Inertial Current Generators of Poynting Flux in MHD Simulations of Black Hole Ergospheres

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ABSTRACT

This Letter investigates the physics that is responsible for creating the current system that supports the outgoing Poynting flux emanating from the ergosphere of a rotating black hole in the limit that the magnetic energy density greatly exceeds the plasma rest mass density (magnetically dominated limit). The underlying physics is derived from published three-dimensional simulations that obey the general relativistic equations of perfect magnetohydrodynamics (MHD). It is found that the majority of the Poynting flux emitted from the magnetically dominated regions of the ergosphere has a source associated with inertial effects outside of the event horizon.

Subject headings: Black hole physics - magnetohydrodynamics - galaxies: jets — galaxies: active — accretion disks

1. Introduction

There are two known theoretical mechanism for producing field-aligned outgoing poloidal Poynting flux, $S^P$, at the expense of the rotational energy of a black hole in a magnetosphere that is magnetically dominated. There are electrodynamic processes collectively called Blandford-Znajek mechanisms in which currents flow virtually parallel to the proper magnetic field direction (force-free currents) throughout the magnetically dominated zone all the way to the event horizon (Blandford and Znajek 1977; Phinney 1983; Thorne et al 1986). Therefore, these electrodynamic currents have no source within the magnetically dominated black hole magnetosphere. Alternatively, there is the GHM (gravito-hydromagnetic) dynamo in which large relativistic inertia is imparted to the tenuous plasma by black hole gravity that in turn creates a region of strong cross-field currents (inertial currents), $J^\perp$, that provide the
source of the field-aligned poloidal currents, $J^P$, that support $S^P$ in an essentially force-free outgoing wind, i.e., $\nabla \cdot \mathbf{J} \approx \partial J^\perp / \partial X^\perp + J^P / \partial X^P \approx 0$ at the source (Punsly 2001). The force-free electrodynamic current flow is defined in terms of the Faraday tensor and the current density as $F^{\mu \nu} J_\nu = 0$. As a consequence, $\mathbf{J} \cdot \mathbf{E} = 0$, so $S^P$ in a force-free magnetosphere must be injected from a boundary surface. The two types of sources associated with these two types of Poynting fluxes are quite distinct in the ergosphere: the inertial current provides the $\mathbf{J} \cdot \mathbf{E}$ source in Poynting’s Theorem and the force-free (electrodynamic) component of $S^P$ emerges from a boundary source at the event horizon.

Numerical models can be useful tools for understanding the source of $S^P$ emerging from the ergosphere of a black hole magnetosphere. Some recent three-dimensional simulations in De Villiers et al (2003); Hirose et al (2004); De Villiers et al (2005a,b); Krolik et al (2005) show $S^P$ emanating from magnetically dominated funnels inside of the vortices of thick accretion flows. In these simulations, $J^\perp \approx J^\theta$ (in Boyer-Lindquist coordinates which are used throughout the following) inside of the ergosphere, since the poloidal field settles to a nearly radial configuration early on in the simulation (Hirose et al 2004). In principle, one can clearly distinguish the amount of $S^P$ emerging form the ergosphere in a simulation that is of electrodynamic origin (as proposed in Blandford and Znajek (1977)) from the amount due inertial effects (the GHM theory of Punsly (2001)) by quantifying the relative strengths of $S^P$ emerging from the inner boundary (the asymptotic space-time near the event horizon) with the amount created by sources within the ergosphere. In the high spin rate simulations in questions, over 70% of $S^P$ emerging from the ergospheric funnel is created outside of the inner boundary.

