Internal structure of exotic hadrons by high-energy exclusive reactions

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We propose to use high-energy exclusive reactions for probing internal structure of exotic hadron candidates. First, the constituent counting rule of perturbative QCD can be used for finding internal configurations of an exotic hadron candidate. It is because the number of constituents ($n$), which participate in the exclusive reaction, is found by the scaling behavior of the cross section $d\sigma/dt \propto 1/s^{n-2}$ at large momentum transfer, where $s$ is the center-of-mass energy squared. As an example, we show that the internal structure of $\Lambda(1405)$ should be found, for example, by the reaction $\pi^- + p \rightarrow K^0 + \Lambda(1405)$. Second, the internal structure of exotic hadron candidates should be investigated by hadron tomography with generalized parton distributions (GPDs) and generalized distribution amplitudes (GDAs) in exclusive reactions. Exotic nature should be reflected in the GPDs which contain two factors, longitudinal parton distributions as indicated by the constituent counting rule and transverse form factors as suggested by the hadron size. The GDAs should be investigated by the two-photon process $\gamma^* \gamma \rightarrow hh$, for example $h = f_0$ or $a_0$, in electron-positron annihilation. Since the GDAs contain information on a time-like form factor, exotic nature should be found by studying the $hh$ invariant mass dependence of the cross section. The internal structure of exotic hadron candidates should be clarified by the exclusive reactions at facilities such as J-PARC and KEKB.

KEYWORDS: hadron, exotic, exclusive, QCD, quark, gluon

1. Introduction

Existence of new hadrons has been investigated for a long time, and a few hundred hadrons have been discovered [1]. Almost all the hadrons are understood by the internal configurations of $q\bar{q}$ and $qqq$ as proposed in the original quark model. Exotic hadrons, such as tetraquark $(qq\bar{q}\bar{q})$, pentaquark $(qqqq\bar{q})$, and glueball $(gg)$, have been investigated since the quark-model proposal in 1964. For example, $f_0(980)$ and $a_0(980)$ are known as exotic hadron candidates. In a native quark model, scalar mesons in the 1 GeV region could be interpreted as $\sigma = f_0(600) = (u\bar{u} + d\bar{d})/\sqrt{2}$, $f_0(980) = s\bar{s}$, $a_0(980) = ud$, $(u\bar{u} - d\bar{d})/\sqrt{2}$, $d\bar{u}$. Since the strange quark is heavier than up and down quarks, these quark configurations suggest the mass hierarchy $m(\sigma) \sim m(a_0) < m(f_0)$, which is in contradiction to the experimental one $m(\sigma) < m(a_0) \sim m(f_0)$, where $f_0(980)$ is simply denoted as $f_0$. Furthermore, the strong-decay width $f_0 \rightarrow 2\pi$ in the quark model largely overestimates the experimental data [2]. Radiative and two-photon decay widths also indicate multiquark structure [3]. Therefore, $f_0$ and $a_0$ could be considered as exotic tetraquark hadrons or $K\bar{K}$ molecules. The $a_0-f_0$ mixing intensity also provides a clue for their $K\bar{K}$ compositeness [4].

Furthermore, $\Lambda(1405)$ has been considered as an exotic hadron because the $\Lambda(1405)$ mass is
anomalously light. Although the ground-state $\Lambda$ is heavier than the nucleon due to the heavier strange-quark mass, their lowest excitation states with $\left(1/2\right)^{-}$ show the reversed mass relation $m_{\Lambda(1405)} < m_{N(1535)}$, which is difficult to be understood within the quark model. It is likely to be a $\bar{K}N$ state. It was pointed out in Ref. [4] that a future measurement of the $\Lambda(1405)$ radiative decay width should constrain the $\bar{K}N$ compositeness.

These hadrons have been investigated in global observables such as spins, parities, masses, and decay widths. For finding clear evidences of their exotic signatures, we consider to use high-energy reactions. Because quark and gluon degrees of freedom are relevant at high energies, internal configurations are expected to become apparent. Parton distribution functions are not directly measured because unstable exotic hadrons cannot be used as fixed targets. Then, it was studied that fragmentation functions could be used for such a purpose by considering the difference between favored and disfavored functions [5]. Here, we propose to use high-energy exclusive reactions for probing the internal structure of exotic hadron candidates. First, we explain that the constituent counting rule can be used for exotic hadron candidates in Sec. 2 by the scaling behavior of exclusive cross sections [6]. Second, we show in Sec. 3 that the generalized parton distributions (GPDs) and generalized distribution amplitudes (GDAs) can be used for tomography of exotic hadron candidates [7].

