BOHMIAN PARTICLE TRAJECTORIES IN RELATIVISTIC
BOSONIC QUANTUM FIELD THEORY

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We study the de Broglie–Bohm interpretation of bosonic relativistic quantum mechanics and argue that the negative densities and superluminal velocities that appear in this interpretation do not lead to inconsistencies. After that, we study particle trajectories in bosonic quantum field theory. A new continuously changing hidden variable - the effectivity of a particle (a number between 0 and 1) - is postulated. This variable leads to a causal description of processes of particle creation and destruction. When the field enters one of nonoverlapping wave-functional packets with a definite number of particles, then the effectivity of the particles corresponding to this packet becomes equal to 1, while that of all other particles becomes equal to 0.

Key words: de Broglie–Bohm interpretation, relativistic quantum mechanics, quantum field theory.

1 INTRODUCTION

The de Broglie–Bohm (dBB) interpretation of nonrelativistic quantum mechanics (QM) and relativistic quantum field theory (QFT) [1, 2, 3, 4, 5] offers a clear answer (see, e.g., Refs. [6, 7, 8]) to notoriously difficult ontological questions that arise from the conventional interpretation of QM and QFT. Yet, the current form of the dBB theory of motion is still not completely satisfactory. Relativistic wave equations of bosonic particles lead to superluminal velocities and motions backwards in time [9, 10, 11]. Similarly, the particle density of relativistic bosonic fields may be negative [4, 5]. There exist formal ways to overcome these problems by linearizing the Klein-Gordon equation [12] or by using the energy-momentum tensor to define timelike particle trajectories [13], but the resulting particle densities are not in correspondence with the conventional notion of particle in QFT [5]. This led to a conclusion that bosons do not have particle trajectories, i.e., that bosons are causally evolving fields [14, 5]. Thus, in contrast with the conventional QFT, according to the current version of the dBB theory of motion there is a great asymmetry between bosons and fermions because relativistic fermions do have particle trajectories and are not described by quantum fields [5, 11, 15].

One of the arguments in favor of the nonexistence of particle trajectories for bosons is a difficulty with a causal description of trajectories for particles that are created or destroyed or for states with an indefinite number of particles [5]. However, the same problem remains for fermion trajectories as well, so the current version of the dBB theory of fermions cannot describe observed effects of fermion creation and destruction. There is also an alternative version of the dBB theory of fermions that describes creation and destruction of fermions [14], but this version
does not incorporate particle trajectories. It has been argued that processes of particle creation and destruction cannot be described in a deterministic way \[15, 16\]. There is an attempt to describe that in a deterministic way by introducing a direct particle interaction \[17\], but it does not seem that it leads to the same statistical predictions as the conventional QFT. A recent version of the dBB theory of fermions \[18\], based on earlier work \[19, 20\], describes creation and destruction of fermions and incorporates particle trajectories.

In this paper we propose a solution of the problems with the dBB interpretation of relativistic bosons discussed above. We argue that superluminal velocities, motions backwards in time and negative densities do not lead to any inconsistencies. Moreover, it seems that these properties are desirable for a causal description of some QFT effects. After that, we propose a causal interpretation of multiparticle wave functions that result from QFT. In this interpretation, all particles that may exist in a state with an indefinite number of particles do actually exist for all time. Particles are never really created or destructed. However, to each particle, we attribute a new deterministic continuously evolving nonlocal hidden variable - the effectivity \(e\).

A particle with \(e = 0\) has the effects as if it did not exist, while that with \(e = 1\) has the effects as a particle in the usual sense. We explain how in the process of measurement all effectivities take values equal to either 0 or 1, which has the same effect as if the wave functional had “collapsed” to a state with a definite number of particles.

The paper is organized as follows. In Sec. 2 we study the Bohmian particle trajectories in “one-particle” relativistic QM. In Sec. 3 we study many-particle wave functions in interacting QFT and give a causal interpretation of them in terms of Bohmian particle trajectories. A critical discussion of our results is given in Sec. 4. In the Appendix, we present a short review of the general theory of quantum measurements in the dBB interpretation.

## 2 PARTICLE TRAJECTORIES IN RELATIVISTIC QM

### 2.1 Basic Equations

Consider a real scalar field \(\phi(x)\) satisfying the Klein-Gordon equation (in a Minkowski metric \(\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)\))

\[
(\partial_0^2 - \nabla^2 + m^2)\phi = 0.
\]

Let \(\psi = \phi^+ (\psi^* = \phi^-)\) be the positive (negative) frequency part of the field \(\phi = \phi^+ + \phi^-\). The particle current is \[21, 22, 23\]

\[
j_\mu = i\psi^* \partial_\mu \psi,
\]

where \(A \partial_\mu B \equiv A\partial_\mu B - B\partial_\mu A\). The quantity

\[
N = \int d^3x j_0
\]

represents the positive-definite number of particles (not the charge!). This is most easily seen from the plane-wave expansion \(\phi^\pm(x) = \int d^3k a(k) e^{-ikx}/\sqrt{(2\pi)^32k_0}\), because then \(N = \int d^3k a^\dagger(k)a(k)\). (For more details, see Refs. \[21, 22, 23\], where it is shown that the particle current and the decomposition \(\phi = \phi^+ + \phi^-\) makes sense even when a background gravitational field or some other potential is present.) The particle density \(j_0\) can also be written as

\[
j_0 = i(\phi^- \pi^+ - \phi^+ \pi^-)\) (where \(\pi = \pi^+ + \pi^-\) is the canonical momentum), which is the form used in Refs. \[4, 5\].

