Flux flow resistivity in the system with multiple superconducting gaps

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Abstract

We calculate the flux flow resistivity in a superconductor with multiple s-wave superconducting gaps. Our result agrees well with anomalous field dependence of the resistivity recently observed in the two-gap superconductor MgB$_2$.

Key words: flux flow, multi-gap structure, MgB$_2$

The vortex lines under an applied current catch the Lorentz force and move perpendicular to the current and magnetic field when the Lorentz force becomes larger than the pinning force. This is the flux flow state and the finite resistivity arises[1]. The essence of the flux flow resistivity is the presence of the bound states inside the vortex core with small energy gap, so that the conductivity in the core is practically normal in the usual temperature range. Such states were found by Caroli, de Gennes and Matricon by using the microscopic methods[1]. Bardeen and Stephen found that the flux flow resistivity is basically proportional to the magnetic field $H$ in both of the dirty and moderately clean s-wave superconductors[1, 2]. This relation holds well almost all of the field range in the vortex state.

The superconductivity of MgB$_2$ has the highest transition temperature ($T_c \simeq 39K$) in the metallic materials at present[3] and the great amount of investigations has been done[4]. As one of the characteristic features in this superconductor, we can point out the fact that two different superconducting gaps with s-wave pairing symmetry exist on the two separated Fermi surfaces having roughly equal density of states (DOS) [4]. Recently, the measurement

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of the flux flow resistivity in MgB$_2$ was reported by Matsuda’s group[5]. Large deviations from the $H$-linear dependence for the flux flow resistivity has been found, in spite of the fact that the pairing symmetry of MgB$_2$ is $s$-wave[4]. Such anomalous behavior will be related to the multi-gap structure, but a clear explanation has not been shown so far. In this paper, we investigate the flux flow state in the multi-gap model and propose new mechanism for the anomalous flux flow resistivity.

First, let us remind the flux flow in the system with single superconducting band with a $s$-wave gap[1]. We suppose that the magnetic field $\mathbf{H} = H\hat{z}$ and that the pinning force is negligible[5]. The velocity of flux lines $\mathbf{v}$ is related to the applied current $\mathbf{J}$ by the balance equation

$$\mathbf{J} \times \hat{z} \Phi_0 / c = \eta \mathbf{v},$$

(1)

where $\Phi_0 = hc/2e$ is the unit flux and the l.h.s. and the r.h.s of Eq. (1) denote the Lorentz force and viscous force, respectively. The origin of viscosity $\eta$ is the dissipation caused by the Calori-de Gennes-Matricon bound states inside the core with small energy gap[1]. The dissipation induces the electric field and its macroscopic value is $\mathbf{E} = \mathbf{H} \times \mathbf{v}/c$. By using this equation and Eq. (1), we can derive $\mathbf{J} = \mathbf{E}/\rho_f$, where $\rho_f = \Phi_0 H/\eta c^2$ denotes the flux flow resistivity. As we mentioned, Bardeen and Stephen found that $\rho_f$ is proportional to the magnetic field, namely, $\rho_f = \rho_n H/Hc_2$, where $\rho_n$ is the normal state resistivity[1]. Then, we obtain $\eta$, which is independent of $H$, and $\mathbf{v}$ is determined by Eq. (1).

Next, we discuss the multi-gap system. We suppose that there are two superconducting bands with different-sized $s$-wave gaps in the system. The vortex states appear for each bands when the magnetic field greater than the lower critical value is applied to the system. The applied current $\mathbf{J}$ is divided and the flux flow states arise for each two bands. Let $\mathbf{J}_1$ and $\mathbf{J}_2$ be the divided current in 1st-band and that in 2nd-band, respectively. We suppose that vortices in the 1st-band and that in the 2nd-band have the same velocity, since two kind of the cores would be overlapped because of the nature of the superconductivity.

We extend the balance equation Eq. (1) to the two-gap system, that is

$$\mathbf{J}_i \times \hat{z} \Phi_0 / c = \eta_i \mathbf{v},$$

(2)

where $\eta_i$ ($i = 1, 2$) is the viscosity for the $i$-th band vortices. By using the equation $\mathbf{E} = \mathbf{H} \times \mathbf{v}/c$, we can derive $\mathbf{J}_i = \mathbf{E}/\rho_f^{(i)}$, where $\rho_f^{(i)} = \Phi_0 H/\eta c^2$.
denotes the flux flow resistivity in \( i \)-th band. Then,

\[
\mathbf{J} = \sum_{i=1}^{2} \mathbf{J}_i = \sum_{i=1}^{2} \frac{1}{\rho_f^{(i)}} \mathbf{E},
\]  

and this equation indicates that the total flux flow resistivity in this system is

\[
\frac{1}{\rho_f^{\text{two}}} = \sum_{i=1}^{2} \frac{1}{\rho_f^{(i)}},
\]

which reminds us the parallel connection between two different resistors.

