Gravitational Radiation Background from Boson Star Binaries

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Abstract
We calculate the gravitational radiation background generated from boson star binaries formed in locally dense clusters with formation rate tracked by the regular star formation rate. We compute how the the frequency window in gravitational waves is affected by the boson field mass and repulsive self-coupling, anticipating constraints from EPTA and LISA. We also comment on the possible detectability of these binaries.

1. Introduction
The recent detection of gravitational waves (GW) by LIGO and VIRGO have opened up a new window for our understanding of the physical properties of the universe [1]. Probing the energy density of the stochastic Gravitational Wave Background (GRB) formed by the superposition of a large number of individual gravitational wave merger events is a long term goal of the next generation of GW detectors. It is thus of great interest to investigate different potential sources of GRBs and how to distinguish between their potential observational signatures. In this letter, we compute the GRB of an important class of hypothetical objects, merging binaries of Exotic Compact Objects (ECOs) composed of self-interacting scalar field configurations known as boson stars (BSs). Such objects were first proposed in the late 1960s [2] and further studied in the 1980s and 1990s [3, 4, 5, 6], but are now experiencing a revival due to their potential role as dark matter candidates [7] and as remnants of early universe physics [8]. The gravitational wave production from individual events of the merger of two

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boson stars has been studied in \cite{9} and \cite{10}, for example. A preliminary estimate of the GRB in boson-star binary mergers was given in \cite{11}.

The success of inflationary cosmology \cite{12} and the discovery of the Higgs Boson \cite{13}–\cite{14} have opened up the possibility that different self-interacting scalar fields might exist in nature. The presence of such fundamental scalar fields in the early universe, maybe in dark matter clusters, could have led to their condensation into self-gravitating compact objects \cite{15,16,17}. It is quite remarkable that for a repulsive self-interaction $\lambda |\phi|^4$ and a scalar field mass $m$, such objects have masses $M_{\text{BS}} \sim \sqrt{\lambda} M_{\text{Pl}}^3/m^2$, which, for $m/\lambda^{1/4} \sim m_p$, where $m_p$ is the proton mass, are parametrically equivalent to the Chandrasekhar mass \cite{18}.

Indeed, even a free, massive scalar field can generate a self-gravitating object, supported against gravitational collapse solely by quantum uncertainty \cite{2}. This distinguishes them from fermionic compact objects such as neutron stars (NS) and white dwarfs, which are prevented from collapse due to degeneracy pressure \cite{19}. Another key difference, important observationally to distinguish the two classes of compact objects, is that the simplest BSs do not radiate electromagnetically.

In $\Lambda$CDM cosmology, using certain FDM models, the first star formation in the center of spherically-symmetric dark matter mini-halos have been found to be around $z \sim 20 – 30$ \cite{20}, \cite{21}. Given the uncertainty in the properties of such primordial scalar fields, and to provide a more general analysis, we assume here that BSs were formed at a rate that tracks the regular star formation rate, in locally-dense dark matter clusters. We will thus adopt this initial range of redshifts as a benchmark for our analysis. Our results can be extended to arbitrarily large redshifts.

As with their fermionic counterparts, BSs have a critical maximum mass against central density beyond which they are unstable to gravitational collapse into black holes (BHs) \cite{3}–\cite{22}. In this paper, we treat the two stars in the binary BS system as having the same maximum mass and radius, which leads to the two objects having the same compactness, defined as $C = G_N M/R$. The GRB is typically characterized by the dimensionless quantity $\Omega_{\text{GW}}(f)$, the contribution in gravitational radiation in units of the critical density in a frequency window $f$ and $f + \delta f$ to the total energy-density of the universe in a Hubble time. By studying their gravitational imprints, we hope to gain insight on the properties of these exotic objects, expanding the results of \cite{11} and bringing them closer to current and planned observations.

