A Review of the Discrete Element Method Application on Concrete Materials

F Ma’arif¹,², Z Gao¹ and F Li¹

¹Department of Transportation and Civil Engineering, Beihang University, Beijing - People's Republic of China
²Department of Civil Engineering, Universitas Negeri Yogyakarta - Indonesia
Corresponding author: faqihmaarif07@buaa.edu.cn

Abstract. This paper describes an attempt to analyze each methodology and bring it all together in one place for crossreferencing expert analysis. The advantage of using DEM compared to continuum-based techniques is in cracking and fragmentation starting and spreading because the DEM system is naturally disconnected. Concrete is a non-homogeneous composite material with significant heterogeneity of quasi-brittle characters. Structural failure behavior in concrete usually preceded by initial cracking and influenced by the mesoscale structure, interface between aggregate and mortar matrix, especially under complex pressure conditions. The ability of the DEM to produce 3-phase material representation in concrete will significantly depend on the application of the contact model between particles that have bound during the simulation. The main development area in this paper is related to stages (particle contact, contact model), parameters, and implementation that are entirely established in the simulation phase of materials. It will assist future researchers in improving analysts when modeling concrete to select appropriate methods and procedures, and to identify critical parameters for specific applications.

Keywords: Concrete, Discrete Element Method

1. Introduction

The discrete element method (DEM) was first developed by [1-2] for the analysis of rock mechanics problems. The basic of DEM using spherical or cylindrical particles was proposed by [1] to investigate the constitutive laws for soil. In [2] show that DEM has better results in modeling material than finite element method. Currently, the Discrete Element Method has been widely used to investigate the micromechanical features of granular assemblies and the field of civil engineering related to structural behavior [3-4] and material [5-6].

Further development of DEM [7] produced new work by [8-11], which became the basis for the progress of the DEM in concrete technology. In [12-13], who introduced the concept of friction relations in brittle materials, also declared Identification and Validation of a DEM for Concrete. The idea of particle contact model [14-15], composite materials [16-19], reported Fracture in concrete. The concrete tensile and compressive relationships at the mesoscale [20-21] for the effect of specimen size as well as specific modeling using the RBSM method [22-28].

The main problem in DEM modeling is the use of tens of thousands of particles, using simulations that require thousands of steps of analysis time. In the two-dimensional analysis, particles modeled as simple or geometric discs, which are assumed to be circular. This selection of circular particles has the advantage of simplifying computing but also a tendency to roll. The strength of reasonable force does not contribute to the moment, because the contact force will invariably pass through its particle centroid.
In solid modeling, it is difficult to declare the real material simulation because the material represented by the continuum converted to discrete [29]. Based on the mesoscale concept, the concrete considered as a composition of a continuous binding matrix (usually hardened cement paste) with inserted aggregates (such as crushed stone, gravel, or sand) and air holes (pores). For modeling, the interface transition zone (ITZ) between the aggregate and the matrix plays a special role.

The algorithm has an essential role because it will affect the sensitivity of time analysis or computation. There are at least two main groups in determining the algorithm, including hierarchical and the Flat Algorithm [30]. Furthermore, algorithms in particle contact simulation consist of three types [30-31], namely the sweep and prune algorithm, neighboring cell, nearest neighbor. In the sweep and prune algorithm, the adjacent aligned axis squares become overlapping particles operating areas, if and only if the particles overlap along all axes. In contrast, the key to neighboring cells or nearest neighbor (grid algorithm) is a high-speed search for neighbors to achieve particle complexity.

However, the size of the simulation and time determines the accuracy and stability of analysis. The small particle sizes produce a consumes of energy and unnecessary. Meanwhile, if the size and selection of particles too large, it contributes to the wrong simulation and causes instability in the analysis or simulation.

Finally, a cross-analysis carried out to find the best formulation for the contact model of particles and elements. The new alternative methods of contact detection considered and the opportunities for the constitutive materials presented to develop in more detail.

2. Discrete Element Method

2.1 Particle modelling in concrete

In DEM analysis, spherical particles frequently used because of the smaller internal friction angles compared to the original aggregate material. In [1] has introduced algorithms for calculating ellipse-ellipse and ellipse wall particles. As preliminary results, researchers encoded simulations in two-dimensional particles and namely BALL and TRUBAL.

