Fowler-Nordheim Electron Cold Emission Formalism in Presence of Strong Magnetic Field

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ABSTRACT
Formalisms for both non-relativistic as well as relativistic versions of field emission of electrons in presence of strong quantizing magnetic field, relevant for strongly magnetized neutron stars or magnetars are developed. In the non-relativistic scenario, where electrons obey Schrödinger equation, we have noticed that when Landau levels are populated for electrons in presence of strong quantizing magnetic field the transmission probability exactly vanishes unless the electrons are spin polarized in the opposite direction to the external magnetic field. On the other hand, the cold electron emission under the influence of strong electrostatic field at the poles is totally forbidden from the surface of those compact objects for which the surface magnetic field strength is \( \gg 10^{15} \) G (in the eventuality that they may exist). Whereas in the relativistic case, where the electrons obey Dirac equation, the presence of strong quantizing magnetic field completely forbids the emission of electrons from the surface of compact objects if \( B > 10^{13} \) G.

Key words: magnetars, magnetic field, dense matter, atomic processes, relativistic processes, neutron stars

1 INTRODUCTION

There are mainly three kinds of electron emission processes from metal surface, they are: (i) thermionic emission, (ii) photoelectric emission and (iii) cold emission or field emission.

The field emission or cold emission, which we have investigated in the present article in the context of strongly magnetized neutron stars or magnetars, is an electron emission process induced by strong external electrostatic field at zero or extremely low temperature. Field emission can happen from solid and liquid surfaces, or from individual atoms. It has been noticed that the field emission from metals occurs in presence of high electric field: the gradients are typically higher than 1000 volts per micron and the emission is strongly dependent upon the work function of the material. Unlike the thermionic emission and photo-emission of electrons, the field emission process can only be explained by quantum tunneling of electrons, which has no counter classical explanation. However, for general type surface barrier, this purely quantum mechanical problem can not be solved exactly, a semi-classical approximation, known as WKB (the name is an acronym for Wentzel-Kramers-Brillouin) is needed to get tunneling coefficients. Now to explain cold electron emission from metals, one may assume that because of quantum fluctuation, electrons from the sea of conduction electrons (degenerate electron gas) always try to tunnel out through the metallic surface (surface barrier). However, as soon as an electron comes out, it induces an image charge on the metal surface, which pulls it back and does not allow this emitted electron to move far away from metal surface in the atomic scale. But if some strong attractive electrostatic field is applied near the metallic surface, then depending on the Fermi energy of electrons, the height of the surface barrier and the local work function, the electrons may overcome the effect of image charge and get liberated. Since the external strong electric field is causing such emission and does not depend on the thermal properties of the metal, even the metal can be at zero temperature, it is called field emission or cold emission.

The theory of field emission from bulk metals was first proposed by Fowler and Nordheim in an epoch making paper in the proceedings of Royal Society of London in the year 1928 [Fowler & Nordheim 1928] (see also Stern et al. 1929).
the non-relativistic case. In section-3, we have repeated the same calculation for the relativistic scenario. In the last section we have given the conclusions and future prospects of this work.

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changing linearly with $z$-coordinate. Here $C$ is the constant surface barrier and $F$ (absorbing the magnitude of electron charge

Moreover, to the best of our knowledge, neither the non-relativistic nor the relativistic version of cold emission processes in presence of strong quantizing magnetic field, relevant for electron emission from the poles of strongly magnetized neutron stars/magnetars, even with simple type potential barriers have been properly investigated. In the conventional pulsar model it is generally assumed that the emission of electrons and thereby formation of magnetosphere is mainly caused by strong electric field at the polar region which is produced by the strong magnetic field of rotating neutron stars. The flow of high energy electrons along the direction of magnetic lines of forces and their penetration through the light cylinder is conventionally pictured with the current carrying conductors. Naturally, if the conductor is broken near the pulsar surface the entire potential difference will be developed across a thin gap, called polar gap. This is of course based on the assumption that above a critical height from the polar gap, because of high electrical conductivity of the plasma, the electric field $F$, parallel to the magnetic field near the poles is quenched. Further, a steady acceleration of electrons originating at the polar region of neutron stars, travelling along the field lines, will produce magnetically convertible curvature $\gamma$-rays. If these curvature $\gamma$-ray photons have energies $> 2m_e c^2$ (with $m_e$ is the electron rest mass and $c$ is the velocity of light), then pairs of $e^- - e^+$ will be produced in enormous amount with very high efficiency near the polar gap. These produced $e^- - e^+$ pairs form what is known as the magneto-spheric plasma. The cold emission, therefore plays a significant role in magneto-spheric plasma formation. In turn, the motion of charged particles in the magnetosphere in presence of strong magnetic field causes pulsar emission in the form of synchrotron radiation. Therefore the cold emission process indirectly also affects the intensity of synchrotron radiation.