2. Boson Star properties

2.1. Isolated Boson Stars

Very light bosons could form a Bose-Einstein condensate (BEC) in the early or late universe through various mechanisms \cite{15}–\cite{16}–\cite{17}. Such objects are macroscopic quantum states that are prevented from collapsing gravitationally by the Heisenberg uncertainty principle in the non-interacting \cite{2} and attractive self-interaction case \cite{15}, or, in another possibility, through a repulsive self-interaction that could balance gravity’s attraction \cite{18}. In this Letter, we study an Einstein-Klein-Gordon system with the following Lagrangian,

$$\mathcal{L} = \sqrt{-g} \left[ |(\partial \phi)|^2 - m^2 |\phi|^2 - \frac{1}{2} \lambda |\phi|^4 \right], \quad (1)$$
where $\phi$ is a complex scalar field carrying a global $U(1)$. Real scalar fields can also form gravitationally-bound states, but these are time-dependent and have different properties [23]. Colpi et al showed that the maximum mass of a spherically-symmetric BS with repulsive self-interaction is given by [18]

$$M_{*\max}^\ast \sim \frac{0.22\, M_p^2}{m} \alpha^{1/2} \approx \frac{0.06\sqrt{\lambda} M_p^3}{m^2},$$

(2)

where the rescaled coupling $\alpha$ is defined as $\alpha \equiv \lambda M_p^2/(4\pi m^2)$. For a boson star with a repulsive self-interaction, the radius can be estimated to be

$$R_\ast \sim \frac{\sqrt{\lambda}}{\sqrt{G_N m^2}}.$$  

(3)

The compactness of boson stars is discussed in many references such as [24, 7]. We note that the compactness and mass of the stars are especially relevant for binary GW events. Different formation mechanisms have been discussed in Refs. [15, 16, 17]. However, since we are focussing here on the gravitational radiation background, we need not worry about specific formation mechanisms that lead to highly compact BSs. We will assume they exist and compute their contribution to the GRW. We also note that if one assumes the complex scalar $\phi$ to be responsible for the dark matter in the Bullet Cluster, Ref. [25] shows that the constraint on the dark matter cross section [26, 27, 28] can be translated into a bound on the boson’s self-coupling, because the relative velocity of the Bullet Cluster is higher than the sound speed of the condensate. The translated bound on the self-interaction strength is

$$\lambda \lesssim 10^{-11} \left(\frac{m}{eV}\right)^{3/2}.$$  

(4)

We note in passing that Ref. [25] shows that BEC requires light scalars $m < 1eV$. However, the bound is based on the inter-particle spacing estimated from the average density of dark matter in the Universe. Since in the absence of a fundamental theory the exact formation process of boson stars remain unclear, we consider the possibility of their formation due to a large local density fluctuation. Therefore, we do not worry about the bound on the scalar mass. In what follows, we saturate the Bullet Cluster bound and parametrize the boson star mass effectively as

$$M_* = x M_{*\max}^\ast = 2.5 \times 10^9 \, x \left(\frac{eV}{m}\right)^{5/4} \, M_\odot,$$

(5)

where $x$ is the fraction between boson star mass and the maximum stable mass, and the radius as

$$R_* = y \frac{\sqrt{\lambda}}{\sqrt{G_N m^2}} = 1.1 \times 10^7 \, y \left(\frac{eV}{m}\right)^{5/4} \, R_\odot,$$

(6)

where $y$ is the fraction or multiple of the star radius from Eq. (3).
2.2. Boson Star Binaries

We briefly describe the properties of boson star binaries that are relevant for the calculation of gravitational radiation. In what follows, we assume a conservative model for the estimation of the binary formation rate, which tracks the star formation rate (SFR) of luminous stars. Empirically, the luminous star-formation rate can be parametrized as a function of redshift \( z \) and stellar mass \( M \) \(^{[29]}\), in units of \( \text{yr}^{-1}\text{Mpc}^{-3} \) as

\[
SFR(z, M) = SFR_0 \left( \frac{M_\odot}{M} \right) \frac{a e^{b(z-z_m)}}{a - b + b e^{a(z-z_m)}}. \tag{7}
\]

