Impurity Effects in Nodal Extended $s$- and Nodeless $d$-Wave Superconductors: 
Gap Symmetry of BiS$_2$-Based Layered Superconductors
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In BiS$_2$-based layered superconductors, the existence of gap nodes on Fermi-surface curves has been suggested from angle-resolved photoemission spectroscopy measurements, whereas the conventional $s$-wave gap has been proposed from measurements of superfluid density and thermal conductivity. To reconcile these two distinct experimental results of the gap node, we investigate nonmagnetic impurity effects in the superconductor with a disconnected pocketlike Fermi-surface structure. Then, we claim that the seemingly contradictory situation concerning the gap node is resolved by a concept of dirty nodal extended $s$-wave superconductivity. Provided that it is unnecessary to consider the nodes of the gap, at first glance, the conventional $s$-wave gap seems to be a unique solution, but in the pocketlike Fermi-surface topology, a nontrivial possibility of a nodeless $d$-wave superconductor is pointed out. To clarify the gap symmetry, we propose to perform experiments on nuclear magnetic relaxation rate $T_1^{-1}$ in BiS$_2$-based layered superconductors.

1. Introduction

In 2012, Mizuguchi et al. discovered the new BiS$_2$-based layered superconductor LaO$_{1-x}$F$_x$BiS$_2$. The mother compound LaOBiS$_2$ is insulating, while owing to the substitution of F for O, electrons are doped into the BiS$_2$ layer. Then, the system becomes metallic and superconductivity occurs at low temperatures. It has been experimentally known that the superconducting transition temperature $T_c$ is maximum at $x = 0.5$. In the sample synthesized under high pressures, the highest $T_c$ among the BiS$_2$ family has been obtained. The onset $T_c$ is 11.1 K and the temperature at which the resistivity becomes zero is 8.5 K.

Concerning the mechanism of superconductivity in new materials, an important clue has been frequently obtained from the information on the gap symmetry revealed by the measurement of physical quantities and theoretical research on the gap structure. In the case of BiS$_2$-based layered superconductors, we notice that the results are consistent with $s$- or extended $s$-wave pairing suggested from the temperature dependence in the superfluid densities of Bi$_4$O$_3$S$_3$, LaO$_{0.5}$F$_{0.5}$BiS$_2$, and NdO$_{1.4}$BiS$_2$. Among them, here, we focus on NdO$_{1.4}$F$_2$BiS$_2$. As mentioned above, it has been reported that the gap symmetry is consistent with $s$- or extended $s$-wave for both $x = 0.3$ and 0.5 in the measurement of superfluid density. Also, from the thermal transport measurements, a conventional $s$-wave gap has been considered to be realized in NdO$_{0.71}$F$_{0.29}$BiS$_2$. On the other hand, a different result of the gap structure has been reported for NdO$_{0.71}$F$_{0.29}$BiS$_2$. Namely, the existence of the gap node on the Fermi-surface curves has been suggested from angle-resolved photoemission spectroscopy measurements. At first glance, these results seem to contradict each other. It is an important and challenging task to find a way to reconcile those results of the symmetry of the gap function from a theoretical viewpoint.

In this paper, we investigate nonmagnetic impurity effects on BiS$_2$-based layered superconductors by evaluating the density of states (DOS), nuclear magnetic superrelaxation rate $T_1^{-1}$, and the superfluid density $\rho_s$ for both nodal extended $s$-wave and nodeless $d$-wave gap functions within a self-consistent $T$-matrix approximation. On the basis of a concept of dirty nodal extended $s$-wave superconductors, it is claimed that the existence of the node of the gap on the Fermi-surface curve does not contradict the $s$-wave-like temperature dependence of $\rho_s$. Provided that the gap nodes on the Fermi-surface curves are ignored, the nodeless $d$-wave superconductor becomes another candidate in the present Fermi-surface topology, in addition to the conventional $s$-wave gap.

The paper is organized as follows. In Sect. 2, we show our model and the formulation for the impurity effects in the superconducting state. We also explain the $T_c$ reduction due to nonmagnetic impurities and the Fermi-surface structure of BiS$_2$-based layered superconductors. In Sect. 3, we show our calculation results of the physical quantities for both extended $s$- and nodeless $d$-wave superconductors. Finally, in Sect. 4, we summarize this paper and discuss the possible relationship of our scenario on dirty superconductors with the experimental results. We also propose the experimental measurement of $T_1^{-1}$ in BiS$_2$-based layered superconductors. Throughout this paper, we use such units as $h = k_B = 1$.

2. Model and Formulation

2.1 Model Hamiltonian

In this paper, we consider the Hamiltonian

$$H = H_0 + H_{\text{imp}},$$

where $H_0$ denotes the effective Hamiltonian without impurities, whereas $H_{\text{imp}}$ indicates the non-magnetic impurity term. The first term $H_0$ is given by

$$H_0 = \sum_{k\sigma} \varepsilon_k c^\dagger_{k\sigma} c_{k\sigma} + \sum_{k,k',\sigma} V_{k,k'} c^\dagger_{k\sigma} c^\dagger_{k'\sigma} c_{k'\sigma} c_{k\sigma},$$

where $\varepsilon_k$ is the kinetic energy of electrons with the wave vector $k$, $c_{k\sigma}$ is the annihilation operator of electrons with $k$ and spin $\sigma$, and $V_{k,k'}$ is the effective attraction between electrons. The second term $H_{\text{imp}}$ is given by

$$H_{\text{imp}} = U \sum_{i,\sigma} c^\dagger_{i\sigma} c_{i\sigma},$$

where $U$ is the on-site repulsion energy between electrons.
where $U$ denotes the potential due to nonmagnetic impurities, $r_i$ indicates the position of the $i$-th impurity, and $c_{r_i}$ indicates the electron annihilation operator at position $r_i$.

We solve this system by the following procedure. First, we apply the Hartree–Fock–Gor'kov approximation to $H_0$ by introducing the superconducting gap. Then, we include the effect of impurity scattering on the superconducting electrons in a framework of the perturbation theory concerning $U$, called a self-consistent $T$-matrix approximation.\(^{10-15}\)

2.2 Hartree–Fock–Gor'kov approximation

Let us first consider the mean-field approximation extended to the superconducting state, i.e., the Hartree–Fock–Gor'kov approximation. Then, we obtain

\[
H_{0\text{MF}} = \sum_{\sigma} \varepsilon_{k} c_{k\sigma}^\dagger c_{k\sigma} - \sum_{k} \left( \Delta_k c_{k\uparrow} c_{-k\downarrow} + \Delta_k^* c_{k\downarrow}^\dagger c_{-k\uparrow} \right),
\]

(4)

where the gap function $\Delta_k$ is defined by

\[
\Delta_k = -\sum_{k'} V_{k,k'} (c_{k'} c_{k})\gamma.
\]

(5)

Here, $\langle \cdots \rangle$ denotes the operation to take the thermal average using $H_{0\text{MF}}$. Note that the Hartree–Fock terms are already included in the electron energy $\varepsilon_k$, and the constant energy shift including the gap function is ignored in $H_{0\text{MF}}$.