Further the exactly solvable models with simple type tunneling barrier lead to equations that underestimate the emission current density by a factor of 1000 or more. If a more realistic type barrier model is used by inserting an exact surface potential in the simplest form of the Schrödinger equation, then a complicated mathematical problem arises over the resulting differential equation. It is in principle therefore mathematically impossible to solve the equation exactly in terms of the usual functions of mathematical physics, or in any simple way.

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NON-RELATIVISTIC SCENARIO 

To develop the modified version of field emission process for the electrons in the non-relativistic scenario in presence of a strong quantizing magnetic field, we have followed the basic calculation presented in the seminal Royal Society paper by Fowler & Nordheim (1928). In the modified version, we assume a cylindrical co-ordinate system $(\rho, \theta, z)$ and the constant magnetic field $\vec{B}$ is along positive $z$-direction, with the usual gauge for the vector potential $A = (\vec{B} \times \vec{\rho})$. Following Fowler & Nordheim (1928), we assume that the triangular shape surface potential is given by $V(z) = C - Fz$, which is changing linearly with $z$-coordinate. Here $C$ is the constant surface barrier and $F$ (absorbing the magnitude of electron charge

In the next section we have studied the effect of strong quantizing magnetic field on the field emission of electrons for the non-relativistic case. In section-3, we have repeated the same calculation for the relativistic scenario. In the last section we have given the conclusions and future prospects of this work.
e, we replace \( eF \) by \( F \) is the driving field for electron emission from the poles. Consideration of linear type surface barrier potential has no-doubt some historical importance. This type of potential barrier was first used in the original work by Fowler and Nordheim. However, there are two other important reasons: since this is the first time the problem is solved in presence of strong quantizing magnetic field, therefore to get an analytical solution, we have considered such simplest triangular type potential barrier. Our intension is to solve the problem analytically in a way in which the physical meaning of the problem is not lost. The other reason behind such a choice is that, in the case of strongly magnetized rotating neutron stars / magnetars, the produced electric field at the polar region is approximately constant for uniform magnetic field strength at that region and constant rotational period of the object. We believe that for the potential barrier at the poles both assumptions are approximately valid. Then assuming the conservative force field relation \( dV/dz = -eF \) we get the triangular type potential barrier at the poles. Under such situation, the Schrödinger equation satisfied by the electrons which are confined within the matter (in this case within the neutron star/magnetar crustal matter) is given by (throughout the paper for the sake of convenience we assume natural units, i.e., \( \hbar = c = 1 \))

\[
- \frac{1}{2m} \left[ \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial \psi}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 \psi}{\partial \theta^2} + \frac{\partial^2 \psi}{\partial z^2} \right] - \frac{ieB}{2m} \frac{\partial \psi}{\partial \theta} + \left( \frac{e^2 B^2 \rho^2}{8m_e} - E \right) \psi = 0
\]  

Whereas for the electrons just liberated out through quantum mechanical tunneling, one has to consider the potential \( V(z) \) along with \( E \). The relevant equation is given by

\[
- \frac{1}{2m} \left[ \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial \psi}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 \psi}{\partial \theta^2} + \frac{\partial^2 \psi}{\partial z^2} \right] - \frac{ieB}{2m} \frac{\partial \psi}{\partial \theta} + \left( \frac{e^2 B^2 \rho^2}{8m_e} - E + V(z) \right) \psi = 0
\]  

where the energy eigen value \( E \) is given by the eqns.(4) and (5) for two different physical situations. If we assume a separable solution for the eqns.(1) and (1a), satisfied by freely moving electrons and moving under the potential \( V(z) \), the wave functions can be represented by

\[
\psi(\rho, \theta, z) = \phi_{n_\rho, m}(\rho, \theta)f_\nu(z)
\]  

where for eqn.(1), the longitudinal part is plane wave type, whereas for eqn.(1a), we shall evaluate \( f_\nu(z) \) using the technique as discussed below. Since there is no potential associated with the transverse motion for the electrons, the free transverse part of the wave function is given by

