Research Article

In-Plane Instability of Parabolic Arches under Uniformly Distributed Vertical Load Coupled with Temperature Gradient Field

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1.Introduction

Long-span steel arch structures are widely applied to engineering, for instance, long-span steel roofs of the terminal, long-span steel arch bridge, etc. For large-span steel roofs, the temperature inside and outside of the roof is different due to sun exposure. In summer, the temperature inside the structure is lower than that outside the structure, while in winter, the temperature inside the structure is higher than that outside the structure. The inconsistence of internal and external temperature will produce linear gradient temperature field and internal force to the structure, and the linear gradient temperature field and internal force affect the bearing capacity of the structure. Except for the linear gradient temperature field caused by solar irradiation, the fire inside the structure will cause the internal temperature of the structure to be higher than the external temperature and then generates the linear gradient temperature field. Hence, the study of the steel arch structure under linear temperature gradient field has a profound significance on the fire resistance design of the steel arch structure.

Many scholars have conducted a series of research studies on the buckling of steel arch structures. Pi and Trahair. [1] studied the inelastic lateral buckling strength and design of steel arches under general loading using an advanced nonlinear inelastic finite element method of analysis. Mallon et al. [2] researched the influence of the initial curvature of thin shallow arches on the dynamic pulse
buckling load. Moon et al. [3] investigated the geometrically nonlinear behavior of pin-ended shallow parabolic steel arches subjected to a vertically distributed load for assessing the buckling load. Pi and Bradford. [4] researched the dynamic in-plane buckling of a shallow pin-ended circular arch under a central radial load that is applied suddenly with infinite duration. Bradford et al. [5] researched the pre-buckling behavior of a pin-ended circular arch under a uniform radial load. Han et al. [6] researched the in-plane nonlinear behavior and stability of shallow circular arches with elastic horizontal supports under a uniform radial load by the principle of virtual work. Zeng et al. [7] studied the stability analysis of elastic restrained arc steel arch under concentrated load. Kang et al. [8] conducted a dynamic response analysis of an elastically supported arc steel arch structure under blast impact loading. Yan et al. [9] researched a nonuniform shallow arch characterized by three constant stiffness regions under a central concentrated load. Li et al. [10] established an analytical solution to predict the buckling load of the thin-walled arch under a point load at midspan position. Fan et al. [11] carried out an analytical study and a numerical simulation on the nonlinear in-plane buckling behavior of the shallow parabolic steel arch with tension cables and pin joints. Pi and Bradford [12] researched the nonlinear thermoelastic buckling behavior of articulated shallow arc steel arches under the impact of linear temperature gradient field. Bouras and Vrcelj. [13] carried out the nonlinear elastic prebuckling and in-plane buckling analysis of the circular shallow arches under a uniformly distributed load and time-varying uniform temperature field. Song et al. [14] analyzed an in-plane jump buckling and bending behavior for arc steel arches subjected to fire. Asgari et al. [15] theoretically researched the nonlinear thermoelastic behavior of pin-ended functionally graded material (FGM) circular shallow arches. Li and Zheng, [16] investigated the buckling of confined thin-walled functionally graded material (FGM) arch subjected to external pressure. Tang et al. [17] researched the in-plane asymmetric buckling of the heated functionally graded material (FGM) circular arches under uniform pressure fields. Li et al. [18] researched the analytical process of the functionally graded porous (FGP) arch structure in an elevated thermal field. Cai et al. [19] researched the in-plane stability of rotationally restrained parabolic shallow steel arches under a vertical uniform load and temperature changes below 100°C and used the virtual work principle method to establish the nonlinear equilibrium and buckling equations. Pi and Bradford [20] researched nonlinear in-plane buckling of circular arches being subjected to uniform radial and thermal loading. However, only a few scholars have researched the parabolic steel arches under the temperature field, especially the effect of temperature gradient field. Parabolic arches are widely used in practical engineering. The internal force analysis and stability analysis of parabolic arches are important parts of arch design, construction and maintenance, etc. However, the analytical solutions of in-plane instability of parabolic arches being subjected to uniformly distributed vertical load and temperature gradient field are not solved in the opening.

