Momentum Resolved tunneling in a Luttinger Liquid

S. A. Grigera,1 A. J. Schofield,1 S. Rabello,2 and Q. Si2

1School of Physics and Astronomy, University of Birmingham, Birmingham B15 2TT, United Kingdom.
2Department of Physics, Rice University, Houston, TX 77251-1892

We consider momentum resolved tunneling between a Luttinger liquid and a two dimensional electron gas as a function of transverse magnetic field. We include the effects of an anomalous exponent and Zeeman splitting on both the Luttinger liquid and the two dimensional electron gas. We show that there are six dispersing features that should be observed in magneto-tunneling, in contrast with the four features that would be seen in a non-interacting one dimensional electron gas. The strength of these features varies with the anomalous exponent, being most pronounced when \( \gamma_\rho = 0 \). We argue that this measurement provides an important experimental signature of spin-charge separation.

I. INTRODUCTION

Haldane’s Luttinger-liquid hypothesis— that all one-dimensional metals are adiabatically continuous with the Tomonaga-Luttinger model— has underpinned our current understanding of the metallic state in one-dimension. The low-energy properties of the metal are characterized by separate spin and charge velocities \((v_\sigma \text{ and } v_\rho)\) and, at most, two further anomalous exponents \((\gamma_\sigma \text{ and } \gamma_\rho)\). The low-energy excitations are completely different from those of the non-interacting electron gas. The one-dimensional metal is described in terms of spinons and holons rather than quasi-electron-like excitations. As a result the low energy spectrum has no overlap with the corresponding non-interacting one and the metal is therefore a non-Fermi liquid.

Although much is known theoretically about the properties of a Luttinger liquid (see, for example, Voit in Ref. 5), experimental verification of these ideas is ongoing. A wide variety of measurements have been performed and interpreted within the Luttinger-liquid framework. These include work on the quasi-1D organics, inorganic charge-density wave materials, semiconductor quantum wires and edge states in the fractional quantum Hall regime. However, most of these experiments have focused on identifying the anomalous exponents. Experiments which directly probe the separation of charge and spin in one dimension have proved to be more challenging. Arguably the most convincing measurements have been those of angle-resolved photoemission in metals and insulators. Nevertheless, there remains a need for a low-energy probe of the excitation spectrum of the Luttinger liquid.

In a recent Letter, Altland et al. proposed a novel spectroscopy of the Luttinger liquid state using magneto-tunneling. They showed how the tunneling conductance between a quasi-1D metal and a two-dimensional electron gas (the spectrometer) responds to a transverse magnetic field and allows features associated with the spinon and holon dispersion to be resolved. This momentum-conserving tunneling spectroscopy then provides a method of determining the low energy spectrum and identifying features associated with separate spin and charge excitations. In that Letter the authors considered, for simplicity, the special case where the anomalous exponents \( \gamma_\rho \) and \( \gamma_\sigma \) were both equal to 0. They also ignored the effect of the magnetic field on the spectral functions. The magnetic field was assumed to simply tune the relative momentum of the tunneling electron as it moves between the one-dimensional metal and the two dimensional electron gas. This same tuning can also be achieved by changing the carrier density (and hence \( k_F \)) in either the wire or the two-dimensional electron gas. Experimentally, using a transverse magnetic field is likely to be by far the easiest way of tuning the intra-chain momentum. The work of Altland et al. raised two further interesting questions that we will address here. Firstly, how sensitive are the tunneling results to the value of the anomalous exponent? Perhaps more importantly, would the Zeeman splitting of Fermi-liquid quasiparticles give rise to two features and thereby mimic the spin-charge separation that the experiment was supposed to resolve.

In this paper we revisit the idea of momentum dependent tunneling and solve for the tunneling conductance for arbitrary anomalous exponent \( \gamma_\rho \). (The other exponent, \( \gamma_\sigma \) is equal to 0 in any rotationally symmetric system.) Recently two of the present authors have also calculated the change in the spectral function Luttinger liquid spectral function due to a magnetic field. Using this result, we have now computed the tunneling conductance beyond the restrictions of Ref. 11. We find that the signature of magneto-tunneling into a Luttinger liquid is radically different from that in a non-interacting one dimensional metal and the magnetic field reinforces this difference. The dispersion of spinons and holons may be separately identified from sharp features in the tunneling conductance. However, these features become less singular as the anomalous exponent varies away from unity.

