POLARIZED DEFORMED NUCLEI STUDIED VIA COINCIDENCE POLARIZED ELECTRON SCATTERING: The case of $^{21}\text{Ne}^*$

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ABSTRACT

Coincidence reactions of the type $\vec{A}(\vec{e},e'N)B$ involving the scattering of polarized electrons from deformed polarized targets are discussed within the context of the plane–wave impulse approximation. A general expression for the polarized spectral function for transitions leaving the residual nucleus in discrete states is presented. General properties and angular symmetries exhibited by the polarization observables are discussed in detail. Results for unpolarized cross sections as well as for polarization ratios (asymmetries) are obtained for typical quasi–free kinematics. The dependences of the polarization observables on the bound neutron momentum, target polarization orientation, nuclear deformation and value of the momentum transfer $q$ are discussed in detail for various different kinematical situations.

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1. INTRODUCTION

This work is based on a study of polarization observables in coincidence electron scattering reactions within the context of the Plane–Wave Impulse Approximation (PWIA) or factorized Distorted–Wave Impulse Approximation (DWIA), where the cross section can be factorized into two basic terms, the electron–nucleon cross section and the spectral function.\(^{[1–4]}\) We shall consider only the PWIA in detail in the present work. The former factor deals directly with the interaction between the incident electrons and the bound nucleons inside the nucleus, while the latter gives us the probability that a nucleon is to be found in the nucleus with given energy and momentum. Although the PWIA is evidently an oversimplification in the description of the reaction mechanism, it gives us a very clear picture of the physics contained in the problem and has proven to be quite useful in studying the single–nucleon content of the nucleus under appropriate kinematical conditions (specifically, high enough values of \(q\) where the scattering process is expected to be “quasi–free” and hence only mildly influenced by final–state interactions and various exchange effects which are usually neglected or, at best, only treated approximately\(^{[1,2]}\)).

The study of the electron–nucleon cross section for polarized incident electron and polarized target has already been presented in detail in Ref. \([3]\). A very important advantage of the PWIA is the ability to treat some of the relativistic aspects of the reaction in a complete way. At high values of the momentum transfer and quasi–free kinematics, the ejected nucleon becomes relativistic and hence any fully non–relativistic treatment of the reaction must be viewed with caution. In this paper we place our focus on the second of the factors above, namely, on the nuclear spectral function. Our approach for this quantity is non–relativistic; however, typically the most important contributions to the cross section come from struck nucleons within the Fermi sea (\(p < p_F \approx 200 – 250\) MeV/c) and consequently non–relativistic approximations for the initial–state structure (and therefore for the spectral function) may be expected to be approximately valid.

As previously stated, the analysis within the context of the PWIA/DWIA of coincidence polarized electron scattering from polarized nuclei leads to factorized expressions for the differential cross sections. These involve the polarized single–nucleon cross section\(^{[3]}\) multiplied by a polarized spectral function. The latter provides the probability of finding a nucleon in the nucleus having given energy and momentum and, importantly, given spin projection. Specifically, we consider in detail the case of particular transitions (to ground and low–lying excited states) involving the polarized nucleus \(^{21}\text{Ne}\). This target constitutes an interesting case, being a deformed nuclear system with a well–established rotational spectrum\(^{[5]}\) and yet is light enough that electron distortion effects can be neglected, at least at this early stage of the theoretical analysis. Of considerable importance for practical experiments is the fact that, as for \(^3\text{He}\), the case of polarized neon involves a noble gas and hence depolarization effects from collisions with target–container walls are minimized, permitting high–density polarized targets to be constructed.

Using the nuclear rotational model of Bohr and Mottelson\(^{[6]}\) to describe polarized neon, the present studies provide a starting point for investigating spin degrees of freedom in coincidence electron scattering from such nuclei; our present interest is focused on providing a guide to where the greatest sensitivities to particular aspects of the polarized nuclear spectral functions are to be found. We present results for asymmetries and/or polarization ratios as well as for totally unpolarized cross sections. In so–doing we stress the potential importance
of studying ratios of observables which turn out to be less sensitive to the nature of the underlying dynamical assumptions made (at least they will be shown to have different sensitivities). We have examined the behaviour of the asymmetries as functions of the struck–nucleon momentum \( p \) as well as versus the angles defining the direction of the target polarization vector for different kinematics. The effects introduced into the different responses by the deformation of the target and its connection with the polarizations, as well as the behaviour of the various responses in regions where relativistic effects in the currents may be expected to play a significant role (\( q = 1 \) GeV), have been also considered and the results are discussed in detail in the present work.

The study of deformed nuclear systems through electron scattering reactions started some years ago.\(^7,8\) In the particular case of inclusive (single–arm) processes, the predictions of various models for the different components of the electromagnetic current of the ground–state band in even–even and odd–\( A \) deformed nuclei in the rare–earth region have been already presented in previous papers.\(^9,10\) These ideas have also been applied to lighter nuclear systems where one can neglect the Coulomb distortion effects which otherwise complicate the analysis;\(^11\) in particular, the case of \( ^{21}\text{Ne} \) has been treated in detail in Ref. \(^{12}\). For coincidence reactions of the type \( A(e, e'N)B \), a great deal of effort has been expended in recent years to measure momentum distributions in nuclei. An interesting question that arises is whether momentum distributions in deformed nuclei may look different from those in the spherical case, \textit{i.e.}, whether nuclear deformation may lead to observable effects in coincidence quasielastic electron scattering reactions. This subject was first studied in the totally unpolarized situation (incident electron and target not polarized) some years ago.\(^8,13–15\) In this work our interest has been to generalize such studies when polarization degrees of freedom are taken into account and to provide a new probe of the deformed, spin–dependent spectral function of rotational nuclei. In the future we intend to explore some natural extensions of the present work — in particular to include treatments beyond the PWIA and to describe polarized coincidence reactions at large inelasticity.

This paper is organized as follows: in Sect. 2 we present a brief summary of the general formalism needed in treating the process \( \vec{A}(\vec{e}, e'N)B \) within the context of PWIA, including a summary of the basic ingredients entering into the single–nucleon response functions drawn from our previous work.\(^3\) The calculation and discussion of the general properties of the polarized spectral function for deformed nuclear systems are discussed in Sect. 3 (with some developments given in Appendix A) and in Sect. 4 we present and analyze the general expressions obtained for the asymmetries. Results are presented in Sect. 5: specifically, details concerning the description of the target \( ^{21}\text{Ne} \) are presented in Sect. 5.1, while in Sect. 5.2 we discuss the kinematics used in the calculations, and then in Sects. 5.3–5.6 results for the cross sections, asymmetries and response functions are presented. Finally in Sect. 6 we give a summary of what has been learned from these investigations and present our conclusions.

2. COINCIDENCE ELECTRON SCATTERING FORMALISM

2.1 General Exclusive Electron Scattering

Following Refs. \(^3,16\), in this section we briefly summarize the essential parts of the general formalism involved in describing reactions of the type \( \vec{A}(\vec{e}, e'N)B \), where incident electron and target nucleus (\( A \)) are polarized. We restrict our attention to the extreme relativistic limit (ERL) for the electrons and to the case where the residual nucleus \( B \) is left in a bound state.
The process in the Born approximation is represented using the Feynman diagrams in Fig. 1. The different kinematic variables in the laboratory frame are the following: $K^\mu \equiv (\epsilon, k)$ and $K'^\mu \equiv (\epsilon', k')$ are the four–momenta of the incident and scattered electrons, respectively, while the hadronic variables $P^A_\mu \equiv (M_A, 0)$, $P^B_\mu \equiv (E_B, \mathbf{p}_B)$, $P^N_\mu \equiv (E_N, \mathbf{p}_N)$ are the four–momenta of the target, residual nucleus and emitted nucleon in the laboratory frame, respectively. We have the relationships $E_B = \sqrt{p_B^2 + M_B^2}$ and $E_N = \sqrt{p_N^2 + M_N^2}$, where $M_N$ is the nucleon mass, $M_B$ is the rest–mass of the residual nucleus (and includes any internal excitation energy in that system), $p_B \equiv |\mathbf{p}_B|$ and $p_N \equiv |\mathbf{p}_N|$. The four–momentum transferred by the virtual photon is given by $Q^\mu \equiv (\omega, q) = (K - K')^\mu = (P_N + P_B - P_A)^\mu$. Here the energy transfer is $\omega = \epsilon - \epsilon'$ and the three–momentum transfer is $q = k - k'$, with the magnitude of the latter being denoted $q \equiv |q|$.

Using the property of conservation of the nuclear electromagnetic current and after integrating over the nucleon energy $E_N$ one can write the general expression for the differential cross section for exclusive electron scattering:[3,4,16]

$$
\frac{d\sigma^h}{d\epsilon' d\Omega e d\Omega_N} = \frac{p_N M_N M_B}{(2\pi)^3 M_A} \sigma_{\text{Mott}} f_{\text{rec}}^{-1} \{R + hR'\}
$$

$$
\equiv \Sigma + h\Delta ,
$$

(1)

containing the helicity–sum (electron unpolarized) and helicity–difference (electron polarized) cross sections, $\Sigma$ and $\Delta$, respectively. The other quantities in Eq. (1) are defined in Ref. [3,16]. Note in the above expression that integration over the emitted nucleon energy $E_N$ actually means that the value of the momentum $p_N$ is completely specified by solving the energy balance equation

$$
\sqrt{p_N^2 + M_N^2} + \sqrt{p_N^2 - 2p_N q \cos \theta_N + q^2 + M_B^2} = M_A + \omega ,
$$

(2)

with $\theta_N$ the relative angle between the momenta $\mathbf{p}_N$ and $\mathbf{q}$. The functions $R$ and $R'$ represent the hadronic responses involving unpolarized or polarized electrons, respectively. Both classes of responses may in general have contributions due to the orientation of the target. They can be decomposed, as usual, into six general classes of response labelled $L$, $T$, $TL$ and $TT$ for unpolarized ($T'$ and $TL'$ for polarized) electron scattering. Each response is multiplied by its corresponding lepton kinematical factor, $v_L$, $v_T$, etc., where expressions for the six response functions $R^K$ and kinematical factors $v_K$ are given in Ref. [16]. We consider the case where only the target and/or the initial electrons are polarized, while final polarization is not observed. It can be shown that the dependence on the azimuthal angles $\phi_N$ and $\phi^*$ corresponding to the emitted nucleon and target polarization directions, is the following:[17]

$$
\begin{align}
\Sigma &\sim v_L W^L (\Delta \phi) + v_T W^T (\Delta \phi) \\
&\quad + v_{TL} \left( \cos \phi_N W^{TL} (\Delta \phi) + \sin \phi_N \tilde{W}^{TL} (\Delta \phi) \right) \\
&\quad + v_{TT} \left( \cos 2\phi_N W^{TT} (\Delta \phi) + \sin 2\phi_N \tilde{W}^{TT} (\Delta \phi) \right) \\
\text{(3a)}
\end{align}
$$

and

$$
\begin{align}
\Delta &\sim v_T \tilde{W}^{T'} (\Delta \phi) \\
&\quad + v_{TL'} \left( \sin \phi_N W^{TL'} (\Delta \phi) + \cos \phi_N \tilde{W}^{TL'} (\Delta \phi) \right) , \\
\text{(3b)}
\end{align}
$$

[17]
where each response depends on \( q, \omega, p_N \) and \( \theta_N \) as well as on the target polarization angles \( \theta^* \) and \( \Delta \phi \). It should be noted that these results have been expressed in terms of \( \Delta \phi \equiv \phi^* - \phi_N \) (see Fig. 2).

In the particular case of the PWIA to be discussed below, the terms \( \tilde{W}^{TLL}(\Delta \phi) \) and \( \tilde{W}^{TT}(\Delta \phi) \) do not appear. Furthermore, for situations where the target nucleus is unpolarized, all responses with tildes vanish; in PWIA the response \( W^{TLL}(\Delta \phi) \) also vanishes. All of the comments made in the case of PWIA can be also applied to the factorized DWIA and the fact that certain (time–reversal odd, see Refs. [17]) responses are absent in the PWIA or factorized DWIA reflects the nature of these approximations. To the extent that experimental studies yield nonzero results for these responses, it will be possible to evaluate the quality of these (factorized) approximations. In Sect. 4 the dependence on \( \Delta \phi \) of the different responses within the PWIA will be made explicit.

2.2 The Plane–Wave Impulse Approximation

From now on we will focus on coincidence electron scattering within the context of the PWIA. Here, as is well known,[1–4] in addition to restricting the currents to one–body operators (Impulse Approximation), one makes several more stringent assumptions. Firstly, one takes the emitted nucleon to be a plane wave, i.e., the nucleon is ejected from the nucleus without any further interaction with the residual nuclear system. Secondly, one assumes that the nucleon detected in the coincidence reaction is the one to which the virtual photon is attached (see Fig. 3) and thus neglects various classes of exchange effects. Within the PWIA, the general expression for the six–fold differential cross section can be written as the following:[3]

\[
\frac{d\sigma}{d\Omega_e d\Omega_1 d\Omega_2 dE_N} = p_N \frac{M_N M_B}{E_B} \sum_{mm'} \sigma_{mm'}^{eN} S_{mm'}(p, E, \Omega^*)
\]

\[
= p_N E_N \sum_{mm'} \tilde{\sigma}_{mm'}^{eN} \tilde{S}_{mm'}(p, E, \Omega^*)
\]

where \( \sigma_{mm'}^{eN} \) is the “off–shell polarized electron–nucleon cross section” and \( S_{mm'}(p, E, \Omega^*) \) the polarized spectral function whose diagonal components \( (m = m') \) give the probability to find a nucleon in the target with momentum \( p \), energy \( E \) and spin projection \( m \). For convenience and to connect to previous work[4] in Eq. (4b) we have introduced a secondary single–nucleon cross section and a secondary spectral function via the following:

\[
\tilde{\sigma}_{mm'}^{eN} \equiv \left( \frac{M_N^2}{EE_N} \right) \sigma_{mm'}^{eN}
\]

\[
\tilde{S}_{mm'}(p, E, \Omega^*) \equiv \left( \frac{M_B}{E_B} \right) \left( \frac{\bar{E}}{M_N} \right) S_{mm'}(p, E, \Omega^*) \approx S_{mm'}(p, E, \Omega^*)
\]

where \( \bar{E} \equiv \sqrt{p^2 + M_N^2} \). The energy of the struck nucleon is given by \( E = M_A - E_B \) and in general \( \bar{E} \neq E \), namely, the kinematics are off–shell for this particle. Another energy commonly used in discussions of coincidence electron scattering is the missing energy, defined as \( E_m \equiv M_N + M_B - M_A = \omega - T_N - T_B \), where \( T_N \) and \( T_B \) are the kinetic energies of the ejected nucleon and the (excited) residual nucleus, respectively. Clearly one may use \( E \) or \( E_m = M_N + M_B - E_B - E \) as an argument in the spectral function — in this work we
shall use $E_m$ rather than $E$. Alternatively, it has proven useful\cite{ref3} to use still another energy \( E \equiv E_B - E_B^0 \), where $E_B^0$ is the energy of the daughter nucleus in its ground state. This has the merit of involving a simple bound, viz. \( E \geq 0 \), by construction. Naturally one can write expressions for \( E = E(\mathcal{E}, p) \) and \( E_m = E_m(\mathcal{E}, p) \) and thus can write $S_{mm'}$ as a function of $\mathcal{E}$ and $p$.

