Infrared signature modelling of a rocket jet plume – comparison with flight measurements

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Abstract. The infrared signature modelling of rocket plumes is a challenging problem involving rocket geometry, propellant composition, combustion modelling, trajectory calculations, fluid mechanics, atmosphere modelling, calculation of gas and particles radiative properties and of radiative transfer through the atmosphere. This paper presents ONERA simulation tools chained together to achieve infrared signature prediction, and the comparison of the estimated and measured signatures of an in-flight rocket plume. We consider the case of a solid rocket motor with aluminized propellant, the Black Brant sounding rocket. The calculation case reproduces the conditions of an experimental rocket launch, performed at White Sands in 1997, for which we obtained high quality infrared signature data sets from DRDC Valcartier. The jet plume is calculated using an in-house CFD software called CEDRE. The plume infrared signature is then computed on the spectral interval 1900-5000 cm⁻¹ with a step of 5 cm⁻¹. The models and their hypotheses are presented and discussed. Then the resulting plume properties, radiance and spectra are detailed. Finally, the estimated infrared signature is compared with the spectral imaging measurements. The discrepancies are analyzed and discussed.

1. Introduction

Infrared space or airborne surveillance sensors are the most effective systems to detect and track proliferating ballistic missiles [1]. These sensors have to be designed and optimized upon realistic infrared signatures of plume rockets. The lack of observations in operational conditions leads to the use of simulations to complete ground measurements. The infrared signature modeling of rocket plumes is a challenging problem involving rocket geometry, propellant composition, combustion modeling, trajectory calculations, fluid mechanics, atmosphere modeling, gas and particles radiative properties, and radiative transfer through the atmosphere. ONERA decided to tackle this complex problem through a common project involving several research teams. This paper presents the simulation tools chained together to achieve the infrared signature prediction and the comparison of the estimated and measured signatures of an in-flight rocket plume.

We considered the case of a solid rocket motor with aluminized propellant, the Black Brant sounding rocket (maximum thrust 77 kN). The calculation case reproduces the conditions of an experimental rocket launch, performed at White Sands in 1997, during which DRDC Valcartier
recorded high quality infrared signature data. Records were collected by a spectral imager of 8 by 8 elements, associated with a tracking system, with a spectral resolution of 4 cm\(^{-1}\) on the spectral interval 1900-5000 cm\(^{-1}\) and a time interval of about 420 ms. The Black Brant was the second stage of the sounding rocket. The measurements encompass 51 frames acquired over 20 seconds of the Black Brant propulsion phase, from altitude 5 km to 23 km. The distance between the observer and the missile launch site is about 13.5 km. A test point is chosen at altitude 8 km. This point was selected as our first calculation point in order to limit the effects of high altitude on the plume physics and because it corresponds to an acquisition where the whole plume is visible in the frame. At this altitude, the rocket is in a stabilized phase of thrust.

![Figure 1: Examples of DRDC Valcartier spectral imager measurements for Black Brant launch in White Sands (1997)](image)

2. Models and hypotheses

2.1. Restitution
A preliminary work (not detailed here) was conducted in order to retrieve the propulsion characteristics of the sounding rocket Black Brant launched on November 18\(^{th}\) 1997 at White Sands Missile Range. The first stage (not studied) was a Terrier MK 70, while the second stage was a Black Brant VC MOD2 [2][3]. Insights about propellant composition and shape, nozzle geometry and propulsion elements were inferred from references [4][5][6].

2.2. Combustion and CFD models
The modeling of the infrared signature (IRS) of the Black Brant plume is obtained using a chain of multi-physics codes developed at ONERA. The computational flow starts from the combustion chamber and nozzle properties, whose results are used both for the trajectory calculation and for the plume flowfield computation, and ends with the radiative transfer computation to retrieve the infrared signature as seen by the spectro-imaging sensor.

