Dark matter annihilation near a naked singularity

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We investigate here the dark matter annihilation near a Kerr naked singularity. We show that when dark matter particles collide and annihilate in vicinity of the singularity, the escape fraction to infinity of particles produced is much larger, at least $10^2 - 10^3$ times the corresponding black hole values. As high energy collisions are generically possible near a naked singularity, this provides an excellent environment for efficient conversion of dark matter into ordinary standard model particles. If the center of galaxy harbored such a naked singularity, it follows that the observed emergent flux of particles with energy comparable to mass of the dark matter particles is much larger compared to the blackhole case, thus providing an intriguing observational test on the nature of the galactic center.

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Naked singularities are one of the most exotic predictions of classical general relativity, which are in principle directly observable by external observers in the universe, unlike their black hole siblings. Many investigations have shown that given regular initial density and pressure profiles, there are classes of gravitational collapse evolutions that give rise to naked singularity as final state of collapse for a massive star, subject to an energy condition and with astrophysically reasonable equations of state such as dust, perfect fluids and such others. There are many static and stationary solutions also of the Einstein equations containing naked singularities, such as the Kerr and Reissner-Nordstrom geometries (see. e.g. [1] and references therein). A popular idea to avoid classical nakedness has been the cosmic censorship conjecture [2], which has given rise to extensive debates on their existence. As naked singularities arise in a regime where quantum gravity should replace the classical general relativity, singularities may be resolved using an appropriate quantum theory of gravity [3]. Also, as we have no observable signatures of the Planck scale physics as yet, naked singularities could be in fact a boon for a quantum theory of gravity. The super-ultra-dense regions of extreme curvatures being visible in this case, the quantum gravity signatures in their vicinity are in principle observable, providing a rare test of this ultra-high energy regime.

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Naked singularities could give rise to intriguing physical effects, and particle acceleration by naked singularities formed in gravitational collapse was investigated recently [4].

We investigate here the dark matter annihilation near the Kerr naked singularity. The process of high energy collisions between particles around extremal Kerr blackhole was examined, and the emergent flux from dark matter annihilation was estimated in [5,6]. We show here that the results for the naked singularity case differ importantly from the blackhole case in two crucial aspects. Firstly, the collisions at large center of mass energies occur generically in naked singularity geometry, so the conversion of dark matter into ordinary particles is rather efficient. Secondly, it turns out that the escape fraction for the particles formed in the dark matter annihilations is much larger. We focus on the case where the Kerr spin parameter is slightly larger than unity. This is naked singularity, in which case we get highly energetic relativistic collisions occurring generically, unlike the black hole case where extreme fine-tuning of angular momentum of colliding particles is necessary. We thus see that the observational and astrophysical features of naked singularities if they occur in nature would be indeed significantly different from the blackholes.

As we understand today, galaxies formed within dark matter halos and the supermassive compact astrophysical blackholes, also referred to as blackhole candidates, are formed at the center of galaxies. In the hypothetical situation of a Kerr naked singularity located at the center of galaxy as supermassive astrophysical blackhole, the dark matter is accreted onto it due to gravitational pull, forming an overdense region. Dark matter particles interact relativistically with large center of mass energy of collision generically, near the central compact object. The annihilation crosssection of dark matter would be large at high center of mass energy of collision, and due to large density the annihilation rate is enhanced significantly. We calculate here the escape fraction for the standard model particles to infinity in the naked singularity background, created by the dark matter annihilation, which is shown to be large. Thus the emergent flux at infinity would be much larger compared to Kerr blackholes, where the high energy collisions happen near the horizon for particles with highly finetuned parameters, and escape fractions are significantly smaller. Such a scenario provides an intriguing observational test to understand the nature of the galactic center better. We note that Kerr super-spinars were discussed in the context of the galactic center recently [7].

We work in the units where the gravitational radius of the central object and the mass of the dark matter particle are set to unity, \( r_g = \frac{GM}{c^2} = \mu = 1 \). The dark matter is taken to be nonrelativistic faraway. A collision between two dark matter particles accreted from the halo is considered, assuming that the particles follow geodesic motion in the equatorial plane. The physical process is discussed when two dark matter particles interact and decay into two massless particles.
FIG. 1: The variation of center of mass energy of collision between the ingoing and outgoing particles with angular momenta $L_1 = -0.4, L_2 = 1.4$ with radius around $r = 1$ in a spacetime containing naked singularity with spin parameter close to unity, $a - 1 = \epsilon = 10^{-8}$.

