DENSITY MATRIX KINETIC EQUATION DESCRIBING A PASSAGE OF FAST ATOMIC SYSTEMS THROUGH MATTER

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Abstract. The quantum-mechanical consideration of a passage of fast dimesoatoms through matter is given. A set of quantum-kinetic equations for the density matrix elements describing their internal state evolution is derived. It is shown that probabilistic description of internal dynamics of hydrogen-like atoms is impossible even at sufficiently low energies because of the “accidental” degeneracy of their energy levels.

1 Introduction

In the last years interest in studying elementary hadronic atoms (hydrogen-like atoms formed by two oppositely charged hadrons) has been growing [1].

The basic properties of these atoms are governed by the Coulomb interaction between their constituents. The strong hadron-hadron interaction causes only shifts of energy levels and increase of widths of s-states of hadronic atoms which are related in a simple way to the values of strong scattering lengths of hadrons.

Thus, the study of hadronic atoms provides valuable information about the low-energy hadronic interactions. At present a number of ongoing or planned experiments are aimed to measure strong energy level shifts and (or) widths of the ground states of the atoms \( A_{\pi^-p} \), \( A_{K^-p} \), \( A_{\pi^-\pi^+} \), \( A_{\pi^\pm K^\mp} \) with a high precision [2, 3, 4].

The \( A_{\pi^-p} \) and \( A_{K^-p} \) atoms can be produced at rest in the laboratory frame, and a well-developed technique of X-ray spectroscopy can be applied for a precise measurement of their radiative spectra.

The study of the atoms \( A_{\pi^-\pi^+} \) and \( A_{\pi^\pm K^\mp} \) (dimesoatoms) is a much more complicate problem because they practically cannot be produced at rest. The only available way to produce a reasonable amount of dimesoatoms is the interaction of high-energy projectiles with fixed nuclear targets. The dimesoatoms produced in this way move with high (relativistic, as a rule) velocities which determines the choice of methods for investigation of their properties.

For example, the method for measurement of \( \tau_0 \) (the pionium ground state lifetime) used in the DIRAC experiment at CERN [4] is based on comparison of the observed rate of \( \pi^+\pi^- \)-pairs from the break-up of pionium atoms in the Coulomb field of target atoms with the theoretical calculations of this rate which include the \( \tau_0 \) as a parameter.

It is clear that the error in the value of \( \tau_0 \) resulting from such indirect measurement includes not only experimental errors but also uncertainties of the approx-
imations used in the theoretical description of the pionium production and their subsequent passage through the target matter.

Thus, any improvement of the theoretical treatment of the problems related to the DIRAC experiment helps to reduce the resulting error of $\tau_0$ measurement. Some important results in this direction have been obtained in [5, 6, 7, 8] dealing with the theoretical description of the production of dimesoatoms and their interactions with separate target atoms.

In this paper, we will touch on a problem of describing the internal dynamics of dimesoatoms moving through matter and interacting with its atoms. It is commonly supposed [9] that the dimesoatomic dynamics can be described with the help of a set of kinetic equations for the probabilities $P_i(z)$ to find the dimesotom in the definite quantum state $|i\rangle$ at the distance $z$ from the production point.

Although the form of these equations is similar to the Pauli quantum kinetic equations [10], we will refer to them as the classical (probabilistic) ones because they ignore interference between different states of dimesoatoms during their passage through the matter which is pure quantum effects.

It is well known that these interference effects play a decisive role in giving rise to the so-called superpenetration phenomena [11] at ultrarelativistic energies. However, at the typical dimesoatom energies in DIRAC most interference effects are strongly suppressed by the time-formation effects.

But in the case of dimesoatoms some of interference effects occur even at sufficiently low energies because of accidental degeneracy of energy levels of hydrogen-like atoms [12]. Since these effects cannot be treated in the classical approach of the paper [9], one have to use a more sophisticated approach for consideration of internal dimesoatom dynamics based on the density-matrix description of quantum systems.

The main goal of this paper is to derive a set of equations for the density matrix elements describing the internal dynamics of multilevel atoms moving through matter and interacting with its atoms. Although for simplicity the derivation is restricted to the case of elementary (two-particle) atoms, the method can be easily generalized to a more complicated case of arbitrary composed system.

