Notes on multiple superconducting phases in UTe$_2$ — Third transition —

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(Dated: April 21, 2020)

A three-component Ginzburg-Landau theory for a triplet pairing is developed to understand the observed multiple phases in a new superconductor UTe$_2$ under pressure. Near the critical pressure $P_c=0.2\text{GPa}$ where all components are perfectly degenerate the three successive superconducting transitions are predicted to occur. The $p$-wave pairing symmetry realized in UTe$_2$ is characterized by non-unitarity and chirality with point nodes, thus time reversal symmetry spontaneously broken.

Much attention has been focused on ferromagnetic superconductors (SC) [1], UGe$_2$, URhGe, and UCoGe, including a new spin-polarized heavy Fermion material UTe$_2$ [2, 3] which is believed to belong to the same category to the former three compounds. A series of experiments on UTe$_2$ indicates that (1) a spin-triplet pairing is realized, (2) a pair of point nodes is orientated along the $a$-axis in orthorhombic crystal (D$_{2h}$) [4], (3) it is chiral where the time reversal symmetry is spontaneously broken and characterized by edge current. Thus the pairing symmetry is best described by the pair function $(b+ic)(p_b+ip_c)$ with double chiral non-unitary state both in the $d$-vector of the spin space and in the orbital space. This pairing function analogous to the superfluid $^3$He $\Lambda_1$ and $\Lambda_2$ phases explains several outstanding experimental facts, including (1), (2) and (3) mentioned above.

Recently, Aoki, et al [5] and Braithwaite, et al [6] have discovered multiple phases in UTe$_2$ under pressure ($P$). They enumerate four different SC phases in the $T$-$H$-$P$ space at least with $T$ and $H$ being temperature and applied field. Since our previous theory [7] can provide three phases, that is, the $\Lambda_1$, $\Lambda_2$ and $\Lambda_3$ phases, it needs to be extended to accommodate the additional phases. This can be done quite naturally because our theoretical framework is based on the overall symmetry SO(3)×D$_{2h}$×U(1) with the spin, orbital and gauge symmetry respectively.

This triple spin symmetry of the pairing function is expressed by a complex three component vectorial order parameter $\tilde{\eta} = (\eta_a, \eta_b, \eta_c)$. Before [5] we assumed the only two components are active, mainly aiming at explaining the $H_{c2}$ phase diagrams, consisting of a reentrant SC in URhGe, an S-shape one in UCoGe, and an L-shape one in UTe$_2$. Now the multiple phase diagram found requires clearly that all the three components should be active afterall.

Under D$_{2h}$ symmetry the most general Ginzburg-Landau free energy functional up to the quadratic order is expressed by

$$F^{(2)} = \alpha_0(T - T_{c0})|\tilde{\eta}|^2 + b|\tilde{M} \cdot \tilde{\eta}|^2 + cM^2|\tilde{\eta} \cdot \tilde{\eta}|^2 + ikM \cdot \tilde{\eta} \times \tilde{\eta}^*$$

with $b$ and $c$ positive constants. We set $c=0$ without loss of generality. The last term arises due to the non-unitarity of the pairing function in the presence of the moment $\tilde{M}$ which is to break the SO(3) spin symmetry. While the spontaneous ferromagnetic moment is established below the Curie temperatures in UGe$_2$, URhGe, and UCoGe, it is slowly fluctuating in UTe$_2$ which is detected by the NMR experiment by Tokunaga, et al [8]. Upon lowering $T$ the fluctuations are wiped out via the $T_2$ measurement window, implying that the time scale of the moment fluctuations is so slow, an order of MHz $\sim 10^{-5}$K. For the superconducting electrons with $T_c=1.6$K this time scale is slow enough to regard it as a symmetry breaker as explained before [5].

It is convenient to introduce $\eta_k = \frac{1}{\sqrt{2}}(\eta_b \pm i\eta_c)$ for $\mathbf{M} = (M_a, 0, 0)$. From the above equation the quadratic term $F^{(2)}$ becomes

$$F^{(2)} = \alpha_0 \left((T - T_{c1})|\eta_+|^2 + (T - T_{c2})|\eta_-|^2 + (T - T_{c3})|\eta_a|^2 \right)$$

with

$$T_{c1, 2} = T_{c0} \pm \frac{\kappa}{\alpha_0} M_a$$

$$T_{c3} = T_{c0} - \frac{b}{\alpha_0} M_a^2$$

The actual second transition temperature may be modified because of the fourth order GL terms. Here we note
Thus away from the degenerate point at \( M \equiv 0 \), the A phase starts at \( T_{c3} \), denoted symbolically by \( A \). The gradient GL energy is given

\[
F_{\text{grad}} = \sum_{\nu=a,b,c} \left( K_a |D_x \eta_\nu|^2 + K_b |D_y \eta_\nu|^2 + K_c |D_z \eta_\nu|^2 \right)
\]

where \( D_j = -i\hbar \partial_j - 2eA_j/c \) and the mass terms are characterized by the coefficients \( K_j \) \((j = a, b, c)\) in \( D_{2k} \). It is seen from this form that \( H_{c2} \)'s for the three components each starting at \( T_{c3} \) \((j = 1, 2, 0)\) with different slopes intersect each other, never avoiding or leading to a level repulsion.

