E$_6$ unification in a model of dark energy and dark matter

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A model of dark energy and dark matter was proposed earlier by one of us (PQH) which involved an unbroken gauge group SU(2)$_{Z}$ whose coupling $\alpha_Z \equiv g_Z^2/4\pi \sim O(1)$ at a scale $\Lambda_Z \sim 3 \times 10^{-3}$ eV starting from a value within the range of the Standard Model (SM) couplings at a high energy scale $\sim 10^{16}$ GeV. In that model, the universe is assumed to be presently trapped in a false vacuum with an energy density $\sim \Lambda_Z^{4}$. In this paper, we present a scenario in which SU(2)$_{Z}$ is unified with the SM through several steps: $E_6 \rightarrow SU(2)_Z \otimes SU(6) \rightarrow SU(2)_Z \otimes SU(3)_c \otimes SU(3)_L \otimes U(1) \rightarrow SU(2)_Z \otimes SU(3)_c \otimes SU(2)_L \otimes U(1)\gamma$. This unification provides a rationale for why the value of the SU(2)$_{Z}$ coupling is within the range of the SM couplings at high energies. The particle content and the route of symmetry breaking in this model is very different from the usual $E_6$ unification encountered in the literature. Several implications, in addition to the dark energy, include the existence of heavy mirror particles which could be searched for at future colliders such as the LHC.

I. INTRODUCTION

The origin of the so-called dark energy responsible for the acceleration of the present universe is one of the deepest problems for at least the next several decades. In the quest for an understanding of that origin, there are several approaches that are worth mentioning. One is the improvement of cosmological and astrophysical observations which have become increasingly precise. This will allow us to study, among other things, the equation of state of the dark energy $p = w \rho$ as a function of redshift, for example.

Somewhat coupled to the observational effort is the various theoretical enterprises aimed at trying to make sense out of the discovery of the accelerating universe. In particular, several models were constructed: quintessence, modified gravity, etc... However, the most recent determination of the equation of state $w \sim -1$ appears to be consistent with the present universe which is dominated by a cosmological constant and cold dark matter, the so-called $\Lambda$CDM scenario [1]. Although it is probably too early to decide which scenario is the most plausible one, it might be interesting to build a dynamical model which can practically mimic the $\Lambda$CDM scenario at the present time.

In [2], such a model was constructed in which it was proposed that the present universe is trapped in a false vacuum characterized by an energy density $\rho_V \sim (3 \times 10^{-3}$ eV$)^4$. A detailed description of the origin of the potential which describes this false vacuum was given in [3]. In a nutshell, this is a potential of a pseudo-Nambu-Goldstone (PNG) boson $a_Z$ (the imaginary part of a singlet scalar field $\phi_Z$) coming from a spontaneously broken global symmetry $U(1)_A^{(Z)}$ present in the model. It is induced by instantons of a new gauge group SU(2)$_{Z}$ which grows strong at a scale $\sim 3 \times 10^{-3}$ eV [4]. In [5], it was shown how $a_Z = g_Z^2/4\pi$ becomes order of unity at $\Lambda_Z \sim 3 \times 10^{-3}$ eV starting from an initial value at $M \sim 10^{16}$ GeV of the order of the Standard Model (SM) couplings at a similar scale. It was suggested in [6] that this behavior could arise from some form of unification of SU(2)$_{Z}$ with the SM into the gauge group $E_6$ with the following suggested symmetry breaking pattern: $E_6 \rightarrow SU(2)_Z \otimes SU(6) \rightarrow SU(2)_Z \otimes SU(3)_c \otimes SU(3)_L \otimes U(1)_b \rightarrow SU(2)_Z \otimes SU(3)_c \otimes SU(2)_L \otimes U(1)\gamma$. (Notice that the group $SU(3)_c \otimes U(1)_b$ was discussed by several authors in the early nineties and an incomplete list of references is given in [6].) In this sense, the emergence of a new (unbroken) strong gauge group SU(2)$_{Z}$ from the Grand Unified Theory (GUT) group $E_6$ and its subsequent evolution is well motivated. In this paper, we will present a detailed discussion of this unification scenario.

In passing, it is worth mentioning that the model presented in [6] also contained several interesting implications such as the presence of cold dark matter candidates which are the fermions transforming as adjoints of SU(2)$_{Z}$ and are SM singlets; and a new mechanism of leptogenesis [7] involving the decay of a “messenger” scalar field $\phi^{(Z)}$ which carries the quantum numbers of both SU(2)$_{Z}$ and SU(2)$_{L}$ and its subsequent evolution is crucial both in generating SM particles at the end of inflation through thermalization and in the subsequent leptogenesis (above the electroweak scale) which converts a net SM lepton number into a net baryon number through the electroweak sphaleron. In addition, it was emphasized in [6] and [7] that one actually look for the messenger field- called the “lepton number progenitor” in [6]-itself at future colliders such as the LHC (or even the proposed ILC) since its mass is constrained by...
the leptogenesis scenario to be less than $1 \, TeV$ \cite{6}.

The plan of the paper will be as follows. First, we will list the particle content of the model in \cite{2,3} and show how it fits into representations of $E_6$. Next, we discuss the pattern of symmetry breaking of the model, including discussions on fermion and scalar masses. We then follow with an analysis of the renormalization-group evolution of the various gauge couplings. In particular we will compute the two relevant unification scales: $M_6$ where $SU(6)$ is broken down and $M_{GUT}$ where $E_6$ is broken down. It is the value of $M_6$ which is most relevant to the proton lifetime as we shall see. We will end with some brief comments in the conclusion on the experimental implications of our model, in particular the possible detection of various particles which are present in the model such as mirror fermions.

II. FITTING THE PARTICLE CONTENT OF THE DARK ENERGY MODEL INTO $E_6$

We now list the particle content of the dark energy model of \cite{2,3}. (The latter reference contains extensive details of that model.) As we shall see below, the effective gauge group just above the electroweak breaking scale is $SU(2)_Z \otimes SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$. We concentrate primarily on the content of particles which are non-singlet under $SU(2)_Z$ and on a complex scalar field which is singlet under both sectors. SM particles are all $SU(2)_Z$ singlets and we shall not need to list them explicitly here.

A. Particle content (other than the SM one) of the dark energy model \cite{3}

I) $SU(2)_Z$-non-singlet fermions: Under $SU(2)_Z \otimes SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$, these transform as

$$\psi^{(Z)}_{L,R} = (3,1),$$

(1)

where $i = 1,2$. The fermions $\psi^{(Z)}_i$ were shown to have the appropriate masses ($O(100-200 \, GeV)$) and annihilation cross sections (typical of a weak cross section) to be considered as candidates for the WIMP cold dark matter \cite{2,3}.

II) $SU(2)_Z$-non-singlet scalars:

$$\varphi^{(Z)}_i = (\varphi^{(Z)}_i,0,\varphi^{(Z)}_i,-) = (3,1,2,Y_\varphi = -1),$$

(2)

where $i = 1,2$. These are the so-called messenger fields since they carry quantum numbers of both the SM and $SU(2)_Z$ sectors. (One of them, $\varphi^{(Z)}_2$, is constrained in \cite{3} and \cite{6} to be much heavier than the other $\varphi^{(Z)}_1$ whose mass is constrained to be around $\sim 300 - 1000 \, GeV$.) As such, they have Yukawa couplings with SM leptons and $\psi^{(Z)}_i$. It was shown in \cite{6} that it is this kind of SM-lepton-number-violating coupling which can give rise to

a new mechanism of leptogenesis through a CP-violating decay of the lightest of the two messenger fields, namely $\varphi^{(Z)}_i$.

III) $SU(2)_Z \otimes SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$-singlet scalar: $\phi_Z = (1,1,1,0)$. In \cite{2}, this complex singlet scalar field is written as

$$\phi_Z = (v_Z + \sigma_Z) \exp(i a_Z/v_Z),$$

(3)

where $\langle \sigma_Z \rangle = 0$ and $\langle a_Z \rangle = 0$ with $\langle \phi_Z \rangle = v_Z$. The “angular” part of this field, $a_Z$, which is an axion-like particle, plays the role of the acceleron in the model of dark energy of \cite{3}. The “radial” part of the field, $\sigma_Z$, could play the part of the inflaton in a “low-scale” (i.e. a scale which is less than a typical GUT value) inflationary scenario \cite{7}.

As shown in \cite{3}, the model exhibits a global $U(1)_A$ symmetry which is spontaneously broken by $\langle \phi_Z \rangle = v_Z$ and explicitly broken by the $SU(2)_Z$ instanton effects (making $a_Z$ a Pseudo-Nambu-Goldstone (PNG) boson instead of a NG boson). The fermions $\psi^{(Z)}_i$ acquire masses through the couplings with $\phi_Z$ \cite{5}.

The evolution of the $SU(2)_Z$ gauge coupling, $g_Z$, depends on the particle content listed in (I) and (II). As shown in \cite{3}, starting with an initial value at a scale $\sim 10^{16} \, GeV$ within a factor of two or three of the corresponding SM values for the gauge couplings, $\alpha_Z = g^2_Z/4\pi$, remains relatively flat down to $O(100 \, GeV)$, when $\psi^{(Z)}_i$ decouple, and starts to rise until $\alpha_Z = 1 = \sim 3 \times 10^{-3} \, eV$. This is the scale when $SU(2)_Z$ grows strong and where the instanton-induced $a_Z$-potential used in the dark energy model is generated.