The plan of the paper is as follows. In Section 2, the main approximations are described. In Section 3, a set of equations for density matrix elements is derived. In Section 4, the correspondence between density-matrix approach and probabilistic approach of the paper [9] is discussed. In Section 5, a brief summary of the main results is given.

## 2 Main approximations

Any discussion of relativistic dimesotom interactions faces, first of all, the problem of the relativistic description of a bound system. Application of usual methods based on the Bethe-Salpeter equation [13] would lead to overcomplication of the technical part of the problem under consideration, which may mask the main goal of this work.

To avoid this difficulty, we will use the following trick. We will first consider the interaction of fast but still nonrelativistic dimesoatoms with the target. In this case the relatively simple mathematical formalism of nonrelativistic quantum mechanics can be used for the description of both internal and external dynamics.
of dimesoatoms. This results in simple energy dependence of the density matrix.

The next step is the “analytical” extension of this nonrelativistic result to relativistic energies appealing to the physical intuition. Although this way of derivation cannot be considered as rigorous we believe that it allows one to get the most essential features of the desired result.

We restrict our consideration to the case of rather high dimesoatom velocities and rather thin targets so that the “frozen” target approximation can be used. In other words, we will suppose that the positions of target atoms practically do not change during the passage of a dimesoatom through the target.

In this case, the stationary Schrödinger equation can be used to describe the interaction between the dimesoatom and the target:

\[ H\psi = E\psi, \]  
\[ H = H_{int} + H_{ext}, \]  
\[ H_{int} = -\frac{1}{2\mu}\Delta \vec{r} + V_{int}(\vec{r}), \]  
\[ H_{ext} = -\frac{1}{2M}\Delta \vec{R} + V_{ext}(\vec{r}, \vec{R}; \{\vec{\rho}\}). \]

Here \( \psi \equiv \psi(\vec{r}_+, \vec{r}_-), \) \( \vec{r}_+ \) and \( \vec{r}_- \) are the radius-vectors of the positively and negatively charged mesons respectively, \( m_+ \) and \( m_- \) are their masses; \( M = m_+ + m_- \), \( \mu = m_+ m_- / M \); \( \vec{R} = (m_+ \vec{r}_+ + m_- \vec{r}_-)/M \) is the radius-vector of the dimesoatom center-of-mass; \( \vec{r} = (\vec{r}_+ - \vec{r}_-) \) is the radius-vector of relative motion of dimesoatom constituents; \( \{\vec{\rho}\} \) is the set of the radius-vectors of the target atom center-of-mass; \( V_{int}(\vec{r}) \) is the potential energy of interaction between dimesoatom constituents; \( V_{ext}(\vec{r}, \vec{R}; \{\vec{\rho}\}) \) is the potential energy of interaction between the dimesoatom and the target atom:

\[ V_{ext} = e \sum_k [\Phi(\vec{r}_+ - \vec{\rho}_k) - \Phi(\vec{r}_- - \vec{\rho}_k)], \]

\[ \vec{r}_+ = \vec{R} + \xi \vec{r}, \quad \vec{r}_- = \vec{R} - \eta \vec{r}, \]

\[ \xi = \frac{m_-}{M}, \quad \eta = \frac{m_+}{M} = 1 - \xi, \]

\( \Phi \) is the electrostatic potential of the target atom at rest; \( e \) is the elementary charge.

Let us consider that the beam of dimesoatoms is in a definite state of the discrete spectrum \( |i\rangle \) and moves in the positive direction of the \( Oz \)-axis \( (\vec{p} = (0, 0, p) \) is the beam momentum). The target is supposed to be a plate of thickness \( L \) and of the infinite transverse size, placed perpendicular to the beam, so that one surface has \( z = 0 \) and the other has \( z = L \). Then, it the \( z \leq 0, \)

\[ \psi(\vec{r}_+, \vec{r}_-) = \exp(\vec{p}\vec{R}) \cdot \phi_i(\vec{r}), \]

\[ H_{int}\phi_i(\vec{r}) = \varepsilon_i\phi_i(\vec{r}), \]

where \( \varepsilon_i \) is the energy of the state \( |i\rangle \).

