Spin and Charge Excitations in the Two-Dimensional \(t\)-\(J\) Model: Comparison with Fermi and Bose Systems

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Using high temperature series we calculate temperature derivatives of the spin-spin and density-density correlation functions to investigate the low energy spin and charge excitations of the two-dimensional \(t\)-\(J\) model. We find that the temperature derivatives indicate different momentum dependences for the low energy spin and charge excitations. By comparing short distance density-density correlation functions with those of spinless fermions and hard core bosons, we find that the \(t\)-\(J\) model results are intermediate between the two cases, being closer to those of hard core bosons. The implications of these results for superconductivity are discussed.

The nature of the ground state and low energy excitations for two-dimensional strongly correlated electrons doped slightly away from half-filling is of considerable interest for understanding the properties of high temperature superconductors \([1]\). The \(t\)-\(J\) model on a square lattice is a widely studied model used to investigate these problems. While the properties of a single hole introduced into an antiferromagnet are well understood \([2]\), the spectral function of the 2D \(t\)-\(J\) model directly. From the high temperature series for the momentum distribution \(\delta n_k\) we calculated \([3]\) the temperature derivative \(dn_k/dT\) which we used as a proxy for the momentum dependence of the low energy part of the spectral function. Our results for \(dn_k/dT\) showed that the low energy excitations of the 2D \(t\)-\(J\) model are spread throughout the Brillouin zone and are in general not conventional quasiparticles.

A consequence of this result is that \(dn_k/dT\) does not completely determine the momentum dependence of the low energy elementary excitations in the 2D \(t\)-\(J\) model. To further investigate the nature of the low energy elementary excitations, we have extended our calculations to the equal time spin-spin and density-density correlation functions, \(S(q)\) and \(N(q)\) respectively, and their temperature derivatives \(dS(q)/dT\) and \(dN(q)/dT\).

We have calculated high temperature series for \(S(q)\) and \(N(q)\) of the 2D \(t\)-\(J\) model to twelfth order in inverse temperature \(\beta = 1/k_BT\). Our calculations extend previous series calculations \([4,5]\) for the correlation functions of the \(t\)-\(J\) model. The \(t\)-\(J\) Hamiltonian is given by

\[
H = -tP \sum_{\langle ij \rangle, \sigma} \left( c_{i\sigma}^\dagger c_{j\sigma} + c_{j\sigma}^\dagger c_{i\sigma} \right) P + J \sum_{\langle ij \rangle} S_i \cdot S_j,
\]

where the sums are over pairs of nearest neighbor sites and the projection operators \(P\) eliminate from the Hilbert space states with doubly occupied sites. The definitions of the spin-spin and density-density correlation functions are

\[
S(q) = \sum_r e^{iqr} \langle S_r^z S_r^z \rangle \tag{2}
\]

\[
N(q) = \sum_r e^{iqr} \langle \Delta n_R \Delta n_r \rangle, \tag{3}
\]

where \(S_r^z = \frac{1}{2} \sum_{\alpha\beta} c_{r\alpha}^\dagger c_{r\beta}^\dagger c_{r\beta} c_{r\alpha}\) and \(\Delta n_r = \sum_\sigma c_{r\sigma}^\dagger c_{r\sigma} - n\). The series are extrapolated to \(T = 0.2J\) by Padé approximants and a ratio technique used previously \([6]\) for \(n_k\).

