The Distribution of Multiple Energy Minima and Hysteresis in Zero Temperature Random Field Ising Model

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Abstract. The Random Field Ising Model (RFIM) is the simplest physical model reflecting effect of quenched disorder on the different types of phase transitions in solids. The presence of multiple energy minima in the RFIM is an important feature determining main physical properties of the systems with a field type of quenched disorder. In particular, according to computer simulations of Sethna at al. performed for a zero temperature RFIM, the irreversible loop of magnetization appears as direct result of their presence. In the present paper the analytic method based on the total number of energy minima estimation is developed to study the hysteretic behavior of a zero temperature RFIM. As a result, the magnetization hysteresis loop is obtained as a marginal curve separating metastable and completely unstable states in the magnetization-external field plane.

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1. Introduction

The Random Field Ising Model (RFIM) is the simplest physical model reflecting the influence of different type of defects or quenched disorder on the different types of phase transitions in solids. Starting from the pioneering work of Imry and Ma [1] a great number of investigations have been made to understand the nature of different phase transitions in presence of random bond or field type of quenched disorder [2, 3, 4, 5, 6, 7, 8]. The literature contains several reviews on the theory and experiments [9, 10, 11, 12, 13, 14, 15]. The investigations developed in time from the equilibrium properties and lower critical dimensionality to dynamics of domain walls, metastability and hysteretic properties. The RFIM is now also used to explain a role of impurities in displacive transitions and random fields effect in charge-density waves. The multiple energy minima in the RFIM is an important feature determining main equilibrium and non-equilibrium physical properties of these systems. The most important effect resulting from presence of the infinite number of local energy minima in RFIM can be expected out of equilibrium (for example, in zero temperature case) when the thermal
fluctuations become negligibly small. In particular, the irreversible magnetization loop appears as a direct result of multiple energy minima in RFIM. This is confirmed by the direct computer simulations of magnetization processes reported in several publications (see [16, 17]). Due to a great number of possible stationary microscopic configurations, the magnetization process can be considered as an infinite bifurcation sequence of transitions between the multiple stationary (metastable) states when the external field is changed. Such transitions occur each time as the external field approaches the corresponding critical value where the current spin configuration becomes unstable. Then, this unstable configuration is changed at a fixed external field through a sequence of spin-by-spin flip transitions until a new stationary state is achieved. The main topic of this paper is to represent the alternative analytic method that gives a possibility to establish the direct connection between the infinite number of metastable states in RFIM and magnetization hysteresis loop. The method is based on the calculations of the total number of energy minima for the random field Ising Hamiltonian corresponding to different fixed values of the external field and specific magnetization per site. The hysteresis loop appears as a marginal curve separating \( m - h \) plane into the metastability region inside of the loop and the region of completely unstable states outside of it.

2. Local Energy Minima in RFIM

2.1. Stationary States of Zero Temperature RFIM Hamiltonian

Consider the Ising spin system \( \sigma_i = \pm 1 \) defined on a regular \( N \)-site lattice and interacting with the nearest neighbors, external uniform field \( h \) and irregular \( \tau_i \) (random) fields (here, \( i \in 1, ..., N \) is a lattice site number). The energy of such a system is known to be represented as follows:

\[
H\{s\} = -\frac{1}{2} J \sum_{ij} v_{ij} \sigma_i \sigma_j - \sum_i (h + \tau_i) \sigma_i
\]

(1)

where, the first term represents the nearest neighbors interaction energy (\( v_{ij} = 1 \neq 0 \) only for nearest neighbors), \( J \) is a spin-spin interaction constant and the second one reflects the influence of both the external and the random fields; \( \{s\} \equiv \{\sigma_1, \sigma_2, ..., \sigma_N\} \) denotes \( N \)-spin configuration vector. The random field values \( \tau_i \) are assumed to be non-correlated at the different lattice sites and characterized with the probability distribution function \( P(\tau) \) that is the same for all the lattice sites. One can also define the possible local minima of RFIM Hamiltonian as a set of Ising spin configurations \( s \) satisfying the following conditions:

\[
H\{s\} < H\{s^{(i)}\} \quad \forall i \in 1, ..., N
\]

