The $Z \rightarrow \gamma\gamma, \, gg$ Decays in the Noncommutative Standard Model

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(Dated: October 24, 2018)

On noncommutative spacetime, the Standard Model (SM) allows new, usually SM forbidden, triple gauge boson interactions. In this letter we propose the SM strictly forbidden $Z \rightarrow \gamma\gamma$ and $Z \rightarrow gg$ decay modes coming from the gauge sector of the Noncommutative Standard Model (NCSM) as a place where noncommutativity could be experimentally discovered.

PACS numbers: 12.60.Cn, 13.38.Dg, 02.40.Gh

In this article we consider strictly SM forbidden decays coming from the gauge sector of the NCSM which could be probed in high energy collider experiments. This sector is particularly interesting from the theoretical point of view. It is the place where different models show the greatest differences. In particular there are models that do not require any new triple gauge boson interactions. This depends on a choice of representation. It is, however, important to emphasize that generically one should expect triple boson interactions. We will in particular argue that a model that does have new triple gauge boson interactions is natural as an effective theory of noncommutativity. Our main results are summarized in equations (17) to (19).

The idea that coordinates may not commute can be traced back to Heisenberg. A simple way to introduce a noncommutative structure into spacetime is to promote the usual spacetime coordinates $x$ to noncommutative (NC) coordinates $\hat{x}$ with

$$[\hat{x}^\mu, \hat{x}^\nu] = i \theta^{\mu\nu}, \quad [\theta^{\mu\nu}, \hat{x}^\rho] = 0, \quad \theta^{\mu\nu} = \text{constant, real, antisymmetric matrix}$$

(1)

were $\theta^{\mu\nu}$ is a constant, real, antisymmetric matrix. The noncommutativity scale $\Lambda_{NC}$ is fixed by choosing dimensionless matrix elements $\Lambda_{NC}^n = \Lambda_{NC}^2 \theta^{\mu\nu}$ of order one. The original motivation to study such a scenario was the hope that the introduction of a fundamental scale could deal with the infinities of quantum field theory in a natural way. The simple commutation relation (1) with constant $\theta^{\mu\nu}$ fails to provide a complete regularization. But more complicated noncommutative structures can indeed introduce spacetime lattice structures into the theory that are compatible with a deformation of continuous spacetime symmetries (see, e.g., [3]). This is in contrast to the situation in ordinary lattice field theory, where only discrete translation symmetries survive. Aside from these technical merits, the possibility of a noncommutative structure of spacetime is of interest in its own right and its experimental discovery would be a result of fundamental importance.

Noncommutative gauge theory has become a focus of interest in string theory and M-theory with the work given in Ref.[1]. Noncommutativity of spacetime is very natural in string theory and can be understood as an effect of the interplay of closed and open strings. The commutation relation (1) enters in string theory through the Moyal-Weyl star product

$$f \star g = \sum_{n=0}^{\infty} \frac{\theta_{\mu_1\nu_1} \cdots \theta_{\mu_n\nu_n}}{(2\pi)^n n!} \partial_{\mu_1} \cdots \partial_{\mu_n} f \cdot \partial_{\nu_1} \cdots \partial_{\nu_n} g.$$  

(2)

For coordinates: $x^\mu \star x'^\nu = x'^\nu \star x^\mu = i \theta^{\mu\nu}$. The tensor $\theta^{\mu\nu}$ is determined by a NS $B^{\mu\nu}$-field and the open string metric $G^{\mu\nu}$, which both depend on a given closed string background. The effective physics on D-branes is most naturally captured by noncommutative $U(N)$ gauge theory, but it can also be described by ordinary gauge theory. Both descriptions are related by the Seiberg-Witten (SW) map, which expresses noncommutative gauge fields in terms of fields with ordinary “commutative” gauge transformation properties.

