Extended JC-Dicke model for a two-component atomic BEC inside a cavity

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We consider a trapped two-component atomic Bose-Einstein condensate (BEC), where each atom with three energy-levels is coupled to an optical cavity field and an external classical optical field as well as a microwave field to form the so-called ∆-type configuration. After adiabatically eliminating the atomic excited state, an extended JC-Dicke model is derived under the rotating-wave approximation. The scaled ground-state energy and the phase diagram of this model Hamiltonian are investigated in the framework of mean-field approach. A new phase transition is revealed when the amplitude of microwave field changes its sign.

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I. INTRODUCTION

The so-called Dicke model [1] describes the interaction of a large number of two-level systems (e.g., atoms) with a single optical mode. Since the effective light-matter coupling strength is dependent on the number of atoms \( N (\propto \sqrt{N}) \), a sufficiently large \( N \) would lead to a classical phase transition \([2, 3]\) (at finite temperatures) from the normal state, which corresponds to the atomic ground state associated with the vacuum state of the optical mode, to the superradiant-phase state, where the phenomenon of superradiance occurs with the finite scaled mean numbers of both photons and excited-state atoms. Recently, exploration of quantum phase transitions in the Dicke model at zero temperature has attracted significant attentions \([4–22]\). The drastic change across the critical point due to a qualitative change of the ground state of the Dicke model has been investigated, in the frameworks of the scaled ground state energy, macroscopic atomic excited-state population, quantum entanglement, Berry phase, quantum chaos and so on.

Apart from the standard Dicke model, several more generalized Dicke models have also been proposed and studied \([7–9, 14–20]\). In some of them \([15, 16]\), the free atoms in the standard Dicke models are replaced by the atomic Bose-Einstein condensate (BEC). A BEC describes a collective quantum state of a large number of atoms and may be used to generate a macroscopic quantum object with a longer lifetime compared to the free atoms. It has attracted much interest to combine the two condensated states (i.e. the two condensed states) after adiabatically eliminating the atomic excited state under the large single-photon detuning condition. Under the so-called rotating-wave approximation (RWA), we derive an effective, extended JC-type \([29]\) Dicke model for such a two-component BEC. The phase diagram for the derived extended JC-Dicke model is also investigated in detail.

II. MODEL AND HAMILTONIAN

The setup of the JC-Dicke model under consideration is depicted in Fig. 1. An optically-trapped Rb atomic BEC under the two-mode approximation with the atomic states \( |F = 1, m_f = -1\rangle \) (ground state \( |1\rangle \)) and \( |F = 1, m_f = 0\rangle \) (metastable state \( |2\rangle \)) is placed in a single-mode quantized optical cavity. These two states of the atomic BEC are coupled via both an external microwave field and a two-photon process \([27]\) mediated by an ancillary excited state \( |3\rangle \) (from \( 5^2P_{3/2} \)), where the single-photon transition between \( |3\rangle \) and the ground state \( |1\rangle \) (or the metastable state \( |2\rangle \)) is coupled to the quantized optical cavity field (or the external classical optical field). Here we assume that both the corresponding single-photon detunings are large and the corresponding two-photon detuning is very small. For such a case of two-photon Raman process \([28]\), the ancillary excited state can be adiabatically eliminated and the effective Hamiltonian of the two-component BEC reads \((\hbar = 1 \text{ hereafter})\)

\[
H_{\text{eff}} = H_{at} + H_{ph} + H_{at-ph} + H_{at-cl} + H_{at-at}
\]  

(1)
\[ H_{at} = \nu_1 c_1^+ c_1 + (\nu_2 + \omega_2 - \omega_1) c_2^+ c_2, \]  
\[ H_{ph} = \omega a^+ a, \]  
\[ H_{at-op} = \lambda_{\text{eff}} e^{i\omega_{\text{at}} t} c_1^+ a + \text{h.c.}, \]  
\[ H_{at-mw} = \Omega e^{-i\omega_{\text{mw}} t} c_1^+ c_1 + \text{h.c.}, \]  
\[ H_{at-at} = \frac{\eta_1}{2} c_1^+ c_1 c_1 + \frac{\eta_2}{2} c_2^+ c_2 c_2 + \eta_{12} c_1^+ c_1 c_2^+ c_2, \]

