Massive $\mathcal{N} = 1$ supermultiplets with arbitrary superspins

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Abstract

In this paper we give explicit construction of massive $\mathcal{N} = 1$ supermultiplets in flat $d = 4$ Minkowski space-time. We work in a component on-shell formalism based on gauge invariant description of massive integer and half-integer spin particles where massive supermultiplets are constructed out of appropriate set of massless ones.

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**Introduction**

In a flat space-time massive spin $s$ particles in a massless limit decompose into massless spin $s$, $s-1$, ... ones. This, in particular, leads to the possibility of gauge invariant description of massive spin $s$ particles, e.g. [1]-[13]. In this, two different approaches could be used. From one hand, one can start with usual non gauge invariant description of massive particle and achieve gauge invariance through the introduction of additional fields (thus promoting second class constraints into the first class ones). From the other hand, one can start with the appropriate set of massless particles having gauge invariance from the very beginning and obtain massive particle description as a deformation of massless theory. This last approach closely mimic situation in spontaneous gauge symmetry breaking where gauge field has to eat some Goldstone field(s) to become massive.

In the supersymmetric theories all particles must belong to some supermultiplet, massive or massless. Till now most of investigations in supersymmetric theories where bounded to massless supermultiplets. Quite a few results on massive supermultiplets mainly devoted to superspins 1 and 3/2 exist [14]-[21]. The aim of this paper is to extend these results to include massive $N = 1$ supermultiplets with arbitrary superspins. Certainly, it would be nice to have superfield off-shell description of such supermultiplets, but as previous results clearly show it is a highly non-trivial task. So in this paper we restrict ourselves with component on-shell formalism in terms of physical fields. The same reasoning on the massless limit means that massive supermultiplets could (should) be constructed out of the massless ones in the same way as massive particles out of the massless ones. So our approach will be supersymmetric generalization of the second approach to massive particle description mentioned above. Namely, we will start with appropriate set of massless supermultiplets and obtain massive one as a smooth deformation.

The paper is organized as follows. Though our previous examples on massive superspin 1 [21] and superspin 3/2 [15] supermultiplets already give important hints on how general case of arbitrary superspin could looks like, due to peculiarities of lower spin fields they are not enough to achieve such generalization. Thus, in the first two sections we give two more concrete examples, namely massive supermultiplets with superspin 2 and 5/2, correspondingly. All these and subsequent results heavily depend on the gauge invariant description of massive particles with integer [2, 3] and half-integer [10] spins as well as on the known form of massless supermultiplets [22]. For reader convenience and to make paper self-contained, in the next two sections we give all necessary formulas in compact condensed notations. One of the lessons from previous investigations is that the structures of massive supermultiplets with integer and half-integer superspins are different, so in the last two sections we consider these two cases separately. We will see that, in spite of large number of fields, all calculations are pretty straightforward and mainly combinatorical.

1 **Superspin 2**

Massive superspin 2 supermultiplet contains four massive particles with spins 5/2, 2, 2’ and 3/2, correspondingly. In the massless limit massive supermultilets must decompose into the appropriate set of massless ones in the same way as massive spin $s$ particles — into massless
spin $s$, $s-1$, ... ones. Simple counting of physical degrees of freedom immediately gives:

\[
\begin{pmatrix} 2 & 5/2 & 2' \\ 3/2 & & \end{pmatrix} \Rightarrow \begin{pmatrix} 5/2 & \end{pmatrix} \oplus \begin{pmatrix} 2 & \end{pmatrix} \oplus \begin{pmatrix} 3/2 & \end{pmatrix} \oplus \begin{pmatrix} 1 & \end{pmatrix} \oplus \begin{pmatrix} 1/2 & \end{pmatrix} \oplus \begin{pmatrix} 1/2' & \end{pmatrix}
\]

So we will start with five massless supermultiplets ($\Phi_{\mu\nu}, h_{\mu\nu}$), ($f_{\mu\nu}, \Psi_{\mu}$), ($\Omega_{\mu}, A_{\mu}$), ($B_{\mu}, \psi$) and ($\chi, z$). From our previous experience with massive superspin 1 and superspin 3/2 supermultiplets we know that it is crucial for the construction of massive supermultiplets to make dual rotation of vector $A_{\mu}$ and axial-vector $B_{\mu}$ fields mixing massless supermultiplets containing these fields. But now we have two tensor fields $h_{\mu\nu}$ and $f_{\mu\nu}$ as well, moreover they necessarily must be tensor and pseudo-tensor ones. Thus we have to consider the possibility to mix massless supermultiplets with these fields as well and the real structure of massless supermultiplets we are going to work with looks like:

\[
\begin{pmatrix} h_{\mu\nu} & \Phi_{\mu\nu} & f_{\mu\nu} \\ \Psi_{\mu} & \Omega_{\mu} & B_{\mu} \\ \chi & z & \end{pmatrix}
\]

Then, introducing a sum of the massless Lagrangians for bosonic fields:

\[
\mathcal{L}_0 = \frac{1}{2} \partial^\alpha h_{\mu\nu} \partial_\alpha h_{\mu\nu} - (\partial h)^\mu (\partial h)_\mu + (\partial h)^\mu \partial_\mu h - \frac{1}{2} \partial^\mu h \partial_\mu h + \\
+ \frac{1}{2} \partial^\alpha f_{\mu\nu} \partial_\alpha f_{\mu\nu} - (\partial f)^\mu (\partial f)_\mu + (\partial f)^\mu \partial_\mu f - \frac{1}{2} \partial^\mu f \partial_\mu f - \\
- \frac{1}{4} A_{\mu\nu}^2 - \frac{1}{4} B_{\mu\nu}^2 + \frac{1}{2} (\partial_\mu \phi)^2 + \frac{1}{2} (\partial_\mu \pi)^2
\]

as well as sum of the massless Lagrangians for fermionic fields:

\[
\mathcal{L}_0 = \frac{i}{2} \bar{\Phi}^\mu \hat{\partial} \Phi_{\mu} - 2i (\bar{\Phi} \gamma)^\mu (\partial \Phi)_\mu + i (\bar{\Phi} \gamma)^\mu \hat{\partial} (\gamma \Phi)_\mu + i (\bar{\Phi} \gamma \bar{\partial}) \Phi - \frac{i}{4} \bar{\Phi} \hat{\partial} \Phi - \\
- \frac{i}{2} \bar{\Psi}^\mu \hat{\partial} \Psi_{\mu} + i (\bar{\Psi} \gamma) (\partial \Psi) - \frac{i}{2} (\bar{\Psi} \gamma) \hat{\partial} (\gamma \Psi) + \frac{i}{2} (\bar{\psi} \gamma \bar{\partial} \psi - \\
- \frac{i}{2} \bar{\Omega}^\mu \hat{\partial} \Omega_{\mu} + i (\bar{\Omega} \gamma) (\partial \Omega) - \frac{i}{2} (\bar{\Omega} \gamma) \hat{\partial} (\gamma \Omega) + \frac{i}{2} (\bar{\chi} \gamma \bar{\partial} \chi)
\]

it is not hard to check that the most general supertransformations leaving sum of massless Lagrangians invariant have the form (round brackets denote symmetrization):

\[
\delta \Phi_{\mu\nu} = -\frac{i}{\sqrt{2}} \sigma^{\alpha\beta} \partial_\alpha (\cos(\theta_2)h_{\beta(\mu} \gamma_{\nu)} - \sin(\theta_2)\gamma_5 f_{\beta(\mu} \gamma_{\nu)}) \eta
\]

\[
\delta h_{\mu\nu} = \sqrt{2} \cos(\theta_2) (\bar{\Phi}_{\mu\nu} \eta) + i \sin(\theta_2) (\bar{\Psi}_{(\mu \gamma_{\nu)}} \eta)
\]

\[
\delta f_{\mu\nu} = \sqrt{2} \sin(\theta_2) (\bar{\Phi}_{\mu\nu} \gamma_5 \eta) + i \cos(\theta_2) (\bar{\Psi}_{(\mu \gamma_{\nu)}} \gamma_5 \eta)
\]

\[
\delta \Psi_{\mu} = -\sigma^{\alpha\beta} \partial_\alpha (\sin(\theta_2)h_{\beta\mu} + \cos(\theta_2)f_{\beta\mu} \gamma_5) \eta
\]

