Observation of nonlocality sharing via “strong” weak measurements

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Nonlocality plays a fundamental role in quantum information science. Recently, it has been theoretically predicted and experimentally demonstrated that the nonlocality from an entangled pair can be shared among multiple observers using weak measurements with moderate strength. Here we report an observation of a counterintuitive result that nonlocality sharing can be achieved using weak measurements with very strong measurement strength. Our result not only sheds light on the interplay between nonlocality and quantum measurements, but also may find applications in quantum steering, unbounded randomness certification and quantum communication network.

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To date, however, most discussions of nonlocality scenario focus on one pair of entangled qubits distributed to only two separated observers. Recently, a surprising result that nonlocality can actually be shared among more than two observers using weak measurements, has been reported by Silva et al. [24]. In Silva’s scenario, a pair of maximally-entangled qubits are distributed to three observers Alice, Bob1 and Bob2, in which Alice accesses one qubit and the two Bobs access the other qubit. While Bob1 performs a weak measurement on his qubit and then passes it to Bob2, Alice and Bob2 perform projective measurements on their own qubits respectively. Nonlocality sharing, which is certified by a double violation of Bell-CHSH inequality among Alice-Bob1 and Alice-Bob2, can be observed when the strength of Bob1’s measurement is within a reasonable range [24–26]. This can be understood as follows: The strength of Bob1’s measurement has to be on one hand strong enough for Bob1 to retrieve enough information from his qubit so as to establish quantum correlation between Alice and Bob1, and on the other hand not too strong for Bob2 to retain some quantum correlation with Alice.

Recently, we have extended Silva’s scheme and proven a counterintuitive result that nonlocality sharing can actually be achieved even with very “strong” weak measurements [27]. In our protocol, the strength of the measurement performed by Bob1 can be increased to the extreme as long as it is still a weak measurement. In this Letter, we reported the observation of such phenomenon in a photonic system using optimal weak measurements. In our experiment, we have observed a double violation of Bell-CHSH inequality in a region which is not allowed for nonlocality sharing in the original scheme [24–26].

Let us first briefly review Silva’s scheme that allows double violation of Bell-CHSH inequality [24]. As shown in Fig. 1, three observers, Alice, Bob1 and Bob2, perform measurements on a two-qubit entangled state, where Alice measures qubit 1 and the two Bobs measure qubit 2 sequentially. Each observer randomly chooses one of two observables to measure. The two observables for Alice (Bob1/Bob2) are \( \hat{\omega}/\hat{\nu} \) and \( \hat{\omega}'/\hat{\nu}' \). Alice and Bob1 use their measurement results to construct a CHSH parameter \( I_{\text{CHSH}}^{(1)} \), which is defined as

\[
I_{\text{CHSH}}^{(1)} = \langle \hat{\omega} \otimes \hat{\mu} \rangle + \langle \hat{\omega}' \otimes \hat{\mu}' \rangle + \langle \hat{\omega} \otimes \hat{\mu}' \rangle - \langle \hat{\omega}' \otimes \hat{\nu} \rangle \tag{1}
\]

In the same way, Alice and Bob2 construct another CHSH parameter \( I_{\text{CHSH}}^{(2)} \) defined as

\[
I_{\text{CHSH}}^{(2)} = \langle \hat{\omega} \otimes \hat{\nu} \rangle + \langle \hat{\omega}' \otimes \hat{\nu}' \rangle + \langle \hat{\omega} \otimes \hat{\nu}' \rangle - \langle \hat{\omega}' \otimes \hat{\nu} \rangle \tag{2}
\]

Nonlocality, the phenomenon that the results of local measurements performed on distant parties of a composite system can not be explained by local hidden variable theories, is one of the most fascinating characteristics of quantum mechanics [1]. Since its derivation by Bell in 1964 [2], nonlocality has been studied extensively from various perspectives [3–9] and verified in many different quantum systems [10–18]. Not only is nonlocality important for our understanding of quantum mechanics, but it also serves as a necessary resource in device-independent quantum information protocols, such as quantum key distribution [23].
A CHSH parameter greater than 2 cannot be explained by any local hidden variable theory. According to the monogamy property of nonlocality \cite{28,29}, it is impossible to achieve a double Bell-CHSH violation, namely to have both $I_{\text{CHSH}}^{(1)}$ and $I_{\text{CHSH}}^{(2)}$ greater than 2, on condition that the measurements performed by the three observers are all projective measurements. However, if Bob1’s measurement is a weak measurement, which means non-signalling hypothesis is dropped, this restriction no longer holds. As shown in Fig. 2(a), by setting $\hat{\omega} = X$, $\hat{\omega}' = Z$, $\hat{\mu} = \frac{X + Z}{\sqrt{2}}$, $\hat{\mu}' = \frac{X - Z}{\sqrt{2}}$, one can get

