CRITICAL SCATTERING AT THE CHIRAL PHASE TRANSITION AND LOW-$p_T$ ENHANCEMENT OF MESONS IN ULTRA-RELATIVISTIC HEAVY-ION COLLISIONS

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Abstract: The enhancement of pions and kaons observed at small transverse momenta in ultra-relativistic heavy-ion collisions may at least partly reflect critical scattering expected to occur in the neighborhood of a second order phase transition. Kinetic equations in the relaxation time approximation are proposed for the time evolution of the quark distribution function into that of the pions. Relaxation times for thermalization and hadronization processes are functions of momenta and approach zero in the limit $p \to 0$, a consequence of criticality at the phase transition. Data can be reproduced for suitably chosen parameters.

HEIDELBERG, OCTOBER 1994

1 Research supported in part by the Federal Minister for Research and Technology (BMFT) grant number 06 HD 742 (0), and the Deutsche Forschungsgemeinschaft, grant number Hu 233/4-3.

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Introduction

The low-\(p_T\) enhancement observed in ultra-relativistic heavy-ion collisions is one of the unexpected results in this domain of physics (c.f. the review [1]): At small values of the transverse mass, \(m_T = \sqrt{m^2 + p_T^2}\), the observed distributions of pions and kaons, \(dN/dm_T\), considerably exceed the thermal distribution \(\exp(-m_T/T)\) [2]. Various explanations have been proposed: e.g. existence of the transverse flow [3, 4], creation of small plasma droplets [5], decays of resonances [6], formation of the pion system out of chemical equilibrium [7, 8], or the medium modification of the pion dispersion relation [9]. In this paper we propose yet another explanation, namely that the low-\(p_T\) enhancement reflects critical scattering.

Critical scattering or critical opalescence is a rather general phenomenon observed in the neighborhood of a second order phase transition [10]: The cross section \(d^3\sigma/dq\) for the scattering of light, X-rays or neutrons on a medium is given by the expression

\[
\frac{d^3\sigma}{dq} \propto \int d^3r \int d^3r' e^{iq(r-r')} \langle n(r)n(r') \rangle_T,
\]

where \(q\) is the momentum transfer and \(\langle n(r)n(r') \rangle_T\) is the density-density correlation function for the particles in the medium at the temperature \(T\). In the neighborhood of a phase transition a long-range ordering develops and the cross section for small values of \(q\) has the form

\[
\frac{d^3\sigma}{dq} \propto \frac{1}{k^2(T) + q^2},
\]

where

\[
k^2(T) \propto \left| 1 - \frac{T}{T_c} \right|^\gamma
\]

is the inverse correlation length with \(\gamma\) being the critical exponent.

This kind of experiment, where an external test particle is scattered on the system in its critical state, cannot be performed for a quark-gluon plasma which only lives for \(10^{-23}\) s. However, also particles which are created inside the plasma should experience critical scattering and their observed momentum distribution may bear signal of this phenomenon. In the present paper we discuss one possible realization of such a scenario.

In explicit calculations [11, 12], using the Nambu – Jona-Lasinio (NJL) model for a description of the quark-meson plasma (no gluons), it has been shown that singularities really occur in the cross sections at small center-of-mass energies \(\sqrt{s}\) of colliding particles,
when the phase transition is approached. In particular, the integrated elastic quark-antiquark cross-section $\sigma_{q\bar{q} \rightarrow q\bar{q}}(s, T)$ diverges like $s^{-1}$ for $T \rightarrow T_c$. A singularity is also found for the hadronization cross-section $q\bar{q} \rightarrow \pi\pi$. In both cases the singularity arises because the quark condensate $\langle q\bar{q} \rangle_T$, which is the order parameter of the chiral phase transition, goes to zero at the critical temperature.

May the observed low-$p_T$ enhancement originate from the low $\sqrt{s}$ singularity in the cross sections? We think this is so and demonstrate it using a toy model. Kinetic equations are proposed for the time evolution of a spatially homogeneous quark plasma (described by the single particle quark distribution function $f_q(p, t)$, $p = |\mathbf{p}|$) into a pion gas (with a corresponding distribution function $f_\pi(p, t)$). Criticality of the system enters via the singularities in the thermalization and hadronization cross sections.

In order to keep the model as transparent as possible we work in the relaxation time approximation, thus reducing the kinetic equations to a coupled system of first order integro-differential equations for $f_q(p, t)$ and $f_\pi(p, t)$. Contrary to the usual approaches, the relaxation times $\tau(p, t)$ depend on momenta via the energy dependence of the cross sections. The singularities in the cross sections due to criticality of the medium translate themselves into singularities of the inverse relaxation times $1/\tau(p, t)$ in the momentum $p$.

