Anomaly Cancellation in Six Dimensions*

Jens Erler

David Rittenhouse Laboratory
University of Pennsylvania
Philadelphia, PA 19104 (USA)

Abstract

I show that anomaly cancellation conditions are sufficient to determine the two most important topological numbers relevant for Calabi-Yau compactification to six dimensions. This reflects the fact that K3 is the only non-trivial CY manifold in two complex dimensions. I explicitly construct the Green-Schwarz counterterms and derive sum rules for charges of additional enhanced U(1) factors and compare the results with all possible Abelian orbifold constructions of K3. This includes asymmetric orbifolds as well, showing that it is possible to regain a geometrical interpretation for this class of models. Finally, I discuss some models with a broken $E_7$ gauge group which will be useful for more phenomenological applications.

*Work supported by Deutsche Forschungsgemeinschaft.
1 Introduction

It has been clear that anomaly cancellation belongs to the outstanding properties of superstrings, ever since Green and Schwarz [1] showed that the zero slope limit of the SO(32) superstring theory gives rise to an anomaly free $D = 10$ supergravity theory and evades the no-go theorem of Alvarez-Gaumé and Witten [2]. On general grounds [3, 2] this property persists in any approximation of the theory and in particular in an effective supergravity theory in a lower number of dimensions. Such a theory can be obtained by direct construction of strings in $D < 10$ or by geometrical compactifications on Calabi-Yau manifolds [4].

In this paper I reverse the direction and ask to what extent the conditions of absence of anomalies can give us information about the existence of Calabi-Yau (CY) manifolds. This is in the spirit of a paper by Seiberg [5], where the number of moduli fields was derived using the absence of anomalies in type IIB supergravity in six dimensions. Here I discuss the heterotic string and show in section 2 that the requirements leading to a six-dimensional supergravity theory which is free of gauge and gravitational anomalies turn out to be sufficient to determine the main topological data of Kummer’s third surface (K3). These are the number of independent (1,1)-forms ($h_{1,1}$) and of (0,1) forms with values in the endomorphisms of the tangent bundle, i.e. $\dim H^1(End T)$. In physical terms, they determine the number of generations transforming in the 56 of $E_7$ and the number of singlet fields. In section 2 I also derive the explicit Green-Schwarz counterterms for six-dimensional superstring models as well as the transformation rules for the antisymmetric tensor field. Again, this is done by using arguments related to anomalies only.

Of course, it would be of great importance to obtain a similar result for the much more complicated case of four-dimensional theories. Here a huge number of CY manifolds could be constructed [6], but the complete classification is still an open problem. At the same time the absence of gauge anomalies in $D = 4$ supergravity theories can be achieved for an arbitrary number of generations transforming in the 27 or 27 of $E_6$. Thus, a straightforward generalization of the $D = 6$ case is not possible. On the other hand, considerations of other kinds of anomalies might help to improve the situation. However, this is outside the scope of this paper and the four-dimensional case must be treated elsewhere.

In section 3 I find all possible $Z_N$ orbifold limits of K3. In particular, by noticing the uniqueness of K3, it becomes clear that even for asymmetric orbifolds [7] we regain an unambiguous geometrical interpretation [1]. In contrast to symmetric orbifolds [8, 9, 10], these models cannot be obtained by using identical shift vectors acting in the gauge lattice and in the lattice of bosonized NSR fermions. In this publication I introduce this type of construction and point out the similarities to and differences from the symmetric case. Compactifications to four dimensions will

\footnote{Here I assume Gepner's conjecture [8] to be true, which states that all (4,4) supersymmetric conformal field theories correspond to $\sigma$-models on K3.}
be treated in \[12\], where I will present the resulting (2,2) models.

The symmetric orbifold limits of K3 were already discussed in \[13\]. However, the computation of singlet fields in \[13\] is incorrect for non-prime \(Z_N\) orbifolds. In fact, this determination is somewhat involved and special care is needed for the projection onto twist invariant states. Fortunately, the anomaly cancellation conditions of section 2 supply us with a perfect check. Moreover, they give us a number of sum rules for charges under the additional \(U(1)\)’s, which serve as additional tests.

Restrictions and relations coming from anomaly cancellation, which are often related to index theorems, are also useful for more phenomenological questions. For the heterotic string to be of any relevance to describe nature, almost all gauge symmetries of the 496-dimensional gauge group in \(D = 10\) must be broken at low energies. Significant progress has been made in the construction of vacuum configurations coming very close to what we find phenomenologically. Here it is of particular importance to break the rank of the gauge group. This is usually achieved by a variety of mechanisms, which all have in common the phenomenon that vacuum expectation values (vev’s) are given to scalar (moduli) fields connecting degenerate string vacua. Presently we do not understand the dynamics which determines the values of the moduli, but it is evident that it has to be very efficient. Therefore it appears quite remarkable that an \(SU(3) \times U(1)\) subgroup remains unbroken. If the dynamically preferred values of moduli do not correspond to enhanced gauge symmetries, which is also suggested by discussions of non-perturbative potentials \[14\], why then, is the gauge symmetry not completely broken? A possible explanation would be that there is no moduli direction to break \(SU(3) \times U(1)\). The reason for this in turn, should then be some kind of index. Merely imposing some global symmetry would not improve the situation, since in general the moduli vev’s would break it as well.

I illustrate the above mentioned efficiency of symmetry breaking by means of a simple example using continuous Wilson lines in the \(Z_3\) orbifold.

In appendix A, I present group theoretical identities relevant for anomaly cancellation in four and six dimensions.

2 Anomaly cancellation in six dimensions

In this section, I discuss some facts about anomaly cancellation in six-dimensional supergravity theories. The basic diagram to be examined is the box with an even number of external gravitons and gauge fields \[2\]. The resulting anomalous diagrams can be classified as purely gravitational, purely gauge or mixed gauge and gravitational. The pure gauge anomalies will be referred to as quartic if all external gauge fields belong to one group factor, and as cubic if three gauge fields belong to one group factor and the fourth one to a \(U(1)\).

For the theory to be anomaly free, one of two conditions have to be met for each type of anomaly. Either the coefficient of the respective anomaly vanishes or the one loop anomaly can be cancelled by the variation of a Green-Schwarz tree-
level counterterm involving an antisymmetric tensor field. The latter option requires a peculiar factorization of the non-vanishing anomaly into two expressions each quadratic in the gravitational \((R)\) resp. gauge field strengths \((F)\).

