Abstract

Adversarial training is arguably an effective but time-consuming way to train robust deep neural networks that can withstand strong adversarial attacks. As a response to the inefficiency, we propose the Dynamic Efficient Adversarial Training (DEAT), which gradually increases the adversarial iteration during training. Moreover, we theoretically reveal that the connection of the lower bound of Lipschitz constant of a given network and the magnitude of its partial derivative towards adversarial examples. Supported by this theoretical finding, we utilize the gradient’s magnitude to quantify the effectiveness of adversarial training and determine the timing to adjust the training procedure. This magnitude based strategy is computational friendly and easy to implement. It is especially suited for DEAT and can also be transplanted into a wide range of adversarial training methods. Our post-investigation suggests that maintaining the quality of the training adversarial examples at a certain level is essential to achieve efficient adversarial training, which may shed some light on future studies.

1. Introduction

Although Deep Neural Networks (DNNs) have achieved remarkable success in a wide range of computer vision tasks, such as image classification [11, 13, 14], object detection [19, 20], identity authentication [24], and autonomous driving [9], they have been demonstrated to be vulnerable to adversarial examples [3, 8, 26]. The adversarial examples are maliciously perturbed examples that can fool a neural network and force it to output arbitrary predictions. There have been many works on crafting various types of adversarial examples for DNNs, e.g., [3, 16, 26–28]. In the meanwhile, diverse defensive strategies are also developed against those adversarial attacks, including logit or feature squeezing [34], Jacobian regularization [4], input or feature denoising [29, 33], and adversarial training [10, 16, 22]. However, most of these solutions have been shown to either provide “a false sense of security in defenses against adversarial examples” and be easily defeated by advanced attacks [1]. Athalye et al. [1] demonstrate that adversarial training is an effective defense that can provide moderate or even strong robustness to advanced iterative attacks.

The basic idea of adversarial training is to utilize adversarial examples to train robust DNN models. Goodfellow et al. [10] showed that the robustness of DNN models could be improved by feeding FGSM adversarial examples during the training stage. Madry et al. [16] provided a unified view of adversarial training and mathematically formulated it as a min-max optimization problem, which provides a principled connection between the robustness of neural networks and the adversarial attacks. This indicates that a qualified adversarial attack method could help train robust models. However, generating adversarial perturbations requires a significantly high computational cost. As a result, conducting adversarial training, especially on deep neural networks, is time-consuming.

In this paper, we aim to improve the efficiency of adversarial training while maintaining its effectiveness. To achieve this goal, we first analyze the robustness gain speed per epoch with different adversarial training strategies, i.e., PGD [16], FGSM with uniform initialization [32], and FREE [22]. Although FREE adversarial training makes better use of backpropagation via its batch replay mechanism, the visualization result shows that it is delayed by redundant batch replays, which sometimes even cause catastrophic forgetting and damage the trained model’s robustness. Therefore we propose a new adversarial training strategy, called Dynamic Efficient Adversarial Training (DEAT). DEAT begins training with one batch replay and increases it if the pre-defined criterion is stratified. The effectiveness of DEAT has been verified via different manual criteria. Although manual approaches show considerable performance, they heavily depend on prior knowledge about the training task, which may hinder DEAT to be deployed on a broad range of datasets and model structures directly. So this leads to an interesting research question:

- How to efficiently quantify the effectiveness of adversarial training without extra prior knowledge?

To answer this question, we theoretically analyze the
connection between the Lipschitz constant of a given network and the magnitude of its partial derivative towards adversarial perturbation. Our theory shows that the magnitude of the gradient can be viewed as a reflection of the training effect. Based on this, we propose a magnitude based approach to guide DEAT automatically, which is named M-DEAT. In practice, it only takes a trivial additional computation to implement the magnitude based criterion because the gradient has already been computed by adversary methods when crafting perturbation. Compared to other state-of-the-art adversarial training methods, M-DEAT is highly efficient and remains comparable or even better robust performance on various strong adversarial attack tests. Besides our theoretical finding, we also explore how the magnitude of gradient changes in different adversarial training methods and provide an empirical interpretation of why it matters. The takeaway here is that maintaining the quality of adversarial examples at a certain level is more helpful to adversarial training than using adversarial examples generated by any particular adversaries all the time.

In summary, the key contributions of this paper lie in three aspects:

• We analyze adversarial training methods from a perspective on the tendency of robustness gain during training, which has rarely been done in previous studies.

• By a theoretical analysis of the connection between the Lipschitz constant of a given network and its gradient towards training adversarial examples, we propose an automatic adversarial training strategy (M-DEAT) that is highly efficient and achieves the-state-of-the-art robust performance under various strong attacks.

• Through comprehensive experiments, we show that the core mechanism of M-DEAT can directly accelerate existing adversarial training methods and reduce up to 60% of their time-consumption. An empirical investigation has been done to interpret why our strategy works.

2. Related Work

Since Szegedy et al. [26] identified that modern deep neural networks (DNNs) are vulnerable to adversarial perturbation, a significant number of works have been done to analyze this phenomena and defend against such a threat [12]. Among all these defenses, adversarial training is currently the most promising method [1]. Nowadays, PGD adversarial training [16] is arguably the standard means of adversarial training, which can withstand strong adversarial attacks but is extremely computationally expansive and time-consuming. Its efficiency can be improved from different perspectives. For example, Zheng et al. [39] utilized the transferability of training adversarial examples between epochs, and YOPO [36] simplifies the backpropagation for updating training adversarial examples. A more direct way is simply reducing the number of iteration for generating training adversarial examples. Like in Dynamic Adversarial Training [30], the number of adversarial iteration is gradually increased during training. On the same direction, Friendly Adversarial Training (FAT) [38] carries out PGD adversaries with an early-stop mechanism to reduce the training cost. Although FGSM adversarial training is not effective, Wong et al. showed that FGSM with uniform initialization could adversarially train models with considerable robustness more efficiently than PGD adversarial training. They also reported that cyclic learning rate [25] and half-precision computation [17], which are standard training techniques and summarized in DAWN-Bench [5], can significantly speed up adversarial training. We refer the combination of these two tricks as FAST, and FGSM adversarial training with uniform initialization as U-FGSM in the rest of this paper. Besides improvements on PGD adversarial training, Shafahi et al. [22] proposed a novel training method called FREE, which makes the most use of backpass by updating the model parameters and adversarial perturbation simultaneously. This mechanism is called batch replay, and we visualize the process in Fig. 1(a). In parallel to efficiency, some works focus on enhancing the effectiveness of adversarial training. The main direction is replacing the cross-entropy loss with advanced surrogate loss functions, e.g., TRADE [37], MMA [7], and MART [31]. Meanwhile, previous studies have shown that some of these loss augmentations can also be accelerated [38].
3. Preliminaries

