Nutational oscillations suppression in attitude dynamics of spacecraft by relative motion of its movable module

A V Doroshin¹, A V Eremenko¹

¹Samara National Research University, Moskovskoe shosse34, Samara, Russia, 443086

e-mail: yeryomenko.a@bk.ru, doran@inbox.ru

Abstract. Attitude dynamics of a spacecraft with a movable module on retractable beams are considered. The movable module is attached to the main body of the spacecraft by retractable beams of variable length. The spacecraft main body contains a simple jet-engine with constant thrust. The retractable beams can tilt the movable module relatively the main part of the spacecraft, which shifts the position of the mass center. The control scheme of the nutation oscillations suppressing is constructed. This control scheme uses the impulses of the jet-engine on intervals of positive values of the equatorial component of the spacecraft angular velocity at the constant angular displacement of the movable unit. The main aim of this work is to develop the simplest control algorithm, which can be applied in cases of simple constructional forms of the small spacecraft with primitive equipment, including nano-satellites, that is economically profitable and can be used in more space missions.

1. Introduction

Research of the attitude dynamics of the spacecraft still is the important task of the modern space flight dynamics, which topicality increases due to developing the modern schemes of the nanosatellites with simplest control systems, including the control systems with the movable internal masses [1-5]. In this paper the attitude dynamics spacecraft (SC) with a movable module on retractable beams of variable length (figure 1) is considered [1, 2]. The attitude control is fulfilling by the length of the beams changing that tilts the movable module and translates the position of the mass center and creates the torque from the jet-engine with constant fixation relative the main body of the spacecraft. The movable module can represent the multifunctional element, e.g. a telescope, radiometer, etc. Such a control scheme allows using the functional a module as the simple actuator of the control system. Therefore, in this case the control does not need the presence of special executive bodies. The main aim of this research is to develop the control scheme to suppressing the nutational oscillations with the help of impulses of jet-engine at the the constancy of the angular displacement of the movable module relatively the main part of the SC.

2. The mathematical model

Let us consider the compound structure of the SC [1, 2] with one movable module (Figure 1), which is connected by the retractable beams with variable length (1 - the main body of SC, 2 - the movable module, 3 - the jet-engine, 4 - retractable beams). The sequence of rotation angles of the main body of the SC relative the inertial space is following: \( Z \to X \to Z \), by angles \( \psi, \theta, \phi \).

The coordinates systems are:
- OXYZ – the coordinate frame with the origin in the mass center, which axes are parallel to the main axes of the main body;
- O₁X₁Y₁Z₁ – the frame with the origin in mass center of the main body, which axes are parallel to the main axes of the main body;
- O₂X₂Y₂Z₂ – the main connected frame of the movable module.

Figure 1. The spacecraft with the movable module.

At small angles of the rotation of the movable module it is possible to neglect the terms from the curvature of retractable beams. In this case, we can obtain the mathematical model with the help of the angular momentum law [1, 2]:

$$\frac{d\mathbf{K}}{dt} = \frac{d\mathbf{K}}{dt} + \mathbf{\omega}_1 \times \mathbf{K} = \mathbf{M}^e,$$

where \(\mathbf{K}\) is the angular momentum of the system, \(\mathbf{\omega}_1\) is the angular velocity of the NS main body, \(\mathbf{M}^e\) is the external torque. In this work \(\mathbf{M}^e\) equals zero. The angular momentum is the sum of the angular momentum of the main body \(\mathbf{K}_1\) and the moment on movable module \(\mathbf{K}_2\):

$$\mathbf{K} = \mathbf{K}_1 + (\mathbf{\sigma}_1 \mathbf{K}_2) ,$$

where \(\mathbf{\sigma}_1\) is the translation matrix into the frame \(\mathbf{C}_{2}\mathbf{X}_{2}\mathbf{Y}_{2}\mathbf{Z}_{2}\) from the frame \(\mathbf{O}_{1}\mathbf{X}_{1}\mathbf{Y}_{1}\mathbf{Z}_{1}\). The movable modus has two degrees of freedom, which correspond to two relative rotations around axes \(X, Y\) on angles \(\alpha\) and \(\beta\).

