Dynamically Induced Multi-Channel Kondo Effect

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We study how the multi-channel Kondo effect is dynamically induced to affect the photoemission and the inverse photoemission spectrum when an electron is emitted from (or added to) the completely screened Kondo impurity with spin $S > 1/2$. The spectrum thereby shows a power-law edge singularity characteristic of the multi-channel Kondo model. We discuss this anomalous behavior by using the exact solution of the multi-channel Kondo model and boundary conformal field theory. The idea is further applied to the photoemission for quantum spin systems, in which the edge singularity is controlled by the dynamically induced overscreening effect with a mobile Kondo impurity.

I. INTRODUCTION

The multi-channel Kondo effect has been the subject of intensive theoretical and experimental studies, which is characterized by unusual non-Fermi liquid behaviors. Its applications are now extended not only to standard dilute magnetic alloys, but also to quantum dots, etc. Thus far, theoretical and experimental studies on the multi-channel Kondo effect have been focused on a static Kondo impurity, which has been related to the measurements of the specific heat, the spin susceptibility, the resistivity, etc. This naturally motivates us to address a question whether such a nontrivial phenomenon can be observed in dynamically generated situations.

The photoemission and the inverse photoemission may be one of the key experiments to study non-Fermi liquid behaviors, which reveal the dynamics of a single hole or electron suddenly created in the system. Here we propose the dynamically induced multi-channel Kondo effect, when an electron is emitted from (or added to) the Kondo impurity by the photoemission (inverse photoemission). A remarkable point is that the ground state of the system is assumed to be a completely screened Kondo singlet, and non-Fermi liquid properties are generated by an electron or hole suddenly created. We study low-energy critical properties of the spectrum by using the exact solution of the multi-channel Kondo model combined with boundary conformal field theory (CFT). We analyze the one-particle Green function for the impurity to show its typical non-Fermi liquid behavior. It is further demonstrated that this effect can be observed even in a homogeneous system without impurities. To show this explicitly, we apply the analysis to the photoemission spectrum in a quantum spin chain with spin $S > 1/2$.

This paper is organized as follows. In §2 we briefly illustrate the idea of the dynamically induced multi-channel Kondo effect, and derive low-energy scaling forms of the one-particle Green function. We discuss non-Fermi liquid properties in the spectrum by exactly evaluating the critical exponents. In §3 the analysis is then applied to the photoemission spectrum for a quantum spin chain. Brief summary is given in §4. We note that preliminary results on this issue have been reported in Ref. 11.

II. LOW-ENERGY DYNAMICS IN THE PHOTOEMISSION SPECTRUM

A. dynamically induced Kondo effect

Let us consider the spin-$S$ Kondo impurity which is completely screened by conduction electrons with $n(=2S)$ channels. The impurity spin is assumed to be composed of $n$ electrons by the strong Hund coupling. To study the core-electron photoemission spectrum, we start with spectral properties of the impurity Green function,

$$G(t) = -i < T \left[ d(0) d^\dagger(t) \right> = G^>(t) + G^<(t),$$

where $d$ is the annihilation operator for one of core electrons which compose the impurity spin and $T$ is the conventional time-ordered product. Here, $G^>(t)$ ($G^<(t)$) is the Green function, which is restricted in $t > 0$ ($t < 0$). For the photoemission, we consider $G^<(t)$. To be specific, we discuss the case that a core electron is emitted as depicted in Fig. 1 (a), for which the binding energy $-\omega_\alpha$ (measured from the Fermi energy) is assumed to be larger than the band width $D$. Then in the excited state the overscreening system is generated, which is referred to as the dynamically induced overscreening Kondo effect. At the low-energy regime around $-\omega_\alpha$, we may express the operator as $d(t) \simeq e^{i\omega_\alpha t} \phi(t)$ where $\phi(t)$ is the corresponding boundary operator in boundary CFT, which characterizes the boundary critical phenomena. It is known that the Fermi-edge singularity is formulated by the boundary operator in which nontrivial effects for the overscreening Kondo effect are incorporated in $\phi_\alpha(t)$. We write down the one-particle Green function $G^<(t)$ as,