Given a training set with $N$ pairs of example and label $(x_i, y_i)$ that are drawn i.i.d. from a distribution $\mathcal{D}$, a classifier $F_\theta : \mathbb{R}^n \rightarrow \mathbb{R}^c$ can be trained by minimizing a loss function $\sum_{i=1}^N L(F_\theta(x_i))$, where $n$ (resp. $c$) is the dimension of the input (resp. output) vector.

For the image classification task, the malevolent perturbation can be added into an input example to fool the target classifiers [26]. As a defensive strategy, adversarial training can be added into an input example to fool the target classifier $F_\theta$.

The inner maximization problem in Equ. (1) can be approximated by adversarial attacks, such as FGSM [10] and PGD [16]. The approximation calculated by FGSM can be written as

$$\delta = \epsilon \cdot \text{sign}(\nabla_x F_\theta(x)).$$

PGD employs FGSM with smaller step size $\alpha$ multiple times and projects the perturbation back to $B_\epsilon$ [16], the procedure can be described as

$$\delta^i = \delta^{i-1} + \alpha \cdot \text{sign} (\nabla_x L(F_\theta(x + \delta^{i-1}))).$$

So it can provide a better approximation to the inner maximization.

The computational cost of adversarial training is linear to the number of backpropagation [32]. When performing regular PGD-$i$ adversarial training as proposed in [16], the adversarial version of natural training examples is generated by a PGD adversary with $i$ iterations. As a result, its total number of backpropagation is proportional to $(i+1)MT$, where $M$ is the number of mini-batch, and $T$ represents the number of training epochs. By contrast, U-FGSM [32] only uses a FGSM adversary and requires $2MT$ backpropagation to finish training. So from the perspective of computational cost, U-FGSM is much cheaper than PGD adversarial training. As regarding to FREE-$r$ adversarial training [22], which updates the model parameters and adversarial examples simultaneously, it repeats training procedure $r$ times on every mini-batch and conducts $rMT$ backpropagation to train a robust model.

4. Proposed Methodology

To further push the efficiency of adversarial training, we try to boost the robustness gain with as few backpropagations as possible. Previous studies [2, 30] implicitly show that adversarial training is a dynamic training procedure.

Thus, we take a closer look at the robustness gain per epoch during FREE adversarial training. As shown in the left column of Fig. 2, the evaluation performance of PGD-7 and U-FGSM grows linearly through the whole training process. It is as expected that the robustness of models trained by these two methods is enhanced monotonously because they only employ a specific adversary to craft training adversarial examples. The training cost here, represented by the number of backpropagation, are $20M$ and $80M$ for U-FGSM and PGD-7, respectively. By contrast, the performance of trained models does not increase linearly during FREE adversarial training, which is shown in the middle column of Fig. 2. FREE-8 spends the same amount of computational cost as PGD-7 in Fig. 2, while FREE-4 is half cheaper (40M). We noticed that in the FREE method, the network is trained on clean examples in the first turn of batch replay, and the actual adversarial training starts at the second iteration. However, if the model is adversarially trained repeatedly too many times on a mini-batch, catastrophic forgetting would occur and influence the training efficiency, which can be demonstrated by the unstable grown pattern of FREE-8.

4.1. Dynamic Efficient Adversarial Training

These observations motivate us to introduce a new adversarial training paradigm named Dynamic Efficient Adversarial Training (DEAT). As shown in Algorithm 1, we start the training on natural examples via setting the number of
batch replay \( r = 1 \) and gradually increase it to conduct adversarial training in the later training epochs. An illustration showing how DEAT works is presented in Fig. 1(b).

A pre-defined criterion controls the timing of increasing \( r \), and there are many possible options for the increase criterion. Here we first discuss two simple yet effective ways to achieve this purpose. Apparently, increasing \( r \) every \( d \) epochs is a feasible approach. In the right column of Fig. 2, DEAT is first carried out at \( d = 3 \), denoted by DEAT-3, and its computation cost is \( 22M \) that is given by \( [T(T + d)/2d] \). We can see that DEAT-3 only takes half the training cost and achieves a comparable performance to FREE-4. Although there is no adversarial robustness shown at the early epoch, the DEAT-3 trained model’s robustness shoots up significantly after training on adversarial training examples. Besides this naive approach, although the adversarially trained models’ robustness rises remarkably during adversarial training procedures, we observed that their training accuracy is only slightly above 40%. So, intuitively, 40% can be defined as a threshold to determine when to increase \( r \). Under this setting, denoted by DEAT-40%, the number of batch replay will be added one if the model’s training accuracy is above the pre-defined threshold. As we can see in the right column of Fig. 2, DEAT-40% achieves a considerable performance with a smoother learning procedure compared to DEAT-3. The training cost of DEAT-40% is \( 25M \). However, both DEAT-3 and DEAT-40% have the same limitation, i.e., they need prior knowledge to set up a manual threshold, which could not be generalized directly to conduct adversarial training on different datasets or model architectures.

### 4.2. Gradient Magnitude Guided Strategy

To resolve the dependency on manual factors, we introduce a gradient-based approach to guide DEAT automatically. We begin with the theoretical foundation of our approach.

