INTRODUCTION

Combined cycle power plants (CCPPs) are widely used in the electric power generation. They owe their popularity to their high efficiency, great flexibility, short installation time, and low installation cost. In a CCPP, modeling, control, stability analysis, and reliability of a gas turbine unit are a challenging problem. This is due to the fact that the traditional PID-based controllers do not have proper performance due to parameter variations, unmodeled dynamics, modeling errors, modeling simplifications, or neglected nonlinearity, and may even result in system instability. Hence, robustness should be a major design feature when controlling a gas turbine power plant (GTPP). Various models have been proposed in the literature to accurately represent the dynamics of CCPP. A dynamic model for a single-shaft combined cycle plant including a supervisory control was proposed. The proposed model was assessed in terms of its stability and response to frequency transients during the design phase.
full-load operations. A comparative study between different gas turbines and combined cycle models can be found in. The authors of paper compared the performance of a detailed model, GAST2A model, and GGOV1 model in terms of their responses regarding electric load and frequency transients. A verified combined cycle gas turbine model was considered in the study reported in. The impact of combined cycle gas turbine dynamics on the frequency control of an island electricity system was also considered in that study. In, Kakimoto et al investigated several combined cycle plants models to build a new one and then studied the dynamic behavior of the plant including temperature and speed control, fuel flow command, inlet guide vanes (IGV), and output power in the presence of frequency drops. The work reported in focused on the frequency regulation issue and aimed at improving the combined cycle dynamic response and frequency regulation performance and maintaining the machine variables within a safe operational level. A combined cycle simple model for the investigation of abnormal frequency conditions can be found in.

Likewise, various control approaches were proposed in the literature for the gas and steam turbines in a CCPP. Saikia et al proposed a PID controller for the combined cycle gas turbine plant and used a firefly algorithm to optimize the controller gains. The performance and robustness of the proposed controller were investigated using the sensitivity analysis. A fractional-order fuzzy PID controller based on the particle swarm optimization algorithm was proposed for CCPP in the study by Haji-Haji and Monje. The proposed controller was implemented in the speed control loop to improve the frequency response during any change in power demand or frequency deviation. Jadhav et al took advantage of Bode's ideal loop transfer function to design a robust fractional-order controller for the speed loop to improve the frequency response of a gas turbine plant. In the study reported in, a neuro-fuzzy—based controller was proposed and evaluated for set-point variations in a grid-connected heavy-duty gas turbine plant. Most of the above control approaches, however, lacked robustness to parameter variations and dynamic system mismatches. Gorbani et al presented a model predictive controller (MPC) for a MS9001E gas turbine mounted in Montazer Ghaem Power Plant.

The performance of the GTPP is highly dependent upon the control strategy considered. Failure to provide proper control to any unwanted frequency drop or unit over temperature can negatively affect the power plant and finally lead to serious damage to the gas turbine components, and might even lead to system shutdown. Any frequency load fluctuation or set-point variation in power plants can cause instability in power grids and potentially lead to blackouts.

Furthermore, there are lots of discrepancies between the mathematical models (boiler, combustor, compressor, fuel system, air system, etc) used for design and the actual gas turbines’ dynamics in practice. Therefore, designing robust controllers for such systems can have a great impact on their performance and life span. The proposed algorithms should not only be able to make the closed-loop power plant stable but also provide acceptable performance levels (tracking capability, transient and steady-state performance, etc) in the presence of frequency disturbance, unmodeled power plant dynamics, and measurement noise.

Due to the fact that robust control theory explicitly deals with uncertainty in its approach to control design, it has long been considered in controlling various systems. In, a $H_\infty$ controller was designed for the speed and temperature control of a power plant gas turbine. It was shown that $H_\infty$ resulted in improvements in the performance compared to MPC and PID controllers for the same conditions. $\mu$-synthesis was considered in to guarantee the robust stability (RS) and performance of an industrial hydraulic excavator. A multi-objective robust $H_2/H_\infty$ fuzzy tracking approach was proposed in to control the nonlinear superheat temperature system in a power plant.

In this study, we design, implement, and compare the performance of four robust control approaches for both temperature and speed control of a V94.2 gas turbine mounted in Damavand combined cycle plant. The proposed $H_2$, $H_\infty$, $H_2/H_\infty$, and $\mu$-synthesis controllers aim at (a) guaranteeing the RS and performance of the closed-loop system in the presence of model uncertainties, (b) maintaining the temperature output within a desired level, and (c) improving the speed loop response for any frequency drop caused by a change in the power demand or output disturbances.

## 2 | NONLINEAR GAS TURBINE POWER PLANT MODEL

A simplified block diagram of a gas turbine plant including gas turbine, rotor inertia dynamics, fuel and air system, and temperature transducer is illustrated in Figure 1, where air flow $W_a$ and fuel flow $W_f$ are the main manipulated signals, and rotor speed (frequency) $N$, power $P_m$, and exhaust temperature $T_e$ are the main output signals. In Figure 1, the value of $T_{off} = 0.01$ (pu) refers to temperature offset; $K_W = 2.1281$ is the inverse of air control time constant; for the overheat control, 3.3 and 0.4699 are the time constant of overheat control and the overheat control integration rate, whereas 2.49 and 0.12 refer to the time constant of the speed control and the speed control integration rate, respectively. The description of other input/output signals used in Figure 1 is summarized in Table 1.

