Spin-Spin Interactions in Gauge Theory of Gravity, Violation of Weak Equivalence Principle and New Classical Test of General Relativity

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Abstract

For a long time, it is generally believed that spin-spin interactions can only exist in a theory where Lorentz symmetry is gauged, and a theory with spin-spin interactions is not perturbatively renormalizable. But this is not true. By studying the motion of a spinning particle in gravitational field, it is found that there exist spin-spin interactions in gauge theory of gravity. Its mechanism is that a spinning particle will generate gravitomagnetic field in space-time, and this gravitomagnetic field will interact with the spin of another particle, which will cause spin-spin interactions. So, spin-spin interactions are transmitted by gravitational field. The form of spin-spin interactions in post Newtonian approximations is deduced. This result can also be deduced from the Papapetrou equation. This kind of interaction will not affect the renormalizability of the theory. The spin-spin interactions will violate the weak equivalence principle, and the violation effects are detectable. An experiment is proposed to detect the effects of the violation of the weak equivalence principle.

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1 Introduction

It is known that the source of gravitational interactions is energy-momentum. In Newton’s classical theory of gravity[1], the motion of a test particle in gravitational field is driven by the gravitational force on the particle, and the spin of the particle will not affect its motion in gravity. In general relativity[2, 3], the motion of a test particle in gravitational field is determined by geodesic equation. But the motion of a spinning particle is not given by geodesic equation. In general relativity, the motion of a spinning particle is given by the Papapetrou equation, which is deduced from the Bianchi identities and under the assumption of pole-dipole approximation[4, 5, 6, 7].

Quantum Gauge Theory of Gravity(QGTG) is first proposed in 2001[8, 9, 10, 11]. It is a quantum theory of gravity proposed in the framework of quantum gauge field theory. In 2003, Quantum Gauge General Relativity(QGGR) is proposed in the framework of QGTG[12, 13, 14]. Unlike Einstein’s general theory of relativity, the cornerstone of QGGR is the gauge principle, not the principle of equivalence, which will cause far-reaching influence to the theory of gravity. In QGGR, the field equation of gravitational gauge field is just the Einstein’s field equation, so in classical level, we can set up its geometrical formulation[15], and QGGR returns to Einstein’s general relativity in classical level. The field equation of gravitational gauge field in QGGR is the same as Einstein’s field equation in general relativity, so two equations have the same solutions, though mathematical expressions of the two equations are completely different. For classical tests of gravity, QGGR gives out the same theoretical predictions as those of GR[16], and for non-relativistic problems, QGGR can return to Newton’s classical theory of gravity[17]. Based on the coupling between the spin of a particle and gravitoelectromagnetic field, the equation of motion of spin can be obtained in QGGR. In post Newtonian approximations, this equation of motion of spin gives out the same results as those of GR[18]. QGGR is a perturbatively renormalizable quantum theory, and based on it, quantum effects of gravity[19, 20, 21, 22] and gravitational interactions of some basic quantum fields [23, 24] can be explored. Unification of fundamental interactions including gravity can be fulfilled in a semi-direct product gauge group[25, 26, 27, 28]. If we use the mass generation mechanism which is proposed in literature [29, 30], we can propose a new theory on gravity which contains massive graviton and the introduction of massive graviton does not affect the strict local gravitational gauge symmetry of the action and does not affect the traditional long-range gravitational force[31]. The existence of massive graviton will help us to understand the possible origin of dark matter.

The equivalence principle is generally believed to be one of the milestone of Einstein’s general relativity. It has been tested by many experiments[32, 33, 34, 35, 36]. All these experiments give out null result for the violation of equivalence principle. On the other hand, it is generally believed that spin-spin interaction between rotating bodies will violate equivalence principle. Zhang et al. proposed a phenomenological model for spin-spin interactions[37]. Later, Zhang and his collaborator design a double free fall experiment to test equivalence principle with free falling gyroscopes[38, 39]. There has also been an interest in spin-gravitational coupling for a long time[40, 41, 42, 43, 21, 44].
In a previous paper, the equation of motion of a spinning test particle in gravitational field is deduced based on the coupling between the spin of the particle and the gravitomagnetic field[44]. In this paper, we will study the post Newtonian approximation of that equation. The post Newtonian approximation of the Papapetrou equation is also studied. Then, we apply the equation to the problem of gyroscope which is moving around the earth. Finally, qualitative results on the motion of the gyroscope are given, and based on this calculation, an experiment is proposed to detect the effects of the violation of the weak equivalence principle.

2 Equation of Motion of a Spinning Particle

For the sake of integrity, we give a simple introduction to QGGR and introduce some basic notations which is used in this paper. Details on QGGR can be found in literatures [8, 9, 10, 11, 12, 13, 14, 16]. In gauge theory of gravity, the most fundamental quantity is gravitational gauge field \( C_\mu(x) \), which is the gauge potential corresponding to gravitational gauge symmetry. Gauge field \( C_\mu(x) \) is a vector in the corresponding Lie algebra, which is called gravitational Lie algebra. So \( C_\mu(x) = C_\alpha^\mu(x) \hat{p}_\alpha(\mu, \alpha = 0, 1, 2, 3) \), where \( C_\alpha^\mu(x) \) is the component field and \( \hat{p}_\alpha = -i \frac{\partial}{\partial x^\alpha} \) is the generator of global gravitational gauge group. The gravitational gauge covariant derivative is given by

\[
D_\mu = \partial_\mu - igC_\mu(x) = G_\mu^\alpha \partial_\alpha, \tag{2.1}
\]

where \( g \) is the gravitational coupling constant and matrix \( G = (G_\mu^\alpha) = (\delta_\mu^\alpha - gC_\mu^\alpha) \). Its inverse matrix is \( G^{-1} = \frac{1}{L-gC} = (G^{-1}_\mu^\alpha) \). Using matrix \( G \) and \( G^{-1} \), we can define two important composite operators

\[
g^{\alpha\beta} = \eta^{\mu\nu}G_\mu^\alpha G_\nu^\beta, \tag{2.2}
\]
\[
g_{\alpha\beta} = \eta^{\mu\nu}G^{-1}_\mu^\alpha G^{-1}_\nu^\beta. \tag{2.3}
\]

which are widely used in QGGR. In QGGR, space-time is always flat and space-time metric is always Minkowski metric, so \( g^{\alpha\beta} \) and \( g_{\alpha\beta} \) are no longer space-time metric. They are only two composite operators which consist of gravitational gauge field. The field strength of gravitational gauge field is defined by

\[
F_{\mu\nu}(x) \triangleq \frac{1}{g}[D_\mu, D_\nu] = F^\alpha_{\mu\nu}(x) \cdot \hat{p}_\alpha \tag{2.4}
\]

where

\[
F^\alpha_{\mu\nu} = G_\mu^\beta \partial_\beta C^\alpha_\nu - G_\nu^\beta \partial_\beta C^\alpha_\mu. \tag{2.5}
\]

