Non-monotonic energy dependence of net-proton number fluctuations

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Non-monotonic variation with collision energy (\(\sqrt{s_{\text{NN}}}\)) of the moments of the net-baryon number distribution in heavy-ion collisions, related to the correlation length and the susceptibilities of the system, is suggested as a signature for the Quantum Chromodynamics (QCD) critical point. We report the first evidence of a non-monotonic variation in kurtosis times variance of the net-proton number (proxy for net-baryon number) distribution as a function of \(\sqrt{s_{\text{NN}}}\) with 3.1\(\sigma\) significance, for head-on (central) gold-on-gold (Au+Au) collisions measured using the STAR detector at RHIC. Data in non-central Au+Au collisions and models of heavy-ion collisions without a critical point show a monotonic variation as a function of \(\sqrt{s_{\text{NN}}}\).
One of the fundamental goals in physics is to understand the properties of matter when subjected to variations in temperature and pressure. Currently, the study of the phases of strongly interacting nuclear matter is the focus of many research activities worldwide, both theoretically and experimentally [1, 2]. The theory that governs the strong interactions is Quantum Chromodynamics (QCD), and the corresponding phase diagram is called the QCD phase diagram. From different examples of condensed-matter systems, experimental progress in mapping out phase diagrams is achieved by changing the material doping, adding more holes than electrons. Similarly it is suggested for the QCD phase diagram, that adding more quarks than antiquarks (the energy required is defined by the baryonic chemical potential, $\mu_B$), through changing the heavy-ion collision energy, enables a search for new emergent properties and a possible critical point in the phase diagram. The phase diagram of QCD has at least two distinct phases: a Quark Gluon Plasma (QGP) at higher temperatures, and a state of confined quarks and gluons at lower temperatures called the hadronic phase [3–5]. It is inferred from lattice QCD calculations [6] that the transition is consistent with being a cross over at small $\mu_B$, and that the transition temperature is about 155 MeV [7–9]. An important predicted feature of the QCD phase structure is a critical point [10, 11], followed at higher $\mu_B$ by a first order phase transition. Attempts are being made to locate the predicted critical point both experimentally and theoretically. Current theoretical calculations are highly uncertain about the location of the critical point. Lattice QCD calculations at finite $\mu_B$ face numerical challenges in computing [12, 13]. Within these limitations, the current best estimate from lattice QCD is that if there is a critical point, its location is likely above $\mu_B \sim 300$ MeV [12, 13]. The goal of this work is to search for possible signatures of the critical point by varying the collision energy in heavy ion collisions to cover a wide range in effective temperature ($T$) and $\mu_B$ in the QCD phase diagram [14].

Another key aspect of investigating the QCD phase diagram is to determine whether the system has attained thermal equilibrium. Several theoretical interpretations of experimental data have the underlying assumption that the system produced in the collisions should have come to local thermal equilibrium during its evolution. Experimental tests of thermalization for these femto-scale expanding systems are non-trivial. However, the yields of produced hadrons and fluctuations of multiplicity distributions related to conserved quantities have been studied and shown to have characteristics of thermodynamic equilibrium for higher collision energies [12, 15–20].

Upon approaching a critical point, the correlation length diverges and thus renders, to a large extent, microscopic details irrelevant. Hence observables like the moments of the conserved net-baryon number distribution, which are sensitive to the correlation length, are of interest when searching for a critical point. A non-monotonic variation of these moments as a function of $\sqrt{s_{NN}}$ has been proposed as an experimental signature of a critical point [10, 14]. However, considering the complexity of the system formed in heavy-ion collisions, signatures of a critical point are detectable only if they can survive the evolution of the system, including the effects of finite size and time [21]. Hence, it was proposed to study higher moments of distributions of conserved quantities ($N$) due to their stronger dependence on the correlation length [11]. The promising higher moments are the skewness, $S = \langle (\delta N)^3 \rangle / \sigma^3$, and kurtosis, $\kappa = \langle (\delta N)^4 \rangle / \sigma^4 - 3$, where $\delta N = N - M$, $M$ is the mean and $\sigma$ is the standard deviation. The magnitude and the sign of the moments, which quantify the shape of the multiplicity distributions, are important for understanding the critical point [14, 22]. An additional crucial experimental challenge is to measure, on an event-by-event basis, all of the baryons produced within the acceptance of a detector [23–25]. However, theoretical calculations have shown that the proton-number fluctuations can also reflect the baryon-number fluctuations at the critical point [23, 26].

