Investigation of $^{23}$N in a three-body model

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Abstract

The neutron-drip-line nucleus $^{23}$N is investigated in a three-body model consisting of a $^{21}$N core and two valence neutrons. Using the Faddeev formalism with the realistic neutron-neutron potential and the neutron-core potential, we calculate the ground-state properties of $^{23}$N including its two-neutron separation energy, and obtain good agreement with the experiments. We also find an excited $^{23}$N state with a shallow two-neutron separation energy at about 0.18 MeV. By evaluating the root-mean-square matter radii, the average distances between the two valence neutrons, and the average distances from the core to the center-of-mass of the valence-neutron pair, we show that the excited state of $^{23}$N has a distinct halo structure. Through calculating the correlation density distributions of the $^{23}$N three-body system in configuration space, we find that the excited state of $^{23}$N has a triangular shape, which is similar to but much more extended than the ground state. This scaling symmetry between the ground and excited states indicate the existence of an Efimov state in $^{23}$N.

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I. INTRODUCTION

The properties of neutron-rich nuclei close to the drip-line are crucial for understanding fundamental mechanism governing the stability and formation of nuclei. In recent years, tremendous experimental and theoretical efforts have been devoted to the investigation of neutron-rich nuclei [1–5]. The discoveries of light nuclei with halo structures have attracted great attention because of their exotic properties [6–10] and their important role in the big bang nucleosynthesis [11]. For the nitrogen isotopes, many studies have focused on neutron-rich isotopes with mass number ranging from \( A = 17 \) to \( A = 22 \) [12–17]. However, \( ^{23}\text{N} \) has rarely been investigated since its first production in 1998 [18, 19]. As a newly synthesized drip-line nucleus, many physical properties of \( ^{23}\text{N} \) such as the energy spectrum and the structure configuration have not been observed or predicted yet except for the one-neutron separation energy \( S_n \). \( S_n \)s in \( ^{22}\text{N} \) and \( ^{23}\text{N} \) are respectively 1.28 MeV and 1.79 MeV, which are much smaller than that in \( ^{21}\text{N} \) of 4.59 MeV [20]. Therefore, in a first-order approximation, one can treat the \( ^{23}\text{N} \) nucleus as a three-body system composed by an inert \( ^{21}\text{N} \) core, which is tightly bound, and two valance neutrons moving at a relatively large distance away from the core.

Various few-body methods have been adopted to calculate different three-body quantum systems, such as the Faddeev formalism [21], the equivalent two-body method [22, 23], the Green’s Function Monte Carlo [24], and the THSR (Tohsaki-Horiuchi-Schuck-Röpke) wave function [25, 26]. Among these different approaches, the Faddeev formalism has a specific advantage that it can also describe the three-body mechanism in a heavier system, which is normally encoded in microscopic calculations, in a computationally simpler way [21]. Various neutron-rich nuclei, including \( ^{6,8}\text{He}, ^{11}\text{Li}, ^{12,14}\text{Be}, ^{17}\text{B}, \) and \( ^{22}\text{C} \) have been investigated with the Faddeev formalism [27–36]. This approach has also been applied recently to proton-rich nuclei, such as \( ^{17}\text{Ne}, ^{18}\text{Ne}, \) and \( ^{28}\text{S} \) [37, 38]. In this work, we have applied the Faddeev equations to the \( ^{21}\text{N}+n+n \) three-body system. By solving these equations, we obtain various properties of the \( ^{23}\text{N} \) nucleus, which are essential for understanding the halo structure inside the nucleus.

In Sec. II, we provide a brief introduction to the Faddeev equations and the two-body interactions adopted in our calculations. In Sec. III, we present the numerical results of the three-body calculation and analyze the halo structure of the ground state and our newly
discovered excited state in \(^{23}\)N. At last, the conclusion is given in Sec. IV.

