Instability of relativistic shock waves: Numerical study on the basis of model equation of state

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Abstract. The behavior of unstable relativistic shock waves is studied with the use of specially developed model equation of state (EOS). The EOS admits the Taub–Hugoniot adiabats with segments on which the criteria of the relativistic shock wave stability are violated. The instability segments are overlapped by the regions with ambiguous representation of the shock-wave discontinuity. The simulations are fulfilled for $L < -1$ and $L > (1 + 2M + v_0v_1)/(1 - v_0v_1)$ instability conditions, where $L$ is relativistic analog of Dyakov parameter, $M$ is post-shock Mach number, $v_0$ and $v_1$ are pre- and post-shock velocities in the shock attached reference frame. Under the condition of ambiguous representation of the shock-wave discontinuity in the former case the splitting of the unstable shock with formation of a composite compression wave with Lorentz factor dependent structure is observed. It is shown that the latter condition leads to two-dimensional non-stationary solutions characterized by presence of strong transverse waves.

1. Introduction
The linear theory of the relativistic shock-wave stability was developed by V M Kontorovich, G Russo and A M Anile [1–3]. In accordance with this theory, shock waves are unstable with respect to small periodic perturbations if either of the two conditions is fulfilled:

$$L < -1,$$

$$L > (1 + 2M + v_0v_1)/(1 - v_0v_1),$$

where

$$L = \frac{p_1 - p_0}{\tau_0 - \tau_1} \left( \frac{d\tau}{dp} \right)_H$$

is a relativistic analog of the Dyakov parameter, $M$ is post-shock Mach number, $\tau$ is generalized volume, $v_0$ and $v_1$ are pre- and post-shock velocities in the shock attached reference frame, and the derivative is taken along the Taub–Hugoniot ($p-\tau$) curve. Here and below, non-dimensional variables are used, which are defined by the following scales: velocity scale is the speed of light $c$, energy scale is $c^2$, a scale for pressure and energy density is $\rho_{mc^2}$, where $\rho_m$ is the characteristic density, the value of which is defined at construction of the model equation of state (EOS).

Previously, the nonlinear behavior of non-relativistic shock waves, unstable from the point of view of the linear theory, have been considered in [4,5] using the proposed in [6] model equation of state and its modified variant [5]. In accordance with the conclusion of the fundamental work...
[7] the segments of the Hugoniot which correspond to the unstable shocks are overlapped by regions with the ambiguous representation of the shock-wave discontinuity. If such the region overlaps the shock adiabat segment with fulfillment of the condition $L < -1$, a composite compression wave (CCW), including two shock waves of the same direction and an isentropic compression wave between them, is realized [4] instead of the single shock wave. If the condition $L > 1 + 2M$ [non-relativistic analog of the condition (2)] is fulfilled, shocks belonging to the ambiguous representation region acquire two-dimensional non-stationary structure characterized by the presence of strong transverse secondary waves [5].

The manifestations of the shock-wave instabilities can be of importance in the physics of hot nuclear matter where an appearance of specific wave configurations may serve the signal of direct or reverse quark–hadron phase transition. According with present concepts, the phenomena occurring after the quark–gluon plasma (QGP) thermalization can be described in the framework of relativistic hydrodynamics (see, e.g., [8, 9]). Attempts to find the signals mentioned above have been made in a number of works (for example, [10] and the references cited in this paper). The shock-wave compression with the first-order quark–hadron phase transition has been mainly considered. In the case considered the Taub–Hugoniot adiabats crossing the phase boundary have the kink, and the CCW of two-wave structure instead the primary shock wave is generated (see also [11]).

At the same time, because of the existing uncertainty in the modeling of the thermodynamic properties of hot nuclear matter, one can propose a continuous change of the thermodynamic properties in the phase transition region that leads to shape a smooth inflection of shock curve instead the kink. If the inequality (1) is fulfilled for such the region, the CCW consisting of three elements (two shock waves moving in the same direction and an isentropic compression wave separating them) is expected to form.

Besides, even the existing and using hot matter equations of state, built on the basis of the hadronic gas EOS and the bag model for the QGP, may give a positive slope of the Taub–Hugoniot adiabats ($\frac{dp}{d\tau}^\text{H} > 0$) considered in the ($p-\tau$) plane (the entrance to the QGP region). This condition is necessary (although is not sufficient) for the fulfillment of the inequality (2). If the further EOS improvement makes the shock curve slope to be sufficient, it may cause to the formation of the wave structure analogous to the described above for non-relativistic case [5] and being also of theoretical interest.