In QGGR, gravitational gauge field \( C_\mu^\alpha \) is a spin-2 tensor field. The field equation of gravitational gauge field given by the least action principle is equivalent to the Einstein’s field equation in General Relativity[12, 13, 14].
According to literature [44], the equation of motion of a spinning test particle in gravitational field is

\[
\frac{Dp^\gamma}{D\tau} = g^{\gamma\alpha}\left[\nabla_\alpha\left(\frac{g}{2}g_{\beta\delta}J^{\rho\sigma}F^\delta_{\rho\sigma}\right) - \nabla_\beta\left(\frac{g}{2}g_{\alpha\delta}J^{\rho\sigma}F^\delta_{\rho\sigma}\right)\right] \frac{dx^\beta}{d\tau}, \tag{2.6}
\]

where \(J^{\rho\sigma}\) is the spin tensor of the particle, and

\[
\frac{Dp^\gamma}{D\tau} = \frac{dp^\gamma}{d\tau} + \Gamma^\gamma_{\alpha\beta}p^\alpha \frac{dx^\beta}{d\tau}, \tag{2.7}
\]
or equivalently

\[
\frac{D}{D\tau}\left(p^\gamma + \frac{g}{2}J^{\rho\sigma}F^\gamma_{\rho\sigma}\right) = \frac{g}{2}g^{\gamma\alpha}g_{\beta\delta}\nabla_\alpha\left(J^{\rho\sigma}F^\delta_{\rho\sigma}\right) \frac{dx^\beta}{d\tau}. \tag{2.8}
\]

Equation (2.6) can be written into the following form

\[
\frac{Dp^\alpha}{D\tau} = f_s^\alpha, \tag{2.9}
\]

where \(f_s^\alpha\) is the interaction force originated from the coupling of spin and gravitomagnetic field,

\[
f_s^\alpha = g^{\alpha\gamma}\left[\partial_\gamma\left(\frac{g}{2}g_{\beta\delta}J^{\rho\sigma}F^\delta_{\rho\sigma}\right) - \partial_\beta\left(\frac{g}{2}g_{\gamma\delta}J^{\rho\sigma}F^\delta_{\rho\sigma}\right)\right] \frac{dx^\beta}{d\tau}. \tag{2.10}
\]

## 3 Post Newtonian Approximation

Now, let's discuss its the post Newtonian approximation. The standard procedure of the post Newtonian approximation can be found in text book[45, 46]. In gauge theory of gravity, post Newtonian approximation is done in a similar way. Let's consider the case that a gyroscope moving around the earth. Then its post Newtonian approximations give

\[
\begin{align*}
g_{00} & = \eta_{00} - 2\phi + o(\bar{v}^4) \\
g_{ij} & = \eta_{ij} - 2\delta_{ij}\phi + o(\bar{v}^4) \\
g_{i0} & = \eta_{i0} = \zeta_i + o(\bar{v}^5) \\
U^0 & = 1 + o(\bar{v}^2) \\
U^i & = v^i + o(\bar{v}^3) \\
\frac{\partial}{\partial x^i} & \sim \frac{1}{\bar{r}} \\
\frac{\partial}{\partial \tau} & \sim \frac{\bar{v}}{\bar{r}}
\end{align*} \tag{3.1}
\]
where $\phi$ and $\zeta_i$ are given by the following relations

$$\phi = -\frac{GM_{\odot}}{r},$$  \hspace{1cm} (3.2)

$$\zeta = \frac{2G}{r^3}(\vec{x} \times \vec{J}_{\odot}).$$  \hspace{1cm} (3.3)

In above relations, $M_{\odot}$ and $\vec{J}_{\odot}$ are mass and spin angular momentum of the source. The leading terms of gravitational gauge fields are

$$gC_i^j = gC^i_j = -\delta_{ij}\phi + o(\vec{v}^4)$$  \hspace{1cm} (3.4)

The gravitational gauge field strength $F_{\mu\nu}^{\alpha}$ have the expansion

$$gF_{ij}^k = \delta_{ik}(\partial_j\phi) - \delta_{jk}(\partial_i\phi) + o(\vec{v}^4/r)$$

$$gF_{ij}^0 = -\frac{1}{2}(\partial_i\zeta_j - \partial_j\zeta_i) + o(\vec{v}^5/r)$$

$$gF_{i0}^k = -gF_{0i}^k = \frac{1}{2}\partial_i\zeta_k + \delta_{ik}\phi + o(\vec{v}^5/r)$$

$$gF_{i0}^0 = -gF_{0i}^0 = \delta_i\phi + o(\vec{v}^4/r)$$  \hspace{1cm} (3.5)

If we use the following relations,

$$B_i^{\alpha} = \frac{1}{2}\varepsilon_{ijk}F_{jk}^{\alpha},$$  \hspace{1cm} (3.6)

$$E_i^{\alpha} = F_{0i}^{\alpha},$$  \hspace{1cm} (3.7)

we find that the gravitoelectromagnetic fields have the following approximations

$$gB_i^k = -\frac{1}{2}\varepsilon_{ikm}(\partial_m\phi) + o(\vec{v}^4/r)$$

$$gB_i^0 = \frac{1}{2}(\nabla \times \zeta)_i + o(\vec{v}^5/r)$$

$$gE_i^0 = -(\nabla \phi)_i + o(\vec{v}^4/r)$$

$$gE_i^k = -(\frac{1}{2}\partial_i\zeta_k + \delta_{ik}\phi) + o(\vec{v}^5/r)$$  \hspace{1cm} (3.8)

For spin tensor $J^{\mu\nu}$, we have

$$J^{ij} \sim \vec{J}$$

$$J^{i0} \sim \vec{v}\vec{J}$$  \hspace{1cm} (3.9)
The quantity $\Gamma_{\alpha\beta}^\gamma$ is defined by

$$\Gamma_{\alpha\beta}^\gamma = \frac{1}{2} g^{\gamma\delta} \left( \frac{\partial g_{\alpha\delta}}{\partial x^\beta} + \frac{\partial g_{\beta\delta}}{\partial x^\alpha} - \frac{\partial g_{\alpha\beta}}{\partial x^\delta} \right).$$

(3.10)

In post Newtonian approximations, its leading contributions are

$$
\begin{align*}
\Gamma_{00}^i &= \frac{\partial \phi}{\partial x^i} + \partial_i (2\phi^2 + \psi) + \frac{\partial \phi}{\partial x^i} + o(\bar{v}^6/\bar{r}) \\
\Gamma_{0j}^i &= \frac{1}{3} (\partial_j \zeta_i - \partial_i \zeta_j) - \delta_{ij} \frac{\partial \phi}{\partial x^i} + o(\bar{v}^2/\bar{r}) \\
\Gamma_{jk}^i &= -\delta_{ij} \partial_k \phi - \delta_{ik} \partial_j \phi + \delta_{jk} \partial_i \phi + o(\bar{v}^4/\bar{r}) \\
\Gamma_{00}^0 &= \frac{\partial \phi}{\partial t} + \frac{\partial \phi}{\partial x^0} + \frac{\partial \phi}{\partial t} + o(\bar{v}^2/\bar{r}) \\
\Gamma_{0i}^0 &= \partial_0 \phi + \partial_0 \psi + o(\bar{v}^2/\bar{r}) \\
\Gamma_{ij}^0 &= -\frac{1}{2} (\partial_i \zeta_j + \partial_j \zeta_i) - \delta_{ij} \frac{\partial \phi}{\partial x^i} + o(\bar{v}^3/\bar{r})
\end{align*}
$$

