JETS FROM TIME-DEPENDENT ACCRETION FLOWS ONTO A BLACK HOLE

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ABSTRACT

We investigate time-dependent inviscid hydrodynamical accretion flows onto a black hole using numerical simulations. We consider accretion that consists of hot tenuous gas with low specific angular momentum and cold dense gas with high specific angular momentum. The former accretes continuously and the latter highly intermittently as blobs. The high specific angular momentum gas blobs bounce at the centrifugal barrier and create shock waves. The low specific angular momentum gas is heated at the shock fronts and escapes along the rotation axis. The outgoing gas evolves into pressure-driven jets. Jet acceleration lasts until the shock waves fade out. The total amount of the mass ejection is about 1%–11% of the mass of the blobs. The jet mass increases when the gas blobs are more massive or have larger specific angular momentum. We get narrower, well-collimated jets when the hot continuous flow has a lower temperature. In the numerical simulations we used a finite difference code based on the total variation diminishing scheme. It is extended to include the blackbody radiation and to apply a multi-time-step scheme for time marching.

Subject headings: accretion, accretion disks — black hole physics — galaxies: jets — galaxies: nuclei — methods: numerical

1. INTRODUCTION

It has been widely believed that accreting black holes are the central engines of active galactic nuclei (AGNs). The accretion is thought to be the origin of the X-ray and γ-ray emission and jets emerging from AGNs. This has been investigated with various types of numerical simulations. Wilson (1972) initiated hydrodynamical simulation of gas accretion onto a black hole. His simulation was followed by Hawley, Smarr, & Wilson (1984a, 1984b) and Clarke, Karpik, & Henriksen (1985). Uchida & Shibata (1985) and Shibata & Uchida (1986) initiated magnetohydrodynamic simulations of accretion disks threaded by magnetic fields. In their simulations a part of the disk is accelerated by magnetocentrifugal force and evolves into conical bipolar jets. Egedal, Coroniti, & Katz (1988) investigated super-Eddington accretion flow taking account of radiative force. The radiation pressure accelerates jets in their simulation.

Although some of the previous simulations succeeded for jet formation, they have not fully taken account of the relation between the jets and time variability. Recent observations suggest a close link between the jets and time variability. Some AGNs associated with jets show intense flares of very short duration. The BL Lac objects Mrk 421 and Mrk 501 showed X-ray and γ-ray flares of only 30 minutes to several hours duration (Kerrick et al. 1995 and Macomb et al. 1995 for Mrk 421; Catanese et al. 1997 for Mrk 501). Since the γ-rays are nonthermal, the flares are likely to be generated by the shock waves. The short duration of flares indicates that the shock waves generating γ-rays are transient and formed near the black hole. If a transient shock wave forms near the black hole, it will affect the mass ejection, i.e., jets.

Since the X-rays and γ-rays are highly variable on the timescale of hours, the accretion is also expected to be highly variable. Although many simulations have been performed for time-dependent accretion onto a black hole, most of these take a steady state approach in which the variability of the flow is mild (Molteni, Lanzafame, & Chakrabarti 1994). In this paper we investigate highly variable accretion onto a black hole with numerical simulations.

Using numerical simulations, we study accretion of dense gas clouds onto a black hole. We assume that the dense gas clouds are formed by some unknown instability or by the disruption of a larger cloud. They will spiral during the accretion and will be stretched in the azimuthal direction because of differential rotation. The stretched gas cloud becomes a tightly wound spiral similar to a torus around the black hole. We approximate the stretched spiral cloud as a gas torus in our simulations, assuming the axial symmetry to simplify the models and to save computation time.

Our numerical simulation code has several advantages over those of the predecessors. First, we employ the total variation diminishing scheme (TVD; see, e.g., Hirsch 1988, 460) to compute a shock wave without numerical oscillations. We modified a typical TVD scheme, the Roe (1981) scheme, for a general equation of state. Second, our simulations cover a wide region, with r = 2r_s to 10^4r_s and 100 or more grid points in the r direction and 100 or more grid points in the θ direction, where r_s denotes the Schwarzschild radius of the black hole. The dynamic range is very large in the r direction, and the spatial resolution is very high (Δr/r ≤ 9.5 × 10^{-3} and Δθ ≤ 1.6 × 10^{-2}). To follow the time evolution accurately we employed the multigrid scheme of Chiang, van Leer, & Powell (1992), which achieves second-order accuracy and the conservation of mass, momentum, and energy on a nonuniform numerical grid. These improvements contribute much to the quality of our numerical simulations. As shown later, we find jets emanating along the rotation axis in our numerical simulations. The jet formation and the propagation are simulated without numerical oscillations and strong numerical dampings.

In § 2 we describe our models and the methods of computations. The results of numerical simulations are shown in § 3. We find jets accelerated by shock waves in most of our numerical simulations. We discuss the formation of jets in...
§ 4. Comparisons with preceding numerical simulations and observations are also given in § 4. A short summary is given in § 5. In the Appendix we present a Roe (1981) type scheme for a general equation of state.

2. MODELS

We investigate time-dependent accretion flows onto a black hole having the mass $M$. For simplicity, the gravity of the black hole is approximated by the pseudo-Newtonian potential (Paczynski & Wiita 1980)

$$\Phi(r) = -\frac{GM}{r - r_g},$$

where $G$ denotes the gravitational constant. The Schwarzschild radius, $r_g$, is defined as

$$r_g = \frac{2GM}{c^2},$$

where $c$ is the speed of light. This pseudo-Newtonian potential reproduces salient features of the Schwarzschild black hole. A particle having a small specific angular momentum enters into the black hole without centrifugal bounce both in the Schwarzschild metric and in the Newtonian dynamics with the pseudo-Newtonian potential. The critical specific angular momentum is the same, and $\ell_{cr} = 2cr_g$. With the pseudo-Newtonian potential the hydrodynamical equations are expressed as

$$\frac{\partial \rho}{\partial t} + \mathbf{V} \cdot (\rho \mathbf{v}) = 0,$$

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \rho (\mathbf{v} \cdot \nabla) \mathbf{v} + \nabla P = -\rho \nabla \Phi,$$

$$\frac{\partial P}{\partial t} + \mathbf{V} \cdot (P \mathbf{v}) = \rho \mathbf{v} \cdot \left( \rho \frac{\partial \mathbf{v}}{\partial t} + \frac{\partial \Phi}{\partial x} \mathbf{v} \right),$$

$$\frac{\partial T}{\partial t} + \mathbf{V} \cdot (T \mathbf{v}) = \frac{1}{\gamma - 1} \frac{\partial}{\partial x} \left( \frac{\partial P}{\partial x} + \Gamma \mathbf{v} \cdot \nabla \Phi \right),$$

$$\frac{\partial S}{\partial t} + \mathbf{V} \cdot (S \mathbf{v}) = \frac{1}{\gamma - 1} \frac{\partial}{\partial x} \left( \frac{\partial T}{\partial x} + \Gamma \mathbf{v} \cdot \nabla \Phi \right),$$

where $\rho$ is the density, $P$ is the pressure, $v$ is the velocity, $E$ is the specific internal energy, and $H$ is the specific enthalpy. For simplicity, in the above equations we did not take account of the radiative and the viscous processes. This simplification may be validated, since we restrict ourselves to the dynamical processes. We assume that the flow consists of the ideal gas and blackbody radiation. Accordingly, the pressure, specific internal energy, specific enthalpy, and specific entropy are expressed as a function of temperature, $T$, and the density,

$$P = \frac{\rho kT}{\mu m_H} + \frac{aT^4}{3},$$

$$E = \frac{1}{\gamma - 1} \frac{kT}{\mu m_H} + \frac{aT^4}{\rho},$$

$$H = \frac{\gamma}{\gamma - 1} \frac{kT}{\mu m_H} + \frac{4aT^4}{3\rho},$$

$$S = \frac{k}{\mu m_H} \left( \frac{1}{\gamma - 1} \log T - \log \rho \right) + \frac{4aT^3}{3Î}$$

respectively. Here $\mu$ is the mean molecular weight, $k$ is the Boltzmann constant, $m_H$ is the mass of the hydrogen atom, $a$ is the Stefan-Boltzmann constant, and $Î$ is the ratio of specific heats. In numerical simulations we set $M = 10^7 M_\odot$, $\mu = 0.6$, and $Î = 5/3$.

