Ultra-High Energy Quenching of the LPM Effect: Implications for GZK-Violating Events

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Cosmic rays in the regime of $10^{19} - 10^{20}$ eV should not exist, according to the GZK bound. Conflicts between data and the bound have led to suggestions that laws of physics must be modified. We find that physical processes neglected in the early history terminate the LPM effect, important in calibrating the energy of electromagnetic and hadronic showers in the GZK energy regime. The processes neglected are the direct losses of electrons and positrons, overlooked by LPM and greatly augmented by ultra-high energy circumstances. We present an exact formula for direct energy losses of $e^+e^-$ pairs in electromagnetic showers. Numerical integration in current hadronic models shows that actual energy losses are always substantially larger than LPM estimates, but the energy at which crossover occurs depends on models.

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Some thirty years ago, Greissen, Zatsepin and Kuzmin (GZK) found that cosmic rays above about $5 \times 10^{19}$ eV should not be observed. Yet since its inception the GZK bound has never been confirmed, as evidenced by observation of dozens, if not hundreds of anomalous events. Extraordinary contradictions are indicated by angular correlations of GZK-violating showers and cosmologically distant sources. Experimental facilities such as HiRES and AUGER are poised to explore the question with vastly more data. The conflict between the data and the bound has led many theoretical workers to suggest that laws of physics must be modified. Suggestions of new physics include exotic particles, magnetic monopoles, neutrinos with large cross sections from low-scale gravity, and even the rejection of Lorentz invariance.

High energy air showers tend to be measured better with increasing energy, due to increased signal-to-noise. However the calibration of shower energies depends on details of the shower evolution. Hadronic showers have some model dependence, but the electromagnetic components are thought to be well-understood. Many years ago, Landau and Pomeranchuk (LP) showed that high energy electromagnetic interactions in matter destructively interfere, effectively increasing the interaction length. Migdal (M) gave formulas smoothly interpolating between different coherence regimes. Monte Carlo codes incorporate these effects, currently believed to extend to asymptotically high energies. The question of ultra-high energy calibration involves what and how much to believe from LPM.

The LPM calculations hinge on dominance by a concept called the “formation length” of bremsstrahlung. In a peculiar remark, Galitsky and Guerevich (GG) noted an instance where the absorption length might be less than the coherence length. Then photons would be absorbed before they were fully emitted. GG were discussing $e^+e^-$ pair production via the Bethe-Heitler (BH) cross section. Apparently GG missed the suppression of pair absorption by LPM effects, which decreases the pair-production rate self-consistently. In fact the LPM regime extends to arbitrarily high energies, if treated as an electrodynamics problem in isolation. Yet nobody working before 1960 could anticipate the evolution of high energy hadronic physics.

We highlight a rather subtle point: the hadronic component of electrons, positrons and photons. High energy electrons undergo two independent causes of electromagnetic energy loss: the radiative losses and the direct losses. Radiative losses generate amplitudes proportional to the acceleration $\dot{\beta}$. Direct losses exist as $\dot{\beta} \to 0$, and were not mentioned by LPM. Direct losses traditionally start with the equivalent photon density $dN_\gamma/dx_\gamma$ per Feynman $x_\gamma$ in the uniformly moving electron’s fields. When such photons interact with cross section $\sigma_{tot}$, then the equivalent energy loss per unit distance $z$ is given by

$$\frac{dE_{\text{equivalent}}}{dz} = E \int dx x \frac{dN_\gamma}{dx} n\sigma_{tot}(xE).$$

(1)

The electron then has hadronic interactions because $\sigma_{tot}$ has hadronic contributions, commonly called photonic reactions. That is, the physical electron cannot be separated from a comoving cloud of photons, and the physical comoving photons cannot be separated from their comoving component of hadrons.

Numerous hadronic interactions are carefully implemented in current air-shower Monte carlo codes. For muons and tau leptons the direct photonic losses are larger than the radiative losses, and are included via equivalent photons in current codes: but not for the electron. Meanwhile the photonic reactions are not LPM-suppressed because they occur on very short
hadronics distances and time scales. Absorption of photons also affects the LPM radiative losses. Since electron and positron pairs, plus photons, are the primary component of electromagnetic shower cascades, the photonic nuclear losses by electrons may have an observable effect in shower evolution.

