The $1/N_c$ expansion ($N_c$ being the number of QCD colors) has been applied in recent papers to the phenomenology of excited baryon resonances. This talk surveys the work done to date, and discusses its successes and remaining challenges.

1. Introduction

Baryon resonances represent a particularly striking example of “physics in our own backyard.” The technology exists to carry out a vast array of interesting and incisive experiments to uncover precise information about their nature, and researchers are limited only by access to financial and human resources to accomplish this program. From the theoretical point of view, however, $N^*$’s are rather peculiar objects; indeed, for anyone approaching QCD as a pure gauge theory of quarks confined by the gluon field, the whole hadron spectrum is enigmatic. One certainly expects bound quark states to exist in some form, but fundamental (current) quarks account for only a small portion of the hadron wave function. Why the simple quark model should be so successful in identifying the quantum numbers of not only ground-state hadrons but excited ones as well is a long-standing mystery.

In the version of this talk presented at NStar 2002, I recognized that large $N_c$ QCD is still a rather exotic topic for most people in our field, and raced through a 15-minute introduction to the $1/N_c$ expansion before getting to the central issue of baryon phenomenology. Here I have the luxury of simply pointing to a set of summer school lectures\footnote{presented some years ago that contain all the introductory material, and that you may peruse at your leisure. For the purpose of these Proceedings I reprise only the central points:}

- There is nothing intrinsically crazy about letting the number $N_c$
of QCD color charges be some number $> 3$. QCD, it turns out, would not be qualitatively radically different were $N_c$ odd and $> 3$. Baryons would then be fermions carrying the quantum numbers of $N_c$ quarks, and hence would be much heavier than $q\bar{q}$ mesons.

- The $1/N_c$ expansion organizes the infinite number of Feynman diagrams for a given process into distinct classes based on the power of $N_c$ arising in each. These $N_c$ factors arise from the 't Hooft scaling\(^2\) of the strong coupling constant, $\alpha_s \propto 1/N_c$ (which, it turns out, is the unique sensible way to take the large $N_c$ limit), and combinatoric factors from closed color loops. The suppression of a class of diagrams by fewer powers of $1/N_c$ means greater physical significance.

- A number of phenomenologically observed results in meson physics follow directly from the large $N_c$ limit. These include the decoupling of glueballs from ordinary mesons, the OZI rule, and the apparent dominance of heavy meson resonances over multi-pion states (e.g., vector meson dominance), even when the latter are greatly favored by phase space.

- In the large $N_c$ limit, spin and flavor symmetries for baryons combine into a spin-flavor symmetry.\(^3,^4\) When 3 light flavors are included, this is the famous SU(6) symmetry. That is to say, SU(6) is an approximate symmetry for baryons, broken by effects of $O(1/N_c)$. The baryon ground states fill a multiplet that generalizes the SU(6) 56-plet and contains the $N$ and $\Delta$. The $1/N_c$ expansion thus gives a field-theoretic explanation for the successes of 1960's-vintage SU(6) results: For example, $\mu_p/\mu_n = -3/2 + O(1/N_c)$.

- Baryons in $1/N_c$ may be considered in a Hartree approach, i.e., each quark sees (to lowest order in $1/N_c$) the collective effect of the other $N_c - 1$.\(^5\) Using this and the 't Hooft scaling, it is possible to show that baryons have a characteristic size of $O(N_c^0)$; they do not grow to arbitrarily large dimensions as $N_c \to \infty$.

- It is possible to study baryon observables in the $1/N_c$ expansion by studying operators that break the spin-flavor symmetry.\(^5,^7,^8\) Each such operator has a well defined $1/N_c$ power suppression (from counting the minimum number of gluons necessary for such an operator to appear in an interaction), and a possible enhancement from combinatoric powers of $N_c$ if the $N_c$ quarks contribute coherently to the operator’s matrix element. Since the number of baryons in a given spin-flavor representation is finite, the number
of operators that can give linearly independent matrix elements, just like the basis of a vector space, is also finite.