The parameters \( SFR_0, z_m, a, \) and \( b \) are all determined by fitting to observations such as gamma-ray burst rates and the galaxy luminosity function. We adopt the fit from gamma-ray bursts from \(^{[30]}\). We further parameterize the efficiency of the binary boson star formation as a fraction of \( SFR(z, M) \), denoted as \( f_{\text{BBS}} \leq 1 \). The boson star binary formation rate is, for a boson star of mass \( M_* \) and formation redshift \( z_f \),

\[
R_{\text{BBS}}(z_f, M_*) = f_{\text{BBS}} \times SFR(z_f, M_*). \tag{8}
\]

Since we do not need all of the binaries to survive today to leave their gravitational radiation imprint, we calculate the merger rate at redshift \( z \), which is mainly determined by the binary formation rate at redshift \( z_f \). On the other hand, the larger the binary separation at formation, the less likely they would have successfully merged, due to gravitational perturbations from other sources. Following Ref. \(^{[31]}\), we use an appropriately normalized weight function \( p(\Delta t) \) to account for the merger efficiency, where \( \Delta t \) is the time delay from formation of the binary to coalescence,

\[
R_m(t, M_*, f_{\text{BBS}}) = \int_{\Delta t_{\text{min}}}^{\Delta t_{\text{max}}} R_{\text{BBS}}(t - \Delta t, M_*) p(\Delta t) \, d\Delta t. \tag{9}
\]

Here, \( \Delta t_{\text{min}} \) is the minimum time between formation and coalescence, and \( \Delta t_{\text{max}} \) is determined by the maximum initial separation which allows for binary formation. As we will see below, the result is not sensitive to the precise choice of \( \Delta t_{\text{max}} \). We will comment on a suitable \( \Delta t_{\text{min}} \) for this integral in the following section. We relate redshift to cosmic time with the approximate formula from Ref. \(^{[32]}\),

\[
t(z) = \frac{2/H_0}{1 + (z + 1)^2}, \tag{10}
\]

where \( H_0 \) is the Hubble constant today. Next, let us estimate \( p(\Delta t) \). For a pair of stars A and B, their initial separation \( a \) defines a sphere inside which the number of stars is \( N(a) = \rho \pi a^3 / 6 \). Assuming that the chance of any pair of stars forming a binary is roughly the same inside the sphere, the probability that stars A and B are bounded is

\[
p(a) = \left( \frac{N(a)}{2} \right)^{-1} = \frac{2}{N(a)(N(a) - 1)} \propto a^{-6}. \tag{11}
\]
This simple model captures the sharp decrease in the binary population as the pair separation increases. We note that the difficulty for binaries with initial large separation to form is not from perturbations that rip the two stars apart. Instead, the many ‘inbetweener’ are likely to form binaries with each of the two stars separately. Since gravitational radiation is the only channel for energy release, and since most of the initial binding and inspiraling process can be described by Newtonian dynamics, we use the merging time as in Ref. [33],

$$\Delta t \sim a^4.$$  \(12\)

This gives a weight function $p(\Delta t) \sim 1/\Delta t^{3/2}$. This weight function also implies that the result is not sensitive to $\Delta t_{\text{max}}$ and the precise determination of the initial separation. The boson star formation rate and merger rate are shown in Fig. 1. As one can see, the merger rate is not very sensitive to $\Delta t_{\text{min}}$. The magnitude of the merger rate is controlled by $f_{\text{BBS}}$, which will be constrained together with their mass and radius.

3. Gravitational Waves from Boson Stars

3.1. Gravitational Waves from Single Binaries

The most important contribution to the stochastic background comes from the inspiral phase of the binary mergers. In this stage, the calculation can be done analytically. The system can be approximated by a pair of purely self-gravitating point masses emitting mostly gravitational quadrupole radiation. The radiation power is

$$P = \frac{32}{5} G N \mu^2 \omega^6 r^4.$$

\(13\)

