Unitarity: confinement and collective effects in hadron interactions

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Abstract

We discuss how saturation of unitarity would change phase structure of hadronic matter at very high temperatures emphasizing the role of the vacuum state with spontaneously broken chiral symmetry.
Introduction

It is widely known, that the main fundamental problems of QCD are related to confinement and spontaneous chiral symmetry breaking phenomena. Those phenomena are associated with collective, coherent interactions of quarks and gluons, and result in formation of the asymptotic states, which are the colorless, experimentally observable particles. Owing to experimental efforts during recent decades it has become evident that the coherent collective dynamics survives at high energies.

The hypothesis on the completeness of the set of asymptotic states plays an important role (cf. [1, 2]) and leads, e.g. due to unitarity of scattering matrix to the optical theorem relating the total cross-section with the forward elastic scattering amplitude. Thus, the general principles play a guiding role in hadron interaction studies, and, in particular, unitarity which regulates the relative strength of elastic and inelastic processes is the most significant one. It is important to note here again that unitarity is formulated for the asymptotic colorless hadron on-mass shell states and is not directly connected to the fundamental fields of QCD — color fields of quarks and gluons.

The Hilbert space corresponds to colorless hadron states, it is constructed using vectors obtained by acting with the relevant creation operators on the physical vacuum. The state of physical vacuum as it is defined in the axiomatic field theory is a state without particles, annihilation operator when acts on it produces zero. It is the state of lowest energy and invariant under Lorentz transformations. Nowadays it is accepted that this vacuum state is not unique. Indeed, colored current quarks and gluons are the degrees of freedom related to the perturbative vacuum which is different from physical one. According to the confinement property of QCD, isolated colored objects cannot exist in the physical vacuum. Transition from physical vacuum to the perturbative one occurs in the process of deconfinement and results in quark-gluon plasma formation, i.e. gaseous state of free colored quarks and gluons. It is clear that hadrons and free quarks and gluons cannot coexist together since they live in different vacua [3] and there is no room for objects like quark-proton scattering amplitude. This fact provides important restrictions on the possible mechanisms of deconfinement.

The picture described above is commonly accepted in the theory of strong interactions. The main ingredient here is the assumption on the same scale of transitions confinement-deconfinement and chiral restoration. This assumption has a theoretical ground in some of lattice calculations (cf. e.g. [5]).

However, it is often assumed that the scales relevant to confinement and chiral symmetry breaking are different [6], scale of confinement is $\Lambda_{QCD} = 100 - 300$ MeV while chiral symmetry breaking scale — $\Lambda_\chi \simeq 1$ GeV. Thus, in the range between these two scales the matter is in a deconfined state but chiral symmetry
is spontaneously broken there. In the line with this picture, which can be treated as a posteriori justification, long time ago, in the pre-QCD era, it was supposed that hadrons have a simple structure and nonrelativistic quark model has been commonly adopted. During recent time such a model has evolved and obtained much more solid theoretical ground [6, 7, 8]. As it will be discussed further, one can assume existence inside the hadron of the third (nonperturbative) vacuum state with colored constituent quarks and pions as relevant degrees of freedom.

In this note we would like to consider how unitarity can constrain dynamics of the confined objects. We look into a possible mechanism of deconfinement assuming existence of a nonperturbative vacuum in addition to perturbative and physical ones and study the role of unitarity saturation in the process of deconfinement. It will be shown that saturation of unitarity is acting as a confinement restoration process. This is not surprising, since unitarity implies completeness of asymptotic colourless states. In this case it leads to appearance of a new state of a hadron matter at superhigh temperatures.

1 Nonperturbative vacuum and effective degrees of freedom

The origin of the nonperturbative vacuum and relevant effective degrees of freedom are related to the mechanism of spontaneous chiral symmetry breaking ($\chi$SB) in QCD [9], which leads to generation of quark masses and appearance of quark condensates. This mechanism describes transition of the current into constituent quarks. Massive constituent quarks appear as quasiparticles, i.e. current quarks and the surrounding clouds of quark–antiquark pairs.

Collective excitations of the condensate are the Goldstone bosons, and the constituent quarks interact with each other via exchange of the Goldstone bosons; this interaction is mainly due to pion field. Pions themselves are the bound states of massive quarks. Thus, the effective interaction of constituent quarks proceeds through exchange of Goldstone bosons. Constituent quark interactions with Goldstone bosons is strong and could have the following form [8]:

$$L_I = \bar{Q} [i\not\partial - M \exp(i\gamma_5 \pi^A F_{\pi})] Q, \quad \pi^A = \pi, K, \eta.$$  \hspace{1cm} (1)

For simplicity, in what follows we will refer to pions only, denoting by this generic word all Goldstone bosons, i.e. pions themselves, kaons and $\eta$-mesons.