**II. FORMALISM**

In the Faddeev formalism, the full wave function \(\Psi^{JM}\) for a three-body system can be decomposed into three components with respect to different sets of Jacobi coordinates \[39\]:

\[
\Psi^{JM} = \Psi_1^{JM}(x_1, y_1) + \Psi_2^{JM}(x_2, y_2) + \Psi_3^{JM}(x_3, y_3),
\]

where \(\Psi_i^{JM}\) is the \(i\)-th component of the full wave function with total angular momentum \(J\) and its \(z\)-component \(M\). The \(i\)-th set of the Jacobi coordinates corresponding to this component is shown in Fig. 1 for \(i = 1, 2, 3\) \[21\].

**FIG. 1.** Three sets of Jacobi coordinates in the Faddeev formalism for the core+n+n three-body system.

The coupled-channel Faddeev equations for the core+n+n system are expressed as

\[
(T_1 + V_1 - E)\Psi_1^{JM} = -V_1(\Psi_2^{JM} + \Psi_3^{JM})
\]
\[
(T_2 + V_2 - E)\Psi_2^{JM} = -V_2(\Psi_3^{JM} + \Psi_1^{JM})
\]
\[
(T_3 + V_3 - E)\Psi_3^{JM} = -V_3(\Psi_1^{JM} + \Psi_2^{JM})
\]

where \(T_i = T_{xi} + T_{yi}\) is the relative kinetic energy in \(i\)-th coordinate set. \(V_i \equiv V_{jk}\) represents the effective two-body nuclear force between particles \(j\) and \(k\). The indices \((i, j, k) \equiv (1, 2, 3)\) run through cyclic permutation \[21\].
We then introduce the Jacobi coordinates, which include the hyper radius $\rho$ and the hyper-angle $\theta_i$. They are related to the Jacobi coordinates by \[ \rho^2 = x_i^2 + y_i^2, \quad \theta_i = \arctan\left(\frac{x_i}{y_i}\right). \] (3)

One should notice that the hyper-radius is the same in different sets of Jacobi representation, but the hyper-angles are different. To solve the Faddeev equations, we adopt the hyperspherical harmonic method. Using this approach, one can conveniently separate the hyper-angle and hyper-radial dependencies of the wave function. The hyperspherical decomposition of the i-th component of the wave function is defined as \[ \Psi^{JM}_i = \rho^{-5/2} \sum_{\alpha_i} \sum_{K_i} \chi^{i,J}_{\alpha_i,K_i}(\rho) \phi^{l_{xi} l_{yi}}_{K_i}(\theta_i) |i : \alpha_i\rangle, \] (4) where $\alpha_i \equiv \{l_{xi}, l_{yi}, L_i, s_j, s_k, S_{xi}\} J_i$ and $|i : \alpha_i\rangle$ represent the quantum numbers in the specific partial-wave channel occupied by the three-body system. The hyper-angular function $\phi^{l_{xi} l_{yi}}_{K_i}(\theta_i)$, which are the eigen-solution of the hyper-angular equation, is expanded in terms of Jacobi polynomials as
\[ \phi^{l_{xi} l_{yi}}_{K_i}(\theta_i) = N^{l_{xi} l_{yi}}_{K_i} (\sin \theta_i)^{L_{xi}} (\cos \theta_i)^{L_{yi}} P^{l_{xi}+1/2,l_{yi}+1/2}_{n_i} (\cos 2\theta_i). \] (5)

Here $P^{l_{xi}+1/2,l_{yi}+1/2}_{n_i} (\cos 2\theta_i)$ denotes the Jacobi polynomial with $n_i = 0, 1, 2, \ldots$. $N^{l_{xi} l_{yi}}_{K_i}$ is the normalization coefficient, and $K_i = l_{xi} + l_{yi} + 2n_i$ indicates the hyper-angular momentum with respect to the corresponding Jacobi polynomial.

