Scalar Glueball Decay Into Pions In Effective Theory

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We discuss the mixing between the sigma meson $\sigma$ and the "pure" glueball field $H$ and study the decays of the scalar glueball candidates $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ (a linear combination of $\sigma$ and $H$) into two pions in an effective linear sigma model.

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Quantum Chromo Dynamics (QCD), as the fundamental theory of the strong interaction predicts the existence of exotic mesons made of gluons. Observations of these gluonium will provide a direct confirmation on the special feature of non-abelian gauge theory.

Lattice QCD [1] and various phenomenological models, such as the potential models [2] flux tube [3] all predict the lowest-lying glueball is $0^{++}$ state, whose mass is around 1.6GeV. However it is very difficult to identify glueballs experimentally and distinguish them from the normal $\bar{q}q$ states. So far five isoscalar states, $\sigma(400-1200)$, $f_0(980)$, $f_0(1370)$, $f_0(1500)$, $f_0(1710)$ confirmed experimentally as $0^{++}$ states in the region 1000GeV-1700GeV [4]. It is beyond the accommodation of the naive quark model if there is only one nonet in this energy region. The excited states are generally expected to be much heavier. Recently, some new approaches have been proposed, which tried to include these states into two nonets [5] [6]. These models should be confirmed further by the experiments. Now let us consider these states in more detail.

The identification of the sigma particle $\sigma$ is highly controversial. It is listed in the Particle Data Tables [4] as a very broad meson with mass around 400$-1200$ MeV and full width 600$-1200$ MeV. This particle, needed in the linear sigma model, has been argued to play an important role in nuclear physics [7] and in the study on the chiral phase transition [8]. Recently, experiments on $D \rightarrow 3\pi$ in E791 [9] and $J/\psi \rightarrow \omega \pi \pi$ in BES [11] provide strong evidences on the existence of $\sigma(400-1200)$. And the theoretical studies on these processes [10] [12] show that the linear $\sigma$ model gives rise to a reasonable description of the $\sigma$ decay into $\pi$'s.

It should be pointed out that the authors of Refs. [13] [14] take $\sigma(400-1200)$ as a glueball, because $\sigma(400-1200)$ does not appear in $\gamma \gamma \rightarrow \pi \pi$. Pennington later pointed out that the gauge invariance requires a mass-dependent fit in $\gamma \gamma \rightarrow \pi \pi$, then the appearance of $\sigma(400-600)$ is still possible. Törnqvist used linear sigma model to give a very good fit to the light meson spectrum [15]. The most recently, Pennington et. al. showed that $\sigma$ has a large content of $\bar{u}u + \bar{d}d$ by the finite energy QCD sum rule approach [17]. Therefore, in this paper we take $\sigma(400-1200)$ as a lightest $0^{++} \bar{q}q$ meson. $f_0(980)$ is often interpreted as a multiquark state, i.e. $\bar{K}K$ bound state or $\bar{q}q$ [4], because it strongly couples to $\bar{K}K$. $f_0(1370)$ has a very broad decay width and is generally considered as a $\bar{q}q$ state, however, a recent fit indicated that it has a large gluon content [18]. Therefore, we will take it as a glueball candidate in this paper. $f_0(1500)$ and $f_0(1710)$ are the most possible candidates for the glueballs up to now, not only because their masses are very close to the predictions of Lattice QCD, but also because they have very narrow widths and couple to photons weakly. Besides, $f_0(1500)$ also has enhanced production rate at low transverse momentum transfer in central collisions [4]. Regarding the role of the sigma particle in the glueball physics, BES [19] and MKIII [20] data on $B(J/\Psi \rightarrow \gamma f_0(1500) \rightarrow \gamma 4\pi)$ and $B(J/\Psi \rightarrow \gamma f_0(1710) \rightarrow \gamma 4\pi)$, Crystal Barrel data [21] on $\bar{p}p \rightarrow \pi^0 f_0(1500) \rightarrow \pi^0 (2\pi, \bar{K}K, 2\eta, \eta', 4\pi)$ show $f_0(1500)$ decays into $4\pi$ dominantly through sigma channel(S partial wave), and also hints it strongly couples to $2\sigma$.