The solution of (1) in the region \( z > 0 \) will be sought in the form

\[ \psi(\vec{r}_+, \vec{r}_-) = \exp(i\vec{p}\vec{R}) \cdot F(\vec{R}, \vec{r}; \{\vec{\rho}\}). \]
Then the equation for $F$ following from (1) is conveniently written as

$$
\frac{1}{2M} \left( \frac{\partial^2}{\partial z^2} + 2ip \frac{\partial}{\partial z} \right) F \left( z, \vec{b}, \vec{r}, \{ \vec{\rho} \} \right) = \left( -\frac{\Delta_{\vec{b}}}{2M} + H_{int} + V_{ext} \right) F \left( z, \vec{b}, \vec{r}, \{ \vec{\rho} \} \right),
$$

(11)

where $z$ and $\vec{b}$ are the longitudinal and transverse parts of the radius-vector $\vec{R} = \{ \vec{b}, z \}$.

At rather high dimesoatomic energies, such that the condition

$$
p a \gg 1, \quad a \sim (m_e \cdot \alpha \cdot Z^{1/3})^{-1}
$$

(12)
is satisfied, the term $\partial^2 F/\partial z^2$ on the left side of (11) can be neglected [12] (in (12) $a$ is the screening radius of the target atom, $Z$ is its atomic number and $m_e$ is the electron mass).

In this approximation the quantity

$$
\tilde{F} = \exp \left( \frac{iM}{p} \varepsilon_i z \right) \cdot F
$$

(13)
obey the equation

$$
-i \frac{\partial \tilde{F}}{\partial z} = \tilde{H} \cdot \tilde{F},
$$

(14)

$$
\tilde{H} = -\frac{1}{2p} \Delta_{\vec{b}} + \frac{M}{p} H_{int} + \frac{1}{v} V_{ext}
$$

(15)

which looks like the nonstationary Schrödinger equation.

In (15) $v$ is the dimesoatom velocity, which in the nonrelativistic limit coinsides with the ratio $p/M$. But keeping in mind the subsequent “analytical extension” of the below results to the region of relativistic energies, we will assume not the equality $p/M = v$ in the general case but the relativistic relationship $p = Mv \gamma$. With this prescription the right energy dependence of the time-formation effects is reproduced (see also [14]).

## 3 Density matrix kinetic equation

Let us write the expression for the density matrix describing the dimesoatom state after covering the distance $z$ in the matter

$$
\rho(z; \vec{r}_1, \vec{r}_2) = \Sigma^{-1} \int d\vec{b}d\vec{b}_1d\vec{b}_2d\vec{r}_1d\vec{r}_2d\vec{r}_2' \langle G(z; \vec{b}, \vec{r}_1; \vec{b}_1', \vec{r}_1'); \{ \vec{\rho} \} \rangle \\
\times G^*(z; \vec{b}, \vec{r}_2; \vec{b}_2', \vec{r}_2'; \{ \vec{\rho} \}) \langle \phi_i(\vec{r}_1')\phi_i^*(\vec{r}_2') \rangle.
$$

(16)

Here $G$ is the Green function of (14), the solution of which is given by the expression

$$
\tilde{F}(z, \vec{b}, \vec{r}, \{ \vec{\rho} \}) = \int G(z; \vec{b}, \vec{r}; \vec{b}', \vec{r}'; \{ \vec{\rho} \}) \phi_i(\vec{r}') d\vec{r}' d\vec{b}'.
$$

(17)

$\Sigma$ is the beam cross section area, and the symbol $\langle \rangle_{\{ \vec{\rho} \}}$ means averaging over all possible states of target atoms.
To perform this averaging in the explicit form, we will use the path integral representation \[15\] of the Green function:

\[
G(z; \vec{b}, \vec{r}; \vec{b}', \vec{r}'; \{\vec{\rho}\}) = \int D\vec{b}(\tilde{z})D\vec{r}(\tilde{z}) \exp(iS),
\] (18)

where the action \(S\) of the dimesoatom interacting with the matter is

\[
S = \int_0^z dz' \left[ \frac{p}{2} \left( \frac{d\vec{b}(z')}{dz'} \right)^2 + \frac{v\gamma\mu}{2} \left( \frac{d\vec{r}(z')}{dz'} \right)^2 
- \frac{1}{v\gamma} V_{\text{int}}(\vec{r}(z')) - \frac{1}{v} V_{\text{ext}}(z'; \vec{b}(z'), \vec{r}(z')) \right].
\] (19)

The path integration in (18) is performed over all trajectories \(\{\vec{b}(\tilde{z}), \vec{r}(\tilde{z})\}\) beginning with the point \(\{\vec{b}(0), \vec{r}(0)\} = \{\vec{b}', \vec{r}'\}\) and ending with the point \(\{\vec{b}(z), \vec{r}(z)\} = \{\vec{b}, \vec{r}\}\).

