PQCD study of the $B_{SL}$ and $n_c$ controversy in inclusive $B$ decays

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Abstract  
We calculate the semileptonic branching ratio and the charm counting of inclusive $B$ meson decays using the perturbative QCD formalism. For the nonleptonic decays, we employ the modified factorization theorem, in which Wilson coefficients evolve with the characteristic scales of the decay modes. It is found that the decay rate of the single-charm mode $b \to c \bar{u}d$ is enhanced, and a lower $B_{SL}$ is obtained without increasing $n_c$. We predict a larger $b \to c\tau\bar{\nu}$ branching ratio compared to that from the conventional heavy-quark-effective-theory based operator product expansion. Our result of the $B$ meson lifetime is also consistent with the data.
With rapid theoretical and experimental developments, the heavy flavor physics has reached the level of accuracy that the data can be used to test the standard model. Especially, the various quantities related to the heavy meson decays have been calculated and compared with experimental data. In spite of many successes, a discrepancy exists consistently between the measurements and the theoretical predictions of the inclusive semileptonic branching ratio \( B_{\text{SL}} \) and the charm counting \( n_c \) of the \( B \) meson decays: The latest results from CLEO, measured at the \( \Upsilon(4s) \) resonance, are \( B_{\text{SL}} = (10.19 \pm 0.37\%) \) and \( n_c = 1.12 \pm 0.05 \) \cite{1}, while LEP reports, at the \( Z \) resonance, \( B_{\text{SL}} = (11.12 \pm 0.20\%) \) and \( n_c = 1.20 \pm 0.07 \) \cite{2}. The naive parton model, which coincides with the leading-order heavy-quark-effective-field-theory (HQEFT) calculation, gives \( B_{\text{SL}} \approx 13\% \) \cite{3}. Conventionally, the inclusive \( B \) meson decays are described by a systematic HQEFT based Euclidean space operator product expansion (OPE) of relevant hadronic matrix elements in the inverse power of the \( b \) quark mass \( m_b \). The assumption of global and local quark-hadron dualities is employed to justify this approach. The \( 1/m_b^2 \) corrections (there are no \( 1/m_b \) corrections) have been investigated and found to be less than 5% of the leading-order predictions \cite{4}. Hence, it was argued that the values of \( B_{\text{SL}} \) < 12.5% can not be accommodated by QCD theory \cite{5}, and this discrepancy could be an indication of “new physics” beyond the standard model. For example, an enhanced \( b \to s g \) vertex has been proposed to explain the anomaly \cite{6}.

It is quite possible that the assumption of the local duality fails at the \( 1/m_b \) level. Recently, there was theoretical indication that \( 1/m_b \) corrections might appear in nonleptonic decays \cite{7}. Altarelli et al. has observed that if \( m_b \) in the phase space factor was replaced by the \( B \) meson mass \( M_B \), and by \( M_{\Lambda_b} \) in the \( \Lambda_b \) decays, due to the breakdown of the local duality, the predictions agree well with the data of \( B_{\text{SL}} \) and of the lifetime ratio \cite{8}. This naive modification increases the nonleptonic decay rates, and thus reduces \( B_{\text{SL}} \). On the other hand, though the nonperturbative corrections are small, it was pointed out that perturbative corrections may be significant \cite{8}. Bagan et al. have computed the \( O(\alpha_s) \) corrections with the effects of the charm mass included \cite{9}, and found that \( B_{\text{SL}} \) is indeed lowered with the increase of the \( b \to c\bar{c}s \) rate. However, the predictions are sensitive to the choice of the renormalization scale \( \mu \) \cite{10}: \( B_{\text{SL}} = 12.0 \) for \( \mu = m_b \), and \( B_{\text{SL}} = 10.9 \). 

for $\mu = m_b/2$. Furthermore, an enhanced $b \to c\bar{c}s$ mode, while lowering $B_{SL}$, enlarges $n_c$ to around 1.20, which, though consistent with the LEP data, becomes significantly higher than the CLEO data. This is sometimes called the “missing charm puzzle”.