denoting the energies of the free atoms in the BEC, the free cavity field, the reduced effective interaction between the BEC with the optical fields, the interaction of the BEC with the microwave field, and the atom-atom collision interaction of the BEC, respectively. Here, \( c_{1,2} \) (\( c_1^+, c_2^+ \)) are the annihilation (creation) bosonic operators for \( |1 \rangle \) and \( |2 \rangle \), respectively; \( \omega_i \) (\( i = 1, 2, 3 \)) is the corresponding internal level-energy for atomic state \( |i \rangle \); \( \nu_l = \int d^3r \phi_l^*(r) [-\nabla^2/2m + V(r)]\phi_l(r) (l = 1, 2) \) is the trapped frequency for the states \( |1 \rangle \) and \( |2 \rangle \) with \( V(r) \) being the trapped potential, \( m \) the atomic mass, and \( \phi_l(r) \) the corresponding condensate wavefunction; \( a \) (\( a^+ \)) is the annihilation (creation) operator of the cavity mode with the frequency \( \omega_c = \lambda_{\text{eff}} q_{23}/\Delta \) is the reduced effective coupling strength for two-photon Raman process, where \( q_{13} (q_{23}) \) is the corresponding coupling strength between the BEC and the quantized cavity field (classical optical field), \( \Delta \) is the large single-photon detuning: \( \Delta = \omega_3 - (\nu_2 + \omega_2) - \omega_1 \gg \{q_{13}, q_{23}\} \) with \( \omega_1 \) the frequency of the classical optical field; \( \Omega \) is the corresponding coupling strength between the BEC and the microwave field (with the frequency \( \omega_{\text{mw}} \)): \( \eta_1 = (4\pi\rho_1/m) \int d^3r |\phi_1(r)|^4 \) and \( \eta_{12} = (4\pi\rho_{12}/m) \int d^3r |\phi_1^*(r)\phi_2(r)|^2 \) with \( \rho_1 \) and \( \rho_{12} \) \((= \rho_2) \) the intraspecies and the interspecies s-wave scattering lengths, respectively. It is remarked that the RWA has been used for all optical/microwave fields coupling to the atomic BEC.

By using the Schwinger relations
\[ J_+ = c_1^+ c_1, \quad J_- = c_2^+ c_2, \]  
\[ J_z = \frac{c_2^+ c_2 - c_1^+ c_1}{2}, \]
which fulfill
\[ [J_+, J_-] = 2J_z, \quad [J_z, J_{\pm}] = \pm J_{\pm}, \]
the Hamiltonian \( H \) can be written as
\[ H_{\text{eff}} = \omega a^+ a + \omega_0 J_z + \frac{\eta}{N} J_z^2 \]
\[ + [\Omega e^{-i\omega_{\text{mw}} t} + \frac{\lambda}{\sqrt{N}} e^{i\omega_{\text{at}} t}] J_z + \text{h.c.}, \]
where
\[ N = c_2^+ c_2 + c_1^+ c_1 \]
is the number of the atoms,
\[ \omega_0 = \nu_2 + \omega_2 - \nu_1 - \omega_1 + \frac{N - 1}{2} (\eta_2 - \eta_1), \]  
\[ \eta = (\eta_1 + \eta_2 - \eta_{12})N, \]  
\[ \lambda = \lambda_{\text{eff}} \sqrt{N}, \]
and the constant term
\[ \text{const} = \frac{N}{2} \left[ (\nu_2 + \omega_2 - \omega_1 - \frac{\eta_2}{2} + \gamma_2 N) + (\nu_1 - \frac{\eta_1}{2} + \gamma_1 N) \right] \]
can be neglected in the following consideration.

For \( \Omega \neq 0 \), to eliminate the time-dependence in Hamiltonian \( H \), we perform a unitary transformation \( U = \exp[-i\omega_{\text{mw}} J_z t - i(\omega_{\text{mw}} + \omega_1) a^+ a t] \) and obtain an effective Hamiltonian
\[ H = \omega_0 a^+ a + \omega_0 J_z + \frac{\eta}{N} J_z^2 \]
\[ + \left( \frac{\lambda}{\sqrt{N}} a J_+ + \Omega J_z + \text{h.c.} \right), \]
where \( \omega_0 = (\omega - \omega_{\text{mw}} - \omega_1) \) and \( \omega_0 = (\omega - \omega_{\text{mw}} - \omega_1) \) are the effective frequencies in the rotating frame, in which \( H \) is independent of time. To the best of our knowledge, Hamiltonian \( H \) appears to be a new one in literatures, and thus we call it as the extended JC-Dicke model.

