Abstract

We study the idea of a composite Higgs in the framework of a five-dimensional AdS theory. We present the minimal model of the Higgs as a pseudo-Goldstone boson in which electroweak symmetry is broken dynamically via top loop effects, all flavour problems are solved, and contributions to electroweak precision observables are below experimental bounds. Since the 5D theory is weakly coupled, we are able to fully determine the Higgs potential and other physical quantities. The lightest resonances are expected to have a mass around 2 TeV and should be discovered at the LHC. The top sector is mostly composite and deviations from Standard Model couplings are expected.
1 Introduction and Motivation

One of the most elegant proposals to explain the origin of electroweak symmetry breaking (EWSB) is technicolor (TC). In technicolor theories the breaking of the EW symmetry arises from a strongly interacting sector, similarly as the chiral symmetry breaking occurs in QCD. Simple TC models, however, do not pass the electroweak precision tests (EWPT) at LEP and SLAC colliders. The main problem is their contribution to the Peskin-Takeuchi \( S \) parameter that, when possible to estimate, is larger than allowed by the experimental data.

An interesting and still economical variation is to have a Higgs arising as a composite pseudo-Goldstone boson (PGB) from the strongly interacting sector. In this case, the Higgs mass is protected by an approximate global symmetry and is only generated via quantum effects. The electroweak scale is of order \( f_\pi \), where \( f_\pi \sim m_\rho/(4\pi) \) is the analog of the pion decay constant and \( m_\rho \) is the scale of the new resonances. Although this implies a contribution to \( S \) similar to TC models, one needs to calculate its precise value to know whether these models pass the EWPT or not. This has not been possible so far, mainly due to the difficulty of treating strongly interacting theories. Another problem of these models is explaining the origin of fermion masses.

Here we want to pursue the idea of the Higgs as a composite PGB by studying it in the framework of five-dimensional models defined on a slice of AdS spacetime. Models with an extra dimension have a 4D holographic description that resembles strongly coupled conformal theories (CFT) with a large number of “colors” \( N \). This description consists in separating the 5D fields in a bulk piece and a boundary variable, treating them as distinct degrees of freedom. From the point of view of the 4D effective theory on the boundary, the bulk acts similarly to a strongly interacting sector, whose global symmetries are determined by those of the bulk. Boundary variables are instead equivalent to fields external to the strong sector and coupled to it. Based on this observation, one can construct 5D models that mimic strongly coupled theories. This correspondence is clearly qualitative in the sense that we do not know the microscopic dynamics of the strong sector, but it proves extremely useful to have a quick understanding of the 5D physics and shape the low-energy “chiral” Lagrangian using symmetry considerations. Physical quantities can be computed resorting to the full 5D theory, since it is weakly coupled. In particular, an expansion in the 5D gauge coupling plays the role of the \( 1/N \) expansion of the 4D strongly coupled theory.

In ref. a first example of holographic PGB Higgs was given. Fermion masses were easily incorporated and a calculation of the Higgs potential was performed. Nevertheless, it lacked a custodial symmetry needed to prevent large contributions to the Peskin-Takeuchi \( T \) parameter.
In this paper we consider an alternative model of Higgs as a holographic PGB based on a global $SO(5) \times U(1)_{B-L}$ invariance. This is the minimal symmetry group which contains the EW gauge group, can deliver a Higgs as a PGB, and has an unbroken $SO(3)$ custodial symmetry. We are able to fully determine the Higgs potential arising from one-loop diagrams involving Standard Model (SM) fields and show that EWSB is triggered by the top quark contribution. The Higgs vacuum expectation value (VEV) is given by $v = \epsilon f_\pi$, where $\epsilon$ is a model-dependent parameter. We can accommodate a value of $S$ and $T$ consistent with the EWPT for $\epsilon \lesssim 0.4$, which can be obtained by a mild tuning in the parameters of the model ($\sim 10\%$). The theory is therefore fully realistic. We find that the Higgs is always very light, $m_{\text{Higgs}} \lesssim 140$ GeV (see fig. 1), one of the most important predictions of the model. The mild tuning in the parameter space can be completely eliminated by introducing an extra source of $SO(5)$ breaking. In this case we find that the Higgs is generally heavier (see fig. 3). The model also predicts extra gauge and fermionic resonances with masses $\sim 1 - 3$ TeV. All these new particles come in complete $SO(5)$ multiplets and have, with the exception of those with top quantum numbers, an approximate $SO(4)$ degeneracy.

We proceed as follows. First (section 2), we describe a 4D model based only on symmetry principles. The Higgs is assumed to be a composite PGB of a strongly coupled sector that is conformal at high energies. By integrating out the CFT dynamics one obtains an effective Lagrangian for the SM external fields that can be parametrized in terms of a set of form factors. The value of these form factors depends on the strong dynamics and cannot be determined from the 4D theory. Hence we take a second step (section 3): we show the corresponding 5D AdS theory that leads to the same effective Lagrangian as the 4D model described above. Since the 5D theory is weakly coupled, we are able to compute the precise value of the form factors. This allows us to calculate the $S$ parameter, the Higgs potential and other important physical quantities. In section 4 estimates of the $T$ parameter and of the correction to $Z \to b_L \bar{b}_L$ are also given using Naive Dimensional Analysis (NDA), and shown to be close to the experimental limits. We conclude (section 5) comparing our model with other popular schemes of EWSB based on a PGB Higgs.

2 The minimal 4D model of PGB Higgs with custodial symmetry

Let us consider a 4D theory that contains a strongly interacting sector with the following properties. It has a large number of “colors” $N$, a mass gap at the infrared scale $\mu_{\text{IR}} \sim \text{TeV}$, and it is conformal at high energies. The mass gap is responsible for the formation of a tower of bound states with lowest mass of order $m_\rho \sim \mu_{\text{IR}}$. A global symmetry $SU(3)_c \times SO(5) \times U(1)_{B-L}$ is spon-
taneously broken down to $SU(3)_c \times SO(4) \times U(1)_{B-L}$ at a scale $f_\pi \sim (\sqrt{N/4\pi}) m_\rho$. The operator responsible for this breaking will be assumed to have a large dimension. The SM gauge bosons and fermions are elementary fields external to the strongly interacting CFT. The top quark constitutes an exception and it will be mostly composite, as we will see later. The SM gauge bosons couple to the CFT through its conserved currents, gauging an $SU(3)_c \times SU(2)_L \times U(1)_{Y}$ subgroup of the global invariance. In the following we will neglect the $SU(3)_c$ color group since it plays no role in the mechanism of EWSB. Due to the non-linear realization of $SO(5)$, the theory at tree level has a large set of degenerate vacua, some of which preserve $SU(2)_L \times U(1)_Y$, while others do not. Thus, whether the electroweak symmetry is broken or not is a dynamical issue. By expanding around an $SU(2)_L \times U(1)_Y$–preserving vacuum, so that $SO(4) \sim SU(2)_L \times SU(2)_R$ and hypercharge is realized as $Y = T_{3R} + (B - L)/2$, all fields can be classified according to their electroweak quantum numbers. In particular, there is a Goldstone boson transforming as a $4$ of $SO(4)$, a real bidoublet of $SU(2)_L \times SU(2)_R$; it is a composite state of the CFT and we will identify it with the Higgs boson.

As long as no explicit breaking of the global $SO(5)$ symmetry is introduced into the theory, the Higgs field is an exact Goldstone, and as such it has vanishing potential at any order in perturbation theory. However, interactions between the CFT and the SM fields explicitly violate $SO(5)$ and will give rise, at the one-loop level, to a non-vanishing Higgs potential. In other words, the degeneracy of classical vacua is lifted by quantum effects and the Higgs becomes a composite PGB. The potential generated by gauge interactions alone does not trigger EWSB, since gauge forces will tend to align the vacuum along the $SU(2)_L \times U(1)_Y$–preserving direction. A further contribution to the potential, which can “misalign” the vacuum, comes however from the fermions living in the elementary sector, in particular, the top. Fermions will be assumed to couple linearly to the strong sector through operators $\mathcal{O}$ made of CFT fields: $\mathcal{L} = \lambda \bar{\psi} \mathcal{O}$. The running coupling $\lambda(\mu)$ obeys the RG equation

$$
\frac{d \lambda}{d \mu} = \gamma \lambda + a \frac{N}{16\pi^2} \lambda^3 + \cdots,
$$

where the dots stand for terms subleading in the large-$N$ limit, and $a$ is an $\mathcal{O}(1)$ positive coefficient. The first term in eq. (1) drives the energy scaling for $\lambda$ as dictated by the anomalous dimension $\gamma = \text{Dim}[\mathcal{O}] - 5/2$, $\text{Dim}[\mathcal{O}]$ being the conformal dimension of the operator $\mathcal{O}$. The second term originates instead from the CFT contribution to the fermion wave function renormalization. The low-energy value of $\lambda$ is determined by $\gamma$. For $\gamma > 0$, the coupling of the elementary fermion to the CFT is irrelevant, and $\lambda$ decreases with the energy scale $\mu$. Below $\mu_{IR}$, we have

$$
\lambda \sim \left( \frac{\mu_{IR}}{\Lambda} \right)^\gamma,
$$

(2)
where $\Lambda \sim M_{Pl}$ is the UV cutoff of the CFT. Therefore fermions $\psi$ with $\gamma > 0$ will have a small mixing with the CFT bound-states and thus a small Yukawa coupling. For $\gamma < 0$, the coupling is relevant and $\lambda$ flows at low energy towards the fixed-point value

$$\lambda = \frac{4\pi}{\sqrt{N}} \sqrt{-\gamma} / a.$$ (3)

In this case the mixing between the fermion $\psi$ and the CFT is large, and sizable Yukawa couplings can be generated.

