Scheduling with Rate Adaptation under Incomplete Knowledge of Channel/Estimator Statistics

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Abstract

In time-varying wireless networks, the states of the communication channels are subject to random variations, and hence need to be estimated for efficient rate adaptation and scheduling. The estimation mechanism possesses inaccuracies that need to be tackled in a probabilistic framework. In this work, we study scheduling with rate adaptation in single-hop queueing networks under two levels of channel uncertainty: when the channel estimates are inaccurate but complete knowledge of the channel/estimator joint statistics is available at the scheduler; and when the knowledge of the joint statistics is incomplete. In the former case, we characterize the network stability region and show that a maximum-weight type scheduling policy is throughput-optimal. In the latter case, we propose a joint channel statistics learning - scheduling policy. With an associated trade-off in average packet delay and convergence time, the proposed policy has a stability region arbitrarily close to the stability region of the network under full knowledge of channel/estimator joint statistics.

1 Introduction

Scheduling in wireless networks is a critical component of resource allocation that aims to maximize the overall network utility subject to link interference and queue stability constraints. Since the seminal paper by Tassiulas and Ephremides (1), maximum-weight type algorithms have been intensely studied (e.g., [2]-[8]) and found to be throughput-optimal in various network settings. The majority of existing works employing maximum-weight type schedulers are based on the assumption that full knowledge of channel state information (CSI) is available at the scheduler. In realistic scenarios, however, due to random variations in the channel, full CSI is rarely, if ever, available at the scheduler. The dynamics of the scheduling problem with imperfect CSI is, therefore, vastly different from the problem with full CSI in the following two ways (1) a non-trivial amount of network resource, that could otherwise be used for data transmission, is spent in learning the channel; (2) the acquired information on the channel is potentially inaccurate, essentially underscoring the need for intelligent rate adaptation and user scheduling. Realistic networks are thus characterized by a convolved interplay between channel estimation, rate adaptation, and multiuser scheduling mechanisms.

These complicated dynamics are studied under various network settings in recent works ([9]-[15]). In [9], the authors study scheduling in single-hop wireless networks with Markov-modeled binary ON-OFF channels. Here scheduling decisions are made based on cost-free estimates of the channel obtained once every few slots. The authors show that a maximum-weight type scheduling policy, that takes into account the probabilistic inaccuracy in the channel estimates and the memory in the Markovian channel, is throughput-optimal. In [10], the authors study decentralized scheduling under partial CSI in multihop wireless networks with Markov-modeled channels. Here, each user knows its channel perfectly and has access to delayed CSI of other users’ channels. The authors characterize the stability region of the network and show that a maximum-weight type threshold policy, implemented in a decentralized fashion at each user, is throughput optimal.

In [12], the authors study scheduling under imperfect CSI in single-hop networks with independent and identically distributed (i.i.d.) channels. They consider a two-stage decision setup: in the first stage, the scheduler decides whether to estimate the channel with a corresponding energy cost; in the second stage, scheduling with rate adaptation is performed based on the outcome of the first stage. Under
this setup, the authors propose a maximum-weight type scheduling policy that minimizes the energy consumption subject to queue stability.

While studying scheduling under imperfect CSI is a first step in the right direction, these works assume that complete knowledge of the channel/estimator joint statistics, which is crucial for the success of opportunistic scheduling, is readily available at the scheduler. This is another simplifying assumption that need not always hold in reality. Taking note of this, we study scheduling in single-hop networks under imperfect CSI, and when the knowledge of the channel/estimator joint statistics is incomplete at the scheduler. We propose a joint statistics learning-scheduling policy that allocates a fraction of the time slots (the exploration slots) to continuously learn the channel/estimator statistics, which in turn is used for scheduling and rate adaptation during data transmission slots. Note that our setup is similar to the setup considered in [15]. Here the author considers a two-stage decision setup. When applied to the scheduling problem, this work can be interpreted as follows. One of \( K \) estimators is chosen to estimate the channel in the first stage, with unknown channel/estimator joint statistics. The second stage decision is made to minimize a known function of the estimate obtained in the first stage. Our problem is different from this setup in that the channel/estimator joint statistics is important to optimize the second stage decision in our problem - i.e., scheduling with rate adaptation. This is not the case in [15] where a known function of the estimate is optimized and the channel/estimator joint statistics is helpful only in the first stage that decides one of \( K \) estimators. Our contribution is two-fold:

- When complete knowledge of the channel/estimator joint statistics is available at the scheduler, we characterize the network stability region and show that a simple maximum-weight type scheduling policy is throughput-optimal. It is worth contrasting this result with those in [9]-[12]. In these works, imperfection of CSI is assumed to be caused by specific factors like delayed channel feedback, infrequent channel measurement, etc, whereas, in our model, since the channel/estimator joint statistics is unconstrained, the CSI inaccuracy is captured in a more general probabilistic framework.

- Using the preceding system level results as a benchmark, we study scheduling under incomplete knowledge of the channel/estimator joint statistics. We propose a scheduling policy with an in-built statistics learning mechanism and show that, with a corresponding trade-off in the average packet delay before convergence, the stability region of the proposed policy can be pushed arbitrarily close to the network stability region under full knowledge of channel/estimator statistics.

The paper is organized as follows. Section II formalizes the system model. In Section III, we characterize the stability region of the network and propose a throughput-optimal scheduling policy. In Section IV, we study joint statistics learning-scheduling and rate adaptation when the scheduler has incomplete knowledge of channel/estimator statistics. Concluding remarks are provided in Section V.

## 2 System Model

We consider a wireless downlink communication scenario with one base station and \( N \) mobile users. Data packets to be transmitted from the base station to the users are stored in \( N \) separate queues at the base station. Time is slotted with the slots of all the users synchronized. The channel between the base station and each user is i.i.d. across time slots and independent across users. We do not assign any specific distribution to the channels throughout this work. The channel state of a user in a slot denotes the number of packets that can be successfully transmitted without outage to that user, in that slot. Transmission at a rate below the channel state always succeeds, while transmission at a rate above the channel state always fails. We assume the channel state lies in a finite discrete state space \( \mathcal{S} \). Let \( C_i[t] \) be the random variable denoting the channel state of user \( i \) in slot \( t \). The channel state of the network in slot \( t \) is denoted by the vector \( \mathbf{C}[t] = [C_1[t], C_2[t], \ldots, C_N[t]] \in \mathcal{S}^N \). In each slot, the scheduler has access to estimates of the channel states, i.e., \( \hat{\mathbf{C}}[t] = [\hat{C}_1[t], \hat{C}_2[t], \ldots, \hat{C}_N[t]] \in \mathcal{S}^N \). The estimator is fixed for each user and the estimates are independent across users. The channel/estimator joint statistics for user \( i \) is given by the \(|\mathcal{S}|^2 \) probabilities \( P(C_i = c_i, \hat{C}_i = \hat{c}_i), \forall c_i \in \mathcal{S}, \hat{c}_i \in \mathcal{S} \).

We adopt the one-hop interference model, where, in each slot, only one user is scheduled for data transmission. The scheduler (base station), based on the channel estimate and the queue length infor-
mation, decides which user to schedule and performs rate adaptation in order to maximize the overall network stability region. Let $I[t]$ and $R[t]$ denote the index of the user scheduled to transmit and the corresponding rate of transmission, respectively, at slot $t$. Due to potential mismatch between the channel estimates and the actual channels, it is possible that the allocated rate is larger than the actual channel rate, thus leading to outage. In this case, the packet is retained at the head of the queue and a retransmission will be attempted later. Let $Q_i[t]$ denote the state (length) of queue $i$ at the beginning of slot $t$. Let $A_i[t]$ denote the number of exogenous packet arrivals at queue $i$ at the beginning of slot $t$ with $E[A_i[t]] = \lambda_i$. The queue state evolution can now be written as a discrete stochastic process: 

$$Q_i[t+1] = [Q_i[t] - 1(I[t] = i)R[t] \cdot 1(R[t] \leq C_i[t])]^+ + A_i[t],$$

where $[\cdot]^+ = \max\{0, \cdot\}$. We adopt the following definition of queue stability [2]: Queue $i$ is stable if there exists a limiting stationary distribution $F_i$ such that $\lim_{t \to \infty} P(Q_i[t] \leq q) = F_i(q)$.

### 3.1 Network Stability Region

Consider the class of stationary scheduling policies $G$ that base their decision on the current queue length information $[Q_1, \ldots, Q_N]$, the channel estimates $[\hat{C}_1, \ldots, \hat{C}_N]$, and full knowledge of channel/estimator joint statistics. Define the network stability region as the closure of the arrival rates that can be supported by the policies in $G$ without leading to system instability. Let $P_C(\hat{c} = [\hat{c}_1, \ldots, \hat{c}_N])$ denote the probability of the channel estimate vector. Thus,

$$P_C(\hat{c} = [\hat{c}_1, \ldots, \hat{c}_N]) = \prod_{i=1}^N P(\hat{C}_i = \hat{c}_i),$$

where the probabilities $P(\hat{C}_i = \hat{c}_i)$ are evaluated from the knowledge of the channel/estimator joint statistics. Defining $\mathcal{CH}[\bar{A}]$ as the convex hull (17) of set $\bar{A}$ and $\bar{I}_i$ as the $i$th coordinate vector, we record our result on the network stability region below.