We discuss the \( H_{c2} \) data of UTe\(_2\) under pressure in light of this general rule. As shown in Fig. 2 where at \( P=0.7\text{GPa} \) for \( H \parallel a \) the \( H_{c2} \) data points are quoted from Ref. [2]. In addition to \( T_{c1} \) and \( T_{c2} \) we identify \( T_{c3} \). Five phases are enumerated.

that among the GL fourth order terms, \( \text{Re}(\eta_+^2 \eta_-) \) becomes important in interpreting the \( H_{c2} \) data later because it is independent of the signs of the GL parameters.

Figure 1 shows that as a function of moment \( M \), \( T_{c1} \) \((T_{c2})\) increases \( (\) decreases \( )\) linearly, while \( T_{c3} \) always decreases as \( M \) grows. The three transition lines meet at \( M=0 \) where the three components \( \eta \) are all degenerate. Thus away from the degenerate point at \( M=0 \), the \( A_0 \) phase starts at \( T_{c3} \) quickly disappears from the phase diagram. Below \( T_{c2} \) the two components \( \eta_+ \) and \( \eta_- \) coexist, symbolically denoted by \( A_1+A_2 \). Note that because their transition temperatures are different, \( A_1+A_2 \) is not the so-called A-phase which is unitary, but generically non-unitary except at the degenerate point \( M=0 \) where the totally symmetric phase is realized with time reversal symmetry preserved. Likewise below \( T_{c3} \) all the components coexist; \( A_1+A_2+A_0 \) realizes.

Under an applied field with vector potential \( A \), the gradient GL energy is given

\[
F_{\text{grad}} = \sum_{\nu=a,b,c} \left( K_a |D_x \eta_\nu|^2 + K_b |D_y \eta_\nu|^2 + K_c |D_z \eta_\nu|^2 \right)
\]

where \( D_j = -i\hbar \partial_j - 2eA_j/c \) and the mass terms are characterized by the coefficients \( K_j \) \((j = a, b, c)\) in \( D_{2k} \). It is seen from this form that \( H_{c2} \)'s for the three components each starting at \( T_{c3} \) \((j = 1, 2, 0)\) with different slopes intersect each other, never avoiding or leading to a level repulsion.

We discuss the \( H_{c2} \) data of UTe\(_2\) under pressure in light of this general rule. As shown in Fig. 2 where at \( P=0.7\text{GPa} \) for \( H \parallel a \) the \( H_{c2} \) data points are quoted from Ref. [3]. We draw the three continuous lines to connect those points, finding the missing third transition along the \( T \) axis. Note that the tricritical point with three second order lines is thermodynamically forbidden [4]. We name each region A, B, C, D, and E. Those are characterized by \( A=A_1 \) at \( T_{c1} \), \( B=A_1+A_2 \) at \( T_{c2} \), \( C=A_1+A_2+A_0 \) at \( T_{c3} \), \( D=A_1+A_0 \), and \( E=A_0 \). It is understood that this phase diagram is quite exhaustive, no further state expected in our framework. At the intersection points in Fig. 2 the four transition lines should always meet together according to the above general rule and thermodynamics. The lines indicate how those three phases interact each other, by enhancing or suppressing. \( T_{c3} \) could be raised by the presence of \( A_1 \) and \( A_2 \) phases due to the fourth order term \( \text{Re}(\eta_+^2 \eta_-) \) mentioned.

Figure 3 shows the phase diagram in the \( P-T \) plane where \( T_{c3} \) determined thus for \( P=0.40, 0.54 \) and \( 0.70\text{GPa} \) is displayed by triangle symbols. The identification for each phase is done reasonably. The critical pressure \( P_{c3}=0.2\text{GPa} \) where all phase transition lines meet is identified. Away from \( P_{c3} \) the three transition lines split linearly for \( T_{c1} \) and \( T_{c2} \), and quadratically for \( T_{c3} \). Note that at ambient pressure the second transition \( T_{c2} \) is probed by NMR experiment [10].

In conclusion, we have identified the third SC phase \( A_0 \) in addition to \( A_1 \) and \( A_2 \) through the GL analysis of the \( H_{c2} \) measurements under pressure. The chiral-p-wave pairing with non-unitary triplet state is found to be realized in UTe\(_2\) analogous to superfluid \( ^3\text{He} \) A-phase under fields, thus time reversal symmetry is spontaneously broken.

Acknowledgments The author thanks D. Aoki for sharing experimental data prior to publication. This work was supported by JSPS KAKENHI No.17K05553.

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