based on the plausibility of Eq. (17). We define an “average” potential $U_{v,s}(X)$ and a residual interaction $\Delta U_{v,s}(r, X)$ such that

$$V_{v,s}^q(r) = U_{v,s}(X) + \Delta U_{v,s}(r, X). \quad (18)$$

This equation is clearly a tautology. However, we may choose the potentials $U_{v,s}(X)$ so as to make the first-order perturbation theory evaluation of $\Delta U_{v,s}(r, X)$ vanish. This suggests that we can develop a convergent perturbative treatment of $\Delta U_{v,s}$.

The feature that $U_{v,s}$ depends only on $X$ allows us to simplify the solution of the Dirac equation (16). We take

$$\psi_q(t, r - X, r) = \sum_n c_n e^{-i[\epsilon_n^s(X) + U_{v}(X)]t} \phi_n(r - X; X), \quad (19)$$

where the quark energies $\epsilon_n^s(X)$ and wave functions $\phi_n(r - X; X)$ of quarks with quantum numbers $n$ are determined by solving the equation

$$[-i\gamma \cdot \nabla_r + m_q - U_s(X) + V_{con}(r - X)] \phi_n(r - X; X) = \epsilon_n^s(X) \gamma^0 \phi_n(r - X; X). \quad (20)$$

The quantity $c_n$ is to be determined by perturbative calculations with the residual interaction $\Delta U_{v,s}(r, X)$.

Comparing Eq. (20) with the equation for the free nucleon (15), it is clear that the quark wave function $\phi$ in the medium is of the same form as the free quark wave function $\phi_f$, but with the bare quark mass $m_q$ replaced by an effective mass.
\[ m_q^*(X) \equiv m_q - U_s(X). \] (21)

Thus the quark energies and densities (or wave functions) are implicitly functions of \( X \) only via \( m_q^*(X) \). This is properly expressed by the notation \( \epsilon^*(m_q^*(X)) \) and \( \rho_{v,s}^q(r - X; m_q^*(X)) \).

We will continue to use the simpler notation \( \epsilon^*(X) \) and \( \rho_{v,s}^q(r - X; X) \) with this understanding. We emphasize that the vector interactions have no effect on the nucleon properties in the medium other than an overall phase in the wave function, which results in a shift in the nucleon energies.

As a first step let us assume that the nucleon consists of three quarks each in a state with \( \kappa = -1 \). This state is treated in perturbation theory, so that state is simply \( \phi_0 \), which we shall simply denote as \( \phi \), i.e. \( \phi \equiv \phi_0 \). Similarly \( \epsilon \equiv \epsilon_0 \). (In this case \( c_n = c_n^{(0)} = \delta_{n,0} \).) These ground state quark wave functions then determine the quark vector and scalar densities

\[ \rho_v^q(\tilde{r}; X) = \phi^\dagger(\tilde{r}; X)\phi(\tilde{r}; X), \] (22)
\[ \rho_s^q(\tilde{r}; X) = \bar{\phi}(\tilde{r}; X)\phi(\tilde{r}; X), \] (23)

subject to the normalization condition

\[ \int d^3\tilde{r} \rho_v^q(\tilde{r}; X) = 1. \] (24)

We also introduce for convenience the definition

\[ \int d^3\tilde{r} \rho_s^q(\tilde{r}; X) \equiv S(X). \] (25)

\( U_{v,s}(X) \) is chosen according to the criterion that the first-order perturbation theory evaluation of \( \Delta U_{v,s}(r, X) \) vanishes. This is achieved with the definitions

\[ U_v(X) \equiv \int d^3r \rho_v^q(r - X; X)V_v^q(r), \] (26)
\[ U_s(X) \equiv \int d^3r \rho_s^q(r - X; X)V_s^q(r) / S(X). \] (27)

Explicitly then, the first-order corrections to the potential \( U_v(X) \) and the quark energy \( \epsilon^*(X) \) are
\begin{equation}
U^{(1)}_v(X) \equiv \int d^3r \, \phi^3(r-X;X) \Delta U_v(r,X) \, \phi(r-X;X) = 0,
\end{equation}
\begin{equation}
\epsilon^{(1)}(X) \equiv \int d^3r \, \bar{\phi}(r-X;X) \Delta U_s(r,X) \, \phi(r-X;X) = 0.
\end{equation}

The quantity \(\Delta U_{v,s}\) does cause a first-order change in the quark wave function and a second order change in the potentials. This is discussed more fully in Appendix B.

The net result is that Eqs. (26) and (27) determine the mean fields that act on the quarks. Our mean fields are computed by taking the average over the relevant quark densities, whereas in Ref. [7] these fields are evaluated at the center of the nucleon. This difference has a modest effect on the properties of finite nuclei, as discussed in the results described in the next section.

We may now obtain the nuclear vector and scalar potentials to be used in Eq. (1). The term \(U_v\) is present for each quark. Hence the nuclear vector potential is three times the shift in the quark energy caused by \(U_v(X)\), i.e.
\begin{equation}
V_v(X) = 3 \, U_v(X).
\end{equation}

The form of the effective mass \(M^*(X)\) depends on whether one uses the relativistic potential models or the MIT bag model. In the relativistic potential models we expect the mass of the nucleon to be just the sum of the energies of the three quarks. Hence, in the absence of any correction for center-of-mass effects,
\begin{equation}
M^*(X) = 3 \, \epsilon^*(m^*_q(X)),
\end{equation}
where \(\epsilon^*(m^*_q)\) is the energy eigenvalue of Eq. (20). It is convenient to introduce a nuclear scalar potential \(V_s(X)\) to be used in Eq. (1), with
\begin{equation}
V_s(X) \equiv M - M^*(X) = 3 \left( \epsilon(m_q) - \epsilon^*(m^*_q) \right).
\end{equation}
This facilitates a comparison of QMC models with QHD [6], and also allows us to use the experimental proton and neutron masses in Eq. (1) with the same scalar potential.

