Quantum Hall effect in a high-mobility two-dimensional electron gas on a cylindrical surface

K.-J. Friedland, R. Hey, H. Kostial, A. Riedel, D. Maude†
Paul-Drude-Institut für Festkörperlektronik, Hausvogteiplatz 5–7, 10117 Berlin, Germany,
†High Magnetic Field Laboratory, Centre National de la Recherche Scientifique, 25 Avenue des Martyrs, 38042 Grenoble, France
E-mail: kjf@pdi-berlin.de

Abstract. We study the quantum Hall effect in a high-mobility, two-dimensional electron gas on a cylindrical surface, where a strong magnetic field gradient is formed along the cross section of the Hall bar. The heterostructure is rolled-up as a single tube and despite the formation of a new surface the mobility remains high so that the electrons mean free path is as large as the radius of the tube, \( R = 20 \ \mu \text{m} \). This results in new trajectories, which are bent in space, while being still confined in the quantum well. We show that even for a large magnetic field gradient the Hall resistance shows pronounced plateaus, indicating that the conductance is fully determined by one-dimensional channels corresponding to Landau states including Zeeman splitting. These channels split off the edge into the bulk due to magnetic barriers. We investigate the important case when stripes are formed, for which the magnetic field is oriented nearly 'in-plane'. So called 'snake'-like trajectories with backwards directed velocity exist within these stripes, which do not contribute to the main charge transport.

1. Introduction
The self-rolling of thin pseudomorphically strained semiconductor bilayer systems based on epitaxial heterojunctions grown by molecular-beam epitaxy (MBE) as proposed by Prinz and coworkers [1] allows to realize electron motion on curved surfaces. The interesting question is, how the electron motion is affected by the local curvature of the surface and how the global topology influences quantum-mechanical wave functions [2]. The most dominant modification of the electron motion on curved surfaces is the effectively nonuniform normal-to-surface component of the magnetic field.

Recently, high mobility two-dimensional electron gases (HM2DEG) were fabricated on freestanding rolled (Al,Ga)As heterostructures [3, 4]. A new class of electron trajectories, such as 'snake'-like trajectories, were proposed to exist in ballistic mesoscopic structures with non-uniform magnetic fields [5, 6]. In fact, for curved HM2DEG with a low-temperature mean free path of the electrons \( l_\perp \) being comparable with the curvature radius \( R \), we were able to demonstrate extended trochoid-trajectories, which move oppositely to the direction of the other guided trajectories [3].

Here we investigate the quantum Hall effect with the conductance along parallel channels with distinctively different normal-to-surface component of the magnetic field \( B_\perp \). To realize this, we align the Hall bar with the HM2DEG on a tube with the current flowing parallel to the tube axis. The width of the Hall bar of 20 \( \mu \text{m} \) is as large as the tube radius \( R \). Therefore,
the cross section of the Hall bar spans over the arc angle \( \varphi_{Hb} \simeq 60^\circ \) on the cylinder segment, which results in a large gradient of \( B_\perp \). Here, we present results that demonstrate exclusively one-dimensional quantized transport, even when parallel channels with 'snake'-like trajectories are present.

2. Experimental
We use a particular heterojunction grown by MBE, where the HM2DEG is located in a 13-nm-wide GaAs single quantum well with barriers consisting of short-period AlAs/GaAs superlattices [7]. This structure was proven to effectively smooth the potential fluctuations from remote impurity doping and from surface states. As a result, the electron mobility remains high even after the lift-off process so that the low-temperature mean free path of the electrons \( l_S \) is as long as the curvature radius \( R \) of the tube [3].