\[
\phi_{n_\rho, m}(\rho, \theta) = \frac{\exp(i m \theta)}{(2\pi)^{1/2} \rho_0} \frac{1}{\rho_0^{1-|m|}} \left[ \left( \frac{|m| + n_\rho!}{2^{n_\rho} n_\rho! \ |m| !} \right) \right] 
\times \rho^{|m|} \exp \left( -\frac{\rho^2}{4\rho_0^2} \right) L_{n_\rho, m} \left( \frac{\rho^2}{2\rho_0^2} \right)
\]  

with \( \rho_0 = (2/eB)^{1/2} \) is the Larmor radius, the radius of the lowest Landau orbit and \( L_{n_\rho, m} \) is the Laguerre polynomial. The energy eigen value is given by

\[
E = \frac{\rho^2}{2m_e} + \mu_B B(2n_\rho + \nu \pm \nu + 1)
\]  

without the electron spin and with the inclusion of the electron spin, it will be

\[
E = \frac{\rho^2}{2m_e} + \mu_B B(2n_\rho + \nu \pm \nu + 1) + \mp \mu_B B
\]  

where \( \mu_B = e/2m \), the Bohr magneton. For the motion of electrons along \( z \)-direction, the Schrödinger equation satisfied by \( f_\nu(z) \) with and without potential \( V(z) \) can be obtained by averaging eqns.(1) and (1a) respectively over the transverse wave function \( \phi_{n_\rho, m}(\rho, \theta) \). Then for just tunnelled out electrons in the lowest Landau level with \( n_\rho = \nu = m = 0 \), moving along \( z \)-axis under the influence of the potential \( V(z) \) and having no spin contribution, we have

\[
- \frac{1}{2m} \frac{d^2 f_0}{dz^2} + V(z) f_0 = (E - \mu_B B) f_0 = w f_0
\]  

Whereas for the same kind of liberated electrons with spin polarization in the negative direction of \( z \)-axis, we have \( w = E \) and for the polarization along the direction of magnetic field, \( w = E - 2\mu_B B \). On the other hand, for the electrons confined within the crustal matter of magnetars and moving freely, we have \( V(z)=0 \). The corresponding Schrödinger equation is given by

\[
\frac{d^2 f_0}{dz^2} + w^2 f_0 = 0
\]  

Where \( w_0 = (2m w)^{1/2} \), is the equivalent electron momentum along \( z \)-axis and \( w \) can have three possible values as mentioned above. Then following eqn.(10) of Fowler & Nordheim [1928], we can write down the solution of eqn.(7) for free electrons inside the crustal matter in the form

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In absence of electron spin term or polarization along the direction of magnetic field, the factor $Q$ in our formalism is defined as

$$Q = \frac{2}{3}(2m_eF)^{1/3} \left( \frac{C - w}{F} \right)^{3/2}$$

and is related to the Hankel function argument by the relation

$$H_{1/3}^{(2)} \left[ \exp \left( -\frac{3}{2} \pi \right) Q \right] \text{ at } z = 0$$

where $H_{1/3}^{(2)}(x)$ is the Hankel function of second kind of order 1/3 with argument $x$. As we shall see below the quantity $Q$ defined in Fowler & Nordheim (1928) after eqn.(12) is much larger in our formalism in presence of strong quantizing magnetic field, unless the direction of spin polarization for the emitted electrons are opposite to the direction of external magnetic field. In absence of electron spin term or polarization along the direction of magnetic field, the factor $Q$ is virtually infinitely large ($Q \approx \infty$). We shall further show that this infinitely large $Q$ will make the transmission probability of electrons vanishingly small. In our formalism the factor $Q$ is defined as

$$Q = \frac{2}{3}(2m_eF)^{1/3} \left( \frac{C - w}{F} \right)^{3/2}$$

and is related to the Hankel function argument by the relation

$$H_{1/3}^{(2)} \left[ \exp \left( -\frac{3}{2} \pi \right) Q \right] \text{ at } z = 0$$

