Witnessing Quantum Aspects of Gravity in a Lab

Underground test of gravity-related wave function collapse

Sandro Donadi

Centre for Quantum Materials and Technologies, School of Mathematics and Physics

24/09/2024
THE MEASUREMENT PROBLEM

The Schrödinger equation:

- Linear
- Deterministic

\[ i\hbar \frac{d}{dt} |\Psi(t)\rangle = H |\Psi(t)\rangle \]

What exactly qualifies some physical systems to play the role of “measurer”? Was the wave function of the world waiting to jump for thousands of millions of years until a single-celled living creature appeared? Or did it have to wait a little longer for some better qualified system... with a PhD?

J. S. Bell

The wave packet reduction postulate:

- Non Linear
- Stochastic

\[ \frac{|a_1\rangle + |a_2\rangle}{\sqrt{2}} \xrightarrow{\text{measurement}} \begin{cases} |a_1\rangle & \text{half of total cases} \\ |a_2\rangle & \text{half of total cases} \end{cases} \]

There are two different laws for the evolution of the state vectors but it is not clear when to use which one.
IDEA: to merge the Schrödinger evolution and the wave function collapse into a unified dynamics.

The new dynamics must be:

1) Close to Schrödinger equation for microscopic systems but collapsing efficiently macroscopic systems.

2) **Non linear**, otherwise there is no collapse;

3) **Stochastic**, otherwise there can be faster than light signaling. (N. Gisin, Helv. Phys. Acta **62.4**, 363-371 (1989))

For example the Schrödinger-Newton equation suffers of this problem. (M. Bahrami, A. Großardt, S. Donadi, A. Bassi, New J. Phys. **16(11)**, 115007 (2014))
THE CSL MODEL
(CONTINUOUS SPONTANEOUS LOCALIZATIONS MODEL)

G.C. Ghirardi, P. Pearle and A. Rimini, Phys. Rev. A 42, 78 (1990).

\[
\frac{d}{dt} \psi_t = \left[ -\frac{i}{\hbar} \hat{H} - \frac{\sqrt{\lambda}}{m_0} \int d\mathbf{x} \left( \hat{\mu}(\mathbf{x}) - \langle \hat{\mu}(\mathbf{x}) \rangle_{\psi_t} \right) \right] dW_t(\mathbf{x})
\]

\[-\frac{\lambda}{2m_0^2} \int d\mathbf{x} \int d\mathbf{y} g(\mathbf{x} - \mathbf{y}) \left( \hat{\mu}(\mathbf{x}) - \langle \hat{\mu}(\mathbf{x}) \rangle_{\psi_t} \right) \left( \hat{\mu}(\mathbf{y}) - \langle \hat{\mu}(\mathbf{y}) \rangle_{\psi_t} \right) dt \right] |\psi_t\rangle
\]

\[
\mathbb{E} [dW_t(\mathbf{x})dW_t(\mathbf{y})] = g(\mathbf{x} - \mathbf{y}) dt \quad g(\mathbf{x} - \mathbf{y}) = e^{-\frac{(\mathbf{x} - \mathbf{y})^2}{4r_C^2}}
\]

- Localization in space;
- Amplification mechanism.

THE MASTER EQUATION

\[
\frac{d\hat{\rho}_t}{dt} = -\frac{i}{\hbar} \left[ \hat{H}, \hat{\rho}_t \right] - \frac{\lambda}{2m_0^2} \int d\mathbf{x} \int d\mathbf{x}' e^{-\frac{(\mathbf{x} - \mathbf{x}')^2}{4r_C^2}} \left[ \hat{\mu}(\mathbf{x}), [\hat{\mu}(\mathbf{x}'), \hat{\rho}_t] \right]
\]
Underground test of gravity-related wave function collapse