(3.11)

where $\psi$ is determined by the following equation

$$\nabla^2 \psi = \frac{\partial^2 \phi}{\partial t^2} + 4\pi G \left( \frac{1}{2} T_{00} + T_{ii} \right).$$

(3.12)

Using all above approximations, we can calculate the leading contribution of the force $f^\alpha_s$ from equation (2.10), which is

$$f_s^i = \partial_i \left[ \frac{1}{2} \overrightarrow{J} \cdot (\nabla \times \overrightarrow{\zeta}) \right] + \left( \overrightarrow{J} \times \nabla \right)_i \left[ \partial_0 \phi + \frac{\partial}{\partial x^i} \cdot \nabla \phi \right] + 2 \overrightarrow{J} \cdot (\overrightarrow{v} \times \nabla) (\partial_i \phi),$$

(3.13)

$$f_s^0 = \overrightarrow{J} \cdot (\overrightarrow{v} \times \nabla) \left[ -\partial_0 \phi + \frac{\partial}{\partial x^i} \cdot \nabla \phi \right] + \frac{1}{2} (\overrightarrow{v} \cdot \nabla) \left[ \overrightarrow{J} \cdot (\nabla \times \overrightarrow{\zeta}) \right].$$

(3.14)

We can see that $f_s^i$ is of order $\bar{v}^3 J/\bar{r}^2$, and $f_s^0$ is of order $\bar{v}^4 J/\bar{r}^2$.

In general relativity, under pole-dipole approximation, the equation of motion of a spinning particle is given by the following Papapetrou equation

$$\frac{D}{D\tau} \left( m u^\alpha + u_\beta \frac{D J^{\alpha\beta}}{D\tau} \right) - \frac{1}{2} J^{\beta\gamma} u^\delta R^\alpha_{\delta\beta\gamma} = 0,$$

(3.15)

where $u^\alpha = \frac{dx^\alpha}{d\tau}$ is the four-velocity, and $R^\alpha_{\delta\beta\gamma}$ is the curvature tensor. Now, let’s discuss its post Newtonian approximation. In most cases,

$$\frac{D J^{\alpha\beta}}{D\tau} \ll m,$$

(3.16)
so,
\[ u_\beta \frac{D}{D\tau} J^{\alpha\beta} \ll \mu u^\alpha. \] (3.17)

Therefore, in leading order approximation, the term \( u_\beta \frac{D}{D\tau} J^{\alpha\beta} \) in the Papapetrou equation can be omitted. Then in post Newtonian approximation, equation (3.15) simplifies to
\[
\frac{Dp^\alpha}{D\tau} = g J^{\alpha\mu} u_\beta \left( \partial_\beta \partial_\sigma C_\rho^\alpha - \eta_\alpha^\rho \eta_\beta^\sigma \partial_\alpha \partial_\sigma C_\beta^\beta \right). \] (3.18)

Compare it with equation (2.9), we find that, in the present case, the interaction force originated from the coupling between the spin and gravitomagnetic field is
\[
f_s^{\alpha i} = g J^{\alpha\mu} u_\beta \left( \partial_\beta \partial_\sigma C_\rho^\alpha - \eta_\alpha^\rho \eta_\beta^\sigma \partial_\alpha \partial_\sigma C_\beta^\beta \right). \] (3.19)

After applying results (3.1) and (3.4), we can prove that
\[
f_s^n = \partial_\ell \left[ \frac{1}{2} \vec{J} \cdot (\nabla \times \vec{\zeta}) \right] + \left( \vec{J} \times \nabla \right)_i \left[ \partial_0 \phi + \vec{v} \cdot \nabla \phi \right] + 2 \vec{J} \cdot (\vec{v} \times \nabla) (\partial_i \phi), \] (3.20)
\[
f_s^0 = \vec{J} \cdot (\vec{v} \times \nabla) \left[ - \partial_0 \phi + \vec{v} \cdot \nabla \phi \right] + \frac{1}{2} (\vec{v} \cdot \nabla) \left[ \vec{J} \cdot (\nabla \times \vec{\zeta}) \right]. \] (3.21)

Compare them with equations (3.13) and (3.14), we find that, in post Newtonian approximation, the force \( f_s^{\alpha i} \) is the same as \( f_s^{\alpha} \). So, the equation of motion (2.6) and the Papapetrou equation give out the same results in the post Newtonian approximation.

## 4 Spin-Spin Interactions Force

Now, let’s change the forms of equations (3.13) and (3.14) into more explicit forms. When we discuss the case that a gyroscope moves around the earth, the post Newtonian field \( \phi \) and \( \vec{\zeta} \) are given by equation (3.2) and (3.3) respectively. Then in leading term approximations, equation (3.13) is changed into
\[
f_s^i = -3G \left( \vec{J} \cdot \vec{J} \right) \frac{\partial}{\partial r} - 3G \left( \vec{r} \cdot \vec{J} \right) \frac{\partial}{\partial r} - 3G \left( \vec{r} \cdot \vec{J} \right) \frac{\partial}{\partial r} + 15G \left( \vec{r} \cdot \vec{J} \right) \frac{\partial}{\partial r} + 3G \frac{\left( \vec{J} \times \vec{r} \right)}{r^2} - 3G \frac{\left( \vec{v} \cdot \vec{J} \right) \frac{\left( \vec{J} \times \vec{r} \right)}{r^2}}{r^2} - 6G \frac{\left( \vec{J} \cdot (\vec{v} \times \vec{r}) \right)}{r^4} + (\vec{J} \times \nabla) \frac{\partial \phi}{\partial r}, \] (4.1)

and (3.14) is changed into
\[
f_s^0 = -3G \left( \vec{v} \cdot \vec{J} \right) \frac{\partial}{\partial r} - 3G \frac{\left( \vec{v} \cdot \vec{J} \right) \left( \vec{r} \cdot \vec{J} \right)}{r^2} - 3G \frac{\left( \vec{J} \cdot \vec{J} \cdot \vec{r} \right)}{r^2} - 3G \frac{\left( \vec{J} \cdot (\vec{v} \times \vec{r}) \right) \left( r \cdot \vec{v} \right)}{r^2} - 15G \frac{\left( \vec{r} \cdot \vec{J} \right) \left( \vec{r} \cdot \vec{J} \right)}{r^2} \frac{\partial \phi}{\partial r}, \] (4.2)
In equation (4.1), the first four terms are from spin-spin interactions, the next two terms are from spin-orbit interactions. We can see that the spin-spin interaction force has the following form

