About Geometry and Initial Phase of Cloud-to-Ground Lightning

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Abstract

Cloud-to-ground lightning is the most common among atmospheric discharges. Since electric fields in the vicinity of a thunder-cloud do not exceed 250 kV/m the physical process that triggers the lightning remains unexplained. Recent measurements established a weak correlation between solar wind and incidence of lightning. Here we show, that if an ionized path created by cosmic rays provides a trigger, the distribution of lengths between two successive forking points in a lightning channel (internodes) closely resembles the exponential distribution with average length between 415 m and 510 m. The results, if confirmed, imply that a thunder-cell may be an additional source of fast elementary particles that initiate lightning process.

1 Reaction types and lightning topology

A charged particle creates an ionized path when passing through the atmosphere. Any collisions of the projectile with nuclei in the atmosphere may produce additional projectiles creating a fork in the path. The result of this process is a tree structure of ionized paths, whose geometry matches the geometry of the stepped leader and that of subsequent lightning.

The charged projectiles in cosmic showers are protons $p^\pm$, pions $\pi^+/\pi^-$, kaons $K^+/K^-$, and muons $\mu^+/\mu^-$. The roles of electrons and positrons are here neglected due to at least four orders of magnitude lower ionization rates.

We group the reactions of projectiles with nuclei $X$ in air according to their correspondence to parts of a lightning channel. Notation $\{a,b,c\}$ here refers to ",a, b, or c", and symbols for particles denote their charged variants, for example $\pi$ denotes either $\pi^+$ or $\pi^-$. 

a) stem, end of a channel in mid-air, corresponds to one of the following cases:

- particle $\{p,\pi,K,\mu\}$ coming to rest, or
- capture of the charged projectile $\{p,\pi,K\} + X \rightarrow X^* + \text{neutals}$, or
- decay $\mu \rightarrow e + \nu + \bar{\nu}$

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b) non-forked part corresponds to one of the following cases:
   − passage of \{p,\pi,K,\mu\} without decay or collision, or
   − collision of swap type \{p,\pi,K\} + X \rightarrow X^* + \{p,\pi,K,\mu\} +
     neutrals, or
   − decays
     \[ \pi \rightarrow \mu + \gamma \]
     \[ K \rightarrow \pi + \text{neutrals} \]

   c) forking with \( N = 2 \) prongs corresponds to
   collision \{p,\pi,K\} + X \rightarrow X^* + n_p \cdot p + n_\pi \cdot \pi + n_K \cdot K + n_\mu \cdot
eutrals, where \( n_p + n_\pi + n_K + n_\mu = 2 \)

d) forking with \( N = 3 \) prongs corresponds to either
   − collision \{p,\pi,K\} + X \rightarrow X^* + n_p \cdot p + n_\pi \cdot \pi + n_K \cdot K +
     n_\mu \cdot \mu + \text{neutrals}, where \( n_p + n_\pi + n_K + n_\mu = 3 \), or
   − decay \( K \rightarrow \pi + \pi + \pi \) (charged pions)

e) forking with \( N \geq 4 \) prongs corresponds to
   collision \{p,\pi,K\} + X \rightarrow X^* + n_p \cdot p + n_\pi \cdot \pi + n_K \cdot K + n_\mu \cdot
     \mu + \text{neutrals}, where \( n_p + n_\pi + n_K + n_\mu \geq 4 \)

A fork can be produced only by collision of a charged hadron with a nucleus or, in case of kaon (\( N = 3 \) case), a decay into three charged pions. An internode part of the channel between two subsequent forking points can only be created by one of the charged hadrons \( p, \pi, \) or \( K \).

## 2 Range and ionization rate

The charged projectiles interact with electrons in the surrounding media loosing their kinetic energy according to Bethe formula. \[6\] In non-dimensional form with energy normalized as \( y = \gamma - 1 = W_k/(m_0c^2) \) and distance \( l \) normalized as \( x = l/\lambda \) the relativistic variant of the formula reads:

\[
\frac{dy}{dx} = -\frac{\lambda \kappa}{m_0 c^2} \left[ \frac{(y+1)^2}{y(y+2)} \cdot \ln \frac{\alpha y(y+2)}{\sqrt{1+2\mu(y+1)}} - 1 \right] \quad (1)
\]

where \( m_0 \) is the rest mass of the projectile, \( \mu \) is ratio of electron mass to projectile mass, and \( \lambda \) is its mean free path. The mean free paths for the projectiles modelled as electrically neutral rigid balls with dimensions corresponding to their charge radii give \( \lambda_p = 417 \text{ m} \), \( \lambda_\pi = 444 \text{ m} \), and \( \lambda_K = 507 \text{ m} \). The parameters \( \kappa \) and \( \alpha \) are