$$I_{\text{CHSH}}^{(1)} = 2\sqrt{2}G, I_{\text{CHSH}}^{(2)} = \sqrt{2}(1 + F),$$

(3)

where $G$ is the precision factor of Bob1’s measurement, which denotes the strength of the measurement and how much information is gained through such measurement, and $F$ is the quality factor of Bob1’s measurement, which corresponds to the coherence remained after the measurement. There is a trade-off relationship between $G$ and $F$ ($G^2 + F^2 \leq 1$). When Bob1’s weak measurement is optimal, namely $G^2 + F^2 = 1$, the two CHSH parameters can be plotted as functions of $G$, where $I_{\text{CHSH}}^{(1)}$ ($I_{\text{CHSH}}^{(2)}$) is represented by the blue line (red curve) as shown in Fig. 2(c). One can see that in this protocol, only when the strength of the weak measurement is moderate, namely $G \in (0.707, 0.910)$, a double Bell-CHSH violation can be achieved.

The reason that a double Bell-CHSH violation can only be achieved with a moderate-strength weak measurement in this protocol was explained previously in the following way \cite{24}. If the strength of Bob1’s measurement is too weak, Bob1 cannot gain enough information about qubit 2 to make $I_{\text{CHSH}}^{(3)} > 2$. If the strength of Bob1’s measurement is too strong, it destroys the quantum correlation between the two qubits so badly that the remaining correlation is not strong enough for Alice and Bob2 to make $I_{\text{CHSH}}^{(2)} > 2$. Here we point out that such argument is not valid and a double Bell-CHSH violation can actually be achieved with a very “strong” measurement as long as it is still a weak measurement.

Let us investigate the original protocol and explain why it can be improved. In Silva’s scheme, Alice’s observables are $X$ and $Z$. Based on Alice’s observables, Bob1 set his observables to be $\hat{\mu} = \frac{X + Z}{\sqrt{2}}$ and $\hat{\mu}' = \frac{X - Z}{\sqrt{2}}$, which renders the maximum value of $I_{\text{CHSH}}^{(1)} = 2\sqrt{2}G$. However, the choice of Bob1’s observables only considers the maximization of $I_{\text{CHSH}}^{(1)}$, which will unavoidably lower the upper bound of $I_{\text{CHSH}}^{(2)}$. To achieve a double Bell-CHSH violation, one needs to make sure that both CHSH parameters are greater than 2. As a result, if the aim is to widen the double Bell-CHSH violation region as much as possible, a better choice is to maximize the value of $\min(I_{\text{CHSH}}^{(1)}, I_{\text{CHSH}}^{(2)})$ instead of $I_{\text{CHSH}}^{(1)}$ or $I_{\text{CHSH}}^{(2)}$ alone.

We now present our scheme of nonlocality sharing. In our protocol, Alice uses the same fixed observables $X$ and $Z$. The choice of Bob1’s and Bob2’s observables depends on the precision factor $G$ of Bob1’s measurement.

When $G \leq 0.8$, Bob1 and Bob2 will use the same fixed observables $\frac{X + Z}{\sqrt{2}}$ and $\frac{X - Z}{\sqrt{2}}$ as the original scheme to maximize $\min(I_{\text{CHSH}}^{(1)}, I_{\text{CHSH}}^{(2)})$. The reason that the observables used for maximizing $I_{\text{CHSH}}^{(1)}$ can also maximize $\min(I_{\text{CHSH}}^{(1)}, I_{\text{CHSH}}^{(2)})$ is that, $I_{\text{CHSH}}^{(2)}$ is always less than or equal to $I_{\text{CHSH}}^{(2)}$ when $G \leq 0.8$, which means $\min(I_{\text{CHSH}}^{(1)}, I_{\text{CHSH}}^{(2)}) = I_{\text{CHSH}}^{(1)}$ under the condition of $G \leq 0.8$.

