Validity and reliability of estimated modulus of elasticity of cementitious materials

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Abstract. Modulus of elasticity is a vital property in cementitious material (CM) design and analysis. A better understanding of the relationship between density, strength and stiffness is needed to construct proper structures especially with reinforced cementitious materials. For this purpose, modulus of elasticity of CMs with different waste aggregates was analyzed and the reliability of estimated values of various equations proposed by standards and sources in literature, suggested in this investigation and derived by a software chosen was discussed. The results demonstrated that as compared to experimental results, the model developed by software was the most accurate as the percentages of error in prediction were in a good agreement.

1. Introduction
The estimation of the hardened characteristics of CM by evaluating its properties has been of deep interest. Among these hardened characteristics, to design plain, reinforced, and prestressed structures, the elastic modulus E is a fundamental parameter that needs to be defined [1]. The elastic modulus of CM is very difficult to predict because the properties of CM is affected by the features and amounts of ingredients [2]. In addition to all these different ingredients, the differences in features of the different ingredients at the various scales further enhance the inhomogeneity of cementitious materials [3]. Differences in rigidity and strength of the ingredients possess the effect on the overall rigidity and failure behavior of the material. Particularly, E-modulus and shape of aggregate and cement paste structure have an important effect on the results [4]. Phase and its volumetric ratio considerably affect the CM strain due to the anisotropic structure. The transition zone between aggregate and cement paste also has a vital significance on elastic behavior as well as the phases. The non-homogeneity and anisotropy have made CM nearly impossible to determine its modulus of elasticity by either a theoretical approach or a directly experimental procedure [5]. The modulus of elasticity of CM is commonly defined in terms of compressive strength [6]. The equations derived from experimental investigations are present to predict the modulus of elasticity from the compressive strength, which is a conventional evaluation for determining the properties of CM [7]. Although most empirical expressions for estimating the modulus of elasticity have been suggested by some researchers, very few expressions are corresponded to all the data because the mechanical features of CM are remarkably related to the features and amounts of binders and aggregates [6]. Aggregates play an important role on both fresh and hardened features of CM [8]. The selection of the suitable fine
aggregate for a specified implementation is of major significance as far as features of CM are taken into consideration.

Different formulas are proposed by many researchers to compute the modulus of elasticity of CM. Most of them based on the compressive strength are not suitable for both normal and high strength CMs. Many models suggested in the literature could not precisely predict the modulus of elasticity of CM specimens made with different types of aggregates.

The aim was to study the relationship between main parameters namely the density and the compressive strength that had an important influence on the modulus of elasticity and to find the most reliable formula which can be applicable to CMs containing different types of aggregates. Therefore, the experimentally obtained compressive strength, modulus of elasticity and density values were analyzed, and the coefficients of the parameters were arrived at and given by the regression equations that give the predicted values.

2. Experimental data and details

The models presented in this study are validated on experimental data from a previous research [9]. In the pronounced investigation, the materials used, and procedure applied on specimens for determination of E-modulus, density and compressive strength are as in the following.

CEM II/A-M (P-L) 42.5N was used as cement according to TS EN 197-1 Brick, marble and ceramic were collected from construction sites and ground until a similar grading as of natural crushed aggregate was obtained. Natural crushed fine aggregate utilized in the study was a calcareous aggregate. Super-plasticizer was used for providing flow diameter of CM in a constant range in order to prevent consistency difference effect on specimen properties.

The calculation of density of hardened CM is given in equation (1).

\[
density = \frac{A}{B - C}
\]

Where A is the dry mass in the air (g), B is the saturated surface-dry mass in the air (g), and C is the saturated surface-dry mass in the water.

The modulus of elasticity of the ø50x100 mm cylinder specimens was measured according to ASTM C 469-02 at the age of 28 days and calculated by using the equation (2):

\[
E = \frac{\sigma_2 - \sigma_1}{\varepsilon_2 - 0.000050}
\]

Where: \(E\) = chord modulus of elasticity, MPa, \(\sigma_2\) = stress at 40% of ultimate load, \(\sigma_1\) = stress at a longitudinal strain, \(\varepsilon_1\), of 50 millionths, MPa, \(\varepsilon_2\) = longitudinal strain produced by stress \(\sigma_2\).

3. Tools used for modulus of elasticity estimation

As also stated in another study [6], CM with a modulus of elasticity of 0 MPa possesses no compressive strength, therefore, the equations except for software is expressed as in equation (3).

\[
E = a* \sigma^b* \gamma^c
\]

3.1. Models specified by codes

ACI 318 and BS 8110 propose the equations (4) and (5), respectively to calculate the modulus of elasticity values of specimens.

\[
E_c = \gamma^{1.5} * 0.043 * \sqrt{\sigma}
\]

\[
E_c = \gamma^2 * 0.0017 * \sigma^{0.33}
\]
Where: \( E_c \) = modulus of elasticity, \( \gamma \) = air dry density of the specimen, \( \sigma \) = compressive strength.