In this paper we take an effective lagrangian approach and discuss the glueball decay into two-pion final states. Particularly we will pay attention to the mixing between the glueball and the sigma meson. The
mixing angle is correlated with the decay width of the glueball into two pions. On the other hand, by using the low energy theorem and QCD Sum Rule, we can estimate the mixing angle. Then, some very useful informations for those candidates can be obtained.

To begin with, we consider, for simplicity, the linear sigma model for two flavors. As usual we introduce the field

\[ \Phi = \sigma \tau^0 + i \vec{\pi} \vec{\tau}, \]

where \( \tau^0 \) is unity matrix and \( \vec{\tau} \) the Pauli matrices with normalization condition \( Tr \tau^a \tau^b = 2 \delta^{ab} \). Under a \( SU_L(2) \otimes SU_R(2) \) chiral transformation, \( \Phi \) transforms as

\[ \Phi \rightarrow L^\dagger \Phi R. \]

A renormalizable lagrangian of linear sigma model is given now by

\[ L_{\Phi - \Phi} = Tr \{ \partial_\mu \Phi^\dagger \partial^\mu \Phi \} - \lambda [Tr \{ \Phi^4 \Phi \} - \frac{f_\pi^2}{2}], \]

where \( f_\pi \) is the vacuum expectation value of the sigma field and \( \lambda \) the self coupling constant.

Let's now add the \( O^{++} \) "pure" glueball state \( H \) on the lagrangian. Given that the \( H \) is made of gluons and singlet under the chiral symmetry, there are only two terms which give rise to the interaction among \( H \), the sigma and pions,

\[ L_{H - \Phi} = g_1 f_\pi H [Tr \{ \Phi^4 \Phi \} - \frac{f_\pi^2}{2}] + g_2 H^2 [Tr \{ \Phi^4 \Phi \} - \frac{f_\pi^2}{2}], \]

where two free parameters, \( g_1 \) and \( g_2 \) are introduced to describe the strength of couplings of one and two \( H \) to the sigma or pions.

For the self interaction of glueballs, the lagrangian is given by

\[ L_{H - H} = \frac{1}{2} \partial_\mu H \partial^\mu H + \frac{1}{2} m_H^2 H^2 + f_3 H^3 + f_4 H^4, \]

where \( m_H^2 \) is the mass of glueball, \( f_3, f_4 \) the self coupling constants.

After the chiral symmetry is broken by non-vanishing vacuum expectation value of the sigma field, lagrangian in (4) generates not only the interaction for glueball decay, but also a mixing between the glueball \( H \) and the sigma field \( \sigma \). The mass matrix for \( (H, \sigma) \) is

\[ m^2 = \begin{pmatrix} m_H^2 & g_1 f_\pi^2 \\ g_1 f_\pi^2 & m_\sigma^2 \end{pmatrix}, \]

where \( m_\sigma^2 = 2 \lambda f_\pi^2 \).

To diagonalize the mass matrix, we introduce an unitary matrix

\[ V = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \sim \begin{pmatrix} 1 & \theta \\ -\theta & 1 \end{pmatrix}. \]
Then the mass eigen-states are given by

$$\begin{pmatrix} H' \\ \sigma' \end{pmatrix} = \begin{pmatrix} 1 & \theta \\ -\theta & 1 \end{pmatrix} \begin{pmatrix} H \\ \sigma \end{pmatrix},$$

(8)

with $\theta = g_1 f_\pi^2 / (m_H^2 - m_\sigma^2)$.

