Quasiclassical Approach to Two-level Systems With Dissipation

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Abstract

The quantum dynamics of two-level systems under classical oscillator heat bath is mapped to the classical one of a charged particle under harmonic oscillator potential plus a magnetic field in a plane. The behavior of eigenstates and tunneling and localization are studied in detail. The broken symmetry condition and Langevin-like dissipative equation of motion are obtained. Some special dynamic features are considered.

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1 Introduction

Tunneling and localization in dissipative system have attracted much attention for their theoretical and experimental studies have much enriched our understanding on the real physical world [1-12]. For macroscopic and mesoscopic quantum tunneling phenomena two classes of processes are interested: the decay from metastable states and quantum coherence tunneling. The former had been extensively investigated by many authors [1, 6]. The studies indicate that the tunneling probability is significantly suppressed at strong dissipation if we require the dissipative Langevin equation

\[ M\ddot{q} + \eta \dot{q} + \frac{dV}{dq} = F_{\text{ext}} \]  

(1.1)

to be valid in semiclassical regime. The latter is a more difficult problem. A remarkable two-level model was given by Leggett et al. [1, 2, 3], and the possibility of localization in the presence of dissipation was intensively studied. However, it is difficult to give a intuitive and uniformed description to shed light the interaction between two-level system and environment. In this paper, we shall study the behavior of two-level model of Leggett et al. by reducing the quantum dynamics of the two-level system to the movement of a charged particle in a harmonic oscillator potential under a magnetic field in a plane through a mapping. The method is the generalization of that discussed in ref. [14]. In this approach the quasiclassical dissipative dynamics for two-level quantum evolution is mapped to a classical mechanical problem. Here “quasiclassical” means that we shall treat the environment oscillators classically and the two-level system interacted with the environment oscillators by quantum mechanics.

Our main conclusion is the following:

(1) When the acceleration of environment oscillators is small, the total energy values(system plus environment) \( E_c < -\hbar \omega_0 \) as increasing of the interaction between
the system and environment. The states are localized in the left or right of a symmetric double-well.

(2) The solution is stable.

(3) The dynamic equation for $S(t) = \langle \sigma_3 \rangle$ is obtained that explores the main physical picture of the system interacted with environment.

## 2 Ground State

The Hamiltonian for spin-boson is given by

$$\hat{H} = -\frac{1}{2} \hbar \Delta \sigma_1 + \frac{1}{2} \sum_{j=1}^{N} \left( \frac{p_j^2}{m_j} + m_j \omega_j^2 x_j^2 \right) + \frac{q_0}{2} \sigma_3 \sum_{j=1}^{N} c_j x_j,$$

where $\Delta = 2 \omega_0$ is the energy splitting between the two levels, $\sigma_1, \sigma_3$ are Pauli matrices and $x_i, p_i$ are the coordinate and momentum of $i$-th oscillators, respectively. Eq (2.1) can be recast to the form

$$\hat{H} = -\frac{1}{2} \hbar \Delta \sigma_1 + \frac{q_0}{2} \sigma_3 \sum_{j=1}^{N} c_j \left( \frac{\hbar}{2m \omega_j} \right)^{1/2} (a_j + a_j^+) + H_R,$$

where $H_R = \sum_{j=1}^{N} (a_j^+ a_j + \frac{1}{2}) \omega_j \hbar$, and

$$[a_j, a_i^+] = \delta_{ij}, \quad a_j = \frac{1}{\sqrt{2}} \left( \sqrt{\frac{m \omega}{\hbar}} x_j + i \frac{1}{\sqrt{m \omega \hbar}} p_j \right),$$

The wave function obeys

$$i \hbar \frac{\partial \Psi}{\partial t} = H \Psi.$$

Making the transformation

$$\Psi' = e^{-\frac{i}{\hbar} H_R t} \Psi,$$

one obtains

$$i \hbar \frac{\partial \Psi'}{\partial t} = \left\{ -\frac{1}{2} \hbar \Delta \sigma_1 + \frac{q_0}{2} \sigma_3 \sum_{j=1}^{N} c_j (e^{i \hbar \omega_j t} a_j^+ + e^{-i \hbar \omega_j t} a_j) \right\} \Psi'.$$

Obviously the non-commutativity between $x_i$ and $H_R$ only gives rise to the canonical transformation by $b_j^+ = e^{i \omega_j t} a_j^+$ that preserves $H_R = \sum_{j=1}^{N} (b_j^+ b_j + \frac{1}{2}) \omega_j \hbar$. 