**Assumption 1.** Let \( f(\theta; \cdot) \) be a shorthand notation of \( L(F_0(\cdot)) \), and \( \| \cdot \|_p \) be the dual norm of \( \| \cdot \|_q \). We assume that \( f(\theta; \cdot) \) satisfies the Lipschitzian smoothness condition

\[
\| \nabla_x f (\theta; x) - \nabla_x f (\theta; \hat{x}) \|_p \leq l_{xx} \| x - \hat{x} \|_q, \quad (4)
\]

where \( l_{xx} \) is a positive constant, and there exists a stationary point \( x^* \) in \( f (\theta; \cdot) \).

The Lipschitz condition (4) is proposed by Shaham et al. [23], and Deng et al. [6] proved that the gradient-based adversary can find a local maximum that supports the existence of \( x^* \). Then the following proposition states the relationship between the magnitude of \( \nabla_x f(\theta; x) \) and the Lipschitz constant \( l_{xx} \).

**Theorem 1.** Given an adversarial example \( x' = x + \delta \) from the \( \ell_\infty \) ball \( B_\epsilon \). If Assumption 1 holds, then the Lipschitz constant \( l_{xx} \) of current \( F_0 \) towards training example \( x \) satisfies

\[
\frac{\| \nabla_x f (\theta; x') \|_1}{2\epsilon |x|} \leq l_{xx}. \quad (5)
\]

**Proof.** Substituting \( x' \) and the stationary point \( x^* \) into Equ. (4) gives

\[
\| \nabla_x f(\theta; x') - \nabla_x f(\theta; x^*) \|_1 \leq l_{xx} \| x' - x^* \|_\infty. \quad (6)
\]

Because \( x^* \) is a stationary point and \( \nabla_x f (\theta; x^*) = 0 \), the left side of Equ. (6) can be written as \( \| \nabla_x f (\theta; x') \|_1 \). The right side of Equ. (6) can be simplified as

\[
l_{xx} \| x' - x^* \|_\infty = l_{xx} \| x' - x + x - x^* \|_\infty \\
\leq l_{xx} (\| x' - x \|_\infty + \| x - x^* \|_\infty) \\
\leq l_{xx} (2\epsilon |x|),
\]

where the first inequality holds because of the triangle inequality, and the last inequality holds because both \( \| x' - x \|_\infty \) and \( \| x - x^* \|_\infty \) are upper bounded by \( B_\epsilon \), that is given by \( \epsilon |x| \), where \( | \cdot | \) returns the size of its input. So Equ. (6) can be written as Equ. (5). \( \square \)

Through Theorem 1, we can estimate a local Lipschitz constant \( l_{xx} \) on a specific example \( x \). Note that once the training set \( X \) is fixed, the \( 2\epsilon |x| \) is a constant, too. Then the current adversarial training effect can be estimated via \( \sum_{x \in X} \| \nabla_x f (\theta; x') \|_1 \) or \( l_X \) for short. Intuitively, a small \( \| \nabla_x f (\theta; x') \|_1 \) means that \( x' \) is a qualified adversarial example that is close to a local maximum point. On the other hand, Theorem 1 demonstrates that \( \| \nabla_x f (\theta; x') \|_1 \) can also

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**Algorithm 1** Dynamic Efficient Adversarial Training

**Require:** Training set \( X \), total epochs \( T \), adversarial radius \( \epsilon \), step size \( \alpha \), the number of mini-batches \( M \), and the number of batch replay \( r \)

1: \( r \leftarrow 1 \)
2: for \( t = 1 \) to \( T \) do
3:     for \( i = 1 \) to \( M \) do
4:         Initialize perturbation \( \delta \)
5:         for \( j = 1 \) to \( r \) do
6:             \( \nabla_x, \nabla_{\theta} \leftarrow \nabla L (F_0 (x_i + \delta)) \)
7:             \( \theta \leftarrow \theta - \nabla_{\theta} \)
8:             \( \delta \leftarrow \delta + \alpha \cdot \text{sign}(\nabla_x) \)
9:             \( \delta \leftarrow \max(\min(\delta, \epsilon), -\epsilon) \)
10:        end for
11:     end for
12:     if meet increase criterion then
13:         \( r \leftarrow r + 1 \)
14: end if
15: end for
reflect the loss landscape of current $F_0$, while a flat and smooth loss landscape has been identified as a key property of successful adversarial training [22].

From these two perspectives, we wish to maintain the magnitude of the gradient at a small interval via increasing the number of batch replay. Specifically, we initialize the increasing threshold in the second epoch, which is also the first training epoch on adversarial examples (See line 10 in Algorithm 2). The introduced parameter $\gamma$ aims to stabilize the training procedure. By doing so, $r$ is only increased when the current $l_X$ is strictly larger than in previous epochs. In the following rounds, if $l_X$ is larger than the current threshold, DEAT will increase $r$ by one and update the threshold. Please note that $\nabla_x f(\theta; x')$ has already been computed in ordinary adversarial training. Therefore M-DEAT only takes a trivial cost, i.e., compute $\|\nabla_x\|_1$ and update threshold, to achieve a fully automatic DEAT. In the next section, we will provide a detailed empirical study on the proposed DEAT and M-DEAT methods.

5. Empirical Study

In this section, we report the experimental results of DEAT on CIFAR-10/100 benchmarks. All experiments are built with Pytorch [18] and run on a single GeForce RTX 2080Ti. For all adversarial training methods, we set the batch size at 128, and use SGD optimizer with momentum 0.9 and weight decay $5 \cdot 10^{-4}$. We use PGD-$i$ to represent a $\ell_{\infty}$ norm PGD adversary with $i$ iterations, while CW-$i$ adversary optimizes the margin loss [3] via $i$ PGD iterations. All adversaries are carried out with step size $2/255$, and the adversarial radius $\epsilon$ is $8/255$.