**Proposition 1.** The stability region of the network is given by

$$\Lambda = \sum_{\hat{c} \in \mathcal{S}^N} P_C(\hat{c}) \cdot \mathcal{CH}\left[0, P(C_i \geq r^*_i(\hat{c}_i) | \hat{C}_i = \hat{c}_i) : I_i = i, \ldots, N\right],$$

where $r^*_i(\hat{c}_i) = \arg \max_{r \in \mathcal{S}} \{P(C_i \geq r | \hat{C}_i = \hat{c}_i) : I_i = i\}$ and the conditional probabilities $P(C_i \geq r^*_i(\hat{c}_i) | \hat{C}_i = \hat{c}_i)$ are evaluated from the knowledge of the channel/estimator joint statistics.

**Proof Outline:** The proof contains two parts. We first show that any rate vector $\lambda$ strictly within $\Lambda$ is stably supportable by some randomized stationary policy. In the second part, we establish that any arrival rate $\lambda$ outside $\Lambda$ is not supportable by any policy. We show this by first identifying a hyperplane that separates $\lambda$ and $\Lambda$ using the strict separation theorem (17). We then define an appropriate Lyapunov function and show that, for any scheduling policy, there exists a positive drift, thus rendering the queues unstable (18). Details of the proof are available in Appendix A.

### 3.2 Optimal Scheduling and Rate Allocation

In this section, we propose a maximum-weight type scheduling policy with rate adaptation and show that it is throughput-optimal, i.e., it can support any arrival rate that can be supported by any other policy in $G$. The policy is introduced next.
Scheduling Policy $\Psi$

At time slot $t$, the base station makes the scheduling and rate adaptation decisions based on the channel/estimator joint statistics and the channel estimate vector $\hat{C} = \hat{c}$ (the time index is dropped for notational simplicity).

1. Rate Adaptation:

   For each user $i$, assign rate $R_i$ such that,
   
   $$R_i = \arg \max_{r \in S} \{ P(C_i \geq r | \hat{C}_i = \hat{c}_i) \cdot r \}$$

2. Scheduling Decision:

   Schedule the user $I$ that maximizes the queue-weighted rate $R_i$, as follows:
   
   $$I = \arg \max_i \{ Q_i \cdot P(C_i \geq R_i | \hat{C}_i = \hat{c}_i) \cdot R_i \}$$

Note that when the channel state estimation is accurate, the conditional probability $P(C_i \geq r | \hat{C}_i = \hat{c}_i)$ will be a step function, with $\Psi$ essentially becoming the classic maximum-weight policy in [2]. The next proposition establishes the throughput optimality of policy $\Psi$. Details are provided in Appendix B.

**Proposition 2.** The scheduling policy $\Psi$ stably supports all arrival rates that lie in the interior of the stability region $\Lambda$.

*Proof Outline:* The proof proceeds as follows. Consider a Lyapunov function $L(Q[t]) = \sum_{i=1}^{N} Q_i^2[t]$. For any arrival rate $\lambda$ that lies strictly within the stability region $\Lambda$, we know it is stably supportable by some policy $G_0$. Under $G_0$, we show that the corresponding Lyapunov drift is negative. We then show that policy $\Psi$ minimizes the Lyapunov drift and hence it will have a negative drift, thus establishing the throughput optimality of $\Psi$.

The results obtained thus far when the channel/estimator joint statistics is available at the scheduler are along expected lines. Nonetheless, they serve as a benchmark to the rest of the work under incomplete knowledge of the channel/estimator joint statistics, which is the main focus of the paper.

4 Incomplete Knowledge of Channel/Estimator Joint Statistics

In this section, we study scheduling with rate adaptation when the scheduler only has knowledge of the marginal statistics of the estimator, i.e., $P(\hat{C}_i = \hat{c}_i), \forall c_i \in S, i \in \{1, \ldots, N\}$, and hence, the knowledge of the channel/estimator joint statistics is incomplete at the scheduler. We first illustrate, with a simple example, that significant system level losses are incurred when no effort is made to learn these statistics, and hence no rate adaptation is performed.

4.1 Illustration of the Gains from Rate Adaptation

With incomplete information on the channel-estimator joint statistics, the scheduler naively trusts the channel estimates to be actual channel states and transmits at the rate allowed in this state. Under this scheduling structure, for the single-hop network we consider, the stability region is given in Appendix C by

$$\tilde{\Lambda} = \sum_{\hat{c} \in S^N} P_{\hat{C}}(\hat{c}) \cdot C_H \left[ 0, P(C_i \geq \hat{c}_i | \hat{C}_i = \hat{c}_i) \hat{c}_i \cdot \mathbf{1}_i; i = 1, \cdots, N \right].$$  \(3\)
Joint statistics learning-scheduling policy (parameterized by $\gamma$)

1. In each slot, the scheduler first decides whether to explore the channel of one of the users or transmit data to one of the users. Specifically, it randomly decides to explore the channel of user $i$ with probability $x_i \hat{C}_i / N$ where $\sum_{i=1}^N x_i \hat{C}_i / N < 1$. The quantity $x_i \in (0,1]$ is a function of $\gamma$ and the channel estimate, $\hat{C}_i$, of user $i$. It is optimized to maximize the worst-case rate of convergence of the statistics learning mechanism subject to the $\gamma$ constraint. We postpone the discussion on this optimization to Proposition 4. Note that, we have dropped the time index from the estimates for ease of notation.

2. If a user is chosen for exploration, this time slot becomes an observing slot. Call the chosen user as $e$. The scheduler now sends data at a rate $r$ that is chosen uniformly at random from the set $S$. Let the quantity $\xi(t)$ indicate whether the transmission was successful or not:

$$\xi(t) = 1(c_e \geq r),$$

For a two-user single-hop network, this region is plotted in Fig. 1 along-side the network stability region when full knowledge of the channel/estimator joint statistics is available at the scheduler and hence rate adaptation is performed. The channel between the base station and each user is independent and binary ($S = \{0.2, 1\}$) with $P(C_i = 1) = 0.8$, for $i = 1, 2$. For different mismatch between the channel and the estimate, Fig. 1 plots the stability region of the system when rate adaptation is performed and when it is not. Note the significant reduction in the stability region when rate adaptation is not performed. This loss increases with increase in the degree of channel-estimator mismatch. The preceding example underscores the importance of rate adaptation and hence the need to learn the channel/estimator joint statistics. We now proceed to introduce our joint statistics learning-scheduling policy.

4.2 Joint Statistics Learning - Scheduling Policy

We design the policy with the following main components: (1) The fraction of time slots the policy spends in learning the channel/estimator joint statistics is fixed at $\gamma \in (0, 1)$, (2) The worst-case rate of convergence of the statistics learning process is maximized. We formally introduce the policy next, followed by a discussion on the policy design.
where, recall, $c_e$ denotes the current channel state of user $e$. Let $\Theta_{e,c_e,r}$ denote the set of exploration time slots when the channel estimate of user $i$ was $\hat{c}$ and user $i$ was explored with rate $r$. Thus, the current slot is added to the set $\Theta_{e,c_e,r}$. Now, an estimate of the quantity $P(C_e \geq r | \hat{C}_e = \hat{c}_e)$ is obtained using the following update:

$$
\hat{P}_t(C_e \geq r | \hat{C}_e = \hat{c}_e) = \frac{\sum_{k \in \Theta_{e,c_e,r}} \xi(k)}{|\Theta_{e,c_e,r}|}
$$

where $|\mathcal{V}|$ denotes the cardinality of set $\mathcal{V}$. We assume $\hat{P}_t(C_e | \hat{C}_e)$ to be uniform when $\Theta_{e,c_e,r} = \emptyset$, i.e., $\hat{P}_t(C_e \geq r | \hat{C}_e = \hat{c}_e) = 1 - r/|S|$.

(3) With probability $1 - \sum_{i=1}^{N} \frac{x_i}{N}$, no user is chosen for exploration and the slot is used for data transmission. The scheduler follows policy $\Psi$ introduced in the previous section with $P(C_i \geq r | \hat{C}_i = \hat{c}_i)$ replaced by the estimate $\hat{P}_t(C_i \geq r | \hat{C}_i = \hat{c}_i)$.

An illustration of the proposed policy is provided in Fig. 2. We now discuss the design of the quantities $x_i^j, \hat{c}_i \in S, i \in \{1, \ldots, N\}$. Let $\eta_{\hat{c}_i} = P(\hat{C}_i = \hat{c}_i)$ be a measure of how often the channel of user $i$ is explored when the estimate is $\hat{c}_i$. For fairness considerations, we impose the following constraint in addition to the $\gamma$-constraint discussed earlier:

$$
\sum_{\hat{c}_i \in S} \eta_{\hat{c}_i} = \gamma/N.
$$

The preceding constraint ensures that each user’s channel is explored for an equal fraction, $\gamma/N$, of the total time slots. From strong law of large numbers, with probability one, $\hat{P}_t(C_i \geq r | \hat{C}_i = \hat{c}_i)$ will converge to $P(C_i \geq r | \hat{C}_i = \hat{c}_i)$ as $t$ tends to infinity. The rate of convergence of the channel/estimate
joint statistics, parameterized by the user and the channel estimate, is given by the following lemma. Henceforth, we drop the suffix $i$ from $\hat{c}_i$ for notational convenience.

