CFO Estimation Based on Auto-Correlation With Flexible Intervals for OFDM Systems With 1-bit ADCs

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This work was supported in part by the Institute of Information and Communications Technology Planning and Evaluation (IITP) grant funded by the Korea government (MSIT), Development on the disruptive technologies for beyond 5G mobile communications employing new resources, under Grant 2018-0-00809, and in part by the Brain Korea 21 Plus (BK21+) Project for the School of Electrical Engineering, Korea Advanced Institute of Science and Technology (KAIST).

ABSTRACT For orthogonal frequency division multiplexing (OFDM) systems, the carrier frequency offset (CFO) estimation is required for the frequency synchronization before the data reception. When 1-bit ADCs are employed in OFDM systems, the available information is limited to the sign of the received signal, and the acquisition of the CFO is not easy problem. To overcome this limitation, we utilize the extended preamble and construct the bank of the CFO estimators based on auto-correlation (AC) taking the samples from the preamble with flexible intervals, which produces the candidates for CFO estimate. We propose the CFO estimation algorithms using the normalized squared errors (NSEs) as the concept to check the quality of the candidates of CFO estimate. The simulation results show that the proposed algorithm works well in the multipath fading channel, and is superior to the conventional CFO estimation method based on AC with fixed interval.

INDEX TERMS Carrier frequency offset, OFDM, synchronization, 1-bit ADC.

I. INTRODUCTION
Orthogonal frequency division multiplexing (OFDM) as the waveform for the multicarrier transmission has been used in wireless communication systems [1]–[3]. In practical OFDM systems, the time and frequency synchronization procedures are important to mitigate intersymbol and inter-carrier interferences, and performed by utilizing preambles or pilots as reference signal. In [4]–[6], the carrier frequency offset (CFO) estimation is dealt with, and based on auto-correlation (AC) using the repetition structure of the preambles.

Also, the OFDM is suitable for the wideband communication systems. However, the expanded bandwidth causes the growth of the sampling rate of the analog-to-digital converters (ADCs), which poses problems for high power consumption and hardware complexity. Thus, 1-bit ADCs as the extreme case of low resolution ADCs can be considered as the solutions for low power consumption and hardware complexity [7]. The OFDM is employed in massive MIMO (multiple input multiple output) [8], [9] and millimeter wave (mmWave) [10] with low resolution ADCs.

In [11]–[13], the channel estimation is considered for the OFDM systems with low resolution ADCs. In [12], the deep learning based methods are utilized to estimate channel, and the autoencoder for data symbol detection is also proposed. In [13], the Turbo-like channel estimation is proposed, and the prototyping system is presented for the reliable data transmission.

When 1-bit ADCs are deployed in the OFDM systems, the available information is only the sign of received signal. Thus, the phase differences from the received signal are distorted, so the CFO estimation is difficult in the OFDM receiver with 1-bit ADCs. In addition, there are several works dealing with the synchronization issues in the mmWave systems with 1-bit ADCs [14]–[16]. The directional frame synchronization method is developed in [14], and the message passing based algorithm is proposed to jointly estimate CFO and channel in [15]. In [16], the synchronization sequences is designed, and the CFO estimation methods based on the ratio metric exploiting those sequences proposed. However, previous works for the CFO estimation are not addressed well in OFDM systems with 1-bit ADCs.

In this paper, we focus on the CFO estimation based on AC in the OFDM systems with 1-bit ADCs. The contributions of this paper are summarized as follows.
We present the relation between CFOs and flexible intervals of repetition parts in the extended preamble used to overcome the lack of the available information. We suggest the feasibility of the perfect CFO estimation from the relation that makes the phase difference between repetition parts undistorted.

We propose the CFO estimation algorithm to be able to perfectly estimate CFO corresponding to the relation by using the AC with the flexible intervals. By adopting the normalized squared error (NSE) evaluating the candidates of the CFO estimate, the proposed algorithm can determine the final CFO estimate for the fractional CFO range by comparing the NSEs.

**Notations:** For a complex number \( x \), \( x_R \) and \( x_I \) denote the real and imaginary parts of \( x \), and \( \angle x \) denotes the phase of \( x \). For a real number \( x \), the sign function is slightly modified and defined as \( S(x) = \{ 1, x \geq 0 \}, \text{or} -\frac{1}{2}, x < 0 \). For a real number \( x \) and a positive real number \( y \), \( (x)_y \) denotes the limited value \( x \in [-y, y] \). For a set \( X \), \( |X| \) denotes the cardinality of \( X \). \( Z \) is the set of integer number. \( Z_+ \) is the set of positive integer number. \( \mathbb{E}\{ \cdot \} \) is the expectation operation.