B. Representations of $E_6$

How do the above particle content along with the SM particles fit into representations of a possible GUT group $E_6$? We first notice that the fermions $\psi^{(Z)}_i$ and the messenger scalar fields $\varphi^{(Z)}_i$ transform as adjoints of $SU(2)_Z$. As it will become clear below, $\psi^{(Z)}_i$ would fit into adjoint representations, 78, of $E_6$. This is in contrast with SM particles which, as we shall see, are grouped in fundamental representations 27. In order to see what one really needs, we first observe that the maximal subgroup of interest in this paper is the following:

$$E_6 \supset SU(2)_Z \otimes SU(6).$$

(4)

Notice that the most frequent embedding that one encounters in GUT models is $E_6 \supset SO(10) \otimes U(1)$, where $SO(10)$ represents the popular GUT model \cite{8}. Our symmetry breaking path is different here. Since the dark energy model involves an unbroken $SU(2)_Z$, it is the above maximal subgroup of $E_6$ that we will be concerned with, namely $SU(2)_Z \otimes SU(6)$. As we shall
see below, the subgroup $SU(6)$ contains the SM and is broken in several steps. At this point, it is useful to list the maximal subgroups of $SU(6)$, namely $SU(6) \supset SU(5) \otimes U(1)$, $SU(6) \supset SU(2) \otimes SU(4) \otimes U(1)$, and $SU(6) \supset SU(3) \otimes SU(3) \otimes U(1)$. It is this last embedding which forms the core of our model. In the next section, we will discuss the symmetry breaking of $E_6$ and of its subgroups. We will focus in this paper on a particular model

$$E_6 \xrightarrow{M_{G/6}} G_1 \xrightarrow{M_6} G_2 \xrightarrow{M_6} G_3 \xrightarrow{M_W} G_4$$

(5)

where,

$$G_1 = SU(2)_Z \otimes SU(6),$$

(6a)

$$G_2 = SU(2)_Z \otimes SU(3)_c \otimes SU(3)_L \otimes U(1)_6,$$

(6b)

$$G_3 = SU(2)_Z \otimes SU(3)_c \otimes SU(2)_L \otimes U(1)_Y,$$

(6c)

$$G_4 = SU(2)_Z \otimes SU(3)_c \otimes U(1)_{em}.$$  

(6d)

For the moment, we first focus on the matter representations under $E_6$ and its subgroups.

Let us first look at the decomposition of the two $E_6$ representations which are of interest to us: $27$ and $78$, under $SU(2)_Z \otimes SU(6)$. What follows is a general discussion which, for the moment, ignores chirality issues to be dealt with subsequently. One has

$$27 = (2, 6) + (1, 15),$$

(7a)

$$78 = (3, 1) + (1, 35) + (2, 20).$$

(7b)

A preliminary look at (7b) reveals the fact that the $SU(2)_Z$ fermions $\psi_i^{(Z)}$ should fit into the $78$ representations, being a triplet under $SU(2)_Z$ and a singlet under $SU(6)$ (which contains the SM). The next question concerns the SM fermions and how they fit. As stressed above, one feature of the dark energy model is the fact that SM particles are singlets under $SU(2)_Z$. From (7b), one has $(1, 15)$ and from (7a), one has $(1, 35)$. Since $SU(6)$ will be subsequently broken down to the SM (in several steps) and since SM particles are in fundamental representations of $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$, it can be seen that $(1, 35)$ (which belongs to $78$) cannot contain the SM particles. (In particular, under $SU(3)_c \otimes SU(3)_L \otimes U(1)$, $35 = (1, 1)(0) + (8, 1)(0) + (1, 8)(0) + (3, 3)(-1) + (3, 3)(1)$, which does not contain SM leptons among others.)

We then conclude that $\psi_i^{(Z)}$ and SM fermions fit into $78$ and $27$ representations respectively. We next present in detail this embedding.

C. Fermion embedding into $E_6$ representations and those of its subgroups

The fermions of the model of $\text{[3]}$ can now be classified under $E_6$ and its subgroup $SU(2)_Z \otimes SU(6)$ as follows.

- $\psi_i^{(Z)}$: Since $SU(2)_Z$ is an unbroken, vector-like gauge group (just like QCD), both left- and right-handed $\psi_i^{(Z)}$ transform in the same way (triplets) under $SU(2)_Z$ as listed in (1). In consequence, $\psi_i^{(Z)}$ is part of $78_{L,R}$ which is

$$78_{L,R} = (3, 1)^i_{L,R} + (1, 35)^i_{L,R} + (2, 20)^i_{L,R},$$

(8)

with $i = 1, 2$. From this, one can readily make the identification $\psi_i^{(Z)} \equiv (3, 1)^i_{L,R}$. We will discuss the masses of these fermions below, in particular we will show how these fermions can avoid bare masses (being vector-like).

- SM fermions:

SM fermions transform under $SU(2)_Z \otimes SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ as $q_L = (1, 3, 2; 1/3)$, $u_R = (1, 3, 1, 2/3)$, $d_R = (1, 3, 1, -1/3)$, and $L_L = (1, 1, 2, -1/2)$, with the last entries denoting $Y/2$, the $U(1)_Y$ quantum numbers. Can they fit into a single $27$ representation of $E_6$?

First, we notice that the parts $(2, 6)$ of $27$ under $SU(2)_Z \otimes SU(6)$ cannot contain SM particles. (They will acquire a large mass as shown below.) This leaves us with $(1, 15)$. As stated above the path that we chose for the breaking of $SU(6)$ is $SU(6) \rightarrow SU(3) \otimes SU(3) \otimes U(1)$, with $15$ transforming under $SU(3) \otimes SU(3)$ as $15 = (3, 1) + (1, 3) + (3, 3)$. To see explicitly how this might represent the usual SM particles, we rewrite $SU(3) \otimes SU(3)$ as $SU(3)_c \otimes SU(3)_L$. Let us first start with $15_L$ (which is part of $27_L$). We have

$$15_L = (3, 1)_L + (1, 3)_L + (3, 3)_L.$$

(9)

Using the well-known Weyl two-component spinors, one has $\psi_L = \sigma_\alpha \bar{\psi}_R^\alpha$. This means that $3_L$ comes from $3_R$. Hence $(9)$ contains a colored left-handed triplet and a color-singlet right-handed triplet under $SU(3)_L$. In the next section where we will discuss the symmetry breaking of the model, it will be seen that $SU(3)_L \supset SU(2)_L$. Therefore under $SU(2)_L$, $(9)$ contains a left-handed “quark” doublet and a right-handed “lepton” doublet.

The above discussion reveals two important points. The first is that $(9)$ (or $27_L$) cannot alone accommodate SM leptons. The second point is that it contains “mirror fermions” in the form of the right-handed “lepton” doublet. We are now forced to in-
D. Embedding of the messenger scalar fields $\tilde{\varphi}^{(Z)}$ in $E_6$ representations

- $\tilde{\varphi}^{(Z)}_1$:
  The messenger scalar fields $\tilde{\varphi}^{(Z)}_1$ of $\tilde{3}$ transform as $(3, 2)$ under $SU(2)_Z \otimes SU(2)_L$ (see (2). (It is a color-singlet.) The representation with the lowest dimension that contains a triplet of color-singlet.) The representation with the lowest dimension that contains a triplet of color-singlet. The representation with the lowest dimension that contains a triplet of color-singlet. The representation with the lowest dimension that contains a triplet of color-singlet.

- $\tilde{\varphi}^{(Z)}_2$:
  $\tilde{\varphi}^{(Z)}_2$ as presented in $\tilde{3}$ is a singlet under both $SU(2)_Z$ and the SM. The most economical way is for $\phi_Z$ to be a singlet of $E_6$ as well.

III. PATTERN OF SYMMETRY BREAKING OF $E_6$, FERMION Masses, PROTON DECAY

A. Breaking of $E_6$

The pattern of symmetry breaking of $E_6$ that we discuss in this section is given in (5). We now discuss each step of the breaking chain. In this discussion, we will keep in mind that $SU(2)_Z$ remains unbroken.

- $E_6 \to SU(2)_Z \otimes SU(6)$:
  To achieve the above breaking, one should find a Higgs representation which contains a singlet under $SU(2)_Z \otimes SU(6)$.

$$650 = (1, 1) + (1, 35) + (2, 20) + (3, 35) + (2, 70) + (2, 70) + (1, 189).$$  

(12)

From (12), it follows that

$$\langle 650 \rangle = \langle (1, 1) \rangle \neq 0 \quad (13)$$

achieves the desired breaking $E_6 \to SU(2)_Z \otimes SU(6)$. In this step, the $E_6/SU(2)_Z \otimes SU(6)$ gauge bosons acquire a mass of order $M_{GUT}$.

- $SU(2)_Z \otimes SU(6) \to SU(3)_c \otimes SU(3)_L \otimes U(1)_Y$:
  Notice that, in what follows, our normalization for the $SU(2)$ quantum numbers which will be used in Section (IV) differs from the ones used in [9] by a factor of $1/(2\sqrt{3})$. The breaking of $SU(6)$ down to $SU(3)_c \otimes SU(3)_L \otimes U(1)_Y$ is achieved by looking at the decomposition:

$$35 = (1, 1)(0) + (8, 1)(0) + (1, 8)(0) + (3, 3)(-1/\sqrt{3}) + (3, 3)(1/\sqrt{3}).$$  

(14)

So

$$\langle 35 \rangle = \langle (1, 1)(0) \rangle \neq 0, \quad (15)$$

will give $SU(6) \to SU(3)_c \otimes SU(3)_L \otimes U(1)_Y$.

Since $(1, 35)$ is also contained in $650$, one might use the same field to achieve both breaking with $\langle (1, 35) \rangle \sim M_6 < \langle (1, 1) \rangle \sim M_{GUT}$. We now have

$$\langle 650 \rangle = \langle (1, 1) \rangle (\sim M_{GUT}) + \langle (1, 35) \rangle (\sim M_6).$$  

(16)

The $SU(6)/SU(3)_c \otimes SU(3)_L \otimes U(1)_Y$ gauge bosons acquire a mass of order $M_6$.

Since SM fermions and their mirror counterparts belong to a $15$ of $SU(6)$, the $U(1)_Y$ generator, which is a diagonal generator of $SU(6)$, $T_{35}$, has the normalization $Tr T_{35}^2 = 2$ (with the usual convention that, for the fundamental representation, one has $Tr T_i^2 = 1/2$). We will see below that the appropriate coefficient which multiplies $T_{35}$ is $C_6 = 2/\sqrt{3}$ in order to make connection with the SM $U(1)_Y$ quantum numbers.
The main purpose is to know when to include or exclude a certain fermion from the RG evolution of the gauge couplings.