\[ f_{ss} = -3G(\vec{J} \cdot \vec{J}_\oplus) \frac{\vec{r}}{r^5} - 3G(\vec{r} \cdot \vec{J}_\oplus) \frac{\vec{J}}{r^5} - 3G(\vec{r} \cdot \vec{J}) \frac{\vec{J}_\oplus}{r^5} + 15G(\vec{r} \cdot \vec{J})(\vec{r} \cdot \vec{J}_\oplus) \frac{\vec{r}_5}{r^7}, \]  

which is symmetric under the exchange of two spin operators. So, the spin-spin interaction force is proportional to the spin of each particle, but decreases as the inverse quadruplicate of the distances. Equation (4.3) can be changed into another form

\[ f_{ss}^i = G \frac{J_i M_{jk} J_{\oplus k}}{r^4}, \]  

where

\[ M_{jk} = -3\delta_{jk} \hat{r}_i - 3\delta_{ij} \hat{r}_k - 3\delta_{ik} \hat{r}_j + 15\hat{r}_i \hat{r}_j \hat{r}_k. \]  

In above equation, \( \hat{r} = \frac{\vec{r}}{r} \) is a unit vector. It can be seen that \( M_{jk} \) is symmetric under exchange of any two indexes of \( i, j \) and \( k \).

### 5 Test Gravity Theory With a Gyroscope

Now, we discuss a classical test of gravity theory. Suppose that a gyroscope is placed in an orbit around the earth. For the earth, its gravitational field is quite stable, so the time derivative of the earth’s gravitational potential almost vanishes

\[ \frac{\partial \phi}{\partial t} = 0. \]  

In the test, we select a special orbit so that the spin axis of the earth is along the direction normal to the plane of the orbit. in this case

\[ \vec{r} \cdot \vec{J}_\oplus = \vec{v} \cdot \vec{J}_\oplus = 0. \]  

In this case, the time component and the space component of the spin-dependent force \( f_s^0 \) and \( f_s^i \) are respectively simplified into

\[ f_s^i = -3G(\vec{J} \cdot \vec{J}_\oplus) \frac{\vec{r}}{r^5} - 3G(\vec{r} \cdot \vec{J}) \frac{\vec{J}_\oplus}{r^5} + 3GM_\oplus \frac{(\vec{J} \times \vec{v})_i}{r^5}, \]  

\[ f_s^0 = -3G(\vec{J} \cdot \vec{J}_\oplus)(\vec{r} \cdot \vec{v}) \frac{1}{r^5} + 3GM_\oplus \frac{\vec{J} \cdot (\vec{v} \times \vec{r})}{r^5}. \]  

Then, we let the spin axis of the gyroscope be in the plane of the orbit. So, the spin axis of the earth \( \vec{J}_\oplus \) is perpendicular to the spin axis of the gyroscope \( \vec{J} \),

\[ \vec{J}_\oplus \cdot \vec{J} = 0, \]  

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and \( \vec{r} \times \vec{v} \) is perpendicular to the orbit plane, and therefore to the spin axis of the gyroscope

\[
\vec{J} \cdot (\vec{r} \times \vec{v}) = 0.
\] (5.6)

In this case, the time component of the spin-dependent force \( f^0_s \) vanishes, and space-component of the spin-dependent force finally becomes

\[
\vec{f}_s = -3G(\vec{r} \cdot \vec{J}) \frac{\vec{J}}{c^2 r^3} + 3G M_\odot \frac{\vec{J} \times \vec{v}}{c^2 r^3} - 3G M_\odot (\vec{v} \cdot \vec{r}) \frac{(\vec{J} \times \vec{r})}{c^2 r^3}.
\] (5.7)

In post newtonian approximation, \( g_{\alpha\beta} \) is given by (3.1). In spherical coordinate system, we can prove that the non-vanishing components of \( g_{\alpha\beta} \) are

\[
\begin{align*}
g_{tt} &= -B(r) \\
g_{rr} &= A(r) \\
g_{\theta\theta} &= A(r)r^2 \\
g_{\phi\phi} &= A(r)r^2 \sin^2 \theta \\
g_{\phi t} &= g_{t\phi} = C(r) \sin^2 \theta
\end{align*}
\] (5.8)

where

\[
\begin{align*}
A(r) &= 1 - 2\phi = 1 + \frac{2G M_\odot}{r} \\
B(r) &= 1 + 2\phi = 1 - \frac{2G M_\odot}{r} \\
C(r) &= -\frac{2G J_\odot}{c^2 r}
\end{align*}
\] (5.9)

The non-vanishing component of the inverse matrix of \( g_{\alpha\beta} \) is

\[
\begin{align*}
g^{tt} &= -\frac{1}{B(r)} \\
g^{rr} &= \frac{1}{A(r)} \\
g^{\theta\theta} &= \frac{1}{A(r)r^2} \\
g^{\phi\phi} &= \frac{1}{A(r)r^2 \sin^2 \theta} \\
g^{\phi t} &= g^{t\phi} = \frac{C(r)}{r^2}
\end{align*}
\] (5.10)

\( \Gamma^\gamma_{\alpha\beta} \) is defined by (3.10). In spherical coordinate system, its non-vanishing components
are
\[
\begin{aligned}
\Gamma^t_{tr} &= \Gamma^t_{rt} = \frac{B'(r)}{2B(r)} \\
\Gamma^t_{tt} &= \frac{B'(r)}{2A(r)} \\
\Gamma^\theta_{\phi\phi} &= \Gamma^\phi_{\theta\phi} = -\frac{C'(r)}{2A(r)} \sin^2 \theta \\
\Gamma^r_{\theta\phi} &= \Gamma^r_{\phi\theta} = \left[ -\frac{C'(r)}{2B(r)} + \frac{A(r)C(r)}{r} \right] \sin^2 \theta \\
\Gamma^r_{rr} &= \frac{A'(r)}{2A(r)} \\
\Gamma^\phi_{r\phi} &= -\frac{r^2A'(r)}{2A(r)} - r \\
\Gamma^\theta_{r\theta} &= \Gamma^\theta_{\theta r} = \frac{A'(r)}{2A(r)} + \frac{1}{r} \\
\Gamma^\phi_{rr} &= -\sin \theta \cos \theta \\
\Gamma^r_{\theta\phi} &= \Gamma^\phi_{r\theta} = \frac{C(r)}{r^2A(r)} \cot \theta \\
\Gamma^\phi_{\theta\theta} &= \Gamma^\phi_{\phi\theta} = \cot \theta \\
\Gamma^\phi_{\phi\phi} &= \Gamma^\phi_{\phi\phi} = \cot \theta \\
\Gamma_{\phi,\phi} &= \Gamma_{\phi,\phi} = \cot \theta
\end{aligned}
\]