Of course, in coincidence electron scattering the kinematics demand a specific relationship between the energies. Specifically, in specifying the electron scattering kinematics $q$ and $\omega$ are fixed; detecting a nucleon amounts to fixing $p_N$ (or $E_N$) and $\theta_N$ and hence, through Eq. (2), $M_B$ is specified. Finally, since $p = \sqrt{\frac{p_N^2}{2} - 2p_Nq \cos \theta_N + q^2 + M_B^2}$, the struck nucleon momentum $p$ is also specified and therefore $\mathcal{E}$, $E$ and $E_m$ are as well. Introducing the binding energies of the target and daughter nucleus, $\epsilon_A^o \equiv AM_N - M_A$ and $\epsilon_B \equiv (A - 1)M_N - M_B$, respectively, the expressions for $\sigma_{mm'}^{en}$ and $S_{mm'}(p, E_m, \Omega^*)$ are given by\cite{ref3}:

$$\sigma_{mm'}^{en} = \frac{2\alpha^2}{Q^1} \left( \frac{e'}{e} \right) \eta_{\mu\nu} \mathcal{W}_{mm'}^{\mu\nu}(p; q) \quad . \tag{6}$$

and

$$S_{mm'}(p, E_m, \Omega^*) = \sum_A p(A) \sum_B \langle B|a_{pm'}|A\rangle^* \langle B|a_{pm}|A\rangle \delta(E_m + \epsilon_B - \epsilon_A^o) \quad , \tag{7}$$

where $\eta_{\mu\nu}$ is the leptonic tensor (in the ERL with only the incident electron polarized) and \( \mathcal{W}_{mm'}^{\mu\nu}(p; q) \) is the single–nucleon tensor that depends on the $\gamma NN$ vertex (see discussion in Ref. [3]). Since we assume that no final polarizations are measured, in the spectral function the sum over $B$ involves all possible nuclear states including a sum over magnetic substates.

The cross section after integrating over the energy $E_N$ can be expressed as

$$\frac{d\sigma}{d\Omega_e d\epsilon' d\Omega_N} = \frac{p_NM_NM_B}{M_A} f_{rec}^{-1} \sum_{mm'} \sigma_{mm'}^{en} n_{mm'}(p, \Omega^*) \quad , \tag{8}$$

with

$$n_{mm'}(p, \Omega^*) = \sum_A p(A) \sum_B \langle B|a_{pm'}|A\rangle^* \langle B|a_{pm}|A\rangle \quad , \tag{9}$$

the spin–dependent density matrix in momentum space. The single–nucleon tensor and the spectral function (and hence, $n_{mm'}(p, \Omega^*)$) can easily be shown to satisfy the following general symmetries:

$$\mathcal{W}_{mm'}^{\mu\nu}(p; q) = \mathcal{W}_{m'm}^{\nu\mu}(p; q) \quad , \tag{10a}$$

$$S_{mm'}(p, E, \Omega^*) = S_{m'm}^{*}(p, E, \Omega^*) \quad . \tag{10b}$$

In Eqs. (8,9) one should note that when polarization degrees of freedom are taken into account, in contrast with the unpolarized case, the spectral function and electron–nucleon cross section in general will contain both diagonal and off–diagonal spin components. The various elements that make up the PWIA (and also the DWIA) descriptions of $\bar{A}(\bar{e}, \epsilon N)B$ reactions can all be
considered to be $2 \times 2$ hermitian matrices in spin–space, where the four components correspond to all the possible values of the projections of the spin of the bound nucleon, $m, m' = \pm 1/2$ along a specific direction.

Imposing current conservation for the single–nucleon current the “off–shell polarized electron–nucleon cross section” can be decomposed in the following way:

\[
\sigma_{eN}^{mm'} = \sigma_{\text{Mott}} \left\{ \sum_K v_K R^K_{mm'} + \hbar \sum_{K'} v_{K'} R^{K'}_{mm'} \right\},
\]

where the electron kinematical factors $(v_K, v_{K'})$ and single–nucleon response functions $R^K_{mm'}$ are labelled as usual by $K = L, T, TL, TT$ and $K' = T', TL'$. The response functions are given in terms of specific Lorentz components of the single–nucleon tensor $W^{\mu\nu}_{mm'}$, as discussed in Refs. [3,16]. The hadronic response functions entering in the coincidence cross section can now be written in terms of the quantities defined above:

\[
R^K = (2\pi)^3 \sum_{mm'} R^K_{mm'} n_{mm'}(p, \Omega^*) .
\]

One should note that in obtaining the single–nucleon responses (see Refs. [3,4]) the $z$–component is usually eliminated in favor of the zero–component using current conservation (the $z$–axis is along $q$). One could also proceed the other way and eliminate the charge components in favor of the longitudinal components. Both are completely equivalent for an on–shell single–nucleon (conserved) electromagnetic current. However, in the case of off–shell nucleons various prescriptions are usually employed that can lead to different results when the longitudinal or charge components (or neither) are eliminated via the property of current conservation. Detailed discussions of this problem have already been presented in Ref. [3] and, drawing on those, in next section we only summarize the main results and general properties of the single–nucleon response functions that are relevant for this work. As discussed in Sect. 1, our aim in this paper is much more concerned with the spin–dependent nuclear problem contained in the polarized spectral function and its application to the case of deformed systems (in particular $^{21}\text{Ne}$). However, the knowledge of the symmetries displayed by both types of responses (single–nucleon and nuclear) will allow us to discuss in a very straightforward way the general properties and symmetries introduced by the polarization of the target and incident electron.

2.3 Half–off–shell Single–nucleon Responses

In the Impulse Approximation one has to deal with the half–off–shell $\gamma NN$ vertex. At present, there is not yet any rigorous approach to treat the off–shellness property of the bound single–nucleon current.† Therefore, it has become common practice to use specific off–shell extrapolations of the on–shell vertex. The most frequently used in the analysis of experimental data are the ones introduced by de Forest. They are constructed via the following three steps:

a) Treat the spinors as free;
b) Employ the so–called $CC1$ and $CC2$ forms for the vertex operator;

† For a review of this subject, see Refs. [18–22].
c) Impose (or not) current conservation by elimination of the longitudinal contributions in favor of the charge contributions (or vice versa).

Following the work of de Forest for the totally unpolarized situation, some of us (J.A.C., T.W.D. and G.I.P.) generalized those previous studies to the case in which the target and incident electrons are polarized. We showed that, given a specific de Forest–type prescription for the off–shell vertex $\Gamma^\mu$, one can express the spin–dependent single–nucleon tensor in terms of two tensors $S^{\mu\nu}$ and $A^{\mu\nu}_{mm'}$, where $S^{\mu\nu}$ is real and symmetric under the interchange $\mu \leftrightarrow \nu$, whereas $A^{\mu\nu}_{mm'}$ is antisymmetric under the interchange $\mu \leftrightarrow \nu$, and has real diagonal terms and in general complex off–diagonal terms. The cross sections resulting from both recipes for the single–nucleon tensor when current conservation is not imposed (denoted by $NCC1$ and $NCC2$) were compared with the so–called $CC1$/$CC2$ cross sections obtained by enforcing current conservation through elimination of the $\mu = 3$ or $\mu = 0$ components of the current matrix elements by means of the relation $qJ^{(3)} = \omega J^{(0)}$ (denoted by $CC1^{(0)}$/CC2$^{(0)}$ or $CC1^{(3)}/CC2^{(3)}$, respectively).

By enforcing current conservation and using the expansion of the responses in terms of symmetric and antisymmetric parts, one sees that the electron–unpolarized single–nucleon response functions $R^K_{mm'}$, $(K = L, T, TL$ and $TT)$ are diagonal in the spin indices and take the same value for both components $(m = m' = +, -)$. Hence, for simplicity we will denote them by $R^K$. One can also show that $R^L$ and $R^T$ are independent of $\phi_N$, while $R^{TL}$ and $R^{TT}$ are proportional to $\cos \phi_N$ and $\cos 2\phi_N$, respectively. The explicit $CC1^{(0)}$ and $CC2^{(0)}$ expressions of these responses can be found in Ref. [3]. It is interesting to remark that the longitudinal response $R^L$ is the same for these two prescriptions, while this is not the case for $CC1^{(3)}$ and $CC2^{(3)}$ prescriptions. This indicates that the ambiguities introduced by the possible choices of $\Gamma^\mu$ for the off–shell case are minimized when current conservation is imposed to eliminate $J^{(3)}$ in favor of $J^{(0)}$. Thus, we have chosen this prescription ($CC1^{(0)}$) for the calculations presented in this work.

In the case of the electron–polarized single–nucleon responses $R^K_{mm'}$, $(K' = T'$, $TL'$), one can show that for any off–shell vertex operator $\Gamma^\mu$, the $\phi_N$–dependence can be written as follows:[3]

\[
\begin{align*}
R^T_l &= \frac{1}{2}(R^T_{++} - R^T_{--}) = A \\
R^T_s &= \text{Re}(R^T_{+-}) = B \cos \phi_N \tag{13a} \\
R^T_n &= \text{Im}(R^T_{+-}) = B \sin \phi_N \tag{13b} \\
R^{TL}_l &= \frac{1}{2}(R^{TL}_{++} - R^{TL}_{--}) = C \cos \phi_N \tag{13c} \\
R^{TL}_s &= \text{Re}(R^{TL}_{+-}) = \frac{1}{2}(D - E) + \frac{1}{2}(D + E) \cos 2\phi_N \tag{13d} \\
R^{TL}_n &= \text{Im}(R^{TL}_{+-}) = \frac{1}{2}(D + E) \sin 2\phi_N \tag{13e}
\end{align*}
\]

where $A$, $B$, $C$, $D$, $E$ are functions of $\theta_N$. In particular, functions $A$ and $D \pm E$ are symmetric in $\theta_N$, while $B$ and $C$ are antisymmetric. For parallel kinematics, where $\theta_N \to 0$, we find that $A$ and $D - E$ can be nonzero whereas $B$, $C$ and $D + E$ must vanish. Note that a relationship exists involving two of the $T'$ responses: $R^T_n/R^T_s = \tan \phi_N$. In the particular cases of the prescriptions $CC1^{(0)}$ and $CC2^{(0)}$, the explicit expressions for the terms $A, B, C, D, E$ can be found in Ref. [3].
3. POLARIZED SPECTRAL FUNCTION AND DEFORMED NUCLEI

In this section we develop a general expression for the polarized spectral function spin matrix which enters in the analysis of the reaction $\vec{A}(\vec{e}, e'N)B$ in the PWIA. As previously stated, we will restrict our attention to some simplified situations: in particular, we will always consider the final nucleus to be left in a bound state and assume parity conservation. The nuclei involved in the process will be described by the rotational model of Bohr and Mottelson using Nilsson and Hartree–Fock single–particle wave functions. General properties and symmetries will be discussed in Sect. 3.1.

The general expression for the spin–dependent spectral function is

$$S_{mm'}(p, E_m, \Omega^*) = \sum_{M_A} p(M_A) \sum_B <B|a^+_{p m'}|A> <B|a_{p m}|A> \delta(E_m + \epsilon_B - \epsilon_A^*) =$$

$$= \sum_{M_A} p(M_A) \sum_B <A|a^+_{p m'}|B> <A|a^+_{p m}|B> \delta(E_m + \epsilon_B - \epsilon_A^*) ,$$

(14)

where $|B>$ and $|A>$ represent the final and initial nuclear states, respectively, and $a_{p m}$, ($a^+_{p m}$), are annihilation (creation) operators which destroy (create) a nucleon with momentum $p$ and spin projection $m$. Since we are considering the case of discrete nuclear states we have good quantum numbers $J_A$ and $J_B$, respectively, as well as parities $\pi_A$ and $\pi_B$. We will refer all the quantities to the system defined by the axes $x, y, z$ (see Fig. 2). This means that the final unpolarized nucleus will be characterized by state vectors $|J_B M_B>$ defined with respect to the $z$–axis. On the other hand, the target nucleus is polarized, i.e., the target is prepared with magnetic substates $|J_A M_A>$ in the direction $P^*$ populated in a non–uniform manner with probabilities $p(M_A)$. Thus, the target nuclear states are quantized with respect to a fixed quantization axis specified by the spherical coordinates $\Omega^*$. We therefore have,

$$|J_A M_A> = \sum_{M_A'} D^{*J_A}_{M_A M_A'}(\Omega^*) |J_A M_A'> ,$$

(15)

where the eigenstates $|J_A M_A'>$ are referred to the system with axes of quantization along $q$. We follow the convention of Edmonds\cite{edmonds} for rotation matrices.$^\dagger$

Expanding the single–nucleon creation (annihilation) operators over a basis of irreducible tensor operators $a_{\ell j m_\ell}(p)$ ($\tilde{a}_{\ell j m_\ell}(p)$) and after some algebra (see Appendix A for details), the polarized spectral function can be written in terms of different tensor polarization components as

$$S_{mm'}(p, E_m, \Omega^*) = \sum_I S^{(I)}_{mm'}(p, E_m, \Omega^*) ,$$

(16)

where $I$ denotes the polarization rank ($I = 0$ corresponds to the complete unpolarized case, i.e., target and incident electrons unpolarized). Each of the tensor polarization contributions

$^\dagger$ Note that the convention of Brink and Satchler (BS) is related to that of Edmonds (Ed) through the relation $[D^{*I}_{MK}]_{BS} = [D^{I}_{KM}]_{Ed}$.
in the spectral function for parity–conserving electron scattering are given finally by

\begin{equation}
S_{mm'}^{(I)}(p, E_m, \Omega^*) = (-1)^{m-1/2} f_I^{J_A} \sum_B \sum_{\ell, \ell'} \sum_{\ell' J_A} \sum_{\ell' J_B} \sum_{L M} (-1)^{J_A + J_B + j + \ell'} [j][\ell'][\ell'] [L][K]^2 
\times C_{\ell j}^{* J_A J_B}(p) C_{\ell' j'}^{J_A J_B}(p) Y_{I M}^{*} - M(\Omega^*) Y_{L M}^{-} - H(\Omega) \left( \begin{array}{ccc} \ell & \ell' & L \\ 0 & 0 & 0 \end{array} \right) \left( \begin{array}{ccc} L & I & K \\ H & M & N \end{array} \right) 
\times \left( \begin{array}{ccc} K & 1/2 & 1/2 \\ N & m & -m' \end{array} \right) \left( \begin{array}{ccc} J_A & J_A & I \\ J_B & J_B & J_B \end{array} \right) \left( \begin{array}{ccc} \ell & j & 1/2 \\ \ell' & j' & 1/2 \end{array} \right) \delta(E_m + \epsilon_B - \epsilon_A^o),
\end{equation}

where \( f_I^{J_A} \) are the spherical Fano statistical tensors as given in Eq. (A.7) and \( C_{\ell j}^{* J_A J_B}(p) \) [\( C_{\ell' j'}^{J_A J_B}(p) \)] the reduced nuclear matrix elements (see Appendix A). Note that all of the angular dependence is contained in the two spherical harmonics, \( Y_{I M}^{*} - M(\Omega^*) \) and \( Y_{L M}^{-} - H(\Omega) \) with \( \Omega \equiv (\theta, \phi) \), the angles defining the direction of the struck nucleon in the laboratory frame.

A sum over the single–particle quantum numbers \( \ell, \ell' (j, j') \) is involved in the expression for the spectral function and introduces interferences between different single–nucleon orbitals. This arises only when polarization degrees of freedom are taken into account. In the complete unpolarized case (see Appendix A), only the trace of \( S_{mm'} \) enters; in other words the spectral function is reduced to

\begin{equation}
S_{mm'}(p, E_m) = S(p, E_m) \delta_{mm'} = \frac{1}{8\pi} \frac{1}{[J_A]^2} \sum_B \sum_{\ell j} |C_{\ell j}^{* J_A J_B}(p)|^2 \delta(E_m - \epsilon_B + \epsilon_A^o) \delta_{mm'},
\end{equation}

and therefore, the interference terms do not appear. These effects will be studied in detail in a forthcoming work. In the present paper we are interested in studying the general behaviour of the differential cross sections as well as of the hadronic response functions and polarization ratios as functions of the bound nucleon momentum, the target polarization direction and the deformation of the target.

### 3.1 Properties and Symmetries of the Spectral Function

A property that emerges from Eq. (10b) is that

\begin{equation}
S_{mm'}^{(I)}(p, E_m, \Omega^*) = S_{m'm}^{(I)*}(p, E_m, \Omega^*).
\end{equation}

On the other hand, using parity conservation it is straightforward to show that,

\begin{equation}
S_{m'm}^{(I)}(p, E_m, \Omega^*) = (-1)^{J + 1 - m - m'} S_{m'm}^{* (I)}(p, E_m, \Omega^*).
\end{equation}

Combining both relations it is easy to see that the off–diagonal components of the spectral function are zero for \( I = \text{even} \).