PART 1: Introduction to the DP model

**DIÓSI MODEL**
L. Diósi, Phys. Rev. A **40**, 1165–1174 (1989).

\[
\begin{align*}
\frac{d|\psi_t\rangle}{dt} &= \left[-\frac{i}{\hbar} \hat{H} dt + \sqrt{\frac{G}{\hbar}} \int d\mathbf{x} \left( \hat{\mu}(\mathbf{x}) - \langle \hat{\mu}(\mathbf{x}) \rangle \right) dW_t(\mathbf{x}) - \\
&\quad - \frac{G}{2\hbar} \int d\mathbf{x} d\mathbf{y} \frac{\left( \hat{\mu}(\mathbf{x}) - \langle \hat{\mu}(\mathbf{x}) \rangle \right) \left( \hat{\mu}(\mathbf{y}) - \langle \hat{\mu}(\mathbf{y}) \rangle \right)}{|\mathbf{x} - \mathbf{y}|} dt \right] |\psi_t\rangle
\end{align*}
\]

\[
\begin{align*}
\frac{d\hat{\rho}(t)}{dt} &= -\frac{i}{\hbar} \left[ \hat{H}, \hat{\rho}(t) \right] - \frac{G}{2\hbar} \int \frac{d\mathbf{x} d\mathbf{y}}{|\mathbf{x} - \mathbf{y}|} \left[ \hat{\mu}(\mathbf{x}), \left[ \hat{\mu}(\mathbf{y}), \hat{\rho}(t) \right] \right]
\end{align*}
\]

\[
\langle \mathbf{a} | \hat{\rho}(t) | \mathbf{b} \rangle = \langle \mathbf{a} | \hat{\rho}(0) | \mathbf{b} \rangle e^{-t/\tau}
\]

\[
\tau^{-1} = \frac{G}{2\hbar} \int d\mathbf{x} d\mathbf{y} \frac{(\mu_a(\mathbf{x}) - \mu_b(\mathbf{x})) (\mu_a(\mathbf{y}) - \mu_b(\mathbf{y}))}{|\mathbf{x} - \mathbf{y}|}
\]
Underground test of gravity-related wave function collapse

PART 1: Introduction to the DP model

**PENROSE PROPOSAL**

R. Penrose, Gen. Relativ. Gravit. 28, 581–600 (1996).
R. Penrose, Found. Phys. 44, 557–575 (2014).

\[
\Delta E_{\text{DP}} = \frac{1}{G} \int \frac{\hbar}{\Delta E_{\text{DP}}} \left( g_a(r) - g_b(r) \right)^2 \\
= 4\pi G \int \mu_a(r) \int \mu_b(r') \\
\mu(x) \text{ cannot be point-like} \rightarrow \text{mass density with extension } R_0
\]

Proton: \( m = 10^{-27} \text{ Kg}, \quad R = 10^{-15} \text{ m} \)
Dust grain: \( m = 10^{-12} \text{ Kg}, \quad R = 10^{-5} \text{ m} \)

G. Ghirardi, R. Grassi, A. Rimini, Phys. Rev. A 42, 1057 (1990).
TESTING THE COLLAPSE MODELS

S. L. Adler and A. Bassi, Science 325, 275 (2009).
M. Carlesso, S. Donadi, L. Ferialdi, M. Paternostro, H. Ulbricht, A. Bassi. Nat. Phys. 18, 243 (2022).

Modified Schrödinger dynamics $\implies$ models can be tested against QM.

Two type of experiments:

1. **Interferometric** experiments: one searches for loss of coherences in spatial superposition;

2. **Non-Interferometric** experiments: collapse being random $\implies$ diffusive effects on the system e.g. heating that can be measured in principle.

Non-interferometric experiments provide better bounds, they do **not require** to prepare the systems in spatial superposition.
TEST OF THE MODEL

Interferometric tests, there are interesting proposals:

1) Optomechanical devices (W. Marshall, et al. Phys. Rev. Lett. 91, 130401 (2003)).

2) B.E.C. (R. Howl, R. Penrose, I. Fuentes, New J. Phys. 21, 043047 (2019)).