- The $1/N_c$ expansion provides a natural way to define in a rigorous way what is meant\(^9\) by a “constituent quark.” Inasmuch as physical baryons fill well-defined spin-flavor representations whose Young tableaux consist of $N_c$ fundamental-representation “boxes,” the full physical baryon wave function (as determined through observable amplitudes) can be chopped in an unambiguous way into $N_c$ quark interpolating fields. That is, each box represents a well-defined field whose quantum numbers are those of a quark, such that when all $N_c$ of them are recombined, the full baryon wave function is recovered. Such a field may rightly be called a constituent quark; in terms of fundamental fields it consists of many Fock components: $q$, $qq$, $qgg$, $qq\bar{q}$, etc.

Using the Hartree picture and the interpretation of quark fields just described, one may suppose that the first orbitally-excited baryons (the ones corresponding to the negative-parity states such as $N(1535)$, $\Lambda(1405)$, etc.) should be treated as a spin-flavor symmetrized “core” of $N_c-1$ quarks and a single quark excited to a relative orbital angular momentum $\ell = 1$. Does this picture produce a phenomenology in agreement with experiment? Certainly when $N_c = 3$ it generates the same quantum numbers for $N^*$’s as seen in the conventional quark model. However, before examining the quantitative results, let us digress briefly to see what happens with $1/N_c$ analysis for the ground-state baryons.

The operator analysis itself is essentially a complicated version of the Wigner-Eckart theorem. One writes down an effective Hamiltonian consisting of a sum over all possible linearly-independent spin-flavor operators, including their $1/N_c$ and other [e.g., SU(3) flavor symmetry-breaking $\epsilon \approx 0.3$] suppressions:

$$\mathcal{H} = \sum_i \frac{c_i}{N_c^{3\ell}} \mathcal{O}_i,$$  \hspace{1cm} (1)

where $\mathcal{O}_i$ are spin-flavor operators whose matrix elements are determined entirely by group theory (Clebsch-Gordan coefficients), and $c_i$ are unknown numerical coefficients (reduced matrix elements) that could be calculated from the dynamics of nonperturbative QCD (e.g., on the lattice), but can also be extracted from experiment.

Given a set of observables, one can then determine if the $1/N_c$ expansion describes the system successfully. Once all dimensionful parameters are
removed (for example, by taking ratios of observables), the $c_i$'s should be of order unity. If they are much larger, then the $1/N_c$ expansion has failed; if much smaller, then some undetermined physics is required beyond the $1/N_c$ expansion. This program was first carried out for the ground-state baryons,\textsuperscript{10} and the results for the isoscalar combinations are presented in Fig. 1. We see there that each suppression by powers of $N_c = 3$ (as well as $\epsilon$) is clearly visible, consistent with the hypothesis that the $c_i$'s are all of a “natural” size; one concludes that the whole ground-state spectrum is given in a natural way by the $1/N_c$ expansion, even for $N_c$ as small as 3.

![Figure 1. Isoscalar mass combinations of the ground-state baryon multiplet in the $1/N_c$ expansion. $\epsilon \approx 0.3$ denotes SU(3) flavor breaking.](image)

For example, the point labeled by $\epsilon^2/N_c^2$ in Fig. 1 measures the amount by which a particular combination of Gell-Mann–Okubo and Gell-Mann decuplet equal-spacing rules [each of which is broken at $O(\epsilon^2)$] is violated.
relative to the averaged mass of ground-state baryons ("experimental accuracy"). The relevant operator is $O_i = \{T_8, T_8\}/N_c$, where $T_8$ is formed by sandwiching the Gell-Mann matrix $\lambda_8$ between the baryon quark fields.