Hence, this paper derives the cross-sectional effective centroid and effective stiffness for parabolic arches under the linear temperature gradient field and obtains the axial and bending actions of parabolic arches under linear temperature gradient field coupled with vertical uniform load. In addition, the analytical solution of the critical load for in-plane instability of the parabolic arches under temperature gradient field coupled with vertical uniform load is also obtained, and it is verified by the numerical simulations software ANSYS.

2. Analysis of Geometrical and Material Properties of Parabolic Arch

2.1. Basic Analysis

2.1.1. Mechanical Model. The fixed parabolic steel arch under linear gradient temperature field coupled with vertical uniformly distributed load is considered as the study object, which is plotted in Figure 1.

\[ o'x' y' z' \] denotes the initial coordinates of the parabolic steel arch, \( o' \) is situated in the cross-sectional geometric center of vault, and \( o'x'_1, o'y'_1, \) and \( o'z'_1 \) are the horizontal coordinate axis, the vertical coordinate axis, and the lateral coordinate axis of the initial coordinates, respectively. \( o's' \) is the geometric centroid axis of the initial coordinates. Figure 1(a) shows that \( f, L, l, S \) are the rise, span, half-span, and length of the parabolic steel arch, respectively. Figure 1(b) shows that \( T_1 \) and \( T_2 \) are top and bottom cross-sectional temperatures, respectively. \( h \) represents the cross-sectional height.

In addition, the equation of geometric coordinates of the parabolic steel arch can be defined based on Figure 1 as

\[ y'^* = \frac{a_1 l^4}{2} x'^*_1, \]  

(1)

where \( a_1 \) represents the shape factor of the parabolic steel arch, which can be given by

\[ a_1 = \frac{2f}{T} \]  

(2)

\( x'_1 \) represents nondimensional coordinate of the \( o's' \) axis, which is denoted as \( x'_1 = x'^*_1/l \), and so the arc differential \( ds \) of the parabolic arch can be calculated by

\[ ds = l\sqrt{1 + (a_1 x'_1)^2} \mathrm{d}x'_1. \]  

(3)

2.1.2. Basic Hypothesis. The in-plane instability analysis for parabolic steel arches under the linear temperature gradient field studied in this paper satisfies the following hypothesis.

(1) The environment temperature is 20°C.

(2) Cross-sectional temperature, parabolic arch deformation, temperature dilatation factor \( a \), and Poisson’s ratio \( \nu \) are independent of time.

(3) The temperature \( T \) of the cross section of the arch varies uniformly along the \( o's' \) axis and the \( o'x' \) axis.
axis and linearly along the $o^*y^*$ axis, as shown in Figure 2, where $T_1 < T_2$.

Hence, the temperature at any point on the cross section of parabolic arch can be calculated by

$$T(y) = T_a + \frac{y \cdot \Delta T_g}{h},$$

where $T_a$ represents the cross-sectional average temperature of the parabolic arch, which can be obtained by

$$T_a = \frac{T_1 + T_2}{2},$$

and $\Delta T_g$ represents temperature difference value between top and bottom of the cross section, which can be calculated by

$$\Delta T_g = T_2 - T_1.$$

2.1.3. Modulus of Elasticity. In this paper, steel is chosen as the material for the arch. The modulus of elasticity of steel $E_T = \xi_T \cdot E_0$, where $E_0$ is the modulus of elasticity of Q235 steel at temperature 20°C, and $\xi_T$ is the temperature affection factor, which can be given by

$$\xi_T = \frac{7T - 4780}{6T - 4760}, \quad 0°C < T \leq 600°C.$$  

Figure 3 shows the influence of temperature on steel elastic modulus. As shown in the picture, steel elastic modulus decreases with the an increase of material temperature, and the value decreases more remarkably with the augment of temperature.

