The characteristics of relational students in understanding the concepts of normal subgroups

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Abstract. The quality of students' responses in understanding the normal subgroup concept can be analyzed through SOLO Taxonomy. One level was relational. The purpose of this study was to describe the characteristics of relational students in understanding the concept of a normal subgroup. This was part of research development. This stage was a needs assessment. The research subjects were 29 undergraduate students of Mathematics Education at the University of Bengkulu. The instrument of this research was the researchers themselves with interview guides. Data collection was done through task-based interviews. Data were analyzed qualitatively through fixed comparison techniques. The results of this study are that the concept of normal subgroup was understood through the relationships between objects in a group. Like the right coset and left coset. The relationship between these objects becomes an integrated entity. The conclusion of this study was that relational students can represent all elements and conduct interdependence with one another, so that it becomes an integrated entity.

Keywords: Relational level, concept understanding, normal subgroup

1. Introduction

Abstract Algebra was a compulsory subject in the undergraduate mathematics education program. That was material that was quite difficult. Our experience teaching the course, students experienced being sequenced at the beginning of the lecture. They are surprised by the deductive structure with abstract objects. Based on Dubinsky, et. al. [1], abstract algebra in general, and group theory in particular, present a serious educational problem. The faculty of mathematics and students generally regard it as one of the most difficult undergraduate subjects. It seems to give students a lot of difficulties, both in terms of dealing with content and developing attitudes towards abstract mathematics [1][2]. Therefore, abstract algebra serve as a vehicle to introduce concepts and arguments that relate in some important way to the discussion [3]. Therefore, abstract algebra plays a role in efforts to improve the quality of students' logical thinking. Abstract algebra learning encourages students to axiomatic deductive thinking, critical and creative thinking [4]. It was very important that every student of the mathematics and mathematics education undergraduate program learns about it. He will become a professional mathematician or become a mathematician and applied mathematics. It was a profession that requires algebraic abilities.

Abstract algebra has become increasingly important not only in mathematics itself, but also in various other disciplines [3]. The application of abstract algebraic concepts and principles benefits the development of science, knowledge and technology. Learning algebra also contributes to basic
knowledge of human thought and to serve certain special purposes [1]. For example, the importance of results and abstract algebraic concepts play a more important role in physics, chemistry, and computer science, to cite some such external fields [3]. We need to develop pedagogical strategies that can improve students' success in learning abstract algebra. It was a long-term development in the learning and teaching of undergraduate mathematics [1].

In abstract algebra learning, educators start with primitive terms. That was the concept of the base as an entity that was not defined. The next structure was a concept that was defined, then axioms. The logical consequence was that there are theorems that must be verified. The proof uses arguments in the form of statements in the structure that have true value. That was a way the lecturer increases the quality of the argument from each prove a theorem. A cognitive process that can be measured through a collection of student mental activities [5]. This collection was in the form of genetic decomposition in a description of the quality of student responses in the taxonomy of SOLO (Structure of Observed Learning Outcomes) [6].

SOLO Taxonomy was designed as an evaluation tool about the quality of student responses to a task. There are five taxonomic levels, namely structural, unstructural, multi-structural, relational, and extended abstract [6] [7]. It was providing a systematic way to describe how students perform in understanding academic assignments. A student can be at a low level and other students can be at a higher level [8]. Biggs & Collis [9] states that pre-structural students cannot carry out tasks using irrelevant data. It was a Level-0 student. Level-1 was uni-structural students. They can use one piece of information in response to a task. Multi-structural students (Level-2) can use several pieces of information but cannot connect them together. While Level-3 was a relational student who can combine separate pieces of information to produce a completion of a task. The highest level was extended abstract students. Students at this level can produce general principles of integrated data that can be applied to new situations. The characteristics of each level are the cognitive processes of students in understanding mathematics. Therefore, we can trace students' mental activities in response to abstract algebraic tasks.

The quality of student responses in understanding the normal subgroup concept can also be analyzed through SOLO Taxonomy [7]. The analysis was carried out through genetic decomposition [10][11]. It was a mental activity and student performance. Sunardi's research shows the existence of a new level in the SOLO Taxonomy. The new level was between the relational level and the extended abstract level. That was a level called the abstract level [2].