Now the transmission probability for electrons as defined in Fowler & Nordheim (1928) is given by

$$D(w) = \left| \frac{a^2 - |a'|^2}{|a|^2} \right|$$

Since the transmission coefficient $D$ related to $Q$ factor by the relation $D \sim \exp(-2Q)$, and the field emission current for the electrons from the zeroth Landau level is related to $D$ by the integral

$$R = \frac{eB}{2\pi} \int_{0}^{\infty} f(w)D(w) d\frac{p_z}{m_e}$$

with $f(w)$ the Fermi distribution function, then it is quite obvious that in the electron field emission current, the polarization effect will come only from the transmission coefficient $D(w)$. In the following, we therefore focus our study in the investigation of the properties of the emission coefficient. For large $Q$, we have from eqn.(18) of Fowler & Nordheim (1928) in our modified form

$$D(w) \approx \frac{[w(C - w)]^{1/2}}{C} \exp \left( -\frac{4}{3}(2m_eF)^{1/2} \left( \frac{C - w}{F} \right)^{3/2} \right)$$

Let us now analyze the argument part of the exponential. Following Fowler & Nordheim (1928), we put $C = \mu_e + w_f$, where $\mu_e$ is the electron Fermi energy and $w_f = w_e \times (B/B_0)^{1/2}$ in eV is the work function (see Ghosh & Chakrabarty 2011) for the emission of electrons along the direction of magnetic field, where $w_e \approx 82.93$ and $B_0^{(c)} \approx 4.43 \times 10^{13}$G, the typical value of magnetic field strength at which the Landau levels for the electrons are populated in the relativistic scenario. Again, as defined before, the quantity $w$ can have three possible values. To get an order of magnitude estimate for the terms containing Bohr magneton and work function, we assume that the emitted electrons carry the maximum possible energy, i.e., the electron Fermi energy for temperature $T \rightarrow 0$ limit. Then $C - w = w_f + \beta \times \mu_B B$, where the parameter $\beta = 0$, or $= 1$ or $= 2$ for the spin polarization opposite to the direction of external magnetic field, i.e., with conventional direction of spin polarization in presence of strong magnetic field, or no spin term or spin polarization along the direction of magnetic field respectively. Taking into account the denominator $F$ of the argument and expressing in terms of magnetic field strength as defined at the beginning of the introduction and using $\mu_B \approx 5.79 \times 10^{-15}$MeV G$^{-1}$, the numerical value for the Bohr magneton, we have approximately from the expression for $Q$ as defined above

$$Q \approx h_0^{1/2} (\beta \times 0.5 + w_e h_0^{-1/2} \times 10^{-6})^{3/2} \times 10^7 = QB + Qw_f$$

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where \( h_f = B/B_0^\beta \), \( Q_B \) and \( Q_{\text{wf}} \) are the contributions from Bohr magneton or spin term and work function part respectively. It is quite obvious that the contribution from spin term \( Q_B \) is extremely large for \( \beta = 1 \) or \( \beta = 2 \) and makes the transmission coefficient exactly zero. Which physically means that if we do not consider electron spin or assume electron spin polarization along the direction of magnetic field, the electron transmission coefficient and in turn the electron transmission current vanishes exactly, i.e., there will be no field emission under such situations. On the other hand, the second term, the work function part, unlike the first term, gives finite contribution to transmission coefficient. The first term, i.e., the spin term will make \( \beta \) exactly, i.e., there will be no field emission under such situations. On the other hand, the second term, the work function along the direction of magnetic field, the electron transmission coefficient and in turn the electron transmission current vanishes exactly zero for the non-zero values of \( \beta \). For the transmission of spin polarized electrons \( \beta = 0 \) and consequently \( Q_B = 0 \). Therefore \( Q = Q_{\text{wf}} \). In this situation the electron energy eigen value obtained from eqn.\((5)\) for \( n_\nu = \nu = m = 0 \) is given by

\[
E = \frac{p_z^2}{2m_e}
\tag{15}
\]