### Table 1

**Summary of Simulations**

| Model | Resolution* ($N_r \times N_\theta$) | $\ell_{torus1}$ (cr) | $M_{torus1}$ ($10^{37} g$) | $\ell_{torus2}$ (cr) | $M_{torus2}$ ($10^{37} g$) | Shape | $P/\rho^*$ | $M_{jet}^d$ ($10^{37} g$) | $\eta^*$ |
|-------|----------------------------------|-----------------------|---------------------|-----------------------|---------------------|--------|------------|---------------------|--------|
| A1... | 800 $\times$ 100                | 1.9                   | 60                  | ...                  | 60                  | Circ   | 0.001     | 0.0                  | 0.00   |
| A2... | 800 $\times$ 100                | 1.98                  | 60                  | ...                  | 60                  | Circ   | 0.001     | 0.0                  | 0.00   |
| A3... | 800 $\times$ 100                | 2.0                   | 60                  | ...                  | 60                  | Circ   | 0.001     | 1.9                  | 0.03   |
| A4... | 800 $\times$ 100                | 2.1                   | 60                  | ...                  | 60                  | Circ   | 0.001     | 2.1                  | 0.03   |
| A5... | 800 $\times$ 100                | 2.2                   | 60                  | ...                  | 60                  | Circ   | 0.001     | 3.0                  | 0.05   |
| A6... | 800 $\times$ 100                | 2.3                   | 60                  | ...                  | 60                  | Circ   | 0.001     | 1.4                  | 0.02   |
| B1... | 800 $\times$ 100                | 2.1                   | 30                  | ...                  | 60                  | Circ   | 0.001     | 0.2                  | 0.01   |
| B2... | 800 $\times$ 100                | 2.1                   | 120                 | ...                  | 60                  | Circ   | 8.5       | 0.07                |
| C1... | 800 $\times$ 100                | 2.1                   | 60                  | ...                  | 60                  | Circ   | 0.001     | 1.9                  | 0.03   |
| C2... | 800 $\times$ 100                | 2.1                   | 60                  | ...                  | 60                  | Circ   | 0.001     | 2.0                  | 0.03   |
| C3... | 800 $\times$ 100                | 2.1                   | 60                  | ...                  | 60                  | Circ   | 0.001     | 3.4                  | 0.04   |
| C4... | 800 $\times$ 100                | 2.1                   | 60                  | ...                  | 60                  | Circ   | 0.001     | 8.2                  | 0.07   |
| D1... | 800 $\times$ 100                | 2.1                   | 60                  | ...                  | 60                  | Rect1  | 0.001     | 13.2                 | 0.11   |
| D2... | 800 $\times$ 100                | 2.1                   | 60                  | ...                  | 60                  | Rect2  | 0.0005    | 9.6                  | 0.09   |
| E1... | 1600 $\times$ 200               | 2.1                   | 60                  | ...                  | 60                  | Circ   | 0.0005    | 2.8                  | 0.04   |
| E2... | 1600 $\times$ 200               | 2.1                   | 60                  | ...                  | 60                  | Circ   | 0.0005    | 9.8                  | 0.09   |
| F1*  | 800 $\times$ 100                | 2.1                   | 60                  | ...                  | 60                  | Circ   | 0.01      | ≥11.5                | 0.04   |
| F2*  | 800 $\times$ 100                | 2.1                   | 2.1                 | 2.1                  | 60                  | Circ   | 0.01      | ≥1.8                 | 0.01   |

* Number of the grid points in the $r$ and $\theta$ directions, respectively.

% Shape of cross section of a gas torus. The code “Circ” denotes that the shape is circular with a radius of 4$r_g$. The code “Rect1” denotes rectangular with a half thickness of $8r_g$ and a width of 6$r_g$. The code “Rect2” denotes rectangular with a half thickness of 12$r_g$ and a width of 6$r_g$.

$^b$ $P/\rho$ evaluated at the outer boundary.

$^c$ Maximum of the jet mass.

$^d$ Efficiency of the energy gain of the jets. See text for the definition.

$^e$ In model F1 gas tori of 6.0 $\times$ 10$^{15}$ g are sequentially submitted with an interval of 2.3 hr.

$^f$ In model F2 a gas having $\ell = 2.1r_g$ inflows continuously from the outer boundary.
We solved the hydrodynamical equations (3)–(5) numerically in the spherical polar coordinates, \((r, \theta, \phi)\). We assume that the flow is axisymmetric and symmetric with respect to the midplane. The computational domain covers from \(r = 2r_g\) to \(r = 104r_g\) and \(\theta = 0\) to \(\theta = \pi/2\). The inner boundary at \(r = 2r_g\) is set so that the gas flows with a supersonic radial velocity and, accordingly, no information can propagate into the computational domain from the boundary. The outer boundary at \(r = 104r_g\) is a fixed one.

The domain of computation is divided into eight concentric shells. In a given shell the grid spacings are uniform in the \(r\) and \(\theta\) directions. In a given shell the radial grid spacing, \(\Delta r\), is twice as large as in the inner adjacent shell. Accordingly, it is 128 times larger in the outermost shell than in the innermost shell. Each shell has 100 grid points in the \(r\) and \(\theta\) directions in all the models except E1 and E2. In models E1 and E2 the grid points are twice as large as the others. In all the shells the radial grid spacing, \(\Delta r\), is smaller than \(\Delta r/r \leq 9.5 \times 10^{-3}\).

We applied the multi-time-step scheme of Chiang et al. (1992) to integrate the hydrodynamical equations. In the multi-time-step method, the time step, \(\Delta t\), is set separately for each shell so that it is proportional to \(\Delta r\) in all the shells. Accordingly, the evolution in a given period is followed by smaller time steps in an outer shell. This method saves computation time and reduces numerical diffusion, especially in the outer shells. If we had used an equal \(\Delta t\) for time marching, the numerical diffusion would have smeared out the shock front in the outer region because of the low CFL number thereof.

To calculate the numerical flux we used the extended Roe (1981) type approximate Riemann solver for an arbitrary equation of state, while Chiang et al. (1992) used the original Roe (1981) Riemann solver. The details of our approximate Riemann solver are given in the Appendix.

Our numerical solutions are second-order accurate in time and space. The total mass, angular momentum, and total energy are conserved in the domain of computation. We confirmed that the specific entropy \(s\), specific angular momentum \(\ell\), and specific total energy \(E_{\text{tot}} = (H + v^2)/2 + \Phi\) of a gas element become stationary along the streamline with errors less than \(4 \times 10^{-11}\) erg cm\(^{-2}\) g\(^{-1}\) K\(^{-1}\), \(2 \times 10^{-4} c r_g\), and \(10^{-2} c^2\), respectively, when the accretion rate is kept constant for a long time. The error of the specific entropy corresponds to that of the temperature of 70 K, while typical temperature of the flow in our simulations is of the order of 10\(^5\) K. These numerical errors come from the finite differencing. The errors decrease in proportion to the square of the grid spacing, \((\Delta r)^2\).

We constructed 18 models having different initial conditions. In most models one or two dense cold gas tori are superimposed on a hot steady flow. The gas tori are in a pressure balance with the surrounding flow at the initial state. The hot steady flow is characterized by \(P/\rho\) evaluated at the outer boundary. The models are summarized in Table 1.

3. RESULTS

3.1. Typical Example

In this subsection we concentrate on model A4, a typical example of our models. In model A4, one dense gas torus accretes toward the black hole, and bipolar jets emanate along the rotation axis with a semirelativistic velocity.

Figure 1 uses cross sections on the meridional plane to show the evolution of model A4. They are denoted in the cylindrical polar coordinates, \((\sigma, z)\). In Figure 1a, gray denotes the initial density distribution and the arrows denote velocity. The density is denoted also by contours, the levels of which are spaced at an interval of \(\Delta (\log_{10} \rho) = 0.5\). The hot gas accretes from the outer boundary at a constant accretion rate, \(0.11 M_\odot\) yr\(^{-1}\). On the outer boundary, the density and the pressure decrease exponentially with increasing \(\theta\),

\[
\rho(r_{\text{out}}, \theta) = \rho(r_{\text{out}}, 0) \exp (-3\theta),
\]

where \(r_{\text{out}}\) is the radius of the outer boundary. The \(r\) and \(\theta\) components of the velocity are set to be \((v_r, v_\theta) = (-2 \times 10^{-2} c, 0)\) on the boundary. The gas inflowing from the outer boundary has specific angular momentum \(\ell = 1.0 c r_g\) in the region of \(75^\circ \leq \theta \leq 90^\circ\) and \(\ell = 0\) in the region of \(0^\circ \leq \theta < 75^\circ\). The ratio of the pressure to the density is \(10^{-3} c^2\) on the boundary.

