Hadronic matrix elements for $B$-mixing in the Standard Model and beyond

C. M. Bouchard, for the Fermilab Lattice and MILC Collaborations

Physics Department, The Ohio State University, Columbus, Ohio 43210, USA

Abstract. We use lattice QCD to calculate the $B$-mixing hadronic matrix elements for a basis of effective four-quark operators that spans the space of all possible contributions in, and beyond, the Standard Model. We present results for the $\text{SU}(3)$-breaking ratio $\xi$ and discuss our ongoing calculation of the mixing matrix elements, including the first calculation of the beyond the Standard Model matrix elements from unquenched lattice QCD.

Keywords: $B$-mixing, lattice QCD, beyond the Standard Model, hadronic matrix elements
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MOTIVATION

Through a combination of the GIM mechanism, Cabibbo suppression, and loop suppression, the Standard Model (SM) contribution to $B$-mixing is small and new physics (NP) effects may be discernible [1]. In fact, there are experimental hints this may be the case. In unitarity triangle analyses [2, 3] a persistent $2 - 3\sigma$ inconsistency is suggestive of NP and points to $B$-mixing as a possible source. DØ's same-sign dimuon charge asymmetry [4] and global analyses by UTfit [5] and CKMfitter [6] reveal $2 - 4\sigma$ discrepancies in SM $B$-mixing. Experimental measurements of the $B$-mixing oscillation frequency [7] have sub-percent precision but cannot be fully leveraged in the search for NP as theory errors, dominated by hadronic uncertainty, are an order of magnitude larger [8].

CALCULATION

To lowest order in the SM, $B$-mixing is described by box diagrams [cf. Fig. 1 (left)]. Under the operator product expansion (OPE) flavor-changing short-distance interactions, of $\mathcal{O}(100 \text{ GeV})$ in the SM and higher in NP scenarios, and long-distance hadronic physics, of $\mathcal{O}(500 \text{ MeV})$, factorize. At the energies relevant to $B$-mixing, of $\mathcal{O}(M_B \sim 5 \text{ GeV})$, flavor-changing physics is described by the local, effective, four-quark interaction of Fig. 1 (right). A commonly used basis of mixing operators is

$$
\mathcal{O}_1 = (\bar{b}^\alpha \gamma_\mu Lq^\alpha) (\bar{b}^\beta \gamma_\mu Lq^\beta) \quad \mathcal{O}_4 = (\bar{b}^\alpha Lq^\alpha) (\bar{b}^\beta Rq^\beta)
$$

$$
\mathcal{O}_2 = (\bar{b}^\alpha Lq^\alpha) (\bar{b}^\beta Lq^\beta) \quad \mathcal{O}_5 = (\bar{b}^\alpha Lq^\beta) (\bar{b}^\beta Rq^\alpha)
$$

$$
\mathcal{O}_3 = (\bar{b}^\alpha Lq^\beta) (\bar{b}^\beta Lq^\alpha)
$$

(1)

where $L/R$ are left/right projection operators and $\alpha, \beta$ are color indices. A calculation of the matrix elements of these operators is sufficient to parameterize the hadronic
FIGURE 1. (Left) A SM contribution to $B$-mixing and (right) an effective four-quark interaction.

contributions to $B$-mixing in and beyond the SM.

Under the OPE, the expression for the oscillation frequency factorizes:

$$\Delta M_q = \sum_i C_i(\mu) \langle B_q^0|\mathcal{O}_i(\mu)|\bar{B}_q^{0}\rangle,$$

(2)

where the short-distance $C_i$ are model dependent and the long-distance hadronic mixing matrix elements $\langle B_q^0|\mathcal{O}_i|\bar{B}_q^{0}\rangle$ must be calculated nonperturbatively using lattice QCD. The phenomenologically-relevant bag parameters $B_{B_q}^{(i)}$ and SU(3)-breaking ratio $\xi$ are related to the matrix elements by

$$\langle B_q^0|\mathcal{O}_i|\bar{B}_q^{0}\rangle = c_i f_{B_q}^2 B_{q} B_{B_q}^{(i)}$$

and

$$\xi = f_{B_s} \sqrt{B_{B_s}^{(1)}/B_{B_d}^{(1)}},$$

(3)

where $f_{B_q}$ is the $B$-meson decay constant and $c_i$ are numerical factors.

The Lattice Calculation. Working in the $B$-meson rest-frame, we generate correlation function data via Monte Carlo evaluation of the path integral representations of the vacuum expectation values

$$C_{2pt}^n(t) = \langle B_q^0(t) B_q^0(0)^\dagger \rangle$$

and

$$C_{3pt}^n(t_1,t_2) = \langle B_q^0(t_2) \mathcal{O}_i(0) B_q^0(t_1) \rangle,$$

(4)

with gauge field integration performed with the MILC ensembles. We simulate with staggered light and Fermilab heavy quarks (for details of gluon and quark discretizations see [9] and references therein). Using Bayesian fitting techniques [10], these data are fit to the ansätze [11],

$$C_{2pt}^n(t) = \sum_{n=0}^N Z_n^2 (-1)^{n(t+1)} \left( e^{-M_n t} + e^{-M_n (T-t)} \right)$$

and

$$C_{3pt}^n(t_1,t_2) = \sum_{n,m=0}^N \langle B_q^0 | \mathcal{O}_i | \bar{B}_q^{0}\rangle \frac{Z_n Z_m}{2\sqrt{M_n M_m}} (-1)^{n(t_1+1)+m(t_2+1)} e^{-M_n t_1 - M_m t_2},$$

(5)

where $Z_n$ is the amplitude and $M_n$ the mass of the $B$-meson $n^{th}$ excited state and $T$ the temporal extent of the lattice. From these fits we extract the matrix elements over

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1 An alternate definition of the bag parameter includes a factor of $M_{B_q}^2/(m_b + m_q)^2$ with $c_i$ for $i \neq 1$. 
a range of valence-quark masses, light sea-quark masses, and lattice spacings. Lattice matrix elements are matched to the continuum at one loop in tadpole-improved lattice perturbation theory [12]. The continuum matrix elements corresponding to physical light (for $B^0_d$) or strange (for $B^0_s$) valence quark, and at physical light sea-quark mass, are then obtained by extrapolation/interpolation with the aid of rooted, staggered, chiral perturbation theory [13, 14].

**Status and Outlook.** We recently completed a calculation of $\xi = 1.268(63)$ using data at lattice spacings down to $\approx 0.09$ fm and valence quarks as light as $0.1 m_s$ [15].

An ongoing calculation includes an update of $\xi$ and the calculation of matrix elements and bag parameters for all five operators in Eq. (1). In addition, it includes several improvements: a three-fold increase in statistics; data at lattice spacings as small as $\approx 0.045$ fm and valence quarks as light as 0.05 $m_s$, to reduce the effect of the continuum-chiral extrapolation; and a more thorough treatment of chiral perturbation theory [14].

Preliminary results for the matrix elements with the new, extended data set can be found in Table 4 of Ref. [16]. These results are based on data at lattice spacings down to $\approx 0.06$ fm and valence quarks as light as 0.1 $m_s$. Initial studies show that our new treatment of chiral perturbation theory has a negligible effect on the values of the matrix elements in this analysis.

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