Suppose now some non-Abelian factor \(G_A\) of the gauge group possesses a fourth-order Casimir invariant. Then the quartic gauge anomaly is of the form

\[ \mu_A \text{tr} F_A^4 + \nu_A (\text{tr} F_A^2)^2. \]  

(1)

Obviously, the above mentioned factorization is impossible unless all coefficients \(\mu_A\) vanish. Likewise, the part of the pure gravitational anomaly which cannot be written as a square necessarily has to vanish.

If the gauge group contains a non-Abelian factor for which a third-order invariant exists and at least one Abelian factor, then a cubic anomaly is possible. Again it is necessary that it vanishes. As is well known from anomaly considerations in four dimensions, only \(SU(N)\) groups with \(N \geq 3\) have third order invariants. However, there is another case where we can meet a cubic anomaly. This happens if we have at least two \(U(1)\) factors and consider the diagram with three photons belonging to one \(U(1)\) and the remaining photon to the other one. In general this kind of cubic anomaly does not vanish. Once complete factorization is achieved, however, it is always possible to find linear combinations of the \(U(1)\)'s such that all cubic anomalies vanish. This then also ensures that anomalies containing gauge fields of three or four different group factors vanish.

The total anomaly originates from four sources \([2, 15, 16]\),

\[ I = I_{3/2}(R) - I_{1/2}(R) + I_{1/2}(R, F) - \sum_i s^i I_{1/2}^i(R, F), \]  

(2)

where the first is due to the gravitino, the second to the dilatino, the third to gauginos and the last to matter fermions\(^2\). The dilatino and matter fermions contribute with a minus sign, since supersymmetry requires them to have opposite chirality as compared to the gravitino and gauginos. The factor \(s^i\) counts the multiplicity of \(^2\)There is also a self-dual antisymmetric tensor field in the supergravity multiplet. Its contribution to the anomaly, however, is cancelled by the anti-self-dual tensor field in the dilaton multiplet. See also the discussion following equation (11).
representation $R_i$. Using results from [3, 13] we find

\[
i(2\pi)^3 I = \frac{1}{3!60}(244 + y - s)\text{tr}R^4 \\
+ \frac{1}{4608}(-44 + y - s)(\text{tr}R^2)^2 \\
- \frac{1}{96}\text{tr}R^2[\text{Tr}F_A^2 - \sum_{i,A} s_i^A(\text{tr}R_iF_A^2)] \\
+ \frac{1}{24}[\text{Tr}F_A^4 - \sum_{i,A} s_i^A(\text{tr}R_iF_A^4)] \\
- \frac{1}{6} \sum_{i,j,A,B} s_{ij}^{AB}(\text{tr}R_iF_A^2)(q_B^IF_B) \\
- \frac{1}{4} \sum_{i,j,A,B} s_{ij}^{AB}(\text{tr}R_iF_A^2)(\text{tr}R_jF_B^2) \\
- \frac{1}{2} \sum_{i,j,k,A,B,C} s_{ijk}^{ABC}(\text{tr}R_iF_A^2)(q_B^IF_B)(q_C^kF_C) \\
- \sum_{i,j,k,l,A,B,C,D} s_{ijkl}^{ABCD}(q_i^AF_A)(q_B^IF_B)(q_C^kF_C)(q_D^lF_D).
\]

(3)

$y$ denotes the dimension of the total gauge group, $s$ the total number of hypermultiplets (matter), $s_i^A$ the number of hypermultiplets transforming in representation $R^i$ of group factor $G_A$, $s_{ij}^{AB}$ the number of hypermultiplets transforming in representation $(R^i, R^j)$ under $G_A \times G_B$, etc. Tr refers to the trace in the respective adjoint representation and $q_A$ to the charge under $U(1)_A$. The trace over curvature matrices in $R$ is in the vector representation of $SO(5,1)$. Notice the different prefactors in front of the various pure gauge terms. They arise because a different number of gauge group factors are involved yielding different combinatorical factors. The last two terms can only arise if there are at least two or four $U(1)$ factors, respectively.

From the previous discussion, it is now clear that the coefficient of the first term must vanish. Thus, as already found in [14, 15, 13], we have to require

\[s - y = 244.\]

(4)

It is important to emphasize that relation (4) has to be strictly satisfied for any $N = 1$ superstring vacuum in six dimensions and in particular for any orbifold model\cite{13}. An immediate application of (4) is its interpretation as a one Higgs rule, i.e., whenever we turn on a non-trivial modulus vev and smoothly break some gauge symmetry, then each gauge boson acquiring a mass is accompanied by one matter field only, which, of course, plays the rôle of the Higgs multiplet. Each hypermultiplet remains exactly massless unless it delivers the relevant degrees of freedom for the supersymmetric Higgs effect. As a consequence, in six dimensions only D-term masses rather than F-term masses are possible.

Let me briefly compare this with the situation in four dimensions. Pure gravitational anomalies do not exist there, so that a relation like (4) cannot in general be deduced. On the other hand, it was observed in [10] that very often in $Z_N$ orbifolds a\footnote{In [13] some of the presented orbifold spectra fail to satisfy (4) and consequently cannot be correct.}
three Higgs rule is in effect, which states that each gauge boson becoming massive is accompanied by three matter fields. This is an obvious generalization of the above strict one Higgs rule, but there are cases where it is violated. Nevertheless the existence of these and other remarkable regularities might result from other kinds of anomalies.

In order to exploit the anomaly further, we have to use the following group theoretical expressions:

\[
\begin{align*}
\text{Tr} F_A^4 &= T_A \text{tr} F_A^4 + U_A (\text{tr} F_A^2)^2 \\
\text{Tr} F_A^2 &= V_A \text{tr} F_A^2 \\
\text{tr}_R F_A^4 &= t_A' \text{tr} F_A^4 + u_A' (\text{tr} F_A^2)^2 \\
\text{tr}_R F_A^3 &= w_A' \text{tr} F_A^3 \\
\text{tr}_R F_A^2 &= v_A' \text{tr} F_A^2.
\end{align*}
\]

If the symbol \( \text{tr} \) is used without specification it refers to the trace in the fundamental representation. Since the coefficients \( \mu_A \) in (11) have to vanish, we can write down the following relations:

\[
T_A = \sum_i s_A^i t_A^i \quad \forall A.
\]

This is the previously mentioned relation which relates the multiplicities \( s_A^i \) to purely group theoretical quantities. If we apply it to the \( E_8 \times E_8 \) theory with gauge and spin connection identified we get an unpleasant surprise. No factor of the resulting \( E_7 \times E_8 \) gauge group has a fourth order Casimir invariant. Thus, relations (10) are trivially satisfied since all \( t_A^i \) vanish. However, if we use the \( SO(32) \) theory instead, we find the required information. In fact, the resulting group is \( SO(28) \times SU(2) \) and for \( SO(N) \) groups we have

\[
\begin{align*}
\text{Tr} F_{SO(N)}^4 &= (N - 8) \text{tr} F_{SO(N)}^4 + 3 (\text{tr} F_{SO(N)}^2)^2, \\
\text{Tr} F_{SO(N)}^2 &= (N - 2) \text{tr} F_{SO(N)}^2.
\end{align*}
\]

Hence, \( T_{SO(28)} = 20 \) so that 20 vector representations are needed to satisfy condition (11). It is easy to convince oneself that this is also the number of moduli multiplets\(^5\), which in turn is given by \( h_{1,1} \).