At the initial stage \((t = 0 \text{ hr})\), one dense gas torus is superimposed, with its center at \(r = 20 r_g\), on the midplane in a steady ambient gas flow. The gas torus has the mass of \(6.0 \times 10^{18}\) g. The density is \(3.5 \times 10^{-13}\) g cm\(^{-3}\) at the center of the torus. The torus is 3.9 times denser than the surrounding ambient gas. The specific angular momentum of the gas in the torus is high, \(\ell = 2.1 c r_g\), while for the ambient gas it is low, \(\ell \leq 1 c r_g\). Both the torus and the ambient gas have negative specific total energy, \(E_{\text{tot}} = -0.002 c^2\).

At \(t = 1.9 \text{ hr}\), the gas torus infalls and is stretched in the \(r\) direction (Fig. 1b). The gas torus extends from \(r = 2.5 r_g\) to \(r = 8.5 r_g\). The gas torus is compressed in the \(z\) direction. The centrifugal force is not effective at this stage. The infall of the gas torus decelerates owing to the centrifugal barrier. At \(t = 2.0 \text{ hr}\), part of it moves outward. This is the collision and the bounce of the torus with the centrifugal barrier (i.e., the funnel wall of Molteni et al. 1994). For the gas with \(\ell = 2.1 c r_g\), the barrier is located at \(r = 2.8 r_g\) on the midplane. At \(t = 2.5 \text{ hr}\), the gas torus splits in two vertically and the ambient gas inflows between the parts. A shock wave forms at the outer edge of the torus. At the shock front, the gas heats up from \(1.0 \times 10^5\) to \(3.0 \times 10^5\) K. The specific total energy becomes positive, \(E_{\text{tot}} = 0.08c^2\). The shock wave is an oblique one, and the postshocked gas flows along the shock front toward the \(z\)-axis.

The shock front is an arc in the \((\sigma, z)\)-plane. The arc grows in the curvature radius and angular extension. At \(t = 3.1 \text{ hr}\), the arc reaches the \(z\)-axis, and the shock waves cross themselves at \((\sigma, z) = (0, 5 r_g)\) (Fig. 1c). At the crossing point, the postshocked gas concentrates and has high pressure. The high pressure pushes gas to outflow along the \(z\)-axis (Fig. 1d). The outflowing gas evolves into bipolar jets. The postshocked gas has a high specific total energy, \(E_{\text{tot}} = 0.05 c^2\) at \((\sigma, z, t) = (0, 10 r_g, 4.2 \text{ hr})\) (Fig. 1b). We define jets as gas components with positive radial velocity and specific total energy.

The gas outflowing as jets has low specific angular momentum (Fig. 2a) and hence was originally of low density. The gas having high specific angular momentum of \(\ell > 2 c r_g\) does not flow near the \(z\)-axis owing to the centrifugal barrier. The gas torus works as a shield to prevent low specific angular momentum gas from accreting onto the black hole. A shock wave forms by the collision of the low specific angular momentum gas with the torus. The post-
shocked gas bypasses the gas torus. Most of the postshocked gas rushes onto the black hole without being sustained by the centrifugal force. A small fraction of the postshocked gas is accelerated again near the axis and evolves into bipolar jets.

At $t = 9.8$ and 18.5 hr the heads of the jets reach $z = 48r_g$ (Fig. 1e) and $z = 80r_g$ (Fig. 1f), respectively. The jets are collimated more as they propagate. At $t = 9.8$ hr, the gas velocity at the head of the jets is $v_r = 0.2c$. The gas velocity is almost radial ($|v_r| \gg |v_r|$), and the sound speed is 3 times smaller than $v_r$. The sound speed decreases owing to the expansion. The collimation will continue, although we stopped this simulation at $t = 18.5$ hr.

Figure 3 shows the structures on the $z$-axis at $t = 9.8$ hr. The radial velocity has two peaks, one at $z = 20r_g$ and one at $z = 48r_g$. The former peak is the shock front propagating outward and the latter is that of the rarefaction wave. The footpoint of the jets ($v_r = 0$) moves gradually outward from...
z = 7r_g (at t = 4.8 hr) to z = 18r_g (at t = 9.8 hr). The evolution of the jets’ mass is shown in Figure 4. The jet mass increases at a constant rate, 1.8 × 10^{23} \text{g s}^{-1}. The constant ejection of the jets starts at t = 3.5 hr and lasts 3.2 hr. The mass of the jets decreases gradually in the period of t ≥ 6.7 hr. The jet energy, which is defined as the positive energy \(E_{\text{tot}}(\text{at } t = 4.8 \text{ hr})\) to \(E_{\text{tot}}(\text{at } t = 9.8 \text{ hr})\), evolves almost proportionally to the jet mass throughout the simulation.

3.2. Effects of the Specific Angular Momentum of the Torus

We constructed models A1, A2, A3, A5, and A6 to study the effects of the specific angular momentum of the torus. The specific angular momentum is \(\ell = 1.9, 1.98, 2.0, 2.2,\) and \(2.3r_g\) in models A1, A2, A3, A5, and A6, respectively, while it is \(\ell = 2.1r_g\) in model A4. The other model parameters are the same as those in model A4.

These models can be classified on the basis of whether \(\ell\) is lower or higher than the critical parameter, \(\ell_{\text{mb}}\). Here \(\ell_{\text{mb}}\) is the specific angular momentum of the marginally bound stable orbit (Abramowicz, Jaroszynski, & Sikora 1978) and is equal to \(2c_r\) in our model.

When the specific angular momentum of the torus is low (models A1 and A2), the gas torus accretes onto the black hole without experiencing the centrifugal barrier. It stretches in the \(\sigma\) direction and is compressed in the \(z\) direction during the infall. At \(t = 0.93\) hr, the torus extends from \(\sigma = 6r_g\) to \(\sigma = 17r_g\) and is compressed within \(z < 2.5r_g\). At \(t = 1.4\) hr, the inner edge of the torus reaches the inner boundary, \(r = 2r_g\). The radial velocity of the torus is \(0.5c\) in model A1 and \(0.6c\) in model A2 at the inner boundary \((\sigma, z) = (2r_g, 0)\). The velocities are slower than the radial velocity of the ambient gas, \(0.85c\), but are still highly super-
sonic. In model A1, the gas torus has accreted onto the black hole by \( t = 2.3 \) hr, and in model A2, by \( t = 2.4 \) hr. The flow becomes steady thereafter. Shock waves and jets are not formed in models A1 and A2.

When the specific angular momentum of the torus is high (models A3, A5, and A6), shock waves and jets are formed. At \( t = 1.9 \) hr the torus extends from \( \varpi = 2.5r_g \) to \( \varpi = 8.5r_g \) and is compressed within \( |z| < 2.5r_g \) in all the models. A part of the torus collides with a centrifugal barrier and moves outward at \( v_r = 2.05, 2.00, 2.00, \) and \( 1.95 \) hr in models A3, A4, A5, and A6, respectively. At the outer edge of the torus, an arc shock wave forms. The arc reaches the \( z \)-axis and the shock waves cross themselves at \( (m, z) = (0, 5r_g) \) in all these models, at \( t = 3.15, 3.1, 3.1, \) and \( 3.05 \) hr in models A3, A4, A5, and A6, respectively. At the crossing point the postshocked gas has high pressure and the pressure-driven jets emanate. At \( t = 7.6 \) hr, the head of the jet reaches \( z = 32r_g, 34r_g, 38r_g, \) and \( 29r_g \) in models A3, A4, A5, and A6, respectively. At the same time, the gas velocities of the jet are \( v_r = 0.20c, 0.25c, 0.28c, \) and \( 0.20c \) in models A3, A4, A5, and A6, respectively.

Figure 4 shows the evolution of the jet mass for models A3, A4, A5, and A6. The jet ejection starts at the same time of \( t = 3.5 \) hr in all the models. In model A5 the mass ejection rate is \( 3.0 \times 10^{23} \) g s\(^{-1}\), 1.3 times larger than that in models A3, A4, and A6. The increase of the jet mass lasts for \( 2.5 \) hr in model A3 and for about \( 3.5 \) hr in models A4, A5, and A6. Accordingly, the resultant jet mass is largest in model A5. The jet mass gradually decreases when \( t > 6 \) hr in model A3 and when \( t > 7 \) hr in models A4, A5, and A6.

