Crack detection in rotating shafts using combined wavelet analysis

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Abstract. There are many rotating mechanical systems on board ships that comprise the main shaft transmission element. Proper functioning of shafts depends on many factors: alignment system, connections with other elements – couplings, gearboxes, bearing elements with ball bearings or bushings, operated regime, and al. All these factors can lead to fatigue of the shaft over time and to appearance of internal/external cracks which can determine a total damage of the shaft at some point in time. To avoid these disasters the operation of these rotating systems is monitored based on vibration analysis of recorded signals (FFT and DFT analysis). Due to the transient modes and the presence of the noise, the processing of the recorded signals must be performed with methods that present a good resolution simultaneously in the time and frequency domain. In the paper, measurements are made on a stand with a rotating system and it is determined the influence of some factors (measuring points and directions, speed, operated mode, crack size, presence of misalignment, noise) on the quality of the Wavelet Transform results. In order to improve the clarity of amplitudes and frequencies changes at the initiation of shaft cracks, the authors are proposing the combined use of Continuous Wavelet Transform (CWT), Discrete Wavelet Transform (DWT) and Wavelet Packet Transform (WPT). The authors propose using power spectral density (PSD) as it is a method with good performance in detecting faults in rotating machines.

1. Review of crack shaft detection

The occurrence of cracks in rotating shaft is caused by fatigue, defects in material structure and surface corrosion in the environment. During rotating, shaft is subjected to a variable load, but especially to torsional loads which cause stresses and strains cycles. The presence of stress concentrators and plastic deformations determines the cracks initiation and propagation, thereof to the failure of the shaft. Cracks in rotating shafts lead to reduced local rigidity due to the deformation energy concentration near the crack. Geometrically, the cracks are classified as transverse cracks (perpendicular to the axis of the shaft), longitudinal cracks (parallel to the axis of the shaft), oblique cracks, cracks that close and open during spinning (“breathing”), cracks permanently opened (“gaping”).

The cross-sectional fracture modeling was performed by introducing an elastic matrix of the cracked section [1]. The Finite Element Method was used by S.El. Arem and H. Maitournam [2] for modeling a rotating shaft with transverse cracks. The classic methods of crack detection (ultrasonic...
testing, lamb wave, X-ray, acoustic emission, etc.) are dependent on amplitude and can only be applied under specific conditions. Methods for identifying cracks in rotating shafts are divided into two categories: modal testing methods and vibration analysis methods. In , On the dynamics of cracked rotors: A literature survey “[3], Wauer presents a review of the dynamic field of cracked rotors, including the modeling of the cracked part of the structure and finding different detection procedures to diagnose fracture damage. Sabnavis et al. [4] reviews the literature on cracked shaft detection and diagnostics published after 1990. Kumar and Rastogi [5] made an inventory of the methodologies adopted by various researchers to investigate a cracked rotor.

The presence of cracks in the rotating shaft causes a decrease/increase of the amplitudes of certain frequencies or harmonics of order 2, 3 or the occurrence of new frequencies in the spectrum.

Due adaptability and processing capabilities of multi-resolution, wavelet transform (WT) is a powerful mathematical tool for diagnosing rotating machinery operation.

Ruqiang Yan et al. [6] provide a review on recent applications of the wavelets: continuous wavelet transform, discrete wavelet transform, wavelet packet transform, and second generation wavelet transform-based fault diagnosis.

For the location of the impulse/transient phenomenon in the time-frequency domain, the Continuous Wavelet Transform (CWT) is applied. The signal that contains the different amplitudes impulses in the time domain is isolated and CWT is calculated. Frequency content in the CWT scale-time provides a clearer interpretation of the vibrational behavior of the shaft.

A method of accurately determining defects in the shafts was proposed by S. Guo et al. [7], using the continuous wavelet transform scalogram (CWTS), which is obtained by decomposing the recorded signal to the rotating shaft at different scales, using wavelet transform.

Nagaraju et al. [8] use the phase angle information (between the two signals cracked/ un-cracked or two transverse vibrations), added to the 3D (CWT) representation for crack detection in the rotor, which can be a better crack indicator because it is much less sensitive to noise disturbance.

Discrete Wavelet Transform (DWT) is successfully used to eliminate the noise and highlights the peak in the recorded vibration signal. Sawicki et al., using DWT, have performed a multiresolution analysis of the measured vibration signal from the healthy rotor and a rotor with a transverse crack [9].

