Identification of microwave heating system with symmetrical octagonal tube cavity using ARX model

D Prastiyanto¹, E Apriaskar¹, P A Handayani², R Destanto¹, M A Malik¹ and A E Ramadhan¹

¹Department of Electrical Engineering, Faculty of Engineering, Universitas Negeri Semarang, Semarang 50229, Indonesia
²Department of Chemical Engineering, Faculty of Engineering, Universitas Negeri Semarang, Semarang 50229, Indonesia

dhidik.prastiyanto@mail.unnes.ac.id

Abstract. As one of the complex technologies in the heating process, microwave heating system has gained attentions in the recent years. A further development of microwave heating control technology becomes necessary considering its superiorities in less energy consumption, volumetric heating ability and satisfying mechanical properties. Several studies on microwave heating system modelling were conducted to encourage the controller design. This work presents the modelling of a microwave heating system using autoregressive with exogenous variables or so-called ARX. The modelling conducted in this research applied the approach of input-output identification method. The generation of input-output datasets for the identification of microwave heating system was carried out in COMSOL simulation environment using symmetrical octagonal tube cavity design with two magnetrons as the inputs and five temperature sensors as the output measurement devices. For the validation and evaluation of the approach, MATLAB identification system toolbox was utilized to find the best ARX model based on the given input-output datasets. The validation test shows that the best ARX model can reach more than 93% in the fit value given all the datasets for all the outputs, while for the evaluation using the perturbed signals from the outside of datasets, the chosen model can obtain 97.35% in the average of the fit value for all the outputs.

1. Introduction

In contrast to conventional heating, microwave heating has an entirely different heating mechanism called volumetric heating ability [1]. Microwave heating system is one of the complex technologies in the heating process that has provided benefits in many industrial process fields, such as food, wood, ceramics, plastics, etc. Microwave radiation has been accepted as a heating source because of several advantages it offers [2]. Microwave processing has benefits, including less energy consumption, volumetric heating, and better mechanical properties [3].

Microwave heating sometimes can causes uneven heat distribution, affecting the internal structure of the heated product, resulting in poor quality of the product [4,5]. An excellent control system for the microwave heating process can avoid or at least minimize the probability that matter occurs [1]. The system dynamics modeling can help creating a controller with a satisfying performance because it allows obtaining information about the temperature distribution of heating using microwaves.
Several studies on microwave heating have been carried out, including modeling microwave heating system. Various methods were discussed, such as multiphysics numerical methods [6], spectral methods [7]; some research modeling microwave heating also uses a differential equation [8]. Most of them implicate the understanding of physical or chemical laws admitted to the dynamics of the microwave heating process. A different approach was conducted by [9] that performed an optimization of microwave-based cellulosic biomass heating. The system was identified by a black-box model using Autoregressive with Exogenous Variables (ARX) method with the results of conformity reaching 99.13%, before completing the optimization using the Taguchi method. Another research conducted by [10] used a black-box system identification with the ARX method to obtain a mathematical equation that represents the relationship between microwave power level and the temperature generated on a probe placed in a microwave applicator. 

Different to [10], this work provides the modeling of microwave heating system with the approach of ARX method, but with five temperature sensors monitored and two magnetron as the microwave power generator. The information about initial dielectric constant and dielectric loss factor are also considered as inputs of the system. Thus, there are 4 inputs and 5 outputs measured in this system. The obtained ARX model is considered to describe the relation between two microwave power levels from the two magnetrons along with the dielectric properties as inputs and five temperature probes as outputs.

The organization of this paper is started with background and motivation of this work in section 1. Section 2 follows with the description of the evaluated system design and the proposed method. Before presenting brief conclusion about the contribution of this work along with possible future works in section 4, there is section 3 which presenting the result details of this work followed with discussions about how the best model for microwave heating system in this work obtained from several evaluations.

2. Design and Method

2.1. Microwave Heating System Design

In general, a microwave heating system consists of a microwave cavity, waveguide, and microwave generator. The microwave generator part consists of a power supply system and a microwave source originating from the magnetron, while the waveguide functions as a liaison as well as a distributor of electromagnetic waves from the magnetron to the microwave cavity. The microwave cavity is the part in which a material processed through microwave heating.

This research uses a symmetric octagonal shape with the main material of aluminium or iron that has good conductivity. Materials with a good conductivity are used so that they can reflect electromagnetic waves from the magnetron inside the microwave cavity properly, so that the heating process is optimal. The cavity is made with a diameter of 30 cm and a height of 37 cm, as shown in figure 1. It is a symmetrical octagonal tube cavity with two ports for magnetrons as the microwave source. This cavity design was actually examined in [11] and found a considerable power efficiency. As for the waveguide, WR340 type with a width of 8.631 cm, a height of 4.318 cm, and a length of 10 cm is utilized. The physical design of the waveguide can be seen in figure 2.