The outline of the paper is as follows. We begin by establishing the formalism for magneto-tunneling when momentum parallel to the wire is conserved. We then introduce the spectral functions for arbitrary \( \gamma_\rho \) but initially ignore any change induced by the magnetic field. Next, we show how the magnetic field can straightforwardly be taken into account within this formalism and we compute the tunneling conductance. Finally we dis-
we know theoretically that there should be features in the electron spectral function related to the underlying excitations: spinons and holons. The dispersion of the spinons and holons can be seen in Fig. 2(a) as two lines of singularities of the spectral function in the $\omega, q$ plane. We assume that there is a two-dimensional Fermi liquid on the other side of the tunnel barrier. In a Fermi liquid there are electron-like quasiparticles which are reflected in the spectral function as a single line of singularities in the $\omega, q$ plane [see Fig. 2(b)].

It is this profound difference in the nature of the excitations of a Luttinger liquid compared to a Fermi liquid that the magneto-tunneling experiment is designed to expose. Essentially the measurement measures the relative dispersion of the singular features in the 1D and 2D spectral functions as follows. The tunnel current is given by the integrated product of the two spectral functions of Fig. 2 with the magnetic field giving a relative offset along the $q$ direction. Fig. 3 shows that this product (and hence the current) divides into four distinct regions (a) to (d) depending on this magnetic field dependent offset and the dispersion of the singular features in the spectral functions. Thus the tunneling conductance shows three abrupt features separating the four regions as a function of applied magnetic field. These features can be seen in Fig. 4. Finally when one includes the Zeeman splitting there are separate tunneling processes for up- and down-spin electrons and the three abrupt features each split in two giving a total of six features in the conductance. This is shown in Fig 5. The dispersion of these features as the magnetic field and applied voltage is changed also allows us to determine the relative velocity of the spinon and holon is shown in Fig. 6. In the absence of spin-charge separation there would be only one dispersing singularity in the 1D spectral function leading to four features in the tunneling conductance as a function of field. This then is the essence of our results and in the rest of the paper we give a more precise derivation of them.

III. FORMALISM

We consider single-electron tunneling between a one-dimensional interacting electron metal, parallel to the $x$ axis, and a two-dimensional electron gas in the $xy$ plane separated from the 1D wire by a distance $d$ along the $z$ direction. A potential difference is applied between the wire and the two-dimensional electron gas and a magnetic field is applied in the plane of the two dimensional electron gas but perpendicular to the wire (along the $y$ direction). The geometry is shown schematically in Fig. 1. The appropriate formalism was first derived in the context of superconductivity. The Hamiltonian for the system may be written as

$$\hat{H} = \hat{H}_{1D}(B) + \hat{H}_{2D}(B) + \hat{H}_T .$$

This differs from the Hamiltonian considered in Ref. [11] since we have explicitly allowed a coupling of the mag-
is smooth—and so may be written as \( t(x - x_{2D}, y_{2D}) \). In momentum space then this coupling may be written as

\[
\hat{H}_T = \sum_k t_k \hat{\phi}_{k_x-x_{2D}/2,k_y,\sigma}^\dagger \hat{\psi}_{k_x+y_{2D}/2,\sigma} + \text{H.c.},\]  

(3)

where \( q_B = eBd + k_{2D}^x - k_{1D}^x \). Thus we see that momentum parallel to the 1D wire is conserved up to the change induced by moving the applied magnetic field. The applied field then tunes the tunneling momentum of the electron. Here \( t_k \) is the Fourier transform of the tunneling matrix element.

Given the tunneling Hamiltonian, it is then straightforward to determine the tunneling current (see, for example, Mahan\(^{14}\)). Note that our starting Hamiltonian neglects any interactions between the two dimensional electron gas and the quantum wire—the only coupling is via single electron tunneling. Thus with this assumption there are no vertex corrections in the tunneling current and it can be written directly in terms of the single electron Greens function for the 2D electron gas and the quantum wire. The form for the current is most intuitively expressed in terms of the electron spectral functions for the 2D and 1D systems respectively

\[
I(B,V) = \int d\omega \sum_k t_k^2 \{ f(\omega) - f(\omega - eV) \}
\]

(4)

For the purposes of this paper we assume that the tunneling is through nearest contact only so that \( t(x - x_{2D}, y_{2D}) = t_0 \delta(x - x_{2D}) \delta(y_{2D}) \). It would be straightforward to relax this assumption by introducing an additional “aperture function” which would need to be included in the momentum convolution.

