Real-time Neural MPC: Deep Learning Model Predictive Control for Quadrotors and Agile Robotic Platforms

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Abstract—Model Predictive Control (MPC) has become a popular framework in embedded control for high-performance autonomous systems. However, to achieve good control performance using MPC, an accurate dynamics model is key. To maintain real-time operation, the dynamics models used on embedded systems have been limited to simple first-principle models, which substantially limits their representative power. In contrast to such simple models, machine learning approaches, specifically neural networks, have been shown to accurately model even complex dynamic effects, but their large computational complexity hindered combination with fast real-time iteration loops. With this work, we present Real-time Neural MPC, a framework to efficiently integrate large, complex neural network architectures as dynamics models within a model-predictive control pipeline. Our experiments, performed in simulation and the real world onboard a highly agile quadrotor platform, demonstrate the capabilities of the described system to run learned models with, previously infeasible, large modeling capacity using gradient-based online optimization MPC. Compared to prior implementations of neural networks in online optimization MPC we can leverage models of over 4000 times larger parametric capacity in a 50Hz real-time window on an embedded platform. Further, we show the feasibility of our framework on real-world problems by reducing the positional tracking error by up to 82\% when compared to state-of-the-art MPC approaches without neural network dynamics.

Framework Code: \url{https://github.com/TUM-AAS/ml-casadi}
Experimental Code: \url{https://github.com/TUM-AAS/neural-mpc}

I. INTRODUCTION

MODEL Predictive Control (MPC) is one of the most popular frameworks in embedded control thanks to its ability to simultaneously address actuation constraints and performance objectives through optimization. Due to its predictive nature, the performance of MPC hinges on the availability of an accurate dynamics model of the underlying system. This requirement is exacerbated by strict real-time constraints, effectively limiting the choice of dynamics models on embedded platforms to simple first-principle models. Combining MPC with a more versatile and efficient dynamics model would allow for an improvement in performance, safety and operation closer to the robot’s physical limits.

Precise dynamics modeling of autonomous systems is challenging, e.g. when the platform approaches high speeds and accelerations or when in contact with the environment. Accurate modeling is especially challenging for autonomous aerial systems, as high speeds and accelerations can lead to complex aerodynamic effects \cite{Kaufmann2015}, and operating in close proximity to obstacles with an aerial vehicle requires modeling of interaction forces, e.g. ground effect. Data-driven approaches, in particular neural networks, demonstrated the capability to accurately model highly nonlinear dynamical effects \cite{Kaufmann2015, Arrizabalaga2018}. However, due to their large computational complexity, the integration of such models into embedded MPC pipelines remains challenging due to high frequency real-time requirements. To overcome this problem prior works have relied on one of two strategies:

(I) Largely reducing the model’s capacity to the point where a lot of the predictive performance is lost but real-time speeds can be achieved \cite{Kaufmann2015, Arrizabalaga2018, Pfaff2018, Kopp2018, Barna2018}. Commonly, the model is reduced to a Gaussian Process (GP) with few supporting points \cite{Kaufmann2015, Arrizabalaga2018} or small neural networks \cite{Pfaff2018, Kopp2018}. Still, these methods are exclusively applied off-device on a powerful CPU.

(II) A control strategy different to online optimized MPC is used which are either non-predictive \cite{Kaufmann2015, Arrizabalaga2018}, do not use online optimization \cite{Pfaff2018, Kopp2018, Barna2018}, or learn the controller end-to-end \cite{Kaufmann2015, Arrizabalaga2018, Pfaff2018, Kopp2018, Barna2018}.
In this paper, we present an efficient framework, **Real-time Neural MPC (RTN-MPC)**, that allows for the integration of large-capacity, data-driven dynamics models in online optimization Model Predictive Control and its deployment in real-time on embedded devices. Specifically, the framework enables the integration of arbitrary neural network architectures as dynamics constraints into the MPC formulation. To this end, **RTN-MPC** leverages CPU or GPU parallelized local approximations of the data-driven model. Compared to a naive integration of a deep network into an MPC framework, our approach allows unconstrained model architecture selection, embedded real-time capability for larger models, and GPU acceleration, without a decrease in performance.

