Realistic Cosmological Constant

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Abstract

Scenarios of supersymmetry breaking at various scales from TeV to GUT to the string are generated. A previous analysis generated the value of the experimentally measured cosmological constant from supersymmetry breaking at the TeV scale. Via a reorganization of the perturbative series, values of the cosmological constant are generically reconcilable with supersymmetry breaking scenarios having scales from the TeV on up to the string. The scenario with only a single supersymmetry breaking scale occurs at the GUT scale, generically.
Introduction

The cosmological constant has been assumed to be zero for the prior three decades, due possibly to the origins of the work in supersymmetry breaking. However, recent astrophysical data indicate that it has a small value, and subsequently there have been many attempts to model its value in string theory and in quantum field theory. In fact the value fits the indicative formula [1],

\[ \int \frac{\Lambda^8}{m_{pl}^4}, \]  

with \( \Lambda \) approximately the TeV scale (e.g. \( 1 - 2 \) TeV). This formula, and its origin, is not emphasized much in the literature. There is also an approximate hierarchy to the mass patterns [2] of the fundamental fermions following from

\[ \int (\psi^\alpha \psi^\alpha + \psi^{\dot{\alpha}} \psi^{\dot{\alpha}}) \left( \frac{\Lambda}{m_{pl}} \right)^{n/16}, \]  

also with \( \Lambda \) of the order of a TeV (and \( m_{pl} = 1.2 \times 10^{28} \)), and \( n \) an integer (the next order correction is presented in [2]). The origin of these formulae could be explained with supersymmetry breaking at a TeV, however, different energies should be more natural given the different scales reflected in geometry.

The nature of the formulae to the patterns of the fundamental masses require a deeper explanation [2], possibly in supersymmetry breaking [3],[4]. In this regard, the supersymmetry breaking with differing higher energy scales is relevant to the parameters listed in (1) and (2), and also to realistic model building. In addition the perturbation theory should be included.

The scenario adopted in [1] uses a supersymmetry breaking involving eight different scales, \( \Lambda_1 \) to \( \Lambda_8 \). Due to the fact that in theory upon three or more scales set to zero the cosmological constant equals zero (D-terms in \( N = 2 \) models allow for a classical cosmological constant), the functional form of the cosmological constant has to be

\[ \frac{\Lambda^8}{m_{pl}^4} \rightarrow \prod \frac{\Lambda_i}{m_{pl}^4}, \]  

and in the work of [1] the scales are taken to be of the TeV scale. A generalization of this work to encompass the string scale, as well as energies between the TeV and the string scale, is relevant to both particle physics phenomenology and cosmology.
'Natural' Value

The perturbative series expansion contributing to the cosmological density is

\[ V(\Lambda) = \sum b_n \Lambda^4 \left( \frac{\Lambda^2}{m_{pl}^2} \right)^n, \tag{4} \]

with the scale \( \Lambda \) in accordance with a supersymmetry breaking scale. For the moment, all supersymmetry breaking scales are taken to be identical and equal to \( \Lambda \). The coefficients \( b_n \) are determined by the loop expansion. This expansion generically produces a \( V(\Lambda) \) which is \( 10^{56} \) greater than the observed value, if the TeV scale is chosen for \( \Lambda \).

A re-ordering of the perturbative series can be easily implemented. For example, the first two terms in the series are grouped in the manner,

\[ b_0 \Lambda^4 + b_1 \Lambda^6/m_{pl}^2 + \ldots = c_0 \Lambda^2 m_{pl}^2 \left( e^{-c_1 \Lambda^2/m_{pl}^2} - 1 \right) + \ldots, \tag{5} \]

so that the expansion of the exponential generates (4). The remainder terms are altered in accordance to agree with (4); they may also be written in terms of exponentials. This rewriting of the series can be done to any order of accuracy.

Physically, there is no reason a priori to suspect that either the form in (4) or (5) is more physical, aside from the physical meaning of the exponential which could be interpreted as an instanton-like effect.

The cosmological data is next used to determined the value of \( \Lambda \). The powers of the scale breaking are determined by \( \Lambda = 10^x \), and \( e^{10} = 10^{4.34} \), as

\[ 8.68(x - 28) + 2x + 56 = -8 \quad 10.7x = 179. \tag{6} \]

In the units of this paper, the ’accepted’ value of the observed cosmological constant is \( 10^{-8} \). This calculation results in the scale of supersymmetry breaking being

\[ x = 15.5 \quad \Lambda = 10^{16.7}. \tag{7} \]

The factors of 100 to 1000 in the coefficients \( c_0 \) and \( c_1 \) change little the \( x \) value (16–17). The scale of \( \Lambda \) is the Grand Unified Theory (GUT) scale. This is interesting, as a supersymmetry breaking scale at the GUT scale will generate the currently ’accepted’ non-vanishing value of the cosmological constant.
The next term in the series has the value,

\[
\frac{\Lambda^8}{m_{pl}^4} \sim 10^{136-112} = 10^{18}
\]  

(8)

and is 28 orders of magnitude too large. Another exponential rewriting can be performed to lower its value in accordance with the previous two terms. Then there is a relative suppression of \( \Lambda^2/m_{pl}^2 \sim 10^{-22} \). Otherwise, fine tuning of the coefficient \( b_2 \) is required to an accuracy of 28 digits. The \( \Lambda^{10}/m_{pl}^4 \) term is of order \( 10^3 \) and requires exponentiation; this term can be packaged with the previous one.