In this case there will be electron field emission with their spins polarized opposite to the direction of external magnetic field. From the above expressions for electron energy eigen value, it is quite obvious that for our model on cold emission of polarized electrons the rest of the mathematical formulation will be almost identical with that of Fowler and Nordheim in presence of strong quantizing magnetic field. We have further noticed that the values for transmission coefficient \( D \) and the transmission current \( R \) remain finite and large enough for the magnetic field strength \( \leq 10^{15} \text{G} \). Of course in our model the magnetic field dependency of \( Q \) will come from the work function \( w_f \) and the field intensity \( F \) and only the polarized electrons are allowed to tunnel through the surface barrier. Then following Fowler and Nordheim we have obtained the cold electron emission current from eqn.\((12a)\) at \( T \rightarrow 0 \) using Sommerfeld’s lemma. While calculating emission current numerically, we have used the exact form of \( D \) as given in Fowler & Nordheim \((1928)\) after eqn.\((16)\). In fig.\((1)\) we have shown schematically the variation of cold electron current with the strength of electric field intensity at the poles and is represented by the solid curve. Since the electric field at the poles is produced by the rotating magnetic field, we have also shown in the same graph with dashed curve the variation of electron cold current with the intensity of magnetic field. Since the electric field intensity varies linearly with the magnetic field strength, the qualitative nature of the curves are almost identical. For the neutron stars with very low surface magnetic field, the produced electric field, which acts as driving force, is also small enough, therefore the electron field emission current will be extremely small as shown in the curve. For the neutron stars with moderate surface magnetic field strength \( (B \sim 10^{12} - 10^{15} \text{G}) \) the field current is quite high. Now for the objects with large surface magnetic field \( (B \gg 10^{15} \text{G}) \), the work function will also become large. Therefore beyond some maximum value for electron field current, since the work function part dominates over the driving electric force at the poles, the electron field current will decrese and will become vanishingly small. This is the case for the objects with ultra-high surface magnetic field \( (\gg 10^{15} \text{G}) \). In the figure, we have not shown the variation of electron field current with the magnetic field strength (computed at the surface) for a particular neutron star. Therefore each magnetic field / electric field points corresponds a particular type of compact magnetized object, with surface magnetic field from very low to ultra-high values. From the curves it is quite obvious that for magnetars with surface magnetic field \( (\sim 10^{15} \text{G}) \), the electron field current is quite high and very close to the peak value. Unlike the original work of Fowler and Nordheim the tunneling coefficient does not follow exponential law. However, the variation of cold current with the electric field strength can be obtained numerically. The numerically fitted functional form is given by

\[
R = 0.26F_{24}^{1/2}\exp(-9.8 \times F_{14})
\tag{16}
\]

where \( R \) is the field current in Amp/cm\(^2\), \( F_{14} = 10^{-14} F \) and \( F_{24} = 10^{-24} F \).

### 3 EFFECT OF STRONG QUANTIZING MAGNETIC FIELD ON COLD EMISSION: RELATIVISTIC SCENARIO

In the relativistic scenario we have repeated the non-relativistic calculation for the cold emission transmission co-efficient. In this section we have considered emission of high energy electrons from the polar region of strongly magnetized neutron stars with magnetic field \( 10^{12} \leq B \leq 10^{17} \) in Gauss. The potential is introduced by hand in the Dirac equation using standard relativistic hadro-dynamic technique. Following Fowler and Nordheim, here also we have considered a triangular type potential barrier at the polar region. Like the previous case, we have considered cylindrical coordinate system and the choice of gauge is same for \( \hat{A} \). The radial part of upper component satisfies the equation

\[
\left[ \beta_\lambda + \frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho} - k^2 \rho^2 \right] R(\rho) = 0
\tag{17}
\]

where \( \beta_\lambda = E^2 - m^*{}^2 - p_z^2 + 2\lambda k \) with \( \lambda = \pm 1 \) for up and down spin states respectively, \( k = eB/2 \), \( E \) is the energy eigen value for the Dirac equation and \( m^* = m + V(z) = m + C - Fz \) is the effective electron mass and \( m^* = m \) for free electrons when \( V(z) = 0 \). The solution of the above equation is given by
\[ R(\rho) = N \exp\left(-\frac{\rho}{2}\right) L_n(\rho) \]  

where \( L_n(\rho) \) is the Legendre polynomial of order \( n \), \( \rho = k^2 \rho^2 \) and \( N \) is the normalization constant. For up and down spin states, the energy eigenvalues are given by \( E_\uparrow = [p_\uparrow^2 + m^2 + 2neB]^{1/2} \) and \( E_\downarrow = [p_\downarrow^2 + m^2 + 2(n + 1)eB]^{1/2} \) respectively. In general we may write \( E_\nu = (p_\nu^2 + m^2 + 2\nu eB)^{1/2} \). Then following the same averaging technique as discussed in the previous section for the non-relativistic case, the wave function of electrons corresponding to the motion along \( z \)-direction is given by

\[ \alpha \frac{d^2 \nu}{du^2} + 2(\nu - u^2) \nu = 0 \]  