To place the object to be heated inside the cavity, there is a box with 12 cm in length, 5 cm in height, and 12 cm in width, as shown in figure 3. The material of the box uses a microwave-transparent type material, one of which is Teflon. Four sensors are placed on each side and one on its center to allow monitoring of the temperature of the polymer material being processed in real-time.
2.2. ARX Model Structure

ARX structure is one of the most common and simplest model structure to represent a system. The model is constructed by the Autoregressive (AR) part, \( A(q)y(t) \), and the auxiliary part (X), \( B(q)u(t) \). The structure of the model is based on the input-output relationship as shown in equation (1)

\[
y(t) = a_1 y(t - 1) + \cdots + a_{n_a} y(t - n_a) = b_1 u(t - 1) + \cdots + b_{n_b} u(t - n_b) + e(t) \quad (1)
\]

Where \( a_1, a_2, \ldots \in \mathbb{R}^{n_y \times n_y}, b_1, b_2, \ldots \in \mathbb{R}^{n_y \times n_u} \) is coefficient matrix, \( u(t) \in \mathbb{R}^{n_u} \) dan \( y(t) \in \mathbb{R}^{n_y} \) are the measured inputs and outputs at instant sampling \( k \) respectively, \( e(t) \) and \( d \) are the input and output delays. The ARX model has the following structure:

\[
A(z)y(t) = B(z)u(t) + e(t) \quad (2)
\]

3. Results and Discussions

3.1. Generating input-output datasets

Before modeling the microwave heating system, simulation is needed to get datasets of certain input and output pairs. Figure 1 shows how the simulation conducted using COMSOL to obtain the temperature inside the system. In this study, several variations of magnetron power level were used as perturbation signals, specifically in 20%, 40%, 60%, 80%, and 100% of the maximum power that the magnetron could produce. The frequency used in the simulation was 2.45 GHz (Magnetron Frequency) and the parameters of the heated material were DGEBA with a dielectric constant of 3.73 and a dielectric loss factor of 0.68.
Figure 4. Simulation Microwave Heating in Comsol Multiphysics 5.6

From table 1 it can be seen that there were five datasets produced through simulation. There were four parameters involved in the simulation, including dielectric constants, dielectric loss factor, magnetron power-1, and magnetron power-2. For the five datasets, dielectric constants and dielectric loss factor were permanent with 3.73 and 0.68, respectively, due to the similarity of the heated object, which was DGEBA. As for the magnetron power-1 and 2, they were classified into five different values with gradual increasing from one dataset to another. Dataset IO-20 represents the dataset generated by given dielectric properties with magnetron power-1 and 2 in 180 W or 20% of the allowed maximum value 900 W. For 40% of the maximum value or 360 W, the generated dataset is named IO-40. It follows with IO-60, IO-80, and IO-100 which are named based on the perturbation signal in 60%, 80%, and 100%, consecutively, of the allowed maximum value. The assignment of power level with the same values for both magnetrons were not coincidental, but to produce a well-distributed temperature for each side of the heated material. Different values for both magnetrons could ensure the heterogeneity of the temperature of the heated material that could affect to the worsening of the mechanical properties.

| Dataset | Dielectric Properties | Magnetron Power-1 (W) | Magnetron Power-2 (W) |
|---------|-----------------------|-----------------------|-----------------------|
| IO-20   | 3.73 0.68             | 180                   | 180                   |
| IO-40   |                      | 360                   | 360                   |
| IO-60   |                      | 540                   | 540                   |
| IO-80   |                      | 720                   | 720                   |
| IO-100  |                      | 900                   | 900                   |

For each dataset, there are five outputs representing temperature sensor probes generated through simulation in COMSOL. Figure 5-9 show the examples of the output produced by perturbation signals as set in the table 1 for dataset IO-40. Given a constant 360 W for the two magnetrons, the increasing trend of the temperature output for all probes illustrated in the figures. All probes, except probe-5 have an exponential increasing trend, while probe-5 has a linear one. Probe-5 has a high rise compared to other sensors because of its position where is in the middle of the object, in accordance with the concept of microwave heating that heat starts from the inside of the material towards the surface. As occurred in [12], the maximum temperature of the microwave heating process on a certain surface is
laid on its center. It makes the temperature increases faster in probe-5 rather than the others. It can also be shown from the final value of the temperature for all the probes. Probe-5 has the highest value for the temperature with more than 90 degrees in Celsius for 120 seconds, while the others are only below 70 degrees in Celsius, despite having started in the same value at 25 degrees.