**Contribution**

Our contribution is threefold: First, we formulate the computational paradigm for **RTN-MPC**, an MPC framework which uses deep learning models in the prediction step. By separating the computationally heavy data-driven model from the MPC optimization we can leverage efficient online approximations which allow for larger, more complex models while retaining real-time capability. Second, we compare and ablate the MPC problem with and without our **RTN-MPC** paradigm demonstrating improved real-time capability on CPU, which is further enhanced when GPU processing is available. Finally, we evaluate our approach on multiple simulation-based and real-world experiments using a high speed quadrotor in aggressive and close-to-obstacle maneuvers. All while running large models, multiple magnitudes higher in capacity compared to state-of-the-art algorithms, in a real-time window.

To the best of the authors’ knowledge, this is the first approach enabling data-driven models, in a real-time **on-board** gradient-based MPC setting on agile platforms. Further, it scales to large models, vastly extending simple two or three-layer networks, on an off-board CPU or GPU enabling model sizes deemed unfit for closed-loop MPC [1], [2]. The introduced framework, while demonstrated on agile quadrotors, can be applied broadly and benefit any controlled agile system such as autonomous vehicles or robotic arms.

**II. RELATED WORK**

With the advent of deep learning, there has been a considerable amount of research that aims to combine the representational capacity of deep neural networks with system modeling and control. In the following, we provide a brief overview of prior work that focuses on learning-based dynamics modeling, and data-driven control.

**Data-driven Dynamics Models.** Thanks to their ability to identify patterns in large amounts of data, deep neural networks represent a promising approach to model complex dynamics. Previous works that leverage the representational power of deep networks for such modeling tasks include aerodynamics modeling of quadrotors [1], [2], [13] and helicopters [19], turbulence prediction [20], tire friction modeling [21], and actuator modeling [22]. Although these works demonstrated that neural networks can learn system models that are able to learn the peculiarities of real-world robotic systems, they were restricted to simulation-only use cases or employed the network predictions as simple feedforward components in a traditional control pipeline: Saviolo et al. [2] had to revert their accurate physics-inspired model to a simple multi-layer-perceptron for closed-loop control.

**Data-Driven Control.** Leveraging the power of learned models in embedded control frameworks has been extensively researched in recent years. Most approaches have focused on combining the learned model with a simple reactive control scheme, such as the “Neural Lander” approach [3]. Neural Lander uses a learned model of the aerodynamic ground-effect to substantially improve a set-point controller in near-hover conditions. In [23], a learned recurrent dynamics model formulates a model-based control problem. While this approach allowed the system to adapt online to changing operating conditions, it cannot account for system constraints such as limited actuation input. Recent approaches that integrate the modeling strengths of data-driven approaches in the MPC framework propose the use of Gaussian Processes (GP) as a learned residual model for race cars [4] and quadrotors [5]. For Gaussian Processes, both their complexity and accuracy scale with the number of inducing points and with their dimensionality, limiting their performance on embedded systems. The approaches of Chee et al. [5], Williams et al. [7] and Spielberg et al. [6] follow the approach of [3] but model the quadrotor’s dynamics residual using a small neural network for different applications. In Table I, we compare existing data-driven MPC approaches based on their modeling capacity. All state-of-the-art models are severely limited by the small modeling capacity of either GPs with a small number of supporting points or small two- or three-layer neural networks.

| Model Architecture | RT Complexity Parameters | RT Platform |
|--------------------|--------------------------|-------------|
| DD-MPC [3]         | Gaussian Process          | 20 Sup. Points | 120 | Intel i7 |
| KNODE-MPC [5]      | MLP                       | 2 Layer (64x64) | 4096 | Intel i7 |
| NNMPC [6]          | MLP                       | 1 Layer (32)  | 32  | Intel i7 |
| PL-TCN [7]         | MLP                       | 3 Layer(64x32x32) | 3072 | Jetson ARM/GPU |

