Evidence for non-linear quasiparticle tunneling between fractional quantum Hall edges

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Abstract

Remarkable nonlinearities in the differential tunneling conductance between fractional quantum Hall edge states at a constriction are observed in the weak-backscattering regime. In the $\nu = 1/3$ state a peak develops as temperature is increased and its width is determined by the fractional charge. In the range $2/3 \leq \nu \leq 1/3$ this width displays a symmetric behavior around $\nu = 1/2$. We discuss the consistency of these results with available theoretical predictions for inter-edge quasiparticle tunneling in the weak-backscattering regime.

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Under the application of intense perpendicular magnetic fields, the kinetic energy spectrum of the two dimensional electron gas (2DEG) becomes quantized and breaks into a sequence of macroscopically degenerate Landau levels. The properties of electron states in this regime are largely driven by Coulomb interactions. At some “magic” fractional values of the filling factor $\nu$, in particular, the 2DEG condenses into collective phases: the fractional quantum Hall (FQH) effect is the most remarkable fingerprint of such incompressible quantum fluids \cite{1, 2}. Laughlin showed \cite{3} that the emergence of such collective states implies the existence of new quasiparticles carrying fractional charge. Additionally, at the strong magnetic fields characteristic of the FQH effect the lowest-energy charged excitations are confined at the edge of the sample in one-dimensional branches whose excitations are forced to propagate only in one direction. Wen \cite{4} was the first to describe such one-dimensional modes in terms of a chiral Luttinger liquid. Wen’s proposal stimulated a considerable amount of work aimed at understanding the properties of this non-Fermi liquid state \cite{5}.

The departure from the Fermi-liquid behavior is predicted to show up in many properties of an interacting one-dimensional system \cite{6}. Most of the experiments in quantum Hall systems concentrated on the observation of the power-law behavior of the electron tunneling from a metal to the FQH edge \cite{7, 8, 9} and between two FQH edges \cite{10, 11} in the strong backscattering limit. Resonant tunneling in this regime was also explored \cite{12}. The inter-edge tunneling process, in particular, can be experimentally induced at a quantum point contact (QPC) constriction. In the strong backscattering regime one observes the tunneling of electrons between two quantum Hall fluids separated by the QPC. For simple fractions (i.e. $\nu = 1/q$, where $q$ is an odd integer), this leads to a dc tunneling current at temperature $T=0$ given by $I_T \propto V_T^{(2/\nu-1)}$. Notably $I_T$ vanishes when the bias voltage $V_T$, with $V_T$ labeling the potential difference between the two edges, tends to zero \cite{5}.

In the opposite limit of weak backscattering the quantum Hall fluid is weakly perturbed by the QPC constriction. In this case the inter-edge tunneling current (again, at $\nu = 1/q$) consists of Laughlin quasiparticles of charge $e^* = \nu e$ that scatter between the edges through the quantum Hall fluid (see Fig.1 panel (a)). At $T=0$ the quasiparticle tunneling rate is predicted to grow at low voltages as $I_T \propto V_T^{(2\nu-1)}$ in contrast to the electron-tunneling case discussed above. This remarkable nonlinear behavior is removed at finite temperature. When $V_T$ falls below a critical value $V_{T,max}$ of the order of $k_B T / e^*$, the tunneling current reverts to the linear ohmic behavior $I_T \propto V_T$. In the differential tunneling characteristics
(dI_T/dV_T) this leads to a peak centered at V_T = 0 with a width ∆V_T ≃ 2V_{T,max}. This phenomenology was first predicted by Wen [13] who showed, in particular, that the width of the finite-temperature peak is related to the fractional charge of the quasiparticle. The complex phenomena associated to non-linear quasiparticle tunneling, however, are still largely experimentally unexplored. During recent years magneto-transport experiments in the weak backscattering regime were reported and concentrated on shot noise measurements aimed at detecting the fractional charge of the quasiparticle [14].

In this Letter we report the observation of non-linear inter-edge tunneling in the weak backscattering regime. We present the differential tunneling conductance as a function of bias voltage in a wide range of temperatures and filling factors. In the FQH regime our data cannot be described by electron tunneling but display the features of Wen’s theory of quasiparticle tunneling. We demonstrate that while the differential tunneling characteristic shows the tendency towards a diverging behavior as the temperature is lowered at ν = 1/3, it develops a peak centered at V_T = 0 as the temperature is increased. Width and shape of this zero-bias peak are determined by the fractional charge of the quasiparticle, consistently with Wen’s predictions. We also discuss the results obtained for filling factors between ν = 2/3 and ν = 1/3 where the zero-bias peak is observed even at the lowest probed temperature of 30 mK. Finally, we show that the evolution of the width of the differential conductance peak displays an unexpected symmetry around ν = 1/2 not explained by current theories. We believe that these results combine to provide the first evidence of non-linear quasiparticle tunneling in FQH systems.

