Experimental and Theoretical Analysis of the Factors Influencing the Flow-out Rate from a Bottle

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Abstract. While looking for parameters that may affect the experiment results, one of the basic experimental principles can be adopted: the principle of controlling a single variable. This experiment is centered on the flow rate of the liquid in the plastic bottle, and the fixed value that remains unchanged is the physical properties of the bottle itself and the liquid. The density of the liquid and the angle of the bottle are two factors that could affect the flow-out rate. The ten different angles from 0° to 90° are chosen to be observed. And five different densities of alcohol are chosen to be tested as well. Under reasonable error control, the angle of the bottle body can be obtained when the liquid flow rate is the maximum. Based on the experimental derivation process recorded in this article, the basic knowledge of statistics is used, combining with as much precision measuring equipment as possible. Then, the Bernoulli equation is also used to calculate the ideal value, which is also compared with the actual value, to get the impact factors. The experimental results are scientifically analysed and deduced, and finally, substantial results are obtained.

Keywords: angle, bottle, density, single variable, Bernoulli equation.

1. Introduction

1.1. Problem statement

Liquid is one of the three major material forms. It does not have a definite shape, and it is often affected by the container. However, its volume is constant under the environment of constant pressure and temperature. In a transparent water bottle, the speed at which the liquid is poured may be affected by many factors of the object, the most obvious being the angle of the bottle or the viscosity of the liquid.

The relationship between fluid viscosity and density is important in this experiment. In particular, when the phase difference between the force applied on the piezoelectric tuning fork and its vibration speed is $4\pi+$, the viscosity and density of the liquid have a linear relationship. This linear separation of viscosity and density is very important to solve the fluid's viscosity and density. In other words, there is a specific relationship between the viscosity and density of the liquid. Therefore, in this experiment, it is assumed that there is a certain relationship between the density of the liquid and the flow rate.
1.2. Literature review

Many scholars have already done extensive in-depth research on the issue of liquid flow rate in the bottle. Intuitively, bubbles will appear in the bottle when the bottle is poured. Hundreds of years ago, Leonardo Da Vinci made a famous discovery that bubbles rise sinusoidally. The kinetics of bubble formation at the bottle mouth depends on the thermophysical properties of the liquid, the geometry of the bottle, and its inclination. These parameters are inevitably intertwined, making bottle-emptying dynamics the next frontier research direction for bubble physicists. In the recent (AIP) journal Physics of Fluids, Lokesh Rohilla and Arup Kumar Das have analyzed the bubble dynamics in commercial beverage bottles through high-speed photography [1]. Their experiment showed that there is a critical angle of inclination. After that, even if the inclination angle of the bottle continues to increase, the emptying time of the bottle will not decrease. This phenomenon is caused by the saturation of the gap. At the mouth of the bottle, the air around the liquid occupies the space at an inclined angle.

The effect of liquid viscosity on the rise rate of a large gas bubble through the stagnant liquid in a tube is shown to be limited to that on the film flow past the bubble on the tube wall. The independent variables in the experiment are the different kinds of liquids, and the dependent variables are assumed to be the shape of bubbles. The experimental results show that a criterion of similarity between the shapes of bubbles in different liquids can be obtained experimentally, which enables the development of a correlation for the velocities of rise. There are also many studies on Taylor Bubbles, such as the shape of Taylor bubbles in Newtonian liquid in a stagnant state [2]. S. Nogueria and M. L. Riethmuller studied the flow in the wake and near-wake regions of individual Taylor bubbles rising through stagnant and co-current vertical columns of Newtonian liquids, employing simultaneously particle image velocimetry (PIV) and pulsed shadowgraphy techniques (PST) [3]. Experiments were made with water and aqueous glycerol solutions covering a wide range of viscosities [Math Processing Error], in an acrylic column of 32 mm (ID). The detailed study of the flow in the wake and near-wake regions contributes to a better understanding of the interaction and coalescence mechanisms between Taylor bubbles.

In order to reduce the gross errors caused by humans, some scholars have chosen to use 3D modeling to simulate the scene where the water in the bottle completely flows out [4]. A 3D Computational Fluid Dynamics (CFD) approach is used to investigate the effect of parameters, such as bottle geometry, surface tension, or inclination on bottle emptying processes [4]. The method can be used to assess changes in bottle emptying times induced by new bottle shapes, and it is shown to be much more accurate than the existing empirical models. This method is an effective solution to the difficulty of a poor experimental environment.

