Electron capture in GaAs quantum wells via electron-electron and optic phonon scattering

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Abstract
Electron capture times in a separate confinement quantum well (QW) structure with finite electron density are calculated for electron-electron (e-e) and electron-polar optic phonon (e-pop) scattering. We find that the capture time oscillates as function of the QW width for both processes with the same period, but with very different amplitudes. For an electron density of $10^{11} cm^{-2}$ the e-e capture time is $10^{1} - 10^{3}$ times larger than the e-pop capture time except for QW widths near the resonance minima, where it is only $2 - 3$ times larger. With increasing electron density the e-e capture time decreases and near the resonance becomes smaller than the e-pop capture time. Our e-e capture time values are two-to-three orders of magnitude larger than previous results of Blom et al. [Appl. Phys. Lett. 62, 1490 (1993)]. The role of the e-e capture in QW lasers is therefore readdressed.
The electron capture in a quantum well plays an important role in optimizing the performance of separate confinement heterostructure quantum well (SCHQW) lasers [1]. Quantum calculations [2] of polar optic phonon (pop) emission induced capture in GaAs QW predicted oscillations of the capture time versus the QW width, which have been observed [3]. The minima of the oscillations provide the optimum well and barrier width for an optimized capture efficiency, resulting in an improved modulation response and a reduced threshold current of the laser. At high electron densities the electron-electron (e-e) scattering induced capture is expected to play an important role. Blom et al. [4] predicted that the e-e capture time in a GaAs QW with electron density of $10^{11} \text{cm}^{-2}$ oscillates with approximately the same amplitude (and the same period) as the e-pop mediated capture time. Away from the oscillation minima the e-e capture was weak and the e-e capture was expected to increase the threshold current in the SCHQW laser via excess carrier heating in the QW [4]. The minima of the e-e capture time oscillations were found to be below 1 ps, which is a promising value for efficient capture. This result is surprising, because subpicosecond times are typical for intrasubband e-e scattering [5, 6], rather than for intersubband e-e scattering [7]. Further work is also necessary to clarify the density dependence of the e-e capture.

In this letter the e-e and e-pop scattering induced capture times are recalculated for the same SCHQW as in the letter by Blom et al. [4]. We find that for an electron density of $10^{11} \text{cm}^{-2}$ the e-e capture time is typically $10^1 - 10^3$ times larger except for QW widths near the resonance minima, where it is only 2 – 3 times larger. For densities above $\sim 5 \times 10^{11} \text{cm}^{-2}$ the resonant e-e capture time is smaller than the e-pop capture time. The e-e capture is found to be too weak to cause a significant excess carrier heating which is in contrast to the conclusion of Ref. [4]. Compared to the e-pop scattering limited capture, the e-e capture decreases the total capture time for an optimized (resonant) QW width, with a factor of about 2.9 – 4.2 at density of $10^{12} \text{cm}^{-2}$.

The analyzed SCHQW consists of the $\text{Al}_x\text{Ga}_{1-x}\text{As}/\text{GaAs}/\text{Al}_x\text{Ga}_{1-x}\text{As}$ QW with 500 $\text{Å}$ $\text{Al}_x\text{Ga}_{1-x}\text{As}$ barriers, embedded between two semi-infinite $\text{AlAs}$ layers. The e-e scattering is treated following the approach of Ref. [5]. When two electrons in subbands $i, j$ with wave vectors $\mathbf{k}$ and $\mathbf{k}_0$ are scattered to subbands $m, n$ with wave vectors $\mathbf{k}'$ and $\mathbf{k}'_0$, the e-e scattering rate of an electron with wave vector $\mathbf{k}$ from subband $i$ to subband $m$ is given by

$$\lambda_{im}(\mathbf{k}) = \frac{1}{N_S A} \sum_{j, n, \mathbf{k}_0} f_j(\mathbf{k}_0) \lambda_{ijmn}(g),$$

where $g = |\mathbf{k} - \mathbf{k}_0|$,\n
$$\lambda_{ijmn}(g) = \frac{N_S m^* \epsilon^4}{16 \pi \hbar^2 \kappa^2} \int_0^{2\pi} d\theta \frac{F_{ijmn}^2(q)}{q^2 c^2(q)} ,$$

$$q = \frac{1}{2} \left[ 2g^2 + \frac{4m^*}{\hbar^2} E_S - 2g \left( g^2 + \frac{4m^*}{\hbar^2} E_S \right)^{1/2} \cos \theta \right]^{1/2} ,$$

$$F_{ijmn}(q) = \int_{-\infty}^{\infty} dz \int_{-\infty}^{\infty} d\chi_0(z) \chi_i(z) \chi_j(z_0) e^{-q|z-z_0|} \chi_m(z) \chi_n(z_0).$$

