Frequency Response Evaluation by a Scaling Method Considering the Structure Change

Ping-jia JIA\textsuperscript{1,2,*} and Long CHEN\textsuperscript{1,2,3}

\textsuperscript{1}State Key Lab of Digital Manufacturing Equipment and Technology, Huazhong University of Science and Technology, Wuhan, China
\textsuperscript{2}School of Mechanical Science and Engineering, Huazhong University of Science and Technology, Wuhan, China
\textsuperscript{3}Guangdong Intelligent Robotics Institute, Dongguan, China

*Corresponding author

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Abstract. Under operating conditions of machine tool, the structure’s dynamic is usually evaluated by operational modal (OM) analysis that is difficult to be used to estimate the frequency response function (FRF) because of the unknown state of complex input forces. In this paper, the scaling factor of OM shape was evaluated by the proposed scaling method which was based on the structure change. Firstly, OM parameters of the selected machine tool structure were estimated at four different positions. Results indicated that the mode shapes and the damping ratios were not influenced obviously by the structural change, but the natural frequency can be affected evidently. Secondly scaling factors were calculated by the frequency changes of the machine tool and the mode shapes of the movable components such as the worktable and the stocks, using the proposed method. Thirdly, machine tool's FRF was calculated from the scaled operational modal parameters. Besides, it has been found an interesting phenomenon that different natural frequencies were sensitive to different movable components of the machine tool.

Introduction

Dynamic behavior of machine tool is directly influenced by its structure type and distribution. Usually, dynamic characteristics are evaluated by computer-aided engineering and experiment-based modal analysis. For the front method, such as finite element analysis [1, 2], this is good to develop an accurate model, but difficult to establish, because dynamic stiffness (~ 60 \%) and damping (~ 90 \%) of the whole machine tool are introduced by joints [3]. The later method, including experimental modal analysis (EMA) and operational modal analysis (OMA), is mainly based on experimental procedure. Three steps (artificial excitation, measurement of input forces, and output responses) are necessary to be performed in EMA [4, 5], while dynamics of machine tool structure cannot be expressed by its modal parameters under operating conditions. Conversely, OMA is available to acquire modal parameters from the structural responses when the input signals are unknown. A challenge of OMA is that the input loads of machine tool are not clear and further mode shapes cannot be normalized. The input-output relations of a system, which can be represented by frequency response function (FRF), are essential information in various applications such as response prediction and stress analysis [6, 7, 8]. If the FRF are obtained from modal parameters, the following information is needed for each mode: the natural frequency, the damping factor and the mass normalized (scaled) mode shape. Therefore, an extra method to calculate the scaling factors is needed.

Especially for a complete machine tool, it is impossible to simplify a machine tool by adding virtual mass or spring with different mechanical conditions (stiffness, mass, operating modal testing, etc.). In this paper, a structural change method ignoring any additional mass and stiffness was proposed to analyze the scaling factor of the operational mode shape. By using this method, the FRF of the machine tool structure under operational condition can be acquired automatically without any manual work.
Numerical Model Validation

In this paper, the common mass-change or stiffness-change method is used to analyze the structural change modal, and this is not necessary to describe. In order to validate the proposed method, a numerical investigation of a system was performed. The system includes 12 degree of freedoms (DOFs) that represent the immovable component and 4 DOFs represent movable component. For simplifying the analysis, the axial stiffness and mass of each DOF of the immovable component were chosen to have the same values \( K_1 = K_2 = \ldots = K_{12}, \quad M_1 = M_2 = \ldots = M_{12} \), and the given value were 4000 N/m and 200kg. The axial stiffness and the mass of each DOFs of the movable component are the same too and the values are 10000N/m and 20kg, respectively.

To examine the effect of the position change on the natural frequency, we compare the dynamic responses of the structure at different positions (position #1 and position #2). For position #1, the movable component was connected to the fifth DOF of the immovable component. For position #2, the movable component was connected to the eighth DOF of the immovable component. Fig. 1 indicates the frequency shifts of the system at different positions in the range of 0-45Hz. The red curve in the figure indicates the frequency response of structure at position #1 and the blue curve indicates the frequency at position #2. From the figure, it can be noted that there are four modes for the structure and the natural frequency of each mode shifts as the movable component at different positions.

The modal shape vectors corresponding to the structure at different positions are list in table 1. The values in gray color indicate the mode shape of the movable component and the mode shapes of the DOF of the immovable attached by the movable component. It can be noted that the modal shapes of the movable component are nearly the same as those of the DOF of the immovable attached by the movable component. These illustrate that the assumption represented by equations (7) and (8) in section 2 are correct and validate.