(2)

where, each local minimum \( s \) is a spin configuration having lower energy in respect to all other \( N \) configurations \( s^{(i)} \) that differ from the initial one \( s \) by a single spin at \( i \)-th
site flipped in opposite direction: $\sigma_i \rightarrow -\sigma_i$. In other words, the system of inequalities:

$$u_i(s) = \frac{1}{2}[H(s^{(i)}) - H(s)] = \sigma_i \left(\tau_i + h + J \sum_j v_{ij}\sigma_j\right) > 0; \quad \forall i \in 1, ..., N$$ (3)

define all the possible stationary spin configurations for zero temperature RFIM. It should be noted that these inequalities can be satisfied in the unique case when both factors in the product $\sigma_i \left(\tau_i + h + J \sum_j v_{ij}\sigma_j\right)$ will have the same signs. Therefore, the solution of inequalities is completely equivalent to the following equations:

$$\sigma_i = \text{sign} \left(\tau_i + h + J \sum_j v_{ij}\sigma_j\right)$$ (4)

One can interpret these equations as local equilibrium conditions for each site spin in the local self-consistent field

$$h_i = \tau_i + h + J \sum_j v_{ij}\sigma_j$$ (5)

Accordingly, each site spin is oriented such a way to have the same direction as the local field applied. This equation also represents a zero temperature limit of the mean field equation that is often used in different studies of the equilibrium properties of RFIM. In particular case $J = 0$ the Eq.(4) immediately gives the only possible solution:

$$\sigma_i = \text{sign} (\tau_i + h)$$ (6)

that corresponds to a single minimum of RFIM in this case. The total magnetization per site in this case can be expressed as

$$m_0(h) = \frac{1}{N} \sum_i \sigma_i = \int d\tau P(\tau)\text{sign}(h + \tau)$$ (7)

where, denoting the Dirac’s function as $\delta(t)$ one can define the probability distribution function of the random field values:

$$P(t) = \frac{1}{N} \sum_i \delta(t - \sigma_i)$$ (8)

$m_0(h)$ will therefore represent a single magnetization curve in case of non-interacting Ising spin system in a random field.

### 2.2. Numerical Simulation of Magnetization Curves

The most important effect resulting from presence of the infinite number of local energy minima in RFIM can be expected for the magnetization properties of Ising spin system in zero temperature case when thermal fluctuations do not play any role.
In absence of thermal fluctuations each stationary state will still remain the same as long as the first external field value is achieved where this state becomes unstable and pinning equations Eq.(3) or Eq.(4) can not be more satisfied. Then, the unstable spin configuration is changed at fixed external field through a sequence of single spin flip transitions until a new stationary state to be achieved. Therefore, due to great number of possible stationary microscopic configuration consistent with the pinning conditions the magnetization process can be considered as a bifurcation sequence of transitions between the different stationary states as the external field is changed. The necessary condition for a single spin transition can be represented as follows:

\[ \sigma_i \rightarrow -\sigma_i \quad \text{if only} \quad u_i(s) < 0 \quad \& \quad u_i(s) = \min \]  

It is denotes that new stable spin configuration at each fixed external field is achieved through a sequence spin by spin flip processes running along a single possible configuration path providing the maximal energy gain at each elementary stage of transition. The configuration path satisfying conditions Eq.(3) provides the maximal rate of the free energy decrease in agreement with usual requirements of non-linear thermodynamics and give necessary algorithm for the computer simulation methods and modeling of magnetization processes in framework of RFIM. Fig.1 shows the typical magnetization loop resulting from such a type simulation procedure performed for the one-dimensional RFIM. Numerically simulated gaussian random field values on the one-dimensional lattice with 1000 sites were used for calculations.

3. Analytic Accounting of the number of local minima

3.1. General expressions

By using the definition from Eq.(3)one can propose the following general way for evaluation of the total number of local energy minima in RFIM. For this aim, denote initially the half of energy difference as: And then, one can easily construct the characteristic functional containing information on the all possible local minima in RFIM that can be written as follows:

\[ L(s) = \prod_i \eta(u_i(s)) \]  

where, \( \eta(t) \) is the Heaviside function \( \eta(t) = 1 \), if \( t > 0 \) and \( \eta(t) = 0 \), if \( t < 0 \). One can easily check that the above introduced characteristic functional takes the unit value only if any spin configuration is a local minimum of RFIM Hamiltonian and zero in other case. Therefore, taking the sum over all possible \( 2^N \) Ising spin configurations one can obtain the following general representation for the total number of local energy
minima in RFIM:

\[ Z(h) = \sum_{\{s\}} L\{s\} \]  

(11)