### 3.3. Effects of the Torus Mass on Jets

We constructed models B1 and B2 to study the effects of the torus mass on the jets. These models are the same as model A4 except for the density in the tori. The torus mass is \( 3.0 \times 10^{28} \) g in model B1 and \( 1.2 \times 10^{28} \) g in model B2, while it is \( 6.0 \times 10^{28} \) g in model A4.

Early stages of models B1 and B2 are similar to those of model A4. At \( t = 2.0 \) hr the gas torus collides with the centrifugal barrier and a part of the gas torus moves outward in all the models. A shock wave forming at the outer edge of the torus has a larger phase velocity in model B2 and a smaller one in model B1 than in model A4. At \( t = 3.5 \) hr the shock waves created at the outer edge of the torus reach \( \varpi = 8.5r_g, 11.5r_g, \) and \( 12.5r_g \) in models B1, A4, and B2, respectively. The shock is stronger in model B2 and weaker in model B1 than in model A4. The maximum value of the specific total energy in the postshocked gas is \( 0.02c^2, 0.04c^2, \) and \( 0.1c^2 \) in models B1, A4, and B2, respectively. The shock waves cross on the \( z \)-axis and jets form at \( t = 3.5 \) hr in model B1 and at \( t = 3.1 \) hr in model B2. The shock waves cross at \( (m, z) = (0, 5r_g) \) in all the models.

In model B1 the outflowing gas is decelerated by colliding with the infalling hot gas. At \( t = 6.3 \) hr, the head of the outflowing gas reaches \( z = 10r_g \). Thereafter, a part of the gas becomes inflow toward the black hole. At \( t = 6.7 \) hr, the radial velocity is negative \( (v_r < 0) \) everywhere in the computational domain. By \( t = 7.7 \) hr most of the hot gas components that were the outflow have accreted onto the black hole.

In model B2 the bipolar outflows evolve into jets. At \( t = 6.9 \) hr the jets reach \( 50r_g \). The gas velocity is \( v_r = 0.37c \), larger than that in model A4. The radial velocity of the jets has two peaks, as in model A4.

Figure 5a shows the evolution of the mass of the jets in models B1, A4, and B2. When the torus mass is larger, the mass of the jet increases at a higher rate and the jet ejection lasts longer. The rate of the mass increase is proportional to the torus mass, as shown in Figure 5b. In model B1 the jet mass is very small and the jets have a very short lifetime. There should be a critical mass of \( \sim 10^{28} \) g for the torus to induce jets through the shock wave formation.

The jet ejection lasts for \( 4 \) hr and \( 6.5 \) hr in models A4 and B2, respectively. Thereafter, the jet mass decreases owing to the collision with the ambient inflow. At the final stage of the simulation in model B2, \( t > 12 \) hr, the jets begin to grow in mass again. The increase of the mass is due to the snowplow of the ambient gas by strong jets. The gas torus stays around \( (m, z) = (5r_g, 0) \) at the final stages of the simulation.

### 3.4. Accretion of Two Tori

We constructed models C1, C2, C3, and C4, in which two dense gas tori are imposed at the initial stage. The inner gas torus is at \( (m, z) = (20r_g, 0) \) and the outer torus is at \( (m, z) = (30r_g, 0) \). Each gas torus has a mass of \( 6 \times 10^{28} \) g. The inner gas torus has a specific angular momentum of \( 2.1c r_g \) in all the models. The outer torus has specific angular momenta of \( 1.0, 1.7, 1.9, \) and \( 2.1c r_g \) in models C1, C2, C3, and C4,
respectively. The other model parameters are the same as those in model A4.

Figure 6 shows the evolution of model C4. All the models resemble each other by $t = 3.8$ hr, i.e., when the inner torus has collided with the outer one at $(\sigma, z) = (10r_g, 0)$. At that time the head of the jet reaches $z = 11r_g$. Model C2 is very similar to model C1 even at later stages, up to the end of the simulations. The jet mass is only 5% different in these models. The velocity distribution of the jets is almost the same in these models.

The collision of the outer torus with the inner one strengthens the shock wave appreciably in models C3 and C4. The shock wave forming at the outer edge of the torus has a larger phase velocity in models C3 and C4 than in models C1 and C2. At $t = 4$ hr, the outer edge of the shock reaches $r = 16r_g$ and $21r_g$. The specific total energy of the postshocked gas is larger ($0.02c^2$ in model C3 and $0.05c^2$ in model C4) than in models C1 and C2. After the collision with the outer torus, the first torus splits into two in the vertical direction. Each of the two leaves the midplane moving outward in the radial direction.

The outer torus forms another shock wave after its collision with the centrifugal barrier in models C3 and C4. This shock wave crosses at $5r_g$ on the $z$-axis and creates second jets at $(\sigma, z, t) = (0, 8r_g, 6.4)$ hr. At $t = 7.5$ hr, the radial velocity of the jet has four peaks, at $9r_g, 16r_g, 27r_g, 42r_g$ in model C3 and at $10r_g, 17r_g, 27r_g, 42r_g$ in model C4. The outer two peaks are the shock and rarefaction waves due to the inner torus, while the inner two peaks are those due to the outer torus. The second jets have a higher outgoing velocity and overtake the first jets. The second jet in model C4 is stronger than that in model C3. The gas velocity of the second jet is $0.20c$ in model C3 and $0.35c$ in model C4. When the outer torus has higher specific angular momentum, $\ell \geq \ell_{mb}$, it strengthens the jets by the formation of the second shock wave.

Figure 7 shows the mass of the jets in models C1, C2, C3, and C4 as a function of $t$. By $t = 6.4$ hr the mass of the jets increases at a constant rate in all the models. In model C4 the rate increases by a factor of 2 in the period of 6.4 hr. This enhancement is ascribed to the infall of the outer torus of high specific angular momentum. In model C3 the rate increases slightly in the period of $6.4 \text{ hr} < t < 7.8$ hr. In models C1 and C2, the evolution of the jet mass is almost the same and is also similar to that in model A4. This implies that a gas blob with low specific angular momentum, $\ell < \ell_{mb}$, does not increase the jet mass.

The shock waves that have propagated outward begin to propagate inward around $t \approx 6$ hr in models C1 and C2, $t \approx 8$ hr in model C3, and $t \approx 11$ hr in model C4. Thereafter the jet mass remains constant. The slight decrease of the jet mass is due to the deceleration by the interaction with the ambient inflow. As in model B2, another increase of the mass in model C4 in the period of $t > 14$ hr is due to snowplow of the ambient gas by the strong jets.

3.5. Effects of the Shape of the Tori

We constructed models D1 and D2 to study the effects of the shape of the gas tori. Models D1 and D2 are the same as model C4 except for the shape of the gas torus at $t = 0$ hr. In model C4 each gas torus has a circular cross section with a radius of $4r_g$ in the $(\sigma, z)$-plane. In models D1 and D2 each gas torus has a rectangular cross section with a width of $6r_g$. Each torus has a half thickness of $8r_g$ and $12r_g$ in the $z$ direction in models D1 and D2, respectively. The mass of the each torus is the same, as are the pressures in the torus in models C4, D1, and D2. The torus density is lowest in model D2 and lower in model D1 than in model C4.

While the gas tori move inward, they are stretched in the $\sigma$ direction and compressed a little in the $z$ direction in models D1 and D2, as well as in model C4. The inner gas torus collides with the centrifugal barrier at $t = 2.0$ hr and returns outward. A shock wave forms at the outer edge of the inner torus, and the shock front has a larger vertical extent in models D1 and D2 than in model C4.

The inner torus collides with the outer torus at $t = 2.5$ hr in models D1 and D2. The collision has the largest cross section in model D2 and a larger one in model D1 than in model C4. The inner torus is decelerated more by the collision in models D1 and D2 than in model C4. Also the shock front expands relatively slowly in models D1 and D2. In models D1 and D2 the shock waves have a larger extent in the $z$ direction than in model C4. This is because the gas tori have a larger height in models D1 and D2 than in model C4. Jets start to emerge in models D1 and D2 at $t = 4.5$ hr, i.e., 1.0 hour later than in model C4. Figure 8
shows the evolution of the jet mass for models D1, D2, and C4. At the early stages of the jet ejection, $3 \text{ hr} < t < 10 \text{ hr}$, the mass ejection rates are larger in models D1 and D2 by a factor of 1.5 than that in model C4. In all the models the mass ejection rate increases slightly at $t = 7 \text{ hr}$, when the second shock wave formed by the initially outer gas torus reaches the $z$-axis. The mass of the jets reaches a peak at $t = 10 \text{ hr}$ in models D1 and D2.