The process of the reorientation of the SC with the help of simple impulse of jet-engine at tilting movable module was considered in [1], and the complex attitude dynamics of the system with the generation of complex attractors was investigated in [2]. So, the main description of the motion equations can be found in [1, 2]. In the next description the components of the angular velocity of the SC main body in its own connected frame are denoted as \(\mathbf{\omega}_1 = [p \quad q \quad r]^T\), and the general tensors of inertia of the main body and movable unit have the diagonal form:

$$\mathbf{I}_1 = \text{diag}(A_x, B_x, C_x), \quad \mathbf{I}_2 = \text{diag}(A_x, A_y, C_y)$$

3. The suppression of the nutational oscillations and the stabilization of the longitudinal rotation

Using the torque from the jet-impulse at the constant tilting \((\alpha_z = \text{const})\) movable module, it is possible to fulfill the suppression of oscillations of the equatorial components of the angular velocity. The corresponding control law for jet-impulses can be selected in the form:

$$p = \begin{cases} 0, & p < 0 \\ c_p, & p > 0 \end{cases}$$

(3)
where \( P \) is the piecewise constant engine thrust \((c_\text{p} = \text{const})\).

Let us show the modeling results for the simplest planar type of motion \((q = r = 0)\). The inertia-mass parameters are presented at the table 1.

| Inertia-mass parameters       | Values       |
|-------------------------------|--------------|
| Mass of the main body [kg]    | 3            |
| Mass of the movable module [kg]| 2           |
| \( A_b \) [kg\*m^2]          | 0.013        |
| \( B_b \) [kg\*m^2]          | 0.01         |
| \( C_v \) [kg\*m^2]          | 0.005        |
| \( A_v \) [kg\*m^2]          | 0.0025       |
| \( C_v \) [kg\*m^2]          | 0.0025       |
| \( O_1 C_1 \) [m]            | 0.1          |
| \( O_2 C_2 \) [m]            | 0.1          |

The initial conditions for modeling are: \( \theta = 0 \) [rad]; \( \alpha = 0.2 \) [rad]; \( P = 2 \) [N]; \( p = 1 \) [rad/s], \( q = r = 0 \) [rad/s]). The simplest modeling results for the planar case of the angular motion are presented at figures 2-3.

![Figure 2. The angular velocity p(t).](image1)

![Figure 3. The angle of the main body rotation \( \theta \).](image2)
Figure 4. The value of the jet-thrust.

As can we see from (figure 3-4) the value of thrust has the single-step form. Now let us consider the more complex case of the spatial motion at the constancy of angle $\alpha$. The equations (1) should be expressed in the form:

$$\begin{align*}
\dot{p} &= f_{11} q^2 + f_{12} r^2 + f_{13} q r + f_{14} P \sin(\alpha) \\
\dot{q} &= f_{21} p q + f_{22} p r \\
\dot{r} &= f_{31} p q + f_{32} p r
\end{align*}$$

(4)

where $f_{ij}$ will be constants depending from inertia-mass parameters, $f_{14} \sin(\alpha)$ corresponds to the arm of the thrust, and, e.g., at $\alpha = 0$ we present the partial shape:

$$f_{13} = \frac{B - C}{A}; \quad f_{22} = \frac{C - A}{B}; \quad f_{31} = \frac{A - B}{C};$$

(5)

where $A, B, C$ are the general inertia moments of SC.

From (4) it is possible to see, that at the opposite direction of the angle $\alpha$ relative to the component of the angular velocity $p$, the stopping torque will be created. This fact explains the form of law (6), which is defined as dependencies from the sign($p$).

As a result the following evolutions of the motion parameters are actual (figure 5-9) at the initial values $p = q = r = 1$ [rad/s], $\alpha = 0.2$, $P = 2$ [N].