5.1. Under FAST Training Setting

We first provide our evaluation result under FAST training setting, which utilizes the cyclic learning rate [25] and half-precision computation [17] to speed up adversarial training. We set the maximum of the cyclic learning rate at 0.1 and adopt the half-precision implementation from [32]. On the CIFAR-10/100 benchmarks, we compare our methods with FREE-4, FREE-8 [22], and PGD-7 adversarial training [16]. We report the average performance of these methods on three random seeds. FREE methods and PGD-7 are trained with 10 epochs, which is the same setting as in [32]. As a reminder, DEAT-3 increases the number of batch replay every three epochs, and DEAT-40% determines the increase timing based on the training accuracy. DEAT-3 is carried out with 12 training epochs, while DEAT-40% and M-DEAT run 11 epochs, because their first epoch are not adversarial training. All DEAT methods are evaluated at step size $\alpha = 10/255$. We set $\gamma = 1$ for PreAct-ResNet and $\gamma = 1.01$ for Wide-ResNet.

CIFAR-10 As shown in Tab. 1, all DEAT methods perform properly under FAST training setting. Among three DEAT methods, models trained by M-DEAT show the highest robustness. Because of the increasing number of batch replay at the later training epochs, M-DEAT conducts more backpropagation to finish training, so it spends slightly more on the training time. FREE-4 achieves a comparable robust performance to M-DEAT, but M-DEAT is 20% faster and has higher classification accuracy on the natural test set. The time consumption of FREE-8 almost triple those two DEAT methods, but it only achieves a similar amount of robustness to DEAT-3. PGD-7 uses the same number of backpropagation as FREE-8 but performs worse.

The comparison on wide-resnet model also provides a similar result. Evaluation results on Wide-ResNet 34 architecture are presented in Tab. 2. We can see that most methods perform better than before because Wide-ResNet 34 has more complex structure and more parameters than PreAct-ResNet 18. Although FREE-8 achieves the highest overall robustness, its training cost is more expensive than others. FREE-4 also performs considerably, even it is half cheaper than FREE-8. M-DEAT is 22% faster than FREE-4 with comparable robustness and achieve 97% training effect of FREE-8 with 40% time consumption. Other DEAT methods are faster than M-DEAT due to the reason that less backpropagation has been conducted. Models trained by them show reasonable robustness on all tests, but their performance is not as good as M-DEAT. We can see that DEAT can achieve a considerable amount of robust performance.

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1The publicly available training code can be found at https://github.com/locuslab/fast_adversarial/tree/master/CIFAR10.
step learning rate schedule, where the learning rate starts  

and all approaches are trained for 50 epochs with a multi- 

ciency, we choose the PreAct-ResNet as the default model.  

5.2. Under Standard Training Setting  

for most existing adversarial training methods.  

experiments on a standard training setting, which is suitable  

reasonable robustness, while PGD-7 does not fit the FAST  

what we did on DEAT-20%.  

two DEAT methods that based on manual criterion show  

overall performance and training cost to FREE-4. Both M-  

DEAT and FREE-4 outperform FREE-8, whose computa-  

tional cost is doubled. On the other hand, although those  

two DEAT methods that based on manual criterion show  

reasonable performance, they may need additional adjust-  

ment to be carried out on different training settings, like  

what we did on DEAT-20%.  

In above experiments, we can see most methods provides  

reasonable robustness, while PGD-7 does not fit the FAST  

training setting and performs poorly. So we also conduct  

experiments on a standard training setting, which is suitable  

for most existing adversarial training methods.  

5.2. Under Standard Training Setting  

The experimental result under a regular training setting  

is summarized in Tab. 4. We  

CIFAR-100  

Because the classification task on CIFAR-100 is more difficult than on CIFAR-10, we test all meth-  

ods on Wide-ResNet. All methods are carried out under  

the same setting except for DEAT-40%, where we man-  

ually change its threshold from 40% to 20%. Thus, as  

shown in Tab. 3, it can be observed that all baseline meth-  

ods spend similar training time besides DEAT-20% and M-  

DEAT, which is fully automatic. M-DEAT has a similar  

overall performance and training cost to FREE-4. Both M-  

DEAT and FREE-4 outperform FREE-8, whose computa-  

tional cost is doubled. On the other hand, although those  

two DEAT methods that based on manual criterion show  

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ment to be carried out on different training settings, like  

what we did on DEAT-20%.  

In above experiments, we can see most methods provides  

reasonable robustness, while PGD-7 does not fit the FAST  

training setting and performs poorly. So we also conduct  

experiments on a standard training setting, which is suitable  

for most existing adversarial training methods.  

Table 1. Evaluation on CIFAR-10 with PreAct-ResNet 18. All methods are carried out with cyclic learning rate and half-precision.  

| Method    | Natural | FGSM | Adversarial PGD-100 | Adversarial CW-20 | Time (min) |
|-----------|---------|------|---------------------|-------------------|------------|
| DEAT-3    | 80.09%  | 49.75% | 42.94%              | 42.26%            | 5.8        |
| DEAT-40%  | 79.52%  | 48.64% | 42.81%              | 42.08%            | 5.6        |
| M-DEAT    | 79.88%  | 49.84% | 43.38%              | 43.36%            | 6.0        |
| FREE-4    | 78.46%  | 49.52% | 43.97%              | 43.16%            | 7.8        |
| FREE-8    | 74.56%  | 48.60% | 44.11%              | 42.49%            | 15.7       |
| PGD-7     | 70.90%  | 46.37% | 43.17%              | 41.01%            | 15.1       |

Table 2. Evaluation on CIFAR-10 with Wide-ResNet 34. All methods are carried out with cyclic learning rate and half-precision.  

| Method    | Natural | FGSM | Adversarial PGD-100 | Adversarial CW-20 | Time (min) |
|-----------|---------|------|---------------------|-------------------|------------|
| DEAT-3    | 81.58%  | 50.53% | 44.39%              | 44.43%            | 41.6       |
| DEAT-40%  | 81.49%  | 51.08% | 45.03%              | 44.49%            | 38.9       |
| M-DEAT    | 81.59%  | 51.29% | 45.27%              | 45.09%            | 43.1       |
| FREE-4    | 80.34%  | 51.72% | 46.17%              | 45.62%            | 55.4       |
| FREE-8    | 77.69%  | 51.45% | 46.99%              | 46.11%            | 110.8      |
| PGD-7     | 72.12%  | 47.39% | 44.80%              | 42.18%            | 109.1      |

However, the manual criterion like DEAT-40% cannot be generalized properly, while M-DEAT can still perform well without any manual adjustment.  