To interpret our results for the \(t\)-\(J\) model we examine \(dN(q)/dT\) for the tight-binding and spinless fermion models. For these non-interacting models \(dN(q)/dT\) is given by

\[
\frac{dN(q)}{dT} = -g \int \frac{dk}{(2\pi)^2} \left( n_k \frac{dn_k+q}{dT} + n_k \frac{dn_k-q}{dT} \right), \tag{4}
\]

where \(g = 2\) for tight binding and \(g = 1\) for spinless fermions. The properties of \(dN(q)/dT\) for the non-interacting models are determined by the convolution of \(n_k\) with \(dn_k+q/dT\). At low temperatures, due to Fermi statistics \(n_k\) is large only inside the Fermi surface and \(dn_k+q/dT\) is negative just inside the Fermi surface, positive just outside and close to zero elsewhere. The convolution will then give a significant contribution to the integral in Eq. 4 when \(q\) is such that only one of the positive or negative parts of \(dn_k+q/dT\) overlaps \(n_k\). This occurs for \(q \approx 0\) and \(q \approx 2k_F\), as demonstrated in Fig. 1 where we plot \(dN(q)/dT\) for the tight binding and spinless fermion models. The main features of these plots are a large positive spike at \(q \approx 0\) and a smaller but more extended negative dip located at \(q \approx 2k_F\). The shape
of the $2k_F$ line depends on the nature of the Fermi surface (hole like or electron like) but in both cases $2k_F$ is a continuous curve in the Brillouin zone.

For the $t$-$J$ series calculations the $q \approx 0$ (long range) parts of the correlation functions have the least accuracy, while we expect the correlation functions at larger wavevectors (short range) to be well determined. Consequently, we concentrate in our analysis on the locations and properties of the $2k_F$ features in the $t$-$J$ model correlation functions. Using the non-interacting models as guides, we search for the $2k_F$ features in the $t$-$J$ correlation functions by looking for the largest negative values of $dN(q)/dT$ and $dS(q)/dT$.

Results for $N(q)$ and $S(q)$ with electron density $n = 0.8$, $J/t = 0.4$ and $T = 0.2J$ are shown in Fig. 2. Our data are in good agreement with previous calculations at higher temperatures [6,7]. The Brillouin zone sums of the correlation functions agree with their respective sum rules to within 0.5%. Using data at $T = 0.2J$ and $T = 0.4J$ we calculate $\Delta N(q)/\Delta T$ and $\Delta S(q)/\Delta T$ as approximations for the temperature derivatives at $T = 0.3J$.

Results for $\Delta S(q)/\Delta T$ are shown in Fig. 3. The peak at $q \approx 0$ is considerably broader than for the non-interacting models, in agreement with the temperature dependence observed in previous calculations [6]. The only part of the Brillouin zone where $\Delta S(q)/\Delta T$ is negative is a roughly circular region of approximate radius $0.6\pi$ centered on $(\pi, \pi)$. We note that even at the lowest temperature accessible to us, the spin correlations are still

FIG. 1. (color) Full Brillouin zone plots of $dN(q)/dT$ for left, the tight binding model and right, the spinless fermion model. The parameters for both models are $n = 0.8$ and $T = 0.08t$. Large, positive central spikes show up as red and dark blue, while the negative features at $2k_F$ are orange and yellow.

FIG. 2. (color) Three dimensional full Brillouin zone plots of correlation functions for the $t$-$J$ model. Top, $S(q)$ and bottom, $N(q)$. The parameters for both plots are $n = 0.8$, $J/t = 0.4$ and $T = 0.2J$.

FIG. 3. (color) Two views of the full Brillouin zone plot of $\Delta S(q)/\Delta T$ in units of $J^{-1}$ for the $t$-$J$ model. The data and color coding are the same for both plots, with parameters $n = 0.8$, $J/t = 0.4$ and $T = 0.3J$.

FIG. 4. (color) Two views of the full Brillouin zone plot of $\Delta N(q)/\Delta T$ in units of $J^{-1}$ for the $t$-$J$ model. The central peak with a maximum value of $0.08J^{-1}$ has been truncated at $0.01J^{-1}$ in the lower plot to better show the behavior at large wavevectors. The data and color coding are the same for both plots, with parameters $n = 0.8$, $J/t = 0.4$ and $T = 0.3J$. 