With the rise of deep reinforcement learning (RL), a new class of controllers for robotic systems has emerged that directly maps sensory observations to actions. Popular instances of such RL controllers are imitation learning of an expert controller [12], [13], [16] as well as Model-free and Model-based reinforcement learning [7], [10], [11], [17], [24]. Although such approaches achieve high control frequencies and may outperform online MPC approaches, they commonly require training in simulation, do not allow for tuning without costly retraining, and often discard the optimality, robustness and generalizability of an online optimized MPC framework.

Our work is inspired by [2]–[7] but replaces the Gaussian Process dynamics of [3], [4] or the small neural networks of [5], [6] with networks of higher modeling capacity [1], [2] and uses gradient-based optimization as opposed to a sampling-based scheme [7]. The resulting framework allows a combination of the versatile modeling capabilities of deep neural networks with state-of-the-art embedded optimization software without tightly constraining the choice of network architecture.
III. Problem Setup

In its most general form, MPC solves an optimal control problem (OCP) by finding an input command $u$ which minimizes a cost function $L$ subject to its system dynamics model $\dot{x} = f(x, u)$ while accounting for constraints on input and state variables for current and future timesteps. Traditionally, the model $f$ is manually derived from first principles using “simple” differential-algebraic equations (DAE) which often neglect complicated dynamics effects such as aerodynamics or friction as they are hard or computationally expensive to formalize. Following prior works [2]–[5], we partition $f$ into a mathematical combination of first principle DAEs $f_R$ and a learned data-driven model $f_D$. This enables more general models extending the capability of DAE dynamics models. To solve the aforementioned OCP, we approximate it by discretizing the system into $N$ steps of step size $\delta t$ over a time horizon $T$ using direct multiple shooting [25] which leads to the following nonlinear programming (NLP) problem

$$
\min_{\omega} \sum_{k=0}^{N-1} L(x_k, u_k)
$$

subject to

$$
x_{k=0} = x_0
$$

$$
x_{k+1} = \phi(x_k, u_k, f, \delta t)
$$

$$
f(x_k, u_k) = f_R(x_k, u_k) + f_D(x_k, u_k)
$$

$$
g(x_k, u_k) \leq 0
$$

where $x_0$ denotes the initial condition and $g$ can incorporate (in-)equality constraints, such as bounds in state and input variables. $\phi$ is the numerical integration routine to discretize the dynamics equation where commonly a 4th order Runge-Kutta algorithm is used involving $E = 4$ evaluations of the dynamics function $f$. To leverage advancements in embedded solvers, the NLP is optimized using sequential quadratic programming (SQP) with $\omega$ being the SQP iterate $\omega^i = [x_0^i, u_0^i, \ldots, x_{N-1}^i, u_{N-1}^i]$.

IV. Bringing Neural MPC to Onboard Real-time

In this section, we lay down the key concepts to speed up the optimization times of MPC control with neural networks. The key insight in Section IV-A is that local approximations of the learned dynamics are sufficient to keep alike performance while drastically improving the generation process of the optimization problem. This insight is utilized in a three-phased embedded real-time optimization procedure in Section IV-B.

A. Locally Approximated Continuity Quadratic Program

Due to advances in embedded optimization solvers, SQP has become a well-suited framework to efficiently solve NLPs resulting from multiple shooting approximations of OCPs. This involves repetitively approximating and solving Eq. (1) as a quadratic program (QP). The solution to the QP leads to an update on the iterate $\omega^{i+1} = \omega^i + \Delta \omega^i$ where the step $\Delta \omega^i$ is given by solving the following QP

$$
\min_{\Delta \omega^i} \sum_{k=0}^{N-1} \frac{q_k}{r_k}^T \Delta x_k + \frac{\Delta x_k}{\Delta u_k}^T H_k \frac{\Delta x_k}{\Delta u_k}
$$

