An optical diode and magnifier from a general function photonic crystals

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We have presented a new general function photonic crystals (GFPCs), which refractive index is a function of space position. Based on Fermat principle, we achieve the motion equations of light in one-dimensional general function photonic crystals, and calculate its transfer matrix. In this paper, we choose the line refractive index function for two mediums \( A \) and \( B \), and obtain new results: (1) when the line function of refractive indexes is up or down, the transmissivity can be far larger than 1. (2) when the refractive indexes function increase or decrease at the direction of incident light, the light intensity should be magnified or weaken, which can be made light magnifier or attenuator. (3) The GFPCs can also be made optical diode. The new general function photonic crystals can be applied to design more optical instruments.

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1. Introduction

Photonic crystals are artificial materials with periodic variations in refractive index that are designed to affect the propagation of light [1-4]. An important feature of the photonic crystals is that there are allowed and forbidden ranges of frequencies at which light propagates in the direction of index periodicity. Due to the forbidden frequency range, known as photonic band gap (PBG) [5-6], which forbids the radiation propagation in a specific range of frequencies. The existence of PBGs will lead to many interesting phenomena, e.g., modification of spontaneous emission [7-9] and photon localization [10]. Thus numerous applications of photonic crystals have been proposed in improving the performance of optoelectronic and microwave devices such as high-efficiency semiconductor lasers, right emitting diodes, wave guides, optical filters, high-Q resonators, antennas, frequency-selective surface, optical limiters and amplifiers [11-18].

In Ref. [19], we have proposed special function photonic crystals, which the medium refractive index is the function of space position, but the function value of refractive index is equal at two endpoints of every medium \( A \) and \( B \). In this paper, we present a new general function photonic crystals (GFPCs), which refractive index is an arbitrary function of space position. Unlike conventional photonic crystals (PCs), which structure grow from two materials, \( A \) and \( B \), with different dielectric constants \( \varepsilon_A \) and \( \varepsilon_B \). Firstly, we give the motion equation of light in one-dimensional GFPCs based on Fermat principle. Secondly, we calculate the transfer matrix for the GFPCs, which is different from the transfer matrix of the conventional PCs. Thirdly, we give the band gap structure and transmissivity. Finally, we choose the linearity refractive index functions for two medium \( A \) and \( B \), and give the light field distribution in the GFPCs. We obtain some new results: (1) when the line function of refractive indexes is up, the transmissivity can be far larger than 1. (2) when the line function of refractive indexes is down, the transmissivity can be far smaller than 1. (3) when the refractive indexes function increase at the incident direction of light, the light intensity should be magnified, which can be made light magnifier. (4) when the refractive indexes function decrease at the incident direction of light, the light intensity should be weaken, which can be made light attenuator. (5) The GFPCs can be made optical diode, which transmits light from an input to an output, but not in reverse direction.

2. The light motion equation in general function photonic crystals

For the general function photonic crystals, the medium refractive index is a periodic function of the space position, which can be written as \( n(z) \), \( n(x, z) \) and \( n(x, y, z) \) corresponding to one-dimensional, two-dimensional and three-dimensional function photonic crystals. In the following, we shall deduce the light motion equations of the one-dimensional general function photonic crystals, i.e., the refractive index function is \( n = n(z) \), meanwhile motion path is on \( xz \) plane. The incident light wave strikes plane interface point \( A \), the curves \( AB \) and \( BC \) are the path of incident and reflected light respectively, and they are shown in FIG. 1.

The light motion equation can be obtained by Fermat principle, it is

\[
\delta \int_A^B n(z) ds = 0.
\]

(1)

In the two-dimensional transmission space, the line element \( ds \) is

\[
ds = \sqrt{(dx)^2 + (dz)^2} = \sqrt{1 + \frac{2}{z^2} dx},
\]

(2)
where \( \dot{z} = \frac{dz}{dt} \), then Eq. (1) becomes

\[
\delta \int_A^B n(z) \sqrt{1 + (\dot{z})^2} dx = 0. \tag{3}
\]

The Eq. (3) change into

\[
\int_A^B \left( \frac{\partial(n(z) \sqrt{1 + \dot{z}^2})}{\partial z} \right) \delta z + \frac{\partial(n(z) \sqrt{1 + \dot{z}^2})}{\partial z} \delta z dx = 0, \tag{4}
\]