**II. PRELIMINARIES**

**A. SIGNAL MODEL**

The received signal distorted by CFO in OFDM system is written by

\[
y[n] = e^{j2\pi n/N} \sum_{i=0}^{L-1} h[i]x[n-i] + w[n].
\]

(1)

where \( -N_{cp} \leq n \leq N - 1 \). \( x[n] \) is the \( n \)-th sample of OFDM symbol with \( |x[n]|^2 = 1 \), \( h[i] \) is the \( i \)-th tap of multipath fading channel with the length \( L \), \( w[n] \) is additive white Gaussian noise (AWGN) with zero mean and variance \( \sigma_w^2 \). \( N \) is OFDM symbol length, and \( N_{cp} \) is cyclic prefix length. We define the signal-to-noise ratio (SNR) by \( \text{SNR} = 1/\sigma_w^2 \). \( \varepsilon \) is a CFO normalized by subcarrier spacing. For the description, \( y[n] \) is simply represented as

\[
y[n] = e^{j2\pi n/N} y_0[n] + w[n].
\]

(2)

where \( y_0[n] = \sum_{i=0}^{L-1} h[i]x[n-i] \) is the received signal without CFO and AWGN. The quantized received signal is given by

\[
y_{\text{ADC}}[n] = S(y_R[n]) + jS(y_I[n]).
\]

(3)

Fig. 1 shows the block diagram of the OFDM system with 1-bit ADCs for CFO estimation, and \( y_{\text{ADC}}[n] \) is the output of the 1-bit ADCs. We assume the perfect time synchronization.

**B. LIMITATIONS OF CONVENTIONAL CFO ESTIMATION METHODS AND MOTIVATIONS**

Among the conventional CFO estimation methods as mentioned above, we focus on the simple and efficient methods based on AC, which use preambles as reference signals [4]–[6]. Assuming one OFDM symbol containing \( B \) identical repetition parts as the preamble, the samples of \( y[n] \) with the fixed interval \( N/B \) are taken to calculate AC, and then the CFO \( \varepsilon \) in (1) can be estimated from the range of \([-B/2, B/2]\) [4]–[6] as,

\[
\hat{\varepsilon} = \frac{B}{2\pi} \angle \left( \sum_{m=0}^{B-2} \sum_{n=0}^{N-1} y[n + Bm] \cdot e^{j\varepsilon n} \right).
\]

(4)

In the OFDM system with 1-bit ADCs, the \( n \)-th sample of \( y_{\text{ADC}}[n] \) instead of \( y[n] \) is used and represented as the QPSK symbols, so the phase differences between samples of \( y_{\text{ADC}}[n] \) are limited to multiple of \( \pi/2 \) as distorted values. There is limitation in estimating \( \varepsilon \) by CFO estimation methods based on the AC taking the samples of \( y_{\text{ADC}}[n] \) with fixed interval \( N/B \) as (4). However, if the phase differences between the samples of \( y_{\text{ADC}}[n] \) are multiple of \( \pi/2 \) as undistorted values, it is possible to estimate \( \varepsilon \) by using (4). In other words, if we can take the samples of \( y_{\text{ADC}}[n] \) with flexible intervals to make the phase difference caused by \( \varepsilon \) be multiple of \( \pi/2 \) as an undistorted value, \( \varepsilon \) can be estimated by CFO estimation method based on AC.

Let \( E \) be the set of CFOs producing the phase difference of multiple of \( \pi/2 \) as an undistorted value, and \( \varepsilon_i \) be the \( i \)-th element of \( E \). If the phase differences \( \frac{2\pi |\varepsilon_i| n}{N} = \frac{\pi}{2} \) where \( q \in \{1,2\}, \)

\[
|\varepsilon_i| n_i = \frac{qN}{4}.
\]

(5)

This indicates the specific corresponding relation of CFO \( \varepsilon_i \) and interval \( n_i \) denoted by \( (\varepsilon_i, n_i) \) for convenience. For \( \varepsilon_i \), the undistorted phase difference is obtained by taking the samples of \( y_{\text{ADC}}[n] \) with the flexible interval \( n_i \) from the relation of \( (\varepsilon_i, n_i) \). Hence, \( \varepsilon_i \) can be directly estimated by using the conventional CFO estimation method based on the AC with the flexible interval \( n_i \). In (4), \( B/\pi \) is replaced by \( \frac{N}{\pi n_i} \) from \( (\varepsilon_i, n_i) \) to estimate \( \hat{\varepsilon}_i \). To deal with an unknown \( \varepsilon \), we adopt the NSE defined as \( e_i = |(\hat{\varepsilon}_i - \varepsilon_i)/\varepsilon_i|^2 \) where \( \hat{\varepsilon}_i \) is the estimate of \( \varepsilon_i \). Since the NSE \( e_i \) can evaluate the similarity between \( \hat{\varepsilon}_i \) and \( \varepsilon_i \), i.e., the quality of \( \hat{\varepsilon}_i \), we consider the CFO estimation method for the unknown \( \varepsilon \) by utilizing the NSE \( e_i \) as the additional information.

