Analytical scannable-shaped beam pattern synthesis via superposition principle

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Abstract
A shaped beam pattern (SBP) synthesis algorithm, which obtains the desired SBP in an analytical way, is proposed. The new algorithm describes the SBP with the weighted superposition of a set of pencil beam patterns (PBPs). First, the array weight of the PBP is obtained via analytical algorithms, such as; Chebychev and Taylor, etc. Then, by computing the weights of the PBPs with the least-square method, the synthesized SBP is expressed explicitly. Different from the state-of-the-art algorithms, the proposed algorithm can obtain the desired SBP analytically, avoiding the time-consuming iteration processes. Moreover, the proposed algorithm enables the array antenna to scan the beam in different directions. Simulations with linear and planar array antennas are provided to assess the superiority and effectiveness of the proposed algorithm.

1 | INTRODUCTION
Beam pattern synthesis (BPS) with array antenna has attracted extensive attention for its flexibility in generating the desired beam pattern by optimally selecting the array weight or the array position [1–3]. The BPS problem is mainly divided into two categories: (1) the pencil beam pattern (PBP) synthesis (PBPS) [4–6] and (2) the shaped beam pattern (SBP) synthesis (SBPS) [7–9]. This article investigates the SBPS problem by optimizing the array weight. Two typical SBPs, including the flat-top beam pattern [10–12] and the cosecant square beam pattern [13–15] would be discussed. These two types of patterns are commonly used in 4G communication and satellite communication. For instance, the satellite signal receiving with antenna system on mobile vehicles requires a flap-top beam pattern to provide the well signal reception even the vehicles are randomly moving, while for a 4G base station, it requires a cosecant square beam pattern to ensure its radiation powers in its effective coverage area are equal.

Lots of algorithms were proposed for solving the SBPS problem. The evolutionary algorithms, including the genetic algorithm [16], the particle swarm optimization (PSO) [17] and the ant colony optimization [18] etc., appeared the earliest. These algorithms are effective for solving the SBPS problem with moderate array size. However, they often suffer intensive computational burden when the array size becomes large. To overcome these drawbacks, the convex-optimization-problem-based (COPB) algorithms were proposed [19–20]. With the convex structure, the COPB algorithms can solve the SBPS problem with the existing toolbox, such as CVX and SeDumi. It needs to be noted that the aforementioned algorithms are applicable to both the uniformly and non-uniformly distributed array antenna. However, all those algorithms are based on the iterative strategy, which indicates that the optimal or the sub-optimal solutions are obtained by searching the possible solution space with polynomial complexity. When the array size becomes large, such as the upcoming 5G communication and the Low-Earth-Orbit satellite constellation communication applications, even the COPB algorithms would become slow or even are intractable.

The article proposes an analytical algorithm for the SBPS with a uniformly distributed array antenna. This algorithm describes the SBP as a weighted superposition of a set of PBPs. The idea is inspired by [9] and [21]. In these two references, the
SBPS problem is solved with an analytical algorithm that is super simple and effective. One obvious disadvantage of this existing algorithm is that its performance would degenerate when the center of the SBP beam is far away from the broadside axis (Figure 1). To make the algorithm scannable, this article determines the weighted superposition weight of the PBPs via the least-square method (LSM). With the LSM, the proposed algorithm would minimize the total power difference between the desired SBP and the obtained SBP. In such a way, the proposed algorithm is expected to scan in different directions with a low sidelobe level (SLL). Two advantages of the proposed algorithm are (1) its computation complexity is decided by the required angular resolution of the beam pattern which, therefore, makes the proposed algorithm work much faster than the existing algorithms. Its advantage would be more obvious when the array size becomes large because the computation complexity of the most existing synthesis algorithms is polynomially proportional to the array size and (2) the array weight can be analytically obtained by giving the array layout and does not require any iteration process as the existing algorithms do.

The remainder of the article is organized as follows: the SBPS problem expressed by the weighted superposition of a set of PBPs is introduced in Section 2; the analytical proposed algorithm is proposed in Section 3; simulations and discussion are carried out in Section 4 and the conclusions are drawn in Section 5.