For convenience, we introduce the components \( S_0, S_t, S_s \) and \( S_n \) which are defined to be real (for clarity, here we do not specify dependence of the spectral function on momentum, energy and angular variables),

\begin{align}
S_0 & \equiv S_{++} + S_{--} & (21a) \\
S_t & \equiv S_{++} - S_{--} & (21b) \\
S_s & \equiv S_{+-} + S_{-+} = 2 Re(S_{+-}) & (21c) \\
S_n & \equiv -i(S_{+-} - S_{-+}) = 2 Im(S_{+-}) & (21d)
\end{align}
Using the properties given by Eqs. (19,20), it follows that $S_0$ only has contributions from even $I$–values, whereas $S_l$, $S_s$ and $S_n$ only have contributions from odd $I$–values. One can also see that

$$S_0 = \sum_{I=\text{even}} S_0^{(I)}(p, E_m, \Omega^*) = 2 \sum_{I=\text{even}} S_m^{(I)}(p, E_m, \Omega^*)$$ (22a)

$$S_l = \sum_{I=\text{odd}} S_l^{(I)}(p, E_m, \Omega^*) = 2(-1)^{1/2-m} \sum_{I=\text{odd}} S_m^{(I)}(p, E_m, \Omega^*)$$ (22b)

$$S_s = \sum_{I=\text{odd}} S_s^{(I)}(p, E_m, \Omega^*) = 2 \sum_{I=\text{odd}} \text{Re} \left[ S_m^{(I)}(p, E_m, \Omega^*) \right]$$ (22c)

$$S_n = \sum_{I=\text{odd}} S_n^{(I)}(p, E_m, \Omega^*) = 2(-1)^{1/2-m} \sum_{I=\text{odd}} \text{Im} \left[ S_m^{(I)}(p, E_m, \Omega^*) \right].$$ (22d)

With this new notation, one should remember that the 0–component is the only one that contributes in the electron–unpolarized responses (it may include contributions from target polarization, $I \geq 2$). The other three components $l$, $s$ and $n$ enter in the two electron–polarized responses (electrons and target polarized).

The electron–unpolarized (helicity sum) and electron–polarized (helicity difference) cross sections (see Sect. 2.1) which enter in the definition of the global asymmetry $A = \Delta/\Sigma$ can now simply be written

$$\Sigma = \frac{p_N M_N M_B}{M_A} f_{\text{rec}} \sigma_0^{eN} n_0(p, \Omega^*)$$

$$\Delta = \frac{p_N M_N M_B}{M_A} f_{\text{rec}} \left[ \sigma_l^{eN} n_l(p, \Omega^*) + \sigma_s^{eN} n_s(p, \Omega^*) - \sigma_n^{eN} n_n(p, \Omega^*) \right],$$ (23)

where $\sigma_i^{eN}$, $(i = 0, l, s, n)$ are the components for the “off–shell electron–nucleon cross section” defined in terms of $\sigma_{mm'}^{eN}$ in analogy to Eqs. (21) but divided by a factor 2 (see Ref. [3]), and $n_i(p, \Omega^*)$ are the different components of the spin–dependent momentum distribution (Eq. (9)) defined in the same way as the components of the polarized spectral function. The hadronic response functions (Eq. (12)) are then given by

$$R^K = (2\pi)^3 R^K n_0(p, \Omega^*)$$ (24a)

$$R^{K'} = (2\pi)^3 \left[ R_l^{K'} n_l(p, \Omega^*) + R_s^{K'} n_s(p, \Omega^*) - R_n^{K'} n_n(p, \Omega^*) \right],$$ (24b)

where $K = L, T, TL, TT$ and $K' = T', TL'$. The single–nucleon response functions, $R_i^{(K/K')}$ are specified in the way discussed in Sect. 2.3 (see Ref. [3] for details).

In connection with the angular dependence in the polarized spectral function, one should note that the whole dependence in Eq. (17) is given through the two spherical harmonics, $Y_L^{-M}(\Omega^*)$ and $Y_L^{-H}(\Omega^*)$, with $\Omega \equiv \{\theta, \phi\}$ and $\Omega^* \equiv \{\theta^*, \phi^*\}$ being the angles defining the direction of the struck nucleon momentum $p$ and the direction of the target polarization $P^*$, respectively. From Eqs. (17,21) the different components of the spin–dependent momentum distribution for a fixed $J_B$–value can be written as follows:
a) The component entering in the electron–unpolarized cross section is given by

\[ n_0^{J_B} = \sum_{I=\text{even}} [I] f_I^{J_A} J(J_A, J_B, I; p) P_I(\cos \xi) , \tag{25} \]

where \( J(J_A, J_B, I, p) \) contains all of the dependence on the model used in the evaluation of the nuclear wave functions. As noted above, it depends on the total angular momenta of the target and residual nucleus, on the rank of the polarization tensor and on the magnitude of the struck–nucleon momentum. Its explicit expression in terms of the nuclear reduced matrix elements is given by

\[ J(J_A, J_B, I, p) = \frac{(-1)^{J_A+J_B+1/2}}{4\pi} \sum_{\ell j^\prime} \sum_{j j^\prime} (-1)^{2j} \frac{1}{[\ell][j^\prime][j][\ell^\prime]} C_{\ell j}^{J_A J_B}(p) C_{\ell^\prime j^\prime}^{J_A J_B}(p) \]

\[ \times \left\{ \begin{array}{c} J_A \ J_A \ I \\ j \ j^\prime \ 0 \ 0 \ 0 \end{array} \right\} \left\{ \begin{array}{c} j \ j^\prime \ I \\ \ell \ \ell^\prime \ 1/2 \ end{array} \right\} , \tag{26} \]

The angle \( \xi \) entering in the Legendre polynomial \( P_I(\cos \xi) \) is the relative angle between the direction of the target polarization and the momentum of the bound nucleon. It is given through the relation

\[ \cos \xi = \cos \theta^* \cos \theta + \sin \theta^* \sin \theta \cos \Delta \phi , \tag{27} \]

where

\[ \Delta \phi = \phi^* - \phi_N = \phi^* - \phi \tag{28} \]

is the angle between the planes \((q, P^*)\) and \((q, p_N)\) [or equivalently \((q, p)\)].

b) The three components that enter in the electron–polarized cross section are given by

\[ n_l^{J_B} = \sum_{I=\text{odd}} [I] f_I^{J_A} \sum_{L=I-1}^{I+1} K(J_A, J_B, I; p) \]

\[ \times \sum_{M=-I}^{I} X(I, L, M, \theta) \sqrt{\frac{(I+M)!}{(I-M)!}} P_{I-M}^{-M}(\cos \theta^*) \cos M\Delta \phi , \tag{29a} \]

\[ n_s^{J_B} = \sum_{I=\text{odd}} [I] f_I^{J_A} \sum_{L=I-1}^{I+1} K(J_A, J_B, I; p) \sum_{M=-I}^{I} Z(I, L, M; \theta) \]

\[ \times \sqrt{\frac{(I+M)!}{(I-M)!}} P_{I-M}^{-M}(\cos \theta^*) \cos (M\Delta \phi + \phi_N) \tag{29b} \]
\[ n_n^{J_B} = - \sum_{l=\text{odd}} \left[ I \right] f^{J_A}_l \sum_{L=I-1}^{I+1} K(J_A, J_B, I, L; p) \sum_{M=-I}^{I} Z(I, L, M; \theta) \]
\[ \times \sqrt{\frac{(I + M)!}{(I - M)!}} P^M_I (\cos \theta^*) \sin (M \Delta \phi + \phi_N). \tag{29c} \]

The common function \( K(J_A, J_B, I, L; p) \), with a form similar to \( J(J_A, J_B, I; p) \), contains all of the nuclear structure dependence. Its explicit expression is

\[ K(J_A, J_B, I, L; p) = \frac{\sqrt{3}}{2\pi} (-1)^{J_A+J_B} [L^2 \sum_{\ell, \ell'} \sum_{j, j'} (-1)^{\ell+\ell'} [j][j'][\ell][\ell'] C^{J_A J_B}(p) C^{J_A J_B}_{\ell \ell'} (p) \]
\[ \times \begin{pmatrix} \ell & \ell' & L \\ 0 & 0 & 0 \end{pmatrix} \{ J_A & J_A \\ j & j' \} \{ I & 1/2 \} \begin{pmatrix} L & I \\ \ell & j' \end{pmatrix} \begin{pmatrix} I & 1 \\ j' \ell' \end{pmatrix} \} . \tag{30} \]

The functions \( X(I, L, M; \theta) \) and \( Z(I, L, M; \theta) \) are given by

\[ X(I, L, M; \theta) = \frac{1}{\sqrt{2}} \sqrt{\frac{(L - M)!}{(L + M)!}} \begin{pmatrix} L & I & 1 \\ -M & M & 0 \end{pmatrix} P^M_L (\cos \theta) \tag{31a} \]
\[ Z(I, L, M; \theta) = \sqrt{\frac{(L - M + 1)!}{(L + M - 1)!}} \begin{pmatrix} L & I & 1 \\ 1 - M & M & -1 \end{pmatrix} P^{M-1}_L (\cos \theta) . \tag{31b} \]

Note that the relation, \( X(I, L, -M; \theta) = (-1)^M X(I, L, M; \theta) \) holds. With regards to the dependence on the azimuthal angles, \( n_o^{J_B} \) and \( n_l^{J_B} \) depend only on \( \Delta \phi \), while \( n_s^{J_B} \) and \( n_n^{J_B} \) depend on both \( \Delta \phi \) and \( \phi_N \).

### 3.2 Application to Deformed Nuclear Systems

In the general expression for the polarized spectral function, one should note that the dependence on the nuclear model is contained in the reduced nuclear matrix elements \( C^{J_A J_B}_{\ell \ell'} (p) \). As previously stated, we focus on the case of rotational nuclei and use the factorization approximation of Bohr and Mottelson\(^6\). The wave function for axially–symmetric deformed nuclei in the laboratory system is written in terms of the relative orientation of the body–fixed system and the intrinsic wave function. For a given \( K_A \)–band in the \( A \)–body nucleus with intrinsic (Slater determinant or BCS) wave function \( \Phi_{K_A} \), the nuclear wave function is given by\(^6\)

\[ |\Psi_A J_A M_A\rangle \equiv |J_A K_A M_A\rangle = \left( \frac{2J_A + 1}{16\pi^2 (1 + \delta_{K_A,0})} \right)^{1/2} \]
\[ \times \left[ D^{J_A}_{K_AM_A} \Phi_{K_A} + (-1)^{J_A+K_A} D^{J_A}_{-K_AM_A} \Phi_{-K_A} \right] \tag{32} \]
and a similar expression holds for the wave function corresponding to the residual nucleus $B$. With these nuclear wave functions, the reduced nuclear matrix elements that result are given by

$$C_{\ell j}^{J_A J_B}(p) = \frac{(-1)^{J_A - K_A} [J_A][J_B]}{\sqrt{(1 + \delta_{K_A,0})(1 + \delta_{K_B,0})}} \sum_{\mu} \left[ \begin{array}{ccc} J_A & j & J_B \\ -K_A & \mu & -K_B \end{array} \right] \langle \Phi_{K_A} | a_{\ell j \mu}^+(p) | \Phi_{K_B} \rangle +$$

$$(-1)^{J_B + K_B} \left[ \begin{array}{ccc} J_A & j & J_B \\ -K_A & \mu & -K_B \end{array} \right] \langle \Phi_{K_A} | a_{\ell j \mu}^+(p) | \Phi_{K_B} \rangle ,$$

where $a_{\ell j \mu}^+(p)$ is the creation operator for a nucleon with momentum $p$ and quantum numbers $\ell, j, \mu$. The process considered involves a polarized odd–$A$ target and therefore $J_A$ and $K_A$ take on half–integer values. We only consider transitions from the ground state of the target nucleus to states in the residual nucleus within the ground–state band, $K_B = 0$. In such a case, the reduced matrix elements are then just given by

$$C_{\ell j}^{J_A J_B}(p) = \sqrt{2} ( -1)^{J_A - K_A} [J_A][J_B] \left[ \begin{array}{ccc} J_A & j & J_B \\ -K_A & \mu & -K_B \end{array} \right] \langle \Phi_{K_A} | a_{\ell j \mu}^+(p) | 0 \rangle ,$$

with

$$\langle 0 | a_{\ell j K_A}(p) | \Phi_{K_A} \rangle = \sqrt{2 v_{K_A}^2 \sum_n c_{n \ell j K_A}^2} \psi_{n \ell j}(p) .$$

Here $v_{K_A}^2$ is the probability for the orbit $K_A$ to be occupied in the target nucleus. Calculations for the intrinsic state of the odd–$A$ nucleus are done in the pair–filling approximation, fixing the $K_A$–orbital of the odd–nucleon, in which case $2 v_{K_A}^2 = 1$. Finally, $c_{n \ell j K_A}^2$ are the amplitudes of the single–particle deformed state $| \Phi_{K_A} \rangle$ in the spherical basis and $\psi_{n \ell j}(p)$ is the Fourier transform of the $n \ell j$ radial wave function in such a basis.

4. RESPONSE FUNCTIONS AND POLARIZATION RATIOS

In this section we obtain the final expressions for the different hadronic response functions and asymmetries and/or polarization ratios. The differential cross section (Eq. (1)) can be written in the following way

$$\frac{d\sigma^h}{d\epsilon' d\Omega_e d\Omega_N} = \left[ \frac{d\sigma}{d\epsilon' d\Omega_e d\Omega_N} \right]_0 [1 + \mathcal{P}_\Sigma + h\mathcal{P}_\Delta] ,$$

where $\left[ \frac{d\sigma}{d\epsilon' d\Omega_e d\Omega_N} \right]_0$ is the differential cross section for target and electron unpolarized. In terms of the single–nucleon responses ($R^K$) and reduced nuclear matrix elements ($C_{\ell j}^{J_A J_B}(p)$), it is given by

$$\left[ \frac{d\sigma}{d\epsilon' d\Omega_e d\Omega_N} \right]_0 = \frac{p_N M_N M_B}{(2\pi)^3 M_A} \sigma_{\text{Mott}} f_{\text{rec}}^{-1} [R]_0$$

$$= \frac{p_N M_N M_B}{4\pi M_A} \sigma_{\text{Mott}} f_{\text{rec}}^{-1} \left[ \sum_{\ell j} [C_{\ell j}^{J_A J_B}(p)]^2 \right] \sum_K v_K R^K ,$$

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where the sum extends over $K = L, T, TL, TT$. The terms $P_\Sigma$ and $P_\Delta$ are the polarization ratios. The fraction, $P_\Delta/(P_\Sigma + 1)$ measures the relationship between the helicity–difference (electron–polarized) and the helicity–sum (electron–unpolarized) cross sections. The term $P_\Sigma$ gives the target asymmetry when the electron beam is unpolarized, whereas $P_\Delta$ takes into account as well the contribution coming from the polarization of the incident electrons.

Let us first discuss some general properties of $P_\Sigma$. It is important to note that $P_\Sigma$ is independent of the single–nucleon cross section ($\sigma_{eN}^{\text{mm}}$) (Eq. (23)). Therefore, it does not depend on the single–nucleon current and is free from the ambiguities introduced by the various choices ($CC1^{(0)}, CC2^{(0)}, CC1^{(3)}, CC2^{(3)}, NCC1$ and $NCC2$) discussed in Ref. [3]. In this respect, $P_\Sigma$ may be an ideal tool for studying nuclear structure, in particular the spin–dependent momentum distribution of bound nucleons.