In the following examples, it is shown that there is the limitation of CFO estimation method based on the AC with fixed interval, and the effectiveness of the corresponding

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1Due to the periodicity of the phase, it is possible to represent the valid phase in 1-bit quantization case for \( q = 1 \) and 2. When \( \varepsilon_i > 0 \), the phase difference is \( \pi/2 \) and \( \pi \) for \( q = 1 \) and 2, respectively. When \( \varepsilon_i < 0 \), the phase difference is \( -\pi/2 \) and \( -\pi \) for \( q = 1 \) and 2, respectively. Therefore, for \( q = 1 \) and 2, the all possible phases can be represented.
relation in (5). In addition, the meaning of the NSE for each CFO $\varepsilon_i$ is presented.

EXAMPLE

Assuming $N = 4$, $B = 2$, and $\varepsilon = \frac{1}{6}$, let $y_0 = \{y_0[0], \ldots, y_0[11]\} = \{b, b, b, b, b, b\}$ where $b = [e^{j\pi/3}, e^{j2\pi/3}]$ as the identical repetition part, be the received signal vector without CFO $\varepsilon$. Let $\varepsilon = [\varepsilon_0, \ldots, \varepsilon_{11}]$ where $\varepsilon_i = e^{j\frac{\pi}{N}}$, be the phase rotation vector due to $\varepsilon$. When $y = e \otimes y_0$ where $\otimes$ is the element-wise multiplication, the quantized version of $y$ is $y_{ADC}$.

**III. PROPOSED CFO ESTIMATION**

In the OFDM systems with 1-bit ADCs, we employ the CFO estimation methods based on AC using a preamble. However, since the available information is limited to the quantized received signal, we use the extended preamble to take the samples with the flexible intervals. Then, $E$ is specifically defined from the extended preamble by using the corresponding relation $(\varepsilon_i, n_i)$.

In order to estimate not only $\varepsilon_i \in E$ but also the unknown $\varepsilon$, we propose the CFO estimation algorithm adopting the NSE. In consideration of the inherent convexity of the NSE\(^2\) and the periodicity of phase, $\varepsilon_i$ are properly compared to estimate the unknown $\varepsilon$.

**A. TRAINING SYMBOL**

As shown in Fig. 3, the extended preamble of length $N_T$ contains multiple OFDM symbols of length $N$, which consist of the repetition parts. Each OFDM symbol includes $B$ identical repetition parts, and each repetition part denoted by $A$ is the basic unit of length $\frac{N}{B}$. $n_i$ is the flexible interval between repetition parts for AC, and multiple of $\frac{N}{B}$. $W$ is the size of the window for AC. The maximum value of $n_i$ is denoted by $n_{max}$. For example, in Fig. 3, the two grayed repetition parts of the length $W$ are the interval as $n_i$ apart, and used for AC to obtain a phase difference. For AC, the windows of size $W$ slide while maintaining the interval $n_i$, and sequentially cover all the repetition parts.

\[ E = \{e_1, e_2, \ldots, e_{2K}\} = \{B, B/4, \ldots, B/4(k-1), -B/4, \ldots, -B, -B/2\}. \]  

Specifically, $e_i$ is constructed for the cases of $q = 1$ and $q = 2$ in (5) as follows.

- $|e_i|n_i = \frac{N}{4}$ for $q = 1$ in (5)
- For $2 \leq i \leq K$, $e_i = \frac{B}{4(k-1)}$ from $n_i = \frac{N}{2}(i-1)$

\(^2\)The details are explained in 1) Convexity of NSE at the end of this section.
– For \( K + 1 \leq i \leq 2K - 1 \), \( \varepsilon_i = -\frac{B}{N} i \) from
\[ n_i = \frac{N}{2}(2K - i) \]
- \( |\varepsilon_i| n_i = \frac{N}{2} \) for \( q = 2 \) in (5)
- For \( i = 1 \), \( \varepsilon_1 = \frac{B}{N} \) from \( n_1 = \frac{N}{2} \)
- For \( i = 2K \), \( \varepsilon_{2K} = -\frac{B}{N} \) from \( n_{2K} = \frac{N}{2} \)

\( \varepsilon_1 \) and \( \varepsilon_{2K} \) correspond to the boundaries of the CFO range of \([-\frac{B}{N}, \frac{B}{N}]\).