2. Physical Quantities in Boyer-Lindquist Coordinates

The Kerr metric (that of a rotating uncharged black hole), $g_{\mu \nu}$, in Boyer-Lindquist coordinates $(r, \theta, \phi, t)$, is given by a line element that is parameterized by the black hole mass, $M$, and the angular momentum per unit mass, $a$, in geometrized units (Thorne et al 1986). We use the standard definitions, $\rho^2 = r^2 + a^2 \cos^2 \theta$ and $\Delta = r^2 - 2Mr + a^2$, where $\Delta = 0$ at the event horizon, $r_+ = M + \sqrt{M^2 - a^2}$. The "active" region of space-time is the ergosphere, where black hole energy can be extracted, $r_+ < r < r_\text{as} = M + \sqrt{M^2 - a^2 \cos^2 \theta}$ (Penrose 1969).
2.1. The Toroidal Magnetic Field Density and the Cross-Field EMF

The flux of electromagnetic angular momentum along the poloidal magnetic field direction is the component of the stress-energy tensor, \( T^{r}_{\phi} = \frac{1}{(4\pi)}F_{\phi\alpha}F^{\alpha r} \), in the approximation that the field is radial. In steady state, the electromagnetic angular momentum flux per unit poloidal magnetic flux is the toroidal magnetic field density:

\[ -B^T \equiv \sqrt{-g} F^{\phi r}, \]

where \( g = -\rho^4 \sin^2 \theta \) (Phinney 1983; Punsly 2001). Similarly, the electromagnetic energy flux along the poloidal magnetic field direction, \( S^P \), is the component, \( T^{r}_{t} = \frac{1}{(4\pi)}F_{t\alpha}F^{\alpha r} \), in the approximation that the poloidal field is radial (Thorne et al 1986). In steady state, the energy flux per unit poloidal magnetic flux is \(-\Omega F/c B^T\), where \( \Omega \) is the field line angular velocity (Phinney 1983; Punsly 2001). Consequently, \( B^T \) is useful for quantifying the energy and angular momentum fluxes as steady state is approached. From Ampere’s law,

\[ \sqrt{-g} J^\theta = B^T, + (\sqrt{-g} F^{\theta r}) \cdot \hat{t}. \] (2-1)

At late times, as an approximate steady state is reached, one expects \( \sqrt{-g} J^\theta \approx B^T, \). Therefore, at late times, \( J^\theta \) is a potential source for the current system that supports \( S^P \).

Even when a system has not reached a time stationary state, one can introduce a well-defined notion of \( \Omega \) that becomes the field line angular velocity in the steady state. If the field is nearly radial one can simply define the expression, \( F_{\theta\phi} = -\Omega F_{\theta\phi}. \) With this definition, \( \Omega \) is a function of space and time and in steady state it becomes a constant along a perfect MHD flux tube. Therefore, the EMF across the magnetic field is \(-\Omega F_{\theta\phi} \) by definition.

2.2. The Source of Poloidal Poynting Flux

In Boyer-Lindquist coordinates, the curved space-time equivalent of the "J·E" source of \( S^P \) is the term \( F_{t\alpha}J^\alpha \). This notion is described by the integral version of Poynting’s Theorem which is the integral of \( T^{\nu}_{\nu} = F_{\nu\mu}J^{\mu} \) combined with Stokes’ theorem (Thorne et al 1986; Phinney 1983)

\[ \int [F_{t\phi}J^\phi + F_{tr}J^r + F_{t\theta}J^\theta] dV - \frac{d}{dt} \int T^1_i dV = [1/(4\pi)] \oint \sqrt{-g} F_{t\alpha}F^{\alpha n} dA; \] (2-2)

where \( dV = \sqrt{-g}drd\phi d\theta \) is the spatial volume element of a section of a thick spherical shell and \(-T^1_i\) is the energy density of the field and \( n \) is the normal direction to the surface area element, \( dA \), of the Gaussian pillbox. In steady state, the source of \( S^P \) is \(-\Omega F_{\theta\phi}J^\theta \) in the approximation of a radial field. Without the radial approximation, one needs to define a
poloidal field direction, $B^p$, and a cross-field poloidal EMF, $E_\perp \equiv -\Omega_\times B^p$ then the source of $S^p$ is $E_\perp J_\perp$. This can all be setup with great mathematically complexity (see Punsly (2001) for sample calculations) and no additional physical insight. The reader should remember that the poloidal field is not exactly radial in the following and the simple notion of $J^\theta$ is used to approximate $J_\perp$.

