Measurement of the Strong Coupling Constant from Inclusive Jet Production at the Tevatron \( \overline{p}p \) Collider

We report a measurement of the strong coupling constant, \( \alpha_s(M_Z) \), extracted from inclusive jet production in \( \overline{p}p \) collisions at \( \sqrt{s} = 1800 \) GeV. The QCD prediction for the evolution of \( \alpha_s \) with jet transverse energy \( E_T \) is tested over the range \( 40 < E_T < 450 \) GeV using \( E_T \) for the renormalization scale. The data show good agreement with QCD in the region below 250 GeV. In the text we discuss the data-theory comparison in the region from 250 to 450 GeV. The value of \( \alpha_s \) at the mass of the \( Z^0 \) boson averaged over the range \( 40 < E_T < 250 \) GeV is found to be \( \alpha_s(M_Z) = 0.1178 \pm 0.0001(\text{stat}) \pm 0.0025(\text{exp, syst}) \). The associated theoretical uncertainties are mainly due to the choice of renormalization scale (5%) and input parton distribution functions (5%).

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Jet production at hadron colliders provides an excellent opportunity for testing the theory of strong interactions. Quantum Chromodynamics (QCD) has achieved remarkable success in describing hadron interactions at short distances (large momentum transfers), owing to the property of asymptotic freedom. Asymptotic freedom predicts a logarithmic decrease of the coupling strength, \( \alpha_s(\mu) \), as the momentum scale \( \mu \) characterizing a process increases. Processes with large momentum transfer can then be described by an expansion in powers of \( \alpha_s(\mu) \). The value of \( \alpha_s \), a free parameter of QCD, is one of the fundamental constants of nature. Its determination is the essential measurement of QCD, and the observation of its evolution, or running, with momentum transfer is one of the key tests of the theory. At \( e^+e^- \) collisions \( \alpha_s \) has been measured from the fragmentation functions, event shapes, jet production rates, and \( \tau \) lepton decay properties. In lepton-hadron collisions, \( \alpha_s \) has been measured from scaling violations, jet production rates, and momentum sum rules. A precise value for \( \alpha_s \) has also been obtained from a global fit to properties of the \( Z^0 \) boson measured at the CERN LEP and the SLAC SLC \( e^+e^- \) colliders and the \( W \) boson and top quark masses. A review of these and other \( \alpha_s \) measurements can be found in [1]. In this letter, we present a measurement of \( \alpha_s \) from the inclusive jet cross section in \( \overline{p}p \) collisions over the jet transverse energy \( (E_T) \) range from 40 to 450 GeV.

This measurement is based on a data sample of integrated luminosity 87 pb\(^{-1}\) collected by the Collider Detector at Fermilab (CDF) during the 1994-95 run (Run 1b) of the Fermilab Tevatron \( \overline{p}p \) collider operating at \( \sqrt{s} = 1.8 \) TeV. The CDF detector is described elsewhere [2]. Details of the measurement of the inclusive jet differential cross section can be found in [3]. Briefly, jets are reconstructed using an iterative fixed cone algorithm with a radius \( R = (\Delta \eta^2 + \Delta \phi^2)^{1/2} = 0.7 \), where \( \eta \equiv -\ln(\tan(\theta/2)) \) is the pseudorapidity, evaluated from the angle \( \theta \) between the centerline of the jet cone and the proton beam line, and \( \phi \) is the azimuthal angle. The inclusive jet cross section includes all jets in an event in the pseudorapidity range \( 0.1 < |\eta| < 0.7 \). The measured spectrum is corrected for the calorimeter response, resolution and the underlying event energy using an iterative unsmearing procedure which changes both the energy scale and the normalization simultaneously. The value of \( \alpha_s \) is determined by comparing the jet cross section with the next to leading order (NLO) perturbative QCD predictions [4]. In the \( E_T \) region studied, the non-perturbative contributions to the inclusive jet cross section are estimated to be negligible [5]. The procedure of extracting \( \alpha_s \) can be summarized by the equation

\[
\frac{d\sigma}{dE_T} = \alpha_s^2(\mu_R)\hat{X}(0)(\mu_F, E_T)\left[1 + \alpha_s(\mu_R)k_1(\mu_R, \mu_F, E_T)\right]
\]

