Conspirative cosmology with variable constants

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Abstract

We have investigated a cosmological model with variable speed of light ($c$), gravitational constant ($G$) and cosmological constant ($\Lambda$). The model is shown to solve the horizon, flatness and monopole problems of the early universe. We have found that with a certain variation of these parameters the model predicts a cosmic acceleration. The model also predicts that for a flat universe $\Lambda$ vanishes in both radiation and matter phases. If the gravitational constant is allowed to increase then we might not need the existence of dark matter.

Key Words: Cosmology: inflation, early universe, variable constants, cosmological parameters

1 Introduction

The idea of change fundamental constant of physics was first suggested by Dirac (Dirac, 1937, 1938). He postulated that the gravitational constant ($G$) decreases with time ($t$) as, $G \propto \frac{1}{t^6}$. It was thought such a variation would help understand the existence of very large numbers appearing when one compares atomic physics with cosmology. But very recently Moffat (1993), Albrecht and Magueijo (1999), Barrow (1999, 2003), Avelino & Martins (1999) have conjectured that if the speed of light falls at certain rate then the horizon, flatness and monopole problems of the standard model can be solved without recourse

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to inflationary paradigm. However, the variation of the speed of light will have important consequences especially during the nucleosynthesis era. Besides this, the variation of the speed of light will require a critical revision of the theory of relativity which relies upon the constancy of speed of light. The implication of this variation will be immense in the arena of thermodynamics, astrophysics and cosmology. This variation will also alter our basic principle and laws in physics, e.g., Lorentz invariance and general covariance. It seems plausible, that the force of gravity can be integrated into the theory of electrodynamics if the velocity of light changes according to some cosmological law.

We will postulate here in this letter that such a change is countered by a change in the cosmological (or gravitational) constant, in such a way that the usual energy conservation still holds. With this minimal change of cosmology in mind we investigate the consequences of this variation.

2 The model

Solution of Einstein field equations for a universe with cosmological constant $\Lambda$ yields the two equations

\[
\left( \frac{\dot{R}}{R} \right)^2 = \frac{8\pi G \rho}{3} - \frac{k c^2}{R^2} + \frac{\Lambda c^2}{3},
\]

and

\[
\left( \frac{\ddot{R}}{R} \right) = -\frac{4\pi G}{3} \left( \rho + \frac{3p}{c^2} \right) + \frac{\Lambda c^2}{3}.
\]

The cosmic fluid is characterized by the equation of state

\[
p = (\gamma - 1)\rho c^2, \quad 1 \leq \gamma \leq 2.
\]

It is assumed by Barrow (2003) that the Friedmann cosmology in the presence of time dependent speed of light $c(t)$ is described by the same Friedmann equation as those of constant $c$. Therefore, in what follows we will consider $c$ and $\Lambda$ as functions of time (or scale factor). We employ the following ansatzs for $\Lambda$ and $c$ as

\[
c = c_0 R^n, \quad \Lambda = \Lambda_0 R^m,
\]

where $n, m, c_0, \Lambda_0$ are undetermined constants. With this prescription, eqs.(1) and (2) yield

\[
\left( \frac{\dot{R}}{R} \right)^2 = \frac{k c_0^2(2 - 3\gamma)}{2n + 3\gamma - 2} R^{2n-2} + \frac{\gamma \Lambda_0 c_0^2}{m + 2n + 3\gamma} R^{m+2n} + AR^{-3\gamma},
\]
where $A$ is an integration constant. Now consider the following cases:

If $\Lambda \neq 0$. We see from eq.(5) that whenever the term

$$n < \frac{1}{2}(2 - 3\gamma) \quad \text{and} \quad m < -2 ,$$

first term in LHS of eq.(5) becomes negligibly small for large $R$ in comparison with the other two terms. If this happens the curvature term will be insignificantly small and the flatness problem is automatically solved, as well as the horizon and monopole problems. We further admit that if the universe is dominated by strings ($\gamma = \frac{2}{3}$), then there will be no flatness or horizon problems, as evident from eq.(5). Inflationary solution is known to require the condition $\gamma < -\frac{2}{3}$ implying an exotic equation of state to solve these problems. We remark here that our eqs.(5) and (6) generalize Barrow (2003) equations to include a variable $\Lambda$ term. We arrive at eqs.(5) and (6) using Friedmann equations directly, in comparison with Barrow (2003) method which employs matter conservation law [see eq.(3)].

In the radiation dominated phase $\gamma = \frac{4}{3}$ so that eq.(6) reduces to

$$n < -1$$

which implies a decaying speed of light.