3. The Source of Ergospheric Poynting Flux in the KDE Simulation

The simulation "KDE" of De Villiers et al (2003); Hirose et al (2004); De Villiers et al (2005a,b); Krolik et al (2005) is of the most interest since it generates an order of magnitude more $S^p$ than any of the other simulations (Krolik et al 2005). The magnetically dominated funnel spans the latitudes $0^\circ < \theta < 55^\circ$ at the inner calculational boundary, near $r_+$. Quantifying the magnetic dominance in the funnel is the pure Alfvén speed, $U_A = B^p/(\sqrt{4\pi n\mu c})$, where $n$ is the proper number density, $\mu$ is the specific enthalpy of the plasma and $B^p$ is the poloidal field strength. Within the funnel $10 < U_A^2 < 10^4$. The funnel $S^p$ averaged over time and azimuth in KDE near $r_+$ is shown in figure 1.

This contour map indicates a region of strong outgoing $S^p$ in the evacuated funnel, $30^\circ < \theta < 55^\circ$, $r > r_+$. It is clear that $S^p$ suddenly diminishes close to $r_+$ at $r \approx 1.3M - 1.5M$. Inspection of the contour map indicates that over 72% of $S^p$ is created within a thin layer near $r \approx 1.4M$. Because of the saturation of the dark red color in the plotting routine, $S^p$ might be even larger above the switch-off layer than indicated in the contour map. Thus, we only have a lower bound on the strength of $S^p$ above the switch-off layer and it is likely that more than 72% of the energy flux is created within this thin layer. This effect even continues into the weak $S^p$ region closer to the pole at $20^\circ < \theta < 30^\circ$, $r > r_+$.

In order to investigate possible source terms for $S^p$, in (2.2), one needs to be explicit about what is plotted in figure 1. Whenever a time lapse of 80 M occurs within the high time resolution simulation, data is stored for a time snapshot. There are 75 of these snapshots that are averaged in figure 1, steps 26 through 100. Thus, the discrete time average of (2.2) is relevant,

$$
\frac{1}{75} \sum_{i=26}^{100} \left[ \int F_{\alpha\alpha} J^\alpha \, dV \right]_i - \frac{1}{75} \sum_{i=26}^{100} \left[ \frac{d}{dt} \int T_i \, dV \right]_i 
= \frac{1}{12(4\pi)} \int_{1+3} 3 \sqrt{-g} F_{\alpha\beta} F^{\alpha\beta} \, d\theta \, d\phi + \int_{2+4} 4 \sqrt{-g} F_{\alpha\beta} F^{\alpha\beta} \, dr \, d\phi,
$$

where the four sides of the Gaussian pillbox of integration in figure 1 are the four curves labelled ”1 - 4”. The result of significance from this plot is that $\sum_{i=26}^{100} \int_3 \sqrt{-g} F_{\alpha\beta} F^{\alpha\beta} \, d\theta \, d\phi >
2 | \sum_{i=26}^{100} \int_{1}^{2+4} \sqrt{-g} F_{\alpha} F^{\alpha \nu} d\theta d\phi |, \text{ i.e. the time averaged } S^{P} \text{ increases across this thin volume. There are three possible source terms for this increase in } S^{P} \text{ when (3.1) is applied to the pillbox in figure 1. The time averaged field decay from a finite spatial region (the second term on the LHS) is not a physically viable source of long term } S^{P} \text{ (unless the simulation is pathological and keeps creating local field energy from numerical artifacts). The only reasonable choices are the } \mathbf{J} \cdot \mathbf{E} \text{ term and the conversion of latitudinal Poynting flux, } S^{\perp}, \text{ from the surface terms } \sum_{i=26}^{100} \int_{2+4} \sqrt{-g} F_{\alpha} F^{\alpha \theta} d\theta d\phi. \text{ We don’t have time averaged plots of } S^{\perp}, \text{ so we don’t know how the magnitude of this term. Because of the symmetry at the pole, the only viable scenario is that } S^{\perp} \text{ radiates poleward from its source in the funnel wall and somehow converts to } S^{P} \text{ in the pillbox. One thing that is unequivocal from figure 1, is that during the lifetime of the simulation, less than 30\% of } S^{P} \text{ that reaches emerges from the ergospheric funnel came from the inner boundary, the rest was created external to the boundary.}