The measurements reported here are from Au+Au collisions recorded by the STAR detector [27] at RHIC from the years 2010 to 2017. The data is presented for $\sqrt{s_{NN}} = 7.7$, 11.5, 14.5, 19.6, 27, 39, 54.4, 62.4 and 200 GeV as part of phase-I of the Beam Energy Scan (BES) program at RHIC [15]. These $\sqrt{s_{NN}}$ values correspond to $\mu_B$ values ranging from 420 MeV to 20 MeV at chemical freeze-out [15]. All valid Au+Au collisions occurring within 60 cm (80 cm for $\sqrt{s_{NN}} = 7.7$ GeV) of the nominal interaction point along the beam axis are selected. For the results presented here, the number of minimum bias Au+Au collisions ranges between 3 million for $\sqrt{s_{NN}} = 7.7$ GeV and 585 million at $\sqrt{s_{NN}} = 54.4$ GeV. These statistics are found to be adequate to make the measurements of the moments of the net-proton distributions up to the fourth order [28]. The collisions are further divided into centrality classes characterised by their impact parameter, which is the closest distance between the centroid of two nuclei passing by. In practice, the impact parameter is determined indirectly from the measured multiplicity of charged particles other than protons ($p$) and anti-protons ($\bar{p}$) in the pseudo-rapidity range $|\eta| < 1$, where $\eta = -\ln[\tan(\theta/2)]$, with $\theta$ being the angle between the momentum of the particle and the positive direction of the beam axis. We exclude $p$ and $\bar{p}$ while classifying events based on impact parameter specifically to avoid self-correlation effects [29]. The effect of self-correlation potentially arising due to the decay of heavier hadrons into $p(\bar{p})$ and other charged particles has been checked to be negligible from a study using standard heavy-ion collision event generators, HIJING [30] and UrQMD [31]. The effect of resonance decays and the pseudo-rapidity range for centrality determination have been understood and optimized using model calculations [32, 33]. The results presented here correspond to two event classes: central collisions (impact parameters $\sim 0$–3 fm, obtained from the top 5% of the above-mentioned multiplicity distribution) and peripheral collisions (impact parameters $\sim 12$–13 fm, obtained from the 70–80% region of the multiplicity distribution).

The protons and anti-protons are identified, along with their momenta, by reconstructing their tracks in the Time Projection Chamber (TPC) placed within a solenoidal magnetic field of 0.5 Tesla, and by measuring their ionization energy loss $(dE/dx)$ in the sensitive gas-filled volume of the chamber. The selected kinematic region for protons covers all azimuthal angles for the rapidity range $|y| < 0.5$, where rapidity $y$ is the
inverse hyperbolic tangent of the component of speed parallel to the beam direction in units of the speed of light. The precise measurement of $dE/dx$ with a resolution of 7% in Au+Au collisions allows for a clear identification of protons up to 800 MeV/c in transverse momentum ($p_T$). The identification for larger $p_T$ (up to 2 GeV/c, with purity above 97%) is made by a Time Of Flight detector (TOF) [34] having a timing resolution of better than 100 ps. A minimum $p_T$ threshold of 400 MeV/c and a maximum distance of closest approach to the collision vertex of 1 cm for each $p(\bar{p})$ candidate track is used to suppress contamination from secondaries and other backgrounds [15, 35]. This $p_T$ acceptance accounts for approximately 80% of the total $p + \bar{p}$ multiplicity at mid-rapidity. This is a significant improvement from the results previously reported [35] which only had the $p + \bar{p}$ measured using the TPC. The observation of non-monotonic variation of the kurtosis times variance ($C_4\sigma^4$) with energy is much more significant with the increased acceptance. For the rapidity dependence of the observable see Supplemental Material [34].

