Cosmology with bulk viscosity and the gravitino problem

Consequences of imperfect fluids on gravitino production

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Abstract The gravitino problem is revisited in the framework of cosmological models in which the primordial cosmic matter is described by a relativistic imperfect fluid. Dissipative effects (or bulk viscosity effects) arise owing to the different cooling rates of the fluid components. We show that the effects of the bulk viscosity allow one to avoid the late abundance of gravitinos. In particular, for particular values of the parameters characterizing the cosmological model, the gravitino abundance turns out to be weakly depending on the reheating temperature.

1 Introduction

Imperfect fluids in cosmology are characterized by the fact that the different components of cosmic fluids are coupled and, having different internal equation of states, their cooling rates turn out to be different as the Universe expands. As a consequence, a deviation of the system from equilibrium occurs. The different cooling rates of the components are responsible for the presence of a bulk viscous pressure of the cosmic medium as a whole. The latter is the only possible dissipative phenomenon for an homogeneous and isotropic Universe (see [1-42] and references therein). In particular, in the Friedman–Robertson–Walker (FRW) Universe, the dissipation is described by a scalar quantity (the bulk viscosity as referred to the thermodynamical approach [42]). Bulk viscosity enters Einstein’s field equations, altering the thermal histories of (relic) particles, compared with cosmology with perfect fluids, and therefore their abundance.

A comment is in order. In standard cosmology, the bulk viscosity effects are absent at high temperatures because, essentially, the bulk viscosity coefficient \( \zeta \) is proportional to \( \delta \equiv (1 - 3w)^2 \), where \( w \) is the adiabatic index (with \( w = \partial P/\partial \rho = 1/3 \) in a Universe radiation dominated). However, such a value of \( w \) is relaxed whenever one considers the interactions among massless (relativistic) particles. These lead to running coupling constants, and hence to a trace anomaly \([43,44]\) \( T_{\mu\nu} \propto \beta(\rho)F^{\mu\nu}F_{\mu\nu} \neq 0 \). For SU(5) gauge theory, characterized by a coupling \( g \) and \( N_f \) flavors, the effective equation of state turns out to be modified as \( 1 - 3w = 56\pi^4 N_f / (96\pi^2) \), whose numerical value may lie in the range 1–3×10^{-2}–10^{-1} \([43]\).

To give an estimation of the role of the bulk viscosity, we recall that \( \zeta \) is related to the scalar pressure \( \Pi \) [that enters directly into cosmological equations; see (2.2)] through the constitutive equations for dissipative quantities \( \Pi = -3\zeta H \). For radiative fluids, i.e. fluids consisting of interacting massless and massive particles, kinetic theory or fluctuation theory arguments allow one to derive the dissipative coefficients \( \zeta \) in terms of the relaxation time \( \tau: \zeta = 4\alpha_0 T^4 \tau \delta \), where (in units \( \hbar = 1 = c \)) \( \alpha_0 = \pi^2 k_B^4 /15 \approx 0.65 \) is the radiation constant and \( k_B \) the Boltzmann constant (of course the expression for \( \zeta \) changes for different fluids \([42]\)). Comparing the scalar pressure to the radiation energy density \( \rho = (\pi^2 g_*) T^4 / 30 \) one gets \( \Pi / \rho = 24\pi^4 / (\sqrt{3}) (\zeta / \rho_0) \lesssim 2.4 \times 10^{-2} \) for \( \tau H < 1 \) and \( \delta \lesssim 10^{-2} \). Another interesting possibility to have \( w \neq 1/3 \) is to consider quantum fluctuations of primordial fields \([45,46]\) that lead to \( p = (\rho - (T_{\mu\nu}^\mu) / 3) \), where \( (T_{\mu\nu}^\mu) = k_3 (\delta_R + R_{\alpha\beta} R^\alpha\beta) \) and \( \delta_R \) is the number of quantum fields (for example, \( k_3 = 1/144\pi^2) (N_0 + 31N_1 + 11N_2 / 2) \sim 0.07 \) for SU(5) model, and similarly for \( k_1 \), \( R = -6(H + 2H^2) \), \( R_k^0 = -3(H + H^2) \) and \( R_k^1 = -(H + 3H^2) \delta_R \).

