Temperature dependence of an Efimov resonance in $^{39}\text{K}$

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Ultracold atomic gases are an important testing ground for understanding few-body physics. In particular, these systems enable a detailed study of the Efimov effect. We use ultracold $^{39}\text{K}$ to investigate the temperature dependence of an Efimov resonance. The shape and position of the observed resonance are analyzed by employing an empirical fit, and universal finite-temperature zero-range theory. Both procedures show that the resonance position shifts towards lower absolute scattering length when approaching the zero-temperature limit. We extrapolate this shift to obtain an estimate of the three-body parameter at zero temperature. A surprising (and currently outstanding) finding of our study is that the resonance becomes less prominent at lower temperatures. Finally, we present measurements performed near the Feshbach resonance center and discuss the prospects for observing the second Efimov resonance in $^{39}\text{K}$.

I. INTRODUCTION

Three particles interacting via short-range two-body potentials possess an intricate universal spectrum of three-body bound states while the two-body subsystems are unbound \[ [16] \]. This feature is a cornerstone of few-body physics and is known as the Efimov effect.

Atomic vapors cooled to ultracold temperatures are an important tool for studying three-body systems. They provide unprecedented control and flexibility, e.g., Feshbach resonances allow the two-body scattering length $a$ to be tuned to arbitrary values \[ [7] \]. By choosing a near the appearance of an Efimov state, three-body recombination losses are enhanced, and an Efimov resonance can be observed through loss spectroscopy \[ [2] [8] [9] \]. This loss signature of the Efimov physics allowed for the first unambiguous observation of an Efimov resonance \[ [10] \], and has since become the primary method of studying resonantly interacting three-body systems experimentally in both homo- and heteronuclear systems \[ [11] [29] \].

A central property of Efimov states is their universal behavior across different atomic species and Feshbach resonances \[ [2] [6] [13] [19] [26] [29] \]. The universal limit is reached in the ideal case of zero temperature and zero-range interactions. Here, the entire energy spectrum of three identical bosons is determined by the three-body parameter which fixes the location of the Efimov ground state, and by the universal scaling factor of approximately 22.7 which determines the spacing between the Efimov states. However, finite-temperature effects and finite-range interactions introduce modifications to this universal behavior. They drastically influence the appearance of Efimov resonances, hindering observations of consecutive resonances, and challenging the applicability of universality in few-body systems.

Several previous studies have considered the temperature dependence of Efimov resonances \[ [10] [21] [25] [27] [30] [34] \], but so far, the extent of systematic experimental investigations with focus on the temperature dependence is limited to a single study in a Cs ensemble \[ [25] \]. By analyzing loss spectra obtained at different temperatures, it was found that the obtained Efimov resonance position has a temperature dependence, which cannot be accounted for by zero-range theory.

Within this work, we study the temperature dependence of an Efimov resonance in a $^{39}\text{K}$ sample. We use a novel preparation technique to ensure that the initial temperature of the ensemble is independent of the chosen interaction strength. The observed Efimov resonance changes its character with the temperature, and we analyze its appearance with two different methods. We observe that the apparent position of the resonance shifts towards smaller absolute values of the scattering length as the temperature is decreased. In contrary to theoretical expectations, the resonance becomes less pronounced at lower temperatures. By extrapolating the resonance position to zero temperature, we obtain an estimate of the three-body parameter for $^{39}\text{K}$. Finally, we present measurements performed near the Feshbach resonance center and discuss the prospect for observing the second Efimov resonance in $^{39}\text{K}$.

The rest of the paper is structured as follows. In Sec. \[ II \] we introduce the finite-temperature theory which will be used for characterizing the experimentally observed Efimov resonance. Section \[ III \] describes the experimental procedure for obtaining ultracold thermal samples. The method for evaluating the losses in these samples is presented in Sec. \[ IV \] and the obtained results are provided in Sec. \[ VI \]. Furthermore, in Sec. \[ VII \] we present measurements obtained at strong interactions and discuss the second Efimov resonance. Finally, we draw conclusions in Sec. \[ VIII \].

II. THEORY OF EFIMOV RESONANCES AT FINITE TEMPERATURES

The three-body loss of particles is described by the equation $dn/dt = -\alpha n^3$, where $n$ is the density of particles
and $\alpha$ is the three-body recombination coefficient, which determines the probability for three particles to recombine [2]. If the density is known, this equation can be used to extract $\alpha$ from experimental data as described in Sec. IV. Here we briefly review how to relate $\alpha$ to the microscopic parameters. The analysis is based on the theory developed in Refs. [35] [36]. However, instead of employing momentum space for calculations, we consistently use coordinate space.