2.1.4. Effective Stiffness and Effective Centroid. The I section is taken as the cross section of parabolic steel arch studied in this paper, and when the parabolic steel arch is under the linear gradient temperature field, the elastic modulus changes along the $o^*y^*$ axis, and the vertical coordinate of effective centroid also changes. Therefore, the effective coordinate system $oxyz$ can be determined by the location of the effective centroid, which is shown in Figure 4.
Accordingly, the vertical coordinate of effective centroid of the \( o \ast y \ast \) axis is given by

\[
y_c^\ast = - \frac{\int_A E_T(y^\ast)y^\ast \, dA}{\int_A E_T(y^\ast) \, dA}.
\]  

(9)

where \( y_c \) is the vertical coordinate of effective centroid of the \( oy \) axis.

The effect of linear gradient temperature field on the effective center of parabolic arch cross section is shown in Figure 5, and it can be seen that the effective center of shape is shifted to the side with lower temperature under linear gradient temperature field, and the effect of gradient temperature on the I section is greater than that on the rectangular section.

\[
E_T(y^\ast) = E_0 \left( \frac{7T_a - 4780}{6T_a - 4760} \right) h + 7 \Delta T_g y^\ast.
\]  

(8)

In addition, \( T_o \) is the temperature of the effective centroid, which can be given by

\[
T_o = T_a + \frac{\Delta T_g y_c^\ast}{h}.
\]  

(10)

By replacing the vertical coordinate \( y^\ast \) with \( y^\ast = y - y_c \), the elastic modulus along the \( oy \) axis can be given by

\[
E_T(y) = E_0 \left( \frac{7T_a - 4780}{6T_a - 4760} \right) h + 7 \Delta T_g (y - y_c).
\]  

(11)

For ensuring that the analysis of the internal forces of the parabolic arch under gradient temperature is precise, the effective stiffnesses \( EI \) and \( EA \) of the arch section are derived, which can be given separately by

\[
EI = \int_A E_T(y)y^2 \, dA,
\]

\[
= \frac{2320T_hE_0}{27\Delta T_g^2} \left[ \frac{1}{2}t^3 + t_f (b - t_w) \right] + \frac{7E_0t_wh^3}{72\Delta T_g}
\]

\[
+ \frac{7E_0t_f}{12\Delta T_g} \left( h^2 - 2t_f h + \frac{4}{3}t_f^3 \right) (b - t_w) - \frac{y_c^2EA}{2}
\]

\[
+ \frac{1160h^3T^2_eE_0}{81\Delta T_g^3} [(b - t_w) \Xi_1 + b \Xi_2],
\]  

(12)

\[
EA = \int_A E_T(y) \, dA = \frac{1160E_0}{9\Delta T_g} [ (b - t_w) \Xi_1 + b \Xi_2 ] + \frac{7E_0t_wh}{3} \left[ \frac{1}{2}t_f (b - t_w) \right] ,
\]
where \( t_f \) represents the thickness of the flange plate of the I section, \( t_w \) represents the thickness of the web of the I section, and \( b, h, \) and \( A \) represent the width, height, and area of the I section, respectively. Beyond that, the parameters \( T_s, \) \( \Xi_{12} \) can be mathematically expressed as

\[
T_s = 2380 - 3T_a,
\]

\[
\Xi_1 = \ln \left( \frac{2T_s h + 3(h - 2t_f)\Delta T_g}{2T_s h - 3(h - 2t_f)\Delta T_g} \right),
\]

\[
\Xi_2 = \ln \left( \frac{2T_s - 3\Delta T_g}{2T_s + 3\Delta T_g} \right).
\]