**Lemma 3.**

$$
\limsup_{t \to \infty} \frac{\hat{P}_t(C_i \geq r | \hat{C}_i = \hat{c}) - P(C_i \geq r | \hat{C}_i = \hat{c})}{\sqrt{\frac{2 \log \log(\frac{N_{\hat{c}, t}/|S|}{N_{\hat{c}, t}/|S|})}{(\log \log(\frac{N_{\hat{c}, t}/|S|}{N_{\hat{c}, t}/|S|}) \log \log(\frac{N_{\hat{c}, t}/|S|}{N_{\hat{c}, t}/|S|})}}} = \sqrt{2}\sigma
$$

almost surely (a.s.), where

$$
\sigma = \sqrt{P(C_i \geq r | \hat{C}_i = \hat{c}) (1 - P(C_i \geq r | \hat{C}_i = \hat{c}))}.
$$

**Proof.** We use $N_r[t]$ to denote the number of exploration slot corresponding to estimated channel $\hat{C}_i = \hat{c}$ and rate $r$. We express the left hand side of the equation in the lemma as follows.

$$
\limsup_{t \to \infty} \frac{\hat{P}_t(C_i \geq r | \hat{C}_i = \hat{c}) - P(C_i \geq r | \hat{C}_i = \hat{c})}{\sqrt{2 \log \log(\eta_{\hat{c}, \hat{c}} t/|S|)/N_r[t]}} = \limsup_{t \to \infty} \frac{\hat{P}_t(C_i \geq r | \hat{C}_i = \hat{c}) - P(C_i \geq r | \hat{C}_i = \hat{c})}{\sqrt{2 \log \log N_r[t]/N_r[t]}}.
$$

From Law of Iterated Logarithm (LIL), we get

$$
\limsup_{t \to \infty} \frac{\hat{P}_t(C_i \geq r | \hat{C}_i = \hat{c}) - P(C_i \geq r | \hat{C}_i = \hat{c})}{\sqrt{2 \log \log N_r[t]/N_r[t]}} = \sigma
$$

almost surely. We also have

$$
\frac{\log \log N_r[t]}{\log \log(\eta_{\hat{c}, \hat{c}} t/|S|)} = 1 + \frac{\log \log N_r[t] - \log \log(\eta_{\hat{c}, \hat{c}} t/|S|)}{\log \log(\eta_{\hat{c}, \hat{c}} t/|S|)} = 1 + \frac{\log(1 + \frac{\log(1 + \log N_r[t]/|S|)}{\log(\eta_{\hat{c}, \hat{c}} t/|S|)})}{\log \log(\eta_{\hat{c}, \hat{c}} t/|S|)}.
$$

(6)

Because $\{N_r[t]\}$ is a renewal process (LIL) with inter-renewal time $(\eta_{\hat{c}, \hat{c}} t/|S|)^{-1}$, we will have

$$
\lim_{t \to \infty} N_r[t]/(\eta_{\hat{c}, \hat{c}} t/|S|) = 1 \text{ almost surely.}
$$

Hence (6) tends to 1 almost surely. Substituting equation (5) and (6) into (4) we get

$$
\limsup_{t \to \infty} \frac{\hat{P}_t(C_i \geq r | \hat{C}_i = \hat{c}) - P(C_i \geq r | \hat{C}_i = \hat{c})}{\sqrt{2 \log \log(\eta_{\hat{c}, \hat{c}} t/|S|)/N_r[t]}} = \sigma,
$$

almost surely. \qed

Note from the preceding lemma that, for each $\{i, \hat{c}\}$, the higher the quantity $\eta_{\hat{c}, \hat{c}}$, the faster the convergence of $\hat{P}_t(C_i \geq r | \hat{C}_i = \hat{c})$. Also note that, for each user $i$, the channel estimate $\hat{c}$ with the slowest convergence affects the overall convergence performance for that user $i$. Taking note of this, we proceed to design $x^*_\hat{c}$ that maximizes the lowest convergence rate – the bottleneck.
The optimization problem \((U)\) for user \(i\) is given by
\[
\max_{x_i^\hat{c}} \min_{\hat{c}} \eta_{i,\hat{c}} = \frac{1}{N} P(\hat{C}_i = \hat{c}) x_i^\hat{c}
\]
s.t. \[
\sum_{\hat{c} \in S} \eta_{i,\hat{c}} = \frac{\gamma}{N},
\]
\[
0 < x_i^\hat{c} \leq 1, \quad \text{for all } \hat{c} \in S
\]

For ease of exposition, we assume, without loss of generality, that \(P(\hat{C}_i = s_1) \leq P(\hat{C}_i = s_2) \leq \cdots \leq P(\hat{C}_i = s_{|S|})\). Let \([x_i^{s_1^*}, x_i^{s_2^*}, \ldots, x_i^{s_{|S|}^*}]\) be the optimal solution to the above problem. We now record the structural properties of the optimal solution.

**Proposition 4.** The solution \(x_i^{s_k^*}, \forall k \in \{1, \ldots, |S|\}\), to the optimization problem \((U)\) can be obtained with the following algorithm:

1. **Initialization:** Let \(k = 1, \Gamma = \emptyset, \omega = 0;\)

2. **If** \(P(\hat{C} = s_k) \geq \frac{\gamma - \sum_{s_j \in \Gamma} P(\hat{C}_i = s_j)}{|S| - \omega}\), **then**, \(\forall l \geq k, x_i^{s_l^*} = \frac{\gamma - \sum_{s_j \in \Gamma} P(\hat{C}_i = s_j)}{|S| - \omega} \cdot P(\hat{C} = s_l)\).

   **Algorithm terminates.**

3. **Otherwise** \(x_i^{s_k^*} = 1, \Gamma = \Gamma \cup s_k, \omega = \omega + 1, k = k + 1.\) **If** \(\Gamma = S, \text{ algorithm terminates, otherwise repeat Step (2).}\)

**Proof Outline:** The proof proceeds by establishing two crucial properties of the optimal solution. First, define \(\Omega_i\) as the set of all channel estimates \(s_k\) such that the optimal \(x_i^{s_k^*} = 1\). Thus \(\Omega_i = \cup_k \{s_k : x_i^{s_k^*} = 1\}\). If no such estimate exists, \(\Omega_i = \emptyset\). The optimal solution has the following properties:

1. **If** \(\Omega_i = \emptyset\) **then** \(P(\hat{C}_i = s_k) x_i^{s_k^*} = \gamma/|S|, \forall k.\)

2. **If** \(\Omega_i \neq \emptyset,\) then \(x_i^{s_1^*} = 1.\)
Recall that the channel states are ordered such that $P(\hat{C}_i = s_1) \leq P(\hat{C}_i = s_2) \leq \cdots \leq P(\hat{C}_i = s_{|S|})$. The first property essentially says that if there does not exist a channel estimate $s$, for which $x^{s*}_s = 1$, then the optimal solution is such that the learning rate $\frac{P(\hat{C}_s = s_k)x^{s*}_s}{N}$ is uniform across all $s_k$, for all $k \in \{1, \ldots, |S|\}$. Because, otherwise, there is always room to improve the bottleneck convergence rate by redesigning the quantities $x^{s*}_s$. The second property says that whenever there exists an estimate $s_{k, k \neq 1}$ for which $x^{s*}_s = 1$, the estimate $s_1$ acts as a bottleneck, and the optimal value of $x^{s*}_s$ must be 1. The proposed algorithm now checks whether a solution yielding uniform convergence rate is feasible. If so, the solution is trivially given by $x^{s*}_s = \frac{1}{P(\hat{C}_s = s_k)}$ for all $k \in \{1, \ldots, |S|\}$. Otherwise, using the preceding properties, the algorithm assigns $x^{s*}_s = 1$ and goes on to solve the reduced optimization problem over $x^{s*}_2, \ldots, x^{s*}_{|S|}$, iteratively. Details of the proof can be found in Appendix D.

The proposed algorithm is illustrated in Fig. 3 when $\gamma = 0.2$, $N = 2$ and $S = \{1, \ldots, 6\}$. Focusing on User 1, Fig. 3(a) plots the probability of the estimated channels and the optimal values of $x^{s*}_s$, $s \in S$. Note that, the lower the value of $P(\hat{C}_1 = s)$, the higher the assigned $x^{1*}_s$, since the algorithm maximizes the bottleneck convergence rate $\frac{P(\hat{C}_s = s)x^{1*}_s}{N}$. This is further illustrated in Fig. 3(b) where the optimized convergence rate is shown to be ‘near uniform’, underlining the minmax nature of the optimization. Note that the structure of the minmax algorithm bears some similarity with the water-filling algorithm used in power allocation across parallel channels. There the algorithm tries to ‘equalize’ the sum of two components (signal and noise powers) across channels, while the minmax algorithm we propose tries to ‘equalize’ the product of two components ($P(\hat{C}_i = s)$ and $x^{s*}_s$).

We now perform a stability region analysis of the proposed policy. Define the stability region of a policy as the exhaustive set of arrival rates such that the network queues are rendered stable under the policy. The stability region of the proposed policy, parameterized by $\gamma \in (0, 1)$, is recorded below.

**Proposition 5.** The stability region $\Lambda'_\gamma$ of the proposed policy is given by

$$\Lambda'_\gamma = \{\lambda \ s.t. \ \frac{\lambda}{1 - \gamma} \in \Lambda\} \triangleq (1 - \gamma)\Lambda.$$

where $\Lambda$ is the stability region of the network when complete channel/estimator joint statistics is available at the scheduler.