The model is then described by the Lagrangian:

$$\mathcal{L} = \mathcal{L}_{\text{CFT}} + \mathcal{L}_{\text{SM}} + J^aL^\mu W^a_L + J^\mu B_\mu + \sum_r \lambda_r \bar{\psi}_r O_r + \text{h.c.}. \quad \text{(4)}$$

The sum runs over all SM fermionic representations $\psi_r = \{q_L, u_R, d_R, l_L, e_R\}$, (a family index is understood), and $W^a_L$, ($a_L = 1, 2, 3$). $B_\mu$ stand for SU(2)$_L$ and U(1)$_Y$ gauge bosons respectively. At tree level the massless spectrum of the theory is that of the SM. The Higgs is the Goldstone boson and can be parametrized by the fluctuations along the broken generators $T^\hat{a}$, $\hat{a} = 1, 2, 3, 4$:

$$\Sigma = \Sigma_0 e^{\Pi / f_\pi} , \quad \Sigma_0 = (0, 0, 0, 0, 1) , \quad \Pi = -i T^\hat{a} h^\hat{a} \sqrt{2}. \quad \text{(5)}$$

Using the SO(5) generators given in appendix C one easily finds the explicit expression for $\Sigma$:

$$\Sigma = \frac{\sin h / f_\pi}{h} (h^1, h^2, h^3, h^4, h \cot h / f_\pi) , \quad h = \sqrt{(h^\hat{a})^2}. \quad \text{(6)}$$

The vacuum is characterized by the angular variable $\langle h \rangle / f_\pi$. Defining $\epsilon = \sin \langle h \rangle / f_\pi$, we have

$$\langle \Sigma \rangle = (0, 0, \epsilon, 0, \sqrt{1 - \epsilon^2}) , \quad \text{(7)}$$

where the value of $\epsilon$ can range between 0 (no EWSB) and 1 (maximal EWSB), depending on the effective potential of $h$ as we will discuss later.

By integrating out the CFT dynamics, one can write an effective Lagrangian for the external fields. It is convenient to express this Lagrangian in an SO(5)-symmetric way. To do so, we promote the elementary fermions to fill complete spinorial representations of SO(5). A spinorial representation of SO(5), a 4 of SO(5), contains two (complex) doublets, one transforming under SU(2)$_L$, the other transforming under SU(2)$_R$. We then embed $q_L, u_R, d_R$ as

$$\Psi_\ell = \begin{bmatrix} q_L \\ Q_L \end{bmatrix} , \quad \Psi_u = \begin{bmatrix} q^u_R \\ u^u_R \end{bmatrix} , \quad \Psi_d = \begin{bmatrix} q^d_R \\ u^d_R \end{bmatrix}. \quad \text{(8)}$$
The additional components $Q_L$, $q_R^u$, $d_R^u$, $q_R^d$, $u_R^u$ must be considered as non-dynamical external sources to the CFT, and they do not play any physical role. They are simply a useful tool for embedding the SM fermions in multiplets of SO(5) and thus being able to write the effective Lagrangian in a compact SO(5)-invariant fashion. Leptons can be promoted to spinorial representations in a similar way, whereas in the gauge sector we introduce extra non-dynamical vectors to obtain complete adjoint representations $A_\mu$, $B_\mu$ of SO(5) $\times$ U(1)$_{B-L}$.

We can now write the effective Lagrangian using an SO(5) $\times$ U(1)$_{B-L}$-invariant notation. After integrating out all CFT states at tree level, including fluctuations of the Higgs field around a constant classical background $\Sigma$, the most general effective Lagrangian for the external fields is, in momentum space and at the quadratic level,

$$L_{\text{eff}} = \frac{1}{2} P_{\mu\nu} \left[ \Pi_0^B(p) B^\mu B^\nu + \Pi_0(p) \text{Tr} [A^\mu A^\nu] + \Pi_1(p) \Sigma A^\mu A^\nu \Sigma^T \right]$$

$$+ \sum_{r=q,u,d} \bar{\Psi}_r p \left[ \Pi_0^r(p) + \Pi_1^r(p) \Gamma^i \Sigma_i \right] \Psi_r$$

where $P_{\mu\nu} = \eta_{\mu\nu} - p_{\mu} p_{\nu}/p^2$ and $\Gamma^i$, $i = 1, \ldots, 5$, are the gamma matrices for SO(5) (see appendix C):

$$\Gamma^i \Sigma_i = \begin{pmatrix} 1 & \hat{\sigma} \sin h/f \pi \\ \hat{\sigma}^\dagger \sin h/f \pi & -1 \cos h/f \pi \end{pmatrix}, \quad \hat{\sigma} \equiv \sigma^\dagger h \hat{\sigma} / h$$

$$\sigma^\dagger = \{\bar{\sigma}, -i1\}.$$  \hspace{1cm} (10)

The form factors $\Pi(p)$, $M(p)$ encode the effect of the strong dynamics, and cannot be determined perturbatively in the 4D theory. Their poles match with the CFT spectrum. A possible mixing term between $\Psi_u$ and $\Psi_d$ in eq. (9) has been neglected since it does not play any role in our calculations. Also, we have not written down possible bare kinetic terms and gauge-fixing terms for the external fields, i.e. terms not induced by the strong dynamics. They can be included in a straightforward way. We are only interested in two-point functions since, as we will see below, these are the only ones needed for the calculation of the $S$ parameter and the Higgs potential.

From the form factors of eq. (9) one can derive the low-energy effective theory. This is the theory of the light states, the SM fields and the Higgs (the equivalent of the chiral theory in QCD). It is obtained by performing an expansion in derivatives and light fields over $m_\rho$:

$$L = L_{\text{kin}} + L_{\text{yuk}} - V(\Sigma) + \Delta L.$$  \hspace{1cm} (11)

\footnote{However, choosing a specific representation for the fermions does have some physical consequence in our theory. Indeed, we are implicitly demanding for the composite fermionic operators $\mathcal{O}$ to come in complete multiplets of that particular representation (a spinorial representation in our specific case). In our class of theories, these operators have the same quantum numbers of the CFT fermionic bound states, so that we are implicitly making a specific choice upon the strong sector.}
The term $L_{\text{kin}}$ contains the kinetic terms of the dynamical fields

$$L_{\text{kin}} = \frac{f_\pi^2}{2} (D_\mu \Sigma) (D^\mu \Sigma)^T + \sum_r Z_r \bar{\psi}_r \gamma_\mu \psi_r - \frac{1}{4g^2} W_\mu^a W^{a \mu} - \frac{1}{4g'^2} B_\mu B^{\mu},$$

(12)

where $Z_q = \Pi^q_0(0) + \Pi^q_1(0)$, $Z_{u,d} = \Pi^u,d_0(0) - \Pi^u,d_1(0)$, $f_\pi^2 = \Pi_1(0)$, $1/g^2 = -\Pi_0'(0)$, $1/g'^2 = -(\Pi_0''(0) + \Pi_0'(0))$. The Higgs potential $V(\Sigma)$ is generated at one loop by gauge and fermion interactions. We will show below that the top contribution can trigger EWSB and the Higgs field $h$ acquires a VEV, breaking SO(4) down to the custodial SO(3) group. From the kinetic term for $\Sigma$ we obtain $M_W^2 = g^2 v^2 / 4$, where we have defined the EWSB scale

$$v \equiv \epsilon f_\pi = f_\pi \sin \langle h \rangle = 246 \text{ GeV}.$$

(13)

The term $L_{\text{yuk}}$ contains the Yukawa couplings between the Higgs and the elementary fermions and comes from the expansion of the last term of eq. (9):

$$L_{\text{yuk}} = \sin h / f_\pi \left[ M_1^u(0) \bar{q}_L h \sigma \bar{a} \left( u_R \right) + M_1^d(0) \bar{q}_L h \sigma \bar{a} \left( d_R \right) + \text{h.c.} \right].$$

(14)

When the Higgs acquires a VEV, the fermions get a mass

$$m_{u,d} = \frac{M_1^{u,d}(0)}{\sqrt{Z_q Z_{u,d}}} v \equiv y_{u,d} v,$$

(15)

where by NDA $y_{u,d} \sim \lambda_{u,d} \lambda_q \sqrt{N}/4\pi$. By choosing $\gamma_{q,u,d} > 0$, we have, according to eq. (2), that $\lambda_{q,u,d}$ are strongly suppressed at low energies, and the fermions are weakly coupled to the CFT. This can be used to explain in a natural way the smallness and the hierarchical structure of the masses of the light fermions [4, 5]:

$$m_{u,d} \sim \sqrt{N} \left( \frac{\mu_{\text{IR}}}{\Lambda} \right)^{\gamma_q + \gamma_{u,d}} v.$$

(16)

It is interesting to notice that this theory has a GIM mechanism, since flavour changing neutral current (FCNC) effects involving light fermions are also suppressed by the couplings $\lambda_{u,d,q}$ (see, for example [5, 6, 7, 8]). In order to have a large top mass, we will require $\gamma_u < 0$ and $\gamma_q \approx 0$ for the third quark generation. This choice is compatible with EWPT, as we will discuss in detail, and implies that the physical right-handed top quark is mostly composite.

