Sensitivity Study on Influencing Factors of Coupled Motion Characteristics for Truss Spar

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Abstract. In the absence of mooring condition, the heave and pitch coupled equations are established considering the effect of static stability and wave surface elevation for a truss spar platform. The influence of the change of the pitch period and the external excitation force (combined wave loads and wind loads together) on the stability of the heave-pitch coupling motion is studied by means of numerical iteration method. The results show that when the wave frequency is close to the natural heave frequency besides the heave-pitch frequency ratio is close to 2:1, both of the amplitude of heave and pitch motion are increased considering the wind loads. It is about 12 percent increase for heave motion, while 8 percent for pitch motion. Meanwhile, there is a penetration phenomenon between heave and pitch. And the wind load play a more important role in pitch motion, the characteristic frequency of pitch motion will have an obvious change between 1/2 times sub harmonic motion frequency component and 1 time forced motion frequency component considering the wind load.

1. Introduction

Truss spar platform is the one that improved on the basis of the classic spar, and it can avoid the risk that the first generation of classical spar platform will be broken down during the process of assembling, transporting and erecting. In 2002, Nansen platform was installed in the East Breaks 602 area by Ker-McGee Company, which indicated the birth of second generation of Spar platform. The main body of the platform is a large draft and large diameter floating column structure, the difference of the internal oil storage will make the gravity center and the buoyant center adjust, so the natural period of the platform will change accordingly. Under normal sea conditions, the effect of this slight change on the platform body may not be obvious, but considering the particularity of heave-pitch coupled inner resonance, the change of the period value may have a great influence on the motion results, for example, in some marine environments (eg, West Africa and Indonesia) the long-period conditions may occupy a certain proportion, and these surge peak period can reach 30s.

In this paper, combined the wind and wave loads, the dynamic response of the Truss Spar platform is studied. The effect of wind load on the heave-pitch motion will be tested in the simulation. Furthermore, keep the incident wave frequency invariable, and change the pitch period, the sensitivity effects on the heave-coupled motion will be found.
2. The establishment of motion equation

\[
\begin{align*}
(m + m_{33}) \ddot{\xi} + B_3 \dddot{\xi} + B_{32} \dddot{\xi} + \rho g A_w \left( \zeta_3 - \frac{\xi_3^2}{2} H_g + \frac{\xi_3^2}{2} \zeta_3 \right) &= F_3 (\omega + \theta) \\
(I + I_{55}) \dddot{\xi} + B_{31} \dddot{\xi} + B_{32} \dddot{\xi} + \rho g \sqrt{\frac{G M}{\rho h}} \dot{\xi} + \frac{1}{2} \rho g V + \rho g A_w \sqrt{\frac{G M}{\rho h}} \eta &= -\left( \frac{1}{2} \rho g V + \rho g A_w \sqrt{\frac{G M}{\rho h}} \right) \dot{\xi} + \frac{1}{4} \rho g V H_g + \frac{1}{2} \rho g A_w \sqrt{\frac{G M}{\rho h}} H_g \dot{\xi} = M_5 + M_{wind}
\end{align*}
\]

Where, \( m \) and \( m_{33} \) are the body mass and body’s added mass, respectively. \( \xi_5 \) is the pitch amplitude. \( I \) and \( I_{55} \) are pitch moment and added moment of inertia, respectively. \( F_3 \) and \( M_5 \) are stochastic wave force and stochastic wave moment, respectively. \( M_{wind} \) is wind heeling moment. \( B_{31} \) and \( B_{32} \) are linear damping coefficient and quadratic damping coefficient in heave motion. \( B_{33} \) and \( B_{52} \) are linear damping coefficient and quadratic damping coefficient in pitch motion. \( H_g \) is the distance from the center of gravity to the undisturbed water surface. \( \sqrt{G M} \) is the metacentric height. \( A_w \) is water plane area. \( \eta \) is wave elevation.