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Keeping this canonical transformation in mind and still denoting the transformed coordinates by \( x_j \) we have

\[
\frac{i\hbar}{\partial} \Psi' = \left\{ -\frac{1}{2} \hbar \Delta \sigma_1 + \frac{q_0}{2} \sigma_3 \sum_{j=1}^{N} c_j x_j \right\} \Psi'. \tag{2.7}
\]

Making further transformation

\[
\Psi' = \exp\{-i\xi(t)\sigma_3\} \phi \tag{2.8}
\]

where

\[
\xi(t) = \frac{q_0}{2\hbar} \int_0^t \sum_i c_i x_i(\tau) d\tau = \frac{q_0}{2\hbar} \int_0^t d\tau X(\tau), \tag{2.9}
\]

we have

\[
\frac{i\hbar}{\partial} \frac{\partial \phi}{\partial t} = \left( -\frac{\Delta}{2} \right) (\sigma_+ e^{iq_0 \xi} + \sigma_- e^{-iq_0 \xi}) \phi \tag{2.10}
\]

or

\[
\frac{i}{\partial} \frac{\partial \alpha}{\partial t} = \left( -\frac{\Delta}{2} \right) e^{iq_0 \xi} \beta, \\
\frac{i}{\partial} \frac{\partial \beta}{\partial t} = \left( -\frac{\Delta}{2} \right) e^{-iq_0 \xi} \alpha, \tag{2.11}
\]

where \( \phi = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \). The normalization condition is

\[
|\alpha|^2 + |\beta|^2 = 1. \tag{2.12}
\]

Defining planar vectors

\[
\vec{r} = \text{Re} \alpha \vec{e}_x + \text{Im} \alpha \vec{e}_y, \quad \alpha = re^{-i\theta}, \\
\vec{\rho} = \text{Re} \beta \vec{e}_x + \text{Im} \beta \vec{e}_y, \quad \beta = \rho e^{-i\varphi} \tag{2.13}
\]

and

\[
\vec{B}(t) = \frac{q_0}{\hbar} \left( \sum_j c_j x_j \right) \vec{e}_z = \sum_j c_j \vec{B}_j. \tag{2.14}
\]
Eq. (2.11) can be recast to the form:

\[
\frac{d^2}{dt^2} \vec{r} = -\omega_0^2 \vec{r} - \frac{d\vec{r}}{dt} \times \vec{B}, \quad (2.15)
\]

\[
\frac{d^2}{dt^2} \vec{\rho} = -\omega_0^2 \vec{\rho} + \frac{d\vec{\rho}}{dt} \times \vec{B}. \quad (2.16)
\]

Henceforth the environment is viewed to be classical, namely any oscillator in the heat bath is treated as one of the driven harmonic oscillator with the driven force proportional to the time-dependent average value of \(\sigma_z\) through the canonical equation of motion:

\[
\frac{d^2}{dt^2} B_i + \omega_i^2 B_i + \frac{q_0 c_i^2}{2m_i \hbar} S(t) = 0 \quad (2.17)
\]

where

\[
S(t) = \langle \sigma_z \rangle = \langle \Psi | \sigma_3 | \Psi \rangle = \langle \phi | \sigma_3 | \phi \rangle. \quad (2.18)
\]

Because of eqs. (2.12), it is easy to know that

\[
S(t) = 2r^2 - 1 = 1 - 2\rho^2, \quad (2.19)
\]

\[
\langle \sigma_1 \rangle = r^2 \hat{\theta}, \quad (2.20)
\]

\[
\dot{\vec{r}}^2 = \omega_0^2 \rho^2, \quad \dot{\vec{\rho}}^2 = \omega_0^2 r^2. \quad (2.21)
\]

The energy conservation reads

\[
\frac{1}{2} \dot{r}^2 + \frac{1}{2} \omega_0^2 r^2 = \frac{1}{2} \omega^2, \quad (2.22)
\]

\[
\frac{1}{2} \dot{\rho}^2 + \frac{1}{2} \omega_0^2 \rho^2 = \frac{1}{2} \omega^2. \quad (2.23)
\]

The total energy is

\[
E_{\text{total}} = \langle \Psi | H | \Psi \rangle = -\hbar r^2 \hat{\theta} + q_0 \hbar (r^2 - \frac{1}{2}) \sum_{i=1}^{N} c_i x_i + \sum_{i=1}^{N} m_i \frac{1}{2} (\dot{x}_i^2 + \omega_i^2 x_i^2). \quad (2.24)
\]

Noting that the energy \(-\omega_0 \hbar \langle \sigma_1 \rangle\) is precisely the minus sign of angular momentum of charged particle moving in a applied magnetic field \(\vec{B}(t)\) shown by eq. (2.13).
Now the two-level system obeys eq. (2.15) or (2.16) which looks like the dynamics of charged particle experiencing harmonic force in a magnetic field perpendicular to the plane in which the particle moves. This picture is similar to Feynman-Vernon-Hellwarth’s Bloch vector stratagem [13]. The dynamics of oscillators in heat bath is described by eq. (2.17).