In this paper we shall analyze the inclusive $B$ meson decays based on perturbative QCD (PQCD) factorization theorems [1]. It will be shown that the experimental data of $B_{SL}$ and $n_c$ can be accommodated simultaneously within the framework of the standard model, without resort to new physics. In a series of recent works [12, 13] we have developed a PQCD formalism for the inclusive semileptonic decay $B \to X_u e\bar{\nu}$, whose decay rate is written as the convolution of a hard subamplitude with the $B$ meson distribution function and the $u$-quark jet function. The idea is briefly explained as follows: Loop corrections produce infrared divergences, which signal the sensitivity to long-distance physics. The soft divergences associated with vanishing loop momenta survive after summing over all diagrams of the same order of $\alpha_s$. These divergences are absorbed into the $B$ meson distribution function, which describes the probability that the $b$ quark carries some fraction of the $B$ meson momentum. This function has been identified as the soft function associated with the Fermi motion of the heavy quark inside the heavy hadron, and as the outcome of the resummation of nonperturbative power corrections in HQEFT [14]. Near the end points of the decay spectra, the invariant mass of the $u$ quark vanishes, and the collinear divergences due to radiative gluons parallel to the $u$ quark become important. Large double logarithms from the overlap of the soft and collinear divergences then arise, and are absorbed into the jet function. The remaining finite piece of the radiative corrections is grouped into the hard subamplitude.

The hard subamplitude, dominated by short-distance contributions, is evaluated using perturbation theory. The end-point double logarithms contained in the jet function are systematically resumed into a Sudakov factor. It turns out that the Sudakov factor suppresses the long-distance contributions, and renders the perturbative calculation more reliable and self-consistent. The nonperturbative $B$ meson distribution function is universal, and determined by other experiment data, such as the spectrum of the inclusive radiative decay $B \to X_s \gamma$ [15]. Note that the analysis of inclusive heavy meson decays suffers the infrared renormalon ambiguity, which appears as power corrections starting with $O(1/m_b^2)$. This
ambiguity can be parametrized as an exponential factor associated with the distribution function, which has also been fixed from the best fit to the \( B \to X_s \gamma \) spectrum \[13\]. The above prescriptions specify clearly the treatment of each subprocess in the formalism. The advantage of our approach lies in that it is formulated according to the meson kinematics, instead of the parton kinematics as used in HQEFT. Hence, the replacement of \( m_b \) by \( M_B \) postulated in \[8\] is implemented naturally. The kinematic gap between the parton picture and the hadron picture is also removed.

We apply the above PQCD factorization theorem to the charmed decays,

\[
B(P_B) \to X_c + l(p_l) + \bar{\nu}(p_\nu),
\]

which dominate the inclusive semileptonic \( B \) meson decays. The momenta are expressed, in terms of light-cone coordinates, as

\[
P_B = \left( \frac{M_B}{\sqrt{2}}, \frac{M_B}{\sqrt{2}}, 0_\perp \right), \quad p_l = (p_l^+, p_l^-, 0_\perp), \quad p_\nu = (p_\nu^+, p_\nu^-, 0_\perp),
\]

where the minus component \( p_l^- \) vanishes for massless leptons. We choose \( E_l, q^2 \) and \( q_0 \), or equivalently, the three dimensionless quantities,

\[
x = \frac{2E_l}{M_B}, \quad y = \frac{q^2}{M_B^2}, \quad y_0 = \frac{2q_0}{M_B},
\]

as the independent variables, \( E_l \) being the lepton energy and \( q \equiv p_l + p_\nu \) the momentum of the lepton pair. Their kinematic ranges are

\[
\frac{2m_l}{M_B} \leq x \leq 1 - \beta + \alpha,
\]

\[
\alpha \leq y \leq \alpha + (1 + \alpha - \beta - x) \frac{x + \sqrt{x^2 - 4\alpha}}{2 - x - \sqrt{x^2 - 4\alpha}},
\]

\[
x + \frac{2(y - \alpha)}{x + \sqrt{x^2 - 4\alpha}} \leq y_0 \leq 1 + y - \beta,
\]

with the constants

\[
\alpha \equiv \frac{m_l^2}{M_B^2}, \quad \beta \equiv \frac{M_D^2}{M_B^2},
\]

