Design of Compact Huygens’ Metasurface Pairs With Multiple Reflections for Arbitrary Wave Transformations

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Abstract—Huygens’ metasurfaces (HMSs) have demonstrated a remarkable potential to perform wave transformations within a subwavelength region. In particular, omega-bianisotropic HMSs have allowed for the passive implementation of any wave transformation that conserves real power locally. Previous reports have also shown that HMS pairs are capable of realizing transformations that break the local power conservation requirement by redistributing the total power, while the wave propagates between the two metasurfaces. However, the required separation distance overshadows the low-profile characteristics of the individual metasurfaces and leads to bulky designs, especially for lower frequencies. In this article, we develop a method of designing omega-bianisotropic HMS pairs, relying on a point-matching process of the real power at the two metasurfaces. We highlight the versatility of our method by presenting two variations of the configuration, depending on whether the electromagnetic source is located within or outside the metasurface pair. Based on the examples of a cylindrical-wave to plane-wave transformation and a beam expander, we examine the impact of multiple reflections, as a way to overcome the size limitations and design compact structures. Moreover, we explore possible beamforming applications through an example of a Taylor-pattern antenna with a single feed-point between the two metasurfaces.

Index Terms—Beamforming, Huygens’ metasurfaces (HMSs), multiple reflections, power matching, wave transformations.

I. INTRODUCTION

HUYGENS’ metasurfaces (HMSs) are electrically thin devices that have attracted considerable attention as an efficient tool to manipulate electromagnetic waves at will [1]–[3]. In their passive form, they consist of subwavelength elements (unit cells) arranged in a thin sheet, which induce equivalent electric and magnetic currents, when excited by an incident wave. The current densities that support the desired electric and magnetic fields at the two sides of the metasurface are calculated using the generalized sheet transition conditions (GSTCs) [4]. Subsequently, the current densities are discretized across the metasurface and the response of each individual unit cell is engineered in terms of polarizabilities, susceptibilities, or surface impedances/admittances [4]–[6]. This approach has led to the design and experimental demonstration of passive HMSs for numerous electromagnetic applications, such as engineering refraction, reflection and absorption, beam focusing and polarization control among others [2], [3], [7]–[12].

In the effort to realize more complicated wave transformations, omega-bianisotropy has emerged as a way to introduce another degree of freedom in the design process, accounting for magnetoelectric coupling [13]–[15]. Specifically, in omega-bianisotropic HMSs, the electric and magnetic fields both excite orthogonally polarized equivalent electric and magnetic currents. Under this condition, it has been theoretically proven that a passive and lossless HMS can be designed for any transformation, as long as the real power density propagating normal to the boundary is locally conserved [13]. Previously impossible to realize wave transformations, such as perfect wide-angle reflectionless refraction, were successfully demonstrated with the use of omega-bianisotropic HMSs [16]–[18].

Despite the extra degree of freedom provided by the omega-bianisotropy, the condition of local power conservation along the metasurface places constraints on the form of the input and output electric and magnetic fields. Several solutions have been proposed as a way to overcome these limitations and achieve nonlinear power conserving transformations. For instance, surface waves have been used either as auxiliary fields to restore power matching locally without interfering with the far-field characteristics of the transformation or as a way to redistribute the power within metasurface systems that are based on the conversion of propagating waves to surface waves and vice versa [19]–[21]. However, the use of auxiliary surface waves requires specifying their form, which can be a nontrivial task for some applications (e.g., beamforming) [19]. On the other hand, metasurface systems usually suffer from low conversion efficiency between surface and propagating waves, as well as from losses and distortion at the discontinuities between the different regions of the system [21].

Recently, two cascaded omega-bianisotropic HMSs have been studied for performing wave transformations that break the local power conservation condition [22], [23]. As the wave propagates between the two metasurfaces, the power density
profile is reshaped. Therefore, it is possible to satisfy local power conservation for the two metasurfaces individually, even if the input and output power density profiles of the total structure are different. One major disadvantage of this approach is the size requirements for the metasurface pair, as a substantial separation may be required, especially for the cases that the input and output power density profiles differ significantly. Although in [23] it was stated that the introduction of multiple reflections may reduce the size requirements, their effect was not investigated.

In this article, we design pairs of omega-bianisotropic HMS for arbitrary wave transformations that do not conserve local power. As in [23], the design method relies on determining the fields in the region between the two metasurfaces, so that the power density is matched at both metasurfaces simultaneously. In Section II, we reformulate the theoretical framework by expanding the fields at the inner boundaries of the metasurface pair into two summations of spatially shifted basis functions that propagate in opposite directions. By defining the basis functions expansion in the spatial domain, we avoid the discretization of fields in terms of modes (wavevectors) and angles of propagation, as done in [23]. In addition, the use of two counter-propagating field distributions allows for easily introducing multiple reflections in the design. The proposed design method can be adjusted to handle two geometry variations, depending on whether the source is placed outside or within the metasurface pair. To validate our formulation and suggest possible applications, we present several examples of wave transformations that break the local power conservation condition in Section III. For the first geometry variation with the source outside the metasurface pair, we design a cylindrical-wave to plane-wave transformation and a Gaussian beam expander. Through these examples, we examine the usefulness of multiple reflections in reducing the separation distance between the two metasurfaces. For the second scenario with the source located within the metasurface pair, we show that both sides can be treated as independent output apertures and the incident power from the source can be arbitrarily split between them. Based on this configuration, we present a beamforming example featuring a single-sided or a double-sided Taylor distribution output. In Section IV, we comment on the influence of cavity effects to the sensitivity and the bandwidth of the proposed design and we investigate possible ways to render it less susceptible to geometrical and frequency variations. Lastly, we conclude our work in Section V.