subject to

$$
\Delta x_{k+1} = A_k \Delta x_k + B_k \Delta u_k + \Delta x_{k+1}, \quad k = 0, \ldots, N - 1,
$$

$$
-g_k \geq G_{k}^{\omega} \Delta x_k + G_{k}^{u} \Delta u_k, \quad k = 0, \ldots, N,
$$

where $q_k = \frac{\delta}{\delta x_k} L(x_k^i, u_k^i, f, \delta t)$ and $r_k = \frac{\delta}{\delta u_k} L(x_k^i, u_k^i)$ linearize the cost function and, under given circumstances, the hessian $H_k$ can be approximated by the Gauss-Newton algorithm. $\phi_k$ and $\bar{g}_k$ are shorthand notations for the function evaluations $\phi(x_k^i, u_k^i, f, \delta t)$ and $g(x_k^i, u_k^i)$. The main computational burden lies in the parameter computation of the continuity condition Eq. (2). Specifically for each shooting node $k = 0, \ldots, N - 1$ we need to compute

$$
A_k = \frac{\delta}{\delta x_k} \phi(x_k^i, u_k^i, f, \delta t), \quad B_k = \frac{\delta}{\delta u_k} \phi(x_k^i, u_k^i, f, \delta t),
$$

Leading to $N + E \times 2$ evaluations of the partial differentiations

$$
\delta f(x, u) = \delta f_N(x, u) + \delta f_D(x, u)
$$

and $N + E$ function evaluations

$$
f(x, u) = f_N(x, u) + f_D(x, u)
$$

of the dynamics equation. For computational heavy data-driven dynamics models $f_D$ this leads to extensive processing times generating the QP.

The learned data-driven dynamics $f_D$ are assumed to be accurate over the entire input space of states and controls present in the training dataset. However, to create the QP continuity condition we only require the model and its differentiations to be accurate in and around specific input values $\omega^i$. Thus, to speed up the QP generation we replace the computationally heavy globally valid data-driven dynamics equation $f_D$ with a computationally light locally valid approximation up to second order around the current iterate

$$
f_D(x, u) \approx f_D^i + J_{D,k}^i \left[ x - x_k^i \right] + \frac{1}{2} H_{D,k}^i \left[ x - x_k^i \right] \left[ u - u_k^i \right].
$$

(4)

The required differentiations are readily available as submatrices of $J_{D,k}$ for first-order approximations or as submatrices of a Tensor multiplication and sum for second-order approximations. The induced error of this computational simplification is of second order for a first-order approximation and of third order for a second-order approximation in the size of state and control changes between nodes. We will experimentally demonstrate this error to be neglectable for agile platforms where $\delta t$ is small in Section VII.

Applying Eq. (4), the QP creation becomes independent of the complexity and architecture of the data-driven dynamics model. Further, with $J_{D,k}$ and $H_{D,k}$ being the single interfaces between the SQP optimization and the data-driven dynamics model, we are free to optimize the approximation
linearizing around \( x \in \omega \) response phase solves the pre-constructed QP using the dis-

\[ f(x, u) = f(x, u) + f_D(x, u). \]

Fig. 2: Data flow for our RTN-MPC algorithm. The data-
driven (DD) preparation phase is performed efficiently using optimized machine learning batch-differentiation tools on CPU or GPU.

process independent of the NLP framework; passing them as parameters to the continuity condition procedure of the QP generation. As \( f_D \) is a neural network model commonly consisting of large matrix multiplications we are therefore free to use algorithms and hardware optimized for neural network evaluation and differentiation. Those capabilities are readily available in modern machine learning tools such as PyTorch and TensorFlow. This enables us to calculate the Jacobians and Hessians for all shooting nodes \( N \) as a single parallelized batch on CPU or GPU.