When $G > 0.8$, $I_{\text{CHSH}}^{(2)}$ is not necessarily greater than $I_{\text{CHSH}}^{(1)}$ anymore. In this case, to maximize $\min(I_{\text{CHSH}}^{(1)}, I_{\text{CHSH}}^{(2)})$, one need to increase the value of $I_{\text{CHSH}}^{(2)}$, namely raising the quantum correlation between Alice and Bob2. We find that, given a certain $G$ value, the smaller the difference between Bob1’s two observables $\hat{\mu}$ and $\hat{\mu}'$, the more the quantum correlation will be left for Alice and Bob2 after Bob1’s measurement. As a result, we set Bob1’s and Bob2’s observables (as shown in Fig. 2(b)) to be

$$\hat{\mu} = \cos \gamma X + \sin \gamma Z, \quad \hat{\mu}' = \cos \gamma X - \sin \gamma Z,$$

$$\hat{\nu} = \cos \delta X + \sin \delta Z, \quad \hat{\nu}' = \cos \delta X - \sin \delta Z,$$

(4)

where $\gamma$ and $\delta$ are angles between 0 and $\pi/4$, whose values would be determined by $G$. One can see that, while the value of $I_{\text{CHSH}}^{(1)}$ is a function of two parameters $G$ and $\gamma$, the value of $I_{\text{CHSH}}^{(2)}$ is a function of all three parameters $G$, $\gamma$ and $\delta$. Given a certain $G$, once $\gamma$ is set, not only $I_{\text{CHSH}}^{(1)}$ can be calculated, but also a proper value of $\delta$ can be chosen to maximize the value of $I_{\text{CHSH}}^{(2)}$. As a result, given a certain $G$ greater than 0.8, if we start with $\gamma = \pi/4$ and then gradually decrease its value, the value of $I_{\text{CHSH}}^{(1)}$ will decrease, and in the meantime the
value of $I_{\text{CHSH}}^{(2)}$ will increase assuming $\delta$ is updated to maximize $I_{\text{CHSH}}^{(2)}$. By decreasing the value of $\gamma$, the value of $\min(I_{\text{CHSH}}^{(1)}, I_{\text{CHSH}}^{(2)})$ can be continuously increased until we get to the point that $I_{\text{CHSH}}^{(1)} = I_{\text{CHSH}}^{(2)}$. For any given $G$ greater than 0.8, we can always find numerical solutions for $\gamma$ and $\delta$ to make sure that $I_{\text{CHSH}}^{(1)} = I_{\text{CHSH}}^{(2)}$ and thus obtain the maximum value of $\min(I_{\text{CHSH}}^{(1)}, I_{\text{CHSH}}^{(2)})$.

As shown in Fig. 2(c), the brown curve indicates the values of both $I_{\text{CHSH}}^{(1)}$ and $I_{\text{CHSH}}^{(2)}$ in the region of $G > 0.8$. One can see that the two CHSH parameters are always greater than 0.8, we can always find numerical solutions for $\gamma$ and $\delta$ to make sure that $I_{\text{CHSH}}^{(1)} = I_{\text{CHSH}}^{(2)}$ and thus obtain the maximum value of $\min(I_{\text{CHSH}}^{(1)}, I_{\text{CHSH}}^{(2)})$.

Optical realization of optimal weak measurement - The key to experimentally demonstrate our scheme is to realize an optimal weak measurement with tunable strength. Let us first explain how to realize an optimal weak measurement with an ancillary qubit. As shown in Fig. 3(a), an arbitrary single qubit state $|\psi\rangle$, defined as $|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$ ($|\alpha|^2 + |\beta|^2 = 1$), is to be weak-measured. Let $|\psi\rangle$ and an ancillary qubit $|0\rangle$ go through a two-qubit unitary $U$, where $U$ is defined as

$$U = \begin{pmatrix} \cos \theta & -\sin \theta & 0 & 0 \\ \sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & \sin \theta & -\cos \theta \\ 0 & 0 & \cos \theta & \sin \theta \end{pmatrix}. \quad (5)$$

After passing through $U$, the two-qubit state becomes

$$\alpha |0\rangle \otimes (\cos \theta |0\rangle + \sin \theta |1\rangle) + \beta |1\rangle \otimes (\sin \theta |0\rangle + \cos \theta |1\rangle). \quad (6)$$

A projective measurement on 0/1 basis is then applied to the ancillary qubit, which would effectively realize a weak measurement on $|\psi\rangle$. By easy calculation, one can find $G = \cos 2\theta$ and $F = \sin 2\theta$ in this weak measurement, which satisfies the optimal weak measurement condition [24].