Quantum field theory on non-commutative space-time can be studied also independently of string theory. There are two major approaches. The original one based on actions that resembles that of Yang-Mills theory with matrix multiplication replaced by the Moyal-Weyl star product and a more recent one that utilizes the so-called Seiberg-Witten map to express non-commutative fields in terms of physical (commutative) fields. Both have their advantages and limitations. In the original approach unusual non-perturbative effects like UV/IR mixing can be studied, but gauge theories are limited to the gauge group $U(N)$ in the fundamental representation. There are also indications of more fundamental problems in the rigorous definition of the S-matrix. The second approach treats non-commutativity strictly perturbatively via Seiberg-Witten map expansion in terms of $\theta$. A major advantage of the second approach is that models with any gauge group – including the one of the standard model – and any particle content can be con-
yielded. Further problems that are solved in this approach include the charge quantization problem of NC Abelian gauge theories and the construction of covariant Yukawa couplings. The action is manifestly gauge invariant. It is written in terms of physical fields and their derivatives and should be understood as an effective model describing non-commutative effects in particle physics, see [14, 15, 17] and references therein.

Experimental signatures of noncommutativity have been discussed from the point of view of collider physics [11-14] as well as low-energy non-abelian experiments [14, 15, 17]. Two widely disparate sets of bounds on $\Lambda_{NC}$ can be found in the literature: bounds of order $10^{11}$ GeV [15] or higher [14], and bounds of a few TeV's from colliders [11-13]. All these limits rest on one or more of the following assumptions which may have to be modified: (1) $\theta$ is constant across distances that are very large compared with the NC scale; (2) unrealistic gauge groups; (3) noncommutativity down to low energy scales. The decay of the Z-boson into two photons was previously considered in [14], where the authors rely on a noncommutative U(1) model, i.e., not yet a bonafide noncommutative model of the electroweak sector or the standard model.

There are two essential points in which NC gauge theories differ from standard gauge theories. The first point is the breakdown of Lorentz invariance with respect to a fixed non-zero $\theta^{\mu\nu}$ background (which obviously fixes preferred directions) and the other is the appearance of new interactions (three-photon coupling, for example) and the modification of standard ones. Both properties have a common origin and appear in a number of phenomena.

The action of NC gauge theory resembles that of ordinary Yang-Mills theory, but with star products in addition to ordinary matrix multiplication. The general form of the gauge-invariant action for gauge fields is [17]

$$S_{\text{gauge}} = -\frac{1}{4} \int d^4 x \mathbf{T} \frac{1}{G^2} \tilde{F}_{\mu\nu} \ast \tilde{F}^{\mu\nu}. \quad (3)$$

Here $\mathbf{T}$ is a trace and $G$ is an operator that encodes the coupling constants of the theory. Both will be discussed in detail below. The NC field strength is

$$\tilde{F}_{\mu\nu} = \partial_\mu \tilde{V}_\nu - \partial_\nu \tilde{V}_\mu - i[\tilde{V}_\mu, \tilde{V}_\nu] \quad (4)$$

and $\tilde{V}_\mu$ is the NC analog of the gauge vector potential. The Seiberg-Witten maps are used to express the non-commutative fields and parameters and their derivatives. This automatically ensures a restriction to the correct degrees of freedom. For the NC vector potential the SW map yields

$$\tilde{V}_\xi = V_\xi + \frac{1}{4} \theta^{\mu\nu} (V_\xi, (\partial_\mu V_\xi + F_{\mu\xi})) + \mathcal{O}(\theta^2), \quad (5)$$

where $F_{\mu\nu} \equiv \partial_\mu V_\nu - \partial_\nu V_\mu - i[V_\mu, V_\nu]$ is the ordinary field strength and $V_\mu$ is the whole gauge potential for the gauge group $G_{SM} \equiv SU(3)_C \times SU(2)_L \times U(1)_Y$

$$V_\mu = g' A_\mu(x) Y + g \sum_{a=1}^{3} B_{\mu,a}(x) T^a_L + g_s \sum_{b=1}^{8} G_{\mu,b}(x) T^b_S. \quad (6)$$