Alternatively, \(\phi\) may be interpreted not as a field containing an arbitrary number of particles, but rather as a one-particle wave function. Historically, this later interpretation was attempted...
before the former one. Contrary to a field, a wave function is not an observable. In this later interpretation, which we employ in the rest of this section, it is convenient to normalize the wave function $\phi$ such that $N = 1$.

The current (2) is conserved:

$$\partial_\mu j^\mu = 0,$$

which implies that (3) is also conserved: $dN/dt = 0$. In the causal interpretation, we postulate that the particle has the trajectory determined by

$$\frac{dx^\mu}{d\tau} = \frac{j^\mu}{2m\psi^*\psi}.$$  \hspace{1cm} (5)

The affine parameter $\tau$ may be eliminated by writing the equation for the trajectory as

$$\frac{dx}{dt} = \frac{j(t, x)}{j_0(t, x)},$$  \hspace{1cm} (6)

where $t = x^0$, $x = (x^1, x^2, x^3)$, $j = (j^1, j^2, j^3)$. By writing $\psi = Re^{iS}$, where $R$ and $S$ are real functions, we can also write all the equations above in the Hamilton–Jacobi form. Eq. (5) can be written as

$$\frac{dx^\mu}{d\tau} = -\frac{1}{m} \partial_\mu S.$$  \hspace{1cm} (7)

The Klein-Gordon equation (1) is equivalent to a set of two equations

$$\partial^\mu (R^2 \partial_\mu S) = 0,$$

$$- \frac{\partial^\mu S}{2m} \frac{\partial S}{\partial S} + \frac{m}{2} + Q = 0,$$

where (8) is the conservation equation (4), (9) is the Hamilton–Jacobi equation, and

$$Q = \frac{1}{2m} \frac{\partial^\mu \partial_\mu R}{R}$$  \hspace{1cm} (10)

is the quantum potential. From (7), (9), and the identity

$$\frac{d}{d\tau} = \frac{dx^\mu}{d\tau} \partial_\mu,$$

we find the equation of motion

$$m \frac{d^2 x^\mu}{d\tau^2} = \partial_\mu Q.$$  \hspace{1cm} (12)

It is easy to show that all the equations above have the correct nonrelativistic limit. In particular, by writing

$$\psi = e^{-imt/\sqrt{2m}} \chi$$

and using $|\partial_t \chi| \ll m|\chi|$, $|\partial^2 \chi| \ll m|\partial_t \chi|$, from (2) and (11) we find the approximate equations

$$j_0 = \chi^* \chi,$$

$$\frac{\nabla^2}{2m} \chi = i\partial_t \chi,$$

which are the usual nonrelativistic equations for the conserved probability density and for the evolution of the wave function $\chi$, respectively.

When the nonrelativistic limit cannot be applied, then the quantity $j_0$ is not positive definite, so in that case it cannot be interpreted as a probability density. For such a general situation, we give the interpretation of $j_0$ in the following subsection.
2.2 Physical Interpretation

Figure 1: A part of a boson-particle trajectory. The particle moves backwards in time from A to C. The dashed line represents a spacelike hypersurface Σ intersected by the trajectory at 3 points. The particle density \( j_0 \) is positive at the points marked by 1 and negative at the point marked by -1. The number of particles at Σ is equal to 1 because \( 1 + (-1) + 1 = 1 \). In the reinterpretation of negative densities, all particles move forwards in time and the physical number of particles at Σ is equal to 3. A pair of particles is created at C and annihilated at A. At these two points, \( j_0 = 0 \).

A typical trajectory that may arise as a solution of (6) is sketched in Fig. 1. The velocity of a particle may be superluminal. Moreover, the particle may move backwards in time, which occurs at the points where \( j_0 < 0 \). Of course, superluminal velocities and negative densities are closely related, because any vector \( u_\mu \) with \( u_0 > 0 \) may be transformed by a Lorentz transformation to \( u'_\mu \) with \( u'_0 < 0 \), if and only if \( u_\mu \) is spacelike. The 3-velocity is infinite at the points where \( j_0 = 0 \) and \( j \neq 0 \).

The motion backwards in time does not lead to causal paradoxes when one realizes that it is physically indistinguishable from a motion forwards in time but with negative energy [24]. Therefore, it is natural to reinterpret the negative densities by introducing the physical number of particles

\[
N_{\text{phys}} = \int d^3x |j_0|, \tag{16}
\]

where the physical particle density \(|j_0|\) is nonnegative. Contrary to [14], the physical number of particles is not conserved. A pair of particles, one with positive and the other with negative energy, may be created at one point and annihilated at another point. (Recall that this is not a particle-antiparticle pair because we are studying a real field \( \phi \)). Needless to say, this creation and annihilation of highly off-shell particles resembles the behavior of “virtual” particles in the conventional QFT.