II. Method of Moments Approximation

In this section, the theoretical formulation for designing a pair of passive and lossless omega-bianisotropic HMSs is developed. Two configurations are considered depending on the location of the source, as illustrated in Fig. 1. In the first one, the electromagnetic source is placed before the first metasurface (M1) and the input field distribution \( \{E_{\text{in}}, H_{\text{in}}\} \) is transformed to an arbitrary output field distribution \( \{E_{\text{out}}, H_{\text{out}}\} \) at the output of the second metasurface (M2). On the contrary, in the second configuration the source is placed between the two metasurfaces and two arbitrary output field distributions, represented by \( \{E_{\text{out}}^{(1)}, H_{\text{out}}^{(1)}\} \) and \( \{E_{\text{out}}^{(2)}, H_{\text{out}}^{(2)}\} \), can be supported at the two sides of the metasurface pair. For both geometries, it is assumed that the metasurfaces extend infinitely along the \( z \)-direction for simplicity. The distance between them is denoted by \( d \), while \( L_{\text{tot}} \) stands for the total width of each metasurface. Furthermore, the electric field is assumed to be transverse electric (TE) polarized with the only nonzero component being along \( \hat{z} \).

A. Calculation of Real Power at the Boundaries of the Two Metasurfaces

As observed in Fig. 1, the fields at the outer boundaries of the metasurface pair (i.e., \( y = 0^- \) for M1 and \( y = d^+ \) for M2) are known for both configurations, as long as the input fields (or the type of the source) and the desired output fields are specified. Therefore, the real power density flowing normal to the metasurfaces at the planes \( y = 0^- \) and \( y = d^+ \) can be calculated through the Poynting vector. Then, the aim is to properly define the electromagnetic fields between the two metasurfaces in order to satisfy local power conservation at both boundaries simultaneously, while obeying Maxwell’s equations. To this purpose, two field distributions are introduced propagating to the forward (+\( \hat{y} \)) and to the backward (−\( \hat{y} \)) directions. When the source is placed before M1, the summation of the two counter-propagating field distributions equals the total fields between the two metasurfaces, whereas the incident fields from the source should also be added in the case that this is located within the metasurface pair.

The electric field of the forward-propagating wave \( E_{\text{z}}^{\text{forw}}(x, y) \) is approximated at M1 (\( y = 0^+ \)) as a summation of \( 2N + 1 \) terms

\[
E_{\text{z}}^{\text{forw}}(x, y = 0^+) = \sum_{n=-N}^{n=N} A_{f,n} g_n(x) \tag{1}
\]

where \( A_{f,n} \) are the unknown complex weights, still to be determined. The basis functions \( g_n(x) \) in (1) are defined as

\[
g_n(x) = \frac{\sin(k(x - nL))}{k(x - nL)} \tag{2}
\]

where \( k = 2\pi/\lambda \) is the free-space wave-vector (\( \lambda \) standing for the wavelength of operation) and \( L = L_{\text{tot}}/(2N + 1) \).
represents the spatial shift between two adjacent functions \( g_n(x) \), as illustrated in Fig. 1(a). By taking the Fourier transform of (1) with respect to the \( x \)-coordinate, we can write the spatial spectrum of the forward-propagating electric field after the first metasurface

\[
E_{f,0}^{\text{form}}(k_x, y = 0^+) = \left\{ \begin{array}{ll}
\prod_{n=N}^{n=n} A_{f,n} e^{jk_n x}, & |k_x| \leq k \\
0, & |k_x| > k.
\end{array} \right.
\]  

(3)

As observed from (3), the particular choice of basis functions makes it possible to have a continuous band-limited spectrum in terms of wavenumbers \( k_x \). Specifically, the spatial width of the basis functions \( g_n(x) \) has been adjusted, so that each one exhibits a constant spectral amplitude for the propagating components \(|k_x| \leq k\), while there is no evanescent spectrum \(|k_x| > k\) that may be difficult to be excited passively from the metasurface. Therefore, a continuous propagating spectrum is formed in (3), as opposed to the plane-wave expansion that is usually encountered in the literature and results in a discrete set of wavenumbers \( k_x \) (or, equivalently, a discrete set of angles of propagation for the constituent plane waves) [23]. It should also be noted that in the case of relatively smooth input and output power distributions the absence of evanescent waves does not restrict the ability to achieve power matching at the two metasurfaces. Lastly, any evanescent spectrum created by the two metasurfaces, due to the nonperfect local power mismatch or the discretization, are quickly decayed away from them; thus, they do not affect the wave transformation performed by the other metasurface.

Each of the spectral components of the wave propagates along \(+\hat{y}\) with a wavenumber \( k_y = \sqrt{k^2 - k_x^2} \). Subsequently, the forward-propagating electric field at any given \( y \)-plane is given as the following inverse Fourier transform:

\[
E_{x}^{\text{form}}(x,y = 0^+) = \frac{1}{2\pi} \int_{-k}^{k} E_{x}^{\text{form}}(k_x, y = 0^+) e^{-jk_y y} e^{-jk_x x} dk_x
\]

\[
= \sum_{n=-N}^{N} A_{f,n} \frac{1}{2k^2} \int_{-k}^{k} e^{-j\sqrt{k^2 - k_x^2} y} e^{-jk_x (x-nL)} dk_x.
\]  

(4)

From Maxwell’s equations, the magnetic field component tangential to the metasurface can be calculated as

\[
H_x^{\text{form}}(x,y) = \frac{j}{\omega \mu} \frac{\partial E_x^{\text{form}}}{\partial y}
\]

\[
= \sum_{n=-N}^{N} A_{f,n} \frac{1}{2k^2} \int_{-k}^{k} \sqrt{k^2 - k_x^2} e^{-j\sqrt{k^2 - k_x^2} y} e^{-jk_x (x-nL)} dk_x
\]  

where \( \eta = \sqrt{\mu/\varepsilon} \) is the characteristic impedance of the medium.