CIFAR-100  

Because the classification task on CIFAR-100 is more difficult than on CIFAR-10, we test all meth- 

ods on Wide-ResNet. All methods are carried out under the same setting except for DEAT-40%, where we man- 

ually change its threshold from 40% to 20%. Thus, as 

shown in Tab. 3, it can be observed that all baseline meth- 

ods spend similar training time besides DEAT-20% and M- 

DEAT, which is fully automatic. M-DEAT has a similar 

overall performance and training cost to FREE-4. Both M- 

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tional cost is doubled. On the other hand, although those 

two DEAT methods that based on manual criterion show 

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In above experiments, we can see most methods provides 

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experiments on a standard training setting, which is suitable 

for most existing adversarial training methods.  

5.2. Under Standard Training Setting  

The experimental result under a regular training setting is 

presented in this section. For the sake of evaluation effi- 

ciency, we choose the PreAct-ResNet as the default model and all approaches are trained for 50 epochs with a multi- 

step learning rate schedule, where the learning rate starts 

at 0.05 and is decayed at 25 and 40 epochs. In this exper- 

iment, we add FAT [38], TRADES [37], and MART [31] into our comparison. FAT introduces an early stop mechanism into the generation of PGD training adversarial examples to make acceleration. Its original version is carried out with 5 maximum training adversarial iterations. While both original TRADES and FAT with TRADES surrogate loss, denoted by F+TRADES, use 10 adversarial steps, which are the same settings as in [37, 38]. The same number of adversarial steps is also applied in the original MART and FAT with MART surrogate loss (F+MART), which is the same settings reported in [31]. As for the hyper-parameter τ that controls the early stop procedure in FAT, we conduct the original FAT, F+TRADES and F+MART at τ = 3, which enables their highest robust performance in [35]. FREE adversarial training is conducted with three different numbers of batch replay for a comprehensive evaluation. We only evaluate M-DEAT under this setting because those two DEAT methods cannot be deployed easily. In addition, we also introduce our magnitude based strategy into PGD, TRADE, and MART, namely M+PGD, M+TRADES, and M+MART. To be specific, we let the number of training adversarial iterations start at 1 and compute the l∞ during training. The procedure described in lines 8–14 of Algorithm 2 has been transplanted to control the increase of the adversarial iterations². All magnitude based methods are evaluated at step size α = 10/255 and γ = 1.01 if applied. As suggested in [21], each adversarial trained model is tested by a PGD-20 adversary on a validation set during training, we save the robustest checkpoint as for the final evaluation.  

The experimental results are summarized in Tab. 4. We can see that although FREE-4 and FAT finish training marginally faster than others, their classification accuracy on adversarial tests are substantially lower as well. Compared to FREE-6, which is more efficient than FREE-8 and shows better robustness, M-DEAT is about 16% faster and has higher accuracy on all evaluation tasks. PGD-7 is a strong baseline under the standard training setting, it achieves the second robustest method when there is no surrogate loss function. M+PGD outperforms PGD-7 on
all adversarial tests and takes 8% less time-consumption. While M-DEAT is about 36% faster than PGD-7 and remains a comparable robustness against the CW-20 adversary. FAT shows highly acceleration effect on both PGD and TRADE. Although FAT-5 only achieves a fair robustness, F+TRADE’s performance is remarkable. It can be seen that TRADE boosts the trained model’s robustness, but also significantly increases the computational cost. F+TRADE obtains a comparable robustness in half of TRADE’s training time. By contrast, M+TRADE is 26% faster than TRADE and also shows a similar overall performance. Although there is no substantial improvement on the CW-20 test, MART achieves the highest robust performance on FGSM and PGD-100 tests but has the lowest accuracy on natural examples. Our M+MART reduces the time consumption of MART almost by a factor of three, and remains a considerable amount of robustness to FGSM adversary. However, F+MART is even slower than the original MART and only achieves a similar robustness to M+MART.

5.3. Comparison and Collaboration with U-FGSM

We report the competition between DEAT and U-FGSM and demonstrate how the magnitude of gradient can help FGSM training to overcome the overfitting problem, which is identified in [32]. The comparison is summarized in Tab. 5. It is observed that the differences of robust performance between U-FGSM and DEAT methods are subtle under FAST training setting. They achieve a similar balance between robustness and time consumption, while M-DEAT performs better on most tests but also takes slightly more training time. When using the standard training setting, U-FGSM is faster but only achieves a fair robust performance. Meanwhile, U-FGSM adversarial training also suffers from the catastrophic overfitting when its step size \( \alpha \) is larger than 10/255 [32]. This means the robust accuracy of U-FGSM adversarial training with respect to a PGD adversary may suddenly and drastically drop to 0% after several training epochs. To address this issue, Wong et al. [32] randomly save a batch of training data and conduct an additional PGD-5 test on the saved data. The training process will be terminated if the trained model lost robustness in the validation. We reproduce this phenomena by carrying out U-FGSM at \( \alpha = 12/255 \) with a flat learning rate (0.05) for 30 epochs and record \( l_X \) during training. It can be observed in Fig. 3 that \( l_X \) surges dramatically as soon as the catastrophic overfitting occurs. By monitoring the changing ratio of \( l_X \), we can achieve early stopping without extra validation. Specifically, we record \( l_X \) at each epoch and begin to calculate its changing ratio \( l'_X \) since epoch \( m \). Let \( l'_X \) represent \( l_X \) at epoch \( t \), where \( t > m \), the corresponding

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Table 4. Evaluation on CIFAR-10 with PreAct-ResNet 18. All methods are carried out with multi-step learning rate and 50 training epochs.