Note that superluminal velocities do not contradict existing experiments because, according to the general theory of quantum measurements [1] [2] [5] (see also the Appendix), the outcome of an ideal measurement of the 4-momentum may only be an on-shell eigenvalue \( p_\mu \) attributed to the eigenfunction \( \psi_p(x) = \exp(-ip_\mu x^\mu) \) that solves (1). This is because the total final wave function that describes the entanglement between the measured particle and the measuring apparatus during an ideal quantum measurement can be written as

\[
\psi(x, y) = \int d^3p \, c_p \psi_p(x) \chi_p(y), \tag{17}
\]

where \( y \) denotes the coordinates of the measuring apparatus. The wave functions \( \chi_p(y) \) do not overlap, in the sense that \( \chi_p(y) \chi_{p'}(y) = 0 \) for \( p \neq p' \). Therefore, as explained in Refs. [1] [2] [5]
and the Appendix, the wave functions $\psi_p(x)\chi_p(y)$ constitute a set of nonoverlapping “channels”, so a particle in a “channel” behaves as if the other “channels” did not exist.

The creation and annihilation of free particles described above does not occur during an ideal measurement of a particle momentum because the trajectories are straight lines when $\psi(x) = \psi_p(x)$. Similarly, when a position of a particle is measured with good accuracy, then $\psi_p(x)$ are replaced by wave functions well localized in space, so multiple particles created during the measurement are confined inside a volume too small to allow the experimental distinction between the particles at different positions. This explains why the creation and annihilation of free particles is not seen in experiments. However, in principle, it would not be impossible to see such creations and annihilations if we knew how to perform measurements radically different from the ideal ones. Thus, our theory leads to predictions that differ from those of the conventional approach and might be observable in future.

There are also no causal paradoxes because the trajectories of particles, including those that are created or annihilated, are uniquely and self-consistently determined by specifying the fields $\phi(x)$ and $\pi(x)$ at an initial spacelike Cauchy hypersurface and one initial position of the particle at that hypersurface. It may appear, as in Fig. 1, that the specification of one particle position implies the simultaneous existence of other particles at different positions, but this nonlocal feature does not contradict the causality in the sense above.

Finally, note that the existence of motions backwards in time, which may be reinterpreted as motions with negative energy, may be regarded as a desirable property, because the black-hole evaporation is often viewed as a process in which a pair of particles is produced at the horizon, so that the particle with positive energy escapes from the horizon, while the particle with negative energy is absorbed by the black hole [25]. However, in this paper, we do not further explore the causal description of a specific process of particle creation such as the black-hole evaporation. Instead, we give a general formalism that describes the particle creation in the dBB interpretation of interacting QFT.

3 PARTICLE TRAJECTORIES IN RELATIVISTIC QFT

3.1 Wave Functionals and Many-Particle Wave Functions

In the Heisenberg picture, the hermitian field operator $\hat{\phi}(x)$ satisfies the equation of motion

$$ (\partial_0^2 - \nabla^2 + m^2)\hat{\phi} = J(\hat{\phi}), $$

where $J(\hat{\phi})$ is a nonlinear function that describes the interaction. In a more general case, $J$ may also be a function of other quantum fields or a function of background classical fields. In the Schrödinger picture, the time evolution is determined by the Schrödinger equation

$$ H[\phi, -i\delta/\delta\phi]\Psi[\phi, t] = i\partial_t\Psi[\phi, t], $$

where $\Psi[\phi, t]$ is a functional with respect to $\phi(x)$ and a function with respect to $t$. A normalized solution of (19) can be expanded as

$$ \Psi[\phi, t] = \sum_{n=0}^{\infty} \tilde{\Psi}_n[\phi, t], $$

where $\tilde{\Psi}_n$ are unnormalized $n$-particle wave functionals. Since any (well-behaved) function $\phi(x)$ can be Fourier expanded, the functionals $\tilde{\Psi}_n$ can be further expanded as [5] [26]

$$ \tilde{\Psi}_n[\phi, t] = \int d^3k_1 \cdots d^3k_n c_n(\vec{k}^{(n)}, t)\Psi_{n,\vec{k}^{(n)}}[\phi], $$

5
where \( \mathbf{k}^{(n)} = \{ k_1, \ldots, k_n \} \). The functionals \( \Psi_{n,\mathbf{k}^{(n)}}[\phi] \) constitute a complete orthonormal basis for the expansion of an arbitrary functional \( \Psi[\phi] \). (This basis generalizes the complete orthonormal basis consisting of the Hermite functions \( h_n(x) = (\sqrt{2^n n!})^{-1/2} e^{-x^2/2} H_n(x) \), where \( H_n(x) \) are the Hermite polynomials. For more details, see Ref. [26]). They have a property

\[
\int D\phi \Psi_{n,\mathbf{k}^{(n)}}[\phi] \phi(x_{1}) \cdots \phi(x_{n'}) \Psi_{n',\mathbf{k}^{(n')}}[\phi] = 0 \quad \text{for} \quad n' \neq n. \tag{22}
\]