At the two end points A and B, their variation is zero, i.e., \( \delta z(A) = \delta z(B) = 0 \). For arbitrary variation \( \delta z \), the Eq. (4) becomes

\[
\frac{dn(z)}{dz} \sqrt{1 + \dot{z}^2} - \frac{dn(z)}{dz} \dot{z}^2 \sqrt{1 + \dot{z}^2} - \frac{\dot{z} \dot{\delta z}}{1 + \dot{z}^2} = 0, \tag{5}
\]

simplify Eq. (5), we have

\[
\frac{dn(z)}{n(z)} = \frac{\dot{z} \dot{\delta z}}{1 + \dot{z}^2}. \tag{6}
\]

The Eq. (6) is light motion equation in one-dimensional function photonic crystals.

### 3. The transfer matrix of one-dimensional general function photonic crystals

In this section, we should calculate the transfer matrix of one-dimensional general function photonic crystals. In fact, there is the reflection and refraction of light at a plane surface of two media with different dielectric properties. The dynamic properties of the electric field and magnetic field are contained in the boundary conditions: normal components of \( D \) and \( B \) are continuous; tangential components of \( E \) and \( H \) are continuous. We consider the electric field perpendicular to the plane of incidence, and the coordinate system and symbols as shown in FIG. 2.

On the two sides of interface I, the tangential components of electric field \( E \) and magnetic field \( H \) are continuous, there are

\[
\begin{cases}
E_0 = E_I = E_{i1} + E'_{i2} \\
H_0 = H_I = H_{i1} \cos \theta_i^I - H_{r2} \cos \theta_i^I .
\end{cases} \tag{7}
\]

On the two sides of interface II, the tangential components of electric field \( E \) and magnetic field \( H \) are continuous, and give

\[
\begin{cases}
E_{II} = E'_{I2} = E_{i2} + E_{r2} \\
H_{II} = H'_I = H_{i2} \cos \theta_i^I - H_{r2} \cos \theta_i^I .
\end{cases} \tag{8}
\]

the electric field \( E_{i1} \) is

\[
E_{i1} = E_{10} e^{i(k_{xA} + k_{zA})} |_{z=0} = E_{10} e^{i \theta_{iA} x_A}, \tag{9}
\]

and the electric field \( E_{i2} \) is

\[
E_{i2} = E_{10} e^{i(k_{xB} + k_{zB})} |_{z=b} = E_{10} e^{i \theta_{iB} x_B + \cos \theta_i^B}. \tag{10}
\]

Where \( x_A \) and \( x_B \) are \( x \) component coordinates corresponding to point \( A \) and point \( B \). We should give the relation between \( E_{i2} \) and \( E_{i1} \). By integrating the two sides of Eq. (6), we can obtain the coordinate component \( x_B \) of point \( B \)

\[
\int_{n(0)}^{n(z)} \frac{dn(z)}{n(z)} = \int_{k_0}^{k_z} \frac{\dot{z} \dot{\delta z}}{1 + \dot{z}^2}, \tag{11}
\]

to get

\[
k_z^2 = (1 + k_0^2) \left( \frac{n(z)}{n(0)} \right)^2 - 1, \tag{12}
\]

and

\[
dx = \frac{dz}{\sqrt{(1 + k_0^2) \left( \frac{n(z)}{n(0)} \right)^2 - 1}}. \tag{13}
\]
where \( k_0 = \cot \theta^I \) and \( k_z = \frac{\omega}{c} \). From Eq. (12), there is \( n(z) > n(0) \sin \theta^I \) and the coordinate \( x_B \) is

\[
x_B = x_A + \int_0^b \frac{dz}{\sqrt{1 + k_0^2 n(z)^2}}.
\]

(14)

where \( b \) is the medium thickness of FIG. 1 and FIG. 2. By substituting Eqs. (9) and (14) into (10), and using the equality

\[
n(0) \sin \theta^I = n(b) \sin \theta^I,
\]

we have

\[
E_{i2} = E_{a1} e^{i\delta_b},
\]

(16)

where

\[
\delta_b = \frac{\omega}{c} n_b(b) (\cos \theta^I_b + \sin \theta^I_b) \int_0^b \frac{dz}{\sqrt{\frac{n_b^2(z)}{n_0^2} \sin^2 \theta^I_a - 1}}.
\]

