Noncommutative approach to diagnose degenerate Higgs bosons at 125 GeV

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ABSTRACT: We propose a noncommutative (NC) version for a global O(2) scalar field theory, whose damping feature is introduced into the scalar field theory through the NC parameter. In this context, we investigate how noncommutative drives spontaneous symmetry breaking (SSB) and Higgs-Kibble mechanisms and how the damping feature workout. Indeed, we show that the noncommutativity plays an important role in such mechanisms, i.e., the Higgs mass and VEV dependent on NC parameter. After that, it is explored the consequences of noncommutativity dependence of Higgs mass and VEV: for the first, it is shown that there are a mass-degenerate Higgs bosons near 126.5 GeV, parametrized by the noncommutativity; for the second, the gauge fields gain masses that present a noncommutativity contribution.

KEYWORDS: Spontaneous symmetry breaking, Higgs-Kibble mechanism, Noncommutative Theory.
1 Introduction

Noncommutativity has been extensively investigated in different contexts: quantization procedure\cite{1–11}, the Yang-Mills theory on a NC torus\cite{12, 13}, matrix model of M-theory\cite{14–17}, string theory\cite{18–25} and D-brane\cite{26–31}, SSB and Higgs-Kibble mechanisms\cite{32–36}. At last scenario, it was investigated how noncommutativity affects the IR-UV mixing and the appearance of massless excitations\cite{32, 33}, the relation between symmetry breaking in NC cut-off field theories\cite{34, 35} and the role played by the noncommutativity in the masses generation of new bosons\cite{36}. Despite of all this extensive research, the whole role played by noncommutativity in mass generation in the global $O(2)$ scalar field theory was not properly investigated. In order to fill some gap into this matter, that problem must be investigated from an alternative point of view, precisely based in the induction of a damping feature into the theory\cite{31}. At this context, we show how noncommutativity affects the conception for spontaneous symmetry breaking\cite{37–40} and Higgs-Kibble mechanisms\cite{41–43} and, consequently, it is shown that noncommutativity might explain a mass-degenerate Higgs bosons\cite{44–50} near 126.5 GeV and that the noncommutativity contribution arises into the masses gained by gauge fields.

This work is organized as follows. In section 2, we explore Noncommutative Mapping\cite{11} in the global $O(2)$ scalar field theory. In section 3, the contribution of noncommutativity in the idea of spontaneous symmetry breakdown mechanism is shown and, as well as, it is also discussed how mass-degenerate Higgs bosons near 126.5 GeV arise due to noncommutativity. Further, it is also shown how it can be related with the damping phenomenon. In section 4, the noncommutative contribution into Higgs-Kibble mechanism is investigate and its consequence in masses gained by gauge fields is discussed. At the end, some conclusions are presented.
2 NC scalar field theory

In order to illustrate the contribution of noncommutativity in the context of field theory, we chose a simplest scalar field in four space-time dimensions, namely, a global $O(2)$ scalar theory, whose dynamics is governed by

$$
\mathcal{L} = \frac{1}{2} (\partial_{\mu} \phi_i)(\partial^{\mu} \phi_i) - \frac{\mu^2}{2} \phi_i \phi_i - \frac{\lambda}{4} (\phi_i \phi_i)^2 ,
$$

(2.1)

where $\lambda$ is a positive number, $\mu^2$ can be either positive or negative and the field $\phi_i$ transforms as a 2-vector. The corresponding Hamiltonian is

$$
\mathcal{H} = \frac{\pi_i \pi_i}{2} + \frac{(\nabla \phi_i)(\nabla \phi_i)}{2} + U ,
$$

(2.2)

the potential is given by

$$
U = \frac{\mu^2}{2} \phi_i \phi_i + \frac{\lambda}{4} (\phi_i \phi_i)^2 .
$$

(2.3)

It is well know that if $\mu^2 > 0$, then the vacuum is at $\phi_i = 0$ and the symmetry is manifest, and $\mu^2$ is the mass of the scalar modes. On the other hand, if $\mu^2 < 0$, then there is a new vacuum solution given by $(\phi_i \phi_i) = -\frac{\mu^2}{\lambda}$, which has an infinite number of possible vacua.