Thus, we will assume that vacuum state $V_{pt}$ has a perturbative nature at short distances with current quarks and gluons as degrees of freedom, at large distances the physical vacuum state $V_{ph}$ has relevant colorless hadrons as degrees of freedom, and inside a hadron the vacuum $\tilde{V}_{np}$ has a nonperturbative origin with
constituent quarks and Goldstone pions being relevant degrees of freedom. We suppose the picture of a hadron consisting of constituent quarks embedded into quark condensate and interacting with pions which have a dual role: Goldstone and physical particles.

There are different approaches to the deconfinement dynamics, e.g. the deconfinement mechanism can be formulated in terms of percolation theory. It was recently [10, 11] proposed to use it as a candidate for the mechanism of deconfinement in the form of analytical crossover (without first and second order phase transitions). This form of deconfinement was found in the experimental studies at RHIC. Evidently such purely geometrical approach should be amended by a dynamical mechanism and indeed color dynamics of deconfinement due to formation of molecular-like aggregations was proposed in [3]. The vacuum inside the hadron was taken to be a perturbative one and quark interactions have origin in the color dynamics. It seems, however, that for crossover nature of deconfinement dynamics it is more natural to expect transition \( V_{ph} \rightarrow V_{np} \) instead of transition \( V_{ph} \rightarrow V_{pt} \). Indeed, using effective quark-pion interaction inside hadron and hadron-pion interaction outside hadron, we have a pion field as an universal interaction agent for both confined and deconfined states and this could serve as a natural explanation of deconfinement as a cross-over transition.

Experimentally deconfined state of matter has been discovered at RHIC where the highest values of energy and density have been reached. This deconfined state appears to be strongly interacting collective state with properties of perfect liquid. It is interesting to note that phase transition from parton gas to liquid could explain saturation phenomena in deep inelastic processes [4]. The importance of the experimental discoveries at RHIC is that the matter is strongly correlated and reveals high degree of coherence when it is well beyond the critical values of density and temperature. In the framework of the approach under consideration this state can be interpreted as a quark (constituent)-pion liquid in the nonperturbative vacuum \( V_{np} \).

The following question arises: what one should expect at higher temperatures, e.g. at the LHC energies, i.e. would one observe transition \( V_{np} \rightarrow V_{pt} \) finally, or other possibilities exist? Indeed, due to large kinetic energy of the constituent quarks in the nonperturbative vacuum there should be a finite probability to form colorless clusters again, i.e. confinement mechanism could take place, transition \( V_{np} \rightarrow V_{ph} \) would happened instead of \( V_{np} \rightarrow V_{pt} \) and hadrons would reappear. This hypothetical possibility obtain support from unitarity saturation at very high energies. In the next section we discuss this problem assuming that unitarity is saturated at very high energies.
2 Deconfinement and saturation of unitarity (reflective scattering)

The elastic scattering \( S \)-matrix (i.e. the \( 2 \rightarrow 2 \) scattering matrix element) in the impact parameter representation can be written (in the rational unitarization scheme) in the form of linear fractional transform (cf. \[12\] and references therein):

\[
S(s, b) = \frac{1 + iU(s, b)}{1 - iU(s, b)},
\]

(2)

where \( U(s, b) \) is the generalized reaction matrix. It is considered to be an input dynamical quantity. The explicit form of the function \( U(s, b) \) and numerical predictions for the observable quantities depend on the particular model used for hadron scattering description. For the qualitative purposes it is sufficient that this function increases with energy in a power-like way and decreases with impact parameter like a linear exponent or Gaussian.\(^1\) Also for simplicity consider for the time being the case of pure imaginary \( U \)-matrix and make the replacement \( U \rightarrow iU \), i.e.

\[
S(s, b) = \frac{1 - U(s, b)}{1 + U(s, b)}.
\]

(3)

It can easily be seen that the new scattering mode, reflective scattering (when \( S(s, b) < 0 \)) starts to appear at the energy \( s_R \), which is determined as a solution of the equation

\[
U(s_R, b = 0) = 1.
\]

Indeed, the unitarity relation written for the elastic scattering amplitude \( f(s, b) \) in the high energy limit has the following form

\[
\text{Im} f(s, b) = h_{el}(s, b) + h_{inel}(s, b).
\]

(4)

Inelastic overlap function \( h_{inel}(s, b) \) is connected with \( U(s, b) \) by the relation

\[
h_{inel}(s, b) = \frac{U(s, b)}{[1 + U(s, b)]^2},
\]