It is important to realize that the choice of the representation in the definition of the trace $\mathbf{T}$ has a strong influence on the theory in the noncommutative case. The reason for this is, that owing to the Seiberg-Witten map, terms of higher than quadratic order in the Lie algebra generators will appear in the trace. The choice of the trace corresponds to a choice of the representation of the gauge group. The adjoint representation would not lead to new triple gauge boson interactions and, in particular, show no triple-photon vertices [17]. This, however, would be an ad hoc choice (unless we are discussing a GUT scenario.) Let us emphasize again that the action that we present here should be understood as an effective theory. From this point of view, all representations of gauge fields that appear in the SM have to be considered in the definition of the trace. Consequently, according to [17], we choose a trace over all particles with different quantum numbers in the model that have covariant derivatives acting on them. In the SM, these are, for each generation, five multiplets of fermions and one Higgs multiplet. The operator $G$, which determines the coupling constants of the theory, must commute with all generators $(Y, T^g_L, T^b_S)$ of the gauge group, so that it does not spoil the trace property of $\mathbf{T}$. This implies that $G$ takes on constant values $g_1, \ldots, g_6$ on the six multiplets (Table 1 in Ref. [17]). The operator $G$ is in general a function of $Y$ and the casimirs of $SU(2)$ and $SU(3)$. However, because of the special assignment of hypercharges in the SM it is possible to express $G$ solely in terms of $Y$.

The action up to linear order in $\theta$ allows new triple gauge boson interactions that are forbidden in the SM and has the following form

$$S_{\text{gauge}} = -\frac{1}{4} \int d^4 x f_{\mu\nu} f^{\mu\nu}$$

$$- \frac{1}{2} \int d^4 x \mathbf{T} (F_{\mu\nu} F^{\mu\nu}) - \frac{1}{2} \int d^4 x \mathbf{T} (G_{\mu\nu} G^{\mu\nu})$$

$$+ g \theta^{\rho\tau} \int d^4 x \mathbf{T} \left( \frac{1}{4} G_{\rho\tau} G_{\mu\nu} - G_{\rho\mu} G_{\nu\tau} \right) G^{\mu\nu}$$

$$+ g' \theta^{\rho\tau} \int d^4 x \left( \frac{1}{4} f_{\rho\tau} f_{\mu\nu} - f_{\mu\rho} f_{\nu\tau} \right) f^{\mu\nu}$$

$$+ g' \theta^{\rho\tau} \int d^4 x \left( \frac{1}{4} f_{\rho\tau} F^{a}_{\mu\nu} - f_{\mu\rho} F^{a}_{\nu\tau} \right) F^{\mu\nu,a} + \text{c.p.}$$

$$+ g' \theta^{\rho\tau} \int d^4 x \left( \frac{1}{4} f_{\rho\tau} G^{b}_{\mu\nu} - f_{\mu\rho} G^{b}_{\nu\tau} \right) G^{\mu\nu,b} + \text{c.p.},$$

where c.p. means cyclic permutations in $f$. Here $f_{\mu\nu}$, $F^{a}_{\mu\nu}$, and $G^{b}_{\mu\nu}$ are the physical field strengths corresponding to the groups $U(1)_Y$, $SU(2)_L$, and $SU(3)_C$, respec-
tively. The constants $\kappa_1$, $\kappa_2$, and $\kappa_3$ are parameters of the model. They are functions of $1/g_i^2$, ($i = 1, \ldots, 6$) and have the following form:

$$
\kappa_1 = -\frac{1}{g_1^2} + \frac{8}{9g_4^2} - \frac{1}{9g_1^2} + \frac{1}{4g_5^2} + \frac{1}{4g_6^2},
$$

$$
\kappa_2 = -\frac{1}{4g_2^2} + \frac{1}{4g_5^2} + \frac{1}{4g_6^2},
$$

$$
\kappa_3 = +\frac{1}{3g_3^2} - \frac{1}{6g_4^2} + \frac{1}{6g_5^2}.
$$

(8)