It is possible to solve $H_{0\text{MF}}$ by a couple of methods, but here, we employ the Green’s function technique,\(^{16}\) not the Bogoliubov canonical transformation method. Let us now define the Green’s function $G_0(k,i\omega_n)$ as

\[
G_0(k,i\omega_n) = -\int_{0}^{1/T} d\tau e^{i\tau\omega_n} (\Psi_k(\tau)\Psi_k^\dagger(0)),
\]

(6)

where the boldface character represents the matrix in Nambu space, $T$ is temperature, and $\omega_n$ denotes the fermion Matsubara frequency, given by $\omega_n = \pi T (2n+1)$ with an integer $n$.

\[
\Psi_k(\tau) = e^{iH_0 t \psi_k e^{-\tau H_0}}.
\]

We introduce the two-component field operator $\Psi_k$ as

\[
\Psi_k = \begin{pmatrix} c_{k\uparrow} \\ c_{-k\downarrow}^\dagger \end{pmatrix}, \quad \Psi_k^\dagger = \begin{pmatrix} c_{k\downarrow}^\dagger \\ c_{-k\uparrow} \end{pmatrix}.
\]

(7)

After some algebraic calculations, we obtain $G_0$ as

\[
G_0^{-1}(k,i\omega_n) = i\omega_n \sigma_0 - \Delta_k \sigma_1 - e_k \sigma_3,
\]

(8)

where $\sigma_0$ is a $2 \times 2$ unit matrix, while $\sigma_1$ and $\sigma_3$ are respectively given by

\[
\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.
\]

(9)

After performing the summation in terms of the Matsubara frequency, we obtain the gap equation as

\[
\Delta_k = -\sum_{k'} V_{k,k'} \frac{\Delta_{k'}}{2E_{k'}} \tanh \frac{E_{k'}}{2T}.
\]

(10)

To solve the gap equation, we assume the separable-type attractive interaction given by

\[
V_{k,k'} = \begin{cases} -V |\varepsilon_{k}| < \omega_c & \text{and} |\varepsilon_{k'}| < \omega_c, \\ 0 & \text{otherwise,} \end{cases}
\]

(11)

where $V > 0$ is the magnitude of the attraction and the cut-off frequency $\omega_c$ is assumed to be much less than the Fermi energy. Accordingly, the gap function is given by

\[
\Delta_k = \Delta \psi_k,
\]

(12)

where $\Delta$ denotes the magnitude of the gap and, in general, it depends on temperature and impurity concentration.

In the weak-coupling limit, we solve the gap equation to obtain the superconducting transition temperature $T_c$ and the gap at absolute zero temperature, $\Delta_0(0)$, for the case without impurities. Then, we obtain the ratio as

\[
\frac{\Delta_0(0)}{T_c} = \pi e^{-\gamma - \eta},
\]

(13)

where $\gamma$ denotes the Euler’s constant and $\eta$ is given by

\[
\eta = \frac{(\psi_{\pi/2}^2 \log |\psi_{\pi/2}|)_{\text{FS}}}{(\psi_{\pi/2}^2)_{\text{FS}}}.
\]

(14)

Here “FS” indicates the abbreviation of Fermi surface and $\langle \cdots \rangle_{\text{FS}}$ denotes the operation to take the average over the Fermi-surface curve. For the anisotropic gap, $\eta$ becomes negative, leading to the enhancement of the ratio.

2.3 Self-consistent $T$-matrix approximation

Next, we include the nonmagnetic impurity effect,\(^{10-15}\) which is considered through the self-energy $\Sigma$. The Green’s function obeys the Dyson’s equation

\[
G^{-1}(k,z) = G_0^{-1}(k,z) - \Sigma(k,z),
\]

(15)

where we use $z$ for the frequency, which is obtained by the analytic continuation of $z = i\omega_n$, and the impurity self-energy $\Sigma$ should be expressed as

\[
\Sigma = \Sigma_0 \sigma_0 + \Sigma_1 \sigma_1 + \Sigma_3 \sigma_3.
\]

(16)

Then, we obtain

\[
G^{-1}(k,z) = \tilde{\varepsilon} \sigma_0 - \tilde{\Delta} \sigma_1 - \tilde{\varepsilon} \sigma_3,
\]

(17)

where

\[
\tilde{\varepsilon} = z - \Sigma_0, \quad \tilde{\Delta} = \Delta_0 + \Sigma_1, \quad \tilde{\varepsilon} = \varepsilon_k + \Sigma_3.
\]

(18)

Hereafter, we simply ignore $\Sigma_3$, since this term is not essentially important in the following discussion. In fact, the correction to one-electron energy is considered to be included at the level of Hartree–Fock approximation.

Let us evaluate $\Sigma$ in a single-site approximation given by

\[
\Sigma(z) = n_{\text{imp}} T(z),
\]

(19)

where $n_{\text{imp}}$ indicates the impurity concentration and $T$ denotes the $T$-matrix for impurity scattering. The $T$-matrix obeys the equation given by

\[
T(z) = U + U \sum_k G(k,z) T(z),
\]

(20)

where $U = U \sigma_3$. In the unitarity limit, i.e., $U N_0(0) \gg 1$, where $N_0(0)$ denotes the DOS at the Fermi level in the normal state, we obtain

\[
\Sigma_0 = -\alpha \frac{g_0}{g_0^2 - g_1^2}, \quad \Sigma_1 = \alpha \frac{g_1}{g_0^2 - g_1^2}, \quad \Sigma_3 = \alpha \frac{g_1}{g_0^2 - g_1^2},
\]

(21)

where $\alpha$ is a pair-breaking parameter given by

\[
\alpha = \frac{n_{\text{imp}}}{\pi N_0(0)}.
\]

(22)

The averaged Green’s functions $g_0$ and $g_1$ are respectively
Given by
\[ g_0 = -\left( \frac{z}{\sqrt{\Delta_k^2 - z^2}} \right)_{FS}, \quad g_1 = -\left( \frac{\Delta_k}{\sqrt{\Delta_k^2 - z^2}} \right)_{FS} \]  
(23)

The normalized DOS $N(z)$ in the superconducting state affected by impurity scattering is given by
\[ N(z) = -\text{Im} g_0. \]  
(24)

Note that $N(z)$ is normalized by $N_0(0)$ and $N(\infty) = 1$.