Likewise, the reflected wave is expanded at the second metasurface \((y = d^-)\) as

\[
E_{x}^{\text{ref}}(x,y = d^-) = \sum_{n=-N}^{N} A_{r,n} g_n(x)
\]  

(6)

where the complex weights \( A_{r,n} \) form another set of unknowns. By following a similar approach for the reflected fields (defining in this case the zero-phase plane at \( y = d \) and propagating each Fourier component at the \(-\hat{y}\) direction), we arrive at the following expression for the reflected electric field at any \( y \)-plane:

\[
E_{x}^{\text{ref}}(x,y) = \sum_{n=-N}^{N} A_{r,n} \frac{1}{2k^2} \int_{-k}^{k} e^{j\sqrt{k^2 - k_x^2} (y-d)} e^{-jk_x (x-nL)} dk_x.
\]  

(7)

Then, the tangential component of the reflected magnetic field is obtained directly from Maxwell’s equations as

\[
H_x^{\text{ref}}(x,y) = -\sum_{n=-N}^{N} A_{r,n} \frac{1}{2k^2} \int_{-k}^{k} \sqrt{k^2 - k_x^2} e^{j\sqrt{k^2 - k_x^2} (y-d)} e^{-jk_x (x-nL)} dk_x
\]  

(8)

As stated above, in the case that the source is placed before M1, as shown in Fig. 1(a), the total electric and magnetic fields between the two metasurfaces are found as a superposition of the forward wave in (4)–(5) and the reflected wave in (7)–(8). By substituting \( y = 0 \) and \( y = d \) in the above expressions, the total tangential electric and magnetic fields are calculated at the inner boundaries of the two metasurfaces

\[
E_x(x,y = 0^+) = \sum_{n=-N}^{N} A_{f,n} I_n + \sum_{n=-N}^{N} A_{r,n} I'_n
\]  

(9a)

\[
H_x(x,y = 0^+) = \sum_{n=-N}^{N} A_{f,n} J_n - \sum_{n=-N}^{N} A_{r,n} J'_n
\]  

(9b)

\[
E_x(x,y = d^-) = \sum_{n=-N}^{N} A_{f,n} I_n' + \sum_{n=-N}^{N} A_{r,n} I_n
\]  

(9c)

\[
H_x(x,y = d^-) = \sum_{n=-N}^{N} A_{f,n} J_n' - \sum_{n=-N}^{N} A_{r,n} J_n
\]  

(9d)

where \( I_n, I'_n, J_n, \) and \( J'_n \) are coefficients that depend on the \( x \)-coordinate according to the expressions

\[
I_n = \frac{1}{2k} \int_{-k}^{k} e^{-jk_x (x-nL)} dk_x
\]  

(10a)

\[
I'_n = \frac{1}{2k} \int_{-k}^{k} e^{-j\sqrt{k^2 - k_x^2} (x-nL)} dk_x
\]  

(10b)

\[
J_n = \frac{1}{2k^2} \int_{-k}^{k} \sqrt{k^2 - k_x^2} e^{-jk_x (x-nL)} dk_x
\]  

(10c)

\[
J'_n = \frac{1}{2k^2} \int_{-k}^{k} \sqrt{k^2 - k_x^2} e^{-j\sqrt{k^2 - k_x^2} (x-nL)} dk_x.
\]  

(10d)
The real power density is then determined as

$$P_y(x, y = 0^+) = \frac{1}{2} \text{Re} \left\{ \sum_{n=-N}^{N} \sum_{m=-N}^{N} \left( I_n J_m^* A_{f,n} A_{r,m} + I_n^* J_m A_{f,n}^* A_{r,m}^* \right) - I_n J_m^* A_{f,n} A_{r,m}^* - I_n^* J_m A_{f,n}^* A_{r,m} \right\} \quad (11a)$$

$$P_y(x, y = d^-) = \frac{1}{2} \text{Re} \left\{ \sum_{n=-N}^{N} \sum_{m=-N}^{N} \left( I_n J_m^* A_{f,n} A_{r,m} + I_n^* J_m A_{f,n}^* A_{r,m}^* \right) - I_n J_m^* A_{f,n} A_{r,m}^* - I_n^* J_m A_{f,n}^* A_{r,m} \right\} \quad (11b)$$

For a given pair of metasurfaces (fixed separation $d$, width $L_{\text{tot}}$ and discretization $N$), the coefficients in (10) can be numerically determined and the real power density, calculated in (11), solely depends on the two sets of unknown weights $A_{f,n}$ and $A_{r,n}$.

On the other hand, when the source is placed between the two metasurfaces, the total field expressions in (9) should be supplemented by the incident fields produced directly from the source. The expressions for the real power density at the two boundaries are, then, modified to

$$P_y(x, y = 0^+) = \frac{1}{2} \text{Re} \left\{ \sum_{n=-N}^{N} \sum_{m=-N}^{N} \left( I_n A_{f,n} + I_n^* A_{r,n} + E_{\text{inc},z}^{(1)} \right) \right\} \quad (12a)$$

$$P_y(x, y = d^-) = \frac{1}{2} \text{Re} \left\{ \sum_{n=-N}^{N} \sum_{m=-N}^{N} \left( I_n A_{f,n} + I_n^* A_{r,n} + E_{\text{inc},z}^{(2)} \right) \right\} \quad (12b)$$

where $\{E_{\text{inc},z}^{(1)}, H_{\text{inc},z}^{(1)}\}$ and $\{E_{\text{inc},z}^{(2)}, H_{\text{inc},z}^{(2)}\}$ are the profiles of the incident tangential electric and magnetic fields at M1 and M2, respectively, as calculated from the source in the absence of the two metasurfaces.

Regarding the configuration in Fig. 1(a) with the source located before M1, the use of two counter-propagating waves allows for the handling of multiple reflections between the two metasurfaces. Certainly, in the case that the weights of the reflected wave $A_{r,n}$ are set to zero, multiple reflections are not considered and the field transformation is designed to be performed by two purely transmissive omega-bianisotropic HMSs, as previously shown in [22], [23]. The feasibility of each transformation is dependent on how closely the power distributions in (11) or (12) can approximate the required power profiles at the outer boundaries of M1 and M2. Under satisfactory local power matching, the two metasurfaces can be designed as passive and lossless. For a given transformation and separation distance, there is no assurance that there exist some set of complex weights $A_{f,n}$ and $A_{r,n}$, so that the power is locally matched simultaneously at M1 and M2. The restrictions stem from the equations of propagation of the electric and magnetic fields between the two planes, as given in (4)–(5) and (7)–(8). However, the hypothesis here is that the use of multiple reflections between the two metasurfaces allows to reduce the separation of the two metasurfaces for a desired field transformation. This is theoretically justified, since the propagation between the two metasurfaces is the mechanism to redistribute the power; thus, multiple reflections effectively increase the propagation length, while maintaining the compactness of the design. Needless to say, the use of a backward-propagating wave is also advantageous in the second configuration of Fig. 1(b), as multiple reflections allow for more accurate redistribution of the total power and improved local power matching at the two sides of each metasurface.

