The colormagnetic confinement in QCD

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Abstract

Colormagnetic confinement as a natural component of the QCD confinement is explained and treated in the framework of the Field Correlator Method. For quarks and gluons in hadrons the effects of the colormagnetic confinement are discussed at zero temperature, where it contributes to the spectrum properties and can create its own bound states, while at nonzero temperature in the EoS of the quark gluon plasma the colormagnetic confinement plays a dominating role. Its properties in the QCD thermodynamics are discussed in detail. In particular the CM string tension and the Debye screening mass calculated in FCM are compared with lattice data.

1 Introduction

The confinement in QCD is a general phenomenon which establishes main features of our Universe, yielding more than 90 percent of its visible mass. The theory of colormagnetic confinement (CMC) and colorelectric confinement (CEC) based on the Field Correlator Method (FCM) has been formulated in the form of analytical approach [1, 2, 3, 4, 5] and studied numerically by the lattice data [6, 7, 8, 9, 10], which support the good convergence of the method. Since that time the CEC was studied in detail and its basic mechanism -FCM where correlators of field strength (FS) are calculated selfconsistently via integrals of FS – was exploited in numerous analysis of experimental and lattice data -see [5] for recent review. The role of CMC
is less evident since at $T = 0$ it appears as the spin-dependent corrections in hadron spectra and reactions. Moreover, the CMC also defines the basic interaction of quarks and gluons at high temperature and in the quark-gluon plasma. The CMC is provided by the colormagnetic field correlators in the same way as the standard CEC arises from the colorelectic correlators and at zero temperature both correlators coincide. However, they are yielding completely different contributions to the hadron dynamics: the CEC establishes the main part of the visible hadron mass of the Universe, while at zero temperature the main role of the CMC is providing one half of the vacuum field energy and establishing the spin and the momentum-dependent terms in the hadron Hamiltonian. This provides important corrections in the hadron spectra as will be discussed below. With increasing temperature the roles of both confining forces change drastically: the CEC is decreasing and finally disappears at the critical temperature, $T \geq T_c$, while the CMC grows (with the CMC string tension increasing as $T^2$) and plays an important role in the quark-gluon dynamics. For that reason the analysis of the quark-gluon plasma requires the account of the CMC.

The important role in the analysis of the CMC was always played by lattice analysis \[11, 12, 13\] which revealed from the very beginning that CMC is not like CEC for temperatures $T > T_c$ and moreover the colormagnetic string tension $\sigma_s$ at large $T$ is proportional to $T^2$ \[12, 13\]. It was understood that CMC could be analyzed in the 3d model with the adjoint Higgs field \[14, 15, 16, 17\]. Moreover the analysis of the gluon screening mass has allowed the lattice measurement of the nonperturbative Debye mass $m_D(T)$ \[18, 19, 20\].

We shall demonstrate below the analytic calculation of both CMC string tension $\sigma_s$ and $m_D(T)$ in the framework of FCM and display a good agreement with lattice data. At this point it is important to stress that FCM enables one to calculate the field correlators (both CM and CE) as two-gluon Green’s functions (gluelumps) $G_{gl}$ where gluons interact via CM and CE confinement and the resulting equation for the string tension is an integral of $G_{gl}$ with gluons interacting via the same $\sigma$. This gives a check of selfconsistency of the whole method and as we shall show below it enables one to calculate $\sigma_s(T)$ without extra parameters in agreement with lattice data. Summarizing the additional features of the CMC (being the important part of the general nonperturbative FCM method), one discovers the strong spin-orbit force (“the Thomas term”) in hadron spectroscopy, the strong coupling effect in the qgp, the origin of the effective screening mass at $T > T_c$, the resolution
of the Linde problem in the high $T$ perturbation theory. It is the purpose of the present paper to summarize the existing knowledge of the CMC and to propose possible developments in this field, which can be checked both numerically and experimentally. For many years the confinement theory was also using different ideas based on the geometrical or quasiclassical objects in the QCD vacuum, such as monopoles or center vortices (see \cite{21, 22} and \cite{23, 24} as reviews). In principle this can be accommodated in the FCM as an additional (might be unnecessary) detailisation of the FCM correlators, whereas the method can keep its form. In this sense one consider this approach as an attempt to understand why at all field strength correlators have nonzero vacuum averages in the confining phase. The present paper gives an answer to this question and predict CM correlators at low and high temperatures both in the confining and deconfined phase. The plan of the paper is as follows. In the next section we introduce the field correlators responsible for the colorelectric (CE) and colormagnetic (CM) confinement and construct the hadron Hamiltonian containing both effects. In section 3 we specifically study the CM effects in hadrons in the phase of the CE confinement. The section 4 is devoted to the CM interactions in the CE deconfined phase where we discuss the analysis of the CM effects in quark-gluon plasma which yield the growing as $T^2$ the CMC string tension. We also analyze the standard perturbative theory in qgp and using the CMC, we distinguish and resolve the famous Linde problem. The concluding section contains the overall discussion of the results and an outcome.