The devices here studied were realized starting from a high-mobility GaAs/Al_{0.3}Ga_{0.7}As two-dimensional electron gas (2DEG) with low-temperature mobility μ ∼ 1 × 10^6 cm^2/Vs and two-dimensional electron density n ∼ 5×10^{10} cm^{-2}. The 2DEG was located 140 nm from the surface. The measurements discussed below were performed in a dilution refrigerator. The QPC constriction was nanofabricated on Hall-bar mesas (width of 80 μm) using e-beam lithography and Al metallization. The width and length of the QPC constriction were 300 nm and 600 nm, respectively.

Differential inter-edge tunneling conductance as a function of bias voltage V_T was controlled by exploiting the QPC (see Fig.1 panel (a)) in order to force edge states to flow close to each other in the constriction. In the measurements presented here the split-gate was biased at V_g = −0.4 V which is just below the 2DEG depletion leading to 2D-1D thresh-
old at zero magnetic field. At these bias conditions therefore the conduction takes place through the point contact only. This ensures that edge states indeed propagate inside the QPC while they are still separated enough to avoid strong interactions. A current $I$ with both dc and ac components was supplied to the Hall bar. This current causes a Hall voltage drop $V_H = \rho_{xy} I$ (with dc and ac components) between the counter-propagating edge channels within the constriction \cite{[17]}. In the weak backscattering regime $V_H$ coincides with the tunneling bias i.e., $V_T = V_H$. Four-wire differential-resistance measurements were carried out using an ac lock-in technique (see Fig.1 panel (a)) with $0 \leq I_{dc} \leq 45 \text{ nA}$ and $I_{ac} = 250 \text{ pA}$. The latter value was chosen in order to have an acceptable signal-to-noise ratio without introducing unphysical nonlinearities in the tunneling current. The quantity that is measured in our configuration is the resistivity drop at the constriction (the differential longitudinal resistance $dV/dI$). In the weak backscattering regime $dV/dI$ is directly related to the differential tunneling conductance by:

$$dV/dI = \rho_{xy}^2 dI_T/dV_T.$$  \hspace{1cm} (1)

The presence of residual backscattering outside the constriction leads to a background signal superimposed to (1). In the present devices this background is significant at $\nu = 1/3$. Even away from the constriction the longitudinal resistivity is around $4k\Omega$, i.e. about 30\% of the value measured at the constriction in the bias range of interest for the weak backscattering regime. However it does not display a sizeable variation as a function of the tunneling bias (data not shown) at least for low values of the tunneling bias. This and additional control experiments carried out at lower values of $V_g$ allow us to unambiguously attribute the observed structures in $dV/dI$ to quasiparticle tunneling at the QPC.

Panel (b) in Fig.1 shows the longitudinal resistance $\rho_{xx}$ at $I_{dc} = 0$ as a function of magnetic field at $V_g = -0.4 \text{ V}$. This measurement displays Shubnikov-de-Haas oscillations associated to the quantum Hall states in the presence of the constriction. At $I_{dc} \neq 0$ marked nonlinearities are observed in the FQH regimes. An example is reported in the main panel of Fig.1 where two representative differential conductance curves at filling factors 2 and 1/3 are shown. As expected, the behavior in the integer regime is linear even at the lowest temperatures explored. In order to emphasize the differences observed in the fractional regime, the marked nonlinear behavior measured in the case $\nu = 1/3$ is shown at the comparatively high temperature of $T = 400\text{mK}$.
Figure 2 (panel a) shows the temperature evolution at $\nu = 1/3$ for temperatures up to 900 mK. At this FQH state, the lowest-temperature curve (at 30 mK) shows a minimum at zero bias. The zero-bias peak, however, is recovered at higher temperatures. It develops above 400 mK and tends to disappear as temperature is increased above 900 mK. We can understand the data in Fig. 2 in the framework of the weak-backscattering theory for inter-edge tunneling originally proposed by Wen [13]. We recall that in the weak-backscattering regime the constriction is a small perturbation for edge-state propagation and induces a limited and localized backscattering. This limit can be quantified comparing the tunneling current ($I_T$) to the total current ($I$) flowing through the device. We define the scattering to be weak when $I_T \ll I$ (this condition could also be stated as $dV/dI \ll h/\nu e^2 \sim 75k\Omega$). In this regime we expect,