for the supermultiplets containing spin 2 fields and

\[
\delta \Omega_{\mu} = -\frac{i}{2\sqrt{2}} \sigma^{\alpha\beta} (\cos(\theta_1)A_{\alpha\beta} - \sin(\theta_1)\gamma_5 B_{\alpha\beta}) \gamma_{\mu} \eta
\]
\[
\delta A_\mu = \sqrt{2} \cos(\theta_1) (\Omega_\mu \eta) + i \sin(\theta_1) (\bar{\psi} \gamma_\mu \eta) \\
\delta B_\mu = \sqrt{2} \sin(\theta_1) (\bar{\Omega} \mu \gamma_5 \eta) + i \cos(\theta_1) (\bar{\psi} \gamma_\mu \gamma_5 \eta) \\
\delta \psi = -\frac{1}{2} \sigma^{\alpha \beta} (\sin(\theta_1) A_{\alpha \beta} + \cos(\theta_1) \gamma_5 B_{\alpha \beta}) \eta
\]

for those with (axial)vector ones. The last supermultiplets is simple:

\[
\delta \chi = -i \gamma^\mu \partial_\mu (\varphi + \gamma_5 \pi) \eta, \quad \delta \varphi = (\bar{\chi} \eta), \quad \delta \pi = (\bar{\chi} \gamma_5 \eta)
\]

To construct massive supermultiplet we have to add mass terms for all fields as well as appropriate corrections to fermionic supertransformations. In this, the most important question is which lower spin fields play the roles of Goldstone ones and have to be eaten to make main gauge fields massive. For the bosonic fields (taking into account parity conservation) the choice is unambiguous: vector \(A_\mu\) and scalar \(\varphi\) fields for tensor field \(h_{\mu \nu}\) and axial-vector \(B_\mu\) and pseudo-scalar \(\pi\) — for pseudo-tensor \(f_{\mu \nu}\). Thus bosonic mass terms will be:

\[
\frac{1}{m} \mathcal{L}_1 = \sqrt{2} [h_{\mu \nu} \partial_\mu A_\nu - h(\partial A)] - \sqrt{3} A^\mu \partial_\mu \varphi + \sqrt{2} [f_{\mu \nu} \partial_\mu B_\nu - f(\partial B)] - \sqrt{3} B^\mu \partial_\mu \pi \\
\frac{1}{m^2} \mathcal{L}_2 = -\frac{1}{2} (h_{\mu \nu} h_{\mu \nu} - h^2) - \sqrt{3} h \varphi + \varphi^2 - \frac{1}{2} (f_{\mu \nu} h_{\mu \nu} - f^2) - \sqrt{3} f \pi + \pi^2
\]

But for fermions we have two spin 3/2 and two spin 1/2 fields and there is no evident choice. Thus we introduce the most general mass terms for the fermions:

\[
\frac{1}{m} \mathcal{L}_m = -\frac{1}{2} \bar{\Phi}^{\mu \nu} \Phi_{\mu \nu} + (\bar{\Phi} \gamma)^{\mu} (\gamma \Phi)_{\mu} + \frac{1}{4} \bar{\Phi} \Phi - \\
-i \alpha_1 [\bar{\Phi}^{\mu \nu} \gamma_\mu \Psi_\nu - \frac{1}{2} \bar{\Phi} (\gamma \Psi)] - i \alpha_2 [\bar{\Phi}^{\mu \nu} \gamma_\mu \Omega_\nu - \frac{1}{2} \bar{\Phi} (\gamma \Omega)] + \\
+ a_1 [\bar{\Psi}^{\mu} \Psi_\mu - (\bar{\Psi} \gamma_5) (\gamma \Psi)] + a_2 [\bar{\Omega}^{\mu} \Omega_\mu - (\bar{\Omega} \gamma_5) (\gamma \Omega)] + a_3 [\bar{\Psi}^{\mu} \Omega_\mu - (\bar{\Psi} \gamma_5) (\gamma \Omega)] + \\
+ i a_4 (\bar{\Psi} \gamma_5) \psi + i a_5 (\bar{\Psi} \gamma_5) \chi + i a_6 (\bar{\Omega} \gamma_5) \psi + i a_7 (\bar{\Omega} \gamma_5) \chi + \\
+ a_8 \bar{\psi} \psi + a_9 \bar{\psi} \chi + a_{10} \bar{\chi} \chi
\]

and proceed with calculations. Cancellation of variations with one derivative gives:

\[
\sin(\theta_2) = \cos(\theta_2) = \frac{1}{\sqrt{2}} \quad \alpha_1 = \frac{1}{\sqrt{2}} \\
\sin(\theta_1) = \cos(\theta_1) = \frac{1}{\sqrt{2}} \quad \alpha_2 = \sqrt{2} \\
- \frac{1}{4} \quad a_3 = 1, \quad a_4 = \sqrt{2}, \quad a_5 = 0 \\
a_6 = a_2 \sqrt{2}, \quad a_7 = \sqrt{3}, \quad a_8 = 0, \quad a_9 = -\sqrt{6}
\]

while variations without derivatives give:

\[
a_2 = \frac{1}{2}, \quad a_{10} = -\frac{1}{2}
\]
we can write final form of fermionic supertransformations as:

\[ \frac{1}{m} \mathcal{L}_m = -\frac{1}{2} \Phi_{\mu\nu} \Phi_{\mu\nu} + (\Phi_{\gamma})^\mu (\gamma_{\Phi})_\mu + \frac{1}{4} \Phi \Phi - \\
- \frac{i}{\sqrt{2}} \Phi_{\mu\nu} \gamma_\mu \Psi_\nu + \frac{i}{2 \sqrt{2}} \Phi (\gamma \Psi) - i \sqrt{2} \Phi_{\mu\nu} \gamma_\mu \Omega_\nu + \frac{i}{\sqrt{2}} \Phi (\gamma \Omega) + \\
- \frac{1}{4} \bar{\Psi}_{\mu} \Psi_{\mu} + \frac{1}{4} (\bar{\Psi}_{\gamma}) (\gamma \Psi) + \frac{1}{2} \bar{\Omega}_{\mu} \Omega_\mu - \frac{1}{2} (\bar{\Omega}_{\gamma}) (\gamma \Omega) + \bar{\Psi}_{\mu} \Omega_\mu - (\bar{\Psi}_{\gamma}) (\gamma \Omega) + \\
+ \frac{i}{\sqrt{2}} (\bar{\Psi}_{\gamma}) \psi + \frac{i}{\sqrt{2}} (\bar{\Omega}_{\gamma}) \psi + i \sqrt{3} (\bar{\Omega}_{\gamma}) \chi - \sqrt{6} \bar{\psi} \chi - \frac{1}{2} \bar{\chi} \chi \] (7)
correspond to invariance of the Lagrangian (besides global supertransformations) under three local spinor gauge transformations:

\[ \delta \Phi_{\mu\nu} = \partial_{(\mu} \xi_{\nu)} + \frac{im}{2} \gamma_{(\mu} \xi_{\nu)} + \frac{m}{4 \sqrt{2}} g_{\mu\nu} \xi_1 + \frac{m}{2 \sqrt{2}} g_{\mu\nu} \xi_2 \]
\[ \delta \Psi_\mu = \partial_\mu \xi_1 + \frac{m}{\sqrt{2}} \xi_\mu - \frac{im}{4} \gamma_\mu \xi_1 + \frac{im}{2} \gamma_\mu \xi_2 \]
\[ \delta \Omega_\mu = \partial_\mu \xi_2 + m \sqrt{2} \xi_\mu + \frac{im}{2} \gamma_\mu \xi_1 + \frac{im}{2} \gamma_\mu \xi_2 \]
\[ \delta \psi = m \sqrt{2} \xi_1 + m \frac{\xi_2}{\sqrt{2}} \]
\[ \delta \chi = m \sqrt{3} \xi_2 \] (8)

From these formula one can easily determine which combination of spin 3/2 fields \( \Psi_\mu \) and \( \Omega_\mu \) plays the role of Goldstone field for spin 5/2 field \( \Phi_{\mu\nu} \). Indeed, if one introduces two orthogonal combinations:

\[ \tilde{\Psi}_\mu = \frac{1}{\sqrt{5}} \Psi_\mu + \frac{2}{\sqrt{5}} \Omega_\mu, \quad \tilde{\Omega}_\mu = -\frac{2}{\sqrt{5}} \Psi_\mu + \frac{1}{\sqrt{5}} \Omega_\mu \]
then after diagonalization of mass terms one finds that \( \tilde{\Psi}_\mu \) is a Goldstone field, while \( \tilde{\Omega}_\mu \) — physical field with the same mass as \( \Phi_{\mu\nu} \).