2 The colormagnetic and colorelectric correlators and the QCD Hamiltonian

The CEC in the framework of FCM was exploited as a basic dynamical theory for the hadron spectra and wave functions \cite{25, 26, 27, 28, 29} with numerous applications \cite{31, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44}. To give a simple idea of the FCM we can describe the following picture of the hadron in the QCD. In QCD the quarks and gluons propagate along Wilson lines and the propagation of all hadrons can be described by the corresponding Wilson loops, which according to the nonabelian Stokes theorem contain inside numerous field fluxes which in the certain gauge (“the generalized contour gauge”, see \cite{3} for details and discussion) can be written simply as
$F_{\mu,\nu}(z)d\sigma_{\mu,\nu}(z)$, actually, the integral of those.

In the FCM one considers these fluxes, with all $z$ inside the Wilson loop, as a statistical medium with the field correlators, defined by the average values of $\langle F(x)F(y)\rangle, < F(x)F(y)F(z) >$. It was proved that in FCM the lowest correlators $\langle F(x)F(y)\rangle$ are dominant, while the higher ones contribute less than 5 percent in agreement with detailed lattice data $[5]$. This result refers to the time-like $F_{i4} = E_i$, as well as to the space-like $F_{ik} = e_{iki}H_l$ field strengths. This stochastic concept, fully supported by existing data, will be the basis of our analysis here, mainly devoted to the CMC, described by the colormagnetic field correlators $\langle H_i(x)H_k(y) \rangle$, and the resulting physical phenomena.

We start with the definition of the field correlators, both colorelectric and colormagnetic.

\[
\frac{g^2}{N_c} \langle \langle T r E_i(z) \Phi(z, z')E_j(z') \Phi(z', z) \rangle \rangle = \delta_{ij} \left[ D^E(u) + D_1^E(u) + u^2 \frac{\partial D^E}{\partial u^2} \right] + u_i u_j \frac{\partial D^E}{\partial u^2},
\]

(1)

\[
\frac{g^2}{N_c} \langle \langle T r H_i(z) \Phi(z, z')H_j(z') \Phi(z', z) \rangle \rangle = \delta_{ij} \left[ D^H(u) + D_1^H(u) + u^2 \frac{\partial D^H}{\partial u^2} \right] - u_i u_j \frac{\partial D^H}{\partial u^2},
\]

(2)

\[
\frac{g^2}{N_c} \langle \langle T r H_i(z) \Phi(z, z')E_j(z') \Phi(z', z) \rangle \rangle = e_{ijk} u_4 u_k \frac{\partial D^{EH}}{\partial u^2}.
\]

(3)

Here the resulting correlators $D^E(u), D^H(u)$ define the confinement interaction – the string tensions- in the planes $(i4), (ik)$, namely,

\[
\sigma^{E(H)} = \frac{1}{2} \int d^2 z D^{E(H)}(z).
\]

(4)

It is important that at zero temperature all Euclidean planes are equivalent and both colormagnetic (CM) and colorelectric (CE) correlators coincide, as well as the string tensions, and each hadronic system is under the action of both colorelectric and colormagnetic forces. However, above the critical temperature $T_c$ the colorelectric correlators vanish and the QCD vacuum is fully in the realm of the CM correlators (apart from the perturbative
interactions). It is the purpose of the present paper to study specifically the
effects of the colormagnetic interactions both, below $T_c$ – the colorelectric
confinement region, and above $T_c$ – in the CMC region. In this section
we derive the Hamiltonian with the CEC and CMC in the quark-antiquark
systems.

The Hamiltonian for heavy quarks in terms of the field correlators was
written in [25]. To derive the Hamiltonian in the case of light quarks one
can use the relativistic Fock–Feynman–Sawinger path integral method [26],
which relates the integral representation of the $q\bar{q}$ Green’s function with the
Hamiltonian in terms of the virtual quark (antiquark) energies $\omega_1$ ($\omega_2$). Its
general form was elaborated in [29, 33]. We follow below the form of [31],
where the result is presented in terms of $\sigma_H, \sigma_E$. The general form of the
Hamiltonian consists of the radial kinetic term $H_0$, the orbital motion term
$H_l$, the spin-dependent term $H_{sd}$, the perturbative contribution $H_{pert}$ and the
self-energy term $H_{se}$. To make the complicated general form of the Hamilto-
nian more simple it is convenient to introduce the extra parameters (called
“einbeins”), which are defined via the solution of the subsidiary equations
for the resulting energy (mass) eigenvalues,

$$\frac{\partial M(\lambda_i)}{\partial \lambda_i} = 0, \lambda_i = \omega_1, \omega_2, \nu(\beta), \eta. \quad (5)$$