In order to match the SM action at zeroth order in $\theta$, three consistency conditions have been imposed in (7):

$$
\frac{1}{g'^2} = \frac{2}{g_1^2} + \frac{1}{g_2^2} + \frac{8}{3g_3^2} + \frac{2}{3g_4^2} + \frac{1}{3g_5^2} + \frac{1}{g_6^2},
$$

$$
\frac{1}{g'^2} = \frac{1}{3} \frac{1}{g_4^2} + \frac{1}{g_5^2},
$$

$$
\frac{1}{g'^2} = \frac{1}{g_3^2} + \frac{2}{g_4^2} + \frac{1}{g_5^2}.
$$

(9)

These three conditions together with the requirement that $1/g_i^2 > 0$, define a three-dimensional simplex in the six-dimensional moduli space spanned by $1/g_1^2, \ldots, 1/g_6^2$.

From the action (7) we extract the neutral triple-gauge boson terms which are not present in the SM Lagrangian. In terms of physical fields $(A, Z, G)$ they are

$$
\mathcal{L}_{\gamma\gamma\gamma} = \frac{e}{4} \sin 2\theta_W K_{\gamma\gamma\gamma} \theta^\rho A^\mu (A^\rho A^\nu - 4A^\mu A^\nu),
$$

$$
K_{\gamma\gamma\gamma} = \frac{1}{2} gg'(\kappa_1 + 3\kappa_2); \quad (10)
$$

$$
\mathcal{L}_{Z\gamma\gamma} = \frac{e}{4} \sin 2\theta_W K_{Z\gamma\gamma} \theta^\rho [2Z^\mu (2A^\rho A^\nu - A^\mu A^\nu) + 8Z_{\mu\rho}A^\nu A^\tau - Z_{\nu\sigma}A^\mu A^\tau],
$$

$$
K_{Z\gamma\gamma} = \frac{1}{2} \left[ g'^2 \kappa_1 + \left( g'^2 - 2g^2 \right) \kappa_2 \right]; \quad (11)
$$

$$
\mathcal{L}_{ZZ\gamma} = \mathcal{L}_{Z\gamma\gamma}(A \leftrightarrow Z),
$$

$$
K_{ZZ\gamma} = -\frac{1}{2g g'} \left[ \frac{1}{2} \left( g'^4 \kappa_1 + 3g^4 \kappa_2 \right) \right]; \quad (12)
$$

$$
\mathcal{L}_{ZZZ} = \mathcal{L}_{\gamma\gamma\gamma}(A \rightarrow Z),
$$

$$
K_{ZZZ} = -\frac{1}{2g^2} \left[ g'^4 \kappa_1 + 3g^4 \kappa_2 \right]; \quad (13)
$$

$$
\mathcal{L}_{Zgg} = \mathcal{L}_{Z\gamma\gamma}(A \rightarrow G^b),
$$

$$
K_{Zgg} = \frac{g_3^2}{2} \left[ 1 + \left( \frac{g}{g'} \right)^2 \right] \kappa_3; \quad (14)
$$

$$
\mathcal{L}_{gg} = \mathcal{L}_{Zgg}(Z \rightarrow A),
$$

$$
K_{gg} = -\frac{g_3^2}{2} \left[ \frac{g}{g'} + \frac{g'}{g} \right] \kappa_3,
$$

(15)

where $A_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}$, ect.

![Fig. 1](image-url)  

**FIG. 1:** The three-dimensional simplex that bounds possible values for the coupling constants $K_{\gamma\gamma\gamma}$, $K_{Z\gamma\gamma}$, and $K_{Zgg}$ at the $M_Z$ scale. The vertices of the simplex are: $(-0.184, -0.333, 0.054), (-0.027, -0.340, -0.108), (0.129, -0.254, 0.217), (-0.576, 0.010, -0.108), (-0.497, -0.133, 0.054),$ and $(-0.419, 0.095, 0.217)$.