Similar to the case of massive supermultiplet with superspin 1, both mixing angles have been fixed: \( \theta_1 = \theta_2 = \pi/4 \) and this, in turn, means that all bosonic fields enter through the complex combinations only:

\[ H_{\mu\nu} = h_{\mu\nu} + \gamma_5 f_{\mu\nu}, \quad C_\mu = A_\mu + \gamma_5 B_\mu, \quad z = \varphi + \gamma_5 \pi \]

Introducing gauge covariant derivatives:

\[ \nabla_\mu H_{\alpha\beta} = \partial_\mu H_{\alpha\beta} - \frac{m}{\sqrt{2}} C_\mu g_{\alpha\beta}, \quad \nabla_\mu z = \partial_\mu z - m \sqrt{3} C_\mu \]
we can write final form of fermionic supertransformations as:

\[ \delta \Phi_{\mu\nu} = [-\frac{i}{2} \sigma^{\alpha\beta} \nabla_\alpha \bar{H}_{\beta(\mu} \gamma_{\nu)} - m H_{\mu\nu} + \frac{m}{4} \gamma_{(\mu} (\gamma H)_{\nu)} + \frac{m}{4 \sqrt{2}} g_{\mu\nu} z] \eta \]
\[ \delta \Psi_\mu = [-\frac{1}{\sqrt{2}} \sigma^{\alpha\beta} \nabla_\alpha \bar{H}_{\beta\mu} - \frac{im}{2 \sqrt{2}} (\gamma H)_\mu + \frac{1}{\sqrt{3}} \nabla_\mu z + \frac{im}{4 \sqrt{3}} \gamma_\mu z] \eta \]
\[ \delta \Omega_\mu = [-\frac{i}{4} \sigma^{\alpha\beta} \bar{C}_{\alpha\beta} \gamma_\mu + \frac{im}{2 \sqrt{2}} (\gamma H)_\mu + \frac{1}{\sqrt{3}} \nabla_\mu z - \frac{im}{2 \sqrt{3}} \gamma_\mu z] \eta \]
\[ \delta \psi = -\frac{1}{2 \sqrt{2}} \sigma^{\alpha\beta} \bar{C}_{\alpha\beta} \eta \quad \delta \chi = -i \gamma_\mu \nabla_\mu z \eta \] (9)
Note also that due to complexification of bosonic fields the Lagrangian and supertransformations are invariant under global axial $U(1)_A$ symmetry, axial charges for all fields being:

| field      | $q_A$ | $\eta$ | $\Phi_{\mu\nu}$, $\Psi_{\mu}$, $\Omega_{\mu}$, $\psi$, $\chi$ | $H_{\mu\nu}$, $C_{\mu}$, $\zeta$ |
|------------|------|-------|-------------------------------------------------|------------------|
|            | 1    | 0     |                                                 | -1               |

## 2 Superspin 5/2

Our next example — massive supermultiplet with superpin 5/2. It also contains four massive fields: with spin 3, 5/2, 5/2' and 2 and in the massless limit it should reduce to six massless supermultiplets:

\[
\begin{pmatrix} 5/2 & 3 \\ 2 & 5/2 \end{pmatrix} \Rightarrow \begin{pmatrix} 3 & 5/2 \\ 2 & 3/2 \end{pmatrix} \oplus \begin{pmatrix} 3/2 & 1 \\ 1/2 & 0 \end{pmatrix} \oplus \begin{pmatrix} 1/2 & 0, 0' \end{pmatrix}
\]

By analogy with all previous cases we will take into account possible mixing for bosonic tensor and vector fields, so we will start with the following structure of massless supermultiplets:

\[
\begin{pmatrix} \Phi_{\mu\nu\lambda} \\ \Psi_{\mu\nu} \end{pmatrix} \oplus \begin{pmatrix} \Omega_{\mu\nu} \\ f_{\mu\nu} \end{pmatrix} \oplus \begin{pmatrix} A_{\mu} \\ \Omega_{\mu} \end{pmatrix} \oplus \begin{pmatrix} B_{\mu} \\ \chi \end{pmatrix} \oplus \begin{pmatrix} \psi \end{pmatrix} \oplus \begin{pmatrix} \chi \end{pmatrix}
\]

So we introduce sum of the massless Lagrangians for bosonic fields:

\[
\mathcal{L}_0 = -\frac{1}{2} \partial^\mu \Phi_{\nu\lambda} \partial_\nu \Phi_{\mu\lambda} + \frac{3}{2} (\partial \Phi)^{\mu\nu}(\partial \Phi)_{\mu\nu} - 3 (\partial \Phi)^{\mu\nu} \partial_\nu \Phi_{\mu} + \frac{3}{2} \partial^\mu \Phi^{\nu\rho} \partial_\nu \Phi_{\rho} + \frac{3}{4} (\partial \Phi)^2 \\
+ \frac{1}{2} \partial^\mu h^{\mu\nu} \partial_\nu h_{\mu\nu} - (\partial h)^{\mu} (\partial h)_{\mu} + (\partial h)^{\mu} \partial_\mu h - \frac{1}{2} \partial^\mu \partial_\nu h^{\mu\nu} - \frac{1}{4} A_{\mu\nu}^2 + \frac{1}{2} (\partial \mu \pi)^2
\]

as well as sum of the massless Lagrangians for fermionic fields:

\[
\mathcal{L}_0 = \frac{i}{2} \bar{\Psi}^{\mu\nu} \partial_\nu \Psi_{\mu} - 2 i (\bar{\Psi} \gamma)^{\mu} (\partial \Psi)_{\mu} + i (\bar{\Psi} \gamma)_{\mu} \partial (\gamma \Psi)_{\mu} + i (\bar{\Psi} \gamma \partial) \Psi - \frac{i}{4} \bar{\Psi} \partial \Psi + \\
+ \frac{i}{2} \bar{\Omega}^{\mu\nu} \partial_\nu \Omega_{\mu} - 2 i (\bar{\Omega} \gamma)^{\mu} (\partial \Omega)_{\mu} + i (\bar{\Omega} \gamma)^{\mu} \partial (\gamma \Omega)_{\mu} + i (\bar{\Omega} \gamma \partial) \Omega - \frac{i}{4} \bar{\Omega} \partial \Omega - \\
- \frac{i}{2} \bar{\Psi}^{\mu} \partial_\mu \Psi + i (\bar{\Psi} \gamma)(\partial \Psi) - \frac{i}{2} (\bar{\Psi} \gamma) \partial (\gamma \Psi) + \frac{i}{2} \bar{\Psi} \partial \Psi - \\
- \frac{i}{2} \bar{\Omega}^{\mu} \partial_\mu \Omega + i (\bar{\Omega} \gamma)(\partial \Omega) - \frac{i}{2} (\bar{\Omega} \gamma) \partial (\gamma \Omega) + \frac{i}{2} \bar{\Omega} \partial \Omega
\]

and start with the following global supertransformations:

\[
\delta \Phi_{\mu\nu\lambda} = i (\bar{\Psi}_{(\mu\nu\lambda)} \eta) \quad \delta \Psi_{\mu\nu} = [-\sigma^{\alpha\beta} \partial_\alpha \Phi_{\beta\mu\nu} + \frac{1}{4} \partial_{(\mu\nu)}(\gamma \Phi)] \eta
\]
for the supermultiplet \((3, 5/2)\),

\[
\delta \Omega_{\mu \nu} = -\frac{i}{\sqrt{2}} \sigma^{\alpha \beta} \partial_\alpha (\cos(\theta_2) h_{\beta \gamma} - \sin(\theta_2) f_{\beta \gamma} \gamma) \eta \\
\delta h_{\mu \nu} = \sqrt{2} \cos(\theta_2) (\bar{\Omega}_{\mu \nu} \eta) + i \sin(\theta_2) (\bar{\Psi}_{\mu \gamma} \gamma) \eta \\
\delta f_{\mu \nu} = \sqrt{2} \sin(\theta_2) (\bar{\Omega}_{\mu \gamma} \eta) + i \cos(\theta_2) (\bar{\Psi}_{\mu \gamma} \gamma) \eta \\
\delta \Psi_\mu = -\sigma^{\alpha \beta} \partial_\alpha (\sin(\theta_2) h_{\beta \gamma} + \cos(\theta_2) f_{\beta \gamma} \gamma) \eta 
\]

