Visualizing Hierarchical Social Networks

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Abstract
The authors propose a novel technique for the visualization of networks that contain a hierarchical structure: networks in which certain nodes and groups of nodes can be classified through a relation of precedence. Networks with a hierarchical structure frequently arise in sociology and various other disciplines, but the existing methods for visualizing such networks leave much to be desired. The method developed in this work builds on the tradition of visualization in social network analysis; it aims to simultaneously represent the positions of different nodes and the relationships between groups containing the nodes in the network. As such, the proposed visualization method facilitates theoretical and empirical analysis of social structures by algorithmically combining information from the underlying network with the information from the hierarchical structure of the network. The authors illustrate the proposed method with social networks examined through cohesive blocking and k-core decomposition.

Keywords
visualization, social networks, cohesive blocking, financial networks, hierarchical networks

Networks that contain a hierarchical structure—networks in which certain nodes and groups of nodes can be classified through a relation of precedence—are ubiquitous in structures observed in various fields, including domains as different as firm ownership relations, scientific collaboration, and protein interaction chains (Clauset, Moore, and Newman 2008; Girvan and Newman 2002; Grabowski and Kosiński 2004; López, Mendes, and Sanjuán 2002; Mani and Moody 2014; Newman 2006). In this article, we contribute to social network analysis and data visualization in sociology (Correa and Ma 2011; Freeman 2000, 2005; Healy and Moody 2014; Krempel 2009; Moody, McFarland, and BenderdeMoll 2005) through a novel algorithm for visualizing hierarchical networks. In developing this method of visualization, we aim to simultaneously represent the positions of different nodes and the relationships between groups containing the nodes in the network. We illustrate, through substantive examples, that the algorithm we develop is an effective tool in the exploration and explanation of patterns (Brandes, Kenis, and Raab 2006; Crosby 1997; Tufte 2001). Visualization has a long tradition in social network analysis (Freeman 2000), going back to Moreno’s (1932, 1934) work during the 1930s and his use of sociograms to represent connections among various actors. Over the years, visualization has developed in tandem with social network analysis. Some of the most illuminating applications of visualization, such as Kadushin’s (1974) work on intellectual elites and Freeman and White’s (1993) use of Galois lattices, were concerned with simultaneous representation of social positions and social groups. Second, as we illustrate below, the simultaneous visualization of a social network and the hierarchical structure of the network is an effective tool in capturing the complex interplay between social positions, groups, and the relationships among groups. Third, at the most practical level, we aim to understand the structure of financial networks around the world. The method we employ to examine groups and positions of different financial actors is cohesive blocking (Moody and White 2003; White and Harary 2001), which provides a rich set of information on the

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structural cohesion and embeddedness patterns in financial networks. In our experience, visualization is an indispensable tool in understanding and explaining the multidimensional structure of networks revealed by the cohesive blocking analysis.

The technique we develop in this article builds on a voluminous literature in computer science and social network analysis (Ahajian, El Haddad, and Badir 2018; Holten 2006; Nikolaev, Razib, and Kucheriya 2015). Harel (1988) is one of the first works to formalize the simultaneous representation of a relation between nodes and structural relations between sets of nodes in a graph. Since then, various authors have offered solutions such as elastic hierarchies (Storey and Muller 1995) and hierarchical multiperspective views (Zhao, McGuffin, and Chignell 2005) to this visualization problem. A recent influential approach is hierarchical edge bundling (Holten 2006), which bends and groups adjacency relations by inclusion relations among sets of nodes in a graph. Edge bundling in radial layouts is particularly effective in representing large graphs, and it has already found sophisticated application in social network analysis (Crnovrsanin et al. 2014). Our contribution differs from these approaches in its emphasis on representing inclusion and exclusion relations that are essential to understanding the structure of a social network without losing sight of the overall distribution of relations among nodes. As we show, the method we offer is particularly effective in visualizing hierarchical graphs produced by techniques such as k-core and k-shell decomposition (Alvarez-Hamelin et al. 2005; Carmi et al. 2007; Miorandi and de Pellegrini 2010), community detection (Girvan and Newman 2002), and cohesive blocking (Moody and White 2003).

