The Dead Cryptographers Society Problem

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Abstract. This paper defines a new problem, The Dead Cryptographers Society Problem – DCS (where several great cryptographers created many polynomial-time Deterministic Turing Machines (DTMs), ran them on their proper descriptions concatenated with some arbitrary strings, deleted them and leaved only the results from those running, after they dyed: if those DTMs only permute the bits on input, is it possible to decide the language formed by such resulting strings within polynomial time?), proves some facts about its computational complexity, and discusses some possible uses on Cryptography, such as into distance keys distribution, online reverse auction and secure communication.

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1. Introduction

In [1], I have promised some practical applications for some new concepts over there introduced to the Theoretical Computer Science community.

In this little paper, I hope I begin to pay that promise.

2. The Dead Cryptographers Society Problem

Definition 2.1. Permuting DTM. A DTM (deterministic Turing Machine, or an algorithm or computer program with unbounded memory) \( M \) is a permuting DTM iff its only function is to take a [finite but unbounded] string of \( n \) input bits and produce from them a string of \( n \) output bits that forms a permutation of the input: \( M(b_1b_2b_3...b_n) = b_{\sigma(1)}b_{\sigma(2)}b_{\sigma(3)}...b_{\sigma(n)} \), where each \( b_i \) is an individual bit, \( 1 \leq \sigma(i) \leq n \), and \( \sigma(i) = \sigma(j) \iff i = j. \)
Definition 2.2. The Dead Cryptographers Society Problem (DCS). The DCS is a formal language over the alphabet $\Sigma = \{0, 1\}$ formed by all words (or binary strings) $w$ generated by some polynomial-time permuting DTM $M$ running on its proper binary description or code $\langle M \rangle$ concatenated with some arbitrary binary string $s$: $w = M(\langle M, s \rangle)$.

Theorem 2.1. The DCS is in NP.

Proof. See that the DCS is in NP \cite{2}, evidently, since if $w \in \text{DCS}$, then it is verifiable within polynomial time whether or not a [guessed] poly-time DTM $M$ on the [guessed] input $\langle M, s \rangle$ writes $w$ on its tape and then halts (where $|w| = |\langle M, s \rangle|$): it suffices to simulate the running of that DTM $M$ on $\langle M, s \rangle$ and then to check the contents of the tape after the simulation has finished, in order to see whether it is equal to $w$. Note that simulating the running of any polynomial-time DTM on an arbitrary input can be done within polynomial time too (as a polynomial is a time-constructible function \cite{3}).

Theorem 2.2. The DCS is not in P.

Proof. Can the DCS is in P? No, at all, plainly, since there is no polynomial that bounds the running time of the all polynomial-time permuting DTM (and the polynomial that bounds the running time of a specific one is not known neither given for us), and then it is impossible to decide or to reduce the DCS into another NP problem within deterministic polynomial time. See \cite{1} and \cite{2} for details of the demonstration of this amazing fact, and the Sections 2.1 and 2.2 below, for some simple examples of possible applications.

2.1 Online reverse auction mathematically proven poly-unbreakable

We can now construct a protocol for online [virtual] reverse auction \cite{4} mathematically proven unbreakable into polynomial time, by the Theorem 2.2:

It suffices the bidder at the reverse auction to construct a permuting DTM $M$, to run it on $\langle M, b \rangle$, where $b$ is a string representing its bid (into some codification form or standard ruled by the auctioneer), to concatenate the output with $H(\langle M, M' \rangle)$ (where $M'$ is the inverse machine of that $M$ ($M'(M(x)) = x$) and $H$ is a cryptographic hash function \cite{6} ruled by the auctioneer too, whom provides it to all the bidders), to store the generated string $w$ in a trusted third, sending it to the reverse auctioneer too, and then to hide the DTMs $M$ and $M'$ from the external world, which assures the confidentiality of that bid.

After, in a virtual public session, all the bidders present to the reverse auctioneer their respective DTMs $M$ and $M'$ in order to recuperate their bids (the winner is who has made the lowest one), where obviously there cannot be fraud, because running that $M$ on $M'(M(\langle M, b \rangle))$ and concatenating the output with $H(\langle M, M' \rangle)$ must result in that same string $w$ sent to the auctioneer, and any other input must result in another [different] output $w' \neq w$, since $M$ and $M'$ must be permuting [bijective] machines. Remember yet that $M(\langle M, b \rangle)$ is part of $w$, thus it must be right. Finally, in case of doubts that trusted third can guarantee the authenticity of $w$.