$E_S = E_i + E_j - E_m - E_n$, the summation over $\mathbf{k}_0$ includes both spin orientations, $m^*$ is the electron effective mass in $\text{GaAs}$, $\kappa$ the static permittivity, $A$ the normalization area, $E_j$ the subband energy and $f_j(\mathbf{k})$ the electron distribution in subband $j$. Wave functions $\chi_i$ are obtained assuming the $x$-dependent effective mass and flat $\Gamma$-band with parabolic energy dispersion, both interpolated [8] between the $\text{GaAs}$ and $\text{AlAs}$. To deal with the 0.3-eV $\text{GaAs}$ QW [8] we take $x = 0.305$. The e-e capture time $\tau_{e-e} = \sum_{i, k} f_i(\mathbf{k}) / \sum_{i, k, m} f_i(\mathbf{k}) \lambda_{i,m}(\mathbf{k})$, where the summation over $i$ ($m$) includes the subbands above (below) the $\text{AlGaAs}$ barrier, and summation over $j, n$ in [8] involves the subbands below the $\text{AlGaAs}$ barrier. $f_i(\mathbf{k})$ is the Fermi function taken at temperature $8K$ and for an electron density $N_S = 10^{11} \text{cm}^{-2}$. $\epsilon(q) = 1 + (qs/q)F_{1111}(q)$ is the static screening function due to the electrons in the lowest subband [9], where $qs = \epsilon^2 m^*/(2\pi \kappa \hbar^2)$.

Full circles in Fig. 1 show $\tau_{e-e}$ versus the QW width for $f_i(\mathbf{k})$ taken as a constant distribution up to 36.8 meV above the $\text{AlGaAs}$ barrier, which roughly models the injected "barrier" distribution after a rapid phonon cooling [3, 9]. In the inset our calculation is compared with the result (crosses) of Ref. [4]. Both $\tau_{e-e}$ curves oscillate with the QW width and reach a resonant minimum, whenever a new bound state merges into the QW (the shift of our resonance minima to slightly lower QW widths is due to different effective
masses in GaAs, AlGaAs and AlAs, which we considered when we calculated the electron wave functions. However, our $\tau_{e-e}$ is two-to-three orders of magnitude larger. The difference of a factor of 4 is due to the missing factor of 1/4 in the e-e scattering rate of Ref. 4 (see Ref. 3 for details). When the $\tau_{e-e}$ values from Ref. 4 are multiplied by a factor of 4, our $\tau_{e-e}$ is still $\sim 100$ times larger.

In order to provide insight we consider the QW with width $w = 49$ Å. To demonstrate how the form factor (see Fig. 2a) affects the e-e scattering rate, we compare in Fig. 3a $\lambda_{ijmn}(g)$ as obtained using $F_{ijmn}^2(g)$, shown in Fig. 2a, with $\lambda_{ijmn}(g)$ obtained with $F_{ijmn}^2(q) = 1$. The latter is typically between $\sim 10^{12}$s$^{-1}$ and $\sim 4 \times 10^{12}$s$^{-1}$ for all capture transitions and its dependence on $i, j, m, n$ is simply manifested through $E_S$. Figure 3a shows a quite different behavior and the relative importance of the individual capture transitions is determined by the behavior of the form factors (Fig. 2a). The individual capture times are at least two orders of magnitude larger than the subpicosecond capture times shown in Fig. 3b. Subpicosecond e-e scattering is characteristic for intrasubband transitions as illustrated in Fig. 3 for $\lambda_{1111}(g)$. The form factor $F_{1111}$ reduces $\lambda_{1111}(g)$ only insignificantly and our $\lambda_{1111}(g)$ values are close to similar calculations of Refs. 3 and 4. We believe that Ref. 4 predicts much smaller e-e capture times due to a numerical error. It is straightforward to verify Fig. 3b quantitatively, because formula (2) is reduced to a simple single-integral for $F_{ijmn}^2$. For the form factors, calculations are also relatively simple and we can reproduce those published in Ref. 4. It can be seen without calculation that the results in Fig. 3a have correct order of magnitude, since they naturally follow from Figs. 3b and 2a.

