Universal Aspects of $U(1)$ Gauge Field Localization on Branes in $D$-dimension

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Abstract

In this work we study the general properties of the vector field localization on $(D - d - 1)$-brane with co-dimension $d$ in asymptotically AdS spacetime. We consider a conformally flat metric with the warp factor depending only on the transverse extra dimensions. It is known that the free gauge field cannot be confined in co-dimension one braneworlds. Also, in most of the higher co-dimension braneworlds a general mechanism to solve this is lacking. For the co-dimension one case such problem of localization can be solved by using non-minimal couplings with gravity. Here we apply this mechanism to the case of generic braneworld with arbitrary co-dimension stressing the universal aspects that lead to localization of gauge field. We show with this that an analytical solution to the $U(1)$ gauge field (zero-mode) sector over the brane can be always obtained and it is valid for any warp factor. The existence of this zero-mode solution has as first consequence to exclude any possible tachionic modes of the theory. Beyond this a localization condition for this zero-mode was obtained by setting up the bulk geometry as asymptotically AdS. Thus our solution has an universal validity independent of the particular model considered. We also consider the scalar components of the vector field. A drawback in the co-dimension one case is that these components are never localized simultaneously with the gauge field. We show that when more co-dimensions are considered there is an indication that both sectors can be localized. To conclude we obtain some conditions in which the localization of gravity ensure localization of gauge field in this scenarios propose by us.

Keywords: Fields Localization, Geometrical Coupling, $U(1)$ Gauge Fields.
I. INTRODUCTION

The formulation of theories in spacetimes with more than 4-dimensions as a tool to solve problems in physics is not new \[1, 2\]. However, only after the development of string theory and the compactification mechanisms of extra dimensions at the end of the last century, that tool began to be regarded a possible real description of nature \[3\]. A feature of these higher dimensional theories was the need of these extra dimensions to be compacted into a very small spatial volume inaccessible in the available energy range. This because the Newton’s gravitational law depend explicitly of the number of spatial dimensions, and it indicates the presence of only three large spatial dimensions. The first to speculate about the possibility of these extra dimensions being non-compact were Rubakov and Shaposhnikov \[4\]. The authors showed that such a large extra dimension can occur as long as the fields of the Standard Model (SM), thus as the gravity, are confined to a 4-dimensional hypersurface. In order that our energy scale (TeV) does not allows us to access such extra dimensions.

In this direction, L. Randall and R. Sundrum (RS) proposed two models with warped geometry in an $AdS_5$ spacetime with delta-like 3-branes \[5, 6\]. The RS-I model, proposed to solve the Higgs hierarchy problem, consider a 5-dimensional universe $(x^\mu, \phi)$ with the spatial dimension $\phi$ compacted under a circle with an orbifold symmetry $S_1/Z_2$. At the fixed points ($\phi = 0, \pi$) are localized two delta-like 3-branes, and the 3-brane at $\phi = \pi$ would correspond to our universe with all fields of the Standard Model (SM) confined. The RS-II model consider only one delta-like 3-brane with a non-compact and infinite extra dimension $(x^\mu, y)$, and it was proposed as an alternative to the compactification. On both models, the gravity is localized on the 3-brane reproducing a 4D gravitational theory consistent with the experimentally observed. Although the gravity is localized on the brane and it is possible to show that the scalar field is also, the other fields of SM are not confined as imagined by RS \[7–9\]. After the successful RS models other proposals of braneworld with localized gravity are developed. Some these in five dimensions (5D) as the smooth versions of RS-II (thick branes) in Ref. \[10, 11\]; thick branes RS-II-type with inner structure \[12, 13\]; cosmological models, where the metric of the braneworld is like Robertson-Walker \[14, 15\]. Beyond others proposals in spacetimes with more than 5D as: 3-brane generated by topological defect string-like or vortex in 6D \[16, 17\]; or braneworld models generated by the intersection of delta-like branes \[18, 19\], and others \[20, 22, 25, 60\]. In all this models, the issue of
localization of the SM fields are always important points to be verified \[26\]–\[30\].

Among then the localization of \(U(1)\) gauge field, a particular Yang-Mills field and one of the pillars for the construction of the SM, has a considerable importance. It is a known fact that the free abelian gauge field is not confined on the braneworld models \[8\]–\[9\], \[32\]–\[35\], \[60\]. Some attempts to solve this problem were realized. In most cases introducing new degree of freedom, such as interaction terms with fermionic or scalar (\textit{dilatonic}) fields \[11\], \[36\]–\[43\]. Although these mechanisms allow us to confine the \(U(1)\) gauge field (zero-mode), arise other questions about the meaning of these new fields for the theory and also such mechanisms are particular to each model. The authors K. Ghoroku and A. Nakamura developed a localization mechanism in RS-II model without the need of introducing new degrees of freedom in the theory \[44\]. They introduce a mass term and an interaction term non-covariant between the vector field and the 3-brane. This mechanism works, however they still introduce a new parameter in the theory. Furthermore, there is no solid motivation for the introduction of a coupling with the 3-brane. Based on this mechanism, a purely geometric localization mechanism was proposed in Ref. \[18\], \[45\], \[46\], where an interaction term of the \(U(1)\) gauge field with the Ricci scalar is added. This \textit{Yukawa geometric coupling} allows us to localize the massless mode of abelian gauge field and has the advantage of to be covariant and does not introduce new degrees of freedom or free parameter in the theory. Beyond that, as it will be shown latter, the interaction with the 3-brane arises as a consequence of the coupling of the vector field with gravity only. Afterward, this mechanism was applied to the massive modes leading to study of the resonant modes of vector and \(p\)–form fields in thick brane models \[17\], \[49\]; also by looking for evidences of a non-zero mass to photon as a consequence of the existence of extra dimensions \[50\]; beyond of application this non-minimal coupling with gravity for provide the localization of other fields \[51\], \[52\]. All this questions was developed in type RS-II models with only \textit{one} large extra dimension.

Despite of the above results a generalization of the geometrical coupling mechanism to more than one transverse extra dimensions is lacking. As presented above many others scenarios of braneworld with more extra dimensions were proposed allowing a more rich configuration gravitational. Besides this the vector field will have more scalar components which can play an important role over the brane. Therefore in this manuscript we study vector field localization in a spacetime with an arbitrary number of extra dimensions. We look for universal aspects of the gauge field localization and to the possibility that this
field can be simultaneously localized with the scalar field components for some range of parameters of the model. This work is organized as follows. In section (II), a review of the main aspects of gravity localization on generic braneworld model is made. It is also presented some example of specific cases as RS-II model with delta-like and smooth brane in 5D, and the brane intersection braneworld in $D$-dimensions. In section (III), the general aspects of the confinement of the $U(1)$ gauge field on a general braneworld scenario are discussed. It is also realized a study about the localization of scalars components in this generic model. The conclusions are left for section (IV).

II. REVIEW ABOUT GRAVITY AND $U(1)$ GAUGE FIELD LOCALIZATION

In this section the main ideas about the universal aspects of gravity localization and its applications in the particular cases of the RS-II and of the delta-like branes intersection models are reviewed. Beyond that, it is presented some of results about localization of the $U(1)$ gauge field with geometric coupling mentioned in previous section.