3 Early acceleration

If the universe was dominated by strings and that $\Lambda \neq 0$, then irrespective of the value of $k$ (i.e., $0, 1, -1$), one would have

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{\gamma \Lambda_0 c_0^2}{m + 2n + 3\gamma} R^{m + 2n} ,$$

if $A = 0$. We see that the curvature term vanishes identically. It is evident from the above equation that, when $\Lambda_0 > 0$ one has

$$m + 2n + 2 > 0$$

With this constraint eq.(8) can be easily solved to give

$$R = B t^{-\frac{2}{m + 2n}}$$

where $B =$ constant. The constraint in eq.(9) implies that

$$\ddot{R} > 0 .$$

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Thus one gets a power law inflation in comparison with the problematic inflationary solution which normally requires an existence of an unseen scalar particles.

Now if $A \neq 0$ but $k = 0, \Lambda = 0$, one has for radiation dominated (RD) phase the equation

$$\left(\frac{\dot{R}}{R}\right)^2 = AR^{-4},$$

which can be solved to give

$$R = D t^{\frac{1}{2}},$$

where $D = \text{const}$. This is the usual Einstein-de Sitter solution for RD phase.

4 Matter dominated (MD) phase

For matter dominated phase one has $\gamma = 1$. For a flat universe ($k = 0$) with $\Lambda = 0, A \neq 0$, one has

$$\left(\frac{\dot{R}}{R}\right)^2 = AR^{-3},$$

which is solved to give

$$R = D' t^{\frac{2}{3}},$$

which is the usual Einstein-de Sitter solution. We observe that this cosmology is determined by the constants $k, c, \Lambda$ and does not depend on $G$. However, we can not here determine the time variation of $c$ and $\Lambda$ completely.

Now if $A = 0$ and $k = 0, \Lambda > 0$ one has the constraint

$$m + 2n + 3 > 0.$$  \hfill (16)

Applying the above constraint in eq.(5) one obtains

$$R = B't^{-\frac{2}{m+2n}},$$

where $B' = \text{constant}$. With the aid of eq.(16), eq.(17) implies that

$$\ddot{R} > 0$$

as long as

$$m + 2n > -2.$$  \hfill (19)
Thus, we see that with a variable speed of light one can solve the horizon and flatness problems of the early universe; and justify the present cosmic acceleration. Since our model does not determine \( m \) and \( n \) uniquely, their values has to be found from the present observational data.

5 Conspirative mechanism (I)

One can derive the conservation equation from eqs.(1) and (2). This is done by differentiating eq.(1) and eliminating \( \ddot{R} \) between eqs.(1) and (2). We treat here \( c \) and \( \Lambda \) as variable but \( G = \text{constant} \). Consequently, one obtains the following equation

\[
\dot{\rho} + 3 \frac{\dot{R}}{R} \left( \rho + \frac{p}{c^2} \right) = -\frac{\dot{\Lambda}c^2}{8\pi G} - \frac{\Lambda \dot{c} \dot{c}}{4\pi G} + \frac{3k \dot{c} \dot{c}}{4\pi GR^2} \tag{20}
\]

We assume here that the usual energy conservation hold so that

\[
\dot{\rho} + 3 \frac{\dot{R}}{R} \left( \rho + \frac{p}{c^2} \right) = 0 \tag{21}
\]

provided that the other scalars \( (c, \Lambda) \) conspire to satisfy it, i.e.,

\[
\frac{\dot{\Lambda}c^2}{8\pi G} + \frac{\Lambda \dot{c} \dot{c}}{4\pi G} - \frac{3k \dot{c} \dot{c}}{4\pi GR^2} = 0 \tag{22}
\]

In this case one assumes, not only Friedmann equation to be the same for variable \( c \) and \( \Lambda \), but also the usual energy conservation to hold too. With this assumption one can solve eq.(22) to study the effect of the variation of \( c \) on this cosmology. Again assume the speed of light to have the form

\[
c = c_0 R^n, \tag{23}
\]

where \( c_0 \) and \( n \) are undetermined constants. Integrating eq.(21) using eq.(3) to obtain

\[
\rho = A' R^{-3\gamma}, \tag{24}
\]

where \( A' = \text{constant} \). Integrating eq.(22) using eq.(23) one gets

\[
\Lambda = \frac{3kn}{(n-1)} R^{-2} + FR^{-2n}, \quad F = \text{constant} \tag{25}
\]