The speed and temperature control loops of the GTPP are shown in Figure 1. As Figure 1 shows (red line), the speed (frequency) control loop includes the speed controller, fuel limiter, fuel system, gas turbine dynamics, and rotor inertia. The main objective of the speed control loop is to act through minimum value selection and fuel system to compensate any difference between generation and load in the frequency deviation.
The temperature control loop, on the other hand, includes the normal and overheat temperature control branches (Figure 1, green line). The main purpose of the overheat control branch is to reduce the fuel flow in order to prevent turbine overloading in extreme overheat cases. This loop includes fuel limits, fuel system, gas turbine dynamics, temperature transducer, and overheat controller. Note that, for a CCPP, a common practice in the industry consists on using the bottom steam cycle when the exhaust temperature exceeds its limit.

The normal temperature branch acts through the air system or IGV, and includes air control, gas turbine equations, and temperature transducer. This branch prevents the exhaust gas temperature from exceeding the reference value $T_r$. Note that the frequency (speed) and normal temperature loops are the main purpose of this paper; therefore, it is assumed that the overheat control temperature is not active all the time.

Besides, there is an anti-windup mechanism included in temperature control loop to prevent the controller from saturation and to make sure that the controller can react properly to control GTTP. For a gas turbine process, the relationship between the inputs $W_f$ and $W_a$ and output $T_i$ is:

$$T_i = T_d + (T_{i0} - T_{d0}) \frac{W_f}{W_a}, \quad (1)$$

$$x = \left( \frac{Q_{r0} W_a}{T_{r0}} \right)^{\frac{1}{\gamma - 1}}, \quad (2)$$

$$T_d = T_a \left( 1 + \frac{x - 1}{\eta_c} \right), \quad (3)$$

where $T_{i0}$ is the rated value for the gas turbine inlet temperature, $T_{d0}$ is the rated value for the compressor discharge temperature, $x$ is the compressor temperature ratio, $P_{r0}$ is the nominal compressor pressure ratio, $\gamma$ is the ratio of specific heats, and $\eta_c$ represents the compressor efficiency. The gas turbine inlet temperature $T_i$ can be expressed as follows:

$$T_i = T_a \left[ 1 - \left( 1 - \frac{1}{x} \right) \eta_i \right]. \quad (4)$$

where $\eta_i$ is the turbine efficiency. The energy supplied to gas turbine is given by:

$$E_g = K_0 \left( (T_i - T_c) - (T_d - T_a) \right) W_a, \quad (5)$$

where $K_0$ is the gas turbine coefficient. Further details about the parameters and specifications of the GTPP model can be found in.1,3,4

### 3 | GAS TURBINE POWER PLANT SYSTEM IDENTIFICATION AND VALIDATION

According to the ARX structure, the relationship between the estimated output $y(t)$, measured data input $u(t)$, and white-noise disturbance value $e(t)$ can be described as:

$$y(t) + a_1 y(t-1) + \cdots + a_n y(t-na) = b_1 u(t-nk) + \cdots + b_{nb} u(t-nb-nk+1) + e(t), \quad (6)$$
where \( u(t – nk) \) and \( y(t – na) \) refer to the past inputs and outputs, respectively, \( na \) is the number of poles, \( nb \) is the number of zeroes plus 1, and \( nk \) is the dead time. Then, \( a \) and \( b \) are constant factors that could be estimated using the least square error method (LSM).

As Figure 1 shows, the airflow \( W_a \) is obtained by multiplying \( N \) by IGV. This means that any change in IGV can not only affect the exhaust temperature but also impact the power and frequency outputs. Thus, the IGV signal needs to be considered as an input in the estimation process. According to Figure 1, in order to estimate a linear ARX model for the speed loop, the fuel flow Fuel, output frequency \( N \), ambient temperature \( T_a \), and IGV should be considered as input variables, whereas the power demand \( P_m \) should be considered as output signal. The equation relating the frequency (speed) output \( N \) with the inputs Fuel, \( T_a \), and IGV can be determined using the rotor dynamics and \( P_m \). Note also that the input signal Fuel is equal to fuel demand signal (Fd) for a power plant under normal operating conditions.

Similarly, based on Figure 1 for the temperature loop, the variables Fuel, \( N \), \( T_a \), and IGV are considered as inputs, and the exhaust temperature \( T_e \) as output. In this paper, the considered base load for the V94.2 GTTP is 115MW and the atmospheric pressure is around 896.5 (mbar). The modeling and identification has been done around V94.2 gas turbine operating point. \( T_a \) is 30°C or 303 K, IGV is [0.52, 1], \( N \) is [0.95, 1], and Fuel is [0, 1.0]. Figure 2 shows the real data for V94.2 gas turbine in Damavand CCPP. The data are collected from no-load to full-load conditions with sampling time equals to 1 seconds. Note that, in the case of control loops, the main focus is on frequency and normal temperature loops; hence, the overheat control loop is not active during the simulation process and the Fuel signal is equal to \( F_d \).