Eqs. (2.15) and (2.17) allows to describe the shift of ground state of two-level system when the interaction between the system and environment is getting large.

Looking at eq. (2.15), we see that the circular movement orbit of charged particle can be verified as the ground state when \( \ddot{B}_i \) does not vary with time at \( \vec{r} = \vec{R} \):

\[
B = \sum_j B_j = \left( \sum_j \frac{q_0^2 c_j^2}{2\hbar m_j \omega_j^2} \right) (1 - 2R^2). \tag{2.25}
\]

Denoting \( \vec{R} = R\vec{e}_r, \vec{r}|_{\vec{r} = \vec{R}} = v\vec{e}_\theta \), we obtain from eqs. (2.25) and (2.15)

\[
(2R^2 - 1)(1 - \frac{q_0^2}{2\hbar \omega_0} vR \sum_{i=1}^N \frac{c_i^2}{m_i \omega_i^2}) = 0. \tag{2.26}
\]

It is easy to know that

\[
vR = \omega_0 R\sqrt{1 - R^2} = \frac{2\hbar \omega_0^2}{q_0^2} \left[ \sum_j \frac{c_j^2}{m_j \omega_j^2} \right]^{-1}. \tag{2.27}
\]

Hence eq. (2.26) becomes

\[
R^4 - R^2 + \frac{4\hbar^2 \omega_0^2}{q_0^4} \left( \sum_j \frac{c_j^2}{m_j \omega_j^2} \right)^{-2} = 0. \tag{2.28}
\]

The solution of eq. (2.28) corresponds to the lowest-energy state of \( \phi \).

Denoting

\[
D = 1 - \frac{16\hbar \omega_0^2}{q_0^2} \left( \sum_j \frac{c_j^2}{m_j \omega_j^2} \right)^{-2}, \tag{2.29}
\]

we find

(1) for \( D < 0 \), no solution.

(2) for \( D = 0 \),

\[
R = \frac{\sqrt{5}}{2}, v = \pm \frac{\sqrt{5}}{2} \omega_0, B = 0. \tag{2.30}
\]
for $D > 0$, two solutions:

$$R^2 = \frac{1}{2} \left\{ 1 \pm \sqrt{1 - \frac{16\hbar^2 \omega_0^2}{q_0^4} \left( \sum_{i=1}^{N} \frac{c_i^2}{m_i \omega_i^2} \right)^{-2}} \right\}^{1/2}. \quad (2.31)$$

In the case (2) $B = 0$, $|\phi\rangle = \frac{1}{\sqrt{2}} e^{\pm i\omega_0 t} \left( \begin{pmatrix} 1 \\ \pm 1 \end{pmatrix} \right)$, the energy of system is just the total energy (system plus invariant):

$$E_{\text{total}} = \langle \Phi | \hbar \frac{d}{dt} | \Phi \rangle = \mp \hbar \omega_0. \quad (2.32)$$

It is obvious that for the absence of interaction between the system and environment, i.e. the magnetic field vanishes, the system is nothing but a bare two-level one, namely, the total energy system plus environment is equal to $-\hbar \omega_0$, and the excitation state has the energy $\hbar \omega_0$.

In the case (3) $D > 0$ means that

$$\frac{q_0^2}{4} \sum_{j=1}^{N} \frac{c_j^2}{m_j \omega_j^2} > \hbar \omega_0. \quad (2.33)$$