Shower energy calibration also involves detailed features of the detector response, which is not our task. Our task here is the termination of the LPM regime above a certain energy. The exact electromagnetic energy loss comes from the Poynting vector,

$$\frac{dE_{tot}}{dz} = \frac{b_{min}}{2\pi} \int d\phi d\omega \left[ \hat{E}_\omega (b_{min}) \times \hat{B}_\omega (b_{min}) \right]$$  \hspace{1cm} (2)

Here $\hat{E}_\omega (b)$, $\hat{B}_\omega (b)$ are the electric and magnetic fields in Fourier $\omega$ space evaluated at transverse spatial coordinate $\hat{b}$. The trajectory of a fast charge has a series of small angle kinks distributed along a nearly straight path. Up to a phase, the radiation fields from a particular kink are given by

$$\hat{R}E_{rad} = Q \frac{\hat{n} \times \hat{n} \times \Delta \hat{b}}{1 - \hat{n} \cdot \hat{b}}$$  \hspace{1cm} (3)

where $\hat{n}$ is a unit vector pointing to the observation point and $\Delta \hat{b}$ is the change in velocity at the kink in units of $c$. Kinks separated by less than the formation length of radiation add their fields with destructive interference. In free space the formation length $L_o \sim \gamma^2 / \omega$ for a charge with Lorentz boost factor $\gamma$, while in the LPM regime $L \sim \sqrt{\omega} L_o / \gamma$.

The kink field [Eq. (3)] transforms like a boosted dipole. Meanwhile the “kinematic” monopole fields of the boosted Coulomb charge are azimuthally symmetric about the trajectory. By orthogonality over the azimuthal integral, there is zero net interference between the energy losses of the boosted Coulomb fields and the radiation fields:

$$\frac{dE}{dz} = \frac{dE_{radiative}}{dz} + \frac{dE_{direct}}{dz}$$  \hspace{1cm} (4)

We set aside the radiative losses for now, and turn to the direct term. The fields of a uniformly moving charge are given by

$$|E_\perp \omega| = \frac{q \omega}{v^2 \epsilon \omega \gamma m} \sqrt{\frac{2}{\pi}} K_1 \left( \frac{b \omega}{v^2 \gamma m} \right)$$

$$|E_\parallel \omega| = \frac{-i q \omega}{v^2 \epsilon \omega \gamma m} \sqrt{\frac{2}{\pi}} K_0 \left( \frac{b \omega}{v^2 \gamma m} \right)$$  \hspace{1cm} (5)

where $K_0$ and $K_1$ are modified Bessel functions. The effective Lorentz boost factor in the medium, denoted by $\gamma_m$, is defined as

$$\frac{1}{\gamma_m} = \sqrt{1 - \beta^2 \epsilon \omega} = \sqrt{\frac{1}{\gamma^2} + 1 - \epsilon \omega}.$$  \hspace{1cm} (6)

The corresponding formula for the energy loss is

$$\frac{dE}{dz} = \frac{2}{\pi} \alpha \text{Re} \int_0^\infty d\omega \frac{i \omega^2 b_{min}}{\epsilon \omega \gamma_m^2} \left[ \frac{1}{\epsilon^2} - b^2 \right] \times K_0 \left( \frac{b_{min} \omega}{\gamma_m^2} \right)$$

$$K_1 \left( \frac{b_{min} \omega}{\gamma_m^2} \right) K_0 \left( \frac{b_{min} \omega}{\gamma_m^2} \right),$$  \hspace{1cm} (7)

attributed to Fermi $\gamma_0$. Here $b_{min}$ is the minimum impact parameter for which the loss subprocess proceeds.