In the MIT bag model with quarks of effective mass \(m^*_q\), the ground state solution to Eq. (20) is [2]
\( \phi(r) = \mathcal{N}_q \left( \begin{pmatrix} j_0(x_qr/R) \\ \mathbf{\sigma} \cdot \hat{r} \beta_q j_1(x_qr/R) \end{pmatrix} \right) \frac{\chi_q}{(4\pi)^{1/2}}, \) \hspace{1cm} (33a)

where

\( e^* = \Omega^*_q/R, \) \hspace{1cm} (33b)

\( \Omega^*_q = \left[ x^2_q + (Rm^*_q)^2 \right]^{1/2}, \) \hspace{1cm} (33c)

\( \mathcal{N}_{q}^{-2} = 2R^3 j^2_0(x_q) \left[ \Omega^*_q(\Omega^*_q - 1) + Rm^*_q/2 \right] / x^2_q, \) \hspace{1cm} (33d)

\( \beta_q = \left[ (\Omega^*_q - Rm^*_q)/(\Omega^*_q + Rm^*_q) \right]^{1/2}, \) \hspace{1cm} (33e)

and \( \chi_q \) is the quark spinor. The eigenvalue \( x_q \) is determined by satisfying the linear boundary condition \( j_0(x_q) = \beta_q j_1(x_q) \) at the bag surface.

We take the nucleon mass to be

\[ M^*(X) = \frac{3\Omega^*_q - z}{R} + \frac{4}{3}\pi R^3 B. \] \hspace{1cm} (34)

Here \( B \) is the bag constant and \( z \) is a free parameter that is supposed to account for zero-point motion, vacuum corrections, etc. Our definition is different than Fleck et al. [4] and the initial work of Saito and Thomas [2], who include a correction for spurious center-of-mass motion. The parameters \( B \) and \( z \) are determined by the free nucleon mass \( M \) for a given bag radius \( R_0 \), and are given in Table I. A possible medium-dependence of \( B \) and \( z \) is considered in section IV.

Following the Born-Oppenheimer approximation, the bag is assumed to respond instantaneously to changes in the nuclear environment. Hence the equilibrium condition,

\[ \frac{\partial M^*}{\partial R} \bigg|_{R=R^*} = 0, \] \hspace{1cm} (35)

also applies in the medium [2]. In practice, for the models considered in this paper, \( R^* \) at nuclear matter densities is not very different from the radius \( R_0 \) of a free nucleon bag. Thus the dominant contribution to \( V_s(X) \) is given by the single particle energies, consistent with Eq. (32).
The desired results of this section are the vector and scalar potentials that act on a nucleon, contained in Eqs. (30) and (32). These in turn depend on the mean vector and scalar quark potentials, given in Eqs. (26) and (27). The next step is to obtain more specific forms for these potentials for use in our calculations. This is done in the next section.

C. Specific Evaluation of Nucleonic Potentials

We wish to obtain the external vector and scalar mean-fields which act on quarks in a nucleon centered at \( X \). We shall first consider the vector potential \( V_v(r) \). The formal solution to Eq. (13) is

\[
V_v^q(r) = 3(g_v^q)^2 \int d^3X' \rho_v(X') \int d^3r' \frac{e^{-m_v|\mathbf{r} - \mathbf{r}'|}}{4\pi|\mathbf{r} - \mathbf{r}'|} \rho_v(q)(\mathbf{r}' - X'; X').
\]  

(36)  

This is easily evaluated in momentum space. Using the relation

\[
\frac{e^{-m_v|\mathbf{r} - \mathbf{r}'|}}{4\pi|\mathbf{r} - \mathbf{r}'|} = \int \frac{d^3q}{(2\pi)^3} \frac{e^{i\mathbf{q} \cdot \mathbf{r}}}{q^2 + m_v^2},
\]  

(37)  

and changing variables to \( \tilde{\mathbf{r}} = \mathbf{r}' - X' \), we can rewrite Eq. (36) as

\[
V_v^q(r) = 3(g_v^q)^2 \int \frac{d^3q}{(2\pi)^3} \frac{e^{i\mathbf{q} \cdot \mathbf{r}}}{q^2 + m_v^2} \int d^3X' e^{-i\mathbf{q} \cdot \mathbf{X}'} \rho_v(X') \int d^3\tilde{\mathbf{r}} e^{-i\mathbf{q} \cdot \tilde{\mathbf{r}}} \rho_v^q(\tilde{\mathbf{r}}; X').
\]  

(38)  

The last integral above suggests that one may define a vector form factor for bound nucleons:

\[
v(q; X) \equiv \int d^3\tilde{\mathbf{r}} e^{-i\mathbf{q} \cdot \tilde{\mathbf{r}}} \rho_v^q(\tilde{\mathbf{r}}; X).
\]  

(39)  

Note that the normalization condition (24) ensures that \( v(0; X) = 1 \). The use of the definition (39) leads to the result

\[
V_v^q(r) = 3(g_v^q)^2 \int \frac{d^3q}{(2\pi)^3} \frac{e^{i\mathbf{q} \cdot \mathbf{r}}}{q^2 + m_v^2} \int d^3X' e^{-i\mathbf{q} \cdot \mathbf{X}'} \rho_v(X')v(q; X').
\]  

(40)  

The external vector potential felt by a nucleon centered at \( X \) is given according to Eqs. (26) and (30) by the convolution

\[
V_v(X) = 3 \int d^3r \rho_v^q(r - X; X)V_v^q(r)
\]  

\[
= (3g_v^q)^2 \int \frac{d^3q}{(2\pi)^3} \frac{e^{i\mathbf{q} \cdot \mathbf{X}}}{q^2 + m_v^2} v(q; X) \int d^3X' e^{-i\mathbf{q} \cdot \mathbf{X}'} \rho_v(X')v(q; X').
\]  

(41)
This quantity is easily recognized as the mean-field vector potential for nucleons interacting via meson exchange with (position-dependent) form factors. For the vector meson, the form factors are all that distinguishes the present model from the QHD model [6,10].

The quark scalar potential $V^q_s(r)$ follows analogously from Eq. (40). We find

$$V^q_s(r) = 3(g^q_s)^2 \int \frac{d^3q}{(2\pi)^3} \frac{e^{iq \cdot X}}{q^2 + m^2_s} \int d^3X' e^{-iq \cdot X'} \rho_s(X')s(q; X'),$$

where the scalar form factor $s(q; X)$ for bound nucleons is defined by the equation

$$s(q; X) \equiv \int d^3\tilde{r} e^{-iq \cdot \tilde{r}} \rho^q_s(\tilde{r}; X).$$

Unlike the quark vector form factor, there is no normalization constraint on $s(0; X)$, which is just the quantity $S(X)$ of Eq. (25).

In analogy to Eq. (41), the mean quark scalar potential of Eq. (27) is given by the convolution

$$U_s(X) = \int d^3r \rho^q_s(r - X; X)V^q_s(r) / S(X)$$

$$= \frac{3(g^q_s)^2}{S(X)} \int \frac{d^3q}{(2\pi)^3} \frac{e^{iq \cdot X}}{q^2 + m^2_s} s(q; X) \int d^3X' e^{-iq \cdot X'} \rho_s(X')s(q; X').$$

This is the quantity used in the Dirac equation (20) that determines the quark energies $\epsilon^*$, which in turn determines the external nuclear scalar potential $V_s(X)$.

