Investigating the effects of a non-uniform magnetic field on heat and flow characteristics of a ferrofluid

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Abstract. A numerical study is performed to investigate the effect of a non-uniform magnetic field from a current carrying wire on the ferrofluid flow. The analysis is carried out for a semi-circular annulus with three different locations of wire relative to it, by solving coupled set of flow field equations, energy equations and the Maxwell's magnetostatics equations. Results from the present study offers better insight about the ferrofluid behaviour and heat transfer mechanism. It also explains the dependency of flow distribution on the location of the electric wire and the magnitude of current flowing through it.

1. Introduction
Because of their distinct thermophysical and magnetic properties, ferrofluids are being widely used to regulate as well as to enhance the heat and flow characteristics [1, 2]. Various studies [3] have been performed by researchers to investigate the mechanism of Thermomagnetic Convection (TMC) using a non-uniform magnetic field from different sources such as permanent magnet, electromagnet, and electric wires. Aminfar et al. [4, 5] carried out a numerical analysis to study the effect of magnetic field from a wire on ferrofluid flow within a rectangular and a helical duct respectively. Vatani et al. [6] did experimental as well as numerical analysis of ferrofluid surrounding an electrically heated vertical wire. Additionally, many authors also discussed the influence of non uniform magnetic fields from a current carrying wire on biomagnetic fluid flows [7, 8, 9]. In general, all the above studies concluded that non-uniform magnetic field alters the velocity profile which ultimately affects the Nusselt number distribution inside the ferrofluids.

The present study aims to investigate the influence of a non-uniform magnetic field distribution on combined natural convection and TMC within a semi-circular annulus. For observing the flow behaviour, streamlines are compared along with averaged Nusselt number (Nu$_{avg}$) magnitude among all the configurations.

2. Problem Description and Computational Methodology
For the present analysis, a water based colloidal suspension of 5% of volume fraction ($\phi$) of $Fe_3O_4$ particles is considered as the working fluid. This ferrofluid is present within a 2-D semi-circular annulus of a fixed $L/D$ ratio of 0.8, where $L$ is the width of annulus ($L = Ro - Ri$) and $D$ represents the internal diameter. An electric wire is assumed to carry a constant current of known magnitude in the perpendicular plane and is placed at a radial distance of 0.75$R_i$ in three different orientations ($\theta = 60^\circ$) as shown in Figure 1. All the thermophysical
properties for ferrofluid \((\rho_{nf}, C_{pnf}, \mu_{nf}, \text{and } k_{nf})\) are modelled in accordance with the equations and values mentioned in Fadaei et al. [10]. The simulations are carried out for a fixed \(Ra\) \((Ra = g\beta(T_H - T_C)L^3_c/\nu_{nf}\alpha_{nf})\) of \(10^6\) and three different magnitudes of \(Mn\) \((\mu_0 H_r^2 L^2_c/\rho_{nf}\alpha_{nf}^2)\) where \(H_r = I/(2\pi b)\). The domain is discretised into a structured non-uniform grid with fine meshing near both the walls (Figure 2). The non-dimensionalised distributions of the strength of magnetic field \((H^* = H/H_{max})\) for all three locations are shown in Figures 3-5.

The components of \(H\) are modelled as a function of their coordinates in Cartesian form [7]. For the sake of brevity, only momentum equation of a laminar, incompressible, and transient Newtonian fluid is mentioned here (Equation 1) and the boundary conditions are provided in Table 1. The continuity equation, energy equation, and Maxwell’s equation for an electrically non-conducting medium can be referred from Vatani et al. [6].

\[\rho_{nf} \frac{D\mathbf{U}}{Dt} = -\nabla p + \mu_{nf} \nabla^2 \mathbf{U} + \mu_0 (\mathbf{M} \cdot \nabla) \mathbf{H} + g(\rho_{nf} - \rho_\infty)\]  

To calculate the Kelvin body force \((\mu_0 (M \cdot \nabla) H)\), the magnetisation of ferrofluid is modelled with the superparamagnetic magnetisation law [11] as shown in Equation 2.

\[\mathbf{M} = M_d \phi \left[ \coth(\alpha) - \frac{1}{\alpha} \right] \quad \text{where } \alpha = \frac{\mu_0 m_H}{K_\beta T} \]  

As the present analysis deals with the influence of non-uniform magnetic field on flow and heat characteristics, a validation study is carried out to compare the local Nusselt number \((Nu_{local})\) distribution with Ganguly et al. [12]. It can be clearly observed from Figure 6 that, the present numerical model shows a nice agreement with the published results. Also, to make sure that our model is free of discretization errors, results from five different grid sizes are compared for a case of \(Ra = 10^6\) and \(Mn = 2.573 \times 10^{11}\), at the location B. It can be seen from Table 2, even for a considerable change in mesh count, domain averaged magnitudes of temperature \((T_{avg})\) and velocity \((U_{avg})\) shows small differences, thus Grid 4 is finalised. All the computations for present work are performed within a C++ based open-source framework OpenFOAM 5.0. [13] where PIMPLE algorithm is used for pressure-velocity coupling.
3. Results and Discussion