It was shown in \[16\] that

\[
\int_0^z dz' \frac{1}{v} V_{\text{ext}}(z'; \vec{b}(z'), \vec{r}(z')) = \sum_k \vartheta(z - z_k) \left\{ \chi(\vec{b}(z_k) + \xi \vec{s}(z_k) - \vec{\tau}_k) 
- \chi(\vec{b}(z_k) - \eta \vec{s}(z_k) - \vec{\tau}_k) \right\},
\] (20)

where the Heavyside step-function \(\vartheta(z)\) is 0 for \(z < 0\) and 1 for \(z > 0\); \(\vec{s}\) and \(\vec{\tau}_k\) are the transverse components of the vectors \(\vec{r}\) and \(\vec{\rho}_k\); \(z_k\) is the longitudinal component of the vector \(\vec{\rho}_k\).

The quantity

\[
\chi(\vec{B}_{\xi(\eta)}) = \frac{e}{v} \int_{-\infty}^{\infty} \Phi \left( \sqrt{\vec{B}_{\xi(\eta)}^2 + z^2} \right) dz,
\] (21)

\[
\vec{B}_k = \vec{b} + \xi \vec{s} - \vec{\tau}_k, \quad \vec{B}_n = \vec{b} - \eta \vec{s} - \vec{\tau}_k,
\]

is the phase shift acquired by the wave function of the hadron while it moves through the field of the isolated target atom.

Let us use the relation \[11\]

\[
\left\langle \exp \left[ \sum_k f(\vec{\rho}_k, \{x\}) \right] \right\rangle_{\{\vec{\rho}\}} = \exp \left\{ -n_0 \int d\vec{\rho} (1 - f(\vec{\rho}, \{x\})) \right\},
\] (22)

where \(n_0\) is the number of target atoms in the unite volume; \(f(\vec{\rho}_k, \{x\})\) is the arbitrary function of the target atom coordinates and of the set \(\{x\}\) of others variables.

Then the average of the product of two Green functions over the target atom coordinates is

\[
\left\langle G(z; \vec{b}_1, \vec{r}_1; \vec{b}'_1, \vec{r}'_1; \{\vec{\rho}\})G^*(z; \vec{b}_2, \vec{r}_2; \vec{b}'_2, \vec{r}'_2; \{\vec{\rho}\}) \right\rangle_{\{\vec{\rho}\}}
= \int D\vec{b}_1(\tilde{z})D\vec{b}_2(\tilde{z})D\vec{r}_1(\tilde{z})D\vec{r}_2(\tilde{z}) \exp \left( iS_1 - iS_2 - n_0 \int_0^z dz' \int \omega d\vec{r} \right),
\] (23)
\[ \bar{S}_{1(2)} = \int_0^z dz' \left[ \frac{p}{2} \left( \frac{d\bar{b}_{1(2)}(z')}{dz'} \right)^2 + \frac{v\gamma\mu}{2} \left( \frac{d\bar{r}_{1(2)}(z')}{dz'} \right)^2 - \frac{1}{v\gamma} V_{\text{int}} \left( \bar{r}_{1(2)}(z') \right) \right], \quad (24) \]

\[ \omega = 1 - \exp[i\Delta\chi_1 - i\Delta\chi_2], \quad (25) \]

\[ \Delta\chi_{1(2)} = \chi \left[ \bar{b}_{1(2)}(z') + \xi\bar{s}_{1(2)}(z') - \bar{\tau} \right] - \chi \left[ \bar{b}_{1(2)}(z') - \eta\bar{s}_{1(2)}(z') - \bar{\tau} \right]. \quad (26) \]

With the replacement
\[ \bar{\tau} \rightarrow \bar{\tau} + \bar{b}_+(z'), \quad (27) \]

where the \( \bar{\tau} \) is impact vector and
\[ \bar{b}_+(z') = \left( \bar{b}_1(z') + \bar{b}_2(z') - \bar{\tau} \right), \quad (28) \]

it is easy to see that \( \omega \) depends only on
\[ \bar{b}_-(z') = \left( \bar{b}_1(z') - \bar{b}_2(z') - \bar{\tau} \right) \quad (29) \]

and does not depend on \( \bar{b}_+(z') \).

