Abstract

We present the latest Next-to-Next-Leading order strong coupling constant, $\alpha_s$, extracted from Gross-Llewellyn-Smith sum rule. The value of $\alpha_s$ from this measurement, at the mass of Z boson, is $\alpha_{s}^{NNLO}(M_{Z}^{2}) = 0.114^{+0.009}_{-0.012}$. We discuss the previous strange sea quark measurement from the CCFR experiment and the prospects for improvements of the measurement in the NuTeV experiment.
Introduction

Neutrino-nucleon ($\nu$-N) deep inelastic scattering (DIS) experiments provide a good testing field for Quantum Chromo-Dynamics (QCD), the theory of strong interactions. The $\nu$-N DIS experiments probe the structure of nucleons and provide an opportunity to test QCD evolutions and to extract the strong coupling constant, $\alpha_s$. They are complementary measurements to charged lepton DIS experiments of nucleon structure functions. The advantage of $\nu$-N DIS measurements over charged lepton experiments is that $\nu$-N experiments can measure both nucleon structure functions, $F_2(x, Q^2)$ and $xF_3(x, Q^2)$, due to pure V-A nature. The $\nu - N$ differential cross sections are written, in terms of the structure functions:

$$\frac{d^2\sigma^{\nu(\overline{\nu})}}{dx dy} = \frac{G_F^2 ME_\nu}{\pi} \left[ \left(1 - y - \frac{Mxy}{2E_\nu} + \frac{y^2}{2} \frac{1 + 4M^2x^2/Q^2}{1 + R(x, Q^2)} \right) F_2^{\nu(\overline{\nu})} \pm \left(y - \frac{y^2}{2}\right) xF_3^{\nu(\overline{\nu})} \right]$$

(1)

where $R(x, Q^2) = \sigma_L/\sigma_T$, the ratio of longitudinal to transverse absorption cross sections. The structure functions $F_2(x, Q^2)$ and $xF_3(x, Q^2)$ are extracted from fitting Eq. 1 to the measured differential cross sections.

In this paper, we present the next-to-next leading order (NNLO) determination of $\alpha_s$ from the Gross-Llewellyn-Smith (GLS) sum rule which states that the total number of valence quarks are given by the integration of the non-singlet structure function $xF_3$ over entire regions of $x$, the momentum fraction carried by the struck quark. We also discuss the measurements of strange sea quark distributions from CCFR experiment and the expected improvements of this measurement in NuTeV.

The Experiment

CCFR (Columbia-Chicago-Fermilab-Rochester)/NuTeV experiment is a $\nu - N$ DIS experiment at the Tevatron in Fermilab. The CCFR experiment used broad momentum beam of mixed neutrinos ($\nu_\mu$) and antineutrinos ($\overline{\nu}_\mu$) from decays of the secondary pions and kaons, resulting from interactions of 800 GeV primary protons on a Beryllium-Oxide (BeO) target. The NuTeV experiment, successor of CCFR, used separate beams of $\nu_\mu$ or $\overline{\nu}_\mu$ during a given running period, using a Sign-Selected-Quadrupole-Train (SSQT) [1].

The CCFR/NuTeV detector [2] consists of two major components : target calorimeter and muon spectrometer. The target calorimeter is an iron-liquid-scintillator sampling calorimeter, interspersed with drift chambers to provide track information of the muons resulting from charged-current (CC) interactions where a charged weak boson ($W^+$ or $W^-$) is exchanged between the $\nu_\mu$ ($\overline{\nu}_\mu$) and the parton. The calorimeter provides dense material in the path of $\nu$ ($\overline{\nu}$) to increase the rate of neutrino interactions. The hadron energy resolution of the calorimeter is measured from the test beam and is found to be : $\sigma/E_{Had} = (0.847 \pm 0.015)/\sqrt{E_{Had}(GeV)} + (0.30 \pm 0.12)/E_{Had}$.