A. Universal Aspects of Gravity Localization and Type RS-II Models

In Ref. [53] Csaba Csáki et al. make a study about the universal aspects of gravity localization in braneworld models. They split the study in one with a conformally-flat metric in $D$-dimensions and other with a non-conformal metric. As we will deal with the universal aspects of vector field localization in the first scenario, we will restrict ourselves to present only that case. Let us start proposing a $D$-dimensional conformally-flat metric in the form,

$$ds^2 = e^{2\sigma(y)}\eta_{MN}dx^Mdx^N = e^{2\sigma(y)}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dy^j dy^j\right),$$

with $\eta_{\mu\nu} = \text{diag}(-1,1,1,1)$ and the warp factor $e^{2\sigma(y)}$ depending only the transverse extra dimensions to the brane $y^j$. Here and through all the work, the capital labels ($M,N...$) assume value on all $D$-dimensions; the greek labels ($\mu,\nu...$) = 1, 2, 3, 4 are related to brane; and the latin labels ($j,k..$) = 1, 2...$d = (D - 4)$, where $d$ is the number of extra dimensions, are related the transverse extra dimensions to the brane. In this generic background the authors consider the fluctuations of the metric in the form, $\eta_{\mu\nu} \rightarrow \eta_{\mu\nu} + h_{\mu\nu}$, with the transverse gauge condition, $\partial_\mu h^{\mu\nu} = 0$, and the traceless condition, $h^{\mu}_{\mu} = 0$. Using the
expressions for $R$ and $R_{MN}$ in appendix (A1) we can write the Einstein tensor after a conformal transformation and considering an action in the form,

$$S = \int \! d^D x \left[ \sqrt{-g(x)} \left( \kappa^2 R + \Lambda \right) + \sqrt{-g} V(y) \right].$$

(2)

Where $\Lambda$ is the cosmological constant and $V(y)$ is a function that give the energy distribution of the 3-brane. We get the follow equation for the gravitational fluctuations $h_{\mu \nu}$,

$$- \partial_M \partial^M h_{\mu \nu} - (D - 2) \partial_M \sigma \partial^M h_{\mu \nu} = 0.$$ 

(3)

Proposing the ansatz $h_{\mu \nu}(x, y) = \tilde{h}_{\mu \nu}(x) \psi(y) e^{-\frac{D-2}{2} \sigma}$, we obtain the follow type-Schrödinger equation,

$$- \frac{1}{2} \nabla_y^2 \psi(y) + U(y) \psi(y) = \frac{1}{2} m^2 \psi(y),$$

(4)

with,

$$U(y) = \frac{1}{2} \left[ \left( \frac{D-2}{2} \right)^2 \nabla_y \sigma \cdot \nabla_y \sigma + \left( \frac{D-2}{2} \right) \nabla^2_y \sigma \right].$$

(5)

where $\square \tilde{h}_{\mu \nu}(x) = \eta_{\rho \lambda} \partial_\rho \partial^\lambda \tilde{h}_{\mu \nu}(x) = m^2 \tilde{h}_{\mu \nu}(x)$ and $\nabla^2_y = \sum_j \partial_j \partial_j$ are derivatives in extra dimensions. And also the action for zero-mode gravitational,

$$S_0 \sim \int \! d^{D-4} y \psi_0^2(y) \int \! d^4 x \partial_\rho \tilde{h}_{\mu \nu}(x) \partial^\rho \tilde{h}_{\mu \nu}(x).$$

(6)

A well defined theory on the 3-brane is obtained if the integral in extra dimensions are finite. We must make it clear that Eq. (4) do not specify the kind of 3-brane where the analysis of localization of gravity will made, if it is delta-like or smooth brane. Beyond that, there is not a restriction on the number of extra dimensions or if these are infinitely large or compacted.

The form of Eq. (4) is very convenient because it allows us to obtain qualitative information of the system by the analysis of the potential-type term beyond other useful information as: first, Eq. (4) can be written in the form, $\hat{Q}_y^\dagger \cdot \hat{Q}_y \psi(y) = m^2 \psi(y)$, where $\hat{Q}_y = \nabla_y - \left( \frac{D-2}{2} \right) \nabla_y \sigma$. In quantum mechanic, this is like a supersymmetric quantum mechanic problem. And, as the “Hamiltonian” $\hat{Q}_y^\dagger \cdot \hat{Q}_y$ is a positive definite Hermitian operator, this imply that there are not gravitational tachyonic modes, as required for stability of the gravitational background. Beyond that, the zero-mode is obtained by solve the equation, $\hat{Q}_y \psi_0(y) = 0$, that has a solution in the form, $\psi_0(y) = \exp \left[ \left( \frac{D-2}{2} \right) \sigma(y) \right]$; second, the asymptotic behavior of potential-type term determine if the states $\psi(y)$ can be normalized. If
\( U(y) \to +\infty \), when \( |y^j| \to \infty \), then we have a confining system and all states can be normalized; if \( U(y) \to -\infty \), when \( |y^j| \to \infty \), then the states cannot be normalized; and the case where \( U(y) = u_0 \) (constant), when \( |y^j| \to \infty \), only allow normalized modes for these with value of \( m_n^2 < u_0 \). For the case where \( m_n^2 = u_0 \), the analysis must be realized case by case.

For conclude we will make a different approach from C. Csáki et.al. and assume that the background is asymptotically an AdS spacetime. This feature evidently does not define the background metric however, it restricts the shape of the warp factor when \( |y^j| \to \infty \). In this case, when we consider asymptotically the equation for the Ricci scalar \((A3)\), then we should have \( R(\infty) = -\kappa \), with \( \kappa > 0 \) and constant. This leads to the following asymptotic behavior for \( \sigma(y) \),

\[
\sigma(|y^j| \to \infty) = -\ln \left( \sum_j \beta_j |y^j| \right),
\]

with \( \beta_j \)'s constants. If we are assuming that \( \psi_0(y) \) is regular in all range of integration, than the asymptotic behavior of \( \psi_0(y) \) define the convergence of the integral \( \int d^{D-4}y\psi^2(y) \) and the expression \((7)\) lead always to the massless mode \( \psi_0(y) = \exp \left[ (\frac{D-2}{2})\sigma(y) \right] \) localized on the 3-brane. Some examples are: (a) the RS-II model \([6]\), where we have \( D = 5 \), \( \sigma(y) = -\ln(1 + k|y|) \) and the zero-mode stay in the form, \( \psi_0(y) = (1 + k|y|)^{-\frac{3}{2}} \); (b) yet in \( D = 5 \), but now with a smooth model, \( \sigma(y) = -\ln(1 + k^2y^2) \) \([10]\). The zero-mode stay, \( \psi_0(y) = (1 + k^2y^2)^{-\frac{3}{2}} \); (c) for the brane intersection model of Arkani-Hamed \([19]\), \( D \) is arbitrary, \( \sigma(y) = -\ln \left( 1 + k \sum_j |y^j| \right) \) and the zero mode stay, \( \psi_0(y) = \left( 1 + k \sum_j |y^j| \right)^{-\frac{D-2}{2}} \).

It is easy to show that all these models have the integral \( \int d^{D-4}y\psi^2(y) \) finite therefore, the gravitational massless mode localized on the 3-brane.

### B. Remarks on \( U(1) \) Gauge Field Localization Through Geometric Coupling

When we talk about localization of fields in RS-type braneworlds, we usually want to factor an action in the form,

\[
S = \int d^4x d^{D-4}z \sqrt{-g^{(D)}L^{(D)}_{\text{matter}}} ,
\]

into a sector concerning an effective field theory on the 3-brane and an integral in the coordinates of the extra dimensions in the form, \( K = \int d^{D-4}zf(z) \) as follows,

\[
S = \int d^{D-4}zf(z) \int d^4x \sqrt{-g^{(4)}L^{(4)}_{\text{matter}}} = K \int d^4x \sqrt{-g^{(4)}L^{(4)}_{\text{matter}}} .
\]
Here, we have two ways to treat the problem: (i) In RS-II type models, the extra dimension is non-compact and infinite, so a well-defined theory on the 3-brane is obtained when $K$ is a finite quantity; (ii) In RS-I type models, the extra dimension is compact, so if $f(z)$ is a regular function in the integration range, then $K$ will already be a finite quantity. However, this is not enough to guarantee a consistent effective field theory on the visible 3-brane \cite{32, 33, 57, 58}.