Normalize the equation (1) and (2), the following equation can be obtained,

\[
\begin{align*}
\ddot{\xi} + a_{11} \dddot{\xi} + a_{12} \dddot{\xi} + \omega_A^2 \xi_3 - a_{13} \zeta_3^2 + a_{14} \xi_3 \zeta_3 &= F_c \\
\dddot{\xi} + b_{11} \dddot{\xi} + b_{12} \dddot{\xi} + \omega_B^2 \xi_3 - a_{23} \zeta_3^2 + a_{24} \xi_3 \zeta_3 &= \overline{M}_c + \overline{M}_{wind}
\end{align*}
\]

Where, \( a_{11} = \frac{B_{31}}{m + m_{33}} \), \( a_{12} = \frac{B_{32}}{m + m_{33}} \), \( a_2 = \frac{\rho g A_w H_g}{2(m + m_{33})} \), \( a_3 = \frac{\rho g A_w}{2(m + m_{33})} \), \( F_c = \frac{F_3}{m + m_{33}} \), \( b_{11} = \frac{B_{31}}{I + I_{55}} \), \( b_{12} = \frac{B_{32}}{I + I_{55}} \), \( b_2 = \frac{\rho g (V + 2A_w \sqrt{G M})}{2(I + I_{55})} \), \( b_3 = \frac{\rho g H_g (V + 2A_w \sqrt{G M})}{4(I + I_{55})} \), \( \overline{M}_c = \frac{M_5}{I + I_{55}} \).

It can be concluded that there is an obvious coupling relationship between the vertical motion equation and the pitch motion equation of Truss Spar platform.

3. The calculation of environmental loads

The main particulars of Truss Spar are according to the “Horn Mountain” platform, which are given in Table 1.

| Table 1. Main particulars of the Truss Spar. |
|---------------------------------------------|
| Body diameter | 32.31m |
| Draft         | 153.924m |
| Center of gravity | 90.39m |
| Length of hard tank | 68.88m |
| Heave plate   | 32.31m x 32.31m |
| Spacing of heave plates | 23.8 m |
| Displacement | 56401.45 t |
| Number of heave plates | 3 |
| Pitch gyradius | 60.96m |
| Linear damping of heave | 0.0379 |
| Quadratic damping of heave | 0.0186 |
| Linear damping of pitch | 0.0145 |
| Quadratic damping of pitch | 0.0154 |
Wind loads calculation: The upper module of the platform is shown in Figure 1, including the cylinder sleeve at the bottom, double deck at the middle, the buildings at the uppermost deck, two cranes and crane at the platform side etc. According the recommendation in "Rules for mobile offshore drilling units by CCS", projected are for all columns are included without considering the shading effect, and the surface wind load on the platform is calculated.

![Figure 1. The simplified diagram.](image)

Wave load calculation: The first-order wave load is calculated based on the three-dimensional diffraction and radiation theory. The wave force acting on the vertical cylinder can be obtained according literature "Weggel D, Roesset J (1994) Vertical hydrodynamic forces on truncated cylinders".

\[
F_x = \rho g H \pi R^2 \left[ 1 - \frac{1}{2} \sin(kR) \left( \frac{J_1(kR)}{kR} \right) e^{-kd} \cos(\alpha t - \alpha_1) \right]
= 2 \rho g A \pi R^2 \left[ 1 - \frac{1}{2} \sin(kR) \left( \frac{J_1(kR)}{kR} \right) e^{-kd} \cos(\alpha t - \alpha_1) \right]
\]

(4)

\[
M_x = \int_0^\infty \frac{4 \rho g A}{k^2} \frac{\cos(kz)}{\cosh(kd)} \left[ \frac{1}{J_1(kR)} \right]^2 + \left( \frac{Y_1(kR)}{Y_1(kR)} \right)^2 \cos(\alpha t - \alpha_1)(z_0 - z) \, dz
= \frac{4 \rho g A}{k^2} \frac{1}{\cosh(kd)} \left[ \frac{1}{J_1(kR)} \right]^2 + \left( \frac{Y_1(kR)}{Y_1(kR)} \right)^2 \cos(\alpha t - \alpha_1)
\]

(5)