Three spinless bosons are conveniently studied in the hyperspherical formalism [37], where all relevant dynamics at low energies is described using a single differential equation

$$\left( \frac{1}{2} \frac{d^2}{d\rho^2} + \frac{\nu^2(\rho) - 1/4}{2\rho^2} - k^2 \right) f(\rho) = 0,$$  \tag{1}

where $k^2 = mE/\hbar^2$ with $E$ the energy of the system and $m$ the mass of a particle; $f(\rho)$ is the three-body wave function described in hyperspherical coordinates with $\rho = \sqrt{2/3}(r_1 + r_2 + r_3 - r_1 \cdot r_2 - r_2 \cdot r_3 - r_1 \cdot r_3}$ (here $r_1$ is the coordinate of the $i$th particle). The function $\nu(\rho)$ contains information about the two-body interactions. For a zero-range interaction potential [38] the function $\nu(\rho)$ solves the transcendental equation

$$\frac{\nu \cos \left( \frac{\nu \rho}{2} \right) - \frac{s_0}{\sqrt{3}} \sin \left( \frac{\nu \rho}{2} \right)}{\sin \left( \frac{\nu \rho}{2} \right)} = -\sqrt{2} \frac{\rho}{|a|},$$  \tag{2}

where $a$ is the scattering length. Note that within this work we only consider $a < 0$.

The Schrödinger equation (1) reduces the complexity of the Efimov effect to the investigation of a simple one-body problem – a particle in the $a^2(\nu^2 - 1/4)/(2\rho^2)$ potential. This potential is shown in Fig. 1. It contains a barrier whose maximum $\simeq 0.14$ is located at $\rho/|a| \simeq 1.46$. It is repulsive for $\rho \to \infty$, whereas for $\rho \to 0$ it is attractive.

We now discuss the attractive region in more detail. For $\rho \to 0$ one of the solutions to Eq. (2) is imaginary $\nu_s = is_0$, where $s_0 \simeq 1.00624$. It leads to a (super) attractive $-1.2625/r^2$ potential in Eq. (1), which supports an infinite number of bound states with the ground state of infinite energy [39] – the Thomas collapse [40]. This collapse is due to the vanishing effective range parameter of the two-body potential, hence, for small values of $a$ it is unphysical. For $|a| \to \infty$ the infinite tower of bound states is called the Efimov effect [41]; the barrier is extended well beyond the two-body interaction range and Efimov states may be detected [42].

The existence of the Thomas collapse means that the Schrödinger equation (1) is ill-defined: It has to be regularized at short-distances. We do so by parameterizing the behavior of the three-body wave function at $\rho \to 0$ [43] for all $k$ as

$$f(\rho \to 0) \sim \sqrt{\rho} \left( A\rho^{\nu_s} + \rho^{-\nu_s} \right),$$  \tag{3}

where the parameter $A$ determines the short-range three-body physics at $k = 0$. Note that the momentum $k$ plays a marginal role at $\rho \to 0$ (compared to the potential), and, therefore, we omit its effect here. All scattering observables can be now calculated from the wave function at large distances

$$f(\rho \to \infty) = He^{-i\sqrt{3}k\rho} + Ge^{i\sqrt{3}k\rho},$$  \tag{4}

To obtain the scattering amplitudes $H$ and $G$ numerically, one solves Eq. (1) with the conditions (3) at short distances and then fits the solution to the large-distance form given by Eq. (4). To optimize this approach one can first solve the Schrödinger equation with the following boundary conditions (cf. [36]),

$$f(\rho \to 0) \sim \sqrt{\rho} \left( (k\rho)^{\nu_s} + s_{11}(k\rho)^{-\nu_s} \right),$$  \tag{5}

$$f(\rho \to \infty) = e^{i\sqrt{3}k\rho},$$  \tag{6}

thus, determining the function $s_{11}(k|a|)$. Note that the function $f$ here only has an outgoing flux at $\rho \to \infty$, whereas $f^*$ only has an incoming. Once the function $s_{11}$ is known, the scattering amplitudes are easily computed for every value of the parameter $A$

$$\frac{G}{H} = \frac{s_{11}k^{\nu_s} - Ak^{-\nu_s}}{s_{11}Ak^{\nu_s} - k^{\nu_s}}.$$  \tag{7}