We now show how to experimentally realize a weak measurement on a polarization-encoded photonic qubit in a specific basis $\{|\phi\rangle, |\phi^+\rangle\}$, where $|\phi\rangle = \cos \phi |H\rangle + \sin \phi |V\rangle$ and $|\phi^+\rangle = \sin \phi |H\rangle - \cos \phi |V\rangle$ ($H/V$ denotes horizontal/vertical polarization). As shown in Fig. 3(b), the optical weak measurement setup consists of three parts. The first part is a half-wave plate (HWP1) setting at $\varphi/2$, which is used to transform the measurement basis from $\{|\phi\rangle, |\phi^+\rangle\}$ to $\{|H\rangle, |V\rangle\}$. The second part, composed of the devices between HWP1 and HWP4, HWP4’, is used to implement a weak measurement in $\{|H\rangle, |V\rangle\}$ basis. The third part, consisting of HWP4 and HWP4’ both setting at $\varphi/2$, is used to transform the measurement basis from $\{|H\rangle, |V\rangle\}$ back to $\{|\phi\rangle, |\phi^+\rangle\}$.

The essential part of this setup is the second part, which realizes a weak measurement in $\{|H\rangle, |V\rangle\}$ basis by utilizing spatial degree of freedom as the ancillary qubit. Suppose the qubit entering the second part is $|\psi\rangle = \alpha |H\rangle + \beta |V\rangle$. By setting HWP2 and HWP3 at $\theta/2$, the state exits part 2 will become

$$\alpha |H\rangle (\cos \theta |l\rangle + \sin \theta |u\rangle) + \beta |V\rangle (\sin \theta |l\rangle + \cos \theta |u\rangle), \quad (7)$$

where $|l\rangle$ and $|u\rangle$ denote the lower and upper spatial modes, respectively (See Methods). This state has the same form as the one shown in equation 6, which can be used for optimal weak measurement. As a result, by measuring the spatial qubit, one can realize an optimal weak measurement on the polarization qubit in $H/V$ basis with a precision factor $G = \cos 2\theta$.

In practice, one can use a simplified setup to realize the same optimal weak measurement [25]. As shown in Fig. 3(c), when the input photon exits at the middle spatial mode, it indicates an optimal weak measurement in
FIG. 3: Experimental implementation of optimal nonlocality sharing. (a) The circuit of optimal weak measurement using an ancillary qubit. (b) Optimal weak measurement realized in a photonic system. (c) The simplified optical setup for optimal weak measurement. (d) Experimental setup. Polarization-entangled photon pairs degenerated at 808nm are produced by pumping a type-II beta barium borate (BBO) crystal with an ultraviolet laser centered at 404nm. HWP6 (HWP5) is used for Alice (Bob2) to set the observable of her (his) projective measurement. The orange region shows Bob1’s weak measurement setup. HWP1 and HWP4, setting at the same angle, are used to set the observable of Bob1’s weak measurement. HWP2 and HWP3, setting at $\frac{\pi}{2} - \frac{\theta}{2}$ and $\frac{\theta}{2}$ respectively, set the precision factor $G$ of Bob1’s measurement to $\cos 2\theta$.

the basis $\{|\varphi\rangle, |\varphi^\perp\rangle\}$ with +1 outcome has been implemented. By changing the angles of HWP1 and HWP4 from $\varphi/2$ to $\varphi/2 + \pi/4$, the same setup effectively implements an optimal weak measurement with -1 outcome.

