Higher derivative correction to Kaluza-Klein black hole solution

Hossein Yavartanoo 1* and Sangheon Yun 2†

1Center for Theoretical Physics and BK-21 Frontier Physics Division, Seoul National University, Seoul 151-747 KOREA

2School of Physics and Astronomy, Seoul National University, Seoul 151-747 KOREA

ABSTRACT: We investigate the attractor mechanism in Kaluza-Klein black hole solution in the presence of higher derivative terms. In particular, we discuss the attractor behavior of static black holes by using the effective potential approach as well as entropy function formalism. We consider different higher derivative terms with a general coupling to moduli field. For the $R^2$ theory, we use effective potential approach, looking for solutions which are analytic near the horizon and show that they exist and enjoy the attractor behavior. The attractor point is determined by extremization of the modified effective potential at the horizon. We study the effect of the general higher derivative corrections of $R^n$ terms. Using the entropy function we define the modified effective potential and we find the conditions to have the attractor solution. In particular for a single charged Kaluza-Klein black hole solution we show that higher derivative correction dresses the naked singularity for an appropriate coupling, and we can find the attractor solution.

KEYWORDS: Attractor mechanism, Entropy function, Kaluza-Klein black holes.

*yar [at] phya.snu.ac.kr
†sanhan1 [at] phya.snu.ac.kr
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1. Introduction

One of the main achievements of string theory has been a successful explanation of black hole entropy [1]. For a large class of BPS black hole solutions, we are able to identify the black hole entropy as the logarithm of the degeneracy of states belonging to the microstates of the corresponding black hole. This prediction includes not only the leading entropy formula of Bekenstein and Hawking (see [2] for a recent review) but also all the subleading quantum gravitational corrections which was proposed in [3], building on the work of [4] (see also [5] and references there).

BPS black holes are known to exhibit an attractor mechanism, whereby the values of scalar fields at the horizon are determined only in terms of the charges carried by the black hole and are independent of the asymptotic values of the scalar fields. As a result, the black hole solution at the horizon and the resulting entropy turn out to be determined completely in terms of the conserved charges. This phenomenon plays an important role in understanding the entropy of asymptotically flat non-supersymmetric extremal black holes in string theory [6].

This phenomenon has been discovered in the context of $\mathcal{N} = 2$ supergravity [7], then extended to other supergravity theories [8]. It is well understood that supersymmetry does not really play an essential role in the attractor mechanism. The attractor phenomenon works as a consequence of the enhanced symmetry of the near horizon geometry of an extremal black hole, which is $AdS_2 \times S^p$ for a static spherically symmetric black hole in $p + 2$ dimensions [9]. In fact, the ‘long throat’ of $AdS_2$ is at the basis of attractor mechanism [9, 10].

Over the last few decades, Kaluza-Klein black holes have attracted considerable attention. The original Kaluza-Klein black hole solution in four dimensions was obtained by compactification of five-dimensional pure gravity on a circle [12]. The field contents of this theory are a $U(1)$ gauge field, a scalar field, and gravity. Stationary black hole solutions are parametrized by their electric $Q$ and magnetic $P$ gauge charges, as well as their mass $M$ and angular momentum $J$. There is a simple embedding of the system into string theory. Taking the product of the original five dimensional solution with a flat $T^6$ (or a general Calabi-Yau three-fold), we obtained a solution to $M$ theory, whose $IIA$ reduction has $D0$ and $D6$ charges. Even in the extremal limit, this black hole is not supersymmetric [13], due to the absence of supersymmetric bound states of $D0$ and $D6$ branes. However, there is a quadratically stable non-supersymmetric, $D0 − D6$...
bound states [14], and this will serve as a basis to the microscopic picture, which have been discussed in [15]. Having a successful statistical description of the leading entropy formula of the Kaluza-Klein black hole, one natural question is whether one can consider the subleading quantum gravitational corrections in this picture.

During the last several years, study of higher derivative corrections to the entropy of supersymmetric black holes has provided successful results in string theory. In many examples these corrections match the appropriate corrections to the statistical entropy of the corresponding microscopic system. While studying the higher derivative corrections to the entropy of a generic black hole is a difficult problem, a general method for computing the entropy of extremal black holes was developed in [9]. This method does not provide an explicit construction of the full black hole solution, but it gives a way to compute the near horizon field configuration and entropy of an extremal, but not necessarily supersymmetric black hole, with a given set of charges.

In this paper we examine the effect of higher derivative corrections to the Kaluza-Klein black hole near horizon solution. Following [16] and [17] (for some related works, see also [19]), we use perturbative methods and numerical analysis to show that, the horizon of extremal Kaluza-Klein black hole in the presence of a general higher derivative correction $R^n$, is attractor. This analysis supports the existence of the attractor mechanism for Kaluza-Klein black hole with higher derivative corrections.

We start with a simple example of such corrections, Gauss-Bonnet correction. Although in four dimensions this term is a total derivative, when it coupled to moduli field, it cannot be integrated out. To study the effect of this term in attractor solution, we use the near horizon analysis [16]. We will show that the condition to have attractor solution are succinctly stated in terms of a “modified effective potential” $W$ for the scalar field. The condition is that, $W(\phi)$ as a function of the moduli field, must have a minimum, which means $\partial W(\phi_0) = 0$, and $\partial^2 W(\phi_0)$ is positive. The resulting attractor value for the moduli is the critical value, $\phi_0$. Moreover, the entropy of the black hole is proportional to $W(\phi_0)$, and is thus independent of the asymptotic values for the moduli. By using the entropy function method we will find the modified effective potential for the general $R^2$ correction. We also study higher derivative corrections of the form $R^n$ as well and we will introduce the modified effective potential for these general higher derivative corrections.

Although in this paper we are mainly interested in Kaluza-Klein black holes, but the results we have found is applicable to all extremal black hole solutions in Einstein-Hilbert gravity coupled to arbitrary number of moduli fields and $U(1)$ gauge fields in
the presence of higher derivative corrections. The only difference is we need to use general form for the effective potential $V_{\text{eff}}$, introduced in [16].

The paper is organized as follows: In section 2 we review Kaluza-Klein Black hole solution. In section 3 we will consider $R^2$ correction to Kaluza-Klein black hole solution. We restrict ourselves to Gauss-Bonnet term. Using entropy function method, we generalize our results to a general $R^2$ correction. In section 4 we study general $R^n$ corrections. The last section is devoted to discussion.