To simulate the loss of particles we assume [42] that $|A| < 1$ (see [2] and references therein), which means that some particles are lost close to $\rho = 0$. The recombination coefficient for a given momentum is then (cf. [43])

$$\alpha_k = 36(2\pi)^2 \sqrt{\frac{\hbar}{mk^4}} \left( 1 - \frac{|G/H|^2}{1 - |A|^2} \right),$$  \tag{8}

where $1 - |G/H|^2$ determines the number of particles lost in the scattering governed by Eq. (1). The prefactor connects this one-body problem to the three-body one. In terms of $A$, the parameter $\alpha_k$ is given by

$$\alpha_k = 36(2\pi)^2 \sqrt{\frac{\hbar}{mk^4}} \left( 1 - \frac{|A|^2}{1 - (1 - |s_{11}|^2)/|1 - k^{-2\nu_s}s_{11}|} \right).$$  \tag{9}
theories that relate $\alpha$ at which effect in ultracold atoms. It defines the scattering lengths $|a_0|$ from one-dimensional scattering theory. The behavior for infinitely high energies. These limits follow directly from Eq. (9) to the standard three-body parameter $a_0$ derived in [2, 48] as discussed in [36]. Here we match ideas of [46, 47] as discussed in [36].

To obtain the recombination coefficient for a fixed temperature we thermally average it assuming the Boltzmann distribution

$$\alpha = \frac{1}{2(k_B T)^3} \int \alpha_k E^2 e^{-E/k_B T} dE,$$

where $k_B$ is Boltzmann’s constant.

To calculate $\alpha$ we hence need to compute $s_{11}$ and specify $A$. The latter we write as $A = -R_0^{-2\nu \nu'} e^{-2\eta_-} (\text{cf.} [20])$, where $\eta_-$ defines the recombination rate; note that the wave function vanishes at $\rho = R_0$ for $\eta_- = 0$. The function $s_{11}(k|a|)$ has previously been calculated [35, 36] using the Skornikov-Ter-Martirosyan equation [49]. Here we calculate it directly using the Schrödinger equation [1] – we fix the boundary conditions at $\rho \to \infty$ and use a finite-difference method to calculate the function at $\rho \to 0$, which determines $s_{11}$. The function $s_{11}$ calculated in this way agrees well with previous results [35]. To give some insight into $s_{11}$, we plot $|s_{11}|$ in Fig. 2. Note that $|s_{11}|(0) = 1$ and $|s_{11}|(\infty) = 0$, i.e., transmission is not possible at zero energy and it is perfect for infinitely high energies. These limits follow directly from one-dimensional scattering theory. The behavior beyond these trivial limits can be obtained using the ideas of [46, 47] as discussed in [36].

To relate $R_0$ to the standard three-body parameter $a_-$ we match $\alpha_k$ from Eq. (9) to $a_0$ derived in [2, 48]

$$a_0 = \frac{3 \times 4590 \sinh(2\eta_-) \ h a^4}{\sin^2(\eta_0 \ln(a/a_-)) + \sinh^2(\eta_-)}.$$

We obtain $|a_-| = e^{(\delta - \pi n/2)/s_0 R_0}$ and choose $n = 1$, so $|a_-| \approx 1.017 R_0$ (cf. [36, 39]). Note that we used $s_{11}(x \to 0) = \frac{1}{2} e^{-2i\delta}$, where $\delta \approx 1.588$ [36, 46], moreover we derived $|s_{11}(k|a|)|^2 \approx 1 - 22.37 (ka)^4$.

The parameter $|a_-|$ is central to understand the Efimov effect in ultracold atoms. It defines the scattering lengths at which $a_0/a^4$ is maximal, at zero temperature. The universal properties of Efimov physics predicts that $|a_-|$ lies within the interval $[8.27 R_{\text{vdW}}, 11.19 R_{\text{vdW}}]$ [50], where $R_{\text{vdW}}$ is the van der Waals length. To date, there are no theories that relate $\eta_-$ to other microscopic parameters.