In this paper, we evaluate the nuclear magnetic relaxation rate $T_1^{-1}$ in the superconducting state, since it is sensitive to the change in DOS in the low-energy region. When we define $R = (T/T_s)^{-1}$, we discuss the temperature dependence of the ratio $R_s(T)/R_n$, where the subscripts “s” and “n” indicate the superconducting and normal states, respectively. By following the Bardeen–Cooper–Schrieffer theory,\(^{17}\) we obtain $R_s(T)/R_n$ as
\[ \frac{R_s(T)}{R_n} = 2 \int_0^\infty dz \left( -\frac{\partial f}{\partial z} \right) \left[ N^2(z) + M^2(z) \right], \]  
(25)

where $f = 1/(e^{z/T} + 1)$ and $M(z)$ is given by
\[ M(z) = -\text{Im} g_1. \]  
(26)

We are also interested in the temperature dependence of superfluid density, $\rho_s(T)$, which is evaluated from results of the penetration depth experiment. It is given by
\[ \frac{\rho_s(T)}{\rho_n} = 1 - 2 \int_0^\infty dz \left( -\frac{\partial f}{\partial z} \right) N(z), \]  
(27)

where $\rho_n$ indicates the electron density in the normal state.

Here, we comment on the assumption when we will perform numerically the above integrals. It is assumed that the ratio of $T_c$ to the gap at $T = 0$ is not affected by impurity scattering. Namely, we use the relation $\Delta(0)/T_c = \Delta_0(0)/T_{c0}$ and the ratio is given by Eq. (13).

2.4 Fermi-surface structure and gap functions

The canonical model for Bi$_2$S$_3$-based layered superconductors was proposed by Usui and coworkers,\(^{18,19}\) just after the discovery of Bi$_2$S$_3$-based layered superconductors. The minimal model to describe the electronic structure of the Bi$_2$S$_3$ layer is the two-band Hamiltonian composed of Bi 6$p_x$ and 6$p_y$ orbitals on the two-dimensional square lattice. In Fig. 1(a), we show the Fermi-surface curves for $x = 0.3$ of the minimal model. We point out a characteristic issue that pocketlike disconnected Fermi-surface curves appear around $k = (\pm \pi, 0)$ and $(0, \pm \pi)$. Note that for $x < 0.52$, we find the Fermi-surface curve originating from the lower-energy band.\(^{18-20}\) Thus, for $x = 0.3$, we consider effectively only the band forming the Fermi-surface curves for the appearance of superconductivity.

In order to concentrate on the impurity effect in this case, we simplify the Fermi-surface structure by maintaining its topology. As shown in Fig. 1(b), we consider four semicircle Fermi-surface curves at around $k = (\pm \pi, 0)$ and $(0, \pm \pi)$, in order to reproduce the pocketlike disconnected Fermi-surface structure of the effective model for Bi$_2$S$_3$-based layered materials. For the operation to take the average over the Fermi-surface curve, it is useful to define the angle $\phi$ to specify the position on the Fermi-surface curve, as shown in Fig. 1(b).

In this paper, we consider both extended $s$- and $d$-wave gap functions. Note that we do not consider the $s$-wave gap here, since it is well known that nonmagnetic impurities do not affect it. When we consider the extended $s$-wave gap $\Delta_k \propto \cos k_x + \cos k_y$, the nodal lines cross the Fermi-surface curves, as shown in Fig. 2(a). When we use the angle $\phi$ defined in Fig. 1(b), the gap is well approximated by $\Delta(\phi) = \Delta \cos(2\phi)$ on the Fermi-surface curves. This is essentially the same function as the $d$-wave gap on the large Fermi-surface curve with the center at the $\Gamma$ point for high-$T_c$ cuprates. Note, however, that the extended $s$-wave gap is allowed to have a constant component. Thus, in general, the extended $s$-wave gap is written as
\[ \Delta(\phi) = \Delta |p + \cos(2\phi)|, \]  
(28)

where $\Delta$ in this case is the gap of the anisotropic part and $p$ denotes the ratio of isotropic to anisotropic gaps. In the present Fermi-surface structure, the node positions move from $\phi = \pi/4$ and $3\pi/4$ for $p = 0$ to $\phi = \pi/2$ for $p = 1$. For $p > 1$, the nodes do not appear on the Fermi-surface curves. The impurity effect should be different depending on the gap ratio $p$. This point will be discussed in detail later.

Next, we consider the $d$-wave gap. In sharp contrast to the case with a large Fermi-surface curve with the center at the $\Gamma$ point, the present Fermi-surface curves do not cross the lines of $k_x = \pm k_y$. Thus, when we assume the $d$-wave gap, $\Delta_k \propto \cos k_x - \cos k_y$, the nodes do not appear on the Fermi-surface curves, as shown in Fig. 2(b). In this sense, it can
be called the nodeless $d$-wave gap. Note, however, that the gap has the same sign on the Fermi-surface curve, while the sign change occurs between the gap functions on the different pocketlike Fermi-surface curves. Thus, when we consider the gap on the pocketlike Fermi-surface curve, it looks like a simple $s$-wave gap at first glance, but the average of the gap over the whole Fermi-surface curves becomes zero. This fact has a remarkable impact on the impurity effect on nodeless $d$-wave superconductors. In the following calculations, we consider the nodeless $d$-wave gap as

$$\Delta_k = \begin{cases} -\Delta & \text{on the FS A}, \\ +\Delta & \text{on the FS B}, \end{cases}$$

(29)

where $\Delta$ is the magnitude of the gap, and the Fermi-surface curves A and B are defined in Fig. 2(b). Note here that for simplicity, we ignore the $k$ dependence of the gap on the Fermi-surface curves, but it is easy to check the validity of this approximation.

2.5 Reduction in $T_c$

Before showing our calculation results concerning the impurity effect on physical quantities, let us briefly discuss the reduction in the transition temperature $T_c$ due to nonmagnetic impurity scattering.

First, we consider the extended $s$-wave gap. Except for the case of $p = 0$, the well-known Abrikosov–Gor’kov formula for the $T_c$ reduction cannot be simply used, since there exists a constant component $p$. As pointed out by Tsuneto, it is necessary to consider the change in the pairing interaction due to the nonmagnetic impurity scattering. After some algebraic calculations, we obtain the generalized formula

$$\log \left( \frac{T_{c0}}{T_c} \right) = \beta \left[ \psi \left( \frac{1}{2} + \frac{\alpha}{2\pi T_c} \right) - \psi \left( \frac{1}{2} \right) \right],$$

(30)

where $T_{c0}$ indicates the superconducting transition temperature without nonmagnetic impurity, $\psi$ is the di-gamma function, and $\beta$ is given by

$$\beta = 1 - \frac{\langle \Delta_k \rangle^2_{FS}}{\langle \Delta_k \rangle^2_{FS}}.$$  

(31)

In the case of the simple $s$-wave gap, $\Delta_k$ is constant and thus, $\beta = 0$, indicating that $T_c$ is not affected at all by the nonmagnetic impurity. On the other hand, for the $d$-wave gap, since $\langle \Delta_k \rangle_{FS} = 0$, we obtain $\beta = 1$, leading to the well-known Abrikosov–Gor’kov formula.