The identified block diagram of the gas turbine plant CP(s) is shown in Figure 3, where \( P_d \) represents the demand power (per unit (pu)). Based on Equation 6 and the ARX structure, the transfer functions presented in Figure 3 are as follows:

\[
\begin{align*}
G_1(s) &= \frac{P_m(s)}{Fuel(s)} = \frac{0.65s^3 + 0.46s^2 + 0.569s + 0.0412}{s^4 + 0.988s^3 + 2.208s^2 + 0.645s + 0.033}, \\
G_2(s) &= \frac{P_m(s)}{N(s)} = \frac{-2.12s^3 - 2.896s^2 - 1.94s - 0.017}{s^4 + 0.988s^3 + 2.208s^2 + 0.645s + 0.033}, \\
G_4(s) &= \frac{P_m(s)}{IGV(s)} = \frac{-0.008s^2 + 0.001s - 0.003 + 2.756 \times 10^{-05}}{s^4 + 0.988s^3 + 2.208s^2 + 0.645s + 0.033}, \\
G_5(s) &= \frac{P_m(s)}{Fuel(s)} = \frac{0.061s^3 + 0.071s^2 + 0.115s + 0.00071}{s^4 + 0.988s^3 + 2.208s^2 + 0.645s + 0.033}, \\
G_6(s) &= \frac{T_e(s)}{N(s)} = \frac{13.22s + 0.050}{s^2 + 6.31s + 0.096}, \\
G_7(s) &= \frac{T_e(s)}{Fuel(s)} = \frac{-0.025s - 5.32 \times 10^{-05}}{s^2 + 6.31s + 0.096}, \\
G_8(s) &= \frac{T_e(s)}{IGV(s)} = \frac{-1.34s - 0.028}{s^2 + 6.31s + 0.096}, \quad (14)
\end{align*}
\]

\[
G_3(s) = \frac{1}{14.5s}, \quad (15)
\]

In a time series format, the transfer functions are as follows:

\[
\begin{align*}
G_1 &= (0.198 \pm 0.005)e^{(0.334 \pm 1.36)i} + 0.0157e^{-0.0661i} + 0.239e^{0.255i}, \\
G_2 &= (-0.737 \pm 0.552)e^{(0.334 \pm 1.36)i} + 0.272e^{-0.0661i} - 0.92e^{-0.255i}, \\
G_3 &= 0.069, \\
G_4 &= (-0.003 \pm 0.002)e^{(0.334 \pm 1.36)i} + 0.0007e^{-0.0661i} - 0.003e^{-0.255i}, \quad (19)
\end{align*}
\]

Table 1: Main input/output signals of the GTPP model

| Signals  | Definition                                      |
|---------|------------------------------------------------|
| \( P_m \) | Gas turbine generated power (pu)                |
| \( E_g \) | Thermal power converted by the gas turbine     |
| \( T_e \) | Exhaust temperature (pu)                       |
| \( T_i \) | Reference temperature (pu)                     |
| \( T_a \) | Ambient temperature (°C)                       |
| \( T_l \) | Gas turbine inlet temperature (pu)             |
| \( F_d \) | Fuel demand signal (pu)                        |
| \( N \)  | Rotor speed (frequency)                        |
| \( W_a \) | Air flow                                       |
| \( W_f \) | Fuel flow (pu)                                  |
| IGV     | Inlet guide vanes                               |
| \( T_{i0} \) | Rated value for gas turbine inlet temperature (°C) |
| \( T_{d0} \) | Rated value for the compressor discharge temperature (°C) |
| \( x \)   | Compressor temperature ratio                   |
| \( \eta_t \) | Turbine efficiency                             |
Figure 4 provides the steady-state diagram for the identified ARX model, where the power plant variables $T_e$, IGV, and Fuel are plotted versus changes in the power demand (pu). Two operating regions can be distinguished in Figure 4. In the first (I) operating region, the temperature $T_e$ is lower than the set point; therefore, IGV is set to its minimum value (0.52 (pu)). When increasing the fuel flow, the exhaust temperature rises to the reference ($T_r = 1$) and the power demand to 0.7917, and then, the power plant enters the operating region II. In this region, the temperature control loop acts via the IGV control to maintain $T_e$ at its reference value.

The estimated linear model specifications and measured and simulated model outputs for the speed and temperature loops are illustrated in Table 2 and Figure 5, where MSE refers to the mean square normalized error performance function, FPE is Akaike’s final prediction error, and Fit is fitness percentage. As Figure 5 shows, the simulated outputs closely follow GTPP power and temperature outputs. Based on the residual analysis tests, and measured and simulated model outputs, it was found that the estimated linear ARX model is accurate enough to describe the behavior of the gas turbine plant. This can also be verified by the fitness results illustrated in Table 2, which are 94.7% for the temperature loop and 92.72% for speed loop. Additional assessment of the identified model including the speed $C_{\text{Speed}}$ and temperature $C_{\text{Temp}}$ loops will be performed in Simulation section.

### 4 ROBUST CONTROL FORMULATION

The standard block diagram of the robust control method including external inputs $w$, control inputs $u$, output signals $z$, and measured outputs $y$ is presented in Figure 6. In this figure, $M(s)$ and $K(s)$ refer to the plant model and controller, respectively. The main control objective is to find a controller $K(s)$ that would stabilize plant $M(s)$ and minimize a norm of the transfer function from input $w$ to output $z$. In this paper, the objective function that would maintain the turbine speed and the exhaust temperature within their desired interval is $S/KS$, with $S$ the sensitivity function and $K$ the controller function.