Results for $\Delta S(q)/\Delta T$ are shown in Fig. 3. The peak at $q \approx 0$ is considerably broader than for the non-interacting models, in agreement with the temperature dependence observed in previous calculations [6]. The only part of the Brillouin zone where $\Delta S(q)/\Delta T$ is negative is a roughly circular region of approximate radius $0.6\pi$ centered on $(\pi, \pi)$. We note that even at the lowest temperature accessible to us, the spin correlations are still
peaked at \((\pi, \pi)\). The correlation length, around \((\pi, \pi)\) appears to be saturating or perhaps even decreasing with lowering \(T\). This could be taken as an indication for incommensurate correlations at still lower temperatures, as the peak at \((\pi, \pi)\) splits into several distinct peaks. Experimentally, for the high temperature superconducting materials, incommensurate spin-correlations only arise below about 100 K.

The negative feature in \(\Delta S(q)/\Delta T\) does not form a closed curve in the Brillouin zone. In particular, \(\Delta S(q)/\Delta T\) remains positive from \((0, 0)\) to \((\pi, 0)\), with no indication of a Fermi surface in the low energy spin excitations centered near \(\pi/2, \pi/2\) and extending perpendicular to the zone diagonals. These features are located near the peaks observed in \(dn_k/dT\) and \(\nabla_k n_k\), consistent with the strongest features in \(n_k\) being due to an underlying spinon Fermi surface.

![Graph of \(N(\pi, 0)\) versus \(T/J\)](image1)

![Graph of \(N(\pi, \pi)\) versus \(T/J\)](image2)

**FIG. 5.** Top, \(N(\pi, 0)\) and bottom, \(N(\pi, \pi)\) versus temperature. Solid line: spinless fermions, squares: hard core bosons and triangles: \(t-J\) model. The density is \(n = 0.8\) for all of the data sets and \(J/t = 0.4\) for the \(t-J\) model.

Results for \(\Delta N(q)/\Delta T\) are shown in Fig. 4. The peak for \(q \approx 0\) is much sharper than for \(\Delta S(q)/\Delta T\) and more like the non-interacting models. The negative feature in \(\Delta N(q)/\Delta T\) does make a closed curve in the Brillouin zone, in contrast to \(\Delta S(q)/\Delta T\). The location and shape of the negative feature in \(\Delta N(q)/\Delta T\) are similar to the \(2k_F\) line in \(dN(q)/dT\) of the spinless fermion model at the same density. The similarity extends to having the strongest negative feature in both models near \((\pi, 0)\). In the spinless fermion model this is due to parts of the \(2k_F\) curve overlapping after being translated back into the first zone. The momentum width of the negative feature in \(\Delta N(q)/\Delta T\) for the \(t-J\) model is considerably broader than the \(2k_F\) line for spinless fermions and this width is not temperature dependent down to \(T = 0.2J\). Interpreting the negative feature in \(\Delta N(q)/\Delta T\) for the \(t-J\) model as arising from an underlying momentum distribution for the charge degrees of freedom gives low energy charge excitations smeared out over a range of momenta near \(k_F\) of spinless fermions at the same density. For \(n = 0.8\) the charge excitations are centered on the zone diagonals and away from \((\pi, 0)\).

The low temperature momentum dependence of \(N(q)\) for the \(t-J\) model, while in general similar to spinless fermions, has important differences from spinless fermions at large momenta. As shown in Fig. 2, \(N(q)\) for the \(t-J\) model near \((\pi, \pi)\) and \((\pi, 0)\) is slightly smaller than 0.2, the value of \(N(q)\) for spinless fermions when \(q > 2k_F\). This means that holes in the \(t-J\) model have a greater tendency to be nearest or next-nearest neighbors than in the spinless fermion model, with the effect largest for nearest neighbors. This tendency has also been observed in exact diagonalization calculations and Green’s function Monte Carlo calculations, in both of which the effect is much larger than observed in the series calculation. This enhancement is probably due to finite size effects, though more work is needed to fully understand the differences between the series calculations and the Green’s function Monte Carlo and exact diagonalization calculations.