- SU(3) \( \otimes U(1)_6 \ \cdot \) \( \otimes U(1)_Y \)

In this breaking, the generator of \( U(1)_Y \) is a linear combination of \( T_{35} \) and a diagonal generator of \( SU(3)_L \), namely \( T_{8L} \), which is actually one of the diagonal generators of the original \( SU(6) \). It is

\[
Y/2 = C_6 T_{35} + C_{3L} T_{8L} = (2/\sqrt{3}) T_{35} + (1/\sqrt{3}) T_{3L},
\]

(17)

in order to get the correct weak hypercharge quantum numbers for the SM particles as we shall see below. The Higgs representation that can accomplish the above breaking should be a singlet under \( SU(2)_L \) and should have \( Y/2 = 0 \).

The Higgs representation which satisfies the above criterion is the following

\[
21 = (6, 1)(-1/\sqrt{3}) + (1, 6)(1/\sqrt{3}) + (3, 3)(0).
\]

(18)

The component \((1, 6)(1/\sqrt{3})\) is the one that satisfies the desired requirement. Since \( 6 \) is a symmetric second-rank tensor of \( SU(3)_L \), it is straightforward (see e.g. [11]) to obtain the desired breaking \( SU(3)_L \otimes U(1)_6 \ \cdot \) \( \otimes U(1)_Y \) when \( \langle (1, 6)(1/\sqrt{3}) \rangle \neq 0 \). One can also see that the \( U(1)_Y \) gauge boson \( B^\mu \) is found to be (see e.g. [12] and [13])

\[
B^\mu = \cos \theta_L A_{35}^\mu + \sin \theta_L A_{3L}^\mu,
\]

(19)

where

\[
\cos \theta_L = \frac{g_3 L C_6}{\sqrt{g_{3L}^2 C^2_6 + g_6^2 C^2_{3L}}}; \quad \sin \theta_L = \frac{g_6 C_{3L}}{\sqrt{g_{3L}^2 C^2_6 + g_6^2 C^2_{3L}}}.
\]

(20)

The following gauge bosons obtain masses of \( O(M_3) \): an \( SU(2)_L \) doublet (which absorbs the doublet Nambu-Goldstone (NG) bosons of the \( 6 \) scalar) and the combination which is orthogonal to \( B^\mu \) (which absorbs the imaginary part of \( SU(2)_L \) singlet scalar).

- \( SU(2)_L \otimes U(1)_Y \xrightarrow{MW} U(1)_{em} \)

This last step is accomplished in the usual manner, namely by the use of a \( SU(2)_L \) complex Higgs doublet belonging to \((1, 3) \subset (1, 15) \subset 27 \).

\[27: \]

Let us remember that a mass term can be written in terms of two-component Weyl spinors as \( \psi^T_{L, (L,R)} \sigma_2 \psi_{2,(L,R)} \). Also, one can solely use left-handed Weyl spinors by recalling that \( \psi^c_L = \sigma_1 \psi^r_R \). Since our model contains \( 27^c_{L,R} \) with now \( 27^c_{L} = \sigma_2 27^c_{R} \), we obtain the following Lorentz-invariant combinations

\[
27^c_{L} \sigma_2 27_{L} \sim 27 + 351 + 351', \quad (21a)
\]

\[
27^c_{L} \sigma_2 27_{L} \sim 27 + 351 + 351', \quad (21b)
\]

\[
27^c_{L} \sigma_2 27_{L} \sim 1 + 78 + 650, \quad (21c)
\]

where the right-hand sides of Eqs. (21a, 21b, 21c) denote the resulting representations under \( E_6 \).

Since (21c) contains a singlet, one can avoid a gauge-invariant bare mass term by having e.g. a discrete symmetry such that one can assign \( 27^c_{L} \rightarrow 27^c_{L} \) and \( 27^c_{R} \rightarrow -27^c_{R} \). In consequence, (21a) and (21b) are even while (21c) is odd under that symmetry. This prevents \( 27 \) from having a bare mass term. As a result, the possible Higgs representations that appear on the right-hand side of (21a) and (21b) have even “parity” while those on the right-hand side of (21c) should possess odd “parity”.

The next step is to examine which Higgs representation will be appropriate to use to give masses to the fermions which belong to \( 27_{L,R} \). We must first understand which fermion bilinears (relevant for the mass terms) are contained in (21a, 21b, 21c).

In order to do so, we must first recall the particle contents of \( 27_{L,R} \). In particular, we would like to know the \( SU(2)_L \otimes U(1)_Y \) quantum numbers of the SM fermions and their mirror counterparts contained in \( 27_{L,R} \) namely \((1, 15)_{L,R} \subset 27_{L,R} \).

From hereon, we will list the particle contents of \( 15_{L,R} \) under \( SU(2)_L \otimes U(1)_Y \), making use of \( \psi^c_{L,R} = \sigma_2 \psi^r_{L,R} \). We have

\[
15_{L} = (3, 1)_L + (1, 3)_L + (3, 3)_L
\]

\[
= (u^c_L, l^M_{L,c} + e^c_L, q^L + d^M_L),
\]

(22)

where the particle identifications follow from the quantum numbers coming from the pattern of symmetry breaking of the last section and where \( u^c_L \) represents an \( SU(2)_L \) singlet right-handed up-quark, \( l^M_{L,c} \) a mirror right-handed lepton doublet, \( e^c_L \) a \( SU(2)_L \) singlet right-handed charged lepton, \( q^L \) a left-handed quark doublet, and \( d^M_L \) a mirror \( SU(2)_L \) singlet right-handed down-quark. Similarly, one can carry out the same exercise with (10) to get

\[
15^c_{L} = (u^M_L, l^L + e^M_c, q^M_{L,c} + d^c_L).
\]

(23)

\[\text{B. Fermion masses}\]

The main focus in this section is to discuss which fermion might be “heavy” and which might be “light”. The main purpose is to know when to include or exclude a certain fermion from the RG evolution of the gauge couplings.

- \( 27: \)
From (22,23), it is straightforward to see that the following fermion bilinears which are relevant for the mass terms:

- SM fermion bilinears:
  1. Up-quark mass term \( u_{L}^{c,T} \sigma_{2} q_{L} \) from \( 15_{L}^{T} \sigma_{2} 15_{L} \).
  2. Down-quark mass term \( d_{L}^{c,T} \sigma_{2} q_{L} \), charged lepton mass term \( e_{L}^{c,T} \sigma_{2} l_{L} \) from \( 15_{L}^{c,T} \sigma_{2} 15_{L} \) and \( 15_{L}^{T} \sigma_{2} 15_{L} \).

- Mirror fermion bilinears:
  1. Mirror Up-quark mass term \( u_{M}^{c,T} \sigma_{2} q_{M} \) from \( 15_{L}^{c,T} \sigma_{2} 15_{L} \).
  2. Mirror Down-quark mass term \( d_{M}^{c,T} \sigma_{2} q_{M} \), charged lepton mass term \( e_{L}^{c,T} \sigma_{2} l_{L} \) from \( 15_{L}^{c,T} \sigma_{2} 15_{L} \) and \( 15_{L}^{T} \sigma_{2} 15_{L} \).

Notice that we have not touched the issue of neutrino masses in the above discussion. This will be dealt with at the end of this section. We first examine the Higgs scalars which can couple to the above fermion bilinears.

The Higgs representation that can give masses to the (SM and mirror) Up-quarks can be inferred by looking at the product \((3,1)(-1/\sqrt{3}) \times (3,3)(0) \supset (1,3)(-1/\sqrt{3})\), where the last entries refer to the \( U(1)_{6} \) quantum numbers. In consequence, one needs a Higgs scalar which transforms as \((1,3)(1/\sqrt{3})\) \( \subset 15 \subset 27 \). We shall call this field \( \phi(27) \) whose \( SU(2)_{L} \) doublet which belongs to \((1,3)(1/\sqrt{3})\) is the one that develops a non-vanishing V.E.V. resulting in the breakdown of \( SU(2)_{L} \otimes U(1)_{Y} \). We shall assume that the mirror fermions all have masses of \( O(>200 \text{ GeV}) \). Notice that \( \phi(27) \) has even “parity” as discussed above.

For the (SM and mirror) charged leptons and Down-quarks, one looks at the product \((1,3)(1/\sqrt{3}) \times (3,3)(-1/\sqrt{3}) \supset (1,8)(0)\) and \((3,3)(0) \times (3,3)(0) \supset (1,8)(0)\). This comes from \( 15_{L}^{c,T} \sigma_{2} 15_{L} \), and contains terms such as \( e_{L}^{c,T} \sigma_{2} l_{L} \), etc... The obvious choice of the Higgs representation which can couple to these bilinears is \((1,8)(0) \subset (1,35) \subset 78 \). This Higgs field is denoted by \( \phi(78) \). The extra \( SU(2)_{L} \) Higgs doublet is contained in \( 8 = 1 + 2 + 2 + 3 \). Again we shall assume that the mirror fermions all have masses of \( O(>200 \text{ GeV}) \). According to the discussion above, \( \phi(78) \) has odd “parity”.

The above discussions point to the fact that we need at least two Higgs doublets in our model: one for the Up sector and one for the Down sector.

The next question concerns the mass of \((2,6)_{L,R} \in 27_{L,R} \). Notice that \( (2,6)_{L} \times (2,6)_{L} \supset (1,1) \). Looking at (24), one notices that \((1,1) \subset 650 \) and it is the V.E.V. of this component that breaks \( E_{6} \) down to \( SU(2)_{Z} \otimes SU(6) \) at \( M_{GUT} \). Therefore, \((2,6)_{L,R} \) can obtain a mass of \( O(M_{GUT}) \) by coupling to the 650 Higgs field which has odd “parity.”