Throughout this article, we use several conventions to refer to the various objects under consideration. We define a graph \( N \) as the unordered pair \((V, E)\) where \( V \) is the set of vertices and \( E \) is the set of edges in \( N \). Each edge \( e \in E \) belongs to two-element subsets of \( V \). We denote the edge from vertex \( i \) to vertex \( j \) as \( e_{ij} \). In social network analysis, a graph \( N \) is often discussed by using the terminology of network, nodes (e.g., actors), and ties (e.g., social relations between actors) (Diestel 2005; Wasserman and Faust 1994). We predominantly use the social network terminology. A trivial network is a network consisting of a single node. Take any two nodes \( v_i \) and \( v_j \) in a network \( N \). An alternating sequence of nodes and edges in \( N \) connecting \( v_i \) and \( v_j \) constitutes a walk. When the nodes and edges are distinct, it is called a path. A cycle is a walk where the beginning and end nodes are the same. Disjoint paths in \( N \) are paths where no edges and no nodes except \( v_i \) and \( v_j \) are common. A cutset \( C \) of \( N \) is defined as the set of nodes that separates \( N \) into two disjoint networks \( X \) and \( \bar{X} \), which are called the cuts of \( N \) induced by \( C \). Following Harary (1969), we define \( k \)-connectivity of a network \( N \) as the minimum number of nodes whose removal will result in a disconnected or trivial network. A tree is a network that does not contain any cycles and where each pair of nodes are connected by a unique path.

In analyzing hierarchical networks, we restrict our focus to hierarchies in which the relationship is containment. Thus, the nodes of the hierarchy (i.e., supernodes) in a branch are distinct. For two nodes that are related to each other, one of them is a strict subset of the other. We refer to the primary network as \( G \) and to an individual node and the set of nodes as \( v \) and \( V \), respectively. We refer to the associated hierarchy as \( T \), with nodes (or supernodes) denoted by \( t \). Each node in \( G \) belongs to some supernode in \( T \); the membership of a node \( v \) is given by \( \tau(v) \). The members of \( t \) are the nodes in \( G \) belonging to \( t \), that is, all \( v \)'s for which \( \tau(v) = t \); the children of \( t \) are the other nodes in \( T \) that are direct descendants of \( t \).

This article is structured as follows. We first describe the cohesive blocking technique, a network analysis method for which this visualization is particularly helpful. The next section describes the layout algorithm in detail. In the following section we present several examples that illustrate the visualization technique at work. The final section concludes the article with a discussion of the uses of this technique and avenues for further research.

**Cohesive Blocking**

Social structure as the patterned “crystallization of relationships”—and, once crystallized, a sui generis entity shaping social action—is a fundamental notion in sociology (Durkheim [1893] 1964; Giddens 1979; Lizardo 2010; Martin 2009:1–2; Simmel [1908] 1950:94–95). Cohesive blocking (Moody and White 2003) is a technique to formally analyze social structures through social network data. This technique builds on two concepts with deep roots in sociological thought, social cohesion—the binding of social actors into a collectivity (Durkheim 1964; Fantasia 1988; Hechter 1987; Meyer and Kimeldorf 2015; Simmel [1922] 1964)—and embeddedness (Granovetter 1985; Zukin and DiMaggio 1990). Classical sociologists such as Durkheim (1964), Weber (1978), and Tönnies ([1887] 1957) emphasized the structural, ideational, and affective elements in their approaches to social cohesion. The problem is that these different dimensions of social cohesion are analytically separate and can at times exercise effects in opposite directions. Instead of such an approach, Moody and White (2003) focused on structural cohesion by defining it through the relational togetherness of a group. Apart from gains in analytical precision, such a focus also enables an effective correspondence between the substantive sociological idea (i.e., relational togetherness) and the mathematical formalism used to examine social cohesion. The key to this effective correspondence is Menger’s theorem.