The Hamiltonian can be written as

$$H(\omega_1, \omega_2, \nu) = H_0 + H_{int} + H_l + H_{sd} + H_{pert} + H_{se}. \quad (6)$$

Here $H_0$ contains only the radial kinetic motion and is written in terms
of quark and antiquark effective energies $\omega_1, \omega_2$. In what follows we shall
discuss the case of the equal masses, with correspondingly $\omega_1 = \omega_2 = \omega$. The
case of general mass relations can be found in [29].

$$H_0 = \omega + \frac{p_r^2 + m^2}{\omega}. \quad (7)$$

$$H_{int} = \int_0^1 d\beta \left[ \frac{\sigma_1^2 r^2}{2\nu} + \frac{\nu}{2} + \sigma_2 r \right]. \quad (8)$$

Here $\sigma_1 = \sigma_H + \eta^2 (\sigma_H - \sigma_E), \sigma_2 = 2\eta (\sigma_E - \sigma_H)$. The orbital part of the
Hamiltonian, $H_l$ depends not only on the effective energies but also on the
colorelectric and colormagnetic string tensions, expressed via the einbein factor \( \nu(\beta) \), see \([29, 30]\),

\[
H_l = \frac{L^2}{r^2[\omega + 2 \int_0^1 d\beta (\beta - 1/2)^2 \nu(\beta)]}. \tag{9}
\]

The most complicated term of the Hamiltonian is the spin-dependent part, derived in \([25, 37, 39, 40]\),

\[
H_{sd} = \left( \frac{\sigma_1^2}{4\omega_1^2} + \frac{\sigma_2^2}{4\omega_2^2} \right) L_i \frac{1}{r} (V_0'(r) + 2V_1'(r)) + \frac{\sigma_1^4 + \sigma_2^4}{2r\omega_1\omega_2} V_2'(r) + \\
+ \frac{3\sigma_1 n_i \sigma_2^2 n_k - \sigma_1^4 \sigma_2^2}{12\omega_1\omega_2} V_3(r) + \frac{\sigma_1^4 \sigma_2^2}{12\omega_1\omega_2} V_4(r). \tag{10}
\]

Here the spin-dependent potentials are expressed via the field correlators \( D^E, D^I_{1}, D^H, D^I_{1} \), where the last two correlators appear due to the CMC, namely,

\[
V_0'(r) = 2 \int_0^\infty d\nu \int_0^r d\lambda D^E(\lambda, \nu) + r \int_0^\infty d\nu D^E_1(r, \nu), \tag{11}
\]

\[
V_1'(r) = -2 \int_0^\infty d\nu \int_0^r d\lambda (1 - \lambda/r) D^H(\lambda, \nu), \tag{12}
\]

\[
V_2'(r) = \frac{2}{r} \int_0^\infty d\nu \int_0^r \lambda d\lambda D^H(\lambda, \nu) + r \int_0^\infty d\nu D^H_1(r, \nu), \tag{13}
\]

\[
V_3(r) = -2r^2 \frac{\partial}{\partial r^2} \int_0^\infty d\nu D^H_1(r, \nu), \tag{14}
\]

\[
V_4(r) = 6 \int_0^\infty d\nu \left[ D^H(r, \nu) + (1 + \frac{2r^2}{3} \frac{\partial}{\partial \nu^2} D^H_1(r, \nu)) \right]. \tag{15}
\]

Here the field correlators depend on only one variable: \( D(x, y) = D(\sqrt{x^2 + y^2}) \). One can see important contribution of the CMC terms, \( D^H \) and \( D^I_1 \), which define the spin-spin forces, and one may wonder what is the contribution of their purely nonperturbative parts. To this end we are using \([8]-[12]\) in the large \( r \) region and obtain the estimates,

\[
V_0'/r = 1/r \sigma^E + O(1/r^2), \tag{16}
\]

\[
V_1'/r = -1/r \sigma^H + O(1/r^2), \tag{17}
\]
while $V_3, V_4$ terms decay exponentially at large $r$. As a result at large $r$ one obtains the dominant contribution for the spin-orbit force $V_{ls}$ in the case of equal quark and antiquark mass (the first two terms in (7),

$$V_{ls} = \frac{SL}{2\omega^2 r}(\sigma^E - 2\sigma^H) + O(1/r^2). \tag{18}$$

This expression can be compared with purely perturbative contributions to the spin-dependent interactions, where the CMC and CEC do not appear, which, however, can be derived from the correlators $D_{1E,H}$. To this end we can identify the purely perturbative spin-dependent contributions $V_{ip}(i = 1, 2, 3, 4)$, namely [32],

$$V_{1p} = 0, \quad V_{2p}' = \frac{4\alpha_s}{3r^2}, \quad V_{3p} = \frac{4\alpha_s}{r^3}, \quad V_{4p} = \frac{32\pi\alpha_s\delta^3(r)}{3}. \tag{19}$$