$$\text{Im}G^<(\omega) = \frac{1}{2} \int_{-\infty}^{\infty} <\phi_\alpha^\dagger(0)\phi_\alpha(t) > e^{i\omega t} e^{i\omega t} dt.$$
form higher spin \( S + 1/2 \) by the strong Hund-coupling, as shown in Fig. 1 (b). Then we may write \( d(t) \simeq e^{-i\omega^\beta t}\phi^\beta(t) \), where \( \omega^\beta > D \) is the energy cost to make \( S + 1/2 \) spin, and \( \phi^\beta(t) \) is another boundary operator which controls the underscreening Kondo effect induced by the inverse photoemission. We have

\[
\text{Im}G^\gamma(\omega) = -\frac{1}{2} \int_{-\infty}^{\infty} <\phi^\gamma(t)\phi^\gamma(0)> e^{-i\omega t}e^{i\omega t} dt. \tag{3}
\]

In order to evaluate the critical exponents, we now employ the idea of finite-size scaling in CFT. The scaling form of the correlators \(<\phi^\gamma(0)\phi^\gamma(t)>\) and \(<\phi^\gamma(t)\phi^\gamma(0)>\) are given by

\[
<\phi^\gamma_{\alpha}(0)\phi^\gamma_{\alpha}(t)> = \sum_{N=0}^{\infty} |<0|\phi^\gamma_{\alpha}(0)|N>|^2 e^{2\pi F(\Delta_{\alpha} + N)t},
\]

\[
<\phi^\gamma(0)\phi^\gamma_{\beta}(t)> = \sum_{N=0}^{\infty} |<N|\phi^\gamma_{\beta}(0)|0>|^2 e^{-2\pi F(\Delta_{\beta} + N)t},
\]

in the long-time asymptotic region. According to the finite-size scaling, the boundary dimensions \( \Delta_{\alpha} \) and \( \Delta_{\beta} \) are read from the lowest excitation energy \( \Delta E \),

\[
\Delta E = \frac{\pi F}{l} \Delta_{\gamma}, \tag{4}
\]

with \( \gamma = \alpha, \beta \), where \( l \) corresponds to the system size of one dimension in the radial direction. We thus end up with the relevant scaling forms as

\[
\text{Im}G^\gamma_c(\omega) = \frac{\pi}{\Gamma(2\Delta_{\alpha})v_F 2\Delta_{\alpha}} \theta(-\omega - \omega_{\alpha})(-\omega - \omega_{\alpha})^X_{\alpha},
\]

\[
\text{Im}G^\gamma(\omega) = \frac{-\pi}{\Gamma(2\Delta_{\beta})v_F 2\Delta_{\beta}} \theta(\omega - \omega_{\beta})(\omega - \omega_{\beta})^X_{\beta}, \tag{5}
\]

where \( X_{\gamma} = 2\Delta_{\gamma} - 1 \) that \( \gamma \) represents \( \alpha \) and \( \beta \).

In both cases, the spectral functions have power-law edge singularity due to the dynamically induced multi-channel Kondo effect, which will be shown to exhibit non-Fermi liquid properties.

**B. exact critical properties**

We now discuss low-energy critical properties by exactly evaluating \( \Delta_{\alpha} \) and \( \Delta_{\beta} \). To this end, we consider the multi-channel Kondo model,

![FIG. 1. Over screening and underscreening systems dynamically generated by the photoemission and the inverse photoemission, for which the impurity spin is assumed to be completely screened in the ground state.](image)

\[
\mathcal{H} = -i \sum_{a,m} \int dx \psi^\dagger_{am}(x)\partial_t \psi_{am}(x) + 2J \sum_{a,b,m} \psi^\dagger_{am}(0)\sigma^\nu_{ab}\psi_{bm}(0)S^\nu - HS^z, \tag{6}
\]

where \( \psi^\dagger_{am} \) is the creation operator for conduction electrons with spin \( a = \uparrow, \downarrow \) and orbital indices, \( m = 1, \ldots, n \).