2. Review of Kaluza-Klein black hole solution

We consider four-dimensional Kaluza-Klein black hole solution. This solution has been found by reducing five-dimensional Kerr black hole to three dimensions, applying $SO(2,1)$ boosts and oxidizing up to four dimensions. It is the solution of five-dimensional Einstein-Hilbert action reduced to four dimensions:

$$I = \frac{1}{16\pi G_4} \int d^4x \sqrt{-G_4} \left( R_{(4)} - \frac{2}{3} \partial^\mu \Phi \partial_\mu \Phi - e^{2\Phi} F_{\mu\nu} F_{\mu\nu} \right)$$ (2.1) \{action0\}

The original five-dimensional metric is given by

$$ds^2_{(5)} = e^{4\Phi/3}(dy + 4A_\mu dx^\mu)^2 + e^{-2\Phi/3}ds^2_{(4)}$$ (2.2)

and the four-dimensional metric is

$$ds^2_{(4)} = -\frac{H_3}{\sqrt{H_1 H_2}}(dt + \omega d\phi)^2 + \frac{\sqrt{H_1 H_2}}{\Delta}d\rho^2 + \sqrt{H_1 H_2}d\theta^2 + \frac{\Delta \sqrt{H_1 H_2}}{H_3} \sin^2 \theta d\phi^2$$ (2.3)

where

$$H_1 = \rho^2 + a^2 \cos^2 \theta + \rho(p - 2m) + \frac{p}{p + q} \frac{(p - 2m)(q - 2m)}{2}$$

$$- \frac{p}{2m(p + q)} \sqrt{(p^2 - 4m^2)(q^2 - 4m^2)} a \cos \theta ,$$ (2.4)

$$H_2 = \rho^2 + a^2 \cos^2 \theta + \rho(q - 2m) + \frac{q}{p + q} \frac{(p - 2m)(q - 2m)}{2}$$

$$+ \frac{q}{2m(p + q)} \sqrt{(p^2 - 4m^2)(q^2 - 4m^2)} a \cos \theta ,$$ (2.5)

$$H_3 = \rho^2 + a^2 \cos^2 \theta - 2m \rho ,$$ (2.6)

$$\Delta = \rho^2 + a^2 - 2m \rho ,$$ (2.7)

$$\omega = \sqrt{pq}(pq + 4m^2) \rho - m(p - 2m)(q - 2m)$$

$$2m(p + q) H_3 \ a \sin^2 \theta d\phi ,$$ (2.8)
The solution for the dilaton is given by

\[ e^{4\Phi/3} = \frac{H_2}{H_1} \]  

(2.9) \{\text{dilaton}\}

where the dilaton has been asymptotically set to zero. Finally the gauge field is given by

\[ A_\mu dx^\mu = - \left[ 2Q\left(\rho + \frac{p - 2m}{2}\right) + \sqrt{\frac{q^3(p^2 - 4m^2)}{4(p + q)} m} a \cos \theta \frac{H_2}{H_1} \right] dt 
\]

\[ - H_2^{-1} \left[ 2P(H_2 + a^2 \sin^2 \theta) \cos \theta + \sqrt{\frac{p(q^2 - 4m^2)}{4(p + q)^3}} \times \right. \]

\[ \times \left[ (p + q)(p\rho - m(p - 2m)) + q(p^2 - 4m^2) \right] \frac{a \sin^2 \theta}{m} \left. \right] d\phi, \]  

(2.10)

The four parameters \((m, a, q, p)\) appearing in the solution are related to the physical mass \(M\), angular momentum \(J\), electric charge \(Q\), and magnetic charge \(P\) through:

\[ 2G_4 M = \frac{p + q}{2}, \]  

(2.11)

\[ G_4 J = \frac{\sqrt{pq(pq + 4m^2)}}{4(p + q)} a, \]  

(2.12)

\[ Q^2 = \frac{q(q^2 - 4m^2)}{4(p + q)}, \]  

(2.13)

\[ P^2 = \frac{p(p^2 - 4m^2)}{4(p + q)}. \]  

(2.14)

By eliminating conical singularity in the Euclidean \((\tau = it, \rho)\) sector, we obtain the temperature

\[ T = \frac{m}{\pi \sqrt{pq}} \left[ \frac{pq + 4m^2}{p + q} + \frac{2m^2}{\sqrt{m^2 - a^2}} \right]^{-1}, \]  

(2.15) \{\text{T}\}

where \(P\) and \(Q\) are electric and magnetic charges carried by black hole. The area of the black hole can be determined from the four-dimensional Einstein metric and it gives the black hole entropy:

\[ S = \frac{A}{4G_4} = \frac{\pi pq}{G_4} \left[ m + \frac{pq + 4m^2}{2m(p + q) \sqrt{m^2 - a^2}} \right]. \]  

(2.16)

Next we note that the action (2.1) has a scaling symmetry:

\[ \Phi \rightarrow \Phi + \Phi_\infty, \quad F_{\mu\nu} \rightarrow e^{\Phi_\infty} F_{\mu\nu}, \]  

(2.17)
for a constant $\Phi_\infty$. Therefore we can generate one parameter family of solutions carrying fixed electric and magnetic charges by using the transformation:

$$\Phi \to \Phi + \Phi_\infty, \quad F \to e^{-\Phi_\infty} F, \quad Q \to e^{\Phi_\infty} Q, \quad P \to e^{-\Phi_\infty} P \quad (2.18)$$

The horizon and ergoregion are given by:

$$\Delta = 0 \implies \rho_\pm = m \pm \sqrt{m^2 - a^2}$$

$$g_{tt} = -\frac{H_3}{\sqrt{H_1 H_2}} = 0$$

The extremal limit is defined as the limit of degenerate horizon and zero temperature. From (2.15) it is clear that it can be achieved in two ways, going to two distinct branches of solution:

- **Ergo-free branch:**

  Consider the limit: $m, a \to 0$ with $a/m, p$ and $q$ held finite. The horizon is located at $\rho = 0$. In this limit $p, q$ and $a/m$ can be taken as the independent parameters labeling the solution. In this limit it is easy to see that the angular velocity of the horizon vanishes and there is no ergosphere. The mass and the black hole entropy in this case are given by:

$$2G_4 M = \left( Q^{2/3} + P^{2/3} \right)^{3/2}, \quad S = 2\pi \sqrt{\frac{P^2 Q^2}{G_4^2} - J^2} \quad (2.19)$$

where $G_4 |J| < |PQ|$.

- **Ergo branch:**

  The extremal limit on this branch is given by taking $a = m$ in the black hole solution. The horizon is located at $\rho = m$. In this case we note that $H_3$ changes from being positive at large distance to negative at horizon, therefore $g_{tt}$ changes sign as we go from the asymptotic region to the horizon and the solution has an ergosphere. In this branch the entropy is given by

$$S = 2\pi \sqrt{J^2 - \frac{P^2 Q^2}{G_4^2} \quad (2.20)}$$

where $G_4 |J| > |PQ|$.