The Fermi formula is rather more sophisticated than the equivalent photon approximation. Replacement of $1/\epsilon - 1 - (\epsilon - 1) - \epsilon \omega$ plus the limit $\omega \to 0$, are needed to find the approximation Eq. [8]. Equivalent photons can be used to calculate various subprocesses $d\sigma$, but only at leading-log order. When subprocesses are pursued beyond leading order certain re-summations are needed, which depend on the process. But there are no restrictions on Poynting’s theorem, even in the quantum domain. All the terms needed for energy loss are summed in the “Fermi formula” Eq. [8], which is deeper than it may appear at first sight.

As a consequence, there are two distinct regimes:

$$\frac{1}{\gamma^2} \gg |\epsilon \omega - 1| \to \frac{\omega^2}{\gamma^2} < \frac{1}{b_{min}};$$  \hspace{1cm} (8)

$$\frac{1}{\gamma^2} \ll |\epsilon \omega - 1| \to \frac{\omega^2}{\gamma^2} |\epsilon \omega - 1| < \frac{1}{b_{min}}.$$  \hspace{1cm} (9)

We use the “absorption length” $1/(\omega \text{Im} [\epsilon_\omega]) = L_{abs}$ as a characteristic length, of order $1/n \sigma_{tot}$. The regime of Eq. [8] corresponds to free space, or any other cases of negligible medium effects. The Bessel functions then permit $\omega < m \gamma$, which is the kinematic maximum. Then $dE/dz \sim E$ for the familiar case of $L_{abs}$ being constant. In the LPM model $1/L_{abs} \sim \sqrt{\omega}/E$, $\omega_{max}$ is the kinematic maximum at high enough energies, and $dE/dz \sim \sqrt{E}$. In contrast, the other regime of Eq. [9] is dominated by absorption in the medium, and $\omega_{max} \sim m/\sqrt{\epsilon \omega - 1}$. The corresponding loss integral is constant if (say) $L_{abs}$ is constant. The crossover between the two regimes occurs when the absorption maximum falls below the kinematic maximum, namely

$$\gamma^2 > \frac{1}{|\epsilon \omega - 1|} \to \text{absorption dominates.}$$  \hspace{1cm} (10)

Multiplying both sides of Eq. [10] by wavelength $\lambda$, the regime of absorptive dominance corresponds to $\gamma^2 \lambda > L_{abs}$. The left-hand side is the free-space formation length, which ceases to apply when it exceeds the absorption length. This is just the criterion for absorption dominance cited by order of magnitude formulas by GG $\gamma_0$.

We continue by noting that absorption and propagation are linked by unitarity and dispersion relations relating the real and imaginary parts of propagators. Indeed
the renormalized photon propagator in a medium is elegantly coded into the 
permittivity $\epsilon_\omega$ appearing in $\gamma_m$ and Eq. \[\text{[7]}\]. By definition

$$
\frac{1}{\epsilon_\omega \omega^2 - k^2} = \frac{1}{\omega^2 - k^2 - \Sigma},
$$

(11)

where $\Sigma$ is the irreducible self-energy from summing bubble diagrams. (The “pole model” for $\epsilon_\omega$ of non-relativistic tradition is one example.) Either symbol, $\epsilon_\omega$ or $\Sigma$ is equally valid to parametrize the propagator, with the identity relating them $\epsilon_\omega - 1 = -\frac{\Sigma}{\omega^2}$. In this form the optical theorem is

$$
\text{Im}[\epsilon_\omega] = \frac{n \sigma_{\text{tot}}(\omega)}{\omega},
$$

(12)

where $n$ is the number density of scatterers with photon total cross section $\sigma_{\text{tot}}(\omega)$. Given $\sigma_{\text{tot}}$, then the full dielectric constant follows by dispersion relations,

$$
\text{Re}[\epsilon(\omega)] = 1 + \frac{1}{\pi} \mathcal{P} \int d\omega^\prime \frac{\text{Im}[\epsilon_\omega]}{\omega^\prime - \omega},
$$

where $\mathcal{P}$ is the principal value.