**IV. TUNNELING WITH GENERAL \( \gamma_\mu \)**

We start by specifying the spectral functions used in the calculation of the current. For the two-dimensional system the spectral function \( A_{2D,\gamma} \) for energies close to the Fermi energy (and integrated over the momentum component transverse to the wire) is given by\(^{15}\) (see Fig. 2)

\[
A_{2D}(q,\omega) = \sqrt{2m} \frac{\Theta(\omega - q v_F)}{\sqrt{\omega^2 - q v_F^2}}.
\]

(5)

The spectral function of a Luttinger liquid with spin rotation-invariant interactions \( (\gamma_\mu = 0) \) is non-trivial (see Fig. 2), and a closed and tractable analytic expression is still lacking in the literature. The asymptotic behavior, on the other hand, is well characterized\(^{15}\). At very small \( q \), the function looks very similar to a spinless fermions’ function. As \( q \) is increased two peaks become apparent, a reflection of spin-charge separation, one at \( v_\rho q \) and another at \( v_\sigma q \), where \( v_\rho \) and \( v_\sigma \) are the velocities of charge
and spin density waves, respectively. The exponent of the singularity at \( v_\sigma q \) is \( 2\gamma_\rho - \frac{1}{2} \) and the corresponding one at \( v_\rho q \) is \( \gamma_\rho - \frac{1}{2} \). The function terminates at negative \( q \) at \(-v_\rho q\) with a non-singular exponent \( \gamma_\rho \). The parameter \( \gamma_\rho \), which characterizes the interactions between left and right movers is always positive. The case of non-interacting left and right branches, which was studied in reference [11] corresponds to the case \( \gamma_\rho = 0 \). As \( \gamma_\rho \) increases, the power law divergences gradually weaken into cusp singularities, and the spectral weight, which for \( \gamma_\rho = 0 \) is confined within \( v_\sigma q \) and \( v_\rho q \) is gradually transferred by the electronic correlations toward higher values of \( q \) and \( \omega \). The spectral function should also be invariant to the transformations \((p \rightarrow -p; q \rightarrow -q)\) where \( p = \pm \) labels left and right movers, and \((\omega \rightarrow -\omega; q \rightarrow -q)\).

For \( \omega > 0 \), the case of our interest, a function that has the correct singularities and asymptotic behavior can be written as

\[
A_{1D}(q, \omega) = \frac{W(q, \omega) \Theta(\omega - qv_\rho)}{|\omega - v_\sigma q|^{\frac{1}{2} - 2\rho} |\omega - v_\rho q|^{\frac{1}{2} - \rho}} \quad \text{where} \quad W(q, \omega) = \begin{cases} \Theta(qv_\rho - \omega) + \Theta(\omega - qv_\rho)c(\gamma_\rho) & \text{if } \gamma_\rho \neq 0 \\ \Theta(qv_\rho - \omega) & \text{if } \gamma_\rho = 0 \end{cases}
\]

for \( q > 0 \) plus a non-divergent term for \( q < 0 \) and \( \gamma_\rho \neq 0 \) which is proportional to

\[
\Theta(\omega + v_\rho q)c(\gamma_\rho)(\omega + v_\rho q)^{\gamma_\rho}.
\]

This function is not normalizable for \( \gamma \neq 0 \). In our case the consequences of this are merely reduced to an undetermined constant that multiplies the conductivity for each \( \gamma_\rho \). In this paper we will consider the case \( v_F > v_\rho > v_\sigma \), but it should be noted that the results can be trivially extended to consider the other possible cases.

Substituting equations \[3\] and \[4\] into \[4\] we find for the tunneling current at \( T = 0 \)

\[
I(V, B) = \frac{4\sqrt{2}I_0(eV)\frac{1}{2} + 3r}{\pi \sqrt{m v_F}} \sum_{\alpha} \int_{l_\alpha}^{u_\alpha} dx \int_{L_\alpha}^{U_\alpha} ds \frac{1}{|sa_\sigma - x|^2 |sa_\rho - x|^{\gamma_\rho} |s - x + (r - 1)|^{\gamma_\sigma}}.
\]