Figure 3 shows a calculated for $^{39}$K at different temperatures. For the sake of argument, we have chosen $|a_-| = 600 a_0$ ($a_0$ is the Bohr radius) and $\eta_- = 0.2$. At small values of $|a|$ the temperature effects are negligible and all curves coincide. At larger values of $|a|$ the finite-temperature effects become influential, they significantly alter $\alpha$ and the appearance of Efimov resonances, which are observable only for temperatures that allow a sizable portion of atoms to scatter at the energies below the height of the three-body barrier, i.e., $0.14 \ h^2/(m |a|^2) \approx k_B \times 1700 \text{K}$. It is worth noting that for increasing temperatures, the position of the recombination maximum shifts towards smaller $|a|$ [10, 30, 41, 51], which was also observed previously [25]. However, in Ref. [25], this behavior had to be slightly corrected due to unknown finite-range effects, which led to the dependence of the three-body parameter $a_-$ on temperature. In future work, it will be interesting to incorporate finite-range corrections [52] into the theory to understand existing experimental data.

### III. PREPARATION AND LOSS SPECTROSCOPY OF ULTRACOLD $^{39}$K ATOMS

We study Efimov resonances experimentally by performing loss spectroscopy across a range of interaction strengths with $^{39}$K atoms prepared at different initial temperatures.

The experiments were conducted using apparatus previously described in [53]. Briefly summarized, a dual-species magneto-optical trap captures and cools $^{39}$K and $^{87}$Rb simultaneously in a glass cell. Subsequently, optical molasses and pumping is applied to both species, and they are captured in the $F = 2, m_F = 2$ state by a magnetic quadrupole trap. This trap mechanically transports the atoms to a different chamber, where they are loaded.
An inherent experimental problem when accessing strong interactions is the finite speed at which a magnetic field can be changed. Before the target scattering length is reached, losses and dynamical processes can take place and introduce errors. To circumvent this inherent issue, we have developed a preparation procedure, which is shown schematically in Fig. 4.

When the evaporation in the $|1, -1\rangle$ state is complete and sufficiently low temperatures are reached, the atoms are transferred to the $|1, 0\rangle$ state, which has a small negative scattering length. The magnetic field is then adjusted to a target value, and subsequently a wait time of 0.5 s is added to ensure a stable field and complete rethermalization. Finally, the atoms are transferred back to the $|1, -1\rangle$ state, which initiates a loss measurement. This procedure avoids a direct exposure of the atoms to very large scattering lengths prior to the measurement and is essential to preserve the low temperatures achieved by evaporative cooling.

A loss measurement is performed by holding the sample for a variable time at a chosen interaction strength and releasing it from the trap afterwards. The magnetic field is turned off simultaneously with the release of the cloud. An absorption image is recorded after a total expansion time of 20 ms, which allows the temperature and number of particles to be obtained.

To characterize the three-body loss, a series of decay measurements covering a range of interactions is performed. Multiple data series were acquired at various initial temperatures, which allows the temperature dependence of the Efimov resonance to be studied. The different initial temperatures are reached by evaporating and holding the atoms using various dipole trap configurations. In addition, the state preparation procedure into the interacting state was varied to test whether it had an influence on the observed Efimov resonance. The essential information on each data series is given in Tab. I. In Fig. 5 we show the initial temperature for all data series, obtained by performing a fit as explained in Sec. IV.

In Fig. 5 we show the initial temperature for all data series, obtained by performing a fit as described in the following section. The preparation of the ensemble through the non-interacting state clearly ensures a constant initial temperature across all interactions.

### IV. LOSS EVALUATION

An Efimov state manifests itself experimentally as an increase of the three-body recombination coefficient $\alpha$ at a specific interaction strength. It is therefore necessary to carefully analyze atomic losses to characterize an Efimov resonance.

In a harmonic trap, three-body losses preferentially occur in the dense center of the ensemble. Since the average potential energy of atoms is lower here, three-body losses result in heating. At a specific interaction strength, both the change in the temperature $T$ and atom number $N$ thus have to be analyzed to obtain $\alpha$. 
TABLE I. Sample parameters for all data sets. The symbols refer to Fig. 5. The parameters $T_0$, $N_0$, and $\pi_0$ refer to the initial temperature, atom number and density averaged across all decay measurements in a given data set. The state preparation procedure into the interacting state is also given: ‘magnetic field ramp’ refers to the experiment in which the target scattering length is reached through a ramp of the magnetic field, instead of preparation from a weakly-interacting state.

