Is cosmology compatible with sterile neutrinos?

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By combining data from cosmic microwave background (CMB) experiments (including the recent WMAP third year results), large scale structure (LSS) and Lyman-α forest observations, we constrain the hypothesis of a fourth, sterile, massive neutrino. For the 3 massless + 1 massive neutrino case we bound the mass of the sterile neutrino to $m_s < 0.26 eV (0.44 eV)$ at 95% (99.9%) c.l.. These results exclude at high significance the sterile neutrino hypothesis as an explanation of the LSND anomaly. We then generalize the analysis to account for active neutrino masses (which tightens the limit to $m_s < 0.23 eV (0.42 eV)$) and the possibility that the sterile abundance is not thermal. In the latter case, the constraints in the (mass, density) plane are non-trivial. For a mass of $> 1 eV$ or $< 0.05 eV$ the cosmological energy density in sterile neutrinos is always constrained to be $\omega_s < 0.003$ at 95% c.l. However, for a sterile neutrino mass of $\sim 0.25 eV$, $\omega_s$ can be as large as 0.01.

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Introduction.— Recent cosmological data coming from measurements of the Cosmic Microwave Background (CMB) anisotropies (see e.g. [1 2]), galaxy clustering (see e.g. [3]) and Lyman-alpha forest clouds [4] are in a spectacular agreement with the expectations of the so-called standard model of structure formation, based on primordial adiabatic inflationary perturbations and a cosmological constant.

Since the model works so well, the ambitious idea of using cosmology to test aspects of particle physics is becoming a reality. An excellent example of this comes from the new cosmological constraints on neutrino physics.

Cosmological neutrinos have a profound impact on cosmology since they change the expansion history of the universe and affect the growth of perturbations from measurements of the so-called standard model of structure formation, based on primordial adiabatic inflationary perturbations and a cosmological constant.

Results from the Liquid Scintillator Neutrino Detector (LSND) [8] challenge the simplicity of this picture. The LSND experiment reported a signal for $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillations in the appearance of $\bar{\nu}_e$ in an originally $\bar{\nu}_\mu$ beam. To reconcile the LSND anomaly with results on neutrino mixing and masses from atmospheric and solar neutrino oscillation experiments, one needs additional mass eigenstates. One possibility is that these additional states are related to right-handed neutrinos, for which bare mass terms ($M_{\nu R\nu_R}$) are allowed by all symmetries. These would be sterile, i.e. not present in $SU(2)_L \times U(1)_Y$ interactions. The “$3+1$ sterile” neutrino explanation assumes that the $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillation goes through $\bar{\nu}_s \rightarrow \bar{\nu}_e$. The additional sterile state is separated by the three active states by a mass scale in the range $0.17 eV < m_{\beta\beta} < 2.0 eV$ at 99% c.l. [9] from the Heidelberg-Moscow double beta decay experiment is at odds with the cosmological bound.

While the neutrino masses are very difficult to measure experimentally, mass differences between neutrino mass eigenstates ($m_1, m_2, m_3$) have now been measured in oscillation experiments. Observations of atmospheric neutrinos suggest a squared mass difference of $\Delta m^2 \sim 3 \times 10^{-3} eV^2$, while solar neutrino observations, together with results from the KamLAND reactor neutrino experiment, point towards $\Delta m^2 \sim 5 \times 10^{-5} eV^2$. The two measured mass differences are easily accomodated in simple extensions of the Standard Model by giving masses to at least two of the neutrinos. If these masses are greater than $\sim 0.1 eV$, all three neutrinos must be nearly degenerate, with small differences accounting for the observations.

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the first results at the beginning of the next year.