where we have introduced the dimensionless parameters \( r = g_B v_F/eV, a_\rho = v_F/v_\rho \) and \( a_\sigma = v_F/v_\sigma \) and the dimensionless one-dimensional variables \( x = qv_F/eV \) for the 1D wire wave-vector and \( s = \omega/eV \) for the frequency. \( I_0 = e|t|^2 m/\pi \) is the natural unit for current in this problem. From this integral we identify four different regions with different qualitative behavior, \( R_j, j = 1 \ldots 4 \), corresponding to different situations of overlap between the one- and two-dimensional spectral functions. Figure \[3\] is an schematic representation of the relative positioning between \( A_{1D} \) and \( A_{2D} \) in the four regions. In terms of the dimensionless parameters these are given by \( R_3 \): \( r; R_2: 1 \leq r \leq a_\rho; R_3: a_\rho \leq r < a_\sigma; \) and \( R_4: r > a_\sigma \). In each region, and for practical reasons only, the calculation is in turn split into different integrals of the same integrand. Table \[1\] lists the upper and lower limits corresponding to each different region. Although the majority of these integrals cannot be integrated analytically, the asymptotic behavior in the different regions can be obtained by standard calculus procedures. The \( q < 0 \) non-singular part of the spectral function for \( \gamma \neq 0 \) (equation \[7\]) only contributes a small featureless onset of conductivity at \( r = -a_\rho \), and a background of finite conductivity noticeable only for small values of \( r \) and big values of \( \gamma_\rho \).

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Figure \[3\] shows the differential conductance \( G = dI/dV \) as a function of the dimensionless parameter \( r \) at \( T = 0 \). In the following we discuss the behavior of \( G \)

| Region | \( \alpha \) | \( l_\alpha \) | \( u_\alpha \) | \( L_\alpha \) | \( U_\alpha \) |
|--------|-------------|-------------|-------------|-------------|-------------|
| \( R_1 \) | 1 | 0 | \( r \) | \( x - (r - 1) \) | 1 |
| \( R_2 \) | 2 | \( \frac{r - 1}{a_\rho - 1} \) | \( sa_\rho \) | \( sa_\rho \) | \( sa_\rho \) |
| \( R_3 \) | 3 | \( \frac{r - 1}{a_\sigma - 1} \) | \( sa_\rho \) | 0 | \( sa_\rho \) |
| \( R_4 \) | 4 | \( \frac{r - 1}{a_\rho - 1} \) | 1 | 0 | \( sa_\rho \) |
| \( R_5 \) | 5 | \( \frac{r - 1}{a_\sigma - 1} \) | \( sa_\rho \) | \( sa_\rho \) | \( sa_\rho \) |
| \( R_6 \) | 6 | 1 | 0 | \( sa_\rho \) | \( sa_\rho \) |
| \( R_7 \) | 7 | 0 | \( 1 \) | \( 0 \) | \( sa_\rho \) |

in each region.

\( R_1 \) For \( r < 1 \) and \( \gamma_\rho \neq 0 \) \( A_{2D} \) overlaps with the non-divergent part of \( A_{1D} \), leading to a finite but non-singular flow of current, with negative differential conductance. When \( r \) reaches the value of \( 1 \), the conductance diverges as \( y \sim -(1 - r)^{-1/2 + 3\gamma_\rho} \), where \( y = \)
FIG. 3: The tunnel current is determined by the product of the spectral functions of Fig. 2(a) and (b) offset in momentum by $q_B$, an amount proportional to the transverse magnetic field. As the field is increased the region of overlap goes through four stages (a) to (d) labeled $R_1$ to $R_4$ in the text. The light dark and shaded areas represent, respectively, the areas where the one-dimensional and the two-dimensional spectral functions have a finite non-zero value; the thick lines represent the lines of singularities.

$$G \sqrt{E_F} e^{\frac{1}{4} + 3\gamma_\rho} V^{-\frac{1}{4} + 3\gamma_\rho} / I_0$$ is a dimensionless measure of the conductance. For $\gamma_\rho = 0$ the two areas do not overlap, implying that the current vanishes.