Finally, we now come to the topic of neutrino masses. First, we notice that the mirror lepton doublet contains a right-handed neutral lepton. Could it be the right-handed neutrino? Normally, the right-handed neutrino is viewed as a singlet of \( SU(2)_{L} \otimes U(1)_{Y} \). For example, in left-right symmetric models \([14]\), the right-handed neutrino is a member of a right-handed doublet of the gauge group \( SU(2)_{R} \) and therefore is naturally a \( SU(2)_{L} \otimes U(1)_{Y} \) singlet. In our case, the right-handed neutral lepton is a member of an \( SU(2)_{L} \) doublet but its partner is however not the SM charged lepton but its mirror counterpart. Since it does not interact at tree level with a SM charged lepton, it could play the role of the right-handed neutrino. If this is the case, it follows that the Dirac neutrino mass term cannot come from a coupling to the SM Higgs doublet(s), the reason being that \((\bar{\nu}_{R}, \bar{e}_{R}^{M}) \times (\nu_{L}, e_{L}) \) is an \( SU(2)_{L} \) singlet.

From (24), we notice that the previous term comes from \( 27_{L}^{c,T} \sigma_{2} 27_{L} \) which contains an \( E_{6} \) singlet. A Dirac neutrino mass term can then arises from \( 27_{L}^{c,T} \sigma_{2} 27_{L} \phi_{S}(1) \), where \( \phi_{S}(1) \) is an \( E_{6} \) singlet and has odd “parity.” The Dirac neutrino mass is then found to be \( m_{\nu} = m_{\nu}, \phi_{S}(1) \). The interesting fact is that Dirac neutrino masses can be small not by fine-tuning the Yukawa coupling in the case where one simply adds a right-handed neutrino to the SM, but by having a small V.E.V. \( \phi_{S}(1) \). The implication of this scenario in terms of the see-saw mechanism will be dealt with elsewhere \([13]\). Notice that the aforementioned coupling also gives a mixing between the mirror quarks and their SM counterparts, and similarly between the charged mirror and SM leptons. However, this mixing is highly suppressed with an angle being proportional to \( m_{\nu}/(m_{q} - m_{d,M}, m_{l} - m_{l,M}) \).

- 78:
  The fermions \( 78_{L,R}^{i} \) decompose under \( SU(2)_{Z} \otimes SU(6) \) as shown in (3). Let us recall that \( \psi_{i}^{(Z)} = (3,1)^{i} \in 78^{i} \) where \( i = 1, 2 \). Let us also recall that the model proposed in (3) contains a global \( U(1)_{A} \) symmetry under which one has the following transformations: \( \psi_{i}^{(Z)} \rightarrow e^{-i\alpha} \psi_{i}^{(Z)} \), \( \psi_{i}^{(Z)} \rightarrow e^{i\alpha} \psi_{i}^{(Z)} \), which also apply to \( 78_{L,R}^{i} \). A Dirac mass term of the form \( 78_{L}^{c,T} \sigma_{2} 78_{L}^{i} \) would carry a phase \( e^{-2i\alpha} \) under that transformation. It follows that the Higgs field which couples to the aforementioned fermion bilinear should have a phase \( e^{i\alpha} \) under
the same transformation. This cannot be the Higgs fields that couple to the 27 as we have discussed above. What could be the minimal Higgs additions to the previous choices be? First notice that

\[ 78_{c,T} \sigma_2 78_{L,\bar{L}} \sim 1 + 78 + 650, \quad (24) \]

and

\[ (2, 20) \times (2, 20) \supset (1, 1) + (1, 35), \quad (25a) \]

\[ (1, 35) \times (1, 35) \supset (1, 1) + (1, 35), \quad (25b) \]

\[ (3, 1) \times (3, 1) = (1, 1) + (3, 1) + (5, 1). \quad (25c) \]

In (25), the (1, 1) can be most conveniently the E_6 singlet 1, and (1, 35) can be part of the 78 on the right-hand side of Eq. (24). We shall denote these scalars by \( \phi(1) \) and \( \phi(78) \), both of which acquire a phase \( e^{\pm i2\pi/3} \) under the \( U(1)_A \) transformations.

In this context, we readily identify \( \phi(1) \equiv \phi_Z \) with \( \phi_Z \) being the singlet scalar field used in the model of dark energy and dark matter of \( 2,3 \). Let us recall that the imaginary part of \( \phi_Z \) plays the role of the accelerator \( \bar{3} \) and the real part plays the role of the inflaton \( 3 \). Its V.E.V. \( \langle \phi_Z \rangle = v_Z \) gives a common mass \( O(m_{\psi(2)}) \sim \phi(2, 20), (1, 35), \) and \( (3, 1) \equiv \psi(z) \) through a coupling \( 78_{L,\bar{L}} \sigma_2 78_{L,\bar{L}} \phi_Z \). Furthermore, a coupling of the form \( 78_{L,\bar{L}} \sigma_2 78_{L,\bar{L}} \phi(78) \) gives a common mass to \( (2, 20) \) and \( (1, 35) \) when \( \langle (1, 35) \rangle \sim O(M_6) \). From \( 3 \), one expects \( m_{\psi(2)} \sim O(200 \text{ GeV}) \) which is much smaller than \( M_6 \). In consequence, the fermions \( (2, 20) \) and \( (1, 35) \) of 78 having a mass of order \( M_6 \), are much heavier than \( (3, 1) \equiv \psi(z) \).

In summary, we have shown above how the \( (2, 6) \in 27, (2, 20) \in 78, \) and \( (1, 35) \in 78 \) can become very heavy while the SM particles and their counterparts as well as \( \psi(z) \) can remain “light”.

C. Proton decay

The spontaneous breakdown of the subgroup \( SU(6) \) will in general induce proton decays with a lifetime being proportional to \( M_6^2/\alpha_6^2 \). But one first has to make sure from a group-theoretical point of view that operators responsible for the proton decay exist in the model.

First, let us recall that the V.E.V. of a 35 Higgs breaks \( SU(6) \) down to \( SU(3)_c \otimes SU(3)_L \otimes U(1)_6 \) giving masses of \( O(M_6) \) to the \( (3, 3)(-1/\sqrt{3}) \) and \( (3, 3)(1/\sqrt{3}) \) gauge bosons (see Eq. (13)). Under \( SU(2)_L \), these gauge bosons transform as \( (3, 3)(-1/\sqrt{3}) = (3, 2)(-1/\sqrt{3}) + (3, 1)(-1/\sqrt{3}) \) and \( (3, 3)(1/\sqrt{3}) = (3, 2)(1/\sqrt{3}) + (3, 1)(1/\sqrt{3}) \). For definiteness, let us denote these gauge bosons by \( U_{\mu} \equiv (3, 3)(-1/\sqrt{3}) \) and \( \bar{U}_{\mu} \equiv (3, 3)(1/\sqrt{3}) \). From Eqs. (9, 22), one can see that the relevant transitions for proton decay are the following: \( q_L \rightarrow e_L^\mu, u_L^\mu \). This means that, in terms of representations, one has \( (3, 3)(0)_L \rightarrow (3, 1)(1/\sqrt{3})_L, (3, 1)(-1/\sqrt{3})_L \).

- \( \psi_L^1 \equiv (3, 3)(0)_L \), \( \psi_L^2 \equiv (3, 1)(1/\sqrt{3})_L \):
  \( \psi_L^2 \gamma_\mu \psi_L^1 \sim (3, 3)(1/\sqrt{3})_L \)
  \( U^\mu \). Here \( g_6 \psi_L^2 \gamma_\mu \psi_L^1 \) contains the coupling of \( q_L \) to \( e_L^\mu \).
- The above interactions describe for instance a proton decay process such as \( p \rightarrow e^+\pi^0 \) by the exchange of the U-boson with mass of \( O(M_6) \). Some numerical estimates are given in the next section.

IV. RENORMALIZATION GROUP ANALYSIS

In this section we study the evolution of the coupling constants, associated to the symmetry breaking scheme introduced in section III. We split our analysis into three parts. The former contains the general equations describing the evolution of the couplings at each step of the symmetry breaking process, along with the generators of the \( U(1) \) groups. A formal expression for \( \sin^2\theta_W \) is also derived. In the second part, we expose our numerical analysis of the problem, where threshold effects are properly taken into account. In the last part we present our results.

A. General analysis

In this section, we focus on the gauge group \( SU(6) \) and its breaking down to the standard model subgroups, according to the scheme

\[ SU(6) \overset{M_6}{\longrightarrow} SU(3)_c \otimes SU(3)_L \otimes U(1)_6 \overset{M_3}{\longrightarrow} SU(3)_c \otimes SU(3)_L \otimes U(1)_Y \overset{M_2}{\longrightarrow} SU(3)_c \otimes U(1)_{em}. \quad (26) \]

- At mass scale \( M_6 \), the couplings satisfy the following condition

\[ g_3^2(M_6) = g_3^2L(M_6) = g_1^2(M_6) = g_3^2(M_6), \quad (27) \]

where, with an obvious notation, \( g_3, g_3L, \) and \( g_1 \) denote the couplings corresponding respectively to \( SU(3)_c, SU(3)_L \) and \( U(1)_Y \).

The abelian \( U(1)_6 \) group is associated to the unbroken diagonal generator, \( T_3 \), of \( SU(6) \), which does not belong to \( SU(3)_c \times SU(3)_L \).
For $M_{3L} \leq E \leq M_6$, the solutions to renormalization group equations read
\begin{align}
1 & \frac{g_3^2(M)}{g_3^2(M_{3L})} = \frac{1}{g_5(M_6)} + 2 b_3 \ln \frac{M}{M_6} \tag{28a} \\
1 & \frac{g_{1,6}^2(M)}{g_{1,6}^2(M_{3L})} = \frac{1}{g_5^2(M_6)} + 2 b_{1,6} \ln \frac{M}{M_6} \tag{28b} \\
1 & \frac{g_3^2(M)}{g_3^2(M_{3L})} = \frac{1}{g_5^2(M_6)} + 2 b_3 \ln \frac{M}{M_6}, \tag{28c}
\end{align}
where the coefficients $b_i$ are related to the beta function and their explicit expression will be given in the next subsection.

- At mass scale $M_{3L}$, the gauge group $SU(3)_L$ breaks down to $SU(2)_L$ and recombines with $U(1)_6$, in order to give the weak hypercharge $U(1)_Y$ group.