Menger’s theorem states that there is a crucial relationship between the number of nodes separating any two nodes \( s \) and \( r \) in a network and the number of disjoint \( s \)-\( t \) paths in \( N \) (Bondy and Murty 1976). Whitney (1932a, 1932b) offered a criterion for connecting the notion of \( k \)-connectivity to the number of disjoint paths in a network, which can be generalized to the
whole network. Formally, \( N \) is \( k \)-connected if and only if it contains \( k \) independent paths between any two nodes (Diestel 2005). Now, denote any group of nodes connected by at least \( k \) paths in \( N \) as \( k \)-connected and a \( k \)-component of \( N \). Menger’s theorem and its extensions imply that a \( k \)-component is a component with no cutset fewer than \( k \) nodes (Diestel 2005; Harary 1969; Moody and White 2003).

The mathematical formalism of graph connectivity, encapsulated in Menger’s theorem and its extensions by various authors, offers a rich opportunity to express the concept of social cohesion with reference to the cohesion of a social structure. Namely, as Moody and White (2003:109) suggested, the concept of structural cohesion can be defined with reference to the minimum number of nodes that constitute a cutset and the minimum number of “independent relational paths” connecting any pair of nodes in a social structure. Then, each set of nodes with \( k \)-connectivity in a network, a \( k \)-component, constitutes a group with a defined degree of structural cohesion. Furthermore, each \( k \)-component in a network may contain other groups that possess their own degrees of cohesion. Moody and White called such groups, each of which possesses a measurable degree of cohesion and embedded in other groups with varying degrees of cohesion, the cohesive blocks of a network.

It can be shown that the nestedness of such groups, the embeddedness of groups in a network (Granovetter 1985), generates a tree, in which each node is a cohesive block consisting of nodes held together through \( k \) independent relational paths and in which the children of the block are cohesive groups nested in that block.1 Nestedness follows from the fact that the components generated by a \( k \)-cutset might have stronger cohesion and thus higher \( k \)-connectivity. The tree structure generated by the nestedness relationship between cohesive blocks, \( T \), represents the hierarchy that we aim to visualize. The embeddedness of a cohesive block is defined as the depth level of the block in \( T \).

The tree structure obtained by the cohesive blocking technique can be illustrated with the karate club social network of Zachary (1977). The karate club social network data come from Zachary’s ethnographic research on the interactions between members of a voluntary karate club. In this network, each tie represents two members who interact on a repeated basis outside the club itself (see Figure 1). In the original study, Zachary studied the conflict within the club between the karate instructor (node 1) and the club president (node 34) by assigning a numerical value (“capacity”) to each tie on the basis of the number of distinct contexts in which two

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1For details, refer to Moody and White (2003:123–24), who offered an algorithm to generate all the cohesive blocks in a network. Their algorithm extends the works of Kanevsky (1993), who combined ideas from Even and Tarjan (1975) and Provan and Shier (1996) for finding all minimum (\( s-t \))-cutsets in a network.
underlying network data with the information from the hierarchy tree.

**Layout Algorithm**

Our goal is to simultaneously visualize both the hierarchical structure of $T$ as well as the structure of the underlying network $G$. Although many algorithms exist for laying out networks (Fruchterman and Reingold 1991; Jacoby et al. 2014; Kamada and Kawai 1989) and for laying out hierarchies (Abello 2004; Abello, Kobourov, and Yusufov 2004; Eades and Feng 1996; Frishman and Tal 2004; Galili 2015; Johnson and Shneiderman 1991; Urbanek 2008), our review of the literature suggests no standard algorithms for simultaneous visualization of both aspects of a network (Chen, Härdle, and Unwin 2008; Di Giacomo and Lubiw 2015; Tamassia 2013). The hierarchical edge bundling approach of Holten (2006), despite its utility, does not meet one of our main requirements: representing the inclusion-exclusion patterns at different levels of the hierarchy. In addition, we discovered no standard ways to extend existing algorithms to fulfill this goal. In particular, because a key goal of our visualization is to ensure that the layout of the network respects the hierarchical boundaries, we needed an algorithm that separated distinct components of the hierarchy and ensured that nodes stayed inside hierarchical boundaries. We found no standard algorithm that could impose such geometric constraints. In fact, many layout algorithms impose no particular constraint on the drawing area at all. The approach we ultimately adopt does, however, extend prior ideas in this field, particularly the boundary-repulsive force of Davidson and Harel (1996).