In a similar approach\(^4\), the number of moduli fields could be determined using the fact that type IIB superstrings can also be compactified on CY manifolds. In this case, the anomalies from the supergravity multiplet are cancelled by the dilaton multiplet and 20 further matter multiplets. In type IIB supergravity theories in ten and six dimensions (anti)-self-dual antisymmetric tensor fields play an important

---

\(^4\)I thank Hans Peter Nilles for an e-mail discussion about this point.

\(^5\)The simplest way is to take a four-dimensional point of view upon dimensional reduction and to count the number of vector representations of \( SO(26) \), which are known to correspond to the number of complex moduli.
rôle and contribute to anomalies as the only bosonic fields. The reason for this is analogous to the reason why chiral fermions contribute. The chiral respectively (anti)-duality constraints in general give rise to a failure of gauge and Lorentz invariance at the quantum level \[4\]. Antisymmetric tensor fields also play a prominent rôle in the Green-Schwarz mechanism. Here, however, the cancellation procedure involves one-loop and tree diagrams and should not be confused with the type IIB case, where the anomaly is purely one-loop. In fact, anomaly freedom for type IIB supergravity in \(D = 10\) was already observed in \[4\]; it is not necessary to assume that it arises as the zero slope limit of a string theory, and no (string motivated) counterterms are needed.

Using the fact that the \(SO(28) \times SU(2)\) theory has, like \(E_7 \times E_8\), \(y = 381\) vector fields we find with help of eq. (4) the number of hypermultiplets to be \(s = 625\). Subtracting the 10 generations of \((28, 2)\), or of 56 of \(E_7\), we are left with 65 singlet states out of which 20 are moduli fields and the remaining 45 are due to \(H^1(\text{End} T)\).

To summarize, each two-complex-dimensional CY manifold must necessarily have

\[ h_{1,1} = 20, \]

\[ \dim H^1(\text{End} T) = 90. \]

Of course, these equations must also be satisfied in case Gepner’s conjecture\[^7\] fails to be true.

The question arises as to whether it is really necessary to take a detour in order to arrive at the number of 56-plets of \(E_7\), such as going to the type IIB string theory or a different vacuum state of the heterotic string. I will show later in this section that the mere existence of the hidden \(E_8\) in fact gives another proof of eq. (12).

I proceed by stating another necessary condition for arriving at an anomaly free result, namely the vanishing of non-Abelian cubic anomalies,

\[ \sum_{i,j,A,B} s_{AB}^{ij} u_A^i v_B^j = 0. \]

If, for simplicity, we further assume that we have no more than one \(U(1)\) factor in the gauge group, we can also drop the last two terms in expression (3). The remaining part is

\[ i(2\pi)^3 I = -\frac{1}{16}[(\text{tr} R^2)^2 + \frac{1}{6}(\text{tr} R^2) \sum_A (V_A - \sum_i s_A^i u_A^i)(\text{tr} F_A^2) - \frac{2}{3} \sum_A (U_A - \sum_i s_A^i u_A^i)(\text{tr} F_A^2)]^2 + 4 \sum_{i,j,A,B} s_{AB}^{ij} u_A^i v_B^j (\text{tr} F_A^2)(\text{tr} F_B^2). \]

We must require that it can be written in the form

\[ i(2\pi)^3 I = -\frac{1}{16} [\text{tr} R^2 - \sum_A \alpha_A^{(1)} \text{tr} F_A^2] \times [\text{tr} R^2 - \sum_B \alpha_B^{(2)} \text{tr} F_B^2]. \]

\[^6\]Each hypermultiplet involves two complex scalars.

\[^7\]See footnote on page 1.
The coefficients are determined by
\[
\begin{align*}
\alpha_A^{(1)} + \alpha_A^{(2)} &= \frac{1}{6} \left( \sum_i s_A^i v^i_A - V_A \right) =: \tilde{V}_A, \\
\alpha_A^{(1)} \alpha_A^{(2)} &= \frac{2}{3} \left( \sum_i s_A^i v^i_A - U_A \right) =: \tilde{U}_A,
\end{align*}
\]
yielding
\[
\alpha_A^{(1,2)} = \frac{\tilde{V}_A}{2} \pm \frac{1}{2} \sqrt{\tilde{V}_A^2 - 4 \tilde{U}_A}.
\]
The non-trivial conditions come along with the cross terms, i.e.
\[
\alpha_A^{(1)} \alpha_B^{(2)} + \alpha_A^{(2)} \alpha_B^{(1)} = 4 \sum_{i,j} s_{AB}^{i j} v^i_A v^j_B =: \kappa_{A, B} \quad \forall A, B
\]
must be satisfied identically. In order to illustrate the above formulae, let me complete the \(SO(28) \times SU(2)\) example. With help of eqs. (11) we read off \(V_{SO(28)} = 26\) and \(U_{SO(28)} = 3\), and hence
\[
\begin{align*}
\tilde{V}_{SO(28)} &= -1, \\
\tilde{U}_{SO(28)} &= -2, \\
\alpha_{SO(28)}^{(1,2)} &= 1, -2.
\end{align*}
\]
As for \(SU(2)\) we find
\[
\begin{align*}
\tilde{V}_{SU(2)} &= 46, \\
\tilde{U}_{SU(2)} &= 88, \\
\alpha_{SU(2)}^{(1,2)} &= 2, 44.
\end{align*}
\]
Only with this relative assignment of \(\alpha^{(1)}\) and \(\alpha^{(2)}\) is condition (19) satisfied, since we have 10 representations of \((28, 2)\) and \(\kappa_{SO(28), SU(2)} = 40\). Therefore,
\[
i(2\pi)^3 I = -\frac{1}{16} \left[ \text{tr} R^2 - \frac{1}{26} \text{Tr} F_{SO(28)}^2 - \frac{1}{2} \text{Tr} F_{SU(2)}^2 \right] \times \left[ \text{tr} R^2 + \frac{1}{13} \text{Tr} F_{SO(28)}^2 - 11 \text{Tr} F_{SU(2)}^2 \right],
\]
where traces in the adjoint representations of the gauge groups are used. Note that then the coefficients in the first factor are simply given by \(\frac{1}{k_A}\), where \(k_A\) is the dual Coxeter number.