For free fields, i.e., when \( J = 0 \) in (18), the coefficients \( c_n(\mathbf{k}^{(n)}, t) \) have a simple oscillating behavior of the form

\[
c_n(\mathbf{k}^{(n)}, t) = c_n(\mathbf{k}^{(n)}) e^{-i\omega_n(\mathbf{k}^{(n)}) t}, \tag{23}
\]

where

\[
\omega_n(\mathbf{k}^{(n)}) = E_0 + \sum_{j=1}^{n} \sqrt{k_j^2 + m^2} \tag{24}
\]

and \( E_0 \) is the vacuum energy. In this case, the quantities \( |c_n(\mathbf{k}^{(n)}, t)|^2 \) do not depend on time, which means that the number of particles (corresponding to the quantized version of (3)) is conserved. In a general case with interactions, the Schrödinger equation (19) leads to a more complicated time dependence of the coefficients \( c_n(\mathbf{k}^{(n)}, t) \), so the number of particles is not conserved.

For free fields, the (unnormalized) \( n \)-particle wave function is [27]

\[
\psi_n(\mathbf{x}^{(n)}, t) = \langle 0| \hat{\phi}(t, x_1) \cdots \hat{\phi}(t, x_n)|\Psi \rangle, \tag{25}
\]

where \( \mathbf{x}^{(n)} = \{ x_1, \ldots, x_n \} \). (The multiplication of the right-hand side of (25) by \( (n!)^{-1/2} \) would lead to a normalized wave function only if \( \Psi = \tilde{\Psi}_n \) in (21).) The generalization of (25) to the interacting case is not trivial because, in systems with an unstable vacuum, it is not obvious what the analog of the state \( \langle 0 \rangle \) in (25) is. To treat this problem correctly, we find the Schrödinger picture more convenient. Using the Schrödinger picture, (25) becomes

\[
\psi_n(\mathbf{x}^{(n)}, t) = \int D\phi \Psi_{0}[\phi] e^{-i\varphi(t)} \phi(x_1) \cdots \phi(x_n) \Psi[\phi, t], \tag{26}
\]

where \( \varphi(t) = -E_0 t \). For the interacting case, we define the wave function to be given by (25), but with a different phase \( \varphi(t) \). This phase is defined by an expansion of the form of (20):

\[
\hat{U}(t)\Psi_{0}[\phi] = r_0(t) e^{i\varphi_0(t)} \Psi_{0}[\phi] + \sum_{n=1}^{\infty} \cdots, \tag{27}
\]

where \( r_0(t) \geq 0 \) and \( \hat{U}(t) = U[\phi, -i\delta/\delta\phi, t] \) is the unitary time-evolution operator. From (20), (21), and (22) we see that, even in the interacting case, only the \( \tilde{\Psi}_n \)-part of \( \Psi \) contributes to (26), which justifies to call \( \tilde{\Psi}_n \) the \( n \)-particle wave functional.

The wave function (25) can also be generalized to a nonequal-time wave function

\[
\psi_n(\mathbf{x}^{(n)}) = S_{\{x_j\}} \langle 0| \hat{\phi}(x_1) \cdots \hat{\phi}(x_n)|\Psi \rangle. \tag{28}
\]

Here \( S_{\{x_j\}} \) denotes symmetrization over all \( x_j \), which is needed because the field operators do not commute for nonequal times. For the interacting case, the nonequal-time wave function is defined as a generalization of (26) with the replacements

\[
\begin{align*}
\phi(x_j) & \rightarrow \hat{U}^\dagger(t_j)\phi(x_j)\hat{U}(t_j), \\
\Psi[\phi, t] & \rightarrow \hat{U}^\dagger(t)\Psi[\phi, t] = \Psi[\phi], \\
e^{-i\varphi_0(t)} & \rightarrow e^{-i\tilde{\varphi}_0(t_1)}\hat{U}(t_1),
\end{align*} \tag{29}
\]
followed by symmetrization.

For free fields, the wave function (28) satisfies the equation
\[ \sum_{j=1}^{n} \left( (\partial_{\mu} \partial_{\mu})_{j} + m^{2} \right) \psi_{n}(\vec{x}^{(n)}) = 0. \] (30)

Taking the nonrelativistic limit first and then putting \( t_{1} = \cdots = t_{n} = t \) in (30), we find the multiparticle generalization of (15)
\[ \sum_{j=1}^{n} -\frac{\nabla_{j}^{2}}{2m} \chi_{n}(\vec{x}^{(n)}, t) = i\partial_{t} \chi_{n}(\vec{x}^{(n)}, t). \] (31)

This is the standard nonrelativistic multiparticle Schrödinger equation, usually postulated without reference to QFT.