3) Experiments in space: no gravity ---> more time (MAQRO, CAL, etc..).
(A. Belenchia, et al. Nature 596, 32–34 (2021), G. Gasbarri et al. Commun. Phys. 4, 155 (2021))

…but they are still hard to perform.

Non interferometric tests does not require to create large superpositions!

1. LISA Pathfinder (B. Helou, B. Slagmolen, D. E. McClelland, and Y. Chen, Phys. Rev. D 95, 084054 (2017)).

2. Spontaneous heating (A. Vinante, H. Ulbricht., AVS Quantum Science 3, 4 (2021));

3. When the system is charged, the master equation implies radiation emission, even when it is well localized!
THEORETICAL CALCULATIONS: AN OUTLINE
S. Donadi, K. Piscicchia, C. Curceanu, L. Diósi, M. Laubenstein, and A. Bassi, Nat. Phys. 17, 74 (2021).

Radiation emission rate:

\[ \frac{d\Gamma_t}{d\omega_k} = \frac{k^2}{c} \sum_{\nu} \int d\Omega_k \frac{d}{dt} \langle a_{k\nu}^\dagger a_{k\nu} \rangle_t \]

Adjoint Master Equation:

\[ \frac{d}{dt} O(t) = \frac{i}{\hbar} [H, O(t)] + \int dQ \sum_{k,k'} \tilde{\Gamma}_{k,k'}(Q) \left( e^{-i\frac{Q \cdot x_k}{\hbar} O(t)} e^{i\frac{Q \cdot x_k}{\hbar}} - \frac{1}{2} \left\{ O(t), e^{-i\frac{Q \cdot x_k'}{\hbar}} e^{i\frac{Q \cdot x_k}{\hbar}} \right\} \right) . \]

\[ \tilde{\Gamma}_{n,n'}(Q) = \frac{4G \tilde{\mu}_n(Q) \tilde{\mu}^*_n(Q)}{\pi \hbar^2 Q^2} . \]

Perturbative treatment of the collapse and the EM interaction, calculation ….. and more calculation….. and finally:

\[ \frac{d\Gamma_t}{d\omega_k} = \frac{2 Ge^2 N^2 N_a}{3 \pi^{3/2} \varepsilon_0 c^3 R_0^3 \omega_k} \]

\[ 10^{-4} \lesssim \lambda \lesssim 10^{-1} \text{Å} \]

N = atomic number; Na = number of atoms; R₀ = mass density size.
AN EASIER APPROACH

L. Diósi and B. Lukács, Phys. Lett. A 181, 366–368 (1993); S. L. Adler. J. Phys. A 40, 2935–2957 (2007).
S. Donadi, K. Piscicchia, R. Del Grande, C. Curceanu, M. Laubenstein, and A. Bassi, Eur. Phys. J. C 81, 773 (2021).

\[ i\hbar \frac{d|\psi_t\rangle}{dt} = \left[ \hat{H} + V_{cm}(t) \right] |\psi_t\rangle \]

\[ V_{CSL}(t) = -\frac{\hbar\sqrt{\lambda}}{m_0} \int d\mathbf{y} \mu(\mathbf{y}) w(\mathbf{y}, t) \]

\[ \mathbb{E}[w(\mathbf{y}, t)w(\mathbf{y}', t')] = e^{-\frac{(\mathbf{y}-\mathbf{y}')^2}{4r^2c}} \delta(t - t') \]

\[ V_{DP}(t) = \sqrt{8\pi G\hbar} \int d\mathbf{y} \mu(\mathbf{y}) \xi(\mathbf{y}, t) \]

\[ \mathbb{E}[\xi(\mathbf{y}, t)\xi(\mathbf{y}', t')] = \frac{\delta(t - t')}{|\mathbf{y} - \mathbf{y}'|} \]

Lead to the same master equation of the collapse equations —> same radiation emission

\[ a_{cm}(t) = -\frac{\nabla V_{cm}}{m} \]

\[ P(t) = \frac{e^2}{6\pi\varepsilon_0 c^3} a_{cm}^2(t) = \int_0^{+\infty} d\omega \ \hbar \omega \frac{d\Gamma(t)}{d\omega} \]
**EXPERIMENTAL PART: SETUP AND MEASURED SPECTRUM**