The generator of $U(1)_Y$ is given by the linear combination
\begin{equation}
T_Y = C_6 T_{35} + C_{3L} T_{3L}, \tag{29}
\end{equation}
where $T_{35}$ has been introduced before and $T_{3L}$ is the diagonal generator, $T_6$, of $SU(3)_L$. The coefficients appearing in the expression assume the explicit values
\begin{equation}
C_6 = 2/\sqrt{3} \quad \text{and} \quad C_{3L} = 1/\sqrt{3}. \tag{30}
\end{equation}

Therefore, the matching conditions for the couplings at $M_{3L}$ read
\begin{equation}
g_3^2(M_{3L}) = g_{3L}^2(M_{3L}), \tag{31a}
\end{equation}
\begin{equation}
1 \frac{1}{g_Y^2(M_{3L})} = \frac{1}{g_3^2(M_{3L})} + \frac{4}{g_{1,6}^2(M_{3L})}, \tag{31b}
\end{equation}
while their evolution, for $M_Z \leq E \leq M_{3L}$, is governed by
\begin{align}
1 & \frac{1}{g_Y^2(M)} = \left[ \frac{3}{g_{3L}^2(M_{3L})} + \frac{4}{g_{1,6}^2(M_{3L})} \right] + 2 b_Y \ln \frac{M}{M_{3L}} \tag{32a} \\
1 & \frac{1}{g_Y^2(M)} = \frac{1}{g_3^2(M_{3L})} + 2 b_3 \ln \frac{M}{M_{3L}} \tag{32b} \\
1 & \frac{1}{g_Y^2(M)} = \frac{1}{g_5^2(M_6)} + 2 b_3 \ln \frac{M}{M_6}. \tag{32c}
\end{align}

- Finally, at the electro-weak scale $M_Z$ the standard model groups break down to $SU(3)_c$ and $U(1)_{em}$, with the well known relations
\begin{equation}
Q = T_Y + T_{2L}, \tag{33}
\end{equation}
where $T_{2L}$ being the diagonal generator $T_3$ of $SU(2)_L$ for the electromagnetic charge generator and
\begin{equation}
1 \left[ e^2(M_Z) \right] = \frac{1}{g_2^2(M)} + \frac{1}{g_Y^2(M)}, \tag{34}
\end{equation}
Combining Eqs. (27, 31, 34, 28, 32) and using the standard MS definition for $\sin^2 \theta_W$, i.e.
\begin{equation}
\sin^2 \theta_W(M_Z) = \frac{e^2(M_Z)}{g_Y^2(M_Z)}, \tag{35}
\end{equation}
we obtain the following formula
\begin{equation}
\sin^2 \theta_W(M_Z) = \frac{3}{8} \left\{ 1 - 8 \pi \alpha(M_Z) \left[ K \ln \frac{M_Z}{M_{3L}} + K' \ln \frac{M_{3L}}{M_6} \right] \right\}, \tag{36}
\end{equation}
where
\begin{align}
\alpha(M_Z) &= \frac{e^2(M_Z)}{4\pi}, \tag{37a} \\
K &= b_Y - \frac{5}{3} b_2, \tag{37b} \\
K' &= \frac{4}{3} (b_{1,6} - b_{3L}). \tag{37c}
\end{align}

The overall factor $3/8$ represents the value of $\sin^2 \theta_W$ at the mass scale $M_{3L}$, where the weak hypercharge $U(1)_Y$, first appears in the symmetry breaking process. The fact that we obtain the same value $3/8$ as in the SU(5) GUT theory (though at a different mass threshold) depends on the particle contents of the two models: indeed, the mirror fermions have identical quantum numbers to the SM fermions. (One example of how unconventionally charged particles affect $\sin^2 \theta_W$ can be found in [12]).

**B. Numerical analysis**

The behavior of the couplings is analyzed, as the energy increases, starting from $\Lambda_Z \sim 3 \times 10^{-3} \text{eV}$, till almost the Planck scale $M_{\text{Planck}} \sim 1.2 \times 10^{19} \text{GeV}$. In the following, we first define the equations used in our analysis, then we summarize the particle content and show how it affects the evolution, leaving the results and their discussion to the next section.

The evolution of all the couplings (except from $g_Z$, which we are going to discuss separately) is well described by the RG equation at one-loop $[16]$
\begin{equation}
\frac{d\alpha_i}{dt} = -\frac{b_i}{2\pi} \alpha_i^2 \tag{38}
\end{equation}
where $t \equiv \ln \mu$ and $\alpha = g^2/4\pi$. For a general product of gauge groups $G_1 \otimes G_2 \otimes \ldots$, the coefficients $b_i$ read
\begin{equation}
b_i = \frac{11}{3} C_2(G_i) - \frac{2}{3} T(F_i) \prod_{j \neq i} d(F_j) - \frac{1}{3} T(S_i) \prod_{j \neq i} d(S_j). \tag{39}
\end{equation}

with $C_2$ the quadratic Casimir operator of the group $G_i$, acting on the adjoint representation, and $T$ and $d$ being, respectively, the Dynkin index and the dimension of
the chiral fermion \((F)\) and complex scalar \((S)\) representations. Since at very low energies and till roughly the electro-weak scale, only \(\alpha_Z\) is running, our initial inputs are those associated to the Standard Model couplings at mass scale \(M_Z\), given by the experimental data

\[
\alpha_Y(M_Z^2) = \frac{\alpha(M_Z^2)}{\cos^2 \theta_W(M_Z^2)},
\]

\[
\alpha_2(M_Z^2) = \frac{\alpha(M_Z^2)}{\sin^2 \theta_W(M_Z^2)},
\]

\[
\alpha_3(M_Z^2),
\]

with the MS values \([17]\)

\[
1/\alpha(M_Z^2) = 127.906(19), \quad \alpha_3(M_Z^2) = 0.1213(18)
\sin^2 \theta_W(M_Z^2)_{\text{exp}} = 0.23120(15).
\]

We now consider \(\alpha_Z = g_Z^2/4\pi\), where \(g_Z\) is the \(SU(2)_Z\) gauge coupling. Its evolution has been already studied in \([3]\), for a range of energies, roughly covering \(\Lambda_Z \leq E \leq M_6\). In this interval, which extends to very low values, the two-loop approximation turns out to be more accurate. Therefore,

\[
\frac{d\alpha_Z}{dt} = \frac{b_0}{2\pi} \alpha_Z^2 - \frac{b_1}{8\pi^2} \alpha_Z^3,
\]

where general expressions for the coefficients \(b_0\) and \(b_1\) can be found in \([16]\) and the initial value is assumed to be \(\alpha_Z(\Lambda_Z) \sim 1\). As concerns higher energies, above \(M_6\), the one-loop approximation is sufficiently reliable and we will use Eq. \([38, 39]\), taking, as an input, the value \(\alpha_Z(M_6)\) resulting from Eq. \((41)\).

The next step consists in calculating explicitly the coefficients \(b_i\). In order to accomplish this, we need to know the transformation properties, under the gauge groups, of all the particles involved at each step of the symmetry breaking process. First of all, we list all the \(E_6\) representations which enter our analysis:

- fermions: three \(27_{L,R}\) and two \(78_{L,R}\);
- scalars:
  - Higgs fields: one \(650\), two \(78\), one \(351\) and one \(27\);
  - messenger fields: two \(351\).

Next, we identify four regions, which are characterized by different symmetry groups and whose boundaries are determined by Eq. \((26)\):

1. \(\Lambda_Z \leq E \leq M_Z\),
2. \(M_Z \leq E \leq M_{3L}\),
3. \(M_{3L} \leq E \leq M_6\),
4. \(M_6 \leq E \leq M_{\text{GUT}}\).

Within each of them, we will analyze in detail the threshold effects, due to the presence of particles with different masses. (The behavior of \(\alpha_Z\) in the interval of energies between \(\Lambda_Z\) and \(M_6\) has been discussed at length in \([3]\), therefore we will refer to it for the details.)

1. For \(\Lambda_Z \leq E \leq M_Z\), the coefficients \(b_0^Z\) and \(b_1^Z\), appearing in Eq. \((41)\), read explicitly

\[
b_0^Z = \frac{22}{3} - \frac{8}{3} n_\psi, \quad b_1^Z = \frac{4}{3} (34 - 32 n_\psi),
\]

where \(n_\psi\) denotes the number of fermions \(\psi^{(Z)}\) (no messenger fields are present at this stage). Choosing the mass of the fermions to be \(m_1 = 50\) GeV and \(m_2 = 100\) GeV, we can identify two sub-regions:

(a) \(\Lambda_Z \leq E \leq m_1\), with \(n_\psi = 0\),
(b) \(m_1 \leq E \leq M_Z\), with \(n_\psi = 1\).

2. Between \(M_Z\) and \(M_{3L}\), the symmetry is

\[
SU(2)_Z \otimes SU(3)_c \otimes SU(2)_L \otimes U(1)_Y
\]

and the particle content, along with the pattern of thresholds, becomes richer. At this stage we have:

- the SM and the mirror fermions (the latter with a mass scale assumed to be \(M_{M} \sim 250\) GeV and the number of families \(n_{MF} = 3\)),
- \(n_\psi = 1, 2\) fermionic fields \(\psi^{(Z)}_{R,L} \sim (3, 1, 1, 0)_{R,L} \subset 78\),
- one messenger field \(\varphi_1^{(Z)} \sim (3, 1, 2, -1/2) \subset (3, 1, 3, -1/\sqrt{3}) \subset (3, 15) \subset 351\), with a mass \(M_{\varphi_1} \sim M_{M} \sim 250\) GeV,
- two electro-weak Higgs doublets
  - \(\varphi_1 \sim (1, 1, 2, 1/2) \subset (1, 1, 3, 1/\sqrt{3}) \subset (1, 15) \subset \phi(27)\),
  - \(\varphi_2 \sim (1, 1, 2, 1/2) \subset (1, 1, 8, 0) \subset (1, 35) \subset \phi(78)\),

which we assume to enter the renormalization group equations at the scales \(\Lambda_{\varphi_{1,2}} \sim 150 \div 500\) GeV.

