Evaluation of Rock Joint Coefficients

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Abstract. A computer method for evaluation of rock joint coefficients is described and several applications are presented. The method is based on two absolute numerical indicators that are formed by means of the Fourier replicas of rock joint profiles. The first indicator quantifies the vertical depth of profiles and the second indicator classifies wavy character of profiles. The absolute indicators have replaced the formerly used relative indicators that showed some artificial behavior in some cases. This contribution is focused on practical computations testing the functionality of the newly introduced indicators.

1. Introduction

The concept of joint rock coefficients (JRC) was introduced by Barton and Choubey [1, 2] forty years ago. Yet, it has been used in geotechnical practice up to present days. Soon after introducing this concept, many authors tried to express the coefficients \( R_{\text{JRC}} \) in various analytical forms.

Thse and Cruden [3] investigated eight different parameters as potential candidates for JRC. These eight parameters include two classical parameters \( R_a \) and \( R_q \) taken from the arsenal of roughness metrology of metallic surfaces, and root mean square values \( Z_2 \), \( Z_3 \) computed from the first and second derivatives of the measured profiles, respectively. The eight parameters also include the parameter \( Z_4 \) that represents the percentage excess of the distance measured along the profile where the slope is positive over the distance where the slope is negative; two statistical parameters called mean square values (MSV) and the autocorrelation function (ACF); and, finally, the structure function (SF). Thse and Cruden [3] computed all eight parameters from the measured two-dimensional (2D) profiles \( y(x) \) within the length interval \((0, L)\) according to the following formulae

\[
R_a = \frac{1}{L} \int_0^L |y| \, dx, \quad R_q = \frac{1}{L} \int_0^L y^2 \, dx \quad \text{(root mean square)} \tag{1}
\]

\[
Z_2 = \frac{1}{L} \left[ \left( \frac{dy}{dx} \right)^2 \right] \, dx, \quad Z_3 = \frac{1}{L} \left[ \left( \frac{d^2 y}{dx^2} \right)^2 \right] \, dx, \quad Z_4 = \frac{1}{L} \left[ \sum (x_i)_{\text{positive}} - \sum (x_i)_{\text{negative}} \right] \tag{2}
\]

\[
\text{MSV} = \frac{1}{L} \int_0^L y^2 \, dx, \quad \text{ACF} = \frac{1}{L} \int_0^L y(x) \cdot y(x + \Delta x) \, dx, \quad \text{SF} = \frac{1}{L} \left[ |y(x) - y(x + \Delta x)|^2 \right] \, dx \tag{3}
\]
Thse and Cruden [3] found that only two of the eight parameters, namely $Z_2$ and $SF$, are strongly correlated with Barton’s joint roughness coefficient $JRC$, and suggested the corresponding regression relations

$$JRC = 32.2 + 32.47 \log(Z_2), \quad R_{JRC} = 37.28 + 16.58 \log(SF)$$ (4)

Maerz et al. [4] measured large areas of joint rock surfaces using an interesting technique called shadow profilometry and, as a descriptor of the joint roughness coefficient, proposed the so-called roughness profile index $R_p$, related to the so-called micro-average angle $i$, defined as an average of inclination angles $I_j$ between adjacent points along the sampling line

$$i = \frac{1}{n} \sum_{j=1}^{n} |I_j| = \arccos \left( \frac{1}{R_p} \right)$$ (5)

Maerz et al. [4] also suggested a regression pattern $R_{JRC}(R_p)$ in the following form

$$JRC = 411(R_p - 1)$$ (6)

Hong et al. [5], [6] noticed that the conventional methods for digitized surface profiles usually underestimate the surface roughness parameters due to aliasing and the undetected dead zones of surface texture.

Lee et al. [7] proposed the fractal dimension as a tool for an objective quantification of roughness profiles of rock discontinuities. They computed dimension $D$ from two-dimensional profiles by the yard-stick method, and suggested a regression equation as follows:

$$JRC = -0.87804 + 37.7844 \left( \frac{D-1}{0.015} \right) - 16.9304 \left( \frac{D-1}{0.015} \right)^2$$ (7)

Kulatilake et al. [8] used a spectral function $S(f)$

$$S(f) = K_s f^{5-2D}$$ (8)

where $f$ is the spatial frequency of the two-dimensional profile and $K_s$ is a constant. They classified the roughness of joint surfaces by the three parameters $(D, I, K_s)$; the symbol $I$ represented the inclination angle of the asperities.

Although many of the suggested regression patterns mentioned above provide good results, the Barton visual assessment of joint roughness coefficients remains the most frequently used method in geotechnical practice. The visual comparative method proposed by Barton [1] employs the measured roughness coefficient $JRC$ assigned to each standard database 2D profile. The measured $JRC$ values represent one great advantage of this method, since measured data are usually closer to reality than those generated by theoretical models. However, visual comparison of surfaces is rather subjective method. For this reason, it would be useful to develop a fully computerized objective procedure that
would replace the subjective visual comparison of the investigated rock joint profiles with Barton's standard profiles. Developing such a computerized procedure is a goal of the present contribution.