The main purpose of this work is to extend the nonlinear analysis of the behavior of unstable shock waves performed in [4, 5] to relativistic case. In the calculations, a modification of the model EOS [6], satisfying the relativistic causality principle and allowing to consider indicated above anomalies of the Taub–Hugoniot adiabats, has been used.

2. Model relativistic EOS
Consider the main points of the model EOS construction. To calculate adiabatic flows, it is sufficient to have the caloric equation of state. Let us start from the equation of state with conventional convex shape of isentropes, but with rapid diminishing of compressibility of the matter at $p \approx 1$,

$$V = V_0(S) - C\sqrt{\pi} \text{erf}(p),$$

where

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

is the error function, $C = \text{const}$, $p$ and $V$ are the dimensionless pressure and specific rest mass volume, accordingly. The internal energy, satisfying the condition $\varepsilon = 0$ at $p = 0$, is given by $\varepsilon = C[1 - \exp(-p^2)]$. In accordance with [6], the anomalies of the shock curves, which lead to
the shock-wave instabilities, may be achieved perturbing (3) by a factor $C = 4 - \exp(-(V - 4)^2)$. In this form, the EOS admits Taub–Hugoniot adiabats with segments on which the relativistic shock-wave instability criteria are satisfied, but it can not be applied in the relativistic case due to violation of the relativistic causality principle. Namely, the speed of sound, corresponding to the EOS, grows as $\exp(p^2)$ with increase of $p$ and exceeds the speed of light at strong compression. To correct this, it is sufficient to improve the $p \to \infty$ asymptotics in the expression for internal energy. Finally, the relativistic consistent model equation of state is written as follow:

$$\varepsilon(V, p) = \xi(1 - \exp(-(p/\xi)^2 + k(p/\xi)^n V)(4 - \exp[-(V - 4)^2]),$$  \hfill (5)

where $k, \xi$ are some constants. In particular case $k = 0, \xi = 1$, this equation reduces to model equation of state [6]. Introduction to the equation of the additional term $k(p/\xi)^n V$ allows to guarantee the fulfillment of the relativistic causality condition $c_s < 1$. The parameter $n$ determines the behavior of the sound velocity at high pressures. The speed of sound is bounded, if $n \geq 1$. In the calculations presented in this paper, value $n$ is taking to be 2. The parameter $\xi$ scales the internal energy, which makes it possible to vary the Lorentz factor of the shock wave $1/(1 - v_0^2)^{1/2}$ and to investigate relativistic effects in the framework of a special problem formulation. At $\xi \to 0$ the solution of the relativistic hydrodynamics equations written in variables $(v/\sqrt{\xi}, p/\xi, \rho)$ tends to its classical non-relativistic limit, independent of the parameter $\xi$ (here $v$ is the velocity vector, $p$ is the pressure, $\rho$ is the rest mass density). The dependence of the solution, including the shape of the shock adiabats, the boundaries of the shock wave splitting, the structure of the composite compression waves, etc., on this parameter is a manifestation of relativistic effects. The dependence of speed of sound on the specific volume being the inverse of the rest mass density, and pressure for $\xi = k = 0.003$ is shown in figure 1.
3. Equations
The equations of relativistic fluid dynamics include the energy-momentum conservation equation

$$\nabla_\beta T^{\alpha\beta} = 0, \quad (6)$$

where $T^{\alpha\beta}$ is the energy-momentum tensor. The right eigenvectors for the Jacobian matrix are as follows

$$F^{\alpha} = (Dv_i, S_1v_i + p\delta_{1i}, S_2v_i + p\delta_{2i}, S_1v_i - Dv_i)^T. \quad (8)$$

The system is complemented by the conservation law

$$\nabla_\alpha (\rho u^\alpha) = 0. \quad (7)$$

4. Numerical method
The set of equations (6)–(8) can be represented in the form of hyperbolic conservation laws in the Cartesian coordinates $(t, x, y)$:

$$\frac{\partial U}{\partial t} + \frac{\partial F_1(U)}{\partial x} + \frac{\partial F_2(U)}{\partial y} = 0, \quad (9)$$

where $U = (D, S_1, S_2, \tau_e)^T$ is the vector of conservative variables, the fluxes of which have the form $F^i = (Dv_i, S_1v_i + p\delta_{1i}, S_2v_i + p\delta_{2i}, S_1v_i - Dv_i)^T$. The conservative variables $D, S_1, S_2$ and $\tau_e$ are related to the density, velocity components and relativistic enthalpy $h = (e + p)/\rho$ by the following relations

$$D = \rho \Gamma, \quad S_1 = \rho \Gamma^2 v_i, \quad \tau_e = \rho \Gamma^2 - p - \rho \Gamma. \quad (10)$$