With the terms of the strain energy can be simplified as

\[
U_{s0} = \frac{1}{2} \int_{-S/2}^{S/2} \int_A E_T \epsilon_{xx0}^2 dA d_{ds} - \int_{-S/2}^{S/2} \int_A E_T \alpha \left( \Delta T_a - \frac{y\Delta T_g}{h} \right) \epsilon_{xx0} dA d_{ds},
\]

3. Results and Discussion

3.1. Internal Force Analysis. As the parabolic steel arches are linear elastic, their strain energy in the preinstability state under linear temperature gradient field coupled with vertical uniformly distributed load \( q \) can be given by

\[
N = -\int_A E_T \epsilon_{xx0} dA = -\int_A E_T (y) \left[ \frac{ds_1 - ds}{ds} + \left( \frac{1}{\rho_1} - \frac{1}{\rho} \right) \right] dA = -AE \epsilon_{m0},
\]

\[
M = -\int_A E_T y \epsilon_{xx0} dA = -\int_A E_T (y) y \left[ \frac{ds_1 - ds}{ds} + \left( \frac{1}{\rho_1} - \frac{1}{\rho} \right) \right] dA = -EI \epsilon_{\rho 0}.
\]

Accordingly, by substituting (16) and (17) into (14), the term of the strain energy can be simplified as

\[
\int_A E_T \epsilon_{xx0}^2 dA = \int_A E_T (y) \left[ \frac{ds_1 - ds}{ds} + \left( \frac{1}{\rho_1} - \frac{1}{\rho} \right) \right]^2 dA = \frac{N^2}{EA} + \frac{M^2}{EI},
\]

where \( ds_1 \) is the arc differential after deformation and \( \rho_1 \) and \( \rho \) are radii of curvature of the parabolic steel arches before and after deformation, respectively. In addition, the axial force \( N \) and bending moment \( M \) can be expressed as

\[
N = -E_T \int_A \epsilon_{xx0} dA = -E_T \epsilon_{m0},
\]

\[
M = -E_T y \int_A \epsilon_{xx0} dA = -E_T \epsilon_{\rho 0}.
\]

By substituting equations (3), (16), (17), and (18) into equation (14), the strain energy can be further simplified as

\[
U_{s0} = \frac{1}{2} \int_{-1}^{1} \left[ \frac{N^2}{EA} + \frac{M^2}{EI} \right] dV_1 d\Delta T_0 + \frac{N}{h} \int_{-1}^{1} NIV_1 d\Delta T_0 + \frac{M}{h} \int_{-1}^{1} MIV_1 d\Delta T_0,
\]

where

\[
V_1 = \sqrt{1 + (a_1 x_1)^2}.
\]
subjected to linear temperature gradient field and vertical uniformly distributed load $q$ can be obtained in the opening literature. The internal force of the arch can be solved by the force method which is based on Castigliano’s theorems, and when an arch is under linear temperature gradient field coupled with vertical uniformly distributed load $q$, the internal forces for the arch can be herein calculated by the force method in this paper.

Based on the force method, the parabolic steel arch is cut into two pieces at the top, which is shown in Figure 6. These two parts are the statically determinate structures with unknown additional axial forces ($N_c$), bending moment ($M_c$), and shear force ($Q_{yc}$). However, according to the principle of structural symmetry, the unknown additional shear force ($Q_{yc}$) is equal to zero.

In addition, to make the cut arch equivalent to the original arch, the relative axial displacement $\Delta Nc$ and the relative rotation $\Delta MC$ at the cutting location should be equal to zero. Based on Castigliano’s theorems, the relative axial displacement $\Delta Nc$ and the relative rotation $\Delta MC$ can be, respectively, given by

$$\Delta Nc = -\frac{\partial U_{x0}}{\partial Nc} = 0, \quad (21)$$
$$\Delta MC = -\frac{\partial U_{x0}}{\partial Mc} = 0. \quad (22)$$