If one sets \( F = 0 \), we get

\[
\Lambda = \frac{3kn}{(n-1)} R^{-2}, \tag{26}
\]
a variation law that already suggested by several authors (Ozer & Taha, 1986, 1987; Chen & Wu, 1990) with different motivation. It is interesting to link the variation of $\Lambda$ to the variation of the speed of light. We observe that if $n = 0$ then $\Lambda = 0$. This implies that if the speed of light is constant then the cosmological constant vanishes. However, if $k = 0$ then $\Lambda = 0$ whether $c$ is constant or not. Thus one can solve the cosmological constant and flatness problems simultaneously, in radiation and matter dominated phases. These problems have preoccupied many scientists for a long time. Some string theorists believe that $\Lambda = 0$ according to string theory. We therefore, provide a viable model realizing this belief.

One then would attribute a non-zero cosmological constant to a variation of $c$. Cosmologists have been seeking some symmetry that dictates $\Lambda$ to vanish. The existence of a non-zero $\Lambda$ in the early universe is needed for inflationary solution. The present observations coming from type Ia supernovae suggest that our universe is accelerating at the present time. With the present model we have shown that this cosmic expansion is also featured. Thus our model could give a plausible answer to this enigma. However, we have shown that the cosmological problems that inflation thought to solve can be solved in the framework of the present cosmology. We come to the conclusion that a vanishing cosmological constant follows from either the constancy of the speed of light and/or flatness of the universe.

Again, this setting solves the horizon and flatness problems in the early universe with the same condition as the one in eq.(6). Substituting eqs.(24) and (26) in eq.(1) one gets

$$\left( \frac{\dot{R}}{R} \right)^2 = \frac{8\pi G \Lambda}{3} R^{-3\gamma} + \frac{(2n + 1)}{(n - 1)} k c_2^2 R^{2n - 2}.$$  \hspace{1cm} (27)

This time the flatness and horizon problems can be solved with

$$n = -\frac{1}{2} \quad \text{or} \quad n < \frac{1}{2} (2 - 3\gamma).$$  \hspace{1cm} (28)

For the radiation ($\gamma = \frac{4}{3}$) and matter ($\gamma = 1$) dominated phases, we obtain the usual Einstein-de Sitter solutions, viz.,

$$R \propto t^{\frac{1}{2}}, \quad R \propto t^{\frac{3}{2}}, \quad \Lambda = 0.$$  \hspace{1cm} (29)

6 Variable $c, \Lambda$ and $G$

We will consider here, for completeness,, a model in which $c, G$ and $\Lambda$ vary with scale factor ($R$). Accordingly, eqs.(1) and (2) yield (by differentiating eq.(1) and substituting it eq.(2) to eliminate $R, \dot{R}$

$$\rho' + \frac{3}{R} \left( \rho + \frac{p}{c^2} \right) + \rho \frac{G'}{G} - \frac{3k c c'}{4\pi G R^2} + \frac{N c^2}{8\pi G} + \frac{\Lambda c c'}{4\pi G} = 0,$$  \hspace{1cm} (30)
where the prime $'$ denotes derivative with respect to the scale factor ($R$). A simplest solution of eq.(30) is to consider the power law for $c$, $\Lambda$ and $G$ of the form

$$c = c_0 R^n, \quad \Lambda = \Lambda_0 R^m, \quad G = G_0 R^\alpha.$$  \hspace{1cm} (31)

Applying eq.(31) to eq.(30) using eq.(3), one obtains

$$\rho = AR^{3\gamma - \alpha} + \frac{c_0^2 (3nk - n\Lambda_0 + \Lambda_0)}{4\pi G_0 (2n + 3\gamma - 2)} R^{2n - \alpha - 2},$$  \hspace{1cm} (32)

where for consistency we take $m = -2$, and $A = \text{const.}$ Substituting eq.(32) in eq.(1) using eq.(31) we get

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G_0 A}{3} R^{-3\gamma} + \frac{c_0^2 (2k - 3\gamma k + \gamma \Lambda_0)}{(2n + 3\gamma - 2)} R^{2n - \alpha - 2}.$$  \hspace{1cm} (33)