and the lepton mass \( m_l \) and the \( D \) meson mass \( M_D \). \( M_D \) is introduced as the minimal invariant mass of the decay product \( X_c \). It is easy to check that the above ranges reduce to those for the decay \( B \to X_u e\bar{\nu} \) as \( m_l, M_D \to 0 \) \[12\].
In the parton picture the $B$ meson is composed of a $b$ quark with momentum $p_b = P_B - p$, where $p = (p^+, 0, p_\perp)$ is the momentum carried by other light degrees of freedom. Since the $c$ quark is massive, there are no collinear and thus no large double logarithms associated with it, differing from the decay $B \to X_u e\bar{\nu}$ \cite{12}. That is, QCD corrections to the charmed semileptonic decays are weaker. A convenient factorization scale is the transverse momentum $p_\perp$, or its conjugate variable $b$, of the $b$ quark inside the $B$ meson \cite{16}. $1/b$ marks the scale separation, below which physics is absorbed into the $B$ meson distribution function $f$, and above which physics is absorbed into the hard $b$ quark decay subamplitude $H$. Therefore, we write the decay rates as,

$$\frac{1}{\Gamma^{(0)}_\ell} \frac{d^3\Gamma}{dx dy dy_0} = \frac{M_B^2}{2} \int_{z_{\text{min}}}^1 dz \int_0^\infty db f(b, z, b) \tilde{J}_c(b) H(z),$$

with $\Gamma^{(0)}_\ell \equiv (G_F^2/16\pi^3)|V_{cb}|^2 M_B^5$. The momentum fraction $z$ is defined as $z \equiv p_b^+/P_B^+ = 1 - p^+/P_B^+$. $z$ is equal to the maximum 1 as the $b$ quark takes all the $B$ meson momentum in the plus direction. The minimum of $z$ is determined by the condition $p_c^2 > m_c^2$, $p_c$ and $m_c$ being the $c$ quark momentum and mass, respectively, which leads to

$$z_{\text{min}} = \frac{\frac{y_0}{2} - y + \frac{m_c^2}{M_B^2}}{1 - \frac{y_0}{2} + \frac{x}{\sqrt{x^2 - 4\alpha}} \left[ -\frac{y_0}{2} + \frac{y}{x} + \frac{\alpha}{x} \right]}.$$

The function $\tilde{J}_c$ denotes the final-state cut, which is in fact part of the hard subamplitude.

We factorize it out in order to compare Eq. (6) with the formulas in the nonleptonic case below. $J_c$ and $H$ in momentum space are given by

$$J_c = \delta \left( p_c^2 - m_c^2 \right)$$

$$= \delta \left( M_B^2 \left\{ z - (1 + z) \frac{y_0}{2} + y + \frac{x(1-z)}{\sqrt{x^2 - 4\alpha}} \left[ -\frac{y_0}{2} + \frac{y}{x} + \frac{\alpha}{x} \right] - \frac{m_c^2}{M_B^2} \right\} \right) - p_\perp^2 - 2p_\nu \cdot p_\perp,$$

$$H \propto (p_b \cdot p_\nu)(p_\ell \cdot p_c)$$

$$\propto \left( (y_0 - x) \left\{ 1 - \frac{(1-z)}{2} \left( 1 - \frac{x}{\sqrt{x^2 - 4\alpha}} \right) \right\} - \frac{(1-z)}{\sqrt{x^2 - 4\alpha}} (y - \alpha) \right)$$

$$\times \left( \frac{x}{2} \left\{ 1 + z + (1-z) \sqrt{x^2 - 4\alpha} \sqrt{x} \right\} - y - \alpha \right).$$

(9)
The universal distribution function determined from the decay spectrum of $B \to X_s\gamma$ is written as \[15\],

$$f(z, b) = \frac{0.02647 \cdot 0.02647 \cdot (1 - z)^2}{[(z - 0.95)^2 + 0.0034z]^2} \exp(-0.0125 M_B^2 b^2),$$

where the exponential parametrizes the infrared renormalon contribution at large $b$.