S. Donadi, K. Piscicchia, C. Curceanu, L. Diósi, M. Laubenstein, and A. Bassi, Nat. Phys. 17, 74 (2021).

Data collected in 62 days:
- Grey line: measured, tot= 576
- Green line: simulated, tot= 506

**R_0 > 0.54 \times 10^{-10} \text{ m} (95\% \text{ c.l.})**

\[
\mu P \sim \left| \psi \right|^2 \\
\langle u^2 \rangle = \frac{B}{8\pi^2} R_0 = 0.05 \times 10^{-10} \text{ m}
\]

---

1, germanium crystal; 2, electric contact; 3, plastic insulator; 4, copper cup; 5, copper end-cup; 6, copper block and plate; 7, inner copper shield; 8, lead shield.

---

Underground test of gravity-related wave function collapse

**PART 2: experimental tests of the DP model**

---

I. J. Arnquist, et al. (Majorana Collaboration), Phys. Rev. Lett. **129**, 080401 (2022).
When the wavelength of the emitted photon is larger than 0.1 Å (~123 keV), the emission rate depends on the atomic structure in a non-trivial way. DP and CSL predictions become qualitatively different.
ARE ALL COLLAPSE DYNAMICS DIFFUSIVE?

- **Non-interferometric tests** provide a powerful way to test collapse theories.

- They are based on the **diffusive motion** induced by the noise responsible for collapse to systems.

TWO INTERESTING QUESTIONS

- Is this a feature of collapse models or something more general?

- Is it possible to have collapse without diffusion?

This is **interesting** from a **phenomenological** (non-interferometric tests) as well as from a **more fundamental** (diffusion is there or not?) point of view.
COLLAPSE DYNAMICS ARE DIFFUSIVE
S. Donadi, L. Ferialdi, A. Bassi, Phys. Rev. Lett. 130, 230202 (2023).

There is a dynamics describing the collapse. Under the assumptions:

1) No faster than light signalling. This implies there is a linear map for $\hat{\rho}$.
(N. Gisin, Helv. Phys. Acta 62.4, 363-371 (1989)).

2) The map describing the collapse being completely positive $\rightarrow$ Kraus form:
\[
\Phi[\hat{\rho}] = \sum_k \hat{A}_k \hat{\rho} \hat{A}_k^\dagger
\]
\[
\hat{A}_k = \hat{A}_k(\hat{q}, \hat{p})
\]

3) The map being space-translational covariance:
\[
e^{-\frac{i}{\hbar} \hat{p} \cdot \hat{x}} \Phi[\hat{\rho}] e^{\frac{i}{\hbar} \hat{p} \cdot \hat{x}} = \Phi \left[ e^{-\frac{i}{\hbar} \hat{p} \cdot \hat{x}} \hat{\rho} e^{\frac{i}{\hbar} \hat{p} \cdot \hat{x}} \right]
\]

Collapse in space $\implies$ Diffusion in momentum
Consider a graphene plate in a superposition:

\[ \tau_{obs} = 0.01s \quad L_{obs} = 25\mu m \]

\[ d = 4L \]
It seems reasonable to set an upper bound at $R_0 \lesssim 1 \text{ cm}$.
CONCLUSIONS

- In the DP model the spontaneous collapse is related to gravity;

- Under general assumptions the spontaneous collapse implies diffusive effects, that leads to emission of radiation;

- The experiments based on radiation emission set the strongest lower bound on the parameter $R_0$ (for markovian models). Moving to lower energies, the emission of the DP model is qualitatively different from that of other collapse models.