Experiment - After explaining how to realize an optimal weak measurement on a polarization-encoded qubit, we now report our experiment of nonlocality sharing in a photonic system. As shown in Fig. 3(d), a standard type-II spontaneous parametric down-conversion source is used to produce a two-photon maximally entangled state $\frac{1}{\sqrt{2}}(|H_1\rangle|H_2\rangle - |V_2\rangle|V_2\rangle)$, where photon 1 is designated to Alice and photon 2 to Bob1 and Bob2. On Alice’s side, photon 1 pass through HWP6 and PBS1, which together implement a projective measurement of $\omega = X$ or $\omega' = Z$. Bob1 lets photon 2 go through the weak measurement setup (the orange region), which implements an optimal weak measurement of $\mu = \cos \gamma X + \sin \gamma Z$ or $\mu' = \cos \gamma X - \sin \gamma Z$ with a precision factor $G$, where $\gamma$ is controlled by HWP1 and HWP4, and $G$ by HWP2 and HWP3. After the weak measurement, photon 2 is then passed to Bob2, who implements a projective measurement of $\nu = \cos \delta X + \sin \delta Z$ or $\nu' = \cos \delta X - \sin \delta Z$ with HWP5 and PBS2, where $\delta$ is controlled by the angle of HWP5. Both $\gamma$ and $\delta$ are properly chosen to suit the precision factor $G$ so as to maximize $\min(I^{(1)}_{\text{CHSH}}, I^{(2)}_{\text{CHSH}})$ given a certain $G$ value.

In our experiment, we have chosen 9 different values of $G$ and set the measurement bases accordingly. The two CHSH parameters $I^{(1)}_{\text{CHSH}}$ and $I^{(2)}_{\text{CHSH}}$, which are constructed using the measurement results, are shown in Fig. 4. Theoretically, $\min(I^{(1)}_{\text{CHSH}}, I^{(2)}_{\text{CHSH}})$ reaches its maximal value 2.263 when $G = 0.8$. Experimentally, we get $I^{(1)}_{\text{CHSH}} = 2.214 \pm 0.011$ and $I^{(2)}_{\text{CHSH}} = 2.269 \pm 0.019$, which are both more than 10 standard deviations above the classical bound. Even when the strength of Bob1’s measurement reaches a very high level, say $G = 0.96$, we can still observe a double Bell-CHSH violation with $I^{(3)}_{\text{CHSH}} = 2.028 \pm 0.024$ and $I^{(4)}_{\text{CHSH}} = 2.047 \pm 0.020$, which is impossible using the original protocol. In general, our experimental results are in good agreement with the theoretical predictions. The imperfections of the results mainly due to the imperfect photon source and the non-ideal interferometer.

Conclusions - In summary, we have experimentally demonstrated nonlocality sharing among three observers using weak measurement with a wide range of measurement strength. Unlike previous experiments, where double violations of Bell-CHSH inequality can only be
observed using a weak measurement with moderate strength, we have demonstrated that double violations can be achieved with very “strong” measurement as long as it is still a weak measurement. Our results not only can shed new light on the interplay between nonlocality and quantum measurements, especially the realization of nonlocality sharing via weak measurements, but can also find applications in unbounded randomness certification, quantum coherence, quantum steering and quantum communication network.

METHODS

Optical weak measurement in H/V basis As shown in Fig. 3(b), suppose the state after HWP1 and before the first beam displacer (BD) is $|\psi\rangle = \alpha|H\rangle + \beta|V\rangle$. After passing through the first BD, the state can be written as $\alpha|H\rangle|l\rangle + \beta|V\rangle|u\rangle$, where $|u\rangle$ and $|l\rangle$ denote the upper and lower spatial modes, respectively. HWP3 is used to converts $|H\rangle$ to $\cos \theta |H\rangle + \sin \theta |V\rangle$. HWP2 and the 0° half-wave plate can convert $|V\rangle$ to $\sin \theta |H\rangle + \cos \theta |V\rangle$. As a result, after passing through HWP2, the 0° half-wave plate and HWP3, the state becomes $\alpha(\cos \theta |H\rangle + \sin \theta |V\rangle)|l\rangle + \beta(\sin \theta |H\rangle + \cos \theta |V\rangle)|u\rangle$. The following three BDs and four 45° HWPs realize a swapping operation between the polarization and the spatial state, i.e., $|H\rangle|l\rangle \rightarrow |H\rangle|l\rangle$, $|H\rangle|u\rangle \rightarrow |V\rangle|l\rangle$, $|V\rangle|l\rangle \rightarrow |H\rangle|u\rangle$ and $|V\rangle|u\rangle \rightarrow |V\rangle|u\rangle$. As a result, the final output state becomes $\alpha|H\rangle(\cos \theta |l\rangle + \sin \theta |u\rangle) + \beta|V\rangle(\sin \theta |l\rangle + \cos \theta |u\rangle)$. By measuring the spatial qubit, one can realize an optimal weak measurement on the polarization qubit in $H/V$ basis with a precision factor $G = \cos 2\theta$.

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