### 3.2 Causal Interpretation

In the dBB interpretation, the field \( \phi(x) \) has a causal evolution determined by \[ (\partial_{0}^{2} - \nabla^{2} + m^{2}) \phi(x) = J(\phi(x)) - \left( \frac{\delta Q[\phi,t]}{\delta \phi(x)} \right)_{\phi(x)=\phi(x)}, \] (32)

where
\[ Q = -\frac{1}{2|\Psi|} \int d^{3}x \frac{\delta^{2} |\Psi|}{\delta \phi^{2}(x)} \] (33)
is the quantum potential. However, the \( n \) particles attributed to the wave function \( \psi_{n} \) also have causal trajectories. They are determined by a generalization of (11) as
\[ \frac{d\vec{x}_{n,j}}{dt} = \left( \frac{\psi_{n}^{*}(\vec{x}^{(n)}) \nabla_{j} \psi_{n}(\vec{x}^{(n)})}{\psi_{n}^{*}(\vec{x}^{(n)}) \partial_{t,j} \psi_{n}(\vec{x}^{(n)})} \right)_{t_{1}=\cdots=t_{n}=t}, \] (34)

for \( j = 1, \ldots, n \). The norms of the wave functions \( \psi_{n} \) change with time because the coefficients \( |c_{n}| \) change with time. However, in (34), the norms are irrelevant. Even when \( c_{n} \to 0 \) (for some, but not all \( n \)) during the evolution (for example, this may occur for \( t \to \pm \infty \) in a scattering process), the ratio on the right-hand side of (34) is well defined. Of course, there may exist isolated points at which the ratio diverges, but we already know how to physically interpret these points as points at which the velocity is infinite. This means that these \( n \)-particles have well-defined trajectories even when the probability (in the conventional interpretation of QFT) of their experimental detection is equal to zero. In the dBB interpretation of QFT, we can introduce a new causally evolving parameter \( e_{n}[\phi,t] \) defined as
\[ e_{n}[\phi,t] = \frac{|\tilde{\Psi}_{n}[\phi,t]|^{2}}{\sum_{n'=0}^{\infty} |\tilde{\Psi}_{n'}[\phi,t]|^{2}}. \] (35)
The evolution of this parameter is determined by the evolution of \( \phi \) given by (32) and by the solution (20) of (19), so one does not need a separate evolution equation for \( e_{n}[\phi,t] \). This parameter might be interpreted as a probability that there are \( n \) particles in the system at the time \( t \) if the field is equal (but not measured!) to \( \phi(x) \) at that time. However, in the dBB interpretation, we do not want an intrinsically stochastic interpretation. Therefore, we
postulate that $e_n$ is an actual property of the particles guided by the wave function $\psi_n$. We call this property the effectiveness of these $n$ particles. From the point of view of the conventional interpretation of QFT, this is a nonlocal hidden variable attributed to the particles. We introduce this parameter in order to provide a deterministic description of the creation and destruction of particles. We postulate that the effective mass of a particle guided by $\psi_n$ is $m_{\text{eff}} = e_n m$, and similarly for the energy, momentum, charge, and other measurable quantities that are proportional to the number of particles. This is achieved by postulating that the mass density $\rho_{\text{mass}}$ is given by

$$\rho_{\text{mass}}(x,t) = m \sum_{n=1}^{\infty} e_n \sum_{j=1}^{n} \delta^3(x - x_{n,j}(t)),$$

and similarly for the other quantities. Therefore, if $e_n = 0$, then these $n$ particles are ineffective, i.e., their effect is as if they did not exist. Similarly, if $e_n = 1$, then their effect is as they exist in the usual sense. However, since the trajectories are defined even for the particles for which $e_n = 0$, the initial condition for particle positions contains one initial condition for the particle guided by $\psi_1$, two initial conditions for the particles guided by $\psi_2$, and so on, which leads to an infinite number of initial positions. In this way, QFT is really a theory of an infinite number of particles, although some of them may be ineffective. (This resembles the conventional picture of QFT as a theory of an infinite number of particles, although some of them may be “virtual”.)

The formalism may also be generalized to a case with many different particle species described by various bosonic fields. The wave functional $\tilde{\Psi}_n$ generalizes to $\tilde{\Psi}_{\{n\}}$, where $\{n\} = \{n_1, \ldots, n_{N_s}\}$ and $N_s$ is the number of different particle species. Equation (35) generalizes to

$$e_{\{n\}}[\{\phi\}, t] = \frac{[\tilde{\Psi}_{\{n\}}[\{\phi\}, t]]^2}{\sum_{\{n'\}} [\tilde{\Psi}_{\{n'\}}[\{\phi\}, t]]^2},$$

where $\{\phi\} = \{\phi_1, \ldots, \phi_{N_s}\}$.

In experiments in which the number of particles is measured, one finds that a particle either exists or does not exist. In other words, the measured effectiveness is either 0 or 1. At first sight, this is in contradiction with our theory that allows effectivities to take any value from the compact interval $[0, 1]$. However, there is no contradiction! If different $\tilde{\Psi}_n$’s in the expansion (20) do not overlap in the $\phi$ space, then these $\tilde{\Psi}_n$’s constitute a set of nonoverlapping “channels” for the causally evolving field $\phi$. The field necessarily enters one and only one of the “channels”. From (35) we see that $e_n = 1$ for the “channel” $\tilde{\Psi}_n$ that is not empty, while $e_n' = 0$ for all other empty “channels” $\tilde{\Psi}_n'$. (This is because, owing to the assumption that different $\tilde{\Psi}_n$’s do not overlap, $\tilde{\Psi}_n' = 0$ at the configuration $\phi$ which is from the support of $\tilde{\Psi}_n$.) The effect is the same as if the wave functional $\Psi$ “collapsed” into one of the states $\tilde{\Psi}_n$ with a definite number of particles.