**Proof Outline:** The proof proceeds by showing that, under the proposed joint statistics learning - scheduling policy, the instantaneous maximal sum of the queue weighted achievable rates, with sufficient time, can be arbitrarily close to the case when perfect knowledge of the statistics is available. Details are provided in Appendix E.

### 4.3 Throughput - Delay Tradeoff

As $\gamma \to 0$, the proposed policy has a stability region that can be arbitrarily close to the system stability region $\Lambda$. The trade-off involved here is the speed of convergence and hence queueing delays before convergence. Since an analytical study of this trade-off appears complicated, we proceed to perform a numerical study. The simulation setup is described next.

We use i.i.d. Rayleigh fading channels with minimum mean square error (MMSE) channel estimator as seen in [22] and [23]. The channel model is given by

$$Y = \sqrt{\rho}hX + \nu,$$

where $X, Y$ correspond to transmitted and received signals, $\rho$ is the average SNR at the receiver, and $\nu$ is the additive noise. Both $h$ and $\nu$ are zero-mean complex Gaussian random variables, i.e., with probability density $CN(0, 1)$. Let $\hat{h}$ denote the estimate of the channel and $\hat{h}$ denote the estimation error. Under the channel statistics assumed, $\hat{h}$ is zero-mean complex Gaussian with variance $\beta$, where the value of $\beta$ depends on the resources allocated for estimation (\textsuperscript{24}). Given the value of $\hat{h}$, the channel rate is $R = \log(1 + \rho|h|^2)$. We quantize the transmission rate to make the channel state space to be discrete and
finite. We assume a two-user network and fix $\beta = 0.1$ and $\rho = 50$ for both users’ channels. We study the average behavior of the proposed policy by implementing it over 10000 parallel queuing systems.

We first study the time evolution of the probability of transmission success for different values of $\gamma$. Fig. 4(a) shows that, for any $\gamma$, the probability of successful transmission increases as the accuracy of the estimate of the channel/estimator joint statistics improves with time. Also, as expected, the larger the value of $\gamma$ is, the faster is the improvement in the probability of successful transmission. Note that higher transmission success probability essentially means less number of retransmissions. This is illustrated in Fig. 4(b).

In Fig. 5, we study the time evolution of the average packet delay - the delay between the time a packet enters the queue and the time it leaves the head of the queue - for various values of $\gamma$. Note that $\gamma$ influences the average delay through (1) the average number of retransmissions and (2) the fraction of time slots available for transmissions. It is expected that the nature of the influence of $\gamma$ on the average delay depends on whether the estimate of the channel/estimator joint statistics has reached convergence or not. After convergence, the average delay is influenced by $\gamma$ solely through the fraction of time slots available for transmissions. Thus, after convergence, the higher the value of $\gamma$, the higher the average delay. This is illustrated in Fig. 5. Before convergence, however, the effect of $\gamma$ on the average delay is not straightforward. Fig. 5, along with the fact that higher $\gamma$ results in faster convergence, suggests the following: before convergence, $\gamma$ influences the average delay predominantly through the average number of retransmissions, resulting in decreasing average delay for increasing $\gamma$. In fact, Fig. 5 suggests the existence of a larger phenomenon: the trade-off between throughput (the stability region) and the delay before convergence.

5 Conclusion

We studied scheduling with rate adaptation in single-hop queueing networks, under imperfect channel state information. Under complete knowledge of the channel/estimator joint statistics at the scheduler, we characterized the network stability region and proposed a maximum-weight type scheduling policy that is throughput optimal. Under incomplete knowledge of the channel/estimator joint statistics, we designed a joint statistics learning - scheduling policy that maximizes the worst case rate of convergence of the statistics learning mechanism. We showed that the proposed policy can be tuned to achieve a stability region arbitrarily close to the network stability region with a corresponding trade-off in the average packet delay before convergence and the time for convergence.
Figure 5: Illustration of the average packet delay over time for various values of $\gamma$.

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Appendices

A Proof of Proposition 1

Proof. (Sufficiency) Define the Lyapunov function $L(Q[t]) = \sum_{i=1}^{N} Q_i^2[t]$. Recall the queue dynamic equation given by Equation (1), the Lyapunov drift can be written as

$$
\Delta L[t] = E\left[ \sum_{i=1}^{N} Q_i^2[t+1] - Q_i^2[t] \mid Q[t] \right] 
\leq B + 2 \sum_{i=1}^{N} Q_i[t] \left( \lambda_i - E\left[ \mathbf{1}(I[t] = i) \cdot R[t] \cdot \mathbf{1}(R[t] \leq C_i[t]) \mid Q[t] \right] \right) 
\leq B + 2 \sum_{i=1}^{N} Q_i[t] \left( \lambda_i - E[1(I[t] = i) \cdot R[t] \mid Q[t]] \right) + A_i^2[t] \mid Q[t].
$$

where

$$
B = \sum_{i=1}^{N} E[1(I[t] = i) \cdot R^2[t] \mid R[t] \leq C_i[t]] + A_i^2[t] \mid Q[t].
$$
Noting that $B$ is bounded. Let $r_i^*(c_i) = \arg \max_{r_i} P(C_i \geq r_i \mid \hat{C}_i = c_i) \cdot r_i$. Consider any arrival vector $\lambda$ strictly within the interior of $\Lambda$. For each channel state $\hat{c}$, there exist scaling vector $\alpha^\hat{c}$ and $\delta > 0$ associated with it such that

$$\lambda_i + \delta < \sum_{\hat{c} \in S^N} \pi_{\hat{c}} \cdot \alpha^\hat{c}_i \cdot P(C_i \geq r_i^*(\hat{c}_i) \mid \hat{C}_i = \hat{c}_i) \cdot r_i^*(\hat{c}_i),$$

for any user $i$, where $\sum_{i=1}^N \alpha_i^\hat{c} = 1$ for $\forall \hat{c}$.

Therefore, we can design a scheduling policy that does the following: At the channel estimation $\hat{C}[t] = \hat{c}[t]$, channel $i$ is activated with probability $\alpha_i^\hat{c}$. The rate allocated to channel $i$ will be $r_i^*(\hat{c}_i[t])$.

Then the service rate of user $i$ will be:

$$\mu_i = E[1(I[t] = i) \cdot R_i[t] \cdot 1(R_i[t] \leq C_i[t])] = \sum_{\hat{c} \in S^N} \pi_{\hat{c}} \cdot \alpha^\hat{c}_i \cdot P(C_i \geq r_i^*(\hat{c}_i) \mid \hat{C}_i = \hat{c}_i) \cdot r_i^*(\hat{c}_i)$$

Substitute (9) into (8), the Lyapunov drift function now becomes

$$\lambda_i - \mu_i < -\delta.$$  

Noting that in this policy the rate adaptation and link activation is completely determined by the channel estimation, and does not rely on queue length information, and therefore in this case

$$E\left[1(I[t] = i) \cdot R[t] \cdot 1(R[t] \leq C_i[t]) \mid Q[t] \right] = E\left[1(I[t] = i) \cdot R[t] \cdot 1(R[t] \leq C_i[t]) \right] = \mu_i$$

Substitute (10) into (7), the Lyapunov drift expression now becomes

$$\Delta L[t] \leq B - 2\delta \sum_{i=1}^N Q_i[t].$$

Because the scheduling and rate adaptation decision only depends on the current queue length and current channel estimate state, the queue evolves as a Markov Chain. According to Foster-Lyapunov Stability criterion [15, 16], the queues will be stable.

*(Necessity)* From strict separation theorem [17], for any arrival rate vector out side the proposed region $\Lambda$, there exist $\beta$, $\delta > 0$, such that for any vector $\tilde{\nu}$ inside the stability region $\Lambda$

$$\sum_{i=1}^N \beta_i (\lambda_i - \nu_i) \geq \delta$$

Define Lyapunov function $L(Q[t]) = \sum_{i=1}^N \beta_i Q_i[t]$. For any stationary scheduling policy that makes $I[t]$ and $R[t]$ decisions, we will have the following Lyapunov drift expression

$$E\left[L(Q[t+1]) - L(Q[t]) \mid Q[t] \right] = \sum_{i=1}^N \beta_i E\left[A_i[t] - 1(I[t] = i) \cdot 1(R_i[t] \leq C_i[t]) \cdot R_i[t] \mid Q[t] \right]$$

$$= \sum_{i=1}^N \beta_i \left[\lambda_i - E\left[1(I[t] = i) \cdot 1(R_i[t] \leq C_i[t]) \cdot R_i[t] \mid Q[t] \right] \right]$$

Let $\mu_i = E\left[1(I[t] = i) \cdot 1(R_i[t] \leq C_i[t]) \cdot R_i[t] \mid Q[t] \right]$.
Next we are going to show that \( \bar{\mu} \in \Lambda \). Consider
\[
E \left[ I(t) = i \cdot 1(R_i[t] \leq C_i[t]) \cdot R_i[t] \Big| Q[t] \right]
\]
\[
= E \left[ E \left[ I(t) = i \cdot 1(R_i[t] \leq C_i[t]) \cdot R_i[t] \Big| \hat{C}[t] \right] \right]
\]
\[
= \sum_{c_i \in S_N} \pi_{c_i} \cdot E \left[ I(t) = i \cdot 1(R_i[t] \leq C_i[t]) \cdot R_i[t] \Big| \hat{C}[t] \right] = \hat{c}_i
\]
\[
= \sum_{c_i \in S_N} \pi_{c_i} \cdot 1(I[t] = i) \cdot R_i[t] \cdot \Pr(R_i[t] \leq C_i[t]) \hat{C}_i[t] = \hat{c}_i
\]
\[
\leq \sum_{c_i \in S_N} \pi_{c_i} \cdot 1(I[t] = i) \cdot r_i^*(c_i[t]) \cdot \Pr(r_i^*(c_i[t]) \leq C_i[t]) \hat{C}_i[t] = \hat{c}_i[t].
\]