Figure 1 shows the event-by-event net-proton ($N_p - N_{\bar{p}} = \Delta N_p$) distributions obtained by measuring the number of protons ($N_p$) and anti-protons ($N_{\bar{p}}$) at mid-rapidity ($|y| < 0.5$) in the transverse momentum range $0.4 < p_T$ (GeV/c) $< 2.0$ for Au+Au collisions at various $\sqrt{s_{NN}}$. To study the shape of the event-by-event net-proton distribution in detail, cumulants ($C_n$) of various orders are calculated, where $C_1 = M$, $C_2 = \sigma^2$, $C_3 = 3\sigma^3$ and $C_4 = \sigma^4$.

Figure 2 shows the net-proton cumulants ($C_n$) as a function of $\sqrt{s_{NN}}$ for central and peripheral (see Supplemental Material [34] for a zoomed version). Au+Au collisions. The cumulants are corrected for the multiplicity variations arising due to finite impact parameter range for the measurements [32]. These corrections suppress the volume fluctuations considerably [32, 36]. A different volume fluctuation correction method [37] has been applied to the 0-5% central Au+Au collision data and the results were found to be consistent with those shown in Fig 2. The cumulants are also corrected for finite track reconstruction efficiencies of the TPC and TOF detectors. This is done by assuming a binomial response of the two detectors [35, 38]. A cross-check using a different method based on unfolding [34] of the distributions for central Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV has been found to give values consistent with the cumulants shown in Fig. 2. Further, the efficiency correction method used has been verified in a Monte Carlo calculation. Typical values for the efficiencies in the TPC (TOF-matching) for the momentum range studied in 0-5% central Au+Au collisions at $\sqrt{s_{NN}} = 7.7$ GeV are 83%(72%) and 81%(70%) for the protons and anti-protons, respectively. The corresponding efficiencies for $\sqrt{s_{NN}} = 200$ GeV collisions are 62%(69%) and 60%(68%) for the protons and anti-protons, respectively. The statistical uncertainties are obtained using both a bootstrap approach [28, 38] and the Delta theorem [28, 38, 39] method. The systematic uncertainties are estimated by varying the experimental requirements to reconstruct $p(\bar{p})$ in the TPC and TOF. These requirements include the distance of the proton and anti-proton tracks from the primary vertex position, track quality reflected by the number of TPC space points used in the track reconstruction, the particle identification criteria passing certain selection criteria, and the uncertainties in estimating the reconstruction efficiencies. The systematic uncertainties at different collision energies are uncorrelated.

The large values of $C_3$ and $C_4$ for central Au+Au collisions show that the distributions have non-Gaussian shapes, a possible indication of enhanced fluctuations arising from a possible critical point [11, 22]. The corresponding values for peripheral collisions are small and close to zero. For central collisions, the $C_1$ and $C_3$ monotonically decrease with increasing $\sqrt{s_{NN}}$.

We employ ratios of cumulants in order to cancel volume variations to first order. Further, these ratios of cumulants are related to the ratio of baryon-number susceptibilities. The
latter are $\chi_n^B = \frac{p^n}{\mu_B}$, where $n$ is the order and $P$ is the pressure of the system at a given $T$ and $\mu_B$, computed in lattice QCD and QCD-based models [40]. The $C_3/C_2 = \frac{\Sigma}{\kappa^2} = (\chi_3^B/T)/(<\chi_3^B>/T^2)$ and $C_t/C_2 = \frac{\kappa^2}{\sigma^2} = (<\kappa^t>/T^2)$. Close to the critical point, QCD-based calculations predict the net-baryon number distributions to be non-Gaussian and susceptibilities to diverge, causing moments, especially higher-order susceptibilities, to have a non-monotonic variation as a function of $\sqrt{s_{NN}}$ [40, 41].