Under certain circumstances, these two layout goals, representing the hierarchical structure as well as the underlying network, can conflict. A hierarchy tree in which nodes are related to one another through subset or superset relationship can be directly represented on the Cartesian plane. However, nodes of the underlying network $G$ might belong to multiple supernodes in $T$. For instance, in cohesive blocking, the nodes in a cutset end up on both sides of the cut induced by the cutset (Harary 1969). Sociologically, the overlaps between the different supernodes of the hierarchy tree are what gives cohesive blocking its relational focus (Emirbayer 1997) because it enables the identification of “groups in terms of sets of relationships” rather than simply “sets of individuals” (Moody and White 2003:111). These nodes, which act as articulation points between the hierarchy branches, have important substantive implications in other disciplines as well. For example, they are crucial in the study of transportation networks and reliability analysis (Gibbons 1985:58; Jensen and Bellmore 1969:171–72; Nagamochi and Ibaraki 2008).

In these cases, we opt to duplicate nodes that fall into multiple blocks, placing a copy into each block along with that node’s connections to the rest of the network. Admittedly, we are motivated primarily by the cohesive blocking technique, where the cutset nodes are key determinants of structural cohesion (Moody and White 2003). However, we also believe that this approach is sufficiently general because it emphasizes that such nodes play an important bridging role in the network, helping bind otherwise distant branches of the hierarchy. Nonetheless, it should be noted that this approach would break down for networks where such nodes are the rule rather than the exception.

We combine three techniques to simultaneously represent the hierarchical structure as well as the underlying network. First, we use a standard hierarchy visualization algorithm—the “squarified treemap” algorithm (Bruls, Huizing, and Van Wijk 2000)—to partition the visual area for the hierarchy. Second, we use a nesting offset (Johnson and Shneiderman 1991), geometric subsets, and shading to represent hierarchy. Third, we construct and optimize an energy function to lay out network nodes within these partitions. The combination of these techniques produces graphs that reveal how network structure interacts with the hierarchical structure. Below, we discuss the algorithmic details of our approach.

**Hierarchy Layout: Treemaps**

Visualizing complex and multidimensional information such as a hierarchical network requires close attention to several issues. First, space should be used efficiently, which implies that empty areas of the visual representation should be minimized without compromising informational accuracy. Second, the chosen method should offer means to represent the order relationship among the different components of the network in an unequivocal manner. Third, it is often desirable to represent the importance of different components (e.g., size of a network component) in a way that does not hinder the representation of the hierarchical structure.

Meeting all the three requirements is surprisingly hard. The most common way of representing trees as a directed graph growing in a single direction starting from a root node is space inefficient, as can be seen in Figure 2. Furthermore, it is difficult to parse when the tree is large.

Treemaps (Johnson and Shneiderman 1991; Shneiderman 1992; Shneiderman and Wattenberg 2001; Wood and Dykes 2008) offer a solution that meets all of these requirements. Since their introduction (Johnson and Shneiderman 1991), treemaps have become a standard tool for illustrating hierarchically structured information (e.g., file sizes within a hierarchy such as folders on a hard drive), and the popularity of the original algorithm has generated a family of algorithms

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2Moody and White (2003) visualized cohesive blocks by first laying out nodes and then drawing a border around nodes in the same cohesive block. Although effective in visualizing small networks manually, their approach has two shortcomings when it comes to visualizing complex networks. First, standard layout algorithms cannot guarantee that vertices in the same block stick together. Second, the overlaps between multiple blocks create considerable visual noise in most networks with more than 20 nodes.
inspired by the treemap approach. This family of algorithms takes as input a tree and recursively partitions a visual area into regions whose areas are proportional to the size of their corresponding subtrees. The key idea is to map “the display space into a collection of rectangular bounding boxes representing the tree structure” (Johnson and Shneiderman 1991:284). The most important advantage of the treemap family of algorithms is that they are space-filling: these algorithms use all the available two-dimensional space (Shneiderman 1992).