The horizon, flatness and cosmological constant problems will be solved with the condition

$$n < \frac{1}{2} (2 - 3\gamma),$$  \hspace{1cm} (34)

which is the same as eqs.(6) and (28) (except here $m = -2$). Equations (32) and (33) generalizes eqs.(8) and (9) of Barrow & Magueijo (1999) in their attempt to solve the quasi-flatness and quasi-lambda problems. Thus, whether $G$ varies or not, the solution of the horizon and flatness problems is not affected. This solution depends only on the variation that $\Lambda$ and $c$ assume. This is also evident as $\alpha$ does not enter in the constraint in eq.(34), and from eq.(33). The effect of a positive $\alpha$ is to increase the energy density of the universe in the early times and decrease (dilute) it at late times. This may help explain why, today, we observe the universe to have a deficit in its anticipated energy density (as favored by inflationary models). Hence, one need not assume any dark matter to scale up the energy density to the required level. A negative $\alpha$ will have the opposite contribution. For a more realistic model of the universe, one should consider the effect of bulk viscosity (since it is a basic property of any real fluid) on the evolution of the universe. In an earlier work (Arbab, 1997), we have shown that in a viscous universe with variable $G$ and $\Lambda$ a de-Sitter inflationary solution arises naturally without being imposed. We will tackle this problem in the forthcoming letter.

### 6.1 Radiation and Matter dominated phases

We will study here the following two cases:
6.1.1 \( A = 0 \).

To solve eq.(33) for matter dominated phase we set \( \gamma = 1 \), so that for \( \Lambda_0 > 0 \), one obtains
\[
R = N_m t^{\frac{1}{(1-n)}} , \quad N_m = c_0 \sqrt{\frac{(\Lambda_0 - k)}{(2n + 1)}} , \quad n \neq 1 , -\frac{1}{2} .
\) (35)

This solution is also obtained by Barrow and Magueijo (1999) with \( G \) constant [see eq.(15)].

To solve eq.(33) for radiation dominated phase we set \( \gamma = \frac{4}{3} \), so that that for \( \Lambda_0 > 0 \), one obtains
\[
R = N_r t^{\frac{1}{(1-n)}} , \quad N_r = c_0 \sqrt{\frac{(2\Lambda_0 - 3k)}{3(n + 1)}} , \quad n \neq 1 , -1 .
\) (36)

We see that the scale factor \( R \) has the same form but \( n \) assumes different values in the two eras (RD & MD). This really shows that the way \( c \) varies affect the expansion of the universe that result in solving may of the cosmological problems.

6.1.2 \( A \neq 0 \).

In this case we will assume that
\[
(2k - 3\gamma k + \gamma \Lambda_0) = 0 , \quad \text{and} \quad 2n + 3\gamma - 2 \neq 0 ,
\] (37)
in both eras (RD & MD). This is guaranteed if one takes \( k = 0 \) and \( \Lambda_0 = 0 \). For this situation the solution of eq.(33) reduces to the Einstein -de Sitter solution, viz.,
\[
R \propto t^{\frac{1}{2}} \quad \text{(RD)} \quad \text{and} \quad R \propto t^{\frac{2}{3}} \quad \text{(MD)}.
\) (38)

7 Conspirative mechanism (II)

We assume again that the usual energy conservation
\[
\dot{\rho} + \frac{3}{R} \frac{\dot{R}}{R} \left( \rho + \frac{p}{c^2} \right) = 0
\] (39)

to hold. Integrating eq.(39) one obtains
\[
\rho = R^{-3\gamma} , \quad A = \text{const.}
\] (40)

Using eq.(39) in eq.(30) one gets
\[
\rho \frac{G'}{G} - \frac{3kcc'}{4\pi GR^2} + \frac{N' c^2}{8\pi G} + \frac{\Lambda cc'}{4\pi G} = 0 ,
\] (41)
As before we adopt the power law for $c$ and $\Lambda$ of the form

$$c = c_0 R^n, \quad \Lambda = \Lambda_0 R^m.$$  \hfill (42)

Using eqs.(42) and (40), eq.(41) can be integrated to give

$$G = \frac{c_0^2}{4\pi A} \frac{(3nk - n\Lambda_0 + \Lambda_0)}{(2n + 3\gamma - 2)} R^{2n-2} + B, \quad m = -2, \quad B = \text{const.}$$  \hfill (43)

Substituting eqs.(40), (42) and (43) in eq.(1) yields

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{c_0^2 (\gamma \Lambda_0 - 3k\gamma + 2k)}{(2n + 3\gamma - 2)} R^{2n-2} + DR^{-3\gamma}, \quad D = \frac{8\pi AB}{3}.$$  \hfill (44)

We observe that

$$\Lambda \propto R^{-2},$$  \hfill (45)

a variation that is favored by several cosmologists (e.g., Chen & Wu, Ozer & Taha).