**Kinetic description of the hadronization of the quark plasma**

The above ideas are translated into the following set of kinetic equations

$$\frac{df_q(p, t)}{dt} = -\frac{f_q(p, t)}{\tau_{\text{th}}(p, t)} - \frac{f_q(p, t)}{\tau_{\text{had}}(p, t)} + \frac{f_\pi(p, t)}{\tau_{\text{dec}}(p, t)}$$ (4)

$$\frac{df_\pi(p, t)}{dt} = -\frac{f_\pi(p, t)}{\tau_{\text{th}}(p, t)} + \frac{f_q(p, t)}{\tau_{\text{had}}(p, t)} - \frac{f_\pi(p, t)}{\tau_{\text{dec}}(p, t)} - \frac{f_\pi(p, t)}{\tau_{\text{em}}(p)}$$, (5)

$$\frac{df_{\text{em}}(p, t)}{dt} = \frac{f_\pi(p, t)}{\tau_{\text{em}}(p)}$$, (6)

Eq. (4) determines the time evolution of the quark distribution function $f_q(p, t)$ (it describes both quarks and antiquarks, i.e. $f_q(p, t) = f_{\text{quarks}}(p, t) + f_{\text{antiquarks}}(p, t)$). The first term on the r.h.s. of Eq. (4) is the collision term written in the relaxation time approximation; it is responsible for the thermalization of quarks since the distribution function $f_q(p, t)$
is always attracted to the thermal one \( f_{q,\text{th}}(p, t) \). The second (third) term describes the loss (gain) of quarks due to the hadronization (deconfinement) process where we limit ourselves to the reaction \( q\bar{q} \to \pi\pi \) (\( q\bar{q} \leftrightarrow \pi\pi \)). Of the possible reaction channels \( q\bar{q} \to n\pi \) (\( n \geq 2 \)), the case \( n = 2 \) may be the dominant one for phase space reasons at moderate energies in the \( q\bar{q} \) system, and we assume that this is so.

In Eq. (3), for the time evolution of the distribution functions of pions, the first three terms on the r.h.s. describe: the thermalization of pions, appearance of pions due to hadronization of quarks, and the two-pion reaction into quark-antiquark pairs, respectively. The last (fourth) term accounts for the emission of pions from the plasma, which in a realistic system proceeds via the surface, while we simulate it via a homogeneously distributed sink. The emission of pre-equilibrium pions is crucial in order to observe deviations from thermal equilibrium.

Eqs. (4) - (6) can be solved for any initial conditions (i.e. assuming some particular form of the distribution functions at the initial time \( t = 0 \)) provided the thermal distribution functions \( f_{q,\text{th}}(p, t) \) and are known at all times. Since the first terms on the r.h.s. of Eq. (4) and (5) represent the collision terms (in the relaxation time approximation) they must obey the symmetries leading to particle and energy conservation. This gives the following constraints for \( f_{q,\text{th}}(p, t) \) and at each time \( t \)

\[
\int \frac{d^3p}{(2\pi)^3} \frac{f_i(p, t) - f_{i,\text{th}}(p, t)}{\tau_{\text{th}}(p, t)} = 0, \tag{7}
\]

\[
\int \frac{d^3p}{(2\pi)^3} \sqrt{p^2 + m_i^2} \frac{f_i(p, t) - f_{i,\text{th}}(p, t)}{\tau_{\text{th}}(p, t)} = 0 \tag{8}
\]

(here \( i = q \) or \( \pi \)). Eqs. (7) and (8) determine the temperature \( T_i(t) \) and the chemical potential \( \mu_i(t) \) appearing in the thermal distributions. For simplicity we assume the low-density high-temperature form of these functions, namely a Boltzmann distribution

\[
f_{i,\text{th}}(p, t) = g_i \exp \left[ -\frac{\sqrt{p^2 + m_i^2} - \mu_i(t)}{T_i(t)} \right], \tag{9}
\]

where \( g_i \) are the degeneracy factors: \( g_q = 24 \) (quarks and antiquarks having two different spin projections, 3 colors and 2 flavors) and \( g_\pi = 3 \) (three different values of the isospin).
Relaxation times and cross sections

The analysis of the exact collision term in the Boltzmann kinetic equation (analogous to that in [13]) leads to an expression for the average time for hadronization of two quarks into two pions

\[
\frac{1}{\tau_{\text{had}}(p, t)} = \frac{1}{2\sqrt{p^2 + m_q^2}} \int \frac{d^3p_1}{(2\pi)^3 \sqrt{p_1^2 + m_q^2}} f_q(p_1, t) F_{qq}(s) \sigma_{q\bar{q} \rightarrow \pi\pi}(s),
\]

where \(\sigma_{q\bar{q} \rightarrow \pi\pi}(s)\) denotes the total cross section for this process. The relativistic flux factor of incoming quarks is

\[
F_{qq}(s) = \frac{1}{2} \frac{1}{\sqrt{s}} \left(1 + \frac{s_0}{s}\right),
\]

where \(s\) is the center-of-mass energy of the quarks with momenta \(p\) and \(p_1\), respectively. The expression giving the deconfinement relaxation time, \(\tau_{\text{dec}}(p, t)\), has the form analogous to Eq. (10).