Experimental evidence for noncommutativity coming from the gauge sector, should be searched for in processes which involve the above vertices. The simplest and most natural choice are the $Z \rightarrow \gamma\gamma$, $gg$ decays, allowed for real (on-shell) particles. All other simple processes, such as $\gamma \rightarrow \gamma\gamma$, $gg$, and $Z \rightarrow Z\gamma$, $ZZ$, are on-shell forbidden by kinematics. The $Z \rightarrow \gamma\gamma$, $gg$ decays are strictly forbidden in the SM by angular momentum conservation and Bose statistics (Yang Theorem) [14, 20], therefore they both could serve as a clear signal for the existence of spacetime noncommutativity [31].
annihilation, for \( \Gamma_{Z \rightarrow \gamma \gamma} \) is \(< 1.3 \times 10^{-4} \text{GeV} \) \cite{23}.

The \( Z \rightarrow gg \) decay mode should be observed in \( Z \rightarrow 2 \) jets processes. However, it could be smothered by the strong \( Z \rightarrow q \bar{q} \) background, i.e. by hadronization, which also contains NC contributions. Since the hadronic width of the \( Z \) is in good agreement with the QCD corrected SM, the \( Z \rightarrow gg \) can at most be a few percent. Taking into account the discrepancy between the experimentally observed hadronic width for the \( Z \)-boson and the theoretical estimate based on the radiatively corrected SM, we estimate the upper bound for any new hadronic mode, like \( \Gamma_{Z \rightarrow gg} \), to be \(~ 10^{-3} \text{GeV} \) \cite{23}.

We now derive the partial widths for the \( Z(p) \rightarrow \gamma(k) \gamma(k') \) decay. Care has to be taken when one tries to compute matrix elements in NCGFT. In our model, the \( \text{in} \) and \( \text{out} \) states can be taken to be ordinary commutative particles. Quantization is straightforward to the order in \( \theta \) that we have considered; Feynman rules can be obtained either via the Hamiltonian formulation or directly from the Lagrangian; a rather convenient property of the action, relevant to computations, is its symmetry under ordinary gauge transformations in addition to noncommutative ones. From the Lagrangian \( \mathcal{L}_{Z \gamma \gamma} \), it is easy to write the gauge-invariant amplitude \( \mathcal{M}_{Z \rightarrow \gamma \gamma} \) in momentum space. Since we are dealing with a SM forbidden process, this is essentially done using distorted wave Born approximation. It gives:

\[
\sum_{\text{spins}} |\mathcal{M}_{Z \rightarrow \gamma \gamma}|^2 = -\theta^2 + \frac{8}{M_Z^2} (p \theta^2 p) - \frac{16}{M_Z^2} (k \theta k')^2.
\]  (16)

From above equation and in the \( Z \)-boson rest frame, the partial width of the \( Z \rightarrow \gamma \gamma \) decay is

\[
\Gamma_{Z \rightarrow \gamma \gamma} = \frac{\alpha}{12} M_Z^5 \sin^2 2\theta_W K_{Z \gamma \gamma} \left[ \frac{7}{3} \left( \theta_T^2 \right)^2 + \left( \theta_S \right)^2 \right],
\]  (17)

where \( \theta_T = \{ \theta^{01}, \theta^{02}, \theta^{03} \} \) and \( \theta_S = \{ \theta^{23}, \theta^{13}, \theta^{12} \} \), are responsible for time-space and space-space noncommutativity, respectively. This result differs essentially from that given in \cite{14} where the \( \Gamma_{Z \rightarrow \gamma \gamma} \) partial width depends only on time-space noncommutativity.