After substituting Eq. (5) into Eq. (4), the hyper-radial function $\chi^{i,J}_{\alpha_i,K_i}(\rho)$ and the hyper-angular function $\phi^{l_{xi} l_{yi}}_{K_i}(\theta_i)$ are obtained using the numerical program to solve the Faddeev equations \[.] (21)

In the calculations of $^{23}\text{N}$ as the $^{21}\text{N}+n+n$ system, the core-neutron and the neutron-neutron interactions can be determined phenomenologically by fixing the experimental data. In this paper, we adopt the well-known GPT potential neglecting only the spin-spin term for the n-n interaction and keeping the central, tensor and spin-orbit parts. This potential provides good fits to the low-energy properties of the low-energy n-n scattering \[.] (31, 40). For the core-neutron interaction, we adopt a Woods-Saxon form \[.] (34, 37, 41)\n\[ V_{n-core}(r) = \frac{V_0}{1 + \exp\left(\frac{r-a}{\alpha}\right)} + \frac{V_{so}}{ra} \exp\left(\frac{r-a}{\alpha}\right) (1 + \exp\left(\frac{r-a}{\alpha}\right))^2 \cdot \mathbf{L} \cdot \mathbf{S}, \] (6)
where \( r_0 = 1.25A^{1/3} \text{ fm} \) and \( a = 0.65 \text{ fm} \). \( V_0 \) is the depth of the Woods-Saxon potential, and \( V_{so} \) represents the strength of the spin-orbit coupling. \( V_0 \) and \( V_{so} \) can be determined by fixing the binding energies of the core-neutron two-body subsystem. We then apply the super symmetric transformation to this interaction to eliminate the spurious bound states which are forbidden by the Pauli principle.

III. NUMERICAL RESULTS

We discuss in this section the numerical results for calculating \(^{23}\text{N}\) in a three-body model. We firstly discuss the physical model for the core and explain our choices of parameterization for the effective n-\(^{21}\text{N}\) interaction, which are suitable to reproduce the experimental data. Then we solve the Faddeev equations and calculate the neutron-separation energies and configuration-space wave functions of \(^{23}\text{N}\). These observables can be utilized to analyze the halo structure of \(^{23}\text{N}\).

To construct the effective n-\(^{21}\text{N}\) interaction, information about the shell-model occupations of the \(^{21}\text{N}\) core and valance neutrons, which can be determined from experiments, needs to be known. Here we make two assumptions in the shell-model description. Firstly, we assume that the neutrons inside the \(^{21}\text{N}\) core occupy the \((0s_{1/2})^2\), \((0p_{3/2})^4\), \((0p_{1/2})^2\) and \((0d_{5/2})^6\) orbits, which are forbidden to be occupied by the valence neutrons due to the Pauli principle. With the lack of the experimental information about the excited states in \(^{21}\text{N}\), we simply neglect these excited states in our calculations as a first-order approximation. Similar assumptions are also made in the work by other groups, e.g., in Refs. [37, 41, 42]. With these limitations, we assume that the valence neutron occupies the \(1s_{1/2}\) state in the n-core sub-system of \(^{22}\text{N}\).

Using the above assumptions and the Wood-Saxon potential form, i.e., Eq. (6), we determine the core-n interaction by reproducing two experimental results. The first is the one-neutron separation energy of \(^{22}\text{N}\), corresponding to the binding of the \(1s_{1/2}\) orbit, i.e., \( S_n = \epsilon(1s_{1/2}) = 1.28 \pm 0.21 \text{ MeV} \). We also take into account the one-neutron separation energy of \(^{21}\text{N}\), which is related to the binding of the \(0d_{5/2}\) orbit, i.e., \( \epsilon(0d_{5/2}) = 4.59 \pm 0.11 \text{ MeV} \). The experimental error of \( \epsilon(1s_{1/2}) \) is considered in our calculations; while the error bar of \( \epsilon(0d_{5/2}) \) is neglected due to its relatively small size. By reproducing the upper bound, lower bound and mean value of \( \epsilon(1s_{1/2}) \), and also fixing the mean value of \( \epsilon(0d_{5/2}) \), we de-
termine three sets of parameters of the core-n interactions and list them in Table I. The corresponding root-mean-square (r.m.s.) radii of the core, the valence neutron and total matter in the $^{21}$N-n two-body system are calculated with respect to different parameterizations and shown in Table II. The neutron density distributions of $^{22}$N are shown in Fig. 2, where the probability density of the neutrons inside core and of the last neutron occupying the $1s_{1/2}$ state, are shown respectively. We find that the last neutron has a much more extended matter density distribution than the neutrons inside the core do. This clearly indicates a halo structure in the $^{21}$N-n system.