Note that, according to Figure 1, the overheat control branch only acts in the extreme overheat cases to prevent

\[
G_e = (0.004 \pm 0.0111) e^{0.535 s + 1.364} - 0.0181 e^{-0.0061 s} + 0.071 e^{-0.255s}, \quad (20)
\]

\[
G_6 = 13.2 e^{-3.16 s} (\cosh (3.14 t) - 1.0 \sinh (3.14 t)), \quad (21)
\]

\[
G_7 = -0.0247 e^{-3.16 s} (\cosh (3.14 t) - 1.0 \sinh (3.14 t)), \quad (22)
\]

\[
G_8 = 1.45 e - 3.16 s (\cosh (3.14 t) - 0.984 \sinh (3.14 t)), \quad (23)
\]

\[
G_9 = -1.34 e^{-3.16 s} (\cosh (3.14 t) - 0.998 \sinh (3.14 t)). \quad (24)
\]
turbine overloading. Thus, this control branch is not active during the normal operation and there is no direct feedback from the output temperature error $e_T$ to the speed controller. Note also that, based on Equation 5, the output power can be affected by the air flow or IGV control, and this impact can be considered as an input disturbance in the robust control design procedure. Similarly, according to Figure 1 (green line) for temperature control loop, there is no direct feedback from the output speed error $e_N$ to the temperature controller. Thus, the speed and temperature loops could either be controlled using a multi-inputs, multi-outputs (MIMO) controller or two single-input, single-output (SISO) controllers.

Figures 7 and 8 represent the linear estimated model of the speed and temperature loops of the GTPP, where Fuel and IGV are the manipulated variables, whereas $N$ and $T_e$ are the measured outputs. $N_{\text{ref}}$, $P_d$, $T_{\text{ref}}$, and $T_a$ are the reference inputs, and $Z_N$, $Z_{\text{Temp}}$, $Z_{\text{Fuel}}$, and $Z_{\text{IGV}}$ are the weighted output signals. Two variables $e_N$ and $e_T$ are used to show the error of the speed and temperature loops, respectively. Here, the speed error is defined as the difference between speed reference $N_{\text{ref}}$ and the actual speed $N$. Similarly, the temperature error is computed using the difference between temperature reference $T_{\text{ref}}$ and the actual temperature $T_e$. Besides, $W_N$, $W_{\text{Temp}}$, $W_{\text{Fuel}}$, and $W_{\text{IGV}}$ refer to the speed, temperature, fuel,
and IGV weighting functions, respectively. $M_{\text{Speed}}$ and $M_{\text{Temp}}$ refer to the weighted transfer functions for the speed and temperature loops, respectively. In this paper, for the sake of simplicity, we will design two SISO robust controllers for the combined cycle plant.

According to Figures 6 and 7, the relationship between the inputs $w(s)$ and $u(s)$ and the outputs $z(s)$ and $y(s)$ can be defined by:

$$
\begin{bmatrix}
  z(s) \\
  y(s)
\end{bmatrix} = M_{\text{Speed}}(s) \begin{bmatrix}
  w(s) \\
  u(s)
\end{bmatrix},
$$

where $w$, $u$, $z$, and $y$, and $M_{\text{Speed}}$ can be defined as follows:

$$
\begin{cases}
  w = \begin{bmatrix}
    N_{\text{ref}} \\
    P_d \\
    T_a \\
    IGV
  \end{bmatrix},
  u = \text{Fuel},
  z = \begin{bmatrix}
    Z_N \\
    Z_{\text{Fuel}}
  \end{bmatrix},
  y = e_N,
\end{cases}
$$

$$
M_{\text{Speed}} = \begin{bmatrix}
  M_{11}(s) & M_{12}(s) \\
  M_{21}(s) & M_{22}(s)
\end{bmatrix},
$$

where

$$
M_{21}(s) = \begin{bmatrix}
  1 & -G_1G_2, & -G_1G_3, & -G_1G_2G_3 \\
  0 & 1-G_2G_3, & 1-G_1G_3, & 1-G_2G_3
\end{bmatrix},
$$

$$
M_{22}(s) = \begin{bmatrix}
  -G_1G_3 \\
  1-G_2G_3
\end{bmatrix},
$$

$$
M_{12}(s) = \begin{bmatrix}
  W_NM_{22} \\
  W_{\text{Fuel}}
\end{bmatrix}.
$$

The transfer function ($T_{zw}$) from $w$ to $z$ can be written as follows:

$$
F_I(M_{\text{Speed}}, K_{\text{Speed}}) = M_{11} + M_{12}K_{\text{Speed}}(I - M_{22}K_{\text{Speed}})^{-1}M_{21}. 
$$

Similarly, for temperature control loop and from Figures 6 and 8, the relationship between inputs $w'(s)$ and $u'(s)$ and outputs $z'(s)$ and $y'(s)$ can be described by:

$$
\begin{bmatrix}
  z'(s) \\
  y'(s)
\end{bmatrix} = M_{\text{Temp}}(s) \begin{bmatrix}
  w'(s) \\
  u'(s)
\end{bmatrix},
$$

where $w'$, $u'$, $z'$, and $y'$, and $M_{\text{Temp}}$ are:

$$
\begin{cases}
  w' = \begin{bmatrix}
    T_{\text{ref}} \\
    N \\
    T_a \\
    \text{Fuel}
  \end{bmatrix},
  u' = \text{IGV},
  z' = \begin{bmatrix}
    Z_{\text{Temp}} \\
    Z_{\text{IGV}}
  \end{bmatrix},
  y' = e_T,
\end{cases}
$$

$$
M_{\text{Temp}}(s) = \begin{bmatrix}
  M_{11}'(s) & M_{12}'(s) \\
  M_{21}'(s) & M_{22}'(s)
\end{bmatrix}.
$$