$R_2$.— For $r > 1$, the spectral functions for $\gamma_\rho = 0$ start overlapping as well, and the conductance diverges as $g \sim -(r-1)^{-1/2+3\gamma_\rho}$. For finite $\gamma_\rho$, however, the cusp is also not symmetric (see Fig. 4) since the spectral weight of the singularity is different at either side of $qv_\rho$. The behavior of the conductance is unaltered up to the boundary to $R_3$, where

$$g(a_\rho^+) - g(a_\rho^-) = \frac{a_\rho}{a_\rho - 1} \frac{1/2 - 2\gamma_\rho}{1/2 + 2\gamma_\rho} a_\rho \lim_{r \to a_\rho} (r - a_\rho)^{\gamma_\rho},$$

$$a_\rho^* = a_\rho, \pm \delta, \delta$$ infinitesimal and positive. This implies that for $\gamma_\rho = 0$, the conductance exhibits a discontinuity $\Delta$, the magnitude of which is

$$\Delta = \frac{a_\rho}{a_\rho - 1} \sqrt{\frac{a_\sigma a_\rho}{a_\sigma - a_\rho}}. \quad (10)$$

For every non-zero value of $\gamma_\rho$ this step is rounded off (see Fig. 4) into a continuous function with a pronounced change at $a_\rho$, the change decreasing progressively as $\gamma_\rho$ is increased.

$R_3$.— In this region the differential conductance becomes positive. As $r$ approaches the singularity line corresponding to $a_\sigma$, the boundary with $R_4$, the conductance shows a pronounced increase. Again the case of $\gamma_\rho = 0$ is unusual, since the boundary between $R_3$ and $R_4$ shows a singularity, which is of logarithmic type:

$$g(r, \gamma_\rho = 0) \to \lim_{r \to a_\sigma} - \frac{a_\sigma}{a_\sigma - 1} \sqrt{\frac{a_\rho a_\sigma}{a_\rho - a_\sigma}} \ln(a_\sigma - r). \quad (11)$$

On the other hand, for any non-zero $\gamma_\rho$, the behavior is found to be

$$g(r) \to \lim_{r \to a_\sigma} - \frac{a_\sigma}{a_\sigma - 1} \sqrt{\frac{a_\rho a_\sigma}{a_\rho - a_\sigma}} \ln(a_\sigma - r).$$

which is a non-divergent peak (see Fig. 4).

$R_4$.— The boundary is symmetric in $r$ around $a_\sigma$. The asymptotic behavior is $g \sim r^{-1/2}$ for all values of $\gamma_\rho$. 

FIG. 4: (color online) Differential tunneling conductance in the absence of Zeeman splitting shown as a function of dimensionless magnetic field $r = q_0 v_F / eV$ for different values of the anomalous exponent, $\gamma_\rho$, and for dimensionless holon velocity $a_\rho = v_F / v_\rho = 2$ and spinon velocity, $a_\sigma = v_F / v_\sigma = 3$. The graphs have been shifted and re-scaled for clarity. The arrows mark divergences. Notice how increasing the anomalous exponent from the non-interacting value of zero reduces the three features in the differential conductance.
V. ADDING A ZEEMAN SPLITTING

In this section we consider the effects on the spectral functions of a Zeeman coupling to the magnetic field.

\[ A_{1D}(q, \omega, \varsigma) = \frac{W(q, \omega)\Theta(\omega - qv_\sigma - \varsigma B)}{\sqrt{\omega - tv_\sigma q - \varsigma B} - 2\gamma B} \]  
\[ \text{where,} \]
\[ W(q, \omega) = \begin{cases} \Theta(qv_\rho - \varsigma Bv_\rho/v_\sigma - \omega) + \Theta(\omega - qv_\rho - \varsigma Bv_\rho/v_\sigma)c(\gamma_\rho) & \text{if } \gamma_\rho \neq 0 \\ \Theta(qv_\rho - \varsigma Bv_\rho/v_\sigma - \omega) & \text{if } \gamma_\rho = 0 \end{cases} \]  
(14)

where \( \varsigma \) takes the value of the spin \((\pm 1/2)\) times the Zeeman coupling factor. We have considered only positive \( q \) and \( \omega \), since as we discussed in the previous section the contribution of \( q < 0 \) is merely to add a finite background of conductivity. (This background is very small for \( \gamma_\rho \approx 0 \).) Furthermore, we have seen that the region of interest, where the features in the spectral functions are easily distinguished in the differential conductivity, is restricted to small values of the anomalous exponent (below \( \gamma_\rho \approx 0.2 \)); for these values of \( \gamma_\rho \) the spectral weight outside the \( qv_\rho, qv_\sigma \) region is so small that all non-divergent contributions from \( A_{1D} \) outside this interval are negligible. On the other hand, the effect of the magnetic field in the two dimensional system is taken into account by writing the spectral function as

\[ A_{2D}(q, \omega, \varsigma') = \frac{\sqrt{m} \Theta(\omega - qv_F - \varsigma' B)}{\sqrt{\omega - qv_F - \varsigma' B}} \]  
(15)

where \( \varsigma' \) is the equivalent of \( \varsigma \) for the two dimensional system.