3.2. Validation tests of the model candidates
From each dataset, we obtained five candidates in the form of ARX model as the mathematical equations to represent the dynamics of microwave heating system with a symmetrical octagonal tube cavity. The obtained models are then mentioned based on the name of the dataset. For example, ARX20 is the model candidate generated by dataset IO-20, and it also admit for the rest of datasets.

The obtained candidate models were then validated based on the fit value as shown in equation (3), with \( y \) as the tested model output, \( \bar{y} \) as its average, and \( \hat{y} \) as the validation data output. It is common to find the fit value of a certain model from its working dataset. In this work, the evaluation utilized the validation concept as conducted in [13] that involved validation data from one to another. For example, it allowed to check the fit value of ARX20 when perturbed by input signals from dataset IO-40 or another dataset, except from IO-20. Table 2 shows the complete validation for all candidates using given datasets.

\[
\% \text{Fit value} = 100 \times \left[ 1 - \frac{\text{norm}(y - \bar{y})}{\text{norm}(y - \hat{y})} \right] \tag{3}
\]

As shown in table 2, the highest fit value occurred when a certain model was validated using its working dataset with average fit value of all sensors for all candidates reached more than 99.9%. As for validation which involves different dataset from its working one, ARX100 performed the highest when validated using IO-80 dataset. Unfortunately, ARX100 might not be chosen as the best model candidate since it almost reached the lowest average fit value of all sensors before ARX20 with IO-100 dataset with 88.278%. Thus, it is sure that ARX20 is also not the best candidate since it had the lowest value in 88.212%. The other model that generated average fit value for all sensors close to the
lowest one is ARX80, so the remaining options are ARX40 and ARX60 that obtained more than 90% for all datasets.

| Datasets | % Average fit value of all sensors for model candidates |
|----------|--------------------------------------------------------|
|          | ARX20 | ARX40 | ARX60 | ARX80 | ARX100 |
| IO-20    | 99.974| 93.048| 90.68 | 88.948| 88.278 |
| IO-40    | 92.632| 99.98 | 97.68 | 96.322| 95.61  |
| IO-60    | 90.182| 97.684| 99.98 | 98.772| 98.052 |
| IO-80    | 88.95 | 96.516| 98.824| 99.98 | 99.274 |
| IO-100   | 88.212| 95.82 | 98.126| 99.258| 99.972 |

To determine which one is better between ARX40 and ARX60, the evaluation was continued with validation data from the outside of datasets. Two simulations were conducted to generate two new datasets named IO-30 and IO-70, which illustrated the use of 30% and 70%, respectively, of the allowed maximum power for both magnetrons as the perturbation signals. Table 3 shows the resulted validation using the new datasets.

In table 3, it can be confirmed that ARX40 and ARX60 still performed well despite having been validated with the new datasets. Fit value for all sensors reached more than 95%, except for sensor from Probe-5 (Y5) of ARX60 that only had 93.09% when validated with dataset IO-30. However, it is also ARX60 which generated the highest fit value with 99.49% when validated with dataset IO-70 for sensor Y4. Despite reaching only 98.21% as its highest fit value when validated with dataset IO-30 for sensor Y4, ARX40 looked more consistent around 97%. As for average fit value for all sensors given two datasets, ARX40 and ARX60 resulted in 97.35% and 97.34%, respectively. It shows that ARX40 can be considered as the best candidate due to its consistency and the average fit value from the two datasets.

| Datasets | Sensors | %Fit value generated by |
|----------|---------|-------------------------|
| IO-30    | Y1      | 97.87                   |
|          | Y2      | 97.87                   |
|          | Y3      | 97.88                   |
|          | Y4      | 98.21                   |
|          | Y5      | 96.6                    |
| IO-70    | Y1      | 97.26                   |
|          | Y2      | 97.26                   |
|          | Y3      | 97.27                   |
|          | Y4      | 97.69                   |
|          | Y5      | 95.61                   |

From several validation tests conducted it is confirmed that ARX40 performed the best, whether being validated with given datasets from identification process or the new datasets. Figure 10 shows the samples of ARX-40 performances when validated using IO-60 as the representative of the given datasets and IO-30 as of the new datasets. The samples were measured from sensor Y4 but considering validation data presented in table 2 and 3, the same trend must have also occurred for the other sensors. Equation (4) – (8) are the ARX40 mathematical models for all output sensors following the structure written in equation (2).
\[ y_1(t) = 0.6807y_1(t-1) - 0.04494y_1(t-2) + 0.1618y_2(t-1) - 0.08117y_2(t-2) + 0.1638y_3(t-1) + 0.06229y_3(t-2) + 0.388y_4(t-1) - 0.3347y_4(t-2) + 0.1541y_5(t-1) - 0.1496y_5(t-2) + 0.0001209u_1(t) + e_1(t) \]  