\[
\kappa = \frac{e_0^2 n_e}{2\pi m_e c^2 \epsilon_0^2} \approx 4.77 \cdot 10^{-13} \text{ J/m} \approx 2.98 \text{ MeV/m} \]

\[
\alpha = \frac{2 m_e c^2}{\langle I \rangle} \approx 1.28 \cdot 10^4 \quad (2)
\]

with the following values for the lower Earth atmosphere: \( e_0 = 1.6 \cdot 10^{-19} \text{ A s} \), \( n_e = 4.59 \cdot 10^{26} \text{ m}^{-3} \), \( c_0 = 3 \cdot 10^8 \text{ m/s} \), \( \epsilon_0 = 8.854 \cdot 10^{-12} \text{ A s/(V m)} \), and \( \langle I \rangle = 72.663 \text{ eV} \).
For any type of the projectile the charge deposited on the trajectory at energies above 0.5 GeV is around $10^{-14}$ C/m (see Fig. 1 right). The distance between a thunder-cell and ground is above 2 km. To reach this distance, a projectile requires initial kinetic energy of at least 2 GeV (see Fig. 1 left).

3 Average internode length

The probability that a neutral particle collides with a nucleus inside a cube of side length $\Delta l$ is approximately $q_0 = \Delta l/\lambda$ for $\Delta l \ll \lambda$. For charged particles this value changes to $q/q_0 = 1 - W_c/W$ due to Coulomb interactions where $W_c$ is the minimum kinetic energy required for collision (for protons $W_c \approx 3.5$ MeV). If the probability for survival at length $\Delta l$ equals $\Delta P_{\text{col}} = 1 - q$, then for length $l = n \cdot \Delta l$ and small $q \ll 1$ one obtains $P_{\text{col}}(l) = (\Delta P_{\text{col}})^n = (1 - q)^n = \prod \left[1 - q_0(1 - W_c/W(l))\right]$, or

$$\frac{d \ln P_{\text{col}}}{dx} = -1 + \frac{y_c}{y} \tag{3}$$

where $y_c = W_c/(m_0 c^2)$.

Let the probability for survival of unstable particles during the interval $\Delta t$ be $\Delta P_{\text{dec}} \approx 1 - \Delta t/\tau_0$, where $\tau_0$ is the mean life time at rest. For relativistic particle of kinetic energy $W_k = (\gamma - 1)m_0 c^2$ we find

$$\frac{d \ln P_{\text{dec}}}{dx} = -\frac{\lambda}{c \tau_0} \cdot \frac{1}{\sqrt{y^2 + 2y}} \tag{4}$$

The average length of the path is thus

$$\langle l \rangle = \int_0^L \left| \frac{dP}{dl} \right| dl \tag{5}$$

where $P = P_{\text{col}}$ for collision and $P = P_{\text{dec}}$ for decay.
The average length of internodes as a result of a collision and those that are a result of decay $K \rightarrow 3\pi$ are plotted in Fig. 2. Average length for $K$ that decayed into three charged $\pi$ s depends strongly on the initial energy but is less likely to occur as kaons themselves are the rarest products among the three, and since this type of decay has a rate around 6%.

4 Discussion

The average internode length is a mixture of averages for all types of involved hadrons: $\langle l \rangle_p$ for protons, $\langle l \rangle_\pi$ for pions, $\langle l \rangle_K$ for kaons. For relative rates of protons $p_p$, pions $p_\pi$, and kaons $p_K$ our prediction of the average length for a given set of internodes is

$$\langle l \rangle \approx p_p \langle l \rangle_p + p_\pi \langle l \rangle_\pi + p_K \langle l \rangle_K$$

The correctness of this prediction may suffer due to two types of the processes not detected when observing lightning geometry: first, the rare decay of kaons into three charged pions with energy dependent average tends to lower the average length for forking with either two or three prongs, and second, collisions of swap type (see reaction type b) in Section 1.) do not produce a fork in a channel and make the experimental average higher.

Cloud-to-ground lightnings initiated by cosmic rays are at present indistinguishable from the ones that are not. Since the origin of all CG lightning channels is inside a thunder-cell, our result for internode lengths – if confirmed – implies that thunder-cell is a probable source of fast elementary particles. We suggest that freezing of super-cooled water at rates above $100 \, \text{kg/s}$ in a thunder-cell bounded by $0^\circ \text{C}$ and $-40^\circ \text{C}$ isotherms is the most likely process responsible for ejection of the particles. The computed charge densities of the order $10^{-14} \, \text{C/m}$ imply that the majority of the tens of coulombs of the charge...
transported during lightning process originates either from the cloud or from the ground.

References

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