To analyse the configurations, magnitudes of mean velocity ($U_{\text{Mean}} = \sqrt{U_x^2 + U_y^2}$) and $N_{u\text{avg}}$ are compared for fluid domain and inner hot wall respectively. It is observed that, the magnitude of $N_{u\text{avg}}$ as well as the $U_{\text{Mean}}$ increases with the increase in $Mn$ at all the wire locations. To gain further insights, streamlines are plotted for all three positions of wire at highest $Mn$ with a reference case of $Mn = 0$. In the absence of any current, the streamlines displays a typical buoyancy driven flow characteristics where the less denser fluid moves in the upward direction creating a clockwise (CW) plume at the top (Figure 9 (a)). As opposed to this, the non-uniform magnetic field from the current carrying wire will disturb this arrangement and create local vortices. This phenomenon can be clearly noticed from Figure 9 (b) and 9 (d) where small recirculation zones are present at the top (CW) and bottom (ACW) of annulus for location A and C respectively. These recirculations also improve overall heat transfer from the inner hot wall to the surrounding fluid which can be confirmed from Figure 7 where $N_{u\text{avg}}$ has larger magnitudes for location A and C as compared to the location B for all $Mn$. In contrary to the $N_{u\text{avg}}$, maximum values for $U_{\text{Mean}}$ (Figure 8) are observed at location A, which can be attributed to the combined outcome of vertical plume from the buoyancy force (CW) and local recirculations (CW) from the current carrying wire. For location B, alternate CW and ACW

![Figure 6](image)

**Figure 6.** Comparison of $N_{u\text{local}}$ distribution from the present numerical results with Ganguly et al. [12].

![Figure 7](image)

**Figure 7.** Variation of $N_{u\text{avg}}$ for different positions of wire and $Mn$.

![Figure 8](image)

**Figure 8.** Variation of $U_{\text{Mean}}$ for different positions of wire and $Mn$.

### Table 1. Information about Boundary Conditions

| Inner Wall | Outer Wall | Centreline |
|------------|------------|------------|
| $u, v, w = 0$ | $u, v, w = 0$ | $\partial U / \partial n$ |
| $T$ | $T_H$ | $T_C$ | $\partial T / \partial n$ |

### Table 2. Grid Independence Test

| No. of Nodes | $T_{\text{avg}}$ | $U_{\text{avg}}$ |
|-------------|-----------------|-----------------|
| Grid 1  $80 \times 40$ | 300.458 | $1.273 \times 10^{-5}$ |
| Grid 2  $100 \times 50$ | 300.438 | $2.798 \times 10^{-5}$ |
| Grid 3  $120 \times 60$ | 300.411 | $2.473 \times 10^{-5}$ |
| Grid 4  $140 \times 70$ | 300.403 | $2.160 \times 10^{-5}$ |
| Grid 5  $160 \times 80$ | 300.392 | $1.814 \times 10^{-5}$ |
vortices nullify each other's contribution, which results in the lowest values for both $Nu_{avg}$ and $U_{Mean}$.

4. Conclusion
A numerical study is carried out to explore the influence of an electric wire on heat transfer and flow attributes of a ferrofluid within a semi-circular annulus. It is observed that a current-carrying wire can be used as an effective means to control and increase the localised heat transfer within ferrofluids. The magnitudes of both $Nu_{avg}$ and $U_{Mean}$ increase with $Mn$ and their highest values are noticed for location C and location A respectively.

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References
[1] Alsaady M, Fu R, Li B, Boukhanouf R and Yan Y 2015 Applied Thermal Engineering 88 14–21
[2] Nkurikiyimfura I, Wang Y and Pan Z 2013 Renewable and Sustainable Energy Reviews 21 548–561
[3] Afifah A, Syahrullail S and Sidik N 2016 Renewable and Sustainable Energy Reviews 55 1030–1040
[4] Aminfar H, Mohammadpourfard M and Zonouzi S A 2013 Journal of Magnetism and Magnetic materials 327 31–42
[5] Aminfar H, Mohammadpourfard M and Ahangar Zonouzi S 2014 Journal of heat transfer 136
[6] Vatani A, Woodfield P L, Nguyen N T, Abdollahi A and Dao D V 2019 Journal of Magnetism and Magnetic Materials 489 165383
[7] Mousavi S M, Darzi A A R, Ali Akbari O, Toghraie D and Marzban A 2019 Journal of Magnetism and Magnetic Materials 489 165383
[8] Papadopoulos P and Tzirtzilakis E 2004 Physics of Fluids 16 2952–2962
[9] Sharifi A, Yekani Motlagh S and Badfar H 2018 International Journal of Computational Fluid Dynamics 32 248–259
[10] Fadaei F, Shahrakhi M, Dehkordi A M and Abbasi Z 2017 Journal of Magnetism and Magnetic Materials 429 314–323
[11] Rosensweig R E 2013 Ferrohydrodynamics (Courier Corporation)
[12] Ganguly R, Sen S and Puri I K 2004 Journal of Magnetism and Magnetic Materials 271 63–73
[13] The OpenFOAM Foundation, OpenFOAM v5 User Guide