The third equality holds because \( R[t] \) and \( I[t] \) is determined by the current channel estimation and queue length information within the class \( G \) of stationary policies, and also the \( i.i.d. \) channel assumption. The above expression indicates that \( \bar{\mu} \in \Lambda \). Hence from (12)
\[
E \left[ L(Q[t+1]) - L(Q[t]) \Big| Q[t] \right] \geq \delta
\]

The Lyapunov function will always have a positive drift and therefore, the queue is unstable. \( \square \)

**B Proof of Proposition 2**

*Proof.* Assume the arrival rate vector \( \lambda \) is strictly within the interior of stability region, there exists \( \varepsilon > 0 \) such that \( \lambda + \varepsilon \bar{1} \in \text{int}(\Lambda) \). Because \( \lambda \) is strictly within the stability region, similar to the proof of Proposition 1, there exists some randomized scheduling policy \( G_0 \) that stably supports the arrival rate vector \( \lambda + \varepsilon \bar{1} \), and that \( G_0 \) will only depends on the estimated channel state.

Suppose the proposed scheduling policy \( \Psi \) will result in rate allocation \( R_i[t] \) and scheduling decision \( I[t] \) at time \( t \). Consider the policy \( G_0 \) that act at the same time \( t \) with the same channel state estimate and queue lengths knowledge, we denote its rate allocation to be \( \tilde{R}_i[t] \) and link activation decision to be \( \tilde{I}[t] \), therefore we have
\[
\sum_{i=1}^{N} Q_i[t] \cdot E \left[ I(t) = i \cdot \tilde{R}_i[t] \cdot 1(\tilde{R}_i[t] \leq C_i[t]) \right]
\]
\[
= \sum_{i=1}^{N} Q_i[t] \cdot E \left[ 1(\tilde{I}[t] = i) \cdot \tilde{R}_i[t] \cdot 1(\tilde{R}_i[t] \leq C_i[t]) \Big| Q[t], \hat{C}[t] \right]
\]
\[
\leq \sum_{i=1}^{N} Q_i[t] \cdot E \left[ 1(I[t] = i) \cdot R_i[t] \cdot 1(R_i[t] \leq C_i[t]) \Big| Q[t], \hat{C}[t] \right].
\]

The last inequality holds because policy \( \Psi \) maximizes the left hand side of the above inequality at every time slot. Also because the queue of each user is stable under policy \( G_0 \), we have
\[
\lambda_i + \varepsilon \leq E \left[ 1(\tilde{I}[t] = i) \cdot \tilde{R}_i[t] \cdot 1(\tilde{R}_i[t] \leq C_i[t]) \right]
\]
And therefore
\[
\sum_{i=1}^{N} Q_i[t] \cdot (\lambda_i + \varepsilon) \leq \sum_{i=1}^{N} Q_i[t] \cdot E \left[ 1(\tilde{I}[t] = i) \cdot \tilde{R}_i[t] \cdot 1(\tilde{R}_i[t] \leq C_i[t]) \right]
\]
\[
\leq \sum_{i=1}^{N} Q_i[t] \cdot E \left[ 1(I[t] = i) \cdot R_i[t] \cdot 1(R_i[t] \leq C_i[t]) \Big| Q[t], \hat{C}[t] \right].
\]
Substitute it back to Lyapunov drift expression (7), then we will have:

\[
\Delta \mathbf{L}[t] \leq B + 2 \sum_{i=1}^{N} Q_i[t] \left( \lambda_i - E\left[ R[t] \cdot \mathbf{1}(R[t] \leq C_i[t]) \mathbf{|} \mathbf{Q}[t], \hat{C}[t] \right] \right)
\]
\[
\leq B - 2\delta \sum_{i=1}^{N} Q_i[t]
\]

Because scheduling policy \(\Psi\) only depends on current queue length and channel estimate, and because the channel process is a \(i.i.d.\) across time, the queue evolution under policy \(\Psi\) will be a Markov Chain. From Foster-Lyapunov criterion, the statement is proven.

\(\square\)

## C Proof of Stability Region Without Rate Adaptation

**Proof.** The proof of the statement is somehow similar to proof of proposition 1.

*(Sufficiency)* Define the Lyapunov function \(L(\mathbf{Q}[t]) = \sum_{i=1}^{N} Q_i^2[t]\), the Lyapunov drift can be written as Equation (7). For any arrival vector \(\lambda\) strictly within the interior of \(\hat{\Lambda}\) and each channel state \(\hat{c}\), the vector \(\alpha^\hat{c}\) and \(\delta > 0\) satisfies

\[
\lambda_i + \delta < \sum_{\hat{c} \in S^N} \pi_{\hat{c}} \cdot \alpha_i^\hat{c} \cdot P(C_i \geq \hat{c}| \hat{C}_i = \hat{c}_i) \cdot \hat{c}_i,
\]

for any user \(i\), where \(\sum_{i=1}^{N} \alpha_i^\hat{c} = 1\) for \(\forall \hat{c}\).

The rest of the proof follows similar as in Proposition 1.

*(Necessity)* Similar to the proof of Proposition 1, for any arrival rate vector out side \(\hat{\Lambda}\), there exist \(\beta\), \(\delta > 0\), such that for any vector \(\hat{\nu}\) inside the stability region \(\hat{\Lambda}\), we have \(\sum_{i=1}^{N} \beta_i (\lambda_i - \nu_i) \geq \delta\).

Define Lyapunov function \(L(\mathbf{Q}[t]) = \sum_{i=1}^{N} \beta_i Q_i[t]\), for any stationary policy that makes scheduling decision \(I[t]\) and rate adaptation decision \(R[t]\), again we will have the similar Lyapunov drift expression as in Equation (12),

\[
E\left[ L(\mathbf{Q}[t+1]) - L(\mathbf{Q}[t]) \mathbf{|} \mathbf{Q}[t] \right] = \sum_{i=1}^{N} \beta_i \cdot \left( \lambda_i - E\left[ \mathbf{1}(I[t] = i) \cdot \mathbf{1}(\hat{C}_i[t] \leq C_i[t]) \cdot \hat{C}_i[t] \mathbf{|} \mathbf{Q}[t] \right] \right)
\]

Let \(\mu_i = E\left[ \mathbf{1}(I[t] = i) \cdot \mathbf{1}(\hat{C}_i[t] \leq C_i[t]) \cdot \hat{C}_i[t] \mathbf{|} \mathbf{Q}[t] \right]\). We can show \(\hat{\mu} \in \hat{\Lambda}\) from

\[
E\left[ \mathbf{1}(I[t] = i) \cdot \mathbf{1}(\hat{C}_i[t] \leq C_i[t]) \cdot \hat{C}_i[t] \mathbf{|} \mathbf{Q}[t] \right] = \sum_{\hat{c} \in S^N} \pi_{\hat{c}} \cdot \mathbf{1}(I[t] = i) \cdot \hat{c}_i \cdot \text{Pr(} \hat{c}_i \leq C_i[t]| \hat{C}_i[t] = \hat{c}_i) \]
\[
= \sum_{\hat{c} \in S^N} \pi_{\hat{c}} \cdot \mathbf{1}(I[t] = i) \cdot \hat{c}_i \cdot \text{Pr(} \hat{c}_i \leq C_i[t]| \hat{C}_i[t] = \hat{c}_i).
\]

The rest follows similarly as in proof of Proposition 1. \(\square\)
D Proof of Proposition 4

For notational convenience, we drop user index $i$ in the proof. The optimization problem (U) can be re-written as

$$\max_{x_\hat{c}} \min_{\hat{c}} \Pr(\hat{C} = \check{c}) x_{\hat{c}}$$

subject to

$$\sum_{\hat{c} \in \mathcal{S}} \Pr(\hat{C} = \check{c}) x_{\hat{c}} = \gamma$$

$$\sum_{\hat{c} \in \mathcal{S}} \Pr(\hat{C} = \check{c}) = 1$$

$$0 < x_{\hat{c}} \leq 1.$$

This problem can be transformed into a Linear Programming problem as the following.

$$\max t$$

subject to

$$\Pr(\hat{C} = \check{c}) x_{\hat{c}} \geq t$$

$$\sum_{\hat{c} \in \mathcal{S}} \Pr(\hat{C} = \check{c}) x_{\hat{c}} = \gamma$$

$$\sum_{\hat{c} \in \mathcal{S}} \Pr(\hat{C} = \check{c}) = 1$$

$$0 < x_{\hat{c}} \leq 1.$$

Hence the problem has become a convex optimization problem. We let $[t^*, x_{s_1}^*, x_{s_2}^*, \cdots, x_{s_{|S|}}^*]$ be the optimal solution to the above problem and let $\Omega = \{s_k : x_{s_k}^* = 1\}$.

