We investigate the effects of inhomogeneous scalar field configurations on effective action at finite temperature. Recently, Walliser et al. have calculated the leading perturbative corrections to the wave function correction term $Z(\phi, T)$, for the standard model at finite temperature. Here we follow their results and apply them to calculate the effective energy of the sphaleron solution, including derivative correction term $Z(\phi, T)$. The dominant contribution to $Z(\phi, T)$ comes from vector boson loops. We also study the possible effects of corrected sphaleron energy on the baryon number violation rate in the standard model.

I. INTRODUCTION

Baryon number violation processes in the electroweak theory play an important role in the understanding of baryon asymmetry of the universe. It is known that anomaly induced baryon number violation rate at zero temperature is considerably suppressed. At high temperatures the rate formula is derived by Affleck-Arnold mechanism. At electroweak symmetry breaking temperatures sphaleron configuration separates different Chern-Simons vacua. Each time system makes a transition through sphaleron configuration, anomaly induced baryon number violation occurs. The sphaleron rate is given by,

$$\Delta \propto (\beta E_{sph})^8 \exp[-\beta E_{sph}]$$

Where $\beta = T^{-1}$ and $E_{sph}$ is the energy of the sphaleron solution.

We have reanalyzed this rate formula and found that for all values of Higgs boson mass sphaleron rate is unsuppressed and any net baryon excess generated prior to electroweak phase transition is washed out. This might indicate that present baryon asymmetry can not be explained in the framework of minimal standard model.

In this article we review derivative corrections to the effective action and apply it to calculate corrections to the sphaleron energy. We find that correction to sphaleron energy
is negative and sphaleron rate is strongly unsuppressed. This again indicates complete erasure of baryon asymmetry of the universe in the minimal standard model.

II. THE DERIVATIVE CORRECTION TERM

Effective action can be written as an expansion in terms of fields and its derivatives (momentum space expansion),

\[
\Gamma[\phi(x)] = \int d^4x \left[ \frac{1}{2} \{1 + Z(\phi)\} (\partial^\mu \phi)^\dagger(\partial_\mu \phi) - V(\phi) + \text{higher order derivative terms} \right]
\]

(2)

Here \( Z(\phi) \) is the derivative (wave function) correction term. We will confine to kinetic energy correction term and hence neglect higher order derivative terms. We point out that effective potential can be extracted out by expanding effective action in a constant background field \( \bar{\phi} \). In Euclidean space finite temperature effective action is written as,

\[
\Gamma[\phi, T] = \beta \int d^3x \left[ \frac{1}{2} \{(1 + Z(\phi, T)) (\partial^\mu \phi)^\dagger(\partial_\mu \phi) + V(\phi, T) \} \right]
\]

(3)

Where \( \beta = \frac{1}{T} \) and we have considered time independent fields, \( \phi(x) = \phi(\vec{x}) \). In the following we briefly describe effects of \( Z(\phi,T) \) term for a bubble nucleation theory [6], [7].

The free energy of the bubble solution is of the form,

\[
F[\phi, T] = \int d^3x \left[ \frac{1}{2} |\vec{\nabla} \phi|^2 + V(\phi, T) \right]
\]

(4)

The exact expression for the free energy of the bubble solution is given by \( F[\bar{\phi}(r), T] \), with \( \bar{\phi}(r), r=|\vec{x}| \) representing bubble solution. \( V(\phi,T) \) is temperature dependent effective potential,

\[
V(\phi, T) = \frac{1}{2} DT^2 \phi^2 - ET\phi^3 + \frac{\lambda \phi^4}{4}
\]

Where constants D and E depend only on the scalar self coupling constant \( \lambda \). \( V(\phi,T) \) is shown in Fig. 1. We note that \( V(\phi,T) \) describes first order phase transition with two degenerate minima at critical temperature \( T = T_c \), at \( T < T_c \) there is one local minimum at \( \phi = 0 \) and a global minimum at \( \phi = \phi_{\text{min}} \).

The effect of \( Z(\phi, T) \neq 0 \) term can be understood by considering a scalar field transformation in equation (3),

\[
\tilde{\phi}(r) = \int d\phi \sqrt{1 + Z(\phi, T)}
\]

Now for \( Z(\phi, T) > 0 \),

\[
\frac{\partial \tilde{\phi}(r)}{\partial \phi(r)} = \sqrt{1 + Z(\phi, T)} > 0
\]
This means that free energy $F(\tilde{\phi}, T)$ and $V(\tilde{\phi}, T)$ have the same form and $V(\tilde{\phi}, T)$ is locally rescaled. However the surface energy of the bubble changes significantly which in turn affects free energy, $F(\tilde{\phi}, T)$. A positive value of $Z(\phi, T)$ increases the free energy and negative value of $Z$ decreases the free energy of bubble solution.

Another important consequence of positive $Z$ is that it enhances the strength of phase transition. Since positive $Z$ increases surface energy, to counter effect this enhancement universe would have to supercool further to complete the phase transition. This in turn reduces the end temperature of phase transition $T_e$, thereby increasing the strength of phase transition, $\frac{\phi_{\text{min}}}{T_e}$.