This allows to perform path integration over the variables \( \bar{b}_1(\bar{z}) \) and \( \bar{b}_2(\bar{z}) \) in (23) in the explicit form (see e.g. [16]) and to represent the density matrix (16) in the form
\[ \rho(z; \bar{r}_1, \bar{r}_2) = \int D\bar{r}_1(\bar{z})D\bar{r}_2(\bar{z})d\bar{r}_1'd\bar{r}_2'd\bar{r}' \]
\[ \times \exp \left\{ i\bar{S}_1 - i\bar{S}_2 - n_0 \int_0^z dz' \bar{\Omega} \right\} \phi_i(\bar{r}_1')\phi_i^*(\bar{r}_2'), \quad (30) \]

where
\[ \bar{\Omega} = \int \omega d\bar{\tau} = \Omega(\bar{s}_1(z'), \bar{s}_2(z')), \quad (31) \]

\[ \Omega(\bar{s}_1, \bar{s}_2) = \int d\bar{\tau} \left\{ \Gamma(\bar{\tau}, \bar{s}_1) + \Gamma^*(\bar{\tau}, \bar{s}_2) - \Gamma(\bar{\tau}, \bar{s}_1)\Gamma^*(\bar{\tau}, \bar{s}_2) \right\}, \quad (32) \]

\[ \Gamma(\bar{\tau}, \bar{s}_{1(2)}) = 1 - \exp \left\{ i\chi(\bar{\tau} - \xi\bar{s}_{1(2)}) - i\chi(\bar{\tau} + \eta\bar{s}_{1(2)}) \right\}. \quad (33) \]

\( \Gamma(\bar{\tau}, \bar{s}) \) is the interaction operator of the Glauber theory for interaction of dimesoatoms with target atoms [17]. In particular, the transition amplitudes of the dimesoatoms between the states \( |i \rangle \) and \( |k \rangle \) in the Coulomb field of target atoms are related to \( \Gamma(\bar{\tau}, \bar{s}) \) by the equation
\[ A_{kl}(q) = \frac{i}{2\pi} \int d^2\tau d^3r e^{iq\cdot\bar{\tau}} \psi_k^*(\bar{r})\psi_l(\bar{r})\Gamma(\bar{\tau}, \bar{s}). \quad (34) \]

Path integrations (like the usual ones) in (30) can be performed by numerical methods ([18]).

But for the purpose of the qualitative analysis it is more useful to deal with the kinetic equation for the density matrix
\[ i\frac{\partial \rho(z; \bar{r}_1, \bar{r}_2)}{\partial z} = \frac{1}{v\gamma} \left[ H_{\text{int}}(\bar{r}_1) - H_{\text{int}}^*(\bar{r}_2) \right] \rho(z; \bar{r}_1, \bar{r}_2) \]
\[ -in_0\Omega(\bar{s}_1, \bar{s}_2)\rho(z; \bar{r}_1, \bar{r}_2), \quad (35) \]
which is the consequence of the path integral representation (30) (see, for example, [15]).

Because of the hermiticity of $H_{\text{int}}$ this equation can be rewritten in the operator form

$$i \frac{\partial \rho}{\partial z} = \frac{1}{v \gamma} [H_{\text{int}}, \rho] - i n_0 \Omega \rho,$$

where the first term on the right side (Liouville term) describes the causal part of the internal dimesoatom dynamics. The nature of the second term is pure stochastic because of the stochastic distribution of the atoms in the target.\footnote{For each separate projectile the target “is frozen” in the definite state. But because of the stochastic distribution of the projectiles in the beam these definite “frozen” states of the target are distributed stochastically too.}

In the dimesoatom rest frame Eq. (35) takes the form [14]

$$i \frac{\partial \rho(t; \vec{r}_1, \vec{r}_2)}{\partial t} = H_{\text{int}}(\vec{r}_1) \rho(t; \vec{r}_1, \vec{r}_2) - H_{\text{int}}^*(\vec{r}_2) \rho(t; \vec{r}_1, \vec{r}_2) - iv \gamma n_0 \Omega(\vec{s}_1, \vec{s}_2) \rho(t; \vec{r}_1, \vec{r}_2),$$

which is similar to the form of Eq. (116) of paper [19] for the density matrix of atoms moving in the laser fields. But the meaning of the terms on the right side of this equation and on the right side of our Eq. (37) is different.