Substituting (8) into (4) and (5), we obtain the lagrangian for $H'$ decay

$$L_{H'} = H'\theta \left( \frac{m_H^2}{2f_\pi^2} \pi^2 + \frac{2m_H^2}{2f_\pi^2} - 2g_2 f_\pi \right) \sigma^2 + O(\theta^2).$$

(9)

In (9), the couplings of the glueball to sigma and the pions are proportional to the mixing angle $\theta$. This could be understood that the mixing between $H$ and $\sigma$ is of the same order as that for $H'$ to decay into light hadrons (see figure 1). Furthermore, under the assumption of vector meson dominance [8], $\rho$ meson can be introduced by replacing $\partial^\mu \Phi$ in (3) by $D^\mu \Phi = \partial^\mu \Phi - ig(\rho^\mu \Phi - \Phi \rho^\mu)$. We found that the direct coupling of $H'$ and $\rho\rho$ vanishes at the tree level.

Identifying the glueball state $H'$ with the candidates, the phenomenologies of our model can be summarized as follows:

i) Glueball decay into two pions: The decay width of the glueball into two pions is given by

$$\Gamma(H' \rightarrow 2\pi) = 3g_2^2 \left( \frac{m_{H'}}{2f_\pi^2} \right)^2 \sqrt{m_H^2 - 4m_\pi^2}. $$

(10)

Experimentally, the data of $f_0(1370)$ are [9]

$$\Gamma_{tot} = 200 - 500 \text{MeV} \quad \frac{\Gamma(\pi\pi)}{\Gamma_{tot}} = 0.26 \pm 0.09$$

(11)

Therefore, $\Gamma(\pi\pi) \sim 100 \text{MeV}$. from (10), we obtain $\theta \sim 0.16$.

For $f_0(1500)$, different groups present controversial results. Let us use Crystal Barrel Group data [21], $Br(f_0(1500) \rightarrow 2\pi) : Br(f_0(1500) \rightarrow 4\pi) = 4.39 \pm 0.16 : 14.9 \pm 3.2$ and $\Gamma_{total} = 120 \pm 20 \text{MeV}$. Assuming that $Br(f_0(1500) \rightarrow 4\pi)$ is about 50% one has that $\Gamma(2\pi) = 17.5 \pm 6 \pm 3 \text{MeV}$. Using this value, we obtain $\theta \sim 0.058$.

For $f_0(1710)$, the data are

$$\Gamma_{tot} = 125 \text{MeV}, \quad \frac{\Gamma(\pi\pi)}{\Gamma_{tot}} = 0.039^{+0.002}_{-0.024}.$$ 

(12)

Then, $\theta \sim 0.025$.

ii) Glueball decay into four pions: the decay of the glueball into four pions is through the intermediate two sigma states. In Figure 2, we plot the ratio of glueball decay into $2\sigma$ to $2\pi$ as function of $g_2$. This measurement can be used to determine the parameter $g_2$.

For item i), an interesting application is to correlate it with the estimate of the mixing angle $\theta$ by using QCD sum rule.

The low energy theorem states that [22]
\[ i \int dx <|\mathcal{T} O(x)\mathcal{O}|> = -d_O <|\mathcal{O}(0)|>, \] (13)

where \( d_O \) is the mass dimension of operator \( O \), \( \mathcal{O} \) is the trace of the energy momentum tensor

\[
\theta_\mu = \frac{\beta(\alpha_s)}{4\alpha_s} G^{\mu\nu} G_{\mu\nu} \approx \frac{-b\alpha_s}{8\pi} G^{\mu\nu} G_{\mu\nu},
\]

\( \beta(\alpha_s) \) is Gell-Mann-Low function, \( b=11 \) for pure Yang-Mills QCD) which vanishes in classical level. To obtain the mixing angle \( \theta \), let us firstly choose operator \( O \) to be \( O = (\bar{u}u + \bar{d}d)/\sqrt{2} \). Assuming that the lowest-lying \( 0^{++} \bar{q}q \) state, i.e. \( \sigma \) and the lowest-lying \( 0^{++} \) glueball saturate the correlation function we obtain

\[
\langle |O_\sigma(0)|\sigma \rangle \frac{1}{m_\sigma^2} \langle \sigma|\mathcal{O}|\rangle + \langle |O_\sigma(0)|H \rangle \frac{1}{m_H^2} \langle H|\mathcal{O}|\rangle = -d_\sigma \langle |O_\sigma(0)| \rangle. \] (14)

Similarly, taking now the operator \( O \) to be \( \sigma \), we have

\[
\langle |O(0)|H \rangle \frac{1}{m_H^2} \langle H|\mathcal{O}|\rangle + = -d_G \langle |O(0)| \rangle. \] (15)

In deriving (13), we have also assumed that the ground state of \( 0^{++} \) glueball saturates the l.h.s of (13).