Using the expression given in the previous section for the 0–component of the spectral function (Eqs. (25,26)) we can obtain explicit expressions for the target polarization ratio. This will allow us to study in a very simple way the behaviour of $P_\Sigma$ for the nucleus of interest here ($^{21}$Ne) at different kinematics. The target polarization ratio for a fixed–$J_B$ state can be written

$$P_{\Sigma}^{J_B} = \frac{4\pi[J_A]^2}{\sum_{\ell j}|C_{\ell j}^{J_B}(p)|^2} \sum_{I \geq 2, \text{even}} [I] f_I^{J_A} J(A, J_B, I; p) P_I(\cos \xi),$$

where $J(A, J_B, I; p)$ is defined in Eq. (26) and $f_I^{J_A}$ is the Fano statistical tensor given in Eq. (A.7). Note that $P_{\Sigma}^{J_B}$ depends in general on the nuclear model. $C_{\ell j}^{J_B}(p)$ is the $\ell, j$–component of the bound nucleon that is knocked–out in the transition from $J_A$ to $J_B$ (see Eqs. (33–35)). The only angular dependence of $P_\Sigma$ is given through $\xi$, the relative angle between the direction of the polarization vector and the momentum of the bound nucleon (see Eq. (27)). The particular case in which the final nucleus is considered to be in its ground state, i.e. $J_B = 0^+$, is specially simple: there the values of the single–nucleon angular momenta are fixed and $j$ and $\ell$ take on single values ($j = j' = J_A$ and $\ell = \ell'$). The target polarization ratio is then reduced to

$$P_{\Sigma}^{J_B=0} = (-1)^{J_A+1/2}[J_A]^2[\ell]^2 \sum_{I \geq 2, \text{even}} [I] f_I^{J_A} \left( \begin{array}{ccc} \ell & \ell & I \\ 0 & 0 & 0 \end{array} \right) \left\{ \begin{array}{ccc} J_A & J_A & I \\ \ell & \ell & 1/2 \end{array} \right\} P_I(\cos \xi),$$

where the value of $\ell = J_A \pm 1/2$ is fixed by the parity and angular momentum of the target nucleus. Note that $P_{\Sigma}^{J_B=0}$ is independent of the model used for the description of the single–particle wave functions.

The structure of $P_\Delta$ (electron–target polarization ratio) is much more complex due to the new degrees of freedom introduced by the polarization of the incident electrons. In this situation one needs to evaluate three components of the spectral function ($S_l$, $S_s$, $S_n$), as well as the three components of the single–nucleon responses ($R_l^{K'}, R_s^{K'}, R_n^{K'}$). In order to simplify the discussion, we can decompose the electron–target polarization $P_\Delta$ in the following way:

$$P_\Delta = \sum_{K' = T', TL'} v_{K'} P_{\Delta}^{K'},$$

where $v_{K'}$ are the kinematical factors$^{[16]}$ and $P_{\Delta}^{K'}$ are polarization ratios given by

$$P_{\Delta}^{K'} = \frac{R_{K'}}{|R_0|}$$

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with \( |R|_0 \) as given in Eq. (37). Using the explicit expressions for the various single–nucleon responses and for the components of the spectral function given in Sect. 2.3 and 3, respectively, one finds the following expressions for the two hadronic response functions involved:

\[
R^\prime(T) = (2\pi)^3 \sum_{I=\text{odd}} [I] f_I^{\prime \Lambda} \sum_{L=I-1}^{I+1} \mathcal{K}(J_A, J_B, I, L; p) \sum_{M=-I}^{I} \sqrt{(I+M)!/(I-M)!} P_{I-M}^M(\cos \theta^*)
\times \left[A\chi(I, L, M; \theta) + B\Xi(I, L, M; \theta)\right] \cos M\Delta \phi
\]

\[
R^{TL'} = (2\pi)^3 \sum_{I=\text{odd}} [I] f_I^{\prime \Lambda} \sum_{L=I-1}^{I+1} \mathcal{K}(J_A, J_B, I, L; p) \sum_{M=-I}^{I} \sqrt{(I+M)!/(I-M)!} P_{I-M}^M(\cos \theta^*)
\times \left[C\chi(I, L, M; \theta) + D\Xi(I, L, M; \theta)\right] \cos M\Delta \phi \cos \phi_N
\]

\[
- E\Xi(I, L, M; \theta) \sin M\Delta \phi \sin \phi_N
\]

where again the expressions refer to a fixed \( J_B \)–value of the residual nucleus. The terms, \( A, B, C, D, E \) were introduced in Sect. 2.3. The function \( \mathcal{K}(J_A, J_B, I, L; p) \) that contains the dependence on the \( C_{J_A J_B}^{I}(p) \) nuclear amplitudes, as well as the functions \( \chi(I, L, M; \theta) \) and \( \Xi(I, L, M; \theta) \) are given in Eqs. (30, 31).

Some very general properties for these electron–polarized hadronic responses now become evident. First, we note that the dependence on the single–nucleon responses cannot be factored–out, as it could in the case of the electron–unpolarized response functions. Therefore, \( \mathcal{P}_\Delta \) depends on the choice of the single–nucleon current. On the other hand, Eqs. (41) show the explicit dependence on the azimuthal angles \( \Delta \phi \) and \( \phi_N \) of the \( T' \) and \( TL' \) response functions. These expressions allow us to study some particular situations where such functions can or cannot be probed. For example, it is known that for \( \bar{N} \)–polarized nuclei and co–planar kinematics both electron–polarized responses are zero and no additional information is gained by polarizing the electron. This situation corresponds to \( \theta^* = \phi^* = 90^\circ \) (nucleus polarized in the \( y \)–direction) and \( \phi_N = 0^\circ \) (nucleon detected in the scattering plane). From Eqs. (41) it is easy to see that both responses \( T' \) and \( TL' \) are zero. For \( \phi_N = 0^\circ \) only the terms proportional to \( \cos M\Delta \phi \) may survive. Since \( \Delta \phi = -90^\circ \), only the terms with \( |M| = \text{even} \) may contribute, but the Legendre polynomials are zero for \( I = \text{odd}, |M| = \text{even} \) and \( \theta^* = 90^\circ \). In Table 1 we summarize the situations in which one or both response functions \( (T', TL') \) are zero.

5. RESULTS FOR THE REACTION \( ^{21}\text{Ne}(\vec{e}, e'\vec{n})^{20}\text{Ne} \)

5.1 The Nuclear Model

In this section we present and discuss the results obtained for the cross sections, response functions and polarization ratios introduced in the previous sections for the case of \( ^{21}\text{Ne} \). As discussed in Sect. 3.2 we consider transitions to states in the residual nucleus within the ground–state band, \( K_B = 0 \), and then the ejected nucleon is the last bound neutron. The selection of \( ^{21}\text{Ne} \) as the particular nucleus for which to apply all of the formalism developed above arises for several reasons. Firstly, from an experimental point of view, being a noble
gas $^{21}$Ne is only weakly–reacting chemically and hence does not tend to depolarize when undergoing collisions with the target–container walls. Secondly, it presents a well–established rotational energy spectrum with well–defined bands,[5] that is, the experimental data support the description of such nucleus within the rotational model of Bohr and Mottelson.[6] Finally, its charge is not very large ($Z = 10$) and one does not need to worry unduely about Coulomb distortion of the electron.

The two nuclei involved in the scattering reaction (target and residual system) are taken as deformed nuclear systems with axial symmetry and the same mean field. The deformed single–particle wave functions have been calculated by using (1) the phenomenological potential of the Nilsson model and (2) the more sophisticated self–consistent density–dependent potentials of deformed Hartree–Fock (DDHF) calculations. Cross sections, response functions and polarization ratios obtained with both models are compared in next section. Here we summarize the main results of the Nilsson model and DDHF calculations that are relevant for the discussion in next sections.

Let us start by making some very general remarks in the case of the Nilsson model.[27] In this model one introduces a deformed harmonic oscillator potential with spin–orbit coupling terms and axial symmetry. The parametrization of such a potential for the description of $^{21}$Ne has been taken from the literature[28] and pairing correlations have been omitted, as is the standard procedure for $s$–$d$ shell nuclei. In the case of $^{21}$Ne, the ground state is $J^A = 3/2^+$ and the last unpaired neutron is in the state $K^\pi = 3/2^-$. Eight major shells ($N$) have been used in the diagonalization of the Nilsson hamiltonian and $N$–admixtures have been considered. The equilibrium Nilsson deformation parameter for $^{21}$Ne in its ground state is found to be $\delta_{eq} = 0.31$

In the Hartree Fock model, the calculations have been made using the McMaster version of the deformed Hartree-Fock code[29] that follows closely the method of Ref. [30]. In all the cases 50 iterations have been enough for reaching a good convergence. The effective two-nucleon interaction from which the average one-body field is obtained has been chosen to be the Skyrme-type interaction SKA (Ref. [31]). Tables I and II of Ref. [12] summarize the theoretical results obtained for $^{21}$Ne with different Skyrme interactions using a deformed H.O. basis with $N_0 = 10$, and their comparison to experimental data. We summarize in Table 2 the results obtained with SKA interaction for r.m.s. radii and quadrupole moments in $r$–space and in $p$–space. Also shown in this table are the results obtained with the Nilsson model at the equilibrium deformation ($\delta_{eq} = 0.31$) in terms of $b$ (the harmonic oscillator length parameter in fm units). In this table $Q_0^r$ represents the standard quadrupole moment for protons ($\pi$) or neutrons ($\nu$), and $Q_0^p$ is defined likewise in $p$–space

$$Q_0^r = \int d\mathbf{r} \rho(\mathbf{r}) P_2(\Omega_\mathbf{r})$$

$$Q_0^p = \int d\mathbf{p} \rho(\mathbf{p}) P_2(\Omega_\mathbf{p})$$

where $\rho(\mathbf{r})$ and $\rho(\mathbf{p})$ are the intrinsic one–body densities in $r$–space and in $p$–space, respectively.

The deformation parameters in $r$ and in $p$–spaces are defined following the standard convention[6]

$$\beta^r = \sqrt{\frac{\pi}{5}} \frac{Q_0^r}{A\langle r^2 \rangle}$$

$$\beta^p = \sqrt{\frac{\pi}{5}} \frac{Q_0^p}{A\langle p^2 \rangle}$$
Note that the $\beta$ parameters corresponding to densities in $p$–space are much smaller than those in $r$–space in agreement with the fact that at the equilibrium deformation the equalities $\langle p_x^2 \rangle = \langle p_y^2 \rangle = \langle p_z^2 \rangle$ are approximately satisfied\cite{6,13,14}. It is however interesting to note that in this odd–$A$ nucleus the $\beta^p$ value is larger than in the even–even nuclei discussed in previous papers\cite{13,14}. This is understood from the fact that in the odd–$A$ case the unpaired nucleon tends to increase the anisotropy in $p$–space, which is however smaller than that in $r$–space even for the unpaired nucleon. This is illustrated in Figure 4 where we compare the $\ell = 0, 2$ multipoles of the single–particle density for the unpaired orbital in $r$–space and in $p$–space.

In this figure the single–particle monopole densities in $r$–space and $p$–space are denoted by $\rho_0(r)$ and $n_0(p)$ respectively and the quadrupole densities are denoted by $\rho_2(r)$ and $n_2(p)$. They have been calculated from the expressions

\begin{align}
\rho(r) &= \sum_{\ell} \rho_\ell(r) P_\ell(\Omega_r) \\
n(p) &= \sum_{\ell} n_\ell(p) P_\ell(\Omega_p)
\end{align}

with $\rho(r)$ ($n(p)$) the intrinsic density in $r$–space ($p$–space) corresponding to the deformed orbital $K^\pi = 3/2^+$, occupied by the odd neutron averaged on spin.

The DDHF results (solid lines) are compared to the results of Nilsson model with (dashed lines) and without (short–dashed lines) major shell N–admixtures. It is interesting to note that the nice agreement between DDHF and Nilsson ($\Delta N=2$) results is destroyed when N–admixtures are neglected in the Nilsson model. As seen in the figure in the latter case (Nilsson ($\Delta N=0$)) the monopole and quadrupole densities differ not only quantitatively but also qualitatively from the corresponding DDHF results. This is most noticeable for the quadrupole density in $p$–space ($n_2(p)$) that becomes negative at $p \geq 1$ fm$^{-1}$ in both Nilsson ($\Delta N=2$) and DDHF results. This important feature of $n_2(p)$ is necessary to satisfy the isotropy condition $\beta^p \approx 0$ at equilibrium\cite{6,13,14}. This feature is lost when $\Delta N=2$ admixtures are neglected in the Nilsson model, and this is the reason why $\Delta N=2$ admixtures have to be included when discussing momentum distributions. In Table 3 we give the $\ell j$ contributions to the normalization of the odd–neutron wave function. The tabulated $n_{\ell j}$ weights have been calculated as described in Refs. [13,14] (see in particular Eq. (27) in Ref. [14]). As seen in this table the dominant angular momentum component of the $K^\pi = 3/2^+$ wave function is $d_5/2$. In turn 92\% of this component corresponds to the N=2 shell. On the overall the total contribution from higher N shells (N$>2$) to this state amounts to $\sim 10\%$. The amount of admixtures from higher N shells in the other occupied single–particle states is of this same order. Even though the admixtures from higher N shells are small, they play a crucial role in getting the correct $\beta$–values in $r$ and $p$–spaces simultaneously.

In Figs. 5–8 we show representative results for the unpolarized and polarized momentum distributions given by Eqs. (25) and (29). For brevity only the transition between the $^{21}$Ne and $^{20}$Ne ground states is considered here and the struck–nucleon momentum $p$ is taken to lie in the $xz$–plane, so that the momentum distributions can be displayed as surfaces with $p_x$ and $p_z$ as independent variables. We begin with Figs. 5–7 in which the target nucleus $^{21}$Ne is assumed to be polarized in the $z$–direction (along $q$). The electron–unpolarized momentum distribution $n_0$ for this case is shown in Fig. 5. It exhibits peaks symmetrically placed along the $p_x$–axis and a nodal line lying along the $p_z$–axis. If the target is polarized in the $x$–direction instead
of the $z$–direction, then the same functional behaviour is obtained except that $p_x \leftrightarrow p_z$. The polarized momentum distributions $n_l$ and $n_s$ (for target polarization in the $z$–direction) are displayed in Figs. 6 and 7. In these cases, if the polarization is placed along the $x$–axis, then $n_l$ and $n_s$ are interchanged and as before $p_x \leftrightarrow p_z$. For target polarization along either $z$– or $x$–axes one has $n_n = 0$. Finally, for target polarization placed in the $y$–direction $n_0 = n_n$ is that shown in Fig. 8, while $n_l = n_s = 0$.

Thus, we see considerable richness in the momentum distributions for polarized targets of the types accessible both with and without polarized electrons. For unpolarized targets, on the other hand, the distributions must be averaged over all angles and consequently much of this richness will be lost. In that case, the momentum distribution will depend only on the magnitude of $p$ (see Appendix A.1). Effectively, orienting a polarized deformed nucleus means that the nuclear matter is distributed asymmetrically in coordinate space (and hence in momentum space as employed here). When combined with the polarized single–nucleon cross section to obtain the coincidence cross section (as in the following results and discussions), one will see reflections of these distributions: orienting the target in some specific way one will find more cross section when the outgoing nucleon emerges in some direction for $p_N$ (and hence $p$) than in another.

5.2 Electron Scattering and Polarization Kinematics

Forward– and backward–angle electron scattering situations have been considered and in the following results are shown for both co–planar ($\phi_N = 0^\circ$) and out–of–plane ($\phi_N = 90^\circ$) kinematics. The incident electron beam and nuclear target are assumed to be 100% polarized, i.e., the weighting factor $p(M_A)$ in the expression of the spectral function (Eq. (14)) is given by $p(M_A) = \delta_{J_{A,M_A}}$. The off–shell prescription used in all of the calculations has been the so–called $CC1(0)$ form. This is the most used prescription and moreover, the results obtained with it are not too different from the results obtained with other prescriptions such as $CC2(0)$, $NCC1$ and $NCC2$ for values of the momentum $p$ not too high (for a detailed study of this subject see Ref. [3]). In the $CC1(0)$ prescription current conservation has been imposed by eliminating the longitudinal components of the current in favor of the charge components and the specific form of the single–nucleon current is

$$\Gamma_{CC1}^\mu = (F_1 + F_2)\gamma^\mu - \frac{F_2}{2M_N}(P + P_N)^\mu, \quad (45)$$

with $\bar{P} \equiv (\bar{E}, \mathbf{p})$ the four–momentum for kinematics having the same three–momentum as the struck nucleon ($\mathbf{p}$), but on–shell energy $\bar{E} = \sqrt{\mathbf{p}^2 + M_N^2}$. $F_1$ and $F_2$ are the on–shell Pauli and Dirac form factors, respectively. They are related to the Sachs form factors in the usual way:

$$F_1(\tau) = [G_M(\tau) - G_E(\tau)]/(1 + \tau)$$
$$F_2(\tau) = [G_E(\tau) + \tau G_M(\tau)]/(1 + \tau), \quad (46)$$

where $\tau \equiv -Q^2/4M_N^2$. Simple expressions have been assumed for the single–nucleon form factors, viz., dipole $\tau$–dependences for $G_{Mp}$, $G_{Mn}$ and $G_{Ep}$ together with the Galster$^{[32]}$ parametrization for $G_{En}$ (see Refs. [33] for explicit parametrizations).

