Quantum Chaos, Irreversibility, dissipation and dephasing

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Abstract

The main idea of "Quantum Chaos" studies is that Quantum Mechanics introduces two energy scales into the study of chaotic systems: One is obviously the mean level spacing $\Delta \propto \hbar^d$, where $d$ is the dimensionality; The other is $\Delta_b \propto \hbar$, which is known as the non-universal energy scale, or as the bandwidth, or as the Thouless energy. Associated with these two energy scales are two special quantum-mechanical (QM) regimes in the theory of driven system. These are the QM adiabatic regime, and the QM non-perturbative regime respectively. Otherwise Fermi golden rule applies, and linear response theory can be trusted. Demonstrations of this general idea, that had been published in 1999, have appeared in studies of wavepacket dynamics, survival probability, dissipation, quantum irreversibility, fidelity and dephasing. The following presentation is intended for non-specialists.

Driven quantum systems, described by Hamiltonian $\mathcal{H}(Q, P, x(t))$, where $x(t)$ is a time dependent parameter, are of interest in mesoscopic physics (quantum dots), as well as in nuclear, atomic and molecular physics. Due to the time dependence of $x(t)$, the energy of the system is not a constant of motion. Rather the system makes "transitions" between energy levels. Notions such as "survival probability" and "dissipation", just emphasize particular aspects of the energy spreading process.

The name "Quantum Mechanics" is associated with the idea that the energy is quantized. For generic (chaotic) system the mean level spacing is $\Delta \propto \hbar^d$, where $d$ is the dimensionality of the system. However, one should recognize that there is a second energy scale $\Delta_b \propto \hbar$ which is introduced by Quantum Mechanics. This $\hbar$ energy scale is known in the literature as the "non-universal" energy scale (Berry), or as the "bandwidth" (Feingold and Peres) or as the Thouless energy.

The main focus of Quantum chaos studies (so far) was on issues of spectral statistics. In this context it turns out that the sub-$\hbar$ statistical features of the energy spectrum are "universal", and obey the predictions of random matrix theory. Non universal (system specific) features are
reflected only in the large scale properties of the spectrum (hence we can "hear the shape of the drum"). This is the reason why $\Delta_b$ is known as the "non-universal" energy scale.

Having two quantal energy scales implies the existence of two special QM regimes in the theory of driven systems. The simplest demonstration of this idea is in the context of linear driving ($\dot{x} = V$). The existence of the QM adiabatic regime (very very small $V$) is associated with having finite $\Delta$. The existence of the QM non-perturbative regime (where $V$ is quantum mechanically large, but still classically small) is associated with the energy scale $\Delta_b$.

**Heuristic explanation of having QM adiabatic regime**: If the rate $V$ is very very small, the system remains all the time in the same level. This is because there are QM recurrences that block the attempt to make a transition to any other level. Obviously this effect is related to having finite (non-zero) level spacing.

**Heuristic explanation of having QM non-perturbative regime**: If $V$ is large enough to induce transitions between levels, then we have to ask what is the maximal size of a single "step" in energy space. (Here "step" means first order transition). The answer is that the maximal step is the bandwidth $\Delta_b$. The energy spreading process after many "steps" becomes diffusive. If $V$ is too large, then the breakdown of perturbation theory happens before even a single step is taken. In such case perturbation theory cannot be used in order to analyze the energy spreading process.

The identification of the "non-perturbative" regime is the main observation of [Cohen PRL 1999]. An associated observation is that the semiclassical limit is contained in the non-perturbative regime. The idea has been applied and generalized to the analysis of the energy spreading process (the decay of the survival probability, and the growth of the variance) in case of "wavepacket dynamics" [main collaborator: Kottos]. In these studies the control is over the strength/amplitude $A$ off the perturbation. The idea also has been generalized to the case of periodically driven systems [Cohen and Kottos, PRL 2000], where the control is over both the rate of change $V$ and the amplitude $A$ of the driving.

Recently the idea of having a non-perturbative/semiclassical regime has been adopted [Jacquod, Silvestrov and Beenakker PRE 2001] into the context of quantum-irreversibility studies (also known as "fidelity" or "Loschmidt echo" studies). Also here, if the perturbation strength $A$ is large enough ("large" in a quantum mechanical sense, but still assumed to be small in a classical sense), one gets into a semiclassical regime. This is the same idea as in our "survival probability" studies in the context of "wavepacket dynamics". But in this particular context one can further argue [Jalabert and Pastawski, PRL 2001] that the "semiclassical decay" is in fact a perturbation independent "Lyapunov decay".
Another recent observation [Cohen PRE 2002] is in the context of dephasing due to the interaction with chaotic degrees of freedom. The same idea of having non-perturbative/semiclassical regime is applicable. In fact the study of "dephasing" can be regarded as a generalization of quantum irreversibility studies. The calculation of the dephasing factor reduces to the study of "fidelity" in a scenario where one has control over both the amplitude $A$ and the rate $V$ of the driving.

References:

For a pedagogical presentation, including references, see D. Cohen, "Driven chaotic mesoscopic systems, dissipation and decoherence", lecture notes of the course to be given in the 2002 Wroclaw school. A preliminary version can be found in [http://www.bgu.ac.il/~dcohen](http://www.bgu.ac.il/~dcohen).