The most basic version of the treemap family of algorithms partitions areas into rectangles. However, such an approach results in rectangles with large aspect ratios: a common problem is the representation of various nodes in the tree as rectangles that are too thin. The “squarified treemap” variant (Bruls et al. 2000) avoids this problem. Squarified treemap algorithm attempts to make the rectangles as square as possible. The advantage of the squarified treemap algorithm is that the bounded rectangles representing tree nodes are much easier to parse and compare.

To represent the hierarchy and thus nestedness of a child supernode in parent supernodes, we go back to the method of nested treemaps in the original study of Johnson and Shneiderman (1991). Nested treemaps represent hierarchy by placing nodes into regions corresponding to the deepest level of the hierarchy in which the nodes occur. A nesting offset and geometric subset relationship become the instruments to mark hierarchy and separation between nodes: deeper levels of the hierarchy are laid out strictly inside the boundaries of their parents. Furthermore, shading of deeper blocks can emphasize hierarchical depth. Although a nesting offset and shading are not strictly necessary for small networks, they become crucial tools as the network grows in size and complexity. Thus, we combine the squarified treemap algorithm with nesting and shading in our visualization algorithm; we place duplicated nodes once at the deepest level of each branch in which they occur.

One disadvantage of the nested offset approach is that nodes deep in the hierarchy have less space assigned to them than nodes near the root. In extreme cases, the offset can reduce the space available to position these nodes to zero. As such we augment the nested treemap algorithm by increasing the “weight” of deeper nodes. By making deep nodes take up more space in the tree, we can ensure that nodes throughout the hierarchy have an equal amount of room available to them in the final layout.

Algorithm 1 gives the general description of our procedure. In the situation presented here, we use the squarified treemap algorithm to lay out $T$, with the area of a subtree proportional to the number of members in the subtree. Given that the squarified treemap algorithm is a standard tool and the algorithm is well understood, we assume its availability and refer the reader to Bruls et al. (2000) for the details.

![Image of Algorithm 1](https://bitbucket.org/avashevko/cblayout)

Finally, we ensure that all nodes have an equal amount of space after nesting by repeating LAYOUT to calculate appropriate weights $w(t)$ for each part of the tree. Starting from an equal weight for each subtree, we iterate the layout procedure, updating weights after each iteration to ensure that the available area per node remains constant.

**Network Layout: Energy Minimization**

 Armed with partitions of the display area, we proceed to lay out individual nodes using a modification of a force-directed graph layout algorithm (Davidson and Harel 1996; Fruchterman and Reingold 1991; Kamada and Kawai 1989). We construct an energy function representing two components: the first, repulsion by the boundaries of the partition; the second, forces reflecting the structure of the network.

Formally, energy is a function of node positions in the display area. We denote the position of node $v$ by $x_v$. We lay

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3Other variants exist to lay out the tree with different geometric shapes, as discussed later.

4An example implementation of the algorithm presented here as an open-source Python package can be found at https://bitbucket.org/avashevko/cblayout.

5We use square brackets to either define a list ([ . . .]) or select an item from the list (list[i]), and we use ++ to denote list concatenation.
out the graph by minimizing this energy function through stochastic gradient descent over the set of node positions.