The exact solution of this model is expressed in terms of the Bethe equations for spin rapidities \( \lambda_{\alpha} \) and charge rapidities \( k_j \),

\[
e^{-ik_j L} = \prod_{\alpha=1}^{M} \frac{\lambda_{\alpha} + \text{in}/2}{\lambda_{\alpha} - \text{in}/2} \prod_{\alpha=1}^{M} \frac{\lambda_{\alpha} - \text{in}/2}{\lambda_{\alpha} + \text{in}/2} \left( \frac{\lambda_{\alpha} + \text{in}/2}{\lambda_{\alpha} - \text{in}/2} \right)^N = \prod_{\alpha=1}^{M} \frac{\lambda_{\alpha} - \lambda_{\beta} + i}{\lambda_{\alpha} - \lambda_{\beta} - i}, \tag{7}
\]

where \( N \) is the number of electrons and \( L = 2l \) is the one-dimensional system size. It is assumed that the impurity with spin \( S > 1/2 \) is completely screened in the ground state. Then, the core-level photoemission suddenly reduces the impurity spin, thus inducing the overscreening Kondo effect with \( n - 2S = 1 \). As for the underscreening effect induced by the inverse photoemission, the condition is replaced by \( n - 2S = -1 \). At zero temperature the ground-state properties are described by the \( n \)-th order string solutions.
\[ \lambda_{l}^{n,\alpha} = \lambda_{l}^{n} + \frac{1}{2}(\alpha + 1 - 2\alpha), \]  
(8)

where \( \alpha = 1, \ldots, n \) and \( l = 1, \ldots, M_{n} \) which is restricted by \( M = nM_{n} \). Here, \( n \) represents the number of orbitals.

It is well known that a naive application of finite-size techniques based on the string hypothesis turns out to fail for the overscreening case at zero magnetic field. \[ 3,6 \]

This difficulty comes from an improper treatment of the \( Z_{n} \) symmetry sector in terms of the string solutions. However, as long as the finite magnetic field is concerned, we can use the Bethe equations to describe its critical properties. We will separately discuss the case of zero magnetic field, by incorporating \( Z_{n} \) sector correctly. \[ 3,6 \]

By applying standard procedures to eq. (7), it is straightforward to exactly evaluate the lowest excitation energy in magnetic fields, for which one of the impurity electrons is assumed to be removed from the system,

\[ \Delta E = \frac{\pi \nu_{F}}{l} \left( \frac{\pi^{2}}{4n} + n(n_{\text{imp}})^{2} \right). \]  
(9)

Although the above finite-size correction is apparently similar to that for 1D solvable systems with a static impurity or boundaries, \[ 3,6 \] the final-state interaction induced by photoemission is included in the present case. \[ 3,6 \]

Thus all the features which are governed by the dynamical Kondo effect can be read from this quantity.

By applying the finite-size scaling in eq. (8), the critical exponent \( X_{\gamma} \) in eq. (3) is now obtained as,

\[ X_{\gamma} = \frac{\delta_{\alpha}^{2}}{2n} + 2n(n_{\text{imp}})^{2} - 1, \]  
(10)

where \( \delta_{\alpha} \) is the charge scattering phase shift, which is caused by a created hole as in the ordinary Fermi edge singularity. \[ 3,6 \] This term depends on the detail of potential scattering. It is mentioned that the second term with the phase shift \( n_{\text{imp}} \) is caused by the Kondo effect, which is explicitly evaluated as,

\[ n_{\text{imp}} = \int_{-\infty}^{\lambda_{0}} \sigma_{\text{imp}}(\lambda)d\lambda, \]  
(11)

where

\[ \sigma_{\text{imp}}(\lambda) = \sigma_{\text{imp}}^{0}(\lambda + 1/J) \]  

\[ - \int_{-\infty}^{\lambda_{0}} G_{n}(\lambda - \lambda') \sigma_{\text{imp}}(\lambda')d\lambda'. \]  
(12)

Here \( \sigma_{\text{imp}}^{0}(\lambda) \) and \( G_{n}(\lambda) \) are determined by

\[ \sigma_{\text{imp}}^{0}(\lambda) = \frac{1}{\pi} \sum_{l=1}^{\min(n,2S)} \frac{\alpha}{l^{2} + \frac{1}{4}(n + 2S - 1 - 2l)^{2}}, \]  

\[ G_{n}(\lambda) = \frac{1}{\pi} \frac{n}{\lambda^{2} + n^{2}} + \frac{2}{\pi} \sum_{\alpha=1}^{n-1} \frac{1}{\lambda^{2} + \frac{1}{4}(2n - 2\alpha)^{2}}. \]  
(13)