Substituting (19) into (21) and (22), respectively, the corresponding force method equation can be obtained as

$$\int_{-1}^{1} \left[ \frac{N}{EA} \frac{\partial N}{\partial Nc} + \frac{M}{EI} \frac{\partial M}{\partial Nc} \right] IV_1 \, dx_1 = 0, \quad (23)$$
$$\int_{-1}^{1} \left[ \frac{N}{EA} \frac{\partial N}{\partial Mc} + \frac{M}{EI} \frac{\partial M}{\partial Mc} \right] IV_1 \, dx_1 = 0. \quad (24)$$

Because the segmentations of two part of the parabolic steel arch are the statically determinate structures, based on the principle of force equilibrium, the axial force $N$ and bending moment $M$ can be solved, respectively, as

$$N = \frac{qla_1 x_1^2 - N_c}{a_1 x_1^2 + 1}, \quad (25)$$
$$M = M_c - \frac{q(lx_1)^2}{2} - N_c a_1 l x_1. \quad (26)$$

The solutions of the internal force obtained from (25) and (26) have two unknown values ($N_c$ and $M_c$); to solve them, substituting (25) and (26) into (23) and (24), $N_c$ and $M_c$ can be obtained as

$$N_c = \frac{I_{11} E A a_1 \Delta T_o - I_{21} q l}{\Psi}, \quad (27)$$
$$M_c = \frac{3 l (I_{11} q l - I_{31} E A a_1 \Delta T_o)}{\Psi \left( \Xi - 2 a_1 \sqrt{a_1^2 + 1} \right)} \frac{E I a \Delta T_o}{h}. \quad (28)$$

Figure 6: Parabolic arch force method diagram.
where

\[
\begin{align*}
\Gamma_1 &= 768r_s^2 a_1^4 \left( \Xi_1 - 2a_1 \sqrt{a_1^2 + 1} \right), \\
\Gamma_2 &= 3\Xi \left( 64a_1^2 r_s^2 + \ell^2 \right) - 4a_1 \left( 8a_1^4 + 8a_1^2 \right) \Xi \sqrt{a_1^2 + 1} + 4a_1 \left( a_1 + 1 \right) \left[ 4a_1^4 - 8a_1^2 - 9 \right] \Xi^2 - 192r_s^2 a_1^2, \\
\Gamma_3 &= 32r_s^2 a_1^3 \left[ \Xi^2 + 4a_1^4 \Xi \sqrt{a_1^2 + 1} - 4a_1^2 \left( 2a_1^2 - 3a_1^2 - 1 \right) \right], \\
\Gamma_4 &= 8r^2 \left[ 4a_1^2 \left( 4a_1^2 + 1 \right) (a_1^2 + 1) \Xi - 2a_1 \sqrt{a_1^2 + 1} (2a_1^2 - 1) \Xi^2 \right] - 8a_1^2 \left( 2a_1^2 + 1 \right) \left( \sqrt{a_1^2 + 1} \right)^3, \\
\Psi &= 3\Xi^3 a_1 \left[ 128a_1^2 r_s^2 + \ell^2 \right] - 4a_1^3 \sqrt{a_1^2 + 1} \left[ 8a_1^4 + 8a_1^2 \right] \Xi + 192r_s^2 a_1^2 \Xi + 4a_1^3 \ell^2 \left( 4a_1^4 - 8a_1^2 - 9 \right) (a_1^2 + 1), \\
\Xi &= \ln \left( \sqrt{a_1^2 + 1} + a_1 \right) - \ln \left( \sqrt{a_1^2 + 1} - a_1 \right),
\end{align*}
\]

(i.e., \( q = 10^5 \text{kN/m}, 2 \times 10^5 \text{kN/m}, 3 \times 10^5 \text{kN/m} \)) are plotted in Figures 8(a) and 8(b), respectively, where \( \Delta T_g = 100^\circ \text{C}, T_a = 200^\circ \text{C}, \) and \( L/\gamma_s = 30. \)

According to Figure 8, the central axial and bending actions \( N_c \) and \( M_c \) augment as the uniformly distributed vertical load \( q \) augments.