To construct the repetition part \( A \) of length \( \frac{N}{2} \), the complex Gaussian random vector of length \( \frac{N}{2} \) is generated, and then normalized to make the 2-norm of the vector be \( \sqrt{\frac{N}{2}} \). Also, to generate the OFDM symbols contained in the extended preamble, the pilots can be obtained from the output of FFT of one OFDM symbol with \( B \) identical repetition parts, and then nonzero pilots appear at the subcarrier positions of \( 0, B, 2B, \ldots, N - B \) as the comb-type. The extended preamble is constructed by concatenating \( \frac{N}{2} \) OFDM symbols, and then the CP consisting of multiple of the repetition parts is attached to the extended preamble.

**B. PROPOSED CFO ESTIMATION METHODS**

According to \( E \), the bank of the CFO estimators based on AC with the flexible intervals is represented by

\[
\hat{\varepsilon}_i = \frac{2|\varepsilon_i|}{\pi} \sum_{m=0}^{(N_T-n_i)\frac{B}{2}-1} \sum_{n=0}^{W-1} y_{\text{ADC}}[n + \frac{N}{N} m + n_i] 
\times y_{\text{ADC}}[n + \frac{N}{N} m],
\]

where \( i = 1, \ldots, |E| \), and \( \frac{2N}{\pi} \) replaces \( \frac{B}{\pi} \) in (4) for \( (\varepsilon_i, n_i) \) for \( q = 1 \). The intervals \( n_i \) of the repetition parts can be flexibly selected to obtain the phase differences needed to estimate \( \varepsilon_i \).

When each term of AC
\[ y_{\text{ADC}}[n + \frac{N}{N} m + n_i] \]
achieves \( \varepsilon_i \) for \( \varepsilon_i > 0 \), the \( i \)-th estimator in (7) results in \( \hat{\varepsilon}_i = \varepsilon_i \) as the best performance. The conventional CFO estimator in (4) is equivalent to the CFO estimator corresponding to \((\varepsilon_1, n_1)\) or \((\varepsilon_{2K}, n_{2K})\).

**Lemma 1:** For \( \varepsilon_i \in E \), the CFO estimator corresponding to \((\varepsilon_i, n_i)\) achieves NSE \( \varepsilon_i = 0 \) at high SNR.

**Proof:** If
\[ y_{\text{ADC}}[n + \frac{N}{N} m + n_i] \]
achieves \( \varepsilon_i \) for \( \varepsilon_i > 0 \), the CFO estimator corresponding to \((\varepsilon_i, n_i)\) results in \( \hat{\varepsilon}_i = \varepsilon_i \), and then \( \varepsilon_i = 0 \) is achieved.

Hence, assuming \( \sigma_w = 0 \), we will show that
\[ y_{\text{ADC}}[n + \frac{N}{N} m + n_i] \]
achieves \( \varepsilon_i \) for \( \varepsilon_i > 0 \).

For the description, (2) is simply written by
\[ y[n] = y_{\text{ADC}}[n] + w[n] \]
where \( y_{\text{ADC}}[n] = e^{j2\pi f_s n N} y_0 \).

By replacing the index of \( y[n] \)
\[ y[l + n_i] = y_{\text{ADC}}[l + n_i] + w[l + n_i] \]
achieves \( \varepsilon_i \) due to the repetition parts.

When \( \sigma_w = 0 \), the quantized signal \( y[l] \) and \( y[l + n_i] \) are represented by
\[ y_{\text{ADC}}[l + n_i] = S(\varepsilon_i, n_i l) \]
and
\[ y_{\text{ADC}}[l + n_i] = S(\varepsilon_i, n_i l) \]
Thus, \( \hat{\varepsilon}_k = \varepsilon_k \) for \( \varepsilon_k > 0 \).

Now, for \( \varepsilon_k < 0 \), the valid phase of \( \frac{2\pi e_{\text{ADC}}}{N} \) can become \( \frac{\pi}{2} \) and \( \pi \), and then the valid phase of \( \frac{2\pi e_{\text{ADC}}}{N} \) is \( \pm \frac{\pi}{2} \). The CFO estimator corresponding to \( (\varepsilon_k, n_k) \) can also produce \( \hat{\varepsilon}_k = \varepsilon_k \) for \( \varepsilon_k < 0 \). Hence, there can exist the ambiguity of the CFO estimation for unknown \( \varepsilon \in E \) by using the CFO estimation method comparing with the NSES.

**Lemma 2:** For \( \varepsilon_i \in E \), if \( n_k = (2z + 1) n_i \) where \( z \in \mathbb{Z}_+ \), then the CFO estimator corresponding to \((\varepsilon_k, n_k)\) achieves \( \varepsilon_k = 0 \) at high SNR.