A detailed calculation in this case leads to the coefficient $c_i = 1.09 \pm 0.03$. Similar results obtain for all the other combinations. Had we dismissed the $N_c$ factors as irrelevant, we would then have obtained $c_i \approx 1/9$ and similar power-of-3 deficits in the other mass combinations, indicating that including the factors of $1/N_c$ is essential to understanding the baryon mass spectrum. It is important to note that the old SU(6) or quark-model fits to baryon masses tended to fit each mass individually ($p$, $n$, $\Sigma^+$, etc.), whereas this approach fits to the smallest mass differences available, a much more precise test of the symmetry. Indeed, though not presented here, the successes of $1/N_c$ continue in the isospin-breaking mass differences as well.\textsuperscript{10,11}

Many other ground-state observables, such as axial couplings, magnetic moments, charge radii, quadrupole moments, and the spectrum of baryons containing a heavy quark have been considered in the operator formalism, with a high degree of success. For sake of space, I merely point out a recent list of references.\textsuperscript{12}

This conference, however, is about $N^*$'s. To begin with, what happens when the operator approach is applied to the $N^*$ mass spectrum? For much of the remainder of this talk, let us consider the resonances in the negative-parity multiplet.\textsuperscript{13,14,15,16,17}

It is possible to carry out an operator analysis for the excited states just as we have done for the ground states, although it is a bit more complicated: One must distinguish operators acting upon the $(N_c - 1)$-quark core versus the excited quark and the orbital angular momentum connecting them, and this introduces a larger operator basis. Nevertheless, the calculations have been done and a remarkable result obtains: Whereas the coefficients $c_i$ for the ground states are all $O(1)$, this is true only for a subset of the $c_i$'s in the excited states, the remainder being much smaller. Does this mean that the $1/N_c$ expansion has failed here? Not at all—in fact, it indicates that not only are the appropriate $1/N_c$ suppressions present, but they must be enhanced by some additional dynamical suppression (chiral symmetry, perhaps).

Table 1 demonstrates this point by presenting results of such a fit to coefficients. The labels $S$, $T$, and $G$ refer to spin, flavor, and spin-flavor operators, respectively, uppercase (lowercase) indicate those acting upon the core (excited quark), and $\ell$ is the excited quark relative orbital angular momentum operator. One difference compared with the ground-
state analysis is that the $c_i$’s here have dimensions of mass and should be thought of rather as $c_i \Lambda_{\text{QCD}}$, whose natural magnitude is $\sim 500$ MeV. The coefficients $d_i$ are those of SU(3)-breaking operators, and should have typical sizes $\sim \epsilon c_i \Lambda_{\text{QCD}} \sim 150$ MeV. It is clear that only $c_{1,3,4,6,7}$ and $d_2$ appear to be of a natural size, the remainder rather smaller.

A number of interesting conclusions follow from these results, among which: 1) It is perfectly natural that the $\Lambda(1405)$ is the lightest $N^*$, despite containing a strange quark: The hyperfine operator $O_6$ does not contribute to SU(3) singlet states but pushes all the others up 200–300 MeV. 2) The value obtained for the $N(1535)$-$N(1650)$ and $N(1520)$-$N(1700)$ mixing angles is stable whether one fits the coefficients using either pionic decay, photoproduction, or $N^*$ masses. 3) Most significant to obtain a good fit to mixing angles is the inclusion of the flavor-dependent tensor $[t(2)]^\prime$ operator $O_3$. 4) The spin-orbit coupling $(c_2)$ is not large, but nevertheless explains the $\Lambda(1520)$-$\Lambda(1405)$ splitting.

| $O_i$ | $c_i$ (in MeV) |
|------|----------------|
| $O_2 = \ell_i \cdot s_i$ | $c_2 = 52 \pm 15$ |
| $O_3 = \frac{1}{4} s_i^2 g_{ia} G^\ast_{ja}$ | $c_3 = 116 \pm 44$ |
| $O_4 = \frac{1}{\sqrt{6}} \ell_i \cdot t_a G^\ast_{ia}$ | $c_4 = 110 \pm 16$ |
| $O_5 = \frac{3}{4} s_i S_i^c$ | $c_5 = 74 \pm 30$ |
| $O_6 = \frac{1}{\sqrt{6}} S_i^c S_i^c$ | $c_6 = 480 \pm 15$ |
| $O_7 = \frac{1}{\sqrt{15}} s_i S_i^c$ | $c_7 = 159 \pm 50$ |
| $O_8 = \frac{2}{\sqrt{15}} t_i S_i^c$ | $c_8 = 3 \pm 45$ |
| $O_9 = \frac{1}{\sqrt{15}} \ell_i g_{ja} \{S_i^c, G^\ast_{ja}\}$ | $c_9 = 71 \pm 51$ |
| $O_{10} = \frac{1}{\sqrt{15}} t_a \{S_i^c, G^\ast_{ja}\}$ | $c_{10} = -84 \pm 28$ |
| $O_{11} = \frac{1}{\sqrt{15}} \ell_i g_{ja} \{S_i^c, G^\ast_{ja}\}$ | $c_{11} = -44 \pm 43$ |