In the commutative framework, the symplectic variables are $\xi^\beta = (\phi_i, \pi_i)$ and the symplectic matrix is

$$
f = \left( \begin{array}{cc} 0 & \delta_{ij} \\ -\delta_{ij} & 0 \end{array} \right) \delta^{(3)}(x - y) .
$$

(2.4)

The noncommutativity is introduced into the model changing the brackets among the phase-space variables, given by

$$
\{ \tilde{\phi}_i, \phi_j \} = 0 , \quad \{ \tilde{\phi}_i, \tilde{\pi}_j \} = \delta_{ij} \delta^{(3)}(x - y) , \quad \{ \tilde{\pi}_i, \tilde{\pi}_j \} = \theta \varepsilon_{ij} \delta^{(3)}(x - y) ,
$$

(2.5)

where $\theta$ embraces the noncommutativity. These brackets are comprised by the symplectic matrix in NC basis, namely:

$$
\tilde{f} = \left( \begin{array}{cc} 0 & \delta_{ij} \\ -\delta_{ij} & \theta \varepsilon_{ij} \end{array} \right) \delta^{(3)}(x - y) .
$$

(2.6)

The NC transformation matrix[11], $R = \sqrt{\tilde{f} f^{-1}}$, is written as

$$
R = \left( \begin{array}{cc} \delta_{ij} & 0 \\ \frac{1}{2} \theta \varepsilon_{ij} \end{array} \right) \delta^{(3)}(x - y) .
$$

(2.7)

Since the commutative symplectic variables $\xi^\beta = (\phi_i, \pi_i)$ change to the NC ones $\tilde{\xi}^\alpha = (\tilde{\phi}_i, \tilde{\pi}_i)$ through $d\tilde{\xi}^\alpha = R^\alpha_\beta d\xi^\beta$, it follows that

$$
\tilde{\phi}_i = \phi_i , \quad \tilde{\pi}_i = \pi_i + \frac{1}{2} \theta \varepsilon_{ij} \phi_j .
$$

(2.8)
In agreement with the NC Mapping\cite{[11]} the NC first-order Lagrangian can be read as

\[ \tilde{L}(\phi_i, \dot{\phi}_i) = \pi_i \dot{\phi}_i - H(\phi_i, \pi_i), \] (2.9)

where \( \tilde{H}(\phi_i, \pi_i) = H(\tilde{\phi}_i, \tilde{\pi}_i) \) and the latter one is the NC version of the Hamiltonian, Eq.(2.2), given by

\[ \tilde{H}(\tilde{\phi}_i, \tilde{\pi}_i) = \frac{\tilde{\pi}_i \tilde{\pi}_i}{2} + \left( \nabla \tilde{\phi}_i \right) \left( \nabla \tilde{\phi}_i \right) + \frac{\mu^2}{2} \tilde{\phi}_i \tilde{\phi}_i + \frac{\lambda}{4} \tilde{\phi}_i \tilde{\phi}_i^2. \] (2.10)

The Hamiltonian density in Eq.(2.10), with the help of Eq.(2.8), renders to

\[ \tilde{H}(\phi_i, \pi_i) = \frac{\pi_i \pi_i}{2} + \frac{\theta}{2} \varepsilon_{ij} \phi_j + \left( \nabla \phi_i \right) \left( \nabla \phi_i \right) + \tilde{U}, \] (2.11)

where

\[ \tilde{U} = \frac{\mu^2}{2} \phi_i \phi_i + \frac{\lambda}{4} \phi_i \phi_i^2 + \frac{\lambda}{8} \phi_i \phi_i, \] (2.12)

with \( \tilde{\mu}^2 = \mu^2 + \frac{1}{4} \theta^2 \). Observe that the original model is restored when \( \theta \) is a null quantity.