Using the transformation

\[ z = \frac{m + C - uF^{1/2}}{F} \]

with the new variable \( u \), the above equation for \( \nu \) reduces to

\[ \alpha \frac{d^2 \nu}{du^2} + (\alpha^2 - u^2) \nu = 0 \]

where \( \alpha = (E^2 - 2ueB)^{1/2}/F^{1/2} \), which is the well known form of differential equation for one dimensional quantum mechanical harmonic oscillator. With \( \alpha^2 = 2l \), we have \( E^2 = 2(ueB + 1F) \), and the solution is given by

\[ \nu \nu = i\bar{N} \exp\left(-\frac{u^2}{2}\right) H_l(u) = (\nu \nu)_I \]  

where \( \bar{N} \) is the normalization constant and \( H_l(u) \) is the Hermite polynomial of order \( l \). This spinor solution \( \nu \nu \nu \) is for those electrons which have already been liberated out through the surface into vacuum under the influence of electric field \( F \) (here liberated out from the crustal matter of strongly magnetized neutron stars or magnetars into the magnetosphere through polar region).

The equation satisfied by free electrons bound within the crustal matter by the barrier potential at the surface can be obtained by putting \( V(z) = 0 \) and is given by

\[ \alpha \frac{d^2 \nu}{dz^2} + \alpha^2 \nu = 0 \]  

where in this case, \( \alpha = (E^2 - m^2 - 2\nu eB)^{1/2} \) is the free electron momentum along \( z \)-axis of energy \( E \). Now following the notation of [1928], we express the solution for free electrons within the system, confined by the surface barrier \( V(z) \), in the form

\[ \nu \nu = \frac{1}{\alpha^{1/2}} \left[ a \exp(i\alpha z) + a' \exp(-i\alpha z) \right] = (\nu \nu)_II \]  

where as before \( a \) is the probability amplitude for electrons moving along the positive direction of \( z \)-axis (incident part), whereas \( a' \) is the corresponding quantity for left moving waves (reflected part from the surface barrier). Assuming the interface between the crustal matter of the strongly magnetized neutron stars and the magnetosphere is at \( z = 0 \), the wave function and their derivatives must be continuous at \( z = 0 \), i.e.,

\[ (\nu \nu)(0)_{II} = (\nu \nu)(0)_{II} \quad \text{and} \quad (\nu \nu)'(0)_{II} = (\nu \nu)'(0)_{II} \]

Using the relation \( H_I(u) = 2\nu H_{I-1}(u) \), we have

\[ a + a' = N\alpha^{1/2} \exp\left(-\frac{u_0^2}{2}\right) H_l(u_0) \]  

and

\[ i\alpha^{1/2}(a - a') = N \exp\left(-\frac{u_0^2}{2}\right) [u_0H_l(u_0) - 2lH_{l-1}(u_0)] \]

where \( u_0 = u(z = 0) = m/F^{1/2} \). These two conditions may be rearranged in the following form

\[ a + a' = X \quad \text{and} \quad a - a' = iy \]

where \( X \) and \( Y \) are the two real quantities. Hence it is straightforward to verify from eqn.(12) that in the relativistic scenario the transmission coefficient vanishes exactly. Therefore from the analysis of this section, we may conclude that if the barrier in combination with the external electrostatic driving force behaves like a scalar type potential and is triangular in shape at the surface, then the relativistic electrons cannot tunnel through the surface barrier whatever be their kinetic energies and the strength of external electric field.
4 CONCLUSION

The non-relativistic scenario of cold electron emission in presence of strong magnetic field is believed to be the first attempt in this direction. While obtaining the electron transmission probability in the non-relativistic scenario under the influence of strong electric field at the poles, we have noticed that in our theoretical formalism, the emission is allowed if we take electron in this direction. While obtaining the electron transmission probability in the non-relativistic scenario under the influence of strong magnetic field, we have noticed that in our theoretical formalism, the emission is allowed if we take electron in this direction.

In conclusions, from our relativistic formalism of cold emission of electrons we can state that relativistic electrons populating the neutron star interior can not be extracted from cold emission from the poles of a neutron star, independently from the magnetic field strength. Non-relativistic electrons with anti-parallel spin can be extracted for standard (observed) values of magnetic field strengths, but can not be extracted from the surface of objects with $B \gg 10^{15} \text{G}$ (in the eventuality that such exotic objects can exist).

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Figure 1. (i) With dashed curve the variation of electron field current plotted along left side vertical axis with the neutron star magnetic field plotted along $x$-axis at the bottom and expressed in terms of critical field $B_c^{(e)}$ for electron is shown. (ii) The variation of same quantity plotted along right side vertical axis with the electric field in Volt/cm produced by rotating magnetic field of magnetars, plotted along horizontal axis at the top is shown by solid curve.

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