### III. Mean-Field Ground State Energy

We now look into the ground-state properties of Hamiltonian \( H \) and the corresponding quantum phases as well as their transitions. We here consider the case of positive \( \omega_\alpha \), where the stable ground state is anticipated for
we here introduce the displacements for the two shifting $\alpha$ with the complex displacement parameters $\beta$ as

$$J_+ = b\sqrt{N - b^2}, \quad J_- = \sqrt{N - b^2},$$

$$J_z = b^2 - \frac{N}{2}$$

(9)

with $[b, b^\dagger] = 1$, the effective Hamiltonian reads

$$H = \omega_0 a^\dagger a + \omega_b (b^\dagger b - \frac{N}{2}) + \frac{\eta}{N} (b^\dagger b - \frac{N}{2})^2$$

$$+ [c(\lambda a + \Omega) b] \sqrt{1 - \frac{b^2}{N} + h.c.}.$$  (10)

Similar to that used in the standard Dicke model [4], we here introduce the displacements for the two shifting boson operators as $c^\dagger = a^\dagger - \sqrt{N} \alpha^*$ and $d^\dagger = b^\dagger + \sqrt{N} \beta^*$ with the complex displacement parameters $\alpha$ and $\beta$ describing the scaled collective behaviors of both the atoms and the photons [4, 8, 12, 16]. In fact, the current method of introducing the displacements is equivalent to the mean field approach. In this framework, it is clear that $0 \leq |\beta| \leq 1$.

After expanding the terms in the square in (10), the scaled Hamiltonian can be written up to the order of $N^{-1}$ as

$$H/N = H_0 + N^{-1/2} H_1 + N^{-1} H_2,$$  (11)

where

$$H_0 = \omega_0 a^\dagger a + \omega_b (b^\dagger b - \frac{1}{2}) + \eta (b^\dagger b - \frac{1}{2})^2$$

$$- [c(\lambda a + \Omega) b] \sqrt{1 - \beta^2} + c.c.]$$  (12)

denotes the scaled constant energy, and $H_{1,2}$ denote the linear and bilinear terms, respectively. It is noted that $H_{0,1,2}$ are independent of the number of atoms $N$.

The scaled ground-state energy is just given by the scaled constant energy in the Hamiltonian

$$E^N_g(\alpha, \beta) \equiv \frac{E_g(\alpha, \beta)}{N} = H_0,$$  (13)

where the displacements $\alpha$ and $\beta$ should be determined from the equilibrium condition

$$\partial [E_g(\alpha, \beta)/N] / \partial \alpha^* = 0,$$  (14a)

$$\partial [E_g(\alpha, \beta)/N] / \partial \beta^* = 0.$$  (14b)

After some derivations, we find that $\alpha$ is given by

$$\alpha = \frac{\lambda}{\omega_0} \beta \sqrt{1 - \beta^2},$$  (15)

and $\beta$ satisfies

$$\Omega \sqrt{1 - \beta^2} = \beta \omega_b + \omega (2\beta^2 - 1)$$

$$+ \left( \frac{\Omega \beta^2}{2\sqrt{1 - \beta^2}} + c.c. \right),$$  (16)

where $w = \eta + |\lambda|^2 / \omega_0$.

From the above equation (13), it is obvious that $\Omega/\beta$ should be real since all of the parameters, except for the coupling strengths $\lambda$ and $\Omega$, are real. Here, we assume the case of real $\Omega$ for simplicity [31]. That means, $\beta$ should also be real: $-1 \leq \beta \leq 1$ and satisfy

$$0 = \omega_b \beta \sqrt{1 - \beta^2} + \omega (2\beta^2 - 1) \sqrt{1 - \beta^2}.$$  (17)

For a general real $\Omega$, the scaled ground state energy in Eq. (13) is given by the displacement $\beta$ (by using Eq. (16) to eliminate the displacement $\alpha$) as

$$E^N_g(\beta) = \omega_b (\beta^2 - \frac{1}{2}) - 2\Omega \beta \sqrt{1 - \beta^2} + w (\beta^2 - \frac{1}{2})^2.$$  (18)

The displacement $\beta$ is the non-trivial real solution for equation (17). In general, there are more than one real solutions for Eq. (17), only the one that leads to the minimal scaled ground state energy should be chosen.

We would like to remark that the displacements determined from the equilibrium equations could just make the linear term $H_1$ be 0. At the same time, the bilinear term $H_2$ makes no contribution to the scaled ground state energy. Therefore, the exact forms of $H_{1,2}$ are not needed in our analysis, and thus only the constant term $H_0$ determines fully the scaled ground state energy of the current system.