| Method    | Natural | Adversarial | Time (min) |
|-----------|---------|-------------|------------|
|           | FGSM    | PGD-100     | CW-20      |
| FREE-4    | 85.00%  | 54.47%      | 45.64%     | 46.55%    | 81.7  |
| FREE-6    | 82.49%  | 54.55%      | 48.16%     | 47.42%    | 122.4 |
| FREE-8    | 81.22%  | 54.05%      | 48.14%     | 47.12%    | 163.2 |
| M-DEAT    | 84.57%  | 55.56%      | 47.81%     | 47.58%    | 100.5 |
| PGD-7     | 80.07%  | 54.68%      | 49.03%     | 47.95%    | 158.8 |
| FAT-5     | 84.15%  | 53.99%      | 45.49%     | 46.09%    | 91.8  |
| M+PGD2    | 80.82%  | 55.25%      | 49.12%     | 48.13%    | 144.1 |
| TRADE     | 79.39%  | 55.82%      | 51.48%     | 48.89%    | 456.2 |
| F+TRADE1  | 81.61%  | 56.31%      | 50.55%     | 48.43%    | 233.9 |
| M+TRADE2  | 80.26%  | 55.69%      | 51.19%     | 48.57%    | 336.1 |
| MART      | 75.59%  | 57.24%      | 54.23%     | 48.60%    | 147.7 |
| F+MART1   | 83.62%  | 56.03%      | 48.88%     | 46.42%    | 234.4 |
| M+MART2   | 83.64%  | 57.15%      | 48.98%     | 45.88%    | 52.1  |

1 Variants that are accelerated by FAT.
2 Variants that are guided by the magnitude of gradient.

Figure 3. Visualize the relationship between robustness and the magnitude of gradient \( l_X \) during U-FGSM adversarial training at step size \( \alpha = 12/255 \). The catastrophic overfitting occurs at epoch 23, where \( l_X \) begins to shoot up.

Table 5. Compare M-DEAT to U-FGSM under different environments. The time unit here is minute.

| Set up | Method    | Natural | PGD-100 | CW-20 | Time |
|--------|-----------|---------|---------|-------|------|
|        | PreAct-Res.| M-DEAT  | 79.88%  | 43.38% | 43.36% | 6.0  |
|        | + FAST1   | U-FGSM  | 78.45%  | 43.86% | 43.04% | 5.6  |
|        | Wide-Res. | M-DEAT  | 81.59%  | 45.27% | 45.09% | 43.1 |
|        | + FAST    | U-FGSM  | 80.89%  | 45.45% | 44.91% | 41.3 |
|        | PreAct-Res.| M-DEAT  | 84.57%  | 47.81% | 47.58% | 100.5|
|        | + Standard2 | U-FGSM | 81.99%  | 45.77% | 45.74% | 40.3 |

| Set up | Method    | Natural | PGD-100 | CW-20 | Time |
|--------|-----------|---------|---------|-------|------|
|        | Wide-Res. | M-DEAT  | 53.57%  | 24.33% | 23.08% | 52.9 |
|        | + FAST    | U-FGSM  | 52.54%  | 24.17% | 22.93% | 41.6 |

1 Training with cyclic learning rate and half-precision.
2 Training 50 epochs with multi-step learning rate.
changing ratio $R^t$ can be computed as follow:

$$R^t = \frac{l_X^t - l_X^{t-m}}{m}.$$  

We stop the training process when $R^t > \gamma \cdot R^{t-1}$, where $\gamma$ is a control parameter and plays the same role as in Algorithm 2. Denoted by M+U-FGSM\(^2\), our approach has been evaluated at $m = 2$ and $\gamma = 1.5$. We report the average performance of M+U-FGSM and Wong et al.'s approach on 5 random seeds in Tab. 6.

Compared with the validation based approach, we can see that early stopping based on the changing ratio of $l_X$ is more sensitive and maintains a stable robust performance. The improvement on time-consumption is only trivial because this experiment is done on an advanced GPU, it should be more visible on less powerful machines.

Table 6. Compare different early stopping strategies on preventing the catastrophic overfitting.

| Method       | Natural | PGD-100 | Stop at (-/30) | Time (min) |
|--------------|---------|---------|----------------|------------|
| U-FGSM       | 75.86±1.5% | 40.47±2.4% | 23±2          | 20.2±1.5   |
| M+U-FGSM     | 73.33±1.1% | 41.80±0.7% | 19.4±4        | 16.2±3.1   |

5.4. Why does $l_X$ Matter

In the last part of empirical studies, we explore why this magnitude of gradient based strategy works. It can be seen from previous experiments that the magnitude of gradient $l_X$ can be deployed in different adversarial training methods even with loss augmentation. We then conduct an empirical investigation on how $l_X$ changes in a set of adversarial training algorithms, i.e., FREE, U-FGSM, PGD-7, and M-DEAT. They are conducted with PreAct-ResNet on CIFAR-10 under FAST training setting. Recall that $l_X$ is given by $\sum_{x \in X} ||\nabla_x||_1$, and a small $||\nabla_x||_1$ means the corresponding adversarial example is close to a local minimum point. From this perspective, $l_X$ can be viewed a measurement of the quality of training adversarial examples. It can be observed in Fig. 4, where illustrates that the quality of training adversarial examples is maintained at a certain level in M-DEAT and U-FGSM. As for FREE and PGD, which employ the same adversary at each epoch, the quality of their training adversarial examples gradually decrease when the trained model become more robust. This experiment suggests that maintaining the quality of adversarial examples is essential to adversarial training. DEAT achieve that via increasing the number of batch replay. Because its first training epoch is on natural examples with random initialized perturbation, $l_X$ is high at the beginning and slumps after adversarial examples join the training. U-FGSM has the same effect and does not use multiple adversarial steps under FAST training setting, which also explains why it works so well in Sec. 5.3. An interesting phenomena is that even U-FGSM only uses adversarial examples to train models, $l_X$ in the first epoch is still relatively higher than that in later epochs. It seems that conduct adversarial training on just initialized models directly would not provide a promising effectiveness.