(5)

and the only condition to obey unitarity is \( \text{Im}U(s, b) \geq 0 \). Elastic overlap function is related to the function \( U(s, b) \) as follows

\[
h_{el}(s, b) = \frac{|U(s, b)|^2}{[1 + U(s, b)]^2}.
\]

(6)

\(^1\)In fact, analytical properties of the scattering amplitude imply linear exponential dependence at large values of \( b \).
At sufficiently high energies inelastic overlap function $h_{inel}(s, b)$ would have a peripheral $b$-dependence and will tend to zero for $b = 0$ at $s \to \infty$ cf. e.g. [12]). Therefore, corresponding behavior of elastic scattering $S$-matrix (note that $S(s, b) = 1 + 2i f(s, b)$) can then be interpreted as an appearance of a reflecting ability of scatterer due to increase of its density beyond some critical value. In another words, the scatterer has now not only absorption ability (due to presence of inelastic channels), but it starts to be reflective at very high energies. In central collisions, $b = 0$, elastic scattering approaches to the completely reflecting limit $S = -1$ at $s \to \infty$. As a side remark, it should be noted that absorption is not a result of imaginary nature of scattering amplitude, at small impact parameters reflective scattering mode can exist for the pure imaginary scattering amplitude.

At the energy values $s > s_R$ the equation $U(s, b) = 1$ has a solution in the physical region of impact parameter values, i.e. $S(s, b) = 0$ at $b = R(s)$. The probability of reflective scattering at $b < R(s)$ and $s > s_R$ is determined by the magnitude of $|S(s, b)|^2$; this probability is equal to zero at $s \leq s_R$ and $b \geq R(s)$. The dependence of $R(s)$ is determined then by the logarithmic functional dependence $R(s) \sim \frac{1}{\kappa(s)} \ln s$, this dependence is consistent with analytical properties of the resulting elastics scattering amplitude in the complex $t$-plane and mass $M$ can be related to the pion mass. Thus, at the energies $s > s_R$ reflective scattering will mimic presence of repulsive core in hadron and meson interactions and elastic scattering will be dominating process. This kind of elastic scattering preserves the hadron identities and acts against deconfinement. It would lead to the new phase of hadron liquid at very high temperatures.

The following transitions can be foreseen as the temperatures increases at the constant value of chemical potential $\mu$:

$V_{ph}(\text{Hadron gas}) \rightarrow V_{np}(\text{Quark-pion liquid}) \rightarrow V_{ph}(\text{Hadron liquid})$.

Corresponding phase diagram depicted in Fig. 1.

Presence of the reflective scattering can be accounted for using van der Waals method (cf. [13]). This approach was originally used for description of fluids starting from the gas approximation introducing the nonzero size of molecules into consideration.

The hadronic liquid density $n_R(T, \mu)$ can be connected [14] with the density in the approach without reflective scattering $n(T, \mu)$ by the following relation

$$n_R(T, \mu) = \frac{n(T, \mu)}{1 + \kappa(s)n(T, \mu)},$$

where $\kappa(s) = p_R(s)V_R(s)/2$, $p_R(s)$ is the averaged over volume $V_R(s)$ probability of reflective scattering and the volume $V_R(s)$ is determined by the radius of the reflective scattering. At very high energies ($s \to \infty$)

$$n_R(T, \mu) \sim \frac{1}{\kappa(s)} \sim \frac{M^3}{\ln^3 s}.$$
This limiting dependence for the hadron liquid density appears due to presence of the reflective scattering which resembles in the oversimplified geometrical picture a scattering of hard spheres in head-on hadron collisions. It can also be associated with saturation of the Froissart-Martin bound for the total cross-section. It should be noted that the lower densities of hadron matter are needed for percolation, but percolation in the presence of reflective scattering would not lead to deconfinement and at very high temperatures confined phase corresponding to hadron liquid could exist due to unitarity saturation.

**Conclusion**

Thus, it was conjectured that saturation of unitarity would restore confinement and percolation mechanism alone is not sufficient for deconfinement as it was supposed in [10, 11, 13, 14]. In general, we would like to note that at very high temperatures there is a certain probability that matter would return to confined state if the unitarity saturation would occur there.

The problem of completeness of asymptotic colorless states deserves further discussion [2]. In this connection it is important to study a possible existence of the confined phase where chiral symmetry is restored [15] and inclusion of such confined states into the set of asymptotic states.
Evidently, experimental studies with heavy ions at LHC would be able to reveal new phases of matter.

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