Before entering into a discussion of the different figures to follow, we briefly summarize the kinematics involved in the process. As noted in previous sections, our aim is to explore
the different polarization observables in the quasi–free region where one expects to be probing essentially the single–nucleon content of the nucleus. Furthermore, at sufficiently high momentum transfer the process is expected to be only mildly influenced by final–state interactions and various exchange effects not considered in this work. Therefore, the kinematics are selected first by fixing the value of the momentum transfer \( q \) to be reasonably large. Two cases have been considered: \( q = 500 \text{ MeV}/c \) and \( q = 1 \text{ GeV}/c \). Then for both \( q \)–values the energy transfer \( \omega \) considered is set to the value corresponding to the quasielastic peak

\[
\omega_{QP} \equiv \{ \sqrt{q^2 + M_N^2} - M_N \} + E_s ,
\]  

(47)

where \( E_s \) is the separation energy given by \( E_s \equiv M_N + M_B^0 - M_A \).

Solving the energy–balance equation (Eq. (2)) subject to the condition \( -1 \leq \cos \theta \leq +1 \), one can obtain the range of allowed values of the bound nucleon momentum:

\[
 p_{\text{min}} = \frac{1}{4W^2} \left\{ q(\Lambda_+ + \Lambda_-) - 2\sqrt{q^2 + W^2\sqrt{\Lambda_+ \Lambda_-}} \right\} ,
\]  

(48a)

\[
 p_{\text{max}} = \frac{1}{4W^2} \left\{ q(\Lambda_+ + \Lambda_-) + 2\sqrt{q^2 + W^2\sqrt{\Lambda_+ \Lambda_-}} \right\} ,
\]  

(48b)

where we have introduced the quantities

\[
 W^2 = \left( M_A + \omega \right)^2 - q^2
\]  

(49a)

\[
 \Lambda_\pm = (W \pm M_B^0)^2 - M_N^2 ,
\]  

(49b)

so then \( \Lambda_+ - \Lambda_- = 4WM_B^0 \); here \( W \) is the total CM energy. The momentum of the ejected nucleon \( p_N \), and the angle \( \theta \) defining the direction of \( \mathbf{p} \) are given by

\[
p_N = \sqrt{\left\{ \omega + M_A - \sqrt{p^2 + M_B^0} \right\}^2 - M_N^2} ,
\]  

(50)

\[
\cos \theta = \frac{p_N^2 - q^2 - p^2}{2pq} .
\]  

(51)

Accordingly, for fixed values of \( q, \omega \) and the mass of the daughter nucleus \( (M_B) \) one can change the bound nucleon momentum \( p \) in the interval between \( p_{\text{min}} \) and \( p_{\text{max}} \) by varying the angle \( \theta_N \). Note that as \( p \) varies, the final outgoing nucleon momentum \( p_N \) also varies and so does the angle \( \theta \). However, the variation of \( p_N \) is tiny for the whole range of variation of \( p \) in the case in which \( M_B^0 \) is large compared with \( p^2 \), as is the general case for complex nuclei. Accordingly, we have that \( p_N \) is slightly smaller than \( q \) under typical conditions for the kinematics studied here. On the contrary, the value of \( \theta_N \) given by

\[
\cos \theta_N = \frac{p_N^2 + q^2 - p^2}{2p_Nq} ,
\]  

(52)

varies quite rapidly as \( p \) changes.
As a specific example, let us consider the case in which the residual nucleus is in its ground state. Choosing $\omega = \omega_{QP}$ we get

$$p_{\text{min}} = 0$$
$$p_{\text{max}} = 2q \left[ \frac{1 + \sqrt{q^2 + M_N^2/M_B^0}}{1 + \sqrt{q^2 + M_N^0/M_B^0}^2} \right] \approx 2q ,$$

where the approximate result in Eq. (53b) holds for $q \ll M_B^0$. Thus, $p$ can take all the possible values between 0 and $2q$. In this range of variation of $p$, the values of $p_N$ vary between

$$p_N(p_{\text{min}}) = q$$
$$p_N(p_{\text{max}}) = q \left[ 1 - \mathcal{O} \left( \frac{q}{M_B^0} \right) \right] .$$

For the particular cases considered here, $q \leq 1$ GeV/c and $M_B^0 \approx 19$ GeV/c, the difference between the maximum and the minimum values of $p_N$ amounts to less than 5% and hence, all of the effects related to final–state interactions that depend on $p_N$ should remain essentially constant.

In what follows we will present and discuss the results obtained (cross sections, response functions and asymmetries) from various points of view. First, we study the dependence of the observables on the struck–neutron momentum $p$. Different orientations of the target polarization are considered and a comparison between the results obtained using Nilsson and Hartree–Fock single–particle wave functions is presented. Second, we study the general properties of the observables and their dependence on the target polarization orientation. In the two cases, we discuss the situations in which the final nucleus is in its ground state and also consider the effects of having contributions from excited states. Third, we also study the influence of the nuclear deformation on the cross sections and asymmetries. Finally, we explore the effects when one considers high values of the momentum transfer $q$; specifically, we compare the results obtained for $q = 1$ GeV/c with those for $q = 500$ MeV/c.

### 5.3 The $p$–dependence of the Observables

In this section we study the dependence of differential cross sections and polarization observables on the momentum of the bound neutron $p$. The results are presented in Figs. 9–16. The value of the momentum transfer has been fixed to $q = 500$ MeV/c and three different target polarization directions have been considered (along $x$–, $y$– and $z$–axes) (see Fig. 2). Figures 9–11 correspond to the case in which the residual nucleus $^{20}\text{Ne}$ is in its ground state. As mentioned in Sect. 4, for this case only the $\ell_j = d_{3/2}$ component of the struck–neutron wave function enters. In Figs. 12–16 a sum over all the single–particle quantum numbers allowed by transitions to the ground and first excited state in $^{20}\text{Ne}$ has been performed; the inclusion of higher excited states in the daughter nucleus does not basically change the results shown here. In all of these figures the momentum $p$ covers the interval $0 \leq p \leq 2 \text{ fm}^{-1}$, while for higher values of $p$ the cross section would be too small to be measured and additionally short–range correlations (not included in our analysis) may start to play an important role. For the Nilsson model, the oscillator parameter $b$ has been fixed to reproduce the DDHF radius of the deformed single–nucleon orbital involved in the calculations: the value obtained
is \( b = 1.79 \text{ fm} \), which is a little bit lower than the value \( b = 1.82 \text{ fm} \) required to fit the r.m.s. charge radius of the whole nucleus (see Sect. 5.1). As discussed in Ref. [14] the use of a larger value, \( b = 1.82 \text{ fm} \) in the Nilsson model would produce in the cross sections and response functions higher peaks together with simultaneous displacements to the left (to lower values of the momentum \( p \)).

We begin the discussion by considering the totally unpolarized cross sections (incident electron and target not polarized) for the final nucleus in its ground state (Fig. 9). Forward–angle (\( \theta_e = 30^\circ \)) and backward–angle (\( \theta_e = 150^\circ \)) electron scattering is considered. In the former case, the main contributions come from the longitudinal responses, whereas the contributions of the transverse responses are dominant in the latter. The cross section obtained for forward–angle electron scattering is approximately a factor 10 higher than the cross section obtained in the backward–angle situation, a result that is obviously connected with the very different Mott cross sections obtained in the two situations. As mentioned above, co–planar (\( \phi_N = 0^\circ \)) and out–of–plane (\( \phi_N = 90^\circ \)) kinematics have both been considered and in the latter the longitudinal–transverse interference response is zero. For forward–angle electron scattering there are appreciable differences between the differential cross sections for \( \phi_N = 0^\circ \) and \( \phi_N = 90^\circ \) (the latter being of the order of \( \sim 1.3 \) higher in the maximum), while for backward–angle electron scattering the \( \phi_N = 0^\circ \) and \( \phi_N = 90^\circ \) results are much closer. This can be explained by taking into account the fact that the relative contributions of the responses which depend on the angle \( \phi_N \) (responses \( TL, TT \)), compared with the pure longitudinal and transverse responses (independent of \( \phi_N \)), are different for forward– and backward–angle electron scattering. In the first case (\( \theta_e = 30^\circ \)), such contributions are important, whereas for backward–angle scattering the purely transverse response function strongly dominates and the influence of the interference terms is small. Note that in the case of an ejected neutron the longitudinal response is much smaller than the transverse response at the momentum transfer considered (see Fig. 16).

Also seen in Fig. 9 are the results given by the two different nuclear models — Nilsson and Hartree–Fock. The difference between them is significant, being \( \sim 30\% \) higher for DDHF wave functions in the vicinity of the maximum, due mainly to the different weight, although also partly to the different shape of the \( d_{3/2} \) component of the struck–neutron wave function in the two models (see Table 3). In addition, note that the peak in the cross section obtained with Nilsson wave functions occurs for a \( p \)–value that is slightly higher than the corresponding one in the DDHF case. A slightly larger value of the Nilsson model oscillator parameter would improve the agreement with the DDHF result.

In Fig. 10 we show the results for the target polarization ratio \( P_\Sigma \). As discussed in Sect. 4, for the particular case where the daughter nucleus is in its ground state, \( P_\Sigma \) is independent of the nuclear model used in the calculation. The general expression for \( P_\Sigma \) (Eq. (39)) in the case of a 100\% polarized target with angular momentum \( J_A = 3/2 \) and parity \( \pi = +1 \) is simply reduced to \( P_\Sigma = -P_2(\cos \xi) \). Two orientations of the target polarization (along the \( z \)– and \( x \)–axes) are shown. The case of target polarization along the \( y \)–axis can be related to that along the \( x \)–axis: in particular, for co–planar kinematics and target polarization along the \( y \)–axis, the value obtained for \( P_\Sigma \) is the same as with target polarization along the \( x \)–axis and out–of–plane (\( \phi_N = 90^\circ \)) kinematics. The situation where the target polarization is along the \( y \)–axis and \( \phi_N = 90^\circ \) is also equivalent to that with target polarization along the \( x \)–axis and \( \phi_N = 0^\circ \). In the case of the target polarization along the \( z \)–axis, \( P_\Sigma \) is independent of the azimuthal angle \( \phi_N \). The full line represents the case \( \phi_N = 0^\circ \) and target polarization along the \( z \)–axis,
while the dashed (dotted) line corresponds to $\phi_N = 0^\circ$ ($\phi_N = 90^\circ$) and target polarization along the $x$–axis. In the situation given by $\phi_N = 90^\circ$ and $\mathbf{P}^*$ parallel to the $x$–axis, $\mathcal{P}_\Sigma$ is a constant ($\mathcal{P}_\Sigma = 1/2$) because $\cos \xi = 0$ (see Eq. (27)). This is always the case when the target polarization vector and the momentum of the ejected nucleon are in perpendicular planes and $\mathbf{P}^*$ is perpendicular to $\mathbf{q}$. The response functions $R^K$, $K = L, T, TL, TT$ obtained with 100% polarized $^{21}\text{Ne}$, are a factor 1.5 bigger than the responses obtained if the target was not polarized. In the case where $\phi_N = 0^\circ$ and $\mathbf{P}^*$ is parallel to the $x$–axis, the ratio $\mathcal{P}_\Sigma$ presents a very different behaviour; then one has $\cos \xi = \sin \theta$ and hence $\mathcal{P}_\Sigma$ depends on the value of the momentum $p$ (see Eq. (27)). For the extreme value $p = 0$ one has $\cos \xi = \pm 1$ and consequently $\mathcal{P}_\Sigma = -1$. As the value of $p$ increases, $\mathcal{P}_\Sigma$ decreases slowly in absolute value (its variation is of the order of $\sim 20\%$ for the interval of momenta considered). At $p \approx 0.75$ fm$^{-1}$, where the cross section is maximum, $\mathcal{P}_\Sigma \approx -0.95$. Therefore, the ratio between the electron–unpolarized cross sections $\Sigma$ for polarized and unpolarized targets grows from zero to 0.2 in the interval from $p = 0$ to $p = 2$ fm$^{-1}$. Finally, in the case where $\mathbf{P}^*$ is parallel to the $z$–axis, $\mathcal{P}_\Sigma$ does not depend on $\Delta \phi$ and one has $\mathcal{P}_\Sigma = 1/2$ at $p = 0$, which decreases slowly with $p$, being of the order of 0.255 for $p = 2$ fm$^{-1}$.

In Fig. 11 the results for the electron–target polarization ratio $\mathcal{P}_\Delta$ are shown. Figure 11(a) corresponds to forward–angle electron scattering ($\theta_e = 30^\circ$) and Fig. 11(b) to backward–angle electron scattering ($\theta_e = 150^\circ$). The orientations of the target polarization vector and values of the angle $\phi_N$ have been selected as discussed above. We start with the results shown in Fig. 11(a). When the target polarization is along the $z$–axis, the asymmetry $\mathcal{P}_\Delta$ depends very slightly on the value of $\phi_N$ selected. The difference between the results obtained for $\phi_N = 0^\circ$ and $\phi_N = 90^\circ$ is at most of the order of $\sim 15–20\%$. The behaviour of $\mathcal{P}_\Delta$ is the same in both cases, decreasing (towards zero) as the value of the momentum $p$ increases. For the other two orientations of the target polarization (along $x$– and $y$–axes) the behaviour presented by $\mathcal{P}_\Delta$ depends much more on the value of $\phi_N$ selected. For example, when the target polarization vector is oriented along the $x$–axis and $\phi_N = 0^\circ$ (co–planar kinematics), $\mathcal{P}_\Delta$ is very close to zero for small values of $p$ and increases as one goes to higher values of $p$. The same type of behaviour, especially for backward–angle electron scattering, is obtained when the target polarization points along the $y$–axis and $\phi_N = 90^\circ$. Finally, for the target polarization oriented along the $x$–axis and $\phi_N = 90^\circ$, $\mathcal{P}_\Delta$ is different from zero and almost independent of the value of the momentum $p$. We shall see later that this last result is obtained even when one is not restricted to the case of the residual nucleus in its ground state. The situation with the target polarization along the $y$–axis ($N$–polarized target) and co–planar kinematics gives $\mathcal{P}_\Delta = 0$, as already mentioned in Sect. 4. The discussion of the results for backward–angle electron scattering (Fig. 11(b)) follows the same trend. The only difference is the value of $\mathcal{P}_\Delta$ obtained for some particular situations: for example, when the target polarization is oriented along the $z$–axis, $\mathcal{P}_\Delta$ is approximately twice as small as the value obtained in the previous case, whereas if $\theta^* = \phi^* = \phi_N = 90^\circ$, $\mathcal{P}_\Delta$ is twice as large.

Figures 12–16 show the results obtained when the contributions from the first excited state $J_B^* = 2^+$ in the residual nucleus $^{20}\text{Ne}$ are also included. The allowed quantum numbers of the struck–neutron wave function in the spectral function are $j, j' = 3/2, 5/2, 7/2$ and $\ell, \ell' = 2, 4$. The odd–neutron wave function contains all of these components and they carry practically all of the normalization strength (see Table 3). Figure 12 corresponds to the totally unpolarized cross section. Again, forward– and backward–angle electron scattering, as well as values of $\phi_N = 0^\circ$ and $\phi_N = 90^\circ$, have been considered. The cross sections obtained are approximately
obtained with the two models are very similar for all of the responses. For high momenta where cross sections are very small. For the electron–unpolarized responses daughter nucleus in discrete states fall off rapidly and therefore the response functions and such differences start to increase very quickly, the spectral function for transitions leaving the daughter nucleus in discrete states is almost constant for all ∆p-values and in particular for N–polarized nuclei |PΔ| = 0 as expected. For ∆φ = 0°, |PΔ| increases with the momentum p.