2 | PROBLEM FORMULATION

Without loss of generality, a linear array antenna consisting of \( N \) elements with the position being \( r_n = n, n = 1, \ldots, N \) is considered. Denote \( E_n(\theta), a_n(\theta) \) and \( w_n \) the far-field electric field, the array factor and the element weight of the \( n^{th} \) \((n = 1, \ldots, N)\) element, respectively. The total far-field electric field of the array antenna can be expressed as the sum of those of the elements:

\[
f(\theta, \theta_c) = \sum_{n=1}^{N} a_n(\theta, \theta_c)E_n(\theta)w_n \tag{1}
\]

where \( a_n(\theta, \theta_c) = \exp(jkr_n(\sin \theta - \sin \theta_c)) \) with \( \kappa = \frac{2\pi}{\lambda} \) being the spatial wavenumber, \('\theta'\) being the electromagnetic wavelength and \('\theta_c'\) being the center direction of the beam. By vectorizing, the above electric field can be rewritten as

\[
f(\theta, \theta_c) = \mathbf{a}_E^H(\theta, \theta_c)\mathbf{w}
\]

where

\[
\begin{align*}
\mathbf{a}_E(\theta, \theta_c) &= \mathbf{a}(\theta, \theta_c) \odot \mathbf{E}(\theta) \\
\mathbf{a}(\theta, \theta_c) &= [a_1(\theta, \theta_c), \ldots, a_N(\theta, \theta_c)]^H \\
\mathbf{E}(\theta) &= [E_1(\theta), \ldots, E_N(\theta)]^H \\
\mathbf{w} &= [w_1, \ldots, w_N]^H
\end{align*}
\]

Restricted by the limited resolutions of kinds of hardware, the beam angle is usually discretized. Denote the mainlobe and the sidelobe of the SBP \( \Theta_{ML} \) and \( \Theta_{SL} \), respectively. Denote the angular-step \( \Delta \theta \), the discretized angles in the mainlobe are \( \theta_{l_m} \in \Theta_{ML}, l_m = 1, \ldots, L_M \) with \( L_M = \lfloor \frac{\theta_{\text{top}}}{\Delta \theta} \rfloor \), and those in the sidelobe are \( \theta_{l_s} \in \Theta_{SL}, l_s = 1, \ldots, L_S \) with \( L_S = \lfloor \frac{\theta_{\text{top}}}{\Delta \theta} \rfloor \). To obtain the desired SBP, the PBPs pointing to different directions, i.e. \( f_{\text{PBP}}(\theta, \theta_{l_m}) \), are combined as in [9, 21]:

\[
f_{\text{SBP}}(\theta) = \sum_{l_m=1}^{L_M} f_{\text{PBP}}(\theta, \theta_{l_m})w_{l_m, \text{SBP}} \tag{4}
\]

where

\[
f_{\text{PBP}}(\theta, \theta_{l_m}) = \mathbf{a}_{E}^H(\theta, \theta_{l_m})\mathbf{w}_{\text{PBP}} \tag{5}
\]

with \( w_{l_m, \text{SBP}} \) being the weight of the \( l_m^{th} \) PBP and \( \mathbf{w}_{\text{PBP}} = [w_{1, \text{PBP}}, \ldots, w_{N, \text{PBP}}]^H \).

For uniformly distributed array antenna scenarios, the array weight of the PBP (5) in this article, that is \( \mathbf{w}_{\text{PBP}} \), can easily be obtained via Chebychev or Taylor algorithms. In [9] and [21], \( w_{l_m, \text{SBP}} \) is determined by the amplitude of the desired SBP. Marked the existing algorithm as LQAlg. The LQAlg is clear in theory and simple in applications. However, one of its obvious disadvantages is that the synthesis performance would deteriorate seriously when the SBP scans in different directions, especially when its center is far from the broadside axis. Four different 40°-width flat-top beam patterns with different centre beam directions via a 41-element linear array are synthesized for further explanation in Figure 1. The figure shows the synthesized results via the LQAlg with different centre angles, that is, \( \theta_c = \{0^\circ, 20^\circ, 40^\circ, 60^\circ\} \). When the beam centre

\[\text{The weighed superposition weight of the PBPs is directly determined by the amplitude of the SBP in [9].}\]
scans from 0° to 60°, the beam performance worsens along with the centre angle. Specifically, the LQ_ALG have low SLL and low mainlobe ripple level (MRL) when \( \theta_c = 0° \) and 20°; however, the SLL and MRL would become larger when \( \theta_c \) increases. Worse is that when \( \theta_c = 60° \), the SLL and MRL are almost unacceptable. In this article, a simple scheme based on the LSM for computing \( w_{l,SBP} \) is proposed. The new algorithm is capable of scanning in different directions with slight performance deterioration.