The last term $\Delta L$ in the effective Lagrangian (11) contains all higher-order operators in the chiral expansion. The only one that is relevant for us here is that responsible for the $S$ parameter. It originates from the third term of eq. (9):

$$\Delta L \supset \frac{1}{2} \Pi_1'(0) W_\mu^a B^{\mu} \Sigma T^{aL} Y \Sigma^T,$$

(17)
where $T^a_L$, $Y$ are respectively the generators of SU(2)$_L$ and hypercharge. Eq. (17) gives

$$S = 4\pi\Pi_1'(0)\epsilon^2. \quad (18)$$

The $T$ parameter does not receive any contribution at tree level from the CFT due to the custodial symmetry. Nevertheless, it can be induced at the quantum level due to top interactions. We will discuss in section 4 the size of these contributions. Apart from $S$ and $T$, there are other two parameters constrained by LEP: $W$ and $Y$, defined in [9]. They are however quite small in the present model, since they arise from dimension-six operators and are thus suppressed by a factor $(g^2f_\pi^2/m^2_\rho)$ compared to $S$ and $T$.

2.1 Higgs potential and vacuum misalignment

A virtual exchange of elementary fields can transmit the explicit breaking of SO(5) from the elementary sector to the CFT and generate a potential for the PGB Higgs. The dominant contribution comes at one-loop level from the elementary SU(2)$_L$ gauge bosons and top quark. This is given by the Coleman-Weinberg potential

$$V(h) = \frac{9}{2} \int \frac{d^4p}{(2\pi)^4} \log \Pi_W - (2N_c) \int \frac{d^4p}{(2\pi)^4} \left[ \log \Pi_{b_L} + \log \left( p^2 \Pi_{t_L} \Pi_{t_R} - \Pi_{t_Lt_R}^2 \right) \right], \quad (19)$$

where $\Pi_i(p)$ are the self-energies of the corresponding SM fields in the background of $h$. These can be written as functions of the form factors of eq. (9), by using eq. (10):

$$\Pi_W = \Pi_0 + \frac{\Pi_1}{4} \sin^2 \frac{h}{f_\pi}, \quad \Pi_{b_L} = \Pi_{t_L} = \Pi_0^2 + \Pi_1^2 \cos \frac{h}{f_\pi}, \quad \Pi_{t_Lt_R} = M_1^a \sin \frac{h}{f_\pi}, \quad \Pi_{t_R} = \Pi_0^a - \Pi_1^a \cos \frac{h}{f_\pi}. \quad (20)$$

Apart from a constant piece, the potential of eq. (19) is finite since the form factors $\Pi_1$ and $M_1$ drop with the momentum as $\left|\langle \Phi \rangle\right|^2/p^{2d}$, where $\Phi$ is the CFT operator of dimension $d \gg 1$ responsible for the SO(5) breaking. $^2$ This fast decrease with the momentum allows us to expand the logarithms in eq. (19) and write the approximate formula $^3$

$$V(h) \simeq \alpha \cos \frac{h}{f_\pi} - \beta \sin^2 \frac{h}{f_\pi}, \quad (21)$$

$^2$In fact, in the 5D model the form factors drop exponentially with the momentum, corresponding to $d \to \infty$.

$^3$This approximate formula leaves out the top logarithmic contribution to the Higgs quartic coupling $\propto \log(m_t/m_\rho) \sim \log \epsilon$ since it comes from a subleading term in the expansion. This contribution can be large if $\epsilon$ is very small, and in that case it should be incorporated. For the qualitative discussion presented here, we will neglect it. For the 5D calculation of the next section, however, we will take the full potential eq. (19).
where
\[ \alpha = 2N_c \int \frac{d^4p}{(2\pi)^4} \left( \frac{\Pi^u_1}{\Pi^u_0} - \frac{2\Pi^d_1}{\Pi^d_0} \right), \quad \beta = \int \frac{d^4p}{(2\pi)^4} \left( 2N_c \frac{(M^u_1)^2}{(-p^2)\Pi^u_0\Pi^u_1} - \frac{9\Pi_1}{8\Pi_0} \right). \] (22)

This potential has a minimum at \( \cos h/f_\pi = -\alpha/(2\beta) \), i.e.
\[ \epsilon = \sqrt{1 - \left( \frac{\alpha}{2\beta} \right)^2}. \] (23)

Thus, for suitable values of \( \alpha \) and \( \beta \) the EWSB can occur dynamically. Gauge fields only contribute to the \( \sin^2 \) operator with an overall positive coefficient, \( \beta_{\text{gauge}} < 0 \), and tend to align the vacuum in the \( \text{SU}(2)_L \) preserving direction. A misalignment of the vacuum can only come from top loops, and only if the coefficients \( \alpha \) and \( \beta \) are comparable in size. An NDA estimate of the two coefficients shows that \( \alpha \) is expected to be larger than \( \beta \): \( \alpha \sim m^4_\rho \lambda^2_{q,u} N_c N/(16\pi^2)^2 \), \( \beta \sim m^4_\rho y^2_t N_c N/(16\pi^2)^2 \), where \( y_t \sim \lambda_q \lambda_u \sqrt{N}/4\pi \) is the top Yukawa coupling. However, \( \alpha \) turns out to be generally smaller than its NDA estimate, and the contributions from \( q_L \) and \( t_R \) are always opposite in sign (see eq. (20) and (22)), thus tending naturally to cancel each other. The physical Higgs mass is given by
\[ m^2_{\text{Higgs}} \simeq \frac{2\beta \epsilon^2}{f^2_\pi} \sim \frac{2N_c}{N} y^2_t v^2. \] (24)

We see that for moderate values of \( N \) the Higgs mass can be above the experimental bound \( m_{\text{Higgs}} > 114 \text{ GeV} \). Notice that, despite being generated at one-loop level, the quartic coupling is \( \mathcal{O}(1) \). Indeed, due to the strong dynamics involved in the loop diagrams, the actual loop expansion parameter is \( 1/N \), as it is evident from eq. (24).

Remarkably, this minimal theory can accomplish a realistic EWSB. To quantify this statement we must calculate the precise value of \( \alpha, \beta \) and the \( S \) parameter. This will be done in the 5D theory. We will see that in order to satisfy the experimental constraints coming from \( S, T \) and \( Z \to b\bar{b} \), a value \( \epsilon \lesssim 0.4 \) is necessary. This implies, from eq. (23), that the relation \( \alpha \simeq 2\beta \) must be fulfilled at the 10% level. We will show that this can be accomplished in certain regions of the 5D parameter space.

It is interesting to notice that the mild tuning required in the minimal model can be completely avoided if one introduces new sources of SO(5) breaking that generate extra terms in the Higgs potential. For example, if the Higgs potential has an extra \( \sin^4 \) term
\[ V(h) \simeq \alpha \cos \frac{h}{f_\pi} - \beta \sin^2 \frac{h}{f_\pi} + \gamma \sin^4 \frac{h}{f_\pi}, \] (25)
the minimum, for \( \epsilon < 1 \), is given by

\[
\epsilon^2 = \frac{\alpha}{4\gamma} \left[ \pm 1 + 2\beta/\alpha \right] + \mathcal{O}(\epsilon^4),
\]

(26)

where the difference in sign comes from \( \cos h/f_\pi \simeq \pm (1 - \epsilon^2/2) \). Eq. (26) shows that a coefficient \( \gamma \) slightly larger than \( \alpha \), but still of one loop size, can give \( \epsilon \sim 0.2 - 0.4 \) with virtually no fine tuning in the second factor. New contributions to the potential can come from different sources. In appendix B we give an example of how eq. (25) can be generated by loop effects of additional fermions, or at tree-level by an extra scalar field, following a mechanism already proposed in ref. [3].

Comparing with the original Georgi-Kaplan models where a large hierarchy \( v \ll f_\pi \) (and therefore a large fine-tuning) was required to avoid the strong constraints from FCNC [2], we see that in our case we only need a very small hierarchy between \( v \) and \( f_\pi \). No relevant constraint on \( \epsilon \) comes in our model from FCNC due to the different realization of flavour.