- The model does not collapse effectively some macroscopic systems, but the choice of these systems is arbitrary. Still, it seems reasonable to set an upper bound on $R_0$ such that $4 \times 10^{-10} \text{ m} \lesssim R_0 \lesssim 10^{-2} \text{ m}$. 
The Diósi-Penrose model and its experimental tests

Collaborators

COLLABORATORS

Radiation emission

Kristian Piscicchia
Centro Enrico Fermi (Rome)

Catalina Curceanu
Frascati Laboratories (Rome)

Lajos Diósi
Wigner Research Centre (Budapest)

Matthias Laubenstein
Gran Sasso Laboratories (Gran Sasso)

Angelo Bassi
Trieste University (Trieste)

Simone Manti
Frascati Laboratories (Rome)

Theoretical bound

Laria Figurato
Trieste University (Trieste)

Marco Dirindin
Trieste University (Trieste)

José Luis Gaona Reyes
Trieste University (Trieste)

Matteo Carlesso
Trieste University (Trieste)

Angelo Bassi
Trieste University (Trieste)
The Diósi-Penrose model and its experimental tests

FINANCIAL SUPPORT

THE GROUP AT QUEEN’S UNIVERSITY

THANKS
WHY DECOHERENCE IS NOT ENOUGH
A. Bassi and G.C. Ghirardi, Phys. Rep. 379, 257 (2003).

Why changing the dynamics at the level of the state vectors:

\[ |\psi\rangle = \frac{|+\rangle + |-\rangle}{\sqrt{2}} \quad \text{Collapse} \quad \{50\% |+\rangle, 50\% |-\rangle\} \]

The corresponding dynamics for the density matrix is:

\[ \rho = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \quad \text{Collapse} \quad \rho = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \]

Is the diagonalization of the density matrix a sufficient condition? NO.

\[ \rho = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \]

\[ \{50\% |+\rangle, 50\% |-\rangle\} \]

\[ \left\{ \frac{|+\rangle + |-\rangle}{\sqrt{2}}, \frac{|+\rangle - |-\rangle}{\sqrt{2}} \right\} \]

\[ \{50\% \frac{|+\rangle + |-\rangle}{\sqrt{2}}, 50\% \frac{|+\rangle - |-\rangle}{\sqrt{2}} \} \]
PART 3: Collapse $\Rightarrow$ diffusion

Sandro Donadi

COLLAPSE IMPLIES DIFFUSION

Under the hypothesis:
- The probability the state collapses is Poissonian in time;
- Translational covariance and momentum independence;
- Time of decay is that given by Penrose;

\[
\frac{d \rho(t)}{dt} = -\frac{i}{\hbar} [H, \rho(t)] + \mathcal{L} [\rho(t)]
\]

\[
\mathcal{L} [\rho(t)] = \int d\mathbf{Q} \int_0^\infty \sum_{j=1}^\infty \left[ \left( e^{\frac{\mathbf{i}}{\hbar} \mathbf{Q} \hat{\rho}(t) \mathbf{Q}^\dagger} \hat{\rho}(t) \right) \left( e^{\frac{-\mathbf{i}}{\hbar} \mathbf{Q} \hat{\rho}(t) \mathbf{Q}^\dagger} \hat{\rho}(t) \right) \right] \langle \mathbf{a} \left| \mathbf{Q} \hat{\rho}(t) \mathbf{Q}^\dagger \right| \mathbf{b} \rangle
\]

\[
\langle \mathbf{a} \left| \mathcal{L} [\hat{\rho}(t)] \right| \mathbf{b} \rangle = \left[ -\frac{1}{\tau_{\text{DP}} (\mathbf{a}, \mathbf{b})} \right] \langle \mathbf{a} \left| \hat{\rho}(t) \right| \mathbf{b} \rangle \quad \Rightarrow \quad \hat{\Gamma}(\mathbf{Q}) = \frac{4G}{\pi \hbar^2} \frac{|\hat{\mu}(\mathbf{Q})|^2}{\mathbf{Q}^2}
\]

\[
\frac{d \rho(t)}{dt} = -\frac{i}{\hbar} [H, \rho(t)] - \frac{4\pi G}{\hbar} \int d\mathbf{x} \int d\mathbf{y} \frac{1}{|\mathbf{x} - \mathbf{y}|} [\hat{\mu}(\mathbf{y}), [\hat{\mu}(\mathbf{x}), \rho(t)]]
\]
WHAT I WILL PRESENT

1. Motivations and introduction to the Diósi-Penrose (DP) model;

2. Experimental tests of the model.

3. Theoretical bounds on the DP model.

Main Take-Away

A) These are rival models to Quantum Mechanics, not reinterpretations: they can be tested with experiments.