Now, let's discuss the equation of motion of the gyroscope, which is given by equation (2.9). Using above results on \( \Gamma_{\alpha\beta} \), we can obtain
\[
\frac{d^2 \theta}{d\tau^2} - \frac{C(r)}{r^2} \sin 2\theta \frac{dt}{d\tau} \frac{d\varphi}{d\tau} + \frac{A'(r)}{A(r)} \left[ \frac{2}{r} + \frac{A'(r)}{A(r)} \right] \frac{dr}{d\tau} \frac{d\theta}{d\tau} - \sin \theta \cos \theta \left( \frac{d\varphi}{d\tau} \right)^2 = \frac{f^\theta}{m},
\]
(5.12)
\[
\frac{d^2 \varphi}{d\tau^2} + \frac{C'(r)}{r^2} \sin 2\theta \frac{dt}{d\tau} \frac{d\varphi}{d\tau} + \frac{2}{r} + \frac{A'(r)}{A(r)} \left[ \frac{2}{r} + \frac{A'(r)}{A(r)} \right] \frac{dr}{d\tau} \frac{d\varphi}{d\tau} + 2\cot \theta \left( \frac{d\theta}{d\tau} \right)^2 = \frac{f^\varphi}{m},
\]
(5.13)
\[
\frac{d^2 \varphi}{d\tau^2} + \frac{B'(r)}{B(r)} \frac{dt}{d\tau} + \frac{3C(r)}{r} \sin^2 \theta \frac{dr}{d\tau} \frac{d\varphi}{d\tau} = \frac{f^t}{m},
\]
(5.14)
The spherical coordinate system is specially selected so that the polar angle \( \theta \) of the plane of the orbit is \( \frac{\pi}{2} \). But the plane of the orbit can not always exactly be \( \frac{\pi}{2} \), for there always exists small perturbation to the motion of the gyroscope. If the small perturbation to the polar angle of the plane of the orbit is denoted by \( \delta \theta \), then we have
\[
\theta = \frac{\pi}{2} + \delta \theta,
\]
(5.16)
where \( \delta \theta \) is a first order infinitesimal quantity. If we only keep first order infinitesimal quantity, then equations (5.12), (5.13), (5.14) and (5.15) are changed into
\[
\frac{d^2 r}{d\tau^2} + \frac{A'(r)}{2A(r)} \left( \frac{dr}{d\tau} \right)^2 + \frac{B'(r)}{2A(r)} \left( \frac{dt}{d\tau} \right)^2 - \frac{(r^2A(r))'}{2A(r)} \left( \frac{d\varphi}{d\tau} \right)^2 - \frac{C'(r)}{A(r)} \frac{dt}{d\tau} \frac{d\varphi}{d\tau} = \frac{f^r}{m},
\]
(5.17)
\[
\frac{d^2\delta\theta}{d\tau^2} + \frac{2}{r} \frac{dr}{d\tau} \frac{d\delta\theta}{d\tau} + \delta\theta \left( \frac{d\varphi}{d\tau} \right)^2 = \frac{f_s^\theta}{m},
\]  
(5.18)

\[
\frac{d^2\varphi}{d\tau^2} + \left[ \frac{2}{r} + \frac{A'(r)}{A(r)} \right] \frac{dr}{d\tau} \frac{d\varphi}{d\tau} + \frac{d}{d\tau} \left( \frac{C(r)}{\sqrt{GM_{\odot}L}} \right) \frac{d\varphi}{d\tau} = \frac{f_s^\varphi}{m},
\]  
(5.19)

\[
\frac{d^2t}{d\tau^2} + \left[ \frac{B'(r)}{B(r)} + \frac{3C(r)\sqrt{GM_{\odot}L}}{r^3} \right] \frac{dt}{d\tau} \frac{dr}{d\tau} = \frac{f_s^t}{m}.
\]  
(5.20)

In order to go any further, we need to know the explicit form the the spin dependent force \( \vec{f}_s \), which is given by equation (5.7). In the present coordinate system, the z axis is selected to be along the direction of \( \vec{J}_\oplus \), so

\[
\vec{J}_\oplus = J_\oplus \hat{k},
\]  
(5.21)

\[
\vec{J} \times \vec{v} = \left[ eJ \sqrt{\frac{GM_{\odot}}{L}} \sin \varphi \sin(\varphi - \varphi_0) + J \sqrt{\frac{GM_{\odot}L}{r}} \cos(\varphi - \varphi_0) \right] \hat{k},
\]  
(5.22)

\[
\vec{r} \cdot \vec{J} = rJ \cos(\varphi - \varphi_0),
\]  
(5.23)

where \( \varphi_0 \) is the intersection angle between the spin \( \vec{J} \) and semimajor axis, \( L \) is the semilatus rectum, and \( e \) is the eccentricity. Then the spin-dependent force has the following form

\[
f_t = 0,
\]  
(5.24)

\[
\vec{f}_s = f_s \hat{k},
\]  
(5.25)

where

\[
f_s = \left[ -\frac{3GJJ_\oplus}{c^2r^4} + \frac{3GM_{\odot}J\sqrt{GM_{\odot}L}}{c^2r^4} \right] \cos(\varphi - \varphi_0).
\]  
(5.26)

The above expression for the force \( \vec{f}_s \) is in the Descartes coordinate system. We need to change its form into that in spherical coordinate system. The components of the force \( \vec{f}_s \) in spherical coordinate system are

\[
\begin{aligned}
& f_s^r = 0, \\
& f_s^\theta = -\frac{f_s}{r}, \\
& f_s^\varphi = 0.
\end{aligned}
\]  
(5.27)

Then, equations (5.17), (5.18), (5.19) and (5.20) are changed into

\[
\frac{d^2r}{d\tau^2} + \frac{A'(r)}{2A(r)} \left( \frac{dr}{d\tau} \right)^2 + \frac{B'(r)}{2A(r)} \left( \frac{dt}{d\tau} \right)^2 - \frac{(r^2A'(r))'}{2A(r)} \left( \frac{d\varphi}{d\tau} \right)^2 - \frac{C''(r)}{A(r)} \frac{d\varphi}{d\tau} = 0,
\]  
(5.28)
\[ \frac{d^2 \theta}{d\tau^2} + \frac{2 dr}{r d\tau} \frac{d \theta}{d\tau} + \delta \left( \frac{d\varphi}{d\tau} \right)^2 = -\frac{f_s}{m r}, \quad (5.29) \]

\[ \frac{d^2 \varphi}{d\tau^2} + \left[ \frac{2}{r} + \frac{A'(r)}{A(r)} \right] \frac{dr}{d\tau} \frac{d \varphi}{d\tau} + \frac{d}{d\tau} \left( \frac{C(r)}{\sqrt{GM_\odot L}} \right) \frac{d \varphi}{d\tau} = 0, \quad (5.30) \]

\[ \frac{d^2 t}{d\tau^2} + \left[ \frac{B'(r)}{B(r)} + \frac{3C(r)\sqrt{GM_\odot L}}{r^3} \right] \frac{dt}{d\tau} \frac{dr}{d\tau} = 0. \quad (5.31) \]