2. Constituent-counting rule for hard exclusive production of an exotic hadron

The cross section of a large-angle exclusive scattering $a + b \rightarrow c + d$ is given by $d\sigma_{ab\rightarrow cd}/dt \simeq \sum_{pol}|M_{ab\rightarrow cd}|^2/(16\pi s^2)$, where $s$ and $t$ are Mandelstam variables defined by the momenta $p_h$ ($h = a, b, c, d$) as $s = (p_a + p_b)^2$ and $t = (p_a - p_c)^2$. The matrix element is expressed as [8]

$$M_{ab\rightarrow cd} = \int [dx_a][dx_b][dx_c][dx_d]\phi_c([x_c])\phi_d([x_d]) \times H_{ab\rightarrow cd}([x_a],[x_b],[x_c],[x_d],Q^2)\phi_a([x_a])\phi_b([x_b]),$$

where $H_{ab\rightarrow cd}$ is the partonic scattering amplitude, and $\phi_h$ is the light-cone distribution amplitude of the hadron $h$ as illustrated in Fig. 1. A set of the light-cone momentum fractions, $x_i = p_i^+/p_h^+$ with $i$-th parton momentum $p_i$, is denoted $[x_h]$ for partons in a hadron $h$.

The high-energy behavior of the cross section is described by the constituent counting rule in perturbative QCD. As shown in Fig. 2, quarks should share large momenta, by exchanging hard gluons, so that they should stick together to form a hadron in a large-angle-exclusive reaction. Assigning hard momentum factors for the internal quarks, gluons, and external quarks, we obtain the constituent-counting rule for the cross section in perturbative QCD:

$$\frac{d\sigma_{ab\rightarrow cd}}{dt} = \frac{1}{s^{n-2}} f_{ab\rightarrow cd}(t/s).$$

Here, the factor $n$ is the number of constituents defined by $n = n_a + n_b + n_c + n_d$, and $f(t/s)$ is a scattering-angle dependent function. This expression indicates that the cross section is proportional

Fig. 1. Exclusive process $a + b \rightarrow c + d$ [6].

Fig. 2. Hard gluon exchange process for exclusive reaction [6].
to $1/s^{n-2}$ with the number of constituents. This scaling behavior is called the constituent-counting rule.

The counting rule has been experimentally investigated [9]. In various hadron two-body reactions, BNL measurements support the scaling predicted by the counting rule. Furthermore, it is also indicated in lepton-facility measurements. In Fig. 3, the cross section of $\gamma + p \rightarrow \pi^+ + n$ is shown at $\theta_{cm} = 90^\circ$ as a function of $\sqrt{s}$. Here, the cross section is multiplied by the factor $s^{7}$ because the number is $n - 2 = 7$. The figure shows typical resonance structure at low energies, whereas the cross section is almost constant at high energies. The data indicate that the transition from hadron degrees of freedom to the quark and gluon ones occurs at $\sqrt{s} = 2.5$ GeV.

The scaling of the cross section could be used for probing internal structure of an exotic hadron because the slope is controlled by the number of constituents ($n$) [6]. As an example, we estimate the cross section of $\Lambda(1405)$ production, $\pi^- + p \rightarrow K^0 + \Lambda(1405)$. For this purpose, we first investigate the ground-$\Lambda$ production $\pi^- + p \rightarrow K^0 + \Lambda$, for which there are many available data at $\theta_{cm} = 90^\circ$. The cross section data of $\pi^- + p \rightarrow K^0 + \Lambda$ show the scaling with $n = 10.1 \pm 0.6$ at high energies, and it is consistent with the number of constituents $n = 2 + 3 + 2 + 3 = 10$. Therefore, it is promising to use the counting rule also for $\Lambda(1405)$ for finding whether it is an ordinary $qqq$ state of five-quark one. There is only one experimental data available for the $\Lambda(1405)$ cross section at $\theta_{cm} = 90^\circ$. We use this data together with the counting rule to show the scaling of the cross section at high energies in Fig. 4 by assuming that the perturbative region starts at $\sqrt{s} = 2$ GeV. Two curves are shown by assuming three- or five-quark state for $\Lambda(1405)$. There are large differences between the two curves as the energy becomes larger, which should be used for judging whether $\Lambda(1405)$ is an exotic five-quark state. Such an experiment is possible, for example, at J-PARC.