![Figure 1. The frequency response of the structure at different positions.](image)
Table 1. The mode shapes of the structure at different positions.

| DOF of the Immovable component | DOF of the Movable component |
|--------------------------------|-------------------------------|
|                               | Mode #1                      | Mode #2                      | Mode #3                      |
| 1                             | 0.0027 + 0.0069i             | 0.7022                       | -0.0363                     |
| 2                             | 0.0043 + 0.0122i             | 1.0000                       | -0.0357                     |
| 3                             | 0.0039 + 0.0144i             | 0.7218                       | 0.0013                      |
| 4                             | 0.0015 + 0.0130i             | 0.0280                       | 0.0370                      |
| 5                             | 0.0354 + 0.3325i             | 0.0168                       | 0.9595                      |
| 6                             | 0.0256 + 0.5293i             | -0.0080                      | 0.5017                      |
| 7                             | -0.0254 + 0.5849i            | -0.0281                      | -0.4670                     |
| 8                             | -0.1073 + 0.4809i            | -0.0321                      | -0.9601                     |
| 9                             | -0.1914 + 0.2401i            | -0.0176                      | -0.4755                     |
| 10                            | 0.2394 + 0.0779i             | -0.0071                      | -0.4934                     |
| 11                            | 0.6005 - 0.0900i             | 0.0076                       | -0.0089                     |
| 12                            | 0.8054 - 0.1946i             | 0.0178                       | 0.4847                      |

The mode shapes of the immovable component at position #1 are compared with those at position #2 by the modal assurance criterion (MAC). The MAC is defined as the squared correlation coefficient between two modal vectors \( v_i \) and \( v_j \), and can be expressed as follows:

\[
MAC_{ij} = \frac{|\langle v_i^H v_j \rangle|^2}{\langle v_i^H v_i \rangle \langle v_j^H v_j \rangle}
\]  

(1)

The MAC value is between 0 and 1, where 1 means that the two vectors are estimates of the same physical mode shape and 0 means that they are orthogonal to each other. Fig. 2 presents the MAC of
the mode shapes the immovable component at position #1 and position #2. The values of MAC in the diagonal of the picture represent the comparison of the mode shapes with same orders and the nondiagonal values of MAC represent the comparison of the modal shapes with different orders. As this figure reveals, the mode shapes of the immovable component at different positions are nearly the same.

![Figure 2. MAC diagram of the modal shape vectors.](image)

**Experimental Setup**

Experimental case study for scaling the mode shapes of a CNC machine tool is provided. Operational modal parameters of the machine tool at four different positions (Fig. 3) were estimated by the operational modal analysis. Headstock and worktable were set in the original positions. The details of the four positions of the machine tool are listed in table 2 and the operational modal test can be briefly described below: Machine tool is excited by the inertial force of the accelerating or decelerating worktable. Seven low-mass, wide-bandwidth and three-axis accelerometers were mounted at the measurement points to monitor the vibrations in three directions (accelerometer reference: PCB-356-A15). All accelerometers were connected to an acquisition system (LMS SCADAS Mobile SCM05), and the sampling rate of the system was 1000 Hz. To acquire the mode shapes of the machine tool structure accurately and carefully, 198 measurement points were densely selected (shown by the nodes of the grid in Fig. 4b; the reference point was the black node at the base shown in Fig. 3b.

![Figure 3. Operational modal analysis with (a) different tool positions and (b) measurement method.](image)

**Table 2. The tool positions for the four operational modal analysis experiment.**

| Tool Positions | x | y | z |
|----------------|---|---|---|
| 1              | 550 | 175 | -580 |
| 2              | 550 | 175 | 0   |
| 3              | 1100 | 175 | -580 |
| 4              | 1100 | 175 | 0   |
Results and Discussion

The Results of the OMA of Machine Tool

The stability diagram is a useful method to estimate the modal parameters of a system. Fig. 4a, Fig. 4b, Fig. 4c and Fig. 4d present the frequency stability diagrams of the OMA of the machine tool at position #1, #2, #3 and #4, respectively, and the red curve in each figure represents the auto power spectrum density of the machine tool. The potential natural frequencies of machine tool are shown on the abscissa axis, and the model orders of the OMA algorithm are displayed on the ordinate axis. Theoretically, if a natural frequency were to appear at any order, there would be a high probability that it was a natural system frequency. The asterisk symbols indicate the relationship between the model order and the solutions found for the order. It is observed that there are four clusters of the symbols between 0Hz to 75Hz. All these clusters are aligning with the peaks of the auto power spectrum density except the second ones, which indicate that all of the natural frequencies of the machine tool are precisely estimated. Fig. 5 presents the damping ratio stability diagrams for the four frequency clusters, respectively. In the damping stability diagrams, the model damping ratio is presented on the x-axis and the model order on the y-axis. Theoretically, if the damping ratio converged to a stable value, then the frequency cluster could be considered as a natural frequency, otherwise, the frequency was false. The blue asterisk symbols in the damping stability diagrams represent the damping ratios of the second frequency clusters. It can be observed that they are not convergent as other clusters. The reason for this is that the second frequency clusters are calculated modes caused by the noise. The modal parameters of the entire machine tool estimated by the operational modal analysis are list in Table 3. Headstock position affected the natural frequency of the first mode, while worktable position affected the second and third mode. Therefore, it can be concluded that the natural frequencies of different mode are sensitive to different movable components.