Figures 9 and 10 show the variation of the dimensionless internal force with the rise-span ratio for parabolic arch under the linear gradient temperature field and the vertical uniformly distributed load. Figure 9(a) shows that the central axial force \( N_c \) augments with the augments of the slenderness ratio \( L/r_x \) when temperature difference \( \Delta T_g = 0^\circ \text{C} \) and the average temperature \( T_a = 20^\circ \text{C} \). The central axial force \( N_c \) augments as the rise-span ratio \( f/L \) augments when the rise-span ratio \( f/L \) is less than 0.1 and then decreases as the rise-span ratio \( f/L \) augments when the rise-span ratio \( f/L \) is bigger than 0.1. Figure 9(b) shows that the central axial force \( N_c \) decreases with the augments of the slenderness ratio \( L/r_x \) when temperature difference \( \Delta T_g = 100^\circ \text{C} \) and the average temperature \( T_a = 200^\circ \text{C} \). The central axial force \( N_c \) decreases as the rise-span ratio \( f/L \) augments.

Figure 10(a) shows that the central bending moment \( M_c \) augments with the augments of the slenderness ratio \( L/r_x \) when temperature difference \( \Delta T_g = 0^\circ \text{C} \) and the average temperature \( T_a = 20^\circ \text{C} \). The central bending moment \( M_c \) decreases as the rise-span ratio \( f/L \) augments. Figure 10(b) shows that the central bending moment \( M_c \) decreases with the augments of the slenderness ratio \( L/r_x \) when temperature difference \( \Delta T_g = 100^\circ \text{C} \), the average temperature \( T_a = 200^\circ \text{C} \), and the rise-span ratio \( f/L \) is approximately less than 0.1. The central bending moment \( M_c \) decreases as the rise-span ratio \( f/L \) augments, while the central bending moment \( M_c \) augments with the augments of the slenderness ratio \( L/r_x \) when the rise-span ratio \( f/L \) is approximately bigger than 0.1. The central bending moment \( M_c \) augments as the rise-span ratio \( f/L \).
3.2. In-Plane Instability Analysis. According to the paper of Pi and Bradford [20], the in-plane instability critical load of parabolic arches boundary load can be compared with the relevant equation of circular arc arch. According to paper of Song et al. [14], the critical axial force of a solidly connected circular arch at both ends can be calculated by the following equation:

\[ N_{cr} = \frac{EI (\eta \pi)^2}{(S/2)^2} \frac{EI}{R^2}, \]  

where the parameter \( \eta \pi \) can be expressed as

\[ \eta \pi = 1.403 \pi. \]
For a parabolic arch, the focal collimation distance \( p \) can be calculated by the following equation:

\[
p = \frac{L^2}{8f}.
\]  

For the sake of solving the critical load of instability of the parabolic arch with fixed ends, the axial force term of (30) can be replaced by the axial force at the top of the arch in

\[
R = \left(1 + \frac{y^2}{y^2}ight)^{2/3} = \rho \left(1 + \left(\frac{x}{p}\right)^2\right)^{2/3} 
\]  

\[
\approx \rho.
\]  

In addition, the radius of curvature of the parabolic arch is given by

\[
R = \frac{1}{\rho}.
\]
the analytical solution of the internal force in the previous section [19].

\[ N_{cr} = \frac{\Gamma_2 q l - \Gamma_1 E A \alpha \Delta T_0}{\Psi} \]  
(34)

Substitute (33) and (34) into (30) and obtain

\[ q_{cr} = \frac{\Psi \left[ \frac{E A (\eta \pi)^2}{2} \left( \frac{S}{2} \right)^2 - \frac{EI}{\Gamma^2 l} \right] + \Gamma_1 E A \alpha \Delta T_0}{\Gamma_2 l} \]  
(35)

Equation (35) is the critical in-plane instability load of a fixed parabolic steel arch under the gradient temperature coupled with vertical uniform load.