**Partition Energy.** Each node \(v\) in \(G\) has some corresponding partition, given by \(P(\{v\})\) or more concisely \(P(v)\). Under the treemap algorithm, each partition is rectangular, so we define distances \(\delta_{xv}, \delta_{yv}\) and \(\delta_{yv}\), representing the distance of \(x_v\) from the top, bottom, left, and right walls of the partition, respectively. Then, the partition energy for a node equals

\[
E_p(x_v) = \begin{cases} 
- \log(\delta_{xv}, \delta_{yv}, \delta_{yv}) & \text{if } v \text{ inside } P(v) \\
\infty & \text{if } v \text{ outside } P(v).
\end{cases}
\]

This function pushes nodes to the center of the rectangle. Partition energy for the overall graph is the sum of node energies:

\[
E_p = \sum_{v \in V} E_p(x_v).
\]

Although we define the energy function only for rectangles, the partition energy function trivially extends to other shapes such as circles or toroids. Nonconvex shapes such as toroids, however, perform poorly under stochastic gradient descent.

**Network Energy.** The second energy component represents the energy embedded in the attractive and repulsive forces of the network. We choose to adopt the Kamada-Kawai energy formulation (Kamada and Kawai 1989). In this model, a spring connects each pair of nodes in the network, with the ideal length of the spring proportional to the shortest-path distance between the nodes and the force of each spring given by Hooke’s law. Writing the position of node \(i\) as \(x_i\), this energy component is given by

\[
E_n(x_i) = \sum_{j=1}^{n} \frac{1}{2} k_{ij} \left( |x_i - x_j| - l_{ij} \right)^2.
\]

As before, network energy for the overall graph is the sum of node energies:

\[
E_N = \sum_{v \in V} E_N(x_v).
\]

The two constants \(l_{ij}\), the ideal length of the spring, and \(k_{ij}\), the spring strength constant, both depend on the shortest-path distance between nodes, denoted \(d_{ij}\). For disconnected nodes, we set \(d_{ij}\) equal to the maximum observed shortest-path distance in the network. Following Kamada and Kawai, we set \(k_{ij} = K/d_{ij}^2\), with \(K\) an adjustable parameter; we diverge by setting \(l_{ij} = L_{ij} \cdot d_{ij}\). Kamada and Kawai kept \(L_{ij}\) = \(L\) constant and proportional to the available display area. We instead adjust \(L_{ij}\) to reflect the area of the partition containing the lowest common ancestor (LCA) of \(t(i)\) and \(t(j)\):

\[
L_{ij} = \text{area}(\text{LCA}(i, j)) / \max_{a, b \in \text{LCA}(i, j)} d_{ab}.
\]

Relative to the Kamada-Kawai algorithm, this adjustment lengthens springs in fully connected and completely connected partitions in the network. This allows dense regions of the network to take up more space.

The overall energy function can then be written as the sum of the energy contributions of each vertex:

\[
E(x) = \sum_{v \in V} E_p(x_v) + \sum_{v \in V} E_N(x_v).
\]

To reiterate, energy is a function of the positions of each vertex in \(G\). Because energy can be decomposed into the energy contributions of individual vertices, we minimize the energy of the graph using stochastic gradient descent with backtracking line search (Armijo 1966). Algorithm 2 describes the overall procedure. An initial randomization of positions distributes vertices within their partitions.

**Algorithm 2.** Stochastic Gradient Descent with Backtracking Line Search.

\[
\begin{align*}
& x_v \leftarrow \text{RANDOMIZE}(v) \\
& \text{repeat} \\
& \quad \text{for } v \in \text{SHUFFLE}(V) \text{ do} \\
& \quad \quad V_v \leftarrow VE(v) \\
& \quad \quad \alpha \leftarrow \text{LINESEARCH}(E,V_v) \\
& \quad \quad x_v \leftarrow x_v - \alpha V_v \\
& \text{end for} \\
& \text{until converged}
\end{align*}
\]

In our experience, backtracking line search is necessary to ensure that nodes do not wander outside of their partitions. Backtracking line search looks for a minimum in the direction of the gradient, looking for the furthest point at which the gradient still approximates the energy function. Starting from an optimistically large step, the algorithm checks whether the decrease in energy is close to that expected by extrapolating the gradient; if it is not, the algorithm checks a closer point. Because the energy function becomes infinite if nodes leave their partitions, backtracking line search avoids putting nodes near boundaries.