The muon spectrometer is located immediately downstream of the target calorimeter and consists of three toroidal magnets and five drift chamber stations to provide accurate measurements of muon momenta. The momentum resolution of the spectrometer ($\sigma/p_\mu$) is approximately 10.1% and the angular resolution is $\theta_\mu = 0.3 + 60/p_\mu$(mrad).

$\alpha_s$ from Gross-Llewellyn-Smith Sum Rule

Once the structure function $xF_3$ is extracted, one can use Gross-Llewellyn-Smith (GLS) sum rule [3], which states that $\int xF_3(dx/x)$ is the total number of valence quarks in a nucleon, up to QCD corrections, to extract the strong coupling constant, $\alpha_s$. Since in leading order (LO), the structure function $xF_3$ is $xq - x7$, the valence quark distributions, integrating $xF_3$ over variable $x$, are equal to number of valence quarks.
Since the GLS sum rule is a fundamental prediction of QCD and the integral only depends on valence quark distributions, $\alpha_s$ can be determined without being affected by less well known gluon distributions. Moreover, since there are sufficient number of measurements of $xF_3$ in a wide range of $Q^2$, one can measure $\alpha_s$ as a function of $Q^2$ in low $Q^2$ where the values of $\alpha_s$ depends most on $\Lambda_{QCD}$.

With next-to-next-to-leading order QCD corrections [4], the GLS integral takes the form:

$$\int_0^1 xF_3(x, Q^2) \frac{dx}{x} = 3 \left( 1 - \frac{\alpha_s}{\pi} - a(n_f)(\frac{\alpha_s}{\pi})^2 - b(n_f)(\frac{\alpha_s}{\pi})^3 \right) - \Delta HT$$

where the term $\Delta HT$ is the corrections from higher-twist effects. The higher-twist correction term, $\Delta HT$, is predicted to be significant in some models [5], while others [6] predict negligibly small corrections. We take $\Delta HT$ as one half the largest model prediction with the associated error covering the full range ($\Delta HT = (0.15 \pm 0.15)/Q^2$).

In order to perform the integration of $xF_3$ in the entire ranges of $x$, we use data from other $\nu$-N DIS experiments (WA59, WA25, SKAT, FNAL-E180, and BEBC-Gargamelle [7]), to cover large-$x$ ranges that are not covered by CCFR due to geometric and kinematic acceptances. The CCFR data has a minimum $x$ of roughly $x = 0.002 Q^2$. To extrapolate below this kinematic limit, we fit $xF_3$ to a power law ($Ax^B$) using all the data points in $x < 0.1$.

Since the data points do not populate the region $x > 0.5$ finely, it is necessary to interpolate between the data points. Thus, we use the shape of the charged lepton DIS $F_2$ in $x > 0.5$, because the shapes of $xF_3$ and $F_2$ should be the same in this region of $x$ due to negligible contribution from sea quarks. The charged lepton $F_2$ data are corrected for nuclear effects [8] before obtaining the shapes. Systematic uncertainties due to extrapolation and interpolation take into account the possible variations of the models in the low-$x$ region and the resonance
Figure 1 shows \( xF_3 \) in the four lowest \( Q^2 \) bins as a function of \( x \). The solid circles represent the experimental \( xF_3 \) data, The solid lines represent the power law fit used for extrapolation of \( xF_3 \) outside the kinematic limit of CCFR. The shaded area in each plot shows the region of \( x \) where the extrapolations are used for the integration.

The \( xF_3 \) integrals are estimated in six \( Q^2 \) bins. The results in each \( Q^2 \) bin are fit to the NNLO pQCD function and higher-twist effect in Eq. 2. This procedure yields a best fit of \( \Lambda^{5,NNLO}_{\overline{MS}} = 165 \text{MeV} \). Evolving this result to the mass of the Z boson, \( M_Z \), in NNLO, this corresponds to the value of \( \alpha_s \):

\[
\alpha_s^{NNLO}(M_Z^2) = 0.114^{+0.005}_{-0.006}(\text{stat.})^{+0.007}_{-0.009}(\text{syst.}) \pm 0.005(\text{theory}) \tag{3}
\]

where systematic uncertainty includes the error in the fit and theory error represents the uncertainty due to higher twist effects and higher order QCD contributions. This result is consistent with the value extracted from the CCFR structure function measurement \([9]\).