**FIGURE 4** Steady-state diagram for GTPP and estimated model

**TABLE 2** ARX model specifications and characteristics

|       | MSE     | FPE     | Fit % |
|-------|---------|---------|-------|
| $T_e$ | $1.161 \times 10^{-05}$ | $1.179 \times 10^{-05}$ | 94.7  |
| $P_m$ | $1.917 \times 10^{-04}$ | $1.927 \times 10^{-04}$ | 92.72 |
where
\[ M'_{21}(s) = \begin{bmatrix} 1 & -G_6 & -G_7 & -G_8 \end{bmatrix}, \]
\[ M'_{22}(s) = -G_9, \]
\[ M'_{11}(s) = \begin{bmatrix} W_{\text{IGV}}M'_{21} \end{bmatrix}, \quad M'_{12}(s) = \begin{bmatrix} W_{\text{Temp}}M'_{22} \\ W_{\text{IGV}} \end{bmatrix}. \] (33)

In Equation 31, \( N \) is equal to \( N_{\text{ref}} \), and the transfer function \( T_{\text{zw}} \) from \( w' \) to \( z' \) for the temperature control loop is defined by:
\[ F'(M'_{\text{Temp}}, K_{\text{Temp}}) = M'_{11} + M'_{12}K_{\text{Temp}}(I - M'_{22}K_{\text{Temp}})^{-1}M'_{21}. \] (34)

For the frequency (speed) control loop (Figure 7 and Equation 26), the manipulated variable is fuel flow (Fuel) and the controlled variable is output frequency \( (N) \), whereas \( N_{\text{ref}}, P_{\text{d}}, T_{\text{a}}, \) and IGV represent the reference and disturbance inputs. For the temperature control loop (Figure 8 and Equation 31), the manipulated variable is IGV and the controlled variable is exhausted temperature \( T_{\text{e}} \), whereas \( T_{\text{ref}}, N, T_{\text{a}}, \) and Fuel represent the reference and disturbance inputs.

4.1 | \( H_2 \) control strategy

The main purpose of considering an \( H_2 \) design for the speed and temperature control of GTPP is to find a controller \( K(s) \) that would (a) internally stabilize \( M(s) \) and (b) minimize the second norm of the following transfer function, where \( W_1 \) and \( W_2 \) are the weighting functions\(^{21,22}\):
\[
\min K \quad \text{stabilizing} \quad \| W_1(I+MK)^{-1} \|_2^2.
\] (35)

For the speed loop in the combined cycle plant model, Equation 35 can be rewritten as follows:
\[
\min K_{\text{Speed}} \quad \text{stabilizing} \quad \| W_N(I+M_{\text{Speed}}K_{\text{Speed}})^{-1} \|_2^2.
\] (36)

And for the temperature control loop, we have the following:
\[
\min K_{\text{Temp}} \quad \text{stabilizing} \quad \| W_{\text{Temp}}(I+M_{\text{Temp}}K_{\text{Temp}})^{-1} \|_2^2.
\] (37)

For this \( H_2 \) optimization problem, the weighting functions are as follows:
Similarly, the main goal of using $H_\infty$ for the speed and temperature loops of the power plant is to find a controller $K(s)$ to internally stabilize $M(s)$ and, at the same time, to minimize the infinite norm of Equations 36 and 37. By choosing appropriate weighting functions, $H_\infty$ can provide RS and nominal performance of the final closed-loop system. In this $H_\infty$ problem, the weighting functions are as follows:

\[ W_N = \frac{s + 9.054}{30.18s + 0.00097}, \quad W_{\text{Fuel}} = 0.02, \]
\[ W_{\text{Temp}} = \frac{s + 2}{20s + 0.006}, \quad W_{\text{IGV}} = 0.01. \] (38)

### 4.2 $H_\infty$ control strategy

Similarly, the main goal of using $H_\infty$ for the speed and temperature loops of the power plant is to find a controller $K(s)$ to internally stabilize $M(s)$ and, at the same time, to minimize the infinite norm of Equations 36 and 37. By choosing appropriate weighting functions, $H_\infty$ can provide RS and nominal performance of the final closed-loop system. In this $H_\infty$ problem, the weighting functions are as follows:

\[ W_N = \frac{s + 9.054}{30.18s + 0.00097}, \quad W_{\text{Fuel}} = 0.02, \]
\[ W_{\text{Temp}} = \frac{s + 2}{20s + 0.006}, \quad W_{\text{IGV}} = 0.01. \] (39)

### 4.3 $H_2/H_\infty$ control strategy

Mixed $H_2/H_\infty$ control combines the disturbance rejection properties of $H_\infty$ design with the ability of $H_2$ to improve the transient behavior of the system against random disturbances. In this paper, the weighting functions for the
controller and power plant optimization problem are as follows:

\[
W_N^h = \frac{s + 9.054}{30.18s + 0.00097}, \quad W_{W\text{fuel}}^{h_2} = 0.01, \quad W_{W\text{IGV}}^{h_2} = 0.015,
\]

\[
W_{\text{Temp}}^{h_2} = \frac{s + 8}{20s + 0.016}, \quad W_{W\text{IGV}}^{h_2} = 0.01, \quad W_{W\text{IGV}}^{h_2} = 0.01. \quad (40)
\]

In Equation 40, the superscripts \(h_\infty\) and \(h_2\) refer to the weighting functions for \(H_\infty\) and \(H_2\) problems, respectively.