The coefficient \(b_i\) can be put in the general form

\[
b_0^Z = \frac{22}{3} - \frac{8}{3} n_\psi - \frac{4}{3} n_{\varphi(z)}, \quad b_1^Z = \frac{4}{3} (34 - 32 n_\psi - 28 n_{\varphi(z)}),
\]

\[
(44a) \quad (44b)
\]
| Energy          | $n_F$ | $n_{MF}$ | $n_{\psi}$ | $n_{Z(x)}$ | $n_{\varphi}$ |
|-----------------|------|---------|------------|------------|--------------|
| $M_Z \leq E \leq m_2$ | 2    | 0       | 1          | 0          | 0            |
| $m_2 \leq E \leq m_t \sim 175$ GeV | 2    | 0       | 2          | 0          | 0            |
| $m_t \leq E \leq M_M$ | 3    | 0       | 2          | 0          | 0            |
| $M_M \leq E \leq \Lambda_{\varphi,1,2}$ | 3   | 3       | 2          | 1          | 0            |
| $\Lambda_{\varphi,1,2} \leq E \leq M_{3L}$ | 3   | 3       | 2          | 1          | 2            |

TABLE I: Regions of energy between the mass scales $M_Z$ and $M_{3L}$. The parameters $n_i$ specify the number of particles of different type, present in each interval.

and

$$b_3 = 11 - \frac{4}{3} n_F - \frac{4}{3} n_{MF},$$  \hspace{1cm} (45a)

$$b_2 = \frac{22}{3} - \frac{4}{3} n_F - \frac{4}{3} n_{MF} - \frac{1}{2} n_{Z(x)} - \frac{1}{6} n_{\varphi},$$  \hspace{1cm} (45b)

$$b_Y = - \frac{20}{9} n_F - \frac{20}{9} n_{MF} - \frac{1}{2} n_{Z(x)} - \frac{1}{6} n_{\varphi}.$$  \hspace{1cm} (45c)

Again, we can identify several sub-regions. Choosing for example $\Lambda_{\varphi,1,2} > M_M$, we can summarize them in Table I.

3. Between $M_{3L}$ and $M_0$ the symmetry becomes

$$SU(2)_Z \otimes SU(3)_c \otimes SU(3)_L \otimes U(1)_Y.$$  \hspace{1cm} (46)

The fields which play a role in the evolution of the couplings at this stage are

- the SM, the mirror fermions and the $\psi^{(Z)}_{R,L}$ fermions introduced before,
- the messenger $\varphi^{(Z)}_{1} \sim (3,1,3,-1/\sqrt{3}) \subset (3,\overline{T}5) \subset \overline{35T},$
- two electro-weak Higgs scalars, transforming, respectively, as
  - $\varphi_1 \sim (1,1,3,1/\sqrt{3}) \subset (1,15) \subset \phi(27),$
  - $\varphi_2 \sim (1,1,8,0) \subset (1,35) \subset \phi(78),$
- the Higgs field, breaking $SU(3)_L \otimes U(1)_Y \rightarrow SU(2)_L \otimes U(1)_Y$, namely $\phi_{3L} \sim (1,1,6,1/\sqrt{3}) \subset (1,21) \subset \overline{351}.$

All these particles affect the coefficients $b_i$ in the following way:

$$b_{Z}^0 = 0, \quad b_{Z}^1 = -96, $$  \hspace{1cm} (47)

and

$$b_3 = 3, \quad b_{3L} = \frac{1}{2}, \quad b_{1,6} = -10.$$  \hspace{1cm} (48)

$$G_1 \quad G_2 \quad b_2 \quad b_3 \quad b_{3L} \quad b_{1,6}$$

fermions $\subset 78$

$$\begin{array}{ccccccc}
(1,1,8,0) & 0 & 0 & -2 & 0 \\
(1,8,1,0) & 0 & -2 & 0 & 0 \\
(1,35) & 0 & 0 & 0 & 0 \\
(1,3,3,1/\sqrt{3}) & 0 & -1 & -1 & -2 \\
(1,3,3,1/\sqrt{3}) & 0 & -1 & -1 & -2 \\
\end{array}$$

scalars $\subset 351$

$$\begin{array}{ccccccc}
(3,3,1,-1/\sqrt{3}) & -2 & -\frac{1}{2} & 0 & -1 \\
(3,1,3,-1/\sqrt{3}) & -2 & 0 & -\frac{1}{2} & -1 \\
(3,3,3,0) & -6 & -\frac{3}{2} & -\frac{3}{2} & 0 \\
(1,6,1,-1/\sqrt{3}) & 0 & -\frac{5}{2} & 0 & -\frac{2}{5} \\
(1,1,6,-1/\sqrt{3}) & 0 & 0 & -\frac{5}{2} & -\frac{2}{5} \\
(1,3,3,0) & 0 & -\frac{1}{2} & -\frac{1}{2} & 0 \\
(1,3,3,0) & 0 & -\frac{1}{2} & -\frac{1}{2} & 0 \\
(1,3,3,0) & 0 & -\frac{1}{2} & -\frac{1}{2} & 0 \\
(1,6,3,0) & 0 & -\frac{1}{2} & 0 & -\frac{2}{5} \\
(1,3,6,0) & 0 & -1 & -\frac{2}{5} & 0 \\
\end{array}$$

TABLE II: Values assumed by the beta function coefficients $b_i$, in the interval $M_{3L} \leq E \leq M_0$, for some selected representations (fermions and scalars are weighted in a different way, consistently with Eq. (39)). $G_{1,2}$ denote respectively $G_1 = SU(2)_Z \otimes SU(6)$ and $G_2 = SU(2)_Z \otimes SU(3)_c \otimes SU(3)_L \otimes U(1)_Y.$

Besides them, however, several other fields, which are part of the $E_6$ representations listed at the beginning of the subsection, may have a mass belonging to this region and may therefore affect the renormalization group equations. For definiteness, we will consider

- some fermionic multiplets, originally part of the $E_6$ representations

$$\begin{array}{c}
78_{R,L} \supset (1,35)_{R,L} + \ldots \\
\end{array}$$  \hspace{1cm} (49)

- some scalar multiplets, coming from the representations

$$\begin{array}{c}
351 \supset (3,1) + (1,21) + (1,105) + \ldots \\
\end{array}$$  \hspace{1cm} (50a)

$$\begin{array}{c}
351 \supset (3,1) + (1,21) + (1,105) + \ldots \\
\end{array}$$  \hspace{1cm} (50b)

and collect them in Table II along with their contribution to the coefficients $b_i$. (In the table, we show only the representation $351$, $351$ being just its conjugate.)

In our analysis, we have selected
• among the fermions, \( n_\psi = 2 \) multiplets of \((1, 1, 8, 0)_{R,L} \) and \((1, 8, 1, 0)_{R,L} \).

• among the scalars, one representation \((1, 1, 6, 1/\sqrt{3})\) (coming from a messenger \(351\)) and two \((1, 3, 3, 0)\) (either coming from a \(351\)-dimensional representation of Higgs or messengers).

This choice does not affect the coefficients \( b_{Z}^{0,1} \), but modify the others according to

\[
\begin{align*}
    b_3 &= -6, \\
    b_{3L} &= -\frac{28}{3}, \\
    b_{1,6} &= -\frac{32}{3}.
\end{align*}
\]  

(51)

4. Between \( M_6 \) and \( M_{\text{GUT}} \), the symmetry groups are

\[
SU(2)_Z \otimes SU(6).
\]

(52)

The particles involved at this stage are

• three families of SM and mirror fermions, transforming jointly as the representations \((1, 15)_{L,R} \subset 27_{L,R} \); 

• \( n_\psi = 2 \) fermions \( \psi_{L,R}^{(Z)} \sim (3, 1)_{L,R} \subset 78_{L,R} \); 

• the Higgs scalar breaking \( SU(6) \) down to its subgroups, namely \( \phi_6 \sim (1, 35) \subset 78 \); 

• the two messenger fields \( \varphi_{1,2}^{(Z)} \sim (3, 15) \subset 351 \).

With this particle content, the coefficients \( b_i \) read 

\[
\begin{align*}
    b_Z &= -18 \quad \text{and} \quad b_6 = 8.
\end{align*}
\]  

(53)

As discussed before, some other particles may acquire mass in this region, either scalars, whose mass can take any value in the interval of energies, or fermions, whose mass is constrained by the strength of their Yukawa interaction with the Higgs field \( \phi_6 \) and can be larger than \( M_6 \) within a factor ten. Again, we collect some of them and their effect on the coefficients \( b_i \) in Table III.

In our analysis, we study the interplay of 

• an additional representation \((3, 15) \subset 351\), entering at \( M_6 \) and coming from the same \( 351 \) containing the Higgs field \( \phi_{3L} \) and 

• different fermionic fields, belonging to the representations \( 78_{L,R} \) and \( 27_{L,R} \), entering at a higher mass scale \( M_{\text{Yuk}} > M_6 \).

The resulting coefficients \( b_i \) assume the general form 

\[
\begin{align*}
    b_Z &= -18 - 10 n_{(3, 15)} - \frac{40}{3} n_{(2, 20)} - 6 n_{(2, \bar{6})}, \\
    b_6 &= 8 - 2 n_{(3, 15)} - 8 n_{(1, 35)} - 8 n_{(2, 20)} - 2 n_{(2, \bar{6})}.
\end{align*}
\]  

(54a, b)

| \( E_6 \) | \( G_1 \) | \( b_Z \) | \( b_6 \) |
|---|---|---|---|
| fermions | | | |
| \((1, 35)\) | 0 | -4 |
| \(78\) | \(-\frac{20}{3}\) | -4 |
| \(27\) | -3 | -1 |
| scalars | | | |
| \((3, 15)\) | -10 | -2 |
| \((1, 21)\) | 0 | -\frac{4}{3} |
| \(351\) | \(-2\) | \(-\frac{1}{3}\) |
| \((1, 105)\) | 0 | \(-\frac{3}{3}\) |
| \((2, \bar{35})\) | -14 | \(-\frac{38}{3}\) |

TABLE III: Values assumed by the coefficients \( b_i \) in the interval of energies \( M_6 \leq E \leq M_{\text{GUT}} \), for some selected representations. Fermions and scalars are weighted according to Eq. (50). \( G_1 \) stands for \( SU(2)_Z \otimes SU(6) \).