The total energy $E$ of Eq. (2) can now be given in terms of the quantities we have just determined. Using Eqs. (3), (13), (5), (40) and (41), we find for the vector meson contribution to the total energy

$$E_v = -\frac{1}{2} \int d^3X \rho_v(X)V_v(X).$$

This is the same as in QHD, and follows from the linear dependence of $V_v(X)$ on the omega field $\omega(r)$. The effect is to remove half the vector potential energy contribution contained in the nucleon energies $E_i$ (see Eq. (1)).

For the scalar meson contribution, we use Eqs. (4), (14), (6), (42) and (44) to find

$$E_s = \frac{1}{2} \int d^3X \rho_s(X) 3U_s(X)S(X).$$
Because \( V_s(X) \neq 3U_s(X)S(X) \), this contribution does not remove half the scalar potential energy contribution contained in the nucleon energies. However, this relation is approximately correct, and becomes exact in the limit of large quark mass \( m_q \). In this limit, \( S(X) \rightarrow 1 \), and \( V_s(X) \) becomes a linear function of \( \sigma(r) \). One therefore expects to recover the results of the QHD model.

Finally, we limit our considerations to spherical nuclei, so that both the quark and nucleon densities are spherically symmetric. The equations we need are then:

\begin{align*}
V_v(X) & = g_v^2 \frac{2}{\pi} \int_0^\infty dq \frac{q^2}{q^2 + m_v^2} j_0(qX)v(q;X) \int_0^\infty dX' X'^2 j_0(qX')\rho_v(X')v(q;X'), \quad (47) \\
U_s(X) & = \frac{g_s^2}{3S(X)} \frac{2}{\pi} \int_0^\infty dq \frac{q^2}{q^2 + m_s^2} j_0(qX)s(q;X) \int_0^\infty dX' X'^2 j_0(qX')\rho_s(X')s(q;X'), \quad (48) \\
v(q;X) & = 4\pi \int_0^\infty dr r^2 j_0(qr)\rho_v^q(r;X), \quad (49) \\
s(q;X) & = 4\pi \int_0^\infty dr r^2 j_0(qr)\rho_s^q(r;X), \quad (50)
\end{align*}

together with the Dirac equations for the quarks (20) and for the nucleons (1). We have introduced the meson-nucleon coupling constants \( g_v \equiv 3g_v^q \) and \( g_s \equiv 3g_s^q \).

### D. Nuclear Matter Limit

In the spirit of the usual approach to the QHD model [6], we use the equilibrium properties of isospin symmetric nuclear matter to fix the parameters of the model. If we consider the limit in which the nucleus is very large, the nucleon densities \( \rho_v(X) \) and \( \rho_s(X) \) can be treated as constants (\( \rho_B \) and \( \rho_s \)), as can the vector and scalar fields. Hence \( U_v = V_v^q \) and \( U_s = V_s^q \), so that \( m_v^* = m_q - V_s^q \). Our formalism will yield the nuclear matter results of Refs. [2,4], except for the correction due to the spurious motion of the center-of-mass. According to Ref. [7], this correction should not be included.

The composite nucleon obeys the Dirac equation with an effective mass \( M^* \), determined by Eq. (34), and a vector potential \( V_v = 3V_v^q \). Hence the nucleons have energy eigenvalues

\[ E(k) = (k^2 + M^{*2})^{1/2} + 3V_v^q. \] \quad (51)
The total energy $\mathcal{E}$ is the sum of the energies of all nucleons below the Fermi energy together with the contribution from the meson fields,

$$\mathcal{E} = 4 \int_{k_F}^{k_F} \frac{d^3k}{(2\pi)^3} \left[ (k^2 + M^*)^{1/2} + 3V^q \right] + \frac{1}{2}m^2\sigma^2 - \frac{1}{2}m^2\omega^2. \quad (52)$$

Minimizing $\mathcal{E}$ with respect to $\omega$ and $\sigma$ yields the meson field equations (cf. Eqs. (13) and (14))

$$V^q_v = g^q_v \omega = \frac{3g^2}{m^2_v} \rho_B, \quad \rho_s = 4 \int_{k_F}^{k_F} \frac{d^3k}{(2\pi)^3} \frac{M^*}{(k^2 + M^*)^{1/2}}, \quad \rho_s = 4 \int_{k_F}^{k_F} \frac{d^3k}{(2\pi)^3} \left( \frac{M^*}{(k^2 + M^*)^{1/2}} \right)$$

$$V^q_s = g^q_s \sigma = \frac{3g^2}{m^2_s} \rho_s S, \quad S = \int_{R_0}^{R} d^3r \bar{\psi}_q \psi_q = \frac{\Omega^*_q/2 + Rm^*_q(\Omega^*_q - 1)}{\Omega^*_q(\Omega^*_q - 1) + Rm^*_q/2}.$$

The last result for the quantity $S$ is an MIT bag model result [2]. The nuclear matter results are therefore determined by the solution to the transcendental equation (54). In terms of the redefined constants $g_v = 3g^q_v$ and $g_s = 3g^q_s$, we can write

$$\mathcal{E} = 4 \int_{k_F}^{k_F} \frac{d^3k}{(2\pi)^3} \left( k^2 + M^* \right)^{1/2} + \frac{g^2}{2m^2_v} \rho^2_v S^2 + \frac{g^2}{2m^2_s} \rho^2_s. \quad (57)$$

The expressions (53), (54), and (57) depend only on the ratio of coupling constants to masses for both the omega and sigma mesons. Following Serot and Walecka [6], we introduce the two dimensionless constants

$$C^2_v = \left( \frac{g_v M}{m_v} \right)^2, \quad C^2_s = \left( \frac{g_s MS}{m_s} \right)^2,$$

as free parameters. These are adjusted to reproduce the equilibrium properties of nuclear matter. We note that the present model reduces to the Walecka model in the limit that the quark mass becomes very large while keeping the bag radius $R_0$ fixed. This is a useful check on our numerical codes.
III. RESULTS

In this section, we will use the MIT bag model to describe the nucleon. The two free nuclear matter parameters $C_2^v$ and $C_2^s$ are adjusted to reproduce a binding energy per nucleon of $E/\rho_B - M = -15.75$ MeV at a Fermi momentum $k_F = 1.30$ fm$^{-1}$. This corresponds to a saturation density of $\rho_B = 0.148$ fm$^{-3}$, which is the same as that used by Serot and Walecka [6]. A somewhat larger value of $\rho_B = 0.17$ fm$^{-3}$ ($k_F = 1.36$ fm$^{-1}$) was used by Saito and Thomas [2]. Our choice follows from the results of our finite nucleus calculations. With the larger saturation density we were unable to simultaneously fit the rms charge radii of light and heavy nuclei, whereas with the smaller value we could.