\[ y_2(t) = 0.4563y_1(t-1) - 0.4883y_1(t-2) + 0.6855y_2(t-1) + 0.09162y_2(t-2) + 0.07584y_3(t-1) - 0.009274y_3(t-2) + 0.2637y_4(t-1) - 0.07915y_4(t-2) + 0.1509y_5(t-1) - 0.1472y_5(t-2) + 9.39e - 05u_1(t) + e_2(t) \]  

\[ y_3(t) = 0.4092y_1(t-1) - 0.1671y_1(t-2) + 0.4492y_2(t-1) + 0.2042y_2(t-2) + 0.1809y_3(t-1) + 0.2047y_3(t-2) + 0.02053y_4(t-1) - 0.3145y_4(t-2) + 0.09615y_5(t-1) - 0.08522y_5(t-2) + 0.0006313u_1(t) + e_3(t) \]  

\[ y_4(t) = 0.2791y_1(t-1) - 0.06306y_1(t-2) + 0.4009y_2(t-1) + 0.06544y_2(t-2) - 0.08885y_3(t-1) - 0.3167y_3(t-2) + 0.6284y_4(t-1) + 0.08418y_4(t-2) + 0.08748y_5(t-1) - 0.07874y_5(t-2) + 0.0004898u_2(t) + e_4(t) \]  

\[ y_5(t) = -1.396y_3(t-1) + 0.4018y_3(t-2) = 0.4312y_4(t-1) - 0.2858y_4(t-2) + 0.1776y_5(t-1) + 0.03144y_5(t-2) - 0.2276y_5(t-1) - 0.1344y_5(t-2) + 0.1242y_4(t-1) - 0.1091y_4(t-2) + 0.0006028u_3(t) + e_5(t) \]  

Figure 10. ARX-40 performance when validated using IO-60 and IO-30 datasets
4. Conclusion
Taking everything into considerations, this work has contributed to presenting mathematical model in
the form of ARX model for a microwave heating system with symmetrical octagonal tube cavity. ARX40 as shown in the equation (4) – (8) are the resulted model from this work to represent the
dynamics of the heating process. Several evaluations involving generated datasets were conducted and proved to show the best-performed model among the candidates.
Considering the fit value generated by ARX40, it may be possible to develop another identification
or modeling method to perform better fit value. Since this work only evaluated through simulation
environment, a real experimental process could be another option for future work. However, building
a real system with the ability to measure all the parameters evaluated in this work could be another challenge considering the production cost of the system. That is why research in the simulation
environment could also be an unfilled space for further development, especially regarding the modeling method.

Reference
[1] Sun Y 2016 Adaptive and Intelligent Temperature Control of Microwave Heating Systems with Multiple Sources (Karlsruhe: KIT Scientific Publishing)
[2] Alonso-buenaposada I D, Menéndez J A, Calvo E G, Menéndez J A and Arenillas A 2016 Microwave heating applied to polymer science 189–191
[3] Naik T P, Rana R S, Singh I and Sharma A K 2021 Microwave Processing of Polymer Matrix Composites: Review of the Understanding and Future Opportunities Recent Advances in Mechanical Engineering eds K M Pandey, P K Patowari, R D Misra, U S Dixit (Singapore: Springer) pp 517–529
[4] Mamontov A V, Nefedov V N and Khritkin S A 2019 Meas. Tech. 62 365–370
[5] Sorrentino L, Esposito L and Bellini C 2017 Compos. Part B Eng. 109 187–196
[6] Kubo M T K, Curet S, Augusto P E D and Boillereaux L 2019 J. Food Eng. 263 366–379
[7] Navarro M C and Burgos J 2017 Appl. Math. Model. 43 268–278
[8] Cherbański R and Rudniak L 2013 Int. J. Therm. Sci. 74 214–229
[9] Tseng K H, Shiao Y F, Chang R F and Yeh Y T 2013 Materials (Basel) 6 3404–3419
[10] Yuan Y, Liang S, Zhong J, Xiong Q and Gao M 2015 The 27th Chinese Control and Decision Conference (2015 CCDC) p 4116–4120
[11] Prastiyanto D, Astuti W, Mahmud A and Wijaya B 2020 J. Phys. Conf. Ser. 1444 012016
[12] Binner J G P, Al-Dawery I A, Aneziris C and Cross T E 1992 MRS Proc. 269 357
[13] Fahmizal, Arrofiq M, Apriaskar E and Mayub A 2019 2019 6th International Conference on Instrumentation, Control, and Automation (ICA) p 52–57

Acknowledgments
The authors would like to thank the LPPM UNNES for the financial support of this research project.
We also appreciate the Faculty of Engineering and Department of Electrical Engineering for encouraging this work.