We present a model in which the question about a nature of the dark energy and the recently raised Hubble tension can be addressed at once. We consider the electroweak axion in the minimal supersymmetric standard model where the axion energy density is identified with the observed dark energy. Along with this, imposing a gauged $Z_{10}$ symmetry makes it possible to have a gravitino dark matter whose mass amounts to $\sim O(1)$ GeV. We find that the gravitino with mass $\sim O(1)$ GeV can be a good candidate of a decaying dark matter of which decay after recombination can reconcile discrepancy in local measurements of the Hubble expansion rate $H_0$ and that inferred from the cosmic microwave background observation.

I. INTRODUCTION

One of the surprising observations in physics in the last century is the nonvanishing dark energy (or cosmological constant) [1–3]. If we take seriously a landscape conjecture based on the string theories [4–12], the dark energy (DE) could be an almost static potential energy. Along with this, imposing a gauged $Z_{10}$ symmetry makes it possible to have a gravitino dark matter whose mass amounts to $\sim O(1)$ GeV. We find that the gravitino with mass $\sim O(1)$ GeV can be a good candidate of a decaying dark matter of which decay after recombination can reconcile discrepancy in local measurements of the Hubble expansion rate $H_0$ and that inferred from the cosmic microwave background observation.

Based on the MSSM, we introduce one pseudo Nambu-Goldstone chiral superfield $A$ whose imaginary part of the complex boson component is the axion ($a$). The theory is assumed to have an invariance under the shift of $A$, that is, $A \rightarrow A + i\alpha$ except for the EW $SU(2)_L$ gauge anomaly term ($\alpha$ is a real constant). In the MSSM, there is an accidental global symmetry $U(1)_{B+L}$ besides the shift symmetry which is, however, anomalous for $SU(2)_L$. Thus, we introduce higher dimensional operators to break the $U(1)_{B+L}$ explicitly so that we can generate a mass for the EW $SU(2)_L$ axion [22].

The axion superfield coupling at low energy is given by,

$$L_{\text{eff}} = \int d^2\theta \frac{1}{32\pi^2} \frac{A}{F_A} WW + \text{h.c.},$$

where $W$ is $SU(2)_L$ gauge field strength and $F_A$ is the decay constant of the axion. Hereafter, we omit the gauge and spinor indices for simplicity. We take

$$H_0 = \frac{1}{2} (1 \pm 3\text{meV})^4$$

In fact, almost the present value of the DE $\sim (3 \pm 1\text{meV})^4$ was introduced in [13] to compensate an inconsistency between the stellar age and the measured Hubble constant $H_0$ even before the observation of the DE [1–3].
FIG. 1. One anti-instanton diagram generating the axion potential $e^{-i A/F A}$. Together with the higher dimension operator $m_{3/2} \prod_{i=1}^{3}(q_i q_i l_i)(\chi \chi)^2(\tilde{H}_u \tilde{H}_d)$, we obtain the axion potential given in Eq. (4).

$F_A \sim M_P \sim 2.4 \times 10^{18}$ GeV so that the quintessence mechanism naturally works [22], where $M_P$ is the reduced Planck mass.

The potential of the axion is generated by the EW $SU(2)_L$ instantons and the dynamical scale of the potential is calculated as [22, 30, 31]

$$\Lambda^4 \sim c e^{-\frac{\pi}{\alpha_2(3M_P)} m_{3/2}^3 M_P},$$

where $\alpha_2(M_P)$ is the $SU(2)_L$ gauge coupling constant at the Planck scale, $m_{3/2}$ denotes the gravitino mass, and $c$ is the model dependent constant which is discussed in the following. In order to suppress dangerous dimension 5 operators for the proton decay \(O = QQQQL\) in the superpotential [32, 33], an Abelian flavor symmetry $U(1)_F$ was introduced under which the quarks and leptons are charged [22]. In this case, the numerical constant $c$ becomes extremely small due to the suppression by high powers of $U(1)_F$ breaking parameter, i.e., $c \simeq 10^{-13}$ [22].

Provided the EW axion plays the role of quintessence field for the DE, the gravitino mass $m_{3/2} \simeq O(1)$ TeV is required to explain the observed DE, i.e., $\Lambda_{\text{DE}}^4 \sim (1 \text{meV})^4$ by the axion potential in Eq. (2).