### 4.4 | \(\mu\)-synthesis control strategy

Compared to \(H_\infty\), the \(\mu\)-synthesis method based on the singular value \(\mu\) and \(D-K\) iteration procedure can be proposed to ensure the RS and performance of the closed-loop system.\(^{26}\)

In the \(\mu\)-synthesis strategy, the aim is to find the controller \(K\) that would minimize the following optimization problem\(^{26,27}\):

\[
\min_{K} \inf_{D, D^{-1} \in H_\infty} \|DF_1(M,K)D^{-1}\|_\infty. \quad (41)
\]

For the GTPP speed loop, the above equation can be rewritten as follows:

\[
\min_{K_{\text{Speed}}} \inf_{D, D^{-1} \in H_\infty} \|DF_1(M_{\text{Speed}},K_{\text{Speed}})D^{-1}\|_\infty. \quad (42)
\]

Similarly, for the temperature loop we have the following:

\[
\min_{K_{\text{Temp}}} \inf_{D, D^{-1} \in H_\infty} \|DF_1(M_{\text{Temp}},K_{\text{Temp}})D^{-1}\|_\infty. \quad (43)
\]

In Equations 41-43, \(K(s)\) and \(D(k)\) can be achieved by the \(D-K\) iteration procedure. The weighting functions are as follows:

\[
W_N = \frac{s + 0.73}{24.18s + 0.00007}, \quad W_{\text{fuel}} = 0.01,
\]

\[
W_{\text{Temp}} = \frac{s + 0.6}{20s + 0.00007}, \quad W_{\text{IGV}} = 0.001. \quad (44)
\]

All weighting functions are selected based on the IGV, temperature, frequency, and power limitations in order to provide the best transient and steady-state behavior in terms of tracking capability and disturbance rejection. For the speed control loop, the main purpose is to maintain the output frequency near to 50 Hz and to avoid from oscillations as much as possible. In the case of temperature loop, the aim is to maintain the output temperature \(T_c\) lower than \(T_r = 539^\circ\text{C}\).

### 5 | SIMULATION RESULTS AND COMPARISON ANALYSIS

In this section, after analyzing the accuracy of the ARX linear model, we assess the performance of the robust controllers and compare their performance in terms of tracking capability, robustness, and transient behavior.

In order to design the \(H_2\), \(H_\infty\), \(H_2/H_\infty\), and \(\mu\)-synthesis robust controllers for the GTPP, model uncertainties (up to 5\%) are first included into the estimated transfer functions in Equations 7-15. These latter are then combined with Equations 38-40 and 44 for weighting functions and used to calculate Equations 25-28 for the speed control loop and Equations 30-33 for the temperature control loop. Finally, the \(H_2\), \(H_\infty\), \(H_2/H_\infty\), and \(\mu\)-synthesis methods described in Robust control formulation section are considered to compute the best controller \(K(s)\) that would stabilize \(M_{\text{Speed}}(s)\) in Equation 27 and \(M_{\text{Temp}}(s)\) in Equation 32, and that would also minimize the mixed sensitivity cost function presented in Equations 36 and 37 for the temperature and speed control loops, respectively.

Figure 9 shows the RS and robust performance (RP) of all four approaches. The lower bounds of the singular values \(\mu\) for speed and temperature loops are shown in Table 3. According to Figure 9 and Table 3, all four controllers \(H_2\), \(H_\infty\), \(H_2/H_\infty\), and \(\mu\)-synthesis can provide the RS and performance of the closed-loop gas turbine plant. From Figure 9 and Table 3, for the speed control loop, the \(H_\infty\) and \(\mu\) controllers provide the best RS and RP, respectively, while this latter is achieved by \(H_\infty\) controller for the temperature loop.

The frequency responses of the speed and temperature closed loops are shown in Figures 10 and 11, respectively. Note that \(H_\infty\) and \(\mu\)-synthesis display larger bandwidths compared to the rest of the controllers. Although this property offers a better transient response for the speed and temperature loops, it can result in further noise amplification. As shown in Figure 11, all the controllers are characterized by low gains in high-frequency ranges to deal with unmodeled dynamics, whereas in low-frequency ranges, high gains are required to reduce steady-state tracking errors and properly attenuate disturbances.

The GTPP model uncertainty is increased from 0 to 1 (0\% to 100\%), and the simulation results are provided in Figures 12 and 13 and Table 4. In these figures and table, the maximum gain (Max-Gain), stability margin (Stab-Margin), and performance margin (Perf-Margin) are plotted versus changes in the model uncertainty, and Stab-Uncer and Perf-Uncer refer to the uncertainty points where the stability and performance curves go below 1. From the figures, it is clear that these two last uncertainty numbers are equal to the points where the gain curve goes above 1 and to infinity, respectively. To understand the data provided in the figures and table, the explanation of three aspects is necessary: (a) The maximum gain is the possible largest value for the closed-loop Bode diagram over an uncertainty range, and it is in a close connection with stability and performance margins, where a bigger gain leads to a lower margin. Two numbers are indicated in Table 4
for maximum gain: The first one is for 5% uncertainty for speed and temperature loops, and the second one is for maximum uncertainty just before the gain goes to infinity; (b) in Table 4, the stability margin refers to the point of the stability curve in Figures 12 and 13 where the uncertainty is 5% for speed and temperature loops; (c) similarly, the performance margin is the point where performance curves in Figures 12 and 13 cross the uncertainty 5% for speed and temperature loops.