3.2. Model suggested by software

Analysis of variance (ANOVA) is a method to analyze outputs. The variance of the observations is the sum of the variances of the independent sources when several sources of variation are acting simultaneously on a set of observations [10]. Therefore, regression and graphical analysis of the obtained data were made by a software TableCurve 3D. In order to find the ideal equation to describe three-dimensional empirical data, TableCurve 3D combines a powerful surface fitter with the ability. In TableCurve 3D, the dependent variable is \( Z \). All equations express \( Z \) as a function of the two independent variables \( X \) and \( Y \).

The equation (6) is an empirical relationship between the modulus of elasticity (\( Z \)) and density (\( X \)) and compressive strength (\( Y \)) and was yielded by the application of response surface methodology.

\[
E = a + b\sigma + c\gamma
\]  

(6)

3.3. Model derived in current study

The best regression yielded with an expression of the power type stated by equation (7) gave the relationship between the modulus of elasticity and the compressive strength.

\[
E = 318,8\sigma^{0,863}, R^2 = 0,86
\]  

(7)

In addition, regression analysis was experienced on the modulus of elasticity and density in oven dry condition (OD) using power regression model which presented the best fit against the other models with the equation (8):

\[
E = 6985\gamma^{1,936}, R^2 = 0,79
\]  

(8)

By multiplying equations (7) and (8), modulus of elasticity was calculated in equation (9).

\[
E = 1492,25\sigma^{0,929}\gamma^{1,391}
\]  

(9)

Where \( E \) is the modulus of elasticity (MPa), \( \sigma \) is the compressive strength (MPa) and \( \gamma \) is the density in OD condition (kg/dm³).

4. Results and discussion

The observed results and predicted modulus of elasticity values are shown in table 1 while figure 1 exhibits the relationship between observed and estimated modulus of elasticity and compressive strength of CMs. All models indicated that the \( E \) value can be predicted by considering the compressive strength and density. Although elastic properties are not sufficient to explain mechanical behavior, they can help us understand the effects of microstructural properties more easily, because their prediction does not involve many unknown parameters [11].

| Modulus of Elasticity/GPa | Actual | Software | ACI 318 | BS 8110 | Calculated |
|---------------------------|--------|----------|---------|---------|------------|
| 28,5                      | 29,1   | 27,2     | 25,8    | 29,0    |            |
| 27,6                      | 26,9   | 25,3     | 24,2    | 27,1    |            |
| 27,9                      | 27,6   | 25,5     | 24,1    | 27,6    |            |
| 28,2                      | 28,0   | 25,6     | 24,0    | 27,8    |            |
| 28,1                      | 27,8   | 25,4     | 23,7    | 27,6    |            |
| 24,3                      | 25,3   | 23,3     | 21,9    | 25,5    |            |
| 23,9                      | 23,7   | 21,4     | 19,9    | 23,8    |            |
Power regression models were used to correlate the parameters except software. The modulus of elasticity obtained by all models mostly underestimated the results. However, equations proposed by software and developed in the current study presented close predicted values to the experimental results. In a study [12], the modulus of elasticity values of CMs were overestimated by the models given by standards whereas in another study [13], the obtained modulus of elasticity using code formulae were less than the experimental result. In the determination of modulus of elasticity, γ used was the density (OD) instead of air dry density and σ was the cube compressive strength instead of cylinder strength specified in standards.

When the design of the construction is based on elasticity considerations, the modulus of elasticity of cement-based material is a parameter essential in the structural analysis for the determination of the strain distributions and displacements [7]. The features of cementitious materials remarkably are related to the multi-scale distribution of fine aggregates and voids inside the matrix as well as on the shape and size of aggregates [14]. Hence, manufacturing process of such materials commonly results in a great diversity in the microstructure.

In CM with high strength, the modulus of elasticity of the hardened cement matrix is high and the variation between the modulus of elasticity of the aggregates and the hardened matrix becomes small enough to lead to larger bond strength and monolithic behavior [15]. This can be attributed to the low modulus of elasticity of the aggregate, leading to a smaller variation between its value and that of the hardened matrix, and to the increased aggregate–paste bond due to the rough and porous surfaces of the aggregates. The modulus of elasticity of the aggregate depends not only on the density, on the pore structure as well and the surface texture of aggregate [16]. Therefore, an aggregate with a compact structure and evenly distributed pores will give a higher modulus of elasticity and more CM rigidity than a more porous aggregate. As compared to normal weight CM, a much lower modulus of elasticity would be expected for lighter CM [17]. The lower E-value absorbs energy from impact and cyclic loading, decreasing the occurrence of micro-cracking at the transition zone between cement and aggregate. A lower E-value provides more flexibility because rigidity is the product of the modulus of elasticity and the moment of inertia, EI [18]. Reduced rigidity could be beneficial at times, and utilization of CM with low density should be considered in these cases rather than normal-weight CM. In cases requiring improved impact or dynamic response, where differential settlement of foundation may take place, and in configurations of shell roofs, reduced rigidity may be necessary.

