Differential Parallel-Serial SQIF Structures Providing High Linearity Response

V Kornev\(^1\), I Soloviev\(^1\), N Klenov\(^1\), O Mukhanov\(^2\)
1 Physics Department, Moscow State University, Moscow 119899, Russia, 2 HYPRES, Inc., 175 Clearbrook Road, Elmsford, NY 10523, USA

E-mail: vkornev@phys.msu.ru

Abstract. Possible approaches to designs of the high linearity differential SQUID arrays and SQIF structures are suggested and discussed. The spectrum analysis technique has been applied to the problem solution. Differential scheme of two series arrays of interferometer cells with abs(sin(x))-like voltage response has been suggested to meet the requirement on the highly linear voltage response of the structure. The differential coupling allows an automatic cancellation of the even spectrum components that degrade the linearity of the array voltage response. If interferometer cells are replaced by parallel arrays, the parallel-series structure provides much higher performance of the voltage response. The differential structures allow designing of wide-band high-linear amplifiers for gigahertz frequency range. A traveling wave differential amplifier design is suggested to avoid limitations resulting from distributive character of the long series array.

1. Introduction

We have performed analysis and formulated requirements to the synthesizing highly linear arrays in terms of spectrum characteristics [1]. In particular, the array structure capable of providing periodic triangular voltage response is the structure meeting the sinc-like frequency dependence of the harmonic spectrum components:

\[
a(n\omega_0) = a_0 \frac{\sin^2(n\omega_0\Delta B/2)}{(n\omega_0\Delta B/2)^2},
\]

where \(2\Delta B\) is width of the triangular pulse of the voltage response, \(\omega_0 = 2\pi/B_T\), \(B_T\) -period of the response to magnetic field \(B\).

It is significant that spectrum of the voltage response with minimum period \(B_T = 2\Delta B\) contains only odd harmonics with amplitudes decreasing monotonically:

\[
a(k\omega_0) = \frac{a_0}{(k\omega_0)^2}, \quad k = 2n - 1
\]

We have proposed approach to synthesis of the LRA (Linear Response Array) and LR SQIF (Linear Response SQIF) structures using interferometer cells with sinusoidal response [1]. At the same
time the other approach to the synthesis problem may be also suggested.

Object of the work is analysis of differential array structures of dc interferometers to work out the other circuits capable of providing linear voltage response with period $B_1 = 2\Delta B$.

2. Two-Cell Differential Circuit
In case of vanishing inductance, the dc interferometer voltage response to homogeneous magnetic field $B$ can be expressed analytically in the following way:

$$V(B) = V_c \sqrt{(I_b / I_c)^2 - \cos(\pi B s_0 / \Phi_0)^2},$$

where $s_0$ - area of the interferometer cell, $I_c$ – interferometer critical current, and $I_b$ – current biasing. At $I_b = I_c$ the response (3) can be reduced as follows:

$$V(B) = V_c \left| \sin(\pi B s_0 / \Phi_0) \right|,$$

and spectrum of the response shows monotonic fall with harmonic number:

$$a(n\Phi_0) = \frac{a_0}{n^2 - 1}.$$

If we remove all even harmonics from (5), we come just to spectrum (2), some difference in the harmonic amplitudes takes place only for initial harmonic numbers. It is easily seen, that an applying of half a flux quantum to the dc interferometer cell changes sign of odd harmonic components. This fact leads to an idea of differential scheme of the unbiased and $\Phi_0/2$-biased interferometer cells to form the spectrum containing only odd harmonics. Fig. 1 presents the unbiased voltage responses $V(B)$ (a), the inversed $\Phi_0/2$-biased response [-V(B)] (-b), and the spectra of the responses. Inset shows differential response of the unbiased and $\Phi_0/2$-biased cells (sum of the shown a and -b responses).

3. Differential Array Structure
The proposed differential circuit of two interferometers can be easy transformed into differential array structure consisting of two serial arrays of dc interferometers with biasing $I_b = I_c$, where $I_c$ – critical current of interferometers. In one array, each cell should be biased by $\Phi_0/2$. This differential

**Figure 1.** The voltage response spectra of dc interferometers with vanishing inductances and current biasing $I_b = I_c$. Spectrum (a) relates to the interferometer which is unbiased magnetically; spectrum (b) - the inversed voltage response spectrum of the $\Phi_0/2$- biased interferometer. The inset shows triangular voltage response on magnetic field for differential connection of the interferometers.
array structure is shown schematically in Fig. 2. An additional flux $\Phi_0/4$ is applied to all interferometer cells to set operating point in the middle of the array response leg.