We then extend our formalism to the nonleptonic decays $B \to X_{c\bar{c}}$ and $B \to X_c$, which are dominated by the $b$ quark decays $b \to c\bar{c}s$ and $b \to c\bar{u}d$, respectively. For simplicity, we ignore the pretty small charmless processes. To simplify the calculation, we route the transverse momentum $p_\perp$ of the $b$ quark through the $c$ quark jet as in the semileptonic cases, such that the $c$ quark momentum is the same as before. For kinematics, we make the correspondence with the $\bar{c}$ ($\bar{u}$) quark carrying the momentum of the massive (light) lepton $\tau$ ($e$ and $\mu$) and with the $s$ and $d$ quarks carrying the momentum of $\bar{\nu}$. The dimensionless variables are then defined exactly as Eq. \(3\). The nonleptonic decays involve more complicated strong interactions characterized by three scales: the $W$ boson mass $M_W$, the characteristic scale $t$ of the $B$ meson decay, and the factorization scale $1/b$. The hard gluon exchanges among the quarks generate logarithms of $M_W$, and the associated physics is factorized into a harder function \[17\]. Collinear divergences and soft divergences from radiative corrections exist simultaneously, when the light quarks, such as $\bar{u}$, $d$, and $s$, become energetic. The resultant double logarithms $\ln^2(\bar{\rho} b)$, $\bar{\rho}$ being the jet momentum defined later, are factorized into the corresponding jet functions. This is how the additional scales $M_W$ and $\bar{\rho}$ are introduced into the nonleptonic decays. There are also soft gluon exchanges among the quarks, whose effects are negligible in bottom decays \[18\]. Similarly, physics with the scale below $1/b$ is absorbed into the (soft) distribution function.

We employ the three-scale factorization theorem developed in \[17\], that embodies both the PQCD factorization theorem stated above and effective field theory. According to the above explanation, the decay rates are expressed as the convolution of the harder, hard, jet and soft subprocesses,

$$\frac{\Gamma}{\Gamma_0} = M_B^2 \int dx dy dy_0 dz \int \frac{d^2b}{(2\pi)^2} H_r(M_W, \mu) H(z, t, \mu) \prod_i \tilde{J}_i(b) \prod_j \tilde{J}_j(b, p_j, \mu) \tilde{S}(z, b, \mu),$$

where the renormalization scale $\mu$ denotes the inclusion of QCD corrections. The functions $\tilde{J}_i$, $i = c$ and $\bar{c}$ for the $b \to c\bar{c}s$ mode and $i = c$ for the $b \to c\bar{u}d$ mode, do not contain double
logarithms, and represent the final-state cuts. The light-quark jet functions $\tilde{J}_j$, $j = s$ for the $b \to c\bar{c}s$ mode and $j = \bar{u}, d$ for the $b \to c\bar{u}d$ mode, contain double logarithm. These double logarithms are resummed into a Sudakov factor [12, 13],

$$\tilde{J}_j(b, p_j, \mu) = \exp[-2s(\bar{p}_j, b)]\tilde{J}_j(b, \mu),$$

where the upper bound of the Sudakov evolution is chosen as the sum of the longitudinal components of $p_j$,

$$\tilde{p}_j \equiv (p_j^+ + p_j^-).$$

We refer to [19] for the expression of the exponent $s$. In the numerical analysis below, $\exp(-s)$ is set to unity as $\bar{p}_j < 1/b$.

The single logarithms in the various subprocesses are summed using the RG equations [12, 17]:

$$\tilde{J}_j(b, \mu) = \exp \left[-\int_{1/b}^{\mu} \frac{d\tilde{\mu}}{\mu} \gamma_{J_j}(\alpha_s(\tilde{\mu})) \right] \tilde{J}_j(b),$$

$$S(z, b, \mu) = \exp \left[-\int_{1/b}^{\mu} \frac{d\tilde{\mu}}{\mu} \gamma_{S}(\alpha_s(\tilde{\mu})) \right] f(z, b),$$

$$H_r(M_W, \mu) = \exp \left[-\int_{M_W}^{\mu} \frac{d\tilde{\mu}}{\mu} \gamma_{H_r}(\alpha_s(\tilde{\mu})) \right] H_r(M_W),$$

$$H(z, t, \mu) = \exp \left[-\int_{\mu}^{t} \frac{d\tilde{\mu}}{\mu} \left[ \sum_{j} \gamma_{J_j}(\alpha_s(\tilde{\mu})) + \gamma_{S}(\alpha_s(\tilde{\mu})) + \gamma_{H_r}(\alpha_s(\tilde{\mu})) \right] \right] H(z, t),$$

$$\gamma_{J_j} = -2\alpha_s/\pi, \gamma_{S} = -C_F\alpha_s/\pi \text{ and } \gamma_{H_r} \text{ being the anomalous dimensions of the jet function, the soft function and the harder function, respectively.} \tilde{J}_j(b), f(z, b) \text{ and } H_r(M_W) \text{ on the right-hand sides are the initial conditions of the RG evolution, and do not contain large logarithms. Therefore, } \tilde{J}_j(b) \text{ are approximated by the tree-level delta functions just like } \tilde{J}_i(b), \text{ and } H_r(M_W) \text{ takes its lowest-order expression } H_r(M_W) = 1. \text{ The } B \text{ meson distribution function } f(z, b) \text{ is the same as that employed in the semileptonic decays. The scale } t \text{ should be chosen as the typical scale of the hard subprocess in order to minimize the involved large logarithms. A natural choice is the multiple of the maximal relevant scales, }$$