For the present extended $s$-wave gap Eq. (28), we can take the average over the Fermi-surface curves. Then, we obtain $\beta$ as

$$\beta = \frac{1}{2p^2 + 1}.$$  

(32)

In Fig. 3(a), we show $T_c/T_{c0}$ as functions of $\zeta$ at various values of $p$, where $\zeta$ is defined as

$$\zeta = \frac{\alpha}{2\pi T_{c0}}.$$  

(33)

Here, we briefly explain the choice of $p$. From the form of the gap $p + \cos(2\phi)$, we immediately recognize that the node of the gap disappears at $p = 1$. We also note that the pure anisotropic case, $p = 0$, is exceptional. Namely, it is necessary to consider four regions as $p = 0, 0 < p < 1, p = 1$, and $p > 1$. Thus, we show the results for four values of $p$ as $p = 0, 0.5, 1,$ and $1.5$.

In the curve for $p = 0$, i.e., $\beta = 1$, $T_c$ becomes zero at $\zeta = \zeta_c$, where $\zeta_c$ denotes the critical value of $\zeta$, at which $T_c$ becomes zero. By using the asymptotic form of the di-gamma function $\psi$, given by

$$\psi \left( \frac{1}{2} + x \right) - \psi \left( \frac{1}{2} \right) \approx \log(4e^\gamma x),$$

(34)

for $x \gg 1$, we obtain $\zeta_c = e^{-\gamma}/4 \approx 0.14$.

When the isotropic gap exists for $p > 0$, the $T_c$ reduction is gradual for large $p$ values and there are no critical $\zeta$ value. The anisotropic part of the gap is washed out by the non-magnetic impurity scattering, indicating that the isotropic gap remains.

Next, we consider the case of the nodeless $d$-wave gap. In a previous study on the nonmagnetic impurity effect in nodal $d$-wave superconductors, the reduction in $T_c$ is given by the Abrikosov–Gor’kov formula,

$$\log \left( \frac{T_{c0}}{T_c} \right) = \frac{\alpha}{2\pi T_{c0}} \left[ \psi \left( \frac{1}{2} + \frac{\alpha}{2\pi T_c} \right) - \psi \left( \frac{1}{2} \right) \right].$$

(35)

Also, in the nodeless $d$-wave case, this Abrikosov–Gor’kov formula is available, since the average of the gap over the Fermi-surface curve vanishes.

In Fig. 3(b), we show the curve for $T_c/T_{c0}$ as a function of $\zeta/\zeta_c$ for the extended $s$-wave gap with $p = 0, 0.5, 1,$ and $1.5$. Note that in this case, except for the case of $p = 0$, there exists no critical value of $\zeta$. In the curve for $p = 0$, $\zeta = \zeta_c$, where $\zeta_c$ denotes the critical value of $\zeta$ at which $T_c$ becomes zero. By using the asymptotic form of the di-gamma function $\psi$, given by

$$\psi \left( \frac{1}{2} + x \right) - \psi \left( \frac{1}{2} \right) \approx \log(4e^\gamma x),$$

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for $x \gg 1$, we obtain $\zeta_c = e^{-\gamma}/4 \approx 0.14$. When the isotropic gap exists for $p > 0$, the $T_c$ reduction is gradual for large $p$ values and there are no critical $\zeta$ value. The anisotropic part of the gap is washed out by the non-magnetic impurity scattering, indicating that the isotropic gap remains.

This point has already been emphasized by Tsuneto. Next, we consider the case of the nodeless $d$-wave gap. In a previous study on the nonmagnetic impurity effect in nodal $d$-wave superconductors, the reduction in $T_c$ is given by the Abrikosov–Gor’kov formula,

$$\log \left( \frac{T_{c0}}{T_c} \right) = \frac{\alpha}{2\pi T_{c0}} \left[ \psi \left( \frac{1}{2} + \frac{\alpha}{2\pi T_c} \right) - \psi \left( \frac{1}{2} \right) \right].$$

(35)

Also, in the nodeless $d$-wave case, this Abrikosov–Gor’kov formula is available, since the average of the gap over the Fermi-surface curve vanishes.

In Fig. 3(b), we show the curve for $T_c/T_{c0}$ as a function of $\zeta/\zeta_c$ for the extended $s$-wave gap with $p = 0, 0.5, 1,$ and $1.5$. Note that in this case, except for the case of $p = 0$, there exists no critical value of $\zeta$. (b) $T_c/T_{c0}$ as a function of $\zeta/\zeta_c$ for nodeless $d$-wave gap. Note that the red broken line denotes Eq. (37).
function for $x \ll 1$, given by
\[
\psi\left(\frac{1}{2} + x\right) - \psi\left(\frac{1}{2}\right) \approx \frac{\pi^2}{2} x,
\]
(36)

$T_c/T_{c0}$ is well approximated as
\[
\frac{T_c}{T_{c0}} = 1 - \frac{\pi^2}{8c_0} \frac{\zeta}{\zeta_c},
\]
(37)
in the case of $T_{c0} - T_c \ll T_{c0}$. This is plotted by the red line, which well agrees with the Abrikosov-Gor'kov formula in the region of $\zeta/\zeta_c \ll 1$.

3. Calculation Results

3.1 Extended s-wave gap

3.1.1 Self-consistent equations

Let us briefly explain the equations for the extended s-wave gap. For $\zeta = 0$, $N(\omega)$ is evaluated as
\[
N(\omega) = \text{Im} \left\{ \frac{\omega}{\sqrt{[p + \cos(2\phi)]^2 - \omega^2}} \right\}. \quad (38)
\]

For the case with nonmagnetic impurities, we solve the self-consistent equations Eqs. (18), (21), (22), and (23) for the extended s-wave gap Eq. (28). Here, we note that $g_1$ does not vanish in general. Note also that the effect of the $g_1$ term has a significant contribution to the isotropic part, while the anisotropic part is not affected at all by the nonmagnetic impurity scattering. To obtain $g_0$ and $g_1$ with nonmagnetic impurities, we rewrite the self-consistent equations as
\[
\tilde{\omega} = \omega + \xi \frac{g_0}{g_0 - g_1}, \quad \tilde{p} = p + \xi \frac{g_1}{g_0 - g_1},
\]
(39)
where $\omega = z/\Delta$, $\tilde{\omega} = \tilde{z}/\Delta$, and $\xi = \alpha/\Delta$.

Green’s functions $g_0$ and $g_1$ are given by
\[
g_0 = -\frac{1}{\pi} \int_0^\pi d\phi \frac{\tilde{\omega}}{\sqrt{[\tilde{p} + \cos(2\phi)]^2 - \tilde{\omega}^2}},
\]
(40)
\[
g_1 = -\frac{1}{\pi} \int_0^\pi d\phi \frac{\tilde{p} + \cos(2\phi)}{\sqrt{[\tilde{p} + \cos(2\phi)]^2 - \tilde{\omega}^2}},
\]
respectively.

We solve the above equations self-consistently concerning $\tilde{\omega}$ and $\tilde{p}$. Note that for the anisotropic gap with $p = 0$, we easily find the solution of $\tilde{p} = g_1 = 0$. In this case, it is sufficient to solve the self-consistent equation concerning $\tilde{\omega}$, as in the case of the nodeless $d$-wave gap.