### 7.1 Radiation and matter dominated phases

We study here two cases as in 6.1:

#### 7.1.1 D=0

In this case eq.(44) reads

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{c_0^2 (\gamma \Lambda_0 - 3k\gamma + 2k)}{(2n + 3\gamma - 2)} R^{2n-2}. $$  \hfill (46)

This can be integrated to give

$$R = N_m t^{\frac{1}{1-n}}, \quad N_m = c_0 \sqrt{\frac{(\Lambda_0 - k)}{(2n + 1)}}, \quad n \neq 1, -\frac{1}{2}. $$  \hfill (47)

during matter dominated phase, and

$$R = N_r t^{\frac{1}{1-n}}, \quad N_r = c_0 \sqrt{\frac{(2\Lambda_0 - 3k)}{3(n + 1)}}, \quad n \neq 1, -1, $$  \hfill (48)

during radiation dominated phase. We notice that eqs.(35) and (36) are identical with eqs.(46) and (47), respectively. This shows that our assumption of eq.(39) was correct. We have already applied a similar conservation law for a bulk model (Arbab, 1997).
7.1.2 $D \neq 0$.

In this case we will assume that

$$(2k - 3\gamma k + \gamma \Lambda_0) = 0 \quad \text{and} \quad 2n + 3\gamma - 2 \neq 0,$$

in both eras (RD & MD). This can be satisfied if one takes $k = 0$ and $\Lambda_0 = 0$. For this situation the solution of eq.(33) reduces to the Einstein -de Sitter solution, viz.,

$$R \propto t^{\frac{2}{3}} \quad \text{(RD)} \quad \text{and} \quad R \propto t^{\frac{2}{3}} \quad \text{(MD)}.$$

(50)

Which is a solution we have already obtained in eq.(38). Thus, in an evolving universe particles interact (conspire) in such a way the energy conservation law holds. Hence, energy conservation is indeed a stringent law that governs the dynamic of the interacting particles in our expanding universe.

8 Avelino & Martins cosmology

Avelino & Martins (1999) propose a generalization of general relativity that includes the variation of $c$ and $G$. They however, argued that the this theory is both covariant and Lorentz invariant. This is parameterized by writing

$$\frac{G}{c^2} = \text{const} \equiv A.$$  

(51)

We apply this equation in eq.(20) with the variation

$$c = c_0 R^n.$$  

(52)

We then obtain

$$\rho = \frac{3nk}{4\pi A} \frac{1}{(2n + 3\gamma - 2)} R^{-2} + MR^{-2n-3\gamma}, \quad M = \text{const.},$$  

(53)

where we have taken for simplicity $\Lambda = 0$.

Now substitute eqs.(51)- (53) in (1) to get

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{kc_0^2(2 - 3\gamma)}{(2n + 3\gamma - 2)} R^{2n-2} + D'R^{-3\gamma}, \quad D' = \frac{8\pi c_0^2 AM}{3}.$$  

(54)

It is clear that this solves the flatness and horizon problems if

$$n < \frac{1}{2}(2 - 3\gamma),$$  

(55)
which is the same as eq.(34). However, Avelino and Martins claim that the horizon and flatness problems require similar conditions to the one found in the context of the standard cosmological model. They argued that a theory that reduces to General Relativity in appropriate limit and solves the horizon and flatness problems of the standard model must either violate the strong energy condition or Lorentz invariance, or covariance. They also show that the variation in eq.(51) guarantees conservation of mass energy density \( \rho \). The solution of eq.(54) is already worked out in sec. 5, 6, 7. We notice that Barrow (1999, 2003) has obtained many solutions that we have found in this letter by different methods. We conclude that Avelino & Martins cosmology is equivalent to Albrecht and Magueijo, Barrow, and Moffat models.

9 Conclusion

We have studied here the effect of the variation of speed of light and cosmological constant in cosmology. We have found that such a variation can in principle solve many of the long persisting problems of standard model of cosmology. Of these problems are the flatness, horizon, monopole and cosmological constant problems. A conspiracy between these variable constants can still preserve some of the standard model picture. The variation of the gravitational constant is not dealt with here, but will be one of our future endeavors. These models share some features of the original Moffat, Albrecht and Barrow scenario. Our model provides a good solution to the existence of dark matter in the universe if the gravitational constant varies with time. It also helps understand the cause of the present cosmic acceleration. Moreover, the cosmological constant is found to relax as \( \Lambda \propto R^{-2} \) which is suggested by some cosmologists.

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