Fig. 3. Cross section of $\gamma + p \rightarrow \pi^+ + n$ and scaling at large energies [6].

Fig. 4. Scaling of $\pi^- + p \rightarrow K^0 + \Lambda(1405)$ cross section at high energies [6].

3. Tomography of exotic hadrons by generalized parton distributions and generalized distribution amplitudes

We proposed that the internal quark configuration could be found by looking at the scaling of an exclusive cross section at high energies. For finding another independent evidence at high energies and for probing much details of internal structure in the three-dimensional form, we propose to use hadron tomography by using generalized parton distributions (GPDs) and generalized distribution amplitudes (GDAs) [7]. Recently, three dimensional structure of the nucleon has been investigated by using the GPDs and TMDs (transverse-momentum-dependent parton distributions) [10], particularly for understanding the origin of nucleon spin due to

Fig. 5. Virtual Compton scattering for GPD [7].
orbital angular momenta of partons.

The GPDs are measured by the virtual Compton process in Fig. 5. We define kinematical variable for expressing the GPDs. First, the momenta \( P, q, \) and \( \Delta \) are defined by the nucleon and photon momenta as \( P = (p + p')/2, q = (q + q')/2, \Delta = p' - p = q - q' \). Then, the momentum-transfer-squared quantities are given by \( Q^2 = -q^2, \bar{Q}^2 = -\bar{q}^2, \) and \( t = \Delta^2 \). The generalized scaling variable \( x \) and a skewness parameter \( \xi \) are defined by \( x = Q^2/(2p \cdot q) \) and \( \xi = \bar{Q}^2/(2\bar{P} \cdot \bar{q}) \). The variable \( x \) indicates the lightcone momentum fraction carried by a quark in the nucleon. The skewness parameter \( \xi \) or the momentum \( \Delta \) indicates the momentum transfer from the initial nucleon to the final one or the one between the quarks.

The GPDs for the nucleon are defined by off-forward matrix elements of quark (and gluon) operators with a lightcone separation between nucleonic states:

\[
\int \frac{dy}{4\pi} e^{ixP \cdot y} \left\langle p' \left| \bar{\psi}(y/2)\gamma^+ \psi(y/2) \right| p \right\rangle \bigg|_{y^+=y_0^+=0} = \frac{1}{2P^+} \overline{\pi}(p') \left[ H_q(x, \xi, t)\gamma^+ + E_q(x, \xi, t) \right] \frac{i\sigma^{\alpha\beta} \Delta_\alpha}{2M} u(p). \tag{3}
\]

Here, \( \sigma^{\alpha\beta} \) is \( \sigma^{\alpha\beta} = (i/2)[\gamma^\alpha, \gamma^\beta] \), and the unpolarized GPDs of the nucleon are \( H_q(x, \xi, t) \) and \( E_q(x, \xi, t) \). The quark field is denoted as \( \psi(y/2) \), and \( u(p) \) is the Dirac spinor. The GPDs contain information on both longitudinal momentum distributions of partons and transverse form factors. It can be seen first by taking the forward limit \( (\Delta, \xi, t \to 0): H_q(x, 0, 0) = q(x) \), where \( q(x) \) is an unpolarized parton distribution function (PDF) in the nucleon. Next, their first moments are the form factors of the nucleon:

\[
\int_0^1 dx H_q(x, \xi, t) = F_1(t) \quad \text{and} \quad \int_0^1 dx E_q(x, \xi, t) = F_2(t).
\]

Because of these two ingredients, a useful functional form of the GPDs

\[
H_q^0(x, \xi = 0, t) = f_n(x) F_n^0(t, x), \tag{4}
\]

is often used. Here, \( f_n(x) \) is a longitudinal PDF, and \( F_n^0(t, x) \) is a transverse form factor at \( x \).