Throughout the calculation of the DOS, since the energy unit is set as $\Delta$, we define $\xi$ as $\alpha/\Delta$, but it is different from $\zeta$. The energy unit $\Delta$ is the solution of the gap equation and it depends on the temperature and the impurity concentration, as mentioned above. In the calculation of $R_n/R_n$ and $\rho_s/\rho_n$, we will consider explicitly the temperature dependence of $\Delta$ for a certain value of $\zeta$.

3.1.2 Results for $p = 0$

In Fig. 4(a), we show the results of the DOS for $p = 0$. First, we consider the case of $\xi = 0$. After some algebraic calculations, we obtain the DOS as
\[
N(\omega) = \frac{2}{\pi} \omega K(\omega),
\]
(41)
for $0 \leq \omega < 1$, while for $\omega > 1$, we obtain
\[
N(\omega) = \frac{2}{\pi} K\left(\frac{1}{\omega}\right),
\]
(42)
where $K(k)$ is the complete elliptic integral of the first kind defined as
\[
K(k) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - k^2 \sin^2 \theta}}.
\]
(43)

We remark that $N(\omega) = \omega$ near the Fermi level, which is essentially the same as the $d$-wave gap on the Fermi-surface curves with the center at the $\Gamma$ point. We find the logarithmic divergence at $\omega = 1$, which is characteristic of the two-dimensional case. For $\zeta > 0$, we find the almost constant DOS near the Fermi surface and the logarithmic divergence is smeared out. The behavior due to the resonant impurity scat-
teering is also the same as that for the nodal $d$-wave gap.\(^{13,14}\)

In Figs. (4)(b) and (4)(c), we show the results of the nuclear magnetic relaxation rate and superfluid density for the extended $s$-wave gap with $p = 0$. For $\zeta = 0$, owing to the effect of the nodes on the Fermi-surface curves, we observe the power-law behavior of $T_\perp^{-1} \propto T^3$ and $1 - \rho_s / \rho_n \propto T$, characteristic of the $d$-wave superconductors. When we dope nonmagnetic impurities, a finite DOS appears at the low-energy region, leading to the revival of the Korringa law, i.e., $T_\perp T_{\text{cr}}$=constant, and the $s$-wave-like constant behavior of $\rho_s / \rho_n$ at low temperatures. Note that $\rho_s / \rho_n$ is reduced from unity at $T = 0$, since part of $\rho_s$ is changed to $\rho_n$ owing to the pair-breaking effect.

### 3.1.3 Results for $p = 0.5$

In Fig. 5(a), we show the results of the DOS for $p = 0.5$. For both $0 \leq \omega < 1 - p$ and $\omega > 1 + p$, we obtain the DOS as
\[
N(\omega) = \frac{\omega}{\sqrt{(1 + \omega)^2 - p^2}} K(\omega),
\]
where $A_\omega$ is given by
\[
A_\omega = \frac{2 \omega^2}{(1 + \omega)^2 - p^2}.
\]
For $1 - p < \omega < 1 + p$, we obtain
\[
N(\omega) = \frac{\omega}{\sqrt{2\omega^2 + 2}} K\left(\frac{\sqrt{2\omega^2 + 2}}{\sqrt{2}}\right).
\]

For $0 < p < 1$, since the nodes still exist on the Fermi-surface curves, the DOS is in proportion to $\omega$ near the Fermi level. As denoted by $N(\omega) = \omega / \sqrt{1 - p^2}$ near $\omega = 0$, the slope becomes steep in comparison with that of $p = 0$. We observe two anomalies at $\omega = 1 - p$ and $1 + p$ in the DOS.

When we dope nonmagnetic impurities, both anomalies are washed out and the finite DOS appears at $\omega = 0$. However, in sharp contrast to the case of $p = 0$, we do not observe the almost constant behavior near the Fermi level. For $\xi > 0.5$, the DOS at $\omega = 0$ begins to decrease and eventually, it becomes zero at $\xi \approx 1$. Then, the gap opens at the low-energy region, since the anisotropic part is washed out by non-magnetic impurity scattering. If we increase $\xi$ further up to an unrealistically large value, the DOS approaches the form of $\omega / \sqrt{\omega^2 + p^2}$, where the size of the effective gap is $p\Delta$. Note again that such a situation is realized only mathematically, since it is necessary to dope huge amounts of impurities.

Next, we show the results of nuclear magnetic relaxation rate and superfluid density for the extended $s$-wave gap with $p = 0.5$ in Figs. 5(b) and 5(c). Since the nodes of the gap still exist on the Fermi-surface curves, we observe the power-law behavior of $T_\perp^{-1} \propto T^3$ and $1 - \rho_s / \rho_n \propto T$ for $\zeta = 0$, although the slope is different from that in the case of $p = 0$. For a small $\zeta$, owing to the finite DOS at the Fermi level, we observe the almost constant value of $R_e / R_n$ at low temperatures, as shown in Fig. 5(b). Note, however, that the flat region is apparently narrow in comparison with the case of $p = 0$, since the DOS is not constant at low-energy regions for $p = 0.5$. As shown in Fig. 5(c), this effect can be clearly found in $\rho_s / \rho_n$ for $\zeta < 0.2$, which is not considered to be constant at low temperatures, although $\rho_s / \rho_n$ at $T = 0$ is reduced from unity. When we further increase the value of $\zeta$, as mentioned in the discussion on the DOS, a finite gap begins to open near the Fermi level. This effect appears in the $s$-wave-like behavior of $R_e / R_n$ and $\rho_s / \rho_n$ for large $\zeta$. In particular, for $\zeta = 1$, we observe a large coherence peak just below $T_c$.

### 3.1.4 Results for $p = 1$

In Fig. 6(a), we show the results of the DOS for $p = 1$. For $\xi = 0$, we obtain the analytic form of the DOS. In the region of $0 \leq \omega < 2$, we obtain
\[
N(\omega) = \frac{\sqrt{\omega}}{\pi} K\left(\frac{\sqrt{\omega + 2}}{2}\right),
\]
while for $\omega > 2$, the DOS is given by
\[
N(\omega) = \frac{2}{\pi} \sqrt{\frac{\omega}{\omega + 2}} K\left(\frac{2}{\sqrt{\omega + 2}}\right).
\]
Note that near the Fermi level, $N(\omega)$ is proportional to $\sqrt{\omega}$ and the logarithmic anomaly appears at $\omega = 2$.

When we increase the value of $\xi$, the logarithmic anomaly at $\omega = 2$ is smeared by the nonmagnetic impurity scattering. Note that for $p = 1$, the finite DOS does not appear at the Fermi level owing to the impurity scattering, in sharp contrast to the case of $p = 0$. Rather, the finite gap begins to open near the Fermi level. The size of the gap monotonically increases with the increase in $\xi$. As expected from the curve for $\xi = 10$ in Fig. 6(a), the DOS asymptotically approaches $\omega / \sqrt{\omega^2 - 1}$ with the effective gap of $\Delta$ for $\xi \to \infty$.