Since we can split the current into the separate contributions of the two possible values of the spin,

\[ I = I_\downarrow + I_\uparrow \]  
(16)

the problem is reduced to the calculation of the integral

\[ I_{\downarrow,\uparrow}(V, B) = \frac{4\sqrt{2}I_0(eV)^{\frac{1}{4} + 3\gamma_\rho}}{\pi\sqrt{\hbar v_F}} \int dq \int d\varepsilon [f(\varepsilon - eV) - f(\varepsilon)] A_{1D}(q, \omega, \varsigma)A_{2D}(q, \omega, \varsigma'). \]  
(17)

We can make use of the results of the previous section and simplify the calculation considerably by defining the spin dependent variable \( q' = q - \varsigma B/v_\sigma \), and the spin dependent parameter

\[ r_{\downarrow,\uparrow} = r \pm \frac{\varsigma' v_\sigma - \varsigma v_F}{v_\sigma} \frac{B}{eV}. \]  
(18)

The general expressions for the current and the differential conductance then reduce to

\[ I_\downarrow = \frac{1}{2}[I(r_{\uparrow}) + I(r_{\downarrow})] \]  
(19)
\[ g_\downarrow = \frac{1}{2}[g(r_{\uparrow}) + g(r_{\downarrow})] \]  
(20)

where \( I(r) \) and \( g(r) \) are the functions for the current and the conductance derived in the previous section. Figure 5 shows the differential conductivity as a function of \( r \) for \( \gamma_\rho = 0, a_\rho = 2, a_\sigma = 3, \varsigma = \varsigma' = 1 \) and \( k_f^D = k_f^D \). The different degree of field splitting of the different features in the conductivity can clearly be seen.

VI. CONCLUSIONS

Having calculated the generalized form of the magneto-tunneling conductance we see that the key signature of the new types of excitation in a Luttinger liquid is revealed in the appearance of six features which disperse with applied field. Loosely this may be viewed as the allowed transitions between spin-split spinon and holon excitations and the spin-split electron in the two-dimensional electron gas. However, this differs dramatically from the case of electron-like excitations in the one dimensional metal which would display four features. So this directly addresses the issue as to whether Zeeman splitting and spin-charge separation are distinguishable in this experiment—they are.
FIG. 5: Differential conductance including the effect of Zeeman coupling to a magnetic field as well as the orbital effect. The conductance is shown as a function of dimensionless magnetic field $r$ for the non-interacting exponent, $\gamma_{\rho} = 0$, with dimensionless holon and spinon velocities, $a_{\rho} = 2$ and $a_{\sigma} = 3$. The Zeeman coupling ($g$ factor) and the $k_F$ are assumed to be identical in the 1D wire and the 2DEG ($\zeta = \zeta' = 1$). The features in Fig. 4 appear to spin-split (to different degrees) by the Zeeman coupling.

FIG. 6: (color online) Contour plot of the differential conductance in the presence of a magnetic field as a function of $B$ and $V$ for a small anomalous exponent, $\gamma_{\rho} = 0.05$. Six dispersing features can be seen—indicative of spin-charge separation. The curves do not meet at $B = 0$ because we do not assume the magnitude of $k_F$ is the same in the 1D wire and the 2DEG. The differential conductance will be symmetric under $B \to -B$ as tunneling will then occur via the opposite branch of the Luttinger and 2DEG spectra.

The effect of an anomalous exponent is more subtle. The original proposal of Ref. 11 took the case of $\gamma_{\rho} = 0$ for calculational simplicity. The more general treatment given here shows that this is, in fact, the most singular case and other values for the anomalous exponent leads to less pronounced effects. Nevertheless, if $\gamma_{\rho}$ is not too far from zero, there will still be six clearly distinguishable features. In the Luttinger model, $\gamma_{\rho}$ comes from inter-branch processes, while spin-charge separation is due primarily to forward scattering intra-branch effects. Thus it is possible that spin-charge separation and an anomalous exponent not far from the non-interacting value of one could co-exist in real quantum wires.