Subject abstract level character can explain the relationship statement given with an argument in solving the problem, able to explain the usefulness of each statement used to solve the problem, he tried to make a new statement outside the original statement by referring to the statement, but did not succeed in proving the truth [12]. Students who are at the abstract level can use all the statements given to solve the problem. He can explain the relationship of the statements given to be an argument in solving problems; explain the usefulness of each statement used to solve the problem; trying to make new statements as a result of proven statements; explain the statement prepared was as a result of existing statements using good arguments and withdrawing conclusions that have been made on paper and pencil, but he has not been able to make the proof; trying to make a new statement exceed the original statement by referring to the statements that exist, but failed to prove the truth [2].

According to Collis & Biggs extended abstract students gave many possible conclusions. He uses abstract principles to interpret concrete facts with the right response. That was an activity carried out consistently. Extended abstract students have conceptual thinking, and can generalize to a new domain [13]. It was a character that the response that students build on a structural pattern can be integrated into a new structure. But, what about relational students?

Some studies suggest that relational students can make responses more than one element that was coherently integrated for a particular case [2]. For problems about the nature of algebra in real numbers, students can integrate two or more information provided, but the integration has not been coherent [12]. Relational students write programs using LOGO in the text model, and if something goes wrong, editing was done. The extended abstract students wrote programs with text models, and
edited them if necessary, and were able to enter variables [14]. That means it still requires deepening of the character of relational students. Therefore, we are interested in describing the characteristics of relational students in understanding the normal subgroup concept.

2. Method
We are a research team at the Teacher Training and Education Faculty on the University of Bengkulu, in the field of mathematics education. The team carried out major research through the developmental research. This paper was part of the development research. This stage was a needs analysis. The research subjects were 29 students of the Bengkulu Mathematics Education Undergraduate Program. The instrument of this research was the researchers themselves with interview guides. Data collection was done through task-based interviews. In-depth interviews were conducted with selected students using snowball techniques. The assignment given to the research subject was "Prove that the intersection of two normal subgroups was a normal subgroup." Data are analyzed qualitatively through the constant comparative technique. It was an analysis of action-process-objects and schemes, through interiorization, encapsulation and thematization. Interiorization was a change of activity from a procedural activity to be able to carry out activities in imagining some of the meanings that affect the conditions produced (is a form of action in the process). Encapsulation was the process carried out on an object. Thematization was a construction that links actions, processes, or separate objects to a particular object so as to produce a scheme. It was the implemented of genetic decomposition analysis [10][11].

3. Results and Discussion
Data from research results in the form of genetic decomposition of research subjects. It was a collection of mental and physical activities carried out by subject while working on assignments and during in depth interviews. First, 29 students worked on assignments using paper and pencil. Then, the research team examined the results of the students' work. We hope that students will be at the relational level. The subject was then interviewed in depth. There was a subject that we describe in this paper. The student was Ane. Exposure to interview data between Researchers (= Q) and students (= Ane) was as follows.

Footage 1:
Q: Based on the question I gave, can you explain your answer?
Ane.01: Well... suppose G was a group, and M, N are normal subgroups of G. First I show that the intersections of M and N are non-empty sets. That was quite clear because M, N subgroups... each contains e as the identity element of G, then e was M member as well as N. member (see Figure 1)
Q: Okay... try to continue...
Ane.02: Then I show that M∩N was a subset of G... suppose I take x member M∩N... then x member M and x member N... because and M, N was subgroups of G then x was G. member
Q: ... then what can you say?
Ane.03: Yes... because x member M∩N then x member G... I conclude M∩N the subset of G was true... (see Figure 2)

Based on Footage 1, the subject was able to connect two different objects into one specific object. It was that Ane shows that subsets of a group builder set are not empty. These associations together contain elements of identity. He concluded that the slices contain an element, which means it was not empty either (see Figure 1). Also, I have consistency in the cognitive process. He was able to connect two different sets into a set which was a subset of G (see Figure 4). Thus, the subject was able to connect two specific objects into meaningful entities.

This conclusion was very good. It was supporting of Sunardi’s research results that relational students make more than one coherently integrated response to certain cases [2]. Caniglia & Meadows
declares that relational students can link information together and explain some ideas related to a topic. For example, students can integrate information to produce viable solutions for a task. Students provide explanations that relate and integrate relevant details. They may often express their abstract ideas with concrete facts. Students can use prior knowledge to explain and provide context [15]. We get very clear information from Ane. He also interiorized and encapsulated correctly. Some processes that connect between objects become meaningful entities. That is, you can see the following description.

![Figure 1. Solution from Ane Part 1](image1)

![Figure 2. Solution from Ane Part 2](image2)

**Footage 2:**

Q: After you prove the first two things, do you still want to explain the next ... buckle explain now ...?