In the meantime, given the increased quality in the data, it is timely to test the sterile neutrino hypothesis using cosmological observations. Several recent analyses have already provided interesting cosmological constraints on a fourth massive neutrino. Here we generalize these in several ways: first, while previous work has considered the case of 3 (massless) + 1 (massive) neutrino, here we also allow masses for the standard 3 neutrinos, as required by oscillation experiments. Second, we use updated cosmological datasets, including the new results from the WMAP satellite and BOOMERAG-2K2 experiment. Finally, the strength of the interactions of a neutrino determines its cosmological abundance. Given how little we know about sterile neutrino interactions (or mass mixing), it therefore seems reasonable to allow the sterile abundance to be a free parameter. Of course, if a sterile neutrino can have any abundance (including zero), there is no mass limit. However, we will see that the constraints in the (mass, density) plane are highly non-trivial.

Cosmology.— The three active neutrinos interact via the well-measured weak interactions. These interactions ensure that they were in thermal equilibrium at early times until they decouple from the primordial plasma slightly before electron-positron annihilation. After decouple, they maintain an equilibrium distribution of a massless fermion with a temperature lower than the photon temperature by a factor of \((4/11)^{1/3}\). This introduces a well-known relation between the energy density of the active neutrinos and their total mass:

\[
\omega_\nu \equiv \frac{\rho_\nu}{\rho_{\text{cr}}} = 0.0106 \frac{m_\nu}{\text{eV}} \quad (1)
\]

where \(\rho_{\text{cr}}\) is the critical energy density, \(h\) parametrizes the Hubble constant via \(H_0 = 100h\) km sec\(^{-1}\) Mpc\(^{-1}\), and here and throughout \(m_\nu\) refers to the sum of all active neutrino masses. So, for example, if the three neutrinos are nearly degenerate, they each have a mass approximately equal to \(m_\nu/3\).

While sterile neutrinos, by definition, do not have weak interactions, they are not pure mass eigenstates. As such, oscillations in the early universe can transform the thermal active neutrinos into a sterile neutrino. Thermalization occurs if

\[
\Delta m_{\text{LSND}}^2 \sin^4 \theta > 3 \times 10^{-6}\text{eV}^2, \quad (2)
\]

where \(\theta\) is an effective mixing angle. In the simplest models with one sterile neutrino, this condition is satisfied, so \(\omega_\nu = 0.0106(m_\nu/\text{eV})\), but there are many ways of evading thermalization. Indeed, if one light sterile neutrino exists, there is every reason to expect one or two more and these considerably complicate the thermalization analysis. It is, for example, possible to have super-thermal abundances if a heavier sterile state decays at relatively late times to a lighter state. In short, if a sterile neutrino exists, its cosmological density is much more uncertain than that of the active neutrinos.

Sterile neutrinos influence the development of inhomogeneities and anisotropies in the universe by changing the epoch of equality and by suppressing perturbations via freestreaming. The epoch at which the energy density in non-relativistic matter equals that in radiation dictates when structure begins to grow. This leaves an imprint on the matter power spectrum: there is a peak at the scale equal to the horizon at the epoch of equality. If this epoch is close to recombination, the residual radiation causes gravitational potentials to decay, and this time variation produces an early Intergated Sachs-Wolfe (ISW) effect, boosting the power on scales near the horizon. The main effect of freestreaming is a suppression of power on scales smaller (wavenumber \(k\) larger) than

\[
k_{\text{fs}} = 0.01 \left(\frac{m_s}{\text{eV}}\right)^{1/2} \text{Mpc}^{-1} \quad (3)
\]

with suppression proportional to \(\omega_s/\omega_m\), where \(\omega_m \equiv \Omega_m h^2\), and \(\Omega_m\) is the total energy density of non-relativistic matter (baryons plus cold dark matter) in units of the critical density.