On the other hand, if we impose a discrete gauge symmetry as the flavor symmetry rather than the continuous $U(1)_F$, we obtain a drastically different gravitino mass. As a matter of fact, the discrete $Z_{10}$ is anomaly free [23] with the charge assignment done in Table 2 of Ref. [22]. Thus, we assume the anomaly free gauged $Z_{10}$ symmetry to suppress the dangerous dimension 5 operators for the proton decay. Then, all fermion zero modes are closed by inserting one higher dimensional operator (see Fig. 1),

$$\mathcal{L} = \kappa m_{3/2}^2 m_{3/2}^4 \prod_{i=1}^{3}(q_i q_i l_i)(\chi \chi)^2(\tilde{H}_u \tilde{H}_d),$$

where $q_i$ and $l_i$ denote the quarks and leptons of three families, $\chi$ is the $SU(2)_L$ gaugino, $\tilde{H}_u, \tilde{H}_d$ are the higgsinos, $\kappa$ is an unknown constant which we assume $\kappa \simeq O(1)$. Here and Hereafter, we take a unit of $M_P = 1$ unless otherwise specified. Notice that one insertion of $m_{3/2}$ is necessary to make the operator consistent with $U(1)_R$ symmetry. The coefficient of $m_{3/2}^2 m_{3/2}^4$ comes from the superspace integration of the Kähler potential [30, 31]. The total flavor charge of this operator is zero. Eventually, we obtain

$$\Lambda^4 \simeq \left( \frac{\kappa}{10^{-4}} \right) \left( \frac{m_{3/2}}{1 \text{GeV}} \right)^3 (1 \times 10^{-3} \text{eV})^4,$$

where $\alpha_2(M_P) = 1/23$ was used. Now it becomes clear that matching the EW axion energy scale in Eq. (4) to the cosmological constant requires $m_{3/2} \sim (0.1 - 1)$ GeV.

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2 We impose a $U(1)_R$ symmetry as in Table 1 of Ref. [22].
3 The $U(1)_F$ breaking parameter $\epsilon$ can be determined to be $\epsilon \simeq 1/17$ to explain the quark and lepton mass matrices [34, 35]. Then, in the instanton calculus, the coefficient $c$ in Eq. (2) is estimated as $\epsilon^{10^3} \simeq 10^{-13}$ to close all the fermion zero modes.

4 We thanks to M. Ibe and M. Yamazaki for discussion about this in the private communication.
5 The weak-gravity conjecture [36] is satisfied for $F_A = M_P$ if $\alpha(M_P) \simeq 2\pi$ holds. This condition is easily achieved by introducing massive matter particles at intermediate energy scales. It is surprising that the condition in Eq. (4) does not change due to a miraculous SUSY cancellation as shown in [23].
for $\kappa = 1 - 10^{-4}$. With the decay constant $F_A \sim M_{\rho}$, we obtain the axion mass $m_a \lesssim 10^{-33}$ eV which is less than the current Hubble expansion rate. Indeed, such an EW axion is able to serve as a slowly rolling quintessence dark energy field.

III. RECONCILING HUBBLE TENSION

In this section, we examine how the model presented in the previous section can help us to reconcile the Hubble tension. Our model is a concrete particle physics model for the decaying dark matter (DDM) resolution to the Hubble tension suggested in [29] and therefore its parameter space should be subject to constraints in [29].

The basic strategy taken in [29] for resolving the Hubble tension is to make evolution of the Hubble expansion rate $H(z)$ after recombination different from that in ΛCDM cosmology so that $H_0$ obtained in ADDM becomes greater than that from ΛCDM. To this end, a Monte Carlo Markov Chain (MCMC) was performed based on ADDM model with four free parameters with priors and several data points for the values of Hubble expansion rate at different redshifts within $0 \leq z \leq 2.4$ were used. The four free parameters include a fraction of a parent DM rest mass transferred to a daughter massless particle ($\epsilon$), a life time of the parent DM ($\tau$), a dark matter abundance today $\Omega_{DM}$, and a reduced Hubble parameter $h = H_0/(100\text{km/sec/Mpc})$ which are used to infer the parent DM energy density at recombination via $\rho_{DM}(a_{rec}) = \Omega_{DM}H_0^2a_{rec}^2$. In the ADDM model, $\rho_{ADDM}(a_{rec})a_{rec}^2$ value starts to decrease after onset of decay of DDM instead of remaining conserved in time. This results in an earlier transition from matter dominated era to DE dominated era.