After the collision at $t = 2.5 \text{ hr}$, the initially inner torus goes outward and the initially outer torus goes inward. The initially outer torus splits in two in the vertical direction and merges again after the collision. It collides with the centrifugal barrier and bounces at $t = 4.3 \text{ hr}$. After the bounce, the initially outer torus fragments, and a small fraction of it departs from the midplane. The initially inner torus turns to infall again at $t = 9 \text{ hr}$. The main bodies of the gas tori merge and form a vibrating compound torus.

Third jets emerge at $t = 12 \text{ hr}$ in model D1 and at $t = 12.5 \text{ hr}$ in model D2. The third jets are initiated by the shock waves formed by the vibration of the compound torus. The shock waves cross at $(r, z, t) = (0, 8r_s, 12 \text{ hr})$ in model D1 and at $(0, 8r_s, 12.5 \text{ hr})$ in model D2. The third jets
increase the mass of the jets again. The head of the jet reaches $88r_g$ in model D1 and $78r_g$ in model D2 at $t = 15$ hr. At the last stage of the calculation, $t = 17$ hr, the difference of the mass of the jets among three models is within a factor of 1.5. The larger mass ejection is due to the large shock wave.

### 3.6. Effects of the Sound Speed of the Ambient Flow

We constructed models E1 and E2 to study the effects of the sound speed of the ambient gas. Models E1 and E2 are the same as models C2 and C4, respectively, except for the temperature of the ambient gas. In models E1 and E2, $P/\rho = 5 \times 10^{-4}c^2$ on the boundary, while in models C2 and C4 it is $10^{-3}c^2$. In models E1 and E2 the flow is concentrated more near the midplane than in models C2 and C4. On the outer boundary the density and the pressure decrease exponentially with increasing $\theta$,

$$\rho(r_{out}, \theta) = \rho(r_{out}, 0) \exp(-4.2\theta),$$

in models E1 and E2. The accretion rate is fixed to be the same on the outer boundary in these four models. At $(\sigma, z) = (40r_g, 0)$ the pressure is 35% lower in models E1 and E2 than in models C2 and C4, while the density thereof is 40% higher in models E1 and E2 than in models C2 and C4.

Figure 9 shows the evolution of model E2. At $t = 2.0$ hr the inner gas torus collides with the centrifugal barrier and bounces in models E1 and E2. The total pressure, $P + \rho v^2$, of the ambient gas is 1.3 times stronger in models E1 and E2 than that in models C2 and C4 at $(\sigma, z) = (10r_g, 0)$. The high total pressure works against the torus outward motion and the shock wave propagation. The shock front reaches only $13r_g$ and $16r_g$ at its maximal expansion in models E1 and E2, respectively, while it reaches $15r_g$ and $21r_g$ in models C2 and C4. Jets start to emerge at $t = 3.0$ hr in these four models from the point $(\sigma, z) = (0, 5r_g)$ in which the shock waves cross. Jets have a higher gas velocity in models E1 and E2 than in models C2 and C4. At an early stage of $t = 4.5$ hr the outgoing gas velocity is as large as $v_r = 0.4c$ in model E1 and $0.5c$ in model E2. When the ambient gas has a lower sound speed, the jets have a larger velocity. At $t = 10.3$ hr the head of the jet reaches $100r_g$ in model E1 and $102r_g$ in model E2. Then the maximum velocity is $0.40c$ in model E1 and $0.45c$ in model E2 (Fig. 10). Jets are collimated more in models E1 and E2 than in models C2 and C4. The better collimation is mainly due to the larger Mach number of the jet. If the ambient gas were much colder, the Mach number of the resultant jet would be much larger and thus better collimated jets would be formed. It should be noted that a higher spatial resolution is required to simulate a colder accretion flow. The grid should be

![Fig. 7](image-url)  
**Fig. 7.**—Same as Fig. 4, but for two gas tori models C1, C2, C3, and C4. The evolution of models C1 and C2 is almost identical. The mass ejection is larger when the gas tori have larger specific angular momentum.

![Fig. 8](image-url)  
**Fig. 8.**—Same as Fig. 4, but for models C4, D1, D2, F1, and F2. In models C4, D1, and D2, only the shape of the initial gas tori is different. The difference of the jet mass among three models is within a factor of 1.5. The difference between models F1 and F2 is the variability of the flow. When the variability of the flow is milder, the mass ejection is smaller.
Fig. 9.—Same as Fig. 1, but for model E2. Panels a–f are arranged in the time sequence (a) $t = 0$ hr, (b) $t = 1.9$ hr, (c) $t = 3.1$ hr, (d) $t = 4.8$ hr, (e) $t = 7.3$ hr, and (f) $t = 10.3$ hr. Jets have a higher gas velocity and thus are collimated more in model E2 than in model C4.

3.7. Sequential Accretion of Many Tori

We constructed models F1 and F2 by changing the number of the gas tori. In model F1, gas tori are sequentially submitted from the outer boundary with an interval of 2.3 hr. Each gas torus has the mass of $6 \times 10^{28}$ g. The mass supply by the gas tori amounts to $0.11 M_\odot$ yr$^{-1}$ on average. In model F2, the gas having the specific angular momentum of $2.1 r_g$ flows continuously from the outer boundary. At the initial stage of model F2 the density, temperature, and radial velocity are the same as those of a steady flow except that the specific angular momentum is $2.1 r_g$ in the region of $r \geq 20 r_g$ and $75^\circ \leq \theta \leq 90^\circ$. In model F2 the gas with a low specific angular momentum acquires at the rate of $4.3 \times 10^{-2} M_\odot$ yr$^{-1}$ and that with a high one at the rate of $6.7 \times 10^{-2} M_\odot$ yr$^{-1}$.
In model F1 the jets are formed intermittently. At \( t = 3.5 \) hr the first gas torus reaches the centrifugal barrier and bounces. The gas torus forms shock waves by colliding with the ambient gas. The shock waves cross at \((\sigma, z, t) = (0, 6r_g, 5.6 \) hr), and the jets are formed. The second and the third gas tori trace the evolution of the first one. They are stretched in the \( \sigma \) direction during the infall and bounce at the centrifugal barrier. The shock waves formed by the second and the third gas tori cross at \( t = 8.8 \) hr and 11.8 hr, respectively, and form jets. At \( t = 11.8 \) hr, the three gas tori coalesce with each other and transform into two vertically separated dense tori. They reside in the region of \( 5r_g < \sigma < 15r_g \) and \( 2r_g < z < 15r_g \).

At \( t = 12 \) hr the fourth torus collides with the centrifugal barrier and bounces. The fourth torus not only forms shock waves at its head but also pushes the coalesced gas upward. The shock waves by the fourth gas torus do not cross on the \( z \)-axis. The fifth and the sixth gas tori collide with the barrier at \( t = 13.6 \) hr and 16.0 hr, respectively. They also push the coalesced gas upward. At \( t = 17 \) hr a part of the coalesced gas is expelled along the funnel wall. The shock waves formed by the fifth and the sixth gas tori cross on the \( z \)-axis at \( t = 14.8 \) hr and 17.7 hr, respectively and form the jets.

At \( t = 18 \) hr the head of the jet reaches \( z = 93r_g \). The gas velocity of the jet is \( 0.20c \). The expelled gas blob of large specific angular momentum spreads in the region of \( 12r_g < \sigma < 22r_g \) and \( 20r_g < z < 35r_g \). Its radial gas velocity is \( 0.15c \).

Figure 10 shows the mass of the jets as a function of \( t \) for model F1. The mass ejection rate is \( 5 \times 10^{23} \) g s\(^{-1}\) on average in the period of \( 7 \) hr < \( t \) < 18 hr. The evolution is similar in models F1 and C3 in the period of \( t < 11 \) hr. The mass ejection rate increases at \( t = 8.8 \) hr and 11.8 hr, when the second and the third gas tori form the jets to emanate. From \( t = 12 \) hr to 15 hr the ejection rate is smaller than...
before. In this period the arc shock waves do not cross on
the z-axis, and new components of jets are not formed. At
about $t = 15$ hr the ejection rate increases again. This
increase is in part due to the ejection of large specific
angular momentum gas.