This is a reasonable assumption since the mass of the dark matter particle is assumed to be much larger than the particles it decays into. We also assume for simplicity that the massless particles travel in the equatorial plane, and their distribution in the center of mass frame of collision is isotropic.

The Kerr metric in equatorial plane, in Boyer-Lindquist coordinates is given by,

$$ds^2 = -(1 - \frac{2}{r}) dt^2 - \frac{4ra}{\Delta} dt d\phi + \left(\frac{r^2}{\Delta}\right) dr^2 + r^2 d\theta^2 + \left(r^2 + a^2 + \frac{2a^2}{r}\right) d\phi^2$$

(1)

where $\Delta = r^2 + a^2 - 2r$ and $a$ is the angular momentum parameter. When $a > 1$ the Kerr metric gives a naked singular solution with no event horizon, and $a \leq 1$ corresponds to a blackhole with event horizon at $r = 1$, the extremal case being $a = 1$. The first integral to the geodesic equation can be found from the normalization condition $U \cdot U = -m^2$ and using the conserved quantities $U \cdot \partial_t = -E$, $U \cdot \partial_\phi = L$ of motion, arising from the isometries of the Kerr metric. Here $m = 1$ for timelike and $m = 0$ for null geodesics respectively, and $E, L$ are interpreted as the conserved energy and orbital angular momentum per unit mass. The velocity components are given by,

$$U^t = \frac{1}{\Delta} \left[ \left( r^2 + a^2 + \frac{2a^2}{r} \right) E - \frac{2a}{r} L \right]$$

$$U^\phi = \frac{1}{\Delta} \left[ \left(1 - \frac{2}{r}\right) L + \frac{2a}{r} E \right], \quad U^\theta = 0$$

$$U^r = \frac{u}{\sqrt{E^2 - m^2 + \frac{2}{r} - \frac{(L^2 - a^2 (E^2 - m^2))}{r^2} + \frac{2(L - aE)^2}{r^3}}}$$

(2)
FIG. 2: The variation of angular momentum $b(r) = L_1(r), L_2(r)$ required for the massless particle to turn back from radius $r$. The lower branch of $L_2(r)$ admits a maximum $L_2 = L_{2\text{max}} \approx -7$ around $r = r_{\text{max}} \approx 4$ for Kerr spin parameter $a \approx 1$.

where $u = \pm 1$ stands for radially outgoing and ingoing particle respectively.

We consider a collision between ingoing and outgoing particles around $r = 1$. Both the particles start from rest at infinity and thus the conserved energy per unit mass is given by $E = 1$. One of the particles which is initially ingoing turns back from $r < 1$ and emerges as an outgoing particle. The angular momentum for this particle must be in the range $2 \left( -1 + \sqrt{1 + a} \right) \leq L < \left( 2 - \sqrt{2a^2 - 2} \right)$. The lower limit stands for the minimum angular momentum it must have to turn back at all, and the upper limit corresponds to the value so that it turns back from $r = 1$. The second particle is an ingoing particle and to reach $r = 1$ and participate in the collision the constraint to be imposed is $L < \left( 2 - \sqrt{2a^2 - 2} \right)$ on its angular momentum [8].

The center of mass energy of collision between the particles with velocities $U^1, U^2$ is given by

$$E_{\text{c.m.}}^2 = 2m^2 \left( 1 - g_{\mu\nu}U^\mu_1 U^\nu_2 \right)$$

The center of mass energy in the vicinity of $r = 1$ is extremely large (see Fig.1). The annihilation crosssection for dark matter particles to turn into ordinary matter particles will be large around $r = 1$. Dark matter particles annihilate into two massless particles. The energy momentum conservation implies that the total conserved energy of the dark matter particles ($2\mu = 2$) will be shared by the two massless particles. The conserved energy of each particle will be of the order of unity $E \approx 1$. These particles travel in opposite directions in the center of mass frame. We track the motion of only one of the two particles and analyze under what conditions it escapes to infinity.