At the dividing value $G_4 |J| = |PQ|$ the extremal horizon disappears and the solution has a naked singularity.
3. $R^2$ correction to Kaluza-Klein Black Hole solution

In this section we use the results of [16] to study attractor mechanism for Kaluza-Klein black hole solution with a general $R^2$ correction. We also discuss the attractor mechanism using the entropy function framework [9]. The entropy function approach is based on the near-horizon geometry and its enhanced symmetries. The analysis in [16] is based on investigating the equations of motion of the moduli and finding the conditions satisfied by the effective potential such that the attractor phenomenon occurs. We also use the entropy function method to find a general $R^n$ correction in the next section.

3.1 Effective potential and non-supersymmetric attractor

In this section, we consider four-dimensional, extremal, nonrotating Kaluza-Klein black hole solution with $R^2$ correction. Then the action is given by

$$I = \frac{1}{16\pi G_4} \int d^4 x \sqrt{-G^{(4)}} \left( R^{(4)} - \frac{2}{3} \partial^\mu \Phi \partial_\mu \Phi - e^{2\Phi} F^{\mu\nu} F_{\mu\nu} + \mathcal{L}_2 \right)$$

(3.1)

where $\mathcal{L}_2$ is a general $R^2$ correction with the following form

$$\mathcal{L}_2 = G(\phi) \left( \alpha R^2 + \beta R_{\mu\nu} R^{\mu\nu} + \gamma R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \right).$$

(3.2)

$\alpha$, $\beta$ and $\gamma$ are arbitrary constants and $G(\Phi)$ is a dilaton dependent coupling. Our primary interest is in the study of the case when the $\mathcal{L}_2$ part of the action (3.1) forms the Gauss-Bonnet combination

$$\alpha = 1, \quad \beta = -4, \quad \gamma = 1$$

(3.3)

Theory with Gauss-Bonnet combination is a very special case of higher derivative gravity. Since the Gauss-Bonnet term in four dimensions is a topological term, the coupling to the scalar fields is crucial. Without this coupling, the Gauss-Bonnet term does not contribute to the equations of motion. In that case, we are left with the KK solution reviewed in previous section.

In this section we use the result of [16, 19] to study the Kaluza-Klein black hole solution and we discuss the attractor mechanism in the presence of $R^2$ corrections. We investigate the equations of motion of the moduli and find the conditions satisfied by the modified effective potential such that the attractor phenomenon occurs.

We focus on static and spherically symmetric space-time metric which can be written as:

$$ds^2 = -a(r)^2 dt^2 + \frac{dr^2}{c(r)^2} + b(r)^2 (d\theta^2 + \sin^2 \theta d\varphi^2)$$

(3.4)
Although it is possible to set \( c(r) = a(r) \) by redefinition of the radial coordinate, we keep \( c(r) \) free to derive the complete set of equations, including the Hamiltonian constraint, from the one-dimensional action.

Using the above ansatz, one finds the following one-dimensional action:

\[
I = \frac{1}{2G_4} \int dt dr \left[ -b^2 (a'c)' + \frac{a}{e} (1 - c^2 b'^2) - 2b(acb')' - \frac{1}{3} abc^2 \phi'^2 - \frac{a}{cb^2} V_{eff}(\phi) \\
+ 4G(\phi) (ca'(-1 + c^2 b'^2))^' \right] 
\]

where \( V_{eff}(\phi) \) is given by

\[
V_{eff}(\phi) = e^{-2\phi} Q^2 + e^{2\phi} P^2 
\]

For the action (3.1), the Bianchi identity and equation of motion for the gauge field can be solved by a field strength of the form

\[
F = \frac{Q}{b^2} dt \wedge dr + P \sin \theta d\theta \wedge d\varphi 
\]

where \( Q \) and \( P \) are electric and magnetic charges respectively.

One derives the equations of motion by varying the above action with respect to \( a, b, c \) and \( \phi \). Then we can put \( c(r) = a(r) \) and the equation for \( c \) turns out to be the Hamiltonian constraint. The equations of motion and Hamiltonian constraint following from (3.5) are

\[
\frac{1}{3} \phi'^2 + \frac{b''}{b} = - \frac{2G''}{b^2} + \frac{2a^2}{b^2} (G' b'^2)' \\
(a^2 b'^2)'' = 2 + 8G' a a'(3 a^2 b'^2 - 1) + 8b (G' a^3 a')' \\
\frac{1}{3} (a^2 b'^2 \phi')' = \frac{1}{2b^2} \frac{dV_{eff}}{d\phi} - 2 \frac{dG}{d\phi} (aa'(a^2 b'^2 - 1))' \\
a^2 b'^2 - 1 + 2a a' b b' - \frac{1}{3} a^2 b'^2 \phi'^2 + \frac{V_{eff}}{b^2} = 4G' a a'(3 a^2 b'^2 - 1) 
\]

where prime indicates derivation with respect to \( r \).

For the attractor phenomenon to occur, it is sufficient that the following two conditions are satisfied [16]. First, for fixed charges, as a function of the moduli, the modified effective potential \( W \), must have a critical point. Denoting the critical values for the scalars as \( \phi^i = \phi^i_0 \) we have,

\[
\partial_i W(\phi^i_0) = 0. 
\]
Second, there should be no unstable direction around this minimum, so the matrix of second derivatives of the potential at the critical point,

$$M_{ij} = \frac{1}{2} \partial_i \partial_j W(\phi_0^k)$$  \hspace{1cm} (3.13)

should have no negative eigenvalue. Schematically we can write,

$$M_{ij} > 0.$$  \hspace{1cm} (3.14)

We will sometimes refer to $M_{ij}$ as the mass matrix and its eigenvalues as masses (more correctly mass$^2$ terms) for the fields, $\phi^i$. It is important to note that in deriving the conditions for the attractor phenomenon, one does not have to use supersymmetry at all. The extremality condition puts a strong constraint on the charges so that the asymptotic values of the moduli do not appear in the entropy formula.