The equations were solved iteratively for the primitive variables at each cell interface and the metric tensor (for special theory of relativity) is

$$\Gamma = 1/(1 - v^2)^{1/2}$$

The right eigenvectors for the Jacobian matrix $\partial F_n/\partial U$ for the flux vector $F_n = F^1n_x + F^2n_y$ are

$$\lambda_1 = \lambda_2 = v_n, \quad \lambda_3 = \lambda_m = (v_n(1 - c^2) - c\sqrt{(1 - v^2)(1 - v^2 - v^2(1 - c^2))})(1 - v^2c^2)^{-1}, \quad (11)$$

where the relativistic speed of sound is defined by

$$c^2 = \left( \frac{\partial p}{\partial e} \right)_{s} = \left( \frac{\partial p}{\partial e} \right)_{\rho} + \frac{\rho}{p + e} \left( \frac{\partial p}{\partial \rho} \right)_{e}. \quad (12)$$

The right eigenvectors for the Jacobian matrix are as follows

$$r^1 = \left( \frac{k}{\lambda_1(k-1)}, v_1, v_2, 1 - \frac{k}{\lambda_1(k-1)} \right)^T,$$

$$r^2 = \left( v_2 \Gamma, 2h\Gamma^2 v_1v_2 - h n_y, 2h\Gamma^2 v_1v_2 + h n_x, -v_1 \Gamma + 2hv_1 \Gamma^2 \right)^T,$$

$$r^3 = \left( 1, -(n_y v_1 + n_x \frac{\delta_m}{a_m})\Gamma, (n_x v_1 + n_y \frac{\delta_m}{a_m})\Gamma, -1 + \frac{\delta \Gamma}{a_m} \right)^T,$$

$$r^4 = \left( 1, -(n_y v_1 + n_x \frac{\delta_p}{a_p})\Gamma, (n_x v_1 + n_y \frac{\delta_p}{a_p})\Gamma, -1 + \frac{\delta \Gamma}{a_p} \right)^T. \quad (13)$$
Figure 2. Model shock adiabats of relativistic hydrodynamics in the variables $p-V$ and $p-\tau$, for different values of the parameter $\xi$.

where $k = h/(h - (\partial e/\partial \rho)_p)$. The variations of the local characteristic variables $\alpha_i = \Gamma_i \cdot \Delta U$ are defined at the cell faces by the equations

$$
\alpha_1 = \Gamma(k - 1)[h \Delta D + \Gamma(\mathbf{v} \cdot \Delta \mathbf{S} - \Delta D - \Delta \tau_e)],
$$

$$
\alpha_2 = \frac{v_n}{h(1 - v_n^2)}(v_n \Delta S_n - \Delta D - \Delta \tau_e) + \frac{1}{h} \Delta S_t,
$$

$$
\alpha_3 = \frac{a_m}{h \Gamma(\lambda_p - \lambda_m)}[(v_n - \lambda_p)\Lambda - (a_p + v_n b_p)\Delta S_n + (b_p + v_n a_p)(\Delta D + \Delta \tau_e)],
$$

$$
\alpha_4 = -\frac{a_p}{h \Gamma(\lambda_p - \lambda_m)}[(v_n - \lambda_p)\Lambda - (a_m + v_n b_m)\Delta S_n + (b_m + v_n a_m)(\Delta D + \Delta \tau_e)].
$$

In (13) and (14) the following notations are used:

$$
\Lambda = \Gamma [(k - 1)h \Delta D + (k + 1)\Gamma(\mathbf{v} \cdot \Delta \mathbf{S} - \Delta D - \Delta \tau_e)],
$$

$$
bl = (v_n - \lambda_l)(1 - v_n^2)^{-1}, \quad a_l = (1 - v_n \lambda_l)(1 - v_n^2)^{-1},
$$

$l = m, p$. Here, $v_n, v_t, S_n, S_t$ are normal and tangential components of the velocity and momentum density; $\Delta f$ denotes jump of the variable. Using (11)–(14), any numerical scheme for scalar equation can be applied to the local characteristic fields, in the present work the third order accuracy essentially non-oscillatory (ENO) scheme [13] is used to evaluate flow variables at cell interfaces. The numerical flux vector at the Gaussian integration points on the interfaces was evaluated using HLLC approximate Riemann solver [14]. The solution was advanced in time using the third order accuracy Runge–Kutta TVD scheme [13].