In model F2 jets are formed but weak. At $t = 2.3$ hr the
head of the high specific angular momentum gas reaches
the centrifugal barrier and bounces. The bounce of the gas is
much weaker than that in model F1. The high specific
angular momentum gas piles up near the barrier and forms
a high-density fat torus. The inner edge of the torus is
bounded by the centrifugal barrier. The torus is in a quasi-
stationary equilibrium. The ambient gas collides with the
torus, and the arc shock waves are formed at the interface.

At $t = 3.8$ hr the torus occupies the region of $4r_g < \sigma <
8r_g$ and $|z| < 3.5r_g$. The density of the torus is $8 \times 10^{-13}$ g
cm$^{-3}$ and is denser by a factor of 9 than the ambient gas on
average. The specific total energy of the ambient gas
increases from $-0.002c^2$ to $0.005c^2$ by passing through the
shock front. The energy gain is smaller than that in model
F1. At $t = 6$ hr the torus occupies the larger region of $4r_g <
\sigma < 12r_g$ and $z < 6r_g$. The arc shock waves approach the
z-axis as the torus grows.

The arc shock waves cross at $(\sigma, z, t) = (0, 5r_g, 6$ hr), and
the jets are formed. At $t = 10.8$ hr the head of the jet reaches
$z = 22r_g$. The gas velocity of the jet is $0.08c$ and is slower
than that in model F1. The jets are not collimated; the
radial width is as large as 90% of the height. The specific
total energy in the jet is $0.01c^2$ and is smaller than that in
model F1.

The mass evolution of the jets is shown in Figure 8. The
mass ejection rate of the jets increases monotonically. It is
less than that in model F1.

4. DISCUSSION

In this section we describe the essence of our model,
discuss the acceleration mechanism in our model, compare
our model with other hydrodynamical models, and apply
our model to AGNs and Galactic black hole candidates.

4.1. Essence of Our Model

In § 3 we studied the formation of shock waves and jets in
highly variable flows with various numerical simulations.
Although each simulation has different features, the flow
pattern and its dynamics are qualitatively similar. Figure 12
illustrates the mechanism of jet formation schematically.
Each panel of Figure 12 is based on the density distribu-
tions in model A4 at a given epoch. Figure 12a shows the
initial stage of model A4. The gray denotes the density dis-
tribution. The dotted curve encircles high specific angular
momentum gas. The dashed curve denotes the centrifugal
barrier for the gas having $\ell = 2.1cr_g$ and $E_{\text{tot}} = 0$. Since the
location of the centrifugal barrier depends little on $E_{\text{tot}}$, the
dashed curve can be regarded as the centrifugal barrier for
the torus gas ($\ell = 2.1cr_g$). Figure 12b denotes the bounce of
the torus gas. The gray denotes the density distribution at
$t = 2.9$ hr. The straight and the turning arrows denote the
flow of low and high specific angular momentum gases,
respectively. The high specific angular momentum gas col-
lides with the centrifugal barrier (dashed curve) and bounces.
After the bounce it collides with the low specific angular
momentum gas, and a shock wave forms at the collision
interface. The shock front is denoted by the thick solid
curve in Figure 12b. Figure 12c shows the jet formation
near the axis. The gray denotes the density distribution at
$t = 6.0$ hr. The shock wave intersects itself on the axis and
the shock front (solid curves) has a complicated structure.
The radial flow converging to the axis turns out to be
outflow evolving to jets and inflow absorbed to the black
hole. The outflow forms another shock wave traveling in the
z direction.

It is essential for jet formation that the flow contains two
components, i.e., high and low specific angular momentum
gases. The jets are accelerated by shock waves, and the

![Figure 12](image-url)
shock waves are formed by the collision of the high and the low specific angular momentum gases. The high specific angular momentum gas ($\ell > 2c_r^g$) bounces at the centrifugal barrier. On the other hand, the low specific angular momentum gas ($\ell < 2c_r^g$) tends to accrete onto the black hole without centrifugal bounce. Eventually they collide with each other to form a shock wave. The collision causes the high specific angular momentum gas to lose its kinetic energy, while the low specific angular momentum gas receives it. This collision is the first step of jet formation in our model. The second step is the intersection of the shock wave on the axis. When the shock wave intersects itself, a part of the low specific angular momentum gas gains energy from the rest and evolves into jets. In the next subsection we evaluate the energy gain at each step.

4.2. Acceleration and Energy Gain

As shown in the previous section, high-energy gas is ejected as jets in our numerical simulations. The jets consist of gas that had low energy and was gravitationally bound at the initial state. The jets are accelerated hydrodynamically and gain energy from gas accreting onto the black hole. In this subsection we discuss how the jet gas gains energy. We also evaluate the efficiency of the energy gain.

As shown in Figure 12, the jet gas is accelerated twice at the encounters with the shock waves. The jet gas was originally the ambient gas flowing near the midplane. First the ambient gas passes through the shock wave when it collides with the gas torus bounced at the centrifugal barrier. Later it passes through the shock wave near the $z$-axis again. In model A4, the specific total energy increases from $E_{\text{tot}} = -0.002c^2$ to $E_{\text{tot}} = 0.03c^2$ at the first passage and to $0.06c^2$ at the second passage. The energy gain is due to the variability of the flow. The specific total enthalpy $(H + v^2/2)$ conserves in the shock rest frame. It increases in the frame in which the shock wave travels. The energy gain, $\Delta E$, is proportional to the shock propagation speed and to the velocity difference at the shock,

$$\Delta E = u \cdot (v_2 - v_1),$$

(12)

where $u$ denotes the phase velocity of the shock front, and the subscripts 1 and 2 denote the value in pre- and post-shocked flow. When the shock wave propagates to the upstream (downstream) side, the gas passing through the shock wave gains (loses) energy.

The energy gain is larger when the shock wave is stronger and moves faster. The shock strength and the propagation speed vary from model to model depending mainly on the $\ell$ of a gas torus. When $\ell < \ell_{mb} = 2c_r^g$, the gas torus is absorbed into the black hole without bounce and hence without forming a shock wave. When $\ell$ is only a little larger than $\ell_{mb}$, the gas torus bounces, but only weakly. Hence the shock wave is weak and moves slowly. When $\ell \approx 2.2c_r^g$, the bounce is strongest, and the resultant shock wave is strongest and moves fastest. When $\ell > 2.2c_r^g$, the bounce is weaker for a larger $\ell$. This dependence on $\ell$ can be understood easily if we approximate the gas torus as a test particle moving in the effective potential. When $\ell = 2.2c_r^g$, the effective potential has the deepest local minimum.

In the following we consider the efficiency of acceleration from the opposite side. When the jets are accelerated and gain energy, the shock-producing gas, i.e., the infalling gas torus, loses its energy. The energy gain and loss should balance. The energy loss has an upper bound given by the energy difference between the initial and final states. When the gas torus settles at the bottom of the effective potential, it liberates the maximum energy available. In model A4 the specific total energy of the gas torus is $-0.022c^2$ at the bottom of the effective potential in the final state. Hence, the gas torus can liberate at most $\Delta E = 0.02c^2$ per unit mass. This means that a gas torus of $6 \times 10^{28}$ g can liberate the total energy of $1.2 \times 10^{48}$ ergs. We measure the efficiency of jet formation, $\eta$, by the ratio of the total energy of the jets to the total available energy. The efficiency is 3% in model A4. The efficiency is listed for each model in Table 1.

4.3. Comparison with Other Hydrodynamical Models

In this subsection we compare our numerical simulations with others. A large number of numerical simulations of jets have been published thus far. Some invoke magnetic fields (e.g., Blandford & Payne 1982; Uchida & Shibata 1985) and others high luminosity as jet-driving sources (e.g., Eggum et al. 1988). Some study the formation and propagation of jets (e.g., Hawley et al. 1984a, 1984b; Clarke et al. 1985; Molteni et al. 1994; Yokosawa 1994), while others study only the propagation of jets (e.g., Tenorio-Tagle, Cantó, & Rozyczka 1988). In this subsection we restrict ourselves to hydrodynamical simulations in which jets emerge from accretion onto a black hole. Comparison between MHD models and ours will be discussed in the next subsection.

Hawley et al. (1984b) noticed outflow in their simulations of accretion onto a black hole. In contrast with our model, the outflow is hollow and along the outer edge of the thick accreting disk. The gas density is very low near the $z$-axis, since the accreting gas has large angular momentum, and, accordingly, the centrifugal force is very strong near the $z$-axis.