Another important aspect is specifying the level of the total output power so that maximum power efficiency is ensured for the designed metasurface pair. Since the metasurfaces are considered lossless, the only loss mechanism is the power escaping from the open sides of the structure. To minimize this leakage, the total output power $P_{\text{tot}}$ is made equal with the total incident power $P_{\text{inc}}$ from the source, which is

$$P_{\text{inc}}^{\text{tot}} = \frac{1}{2} \int_{M1} \text{Re} \left\{ E_{\text{inc},z} H_{\text{inc},z}^* \right\} \, dx$$

$$P_{\text{inc}} = \frac{1}{2} \int_{M1} \text{Re} \left\{ E_{\text{inc},z}^{(1)} H_{\text{inc},z}^{(1)*} \right\} \, dx + \frac{1}{2} \int_{M2} \text{Re} \left\{ E_{\text{inc},z}^{(2)} H_{\text{inc},z}^{(2)*} \right\} \, dx$$

when the source is placed outside or inside the metasurface pair, respectively. The incident power represents the part of the input power that can be handled by the metasurface pair, as it is not directly leaked outside of the configuration.

### B. Point Matching of the Real power

To achieve local power conservation at each metasurface, the input/output power densities at $y = 0^-$ and $y = d^+$ are sampled at the locations $x = nL$, $n = -N, \ldots, N$ and they are equated with the power densities at the inner boundaries of the two metasurfaces at $y = 0^+$ and $y = d^-$, respectively, as given by (11) or (12). Conceptually, this process is equivalent to point matching of the power density at two sets of equidistant points along the two metasurfaces. However, unlike the classic Method of Moments formulation, the electromagnetic quantity involved in the point matching is the power density instead of the electric field values. As a result of the point-matching process, a system of $2(2N + 1)$ equations is formed as

$$G = \begin{pmatrix} P_i(y = 0^-) - P_i(y = 0^+) \\ P_i(y = d^-) - P_i(y = d^+) \end{pmatrix} \bigg|_{n=0}^{n=N} = 0$$

with $-N \leq n \leq N$. This system should be solved for the unknown complex weights $A_{f,n}$ and $A_{r,n}$, that determine the power densities at the inner boundaries of the metasurface pair.
Due to the quadratic nature of the expressions in (11) and (12), the system is nonlinear and a gradient descent method is utilized to minimize the total power mismatch at the two metasurfaces. More specifically, the real and the imaginary parts of each weight define the vector of the unknowns, denoted by $\mathbf{x}$, that is optimized in order to minimize the objective function

$$F(\mathbf{x}) = \frac{1}{2}{\mathbf{G}}^T\mathbf{G}.$$  

(16)

The Jacobean matrix $\mathbf{J}_G$, involving the derivatives of each row of the $\mathbf{G}$ vector with respect to each unknown is analytically calculated (as a function of the coefficients $I_a, I'_a, J_a, J'_a$). Then, at every iteration the vector of the unknowns is updated based on the expression

$$\mathbf{x}^{(n+1)} = \mathbf{x}^{(n)} - \gamma \mathbf{J}_G(\mathbf{x}^{(n)})^T \mathbf{G}(\mathbf{x}^{(n)})$$  

(17)

where $\mathbf{x}^{(n)}$ is the vector of the unknowns at the $n$th iteration, and $\gamma$ represents the learning rate of the optimization algorithm, which is carefully selected for each design problem.

### C. Metasurface Macroscopic Design

Once the optimization algorithm has converged to a solution that minimizes the local power mismatch at the two metasurfaces, the tangential fields at the two sides of each metasurface are used for their design. Due to local power conservation, the field transformations at the two boundaries are guaranteed to be possible with reflectionless and lossless omega-bianisotropic HMSs. Moreover, the parameters are guaranteed to be possible with reflectionless and lossless omega-bianisotropic HMSs. The parameters are still applicable for the fields obtained from the optimization method discussed in Section II-B, as long as the power mismatch at the two boundaries is relatively low. It is also worth mentioning that a constant phase can be added to both the tangential electric and magnetic fields at the output of each metasurface. While adding a constant phase does not affect the power density matching, it can be advantageous to avoid extreme values for the metasurface parameters and facilitate the convergence of the simulations.

In order to perform full-wave simulations, the HMSs are modeled in Ansys HFSS using the three-layer impedance sheet approach [13]. Specifically, each unit cell consists of three cascaded lossless impedance sheets that give the necessary degrees of freedom to implement the electromagnetic response described by the metasurface parameters $Z_{se}, Y_{sm}$, and $K_{em}$ at the corresponding sampling point. The impedance values of the three layers for every unit cell are analytically calculated by [13, (8)]. In our simulations, the impedance sheets are separated by extremely thin ($\lambda/800$) air regions, as this was beneficial to handle some convergence issues in ANSYS HFSS. In addition, the simulation domain is confined between two parallel perfect electric conducting (PEC) plates in the $z$-axis to guarantee uniformity along this direction and TE-polarized fields, while the lateral sides are terminated with perfectly matched-layer (PML) boundaries.

In practical designs, the metasurfaces can be implemented with three layers by replacing the above-mentioned abstract impedance sheets with copper traces etched on standard substrates and bonded together. By varying the geometrical characteristics of the individual layers, the corresponding impedance can be adjusted to match the required values. The coupling between the layers, as well as the copper and dielectric losses may be significant and should be taken into account before the fabrication of the metasurfaces [17]. Moreover, other approaches may be preferred for minimizing losses and increasing the bandwidth of the structure, including the addition of a fourth layer [12, [24], or the use of wire-loop omega-bianisotropic unit cells [18]. Although the purpose here is primarily to validate the proposed theoretical framework and the three cascaded impedance sheets are a sufficient model to this objective, estimations of the bandwidth and the losses of some designs are also discussed in Section IV and Appendix A.