The change regular of dimensionless instability critical load with rise-span ratio for parabolic arches having different gradient temperature fields is shown in Figure 11, where \( N_{E2} \) is the second mode flexural instability load of an axially compressive fixed column having the same length of the fixed parabolic steel arch, which can be expressed as

\[ N_{E2} = \frac{(1.4303 \pi)^2 E A I_x}{(S/2)^2} \]  
(36)

According to Figure 11, when the rise-span ratio is less than 0.15, the dimensionless critical in-plane instability load of parabolic arch is obviously influenced by the temperature gradient field and decreases with the augment of gradient temperature difference. However, when the rise-span ratio is larger than 0.15, the dimensionless critical in-plane instability load of parabolic arch is slightly influenced by the gradient temperature field. In addition, the dimensionless critical in-plane instability load of parabolic arch decreases with the increasing rise-span ratio.

The variation of the dimensionless instability critical load with the rise-span ratio for the parabolic arch under the linear gradient temperature field is shown in Figure 12. As seen from Figure 12, the dimensionless critical in-plane instability load of parabolic arch decreases with the augment of slenderness.
3.3. Finite Element Validation Analysis. Finite element software ANSYS was employed to verify the accuracy of the above theoretical research. A biaxially symmetric I-beam section of the arch has a height \( h \approx 250 \text{ mm} \), a width \( b \approx 150 \text{ mm} \), a span length \( L = 5000 \text{ mm} \), a flange thickness \( t_f \approx 10 \text{ mm} \), and a web thickness \( t_w \approx 6 \text{ mm} \), and the material properties of Q235 steel were selected for modeling, which can be shown in Figure 13.

The beam188 element was chosen to build the parabolic arch mode. According to the help file of the beam188 element of ANSYS, the beam188 element can be used to specify temperature gradients that vary linearly both over the cross section and along the length of the element. The model is built and solved numerically for critical loads and then compared and analyzed with the theoretical solution.

The comparison between the theoretical solution and the finite element results for the dimensionless critical in-plane instability load of the parabolic arch under linear gradient temperature field coupled with vertical uniform load can be seen from Figure 14. According to Figure 14, the theoretical solutions agree well with the finite element consequence data, indicating that (35) can accurately predict the instability critical load of the fixed parabolic steel arch under the gradient temperature field coupled with the vertical uniform load.

4. Conclusion

This paper has presented a theoretical study on the internal forces and critical in-plane instability loads for parabolic arches with I section under the linear gradient temperature field and vertical uniform load. The cross-sectional effective centroid and effective stiffness for parabolic arches under the linear temperature gradient field have been derived. The axial and bending actions of the parabolic arch under linear gradient temperature field coupled with vertical uniform load have also been obtained. In addition, the precise analytical solutions of the critical load for in-plane instability of the parabolic arch under gradient temperature field coupled with vertical uniform load have been obtained, and these solutions have been verified by numerical simulations of ANSYS. The conclusions of this article can be summarized as follows:

1. For parabolic arches under the linear gradient temperature field and vertical uniform load, the central axial force and the central bending moment decrease with the augment of slenderness.

2. The central axial force of the arch first augments with the augment of the rise-span ratio and then decreases with the augment of the rise-span ratio while the rise-span ratio reaches a certain value. The central bending moment decreases with the augment of the rise-span ratio.

3. When the rise-span ratio is less than 0.15, the dimensionless critical in-plane instability load of
parabolic arch is obviously influenced by the temperature gradient field and decreases with the augment of gradient temperature difference; however, when the rise-span ratio is larger than 0.15, the dimensionless critical in-plane instability load of parabolic arch is slightly influenced by the gradient temperature field.

(4) The dimensionless critical in-plane instability load of parabolic arches decreases with the increasing rise-span ratio.

(5) The dimensionless critical in-plane instability load of parabolic arch decreases with the augment of slenderness.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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