Figure 2 illustrated the three dimensional view of the estimated values obtained by the software. The actual data of the compressive strength, density and modulus elasticity of CMs were analyzed by applied statistical method. X represented density, Y represented compressive strength and Z represented modulus of elasticity. The value of the determination coefficient ($R^2 = 0.90$) confirms the proper fit of the model, thus indicating a discrepancy of 0.10 % for total variation, which is a normally
acceptable range of experimental error. The value of the adjusted determination coefficient (adjusted \( R^2 = 0.80 \)) is also high which indicates a high significance for the model.

Relationships between experimental and estimated values of modulus of elasticity are shown in figure 3. The evidence for a good relationship is large \( R^2 \) value which indicated that there were negligible differences between the observed and predicted values of modulus of elasticity. The \( R^2 \) values were 0.901, 0.875, 0.845 and 0.895 for estimated values obtained by using the software, ACI 318, BS 8110 and model developed in the current investigation, respectively. All the predicted values were close to each other and exhibited small variations with the actual data. The software suggested the best fit model which possessed the highest R-square value.

![Figure 3. Relationships between experimental values and estimated values of modulus of elasticity.](image)

5. Conclusions

The final models which were empirical relationships between density and compressive strength and modulus of elasticity derived in the current study, developed by software using response surface methodology and suggested by codes were compared with experimental results. According to the obtained results, all developed models were statistically accurate.

The modulus of elasticity of the cementitious materials was related to compressive strength as well as density, a strong dependency being observed. However, the relationship presented by ACI 318 and the suggested one by BS 8110 were similar, and their R-square values were close and relatively low. Besides, the largest R-square values were obtained from the models both given by software and calculated in the current study.

References

[1] Noguchi T, Tomosawa F, Nemati K M, Chiaia B M and Fantilli A P 2009 A practical equation for elastic modulus of concrete *ACI Struct J* **106** 690-6
[2] Yang C C, Lin Y Y and Huang R 1996 Elastic modulus of concrete affected by elastic moduli of mortar and artificial aggregate *J Mar Sci Technol* **4** 43-8
[3] Bernard F, Kamali-Bernard S and Prince W 2008 3D multi-scale modeling of mortar mechanical behavior and effects of microstructural changes *Cement Concrete Res* **38** 449-58
[4] Topçu İ B and Üğurlu A 2007 Elasticity theory of concrete and prediction of static e-modulus for dam concrete using composite models *Teknik Dergi* **18** 4055-67
[5] Nilsen A U, Monteiro P J M and Gjov O E 1995 Estimation of the elastic moduli of lightweight aggregate *Cement Concrete Res* **25** 276-80
[6] Nemati K M 2007 Relationship between the compressive strength and modulus of elasticity of high-strength concrete *CBM-CI International Workshop* (Karachi, Pakistan) 511
[7] Vilardell J, Aguado A, Agullo L and Gettu R 1998 Estimation of the modulus of elasticity for
dam concrete Cement & Concrete Research 38 93-101

[8] Gonçalves J, Tavares L M, Toledo Filho R, Fairbairn E R and Cunha E 2007 Comparison of natural and manufactured fine aggregates in cement mortars Cement Concrete Res 37 924-32

[9] Kockal N U 2015 Behavior of mortars produced with construction wastes exposed to different treatments Indian J Eng Mater S 22 203-14

[10] Chi J M, Huang R, Yang C C and Chang J J 2003 Effect of aggregate properties on the strength and stiffness of lightweight concrete Cement Concrete Comp 25 197-205

[11] Chamrova R 2010 Modelling and measurement of elastic properties of hydrating cement paste (École Polytechnique Fédérale de Lausanne, Switzerland: PhD Thesis)

[12] Nassif H 2003 Development of high-performance concrete for transportation structures in New Jersey (In cooperation with New Jersey Department of Transportation Division of Research and Technology and U.S. Department of Transportation Federal Highway Administration) FHWA NJ 2003-016 (New Jersey)

[13] Al-Khaiat H and Haque M N 1998 Effect of initial curing on early strength and physical properties of a lightweight concrete Cement Concrete Res 28 859-66

[14] Escoda J, Willot F, Jeulin D, Sanahuja J and Toulemonde C 2011 Estimation of local stresses and elastic properties of a mortar sample by FFT computation of fields on a 3D image Cement Concrete Res 41 542-56

[15] Kayali O, Haque M N and Zhu B 2003 Some characteristics of high strength fiber reinforced lightweight aggregate concrete Cement Concrete Comp 25 207-13

[16] Kockal N U and Ozturan T 2011 Strength and elastic properties of structural lightweight concretes Mater Design 32 2396-403

[17] Lo T Y, Cui H Z and Li Z G 2004 Influence of aggregate pre-wetting and fly ash on mechanical properties of lightweight concrete Waste Manage 24 333-8

[18] Committee ACI 213 2003 Guide for structural lightweight aggregate concrete (ACI 213R-03)