Here we have suppressed the indices $E, H$ in the perturbative expressions. Note that the strong coupling constant $\alpha_s$ is well defined in the coordinate space, since the QCD constant $\Lambda_{\overline{MS}}(n_f)$ is now known from experiment [45, 46]. Finally, we need to take into account the self-energy contribution to the Hamiltonian $H_{se}$, which is a definite negative constant, produced by the $\sigma_{\mu\nu}F_{\mu\nu}$ part in the Green’s function [47, 48],

$$H_{se} = -\frac{4\sigma E}{\pi\omega^0}\chi(m_q). \tag{20}$$

where $\omega^0$ is the stationary value of the effective quark energy, obtained as in [7], and $\chi(m_q) = 0.9$ for the zero quark mass and $\chi = 0.80$ for the $s$ quark. At this point we stress that the resulting Hamiltonian (6) does not contain any fitting constants and is fully defined by the field correlators $D^E, D^1_E, D^H, D^1_H$, while its spin-independent part is defined by only $\sigma_E, \sigma_H$. This is specifically true for the FCM Hamiltonian, while all other existing approaches exploit numerical fitting constants or functions. In the next chapter we shall discuss the comparison of our theoretical results with experimental and lattice data, making a special emphasis on the role of the CMC contributions.

3 The colormagnetic interaction in hadrons at zero temperature

We start our analysis of the resulting Hamiltonian (6) with the spin-independent part and firstly consider the case of $L = 0$. At $T = 0$ both string tensions are
equal $\sigma_E = \sigma_H$, giving $H_l = 0$ and varying over $\nu, \eta$, one obtains the simple result,
\[ H_{int} = (\sigma_1 + \sigma_2)r = \sigma_E r. \]  
(21)

However, taking into account that $H_l$ also contains $\nu$ and therefore should participate in the varying (optimization) process, and keeping unequal $\sigma_E, \sigma_H$, one obtains approximately \[ H_{int} + H_l = \eta \sigma_E r + (1/\eta - \eta)\sigma_H r + \omega y^2, \eta = \frac{y}{\arcsin y}, \]  
(22)

where $y$ is a solution of the equation,
\[ \sqrt{L(L+1)} = \frac{1}{4y}(1 + \eta^2(1 - \sigma_E/\sigma_H))(1/\eta - \sqrt{1 - y^2}) + \frac{\omega y}{\sigma_H r}. \]  
(23)

One can see in (23) that for $\sigma_E = \sigma_H$ and $L = 0$ one has $y = 0$ and the resulting $H_{int} + H_l = \sigma r$, while for $L > 0$ the presence of the parameter $\nu$ in the denominator of (9) (which denotes the string contribution to the rotating mass) brings the so-called string correction in the Hamiltonian. For example, in the heavy quark system this gives $\Delta H_l = -\frac{L^2}{6m^2r}$. One can see that $\sigma_H$ plays an important role in the hadron dynamics at $T = 0$.

Turning to the spin-dependent dynamics one can write the most important nonperturbative contribution in (11); the analysis of the resulting expressions for the spin-dependent potentials, made in \[32, 37\], shows that the nonperturbative CMC contributions to the tensor and spin-spin forces are strongly suppressed, while the spin-orbit forces are dominated by them. Indeed, writing the spin-orbit term from (11), (12) and neglecting the terms $D_E^1, D_H^1$,
\[ V_{so}(r) = \left(\frac{S_1L_1}{2\omega_1^2} - \frac{S_2L_2}{2\omega_2^2}\right) = 1/r(V'_0 + 2V'_1), \]  
(24)

then neglecting $D_E^1$, which produces small contribution at low values of $r$, one has
\[ 1/rV'_0 = 2/r \int_0^\infty d\tau \int_0^r d\lambda D_E(\tau, \lambda); 2/rV'_1 = -4/r \int_0^\infty d\tau \int_0^r d\lambda D_H(\tau, \lambda)(1-\lambda/r). \]  
(25)