**Zeroth order analysis**

Let us start by setting the asymptotic values of the scalars equal to their critical values (independent of $r$), $\phi^i = \phi_0^i$. The equations of motion (3.9), (3.8) and (3.10) can be easily solved. First we solve (3.8) and get $b(r) = r$, and then replace this expression in (3.9) — we obtain:

$$a(r)^2 = 1 + \frac{C_1}{r} + \frac{C_2}{r^2}$$  \hspace{1cm} (3.15)

where $C_1$ and $C_2$ are integration constants. We are interested in the extremal solutions and so the integration constants can be calculated from the ‘double horizon’ condition:

$$C_1 = -2r_H, \quad C_2 = r_H^2,$$  \hspace{1cm} (3.16)

where $r_H$ is the horizon radius. Therefore, we can write the solution as

$$a_0(r) = (1 - \frac{r_H}{r}),$$  \hspace{1cm} (3.17) \hspace{1cm} \{a0\}

that describes the extremal RN solution. Then the dilaton equation gives rise to an important equation

$$\frac{dV_{eff}}{d\phi} + 4 \frac{dG}{d\phi} = 0$$  \hspace{1cm} (3.18)

It defines the modified effective potential as $W = V_{eff} + 4G$.

\footnote{The inner and outer horizons coincide and the equation has a double root.}
Figure 1: $\phi(r)$ vs. $r$, for $Q = 0$ and $P = 10$. Different curves represent different asymptotic values for $\phi_\infty$. Horizon is located at $r_H = 0$.

It is important to notice that, for a single charge Kaluza-Klein black hole without any higher derivative correction there is no attractor solution. This comes from the fact that attractor equation $\partial_\phi V_{\text{eff}} = 0$ has no solution with a single charge. In this case as we have mentioned before, horizon vanishes and we have naked singularity. But in the presence of higher order correction we may find the attractor mechanism. In this case the modified attractor equation $\partial_\phi W = 0$ may still have a solution, for an appropriate non-constant coupling $G(\phi)$.

Having a solution for equation (3.18), we will find a non-zero value for the horizon radius, which means that the higher derivative correction stretches the horizon and the naked singularity is dressed. As a result we will find non-zero value for entropy. In addition the attractor behavior is recovered. In figures (1) and (2) we compare the behavior of the scalar field for a single charged black hole, in the presence of higher order correction, with the case without higher derivative correction. In our example we consider the coupling in higher derivative correction such as the equation (3.18) have a solution.

The Hamiltonian constraint evaluated at the boundary provides a constraint on charges. However, we are interested in solving the Hamiltonian constraint at the horizon and in obtaining a relation between the horizon radius and the effective potential. Using the solution for $a$ and $b$, the Hamiltonian constraint is simplified drastically at the horizon. Thus, the horizon radius, $r_H$, is given by $V_{\text{eff}}$ at the minimum of $W$:

$$r_H^2 = V_{\text{eff}}(\phi_0)$$

(3.19)
Figure 2: $\phi(r)$ vs. $r$, for $Q = 0$ and $P = 10$ with Gauss-Bonnet correction. $G(\phi) = 1/4e^{-2\phi}$
Different curves represent different asymptotic values for $\phi_\infty$. The attractor point is $\phi_0 = -\frac{1}{2} \ln 10$ at the horizon $r_H = \sqrt{10}$.

It is important to notice that in this case Bekenstein Hawking entropy is not just the area of the horizon but receives a correction, which can be calculated from Weyl entropy formula. The result is proportional to the value of modified effective potential at its extremum:

$$S_{BH} = \pi W(\phi_0)$$

(3.20)

First order analysis

For the extremal RN black hole solution carrying the charges specified by the parameter $Q$ and $P$ and the moduli taking the critical values $\phi_0$ at infinity, a double zero horizon continues to exist for small deviations from these attractor values for the moduli at infinity. The moduli take the critical values at the horizon and the entropy remains independent of the values of the moduli at infinity [10]. Now, starting with first order perturbation theory

$$\delta \phi = \phi - \phi_0 = \epsilon \phi_1,$$

(3.21)

where $\epsilon$ is small parameter we use to organize the perturbation theory. Using the zeroth order solution for $a$ and $b$, the first order correction to the scalar field $\phi$ satisfies the following equation

$$\frac{1}{3} \partial_r (a_0^2 b_0^2 \partial_r \phi_1) \simeq \frac{\beta^2}{2b_0^2} \phi_1$$

(3.22)
where $\beta^2$ is the second derivative of the modified effective potential $W(\phi)$ at its minimum $\phi_0$. A general solution of the second order differential equation above, is parametrized by two constants. We are interested in a smooth solution that does not blow up at horizon $r_H$. This requirement eliminates one parameter, therefore the general smooth solution is obtained by

$$\phi_1 \simeq C_1(1 - \frac{r_H}{r})^\gamma,$$

where $\gamma$ is the positive root of the following equation

$$\gamma(\gamma + 1) = \frac{3\beta^2}{2r_H^2} \quad (3.24)$$

and $C_1$ is an arbitrary constant, which denotes the asymptotic value of the scalar field.

Asymptotically (as $r \to \infty$) $\phi_1$ takes a constant value, $C_1$, however it vanishes at the horizon and the value of the scalar is fixed at $\phi_0$ regardless of $C$, its value at infinity. It is clear that if the $\beta^2$ is positive, then the solution is regular at the horizon and so the existence of a regular horizon is related to the existence of the attractor mechanism.

We should also notice that, for a general coupling, $\partial_\phi G(\phi_0)$ does not vanish when $W$ and $G$ does not have the same extremum. Therefore from equations (3.8), (3.9) and (3.11) we find that $a(r)$ and $b(r)$ receive a correction of order $\epsilon$. In this case, in order to find the first order correction we need to solve all four equations (3.8)-(3.11) together.

**Higher order analysis**

Going to higher orders in perturbation theory is in principle straightforward. We solve the system of equations (3.8),(3.9),(3.10) and (3.11) order by order in the $\epsilon$ expansion.

To the first order, we find that one variable, say $C_1$, can not be fixed by the equations. Thus we will find $a_1(r), b_1(r), \phi_1(r)$ as a function of $C_1$. One can check that at any order $n > 2$, we can substitute the resulting values of $(a_m,b_m)$, for all $m < n$ from the previous orders. Then equations (3.8)-(3.11) of the current order together, consistently give,

$$a_n = a_n(C_1), \quad b_n = b_n(C_1), \quad \phi_n = \phi_n(C_1) \quad (3.25)$$

as polynomials of order $n$ in terms of $C_1$. It is worth noting that $C_1$ remains a free parameter to all orders in the $\epsilon$-expansion. Owing to the result above, we observe that $(a_{\infty}, b_{\infty}, \phi_{\infty})$ are varying and will take different values, for different choices for $C_1$. The arbitrary value of $\phi$ at infinity is $\phi = \phi_\infty$, while its value at the horizon is fixed to be $\phi_0$. 