5. Results
The Riemann problem for the relativistic hydrodynamics with initial data corresponding to the shock-wave (SW) discontinuity in the region of its ambiguous representation was considered. For small values of $\xi$, the velocities in the problem of the SW decay are small compared with the
Figure 3. The shock-wave splitting in the region of the ambiguous representation of the shock-wave discontinuity for various values of parameter $\xi$: $0.01$ (a), $0.1$ (b) and $0.2$ (c). The pressure $p/\xi$ is shown as a function of the dimensionless coordinate.

Figure 4. Taub–Hugoniot adiabat containing $L < -1$ SW instability region (AC) is shown in $p-\tau$ variables. Point M is the inflection point of an isentrope at which $(\partial^2 p/\partial \tau^2)_S = 0$. Observed splitting structures are $S \rightarrow \langle S \rangle TCS$ on the segment AM and $S \rightarrow \langle S \rangle TSCS$ on the segment MC, where $S$ and $C$ denote shock and isentropic compression waves, respectively; $T$ stands for a contact discontinuity. Brackets enclose a wave of the opposite family.

speed of light, and the solution tends to a non-relativistic limit. The analysis of the anomalous behavior of non-relativistic shock waves was performed in [4, 5]. Here, this analysis extends to the relativistic case.

Case $L < -1$. When the internal energy (in units of $c^2$) becomes comparable with unity as the parameter $\xi$ increases, relativistic corrections become more and more noticeable. Their
influence, in particular, affects the fulfillment of the SW instability conditions [2, 3] and the splitting structure. The relativistic Taub–Hugoniot adiabats [15]

\[ h_1^2 - h_0^2 + (p_1 - p_0)(\tau_1 + \tau_0) = 0 \]  

(16)
in the \((p/\xi, V)\) and \((p/\xi, \tau)\) planes constructed for the initial state \((p_0 = 0.1\xi, V_0 = 0.1821)\) and various values of the parameter \(\xi\) are shown in figure 2. In this section \(\tau = h/p\) denotes generalized volume; the indices 1 and 2 indicate the pre- and post-shock quantities, respectively. The shock adiabat 1 is close to the classical limit. The differences of the curves constructed for various \(\xi\) are caused by relativistic effects. The Riemann problem solutions corresponding to the post-shock pressure \(p_1 = 0.6\xi\) and different values of the parameter \(\xi\) are shown in figure 3. The solutions have been obtained using the equation of state (1) with \(n = 2\) corresponding values \(\xi\).

The example of Taub–Hugoniot adiabat corresponding \(n = 2\) and \(\xi = 0.003\) is shown in figure 4 in which the regions of the SW instability and SW ambiguous representation are indicated. The instability condition (1) is satisfied for the shock waves with the post-shock states that belong to the section AB of the Taub–Hugoniot curve \(H\). This segment is overlapped by the SW ambiguous representation region \(\text{AC}\). The points A, B, and C are defined by construction of tangent lines to \(H\) passing through the initial point O. The points A and B are the tangency points, while C is the point at which the line passing through point A meets the shock curve. The case shown in figure 3(a) qualitatively coincides with the solution in the non-relativistic limit \(\xi \to 0\). The figures 3(b, c) demonstrate amplification of the isentropic compression wave and growth of the velocity difference between the leading and the second shock in the composite compression wave.

Figure 5 shows the splitting of relativistic shock waves at \(\xi = 0.1\) and \(p_1/p_0 = 4.5; 8; 11; 14\). The lower limit of the SW instability with respect to splitting is determined by the tangency of the Michelson–Rayleigh line and the shock adiabat in the variables \(p-\tau\) (point A in figure 4). The segment AB of the shock adiabat corresponds to the SW splitting with formation of the CCW including a shock wave and an isentropic compression wave. Observed wave split configuration is \(S \rightarrow (S)TCS\) [figure 5(a)], where \(S\) and \(C\) denote shock and isentropic compression waves, respectively; \(T\) is a contact discontinuity, while brackets enclose a conjugating wave of the opposite family. The point M in figure 4 is the point in which \((\partial^2p/\partial\tau^2)_S = 0\). The second shock in the CCW appears at transition to the region \((\partial^2p/\partial\tau^2)_S > 0\) above point M. Figure 5(b) meets the SW splitting with the formation of the CCW, which includes two shock waves and an isentropic compression wave between them \(S \rightarrow (S)TSCS\). As the intensity of the primary SW increases approaching point C, the difference in velocity between the first and the second shocks in the CCW as well as the amplitude of the isentropic compression wave tends to 0 [see figure 5(c, d)]. Above the point C, the SW is stable with respect to the splitting. In the classical limit the points on the shock adiabat with \(p_1/p_0 = 11; 14\) are located outside the region of the SW ambiguous representation where the SW splitting is impossible. Relativistic effects shift the upper boundary of the region of the ambiguous representation to higher pressures. In calculations with \(\xi = 0.3\), the velocity of the shock wave exceeds 0.74 of the speed of light.