In a more general situation, different $\tilde{\Psi}_n$’s of the measured particles may overlap. However, the general theory of ideal quantum measurements [12] provides that the total wave functional can be written again as a sum of nonoverlapping wave functionals in the $\{\phi\}$ space, where one of the fields represents the measured field, while the others represent the fields of the measuring apparatus. In this general case, one and only one of $\tilde{\Psi}_{\{n\}}$’s in (37) becomes nonempty, so the corresponding $e_{\{n\}}$ becomes equal to 1, while all other $e_{\{n'\}}$’s become equal to 0.

The essential point is that, from the point of view of an observer who does not know the actual field configurations, the probability for such an effective “collaps” of the wave functional is exactly equal to the usual quantum mechanical probability for such a “collaps”. This is why our theory has the same statistical predictions as the usual theory. In the case in which all the effectivities are smaller than 1, which corresponds to a situation in which the wave functional
has not “collapsed” into a state with a definite number of particles, our theory is neither in agreement nor in contradiction with the standard theory. This is why the effectivity is a hidden variable. This is completely analogous to the Bohmian particle positions, which agree with the standard quantum mechanical predictions only when the wave function effectively “collapses” into a state with a definite particle position, while in other cases it is neither in agreement nor in contradiction with standard QM.

Thus our approach explains why detectors detect integer number of bosonic particles. There are also other attempts to explain this in the framework of causal interpretation of QFT \[3, 28\], but these attempts do not incorporate particle trajectories. Here we repeat that there are also stochastic approaches to explain this for all fields \[19, 21, 16\] and a deterministic approach for fermions \[18\].

Finally, it is fair to note that our approach explains why detectors detect integer number of particles only if an ideal measurement, based on nonoverlapping wave functionals, is assumed. One might consider this as a serious problem and conclude that a stochastic approach \[19, 20, 16\] better explains integer numbers of bosonic particles. However, we note that this problem is analogous to a problem with the Bohmian interpretation of a nonrelativistic particle in a harmonic potential, which will be found to have energy equal to a Hamiltonian eigenvalue $\omega(n + 1/2)$ (with $n$ being an integer) only if the energy is measured through an ideal measurement (see the Appendix for a general argument). If a theory of quantum measurements is not taken into account, then Bohmian mechanics leads to statistical predictions that agree with the conventional quantum-mechanical predictions only when the statistical predictions refer to the preferred observables. The preferred Bohmian observables are particle positions in the case of nonrelativistic QM and field configurations in the case of bosonic QFT. Particle momenta and energy in nonrelativistic QM and number of particles in bosonic QFT are not preferred observables in Bohmian mechanics, so the explanation of the conventional quantum-mechanical rules for statistical distributions of these observables requires a theory of quantum measurements.

\section{DISCUSSION}

The results of this paper offer a solution to the problems of the current version of the dBB interpretation of relativistic bosonic QM and QFT. We believe that they provide a deterministic interpretation of all physical effects of the conventional bosonic QFT, including a deterministic interpretation of the processes of creation and destruction of particles. We have explicitly presented equations for real spin-0 fields, but the generalization to complex fields and other integer spins is straightforward. In particular, particles and antiparticles resulting from a complex field $\phi$ possess the separate particle currents $j^{(P)}_\mu$ and $j^{(A)}_\mu$, respectively, such that the usual charge current is $j_\mu = i\phi^\dagger \overleftrightarrow{\partial}_\mu \phi = j^{(P)}_\mu - j^{(A)}_\mu \ [21, 22, 23]$. This means that particles and antiparticles should be treated as different particle species, even when electromagnetic interactions are present \[22\]. Refs. \[22, 23\] also contain the generalization of \[2\] to spin-1/2 fields, so it is also straightforward to generalize the results of Sec. \[2\] to fermionic particles and antiparticles. However, it is not trivial to generalize the results of Sec. \[3\] to anticommuting fermionic fields, so this will be discussed in a separate paper.

It is also fair to note that the theory proposed in this paper may not be the only consistent solution to the problems of the current version of the dBB interpretation of relativistic bosonic QM and QFT. Moreover, it is possible that some parts of the theory will turn out to be inconsistent, which will require further modifications of the theory.

For example, we propose that both particles and fields objectively exist and that the macroscopic objects are made of both, but the question of consistency of such a picture requires further
research. It is still possible that only particles or only fields should be fundamental objects in a dBB-type theory.