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3.2 Kaluza-Klein Black Holes with general $R^2$ corrections

To study the effect of higher derivative corrections, here we apply the entropy function formalism to static black holes solution. It was shown by Sen that the attractor mechanism is related to the extremality rather than to the supersymmetry property of a given solution. Therefore, the condition for the existence of the attractor mechanism is the existence of an $AdS_2$ as part of the near horizon geometry of an extremal black hole. The entropy function is defined as

$$E(u, v, e, p) = 2\pi \left( eq - f(u, v, e, p) \right) = 2\pi \left( eq - \int d\theta d\varphi \sqrt{-G_L} \right), \quad (3.26)$$

where $Q = \partial f / \partial e$ is the electric charge, $u$ is the value of the moduli at the horizon, and $e$ is the electric field and $v_1, v_2$ are the sizes of $AdS_2$ and $S^2$, respectively. Thus, $E/2\pi$ is the Legendre transform of the function $f$ with respect to the variables $e$. The reason why it is not a Legendre transform with respect to magnetic charge is due to topological character of the magnetic charge. The Bianchi identities do not change when the action is supplemented with $\alpha'$-corrections, but the equations of motion receive corrections. Then it follows as a consequence of the equations of motion that for a black hole carrying electric charge $Q$ and magnetic charge $P$, the constants $v_1, v_2, u$ and $e$ are given by:

$$\frac{\partial E}{\partial u} = 0, \quad \frac{\partial E}{\partial v_1} = 0, \quad \frac{\partial E}{\partial e} = 0. \quad (3.27)$$

Furthermore, the entropy associated with the black hole is given by $S_{BH} = E(u, v, e, p)$ at the extremum (3.27). If $E$ has no flat directions, then the extremization of $E$ determines $u, v, e$ in terms of $Q$ and $P$. Therefore, $S = E$ is independent of the asymptotic values of the scalar fields. These results lead to a generalized attractor phenomenon for both supersymmetric and non-supersymmetric extremal black hole solutions.

We can apply this method to our action (3.1). The general metric of near horizon geometry $AdS_2 \times S^2$ can be written as

$$ds^2 = v_1(-\rho^2 d\tau^2 + \frac{1}{\rho^2} d\rho^2) + v_2 d\Omega_2^2. \quad (3.28)$$

The field strength ansatz in our case is given by

$$F = e d\tau \wedge d\rho + P \sin \theta d\theta \wedge d\varphi. \quad (3.29)$$

For the metric (3.28) Ricci scalar is $R = 2/v_2 - 2/v_1$. By dimensional argument it is clear that for the metric (3.28) a general $R^2$ correction term like (3.2) is given by

$$L_2 = \frac{c_1}{v_1^2} + \frac{c_2}{v_1 v_2} + \frac{c_3}{v_2^2}. \quad (3.30)$$
where comparing this with (3.2) we will find that
\[
c_1 = c_3 = 4\alpha + 2\beta + 4\gamma, \quad c_2 = -8\alpha. \tag{3.31}
\]

The entropy function \( E(u, v_1, v_2, Q, P) \) and \( f(u, v_1, v_2, e, P) \) are given by\(^3\)
\[
E(u, v_1, v_2, Q, P) = 2\pi [Qe - f(u, v_1, v_2, e)], \tag{3.32}
\]
\[
f(u, v_1, v_2, e, P) = \frac{1}{2} \left( v_1 - v_2 - \left( \frac{P^2}{v_2^2} - \frac{e^2}{v_1^2} \right)v_1v_2e^{2u} + G(u) \left( \frac{c_1v_2}{2v_1} + \frac{c_2}{2} + \frac{c_3v_1}{2v_2} \right) \right).
\]

Then the attractor equations are obtained as:
\[
\frac{\partial E}{\partial v_1} = 0 \Rightarrow -1 + \frac{1}{v_2} V_{\text{eff}}(u) + \left( \frac{c_1v_2}{2v_1^2} - \frac{c_3}{2v_2} \right) G(u) = 0, \tag{3.33}
\]
\[
\frac{\partial E}{\partial v_2} = 0 \Rightarrow 1 - \frac{v_1}{v_2^2} V_{\text{eff}}(u) - \left( \frac{c_1}{2v_1} - \frac{c_3v_1}{2v_2^2} \right) G(u) = 0, \tag{3.34}
\]
\[
\frac{\partial E}{\partial u} = 0 \Rightarrow \frac{v_1}{v_2} \frac{dV_{\text{eff}}(u)}{du} - \left( \frac{c_1v_2}{2v_1} + \frac{c_2}{2} + \frac{c_3v_1}{2v_2} \right) \frac{dG(u)}{du} = 0, \tag{3.35}
\]
\[
\frac{\partial E}{\partial e} = 0 \Rightarrow Q - \frac{ev_2}{v_1} e^{2u} = 0. \tag{3.36}
\]

Solving the first two equations gives us \( v_1 = v_2 = V_{\text{eff}} \). Therefore the third equation is:
\[
\frac{dV_{\text{eff}}(u)}{du} - \frac{c}{2} \frac{dG(u)}{du} = 0, \tag{3.37}
\]
where \( c = c_1 + c_2 + c_3 \). This is an important equation. It is equivalent to finding the critical points of the modified effective potential \( W = V_{\text{eff}} - cG \) at horizon. For the Gauss-Bonnet correction where \( c_1 = c_3 = 0, c_2 = -8 \), this is the same potential we have found in our previous analysis. As we have shown for Gauss-Bonnet case, the equation (3.37) is one of the conditions for the existence of attractor mechanism. If this equation has solutions, then the moduli values at the horizon are fixed in term of the charges.

As we have discussed before, while for a single charged Kaluza-Klein horizon vanishes and we do not have the attractor mechanism, in the presence of higher derivative correction moduli may be fixed at horizon due to extra term in modified effective potential: Higher derivative correction stretches the horizon and the naked singularity

\( ^3 \)We set the Newton’s constant \( G_4 = 1 \)
will be dressed. As a result we will find non-zero value for entropy. Although for single charged black hole in this theory, the necessary condition for having a solution for equation (3.37) is having a non-constant coupling $G(\phi)$.

Finally it is important to notice that the existence of a near-horizon geometry when the moduli are not constants does not imply the existence of the whole solution in the bulk (from the horizon to the boundary) this is the disadvantage of the entropy function formalism. However, in the next subsection we will investigate the equations of motion in the bulk and describe the horizon as an IR critical point of the effective potential.

By replacing $v_1 = v_2 = V_{eff}(\phi_0)$ where $\phi_0$ is the solution of equation $\partial W = 0$ into the (3.32), we obtain the value of the entropy function at the extremum:

$$S_{BH} = \pi W(\phi_0).$$

(3.38)

As an example of $R^2$ corrections to Kaluza-Klein black hole, let us consider $R^2$ terms in the dimensional reduction of type IIA string theory. One way to get such correction is starting with five-dimensional original solution, taking the product with a flat $T^6$ we obtain a solution to M-theory whose type IIA reduction has $D0$ and $D6$ charges.