B. Real-time Neural MPC

Even without a data-driven dynamics model, solving the SQP until convergence is computationally too costly in real-time for agile robotic platforms. To account for this shortcoming, MPC applications subjected to fast dynamics are commonly solved using a real-time-iteration scheme (RTI) [28], where only a single SQP iteration is executed - one quadratic problem is constructed and solved as a potentially sub-optimal but timely input command is preferred over an optimal late one. As shown in Fig. 2, RTN-MPC divides the real-time optimization procedure into three parts: QP Preparation Phase, Data-Driven Dynamics Preparation Phase and Feedback Response.

With available iterate \( \omega^i \), the data-driven dynamics preparation phase calculates \( f_D^i \) and \( J_{D,k}^i \) using efficient batched differentiates of the data-driven dynamics on CPU or GPU.

Meanwhile, the QP preparation phase constructs a QP by linearizing around \( x^i \) and control \( u^i \) using a first-order approximation \( f_D^i(x, u) \) for the continuity condition parametrized by the result of the data-driven dynamics preparation phase.

Once a new disturbed state \( x_{k+1} \) is sensed, the feedback response phase solves the pre-constructed QP using the disturbed state as input. The iterate \( \omega \) is adjusted with the QP result and the optimized command \( u \) is sent to the actuators.

C. Implementation

To demonstrate the applicability of the RTN-MPC paradigm, we provide a implementation using CasADi [29] and acados [30] as the optimization framework and PyTorch [26] as ML framework. This enables the research community to use arbitrary neural network models, trainable in PyTorch and usable in CasADi.

Further, we will compare our RTN-MPC approach against a naive implementation of a neural network data-driven MPC as applied in [2], [5], [6]. Here, the learned model is directly constructed in CasADi in the form of trained weight matrices and activation functions. Subsequently, the QP generation and automatic differentiation engine in CasADi has to deal with the full neural-network structure for which it is lacking optimized algorithms while being confined to the CPU.

V. Runtime Analysis

We demonstrate the computational advantage of our proposed RTN-MPC paradigm compared to a naive implementation of a data-driven dynamics model in online MPC. Thus, we construct an experimental problem in which the nominal dynamics is trivial while the data-driven dynamics can be arbitrarily scaled in computational complexity. As such the nominal dynamics model is a double integrator on a position \( p \) while the data-driven dynamics is a neural network of variable architecture. To solely focus on the computational complexity of the data-driven dynamics, rather than modeling accuracy, the networks are not trained but weights are manually adjusted to force a zero output.

\[ \dot{x} = \begin{bmatrix} \dot{p} \\ \dot{u} \end{bmatrix} = f(x, u) = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial u} \end{bmatrix}, \]

\[ f(x, u) = f(x, u) + \frac{\partial f}{\partial x}(x, u). \]

We use an explicit Runge-Kutta method of 4th order \( \phi(x, u, f, \delta t) = RK4(x, u, f, \delta t) \) to numerically integrate \( f \).

In this experiment, we simulate the system without any model-plant-mismatch to focus solely on runtime. The optimization problem is solved by constructing the multiple shooting scheme with \( N = 10 \) nodes.

Fig. 3 compares two-layer networks with increasing neuron count for a naive implementation and our RTN-MPC framework. On an embedded system, such as the Nvidia Jetson Xavier NX, our approach enables larger models of factor 60.
TABLE II: Runtime comparison between naive implementation and RTN-MPC. Model complexity and parameter count are increasing from left to right. The naive approach becomes computationally costly in runtime above ~10K parametric capacity with control frequencies dropping below 50Hz. Our approach can scale to powerful networks and complex network architectures showing real-time control frequencies of over 100Hz for over 50K parameters on an embedded device and over 13M on a GPU.

| Model Configuration | Control Freq. [Hz] |
|---------------------|--------------------|
|                      | Sodium-CPU ARM-CPU | RTN-MPC ARM-CPU |
| Architecture        | 562                | 148 | 109 |
|Layers               | 2 2 5 5 12 12 20 30 | 118 | 107 |
|Neurons              | 16 128 126 32 512 512 | 85 67 | 64 |
|Parameter Count      | 0 | 354 17K 1.2K 67K 12K 182K 500K | 13M 12M |

Fig. 4: Quadrotor model with world and body frames and propeller numbering convention. Grey arrows indicate the spinning direction of the individual rotors.