The layer stack, including the HM2DEG with an overall thickness of 192 nm, was grown on top of a 20-nm-thick In\(_{0.15}\)Ga\(_{0.85}\)As stressor layer, an essential component of the strained multilayered films (SMLF). An additional 50-nm-thick AlAs sacrificial layer is introduced below the SMLF in order to enable the separation of the SMLF from the buffer layer. The buffer layer of 500 nm GaAs is Be-\( \delta \)-doped. The hole density from Be-\( \delta \)-doping is chosen to fully compensate the two-dimensional electrons, which are formed in the In\(_{0.15}\)Ga\(_{0.85}\)As stressor layer, acting electronically as a quantum well on the flat, unrolled part of the structure. This allows to minimize the mesa etching depth, which results in a reduced mechanical stress due to material thickness variations of the freestanding SMLF. This increases the yield of the tubes fabrication.

For the curved HM2DEG’s, we first processed conventional Hall bar structures in the flat heterojunction along the [010] direction. The 20 \( \mu \text{m} \)-wide Hall bar with three opposite 4 \( \mu \text{m} \)-narrow lead pairs – separated by 10 \( \mu \text{m} \) – are connected to large Ohmic contact pads outside of the rolling area. In a next step, the SMLF including the Hall bar was released by selective etching of the sacrificial AlAs layer with a 5% HF acid/water solution at 4 \( ^\circ \text{C} \) starting from an [010] edge. By relaxing the strain, the SMLF rolls-up along the [100] direction forming a complete tube with a radius of about 20 \( \mu \text{m} \).

3. Results and discussion
The quantum Hall effect in curved HM2DEGs is supposed to originate from charge flow along one-dimensional Landau channels (1DLC), which arise at the positions, where the Fermi energy crosses the Landau-level energies [4, 8]. For our Hall bar with a magnetic field gradient along the cross section – directing from terminal 1 to terminal 3 – as demonstrated in the sketch in Fig. 1, we assume a number of \( M \) one-dimensional conducting channels at the low magnetic field side of the Hall bar, while \( K \) channels split-off the high magnetic field side due to magnetic barriers running into the bulk of the Hall bar. We calculate the four-terminal Hall resistance by means of the Landauer-Büttiker formalism:

\[
\begin{pmatrix}
0 \\
I \\
0 \\
-I
\end{pmatrix} = \frac{h}{2e^2} \begin{pmatrix}
M - K & 0 & 0 & M + K \\
0 & M - K & 0 & 0 \\
K & 0 & M - M - K & 0
\end{pmatrix} \begin{pmatrix}
\mu_1 \\
\mu_2 \\
\mu_3 \\
\mu_4
\end{pmatrix},
\]

where \( I \) is the current flowing out of the lead 2 into the lead 4 and \( \mu_i \) are the leads potentials. The resulting Hall resistance is calculated as:

\[
\varrho_{xy} = \frac{\mu_3 - \mu_1}{I} = \frac{h}{2e^2} \frac{1}{M}.
\]

We conclude that the Hall effect will be dominated by the maximum number of channels \( M \) at the low magnetic field side of the Hall bar.
To prove that, we analyze $\varrho_{xy}$ as a function of the magnetic field at different rotation angles of the tube. As can be seen in the Fig. 1, all $\varrho_{xy}$ traces show the same clear plateaus in the quantum Hall effect at values $\varrho_{xy} = \frac{h}{e^2\nu}$ with the integer filling factor $\nu$. The quantum Hall plateaus at $B_\nu$ positions for the filling factor $\nu$ and the corresponding zeros in the sheet resistances $\varrho_{xx}$ (not shown here) shift to higher magnetic fields according to

$$B_\nu \cos(\varphi_{hf}) = \frac{h n_S}{e \nu},$$

where $n_S$ is the density of the HM2DEG. The density $n_S$ is calculated from the classical Hall effect $R_H$ at low magnetic fields $n_S = \frac{r e}{2 e R_H}$, where $r$ is the Hall factor. We assume $r = 1$, which is a very good approximation in the present system. One can easily show that at low classical magnetic fields the Hall resistance averages over the Hall bar arc angles from $-\varphi_{Hb}/2$ to $\varphi_{Hb}/2$, as $\varrho_{xy} = R_H B \cos(\varphi_0)$ for a homogeneous current distribution. In this way, we can also determine the exact rotation angle of the tube, where $\varphi_0$ is the angle between the magnetic field and the normal of the Hall bar at its center position.