The first correction to the effective action of type IIA theory involves terms with eight derivatives including $R^4$ terms, and derivative of dilaton and NS–NS and RR fields. Here we restrict ourselves to the $R^4$ terms. This includes

$$S_{tree} = \frac{1}{16\pi G_{10}} \int dx^{10} \sqrt{-g} e^{-2\phi} \left( R + \frac{\alpha'^2}{3.2\phi} (t_8 t_s + \frac{1}{8} \epsilon_{10} \epsilon_{10}) (R)^4 \right)$$

(3.39)

$$S_{1-loop} = \frac{1}{16\pi G_{10}} \frac{\alpha'^2}{9.2\phi} g_s^2 \int dx^{10} \sqrt{-g} e^{-2\phi} (t_8 t_s - \frac{1}{8} \epsilon_{10} \epsilon_{10}) (R)^4$$

(3.40)

In the equations above we have used the following standard notation to denote index contractions:

$$\epsilon_{10} \epsilon_{10} R^4 \equiv \epsilon^{\alpha_1 \beta_1 \nu_1 ... \mu_4 \nu_4} \epsilon_{\alpha_1 \beta_1 \sigma_1 ... \rho_4 \sigma_4} R^{\mu_1 \nu_1} ... R^{\rho_4 \sigma_4}$$

(3.41)

$$t_8 t_s R^4 \equiv t_{\mu_1 \nu_1 ... \mu_4 \nu_4} t_{\rho_1 \sigma_1 ... \rho_4 \sigma_4} R^{\mu_1 \nu_1} ... R^{\rho_4 \sigma_4}$$

(3.42)

where the $t_8$ tensor is defined as $(F^{(i)}_{\mu \nu}, i = 1, \ldots, 4$ are any anti-symmetric tensors)

$$t_8^{\mu_1 \nu_1 ... \mu_4 \nu_4} F^{(1)}_{\mu_1 \nu_1} ... F^{(4)}_{\mu_4 \nu_4}$$

$$= 8 (F^{(1)}_{\mu \nu} F^{(2)}_{\rho \lambda} F^{(3)}_{\lambda \mu}) + (F^{(1)}_{\mu \nu} F^{(3)}_{\rho \lambda} F^{(2)}_{\lambda \mu}) + (F^{(1)}_{\mu \nu} F^{(2)}_{\rho \lambda} F^{(3)}_{\lambda \mu})$$

$$-2 (F^{(1)}_{\mu \nu} F^{(2)}_{\rho \lambda} F^{(3)}_{\lambda \mu}) + (F^{(1)}_{\mu \nu} F^{(3)}_{\rho \lambda} F^{(2)}_{\lambda \mu}) + (F^{(1)}_{\mu \nu} F^{(2)}_{\rho \lambda} F^{(3)}_{\lambda \mu})$$

(3.43)
Now by reducing the above action on $K^3 \times T^2$ we get both $R^2$ and $R^4$ correction terms in the four-dimensional effective action $[20]$, where $R^2$ term is given by

$$\mathcal{L}_2 = \frac{C_2}{6} g_s^2 \alpha'^3 \frac{T}{T_0} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}. \quad (3.44)$$

The volume of $K^3$ and $T^2$ are $(2\pi)^4 V$ and $(2\pi)^2 T$ respectively and $V_0$ and $T_0$ are the value of $V$ and $T$ at infinity. $C_2$ is the second chern class of $K^3$ which is 24. The ten-dimensional type $IIA$ metric is given by

$$ds_{10}^2 = ds_4^2 + e^{2\phi/3} \sum_{i=1}^{6} dy_i dy^i \quad (3.45)$$

therefore

$$\frac{T}{T_0} = e^{2\phi/3} \quad (3.46)$$

For this correction term, $c_2 = 0, c_3 = c_3 = \frac{16g_s^2 \alpha'^3}{V_0}$ therefore the effective potential is given by

$$W = Q^2 e^{-2\phi} + P^2 e^{2\phi} - \frac{16g_s^2 \alpha'^3}{V_0} e^{2\phi/3} \quad (3.47)$$

Minimizing this potential gives us

$$e^{2\phi_0} = \left| \frac{Q}{P} \right| \left( 1 + \frac{8g_s^2 \alpha'^3}{3V_0 P^{1/3} Q^{2/3}} + O(\alpha'^{6}) \right) \quad (3.48)$$

therefore the first correction to the entropy which is given by (3.38) is

$$S_{BH} = 2\pi \left| PQ \right| - \frac{16g_s^2 \alpha'^3}{V_0} \left| \frac{Q}{P} \right|^{1/3}. \quad (3.49)$$

From (3.33) and (3.34) we find that the radii of $AdS_2$ and $S^2$ do not receive any correction in order $O(\alpha'^3)$.

4. Kaluza-Klein Black Holes with general higher derivative corrections

Our results in the previous section, $R^2$ corrections to the attractor mechanism for Kaluza-Klein, can be generalized to higher order corrections. To start let us consider a general $R^4$ correction to the action (2.1). We are interested in studying static and extremal black hole solution with $SO(2,1) \times SO(3)$ invariant near horizon geometry. Near horizon metric (3.28) and gauge field (3.29) are the most general possible forms,
consistent with this symmetry. Using this near horizon geometry it is easy to show that, at the horizon
the most general $R^4$ correction has the following structure

$$L_4 = \frac{c_1}{v_1^4} + \frac{c_2}{v_1^3 v_2} + \frac{c_3}{v_1^2 v_2^2} + \frac{c_4}{v_1 v_2^3} + \frac{c_5}{v_2^4} \quad (4.1)$$

where constants $c_1$-$c_5$ are fixed in term of numerical factors in the $R^4$ structure. Using the
definition (3.32), it is easy to calculate the entropy function with this additional term. It is given by

$$E = \pi \left[ v_2 - v_1 + \frac{v_1}{v_2} V_{eff}(u) - \frac{\gamma(u)}{2} v_1 v_2 \left( \frac{c_1}{v_1^4} + \frac{c_2}{v_1^3 v_2} + \frac{c_3}{v_1^2 v_2^2} + \frac{c_4}{v_1 v_2^3} + \frac{c_5}{v_2^4} \right) \right] \quad (4.2)$$