Clarke et al. (1985) studied more spherical accretion onto a black hole. They assumed constant density and angular velocity at the outer boundary set at a given radial distance from the center. Their simulations showed shock waves similar to those found in our models. (A similar shock wave is found by Wilson 1972, although he did not find outflows.) Also in their simulations the shock waves intersect themselves on the $z$-axis and form “density knots” propagating outward. The density knots are similar to the jets in our models, although they are too weak to evolve into jets. Since the outer boundary condition is fixed, the accretion approaches a steady state in which the central black hole absorbs almost all the accreting gas.

Molteni et al. (1994) studied accretion onto a black hole using smoothed particle hydrodynamics (SPH). Although their methods are different, their accretion flow is similar to that of Hawley et al. (1984b). The accreting gas has high angular momentum and the density is very low near the $z$-axis. Although the mass ejection rate is high in their model, it is mainly because the accreting gas has high energy and can be ejected without energy gain.

Yokosawa (1994) studied accretion taking account of viscosity. Although the gas is concentrated near the midplane in his initial model, the polar region is filled with low specific angular momentum gas at later stages. The viscosity transfers the angular momentum and produces low specific angular momentum gas. The low specific angular momentum gas is ejected as polar (filled) jets in Yokosawa (1994) also, although the shock waves and jet acceleration are not clearly seen in his simulations.
It has been confirmed from the above comparisons that (1) polar jets consist of low specific angular momentum gas and (2) time variability is essential to produce strong jets.

4.4. Comparison with MHD Models

Simulations of jets from time-dependent MHD accretion flow have been performed by many authors (e.g., Uchida & Shibata 1985; Shibata & Uchida 1986; Kudo & Shibata 1997; Koide, Shibata, & Kudoh 1998). Shibata & Uchida (1986) investigated accretion disks initially threaded by magnetic field lines and found magnetically driven bipolar outflows. The outflows transport angular momentum from the disk and thus form hollow jets, contrary to the filled jets of our model. In Shibata & Uchida (1986) the mass ejected as the jets is about 10% of the original disk mass, and the energy of the jets is about 10% of the potential energy released by the accretion disk. The mass ejection and the energy ejection rates of the MHD jets is about twice as efficient as those in our typical jets. Of our models, only D1 has ejection rates comparable to those of the MHD models.

In MHD models there are no strong shock waves during the jet acceleration processes, while in our model strong shock waves form and accelerate jets. This coexistence of shock waves and jets in our model gives a natural explanation of simultaneous X-ray and γ-ray flares and the following relativistic jet ejections, which are observed in active AGNs and X-ray binaries, as shown in the next subsection.

4.5. Application to AGNs and Galactic Black Hole Candidates

In this subsection we discuss the application of our model to AGNs and Galactic black hole candidates, X-ray binaries. In our model highly variable accretion produces transient shock waves, which accelerate jets. Since the shock waves are strong and semirelativistic, they will produce high-energy cosmic rays, including γ-rays. The shock-heated gas will emit thermal X-rays. The γ-rays and thermal X-rays will be observed as flares, since their production is temporal. Thus our model predicts simultaneous flares of γ- and X-rays and jet formation associated with the flares.

There are some observations indicating simultaneous γ- and X-ray flares. X-rays flared up in the BL Lac object Mrk 421 on 1994 May 16. Its doubling timescale was about 12 hr (Takahashi et al. 1996). The γ-ray flare occurred one day before the X-ray flare (Kerrick et al. 1995).

Positive correlation between the flux and the hardness of the X-ray is found for other BL Lac objects (Sembay et al. 1993). When the source is brighter, the spectrum is harder, i.e., the nonthermal component is more dominant. This correlation supports an idea that transient shock waves cause flares in the high-energy range.

Although association of jets with flares has not been observed for AGNs, it is observed for black hole candidates in our Galaxy. Hjellming & Rupen (1995) reported episodic ejection of relativistic jets by the X-ray transient GRO J1655−40. X-ray flares occurred in GRO J1655−40 in 1994 August. In VLA images the radio bright points seem to be ejected from the black hole candidate with a semi-relativistic speed. The epochs of the ejection coincide with the peaks of the radio emission, with probable delay of several days. This observation is consistent with our model. Similar events are observed for the Galactic superluminal source GRS 1915+105 (Foster et al. 1996).

Our time-dependent model can produce observed semi-relativistic jets. The velocity of jets observed in black hole candidates ranges from subrelativistic \( v_{\text{jet}} \sim 0.26c \) for the X-ray binaries SS 433 (Margon 1984) to ultrarelativistic for the BL Lac object Mrk 421 (Gaidos et al. 1996). In our model the velocity of the jets is typically \( 0.3c−0.5c \). It is larger when the ambient flow is colder. If we reduce the temperature of the ambient gas, we will obtain faster jets. It is not, however, likely that ultrarelativistic jets will be obtained by our mechanism. This is also true for other known mechanisms. Ultrarelativistic jets would require other, unknown acceleration mechanisms.

5. Conclusions

We investigated time-dependent accretion flows onto a massive black hole numerically. We used new techniques in our simulations, the multi− time− step scheme for time integration and the extended Roe (1981) type approximate Riemann solver for general equation of state. These techniques enabled us to perform a numerical simulation with high resolution and to follow long-term evolution. The high spatial resolution was essential to simulate a cold accretion flow.

We considered accretion that consists of hot ambient gas with low specific angular momentum and cold dense gas with high specific angular momentum. The ambient gas accretes continuously and the dense gas highly intermittently as blobs. Our main results are summarized as follows.

1. Bipolar jets emanate when infalling gas blobs have high specific angular momentum \( \ell \geq \ell_{\text{amb}} \). The blobs form shock waves by colliding with the ambient gas after the bounce at the centrifugal barrier. A part of the shock-heated gas outflows along the rotation axis and evolves into pressure-driven jets. The jet mass ejection is largest when the blobs with \( \ell = 2.2cr_{\text{infall}} \) infall.

2. When gas blobs have low specific angular momentum \( \ell < \ell_{\text{amb}} \), shock waves and jets are not formed. The blobs accrete smoothly onto a black hole without being affected by the centrifugal barrier.

3. Jets are more massive when the blobs are more massive. There is a critical mass of blobs, \( \sim 10^{28} \text{ g} \), to emanate jets when the average accretion rate is \( 0.11 M_{\odot} \text{ yr}^{-1} \).

4. The efficiency of the energy gain of the jets is 3%−11%. It is larger when the blobs have higher specific angular momentum or larger mass.

5. When the ambient gas is colder, the jets are better collimated.

Although we assumed strict axisymmetry in our models, we expect that the jet acceleration mechanism will work also in quasi-axisymmetric accretion. When the accretion is not strictly axisymmetric, the shock front also will not be strictly axisymmetric. The shock wave will converge, but with offset, and the jet will be slightly offset from the z-axis. Nonaxisymmetric accretion is a future problem for three-dimensional numerical simulations.

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APPENDIX

APPROXIMATE RIEMANN SOLVER FOR A GENERAL EQUATION OF STATE

Our scheme is essentially based on the multigrid scheme of Chiang et al. (1992), who solved the flow of an ideal gas in Cartesian coordinates. They used the approximate Riemann solver of Roe (1981) to evaluate the numerical flux. We extended the approximate Riemann solver of Roe (1981) to include blackbody radiation. Our Riemann solver can be applied to any equation of state whenever all the thermodynamical variables are expressed as a function of $\rho$ and $E$. In the following we describe the numerical flux we used.