**Case L > (1 + 2M + v_0v_1)/(1 − v_0v_1).** Consider the case when the instability condition (2) is satisfied on a segment of the Taub–Hugoniot adiabat. Such the adiabat (H) shown in figure 6(a) in \(p-u\) variables corresponds to the following parameters: \(n = 2\) and \(\xi = k = 0.003\); the pre-shock state is \(p_0 = 0.1821, p_0 = 0.1\xi\).

The instability condition is satisfied for the shock waves, which post-shock state corresponds to the segment (EF) in figure 6(a). The endpoints of the instability segment are the points of tangency of the Taub–Hugoniot adiabat and isentropes in the \((p-u)\) plane for waves propagating in the opposite direction. In this points the condition of tangency

\[
\left(\frac{\partial p}{\partial v}\right)_C = \left(\frac{\partial p}{\partial v}\right)_H, \quad (17)
\]
Figure 5. The splitting of a shock wave in the region of the ambiguous representation of the SW discontinuity for various values of $p_1/p_0$: 4.5 (a), 8 (b), 11 (c), 14 (d).

takes place. Here, the derivative in the righthand side of the equality is taken along the Taub–Hugoniot adiabat, while the derivative in the lefthand side is taken along the isentrope $C^-$:

$$\left( \frac{\partial p}{\partial \nu} \right)_{C^-} = -\Gamma \rho c_s h,$$

where the variables are evaluated at the post-shock state, and $\Gamma$ is defined in the rest frame of the fluid ahead of the shock (see [2]). Using construction in $(p-u)$ plane one can see that the instability segment EF [in figure 6(a)] belongs to the segment D’G, in which the SW discontinuity is represented by splitting structures connecting the pre- and post-shock state of the primary shock wave. From the direct construction it follows that within the ambiguity region (D’G) there are the sections D’E, EF and FG, in which following splitting structures are possible: $S \rightarrow \langle S \rangle TS$ in D’E, $S \rightarrow \langle R \rangle TS$ or $S \rightarrow \langle S \rangle TS$ in EF, $S \rightarrow \langle R \rangle TS$ in FG. Here $S$ and $R$ denote shock and rarefaction waves, respectively; $T$ stands for a contact discontinuity, while brackets enclose a wave $(S$ or $R)$ of the opposite family. Let us note, that in contrast to $L < -1$ case when shocks and isentropic wave of the same family are involved some of these splitting schemes are reversible, namely, the splitting leaves the leading shock wave in the ambiguity region. For
Figure 6. Taub–Hugoniot adiabat with fulfillment of the SW instability condition (2) is shown in \( p-u \) variables. Reaction of the shock waves on perturbation in the form of small amplitude compression and rarefaction waves is presented for different sections of the SW ambiguous representation region (EF, D’E and FG). Observed splitting structures are \( S \rightarrow \langle S \rangle TS \) in D’E, \( S \rightarrow \langle R \rangle TS \) or \( S \rightarrow \langle S \rangle TS \) in EF, \( S \rightarrow \langle R \rangle TS \) in FG.

For example, splitting of the shock wave with a post-shock state on FG (\( S \rightarrow \langle R \rangle TS \)) gives the leading shock with the post-shock state on the segment D’E, where, in turn, the “reverse” splitting (\( S \rightarrow \langle S \rangle TS \)) producing the leading shock on FG is possible. This property results in complicated behavior of the shock waves, being a marker of \( L > (1 + 2M + v_0v_1)/(1 - v_0v_1) \) instability.
Figure 7. Formation of cellular detonation-like structure of the SW front in presence of periodic perturbation.