The causal part of (116) also includes the term describing interaction of atoms with the laser fields while the stochastic term of this equation describes effects of spontaneous relaxation caused by interaction of atoms with the quantized electromagnetic field.

Strictly speaking, a similar term should be added to the right part of (37) too. But it can be shown that its influence on the internal dynamics of the dimesoatoms is negligible compared with similar effects caused by interaction of dimesoatoms with target atoms.\footnote{Interaction of dimesoatoms with the quantized electromagnetic field causes only deexcitation of excited dimesoatoms through emission of real photons. Interaction of dimesoatoms with target atoms causes both excitation and deexcitation of dimesoatoms through exchange of virtual photons with target atoms.}

For this reason we do not include the effects of spontaneous electromagnetic relaxation in our consideration.

There is effect that play important role in the dimesoatom dynamics which is not taken into account in (37). It is the effect of instability of dimesoatoms caused by the possibility of their annihilation into neutral hadrons. We will include it phenomenologically.

With this aim let us consider the density-matrix elements

$$\rho_{ik}(z) = \int \psi_i^*(\vec{r}) \psi_k(\vec{r}') \rho(z; \vec{r}, \vec{r}') d\vec{r} d\vec{r}' .$$

Taking into account the Schrödinger equation

$$H_{\text{int}} \psi_i = \varepsilon_i \psi_i,$$

it is easy to see that matrix Eq. (35) is equivalent to the following system of equations for the density-matrix elements:

$$\frac{\partial \rho_{ik}}{\partial z} = i \Delta_{ik} \rho_{ik} - n_0 \sum_{l,m} \Omega_{ik,lm} \rho_{lm} .$$
where
\[ \Delta_{ik} = \left( \varepsilon_k - \varepsilon_i \right)/v\gamma, \]
\[ \Omega_{ik,lm} = \int \psi_i^*(\vec{r}) \psi_l(\vec{r}) \psi_k(\vec{r}') \psi_m^*(\vec{r}') \Omega(\vec{s}, \vec{s}') d\vec{r} d\vec{r}'. \]

It is evident that after including the instability effects the solution of this system at \( n_0 = 0 \) (“empty” target)
\[ \rho_{ik}(z) = \rho_{ik}(0) \exp[i\Delta_{ik} \cdot z] \]
is modified to
\[ \rho_{ik}(z) = \rho_{ik}(0) \exp \left\{ \frac{z}{v\gamma} \left[ i(\varepsilon_k - \varepsilon_i) - \frac{1}{2}(\Gamma_i + \Gamma_k) \right] \right\}, \]
where \( \Gamma_{i(k)} \) are the widths of the states \(|i(k)\rangle\).

Hence Eq. (40) must be replaced by
\[ \frac{\partial \rho_{ik}(z)}{\partial z} = \frac{1}{v\gamma} \left[ i(\varepsilon_k - \varepsilon_i) - \frac{1}{2}(\Gamma_i + \Gamma_k) \right] \rho_{ik}(z) - n_0 \sum_{l,m} \Omega_{ik,lm} \rho_{lm}(z). \]

For the subsequent discussion it is convenient to represent (44) in the integral form
\[ \rho_{ik} = \rho_{ik}(0) \exp[-a_{ik} \cdot z] - n_0 \sum_{l,m} \Omega_{ik,lm} \int \exp[-a_{ik} \cdot (z - z')] \rho_{lm}(z') dz', \]
\[ a_{ik} = -\frac{1}{v\gamma} \left[ i(\varepsilon_k - \varepsilon_i) - \frac{1}{2}(\Gamma_i + \Gamma_k) \right]. \]

4 Time-formation effects

It is easy to check that the coefficients \( \Omega_{ik,lm} \) (41) are different from zero if the relations
\[ m_i - m_k - m_l + m_m = 0, \quad l_i - l_k - l_l + l_m = 2s \]
are satisfied. Above, \( m_{i(k,l,m)} \) and \( l_{i(k,l,m)} \) are the magnetic and orbital quantum numbers of the states \(|i(k,l,m)\rangle\) and \( s \) is an arbitrary integral number (the quantization axis is chosen to coincide with the \( Oz \)-axis).