Also, although (34) seems to us to be the most natural generalization of (6), other generalizations are also possible. In general, particles moving according to (34) do not need to be distributed according to \( j_0 \) even if they were distributed so initially. However, this is not in contradiction with the conventional QFT simply because the conventional relativistic QFT, in general, does not make clear probability predictions for distributions of particle positions. Therefore, our theory is able to give testable predictions on phenomena on which the conventional theory is not able to do that.

The remark above on nonexistence of predictions for distributions of particle positions in QFT requires additional explanations. In Refs. [19, 20, 18], the operator of fermion-number density \( \psi^{\dagger} \psi \) is defined, which, in turn, leads to predictions for distributions of particle positions in fermionic QFT. These predictions do not need to agree with the predictions that result from our theory, which might be considered as a problem for our theory. However, we do not consider this as a serious problem because, contrary to the claim in Refs. [19, 20], we do not consider the interpretation of the operator \( \psi^{\dagger} \psi \) above as a part of the conventional interpretation of fermionic QFT. Instead, by the conventional interpretation (see, e.g., Ref. [29]) we understand the interpretation based on taking the normal ordering of the product \( \psi^{\dagger} \psi \). This, owing to the anticommutative nature of fermionic fields, leads to an operator with both positive and negative eigenvalues. Such normal-ordered operator is interpreted as the operator of charge density, which, in turn, leads to predictions for distributions of charge, not of particles. For example, if the probability of finding charge at some point is equal to zero, it tells us nothing about the probability of finding particle-antiparticle pairs at that point. Similarly, for complex bosonic fields, the operator \( i \phi^{\dagger} \frac{\partial}{\partial \phi} \) defines the charge density, not the particle density. For more details on the difference and similarities between the charge density and the particle density in QFT, see Refs. [21, 22, 23].

The conventional QFT has definite predictions on angular distributions of particles produced in a scattering process, assuming that the particles are found in states with definite momenta. This assumption corresponds to wave functions of the form of (17). In such a case, our theory predicts that particles move according to classical trajectories (straight lines) determined by their momenta, so the predictions on angular distributions are identical to those of the conventional QFT. Similarly, in the nonrelativistic limit without quantum field interactions, our theory of particle trajectories reduces to the usual dBB interpretation of QM, for which it is already known that it is in agreement with the predictions of the conventional nonrelativistic QM. Therefore, as far as we can see, all definite predictions of the conventional theory are also the predictions of our theory, provided that measurements are based on ideal quantum measurements (see the Appendix).

Note also that different definitions of the effectivity, replacing the definition (35), are conceivable. However, as already mentioned, the definition (35) corresponds to a theory in which the probability of the particle existence in a stochastic interpretation is equal to the effectivity of particles in a deterministic interpretation. This leads to an appealing ontological picture in which the probability of existence is reinterpreted as a kind of “degree of existence” (called effectivity) which is not a probabilistic quantity.

Finally, note that if all physical meaning of the quantity \( e_n \) is given by Eq. (36), then this equation explains precisely enough the physical meaning of this quantity, so in this case it is not really necessary to use a funny name for it, such as “effectivity”. However, we view Eq. (36) only as the simplest example of a possible precise meaning of \( e_n \). Different realizations of the general idea that (in some way) the effective mass is equal to the product \( e_n m \) are conceivable.
Therefore, to keep in mind the possibility of different realizations of the general abstract idea of a notion of effectivity, we retain the name “effectivity” for the quantity $e_n$.

The theory presented in this paper is certainly opened for further modifications, refinements, and reinterpretations. We hope that new ideas introduced in this paper, such as the abstract (and perhaps still somewhat vague) notion of effectivity, will motivate further research.

Acknowledgements. The author is grateful to S. Goldstein and R. Tumulka for their critical remarks and suggestions. In particular, the basic idea for Eq. (36) arose from a suggestion of S. Goldstein. The author is also grateful to anonymous referees for their constructive critical objections that stimulated a more clear presentation. This work was supported by the Ministry of Science and Technology of the Republic of Croatia under Contract No. 0098002.

APPENDIX:
THE GENERAL THEORY OF QUANTUM MEASUREMENTS

In our experience, many physicists familiar with the dBB interpretation of QM are not familiar with the corresponding general theory of quantum measurements, despite the fact that this theory is explained in often cited works on the dBB interpretation, such as Refs. [1, 2, 5]. More abstract presentations of this theory can also be found in Refs. [30, 31]. Since this theory of quantum measurements is also important for understanding of the present paper, in this Appendix we present a short review of this theory. For simplicity, we present this theory for the case of nonrelativistic QM with only one measured degree of freedom, but the same ideas can be easily adjusted to other quantum theories as well. Although this Appendix is intended to be self-contained, we note that an interested reader can find more details in the references cited above.