Respectively, one can also find the averaged over the random field ensemble number of local minima as follows:

\[ \overline{Z}(h) = \sum_{\{s\}} \langle L\{s\}\rangle_\tau \]  

(12)

Due to statistical independence of the random field values \( \tau_i \) for different lattice sites the averaging procedure in Eq.(12) is transformed into the product of one-site averages:

\[ \overline{Z}(h) = \prod_{i} \left( \frac{1}{2} + \frac{1}{2} \sigma_i m_0 \left( h + J \sum_j v_{ij} \sigma_j \right) \right) \]  

(13)

Here, we used the following evident property of Heaviside function: \( \eta(t) + \eta(-t) = 1 \) and \( \eta(t) - \eta(-t) = sign(t) \) and also \( \eta(\sigma_i t) = \frac{1}{2} (1 + \sigma_i sign(t)) \) where,

\[ m_0(t) = \int sign(t + \tau) P(\tau) d\tau. \]  

(14)

It will be shown below that \( m_0(t) \) represents a magnetization curve of RFIM in case of non-interacting spins when \( J = 0 \). Therefore, the problem of the total local minima number estimation for the Random field Ising model can be always reduced to a standard statistical-mechanical procedure on calculation of the partition function for a generalized Ising model:

\[ \overline{Z}(h) = \sum_{\{s\}} \exp(V\{s\}) \]  

(15)

with some effective Hamiltonian:

\[ V\{s\} = -N \ln 2 + \sum_i \ln \left( 1 + \sigma_i m_0 \left( h + J \sum_j v_{ij} \sigma_j \right) \right) \]  

(16)

It is important that the effective Hamiltonian can be always expressed as a finite order polynomial of Ising spin variables:

\[ V\{s\} = V^0 + \sum_i V_i^1 \sigma_i + \sum_{ij} V_{ij}^2 \sigma_i \sigma_j + \sum_{ijk} V_{ijk}^3 \sigma_i \sigma_j \sigma_k + ... \]  

(17)

Polynomial coefficients are translation invariant functions of their arguments. These take zero values for coinciding pairs of their arguments and contain only the short range nearest neighbor interactions. The maximal order of this polynomial is \( k + 1 \) where, \( k \) is a number of nearest interacting neighbors of the initial RFIM Hamiltonian. As a result, using the analogy with the partition function of equilibrium thermodynamics
one can assume the following universal asymptotic behavior for the average number of local minima when the lattice sites number $N$ becomes infinitely large:

$$\overline{Z}(h) \xrightarrow{N \to \infty} \exp(\mu(h)N)$$

(18)

where $\mu(h)$ is the independent on $N$ function of the external field. Because the total number of local energy minima $1 \leq Z(h) \leq 2^N$, then $0 \leq \mu(h) \leq \ln 2$ and one can make a general conclusion that $\overline{Z}(h)$ must exponentially grow as $N \to \infty$. Therefore, RFIM can have a single minimum if only $\mu(h) \equiv 0$ and infinite number of local minima in other case. Such a case is realized for the non-interacting spin system at $J = 0$. One can easily obtain this result by using Eq.(13) at $J = 0$:

$$\overline{Z}(h)_{J=0} = \left(\frac{1}{2} \sum_{\sigma=\pm1} (1 + \sigma m_0(h))\right)^N = 1$$

(19)

Naturally, in general case both $\overline{Z}(h)$ and $\mu(h)$ are the functions dependent on the parameters of Random Field Ising Hamiltonian such as the external field $h$, spin-spin interaction constant $J$ and also on the distribution function of random field $P(\tau)$. Along with the total number of local energy minima $\overline{Z}(h)$ one can also consider another important quantity: $\overline{Z}(M, h)$ that gives the number of stationary states corresponding to a fixed value of total magnetization $M = \sum_i \sigma_i$. Similar to Eq.(15) this one can be found as follows:

$$\overline{Z}(M, h) = \sum_{\{s; M\}} \exp(V(s))$$

(20)

where, the summation is performed over configurations with a fixed total magnetization value. One can also expect the following universal asymptotics in the thermodynamic limit $N \to \infty$:

$$\overline{Z}(M, h) \xrightarrow{N \to \infty} \exp(\mu(m, h)N)$$

(21)