Nominal Quadrotor Dynamics Model. The nominal dynamics assume the quadrotor to be a 6 degree-of-freedom rigid body of mass \( m \) and diagonal moment of inertia matrix \( J = \text{diag}(J_x, J_y, J_z) \). Our model is similar to \([3, 32, 33]\) as we write the nominal dynamics \( \dot{x} \) up to second order derivatives, leaving the quadrotors individual rotor thrusts \( T_i \) \( i \in (0,3) \) as control inputs \( u \in \mathbb{R}^4 \). The state space is thus 13-dimensional and its dynamics can be written as:

\[
\dot{x} = \begin{bmatrix} p_{WB} \; \dot{q}_{WB} \; \dot{\psi}_B \end{bmatrix} = f(x, u) = \begin{bmatrix} u_B \; \dot{q}_{WB} \circ \omega_B \end{bmatrix} \quad \begin{bmatrix} \sqrt{w^2} / \omega_B/2 \end{bmatrix} 
\]

with \( g_W = [0, 0, -9.81 \text{ m/s}^2]^T \) denoting Earth’s gravity, \( T_B \) the collective thrust and \( \tau_B \) the body torque. Again, an explicit \textit{Runge-Kutta} integration of 4th order is used.

Augmented Aerodynamic Residual Models. Following previous works \([3, 4]\), we use the data-driven model, in the form of a neural network \( N \), to complement the nominal dynamics by modeling a residual. In its full configuration, our residual dynamics model is defined as:

\[
f(x, u) = f_R(x, u) + f_D(x, u),
\]

where we individually account for disturbances in linear and angular accelerations unknown to the nominal dynamics and \( \theta \) and \( \psi \) are the parameters of the neural networks modeling linear and angular disturbances respectively.

We also evaluate two simplified versions of the residual model:

\[
f_{D_0}(x, u) = \begin{bmatrix} 0_v \; D_{0_v}(v) \end{bmatrix}, \quad f_{D_{0_v}}(x, u) = \begin{bmatrix} 0_v \; D_{0_v}(v) \end{bmatrix}
\]

These simplified models only consider residual forces as a function of the platform’s velocity (left), potentially accompanied by the commanded inputs (right).
**Augmented Ground Effect Model.** To show the strength of our approach, leveraging a complex arbitrary high level input, we extend the residual model using a height map under the quadrotor as additional input to model the ground effect.

\[ f_{D_g}(x, u) = \begin{cases} \frac{0.1}{2} & \text{if } |h_i(p_{W_B}, H_W)| \leq 0.1 \\ 0 & \text{otherwise} \end{cases} \]

where \( z_{W_B} \) is the altitude of the quadrotor and \( h_i \) is a mapping \( h_i : \mathbb{R}^3 \times \mathbb{R}^{N \times M} \rightarrow \mathbb{R}^{3 \times 1} \) which takes the quadrotor’s position \( p_{W_B} \) and a fixed or sensed global height map \( H_W \) of size \( N \times M \) as input. The function returns a \( 3 \times 3 \) local patch of the height map around the quadrotor’s position with a resolution of 10 cm.

**MPC Cost Formulation.** We specify the cost in Eq. \( (1) \) to be of quadratic form \( \mathcal{L}(x, u) = \|x - x_r\|^2_Q + \|u - u_r\|^2_R \) penalizing deviations from a reference trajectory \( x_r, u_r \) and account for input limitations by constraining \( 0 \leq u \leq u_{\text{max}} \).

**VII. EXPERIMENTS**

In our experiments we will re-validate the findings of previous works \([2, 3]\) that using neural-network data-driven models in MPC improves tracking performance compared to no data-driven models or Gaussian Processes. More importantly, however, we will demonstrate that RTN-MPC enables the use of larger network capacities to fully exhaust possible performance gains while providing real-time capabilities.