Figure 3 shows the central 0-5% Au+Au collision data for $\Sigma$ and $\kappa^2$ in the collision energy range of 7.7 - 62.4 GeV, fitted to a polynomial function of order five and four, respectively. The derivative of the polynomial function changes sign [34] with $\sqrt{s_{NN}}$ for $\kappa^2$, thereby indicating a non-monotonic variation of the measurement with the collision energy. The uncertainties of the derivatives are obtained by varying the data points randomly at each energy within the statistical and systematic uncertainties separately. The overall significance of the change in the sign of the slope for $\kappa^2$ versus $\sqrt{s_{NN}}$, based on the fourth order polynomial function fitting procedure from $\sqrt{s_{NN}} = 7.7$ to 62.4 GeV, is 3.1$\sigma$. A similar exercise with the UrQMD [31] and HRG model with canonical ensemble [42] as the non-critical baseline yields an overall significance of 3.3$\sigma$. This significance is obtained by generating one million sets of points, where for each set, the measured $\kappa^2$ value at a given $\sqrt{s_{NN}}$ is randomly varied within the total Gaussian uncertainties (systematic and statistical uncertainties added in quadrature). Then for each new $\kappa^2$ versus $\sqrt{s_{NN}}$ set of points, a fourth order polynomial function is fitted and the derivative values are calculated at different $\sqrt{s_{NN}}$ (as discussed above). A total of 1143 sets were found to have the same derivative sign at all $\sqrt{s_{NN}}$. The probability that at least one derivative at a given $\sqrt{s_{NN}}$ has a different sign is found to be 0.998857, which corresponds to 3.1$\sigma$. A similar procedure was applied to the lower-order product of moments. The $\sigma^2/M$ (not shown) strongly favors a monotonic energy dependence excluding the non-monotonic trend at a 3.4$\sigma$ level. Within 1.0$\sigma$ significance the $\Sigma$ allows for a non-monotonic energy dependence. This is consistent with a QCD based model expectation that the higher the order of the moments the more sensitive it is to physics processes such as a critical point [11].

Figure 4 shows the variation of $\Sigma$ (or $C_3/C_2$) and $\kappa^2$ (or $C_t/C_2$) as a function of $\sqrt{s_{NN}}$ for central and peripheral Au+Au collisions. In central collisions, as discussed above, a non-monotonic variation with beam energy is observed for $\kappa^2$. The peripheral collisions on the other hand do not show a non-monotonic variation with $\sqrt{s_{NN}}$ around the statistical baseline of unity, and $\kappa^2$ values are always below unity. It is worth noting that in peripheral collisions, the system formed may not be hot and dense enough to undergo a phase transition or come close to the QCD critical point. The expectations from an ideal statistical model of hadrons assuming thermodynamical equilibrium, called the Hadron Resonance Gas (HRG) model [33], calculated within the experimental acceptance and considering a grand canonical ensemble (GCE), excluded volume (EV) [43], and canonical ensemble (CE) [42], are also shown in Fig. 4. The HRG results do not quantitatively describe the data. Corresponding $\kappa^2$ ($\Sigma$) results for 0-5% Au+Au collisions from a transport-based UrQMD model [31] calculation, which incorporates conservation laws and most of the relevant physics apart from a phase transition or a critical point, and which is calculated within the experimental acceptance, show a monotonic decrease (increase) with decreasing collision energy (see Supplemental Material [34] for a quantitative comparison). Similar conclusions are obtained from JAM [45], another microscopic transport model. Neither of the model calculations explains simultaneously the measured dependence of the $\kappa^2$ and $\Sigma$ of the net-proton distribution on $\sqrt{s_{NN}}$ for central Au+Au collisions. This can be seen from the values of a $\chi^2$ test between the experimental data and various models for $\sqrt{s_{NN}} = 7.7 - 27$ GeV given in Table I. $p$ reflects the probability that a model agrees with the data.