Our nuclear matter results are given as models A1-A3 in Table II, corresponding to bag radii $R_0$ of 0.6 fm, 0.8 fm, and 1.0 fm. The quark mass $m_q$ has been set to 5 MeV for both $u$ and $d$ quarks. The bag radius $R^*$ decreases slightly in-medium, although it should be noted that the rms radius actually increases by about the same margin. In comparison with the results of the QHD model [6], the effective mass is somewhat larger and the compressibility somewhat smaller for QMC models. Models B1 and B2 allow for a variable bag constant, and model C uses relativistic oscillator wave functions instead of bag wave functions. These models are discussed in detail in the next section.

For finite nuclei, the potentials $V_v(X)$ and $V_s(X)$ are used in the Dirac equation (1) for the nucleon to solve for the nuclear vector and scalar densities self-consistently. Our numerical procedure is to follow the standard iterative algorithm used in previous work [10]. Although the in-medium form factors $v(q; X)$ and $s(q; X)$ depend on $m^*_q(X)$, it is unnecessary to compute them at each value of $X$ and again at each iteration. Instead, we pre-calculate these quantities for quark effective masses corresponding to $U_s$ ranging from 0 to 250 MeV in steps of 1 MeV. Intermediate values of $m^*_q$ are then obtained by interpolation.

At this point we extend our discussion to include the $\rho$-meson and the photon (Coulomb interaction). The vector potentials $V_\rho(X)$ and $V_\gamma(X)$ will have the same form as Eq. (47), with the replacement of the isoscalar vector density $\rho_v(X')$ by the corresponding isovector
density for the $\rho$-meson, and the proton density for the photon. Because we have given the $u$ and $d$ quarks the same mass, the quark vector form factor $v(q;X)$ is the same for the $\rho$ and photon as for the $\omega$.

The $\rho$-nucleon coupling constant may be taken from experiment, or from the requirement that we reproduce the symmetry energy in nuclear matter \[6\]. Alternatively, it follows naturally within this model to assume the universal coupling $g_{\rho}^q = g_{\nu}^q$, in which case $g_{\rho} = g_{\nu}/3$ (since the $\rho$ couples to the isospin of the quarks). Because the $\omega$-nucleon coupling constant $g_{\nu}$ is much smaller here than in QHD, all three approaches give a value of $g_{\rho}$ that is roughly the same. We choose to fix $g_{\rho} = 2.63$, which is consistent with the experimental value. (Note that the definition of $g_{\rho}$ used in refs. \[2,6\] is twice as large as ours.)

As a check on our numerical codes, we were able to reproduce exactly the results of the Walecka model \[6,10\] in the limit that the quark mass becomes very large while holding the bag radius fixed at $R^* = R_0$.

Binding energies and rms charge radii for $^{16}$O, $^{40}$Ca, $^{90}$Zr, and $^{208}$Pb are shown in Table III. The scalar meson mass $m_s$ has been adjusted to fit the experimental rms charge radius of $^{40}$Ca. The binding energy per nucleon for all QMC models shows some improvement over QHD. We note the strong positive correlation between binding energy per nucleon and $M^*/M$ at nuclear matter saturation density (see Table II). Charge radii are in reasonably good agreement with experiment for all nuclei. The charge density of $^{16}$O for model A1 is compared with experiment in Fig. 1. Other QMC models and QHD give similar results.

The improvement over QHD in the binding energies comes at the expense of a reduction in the splitting of spin-orbit pairs, as shown in Fig. 2. This is easily understood if we recall that the spin-orbit splitting depends on the sum of the vector and scalar potentials \[6\]. One can therefore expect a strong negative correlation between spin-orbit splitting and $M^*/M$, as seen in Fig. 2. We also note the insufficient binding of deeply bound states.

Thus one cannot fit both the binding energy and single-particle energies within the framework of the Hartree approximation for this class of models. This points to the need for additional contributions to spin-orbit effects, such as tensor correlations.
IV. EXTENSIONS OF THE MODEL

A. Bag Model

One of the possible extensions of the QMC model is to allow the bag constants \( B \) and \( z \) to acquire a medium-dependence. The parameter \( z \) is supposed to incorporate center-of-mass corrections and the Casimir effect, both of which should depend on the quark effective mass \( m_q^* \). Initially we considered making \( z \) an arbitrary linear function of \( U_s \). However, for reasonable parameter ranges changing \( z \) had little effect, and tended to make the model worse instead of better.

Alternatively, we can allow the bag constant \( B \) to vary in the medium. Since quarks presumably become deconfined at high enough densities, this would imply \( B \to 0 \). This suggests a model for which \( B \) decreases with increasing density. A similar consideration has been made in a recent preprint by Jin and Jennings [8], who made \( B \) a function of \( M^* \). These authors also showed that for a particular functional form of \( B(M^*) \), \( M^* \) becomes a linear function of the scalar field \( \sigma \). Hence they obtained exactly the QHD model results for nuclear matter.

For our purposes it is more convenient to assume that \( B \) is a linear function of \( U_s(X) \), such that

\[
B^*(X) = B \left[ 1 - \alpha_B \frac{U_s(X)}{M} \right],
\]

(60)

with \( \alpha_B \) an arbitrary parameter. Although \( B^* \) does not go to zero at high density, we are restricting our consideration here to densities near that of nuclear matter. Typically in nuclear matter, \( U_s \sim 130 - 200 \) MeV for the models considered here.

The additional dependence of \( M^*(X) \) on \( U_s(X) \), and therefore on \( \sigma(r) \), modifies the source term of expression (14) arising from Eqs. (8) and (12). Instead, we have

\[
\frac{\delta M^*(X)}{\delta \sigma(r)} = -3 g_s^q \rho_s^q(r - X; X) \left[ 1 + \frac{4}{3} \pi R^* B \frac{\alpha_B}{3 MS(X)} \right],
\]

(61)
which follows from Eqs. (20), (27), and (34). Thus the effect of this additional term can be incorporated by making the change

\[ s(q; X) \to s(q; X) \left[ 1 + \frac{4}{3} \pi R^* B \frac{\alpha_B}{3MS(X)} \right], \] (62)

to the results of section II C, and a similar change to the corresponding nuclear matter quantity \( S \) appearing in Eq. (54), viz.

\[ S \to S \left[ 1 + \frac{4}{3} \pi R^* B \frac{\alpha_B}{3MS} \right]. \] (63)

The effect of \( B^* \) is to reduce \( M^* \) and increase the bag radius \( R^* \) in-medium. This is reflected in the results shown in Table II for models labeled B1 and B2, corresponding to \( \alpha_B = 1 \) and \( \alpha_B = 2 \), respectively, and \( R_0 = 0.6 \) fm. With increasing \( \alpha_B \) the results and parameters of the QMC models move closer to those of the QHD model [6].