The DDM decay produces a massless and a massive daughter particles of which four momenta are given as $p_{\mu} = (em_{DDM}, \vec{p})$ and $p_{\mu}' = ((1-\epsilon)m_{DDM}, -\vec{p})$, respectively. Interestingly, it was shown in [29] that the massive daughter particle is still distinguished from an ordinary matter in that its equation of state deviates from zero. With the framework described above, ADDM model succeeded in showing that the modified evolution of the Hubble expansion rate can ease the Hubble tension for the reported parameter spaces of the four free parameters aforementioned.

Given the constraints on the free parameters in [29], we can study how those can be applied to the physical picture we presented in Sec. II. For our model, we consider a scenario where the gravitino takes the role of DDM of which decay results in two products including an EW axion and its fermionic superpartner, axino. The former is regarded as a massless particle which inherits the energy of $\epsilon m_{3/2}$ from the gravitino while the later serves as a massive warm daughter particle. Now we go through mapping of the constraints on the four free parameters in [29] to constraints on the gravitino and axino mass, and the reheating temperature below.

Firstly, we notice that the constraint on $-2.88 \leq \log_{10}\epsilon \leq -0.64$ (68% C.L.) in [29] can be converted into the constraint on $m_a/m_{3/2}$ via the dispersion relation of the axino ($\tilde{a}$)

$$E_{\tilde{a}}^2 = m_{\tilde{a}}^2 + |\vec{p}|^2 \leftrightarrow (1-\epsilon)^2m_{3/2}^2 = m_{\tilde{a}}^2 + \epsilon^2 m_{3/2}^2. \quad (5)$$

With this, we apply the constraint on the lifetime of DDM in [29], i.e., $1.3 \leq \log_{10}(\tau/\text{Gyr}) \leq 2.18$ (68% C.L.), to the following decay rate of gravitino ($\tilde{\Psi}_\mu$) [38]

$$\Gamma(\tilde{\Psi}_\mu \rightarrow \tilde{a} + a) = \frac{m_{\tilde{a}}^3}{192\pi M_{\rho}^2} (1-r_m)^2(1-r_m^2)^3. \quad (6)$$

Then, we obtain a constraint on $m_{3/2}$. In Eq. (6), $m_{3/2}$ and $m_{\tilde{a}}$ are the gravitino and axino mass respectively and $r_m = m_{3/2}/m_{\tilde{a}}$ is used. In Fig. 2, we show the allowed parameter space for the gravitino and axino mass so obtained for $m_{3/2}$ near $O(1)\text{GeV}$. The blue and red region is based on the constraints on $\epsilon$ and $\tau$, respectively. The overlapping region is understood as the eventual allowed region for $(m_{3/2}, m_{\tilde{a}})$ to resolve the Hubble tension. The full allowed gravitino mass to resolve the Hubble tension ranges from $O(0.1)\text{GeV}$ to $O(1)\text{TeV}$. Intriguingly, we observe that the $m_{3/2}$ range in Fig. 2 covers the gravitino mass range capable of reproducing the scale of the dark energy via Eq. (4).\(^7\)

Secondly, the individual constraints on $\Omega_{DM}$ and $h$ in [29] gives the constraint on $\Omega_{DM}h^2$, which is $0.099 \leq$ [38] by referring to [39–41]. There, DM population consists of three components including the gravitino, the axino and the axion. With a life time shorter than the age of universe,
\( \Omega_{\text{DM}}h^2 \lesssim 0.137 \) (68% C.L.). Application of this constraint to the following gravitino DM abundance today [38, 42–44]

\[
\Omega_{3/2}h^2 \simeq 0.2 \times \left( \frac{T_R}{10^6 \text{GeV}} \right) \times \left( \frac{1 \text{GeV}}{m_{3/2}} \right) \times \left( \frac{M_3(T_R)}{3 \text{TeV}} \right)^2 \frac{\gamma(T_R)/(T_R^6/M_3^2)}{0.4},
\]