For a massless particle to turn back at $r$ i.e $\dot{r}(r) = 0$, the angular momentum $b(r)$, obtained
from (2) turns out to be \( b(r) = L_1(r) = \frac{1}{r^2} \left( -2a + \sqrt{r^4 + a^2r^2 - 2r^3} \right) \) or \( b(r) = L_2(r) = \frac{1}{r^2} \left( -2a - \sqrt{4r^4 + a^2r^2 - 2r^3} \right) \). \( L_1(r), L_2(r) \) are plotted in Fig.2. The lower branch of \( L_2(r) \) admits a maximum \( L_{2\text{max}} \approx -7 \) around \( r = r_{\text{max}} \approx 4 \) for Kerr spin parameter \( a \approx 1 \). The massless particle produced in a collision at \( r \) could either be ingoing or outgoing corresponding to \( u = \pm 1 \). The particle will escape to infinity if

\[
\begin{align*}
r < r_{\text{max}}, u = +1, & \quad L_{2,\text{max}} < L < L_1(r) \\
r < r_{\text{max}}, u = -1, a < L < L_1(r) \\
r > r_{\text{max}}, u = +1, & \quad L_2 < L < L_1(r) \\
r > r_{\text{max}}, u = -1, a < L < L_1(0) \\
r > r_{\text{max}}, u = -1, & \quad L_2(r) < L < L_{2,\text{max}}
\end{align*}
\]

The conditions given above can be translated to the escape fractions in the following way. We first make transition to the locally nonrotating frame. The components of any vector \( V \) transform as

\[
e_{\mu}^\nu = \left( \begin{array}{cccc}
g_{t\phi}^2 - g_{tt}g_{\phi\phi} & 0 & 0 & 0 \\
g_{\phi\phi} & 0 & \sqrt{g_{rr}} & 0 \\
g_{t\phi} & 0 & 0 & \sqrt{g_{\theta\theta}} \\
g_{\phi\phi} & 0 & 0 & \sqrt{g_{\phi\phi}} \end{array} \right)
\]

The transformation to center of mass frame from locally nonrotating frame can be given as \( V^\mu = \Lambda_{\mu}^\nu V^\nu \) where \( \Lambda = \Lambda_{\text{boost}} \Lambda_{\text{rot}} \). Here \( \Lambda_{\text{rot}} \) stands for the rotation in the \( r - \phi \) plane to orient the net three-velocity of two colliding particles \( U_{\text{tot}} = (U_1 + U_2) \) along the \( \tilde{r} \) direction in the locally nonrotating frame by an angle \( \alpha = \arccos \left( \frac{U_1^\mu}{\sqrt{U_1^\mu U_1^\nu U_2^\mu U_2^\nu}} \right) \). Here \( \Lambda_{\text{rot}} \) stands for a boost along \( \tilde{r} \) direction to make a transition to the center of mass frame where spatial components of total velocity are zero, \( U_{\text{tot}}^i = 0, i = r, \theta, \phi \). The boost parameter is given by \( \beta = \left( \frac{\sqrt{U_{\text{tot}}^r U_{\text{tot}}^r - U_{\text{tot}}^\theta U_{\text{tot}}^\theta + U_{\text{tot}}^\phi U_{\text{tot}}^\phi}}{U_{\text{tot}}^r} \right), \gamma = \frac{1}{\sqrt{1 - \beta^2}} \)

\[
\Lambda_{\mu}^\nu = \left( \begin{array}{cccc}
\gamma & -\beta \gamma \cos \alpha & 0 & -\beta \gamma \sin \alpha \\
-\beta \gamma & \gamma \cos \alpha & 0 & \gamma \sin \alpha \\
0 & 0 & 1 & 0 \\
0 & -\sin \alpha & 0 & \cos \alpha \end{array} \right)
\]

In the center of mass frame, the distribution of the massless particles produced in the collision is assumed to be isotropic in the equatorial plane. The particles escape if the conserved angular
E.F.

FIG. 3: Fig 3a shows the variation of escape fraction with radius. Highly energetic collisions take place around \( r = 1 \). The escape fraction is more or less constant and takes a value \( E.F. \approx 0.5 \), unlike the extremal Kerr blackhole case where it decreases as one approaches the horizon \( r = 1 \). Fig 3b depicts the slight increase in the escape fraction with the center of mass energy of collision, unlike the Kerr blackhole case where there is a sharp fall in the escape fraction with the center of mass energy of collision.

momentum satisfies conditions \( \mathbf{4} \). We compute the three velocity in the center of mass the frame the particle would have for extreme values of angular momenta. The angle between the extreme velocity vectors would yield the escape angle. The escape angle when divided by \( 2\pi \) would be the escape fraction.

\[
E.F(r, a) = \frac{\Theta(r - r_{max}(a))A_1(r, a) + \Theta(r_{max}(a) - r)A_2(r, a)}{2\pi}
\]