Note that this differs from Ref. [32] [31], where a fiducial model is used and the weight function for NSs is chosen to be $p(\Delta t) \sim 1/\Delta t$. For a study of different delay models, please refer to Refs. [35] [36] [37] [38].
Solving the dissipation equation $P = -\dot{E}$ gives us the characteristic $f(t) \sim t^{-3/8}$ relation, and the radius as a function of $t$, with $t$ being the time before coalescence,

$$f(t) = \frac{5^{3/8}}{8\pi} (G_N m_c)^{-5/8} t^{-3/8},$$

$$r(t) = \left( \frac{256}{5} G_N^3 (M_A + M_B) M_A M_B \right)^{1/4} t^{1/4}, \quad (14)$$

where $m_c$ is the chirp mass given by $m_c = (M_A M_B)^{3/5}/(M_A + M_B)^{1/5}$, with $M_A, M_B$ being the masses of the two stars. This approximation holds until the binary evolves beyond its innermost stable circular orbit (ISCO). Inside the ISCO, tidal effects need to be taken into account, and the post-Newtonian expansion breaks down. The frequency of the ISCO is given by [7]

$$f_{\text{ISCO}} = \frac{C^3/2}{3^{3/2} \pi G_N (M_1 + M_2)}, \quad (15)$$

which is a function of the compactness of the stars defined as $C_* \equiv G_N M_*/R_*$. For boson stars with a fraction $x$ of the maximum mass [2], and a fraction or multiple $y$ of the radius [3],

$$f_{\text{ISCO}} \approx \frac{m^2 \sqrt{G_N}}{6 \sqrt{6\pi^{5/4}} \sqrt{\lambda} \sqrt{x/y^3}} \approx 2.02 \times 10^{-15} \text{ Hz} \sqrt{x/y^3} \sqrt{\frac{1}{\lambda}} \left( \frac{m}{\text{meV}} \right)^2. \quad (16)$$

If we saturate the Bullet Cluster bound as in Eq. 4, $f_{\text{ISCO}}$ scales as $\sim m^{5/4}$.

$$f_{\text{ISCO}} \approx 6.4 \times 10^{-10} \text{Hz} \left( \sqrt{x/y^3} \right) \left( \frac{m}{\text{meV}} \right)^{5/4}. \quad (17)$$

We will estimate $\Delta t_{\text{min}}$ in (9) based on the following argument: if the boson star binary is formed at an initial distance inside the ISCO, the binary will not experience an inspiral phase. Therefore we choose $\Delta t_{\text{min}}$ to correspond to $t_{\text{ISCO}}$, the time between entering the ISCO and coalescence. In what follows, we sum up the contributions from individual mergers to get the total gravitational radiation energy density. When we do the summation, we use $f_{\text{ISCO}}$ as the cut off frequency for each binary to guarantee the calculation based on quadrupole radiation is valid.

### 3.2. Gravitational Radiation Energy Density

The energy spectrum of the gravitational radiation from boson stars is defined as,

$$\Omega_{\text{GW}}(f) \equiv \frac{f}{\rho_c} \frac{d\rho_{\text{GW}}}{df}, \quad (18)$$

where $\rho_{\text{GW}}$ is the energy density of the gravitational wave in that frequency range and $\rho_c$ is the critical energy density. Following [34], this can be written using the merger rate per
Figure 2: (Colored plot online.) Plot of (19). Here the fraction of the maximum boson star mass (2) is taken conservatively to be $x = 10^{-1}$, and the fraction or multiple of the radius (3) is taken as $y = 1$. The self-coupling $\lambda$ has been chosen to saturate the Bullet Cluster constraint (4). The upper, lower, and middle lines are chosen for $f_{\text{BBS}} = 1/2$, $f_{\text{BBS}} = 10^{-3}$, and their geometric mean, respectively. Also shown are the EPTA [39] and the LISA [40] exclusion prospects.

unit of comoving volume per source time $R_m(z, M_*)$, and the differential energy emitted by a single source $dE/df_s$ as,

$$\Omega_{\text{GW}}(f, M_*, f_{\text{BBS}}) = \frac{f}{\rho_c H_0} \int_0^{z_{\text{max}}} \frac{R_m(z, M_*, f_{\text{BBS}})}{(1+z)\sqrt{\Omega_M(1+z)^3 + \Omega_\Lambda}} \frac{dE}{df_s} \, dz$$