Particle packing methods for concrete have also been developed by [33]. Simplified simulations of crushed stone for aggregate concrete based on two parameters, namely: the size of the filter and the maximum size of the required surface element. Fluvial gravels and broken stones were representing round and angular shapes. The ellipsoids were chosen to represent gravel particles, while polyhedrons used to simulate crushed grains.

Particle modeling for concrete was carried out by [34]. The concrete material produced by using ball-shaped particles in which connected with cementation bonds of beam elements. It is assumed that the beam elements relating to the centers of adjacent spherical particles. Meanwhile, proposed by [35] found a mathematical model for composite materials (coarse and fines aggregate, matrix and ITZ). Other parameters related to calculating the value of the coordinates, speed, acceleration, and force have generated from the mathematical model.

In general, particle simulation and analysis of the concrete model more complicated than a spherical shape. Three particle shapes, namely round, ellipse, and polyhedron, have their respective characteristics. Spherical particles are the best alternative because of the simplicity of the calculation. Whereas, elliptical particles have a degree of freedom in simulating different particle shapes even though hard and complex polygon shapes of represented mathematically. The test results show that to represent of concrete aggregates, extra energy is needed by sticking to conventional algorithms.

New algorithmic approaches developed to get a realistic particle model. The particle model, which is substituted by an element known as the Rigid Body Spring Model, has been carried out by [18] [36-42] and recently by [43] and [44]. A systematic approach assumes that there are three phases of material, namely mortar, site, and aggregate. The failure occurred when spring elements are broken, which indicates that a crack has developed in the concrete. On the other hand, a mathematical solution presented by [45], who considers that to model bonds between particles in concrete, it represented by the Timoshenko Beam Bonded-Contact Model (TBBM).

2.2 Contact detection
Contact detection is the consuming part of analyzing the DEM. In general, contact detection divided into two kinds of particles, namely soft and hard particles. Whereas, the contact detection in this paper consists of three types, namely the nearest neighbor, neighbor cell, and sweep and prune classified as soft and hard particles. Sweep and prune is a contact detection which generally used for various analyzes, including concrete materials [6] [30] [46].

The general steps of the sweep and prune are as follows: (1) create a bounding box (parallel to the axis) for each particle while keeping in mind the coordinates of the extreme box projection on each axis. (2) construct a list sorted from the bounding box segment (endpoint of pair) for each dimension that has projected later. (3) sweep through each listing, pinpoint where the boxes are overlapping, as contact records only exist if the bounding boxes overlap in all directions of the axis.

In YADE [50], the basic code is written as follows:

**Bo1 functors**

O.engines=[..., InsertionSortCollider([Bo1_Sphere_Aabb(), Bo1_Facet_Aabb()]), ... ]

**Ig2 functors**

InteractionLoop([Ig2_Sphere_Sphere_Dem3DofGeom()],[],[])

**Ip2 functors**

Material combinations within the simulation

Ip2_FrictMat_FrictMat_FrictPhys

IPhys accepted by the constitutive law (Law2 functor)

Law2_ScGeom_FrictPhys_CundallStrack

Example for concrete materials uniaxial test by YADE:

O.engines=[
  ForceResetter(),
  InsertionSortCollider([Bo1_Sphere_Aabb(aabbEnlargeFactor=intRadius, label='is2aabb'),], verletDist=.05*sphereRadius),
  InteractionLoop(
    [Ig2_Sphere_Sphere_ScGeom(interactionDetectionFactor=intRadius, label='ss2sc')],
    [Ip2_CpmMat_CpmMat_CpmPhys()],
    [Law2_ScGeom_CpmPhys_Cpm()]
  ),
]
Further research needs to develop the model of the bonding between particle contacts, which is a representation of cementitious materials. Because the sweep and prune contact model uses the assumption [1], the innovation of the contact model with cementitious materials can refer to [45]. Modified algorithms would be combined using dynamic relaxation procedures.

2.3 Contact Model

Contact models in concrete materials refer to two main methods, including the Particle Contact Method and the Rigid Body Spring Model (RBSM). A simple contact model has been developed by [1] consists of forces in the normal direction (springs and dashpots) and viscoelastic tangential forces with friction restrictions using Mohr-Coulomb's law.

