THE BISPECTRUM OF GALAXIES FROM HIGH-REDSHIFT GALAXY SURVEYS: PRIMORDIAL NON-GAUSSIANITY AND NON-LINEAR GALAXY BIAS

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ABSTRACT

The greatest challenge in the interpretation of galaxy clustering data from any surveys is galaxy bias. Using a simple Fisher matrix analysis, we show that the bispectrum provides an excellent determination of linear and non-linear bias parameters of intermediate and high-z galaxies, when all measurable triangle configurations down to mildly non-linear scales, where perturbation theory is still valid, are included. The bispectrum is also a powerful probe of primordial non-Gaussianity. The planned galaxy surveys at $z \gtrsim 2$ should yield constraints on non-Gaussian parameters, $f_{NL}$ and $\tilde{f}_{NL}$, that are comparable to, or even better than, those from CMB experiments. We study how these constraints improve with volume, redshift range, as well as the number density of galaxies. Finally, we show that a halo occupation distribution may be used to improve these constraints further by lifting degeneracies between gravity, bias, and primordial non-Gaussianity.

Subject headings: cosmology: theory - large-scale structure of the Universe

INTRODUCTION

Why study high-$z$ galaxy surveys? The recognition that baryon acoustic oscillations in the galaxy power spectrum (Cole et al. 2005; Eisenstein et al. 2005; Hu et al. 2006; Percival et al. 2007) are an excellent probe of the nature of dark energy has led to several proposals for large-volume redshift surveys at $z \gtrsim 1$.

Galaxies are a biased tracer of the underlying matter distribution. The use of highly biased tracers, such as luminous red galaxies at lower $z$ and Lyman break galaxies or Lyman-$\alpha$ emitters at higher $z$ requires a reliable modelling of non-linearity and scale-dependence of galaxy bias, even at relatively large spatial scales (Smith et al. 2006, 2007; McDonald 2006).

The distribution of galaxies is non-Gaussian. The galaxy bispectrum, the three-point correlation function in Fourier space, does not vanish. It has been known for more than a decade that the bispectrum is an excellent tool for measuring galaxy bias parameters, independent of the overall normalization of dark matter fluctuations (Fry 1994; Matarrese et al. 1997; Scoccimarro et al. 2001a). This method has been applied successfully to existing galaxy surveys such as the 2dFGRS and the SDSS, yielding constraints on non-linearities in galaxy bias (Verde et al. 2002; Pan & Szapudi 2003; Gaztanaga et al. 2005; Nishimichi et al. 2006) as well as on the Halo Occupation Distribution (HOD) (Kulkarni et al. 2007). Moreover, it has been shown that the galaxy bispectrum contains additional cosmological information that is not present in the power spectrum (Sefusatti & Scoccimarro 2003; Sefusatti et al. 2006).

The galaxy bispectrum on large scales, or other statistical tools that are sensitive to the higher-order correlation of galaxies, are sensitive to statistical properties of primordial fluctuations: primordial non-Gaussianity.

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2.1. Primordial non-Gaussianity

We explore two parametrizations of primordial non-Gaussianity which are motivated by inflationary models. While representing a wide variety of non-Gaussian models, these parametrizations are by no means exhaustive. Our method can be applied to any other functional forms of the bispectrum (e.g., Liguori et al. 2006; Chen et al. 2006) in a straightforward way.

2.1.1. Local model

The first one is described by the local expression for Bardeen’s curvature perturbations during the matter era, \( \Phi(x) \), in position space \( \Phi_G(x) \). For this local model, most which has been hinted by WMAP (Spergel et al. 2006), the Gaussianity. In this case, the leading contribution in the bispectrum (e.g., Liguori et al. 2006; Chen et al. 2000; Komatsu & Spergel 2001)

\[
\Phi(x) = \Phi_G(x) + f_{NL}^G(\Phi_G(x) - \langle \Phi_G^2(x) \rangle),
\]

(1)

where \( \Phi_G(x) \) is a Gaussian field and \( f_{NL}^G \) is a constant characterizing the amplitude of primordial non-Gaussianity. In this case, the leading contribution in the \( f_{NL}^G \) expansion to the bispectrum, \( B_{\Phi}^{local}(k_1, k_2, k_3) \), of the curvature field is given by

\[
B_{\Phi}^{local} \simeq 2 f_{NL}^G \langle \Phi(k_1) \Phi(k_2) + \text{cyc} \rangle
= 2 f_{NL}^G C_2 \Phi \left[ \frac{1}{k_1^{4-n_S} k_2^{4-n_S}} + \text{cyc} \right],
\]

(2)

where we approximate \( P_{\Phi}(k) \approx P_{\Phi_G}(k) \), and

\[
C_\Phi = \frac{P_{\Phi}(k)}{k^{n_s - 4}},
\]

(3)

which quantifies departure from a scale-invariant spectrum. We include \( n_s \) explicitly, as we are interested in determining how a departure from scale invariance, which has been hinted by WMAP (Spergel et al. 2006), would affect detectability of primordial non-Gaussianity in the distribution of galaxies. For this local model, most of the signal is given by squeezed triangular configurations, \( k_1 \ll k_2, k_3 \). The local type of non-Gaussianity described by equation (2) is predicted in models such as the curvaton scenario (Lyth et al. 2003), models with inhomogeneous reheating (Dvali et al. 2003a), multiple field inflationary models (Bernardeau & Uzan 2002) or generically in models where the non-linearities arise from the evolution of perturbations outside the horizon.

The best limits to date on possible values for the \( f_{NL}^G \) parameter come from measurements of the microwave background bispectrum on the WMAP data (Komatsu et al. 2003; Spergel et al. 2006; Creminelli et al. 2007) \(-36 \leq f_{NL}^G \leq 100 \) at 95% C.L., which corresponds to the 1-\( \sigma \) error of

\[
\Delta f_{NL}^G = 34 \quad \text{(WMAP3)},
\]

(4)

which is a factor of 50 better than the limit from COBE (Komatsu et al. 2002). Upon completion, WMAP is expected to reach \( \Delta f_{NL}^G \simeq 20 \), while the Planck satellite would yield \( \Delta f_{NL}^G \simeq 3 \) (Komatsu & Spergel 2001; Babich & Zaldarriaga 2004; Yadav et al. 2007).

Measurements of the galaxy bispectrum in the SDSS main sample are expected to yield \( \Delta f_{NL} \lesssim 150 \) (Scoccimarro et al. 2004). Recently Pillepich et al. (2006) pointed out that a full-sky measurement of the bispectrum of fluctuations in the 21-cm background might reach \( f_{NL}^{loc} \sim 1 \), while a more aggressive analysis by Cooray (2006) shows that the same observations could reach \( \Delta f_{NL}^{loc} \sim 0.01 \) in principle. Other large-scale structure probes such as cluster abundance, on the other hand, is unlikely to improve CMB limits on \( f_{NL}^{loc} \) (Sefusatti et al. 2007); however, it should provide an important cross-check of the results if a significant \( f_{NL}^{loc} \) was detected in the CMB, and it should not be forgotten that the spatial scales probed by the cluster abundance is smaller than those probed by the CMB.

2.1.2. Equilateral model

The second model for primordial non-Gaussianity is given by

\[
B_{\Phi}^{equil.} = 6 f_{NL}^G C_2^G \left[ \frac{1}{k_1^{4-n_S} k_2^{4-n_S} k_3^{4-n_S}} + \frac{2}{k_2^{4-n_S} k_3^{4-n_S}} + \frac{1}{k_1^{4-n_S} k_2^{4-n_S} k_3^{4-n_S}} \right].
\]

(5)

Babich et al. (2004) and Creminelli et al. (2007) have shown that this form provides a good approximation to the bispectra predicted by higher derivatives and DBI inflationary models (Creminelli 2003; Alishahiha et al. 2004).