FIG. 2: (Color online) (a) The atomic displacement $\beta$ for the ground state, (b) the square of atomic displacement $\beta^2$ for the ground state (or the scaled magnetization plus 1/2: $m + 1/2$), and (c) the scaled ground state energy $E^N_g$ versus $\omega_b$ for different coupling strength $\Omega$. The energy and frequencies are in units of $w$ ($w > 0$).
IV. NUMERICAL RESULTS AND ANALYSIS

In this section, we focus on the numerical calculations of the scaled ground state energy, e.g., $E_g^N(\beta)$ in Eq. (13), and the corresponding displacement $\beta$ that makes $E_g^N(\beta)$ minimal. The minimum $E_g^N(\beta)$ (as well as the corresponding $\beta$) is determined by the three parameters: $\omega_b$, $\Omega$, and $w$.

Figure 2 plots $\beta$, $\beta^2$, and $E_g^N(\beta)$ (corresponding to the ground state of the JC Dicke model) as a function of $\omega_b$ for several values of $\Omega$. All the energies/frequencies in Fig. 2 are in units of the positive $w$. We mentioned “positive $w$” here since $w$ may be positive or negative, while the negative $w$ will lead to different results. Seen from Fig. 2 a second-order normal-superradiant phase transition at $\omega_b/w = 1$ should be expected in the absence of microwave field ($\Omega = 0$), as in the standard Dicke model 2; while in the presence of microwave field ($\Omega \neq 0$), the normal phase and the corresponding transition disappear.

FIG. 3: (Color online) The atomic displacement $\beta$ for the ground state versus the coupling strength $\Omega$ for different $\omega_b$. The frequencies are in units of $w$ ($w > 0$).

The displacement $\beta$ for the ground state is plotted as a function of $\Omega$ (in units of positive $w$) for several values of $\omega_b$ in Fig. 3. For a positive/negative $\Omega$, the corresponding $\beta$ is also positive/negative. At the point of $\Omega \rightarrow 0$, the ground-state $\beta$ may have a jump. In order to look the possible transition phenomenon at the point of $\Omega \rightarrow 0$, we plot the ground-state $\beta$ and $E_g^N(\beta)$ as the function of the parameters $\Omega$ and $w$ (in units of positive $\omega_b$) in the 3-dimensional (3D) Fig. 4. In experiments, the parameters $\Omega$ and $w$ are controllable, e.g., the former one can be easily controlled by changing the strength of the microwave field, the latter one can be controlled by adjusting the atom-atom interaction interactions by the magnetic-field or/and optical Feshbach resonance techniques 32–34. From Fig. 4(a), it is clear that the displacement $\beta$ has a jump when $w > \omega_b$ at the point of $\Omega \rightarrow 0$. Notably, the scaled ground-state energy is always continuous at the jump point, but its first derivative with respect to $\Omega$ does not, which implies a new kind of the first order phase transition at the parameter point whenever $\Omega$ changes its sign. Making the replacement: $\Omega \rightarrow -\Omega$, we can find the corresponding replacements $\beta \rightarrow -\beta$ and $E_g^N(\beta) \rightarrow E_g^N(-\beta) = E_g^N(\beta)$ according to Eqs. (17) and (18), just as seen in Fig. 4.

FIG. 5: (Color online) The atomic scaled magnetization $M$ versus $\Omega$ and $w$. The parameters are in units of $|\omega_b|$ for positive (a) and negative (b) $\omega_b$.

FIG. 6: (Color online) The $\Omega$-$w$ phase diagram for (a) the positive $\omega_b$ and (b) negative $\omega_b$.