A new fault diagnosis method based on Wavelet Packet Decomposition (WPD) and Empirical Mode Decomposition (EMD) and BP network is proposed for extracting the feature for weak signals and diagnosing the early fault by Bin et al. [10]. In their work [11], Goomez et al. present studies on vibration signals obtained from a rotating shaft under different crack depths and locations. Signals obtained are analysed by means of energy using the Wavelet theory, specifically the Wavelet Packets Transform.

An innovative method, named Self-adaptive Entropy Wavelet (SEW), is proposed to detect incipient transverse crack faults on rotating shafts by Huo et al. [12]. CWT is applied to obtain optimized wavelet function using impulse modelling and decomposes a signal into multi-scale wavelet coefficients. Dominant features are then extracted from those vectors using Shannon entropy, which can be used to discriminate fault information in different conditions of shafts. Support Vector Machine (SVM) is carried out to classify fault categories which identify the severity of crack faults.

In order to improve the clarity of amplitudes and frequencies changes at the initiation of shaft cracks, the authors are proposing the combined use of Continuous Wavelet Transform (CWT), Discrete Wavelet Transform (DWT) and Wavelet Packet Transform (WPT). It is also proposed to use the concept of power spectral density (PSD), estimated by wavelet transform in spectral because clearly shows those frequent power variations are large.

2. Wavelet Transform theory

2.1 The Continuous Wavelet Transform (CWT)

CWT is a common signal-processing tool for the analysis of nonstationary signals. CWT of a finite energy signal f(t) and is defined by relation [6, 13]:

\[ \text{CWT}(a,b) = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{\infty} f(t) \psi^{*}(a^{-1} (t-b)) \, dt \]

where \( \psi^{*} \) is the complex conjugate of the wavelet function, and a and b are scale and translation parameters, respectively.
\[ CWT(s, \tau) = \frac{1}{\sqrt{s}} \int f(t) \cdot \psi^* \left( \frac{t-\tau}{s} \right) dt, \]  
(1)

where \( s \) and \( \tau \) are real numbers and represents the scale and translation parameter, the \(^*\) symbol represents the complex conjugate operator of the wavelet function \( \psi_{s,\tau}(t) \):

\[ \psi_{s,\tau}(t) = \frac{1}{\sqrt{s}} \psi \left( \frac{t-\tau}{s} \right) \]  
(2)

CWT is the result of repeated evaluation of the intercorrelation between the considered signal \( f(t) \) and the translated versions of an ensemble of wavelet functions affected by various degrees of dilation or contraction. CWT of a signal \( f(t) \) can work as a band-pass filter with the wavelet function itself and its scale parameters control the filtering performance.

The CWT evaluation involves calculating scalar products between the analyzed signal and the wavelet functions collection \( \psi_{s,\tau}(t) \):

\[ CWT(s, \tau) = \langle f(t), \psi_{s,\tau}(t) \rangle. \]  
(3)

The results of this transformation are called wavelet coefficients and show how well the function correlates with the signal. It is possible to identify the particular values of parameters \( s \) and \( \tau \) for which a signal \( \psi_{s,\tau}(t) \) is strongly correlated with the analyzed signal by plotting the function graph \( |CWT(s, \tau)|^2 \) (the graph is called scalogram) and identifying the maximum values. It has been found that damage due to a sudden loss of stiffness and the moment when it occurs, creates wavelet coefficients with large amplitudes like a spike or an impulse.

2.2 The Discrete Wavelet Transform (DWT)

CWT provides a redundant representation of the signal in terms of scaled and translated wavelet functions derived from the continuous mother wavelet function. The scaling parameter \( s \) depends on the mother function and is closely related to the frequency. CWT is calculated for an infinite number of scales and translations, but to apply to discrete finite signals, both scale and translation are discretized. The scale range is set to cover all frequencies present in the signal based on the conversion between scale and frequency. Several methods of discretization can be used: linear, dyadic and exponential.

In dyadic meshing, the scale and translation parameters are expressed through relationships \( s = 2^j, \tau = k2^j \), which shows that the translation step is not fixed and depends directly on the scale.

Mathematically DWT is expressed through the relationship:

\[ DWT(j, k) = \frac{1}{\sqrt{2^j}} \int f(t) \cdot \psi^* \left( \frac{t-k2^j}{2^j} \right) dt, \]  
(4)

DWT is based on the use of digital filters to optimize the computational process, generating information in frequency bands.