**Proof:** When \( \sigma_w = 0 \), if \( n_k = (2z + 1) n_i \) where \( z \in \mathbb{Z}_+ \), then
\[ \frac{2\pi e_{\text{ADC}}}{N} = \frac{\pi}{2} + \pi \] for \( \varepsilon_k > 0 \) and
\[ \frac{2\pi e_{\text{ADC}}}{N} = -\frac{\pi}{2} - \pi \] for \( \varepsilon_k < 0 \).

First, when \( z = 2z_0 \), where \( z_0 \in \mathbb{Z}_+ \) for \( \varepsilon_k > 0 \) the valid phase of \( \frac{\pi}{2} + 2z_0 \pi \) is \( \frac{\pi}{2} \). Thus, the CFO estimator corresponding to \((\varepsilon_k, n_k)\) results in \( \hat{\varepsilon}_k = \frac{2\pi e_{\text{ADC}}}{N} = \frac{\pi}{2} \), so \( \hat{\varepsilon}_k = \varepsilon_k \) for \( \varepsilon_k > 0 \).

Second, when \( z = 2z_1 \) where \( z_1 \in \mathbb{Z}_+ \), for \( \varepsilon_k > 0 \) the valid phase of \( \frac{\pi}{2} + (2z_1 - 1) \pi \) is \( -\frac{\pi}{2} \). Thus, the CFO estimator corresponding to \((\varepsilon_k, n_k)\) results in \( \hat{\varepsilon}_k = \frac{2\pi e_{\text{ADC}}}{N} = -\frac{\pi}{2} \), so \( \hat{\varepsilon}_k = \varepsilon_k \) for \( \varepsilon_k < 0 \).

Therefore, if \( n_k = (2z_1 + 1) n_i \), the CFO estimator corresponding to \((\varepsilon_k, n_k)\) results in \( \hat{\varepsilon}_k = \varepsilon_k \), and then achieves \( \varepsilon_k = 0 \) for \( \varepsilon_k \in E \) at high SNR.
When the ambiguity of the CFO estimation happens for the maximum interval \( n_{\text{max}} \), \( \hat{e} \) should be satisfied with 
\[
\frac{2\pi |e_i| n_{\text{max}}}{N} \geq \frac{\pi}{2} + \pi \text{ at least, i.e., } |e_i| \geq \frac{3N}{4n_{\text{max}}}. 
\]
Thus, Lemma 2 shows that the ambiguity of CFO estimation is unavoidable for \( |e_i| \geq \frac{3N}{4} \). However, taking advantage of the condition on the occurrence of the ambiguity, we can deal with the uncertainty of CFO estimation for \( e \in \mathcal{E} \).

**Theorem 1:** For \( e \in \mathcal{E} \), if \( |e_i| > |e_k| \) and \( e_i = e_k = 0 \), \( e \) is estimated by determining \( \hat{e} = \hat{e}_i \) using the bank of the CFO estimators in (7) at high SNR.

**Proof:** Assuming \( \hat{e} \neq \hat{e}_i \), for the sake of contradiction, \( \hat{e} = \hat{e}_k \) should be satisfied due to \( e_k = 0 \), and then implies \( |e_k| > |e_i| \) due to \( e_1 = 0 \). However, \( |e_k| > |e_i| \) contradicts the assumption as \( |e_i| > |e_k| \). Therefore, if \( |e_i| > |e_k| \) and \( e_i = e_k = 0 \), then \( \hat{e} = \hat{e}_i \) by using the bank of the CFO estimators in (7).

From the Theorem 1, we can estimate the unknown \( e \in \mathcal{E} \) regardless of the ambiguity of the CFO estimation at high SNR. When \( \sigma_e \neq 0 \), for \( e_i \in \mathcal{E} \), \( e_i \) generally has a small enough value as a local minimum rather than zero. Thus, we search local minimum points of \( e_i \) and determine \( \hat{e} \) according to the Theorem 1. In addition, for \( e \in \mathcal{E} \), the CFO estimation based on the NSE is equivalent to select the most suitable CFO estimator from the bank of the CFO estimators.

Finally, although we can try to estimate the unknown \( e \notin \mathcal{E} \), there is no a CFO estimator corresponding to \( e \). However, since the elements of \( \mathcal{E} \) can cover the CFO range of \([-\frac{B}{2}, \frac{B}{2}] \] densely enough, it is feasible to estimate \( e \) by utilizing the bank of the CFO estimators in (7). Therefore, we propose the CFO estimation algorithm based on the NSE, which is depicted as the block diagram in Fig. 4 and summarized in Algorithm 1.