For the \( Z \)-boson at rest and polarized in the direction of the 3-axis, we find that the polarized partial width is

\[
\Gamma_{Z^3 \rightarrow \gamma \gamma} = \frac{\alpha}{4} M_Z^5 \sin^2 2\theta_W K_{Z^3 \gamma \gamma}^2 \left[ \frac{2}{5} \left( (\theta^{01})^2 + (\theta^{02})^2 \right) + \frac{23}{15} (\theta^{03})^2 + (\theta^{12})^2 \right].
\]  (18)

In the absence of time-space noncommutativity a sophisticated, sensibly arranged polarization experiment could in principal determine the vector of \( \theta_S \). A NC structure of spacetime may depend on the matter that is present. In our case it is conceivable that the direction of \( \theta_{T,S} \) may be influenced by the polarization of the \( Z \) particle. In this case, our result for the polarized partial width is particularly relevant.

\[
\frac{\Gamma_{Z \rightarrow gg}}{\Gamma_{Z \rightarrow \gamma \gamma}} = \frac{\Gamma_{Z^3 \rightarrow gg}}{\Gamma_{Z^3 \rightarrow \gamma \gamma}} = \frac{8}{K_{Z \gamma \gamma}}.
\]  (19)

The factor of eight in the above ratios is due to color.

In order to estimate the NC parameter from upper bounds \( \Gamma_{Z^3 \gamma \gamma}^{\exp} < 1.3 \times 10^{-4} \text{GeV} \) and \( \Gamma_{Z^3 \rightarrow gg}^{\exp} < 1 \times 10^{-3} \text{GeV} \) \cite{23} it is necessary to determine the range of couplings \( K_{Z \gamma \gamma} \) and \( K_{Z \gamma \gamma} \). The allowed region for coupling constants \( K_{Z \gamma \gamma} \) and \( K_{Z \gamma \gamma} \) is given in Fig. (2). Since \( K_{Z \gamma \gamma} \) and \( K_{Z \gamma \gamma} \) could be zero simultaneously it is not possible to extract an upper bound on \( \theta \) only from the above experimental upper bounds alone.

We need to consider an extra interaction from the NCSM gauge sector, namely the triple photon vertex, to estimate \( \theta \). The important point is that the triplet of coupling constants \( K_{Z \gamma \gamma}, K_{Z \gamma \gamma} \) and \( K_{Z \gamma \gamma} \), as well as the pair...
it is possible to estimate can be seen from the simplex in Fig.(1). In conclusion, due to the constraint set by the value of the SM sector. A good reason for this is that the sensitivity to mentalists to look for SM forbidden decays in the gauge new physics.

of couplings $K_{\gamma\gamma}$ and $K_{Z\gamma}$ can never vanish simultaneously due to the constraint set by the value of the SM coupling constants at the weak interaction scale. This can be seen from the simplex in Fig. 4. In conclusion, it is possible to estimate $\theta$ from the NCSM gauge sector through a combination of various types of processes containing the $\gamma\gamma$ and $Z\gamma$ vertices. These are processes of the type $2 \rightarrow 2$, such as $e^+e^- \rightarrow \gamma\gamma$, $e\gamma \rightarrow e\gamma$, and $Z \rightarrow e^+e^-$ in leading order. The analysis has to be carried out in the same way as in Ref. 12. Theoretically consistent modifications of relevant vertices are, however, necessary. Finally, we present the allowed region for pair of couplings $K_{\gamma\gamma}$ and $K_{Z\gamma}$ in Fig. 4. Note, that Figs. 4 to 6 represent projections of pairs of coupling constants from the three dimensional simplex spanned by the constants $K_{\gamma\gamma}$, $K_{Z\gamma}$ and $K_{Z\gamma}^g$.

The structure of our main results 16 to 19 remains the same for $SU(5)$ and $SU(3)_C \times SU(3)_L \times SU(3)_R$ GUT’s that embed the NCSM that is based on the SW map 18, 24; only the coupling constants change. Note, in the particular case of $SO(10)$ GUT there is no triple gauge boson coupling 15.