Meanwhile, the RBSM model was successfully introduced by [18] [36-37] [23-24] [26-28] [44]. The Rigid Body model presented by [25] is an improved representation of the previous method by assuming that concrete consists of three composites phases: (1) mortar as a continuous phase; (2) coarse aggregate, and interface is considered to zero.

\[
P_{ij} \left( D < D_{\text{max}} \right) = P_k \left( 1.065d^{0.5} - 0.053d^4 - 0.012d^6 - 0.0045d^8 + 0.0025d^{10} \right)
\]

(1)

Where \( d = D_{ij}/D_{\text{max}} \); \( D_{\text{max}} = \) maximum size of the aggregate; \( P_k = \) aggregate volume

Equation (2) is a fracture criterion in concrete. It has considered the failure due to compression, stress, and shear, which applies to mortar springs while the interface failure at meso-level uses the Mohr-Coulomb theory as in equation (3) (4).

\[
\frac{r_m}{f_m} = 0.092 + 1.181 \frac{\sigma_m}{f_m} - 0.964 \left( \frac{\sigma_m}{f_m} \right)^2, \quad \left( \frac{\sigma_m}{f_m} \leq 0.6 \right)
\]

(2)

\[
\frac{\sigma_m}{f_m} = -0.568 + 3.406 \frac{\sigma_m}{f_m} - 2.838 \left( \frac{\sigma_m}{f_m} \right)^2, \quad \left( 0.6 < \frac{\sigma_m}{f_m} \leq 1 \right)
\]

(3)

\[
\frac{r_s}{f_m} = 0.084 + \tan 35^\circ \frac{\sigma_m}{f_m}, \quad \left( -0.04 < \frac{\sigma_s}{f_m} < 1.0 \right)
\]

(4)

Where: \( k_n, k_s \) springs and \( D_n, D_s \) are the stiffness and deformation of normal and shear springs, respectively; \( l \) is the length between two elements; and, \( h \) and \( h_j \) are the length of the perpendicular line from the centers of the gravity of elements \( i \) and \( j \), respectively.

![Figure 2. normal and tangential component [45]](image1)

![Figure 3. Representative length of interfacial spring; and Connection element [25]](image2)

Gu [25] assumes that rotational displacement and rotation springs in static problems do not consider because of very small. The damping coefficient \( c_n \) and \( c_s \) used to eliminate energy when solving static problems using dynamic relaxation methods. Stress will dissipate when the elements broken, in case the contact element become to initial crack, compressive and sliding pressure will be generated at the general limit by the contact.

RBSM for concrete is one of the new resolutions in the three-phase analysis of composite materials. Hence, difficulties in particle analysis, represented by the contact area of elements to the modeling of
concrete. The three-phase materials in previous researchers using the cohesive particle contact approach (soft sphere and hard-sphere) has resolved by the RBSM method.

Simulation results show that significant computational savings are available for RBSM simulation compared to sphere particle contacts. The proposed model that developed is the addition of roller and slider in normal and tangential directions refers to [45]. It is necessary even though the value is small. It considered having stress that may affect the behavior of spring elements.

2.4 Parameter Calibration and implementation
The calibration of input property parameters is essential aspects in DEM. The multi-level parametric sensitivity method has been used by [47] to investigate the effect of input parameters on DEM. The level of sensitivity parameters in the DEM was analyzed using the Parameter Sensitivity Analysis Method (PSAM). PSAM has introduced three stages, including (1) Parameter estimation, (2) principal component analysis (PCA), (3) Parameter sensitivity matrix analysis.

In parameter estimation, the empirical models are algebraic equations in the structural system, and the estimated parameters are stated as follows:

\[ y_i = f_i(\xi_i, P_k) + e_i \]  \hspace{1cm} (5)

where: \( y_i \) is the model response, \( \xi_i \) is the process setting (input variable), \( P_k \) is the model parameter and \( e_i \) is the model error, where \( i \leq n \).

The main purpose of the PCA method is to investigate the relationship between mass model responses and sensitivity of parameter. The parameter sensitivity of the matrix analysis method is calculated by [32]:

\[ \text{norm} (S_k) = \sqrt{\sum_i \left\{ \frac{\delta f}{\delta P_k} \right\}^2} \]  \hspace{1cm} (6)

where the norm \( (S_k) \) is the parameter sensitivity of matrix for parameters 1, 2, ..., \( k \) and \( i \leq nm \).