As it was already mentioned, we will restrict ourselves to study the localization of the vector field on a generic braneworld scenario like RS-II model. But rather, it is instructive to give a brief review of some results found in the literature. In 5D RS-II model, the action for the free gauge field can be written as \cite{9},

$$S = -\frac{1}{4} \int d^4 x dy \sqrt{-g} g^{MN} g^{PQ} F_{MP} F_{NQ},$$  \hspace{1cm} (10)

where $F_{MP} = \partial_M A_P - \partial_P A_M$ is a field strength-type tensor and $A_M$ is the vector field. Proposing $A_M = (A_\mu, A_5)$ with the gauge condition $\partial_\mu A^\mu = 0$ and choosing $A_5 = 0$, the variation of this action gives us the equations of motion,

$$\partial_\mu F^{\mu \nu}(x) = m^2 A^\nu(x);$$  \hspace{1cm} (11)

and

$$-e^{-\sigma(y)} \partial_5 \left( e^{\sigma(y)} \partial^5 \chi(y) \right) = m^2 \chi(y).$$  \hspace{1cm} (12)

With the contraction of greek labels ($\mu, \nu...$) made by the Minkowski metric. Beyond that, the action (10) can be factored in the form,

$$S = -\sum_n \int dy e^{\sigma(y)} \chi^2_n \int d^4 x \left[ \frac{1}{4} F^{(n)}_{\mu \nu} F^{(n)}_{\mu \nu} + \frac{1}{2} m^2 A^{(n)}_{\mu} A^{(n)}_{\mu} \right].$$  \hspace{1cm} (13)

The equation (12) has a constant solution for the zero-mode $\chi_0$ ($m^2 = 0$). With this the localization of gauge field $A^{\mu}_{(0)}$ (massless mode of vector field $A^\mu$) is established by the integral,

$$\int dy e^{\sigma(y)} \chi^2_0 = \int dy e^{\sigma(y)} = \int_{-\infty}^{\infty} \frac{dy}{1 + k|y|},$$  \hspace{1cm} (14)

that is not convergent for this type-II Randall-Sundrum model. On the other hand, when it is considered a model in 6D as the string-like braneworld with one compacted and one infinitely large extra dimensions \cite{8, 37},

$$ds^2 = e^{2\sigma(r)} \left( \eta_{\mu \nu} dx^\mu dx^\nu + R_0^2 d\theta^2 + dr^2 \right),$$  \hspace{1cm} (15)
it is possible to write the action for the zero-mode of gauge field as,

$$S = - \int_0^{2\pi} R_0 d\theta \int_0^\infty d\tau e^{2\sigma(r)} \chi_0^2(r) \int d^4x \frac{1}{4} F_{\mu\nu} F^{\mu\nu}. \quad (16)$$

where $A_r = 0$, $A_\theta = 0$ and $\partial_\mu A^\mu = 0$. Beyond that, $\chi(r)$ must satisfy,

$$-e^{-2\sigma(r)} \partial_r (e^{2\sigma(r)} \partial_r \chi(r)) = m^2 \chi(r). \quad (17)$$

Again, there is a constant solution for the zero-mode. Now, the integrals in the extra dimensions are both finite and we would have the $U(1)$ gauge field localized on the brane. However, a more careful analysis show that the free gauge field cannot be localized for any co-dimension one braneworld \[60\]. Because this, it is necessary to obtain a mechanism to confine this field on braneworld models. Usually, it is added interaction terms with other fields or, a strategy very interesting and powerful, to use non-minimal couplings with gravity.

In Ref. \[44\] K. Ghoroku and A. Nakamura developed a localization mechanism for this field without the need of introducing new degrees of freedom, \textit{i.e.}, interaction terms with fermionic or scalar fields. The authors proposed an action in 5-dimensions in the follows way,

$$S = - \int d^4x dz \sqrt{-g} \left[ \frac{1}{4} g^{MN} g^{PQ} F_{MP} F_{NQ} + \frac{1}{2} (M^2 + \tau \delta(z)) g^{MN} A_M A_N \right], \quad (18)$$

where $g_{MN}$ is the metric of the RS-II model, $M^2$ is the mass of vector field and $\tau$ is a coupling constant between the vector field and the 3-brane. After propose $A_N = (\hat{A}_T^T, A_\mu, A_5)$, with $\partial_\mu \hat{A}_T^\mu = 0$, they showed that it is possible to confine the transverse sector of $U(1)$ gauge field if $M > 0$ and $\tau = -2k \left( \sqrt{1 + M^2/k^2} - 1 \right)$. The authors, however, do not give a solid motivation because they need to add a interaction term with the 3-brane.

Based on this mechanism, in Ref. \[18, 46\] the authors propose a localization mechanism by addition of an interaction term of the $U(1)$ gauge field with the Ricci scalar $R$,

$$S = - \int d^4x dz \sqrt{-g} \left[ \frac{1}{4} g^{MN} g^{PQ} F_{MP} F_{NQ} + \frac{\gamma_1}{2} R g^{MN} A_M A_N \right], \quad (19)$$

with $\gamma_1$ a parameter fixed by the boundary conditions and $R$, which in the RS-II model is given by $R = 16k\delta(z) - 20k^2$. Such a model, called the geometric Yukawa coupling, has no additional degrees of freedom or free parameters and provides an ‘origin’ for the terms of interaction proposed in \[18\]. Beyond that, we have a simple interpretation for these interaction terms, they are the result of the interaction of the $U(1)$ gauge field with the
cosmological constant of the "Bulk". After to split \( \mathcal{A}_N = (\hat{A}_\mu + \partial_\mu \Phi, \mathcal{A}_5) \), where \( \partial_\mu \hat{A}_\mu = 0 \) define the transverse gauge field \( \hat{A}_\mu \). The localization of the massless mode of \( \hat{A}_\mu \) occurs for \( \gamma_1 = -1/16 \), a valid result for the RS-I and RS-II models and their smooth versions. It makes this mechanism extremely interesting and powerful. In Ref. [45] the localization of scalar component \( \mathcal{A}_5 \) is studied and it is showed that this occurs for a different value of \( \gamma_1 \). Thus, this kind of coupling with Ricci scalar do not allows us to confine both sectors simultaneously.

### III. GENERAL ASPECTS OF VECTOR FIELD LOCALIZATION

We will study the localization of the vector field in a background like that discussed in section (II A). That is with the braneworld metric in generic form,

\[
ds^2 = g_{MN} dx^M dx^N = e^{2\sigma(y)} \left( \eta_{\mu\nu} dx^\mu dx^\nu + \eta_{jk} dy^j dy^k \right),
\]

where \( \eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1) \), \( \eta_{jk} = \delta_{jk} \) (Kronecker Delta) and the warp factor \( \sigma(y) \) depends only on the transverse extra dimensions \( y^j \). Another assumption that will be made is that the background is asymptotically \( (y^j \to \infty) \) an AdS spacetime. As we saw, this last consideration ensure a gravitational theory consistent on the 3-brane.

In order to study the localization of the vector field in this background, let us to start with an action in the form,

\[
S_2 = -\int d^{(D-d)}x dy^i \sqrt{-g} \left[ \frac{1}{4} \mathcal{F}_{MN} \mathcal{F}^{MN} + \frac{\lambda_1}{2} R \mathcal{A}_M \mathcal{A}^M + \frac{\lambda_2}{2} R_{MN} \mathcal{A}^M \mathcal{A}^N \right],
\]

where \( \mathcal{A}_M \) is the vector field, \( \mathcal{F}_{MP} = \partial_M \mathcal{A}_P - \partial_P \mathcal{A}_M \) is a field strength-type tensor, \( R \) and \( R_{NM} \) are the scalar and Ricci tensor, respectively. Using the relations presented in the appendix (A 1), we can write the Ricci tensor for the metric (20) as,

\[
R_{MN} = -\left[ (D-2) \delta_M^i \delta_N^j + \eta_{MN} \eta^{kj} \right] \partial_k \partial_j \sigma(y) + (D-2) \left[ \delta_M^i \delta_N^j - \eta_{MN} \eta^{kj} \right] \partial_k \sigma(y) \partial_j \sigma(y); \tag{22}
\]

and the Ricci scalar as,

\[
R = - (D-1) e^{-2\sigma(y)} \eta^{jk} \left[ 2 \partial_k \partial_j \sigma(y) + (D-2) \partial_k \sigma(y) \partial_j \sigma(y) \right], \tag{23}
\]

with \( \partial_k = \frac{\partial}{\partial y^k} \) the derivative in the coordinates of the transverse extra dimensions.
The action \((21)\) is invariant by general coordinate transformation. Such that, when we realize a Lorentz transformation at the brane,

\[
A^M' = L^M_M A^M \rightarrow \begin{pmatrix} A^\mu' \\ B^j' \end{pmatrix} = \begin{pmatrix} \Lambda^\mu'_\mu & 0 \\ 0 & \delta^j_j' \end{pmatrix} \begin{pmatrix} A^\mu \\ B^j \end{pmatrix},
\]

where \(\Lambda^\mu'_\mu\) is a usually Lorentz transformation in the Minkowski spacetime. We see that this kind of transformation makes it clear that the components \(A^\mu\) will be Lorentz vector at the brane. On the other hand, the components \(B^j\) will be Lorentz scalars on the brane. Thus, let us to split the analysis of localization for this two field and starts by the fields \(A^\mu\), which the transverse sector we wish to describe the effective electromagnetic theory on the brane.