(13)

for the mixed \((5/2, 2)\) and \((2, 3/2)\) supermultiplets,

\[
\delta \Omega_\mu = -\frac{i}{2 \sqrt{2}} \sigma^{\alpha \beta} (\cos(\theta_1) A_{\alpha \beta} - \sin(\theta_1) \gamma_5 B_{\alpha \beta}) \gamma_\mu \eta \\
\delta A_\mu = \sqrt{2} \cos(\theta_1) (\bar{\Omega}_\mu \eta) + i \sin(\theta_1) (\bar{\psi}_\gamma \gamma_\mu \eta) \\
\delta B_\mu = \sqrt{2} \sin(\theta_1) (\bar{\Omega}_\mu \gamma_5 \eta) + i \cos(\theta_1) (\bar{\psi}_\gamma \gamma_5 \eta) \\
\delta \psi = -\frac{1}{2} \sigma^{\alpha \beta} (\sin(\theta_1) A_{\alpha \beta} + \cos(\theta_1) \gamma_5 B_{\alpha \beta}) \eta 
\]

(14)

for the mixed \((3/2, 1)\) and \((1, 1/2)\) supermultiplets and

\[
\delta \chi = -i \gamma_\mu \partial_\mu (\varphi + \gamma_5 \pi) \eta, \quad \delta \varphi = (\bar{\chi} \eta), \quad \delta \pi = (\bar{\chi} \gamma_5 \eta) 
\]

for the last one.

By analogy with the superspin 3/2 case, we will assume that fermionic mass terms are Dirac ones:

\[
\frac{1}{m} \mathcal{L}_m = -\bar{\Psi}_{\mu \nu} \Omega_{\mu \nu} + 2(\bar{\Psi} \gamma)(\gamma \Omega)_\mu + \frac{1}{2} \bar{\Psi} \gamma \Omega + \\
+i \sqrt{\frac{5}{2}} \left[ -\bar{\Psi}_{\mu \nu} \gamma_\mu \Psi_\nu + \frac{1}{2} \bar{\Psi} \gamma \Psi - \bar{\Omega}_{\mu \nu} \gamma_\mu \Omega_\nu + \frac{1}{2} \bar{\Omega} \gamma \Omega \right] + \\
+ \frac{3}{2} \bar{\psi} \mu \Omega - \frac{3}{2} \bar{\Psi} \gamma (\gamma \Omega) + 2i(\bar{\Psi} \gamma) \psi + 2i(\bar{\Omega} \gamma) \chi - 3 \bar{\psi} \chi 
\]

(15)

where all coefficients are completely fixed by the requirement that the Lagrangian has to be invariant not only under the global supertransformations, but under four (by the number of fermionic gauge fields) spinor gauge transformations:

\[
\delta \Psi_{\mu \nu} = \partial_\mu (\xi_\nu) + \frac{im}{2} \gamma_\mu \eta_\nu + m \sqrt{\frac{5}{2}} g_{\mu \nu} \xi_1, \quad \delta \Omega_{\mu \nu} = \partial_\mu (\eta_\nu) + \frac{im}{2} \gamma_\mu \xi_\nu + m \sqrt{\frac{5}{2}} g_{\mu \nu} \xi_2, \\
\delta \Psi_\mu = \partial_\mu (\xi_1) + m \sqrt{\frac{5}{2}} \xi_\mu + \frac{3}{4} m \gamma_\mu \xi_2, \quad \delta \Omega_\mu = \partial_\mu (\xi_2) + \frac{3}{4} m \gamma_\mu \xi_1, \\
\delta \psi = 2m \xi_1, \quad \delta \chi = 2m \xi_2 
\]

As for the bosonic fields, here the roles of the fields are evident (again taking into account parity conservation): we need tensor \(h_{\mu \nu}\), vector \(A_\mu\) and scalar \(\varphi\) fields to make spin 3 field
\( \Phi_{\mu\nu\lambda} \) massive, while pseudo-tensor \( f_{\mu\nu} \) field needs to eat axial-vector \( B_\mu \) and pseudo-scalar \( \pi \) fields. So the bosonic mass terms are also completely fixed:

\[
\begin{align*}
\frac{1}{m} \mathcal{L}_1 &= \sqrt{3}[-\Phi_{\mu\nu\lambda} \partial_\mu h_{\nu\lambda} + 2\Phi^\mu \partial_\mu h] + \frac{1}{2} \Phi^{\mu} \partial_\mu h + \sqrt{5}[h_{\mu\nu} \partial_\mu A_\nu - h(\partial A)] - \sqrt{6}A^\mu \partial_\mu \varphi + \\
&+ \sqrt{2}[f^{\mu\nu} \partial_\mu B_\nu - f(\partial B)] - \sqrt{3}B^\mu \partial_\mu \pi
\end{align*}
\]

(16)

\[
\begin{align*}
\frac{1}{m^2} \mathcal{L}_2 &= \frac{1}{2} \Phi_{\mu\nu\lambda} \Phi_{\mu\nu\lambda} - \frac{3}{2} \Phi^\mu \Phi_\mu + \frac{3}{4} h^2 + \frac{\sqrt{15}}{2} \Phi^{\mu} A_\mu - \frac{3}{4} A_\mu^2 - \sqrt{\frac{15}{2}} h \varphi + \frac{5}{2} \varphi^2 - \\
&- \frac{1}{2} (f^{\mu\nu} f_{\mu\nu} - f^2) - \sqrt{\frac{3}{2}} f \pi + \pi^2
\end{align*}
\]

(17)

Now we require that the whole Lagrangian be invariant under global supertransformations with appropriate corrections to fermionic transformations. This fixes both mixing angles:

\[
\begin{align*}
\sin(\theta_2) &= \sqrt{\frac{5}{6}}, & \cos(\theta_2) &= \frac{1}{\sqrt{6}}, & \sin(\theta_1) &= \sqrt{\frac{2}{3}}, & \cos(\theta_1) &= \frac{1}{\sqrt{3}}
\end{align*}
\]

and gives the following form of additional terms for fermionic supertransformations:

\[
\begin{align*}
\frac{1}{m} \delta \Psi_{\mu\nu} &= [-\frac{\sqrt{3}}{4} \gamma_\mu (\gamma h)_\nu - \frac{\sqrt{5}}{3} f_{\mu\nu} \gamma_5 + \frac{1}{4} \sqrt{\frac{5}{3}} \gamma_\mu (\gamma f)_\nu \gamma_5] \eta \\
\frac{1}{m} \delta \Omega_{\mu\nu} &= \frac{1}{4} g_{\mu\nu} (\gamma \Phi) - \\
&- \frac{1}{2} \sqrt{\frac{5}{3}} \gamma_\mu A_\nu + \frac{1}{4} \sqrt{\frac{5}{3}} g_{\mu\nu} \hat{A} - \frac{1}{2} \sqrt{\frac{5}{6}} \gamma_\mu (B_\nu) \gamma_5 + \frac{1}{2} \frac{\sqrt{5}}{6} g_{\mu\nu} \hat{B} \gamma_5 \eta \\
\frac{1}{m} \delta \psi &= \frac{1}{8} \sqrt{\frac{5}{2}} \gamma_\mu (\gamma \Phi) - \frac{1}{2} \sqrt{\frac{5}{6}} A_\mu - \frac{5}{2} \sqrt{\frac{5}{6}} \gamma_\mu \hat{A} - \frac{5}{2} \sqrt{\frac{5}{3}} B_\mu \gamma_5 - \frac{1}{2} \sqrt{\frac{5}{3}} \gamma_\mu \hat{B} \gamma_5 \eta \tag{18}
\end{align*}
\]