Evolving to the mean value of \( Q^2 = (3 \text{GeV}^2) \) for this analysis results in \( \alpha_s^{NNLO}(3 \text{GeV}^2) = 0.278 \pm 0.035 \pm 0.05^{+0.035}_{-0.03} \). If the higher-twist effect is neglected, the value of \( \alpha_s^{NNLO}(M_Z^2) \) becomes 0.118.

**Strange Sea Measurements**

In \( \nu - N \) DIS, the flavor changing weak CC interactions provide a clean signature for scattering off an \( s \) quark (\( s \rightarrow c \)), resulting in a pair of oppositely signed muons in the final state through the reaction \( \nu_\mu(\overline{\nu}_\mu) + N \rightarrow \mu^- (\mu^+) + c(\overline{c}) + X \) where \( c(\overline{c}) \) subsequently undergoes a semileptonic decay \( c(\overline{c}) \rightarrow s(\overline{s}) + \mu^+ (\mu^-) + \overline{\nu}_\mu(\nu_\mu) \). In contrast, detecting \( s \) quark with NC interactions in a charged lepton DIS is convoluted with \( s \) production and fragmentation of strange mesons, requiring good particle identifications. The nucleon structure function \( F_2 \) from \( \nu - N \) DIS together with that measured from charged lepton DIS could also measure \( s \) quark distributions. In addition, the difference of the \( \nu - N \) structure function \( xF_3 \) between \( \nu \) and \( \overline{\nu} \) measures the nucleon strange quark contents as well.

CCFR experiment performed both LO \([10]\) and NLO analyses \([11]\). Since a neutrino from the semileptonic decay of the charm is involved in the final state, the CCFR measured the distributions of visible physical quantities: \( x_{\text{vis}}, E_{\text{vis}} = E_{\text{Had}} + p^1_\mu + p^2_\mu \), and \( z_{\text{vis}} = \frac{p^2_\mu}{E_{\text{Had}} + p^2_\mu} \) which is the fractional momentum taken by the charm meson fragmentation. The measured distributions were compared to Monte Carlo predictions which incorporate detector acceptance and resolutions.

The Monte Carlo included the singlet \((xqS(x, \mu^2))\), non-singlet \((xqNS(x, \mu^2) = xqv(x, \mu^2))\), and gluon parton distribution functions from CCFR structure function analysis assuming:

\[
xqv(x, \mu^2) = xuV(x, \mu^2) + xdv(x, \mu^2) \tag{4}
\]
\[
xdv(x, \mu^2) = A_d(1 - x)xuv(x, \mu^2) \tag{5}
\]
\[
xu(x, \mu^2) = xuS(x, \mu^2) \tag{6}
\]
\[
xd(x, \mu^2) = xdS(x, \mu^2) \tag{7}
\]

The above assumes symmetric sea quark distributions. We then parameterized the strange sea distribution to:

\[
x_s(x, \mu^2) = A_s(1 - x)^\alpha \left[ \frac{xu(x, \mu^2) + xd(x, \mu^2)}{2} \right] \tag{8}
\]

where, \( A_s \) is defined in terms of level and shape parameters, \( \kappa \) and \( \alpha \). The level parameter \( \kappa \) is defined as:

\[
\kappa = \int_0^1 [xs(x, \mu^2) + x\overline{s}(x, \mu^2)] \]
Table 1: Summary of the CCFR strange sea analysis. The first set of the errors in each item is statistical and the second set is the systematic uncertainties.

| Fragmentation | $\kappa$  | $\alpha$  | $B_c$  | $m_c$(GeV) |
|---------------|-----------|-----------|--------|------------|
| NLO Collins-Spiller $\epsilon = 0.81$ | 0.477 | -0.02 | 0.1091 | 1.70 |
| NLO Peterson $\epsilon = 0.20$ | 0.468 | -0.05 | 0.1047 | 1.69 |
| NLO Peterson $\epsilon = 0.20$ | 0.373 | 2.50 | 0.1050 | 1.31 |
| LO Peterson $\epsilon = 0.20$ | 0.373 | 2.50 | 0.1050 | 1.31 |