Ane.04: ... I continue by showing that ... The operation on $M \cap N$ applies closed properties ...

suppose I take $x, y$ member $M \cap N$ ... then $x, y$ member $M$ and also $x, y$ member $N$ ... it was known that $M$ and $N$ are subgroups of $G$, respectively ... That was fulfilling that $xy$ member $M$, also $xy$ member $N$ ...

Q: Ok ... continue ...

Ane.05: ... means that $xy$ was a member of $M \cap N$ ... the closed nature of $M \cap N$ was filled ... (see Figure 3)
We listen to Footage 2, and the genetic decomposition means that mental actions are a process that was internalized in cognitive processes. He connects two elements in the slices of two different sets. The subject was able to encapsulate so that it produces an object that the slices of the two sets together with a binary operation are closed. That was the character of the research subject that supports the results of other studies. As Chick stated that relational responses are characterized by the synthesis of information, processes, and outcomes between. It was reaching conclusions, the concept was applied to several data, providing temporary results which are then related to other data and/or processes [16]. Thus, relational students are able to process the linking between objects into an entity that was meaningful through encapsulation. Also, I consistently show excellent mathematical abilities.

Figure 3. Solution from Ane Part 3

Footage 3:
Q: What can you say next ...?
Ane.04: ... I will prove first that $M \cap N$ was a subgroup of $G$ ... because the closed nature of $M \cap N$ has been fulfilled ... It was my turn to show that $M \cap N$ contains inverses ...
Q: Good ... Ok ...
Ane.05: ... take any $x$ member $M \cap N$ ... means $x$ member $M$ and $x$ member $N$ ... with the argument that $M, N$ are subgroups of $G$ ... then the inverse of $x$ member $M$, also member $N$ ... therefore the inverse of $x$ was a member of $M \cap N$ ...
Q: what can you conclude?
Ane.06: ... up to the argument above, $M \cap N$ was a subgroup of $G$ ... (see Figure 4)

Figure 4. Solution from Ane Part 4
Based on Footage 3, we analyzed that I had a good ability to interiorize, encapsulate the previous objects. He was able to relate meaningfully the concepts of normal groups, subgroups and subgroups. He understands that to declare a subset with binary operations forms a subgroup if it satisfies the closed properties and has an inverse. Thinking that his analogy was consistent, he was able to interiorize two sets that contain inverses, and with the properties he has. It was producing an object that the slice contains inverses. This object was an encapsulation. Previous research also supported that at the relational level, students can integrate ideas into the whole, recognize relationships and connect ideas to each other. They may understand some meta-connections, find the relationship between theory and practice, purpose, and the importance of ideas more clearly to them. Some students at this level may be able to use this understanding to apply ideas to new situations [17]. Finally, we obtained a more complete understanding of cognitive processes from Ane. It was described in Footage 4.

**Footage 4:**

Q: What was your explanation ...?
Ane.07: ... yes ... I just need to show one argument to complete the proof ...
Q: ... what's the argument?
Ane.08: ... take any member G from G, and any x member from M∩N ... this means that x members M and x members N ... because M, N are each normal subgroup of G ... means gxg^{-1} member of M as well as gxg^{-1} member of N ... that was ensuring that gxg^{-1} was a member of M∩N ... thus I have already proved that M∩N was a normal subgroup of G. (see Figure 5)

![Figure 5. Solution from Ane Part 5](image)

Based on Footage 4, we get complete information about my cognitive process. It was a conceptual understanding through the process of integrating between different concepts and objects. I was able to connect two properties of a normal subgroup to a new normal subgroup. It was an entity that was
connected through the thematization process. The result was a mature scheme about the principle of a normal subgroup. This result also supports previous research that the relational student response will answer the question, explain, interact, and balance [18]. Students at the relational level understand various aspects of the topic, integrate parts of the problem into coherent code structures, and use these structures to solve problems [19]. As such, we are confident of producing a complete description for relational students. Students at this level understand the concept of a normal subgroup coherently through the relationships between objects in a group. Like slices two normal subgroups are normal subgroups too. The relationship between these objects becomes an integrated entity. That was a mature scheme. This scheme was organized based on actions, processes, by encapsulation of objects. Finally, he was able to thematize into a mature scheme.

4. Conclusion
This study concludes that relational students are able to understand problems conceptually through a process of integrating various concepts and objects. Students can link two properties from the normal subgroup to the new normal subgroup. It was an entity that was connected through a thematic process in the form of a mature scheme.

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