(17)

and similarly

\[
E'_{e2} = E_{a2} e^{i\delta_b}.
\]

Substituting Eqs. (16) and (18) into (7) and (8), and using \( H = \sqrt{\frac{\omega}{\mu_0}} n_0 E \), we obtain

\[
\begin{pmatrix}
E_I \\
H_I
\end{pmatrix} = M_B
\begin{pmatrix}
E_{II} \\
H_{II}
\end{pmatrix},
\]

(19)

where

\[
M_B = \begin{pmatrix}
\cos \delta_b & i \sin \delta_b \\
-i n_b(0) \sqrt{\frac{n_0}{n_b}} \cos \theta^I_b \sin \delta_b & \sqrt{\frac{n_0}{n_b}} n_b(0) \cos \theta^I_b \cos \delta_b
\end{pmatrix}
\]

(20)

The Eq. (20) is the transfer matrix \( M \) in the medium of FIG. 1 and FIG. 2. By refraction law, we can obtain

\[
\sin \theta^I = \frac{n_0}{n(0)} \sin \theta^0, \quad \cos \theta^I = \sqrt{1 - \frac{n_0^2}{n^2(0)} \sin^2 \theta^I}.
\]

(21)

where \( n_0 \) is air refractive index, and \( n(0) = n(z)|_{z=0} \). Using Eqs. (15) and (21), we can calculate \( \cos \theta^I \).

4. The transmissivity and light field distribution of one-dimensional general function photonic crystals

In section 3, we obtain the \( M \) matrix of the half period. We know that the conventional photonic crystals is constituted by two different refractive index medium, and the refractive indexes are not continuous on the interface of the two mediums. We could devise the one-dimensional general function photonic crystals structure as follows: in the first half period, the refractive index distributing function of medium \( B \) is \( n_b(z) \), and in the second half period, the refractive index distributing function of medium \( A \) is \( n_a(z) \), corresponding thicknesses are \( b \) and \( a \), respectively. Their refractive indexes satisfy condition \( n_b(0) \neq n_a(0) \), their structure are shown in FIG. 3, and FIG. 4. The Eq. (20) is the half period transfer matrix of medium \( B \). Obviously, the half period transfer matrix of medium \( A \) is

\[
M_A = \begin{pmatrix}
\cos \delta_a & -i \sin \delta_a \\
-i n_a(0) \sqrt{\frac{n_0}{n_a}} \cos \theta^I_a \sin \delta_a & \sqrt{\frac{n_0}{n_a}} n_a(0) \cos \theta^I_a \cos \delta_a
\end{pmatrix}
\]

(22)

where

\[
\delta_a = \frac{\omega}{c} n_a(a) [\cos \theta^I_a] a \cos \theta^I_a \int_0^a \frac{dz}{\sqrt{\frac{n_a^2(z)}{n_0^2} \sin^2 \theta^I_a - 1}},
\]

(23)

\[
\cos \theta^I_a = \sqrt{1 - \frac{n_0^2}{n_a^2(0)} \sin^2 \theta^0_a},
\]

(24)

and

\[
\sin \theta^I_a = \frac{n_0}{n_a(a)} \sin \theta^0_a.
\]

(25)
\[
\cos \theta^I = \sqrt{1 - \frac{n_0^2}{n_2^2(a)}} \sin^2 \theta^0. \quad (26)
\]

In one period, the transfer matrix \( M \) is
\[
M = M_B \cdot M_A
\]
\[
= \begin{pmatrix}
\cos \delta_b & \frac{-i \sin \delta_b}{\sqrt{n_0(a) \cos \theta^I}} \\
-i n_0(0) \sqrt{\frac{\mu_0}{\rho_0}} \cos \theta^I \sin \delta_b & \cos \delta_a \\
\cos \delta_a & \frac{-i \sin \delta_a}{\sqrt{n_0(a) \cos \theta^{II}}} \\
-i n_0(0) \sqrt{\frac{\mu_0}{\rho_0}} \cos \theta^{II} \sin \delta_a
\end{pmatrix} \quad (27)
\]