A. Localization of Transversal Sector of \(U(1)\) Gauge Field \(-\hat{A}_\mu\)

Due the above discussion, and without loss of generality, it is convenient to separate the \(D\)-dimensional vector field in the form, \(A_N = (\hat{A}_\mu + \partial_\mu \phi, B_k)\), where \(\hat{A}_\mu\) is the transverse sector of the vector field on the brane, such that, \(\partial_\mu \hat{A}^\mu = 0\). With this propose and after some conventional manipulations, one can separate the action \((21)\) in two parts (see appendix \((A2)\)),

\[
S = S_\perp[\hat{A}_\mu] + S[\phi, B_k],
\]

where one part contains only the transverse sector as follows,

\[
S_\perp = -\int d^{(D-d)}x d^d y \sqrt{-g} \left\{ \frac{1}{4} g^{\mu\nu} g^{\rho\lambda} \hat{F}_{\mu\rho} \hat{F}_{\nu\lambda} + \frac{1}{2} g^{\mu\nu} g^{jk} \partial_j \hat{A}_\mu \partial_k \hat{A}_\nu + \frac{\lambda_1}{2} R g^{\mu\nu} \hat{A}_\mu \hat{A}_\nu + \frac{\lambda_2}{2} g^{\mu\nu} g^{\rho\lambda} R_{\mu\rho\nu\lambda} \hat{A}_\mu \hat{A}_\nu \right\},
\]

and an other part contains the longitudinal and scalar sectors of the theory, \(\partial_\mu \phi\) and \(B_k\), respectively. The explicit form of \(S[\phi, B_k]\) was not written because it is not necessary for the discussion of this section. In next section, we will study the localization of \(B_k\) components directly from \((21)\) for convenience. From the action \((26)\) and using the metric \((20)\), we can obtain the following equation of motion for the transverse sector \(\hat{A}_\mu\),

\[
-e^{-(D-4)\sigma} \partial_k \left( e^{(D-4)\sigma} \partial^k \hat{A}^{\lambda} \right) + \lambda_1 R e^{2\sigma} \hat{A}^{\lambda} + \lambda_2 R^{\lambda} \hat{A}^\mu = \partial_\nu \hat{F}^{\nu\lambda},
\]
where the Minkowski metric was used to lower and raise the indices by economy and clarity of equation. To solve the equation (27), we will propose the Kaluza-Klein decomposition
\[ \hat{A}_\nu = \sum_n A^{(n)}_\mu (x) \chi_n (y) e^{-\frac{(D-4)}{2} \sigma (y)}, \]
and as we can see from the relation (22),
\[ R_{\mu \rho} = - \left[ \partial_k \partial^k \sigma (y) + (D-2) \partial_k \sigma (y) \partial^k \sigma (y) \right] \eta_{\mu \rho} = - h (y) \eta_{\mu \rho}. \]  
(28)

With this, we can separate the equation (27) in such form,
\[ \partial_\nu F^{\nu \mu} (n) (x) = m_n^2 A^{(n)}_\mu (x). \]  
(29)

and
\[ - \partial_k \partial^k \chi_n + \left[ \frac{(D-4)^2}{4} \partial_k \sigma (y) \partial^k \sigma (y) + \frac{(D-4)}{2} \partial_k \partial^k \sigma (y) + \lambda_1 R e^{2 \sigma} - \lambda_2 h (y) \right] \chi_n = m_n^2 \chi_n. \]  
(30)

Beyond that, we can write the action (26) as,
\[ S_\perp = - \sum_n \int d^d y \chi_n^2 \int d^{D-d} x \left\{ \frac{1}{4} F^{(n)}_{\mu \rho} F^{\mu \rho}_{(n)} + \frac{1}{2} m_n^2 A^{(n)}_\mu A^\mu_{(n)} \right\}. \]  
(31)

Wherein the contractions of greek indices are performed with the Minkowski metric. As we said, a well-defined (localized) theory on the brane require that the integral,
\[ K_n = \int d^d y \chi_n^2, \]  
(32)

be a finite integral. The equation (30) is written in the Schrödinger-type form. Using the relations (23) and (28), we see that the potential-type term has the general form,
\[ U (y^j) = c_1 \partial_k \partial^k \sigma (y) + c_2 \partial_k \sigma (y) \partial^k \sigma (y). \]  
(33)

As it was already discussed in section (II A), the asymptotic behavior of the potential term indicates if the states can be normalized. In a general way the discussion made after Eq. (5) also is valid for this Eq. (33). Thus we are going obtain a solution for the zero-mode and realize the discussion of localization.

1. Localization of Zero-Mode \( m_n^2 = 0 \)

We are going to propose a zero-mode solution for the equation (30) as \( \chi_0 = e^{a \sigma (y)} \). It will be a solution if \( a = c_1 \) and \( a^2 = c_2 \). This condition \( (c_1^2 = c_2) \) always can be made by setting
up the coupling constants $\lambda_1$ and $\lambda_2$ in action \((26)\), furthermore, it has a very interesting and desirable consequence. It allows us to factor Eq. \((30)\) in the following way,

\[
[-\partial_k + c_1 \partial_k \sigma(y)] \left[ \partial^k + c_1 \partial^k \sigma(y) \right] \chi_n = m_n^2 \chi_n. \tag{34}
\]

This is analogous to a supersymmetric quantum mechanic problem, such we can affirm that there are not tachyonic modes in the spectrum, i.e., $m_n^2 \geq 0$. About the normalization of zero-mode $\chi_0(y) = e^{a\sigma(y)}$, we can infer from the equation \((32)\) that the expression $\chi_0^2$ should go to zero faster than $|y|^d$, when $|y| \to \infty$. On the other hand, the discussion up to this point does not mention the specific form of the warp factor. But, it was required that the background be an asymptotically $AdS$ spacetime. In section \((II\ A)\), we discussed that this feature of the background leads to the following asymptotic behavior for $\sigma(y)$,

\[
\sigma(|y| \to \infty) = -\ln \left( \sum_j \beta_j |y|^d \right), \tag{35}
\]

with $\beta_j$’s constants. When we use this asymptotic expression for $\sigma(y)$ in the equation \((33)\), we obtain the asymptotic value of potential $U(y) \to 0$ and the localization of massless mode ($m_n^2 = 0$) cannot be ensured. However, this expression to $\sigma(y)$ has exactly the required behavior for that $\chi_0$ can be normalized, viz., $\chi_0^2(y \to \infty) = \left( \sum_j \beta_j |y|^d \right)^{-2a}$. Thus, the integral \((32)\) will be finite if we have $2a = 2c_1 > d$, where $d$ is the number of extra dimensions.

Another point very interesting arise when we use this asymptotic warp factor for realize a comparative between localization of gravity and localization of gauge field. As we saw in section \((II\ A)\), the Eq. \((4)\) has the follows solution for the massless mode, $\psi_0(y) = e^{\frac{D-2}{4} \sigma(y)}$, that must be normalized by $\int d^d y \psi_0^2(y) = \int d^d y e^{(D-2)\sigma(y)}$. This is always a finite integral, thus, a consistent gravity theory on the brane. When we setting up $2c_1 \geq (D - 2)$, the localization of gravity necessarily implies localization of $U(1)$ gauge field (massless modes) in this generic scenario proposed by us. Even that the localization of gauge field is given by $2c_1 > d$, if $2c_1 \geq (D - 2)$ is satisfied, the localization of gravity ensure the localization of $U(1)$ gauge field on the 3-brane.

We can use the relations \((23)\) and \((28)\) for write the explicit form of $c_1$ and $c_2$,

\[
c_1 = \frac{D - 4}{2} - 2\lambda_1 (D - 1) - \lambda_2 \quad \text{and} \quad c_2 = \frac{(D - 4)^2}{4} - (\lambda_1 (D - 1) + \lambda_2) (D - 2). \tag{36}
\]

and try identify explicitly if there is some scenario where the localization of gravity ensure the gauge field localization. From expressions in Eq. \((36)\) some particular cases can be analyzed:
(i) Free gauge field - In this case, the parameters $\lambda_1$ and $\lambda_2$ are zero in above equation, and we obtain $2c_1 = (D - 4)$. Thus, as we must have $2c_1 > d$ for that the $\hat{A}^\mu$ to be confined, we conclude that for a 3-brane ($d = D - 4$) and all extra dimensions infinitely large, the free gauge field cannot be localized for any number of extra dimensions. On the other hand, if some these dimensions are compacted and the warp factor does not depend of such dimensions there is the possibility of localization. We are being cautious in stating that free case is localized because for some cases where that finite-integral argument lead to localization [8, 34], the analysis of other aspects, like Hodge duality, shown that the free case cannot be localized [60].