The role of the anomalous exponent in weakening the tunneling singularities has implications for other forms of momentum resolved tunneling experiments. Recently Carpentier et al. have analyzed the tunneling conductance between two Luttinger liquids in a magnetic field (though without including the Zeeman effects as is done here). Again the tunneling current can be viewed as a convolution but now of two Luttinger liquid spectral functions. An anomalous exponent which differs from the non-interacting value will weaken the singularities in both functions in the convolution and will be doubly detrimental to features in the conductance. Thus we believe that using a two dimensional Fermi liquid (2DES) as the spectrometer, as described in this paper, optimizes the probability of seeing the dispersing features of the Luttinger liquid. This is because Fermi liquid theory will always guarantee a square-root singularity in its spectral function (after integration over the transverse momentum) which is the best one can do.

The experimental challenges in carrying out this experiment should not be underestimated. We rely on a number of assumptions. The most obvious is that tunneling is occurring uniformly along the 1D to 2D interface rather than via point-like tunneling. The test for whether an experiment is in this regime comes from the magnetic field dependence. With point-like tunneling, one would expect only weak field dependence of the tunneling current since momentum would no longer be conserved along the wire. Experiments using MBE grown interfaces have shown that the tunnel barriers can be sufficiently well controlled to preserve momentum conservation along the wire during tunneling, hence we believe that semiconductor fabricated quantum wire to 2D metal interfaces will be the most promising candidate.

The second assumption is that the two dimensional system is a well-controlled Fermi liquid with a large electron weight in the quasiparticle $Z \sim 1$. This ensures that the overlap between the electron and the excitations in the 2D spectrometer are large. Again, estimates from semiconductor two-dimensional electron gases (2DEGs) suggest that this is not implausible.

Our final assumption is that the rate limiting step in the experiment is the tunneling process between the 1D and 2D systems. This requires a clean quantum wire with no impurities breaking the wire up into smaller
problems in trying to implement this experiment in semiconductor devices. There the quantum wire is made by ‘pinching off’ a channel in a two-dimensional electron gas with an applied gate voltage. This pushes the one-dimensional sub-bands through the chemical potential until only one remains active. Initial experimental results reveal magneto-tunneling occurring when multiple sub-bands are conducting but the wire becomes insulating in the last sub-band. This is presumably due to impurities blocking conduction (an interesting process in itself). Ultimately, we believe that this should be viewed as a challenge rather than a fundamental flaw in the experiment. However, it also suggests that we should look at alternative realizations of this experiment. One possibility is to use carbon nanotubes as quantum wires since these have already been used to demonstrate Luttinger liquid-like behavior via point tunneling. If a suitable interface could be found with a two-dimensional conventional metal this would be a good alternative candidate for the magneto-tunneling measurement.

Finally we should point out that, although we have used a magnetic field for tuning the relative momentum between the wire and spectrometer, this is not the only method. Using a semiconductor 2DEG one could backgate the device and control the carrier concentration, and hence, $k_F$, in the spectrometer. This gate voltage would then provide the momentum tuning via the difference in $k_F$ between the wire and the 2DEG. Such a method could be used in cleaved edge overgrowth devices where the tunnel barrier to the quantum wire is in the plane of the 2DEG. Using $k_F$ to tune the momentum would, of course, mean there is no need to consider Zeeman coupling. However, the results may be complicated by any carrier concentration dependence of the 2DEG on its Fermi liquid properties, or indeed on the parameters of the Luttinger liquid which may be renormalized via screening from the 2DEG.

To summarize, we have considered momentum-conserving tunneling between a Luttinger liquid and a two dimensional conventional metal. We have shown how a transverse magnetic field can be used to tune the relative momentum of the tunneling electron. This then provides a direct measure of the spectral function of a Luttinger liquid via its convolution with that of a conventional Fermi liquid. The signatures of spin-charge separation are revealed as features in the tunneling conductance and we have shown they vary as a function of Zeeman splitting and anomalous exponent. The advantage of this experiment is that it can be performed with high resolution compared to other probes of the spectral function such as angle resolved photo emission. Also the experiment has a very straightforward theoretical interpretation and hence, if successful, is an unambiguous detector of spin-charge separation. We have also discussed the prospects of performing such an experiment.

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