2. Computer method
The computer method for assessing the rock joint profiles requires the rock joint profiles to be digitally scanned in the form of a discrete function \( f(x, y) \). This discrete function can be fitted by the Fourier partial sum \( F_N(x, y) \) as follows

\[
f(x, y) \approx F_N(x, y) = \sum_{k, n=0}^{N-1} \tau_{kn} \left( a_{kn} \cos \frac{k \pi x}{p} \cos \frac{n \pi y}{q} + b_{kn} \sin \frac{k \pi x}{p} \cos \frac{n \pi y}{q} + c_{kn} \cos \frac{k \pi x}{p} \sin \frac{n \pi y}{q} + d_{kn} \sin \frac{k \pi x}{p} \sin \frac{n \pi y}{q} \right)
\]

where

\[
\Omega = \{ x \in (-p, +p) , y \in (-q, +q) \}
\]

\[
a_{kn} = \frac{1}{pq} \int_{\Omega} f(x, y) \cos \frac{k \pi x}{p} \cos \frac{n \pi y}{q} \, dx \, dy
\]

\[
b_{kn} = \frac{1}{pq} \int_{\Omega} f(x, y) \sin \frac{k \pi x}{p} \cos \frac{n \pi y}{q} \, dx \, dy
\]

\[
c_{kn} = \frac{1}{pq} \int_{\Omega} f(x, y) \cos \frac{k \pi x}{p} \sin \frac{n \pi y}{q} \, dx \, dy
\]

\[
d_{kn} = \frac{1}{pq} \int_{\Omega} f(x, y) \sin \frac{k \pi x}{p} \sin \frac{n \pi y}{q} \, dx \, dy
\]

\[
\tau_{kn} = \begin{cases} 
1 & (k > 0, n > 0) \\
1/2 & k = 0, n > 0 \text{ or } k > 0, n = 0 \\
1/4 & k = n = 0 
\end{cases}
\]

The symbol \( N \) defines the upper bound \( (N-1) \) for the subscripts \( n, k \), and restricts the number of terms in the Fourier partial sum (9).

To characterize wavy character of profiles, the Fourier matrix \( M_N(k, n) \) is introduced along with two indicators \( S(N) \) and \( H(N) \)

\[
M_N(k, n) = \sqrt{(\tau_{kn} \cdot a_{kn})^2 + (\tau_{kn} \cdot b_{kn})^2 + (\tau_{kn} \cdot c_{kn})^2 + (\tau_{kn} \cdot d_{kn})^2}
\]

\[ (16) \]
\[ S(N) = \sum_{k=0}^{N-1} \sum_{n=0}^{N-1} |M_N^{(o)}(k,n) - M_N(k,n)| \]  
(similarity indicator)  
(17)

\[ H(N) = |R^{(o)}(N) - R(N)| \]  
(height indicator)  
(18)

\[ R(N) = \sqrt{\frac{1}{K \cdot L} \sum_{i=1}^{K} \sum_{j=1}^{L} [f(x_i,y_j) - F_N^{(low)}(x_i,y_j)]^2} \]  
(horizontal profile width)  
(19)

where  \( K \cdot L \) is a pixel resolution of the used digital images created during scanning procedure. Rock joints usually have a certain wavy base level  \( F_N^{(low)}(x_i,y_j) \) which is not a part of their inherent surface structure and thus it should be removed from the scanned profile, i.e.  \( f(x_i,y_j) - F_N^{(low)}(x_i,y_j) = f^{(corr)}(x_i,y_j) \). So that the Fourier coefficients (11) - (15) as well as the matrix (16) are computed from the corrected profile  \( f^{(corr)}(x_i,y_j) \).

Both the absolute indicators  \( S(N) \) and  \( H(N) \) were discussed in our separate conference contribution [9] and thus the tests concerning their capability of classifying dynamical conformity of profiles can be found there.

3. Results and discussions

Eight specimens of rock surfaces have been gathered. Two of them (Surface 1 and Surface 2) have been chosen as tested surfaces whereas the remaining seven have served as database surfaces. In the first case, when Surface 1 was tested, the database surfaces consisted of surfaces nos. 2 - 8. In the second case, when Surface 2 was tested, the database surfaces consisted of surfaces nos. 1, 3 - 8. The resulted values of both the indicators  \( S \) and  \( H \) have been plotted graphically in figure 1. The bold arrows in this figure marks the total minima of the graphs. The minima indicate couples of the most similar profiles (investigated surface versus database surface). For example, the minimum of  \( S \) indicator associated with Surface 1 is located at the database no. 4 which means that Surface 1 and database surface 4 represent the two most similar surfaces among all the possible couples of surfaces containing Surface 1.

![Figure 1. Resulted values of the similarity and height indicators related to selected two profiles (Surface 1, Surface 2) and database profiles (1 - 8)](image-url)
As seen from figure 1, both the indicators $S$ and $H$ determines the same couples of the most similar surfaces, namely Surface 1 versus database surface no. 4 and Surface 2 versus database surface no. 8. By expecting visually all possible couples of surfaces, we came to the same conclusions. This fact might support the functionality of both the indicators. However, the identical results of these indicators is not a quite natural matter. Occasionally, different results may appear. This is understandable since surface properties related to profile heights and profile shapes are not tightly correlated and thus their various combinations may occur. For example, two profiles may have comparable heights but quite different wavy shapes. In such a case the indicators $H$ and $S$ will inevitably determine different couples of the most similar surfaces. The question is how such a situation should be solved. The answer is easy. Since the comparison is done between an investigated specimen and the standard database profiles, whose $JRC$ are known, the relevant couple will be that associated with the smaller database $JRC$. Such a solution will always be at the side of higher safety.

4. Conclusions
The computerized comparison of rock joint profiles with standard database profile is an objective procedure based on numerical indicators. In this contribution, two indicators of wavy shape and profile height have been introduced and subjected to preliminary tests to verify their functionality. These preliminary tests have indicated that employment of the two indicators could be prospective but a further tests are necessary for a final decision.

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