The bispectrum in equation (5) is normalized in such a way that for equilateral configurations \( (k_1 = k_2 = k_3 = \) eq), it coincides with the local form given in equation (2). The important difference is that this form has the largest contribution from the equilateral configurations, as opposed to the local form in which the largest contribution comes from the squeezed configurations. The current limits from WMAP are \(-256 \leq f_{NL}^{eq} \leq 332 \) at 95% C.L. (Creminelli et al. 2007), which corresponds to the 1-\( \sigma \) error of

\[
\Delta f_{NL}^{eq} = 147 \quad \text{(WMAP3)}.
\]

(6)

2.1.3. The primordial density bispectrum

Density fluctuations in Fourier space, \( \delta_k \), are related to the curvature perturbations, \( \Phi_k \), via the Poisson equation, \( \delta_k = M(k; a) \Phi_k \), where

\[
M(k; a) = \frac{2}{3} \frac{D(a)}{H^2(a) \Omega_m} k^2 T(k).
\]

(7)

Here \( a \) is the scale factor, \( T(k) \) is the matter transfer function, and \( D(a) \) is the growth function \(^3\).

This allows us to write the primordial contribution to a generic \( n \)-point function of the matter density fields in terms of the respective correlator of the curvature perturbations as

\[
\langle \delta_{k_1} \delta_{k_2} ... \delta_{k_N} \rangle_I = M(k_1; a) M(k_2; a) ... M(k_N; a) \langle \Phi_{k_1} \Phi_{k_2} ... \Phi_{k_N} \rangle.
\]

(8)

\(^3\) The function, \( M(k; a) \), uses the same definition as in Verde et al. (2000) for \( \Omega_m = 1 \), but it differs from \( M(k) \) given in Hikage et al. (2006), where the dependence on the growth function has been taken out as \( M(k; a) = M_{HM}(k) D(a) \). Also, the same function is defined in Scoccimarro et al. (2004) in terms of the gravitational potential (i.e., the 0-0 component of the metric perturbations) during radiation domination, so that \( M(k; a) = -\bar{\rho} M_{SSS}^{eq}(k; a) \) leading to a different definition of the non-Gaussian parameter, \( f_{NL} = -\frac{\bar{\rho}}{H^2} \nabla^S S \).
In particular, the initial (primordial) matter bispectrum, $B_I(k_1, k_2, k_3)$, is given by

$$B_I(k_1, k_2, k_3) = M(k_1)M(k_2)M(k_3) \times B_0(k_1, k_2, k_3),$$

where we have omitted for brevity the explicit dependence of $M(k; a)$ on $a$. We can also relate the linear density power spectrum, $P_L(k)$, to the curvature power spectrum, $P_Q(k)$, as

$$P_L(k) = M^2(k)P_Q(k).$$

A hierarchical relation between the scale dependence of the initial bispectrum and the power spectrum such as $B_0(k) \sim fNL^2P_0^2(k)$ with a constant $fNL$ is by no means generic or universal. A $fNL$ with a peculiar scale dependence unrelated to the power spectrum appears, for instance, in string-motivated models such as DBI inflation (Alihashah et al. 2004; Chen 2005). Our analysis can be applied to any models of primordial non-Gaussianity, provided that the bispectrum can be calculated from those models.

Note that a post-Newtonian effect can yield an additional contribution to non-linearity of primordial perturbations and hence to non-Gaussianity (Bartolo et al. 2003). Although we do not include this effect in our analysis, it would be interesting to study how important the post-Newtonian effect would be for the future galaxy surveys.

2.2. Non-Gaussianity from non-linear gravitational evolution

Even if the initial perturbations are Gaussian, the subsequent gravitational evolution makes the evolved density fields non-Gaussian. On large scales one can study the non-linear evolution of matter density fluctuations by means of perturbation theory, and write the solution up to the second order in $\delta$ as

$$\delta_k \approx \delta_k^{(1)} + \int d^3q_1 d^3q_2 \delta_D(k-q_1-2q_2)F_2(q_1, q_2)\delta_k^{(1)}(q_1)\delta_k^{(1)}(q_2),$$

where $\delta_k^{(1)}$ is the linear solution, and $F_2(k_1, k_2)$ is a known mathematical function given by

$$F_2(k_1, k_2) = \frac{5}{7} + \frac{x}{2} \left( \frac{k_1}{k_2} + \frac{k_2}{k_1} \right) + \frac{2}{7} x^2,$$

with $x \equiv k_1 \cdot k_2$. Therefore, one obtains

$$B_G(k_1, k_2, k_3) = 2F_2(k_1, k_2)P_L(k_1)P_L(k_2) + \text{cyc.}$$

The bispectrum of matter density fluctuations (i.e., no galaxies yet) evolved from non-Gaussian primordial fluctuations on large scales is thus given by the sum of equation (11) and (13):

$$B(k_1, k_2, k_3) = B_I(k_1, k_2, k_3) + B_G(k_1, k_2, k_3).$$

As usual, we shall focus on the reduced bispectrum, defined as

$$Q(k_1, k_2, k_3) = \frac{B(k_1, k_2, k_3)}{P(k_1)P(k_2) + \text{cyc.}}$$

which has an advantage of being only mildly sensitive to cosmological parameters. That is to say, the dependence on cosmology has been “factored out” by a product of the power spectra in the denominator and the $Q_G$ component is particularly insensitive to the amplitude of matter fluctuations (e.g., $\sigma_8$). The reduced bispectrum of matter density fluctuations is also given by the sum of two contributions:

$$Q(k_1, k_2, k_3) = Q_I(k_1, k_2, k_3) + Q_G(k_1, k_2, k_3)$$

$$= \frac{B_I(k_1, k_2, k_3)}{P(k_1)P(k_2) + \text{cyc.}} + \frac{B_G(k_1, k_2, k_3)}{P(k_1)P(k_2) + \text{cyc.}}.$$
Fig. 1.— Equilateral configurations of the reduced bispectrum of dark matter distribution at the second order (“tree-level”). The horizontal lines at $Q(k) = 0.57$ show the gravitational contribution only, which corresponds to $f_{NL}^{loc} = 0 = f_{NL}^{eq}$. The solid, long-dashed and short-dashed lines show the gravitational contribution with non-Gaussian initial perturbations at $z = 0, 1,$ and $4$, respectively. Note that $f_{NL}^{loc} = f_{NL}^{eq}$ for equilateral configurations. (Left panel) The curves above $Q(k) = 0.57$ show $f_{NL} = +100$, while the curves below it show $f_{NL} = -100$. (Right panel) The same as the left panel but for the current WMAP3 limits on $f_{NL}^{loc}$ (top) and $f_{NL}^{eq}$ (bottom).

Field, one may expand the galaxy number overdensity, $\delta_g$, in Taylor series of the underlying matter overdensity, $\delta$, as (Fry & Gaztanaga 1993)

$$\delta_g(x) \simeq b_1 \delta(x) + \frac{1}{2} b_2 \delta^2(x),$$

where $b_1$ and $b_2$ are the linear and non-linear bias parameters, respectively. It has been shown that this model describes the bispectrum or three-point correlation functions from the SDSS and 2dFGRS (e.g., Gaztanaga et al. 2005; Nishimichi et al. 2006), as well as from numerical simulations (e.g., Marin et al. 2007). The galaxy bispectrum is given by

$$B_g(k_1, k_2, k_3) = b_1^2 Q(k_1, k_2, k_3)$$

up to the second order in matter density fluctuations. Here, $B(k_1, k_2, k_3)$ is the intrinsic bispectrum of the underlying matter distribution. The reduced galaxy bispectrum is

$$Q_g(k_1, k_2, k_3) \simeq \frac{1}{b_1} Q(k_1, k_2, k_3) + \frac{b_2}{b_1^2},$$

with $Q(k_1, k_2, k_3) = Q_G(k_1, k_2, k_3) + f_{NL} Q_I(k_1, k_2, k_3)$, one obtains

$$Q_g(k_1, k_2, k_3) \simeq \frac{1}{b_1} Q_G(k_1, k_2, k_3) + \frac{b_2}{b_1} + \frac{f_{NL} Q_I(k_1, k_2, k_3) + b_2}{b_1^2},$$

where we have factorized $f_{NL}$ out from $Q_I$ introducing $Q_I \equiv Q_I(f_{NL} = 1)$.