From Eq. (3), we calculate a high-field angle $\varphi_{hf}$, using the zeros in the $\delta \varrho_{xy}/\delta B$ traces. Fig. 2 shows for example $\delta \varrho_{xy}/\delta B$ for the most symmetrical situation, when $\varphi_0 = 0$. In this case, we estimate $\varphi_{hf} = 28^\circ \approx \varphi_{Hb}/2$.

The increase of the angle between the magnetic field and surface normal at out-of-center Hall bar positions can be seen already at low magnetic fields by the decrease of the classical $R_H(B)$ with increasing magnetic field. We interpret the decrease of $R_H(B)$ by squeezing of current paths toward the edges of the Hall bar with lower $B_\perp$. At high fields, this value eventually becomes $B \cos(\varphi_{Hb}/2)$ in accordance with the Landauer-Büttiker analysis, for which we assign the number of channels with the even filling factors $\nu$.

In Fig. 3, we show $\varphi_0$ and $\varphi_{hf}$ as a function of the tic markers of the dilution refrigerators sample rotation stage (SRS). $\varphi_0$ changes nearly linearly with SRS, but $\varphi_{hf}$ remains always below the angle $\varphi_0 + \varphi_{Hb}/2$ as indicated in Fig. 3 by the upper dotted line, while accordingly to the Landauer-Büttiker approach for our geometry, see Fig. 4, we would expect always $\varphi_{hf} = \varphi_0 + \varphi_{Hb}/2$. In order to understand this discrepancy, we take into account that at angles $\varphi_0 \geq 60^\circ$ and starting from the low magnetic field edge of the Hall bar a channel is formed with so-called free-electron states [5], for which the magnetic field is oriented nearly in-plane. Within a classical picture, these states are described as so-called 'snake'-like orbits (SLO). The special property of SLO is that they carry electrons in the opposite direction as compared to all other states surrounding the zero magnetic field channel.
The striking observation is that even for angles $\varphi_0 \geq 60^\circ$ — in the presence of a zero magnetic field channel — the conductance remains quantized, governed by 1DLS. This directly results from the right plateaus values in the quantum Hall effect at values $\rho_{xy} = \frac{h}{e^2}$, as seen in Fig. 1. We therefore conclude that SLO do not contribute to the conductance, as one could argue from its backwards directed velocity contribution.

Starting from $\varphi_0 \simeq 60^\circ$, SLO replace the 1DLS at the low magnetic field side of the Hall-bar, therefore the conducting 1DLS moves away from the sample boundary, which is demonstrated by the low angle $\varphi_{hf} < \varphi_0 + \varphi_{Hb}/2$. Moreover, we observe that $\varphi_{hf} < \varphi_0 + \varphi_{Hb}/2$ holds also for tube orientations for which a SLO channels does not yet exist. Obviously, the edge channel model is oversimplified. Self-consistent charge and potential distribution of the quantum Hall states resulting in alternating compressible and incompressible states should be carried out in order to define the position where the current may flow along the compressible states with partly filled Landau states. It was shown that compressible states may extend several $\mu$m away from the sample edge [9]. For tilted magnetic fields, the field gradient becomes strong, which will enhance the chance for long-range compressible states at a large distance from the edge as suggested from Fig. 3.

4. Conclusion
In conclusion, the quantum Hall effect in a curved high-mobility, two-dimensional electron gas is fully determined by one-dimensional channels corresponding to Landau states including Zeeman splitting which are split-off the edge into the bulk due to magnetic barriers. To determine the position of the 1DLC and its distance from the edge, more detailed models should be used to determine the compressible and incompressible quantum Hall states. We show that 'snake'-like trajectories do not contribute to the main charge transport.

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