Equations (5.30), (5.31) and (5.28) yield three constants of motion, which are

\[ r^2 A(r) \frac{d \varphi}{d\tau} \cdot \exp \left[ \frac{C(r)}{\sqrt{GM_\odot L}} \right] = J_0 \quad (constant), \]

\[ B(r) \frac{dt}{d\tau} \cdot \exp \left[ \frac{2GJ_\odot \sqrt{GM_\odot L}}{c^4 r^3} \right] = 1. \quad (5.33) \]

\[ A(r) \left( \frac{dr}{d\tau} \right)^2 - \frac{1}{B(r)} + \frac{J_0^2}{r^2 A(r)} e^{-2C(r)/\sqrt{GM_\odot L}} - \frac{4GJ_\odot J_0^2}{3c^4 r^3 \sqrt{GM_\odot L}} - \frac{4GJ_\odot J_0^2}{3c^4 r^3} = -E \quad (constant), \quad (5.34) \]

Multiply both sides of equation (5.29) with \( r^2 \), we get

\[ \frac{d}{d\tau} \left( r^2 \frac{d \delta \theta}{d\tau} \right) + \frac{J_0^2}{r^2} \delta \theta = -\frac{f_s}{m r}. \quad (5.35) \]

Because

\[ \frac{d}{d\tau} = \frac{d \varphi}{d\tau} \frac{d}{d\varphi} = \frac{J_0}{r^2} \frac{d}{d \varphi}, \quad (5.36) \]

we can change (5.35) into the following simpler form

\[ \frac{d^2 \delta \theta}{d\varphi^2} + \delta \theta = -\frac{f_s r^3}{J_0^2 m}. \quad (5.37) \]

The above equation (5.37) is the typical equation of motion of a forced oscillator. Applying (5.26), we get

\[ \frac{f_s r^3}{J_0^2 m} = b \cos(\varphi - \varphi_0)(1 + e \cos \varphi), \quad (5.38) \]

with \( b \) a constant, which are defined by

\[ b = \left( \frac{3(GM_\odot)^{3/2}}{c^2 J_0^2 L^{1/2}} - \frac{3GJ_\odot}{c^2 J_0^2 L} \right) \left( \frac{J}{m} \right). \quad (5.39) \]

The general solution of oscillator equation (5.37) is

\[ \delta \theta(\varphi) = \alpha_1 \cos \varphi + \alpha_2 \sin \varphi + \alpha_3(\varphi) \cos \varphi + \alpha_4(\varphi) \sin \varphi, \quad (5.40) \]
where $\alpha_1$ and $\alpha_2$ are two constants, and $\alpha_3(\varphi)$ and $\alpha_4(\varphi)$ are determined by the following two equations

$$\frac{d\alpha_3(\varphi)}{d\varphi} = \frac{f_s r^3}{J_0^2 m} \sin \varphi,$$

(5.41)

$$\frac{d\alpha_4(\varphi)}{d\varphi} = -\frac{f_s r^3}{J_0^2 m} \cos \varphi.$$

(5.42)

So, we have

$$\alpha_3(\varphi) = \frac{b\sin \varphi_0}{2} \varphi + \frac{b\cos \varphi_0}{2} \sin^2 \varphi - \frac{b\sin \varphi_0}{2} \sin \varphi \cos \varphi$$

$$+ \frac{eb \cos \varphi_0}{3} (1 - \cos^3 \varphi) + \frac{eb \sin \varphi_0}{3} \sin^3 \varphi,$$

(5.43)

$$\alpha_4(\varphi) = -\frac{b\cos \varphi_0}{2} \varphi - \frac{b\cos \varphi_0}{2} \sin \varphi \cos \varphi - \frac{b\sin \varphi_0}{2} \sin^2 \varphi$$

$$-eb \cos \varphi_0 (\sin \varphi - \frac{1}{3} \sin^3 \varphi) - \frac{eb \sin \varphi_0}{3} (1 - \cos^3 \varphi).$$

(5.44)

Therefore, the general solution of equation (5.37) becomes

$$\delta \theta(\varphi) = \alpha_1 \cos \varphi + \alpha_2 \sin \varphi - \frac{b}{2} \varphi \sin(\varphi - \varphi_0) - \frac{b\sin \varphi_0}{2} \sin \varphi$$

$$+ \frac{eb}{3} [-\frac{3}{2} \cos \varphi_0 + \cos(\varphi + \varphi_0) + \frac{1}{2} \cos(2\varphi - \varphi_0)].$$

(5.45)

The orbital period $T$ of the gyroscope is

$$T = 2\pi \sqrt{\frac{a_G^3}{GM_\oplus}},$$

(5.46)

with $a_G$ the semimajor axis of the orbit of the gyroscope

$$a_G = \frac{L}{1 - e^2}.$$

(5.47)

Suppose that at the starting point, $\varphi = 0$. Then at an arbitrary time $t$, the value of $\varphi$ angle is about

$$\varphi(t) \simeq \frac{2\pi t}{T}.$$

(5.48)

When the time $t$ is many times larger than the orbital period $T$, we will have the following approximate solutions for $\alpha_3$ and $\alpha_4$

$$\alpha_3(t) \simeq \frac{(GM_\oplus)^{1/2}}{2a_G^{3/2}} bt \sin \varphi_0,$$

(5.49)

$$\alpha_4(t) \simeq -\frac{(GM_\oplus)^{1/2}}{2a_G^{3/2}} bt \cos \varphi_0.$$

(5.50)
If at the starting point, we make the plane of the orbit exact at the plane with $\theta = \frac{\pi}{2}$, which requires that when $t$ is small, or equivalently $\varphi$ is small, $\delta \theta$ should almost vanish. Applying this initial condition to (5.40), we can obtain

$$\alpha_1 = \alpha_2 = 0. \quad (5.51)$$

Therefore, the solution (5.45) becomes

$$\delta \theta(t) = -\theta_m(t) \sin(\varphi - \varphi_0), \quad (5.52)$$

with

$$\theta_m(t) = \left[ \frac{3GM_\oplus}{2c^2(La_G)^{3/2}} - \frac{3J_\oplus(GM_\oplus)^{1/2}}{2c^2M_\oplus L^2a_G^{3/2}} \right] \left( \frac{J}{m} \right) t. \quad (5.53)$$

$\theta_m(t)$ is the amplitude of the oscillation, which is linearly increased with time.

### 6 Observable Effects

In the last chapter, we have discussed the motion of a gyroscope. When the plane of the orbit is perpendicular to the spin axis of the earth, and the spin axis of the gyroscope lies in the orbit plane, the motion of the gyroscope in the $\hat{e}_\theta$ direction obeys the forced oscillator equation of motion. The resonant force $\rightarrow f_s$ originates from the interaction between the spin of the gyroscope and the gravitomagnetic field of the earth. The solution of the equation is given by (5.52). Now, our question is that whether this forced oscillation is observable, or the amplitude $\theta_m(t)$ is large enough to be detectable? And which information can we obtained from this experiment?