Finally, the galaxy power spectrum is given by $P_g(k) \simeq b_1^2 P(k)$, up to the second order in density fluctuations; however, corrections due to non-linear bias appear at the third-order level (Heavens et al. 1998; Taruya 2000; Smith et al. 2006; McDonald 2006). Therefore, the bias parameters, $b_1$ and $b_2$, affect both the galaxy power spectrum and bispectrum. The bispectrum helps us extract the cosmological information from the galaxy power spectrum by providing $b_1$ and $b_2$.

2.4. Redshift space distortion

The bispectrum measured from redshift surveys is distorted along the line of sight direction by radial motion of galaxies. For our analysis in this paper we shall deal only with a spherically averaged power spectrum and bispectrum. The power spectrum in redshift space after averaging over angles in $k$ space, $P_s(k)$, is related to the real space power spectrum by

$$P_s(k) = a_0^B(\beta) P_g(k),$$

while the bispectrum is given by

$$B_s(k_1, k_2, k_3) = a_0^B(\beta) B_g(k_1, k_2, k_3),$$

where (Kaiser 1987; Sefusatti et al. 2006)

$$a_0^B(\beta) = 1 + \frac{2}{3} \beta + \frac{1}{9} \beta^2,$$

$$a_0^P(\beta) = 1 + \frac{2}{3} \beta + \frac{1}{9} \beta^2,$$

with $\beta \equiv \Omega_m^{5/7}/b_1$. The reduced bispectrum in redshift space is thus given by

$$Q_s(k_1, k_2, k_3) = \frac{a_0^B(\beta)}{a_0^P(\beta)} \left[ \frac{1}{b_1} Q(k_1, k_2, k_3) + \frac{b_2}{b_1} \right].$$

We remark that our treatment does not take into account a peculiar scale-dependence of redshift distortions that reduces the amplitude of non-linear corrections.
Fig. 2.— Configuration dependence of the reduced bispectrum of dark matter distribution from Gaussian and non-Gaussian initial conditions, as a function of an angle, $\theta$, between two wave vectors, $k_1$ and $k_2$, where the magnitude satisfies $k_2 = 2k_1 = 0.02\ h\ Mpc^{-1}$ (top panels) and $0.04\ h\ Mpc^{-1}$ (bottom panels). (Left panels) $f_{NL}^{\text{GG}} = \pm 100$. (Right panels) $f_{NL}^{\text{GG}} = \pm 200$. The dotted black line shows a Gaussian case ($f_{NL} = 0$, redshift independent), while the solid, long-dashed and short-dashed lines show non-Gaussian cases at $z = 0$, 1 and 4, respectively.

3. FISHER MATRIX ANALYSIS

3.1. Method

In our analysis we shall consider a set of surveys characterized by their volume, $V$, mean galaxy density, $n_g$, and redshift range. We shall assume that these surveys have a simple survey geometry, i.e., a contiguous hexahedron.

Our bispectrum estimator is given by

$$\hat{B} \equiv \frac{V_f}{V_B} \int_{k_1} d^3 q_1 \int_{k_2} d^3 q_2 \int_{k_3} d^3 q_3 \delta_D(q_1, q_2, q_3) \delta_{q_1} \delta_{q_2} \delta_{q_3},$$

(26)

where the integration is over the bin defined by $q_i \in (k_i - \delta k/2, k_i + \delta k/2)$, $V_f = (2\pi)^3/V$ is the volume of the fundamental cell in Fourier space, and

$$V_B \equiv \int_{k_1} d^3 q_1 \int_{k_2} d^3 q_2 \int_{k_3} d^3 q_3 \delta_D(q_1, q_2, q_3) \simeq 8\pi^2 k_1 k_2 k_3 \Delta k^3,$$

(27)

with $\Delta k$ a multiple of the fundamental frequency, $k_f = 2\pi/L$. We assume that two coincide, i.e., $\Delta k = k_f$,
thereby taking into account all “fundamental” triangular configurations.

The variance for our estimator, to the leading order, is given by a triple product of the power spectra,

$$\Delta B^2 \approx k_j^3 \frac{s_{123}^{123}}{V_B} P(k_1) P(k_2) P(k_3),$$

where $s_{123} = 6, 2, 1$ for equilateral, isosceles and general triangles, respectively. As for variance of the reduced bispectrum, we assume that variance from the bispectrum in the numerator dominates over that from the power spectra in the denominator:

$$\frac{\Delta Q^2}{Q^2} \approx \frac{\Delta B^2}{B^2}.$$  

(29)

We calculate variance of the redshift-space galaxy reduced bispectrum from equations (29) and (28) with $P(k)$ given by

$$P_{\text{tot}}(k) \equiv P_s(k) + \frac{1}{(2m)^3} \frac{1}{n},$$

where the second term accounts for the shot noise. We finally obtain

$$\Delta Q_s^2(k_1, k_2, k_3) \approx \frac{s_{123}^{123} k_j^3}{V_B} \frac{P_{\text{tot}}(k_1) P_{\text{tot}}(k_2) P_{\text{tot}}(k_3)}{[P_s(k_1) P_s(k_2) + \text{cyc.}]^2}.$$  

(31)

Once the variance of the reduced bispectrum is given, the Fisher matrix for a given redshift bin can be expressed as

$$F_{\alpha\beta} \equiv \sum_{k_1, k_2, k_3 \leq k_{\text{max}}} \frac{\partial Q_s(i)}{\partial p_\alpha} \frac{\partial Q_s(i)}{\partial p_\beta} \frac{1}{\Delta Q_s^2(i)},$$

(32)

where the parameters, $p_\alpha$, represent $b_1$, $b_2$ and $f_{NL}$. We use equation (25) for $Q_s$, which is valid only up to the second order in perturbations. Neglecting higher order corrections would introduce systematic errors when dealing with the real data, particularly at low $z$ and at small spatial scales, where non-linearity is substantial and perturbation theory essentially breaks down. Since we consider high-$z$ surveys on large scales, we expect that higher order effects would not affect our results very much.

As a fiducial cosmological model we use a flat $\Lambda$CDM cosmology with matter density $\Omega_m = 0.3$, baryon density $\Omega_b = 0.04$, Hubble parameter $h = 0.7$, spectral index $n_s = 1$ and $\sigma_8 = 0.9$. Of these parameters, $\sigma_8$ and $n_s$ affect our forecast for the projected errors on the bias parameters most. We thus consider also different values such as those suggested by the WMAP 3-yr results, $\sigma_8 = 0.75$ and $n_s = 0.95$ (Spergel et al. 2006).  

3.2. Comments on covariance matrix

We shall not include covariance between the cosmological parameters and $b_1$, $b_2$, and $f_{NL}$. (We do include covariance between $b_1$, $b_2$, and $f_{NL}$.) Sefusatti et al. (2006) have shown that, for the SDSS main sample, an analysis with the full covariance among all parameters with a prior from the WMAP 3-yr results yields an error on $b_1$ that is twice as large as that from a simpler analysis without covariance. On the other hand, an error on $b_2$ is not affected significantly. Note that the effect on $b_1$ that they observed was due mainly to degeneracy between $b_1$ and the amplitude of matter fluctuations as Sefusatti et al. (2006) did not use the reduced bispectrum. We expect that degeneracy would be lifted in our analysis, as we use the reduced bispectrum in which the overall amplitude of matter fluctuations cancels.

More importantly, we shall not include covariance between different triangular configurations in the bispectrum. The covariance arises from both observational selection functions (i.e., survey geometry and mask) and a connected six-point function generated by non-linear gravitational evolution. This is a rather crude approximation. Scoccimarro et al. (2004) have included the full reduced bispectrum covariance plus the peculiar survey geometry, when they calculate the constraints on galaxy bias and primordial non-Gaussianity from the SDSS main sample. Using the same realizations of the survey and the same estimator for the covariance matrix, Sefusatti & Scoccimarro (2005) have compared the analysis with the full covariance matrix (including the observational selection function) and that with an approximate diagonal Gaussian variance. They have found that the latter simplified treatment overestimates the signal-to-noise by a factor of 2 for $k_{\text{max}} \sim 0.1 \ h \ Mpc^{-1}$, and a factor of 8 for $k_{\text{max}} \sim 0.3 \ h \ Mpc^{-1}$ at redshift zero. It is not clear, however, how to separate the contribution from non-linear evolution from the effect of the selection function. One would generically expect the radial contribution to be smaller at high $z$, as the six-point function from gravitational clustering becomes smaller than the non-connected part of six-point function (which consists of power spectra) at higher $z$. In any case, our results should be taken as a guide, and one needs to perform the full analysis including the selection functions peculiar to a given survey design.