### 3 The 5D model

The 4D theory presented above can be obtained as the holographic description of a 5D weakly coupled model. In this section we describe the 5D model and show how to compute the form factors of eq. (25).

The 5D spacetime metric is given by [10]

\[
ds^2 = \frac{1}{(kz)^2} \left( \eta_{\mu\nu} dx^\mu dx^\nu - dz^2 \right) \equiv g_{MN} dx^M dx^N,
\]

(27)

where the 5D coordinates are labeled by capital Latin letters, \( M = (\mu, 5) \), with \( \mu = 0, \ldots, 3 \); \( z = x^5 \) represents the coordinate for the fifth dimension and \( 1/k \) is the AdS curvature radius. This spacetime has two boundaries at \( z = L_0 \equiv 1/k \sim 1/M_{\text{Pl}} \) (UV-brane) and at \( z = L_1 \sim \mu_{\text{IR}} \sim 1/\text{TeV} \) (IR-brane). The gauge symmetry in the 5D bulk is taken to be \( \text{SU}(3)_c \times \text{SO}(5) \times \text{U}(1)_{B-L} \) reduced to \( \text{SU}(3)_c \times \text{SO}(4) \times \text{U}(1)_{B-L} \) on the IR-brane and to \( \text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_{Y} \) on the UV-brane. This can be easily achieved by imposing a Dirichlet boundary condition for the gauge bosons whose generators we want to break. We choose to work in the unitary gauge \( \partial_z (A_5/z) = 0 \) (see ref. [3]), where only physical gauge configurations survive. In this gauge \( A_5 \) is non-vanishing only in its \( \text{SO}(5)/\text{SO}(4) \) components, which are however constrained to have a fixed profile along the fifth dimension: \( A_5(x, z) = \zeta(z) h(x), \zeta(z) = z \sqrt{2/(L_1^2 - L_0^2)} \). Thus, physical fluctuations of \( A_5 \) correspond to a 4D scalar field \( h(x) \) transforming as a 4 of \( \text{SO}(4) \), the Higgs. From the point of view of the 5D theory, a potential for
$A_5$ is forbidden at tree level by gauge invariance, but it is generated radiatively as a finite-volume effect from non-local operators. This is the Hosotani mechanism for symmetry breaking \[11\].

The SM fermions are embedded into 5D Dirac spinors $\xi_i$ that live in the bulk and belong to the $4_{1/3}$ representation of SO(5)×U(1)$_{B-L}$. For each quark family we define

$$\xi_q = \begin{bmatrix} q_L(++) & q_R(--) \\ Q_L(--) & Q_R(++) \end{bmatrix}, \quad \xi_u = \begin{bmatrix} q^u_L(+-) \\ Q^u_L = \begin{bmatrix} u^c_L(+-) \\ d^c_L(+-) \end{bmatrix} \end{bmatrix}, \quad \xi_d = \begin{bmatrix} q^d_L(+-) \\ Q^d_L = \begin{bmatrix} u^c_R(--) \\ d^c_R(--) \end{bmatrix} \end{bmatrix},$$

where leptons are realized in a similar way. Here ($\pm, \pm$) is a shorthand notation to denote a Neumann ($+$) or Dirichlet ($-$) boundary condition on each brane. Chiralities under the 4D Lorentz group have been denoted with $L,R$, while small $q$’s (capital $Q$’s) denote doublets under SU(2)$_L$ (SU(2)$_R$). Massless modes in eq. (28) arise from ($+, +$) fields. These are $q_L, Q_R$ and $d^c_L, u^c_R$. We get rid of the latter two states by adding an extra field on the IR-brane, $\tilde{Q}_R$, which marries them: $[\bar{Q}_u L + \bar{Q}_d L] \tilde{Q}_R$.

Gauge invariance, however, allows one to write mass and kinetic mixing terms among the different multiplets $\xi_q, \xi_u, \xi_d$ in the bulk and on the IR-brane. The bulk kinetic and mass matrices are SO(5)–symmetric and can be always simultaneously diagonalized through a field redefinition. In that basis, the most general SO(4)-invariant set of mass terms one can write on the IR-brane includes

$$[\tilde{M}_u Q^u_L + \tilde{M}_d Q^d_L] Q_R + \bar{q}_L [\tilde{m}_u q^u_R + \tilde{m}_d d^c_R] + h.c.$$  \hspace{1cm} (29)

Therefore, the massless states become a mixture of $q_L$ with $q^u_L$ and $q^d_L$; $u_R$ with the upper component of $Q_R$; $d_R$ with the lower component of $Q_R$. The mixing angle depends on the value of the 5D bulk masses $M_5^i = c_i k$. Yukawa couplings to $A_5$ arise from the 5D covariant derivative in the fermion bulk kinetic terms. This means that $A_5$ can only connect SU(2)$_L$ doublets with SU(2)$_R$ doublets of opposite Lorentz chirality inside the same 5D multiplet (for example $q_L$ with $Q_R$). Yukawas for the physical massless modes then proceed through the mixing and can be suppressed depending on the mixing angle.

### 3.1 The holographic approach: Matching to the 4D theory

The 5D model described above has exactly the same properties as the 4D CFT theory described in section 2. In particular it leads to the same effective Lagrangian eq. (9) and to the same low-energy
theory eq. (11). In order to match the two theories, it is more convenient to follow the holographic approach instead of the more popular Kaluza-Klein (KK) decomposition.

The holographic procedure consists in separating the bulk fields from their UV-brane value and treating them as distinct variables. If we integrate out the bulk with fixed values of the fields on the UV-brane, we obtain a 4D (non-local) theory defined on the UV boundary. This is the theory to be matched with eq. (9). Boundary values of 5D fields with Neumann (Dirichlet) condition on the UV-brane correspond to the external dynamical (non-dynamical) fields of eq. (9), while the bulk plays the role of the CFT sector. The holographic procedure is clearly inspired by the AdS/CFT correspondence [12], although it does not rely in any sense on the validity of that conjecture. It simply represents an alternative way of describing the 5D model, motivated by symmetry principles. The symmetries of the bulk are the same as those of the CFT, while the 4D UV-brane sector respects only the SM gauge invariance (see refs. [13] and [14] for a similar application of the holographic description).

To obtain the 5D prediction for the form factors of eq. (9) we match the two theories on the SO(4)-invariant vacuum: Σ = Σ₀ (i.e. $h = 0$). Let us start with the gauge sector. On the AdS₅ side, after integrating the bulk as a function of the UV-boundary fields, one finds (at the quadratic level),

$$L_{\text{eff}} = \frac{1}{2} P_{\mu \nu} \left[ \Pi_a(p) A^{a \mu} A^{a \nu} + \Pi^\dagger_{\hat{a}}(p) \tilde{A}^{\hat{a} \mu} \tilde{A}^{\hat{a} \nu} \right].$$  

(30)

The indexes $a$ and $\hat{a}$ run respectively over the SO(4) generators (unbroken on the IR-brane) and the SO(5)/SO(4) generators (broken on the IR-brane), and

$$\Pi_{a,\hat{a}}(p) = -\frac{1}{g_5^2 k L_0} \frac{p}{Y_0(pL_0) \tilde{J}_{0,1}(pL_1) - \tilde{Y}_{0,1}(pL_1) J_0(pL_0)} \cdot$$  

(31)

with $\tilde{J}_1(pL_1) = J_1(pL_1)$, $\tilde{J}_0(pL_1) = J_0(pL_1) - \frac{1}{2} \frac{g_5^2}{g_{1R}^2} pL_1 J_1(pL_1)$ and similarly for $\tilde{Y}_{0,1}$. Here $g_5^2$ denotes the SO(5) bulk gauge coupling, and $1/g_{1R}^2$ is the coefficient of the SO(4) boundary kinetic term on the IR-brane. Analogous formulas apply for the U(1)$_{B-L}$ gauge boson, that has been omitted in eq. (30) for simplicity. We have not written down possible boundary kinetic terms on the UV-brane, though they can be included in a straightforward way. Eq. (30) must be matched to eq. (9) after setting Σ → Σ₀. As we said, boundary values of 5D fields with Neumann (Dirichlet) conditions on the UV-brane must be identified with the dynamical (non-dynamical) external fields of the 4D theory. We get

$$\Pi_a(p) = \Pi_0(p), \quad \Pi_{\hat{a}}(p) = \Pi_0(p) + \frac{1}{2} \Pi_1(p),$$  

(32)
that leads to the determination of the gauge form factors:

\[
\Pi_0(p) = \Pi_a(p), \quad \Pi_1(p) = 2\left[\Pi_\delta(p) - \Pi_a(p)\right].
\]  

(33)