All our experiments are divided into two phases: system identification and evaluation. During system identification, we collect data using the nominal dynamics model in the MPC controller. The state-control-timeseries are further processed in subsequent state, control tuples. Each step is then re-simulated using the nominal controller and the error is used as the training label for the residual model.

During evaluation we track two fixed evaluation trajectories, Circle and Lemniscate, and measure the performance based on the reference position tracking error. As such, we report the (Mean) Euclidean Distance between the reference trajectory and the tracked trajectory as error.

To identify model architectures used in the experiments we use a naming convention stating the model type followed by the size and the implementation type where we differentiate between our RTN-MPC approach (-Ours) and a naive integration (-Naive). N-3-2-Ours is a neural network model with 3 hidden layers, 32 neurons each using our RTN-MPC framework and N-3-2-Naive using a naive integration (-Naive). N-3-2-Ours is a neural network model

**Table III: Results for the Simplified Simulation experiment.** We use two simulation environments featuring varying modeling accuracy and real-time requirements to compare against a non-augmented MPC controller, a naive integration of data-driven dynamics \([4, 5, 6]\), and GPs \([7]\) with respect to real-time capability and model capacity.

### A. Simulation

We use the simulation framework described in \([3]\), where perfect odometry measurements and ideal tracking of the commanded single rotor thrusts are assumed. Drag effects by the rotors and fuselage are simulated, as well as zero mean (\( \sigma = 0.005 \)) constant Gaussian noise on forces and torques, and zero mean Gaussian noise on motor voltage signals with standard deviation proportional to the input magnitude \( \sigma = 0.02 \sqrt{u} \). There are no run-time constraints as controller and simulator are run sequentially in simulated time. Using the simplified simulation, we analyze the predictive performance and run-time of our approach for varying network sizes and directly compare to the naive implementation and Gaussian Process approach. We constrain the residual model to linear accelerations \( f_{D_g} \) to facilitate comparison with prior work \([3]\). To fairly evaluate the run-times of our full and distributed approach and considering the limited resources of embedded systems this experiment was performed on a single CPU core. The results are depicted in Table \([III]\). We also compare with a Nominal model where no learned residuals are modeled in the dynamics function and we also compare with an oracle-like Perfect model which uses the same dynamics equations as the simulation (excluding noise). Neural networks which achieve accurate modeling performance on the simulated dynamics are integrated easily with real-time optimization times below 3ms using our approach while they have high optimization times (up to 36ms) when a naive integration approach is used. The local approximations described in Section \([LV-A]\) do not negatively influence performance compared to a naive implementation.

**BEM Quadrator Simulation.** In addition to the simplified simulation setting, we also evaluate our approach in a highly accurate aerodynamics simulator based on Blade-Element-Momentum-Theory (BEM) \([1]\). In contrast to the simplified...
simulation setting, this simulation can accurately model lift and drag produced by each rotor from the current ego-motion of the platform and the individual rotor speeds. The simulator runs in real-time and communicates with the controller via the Robot Operating System (ROS). We target a real-time control frequency of 100 Hz. We want to understand how our approach copes with increasing parameter count and model complexity of the learning task: First, we change the learned dynamics from just modeling linear acceleration residuals \( f_{D,a} \) with velocity as inputs to also accounting for rotor commands \( f_{D,u} \). In a second step we, model the full residual \( f_D \) additionally outputting residuals on angular accelerations. The results obtained in each of these settings are illustrated in Fig. 5. While a naive approach can accurately model the residuals, its control frequency quickly declines for increasingly complex models. For larger networks, it has excessively high optimization times leading to the controller becoming unstable even in simulation. No additional noise is simulated, leading to error standard deviations within 1\( \text{mm} \) over 5 trials per experiment, induced by non-deterministic ROS transportation times.