Solutions of eq. (2.28) reads

$$R_{\pm} = \left\{ \frac{1}{2} \left\{ 1 \pm \left[ 1 - \frac{16\hbar^2 \omega_0^2}{q_0^4} \left( \sum_{i=1}^{N} \frac{c_i^2}{m_i \omega_i^2} \right)^{-2} \right]^{1/2} \right\} \right\}^{1/2}, \quad (2.34)$$

correspondingly,

$$B_{\pm} = \mp \frac{1}{2\hbar} \left\{ q_0^4 \left( \sum_{i=1}^{N} \frac{c_i^2}{m_i \omega_i^2} \right)^2 - 16\hbar^2 \omega_0^2 \right\}^{1/2}. \quad (2.35)$$

which indicates that there exists the interaction between the system and environment. Differing from eq. (2.33) the total energy is

$$E_{\text{total}} = -\hbar \omega_0 \sqrt{1 - \frac{R^2}{R^2}} + \frac{\hbar}{2} B + \frac{1}{2} \sum_{j=1}^{N} m_j \omega_j^2 x_{j}^2$$

$$= -\frac{q_0^2}{8} \sum_{j=1}^{N} \frac{c_j^2}{m_j \omega_j^2} \left[ 1 + \frac{16\hbar^2 \omega_j^2}{q_0^4} \left( \sum_{i=1}^{N} \frac{c_i^2}{m_i \omega_i^2} \right)^2 \right] < -\hbar \omega_0. \quad (2.36)$$
with degeneracy. Therefore, we find the new ground state with the energy lower than \(-\hbar \omega_0\). When the acceleration of \(x_j\) can be neglected, the physical picture is viewed as the following: The two-level system has the ground state with \(E_{\text{total}} = -\hbar \omega_0\) and the excitation state with \(E_{\text{total}} = +\hbar \omega_0\), if the environment is completely “frozen”. Whereas the ground state of the system becomes double-degeneracy with energy lower than \(-\hbar \omega_0\) as increasing of interaction between the system and environment that corresponds to the non-vanishing displacements of the magnetic oscillators from their equilibrium points. We call the degenerated states localized states because in double-well system theory they represent those states which mainly localized in the left or right well with different parities. Later in this paper we shall show that the bare two-level in the system becomes unstable and will decay into one of the localized eigenstates with the energy lower than \(-\hbar \omega_0\) in the presence of dissipation.

3 Stability

Eqs.(2.15), (2.17), (2.19) together with (2.22) form a set of equation determining both the quantum dynamics of the system and semi-classical dynamics of oscillators in the bath. For \(\ddot{B}_i\) can be neglected the eq. (2.25) holds. With this picture eq. (2.13) takes the form:

\[
\ddot{r} - r \dot{\theta}^2 + \omega_0 r = - \frac{q_0^2}{2\hbar} (2r^2 - 1) \dot{r} \theta \sum_j \frac{c_j^2}{m_j \omega_j^2} = 0, \tag{3.1}
\]

\[
\dot{r}^{2} + 2 \dot{r} \dot{\theta} + \frac{q_0^2}{2\hbar} (2r^2 - 1) \dot{r} \sum_j \frac{c_j^2}{m_j \omega_j^2} = 0. \tag{3.2}
\]

Eq.(3.2) leads to

\[
r \dot{\theta} (r^4 - r^2) J = L_0 = \text{constant of motion}, \tag{3.3}
\]

\[
J = \frac{q_0^2}{4\hbar} \sum_j \frac{c_j^2}{m_j \omega_j^2}. \tag{3.4}
\]
\( L_0 \) is nothing but the effective angular momentum for the charged particle moving in the magnetic field and

\[ L_0 = -E_0 \quad (3.5) \]

where \( E_0 \) stands for the interacted energy of the system. With the help of eqs. (2.30) and (2.34), let us discuss the stability of solutions shown by eqs. (2.30) and (2.31).

(1) For \( r \approx \frac{1}{\sqrt{2}} \) we have

\[
\dot{\theta} = -\frac{J}{4r^2} (2r^2 - 1)^2 \pm \frac{\omega_0}{r^2},
\]

\[
\ddot{r} = -\frac{2r^2 - 1}{4r^3} \left[ \omega_0(\omega_0 \mp J)(2r^2 + 1) + (sr^2 - 1)^2 \right] \equiv F_1(r).
\]

Since \( F_1(\frac{1}{\sqrt{2}}) \) = 0

\[ F_1(\frac{1}{\sqrt{2}} + \delta r) \approx -4\omega_0(\omega_0 \mp J)\delta r, \quad (3.7) \]

so that the system is stable if \( \omega_0 > J \), critical if \( \omega_0 = J \) and unstable if \( \omega_0 < J \).