$$I_T (V_T) = \frac{2\pi |t|^2}{\Gamma(2g)} \left| \frac{2\pi T}{T_0} \right|^{2g-1} \left| \Gamma \left( g + \frac{i x}{2\pi} \right) \right|^2 \sinh \left( \frac{x}{2} \right),$$  \hspace{1cm} (2)

where $g = e^{*2}/\nu e^2$, $x = e^*V_T/k_BT$, $\Gamma$ is the Euler Gamma-function, and $t$ is the inter-edge tunneling amplitude. Note that in Wen’s original theory the chiral Luttinger liquid was assumed to exist only at the filling factors of the fractional quantum Hall effect. In (2) however, the filling factor $\nu$ is allowed to vary continuously. Such an extension of the original formulation can be justified on the basis of a hydrodynamic model, which allows us to derive a continuum of Luttinger liquids with continuously varying $\nu$ [18].

The behavior of $I_T$ depends crucially on the relative size of $e^*V_T$ and $k_BT$. At low temperatures (2) predicts the nonlinear behavior $I_T \propto V_T^{2g-1}$, which, for $g < 1/2$, leads to a growing current with decreasing bias: this is the signature of the overlap catastrophe. At higher temperatures (2) predicts an ohmic behavior $I_T \propto V_T$. The crossover between the two regimes occurs at $V_T = V_{T,max}$, where $I_T$ reaches a maximum. Figure 2 (panel b) shows the calculated differential tunneling conductance (the derivative of (2)) at $\nu = 1/3$ ($g = 1/3$) at different temperatures. The peak at zero bias arises from the ohmic region of the $I_T - V_T$ relation. As the temperature lowers this peak is expected to grow in intensity without saturation and to shrink in size. Indeed the width of this peak (defined here as the distance between the two zeroes of the differential conductance) is $2V_{T,max} \sim 4.79k_BT/e^*$ and is directly related to the effective charge of the quasiparticle involved in the tunneling process [13]. For $V_T > V_{T,max}$ the differential conductance becomes negative, and eventually tends to zero as $V_T^{2g-1}$. 

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Figure 2 (panel c) reports the experimental results obtained for temperatures higher than 400 mK. The tunneling conductance in this range of temperatures is characterized by a zero bias enhancement and the observed overall trend is in good agreement with the calculated behavior shown in Fig.2 (panel b) and discussed above. The agreement is particularly satisfactory for what concerns the width of the conductance peak. At higher temperatures the peak broadens and its amplitude decreases following qualitatively the theory. At temperatures higher than 1K the tunneling is completely linear in this voltage range. The data also show evidence of a negative contribution to the tunneling conductance at larger values of the bias voltage: again this is in qualitative agreement with the theory although the large background rises the average value of the conductance above zero.

At temperatures lower than 400 mK (see Fig.2, panel a), however, a crossover to a completely different behavior is observed. The tunneling conductance exhibits a minimum at zero bias. The disappearance of the peak can be related to its progressive shrinking with temperature: for a given current modulation intensity a threshold temperature can be estimated below which the peak cannot be detected. In the weak backscattering limit $V_{T,ac} \sim \rho_{xy}I_{ac} \sim 0.02mV \sim 100mK$. This value is not in agreement with our experimental finding and suggests that the system by lowering the temperature may evolve into the strong backscattering regime [13]. A further mechanism for the zero-bias suppression can be associated to negative interference between spatially separated tunneling events within the QPC constriction. This effect gets stronger with decreasing temperature and can contribute to the suppression at zero bias [20].

Next we examine the dependence of tunneling current on filling factor. Experimental results are summarized in the color plots of Fig.3 for different temperatures (here higher values of the tunneling conductance are in yellow, lower values in black). The measured $I_T - V_T$ relation was linear at integral filling factor (data not shown) and for temperatures above 900 mK. Otherwise, above 400 mK, the zero-bias peak is visible in the whole range $1/3 \leq \nu \leq 2/3$, and presents an interesting evolution: the peak is strongest and narrowest at $\nu = 1/3$ and $\nu = 2/3$ (the two most prominent QHE fractions) and rapidly broadens and loses its strength as $\nu = 1/2$ is approached from either side. Notably its width evolves symmetrically about $\nu = 1/2$. This is shown in more detail in Fig.4 where the differential tunneling conductance is plotted at three representative values of the magnetic field. The inset of Fig.4 reports the width of the zero-bias peak as a function of magnetic field at
T=400 mK (filled circles) and T = 500 mK (open circles). A similar behavior was found also at T = 700 mK (data not shown).

