Low-complexity near-optimal signal detection for uplink large-scale MIMO systems

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The minimum mean square error (MMSE) signal detection algorithm is near-optimal for uplink multi-user large-scale multiple-input–multiple-output (MIMO) systems, but involves matrix inversion with high complexity. It is firstly proved that the MMSE filtering matrix for large-scale MIMO is symmetric positive definite, based on which a low-complexity near-optimal signal detection algorithm by exploiting the Richardson method to avoid the matrix inversion is proposed. The complexity can be reduced from $\mathcal{O}(K^3)$ to $\mathcal{O}(K)$, where $K$ is the number of users. The convergence proof of the proposed algorithm is also provided. Simulation results show that the proposed signal detection algorithm converges fast, and achieves the near-optimal performance of the classical MMSE algorithm.

Introduction: Large-scale multiple-input–multiple-output (MIMO) employing hundreds of antennas at the base station (BS) to simultaneously serve multiple users is a promising key technology for fifth generation wireless communications [1]. It can achieve orders of magnitude increases in spectrum and energy efficiency, and one challenging issue to realise such a goal is the low-complexity signal detection algorithm in the uplink, due to the increased dimension of large-scale MIMO systems [2]. The optimal signal detection algorithm is the maximum-likelihood algorithm, but its complexity increases exponentially with the number of transmit antennas, making it impossible for large-scale MIMO systems. The fixed-complexity sphere decoding [3] and tabu search [4] algorithms have been proposed with reduced complexity, but their complexity is unaffordable when the dimension of the large-scale MIMO system is large or the modulation order is high [5]. Low-complexity linear detection algorithms such as zero-forcing and minimum mean square error (MMSE) with near-optimal performance have been investigated [2], but these algorithms have to use unfavourable matrix inversion, whose high complexity is still not acceptable for large-scale MIMO systems. Recently, the Neumann series approximation algorithm has been proposed to approximate matrix inversion [6], which converts the matrix inversion into a series of matrix–vector multiplications. However, only marginal complexity reduction can be achieved.

In this Letter, we propose a low-complexity near-optimal signal detection algorithm by exploiting the Richardson method [7] to avoid the complicated matrix inversion. We firstly prove a special property of large-scale MIMO systems that the MMSE filtering matrix is symmetric positive definite, based on which we propose to exploit the Richardson method to avoid the complicated matrix inversion. Then we prove the convergence of the proposed algorithm for any initial solution when the relaxation parameter is appropriate. Finally, we prove the convergence of the Richardson method for any initial solution as we will prove in the following Lemma 2. Consequently, the final accuracy will also not be affected by the initial solution if the number of iterations $i$ is large (e.g. $i = 5$), as will be verified later in the simulation results. Since the relaxation parameter $\mu$ in (4) plays an important role in convergence, next we prove the convergence of the Richardson method for any initial solution when the relaxation parameter is appropriately selected.

Large-scale MIMO system model: We consider an uplink multi-user large-scale MIMO system which employs $N$ antennas at the BS to simultaneously serve $K$ single-antenna users. Usually we have $N > K$, e.g. $N = 128$ and $K = 16$ have been considered [1, 2]. For signal detection, the complex-valued system model can be directly converted to a corresponding real-valued system model, then the estimate of $\hat{H}$ is also symmetric positive definite. Thus, the Gram matrix $G = H^* H$, $G$ is also symmetric. Thus, the Gram matrix $G$ is symmetric positive definite. Finally, as the noise power $\sigma^2$ is positive, we can conclude that the MMSE filtering matrix $W = H H^* \sigma^2 I_K$ is also a symmetric positive definite matrix.

Proposed signal detection based on Richardson method: Unlike the conventional (small-scale) MIMO systems with a small number of antennas, large-scale MIMO systems have a special property in that the MMSE filtering matrix $W$ determined by the MIMO channel matrix $H$ is symmetric positive definite, which can be proved as below.

Lemma 1: For signal detection of large-scale MIMO systems, the MMSE filtering matrix $W$ is symmetric positive definite.

Proof: The column vectors of the channel matrix $H$ in large-scale MIMO systems are asymptotically orthogonal (i.e. rank($H$) = $2K$) [2]. Then we have the equation $H^* H = 0$ when and only when $r$ is a $2K \times 1$ zero vector. Thus, for an arbitrary nonzero $2K \times 1$ vector $r$, we have

$$ (H^* H)r = r^*(H^* H)r > 0 $$

which indicates that the Gram matrix $G = H^* H$ is positive definite. In addition, as we have $G^{-1} = (H^* H)^{-1} = G$, $G$ is also symmetric.

Therefore, we prove the convergence of the proposed algorithm for any initial solution as we will prove in the following Lemma 2. Consequently, the final accuracy will also not be affected by the initial solution if the number of iterations $i$ is large (e.g. $i = 5$), as will be verified later in the simulation results. Since the relaxation parameter $\mu$ in (4) plays an important role in convergence, next we prove the convergence of the Richardson method for any initial solution when the relaxation parameter is appropriately selected.