(2) For solution given by eq. (2.34) we have \( r \approx R_\pm \), then

\[
\dot{\theta} = \frac{J}{r^2} (r^2 - r^4 + \frac{\omega_0^2}{4} J^{-2}),
\]

\[
\ddot{r} = \left[ 4J^2 (r^2 - r^4) - \omega_0^2 \right] \frac{4J^2(3r^4 - r^2) - \omega_0^2}{16J^2r^3} \equiv F_2(r),
\]

\[ F_2(R_\pm) = 0, \]

\[ F_2(R_\pm \mp \delta r) \approx -8\omega_0^2 \left[ \frac{q_0^4}{16\hbar^2\omega_0^5} \left( \sum_j \frac{c_j^2}{m_j\omega_j^2} \right)^2 - 1 \right] \delta r = -2(\sum_j B_j)^2 \delta r. \quad (3.9) \]

Therefore the solution \( R_\pm \) are stable (with degeneracy).

### 4 Semiclassical Equation of Motion

To study the dynamic features we follow the standard procedure (see for example [13]) by assuming \( N \), the number of bath oscillators, is large enough so that we can replace the sum over \( j \) by an integration over \( \omega_i \) in our proceeding discussion. The special distribution in their path integral approach to the quantum Brownian motion [13] will be employed. Following their discussion we shall divide the heat bath
effect into three parts, a renormalization part, dissipative part and random force one and look at how these terms affect the dynamics of tunnelling and localization.

Let us first give a formal solution of eq. (2.17) which takes the form for each \( i \) subscript:

\[
B(t) = b(t) + b_0(t) \quad (4.1)
\]

where \( b_0(t) = c_1 \cos \omega t + c_2 \sin \omega t \) and \( b(t) \) is a particular solution which can be expressed by

\[
b(t) = c_1(t) e^{i\omega t} + c_2(t) e^{-i\omega t} \quad (4.2)
\]

To find a particular form of \( b(t) \) the condition

\[
\dot{c}_1(t) e^{i\omega t} + \dot{c}_2(t) e^{-i\omega t} = 0
\]

is taken into account. It leads to the well-known solution of eq. (4.1):

\[
b_0(t) = B_j(0) \cos \omega j t + \left( \frac{\sin \omega_j t}{\omega_j} \right) \dot{B}_j(0), \quad (4.4)
\]

\[
b(t) = -\frac{q_0^2}{2\hbar} \int_0^t S(\tau) \left[ \sum_j \frac{c_j^2}{m_j \omega_j} \sin \omega_j (t - \tau) \right] d\tau \quad (4.5)
\]

or

\[
b(t) = -\frac{q_0^2}{\pi \hbar} \int_0^t d\tau S(\tau) \int_0^\infty d\omega J(\omega) \sin \omega (t - \tau) \quad (4.6)
\]

where

\[
J(\omega) = \frac{\pi}{2} \sum_j \frac{c_j^2}{m_j \omega_j} \delta(\omega - \omega_j). \quad (4.7)
\]

Following ref. [4], if the Ohmic approximation is assumed:

\[
J(\omega) = A\omega^s e^{-\omega/\omega_c}, \quad (4.8)
\]

then

\[
b(t) = -\frac{q_0^2 A}{\pi \hbar} \int_0^t d\tau S(\tau) \int_0^\infty d\omega e^{-\omega/\omega_c} \sin[(t - \tau)\omega] \omega^s
\]

\[
= -\frac{q_0^2 A}{\pi \hbar} \omega_c^{s+2} \Gamma(s+2) \int_0^t d\tau S(\tau)(t - \tau)
\]

\[
F\left( \frac{s+2}{2}, \frac{s+3}{2}, \frac{3}{2}; -\omega_c^2(t - \tau)^2 \right) \quad (4.9)
\]
where \( F(a, b, c; u) \) is hyper-geometric function.

Let us consider a special case where \( s = 1 \), then

\[
B_0(t) = -\frac{q_0^2 A}{\pi \hbar} \omega_c S(t) + \frac{q_0^2 A}{\hbar} \left( \frac{1}{\pi} \frac{\lambda}{\lambda^2 + t^2} \right) S(0) + \frac{q_0^2}{\hbar} \int_0^t d\tau \frac{dS(\tau)}{d\tau} \left[ \frac{1}{\pi} \frac{\lambda}{\lambda^2 + (t - \tau)^2} \right]
\]  

(4.10)

where \( \lambda = \omega_c^{-1} \). When \( \omega_c \to \infty \), eq. (4.10) is reduced to

\[
b(t) = \frac{\eta q_0^2 \hbar}{\Omega} \frac{dS(t)}{dt} - \frac{\eta \Omega}{\pi \hbar} q_0^2 S(t).
\]  

(4.11)

This limit is nothing but the Debye distribution by making \( \sum_j \to \int_0^\infty d\omega \rho(\omega) \)

\[
\rho_D(\omega) \frac{[c(\omega)]^2}{m(\omega)} = \begin{cases} 
2\eta \omega^2 & \omega < \Omega \\
0 & \omega > \Omega 
\end{cases}
\]  

(4.12)

where \( \Omega \) is a high frequency cut-off, \( \eta \) is the phenomenological friction coefficient in the Langevin equation eq. (1.1).