In this article we have propose two SM strictly forbidden decay modes, namely, $Z \rightarrow \gamma\gamma, gg$, as a possible signature of the NCSM. An experimental discovery of $Z \rightarrow \gamma\gamma, gg$ decays would certainly indicate a violation of the presently accepted SM and definitive apperance of new physics.

In conclusion, the gauge sector of the nonminimal NCSM is an excellent place to discover spacetime noncommutativity experimentally but not the best place to find bounds that exclude it. We hope that the importance of a possible discovery of noncommutativity of spacetime will convince experimentalists to look for SM forbidden decays in the gauge sector. A good reason for this is that the sensitivity to the noncommutative parameter $\theta^{\mu\nu}$ could be in a range of the next generation of linear colliders with a c.m.e. around a few TeV’s.

We would like to thank P. Aschieri, B. Jurčo and H. Štefanič for helpful discussions. One of us (NGD) would like to thank the University of Hawaii Theory Group for hospitality. This work was supported by the Ministry of Science and Technology of the Republic of Croatia under Contract No. 0098002, and by the US Department of Energy, Grant No. DE-FG06-85ER 40224.

FIG. 4: The allowed region for $K_{\gamma\gamma}$ and $K_{Z\gamma g}$ at the $M_Z$ scale, projected from the simplex given in Fig. 1. Note that $K_{Z\gamma\gamma}$ is non-zero at the point where both $K_{\gamma\gamma}$ and $K_{Z\gamma g}$ vanish. The vertices of the polygon are: $(-0.108, -0.576), (-0.108, -0.027), (0.217, 0.129), (0.217, -0.419), (0.054, -0.497)$.
[20] K. Hagiwara, et al., Nucl. Phys. B282, 253 (1987); G. J. Gounaris, J. Layssac and F. M. Renard, Phys. Rev. D62, 073013 (2000).
[21] M.Z. Akrawy et al., Phys. Lett. B 257, 531 (1991); P. Abreu et al., Phys. Lett. B 327, 386 (1994); M. Acciarri et al., Phys. Lett. B 353, 136 (1995).
[22] L. Arnelos et al., Nucl. Phys. B196, 378 (1982); E.W.N. Glover et al., in: Z. Phys at LEP 1, CERN report 89-08 Vol. 2, G. Altarelli and C. Verzegnassi eds., (1989).
[23] Rev. of Particle Properties, Eur.Phys.J. C15, 1 (2000).
[24] N. G. Deshpande and X. G. He, Phys. Lett. B 533, 116 (2002) [arXiv:hep-ph/0112320].
[25] A Combination of Preliminary Electroweak Measurements and Constraints on the Standard Model, LEPEWWG/2002-01, May 2002; ALEPH Collaboration, Limits on anomalous ZZg and Zgg couplings using data from ZZ and Zγ production between 183-208 GeV, ALEPH 2001-061 (July 2001) CONF 2001-041.
[26] L3 Collaboration, M. Acciarri et al., Phys. Lett. B436, 187 (1999); L3 Collaboration, M. Acciarri et al., Phys. Lett. B489, 55 (2000); L3 Collaboration, Search for anomalous ZZg and Zgg couplings in the process ee → Zg at LEP, L3 Note 2672 (July 2001).
[27] DELPHI Collaboration, Study of trilinear Gauge Boson Couplings ZZZ, ZZγ and Zγγ, DELPHI 2001-097 (July 2001) CONF 525.
[28] OPAL Collaboration, G. Abbiendi et al., Eur. Phys. J. C17, 13 (2000).
[29] Th. Müller, D. Neuberger and W.H. Thümmel, Sensitivities for anomalous WWγ and ZZγ couplings at CMS, CMS NOTE 2000/017; Th. Müller, private communications.
[30] In terms of the couplings gi, these are complicated equations describing a family of hyper ellipsoids, however, in terms of $1/g_i^2$ they form a set of linear equations.
[31] Z and γ self-couplings vanish identically in the SM if all particle are on-shell. They can, however, appear if one of the photons is considered an off-shell particle in the s-channel [20].