1 Let us point out that bulk viscosity effects account for the rapid expansion/compression of fluids that cease to be in thermal equilibrium. Therefore the bulk viscosity gives a measure of the pressure that is necessary for restoring the equilibrium to an expanding/compressed system, a condition that naturally arises in a cosmological expanding Universe.

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References: [1-42] and references therein.
Here $m_{3/2}$ is the mass of the particle $\sim 10^2$ GeV. Particles with such a long lifetime generate some issues in cosmology since if they decay after the nucleosynthesis, their decay products (which can be gauge bosons and their gauge partners) would destroy light elements, destroying the successful predictions of Big Bang Nucleosynthesis (BBN). This problem can be avoided putting an upper bound on the reheating temperature. In the framework of GR and from the Boltzmann equation it turns out that the gravitino abundance is proportional to the flux of the energy, and finally one finds that the energy-momentum conservation law is isotropic and homogeneous, one gets

$$\dot{\Theta} = \frac{3}{2} \Theta (\rho + p + \Pi).$$

(2.2)

where $\Theta = \dot{a}/a$ is the Hubble parameter and $\kappa = 8\pi G = 8\pi M_{\text{Pl}}^{-2}$ ($M_{\text{Pl}} \simeq 1.22 \times 10^{19}$ GeV is the Planck mass).

In the radiation dominated era, where the energy density is $\rho = \frac{\pi^2}{30} g_\ast T^4$ and the pressure is $p = \frac{\rho}{3} \left( g_\ast \sim 10^2 \right)$ counts the relativistic degrees of freedom, one finds that the Hubble parameter evolves according to the equation [62]

$$\frac{\dot{H}}{H} - \frac{2}{3} \frac{H^2}{H^2} - 6H^2c_b^2 + \frac{1}{\tau} \left( \frac{\dot{H}}{H} + 2H \right) = 0,$$

(2.3)

where $\tau (= \tau (t))$ is the relaxation time (physically it is interpreted as the mean free time of the relativistic particle), which in general is time dependent, while $c_b \equiv \frac{\xi}{\sqrt{\gamma}}$ is the propagation velocity of a viscous pulse ($\xi$ is the bulk viscosity coefficient), which may assume values $0 \leq c_b^2 \leq 2/3$ [3–5,42,66]. These results refer to the case in which the number of particles is conserved. Taking into account the particle production, i.e. $\nu_\mu N^\mu = n$ [66–68], it follows that the viscous pressure assumes the form $\Pi = - (\rho + p) \Gamma/\Theta$, with $\Theta = 3H$ (obtained for isentropic particle production $\delta = 0$), which is entirely determined by the particle production rate. For a radiation dominated era $p = \rho/3$ and $\rho = \rho + p)/T$, so that the evolution of the Hubble parameter is given by

$$\frac{\dot{H}}{H} - \frac{5}{4} \frac{H^2}{H^2} + 3\dot{H} - 6H^2 \left( c_b^2 - \frac{1}{2} \right) + \frac{1}{\tau} \left( \frac{\dot{H}}{H} + 2H \right) = 0.$$  

(2.4)

In what follows we shall refer to (2.3), the results being very similar to the case (2.4).

To solve Eq. (2.3) we look at solutions for $H(t)$ of the form

$$H = u^\alpha a.$$  

In an homogeneous and isotropic (spatially flat) Universe, the Einstein equations read [62]
where $H_L$, $\tau_L$ and $\eta$ are undetermined constants. To have a hot early Universe we confine ourselves to the case $\eta > 0$.

From (2.2) one infers the relation between the cosmic time and the temperature $T$,

$$t = t_L \left( \frac{T}{H_L} \right)^{\frac{1}{3}},$$

which allows one to cast the expansion rate in the following form:

$$H = A(T) H_{GR}, \quad H_{GR} = \frac{1}{2t},$$

where

$$A(T) = \eta \left( \frac{T}{T_{ref}} \right)^{\nu}, \quad \nu = \frac{2(\eta + 1)}{\eta},$$

$$\eta \equiv 2H_L t_L \left[ \left( \frac{4\pi^2 g_s/45}{M_{Pl}^2} \right)^{1/2} \frac{T_{ref}^2}{M_{Pl} H_L} \right]^{\frac{\eta + 1}{3\eta}}.$$ 