TABLE I. The $p$ values of a $\chi^2$ test between data and various models for the $\sqrt{s_{NN}}$ dependence of $\Sigma$ and $\kappa^2$ values of net-proton distributions in 0-5% central Au+Au collisions. The results are for the energy range 7.7 to 27 GeV which is relevant for the search for a critical point [12, 13].

| Moments | HRG GCE | HRG EV | HRG CE | UrQMD |
|---------|---------|---------|---------|--------|
| $\Sigma$ | < 0.001 | < 0.001 | 0.0754  | < 0.001 |
| $\kappa^2$ | 0.00553 | 0.0450  | 0.0145  | 0.0221 |

In conclusion, we have presented measurements of net-proton cumulant ratios with the STAR detector at RHIC over a wide range of $\mu_B$ (20 to 420 MeV) which are relevant to a QCD critical point search in the QCD phase diagram. We have observed a non-monotonic behavior as a function of $\sqrt{s_{NN}}$, in net-proton $\kappa^2$ in central Au+Au collisions with a significance of 3.1$\sigma$ relative to Skellam expectation. In contrast,
monotonic behaviour with $\sqrt{s_{NN}}$ is predicted for the statistical hadron gas model, and for a nuclear transport model without a critical point, as observed experimentally in peripheral collisions. The deviation of the measured $\kappa\sigma^2$ from several baseline calculations with no critical point, and its non-monotonic dependence on $\sqrt{s_{NN}}$, are qualitatively consistent with expectations from a QCD-based model which includes a critical point [11, 14]. Our measurements can also be compared to the baryon-number susceptibilities computed from QCD to understand various other features of the QCD phase structure as well as to obtain the freeze-out conditions in heavy-ion collisions. Higher event statistics will allow for a more differential measurement of experimental observables in $y-p_T$. They will improve the comparison of the measurements with QCD calculations which include the dynamics associated with heavy-ion collisions, and hence they may help in establishing the critical point.

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FIG. 4. $\sigma$ (1) and $\kappa\sigma^2$ (2) as a function of collision energy for net-proton distributions measured in Au+Au collisions. The results are shown for central (0-5%, filled circles) and peripheral (70-80%, open squares) collisions within $0.4 < p_T$ (GeV/c) < 2.0 and $|y| < 0.5$. The vertical narrow and wide bars represent the statistical and systematic uncertainties, respectively. Shown as an open triangle is the result from the HADES experiment [44] for 0-10% Au+Au collisions and $|y| < 0.4$. The shaded green band is the estimated statistical uncertainty for BES-II. The peripheral data points have been shifted along the x-axis for clarity of presentation. Results from different variants (GCE, EV, CE) of the hadron resonance gas (HRG) model [33, 42, 43] and a transport model calculation (UrQMD [31]) for central collisions (0-5%) are shown as black, red, blue bands and a gold band, respectively.

[1] A. Bzdak, S. Esumi, V. Koch, J. Liao, M. Stephanov and N. Xu, Phys. Rept. 853, 1-87 (2020).
[2] X. Luo and N. Xu, Nucl. Sci. Tech. 28, no.8, 112 (2017).
[3] K. Fukushima and T. Hatsuda, Rept. Prog. Phys. 74, 014001 (2011).
[4] P. Braun-Munzinger and J. Wambach, Rev. Mod. Phys. 81, 1031-1050 (2009).
[5] M. Asakawa and K. Yazaki, Nucl. Phys. A 504, 668-684 (1989).
[6] Y. Aoki, G. Endrodi, Z. Fodor, S. D. Katz and K. K. Szabo, Nature 443, 675-678 (2006).
[7] Y. Aoki, S. Borsanyi, S. Durr, Z. Fodor, S. D. Katz, S. Krieg and K. K. Szabo, JHEP 06, 088 (2009).
[8] A. Bazavov, T. Bhattacharya, M. Cheng, C. DeTar, H. T. Ding, S. Gottlieb, R. Gupta, P. Hegde, U. M. Heller, F. Karsch, E. Laermann, L. Levkova, S. Mukherjee, P. Petreczky, C. Schmidt, R. A. Soltz, W. Soeldner, R. Sugar, D. Toussaint,