The resistivity \( \rho_f^{(i)} \) may obey the Bardeen-Stephen relation, i.e.,

\[
\rho_f^{(i)} = \frac{H}{H_{c2}^{(i)} \rho_n^{(i)}},
\]

where \( H_{c2}^{(i)} \) and \( \rho_n^{(i)} \) are the upper critical field for the vortex state and normal-state resistivity in the \( i \)-th band, respectively.

Let us compare \( \rho_f^{\text{two}} \) in Eq. (4) and the experimental results. First we would like to check the field dependence of \( \rho_f^{\text{two}} \). Suppose that \( H_{c2}^{(1)} > H_{c2}^{(2)} \).

By using Eq. (5),

\[
\rho_f^{\text{two}} = \begin{cases} 
\frac{H}{H_{c2}^{(1)} \rho_n^{(1)} + H_{c2}^{(2)} \rho_n^{(2)}} & (\text{lower critical field } H \leq H_{c2}^{(2)}), \\
\frac{H}{H_{c2}^{(1)} \rho_n^{(1)} + H_{c2}^{(2)} \rho_n^{(2)}} & (H_{c2}^{(2)} < H \leq H_{c2}^{(1)}).
\end{cases}
\]

Then, \( \rho_f^{\text{two}} \) is \( H \)-linear in the low field regime, convex in the high field regime and continuously connected to the normal-state resistivity in the two-band system \((1/\rho_n^{(1)} + 1/\rho_n^{(2)})^{-1}\).

The flow resistivity measurement in MgB\(_2\) was demonstrated for the magnetic field parallel to the \( c \)-axes of the crystal and also for the field in \( ab \)-plane and both of which show anomalous behavior[5]. We use the parameters \( H_{c2}^{(2)}/H_{c2}^{(1)} = 0.04 \) and \( \rho_n^{(2)}/\rho_n^{(1)} = 0.26 \) for \( H \parallel ab \) case, and \( H_{c2}^{(2)}/H_{c2}^{(1)} = 0.06 \) and \( \rho_n^{(2)}/\rho_n^{(1)} = 0.5 \) for \( H \parallel c \) case. The order of these values appear to be consistent with the facts that the DOS for each two bands are roughly equal, the energy size of larger gap is 7 mev and that
of smaller gap is 2mev [4], and the carrier scattering rates for each two bands are almost equal[5]. As is given in Fig. 1, our result agrees well with experimental data, especially for $H \parallel c$ case.

Figure 1: We compare the experimental data of flux flow resistivity in MgB$_2$ given by Ref. [5] and our result Eq. (6). Black dots and triangles show the experimental results for $H \parallel ab$ case and $H \parallel c$ case, respectively. Green line shows Eq. (6) with parameters $H_{c2}^{(2)}/H_{c2}^{(1)} = 0.04$ and $\rho_n^{(2)}/\rho_n^{(1)} = 0.26$, and Red line shows that with $H_{c2}^{(2)}/H_{c2}^{(1)} = 0.06$ and $\rho_n^{(2)}/\rho_n^{(1)} = 0.5$.

In summary, we have considered a system with two superconducting bands. The total flux flow resistivity in the system is written as the parallel connection of the flux flow resistivity for the 1st-band vortices and that for the 2nd-band vortices. The result agrees with anomalous field dependence of the flow resistivity recently observed in the two-gap superconductor MgB$_2$. Our discussion would be generalized to the system with $n(>2)$ superconducting bands. The detailed discussion will be published elsewhere.
References

[1] “Introduction to Superconductivity”, M. Tinkham (2nd. ed., McGraw-Hill, Singapore, 1996), and references therein.

[2] Y. Matsuda, et al. Phys. Rev. B 66 (2002) 014527.

[3] Nagamatsu, N. et al. Nature (London) 410 (2001) 63.

[4] See, for the recent review, P. C. Canfield and G. W Crabtree, Physics Today 56 (3), (2003) 34.

[5] A. Shibata et al. Phys. Rev. B 68 (2003) 060501(R).