The existence of this putative source is a highly significant result and it is desirable to check it by other means. For example, since this occurs near the boundary how do we know that this is not an artifact of an inherent error in the plotting routine? Figure 7 of Krolik et al (2005) is a similarly averaged (in time and azimuth) plot of the magnetic angular momentum flux, \( T_{\phi}^{\ast} = \frac{1}{(4\pi)} F_{\phi \alpha} F^{\alpha \nu}. \) Figure 7 shows that \( T_{\phi}^{\ast} \) is created predominantly in the ergosphere, over 70\% of the electromagnetic angular momentum flux in the funnel is created by sources within the ergosphere for \( 25^\circ < \theta < 55^\circ. \) The remaining fraction can be associated with electrodynamic sources (i.e., sourceless and emerging from the inner boundary). A consistent picture emerges from the time average of Ampere’s law in (2.1) if the dominant source term for \( S^{P} \) in (2.2) is \( F_{\alpha \nu} J^{\alpha} \approx F_{\theta \phi} J^{\theta}. \)

4. The Cross-field Current Density in the Ergosphere

In this section, we use a plot of the strong electromagnetic forces within the evacuated funnel to try to understand the physical mechanism that drives the source of \( S^{P} \) in the simulation, KDE. Presently, there is no existing data that has been extracted from the KDE simulation that directly illustrates the electromagnetic forces or currents in the ergosphere. Fortunately, plots of the electromagnetic force already exist for the KDP model of De Villiers et al (2005a) which is characterized by \( a/M = 0.9. \) So, the best we can do is to explore electromagnetic force plots (such as figure 2) from a closely related simulation, KDP. This model has the second highest \( S^{P} \) luminosity within the family of simulations, but it is an order of magnitude weaker than the KDE model. This should still be qualitative adequate for the following analysis, since it is stated in De Villiers et al (2005a) that the azimuthal
force exists in all the models in the same relative location, but it is strength correlates with spin and is the most pronounced in the KDE model.

The evacuated funnel of KDP is dominated by magnetic energy, $10^4 > U^2 > 10$ (according to figure 3 of Hirose et al (2004)) in the ergosphere if $0^\circ < \theta < 65^\circ$. The nonforce-free nature of the current density will be studied by means of a plot of the electromagnetic force (that were presented in De Villiers et al (2005a) and does not appear in De Villiers et al (2005b)), $F_{\phi \nu} J_\nu = F_{\phi t} J_t + F_{\phi r} J_r + F_{\phi \theta} J_\theta$, within the funnel at $t = 8080M$, averaged over azimuthal angle.

One can scale the individual components of the strong azimuthal force that appears at $r \approx 1.5M$ in figure 2 if turbulence does not dominate the dynamics: $F_{\phi t} J_t \sim \alpha^{-2} F_{\phi t}$, $F_{\phi r} J_r \sim \alpha^{-2} F_{rt}$ and $F_{\phi \theta} J_\theta \sim \alpha^{-2} F_{\theta t}$, where the quantity, $\alpha = \sqrt{\Delta \sin \theta / \sqrt{g_{\phi \phi}}}$, is the lapse function that represents a global redshift factor (Thorne et al 1986; Punsly 2001). The lapse function vanishes at the horizon and is therefore useful in expansions in a small dimensionless parameter near the horizon (at $r=1.5M$, $\alpha^{-2} = 58.8$, $\theta = 45^\circ$). These scalings yield the approximation

$$F_{\phi \nu} J_\nu \approx F_{\phi \theta} J_\theta \approx \frac{2Mr}{c \rho^4 \Delta} (\Omega_F - \Omega) F_{\phi \theta} J^\theta,$$  \hspace{1cm} (4-1)