**III. DERIVATIVE CORRECTIONS TO SPHALERON ENERGY**

Sphaleron solution [4] is described by an SU(2) invariant time independent Higgs field theory, without fermions. The energy of the sphaleron solution is given by,

$$E_{\text{sph}} = \int d^3x \left[ \frac{1}{4} W_{ij}^a W_{ij}^a + (D_i \phi)^\dagger(D_i \phi) + V(\phi) \right]$$

(5)
Where $W^a_{ij}$ is SU(2) gauge field tensor for gauge fields $W^a_i(\vec{x})$ and $\phi(\vec{x})$ is Higgs scalar field.

$$(D_i \phi) = \partial_i \phi - \frac{1}{2}ig\sigma^a W^a_i \phi$$

$$V(\phi, T = 0) = \lambda(\phi^\dagger \phi - \frac{1}{2}v^2)^2$$

$g$ is SU(2) gauge coupling constant. $\lambda$ and $v$ represents scalar self coupling constant and zero temperature vacuum expectation value respectively. We know that in the standard model, tree level Higgs boson mass is related to the scalar coupling constant by,

$$M_h^2 = 2\lambda v^2$$

With a derivative correction term we write the effective energy at finite temperatures as,

$$E_{sph} = \int d^3x \left[ \frac{1}{4} W^a_{ij} W^a_{ij} + \{1 + Z_{SM}(\phi, T)\}(D_i \phi)^\dagger (D_i \phi) + V(\phi, T) \right]$$

(6)

Where $Z_{SM}$ is the standard model contribution to wave function correction term, it can be written as,

$$Z_{SM} = Z_{scalar} + Z_{vector} + Z_{fermi}$$

Recently complete calculations for $Z_{SM}$ have been performed by Jungnickel et al. [7]. Their finding is that scalar boson and fermion contributions are considerably suppressed compare to vector boson contributions. At high temperatures, scalar and vector boson contributions at $T \neq 0$ dominate over their respective contributions at $T=0$. Hence we need only write,

$$Z_{T \neq 0}^{scalar}(\phi, T) = \frac{\lambda^2 T \phi^2}{16\pi} \left( \frac{3}{M_\phi^2} + \frac{9\pi_\phi}{2M_\phi^2} + \frac{1}{M_\chi^2} + \frac{3\pi_\chi}{2M_\chi^2} \right)$$

(7)

Where $M_\phi$ and $M_\chi$ are mass matrices for Higgs and Goldstone boson and $\pi_\phi$ and $\pi_\chi$ are corresponding self energy corrections.

$$Z_{T \neq 0}^{vector}(\phi, T) = -\frac{3g^2 T}{4\pi} \left( \frac{m_w^4}{32M_L^5} - \frac{5m_w^2}{96M_L^3} + \frac{5m_w^4}{16M_T^5} - \frac{41m_w^2}{48M_T^3} + \frac{1}{M_T} \right)$$

(8)

Where $m_w$ is zero temperature mass of gauge boson and $M_L$ and $M_T$ are transverse and longitudinal masses for gauge boson.

$$M_L^2 = m_w^2(\phi) + \pi_L(0)$$

$$M_T^2 = m_w^2(\phi) + \pi_T(0)$$

with, $m_w^2(\phi) = \frac{1}{2}g^2\phi^2$. $g$ is SU(2) gauge coupling constant and $\pi_L(0)$, $\pi_T(0)$ are self energy contributions to gauge boson masses.

The full fermionic contribution is given by,

$$Z_{fermi}(\phi, T) = \frac{f^2}{96\pi^2} \left( 7 + 12\gamma_E + 12 \ln \frac{m_{ren}}{\pi T} \right)$$

(9)
Table [1] Shows sphaleron energy corrections for different values of scalar coupling constant.

Where $f_t$ is top quark Yukawa coupling to Higgs boson and $m_{ren}$ is renormalized mass matrix for top quark. $\gamma_E \simeq 0.577$. For details we refer article by Jungnickel et al. [7].

We had already stressed the fact that vector boson contribution to $Z(\phi, T)$ dominates over scalar and fermion contributions hence we need only include vector boson corrections, that is,

$$Z_{SM} = Z_{\text{vector}}$$

**IV. RESULTS**

We have numerically evaluated the expression (6). Table [1] shows sphaleron energy for different values of scalar coupling constant. The correction is negative and for temperature range 80 to 120 GeV, the numerical value is very small. Energy correction is plotted with temperature for different values of $\frac{\lambda}{g^2}$ in Fig. 2. We mention that in the range we have considered for $\frac{\lambda}{g^2}$ (.001-1.0), the Higgs boson mass varies from 10 GeV to 225 GeV. For this Higgs mass the critical temperature for electroweak phase transition varies from 50 GeV to 270 GeV.

We find that numerical value of correction is very small, however the effect is to decrease the total energy of the sphaleron. We have calculated the sphaleron rate, equation (1), with corrected sphaleron energy and found that sphaleron rate is strongly unsuppressed.
Hence our results only support the current status of electroweak baryogenesis that is, present baryon asymmetry of the universe can not be explained in the framework of minimal standard model. We have considered one loop effective action including self energy corrections to calculate derivative correction term $Z(\phi, T)$, it remains to check the effect of two loop corrections on $Z(\phi, T)$ and hence on sphaleron energy.

Acknowledgments
This work partially supported by Department of Science and Technology, India.
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