(ii) Ricci scalar coupling - In this other case, we will setting up the parameter $\lambda_2 = 0$ in equations (36). When we use that $c_1^2 = c_2$, we obtain that $\lambda_1 = \frac{D-6}{4(D-1)}$ and as we must have $2c_1 > d$ for that the gauge field be localized, this give us $2 > d$. If the brane has 4 dimensions and all extra dimensions are in the range $[-\infty, \infty]$, the localization is possible only in five dimensions (5D). But for an arbitrary number of extra dimensions, localization is also possible if there is only an infinitely large extra dimension and the warp factor depends on it, i.e., in co-dimension one. It is important underline that for this and the next cases such argument of Hodge duality does not apply.

(iii) Ricci tensor coupling - In this case, we will setting up the parameter $\lambda_1 = 0$ in Eq. (36), this allows us write $2c_1 = (D - 4) - 2\lambda_2 > d$, again, when we use that $c_1^2 = c_2$, now we obtain $\lambda_2 = -2$ and the localization of transverse sector $\hat{A}^\mu$ occurs for any number of extra dimensions, even if all extra dimensions are infinitely large.

(iv) In this latter situation, there is a particular case where we can impose $\lambda_1 = -\frac{1}{2}\lambda_2$. This constraint allows us to combine the interaction terms with scalar and Ricci tensor in action (26) in the Einstein tensor. With this, it is only possible to localize the field $\hat{A}^\mu$ for the same cases discussed for the free gauge field in (i). On the other hand, if there is not constraint one of the parameter, $\lambda_1$ or $\lambda_2$, is free. Using that $c_1^2 = c_2$, we obtain the relations $\lambda_2 = -2\lambda_1(D - 1) - 1 \pm \sqrt{\lambda_1(D - 1)(D - 2) + 1}$ and $c_1 = \frac{D-2}{2} \pm \sqrt{\lambda_1(D - 1)(D - 2) + 1}$. As we already saw, the massless mode is confined if $2c_1 > d$, which always can be realized by setting up $\lambda_1$. If we make $d = D - 4$, so $\lambda_1$ must live in range $[-\frac{1}{(D-2)(D-1)}, 0]$ for that the zero mode solution can be confined on the brane.
A interesting comparative analysis can be made between the conditions for the localization of gravity and the condition that we find for the localization of transverse sector $\hat{A}_\mu$. Note that $2c_1 \geq (D - 2)$ only can be satisfied in the cases (iii) and (iv), what lead us to infer that the localization of gravity ensure the localization of transverse sector $\hat{A}_\mu$ in this scenarios presented by us when the non-minimal coupling with gravity through Ricci tensor is considered as showed in the Figs. 1 and 2.

B. Localization of the Scalars Components $B_j$

In the previous section we discussed the localization of the transverse sector $\hat{A}_\mu$, which is exactly the sector that leads to an effective theory for 'photon' at the brane. Now, we will study the localization of the components $B_j$, specially if these can be confined on the brane simultaneously with the transverse sector. It is common in the literature to consider the trivial solution for these fields. However, such components could generate an effective theory at the brane that could would be interpreted as Higgs fields or even dark energy.

As usual, let us make the variation of action (21) with respect to the vector field $A_M$, which gives us the equations of motion,

$$\frac{1}{\sqrt{-g}} \partial_N \left( \sqrt{-g} g^{MN} g^{PQ} F_{MP} \right) = \lambda_1 R g^{MQ} A_M + \lambda_2 g^{MQ} g^{NP} R_{MN} A_P,$$

(37)

and due to the anti-symmetry of $F_{MP}$, we can still obtain,

$$\partial_Q \left( \lambda_1 R \sqrt{-g} g^{MQ} A_M + \lambda_2 \sqrt{-g} g^{MQ} g^{NP} R_{MN} A_P \right) = 0.$$

(38)

From these equations, we can write the equation of motion for the fields $B^j$. By separating the indexes $Q = k$ in the relation (37) and using the relation (38), we easily arrive at the equation of motion,

$$\partial_\rho \partial^\rho B^j + e^{-(D-4)\sigma} \partial_k \left[ e^{(D-4)\sigma} B^{kj} \right] - \lambda_1 R e^{2\sigma} B^j - \lambda_2 R^j_k B^k$$

$$+ \partial^j \left\{ \frac{e^{-(D-2)\sigma}}{(\lambda_1 R - \lambda_2 h(y) e^{-2\sigma}) \partial_k \left[ e^{(D-2)\sigma} \left( \lambda_1 R \delta^k_l + \lambda_2 R^k_l e^{-2\sigma} \right) B^l \right]} \right\} = 0,$$

(39)

wherein $B^{kj} = \partial^k B^j - \partial^j B^k$ and the indices are lowered or raised using the Minkowski metric (remember that $\eta_{jk} = \delta_{jk}$). Looking at the equation (39), we can see the non-triviality in treating these scalar components. Unlike the transverse sector, which had no
coupled components in the equation of motion, here we have a coupled partial differential  
equation relatively complicated. Even writing $R_{jk}$ explicitly,

$$R_{jk} = (D - 2) \left[ -\partial_j \partial_k \sigma(y) + \partial_j \sigma(y) \partial_k \sigma(y) \right] - h(y) \eta_{jk} = (D - 2) \Omega_{jk} - h(y) \eta_{jk}, \quad (40)$$

it is not possible to separate the fields $B^k$. In order to obtain at least a asymptotic solution  
for Eq. (39), we will use the asymptotic warp factor (35). This allows us to do some  
simplification in equation (39). The first is that,

$$\Omega_{jm} = -\partial_m \partial_j \sigma(y) + \partial_m \sigma(y) \partial_j \sigma(y) \to 0. \quad (41)$$

Such that, $R_{jk} \to -h(y) \eta_{jk}$ in Eq. (40). Another point is that $g(y) \equiv e^{2\sigma} (\lambda_1 R - \lambda_2 h(y)e^{-2\sigma})$  
has the general behavior $g(y) = C_0 e^{2\sigma}$ with $C_0$ a constant. Thus, the Eq. (39) can be written  
in the form,

$$e^{-(D-4)\sigma} \partial_k \left[ e^{(D-4)\sigma} \partial^kB^j_{\infty} \right] + \partial_\rho \partial^\rho B^j_{\infty} - g(y)B^j_{\infty} + 2\partial^j B^k_{\infty} \partial_k \sigma + (D - 2) B^k_{\infty} \partial^\rho \partial_k \sigma = 0. \quad (42)$$

Now, let us propose the suitable transformation $B^k_{\infty}(x, y) = B^k(x, y)e^{-(D-2)\sigma}$, such that we  
can eliminate the last two terms in (42). Thus, we get,

$$\partial_k \partial^k B^j(x, y) - \left[ \frac{(D - 2)}{2} \partial_k \partial^k \sigma + \frac{(D - 2)(D - 6)}{4} \partial_k \sigma \partial^k \sigma + g(y) \right] B^j(x, y)$$

$$+ \partial_\rho \partial^\rho B^j(x, y) + 2 \left[ \partial^j B^k(x, y) - \partial^k B^j(x, y) \right] \partial_k \sigma = 0. \quad (43)$$

Note that this transformation, besides allowing the elimination of some terms in Eq. (42), it  
generates a very convenient anti-symmetrisation in the last term of equation (43). We can  
realize the contraction of Eq. (43) with the quantity $\partial_j \sigma$. This eliminates the anti-symmetric  
term and using the asymptotic warp factor (35) we obtain the relation,

$$\left[ \frac{(D-2)}{2} \left( \partial_k \partial^k \sigma + \frac{(D-6)}{2} \partial_k \sigma \partial^k \sigma \right) + g(y) \right] \sum_j \text{sgn}(y^j) \beta_j B^j(x, y)$$

$$- \partial_\rho \partial^\rho \sum_j \text{sgn}(y^j) \beta_j B^j(x, y) = \partial_\rho \partial^\rho \sum_j \text{sgn}(y^j) \beta_j B^j(x, y). \quad (44)$$