where $\Omega = -g_{t\phi} / g_{\phi \phi}$ is known as the ZAMO angular velocity and it approaches the horizon angular velocity deep in the ergosphere (see Thorne et al (1986) for more details). Figure 2 indicates large changes in $F_{\phi \nu} J_\nu$ occur on the order of the grid size at $r \approx 1.5M, \sim 0.01M$. Thus, this must be a dynamic effect and does not derive from $\nabla \alpha^{-2}$. A consistent picture can be constructed by a large $J^\theta$, enhanced by a factor of more than 20 compared to the upstream flow, that is responsible for the large force that initiates at $\alpha \approx 0.10 - 0.25$. From Ampere’s law in (2.1), this should be a prestigious source of $B^T$ and by (2.2) this should also be the strong source of $S^\rho$, $F_{\theta \theta} J^\theta$ consistent with the analysis of the KDE simulation in section 2. Note that $F_{\phi \alpha} J^\alpha \approx F_{\phi \theta} J^\theta$ is the force that torques the plasma (Thorne et al 1986; Punsly 2001). So, the Poynting flux generation in this scenario is associated with a strong electromagnetic torque.

A physical explanation of the strong electromagnetic torque is the most basic concept of GHM.

Namely, the field is rotating a rate that is slower than the enforced rotation velocity of the plasma that is dictated by the dragging of inertial frames in the ergosphere. Gravity tries to pull the plasma forward relative to the field as the horizon is approached, the back reaction of the field is to torque the plasma backwards back onto Larmor helices that thread the field lines. In the process, the field is partially overwhelmed by gravity and twisted forward azimuthally creating $B^T < 0$ even though the plasma is tenuous. Note that $J^\theta$
directed equator-ward in the northern hemisphere of the KDE simulation is of the correct orientation to both source $B^T < 0$ and to torque the plasma if $B^P > 0$ (for more details of this process see Punsly (2001); Semenov et al (2004)). The strong force occurs in figure 2 throughout the range $10 < U_A^2 < 10^4$ indicating that the current is not driven across $B^P$ by the rest mass inertia, but by a relativistic effect imposed on the tenuous plasma by the black hole. Finally, GHM naturally explains the current closure within the global wind. Figure 3 shows a remarkable agreement between the current system of the analytical GMH model depicted in figure 9.12 of Punsly (2001) and the KDE simulation. In the dynamo source, $\partial(\sqrt{-g}J^\theta)/\partial\theta + \partial(\sqrt{-g}J^r)/\partial r \approx 0$, where $J^r$ is the field aligned current (dashed contours in figure 3) that support $S^P$.

5. Conclusion

In this paper, we studied simulations of a magnetically dominated funnel of a rapidly rotating black hole, $a/M = 0.998$. A source that is responsible for creating over 70% of $S^P$ transported through funnel during the life of the simulation was found. Similarly, one can conclude that $< 30\%$ of $S^P$ emerging from the ergospheric funnel is from an inner boundary source, near the horizon. The small residual $S^P$ injected from the boundary into the accretion wind can be considered of electrodynamic origin. The distribution of $S^P$ in figure 1 is in contrast to the Blandford-Znajek solution in which essentially all the $S^P$ is of electrodynamic origin, i.e., it emanates from the horizon and passes through the accretion wind with minimal interaction, thereby maintaining a virtually constant value along each poloidal flux tube throughout the ergosphere.

There were two possible sources for $S^P$ in the funnel, there are the GHM inertial currents and $S^\perp$ injected from the funnel wall. The latter would be a new source of $S^P$ in a magnetically dominated magnetosphere that has not been considered in previous literature. The putative $S^\perp$ is created by inertial effects in the funnel wall. We were unable to distinguish between these two inertial sources with the available data. It was demonstrated that a GHM current explains the ergospheric sources of $S^P$, the electromagnetic angular momentum flux, the large azimuthal forces seen in the ergosphere and global current closure in the wind zone.