**Examples**

We present four examples of the algorithm at work. In each of these examples, we represent the embeddedness level and cohesion (in that order) on the lower right corner of each cohesive block. First, Figure 3 presents a visualization of cohesive blocks in the karate club network (Zachary 1977). This figure reveals the nested structure of blocks. The highest level block, at depth 0, contains a single unique member (node 12), and all other nodes fall into subblocks. The main
Our second and third examples are concerned with the social positions and groups in equity capital markets (ECMs). We use a commercial data set provided by Dealogic, one of the leading firms in financial data collection and dissemination. In ECMs, we focus on initial public offerings, follow-on issues, and transactions pertaining to debt instruments convertible to shares. Our analysis pertains to relations financial intermediaries build during underwriting activities in ECMs.6 One of our main objectives is to understand the social positions of different types of financial actors and how these positions change across financial centers and over time. Thus, we are interested in understanding social groups in ECMs and which groups are at the core of financial networks in ECMs. Cohesive blocking is a powerful tool in probing these issues.

Figure 4 presents the visualization of cohesive blocks in Singapore ECMs between 2008 and 2010. Although one of the largest global financial centers, Singapore is not a leading market in equity issues (Lee and Vertinsky 2011; Woo 2016). As a result, the network structure we observe is relatively simple, compared with other financial centers such as Hong Kong (see Figure 5). The social groups we observe here and the relations among them, however, is much richer than in the karate club network (Figure 3). Figure 4 reveals a number of interesting patterns. First, we find predominantly local and regional small institutions at the periphery of the network, comprising blocks at embeddedness levels 0 and 1. Second, as we get to the core of the financial network in Singapore (block at embeddedness level 2), we find a combination of Singaporean, Malaysian, Korean, Chinese, and Japanese financial institutions. Many of these financial intermediaries are midmarket or large firms taking advantage of dense trade and investment ties between Singapore and the rest of Asia. However, it can be seen that their network involvement depends on the core of the network. This is not surprising, as these firms often play a subsidiary role to transactions led by global or regional powerhouses that dominate the market. Finally, the deeply nested blocks at the core of the network are global bulge bracket banks (e.g., Citibank, Credit Suisse, and Goldman Sachs) and regional behemoths such as Singapore’s DBS and Malaysia’s Commerce International Merchant Bankers. At the core of the Singapore’s ECM financial network, we find prominent institutions such as DBS, UBS, Citibank, and Credit Suisse (in green) taking the lion’s share and binding much of the ECMs. Overall, Figure 4 shows that in financial markets such as ECMs cumulative advantage effects are quite strong (Poon 2003) and financial institutions from advanced industrialized countries dominate the market. Nonetheless, regional institutions play a substantial role in Singapore ECMs. This is a pattern that differs from a mature market

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6We restrict our analysis to the most important actors in these transactions: bookrunners, leading managers, and global coordinators.
such as Japan and many emerging markets, as we explore in other parts of our work.

Figure 5 presents the Hong Kong ECM financial network for the period from 2013 to 2015. Compared with Singapore, the financial network in Hong Kong ECMs are much denser, driven by Hong Kong’s role as the main platform for Chinese firms to raise capital (Lee and Vertinsky 2011). Although the Hong Kong financial network has substantially less branching than that in Singapore, its cohesive block tree is much deeper. This network is much more hierarchical than either of the prior examples, with a core of institutions that operate in the deepest, most cohesive part of the network, and a series of institutions that are less and less affiliated with this central core. Because the hierarchy in this network features substantial depth but little branching, relatively few institutions serve as bridges within the network. Although we do not discuss it here, the Hong Kong ECM financial network reveals a highly competitive market in which the Chinese financial intermediaries play an increasingly larger role compared with global bulge bracket banks.