TABLE I. Parameters of the effective $^{21}$N-n interaction. $\epsilon$ is the single-particle energy of the $^{21}$N-n system. The upper bound (1.49 MeV), the mean value (1.28 MeV), and the lower bound (1.07) MeV of $S_n$ in $^{22}$N are adopted for sets A, B, and C, respectively.

| Set  | $r_0$ (fm) | $a$ (fm) | $V_0$ (MeV) | $V_{so}$ (MeV) | $\epsilon(0s_{1/2})$ (MeV) | $\epsilon(0p_{3/2})$ (MeV) | $\epsilon(0p_{1/2})$ (MeV) | $\epsilon(0d_{5/2})$ (MeV) | $\epsilon(1s_{1/2})$ (MeV) |
|------|-------------|----------|-------------|----------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| A    | 3.45        | 0.60     | -41.45      | -46.45         | -25.382                     | -14.939                     | -9.026                      | -4.590                      | -1.490                      |
| B    | 3.45        | 0.60     | -40.77      | -50.51         | -24.813                     | -14.647                     | -8.224                      | -4.590                      | -1.280                      |
| C    | 3.45        | 0.60     | -40.02      | -55.00         | -24.170                     | -14.316                     | -7.333                      | -4.590                      | -1.070                      |

TABLE II. The calculated r.m.s. radii of the core $R_{th}^c$, the valence neutron $R_{th}^n$ and total matter $R_{th}^m$ in $^{22}$N. $S_n$ denotes the corresponding one-neutron separation energy.

| $S_n$ (MeV) | $R_{th}^c$ (fm) | $R_{th}^n$ (fm) | $R_{th}^m$ (fm) |
|-------------|-----------------|-----------------|-----------------|
| Set A       | 1.490           | 3.21            | 5.25            | 3.53                        |
| Set B       | 1.280           | 3.22            | 5.47            | 3.58                        |
| Set C       | 1.070           | 3.24            | 5.75            | 3.65                        |

With the appropriate parameterization for the effective $^{21}$N-n interaction obtained above, we calculate the bound-state observables of $^{23}$N in the $^{21}$N+n+n three-body model using the Faddeev formalism. In these calculations, we use the cutoff $K_{max}=20$ in the hyperangular-momentum expansions in Eq II to provide a proper numerical accuracy. Comparing with
FIG. 2. Density distributions of neutrons in the subsystem $^{22}\text{N}$. The solid curve is the density distribution of the core neutrons, and the dashed curve is the density distribution of the last neutron.