At zero temperature $\sigma_E = \sigma_H$ and due to $D_H$ one obtains the full contents of the famous negative Thomas term \[49\], which was the object of numerous
Indeed the phenomenological Thomas term for heavy quarks \[49\], \(V_{so} = -\frac{\sigma_{SL}}{2m^2r}\), is produced by both the CMC and CEC, connected by the Gromes relation \[51\] at \(T = 0\) with \(\sigma = \sigma^E = \sigma^H\) (see \[25\], \[40\], \[37\] for more details). In this way one can see that the CMC secures the correct behavior of the spin-orbit forces in hadrons. To understand how it works in reality we can calculate the nonperturbative spin-orbit matrix element for the \(nP\)-states with the radial excitation \(n\): \(a_{so}(nP) = -\frac{\sigma_{so}(1/r)}{2\omega^2(nP)}\), where \(\omega(nP)\) is the effective quark energy, defined in \(5\). The analytically computed values of \(a_{so}(nP)\) for the ground \((n = 1)\) states and different \(q\bar{q}\) systems are given below in Table 1.

| \(q\bar{q}\) | \(n\bar{n}\) | \(c\bar{c}\) | \(b\bar{b}\) |
|---|---|---|---|
| \(a_{so}^{nP}(1P)\) (in MeV) | -88 | -13.3 | -2.3 |
| \(<1/r> (1P)\) (in GeV\(^1/3\)) | 0.241 | 0.394 | 0.559 |
| \(r^{-3}\) (in GeV) | 0.0271 | 0.120 | 0.448 |
| \(a_{so}^{tot}(1P)\) (in MeV) | 41 (abs) | 34.0 (35.0 ± 0.2) | 13.3 (13.6 ± 0.7) |

Here one can see strong decrease of the nonperturbative spin-orbit term with the growing quark mass, which is very small in bottomonium, whereas in a light meson its magnitude is large, providing decreasing of the fine-structure splitting, in agreement with the experimental data.

It is now interesting to compare our results for glueballs with the lattice data \[52\], as it was done in \[40\]. For glueballs the total scheme of the spin-dependent forces is the same as for mesons, given above, except that all field correlators and the string tension are \(9/4\) times larger. The comparison of the FCM prediction for the states \(0^{-+}, 2^{-+}\), split by the spin-orbit interaction, is as follows \[40\] (in GeV): \(M_{glb}(0^{-+}) = 2.56, M_{glb}(2^{-+}) = 3.03\), which can be compared with the lattice data \[53\]: 2.59 GeV and 3.1 GeV, respectively. Note that the FCM calculations do not contain any fitting parameters, while the overall negative constant in the Hamiltonian in \(5\) is calculated via the string tension \[17\], \[48\]. Also in the FCM there is no fitting parameters for
all low-lying mesons \[54\] and only highest states need corrections due to so-called “flattening” of the confinement potential, which occurs due to holes in the film, produced by the pair creation process \[54\]. This is in contrast to the well-known calculations of hadron masses \[55\], where multiple fitting constants are used and the overall subtraction constant is introduced. We would like to underline that in the FCM the negative correction \(H_{se}\) is calculated via string tension and the quark kinetic energies.

Summarizing one can say that the CMC defines the important part of the strong spin-orbit interaction in hadrons at zero temperature, while the the CEC defines the linear confinement interaction, and the perturbative QCD is mostly responsible for the short-range spin-spin forces.

4 The CMC at finite temperature and in the quark-gluon plasma

One can consider the \(T > 0\) region in two aspects:

1. as an individual hadron physics in the regions with \(\sigma_E < \sigma_H\) and in the deconfined region \(\sigma E = 0\),

2. the role of the CMC in the thermodynamics of quark-gluon plasma (qgp). Below we discuss these points in this order.

1. It was shown in \[31\] that the resulting spin-orbit potential \(24), (25)\) has the form of the attractive Thomas potential at large \(r\) and strong repulsive core at small distances, which ensures weakly coupled \(q\bar{q}\) bound states for the quark mass \(m_q > 0.22\) GeV (due to the CMC contribution); e.g. for \(s\)-quark with \(m_s = 0.22\) GeV the \(s\bar{s}\) binding energy is \(-45\) MeV and much less for the \(c\) and \(b\)-quarks. The situation for light quarks in the deconfined region is even more complicated and seems to be similar to the \(Z > g137\) critical phenomena in QED, when the central charge \(Z\) is surrounded by the plasma-like vacuum \[31\]. In difficult to develop the quantitative theory of the corresponding medium at the deconfining temperatures around 150 MeV but one can expect that these effects will give relatively small corrections at this temperature.