Occasionally, energy density might be written as being the sum of kinetic and potential energy\cite{[51]},

\[ E = T + V, \] (2.13)

where, in the Eq.(2.11), \( T \) is the two first term and \( V \), as usual, is the term involving no time derivatives, namely,

\[ V = \frac{\left( \nabla \phi_i \right) \left( \nabla \phi_i \right)}{2} + \tilde{U}. \] (2.14)

As a consequence, if the energy is to be bounded below, \( \tilde{U} \) must be also bounded below.

The Hamilton’s equation of motion \( \dot{\phi}_i = \frac{\partial \tilde{H}(\phi_i, \pi_i)}{\partial \pi_i} \) is calculated and the canonical momenta is obtained as being

\[ \pi_i = \dot{\phi}_i - \frac{1}{2} \theta \varepsilon_{ij} \phi_j. \] (2.15)

Inserting Eq.(2.11) and Eq.(2.15) into the NC first-order Lagrangian in Eq.(2.9), we get the NC second-order Lagrangian

\[ \tilde{L} = \frac{1}{2} \left( \partial_{\mu} \phi_i \right) \left( \partial^{\mu} \phi_i \right) - \theta \left( n^\mu \partial_{\mu} \phi_i \varepsilon_{ij} \phi_j - \frac{\mu^2}{2} \phi_i \phi_i - \frac{\lambda}{4} \phi_i \phi_i \right), \] (2.16)

with the time-like vector \( n^\mu = (1, 0) \), which is a normal vector of a noncovariant set of equitemporal surfaces \( (t = \text{constant}) \) where the Hamiltonian analysis is implemented. However, this noncovariance is apparent, because if we consider a larger set of space-like surfaces to develop the Hamiltonian formalism, this obstruction can be removed\textsuperscript{1}. From this point of view, \( \theta \varepsilon_{ij} \) appear as a set of Lagrange multipliers that imposes the velocity dependent constraint \( \left( \partial_{\mu} \phi_i \right) \phi_j \). As pointed out by some authors\textsuperscript{[53–55]}, a Lagrangian, first-order in velocity \( (\dot{\phi}_i) \), can always be considered as arising from a \( U(1) \) background potential in configuration space. At this point, we would like to point out that the middle term of the right hand side of this NC Lagrangian plays the role of damped term\textsuperscript{[31]}.\textsuperscript{1}

\textsuperscript{1}This observation is well clarified by one of us in the appendix A of Ref.[52].
3 Spontaneous symmetry breaking

In order to show the role played by the noncommutativity into the Spontaneous symmetry breaking and Higgs-Kibble mechanisms, consider the potential $\tilde{U}$, given in Eq.(2.12). In this scenario, if $\tilde{\mu}^2 > 0$ then the vacuum is at $\phi_i = 0$, the symmetry $\phi_i \rightarrow -\phi_i$ is manifest and $\tilde{\mu}^2$ is the mass of the scalar modes. On the other hand, if $\tilde{\mu}^2 < 0$, the spontaneous symmetry broken and the vacuum is at $(\phi_i\phi_i) = -\tilde{\mu}^2$. In the NC framework, the usual discussion about the spontaneous symmetry broken is still valid. In order to illustrate the discussion above, we consider the following plane section, $\phi_1 = 0$, namely:

In Fig.(1a), we can infer that the NC potential ($\tilde{U}$) has its value increased when compared with the commutative one ($U$) due to the NC $\theta$-parameter. In Fig.(1b) the spontaneous symmetry broken is affected by the NC $\theta$-parameter: at the bottom of the graph, the depth of the well is smaller than the one given in the commutative framework and, for a given $\phi_2$, the NC potential has its value increased when compared to commutative one. In analogy with what was done to explain the deep inelastic scattering phenomenon through DHO[56, 57], where internal energy of nucleons increase after the collision so that the potential energy also increases, we argue that the NC $\theta$-parameter plays the role of a damping coefficient[31], even in a spontaneous symmetry broken context.