Actually this turns out to be a generic feature for groups realized at level 1 Kac-Moody algebras\(^8\) and we can write
\[
\alpha_A^{(1)} = \frac{V_A}{k_A}.
\]
Thus, with \(V_A\) given in appendix \([A]\), it can be shown, that
\[
\begin{align*}
\alpha^{(1)}_{SU(N)} &= \alpha^{(1)}_{Sp(N)} = 2, \\
\alpha^{(1)}_{SO(N)} &= \alpha^{(1)}_{G_2} = 1, \quad (N \geq 5) \\
\alpha^{(1)}_{F_4} &= \alpha^{(1)}_{E_6} = \frac{1}{3}, \\
\alpha^{(1)}_{E_7} &= \frac{1}{6}, \\
\alpha^{(1)}_{E_8} &= \frac{1}{30}.
\end{align*}
\]
\(^8\)See also appendix \([A]\).
\(^9\)On the other hand higher level string models \([18]\) give rise to larger values of \(\alpha_A^{(1)}\). This is in particular true for the model of reference \([17]\), where a \(912\) representation of \(E_7\) is involved.
As for $U(1)$ factors one can define a *canonical normalization* for the charges (see section 3). Then it turns out that

$$\alpha_{U(1)}^{(1)} = 1. \quad (24)$$

At level 1, I also obtained simple relations\footnote{See appendix A for notation.} for $\alpha^{(2)}$:

$$\begin{align*}
\alpha_{SU(N)}^{(2)} &= s^{ij} + (N - 4)s^{ijk} - 2 \quad (N \geq 4) \\
\alpha_{SO(N)}^{(2)} &= 2(N-6)s^{2(N-1)} - 2 \quad (N \geq 5) \\
\alpha_{SU(2)}^{(2)} &= \frac{s^2}{6} - 16 \\
\alpha_{SU(3)}^{(2)} &= \frac{s^3}{6} - 18 \\
\alpha_{G_2}^{(2)} &= \frac{s^7}{6} - 10 \\
\alpha_{F_4}^{(2)} &= \frac{s^{26}}{6} - 5 \\
\alpha_{E_6}^{(2)} &= \frac{s^{56}}{6} - 4 \\
\alpha_{E_7}^{(2)} &= \frac{s^{78}}{6} - 1 \\
\alpha_{E_8}^{(2)} &= -\frac{1}{5}.
\end{align*}$$

In the first two cases the number of fundamental representations is fixed by eq. (10).

The final step in this discussion is the introduction of a counterterm \footnote{See appendix A for notation.} designed to cancel the factorized anomaly. It can be chosen to be

$$\Delta L_{GS} = \frac{i}{16(2\pi)^3} B [\text{tr}R^2 - \sum_A \alpha_A^{(2)} \text{tr}F_A^2], \quad (26)$$

and cancels the anomaly \footnote{See appendix A for notation.} if $B$ transforms according to

$$B \rightarrow B + [\text{tr}(\omega d\Theta) - \sum_B \alpha_B^{(1)} \text{tr}(A_B d\Lambda_B)]. \quad (27)$$

Here $\omega$ and $A_B$ are the Lorentz and gauge connections and $\Theta$ and $\Lambda_B$ are the respective transformation parameters.

As an example, I will now determine the spectrum of the $E_7 \times E_8$ theory:

1. Since at level 1 there is no matter transforming non-trivially under $E_8$, we find $\alpha_{E_8}^{(1)} = 1/30$ confirming (23) and $\alpha_{E_8}^{(2)} = -1/5$.

2. Similarly, there are only 56-plets of $E_7$ fixing $\alpha_{E_7}^{(1)} = 1/6$, again in accordance with (23).
3. Since the vector multiplet of $E_8$ is neutral under $E_7$ and there are no other non-trivial $E_8$ multiplets, there cannot be mixed $E_7$ and $E_8$ gauge anomalies and the associated cross terms in

$$i(2\pi)^3 I = -\frac{1}{16} [\text{tr} R^2 - \frac{1}{6} \text{tr} F_{E_7}^2 - \frac{1}{30} \text{tr} F_{E_8}^2] \times [\text{tr} R^2 - \alpha_{E_7}^{(2)} \text{tr} F_{E_7}^2 + \frac{1}{5} \text{tr} F_{E_8}^2]$$

must cancel. Hence, $\alpha_{E_7}^{(2)} = 1$ and $s_{E_7}^{56} = 10 = \frac{1}{2} h_{1,1}$.

From a purely field theory point of view we would not allow for a Green-Schwarz counterterm, since it is not supersymmetric [1]. Consequently, we would require the coefficients $\alpha^{(2)}$ to vanish, which is clearly possible for all cases (except for $E_8$ at level 1) and would even give us a non-trivial restriction on the particle content. However, at level 1, there is no way of satisfying the finiteness condition on super-Yang-Mills theories [19]. On the other hand, string models in general have a non-vanishing GS-counterterm and do not respect the YM-finiteness condition.

3 $Z_N$ orbifold limits of $K_3$

In the previous section I have shown that anomaly considerations in six dimensions lead essentially uniquely to the $K_3$ manifold. I want to use this fact to show that even asymmetric orbifolds [7], which seemingly have no obvious geometrical interpretation since left and right moving coordinates are twisted in a different way, are likely to be just singular points on that manifold. Of course, the orbifold construction presents a distinguished place for studying anomalies, especially since they typically lead to enhanced gauge groups and delicate questions concerning $U(1)$ charges and normalizations can be addressed.

The symmetric orbifold limits of $K_3$ have been discussed in [13], but as mentioned in the proceeding section, anomaly cancellation conditions offer an excellent check showing there are some errors for non-prime twists. Here I include asymmetric orbifolds as well and briefly describe, how to construct and classify these models. In an upcoming publication [12] I will present the four dimensional cases completing the list of Abelian (2,2) orbifolds: Symmetric $Z_N$ orbifolds were discussed in [2] [14] and updated and completed in [11]. Symmetric $Z_N \times Z_M$ orbifolds including discrete torsion can be found in [20]. It is clearly desirable to have the complete list, since they all correspond to exactly solvable models and are complements to the Landau-Ginzburg vacua classified in [3].