3.3. Non-linearity and maximum wavenumber

While one can measure the galaxy power spectrum or bispectrum down to very small spatial scales, say, 10 kpc, it is challenging to extract useful cosmological information from such small spatial scales owing to strong non-linearity. Therefore, one has to decide on the maximum wavenumber, $k_{\text{max}}$, below which theory may be trusted. Not surprisingly, since there are many more modes available on smaller spatial scales, the amount of cosmological information one can extract from data grows as $k_{\text{max}}$ increases. It is therefore important to use a realistic $k_{\text{max}}$ in order not to overestimate the statistical power of a given galaxy survey design.

How do we decide on $k_{\text{max}}$? The first obvious thing to do would be to test our theory of the power spectrum and bispectrum against numerical simulations. A value of $k_{\text{max}}$ can be found by comparing perturbation theory predictions with numerical simulations (see e.g., Leung & Komatsu 2006 for an analysis for the matter power spectrum). It is likely that a simple model provided by the second-order (tree-level) bispectrum given by equation (25) breaks down at a relatively small $k$ due to non-linearities of gravitational growth as well as due to non-linear or even non-local bias. Therefore, a model of the bispectrum that takes into account higher-order perturbations would be necessary to push $k_{\text{max}}$ further. New promising techniques such as a renormalized perturbation theory approach (Crocce & Scoccimarro 2006; 2007, McDonald 2007; Matarrese & Pietroni 2007) may be used to obtain bet-
ter predictions for the power spectrum and bispectrum. Further progress is required particularly for understanding redshift distortions (Scoccimarro et al. 2004).

In this paper we use a very simple prescription for getting \( k_{\text{max}} \). We choose \( k_{\text{max}} \) so that \( \sigma(R_{\text{min}},z) = 0.5 \) and \( k_{\text{max}} = \pi/(2R_{\text{min}}) \). The main motivation for this choice being that for small perturbations in the matter distribution, say, \( \sigma(R,z) < 1 \), one may reasonably expect that an analytical model for non-linearities is viable. Note that \( k_{\text{max}} \) derived in this way depends on \( z \), as \( \sigma(R,z) = \sigma(R,0)D(z)/D(0) \).

An alternative, much more conservative estimate of \( k_{\text{max}} \) could be given by requiring that the error on \( b_1 \) derived from the tree-level bispectrum for some \( k_{\text{max}} \) does not exceed the higher-order (“1-loop”, or 4th-order perturbation) corrections in perturbation theory to the reduced matter bispectrum at the same \( k_{\text{max}} \). Specifically, one may use

\[
\Delta b_1 \geq \frac{\Delta Q^{1-\text{loop}}(k_{\text{max}})}{Q^{\text{tree}}(k_{\text{max}})},
\]

(33)

to determine \( k_{\text{max}} \). Here, \( \Delta b_1 \) is computed including all scales down to \( k_{\text{max}} \) and \( \Delta Q^{1-\text{loop}}(k_{\text{max}}) \) is the 1-loop corrections (Scoccimarro et al. 1997; Scoccimarro et al. 1998) to the tree-level reduced bispectrum, \( Q^{\text{tree}} \), evaluated for equilateral configurations with \( k_1 = k_2 = k_3 = k_{\text{max}} \). This approach, however, makes no use of a large amount of information on small scales, and is far from being optimal. Also, the 1-loop correction to the bispectrum, as it is the case for the power spectrum, tends to overestimate the non-linear behaviour measured in simulations, thereby making this approach even more conservative than necessary. In Sec. 4.1.2 we shall compare these two approaches as a function of volume and number density. For the expressions of the 1-loop corrections to the reduced bispectrum see, e.g., (Bernardeau et al. 2002).

3.4. Fiducial values for the galaxy bias parameters

The galaxy bias parameters, \( b_1 \) and \( b_2 \), depend on a number of factors, including galaxy populations, luminosities, and redshifts. On the other hand, the bias of dark matter halos, which can be calculated from \( N \)-body simulations, is understood relatively well. Therefore, the galaxy bias can be calculated from the dark matter halo bias, if we assume that galaxies form in dark matter halos. To do this, one needs (at least) the following information: (i) the halo bias (Mo et al. 1997; Sheth & Tormen 1999), and (ii) how each halo is populated with galaxies, that is, the Halo Occupation Distribution (HOD), \( \langle N \rangle_M \).

We calculate the galaxy bias parameters from the large scales expression

\[
b_i = \frac{1}{n_g} \int_{M_{\text{min}}} M n_h(M,z) b_i^h(M,z) \langle N \rangle_M, \quad (34)
\]

for \( i = 1 \) and 2, where \( n_h(M,z) \) is the mass function of dark matter halos of mass \( M \) at redshift \( z \), \( b_i^h(M,z) \) is the halo bias function, and the HOD, \( \langle N \rangle_M \), is the mean number of galaxies per halo of a given mass, \( M \). We shall use the Sheth & Tormen’s formula for \( n_h(M,z) \) (Sheth & Tormen 1999):

\[
n_h(M,z) = \frac{\rho}{M^2} \frac{d \ln \sigma}{d \ln M} f(\nu) \quad (35)
\]

where \( \nu = \delta_c/\sigma(M,z) \) with \( \delta_c = 1.686 \), and

\[
f(\nu) = A \sqrt{\frac{2q}{\pi}} \left[ 1 + (q\nu^2)^{-p} \right] \nu e^{-q
\nu^2/2} \quad (36)
\]

with \( A = 0.322, \, p = 0.3 \) and \( q = 0.707 \). The halo bias parameters, \( b_1^h \) and \( b_2^h \), are given by (Mo et al. 1997; Scoccimarro et al. 2001b)

\[
b_1^h(M,z|z_f) = 1 + \epsilon_1 + E_1,
\]

\[
b_2^h(M,z|z_f) = \frac{8}{21} (\epsilon_1 + E_1) + \epsilon_2 + E_2, \quad (37)
\]

where \( z \) refers to the redshift of observation, while \( z_f \) refers to the redshift of formation of halos of mass \( M \), and

\[
\epsilon_1 = \frac{q\nu^2 - 1}{\delta_f}, \quad \epsilon_2 = \frac{q
\nu^2 q
\nu^2 - 3}{\delta_f}, \quad (38)
\]

\[
E_1 = \frac{2p/\delta_f}{1 + (q\nu^2)^p}, \quad E_2 = \frac{1 + 2p}{\delta_f} + 2\epsilon_1, \quad (39)
\]

with \( \delta_f = \delta_c D(z)/D(z_f) \), where \( D(z) \) is the linear growth function. In the right panel of Figure 3 we show the galaxy bias functions, \( b_1(M,z) \) and \( b_2(M,z) \), as a function of \( M \) and \( z \), in the approximation that the formation redshift equals the observation redshift, \( z = z_f \). We shall always assume this throughout the paper.

As for the HOD, we adopt the form proposed by (Tinker et al. 2005):

\[
\langle N \rangle_M = 1 + \frac{M}{M_1} \exp \left( -\frac{M_{\text{cut}} - M}{M_1} \right) \quad (40)
\]

for \( M > M_{\text{min}} \) and zero otherwise. The parameter \( M_1/M_{\text{min}} \) represents the minimum mass above which we find a (central) galaxy in the halo, while \( M_1 \) represents the mass above which we can find a second (satellite) galaxy. Measuring the HOD parameters for subhalo populations from several \( N \)-body simulations at different redshifts and densities, (Conroy et al. 2006) found a correlation between \( M_{\text{cut}} \) and \( M_1 \) given by

\[
\log_{10} M_{\text{cut}} = 0.76 \log_{10} M_1 + 2.3. \quad (41)
\]

One also finds from Table 2 in (Conroy et al. 2006) that \( M_1/M_{\text{min}} \) depends on redshift and density only weakly; thus, for simplicity we shall keep this ratio fixed at \( \log_{10}(M_1/M_{\text{min}}) = 1.1 \), and find \( M_{\text{min}} \) from

\[
n_g = \int_{M_{\text{min}}} M n_h(M,z) \langle N \rangle_M, \quad (42)
\]

for a given \( n_g \).