Here $T_{ref}$ is a reference temperature, which can be fixed for example as the BBN temperature $T_{BBN}$ ($T_{BBN} \approx (10^{-2} - 10^{-4})$ GeV). Inserting (2.5) into (2.3) one derives the expression for the characteristic relaxation time,

$$\tau H = \frac{1 + \frac{\eta}{2H_L}}{3c_s^2 + \frac{\eta(1 + \eta)}{2H_L^2}}.$$ 

Before we investigate the gravitino problem, we focus on some aspects related to the cosmological model under consideration:

- Due to the successful predictions of the BBN, we shall refer to the pre-BBN epoch since it is not directly constrained by cosmological observations. We require that at the instant $t_0$ the Universe starts to evolve according to standard cosmological model, i.e. although the bulk viscosity effects are small during the radiation dominated era, we assume that at $t > t_0$ the adiabatic index is exactly $1/3$, so that they vanish. To determine $t_0$, we set $H(t_0) = H_{GR}(t_0)$, which implies

$$t_0 = t_L \left( \frac{1}{2t_L H_L} \right)^{\frac{1}{\eta}}, \quad \eta \neq -1.$$ 

To preserve the BBN predictions, we then require $t_0 \lesssim t_{BBN}$, where

$$t_{BBN} \approx (10^{-2} - 10^{-3}) \times (10^{22} - 10^{27}) \text{ GeV}^{-1}.$$ 

- To get a first insight of the model (that considerably simplify the Boltzmann equation) we work in the regime $|\dot{T}/T| > H$, where

$$\frac{\dot{T}}{T} = \frac{\eta}{2t} = \frac{\eta}{2Ct_L} \frac{T}{M_{Pl}} \left( \frac{2}{3} \right)^{\frac{1}{2}}.$$ 

The condition $|\dot{T}/T| > H$ occurs for

$$t < \bar{t}, \quad \bar{t} \equiv t_L \left( \frac{|\eta|}{2t_L H_L} \right)^{\frac{1}{|\eta|}} = |\eta|^{\frac{1}{|\eta|}} t_L.$$ 

3 Gravitino problem in cosmology with bulk viscosity

As pointed out in the Introduction, gravitino is generated by means of thermal scattering in the primordial plasma. This occurs during the reheating era after Inflation. To describe the gravitino production one makes use of the Boltzmann equation for the number density of species in a thermal bath. The relevant equation for gravitino production is

$$\frac{d n_{3/2}}{dt} + 3H n_{3/2} = \langle \sigma v \rangle n_{rad}^2.$$ 

Here $n_{3/2, rad}$ refers to gravitino and relativistic species, while $\langle \ldots \rangle$ stands for the thermal average of the gravitino cross section $\sigma$ times the relative velocity of scattering radiation ($v \sim 1$), $\sigma v \sim M_{Pl}^2$. In (3.1) the term $\langle \sigma/\langle E \rangle \rangle n_{rad}^2$ has been neglected. Here $\langle E \rangle$ is the average Lorentz factor. Introducing the abundances of the gravitino and of the relativistic particles, $Y_{3/2} = n_{3/2}/s$ and $Y_{rad} = n_{rad}/s$, respectively, the Boltzmann equation (3.1) assumes the form

$$\frac{dY_{3/2}}{dT} + \frac{3}{T} \left( \frac{\dot{T}}{T} + H \right) Y_{3/2} = \frac{s}{T} \frac{\langle \sigma v \rangle}{\langle E \rangle} Y_{rad}^2.$$ 

In the regime $|\dot{T}/T| > H$ the Boltzmann equation (3.2) reduces to the form

$$\frac{dY_{3/2}}{dT} + \frac{3}{T} Y_{3/2} = \frac{s}{T} \frac{\langle \sigma v \rangle}{\langle E \rangle} Y_{rad}^2.$$ 