As Figure 12 and Table 4 show, in the case of speed loop, the μ-synthesis robust controller provides the best maximum gain (0.18 for 5% uncertainty) and performance margin (4.9) compared to the other robust controllers. Similarly, the $H_\infty$ controller presents the best stability margin (16.52). These results are in close connection with the uncertainties of 0.79 and 0.65 (Table 4) for $H_\infty$ and μ-synthesis, respectively, where a bigger GTPP model uncertainty (Stab-Uncer and Perf-Uncer) yields better performance and stability margins.

A big frequency gain (1.0) for $H_2$ controller leads to worse stability and performance margins. As for temperature control loop, based on Figure 13 and Table 4, the $H_2$ robust controller presents the best gain and stability margins, while the best performance margin result is for $H_\infty$ controller. The μ-synthesis robust controller provides the worst performance margin. This can also be corroborated by the results in Figure 13.

In order to assess and compare the performance of the different controllers in terms of tracking capability, robustness, and transient performance, two simulation scenarios illustrating (a) tracking and (b) disturbance rejection are considered:

- In the tracking scenario, the speed reference $N_{\text{ref}}$ is initially set to 1; $T_{\text{ref}} = 1$ pu; after 1450s, demand load $P_d$ changes from 0.65 to 0.78 pu, and after 650s, it decreases to 0.68 pu. Here, the goal is to evaluate the transient and steady-state responses of the gas turbine plant for changes in the power demand. The transient response of the GTPP is illustrated.
in Figure 14 for all four controllers. The results are also compared to those from the PID-based controller, where PID refers to the PI (speed control block) and I (air control block) controllers for the speed and temperature loops in Figure 1. The proportional and integral coefficients of the PI and I controllers are based on the procedure provided in\(^4\) and\(^28\). The optimum time indices (maximum overshoot \(M_p\), rise time \(T_r\) and settling time \(T_s\)) and output errors of integral time absolute error (ITAE), integral absolute error (IAE), integral time squared error (ITSE), and integral squared error (ISE)\(^29\) for all the control strategies are summarized in Tables 5 and 6, respectively.

- In the disturbance rejection scenario, the transient response of the GTPP is evaluated for a 1\% instantaneous frequency drop, where the power and temperature references are fixed to 0.76 and 1 pu, respectively. Figure 15 and Table 7 show the output responses of the GTPP for ISE, IAE, ITSE, ITAE, mean, standard deviation, and maximum deviation of output errors for this frequency disturbance.

The results from Figures 14 and 15 and Tables 3, 4, 5, 6 and 7 clearly show that the behavior of the closed-loop system is in close connection with the selected weighting functions in Equations 38-40 and 44 and the design specifications. In the gas turbine plant model, the fuel system has a direct effect on the speed and temperature control loops. As Figures 14 and 15 show, any change in fuel command results in changes in the temperature, power, and speed outputs. Furthermore, any unwanted drop or change in the frequency output due to the presence of an inappropriate speed control can negatively affect the power plant and potentially damage the gas turbine components, and even lead to a system shutdown. Note that

| Controller | RS(S) | RP(S) | RS(T) | RP(T) |
|------------|-------|-------|-------|-------|
| \(H_2\)    | 14.34 | 1.16  | 20.01 | 5.22  |
| \(H_\infty\) | \textbf{16.30} | 1.73  | \textbf{20.00} | 6.95  |
| \(H_2/H_\infty\) | 15.61 | 1.51  | \textbf{20.00} | 3.76  |
| \(\mu\)    | 14.49 | 4.18  | 19.68 | 2.65  |

The bold values are the best simulation results.
the performance analysis of the speed loop is relatively easier than that of the temperature loop. Analyzing the performance of this latter requires to take into account the effects of IGV control, fuel system, power, and speed outputs at the same time. In the case of IGV control and $T_e$, as Figures 14 and 15 show, IGV controller has an inverse effect on the temperature output and a control signal with a high overshoot and little rise time can lead to a better performance of the temperature response. In fact, at a fixed fuel flow, increasing the IGV signal increases the airflow to the power plant and decreases the gas temperature in the combustor. As can be seen in Figure 15, in the case of the $H_2/H_\infty$ controller, any delay in opening the airflow can lead to an increase in the exhaust temperature beyond the reference $T_r$.

In terms of tracking performance of the speed loop, as Figure 14 and Table 6 show, among all the robust controllers, $H_\infty$ has the lowest deviations and output errors, while $\mu$-synthesis has the highest deviations and output errors. The advantages of $H_\infty$ are in direct connection with the RP (speed loop) presented in Figure 9 and Table 3. In the case of $H_2$ and $H_2/H_\infty$ controllers, the fuel system acts to increase the fuel flow and compensate the sudden reduction in frequency output provided in Figure 14. As Figure 14 and Table 5 show, this can lead to an overshoot in power demand and output temperature. Increasing temperature over $T_r$ moves the power plant from operating region I to II (Figure 4), where the IGV increases the airflow (Figure 14) to control the burning temperature.