Inside the satellite that carries the gyroscope, the quantity $\theta_m(t)$ is not directly measurable, what is directly measurable inside the satellite is the relative displacement of the gyroscope inside the satellite. The relative displacement $\delta r_\theta(t)$ is given by radius $r$ times angular displacement $\delta \theta(t)$, that is

$$\delta r_\theta(t) = r \cdot \delta \theta(t). \quad (6.1)$$

Therefore, we have

$$\delta r_\theta(t) = -w_m(t) \sin(\varphi - \varphi_0), \quad (6.2)$$

with

$$w_m(t) = r \cdot \left[ \frac{3GM_\oplus}{2c^2(La_G)^{3/2}} - \frac{3J_\oplus(GM_\oplus)^{1/2}}{2c^2M_\oplus L^2a_G^{3/2}} \right] \left( \frac{J}{m} \right) t. \quad (6.3)$$

In order that the oscillation can be observed, its amplitude $w_m$ should be large enough. Now, let’s first estimate its magnitude. There are two terms in the right hand of (6.3), which represent two separate contributions. They are denoted by

$$w_1(t) = \frac{3GM_\oplus r}{2c^2(La_G)^{3/2}} \left( \frac{J}{m} \right) t, \quad (6.4)$$
\[ w_2(t) = -\frac{3r J_{\oplus}(GM_{\oplus})^{1/2}}{2c^2M_{\oplus}^2 a_{G}^{3/2}} \left( \frac{J}{m} \right) t, \tag{6.5} \]

where \( w_1 \) represents the spin-orbit interactions, and \( w_2 \) represents the spin-spin interactions. So, we have

\[ w_m(t) = w_1(t) + w_2(t). \tag{6.6} \]

Suppose that the orbit is near the surface of the earth, in this case

\[ L \simeq a_G \simeq r \simeq R_{\oplus} \simeq 6.38 \times 10^6 \text{m}. \tag{6.7} \]

It is known that the spin angular momentum of the earth is about

\[ J_{\oplus} \simeq 5.92 \times 10^{33} \text{ kg} \cdot \text{m}^2/\text{s}. \tag{6.8} \]

Using all these approximations, we have the following estimations

\[ w_1(t) = 5.14 \times 10^{-9} \left( \frac{J}{m} \right) \text{ m/year}, \tag{6.9} \]

\[ w_2(t) = -1.01 \times 10^{-10} \left( \frac{J}{m} \right) \text{ m/year}, \tag{6.10} \]

where the unit of \( J/m \) is in \( \text{m}^2/\text{s} \). If \( J/m \) is about \( 10^3 \text{m}^2/\text{s} \), then

\[ w_1(t) = 5.14 \mu \text{m/year}, \tag{6.11} \]

\[ w_2(t) = -0.101 \mu \text{m/year}, \tag{6.12} \]

Both of them are large enough to be detectable. According to (6.6), we have

\[ w_m(t) = 5.04 \mu \text{m/year}. \tag{6.13} \]

So, if the magnitude of \( J/m \) is about \( 10^3 \text{m}^2/\text{s} \), the oscillation amplitude of the gyroscope in the \( \hat{e}_\theta \) direction increases about 5.04 micrometer per year, which is detectable.

In the above discussions, it is found that, the dominant contribution to the increasing of the oscillation amplitude comes from the spin-orbit contributions, which is about 51 times larger than that of the spin-spin interactions. Now, the question is that, whether can we simultaneous detect the magnitude of both \( w_1 \) and \( w_2 \)? The answer is that, if we want to detect both of them, we need at least two gyroscopes. Suppose that we have another gyroscope, which is in another satellite that is running around the earth in the opposite direction but with the same orbit parameters. In this case, the direction of the spin axis of the earth inverses, so the oscillation of the gyroscope in the \( \hat{e}_\theta \) direction is

\[ \delta r_\theta(t) = -w'_m(t) \sin(\varphi - \varphi_0), \tag{6.14} \]

where

\[ w'_m(t) = w_1(t) - w_2(t). \tag{6.15} \]
After we detect both $w_m(t)$ and $w'_m(t)$, we can determine both spin-orbit contribution $w_1(t)$ and spin-spin contribution $w_2(t)$

$$w_1(t) = \frac{w_m(t) + w'_m(t)}{2}, \quad (6.16)$$

$$w_2(t) = \frac{w_m(t) - w'_m(t)}{2}. \quad (6.17)$$

### 7 Summary and Discussions

In this paper, based on the equation of motion of a spinning particle in gravitational field, spin-dependent gravitational force is discussed. This kind of gravitational force contains two part, one is the spin-orbit interactions, and another is spin-spin interactions. Both of them are spin-dependent force. This force originates from the coupling between the spin of the particle and gravitoelectromagnetic field. So, whether the results in this paper are reliable depends on whether the coupling between spin and gravitoelectromagnetic field is correct. It is known that, another important effects of gravity, the precession of a gyroscope in gravitational field, also originate from the spin-gravitomagnetic coupling\[45, 46, 18\]. That is, the influence of the spin-gravitomagnetic coupling to the motion of the spin is the precession of a gyroscope, while its influence to the motion of particle is what we discussed in this paper. It is known that the effect on the precession of a gyroscope had already been detected by experiments recently[47, 48], and experimental results are quite consistent with theoretical expectations, which tells us that the spin-gravitomagnetic coupling does exist and is reliable. As long as the spin-gravitomagnetic coupling has influences to the motion of the spin, it must have influence to the motion of the particle.

The spin-dependent force is expressed by equations (3.13) and (3.14). The existence of this force explicitly showed that not all gravitational actions are inertial actions, which means the violation of weak equivalence principle[49]. We can consider this violation from a traditional way. We may consider that the satellite which carries the gyroscope is a free falling frame of reference. According to the weak equivalence principle, no objects in the free falling frame of reference can fell gravitational force, and therefore all objects in that frame should be relatively static or uniform rectilinear moving. But now, we know that, owing to the spin dependent force, the gyroscope is neither relatively static nor uniform rectilinear moving, but oscillating with increasing amplitude. The existence of this oscillation means that the gyroscope feels gravitational force in the free falling frame of reference, which violates the weak equivalence principle.

For a long time, it is generally believed that, even if there exist spin-spin interaction, it must be too weak to be detectable. Indeed, the spin-spin or spin-orbit interaction force
is extremely weak. According to equation (5.7), the spin-spin interaction force is about

\[ \frac{-3GJJ_{\parallel}}{c^2r^4}. \]  

(7.1)

If we suppose that the angular momentum of the gyroscope is about \(10^4 kgm^2/s\), then this force is about \(8 \times 10^{-17} N\), which is indeed too weak to be directly detectable, and is many orders smaller than the upper limit given by experiments\[32, 33, 34, 35, 36\]. But, this force will cause a resonance oscillation in the \(\hat{e}_{\theta}\) direction, which is detectable. Direct measurement of this oscillation will be of great importance, for it is the first direct observation of the violation of the weak equivalence principle. There is a long standing dispute on the reliability of the weak equivalence principle. Some treated it as a corner stone of general relativity, while some others criticized that it is not strictly hold\[49, 50, 51\]. Now, the experiment can serve as a direct experimental solution of the dispute.