For each orbifold model we must assure that a number of conditions are met. Since we are interested in $N = 1$ supersymmetric compactifications, we have to

---

9

9

---

11 There are some solutions for low-dimensional representations at higher level, but they are certainly not realistic and one has to include a huge number of singlets to match the constraint (4). E.g. one could take one symmetric and one antisymmetric second rank tensor representation of $SU(N)$ ($N \geq 3$).
require that only one of the two gravitinos is projected out. Thus the two right moving internal complex coordinates have to be twisted in the same way. Likewise, since I want to discuss \((4, 4)\) models, this has to be true for the left movers, as well. On the other hand, we need not insist on treating left and right movers equally.

The next condition uses the fact that the Lefschetz fixed point theorem determines the degeneracy in the first twisted sector of an orbifold. It is given by

\[ n_1 = |(1 - \beta_L)(1 - \beta_R)|, \]  

where the twist eigenvalues \(\beta\) are non-trivial \(N\)th roots of unity. Since \(n_1\) has to be an integer, symmetric \(Z_N\) orbifolds \((\beta_L = \beta_R)\) can only exist for \(N = 2, 3, 4, 6\) and \(n_1 = 4, 3, 2, 1\). But when allowing for \(\beta_L \neq \beta_R\), I find two more solutions. They correspond to \(Z_8\) with \(n_1 = 2\) and \(Z_{12}\) with \(n_1 = 1\). In fact, there are precisely two non-trivial 8th and 12th roots of unity (plus there complex conjugates), one of them corresponding to \(\beta_L\) and the other to \(\beta_R\). Thus let me define \(\beta_L = e^{2\pi i/8}, e^{2\pi i/12}\) and \(\beta_R = e^{6\pi i/8}, e^{10\pi i/12}\) for \(Z_8\) and \(Z_{12}\), respectively.

It is still necessary to show that these orbifolds actually exist. This can be done by explicitly constructing the torus lattice in which the twist can act. As described in \([21, 22]\) I have to find an order \(N\) twist matrix \(\Theta \in O(4, 4; \mathbb{Z})\) acting on winding and momentum quantum numbers and a background metric \(G\) of the form

\[ G = \begin{pmatrix} (g - b)g^{-1}(g + b) & bg^{-1} \\ -g^{-1}b & \frac{1}{g} \end{pmatrix}, \]

such that

\[ [G\Theta - \Theta^T G] = 0. \]

The \(Z_{12}\) orbifold can be constructed as the product of two two-dimensional asymmetric orbifolds with twist matrices

\[ \Theta_{12} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \end{pmatrix} \]

and background fields

\[ g_{12} = \begin{pmatrix} \frac{\sqrt{3}}{2} \\ 0 \end{pmatrix}, \quad b_{12} = \begin{pmatrix} 0 \\ \frac{1}{2} \end{pmatrix}. \]

This two-dimensional \(Z_{12}\) model can be shown to be equivalent to the one discussed in \([23]\). There it was pointed out that it is an example of an irrational

\[12\] The \(4 \times 4\) matrices \(g\) and \(b\) denote constant background fields; for notation and more details see \([21, 22]\).
two-dimensional conformal field theory, which is exactly solvable and possibly not connected to any rational one. In contrast, due to supersymmetry in $D = 6$ there are many exactly marginal operators in the twisted sectors; therefore, this model is by no means located at a single point.

By again referring to the Lefschetz theorem it is clear that a $Z_8$ orbifold cannot be defined in two dimensions. In four dimensions there are two possibilities: One is a symmetric orbifold, whose twist matrix can be written in terms of $4 \times 4$ block matrices as

$$
\Theta^S_8 = \begin{pmatrix} \theta_8 & 0 \\ 0 & \theta_8^{-1} \end{pmatrix},
$$

where

$$
\theta_8 = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}.
$$

It cannot, however, lead to space time supersymmetry. The other twist is asymmetric and given by

$$
\Theta^A_8 = \begin{pmatrix} 0 & \theta_8 \\ \theta_8^{-1} & 0 \end{pmatrix}.
$$

The fact that the twist defined by (35) has an asymmetric form is not enough to ensure that it is not equivalent to the symmetric one given by (33). One way of proving that they do indeed differ, is to show that the condition (30) can be satisfied only for a specific background configuration. The metrical background $g_8$ is just given by the identity matrix and the antisymmetric background $b_8$ must vanish. In contrast, twist (33) allows for deformations of $g_8$ and non-vanishing $b_8$.

Now let the left movers correspond to the bosonic side of the heterotic string. The shift vectors acting in the gauge lattice are then given by $V = \frac{1}{N}(1,1,0^{14})$. The resulting unbroken gauge group is $E_8 \times E_7 \times U(1)$, except for $Z_2$ where $U(1)$ is enhanced to $SU(2)$. The $U(1)$ charge can be determined by requiring that all gauge bosons have to be neutral. Thus we would assign the charge $Q = c(a_1 + a_2)$ to a state corresponding to a vector $\vec{S} = (a_1, a_2, \ldots, a_{16})$. The normalization constant $c$ can be determined by noting that in the $Z_2$ case $Q$ would play the rôle of the third isospin component. In that case, following standard conventions, $c = 1/2$ and, since in string theory all gauge couplings are equal at tree level, this is also the correct normalization for $U(1)$. In other words, the charge of a state vector $\vec{S}$ is given by $\vec{S}Q$ with the charge operator $\vec{Q} = 1/2(1,1,0^{14})$.

More generally, since all generators of $E_8 \times E_8$ or $SO(32)$ are normalized in the same way, we find the correct normalization for any charge operator in any orbifold model to be

$$
\vec{Q}^2 = \frac{1}{2}.
$$
This is the superstring counterpart of the Grand Unification normalization leading to the prediction of the Weinberg angle.

The spectra can be derived using standard orbifold techniques. A subtlety occurs for non-prime orbifolds in higher twisted sectors. Since incorrect results can be found in the literature, I briefly describe the correct procedure.