COPPELIA is a thermodynamic equilibrium code that computes the motor specific impulse, the temperature and the gaseous species composition in the combustion chamber as a function of the propellant composition and the nozzle expansion ratio, using either a frozen or equilibrium expansion model. The propellant composition retained for the Black Brant calculation is: 19% Al, 61% of ammonium perchlorate, 20% polyurethane. COPPELIA outputs are used to compute the Black Brant trajectory in order to retrieve the rocket speed, the distance from the sensor to the target and the aspect angle of the plume with respect to the sensor line of sight. The chamber temperature, pressure and gaseous composition, the rocket speed, altitude and geometry are used to initialize ONERA computational fluid dynamics code CEDRE for the plume flow field computation.
Table 1: Input characteristics of the Black Brant 8km test case

| Characteristic                        | Value       |
|---------------------------------------|-------------|
| Altitude (m)                          | 7900        |
| Distance from sensor (m)              | 14221       |
| Aspect angle (deg)                    | 112.4       |
| Speed (m/s)                           | 646.8       |
| Flight Mach Number                    | 2.11        |
| Atmospheric pressure (Pa)             | 36120       |
| Atmospheric temperature (K)           | 236.8       |
| Chamber pressure (bar)                | 45.2        |
| Chamber temperature (K)               | 3290.8      |
| Nozzle throat area (m²)               | 0.0111      |
| Nozzle exit area (m²)                 | 0.0871      |

| Element     | Mass fraction (%) |
|-------------|-------------------|
| H           | 0.12%             |
| O           | 0.01%             |
| OH          | 0.22%             |
| H₂          | 2.88%             |
| H₂O         | 6.59%             |
| CO          | 28.95%            |
| CO₂         | 1.52%             |
| HCl         | 16.45%            |
| Cl          | 0.93%             |
| N₂          | 7.98%             |
| Al₂O₃       | 34.34%            |

CEDRE [7] is a multiphysics computational tool for numerical simulation in the field of energetics, with particular emphasis on propulsion applications. The code can handle several coupled physical subsystems, each of them being taken into account by a specialized time-dependent solver. The compressible flow module solves the Navier-Stokes equations with any number of species and various possible models for chemical reactions. The simulation space can be 3D or 2D (plane or axisymmetric). For turbulence, RANS 2- or 4-equations models (k-ε or k-ω...) are available, as well as Large-Eddy Simulation (in the MILES approach or with Smagorinsky subgrid models) [8]. Liquid and solid particles can be modelled using a Lagrangian [9] or Eulerian [10] approach depending on applications. With both particle solvers, interactions with the gas include drag forces, heat exchanges and vaporization-condensation.

![Figure 2: Axisymmetric non structured mesh used for the CFD computation (close-up in nozzle region)](image-url)
For the Black Brant test case we have conducted axisymmetric calculations of the flow on an unstructured mesh (500m x 10m domain, 191K cells) refined on boundaries and in the mixing layer (figure 2). The HLLC scheme with 2nd order reconstruction along with the VanLeer slope limiter is used for space discretization of the convective terms. Time integration is performed using a one-step Euler implicit scheme until a steady-state is obtained. A RANS k-ε turbulence approach is used. However, these Reynolds-averaged Navier-Stokes models are not fully adapted to axisymmetric flows. Therefore, some constants of the original model by Menter [11] were modified following a strategy already used in [12][13]. The chemistry is modeled using a semi-detailed kinetic scheme shown in Table 2. The chemical scheme consists of three main sets of reactions: H₂ and O₂ reactions 1 to 8, CO and CO₂ reactions 9 to 11 and chlorine species reactions 12 to 17. The alumina particles are tracked by the Eulerian dispersed phase solver coupled with the Navier-Stokes solver, and are assumed to remain liquid. No coupling between chemistry and turbulence is considered and radiation is not taken into account in the enthalpy balance.

Having no information about particle size distribution, we resorted to Hermsen’s correlation [14] which relates the particle mass weighted average diameter D43 to the rocket nozzle diameter. We assumed that there was only one class of particles with diameter 5.71 µm (every particle in the plume has the same diameter). The particle dispersion is modeled with the Eulerian approach. The complex refractive index of alumina is computed according to Anfimov empirical correlation [15].

| Number | Reaction |
|--------|----------|
| 1      | O₂ + H ↔ O + OH |
| 2      | H₂ + O ↔ H + OH |
| 3      | H₂ + OH ↔ H₂O + H |
| 4      | 2 OH ↔ H₂O + O |
| 5      | 2 H + M ↔ H₂ + M |
| 6      | H + OH + M ↔ H₂O + M |
| 7      | H + O + M ↔ OH + M |
| 8      | 2 O + M ↔ O₂ + M |
| 9      | OH + CO ↔ H + CO₂ |
| 10     | O₂ + CO ↔ O + CO₂ |
| 11     | O + CO + M ↔ CO₂ + M |
| 12     | H + HCl ↔ H₂ + Cl |
| 13     | H + Cl₂ ↔ HCl + Cl |
| 14     | OH + HCl ↔ H₂O + Cl |
| 15     | O + HCl ↔ OH + Cl |
| 16     | 2 Cl + M ↔ Cl₂ + M |
| 17     | H + Cl + M ↔ HCl + M |