Figure 4. The frequency stability diagrams for positions presented in Fig. 3.
Figure 5. The damping ratio stability diagram for position #1.

Table 3. The comparison of the modal parameters of different experiments.

| Mode | Position #1 Freq | Position #2 Freq | Position #3 Freq | Position #4 Freq | Position #1 Damp | Position #2 Damp | Position #3 Damp | Position #4 Damp |
|------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| Mode 1 | 16.586 | 16.456 | 15.590 | 15.666 | 3.43% | 3.40% | 3.36% | 3.54% |
| Mode 2 | 49.806 | 47.393 | 49.691 | 47.305 | 5.88% | 5.08% | 5.66% | 5.46% |
| Mode 3 | 67.659 | 61.817 | 67.527 | 61.757 | 5.49% | 5.32% | 5.13% | 5.71% |

Fig. 6 presents the MAC diagram of the operational mode shapes of immovable component machine tool (column and the base) at position #1 and #4. It can be noticed that the diagonal of the picture is near 1, which means that the mode shapes of the immovable component are almost the same. Therefore, the first assumption for the proposed scaling factor method is satisfied. Fig. 7 presents the modal shapes of the entire machine tool in x-directions estimated by operational modal analysis. From these results, it is obvious that there is no deformation for the black component (the headstock) and the green component (the worktable). The mass of the headstock and the worktable is listed in Table 4, respectively. Using the estimated mode shape vectors of the worktable and the headstock, the scaling factors for the first three modes are listed in Table 4.
Once the scaling factors are calculated, the frequency response function of the machine tool can be synthetized. Fig. 8 presents the frequency response function the machine tool. The red curve in the figure is the real frequency response function acquired by hammer test in x-direction while the blue curve is the synthetized frequency response function calculated by the proposed method. A good similarity was observed between the two frequency response function, and these illustrate that the method proposed in this paper is corrected and validated. Although there is a good similarity, small difference can still be found. There are four peaks for the blue curve while three peaks for the red curve. The second peak of the blue curve represents the first mode of the machine tool in y-direction which excited by the hammer test in x-direction. This phenomenon is due to the non-orthogonal of the exciting force supplied by the force hammer.

![Figure 8](image)

Fig. 9 presents the synthetized frequency response function the machine tool at four different positions. The main difference between each curve in the figure is the natural frequencies for the
peaks. For position #1 and #3 (or position #2 and #4), the first peaks of the FRFs coincide, which means that the movement of the worktable does not influence the first mode of the machine tool. However, the second and third peaks are obviously influenced by the movement of the worktable and the natural frequencies for position #1 (position #2) are smaller than those for position #3 (position #4). For position #1 and #2 (or position #2 and #4), the second and third peaks almost coincide but the first peaks are different, which is caused by the movement of the headstock. Therefore, for the machine tool in our study, different moveable components can influence the modes with different order.

Figure 9. The comparison of the synthesized frequency response functions of the machine tool in x direction at four different positions.

Conclusions

During machining operations, OMA is a powerful tool to analyze the dynamic modal parameter. However, a drawback in OMA is that the mode shapes cannot be scaled because input loads are unknown. Therefore, FRF of the machine tool cannot be calculated by the operational modal parameters. In this paper, a structural change method ignoring any additional mass and stiffness was proposed to analyze the scaling factor of the operational mode shape.

The operational modal parameters of a CNC machine tool structure were estimated at four different positions to validate the proposed method. The results indicated that the mode shapes and the damping ratios were not influenced obviously by the structural change, but the natural frequency can be affected evidently. The scaling factors were calculated by the frequency changes of the machine tool and the mode shapes of the movable components such as the worktable and the stocks, using the proposed method. At last, the frequency response function of the machine tool was calculated from the scaled operational modal parameters.

Besides, it has been found an interesting phenomenon that different natural frequencies of the machine tool were sensitive to different movable components. For example, for the machine tool in our study, the natural frequency of the first mode is only affected by the movement of the headstock while the natural frequencies of the second and third modes are affected by the movement of the worktable. Therefore, more works are needed to study the machine tool dynamics sensibility which falls outside the scope of this paper.

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