Finally, in Fig. 16 we show the results for the six hadronic response functions R^K, R^K′ obtained with Nilsson and DDHF wave functions. Co–planar kinematics and two orientations of the target polarization (along the z– and x–axes) have been selected. Note that the results obtained with the two models are very similar for all of the responses. For high momenta where such differences start to increase very quickly, the spectral function for transitions leaving the daughter nucleus in discrete states fall off rapidly and therefore the response functions and cross section are very small. For the electron–unpolarized responses R^K, K = L, T, TL, TT, the difference between the results obtained for the two orientations of the target polarization

In Fig. 13 we show the results for the target polarization ratio PΣ. The situations selected are the same as discussed in Fig. 10. The value of PΣ for nuclei polarized along the x–axis and φN = 0°(90°) is the same as for N–polarized nuclei and φN = 90°(0°). The differences produced by using Nilsson or DDHF wave functions are almost negligible for p ≤ 1 fm⁻¹. For higher values of p such differences start to increase, being of the order of ∼25% when p = 1.5 fm⁻¹. For p > 1.5 fm⁻¹, the results in Fig. 13 must be viewed with caution because the tails of the presently used single–particle wave functions in momentum space are likely not reliable and, as previously noted, in this region dynamical short–range correlations begin to be important and to produce strong enhancements of the high–momentum tails of the Fourier transforms of the single–nucleon wave functions. Comparing the results in Fig. 13 with the case where the residual nucleus is in its ground state (Fig. 10), one notes that PΣ is now much smaller in absolute value, i.e., the effects introduced in the electron–unpolarized response functions by the polarization of the target are much weaker. In particular, at the cross section peak (p ≈ 0.75 fm⁻¹) these effects are of the order of 8% for target polarization along q or target polarization along the x–axis and φN = 90°, and of the order of 15% for target polarization along the x–axis and φN = 0°.

The results for the asymmetry PΔ are presented in Figs. 14 (forward–angle electron scattering) and 15 (backward–angle electron scattering). Some of the comments previously made in the discussion of Figs. 11 and 13 can also be applied here and consequently we only mention the most noteworthy differences. For the target polarized along the z–axis, the dependence of PΔ in φN is weak, especially for backward–angle electron scattering where there is basically no difference between the results obtained for φN = 0° and φN = 90°. Furthermore, PΔ does not change much with momentum except for p in the vicinity of 2 fm⁻¹ where the Hartree–Fock wave functions have zeros. For the other two selected orientations of the target polarization (Figs. 14(b), 14(c) and 15(b), 15(c)), one sees that the behaviour of PΔ depends mainly on ∆φ = φ* – φN. For |∆φ| = 90°, PΔ is almost constant for all p–values and in particular for N–polarized nuclei PΔ = 0 as expected. For ∆φ = 0°, |PΔ| increases with the momentum p.
is of the order of 25–30% in the region close to the maximum, \( p \approx 0.75 \text{ fm}^{-1} \). From the results in Fig. 13, one sees that for higher values of the struck–neutron momentum \( p \) such difference starts to decrease, whereas for \( p < 0.8 \text{ fm}^{-1} \) the difference increases. The behaviour presented by the two electron–polarized response functions is different: for \( R^T \) the results for the two orientations of the target polarization are much more different, of the order of a factor \( \sim 6 \) smaller (around the peak) for the case where \( P^* \) is parallel to \( q \). The fact that a neutron is ejected in the scattering reaction explains why the purely transverse response \( R^T \) is dominant, \( \text{viz.} \), a factor of the order of \( \sim 30 \) bigger than the \( R^L \)–response in the region close to the peak.

### 5.4 Dependence of the Asymmetries on the Target Polarization Orientation

In Figs. 17–22 we show the results obtained for the asymmetries \( P_\Sigma \) and \( P_\Delta \) as functions of the target polarization angle \( \theta^* \). The value of the momentum transfer has again been fixed to \( q = 500 \text{ MeV/c} \) and the kinematics selected are the same as in the previous figures, co–planar (\( \phi_N = 0^\circ \)) and out–of–plane (\( \phi_N = 90^\circ \)). In all cases, DDHF single–nucleon wave functions have been used, and five values of the bound neutron momentum \( p \) have been selected, \( p = 0.2, 0.6, 1.0, 1.4 \) and \( 1.8 \text{ fm}^{-1} \). The relationship between these \( p \)–values and the angles \( \theta \) and \( \theta_N \) in the case where the residual nucleus \( ^{20}\text{Ne} \) is in its ground state can be seen in Table 4. These \( \theta^* \)– and \( \theta_N^* \)–values would only be slightly different when contributions from excited states in \( ^{20}\text{Ne} \) are included. Also for simplicity in the discussion, in all cases we have restricted the target polarization vector \( P^* \) to the scattering plane, \( i.e. \), \( \phi^* = 0^\circ \). Three possible situations for the residual nucleus are considered, i) ground state \( (0^+) \), ii) ground state and first excited state \( (0^+, 2^+) \) and iii) ground state and first two excited states \( (0^+, 2^+, 4^+) \).

We start the discussion with the case of \( P_\Sigma \). As seen in Eq. (38), the entire dependence of \( P_\Sigma \) on the target polarization direction is given through the Legendre polynomials \( P_I(\cos \xi) \), where \( \xi \) is the relative angle between the target polarization vector \( P^* \) and the momentum of the bound neutron \( p \). \( I \) is the tensor rank which can only take the value \( I = 2 \) in this case. Therefore, the maximum value of \( |P_\Sigma| \) is obtained when \( p \) and \( P^* \) are parallel or antiparallel. Note that \( P_\Sigma \) positive means an increase of the differential cross section due to target polarization, while \( P_\Sigma \) negative means a decrease, and that the sign of \( P_\Sigma \) does not only depend on \( P_2(\cos \xi) \) but on the specific transitions considered. Accordingly, for observational purposes it is interesting to note which are the largest positive values that \( P_\Sigma \) takes on for different situations. To analyze this we consider the conditions for extrema in \( P_\Sigma \): for arbitrary fixed values of \( \Delta \phi \) these are

\[
\tan \theta^* = \tan \theta \cos \Delta \phi \quad (55a)
\]
\[
\cot \theta^* = -\tan \theta \cos \Delta \phi \quad (55b)
\]

For \( \Delta \phi = 0^\circ \) (\( P^* \) and \( p_N \) located in the same plane) condition (55a) corresponds to \( p \) parallel to \( P^* \) which is the condition for \( |P_\Sigma| \) maximum mentioned above; on the other hand, condition (55b) corresponds to \( p \) perpendicular to \( P^* \). For \( \Delta \phi = \pm 90^\circ \) (\( P^* \) and \( p_N \) in perpendicular planes) the extreme conditions are obtained for fixed values of \( \theta^* \) independent of the direction in which the struck neutron is moving. Condition (55a) corresponds to \( \theta^* = 0^\circ, 180^\circ \) and condition (55b) corresponds to \( \theta^* = 90^\circ \). This is illustrated in Figs. 17–19.

Figure 17 corresponds to the situation in which the residual nucleus is in its ground state. In this case one has \( P_\Sigma = -P_2(\cos \xi) \). For \( \phi^* = \phi_N = 0^\circ \) (\( \Delta \phi = 0^\circ \)) (see Fig. 17(a)), the maximum in \( P_\Sigma \) is obtained for \( \theta^* = \theta - 90^\circ \) (see Table 4) and its value (\( P_\Sigma = 1/2 \)) does not
depend on the momentum \( p \). The minimum value of \( P_\Sigma \) (\( P_\Sigma = -1 \)) is also independent of \( p \) and is obtained for \( \theta^* = \theta \) which is the condition for \( |P_\Sigma| \) maximum. This last condition means that when the bound neutron is moving along the direction of the target polarization vector \( \mathbf{P}^* \), the response functions \( R^K \), \( K = L, T, TL, TT \) are zero. In other words, in this case the differential cross section for unpolarized electrons and polarized target is zero. The relation, \( \cos(\theta^* - \theta) = \pm 1/\sqrt{3} \) gives the condition for which the asymmetry \( P_\Sigma \) is zero, \( i.e., \) the polarization of the target does not have any observable effect on the responses \( R^K \). The results in Fig. 17(b) correspond to \( \phi^* = 0^\circ \) and \( \phi_N = 90^\circ \) (\( \Delta \phi = -90^\circ \)). As one can see, the maximum in \( P_\Sigma \) is obtained for \( \theta^* = 90^\circ \) and its value (\( P_\Sigma = 1/2 \)) is constant for all the struck–neutron momenta \( p \). The minimum value of \( P_\Sigma \) corresponds to \( \theta^* = 0^\circ, 180^\circ \) and its value depends on \( p \) (see Eq. (39)) through the Legendre polynomial. Finally, the zeros of \( P_\Sigma \) are given by the relation, \( \cos \theta^* \cos \theta = \pm 1/\sqrt{3} \), which can only be satisfied at \( p \)–values larger than those considered in Fig. 17(b).

Figures 18–19 show the results for \( P_\Sigma \) including the contributions from the first and also from the first and second excited states in the residual nucleus \( ^{20}\text{Ne} \), respectively. Contrary to the previous situation, in these cases the extreme values of \( P_\Sigma \) always depend on the value of the struck–neutron momentum \( p \). The conditions for maximum and minimum depend not only on \( p \) but also on the number of excited states of the residual nucleus considered in the calculations. In this sense, compare for example the results at \( p = 1.4 \text{ fm}^{-1} \) and \( 1.8 \text{ fm}^{-1} \) in Figs. 18(a) and 19(a) or 18(b) and 19(b). As seen in Figs. 18(b) and 19(b), for \( \phi^* = 0^\circ \) and \( \phi_N = 90^\circ \), \( P_\Sigma \) remains small and changes little with \( \theta^* \) for fixed \( p \). As a general rule if one compares the results in Figs. 18–19 with the previous case shown in Fig. 17, one can see that the range of values of \( P_\Sigma \) is now at least a factor 10 smaller. This effect was already discussed in Sect. 5.1 and allows us to conclude that the poorer the energy resolution is (\( i.e., \) when more states of the residual nucleus are involved in the analysis), the less the effects of the target polarization will be. The magnitude of this effect also depends on the nuclear model used to describe the nuclear excitation.

In Figs. 20–22 we present the results for the polarization ratio \( P_\Delta \). Only results for forward–angle electron scattering (\( \theta_e = 30^\circ \)) are shown in the figures, although the results for backward–angle electron scattering are similar except for the fact that the maximum values of \( P_\Delta \) are generally larger (by a factor of the order of \( 2 \) for \( \theta_e = 150^\circ \)), as illustrated previously in Figs. 11, 14 and 15. As seen in Sect. 4, the general expression for \( P_\Delta \) (Eqs. (40, 41)) is much more complex than in the case of \( P_\Sigma \). The latter asymmetry only depends on the relative angle between \( \mathbf{p} \) and \( \mathbf{P}^* \), while \( P_\Delta \) depends in addition on \( \mathbf{p}_N \). Moreover, for \( ^{21}\text{Ne} \) there are two tensor components that contribute to \( P_\Delta \) with ranks \( I = 1 \) and \( I = 3 \), and therefore, there are no simple analytical expressions for the extreme conditions of \( P_\Delta \) or for their zeros. However, when \( |P_\Delta| \) is large the component \( I = 1 \) strongly dominates (in these cases the contribution of \( I = 3 \) is at most of the order of \( \sim 20\% \)) and the extreme conditions obtained considering only the component \( I = 1 \) reproduce the behaviour seen in Figs. 20–22 reasonably well.

Figure 20 corresponds to the case where the residual nucleus is in its ground state. For \( \phi_N = 0^\circ \) (Fig. 20(a)), \( P_\Delta \) reaches its maximum for small values of \( \theta^* \). As the momentum \( p \) increases the maximum of \( P_\Delta \) is slightly displaced to higher values of \( \theta^* \). Note that for \( 0.2 \leq p \leq 1.8 \text{ fm}^{-1} \) the maxima of \( P_\Delta \) are obtained for \( \theta^* < 35^\circ \). As \( \theta^* \) increases, \( P_\Delta \) decreases and becomes very close to zero for \( \theta^* \approx 110^\circ \). In this last situation, the polarization of the incident electrons has no observable effect on the differential cross section. For \( \phi_N = 90^\circ \)
direction. Contributions from the ground and first excited state in the residual nucleus have been included in the calculations. It should be remarked that in the case of the residual angle electron scattering (θ differential cross section (Eq. (1)) as a function of the target polarization angle = 0 δ important for its maximum in the range 130°. It is interesting to note that for θ* = 90° the values of \( \mathcal{P}_\Delta \) are almost the same (≈ −0.1) for all momenta p selected.

In Figs. 21–22 contributions from excited states in the residual nucleus 20Ne have been added to the contributions to the ground state (0+), in particular, the first (2+) excited state in Fig. 21, and the first and second (4+) excited states in Fig. 22. In both cases one sees that \( \mathcal{P}_\Delta \) has a similar dependence on θ* and also takes on similar values. For \( \phi_N = 0° \) (Figs. 21(a)–22(a)) one can clearly observe different regions: for \( \theta^* \leq 60° \), |\( \mathcal{P}_\Delta \)| is rather constant and significant; for \( 60° \leq \theta^* \leq 130° \), |\( \mathcal{P}_\Delta \)| varies quite rapidly with \( \theta^* \) until it reaches its maximum in the range \( 130° \leq \theta^* \leq 150° \). For \( \phi_N = 90° \) (Figs. 21(b)–22(b)), \( \mathcal{P}_\Delta \) has a very regular behaviour that is independent of the p–value: for \( \theta^* \approx 90° \), \( \mathcal{P}_\Delta \) is approximately zero and for \( \theta^* \approx 0°, 180° \), |\( \mathcal{P}_\Delta \)| takes on its maximum value. In summary, in the different kinematical situations considered, the absolute values of the asymmetry \( \mathcal{P}_\Delta \) are always more important for \( \theta^* \)–values close to 0° and 180°.

To finish with the presentation of the results in this section, in Fig. 23 we show the total differential cross section (Eq. (1)) as a function of the target polarization angle \( \theta^* \). Forward–angle electron scattering (\( \theta^* = 30° \)) has been selected and contributions from the ground and first excited state in the residual nucleus have been included in the calculations. The values of the momentum transfer q and the struck–neutron momentum p have been fixed to q = 500 MeV/c and p = 0.75 fm\(^{-1}\), respectively, where, as previously stated, the latter corresponds approximately to the momentum where the peak of the cross section occurs. For the kinematics considered, \( \theta = 100.14° \). Three values of the azimuthal angle \( \phi_N \) have been used, \( \phi_N = 0° \) (Fig. 23(a)), \( \phi_N = 66° \) (Fig. 23(b)) and \( \phi_N = 90° \) (Fig. 23(c)). In each case, the differential cross sections obtained for different orientations of the target polarization are represented and the totally unpolarized cross section is also shown for comparison. Note that the general behaviour of the cross section is similar in the three graphs. From the results obtained, one can see that the maximum in the total cross section is always reached when \( \phi^* = \phi_N \), i.e., when the target polarization vector \( \mathbf{P}^* \) and the ejected neutron momentum \( \mathbf{p}_N \) are located in the same plane. Any other choice for the angles \( \phi^* \) and \( \phi_N \) reduces the maximum of the cross section. These results also allow one to see which are the target polarization and out–going–nucleon directions that produce a higher increment of the differential cross section relative to the unpolarized case. Specifically, for the selected kinematics (in Figs. 23, \( \theta_N \approx 15° \) and \( \theta \approx 110° \)) the highest increment corresponds to \( \phi_N = \phi^* \) and \( \theta^* \approx 130° \).