where, $m = M/N$ is a specific magnetization per site. But unlike to always positive function $\mu(h)$ defined above the new function $\mu(m, h)$ introduced here and dependent on two variables $m$ and $h$ can take the negative values too in the corresponding area of their change. As a result of Eq.(21), the number of possible stationary states corresponding to different fixed values of specific magnetization $m$ becomes infinitely small approaching zero value as $N \to \infty$ in that region of $m - h$ plane where $\mu(m, h) < 0$. Therefore, one can conclude that all possible infinite set of stationary solutions is always located only inside the region $\mu(m, h) > 0$. The marginal curve $\mu(m, h) = 0$ separating these two regions can be therefore associated with a global magnetization loop. Such interpretation is completely consistent with the results of dynamical approach based on the computer simulation of partial magnetization sub-loops that are always locked inside of main loop confirming its marginal character as shown in Fig.1.
Due to above mentioned analogy with the equilibrium statistical mechanics a great number of approximation methods can be applied to calculation of $Z(h)$ and $Z(M, h)$. As an example, the simple perturbation method will be used below to estimate the number of local minima in case of the small interaction constant limit. The main reason of that approach is to show that the infinite number of local minima effect immediately appears as only $J \neq 0$, starting from an arbitrary weak spin-spin interaction.

3.2. Perturbation Theory Expansion

As known the perturbation theory methods in statistical mechanics providing correct behavior in the thermodynamic limit when $N \to \infty$ are realized as a free energy expansion that is proportional to the log value of a partition functional. Similarly, the corresponding expansion in respect to $\ln Z(h) = N \mu(h)$ and $\ln Z(M, h) = N \mu(M, h)$ should be performed in case $J \to 0$. Accordingly, one can easily obtain the Taylor’s series expansion for the effective Hamiltonian defined in Eq.(14) as $V\{s\} = V_0\{s\} + V_1\{s\} + V_2\{s\}$ up to the second order on $J$. Then, using Ising spin algebra rule ($\sigma^2 = \sigma$), a zero order term can be expressed as follows:

$$V_0\{s\} = \sum_i \ln \left(\frac{1 + \sigma_i m_0(h)}{2}\right) = \frac{1}{2} \ln \left(\frac{1 - m_0^2(h)}{4}\right) N + \frac{1}{2} \ln \left(\frac{1 + m_0(h)}{1 - m_0(h)}\right) \sum_i \sigma_i$$  \hspace{1cm} (22)

Others first and second order perturbation contributions are:

$$V_1\{s\} = J \sum_i \frac{m'_0(h) \sigma_i}{1 + \sigma_i m_0(h)} \sum_j v_{ij} \sigma_j = J \frac{m'_0(h)}{1 - m_0^2(h)} \sum_i \sum_j v_{ij} (\sigma_i - m_0(h)) \sigma_j$$  \hspace{1cm} (23)

$$V_2\{s\} = -\frac{1}{2} J^2 \sum_i \frac{(m'_0(h))^2 \sigma_i \sigma_j}{(1 + \sigma_i m_0(h))^2} \left(\sum_j v_{ij} \sigma_j\right)^2 + \frac{1}{2} J^2 \sum_i \frac{m''_0(h) \sigma_i}{1 + \sigma_i m_0(h)} \left(\sum_j v_{ij} \sigma_j\right)^2 =$$

$$-\frac{1}{2} J^2 \frac{(m'_0(h))^2}{(1 - m_0^2(h))^2} \sum_{ijk} v_{ij} v_{ik} (\sigma_i - m_0(h))^2 \sigma_j \sigma_k +$$

$$\frac{1}{2} J^2 \frac{m''_0(h)}{1 - m_0^2(h)} \sum_{ijk} v_{ij} v_{ik} (\sigma_i - m_0(h)) \sigma_j \sigma_k$$  \hspace{1cm} (24)

As a result, the corresponding expansion of both the $\ln Z(h) = N \mu(h)$ and $\ln Z(M, h) = N \mu(m, h)$ up to the second order on the interaction constant $J$ will look like as follows:

$$\ln Z(h) = \ln Z_0(h) + \langle V_1 \rangle_0 + \langle V_2 \rangle_0 + \frac{1}{2} \left(\langle V_1^2 \rangle_0 - \langle V_1 \rangle_0^2\right)$$  \hspace{1cm} (25)

$$\ln Z(M, h) = \ln Z_0(M, h) + \langle V_1 \rangle_M + \langle V_2 \rangle_M + \frac{1}{2} \left(\langle V_1^2 \rangle_M - \langle V_1 \rangle_M^2\right)$$  \hspace{1cm} (26)
Here, the averaging procedures are defined as:

\[
\langle \ldots \rangle_0 = \frac{\sum_{\{s\}}^{} (\ldots) \exp V_0\{s\}}{\sum_{\{s\}}^{} \exp V_0\{s\}} ; \quad \langle \ldots \rangle_M = \frac{\sum_{\{s,M\}}^{} (\ldots) \exp V_0\{s\}}{\sum_{\{s,M\}}^{} \exp V_0\{s\}} \quad (27)
\]

where, \(\{s,M\}\) is a set of spin configurations with a fixed magnetization value \(M\). Due to statistical independence of spin variables in the different sites and evident result: \(\langle \sigma_i \rangle_0 = m_0(h)\) that can be easily found from the Eqns.(22, 27) both the \(\langle V_1 \rangle_0\) and second term from the Eq.(24) contributing into \(\langle V_2 \rangle_0\) take zero values. Taking also into account that according to Eq.(19) \(Z_0(h) = 1\) and \(\ln (Z_0(h)) = 0\) one can obtain the following results for the total number of stationary states \(Z(h)\):

\[
\ln (Z(h)) = -\frac{1}{2} J^2 \left( \frac{m'_0(h)}{1 - m_0^2(h)} \right)^2 \sum_{ijkl} v_{ij} v_{ik} \left\langle \left( \sigma_i - m_0(h) \right)^2 \sigma_j \sigma_k \right\rangle_0 + \frac{1}{2} J^2 \left( \frac{m'_0(h)}{1 - m_0^2(h)} \right)^2 \sum_{ijkl} v_{ij} v_{ik} \left\langle \left( \sigma_i - m_0(h) \right) \left( \sigma_l - m_0(h) \right) \sigma_j \sigma_k \right\rangle_0 \quad (28)
\]

The first term in square braces is completely reduced with a part \(l = i\) of the second sum. One can also check that only the \(k = i \& l = j\) terms from the remaining part of the second sum will give nontrivial contributions. Therefore,

\[
\ln (Z(h)) = \frac{1}{2} J^2 \left( \frac{m'_0(h)}{1 - m_0^2(h)} \right)^2 \sum_{ij} (v_{ij})^2 \left( 1 - \sigma_i m_0(h) \right)_0 \left( 1 - \sigma_j m_0(h) \right)_0 = \frac{1}{2} J^2 \left( \frac{m'_0(h)}{1 - m_0^2(h)} \right)^2 kN \quad (29)
\]

where \(k\) is the number of nearest neighbors. Therefore, the total number of energy minima becomes infinitely large even in case of an arbitrary weak Ising spin interaction and can be expressed in the second order perturbation theory approach as follows:

\[
\overline{Z(h)} = \exp (\mu (h) N) \quad \text{where,} \quad \mu (h) = \frac{1}{2} k J^2 \left( \frac{m'_0(h)}{1 - m_0^2(h)} \right)^2 \quad (30)
\]

### 3.3. Multiple minima and magnetization loop

In this section the relationship between the great number of possible stationary state of RFIM and the irreversible magnetization loop typical for ferromagnetic materials will be a subject for study and discussion. It will be shown that very important information on the magnetization loop can be obtained not only from the direct dynamic simulations of the magnetization process but also by accounting the number of possible stationary solutions and their distribution on the different values of macroscopic magnetization. More exactly, the method is based on the calculation of \(Z(M, h)\) and solution of
characteristic equation for the hysteresis loop: \( \ln \left( Z(M, h) \right) = \mu(m, h) N = 0 \). In the framework of perturbation theory \( \mu(m, h) \) can be found directly from the Eq.(26). But there is a simplest way to do that by using the evident sum rule: \( Z(h) = \sum_M Z(M, h) \) following from definitions of both these quantities. Using this rule and exponential representations (see Eqns.(18) and (21)) one can obtain asymptotically (as \( N \to \infty \)) the following results:

\[
\exp (\mu(h) N) = \sum_M \exp (\mu(m, h) N) \longrightarrow \exp \left( \max_{m \in [-1,1]} (\mu(m, h)) N \right)
\]

Or, equivalent relationship:

\[
\mu(h) = \mu(\tilde{m}(h), h) \quad \text{where,} \quad \frac{\partial}{\partial m} \mu(m, h) = 0 \quad \text{at} \quad m = \tilde{m}(h) \quad \text{(32)}
\]