Consider for definiteness the second twisted sector of $Z_6$. One would first determine the spectrum of the first twisted sector of $Z_3$. After finding the transformation properties of these states w.r.t. $Z_6$, one will encounter twist phases $\pm 1$. Non-oscillator states and in particular all $56$-states of $E_7$ have phases $+1$. Single oscillators $\alpha_{-2/3}$, however, contribute a relative minus sign as compared to double oscillators $\alpha_{-1/3}\alpha_{-1/3}$, both contributing the same energy. This is because, while the $Z_3$ phase contribution in both cases is $e^{4\pi i/3}$, the $Z_6$ phase contribution $e^{\pm 2\pi i/3}$ is a priori ambiguous. This ambiguity can be resolved by the observation that $\alpha_{-1/3}$ and $\alpha_{-2/3}$ respectively correspond to $\alpha_{-1/6}$ and $\alpha_{-5/6}$ in the first sector. On the other hand, only 5 combinations of the 9 $Z_3$ fixed points are fixed under $Z_6$, while the other 4 transform with a sign. Clearly, the overall twist phase must be $+1$, which then determines the degeneracy of the states.

All six $Z_N$ orbifold spectra with all appearing $U(1)$ charges are presented in table 12. Notice that in the asymmetric cases no matter fields come from the untwisted sectors. This could have been anticipated, since, as described above, the background fields $g$ and $b$ are fixed so that they do not correspond to moduli fields, which would in turn give rise to $56$-plets of $E_7$.

In all cases one can verify the charge sum rules

\[
\sum_i Q_i^2 = 42, \quad \sum_i Q_i^4 = 9, \quad (37)
\]

so that in the notation of section 2

\[
\tilde{V}_{U(1)} = 7, \quad \tilde{U}_{U(1)} = 6, \quad \alpha_{U(1)}^{(1,2)} = 1, 6, \quad (38)
\]

in agreement with (24). Moreover, for $56$-plets we see that

\[
\sum_i 56_i Q_i^2 = 56_{1/2}, \quad (39)
\]

so that according to equation (19) the correct mixed $E_7$ and $U(1)$ anomaly occurs, since

\[
i(2\pi)^3 I = -\frac{1}{16} [\text{tr} R^2 - \frac{1}{6} \text{tr} F_{E_7}^2 - \frac{1}{30} \text{tr} F_{E_8}^2 - F_{U(1)}^2] \times [\text{tr} R^2 - \text{tr} F_{E_7}^2 + \frac{1}{5} \text{tr} F_{E_8}^2 - 6 F_{U(1)}^2]. \quad (40)
\]

Finally, the cross terms between $E_8$ and $U(1)$ cancel in (44).

Thus, I could not only derive the number of singlet states with the help of equations (4) and (10), but I also found three independent conditions which have to
Table 1: \( Z_N \) orbifolds in six dimensions. Displayed are the \( E_7 \) quantum numbers and \( U(1) \) charges (\( SU(2) \) quantum numbers in case of \( Z_2 \)). The states are grouped according to the sector from which they arise.

be satisfied by the charges. It should be emphasized that similar stringent sum rules on \( U(1) \) charges can be found in four dimensions as well, regardless of whether the \( U(1) \) is anomalous or not. In fact, the number of such sum rules increases rapidly with the number of \( U(1) \) factors. In fact, many semi-realistic orbifold models have a large number of \( U(1) \)’s and the sum rules can be used, even if most of them are spontaneously broken.

I finally present some orbifold models with non-standard gauge embeddings. Take the \( SO(32) \) model and use the embedding vector \( V = \left( 1, 1, 1, 1, 1, -2, 0 \right) \) for the \( Z_3 \) orbifold. The arising matter content transforming under \( SO(22) \times SU(5) \times U(1)_Q \) is

\[
\begin{align*}
U & : \ (22, 5)_{+1/2} + (1, \overline{10})_{-1} + 2 \ (1, 1)_{0}, \\
T & : \ 9 \ (22, 1)_{+5/6} + 18 \ (1, \overline{5})_{+1/3} + 9 \ (1, 10)_{-2/3}.
\end{align*}
\]

(41)

The properly normalized charge is given by \( Q = \sqrt{\frac{7}{5}} \tilde{Q} \). I mention this model to show how the cubic and the non-factorizable quartic anomalies are cancelled between the
untwisted and twisted sector.

The other examples use the twist embedding formulation \[9\], which permits continuous Wilson lines breaking the rank of the gauge group \[24\]. This is an explicit realization of the flat directions discussed in \[24\]. Flat directions along Wilson lines are especially interesting, because they do not lead out of the orbifold class and can be studied in great detail. For instance, it is a unique place to derive the modular group of the corresponding \((0,2)\) moduli, using methods developed in \[21, 22, 27\].

Consider the \(Z_3\) orbifold with standard twist embedding. The twist acts as a rotation in one factor of the \(SU(3)^4\) subgroup of \(E_8\) and is equivalent to the standard shift embedding. The spectrum is given in table \[12\]. In this formulation there are four smooth Wilson line directions. Turned on, they break \(E_7 \times U(1)\) to \(E_6\). In this process the untwisted \(56\) and the untwisted charged singlet become massive as the longitudinal parts of the \(\frac{E_7 \times U(1)}{E_6}\) gauge bosons except for one \(E_6\) neutral combination, which becomes a massless \((0,4)\) modulus multiplet, whose scalar components correspond to the Wilson line directions. Thus we arrive at a model with 18 \(27\)-plets of \(E_6\).

Next consider the non-standard embedding case, where the \(Z_3\) rotation acts in all the four \(SU(3)\) subgroups simultaneously. This yields an \(SU(9) \times E_8\) gauge group with matter content

\[
U : \quad 84 + 2 \mathbf{1},

T : \quad 9 \mathbf{36} + 18 \mathbf{9}.
\]

In this case, the \(84\) serves as the Higgs representation breaking \(SU(9)\) completely and also leaving four moduli fields. Thus, starting with a quite non-trivial model, the flat directions “trivialize” it leaving only a pure \(E_8\) YM theory with 486 singlets and 6 (untwisted) moduli.

Finally, it is possible to twist three \(SU(3)\) subgroups of the second \(E_8\) as well. I find the gauge group \(SU(9) \times E_6 \times SU(3)\) with matter transforming as

\[
U : \quad (84, 1, 1) + (1, 27, 3) + 2 (1, 1, 1),

T : \quad 9 (9, 1, 3).
\]

The generic gauge group after Wilson line breaking, however, is just \(SU(3)\), and we are left with 81 triplets and 9 (untwisted) moduli. Further symmetry breaking generally occurs after turning on twisted flat directions and we are faced with the situation that the stringy Higgs effect seems to be too efficient. Generally we are left without any charged matter, and the same is true in four dimensions.