For disturbance rejection capability, as Figure 15 and Table 7 show, among all the robust controllers, $H_\infty$ shows more change and maximum deviation in terms of fuel flow (Figure 15) and power demand compared to the other controllers. The overshoot and deviation noted in Figure 15 can be explained by considering the high bandwidth of $H_\infty$ in Figures 10 and 11, where, for $H_\infty$, minimizing the frequency deviation has more priority than saving fuel flow. Besides, this overshoot in fuel flow can increase both temperature (Figure 15) and power output error (Table 7). As Figure 15 for IGV shows, $H_2$ and $H_\infty$ provide the maximum deviation to maintain the output temperature below $T_r$. This overshoot and deviation in Figures 14 and 15 are in close connection with the value of $W_{IGV} = 0.01$ and the best RP
in Figure 9 (temperature loop). Note that a lower weighing function ($W_{IGV}$) means more control energy for a better output performance.

Based on the simulation results and performance analysis provided in this section, we can conclude that among all the robust control approaches, $H_\infty$ can offer the best stability and
**Table 4**  Maximum gain, stability margin, and performance margin for speed and temperature loops

| Controller | Max-gain | Stab-margin | Stab-uncer | Perf-margin | Perf-uncer |
|------------|----------|-------------|------------|-------------|------------|
|             |          | Speed loop  |            |             |            |
| $H_2$      | 1.0-105.6| 14.59       | 0.70       | 1.0         | 0.41       |
| $H_\infty$ | 0.67-59.68| 16.52       | 0.79       | 1.45        | 0.33       |
| $H_2/H_\infty$ | 0.75-14.31| 15.75       | 0.72       | 1.31        | 0.26       |
| $\mu$      | 0.18-3.1 | 15.72       | 0.69       | 4.9         | 0.65       |
|             |          | Temperature loop |          |             |            |
| $H_2$      | 0.099-333.4 | 21.0       | 1.0        | 7.0         | 0.91       |
| $H_\infty$ | 0.48-333.5 | 21          | 1.0        | 8.6         | 0.93       |
| $H_2/H_\infty$ | 0.14-2824 | 21.0        | 1          | 5.42        | 0.87       |
| $\mu$      | 0.22-3500 | 20.6        | 0.982      | 3.83        | 0.78       |

The bold values are the best simulation results.

**Figure 14**  Output and control responses for $H_2$, $H_\infty$, $H_2/H_\infty$, $\mu$-synthesis, and GTPP controllers

**Table 5**  Optimum time indices for $H_2$, $H_\infty$, $H_2/H_\infty$, $\mu$-synthesis controllers and the main GTPP controller

| Output | $M_p$ (%) | $T_r$ (s) | $T_s$ (s) | Output | $M_p$ (%) | $T_r$ (s) | $T_s$ (s) |
|--------|-----------|-----------|-----------|--------|-----------|-----------|-----------|
| $H_2$  |            |           |           | $H_\infty$ |           |           |           |
| Power  | 32.46     | 1.94      | 62.53     | Power  | 24.25     | 1.40      | 62.69     |
| $T_e$  | 1.06      | 76.88     | 154.25    | $T_e$ | 0.96      | 79.24     | 155.59    |
| $H_2/H_\infty$ |           |           |           | $\mu$ |           |           |           |
| Power  | 24.53     | 1.44      | 62.88     | Power  | 21.18     | 2.31      | 93.58     |
| $T_e$  | 12.65     | 78.12     | 504.37    | $T_e$ | 10.24     | 80.21     | 600       |
| PID    | Power  | 40.06     | 1.09      | 82.81  | $T_e$ | 0.35      | 79.18     | 157.23    |

The bold values are the best simulation results.
performance margins for the temperature loop, whereas for the speed loop, although $H_\infty$ provides the best stability margin, it fails in the performance margin aspect compared to the $\mu$-synthesis controller.

On the other hand, $H_2$ controller resulted in the worst stability and performance margins for the speed loop, whereas $\mu$-synthesis yielded the worst stability and performance margins for the temperature loop. The tracking performance results showed that $H_\infty$ and $H_2$ provided the best transient and steady-state behavior. Indeed, based on the results shown in Table 6, $H_\infty$ can minimize the temperature and power outputs, which can save energy and cost and improve GTPP efficiency. The disturbance rejection analysis showed that both $\mu$-synthesis and $H_\infty$ can offer the minimum tracking error; however, when it comes to load frequency control, $H_\infty$ should be the preferred solution.

### 6 CONCLUSIONS

In this paper, $H_2$, $H_\infty$, $H_2/H_\infty$, and $\mu$-synthesis controllers were designed and implemented for the speed and temperature control of a V94.2 gas turbine mounted in Damavand CCPP. The controllers were compared in terms of tracking...
capability, robustness, and transient performance. They were shown to enhance the GTPP output responses for any frequency drop caused by changes in power demand. Additionally, the controllers were able to maintain the RS and performance of the closed-loop system in the presence of model uncertainties, quickly varying parameters, and unmodeled dynamics.

Our comparison study showed that, even if the $H_{\infty}$ approach should be the preferred solution, the design goals for speed, power demand, and temperature outputs and the constraints on fuel flow and IGV control inputs cannot be satisfied at the same time using a single control approach. A compromise between the different objectives and control requirements is unavoidable and necessary.

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