Let \( \Omega \to \infty \) we have

\[
b(t) = \frac{2\eta q_0^2}{\hbar} \vec{r} \cdot \dot{\vec{r}} - \frac{\eta \Omega}{\pi \hbar} q_0^2 (2\vec{r}^2 - 1).
\]  

(4.13)

The second term is much larger than the first one for the usual environment. In comparison to eq. (2.25) we have

\[
b(t) \approx -\sum_j \frac{q_0^2 c_j^2}{2\hbar m_j \omega_j^2} S(t)
\]  

(4.14)

where

\[
\eta \Omega = \pi \sum_j \frac{c_j^2}{2m_j \omega_j^2}.
\]  

(4.15)

With this notation the broken symmetry condition eq. (2.33) is rewritten as

\[
\frac{\eta \Omega}{2\pi} q_0^2 > \hbar \omega_0
\]  

(4.16)

and correspondingly,

\[
R_\pm = \frac{\sqrt{2}}{2} \sqrt{1 \pm \sqrt{1 - \frac{2\pi^2 \hbar^2 \omega_0^2}{\Omega^2 \eta^2 q_0^4}}},
\]  

(4.17)

\[
B_\pm = \pm \frac{2}{\hbar} \sqrt{\frac{\Omega^2 \eta^2 q_0^4}{4\pi^2} - 4\pi^2 \hbar^2 \omega_0^2},
\]  

(4.18)

\[
E_{\text{total}} = -\frac{\hbar \omega_0}{2} \left[ \frac{\eta q_0^2 \Omega}{2\pi \hbar \omega_0} + \frac{2\pi \hbar \omega_0}{\eta q_0^2 \Omega} \right] \leq -\hbar \omega_0.
\]  

(4.19)
Obviously, eq. (4.19) tells that the larger the $\eta\Omega$ is, the lower the energy of degenerated ground state is.

By virtue of eqs. (4.1), (4.4) and (4.5), using the continuous Debye frequency distribution and following [13] we obtain

$$B(t) = \int_0^\infty \rho_D(\omega)B(\omega, t)d\omega$$

$$= B_S(t) + F(t)$$

(4.20)

where

$$F(t) = \sum B_i(0) \cos \omega_i t + \sum \frac{\dot{B}_i(0)}{\omega_i} \sin \omega_i t$$

(4.21)

is the Langevin force and carries all the characteristics of random force of classical Brownian motion if we assume the thermal probability distribution of heat bath as given by eq. (4.12). In fact, it is easy to calculate the average:

$$\langle \cdots \rangle = \int_{-\infty}^{+\infty} d\dot{x}_j \int_{-\infty}^{+\infty} dx_j \cdots \exp\left\{-\frac{m_j}{2kT}(\dot{x}_j^2 + \omega_j^2 x_j^2)\right\}$$

(4.22)

and find random force:

$$\langle F(t) \rangle = 0,$$

$$\langle F(t)F(t') \rangle = 2kT \frac{\eta q_0}{\pi\hbar} \frac{\sin \Omega(t - t')}{t - t'} \xrightarrow{\Omega \to \infty} 2kT \frac{\eta q_0}{\hbar} \delta(t - t')$$

(4.23)

where $F(t)$ corresponds to $b_0(t)$ in eq. (4.1).

With the above knowledge we come to establish the dynamic equation. We have already known that the angular momentum is related to the energy of two-level system based on eq. (3.5). In the existence of the interaction with environment the energy is altered by the magnetic field, namely, the renormalized angular momentum $L_R$ times $(-\hbar)$ can be viewed as the renormalized energy $\varepsilon_R$ of the two-level system. The $L_R$ is given by

$$L_R(t) = r^2 \dot{\theta} + \frac{\Omega}{8\pi} \frac{\eta q_0^2}{\hbar} S^2(t) \quad \text{and} \quad \varepsilon_R = -\hbar L_R.$$  

(4.24)
Taking the time-derivative of $L_R$ (or $\varepsilon_R$) we get a meaningful relation similar to the classical Langevin dissipative dynamics:

$$
\frac{d}{dt} L_R = \frac{\eta q_0^2}{4\hbar} \left( \frac{d}{dt} S(t) \right)^2 + \frac{1}{4} \left( \frac{d}{dt} S(t) \right) F(t).
$$