Let $x$ denote the coordinate of particle position in the configuration space. Let $\psi(x, t)$ be the corresponding wave function. The dBB theory of particle motion is based on the postulate that the velocity $v$ of the particle is given by

$$v = \frac{1}{m} \partial_x S,$$

where $S(x, t)$ is the phase of the wave function. This postulate provides that the statistical distribution of particle positions is given by $|\psi(x, t)|^2$ for any time $t$, provided that this distribution is given by $|\psi(x, t_0)|^2$ for some initial time $t_0$. However, this fact by itself is not sufficient to provide the agreement of the dBB interpretation with the standard interpretation of QM. A simple way to see this is to consider the statistical distribution of momenta, which, according to the dBB interpretation, is

$$\rho(p, t) = \int dx |\psi(x, t)|^2 \delta(p - \partial_x S(x, t)).$$

In general, this distribution is not equal to the quantum mechanical distribution $|\tilde{\psi}(p, t)|^2$ [where $\tilde{\psi}(p, t)$ is the Fourier transform of $\psi(x, t)$]. In order to see how the dBB interpretation recovers all the statistical results of standard QM, it is necessary to understand the general theory of quantum measurements.

Any measurement eventually reduces to an observation of some macroscopic quantity of the measuring apparatus, such as the position of a needle. In fact, this macroscopic observation can always be eventually reduced to an observation of the position (in the configuration space) of something. Let us idealize and simplify the analysis by introducing only one configuration-space
variable \( y \) corresponding to the measuring apparatus. (Introducing a larger number of such variables does not change the conclusions.) Assume that we want to construct a measuring apparatus that measures an observable represented by a hermitian operator \( \hat{A} \) that acts on the \( x \) space. The wave function \( \psi(x,t) \) can be expanded as

\[
\psi(x,t) = \sum_a c_a(t) \psi_a(x),
\]

(40)

where \( \psi_a(x) \) are complete normalized eigenfunctions of the operator \( \hat{A} \):

\[
\hat{A} \psi_a(x) = a \psi_a(x).
\]

(41)

For simplicity, we assume that the spectrum of the eigenvalues \( a \) is not degenerate. According to standard QM, the probability of finding the state to have the value \( a \) of the observable \( \hat{A} \) is equal to \( |c_a(t)|^2 \). On the other hand, Eq. (40) suggests that this may not be the case in the dBB interpretation, unless \( \hat{A} \) is equal to \( x \). To see how this problem resolves, it is essential to realize that, when the system consisting of the variables \( x \) and \( y \) can be considered as a configuration suitable for measurement of the \( x \) subsystem, the total wave function is not of the form \( \psi(x,t) \chi(y,t) \). Instead, the interaction between the \( x \) subsystem and the \( y \) subsystem should be such that the total wave function takes the form

\[
\Psi(x,y,t) = \sum_a c_a(t) \psi_a(x) \chi_a(y),
\]

(42)

where the normalized wave functions \( \chi_a(y) \) with different labels \( a \) do not overlap in the \( y \) space. Therefore, if the position \( y \) is found to have the value in the support of a wave function \( \chi_a(y) \), then this value is not in the support of any other wave function \( \chi_{a'}(y) \). In other words, if the position \( y \) is found to have the value in the support of \( \chi_a(y) \), then, according to the usual rules of QM, we know that the total wave function is, effectively, equal to \( \psi_a(x) \chi_a(y) \). The probability for this to happen is, according to (12), equal to \( |c_a(t)|^2 \), just as it should be without taking into account any theory of quantum measurements.

The discussion of the preceding paragraph is valid without taking into account the dBB interpretation of QM. However, without the dBB interpretation, it is not clear why and how the variable \( y \) takes a definite value. On the other hand, if the variable \( y \) is also described by the dBB interpretation, then it becomes clear why and how it takes a definite value. From the Bohmian mechanics of composed systems and the fact that the wave functions \( \chi_a(y) \) do not overlap, it is easy to show that if the particle describing the measuring apparatus has the position \( y \) in the support of \( \chi_a(y) \), then the measured particle with the position \( x \) moves in the same way as it was described by the wave function \( \psi_a(x) \chi_a(y) \). In such a case, the value of the observable \( \hat{A} \) is a constant of motion equal to \( a \). In this way, the wave function \( \Psi(x,y,t) \) effectively “collapses” into a wave function \( \psi_a(x) \chi_a(y) \), by \( y \) taking a definite value from the support of \( \chi_a(y) \). In the same sense, the wave function \( \psi(x,t) \) effectively “collapses” into a wave function \( \psi_a(x) \). It only remains to see that the probability for this to happen is equal to \( |c_a(t)|^2 \). We know that, in the dBB interpretation, the probability density in the configuration space is

\[
|\Psi(x,y,t)|^2 = \sum_a |c_a(t)|^2 |\psi_a(x)|^2 |\chi_a(y)|^2,
\]

(43)

where the fact that different \( \chi_a(y) \) do not overlap has been used, which has eliminated the nondiagonal terms proportional to \( \chi_a \chi_{a'} = 0 \) for \( a \neq a' \). By averaging over \( x \), we find the probability distribution in the \( y \) space to be

\[
\rho(y,t) = \sum_a |c_a(t)|^2 |\chi_a(y)|^2 = |\chi(y,t)|^2,
\]

(44)
where
\[
\chi(y, t) \equiv \sum_a c_a(t) \chi_a(y).
\] (45)

This shows that the dBB interpretation predicts that the probability for \( y \) to take a value from the support of \( \chi_a(y) \) is equal to \( |c_a(t)|^2 \).

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