### III. DESIGN EXAMPLES

#### A. Uniform Aperture Illumination by a Single Line-Source Excitation

In the first example, the configuration described in Fig. 1(a) is used to transform an incident cylindrical wave to a uniform field distribution with constant phase along the output aperture. For a given aperture length, the uniform distribution exhibits the maximum directivity and the minimum half-power beamwidth (HPBW). An infinite (along the $z$-axis) current-source operating at the frequency $f = 10$ GHz is placed at distance $s = \lambda/3$ before M1, where $\lambda$ is the respective free-space wavelength. The current source radiates a cylindrical wave that partially impinges on the first metasurface, having the following profile for the tangential electric and magnetic fields:

$$E_{\text{in}z}(x) = -\frac{kNl}{4}H_n^{(2)}(k\sqrt{x^2+y^2}),$$  

(18a)

$$H_{\text{in}z}(x) = \frac{jkI}{4\sqrt{x^2+y^2}}H_n^{(2)}(k\sqrt{x^2+y^2}).$$  

(18b)

where $H_n^{(2)}$ is the $n$th order Hankel function of the second kind and $I = 1$ A is an arbitrarily chosen current amplitude. The purpose of the designed metasurface pair is to transform the fields into a truncated plane wave, characterized by a constant amplitude along the output aperture. Both metasurfaces have a width of $L_{\text{tot}} = 6\lambda$ and are discretized with 61 unit cells ($N = 30$).

To verify the applicability of our method, we first design a pair of reflectionless (i.e., $A_{\text{n}} = 0$, for all $n$) metasurfaces with a separation distance of $d = 1.5\lambda$. Even without multiple reflections, this separation distance is sufficient to achieve acceptable power matching at both boundaries simultaneously. The fields between the two metasurfaces are calculated using the optimization method described in Section II-B and the normal real power density at the two sides of each metasurface is depicted in Fig. 2. Since the design process of the
Fig. 2. Local power matching at the two metasurfaces as given by the optimization method for \( d = 1.5\lambda \). The power densities at the inner sides of the configuration (dashed lines) should locally match the defined input and output power distributions (solid lines) at both metasurfaces.

Fig. 3. Real part (absolute values) of the electric field \( E_z \) for a separation distance \( d = 1.5\lambda \) and totally reflectionless metasurfaces. The cylindrical wave (bottom region) transforms to a truncated plane wave propagating at the output (upper region) after passing through the metasurface pair.

two omega-bianisotropic HMSs assumes perfect local power conservation, it is expected that any deviations at either of the two metasurfaces will induce reflections in the region before M1 and perturb the transmitted fields at the output of M2. The real part (in absolute values) of the electric field is depicted in Fig. 3, where it is clear that the wavefronts transform from cylindrical at the input to planar at the output. Moreover, the amplitude of the electric field at the output of M2 is kept nearly constant with the fluctuations attributed to the power mismatch at the two metasurfaces.

It should be noted again that the sides between the two metasurfaces are open boundaries that do not confine the electromagnetic power within the width of the metasurface pair. However, by defining the input and the output power distributions to have the same total power, the real power leaking to the two sides is minimized, and it is solely attributed to the small power mismatch at the two metasurfaces. In the case considered, 96.6% of the incident power, as defined in (13), is transmitted to the output of the second metasurface, while 3.3% escapes from the two sides and the rest is reflected at the input region. The radiation pattern is depicted in Fig. 4 together with the theoretical one for a perfectly uniform output distribution. For the calculation of the radiation pattern, only the boundaries after M2 \( (y > d) \) are considered for the near-field to far-field transformation. It is observed that the directivity and the beamwidth compare well with the predicted values. Specifically, the obtained directivity from the simulation is 12.56 dB that is very close to the theoretical value of 12.73 dB for an aperture width of \( 6\lambda \). Regarding the main lobe beamwidth, a HPBW of 9° is calculated that is relatively close to the predicted value of 8.3° for a uniformly excited aperture.

Further reducing the separation between the two metasurfaces leads to higher power mismatch and gradually degrades the total performance of the field transformation. These effects can be mitigated by allowing multiple reflections between the two metasurfaces. Although the total configuration remains reflectionless, multiple reflections effectively increase the propagation length and lead to better power matching, especially for smaller separation lengths. The same field transformation is examined with a distance \( d = 0.5\lambda \) between the two metasurfaces and the use of multiple reflections. The designed pair of metasurfaces is simulated and the electric field is plotted in Fig. 5. Standing-wave patterns with higher field values can be observed in the region between the two metasurfaces, since these are partially reflective and form a cavity (open to the sides) between them. The electric field at the output aperture still exhibits nearly uniform amplitude and phase despite the reduced size of the metasurface pair.

To better evaluate the usefulness of multiple reflections at reducing the size of our design, the reflectionless scenario is also simulated and the two cases are compared in terms of the far-field characteristics. The radiation pattern is plotted in Fig. 6, where it is evident that the use of multiple reflections facilitates the wave transformation and a nearly uniform aperture is obtained, although the metasurfaces are placed only \( d = 0.5\lambda \) apart. When multiple reflections are present, the obtained directivity is 12.6 dB, the HPBW is 8.8° and the sidelobe level (SLL) is \(-13.2\) dB, which are all very close to the theoretical values for a uniformly illuminated aperture. On the contrary, in the totally reflectionless case the directivity is reduced to 11.37 dB, while the HPBW is increased to 12.2°. Defining the aperture efficiency as the ratio...
of the obtained maximum directivity divided by the theoretical value of a uniform aperture, this results in only 73% in the totally reflectionless case compared to 97% aperture efficiency with the use of multiple reflections in the design.

B. Beam Expansion for an Incident Gaussian Beam

Beam expanders are commonly used in optics to alter the width of collimated light beams [25]. In their traditional form, they consist of two dielectric lenses of appropriate focal lengths to provide the required scaling to the beam radius. Recently, a pair of omega-bianisotropic HMSs has been employed at microwaves to design a beam expander [26]. In [26] both metasurfaces are purely transmissive and act on the incident fields by adding an appropriate phase profile, so that the first serves as a diverging lens and the second as a converging lens with the designed focal lengths. However, this approach has limitations regarding the separation between the two metasurfaces for a desired scaling factor of the beam radius; as the distance between the two metasurfaces becomes smaller, the performance of the transformation is severely degraded. To overcome these limitations, multiple reflections can be utilized, as suggested in our previous example.