2. We shall turn now to the most important topic of the role of the CMC in the quark-gluon thermodynamics at \(T > T_c\) and show that the CMC will provide the following basic features in this region:
A) the growth of $\sigma_H(T)$ with the temperature, $\sigma_H(T) = \text{const} \ T^2$; B) the effects of the CMC on the quark-gluon medium which gives a special CMC factor in the pressure of quarks and gluons; C) the mass correlation parameter (the Debye mass) defining the gluon exchange forces in qgp in the background of CMC vacuum; D) the violation of the standard perturbation theory in the qgp, when the $g^6$ term contains the infinite series of contributions - the Linde problem. We shall below discuss these topics term by term.

4.1 (A). The colormagnetic string tension $\sigma_H$ at nonzero temperature.

As was discussed in the Introduction this topic was actively studied on the lattice [11, 12, 13, 14] where also the model containing an adjoint Higgs was exploited with similar results [15, 16, 17]. On the theoretical side one can express in the framework of FCM the CM string tension via the gluelump Green’s function, where gluelump is the system of 2 gluons and adjoint Wilson line connected by adjoint strings. Actually gluelumps define the confining dynamics in both CE and CM strings in a selfconsistent way since CE and CM string tensions are expressed via integrals of the corresponding gluelump Green’s functions, where interaction is given again by the CE and CM string tensions. The behavior of $\sigma_H$ near $T_c$ was found in [56] in good agreement with lattice data. In the large $T$ region the FCM allows to define it analytically [57] and compare with lattice data [58] in [59] finding a good agreement. We shall be interested in the region of temperatures $T > T_c$ and exploit the standard definition 4 of $\sigma_H$ via the CM correlator, 

\[ \sigma_H = \frac{1}{2} \int d^2 z D_H(z), \]

where $D_H$ is expressed via two-gluon Green’s function and finally via the product of interacting 4d one-gluon Green’s functions

\[ D^H(z) \sim G^{(2g)}_{4d}(z) \sim (G^{(g)}_{4d})^2. \]

It is important [56, 57] that the path integral along the 4-th axis does not contain interaction and therefore at large $T$ one arrives at the result

\[ G^{g}_{4d} = TG^{(g)}_{3d}(z) + K_{3d}(z) : D^H(z) = \frac{g^4(N^2_c - 1)T^2}{2} \langle G^{2g}_{3d}(z) \rangle + ... , \quad (26) \]

where neglected terms are subleading at large $T$. As a result the CMC string tension at large $T$ can be written as

\[ \sqrt{\sigma_H(T)} = g^2 T c_{\sigma} + \text{const} , \quad c^2_{\sigma} = \frac{N^2_c - 1}{4} \int d^2 w \langle G^{2g}_{3d}(w) \rangle , \quad (27) \]
Numerically the lattice data \[58\] yield $c_\sigma = 0.566 + / - 0.013$. In FCM using \[(27)\] the integral was calculated approximately yielding as a lower limit $c_\sigma = 0.47$ in a reasonable agreement with lattice. We now turn to the region $0 < T < T_c$, where one can generalize the form of $D^H(z)$ to the nonzero $T$ region, summing over infinite $n\beta$ series ($\beta = 1/T$) \[56\], which yields at small $T$

$$\sigma_H(T)/\sigma_H(0) = \frac{\sin h(M/T) + M/T}{\cosh(M/T) - 1} = 1 + 2(1 + M/T) \exp -M/T + O(\exp -2M/T). \quad (28)$$

4.2 (B). The CMC pressure in the quark-gluon plasma

Using the relativistic path integral for the quark and gluon pressure $P_q, P_g$ \[60, 28\], one express those via the spatial loop integrals of the thermal Green’s functions of $q, g$ respectively, $G_3(s), S_3(s)$

$$P_{gl} = \frac{N_c^2 - 1}{\sqrt{4\pi}} \int_0^\infty ds \frac{s^{3/2}G_3(s)}{n} \exp \left(-\frac{n^2}{4T^2 s}\right) L^n_{adj}, \quad (29)$$

where $L^n_{adj}$ is the adjoint Polyakov loop and $G_3(s)$ is the 3d closed loop gluon Green’s function as a function of the relativistic square of distance $s$. It is clear that in the 3d closed loop the confinement is colormagnetic and the result for $q$ and $g$ Green’s functions can be written as \[61\]

$$G_3(s) = \frac{1}{(4\pi s)^{3/2}} \left(\frac{(M_{adj})^2 s}{\text{sh}(M_{adj}^2 s)}\right)^{1/2} \quad (30)$$

Here $M_{adj} = 12\sqrt{\sigma_H(T)}$. For the quark function $S_3(s)$ one should replace $M_{adj}$ by $M_f = 1/3 M_{adj}$. Substituting \[(30)\] into \[(29)\] one obtains the gluon pressure as

$$P_{gl} = \frac{2(N_c^2 - 1)}{(4\pi)^2} \sum_{n=\infty, n=1}^{\infty} L^n_{adj} \int_0^\infty ds \frac{1}{s^3} \exp \left(-\frac{n^2}{4T^2 s}\right) \sqrt{\frac{M_{adj}^2 s}{\text{sh}(M_{adj}^2 s)}}. \quad (31)$$

Here $L^n_{adj}$ can be taken from lattice \[62\] or analytic \[61\] expressions.