Lemma 2: For the N-dimensional linear equation $Ax = b$, the necessary and sufficient conditions for convergence of the Richardson method is that the relaxation parameter $\mu$ satisfies $0 < \mu < 2/\lambda_1$, where $\lambda_1$ is the largest eigenvalue of symmetric positive definite matrix $A$.

Proof: We define $D = I - \mu A$ and $c = \mu b$, where $D$ is the iteration matrix. Then the Richardson iteration (3) can be rewritten as

$$ \hat{x}^{(i+1)} = D \hat{x}^{(i)} + c, \quad i = 0, 1, 2, \ldots $$

We call the iteration procedure convergent if $\lim_{i \to \infty} s^{(i)} = \hat{x}$ and $s = B \hat{x} + c$ for any initial solution $s^{(0)}$. The spectral radius of iteration matrix $D \in \mathbb{R}^{N \times N}$ is the non-negative number $\rho(D) = \max_{1 \leq j \leq N} |\mu_j(D)|$, where $\mu_j(D)$ denotes the $j$th eigenvalue of $D$, and the necessary and sufficient conditions for the convergence of (5) is that the spectral radius should satisfy $\rho(D) < 1$ [7, Theorem 7.2.2].

Without loss of generality, we use $A_1 \geq A_2 \geq \ldots \geq A_N > 0$ to denote the $N$ eigenvalues of symmetric positive definite matrix $A$, where $A_1$ is the largest one. Since $D = I - \mu A$, we have $\rho(D) = 1 - \mu A$, where $\lambda_1$ is the $i$th eigenvalue of $A$, which can be substituted into (6), and then we have $0 < \mu < 2/\lambda_1$.\[\square\]
Computational complexity: The complexity in terms of the required number of multiplications is analysed for comparison. It can be found from (4) that the ith iteration of the proposed signal detection algorithm involves one multiplication of a $2K \times 2K$ matrix $W$ with a $2K \times 1$ vector $s^{(i)}$, and one multiplication of a constant relaxation parameter $\mu$ with a $2K \times 1$ vector $y - Ws^{(i)}$, thus the required number of multiplications is $4K^2 + 2K$ for each iteration.

Table 1 compares the computational complexity of the conventional Neumann series approximation algorithm [6] and the proposed algorithm based on the Richardson method. It is well known that the complexity of the classical MMSE algorithm is $O(K^3)$, and Table 1 shows that the conventional Neumann series approximation algorithm can reduce the complexity from $O(K^3)$ to $O(K^2)$ when the number of iterations is $i = 2$. However, the complexity is $O(K^3)$ when $i \geq 3$. Since usually a large value of $i$ is required to ensure the final approximation performance (e.g. $i = 5$ as will be verified later by simulation results), the overall complexity is still $O(K^3)$, which indicates that only marginal complexity reduction can be achieved. On the contrary, the complexity of the proposed algorithm is reduced from $O(K^3)$ to $O(K^2)$ for any arbitrary number of iterations.

Table 1: Computational complexity

| $i$  | Conventional Neumann series approximation [6] | Proposed algorithm based on Richardson method |
|------|---------------------------------------------|---------------------------------------------|
| $i = 2$ | $12K^2 + 4K$ | $6K^2 + 4K$ |
| $i = 3$ | $8K^2 + 4K + 2K$ | $12K^2 + 6K$ |
| $i = 4$ | $16K^2 + 4K$ | $16K^2 + 8K$ |
| $i = 5$ | $24K^2 - 12K^2 + 2K$ | $20K^2 + 10K$ |

Conclusions: By fully exploiting the special property that the MMSE filtering matrix in large-scale MIMO systems is symmetric positive definite, we propose a low-complexity near-optimal signal detection algorithm based on the Richardson method to avoid the complicated matrix inversion, which can reduce the complexity from $O(K^3)$ to $O(K^2)$. We also prove the convergence of the proposed algorithm for any initial solution when the relaxation parameter is appropriate. Simulation results verify that the proposed algorithm outperforms the conventional method, and achieves the near-optimal performance of the classical MMSE algorithm.

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One or more of the Figures in this Letter are available in colour online.

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Fig. 1 shows the BER performance comparison results, where $i$ denotes the number of iterations. It is clear that the BER performance of both the algorithms improves with the number of iterations, but the proposed algorithm outperforms the conventional one when the same number of iterations are used, which indicates that a faster convergence rate can be achieved by the proposed signal detection algorithm. More importantly, when the number of iterations is moderately large (e.g. $i = 5$ in Fig. 1), the proposed algorithm without the complicated matrix inversion can achieve the near-optimal BER performance of the MMSE algorithm with exact matrix inversion.