$$= f^{2/3} f_{\text{BBS}} \left( \frac{M_*}{M_\odot} \right)^{2/3} \left( \frac{\pi^{2/3} G_N^{2/3} M_\odot^{5/3}}{2^{1/3} 3 \rho_c H_0} \right) \int_0^{z_{\text{max}}} \frac{R_m(z, M_\odot, 1)}{(1+z)^{4/3} \sqrt{\Omega_M(1+z)^3 + \Omega_\Lambda}} \, dz$$

$$= 1.91 f_{\text{BBS}} x^{2/3} \left( \frac{f}{1 \text{ Hz}} \right)^{2/3} \left( \frac{eV}{m} \right)^{5/6}, \quad (19)$$

where we have used $f_s = (1+z)f$ for the emitted (source) frequency, and

$$\frac{dE}{df_s} = \frac{\pi^{2/3}}{3} G_N^{2/3} m_e^{5/3} f_s^{-1/3}. \quad (20)$$

$f_{\text{ISCO}}$ works as a cut-off at the high end of the spectrum, which is shown in Eq. (17). The spectrum is shown in Fig 2 for several benchmark scenarios. In this plot, the fraction of the radius (3) is taken as $y = 1$. It is seen that the signal may be within reach of the next generation of gravitational wave interferometer experiments, and pulsar timing arrays. Also, we observe that the high end of the frequency band, determined by $f_{\text{ISCO}}$, is proportional to $m_5^{5/4}$, if we saturate the Bullet Cluster bound, which indicates that boson stars consisting of heavy scalars are more likely to be probed by gravitational wave experiments. We show in Fig. 3 the bound on binary formation efficiency $f_{\text{BBS}}$, star mass, and star radius based on LISA for a few benchmarks of scalar mass.
4. Discussion

As shown in Fig. 2, the gravitational signal from binaries of stars made of light bosons fall within the reach of the next generation of gravitational wave detectors and pulsar timing arrays. Failure to detect such spectra can be interpreted as a bound on the boson star parameters, as illustrated in Fig. 3. Such a bound can in turn be translated to bounds on the boson mass and self-coupling, once a specific formation scenario is assumed.

The most important contribution to the boson star binary spectrum comes from the inspiral phase, which peaks at \( f_{\text{ISCO}} \), the frequency corresponding to the innermost stable orbit. This peak frequency is a function of the compactness of the boson stars, which depends on the scalar mass and self-coupling. This is to be compared with objects of which the compactness is known. The compactness of a BH is 1/2, whereas realistic assumptions on the EOS for NSs would put them in the range \( 0.13 \lesssim C \lesssim 0.23 \). For the boson stars considered here, the compactness saturates at \( C \leq 0.16 \), so close to the lower range of NSs and below that of BHs. We also note that BS mergers are not accompanied by electromagnetic signatures.

It is important to distinguish the stochastic background from boson stars from that due to more conventional binaries, such as BHs and NSs. Such a comparison relies on three main features. The stochastic spectrum is characterized by the fractional energy density \( \Omega_{\text{GW}}(f) \) and the frequency band \( f \). As is shown in equation (19), \( \Omega_{\text{GW}}(f) \) can be written as a function of the formation rate (parametrized by \( f_{\text{BBS}} \)) and the mass of the boson stars (as a function of \( x \) and \( m \)). A fundamental difference is that boson star masses can take on a wide range of values, from that of NSs to that of supermassive BHs. Boson stars with a mass that falls outside the range typical for NSs and BHs are particularly interesting observationally. This corresponds to relatively heavy bosons, with \( m \sim 10^5 \sqrt{x} \) eV. Also, a more exotic formation scenario than the one considered here may distinguish the boson star signal. For example, by considering redshifts different than the ones that track ordinary star
formation. We leave the analysis of how these parameters impact the boson star stochastic background for future work.

Acknowledgements

DC and CS would like to thank JiJi Fan for useful discussions. SM would also like to thank Xavier Calmet for useful feedback and discussions. MG is partially supported by a US Department of Energy grant DE-SC001038. SM is supported by a Chancellor's International Research Scholarship of the University of Sussex and is grateful for the support of the TPP department at the University of Sussex. CS is supported in part by the International Postdoctoral Fellowship funded by China Postdoctoral Science Foundation, and is grateful for the hospitality and partial support of the Department of Physics and Astronomy at Dartmouth College where this work was done.