(1)

where \( \frac{d\sigma}{dE_T} \) is the transverse energy distribution of the inclusive jets, \( \mu_R \) and \( \mu_F \), related to \( E_T \) by a scale factor, are the renormalization and factorization scales, \( \alpha_s^2(\mu_R)\hat{X}(0)(\mu_F, E_T) \) is the leading order (LO) prediction for the inclusive jet cross section, and \( \alpha_s^2(\mu_R)\hat{X}(0)(\mu_F, E_T)k_1(\mu_R, \mu_F, E_T) \) the NLO contribution. Both \( \hat{X}(0)(\mu_F, E_T) \) and \( k_1(\mu_R, \mu_F, E_T) \) are calculated with the JETRAD Monte Carlo program [6] based on the techniques described in [7] and the matrix elements of [8]. NLO QCD predictions for the inclusive jet cross section are also available in [9, 20] and agree well with those of JETRAD. All calculations are performed in the modified minimal subtraction, \( \overline{\text{MS}} \), scheme [21]. The JETRAD Monte Carlo program generates events with weighting factors, so that the jet clustering algorithm and \( E_T \) and \( \eta \) cuts, mimicking the experimental requirements, are directly applied to the final state partons. The jet clustering in JETRAD is governed by a cone radius \( R \) and a phenomenological parameter \( R_{\text{sep}}\) (default value = 1.3), introduced to match the experimental efficiency of identifying overlapping jets [12]. If two partons are more than \( R_{\text{sep}} \times R \) apart, they are identified as two distinct jets, otherwise they are merged into a single jet.

The inclusive jet data are divided into 33 \( E_T \) bins, from which we obtain statistically independent measurements of \( \alpha_s \) for 33 different values of \( \mu_R \). The \( \alpha_s \) values derived for \( \mu_R = \mu_F = E_T \) using CTEQ4M [22] parton distribution functions (PDFs) are presented in Fig. 1. For \( E_T < 250 \) GeV, there is good agreement with QCD predictions for the running of the coupling constant. The behavior of \( \alpha_s \) at higher \( E_T \) values is a direct reflection of the excess observed in the inclusive jet cross section [3].
The discrepancy with the NLO QCD predictions in this region, though not well understood, may be accommodated by the flexibility allowed by the world data in determining the high-$x$ gluon component in the parton distributions [12].

The measured values of $\alpha_s(\mu_R)$ are evolved to the mass of the $Z^0$ boson, $M_Z$, by using the solution to the 2-loop renormalization group equation

$$\alpha_s(M_Z) = \frac{\alpha_s(\mu_R)}{1 - \alpha_s(\mu_R)(b_0 + b_1 \alpha_s(\mu_R)) \ln(\mu_R/M_Z)}$$

where $n_f$ is the number of active flavors, which is equal to five (six) for $\mu_R$ smaller (larger) than the top quark mass. The values of $\alpha_s(M_Z)$ for all 33 measurements are shown in the inset of Fig. 1. Averaging over the range 40-250 GeV, we obtain

$$\alpha_s(M_Z) = 0.1178 \pm 0.0001 \text{ (stat)}.$$ 

Inclusion of the data with $E_T > 250$ GeV results in an increase of the average value by 0.0001.

The running of $\alpha_s$ is tested by verifying if $\alpha_s(M_Z)$ is independent of the energy scale $E_T$ at which the jet cross section is measured. The 27 values of $\alpha_s(M_Z)$ obtained from the data in the jet $E_T$ range 40-250 GeV are fit to the linear function $P_0 + P_1 \times (E_T/E_T^0 - 1)$ with $E_T^0 = 92.8$ GeV. The fit yields $P_0 = 0.1176 \pm 0.0003$ and $P_1 = 0.0003 \pm 0.0003$ with $\chi^2$/d.o.f. = 1.3, showing that $\alpha_s(M_Z)$ is independent of $E_T$ within one standard deviation. This non-trivial result demonstrates the correctness of the QCD prediction for the evolution of $\alpha_s$ over the above range.

The experimental systematic uncertainties on the value of $\alpha_s(M_Z)$ are derived from those on the inclusive jet cross section. For each source of systematic uncertainty described below, except normalization, the inclusive jet cross section was re-evaluated by varying the corresponding parameter in the detector response by 1σ. For the normalization uncertainty it was changed by a scale factor $[13]$. These uncertainties were propagated to $\alpha_s(M_Z)$ by repeating the procedure described above using the spectra given in Table VI of Ref. [12]. The results are shown in Fig. 2. The deviations of $\alpha_s(M_Z)$ for each spectrum from the central value are given in Table 1.