**TABLE IV: Results for the Real-World experiment.** We improve tracking performance up to 82\% compared to the nominal controller and up to 55\% compared to GPs while being real-time capable unlike a naive integration. Result statistics are reported over 5 runs per experiment.

| Track                        | Error [mm] |
|------------------------------|------------|
| \( v_{\text{max}} \) [m/s]  |            |
| Nominal                      | 321        |
| Ours                         | 359        |
| GP-20                        | 66 ± 4     |
| N-3-32-Naive                 | 260 ± 11   |
| N-4-64-Ours                  | crash      |
| N-3-32-Ours                  | 59 ± 6     |
| GP-20                        | 117 ± 9    |

![Fig. 5: Control Frequency over Tracking Error for Lemniscate trajectory in the realistic BEM simulation - top-left is desired. Our approach (blue) can leverage multidimensional inputs and large model capacities while being real-time capable. Increasing the naive approach to a four layer network (orange) leads to the controller becoming unstable even in simulation. No additional noise is simulated, leading to error standard deviations within 1\( \text{mm} \) over 5 trials per experiment, induced by non-deterministic ROS transportation times.](image)

![Fig. 6: (a) Quadrotor overflying the table in close proximity to the plane. (b) Vertical position error over distance. Vertical lines mark the position of the table. Our approach can model the aerodynamic effects in close proximity to the ground, substantially limiting the tracking error in z.](image)

**B. Real World**

Finally, we perform experiments evaluating the real world effectiveness of our approach by performing a set of agile trajectories using the physical quadrotor platform *agilicious* [34]. Control commands in the form of desired collective thrust and body rates are computed on a Jetson Xavier NX and are tracked by a low-level PID controller. All real-world flight experiments are performed in an instrumented tracking arena that provides accurate pose estimates at 400 Hz. As in the simulation experiments, we compare the tracking error along both circle and lemniscate trajectories at speeds up to 14 m s\(^{-1}\). We evaluate our approach against the nominal controller, the naive integration, and the Gaussian Process configuration deployed in [3]. The results of these experiments is depicted in Table IV, where we improve positional tracking error by up to 82\% compared to the nominal controller while the naive integration becomes unstable due to a long optimization time. Furthermore, we outperform Gaussian Processes by up to 55\%.

**Ground Effect.**

Finally, we demonstrate the generalizability of our approach to other use-cases, modeling the complex aerodynamics of the ground effect using a height map as input (See Section VI). We place a table of 70 cm height in the flight arena and collect data by repeatedly flying over the table in close proximity. During evaluation, we fly repeated trajectories over the table with a target altitude of 80 cm of the quadrotor’s center of gravity; leaving approximately 2 cm between the table and the lowest point of the quadrotor (battery). To isolate the performance of our approach, compensating for ground effect, we evaluate the trained model in two configurations. First, in which the height map information is unknown to the model (Baseline), and second where the information is known to the model. On an evaluation trajectory with 8 flyovers we improve the tracking error in z direction by 72\% in close proximity (table plane +10 cm in xy) above the table. A visualization of a single flyover can be seen in Fig. 6.
VIII. Conclusion

In this work we demonstrated an approach to scale the modeling capacity of data-driven MPC using neural networks to larger, more powerful architectures while being real-time capable on embedded devices. Our framework can improve new and existing applications of data-driven MPC by increasing the available real-time modeling capacity; making our approach generalizable to a variety of control applications.

An open challenge, which is not yet considered in this work, but the authors plan to tackle in the future, is to use a historic sequence of states and control input in a learned dynamics model. This would naturally lead to incorporating sequential and temporal models such (LSTMs, GRUs, and TCNs) in the optimization loop using our approach and would give rise to running approaches currently only feasible in simulation in embedded MPC real-time.

We experimentally show that the controller’s performance is not negatively affected by the real-time inducing approximations. Thus, this method overcomes the limitation of having to sacrifice performance for efficiency as described in previous works [2], [5]. We demonstrate its usefulness by evaluating the isolated real-time capability of RTN-MPC on different devices and applying the framework to the challenging problem of trajectory tracking of a highly agile quadrotor; reducing the tracking error substantially while using powerful models on-device.

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