The e-pop scattering rate of an electron with wave vector $k$ from subband $i$ to subband $m$ reads \[ \frac{e^2 \omega m^*}{8\pi\hbar^2} \left( \frac{1}{\kappa_\infty} - \frac{1}{\kappa} \right) \int_0^{2\pi} d\theta \frac{F_{ijmn}(q)}{q \epsilon(q)}, \] (for spontaneous phonon emission only)

\[ q = \left[ 2k^2 + \frac{2m^*}{\hbar^2} P - 2k \left( k^2 + \frac{2m^*}{\hbar^2} P \right)^{1/2} \cos \theta \right]^{1/2}, \]

where $P = E_i - E_m - \hbar \omega$, $\hbar \omega$ is the pop energy and $\kappa_\infty$ is the high frequency permittivity. We calculate the e-pop scattering induced capture time $\tau_{e-pop}$ by averaging (5) as discussed for $\tau_{e-e}$. Figure 4 compares $\tau_{e-pop}$ with $\tau_{e-e}$ for the parameters and the constant distribution $f_1(k)$ from Fig. 1. The $\tau_{e-pop}$ data shown by empty circles are calculated using the same static screening $\epsilon(q)$ as for the e-e scattering, empty squares show $\tau_{e-pop}$ for $\epsilon(q) = 1$. A more accurate calculation with dynamic screening will give results between these two extreme cases. We conclude that $\tau_{e-e}$ is one-to-three orders larger than $\tau_{e-pop}$ except for QW widths near the resonance minima. This conclusion differs from previous analysis [4] which predicts nearly the same oscillation amplitude in both cases. Ref. 4 also predicts that in the SCHQW lasers with a QW width below 40 Å the e-e capture causes significant excess carrier heating in the QW. Figure 4 does not support this conclusion, because the e-e capture is negligible.

It is easy to assess the dependence of both capture times on the electron density $N_S$. For $N_S > 10^{11} cm^{-2}$ and temperature 8 K the static screening $\epsilon(q)$ is independent on $N_S$, because $f_1(0) \approx 1$. Therefore, the $\tau_{e-pop}$ values in Fig. 4 would be the same also for higher $N_S$ and the $\tau_{e-e}$ values would decrease approximately like $N_S^{-1}$ for each QW width. In Fig. 4 we show $\tau_{e-e}$ for $N_S = 2.8 \times 10^{11} cm^{-2}$, $5 \times 10^{11} cm^{-2}$ and $10^{12} cm^{-2}$ only at QW widths of 43 Å and 46 Å in order to save CPU time. At 43 Å $\tau_{e-e}$ is much larger than $\tau_{e-pop}$ even for $N_S = 10^{12} cm^{-2}$ due to the absence of resonance. At 46 Å, when the first excited subband merges into the QW, $\tau_{e-e}$ resonantly decreases about 500 times and becomes smaller than $\tau_{e-pop}$ when $N_S \approx 5 \times 10^{11} cm^{-2}$. When $N_S = 10^{12} cm^{-2}$, the total capture time $\tau_{e-e}/\tau_{e-pop} = (\tau_{e-e} + \tau_{e-pop})$ is 3.8 ps for the unscreened e-pop capture ($\tau_{e-pop} = 11$ ps) and 4.3 ps for the screened e-pop capture ($\tau_{e-pop} = 18$ ps). Thus, compared to the case $\tau_{e-e} = 0$ the capture efficiency of the QW with the optimized (resonant) width can be improved with a factor 2.9 - 4.2 by increasing $N_S$ to $10^{12} cm^{-2}$. At higher densities (not investigated here) the capture time is expected to increase with $N_S$ on the basis of the results of Refs. 4 and 5. For $N_S > 10^{12} cm^{-2}$ it is no longer justified to treat the e-e and e-pop scattering separately [4, 5], because the electrons interact with a coupled system of electrons and phonons.