Because QGGR can return to general relativity in classical level, and the corner stone of QGGR is gauge principle, our conclusion is that the equivalence principle is not the necessary starting point of general relativity. Therefore, the violation of weak equivalence principle does not affect the correctness of general relativity. On the contrary, if equivalence principle is still considered as corner stone of general relativity, the theory will be logically self-contradictory, for in that case, the violation of weak equivalence principle is a result of the equivalence principle. (Because Papapetrou equation is a direct result of Einstein’s field equation.)

References

[1] Isaac Newton, *Mathematical Principles of Natural Philosophy*, (Cambridge University Press, 1934).

[2] Albert Einstein, Annalen der Phys., 49 (1916) 769.

[3] Albert Einstein, Zeits. Math. und Phys. 62 (1913) 225.

[4] M.Mathisson, Acta Phys. Pol., 6, (1937):163.

[5] J.Lubanski, Acta Phys. Pol., 6, (1937):356.

[6] A.Papapetrou, Proc.Roy.Soc., A209, (1951):248; E.Corinaldesi and A.Papapetrou, Proc.Roy.Soc., 209, (1951):259.

[7] A.H.Taub, J. Math. Phys., 5, (1964):112.

[8] Ning Wu, "Gauge Theory of Gravity", hep-th/0109145.

[9] Ning Wu, Commun. Theor. Phys. (Beijing, China) 38 (2002): 151-156.

[10] Ning Wu, "Quantum Gauge Theory of Gravity", hep-th/0112062.
[11] Ning WU, "Quantum Gauge Theory of Gravity", talk given at Meeting of the Devison of Particles and Fields of American Physical Society at the College of William & Mary (DPF2002), May 24-28, 2002, Williamsburg, Virgina, USA; hep-th/0207254; Transparency can be obtained from: http://dpf2002.velopers.net/talks_pdf/33talk.pdf

[12] Ning WU, Commun. Theor. Phys. (Beijing, China) 42 (2004): 543-552.

[13] Ning WU, "Renormalizable Quantum Gauge General Relativity" gr-qc/0309041.

[14] Ning Wu, Quantum Gauge Theory of Gravity, In Focus on Quantum Gravity Research, chapter 4, ed. David C. Moore, pp.121-169, (Nova Science Publishers, Inc., New York, 2006)

[15] Ning WU, Commun. Theor. Phys. (Beijing, China) 40 (2003): 337-340.

[16] Ning WU, "Classical Solution of Field Equation of Gravitational Gauge Field and Classical Tests of Gauge Theory of Gravity", gr-qc/0508009.

[17] Ning WU, Commun. Theor. Phys. (Beijing, China) 44 (2005): 883-886.

[18] Ning WU, "Coupling Between the Spin and Gravitational Field and the Equation of Motion of the Spin", gr-qc/0603104.

[19] Ning WU, Commun. Theor. Phys. (Beijing, China) 41 (2004): 567-572.

[20] Ning WU, "Mechanism of Gravity Impulse", gr-qc/0510010.

[21] Ning WU, Commun. Theor. Phys. (Beijing, China) 45 (2006): 452-456.

[22] Ning WU, Dahua ZHANG, Commun. Theor. Phys. (Beijing, China) 45 (2006): 858-860.

[23] Ning WU, Commun. Theor. Phys. (Beijing, China) 40 (2003): 429-434.

[24] Ning WU, Commun. Theor. Phys. (Beijing, China) 41 (2004): 381-384.

[25] Ning WU, Commun. Theor. Phys. (Beijing, China) 38 (2002): 322-326.

[26] Ning WU, Commun. Theor. Phys. (Beijing, China) 38 (2002): 455-460.

[27] Ning WU, Commun. Theor. Phys. (Beijing, China) 39 (2003): 561-568.

[28] Ning WU, Unified Theory of Fundamental Interactions, In Quantum Gravity Research Trends, chapter 3, ed. Albert Reimer, pp. 83-122, (Nova Science Publishers, Inc., New York, 2006)

[29] Ning WU, Commun. Theor. Phys., (Beijing, China) 36 (2001) 169-172.

[30] Ning WU, Commun. Theor. Phys. (Beijing, China) 38 (2002): 577-582.

[31] Ning WU, Commun. Theor. Phys. (Beijing, China) 39 (2003): 671-674.
[32] R.V.Eötvös, D.Pekar, and E.Fekete, Ann.Phys.(Leipzig) 68 (1922) 11.

[33] Y.Su et al., Phys.Rev.D 50, (1994) 3614.

[34] T.M.Niebauer, M.P.McHugh, and J.E.Faller, Phys.Rev.Lett. 59 (1987) 609.

[35] K.Kuroda and N.Mio, Phys.Rev.Lett. 62 (1989) 1941.

[36] S.Carsotto et al., Phys.Rev.lett. 69 (1992) 1722.

[37] Y.Z.Zhang, J.Luo, and Y.X.Nie, Mod.Phys.Lett.A 16 (2001) 789.

[38] J.Luo, Y.X.Nie, Y.Z.Zhang, and Z.B.Zhou, Phys.Rev.D 65 (2002) 042005.

[39] Z.B.Zhou, J.Luo, Q.Yan, Z.G.Wu, Y.Z.Zhang, and Y.X.Nie, Phys.Rev.D 66 (2002) 022002.

[40] I.Yu Kobzarev and L.B.Okun, Sov. Phys. JETP 16 (1963) 1343.

[41] D.J.Wineland and N.F.Ramsey, Phys.Rev.A 5 (1972) 821.

[42] C.G. de Oliveria and J. Tiomno, Nuovo Cimento 24 (1962) 672.

[43] B.Mashhoon, Phys.Lett.A 198 (1995) 9.

[44] Ning Wu, ”Equation of Motion of a Spinning Test Particle in Gravitational Field”, gr-qc/0608042

[45] S. Weinberg, Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity, (John Wiley, New York, 1972).

[46] N. Straumann, General Relativity and Relativistic Astrophysics, (Springer-Verlag, Berlin Heidelberg New York Tokyo, 1984).

[47] I.Ciufolini, E.C.Pavlis, et al., Science, 279, (1998):2100.

[48] I.Ciufolini, E.C.Pavlis, Nature, 431, (2004):958.

[49] R. Plyatsko, ”Gravitational Ultrarelativistic Spin-Orbit Interaction and the Weak Equivalence Principle”, gr-qc/0507024.

[50] V. Fock, The Theory of Space, Time and Gravitation, (Pergamon, New York, 1959).

[51] J. Synge, Relativity: The General Theory, (North-Holland, Amsterdam, 19660).