In Fig. 1 we show how proceeds the transition of the confined phase of glueballs into the deconfined phase of gluons with the CMC interaction in
Figure 1: Pressure $P(T)$ as function of temperature $T$ for the confined phase (Glueballs) – solid line, and for the deconfined phase (dashed line). The intersection point is at the critical temperature $T_c$.

Figure 2: The pressure $P(T)/T^4$ in the $SU(3)$ theory in the deconfined phase. The solid line is for the modified oscillator confinement Eq. (30), and filled dots are for the lattice data [63].

the gluon plasma in comparison with the lattice data from [63] (Fig. 1 from [61]).

In Fig. 2 we demonstrate the behavior of the gluon plasma at large $T$ vs the lattice data from [63] (Fig. 3 from [61]).
4.3 (C). The Debye mass in the qgp

In this section we shall show that the only gauge-invariant definition of the Debye mass in the qgp is via the CM mass, i.e. via the square root of the CMC string tension $\sigma_H$, and we shall demonstrate a good agreement between the resulting theoretical and lattice data [64, 65]. The problem of the Debye mass in the QCD standard perturbation theory (SPT) is that it cannot be defined in a gauge-invariant way and therefore one is using some approximate definitions, introducing fitting constants, e.g. in [58] the ansatz was exploited

$$m_D(T) = A g(T) T \sqrt{1 + N_f/6} \text{ with } A = 1.51, 1.42 \text{ for } N_f = 0, 2, \text{ respectively.}$$

Instead in the nonperturbative FCM one can calculate Debye mass with a good accuracy [64, 65]. To this end one defines the Debye mass from the gluon-exchange diagram between trajectories of two charges, see Fig 1 from [65]. It is clear that the gluon distorts the Wilson loop surface of two charges and this additional piece (its 3d projection) contributes being multiplied with $\sigma_H$ to the gluon action. In this way one understands that the exchanged gluon, together with its projection on the unperturbed plane of two charges, forms the gluelump [66, 67] – the system of one gluon plus another static with infinite mass. The $T$-dependent Hamiltonian for the gluelumps was derived in [64, 65] as

$$H_n = \sqrt{p_{\text{perp}}^2 + (2\pi n T)^2} + \sigma_H^{\text{adj}} r_{\text{perp}}, n = 0, 1, 2, \ldots \quad (32)$$

The corresponding gluelump screening mass spectrum was found in [64]. The lowest eigenvalue of $H_0$ is equal to $\epsilon_0 = 2.82 \sqrt{\sigma_H}$. In the next approximation one should take into account the OGE interaction in the gluelump which yields $\Delta \epsilon_0 = -5.08 a_s^{\text{eff}} T / \sqrt{\sigma_H}$. As a result one obtains for the Debye mass

$$m_D = \epsilon_0 + \Delta \epsilon_0 = 2.06 \sqrt{\sigma_H} \quad (33)$$

4.4 (D). CMC in perturbative thermodynamics of QCD

In this approach one of the problems is the resummation of the infinite series of infrared divergent gluon-loop diagrams, which are known as hard thermal loops (HTL) resummation. The perturbation theory of the qgp or purely gluon plasma at $T > T_c$ operates with amplitudes $A_n$ corresponding to diagrams with $n$ vertices, which are produced by the term in the Lagrangian

$$L_3 = g \partial_\mu a_\nu f^{abc} a^b_\mu a^c_\nu.$$ 

There are numerous studies in this field, see e.g. [68, 69]
and a recent review [70]. In this perturbative approach one does not exploit
the notion of the CMC in the deconfined phase of QCD, probably, not realiz-
ing that the deconfined phase implies the absence of the CEC but not CMC.
Instead, one can introduce in this area the notion of the “magnetic mass” of
the gluon to prevent the basic divergencies of the theory without CMC. The
latter were designated by Linde [71] and are known as the “Linde problems”.
The resolution of these problems with account of the CMC was given in [57]
and can be described shortly below as follows. The main problem pertur-

bative QCD thermodynamics (PQCDTh), which essentially operates in the
3

space, is the IR or large distance divergence, since the gluon propa-
gator

\[ G(x, y) \sim \frac{T}{|x-y|} \]

is a slowly decreasing function at large distances \( X \). Corre-
spondingly, the \( n \)-th order amplitude behaves as

\[ A_n \sim g^n T^{n/2+1} X^{n/2-3} \]