In the standard cosmology, with three massless neutrinos, the scale factor at equality is \(a_{\text{EQ}} = 2.82 \times 10^{-4}(0.15/\omega_m)\). A sterile neutrino is relativistic until its temperature drops beneath its mass, so masses of order an eV raise the question: what does it count as, matter or radiation? Since the Hubble rate scales as
a^{-2}$ in a radiation dominated universe and $a^{-3/2}$ in a matter dominated universe, we define the epoch of equality as the moment when

$$\frac{d \ln H}{d \ln a}(a_{eq}) = -\frac{7}{4}. \quad (4)$$

This definition agrees well with the standard definition for massless neutrinos. The dependence of $a_{eq}$ on the sterile neutrino parameters $m_s$ and $\omega_s$ is plotted in Figure 1. This figure suggests that in the limit of very small $m_s$, any appreciable $\omega_s$ will be excluded because neutrinos behave essentially as radiation and shift the redshift of matter-radiation equality significantly, producing an unacceptably large ISW effect.

The amount of suppression due to freestreaming increases as the density increases (from top to bottom in Fig. 1), but the large scales (from which constraints derive) cease to be affected as the neutrino mass increases (from left to right). Therefore, at fixed $\omega_s$, constraints from freestreaming are tighter for small neutrino masses. Note that this differs from the thermal case (dashed curve in Fig. 1). In that case, the neutrino density increases with the mass, so there is more suppression at high masses.

**Data analysis and Results** - To obtain constraints on sterile neutrino parameters, we use the publicly available Markov Chain Monte Carlo (MCMC) package CosmoMC 24. The linear perturbations engine CAMB 24 of the software has been generalized in several ways. First, we allow for a non-thermal sterile neutrino density. Second, we allow for the possibility that the active neutrinos have mass different than the sterile neutrino.

In the MCMC, we sample the following 8 dimensional set of cosmological parameters, adopting flat priors on them: The log mass of thermal sterile neutrinos $\log m_s$ and $\omega_s$, the energy density of 3 degenerate standard massive neutrinos $\omega_{\nu} = m_{\nu}/(94.1 eV)$, the physical baryon and CDM densities, $\omega_b = \Omega_b h^2$ and $\omega_c = \Omega_c h^2$, the ratio of the sound horizon to the angular diameter distance at decoupling, $\Theta_s$, the scalar spectral index and the overall normalisation of the spectrum, $n_s$ and $A_s$, and, finally, the optical depth to reionisation, $\tau_r$. We consider purely adiabatic initial conditions, impose flatness, and do not include gravitational waves.

We include the WMAP three-year data 1, 2 (temperature and polarisation) with the routine for computing the likelihood supplied by the WMAP team 25, as well as the CBI 26, VSA 27, ACBAR 28 and BOOMERANG-2k2 18 measurements of the CMB on smaller scales than those sampled by WMAP. In addition to the CMB data, we also consider the constraints on the real-space power spectrum of galaxies from the SLOAN galaxy redshift survey (SDSS) 29 and the 2dF galaxy redshift survey 31 and Lyman-alpha forest clouds 31, 32 from the SDSS, the gold sample of the recent supernova type Ia data 33, the latest SNLS supernovae data 34 and the constraints from the baryonic acoustic oscillations detected in the Luminous Red Galaxies sample of the SDSS 35 1.

The details of the analysis are the same as those in 36 and the reader is invited to check that paper to examine what constraints the above datasets give for other models including standard 3 degenerate massive neutrinos case.

If the active neutrino masses are fixed to zero and

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1 There is a negligible overlap between the constraints from the 2dFGRS, SDSS and SDSS LRG analysis, as there are galaxies in common in all three data sets.
the sterile neutrino abundance is thermal (similar to the assumptions imposed in Ref. [4]), the upper limit on the sterile neutrino mass is 0.26eV (0.44eV) (all at 95% (99.9%) c.l.). Of course the active neutrino masses are not zero. Taking them as a free parameter leads to an upper limit on the sterile neutrino mass of 0.23eV (0.42eV). This is marginally tighter than the $m_\nu = 0$ constraint because the limit is really on the sum of all neutrino masses. Fixing the active masses to zero allows the maximum $m_s$. Relaxing this restriction leaves less room for a large $m_s$. We have found some sensitivity to the mass difference of the sterile and active states (and this might be measurable with future data), but current data really constrain only the sum of all neutrino masses.