We turn to the total cross section $\sigma_{\text{tot}}(\omega)$. In the LPM model the pair production cross section is \[\text{[7]}\]

$$
\sigma_{\text{LPM}}(\omega) = \frac{7}{9} \frac{m_e^2}{n E_s \sqrt{\omega} X_o}
$$

(13)

where $E_s = 21.2$ MeV is the scattering energy scale and $X_o$ is the radiation length. The corresponding dielectric properties are

$$
\epsilon_{\omega,\text{LPM}} = 1 + \frac{7}{9} \frac{m_e^2}{E_s \sqrt{X_o}} \frac{1}{\pi \sqrt{\omega^3}} \ln \left( \frac{1 + \sqrt{\frac{\omega - i \epsilon}{E_{\text{LPM}}}}}{1 - \sqrt{\frac{\omega - i \epsilon}{E_{\text{LPM}}}}} \right),
$$

(14)

where $E_{\text{LPM}}$ is the LPM scale. Note that $\sigma_{\text{LPM}}(\omega)$ decreases with $\omega$, as necessary for the self-consistency of the LPM argument.

Meanwhile photonuclear contributions increase with $\omega$. At some high energy, the LPM-model absorption cross section can only be a small fraction of $\sigma_{\text{tot}}$. This is because hadronic total cross sections typically scale like $\sigma \sim s^a$, where $s = 2m_p E$ is the center of mass energy-squared on a target of mass $m_p$. Two explanations for this dependence are Regge/Pomeron theory, and the observed growth of small-$x$ parton distributions $q(x)$. Data from HERA shows that $xg(x) \sim x^{-0.4}$ to a good approximation, yielding $\sigma_{\text{tot}} \sim E^{0.4}$. Growth with energy is simply due to the increased number of partons, and occurs in just the same way for the neutrino-nucleon cross section \[\text{[7]}\]. The 1998-2001 photon cross section by Donnachie and Landshoff (DL) \[\text{[12]}\] is a highly cited example:

$$
\sigma_{\text{DL}}^x(E) = 0.00016 \left( \frac{2E}{\text{GeV}} \right)^{0.4372} + 0.067 \left( \frac{2E}{\text{GeV}} \right)^{0.8008} + 0.129 \left( \frac{2E}{\text{GeV}} \right)^{-0.4525} \text{mb}.
$$

(15)

We will use DL as an up to date baseline for $\sigma_\gamma$, although all extrapolations into the UHE regime should be used with caution. The corresponding dielectric properties are

$$
\epsilon_{\omega,\text{DL}} = 1 - \sum_j \frac{n C_j}{\pi \omega_{\text{min}}^j} F \left( 1, 1 - d_j, 2 - d_j, \frac{\omega - i \epsilon}{\omega_{\text{min}}} \right),
$$

(16)

where $\omega_{\text{min}} \sim \text{GeV}$, $F(\alpha, \beta, \gamma, \delta)$ is the hypergeometric function, $C_j$ are the prefactors and $d_j$ are the powers of $\omega$ in Eq. \[\text{[13]}\]. Cross sections are expected to grow until parton saturation sets in, after which asymptotic behavior such as the Froissart bound is expected. The DL total cross section reaches a modest geometric value (40 mb) at $E \sim 10^{21}$ eV, and extrapolations far above that value might be slowed to $\log^2(E)$ or similar asymptotic behavior.

We are not aware of a model for $\sigma_{\text{tot}}(\omega)$ which does not rise in the high energy regime. We compared the 1981 fit of Bezrukov and Bugaev (BB) \[\text{[12]}\] such that Froissart bound dependence sets in at about 50 GeV. The BB fit is

$$
\sigma_{\text{BB}}^N(E) = 114.3 + 1.67 \ln^2 \left( \frac{0.0213 E}{\text{GeV}} \right) \mu\text{b}.
$$

(17)

