Theoretical description of the motion of a material particle in pan vibrating batchers for agricultural purposes

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Abstract. One of the main tasks of which is to ensure sustainable feed production using highly efficient resource-saving machines and technologies. It is known that it is recommended to feed agricultural animals and poultry in the form of feed mixtures, balanced in composition. Feed grain can be processed into compound feed, developing its compound feed production in places of direct consumption. In the technology of compound feed preparation, one of the most important links is the dosing process, subject to special requirements for the accuracy of the input of components to obtain a homogeneous feed mixture. It is recognized to consider the use of vibration technologies and machines as promising in the field of dosing, allowing to achieve significant results in improving quality indicators. The paper proposes a theoretical description of the motion of a material particle through vibrating batchers, which can be both single-component and multicomponent. A differential equation for the relative motion of a particle along an inclined plane performing longitudinal nonharmonic oscillations is obtained. The article presents theoretical trajectories of changes in the displacement, velocity, and acceleration of a particle for one period of oscillation of the tray by a vibrating batcher at various values of the generalized coefficient \( K \), obtained on a computer from the results of numerical solutions.

1. Introduction

The national project of Russia in the field of agriculture provides for the intensive development of the livestock sector. One of the main tasks of which is to ensure sustainable feed production using highly efficient resource-saving machines and technologies.

It is known that it is recommended to feed agricultural animals and poultry in the form of feed mixtures, balanced in composition. Feeding grain fodder in the form of turtle is ineffective and economically inexpedient. Complete feed, balanced in basic nutritional elements, microelements, and vitamins are 25–30 % more effective than conventional grain feeds [1].

Feed grain can be processed into compound feed, developing its compound feed production. This allows you to reduce the cost of purchasing raw materials, their transportation, more efficiently use grain fodder, purchased expensive protein and vitamin-mineral components, and continuously provide collective, peasant (farm) farms with their compound feed. Therefore, the production of compound feed in places of direct consumption becomes a condition for the profitable management of the livestock industry.
The cost of the prepared compound feed depends on the correct construction of the technological process, the choice of working equipment, its configuration in the line, and the clarity of the work of the constituent mechanisms.

2. Results
At the Department of Agroengineering of the Omsk State Agrarian University, samples of new feed preparation machines of vibration and shock operating principles have been created [2]. They can be used as separate independent machines and equipment and fit well into the proposed technological scheme of a feed mill for the preparation of bulk feed mixtures (feed). This paper proposes a theoretical description of the movement of a material particle in tray vibrating batchers, which can be both single-component [3–5] and multicomponent [6, 7]. The peculiarity of the proposed tray vibration batchers is that the working body (tray), performs longitudinal oscillations according to an inharmonic law, in which the movement of feed particles (ingredients) occurs at a constant average speed, which is the technological basis for obtaining a small error in dosing.

It is fairly noted that “the problem of the motion of a material point on a vibrating rough surface plays no less role in the theory of vibrational displacement than the equations of motion of an oscillator in the theory of linear oscillations” [8]. When determining the law of motion of bulk feed in a tray vibrating meter, the following assumptions were made:
- A material particle moves along the load-carrying body (tray) of the vibration dosing device.
- Air resistance has no significant effect on feed movement.

Theoretical model is reduced to the analysis of the motion of a material particle lying on a rough surface, which is subjected to kinematic vibration excitation, obeying the law $\Phi = A(\varphi)$:

$$\overline{A}(\varphi) = \left\{ \begin{array}{l}
\frac{R^2}{\sin^2 \varphi_1} \sin(\varphi - \varphi_1) + R_2^2 - S^2 \sin^2 (\varphi - \varphi_1) \end{array} \right\}^\frac{1}{2}, \quad \text{at } \varphi_1 < \varphi < \varphi_2, \quad \frac{R}{\sin \varphi}, \quad \text{at } \varphi_2 < \varphi < \varphi_3. \quad (1)$$

The material particle of bulk feed is in equilibrium under the action of the following forces (Fig. 1):
1. Weights $P = m \cdot g$
2. Normal reaction force of the plane of the tray $N = P \cos \alpha$.
3. Friction forces described by the Amonton – Coulomb law:
   $$F_{FR} = \{ fN \sign \nu, \nu \neq 0 \}$$

**Figure 1.** Scheme of forces acting on a food particle when it moves along an inclined plane, performing longitudinal nonharmonic oscillations
Although it is known that the actual dependences of the sliding friction force on the value of the relative velocity observed in practice are of a more complex nature and can differ significantly from the Coulomb approximation [9].