Considering the CFO range of interest, e.g., \([-0.5, 0.5] \) as the fractional CFO, we define the threshold of \( e_i \) as \( e_{\text{thr}} = |(e_{i_{\text{th}}}-e_{i_{\text{th+1}}})/e_{i_{\text{th}}}|^2 \) where \( e_{i_{\text{th}}} \) is the maximum value among the elements of \( \mathcal{E} \) belonging to the CFO range of interest, and \( e_{i_{\text{th+1}}} \) is the second maximum value. If \( i_1 \) is out of the CFO range of interest, \( \hat{e}_i \) is not selected as \( \hat{e} \). In step 1 and 2, the CFO estimator corresponding to \( (e_i, n_i) \) in (7) yields \( \hat{e}_i \), and then \( e_i \) are calculated for \( i = 1, \ldots, |\mathcal{E}| \). In step 3, the local minimum points of \( e_i \) are searched through comparison with three consecutive values, and \( \mathcal{I} = \{i_1, i_2, \ldots\} \) is defined as the set of indices corresponding to the local minimum points. In step 4, the indices satisfying \( e_{i_{k+1}} > e_{i_k} \) for \( i_k \in \mathcal{I} \) are picked out, and then excluded from \( \mathcal{I} \). In the remaining steps, \( \hat{e} \) is determined according to \( |\mathcal{I}| \).

Additionally, the compensation of the phase rotation caused by the CFO is performed by applying \( e^{-j2\pi \hat{e} n_i/N} \) in the data path of the OFDM receiver in Fig. 1.

1) **CONVEXITY OF NSE**

We consider the repetition parts of length \( N \) in Section III-A, and the phase difference between the repetition parts of the interval \( n_{i+k} \). The phase difference \( \frac{2\pi |e_{i_{n_{i+k}}}| N}{N} = \frac{2\pi |e_{i_{n_{i}}}| n_{i_k}}{N} \) for \( |e_i| > 0 \) becomes \( \frac{\pi}{2} + \frac{\pi}{2} = k \) for \( 2 \leq i + k \leq K \). \( \frac{\pi}{2} \) is the portion of \( \frac{2\pi |e_{i_{n_{i_k}}}| N}{N} \), and \( \frac{\pi}{2} k \) corresponds to the additional phase, which is valid on its own in quantization case. Since \( n_{i+k} = n_{2K+1-i-k} \), the phase difference \( \frac{2\pi |e_{i_{n_{i+k}}}| N}{N} \) is also \( \frac{\pi}{2} + \frac{\pi}{2} k \) due to the symmetry of \( n_i \).

However, in quantization case, since each repetition part contains \( N \) QPSK symbols, the additional phase is not \( \frac{\pi}{2} k \) as nominal value. Thus, the additional phase is dependent on the sample pairs of the interval \( n_{i+k} \) within repetition parts whose phase difference is not \( \frac{\pi}{2} k \). The number of the sample pairs is denoted by \( c_{i,k} \) and determined by the inequality

\[
c_{i,k} \leq \frac{N}{2} k \quad \text{i.e., } c_{i,k} = \lfloor \frac{N}{2} k \rfloor, \quad \text{where} \quad \frac{N}{2} k \text{ means the effective phase difference of two adjacent elements of each repetition part. When} \quad m < |k| \quad \text{and} \quad m \in \mathbb{Z}, \quad \text{the average phase difference between repetition parts with the interval} \quad n_{i+k} \quad \text{is} \quad \frac{\pi}{2} + \frac{\pi}{2} (c_{i,k} - \frac{N}{2} m) + \frac{\pi m}{2} (\frac{N}{2} (m+1) - c_{i,k}), \quad \text{where} \quad c_{i,k} \text{ is limited to} [-\pi, \pi] \quad \text{due to the periodicity of the phase, the average phase difference is represented by} \quad \left(\frac{\pi}{2} + \frac{\pi}{2} c_{i,k}\right)_m \text{to specify the limitation of the}.

![FIGURE 4. The block diagram of the CFO estimation in the OFDM system.](image-url)
phase, and \((1 + \frac{B}{N}c_{1,k})\) is also represented by \((1 + \frac{B}{N}c_{1,k})^2\). Then, the estimate and the NSE of \(e_{i+k}\) are obtained as \(\hat{e}_{i+k} = e_{i+k}((1 + \frac{B}{N}c_{1,k})^2)\) and \(e_{i+k} = |(1 + \frac{B}{N}c_{1,k})^2 - 1|^2\), respectively. In addition, \((1 + \frac{B}{N}c_{1,k})\) is monotonic increasing with regard to \(k\), and thus \((1 + \frac{B}{N}c_{1,k})^2\) is the sawtooth pattern with regard to \(k\). Therefore, \((1 + \frac{B}{N}c_{1,k})^2 - 1\) is also the sawtooth pattern with regard to \(k\), so there exist the local minima at cusps, which are corresponding to the points with the ambiguity of the CFO estimation in Lemma 2. Thus, \(e_{i+k}\) has also the local minima at the same points. Therefore, \(e_{i+k}\) is not only convex around the points of local minima, but also cyclic with regard to \(k\).