The key quantity, \( n_{\text{imp}} \), is obtained by using Wiener-Hopf method. \[ 3,6 \]

\[ n_{\text{imp}} = \frac{S}{n} - \left( \frac{S}{n} \frac{1}{2} \right) \theta \left( \frac{S}{n} - \frac{1}{2} \right) \]  

\[ + \frac{i}{4\pi} \int_{-\infty}^{\infty} \frac{d\omega}{\omega - i0} e^{-i2\omega log \frac{n}{S}} \frac{\Gamma(1 + i\omega) \Gamma(1/2 - i\omega)}{\Gamma(1 + in\omega)} \]  

\[ \cdot \left( \frac{in\omega + 0}{e} \right)^{\alpha} \frac{e^{-\pi |n - 2S|/|\omega|} - e^{-\pi(n + 2S)/|\omega|}}{1 - e^{-2\pi n/|\omega|}}. \]  

In Fig. 2 we display the critical exponent \( X_{\alpha} \) as a function of magnetic fields for \( n = 3 \) and 4. For clarity, we have subtracted the contribution from the ordinary charge scattering, \( \delta_{\alpha}^{2}/2n \).

The energy unit is the magnetic Kondo temperature \( T_{K} \), which is expressed in terms of the ordinary Kondo temperature \( T_{K} \) via \( T_{H} = [2\pi(n/2e)^{n/2}/\Gamma(n/2)]T_{K} \). At zero-magnetic field, the exponents show a discontinuity, which is related to the \( Z_{n} \) symmetric sector, as mentioned in the text. \[ 3,6 \]

In Fig. 2 we display the critical exponent \( X_{\alpha} \) for the overscreening case as a function of the magnetic field. Note that the magnetic-field dependence of \( X_{\alpha} \) without \( \delta_{\alpha}^{2}/2n \) is determined by the dynamical Kondo effect, because the charge scattering phase shift \( \delta_{\alpha} \) does not depend on magnetic fields. Particularly in weak magnetic field, \( H << T_{H} \), the obtained exponent behaves as

\[ X_{\alpha} \rightarrow \frac{\delta_{\alpha}^{2}}{2n} + \left( \frac{S}{n} - \text{const} \cdot \left( \frac{H}{T_{H}} \right)^{\frac{2}{n}} \right)^{2} - 1. \]  
(14)

It is seen that the phase shift, \( n_{\text{imp}} \), gives rise to the anomalous magnetic-field dependence of the exponent. This non-Fermi liquid behavior is characteristic of the overscreening effect. \[ 3,6 \] Another interesting feature in the overscreening effect appears at \( H = 0 \). We recall here that the symmetry is enhanced from U(1) to SU(2) at \( H = 0 \), for which the boundary dimension \( \Delta_{s} \) for the spin sector is analytically obtained by employing fusion rules hypothesis proposed by Affleck and Ludwig. \[ 3,6 \]

\[ \Delta_{s} = \frac{S(S + 1)}{n + 2}, \]  
(15)
which is a typical conformal dimension for level-$n$ SU(2) Kac-Moody algebra. Note that the critical exponent shows a discontinuity at $H = 0$,

$$X_\alpha(H = 0) - X_\alpha(H \to 0) = 2 \cdot \frac{S(S + 1)}{n + 2} - 2 \cdot \frac{S^2}{n},$$

(16)

which is caused by the fact that $\mathcal{Z}_n$ symmetric sector is massless only at $H = 0$, as already mentioned.

We now move to the underscreening case induced by the inverse photoemission. The calculated critical exponent $X_\beta$ is shown as a function of magnetic fields in Fig. 3. For weak magnetic field, the obtained exponent $X_\beta$ behaves as

$$X_\beta \to \frac{\delta^2}{2n} + n \left( \frac{1}{2} - \frac{1}{\log(H/T_H)} \right)^2 - 1,$$

(17)

which is characteristic of the underscreening system. In contrast to the overscreening case, there is no discontinuity in the exponent in this case, $X_\beta(H = 0) = X_\beta(H \to 0)$.

This completes a general description of the dynamically induced Kondo effect. An important point to be emphasized is that this kind of phenomenon may be observed not only for impurity systems but also for other related quantum systems, which will be explicitly discussed in the next section.

![Graph showing critical exponent $X_\beta$ as a function of $H/T_H$](image)

**FIG. 3.** The critical exponent $X_\beta$ for the underscreening Kondo effect, induced by the inverse photoemission, as a function of magnetic fields for $n = 3$ and 4. The charge phase shift $\delta^2/2n$ is subtracted for simplicity, as before.