For the fermions we can proceed in a similar way. We integrate out the bulk fermionic fields as a function of their values on the UV-brane. We are only allowed to fix on the boundary either the left- or the right-handed fermionic component (left- or right-handed source description, see [14]). For \(\xi_q\) we will take the left-handed component, \(\xi_{qL} = (q_L, Q_L)\), while for \(\xi_u\) we will take the right-handed component, \(\xi_{uR} = (q_u, Q_u)\). We omit \(\xi_d\) for simplicity. At the quadratic level we obtain

\[
\mathcal{L}_{\text{eff}} = \Pi_{qL}(p) \bar{q}_L p q_L + \Pi_{Q_L}(p) \bar{Q}_L p Q_L + \Pi_{qR}(p) \bar{q}_R p q_R + \Pi_{Q_R}(p) \bar{Q}_R p Q_R \\
+ M_q(p) \bar{q}_L Q_R^u + M_Q(p) \bar{Q}_L q_R^u.
\]  

(34)

The fermionic self-energies \(\Pi_i(p)\) and \(M_i(p)\) are given in appendix [A] in terms of brane-to-brane 5D propagators. Matching with the 4D CFT theory, we have

\[
\Pi_{0,1}^q(p) = \frac{1}{2} \left[\Pi_{qL}(p) \pm \Pi_{Q_L}(p)\right], \\
\Pi_{0,1}^u(p) = \frac{1}{2} \left[\Pi_{qR}(p) \pm \Pi_{Q_R}(p)\right], \\
M_{0,1}^u(p) = \frac{1}{2} \left[M_q(p) \pm M_Q(p)\right].
\]  

(35)

The anomalous dimensions \(\gamma\) of the 4D CFT theory are related to the 5D fermion masses \(M_5^i = c_i k\) according to

\[
\gamma_q = \left|c_q + \frac{1}{2}\right| - 1, \quad \gamma_u,d = \left|c_{u,d} - \frac{1}{2}\right| - 1.
\]  

(36)

Therefore the requirement \(\gamma_{q,u,d} > 0\) for the light fermions (see above eq. [16]) implies \(c_q > 1/2\) and \(c_{u,d} < -1/2\), while for the top \(\gamma_q \simeq 0\) and \(\gamma_u < 0\) implies \(c_q \simeq 1/2\) and \(|c_u| < 1/2\). It is useful at this point to define the number of CFT colors \(N\) by identifying \(1/N\) with the perturbative expansion parameter in 5D:

\[
\frac{1}{N} \equiv \frac{g_5^2 k}{16\pi^2}.
\]  

(37)

The 5D NDA condition for calculability reads

\[
\frac{\pi k}{\Lambda_S} = \frac{g_5^2 k}{24\pi^2} \ll 1,
\]  

(38)

\footnote{The different relation for \(q\) and \(u,d\) is because we are fixing on the UV-boundary the left-handed component for \(q\) and the right-handed component for \(u,d\). See ref. [14] for details.}
where $\Lambda_S$ is the strong cutoff scale of the 5D theory. Using the relation $1/g^2 = \ln(L_1/L_0)/g_5^2k + 1/g_{UV}^2 + 1/g_{IR}^2$, where $1/g_{UV}$ ($1/g_{IR}$) is the SU(2)$_L$ UV-brane (SO(4) IR-brane) kinetic term and $g \simeq 0.65$ is the effective SU(2)$_L$ coupling, we can derive a rough lower bound for $g_5^2k$:

$$g_5^2k > g^2 \ln(L_1/L_0) \sim 16. \quad (39)$$

Therefore $N$ is restricted in the interval $1 \ll N \ll 10$.

### 3.2 Predictions of the 5D model

Having matched the 5D and 4D theories, we can now derive the predictions of the 5D model. First, we can obtain from eq. (31) the mass spectrum and decay constants of the vector resonances of the theory. For this purpose we decompose $\Pi_a, ˆ\Pi_a(p)$ as an infinite sum over narrow mesons, as prescribed by the large-$N$ limit:

$$\Pi_a(p) = p^2 \sum_n \frac{F_{\rho n}^2}{p^2 + m_{\rho n}^2}, \quad \Pi_\hat{a}(p) = p^2 \sum_n \frac{F_{\hat{a} n}^2}{p^2 + m_{\hat{a} n}^2} + \frac{1}{2}f_\pi^2. \quad (40)$$

Eq. (31) thus gives

$$f_\pi^2 = \frac{4}{g_5^2k} \frac{1}{L_1^2}, \quad m_\rho \equiv m_{\rho 1} \simeq \frac{3\pi/4}{\sqrt{1 + 9\pi^2/32}z_{IR}} \frac{1}{L_1}, \quad m_{a1} \simeq \frac{5\pi}{4} \frac{1}{L_1}, \quad (41)$$

where we have defined $z_{IR} = g_5^2k/g_{IR}^2$. From eq. (18) and (33) we can obtain the prediction for the $S$ parameter:

$$S = \frac{3N}{8\pi} \epsilon^2 \left[1 + \frac{4}{3}z_{IR}\right]. \quad (42)$$

The 99% CL experimental bound from LEP, $S \lesssim 0.3$ [9], requires:

$$\epsilon \lesssim \sqrt{\frac{10}{N}} \frac{1}{1 + 4/3z_{IR}}. \quad (43)$$

We see that in order to remain in the weak coupling regime, $1/N \ll 1$, we need $\epsilon < 1$. In particular, taking $N = 10$ and $z_{IR} = 0$ (1) requires $\epsilon \sim 0.5$ (0.3). A way of recasting the result for $S$ is by fixing $f_\pi \epsilon = v$ according to eq. (13), and using the value of $m_\rho$ as computed in eq. (41). One obtains

$$S \simeq \frac{27\pi^3}{32} \left(\frac{1 + 4/3z_{IR}}{1 + 9\pi^2/32z_{IR}}\right) \frac{v^2}{m_\rho^2}. \quad (44)$$

In order to satisfy $S \lesssim 0.3$, the lowest vector state cannot be lighter than $m_\rho \sim 2.3$ (1.6) TeV for $z_{IR} = 0$ ($\infty$).

---

5It corresponds to an extra contribution to the $\epsilon_3$ parameter [15] $\Delta \epsilon_3 \lesssim 2.5 \cdot 10^{-3}$ relative to the SM value with $m_{Higgs} = 115$ GeV.
To determine whether EWSB is triggered and an $\epsilon < 1$ is generated in our model, one has to compute and minimize the Higgs potential [14]. The potential is completely determined by the self-energies eqs. [41] and [52]–[57], which are in turn functions of few input parameters: the bulk SO(5) gauge coupling $g_5$, the gauge kinetic terms on the UV and IR branes, $1/g_{UV}^2$ and $1/g_{IR}^2$ (respectively for SU(2)$_L$ and SO(4)), and the top bulk and IR-brane masses, $c_q$, $c_u$, $\widetilde{m}_u$ and $\widetilde{M}_u$. For simplicity we have neglected the subleading contribution to the potential coming from the bottom quark and the hypercharge gauge field. Performing a numerical analysis, we found that in a large region of the input parameter space EWSB is indeed triggered and the bound (43) on the $S$ parameter is satisfied. The results are summarized in fig. 1, where $\epsilon$ and the number of colors of the CFT defined by (37), are given as a function of the physical Higgs mass. Only points which satisfy the bound $S < 0.3$ are shown. The values of the input parameters have been chosen as follows. We fixed the UV coupling $g_{UV}$ by requiring that the low-energy SU(2)$_L$ gauge coupling $g$ equals its experimental value, while the IR gauge kinetic term has been set to be of loop order, $1/g_{IR}^2 = 1/16\pi^2$. The value of $N$ is extracted by fixing the top mass to its experimental value $m_t^{\exp} = 178.0 \pm 4.3$ GeV [16]. The top bulk masses $c_q$, $c_u$ must lie in the interval $-1/2 < c_q, c_u < +1/2$, otherwise the coupling between the elementary $q_L$, $t_R$ and the CFT becomes irrelevant, implying a too small top mass. Values of $c_q$ close to $+1/2$ seem to be preferred in order not to have large corrections to $Z \to b\bar{b}$, as we will explain in detail in the next section. Also, a $c_u$ too close to $+1/2$ is disfavored by $Z \to b\bar{b}$, while values of $c_u$ close to $-1/2$ seem to give a smaller $T$ parameter and they are preferred. We thus scan in our numerical analysis over $-0.3 < c_u < +0.3$ and $0.3 < c_q < 0.5$. Finally, for the IR-brane masses we have taken the following values: $\widetilde{m}_u = 0.5, 1.0, 1.5$, and $-2.1 < \widetilde{M}_u < -0.4$. 

Figure 1: Scatter plots in the $(m_{Higgs}, \epsilon)$ plane (left) and in the $(m_{Higgs}, N)$ plane (right), obtained by scanning over the input parameters in the minimal model. In the second plot, blue squares correspond to $\epsilon > 0.4$, green fat dots to $0.2 < \epsilon < 0.4$, small red dots to $\epsilon < 0.2$. 