The form of the GFPCs transfer matrix \( M \) is more complex than the conventional PCs. The angle \( \theta^I, \theta^{II}, \psi^I \) and \( \psi^{II} \) are shown in Fig. 4. The characteristic equation of GFPCs is
\[
\begin{pmatrix}
E_1 \\
H_1
\end{pmatrix} = M_1 M_2 \cdots M_N \begin{pmatrix} E_{N+1} \\ H_{N+1} \end{pmatrix} = M_{N+1} \begin{pmatrix} E_{N+1} \\ H_{N+1} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} E_{N+1} \\ H_{N+1} \end{pmatrix} \quad (28)
\]

Where \( N \) is the period number. With the transfer matrix \( M \) (Eq. (28)), we can obtain the transmission and reflection coefficient \( t \) and \( r \), and the transmissivity and reflectivity \( T \) and \( R \), they are
\[
t = \frac{E_{N+1}}{E_1} = \frac{2\eta_0}{A\eta_0 + B\eta_0 \eta_{N+1} + C + D\eta_{N+1}}. \quad (29)
\]
\[
r = \frac{E_r}{E_1} = \frac{A\eta_0 + B\eta_0 \eta_{N+1} - C - D\eta_0}{A\eta_0 + B\eta_0 \eta_{N+1} + C + D\eta_0}. \quad (30)
\]

The relation between transmissivity and frequency corresponding to the up line function of refractive indexes (Fig. 5(a))
\[
T = \frac{t \cdot t^*}{}, \quad (31)
\]
\[
R = r \cdot r^*. \quad (32)
\]

Where \( \eta_0 = \eta_0{N+1} = \sqrt{\frac{\mu_0}{\rho_0}} \). In the following, we give the electric field distribution of light in the one-dimensional GFPCs. The propagation figure of light in one-dimensional GFPCs is shown in Fig. 9. From Eq. (28), we have
\[
\begin{pmatrix} E_1 \\ H_1 \end{pmatrix} = M_1(d_1) M_2(d_2) \cdots M_{N-1}(d_{N-1}) M_N(\Delta z_N) \begin{pmatrix} E_N(d_1 + d_2 + \cdots + d_{N-1} + \Delta z_N) \\ H_N(d_1 + d_2 + \cdots + d_{N-1} + \Delta z_N) \end{pmatrix} \quad (33)
\]
where \( d_1, d_2 \) are the thickness of first and second period, respectively, \( \Delta z_N \) is the propagation distance of light in the \( N \)-th period, \( E_1 \) and \( H_1 \) are the intensity of incident electric field and magnetic field, and \( E_N(d_1 + d_2 + \cdots + d_{N-1} + \Delta z_N) \) and \( H_N(d_1 + d_2 + \cdots + d_{N-1} + \Delta z_N) \) are the intensity of the \( N \)-th period electric field and magnetic field. The Eq. (34) can be written as
\[
\begin{pmatrix} E_1 \\ H_1 \end{pmatrix} = M_1^{-1}(\Delta z_N) (A(\Delta z_N) B(\Delta z_N) C(\Delta z_N) D(\Delta z_N)) \begin{pmatrix} E_1 \\ H_1 \end{pmatrix} \quad (34)
\]

the electric field \( E_1 \) and magnetic field \( H_1 \) can be written as
\[
E_1 = E_{i1} + E_{r1} = (1 + r) E_{i1}, \quad (35)
\]
\[ H_1 = H_{i1} \cos \theta_i^0 - H_{r1} \cos \theta_r^0 = \sqrt{\frac{\varepsilon_0}{\mu_0}} \cos \theta_i^0 (1 - r) E_{i1}. \]  

From Eqs. (34)-(36), we can obtain the ratio of the electric field \( E_N(d_1 + d_2 \cdots + d_{N-1} + \Delta z_N) \) within the GFPCs to the incident electric field \( E_{i1} \), it is

\[
\frac{E_N(d_1 + d_2 \cdots + d_{N-1} + \Delta z_N)}{E_{i1}} = |A(\Delta z_N)(1 + r) + B(\Delta z_N)\sqrt{\frac{\varepsilon_0}{\mu_0}} \cos \theta_i^0 (1 - r)|^2. 
\]