In the right panel of Figure 3 we show the galaxy bias parameters, \( b_1 \) and \( b_2 \), from equation (34) as a function of redshift for two values of the mean galaxy density, \( n_g = 5 \times 10^{-3} \) and \( 5 \times 10^{-4} \) h\(^3\) Mpc\(^{-3}\). As expected, for a fixed galaxy number density the value of the linear bias, \( b_1 \), increases with redshift. We find that \( b_1 \) and \( b_2 \) are strongly correlated. We shall come back to this point in Sec. 5.

We admit that these values are derived from very simplified models without much justification. We need observational data to determine the true bias parameters for high-z surveys eventually, although we do not have sufficient data for doing so yet. Nevertheless, we find
our approach useful for our purpose of deriving the fiducial values of $b_1$ and $b_2$ with a realistic redshift evolution, particularly for redshift surveys spanning a wide range in redshift, for which one has to consider a set of redshift bins and assume different fiducial values for $b_1$ and $b_2$ at different $z$. We note that $b_1$ obtained from our method agrees with those obtained in the previous work by assuming $\sigma_{g,q} \approx 1$ and $n_g \approx 5 \times 10^{-3} h^3 \text{Mpc}^{-3}$ (Seo & Eisenstein 2003; Takada et al. 2006).

In Sec. 5 we shall show how one can extend this simple picture by introducing a redshift dependence in the HOD, and how one can make use of the information on galaxy bias derived from the bispectrum to constrain the HOD parameters directly.

4. RESULTS

In this section we present the results from our Fisher matrix analysis of the galaxy bispectrum. We first study how the derived constraints on the galaxy bias parameters and primordial non-Gaussianity depend on the choice of $k_{\text{max}}$, taking into account the two approaches discussed above. We then study how the constraints depend on the survey volume and redshift. Finally we shall apply our method to make forecasts for several current and proposed redshift surveys.

4.1. Dependence on $k_{\text{max}}$, volume, redshift and number density

4.1.1. $k_{\text{max}}$

As mentioned in the previous section, constraints on galaxy bias and primordial non-Gaussianity would depend strongly on $k_{\text{max}}$, the smallest scale included in the analysis.

As an example, we consider sample surveys at two redshifts: the median redshifts of (i) $\bar{z} = 1$ and (ii) $\bar{z} = 3$. Each has the volume of $V = 10 h^{-3} \text{Gpc}^3$ and the number density of $n_g = 5 \times 10^{-3} h^3 \text{Mpc}^{-3}$. (The total number of galaxies in the survey volume at each redshift is 50 million galaxies.) The bias parameters are (i) $b_1 = 1.5$ and $b_2 = 0.035$ at $\bar{z} = 1$, and (ii) $b_1 = 2.6$ and $b_2 = 2.1$ at $\bar{z} = 3$.

In the upper panels of Figure 4 we plot the marginalized, 1-$\sigma$, fractional errors on $b_1$ and $b_2$ at $\bar{z} = 1$ and 3, assuming Gaussian initial conditions, i.e., $f_{NL} = 0$. We observe an interesting effect: a fractional error on $b_1$ improves at lower $z$, while that on $b_2$ improves at higher $z$. (Note that this statement is true only when the same $k_{\text{max}}$ is used at both redshifts. See discussion below.) This can be understood as follows. Let us recall the form of the galaxy reduced bispectrum (Eq. [19]):

\[ Q_g(k_1, k_2, k_3) \approx \frac{1}{b_1} Q(k_1, k_2, k_3) + \frac{b_2}{b_1^2}. \]

Now, $Q$ on the right hand side is independent of $z$ at the tree-level when initial fluctuations are Gaussian. Therefore, the first term falls as $1/b_1$ at higher $z$ where $b_1$ is larger (Fig. 3). On the other hand, the second term actually grows as $z$: for the current example $b_2/b_1^2 = 0.016$ at $z = 1$ and 0.31 at $z = 3$. Therefore, our sensitivity to $b_2$ grows with $z$, while our sensitivity to $b_1$ declines with $z$.

Let us study more quantitatively the sensitivity to $b_1$. In the limit of linear bias, $b_2 = 0$, a signal-to-noise of the reduced bispectrum of equilateral configurations is given by

\[ \frac{Q_g^2(k)}{\Delta Q_g^2(k)} \big|_{b_2=0} \approx \frac{\left[a_b^0(\beta)\right]^2}{\left[a_b^0(\beta)\right]^2} V_B \frac{B_{123}^2(k, k, k; z)}{P_L(k; z)} \propto D^2(z). \]  

(43)

We expect, therefore, that a signal-to-noise of the bispectrum from gravitational instability declines with $z$, 

---

**Fig. 3.** (Left panel) The halo bias functions, $b_1^H(M, z)$ (solid lines) and $b_2^H(M, z)$ (dashed lines), as a function of the mass, $M$, for $z = 0$ (thick lines) and $z = 1$ (thin lines) in the approximation that the formation redshift equals the observation redshift, $z = z_f$. (Right panel) The galaxy bias parameters, $b_1$ and $b_2$, for the mean galaxy density of $n_g = 5 \times 10^{-3}$ (continuous lines) and $n_g = 5 \times 10^{-4} h^3 \text{Mpc}^{-3}$ (dashed lines).
Fig. 4.— (Upper panels) Predicted errors on galaxy bias parameters vs the maximum wavenumber, $k_{\text{max}}$. The dashed and solid lines show the prediction for a galaxy survey at $z = 1$ and 3, respectively. Each survey is assumed to have the survey volume of $V = 10 h^{-3} \text{Gpc}^3$ and the number density of $n_g = 5 \times 10^{-3} h^3 \text{Mpc}^{-3}$. The left panel shows the marginalized 1-$\sigma$ errors on the linear bias, $b_1$, while the right panel shows the non-linear bias, $b_2$. Both assume Gaussian initial conditions, $f_{N \text{L}} = 0$. The vertical lines show $k_{\text{max}}$ as determined from $\sigma(R; z) = 0.5$ for each redshift (see Sec. 3.2). (Lower panels) Predicted errors on primordial non-Gaussian parameters vs $k_{\text{max}}$. The left panel shows the marginalized 1-$\sigma$ errors on the local model, $f_{\text{local}}^{N \text{L}}$, while the right panel shows the equilateral model, $f_{\text{eq}}^{N \text{L}}$. The bias parameters have been marginalized.

resulting in an increasing error on $b_1$ at higher $z$.

In practice, however, we predict that galaxy surveys at higher $z$ should result in better determinations of both $b_1$ and $b_2$. The reason is quite simple: $k_{\text{max}}$ at higher $z$ must be larger than at that lower $z$. In the upper left panel of Figure 4 we show $k_{\text{max}}$ as determined from $\sigma(R; z) = 0.5$: $k_{\text{max}} = 0.17 h \text{Mpc}^{-1}$ at $z = 1$ and $k_{\text{max}} = 0.47 h \text{Mpc}^{-1}$ at $z = 3$. The difference is clear: when the modes up to $k_{\text{max}}$ are included, a survey at $z = 3$ yields an error on $b_1$ that is a factor of 5 better than that at $z = 1$. As for $b_2$, a survey at $z = 3$ does better by nearly two orders of magnitude.