C. Results

The next step consists in evolving the coupling constants through the different regions we have identified, taking care of the threshold effects. Our goal is to derive the values of 

• \( M_6 \) and \( \alpha_6^{-1} \), where the SM groups merge into \( SU(6) \) and 

• \( M_{\text{GUT}} \) and \( \alpha_{\text{GUT}}^{-1} \), where \( SU(2)_Z \) and \( SU(6) \) unify into \( E_6 \).

Some parameters of our theory are free (or can vary in some definite range), and can lead to slightly different scenarios. However, we are not interested in studying all the possible cases in depth, but just focus on a couple of them and show their main features. Therefore, let us start by choosing the mass scales of the two electro-weak Higgs doublets (discussed at point 2. of the previous section) to assume the values \( \Lambda_{\varphi_{1,2}} = 300 \text{ GeV} \) and \( \Lambda_{\varphi_{1,2}} = 150 \text{ GeV} \).

1. \( \Lambda_{\varphi_{1,2}} = 300 \text{ GeV} \).

• With this choice and selecting the mass scale \( M_{3L} \sim 10^{13} \text{ GeV} \), the unification scale \( M_6 \) turns out to range approximately between the values \( 2.8 \times 10^{15} \text{ GeV} \) and \( 7.2 \times 10^{15} \text{ GeV} \). The uncertainty takes into account threshold’s effects and corresponds to a variation of the couplings \( \Delta \alpha/\alpha \equiv (\alpha_{\text{larger}} - \alpha_{\text{smaller}})/\alpha_{\text{smaller}} \approx 4\% \div 4.5\% \).

For example, at \( M_6 = 2.8 \times 10^{15} \text{ GeV}, \) we find \( \alpha_3^{-1} = 15.315 \) and \( \alpha_{3L}^{-1} = 16.004 \alpha_{16}^{-1} = \)
15.441, and the error becomes $\Delta \alpha / \alpha \approx 4.5\%$. Analogously, at $M_6 = 7.2 \times 10^{15}$ GeV we obtain $\Delta \alpha / \alpha \approx 4\%$. In Fig. 11 (see the inset), we show the crossing of the curves occurring at $M_6 = 4.75 \times 10^{15}$ GeV, where $\alpha_3^{-1} = 15.146$, $\alpha_3^{1L} = 15.097$ and $\alpha_5^{1} = 15.137$. Therefore, we take the $SU(6)$ coupling to be

$$\alpha_6^{-1} = \alpha(M_6)^{-1} = 15.137$$

at $M_6 = 4.75 \times 10^{15}$ GeV

and the corresponding error $\Delta \alpha_6 / \alpha_6 \sim 0.3\%$. It is straightforward to estimate the proton partial mean lifetime associated to these values. As represented by $\tau_{p \to e+\pi^0}$, it is predicted to be $19$

$$\tau_{p \to e+\pi^0} \approx 3.28 \times 10^{34} \left( \frac{M_6}{3.48 \times 10^{15}} \right)^4 \left( \frac{\alpha_6^{-1}}{36.63} \right)^2 . \quad (55)$$

Therefore, we obtain $\tau_{p \to e+\pi^0} \approx 2 \times 10^{34} \text{yr}$, a value which is larger than the actual lower limit of $1.47 \times 10^{32} \text{yr} \quad (17)$. Corrections to the central value we have just found come from the lower and upper bounds on the $M_6$ mass range and read, respectively, $\tau_{p \to e+\pi^0} \approx 3 \times 10^{33} \text{yr}$ and $\tau_{p \to e+\pi^0} \approx 9 \times 10^{34} \text{yr}$.

- Next, we study the interplay of different particles above the energy threshold $M_6$ and see how they affect the grand-unification scale $M_{GUT}$. For definiteness, we assume the mass threshold, at which the heavy fermions acquire mass, of the order $O(3 \times 10^{16}$ GeV). The results are collected in Table V and some of them are displayed in Fig. 112.

2. $\Lambda_{\varphi_1,\varphi_2} = 150$ GeV.

- Choosing this value for the electro-weak Higgs doublets’ mass, the mass scale $M_{3L}$ shifts to the value $2.5 \times 10^{13}$ GeV and we obtain

$$M_6 = 3.3 \times 10^{15} \text{GeV} \quad \text{and} \quad \alpha_6^{-1} = 16.787,$$

with an uncertainty due to threshold effects given by $\Delta \alpha_6 / \alpha_6 \sim 1\%$. More generally, $M_6$ can vary in the interval between $1.4 \times 10^{15}$ GeV and $5.9 \times 10^{15}$ GeV, corresponding to $\Delta \alpha_6 / \alpha_6 \sim 5 \div 6\%$. Using formula (55), the proton partial mean lifetime reads, in this case, $\tau_{p \to e+\pi^0} \approx 6 \times 10^{35} \text{yr}$. Corrections to the central value, corresponding respectively to the lower and upper bounds on $M_6$, are given by $\tau_{p \to e+\pi^0} \approx 2 \times 10^{32} \text{yr}$ and $\tau_{p \to e+\pi^0} \approx 5 \times 10^{34} \text{yr}$.

- A similar analysis, performed in the range of energies between $M_6$ and $M_{GUT}$, with a slight change in the mass scale at which the heavy fermionic components enter, i.e. $O(2 \times 10^{16}$ GeV), leads to the results summarized in Table V and showed in Fig. 8416.

V. OBELIQUE CORRECTIONS

In this section we analyze the oblique corrections due to the introduction of new extra particles in the spectrum, below the TeV scale. They are expressed in terms of three parameters, $S$, $T$ and $U \quad (20)$, whose values can be extracted from the electro-weak precision measurement data \cite{17}. The new (with respect to the SM reference point) contributions to $S$, $T$ and $U$ come from:

- the three extra families of chiral mirror fermions,

- the two Higgs doublets (see Section III B),

- the messenger field $\varphi(Z) \sim (3,1,2,Y_\varphi = -1)$,

and are additive.

We notice that the interplay between extra families of chiral fermions, whose masses range from $O(50 \text{ GeV})$ to the electro-weak scale, and the Higgs sector, spanning a certain interval of masses, has already been discussed by He et al \cite{21}. In particular, they have shown that three mirror families would be inconsistent with the electro-weak precision data, unless two Higgs doublets were present. This result applies straightforwardly to our situation, which displays a choice of masses (both for the fermions and the for the Higgs) compatible with the analysis performed in \cite{21}.

What remains to be proven is that also the oblique corrections to $S$ and $T$, due to the messenger field $\varphi(Z)$, can be very small. Assuming the three $SU(2)_L$ doublets to be quasi-degenerate, the corresponding contribution to $T$ is indeed negligible. The correction to $S$ can be evaluated by noting that the custodial isospin symmetry is not violated \cite{22}. Assigning a doublet of messenger fields to the representation $(j_L,j_R) = (1/2,1/2)$ of the global symmetry group $SU(2)_L \otimes SU(2)_R \otimes U(1)_Y$, the correction to $S$ can be visualized in Fig. 6. The value of $S$ ranges in the interval $-0.13 \lesssim S \lesssim 0$, where $S = 0$ corresponds to no splitting between the masses of the custodial representations $J = 0$ and $J = 1$, i.e. $m_0^2/m_1^2 = 1$, and $S = -0.13$ refers to the limit $m_0^2/m_1^2 \to 0$.

In summary, the contributions to the $S$, $T$ and $U$ parameters from new particles contained in our model are compatible with the data.

VI. CONCLUSION

We have presented, in this paper, a grand unified model based on the group $E_6 \supset SU(2)_L \otimes SU(6)$. The main rationale for this unification is the presence of an
the evolution of $\psi$ acceleron in that model. Around $\Lambda_{\psi}$ particularly relevant for these particles to be CDM candidates.

The unbroken gauge group $SU(2)_X$ which forms the cornerstone of the dark energy model of $\mathcal{Z}$, $\mathcal{X}$. In that model, the $SU(2)_Z$ gauge coupling was assumed to have an initial value at high energies ($\sim 10^{16}$ GeV) of the same order as the SM couplings at comparable energies and grows strong i.e. $\alpha_Z \equiv g_Z^2/4\pi \approx O(1)$ at a scale $\Lambda_Z \sim 3 \times 10^{-3}$ eV. In that analysis, the masses of the fermions $\psi_i^{(Z)}$ ($i = 1, 2$) were found to be important in the evolution of $\alpha_Z$ with a value $\sim 200$ GeV found to be particularly relevant for these particles to be CDM candidates. Around $\Lambda_Z$, $SU(2)_Z$ instantons induce a potential for an axion-like particle $a_Z$ which plays the role of the acceleron in that model.