![Graph of \(<\delta_0^2(1,0)\) versus \(T/J\)](image3)

**FIG. 6.** Real space, nearest neighbor hole-hole correlation functions. Solid line: spinless fermions, squares: hard core bosons and triangles: \(t-J\) model. The density is \(n = 0.8\) for all three data sets and \(J/t = 0.4\) for the \(t-J\) model.

Fig. 5 shows the temperature dependence of \(N(q)\) at \((\pi, 0)\) and \((\pi, \pi)\) for the \(t-J\), spinless fermion and hard core boson models. All of these models have an infinite
on-site repulsion which sets the overall scale for $N(q)$. The hard core boson results shown in Figs. 5 and 6 are derived from a twelfth order high temperature series for $N(q)$ of hard core bosons. The ground states of spinless fermions and hard core bosons are known: Fermi sea with a well defined Fermi surface for spinless fermions and a superfluid for hard core bosons. The data for the $t$-$J$ model lie between these two cases, leaving open the possibility that the $t$-$J$ model has a superfluid ground state. The real space nearest neighbor hole-hole correlation function $\langle \delta_0 \delta_{1,0} \rangle$ further supports this behavior. Fig. 6 shows the temperature dependence of $\langle \delta_0 \delta_{1,0} \rangle$ for the $t$-$J$, spinless fermion and hard core boson models. Again, at low temperatures the $t$-$J$ data is between the spinless fermion and hard core boson results.

The temperature dependences shown in Figs. 5 and 6 are also interesting. At high temperatures all three models have similar values, with the $t$-$J$ data very close to spinless fermions. As the temperature is lowered below $T \lesssim 1.5J$ (for $J/t = 0.4$) the $t$-$J$ data deviates from spinless fermions towards hard core bosons. This temperature scale is too high to be due to coherent spin fluctuations. Also, the $t$-$J$ results are only weakly dependent on $J/t$ for $J/t \lesssim 0.5$ and persist to $J/t = 0$. This shows that the enhancement of $\langle \delta_0 \delta_{1,0} \rangle$ in the $t$-$J$ model relative to spinless fermions is due to the presence of two spin species, but not due to a direct spin interaction.

For larger $J/t$, in the phase separation parameter regime, the nearest neighbor density correlation grows rapidly as the temperature is lowered. This latter increase is clearly due to an attraction between the holes mediated by $J$. In contrast to this, the high temperature devaition from spinless fermions towards hard core bosons appears to be a statistical effect. This is, perhaps, analogous to the Hanbury-Brown and Twiss correlation well known for quantum particles [11,12]. Thus it appears that strong on-site repulsion tends to make the charge degrees of freedom in the $t$-$J$ model similar to hard core bosons at a high temperature scale [13]. This would suggest a superfluid ground state for the model, where the spin degrees of freedom merely help to choose the symmetry of the superfluid state. For antiferromagnetic spin correlations d-wave might be favored, while for ferromagnetic spin correlations p-wave symmetry could be favored. These ideas find some support in two-hole calculations [14], but require considerable further investigation.

In conclusion, from series calculations we find that $\Delta S(q)/\Delta T$ and $\Delta N(q)/\Delta T$ for the 2D $t$-$J$ model are quite different, giving further support to non-quasiparticle elementary excitations in 2D strongly correlated systems. Interpreting these results as due to itinerant degrees of freedom with underlying momentum distributions gives low energy spin and charge excitations near the zone diagonals, but with different momentum dependences, and absent near $(\pi, 0)$. The charge correlations show considerable similarity with hard-core bosons, which is suggestive of a superfluid ground state.

This work was supported in part by a faculty travel grant from the Office of International Studies at The Ohio State University (WOP), the Swiss National Science Foundation (WOP), an ITP Scholar position under NSF grant PHY94-07194 (WOP), EPSRC Grant No. GR/L86852 (MUL) and by NSF grant DMR-9616574 (RRPS). We thank the ETH-Zürich (WOP) and the ITP at UCSB (WOP, RRPS) for hospitality while this work was being completed.

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