In this example, the two metasurfaces are separated by a distance of \( d = 0.5\lambda \), where \( \lambda \approx 30 \text{ mm} \) is the wavelength at a frequency of 10 GHz. Both metasurfaces are \( L_{\text{out}} = 9\lambda \) wide and are discretized with 101 unit cells (resulting approximately at a \( \lambda/11 \) unit cell size). The input wave is assumed to be a Gaussian beam with its focus at M1; thus, the electric field at M1 is the following:

\[
E_{\text{in},z}(x) = A_{\text{in}} \exp\left(-x^2/w_{\text{in}}^2\right) \quad (19)
\]

where \( w_{\text{in}} = 1.5\lambda \) is the beam waist and \( A_{\text{in}} = 10^3 \text{ V/m} \) is an arbitrarily chosen peak amplitude. By decomposing the electric field in (19) to its plane-wave components, computing the magnetic field for each one of them and summing the individual contributions, we can calculate the total magnetic field at M1 as

\[
H_{\text{in},z}(x) = \frac{1}{2\pi} \int_{-k}^{k} \frac{k_y}{\eta k} \int_{-L_{\text{out}}/2}^{L_{\text{out}}/2} E_{\text{in},z}(x) e^{jk_{y}x} dx e^{-jk_{z}x} dk_x. \quad (20)
\]

As in the case of the unknown forward and reflected waves, it is assumed that the field spectrum is sufficiently decayed for \( |k_x| > k \), and the integration in (20) is constrained only to the propagating plane-wave components. The aim is to get an output Gaussian beam with a waist of \( w_{\text{out}} \approx 3\lambda \) at M2, while conserving the total propagating power. The output tangential fields at M2 are specified similar to (19) and (20) with the output beam waist \( w_{\text{out}} \) and an appropriately chosen amplitude \( A_{\text{out}} \) to ensure that the total real power for the two distributions is conserved.

First, we design the beam expander based on two independently defined metasurfaces that add a phase profile on the transmitted fields, the first acting as a diverging lens with focal length \( f_1 = -d \) and the second as a converging lens with focal length \( f_2 = 2d \). Second, we compare this approach with the proposed optimization method including multiple reflections in the formulation. The use of multiple reflections allows for better power matching at the two metasurfaces, as expected from the higher number of degrees of freedom involved in the optimization process. Consequently, the field transformation is expected to be more accurate for the same separation distance, compared to the case of two totally reflectionless metasurfaces that act solely on the phase similar to traditional lenses. Both designs are simulated and the real part of the electric field is depicted in Fig. 7. It is clearly observed that using purely transmissive phase-changing HMSs results in considerable deviations from the desired output fields and reflections that perturb the fields in the input region before M1.

The same issues are also present for this distance between the two metasurfaces, if the proposed optimization method is used but without introducing multiple reflections. On the contrary, the above-mentioned problems are substantially mitigated when multiple reflections are considered, as it is evident from Fig. 7(b). Specifically, it is evident that M2 is partially reflected and illuminates parts at the edges of M1 that were not initially illuminated from the incident wave, facilitating in that way the broadening of the Gaussian beam over a relatively small separation distance.

To better evaluate the accuracy of the field transformation, the real power density is plotted at two cuts of the
configuration; the first at a distance $\frac{\lambda}{2}$ after the second metasurface. This small offset is introduced so that any rapid field oscillations close to the metasurfaces owing to the discretization and the discontinuity of the fields at the boundary are sufficiently decayed. The normalized power densities are plotted together with the theoretical input and output power distributions in Fig. 8. Both the input and the output power density profiles are close to the theoretical ones for the design that includes multiple reflections between the two metasurfaces. This suggests that the desired field transformation is successfully performed for this case, while noticeable deviations exist in the design involving phase-changing reflectionless metasurfaces. Lastly, it should be noted that the power efficiency with the proposed design method is 94% with only 6% of the incident power being reflected or escaping through the two sides.

C. Single-Sided Beamforming With a Single Line-Source in the Middle of the Metasurface Pair

In various antenna applications, radiation patterns with specific characteristics (directivity, SLL, main lobe beamwidth, etc.) are required. To this purpose, metasurfaces have been used in different configurations to obtain the aperture fields at the output that would produce the desirable radiation pattern in the far-field region [15], [22]. In the following, we demonstrate how the proposed design of a pair of omega-bianisotropic HMSs can be utilized for the realization of a low-profile Taylor (one parameter) antenna that is excited by a single current line-source placed within the metasurface pair, as shown in Fig. 1(b). The metasurfaces are $L_{\text{tot}} = 6\lambda$ wide ($\lambda$ being the free-space wavelength), the operating frequency is $f = 10$ GHz, and the separation distance is set to $d = 0.75\lambda$. The current line-source is placed exactly in the middle of M1 and M2 ($y = d/2$) and radiates a cylindrical wave that has the following tangential components at the two metasurfaces:

$$E_{\text{inc},z}^{(2)}(x) = -\frac{k\eta I}{4} H_0^{(2)}\left(k\sqrt{x^2 + (d/2)^2}\right), \quad (21a)$$

$$H_{\text{inc},x}^{(2)}(x) = \mp\frac{jkd}{4\sqrt{4x^2 + d^2}} H_1^{(2)}\left(k\sqrt{x^2 + (d/2)^2}\right) \quad (21b)$$

where $I = 1$ A is the current amplitude and $p = \{1, 2\}$ refers to the fields at the corresponding metasurface and the minus (plus) sign is taken in (21b) for $p = 1$ ($p = 2$).

Since a single output is desired, the fields $[E_{\text{out},z}^{(1)}, H_{\text{out}}^{(1)}]$ below M1 are set to zero. On the contrary, the electric field at the upper output is defined based on the Taylor distribution for a SLL of $-20$ dB [27], specifically

$$E_{\text{out},z}^{(2)}(x) = AJ_0\left[j\pi B\sqrt{1 - \left(\frac{2x}{l}\right)^2}\right] \quad (22)$$

where $J_0$ is the Bessel function of the first kind and zero order, $l = 6\lambda$ is the total length of the Taylor aperture, $B = 0.7386$ is a parameter that sets the SLL to the desired level of $-20$ dB, and $A$ is a normalization parameter to equalize the total output power at the upper side with the total incident power, as defined in (14). The tangential magnetic field is computed using the plane-wave decomposition, as given in (20).