### III. APPLICATION TO SPIN CHAINS

We now wish to demonstrate that the dynamically induced Kondo effect proposed here may be observed for gapless quantum spin systems which do not possess impurities. As an example, we consider an integrable anti-ferromagnetic spin chain with spin $S > 1/2$, for which the exact solution is available even for the case with doped holes. The photoemission suddenly removes one electron from the spin system, and thus bears an impurity site with spin $S - 1/2$. In the final state, this impurity spin is screened by host spins, and as a result the overscreening Kondo effect may be dynamically induced. It is remarkable that the induced impurity in this case can move through the lattice via the exchange interaction, and the edge singularity is thus governed by a mobile multichannel Kondo impurity.

Let us consider the gapless $S = 1$ spin chain with a mobile $S = 1/2$ impurity as an example. Although in more general situations including non-integrable models, the higher spin should be a half-odd integer, the present treatment can be straightforwardly extended to such cases. We here consider an integrable spin chain derived by the quantum inverse scattering method.

The quantity

$$\mathcal{H} = \sum_{i=1}^{L} -(1 - \delta S_i, S_{i+1}) P_{i,i+1}(S_i \cdot S_{i+1})$$

$$+ \frac{1}{2} \left( \frac{1}{S_i S_{i+1}} - S_i \cdot S_{i+1} - 1 + \delta S_i, S_{i+1} [1 - (S_i \cdot S_{i+1})^2] \right)$$

where the spin $S_i^2 = S_i (S_i + 1)$ with $S_i = 1$ or $1/2$, and $P_{ij}$ permutes the spin on sites $i$ and $j$.

In order to deal with the excited states when an electron is emitted from the spin chain, we write down the Bethe equations for the spin-$S$ chain with one hole being doped,

$$\left( \frac{\lambda_j + i S}{\lambda_j - i S} \right)^L = \frac{\lambda_j - \nu - i/2}{\lambda_j - \nu + i/2} \prod_{k \neq j}^{N_j+1} \frac{\lambda_j - \lambda_k + i}{\lambda_j - \lambda_k - i},$$

$$1 = \prod_{k=1}^{N_j+1} \frac{\nu - \lambda_k + i/2}{\nu - \lambda_k - i/2},$$

(18)

where $N_j$ is the number of down spins and $\lambda_j (j = 1, \cdots, N_j + 1)$ are spin rapidities. Note that the hole rapidity $\nu$ appears in the above equation, which characterizes a massive charge excitation suddenly created. Thus, $\nu$ specifies the hole momentum $q$. It is seen that the above spin-charge scattering term, which describes the final-state interaction, corresponds to the impurity term in eq. (16).

The manipulation illustrated in the previous section enables us to exactly calculate the scaling dimension for the one-particle Green function via the finite-size corrections.

$$x(\nu) = \frac{1}{4\xi_{2S}^2} \left( 1 - n_{\text{imp}}(\nu) \right) + \xi_{2S} \left( \frac{1}{2} - d_{\text{imp}}(\nu) \right)^2.$$  (19)

The quantity $\xi_{2S}(\lambda) \equiv \xi_{2S}(\lambda_0)$ often referred to as the dressed charge, is given by

$$\xi_{2S}(\lambda) = 1 - \int_{-\lambda_0}^{\lambda} K_{2S}(\lambda - \lambda') \xi_{2S}(\lambda'),$$  (20)
where

\[ K_{2S}(\lambda) = \frac{1}{\pi} \frac{\frac{1}{2}(4S')}{\lambda^2 + \frac{1}{4}(4S')^2} + \frac{2}{\pi} \sum_{l=1}^{2S+1} \frac{\frac{1}{2}(4S - 2l)}{\lambda^2 + \frac{1}{4}(4S - 2l)^2}, \tag{21} \]

and the cut-off parameter \( \lambda_0 \) is related to the magnetization \( m \),

\[ S - m = 2S \int_{-\lambda_0}^{\lambda_0} \rho_{2S}(\lambda) d\lambda. \tag{22} \]