References

[1] B. P. Abbott et al. [LIGO Scientific and Virgo Collaborations], Phys. Rev. Lett. 116, no. 6, 061102 (2016) doi:10.1103/PhysRevLett.116.061102 [arXiv:1602.03837 [gr-qc]].

[2] R. Ruffini and S. Bonazzola, Phys. Rev. 187, 1767 (1969). doi:10.1103/PhysRev.187.1767.

[3] M. Gleiser, Phys. Rev. D 38, 2376 (1988) Erratum: [Phys. Rev. D 39, no. 4, 1257 (1989)]. doi:10.1103/PhysRevD.38.2376, 10.1103/PhysRevD.39.1257.

[4] P. Jetzer, Phys. Rept. 220, 163 (1992). doi:10.1016/0370-1573(92)90123-H.

[5] A. R. Liddle and M. S. Madsen, Int. J. Mod. Phys. D 1, 101 (1992). doi:10.1142/S0218271992000057.

[6] F. E. Schunck and E. W. Mielke, Class. Quant. Grav. 20, R301 (2003) doi:10.1088/0264-9381/20/20/201 [arXiv:0801.0307 [astro-ph]].

[7] G. F. Giudice, M. McCullough and A. Urbano, JCAP 1610, no. 10, 001 (2016) doi:10.1088/1475-7516/2016/10/001 [arXiv:1605.01209 [hep-ph]].

[8] E. W. Mielke and F. E. Schunck, Nucl. Phys. B 564, 185 (2000) doi:10.1016/S0550-3213(99)00492-7 [gr-qc/0001061].

[9] C. Palenzuela, L. Lehner and S. L. Liebling, Phys. Rev. D 77, 044036 (2008) doi:10.1103/PhysRevD.77.044036 [arXiv:0706.2435 [gr-qc]].

[10] C. Palenzuela, P. Pani, M. Bezares, V. Cardoso, L. Lehner and S. Liebling, Phys. Rev. D 96, no. 10, 104058 (2017) doi:10.1103/PhysRevD.96.104058 [arXiv:1710.09432 [gr-qc]].

[11] M. Gleiser, Phys. Rev. Lett. 63, 1199 (1989). doi:10.1103/PhysRevLett.63.1199.

[12] A. H. Guth, Phys. Rev. D 23, 347 (1981). doi:10.1103/PhysRevD.23.347.
[13] S. Chatrchyan et al. [CMS Collaboration], Phys. Lett. B 716, 30 (2012) doi:10.1016/j.physletb.2012.08.021 [arXiv:1207.7235 [hep-ex]].

[14] G. Aad et al. [ATLAS Collaboration], Phys. Lett. B 716, 1 (2012) doi:10.1016/j.physletb.2012.08.020 [arXiv:1207.7214 [hep-ex]].

[15] D. G. Levkov, A. G. Panin and I. I. Tkachev, Phys. Rev. Lett. 118, no. 1, 011301 (2017) doi:10.1103/PhysRevLett.118.011301 [arXiv:1609.03611 [astro-ph.CO]].

[16] E. W. Kolb and I. I. Tkachev, Phys. Rev. Lett. 71, 3051 (1993) doi:10.1103/PhysRevLett.71.3051 [hep-ph/9303313].

[17] E. Seidel and W. M. Suen, Phys. Rev. Lett. 72, 2516 (1994) doi:10.1103/PhysRevLett.72.2516 [gr-qc/9309015].

[18] M. Colpi, S. L. Shapiro and I. Wasserman, Phys. Rev. Lett. 57, 2485 (1986). doi:10.1103/PhysRevLett.57.2485

[19] S. L. Shapiro and S. A Teukolsky, Black Holes, White Dwarfs, and Neutron Stars: The Physics of Compact Objects (John Wiley & Sons, New York NY 1983).

[20] S. Hirano, J. M. Sullivan and V. Bromm, Mon. Not. Roy. Astron. Soc. 473, no. 1, L6 (2018) doi:10.1093/mnrasl/slx146 [arXiv:1706.00435 [astro-ph.CO]].