2.3. Radiative transfer models
The aerothermochemical cartography of the plume computed by CEDRE, which is detailed in the next paragraph, is transposed on a structured mesh for the last step of the modeling chain: the radiative transfer computation. Our infrared signature computation tool, named SIR, takes as input the pressure, temperature, mass fraction of H₂O, CO₂, CO, HCl, alumina particle density and temperature. It can perform radiative transfer computation in a non-scattering medium with either a line-by-line model or
a statistical narrow band model called RGM3000. The scattering media are dealt with using the SHDOM solver described below.

RGM3000 is based on Curtis-Godson approximation [16], which takes into account the spectral correlations in case of multiple layers of gas with different physical conditions. Voigt line profiles inside a narrow band are also considered using the empirical formulation of Crisp et al [17], where a Doppler mean broadening [18] in a narrow band is combined to the Lorentz mean broadening. In rocket plume detection situations, where radiation emitted by hot gases is seen through kilometers of cold atmosphere, the Curtis-Godson approximation is not sufficient to treat the temperature and concentration gradients, and it is known to lead to large errors. For such cases, Ludwig et al [19] suggested to separate spectral lines inside a narrow band into different groups. The assumption is made that lines belonging to different groups are uncorrelated and the Curtis-Godson approximation can be applied to each line group. All the lines of a group are considered to belong to a fictitious gas and the original gas is then a mixture of these independent fictitious gases. The concept of fictitious gases has been discussed and applied to cumulative-k models [20][21] and to statistical models [22]. It has been implemented in our infrared signature code SIR with 5 fictitious gases for H$_2$O and CO$_2$. Particles are not taken into account yet in this model, neither in absorption nor in emission.

The Black Brant plume also contains liquid or solid alumina (Al$_2$O$_3$) particles. These particles strongly emit in the infrared region of the spectrum. It forces us to consider the radiation scattering phenomenon as an important part of the model. Computing the RTE solution with scattering along a given line of sight (LOS) involves the computation of multiple other LOS whose contribution is scattered into the first LOS direction. As a consequence, there is no simple analytical solution to the RTE that can be spectrally averaged. The inadequacy between scattering and spectrally-averaged gas absorption description has been underlined by several authors since decades [23][24][25].

The radiative transfer equation solver named SHDOM (Spherical Harmonics Discrete Ordinates Method) has been developed by Evans [26] in order to study the influence of the heterogeneities of the clouds in the transmission and scattering of radiation. Since geometrical problems, such as boundary conditions, are similar between atmospheric clouds and exhaust plumes, it was decided to use this solver in our application, with modifications consisting in taking into account high temperature combustion gas spectral data (and alumina particles optical properties for further calculations). It has to be noted that SHDOM has been already used to compute ultraviolet emission from alumina loaded rocket exhaust plume [27]. The SHDOM solver discretizes the radiance field by representing the source function at grid points with an adaptive spherical harmonics expansion in the angular variables. The solution method is iterative and consists of four steps. First, the spherical harmonics source function is transformed into discrete ordinates. Then, the source function is integrated to obtain the radiance field. The integral form of the radiative transfer equation (RTE) grid (1) is solved at each point of the field:

$$I_{\nu}^{n}(s) = \exp\left[-\int_{0}^{s} K_{ext}(s')ds'\right] I_{\nu}^{n}(0) + \int_{0}^{s} \exp\left[-\int_{0}^{t} K_{ext}(t)dt\right] J^{\nu-1}(s') \cdot K_{ext}(s')ds' ,$$  \hspace{1cm} (1)

where $I$ is the radiance, $J$ the source function and $K_{ext}$ is the extinction coefficient. The subscripts $u$ and $v$ stand respectively for the zenithal angle and the azimuth angle, and the superscript $n$ indicates the iteration step.

In the third step, the radiance field is transformed back to spherical harmonics. The source function is then computed from the radiance field in spherical harmonics. The radiance and source function are initialized with an Eddington radiative transfer solution on independent columns of the base grid.