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Figure 2: Masses of the first vectorial and fermionic resonances $\rho, \tilde{q}_L, \tilde{b}_R, \tilde{t}_R$ obtained by scanning over the input parameters in the minimal model. Blue squares correspond to $\epsilon > 0.4$, green fat dots to $0.2 < \epsilon < 0.4$, small red dots to $\epsilon < 0.2$.

Fig. 1 shows that the model can give moderately small values of $\epsilon$, i.e. $\epsilon \lesssim 0.4$, needed to pass all EWPT. This requires a 10% adjustment in the parameters of the model, as we already explained in the previous section. The value of $N$ can be large enough to guarantee a sensible perturbative expansion, although for $m_{\text{Higgs}} \sim 115$ GeV it tends to be small, $N \sim 4 - 5$, if one requires $\epsilon \sim 0.4$. One of the most important predictions of the model is that the Higgs mass is always light, $m_{\text{Higgs}} \lesssim 140$ GeV, for $0.1 \lesssim \epsilon \leq 1$. This bound can be relaxed only for very small values of $\epsilon$, due to the log enhanced top contribution $\propto \log(m_t/m_\rho) \sim \log \epsilon$. However, a very small value of $\epsilon$ can be only obtained at the price of a large fine tuning in the parameters of the model. It should be stressed that the theoretical error on the Higgs mass is expected to be large if $N$ is small. For example, for $N \sim 5$ the correction to $m_{\text{Higgs}}$ coming from two loop diagrams can be $\sim 20\%$ or even larger.

The spectrum of the lightest resonances, obtained by scanning over the input parameter space, is shown in figure 2. The fermionic states $\tilde{q}_L, \tilde{t}_R, \tilde{b}_R$ tend to be lighter than the gauge resonances.
For values of $m_{\text{Higgs}} \sim 115$ GeV and $\epsilon \sim 0.4$, all fermionic resonances lie around $1.5 - 2$ TeV, while $m_\rho \sim 2 - 3$ TeV.

Finally, in fig. 3 we show the results obtained by adding a new contribution $\Delta V = \xi \sin^4 h/f_\pi$ to the Higgs potential. The value of $\xi$ is taken to be positive and equal in size to the gauge contribution to $\beta$: $\xi = -\beta_{\text{gauge}} > 0$. This example shows that small deformations of the Higgs potential can help not only to eliminate the 10% fine tuning of the minimal model, as discussed in sec. 2.1, but also to increase the range of the Higgs mass. We checked that similar results also hold taking $\Delta V = \xi \sin^4 h/f_\pi + \zeta \sin^2 h/f_\pi$, with $\zeta = -\xi = \beta_{\text{gauge}}$. A possible origin of these new contributions is given in appendix B.

4 Third family EWPT: $Z \to b\bar{b}$ and the $T$ parameter

A large top Yukawa implies that the top must substantially mix with the CFT (diagram fig. 4(a)). This in turn suggests that there could be large deviations from the SM prediction for $Z \to b\bar{b}$. Such corrections are induced from the diagram of fig. 4(b). Sizable one-loop contributions to $T$ are also expected from fig. 4(c) (see also refs. [17, 18] for a similar discussion). In this section we will estimate these contributions using NDA and show that they can be under control for $\epsilon \lesssim 0.4$. 

Figure 3: Scatter plots in the $(m_{\text{Higgs}}, \epsilon)$ plane (left) and in the $(m_{\text{Higgs}}, N)$ plane (right), obtained by scanning over the input parameters with an extra contribution $\Delta V(h) = \xi \sin^4 h/f_\pi$, $\xi = -\beta_{\text{gauge}}$. In the second plot, blue squares correspond to $\epsilon > 0.4$, green fat dots to $0.2 < \epsilon < 0.4$, small red dots to $\epsilon < 0.2$. 


Figure 4: Diagrams in the 4D holographic theory that generate the top Yukawa coupling (a), a correction to $Z \rightarrow b_L \overline{b}_L$ (b), and the $T$ parameter (c). A grey blob represents the 4D CFT dynamics or the 5D bulk. Another possible diagram contributing to $Z \rightarrow b_L \overline{b}_L$, similar to (b) but with two $\Sigma$ fields attached, is not shown.

Denoting with $\delta g_{Lb}$ the shift in the coupling of $b_L$ to $Z$, NDA leads to the estimates\(^6\)

\[
\frac{\delta g_{Lb}}{g_{Lb}} \sim \lambda_q^2 \frac{N}{16\pi^2} \epsilon^2 \sim \left(\frac{1}{2} - c_q\right) \epsilon^2 ,
\]

\[
\alpha T = \Delta \rho \sim \lambda_u^4 \frac{N}{(16\pi^2)^2} \eta^4 \epsilon^2 \sim \left(\frac{1}{2} + c_u\right)^2 \eta^4 \epsilon^2 ,
\]

where we have used eqs. (45) and (46). We have also included a new parameter $\eta$ to take into account a possible deviation from NDA in the coupling of the composite fermions to the Higgs (a chirality flip factor).\(^7\) From eq. (45) we see that the (liberal) bound $\delta g_{Lb}/g_{Lb} \lesssim 1\%$ from LEP is satisfied for values of $c_q$ close to 1/2. For example, $\epsilon \simeq 0.4$ implies $c_q \simeq 0.4$. From eq. (46), on the other hand, we see that the 99\% CL bound on the $T$ parameter, $T \lesssim 0.3\,[9]\(^8\)$, can also be satisfied for reasonable values of the parameters. For example, setting $\eta \sim 1$, then $\epsilon \simeq 0.4$ implies $c_u \simeq -0.1$ for $N \simeq 10$. Thus, both estimates give $\delta g_{Lb}$ and $T$ close to the experimental limit for values of the 5D parameters used in the analysis of section 3.2. This is an indication that our model can succeed in passing all EWPT, although eqs. (45), (46) should not be taken too seriously, being only estimates and not exact results. One can take into account the correlation among $T$, $\delta g_{Lb}$ and the top mass by making use of the NDA estimate for $m_t$

\[
m_t \sim \lambda_q \lambda_u \frac{N}{16\pi^2} m_\rho \epsilon \eta \sim \sqrt{\left(\frac{1}{2} - c_q\right) \left(\frac{1}{2} + c_u\right)} \frac{4\pi}{\sqrt{N}} v \eta ,
\]

\(^6\)If fig. 4 is drawn using resonances, one can show that there are two kind of diagrams contributing to $\delta g_{Lb}$ and $\Delta \rho$ . Either the Higgs couples to a vector resonance, or to a fermionic resonance through a chirality flip. One can show that the dominant contribution to $\delta g_{Lb}$ and $\Delta \rho$ are respectively that with zero and four chirality flips.

\(^7\)This corresponds to the 5D theory to the mass mixing parameters eq. (29).

\(^8\)It corresponds to an extra contribution to the $\epsilon_1$ parameter [15] $\Delta \epsilon_1 \lesssim 2.5 \cdot 10^{-3}$ relative to the SM value with $m_{Higgs}=115 \text{ GeV}$. 

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and combine it with eqs. (45) and (46). Imposing $\delta g_{Lb}/g_{Lb} \lesssim 1\%$ we then obtain

$$\alpha T \gtrsim 0.4 N \epsilon^6.$$  \hfill (48)

This is quite a stringent bound. Models with $\epsilon = 1$ give $T \sim O(100)$ even for small $N$ and are therefore grossly ruled out. Nevertheless, this bound on $T$ is extremely sensitive to $\epsilon$, and a value for $\epsilon$ slightly smaller than 1 can be enough to satisfy the experimental constraint.

Our estimates (45), (46), (47) and (48) apply quite generally to a large class of 5D models where fermions live in the bulk, EWSB is triggered in the IR and custodial symmetry is violated only on the UV-brane. The model of ref. [7], where the Higgs is localized on the IR-brane, belongs to this class. In ref. [7] the $T$ parameter was explicitly computed and found to be in agreement with eqs. (46) and (48). The bound eq. (48) also applies to 5D Higgsless models [19, 13, 20]. In that case the 4D holographic description consists in a walking TC-like theory where the two scales $f_\pi$ and $v$ coincide ($\epsilon = 1$). Thus, 5D Higgsless models are severely constrained by $T$. This is true, we stress, even in the limit of strong bulk gauge coupling, i.e. for a small value of $N$.