2.2 Distance keys distribution mathematically proven poly-unbreakable

Definition 2.2.1. n-permuting DTMs set (n-p-DTM-set). An $n$-p-DTM-set is a set of $n$ permuting DTMs $M_1, M_2, \ldots, M_n$, where $M_i(M_{i-1}(\ldots(M_1(\langle M_1, x \rangle))\ldots)) = \langle M_i, x \rangle$, that is, the result from running $M_1$ on $\langle M_1, x \rangle$, then running $M_2$ on that result, and so on, until $M_n$, is that same string $\langle M_1, x \rangle$. Notice that a 1-p-DTM-set is an identity DTM $I$ ($I(w) = w$).
With the definition above, we can now also construct a system of distance keys distribution \(^5\) mathematically proven unbreakable into polynomial time, by the Theorem 2.2:

It is enough to utilize a 4-p-DTM-set where the user A creates a key k and then sends \(k' = M_1(\langle M_1, k \rangle)\) to the user B, that returns \(k'' = M_2(k')\) to A. Then returns \(k''' = M_3(k'')\) to B, that recuperates that original \(\langle M_1, k \rangle = M_4(k''')\) and then \(k = \text{right}(\langle M_1, k \rangle; |\langle M_1, k \rangle|-|\langle M_1 \rangle|)\) (\(k\) is the string formed by the \(|k|\) most right positions from \(\langle M_1, k \rangle\)). See that B can eventually check whether or not the messages are authentic, for the DTM \(M_i\) from the 4-p-DTM-set is sent encoded into each message and it is recovered at final.

Observe that the user A does not need to have neither to know \(M_2\) and \(M_4\), and B does not need to have neither to know \(M_1\) and \(M_3\).

Verify that eavesdropping those exchange messages with \(k', k''\) or \(k'''\) does not allow to recover that original key \(k\), since these strings are just permutations of \(M_1\) concatenated with \(k\), and the 4-permuting machines set used in that exchanging of messages is utterly hidden from everyone else besides A and B.

See that this method can be used to send any arbitrary string from A to B, or from B to A, even other 4-p-DTM-set, in order to avoid utilizing the same one for many times or during a large term, which would be very harmful to the safety of that protocol.

Note yet that we can construct a very similar protocol with more general DTMs, not just permuting ones, even with non-length-preserving DTMs (where \(|\text{input}| \neq |\text{output}|\)).

### 2.2.1 Example of a 4-p-DTM-set

Let \(f : \{0,1\}^4 \rightarrow \{0,1\}^4\) be a permuting function where:

| Position of the bit into \(x = i\) | \(\sigma(i) = \text{Position of that bit into } f(x) = 2^i \text{mod } 5\) |
|---------------------------------|---------------------------------|
| 1                              | 2                               |
| 2                              | 4                               |
| 3                              | 1                               |
| 4                              | 3                               |

Verify that \(f(f(f(x)))) = x\), hence four equal DTMs \(M\) that split the [e.g. ASCII coded] message into pieces or parts of four bits, apply repeatedly that function on each part, and concatenate that outputs, form a 4-p-DTM-set, as a consequence of \(2^4 = 1\ (\text{mod } 5)\).

In order to illustrate the functioning of this 4-p-DTM-set, we can use it on a little and simple plain message:

| Plain Text | M | A | T | H |
|------------|---|---|---|---|
| **Parts**  | **Part_1** | **Part_2** | **Part_3** | **Part_4** | **Part_5** | **Part_6** | **Part_7** | **Part_8** |
| **Bit Pos.** | 1 2 3 4 | 1 2 3 4 | 1 2 3 4 | 1 2 3 4 | 1 2 3 4 | 1 2 3 4 | 1 2 3 4 | 1 2 3 4 |
| **Bits**   | 0 1 0 0 | 0 1 0 0 | 0 1 0 0 | 0 1 0 0 | 0 1 0 0 | 0 1 0 0 | 0 1 0 0 | 0 1 0 0 |
| **s_i=f(s_j)** | 0 0 0 1 | 0 0 0 1 | 0 0 0 1 | 0 0 0 1 | 0 0 0 1 | 0 0 0 1 | 0 0 0 1 | 0 0 0 1 |
| **Text**  | + | - | 1 | - | 1 | - | 1 | - |
| **s_i=f(s_j)** | 1 0 0 1 | 1 0 0 1 | 1 0 0 1 | 1 0 0 1 | 1 0 0 1 | 1 0 0 1 | 1 0 0 1 | 1 0 0 1 |
| **Text**  | A | - | A | - | A | - | A | - |
| **s_i=f(s_j)** | 0 1 0 0 | 0 1 0 0 | 0 1 0 0 | 0 1 0 0 | 0 1 0 0 | 0 1 0 0 | 0 1 0 0 | 0 1 0 0 |
| **Text**  | M | A | T | H |
Note that replacing above those \( \{0,1\}^t \) by \( \{0,1\}^{p-1} \) (where \( p \) is any odd prime number), that formula \( 2^i \mod 5 \) by \( k^i \mod p \), where \( k \) is not multiple of \( p \), and "groups of four bits" by "groups of \( p-1 \) bits", we can construct in a similar manner an \( \text{ord}_p(k) \)-p-\text{DTM-set}, because \( k^{\text{ord}_p(k)} \equiv 1 \mod p \), that is, the bits return to their original positions after \( \text{ord}_p(k) \) \( M \) runnings, where \( \text{ord}_p(k) \) is the multiplicative order of \( k \) modulo \( p \).[7]

Notice yet that any \( n\)-p-\text{DTM-set} whose construction is publicly known shall be very unsafe for cryptographic purposes, obviously. So, they must be constructed with creativity and originality, if they are intended to be used into some serious cryptographic protocol.

2.3 Secure communication mathematically proven poly-unbreakable

With the protocol introduced in Section 2.2, we can make a secure communication mathematically proven poly-unbreakable, by the Theorem 2.2: it suffices to utilize a \( 2\)-p-\text{DTM-set} where a user \( A \) can send an arbitrary message \( m \) by sending \( m' = M_1(\langle M_1,m \rangle) \) to the user \( B \), that recuperates that original \( \langle M_1,m \rangle = M_2(m') \) and then \( m = \text{right}((\langle M_1,m \rangle; |(\langle M_1,m \rangle)|-|\langle M_1 \rangle|) \) (\( m \) is the string formed by the \( |m| \) most right positions from \( \langle M_1,m \rangle \)).

As in Section 2.2, \( B \) can eventually check the authenticity of the messages and this method can even be used to a user send other \( 2\)-p-\text{DTM-set} to another one, in order to avoid using the same DTMs repeatedly, by safety reasons. If they want it, the user \( A \) can yet send only \( m \) to \( B \), without concatenating with \( \langle M_1 \rangle \) neither with something else at all, or concatenating it with any other arbitrary string.

3. Conclusion

The conclusion is that maybe we should not summarily disregard new ideas into Theoretical Computer Science as weird and irrelevant ones, because they can perhaps be useful in unexpected applications upon inventive and creative minds.

4. Freedom & Mathematics

“– The essence of Mathematics is Freedom.” (Georg Cantor)[8]

5. References

[1] A. L. Barbosa, The Cook-Levin Theorem is False, unpublished, available: http://www.andrebarbosa.eti.br/The_Cook-Levin_Theorem_is_False.pdf

[2] A. L. Barbosa, \( P \neq NP \) Proof, unpublished, available: http://arxiv.org/ftp/arxiv/papers/0907/0907.3965.pdf

[3] From Wikipedia, the free encyclopedia, “Constructible Function”, unpublished, available: http://en.wikipedia.org/wiki/Constructible_function

[4] From Wikipedia, the free encyclopedia, “Online Auction”, unpublished, available: http://en.wikipedia.org/wiki/Online_auction

[5] From Wikipedia, the free encyclopedia, “Key Distribution Center”, unpublished,
available: [http://en.wikipedia.org/wiki/Key_distribution_center](http://en.wikipedia.org/wiki/Key_distribution_center)

[6] From Wikipedia, the free encyclopedia, “Cryptographic Hash Function”, unpublished, available: [http://en.wikipedia.org/wiki/Cryptographic_hash_function](http://en.wikipedia.org/wiki/Cryptographic_hash_function)

[7] From Wikipedia, the free encyclopedia, “Multiplicative Order”, unpublished, available: [http://en.wikipedia.org/wiki/Multiplicative_order](http://en.wikipedia.org/wiki/Multiplicative_order)

[8] From The Engines of Our Ingenuity, site, “Episode nº 1484: GEORG CANTOR”, posted by John H. Lienhard, unpublished, available: [http://www.uh.edu/engines/epi1484.htm](http://www.uh.edu/engines/epi1484.htm)

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