Where $\text{sgn}(y^j)$ is the signal function and $\beta_j$'s are constants. Now, let us set the field  
$\sum_j \text{sgn}(y^j) \beta_j B^j(x, y) \equiv \Phi(x, y)$, and thus we can write the equation for $\Phi(x, y)$ in the  
form,

$$\left[ \frac{(D-2)}{2} \left( \partial_k \partial^k \sigma + \frac{(D-6)}{2} \partial_k \sigma \partial^k \sigma \right) + g(y) \right] \Phi(x, y) - \partial_\rho \partial^\rho \Phi(x, y) = \partial_\rho \partial^\rho \Phi(x, y). \quad (45)$$
The equation above is exactly what we would get if we had treated the system in co-
dimension 1. Proposing \( \Phi(x, y) = \theta(x)\zeta(y) \), it is possible to separate the variables in the
follows way,

\[
\Box_x \theta(x) = M^2 \theta(x),
\]

and

\[
\left[ \frac{(D-2)}{2} \left( \partial_k \partial^k \sigma + \frac{(D-6)}{2} \partial_k \sigma \partial^k \sigma \right) + g(y) \right] \zeta(y) - \partial_k \partial^k \zeta(y) = M^2 \zeta(y).
\]

We will use the follow non-conventional method for obtain the asymptotic solutions \( B^j(x, y) \).

The relation \( \sum_j \text{sgn}(y^j) \beta_j B^j(x, y) = \Phi(x, y) \), with the field \( \Phi(x, y) \) solved from the equations
\ref{eq:46} and \ref{eq:47}, will be used to remove the coupled terms in the equation \ref{eq:43}. A peculiar
aspect of this procedure is the arising of a non-homogeneous source-type term,

\[
\partial_k \partial^k B^j(x, y) - \left[ \frac{(D-2)}{2} \partial_k \partial^k \sigma + \frac{(D-2)(D-6)}{4} \partial_k \sigma \partial^k \sigma + g(y) \right] B^j(x, y)
\]

\[
+ \partial_\rho \partial^\rho B^j(x, y) - 2\partial^k B^j(x, y)\partial_k \sigma = 2e^\sigma \partial^j \Phi(x, y).
\]

Before to study the localization these fields, we will first establish some issues. First,
from the definition of the field \( \Phi(x, y) \) and using the Eq. \ref{eq:46}, we see that \( \Box_x \Phi(x, y) = M^2 \Phi(x, y) = \sum_j \text{sgn}(y^j) \beta_j \Box_x B^j(x, y) \). If we are treating the zero-mode \( (M^2 = 0) \) of the
field \( \Phi(x, y) \), then we should have \( \sum_j \text{sgn}(y^j) \beta_j \Box_x B^j(x, y) = 0 \), i.e., \( \Box_x B^j(x, y) = 0 \) for each
field \( B^j(x, y) \) independently, otherwise, there are tachyonic modes in the theory. Hence,
the analysis of massless mode of \( \Phi(x, y) \) leads to the analysis of the massless modes of the
components \( B^j(x, y) \). Second, the equation \ref{eq:47} is in the Schrödinger-type form, which
allows us to identify a 'potential'. This potential is even by spatial inversion \( (y^j \rightarrow -y^j) \),
and this generates well-defined solutions for the field \( \Phi(x, y) \) (even and odd) under such
a transformation. Since the field \( \Phi(x, y) \) is defined as a linear combination of the fields
\( B^j(x, y) \), these fields should reflect a well-defined behavior by exchange \( y^j \rightarrow -y^j \). This last
conclusion can be obtained more formally by consider that the action has this invariance.

And the analysis of equations \ref{eq:37} and \ref{eq:38} show that behavior of components \( B^j(x, y) \)
by spatial inversion of extra dimensions. Finally, there are two ways of separating the
fields \( B^j(x, y) \); (a) with the index at the coordinates of the brane, \( B^j(x, y) = B^j(x)Z^j(y) \),
which takes all degrees of freedom for the brane and/or (b) with the index at the transverse
coordinates of the brane, \( B^j(x, y) = B(x)Z^j(y) \), which generates a configuration in which all
fields will manifest themselves at the brane as only one degree of freedom. In fact, due to the form of equation (48), especially by the form of non-homogeneous term, the most general possible solution for this equation is a propose in the form, \( B^i(x, y) = \tilde{B}^i(x, y) + \theta(x)Z^i(y) \), where \( \theta(x) \) is the same field that comes from the separation of the field \( \Phi(x, y) \). In this way, this solution must to satisfy the follows equations,

\[
\begin{align*}
\partial_k \partial^k \tilde{B}^j(x, y) &- \left[ \frac{(D - 2)}{2} \partial_k \partial^k \sigma + \frac{(D - 2)(D - 6)}{4} \partial_k \sigma \partial^k \sigma + g(y) \right] \tilde{B}^j(x, y) \\
&+ \partial_\rho \partial^\rho \tilde{B}^j(x, y) - 2\partial^k \tilde{B}^j(x, y) \partial_k \sigma = 0; \quad (49)
\end{align*}
\]

\[
\begin{align*}
\partial_k \partial^k Z^j_i(x, y) &- \left[ \frac{(D - 2)}{2} \partial_k \partial^k \sigma + \frac{(D - 2)(D - 6)}{4} \partial_k \sigma \partial^k \sigma + g(y) \right] Z^j_i(x, y) \\
&+ M^2 Z^j_i(x, y) - 2\partial^k Z^j_i(y) \partial_k \sigma = 2e^\sigma \partial^i \zeta(y); \quad (50)
\end{align*}
\]

\[
\partial_\rho \partial^\rho \theta(x) = M^2 \theta(x); \quad (51)
\]

and the constraint,

\[
\sum_j \text{sgn}(y^j)\beta_j B^j(x, y) = \sum_j \text{sgn}(y^j)\beta_j \left[ \tilde{B}^j(x, y) + \theta(x)Z^j_i(y) \right] = \Phi(x, y). \quad (52)
\]

1. Localization of Zero-Mode

We will return the equation (47) and obtain a solution for the zero-mode \( \zeta_0(y) \) by propose \( \zeta_0(y) = e^{\tilde{b}\sigma(y)} \), which will be a solution when, \( \tilde{b}_\pm = -\frac{1}{2} \pm \frac{1}{2}[(D - 3)^2 - 4(D - 1)(\lambda_2 + \lambda_1 D)]^{\frac{1}{2}} \).

There is no reason to try to confine the field \( \Phi_0(x, y) \) on the brane, because this field does not appear in the action. However, we may require that this solution goes to zero when \( y^j \to \infty \), so we must eliminate the solution \( \tilde{b}_- \) and require that \( \lambda_2 + \lambda_1 D < \frac{(D-2)(D-4)}{4(D-1)} \).

Beyond that, the field \( \Phi_0(x, y) \) is even in the exchange of \( y^i \to -y^i \), so we must have the fields \( \text{sgn}(y^i)B^j_i(x, y) \) even by that transformation. Using the solution for \( \zeta_0(y) \), the equation (50) stay in shape,

\[
\begin{align*}
\partial_k \partial^k Z^j_i_0(y) &- \left[ \frac{(D - 2)}{2} \partial_k \partial^k \sigma + \frac{(D - 2)(D - 6)}{4} \partial_k \sigma \partial^k \sigma + g(y) \right] Z^j_i_0(y) \\
&- 2\partial^k Z^j_i_0(y) \partial_k \sigma = 2\tilde{b}e^{(\tilde{b}+1)\sigma(y)} \partial^i \sigma(y). \quad (53)
\end{align*}
\]