How about primordial non-Gaussianity? In the lower panels of Figure 4 we show the predicted errors on $f_{N \text{L}}^{\text{loc}}$ and $f_{N \text{L}}^{\text{eq}}$, marginalized over $b_1$ and $b_2$. We find that the difference between $z = 1$ and 3 is negligible at the same $k_{\text{max}}$. This is a consequence of the fact that a signal-to-noise for the primordial bispectrum component is not, in the first approximation, redshift dependent. For equilateral configurations one finds

$$\frac{Q_{I}^2(k)}{\Delta Q_{I}^2(k)} \approx \frac{[a_{N \text{L}}^2(\beta)]^2 V_B}{[a_{N \text{L}}^2(\beta)]^4 k_{N \text{L}}^2(k, k, k; z) P_3(k; z)} \propto \text{constant}$$

where we considered only the primordial term, $Q_I$. Nevertheless, we still predict that galaxy surveys at higher $z$ should result in better determinations of both $f_{N \text{L}}^{\text{loc}}$ and $f_{N \text{L}}^{\text{eq}}$, as $k_{\text{max}}$ must be larger at higher $z$ and therefore many more modes are available for the analysis at higher $z$. 
Results from a much more conservative estimate of $k_{\text{max}}$ (eq. [33]) will be given near the end of the next section.

4.1.2. Volume, redshift, and number density of galaxies

Dependence of the predicted errors on volume is straightforward: it depends simply on $1/\sqrt{V}$. Dependence on $z$ is a combination of two effects: (i) how a signal-to-noise for a given $k_{\text{max}}$ grows with $z$, and (ii) how $k_{\text{max}}$ grows with $z$. Finally, the number density of galaxies determines a signal-to-noise on small scales, where the shot noise plays an important role. In particular, very high-$z$ surveys at, e.g., $z > 3$, do not add very much if the number density of galaxies is too low.

In Figure 5 we show how the predicted constraints on the galaxy bias parameters, $b_1$ and $b_2$, improve with the survey volume, as a function of the median redshift, $\bar{z}$.

We used $n_g = 5 \times 10^{-3} h^3 \text{Mpc}^{-3}$ for the number density of galaxies. The fiducial values of $b_1$ and $b_2$ are calculated for each $\bar{z}$ from Figure 3. We used $k_{\text{max}}$ determined from $\sigma(R, \bar{z}) = 0.5$ for a given $\bar{z}$. Since $k_{\text{max}}$ grows as $\bar{z}$ increases, the predicted constraints on $b_1$ and $b_2$ also improve as $\bar{z}$ increases. A spike at $z \sim 0.8$ in $\Delta b_2/b_2$ is a numerical artifact of $b_2$ being very close to zero. The dependence on volume is given simply by $1/\sqrt{V}$.

In the lower panels of Figure 5 we show the predicted constraints on primordial non-Gaussianity, marginalized over $b_1$ and $b_2$. We find that a survey of the size $V \sim 1 h^{-3} \text{Gpc}^3$ at $z \sim 4 - 6$ or $V \sim 10 h^{-3} \text{Gpc}^3$ at $z \sim 1 - 2$ is as sensitive to $f_{\text{NL}}^{\text{loc}}$ as the CMB data from Planck. A more ambitious design, e.g., $V \sim 10 h^{-3} \text{Gpc}^3$ at $z \gtrsim 2$, can achieve $\Delta f_{\text{NL}}^{\text{loc}} \sim 1$, although it depends on the number density quite strongly. An even more ambitious design, $V \sim 100 h^{-3} \text{Gpc}^3$, would enable us to...
detect the primordial bispectrum from ubiquitous non-Gaussianity “floor” from the second-order evolution of primordial fluctuations (Bartolo et al. 2005).

Constraints on the equilateral type of non-Gaussianity suffer from a stronger degeneracy between the primordial bispectrum and the non-linear gravitational evolution as well as non-linear bias, and thus the predicted errors on $f_{NL}^{eq}$ are an order of magnitude larger than those on $f_{NL}^{loc}$. (See Sec. 4.2 for more detail.) Nevertheless, a survey of $V \sim 10 h^{-3} \text{Gpc}^3$ at $z \sim 1$ should provide a constraint that is comparable to that from the WMAP 3-yr data.

We also computed the Fisher matrix for an all-sky survey from $z = 0$ to 5. We divided the entire redshift range in bins of the size $\Delta z = 0.5$, and used $n_g = 5 \times 10^{-4} h^3 \text{Mpc}^{-3}$. We find that such a survey should provide $\Delta f_{NL}^{eq} \sim 0.2$ and $\Delta f_{NL}^{loc} \sim 2$. These values probably represent the best limits on $f_{NL}$ one can ever hope to achieve from galaxy surveys.

How about the number density of galaxies? When the number density is low, the shot noise completely dominates at small scales, and thus one fails to improve a signal-to-noise by increasing $k_{max}$. This suggests that very high-$z$ galaxy surveys do not add much if the number density of galaxies is too low. In Figure 6 we show the case for $n_g = 5 \times 10^{-4} h^3 \text{Mpc}^{-3}$. Clearly, our sensitivity to all of $b_1$, $b_2$, $f_{NL}^{loc}$ and $f_{NL}^{eq}$ does not improve at all beyond $z \sim 3$. Therefore, it makes sense to conduct very high-$z$ surveys, only if one can detect more than $n_g \sim 10^{-3} h^3 \text{Mpc}^{-3}$.

How robust are these results? The most uncertain parameter in our analysis is $k_{max}$. What if $k_{max}$ is significantly lower than that from $\sigma(R, z) = 0.5$? To address this question, we have repeated our analysis using a much more conservative estimate of $k_{max}$ given by equation (33). This estimate was derived by throwing away any information beyond $k_{max}$ at which the tree-level bispectrum becomes inaccurate. This is a conservative estimate because we can certainly improve our theoretical predic-

![Graphs showing bispectrum values for different number densities](https://example.com/grapht.png)

Fig. 6.— Same as Figure 5 but for a smaller number density of galaxies, $n_g = 5 \times 10^{-4} h^3 \text{Mpc}^{-3}$. 
tion by going to the higher order, “1-loop” (4th order) calculations [Scoccimarro 1997; Scoccimarro et al. 1998]. We show the results in Figure 7. We find significantly weaker constraints; for example, fractional errors on \( b_1 \) from a survey of \( V = 10 \, h^{-3} \text{Gpc}^3 \) now go from 9% to a few percent from \( z = 0 \) to \( z = 5 \), or a factor of \( \sim 8 \) and 20 weaker constraints at \( z \sim 1 \) and 5, respectively. For this choice of \( k_{\text{max}} \) the shot-noise is not very important because we are restricted to a fairly small \( k \) already. Therefore we obtain similar results for a lower density, \( n_g = 5 \times 10^{-3} \, h^3 \text{Mpc}^{-3} \).

4.2. Parameter degeneracy

Are galaxy bias and primordial non-Gaussianity independent? In Figure 8 we show the 2-d joint constraints (95% C.L.) on \((b_1, b_2), (b_1, f_{NL}), \) and \((b_2, f_{NL}), \) marginalized over \( f_{NL}, b_2, \) and \( b_1, \) respectively. The survey parameters are \( V = 10 \, h^{-3} \text{Gpc}^3, n_g = 5 \times 10^{-3} \, h^3 \text{Mpc}^{-3}, z = 1 \) (left panels) and 3 (right panels). The fiducial values of bias parameters are \( b_1 = 1.5 \) and \( b_2 = 0.035 \) at \( z = 1 \), and \( b_1 = 2.6 \) and \( b_2 = 2.1 \) at \( z = 3 \).

We find that \( f_{NL}^{\text{loc}} \) is not degenerate with \( b_1 \) or \( b_2 \), which is a very good news; however, \( f_{NL}^{\text{eq}} \) reveals a rather strong degeneracy with both \( b_1 \) and \( b_2 \). Therefore, the equilateral model turns out to be much harder to constrain by CMB or galaxy surveys.