Defining \( \langle |\mathcal{O}|\rangle = f_H \), \( \langle |\sigma|\sigma \rangle = f_\sigma \), we can roughly define the effective field of the "pure" glueball \( \mathcal{H} \) as

\[
\mathcal{H} = \frac{1}{f_H} \mathcal{O} - \frac{1}{f_H} \langle |\mathcal{O}| \rangle. \] (16)

Therefore,

\[
\langle |\sigma|\mathcal{O}| \rangle \approx \theta f_H. \] (17)

Similar,

\[
\langle H|\sigma \rangle \approx -\theta f_\sigma. \] (18)

From eqs (14)-(18), we get the mixing angle

\[
\theta = -\frac{3m_\sigma^2 m_H}{2f_\sigma f_H (m_H^2 - m_\sigma^2)} \langle |O_\sigma| \rangle = -\frac{3m_\sigma^2 m_H}{2f_\sigma (m_H^2 - m_\sigma^2) \sqrt{-\langle |\mathcal{O}| \rangle}} - \langle |O_\sigma| \rangle. \] (19)

Note that the mixing angle \( \theta \) is proportional to the quark’s condense. The parameter \( f_\sigma \) in (19) can be estimated again by the QCD sum rule. Following [23], we have

\[
f_\sigma^2 = \frac{6}{16\pi} M^4 \left( 1 + \frac{13}{3} \frac{\alpha_s}{\pi} + \frac{8\pi^2}{M^2} \langle |\bar{q}q| \rangle + \frac{\pi^2}{3M^4} (\langle \frac{\alpha_s}{\pi} G^{\mu\nu} G_{\mu\nu} \rangle - \frac{1408}{81 \pi^6 M} \langle |\bar{q}q| \rangle) e^{m_\sigma^2/M^2} \right), \] (20)

where \( M \) is a parameter with mass dimension in Borel transformation.

For the quark and gluon condensate, we take the standard values

\[
\langle |\bar{q}q| \rangle = -(0.1 GeV)^4,
\]

\[
\langle |\bar{q}q| \rangle = \langle |\bar{u}u| \rangle = \langle |\bar{d}d| \rangle = -(0.25 GeV)^3,
\]

\[
\langle |\alpha_s G^{\mu\nu} G_{\mu\nu} | \rangle = 0.06 \pm 0.02 GeV^4.
\]
The mass prediction of sigma is in the region 700MeV-1000MeV, where when M=500MeV, \( m_\sigma = 750 \text{MeV} \) and M=1000MeV, \( m_\sigma = 1000 \text{MeV} \). This is consistent with the particle data book. We also note that some recently experimental fits give a lower sigma mass. However, N.A. Törnqvist gave a sigma mass \( m_\sigma = 860 \text{MeV} \) by using the sigma model [15]. Therefore, in our case QCD sum rule’s prediction is not bad. Let us take our discussion at \( m_\sigma = 860 \text{MeV} \). (19) gives
\[
\theta = 0.10, \quad m_H = 1.37 \text{GeV}, \\
\theta = 0.086, \quad m_H = 1.5 \text{GeV}, \\
\theta = 0.068, \quad m_H = 1.71 \text{GeV}
\] (22)