In Figs. 6(b) and 6(c), we show the results of $R_s(T)/R_n$ and $\rho_s(T)/\rho_n$ for $p = 1$. Note that the DOS near the Fermi level behaves as $N(\omega) \propto \sqrt{\omega}$ for $p = 1$, since the node exists only at $\phi = \pi/2$ on the Fermi-surface curve. For $\xi = 0$, owing to this low-energy behavior of the DOS, we find $(T_1 T)^{-1} \propto T$ and $1 - \rho_s/\rho_n \propto \sqrt{T}$. When $\xi$ increases with impurity doping, as mentioned in the discussion on the DOS, a finite gap immediately opens at the Fermi level. Thus, even for a small $\xi$, we observe the reappearance of the $s$-wave-like behavior for both $(T_1 T)^{-1}$ and $\rho_s/\rho_n$.

### 3.1.5 Results for $p = 1.5$

In Fig. 7(a), the results of the DOS for $p = 1.5$ are shown. For $\xi = 0$, in the region of $0 < \omega < p - 1$, we easily obtain $N(\omega) = 0$ owing to the gap with the magnitude of $p - 1$. For $p - 1 < \omega < p + 1$, the DOS is given by Eq. (46), while for $\omega > 1 + p$, the DOS is given by Eq. (44). The gap with the size of $(p - 1)\Delta$ opens near the Fermi level, since the anisotropic part is relatively smaller than the isotropic gap. The anomaly at $\omega = p - 1$ appears as the gap edge, while the logarithmic divergence is found at $\omega = p + 1$. For $\xi > 0$, the anomalies are smeared out and the gap edge is found to be shifted toward the
isotropic gap edge $\omega = p = 1.5$. Since the low-energy part of the DOS is not so affected by the impurity scattering, the impurity effect is expected to be less sensitive for $p > 1$.

In Figs. 7(b) and 7(c), we depict $R_s/R_n$ and $\rho_s/\rho_n$, respectively, for $p = 1.5$. As already emphasized in the discussion on the DOS for $p > 1$, the gap with the magnitude of $(p-1)\Delta$ exists even at $\zeta = 0$. With impurity doping, the anisotropic part of the gap is gradually washed out and then the gap size is changed to $p\Delta$ for a large $\zeta$. Namely, as long as we concentrate on the low-energy region, we always expect the simple $s$-wave-like behavior in physical quantities. In fact, for both $R_s/R_n$ and $\rho_s/\rho_n$, the changes in the temperature dependence are not significant when we increase $\zeta$. In this sense, the temperature dependence is insensitive to the nonmagnetic impurity for $p > 1$.

### 3.2 Nodeless $d$-wave gap

Next, we move onto the nodeless $d$-wave gap function. We solve the self-consistent equations Eqs. (18), (21), (22), and (23) for the nodeless $d$-wave gap eq. (29). Owing to the property of the sign change among the different pocketlike Fermi-surface curves, $g_1 = 0$ is the trivial solution. Thus, we solve the self-consistent equation for $\bar{\omega}$ as

$$\bar{\omega} = \omega - \xi \sqrt{1 - \omega^2}/\bar{\omega}. \quad (49)$$

The DOS $N(\omega)$ is given by

$$N(\omega) = -\operatorname{Im} g_0 = \operatorname{Im} \sqrt{1 - \omega^2}/\bar{\omega}. \quad (50)$$

Note that in general, $\bar{\omega}$ becomes a complex number.

In Fig. 8(a), we show $N(\omega)$ for the nodeless $d$-wave gap for various values of $\xi$. For the case without impurities ($\xi = 0$), the DOS is zero for $0 < \omega < 1$, while for $\omega > 1$, we obtain

$$N(\omega) = \frac{\omega}{\sqrt{\omega^2 - 1}}. \quad (51)$$

which is the same as that for the simple $s$-wave case. For $\xi > 0$, we find the finite DOS due to resonant scattering near the Fermi level. It is well known that the finite DOS appears at the Fermi level when we include the impurity scattering in the self-consistent $T$-matrix approximation in the case of the nodal gap. The present calculations indicate that even for the nodeless $d$-wave gap, a finite DOS appears at the Fermi level. The important point is the phase change of the gap on the Fermi-surface curve, not the existence of the node.

When we increase the impurity concentration, the value of $\xi$ increases. To clarify the change in the gap due to the impurity, we evaluate the values of the gap edges, $\omega_1$ and $\omega_2$. First, we transform the self-consistent equation concerning $\bar{\omega}$ to the quartic equation in terms of $g_0$. Then, we identify the condition to obtain real solutions of $g_0$, corresponding to the region of $N(\omega) = 0$ with the gap edges. Note that $\omega_1 = 0$ and $\omega_2 = 1$ for $\xi = 0$. As observed in the inset of Fig. 8(a), the gap size defined by $\omega_2 - \omega_1$ becomes zero at $\xi = 0.38$. It is emphasized that even if the gap disappears in the DOS, the superconductivity is not perfectly destroyed, leading to the gapless superconductivity. The impurity-induced gapless superconductivity has been found in the $s$-wave gap with paramagnetic impurities. In this sense, the present result provides another example of gapless superconductivity.

Next, we show the calculated results on nuclear magnetic relaxation rate and superfluid density. In Fig. 8(b), we show the temperature dependence of $R_s/R_n$. In the case of $\zeta = 0$, we introduce a cut-off by hand to avoid the divergence in the DOS at $\omega = 1$, since it brings about the divergence in $R_s/R_n$ just below $T = T_c$. In the experiments for actual superconducting materials, it appears as the coherence peak just below $T = T_c$. We also observe the exponential decay in $R_s/R_n$ at low temperatures. These properties are characteristic of the $s$-wave superconductivity.

Now, we consider the impurity effect on $R_s/R_n$. For $\zeta > 0$, owing to the appearance of the finite DOS near the Fermi level, we observe an almost constant behavior in $R_s/R_n$ at low temperatures, suggesting the revival of the Korringa law at low temperatures. This is one of the common characteristic issues for the nonmagnetic impurity effect in unconventional...
superconductivity. $R_s/R_n$ totally increases with the increase of $\zeta$. Here, we note again that there exists an islandlike finite DOS near the Fermi level. When $\zeta$ is smaller than the value at which the gap disappears, the gap exists in the region of $0 < \omega < 1$. Thus, $R_s/R_n$ slightly decreases with the increase in $T$ in the low-temperature region, while for $T$ larger than the lower gap edge, $R_s/R_n$ increases. In the dirty nodeless $d$-wave superconductors, we expect such nonmonotonic temperature dependence in $R_s/R_n$, although this behavior is not so significant.