Lemma 6. The optimal solution to the optimization problem (U) must satisfy the following structural properties:

(i) If $\Omega = \emptyset$ then $\Pr(\hat{C} = s_k) x_{s_k}^* = \gamma/|\mathcal{S}|$ for all $k$.

(ii) Conversely, if $\Pr(\hat{C} = s_k) x_{s_k}^* = \gamma/|\mathcal{S}|$ for all $k$, then $\Omega = \emptyset$, except for when $\Pr(\hat{C} = s_1) = \frac{\gamma}{|\mathcal{S}|}$.

Proof. (i). Suppose $\Omega = \emptyset$ but we don’t have $\Pr(\hat{C} = s_k) x_{s_k}^* = \gamma/|\mathcal{S}|$ for all $k$. Let $n$ be such that $t^* = \Pr(\hat{C} = s_n) x_{s_n}^*$. Because $\Pr(\hat{C} = s_k) x_{s_k}^*$ values are not equal for every $k$, there exists $m \neq n$ such that $t^* = \Pr(\hat{C} = s_m) x_{s_m}^* < \Pr(\hat{C} = s_m) x_{s_m}^*$. Because $\Omega = \emptyset$, $x_{s_n}^* \neq 1$ and $x_{s_m}^* \neq 1$. Let $\Pi = \{x_{s_k}^* : t^* = \Pr(\hat{C} = s_k) x_{s_k}^*\}$ and let $|\Pi| = \pi$. If we set

$$x_{s_l}^+ = x_{s_l}^* + \frac{\delta}{\pi \Pr(\hat{C} = s_l)}, \forall l \in \Pi$$

$$x_{s_m}^- = x_{s_m}^* - \frac{\delta}{\Pr(\hat{C} = s_m)}$$

where $\delta > 0$ is small that will guarantee that $x_{s_m}^-$ stays positive.

We can check that in this case, still $\sum_{\check{c} \in \Pi} \Pr(\hat{C} = \check{c}) x_{\check{c}}^* + \sum_{\check{c} \in \Pi} \Pr(\hat{C} = \check{c}) x_{\check{c}}^* + \Pr(\hat{C} = c_{s_m}) x_{s_m}^- = \gamma$. But in this case the new value of the objective function $t_{\text{new}}^* > t^*$, contradicts to the assumption that $t^*$ is the optimal value. Therefore we must have $\Pr(\hat{C} = s_k) x_{s_k}^* = \gamma/|\mathcal{S}|, \forall i$, establishing the proof of (i).

(ii). When $\Pr(\hat{C} = s_k) x_{s_k}^* = \gamma/|\mathcal{S}|, \forall k$, and $x_{s_k}^* < 1$ for all $k$, we will have $\Omega = \emptyset$.

If $\exists h, \Pr(\hat{C} = s_h) = \gamma/|\mathcal{S}|$, hence $x_{s_h}^* = 1$. By assumption

$$\Pr(\hat{C} = s_1) x_{s_1}^* = \Pr(\hat{C} = s_h) x_{s_h}^* = \Pr(\hat{C} = s_h).$$

Because $\Pr(\hat{C} = s_1) \leq \Pr(\hat{C} = s_h)$ and $x_{s_h}^* \leq 1$, we must have $\Pr(\hat{C} = s_1) = \Pr(\hat{C} = s_h)$ and $x_{s_1}^* = 1$, establishing the proof of (ii).
Lemma 7. If \( \Omega \neq \emptyset \), then \( x^*_{s_1} = 1 \) and \( t^* = Pr(\hat{C} = s_1) \).

Proof. We proof this Lemma by contradiction. Suppose that \( x^*_{s_1} < 1 \).

(Case 1). If \( t^* = Pr(\hat{C} = s_1)x^*_{s_1} \), without loss of generality, suppose \( s_1 \) is the only channel state that results in \( Pr(\hat{C} = s_1)x^*_{s_1} = t^* \). Because \( \Omega \neq \emptyset \), suppose \( x^*_{s_j} = 1 \) for some state \( s_j \). If we set \( x^\text{new}_{s_j} = 1 - \delta / Pr(\hat{C} = s_j) \) and \( x^\text{new}_{s_1} = x^*_{s_1} + \delta / Pr(\hat{C} = s_1) \), where \( \delta \) is small enough, we will get an new value of the objective function strictly larger than \( t^* \) while still satisfy the constraints in (U), which contradicts to the optimality of \( [t^*, x^*_{s_1}, x^*_{s_2}, \ldots, x^*_{s_k}] \).

(Case 2). If \( t^* < Pr(\hat{C} = s_1)x^*_{s_1} \), suppose \( t^* = Pr(\hat{C} = s_m)x^*_{s_m} \) for some \( s_m \) and assume, with no loss of generality, it is the only state of this kind. Because \( Pr(\hat{C} = s_m)x^*_{s_m} < Pr(\hat{C} = s_1)x^*_{s_1} \) and \( Pr(\hat{C} = s_m) = t^* \geq Pr(\hat{C} = s_1) \), we have \( x^*_{s_m} < x^*_{s_1} < 1 \). We can set \( x^\text{new}_{s_m} = x^*_{s_m} + \delta / Pr(\hat{C} = s_m) \) and \( x^\text{new}_{s_1} = x^*_{s_1} - \delta / Pr(\hat{C} = s_1) \) for \( \delta \) small. Again, this change of variables will result in a new objective function value strictly larger than \( t^* \), contradicts to the optimality of \( t^* \).

Therefore, we have \( x^*_{s_1} = 1 \).

Suppose we have \( t^* < Pr(\hat{C} = s_1)x^*_{s_1} \). Similar to case 2, suppose \( t^* = Pr(\hat{C} = s_m)x^*_{s_m} \) for some \( s_m \), and assume such \( s_m \) is unique, we will have \( x^*_{s_m} < 1 \) because \( Pr(\hat{C} = s_m) = t^* \geq Pr(\hat{C} = s_1) \). By letting \( x^\text{new}_{s_m} = 1 + \delta / Pr(\hat{C} = s_m) \) and \( x^\text{new}_{s_1} = 1 - \delta / Pr(\hat{C} = s_1) \) with \( \delta \) small, we can get a strictly larger objective function value, contradicting to the optimality of \( t^* \). Therefore we must have \( t^* = Pr(\hat{C} = s_1)x^*_{s_1} \). \( \square \)

After we have established the above lemmas, we proceed to the proof of Proposition 4.

(Proof of Proposition 4)

Proof.

(Case 1). First consider the case when \( Pr(\hat{C} = s_1) > \frac{2}{|S|} \). If in this case \( \Omega \neq \emptyset \), then from Lemma 7, \( t^* = Pr(\hat{C} = s_1) > \frac{2}{|S|} \). Then \( \sum_{k=1}^{|S|} Pr(\hat{C} = s_k)x^*_{s_k} \geq \frac{|S|}{1} \cdot t^* > \frac{|S|}{1} \cdot \frac{\gamma}{|S|} = \gamma \). Therefore contradict to the constraint \( \sum_{k=1}^{|S|} Pr(\hat{C} = s_k)x^*_{s_k} = \gamma \). So \( \Omega = \emptyset \), from Lemma 6, we have \( Pr(\hat{C} = s_k)x^*_{s_k} = \frac{\gamma}{|S|} \) for all \( i \) and

\[
x^*_{s_k} = \frac{\gamma}{|S| \cdot Pr(\hat{C} = s_k)}.
\]

(Case 2). Consider when \( \frac{\gamma}{|S|} = Pr(\hat{C} = s_1) \). If \( \Omega = \emptyset \), then from Lemma 6 (i), \( x^*_{s_1} = 1 \), contradict to \( \Omega = \emptyset \). So \( \Omega \neq \emptyset \) and from Lemma 7, \( t^* = Pr(\hat{C} = s_1) = \frac{\gamma}{|S|} \). Because

\[
\sum_{i=1}^{|S|} Pr(\hat{C} = s_k)x^*_{s_k} \geq \sum_{i=1}^{|S|} t^* = |S| \cdot \frac{\gamma}{|S|} = \gamma,
\]

we must have \( t^* = Pr(\hat{C} = s_k)x^*_{s_k} \) for all \( k \) in order to satisfy the constraint of \( \sum_{k=1}^{|S|} Pr(\hat{C} = s_k)x^*_{s_k} = \gamma \). Therefore we still have

\[
x^*_{s_k} = \frac{\gamma}{|S| \cdot Pr(\hat{C} = s_k)}.
\]

It is easy to check here that \( 0 < x^*_{s_k} \leq 1 \) in the case 1 and case 2. We hence have justified the step (2) in Proposition 4 when \( k = 1 \).

(Case 3). If we have \( Pr(\hat{C} = s_1) < \frac{\gamma}{|S|} \), then we can not set \( Pr(\hat{C} = s_k)x^*_{s_k} = \frac{\gamma}{|S|} \) for all \( k \) because \( x^*_{s_1} \leq 1 \). So from Lemma 6, \( \Omega \neq \emptyset \). From Lemma 7, \( x^*_{s_1} = 1 \) and \( t^* = Pr(\hat{C} = s_1) \).