B) Several kind of experiments, have been considered to set bounds on the parameters of the models.
COLLAPSE DYNAMICS ARE DIFFUSIVE
S. Donadi, L. Ferialdi, A. Bassi, Phys. Rev. Lett. 130, 230202 (2023).

- We require that collapse cannot be used to do signalling in EPR-like setups. This implies the map must be linear (N. Gisin, Helv. Phys. Acta 62.4, 363-371 (1989)).

- We also require the map implementing the collapse to be completely positive $\rightarrow$ Kraus form:

$$\Phi[\hat{\rho}] = \sum_k \hat{A}_k \hat{\rho} \hat{A}_k^\dagger$$

$$\hat{A}_k = \hat{A}_k(\hat{q}, \hat{p}) \quad \sum_k \hat{A}_k^\dagger \hat{A}_k = 1$$

- We require the map to be space-translational covariant:

$$e^{-i\frac{\hbar}{\mu} \hat{p} \cdot \hat{x}} \Phi[\hat{\rho}] e^{i\frac{\hbar}{\mu} \hat{p} \cdot \hat{x}} = \Phi \left[ e^{-i\frac{\hbar}{\mu} \hat{p} \cdot \hat{x}} \hat{\rho} e^{i\frac{\hbar}{\mu} \hat{p} \cdot \hat{x}} \right]$$
COLLAPSE DYNAMICS ARE DIFFUSIVE

1) The map changes the average momentum: just measure it.

2) The map does not change the average momentum: we focus on the diffusion along the $j$ direction is quantified by:

$$\Delta_j(\hat{\rho}) = \text{Tr} \left[ \hat{p}_j^2 \Phi[\hat{\rho}] \right] - \text{Tr} \left[ \hat{p}_j^2 \hat{\rho} \right]$$

If for all $\hat{\rho}$ we have $\Delta_j(\hat{\rho}) = 0$

$$\Phi[\hat{\rho}] = \sum_k \hat{A}_k(\hat{p}) \hat{\rho} \hat{A}^\dagger_k(\hat{p})$$

This map cannot collapse in position.

No diffusion $\Rightarrow$ No collapse $=$ Yes Collapse $\Rightarrow$ Yes diffusion

Moreover, if the map collapse in space plane waves which then experience diffusion, then all states experience diffusion.

**Conclusion:** Collapse in space comes together with diffusion in momentum.
Underground test of gravity-related wave function collapse

PART 3: recent developments

THEORETICAL BOUND, SOME EQUATIONS

\[
\Delta E(d) = 8\pi G m^2 \sum_{i,j=1}^{N} f(r_{ij}, R_0, d)
\]

\[
f(r_{ij}, R_0, d) := \frac{\text{erf} \left( \frac{r_{ij}}{2R_{\text{eff}}} \right)}{r_{ij}} - \frac{\text{erf} \left( \frac{|d-r_{ij}|}{2R_{\text{eff}}} \right)}{|d-r_{ij}|}
\]
EXCLUSION PLOT

M. Carlesso, S. Donadi, L. Ferialdi, M. Paternostro, H. Ulbricht, A. Bassi. Nat. Phys. 18, 243 (2022).

Collapse models and their experimental tests

PART 2: The CSL model and its experimental tests
Collapse is weak, it takes time for its effects to build up. In space a system can be let free for times of order of 100 s, not possible on Earth.

Interferometric test: masses up to $10^{11}$ amu

Non-Interferometric test measuring wave function spread: the effect goes as $t^3$!

- MAQRO: http://maqro-mission.org/
- COST action QTSpace: http://www.qtspace.eu/