Now, we turn our attention to the superfluid density in Fig. 8(c). For $\zeta = 0$, we find that the temperature dependence of $\rho_s/\rho_n$ is similar to that of the simple $s$-wave superconductor. For $T < 0.3 T_c$, $\rho_s/\rho_n$ is almost unity. When $T$ is further increased, $\rho_s/\rho_n$ begins to decrease and eventually becomes zero at $T = T_c$. For $\zeta > 0$, owing to the appearance of the finite constant DOS near the Fermi level, the magnitude of $\rho_s/\rho_n$ decreases, but we still observe the almost constant value of $\rho_s/\rho_n$ at low temperatures. In this sense, for both clean and dirty cases, the $s$-wave-like temperature dependence of $\rho_s/\rho_n$ is expected in the nodeless $d$-wave superconductivity, when we normalize $\rho_s$ by its value at $T = 0$.

For small $\zeta$, owing to the existence of the gap in the region of $0 < \omega < 1$, we also find the nonmonotonic temperature dependence of $\rho_s/\rho_n$. The origin is considered to be the same as that in $R_s/R_n$. In principle, it is possible to distinguish $\rho_s/\rho_n$ between clean and dirty cases by the temperature dependence, but it seems to be difficult to detect such subtle changes at low temperatures in actual experiments. It is more realistic to check whether the almost constant part appears or not in $R_s/R_n$ at sufficiently low temperatures.

4. Discussion and Summary

In this paper, in order to obtain some hints to clarify the node structure in BiS$_2$-based layered superconductors, we have investigated the nonmagnetic impurity effect within the self-consistent $T$-matrix approximation. We have performed the calculations of the DOS, nuclear magnetic relaxation rate, and superfluid density in the superconducting state.

We have assumed two cases, namely, the extended $s$- and $d$-wave gap functions. Note that in the present material, we have considered the disconnected pocketlike Fermi-surface curves at around $k = (\pm \pi, 0)$ and $(0, \pm \pi)$. Owing to this structure, there exists no node for the $d$-wave gap, since the nodal lines along the lines of $k_z = \pm k_y$ do not cross the Fermi-surface curves. However, a sign change occurs between the gaps on the pocketlike Fermi-surface curves. Thus, nonmagnetic impurities affect seriously the nodeless $d$-wave gap.

On the other hand, for the extended $s$-wave gap, nodes appear on the Fermi-surface curves. In particular, for the case without the isotropic part, the nonmagnetic impurity effect is essentially the same as that in the nodal $d$-wave superconductors. In this work, we have investigated the impurity effects for all the cases with the isotropic part.

We have considered two distinct experimental results concerning the gap nodes on the Fermi-surface curves. When we consider the extended $s$-wave gap with $p = 0$, the node structure is consistent with the ARPES results. Moreover, if BiS$_2$-based layered superconductors are assumed to be dirty owing to the existence of randomness and/or lattice mismatch, even if nonmagnetic impurities are not explicitly doped, the resonant scattering effect induces the finite DOS near the Fermi level, leading to the $s$-wave-like behavior in physical quantities at low temperatures. For instance, the temperature dependence of $\rho_s/\rho_n$ in the low-temperature region is similar to that of the $s$-wave case, if we normalize $\rho_s/\rho_n$ by its value at $T = 0$. In order to confirm the above idea of the dirty nodal extended $s$-wave superconductivity, we propose the measurement of the temperature dependence of $T_c$ in BiS$_2$-based layered superconductors. The detection of the Korringa-like behavior at low temperatures will be a test of the dirty nodal extended $s$-wave superconductivity.

Now, let us consider another possibility that the ARPES experiments do not indicate the existence of the nodes of the gap. If we accept this possibility, readers may be inclined to think that the simple $s$-wave superconductivity is the unique solution. However, as shown in this paper, a possibility of the nodeless $d$-wave superconductivity cannot be excluded in the BiS$_2$-based layered superconductors, since the temperature dependence of physical quantities in the nodeless $d$-wave superconductor seems to be the same as that in the conventional $s$-wave one in the case of the disconnected Fermi-surface structure for both clean and dirty cases.

For the confirmation of the nodeless $d$-wave superconductivity, it is necessary to perform an experiment in which the sign change between the gaps on the Fermi-surface curves is detected. Thus, the non-magnetic impurity effect can be the key experiment. The finite DOS appears due to a small amount of the nonmagnetic impurity or randomness in the nodeless $d$-wave superconductor. Namely, the detection of the Korringa-like behavior at low temperatures in the dirty case will be the test of the nodeless $d$-wave superconductivity, provided that we ignore the existence of the nodes on the gap. Note that the nonmonotonic temperature dependence in $\rho_s/\rho_n$ and $R_s/R_n$ is interesting from a theoretical viewpoint, but in actual experiments, it seems difficult to detect such subtle changes in the temperature dependence of physical quantities.

Concerning the mechanism of superconductivity in BiS$_2$-based layered materials, it has not been confirmed yet, but the gap symmetry has been vigorously investigated. It was suggested by some theoretical researchers that the gap function in BiS$_2$-based layered superconductors could be explained by extended $s$-wave, $d$-wave, triplet or other unconventional pairing scenarios. On the other hand, a possibility of $s$-wave pairing due to electron-phonon interaction has been discussed. The appearance of superconductivity has been discussed from various theoretical viewpoints on the basis of this two-orbital Hubbard model. The effects of electron-phonon interaction and spin-orbit coupling have also been discussed. The relationship between the characteristic change of the Fermi-surface topology and the symmetry of the superconducting gap function has been pointed out in previous works. It has been discussed that the impurity effect can be a probe of the pairing symmetry in BiS$_2$-based layered superconductors.

The gap anisotropy in BiS$_2$-based layered materials has been discussed in the context of multiorbital superconductivity. In this paper, we have not included orbital degrees of freedom in the impurity scattering, but the gap state is considered to be affected, more or less, by the nonmagnetic impurity scattering even in multiorbital superconductors, when the gap
has nodes on the Fermi-surface curve or the sign is reversed between the gaps on the different Fermi-surface curves. Thus, we believe that the impurity effect is also useful for the determination of the gap symmetry of multiorbital superconductors, although it is necessary to develop carefully the discussion on the nonmagnetic impurity scattering in multi-orbital superconductors from a quantitative viewpoint.

Finally, we comment on the nodeless $d$-wave superconductivity in other systems. The nodeless $d$-wave gap was discussed in a quasi-one-dimension al organic superconductor.\(^{38}\) A possible high-$T_c$ mechanism due to spin fluctuations was proposed in a system with a Fermi-surface pocket.\(^{39}\) In electron-doped high-$T_c$ cuprates, nodeless $d$-wave superconductivity was also examined.\(^{40,41}\) Quite recently, the nodeless $d$-wave gap has been considered in monolayer FeSe.\(^{42}\) Here, we do not mention the superconducting mechanism of iron-based superconductors, since it is beyond the scope of this paper, but we refer to the previous papers, Refs. 43 and 44, in which the impurity effects in the $s_d$-wave superconducting state with a multiband structure were discussed.