The original signal is first processed by a pair of filters type low-pass and high-pass \( h(k) \) and \( g(k) = (-1)^k h(1-k) \). The output of the low pass filter is in turn processed again with a pair of filters of the same nature, with the observation that their cutting frequencies are halved from the first pair. The mechanism is then continued for a number of steps. The filters are built from the selected wavelet function \( \psi(t) \) and the corresponding scaling function \( \phi(t) \), expressed by the relationships [14]:

\[
\begin{align*}
\phi(t) &= \sqrt{2} \sum_k h(k)\phi(2t - k) \\
\psi(t) &= \sqrt{2} \sum_k g(k)\psi(2t - k)
\end{align*}
\]  
(5)

with \( \sum_k h(k) = 2 \) and \( \sum_k g(k) = 0 \).

The wavelet filters decompose the recorded signal into low and high-frequency components:

\[
\begin{align*}
a_{j,k} &= \sum_m h(2k - m)a_{j-1,m} \\
d_{j,k} &= \sum_m g(2k - m)a_{j-1,m}
\end{align*}
\]  
(6)

\( a_{j,k} \) is the approximation coefficient, which represents the signal's low-frequency components, and \( d_{j,k} \) is the detail coefficient, which corresponds to the signal's high-frequency components.
2.3 The Wavelet Packets Transform (WPT)

WPT decompose all approximations and detail components to the desired level. Under these conditions wavelet functions and scaling functions (5) become:

\[
\begin{align*}
    u_{2n}(t) &= \sqrt{2} \sum_k h(k)u_{2n}(2t - k) \\
    u_{2n+1} &= \sqrt{2} \sum_k g(k)u_{2n+1}(2t - k)
\end{align*}
\]

The signal is decomposed into two sets of sub-signals [15]:

\[
\begin{align*}
    d_{j+1,2n} &= \sum_m h(m - 2k)d_{j,n} \\
    d_{j+1,2n+1} &= \sum_m g(m - 2k)d_{j,n}
\end{align*}
\]

Where \(d_{j,n}\) denotes the wavelet coefficients at the j level, n sub-band, \(d_{j+1,2n}\) and \(d_{j+1,2n+1}\) denotes the wavelet coefficients at the j+1 level, 2n and 2n+1 are sub-bands and m is the number of the wavelet coefficients.

3. Experiment and results

3.1 Experimental Procedure

The shaft crack experiment was performed on the „PT 500.11 - Crack Detection in the Rotating Shaft Kit“ which allows simulation of the crack in the shaft [16]. The test kit consists of an electromotor (1), a rotary shaft (5) with a flange simulating the crack, two bearing blocks (4) and a drive belt (7) that loads the shaft with a torque. The crack simulation is performed with an asymmetrical flange (6) connected to the shaft. To highlight the crack during rotating the shaft is loaded with bending torque from the belt drive (7). For the test, all six screws (8) of the flange of a healthy shaft are connected without clearance. To simulate the cracks with different depths, one, three or four screws are fitted as a loose connection.

For the diagnosis of cracks in the rotating shaft it was used equipment from Bruel&Kjaer, consisting in LAN-XI Data Acquisition Hardware for PULSE, accelerometer (3) DeltaTron 4506, laptop with PULSE 14 software for measurement, recording and data processing. The two accelerometers were located on the two bearing blocks. Measurements were made in two stages: the first set with the vertical accelerometers; the second set of tests with the transversally horizontally accelerometers. The accelerometers were located on the two bearing blocks. At each set of tests, vibration signals were simultaneously recorded at the two accelerometers, at three shaft speed ranges (600 rpm, 1200 rpm and 2400 rpm) and four shaft situations (healthy shaft, depths of the crack 16%, 50 %, 66%). The electromotor speeds were measured with a tachometer.

**Figure 1.** Experimental setup for “shaft with crack” simulated with elastic rotor [16]

**Figure 2.** Crack simulation in rotating shaft kit
3.2 Results

For greater accuracy in determining the crack in the rotating shaft, the Wavelet Transform (WT) sets provide a powerful technique for characterizing variations in both the time and frequency domains. The work proposes the combined use of the wavelet transforms (Continuous Wavelet Transform (CWT), Discrete Wavelet Transform (DWT) and Wavelet Packets Transform (WPT)) for to decompose signals in certain areas of the frequency spectrum of interest.

