\[ \Delta I = 1/2 \] rule from staggered fermions.

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We present our latest results for the \[ \Delta I = 1/2 \] rule, obtained on quenched ensembles with \[ \beta = 6.0 \] and 6.2, and a set of \[ N_f = 2 \] configurations with \[ \beta = 5.7 \]. The statistical noise is quite under control. We observe an enhancement of the \[ \Delta I = 1/2 \] amplitude consistent with experiment, although the systematic errors are still large.

We also present a non-perturbative determination of \[ Z_P \], \[ Z_S \] and the strange quark mass. We briefly discuss our progress in calculating \[ \epsilon'/\epsilon \].

1. Introduction and methods

It is well-known that the \[ \Delta I = 1/2 \] channel of non-leptonic kaon decays is enhanced compared to the \[ \Delta I = 3/2 \] one. In particular, \[ \omega \equiv \text{Re} A_0/\text{Re} A_2 = 22 \], where \( A_0,2e^{i\delta_{0,2}} \equiv \langle (\pi\pi)_{I=0,2}|H_W|K^0 \rangle \). This talk is a status report on our work in calculating these matrix elements (MEs) using staggered fermions. We have also computed MEs relevant for \[ \epsilon'/\epsilon \].

The effective weak Hamiltonian for this problem is as follows:

\[
H_W^{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{us} V_{ub}^* \sum_{i=1}^{10} \left[ z_i(\mu) + \tau y_i(\mu) \right] O_i(\mu),
\]

where \( \tau = -V_{td}V_{ts}^*/V_{ud}V_{us}^* \), \( z_i \) and \( y_i \) are Wilson coefficients. We work in the standard basis of the 10 four-fermion operators defined in [3].

Our task is to compute \( \langle \pi\pi|O_i|K \rangle \). Putting a two-pion state on the lattice is a well-known technical problem. We use the chiral perturbation theory method [2] to relate \( \langle \pi\pi|H_W|K \rangle \) to \( \langle \pi|H_W|K \rangle \) and \( \langle 0|H_W|K \rangle \). This is equivalent to subtraction of a single lower-dimension operator \( O_{sub} = (m_s + m_d)\vec{\pi}d + (m_d - m_s)\vec{\gamma}_5d \), which is the only operator allowed to mix due to the chiral properties of staggered fermions. This procedure does not take into account the higher order corrections in chiral perturbation theory (ChPT), in particular the final state interactions of the pions. These corrections are known to be large, which introduces a big systematic uncertainty in our results.

We follow the strategy of ME computation with staggered fermions [1] and compute three types of fermion contractions, known as “eight”, “eye” and “annihilation” diagrams. The latter two types are quite noisy, but we gained enough statistics to bring the noise down to an acceptable level for all basic operators \( O_1 - O_{10} \).

Figure 1 shows our simulation parameters. Our lattice is replicated 4 times in time direction to avoid contamination from excited states. We use degenerate mesons \( (m_s = m_d = m_u) \) and gauge-invariant, tadpole-improved operators with staggered fermions.

![Figure 1. \( B_K \) vs. meson mass squared. Vertical line here and in other plots indicates the physical kaon mass. The fit is of the form \( y = a + bx + cx \log x \).](image-url)
Table 1
Simulation parameters

| $N_f$ | $\beta$ | Size          | L, fm | N  |
|-------|---------|---------------|-------|----|
| 0     | 6.0     | $16^3 \times (32 \times 4)$ | 1.6   | 216|
| 0     | 6.0     | $32^3 \times (64 \times 2)$  | 3.2   | 26 |
| 0     | 6.2     | $24^3 \times (48 \times 4)$  | 1.7   | 26 |
| 2     | 5.7     | $16^3 \times (32 \times 4)$  | 1.6   | 83 |

2. \( \text{Re} A_2 \) and \( \langle O_K \rangle \)

\( \text{Re} A_2 \) can be related by ChPT to \( \langle K^0|O_K|K^0 \rangle \), so at the lowest order we just need to compute \( f_K \) and \( B_K \) (defined in \( \langle K^0|O_K|K^0 \rangle = 8/3 \ m_K f_K B_K \), where \( O_K = \bar{\sigma} \gamma_L d \bar{\sigma} \gamma_L d \)). The \( B_K \) parameter involves calculating only “eight” contractions and is now well studied (e.g. [4,5]). The form of the chiral behaviour \( B_K = a + b \ m_K^2 + c \ m_K^4 \log m_K^2 \) produces a reasonable fit and gives a finite non-zero value in the chiral limit (Fig. 1). However, the physical \( \text{Re} A_2 \) (proportional to \( \langle O_K \rangle/m_K^2 = 8/3 \ f_K^2 B_K \)) is very sensitive to the meson mass, contrary to a naive expectation (see Fig. 2). This is due to a sizeable slope in \( f_K \) vs. \( m_K^2 \) (which is of the same order as the physical slope) (Fig. 3). Thus there is a large uncertainty due to unknown higher order terms in ChPT. Naively taking the meson mass