Last, Figure 6 presents an application of this visualization to a different network analysis technique and a larger network. The figure shows a hierarchy of friendship groups within a 2,587 member community in the National Longitudinal Study of Adolescent to Adult Health data set (Moody 2001), with the hierarchy derived through $k$-core decomposition (Alvarez-Hamelin et al. 2005). Instead of finding cohesive blocks, $k$-core decomposition identifies $k$-cores, subgraphs in which all vertices have degree at least $k$; $k$-core decomposition can operate on larger graphs than cohesive blocking, and the visualization can keep up with these larger networks. The underlying Kamada-Kawai or Fruchterman-Reingold network layout algorithms struggle, however, to reveal fine structure at these higher scales. The figures does reveal a deeply nested hierarchy of high school friendships, showing a single, large, and close-knit friendship community at the 7-core, surrounded by a smaller number of increasingly disconnected students in the periphery. The hierarchy shows no branching until the deepest level, with two distinct but small groups at the 8-core.

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We thank an anonymous reviewer for suggesting this comparison.
level, suggesting that most students participate in a common social hierarchy.

**Discussion and Conclusion**

The algorithm we describe in this article is inspired by a long tradition in social network analysis that focuses on visualization as a crucial tool in capturing social positions and social groups (Freeman 2005; Freeman and White 1993; Krempel 2009). It represents a novel approach to the visual representation of hierarchical networks—networks that contain a hierarchical structure. Our method emphasizes the dual aspect of such networks—the positions of different nodes and the relationships between groups containing the nodes in the network—and highlights how these two aspects interact. As such, the method we develop demonstrates how network structure manifests within hierarchical bounds and shows how a hierarchy constrains network interactions. Perhaps most important, our visualization approach makes clear how network ties cut across hierarchical boundaries, showing both how social relations bind social groups and how certain nodes act as bridges and articulation points, binding different parts of the hierarchy together.

An important application of our technique lies in the interpretation and exploration of results following from methods that identify hierarchies within networks. For instance, cohesive blocking (Moody and White 2003; White and Harary 2001) provides a powerful tool for analyzing social structures by focusing on social cohesion and embeddedness of groups. This method yields rich, multidimensional results that particularly benefit from visualization. However, the algorithm we develop is equally applicable to various methods that identify groups and subgroups in networks (Alba and Moore 1978; Alvarez-Hamelin et al. 2005; Girvan and Newman 2002; Richards and Rice 1981; Wasserman and Faust 1994:260–90). Although there are many techniques for the visualization of clusters and communities in networks, our approach offers a tool for researchers examining the nested structure of networks.

Our algorithm makes a number of principled decisions, some consequential, some less so. Our focus on embeddedness and bridge nodes underlies many of our design decisions about the layout algorithm. We use the squarified treemap algorithm to lay out elements of the hierarchy. Alternatives exist, including cushion (van Wijk and van de Wetering 1999) and Voronoi treemaps (Balzer and Deussen 2005). Although we use Kamada and Kawai’s (1989) spring model to lay out the nodes within elements of the hierarchy, any network model that can be expressed in an energy formulation easily plugs into the existing algorithm. In our own experiments, energy formulations of Fruchterman and Reingold’s (1991) and Davidson and Harel’s (1996) force-directed layouts performed well. We note that various aspects of our algorithm are open to easy modification in future work. As such, we believe the visualization algorithm we offer in this article provides a new tool to network researchers, a tool that is particularly tuned to the visual representation of hierarchical networks.

**Acknowledgments**

We wish to thank the editors of *Socius* and two anonymous reviewers for comments that helped transform the article.

**Funding**

The authors disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: This research was supported by the National University of Singapore (NUS) Strategic Research Grant (WBS: R-109-000-183-646) awarded to the Global Production Networks Centre (GPN @ NUS) for the project Global Production Networks, Global Value Chains and East Asian Development.

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