Having determined the incident and the output fields at the two metasurfaces, the proposed optimization method is used to design the metasurface pair and the total structure is simulated in ANSYS HFSS. The real part of the electric field $E_z$ is plotted in Fig. 9, where it is clear that the waves destructively interfere in the lower output resulting in very low transmission to this side, as desired. In addition, most of the power provided from the source escapes from the upper output according to the amplitude and phase profile defined in (22). To verify the accuracy of the transformation, the far-field radiation pattern is calculated and it is compared with the theoretical one, as predicted from the equivalence principle applied at the output of M2. As is observed in Fig. 10, the curve obtained...
from the simulation matches very closely with the theoretical one regarding the maximum directivity, the amplitude of each lobe and the locations of the nulls. In particular, the simulated SLL and HPBW are $-19.91$ dB and $9.02^\circ$ compared to theoretical values of $-19.7$ dB and $8.94^\circ$, respectively. It should be noted that the theoretically predicted SLL is not exactly $-20$ dB, because the equivalence principle takes into account the discretization of the output fields, according to the width of the unit cells. More significant deviations between theory and simulation results can be observed in Fig. 10 for angles far away from broadside. However, the amplitude in these angles ($\phi < 45^\circ$ and $\phi > 45^\circ$) is more than 30 dB lower compared to broadside; thus, even a small power-density mismatch at the boundaries or numerical issues can affect the simulated values. Regarding the power efficiency, it is calculated that 95.4% of the total input power is transmitted to the upper output, while the power leakage below M1 and to the sides is only 0.7% and 3.9%, respectively. It is pointed out that the total power at the upper output slightly exceeds the total incident power (accounting for 92.1 of the input power), as defined in (14). This is attributed to the partial cancellation of the power leaking to the sides, as the cylindrical wave that directly escapes the metasurface pair interferes destructively with the power escaping due to reflections at the inner boundaries of the two metasurfaces.

D. Double-Sided Beamforming With a Single Line-Source in the Middle of the Metasurface Pair

The beamforming example shown in Section III-C can be extended so that both output sides support two independent field distributions. This case can be perceived as a generalization of the cavity-based antenna presented in [15], where the PEC wall used is replaced with a second omega-bianisotropic HMS. For this scenario, the geometry is identical to the one in Section III-C, but the aim is to have a Taylor-antenna output at both sides of the metasurface pair instead of redirecting all the incident power to the upper side. The Taylor antenna characteristics at both sides are the same as in Section III-C (SLL of $-20$ dB and aperture length $l = 6\lambda$) with an additional requirement that the lower-side Taylor antenna has its maximum directivity at the azimuthal angle $\phi = -110^\circ$ (20° off-broadside). Therefore, the output electric field at M1 is not set to zero, but takes the form of (22) with an additional linear phase to account for the tilting of the maximum directivity direction. Lastly, the amplitudes of the field distributions are normalized so that the total incident power is equally divided at the two sides. It is emphasized that any other unequal power splitting would also be possible, as long as it sums up to the total incident power defined in (14).

The real part of the electric field, as given by full-wave simulations, is depicted in Fig. 11. It is clear that the two metasurfaces form a cavity and the power leaks from both output sides according to the specified field distributions. In particular, the output aperture at M2 radiates toward broadside ($\phi = 90^\circ$), while the output aperture at M1 radiates 20° off-broadside ($\phi = -110^\circ$). As expected, the field values are higher within the metasurface pair, where the multiple reflections produce a higher power concentration. However, as we move horizontally away from the center, the values continuously decay and only a relatively small portion of the power leaks from the two open sides.

The radiation pattern is calculated independently for the two output sides, by selecting the respective boundaries to perform the near-field to far-field transformation. The simulated results...
are given in Fig. 12 and they compare well with the theoretical radiation patterns obtained from the equivalence principle. It is clear that the power is radiated at both sides toward the desired angles. The difference between simulation and theory regarding the directivity and the HPBW is less than 0.05 dB and 0.15°, respectively, for both sides. Moreover, the SLL is only 1 dB higher at the lower output and 1.1 dB lower at the upper output compared to the predicted values. Although some deviations from theory exist for the minor lobes at both outputs, it can be seen that they generally decay away from the angle of maximum radiation, as expected. Lastly, regarding the power splitting, it is estimated that 50.9% and 48.5% of the total input power is guided to the lower and upper outputs, respectively, while only 0.6% escapes from the two sides.

IV. CAVITY EFFECTS ON THE SENSITIVITY AND THE BANDWIDTH OF THE METASURFACE PAIR

From the previous examples, it can be observed that the introduction of multiple reflections into the design leads to an energy build-up within the metasurface pair. As demonstrated, this cavity effect is essential to reduce the separation distance between the two HMSs, when designing for wave transformations that do not locally conserve power. However, the internal reflections induced by the two partially reflective HMSs can substantially decrease the bandwidth of the proposed structure and, in general, affect its sensitivity with respect to the geometrical parameters involved. This effect can be interpreted as an outcome of the interaction between the two metasurfaces, as small errors due to variations in the frequency or the geometry will accumulate as the propagating wave reflects multiple times at the two boundaries. Alternatively, perceiving the structure as a cavity, the multiple reflections increase the energy stored within the cavity with respect to the total radiated power, thus resulting in a higher quality factor and a smaller bandwidth.

In this section, the tradeoff between the separation distance and the sensitivity or the bandwidth of the structure is investigated, based on the example of the uniform output aperture presented in Section III-A. First, the sensitivity of the structure is examined by varying the distance \( d \) between the two metasurfaces in a range around its nominal value. In this way, the parameters of the two metasurfaces remain constant, and it is possible to study separately only the effect of multiple reflections. Second, the frequency bandwidth of the structure is estimated in the case of three nondispersive impedance layers for each metasurface, as described in Section II-C.

To begin with, we vary the distance \( d \) for the example including multiple reflections, as designed in Section III-A for a nominal distance \( d = 0.5\lambda \). The reduction in the maximum directivity and the power efficiency, defined as the ratio between the total output power and the total incident power at M1, are calculated for each variation and the results are plotted in Fig. 13. It is clear that the structure is highly sensitive, since only a ±4% variation in \( d \) results in approximately 1 dB drop of the directivity and noticeable reflections, as the power efficiency drops to around 70%–80%. The performance becomes even worse for larger variations of the distance \( d \), implying that an implementation of such a structure would be challenging, especially for higher frequencies. To compensate for the cavity effects, a slightly larger distance may be preferred for certain applications. The wave transformation is redesigned using multiple reflections for a nominal distance \( d = 0.75\lambda \). While the radiation pattern for the designed distance is very close to the ideal, the sensitivity with the relative distance variation \( \Delta d / d \) is milder, as confirmed from Fig. 13.