For \(\epsilon \notin \mathcal{E}\) and \(\epsilon > 0\), assuming \(e_i\) is most similar with \(\epsilon\),
\[
\frac{2N_{\text{envelope}}}{N} = \frac{\epsilon}{\epsilon_i \left(\frac{\pi}{2} + \frac{\pi}{2} \frac{k}{N}ight)}.
\]

In the same manner as above, the average phase difference is \((\hat{\epsilon}_i \left(\frac{\pi}{2} + \frac{\pi}{2} \frac{k}{N}\right))\), so \(\hat{\epsilon}_{i+k} = e_{i+k}((1 + \frac{B}{N}c_{1,k})^2)\) and \(e_{i+k} = |(1 + \frac{B}{N}c_{1,k})^2 - 1|^2\). Therefore, \(e_{i+k}\) is also convex and cyclic with regard to \(k\).

2) COMPUTATIONAL COMPLEXITY

Both the conventional and the proposed CFO estimation methods are based on AC to calculate the phase difference, and the proposed method contains the additional steps in Algorithm 1 to determine the estimate of CFO \(\hat{\epsilon}\).

First, to compare the computational complexity of the AC, we consider the number of complex multiplications. In the conventional method, the \(W\) complex multiplications are repeatedly performed \(\frac{B}{N}(N_T - W)\) times. In the proposed method, the \(W\) complex multiplications are repeatedly performed \(\frac{B}{N}(N_T - n_i)\) times for the flexible interval \(n_i\), and are summed as \(\sum_{i=1}^{\mathcal{E}} \frac{B}{N}(N_T - n_i)\) for all flexible intervals. For \(|\mathcal{E}| = \frac{2N_TB}{N}\), the number of the complex multiplications is simply derived, and the results are listed in Table 1.

As a result, for the AC, the conventional and the proposed methods require the computational complexity of \(O(N_T)\) and \(O(N_T^2)\), respectively.

### TABLE 1. The number of complex multiplications for the AC in both the CFO estimation methods.

|                   | Conventional method                                   | Proposed method                              |
|-------------------|-------------------------------------------------------|----------------------------------------------|
|                   | \(\frac{B}{N}(N_T - W)\)                             | \(\frac{B}{N}(\frac{N_T^2}{N} + \frac{N_T}{2}N_T - 2\frac{N_T^2}{N_T})\) |

### TABLE 2. The computational complexity of Algorithm 1 except for AC.

| Type of computation | Number of repetitions |
|---------------------|-----------------------|
| Calculation of NSEs | \(|\mathcal{E}|\)       |
| Search for local minima of \(e_i\) | \(|\mathcal{E}| - 2\) |
| Comparison between local minima and threshold \(c_0\) | \(|Z|\) |
| Comparison among candidates of CFO estimate | \(|Z|\) or \(|\mathcal{E}|\) |

Moreover, the proposed method requires the additional computational complexity for the steps in Algorithm 1, which listed in Table 2. Each step requires simple operations, which performed repeatedly. For example, the calculation of NSE \(e_i\) needs 1 real subtraction, 1 real division, and 1 real multiplication, which are repeated \(|\mathcal{E}|\) times. The other steps also require very simple operations, which performed repeatedly on \(|\mathcal{E}|\) scale at most. As a result, the steps except for the computation of AC in Algorithm 1 can be handled as the combinations of simple operations, and require the computational complexity of \(O(N_T)\).

Therefore, the conventional and the proposed methods require the computational complexity of \(O(N_T)\) and \(O(N_T^2)\), respectively.

### IV. SIMULATION RESULTS

We verify the performances of the CFO estimation algorithm based on the NSE in the OFDM systems with 1-bit ADCs over the multipath fading channel. Five paths are selected with path delays of 0, 2, 8, 16, 30 dB. The MSEs are evaluated by \(E[|\hat{\epsilon} - \epsilon|^2]\). Fig. 5 shows the mean squared errors (MSEs) of Algorithm 1 and conventional method (denoted by Alg. and Conv.) according to the number of the OFDM symbols in the preamble, which is equivalent to the preamble length \(N_T\), at SNR = 30 dB. The MSEs are evaluated by \(E[|\hat{\epsilon} - \epsilon|^2]\). As \(N_T\) increasing, the MSEs tend to be enhanced, and then converge. This means that there is limitation in strictly measuring the phase difference for \(\epsilon \notin \mathcal{E}\) for even large \(N_T\) due to 1-bit quantization. However, the MSEs of Algorithm 1 are superior to that of the conventional method, since Algorithm 1 can select the CFO estimator more suitable for \(\epsilon\) from the bank of the CFO estimators. We observe that the MSEs are slightly improved for about \(N_T \geq 8N\) for all cases, and

![FIGURE 5. The MSEs of CFO estimation according to the number of OFDM symbols in the preamble for SNR = 30 dB.](image-url)
even for $B = 16$ of Algorithm 1 the decrement of MSE for the increment of OFDM symbol is less than 10%. Thus, the preamble length is selected as $N_T = 8N$ for the following simulations.