There have also been studies of $N^*$ production and decays using the operator approach^{18,19,20} (A very nice review of these works is available^{21}).

For example, one may analyze$^{18} N^* \rightarrow N \gamma$ using the $1/N_c$ expansion,
for which 19 modes have been measured. Operators may be classified according to the number of quark lines they connect. (In the case of Table 1, $O_6$ is a 2-body operator and $O_{10}$ is a 3-body operator). Owing to the possibility (discussed above) of coherent matrix elements it is possible, for instance, for a 2-body operator to have the same overall power of $N_c$ as a 1-body operator. Such is the case for the operators

$$Q_1 \bar{e}_* \cdot \bar{e}_\gamma$$

and

$$\left( \sum_{\alpha \neq *} Q_\alpha \frac{\vec{S}_\alpha}{N_c} \right) \cdot \vec{S}_*(\bar{e}_* \cdot \bar{e}_\gamma),$$

(2)

where $*$ refers to the excited quark. As before, all of the coefficients turn out to be at most of the expected size. However, a detailed fit shows that the 1-body operators by themselves are sufficient to explain the current data; the 2-body operators do not significantly improve the $\chi^2$. One reaches the remarkable conclusion that the $1/N_c$ expansion again is working, but other physics appears to be required to achieve the desired additional suppressions of many possible terms.

Starting with this empirical observation that 1-body operators dominate the $N^* \to N \gamma$ decays, one may now proceed to predict quite a number (24) of $N^* \to \Delta \gamma$ amplitudes. And while reconstructing such a process experimentally may be a challenging task, careful analysis using the huge data set at facilities such as Jefferson Lab can lead to the extraction of the relevant amplitudes, and hence test the 1-body ansatz.

One may also study excited baryons in a completely symmetric spin-flavor multiplet (what for $N_c = 3$ would be called a $56'$). Again using the 1-body approximation, many (22) predictions for partial widths of the processes $56' \to 56+$ meson obtain. Equally interesting are mass predictions of the unobserved strange members of this multiplet, such as $\Sigma^{*'} = 1790 \pm 192$ MeV. One thrust of these studies is directed toward answering the very interesting question of whether the Roper $N(1440)$ is truly a 3-quark state ($N_c$ quarks in large $N_c$, of course), or a mixture with hybrid $qqqg$ states, 5-quark $qqqq\bar{q}$ states, or others. A careful global analysis using mass and decay information within the $1/N_c$ expansion may sort this out.

The conclusion one draws is that there is something special about the $N^*$’s for arbitrary $N_c$, in that not only $1/N_c$ suppression powers are manifest, but some other dynamics is at work minimizing the effects of many of the possible operators. The particular origin of this physics is a topic currently under study.

Much has been made at this meeting about whether the quark inter-
actions giving rise to the baryon spectrum require flavor dependence. Of course, flavor exchange is a natural consequence of meson exchange potentials, while quark potentials traditionally tend to include spin exchange but not flavor exchange. The $1/N_c$ approach includes both flavor-dependent and -independent operators, and simply deduces which ones turn out to be favored (based on the sizes of their coefficients) from fits to data.

Now, in the completely symmetric ground-state baryons (and restricted to a fixed value of strangeness), the group theory is such that the effect of operators with flavor dependence may always be rewritten as arising from equivalent flavor-independent operators. In the mixed-symmetric negative-parity $N^*$'s, however, this is no longer true, since the system is explicitly separated into core and excited parts, and one may follow the flow of flavor between the two in operators such as $\ell^{(2)} g G$.