6. Conclusion

To improve the efficiency of adversarial learning, we propose a Dynamic Efficient Adversarial Training called DEAT, which gradually increases the number of batch re-plays based on a given criterion. Our preliminary verification shows that DEAT with manual criterion can achieve a considerable robust performance on a specific task but need manual adjustment to deployed on others. To resolve this limitation, we theoretically analyze the relationship between the Lipschitz constant of a given network and the magnitude of its partial derivative towards adversarial perturbation. We find that the magnitude of gradient can be used to quantify the adversarial training effectiveness. Based on our theorem, we further design a novel criterion to enable an automatic dynamic adversarial training guided by the gradient magnitude, i.e., M-DEAT. This magnitude guided strategy does not rely on prior-knowledge or manual criterion. More importantly, the gradient magnitude adopted by M-DEAT is already available during the adversarial training so it does not require non-trivial extra computation. The comprehensive experiments demonstrate the efficiency and effectiveness of M-DEAT. We also empirically show that our magnitude guided strategy can easily be incorporated with different adversarial training methods to boost their training efficiency.

\(^2\)The corresponding pseudo code is provided in supplementary material.
An important takeaway from our studies is that maintaining the quality of adversarial examples at a certain level is essential to achieve efficient adversarial training. In this sense, how to meaningfully measure the quality of adversarial examples during training would be very critical, which would be a promising direction that deserves further exploration both empirically and theoretically.

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A. More details about our experimental results

As a supplement to our empirical study in the main submission, we report the impact of the training adversarial step size $\alpha$ in M-DEAT on the trained models under FAST training setting and provide a visualization of different methods’ training process under standard setting. Besides, we also expand the evaluation on M+U-FGSM to different step sizes to test its reliability.

For the purpose of reproducibility, the code of our empirical study is also available in the supplementary.

A.1. The impact of hyper-parameters $\alpha$ and $\gamma$

We evaluate the trained models’ performance with different step sizes at $\gamma = 1.01$ and $\gamma = 1.0$ respectively. Each model is trained 10 epochs under the same FAST training setting, and each combination of $\alpha$ and $\gamma$ was tested on 5 random seeds. To highlight the relationship between models’ robustness and the corresponding training cost, which is denoted by the number of backpropagation (#BP), we use PGD-20 error that is computed as one minus model’s classification accuracy on PGD-20 adversarial examples. The result has been summarized in Fig. 5. This experiment shows that M-DEAT can be carried out at a large range of $\alpha$. Although M-DEAT achieves a slightly better robust performance when carrying out at a large step size, like 14/255, the corresponding training cost is much higher than that at lower step size. Therefore, as we mentioned in the main submission that all DEAT methods are evaluated at step size $\alpha = 10/255$. We set $\gamma = 1$ for PreAct-ResNet and $\gamma = 1.01$ for Wide-ResNet when using the FAST training setting, while all magnitude based methods are carried out at $\gamma = 1.01$ under standard training setting.

A.2. Visualize the training process under standard training

In the main submission, we introduce the magnitude of trained model’s partial derivative towards adversarial examples to guide our DEAT and existing adversarial methods under a standard training setting with the PreAct-ResNet architecture. A complete version of M-DEAT’s pseudo code is shown in Algorithm 3, while M+PGD, M+TRADE, and M+MART can be described by Algorithm 4 via changing loss functions. Note that we adjust the step size $\alpha^*$ per epoch based on the number of adversarial steps $r$ and the adversarial radii $\epsilon$ to make sure that $\alpha^* \cdot r \geq \epsilon$ for the methods that are guided by the gradient magnitude. All models were trained 50 epochs with a multi-step learning rate schedule, where the learning rate starts at 0.05 and is decayed at 25 and 40 epochs. To better present our experiment under this scenario, we visualize the training process of all methods from two perspectives to show the how models gain their robustness toward the PGD-20 adversary and
Figure 5. Visualize the impact of $\alpha$ and $\gamma$. #BP is a short-hand for the number of backpropagation. PGD-20 error is computed as one minus model’s classification accuracy of PGD-20 adversarial examples.

Figure 6. The number of backpropagation w.r.t epochs.

A.3. Evaluate M+U-FGSM with different step sizes

In section, we evaluate M+U-FGSM at step size $\alpha \in \{12, 14, 16, 18\}$ to test its reliability on preventing the catastrophic overfitting. M+U-FGSM achieves early stopping via monitoring the changing ratio of the magnitude of gradient, which is shown in lines 9–14 in Algorithm 5. The experimental result is summarized in Tab. 7, where we can see that M+U-FGSM provides a reasonable performance under different settings and is as reliable as the validation based U-FGSM to avoid overfitting. It can also be observed that a larger step size does not improve the models’ robustness and may lead to an earlier occurrence of overfitting.

We observe that although U-FGSM and FAT are faster than PGD and M+PGD, their robust performance is lower. M+PGD finish training about 10 minutes earlier than PGD and achieve a higher robust performance after the second learning rate decay. In practice, we set an upper bound on the number of adversarial step for M+PGD, which is the same as in PGD. So the acceleration of M+PGD comes from the reduced adversarial steps at early training stage. It can be seen from Fig. 8(b) that the lines corresponding to the loss change of M+PGD and PGD are highly coincident at the late training stage, because M+PGD conducted the same adversarial steps as in PGD during this period. The result of TRADE related adversarial training methods is summarized in Fig. 7(c). M+TRADE performs similar to F+TRADE at the early training stage, but it is less efficient than F+TRADE because of the increased adversarial iterations. As shown in Fig. 8(c), the lines corresponding to the loss change of M+TRADE and TRADE are also coincident at the late training stage. MART adversarial training methods are summarized in Fig. 7(d) and Fig. 8(d). We can observe from Fig. 8(d) that the curves of MART and M+MART fluctuate similarly during the whole training process, while the loss of F+MART decreased steadily.