We may define the scaled ‘magnetization’ as $M \equiv \langle J_z \rangle/N = \beta^2 - 1/2$ for the ground state. A positive magnetization $M$ means the atomic inversion population: more atoms stay in the upper state $|2\rangle$ than those in $|1\rangle$, while a negative $M$ means the opposite case. Especially, $M = -1/2$ corresponds to the normal phase: all atoms stay in the lower states. Figure 5 plots the magnetization against the parameters $\Omega$ and $w$, where Figs. 5(a) and (b) correspond respectively to the positive and negative $\omega_b$. From Fig. 5, we may define phases $P_1, 2, 3, 4$, as denoted in Fig. 5 and Table I. The line $L_9$ of normal phase and $L_{12}$ of the superradiant phase, appearing only in the absence of microwave field, are the board lines of the phases $P_{1, 2, 3, 4}$ in Fig. 5(a); so are $L'_9$ and $L'_{34}$ the board lines of $P_{3, 4}$ in Fig. 5(b). Table I shows the differences of relevant quantities for all phases in Fig. 5 [53].
We now address the points/line of quantum phase transition in the phase diagram. Point $A$ in Fig. 3(a) denotes the well-known normal-superradiant phase transition (along the transverse arrow) in the standard JC Dicke model, while point $D$ represents a similar one (corresponding to point $A$) for the case with the atomic near-inversion population when $\omega b$ is negative. The line $L_{12}$ corresponds to the phase transition line segment from $P_1$ to $P_2$, which is of the first-order, as indicated above. Intuitively, the phases $P_1$ and $P_2$ may be viewed as the para- and dia- “magnetic” phases, because $\partial M/\partial \Omega > 0$ and $< 0$ for $P_1$ and $P_2$, respectively. Correspondingly, the line segment of $L_{34}$ is the first-order phase transition line that separates $P_3$ and $P_4$, which may be viewed respectively as the dia- and para- “magnetic” phases.

### TABLE I: Relevant quantities for different phases $P_{1,2,3,4}$ in the ground state.

| Quantity | $P_1$ | $P_2$ | $P_3$ | $P_4$ |
|----------|-------|-------|-------|-------|
| $\omega b$ | $> 0$ | $> 0$ | $< 0$ | $< 0$ |
| $\Omega$ | $> 0$ | $< 0$ | $> 0$ | $< 0$ |
| $\beta$ | $(0, \frac{\Omega}{2})$ | $(-\frac{\Omega}{2}, 0)$ | $(\frac{\Omega}{2}, 1)$ | $(-1, -\frac{\Omega}{2})$ |
| $M$ | $(-\frac{1}{2}, 0)$ | $(-\frac{1}{2}, 0)$ | $(0, \frac{1}{2})$ | $(0, \frac{1}{2})$ |
| $\frac{\partial E}{\partial M}$ | $< 0$ | $> 0$ | $< 0$ | $> 0$ |
| $\frac{\partial M}{\partial M}$ | $> 0$ | $> 0$ | $< 0$ | $< 0$ |

It is worth pointing that we have neglected the anti-resonant terms and the corresponding $A^2$ terms ($A$ is the vector potential of the optical field) in the original Hamiltonian. In the standard Dicke model, it was pointed out that the anti-resonant terms would also bring an unpleasant influence near the critical point and lead to the modification of the result of the quantum phase transition \cite{4,36}. It was also pointed out that the quantum phase transition in the standard Dicke model happens in the effective ultra-strong matter-light coupling regime, where the $A^2$ term could also become very strong and may not be omitted. Notably, if the effect of the $A^2$ term were not neglected, the quantum phase transition would be impossible to happen \cite{37} in the standard Dicke model. In order to obtain the quantum phase transition in the Dicke model, it was proposed in Ref. \cite{12} to get an effective Dicke model which does not include the $A^2$ term. In the current scheme, similar to Ref. \cite{12}, we have obtained an effective extended JC-Dicke model that does not have the anti-resonant terms and the $A^2$ terms. In our original Hamiltonian, it is assumed that the matter-light couplings are much smaller than the corresponding atomic transition/optical carrier frequencies, then it is safe to neglect the anti-resonant terms and the $A^2$ terms. After the unitary transformation, the time-independent effective Hamiltonian may lead to the quantum phase transition when the matter-light coupling is comparable to the effective carrier frequencies.

### V. CONCLUSION

In conclusion, we have derived an extended JC-Dicke model for a two-component BEC coupled to the quantized optical cavity and the external classical optical field as well as a microwave field. The scaled ground-state energy and the phase diagram of this model Hamiltonian have been investigated in the framework of mean-field approach. A new first-order phase transition has also been revealed when the amplitude of microwave field changes its sign.

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[29] The so-called “JC” model is the abbreviation for Jaynes-Cummings model (see E. T. Jaynes and F. W. Cummings, Proc. IEEE **51**, 89 (1963)), which describes the coherent coupling of a single two-level atom with one mode of the quantized light field by neglecting the anti-resonant terms in the Hamiltonian. Strictly speaking, our current model including $N$ atoms is more related to the Tavis-Cummings model (see M. Tavis and F. W. Cummings, Phys. Rev. **170**, 379 (1968)). For convenience, we keep the well-known name “JC model” here.

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