String model building so far has been concentrated on finding the gauge group of the standard model or some unifying group. Additional \(U(1)\)'s were broken by using mechanisms such as the one just described. However, given the observations above, the fact that some charged fields (the observed quarks and leptons) remain in the massless spectrum appears as a fine tuning if there are further flat directions breaking, e.g. \(U(1)_{EM}\). One possibility is to find a reason as to why the unwanted
flat directions are not “used” by nature. Of course, that would require a much better understanding of string dynamics. It seems much more likely, however, that the solution to the above problem can be found in the non-existence of further flat direction. If this is true, it would be very important to find examples where charged matter remains at a generic point in moduli space. It might even turn out that some of the models presented in the literature possess this property. However, to my knowledge, they have not yet been examined in this respect.

It should be possible to trace back the non-existence of certain flat directions to some kind of index and/or anomaly. Gauge and gravitational anomalies are certainly not sufficient here. If such an index could be found, this would offer an excellent phenomenological opportunity for discarding a large number of string models which lack the protecting index, like the six-dimensional examples above.

Acknowledgements

It is a pleasure to thank Mirjam Cvetič and Albrecht Klemm for many valuable discussions as well as Ramy Brustein and Paul Langacker for helpful suggestions and comments.

A Group theoretical identities

Relations between adjoint and fundamental representations of the classical Lie groups were derived in [28]. This appendix is meant to be a more extended reference and includes spinor and third rank antisymmetric tensor representations as well as all exceptional groups.

Relations defining $V_A$:

\[
\begin{align*}
\text{Tr} F^2_{SU(N)} &= 2N \quad \text{Tr} F^2_{SU(N)} \\
\text{Tr} F^2_{SO(N)} &= (N-2) \quad \text{Tr} F^2_{SO(N)} \\
\text{Tr} F^2_{Sp(N)} &= (N+2) \quad \text{Tr} F^2_{Sp(N)} \\
\text{Tr} F^2_{G_2} &= 4 \quad \text{Tr} F^2_{G_2} \\
\text{Tr} F^2_{F_4} &= 3 \quad \text{Tr} F^2_{F_4} \\
\text{Tr} F^2_{E_6} &= 4 \quad \text{Tr} F^2_{E_6} \\
\text{Tr} F^2_{E_7} &= 3 \quad \text{Tr} F^2_{E_7} \\
\text{Tr} F^2_{E_8} &= \quad \text{Tr} F^2_{E_8}.
\end{align*}
\]

The last equation serves as a definition\(^\text{13}\).

\(^{13}\)In [10] the definition $\text{Tr} F^2_{E_8} = 30 \text{Tr} F^2_{E_8}$ is used.
Relations defining $T_A$ and $U_A$:

$$
\begin{align*}
\text{Tr}F_{SU(N)}^4 &= 2N \quad \text{tr}F_{SU(N)}^4 + 6 (\text{tr}F_{SU(N)}^2)^2 \\
\text{Tr}F_{SO(N)}^4 &= (N - 8) \quad \text{tr}F_{SO(N)}^4 + 3 (\text{tr}F_{SO(N)}^2)^2 \\
\text{Tr}F_{Sp(N)}^4 &= (N + 8) \quad \text{tr}F_{Sp(N)}^4 + 3 (\text{tr}F_{Sp(N)}^2)^2 \\
\text{Tr}F_{G_2}^4 &= 5/2 (\text{tr}F_{G_2}^2)^2 \\
\text{Tr}F_{F_4}^4 &= 5/12 (\text{tr}F_{F_4}^2)^2 \\
\text{Tr}F_{E_6}^4 &= 1/2 (\text{tr}F_{E_6}^2)^2 \\
\text{Tr}F_{E_7}^4 &= 1/6 (\text{tr}F_{E_7}^2)^2 \\
\text{Tr}F_{E_8}^4 &= 1/100 (\text{tr}F_{E_8}^2)^2.
\end{align*}
$$

(45)

For groups with an independent fourth order Casimir invariant, by definition $v^A_f = t^A_f = 1$ and $w^A_f = 0$ for fundamental representations $f$. Otherwise, $t^A_f = 0$ and $w^A_f$ can be extracted from

$$
\begin{align*}
\text{tr}F_{SU(2)}^4 &= 1/2 (\text{tr}F_{SU(2)}^2)^2 \\
\text{tr}F_{SU(3)}^4 &= 1/2 (\text{tr}F_{SU(3)}^2)^2 \\
\text{tr}F_{G_2}^4 &= 1/4 (\text{tr}F_{G_2}^2)^2 \\
\text{tr}F_{F_4}^4 &= 1/12 (\text{tr}F_{F_4}^2)^2 \\
\text{tr}F_{E_6}^4 &= 1/12 (\text{tr}F_{E_6}^2)^2 \\
\text{tr}F_{E_7}^4 &= 1/24 (\text{tr}F_{E_7}^2)^2 \\
\text{tr}F_{E_8}^4 &= 1/100 (\text{tr}F_{E_8}^2)^2.
\end{align*}
$$

(46)

Other representations, which appear in string models realized at level 1 w.r.t. the underlying Kac-Moody-algebra, are totally antisymmetric tensor representations of higher rank for $SU(N)$ and the lowest dimensional spinor representations of $SO(N)$. The second resp. third relations in (14) and (15) serve as formulae for antisymmetric resp. symmetric second rank tensor representations for all the classical Lie groups.

In order to fix $w^A_{SU(N)}$ for second rank antisymmetric tensor representations $a^{ij}$, I note

$$
\text{tr}_{a^{ij}} F^3_{SU(N)} = (N - 4) \quad \text{tr}F^3_{SU(N)} \quad (N \geq 3).
$$

(47)

As for third rank totally antisymmetric tensor representations $a^{ijk}$ it can be shown that

$$
\begin{align*}
\text{tr}_{a^{ijk}} F^2 &= \frac{1}{2} (N^2 - 5N + 6) \quad \text{tr}F^2 \\
\text{tr}_{a^{ijk}} F^3_{SU(N)} &= \frac{1}{2} (N^2 - 9N + 18) \quad \text{tr}F^3_{SU(N)} \\
\text{tr}_{a^{ijk}} F^4 &= \frac{1}{2} (N^2 - 17N + 54) \quad \text{tr}F^4 + (3N - 12) (\text{tr}F^2)^2.
\end{align*}
$$

(48)

Note that in particular the right hand sides vanish for $N = 3$ and that

$$
\begin{align*}
\text{tr}_{a^{ijk}} F^4_{SU(4)} &= \text{tr}F^4_{SU(4)}, \quad \text{tr}_{a^{ijk}} F_{SU(4)}^2 = \text{tr}F_{SU(4)}^2 \quad \text{and} \quad \text{tr}_{a^{ijk}} F_{SU(4)}^3 = -\text{tr}F_{SU(4)}^3,
\end{align*}
$$

16
as is expected, since $a_{SU(3)}^{ijk} = 1$ and $a_{SU(4)}^{ijk} = 3$.