To sum up, the approaches that adopts the magnitude guided strategies can reduce the adversarial steps at the early training stage to accelerate the training process. Fig. 6 visualizes the number of backpropagation w.r.t epochs, where our magnitude-guided strategy shows adaptive capability when accelerating different methods.
Figure 7. An illustration about how adversarially trained models gain robustness. We employ a PGD-20 adversary to test a model on the first 1280 examples of the CIFAR-10 test set after each training epoch. All methods in this part have been divided into four groups. M-DEAT and three FREE methods are plotted in (a), M+PGD, U-FGSM, FAT, and PGD are plotted in (b). TRADES related methods and MART related methods are shown in (c) and (d) respectively.

Table 7. Compare magnitude guided early stopping (M+U-FGSM) to validation based early stopping (U-FGSM) on preventing the catastrophic overfitting at different step sizes over 5 random seeds.

| α  | Natural | PGD-100 | Stop at (×30) | Time (min) |
|----|---------|---------|---------------|------------|
|    |         |         |               |            |
| M+U-FGSM |         |         |               |            |
| 12 | 73.33±1.1% | 41.80±0.7% | 19.4±4 | 16.2±3.1 |
| 14 | 72.25±1.0% | 41.07±0.8% | 17.8±2 | 14.5±1.6 |
| 16 | 70.64±1.5% | 40.64±0.6% | 15.4±1 | 12.6±0.5 |
| 18 | 71.51±1.9% | 40.90±0.9% | 15.8±2 | 12.6±1.4 |
| U-FGSM |         |         |               |            |
| 12 | 75.86±1.5% | 40.47±2.4% | 23±2 | 20.2±1.5 |
| 14 | 72.36±1.0% | 40.77±1.1% | 17.4±2 | 15.9±1.7 |
| 16 | 72.37±0.7% | 40.98±0.5% | 15.6±1 | 13.6±0.7 |
| 18 | 72.92±0.9% | 39.35±4.2% | 16.6±2 | 14.2±1.3 |
Figure 8. An illustration of the training loss of different adversarial training methods. All methods in this part have been divided into four groups. M-DEAT and three FREE methods are plotted in (a). M+PGD, U-FGSM, FAT, and PGD are plotted in (b). TRADES related methods and MART related methods are shown in (c) and (d) respectively.
Algorithm 3 Magnitude Guided DEAT (M-DEAT)

Require: Training set $X$, total epochs $T$, adversarial radius $\epsilon$, step size $\alpha$, the number of mini-batches $M$, the number of batch replay $r$, the evaluation of current training effect $l_X$ and a relax parameter $\gamma$

1: $r \leftarrow 1$
2: for $t = 1$ to $T$ do
3: $l_X \leftarrow 0$
4: for $i = 1$ to $M$ do
5: Initialize perturbation $\delta$
6: for $j = 1$ to $r$ do
7: $\nabla_x, \nabla_\theta \leftarrow \nabla L (F_\theta (x_i + \delta))$
8: $\theta \leftarrow \theta - \nabla_\theta$
9: $\delta \leftarrow \delta + \alpha \cdot \text{sign} (\nabla_x)$
10: $\delta \leftarrow \max (\min (\delta, \epsilon), -\epsilon)$
end for
11: $l_X \leftarrow l_X + ||\nabla_x||_1$
end for
12: if $t = 1$ then
13: $r \leftarrow r + 1$
else if $t = 2$ then
14: Threshold $\leftarrow \gamma \cdot l_X$
15: else if $l_X > \text{Threshold}$ then
16: Threshold $\leftarrow \gamma \cdot l_X; r \leftarrow r + 1$
end if
end if
end for

Algorithm 4 Magnitude Guided PGD (M-PGD)

Require: Training set $X$, total epochs $T$, adversarial radius $\epsilon$, step size $\alpha$, the number of mini-batches $M$, the number of PGD steps $N$, the evaluation of current training effect $l_X$ and a relax parameter $\gamma$

1: $r \leftarrow 1$
2: for $t = 1$ to $T$ do
3: $l_X \leftarrow 0$
4: for $i = 1$ to $M$ do
5: Initialize perturbation $\delta$
6: for $j = 1$ to $N$ do
7: $\nabla_x \leftarrow \nabla L (F_\theta (x_i + \delta))$
8: $\delta \leftarrow \delta + \alpha^* \cdot \text{sign} (\nabla_x)$
9: $\delta \leftarrow \max (\min (\delta, \epsilon), -\epsilon)$
end for
10: $l_X \leftarrow l_X + \||\nabla_x||_1$
11: $\theta \leftarrow \theta - \nabla_\theta L (F_\theta (x_i + \delta))$
end for
12: if $t = 1$ then
13: $r \leftarrow r + 1$
else if $t = 2$ then
14: Threshold $\leftarrow \gamma \cdot l_X$
15: else if $l_X > \text{Threshold}$ then
16: Threshold $\leftarrow \gamma \cdot l_X; r \leftarrow r + 1$
end if
end if
end for

Algorithm 5 M+U-FGSM

Require: Training set $X$, total epochs $T$, adversarial radius $\epsilon$, step size $\alpha$ and number of minibatches $M$. Compute the changing ratio $R$ since epoch $m$, and a relax parameter $\gamma$.  

1: for $t = 1 \ldots T$ do
2: $l_X \leftarrow 0$
3: for $i = 1 \ldots M$ do
4: $\delta = U (-\epsilon, \epsilon) \cdot \text{sign} (\nabla_\theta L_{ce} (x_i + \delta, y_i))$
5: $\delta = \max (\min (\delta, \epsilon), -\epsilon)$
6: $\theta \leftarrow \theta - \nabla_\theta L_{ce} (F_\theta (x_i + \delta), y_i)$
end for
7: if $t > m$ then
8: $R^t = \frac{l_X^t}{l_X^m}$
9: if $R^t > \gamma \cdot R^t-1$ then
10: Stop training
end if
end if
end for