The density function \( \rho_{2S} \) is determined by the following integral equation,

\[ \rho_{2S}(\lambda) = \frac{1}{2\pi} \Theta_{2S,2S}'(\lambda) - \int_{-\lambda_0}^{\lambda_0} K_{2S}(\lambda - \lambda') \rho_{2S}(\lambda') \tag{23} \]

We stress that two key quantities \( n_{\text{imp}}(\nu) \) and \( d_{\text{imp}}(\nu) \), which contain the effect of a mobile impurity, are introduced in eq. (19),

\[ n_{\text{imp}}(\nu) = \int_{-\lambda_0}^{\lambda_0} \rho_{\text{imp}}(\lambda) d\lambda \]

\[ d_{\text{imp}}(\nu) = -\frac{1}{2} \left( \int_{\lambda_0}^{\infty} - \int_{-\infty}^{-\lambda_0} \right) \rho_{\text{imp}}(\lambda) d\lambda \tag{24} \]

where

\[ \rho_{\text{imp}}(\lambda) = \frac{1}{2\pi} \Theta_{2S,1}'(\lambda - \nu) - \int_{-\lambda_0}^{\lambda_0} K_{2S}(\lambda - \lambda') \rho_{\text{imp}}(\lambda'). \]

Here we have introduced the phase function,

\[ \frac{1}{2\pi} \Theta_{n,k}'(\lambda) = \frac{1}{\pi} \sum_{l=1}^{\min(n,k)} \frac{1}{\lambda^2 + \frac{1}{4}(n + k + 1 - 2l)^2}. \tag{25} \]

These quantities are alternatively represented in terms of the phase shifts \( \delta_L \) and \( \delta_R \) at the left and right Fermi points in massless spin excitations:

\[ n_{\text{imp}}(\nu) = \frac{\delta_L + \delta_R}{2\pi}, d_{\text{imp}}(\nu) = \frac{\delta_L - \delta_R}{2\pi}. \]

We mention that the asymmetric phase shift \( d_{\text{imp}}(\nu) \) is inherent in a mobile Kondo impurity, different from a localized impurity in §2. Note that the scaling dimension \( x \) depends on the hole momentum \( q \) through the asymmetric phase shift.

Let us now discuss low-energy critical properties in the photoemission spectra. We write down the one-particle Green function which depends on the momentum \( q \),

\[ \text{Im}G(q, \omega) \propto (\omega - \omega_c(q))^{X(q)}, \tag{26} \]

with \( X(q) = 2x - 1 \), where \( x \) is the scaling dimension in eq. (19) and \( \omega_c(q) \) is the dispersion of the charge excitation generated by the photoemission. In Fig. 4 we show the obtained critical exponent as a function of the momentum \( q \) for the \( S = 1 \) case.

This anomalous power-law behavior and the momentum dependence of \( X(q) \) are caused by a suddenly induced mobile Kondo impurity. As we discussed in §2, the discontinuity of the exponent \( X \) at \( m = 0 \) is caused by the \( Z_n \) symmetric sector.

In this way, the dynamically induced Kondo effect proposed here may be expected to be observed in photoemission experiments for quantum spin chains. In order for this phenomenon to be observed, there may be several problems to be resolved; preparation of a proper 1D sample, experiments with high resolution, etc. Anyway, it is desired to experimentally find or synthesize rather ideal spin chain systems without the magnetic order even at low temperatures.

**IV. SUMMARY**

In summary we have proposed the multi-channel Kondo effect dynamically induced by the photoemission and the inverse photoemission, for which the ground state is a completely screened Kondo singlet. By studying low-energy critical properties in the photoemission spectra, it has been found that the anomalous behavior generated by this effect is indeed characteristic of the multi-channel Kondo system. In particular, we have demonstrated that the idea proposed here can be directly applied to homogeneous quantum spin systems without any impurity. It has been shown in this case that a mobile Kondo impurity suddenly created by the photoemission gives rise to the momentum-dependent anomalous exponent.

Although we have mainly focused on the photoemission spectra for magnetic impurity systems in this paper, it should be noted that the idea can be directly applied to the photoemission spectrum in quantum dot systems, for
which a multilevel quantum dot with the Hund coupling plays a role of the Kondo impurity. In this connection, it is also interesting to apply a similar idea to the optical absorption spectra in a quantum dot, as recently demonstrated. In such cases, not only the linear but also the non-linear optics play an important role, which may provide interesting phenomena related to the dynamically induced Kondo effect.

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