Since now we have identified the optimal value \( t^* \) of the objective function and \( x^*_{s_1} \), we still need to identify the rest of the solution of \( x^*_{s_j} \) for \( j \neq 1 \). Admitting there might be multiple solutions for those...
Let $x_{s_j}^*$, we consider the following relaxed optimization problem $(U^+)$
\[
\begin{array}{ll}
\max & t \\
\text{s.t.} & \Pr(\hat{C} = s_k)x_{s_k} \geq t; \ k \neq 1 \\
& \sum_{k \neq 1} \Pr(\hat{C} = s_k)x_{s_k} = \gamma^+ \\
& x_{s_1} = 1; \ 0 < x_{s_k} \leq 1, k \neq 1.
\end{array}
\]
where $\gamma^+ = \gamma - \Pr(\hat{C} = s_1)$.

It can be readily verified Lemma 7 and Lemma 8 holds for the above optimization problem with $\gamma$ and $S$ substituted by $\gamma^+$ and $S \setminus s_1$, respectively. Let $\hat{x}_{s_j}^*$, $j = 1, \cdots, N$ be the optimal solution for the above optimization problem $(U^+)$. We proceed to show $\hat{x}_{s_j}^*$ is also optimal solution to optimization problem $(U)$, i.e., satisfying all the constraints of $(U)$ and will preserve the optimality of $t^*$ of $(U)$ identified earlier.

Let $\hat{t}^*$ be the optimal objective function to the problem $(U^+)$. To show that optimal solution to $(U^+)$ (i.e., $\hat{x}_{s_j}^*$ for $j \neq 1$) preserve the optimality of $t^*$, we must check that $\hat{t}^* \geq t^*$. This is indeed the case and is explained as follows. Let $\hat{\Omega}$ be the set of estimated channel states $s_k$ such that $\hat{x}_{s_k}^* = 1$ in $(U^+)$. When $\hat{\Omega} \neq \emptyset$, from Lemma 7, $\hat{t}^* = \Pr(\hat{C} = s_2)x_{s_2}^* = \Pr(\hat{C} = s_2) \cdot 1 \geq \Pr(\hat{C} = s_1) \cdot 1 \geq \Pr(\hat{C} = s_1) \cdot x_{s_1}^* \geq t^*$. When $\hat{\Omega} = \emptyset$, from Lemma 6 we have $\forall j \neq 1$,
\[
\hat{t}^* = \hat{x}_{s_j}^* \Pr(\hat{C} = s_j) = \frac{\gamma - \Pr(\hat{C} = s_1)}{|S| - 1} \geq \frac{|S|\Pr(\hat{C} = s_1) - \Pr(\hat{C} = s_1)}{|S| - 1} = \Pr(\hat{C} = s_1) = t^*
\]
where the inequality is from $\frac{1}{|S|} > \Pr(\hat{C} = s_1)$ assumed in the beginning of Case 3. So the optimality of $t^*$ is preserved. It is also clear that the constraint $\Pr(\hat{C} = s_1)x_{s_1}^* + \sum_{j=2}^{n} \Pr(\hat{C} = s_j)\hat{x}_{s_j}^* = \gamma$ is satisfied.

We hence face a reduced optimization problem $(U^+)$, for which the optimal solution will also be optimal for the original optimization problem $(U)$. Problem $(U^+)$ takes the same form of $(U)$ with $\gamma$ and $S$ substituted by $\gamma^+$ and $S \setminus s_1$, respectively. The proposed algorithm solves the reduced optimization problem by conditioning on the reduced settings of $(U)$. Hence similar proof as in (Case 1)-(Case 3) is also applicable for the iterative algorithm. By doing the proof iteratively, the optimality of the algorithm is proved.

\section{Proof of Proposition 5}

We need the following lemmas before proving Proposition 4:

\textbf{Lemma 8.} For any user $i$ and any observed channel state $c_k$, there exist a constant $T$, such that for any time $t > T$, and any channel states $r_1$ and $r_2$, if $P(C_i \geq r_1 | \hat{C}_i = \hat{c}_k)r_1 > P(C_i \geq r_2 | \hat{C}_i = \hat{c}_k)r_2$, then
\[
\hat{P}_i(C_i \geq r_1 | \hat{C}_i = \hat{c}_k)r_1 > \hat{P}_i(C_i \geq r_2 | \hat{C}_i = \hat{c}_k)r_2 \quad \text{almost surely}.
\]

\textbf{Proof.} Let $\Delta = P(C_i \geq r_1 | \hat{C}_i = \hat{c}_k)r_1 - P(C_i \geq r_2 | \hat{C}_i = \hat{c}_k)r_2$. Then there exist $\delta > 0$ and $\epsilon > 0$ such that $\delta r_1 + \epsilon r_2 < \Delta$. From Strong Law of Large Numbers, there exist time $T$ such that for $t > T$,
\[
\hat{P}_i(C_i \geq r_1 | \hat{C}_i = \hat{c}_k) - P(C_i \geq r_1 | \hat{C}_i = \hat{c}_k) < -\delta \quad a.s.,
\]
\[
\hat{P}_i(C_i \geq r_2 | \hat{C}_i = \hat{c}_k) - P(C_i \geq r_2 | \hat{C}_i = \hat{c}_k) < \epsilon \quad a.s.
\]
Hence we have
\[
\hat{P}_i(C_i \geq r_1 | \hat{C}_i = \hat{c}_k)r_1 - \hat{P}_i(C_i \geq r_2 | \hat{C}_i = \hat{c}_k)r_2 > P(C_i \geq r_1 | \hat{C}_i = \hat{c}_k)r_1 - (\delta r_1 + \epsilon r_2)
\]
\[
\Delta - \Delta = 0 \quad a.s.
\]
Remark: This lemma implies that, there will be a time beyond which the allocated rate with empirical knowledge of $\hat{R}_t(C_i \geq r_i | \hat{C}_i = \hat{c}_k)$ is the same as with accurate knowledge. Because both the number of users and the state space is finite, we will chose the right rate with probability one as time is large, which is summarized in the following corollary.

**Corollary 9.** There exist a time $T$ beyond which, with probability 1, the empirical scheme will allocate rate $\hat{R}[t]$ the same as $R[t]$ when the $P(C[t] | \hat{C}[t])$ is perfect known.

**Proof.** This result is immediate from the previous lemma.

**Lemma 10.** At those non-observation slot $t$, let $\hat{I}[t]$ and $\hat{R}_t[I]$ be the scheduling decision by the joint statistics learning-scheduling policy. Let $I[t]$ and $R_t[I]$ be the scheduling decision of $\Psi$ with accurate knowledge. for any $\rho > 0$. There exist time $N$ beyond which, with probability 1,

$$
\sum_{j=1}^{N} Q_i[t]1(I[t]=j)P(C_j[t] \geq R_i[t] | \hat{C}_j[t] = \hat{c}_j)R_i[t] - \sum_{i=1}^{N} Q_i[t]1(\hat{I}[t]=i)P(C_i[t] \geq \hat{R}_i[t] | \hat{C}_i[t] = \hat{c}_i)\hat{R}_i[t] \\
\leq \rho \sum_{i=1}^{N} Q_i[t]
$$

**Proof.** From corollary 9, let $T_1$ be the time beyond which $R[t] = \hat{R}[t]$ a.s.. We consider the time $t > T_1$ and thus $R_t[I] = \hat{R}_t[I] = r^*_c = \arg \max P(C_j[t] \geq r | \hat{C}_j[t] = \hat{c}_j) \cdot r$ almost surely for all $i$. From strong law of large numbers, let $T_2 > T_1$ be such that beyond which $|\hat{P}_t(C_j \geq r^*_c | \hat{C}_j = \hat{c}_j) - P(C_j \geq r^*_c | \hat{C}_j = \hat{c}_j)|r^*_c \leq \rho$ almost surely for all $j$. We henceforth consider $t > T_2$. Given queue length $Q[t]$ and estimated channel state information $\hat{C}_i[t]$, $I[I]$ and $\hat{I}[I]$ are determined by

$$
I[I] = \arg \max \quad Q_i[t]P(C_i \geq r^*_c | \hat{C}_i = \hat{c}_i)r^*_c \quad a.s.
$$

$$
\hat{I}[I] = \arg \max \quad Q_i[t]\hat{P}_t(C_i \geq r^*_c | \hat{C}_i = \hat{c}_i)r^*_c \quad a.s.
$$

If $I[I] = \hat{I}[I]$, the statement will hold. If $I[I] = h$ but $\hat{I}[I] = k$ for $h \neq k$, we have

$$
Q_h[t]\hat{P}_t(C_k \geq r^*_c | \hat{C}_k = \hat{c}_k)r^*_c \geq Q_h[t]\hat{P}_t(C_h \geq r^*_c | \hat{C}_h = \hat{c}_h)r^*_c, \quad (13)
$$

$$
Q_h[t]P(C_h \geq r^*_c | \hat{C}_h = \hat{c}_h)r^*_c \geq Q_k[t]P(C_k \geq r^*_c | \hat{C}_k = \hat{c}_k)r^*_c. \quad (14)
$$