Figure 5. The angular velocity component $p$ (red), the thrust P (blue).
From the figure 5 we can see the multiple short impulses on the last intervals of the process. It is realizing due to the constancy of the value $P$, which is “quite large for the small amplitude of $p$” at the end of the process.

To solve this problem it is possible to input the sensitivity zone $\Delta p$ in the law:

$$P = \begin{cases} 0, & p < \Delta p \\ c_p, & p > \Delta p \end{cases}$$  \hspace{1cm} (6)

Then at the law (6) and at the initial conditions $p = q = r = 1$ [rad/s], $\alpha = 0.2$, $P = 2$ [N], $\Delta p = 0.1$ [rad/s] we obtain dependencies depicted in figure 7-8.

So, we can see the suppression process of the equatorial components of the angular velocity and the aspiration of longitudinal component $r$ to the constant value. There we can reduce in the time the value of trust $P$ to increase the effectiveness of the suppressing process (figure 9), but the duration of the process also will increase.

All of the presented results correspond to the process of the nutation angle damping, and it is possible to calculate the following evolution for the nutation (figure10).

Figure 6. Components of the angular velocity $p$ (red), $q$ (blue), $r$ (green).

Figure 7. The angular velocity component $p$ (red), the thrust $P$ (blue).

Figure 10 describes the evolution of the nutation angle, which is calculated relative to the initial direction of the angular momentum of the system with corresponding initial conditions for the angular velocity and the following classical Euler angle (precession, nutation, intrinsic rotation):
$p(0)=q(0)=r(0)=1 \text{ [rad/s]}; \varphi(0)=0, \theta(0)=1.407, \varphi(0)=0.785 \text{ [rad]},$ since the nutational oscillations at the absence of the external forces are realized around the direction of the angular momentum.

Figure 8. Components of the angular velocity $p$ (red), $q$ (blue), $r$ (green).

Figure 9. Components of the angular velocity $p$ (red), $q$ (blue), $r$ (green) ($p=1 \text{ [N]}, \Delta p=0.03 \text{ [rad/s]}$).

Figure 10. The evolution of the nutation.

As can we see from the figure10, the suggested simplest control scheme allows to decrees the nutation angle from the right angle value to values oscillating relative to the quit small acute angle.
And here it is needed to note, that the suggested control scheme can be improved in the part of monotonous decreasing of the thrust value $P$ at the impulses forming or in the part of decreasing the $\alpha$ value, that provides the near-zero values of the equatorial angular velocities ($p$ and $q$) and the nutation angle with predefined accuracy.

4. Conclusion
In the work, the simplest control scheme of the suppression of the nutational oscillation was considered for the spacecraft with one movable unit. The control law was found in the piecewise form of the value of jet-trust. This simple control law, as well as the simple construction of the spacecraft, allows us to use the obtained results for the design of the nano-satellites with primitive equipment, that is economically profitable and can be used in more space missions.

5. References
[1] Doroshin A V and Eremenko A V 2019 Shilnikov's Homoclinic Loops in Attitude Dynamics of CubeSAT-3U Nanosatellites with One Movable Unit Proceedings of The International MultiConference of Engineers and Computer Scientists 73-76
[2] Doroshin A V 2017 Attitude Dynamics of Spacecraft with Control by Relocatable Internal Position of Mass Center Lecture Notes in Engineering and Computer Science 2227 231-235
[3] Jianqing Li, Changsheng Gao, Chaoyong Li and Wuxing J 2018 A survey on moving mass control technology Aerospace Science and Technology 82 594-606
[4] Liu H, Guo L and Zhang Y 2012 Composite attitude control for flexible spacecraft with simultaneous disturbance attenuation and rejection performance Journal of Systems and Control Engineering 226 154-161
[5] Ruidong Yan and Zhong Wu 2017 Attitude stabilization of flexible spacecrafts via extended disturbance observer based controller Acta Astronautica 133 73-80

Acknowledgments
The work is supported by the Russian Science Foundation (#19-19-00085).