yields a constraint on the reheating temperature for a range of the gravitino mass. In Eq. (7), \( T_R \) is the reheating temperature, \( M_3 \) is the running gluino mass and \( \gamma \) is the gravitino production rate. With exemplary values of \( M_3 \simeq 3 \text{TeV} \) and \( \gamma(T_R)/(T_R^6/M_3^2) \simeq 0.4 \), we show in Fig. 3 the allowed parameter space for the reheating temperature so obtained for the gravitino DM mass range of our interest near \( \sim 1 \text{GeV} \). Remarkably, we realize that the required reheating temperature to accomplish the thermal production of the gravitino mass near \( \sim 1 \text{GeV} \) can be consistent with the non-thermal leptogenesis [45, 46].

Within the picture we discussed so far, one may wonder whether the saxion (the real part of the complex boson component of the chiral superfield \( A \)) can form the other component of DDM than the gravitino. In order to guarantee that the gravitino is the only DDM candidate in the model, we should suppress the primordial production of the relic saxion from its coherent oscillation.\(^8\) For that, we impose a discrete \( Z_2 \) symmetry under which \( A \) is odd [47]\(^9\) and assume that the induced mass of saxion from its coupling to inflaton is larger than the Hubble expansion rate during inflation [48]. On top of this, it is expected that the thermal production of saxion and axino is highly suppressed as well due to the decay constant \( F_A \) comparable to \( M_P \). Thereby, the model contains the gravitino with \( m_{3/2} \sim O(1) \text{GeV} \) as the sole candidate of DDM.

### IV. CONCLUSIONS

In this letter, we have pointed out possible candidates of DE and DM within MSSM with a chiral superfield \( A \) for the EW axion. The model contains \( U(1)_R \times Z_{10} \) symmetry and the shift symmetry of the \( A \) field. The \( Z_{10} \) flavor symmetry is necessary to suppress higher dimensional operators \( \hat{O} = QQQL \) dangerous for the proton decay.

Now encountering the problems for the nature of DE and the recently raised Hubble tension, we addressed the problems in this letter by (1) imposing a gauged \( Z_{10} \) flavor symmetry and (2) taking the decay constant of the EW axion to be \( F_A \simeq M_P \). These enable us to obtain (1) the dynamical scale of the EW axion potential comparable to the current DE density \( \sim (1 \text{meV})^4 \), (2) the EW axion mass around \( 10^{-33} - 10^{-34} \text{eV} \) and (3) the gravitino mass \( \sim O(1) \text{GeV} \). Therefore, we could identify the EW axion as a quintessence field for the DE. Also, by converting the constraints on the ADDM model parameters in [29] to those on the gravitino mass, axino mass and reheating temperature, we showed that the gravitino with \( m_{3/2} \sim O(1) \text{GeV} \) can be a candidate of DDM with the EW axion and axino as the decay products. With such a small mass of the gravitino, the most natural SUSY breaking mediation mechanism to the MSSM sector is the gauge mediation [49–58]. Finally, in this letter we have constructed a ADDM model reconciling the Hubble tension assuming the quintessence axion model. However, it is easily extended to a QCD axion model with a larger decay constant like a string axion model with \( F_A \simeq 10^{16} \text{GeV} \).

Finally, let us comment on the small-scale problems in the cold dark matter paradigm. In Ref. [59],\(^{10}\) a similar setup to ours is discussed as a solution to the small-scale problems (especially too-big-to-fail problem), where the axino dark matter with the lifetime \( \simeq 10^{10} \text{Gyr} \) decays into the slightly lighter gravitino with the kick velocity \( m_3/m_{3/2} \simeq 10^{-3} c \) and the axion. Contrary to this, we find that the decaying gravitino DM discussed in our work is characterized by the longer life time \( \sim 35 \text{Gyr} \) and the larger kick velocity \( \sim 10^{-3} c - 10^{-1} c \) and thus irrelevant to the small scale problems.

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\(^8\) The Kähler potential of the axion superfield is a function of \( (A + A) \)\(^{2n} \) \((n = 1, 2, 3, \ldots)\).

\(^9\) The lifetime greater than the age of universe.

\(^{10}\) See also Refs. [60–67].
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