The value of $v_1$ and $v_2$ at the horizon are determined by extremizing $E$ with respect to $v_1$ and $v_2$. This gives

$$-1 + \frac{1}{v_2} V_{eff}(u) + \gamma(u) \left( \frac{3c_1 v_2}{2v_1^4} + \frac{c_2}{v_1^3 v_2} + \frac{c_3}{2v_1^2 v_2^2} - \frac{c_5}{2v_2^3} \right) = 0 \quad , \quad (4.3)$$

$$1 - \frac{v_1}{v_2^2} V_{eff}(u) + \gamma(u) \left( -\frac{c_1}{2v_1^3} + \frac{c_2}{2v_1^2 v_2} + \frac{c_3}{v_2} + \frac{3c_5 v_1}{2v_2^4} \right) = 0 \quad . \quad (4.4)$$

Now we can solve these two equations to find $v_1$ and $v_2$. For small coupling $\gamma$ we find

$$v_1 = V_{eff} \left( 1 + \frac{5c_1 + 4c_2 + 3c_3 + 2c_4 + c_5}{2V_{eff}^3} \gamma + \mathcal{O}(\gamma^2) \right) \quad , \quad (4.5)$$

$$v_2 = V_{eff} \left( 1 + \frac{3c_1 + 2c_2 + c_3 - c_5}{2V_{eff}^3} \gamma + \mathcal{O}(\gamma^2) \right) \quad . \quad (4.6)$$

Scalar field equation in the near horizon geometry correspond to extremizing the entropy function with respect to $u$, which gives us

$$\frac{v_1}{v_2} \frac{dV_{eff}}{du} - \frac{1}{2} \frac{d\gamma}{du} v_1 v_2 \left( \frac{c_1}{v_1^4} + \frac{c_2}{v_1^3 v_2} + \frac{c_3}{v_1^2 v_2^2} + \frac{c_4}{v_1 v_2^3} + \frac{c_5}{v_2^4} \right) = 0 \quad . \quad (4.7) \{\text{att3}\}$$

Using equations (4.3) and (4.4) we can simplify the scalar equation into the following form

$$\frac{v_1}{v_2} \frac{dV_{eff}}{du} + \frac{v_2 - v_1}{2\gamma} \frac{d\gamma}{du} = 0 \quad , \quad (4.8) \{\text{eff1}\}$$

and finally using the solutions (4.3) and (4.6), for small coupling $\gamma$ we find:
\[
\frac{d}{du} \left( V_{\text{eff}} - \frac{c \gamma}{2 V_{\text{eff}}^2} \right) \simeq 0 ,
\]

where \( c = c_1 + \ldots + c_5 \). This equation is analogous to the attractor equation (4.37), which we have found for \( R^2 \) correction in the previous section. Therefore it suggests defining the modified effective potential at the horizon as:

\[
W = V_{\text{eff}} - \frac{c \gamma}{2 V_{\text{eff}}^2} .
\]

In addition if we consider dilaton equation at the near horizon geometry (3.28), we will find that the similar term appears on the right hand side of the dilaton equation. Therefore our definition for modified effective potential makes sense. By the same analysis we did in the section 3, we can find that the condition of having attractor solution is reduced to having a minimum for the modified effective potential \( W \)

\[
\partial W(\phi_0) = 0 , \quad \partial^2 W(\phi_0) > 0 .
\]

The black hole entropy is given by the value of entropy function at its extremum. It is easy to show that this is proportional to the value of modified effective potential at its minimum

\[
S_{BH} = \pi W(\phi_0) .
\]

As an example for \( R^4 \) correction, let us consider \( R^4 \) terms in the dimensional reduction of type IIA string theory. As we said, the first correction to the effective action of type IIA theory involves terms with eight derivatives including \( R^4 \) terms (3.39, 3.40). Now if we reduce this action on \( T^6 \) we get both \( R^4 \) corrections terms in the four-dimensional effective action. Consider the near horizon geometry (3.28), the effect of \( R^4 \) corrections (3.39, 3.40) to four-dimensional effective action is given by

\[
\mathcal{L}_4 = \frac{\gamma(u)}{108} \left( \frac{35}{v_1^4} - \frac{5}{v_1^3 v_2} + \frac{21}{v_1^2 v_2^2} - \frac{5}{v_1 v_2^3} + \frac{35}{v_2^4} \right) ,
\]

where coupling \( \gamma(u) \) is given by

\[
\gamma(u) = \gamma_0 e^{2u} = 4(\zeta(3) + \frac{\pi^2 g_s^2}{3}) \alpha'^3 e^{2u} .
\]

By minimizing the modified effective potential (4.10) we can find the attractor value \( \phi_0 \)

\[
e^{2\phi_0} = \left| \frac{Q}{P} \right| + \frac{3\gamma_0}{64|QP|^5} + O(\gamma_0^2) ,
\]
and the correction to the entropy from equation (4.12) is given by
\[ S = 2\pi|PQ| - \frac{3\pi\gamma_0}{32|QP|^3} + O(\gamma_0^2). \] (4.16)

Finally the corrections to the $AdS_2$ and $S^2$ radii are given by
\[ v_1 = 2|PQ| + \frac{9\gamma_0}{32P^2Q^2} + O(\gamma_0^2), \quad v_2 = 2|PQ| + \frac{3\gamma_0}{32P^2Q^2} + O(\gamma_0^2). \] (4.17)

It is straightforward to repeat the above calculus for a general $R^n$ correction. By dimensional analysis one can show that, at the near horizon the most general $R^n$ correction has the following structure
\[ \mathcal{L}_n = \frac{c_1}{v_1^n} + \frac{c_2}{v_1^{n-1}v_2} + ... \frac{c_{n+1}}{v_2^n}. \] (4.18)

Then the entropy function is given by
\[ \mathcal{E} = \pi \left[ v_2 - v_1 + \frac{v_1}{v_2} V_{eff}(u) - \frac{\gamma(u)}{2} v_1 v_2 \sum_{i=0}^{n} \frac{c_{i+1}}{v_1^{n-i}v_2^i} \right]. \] (4.19)