| symbol | $\omega_x, \omega_y, \omega_z$ (2π/s) | $T_0$ (nK) | $N_0$ (10^3) | $\pi_0$ (10^{11}/cm^3) | state preparation procedure |
|--------|----------------------------------|-----------|-------------|-----------------|---------------------------|
| ○      | 83, 109, 83                      | 304(9)    | 34          | 7.7             | magnetic field ramp       |
| ■      | 104, 124, 104                    | 286(20)   | 79          | 18.3            | $\pi$-pulse              |
| ▲      | 86, 114, 86                      | 192(9)    | 20          | 10.0            | rapid adiabatic passage   |
| ▲      | 83, 109, 83                      | 178(11)   | 23          | 11.6            | rapid adiabatic passage   |
| ▼      | 104, 124, 104                    | 175(17)   | 35          | 37.0            | $\pi$-pulse              |
| □      | 90, 5, 28, 79                    | 71.7(1.3) | 70          | 37.1            | $\pi$-pulse              |
| □      | 89, 25, 75                       | 60(3)     | 49          | 27.8            | $\pi$-pulse              |
| ●      | 87, 21, 70                       | 44(3)     | 26          | 17.7            | $\pi$-pulse              |

The time evolution of $T$ and $N$ can be described through the coupled differential equations $dN/dt = -\alpha \int n^3(r) dr$ and $dT/dt = \alpha T \int n^2(r) dr / 3N$. By assuming a Gaussian thermal distribution, the equations can be solved analytically to provide [13, 19]

\[
N(t) = N_0 \left(1 + \frac{3\beta^2 N_0^2}{\sqrt{27} \omega_0^2 \alpha t} \right)^{-1/3}, \quad (12)
\]

\[
T(t) = T_0 \left(1 + \frac{3\beta^2 N_0^2}{\sqrt{27} \omega_0^2 \alpha t} \right)^{1/9}, \quad (13)
\]

where $\beta = (m\bar{\omega}^2/2\pi k_B)^{3/2}$, $m$ is the mass of $^{39}$K, $k_B$ is the Boltzmann constant, and $\bar{\omega} = (\omega_x, \omega_y, \omega_z)^{1/3}$ is the geometric mean of trapping frequencies. To obtain the three-body recombination coefficient from the decay measurements, these equations are simultaneously fitted to the atom number and temperature, which yields $\alpha$ as well as the initial atom number $N_0$ and temperature $T_0$. The temperatures shown in Fig. 5 are obtained through this procedure.

The three-body recombination coefficients for four different initial temperatures are shown in Fig. 6. The magnetic field was converted into the scattering length using a previous characterization of the Feshbach resonance [19]. With increasing $|\alpha|$, $\alpha$ tends to increase, as expected. Additionally, the ground-state Efimov resonance is present at approximately $-700a_0$, which provides a local increase of $\alpha$. The position of the Efimov resonance is in close agreement with a previous observation [19].

With decreasing temperature, the observed Efimov resonance changes. The local maximum of $\alpha$ is shifted towards a lower absolute value. Additionally, the resonance becomes less pronounced. At the lowest studied initial temperature of 44 nK, the resonance is hardly distinguishable from the background slope.

The resonance behavior is in apparent disagreement with the zero-range theory presented in Sec. II and shown in Fig. 3. This points towards the presence of physics unaccounted for by the zero-range model, e.g., finite-range and many-body effects, and we will analyze the data from this perspective.

FIG. 6. Three-body recombination coefficient as a function of the scattering length for four different temperatures. The average initial temperature $T_0$ for each of the data sets is shown in the panels. Two different fits are performed to characterize the Efimov resonance: The dashed lines are empirical fits according to Eq. (15) and the solid gray lines are fits using the finite-temperature theory (see Eq. (17) and the text).

In Tab. I information about the data sets is provided. The observed flattening of the resonance is not correlated with the density of the sample or the state preparation procedure, and we attribute this behavior to the change of the temperature.

V. EFIMOV RESONANCE CHARACTERIZATION

In this section we present two approaches to quantitatively analyze the observed Efimov resonances. This al-
lows for a detailed discussion of the shift and the unexpected suppression of the resonance at low temperatures.

A. Analytic empirical fit

The three-body recombination coefficient $\alpha$ can be described analytically in the ideal limit of zero temperature and zero-range interactions \[2\]. However, in practice, finite temperature and finite-range interactions add upper and lower limits, alter the slope and change the Efimov resonance shape and position. Furthermore, systematic errors of the evaluated ensemble density can introduce inaccuracies.