5. Numerical result

In this section, we report our numerical results of transmissivity. We consider refractive indexes of the linearity functions in a period, it is

\[ n_b(z) = n_b(0) + \frac{n_b(b) - n_b(0)}{b} z, \quad 0 \leq z \leq b, \]  

\[ n_a(z) = n_a(0) + \frac{n_a(a) - n_a(0)}{a} z, \quad 0 \leq z \leq a, \]

Eqs. (38) and (39) are the line refractive indexes distribution functions of two half period mediums \( B \) and \( A \). When the endpoint values \( n_b(0), n_b(b), n_a(0) \) and \( n_a(a) \) are all given, the line refractive index functions \( n_b(z) \) and \( n_a(z) \) are ascertained. The main parameters are: the half period thickness \( b \) and \( a \), the starting point refractive indexes \( n_b(0) \) and \( n_a(0) \), and end point refractive indexes \( n_b(b) \) and \( n_a(a) \), the optical thickness of the two mediums are equal, i.e., \( n_b(0)b = n_a(0)a \), the incident angle \( \theta_i^0 = 0 \), the center frequency \( \omega_0 = 1.215 \times 10^{15} \) Hz, the thickness \( b = 280 \) nm, \( a = 165 \) nm and the period number \( N = 16 \).

In FIG. 5(a), we take \( n_b(0) = 1.38, n_b(b) = 1.9 \) for the medium \( B \), and \( n_a(0) = 2.35, n_a(a) = 4.25 \) for the medium \( A \), which are the up line function of refractive indexes. In FIG. 5(b), we take \( n_b(0) = 4.25, n_b(b) = 2.35 \) for the medium \( B \), and \( n_a(0) = 1.9, n_a(a) = 1.38 \) for the medium \( A \), which are the down line function of refractive indexes. By the refractive indexes function, we can calculate the transmissivity. With the FIG. 5(a) and 5(b), we obtain the transmissivity distribution in FIG. 6 and FIG. 7. From FIG. 6 and FIG. 7, we obtain the results: (1) when the line function of refractive indexes is up, the transmissivity can be far larger than 1 (\( T \) maximum is 10^6). (2) when the line function of refractive indexes is down, the transmissivity can be far smaller than 1 (\( T \) maximum is 10^-7). In the following, we shall study the light field distribution of the one-dimensional GFPCs for the light of positive and negative incident. The positive incident is shown in FIG. 9 and the negative incident is shown in FIG. 10. The refractive indexes line function of positive incident is in FIG. 5(a), and then the refractive indexes line function of negative incident is in FIG. 5(b). The FIG. 5(a) is the function of line increasing, and the FIG. 5(b) is the function of line decreasing. From Eq. (37), we can calculate the electric field distribution of light in the GFPCs. For the positive incident, the electric field distribution is shown in FIG. 11(a) and FIG. 11(b) is the electric field distribution for the negative incident. From FIG. 11(a) and (b), we can obtain the result: (1) when the refractive indexes function increase at the incident direction of light, the light intensity should be boosted up or magnified, which can be made light magnifier (magnifying multiple 10^8). (2) when the refractive indexes function decrease at the incident direction of light, the light intensity should be

![FIG. 7: The relation between transmissivity and frequency corresponding to the down line function of refractive indexes (FIG. 5(b)).](image)

![FIG. 8: The input and output light in the GFPCs.](image)

![FIG. 9: The light positive incident to the GFPCs.](image)
FIG. 10: The light negative incident to the GFPCs.

FIG. 11: The light distribution in the GFPCs. Figure (a) is corresponding to the positive incident (FIG. 9), and Figure (b) is corresponding to the positive incident (FIG. 10).

6. Conclusion

In summary, We have theoretically investigated a new general function photonic crystals (GFPCs), which refractive index is a function of space position. Based on Fermat principle, we achieve the motion equations of light in one-dimensional general function photonic crystals, and calculate its transfer matrix. We choose the line refractive index function for two mediums $A$ and $B$, and obtain some results: (1) when the line function of refractive indexes is up, the transmissivity can be far larger than 1. (2) when the line function of refractive indexes is down, the transmissivity can be far smaller than 1. (3) when the refractive indexes function increase at the direction of incident light, the light intensity should be magnified, which can be made light magnifier. (4) when the refractive indexes function decrease at the direction of incident light, the light intensity should be weaken, which can be made light attenuator. (5) The GFPCs can be made light diode. The new general function photonic crystals can be applied to design more optical instruments.

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