Since the dimesoatoms are produced practically only in the \( ns \)-states \((l = 0, m = 0) \) [4], the “selection rules” for the quantities \( \rho_{ik}(z) \)
\[ \rho_{ik}(z) \neq 0 \quad \text{if and only if} \quad m_i = m_k, \quad l_i = l_k + 2s \]
follow from (42), (47).

No restrictions on the values \( n_{i(k)} \) — principal quantum numbers of the states \(|i(k)\rangle\) — are imposed by the dynamics of interaction of dimesoatoms with target atoms represented by the quantities \( \Omega_{ik,lm} \). But they are imposed by the so-called “length (time)-formation effects”,4 represented by the oscillating factors \( \exp[i\Delta_{ik} \cdot (z - z')] \) in (45).

4These effects belong to the wide class of the effects systematic investigation of which began in the papers [20, 21] (further development see in [22]).
At super-high energies, such that $\Delta_{ik}L \ll 1$ ($L$ is the thickness of the target), the factors $\exp[i\Delta_{ik} \cdot (z - z')] \sim 1$ for any values of the indices $i, k$, and the relations between the values of the diagonal and off-diagonal density matrix elements $\rho_{ik}(z)$ are governed only by the properties of the coefficients $\Omega_{ik,lm}$.

For “low” energies (such that $\Delta_{ik}\bar{\lambda} \gg 1$, where $\bar{\lambda}$ is the typical dimesoatom free-path length in matter) the numerical values of the off-diagonal density matrix elements $\rho_{ik}(z) (i \neq k)$ become negligible compared to the diagonal ones if $\varepsilon_i \neq \varepsilon_k$.

If energy levels of the system under consideration are non-degenerate ($\varepsilon_i \neq \varepsilon_k$, if $i \neq k$), only the diagonal density matrix elements $\rho_{ii} = P_i$ survive at “low” energies because of length-formation effects, and the internal dynamics of such systems can be described by a system of kinetic equations for probabilities $P_i(z)$

$$\frac{dP_i}{dz} = \sum_{i,l} c_{il} P_l(z), \quad c_{il} = -n_0 \Omega_{ii,ll} - a_{ii} \delta_{il}$$

which coincide with the system equations of [9].

Taking into account the “selection rules” (48), we clearly see that this statement is also valid when there is only degeneracy with respect to the value of the magnetic quantum number that is, if the hadron-hadron interaction has only the central symmetry.

However, the Coulomb interaction in the hydrogen-like atoms has additional symmetry that causes extra (accidental) degeneracy of the energy levels with respect to the value of the orbital quantum number. Because of this, even at very low energies some off-diagonal density matrix elements $\rho_{ik}(z)$ with

$$\varepsilon_i = \varepsilon_k, \quad l_i = l_k + 2s$$

survive together with diagonal ones. This excludes the possibility of describing of internal dynamics of these atoms in the framework of “classical” (probabilistic) approach [9] at any energies.\(^5\)

### 5 Conclusion

The accurate description of internal dynamics of relativistic dimesoatoms moving through matter requires the density matrix formalism. The kinetic equation for the density matrix could be represented in the form of the Liouville equation only if the positions of the target atoms will change in time according to a definite law. Because of the stochastic change of these positions, the part of the kinetic equation for density matrix describing the interaction of dimesoatoms with target atoms becomes more complicate. The main result of the present paper is the derivation in the explicit form (32) of the term describing this interaction. That fact that we have automatically obtained an expression for (32) in terms of interaction operators of the Glauber theory can be considered as additional confirmation of the validity of the approximations used.

\(^5\)Strictly speaking, strong interaction between dimesoatomic constituents eliminates the accidental degeneracy of the dimesoatomic energy levels. But this effects is so small numerically that it has no significance for the problem under consideration (for example, $(\varepsilon_{2p} - \varepsilon_{2s})/\varepsilon_{2s} \sim 10^{-3}$).
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