The $\tau_{e-pop}$ curve in Fig. 4 does not show a resonant drop for QW widths 46 Å and 88 Å, because the "barrier" electrons occupy the states below the threshold for pop emission and cannot be scattered into the subband which is in resonance with the top of the QW. A further increase of the QW width shifts the resonant subband deeper into the QW and the e-pop scattering into this subband smoothly increases.
Resonant decrease of the e-pop capture time takes place only in the case when the energy distribution of "barrier" electrons is monoenergetic, with energy slightly lower than the optical phonon energy \[1\]. To show the role of form factors, figure 5 compares the unscreened e-pop scattering rates obtained using \(F_{iimm}\) from Fig. 2b with the rates obtained with \(F_{iimm} = 1\). The individual e-pop capture rates in Fig. 5a are governed by the relevant form factors, while for \(F_{iimm} = 1\) (Fig. 5b) one only finds a simple dependence on \(P\). Compared to the e-e scattering rates in Fig. 3a the corresponding rates in Fig. 5a are systematically higher, because the e-pop capture rate (5) depends on \(F_{iimm}\) linearly while the e-e scattering rate (2) depends on \(F_{ijmn}\) quadratically. This fact naturally makes the e-e capture less effective than the e-pop capture except for high electron densities.

In summary, we have compared the e-e and e-pop capture times in the SCHQW. In both cases the capture time oscillates with the same period, but with very different amplitude. The e-e capture time is much larger than the e-pop capture time except for the QW widths near resonances, where it can be even smaller for electron densities close to \(10^{12} \text{cm}^{-2}\) which leads to an improved capture efficiency of the QW. However, an inefficient e-pop capture in the SCHQW laser should not lead to excess carrier heating \[4\] due to e-e scattering induced capture, because away from the resonance it is still much stronger than the e-e capture.

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Figure captions

\textbf{Fig. 1.} E-e capture time \(\tau_{e-e}\) vs. the QW thickness for \(N_S = 10^{11} \text{cm}^{-2}\). Full circles show the results for \(f_1(k)\) taken as a constant distribution up to 36.8 meV above the barrier. In the inset these results are compared with the data (crosses) from Ref. \[3\].

\textbf{Fig. 2.} (a) Square of the e-e scattering form factor \(F_{ijmn}(q)\) as a function of the wave vector \(q\) for a QW with thickness \(w = 49\ \text{Å}\). The indices \(i, j\) and \(m, n\) label the initial and final subband states, respectively.
States 1, 2 are bound in the QW, states 3, 4, . . . , 9 have subband energies above the AlGaAs barrier. Except for the transition 11 − 11 all other transitions are the e-e capture transitions. (b) The e-pop scattering form factors $F_{imn}(q)$ [see the text] are shown for comparison.

Fig. 3. E-e scattering rate $\lambda_{ijmn}$ vs. the relative wave vector size $g$. (a) Calculation with form factors from Fig. 2a. (b) Calculation with $F_{ijmn} = 1$.

Fig. 4. E-pop capture time $\tau_{e-pop}$ and e-e capture time $\tau_{e-e}$ vs. the QW thickness for $N_S = 10^{11} cm^{-2}$. Open circles show $\tau_{e-pop}$ for the statically screened e-pop interaction, open squares show $\tau_{e-pop}$ for the unscreened e-pop interaction and full circles are the $\tau_{e-e}$ data from Fig. 1. Crosses, asterisks and pluses at 43 Å and 46 Å show the $\tau_{e-e}$ data for $N_S = 2.8 \times 10^{11} cm^{-2}, 5 \times 10^{11} cm^{-2}$ and $10^{12} cm^{-2}$, respectively.

Fig. 5. E-pop scattering rate $\lambda_{im}$ vs. the wave vector $k$ for the QW with width $w = 49$ Å. (a) Calculation with form factors from Fig. 2b. (b) Calculation with $F_{imn} = 1$. 
\[
\lambda_{ijmn}(g) \quad [s^{-1}]
\]

(a)

(b)

11-11, \(E_s = 0\)
41-12, \(E_s = 13.7\) meV
61-12, \(E_s = 18.9\) meV
81-12, \(E_s = 27.0\) meV
31-11, \(E_s = 219.2\) meV
51-11, \(E_s = 223.5\) meV
71-11, \(E_s = 230.8\) meV
91-11, \(E_s = 241.0\) meV
$\lambda_{\text{im}}(k)$ [s$^{-1}$]

$k$ [m$^{-1}$]

(a)

(b)

1-1, P = -36.8 meV
3-2, P = -23.3 meV
6-2, P = -17.9 meV
4-2, P = -23.0 meV
7-2, P = -11.7 meV
8-2, P = -9.8 meV
9-2, P = -1.5 meV
3-1, P = 182.4 meV
4-1, P = 182.6 meV
5-1, P = 186.7 meV
6-1, P = 187.7 meV
7-1, P = 194.0 meV
8-1, P = 195.9 meV
9-1, P = 204.2 meV