This method explicitly provides a representation of sensitivity arranged for specific parameters in the dataset. For the \( k \) parameter system, a vector of norm values \( (S) \) k of size \( k \) will be generated, and the vector compared. In the analysis of the parameter sensitivity matrix, each sensitivity parameter size \( nm \) also compared with other columns in the system. The value calculated following equation [47]:

\[ (CC)_{k1,k2} = \frac{(S^T S)_{k1,k2}^{-1} \delta^2 f}{(S^T S)_{k1,k1} \frac{\delta^2 f}{\delta P_k \delta P_k}} \]  \hspace{1cm} (7)

where: CC is the coefficient of cross-correlation (-1 <CC <+1), and superscript T shows that the transpose matrix, \( k1 \), and \( k2 \) are the two parameters considered. Strong correlations between model parameters shown with CC values close to -1 or +1.

Due to geometrical irregularities in modeling [48], the explicit relationship between local and macroscopic parameters are difficult to apply. Therefore, procedures are needed to perform the desired behavior in several models of calibration. Several authors have referenced to this test, such as [14] [36] [45] [48-49], who apply this method to calibrate the problems of their models, especially in cementitious materials. The researcher uses a statistical approach to define the parameters of the mechanical properties of concrete.

Finally, there are several references for the calibration parameters of concrete materials. The calibration parameter method is different for each condition. The development of multi-level PSA methods has the chance to be investigated, especially for the implementation of specific materials.

2.5 Constitutive Model
The constitutive model in the DEM represents material responses to various conditions, which provide stress-strain relationships for formulating governing equations, along with conservation laws and kinematic particle relationships.

The constitutive law in the normal force direction inspired by the mechanical damage and defines the normal stress $\sigma_N$ into the normal strain section $\varepsilon_N$.

$$\sigma_N = [1 - \omega H(\varepsilon_N)]\bar{E}\varepsilon_N$$  \hspace{1cm} (8)

Where $E$ is the normal modulus, and the elastic slope is in the normal direction. $\omega \in [0, 1]$ is the damage variable that affects the stiffness for unloading and reloading in the tensile regime.

The function of Heaviside $H(\varepsilon_N)$ is to deactivate the effect of damage during compressive stress, which is physically related to crack closure. $\omega$ is the damage evolution function defined in terms of maximum normal strain $\varepsilon$ and material parameters $\varepsilon_0$ and $\varepsilon_f$.

Other researchers such as [35] developed the constitutive relationship of the elasto-viscoplastic damage model by referring to the Perzyna formulation [40-42].

$$\{\Delta\sigma\} = [C](\{\Delta\varepsilon\} - \{\dot{\varepsilon}^{vp}\}\Delta t)$$  \hspace{1cm} (9)

Where $\Delta t$, $\{\Delta\sigma\}$, $\{\Delta\varepsilon\}$, $\{\dot{\varepsilon}^{vp}\}$ and $[C]$ denote the time stepping length, stress increment, strain increment, viscoplastic strain rate, and elasticity matrix, respectively.

Concrete damaged gradually at the peak condition, which indicates degradation of mechanical properties (modulus of elasticity, strength, and an increase in the Poisson ratio). The material in the constitutive model of elasto-viscoplastic represent of scalar damage variable $D$ is 0 (for undamaged), and 1 (for damaged). The algorithm proposed by [35] has succeeded in analyzing high volume content, various shapes, and actual size distribution of aggregate and concrete in mesoscale (three-phase materials).

Besides, the modified constitutive model proposed by [25] is emphasizing the spring element. The amount of spring mortar stiffness based on continuous material theory and field stress conditions as follows:

$$k_{n,s} = \frac{E_e}{(1-\nu_e^2)} \frac{l}{h_i+h_j}$$  \hspace{1cm} (10)

$$k_{s,s} = \frac{E_e}{2(1+\nu_e)} \frac{l}{h_i+h_j}$$  \hspace{1cm} (11)

Where $k$, $s$, and $ks$, $s$ is normal, and shear spring stiffness, respectively, and $E_e$ and $\nu_e$ are elastic modulus and Poisson mortar ratio at the mesoscopic level, respectively.

Shear and normal springs expressed to the constitutive model in figure 4 (before fracture). However, when the normal stress reaches the tensile or elemental strength, the shear stress, under a certain normal stress level, the spring group will fail, respectively. The crack will start at the element boundary, and then the interaction of the element will change into a contact action.