The $Y_{3/2}$-term in (3.2) survives because the adiabatic condition is lost in the cosmological model under consideration, contrarily to the standard cosmology. By integrating from $T_R$
(\gg T) to the temperature \( \tilde{T} \), where \( \tilde{T} \gg T_{BBN} \) is the temperature corresponding to the instant \( t \) defined in (2.12), we find that the general solution to (3.3) (with the initial condition \( Y_{3/2}(T_R) = 0 \)) is

\[
Y_{3/2}(T = \tilde{T}) = \mathcal{B} \Theta_{\tilde{T}}(T_R, \tilde{T}) ,
\]

where

\[
\Theta_{\tilde{T}} \equiv \left( \frac{\tilde{T}}{M_{Pl}} \right)^\Delta - \left( \frac{T_R}{\tilde{T}} \right)^3 \left( \frac{T_R}{M_{Pl}} \right)^\Delta
\]

(3.5)

\[
\Delta = 3 + \frac{2}{\sqrt{\alpha}} ,
\]

(3.6)

\[
\mathcal{B} \equiv \alpha L ,
\]

(3.7)

\[
\alpha_0 \equiv \frac{2 \pi^2 g_*}{45 (1 + 3 T)} \left( \frac{4 \pi^2 g_*}{45} \right)^{1/4} \left[ M_{Pl}^2 \langle \sigma v \rangle Y_{rad}^2 \right] ,
\]

(3.8)

\[
\alpha_L \equiv \left( \frac{M_{Pl} t_L}{H_L} \right) ,
\]

(3.9)

This inserted into (3.9) yields

\[
\alpha_L \approx 2 \left( M_{Pl} t_s \right)^{1/2} \sim M_{Pl} t_s \sim 10^{41} \text{ for } t_s = t_{BBN} .
\]

(3.10)

Interestingly, \( \alpha_L \) is independent on free parameter \( t_L \) and \( H_L \).

Let us now determine the values of the gravitino abundance \( Y_{3/2} \) for different values of \( \Upsilon \). First of all, it is simple to show that values of \( \Upsilon > 0 \) yield \( Y_{3/2} < 0 \), which is physically not acceptable. Moreover, Eqs. (3.4)-(3.9) imply that \( \alpha_L \approx 1 \) for \( \Upsilon \) varying in the range \(-1 < \Upsilon < 0\). Consistently with the cosmological model with bulk viscosity here considered, we have also to analyze the behavior of the relaxation time \( \tau H \) during the evolution of the Universe. From (2.9) it turns out to be given by (at \( \Upsilon > t \) in the pre-BBN era)

\[
\tau H = \frac{1 + \frac{\Upsilon}{\sqrt{4 + \Upsilon^2}}}{3 \epsilon_b^2 + \frac{2 \Upsilon (1 + \Upsilon)}{|\Upsilon|^2} \left( \frac{1}{n} \right)} ,
\]

(3.12)

This function is plotted in Fig. 1, with \( \epsilon_b^2 = 2/3 \) and \( \epsilon_b^2 = 10^{-5} \) (see Sect. 2). The parameters \( \{ \Upsilon, x \} \) have to assume values such that\(^5\) \( 0 \ll \tau H < 1 \).

\footnote{If \( \tau \) is of the order of the mean interaction time \( t_c = 1/\sigma v \) (here \( n \) represents the number density of the target particles with which the is weakly depending on the reheating temperature \( T_R \) for a range of values of the parameter \( \Upsilon \). Let us set \( \tilde{T} = 10^{10} \) GeV. In the case \( |\Upsilon| \ll 1 \), the gravitino abundance reads

\[
Y_{3/2} \approx \alpha L \left( \frac{M_{Pl}}{\tilde{T}} \right)^{1/4} \sim \left[ 2 \times 10^{-2(2+\omega)} \right]^{1/4} ,
\]

which implies \( \omega > -2 \) in order that \( Y_{3/2} \ll 1 \) and we solve the late abundance gravitino problem. Taking, for example, \( \omega = -1 \) and \( 10^{-2} \) one gets \( \tilde{T} \simeq 10^5 \) MeV and the gravitino abundance \( Y_{3/2}(\tilde{T}) \) is completely negligible. Moreover, in the regime \( |\Upsilon| \ll 1 \) it follows that the relaxation time is \( \tau H \approx \left| \frac{\Upsilon}{2} \right| \ll 1 \). As a specific example, consider \( \Upsilon = -1/6 \). In such a case the gravitino abundance \( Y_{3/2} \) is again weakly depending on the reheating temperature \( T_R \). Moreover, one has \( t_s = t_L(2 t_L H_L)^{-6/5} \ll t_s/10 \), and the late gravitino abundance turns out to be \( Y_{3/2} \sim 10^{-18} \ll 1 \). From Eqs. (2.5) and (2.10) one obtains