In Fig. 6, the MSEs are observed to be improved as SNR increasing, and then converge at high SNR. This means that there is limitation in strictly measuring the phase difference for $\varepsilon \notin \mathcal{E}$ at even high SNR due to 1-bit quantization. In the case of $B = 16$, the MSE of Algorithm 1 is 22.2 times smaller than that of conventional method at SNR = 30 dB. In the cases of conventional method and Algorithm 1 of $B = 2$ and $B = 4$, the MSEs at the SNR region around SNR = 8 dB are somewhat lower than the saturated MSEs at SNR = 30. This is because the AWGN noise can aid estimating CFO by varying slightly the patterns of the repetition parts favorably.

MSEs appear contiguously in zigzags. The downward peaks correspond to the elements of $\mathcal{E}$, i.e. these peak points can ideally achieve $\varepsilon_i = 0$ for $\varepsilon_i$ by using the CFO estimator oriented to $(\varepsilon_i, n_i)$. The peaks become denser as $B$ increasing, since the elements of $\mathcal{E}$ are defined more densely. In the case of $B = 16$ for Algorithm 1, the peak points correspond to $[0.50, 0.44, 0.40, 0.36, \ldots]$ as the elements of $\mathcal{E}$. Likewise, the upward peaks occur due to the inherent differences between $\varepsilon \notin \mathcal{E}$ and the elements of $\mathcal{E}$. Since the inherent differences becomes smaller as the CFO getting near zero, the value of the upward peaks become smaller. Unusually, the two centered upward peaks happen from the fact that it is hard to estimate the CFO near zero not included in $\mathcal{E}$. Compared with the conventional method, the Algorithm 1 has the better MSE performances except near the two centered peaks.

In the conventional method, the downward peaks happen at $\varepsilon = \pm0.5$ due to the CFO estimators corresponding to $(0.5, \frac{N}{2})$. The MSE is better near $\varepsilon = \pm0.25$, which correspond to $\pm\frac{\pi}{4}$ that can be expressed as the accurate phase difference relatively, even if distorted by quantization.

Fig. 8 shows the bit error rates (BERs) of the coded OFDM systems. The Turbo code with code rate $\frac{1}{3}$ [2] for 80 data bits and BPSK modulation are applied, and log-MAP algorithm is used for the decoding. Assuming the perfect channel, the zero-forcing equalization is applied. To evaluate the BER of the OFDM systems, we consider the frame consisting of the preamble part and the data part. The preamble of length $N_T = 8N$ and $B = 16$ is followed by the zero guard interval of length 16 and the OFDM symbols for the data in serial order. Compared with the BER of the OFDM systems with full precision ADCs (denoted by Full precision), the BER of the OFDM system with 1-bit ADCs (denoted by 1-bit w/o CFO) is degraded due to the distortion from 1-bit quantization. When the CFO is estimated and compensated, the BERs of the OFDM system is additionally deteriorated due the CFO.
estimation error. The BER of the Algorithm 1 (denoted by 1-bit w/ Alg.) is superior to that of the conventional method (1-bit w/ Conv.). In the case of 1-bit w/ Conv., the data are not recovered. It is remarkable that the BERs are degraded for the cases of 1bit w/o CFO and 1-bit w/ Alg. at high SNR region. This is because the SNR = 1/σ^2 becomes more inaccurate as the distortion due to 1-bit quantization becomes more dominant at high SNR region.

V. CONCLUSIONS

To obtain the accurate CFO, we presented the CFO estimation algorithm in the OFDM systems with 1-bit ADCs over the multipath fading channel. We suggest the related pair for (ε, n) based on the structure of the extended preamble, and the feasibility of the perfect CFO estimation. The bank containing the CFO estimators corresponding to (ε, n) produces the candidates of CFO estimate through the AC with flexible intervals. By comparing the NSEs adopted to check the quality of the candidates of the CFO estimate, the proposed algorithm can estimate the CFO with sufficiently high accuracy. The simulation results show that the proposed algorithm is superior to the conventional method, and has the MSEs dependent on the CFO values.

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