4.3. Current and proposed redshift surveys

We are now in a position to apply our Fisher matrix analysis tools to several current and future redshift galaxy surveys, both at low and high redshifts. For the sake of simplicity, and to allow for easier comparison, we shall assume that each survey is characterized uniquely by its survey volume, \( V, \) redshift range, and galaxy number density, \( n_g. \) In other words, we shall ignore complications related to the specific geometry and selection functions.
A galaxy survey of a large volume and a relatively low galaxy density (but not too low — there should be at least \( n_g \sim 1/P(k_{\text{max}}) \) galaxies for the optimal survey efficiency) is necessary to detect the baryon acoustic oscillations (BAO) in the power spectrum, which may be used to constrain the nature of dark energy. We point out that such a survey should also provide competitive constraints on primordial non-Gaussianity.

We shall consider the following survey designs. Table 1 tabulates \( V, n_g, z, k_{\text{max}}, b_1 \) and \( b_2 \) of these surveys. We calculate \( k_{\text{max}} \) from \( \sigma(R, z) = 0.5 \) and \( k_{\text{max}} = \pi/(2R) \) (Sec. 3.4) and \( b_1 \) and \( b_2 \) from a halo approach given in Sec. 3.4.

(i) Low \( z \) surveys include two on-going surveys and one planned survey:

- The Sloan Digital Sky Survey (SDSS) main sample (for which a detailed analysis of the expected constraints on galaxy bias and primordial non-Gaussianity has been given in Scoccimarro et al. (2004)) at \( z = 0 \) and \( V = 0.3 \, h^{-3}\text{Gpc}^3 \).
- The SDSS Luminous Red Galaxy (LRG) sample at \( z = 0.35 \) and \( V = 0.72 \, h^{-3}\text{Gpc}^3 \).
- A proposed extension of SDSS, APO-LSS survey at \( z = 0.35 \) and \( V = 3.8 \, h^{-3}\text{Gpc}^3 \).

(ii) Intermediate \( z \) surveys include three planned surveys:

- The Hobby-Eberly Dark Energy Experiment (HETDEX) (Hill et al. 2004), targeting Lyman-\( \alpha \) emitters (LAE) between \( z = 2 \) and \( z = 4 \). We assume \( V = 2.7 \, h^{-3}\text{Gpc}^3 \) and \( n_g = 5 \times 10^{-4} \, h^{-3}\text{Mpc}^{-3} \), following the “G2” design given in Takada et al. (2006) for comparison.
- Two redshift surveys with the planned Wide-Field Multi-Object Spectrograph (WFMOS), one detecting 2 million galaxies at \( 0.5 < z < 1.3 \) in \( 2,000 \, \text{deg}^2 \) (WFMOS1: \( V = 4 \, h^{-3}\text{Gpc}^3 \)), and the other detecting 600,000 galaxies at \( 2.3 < z < 3.3 \) in \( 300 \, \text{deg}^2 \) (WFMOS2; \( V = 1 \, h^{-3}\text{Gpc}^3 \)) (Glazebrook et al. 2005). To facilitate comparison we assume the same number density of galaxies for both, \( n_g = 5 \times 10^{-4} \, h^{-3}\text{Mpc}^{-3} \).
- The Advanced Dark Energy Physics Telescope (ADEPT) mission, a space-based redshift survey of 100 million galaxies at \( 1 < z < 2 \) in \( 28,600 \, \text{deg}^2 \). We assume \( V = 100 \, h^{-3}\text{Gpc}^3 \) and \( n_g = 10^{-4} \, h^{-3}\text{Mpc}^{-3} \).

(iii) High \( z \) surveys are represented by the Cosmic Inflation Probe (CIP) mission, a space-based redshift survey targeting \( \text{H}\alpha \) emitters at \( 3.5 < z < 6.5 \) (Melnick et al. 2004). We assume \( V = 3.4 \, h^{-3}\text{Gpc}^3 \) and \( n_g = 5 \times 10^{-3} \, h^{-3}\text{Mpc}^{-3} \), following the “SG” design given in Takada et al. (2006).

The 8th and 9th column in Table 1 tabulate \( \Delta b_1 \) and \( \Delta b_2 \) for Gaussian initial conditions (\( f_{NL} = 0 \)), the 10th, 11th, and 12th column tabulate \( \Delta b_1, \Delta b_2, \Delta f_{NL}^{\text{loc}} \), and the 13th, 14th, and 15th column tabulate \( \Delta b_1, \Delta b_2, \Delta f_{NL}^{\text{eq}} \). For HETDEX, WFMOS, ADEPT, and CIP we provide \( \Delta b_1, \Delta b_2, \Delta f_{NL}^{\text{loc}} \), and \( \Delta f_{NL}^{\text{eq}} \) from each redshift bin as well as \( \Delta f_{NL}^{\text{loc}} \) and \( \Delta f_{NL}^{\text{eq}} \) from a combined analysis of all bins.

The predicted constraints on galaxy bias range from a few percent for APO-LSS, WFMOS and HETDEX to less than 1% for ADEPT, the latter being significantly better owing obviously to a larger survey volume. As we have mentioned already in Sec. 4.2 inclusion of \( f_{NL}^{\text{loc}} \)
does not degrade sensitivity to galaxy bias, whereas $f_{NL}^{eq}$ does degrade it significantly.

Our prediction for SDSS should not be compared directly to those derived by Scoccimarro et al. (2004), as we have ignored covariance between different bispectrum configurations due to non-linear effects and survey geometry (Sec. 3.2), which was included in their work. In this sense our analysis is more optimistic; however, in the other sense our analysis is in fact more realistic than theirs. We have used $k_{max} \simeq 0.09 \ h \ Mpc^{-1}$ for SDSS, which is significantly more conservative than their value, $k_{max} = 0.3 \ h \ Mpc^{-1}$. This has made our predicted errors much weaker than theirs: they obtained $\Delta b_1/b_1 = 0.04$ (ours 0.26) and $\Delta f_{NL}^{eq}$ = 145 (ours 256).

How would these results depend on $\sigma_8$ and $n_s$? A lower value for the rms density fluctuations, $\sigma_8 = 0.75$, as recently suggested by the WMAP 3yr data (Spergel et al. 2006), increases $k_{max}$ because non-linearity is weaker for a lower $\sigma_8$, and thus improves the constraints. On the other hand, it follows from equation (43) that the signal-to-noise for a given triangle configuration is proportional to $\sigma_8^2$, and thus a lower $\sigma_8$ results in worse constraints. When combined, these two effects result in 20% and 30% stronger constraints on galaxy bias and primordial non-Gaussianity, respectively, for low-redshift surveys, whereas these effects cancel for intermediate and high redshift surveys. A departure from a scale invariant spectrum has a smaller effect. For $n_s = 0.95$ we find only 10% improvement in bias and non-Gaussianity.

5. CONSTRANING THE HOD

So far we have assumed that the linear and quadratic bias, $b_1$ and $b_2$, are completely free. For surveys spanning a large redshift range, therefore, we had to introduce multiple redshift bins, and, as a result, we had to have an excessive number of free parameters, two times the number of redshift bins.

In this section we attempt to reduce the number of free parameters by using a halo approach. The fiducial values of $b_1$ and $b_2$ were derived from a given form of the HOD (Sec. 3.4). If we can parametrize the HOD by fewer parameters than two times the number of redshift bins, the model has more constraining power.

In the limit that the evolution of bias is given by the mass function, we may approximate that the HOD is independent of $z$. We make a minimal extension of the HOD that we used in Sec. 3.4 namely, instead of fixing a ratio of $M_4$ and $M_{min}$, we let it free:

$$\log_{10} M_4 = a + \log_{10} M_{min}. (45)$$

The fiducial value is $a = 1.1$, as before. We still assume that a relation between $M_{cut}$ and $M_4$ is given by equation (41). We then replace $b_1$ and $b_2$ at different redshift bins with a single parameter $a$ in the Fisher matrix analysis.

The expected 1-$\sigma$ errors on $a$ and primordial non-Gaussian parameters are given in Table 2 in the third to fifth columns. We find that the error on $f_{NL}^{eq}$ is unaffected: this is an expected result because $f_{NL}^{eq}$ is not degenerate with galaxy bias. On the other hand, we find a significant, about a factor of two, improvement in $f_{NL}^{eq}$. This is due to the fact that the analysis in terms of the HOD is equivalent to introducing a theoretical prior on a relation between $b_1$ and $b_2$, lifting the degeneracy. In other words, while in the previous section we allowed $b_1$ and $b_2$ to vary independently, we are now making use of the fact that the Halo Model predicts them to be strongly correlated.