\[ \text{Footnote 5 continued} \]

given species is interacting, \( \sigma \) the interaction cross section, and \( v \) the mean relative speed of interacting particles), then the hydrodynamical description requires \( \tau H \ll 1 \) [42].
The function $\tau H$ vs. $x$ is plotted in Fig. 2 for different values of $c_B^2$. As we can see, the relaxation time is, for all epochs before BBN $t < \tilde{t} < t_\nu$, smaller than the Hubble time, i.e. $\tau < H^{-1}$, as expected for a fluid description.

As seen till now, particular values of $\Upsilon$ imply that the late gravitino abundance $Y_{3/2}$ is independent (or weakly depending) on the reheating temperature $T_R$. The latter, however, influences the evolution of $Y_{3/2}$ for increasing values of $\Upsilon$. Consider, for example, the value $\Upsilon = -1/2$. From Eq. (3.4) it follows

$$Y_{3/2} \simeq 2.5 \times 10^{-21} \frac{\text{GeV}}{T} \left( \frac{T_R}{\tilde{T}} \right)^2,$$

(3.14)

while $\tilde{t} \simeq t_\nu/4$. Notice that $\Theta_\Upsilon = -1/2 < 0$ and $\sigma_0 < 0$, so that the two negative signs compensate to give a positive $Y_{3/2}$ (see (3.5) and (3.8)). The behaviors of $Y_{3/2}$ and $\tau H$ are plotted in Fig. 3.

The above analysis refers to $-2/3 < \Upsilon < 0, (\Delta < 0)$. Let us analyze now the regime $-1 < \Upsilon \leq -2/3 (\Delta \geq 0)$. For $\Upsilon = -2/3$ one obtains $\tilde{t} \simeq 0.3 t_\nu$, and

$$Y_{3/2} \simeq 1.1 \times 10^{-21} \left( \frac{T_R}{\tilde{T}} \right)^3.$$  

(3.15)

Requiring $Y_{3/2} \lesssim 10^{-10}$ one infers $T_R/\tilde{T} \lesssim 4.6 \times 10^3$, i.e. to solve the late gravitino overproduction the temperature $\tilde{T}$ must be closer and closer to the reheating temperature $T_R$, which does not seem a favorable scenario. Moreover, the relaxation time turns out to be $\tau H \gtrsim O(1)$, as arises from Fig. 1, making the model not suitable for the solution of the gravitino abundance, at least in the approximation $|\tilde{T}/T| > H$. A similar unfavorable scenario follows also for the cases $-1 < \Upsilon < -2/3$ and $\Upsilon < -1$.

To summarize, the cosmological model with bulk viscosity provides favorable scenarios for solving the late overproduction of gravitinos in the case the parameter $\Upsilon$ falls down in the range $-2/3 < \Upsilon \lesssim 0$. In this case, in fact, the cosmological evolution of the Universe deviates considerably from the one based on the standard cosmological model (without bulk viscosity effects), as discussed in the previous section. The range of values $-1 < \Upsilon \leq -2/3, \Upsilon > 0$ and $\Upsilon < -1$, instead, is excluded or partially acceptable, at least in the approximation here considered.

4 Conclusions

In this paper, we have reviewed the gravitino problem in a cosmological model in which bulk viscosity effects are taken into account. To avoid the late overproduction of the gravitinos by thermal scattering in the primordial plasma, we have exploited the fact that if the cosmic fluid is imperfect then the cosmic evolution of the Universe gets modified as compared with the case of perfect fluids (the expansion rate of the Universe can be written in the form $H = A(T) H_{GR}$, where the factor $A(T)$ accounts for bulk viscosity effects). This affects the Boltzmann equation, which describes the time evolution of the gravitino abundance. Moreover, for some choice of the parameters, the late gravitino abundance is weakly depending on the reheating temperature. Cosmology with bulk viscosity provides therefore scenarios able to avoid the late overproduction of gravitinos. It will certainly be interesting to extend...
this analysis to the case of non-thermal perturbative gravitino production (see for example [84,85]). This analysis will be addressed elsewhere.

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