Figure 4. Constitutive model for springs: (a) normal springs; and (b) shear springs [25]
Gu [25] present the constitutive models, which is can modeling concrete according to its nature. Other constitutive models that can be use include [45-46], who developed a constitutive approach for particles. Compared to the constitutive model for sphere particles in concrete, the use of the RBSM method is still relatively rare. However, the constitutive model approach based on RBSM has developed by various researchers, such as [18] [35-43].

Future research will develop for Inelastic contact law with cohesion, twist, and bending, dan Generalized Maxwell model of visco-elastoplastic interactions (two springs and two dashpots for each force component) in sphere particle model.

3. Future of Discrete Element Method
The advancement of particle analysis, the development of parallel DEM, and CFD analyzes, the study of the behavior of bulk materials such as mines, coal and soil are the way to open opportunities for new DEM methods.

Analytical methods developed to integrate DEM with various objectives to obtain calibration parameters, contact models, and particle modeling concerning complex behavior in the Discrete Element Method. Many research interest has led to simulations of bulk materials for industrial uses that model the flow of solid fluids at the particle scale. That is a definite trend in the development of discrete analysis that is processed by software systems using advanced algorithms.

4. Conclusion
The conclusions that can be drawn from the discussion are as follows:

a) The accurate results achieved if a set of parameter values for analysis are following the review of the idealization of concrete forming composite materials.

b) Contact detection is one of the essential components in the discrete element analysis because it consumes the most significant computational time. It is around 76.6% of the overall DEM analysis (force calculation, update position, etc.).

c) Particle size, shape distribution, macro-micro relationships, compressive, and tensile rupture axial tests are one of the main input parameters identified in this paper. The PSA method needs to developed further to reveal the response of the bulk model and parameter sensitivity.

d) The behavior of concrete is affected by its viscoplastic properties, and viscoelastic deviatoric used to inform the quantitative level of stress and tension.

e) Advanced adjustments are needed to achieve more accurate results. Due to computational limitations, the dimensions of particles increased with a simpler form. As a consequence, the condition of the property measured sometimes does not produce satisfying behavior.