3 | THE COMPUTATION OF \( w_{l,SBP} \)

Vectorizing (4), it yields

\[
f_{SBP}(\theta) = F_{PBP}(\theta)w_{SBP} \tag{6}
\]

with

\[
\begin{align*}
F_{PBP}(\theta) &= [f_{PBP}(\theta, \theta_1), \ldots, f_{PBP}(\theta, \theta_{N})]^T \\
w_{SBP} &= [w_{1,SBP}, \ldots, w_{l,SBP}]^T
\end{align*}
\tag{7}
\]

In the LQ_ALG, the selection of \( w_{SBP} \), \( n = 1, \ldots, N \) in [9] is only related to the pattern shape which does not consider the influence of the beam centre direction, that is, \( \theta_c \). For example, the LQ_ALG set \( b_n = 1 \) for a normalized flat-top beam pattern. To make the array antenna scannable, an intuition is to select \( w_{SBP} \) so that \( w_{SBP} \) would vary along with different centre angle \( \theta_c \) of \( f_{SBP}(\theta) \). The natural idea is to obtain \( f_{SBP}(\theta) \) as similar as possible to the desired SBP, that is, \( f_{desired,SBP}(\theta) \), which, therefore, can be expressed as the following optimization problem:

\[
\text{minimize } \rho \\
\| F_{desired, SBP} - F_{SBP} \|_2^2 \leq \rho \tag{8}
\]

with \( F_{desired, SBP} = [f_{desired, SBP}(\theta)], F_{SBP} = [f_{SBP}(\theta)] \) and \( l \in \{l_0, l_n\} \). The symbol \( \{'x\}' \) denotes the matrix consisting of the vector \( x \). In this problem, the sidelobe of the SBP is also controllable. Hence, \( F_{PBP}(\theta) \) in (7) should be replaced by \( F_{PBP}(\theta) = [f_{PBP}(\theta, \theta_0), l \in \{l_0, l_n\}] \). Substitute (6) into (8), it yields \( \| F_{desired, SBP} - F_{PBP,SBP} \|_2^2 \), representing the sum of the squared differences of \( L = L_M + L_S \) linear equations. To minimize this difference, the LSM is used to estimate \( w_{SBP} \) with the following equation \( F_{PBP}^H F_{desired, SBP} = F_{PBP}^H F_{PBP} w_{SBP} \). Hence, the weight \( w_{SBP} \) can be obtained as:

\[
w_{SBP} = (F_{PBP}^H F_{PBP})^{-1} F_{PBP}^H F_{desired, SBP} \tag{9}
\]

where the symbol \( \{\cdot\}^{-1} \) represents the pseudo-inverse of matrix.

Algorithm 1 described in the following table gives out the specific details on how to compute the total array weight via the proposed algorithm to obtain scannable SBP.

### Algorithm 1 The analytical scannable SBP algorithm

1. compute the array weight for the PBP (5), i.e. \( w_{PBP} \), via Chebychev or Taylor algorithm in [22]
2. build matrix \( F_{PBP}(\theta) \) in (7)
3. define the desired SBP \( F_{desired, SBP} \)
4. compute the weight \( w_{SBP} \) with (9)
5. return the total array weight \( w \) with (12) in [9]

4 | SIMULATIONS AND DISCUSSION

This section carries out simulations with both linear array and planar array to validate the performance of the proposed algorithm. Since the proposed algorithm is the improvement of LQ_ALG in [9], only the LQ_ALG is compared (the other existing algorithms for SBP are not analytical. Hence, the array weight cannot be explicitly expressed). All simulations are operated on a 64-bit windows personal computer with 3.5 GHz, Intel Core (TM) i3-4150.

4.1 | linear array scenarios

A linear array with 41 half-wavelength uniformly distributed elements is utilized to obtain the flat-top beam pattern and the cosecant square beam pattern.

4.1.1 | Flat-top beam pattern

Set the beam width 40° and the centre beam directions \( \theta_c = 20°, 40° \) and 60°. The desired SLL for the PBPs in (4) is \( -20 \) dB. Taylor algorithm is used to estimate \( w_{PBP} \) in (5). The angular step \( \Delta \theta = 0.5° \). Figure 2 shows the performance difference between the proposed algorithm and the LQ_ALG:

- The proposed algorithm has lower MRLs than the LQ_ALG in all cases, while it has higher SLL in the first two cases. The MRL of the proposed algorithm becomes worse from \( \theta_c = 20° \) to \( \theta_c = 60° \). However, the variation is inner –2 dB, which makes it acceptable in most applications. Comparatively, the MRL of the LQ_ALG deteriorates very fast. Especially, the obtained beam via the LQ_ALG is not a flat-top when \( \theta_c = 60° \) because its MRL is too large.
- In the first two cases, the SLL for the proposed algorithm is lower than \( -20 \) dB, satisfying the desired \( -20 \) dB requirement. In the last case, i.e. \( \theta_c = 60° \), the SLLs for the proposed algorithm and the LQ_ALG are \(-18.2 \) and \(-8.3 \) dB, respectively. It shows when the beam is far away from the broadside axis, the proposed algorithm could obtain a lower SLL than the LQ_ALG.
- One advantage of the proposed algorithm is that it can remain the MRL (varies inner –2 dB) when the SBP scans in different directions while the MRL would become large via the LQ_ALG.
4.1.2 | Cosecant square beam pattern