In summary, to reconcile the existence of the gap nodes on the Fermi-surface curves and the $s$-wave-like temperature dependence of physical quantities in Bi$_2$Se$_3$-based layered superconductors, we have proposed the dirty nodal extended $s$-wave gap without an isotropic part. Note that in this scenario, it is necessary to assume that the sample becomes dirty owing to the existence of randomness and/or lattice mismatch, even if nonmagnetic impurities are not explicitly doped. Provided that the existence of the node can be ignored, we have suggested a nontrivial possibility of nodeless $d$-wave superconductivity for both clean and dirty cases, in addition to the conventional $s$-wave superconductivity. In both scenarios, a key experiment will be the measurement of $T_c^{-1}$ in Bi$_2$Se$_3$-based layered superconductors.

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1) Y. Mizuguchi, S. Demura, K. Deguchi, Y. Takano, H. Fujihisa, Y. Gotoh, H. Izawa, and O. Miura, J. Phys. Soc. Jpn. 81, 114725 (2012).

2) Y. Mizuguchi, H. Fujihisa, Y. Gotoh, K. Suzuki, H. Usui, K. Karoko, S. Demura, Y. Takano, H. Izawa, and O. Miura, Phys. Rev. B 86, 220510(R) (2012).

3) Y. Mizuguchi, J. Phys. Chem. Solid 84, 34 (2015).

4) Y. Mizuguchi, T. Hiroi, J. Kajitani, H. Takatsu, H. Kadowaki, and O. Miura, J. Phys. Soc. Jpn. 83, 053704 (2014).

5) P. Shrutí Srivastava and S. Patnaik, J. Phys.: Condens. Matter 25, 312202 (2013).

6) G. Lamura, T. Shiroma, P. Bonfa, S. Sanna, R. DeRenzi, C. Baines, H. Luetkens, J. Kajitani, Y. Mizuguchi, O. Miura, K. Deguchi, S. Demura, Y. Takano, and M. Patti, Phys. Rev. B 88, 180509 (2013).

7) L. Jiao, Z. Weng, J. Liu, J. Zhang, G. Pang, C. Guo, F. Gao, X. Zhu, H.-W. Wen, and H. Q. Yuan, J. Phys.: Condens. Matter 27, 225701 (2015).

8) T. Yamashita, Y. Tokiwa, D. Terazawa, M. Nagao, S. Watauchi, I. Tanaka, T. Terashima, and Y. Matsuda, J. Phys. Soc. Jpn. 85, 073707 (2016).

9) Y. Ota, K. Okazaki, H. Q. Yamamoto, T. Yamamoto, S. Watanabe, C. Chen, M. Nagao, S. Watauchi, I. Tanaka, Y. Takano, and S. Shin, Phys. Rev. Lett. 118, 167002 (2017).

10) S. Schmitt-Rink, K. Miyake, and C. M. Varma, Phys. Rev. Lett. 57, 2575 (1986).

11) P. J. Hirschfeld, D. Vollhardt, and P. Wölfle, Solid State Commun. 59, 111 (1986).

12) P. J. Hirschfeld, P. Wölfle, and D. Einzel, Phys. Rev. B 37, 83 (1988) 83.

13) T. Hotta, J. Phys. Soc. Jpn. 62, 274 (1993).

14) T. Hotta, Phys. Rev. B 52, 13041 (1995).

15) A. V. Balatsky, I. Vekhter, and J.-X. Zhu, Rev. Mod. Phys. 78, 373 (2006).

16) P. B. Allen and B. Mitrović, Solid State Phys. 37, 1 (1982).

17) J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. 108, 1175 (1957).

18) H. Usui, K. Suzuki, and K. Kuroki, Phys. Rev. B 86, 220501(R) (2012).

19) K. Suzuki, H. Usui, and K. Kuroki, Phys. Procedia 45, 21 (2013).

20) T. Agatsuma and T. Hotta, J. Magn. Magn. Mater. 400, 73 (2016).

21) A. A. Abrikosov and L. P. Gor’kov, Zh. Eksp. Teor. Fiz. 39, 1781 (1961). [Sov. Phys.-JETP 12, 1243 (1961)].

22) T. Tsuneto, Prog. Theor. Phys. 28, 857 (1962).

23) T. Hotta, J. Phys. Soc. Jpn. 64, 2923 (1995).

24) K. Maki, Superconductivity, eds. R. D. Parks (Dekker, New York, 1969) p. 1035.

25) G. Martins, A. Moreo, and E. Dagotto, Phys. Rev. B 87, 081102(R) (2013).

26) T. Zhou and Z. D. Wang, J. Supercond. Nov. Magn. 26, 2735 (2013).

27) Y. Yang, W. S. Wang, Y. Y. Xiang, Z. Z. Li, and Q. H. Wang, Phys. Rev. B 88, 094519 (2013).

28) Y. Liang, X. Wu, W.-F. Tsai, and J. Hu, Front. Phys. 9, 194 (2014).

29) X. Wu, J. Yuan, Y. Liang, H. Fan, and J. Hu, Europhys. Lett. 108, 27006 (2014).

30) T. Yildirim, Phys. Rev. B 87, 205006(R) (2013).

31) X. Wan, H.-C. Ding, S. Y. Savrasov, and C.-G. Duan, Phys. Rev. B 87, 115124 (2013).

32) B. Li, Z. W. Xing, and Q. G. Huang, Europhys. Lett. 101, 47002 (2013).

33) M. Morice, R. Akashi, T. Koretsune, S. S. Saxena, and R. Arita, Phys. Rev. B 95, 180505 (2017).

34) Y. Gao, T. Zhou, H. Huang, P. Tong, and Q.-H. Wang, Phys. Rev. B 90, 054518 (2014).

35) S. L. Liu, J. Supercond. Novel Mag. 26, 3411 (2013).

36) M. A. Griffith, T. O. Puel, M. A. Continentino, and G. B. Martins, J. Phys.: Condens. Matter 29, 305601 (2017).

37) K. Suzuki, H. Usui, K. Kuroki, and H. Ikeda, Phys. Rev. B 96, 024513 (2017).

38) H. Shimahara, Phys. Rev. B 61, R14936 (2000).

39) K. Kuroki and R. Arita, Phys. Rev. B 64, 024501 (2001).

40) T. Das, R. S. Markiewicz, and A. Bansil, Phys. Rev. Lett. 98, 197004 (2007).

41) Q. Yuan, Physica B 403, 1175 (2008).

42) D. F. Agterberg, T. Shishido, J. O’Halloran, P. M. R. Brydon, and M. Weinert, Phys. Rev. Lett. 119, 267001 (2017).

43) Y. Senga and H. Kontani, J. Phys. Soc. Jpn. 77, 113710 (2008).

44) Y. Bang, H. Y. Choi, and H. Won, Phys. Rev. B 79, 054529 (2009).