The LO prediction included a simple parton model which has tree level calculations for scattering of a $W$ off an $s$ quark, resulting in a charm quark final state. On the other hand, the NLO prediction included the ACOT formalism \cite{12} whose calculation is performed in a variable flavor $\overline{MS}$ scheme. Specifically the NLO prediction included the Born and gluon-fusion diagrams. In addition, since the charm quark is massive, the threshold effect was taken into account in the prediction via leading order slow rescaling formalism where the scaling variable $x$ is replaced with $\xi$ defined as:

$$\xi = x \left( 1 + \frac{m_c^2}{Q^2} \right) \left( 1 - \frac{x^2 M^2}{Q^2} \right),$$

where $m_c$ is the mass of the charm quark.

We then fit the Monte Carlo to data distributions of $x_{vis}$, $E_{vis}$, and $z_{vis}$ for $\kappa$, $\alpha$, $B_c$ (the charmed meson semi-leptonic branching ratio), $m_c$, and $\epsilon$ (fragmentation parameter), using the values of CKM matrix elements $|V_{cd}|$ and $|V_{cs}|$, taken from PDG \cite{13}.

Table I summarizes the results of this analysis, comparing various fragmentation functions at LO and NLO. The result shows that the strange sea level parameter, $\kappa$, is in qualitative agreement between LO and NLO analyses while the shape parameter, $\alpha$, is the same as other sea quarks in NLO but softer in LO. The charm quark mass differs between NLO and LO, due presumably to the fact that the parameter $m_c$ is not a pole mass but rather a theoretical parameter that absorbs the lack of higher corrections.

Aside from the large statistical uncertainty, there were two major sources of systematic uncertainties in performing this measurement in CCFR. First and foremost one is the identification of the secondary muons resulting from the decay of charmed mesons. Since the neutrino beam in CCFR was a mixture of $\nu_\mu$ and $\overline{\nu}_\mu$, the experiment relied on transverse momentum of the muons relative to the neutrino beam direction, $P_{\mu T}$, to distinguish the prompt muons from secondary muons. The second systematic uncertainty is the LO slow rescaling formalism to take into account the charm threshold effect in theoretical predictions.

The experimental systematic uncertainty due to the identification of the secondary muon almost completely disappears in the NuTeV experiment due to the use of SSQT whose wrong sign neutrino contamination at a given mode is less than $10^{-3}$. In addition, the NuTeV experiment is working on improving low energy muon energy measurements in the calorimeter using the intensive calibration beam program. In the theoretical predictions, NuTeV is planning...
ning to incorporate more complete NLO calculations and will investigate the dependence of the predictions on factorization scheme and various parton distribution function parameterizations.

While statistical uncertainty dominated in the previous measurement, improving the systematic uncertainty would reduce the total error significantly. We expect the statistical uncertainty in this analysis would reduce by about a factor of 2.2 combining CCFR and NuTeV data. The NuTeV is currently planning to present a LO analysis this summer.

Conclusions

CCFR measured GLS integral in six $Q^2$ bins and extracted the value of NNLO $\alpha_s$ at the mass of the Z boson. The measured value of $\alpha_s$ is:

$$\alpha_s^{NNLO}(M_Z^2) = 1.114^{+0.009}_{-0.012}$$  \hspace{1cm} (11)

which is consistent with the value measured from the structure function analysis [9] and with world average.

NuTeV has finished data taking in September 1997, running with separate beams of $\nu_\mu$ and $\bar{\nu}_\mu$, and a precision calibration (0.3%) is underway. Strange sea measurement at the NuTeV experiment is making progress, taking advantage of the separate beam and the intensive calibration. NuTeV expects a LO result in the strange sea measurement by the summer.

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