In comparison, the following terms are shown to be irrelevant (to this accuracy) in either case, with reordering or without reordering of the series (9). The term \( \Lambda^{12}/m_{pl}^8 = 10^{-20} \) for \( x = 17 \), and subsequent ones are of smaller value.

In effect, the rewriting of the perturbative series in the form (5) naturally predicts a scale of \( 10^{16-17} \), the GUT scale, for the supersymmetry breaking. This prediction is very stable under perturbations of the coefficients \( b_n \).

**Two Scales**

Various degrees of supersymmetry breaking are expected to happen at different scales. The multiple scale scenario can generate breakings at the string scale combined with breaking at the TeV scale. These cases are considered next. The predictions are interesting in that it straightforward to obtain scales in the GeV range on up to the string energy.

First consider the scenario of two scales \( \Lambda_1 \) and \( \Lambda_2 \). In the absence of a potential allowable D-term, which contributes at the classical level a term \( \Lambda^6/m_{pl}^2 \), the cosmological constant should vanish upon taking the scales to zero. This means that in the termwise, taking \( \Lambda_1 \to 0 \) or \( \Lambda_2 \to 0 \), the function to \( V(\Lambda_1, \Lambda_2) \) must go to zero. For this, the function is modeled by,

\[
V = b_0(\Lambda_1\Lambda_2)^2 + b_1(\Lambda_1\Lambda_2)^3/m_{pl}^2 + \ldots .
\]  

(9)

The expansion in (9) depends on there being two scales, and it is indicative of the two scale supersymmetry breaking scenario.

The reordering of the series in (9) can be done in a couple of ways. The first one considered, and with \( \Lambda_2 > \Lambda_1 \), follows by rewriting the first two terms as,

\[
c_1\Lambda_1\Lambda_2 m_{pl}^2(e^{-c_2\Lambda_2^2/m_{pl}^2} - 1) .
\]  

(10)
Consider the case when $\Lambda_2 = \Lambda_1^2$. The effects of the exponent are, with $\Lambda_1 = 10^x$,

$$8.68(x - 28) + 3x + 56 = -8 \quad 11.68x = 179 \quad (11)$$

and generates,

$$x = 15.3 . \quad (12)$$

This is unphysical as the lower scale is an the order of a 1000 TeV, but the higher scale is at $10^{30}$ eV, which is higher than the string scale.

The next possibility is use the higher scale in the exponent. This results in the numbers,

$$8.68(2x - 28) + 3x + 56 = -8 \quad x = 9.1 . \quad (13)$$

In this case, supersymmetry breaking occurs at a 1 GeV and at the approximate GUT scale ($10^{18}$).

The final two scale scenario considered uses the exponential,

$$e^{-\frac{\lambda_1 \lambda_2}{m_{\tilde{b}}^2}} , \quad (14)$$

and results in the numbers,

$$4.34(x - 28) + 4.34(2x - 28) + 3x + 56 = -8 \quad (15)$$

or,

$$16x = 179 \quad x = 11.2 . \quad (16)$$

The supersymmetry breakings are near a TeV and at also at the string scale.

The two scale scenario shows that it is possible to obtain, with the rewriting of the series, supersymmetry breaking at very different scales. Namely, it is possible to obtain the two breaking such that one is near the TeV scale, or in the 100 GeV scale, and the other is near the string scale, or in the GUT scale regime. This happens with a realistic prediction of the cosmological constant.
It should be noted that the higher order terms, such as $\Lambda^8/m_{\text{pl}}^4$, should also be exponentiated. The higher order terms in the case of $\Lambda = 10^{11}$ are,

\[
\lambda_1^4 \lambda_2^4/m_{\text{pl}}^4 \quad 10^{44+88-112} = 10^{20} \quad (17)
\]
\[
\lambda_1^5 \lambda_2^5/m_{\text{pl}}^6 \quad 10^{55+110-168} = 10^{-3} \quad (18)
\]
\[
\lambda_1^6 \lambda_2^6/m_{\text{pl}}^8 \quad 10^{66+132-224} = 10^{-26} . \quad (19)
\]

The first and second terms requires the exponentation, as the first is 28 orders of magnitude too big, and the remaining terms are in agreement with the $10^{-8}$, the ‘accepted’ observed value of the cosmological constant in these units.