5.5 Dependence on the Nuclear Deformation

In this section we briefly discuss the effect that the nuclear deformation has on the various observables above discussed. For this purpose we use the Nilsson model where we can easily see what is the effect of deformation by changing the value of \( \delta \). The results are presented in Fig. 24: for both unpolarized cross sections (Fig. 24(a)) and asymmetries \( \mathcal{P}_\Sigma \) and \( \mathcal{P}_\Delta \) (Fig. 24(b)), we present ratios between the results obtained for different values of the deformation parameter \( \delta \) and the result corresponding to the equilibrium deformation \( \delta = 0.31 \) (see Sect. 4). All of the results correspond to forward–angle electron scattering (\( \theta_e = 30° \)), co–planar kinematics (\( \phi_N = 0° \)) and target polarization \( \mathbf{P}^* \) oriented along \( \mathbf{q} \)–direction. Contributions from the ground and first excited state in the residual nucleus 20Ne have been included in the calculations. It should be remarked that in the case of the residual
nucleus in its ground state the asymmetries $P_{\Sigma}$ and $P_{\Delta}$ are not affected by the nuclear deformation. Figure 24(a) shows the variation of the unpolarized cross section with the nuclear deformation for three different values of the bound neutron momentum, $p = 0.5$ fm$^{-1}$ (full), 0.75 fm$^{-1}$ (short–dashed) and 1.5 fm$^{-1}$ (dashed). As observed in the figure, the weakest dependence on the nuclear deformation is obtained for $p = 0.75$ fm$^{-1}$ which corresponds to the maximum in the cross section. On the contrary, for large $p$–values the cross section falls as low as a factor of four below the equilibrium result for very large deformations ($\delta = 0.7$). Figure 24(b) corresponds to the asymmetries $P_{\Sigma}$ and $P_{\Delta}$. One can observe that the effects of the nuclear deformation lie within a range of $\sim$25–30% in three cases. Only for $p = 1.5$ fm$^{-1}$ and $P_{\Sigma}$ do the results clearly deviate from the equilibrium value — in this case because of a change in the sign of $P_{\Sigma}$ when one goes to large values of the deformation parameter $\delta$.

5.6 Results for High Momentum Transfer

All of the results presented in previous figures have been obtained by fixing the value of the momentum transfer to $q = 500$ MeV/c and fixing the energy transfer $\omega$ to the value given by Eq. (47) corresponding to the quasielastic peak. In this section our aim is to study briefly the possible effects introduced when one considers a larger $q$–value, still working in the region of the quasielastic peak. In particular, we fix $q = 1$ GeV/c where one expects that for such high momentum transfer the assumptions that go into the PWIA will become even more valid. Specifically, in Figs. 25 we show the ratios between the results obtained for $q = 1$ GeV/c and $q = 500$ MeV/c: Fig. 25(a) corresponds to the totally unpolarized cross section and Figs. 25(b) and 25(c) to the asymmetries $P_{\Sigma}$ and $P_{\Delta}$, respectively. Contributions from the ground and first excited state in the residual nucleus $^{20}$Ne have been included in the calculations. As usual, co–planar ($\phi_{N} = 0^\circ$) and out–of–plane ($\phi_{N} = 90^\circ$) kinematics, as well as three orientations — along the $x$–, $y$– and $z$–axes — have been considered for the target polarization. Before entering into a discussion of the results, it is interesting to remark that for a fixed value of the struck–neutron momentum $p$, the corresponding value of the angle $\theta_{N}$ defining the direction of the ejected neutron momentum is much smaller for higher values of $q$ that in the situation treated above (see Eq. (52)) yielding, for example, $\theta_{N} = 47.06^\circ$ for $q = 500$ MeV/c and $\theta_{N} = 22.85^\circ$ for $q = 1$ GeV/c at $p = 2$ fm$^{-1}$.

We start the discussion with the results for the totally unpolarized cross section (Fig. 25(a)). Full and dashed lines correspond to $\phi_{N} = 0^\circ$ and $\phi_{N} = 90^\circ$, respectively. Note that in both cases the results vary only mildly with the struck–neutron momentum $p$, with the variation being at most on the order of 6–7%. These results show that the totally unpolarized cross section is roughly four times smaller for $q = 1$ GeV/c than for $q = 500$ MeV/c at all $p$–values. In Fig. 25(b) we present the results for the target polarization ratio $P_{\Sigma}$. The three curves shown correspond to $\phi_{N} = 0^\circ$ and target polarization vector $P^{*}$ along the $x$– (full line), $y$– (dashed) and $z$–axes (dotted). As discussed in Sect. 5.1, the results for $\phi_{N} = 90^\circ$ and the three orientations selected of the target polarization would be similar. Before entering into the discussion of the different specific situations, one can see by examining the general expressions introduced in Sect. 4 (Eq. (38)) that the difference in the asymmetry $P_{\Sigma}$ obtained for different values of the momentum transfer $q$ comes exclusively through the Legendre polynomials $P_{I}(\cos \xi)$. For $P^{*}$ parallel to the $y$–axis, a constant ratio of unity is obtained in the whole range of $p$–momenta, since $\cos \xi = 0$ and therefore $P_{\Sigma}$ is independent of $q$. We recall that in this situation the response functions $R_{LL}^{T'}$ and $R_{LL}^{T'L'}$ are zero and hence $P_{\Delta} = 0$. For the other two orientations of the target polarization, we have $\cos \xi = \cos \theta$ for $P^{*}$
parallel to \( q \), and \( \cos \xi = \sin \theta \) for \( P^* \) parallel to the \( x \)-axis. Accordingly, the ratio between the asymmetries \( P_\Sigma \) calculated for \( q = 1 \) GeV/c and \( q = 500 \) MeV/c changes with the value of the struck–neutron momentum \( p \). As \( p \) increases, the ratio between the results obtained for the two values of the momentum transfer \( q \) also increases and one sees that this effect is more pronounced in the case when the target polarization vector is parallel to \( q \).

Finally, in Fig. 25(c) we present the results for the asymmetry \( P_\Delta \) where the scattering angle has been fixed to \( \theta_e = 30^\circ \). For \( \phi_N = 0^\circ \) the results shown correspond to \( P^* \) parallel to \( q \) (dashed line) and \( P^* \) parallel to the \( x \)-axis (full line), while for \( \phi_N = 90^\circ \) the curves correspond to the three orientations of the target polarization, along the \( x \)- (dotted), \( y \)- (short–dashed) and \( z \)-axes (dot–dashed). The behaviour presented by the results for \( \phi_N = 0^\circ \) and \( P^* \) parallel to the \( x \)-axis is due to the zero of the asymmetry \( P_\Delta \) for \( p \approx 1.2 \) fm\(^{-1} \) and \( q = 500 \) MeV/c (see Fig. 14(b)). For \( \phi_N = 90^\circ \) and \( P^* \) parallel to the \( y \)-axis (short–dashed), the results obtained are contained in the range \( \sim [0.3–0.8] \). In the region \( p < 0.6 \) fm\(^{-1} \) the ratio decreases with the momentum \( p \), whereas for \( 0.6 < p < 1.0 \) fm\(^{-1} \) the ratio is rather constant (\( \approx 0.3 \)) and starts to increase slowly for higher \( p \)-values. In the case of the target polarized along \( q \), the results obtained for \( \phi_N = 0^\circ \) and \( \phi_N = 90^\circ \) are almost identical and equal to unity, indicating that in this situation for a fixed value of the momentum \( p \) the asymmetry \( P_\Delta \) takes on the same value regardless of the value of the momentum transfer \( q \). Only for very high \( p \)-values, \( p > 1.8 \) fm\(^{-1} \), does the ratio start to deviate slightly from unity. Finally, in the case where \( \phi_N = 90^\circ \) and \( P^* \) is parallel to the \( x \)-axis the ratio (of the order of 1.2) is practically constant for all \( p \)-values. This means that \( P_\Delta \) for \( q = 1 \) GeV/c is of the order of \( \sim 20\% \) bigger than the value of \( P_\Delta \) obtained for \( q = 500 \) MeV/c for a fixed value of \( p \).

### 6. CONCLUSIONS

In this work we have studied polarization degrees of freedom in coincidence electron scattering from deformed nuclei within the context of the Plane–Wave Impulse Approximation (PWIA). In particular, we have focused on \((e, e'N)\) reactions where both the incident electrons and the target nuclei are assumed to be polarized. Our main objective in this work has been centered on a treatment of the polarized spectral function for deformed nuclei, while a study of the electron–nucleon cross section and single–nucleon response functions for polarized electron scattering from bound “off–shell” nucleons has already been presented in Refs. [3] and summarized in Sect. 2. As an initial choice of deformed nucleus we have selected \(^{21}\text{Ne} \) for several reasons: firstly, from an experimental point of view, \(^{21}\text{Ne} \) is a noble gas and, as in the more familiar case of \(^3\text{He} \), is a good candidate for a polarized target; secondly, it presents a well–established rotational energy spectrum;\(^{[5]} \) and finally, its charge \((Z = 10)\) is not large and therefore the electron Coulomb distortion will not be particularly important in the description of the scattering. The PWIA is expected to be a good approximation in the analysis of the reaction for the sufficiently high momentum transfers and quasielastic kinematics chosen throughout this work.

A general expression for the polarized spectral function for transitions leaving the residual nucleus in discrete states has been obtained. General properties and angular symmetries have been discussed in detail in Sect. 3. Explicit expressions for the hadronic response functions and polarization ratios have been presented in Sect. 4 by combining the polarized spectral function with the single–nucleon responses.\(^{[3]} \) In this context, the asymmetry \( P_\Sigma \) (which depends exclusively on the polarization of the target) has been proven not to be affected by the “off–shell” uncertainties in the treatment of the bound nucleons. On the other hand, in
the case of the residual nucleus in its ground state, the two asymmetries introduced, \( P_\Sigma \) and \( P_\Delta \) have been proven to be independent of the nuclear model used in the evaluation of the single–particle wave functions.

In obtaining specific results we have considered different representative ranges of kinematics — moderate and high \( q \)–values although always at the quasielastic peak, forward– and backward–angle electron scattering, coplanar and non–coplanar nucleon detection and finally different orientations of the target polarization axis \( \mathbf{P}^* \). The results presented in Sect. 5 for unpolarized cross sections as well as for polarization observables (asymmetries \( P_\Sigma, P_\Delta \)) have been examined in detail from different points of view. In general, we find that the range of predictions obtained for such asymmetries is smaller than that seen for the individual response functions or cross sections. We have shown the dependence of the observables on the struck–neutron momentum \( p \) and have seen that the results obtained by using Nilsson and DDHF single–nucleon wave functions are very similar for \( p \)–values within the range \([0–2] \text{ fm}^{-1}\). A general study of the dependence of the polarization observables on the polarized target orientation has also been performed and, in particular, we have explored the kinematical situations for which one observes the biggest effects due to the polarization of the target and electrons. It should be noted that, as a general rule, one can conclude that the poorer the energy resolution is (i.e., when more states of the residual nucleus are involved in the analysis), the less will be the effects of the target polarization. Another interesting result obtained indicates that the global maximum in the differential cross section is always reached when the target polarization vector \( \mathbf{P}^* \) and the ejected neutron momentum \( \mathbf{p}_N \) are located in the same plane. Furthermore, the dependence on the nuclear deformation has been shown not to be very important (the effects are bigger for high values of the struck–neutron momentum \( p \)). Finally, we have shown that under some specific kinematics, the polarization observables are almost independent of the value considered for the momentum transfer \( q \).

As already mentioned, in this work we have investigated in detail spin degrees of freedom in coincidence electron scattering from deformed nuclei within the context of the PWIA. In future work, we intend also to present results for other interesting targets, where we will focus on a study of the interferences between different single–nucleon orbitals and their connections with the polarization degrees of freedom, and to extend the results presented here to the high–missing–energy regime for application in studies of polarized \(^3\text{He}\). Our longer–term objective is also to go beyond the PWIA. In this regard, we end by remarking that ultimately, if polarization–coincidence measurements of the type suggested by the present studies were to prove feasible and if sufficiently fine information on the various components of the polarized spectral function could be obtained, then it would be possible to investigate not only the nuclear structure issues involved in arriving at those distributions, but also the nature of the final–state propagation of the outgoing nucleon. Indeed, using a single species of deformed nucleus might provide the ideal situation for such studies: then the particular nuclear configurations could be selected and yet, as the distribution of nuclear matter in this case is asymmetric, different aspects of the final–state interaction could be probed — for instance, the degree of “transparency” in different directions could be explored.

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APPENDIX A. POLARIZED SPECTRAL FUNCTION

In this appendix we present in detail the algebra needed to obtain a general expression for the polarized spectral function in bound nuclear systems. The general expression for the polarized spectral function is given by Eq. (7). There the single–nucleon creation operators \( a^+_{p m} \) can be expanded over a basis of irreducible tensorial operators \( a^+_{\ell j m_j} (p) \) in the following way:

\[
a^+_{p m} = \sum_{\ell j} \sum_{m \ell m_j} <\ell m_{\ell} | j m_j > Y_{\ell}^{* m_{\ell}} (\Omega) a^+_{\ell j m_j} (p) ,
\]

where \( <\ell m_{\ell} | j m_j > \) are the Clebsch–Gordan coefficients; \( \ell, j \) are the single–nucleon quantum numbers and \( m_{\ell}, m_j \) their projections referred to \( \mathbf{q} \)–direction. \( Y_{\ell}^{* m_{\ell}} (\Omega) \) is the spherical harmonic given by the angular variables \( \Omega \equiv \{ \theta, \phi \} \) which define the direction of the momentum \( p \) in the \( xyz \)–system (Fig. 2). In the case of annihilation operators \( a_{p m}, \) the above expansion reads

\[
a_{p m} = \sum_{\ell j} \sum_{m \ell m_j} <\ell m_{\ell} | j m_j > Y_{\ell}^{m_{\ell}} (\Omega) (-1)^{j m_j} \tilde{a}_{\ell j m_j} (p) ,
\]

where \( \tilde{a}_{\ell j m_j} \) are hole creation operators and consequently irreducible tensorial operators. They are related with the annihilation operators \( a_{\ell j m_j} \) by

\[
\tilde{a}_{\ell j m_j} = (-1)^{j + m_j} a_{\ell j m_j}
\]

Introducing these relations into the general expression for the spectral function (Eq. (7)) we obtain

\[
S_{m m'} (p, E_m, \Omega^*) = \sum_{B} \sum_{\ell \ell'} \sum_{j j'} \sum_{m \ell m'} \sum_{m_j m'_j} Y_{\ell}^{m_{\ell}} (\Omega) Y_{\ell'}^{m'_{\ell}} (\Omega) \times \delta (E_m + \epsilon_B - \epsilon_A) ,
\]

Expressing the Clebsch–Gordan coefficients in terms of the 3–j’s and using the Wigner–Eckart theorem and some relations held by the rotation matrices (Ref. [25]) we can write

\[
S_{m m'} (p, E_m, \Omega^*) = \sum_{B} \sum_{\ell \ell'} \sum_{j j'} \sum_{m \ell m'} \sum_{m_j m'_j} Y_{\ell}^{m_{\ell}} (\Omega) Y_{\ell'}^{m'_{\ell}} (\Omega) (-1)^{\ell - \ell' + m_j - m'_j} [j][j']
\]

\[
\times \sum_{I \ell M} \sum_{M_A} p(A) (-1)^{J_A - M_A} \left[ I \right]^2 D^{* I}_{0 M} (\Omega^*) C^{J_A J_B}_{\ell j} (p) C^{J_A J_B}_{\ell' j'} (p)
\]

\[
\times \left( \begin{array}{ccc} \ell & m_{\ell} & j \\ m & m' & -m_j \end{array} \right) \left( \begin{array}{ccc} \ell' & m'_{\ell} & j' \\ m' & m'' & -m'_{m_j} \end{array} \right) \left( \begin{array}{ccc} J_A & J_A & I \\ M_A & -M_A & 0 \end{array} \right)
\]

\[
\times \sum_{M_A' M_B} (-1)^{J_A - M_A'} \left( \begin{array}{ccc} J_A & J_A & I \\ M_A' & -M_A' & M \end{array} \right) \left( \begin{array}{ccc} J_A & j & J_B \\ -M_A' & m_j & M_B \end{array} \right) \left( \begin{array}{ccc} J_A & j' & J_B \\ -M_A' & m'_j & M_B \end{array} \right)
\]

\[
\times \delta (E_m + \epsilon_B - \epsilon_A) ,
\]

(A.5)
where we have introduced the reduced nuclear matrix elements \( C_{\ell j}^{J_A J_B} (p) \) and \( C_{\ell j'}^{J_A J_B} (p) \) with \( C_{\ell j}^{J_A J_B} (p) \equiv \langle J_A | a^+_j (p) | J_B \rangle \) and are using the notation \([j] \equiv \sqrt{2j + 1} \).