The relaxation time for the thermalization of quarks can be written in the form

\[
\frac{1}{\tau_{\text{th}}(p, t)} = \frac{1}{2\sqrt{p^2 + m_q^2}} \int \frac{d^3p_1}{(2\pi)^3 \sqrt{p_1^2 + m_q^2}} f_q(p_1, t) F_{qq}(s) \sigma_{q\bar{q} \rightarrow \pi\pi}(s)
\]

\[
+ \frac{1}{2\sqrt{p^2 + m_{\pi}^2}} \int \frac{d^3p_1}{(2\pi)^3 \sqrt{p_1^2 + m_{\pi}^2}} f_\pi(p_1, t) F_{q\pi}(s) \sigma_{q\pi \rightarrow q\pi}(s),
\]

where the two terms on the r.h.s. of Eq. (11) appear due to the quark-quark and quark-pion elastic collisions respectively. The formula for the thermalization time of pions can be obtained formally from Eq. (11) by the exchange of the indices \(q\) and \(\pi\).

For the sake of simplicity, in what follows we shall treat the quarks and pions as massless particles. We shall also assume that the cross sections: \(\sigma_{q\bar{q} \rightarrow \pi\pi}(s), \sigma_{qq \rightarrow \pi\pi}(s), \sigma_{\pi\pi \rightarrow \pi\pi}(s)\) and \(\sigma_{q\bar{q} \rightarrow q\pi}(s)\) are given by the generic expression

\[
\sigma(s) = \sigma_0 \left[1 + \frac{s_0}{s}\right],
\]

where \(\sigma_0\) represents a constant contribution to the cross sections, and the appearance of the singular term \(s_0/s\) accounts for the critical phenomena. The hadronization and deconfinement cross sections, \(\sigma_{q\bar{q} \rightarrow \pi\pi}(s)\) and \(\sigma_{\pi\pi \rightarrow q\bar{q}}(s)\), are related to each other by the principle of detailed balance.
\[ g_q^2 \sigma_{q\pi \to \pi}(s) = g_q^2 \sigma_{\pi \pi \to q\pi}(s). \]  

(13)

This relation guarantees that in the absence of emission (\(\tau_{em} \to \infty\)), the kinetic equations lead to the chemical equilibrium \(n_q/n_\pi = g_q/g_\pi\).

Since the relaxation time approximation requires that the system is always close to thermal equilibrium, the distribution functions appearing in Eqs. (10–11) can be replaced by the thermal ones and, in this way, we find the approximate form for the hadronization time

\[
\frac{1}{\tau_{had}(p, t)} = \frac{\sigma_0 n_q(t)}{2} \left[ 1 + \frac{s_0}{4pT_q(t)} \right],
\]

(14)

where

\[
n_q(t) = \int \frac{d^3p}{(2\pi)^3} f_q(p, t).
\]

(15)

Eq. (14) can be used to find the characteristic momentum scale, \(\delta p(t) = s_0/4T_q(t)\), below which the hadronization time significantly deviates from a constant. Due to the faster hadronization of low-energy quarks their temperature \(T_q(t)\) increases and, correspondingly, \(\delta p(t)\) decreases in time. Therefore, in the following we shall use the time average of \(\delta p(t)\), i.e. the quantity \(\delta p = \langle \delta p(t) \rangle\), in order to make the estimate of the momentum range, \(0 < p < \delta p\), where the enhancement in pion production is expected.

In our model the pions are emitted from the whole volume of the interacting system. The most natural physical assumption for \(\tau_{em}(p)\) would be, that it is inversely proportional to the velocity of the pion, which for massless particles equals \(c\) for all momenta. For this reason we choose \(\tau_{em}\) to be a constant. In order to study the sensitivity of our results on the emission rate we shall introduce the ratio \(\mathcal{R}\) defined as

\[
\frac{1}{\tau_{em}} = \mathcal{R} \frac{1}{\tau_{th}(p \to \infty, t = 0)}.
\]

(16)

The magnitude of \(\mathcal{R}\) indicates how much faster the evaporation is, in comparison to the rate of the thermalization processes inside the system.
Results

In order to reduce the number of free parameters in our calculation we have set all the cross sections: $\sigma_{qq\rightarrow \pi\pi}(s)$, $\sigma_{qq\rightarrow q\pi}(s)$, $\sigma_{q\pi\rightarrow q\pi}(s)$ and $\sigma_{\pi\pi\rightarrow \pi\pi}(s)$ to be equal (within a factor of 3 such a result is supported by calculations within the NJL model). Then, we are left with three parameters: $\sigma_0$, $s_0$ and $R$. One can notice that $\sigma_0$ sets the overall time-scale and is unimportant if we are interested in the final ($t \rightarrow \infty$) pion distributions.