Another way to mitigate the cavity effects without increasing the separation distance \( d \) would be to modify...
the input power density so that it is more distributed along M1. To explore this possibility, two current line-sources are added at the same distance $s = \lambda/3$ from M1, but displaced by $\pm 1.25\lambda$ in the $x$-direction with respect to the center source. In addition, the current amplitude of the edge sources is set to be three times less than the current amplitude of the center one. The optimization procedure is slightly modified, as the structure is first designed with purely transmissive omega-bianisotropic HMS ($A_{r,n} = 0$). The solution is then used as a starting point for the optimization algorithm including multiple reflections. The outcome of this two-step optimization process is to keep the amplitude of the backward-propagating wave at a relatively low value compared to the forward-propagating wave. The wider illumination of M1 combined with the modification of the optimization process leads to greater immunity of the transformation performance when varying the distance $d$ away from its nominal value, as it is evident for this case in Fig. 13. In particular, it can be observed that for a $\pm 20\%$ variation of the separation distance, the directivity drops less than 0.5 dB, while the power efficiency remains above 60$. While the structure is no longer single-fed and three sources should be controlled independently, this choice can also be considered, if it is necessary to maintain compactness and design a metasurface pair that is less prone to geometrical or frequency variations.

Finally, we study the bandwidth for the above-mentioned three designs for a uniform output aperture. Each metasurface is represented with three dispersionless impedance layers that are separated by two substrates with a dielectric constant $\epsilon_r = 10.7$ (as in Rogers 6010 substrates) and a realistic thickness $t = 1.9$ mm. Perfect mangetic conductor (PMC) walls are also utilized between the unit cells to restore rectilinear propagation within the metasurface, as presented in [28]. Therefore, the frequency analysis captures not only the change of the electrical dimensions of the structure, but also the change of the metasurfaces' response due to the thickness of the substrates between the impedance layers. The maximum directivity as a function of frequency is plotted in Fig. 14. As observed, the single-source design with a separation distance $d = 0.5\lambda$ is the most narrowband with a 1-dB bandwidth of $\Delta f = 0.23$ GHz or 2.3% of the center frequency. However, the bandwidth of the structure can be increased to 3.9% or 8.4% by increasing the separation distance to $d = 0.75\lambda$ or adopting the three-source illumination, respectively. It is also important that within these frequency ranges the power efficiency is maintained above 75%.

V. Conclusion

In conclusion, wave transformations that do not locally conserve real power have been designed using pairs of HMSs. A design method has been developed, based on determining the electric and magnetic fields between the two metasurfaces, so that the power density is matched simultaneously at the two boundaries. The method relies on expanding the field distributions as weighted basis-functions summations and, then, minimizing the power mismatch across the metasurfaces through optimization of the unknown weights. By allowing for multiple reflections, it was shown that the required distance between the two metasurfaces can be significantly reduced, resulting in compact, yet accurate, field transformations. The design method is also adjustable to applications which require the source to be located within the metasurface pair. Through the example of a Taylor-pattern antenna fed by a single line-source, it was shown that a desired output can be supported at one or both sides of the metasurface pair. Finally, the sensitivity with respect to the separation distance between the two metasurfaces was discussed, when multiple reflections are present, revealing a tradeoff between the bandwidth and the size of the structure.

APPENDIX A

Effect of Losses on the Performance of Metasurface Pairs

Although the main purpose of this article is to present a design method for nonlocal power conserving transformations, it is useful that the effects of losses at the two metasurfaces are also assessed. The losses originate from the dielectric losses of the substrates and the ohmic losses of the copper traces at each of the three impedance layers. The first contribution is introduced into the full-wave simulations through a loss tangent $\tan\delta = 0.0023$ (as in standard Rogers 6010 substrates) characterizing the dielectrics between the layers. For the impedance layers, their required values as computed from the transmission line model for a passive and lossless metasurface are purely imaginary. For instance, the admittance profiles of the layers for the case of a uniform output aperture including multiple reflections are given in Fig. 15, when substrates of thickness $t = 1.9$ mm and dielectric constant $\epsilon_r = 10.7$ are considered. In order to account for the copper losses, the admittance values should be supplemented with a corresponding positive real part. To evaluate these real values, dogbone and dipole layers are simulated as the outer and middle layers, respectively, and their admittances (both the real and imaginary parts) are characterized for different geometric characteristics, as described in [17]. Then, the real parts of the admittances are determined based on the imaginary values for
of the middle layers are paired with higher copper losses. Each unit cell and they are included in the simulated model. Naturally, higher required admittance values for the imaginary parts are associated with higher losses (real parts) as a design closer to the resonance of the layer is necessary.

Simulations are performed for the uniform output aperture example including multiple reflections. The values of the metasurface pair gain, defined as the maximum directivity multiplied by the efficiency of the metasurface pair, the SLL and the HPBW are recorded in Table I. In addition to the lossless and lossy designs, a scenario where only dielectric losses are considered is provided to quantify the relative contribution of the two loss mechanisms. As observed, the introduction of losses at the two metasurfaces drops the gain by around 2 dB, while the SLL and HPBW remain close to the lossless model values. It is noted that the estimation of the gain drop is in agreement with previously reported, purely transmissive, metasurfaces at microwave frequencies which showed a power dissipation of around 20% due to losses (corresponding to $\approx 1$ dB) for a single HMS [10], [17].

![Fig. 15. Admittance values (imaginary parts) of the first (up), second (middle), and third (bottom) layers of the two metasurfaces discretized in 61 unit cells for the example of uniform output aperture. The higher values of the middle layers are paired with higher copper losses.](image)

### Table I

| Loss Type          | Gain (dB) | SLL (dB) | HPBW (degrees) |
|--------------------|-----------|----------|----------------|
| Lossless           | 12.33     | 13.2     | 8.8            |
| Only dielectric    | 11.57     | 13.5     | 9.1            |
| Lossy              | 10.37     | 13.4     | 9.4            |

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