Regarding the data calculation and analysis, some scholars also pointed out that emptying the water in the bottle is a common problem. It is the most common two-phase flows. Samuel and his partners reported on a detailed experimental analysis of this flow configuration covering a wide range of neck-to-bottle diameter ratios, $d^*$, and initial filling ratios [5]. The joint use of a pressure sensor at the top of the bottle and a shadowgraph technique to track the evolution of the upper liquid surface allows the average gas volume fraction in the fluid column to be computed throughout the discharge. Variations of the bottle emptying time, gas content, and characteristics of the pressure signal as a function of $d^*$ are analysed, and scaling or evolution laws are derived [6].

The first problem in the area of cavitation and bubble dynamics was solved by Rayleigh (1917). The case in which the medium filling the cavity is essentially a permanent, no condensable gas is considered, taking into account small-amplitude oscillations, nonlinear oscillations, mass-diffusion effects, and acoustic cavitation and applications. The analysis of some features of the bubbles which are composed predominantly of vapor is also discussed, giving attention to the dynamics of a cavitation bubble, the growth of vapor bubbles, vapor-bubble collapse, and the behavior of vapor bubbles in oscillating pressure fields [7].

Bubble dynamics is a very complex field of science, and it shows that the geometry of container and liquid viscosity are also the parameters of liquid flow rate. However, this project is not that difficult. This research tries to assume this problem as easily as possible, and the process is organized and clear.
1.3. Experimental situation
When people are very thirsty, they always hope that the water in the kettle can be poured out quickly. And people always feel that as long as the bottle body is 90° perpendicular to the horizontal surface, the water can be poured out as quickly as possible. In this way, 90° is not a perfect angle. This research tries to find whether there is a direct relation between the angle of the bottle body and the flow-out rate. And for the density of the liquid, alcohol is selected, and its density is changed to do the experiment.

2. Experimental Methods

2.1. Experimental measurement
A single variable is controlled to ensure that only one variable is changed at a time while other variables in the experiment remain unchanged. This can ensure the absolute influence of a single variable on the experiment, which is more helpful to analyze the experimental results. The two important variables studied in this experiment are the liquid concentration and the other is the bottle angle.

Bottle emptying is a two-phase flow phenomenon. When the liquid and gas exchange positions, the pressure will change accordingly. The flow state is too unsteady. In other words, if the pressure error is not eliminated as much as possible, the experimental results may be very not ideal. Therefore, a method is put forward: drilling a hole in the bottom of the bottle, and then inserting a wooden stick with a diameter slightly smaller than the hole; fixing the stick, so that when pouring water, the inside and outside of the bottle are connected to reduce the pressure difference. And at the same time, as a pointer, this wooden stick can also solve the problem of squint error when comparing human eyes to a certain extent.

2.2. Theoretical analysis
According to the Bernoulli equation, it is assumed that the bottom velocity of water is approximately 0. When calculating the water velocity of the bottle mouth, it is assumed that the water velocity at the surface is zero. The point at the surface is “1”, and the point at the bottle mouth is “2”.

\[
P1 + \rho g h1 + \frac{1}{2} \rho V1^2 = P2 + \rho g h2 + \frac{1}{2} \rho V2^2
\]

\[
P1 = P2 = \text{atm}
\]

\[
V1 = 0
\]

Thus,

\[
\frac{1}{2} \cdot V2^2 = gh1 - gh2 = gh (h2=0)
\]

\[
V2 = \sqrt{2gh}
\]

3. Results and Discussion

3.1. Experimental results
The first part of the experiment was to measure the practical flow-out rate at 10 different degrees of inclination of the bottle body, and to find out the specific range of the angle of inclination at which the flow-out rate from the bottle is the fastest. The experimental measurements are repeated three times. Table 1 shows the mean and standard deviation (STD) flow out time at different inclination angles. ‘UN’ means ‘unable to pour out the water completely’. Similarly, each time of the height difference of the liquid level is measured to calculate the theoretical value of the liquid leaving speed.
Table 1. Time & height data

| Angle/degree | 1st/s | 2nd/s | 3rd/s | Average/s | STD  | H/cm |
|--------------|-------|-------|-------|-----------|------|------|
| 0 (UN)       | 15.99 | 14.94 | 15.24 | 15.39     | 4.40 |      |
| 10 (UN)      | 8.62  | 7.91  | 8.36  | 8.30      | 7.40 |      |
| 20 (UN)      | 6.84  | 6.51  | 6.15  | 6.50      | 9.20 |      |
| 30           | 5.37  | 4.95  | 5.19  | 5.17      | 0.2107 | 12.30 |
| 40           | 3.85  | 3.80  | 4.18  | 3.94      | 0.2065 | 15.50 |
| 50           | 4.12  | 4.35  | 4.55  | 4.34      | 0.2152 | 17.40 |
| 60           | 4.25  | 4.07  | 4.00  | 4.11      | 0.1290 | 18.20 |
| 70           | 4.03  | 3.90  | 3.85  | 3.93      | 0.0929 | 18.50 |
| 80           | 4.79  | 4.77  | 4.65  | 4.74      | 0.0757 | 18.90 |
| 90           | 4.92  | 4.85  | 4.70  | 4.82      | 0.1124 | 19.35 |

Notes:
1. (1) Means the height between the water surface and the bottle mouth.
2. (2) Is the fluid velocity when the water is pulling out of the bottle mouth.
3. (3) Practical discharge rate is calculated by dividing the measured time by the volume of 400ml at each degree.
4. (4) ‘UN’ means unable to complete pour out the water.
5. (5) ‘STD’ means standard deviation.