As a result \( \mu(m, h) \) can be expanded near the maximum \( m = \tilde{m}(h) \) as follows:

\[
\mu(m, h) = -\frac{1}{2} \beta(h) (m - \tilde{m}(h))^2 + \mu(h) \quad \text{where,} \quad \beta(h) > 0 \quad \text{(33)}
\]

In the case of weak interaction \( J \to 0 \) this representation can be used to find the magnetization curve as a solution of the equation \( \mu(m, h) = 0 \). According to Eq.(31) \( \mu(h) = 0 \) at \( J = 0 \) and there is a single degenerated solution \( m = \tilde{m}_0(h) \) for a zero order equation:

\[
\mu_0(m, h) = -\frac{1}{2} \beta_0(h) (m - \tilde{m}_0(h))^2 = 0 \quad \text{(34)}
\]

So, there is no hysteresis at \( J = 0 \). But, as follows from the Eq.(33), it is immediately appears as only \( J \neq 0 \) and represented by two branches \( m = m_\pm(h) \) of hysteresis loop close to \( m = \tilde{m}(h) \) and each other, where:

\[
m_\pm(h) = \tilde{m}(h) \pm \sqrt{2 \mu(h) / \beta(h)} \quad \text{where,} \quad \beta(h) > 0 \quad \text{(35)}
\]

Because \( \sqrt{\mu(h)} \) is the first order on \( J \) then only up to the first order calculations for \( \tilde{m}(h) = \tilde{m}_0(h) + m_1(h) \) and zero order for \( \beta(h) = \beta_0(h) \) should be performed. In particular, one can easily show that \( \tilde{m}(h) \) is exactly equal to a single-site average of Ising spin value \( \langle \sigma_l \rangle \) and expanded into the perturbation series as:

\[
\tilde{m}(h) = \langle \sigma_l \rangle = \langle \sigma_l \rangle_0 + (\langle (\sigma_l - \langle \sigma_l \rangle_0) V_1(s) \rangle_0 = m_0(h) + k J m_0'(h) m_0(h) \quad \text{(36)}
\]

\( \beta_0(h) \) can be found from the direct calculation of zero order term \( \mu_0(m, h) = \frac{1}{N} \ln \left( Z_0(M, h) \right) \) according to Eq.(21) at \( J = 0 \) and its expansion near the maximum at \( m = m_0(h) \). Therefore, \( \beta_0(h) = \left( -\partial^2 / \partial m^2 \right) \mu_0(m, h) |_{m=m_0(h)} \). Results of such calculations are represented below:

\[
\overline{Z_0(M, h)} = \sum_{\{s, M\}} \exp \left( V_0(s) \right)
\]
\[
\mu_0 (m, h) = \frac{1}{N} \ln \left( Z_0(M, h) \right) = \\
\frac{1}{2} [(1 + m) \ln \left( \frac{1 + m_0(h)}{1 + m} \right) + (1 - m) \ln \left( \frac{1 - m_0(h)}{1 - m} \right)]
\]

(38)

\[
\beta_0(h) = \left(-\frac{\partial^2}{\partial m^2}\right) \mu_0 (m, h)_{| m=m_0(h)} = \frac{1}{1 - m_0^2(h)}
\]

(39)

The final equation representing the irreversible magnetization loop for a zero temperature RFIM in the weak interaction limit will look like as follows:

\[
m_{\pm}(h) = \left(1 + k J m_0'(h)\right) m_0(h) \pm J m_0'(h) \sqrt{k \left(1 - m_0^2(h)\right)}
\]

(40)

A set of hysteresis loops calculated according to Eq.(40) for different values of the interaction parameter \( J \) is indicated in Fig.2.

Therefore, the present approach makes it possible to perform detailed analytic studies concerning physical nature of hysteresis by using the Random Field Ising based microscopic consideration.

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Figure captions

Fig.1 Magnetization loop and partial sub-loops numerically simulated for the onedimensional RFIM.

Fig.2 Marginal hysteresis loops separating metastable and unstable states in RFIM calculated for the different values of interaction constant in the framework of perturbation method.
unstable states

metastable states

area

J=0.35

magnetization per site

external field

-1.0

0.0

0.5

1.0

-1.0

-0.5

0.0

0.5

1.0

-1.5

-1.0

-0.5

0.0

0.5

1.0

1.5