[21] S. Hirano, T. Hosokawa, N. Yoshida, H. Umeda, K. Omukai, G. Chiaki and H. W. Yorke, Astrophys. J. 781, 60 (2014) doi:10.1088/0004-637X/781/2/60 [arXiv:1308.4456 [astro-ph.CO]].

[22] M. Gleiser and R. Watkins, Nucl. Phys. B 319, 733 (1989). doi:10.1016/0550-3213(89)90627-5

[23] E. Seidel and W. M. Suen, Phys. Rev. Lett. 66, 1659 (1991). doi:10.1103/PhysRevLett.66.1659

[24] P. Amaro-Seoane, J. Barranco, A. Bernal and L. Rezzolla, JCAP 1011, 002 (2010) doi:10.1088/1475-7516/2010/11/002 [arXiv:1009.0019 [astro-ph.CO]].

[25] J. Fan, Phys. Dark Univ. 14, 84 (2016) doi:10.1016/j.dark.2016.10.005 [arXiv:1603.06580 [hep-ph]].

[26] M. Markevitch et al., Astrophys. J. 606, 819 (2004) doi:10.1086/383178 [astro-ph/0309303].

[27] S. W. Randall, M. Markevitch, D. Clowe, A. H. Gonzalez and M. Bradac, Astrophys. J. 679, 1173 (2008) doi:10.1086/587859 [arXiv:0704.0261 [astro-ph]].

[28] V. Springel and G. Farrar, Mon. Not. Roy. Astron. Soc. 380, 911 (2007) doi:10.1111/j.1365-2966.2007.12159.x [astro-ph/0703232 [ASTRO-PH]].
[29] V. Springel and L. Hernquist, Mon. Not. Roy. Astron. Soc. 339, 312 (2003) doi:10.1046/j.1365-8711.2003.06207.x [astro-ph/0206395].

[30] E. Vangioni, K. A. Olive, T. Prestegard, J. Silk, P. Petitjean and V. Mandic, Mon. Not. Roy. Astron. Soc. 447, 2575 (2015) doi:10.1093/mnras/stu2600 [arXiv:1409.2462 [astro-ph.GA]].

[31] B. P. Abbott et al. [LIGO Scientific and Virgo Collaborations], arXiv:1710.05837 [gr-qc].

[32] M. Carmeli, J. G. Hartnett and F. J. Oliveira, Found. Phys. Lett. 19, 277 (2006) doi:10.1007/s10702-006-0518-3 [gr-qc/0506079].

[33] D. Croon, A. E. Nelson, C. Sun, D. G. E. Walker and Z. Z. Xianyu, arXiv:1711.02096 [hep-ph].

[34] B. P. Abbott et al. [LIGO Scientific and Virgo Collaborations], Phys. Rev. Lett. 116, no. 13, 131102 (2016) doi:10.1103/PhysRevLett.116.131102 [arXiv:1602.03847 [gr-qc]].

[35] L. Tornatore, S. Borgani, K. Dolag and F. Matteucci, Mon. Not. Roy. Astron. Soc. 382, 1050 (2007) doi:10.1111/j.1365-2966.2007.12070.x [arXiv:0705.1921 [astro-ph]].

[36] P. Madau, M. Dickinson, Annual Review of Astronomy and Astrophysics 52, 415 (2014)

[37] I. Mandel and S. E. de Mink, Mon. Not. Roy. Astron. Soc. 458, no. 3, 2634 (2016) doi:10.1093/mnras/stw379 [arXiv:1601.00007 [astro-ph.HE]].

[38] B. P. Abbott et al. [LIGO Scientific and Virgo Collaborations], Astrophys. J. 818, no. 2, L22 (2016) doi:10.3847/2041-8205/818/2/L22 [arXiv:1602.03846 [astro-ph.HE]].

[39] M. Kramer and D. J. Champion, Class. Quant. Grav. 30, 224009 (2013). doi:10.1088/0264-9381/30/22/224009

[40] P. Amaro-Seoane et al., GW Notes 6, 4 (2013) [arXiv:1201.3621 [astro-ph.CO]].