By using the CWT method for detecting cracks in rotating shafts, in the represented scalograms the magnitude of the subcritical and critical peaks of the wavelet coefficients are considered as fracture indicators. For example, in figures 3 and 4 are shown CWT coefficients at 2400 rpm shaft speed, horizontal accelerometers, healthy shaft and depth crack 66%. The scalograms in Figures 3 and 4 shows the continuous wavelet transform of the signals with a Morlet wavelet function. From the CWT scalogram of the shaft crack it can be seen that the resonance frequencies and their harmonics are more obvious.

DWT and WPT transforms have been applied combined, directly in signal decomposition, and also as preprocessing techniques in signal analysis, such as band-pass filters and wavelet de-noising tools.

The frequency range of the recorded signal is 0-1600 Hz. By using DWT with the wavelet function Db3, level 5, we produce the decomposition of the signal up to the low-frequency range 0-50 Hz, which includes the three drive shaft frequencies (10 Hz, 20 Hz, and 40 Hz). The discrete wavelet transform (DWT) permits a time-frequency decomposition of the input signal, but the degree of frequency resolution in the DWT is typically considered too coarse for practical time-frequency analysis.

The approximates a5 of DWT is loaded into the WPT toolbox and processed with the Haar wavelet function, level 3. Also, WPT is applied to de-noising signals by means of thresholding methods.

The power spectral density (PSD) is defined as the total energy density of the recorded signal depending on frequency.
Figure 4. Wavelet coefficients (CWT) for a shaft crack of 66% (2400 rpm, 4096 samples)

To estimate the power spectral density is used non-parametric methods through mediation and smoothing operations performed directly on periodogram and autocorrelation function.

Better accuracy in defining a rotating shaft defects are given due to the power spectral density used in the work, by converting the wavelet to the spectral domain to obtain those frequencies at which variations are high. Estimating PSD is done by the wavelet de-noising process of WPT coefficients.

PSD diagrams are presented in the work (Figures 5-11) to highlight the influence of factors in the accuracy of crack detection in rotating shafts: measuring points (channel 1 near the crack, channel 2 next to the electromotor) and directions, speed, operated mode, crack size, the presence of misalignment and noise.

Figure 5. Healthy shaft measured in the horizontal direction at a speed of 2400 rpm

a) Measurement sensor at channel 1  
b) Measurement sensor at channel 2
Figure 6. Shaft crack (66%) measured in the horizontal direction at a speed of 2400 rpm

Figure 7. Shafts at different speeds measured at channel 1
c) Shaft crack of 50 %

**Figure 8.** Shafts with different crack depth, speed 2400 rpm, horizontal measuring, channel 1

a) Healthy shaft

b) Shaft crack of 16%

c) Shaft crack of 50%

d) Shaft crack of 66%

**Figure 9.** Shafts with different crack depth, speed 2400 rpm, horizontal measuring, channel 2

a) Vertical measuring

b) Horizontal measuring

**Figure 10.** Vertical and horizontal sensors, the speed of 2400 rpm, measuring channel 2, the depth of the crack 66%
4. Discussions and conclusions

In order to improve the clarity of amplitudes and frequencies changes at the initiation of shaft cracks, the authors are proposing the combined use of Continuous Wavelet Transform (CWT), Discrete Wavelet Transform (DWT) and Wavelet Packet Transform (WPT).

By processing the signal recorded with wavelet transforms in the time-frequency domain new features of the signal are highlighted, including the presence of new frequencies or the modification of the amplitudes of some of their components or their higher harmonics. Stronger noise than the actual failure of the signal may lead to misrecognition of the useful information for diagnosis.

The authors propose using power spectral density (PSD) as a method with good performance in detecting faults in rotating machines. Energy or power spectral density is concerned with the distribution over the signal energy or power over the frequency domain. MATLAB toolbox uses non-parametric computation of the PSD and drawing of it. The estimating method of the spectral power density used is based on the threshold of the wavelet coefficient of a periodogram. The random signal which is responsible for the variation of the periodogram, can be removed by the wavelet transform coefficient threshold. The coefficients of Wavelet Packets Transform will be calculated. Figures 5-11 shows the samples of PSD/Frequency -Normalized Frequency diagrams of de-noising signals acquired for various experimental conditions of the shaft. According to these diagrams it is obvious that at each working speed the maximum values of frequencies and their harmonics of the shaft from PSD are increased by increasing the severity of cracks in the shaft.

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