We close this section mentioning a case in which the bound (48) can be weakened. This corresponds to the limit $c_u \to 1/2$ in which one can show that a massive $b'_R$ state of the CFT becomes light, and an extra factor $(m_{\nu}/m_{\rho})^2$ suppresses the bound on $T$:

$$\alpha T \gtrsim 0.4 N \frac{m_{\nu}^2}{m_{\rho}^2} \epsilon^6.$$  \hfill (49)

To understand this suppression factor it is convenient to adopt the holographic description in which the left-handed component of $\xi_u$, instead of the right-handed one, is fixed on the UV-brane. In this case the holographic description is the following [13]: the right-handed top quark arises as a massless CFT bound state and, by SU(2)$_R$ symmetry, it comes along with a massless partner $b'_R$. Together they form a doublet $Q_R = (t_R, b'_R)$ of SU(2)$_R$. An additional external field $b'_L$ exists, which marries $b'_R$. The external field $b'_L$ represents the only source of violation of the custodial symmetry. It is coupled to the CFT (and thus to $b'_R$) through a coupling which becomes marginal for $c_u = 1/2$, and irrelevant if $c_u > 1/2$. In the limit $c_u \to +\infty$, $b'_L$ decouples from the CFT, $m_{\nu} \to 0$, the custodial symmetry is restored, and $T$ vanishes. Therefore $T$ must be proportional to $m_{\nu}^2$. The value of $m_{\nu}$, however, cannot be arbitrarily small, since $b'_R$ mixes also with $b_L$ and induces an additional shift in the coupling of $b_L$ to $Z$ of order $\delta g_{Lb}/g_{Lb} \sim m_{\nu}^2/m_{\nu}^2$. By requiring $\delta g_{Lb}/g_{Lb} \lesssim 1\%$, one has $m_{\nu} \gtrsim 10 m_L$. Thus, $T$ can be somewhat reduced, but one still needs $\epsilon < 1$ to be consistent with the experimental constraint.
5 Conclusions and comparison with other models of PGB Higgs

Theories in which the Higgs arises as a PGB provide a rationale for the smallness of the electroweak scale compared to the scale of new physics, a fact that experiments seem to suggest. This provides an important motivation to search for realistic theories of PGB Higgs.

Here we have presented the minimal model in which (i) EWSB occurs dynamically, (ii) the flavour problem is solved, and that (iii) is consistent with EWPT. The Higgs appears as a composite PGB from a 5D bulk with a symmetry breaking pattern $SO(5) \to SO(4)$. EWSB occurs dynamically via top virtual effects, and the Higgs mass can be calculated as a function of the 5D parameters (see fig. 1).

Other approaches to Higgs as PGB have been previously considered in the literature. The Hosotani mechanism for EWSB [11], where a PGB Higgs also appears as the fifth component of the 5D gauge field, has been extensively studied [21]. Nevertheless, none of these models is fully realistic. The two important ingredients in our approach are the custodial symmetry and that fact that we are working in a 5D AdS space, that allows us to extrapolate the SM couplings up to the Planck scale. As a consequence, we can successfully address the flavour issue. Furthermore, the AdS dynamics forces us to work with a moderately large 5D gauge coupling, $g_5 \sqrt{k} \gtrsim 4$ (see eq. (39)), thus implying a large enough Higgs quartic coupling and a small $f_\pi/m_\rho$ as required by EWPT. This has to be compared with 5D flat theories with no large brane kinetic terms. In that case the 5D coupling is smaller, $g_5 \sqrt{L^{-1}} \simeq g \simeq 0.65$ ($L$ being the length of the fifth dimension), and therefore the 1-loop Higgs quartic is too small.

Another approach to Higgs as a PGB has been Little Higgs (LH) models [22]. In these models the electroweak scale is protected from the strong scale $\mu_{IR}$, up to two-loop effects, due to collective breaking. For this purpose, extra vector states $W'$ are required. The main benefit from this approach is calculability. The theory below $\mu_{IR}$ is weakly coupled and therefore predictable. This is different from our approach in that we take a large $N$ limit in the strong sector to obtain calculability. Both approaches have a similar contribution to the $S$ parameter, since the role of $m_\rho$ in our eq. (44) is now played by $m_{W'}$. In LH models, however, the EW scale is more sensitive to $m_{W'}$ than we are to $m_\rho$, due to the fact that the one-loop contribution to the LH potential is logarithmically divergent. In our case the potential is finite and then less sensitive to the scale of $m_\rho$.

Note: Our model at low-energies $\simeq \mu_{IR}$, however, could be realized in flat space if large UV-brane kinetic terms for the SM fields are added [13]. What the AdS$_5$ geometry does is to give a dynamical origin for these large UV-brane kinetic terms.
new physics.  

Phenomenologically there are other important differences. For example, differently from a LH theory, in our model vector and fermion resonances come in complete representations of SO(5). The resonances \( \tilde{q}_L, \tilde{t}_R \) and \( \tilde{b}_R \) are usually lighter than the vectorial ones (see figs. 2), and therefore their detection can be more feasible. Their single production can proceed at the LHC via a Yukawa interaction:

\[
q_L + W_{long.}, Z_{long.} \rightarrow \tilde{t}_R, \tilde{b}_R,
\]

\[
t_R + W_{long.}, Z_{long.} \rightarrow \tilde{q}_L,
\]

where the incoming particles arise from the colliding protons. Although the top quark content of the proton can be small, the production cross-section is enhanced by the large coupling involved in the vertex. This is because the couplings between CFT (5D bulk) states are of order \( g_5 \sqrt{k} \sim 4 \). The decays of these KK fermions are just the inverse of their production process. The \( \tilde{t}_R \) production is similar to that of \( t' \) in LH models. In refs. \cite{23, 24} it has been shown that \( t' \) can be detected at the LHC if its mass is of order of few TeV. For \( \tilde{q}_L \) and \( \tilde{b}_R \), however, there is no analog in typical LH models. The production of these fermionic resonances can also proceed via a virtual KK gluon \( \tilde{g} \) (with no analog in LH models), predominantly for \( \tilde{t}_R \):

\[
q \bar{q} \rightarrow \tilde{g}^* \rightarrow t_R + \tilde{t}_R.
\]

Here \( q \) denotes a light or heavy quark. Finally, the gauge resonances can be produced via a Drell-Yan process, as in eq. 51. For the case of \( W \) and \( Z \) KK, their production can also proceed by \( W_{long.}, Z_{long.} \) scattering (as in TC models) due to the strong coupling between these states. A detailed study of all these processes will be required to fully explore the phenomenology of our model at future colliders.

We think that the model presented here is a serious alternative to supersymmetric models. Several issues, however, must still be addressed. For example, a full calculation of top quark effects on EWPT (\( T \) parameter and \( Z \rightarrow b \bar{b} \)), gauge coupling unification or string embedding. We leave this for the future.

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10 It is interesting to notice that in the limit of large kinetic terms on the IR-boundary, i.e. small \( g_{IR} \), our model resembles a LH model, since one resonance becomes lighter and more weakly coupled than the others (see eq. 41), and plays the role of \( W' \).

11 See ref. \cite{25} for a first string realization of a PGB from an AdS background.
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A Fermionic self-energies

The fermionic self-energies $\Pi_i(p)$ and $M_i(p)$ of section 3.1 are easily computed in the 5D theory by taking the inverse of the propagator of the corresponding field with endpoints attached to the UV-brane. A technical complication comes from the mixing among different 5D bulk multiplets due to the IR boundary masses $\tilde{m}_u$, $\tilde{M}_u$ of eq. (29). Resumming all possible insertions of the IR mass mixing terms and omitting for simplicity boundary kinetic terms, we obtain

$$\Pi_{q_L}(p) = \frac{k}{p^2} \frac{1}{G_{(++)}^{R_q}(L_0, L_0)} \left\{ 1 - \frac{G_{R_q}^{(++)}(L_0, L_1)G_{R_q}^{(++)}(L_1, L_0)}{G_{R_q}^{(++)}(L_0, L_0)} \times \frac{\tilde{m}_u^2 p^2 (kL_1)^2 G_{L_q u}^{(-+)}(L_1, L_1)}{1 - \tilde{m}_u^2 p^2 (kL_1)^2 G_{R_q}^{(-+)}(L_1, L_1)G_{L_q u}^{(-+)}(L_1, L_1)} \right\},$$

$$\Pi_{Q_L}(p) = \frac{k}{p^2} \frac{1}{G_{(+-)}^{R_Q}(L_0, L_0)} \left\{ 1 - \frac{G_{(-)}^{L_Q}(L_0, L_1)G_{R_Q}^{(-)}(L_1, L_0)}{G_{R_Q}^{(-)}(L_0, L_0)} \times \frac{\tilde{M}_u^2 p^2 (kL_1)^2 G_{R_{Q u}}^{(+)}(L_1, L_1)}{1 - \tilde{M}_u^2 p^2 (kL_1)^2 G_{L_{Q u}}^{(+)}(L_1, L_1)G_{R_{Q u}}^{(+)}(L_1, L_1)} \right\},$$

$$\Pi_{q_R}(p) = \frac{k}{p^2} \frac{1}{G_{(++)}^{L_q u}(L_0, L_0)} \left\{ 1 - \frac{G_{L_q u}^{(++)}(L_0, L_1)G_{L_q u}^{(++)}(L_1, L_0)}{G_{L_q u}^{(++)}(L_0, L_0)} \times \frac{\tilde{m}_u^2 p^2 (kL_1)^2 G_{R_q}^{(-+)}(L_1, L_1)}{1 - \tilde{m}_u^2 p^2 (kL_1)^2 G_{L_q u}^{(-+)}(L_1, L_1)G_{R_q}^{(-+)}(L_1, L_1)} \right\}.$$
The leading one-loop potential generated by these extra fermions, the analog of eq. (21), is of the form \( \Delta V_{\text{heavy}} \). For example, one could introduce two extra elementary fermions in the 4D theory, \( \psi \), between two points \( z, z' \) along the fifth dimension (see for example [3, 26]):

\[
S(p, z, z') = (k^2 z z')^{5/2} \left[ \hat{p} + \gamma^5 \left( \partial_z + \frac{1}{2z} \right) + \frac{c}{z} \right] \left[ P_R G_R(p, z, z') + P_L G_L(p, z, z') \right],
\]