Here \( \Theta \) is the step function. The three velocity in the center of frame in terms of velocity in Boyer-Lindquist coordinates is obtained by \( U^i_3 = U^i = \Lambda^i_{\mu} e_\nu U^\nu \). Then the \( A_1(r, a), A_2(r, a) \) are given by,

\[
A_1(r, a) = \left[ \arccos \left( \frac{U_3(r, a, L = L_1(r), u = 1) U_3(r, a, L = L_2(r), u = 1)}{U_3(r, a, L = L_1(r), u = 1) \parallel U_3(r, a, L = L_2(r), u = 1)} \right) \right] \\
\]

\[
+ \left[ \arccos \left( \frac{U_3(r, a, L = a, u = 1) U_3(r, a, L = L_1(0), u = 1)}{U_3(r, a, L = a, u = 1) \parallel U_3(r, a, L = L_1(0), u = 1)} \right) \right] \\
\]

\[
+ \left[ \arccos \left( \frac{U_3(r, a, L = L_2(r), u = 1) U_3(r, a, L = L_{2max}, u = 1)}{U_3(r, a, L = L_2(r), u = 1) \parallel U_3(r, a, L = L_{2max}, u = 1)} \right) \right] \\
\]

\[
A_2(r, a) = \left[ \arccos \left( \frac{U_3(r, a, L = L_1(r), u = 1) U_3(r, a, L = L_{2max}, u = 1)}{U_3(r, a, L = L_1(r), u = 1) \parallel U_3(r, a, L = L_{2max}, u = 1)} \right) \right] \\
\]

\[
+ \left[ \arccos \left( \frac{U_3(r, a, L = L_1(r), u = 1) U_3(r, a, L = a, u = 1)}{U_3(r, a, L = L_1(r), u = 1) \parallel U_3(r, a, L = a, u = 1)} \right) \right] \\
\]
The escape fraction as a function of radial coordinate is plotted in Fig 3a. The high energy collisions take place around \( r = 1 \). The escape fraction in this region takes a value around \( \text{E.F.} \approx 0.5 \). The variation of escape fraction with the center of mass energy of collision is depicted in Fig 3b. The slow increase in escape fraction with the center of mass energy is contrary to the blackhole case where the same falls sharply with the increment in the center of mass energy around the horizon \( r = 1 \). The absolute value of the escape fraction for center of mass energies larger than \( E_{\text{cm}} > 200 \) is smaller than \( \text{E.F.} < 0.002 \) [6]. Around \( E_{\text{cm}} \approx 1000 \), we have \( \text{E.F.} \approx 0.001 \). Thus the escape fraction for large center of mass energies in the case of Kerr naked singularity is at least 2 – 3 orders of magnitude larger than the blackhole. It follows that the outgoing flux of particles will be correspondingly larger in the naked singularity case.

While annihilation crosssection of the dark matter is extremely small at ordinary energies, it is expected to be much larger at large center of mass energy collisions, so the rate of annihilation will be very large. Such collisions are generic in the vicinity of Kerr naked singularity since the parameters of the particle geodesics necessary to have such high energy collisions is finite. In the case of Kerr blackholes, extreme finetuning of the particle parameters is required for them to participate in high energy collisions. Thus such high energy collisions would be rare around the Kerr blackholes. Also, in the black hole case, since the high energy collisions take place around the event horizon, most of the particles are eaten up by the blackhole. However, for naked singularity most of the particles escape to infinity. Thus the emergent flux of the standard model particles will be much larger in the case of naked singularities.

In [6] the calculation of the emergent flux was made with the assumption that the annihilation crosssection does not vary with the center of energy of collision. It was speculated that it might be possible to have appreciable number of detection events, for instance of neutrinos from the dark matter annihilation, with future experiments like Icecube. The simplifying assumption of constancy of annihilation crosssection implies that the contribution of the collisions farway from the horizon \( r = 1 \) to the flux would be larger due to the large escape fraction. However, if there is a sharp rise in the crossection with center of mass energy, then the contribution to flux from the region around horizon where ultrahigh energy collisions take place is highly underestimated. In case of the naked singularity, not only the annihilation rate around \( r = 1 \) is rather large due to high energy collisions, but also the escape fractions would be large. Thus the contribution to the flux from the region around \( r = 1 \) would be extremely large as compared to the corresponding blackhole case.

We assumed here that the Kerr naked singularity at the center of the galaxy is stable. In order
to actually calculate the flux, one needs an input of the variation of annihilation crosssection of the dark matter from the beyond standard model physics. One also needs a very good understanding of the density of the dark matter around the central supermassive object, which may possibly come from simulations. These issues are under consideration.

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