We now generalize further and allow the sterile neutrino abundance $\omega_s$ to vary. Fig. 3 shows the constraints in the $\omega_s$-$m_s$ plane. Note the distinct peak around the region of $m_s \sim 0.25eV$, presenting an allowed region of parameter space with anomalously large values of $\omega_s$. To the left of this peak, $a_{EQ}$ is very large and the resulting ISW effect precludes agreement with CMB data. When $m_s$ is in the allowed regime, $a_{EQ}$ would still be too large for $\omega_m$ fixed. However, a model with larger $\omega_m$ ($\sim 0.18$) leads to an even smaller, acceptable $a_{EQ}$. Fortuitously, the enhanced cold matter density also mitigates the freestreaming suppression (which scales as $\omega_c^{-1}$). At larger neutrino mass ($\sim 1eV$), additional cold matter would make $a_{EQ}$ too small, so $\omega_m$ must be closer to 0.13 and the free-streaming suppression becomes relevant again, preventing agreement with large scale structure. This is illustrated in Fig. 2. Here we show the angular CMB anisotropy and matter power spectrum for different masses at fixed $\omega_s$. The suppression due to free-streaming is evident in the power spectrum and clearly becomes more severe for smaller masses. However, increasing dark matter density to match the epoch of matter-radiation equality opposes this effect. Crucial to this interpretation is the realization that the matter-radiation equality is very thoroughly measured by the present-day experiments with little model-dependence. The constraint can be summarised in $a_{EQ} \sim (2.95 \pm 0.13) \times 10^{-4}$.

**Conclusions**– By combining data from cosmic microwave background experiments, galaxy clustering and Ly-alpha forest observations we have constrained the hypothesis of a fourth, sterile, massive neutrino, as an explanation of the LSND anomaly. For the 3 massless + 1 massive thermal neutrino case we bound the mass of the sterile neutrino to $m_\nu < 0.26eV(0.44eV)$ at 95 (99.9) % c.l. Marginalizing over active neutrino masses improves the limit to $m_\nu < 0.23eV(0.42eV)$. These limits are incompatible at more than 3$\sigma$ with the LSND result $0.6eV^2 < \Delta m^2_{LSND} < 2eV^2$ (95% C.L.). Moreover, our analysis renders the LSND anomaly incompatible at high significance with a degenerate active neutrino scenario and vice versa. If we allow for the possibility of a non-thermal sterile neutrino, we find that the upper limit of allowed energy density in the sterile neutrino is a strong function of mass. In particular, for $m_s < 1eV$ or $\nu_0 > 0.05eV$ the cosmological energy density in sterile neutrinos is always constrained to be $\omega_s < 0.003$, but that for sterile neutrino mass of $\sim 0.25eV$, $\omega_s$ can be as large as 0.01eV.

The results presented in this paper rely on the assumption that systematics in the public datasets we analyzed (WMAP, Lyman-$\alpha$, etc.) are under control. We argue that this is likely: the datasets are large enough that detailed systematics checks – e.g. dividing the data into multiple subsets, constructing quiet channels that should see nothing, and cross-correlating different bands to reduce noise – have been performed. We also checked that if we drop either small scale CMB, LSS or Lyman-$\alpha$ dataset from the analysis, the constraints simply weaken without any systematic change in the results.

The results presented here also rely on the assumption of a theoretical cosmological model based on a large but limited set of parameters. Extensions of the parameter space – e.g., inclusion of isocurvature modes, gravity waves or a different parametrization for the dark energy component – may modify our conclusions. Those modifications are however not needed by current data and some of them may well lead to stronger limits. Indeed the simple cosmological model with only a handful of parameters does an excellent job explaining a wide variety of data. If the LSND anomaly is confirmed by MINIBOONE, we will have been proved wrong, and cosmologists will need to re-examine the entire framework on which these very tight constraints rest.
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