Because $t > T_2$, we further have

$$
\sum_{j=1}^{N} Q_i[t](I[t]=j)P(C_j[t] \geq R_t[I] | \hat{C}_j[t] = \hat{c}_j)R_t[I] - \sum_{i=1}^{N} Q_i[t](\hat{I}[I]=i)P(C_i[t] \geq \hat{R}_t[I] | \hat{C}_i[t] = \hat{c}_i)\hat{R}_t[I] \\
= Q_h[t]P(C_h \geq r^*_c | \hat{C}_h = \hat{c}_h)r^*_c - Q_k[t]P(C_k \geq r^*_c | \hat{C}_k = \hat{c}_k)r^*_c \\
\leq Q_h[t]|\hat{P}_t(C_h \geq r^*_c | \hat{C}_h = \hat{c}_h)r^*_c + \rho - Q_k[t]|\hat{P}_t(C_k \geq r^*_c | \hat{C}_k = \hat{c}_k)r^*_c - \rho \\
\leq \rho(Q_h[t] + Q_k[t])
$$

almost surely, where the first inequity is from the assumption that $|\hat{P}_t(C_j \geq r^*_c | \hat{C}_j = \hat{c}_j) - P(C_j \geq r^*_c | \hat{C}_j = \hat{c}_j)|r^*_c \leq \rho$ almost surely for all $j$, and the last inequality is from (13). Also, we have

$$
\sum_{i=1}^{N} Q_i[t](\hat{I}[I]=i)P(C_i[t] \geq R_t[I] | \hat{C}_i = \hat{c}_i)\hat{R}_t[I] - \sum_{j=1}^{N} Q_i[t](I[I]=j)P(C_j[t] \geq \hat{R}_t[I] | \hat{C}_j = \hat{c}_j)\hat{R}_t[I] \\
= Q_h[t]P(C_h \geq r^*_c | \hat{C}_h = \hat{c}_h)r^*_c - Q_h[t]P(C_h \geq r^*_c | \hat{C}_h = \hat{c}_h)r^*_c \\
\leq 0 \\
\leq \rho(Q_h[t] + Q_k[t])
$$

almost surely, where the first inequality is from (14). We hence proved the Lemma.
Proof. We first prove that for any \( \Lambda \) strictly within \( \Lambda, \gamma \), i.e., \( \Lambda + \epsilon \mathbf{1} \in \text{int}(\Lambda, \gamma) \), there exists a policy \( G_0 \) that makes decision only based on empirical statistics, i.e., \( \hat{R}_t(C_i \geq r|\hat{C}_i) \) and stably supports \( \Lambda \). Because \[
\lambda_i + \varepsilon \leq (1 - \gamma) \sum_{\hat{c} \in S^N} \hat{\pi}_{\hat{c}} \cdot \alpha_i^\hat{c} \cdot P(C_i \geq r_i^\hat{c}(\hat{c}_i)|\hat{C}_i = \hat{c}_i) \cdot r_i^\hat{c}(\hat{c}_i).
\] (15)

Then the randomized policy \( G_0 \) can be: at every non-observing estimated state \( \hat{c} \), activates channel \( \tilde{I}[t] = i \) with probability \( \alpha_i^\hat{c} \) with the allocated transmission rate \( \hat{R}_t = \arg \max_r \hat{R}_t(C[t] \geq r|\hat{C}[t]) \cdot r \).

Define Lyapunov function \( L(Q[t]) = \sum_{i=1}^{N} Q_i^2[t] \), similar to (7), the Lyapunov drift can be written as
\[
\Delta L[t] = E\left[\sum_{i=1}^{N} Q_i^2[t + 1] - Q_i^2[t] \bigg| Q[t]\right] \\
\leq B_1 + 2 \sum_{i=1}^{N} Q_i[t] \left( \lambda_i - (1 - \gamma) E\left[1(\tilde{I}[t] = i) \cdot \hat{R}[t] \cdot 1(\hat{R}[t] \leq C_i[t]) \bigg| Q[t]\right]\right) \\
\text{where } B_1 \text{ is bounded,}
\]
\[
B_1 = (1 - \gamma)^2 \sum_{i=1}^{N} E\left[1(\tilde{I}[t] = i) \cdot \hat{R}^2[t] 1(\hat{R}[t] \leq C_i[t]) + A_i^2[t] \bigg| Q[t]\right].
\]

Then from Corollary 9, for \( t > N \), \( \hat{R}[t] = r_i^\hat{c}(\hat{c}_i) = \arg \max_r P(C_i \geq r|\hat{C}_i = \hat{c}_i) \cdot r \) almost surely. Hence
\[
(1 - \gamma) E\left[1(\tilde{I}[t] = i) \cdot \hat{R}[t] \cdot 1(\hat{R}[t] \leq C_i[t]) \bigg| Q[t]\right] \\
= (1 - \gamma) \sum_{\hat{c} \in S^N} \hat{\pi}_{\hat{c}} \cdot \alpha_i^\hat{c} \cdot P(C_i \geq r_i^\hat{c}(\hat{c}_i)|\hat{C}_i = \hat{c}_i) \cdot r_i^\hat{c}(\hat{c}_i)
\] (17)

Substitute (15) and (17) into equation (16), the Lyapunov drift function will take the form
\[
\Delta L[t] \leq B - 2\varepsilon \sum_{i=1}^{N} Q_i[t].
\]

Hence Queues will be stable.

We next show that the joint statistics learning-scheduling policy will stabilize \( \Lambda \) similar to the proof of Proposition 2. Given queue length information \( Q[t] \) and estimated channel state \( \hat{C}[t] \), suppose the proposed joint statistics learning-scheduling policy will result in rate adaptation \( \hat{R}[t] \) and scheduling decision \( \tilde{I}[t] \) at time \( t \), and suppose the policy \( \Psi \) with perfect CSI will make rate adaptation decision \( R[t] \) and scheduling decision \( I[t] \). Associated with the joint statistics learning and scheduling algorithm, the Lyapunov Drift can be written as
\[
\Delta L[t] = E\left[\sum_{i=1}^{N} Q_i^2[t + 1] - Q_i^2[t] \bigg| Q[t]\right] \\
\leq B_2 + 2 \sum_{i=1}^{N} Q_i[t] \left( \lambda_i - (1 - \gamma) E\left[1(\tilde{I}[t] = i) \cdot \hat{R}[t] \cdot 1(\hat{R}[t] \leq C_i[t]) \bigg| Q[t]\right]\right) \\
= B_2 + 2 \sum_{i=1}^{N} Q_i[t] \left( \lambda_i - (1 - \gamma) E\left[1(\tilde{I}[t] = i) \cdot \hat{R}[t] \cdot 1(\hat{R}[t] \leq C_i[t]) \bigg| Q[t], \hat{C}[t]\right]\right)
\] (18)

where \( B_2 \) is bounded,
\[
B_2 = (1 - \gamma)^2 \sum_{i=1}^{N} E\left[1(\tilde{I}[t] = i) \cdot \hat{R}^2[t] 1(\hat{R}[t] \leq C_i[t]) + A_i^2[t] \bigg| Q[t]\right].
\]
Since we will have:

\[
\sum_{i=1}^{N} Q_i[t] \cdot E\left[1(\tilde{\ell}[t] = i) \cdot \tilde{R}[t] \cdot 1(\tilde{R}[t] \leq C_i[t]) \right] \\
\leq \sum_{i=1}^{N} Q_i[t] \cdot E\left[1(I[t] = i) \cdot R[t] \cdot 1(R[t] \leq C_i[t]) \mid Q[t], \hat{C}[t] \right].
\]

And therefore for any \(0 < \rho < \varepsilon/(1 - \gamma)\), for \(t > \max\{T_1, N\}\), we have

\[
\sum_{i=1}^{N} Q_i[t] \cdot (\lambda_i + \varepsilon) \leq (1 - \gamma) \sum_{i=1}^{N} Q_i[t] \cdot E\left[1(\tilde{\ell}[t] = i) \cdot \tilde{R}[t] \cdot 1(\tilde{R}[t] \leq C_i[t]) \mid Q[t], \hat{C}[t] \right]
\]

\[
\leq (1 - \gamma) \sum_{i=1}^{N} Q_i[t] \cdot E\left[1(I[t] = i) \cdot R[t] \cdot 1(R[t] \leq C_i[t]) \mid Q[t], \hat{C}[t] \right]
\]

\[
= (1 - \gamma) \sum_{i=1}^{N} Q_i[t] \sum_{C_i[t] \in S} 1(I[t] = i) \cdot \hat{R}[t] \cdot P(\hat{R}[t] \leq C_i[t] \mid \hat{C}_i[t])
\]

\[
= (1 - \gamma) \sum_{C_i[t] \in S} \sum_{i=1}^{N} Q_i[t] 1(I[t] = i) \cdot \hat{R}[t] \cdot P(\hat{R}[t] \leq C_i[t] \mid \hat{C}_i[t])
\]

\[
\leq (1 - \gamma) \sum_{C_i[t] \in S} \sum_{i=1}^{N} Q_i[t] 1(\tilde{\ell}[t] = i) \cdot \tilde{R}[t] \cdot P(\tilde{R}[t] \leq C_i[t] \mid \hat{C}_i[t]) + \rho
\]

\[
= (1 - \gamma) \sum_{i=1}^{N} Q_i[t] \cdot E\left[1(\tilde{\ell}[t] = i) \cdot \tilde{R}[t] \cdot 1(\tilde{R}[t] \leq C_i[t]) \mid Q[t], \hat{C}[t] \right] + \rho
\]

\[
(19)
\]

where the last inequality comes from Lemma 10. Substitute (19) into the Lyapunov drift expression (18), we will have:

\[
\Delta L[t] \leq B_2 - 2(\varepsilon - \rho(1 - \gamma)) \sum_{i=1}^{N} Q_i[t]
\]

Since \(\varepsilon - \rho(1 - \gamma) > 0\), the queues will be stable. \(\square\)