To obtain the apparent Efimov resonance location and width, we therefore perform an empirical fit

$$\alpha(a) = \frac{3 \times 4590 \sinh(2\eta_{\text{e}})}{\sin^2(\theta) \ln(a/a_{\text{e}})} + \frac{\hbar a_e^4}{m} \left(\frac{a}{a_{\text{e}}}\right)^{n_{\text{e}}},$$

(14)

which is similar to Eq. [11]. In this equation, it contains the fitting parameters $n_{\text{e}}$ and $a_{\text{e}}$, which allow $\alpha$ to deviate from the predicted $a^4$ dependence and introduce an overall shift. Furthermore, we provide an upper constraint to $\alpha$, by introducing the effective three-body recombination coefficient

$$\alpha_{\text{eff}}(a) = \left(\frac{1}{\alpha(a)} + \frac{1}{\alpha_{\text{max}}}ight)^{-1},$$

(15)

which is limited due to temperature according to

$$\alpha_{\text{max}} = \frac{36\sqrt{3}\pi^2 \hbar^5}{(k_B T_0)^2 m^3},$$

(16)

following previous work \[19\]. In Eq. (15), the effective recombination rate is finite even at $|a| \to \infty$ when $T \neq 0$. Note that unlike \[19\], we do not fit $\alpha_{\text{max}}$. The fitting parameters are thus the resonance position $a_-$, the elasticity parameter of the trimer $\eta_-$, as well as the empirical parameters $n_{\text{e}}$ and $a_{\text{e}}$.

This fit is applied to the obtained data sets, as shown in Fig. 6. The fit describes the obtained data well across all temperatures, including the observed suppression of the resonance at low temperatures. For all of the data sets, we obtain that $n_{\text{e}} \approx 3$ and $a_{\text{e}}$ is of the order of $a_-$. Other relevant scales are much smaller than $|a_-|$: The van der Waals length is $R_{\text{vdW}} = 64.53a_0$ \[55\], and the intrinsic length of the relevant Feshbach resonance is $R^* = 23a_0$ \[19\]. Therefore we use the parameterization in Eq. (10) derived from the microscopic zero-range Hamiltonian to perform a fit. We write $\alpha$ as

$$\alpha = \delta T^2 \frac{\sqrt{3}\pi^2 \hbar(1 - e^{-4\eta_-})}{mk^2 a^2} \times \int_0^\infty \frac{(1 - |s_1|^2)e^{-\eta_+}}{\left|1 + \left(\frac{xR_0}{|a|}\right)^{-2i\eta_1} - 2i e^{-2i\eta_1} s_1\right|^2} dx,$$

(17)

where $t = \sqrt{mk_B T}/\hbar$. This expression is evaluated numerically with fitting parameters $\eta_-, R_0$ and $\delta$. The latter parameter accounts for the systematic errors of the experiment that originate from the evaluated ensemble density. Note that this fit contains less fitting parameters than the empirical fit.

The fit is applied to the data as shown in Fig. 6. To minimize the influence of finite-range effects, we do not include data with $|a| < 500a_0$ in this fit. If a lower threshold is used, the obtained values of $a_-$ for the four hottest samples and of $\eta_-$ for all samples are similar. However, for the coldest samples, $a_-$ changes significantly. Generally, the theoretical model fails to describe the observed three-body recombination for the four coldest samples. Under the conditions obtained at low temperatures, there are important physical effects that are not taken into account, such as finite-range effects.

VI. EVALUATION OF FINITE TEMPERATURE BEHAVIOR

The two different fitting procedures described above provide means for finding the Efimov resonance position $a_-$ and the elasticity parameter $\eta_-$. In this section, we discuss these results to quantify finite-temperature effects. Previous studies of Efimov resonances, were often based on fits that either use analytical expressions at zero temperature as in Eq. (14), or numerical calculations (cf. Eq. (17)) that take finite-temperature effects more formally into account. This section thus provides an inherent comparison of these different approaches to characterize Efimov resonances.

A. Efimov resonance position

Generally, finite-temperature behavior arises when the thermal wavelength $\lambda_\text{th}$ becomes non-negligible in comparison to the three-body parameter $a_-$. In the zero-temperature case, $\lambda_\text{th}$ is infinite and does not influence the observed Efimov resonance, hence the observed resonance position $a_-$ is equivalent to the standard three-body parameter. However, as the temperature is
increased the observed Efimov resonance is modified. Disregarding finite-range effects, the finite-temperature fit takes these modifications into account and should obtain the same $a_-$ independent of temperature. Any observed changes in $a_-$ from this evaluation therefore originate from aspects not taken into account, such as temperature-dependent finite-range effects.

The empirical fit does not inherently take temperature effects into account. Any observed temperature dependence therefore also reflects the temperature-dependent trends shown in Fig. 3.