In order to put our work in an appropriated way, we consider the isomorphism between $SO(2)$ and $U(1)$ group, which is implemented through the following transformation

$$
\begin{align*}
\phi_1 &= \frac{1}{\sqrt{2}} (\phi + \phi^*), \\
\phi_2 &= -\frac{i}{\sqrt{2}} (\phi - \phi^*),
\end{align*}
$$

(3.1)

where $\phi$ is a complex field and $\phi^*$ is its conjugated one. Inserting those transformation above into Eq.(2.16), we get

$$
\tilde{L} = \phi^*\dot{\phi} - (\nabla\phi^*)(\nabla\phi) - \frac{\theta}{2} (\phi\dot{\phi}^* - \dot{\phi}\phi^*) - \mu^2\phi^*\phi - \lambda(\phi^*\phi)^2,
$$

(3.2)
and applying the usual Legendre transformation, the Hamiltonian is computed and its
given by
\[ \tilde{H} = \pi^* \pi + (\nabla \phi^*)(\nabla \phi) - \frac{\theta}{2} i(\pi \phi - \pi^* \phi^*) + \tilde{U}, \]  
(3.3)
where the potential \( \tilde{U} \) is
\[ \tilde{U} = \tilde{\mu}^2 \phi^* \phi + \lambda (\phi^* \phi)^2, \]  
(3.4)
with
\[ \tilde{\mu}^2 = \mu^2 + \frac{\theta^2}{4}. \]  
(3.5)
Thus the VEV for \( \tilde{\mu}^2 > 0 \) is
\[ \langle \phi \rangle_0 = 0, \]  
(3.6)
the symmetry is preserved and the mass of the complex scalar mode is given by Eq.(3.5).

On the other hand, the VEV for \( \tilde{\mu}^2 < 0 \) is given by
\[ \langle \phi \rangle_0 = \frac{v}{\sqrt{2}} \]  
(3.7)
where
\[ v = \sqrt{-\tilde{\mu}^2/\lambda}. \]  
(3.8)
The field redefinition, around \( \langle \phi \rangle_0 \), for SSB is given by
\[ \phi = e^{i \zeta/v} \left( \frac{v + H}{\sqrt{2}} \right). \]  
(3.9)
Therefore, the Lagrangian, given in Eq.(3.2), renders to
\[ \tilde{\mathcal{L}} = \frac{1}{2} \left[ (\partial_\mu H)(\partial^\mu H) + (\partial_\mu \zeta)(\partial^\mu \zeta) \left( 1 + \frac{H}{v} \right)^2 + \frac{\theta}{2} \frac{\dot{\zeta}}{v} (v + H)^2 - \frac{1}{4} \lambda H^4 - \lambda v H^3 \right. \]
\[ \left. - \frac{1}{2} (\mu^2 + 3\lambda v^2) H^2 - (\mu^2 v + \lambda v^3) H - \frac{1}{2} \mu^2 v^2 - \frac{1}{4} \lambda v^4. \]  
(3.10)
The spectra has one massless Goldstone boson \( \zeta \) and one massive Higgs \( H \). Note that for
\( \theta = 0 \), the usual Lagrangian, given in the literature[51, 58–61], is restored. Further, note
that the noncommutative parameter \( \theta \) leaving trace on the VEV and the Lagrangian has
a new damping term[31] dependent of the velocity \( \dot{\zeta} \) and we can read the squared mass of
the Higgs boson as being
\[ m_H^2 = \mu^2 + 3 \lambda v^2, \]
\[ = M^2 - \frac{3}{4} \theta^2, \]  
(3.11)
where \( M \equiv +\sqrt{-2\mu^2} \) is a positive mass parameter and \( \theta^2 \geq 0 \). The spectra has one
massless Goldstone boson \( \zeta \) and one massive Higgs \( H \). Considering the 125 – 128 GeV
mass window for the Higgs boson[44] and associating the values 125, 128 GeV respectively
to the NC parameters $\theta_c, \theta_0$, from the equation Eq.(3.11), a relation concerning the NC $\theta$-parameters can be obtained as

$$\theta_c^2 - \theta_0^2 = 1012 \text{ GeV}^2$$

(3.12)

and, fixing $\theta_0$ as $\theta_0 = 0$, the mass of the Higgs boson can be computed from

$$m_H = \sqrt{(128 \text{ GeV})^2 - \frac{3}{4} \theta^2}$$

(3.13)