Summing over the indices \( M_A', M_B' \) and \( M_B \) and using the relation (6.2.8) in Edmonds,\textsuperscript{[25]} the expression of the spectral function is reduced to

\[
S_{mm'} (p, E_m, \Omega^*) = \sum_B \sum_{\ell \ell'} \sum_{jj'} \sum_{I M} \sum_{m m'} \sum_{m_j m_j'} J^A_{\ell j} (-1)^{\ell - \ell' - m_j + J_A + J_B} \delta (E_m + \epsilon_B - \epsilon_A^0)
\]

with \( f^A_{\ell j} \) being the spherical Fano statistical tensor given by

\[
f^A_{\ell j} = \sum_{M_A} p(A) (-1)^{J_A - M_A} [I] \left( \begin{array}{ccc} J_A & J_A & I \\ M_A & -M_A & 0 \end{array} \right).
\]

Using the relations obeyed by the spherical harmonics and expressing the rotation matrix containing the target polarization angles in terms of spherical harmonics\textsuperscript{[25]} one obtains

\[
S_{mm'} (p, E_m, \Omega^*) = \sum_B \sum_{\ell \ell'} \sum_{jj'} \sum_{I M} \sum_{m m'} \sum_{m_j m_j'} J^A_{\ell j} (-1)^{\ell - \ell' + J_A + J_B} \delta (E_m + \epsilon_B - \epsilon_A^0)
\]

Finally, one can even simplify the sum over the indices \( m_\ell, m'_\ell, m_j \) and \( m_j' \) by using the relation (2.3.1) in Rotenberg\textsuperscript{[35]}. Taking into account selection rules embodied in the 3–j and 6–j coefficients, the final expression obtained for the different tensor components of the polarized spectral function (Eq. (16)) is

\[
S^{(I)}_{mm'} (p, E_m, \Omega^*) = (-1)^{m - 1/2} f^A_{\ell j} \sum_{\ell \ell'} \sum_{jj'} \sum_{L K} \sum_{M} (-1)^{J_A + J_B + j + \ell'} [j][j'][\ell'][\ell'][L][K]^2
\]

\[
\times C_{\ell j}^{J_A J_B} (p) C_{\ell j'}^{J_A J_B} (p) Y^{\bar{M}}_{\ell} (\Omega^*) Y^{H}_{\ell'} (\Omega) \left( \begin{array}{ccc} \ell & \ell' & L \\ 0 & 0 & 0 \end{array} \right) \left( \begin{array}{ccc} L & I & K \\ H & M & N \end{array} \right)
\]

\[
\times \delta (E_m + \epsilon_B - \epsilon_A^0)
\]
A.1. The Unpolarized Case

From the above expression for the polarized spectral function we can easily obtain the results corresponding to unpolarized target and unpolarized electron. In such a case, the only tensor polarization component which contributes in the spectral function is $I = 0$. The Fano statistical tensor is simply $f_{0J} = 1/[J_A]$ regardless of the detailed population of the magnetic substates. Therefore, the spin–components of the spectral function are reduced to

$$S_{mm'}(p, E_m, \Omega^*) = \sum_B \sum_{\ell \ell'} \sum_{j j'} \sum_L (-1)^{j+\ell-\ell'+m'} \frac{[\ell'][\ell][L]}{[J_A]^2 \sqrt{4\pi}} Y_L^{-H}(\Omega)C_{\ell j}^{sJ_A}J_B(p)C_{\ell' j'}^{J_A J_B}(p)$$

$$\times \left( \begin{array}{ccc} \ell & \ell' & L \\ 0 & 0 & 0 \end{array} \right) \left( \begin{array}{ccc} L & 1/2 & 1/2 \\ -H & m & -m' \end{array} \right) \left\{ \begin{array}{ccc} 1/2 & 1/2 & L \\ \ell' & \ell & j \end{array} \right\} \delta(E_m + \epsilon_B - \epsilon_A^0).$$

(A.10)

From the selection rules embodied in the 3–j coefficients one sees that the only possible values of $L$ are 0 or 1. Moreover, from parity considerations (as mentioned above, we consider only parity–conserving processes) the values of $\ell$ and $\ell'$ must be both even or odd, and this means that $L$ must be even or else the spin–component spectral function will be identically zero. Therefore, one has $L = 0$ and $\ell = \ell'$. With these results one can easily see that the dependence on the angular variables $\Omega = \{\theta, \phi\}$ disappears and the spectral function depends only on the magnitude of the struck–nucleon momentum $p$. On the other hand, $m$ and $m'$ must be equal. Finally, using the relations with the 3–j and 6–j coefficients, one obtains the well–known result for the totally unpolarized situation:[2]

$$S_{mm'}(p, E_m) = S(p, E_m)\delta_{mm'} = \frac{1}{8\pi} \frac{1}{|J_A|^2} \sum_B \sum_{\ell j} |C_{\ell j}(p)|^2 \delta(E_m + \epsilon_B - \epsilon_A^0)\delta_{mm'}. \quad (A.11)$$
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FIGURE CAPTIONS

Fig. 1: Feynman diagram for the exclusive electromagnetic $\vec{A}(\vec{e}, e'N)B$ electron scattering process in the one–photon–exchange approximation.

Fig. 2: Kinematics for two–arm coincidence reactions with polarized targets. $\mathbf{u}_z$ is along $\mathbf{q}$, $\mathbf{u}_y$ is normal to the electron scattering plane and $\mathbf{u}_x = \mathbf{u}_y \times \mathbf{u}_z$ lies in the scattering plane. The target polarization vector $\mathbf{P}^*$ is specified by the angles $(\theta^*, \phi^*)$ in this coordinate system. The unit vectors $\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$ are constructed by rotating the previous ones ($\mathbf{u}_x$, $\mathbf{u}_y$, $\mathbf{u}_z$) by an angle $\phi_N$: $\mathbf{u}_3 \equiv \mathbf{u}_z$, $\mathbf{u}_1$ lies in the nucleonic plane and $\mathbf{u}_2$ is perpendicular to it.

Fig. 3: One–photon–exchange diagram for the reaction $\vec{A}(\vec{e}, e'N)B$ in the plane–wave impulse approximation (PWIA).

Fig. 4: Monopole and quadrupole single–neutron densities in (a) $r$–space and (b) $p$–space obtained from DDHF (solid), Nilsson with N–admixtures (dashed) and Nilsson without N–admixtures (short–dashed). The $b$ value for the Nilsson model calculations is $b = 1.79$ in fm units.

Fig. 5: The momentum distribution $n_0$ from Eq. (25) for the $^{21}\text{Ne} \rightarrow ^{20}\text{Ne}$ (g.s.) transition with the target polarized in the $z$–direction (along $\mathbf{q}$) and with $p_y = 0$ (i.e., where the struck nucleon is found in the $xz$–plane: $\phi = 0 \leftrightarrow$ co–planar kinematics). DDHF single–particle wave functions have been used.

Fig. 6: As for Fig. 5, except now for the momentum distribution $n_1$ from Eq. (29a).

Fig. 7: As for Fig. 5, except now for the momentum distribution $n_s$ from Eq. (29b).

Fig. 8: As for Fig. 5, except now for the momentum distributions $n_0 = n_n$ from Eqs. (25) and (29c), respectively, in the situation where the target polarization lies along the $y$–axis.

Fig. 9: Unpolarized differential cross section (Eq. (37)) for the reaction $^{21}\text{Ne}(e, e'n)^{20}\text{Ne}(g.s.)$. Results correspond to different values of the scattering angle $\theta_e$ and the azimuthal angle of the ejected neutron $\phi_N$, obtained by using Nilsson (full) and DDHF (dashed) single–nucleon wave functions.

Fig. 10: Asymmetry $\mathcal{P}_\Sigma$ (Eq. (39)) for the reaction $^{21}\text{Ne}(e, e'n)^{20}\text{Ne}(g.s.)$. Results correspond to different values of $\phi_N$ and different orientations of the target polarization defined by the angles within parentheses ($\theta^*, \phi^*$), in degrees.

Fig. 11: Same as Fig. 10, but for the asymmetry $\mathcal{P}_\Delta$ (Eqs. (40)) for the reaction $^{21}\text{Ne}(\vec{e}, e'n)^{20}\text{Ne}(g.s.)$. Results correspond to forward (a) and backward (b) electron scattering angles. In each case six different situations are shown defined by the target polarization direction ($\theta^*, \phi^*$) and the value of $\phi_N$: i) $\phi_N = 0^\circ$ and the target polarization vector $\mathbf{P}^*$ oriented along the axes $z$ (full), $x$ (short–dashed) and $y$ (dashed), and ii) $\phi_N = 90^\circ$ and $\mathbf{P}^*$ parallel to $z$ (long–dashed), parallel to $x$ (dotted) and parallel to $y$ (dot–dashed).

Fig. 12: Same as Fig. 9, except that now contributions from the first excited state in the residual nucleus $^{20}\text{Ne}$ are also included.

Fig. 13: Same as Fig. 10, except now including the contributions from the first excited state in the residual nucleus $^{20}\text{Ne}$. For the different kinematical situations considered a comparison
between the results obtained with Nilsson (full) and DDHF (dashed) single–nucleon wave functions is presented.

Fig. 14: Same as Fig. 13, but now for the asymmetry \( P_{\Delta} \). Results correspond to \( \theta_e = 30^\circ \) and different orientations of the target polarization: (a) along \( z \), (b) along \( x \) and (c) along \( y \), and different values of \( \phi_N \).

Fig. 15: Same as Fig. 14, except now for backward–angle electron scattering (\( \theta_e = 150^\circ \)).

Fig. 16: Hadronic response functions \( R^{K/K'} \), \( K = L, T, TL, TT, K' = T', TL' \) (see text). Contributions from the ground and first excited state in \( ^{20}\text{Ne} \) are included and the value of \( \phi_N \) is taken to be \( \phi_N = 0^\circ \). A comparison between the results obtained with Nilsson and DDHF wave functions for two orientations of the target polarization is presented: i) \( P^* \) parallel to \( q \) with Nilsson (full) and DDHF (dashed) wave functions, and ii) \( P^* \) parallel to the \( x \)–axis with Nilsson (short–dashed) and DDHF (long–dashed) wave functions.

Fig. 17: Asymmetry \( P_\Sigma \) versus the target polarization angle \( \theta^* \). Results correspond to \( \phi^* = 0^\circ \) and the residual nucleus in its ground state (g.s.). Panel (a) corresponds to \( \phi_N = 0^\circ \) and panel (b) to \( \phi_N = 90^\circ \). In the two cases we show the results obtained for five different values of the struck–neutron momentum: \( p = 0.2 \text{ fm}^{-1} \) (full), 0.6 \text{ fm}^{-1} (short–dashed), 1.0 \text{ fm}^{-1} (dashed), 1.4 \text{ fm}^{-1} (dot–dashed) and 1.8 \text{ fm}^{-1} (dotted).

Fig. 18: Same as Fig. 17, except that now we also include the contributions from the first excited state in the residual nucleus \(^{20}\text{Ne} \).

Fig. 19: Same as Fig. 17, but including the first two excited states of the daughter nucleus \(^{20}\text{Ne} \).

Fig. 20: Same as Fig. 17, except now for the asymmetry \( P_{\Delta} \) and the scattering angle \( \theta_e = 30^\circ \).

Fig. 21: Same as Fig. 20, except now including the contributions from the states \( 0^+ \) and \( 2^+ \) in \(^{20}\text{Ne} \).

Fig. 22: Same as Fig. 21, now also including the contributions from the second excited state \( 4^+ \) in \(^{20}\text{Ne} \).

Fig. 23: Differential cross section (Eq. (1)) versus the target polarization angle \( \theta^* \). Results correspond to \( p = 0.75 \text{ fm}^{-1} \) and \( \theta_e = 30^\circ \). DDHF single–nucleon wave functions have been used and contributions from the ground and first excited state in the residual nucleus \(^{20}\text{Ne} \) are included. Panel (a) corresponds to \( \phi_N = 0^\circ \), panel (b) to \( \phi_N = 66^\circ \) and panel (c) to \( \phi_N = 90^\circ \). In each case the results obtained for different values of \( \phi^* \) are represented. In panels (a) and (c) \( \phi^* = 0^\circ \) (full), \( \phi^* = 30^\circ \) (short–dashed), \( \phi^* = 60^\circ \) (dashed) and \( \phi^* = 90^\circ \) (dot–dashed), while in panel (b) \( \phi^* = 6^\circ \) (full), \( \phi^* = 26^\circ \) (short–dashed), \( \phi^* = 46^\circ \) (dashed), \( \phi^* = 66^\circ \) (dot–dashed) and \( \phi^* = 86^\circ \) (dash–double–dotted). Also for comparison, in the three graphs the results for the totally unpolarized cross section (Eq. (37)) are shown (dotted).

Fig. 24: Unpolarized cross section (a) and asymmetries \( P_\Sigma/P_{\Delta} \) (b) versus the nuclear deformation parameter \( \delta \) divided by the equilibrium results (\( \delta_{eq} = 0.31 \)). Results correspond to forward–angle electron scattering \( \theta_e = 30^\circ \) and co–planar kinematics \( \phi_N = 0^\circ \). Contributions from the ground and first excited state in the residual nucleus \(^{20}\text{Ne} \) are included. Results are shown for various values of the struck–neutron momentum: \( p = 0.5 \text{ fm}^{-1} \) (full), 0.75 \text{ fm}^{-1} (short–dashed) and 1.5 \text{ fm}^{-1} (dashed) in case (a), and \( p = 0.75 \text{ fm}^{-1} \) for \( P_\Sigma \) (solid), \( P_{\Delta} \) (dashed), and \( p = 1.5 \text{ fm}^{-1} \) for \( P_\Sigma \) (short–dash), \( P_{\Delta} \) (long–dash).
Fig. 25: Ratio between the results obtained for $q = 1$ GeV and $q = 500$ MeV at the corresponding quasielastic peaks and $\theta_e = 30^\circ$. DDHF single-nucleon wave functions have been used and contributions from the ground and first excited state in $^{20}\text{Ne}$ have been included. Panel (a) corresponds to the totally unpolarized cross section (Eq. (37)) with results for $\phi_N = 0^\circ$ (full) and $\phi_N = 90^\circ$ (dashed). Panel (b) corresponds to the asymmetry $P_\Sigma$, where the three curves shown correspond to $\phi_N = 0^\circ$ and target polarization along $x$ (full), $y$ (dashed) and $z$ (dotted). Finally, panel (c) shows the ratio for the asymmetry $P_\Delta$ for, i) $\phi_N = 0^\circ$ and $\mathbf{P}^*$ parallel to $x$ (full), parallel to $z$ (dashed), and ii) $\phi_N = 90^\circ$ with $\mathbf{P}^*$ parallel to $x$ (dotted), parallel to $y$ (short–dashed) and parallel to $z$ (dot–dashed).

**TABLE CAPTIONS**

Table 1: Choices of target polarization and outgoing nucleon directions where the electron–polarized response functions $R^T$ and/or $R^{T\ell L}$ vanish.

Table 2: Quadrupole moments, r.m.s. radii and $\beta$–values in $r$ and $p$–spaces for neutrons ($\nu$) and protons ($\pi$) obtained with Nilsson and density dependent DDHF calculations (see text).

Table 3: Weights $n_{ij}$ of the projected angular momentum components of the odd–neutron wave function in $^{21}\text{Ne}$. Nilsson and DDHF results are shown (values lower than $10^{-3}$ have been omitted).

Table 4: Values of the angles $\theta$ and $\theta_N$ that correspond to the five $p$–values used in the calculations (see Eqs. (51) and (52)). The residual nucleus $^{20}\text{Ne}$ is considered in its ground state.