Figure 1 shows the corresponding relationship between the actual speed and angle in Table 1. And with adding both horizontal and vertical error bars, the fluctuation of the data is more straightforward. The magnitude of the vertical error bar depends on the standard deviation values. The setting value of the horizontal error bar is 2° due to the parallax error and the pressure error.

![Figure 1. Discharge time vs. angle.](image)

Original time collected from the flow-out time was shown as the vertical axis, while the angle position is represented as the horizontal axis.

And Table 2 contains the practical flow-out speed, which was the results of the volume of bottle divided by the area of bottle mouth multiple the flow-out time at different degrees. The practical
The discharge rate is calculated by dividing the measured time by the volume of 400ml at each degree. The equation is shown as follows.

$$v_2 = \frac{500\text{ml}}{S \cdot t}$$  \hspace{1cm} (2)

| Table 2. Practical outflow speed. |
|----------------------------------|
| Degree | $v_2 \text{ cm/s}^{-1}$ |
|--------|------------------------|
| 0(UN)  | 9.29                   |
| 10(UN) | 12.05                  |
| 20(UN) | 13.44                  |
| 30     | 15.53                  |
| 40     | 17.44                  |
| 50     | 18.48                  |
| 60     | 18.90                  |
| 70     | 19.05                  |
| 80     | 19.26                  |
| 90     | 19.48                  |

The theoretical outflow velocity at each angle was calculated/obtained by the Bernoulli equation, and the corresponding results are presented in Table 3.

| Table 3. Theoretical flow-out speed calculated from the Bernoulli equation. |
|-----------------|-----------------|
| Angle/degree    | Theoretical flow-out speed $v$/ m/s |
| 30              | 0.07623         |
| 40              | 0.08561         |
| 50              | 0.09071         |
| 60              | 0.09277         |
| 70              | 0.09351         |
| 80              | 0.09454         |
| 90              | 0.09562         |

Based on the first time of the experiment, it is found that the time for all the liquid to flow out was the shortest when the angle of the bottle body is close to 70°. In other words, the value of declination angle for the bottle with the maximum liquid flow rate is about 70°. The angle range continues to be narrowed to find the best outflow angle. In the second data collection, the time measurement at three angles of 45°, 65° and 75° is selected, and the results obtained are shown in Table 4. In order to reduce the error, the measurement of each angle must be performed three times, and finally, the average value of the three times values can be taken. Overall, it can be noted that the liquid flows most quickly when the angle of inclination of the bottle is between 65° and 70°.

| Table 4. Second time of data collection at narrowed range of degrees |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| Degree | 1st t/s | 2nd t/s | 3rd t/s | Average time/s |
|--------|---------|---------|---------|----------------|
| 45     | 3.85    | 4.17    | 3.95    | 3.99           |
| 65     | 3.57    | 3.70    | 3.87    | 3.71           |
| 75     | 3.75    | 3.95    | 4.00    | 3.90           |
The second part of the experiment was carried out to reveal whether the density of liquid will affect the flow-out rate. Specifically, alcohol is mixed with water to adjust the density. And Figure 2 shows the flow-out time about different concentrations of alcohol solutions at the same angle of 65°.

Table 5. Measurement of flow-out time at different density of alcohol solution

| Concentration of alcohol/% | Density/kg·m⁻³ | 1st t/s | 2nd t/s | 3rd t/s | Average t/s | Std |
|----------------------------|-----------------|-------|-------|-------|-------------|-----|
| 0%                         |                 | 2.77  | 2.88  | 2.81  | 2.82        | 6%  |
| 5%                         |                 | 2.89  | 2.71  | 2.93  | 2.84        | 12% |
| 15%                        |                 | 2.91  | 2.75  | 2.83  | 2.83        | 8%  |
| 25%                        |                 | 2.83  | 2.75  | 2.77  | 2.78        | 4%  |
| 35%                        |                 | 2.89  | 2.78  | 2.81  | 2.83        | 6%  |

Figure 2. Flow out rate change with the density of alcohol

3.2. Results from theoretical analysis

Based on the Bernoulli equation, h was measured at the beginning of the experiment. Since other factors are fixed numbers, the theoretical results were calculated and shown in Table 6.