$$t = \kappa \max \left[ \bar{p}_j, \frac{1}{b} \right],$$

with $\kappa$ an adjustable parameter. Though other scales besides $t$ still lurk in the hard part, the logarithms they generate are relatively small. Since $t$ depends on the quark kinematics,
it reflects the details of the meson dynamics. $t$ remains a hard scale as long as the large $b$ region is Sudakov suppressed by $\exp(-s)$, and the perturbative calculation of the initial condition $H(z, t)$ is reliable. Hence, $H$ assumes its lowest-order expression in Eq. (9).

Combining the exponents in Eq. (14), the QCD evolution from $M_W$ to $t$, governed by $\gamma_{H_r}$, is identified as the Wilson coefficients. Conventionally, the summation of the large logarithms $\ln(M_W/\mu)$ is performed in terms of the effective Hamiltonian with $W$ boson integrated out. The short-distance physics related to the $W$ boson are incorporated into Wilson coefficients. For nonleptonic decays, the relevant effective Hamiltonian (ignoring the penguin operators for the moment) is

$$H_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{cb} V_{cs}^* [c_1(\mu) O_1 + c_2(\mu) O_2], \quad (16)$$

with the four-fermion current-current operators

$$O_1 = (\bar{s}_L \gamma_\mu b_L)(\bar{c}_L \gamma^\mu c_L), \quad O_2 = (\bar{c}_L \gamma_\mu b_L)(\bar{s}_L \gamma^\mu c_L). \quad (17)$$

$O_{1,2}$ require renormalization and thus the Wilson coefficients $c_{1,2}(\mu)$ depend on the renormalization scale $\mu$. It is simpler to work with the operators $O_\pm = \frac{1}{2}(O_2 \pm O_1)$ and their corresponding coefficients $c_\pm = c_2(\mu) \pm c_1(\mu)$, because they are multiplicatively renormalized.

At last, the nonleptonic decay rates are written as

$$\Gamma = \frac{M_B^2}{2} \int dx dy dy_0 dz \int_0^\infty db \left( \frac{N_c+1}{2} c_+^2(t) + \frac{N_c-1}{2} c_-^2(t) \right)$$

$$\times H(z, t) \prod_i \tilde{J}_i(b) \prod_j \tilde{J}_j(b) f(z, b) \exp \left[ - \sum_j s(\bar{p}_j, b) - s_{JS}(t, b) \right] \quad (18)$$

with

$$s_{JS}(t, b) = \int_{1/b}^t \frac{d\bar{\mu}}{\bar{\mu}} \left[ \sum_j \gamma_{J_j}(\alpha_s(\bar{\mu})) + \gamma_S(\alpha_s(\bar{\mu})) \right]. \quad (19)$$

The expressions of the Wilson coefficients $c_\pm$ up to next-to-leading order are referred to [20]. Hence, the three-scale factorization theorem contains the two-stage evolutions: from $M_W$ to $t$ due to the summation of the logarithms $\ln(M_W/t)$ organized into the Wilson Coefficients, and from $t$ to $1/b$ due to the summation of the logarithms $\ln(tb)$ into the exponential $\exp(-s_{JS})$. 
We compute the $B$ meson decay rates of the various modes for different choices of the hard scale parameter $\kappa$. The kinematic inputs are $M_B = 5.28$ GeV, $m_b = 4.6$ GeV, which is determined by the first moment $\bar{\Lambda}/M_B = 1 - m_b/M_B$ of the distribution function in Eq. (10), $M_D = 1.87$ GeV, and $m_c = 1.6$ GeV. Our predictions for $B_{SL}$ and $n_c$ are not sensitive to the variation of $m_c$. The difference between the values for $m_c = 1.6$ GeV and for $m_c = 1.5$ GeV is less than 5%. For the next-to-leading-order Wilson coefficients, we adopt the HV scheme. The difference between the HV and NDR schemes has also been examined, and found to be less than 5%. In Table I.1 we list the results in the obvious notations $r$, with the partial decay rates normalized to $B(\bar{B} \to X_c e \bar{\nu})$. This table is compared to the HQEFT predictions in Table I.2 [4]. On the experimental side, the CLEO group reports $B_{SL} = 10.19 \pm 0.37\%$, $n_c = 1.12 \pm 0.05\%$ and recently $B(b \to c\bar{c}s) = 21.9 \pm 3.7\%$ [21], which are only marginally consistent with the LEP measurements $B_{SL} = 11.12 \pm 0.20\%$ and $n_c = 1.20 \pm 0.07\%$.