Where \( \alpha_1 = 31.0 \times \frac{\pi}{180} (kR)^{-1} \), \( \alpha = \arctan \left( \frac{J'_1(kR)}{Y_1(kR)} \right) \), \( J_n(x) = \sum_{k=0}^{n} \frac{(-1)^k}{k! \Gamma(n+k+1)} \left( \frac{x}{2} \right)^{n+k} \),

\[
Y_n(x) = \frac{1}{\sin(n\pi)} \left[ J_n(x) \cos(n\pi) - J_{-n}(x) \right] = \frac{\cos(n\pi) - (-1)^n}{\sin(n\pi)} J_n(x) \quad J'_n(x) = \frac{1}{2} \sum_{k=0}^{n} \frac{(-1)^k}{k! \Gamma(n+k+1)} \left( \frac{x}{2} \right)^{n+k+1} \]

4. Analysis of calculation results
Studies have shown that, when the heave and pitch frequency ratio is 2:1 compared to the internal resonance relationship, the unstable chaotic motion of the platform will occur when the external wave excitation frequency is close to the natural frequency of the heave motion. However, the influence of
wind load was not considered in previous studies, so this study comprehensively considered the combined effects of wind and wave loads on the sensitivity of its motion characteristics. The calculation conditions are as follows:

(1) The heave and pitch frequency ratio is equal to 2:1 (heave natural period is 20s, pitch natural period is 40s), and the spar platform is in condition of the main heave resonance (that is wave frequency is close to or equal to heave natural frequency). The influence of wind load is considered separately, then the motion response of the two degrees of freedom is calculated and the spectra of the data is analyzed.

Figure 2. The spectrograms of heave and pitch motion (pitch period 40s, wind velocity 16.0m/s).

The studies show that, when the wind load considered, both of the amplitude of heave and pitch motion are increased. It is about 12 percent increase for heave motion, while 8 percent for pitch motion. Meanwhile, it can be observed that, there is a phenomenon the energy is transferred from heave motion mode to pitch motion mode. That is the heave motion amplitude is rapidly increasing to a certain value with the time, and then the amplitude decreases, and becomes stable gradually. On the other hand, the pitch motion amplitude is increasing to a certain value with the time, and becomes stable gradually after a small drop.

The wind load play a small role in heave motion. The frequency is the same with the external wave frequency. For pitch motion, when without considering the wind load, the periodic components appear, of which the 1/2 times sub harmonic motion frequency is the main, and then followed the 1 times forced motion frequency. While considering the wind load, the periodic components also appear, however the sub harmonic components are weakened. 1 times forced motion frequency is the main, and then followed the 1/2 times sub harmonic motion frequency. The main reason is that the wind load is constant combined the wave load, 1time forced motion is more pronounced.
(2) The heave and pitch frequency ratio is equal to 2.5:1 (heave natural period is 20s, pitch natural period is 50s), and the spar platform is in condition of the main heave resonance (that is wave frequency is close to or equal to heave natural frequency). The influence of wind load is considered separately, then the motion response of the two degrees of freedom is calculated and the spectra of the data is analyzed.

![Figure 4. The spectrograms of heave and pitch motion (pitch period 50s, wind velocity 16.0m/s).](image)

![Figure 5. The spectrograms of heave and pitch motion (pitch period 50s, without considering wind).](image)

The studies show that, when the platform is in the state of main heave resonance motion, and the heave and pitch frequency ratio is far away from 2:1, then no pitch instability movement will occur whatever considering the wind load or not. When wind load considered, both of the frequency components of heave and pitch motion are forced motion component. That is 1time of wave frequency component, with no characteristics of the periodic components and sub harmonic motion.

5. Conclusions

Based on the numerical study of the wind load effect on Truss Spar platform, the following conclusions can be drawn:

1. Under the regular wave action, considering the influence of wind load, the inner resonant characteristics of the heave - pitch coupled motion are weakened, and the components of the sub harmonic motion are reduced.

2. The heave and pitch frequency ratio is equal to 2:1, and the spar platform is in condition of the main heave resonance when the wind load considered, both of the amplitude of heave and pitch motion are increased. It is about 12 percent increase for heave motion, while 8 percent for pitch motion. Meanwhile, it can be observed that, there is a phenomenon the energy is transferred from heave motion mode to pitch motion mode.
(3) The wind load play a more important role in pitch motion, for considering the wind load, the characteristic frequency of pitch motion will have an obvious change, between 1/2 times sub harmonic motion frequency component and 1 time forced motion frequency component.

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