The complete supertransformations for fermionic fields could be simplified by introduction of gauge invariant derivatives;

\[
\begin{align*}
\nabla_\mu h_{\alpha\beta} &= \partial_\mu h_{\alpha\beta} - \frac{m \sqrt{5}}{2} A_\mu g_{\alpha\beta}, & \nabla_\mu \varphi &= \partial_\mu \varphi - m \sqrt{6} A_\mu \\
\nabla_\mu f_{\alpha\beta} &= \partial_\mu f_{\alpha\beta} - \frac{m}{\sqrt{2}} B_\mu g_{\alpha\beta}, & \nabla_\mu \pi &= \partial_\mu \pi - m \sqrt{3} B_\mu
\end{align*}
\]

This time bosonic fields do not combine into complex combinations, but due to the fact that fermionic mass terms are Dirac ones the Lagrangian and supertransformations are invariant under global axial \( U(1)_A \) transformations, provided axial charges of all fields are assigned as follows:

| field | \( \eta, \Psi_{\mu\nu}, \Psi_\mu, \psi \) | \( h_{\mu\nu}, f_{\mu\nu}, A_\mu, B_\mu, \varphi, \pi \) | \( \Omega_{\mu\nu}, \Omega_\mu, \chi \) |
|-------|--------------------------------|----------------|----------------|
| \( q_A \) | +1 | 0 | -1 |
3 Massive particles

All our previous and subsequent calculations heavily depend on the gauge invariant description of massive high spin particles. For reader convenience and to make paper self-contained we will give here gauge invariant formulations for massive particles with arbitrary integer \[2, 3\] and half-integer \[10\] spins. We restrict ourselves to flat \(d = 4\) Minkowski space but all results could be easily generalized to the case of (A)dS space with arbitrary dimension \(d\).

3.1 Integer spin

The simplest way to describe massless bosonic field with arbitrary spin \(s\) is to use completely symmetric rank \(s\) tensor \(\Phi_{(\alpha_1}\alpha_2...\alpha_s)}\) which is double traceless. In what follows we will use condensed notations where index denotes just number of free indices and not the indices themselves. For example, the tensor field itself will be denoted as \(\Phi_s\), it’s contraction with derivative as \((\partial \Phi)^s\), it’s trace as \(\tilde{\Phi}_s\) and so on. As we will see this does not lead to any ambiguities then working with free Lagrangians quadratic in fields. In these notations the Lagrangian for massless particles of arbitrary spin \(s\) could be written as:

\[
L_0 = (-1)^s \left[ \frac{1}{2} \partial^\mu \Phi^s \partial_\mu \Phi^s - \frac{s}{2} (\partial \Phi)^{s-1}(\partial \Phi)^{s-1} - \frac{s(s-1)}{4} \partial^\mu \tilde{\Phi}^s \partial_\mu \tilde{\Phi}^s \right. \\
+ \left. \frac{s(s-1)}{2} (\partial \Phi)^{s-1}(\partial_1 \Phi_{s-2}) - \frac{s(s-1)(s-2)}{8} (\partial \tilde{\Phi})^{s-3}(\partial \tilde{\Phi})^{s-3} \right] 
\]

(19)

where \(\tilde{\Phi} = 0\). This Lagrangian is invariant under the following gauge transformations:

\[
\delta_0 \Phi_s = \partial_1 (\xi_{s-1}), \quad \tilde{\xi}_{s-3} = 0
\]

where parameter \(\xi_{s-1}\) is completely symmetric traceless tensor of rank \(s - 1\).

To construct gauge invariant Lagrangian for massive particle which has correct (i.e. with right number of physical degrees of freedom) massless limit, we start with the sum of massless Lagrangians with \(0 \leq k \leq s\):

\[
L_0 = \sum_{k=0}^{s} (-1)^k \left[ \frac{1}{2} \partial^\mu \Phi^k \partial_\mu \Phi^k - \frac{k}{2} (\partial \Phi)^{k-1}(\partial \Phi)^{k-1} - \frac{k(k-1)}{4} \partial^\mu \tilde{\Phi}^k \partial_\mu \tilde{\Phi}^k \right. \\
+ \left. \frac{k(k-1)}{2} (\partial \Phi)^{k-1}(\partial_1 \Phi_{k-2}) - \frac{k(k-1)(k-2)}{8} (\partial \tilde{\Phi})^{k-3}(\partial \tilde{\Phi})^{k-3} \right] 
\]

(20)

Then we add the following cross terms with one derivative as well as mass terms without derivatives:

\[
\frac{1}{m} L_1 = \sum_{k=1}^{s} (-1)^k d_k [(\partial \Phi)^{k-1}\Phi_{k-1} + (k - 1)\tilde{\Phi}^{k-2}(\partial \Phi)_{k-2} + \frac{(k-1)(k-2)}{4}(\partial \tilde{\Phi})^{k-3}\tilde{\Phi}_{k-3}] \\
\frac{1}{m^2} L_2 = \sum_{k=0}^{s} (-1)^k [d_k \Phi^k \Phi_k + c_k \tilde{\Phi}^{k-2}\tilde{\Phi}_{k-2} + f_k \tilde{\Phi}^{k-2}\tilde{\Phi}_{k-2}] 
\]

(21)

and try to achieve gauge invariance with the help of appropriate corrections to gauge transformations:

\[
\frac{1}{m} \delta \Phi_k = \alpha_k \xi_k + \beta_k g(2\xi_{k-2})
\]
Straightforward but lengthy calculations give a number of algebraic equations on the unknown coefficients which could be solved (and this is non-trivial because we obtain overdetermined system of equations) and give us:

\[
\alpha_k^2 = \frac{(s - k)(s + k + 1)}{2(k + 1)^2}, \quad \beta_{k+1} = \frac{k + 1}{2k} \alpha_k, \quad 0 \leq k \leq s - 1
\]

\[
a_k = -\sqrt{\frac{(s - k + 1)(s + k)}{2}}, \quad d_k = \frac{(s - k - 1)(s + k + 2)}{4(k + 1)}
\]

\[
e_k = \frac{k(k - 1)}{16(k + 1)} [(s - k + 2)(s + k - 1) + 6]
\]

\[
f_k = -\frac{1}{4}\sqrt{(s - k + 2)(s - k - 1)(s + k)}
\]

### 3.2 Half-integer spin

For the description of massless spin \( s + 1/2 \) particles we will use completely symmetric rank \( s \) tensor-spinor \( \Psi_s \) such that \( (\gamma \Psi)_{s-3} = 0 \) (in the same condensed notations as before). Then Lagrangian for such field could be written as:

\[
L_0 = i(-1)^s \left[ \frac{1}{2} \Psi^* \dot{\Psi}^* - s(\bar{\Psi} \gamma) s^{-1} (\bar{\Psi} \gamma)^{s-1} + \frac{s}{2}(\bar{\Psi} \gamma)^{s-1} \dot{\gamma} (\Psi)^{s-1} + s(s - 1) \bar{\Psi}^{s-2} \dot{\bar{\Psi}}^{s-2} \right]
\]

and is invariant under the following gauge transformations:

\[
\delta_0 \Psi_s = \partial(1 \xi_{s-1}), \quad (\gamma \xi) = 0,
\]

where gauge parameter \( \xi_{s-1} \) is a \( \gamma \)-traceless tensor-spinor of rank \( s - 1 \).

Once again we start with the sum of massless Lagrangians with \( 0 \leq k \leq s \):

\[
L_0 = \sum_{k=0}^{s} i(-1)^k \left[ \frac{1}{2} \bar{\Psi}^k \dot{\bar{\Psi}}^k - k(\bar{\Psi} \gamma)^{k-1} (\bar{\Psi} \gamma)^{k-1} \right]
\]

\[
+ \frac{k(k - 1)}{2}(\bar{\Psi} \gamma)^{k-1} \dot{\bar{\Psi}}^{k-2} - \frac{k(k - 1)}{8} \bar{\Psi}^{k-2} \dot{\bar{\Psi}}^{k-2} \right]
\]

To combine all these massless fields into one massive particle we have to add the following mass terms:

\[
L_m = \sum_{k=0}^{s} (-1)^k \left\{ -\frac{s + 1}{2(k + 1)} [\bar{\Psi}^k \Psi_k^k - k(\bar{\Psi} \gamma)^{k-1} (\bar{\Psi} \gamma)^{k-1}] + \right.
\]

\[
-\frac{k(k - 1)}{4} \bar{\Psi}^{k-2} (\gamma \Psi)^{k-2} \left. \right\}
\]

and corresponding corrections to gauge transformations:

\[
\delta \Psi_k = \alpha_k \xi_k + i \beta_k \gamma(1)(\xi_{k-1}) + \rho_k g(2)(\xi_{k-2})
\]

Then total Lagrangian will be gauge invariant provided:

\[
c_k = \sqrt{\frac{(s + 1)^2 - k^2}{2}}, \quad \alpha_k = \frac{c_{k+1}}{k + 1}, \quad \beta_k = \frac{s + 1}{2k(k + 1)}, \quad \rho_k = \frac{c_k}{2k}
\]
4 Massless supermultiplets

It is not easy to find in the recent literature the explicit component form of massless supermultiplets with arbitrary superspin [22], so for completeness we will give their short description here. As we have already seen on the lower superspin cases, supermultiplets with integer and half-integer superspins have different structure and have to be considered separately.