5. References
[1] P A Cundall and O D L Strack 1979 A Discrete Numerical model for Granular Assemblies Geotechnique 29 no 1 pp 47–65
[2] P A Cundall and R D Hart 1992 Numerical Modelling of Discontinua Engineering Computations emerald insight 9 no 2 pp 101–113
[3] J Tejchman, J Kozicki, D Les, and L Widulin 2011 Discrete Simulations of Shear Zone Patterning in Sand in Earth Pressure Problems of a Retaining Wall International Journal of Solids and Structures 48 pp 1191–1209
[4] P Taylor and J V Lemos 2007 Discrete Element Modeling of Masonry Structures International Journal of Architectural Heritage: Conservation, Analysis, and Restoration 1 pp 37–41
[5] Laurent, Daudeville, Yann Malécot 2011 Concrete structures Under Impact European Journal of Environmental and Civil Engineering Special Issue 15 pp 101 - 140
[6] V T Tran, F V Donze and P Marin 2010 Discrete Element Model of Concrete Under High Confining Pressure Proceedings of FraMCos International Association of Fracture Mechanics for Concrete and Concrete Structures no 1 pp 481–485
[7] M R Kuhn 1999 Structured Deformation in Granular Materials Mechanics of Materials 31 no 6 pp 407–429
[8] G A D Addetta, F Kun, and E. Ramm 2002 On the Application of a Discrete Model to the Fracture Process of Cohesive Granular Materials 4 pp 77–90
[9] S Hentz, L Daudeville, and F V Donzé 2009 Discrete Element Modeling of a Reinforced Concrete Structure Journal of the Mechanical Behavior of Materials 19 Pages 249–258
[10] T Bangash and A Munjiza 2003 Experimental Validation of a Computationally Efficient Beam Element for Combined Finite-Discrete Element Modelling of structures in Distress Computational Mechanics 30 no 5–6 pp 366–373
[11] P W Cleary 2004 Large Scale Industrial DEM Modelling Engineering Computations (Swansea, Wales) 21 no 2–4 pp 169–204
[12] S Hentz, L Daudeville, and F V Donzé 2004 Identification and Validation of a Discrete Element Model for Concrete Journal of Engineering Mechanics ASCE pp 709-719 10.1061/(ASCE) 0733-9399(2004)130:6(709)
[13] A S J Suiker, N A Fleck 2004 Frictional Collapse of Granular Assemblies Journal of Applied Mechanics ASME 71 pp350-358
[14] D André, I Iordano, C Jean-luc, and J Néauport 2012 Discrete Element Method to Simulate Continuous Material by Using the Cohesive Beam Model Comput. Methods Appl. Mech. Engrg. Elsevier BV 213–216 pp 113–125
[15] M Asadzadeh, A Soroush 2017 Macro and Micromechanical Evaluation of Cyclic Simple Shear Test by Discrete Element Method Particuology 31 pp 129–139
[16] W Leclerc 2017 Discrete Element Method to Simulate the Elastic Behavior of 3D Heterogeneous Continuous Media International Journal of Solids and Structures 121 pp 86–102
[17] P Grassl, David Gregoire, Laura Rojas Solano, and Gilles Piaudier-Cabot 2012 Meso-scale Modelling of the Size Effect on the Fracture Process Zone of Concrete International Journal of Solids and Structures 49 Issue 13 pp 1818-1827
[18] Kawai T 1978 New Discrete Model and Their Application to Seismic Response Analysis of Structures Nuclear Engineering and Design 48(1) pp 207-229
[19] Suchorzewski J, J Tejchman, and M Nitka 2017 Discrete Element Method Simulations of Fracture in Concrete Under Uniaxial Compression Based on its Real Internal Structure International Journal of Damage Mechanics 27 pp 1-30
[20] M Nitka, J Tejchman 2015 Modelling of Concrete Behaviour in Uniaxial Compression and Tension with DEM Granular Matter 17 pp 145–164
[21] Siniae 2017 Application of the Discrete Element Method for the Simulation of Size Effects in Concrete Samples International Journal of Solids and Structures 108 pp 244-253
[22] V T Tran, F V Donzé, P Marin 2011 A Discrete Element Model of Concrete Under High Triaxial Loading Cement & Concrete Composites 33 Issue 9 pp 936-948
[23] Nagai K, Sato Y, and Ueda 2004 Mesoscopic Simulation of Failure of Mortar and Concrete by 2D RBSM Journal of Adv Conc and Tech Japan Concrete Institute 2 no 3 pp 359-374
[24] Z Wang, F Lin, and X Gu 2008 Numerical Simulation of Failure Process of Concrete Under Compression Based on Mesoscopic Discrete Element Model Tsinghua Science and Technology 13 no SUPPL 1 pp 19–25
[25] X Gu, L Hong, Z Wang, and F Lin 2013 A Modified Rigid-Body-Spring Concrete Model for Prediction of Initial Defects and Aggregates Distribution Effect on Behavior of Concrete Computational Materials Science 77 pp 355–365
[26] L C Wang 2013 Meso-Scale Numerical Modeling of the Mechanical Behavior of Reinforced Concrete Members International Journal of Engineering and Technology 5 no 6 pp 680–684
[27] T Yagi, N Takeuchi, K Yamamura, and E Hamasaki 2013 Combined Method for Rigid Bodies-Spring Model and Discrete Element Method APCOM & ISCM 2013: 5Th Asia Pacific Congress on Comput Mech & 4Th Int. Symposium on Computat Mech pp 2–7
[28] Y Song, H Wang, and B Wang 2005 Simulation of the Behaviour of Fully-Graded Concrete at a Mesoscopic Level Based on a Rigid Body Spring Discrete Element Model Computational Methods and Experiments in Material Characterisation 51 pp 99–108
[29] Breugnot A, Gotteland P, Villard P, Garcin P 2010 Modelling of Block Impacts with a Combined Discrete-Continuum Approach *Proceedings of the 3rd Euro Mediterranean Symposium on Advances in Geomaterials and Struc Djerba Tunisia* 10–12 May pp 289–295

[30] Šmilauer, Václav 2016 *Cohesive Particle Model using the Discrete Element Method on the YADE Platform* Ph.D Thesis in Computational Mechanics Czech Technical University in Prague Faculty of Civil Engineering

[31] Hiroshi Mio, Atsuko Shimosaka, Yoshiyuki Shirakawa, Jusuke Hidaka 2006 Cell Optimization for Fast Contact Detection in the Discrete Element Method Algorithm *Advanced Powder Technology* 18 no 4 pp 441–453

[32] A Peña, H J Herrmann, P G Lind 2008 Modeling Slow Deformation of Polygonal Particles Using DEM Particuology 6 Issue 6 pp 506-514