One-dimensional calculations of the shock-waves with the post-shock state in the ambiguity region D’G show that the behavior of the shock wave (whether the SW splits) depends on small perturbations. If the shock wave corresponding the segment FG wave interacts with small amplitude compression wave or the shock wave corresponding the segment D’E interacts with small amplitude rarefaction wave coming to the shock from the side of compressed matter, this interactions result in small change of the parameters of the shock wave. Thus, the shock behaves as stable one. On the other hand, the perturbations of opposite “sign” induce the shock wave splitting with the splitting structures (S → ⟨R⟩TS) and (S → ⟨S⟩TS), respectively. The shock waves corresponding to instability region split in all cases irreversibly; the shocks with final state EF do not form in any splitting process. The splitting structure depends on perturbation as shown in figure 6(b–d), where the results of simulation of perturbed shock wave are presented. Pressure versus scaling coordinate is shown; dashed line corresponds to the primary shock.

The behavior of the shock wave in the region of its ambiguous representation containing the Taub–Hugoniot segment with fulfilled condition \( L > (1 + 2M + v_0v_1)/(1 - v_0v_1) \) of shock-wave instability has been studied numerically in two-dimensional formulation. It was shown in [5] that in this case the presence of a periodic two-dimensional perturbation of plane shock wave leads to non-stationary mode with detonation-like cellular structure formation. At least calculations with small values of \( \xi \) in (1) give the same result. The pressure field evolution in the initially perturbed shock-wave is shown in figure 7. At the front concavity sections (with respect to the shock wave propagation direction), the decay of the shock into the configuration ⟨S⟩TS begins under the action of weak perturbations arriving from adjacent front sections. In the front convexity sections at which weak rarefaction waves arrive, decay is not observed at the initial stage. As a result, transverse waves propagating along the shock front from the side of the
Figure 8. Non-linear interaction of secondary waves: strong secondary waves absorb weak ones.

shock compressed matter are formed. The amplitude of pressure perturbations in the limiting non-stationary mode corresponds to the range of the region of ambiguous representation of the shock wave discontinuity.

Here, we show that if the initial perturbation contains multiple wave length, the shock wave perturbation evolves to the large-scale part of spectrum. This phenomenon is shown in figure 8, where the evolution of the shock wave perturbed initially by multi-wave-length perturbation is presented. This behavior can be explained by non-linear interaction of secondary (transverse) waves: strong secondary waves absorb weak ones. It should be noted that unlike unstable shock-wave (for which the condition $L > (1 + 2M + v_0v_1)/(1 - v_0v_1)$ is satisfied) the shock waves corresponding to the segments D'E and FG of the ambiguity region (figure 6), can propagate in metastable mode. In this case a local strong perturbation of appropriate sign leads to formation of secondary (transverse) waves, which propagate along the shock wave front switching it to the non-stationary mode as shown in figure 9. Consider a plane shock with the post-shock state belonging to D'E segment of the Taub–Hugoniot adiabat (figure 6). This shock-wave behaves like stable one, interacting with weak rarefaction wave, which is coming to the shock front from
Figure 9. Density distribution caused by secondary waves induced by local segment DF.

the side of compressed matter (also, see one-dimensional solutions corresponding to D'E segment in figure 6). The metastable behavior is due to absence of the perturbations of “appropriate sign”. In fact, passage of a vortex or an entropy perturbation through the shock-wave front as well as interaction with a compression wave initiates splitting of the primary shock wave accompanied by formation of secondary waves spreading along the SW surface, as it is shown in figure 9, where the rest mass density is presented. Unlike secondary waves in the case of stable or neutrally stable (in linear analysis) shock waves, these secondary waves propagate without a decay. The fluctuations of the front pressure in this waves correspond approximately to the extension of the region of ambiguous representation of the shock-wave discontinuity [D'G in figure 6(a)].

6. Conclusion
The nonlinear analysis of the non-relativistic shock waves unstable in accordance with linear stability theory criteria is extended to the case of relativistic shock waves. The model equation of state developed and used in the simulation is compatible with relativistic theory (does not produce superluminal speed of sound) and meets to the requirements of thermodynamic stability. The Taub–Hugoniot adiabats built on the basis of the model equation of state contain segments of the shock ambiguous representation overlapping the regions of shock wave instability. The ambiguous representation of the shock wave discontinuity determines the character of the shock wave behavior: one-dimensional splitting with formation of a composite compression wave or two-dimensional splitting based non-stationary mode. This behavior is shown to be analogous to the case of non-relativistic shock waves but taking into account the Lorentz factor.
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