with $0 \leq \theta \leq \sqrt{1012} \text{ GeV}$. At this scenario, we get a NC $\theta$-parameter dependent mass Higgs boson and, consequently, it is possible to reproduce, if $\theta = \sqrt{1012} \text{ GeV}$, the typically mass of Higgs boson discovered at the LHC $\simeq 8$ TeV run-I[62–64]. On the other hand, it is possible to investigate a set of Higgs-like particles with different masses. This mass-degenerate Higgs bosons around 126.5 GeV (mean value in the mass range of the Higgs boson), at the NC scenario, can also be interpreted as being quantum interference effect that should be taken into account for the signal rates from two CP-even Higgs bosons[49, 50]. This effect seems significant since the mass splitting are comparable or smaller than the total decay widths of two nearly degenerate Higgs bosons. Indeed, some authors suggested[44–50] that the observed signals at $\approx$125 GeV may arise from two mass-degenerate Higgs bosons, since these signals rates estimations were performed by summing up the cross sections times decay branching fractions of individual Higgs boson. Another point that should be mentioned, it is that, despite of $\gamma\gamma$ rate is not revealed at the LHC run-II[65–69], the mass-degenerate Higgs bosons $\approx$125 GeV should be investigated, because it is a real challenge to distinguish this possibility from the single Higgs boson case by direct measurements of the Higgs boson mass; direct measurements of the Higgs boson(s) at 125 GeV involve their gauge couplings and Yukawa couplings at the leading order and the energy resolutions of photons and leptons are typically of $\mathcal{O}(1)$ GeV at the LHC run-II[65–69]. After that, we would like to argue that the previous discussion can be workout, in a very similar way, as being a deep inelastic phenomenon since a damping term dependent of the velocity $\zeta$ appears in Eq.(3.10).

4 Higgs-Kibble mechanism

The Lagrangian invariant under global gauge transformation is given by

$$\tilde{\mathcal{L}} = (\partial_\mu \phi^*)(\partial_\mu \phi) - \frac{\theta}{2} i(\dot{\phi}\phi^* - \dot{\phi}^*\phi) - \mu^2 \phi^*\phi - \lambda(\phi^*\phi)^2.$$

(4.1)

The VEV for $\tilde{\mu}^2 < 0$ was calculated in Eq.(3.7) and the field redefinition for SSB, taking as starting point $\langle \phi \rangle_0$, is given by

$$\phi = e^{i\zeta/v}\left(\frac{v + H}{\sqrt{2}}\right).$$

(4.2)

We may express the field derivative as

$$\partial_\mu \phi = e^{i\zeta/v}\left[\partial_\mu + i\frac{1}{v}(\partial_\mu \zeta)\right]\left(\frac{v + H}{\sqrt{2}}\right).$$