And we can show that a solution to this equation (53) can be obtained from the 'ansatz', \( Z^j_i_0(y) = \tilde{Z}^j_i_0(y) + K^j_i_0(y) \), such that,

\[
\begin{align*}
\partial_k \partial^k \tilde{Z}^j_i_0(y) &- \left[ \frac{(D - 2)}{2} \partial_k \partial^k \sigma + \frac{(D - 2)(D - 6)}{4} \partial_k \sigma \partial^k \sigma + g(y) \right] \tilde{Z}^j_i_0(y) \\
&- 2\partial^k \tilde{Z}^j_i_0(y) \partial_k \sigma = 0, \quad (54)
\end{align*}
\]
and
\[ \partial_k \partial^k K^j_0(y) - \left[ \frac{(D - 2)}{2} \partial_k \partial_k^\sigma + \frac{(D - 2)(D - 6)}{4} \partial_k \sigma \partial^k \sigma + g(y) \right] K^j_0(y) \]
\[ - 2\partial^k K^j_0(y) \partial_k \sigma = 2\delta^{(b+1)} e^{(1+b)\sigma(y)} \partial^j \sigma(y), \] (55)
where we added the homogeneous part \([54]\) in order to satisfy some suitable 'boundary condition'. Proposing
\[ Z^j_0(y) = c^j_2 \text{sgn}(y^j) e^{c_1 \sigma(y)} \] and
\[ K^j_0(y) = c_3 \text{sgn}(y^j) |y^j| e^{(1+b)\sigma(y)} \] (without sum in \( j \)), where \( c_1, c^j_2 \) and \( c_3 \) are constants. The solution is obtained when, \( c_1^{(\pm)} = \frac{1}{2} \pm \frac{1}{2}[(D - 3)^2 - 4(D - 1)(\lambda_2 + \lambda_1 D)]^{\frac{1}{2}} \) and \( c_3 = 1 \). We are going to determine the constants \( c^j_2 \) for that the restriction \( \sum_j \text{sgn}(y^j) \beta_j B^j_0(x, y) = \Phi_0(x, y) \) reproduce the co-dimension 1 solution as a particular case of the theory (this requirement is not really necessary, but we will impose that like a 'boundary condition'). We can write the solution \( Z^j_0(y) = Z^j_0(y) + K^j_0(y) \) in a convenient way,
\[ Z^j_0(y) = \text{sgn}(y^j) e^{c_1 \sigma(y)} \left[ c^j_2 + |y^j| \right], \] (56)
Note that this solution is odd. So, a priori, it would not satisfy the boundary condition on a delta-like branes. However, we do not know the behavior of the fields \( B^j_0(x, y) \) near of the origin. In fact, the braneworld model was not defined. Moreover, this parity does not forbid that the integral of localization can be well-defined, as we will see later.

About the solution of equation \([49]\), we can realize a very similar treatment to that made for equation \([54]\). With this we get the general solution,
\[ B^j(x, y) = \text{sgn}(y^j) \tilde{B}^j(x) e^{c_1 \sigma(y)} + \theta(x) \text{sgn}(y^j) e^{c_1 \sigma(y)} \left[ c^j_2 + |y^j| \right], \] (57)
and as this solution must satisfy the constraint \([52]\),
\[ \sum_j \beta_j \tilde{B}^j_0(x) e^{c_1 \sigma(y)} + \theta_0(x) \sum_j e^{c_1 \sigma(y)} \beta_j \left[ c^j_2 + |y^j| \right] = \theta_0(x) e^{(c_1-1)\sigma(y)}. \] (58)
We conclude that, \( \sum_j \beta_j \tilde{B}^j_0(x) = 0 \). Now, we can analyze under what conditions we can 'confine' such solutions on the brane.

In the appendix \([A2]\) we calculated the separation of the kinetic part of the action \([21]\) into a sector containing only the transverse part \( \mathcal{A}^\mu \) and another part with the scalar components \( B^j \). In this latter, the kinetic term that we must deal with the localization of the massless mode is in the form,
\[ S_0[B^j] = -\frac{1}{2} \int d^{(D-d)}x d^d y \sqrt{-g} g^{\mu\nu} g^{jk} \partial_\mu B_j \partial_\nu B_k. \] (59)
Using the solution (57) and as we are interested only in the convergence of the integral when \( y^j \to \infty \), then the relevant sector is the square of Eq. (56), where asymptotically we will realize the follow approximation, \( [c_j^2 + |y^j|]^2 \approx |y^j|^2 \). This leads to a convergent integral if \( 2(c_1^{(+)} - 1) \geq d + 3 \), where we eliminate the solution \( c_1^{(-)} \) because it cannot satisfy this condition. With this condition, we can analyze each of the bellow cases:

(i) For the free field (\( \lambda_1 = \lambda_2 = 0 \)) the condition \( 2c_1^{(+)} - 1 \geq d + 4 \) gets \( (D-3) \geq d + 4 = D \), where we already used that \( D = 4 + d \). Here it is evident that it is not possible to localize the scalar components for the free case.

(ii) For the case \( \lambda_2 = 0 \) (interaction only with Ricci scalar) the convergence condition \( 2(c_1^{(+)} - 1) \geq d + 3 \) leads to a constraint on \( \lambda_1 \) given by, \( \lambda_1 \leq -\frac{3}{4} \frac{(2D-3)}{D(D-1)} \). Note that for the transverse sector \( \hat{A}_\mu \), \( \lambda_1 = \frac{D-6}{4(D-1)} \) and this value is not in the region that allows the localization of the two sectors simultaneously. This is clear from the analysis of the Figure (I). The values of \( \lambda_1(D) \) allowed for the localization of scalars components are all these below the dashed line in the Figure (I), as we can note easily the values of \( \lambda_1(D) \) that allowed the localization of the transverse sector are always out of this region (for any number of dimension), thus it is not possible to localize both sectors simultaneously.

(iii) For the case where \( \lambda_1 = 0 \) (coupling only with Ricci tensor) we get the condition, \( \lambda_2 \leq -\frac{3}{4} \frac{(2D-3)}{(D-1)} \), so that the value \( \lambda_2 = -2 \) defined for the confinement of the transverse sector in the section (II A 1) obeys this relation for any number of transverse extra dimensions, there is the possibility to ‘trapping’ simultaneously both sectors with this kind of coupling as showed in the Figure (2).

(iv) Finally, with \( \lambda_1 \) and \( \lambda_2 \) nonzero, we get \( \lambda_1 D + \lambda_2 \leq -\frac{3}{4} \frac{(2D-3)}{D(D-1)} \). Thus, we can still to have both sectors localized simultaneously on the brane. Either with the constraints over the parameters \( \lambda_1 \) and \( \lambda_2 \) obtained in the item (iv) of section (II A 1) where was analyzed the localization of transverse sector \( \hat{A}_\mu \).

Of course, we only can ensure that this scalars components \( B^i \) are localized together with the transverse sector \( \hat{A}_\mu \) if we obtain the complete solution for that components.
Figure 1: Comparative graphic between allow values of $\lambda_1$ obtained for the localization of the transverse sector (solid line), for the localization of the scalars components (below dashed line) and these values that the localization of gravity ensure the localization of transverse sector (below dotted line).

Figure 2: Comparative graphic between allow values of $\lambda_2$ obtained for the localization of the transverse sector (solid line), for the localization of the scalars components (below dashed line) and these values that the localization of gravity ensure the localization of transverse sector (below dotted line).
IV. FINAL REMARKS

In this work the general properties of the vector field localization on braneworlds with co-dimension $d$ in asymptotically AdS spacetime were studied. It was considered a $D$-dimensional bulk with a generic conformally flat metric $e^{2\sigma}(\eta_{\mu\nu}dx^\mu dx^\nu + \delta_{ij}dy^i dy^j)$, with the warp factor depending only on the transverse extra dimensions $\sigma(y)$. In this context, we use a non-minimal coupling between gravity and the vector field $A_N = (A_\mu, B_k)$ as localization mechanism. The study of zero mode localization for the fields $A_\mu$ (gauge field) and $B_k$ (scalar fields) was separated in some particular cases: (a) a non-minimal coupling only with Ricci scalar; (b) only with Ricci tensor; and (c) the case with both the Ricci scalar and Ricci tensor.