Nevertheless, following the established tradition in solving dynamic problems and considering that the main interest is the averaged parameters of the vibration dosing process, we use the concept of the static friction characteristic in relation to our scheme of a tray (single and multicomponent) vibration batcher, which makes it relatively easy to obtain general and visual results.

From the dynamics of relative motion, it follows that when a material particle of mass $m$ moves along a vibrating plane, a transfer force of inertia acts, which is equal to the product of the mass and the acceleration of the plane (the tray of the vibrating batcher).

The inclined plane performs longitudinal nonharmonic oscillations according to the law described by equation (1). Obviously, in this case, the movement of the feed particle occurs without separation from the surface of the tray of the vibrating meter and coincides with the direction of the oscillations.

Based on the foregoing, we will compose the differential equation of the relative motion of the particle in the projection on the $x$-axis in general form:

$$m \ddot{x} = I_m + mg \sin \alpha - fN \left( \dot{x}^2 + \dot{y}^2 \right) \frac{1}{2},$$

(2)

where $m$ is the mass of the particle, kg;
$g$ is acceleration of gravity, m/s$^2$;
$N$ is the force of normal pressure, N;
$f$ is coefficient of friction of the particle against the bottom of the vibrating metering tray;
$\alpha$ is the angle of inclination of the tray to the horizon, degrees;
$\dot{x}$ and $\dot{y}$ are the current value of the particle velocity along the x and y axes, m/s.

In expression (2), considering the above assumptions of the model under consideration, we neglect the gravity component due to its insignificance in comparison with the inertia force (according to our calculations, the value of the gravity component is $<< 0.05 \Sigma F$).

Since a continuous downhill motion is considered, there is no relative motion of the particle in the vertical direction ($y_0$). Hence it follows that the relative motion of the particle will be characterized by the $x$ coordinate along the plane of the tray, i.e., you can write: $\dot{x} > 0$. Considering that $I_m = mA(\varphi)\omega^2$ and introducing the designation $\kappa = \frac{fN(\dot{x}^2 + \dot{y}^2)\frac{1}{2}}{(x^2+y^2)^{\frac{1}{2}}}$ ($x^2+y^2 \neq 0$), we can, after reducing the right and left sides by $m$, write equation (2), like the motion of a particle with viscous friction, in the following form:

$$\ddot{x} + \kappa \dot{x} = A(\varphi)\omega^2 - \kappa \dot{x}$$

(3)

Let us represent equation (3) as

$$\ddot{x} + \kappa \dot{x} = A(\varphi)\omega^2$$

(4)

where $\varphi$ is the generalized angular coordinate;

$\kappa$ is the generalized coefficient of equivalent viscous friction, which takes into account the mechanism of interaction of the particle with the bottom of the tray and the angle of inclination of the tray of the vibrating batcher to the horizon (takes smaller values with increasing particle velocity $\dot{x}$ and vice versa);

$\kappa \dot{x}$ is the dissipative force represented by viscous friction, i.e., the friction force becomes proportional to the particle velocity and depends on the tilt angle of the vibrating metering tray;

$A(\varphi)\omega^2$ is a periodic external action ($A(\varphi)$ is the amplitude of a variable value at any moment in time according to the system of equations (1)).