The dominant experimental systematic uncertainty in the inclusive jet cross section measurement is due to the calorimeter response to jets. The detector response and jet energy corrections are derived from a combination of test-beam data and Monte-Carlo simulations. The calorimeter response to charged pions was evaluated separately for high and low transverse momentum ($P_T$) pions from test beam data and isolated charged tracks from $pp$ data with an uncertainty of ±5% for $P_T \leq 5$ GeV,
The theoretical uncertainties are mainly due to the choice of renormalization and factorization scales and parton distribution functions. The scales $\mu_F$ and $\mu_R$ are expected to be of the same order as the characteristic scale of the process, which in this case is the jet $E_T$. We have evaluated the changes in $\alpha_s(M_Z)$ resulting from independently varying $\mu_F$ and $\mu_R$ from $E_T/2$ to $2E_T$ and found that the largest changes occur for $\mu_R = \mu_F$. For all results presented in this letter the two scales were set equal. The sensitivity of the measured $\alpha_s(M_Z)$ to changes in these scales is indicated by the shaded band in Fig. 3(a). Over the $E_T$ range from 40 to 250 GeV, the shift in $\alpha_s(M_Z)$ induced by changing the scale from $E_T/2$ ($\alpha_s(M_Z) = 0.1129 \pm 0.0001$) to $2E_T$ ($\alpha_s(M_Z) = 0.1249 \pm 0.0001$) is approximately $+6\%$ independent of $E_T$.

The coefficients $X_0$ and $k_1$ in Eq. (1) depend on the PDFs, which are obtained from global fits to deep inelastic scattering (DIS), Drell-Yan production and other collider data, including the inclusive jet results from Tevatron. Each PDF set has an associated strong coupling constant, $\alpha_s^{PDF}$. The gluon PDF ($G(x)$) determined from the fit is highly correlated with $\alpha_s^{PDF}$ because in equations describing all the processes used, the $G(x)$ is always accompanied by $\alpha_s^{PDF}$. To calculate the above coefficients, the PDFs are evolved using $\alpha_s^{PDF}$. For this procedure of measuring $\alpha_s$ to be valid, the extracted value of $\alpha_s$ should be consistent with the input $\alpha_s^{PDF}$, although not necessarily equal. The variation in parton distributions, especially in the gluon distribution, allowed by the world data was studied by the CTEQ Collaboration by fixing the value of $\alpha_s^{PDF}$ to 0.110, 0.113, 0.116, 0.119 or 0.122, with resulting $\chi^2$ of 1388, 1323, 1323, 1388 or 1543 for 1297 non-jet data points. We use the CTEQ4A series to study the $\alpha_s(M_Z)$ dependence on the PDFs. In addition, we have studied $\alpha_s(M_Z)$ using PDF sets which do not include Tevatron jet results, such as the MRST ($x^2$) sets [24], the MRSA' series [25], and two MRST sets [26]. The $\chi^2_{d.o.f.} = 1.38$ is obtained for CTEQ4M ($\alpha_s^{PDF} = 0.116$), therefore we use this PDF in our final fit. Excluding the PDFs which have obvious disagreement ($\chi^2_{d.o.f.} \geq 5$), we estimate the uncertainty on the $\alpha_s(M_Z)$ due to the PDF choice to be $\pm 5\%$.

Finally, the variation of $R_{sep}$, the jet clustering parameter, from 1.3 to 2.0 results in a 5-7% normalisation change of the inclusive jet cross section. The correspond-
ing variation in the $\alpha_s(M_Z)$ measurement is 2-3%.

To explore the flexibility in the gluon distribution at high $E_T$, a special PDF set, cTEQ4HJ, was generated by including CDF jet data in the global fit with higher statistical weight assigned to high $E_T$ points and a new parameterization of the gluon distribution \[2\]. This PDF yields good agreement between Tevatron data and theoretical predictions. Using this set, we obtain $\alpha_s(M_Z) = 0.1185 \pm 0.0001$, averaged over the entire $E_T$ range.

In conclusion, we have tested the evolution of the strong coupling constant $\alpha_s$ using the inclusive jet cross section data from $\bar{p}p$ collisions at $\sqrt{s} = 1800$ GeV. Our results demonstrate that for $E_T$ in the range of 40-250 GeV with $\mu_R = E_T$, the running of $\alpha_s$ is in good agreement with QCD predictions. The value of $\alpha_s$ expressed at the $Z^0$ boson mass is found to be

$$\alpha_s(M_Z) = 0.1178 \pm 0.0001 (\text{stat})^{+0.0081}_{-0.0095} (\text{exp. syst})$$

This value is in good agreement with the world average $\alpha_s(M_Z) = 0.1181 \pm 0.0020$ \[27\]. The theoretical uncertainties associated with the choice of parton distribution functions ($\sim 5\%$) and the choice of the renormalization scale ($\pm 4\%$) are comparable to the experimental systematic uncertainty.

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