When we have a survey that covers a wide range in $z$, we may be able to constrain more-than-one parameters in the HOD. To see how it works we extend the minimal model by introducing one more parameter:

$$\log_{10} M_4 = a + b_2 + \log_{10} M_{min}, (46)$$

and we assume $b = 0$ as the fiducial value. An analysis based on a single redshift bin would naturally lead to a

| $V$ | $n_g$ | $z$ | $k_{max}$ | $b_1$ | $b_2$ | $\Delta b_1$ | $\Delta b_2$ | $\Delta f_{NL}^{eq}$ | $\Delta b_1$ | $\Delta b_2$ | $\Delta f_{NL}^{eq}$ |
|-----|-------|-----|-----------|-------|-------|-------------|-------------|----------------|-------------|-------------|----------------|
| SDSS | 0.3   | 0   | 0.09      | 1.19  | −0.10 | 0.270       | 0.154       | 0.309          | 0.151       | 255.5       | 0.450          |
| LRG  | 0.72  | 1   | 0.35      | 0.11  | 2.14  | 0.209       | 0.348       | 0.223          | 0.353       | 113.4       | 0.338          |
| APO-LSS | 3.8   | 4   | 0.35      | 0.11  | 1.69  | 0.069       | 0.068       | 0.071          | 0.069       | 34.9        | 0.108          |
| WFMOS1 | 1.6   | 5   | 0.7       | 0.14  | 1.87  | 0.076       | 0.095       | 0.080          | 0.096       | 41.0        | 0.123          |
| ADEPT | 2.4   | 5   | 1.1       | 0.18  | 2.16  | 0.047       | 0.081       | 0.048          | 0.081       | 23.1        | 0.076          |
| WFMOS2 | 45    | 1   | 1.25      | 0.20  | 2.97  | 0.020       | 0.063       | 0.021          | 0.063       | 6.1         | 0.031          |
| HETDEX | 0.5   | 5   | 2.55      | 0.38  | 3.27  | 0.058       | 0.220       | 0.060          | 0.223       | 25.7        | 0.094          |
| CIP   | 1.26  | 50  | 4         | 0.71  | 3.16  | 0.010       | 0.036       | 0.010          | 0.037       | 4.7         | 0.016          |
|       | 1.10  | 50  | 6         | 1.46  | 4.26  | 0.011       | 0.066       | 0.012          | 0.066       | 3.8         | 0.016          |
|       | 1.62  | 50  | 6         | 1.46  | 4.26  | 0.011       | 0.066       | 0.012          | 0.066       | 3.8         | 0.016          |
large degeneracy between $a$ and $b$. However, one can lift this degeneracy by including multiple redshift bins.

In Table 2 columns 6th to 13th, we present the expected 1-σ errors on $a$ and $b$ as well as on primordial non-Gaussianity. One can clearly see that $a$ and $b$ are degenerate within a single redshift bin: combined errors are orders of magnitude smaller than those from single bins. The determination of primordial non-Gaussianity is largely unaffected by the extra HOD parameter.

6. CONCLUSIONS

The quest to understand the nature of dark energy has recently provided a further motivation for future large redshift surveys. It is certain that the study of higher order correlation functions of galaxies will be required in order to extract maximum cosmological information from such large data sets. For instance, as it has been recognized for more than a decade, the bispectrum can be used to measure non-linearities in the galaxy-mass relation. Non-linearity in galaxy bias must be understood better to meet the high accuracy required for precision measurements of the baryon acoustic oscillations and their interpretation as a standard ruler, particularly for highly biased tracers such as the luminous red galaxies and Lyman-α emitters.

We have shown that, with the most conservative assumption about the maximum wavenumber, $k_{\text{max}}$, used in the analysis, the bispectrum measured from a galaxy redshift survey should yield a fractional error on the linear bias of order $0.1 \sqrt{h^{-3} \text{Gpc}^3/V}$ at $z = 0$ to $0.05 \sqrt{h^{-3} \text{Gpc}^3/V}$ at $z = 6$. This is an extremely conservative limit, however, as it assumes no understanding of even the mildest non-linearities in the dark matter and galaxy biasing evolution.

An analysis that includes all configurations down to mildly non-linear scales, within which $\sigma(R) \lesssim 0.5$ is satisfied, should yield more than an order of magnitude better determination of bias parameters, both linear and non-linear, owing to the large number of configurations available at smaller scales. This is precisely where intermediate to high-$z$ galaxy surveys play a leading role: non-linearity is much weaker at higher $z$, and therefore we can access to a large number of modes on small scales.

While we can study non-linearities in the matter distribution by means of new techniques based on perturbation theory and of N-body simulations, a simple, local description of galaxy bias may have to be improved further. In this perspective, our results show that a large amount of information can be extracted from higher-order correlations which, in turn, may be used to constrain more sophisticated models of galaxy biasing.

We have also shown that the bispectrum from large-volume, high-redshift surveys is highly sensitive to primordial non-Gaussianity. The CMB observations have been the best probe of the Gaussian nature of primordial perturbations so far, and the Planck satellite would be quite close to the ideal experiment limit. On the other hand, a redshift survey of the large scale structure actually contains much more information than CMB, as the number of modes available from the three-dimensional fluctuations is vastly larger than that from the two-dimensional temperature and polarization anisotropies.

Not only can they provide independent constraints on scales smaller than those probed by CMB, but also their constraints can be comparable to, if not better than, those from CMB. The best limit one can achieve from an all-sky survey up to redshift $z \sim 5$ should reach $f_{NL}^{\text{loc}} \sim 0.2$ and $f_{NL}^{\text{eq}} \sim 2$, an order of magnitude better than the best limits achievable by CMB. The planned surveys such as HETDEX and ADEPT should reach the constraints that are comparable to those from the WMAP and Planck CMB experiments, respectively. It should also be understood that galaxy surveys provide the best limits on

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| Survey | 1-parameter HOD | 2-parameter HOD |
|--------|----------------|----------------|
|        | $\Delta a$ | $\Delta b$ | $\Delta f_{NL}^{\text{loc}}$ | $\Delta a$ | $\Delta b$ | $\Delta f_{NL}^{\text{eq}}$ |
| SDSS   | 0.05  | 0.12  | 0.24  | 0.15  | 0.15  | 0.25  |
| LRG    | 0.00  | 0.01  | 0.02  | 0.01  | 0.01  | 0.03  |
| APO-LSS| 0.25  | 0.06  | 0.07  | 0.35  | 0.06  | 0.12  |
| WFMOS1 | 0.07  | 0.06  | 0.39  | 0.05  | 0.14  | 0.25  |
| ADEPT  | 0.07  | 0.07  | 0.35  | 0.06  | 0.15  | 0.26  |
| WFMOS2 | 0.07  | 0.07  | 0.35  | 0.06  | 0.15  | 0.26  |
| HETDEX | 2.25  | 0.01  | 0.02  | 0.11  | 0.01  | 0.02  |
| CIP    | 4.00  | 0.00  | 0.00  | 0.00  | 0.00  | 0.00  |
|        | 0.00  | 0.00  | 0.00  | 0.00  | 0.00  | 0.00  |

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TABLE 2
Marginalized 1-σ errors on the HOD parameters and primordial non-Gaussianity from the Fisher matrix analysis of the reduced bispectrum in redshift space for $\sigma_8 = 0.9$ and $n_s = 1$. The survey parameters and $k_{\text{max}}$ are the same as in Table 1.
small scales that are not accessible by CMB. This is particularly important when probing scale-dependent non-Gaussian models.

Finally, we have shown that galaxy bias parameters modeled by the halo occupation distribution should be modeled by the halo occupation distribution and not by Gaussianity. In the analysis of future redshift surveys, providing higher order correlation functions will play a crucial role in the analysis of future redshift surveys, providing indispensable information on galaxy bias as well as on the nature of primordial perturbations.

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