Expression (4) is a differential equation of forced vibrations of a system with one degree of freedom in the presence of a dissipative force with linear friction, known from the theory of vibrations [10, 11] and describing our mechanical system, shown in Figure 1.
The resulting differential equation cannot be integrated by quadratures. A numerical method will be used to solve it; the same method that was implemented on a computer using the expansion of functions $x(t)$ and $\dot{x}(t)$ in a Taylor series, limited by the first three terms:

$$X(t) = X_0 + \frac{\dot{x}(t)}{1!} + \frac{\ddot{x}(t)}{2!} + \frac{\dddot{x}(t)}{3!};$$ (5)

$$\dot{X}(t) = \dot{X} + \frac{\ddot{x}(t)}{1!} + \frac{\dddot{x}(t)}{2!}.$$ (6)

The first term in equation (5) is determined from the kinematics of the chute vibrating batcher, for the initial conditions:

$$X_0 = A(\phi) \cos \phi.$$ (7)

Differentiating expression (7) with respect to time, we find the second term in equation (5):

$$\dot{x}(t) = A(\phi) \cos \phi - A(\phi) \dot{\phi} \sin \phi.$$ (8)

The third term is found from the dynamics of the trough vibrating batcher according to the expression (3):

$$\ddot{X}(t) = A(\phi) \sigma^2 - k \ddot{x},$$ (9)

Differentiating (3) with respect to time, we determine the fourth term in equation (5):

$$\dot{X}(t) = A(\phi) \sigma^2 - k \ddot{x},$$ (10)

where $\dot{A}(\phi)$ is the time derivative of expression (1):

$$\dot{A}(\phi) = \begin{vmatrix} -s \phi \sin(\phi - \phi_1) + \frac{s^2 \phi \cos(\phi - \phi_1) \sin(\phi - \phi_1)}{\sqrt{R^2 - (s \sin(\phi - \phi_1))^2}} & R \phi \cos \phi \end{vmatrix} \sin^2 \phi.$$ (11)

Thus, the obtained mathematical model makes it possible to describe the process of moving particles of bulk feed under various operating modes of the trough vibrating batcher. To clarify the physical essence of the vibration displacement process of the obtained mathematical model, a computer program has been compiled.

Based on the results of numerical solutions obtained on a computer, for fixed values of the vibration parameters of the tray ($A = 8$ mm, $\omega = 47.1 \text{ s}^{-1}$) graphical dependencies (Fig. 2, 3, 4) are constructed, clearly illustrating the nature of the behavior of movement, speed, and acceleration of a material particle for one period of oscillation of the vibrating batcher tray at different values of the generalized coefficient of equivalent viscous friction $k$ (in what follows we will simply call the generalized coefficient).

The number of curves shown in these figures has been chosen for clarity reasons only. As can be seen from the graphs, regardless of the numerical value of the generalized coefficient, the behavior of the movement, velocity, and acceleration of the particle in one revolution of the cam vibrator has the same functional form. From Figure 2 it follows that at first the particle displacement grows rapidly up to a certain limit, and then monotonically increases. It is also shown here that to obtain large displacements of a particle in a continuous mode, it is necessary to strive for the selection of smaller values of the generalized coefficient.

As can be seen from Figure 4, the acceleration of the particle sharply increases, therefore, it is legitimate to assume that there are large values of the acceleration of the working body. And that is not a desirable outcome from the point of view of the durability and reliability of the operation of the pan vibrating batcher. The nature of the particle velocity change during one period of oscillation of the chute has the form shown in Figure 3, from which the period of unsteady motion in the chute vibrating batcher is a short-term phenomenon. Then it becomes stable with a constant value of the average speed, regardless of the generalized coefficient and vibration parameters (Fig. 3). Therefore, it is necessary to limit the choice of the lower limit of the value of the generalized coefficient.
Figure 2. The nature of the behavior of the movement of a particle of feed during one period of oscillation of the tray of a vibrating batcher at different values of the generalized coefficient $k$

Figure 3. The nature of the behavior of the speed of a particle of feed for one period of oscillation of the chute of a vibrating batcher at different values of the generalized coefficient $k$
Figure 4. The nature of the behavior of the acceleration of the feed particle during one period of oscillation of the chute of the vibrating batcher at different values of the generalized coefficient $k$

3. Conclusion
It should be emphasized that the similar graphs discussed above have the same development tendency for other values of the vibration parameters ($A$, $\omega$) of the chute vibrating batcher. In other words, for the real process of dosing bulk feeds with the proposed trough vibration dispenser, both single-component and multi-component, there is a range of rational values of the generalized coefficient, which are acceptable both, if possible, to obtain the maximum average speed of particle movement in a continuous mode, and by the values of dynamic loads in the system.

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