[33] Huan H E 2010 *Computational Modelling of Particle Packing in Concrete* (Master of Engineering aan de Beijing University of Technology: Geboren te Zhuji Zhejiang Province, P R China)

[34] M Obermayr, K Dressler, C Vrettos, P Eberhard 2013 A Bonded-Particle Model for Cemented Sand Computers and Geotechnics *Computers and Geotechnics* 49 pp 299–313

[35] Xu Yi, Shenghong Chen 2016 A Method for Modeling the Damage Behavior of Concrete with a Three-Phase Mesosoucture *Construction and Building Materials* 102 pp 26–38

[36] F Camborde, C Mariotti, F V DonzêA 2000 Numerical Study of Rock and Concrete Behaviour by Discrete Element Modeling *Computers and Geotechnics* pp 225-247

[37] G A D Addetta, F Kun, and E Ramm 2002 On the Application of a Discrete Model to the Fracture Process of Cohesive *Granular Materials* pp 77–90

[38] K Nagai, Y Sato, T Ueda, N Š, and H Nžÿ 2004 Mesoscopic Simulation of Failure of Mortar and Concrete by 2D RBSM *Journal of Adv Conc Tech* 2 no 3 pp 359-374 JCI

[39] K Nagai, Y Sato, T Ueda, N Š, and H Nžÿ 2004 Analytical Study on Influence of Mortar-Aggregate Interface Character on Concrete Strength by RBSM *Journal of Adv Conc Tech* 26 no 2 pp 3–8 Technical Paper

[40] Z Wang, F Lin, and X Gu 2008 Numerical Simulation of Failure Process of Concrete Under Compression Based on Mesoscopic Discrete Element Model *Tsinghwa Science and Technology* 13 no SUPPL 1 pp 19–25

[41] X Gu, L Hong, Z Wang, and F Lin 2013 A Modified Rigid-Body-Spring Concrete Model for Prediction of Initial Defects and Aggregates Distribution Effect on Behavior of Concrete *Comput Mat Sci* 77 pp 355–365

[42] T Yagi, N Takeuchi, K Yamamura, and E Hamasaki 2013 Combined Method for Rigid-Bodies-Spring Model and Discrete Element Method *Apcom & ISCM 2013: 5Th Asia Pacific Congress on Comput Mech & 4Th Int Symposium on Computational Mechanics* (Singapore: APCOM & ISCM) pp 2–7

[43] X Gu, Jia J, Wang Z, Hong Li, Lin F 2013 Determination of Mechanical Parameters for Elements in Meso-Mechanical Models of Concrete *Front Struct Civ Eng* 7 pp 391–401

[44] J Y Jia and X Gu 2017 A 3D Mesoscopic Model for Simulating Failure Process of Concrete Based on Discrete Element Method vol 188 *Proceedings of the 7th Int Conf on Discrete Element Methods, Springer Proceedings in Physics*

[45] N J Brown, J F Chen, J Y Ooi 2014 A Bond Model for DEM Simulation of Cementitious Materials and Deformable Structures *Granular Matter Springer-Verlag Berlin Heidelberg* 16 (3) pp 299–311

[46] J Kozicki 2008 *YADE-OPEN DEM*: An Open-Source Software Using a Discrete Element Method to Simulate Granular Material *International Journal for Computer Aided Engineering and Software* 26 no 7 pp 786-805

[47] Z Yan, S K Wilkinson, E H Stitt, M Marigo 2015 Discrete Element Modelling (DEM) Input Parameters: Understanding Their Impact on Model Predictions Using Statistical Analysis *Comp Part Mech* 2 pp 283–299

[48] S A Magnier and F V Donze 1998 Numerical Simulations of Impacts Using a Discrete Element Method *Mechanics of Cohesive-Frictional Materials* 3 pp 257-276
[49] D André, I Iordano, J L Charles, J Néauport 2012 Discrete Element Method to Simulate Continuous Material by Using the Cohesive Beam Model *Applied Mechanics and Engineering* **213** pp113–125

[50] V Šmilauer, A Gladky, J Kozicki, C Modenese, J Stránský 2010 *Yade, Using and Programming*. In Yade Documentation (V. Šmilauer, ed.), The Yade Project, 1st ed.

**Acknowledgments**
The Financial Supports from the National Natural Science Foundation of China (11472029), (11872092) and Beihang University, are sincerely acknowledged by the authors.