The main purpose of the present paper is to show that, with the desired values of the parameters presented in $\mathcal{Z}$, one can indeed find a scenario in which $SU(2)_Z$ is unified with the SM in a way that was described above. In a nutshell, we found that $SU(2)_Z$ can be embedded in $E_6$ via the route $E_6 \to SU(2)_Z \otimes SU(6)$. As we have discussed in the previous sections, one of the symmetry breaking route that we choose to present here for $SU(6)$ is $SU(6) \to SU(3)_c \otimes SU(3)_L \otimes U(1)_Y \to SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$. Some of the highlights of this unification scenario are the following:

1) The presence of heavy right-handed mirror quarks and leptons which could be produced and searched for at future colliders such as the Large Hadron Collider (LHC): the lightest among them can decay into a SM fermion and $W$ through a mixing between the $SU(3)_L/SU(2)_L$ gauge

| fermions | $n_{(3,15)} = 0$ | $n_{(3,15)} = 1$ |
|----------|-----------------|-----------------|
| $(1,35) \subset 78$ | no crossing | $M_{GUT} = 7.4 \times 10^{18}$ GeV; $\alpha^{-1}_{GUT} = 8.06$ (Fig.1) |
| $(1,35) + (2,20) \subset 78$ | no crossing | $M_{GUT} = 9.6 \times 10^{17}$ GeV; $\alpha^{-1}_{GUT} = 2.51$ (Fig.2) |
| $(1,35) + (2,20) \subset 78$ | $M_{GUT} = 1.5 \times 10^{18}$ GeV; $\alpha^{-1}_{GUT} = 0.30$ |
| $(2,6) \subset 27$ | $M_{GUT} = 4.5 \times 10^{17}$ GeV; $\alpha^{-1}_{GUT} = 3.92$ |

**Table IV:** Values assumed by $M_{GUT}$ and $\alpha^{-1}_{GUT}$, for $\Lambda_{\psi_{1,2}} = 300$ GeV and in the presence of different fermionic fields. Two different scenarios are showed, corresponding to two ($n_{(3,15)} = 0$) and three ($n_{(3,15)} = 1$) scalar representations transforming as $(3,15)$ under the symmetry groups $SU(2)_Z \otimes SU(6)$. For completeness, we quote all the values obtained from the numerical analysis, but we restrict only to $\alpha^{-1}_{GUT} \gtrsim 1$ for our physical discussion.

| fermions | $n_{(3,15)} = 0$ | $n_{(3,15)} = 1$ |
|----------|-----------------|-----------------|
| $(1,35) \subset 78$ | no crossing | $M_{GUT} = 3.0 \times 10^{18}$ GeV; $\alpha^{-1}_{GUT} = 10.52$ (Fig.3) |
| $(1,35) + (2,20) \subset 78$ | $M_{GUT} = 2.9 \times 10^{18}$ GeV; $\alpha^{-1}_{GUT} = 0.36$ | $M_{GUT} = 4.6 \times 10^{17}$ GeV; $\alpha^{-1}_{GUT} = 5.48$ (Fig.4) |
| $(1,35) + (2,20) \subset 78$ | $M_{GUT} = 7.6 \times 10^{17}$ GeV; $\alpha^{-1}_{GUT} = 2.82$ (Fig.5) | $M_{GUT} = 2.3 \times 10^{17}$ GeV; $\alpha^{-1}_{GUT} = 6.75$ |

**Table V:** Values assumed by $M_{GUT}$ and $\alpha^{-1}_{GUT}$, for $\Lambda_{\psi_{1,2}} = 150$ GeV and in the presence of different fermionic fields. Two different scenarios are showed, corresponding to two ($n_{(3,15)} = 0$) and three ($n_{(3,15)} = 1$) scalar representations transforming as $(3,15)$ under the symmetry groups $SU(2)_Z \otimes SU(6)$.
bosons with $W$'s, or it can decay into a SM fermion and $\phi_S(1)$ if the latter is light enough. Finally, as discussed in \[ \phi_Z \), the so-called “progenitor of SM lepton numbers”, the messenger scalar field $\phi_Z$, can be produced at the LHC e.g. via electroweak gauge boson fusion processes (since it does not carry color), and subsequently decays into a SM lepton and $\psi_i(Z)$ with interesting signatures. The full discussion is beyond the scope of this paper.

2) The SM gauge couplings eventually merge (through various steps) into the $SU(6)$ coupling at $\sim 5 \times 10^{15} \text{GeV}$. The value of $SU(6)$-GUT coupling is generally higher than a typical GUT value (even for supersymmetric GUT): $\alpha_6^{-1}(M_6) \sim 20$ as opposed to a non-supersymmetric GUT value $\sim 40$. Since one expects the lifetime of the proton to be proportional to $M_6^4/\alpha_6^2$, it is shown in Section \[ \text{IV} \] that the estimated proton lifetime is well above the current lower bound;

3) The $E_6$ GUT scale, $M_{GUT}$ is shown in Section \[ \text{IV} \] to be at least two orders of magnitude higher than $M_6$.

We found that typically $\alpha_E^{-1} \lesssim 10$. It is interesting to note that the $E_6$ coupling at the unification scale $M_{GUT}$ is not too far from the strong coupling regime $\alpha_{QCD} \sim 1$;

4) As shown in Section \[ \text{III} \], the masses of the “up” and “down” fermion sectors automatically come from couplings to two different Higgs sectors.

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FIG. 1: $\alpha_i(E)^{-1}$ versus $t = \ln(E/\Lambda_Z)$ for $\Lambda_{\phi 1,2} = 300$ GeV and $0 < E < M_{Planck}$ and for a restricted range of energies. Heavy particles entering above $M_6$ include the fermions $(1,35) \subset 78$ and the scalar components $(3,15) \subset 351$. The different indices denote: $a = SU(6)$, $b = SU(2)_Z$, $c = U(1)_a$, $d = SU(3)_c$, $e = SU(3)_L$ for $M_{3L} < E < M_6$ and $SU(2)_L$ for $E < M_{3L}$, and $f = U(1)_Y$. The black bubble stands for the matching condition (31) at the threshold $M_{3L}$. The mass scales read explicitly $M_{3L} = 10^{13}$ GeV, $M_6 = 4.75 \times 10^{18}$ GeV and $M_{GUT} = 7.4 \times 10^{18}$ GeV.
FIG. 2: $\alpha_i(E)^{-1}$ versus $t = \ln(E/\Lambda_Z)$ for $\Lambda_{\phi_{1,2}} = 300 \text{ GeV}$ and $0 < E < M_{\text{Planck}}$ and for a restricted range of energies. Heavy particles entering above $M_6$ include the fermions $(1, 35) + (2, 20) \subset 78$ and the scalar components $(3, 15) \subset 351$. The different indices denote: $a = SU(6)$, $b = SU(2)_Z$, $c = U(1)_6$, $d = SU(3)_c$, $e = SU(3)_L$ for $M_{3L} < E < M_6$ and $SU(2)_L$ for $E < M_{3L}$, and $f = U(1)_Y$. The black bubble stands for the matching condition (31) at the threshold $M_{3L}$. The mass scales read explicitly $M_{3L} = 10^{13} \text{ GeV}$, $M_6 = 4.75 \times 10^{15} \text{ GeV}$ and $M_{GUT} = 9.6 \times 10^{17} \text{ GeV}$.
FIG. 3: $\alpha_i(E)^{-1}$ versus $t = \ln(E/\Lambda_Z)$ for $\Lambda_{\varphi_1,2} = 150\,\text{GeV}$ and $0 < E < M_{\text{Planck}}$ and for a restricted range of energies. Heavy particles entering above $M_6$ include the fermions $(1,35) + (2,20) \subset 78$ and $(2, \bar{6}) \subset 27$. The different indices denote: $a = SU(6)$, $b = SU(2)_Z$, $c = U(1)_a$, $d = SU(3)_c$, $e = SU(3)_L$ for $M_{3L} < E < M_6$ and $SU(2)_L$ for $E < M_{3L}$, and $f = U(1)_Y$. The black bubble stands for the matching condition (31) at the threshold $M_{3L}$. The mass scales read explicitly $M_{3L} = 2.5 \times 10^{13}\,\text{GeV}$, $M_6 = 3.3 \times 10^{15}\,\text{GeV}$ and $M_{\text{GUT}} = 7.6 \times 10^{17}\,\text{GeV}$. 
FIG. 4: $\alpha_i(E)^{-1}$ versus $t = \ln(E/\Lambda_Z)$ for $\Lambda_{\phi_1,2} = 150\,\text{GeV}$ and $0 < E < M_{\text{Planck}}$ and for a restricted range of energies. Heavy particles entering above $M_6$ include the fermions $(1,35) \subset 78$ and the scalar components $(3,15) \subset 351$. The different indices denote: $a = SU(6)$, $b = SU(2)_Z$, $c = U(1)_c$, $d = SU(3)_c$, $e = SU(3)_L$ for $M_{3L} < E < M_6$ and $SU(2)_L$ for $E < M_{3L}$, and $f = U(1)_Y$. The black bubble stands for the matching condition (31) at the threshold $M_{3L}$. The mass scales read explicitly $M_{3L} = 2.5 \times 10^{13} \,\text{GeV}$, $M_6 = 3.3 \times 10^{15} \,\text{GeV}$ and $M_{\text{GUT}} = 3.0 \times 10^{18} \,\text{GeV}$. 


FIG. 5: \( \alpha_i(E)^{-1} \) versus \( t = \ln(E/\Lambda_Z) \) for \( \Lambda_{\psi,2} = 150 \) GeV and \( 0 < E < M_{\text{Planck}} \) and for a restricted range of energies. Heavy particles entering above \( M_6 \) include the fermions \((1, 35) + (2, 20) \subset 78 \) and the scalar components \((3, 15) \subset 351 \). The different indices denote: \( a = SU(6) \), \( b = SU(2)_L \), \( c = U(1)_c \), \( d = SU(3)_c \), \( e = SU(3)_L \) for \( M_{3L} < E < M_6 \) and \( SU(2)_L \) for \( E < M_{3L} \), and \( f = U(1)_Y \). The black bubble stands for the matching condition (31) at the threshold \( M_{3L} \). The mass scales read explicitly \( M_{3L} = 2.5 \times 10^{13} \) GeV, \( M_6 = 3.3 \times 10^{15} \) GeV and \( M_{\text{GUT}} = 4.6 \times 10^{17} \) GeV.

FIG. 6: Correction to the \( S \) parameter produced by the messenger field \( \tilde{\varphi}^{(2)} \), where \( m_{0,1} \) denote, respectively, the mass of the custodial representations \( J = 0 \) and \( J = 1 \). \( S \) ranges in the interval \(-0.13 < S < 0 \), where \( S = 0 \) corresponds to no splitting between the masses of the custodial representations \( J = 0 \) and \( J = 1 \), and \( S = -0.13 \) refers to the limit \( \frac{m_0^2}{m_1^2} \rightarrow 0 \).