In section (III A) we analyzed the gauge field problem, where the features of the background geometry allowed us to obtain a Schrödinger-like equation with the potential given by Eq. (33). Such equation has a general analytic solution for the massless mode given by $\chi_{0}^{(gauge)} = e^{a\sigma(y)}$ if $a^2 = c_1^2 = c_2$, where $c_1$ and $c_2$ are given in Eq. (36) and depends on the kind of coupling used. This solution is valid for any warp factor, either for delta-like or smooth branes. Furthermore the existence of this zero mode solution exclude any possible tachionic modes of the theory. With this general solution, universal aspects of the gauge field localization can be attained. One of the main aspects is that the gauge field can always be confined by imposing only two general conditions: that the spacetime is asymptotically AdS and $(2c_1 > d)$. In section (III A 1) we give a detailed analysis of the cases (a)-(c) above mentioned: (a) For the coupling with only the Ricci scalar, the analytic solution is given by $\chi_{0}^{(gauge)} = e^{a\sigma(y)}$, with $a = c_1 = 1$ and the coupling constant given by $\lambda_1 = \frac{D-6}{4(D-1)}$. Therefore, by the localization condition, such zero-mode is confined if $d < 2$, i.e., in co-dimension one models; (b) For the coupling with only the Ricci tensor, the analytic solution is $\chi_{0}^{(gauge)} = e^{\frac{D}{2}\sigma(y)}$ and the coupling constant $\lambda_2 = -2$. This solution is always localized for any number of transverse dimensions since the localization condition above gives us $d < D$; (c) Finally, the localization of transverse sector of $A_\mu$ with both interaction terms of the cases (a) and (b) was analyzed. The analytic solution in this last case is obtained with $a = c_1 = \frac{D-2}{2} \pm \sqrt{\lambda_1(D-1)(D-2)+1}$, where $\lambda_1$ is a 'free' parameter. This solution is also localized on a 3-brane for any number of extra dimensions, provided that the parameter $\lambda_1$ belongs to the range $\left(-(D-1)^{-1}(D-2)^{-1}, 0\right)$. To conclude we obtain some conditions for
which the localization of gravity ensure localization of gauge field in this scenarios. As seem in section (II A), the zero mode solution for gravitational field in the generic braneworlds considered here is given by \( \psi_0^{(\text{gravity})} = e^{(D-2)\sigma} \). If we compare this to the above analysis for the gauge field we can see that when the coupling of vector field with the Ricci tensor is present in the theory both fields will be localized. This is a very important result, since the entire consistence of the model will depends only on the fact that gravity is consistent. We should stress that the important point of our mechanism is that the zero-mode analysis of the transverse sector of \( A_\mu \) is very general, and it is valid for any braneworld model with the conditions mentioned above. As said early, this is because the solution is given by \( \chi_0^{(\text{gauge})} = e^{a\sigma} \) for any \( \sigma(y) \). Therefore it was not necessary to specify the explicit form of the warp factor to obtain the analytic solution of this sector in section (III A 1). Thus, this confinement mechanism must provide a consistent massless theory for the \( U(1) \) gauge field on the brane for any new warped braneworld model with that features.

We also consider the localization of the scalar components \( B^j \) of that vector field \( A_M \). As said in the introduction, in the co-dimension one case the scalar component is never localized simultaneously with the gauge field. This is a drawback since the backreaction of this field could alter the AdS vacuum. However we showed in section (III B) that when more co-dimensions are considered there is an indication that such components can be localized simultaneously with the gauge field component \( A_\mu \). Differently than the transverse field component, a general analytical treatment of the \( B^j \) was not found. This is due to the fact that the equations of motion (39) can not be diagonalized and therefore are always coupled. However, as we are interested in convergence conditions of the solutions, an asymptotic treatment was realized for the cases (a)-(c) above. With this in mind the asymptotic solutions was found in Eqs. 57 and 59. These solutions indicates that the localization of the gauge and scalar components of the vector field can be simultaneously obtained only for the cases (b) and (c) above mentioned. Therefore only when the interaction with the Ricci tensor is switched on, as showed in Fig. 1 and 2. We should stress that this is another important result, since the localization of both components ensures that the backreaction of \( B^j \) will not jeopardise the AdS feature of the vacuum. However, we could not ensure that these components are really confined because there is not guarantee that the solutions will be regular in all range of integration. In order to fully solve the scalar components \( B^j \), a specific background should be considered. The existence of scalar fields components
localized over the brane is very interesting. These fields play important roles in cosmology and particle physics and in principle can provides phenomenological consequences of the geometrical localization mechanism. However this is beyond the scope of this paper and must be treated elsewhere.
Appendix A: Appendix

1. Some Definitions

The metric used in the section (III) has a very convenient feature for the development of the work, it is a conformal flat metric. This appendix aims to list useful relationships present in any good book of General Relativity for some quantities that depend on the metric. Let us consider a general conforming transformation in the metric $g_{MN} \rightarrow \tilde{g}_{MN} = \Omega^2 g_{MN}$. This transformation takes the following modifications to the geometric quantities $\Gamma^A_{BC}, R, R_{MN}$ related to the old metric $g_{MN}$:

- Christoffel’s symbol,

$$\tilde{\Gamma}^Q_{MN} = \Gamma^Q_{MN} + \delta^Q_M \partial_N \ln \Omega + \delta^Q_N \partial_M \ln \Omega - g_{MN} \partial^Q \ln \Omega.$$  \hspace{1cm} (A1)

- The Ricci tensor,

$$\tilde{R}_{MN} = R_{MN} - \left[(D - 2) \delta^P_M \delta^Q_N + g_{MN} g^{PQ}\right] \frac{1}{\Omega} \nabla_P \nabla_Q \Omega + \left[2(D - 2) \delta^P_M \delta^Q_N - (D - 3) g_{MN} g^{PQ}\right] \frac{1}{\Omega^2} (\nabla_P \Omega) (\nabla_Q \Omega).$$ \hspace{1cm} (A2)

- The Ricci scalar,

$$\tilde{R} = \frac{R}{\Omega^2} - \frac{2(D - 1)}{\Omega^3} g^{PQ} \left[\nabla_P \nabla_Q \Omega + \frac{(D - 4)}{2} \frac{1}{\Omega} (\nabla_P \Omega) (\nabla_Q \Omega)\right].$$ \hspace{1cm} (A3)

In these expressions, $D$ is the dimension of space-time, $\partial_N$ is a partial derivated and $\nabla_N$ is a covariant derivated with the old metric $g_{MN}$.

2. Split of the Action $S[A_N]$ in a Sector $S_{\perp}[^{\hat{\Delta}}_{\mu}]$ and a Sector $S[\phi, B_k]$

In section (III A), we proposed the separation $A_N = \left(\hat{A}_\mu + \partial_\mu \phi, B_k\right)$, where $\partial_\mu \hat{A}_\mu = 0$ is the transverse sector of the abelian gauge field on the 3-brane. This propose allowed to do the separation of the action (21) in one part containing only the transverse sector and another part with the longitudinal sector and the scalar components $B_k$. Here, we will only clarify this procedure. We will start of the action,

$$\frac{1}{4} F_{MN} F^{MN} = \frac{1}{4} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{1}{4} B_{jk} B^{jk} + \frac{1}{2} F_{\mu j} F^{\mu j},$$ \hspace{1cm} (A4)
wherein the first term is already written as a function only of the transverse sector $\hat{A}^\mu$. The last term, which is still with the sectors coupled, can be written as,

$$\frac{1}{2} F_{\mu j} F^{\mu j} = \frac{1}{2} g^{\mu \nu} g^{\nu k} \left( \partial_\mu B_j - \partial_j \hat{A}_\mu - \partial_j \partial_\mu \phi \right) \left( \partial_\nu B_k - \partial_k \hat{A}_\nu - \partial_k \partial_\nu \phi \right)$$

$$= \frac{1}{2} g^{\mu \nu} g^{\nu k} \partial_\mu B_j \partial_\nu B_k - g^{\mu \nu} g^{\nu k} \partial_\mu B_j \partial_k \partial_\nu \phi + \frac{1}{2} g^{\mu \nu} g^{\nu k} \partial_j \partial_\mu \phi \partial_k \partial_\nu \phi$$

$$+ \frac{1}{2} g^{\mu \nu} g^{\nu k} \partial_j \hat{A}_\mu \partial_k \hat{A}_\nu + g^{\mu \nu} g^{\nu k} \partial_j \hat{A}_\mu \partial_k \partial_\nu \phi - g^{\mu \nu} g^{\nu k} \partial_\mu B_j \partial_k \hat{A}_\nu. \quad (A5)$$

The last two terms in this relation can be converted in terms of surface (in the coordinates of the 3-brane!) due to the transversality condition $\partial_\nu \hat{A}_\nu = 0$. Thus, if we assume that such surface terms are null, then we get the action (21) in the form (25).
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