The value for $K_{ext}$ is determined for each gas cell by calculating the mean transmissivity of the cell in one direction by the Malkmus function for each narrow band:

$$\tau(l) = \exp\left[\phi \left(1 - \left(1 + \frac{2Kl}{\phi}\right)^{1/2}\right)\right] \hspace{1cm} \phi = 2\frac{\gamma}{\delta} ,$$  \hspace{1cm} (2)
where \( \bar{\tau}(l) \) is the mean transmissivity on a narrow band, for gas layer of thickness \( l \), \( \kappa, \bar{\gamma}, \bar{\delta} \) are respectively the mean absorption coefficient, the mean half-width, and the mean line spacing in the narrow band. \( K_{\text{ext}} \) is then obtained simply by:

\[
K_{\text{ext}} = -\frac{\log(\bar{\tau}(l))}{l}. \tag{3}
\]

The choice for the length \( l \) depends on each cell and on the configuration of the medium. For a plane parallel medium, \( l \) is simply the height of each layer. This approach is retained for further complete calculations on a jet plume containing gas and particles because of his ability to easily take scattering into account [28]. But it is not able to take spectral correlations into account.

For scattering media, SIR computes the medium optical properties: gas and particle temperatures, phase function, extinction coefficient (with formula (2)) and albedo for every cell. The particle temperature is different from the gas temperature because particles have higher thermal inertia. Their temperature also depends on their diameter. The extinction coefficient for particles and the phase function are computed according to Mie theory. The phase function is given with Henyey Greenstein formula. These data are sent to SHDOM. SHDOM outputs the spectral radiance over a 2D grid that samples the axisymmetric plume for the user-specified aspect angles. SIR performs the spectral and spatial integration to obtain the source intensity in the required spectral band. The infrared signature is computed with no transmission on the spectral interval 1900-5000 cm\(^{-1}\) using a step of 5 cm\(^{-1}\), on 2D grid with 0.2x0.2 m\(^2\) pixels, with an aspect angle of 112° corresponding to the measurements. The atmospheric transmission from the target to the sensor is computed using the US Standard model and multiplied with the plume spectrum.

![Figure 3: Atmospheric transmission from the plume to the sensor](image)
3. Simulation results

Some of CEDRE outputs are displayed on figure 4. The Mach number plot shows the potential core of the round jet, which corresponds to the red area on the top left side. The plume length is about 300m long whereas the flame, which can be seen on the OH mass fraction plot, is only 40m long. The afterburning maximum is located roughly at 18m from the nozzle exit plane on the temperature plot. The particle number plot shows that particles are mostly gathered near the nozzle exit.

Figure 5 shows the evolution of temperature and radiative species mass fractions along the plume axis. A single shock cell is present in this flow, located 70 cm downstream the nozzle. Carbon monoxide is the most abundant radiative species in the nozzle exit plane. CO mass fraction then decreases as it oxides into CO$_2$, which is maximum near the afterburning maximum. Water vapor is the product of OH hydrogenation, and HCl turns into H$_2$O and Cl$_2$.

![Figure 4: CEDRE outputs: Mach number, OH mass fraction, gas temperature (K), alumina particles number (m$^{-3}$)](image-url)
Figure 5: CEDRE outputs: gas temperature and radiative species mass fraction along the plume axis.

Figure 6: Black Brant spectral intensity at altitude 7.9 km, spectral resolution 5 cm\(^{-1}\).
Figure 6 shows the Black Brant plume spectral intensity computed by SHDOM and RGM3000 at the source. CO emission lines can be seen on 1900-2000 cm$^{-1}$ spectral interval. HCl emission lines on 2500-3000 cm$^{-1}$ and water emission lines on 3250-4000 cm$^{-1}$ are overestimated by SHDOM. This results from the uncorrelated calculation of the transmittivity as an input of SHDOM. The particle emission continuum computed by SHDOM is only slightly above the intensity computed by RGM3000 on an unscattering medium around 4500 cm$^{-1}$.

Figure 7 represents the radiance cartography of the Black Brant plume computed by SHDOM with an aspect angle of 112°: 0° is the front and 180° is the back of the rocket. The maximum intensity is located between 18 an 20 m from the nozzle exit plane. It coincides with the temperature maximum, and is close to H$_2$O and CO$_2$ maxima, the main radiative contributors in this large spectral band (1900-5000 cm$^{-1}$).