One can increase the linearity of array voltage response. For this purpose, we should add to the array a few cells with sinusoidal responses. The cells are dc interferometer biased well above critical current ($I_b > 2I_c$). These additional cells are to correct the initial spectral components in (5) in order to approach the desired spectrum (2).

If the interferometer cells in both serial arrays are replaced by parallel arrays, we come to a high-performance parallel-series differential structure. As an example, Fig. 3 shows voltage response of the differential scheme of two parallel arrays of $N = 9$ cells to the applied homogeneous magnetic flux. In one array, each cell is biased by $\Phi_0/2$. Increasing number of the parallel arrays in each shoulder of the differential structure, we come to both the high voltage response amplitude and the high dynamic range.

Figure 2. Differential array structure consisting of two series arrays of dc interferometers with biasing $I_b = I_c$, where $I_c$ – critical current of the interferometers. In one array each cell is biased by magnetic flux $\Phi_0/2$. Additional flux $\Phi_0/4$ is applied to all the cells to set the operating point.

Figure 3. Voltage response of the differential scheme of two parallel arrays of $N = 9$ cells to the applied homogeneous magnetic flux (thick line). In one array, each cell is biased by $\Phi_0/2$. Voltage response of the unbiased parallel array is shown by thin line.
4. Inductance Influence

Fig. 4 shows voltage response of dc interferometer with biasing $I_b = I_c$ at different normalized inductances $l = \pi I_c L / \Phi_0$. We have found that the finite inductance $l$ gives some additional decrease in amplitudes of only initial spectral components of the spectrum (5), while the high harmonics remain the same and decrease as $1/n^2$. Therefore we need to correct only these initial spectral components. As stated above, this can be done by means of additional cells with sinusoidal responses.

5. Travelling Wave Differential Amplifier

To avoid limitations resulting from distributive character of LRA constituting a long serial array we have proposed travelling wave amplifier [1]. In case of the discussed differential array structure we come to differential travelling wave amplifier shown schematically in Fig. 5.

The amplifier design includes input microwave line coupled with two output microwave lines via interferometer cells of the two serial arrays presenting differential structure. Input wave signal propagates along input line and then is absorbed by a matched load. The propagating signal acts

![Figure 5](image-url)
magnetically on the interferometer cells inserted in output microwave lines. Voltage responses of the cells induce output wave running in the output lines. Equal velocities of both input and output waves provide just in-phase process and hence the effective amplification of the output signal. This approach allows using arbitrary number of the cells and thereby achieving of any desired linearity and voltage response amplitude.

In such a way the synthesized structures allow designing of wide band and high linear amplifiers for gigahertz frequency range. Moreover the differential scheme can easy provide NRZ output signal.

6. Conclusion

The problem of synthesis of array structures capable of providing highly linear voltage response has been studied and the differential array structures have been proposed. The travelling wave differential amplifier design for the array structure is suggested to avoid limitations resulting from the distributed nature of long series arrays.

The developed array structures lead to the design of wide band, highly linear amplifiers for gigahertz frequency range. The array-based amplifiers seem being capable of providing essentially higher gain, linearity and dynamical range than the ones shown by SQUID-amplifier [5, 6]. In fact, transfer factor dV/dB is proportional to number N of the interferometer cells and dynamic range increases as \( \sqrt{N} \) for both parallel and series arrays. This results from the fact that in case of parallel array the output voltage noise decreases as \( \sqrt{N} \) at fixed voltage response amplitude; in case of series array the output voltage noise increases as \( \sqrt{N} \) and voltage response increases as N.

In such a way, the use of high enough number N of interferometer cells provides required dynamic range, and the developed design approaches make the arrays the LRAs with extremely high linearity of voltage response.

Acknowledgments

This work was supported in part by CRDF Grant RUP1-1493-MO-05, Russian FASI grant 2.514.11.4012 and Russian Grant for Scientific School 1344.2003.2.

7. References

[1] Kornev V, Soloviev I, Klenov N, Mukhanov O How to Build up the High Linearity SQIF Structure (this conference).