We solved the hydrodynamical equations (3)-(5) in spherical polar coordinates assuming axisymmetry. The equations in the conservation form can be expressed as

$$\frac{\partial}{\partial t} (TU) + \frac{\partial}{\partial r} (TF) + \frac{1}{r} \frac{\partial}{\partial \theta} (TG) = S,$$

where

$$U = \left( \begin{array}{c} \rho \\ \rho v_r \\ \rho v_\theta \\ \rho v_{r\phi} \\ \rho E \end{array} \right), \quad F = \left( \begin{array}{c} \rho v_r \\ \rho v_r^2 + P \\ \rho v_r v_\theta \\ \rho v_r v_{r\phi} \\ \rho v_r H \end{array} \right), \quad G = \left( \begin{array}{c} \rho v_\theta \\ \rho v_\theta v_r \\ \rho v_\theta v_\phi \\ \rho v_\theta v_{r\phi} \\ \rho v_\theta H \end{array} \right),$$

$$T = \left( \begin{array}{cccc} r^2 \sin \theta & 0 & 0 & 0 \\ 0 & r^2 \sin \theta & 0 & 0 \\ 0 & 0 & r^3 \sin \theta & 0 \\ 0 & 0 & 0 & r^3 \sin^2 \theta \\ 0 & 0 & 0 & r^2 \sin \theta \end{array} \right),$$

$$S = \left( \begin{array}{c} 0 \\ r \sin \theta (\rho v_r^2 + \rho v_\theta^2 + 2P) + r^2 \sin \theta \rho g_r \\ -r^2 \cos \theta (\rho v_\theta^2 + P) \\ 0 \\ r^2 \sin \theta \rho v_r g_r \end{array} \right),$$

and

$$g_r = -\frac{GM}{(r - r_0)^2}.$$  \hspace{1cm} (A3)

The conservative variable vector, $U$, is evaluated on the grid points, $(r_i, \theta_j)$, where $i$ and $j$ denote the grid numbers in the $r$ and the $\theta$ directions, respectively.

The numerical flux between the grid points $(r_i, \theta_j)$ and $(r_{i+1}, \theta_j)$ is evaluated as

$$F_{i+1/2,j} = \frac{1}{2} \left( F_{i,j} + F_{i+1,j} - \sum_{k=1}^{s} \lambda_k \delta w_k R_k \right),$$

where

$$R_1 = \left( \begin{array}{c} \bar{v}_r \\ \bar{v}_\theta \\ \bar{v}_\phi \\ \bar{v}_r \bar{v}_\phi \\ (\bar{v}_r^2/2 + \bar{v}_\phi) \end{array} \right), \quad R_2 = \left( \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \right), \quad R_3 = \left( \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{array} \right),$$

$$\lambda_k = \frac{1}{2} \left( -\frac{\rho_2}{\rho_1} + \frac{\rho_1}{\rho_2} + 2 \frac{P_2}{P_1} \right) \left( \frac{v^2}{2} + \rho g_r \right),$$

$$\delta w_k = \frac{1}{\sqrt{\rho}} \left( \frac{v^2}{2} + \frac{P}{\rho_1} \right).$$
\[
\begin{align*}
R_4 &= \frac{\dot{\rho}}{2c_s} \begin{pmatrix} 1 & \bar{v}_r + \bar{c}_s \\ \bar{v}_\theta & \bar{v}_\phi \end{pmatrix}, \\
R_5 &= \frac{\dot{\rho}}{2c_s} \begin{pmatrix} 1 & \bar{v}_r - \bar{c}_s \\ \bar{v}_\theta & \bar{v}_\phi \end{pmatrix}, \\
\end{align*}
\]
(A5)

\[
\lambda_1 = \bar{v}_r, \quad \lambda_2 = \bar{v}_r, \quad \lambda_3 = \bar{v}_r, \quad \lambda_4 = \bar{v}_r + \bar{c}_s, \quad \lambda_5 = \bar{v}_r - \bar{c}_s,
\]
(A6)

\[
\delta w_1 = \rho_{i+1,j} - \rho_{i,j} - \frac{P_{i+1,j} - P_{i,j}}{\rho_{i,j}^2 c_s^2},
\]
(A7)

\[
\delta w_2 = v_{\phi,i+1,j} - v_{\phi,i,j},
\]
(A8)

\[
\delta w_3 = -v_{\theta,i+1,j} + v_{\theta,i,j},
\]
(A9)

\[
\delta w_4 = v_{r,i+1,j} - v_{r,i,j} + \frac{P_{i+1,j} - P_{i,j}}{\rho_{i,j} c_s},
\]
(A10)

\[
\delta w_5 = -v_{r,i+1,j} + v_{r,i,j} + \frac{P_{i+1,j} - P_{i,j}}{\rho_{i,j} c_s},
\]
(A11)

\[
\tilde{\rho} = \sqrt{\rho_{i,j} \rho_{i+1,j}},
\]
(A12)

\[
\bar{v}_r = \frac{\sqrt{\rho_{i,j} v_{r,i,j} + \sqrt{\rho_{i+1,j}} v_{r,i+1,j}}}{\sqrt{\rho_{i,j} + \sqrt{\rho_{i+1,j}}}},
\]
(A13)

\[
\bar{v}_\theta = \frac{\sqrt{\rho_{i,j} v_{\theta,i,j} + \sqrt{\rho_{i+1,j}} v_{\theta,i+1,j}}}{\sqrt{\rho_{i,j} + \sqrt{\rho_{i+1,j}}}},
\]
(A14)

\[
\bar{v}_\phi = \frac{\sqrt{\rho_{i,j} v_{\phi,i,j} + \sqrt{\rho_{i+1,j}} v_{\phi,i+1,j}}}{\sqrt{\rho_{i,j} + \sqrt{\rho_{i+1,j}}}},
\]
(A15)

\[
\bar{v}^2 = \bar{v}_r^2 + \bar{v}_\theta^2 + \bar{v}_\phi^2,
\]
(A16)

\[
\bar{H} = \frac{\sqrt{\rho_{i,j} H_{i,j} + \sqrt{\rho_{i+1,j}} H_{i+1,j}}}{\sqrt{\rho_{i,j} + \sqrt{\rho_{i+1,j}}}},
\]
(A17)

\[
\bar{c}_s^2 = (\gamma - 1)(\bar{H} - \bar{v}^2/2),
\]
(A18)

\[
\frac{1}{\gamma - 1} = \frac{\sqrt{\rho_{i,j} h_{i,j} + \sqrt{\rho_{i+1,j}} h_{i+1,j}}}{\sqrt{\rho_{i,j} (\partial P/\partial \rho)_{i,j} + \sqrt{\rho_{i+1,j} (\partial P/\partial \rho)_{i+1,j}}}},
\]
(A19)
and

\[ \varepsilon = \frac{(\rho_{i+1,j}E_{i+1,j} - \rho_{i,j}E_{i,j}) - (P_{i+1,j} - P_{i,j})/(\gamma - 1)}{(\rho_{i+1,j} - \rho_{i,j}) - (P_{i+1,j} - P_{i,j})/\bar{c}_s^2}. \]  

(A20)

This numerical flux is based on the flux vector splitting. The differences in \( U \) and \( F \) are expressed as the linear combination of the simple waves,

\[ U_{i+1,j} - U_{i,j} = \sum_{k=1}^{5} \delta w_k R_k \]  

(A21)

and

\[ F_{i+1,j} - F_{i,j} = \sum_{k=1}^{5} \lambda_k \delta w_k R_k. \]  

(A22)

This numerical flux reduces to that of Roe (1981) when \( \gamma \) is constant. The numerical flux in the \( \theta \) direction is evaluated in the same way.

Our scheme is similar to that of Glaister (1988). The main difference is the choice of sound speed. He uses the average pressure derivative to derive the sound speed, i.e.,

\[ \bar{c}_s^2 = \frac{\bar{p}_E}{\bar{\rho}^2} + \bar{p}_\rho, \]  

(A23)

where

\[ \bar{p}_E = \frac{(1/2)[P(\rho_{i+1,j}, E_{i+1,j}) + P(\rho_{i,j}, E_{i,j})] - (1/2)[P(\rho_{i+1,j}, E_{i+1,j}) + P(\rho_{i,j}, E_{i,j})]}{E_{i+1,j} - E_{i,j}}, \]  

(A24)

and

\[ \bar{p}_\rho = \frac{(1/2)[P(\rho_{i+1,j}, E_{i+1,j}) + P(\rho_{i,j}, E_{i,j})] - (1/2)[P(\rho_{i+1,j}, E_{i+1,j}) + P(\rho_{i,j}, E_{i,j})]}{\rho_{i+1,j} - \rho_{i,j}}. \]  

(A25)

while we use the average sound speed, \( \bar{c}_s \). Our scheme provides a better approximation to the sound speed, especially when the gas transits from one phase to another between the two adjacent points. Note that \( \bar{c}_s \) is real in our scheme as far as \( \partial P/\partial \rho \) is positive on the grid points. The sound speed derived from the average pressure derivative can be imaginary. Shimizu (1995) adopted our approximate Riemann solver in his numerical simulation of core collapse of supernovae and succeeded in capturing shock waves without numerical oscillations.

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