(58)

where \( P_R, L = (1 \pm \gamma^5)/2 \). We have also defined

\[
\tilde{G}_{R,L}(z, z') = \left[ \pm \partial_z + \frac{(c \pm 1/2)}{z} \right] G_{R,L}(z, z').
\]

(59)

B A deformation of the Higgs potential

There are many possible sources of new contributions to the Higgs potential which can lead to the form of eq. (25). For example, one could introduce two extra elementary fermions in the 4D theory, \( \psi^{(T)}_L \) and \( \psi^{(S)}_R \), the first transforming as a triplet under SU(2)_L, the latter as a singlet. They can acquire a mass \( \sim \mu_{IR} \) before EWSB by marrying some massless composite fermion, thus becoming heavy. The leading one-loop potential generated by these extra fermions, the analog of eq. (21), is of the form \( \Delta V = \tilde{a} \sin^2 h/f \pi + \tilde{b} \sin^4 h/f \pi \). This 4D modification of the minimal model can be obtained in the 5D theory by adding two bulk fermions in the antisymmetric representation of SO(5) with \( (\pm, \mp) \) boundary conditions for the various components.
In what follows we describe how the mechanism of ref. [3] can be applied to the present model to generate a new contribution at tree level. The idea is to introduce new scalar operators $O_{\varphi_i}$ of dimension 2, coupled to external scalar sources $\varphi_i$ according to

$$\Delta L = k \varphi_i O_{\varphi_i} - \frac{1}{2} m_i^2 \varphi_i^2,$$

with $k \sim M_{Pl}$. The scalar sources do not need to be dynamical, but just auxiliary fields that parametrize the breaking of the CFT global symmetry ($i$ labels the different components of an SO(5) representation). Integrating out these scalars one obtains a marginal deformation of the CFT:

$$\Delta L = \lambda_i O_{\varphi_i}^2,$$

where the coupling $\lambda_i$ runs logarithmically with energy. At the scale $k$, $\lambda_i(k) = k^2/(2m_i^2) \sim 1$ for $m_i \sim k$. This SO(5)-breaking contribution will generate new terms in the PBG potential.

Let us consider for example the case in which the fields $\varphi_i$ fit into a traceless symmetric representation of SO(5), a $14_0$ of SO(5) $\times$ U(1)$_{B-L}$. A $14$ decomposes as $14 = 9 + 4 + 1$ under SU(2)$_L \times$ SU(2)$_R$. The different components can have different mass terms, $m_i^2 (i = 9, 4, 1)$, because SO(5) is not a symmetry of the external sector. We are considering, for simplicity, the case in which these masses respect an SO(4) symmetry. Integrating out the scalars at tree-level, one obtains the following contributions to the Higgs potential:

$$\Delta V(\Sigma) = -\frac{\lambda^2}{2} m^4 \rho \left[ \frac{1}{m_1^2} (\Sigma \tilde{T}^0 \Sigma^T)^2 + \frac{1}{m_4^2} (\Sigma \tilde{T}^a \Sigma^T)^2 + \frac{1}{m_9^2} (\Sigma \tilde{T}^n \Sigma^T)^2 \right],$$

where $\tilde{T}^{A=0,a,n}$ are a set of traceless symmetric matrices given in appendix C. The coupling $\lambda_\varphi$ runs logarithmically with energy. Using eq. (6) in eq. (62) one has:

$$\Delta V(h) = -\lambda^2 m^4 \rho \left[ \left( \frac{5}{8} \frac{1}{m_1^2} - \frac{1}{m_4^2} + \frac{3}{8} \frac{1}{m_9^2} \right) \sin^4 \frac{h}{f_\pi} + \left( \frac{1}{m_4^2} - \frac{1}{m_1^2} \right) \sin^2 \frac{h}{f_\pi} \frac{2}{5} \frac{1}{m_1^2} \right].$$

The overall coefficient $\lambda_\varphi$ can be easily of loop size if the coupling of $\varphi_i$ to the CFT is not exactly maximally relevant. It is possible to generate only a $\sin^4 h/f_\pi$ term if the $9$ is the only external source present ($m_{1,4} \rightarrow \infty$). To have a positive coefficient, $m_9^2$ must be negative, which implies that the $9$ should not propagate (otherwise we have a tachyon). It must be considered an auxiliary field, like a D-term in supersymmetry. Another possibility is to assume $m_4 \simeq m_1$ as the result of some symmetry of the UV physics. The other particular form of the extra contribution to the potential mentioned in the text, $\Delta V = \xi (\sin^4 h/f_\pi - \sin^2 h/f_\pi)$, can be obtained if the $4$ is the only external source present ($m_{1,9} \rightarrow \infty$).
The 5D realization of this 4D mechanism follows straightforwardly through holography. An additional scalar field $\Phi$ of 5D mass $M_\Phi^2 = -4k^2$ lives in the bulk and transforms as a $\mathbf{14}$ of SO(5). If its singlet component under SO(4) has a tadpole on the IR-brane, the potential will be generated at tree level. Finally, dynamical (non-dynamical) fields of the 4D external sector, $\varphi_i$, correspond to the components of $\Phi$ on the UV-brane with a Neumann (Dirichlet) boundary condition.

C SO(5) generators

We collect here the SO(5) generators used in the text. A suitable basis for the vectorial representation is

$$T_{ij}^{aL,R} = -\frac{i}{2} \left[ \frac{1}{2} \epsilon^{abc} \left( \delta_i^b \delta_j^c - \delta_i^c \delta_j^b \right) \pm \left( \delta_i^a \delta_j^4 - \delta_j^a \delta_i^4 \right) \right]$$

$$T_{ij}^a = -\frac{i}{\sqrt{2}} \left( \delta_i^a \delta_j^5 - \delta_j^a \delta_i^5 \right)$$

where $i, j = 1, \ldots, 5$ and $T^a (a = 1, \ldots, 4)$, $T^{aL,R} (a_{L,R} = 1, 2, 3)$ are respectively the generators of SO(5)/SO(4) and SO(4)$\sim SU(2)_L \times SU(2)_R$. The spinorial representation of SO(5) can be defined in terms of the Gamma matrices

$$\Gamma^a = \begin{bmatrix} 0 & \sigma^a \\ \sigma^{a\dagger} & 0 \end{bmatrix}, \quad \Gamma^5 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad \sigma^a = \{\bar{\sigma}, -i1\},$$

as

$$T^{aL,R} = \frac{i}{2\sqrt{2}} \left[ \frac{1}{2} \epsilon^{abc} [\Gamma^b, \Gamma^c] \pm [\Gamma^a, \Gamma^4] \right], \quad T^a = \frac{i}{4\sqrt{2}} [\Gamma^a, \Gamma^5],$$

so that

$$T^{aL} = \frac{1}{2} \begin{bmatrix} \sigma^a & 0 \\ 0 & 0 \end{bmatrix}, \quad T^{aR} = \frac{1}{2} \begin{bmatrix} 0 & 0 \\ 0 & \sigma^a \end{bmatrix}, \quad T^a = \frac{i}{2\sqrt{2}} \begin{bmatrix} 0 & \sigma^a \\ -\sigma^{a\dagger} & 0 \end{bmatrix}.$$ (67)

Finally, we give the expression of the orthogonal basis of traceless symmetric 5x5 matrices used in the previous appendix. Denoting with $\hat{T}_0^a, \hat{T}_a (a = 1, \ldots, 4), \hat{T}_n (n = 1, \ldots, 9)$ respectively the $1, 4$ and $9$ representations of the SO(4) subgroup, one has:

$$\hat{T}_0_{ij} = \frac{1}{2\sqrt{5}} \text{diag}(1,1,1,1,-4), \quad \hat{T}_a_{ij} = \frac{1}{\sqrt{2}} (\delta_i^a \delta_j^5 + \delta_j^a \delta_i^5),$$

$$\hat{T}_n_{ij} = \left\{ t_{ij}^{bc} ; \frac{1}{\sqrt{2}} \text{diag}(-1,0,1,0,0) ; \frac{1}{2\sqrt{3}} \text{diag}(1,1,1,-3,0) ; \frac{1}{\sqrt{6}} \text{diag}(-1,2,-1,0,0) \right\},$$

where $t_{ij}^{bc} = (\delta_i^b \delta_j^c + \delta_j^b \delta_i^c)/\sqrt{2}, c > b$ $(b, c = 1, \ldots, 4)$ is a collection of six matrices.
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