Further analysis of three-dimensional simulations about rapidly rotating black holes ($a/M = 0.998$) are needed to clarify the physics that creates $S^P$. At least three consecutive time snapshots are needed in order to find the current distribution from Ampere’s Law at each coarse time step data dump. Since the flow is highly turbulent near the event horizon, it would be important to then time average the current distribution. Higher resolution might
be required to resolve the issue of whether the source in figure 1 is from $J^\perp E_\perp$, $S^\perp$ or some unexpected numerical error.

6. Figure Captions

Figure 1. The azimuthally averaged and time averaged (over 75% of the simulation that ends at $t = 8080/M$) Poynting flux from the model KDE ($a/M = 0.998$) of Krolik et al. (2005). The figure is a magnification of the inner region of figure 10 of Krolik et al. (2005). It is an excision of a region, $0^\circ < \theta < 65^\circ, r \gtrsim r_+$ that is a little larger than the ergospheric portion of the magnetically dominated funnel, $0^\circ < \theta < 55^\circ, r \gtrsim r_+$. The majority of $S^P$ switches-off in a thin layer near $r = 1.3M - r = 1.5M$ (see Krolik et al. (2005) for a description of the units on the color bar). This region is a source for the majority of the outgoing $S^P$ emerging from the funnel. A Gaussian pillbox is drawn as a dashed contour for use in Poynting’s Theorem. There are 26 grid zones between the inner boundary, $r = 1.175M$ and $r = 1.5M$. The plot is provided courtesy of John Hawley

Figure 2. The azimuthally averaged azimuthal electromagnetic force in the evacuated funnel of the model KDP from the central region of figure 7 from De Villiers et al. (2005a) at time $t = 8080/M$ in geometrized units. Magnitude information is expressed by color in code units (see scale above). The funnel wall is marked by the solid white line. The azimuthal gas pressure forces are much smaller in this region (De Villiers et al. 2005a). Note that there is a strong electromagnetic torque between the boundary at $r = 1.45M$ and $r = 1.75M$, an order of magnitude stronger than anywhere else in the ergosphere and the adjacent space-time. The plot only covers the funnel in the ergosphere in the restricted span, $45^\circ < \theta < 65^\circ$, yet the full range of Alfven speeds is captured, $10^4 > U_\perp^2 > 10$. The plot is provided courtesy of John Hawley

Figure 3. An overlay of the time stationary current system from a GHM model reproduced from figure 9.12 of Punsly (2001) and the Poynting flux distribution of KDE. The relative scales are set by the black hole (gray) radius, $r_+ = 1.06M$, and the inner boundary of KDE at 1.175 M. The GHM model assumes very different initial conditions than KDE, but the basic current topology should be common to all GHM magnetospheres: an enhanced strong cross-field current density (which increases equator-ward) in the ergosphere that is the source of a field aligned current system in a nearly force-free wind zone and a strong return current flow at the edge of the wind zone
REFERENCES

Blandford, R. and Znajek, R. 1977, MNRAS. 179, 433

De Villiers, J., Hawley, J., Krolik, 2003, ApJ 599 1238

De Villiers, J., Hawley, J., Krolik, K., Hirose, S. 2005, astro-ph/0407092

De Villiers, J., Hawley, J., Krolik, K., Hirose, S. 2005, ApJ 620 878

Hirose, S., Krolik, K., De Villiers, J., Hawley, J. 2004, ApJ 606, 1083

Krolik, K., Hawley, J., Hirose, S. 2005, ApJ 622, 1008

Penrose, R. 1969, Nuovo Cimento 1, 252

Phinney, E.S. 1983, PhD Dissertation University of Cambridge.

Punsly, B. 2001, Black Hole Gravitohydromagnetics (Springer-Verlag, New York)

Semenov, V., Dyadechkin, S. and Punsly, B. 2004, Science 305 978

Thorne, K., Price, R. and Macdonald, D. 1986, Black Holes: The Membrane Paradigm (Yale University Press, New Haven)