(\textbf{s, s+1/2}). Supermultiplet with integer superspin \(s\) contains bosonic spin \(s\) field and fermionic spin \(s+1/2\) one. In this and in two subsequent sections we will use the same condensed notations as in the previous one. By analogy with superspins 1 and 2 supermultiplets, we start with the following ansatz for supertransformations:

\[
\delta \Psi_s = i\alpha_1 \sigma^{\mu\nu} \partial_\mu \Phi_{\nu(s-1)\gamma_1} \eta, \quad \delta \Phi_s = \beta(\Psi_s \eta)
\]

Indeed, calculating variations of the sum of two massless Lagrangians one can see that most of variations cancel, provided one set \(\alpha_1 = -\frac{\beta}{2}\). The residue:

\[
\delta \mathcal{L} = -(s-1)^s \beta \frac{(s-1)(s-2)}{4} [2(\Psi \partial \partial)^{s-3} \Phi_{s-2} - \bar{\Psi}^{s-2} (\partial \partial \partial \Phi_{s-2}) - (s-2)(\bar{\Psi} \partial)^{s-3} \Phi_{s-3} - 2(\bar{\Psi} \gamma \partial)^{s-3} (\gamma \Phi_{s-3} - (\bar{\Psi} \partial)^{s-3} \Phi_{s-3})]
\]

contains terms with \(\Phi_{s-2}\) only, so we proceed by adding to fermionic supertransformations one more term:

\[
\delta' \Psi_s = i\alpha_2 \partial_{(1 \gamma_1} \Phi_{s-2)}
\]

Then the choice \(\alpha_2 = \frac{(s-1)(s-2)\beta}{4s}\) leaves us with:

\[
\delta \mathcal{L} = -(s-1)^s \beta \frac{(s-1)(s-2)}{4} [2(\bar{\Psi} \gamma \partial)^{s-3} (\gamma \Phi_{s-3} - (\bar{\Psi} \partial)^{s-3} \Phi_{s-3})]
\]

where the only terms are whose with \((\gamma \Phi)_{s-3}\). So we make one more (last) correction to supertransformations:

\[
\delta'' \Psi_s = i\alpha_3 g_{(2 \partial_{1}(\gamma \Phi)_{s-3})}
\]

and obtain full invariance with \(\alpha_3 = -(s-1)(s-2)\beta\). To fix concrete normalization we will use closure of the superalgebra. Calculating the commutator of two supertransformations we obtain:

\[
[\delta_1, \delta_2] \Phi_s = -i \beta^2 (\bar{\eta}_2 \gamma^\nu \eta_1) \partial_\mu \Phi_s + \ldots
\]

where dots mean “up to gauge transformation”. So we set \(\beta = \sqrt{2}\) and our final result looks like:

\[
\delta \Psi_s = -\frac{i}{\sqrt{2}} \sigma^{\mu\nu} \partial_\mu \Phi_{\nu(s-1)\gamma_1} \eta + \frac{i(s-1)(s-2)}{2\sqrt{2}s} [\partial_{(1 \gamma_1} \Phi_{s-2)} - g_{(2 \partial_{1}(\gamma \Phi)_{s-3})}] \eta
\]

\[
\delta \Phi_s = \sqrt{2}(\bar{\Psi}_s \eta)
\]

(\textbf{s+1/2, s+1}). Half-integer superspin multiplet contains fermionic spin \(s + 1/2\) fields and bosonic spin \(s + 1\) one. Again by analogy with lower superspin case we will make the following ansatz for supertransformations:

\[
\delta \Psi_s = \alpha_1 \sigma^{\mu\nu} \partial_\mu \Phi_{\nu(s)} \eta, \quad \delta \Phi_{s+1} = i\beta(\bar{\Psi}_{(s+1)\gamma_1} \eta)
\]
This time most of the variations cancel if one set $\alpha = -\beta$ leaving us with:

$$
\delta \mathcal{L} = i(-1)^s \beta \frac{s(s-1)}{4} [2(\bar{\Psi} \partial \partial)^s-2(\gamma \bar{\Phi})_{s-2} - \bar{\Psi}^s-2 \partial^2(\gamma \bar{\Phi})_{s-2} - 2(\bar{\Psi} \gamma \partial)^s-2 \partial(\gamma \bar{\Phi})_{s-2} - (s - 2)(\bar{\Psi} \partial)^s-3(\gamma \partial \bar{\Phi})_{s-3}]$$

Then the full invariance could be achieved with the following correction to supertransformations:

$$
\delta' \Psi_s = \alpha_2 \partial(1 \gamma_1 (\gamma \bar{\Phi})_{s-2})
$$

provided $\alpha_2 = \frac{s-1}{4}$. To check the closure of superalgebra and to choose normalization we calculate commutator of two supertransformations:

$$
[\delta_1, \delta_2] \Phi_{s+1} = -2i \beta^2 (\bar{\eta}_2 \gamma^\mu \eta_1) \partial_\mu \Phi_{s+1} + \ldots
$$

Then our choice will be $\beta = 1$ and our final form:

$$
\delta \Psi_s = -\sigma^{\mu \nu} \partial_\mu \Phi_{(s)} \eta + \frac{s - 1}{4} \partial(1 \gamma_1 (\gamma \bar{\Phi})_{s-2}) \eta, \quad \delta \Phi_{s+1} = i(\bar{\Psi}(s \gamma_1) \eta)
$$

Note that starting with superspin 2 the structure of supertransformations are defined up to possible field dependent gauge transformations and our choice differs from that of [22]. It makes no difference for massless theories but for massive case the structure of corrections for fermionic supertransformations depends on the choice made.

## 5 Integer superspin

Now, having in our disposal gauge invariant description of massive particles with arbitrary (half-)integer spins, known form of supertransformations for massless arbitrary superspin supermultiplets and concrete examples of massive supermultiplets with lower superspins, we are ready to construct massive arbitrary superspin supermultiplets. As we have seen, integer and half-integer cases have different structures and have to be considered separately.

In this section we consider massive supermultiplet with integer superspin. Such supermultiplet also contains four massive fields: two bosonic spin $s$ fields (with opposite parity) and fermionic spin ($s+1/2$) and ($s-1/2$) ones. Calculating total number of physical degrees of freedom and taking into account possible mixing of supermultiplets containing bosonic fields with equal spins and opposite parity, we start with the following structure of massless supermultiplets:

$$
\begin{pmatrix}
A_s & \Phi_s \\
\Psi_{s-1} & B_s
\end{pmatrix} \quad \Rightarrow \quad \sum_{k=1}^{s} \begin{pmatrix}
A_k & \Phi_k \\
\Psi_{k-1} & B_k
\end{pmatrix} \oplus \begin{pmatrix}
\Phi_0 \\
0
\end{pmatrix}
$$

By analogy with superspin 1 and 2 cases, we will assume that all bosonic fields enter through the complex combinations $C_k = A_k + iB_k$ only (so that all possible mixing angles are fixed
and equal \( \pi/4 \). Thus we choose the following form of supertransformations for massless supermultiplets with \( 1 \leq k \leq s \):

\[
\delta \Phi_k = -i \frac{\sigma^{\mu\nu} \partial_\mu \bar{C}(k-1) \gamma_1 \eta}{24k} [\partial(\gamma_1 \bar{C})_{k-2}] - g(2 \partial_1 (\gamma \bar{C})_{k-3}] \eta
\]

\[
\delta C_k = 2(\Phi_k \eta) + i \sqrt{2}(\Psi_{(k-1) \gamma_1} \eta)
\]

\[
\delta \Psi_{k-1} = -\frac{1}{\sqrt{2}} \sigma^{\mu\nu} \partial_\mu \bar{C}(k-1) \eta + \frac{k-2}{4\sqrt{2}} \partial(\gamma_1 \bar{C})_{k-3} \eta
\]

and also

\[
\delta \Phi_0 = -i \hat{\partial} \bar{z} \eta, \quad \delta \bar{z} = 2(\bar{\Phi}_0 \eta)
\]