The values of $a_-$ obtained through the two fitting procedures are shown in Fig. 7(a-b). The temperature is converted into the dimensionless parameter $R_{vdW}/\lambda_{th}$, which compares the relevant thermal wave length to $R_{vdW}$, where $\lambda_{th}$ is calculated using the average initial temperatures $T_0$. For both evaluation methods, the value of $|a_-|$ tends to decrease when the temperature is lowered. Note that this behavior is opposite to the apparent loss peak position shown in Fig. 3 predicted without finite-range effects.

We estimate the zero-temperature value of the three-body parameter by performing a linear extrapolation to $\lambda_{th} \to \infty$ \cite{25}. For the data obtained through the empirical fit, this procedure provides $-587(86)a_0$, whereas the finite-temperature fit yields $-509(54)a_0$. The two results are within the errors of each other. We estimate a systematic error of the order of 50$a_0$ due to an imprecision of the scattering length determination. The slopes obtained from the fits are similar, with 36 $\times$ $10^3a_0$ for the empirical fit, and 47 $\times$ $10^3a_0$ for the finite-temperature fit.

Universality predicts a three-body parameter within the interval $[8.27R_{vdW}, 11.19R_{vdW}]$ corresponding to $|a_-| \in [534a_0, 722a_0]$. This is in agreement with both our experimental zero-temperature estimates, within the uncertainties.

In Fig. 7(c), we compare the obtained results to the previous characterizations of Efimov resonances in $^{39}$K \cite{39}. The Feshbach resonance strength $s_\nu$ influences the location of the observed Efimov resonance \cite{19, 29}, which is determined by the empirical fit (see Eq. 15), whereas the theoretical fit (see Eq. 17). The two fitting procedures are used to estimate the zero-temperature three-body parameter, shown as solid and dashed lines in (a) and (b), respectively. The corresponding standard errors are shown as faint lines. In (c), we compare the obtained fits to previous studies of Efimov resonances in $^{39}$K that used Feshbach resonances of similar strength $s_\nu$, \cite{19}. The legend provides the Feshbach resonance strength as well as the hyperfine state in which these measurements were performed.

In Ref. \cite{25} a similar linear trend was observed for $|a_-|$, which decreased when the temperature was lowered. However, the slope is significantly stronger in our observations with $^{39}$K, indicating stronger finite-range effects. The observation in Cs is in closer agreement with the universal predictions of a three-body parameter, than our observations in $^{39}$K. It is possible that this is due to finite-range physics or the nature of the employed Feshbach resonance.

### B. Elasticity parameter and suppression of the Efimov resonance

In Fig. 8, we show the elasticity parameters $\eta_-$ at various initial sample temperatures, obtained from the two fitting procedures. The values of $\eta_-$ obtained from the empirical fit show an unexpected growth at low temperatures, which reflects the suppression of the resonance. The results obtained from the finite-temperature fit do not show the increase of $\eta_-$ at low temperatures. However, the finite-temperature fit generally agrees less well with the experimental data at low temperatures. These observations indicate that the suppression effect cannot be accounted for by the physics included in the finite-temperature theory.

There is no few-body mechanism, which is sensitive to the small temperature variation in the limit when the temperature is much smaller than any other energy scale of the problem. It is therefore relevant to consider many-
body mechanisms to explain the suppression. In the experimental realization, the interparticle spacing is of the order of the thermal wavelength $\lambda_{th}$, and quantum statistics is therefore important. For the colder experimental samples, the critical temperature for Bose-Einstein condensation is in fact slightly above the actual initial temperatures. Due to the experimental preparation through a weakly-interacting state with negative scattering length, it is only possible that small Bose-Einstein condensates or solitons exist in the sample as the loss measurement is initiated, even after the hold time of 0.5 s. We speculate that these many-body processes could influence the loss dynamics.

Another source of error which could potentially influence the loss dynamics is the presence of a few atoms not transferred into the target hyperfine state during the state-preparation procedure. However, it is not clear how the presence of a few weakly-interacting impurity atoms can significantly alter the rapid loss dynamics near the Efimov resonance.