The related parameters are set similar to those in the previous sub-part. Figure 3 shows the synthesized cosecant square beam pattern via the proposed algorithm and the LQA_{Alg}. The desired normalized cosecant square beam pattern is defined by

$$f_{\text{csc}}(\theta) = \begin{cases} 1, & \theta \in [\theta_0, \theta_1] \\ \text{norm}(\text{csc}^2(\theta)), & \theta \in [\theta_1, \theta_2] \end{cases}$$

(10)

where $[\theta_0, \theta_2] \in \Theta_{\text{Ali}}$ and norm($\text{csc}^2(\theta)$) represent the normalized form of csc(\theta). The figure shows that the obtained cosecant square beam pattern matches the desired one better than the LQA_{Alg}. The advantage of the proposed algorithm is more obvious when the centre of the mainlobe scans to a bigger angle. It is worth mentioning that the SLL via the proposed algorithm is higher than that via the LQA_{Alg}. However, the SLLs obtained via both algorithms are below $-20 \, \text{dB}$, which satisfies the designed requirement. When the MRL is the more important factor, the proposed algorithm would be more applicable; otherwise, the LQA_{Alg} is preferred.

4.2 | Planar array scenarios

A 21 × 21 half-wavelength distributed planar array antenna is simulated. The desired SLL is $-20 \, \text{dB}$. Figure 4 shows the synthesized results of generating a two-dimensional flat-top beam with the mainlobe being $\theta \times \phi = [40^o, 80^o] \times [40^o, 60^o]$. The red square in the two sub-figures is the desired mainlobe. Both algorithms can obtain the peak SLL around $-20 \, \text{dB}$. However, the proposed algorithm has a smaller mainlobe region which means that the power concentrated in the mainlobe via the proposed algorithm is stronger than that via the LQA_{Alg}. In this sense, the proposed algorithm matches the designed requirement better than the LQA_{Alg} does. Some other simulations, with different mainlobe regions, such as $\theta \times \phi = [40^o, 80^o] \times [40^o, 70^o]$ and $\theta \times \phi = [40^o, 80^o] \times [40^o, 80^o]$, are also carried out. All the results (not shown in the article), indicate that the proposed algorithm can obtain better mainlobe than the LQA_{Alg}. For some of them, the obtained SLLs via the proposed algorithm are worse than the LQA_{Alg} but they are only 2 dB variation around the desired value, i.e. $-20 \, \text{dB}$.

4.3 | Discussion

In the two previous parts, both linear array antenna and planar array antenna are simulated to assess the performance of the proposed algorithm. The advantage of these two algorithms is that both are analytical. Their array weights can be computed directly given the array layouts. Since the performance of the LQA_{Alg} would deteriorate when the mainlobe scans in different directions, the proposed algorithm tries to make an
improvement. By updating the superposition weight of a set of PBPs with the LSM, the proposed algorithm enables the beam to scan in different directions with tiny mainlobe degeneration. Meanwhile, the SLL obtained via the proposed algorithm is higher than the LQA⃗. Gratifying is that the SLL via both algorithms satisfies the designed requirement for most scenarios.

To further validate the performance in the real applications, data including the element pattern and the mutual coupling among elements is considered using the active element pattern (AEP) [23–24]. The AEP of an element can be obtained by exciting it, while the others are connected to matching loads. Figure 5 shows the synthesized flat-top beam pattern and cosecant square beam pattern via the proposed algorithm. Both results are similar to those in Figures 2 and 3. These results further validate the effectiveness of the proposed algorithm for its robust scanning ability.

5 | CONCLUSIONS

The article proposes an analytical SBPS algorithm for uniformly distributed array antenna, which can obtain the array weight analytically. It is an improvement algorithm for the LQA⃗ in [9]. Instead of determining the superposition weight of a set of PBPs by the amplitude of the SBP as in [9], the proposed algorithm computes the superposition weight with the LSM. In such a way, the beam direction is also considered updating the superposition weight, which makes the proposed algorithm to be able to scan its beam with tiny MRL degeneration. It is important to note that the SLL via the proposed algorithm is worse than the LQA⃗ when the mainlobe is close to the broadside direction even it still satisfies the desired value. Besides, compared with the other existing algorithm for the SBP, which solves the SBPS problem with iterative processes, both the proposed algorithm and the LQA⃗ would operate faster when the array size is large.

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