Because there is no stable exotic hadron, the GPDs cannot be directly measured except for transition GPDs, for example, for \( p \to \Lambda(1405) \). However, we consider a gedankenexperiment by which the PDFs or the GPDs of the exotic hadrons can be obtained by assuming a stable target. A simple form of the PDFs is given by \( f_n(x) = C_n x^{\alpha_n} (1 - x)^{\beta_n} \), where the parameters are constrained by the valence-quark number \( \int_0^1 dx f_n(x) = n \) and the quark momentum \( \int_0^1 dx x f_n(x) = \langle x \rangle_q \). The parameter \( \beta_n \) could be determined by the constituent counting rule as \( \beta_n = 2n - 3 + 2\Delta S \) with the spin factor \( \Delta S = \frac{1}{2} \) because the factor \( (1 - x)^{\beta_n} \) controls the behavior in the elastic limit \( x \to 1 \). Using this

![Fig. 6. PDFs of exotic hadrons in comparison with parametrizations of pion and proton PDFs. The solid curves are calculated by using a simple function suggested by the constituent counting rule [7].](image)

![Fig. 7. Transverse form factors for \( x = 0.2, 0.4 \) and \( \Lambda = 0.5, 1.0 \text{ GeV} \) [7].](image)
\[ \beta_n, \text{the number of constituents } n, \text{ and the momentum fraction } \langle x \rangle_q, \text{ we obtain } C_n \text{ and } a_n. \]

The calculated PDFs are shown in Fig. 6 for \( n = 2 \) (meson), 3 (baryon), 4 (tetraquark), and 5 (pentaquark). In comparison, typical PDFs determined from experimental data are shown by the dashed curves for the pion and nucleon at \( Q^2 = 2 \text{ GeV}^2 \). Although differences between the dashed and solid curves vary depending on the \( Q^2 \) value, we obtain a reasonable agreement. It means that the PDFs obtained by the counting rule are reasonable magnitude estimates. As the valence-quark number becomes larger \( (n = 4, 5) \), the distribution shifts toward the small-\( x \) region. In Fig. 7, transverse form factors are shown by assuming the exponential form \( F^h(t, x) = e^{(1-x)/s(x\Lambda^2)} \) with the cutoff parameter constrained by the transverse size as \( \langle r^2_\perp \rangle = 4(1-x)/(x\Lambda^2) \). The \( q^2 \) dependence changes significantly whether the hadron has compact \( q\bar{q} \) (\( qqq \))-like structure or diffuse tetraquark (pentaquark, hadron molecule) one.

The GDAs are defined in the same way with the GPDs in the \( s-t \) crossed channel as shown in Fig. 8. They describe the production of a hadron pair \( hh \), \( \gamma^* \gamma \rightarrow hh \). The final-hadron momenta are denoted \( p \) and \( p' \), the initial photon momenta are \( q (Q^2 = -q^2) \) and \( q' (q'^2 = 0) \). \( P \) is the total momentum \( P = p + p' \), and \( k \) is the quark momentum. The center-of-mass (c.m.) energy squared \( s \) is equal to the invariant-mass squared \( W^2 \) of the final hadron pair, \( s = (q + q')^2 = (p + p')^2 = W^2 \). Then, the variable \( \zeta \) is defined by \( \zeta = p \cdot q' / (P \cdot q') = p^+ / P^+ = (1 + \beta \cos \theta) / 2 \), where \( \beta \) is the velocity of a hadron, \( \beta = |\vec{p}| / p^0 = (1 - 4m_h^2 / W^2)^{1/2} \), with the final hadron mass \( m_h \), and \( \theta \) is the scattering angle in the c.m. frame. The GDAs are expressed by these three variables \((z, \zeta, s = W^2)\).