(4.25)

The first term on the right hand of eq. (4.25) is the dissipative term which always make $L_R$ increase, or in other words, decreased $\varepsilon_R$. The second term represents the work done by the Langevin force. In comparison to eq. (1.1) $S(t)$ plays the role similar to $q(t)$ so we can expect a Langevin-like dynamic equation for $S(t)$ on the basis of eqs. (2.15) and (2.17), that can be written in the form:

\begin{align}
\ddot{r} - r\dot{\theta}^2 + \omega_0^2 r &= -r\dot{\theta} B(t), \\
2r\ddot{\theta} + 2r\dot{\theta} &= \dot{r} B(t).
\end{align}

(4.26, 4.27)

The straight calculation gives

$$
\frac{d^2}{dt^2} S(t) = -4\omega_0^2 S(t) + 4\alpha L_R(t) S(t) - \frac{1}{2} \alpha^2 S^3(t) - 4\left( \frac{\eta q_0^2}{\hbar} \right) L_R(t) \frac{d}{dt} S(t) \\
+ \frac{1}{2} \alpha \left( \frac{\eta q_0^2}{\hbar} \right) S^2(t) \frac{d}{dt} S(t) - 4L_R(t) F(t) + \frac{1}{2} \alpha S^2(t) F(t)
$$

(4.28)

where $\alpha = \frac{\Omega q_0^2}{\pi \hbar}$. In comparison with eq. (2.1), the physical meaning of the RHS of eq. (4.28) can be explained as follows:

The first three terms are the driven forces, the fourth and fifth terms are friction force although they do not always make the motion retarded and the last two terms are random force.

Equations (4.25), (4.28) are the central results of our quasiclassical investigation of dissipative dynamics of two-level system. Although the equations are still hard to be solved, a qualitative consideration can help us insight into some physics of this quantum coherent dynamics. It is not difficult to find that the renormalized angular momentum $L_R$ plays important roles here. It first appears in eq. (4.24) representing the renormalized energy of the system, then in eq. (4.28) acting as a time dependent
factor to adjust the forces acting on the charged particle. As we have studied, for its own properties we can find that when the broken symmetry condition is not satisfied, $L_R$ reaches its minimum value $-\frac{\omega_0}{2}$ and the maximum value $\frac{\omega}{2}$. On the opposite case in which the broken symmetry condition is satisfied $L_R$ still reaches its minimum value $-\frac{\omega_0}{2}$ at the eigenstate \((2.30)\), but now eigenstate with $L_R = \frac{\omega_0}{2}$ becomes a saddle point and $L_R$ reaches its maximum value $\frac{\omega}{2}$ at the true degenerate ground states \((2.34)\). Because the dissipation always makes $L_R$ increase, the dynamics with any initial state will tend to ground state \((2.30)\) at weak coupling situation and to one of the localized ground states at strong coupling situation. This is consistent with our basic knowledge on the broken symmetry picture.

From classical mechanics we know that any velocity independent force $f$ can be written as $f = -\frac{dV}{dq}$. For the first three driven force terms in \((4.28)\) we can do the same thing. The explicative potential can be written in terms of $S$ and $L_R(t)$:

$$V_R(S, t) = \left[2\omega_0^2 - 2\alpha L_R(t)\right]S^2(t) + \frac{1}{8}\alpha^2 S^4.$$  \((4.29)\)

Here $V_R(S, t)$ is time-dependent and just this time dependence exhibits the novel dynamic features whether or not the two-level system processes the broken symmetry. When $\Omega\eta q_0^2/2\pi \leq \hbar \omega_0$, the time dependent coefficient of $S^2$ is never negative and $V_R(S, t)$ is positive everywhere except at $S = 0$, its only minimum point $V_R(S, t) = 0$. When $\Omega\eta q_0^2/2\pi > \hbar \omega_0$ we have a critical value $\hbar \omega_0^2/\Omega\eta q_0^2\pi$ for $L_R(t)$. Under the circumstance of $L_R(t) \leq \pi \hbar \omega_0^2/\Omega\eta q_0^2$ $V_R(S, t)$ has the similar behavior. As soon as $L_R(t)$ exceed this value, the figure of $V_R(S, t)$ will be changed to double well form. At this situation $S = 0$ becomes an unstable equilibrium point and the system at this point will decay due to external perturbation. At the same time two new degenerate minimum points appear and they will tend to
\[ r_\pm = \frac{\sqrt{2}}{2} \sqrt{1 \pm \sqrt{1 - 2\pi^2 h^2 \omega_0^2 / \Omega^2 \eta^2 q_0^4}} \] with the increasing of \( L_R(t) \). These behaviors of \( L_R(t) \) agree with the former results of this paper.