For the basic spinor representations I find,

$$
\begin{align*}
\text{tr}_{2(N-1)} F_{SO(2N)}^2 & = 2^{(N-4)} \text{tr} F_{SO(2N)}^2 \\
\text{tr}_{2(N-1)} F_{SO(2N)}^4 & = -2^{(N-5)} \text{tr} F_{SO(2N)}^4 + 3 2^{(N-7)} (\text{tr} F_{SO(2N)}^2)^2,
\end{align*}
$$

and these relations continue to hold after replacement of $SO(2N)$ with $SO(2N-1)$.

I end this appendix by showing how relations for exceptional groups can be found. For instance, the last relation in (49) can be proved as follows: Under $SU(9)$ the fundamental $248$ of $E_8$ has the decomposition

$$
248 \rightarrow 80 + 84 + \overline{84}.
$$

For the reducible $248$ representation of $SU(9)$ I find, with help of eqs. (45) and (48),

$$
\text{tr}_{248} F_{SU(9)}^4 = \text{tr}_{80} F_{SU(9)}^4 + \text{tr}_{84} F_{SU(9)}^4 + \text{tr}_{\overline{84}} F_{SU(9)}^4
$$

$$
= 18 \text{tr} F_{SU(9)}^4 + 6 (\text{tr} F_{SU(9)}^2)^2 + 2 \left[-9 (\text{tr} F_{SU(9)}^2)^2 + 15 \text{tr} F_{SU(9)}^4 \right] = 36 (\text{tr} F_{SU(9)}^2)^2.
$$

Note, how the independent fourth order invariant cancels out. Similarly,

$$
\text{tr}_{248} F_{SU(9)}^2 = 60 \text{tr} F_{SU(9)}^2,
$$

and so

$$
\frac{\text{tr}_{248} F_{SU(9)}^4}{(\text{tr}_{248} F_{SU(9)}^2)^2} = \frac{\text{tr} F_{Es}^4}{(\text{tr} F_{Es}^2)^2} = \frac{1}{100}.
$$

References

[1] M. B. Green and J. H. Schwarz, *Phys. Lett.* **149B** (1984) 117.

[2] L. Alvarez-Gaumé and E. Witten, *Nucl. Phys.* **B234** (1983) 269.

[3] G. ’t Hooft, in *Recent Developments in Gauge Theories*, G. ’t Hooft et. al. eds., Plenum, New York (1980).

[4] P. Candelas, G. T. Horowitz, A. Strominger and E. Witten, *Nucl. Phys.* **B258** (1985) 46.

[5] N. Seiberg, *Nucl. Phys.* **B303** (1988) 286.

[6] M. Kreuzer and H. Skarke, *Nucl. Phys.* **B388** (1992) 113;
A. Klemm and R. Schimmrigk, *Landau-Ginzburg String Vacua*, preprint HDTHEP-92-13 (TUM–TH–142/92, CERN–TH–6459/92).

[7] K. S. Narain, M. H. Sarmadi and C. Vafa, *Nucl. Phys.* **B288** (1987) 551.
[8] D. Gepner, *Phys. Lett.* 199B (1987) 380.

[9] L. Dixon, J. A. Harvey, C. Vafa and E. Witten, *Nucl. Phys.* B261 (1985) 678; *Nucl. Phys.* B274 (1986) 285.

[10] L. E. Ibáñez, J. Mas, H. P. Nilles and F. Quevedo, *Nucl. Phys.* B301 (1988) 157.

[11] J. Erler and A. Klemm, *Comment on the Generation Number in Orbifold Compactifications*, Munich preprint MPI–Ph/92-60 (TUM–TH–146/92), to appear in *Comm. Math. Phys.*

[12] J. Erler, in preparation.

[13] M. A. Walton, *Phys. Rev.* D37 (1988) 377.

[14] S. Ferrara, D. Lüst, A. Shapere and S. Theisen, *Phys. Lett.* 225B (1989) 363; S. Ferrara and S. Theisen, *Moduli Spaces, Effective Actions and Duality Symmetry in String Compactifications*, CERN preprint TH.5652/90, (UCLA/90/TEP/8), lectures given at the 3rd Hellenic Summer School, Corfu, Greece, Sep. 13–23, 1989.

[15] M. B. Green, J. H. Schwarz and P. C. West, *Nucl. Phys.* B254 (1985) 327.

[16] M. B. Green, J. H. Schwarz and E. Witten, *Superstring Theory*, Vol. 2, Cambridge University Press, Cambridge (1987).

[17] S. Randjbar-Daemi, A. Salam, E. Sezgin and J. Strathdee, *Phys. Lett.* 151B (1985) 351.

[18] D. C. Lewellen, *Nucl. Phys.* B337 (1990) 61.

[19] P. Howe, K. Stelle and P. C. West, *Phys. Lett.* 124B (1983) 55.

[20] A. Font, L. E. Ibáñez and F. Quevedo, *Phys. Lett.* 217B (1989) 272.

[21] J. Erler, D. Jungnickel and H. P. Nilles, *Phys. Lett.* 276B (1992) 303.

[22] J. Erler, *Investigation of Moduli Spaces in String Theories*, thesis at the Technical University of Munich (in German language), preprint MPI-Ph/92-21.

[23] J. Harvey, G. Moore and C. Vafa, *Nucl. Phys.* B304 (1987) 269.

[24] L. E. Ibáñez, H. P. Nilles and F. Quevedo, *Phys. Lett.* 187B (1987) 25.

[25] L. E. Ibáñez, H. P. Nilles and F. Quevedo, *Phys. Lett.* 192B (1987) 332.
[26] M. Cvetič, *Phys. Rev. Lett.* **59** (1987) 2829;  
A. Font, L. E. Ibáñez, H. P. Nilles and F. Quevedo, *Nucl. Phys.* **B307** (1988) 109.

[27] M. Spaliński, *Nucl. Phys.* **B377** (1992) 339;  
M. Spaliński, *Phys. Lett.* **275B** (1992) 47;  
J. Erler and M. Spaliński, *Modular Groups for Twisted Narain Models*, Munich preprint MPI–Ph/92–61 (TUM-TH-147/92).

[28] P. van Nieuwenhuizen, *Anomalies in Quantum Field Theory: Cancellation of Anomalies in d=10 Supergravity*, Leuven University Press, Leuven (1988).