In general QCD sum rule’s calculation is reliable within 30 percent uncertainty. Comparing the values in (22) and the mixing angles obtained by Eq.(11) in i), we will be able to exclude \( f_0(1710) \) as a glueball. It looks more like a \( \bar{s}s \) meson. Both \( f_0(1500) \) and \( f_0(1370) \) are possible glueball states, but we incline to \( f_0(1500) \) is a glueball, because there is still large uncertainty in its experimental data. It is interesting to compare our result with other approaches. In reference [18], authors analyzed states \( f_0(1370), f_0(1500) \) and \( f_0(1710) \) by combining the various experimental data and obtained a mixing pattern in terms of pure glueball and other pure quark bound states to be
\[
\begin{align*}
&f_{i1}^{G}, & f_{i2}^{S}, & f_{i3}^{(N)} \\
&f_0(1710) & 0.39 \pm 0.03 & 0.91 \pm 0.02 & 0.15 \pm 0.02 \\
&f_0(1500) & -0.65 \pm 0.04 & 0.33 \pm 0.04 & -0.70 \pm 0.07 \\
&f_0(1370) & -0.69 \pm 0.07 & 0.15 \pm 0.01 & 0.70 \pm 0.07
\end{align*}
\] (23)

One can find our result by a simple approach is quite consistent with their analysis. In a more recent paper [24], it was pointed out that \( f_0(1500) \) is a gluonium state and \( f_0(1710) \) is a \( \bar{s}s \) state because it is absent in \( \bar{p}p \) annihilation. Meanwhile, \( f_0(1710) \)'s \( \bar{s}s \) dominance is consistent with \( \gamma\gamma \) data since only few decay branching ratios have been measured for this state.

Before concluding our paper, we make two remarks:

1) In the chiral limit which we work on in this paper, the mixing of glueball with sigma is a consequence of the spontaneous chiral symmetry breaking. However, when including the explicit symmetry breaking effects by the quark mass, especially when extending the lagrangian for two flavors to three flavors, there would be one term proportional to the explicit symmetry breaking which also generates mixing of the glueball with the sigma meson as shown in Fig. 3;

2) Other \( \bar{q}q \) scalar mesons are possibly included in our Lagrangian. There will be the mixing between the "pure" glueball with these states. If these new scalar mesons are \( \bar{u}u + \bar{d}d \) mesons, the decay of glueball into two pions can be via these states, therefore the mixing angle between the glueball and sigma determined by (10) will be reduced. On the other hand, in (14) we also need insert these states into the correlator, then mixing angle between the glueball and sigma estimated by the low energy theorem is also reduced. We cannot say these two effects cancel each other completely, however, since we assume that sigma is the lightest.
\( \bar{u}u + \bar{d}d \) meson, the glueball’s decay inclines to via the \( \sigma \) channel. The more interesting result is that, the glueball’s decay width can be reasonable with a small mixing with \( \bar{q}q \) meson. That hints the glueball is still probably "pure". The mixing between the glueball and \( \bar{s}s \) is not relevant to the decay of glueball into pions, it can be included in SU(3) model.

In conclusion, we have studied the decay of the scalar glueball into pions and its mixing with the sigma particle in effective theory. We fit our model to the experimental data on the decay of glueball candidates, \( f_0(1370) \), \( f_0(1500) \) and \( f_0(1710) \) into two pions and obtain the corresponding mixing angle of glueball with sigma. By using our estimation for the mixing angle based on low energy theorem and QCD sum rule, we conclude that \( f_0(1710) \) is not a glueball and the most possible glueball candidate is \( f_0(1500) \). We also show that a glueball decays into four pions through 2 \( \sigma \) intermediate states. The \( \rho \) can contribute to the four-pion decay mode at one-loop level with branch ratio

\[
\frac{\Gamma(2\rho)}{\Gamma(2\sigma)} \sim 0.01. \tag{24}
\]

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Figure Captions

Fig.1 (a) Illustration of glueball decay into $2\sigma$ or $2\pi$; (b) Illustration of glueball mixing with $\sigma$ induced by the quark condense.

Fig.2 Ratio of $f_0(1500)$ decay into $2\sigma$ to $2\pi$ as function of $g_2$. In the numerical calculation, we take $m_\sigma = 500\text{GeV}$

Fig.3 Illustration of glueball mixing with $\sigma$ induced by the quark masses.
Fig. 2

$g_2$ (unit=$\lambda$)
Fig. 3