[Figure 3. \( f_K \) vs. meson mass squared.]

\[ M^2 = (m_K^2 + m_\pi^2)/2 \] and using our quenched value of \( B_K \) in continuum limit yields: \( \text{Re} A_2 = (1.67 \pm 0.02) \cdot 10^{-8} \text{ GeV} \), to be compared with experimental \( 1.25 \cdot 10^{-8} \text{ GeV} \).

3. \( \text{Re} A_0 \) and \( \Delta I = 1/2 \) rule

Results for \( \text{Re} A_0 \) for quenched \( \beta = 6.0 \) and 6.2 ensembles are shown in Fig. 4. The dependence on the meson mass is much smaller than that of \( \text{Re} A_2 \). We have checked the lattice volume dependence and found it to be small for lattice sizes 1.6 fm and above. However, the results significantly depend on the lattice spacing (see Fig. 5). The \( \beta = 6.2 \) point seems to bring the continuum value below the experiment. However, final state interactions could additionally raise it by as much as 100%. Finally, we checked the effect of unquenching and found it to be small compared to noise (see Fig. 6).

We show the ratio \( \text{Re} A_2/\text{Re} A_0 \) for quenched \( (\beta = 6.0) \) and dynamical data sets in Fig. 7. The result is of the same order of magnitude as experiment. However, the dependence on the meson mass is so large that it prevents us from a more
Figure 4. $\text{Re}A_0$ vs. meson mass squared for quenched $\beta = 6.0$ and 6.2. Finite volume study for $\beta = 6.0$ is also shown.

Figure 5. $\text{Re}A_0$ vs. lattice spacing squared. The cross is the result from the larger volume $\beta = 6.0$ ensemble.

One-loop perturbative matching works well for operators $O_1$ and $O_2$ (relevant for $\text{Re}A_0$): the corrections are small, and so is the $q^*$ dependence. However, for operators $O_5 – O_8$ (relevant for $\varepsilon'/\varepsilon$) the situation is much worse: several of the perturbative coefficients have not yet been calculated, while others (notably $C_{PP}$) are too large at one-loop order to trust the perturbation theory. Thus, a non-perturbative matching procedure is necessary to calculate $\varepsilon'/\varepsilon$.

As a first step towards this procedure we have computed bilinear renormalization factors $Z_S$ and $Z_P$ using a non-perturbative method based on the strategy by Martinelli et al. [7] (see Fig. 8). Our best value for the strange quark mass in continuum limit obtained with non-perturbative $Z_S$ in MS at 2 GeV is $103 \pm 8$ MeV, which is quite close to the one-loop result.

$Z_P$ can be used to get a reasonable estimate of the renormalization of operators $O_6$ and $O_8$. The value of $\langle O_6 \rangle$ obtained in this way is very different from the tree-level value: it is much smaller, close to zero. This would produce a negative $\varepsilon'/\varepsilon$, contrary to experiment. However, to give a more definite prediction we need to perform a full non-perturbative renormalization procedure.

5. Summary

We have obtained a reasonable statistical precision in studying $\text{Re}A_0$ and $\text{Re}A_2$ as well as all
Figure 7. Isospin amplitude ratio vs. meson mass squared for quenched and dynamical ensembles with comparable lattice spacings. This enormous dependence on the meson mass comes entirely from the behavior of $\text{Re}A_2$ (see Fig. 2).

Figure 8. Non-perturbative renormalization factors $Z_S$ and $Z_P$ in $\overline{\text{MS}}$ vs. momentum scale $\mu$ for $\beta = 6.2$. The curve is the one-loop perturbation theory expectation (which does not distinguish between $Z_S$ and $Z_P$ in the chiral limit).

matrix elements needed for $\varepsilon'/\varepsilon$. The biggest uncertainties in our prediction of the $\Delta I = 1/2$ rule are higher order ChPT terms and continuum extrapolation. In addition, a non-perturbative operator matching needs to be done for $\varepsilon'/\varepsilon$.

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