For finite nuclei, the results also move towards those of the QHD model [6,10]. There is an improvement in spin-orbit splitting and in the binding energy of deeply bound states, as shown in Fig. 2. Model B2, with the largest change in the bag constant \( B^* \), does particularly well for the single particle energies and the charge density of \( ^{16}O \), shown in Fig. 1. As noted in the previous section, an improvement in spin-orbit splitting comes at the expense of a modest reduction in total energy (Table III).

There are, however, differences between this model and QHD. All QMC models will have electromagnetic form factors that change in the medium. Indeed, the charge density, or at least the quark-core contribution to the charge density, is just the quark vector form factor \( v(q; X) \) (see Appendix A). This change arises principally from two sources. First there is a relative shift in the importance of the upper and lower Dirac wave function components for quarks due to the change in effective mass \( m_q^* \). Secondly, there is a change due to the change in bag radius \( R^* \). For models B1 and B2, the change in \( R^* \) is quite large. We will explore the implications of medium-dependent electromagnetic form factors in a subsequent publication.
B. Relativistic Oscillator Model

As an alternative to the bag model, we consider a relativistic oscillator model with a (scalar) confining potential $V_{\text{con}}(r) = Cr^2$, with $C = 830 \text{ MeV/fm}^2$. This model was used by Tegen et al. [9] to investigate the electromagnetic properties of the quark-core in nucleons. We refer the reader to this paper for further details.

Our nuclear matter results given in Table II are quite similar to those of bag model B1. In particular, the increase in the rms radius of the nucleon is of the same order as that for model B1, which in the latter case is due predominantly to the decrease in the bag constant $B$. The results for the binding energies and rms charge radii are given in Table III. These results are again similar to those of model B1, as is the spin-orbit splitting shown in Fig. 2.

The relativistic oscillator model therefore offers an alternative to the bag models, and has one less free parameter than bag models with a medium-dependent bag constant $B$. Of course, we could also consider making the oscillator constant $C$ medium-dependent. Another advantage of the oscillator models is that center-of-mass effects (which were considered by Tegen et al. [9]) are easier to handle than in bag models.

V. SUMMARY AND OUTLOOK

In this paper we have extended previous work on Quark-Meson Coupling models [2–4] in nuclear matter to finite nuclei. A generalized and systematic method for corrections to the model arising from the spatial nonuniformity of the meson mean fields over the volume of the nucleon is presented. Our procedure is to define the nucleonic potentials in terms of averages over the volume of the nucleon, as in Eqs. (26) and (27). For this definition, the polarization corrections are very small.

The most basic versions of this model, based on either the MIT bag model [5] with a fixed bag constant $B$, or a relativistic oscillator model [9], have a small compressibility $K_V^{-1}$ compared with the QHD model [6], indicating a softer equation of state. While this has
experimental support, the drawback for these models is that this decrease in the compressibility is accompanied by an increase in the effective mass ratio $M^*/M$, which significantly reduces the spin-orbit component of the nuclear mean field and results in insufficient binding for deeply bound states.

One possibility we have explored is to allow the bag constant $B$ to decrease in the medium, reducing the effective mass $M^*$ and restoring much of the phenomenological success of the QHD model for finite nuclei [6, 10]. The accompanying increase in the bag radius has implications for a number of properties of the nuclei, including electromagnetic effects such as charge densities, form factors, and the EMC effect [1].

In this work, we have ignored any effect of a decrease in the quark effective mass on the masses of the virtual mesons. Because the parameters we use depend only on the ratio of coupling constants to meson masses, a change in the meson mass will only result in a rescaling of the coupling constant, without affecting the nuclear matter results. However, there may be significant effects for finite nuclei, particularly at the nuclear surface, and we will explore these in the future.

We have completely neglected the center-of-mass effects for quarks in the nucleon. It is important to handle this problem. In the light front formalism, the nucleon wave function is defined so that the intrinsic wave function is independent of the total momentum of the nucleon. Therefore, constructing a light front version of this model is a useful goal.

There are other possible extensions of the quark coupling model. Aspects of chiral symmetry have been ignored here. It should be possible to construct a chiral version by using the cloudy bag model of the nucleon [12] in which the three-quark bag is surrounded by an evanescent cloud of pions. Yet another aspect is that correlations between nucleons are known to be important. Hence the Quark-Meson Coupling model should be extended beyond the mean field approximation.

The construction of a chiral version of the quark meson coupling model, including physics beyond the mean-field approximation, and in which the nucleon center-of-mass degree of freedom is treated properly, is a useful and important goal. One would have a model
consistent with conventional nuclear physics, which embodies a reasonable treatment of the quark degrees of freedom.

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Consider the coupling of a vector meson to a quark current of the form $j_\mu^i = g_i^q \bar{\psi}_q \gamma^\mu \psi_q$, governed by some generalized quark-meson coupling constant $g_i^q$ to a quark of flavor $i$. We are interested in computing the effective “charge” and “anomalous” form factors governing the coupling of this meson to the composite nucleon. In the momentum representation, the matrix elements of the quark current between nucleon states $|N(p)\rangle$ may be written as

$$\left\langle N(p') \left| \sum_{i=1,3} j_\mu^i(i) \right| N(p) \right\rangle = g \bar{U}(p') \Lambda^\mu(p' - p) U(p), \quad (A1)$$

where $g$ is the meson-nucleon coupling constant, $U(p)$ denotes a nucleon spinor of four-momentum $p$, and $\Lambda^\mu$ is a vertex function of the form

$$\Lambda^\mu(q) = \left[ F_1(q^2) \gamma^\mu + \frac{F_2(q^2)}{2M} i\sigma^{\mu\nu} q_\nu \right]. \quad (A2)$$