Table 6. Comparison between practical and theoretical outflow speed

| angle/degree | Practical outflow speed v₂/m·s⁻¹ | Theoretical outflow speed v/ m·s⁻¹ | Difference between v₂ and v |
|--------------|----------------------------------|-----------------------------------|-----------------------------|
| 30           | 15.53                            | 7.623                             | 7.907                       |
| 40           | 17.44                            | 8.561                             | 8.879                       |
| 50           | 18.48                            | 9.071                             | 9.409                       |
| 60           | 18.90                            | 9.277                             | 9.623                       |
| 70           | 19.05                            | 9.351                             | 9.699                       |
| 80           | 19.26                            | 9.454                             | 9.806                       |
| 90           | 19.48                            | 9.562                             | 9.918                       |

According to the measured results, the rough range of the optimal angle is around 65 degree and 70 degree. At the same time, the density of alcohol solution was not showing any specific relation with flow-out rate. However, another possibility is that due to the accuracy of the measurement’s units were not high enough, the influence of the liquid concentration on the liquid outflow rate is not large enough,
so the existing results cannot accurately reflect the relationship between the alcohol concentration and the outflow rate.

3.3. Comparison and discussion
It is found that the distance between the theoretical value and the actual value is very large. The reason for the extremely big gap can be analyzed from the following aspects.

Firstly, one of the cores of this experiment was to use the measured height value combined with Bernoulli equation to derive the liquid out-flow velocity under different conditions. In theory, the application of the Bernoulli equation needs to meet the following conditions: (1) steady flow; (2) incompressible flow; (3) friction-free flow; (4) the Fluid flows should go along streamlines.

If the above conditions are not met, the accuracy of the calculation results will be affected. Therefore, it is better to use large-caliber and large-volume containers for testing. Thus, the bottle is a standard cylindrical shape, and the diameter of the bottle mouth is large to ensure smooth water flow. Even so, there will still be some inevitable errors.

The methods used to eliminate errors could affect results. Bottle emptying is a two-phase flow phenomenon. When the liquid and gas exchange positions, the pressure will change accordingly. In other words, if the pressure error is not eliminated as much as possible, the experimental results may not be very ideal. Thus, the following strategy is utilized: a hole is drilled in the bottom of the bottle, and then a wooden stick with a diameter slightly smaller than the hole is inserted, which could connect the inside and outside of the bottle to reduce the pressure difference when pouring out water. At the same time, as an indicator, this wooden stick can also solve the problem of squint error when comparing human eyes to a certain extent.

Secondly, regarding the influence of alcohol concentration on liquid flow rate (firstly because the time of each measurement was short), both the measurement error will have a certain impact on the experimental results. Furthermore, the factor of density may not be used as an absolute reference variable. For other types of liquids, if the viscosity of the liquid itself changes significantly with the increase in density, then the outflow rate of this liquid must be affected by its own density. In other words, the results of this experiment cannot decisively explain whether there is a correlation between the concentration of alcohol and the flow-out speed. It can only be tentatively determined on a macro level that the concentration of alcohol is not significant enough or even negligible for the fluid velocity.

4. Conclusion and Future Works

4.1. Conclusion
First, except for the Bernoulli equation, another important part was error analysis.

Second, some errors are inevitable. For example, it is not accurate enough to assume that the liquid flow rate at the bottom of the bottle is zero. It is just an approximation in order to calculate the speed of the bottle mouth more conveniently. Even though a stick is used as an indicator to reduce parallax error, there will definitely be residual errors that cannot be eliminated. On the one hand, part of the error is the lack of experimental equipment. On the other hand, the error was caused by the Bernoulli equation, as the application of the Bernoulli equation has some requirements. It is better to use steady and incompressible flow, friction-free flow, and fluid lows along streamlines.

4.2. Future works
Simulating gas-liquid flows involves a wide range of spatial and temporal scales. Multiple topological changes remain a major challenge nowadays, as the computational cost associated with direct numerical simulation still makes this approach unaffordable. A hydrodynamic model of using quartz tuning forks (QTFs) for density and viscosity sensing, by measuring the resonance frequency and quality factor, has been established based on the cantilever beam theory applied to the atomic force microscope (AFM) [8].

Accurate and robust modeling of compressible two-phase flow is crucial for many engineering applications, such as fuel injectors, nuclear reactors, rocket motors, as well as gas turbines, and heat
pumps. The involved two-phase flow maybe subcritical, transcritical, or supercritical depending on the pressure and temperature operating conditions. Some subsonic, sonic, or supersonic regions may appear due to shock and expansion waves [9].

That is to say, the primary problem to be solved in the future is to eliminate all the interference or to study the interference itself-bubble dynamics. A critical step to achieve these is simulation, which becomes an experiment in an ideal state.

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