The results of $K_{\text{max}}=10$, the difference between the two-body separation energies is about 4%, which is much smaller than the relative error of about 30% due to different choices of parameters. Thus, this cutoff $K_{\text{max}}=20$ is adequate for our Faddeev calculations of the three-body system. We calculate the two-neutron separation energies $S_{2n}$ and r.m.s. matter radii of the ground and excited states of $^{23}\text{N}$ using different sets of parameterization, and list the results in Table III. We observe that both the the ground- and excited-state $S_{2n}$ decrease when the depth of the $^{21}\text{N}$-n potential increases. We also find that the r.m.s. matter radii of the two states increase respectively with the decrease of $S_{2n}$ in the corresponding states. This correlation which show that $^{23}\text{N}$ is a standard halo nucleus, as discussed in [43]. Our obtained two-neutron separation energy of the $\frac{1}{2}^-$ ground state, $S_{2n} = 3.6 \pm 0.5$ MeV, agrees well with the experimental result, i.e., $S_{2n} = 3.07 \pm 0.31$ MeV [20]. The r.m.s. matter radius of the ground-state $^{23}\text{N}$ is about 3.0 fm, which corresponds to the size of a stable nucleus with mass number $A \approx 27$. This radius is much smaller than $^{22}\text{C}$, whose r.m.s. radius is about $5.4 \pm 0.9$ fm [44]. Therefore, although loosely-bound compared with stable nuclei, the halo structure of the $^{23}\text{N}$ ground state may not be as clear as its isotope halo nucleus $^{22}\text{C}$. 


The two-neutron separation energies $S_{2n}$ and r.m.s. matter radii $r_m$ of the ground and excited states of $^{23}$N, calculated with three sets of parametrization and from the experiments. The superscript $^*$ denotes the excited state.

|       | $S_{2n}$ | $r_m$ | $S_{2n}^*$ | $r_m^*$ |
|-------|----------|-------|------------|--------|
|       | (MeV)    | (fm)  | (MeV)      | (fm)   |
| Set A | 4.13     | 2.969 | 0.315      | 4.272  |
| Set B | 3.64     | 2.985 | 0.185      | 4.358  |
| Set C | 3.13     | 3.004 | 0.069      | 4.476  |
| Exp [20] | 3.07±0.31 | -     | -          | -      |

The $\frac{1}{2}^-$ excited state of $^{23}$N, which has not been discovered in experiments yet, is predicted in this work to have an extremely shallow two-neutron separation energy $S_{2n}^*$. With different parameterization, we obtain $S_{2n}^*$ to be in the region of 0.069–0.315 MeV, with a mean value of 0.185 MeV. The r.m.s. radii of this excited state $r_m^*$ is about 4.3 fm, which corresponds to the size of a stable nucleus with $A \approx 80$. Therefore, the excited state of $^{23}$N can be unambiguously described as a giant halo state.

To illustrate the structure of $^{23}$N, we calculate the average distances between the two valence neutrons $r_{nn}$ and the average distances from the core to the center-of-mass of the valence-neutron pair $r_{C(nn)}$ in the $^{21}$N+n+n system. The results in Table IV indicate that both ground and excited states of $^{23}$N have triangular shapes. By calculating the ratios of $r_{nn}$ and $r_{C(nn)}$ respectively in the ground and excited states, we find that

$$\frac{r_{nn}}{r_{C(nn)}} \approx \frac{r_{nn}^*}{r_{C(nn)}^*} \approx 1.9.$$  (7)

The same ratio obtained in both states suggests that these two states have similar geometric structures and are mainly different by a spatially discrete scaling factor.

Furthermore, we also analyze the correlation density distributions of the ground and excited states of $^{23}$N in configuration space. These quantities are evaluated in the Jacobi-coordinate representation with $^{21}$N as the spectator particle, i.e., in the third diagram in Fig. II. The spatial correlation density distribution is calculated as

$$P(r_{nn}, r_{C(nn)}) = x^2 y^2 \int |\Psi_{JM}^m(x, y)|^2 d\Omega_x d\Omega_y.$$  (8)
TABLE IV. The average distances between the two valence neutrons $r_{nn}$ and the average distances from the core to the center-of-mass of the valence-neutron pair $r_{C(nn)}$ in the $^{23}$N ground and excited states. The superscript $\ast$ denotes the excited state.