We start solving our kinetic equations by assuming that the initial quark distribution is a thermal one and that there are no pions in the system. After integrating Eqs. (4) – (6) till the time when all quarks are hadronized and all pions are emitted, one obtains the distribution function $f_{em}(p, t \rightarrow \infty)$ of the observed pions for a set of parameters $s_0$ and $R$.

In Fig. 1 we show our results for the case when the initial temperature of the quarks $T_q(t = 0) = 140$ MeV. The dashed lines represent the initial quark distribution functions, whereas the solid ones represent the distribution functions of the emitted (observed) pions for four different choices of the parameters: (a) $R = 33$, $s_0 = 0.48 \text{ GeV}^2$; (b) $R = 11$, $s_0 = 0.48 \text{ GeV}^2$; (c) $R = 33$, $s_0 = 0.16 \text{ GeV}^2$; and (d) $R = 11$, $s_0 = 0.16 \text{ GeV}^2$. The corresponding values of the momentum scale $\delta p$ are: 210, 260, 90 and 95 MeV, respectively.

In all the considered cases one can clearly see the enhancement in the pion production for $p < \delta p$. During the evolution of our system the low energy quarks hadronize more effectively because of the $p$-dependence of $\tau_{\text{had}}$ and, correspondingly, the low energy pions are preferably produced. Our results also show that the effect depends strongly on the pion emission rate: for larger values of $R$ the effect is more significant. Such a dependence can be easily understood since the thermalization leads always to the distribution functions of the form (1), and in the limiting case $R \rightarrow 0$ no effect could be seen. The last fact indicates also that our mechanism for obtaining the low-$p_T$ enhancement is basically a non-equilibrium one: The excess can be observed only if the pions are emitted from a non-equilibrium system.

In Fig. 2 we plot the ratio $r(p)$ obtained by normalization of the distribution function of the observed pions to the Boltzmann distribution. In this case the latter is obtained by fitting the exponent function to $f_{em}(p, t \rightarrow \infty)$ in the region $0.5 \text{ GeV} < p < 1 \text{ GeV}$. The values of the parameters are the same as those assumed in Fig. 1a. Our result is shown together with the data [14] (14.6 A GeV Si + Pb $\rightarrow \pi^-$, rapidity interval $y = 3.4 – 3.6$). Because we did the calculations in the special case $m_{\pi} = 0$, we treat the data as if they were obtained also for massless pions and identify the quantity $m_T - m_{\pi}$ (appearing in [14]) with the transverse momentum $p_T$. Moreover, using the relation $p_T = \sqrt{2p/3}$ (valid for isotropic
systems) we find \( m_{T} - m_{\pi} = \sqrt{2p/3} \).

Fig. 2 shows that our model is able to explain a characteristic shape of the observed enhancement, if the two parameters \( s_{0} \) and \( R \) are adjusted. Nevertheless, we are of the opinion that the low-\( p_{T} \) enhancement in pion production is caused by the superposition of various mechanisms. In Ref. [14] it is argued that the decay of \( \Delta \) resonances can be responsible for the excess of low-\( p_{T} \) pions, although in different rapidity intervals different ratios of pions from \( \Delta \) decay to direct pions must be assumed to fit well the data. This uncertainty leaves room for other effects. Also the origin of the enhancement in kaon production, starting at much smaller values of \( p_{T} \), is still not clear. Therefore, we think that our mechanism for obtaining the low-\( p_{T} \) enhancement is probably only one of a few contributions leading to this effect.

Acknowledgments: We thank Sandy Klevansky for critical comments concerning the manuscript.
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Figure Caption

**Fig. 1** The initial quark distribution functions (dashed lines) and the distribution functions of the emitted pions (solid lines) for four different choices of the parameters: (a) \( R = 33, s_0 = 0.48 \text{ GeV}^2 \); (b) \( R = 11, s_0 = 0.48 \text{ GeV}^2 \); (c) \( R = 33, s_0 = 0.16 \text{ GeV}^2 \); and (d) \( R = 11, s_0 = 0.16 \text{ GeV}^2 \).

**Fig. 2** The distribution function of the observed pions normalized to the Boltzmann distribution. The parameters as in Fig. 1a. The diamonds show the data [14].
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This figure "fig1-2.png" is available in "png" format from:

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Fig. 2

\[ r(p) \]

\[ R = 33.0 \]

\[ s_0 = 0.48 \text{ GeV}^2 \]