For quarks of equal mass, we can relate the coupling of the omega and rho mesons to the corresponding isoscalar and isovector parts of the familiar electromagnetic nuclear current. It is convenient to introduce the Sach’s form factors

$$G_E(q^2_\mu) = F_1(q^2_\mu) - \eta F_2(q^2_\mu), \quad (A3)$$
$$G_M(q^2_\mu) = F_1(q^2_\mu) + F_2(q^2_\mu), \quad (A4)$$

with $\eta \equiv -q^2_\mu/4M^2$, and $q^2_\mu$ the square of the four-momentum transfer. For individual quark wave functions of the form

$$\phi(r) = \begin{pmatrix} \eta g(r) \\ \sigma \cdot \hat{r} f(r) \end{pmatrix} \frac{\chi_q}{(4\pi)^{1/2}}, \quad (A5)$$

the expressions for the proton form factors $G_{E,M}^p$ are easily derived to leading order in $q^2 \equiv |q|^2$:

$$G_E^p(q^2) = \int_0^\infty dr r^2 j_0(qr) \left[ g^2(r) + f^2(r) \right], \quad (A6)$$
$$G_M^p(q^2) = -\frac{4M}{q} \int_0^\infty dr r^2 j_1(qr) g(r) f(r). \quad (A7)$$
Note that \( G_E^p(0) = \frac{1}{2} \left( G_E^{(0)}(0) + G_E^{(1)}(0) \right) = 1 \) and \( G_E^n(0) = \frac{1}{2} \left( G_E^{(0)}(0) - G_E^{(1)}(0) \right) = 0 \), so that \( G_E^{(0,1)}(0) = F_1^{(0,1)}(0) = 1 \) for both the isoscalar and isovector form factors. The magnetic moment of the proton due to the quark core is given by

\[
\mu_p = \frac{e}{2M} G_M^p(0) = -\frac{2e}{3} \int_0^\infty dr r^3 g(r) f(r), \tag{A8}
\]

and the standard quark model ratio \( \mu_n = -\frac{2}{3} \mu_p \) gives the magnetic moment of the neutron.

The form factor \( G_E^p(q) \) is just the quark vector form factor \( v(q) \) defined in Eq. (39). We expect \( g_\omega = 3g_\omega^q \) for the (isoscalar) \( \omega \)-nucleon coupling constant, and \( g_\rho = g_\rho^q \) for the (isovector) \( \rho \)-nucleon coupling constant.

Experimentally, \( F_2^{(0)}(0) = -1.1203 \) and \( F_2^{(1)}(0) = 3.706 \) for the electromagnetic form factors. Although this includes contributions besides the quark core, such as the pionic cloud, we expect the “anomalous” coupling of the vector mesons to the quark core to be of the same order as \( F_2^{(0,1)} \). Because the isoscalar anomalous coupling term is so small, we have chosen to ignore it in the calculations presented here.

For completeness, we give the appropriate modifications of our formalism when the anomalous tensor terms are included. With the introduction of a “tensor” potential \( V_T(X) \), the Dirac equation (1) for the nucleon becomes

\[
\left[ -i\gamma \cdot \nabla + M^*(X) + \gamma^0 V_v(X) + i\gamma^0 \gamma \cdot \hat{X} V_T(X) \right] \psi_i(X) = E_i \gamma^0 \psi_i(X). \tag{A9}
\]

It is convenient to introduce the “tensor” density

\[
\rho_T(X) = \sum_{i=occ} n\bar{\psi}_i(X) \gamma^0 \gamma \cdot \hat{X} \psi_i(X). \tag{A10}
\]

Assuming the nucleon wave functions are defined analogously to Eq. (A5), we have \( \rho_T(X) = 2 \sum_i G_i(X) F_i(X) \).

For spherical nuclei, the vector potential \( V_v(X) \) of Eq. (47) becomes

\[
V_v(X) = g_v^2 \frac{2}{\pi} \int_0^\infty dq q^2 \frac{q^2}{q^2 + m_\pi^2} j_0(qX) F_1(q; X) \int_0^\infty dX' X'^2 \left[ j_0(qX') \rho_v(X') F_1(q; X') \right. \\
+ q j_1(qX') \rho_T(X') \left. \frac{F_2(q; X')}{2M} \right], \tag{A11}
\]
and the potential $V_T(X)$ is

$$V_T(X) = g_v^2 \frac{2}{\pi} \int_0^\infty dq \frac{q^3}{q^2 + m_v^2} j_1(qX) \frac{F_2(q; X)}{2M} \int_0^\infty dX' X'^2 \left[ j_0(qX') \rho_v(X') F_1(q; X') \right. $$

\[ + \left. q j_1(qX') \rho_T(X') \frac{F_2(q; X')}{2M} \right]. \tag{A12} \]

Note that $V_T(X) \propto -dV_v(X)/dX$ in the absence of a medium-dependence to the form factors. Finally, the contribution of the vector meson field to the total energy is modified from Eq. (45) to

$$E_v = -\frac{1}{2} \int d^3X \left[ \rho_v(X) V_v(X) + \rho_T(X) V_T(X) \right], \tag{A13}$$

which again removes half of the contribution of the vector potential contained in the nucleon energies.

**APPENDIX B: POLARIZATION CORRECTIONS**

Here we consider the second order perturbative corrections to the vector potential $U_v(X)$ and the quark energy $\epsilon^v(X)$ arising from the residual interaction $\Delta U_{v,s}(r,X)$. We also examine the first order corrections to matrix elements. All of these corrections involve intermediate excited states of the nucleon, hence the title of this Appendix.

We study the corrections involving only the vector interaction here, and show that polarization effects are vanishingly small. The formalism and physics of the scalar terms is similar, so we expect the same negligible effects.

It is useful to introduce the braket notation, so that

$$< r | \phi(X) > \equiv \phi(r - X, X), \tag{B1}$$

in the coordinate space representation. With this notation, the residual interaction can be expressed as (see Eqs. (18) and (27))

$$\Delta U_v(r, X) \equiv V_v^q(r) - < \phi(X) | V_v^q(r) | \phi(X) >, \tag{B2}$$
so that the first order perturbation theory correction to $U_v(X)$ vanishes:

$$\delta U^{(1)}_v(X) = \langle \phi(X) | \Delta U_v(r, X) | \phi(X) \rangle = 0.$$  \hspace{1cm} \text{(B3)}$$

The second order correction is given from standard Rayleigh-Schrödinger perturbation theory as

$$\delta U^{(2)}_v(X) = \sum_{n \neq N} \frac{\left| \frac{\langle N(X) | \sum_{i=1,3} \Delta U_v(r_i, X) | n(X) \rangle}{E_N(X) - E_n(X)} \right|^2}{s^{(i)} \equiv r_i - X,}$$