VII. SECOND EFIMOV RESONANCE

The results presented above provide a foundation for discussing the prospect of studying the resonance of the first excited Efimov state in $^{39}$K. For a single component quantum gas, this resonance has only been observed in Cs [21]. This observation was performed at a temperature of approximately 9 nK, which corresponds to $|a^{(1)}|/\lambda_{th} \approx 0.67$, where $a^{(1)}$ is the location of the excited state resonance. By considering $^{39}$K and assuming $-a^{(1)} = 22.7 \times 9.73 R_{vdW} = 14.2 \times 10^4 a_0$, a similar value of $|a^{(1)}|/\lambda_{th}$ is obtained at a temperature of approximately 60 nK. Naively, it should therefore be possible to observe the excited state resonance in $^{39}$K at temperatures up to approximately 60 nK.

Based on our measurement of the Efimov ground state resonance, we model the first excited state Efimov resonance. The finite-temperature theory applied to Cs and $^{39}$K is shown in Fig. 9 to compare the visibility of the previously observed Efimov resonance in Cs with a potential resonance in $^{39}$K. The theory predicts that under similar conditions the Cs resonance is the most distinct of the two. The $^{39}$K Efimov resonance is nevertheless distinguishable from a flat curve.

In an attempt to observe the excited state Efimov resonance, we performed a series of decay measurements near the Feshbach resonance center. The experiments were performed according to the description in Sec. IV and the three-body recombination coefficient was obtained by fitting decay curves as described in Sec. IV. The initial average sample temperature across the range of magnetic fields was approximately 20 nK rising to roughly 42 nK during the measurement.

The obtained three-body recombination coefficients $\alpha$ are shown in Fig. 10. Since an accurate calibration of the scattering length is not available near the Feshbach resonance center, we show $\alpha$ versus magnetic field. A loss maximum is observed at magnetic fields larger than the previously reported Feshbach resonance center. At strong positive scattering lengths, the presence of a weakly bound dimer state alters the loss dynamics and can indeed lead to a loss maximum not located at the resonance center, depending on experimental conditions [56, 57]. In fact, the Feshbach resonance center was previously measured to be 33.64(20) G by locating the loss maximum [19], but our data illustrates the deficiency of this method for accurate Feshbach resonance characterization.

We now analyze the data in the context of observing an excited state Efimov resonance. In Fig. 10 we show the theoretical prediction of the first excited state Efimov resonance, assuming the Feshbach resonance center to be at
FIG. 10. Three-body recombination coefficient $\alpha$ as a function of the magnetic field near the Feshbach resonance center. A theoretically predicted curve displaying the shape of the second Efimov resonance is shown (same as in Fig. 9), assuming the Feshbach resonance center to be at the vertical dashed gray line. This assumption also provides the scattering length axis, shown near to the theoretical prediction.

33.64 G. This allows us to calculate the scattering length axis, which is also shown. A different assumption about the Feshbach resonance center will shift the theoretical curve horizontally on the magnetic field axis. Based on a visual comparison between the data and the theoretical prediction, we do not observe any signatures of the second Efimov resonance.

There are several possible explanations as to why we do not observe the second Efimov resonance. Since we have shown that the first resonance cannot be fully understood by finite-temperature theory, the applicability of the theory at large scattering lengths is unclear. In particular, the suppression of the ground state Efimov resonance is not understood, since it cannot be accounted for by universal zero-range theory. Another possible explanation is the presence of higher-body processes, which become significant compared to three-body recombination close to the resonance center. If four- or higher-body losses are more rapid than three-body losses, the three-body Efimov resonance is not visible. Moreover, the size $\sim 0.7 \mu$m of the excited state Efimov trimer may affect the dynamics in the trap, which on the smallest axis has a characteristic length of 1.7 $\mu$m.

VIII. CONCLUSION

We have studied the ground-state Efimov resonance in $^{39}$K at various temperatures and observed that with decreasing temperature, the obtained value of $|a_-|$ becomes smaller, and the resonance becomes less prominent. The former is attributed to strong finite-range effects; the change in $a_-$ is far more dramatic than in similar measurements performed in Cs [25]. The flattening of the resonance is still an outstanding problem: The observed behavior arises due to effects not included in the simple zero-range model, e.g., finite-range effects or many-body physics, and theoretical calculations beyond the existing models are required to understand our data.

Moreover, we have performed measurements close to the Feshbach resonance center to investigate the prospects of observing a second Efimov resonance. However, we do not observe any resonance feature connected to an excited Efimov state. We believe that this observation is connected to incomplete understanding of the observed first Efimov resonance: Since the ground-state resonance is not fully understood, it is difficult to reliably make predictions about the excited states.

Our measurements show that certain aspects of few-body physics are yet to be understood, and encourage deeper investigations of finite-range and many-body effects on three-body loss measurements.

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