As a result of our assumption mass terms for bosonic fields are completely fixed:

\[
\mathcal{L}_1 = \sum_{k=0}^{s} (-1)^k c_k \bar{C}(k-1) \partial(\gamma_1 \bar{C})_{k-2} + \frac{(k-1)(k-2)}{4} \bar{C}^2 \partial(\gamma_1 \bar{C})_{k-3} + h.c.)
\]

\[
\mathcal{L}_2 = \sum_{k=0}^{s} (-1)^k [d_k \bar{C} \bar{C} + e_k \bar{C} \tilde{C}_{k-2} + f_k (\bar{C} \bar{C})_{k-2} + h.c.)]
\]

where

\[
c_k = \frac{1}{2} \sqrt{\frac{(s+k)(s-k+1)}{2}}
\]

\[
d_k = \frac{(s-k-1)(s+k+2)}{4(k+1)}
\]

\[
e_k = \frac{k(k-1)}{16(k+1)} [(s-k+2)(s+k-1) + 6]
\]

\[
f_k = -\frac{1}{8} \sqrt{(s-k+2)(s+k-1)(s-k+1)(s+k)}
\]

As for the fermionic mass terms, a priori we don’t have any restrictions on them so we have to consider the most general possible form:

\[
\frac{1}{m} \mathcal{L}_m = \sum_{k=0}^{s} (-1)^k \left[ a_{1k} \bar{\Phi}^{k} \Phi_k - k(\bar{\Phi} \gamma)^{k-1} (\gamma \Phi)_{k-1} - \frac{k(k-1)}{4} \bar{\Phi} \bar{\Psi}_{k-2} + \right.
\]

\[
+ a_{2k} \bar{\Phi}^{k} \ Psi_k - k(\bar{\Phi} \gamma)^{k-1} (\gamma \Psi)_{k-1} - \frac{k(k-1)}{4} \bar{\Phi} \bar{\Psi}_{k-2} + \right.
\]

\[
+ a_{3k} \bar{\Psi}^{k} \Psi_k - k(\bar{\Psi} \gamma)^{k-1} (\gamma \Psi)_{k-1} - \frac{k(k-1)}{4} \bar{\Psi} \bar{\Psi}_{k-2} + \right.
\]

\[
+ i b_{1k} (\bar{\Phi} \gamma)^{k-1} \Phi_{k-1} - \frac{k-1}{2} \bar{\Phi} \bar{\Psi}_{k-2} (\gamma \Psi)_{k-2} + \right.
\]

\[
+ i b_{2k} (\bar{\Psi} \gamma)^{k-1} \Psi_{k-1} - \frac{k-1}{2} \bar{\Psi} \bar{\Psi}_{k-2} (\gamma \Psi)_{k-2} + \right.
\]

\[
+ i b_{3k} (\bar{\Psi} \gamma)^{k-1} \Psi_{k-1} - \frac{k-1}{2} \bar{\Psi} \bar{\Psi}_{k-2} (\gamma \Psi)_{k-2} + \right.
\]

\[
+ i b_{4k} (\bar{\Psi} \gamma)^{k-1} \Psi_{k-1} - \frac{k-1}{2} \bar{\Psi} \bar{\Psi}_{k-2} (\gamma \Psi)_{k-2} \right]
\]

(28)

Where:

\[
a_{i} = \frac{1}{2}, \quad a_{2s} = a_{3s} = b_{3s} = b_{4s} = 0
\]

12
The requirement that total Lagrangian be invariant under (appropriately corrected) supertransformations gives:

\[
a_{1k} = -\frac{1}{2}, \quad a_{2k} = -\frac{2\sqrt{2}}{k+1}c_{k+1}, \quad a_{3k} = \frac{k}{2(k+1)}
\]

\[
b_{1k} = -2c_k, \quad b_{2k} = -\frac{1}{\sqrt{2}}, \quad b_{3k} = 0, \quad b_{4k} = -2c_{k+1}
\]

In this, additional terms for fermionic supertransformations look like:

\[
\frac{1}{m}\delta'\Phi_k = \frac{2ic_{k+1}}{k+1}\left[(\gamma C)_k + \frac{k(k-1)}{4(k+1)}\gamma(1\tilde{C}_{k-1}) - \frac{(k-1)(2k+1)}{8k(k+1)}g(2(\gamma\tilde{C})_{k-2}) \right] - \\
-C_k + \frac{k-1}{2k}\gamma(1(\gamma C)_{k-1}) - \frac{(k-1)(k-2)}{8k^2}g(2\tilde{C}_{k-2}) - \\
-\frac{ic_k}{k}[\gamma(1C_{k-1}) - g(2(\gamma C)_{k-2})]
\]  

(29)

\[
\frac{1}{m}\delta'\Psi_k = \frac{kC_{k+2}}{2\sqrt{2}(k+1)}(\gamma(1\gamma\tilde{C})_{k-1}) - \\
-\frac{ik}{\sqrt{2}(k+1)}\left[(\gamma C)_{k} - \frac{k(k-1)}{4(k+1)}\gamma(1\tilde{C}_{k-1}) + \frac{(k-1)(3k+1)}{8k(k+1)}g(2(\gamma\tilde{C})_{k-2}) \right] - \\
-\sqrt{2}c_{k+1}\left[C_k + \frac{1}{k}\gamma(1(\gamma C)_{k-1}) + \frac{(k-1)(k-2)}{4k^2}g(2\tilde{C}_{k-2}) \right]
\]  

(30)

Here the supertransformations for \(\Phi_k\) field contain terms with \(C_{k+1}\), \(C_k\) and \(C_{k-1}\) fields in the first, second and third lines correspondingly, while that of \(\Psi_k\) contain terms with \(C_{k+2}\), \(C_{k+1}\) and \(C_k\) fields.

6 Half-integer superspin

Next we turn to the half-integer superspin case. This time we have two fermionic spin \((s+1/2)\) fields and bosonic ones with spins \((s+1)\) and \(s\). Usual reasoning on physical degrees of freedom and possible mixings leads us to the following structure of massless supermultiplets we will start with:

\[
\begin{pmatrix}
\Phi_s \\
A_{s+1} \\
B_s \\
\Psi_s
\end{pmatrix} 
\Rightarrow 
\begin{pmatrix}
A_{s+1} \\
\Phi_k \\
B_k \\
\Psi_{k-1}
\end{pmatrix} \oplus 
\begin{pmatrix}
A_k \\
\Phi_0
\end{pmatrix}
\]

We see that this structure is rather similar to that of integer superspin case. The main difference (besides the presence of \(A_{s+1}, \Psi_s\) supermultiplet) comes from the mixing of bosonic fields. We have no reasons to suggest that all mixing angles could be fixed from the very beginning so we have to consider the most general possibility here. Let us denote:

\[
C_k = \cos(\theta_k)A_k + \gamma_5 \sin(\theta_k)B_k, \quad D_k = \sin(\theta_k)A_k + \gamma_5 \cos(\theta_k)B_k
\]
In these notations supertransformations for massless supermultiplets could be written as follows. Highest supermultiplet:

\[ \delta A_{s+1} = i(\bar{\Psi}_s \gamma_1 \eta) \quad \delta \Psi_s = -\sigma^{\mu\nu} \partial_\mu A_\nu(s) \eta + \frac{s-1}{4} \partial_{(1}\gamma_1(\gamma A)_{s-2)} \eta \]