\hspace{1cm} \text{where } N \text{ and } n \text{ represent the nucleon and excited baryon states, respectively. The sum on } i \text{ is over the three quarks in the baryon. The confinement of the quarks to the region near } X \text{ enables a Taylor series expansion of } \Delta U_v(r_i, X) \text{ about the point } r_i = X. \text{ Let us define a variable}

$$\sum_{\alpha, \beta = 1,3} s^{(i)}_\alpha s^{(i)}_\beta \frac{\partial^2 V^q_v(X)}{\partial X_\alpha \partial X_\beta}.$$ \hspace{1cm} \text{(B6)}$$

The first three terms of Eq. (B6) do not lead to an excitation of the nucleon and do not contribute to the sum on $n$ of Eq. (B4). This is because the first two terms do not contain any dependence on $s^{(i)}$, and the third term is a center-of-mass operator. The last term of Eq. (B6) can be simplified by regrouping the term $s^{(i)}_\alpha s^{(i)}_\beta$ into quadrupole and monopole terms:

$$s^{(i)}_\alpha s^{(i)}_\beta = s^{(i)}_\alpha s^{(i)}_\beta - \delta_{\alpha, \beta} \frac{1}{3} s^{(i)} \cdot s^{(i)} + \delta_{\alpha, \beta} \frac{1}{3} s^{(i)} \cdot s^{(i)} \equiv Q^{(i)}_{\alpha \beta} + \delta_{\alpha, \beta} \frac{1}{3} s^{(i)} \cdot s^{(i)}. \hspace{1cm} \text{(B7)}$$

The quadrupole term, when combined with the spherically symmetric nature of $V^q_v(X)$, leads to a quadrupole dependence on the nuclear coordinates. The average of this vanishes

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for our spherical shell model. Using the monopole term of Eq. (B7) in Eqs. (B6) and (B4)
leads to the result

\[ \delta U^{(2)}_{v}(X) = \sum_{n \neq N} \left| < N(X) | \sum_{i=1,3} \frac{1}{2} s^{(i)} \cdot s^{(i)} | n(X) > \right|^{2} \left( \nabla^{2} U_{v}(X) \right)^{2}. \]  
(B8)

This is an attractive second order contribution.

We shall proceed to obtain a numerical estimate of the size of the correction given by
Eq. (B8). First we shall assume that the isoscalar monopole excitation strength is taken up
by a single state — the Roper resonance. This allows us to replace the different values of
\( E_{N}(X) - E_{n}(X) \) by a single one, \( \Delta E_{R} \approx -500 \text{ MeV} \). The sum over \( n \) can then be performed
using closure, so that

\[ \delta U^{(2)}_{v}(X) \approx \frac{1}{36} \frac{\nabla^{2} U_{v}(X)^{2}}{\Delta E} < N(X) | (\rho^{2} < - \rho^{2} > + \lambda^{2} - < \lambda^{2} >)^{2} | N(X) > , \]  
(B9)

in which the Jacobi coordinates \( \rho \equiv \frac{1}{\sqrt{2}} (s^{(1)} - s^{(2)}) \) and \( \lambda \equiv \frac{1}{\sqrt{6}} (s^{(1)} + s^{(2)} - 2 s^{(3)}) \) are used,
and \( < \rho^{2} \lambda^{2} > \) represents the nucleonic matrix element of \( \rho^{2} \lambda^{2} \). Simple algebra, and
the reasonable assumption that \( < \rho^{2} > = < \lambda^{2} > \), which holds exactly in the hyperspherical
approximation, leads to the expression

\[ \delta U^{(2)}_{v}(X) \approx \frac{1}{18} \frac{\nabla^{2} U_{v}(X)^{2}}{\Delta E} \left( < \rho^{4} > - < \rho^{2} >^{2} \right). \]  
(B10)

Dimensional analysis yields the result that \( < \rho^{4} > - < \rho^{2} >^{2} \propto r_{N}^{4} \), where \( r_{N} \) is the root
mean square radius of the nucleon. We proceed by using a simple Woods-Saxon form for
\( U_{v}(X) \),

\[ U_{v}(X) = V_{0} / \left[ 1 + e^{(X-R_{A})/a} \right]. \]  
(B11)

Typically, \( R_{A} = r_{0} A^{1/3} \), with \( r_{0} \approx 1.1 - 1.2 \) and \( a = 0.54 \text{ fm} \).

The density is approximately constant near the center of the nucleus, so that \( \nabla^{2} U_{v}(X) \)
can be effective only near the nuclear surface. For example, at \( X = R_{A} \)

\[ \delta U^{(2)}_{v}(R_{A}) \approx \frac{1}{18} \frac{V_{0}^{2}}{(2aR_{A})^{2} \Delta E} \left( < \rho^{4} > - < \rho^{2} >^{2} \right). \]  
(B12)
If one takes $a$ and $r_N$ to each be of the order of the nucleon size, one sees that $\delta U^{(2)}_\rho(R_A)$ is proportional to $(r_N/R_A)^2$, which is very small, a result that has been obtained without using a specific model of confinement. One may get a specific estimate by using the non-relativistic harmonic oscillator quark model of the nucleon. In this case, the closure approximation used above is true exactly, and $<\rho^4> - <\rho^2>^2 = 1.5 r_N^4$. Then, using typical values $r_N = 0.83$ fm, $r_0 = 1.15$ fm, $a = 0.54$ fm, and $V_0 = 250$ MeV, one obtains the result

$$\delta U^{(2)}_\rho(R_A) \approx -\frac{3}{A^{2/3}} \text{MeV},$$

which ranges from $-0.5$ MeV for $^{16}$O to $-0.1$ MeV for $^{208}$Pb. These effects are truly small.