But these operators have the same formal composition as the sort that one could write down in a quark model. For example, $\ell^{(2)} g G$ represents a tensor coupling between the excited quark and the core, where not only spin but isospin is exchanged between the two. This can be accomplished by the excited quark trading places with a quark in the core, a perfectly valid event in the quark model. The standard tensor operator in the quark model would be represented as $\ell^{(2)} s S$ in this notation. If one simply includes both operators and lets the $\chi^2$ fit to the spectrum pick its favorite, one finds that the former is preferred to the latter, meaning that flavor exchange rightfully belongs in the phenomenological quark model for these states.

On the other hand, $\ell^{(2)} g G$ can occur through the exchange of a quark-antiquark pair between the excited and core systems; a quark moving from left to right and an antiquark moving from right to left have the same Feynman diagram representation. This immediately suggests a meson exchange; however, that conclusion only holds if the $q\bar{q}$ pair is correlated in a very particular way. If the time ordering of the two quarks is not so tightly constrained (e.g., the $q$ is emitted by the core long before the $\bar{q}$), the exchange in this single event can only be represented properly as a linear combination involving the overlap of many meson exchanges.

So one sees that the $1/N_c$ expansion accommodates both quark and meson pictures, and there are no contradictions between the two, if only each picture allows for a more expansive definition of the possible phenomena available to each.

Finally, I would like to draw your attention to brand-new work done with Tom Cohen. Note that the sort of analysis used above relies on the assumption that the first band of excited baryons consists solely of single-
quark excitations of ground-state baryons; that is, all forms of configuration mixing are assumed suppressed. Moreover, since real resonances are of course unstable states with appreciable widths while the Hamiltonian used above contains no coupling to decay channels, this analysis can strictly only teach one about the real parts of resonant pole positions.

In fact, it is possible to study scattering partial-wave amplitudes (wherein $N^*$’s are observed in the first place) in the context of $1/N_c$. It has been known for 20 years\cite{1} that a number of linear relations intertwine meson-baryon scattering amplitudes at their leading order, $O(N_c^0)$; a simple example is $S^{\pi N}_{11} = S^{\pi N}_{31}$. Since the $N^*$’s represent poles in these amplitudes, the pole positions themselves must also be equal up to $O(N_c^0)$. That is, every $N_{1/2}$ state that couples to $\pi$-$N$ must be degenerate with a similar $\Delta_{1/2}$ state, up to $O(1/N_c)$ corrections.

Naturally, this begs the question of whether the operator analysis of $N^*$ masses is completely compatible with the full set of relations among the partial-wave amplitudes. A priori one might think that our picture of $N^*$’s has been too naive, that contradictions might arise and that would only be resolved by the inclusion of some complicated form of configuration mixing dictated by the $1/N_c$ expansion. But in fact the two pictures combine seamlessly\cite{2} and complement each other: The amplitude relations never demand any resonances at all, but once resonances are deemed to exist, they must obey certain degeneracies; and the operator approach gives no indication that there are any degeneracies at all between the given states before the matrix elements are computed, but they nevertheless appear and must be explained.

A remarkable result of these degeneracies is that some of the resonances couple to certain meson-baryon channels and not others at leading $1/N_c$ order. For example, the state corresponding to $N(1535)$ decays at leading order exclusively to $\eta$-$N$ rather than $\pi$-$N$, and vice-versa for the $N(1650)$. As experts of $N^*$ physics are well aware, the strong $\eta$-$N$ coupling for $N(1535)$ and weak one for $N(1650)$ have always been among the resonances’ most remarkable features. Furthermore, the mixing angles between resonances of the same $I, J$ values are predicted as simple pure numbers at leading order ($N_c^0$). Work in this area continues,\cite{3,4} with a full treatment of $1/N_c$ corrections next on the agenda.
Acknowledgments

I am grateful to Jefferson Lab for travel support to the conference, and to the organizers of the Conference for local support. This work was funded in part by the National Science Foundation under Grant No. PHY-0140362.

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