The GDAs are defined by the same lightcone operators between the vacuum and the hadron pair \( hh \):

\[
\Phi^h_q(z, \zeta, s) = \int \frac{dy}{2\pi} \frac{e^{(2z-1) P^+ y^-}}{(2\pi)^3} \langle h(p) \bar{h}(p') | \bar{\psi}(-y/2) \gamma^+ \psi(y/2) | 0 \rangle \bigg|_{ly^+ = y^- = 0},
\]

for a quark. The gluon GDA is defined in the similar way. The GDAs are related to the GPDs by the \( s-t \) crossing if the factorizations can be applied as shown in Figs. 5 and 8. The crossing means to move the final state \( h(p') \) to the initial \( h(p) \). It indicates that the momenta \((p, p')\) of the GDAs should be replaced by \((p', -p)\) in the GPDs. Then, the relations between the variables are given by \( z \leftrightarrow (1 - x / \xi)/2, \zeta \leftrightarrow (1 - 1 / \xi)/2, \) and \( W^2 \leftrightarrow t \), which indicates that the GDAs are related to the GPDs by

\[
\Phi^h_q(z, \zeta, W^2) \leftrightarrow H^h_q(x = \frac{1 - 2z}{1 - 2\xi}, \xi = \frac{1}{1 - 2\xi}, t = W^2).
\]

Because there are many studies on the GPDs, this relation seems to be useful for estimating the GDAs. However, we find that the relevant kinematical regions are \( 0 \leq |x| < \infty, 0 \leq |\xi| < \infty, |x| \leq \xi \), \( t \geq 0 \), which are not necessarily the usual physical regions of the GDAs, so that the information of the GPDs is not used directly for the GDAs.

The cross section for \( e\gamma \rightarrow ehh \) is given by [11]

\[
\frac{d\sigma}{dQ^2 dW^2 d\cos \theta} = \frac{\beta \alpha^3}{8 \pi^2} \frac{e^2}{(1-e)} |A_{++}(\zeta, W^2)|^2, \quad A_{++} = \sum_q \epsilon^2_q / 2 \int_0^1 dz \frac{2z - 1}{z(1-z)} \Phi^h_q(z, \zeta, W^2),
\]

in the leading order of \( \alpha_s \) by neglecting gluon GDAs. Here, \( A_{++} \) is the helicity amplitude \( A_{ij} = e^{(*)}_\mu(q) e^{(*)}_\nu(q') T^{\mu\nu} / e^2 \) for the hadron tensor \( T^{\mu\nu} \) of \( \gamma^* \gamma \rightarrow hh \), and \( \Phi^h_q \) is a quark GDA. In order to find
possible signatures of exotic hadrons, we use a simple function

\[ \Phi_q^{h(I=0)}(z, \zeta, W^2) = N_{h(q)} z^\alpha (1 - z) \beta (2z - 1) \zeta (1 - \zeta) F_{h(q)}(W^2), \]  

which satisfies the sum rules for the quark GDAs for the isospin \( I = 0 \) two-meson final states:

\[ \int_0^1 dz \Phi_q^{h(I=0)}(z, \zeta, W^2) = 0, \quad \int_0^1 dz (2z - 1) \Phi_q^{h(I=0)}(z, \zeta, W^2) = -2M_{2(q)}^h \zeta (1 - \zeta) F_{h(q)}(W^2), \]  

where \( M_{2(q)}^h \) is the momentum fraction carried by quarks. The function \( F_{h(q)}(W^2) \) is a form factor of the quark part of the energy-momentum tensor, and it may be taken in the form suggested by the counting rule: 

\[ F_{h(q)}(W^2) = 1/[1 + (W^2 - 4m_h^2)/\Lambda^2]^{n-1}, \]  

where \( n = 2 \) for ordinary \( q\bar{q} \) mesons and \( n = 4 \) for tetraquark hadrons. Using the form factor together with the GDA expression in Eq. (8), we obtain the cross section for \( e + \gamma \rightarrow e + h + \bar{h} \) as the function of \( W^2 \) in Fig. 9 by considering \( h = f_0(980) \) or \( a_0(980) \) \([7]\). Depending on the size and the constituent number, the cross section varies much as the function of \( W^2 \). Therefore, measurements of the \( e\gamma \rightarrow e' h \bar{h} \) cross section should be valuable for determining internal structure of exotic hadrons through the GDAs.

4. Summary

Exotic hadron candidates have been investigated in hadron spectroscopy and their decays; however, the high-energy processes can probe their internal structure. In this work, we studied exclusive reactions by using the idea of constituent counting rule and by the hadron tomography with the GPDs and GDAs. Because the quark and gluon degrees of freedom are relevant at high energies, it should be more appropriate to use the high-energy exclusive reactions for probing the internal configurations of exotic hadron candidates.

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