Our predictions show a tendency that the experimental data can be accommodated if the hard scale is chosen properly. Unlike the next-leading-order HQEFT calculation, we can lower the semileptonic branching ratio into the range of experimental data with the charm counting almost remaining fixed. The value of $B(b \to c\bar{c}s)$ in Table I.1 is consistent with the CLEO measurement. The $b \to c\bar{d}d$ mode gains larger enhancement than the $b \to c\bar{c}s$ mode in our approach, because of the larger available phase space for the QCD evolutions. Therefore, we predict a larger $r_{ud}$ than that from HQEFT. On the contrary, the calculation of Bagan et al. shows that the increase of the mode $b \to c\bar{c}s$ from $O(\alpha_s)$ QCD corrections dominates [3]. Though there is an experimental argument for $r_{ud}$ to be about 4.0 [22], the scene is still problematic [21]. Indeed, if $r_{ud}$ is about 4.0 and $B(b \to c\bar{c}s)$ is 21.9%, which is determined more accurately than $r_{ud}$ by CLEO, the branching ratios of the various modes will not add up to unity [21]. To render the total branching ratios equal to 100%, $r_{ud}$ would be $5.2 \pm 0.6$, consistent with our predictions. Our results of the $B$ meson lifetime $\Gamma$, whose errors come from the uncertainty of $|V_{cb}| = 0.041 \pm 0.005$, are also consistent with the data ($1.62 \pm 0.06) \times 10^{-12}$ sec.

Note that our calculation leads to $r_{\tau\nu} = 0.378$, which is much larger than the HQEFT result 0.23. A simple investigation indicates that $r_{\tau\nu}$ depends on the $B$ meson distribution
function. By varying the distribution function, a smaller $r_{\tau \nu}$ can be obtained. However, we stress that the distribution function employed here is determined from the decay spectrum of $B \rightarrow X_s \gamma$. Therefore, we urge experimental measurements to decide whether the PQCD approach or the conventional HQEFT approach gives better results.

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Table I.1 PQCD predictions of the various decay modes with the notations $r_{\tau\nu} = B(\bar{B} \to X_c\tau\bar{\nu})/B(\bar{B} \to X_c e\bar{\nu})$, $r_{ud} = B(\bar{B} \to X_c)/B(\bar{B} \to X_c e\bar{\nu})$, and $r_{cs} = B(\bar{B} \to X_{cs})/B(\bar{B} \to X_c e\bar{\nu})$.

| $\kappa$ | $r_{\tau\nu}$ | $r_{ud}$ | $r_{cs}$ | $B_{c\bar{c}s}$ | $B_{\text{SL}}$ | $n_c$ | $\Gamma \ (10^{-12} \text{ sec})$ |
|----------|----------------|----------|----------|----------------|----------------|-------|------------------|
| 1.0      | 0.378          | 4.05     | 1.55     | 19.41          | 12.52          | 1.195 | 1.50 ± 0.37      |
| 1.5      | 0.378          | 4.76     | 1.74     | 19.59          | 11.26          | 1.196 | 1.35 ± 0.33      |
| 2.0      | 0.378          | 5.44     | 1.88     | 19.36          | 10.30          | 1.194 | 1.24 ± 0.30      |

Table I.2 HQEFT predictions of the various decay modes.

| $\mu/m_b$ | $r_{\tau\nu}$ | $r_{ud}$ | $r_{cs}$ | $B_{c\bar{c}s}'$ | $B_{\text{SL}}$ | $n_c$ |
|-----------|----------------|----------|----------|----------------|----------------|-------|
| 1.0       | 0.22           | 4.21     | 1.89     | 22.68          | 12.0           | 1.20  |
| 0.5       | 0.23           | 4.75     | 2.20     | 23.98          | 10.9           | 1.21  |