Interesting features also occur in the dissipative force terms in (4.28). The fifth term, contrary to the ordinary understanding of dissipative force, behaves as an advance force and the forth term plays either a retardative or advance role depending on the positive or negative sign of \( L_R(t) \)’s value. In other words, the dissipative force may not only decrease tunneling dynamics but also increase it depending on the initial condition of the system.

In applying above results, we would like to discuss some special dissipative dynamic processes in strong coupling limit. Here we only give qualitative consideration. Detail calculations and comparison with other approaches will appear in elsewhere.

First we consider the most interesting case with a fully localized initial wave function, saying \( S_I(0) = 1 \). Then from the definition of \( L_R \) we have \( L_{R,I}(0) = \Omega \eta q_0^2 / 8\pi \hbar \) and the radial potential felt by the charged particle likes double well form with the minimum points very near \( \pm 1 \). So we can see that the motion can not leave far away from the edge of the unit circle. Since the velocity \( dS/dt \) is very small, the dissipative force terms in (4.28) almost do not affect the motion during any short time period. The particle moves like a damped oscillator and loses its kinetic energy with the change of \( L_R(t) \). As \( L_{R,I}(t) \) reaches its maximum value, the particle will sit at the minimum point of the potential.

Second let us discuss the case where the eigenstate with \( R = \sqrt{2} \), \( v = \sqrt{2} \), \( B = 0 \) is taken as the initial state. In this case we can show that \( S_{II}(0) = 0 \), and \( L_{R,II}(0) = \omega_0 / 2 \). Now the particle stands on the metastable point and will go out of it because of thermal fluctuation.

At last suppose the eigenstate with \( R = \sqrt{2} \), \( v = -\sqrt{2} \), \( B = 0 \) is taken as the
initial state. As discussed above we have $S_{III}(0) = 0$, and $L_{R,III}(0) = -\omega_0/2$. The radial potential in this case has only one minimum and the particle is at the equilibrium point initially. Because both the two dissipative forces are advance ones now, so the range of motion will become larger and larger in the early period of dynamics. Therefore the particle will escape this potential well and the shape of potential will be changed into the double well form.

In conclusion we have studied the quasiclassical dissipative dynamics of a two-level system and compared it with the ordinary Langevin description. The further investigation along this direction and fully quantum mechanical treatment are deserved.

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References

[1] A.O. Calderia and A.J. Leggett, *Phys. Rev. Lett.* **46**, 211(1981); *Ann. Phys. (N.Y.)* **149**, 374(1983).

[2] A.J. Leggett, S. Chakravarty, A.T. Dorsey, Matthew P.A. Fisher, Anupam Garg and W. Zwerger, *Rev. Mod. Phys.* **59**, 1(1987), and the references cited therein.

[3] S. Chakravarty and A. J. Leggett, *Phys. Rev. Lett.* **52**, 5(1984).

[4] A.J. Bray and M.A. Moore, *Phys. Rev. Lett.* **49**, 1546(1982).

[5] W. Zwerger, *Z. Phys.* **B53**, 53(1983); *Z. Phys.* **B54**, 87(1983).

[6] P. Hanggi, *J. Statist. Phys.* **42**, 105(186).

[7] C. Aslangul, N. Pottier and D. Saint-James, *J. Phys.* (Paris) **46**, 2301(1985).

[8] H. Dekker, *Physica*, **141A**, 570(1987).

[9] M. Razavy, *Phys. Rev.* **A41**, 6668(1990).

[10] T. Tsuzuki, *Prog. Theor. Phys.* **81**, 770(1989).

[11] T. Dittrich, B. Oelschlagel and P. Hanggi, *Europhys. Lett.* **22**, 5(1993).

[12] S. Han, J. Lapionte and J.E. Lukens, *Phys. Rev. Lett.* **66**, 810(1991).

[13] A.O. Calderia and A.J. Leggett, *Physica* **121A**, 587(1983).

[14] Lei Wang and Jiushu Shao, *Phys. Rev. A* **49**, R637(1994).

[15] R.P. Feynman, F.L. Vernon, and R.W. Hellwarth, *J. Appl. Phys.* **28**, 49 (1957).