By extremizing above entropy function, we will find the values of $v_1$ and $v_2$ as follows
\[ v_1 = V_{eff} \left[ 1 + \frac{1}{2V_{eff}^{n-1}} \left( (2n - 3)c_1 + (2n - 4)c_2 + ... (n - 3)c_{n+1} \right) \gamma + O(\gamma^2) \right], \] (4.20)
\[ v_2 = V_{eff} \left[ 1 + \frac{1}{2V_{eff}^{n-1}} \left( (n - 1)c_1 + (n - 2)c_2 + ... + c_{n-1} - c_{n+1} \right) \gamma + O(\gamma^2) \right], \] (4.21)
and the resulting scalar field equation is given by
\[ \frac{d}{du} \left( V_{eff} - \frac{c \gamma}{2V_{eff}^{n-2}} \right) \simeq 0, \] (4.22)
where we defined $c = c_1 + ... + c_{n+1}$ which is the correction Lagrangian (4.18) evaluated for the geometry (3.28) with $v_1 = v_2 = 1$. Finally we can define the modified effective potential $W$ for a general $R^n$ correction as follows
\[ W = V_{eff} - \frac{c \gamma}{2V_{eff}^{n-2}}. \] (4.23)
By the same argument we can find that the attractor conditions are \((4.11)\). The entropy of the black hole is given by the entropy function at its extremum. If we denote the solution of equation \((4.22)\) by \(\phi_0\), the black hole entropy which is the value of entropy function at its extremum is given by

\[
S_{BH} = \pi \left[ V_{\text{eff}}(\phi_0) - \frac{c \gamma(\phi_0)}{2 V_{\text{eff}}^n(\phi_0)} \right],
\]

which is equal to the value of modified effective potential at the horizon. Our discussion can be easily generalized for any extremal black hole solution in Einstein-Hilbert gravity coupled with arbitrary number of \(U(1)\) gauge fields and arbitrary number of moduli fields with a general coupling between gauge field and scalar. It can be described by the following action

\[
I = \frac{1}{16\pi} \int d^4x \sqrt{-G} \left( R - 2\partial^\mu \Phi_i \partial_\mu \Phi^i - f_{ab}(\phi_i) F_a^{\mu\nu} F_b^{\mu\nu} - \frac{1}{2} \tilde{f}_{ab}(\phi_i) e^{\mu\alpha\beta} F_a^{\mu\nu} F_b^{\mu\nu} + \gamma(\phi_i) L_n \right),
\]

where \(f_{ab}(\phi_i)\) and \(\tilde{f}_{ab}(\phi_i)\) determine the general moduli dependent gauge couplings. Then one can show that the effective potential is given as follows \([16]\):

\[
V_{\text{eff}}(\phi_i) = f^{ab}(Q_a - \tilde{f}_{ac} P^c)(Q_b - \tilde{f}_{bd} P^d) + f_{ab} P^a P^b
\]

where \(Q^a\) and \(P^a\) are denoted electric and magnetic charges. In the absence of higher derivative corrections, it has been shown that there are two conditions which are sufficient for the existence of an attractor solution \([16]\). First, the charges should be such that the resulting effective potential, \(V_{\text{eff}}(\phi_i)\), has a critical point. We denote the critical values for the scalars as \(\phi_i = \phi_{i0}\). So that,

\[
\partial_i V_{\text{eff}}(\phi_{i0}) = 0
\]

Second, the matrix of second derivatives of the potential at the critical point,

\[
M_{ij} = \frac{1}{2} \partial_{ij} V_{\text{eff}}(\phi_{0i})
\]

should have positive eigenvalues. Once these two conditions hold, we show below that the attractor phenomenon results. Since our results in the previous section are independent of explicit form of effective potential \(V_{\text{eff}}\), we can simply generalize our
results to the extremal black hole solution of the action (4.29). Therefore the modified
effective action is given by

\[ W(\phi_i) = V_{\text{eff}}(\phi_i) - \frac{c \gamma(\phi_i)}{2V'_{\text{eff}}(\phi_i)}, \tag{4.29} \]

The conditions of having attractor solution are changed to having a critical point for
modified effective potential \( W(\phi_i) \) and positivity of the matrix of second derivatives of
the modified potential at the critical point,

\[ \partial_i \partial_j W(\phi_{0i}) > 0. \tag{4.30} \]

And finally the entropy which is given by the value of the entropy function at its extremum is

\[ S_{BH} = \pi W(\phi_{0i}), \tag{4.31} \]

which is equal to the value of the effective potential \( W \) at its critical point \( \phi_{0i} \).

5. Discussion

In this paper, we have investigated the attractor solution for Kaluza-Klein black hole in
the presence of higher derivative terms. We started with the simplest higher derivative
correction, i.e. Gauss-Bonnet term. Although in four dimensions this is a total derivative
but since we considered it coupled to the scalar field we can not integrate it out
and it affects the solution. By investigating solutions of the equations of motion, we
observed the attractor behavior explicitly. We looked for all possible solutions which
admit the criteria of being regular at the horizon and free in the asymptotic region.
The near horizon analysis shows the criteria for attractor behavior. We introduced
the modified effective potential \( W \), which plays the role of the scalar effective potential introduced in \([14]\), in the presence of higher derivative correction. The conditions
for having the attractor solution can be expressed in terms of two conditions on this function. Moreover, the entropy is proportional to this function at its extremum. We
extended our studying to the more general higher derivative corrections \( R^n \). For small scalar coupling \( \gamma \), we have found the modified the effective potential as well.

There is a simple embedding of the Kaluza-Klein black hole into string theory.
Taking the product of extremal Kaluza-Klein black hole solution with a flat \( T^6 \) (or a
general Calabi-Yau three-fold), we obtain a solution of M theory, whose IIA reduction
has \( D0 \) and \( D6 \) charge. In \([15]\) authors have found the microscopic origin for Kaluza-
Klein black hole entropy. The idea is based on the fact this some neutral black holes
can be lifted to M-theory in such a way that the reduction to IIA string theory has both $D0$ and $D6$ charge. Then one can count the number of $D0 - D6$ bound states. To count the bound states, suppose $N0 = 4k^3N$, $N6 = 4l^3N$. If we consider the $T^6$ as a product of three $T^2$s and T-dualize along one cycle of each $T^2$, we get a configuration of four stacks of $D3$-branes wrapping the diagonal cycles of the $T^2$s. There are $N$ branes in each stack. If the $D3$-branes were wrapping the fundamental cycles instead, this configuration would be equivalent to a four charge black hole whose microscopic entropy per intersection is known to be $2\pi N^2$. There are now $(2kl)^3$ intersection points in total, therefore the entropy is given by $2\pi(2kl)^3N^2$. Using the relation between electric and magnetic charges of KK black and number or $D0$ and $D6$-branes we get the correct expression for the black hole entropy. This was shown to work for both static and rotating Kaluza-Klein black holes as long as they had sufficient $D0$ and $D6$ charges after dimensional reduction. Having a successful statistical description for Kaluza-Kleins from string theory, it would be quite interesting if one can find the leading order correction to statistical entropy of the Kaluza-Klein and compare it with our results from gravity side.

**Acknowledgments**

We would like to thank Dumitru Astefanesei, Kevin Goldstein, Sangmin Lee and Soo-Jong Rey for discussions. This work is supported by the Korea Research Foundation Leading Scientist Grant (R02-2004-000-10150-0) and Star Faculty Grant (KRF-2005-084-C00003).

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