(4.3)
The change to a new Lagrangian which is invariant under local gauge transformations is implemented by
\[ \tilde{\mathcal{L}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \phi)^* (D_\mu \phi) - \frac{\theta}{2} i \left[ (D_0 \phi) \phi^* - (D_0 \phi)^* \phi \right] - \mu^2 \phi^* \phi - \lambda (\phi^* \phi)^2, \] (4.4)
where \( F_{\mu\nu} \) is the field strength and \( D_\mu \) is the Abelian covariant derivative defined as
\[ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \] (4.5)
\[ D_\mu \phi = (\partial_\mu + ieA_\mu)\phi. \] (4.6)
We can readily verify with the help of the Eq.(4.3) and Eq.(4.6), that
\[ D_\mu \phi = e^{i\zeta/v} \left[ \partial_\mu + ie \left( A_\mu + \frac{1}{ev} \partial_\mu \zeta \right) \right] \left( \frac{v + H}{\sqrt{2}} \right). \] (4.7)
To demonstrate that the Lagrangian given in Eq.(4.4) is invariant under local gauge transformation:
\[ \phi \to \phi' = e^{-i\zeta/v} \phi \quad \text{and} \quad A_\mu \to A'_\mu = A_\mu + \frac{1}{ev} \partial_\mu \zeta, \] (4.8)
consider the gauged covariant derivative and field
\[ D'_\mu = \partial_\mu + ieA'_\mu, \] (4.9)
\[ \phi' = \frac{v + H}{\sqrt{2}}. \] (4.10)
With these latter, the Eq.(4.7) can be readily rewritten as
\[ D_\mu \phi = e^{i\zeta/v} D'_\mu \phi'. \] (4.11)
That is, the covariant derivative of the field undergoes exactly the same transformation of the field, shown in the first Eq.(4.8), which guarantees the invariance of the Lagrangian given in Eq.(4.4) under local gauge transformation. Since the invariance under local gauge was guaranteed, we can, alternatively, to express the Lagrangian in terms of the primed fields or non-primed fields. Choosing express it in terms of the primed fields:
\[ \phi' = \frac{v + H}{\sqrt{2}}, \] (4.12)
\[ D'_\mu \phi' = \frac{1}{\sqrt{2}} \left[ \partial_\mu H + ieA'_\mu(v + H) \right], \] (4.13)
the Lagrangian, given in Eq.(4.4), renders to
\[ \tilde{\mathcal{L}} = -\frac{1}{4} F'_{\mu\nu} F'^{\mu\nu} + \frac{1}{2} e^2 (v + H)^2 A'_\mu A'^{\mu} + \frac{\theta}{4} e A'_0 (v + H)^2 + \frac{1}{2} (\partial_\mu H)(\partial^\mu H) \]
\[ - \frac{1}{4} \lambda H^4 - \lambda v H^3 - \frac{1}{2} \left( \mu^2 + 3\lambda v^2 \right) H^2 - \left( \mu^2 v + \lambda v^3 \right) H - \frac{1}{2} \mu^2 v^2 - \frac{1}{4} \lambda v^4. \] (4.14)
The degree-of-freedom previously associated with the massless Goldstone boson was transferred to the longitudinal sector of the gauge field making the latter massive. Note that
for $\theta = 0$, the usual Lagrangian given in the literature is restored. Further, note that the noncommutative parameter $\theta$ leaving trace on the VEV and the Lagrangian has a new “damping” term dependent of the scalar potential $A_0'$. We also can read the squared mass of the gauge field and of the Higgs boson as respectively being

$$m_A^2 = v^2 e^2 = \left(M^2 - \frac{\theta^2}{2}\right) \frac{e^2}{2\lambda}. \quad (4.15)$$

Due to the mass-degenerate Higgs bosons, Eq.(3.13), with $M = 128$ GeV and $0 \leq \theta \leq \sqrt{1012}$ GeV, the massive gauge field $A_\mu$ is also mass-degenerate and we can assign the following range for the mass of the gauge field, namely:

$$126.008 \text{ GeV} \cdot \frac{e}{\sqrt{2\lambda}} \leq m_A \leq 128 \text{ GeV} \cdot \frac{e}{\sqrt{2\lambda}}. \quad (4.16)$$

5 Conclusion

We would like to point out that the proposed NC scalar field theory works as being a damped field theory. Moreover, the NC $\theta$-parameter affects the spontaneous symmetry breaking and Higgs-Kibble mechanisms, vide section 3. Here, it was revealed an astonishing feature about the role played by noncommutativity in the spontaneous symmetry breakdown and Higgs-Kibble mechanisms: first, the Higgs boson mass is parametrized by the NC $\theta$-parameter and, therefore, we get a NC $\theta$-parameter dependent mass-degenerate Higgs bosons near 126.5 GeV, namely, $m_H = \sqrt{(128 \text{ GeV})^2 - \frac{3}{4} \theta^2}$ with $0 \leq \theta \leq \sqrt{1012}$ GeV; second, the vacuum state was altered with the introduction of NC $\theta$-parameter, which drives the mass of gauge field to be parametrized by the NC $\theta$-parameter and, therefore, we get a NC $\theta$-parameter dependent mass-degenerate gauge fields, vide Eq.(4.15), that can be adjusted to experimental results by simply tuning the NC $\theta$-parameter.

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