One can see that the diagrams with \( n > g6 \) gluon vertices diverge at large \( X \) –
this is just the Linde problem 1. One can see that the CMC easily solves this
problem. Indeed in 3d the Wilson loops, which cover all the diagram surface,
obey the screening law:

\[ W(C) = \exp(-\sigma_H S_{\text{min}}) \]

where \( S_{\text{min}} \) is the area of

the surface, and \( S_{\text{min}} \sim X^2 \). As a result the amplitude acquires the form

\[ A_n^{\text{conf}} = g^n T^{n/2+1} \int (dX)^{n/2-3} \exp(-\sigma_H X^2) \sim g^n T^{n/2+1} (\sqrt{\sigma_H})^{-\left(\frac{n}{2}-3\right)} \text{const.} \] (34)

Now taking into account \[ g^2 T = g^2 T \text{const.} \], one comes to the conclusion
that all diagrams with \( n > 6 \) yield \( A_n^{\text{conf}} = g^6 T^4 c_n \) (the Linde problem 2).
As a result one should sum up all the diagrams with \( n > 6 \), as it is shown in [57],
which are made finite due to the CMC.

5 Conclusions

We have considered the basic picture of the confined and deconfined matter
which is well described in terms of the colorelectric and colormagnetic field
correlators. The latter are obtained selfcosistently from the nonperturbative
QCD vacuum with the basic characteristics – the gluon condensate, \( G_2 = \frac{\alpha_s}{\pi} \langle (F_{\mu\nu})^2 \rangle \), which can be taken at the standard value, \( G_2 = 0.012 \text{ GeV}^4 [72] \).
As it was shown in [73], \( G_2 \) defines confinement characteristics in the confined
phase (CEC and CMC) with the energy density [72]
\[ \epsilon_{\text{vac}} = -\frac{11-2/3n_f}{32} G_2 \]
and the energy density in the deconfined (only the CMC) area is \( 1/2 \epsilon_{\text{vac}} \).
The corresponding pressure \( P = -F \) in the confined phase can be written
as \( P(\text{conf}) = |\epsilon_{\text{vac}}| + T^4 p_{\text{hadr}} \) and the pressure in the deconfined region is
\[ P(\text{deconf}) = \frac{1}{2} |\epsilon_{\text{vac}}| + T^4 (p_q + p_g) \]. Now from the relation \( P_{\text{conf}}(T_c) = P_{\text{deconf}}(T_c) \) one obtains the equation for the transition temperature \( T_c \) via standard expressions of \( p_h, p_q, p_g \) (with or without additional interactions, which will induce small corrections in the \( T_c \) values since \( \epsilon_{\text{vac}} \) is a dominant magnitude),

\[ T_c = \left( \frac{1/2 |\epsilon_{\text{vac}}| + p_{\text{hadr}}}{p_q + p_g} \right)^{1/4} \]  

(35)

As a result (taking free the quark, gluon, hadron pressures), one obtains in \( T_c = 240, 150, 134 \) MeV for \( n_f = 0, 2, 4 \), which is very close to the lattice data \( T_c = 240, 146, 131 \) MeV. One can see that \( \epsilon(CMC) = 1/2 \epsilon_{\text{vac}} \) plays the main role in the definition of the deconfined phase transition. As it was shown above, the role of the CMC is even more important. Namely, as it was shown above in the section 3, in the confined region the CMC ensures an important part of the interaction, (1) without CMC the sign of the nonperturbative part of spin-orbit force (the Thomas term) would have the opposite sign (see (18,25)), (2) The CMC yields the important string correction \( \Delta H = -\frac{eL^2}{6\pi r} \). We also discussed the possibility of the weakly bound hadrons due to CMC above \( T_c \) (in section 3). Finally in the deconfined region the CMC ensures 3 major effects of the qgp physics:

I. The CMC creates its own factor \( G_3(s) \), which is the main contribution (along with the Polyakov line) to the QCD thermodynamics, which is supported by the lattice calculations (Figs. 1, 2 in section 4).

II. The CMC \( (\sigma_H(T) = \text{const}T^2) \) creates the Debye mass \( m_D = 2.06\sqrt{\sigma_H} \). Finally, the CMC solves the Linde problem \( [57] \) which allows to summarize the infinite set of graphs and make the total sum finite.

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