We use the BB model to compare model-dependence. We found $\epsilon_{\omega}$ analytically by dispersion relations in terms of poly-log functions. A more useful representation as asymptotic series is:

$$
\epsilon_{\omega,\text{BB}} = 1 + \frac{114.3 n}{\pi \omega} \ln \left( \frac{-\omega_{\text{min}}}{\omega - i \epsilon} \right) + \frac{1.67 n}{3 \pi \omega} \left[ 95.01 + 54.32 \ln \left( \frac{-1}{\omega - i \epsilon} \right) + 11.55 \ln^2 \left( \frac{-1}{\omega - i \epsilon} \right) + \ln^3 \left( \frac{-1}{\omega - i \epsilon} \right) \right].
$$

(18)

Potential effects for GZK are illustrated in Fig. \[\text{[1]}\] based on numerical integration of Eq. \[\text{[6]}\] for air at sea level ($X_o \sim 30420$ cm; $\omega_{\text{min}} = 1/m_p$). Conventional radiative losses (also shown) reveal the LPM downturn above $E_{\text{LPM}} \sim 2.3 \times 10^{11}$ MeV. The DL-based calculation becomes non-negligible above about $10^{19}$ eV and exceeds the radiative losses above $5 \times 10^{20}$ eV. The BB-based losses are negligible through this energy range, but also exceed the radiative losses above $10^{23}$ eV. Eventually any $\sigma_{\text{tot}}(\omega)$ model on the market will terminate the LPM regime.

The onset of large direct losses of the DL-model (Fig. \[\text{[1]}\]) coincides alarmingly with the energy range where the GZK bound is confronted. As far as known, the actual cross sections could easily be larger than DL’s, causing the regime to occur even earlier. However we have not finished with the repercussions, because there remain the radiative losses postponed earlier. For any given $x_\gamma = \omega/E$ fraction, the LPM coherence length $L_{\text{LPM}} \sim \sqrt{\gamma/x_\gamma}$ increases indefinitely with $\gamma$. Meanwhile
When a regime of large enough physics as the experimentally observed [14] low energy rect losses become noticeable [13]. This is just the same quenching of radiative LPM occurs just where the di-longer comoving and superposing “on top of one another” showers at 3\times 10^6 MeV. The direct losses with the DL model (long-dashed) shown for comparison. Short-dashed line shows the LPM-suppressed radiative losses, which turn over at E_{LPM} \sim 2.3 \times 10^{11} \text{MeV}. The direct losses of both the DL- and BB-models terminate the LPM regime (radiative losses) above 5 \times 10^{20} and 10^{23} \text{eV}, respectively.

FIG. 1: Total energy loss (dE/dz) of electrons in air at sea level (X_0 = 30420 cm) assuming standard LPM radiative losses. Losses with the DL model are indicated by the solid line, with the BB model (long-dashed) shown for comparison. Unfortunately the uncertainties of formulas are an overestimate. The crossover from LPM (say) above the crossover point, the direct losses dominate anyway, so that it does not matter that the LPM suppression, and thereby the effects we study apply not only to the electron, but to every single charged particle!

We conclude that extrapolations of energy calibra-

1. We have nevertheless prepared Fig. 4 using the standard LPM radiative losses to facilitate comparison. At an order of magnitude energy (say) above the crossover point, the direct losses dominate anyway, so that it does not matter that the LPM formulas are an overestimate. The crossover from LPM radiative to direct absorptive regime is inevitable. Unfortunately the uncertainties of make the onset energy of absorptive dominance very uncertain.

Figure 4 has implications for the calibration of air shower energies. The maximum number of shower particles N_{max} increases with primary energy. The effects of LPM are to slow this increase: for photon-initiated showers at 3 \times 10^{20} \text{eV}, N_{max} \sim 1.3 \times 10^{11} with LPM suppression, and N_{max} \sim 2.2 \times 10^{11} with LPM turned off [17]. Adding direct losses is akin to turning LPM off (Fig. 4). However observational constraints on shower energies include not only N_{max}, but also the shape of the shower based on Greisen-type shower profiles. There is no direct way to go from a particular cross section to the entire shower profile, which incorporates every possible particle reaction. Indeed, the direct losses we study apply not only to the electron, but to every single charged particle!

We conclude that extrapolations of energy calibra-

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