We next consider the effects of $\rho$ meson exchange. The equation that is analogous to Eq. (B6) is

$$\delta U^{(2)}_\rho(s^{(i)}, X) \approx V^q_\rho(X) - <\phi(X)|V^q_\rho(r)|\phi(X)> + \tau^{(i)}_3 s^{(i)} \cdot \nabla V^q_\rho(X),$$

in which the term $\sum_i \tau^{(i)}_3 s^{(i)}$ is the dipole operator $D^{(i)}$. Now the term linear in $s^{(i)}$ does not contribute. We thus obtain the second order correction to the $\rho$ exchange contribution to the vector potential:

$$\delta U^{(2)}_\rho(X) = \sum_{n \neq N} \left| <N(X)|\sum_{i=1,3} D^{(i)}|n(X)> \cdot \nabla U_\rho(X) \right|^2.$$  

This term can be related to the experimentally measured dipole polarizability $\alpha$, which is given by

$$\alpha \equiv 2e^2 \sum_{n \neq N} \left| \frac{<N(X)|\sum_{i=1,3} D^{(i)}|n(X)>}{E_N(X) - E_n(X)} \right|^2.$$  

The use of the Wigner-Eckart theorem allows one to obtain the relation between $\delta U^{(2)}_\rho$ and the measured quantity $\alpha$:

$$\delta U^{(2)}_\rho(X) = -\frac{\alpha}{e^2} |\nabla U_\rho(X)|^2.$$  

The use of the value $\alpha = 12 \times 10^{-4}$ fm$^3$ (as explained in the review [11]) and the Woods-Saxon form of the potential, along with the coupling constants discussed above, leads to the estimate $\delta U^{(2)}_\rho(X = R_A) = -0.2$ MeV, so that this effect is negligible.
The previous examples show that the second order contributions to the vector potentials are very small. The use of the same formalism leads to the result that the second order contributions to the scalar potential are also negligible.

Our final example concerns the computations of matrix elements. Consider some general observable $O$. The first order correction supplied by the vector potential is given by

$$\delta O_v = 2 \sum_{n \neq N} \frac{\langle N(X) |O| n(X) \rangle \langle n(X) |\Delta U_v| N(X) \rangle}{E_N(X) - E_n(X)}.$$  

(B18)

To be specific, consider the case $O = \sum_i s^{(i)} \cdot s^{(i)} \equiv s^2$. In this case we obtain

$$\delta s^2_v(X) = \frac{1}{3} \sum_{n \neq N} \frac{|\langle N(X) |s^2| n(X) \rangle|^2}{E_N(X) - E_n(X)} \nabla^2 U_v(X),$$

(B19)

or, using the approximations used to obtain the corrections to the energy,

$$\delta s^2_v(X) = \frac{2}{3} \frac{<\rho^4> - <\rho^2>^2}{\Delta E} \nabla^2 U_v(X).$$

(B20)

In this case the contribution of the attractive scalar potential tends to cancel that of the repulsive vector potential. We account for this in a simple way by taking a net potential of depth 50 MeV. The net result is that

$$\frac{\delta s^2(X = R_A)}{<s^2>} \approx -\frac{0.01}{A^{1/3}},$$

(B21)

which is again a negligible effect.

The final result is that the dominant effects of the composite nature of the nucleon are contained in the definitions (26) and (27) of the mean quark vector and scalar potentials as expectation values of the quark vector and scalar potentials in the nucleon.
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TABLES

TABLE I. Nucleon bag model parameters corresponding to $m_q = 5$ MeV and a free nucleon mass of $M = 938.9$ MeV.

| $R_0$ (fm) | $\frac{4}{3} \pi R_0^3 B$ (MeV) | $z$       |
|------------|---------------------------------|-----------|
| 0.6        | 232.918                         | 4.00367   |
| 0.8        | 232.915                         | 3.29545   |
| 1.0        | 232.912                         | 2.58724   |

TABLE II. Model parameters corresponding to a nuclear matter binding energy per nucleon of $-15.75$ MeV at $k_F = 1.30$ fm$^{-1}$. Nuclear matter results are given for the compressibility, nuclear effective mass, and in-medium bag radius (rms radius for the oscillator model).

| Model      | $R_0$ (fm) | $C_s^2$ | $C_v^2$ | $K^{-1}_V$ (MeV) | $M^*/M$ | $R^*$ (fm) |
|------------|------------|---------|---------|------------------|---------|------------|
| QHD        |            | 357.62  | 274.00  | 547              | 0.541   |            |
| A1         | 0.6        | 154.94  | 117.46  | 292              | 0.775   | 0.596      |
| A2         | 0.8        | 132.93  | 98.41   | 279              | 0.803   | 0.795      |
| A3         | 1.0        | 117.15  | 84.57   | 266              | 0.823   | 0.993      |
| B1 ($\alpha_B = 1$) | 0.6        | 187.98  | 145.50  | 321              | 0.734   | 0.624      |
| B2 ($\alpha_B = 2$) | 0.6        | 289.53  | 226.11  | 364              | 0.614   | 0.676      |
| C (Rel. Osc.) | 0.642$^a$ | 190.09  | 147.26  | 326              | 0.731   | 0.689$^a$ |

$^a$For the relativistic oscillator we give the rms radius.
TABLE III. Binding energy per nucleon (in MeV), and rms charge radii (in fm) for several closed shell nuclei. We have taken $m_v = 783$ MeV, $m_\rho = 770$ MeV, and $g_\rho = 2.63$. The scalar meson mass $m_s$ has been adjusted to fit the rms charge radius of 3.48 fm in $^{40}$Ca.

| Model | $m_s$ (MeV) | $^{16}$O $\langle E/A \rangle$ | $^{40}$Ca $\langle r^2 \rangle^{1/2}_{ch}$ | $^{90}$Zr $\langle E/A \rangle$ | $\langle r^2 \rangle^{1/2}_{ch}$ | $^{208}$Pb $\langle E/A \rangle$ | $\langle r^2 \rangle^{1/2}_{ch}$ |
|-------|---------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| QHD   | 520     | -4.88           | 2.74            | -6.30           | 3.48            | -7.07           | 4.27            |
| A1    | 450     | -6.22           | 2.75            | -7.46           | 3.48            | -7.84           | 4.32            |
| A2    | 415     | -5.98           | 2.75            | -7.30           | 3.48            | -7.70           | 4.32            |
| A3    | 385     | -5.72           | 2.75            | -7.12           | 3.48            | -7.56           | 4.32            |
| B1    | 460     | -5.93           | 2.75            | -7.23           | 3.48            | -7.68           | 4.31            |
| B2    | 480     | -5.57           | 2.75            | -6.94           | 3.48            | -7.55           | 4.29            |
| C     | 455     | -5.51           | 2.75            | -6.94           | 3.48            | -7.46           | 4.31            |
| Expt  |         | -7.98           | 2.75            | -8.55           | 3.48            | -8.71           | 4.28            |

$^a$Fit
FIGURES

FIG. 1. The charge density of $^{16}$O for bag models A1 and B2, which both have $R_0 = 0.6$ fm. Model B2 has a medium-dependent bag constant. The solid line is the experimental parameterization of H. de Vries, C. W. de Jager and C. de Vries, At. Data Nucl. Data Tables 36, 495 (1987).

FIG. 2. Proton single particle energies in $^{16}$O for all models.