**Three Scales**

The supersymmetry breaking with three scales also has some ambiguity in the rewriting of the perturbative series, and allows for much space between the energy scales. Consider the series for $V(\Lambda_i)$ of the form,

\[
V(\Lambda_i) = b_0 \sum_{i=1}^{4} \Lambda_1 \Lambda_2 \Lambda_3 \Lambda_i + b_1 (\Lambda_1 \Lambda_2 \Lambda_3)^2/m_{\text{pl}}^2 + \ldots , \quad (20)
\]

with $\Lambda_3 > \Lambda_2 > \Lambda_1$.

Upon rewriting the first two terms as

\[
\sum_{i=1}^{4} c_0 \Lambda_3 \Lambda_i m_{\text{pl}}^2 \left( e^{-e_1 \Lambda_1 \Lambda_2/m_{\text{pl}}^2} - 1 \right) , \quad (21)
\]

the counting of the exponents with $i = 3$ (the largest contribution) generates with $\Lambda_i = 10^{e_i}$,

\[
2x_3 + 56 + 4.34(x_1 - 28) + 4.34(x_2 - 28) = -8 . \quad (22)
\]

This equation is not ambiguous, but gives a scale relation between the $\Lambda_i$. Note that the form in (21) does not have the quartet term matching, but rather is a bound
because the largest symmetry breaking scale $\Lambda_3$ is used; a factor of three in the coefficient $c_0$ absorbs this, but other forms may also be used.

Consider the example of $x_1 = 12$ ($\Lambda = 10^{12}$) and $x_2 = x_3 = x$. The relevant numbers are,

$$6.34x = 127 \quad x = 20 . \quad (23)$$

In this case $\Lambda_1 = 10^{12}$ which is the TeV scale, and two more closer to the string scale at $\Lambda_2 = \Lambda_3 = 10^{20}$. The latter should be spaced a bit as three scales are being considered.

The second scenario is

$$\sum_{i=1}^{4} c_0 \Lambda_1 \Lambda_i m_{\text{pl}}^2 \left( e^{-c_1 \Lambda_2 \Lambda_3 / m_{\text{pl}}^2} - 1 \right), \quad (24)$$

the counting of the exponents with $i = 3$ (the largest contribution) generates,

$$x_1 + x_3 + 56 + 4.34(x_2 - 28) + 4.34(x_3 - 28) = -8 . \quad (25)$$

Again, the quartet term can be bounded by a factor of three due to the choice of exponentiation. With the same breaking, $x_1 = 12$ and $x_2 = x_3 = x$, the numbers are,

$$9.68x = 167 \quad x = 17 . \quad (26)$$

The three scales are then at the TeV scale together with another two at the GUT scale. There is considerable flexibility though in the examples, when two of the scales are not chosen together.

To demonstrate the flexibility consider the final example, with $x_1 = 12$ and $x_3 = \frac{3}{2}x_2$. Take the first rewriting in the previous, and the numbers are

$$12.4x_2 = 167 \quad x_2 = 14 . \quad (27)$$

In this case, $\Lambda_1 = 10^{12}$, $\Lambda_2 = 10^{14}$, and $\Lambda_3 = 10^{21}$. The scales are at a TeV, the GUT energy, and also at the string scale. Again, there is much freedom in arranging the scales, and there are potentially other interesting cases.
In the previous examples, the higher order terms must also be arranged in exponential form in order to not spoil the value of the cosmological constant. It is clear, however, that scenarios with various scales of disparate supersymmetry breaking and that are realistic, may be obtained. This is without fine-tuning, and without any real model specificity, unless actual values of the supersymmetry breaking are used.

**Conclusion**

The cosmological constant problem is typically very difficult to solve without some fine-tuning. However, it is possible to use a simple 'rearrangement' of the perturbative series to obtain realistic values, at least in accord with the current supernovae data.

The simplest scenario, requiring only one supersymmetry breaking scale, indicates that with the current 'accepted' value of the cosmological constant, that the supersymmetry breaking is at the scale of $10^{16-17}$ eV. This is interesting for a variety of reasons, not only because the prediction is generic, but also due to the one-loop coupling unification of the standard model at this scale.

More interesting scenarios of supersymmetry breaking can be achieved if the models possess more than one scale. Three or four scales can be used in a direct fashion to obtain supersymmetry breaking at the TeV scale, the GUT scale, and the string scale, in one model. The typical fine-tuning problem of supersymmetry breaking is compounded many fold with supersymmetry breaking at the string scale. This work shows that no real fine-tuning is required even with scales of supersymmetry breaking located near the string as well as in current phenomenological experiments.

The dynamical origin of the possible running of the scales would be interesting to further understand due to the perturbative series used in this work.
References

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