Main set \((1 \leq k \leq s)\):

\[
\begin{align*}
\delta \Phi_k &= -i \frac{k}{\sqrt{2}} \sigma^{\mu\nu} \partial_\mu C_{(k-1)\gamma_1} \eta + \frac{i(k-1)(k-2)}{2\sqrt{2}k} [\partial_{(1}\gamma_1 C_{k-2}) - g_{(2}\partial_{(1}\gamma C)_{k-3}] \eta \\
\delta A_k &= \sqrt{2} \cos(\theta_k)(\bar{\Phi}_k \eta) + i \sin(\theta_k)(\bar{\Psi}_{(k-1)\gamma_1} \eta) \\
\delta B_k &= \sqrt{2} \sin(\theta_k)(\bar{\Phi}_k \gamma_5 \eta) + i \cos(\theta_k)(\bar{\Psi}_{(k-1)\gamma_1} \gamma_5 \eta) \\
\delta \Psi_{k-1} &= -\sigma^{\mu\nu} \partial_\mu D_{(k-1)\eta} + \frac{k-2}{4} \partial_{(1}\gamma_1(\gamma D)_{k-3} \eta}
\end{align*}
\]

and the last supermultiplet:

\[ \delta \Phi_0 = -i \bar{\Psi} \zeta, \quad \delta \bar{\zeta} = 2(\bar{\Phi}_0 \eta) \]

By analogy with superspins 3/2 and 5/2 cases we will assume that fermionic mass terms are Dirac ones. This immediately gives:

\[
\mathcal{L}_f = \sum_{k=0}^{s} (-1)^k \left\{ -\frac{s+1}{k+1} [\bar{\Psi}^k \Phi_k - k(\bar{\Psi} \gamma)^{k-1}(\gamma \Phi)_{k-1} - \frac{k(k-1)}{4} \bar{\Psi} - 2 \Phi_{k-2} - k \bar{\Phi}_{k-2}] - i c_k [(\bar{\Psi} \gamma)^{k-1} \Psi_{k-1} - \frac{k-1}{2} \bar{\Psi} (\bar{\Psi} \gamma)_{k-2} + (\bar{\Psi} \rightarrow \Phi)] \right\}
\]

where:

\[ c_k = \sqrt{\frac{(s+k+1)(s-k+1)}{2}} \]

The choice for the bosonic mass terms (taking into account parity) is also unambiguous:

\[
\mathcal{L}_1 = \sum_{k=0}^{s+1} (-1)^k a_k [A^k \partial_{(1} A_{k-1)} - (k-1) \tilde{A}^{k-2} (\partial A)_{k-2} + \frac{(k-1)(k-2)}{4} \tilde{A}^{k-2} \partial_{(1} \tilde{A}_{k-3)}] + \sum_{k=0}^{s} (-1)^k b_k [B^k \partial_{(1} B_{k-1)} - (k-1) \tilde{B}^{k-2} (\partial B)_{k-2} + \frac{(k-1)(k-2)}{4} \tilde{B}^{k-2} \partial_{(1} \tilde{B}_{k-3)}]
\]

for the terms with one derivative, where:

\[ a_k = \sqrt{\frac{(s+k+1)(s-k+2)}{2}}, \quad b_k = \sqrt{\frac{(s+k)(s-k+1)}{2}} \]

and the following terms without derivatives:

\[
\frac{1}{m^2} \mathcal{L}_2 = \sum_{k=0}^{s+1} (-1)^k [\hat{d}_k A^k A_k + \hat{e}_k \tilde{A}^{k-2} \tilde{A}_{k-2} + \hat{f}_k \tilde{A}^{k-2} A_{k-2}] + \sum_{k=0}^{s} (-1)^k [d_k B^k B_k + e_k \tilde{B}^{k-2} \tilde{B}_{k-2} + f_k \tilde{B}^{k-2} B_{k-2}]
\]

\[ \text{Page } 14 \]
Here:
\[
\hat{d}_k = \frac{(s-k)(s+k+3)}{4(k+1)}, \quad \hat{e}_k = \frac{k(k-1)}{16(k+1)}[(s-k+3)(s+k)+6]
\]
\[
\hat{f}_k = -\frac{1}{4}\sqrt{(s-k+3)(s+k)(s-k+2)(s+k+1)}
\]
\[
d_k = \frac{(s-k-1)(s+k+2)}{4(k+1)}, \quad e_k = \frac{k(k-1)}{16(k+1)}[(s-k+2)(s-k-1)+6]
\]
\[
f_k = -\frac{1}{4}\sqrt{(s-k+2)(s+k-1)(s-k+1)(s+k)}
\]
Note that hatted coefficients differ from the unhatted ones by replacement \( s \to s+1 \).

Now we require that total Lagrangian be invariant under the supertransformations. First of all this fixes all mixing angles:
\[
\sin(\theta_k) = \sqrt{\frac{s+k+1}{2(s+1)}}, \quad \cos(\theta_k) = \sqrt{\frac{s-k+1}{2(s+1)}}
\]
and gives us additional terms for fermionic supertransformations:
\[
\frac{1}{m}\delta'\Psi_k = \alpha_1 A_k + \alpha_2 \gamma_1(\gamma A)_{k-1} + \alpha_3 g_{(2\tilde{A}k-2)} +
+ \beta_1 B_k + \beta_2 \gamma_1(\gamma B)_{k-1} + \beta_3 g_{(2\tilde{B}k-2)} +
+ \frac{k c_{k+1}}{4(k+1)}[\sin(\theta_{k+2})\gamma_1(\gamma \tilde{A})_{k-1} + \cos(\theta_{k+1})\gamma_1(\gamma \tilde{B})_{k-1}]
\]
\[
\frac{1}{m}\delta'\Phi_k = \alpha_4 (\gamma A)_k + \alpha_5 \gamma_1(\tilde{A})_{k-1} + \alpha_6 g_{(2\tilde{A}k-2)} +
+ \beta_4 (\gamma B)_k + \beta_5 \gamma_1(\tilde{B})_{k-1} + \beta_6 g_{(2\tilde{B}k-2)} -
- \frac{a_k}{k\sqrt{2}} \cos(\theta_k)[\gamma_1 A_{k-1} - g_{(2\gamma A)k-2}] -
- \frac{b_k}{k\sqrt{2}} \sin(\theta_k)[\gamma_1 B_{k-1} - g_{(2\gamma B)k-2}]
\]

Where:
\[
\alpha_1 = -\frac{s-k}{\sqrt{2(k+1)}} \cos(\theta_k), \quad \alpha_2 = -\frac{k^2+s+k+1}{\sqrt{2k(k+1)}} \cos(\theta_k)
\]
\[
\alpha_3 = -\frac{(s+1)(k-1)(k-2)}{4\sqrt{2k^2(k+1)}} \cos(\theta_k)
\]
\[
\beta_1 = -\frac{s+k+2}{\sqrt{2(k+1)}} \sin(\theta_k), \quad \beta_2 = \frac{k^2-s+k-1}{\sqrt{2k(k+1)}} \sin(\theta_k)
\]
\[
\beta_3 = -\frac{(s+1)(k-1)(k-2)}{4\sqrt{2k^2(k+1)}} \sin(\theta_k)
\]
\[
\alpha_4 = \frac{s+1}{k+1} \sin(\theta_{k+1}), \quad \alpha_5 = \frac{k(k-1)(s-k)}{4(k+1)^2} \sin(\theta_{k+1})
\]
\[
\alpha_6 = \frac{(k - 1)[(k + 1)(s + 1) - 2k^2(s - k)]}{8k(k + 1)^2} \sin(\theta_{k+1})
\]

\[
\beta_4 = \frac{s + 1}{k + 1} \cos(\theta_{k+1}), \quad \beta_5 = \frac{k(k - 1)(s + k + 2)}{4(k + 1)^2} \cos(\theta_{k+1})
\]

\[
\beta_6 = \frac{(k - 1)[(k + 1)(s + 1) - 2k^2(s + k + 2)]}{8k(k + 1)^2} \cos(\theta_{k+1})
\]

We have explicitly checked that (rather complicated) formulas from this and previous sections correctly reproduce all lower superspins results.

**Conclusion**

Thus, using supersymmetric generalization of gauge invariant description for massive particles, we managed to show that all massive \(N = 1\) supermultiplets could be constructed out of appropriate set of massless ones. In this, in spite of large number of fields involved, all calculations are pretty straightforward and mainly combinatorical. Certainly, using gauge invariance one can fix the gauge where all but four physical massive fields are equal to zero. But in this case all supertransformations must be supplemented with field dependent gauge transformations restoring the gauge. So the structure of resulting supertransformation become very complicated and will contain higher derivative terms.

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