At the moment, a convincing theory of this is not available. Equation 2 is not particle-hole symmetric under any reasonable assumption. Setting $g = \nu$ yields a tunneling conductance curve with a peak at zero bias: however the width of the peak increases monotonically with $\nu$, while its amplitude decreases. It should be pointed out that, at $\nu = 2/3$ (and more generally at filling factors of the Jain sequence $np/(np + 1)$), the result of (2) with $g = \nu$ is in agreement with the weak backscattering theory of composite edge states developed by Kane and Fisher [1]. Finally, we should like to point out that also the width of the zero-bias peak observed at $\nu > 1/3$ and low temperature is not compatible with (2). One is tempted to associate the non-monotonic behavior of the peak with the different structure of the zero-temperature fixed points controlling the charge mode for the Jain sequences with positive and negative $p$. However, additional theoretical analysis of quasiparticle tunneling at non-quantized filling factor values is needed [21].

In conclusion, we reported a marked non-linear behavior in the inter-edge tunneling in the fractional quantum Hall regime in the weak-backscattering limit. The observation of a zero-bias peak in the differential tunneling conductance has been interpreted as evidence for quasiparticle tunneling between fractional edge states. In selected ranges of temperatures and filling factors our data are consistent with Wen’s theory of quasiparticle tunneling. Results as a function of filling factors reveal intriguing features not predicted by current theories.

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FIG. 1: Main panel: differential tunneling conductance \( (dI_T/dV_T) \) as a function of driving current \( I_{dc} \) at the constriction for filling factors \( \nu = 2 \) at \( T = 30 \) mK and \( \nu = 1/3 \) at \( T = 400 \) mK. Panel (a): sketch of the device and experimental set-up. Panel (b): longitudinal resistivity \( \rho_{xx} \) as a function of magnetic field at \( I_{dc} = 0 \) and \( T = 30 \) mK.

FIG. 2: Panel (a): differential tunneling conductance \( (dI_T/dV_T) \) for filling factor \( \nu = 1/3 \) at different temperatures (30 mK, 100 mK, 200 mK, 300 mK, 400 mK, 500 mK, 700 mK, 900 mK from bottom to top). Panel (b): Calculated \( dI_T/dV_T \) (derivative of Eq. 2) in the weak-backscattering regime at \( T = 500 \) mK, 700 mK and 900 mK. Panel (c): selected differential tunneling conductance curves at the same temperatures of panel (b).

FIG. 4: Representative differential tunneling conductance \( (dI_T/dV_T) \) at three values of magnetic field and \( T = 500 \) mK. The inset reports the evolution of the peak width (as derived from a Lorentzian best-fit procedure) as a function of magnetic field for \( T = 400 \) mK (filled circles) and \( T = 500 \) mK (open circles). (see Fig.1 panel (b) to relate magnetic field to the filling factor.)

FIG. 3: Color plots of the differential tunneling conductance \( (dI_T/dV_T) \) as a function of the driving current \( I_{dc} \) and magnetic field \( V_T = \rho_{xy}(B)I_{dc} \) at different temperatures. \( \nu = 1/3 \) occurs at \( B \approx 6T \). Bright yellow regions correspond to high value of the tunneling conductance.
(a) \( \nu = 1/3 \quad T = 400 \text{ mK} \)

(b) \( \nu = 2 \quad T = 30 \text{ mK} \)

\[ \frac{dI_T}{dV_T} (\mu \text{S}) \]

\[ T = 400 \text{ mK} \]

\[ T = 30 \text{ mK} \]

\[ \rho_{xx} (k\Omega) \]

\[ I_{dc} (\text{nA}) \]

Magnetic field (T)
\[ \frac{dI}{dV_T} (\mu S) \]

(a) 900 mK
(b) 500 mk, 700 mk, 900 mk
(c) 30 mK

\[ V_T (mV) \]

\[ dI_T/dV_T (arb. units) \]

\[ dI_T/dV_T (\mu S) \]

(a) 900 mK
(b) 500 mk, 700 mk, 900 mk
(c) 30 mK
$B = 3.2 \, T$
$\nu = 0.62$

$B = 4 \, T$
$\nu = 0.49$

$B = 6 \, T$
$\nu = 0.34$

$T = 500 \, \text{mK}$