4. Comparison with spectro-imaging measurements
To perform the comparison between the spectro-imaging measurements and the simulation results, we applied to the SHDOM results for the source the atmospheric transmission from the plume to the sensor presented on Figure 3. We then computed the plume apparent intensity with RGM3000 model taking into account the atmospheric path from the sensor to the plume. Figure 8 compares the spatially integrated spectral measurements to the apparent spectral intensity computed by SHDOM and RGM3000. There is a good agreement between measurements and calculations, except on the emission line at 2200 cm$^{-1}$, which is slightly overestimated by RGM3000. The overestimation of HCl spectral lines by SHDOM is clearly visible on 2500-3000 cm$^{-1}$, whereas RGM3000 results are closer to the measurements on that spectral band. On the contrary, the particle emission continuum computed by SHDOM around 4500 cm$^{-1}$ is closer to the measurements even if lower.

In order to take advantage of the spectro-imagery, we integrated the apparent radiance over the pixel surface, which gives an apparent intensity per pixel in W.sr$^{-1}$.m$^{-2}$, then summed the pixels along the direction orthogonal to the plume axis. This spatial information gives us insight on the spectral behavior on different stations along the plume axis. Figure 9 compares the measured station radiation integrated over the whole instrument spectral band 1900-5000 cm$^{-1}$ with SHDOM results (computed on a larger spatial domain). A very positive point is that the computed and the measured plumes have same length, which validates our choice for the RANS turbulence model used in CEDRE. One can see
that the computed station radiance is close to the measured station radiance for stations farther than 20 m from the nozzle exit plane. The first two closest stations are overestimated by our simulation. Overall, we obtain a good agreement between calculations and measurements.

We can also analyze whether the discrepancies between our computations and the measurements originate either from the gas or from the particle radiation computations. On figure 10, we integrated the station radiation on spectral bands 1900-2100 cm$^{-1}$, which encompasses the CO$_2$ emitting line, and on 4400-4600 cm$^{-1}$, which is located in the particle emission continuum. The left graph, which shows the gas spectral band, is very similar to the whole spectral band integration represented on figure 9: the CO$_2$ emission close to the nozzle exit plane is overestimated in our simulations. On the right graph, which shows the particle spectral band, one can see that the particle emission continuum is underestimated by CEDRE and SHDOM, except for the first station located about 3 m from the nozzle exit plane. This may indicate that the alumina particle properties and behavior in the plume are not well modeled far from the nozzle exit.

![Figure 8: Black Brant apparent spectral intensity at 7.9 km as seen by the sensor](image)

Spectral resolutions: 5 cm$^{-1}$ for calculations, 4 cm$^{-1}$ for measurements

![Figure 9: Plume radiation at 7.9 km – spectral band 1900-5000 cm$^{-1}$ (nozzle on the left side)](image)
5. Conclusion

We have developed a chain of simulation tools in order to predict rocket plume infrared signatures. This chain involves multiphysics codes performing combustion computation, compressible, turbulent and multiphase flow simulation, and radiative transfer calculation. We have made use of this chain to simulate a Black Brant sounding rocket plume and our computations are compared to infrared spectral imaging measurements. The infrared signature is computed with two radiative transfer codes: the SHDOM solver which takes into account scattering on alumina particles, and RGM3000 which is a statistical narrow-band model using the Curtis-Godson approximation with fictitious gases. The simulated spectrum is close to the measured spectrum: RGM3000 is closer on gas emission lines than SHDOM, whereas SHDOM better reproduces particle emission continuum. However the particle radiation is underestimated by our simulation tools, especially in the region far from the nozzle exit. The calculated and measured plumes have the same length which validates the RANS turbulence model used in CEDRE. Given the uncertainties on the restitution of the motor parameters, on modelling, on weather and experimental conditions, we believe that we have achieved a very good agreement between simulations and measurements.

A sensitivity analysis around the 8km calculation point will be conducted in order to consolidate these first results. It will also help us finding out which modeling features we must improve, such as particle dynamics in the CEDRE solver or spectral correlations in scattering media. In particular, the Radiance Splitting Method described in [28] will be implemented in 3D scattering media. We will then address calculation points at higher altitudes and speed.

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