|         | $r_{nn}$ | $r_{C(nn)}$ | $r_{nn}^\ast$ | $r_{C(nn)}^\ast$ |
|---------|----------|-------------|---------------|-----------------|
| Set A   | 6.415    | 3.357       | 14.600        | 7.640           |
| Set B   | 6.681    | 3.496       | 16.326        | 8.543           |
| Set C   | 6.878    | 3.599       | 17.059        | 8.927           |

The correlation densities for the ground and excited states are shown in Figs. 3 and 4, respectively. On the one hand, we find that the ground state has the largest probability density when the two neutrons are at a distance of about 3.5 fm from the core. On the other hand, the excited state has the largest probability density when the two neutrons are far away from the core with a distance of about 8.5 fm, and are separated from each other with a distance of about 16 fm. The spatial separation in the excited state of $^{23}$N supports the halo structure suggested above. Furthermore, in Fig. 4, a second peak with a smaller amplitude, which mainly comes from the $(0d)^2$ component, also appears in the excited state. The occupancies of the valence neutrons are also calculated. For the ground state, the occupancies of the two valence neutrons in the $(1s)^2$ and $(0d)^2$ states are 95% and 5%, respectively. For the excited state, the occupancies of the two valence neutrons in the $(1s)^2$ and $(0d)^2$ states are 77% and 23%, respectively.

Since the excited state has a much shallower two-neutron separation energy, a much larger r.m.s. radius, and much more extended spatial distribution than the ground state, one may be able to describe the excited state as an Efimov state in the halo nucleus [23, 45, 46]. In fact, one can utilize the geometrical similarity between the ground- and excited-state configurations to distinguish Efimov states [47]: One of the magic properties of Efimov physics is that two consecutive Efimov states can be related to each other by a discrete spatial scaling factor [47]. In $^{23}$N, the dominant parts of the spatial density distribution of the ground and excited states (Figs. 3 and 4) indicate that the configurations of $^{23}$N in these two states have very similar shapes. Moreover, the same ratio between $r_{nn}$ and $r_{C(nn)}$ in the ground and excited states (Eq. (7)) suggests that a discrete scaling symmetry exists.
FIG. 3. Contour diagram for the spatial correlation density distribution of the ground state of $^{23}$N with parameters of Set B. (Color online)

FIG. 4. Contour diagram for the spatial correlation density distribution of the $\frac{1}{2}^-$ excited state of $^{23}$N with parameters of Set B. (Color online)
between the two states. These features are highly consistent with the explanation of Efimov states. However, further theoretical or experimental investigations are needed to confirm the excited state in $^{23}$N.

IV. CONCLUSION AND DISCUSSIONS

We have investigated the properties of $^{23}$N as a three-body system consisting of an inert $^{21}$N core and two valence neutrons. We apply the Woods-Saxon potential form to represent the effective $^{21}$N-n interaction, and reproduce the basic characteristics of the $^{22}$N. By solving the Faddeev equations of the $^{21}$N+n+n three-body system, implemented with the $^{21}$N-n and n-n potentials, we calculate the bound-state observables in the $^{23}$N ground state, where we obtain the two-neutron separation energy $S_{2n}$ consistent with the experimental result. We also discover an excited $^{23}$N with a much shallower two-neutron separation energy of 0.18 MeV. We also calculate the r.m.s. matter radii for the two states. The obtained results suggest a relatively small halo structure for the ground state. On the other hand, a more extended distribution of the valance neutrons is unveiled in the excited state, which indicates an distinct halo structure. The average distances between the valence neutrons, the average distances from the core to the center-of-mass of the valence-neutron pair, and the spatial correlation density distributions of the two states in $^{23}$N are also calculated to investigate the geometric structure of the three-body system. These results indicate that the two states have very similar triangular shapes and can be related to each other by a discrete scaling symmetry. These features suggest that the excited state of $^{23}$N can be an Efimov state. Our calculations of $^{23}$N with the Faddeev formalism enable a new exploration toward the neutron drip line, and urge further experimental investigation of the $^{23}$N ground state and the potential discovery of the $^{23}$N excited state.

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