Switching Nano-Device Based on Rabi Oscillations

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We describe a switching device based on Rabi oscillations. The device consists of a well region separated from a free region by a potential barrier. The potential landscape is adjusted so that one bound state and one quasi level are present. By applying a microwave field with a driving frequency close to the separation between the two levels, a particle initially in the ground state can be activated to the quasi level, and subsequently tunnel to the free region. The probability for tunneling in the free region exhibits a plateau structure, as the wave function is emitted by bursts after each Rabi oscillations. The leakage current can then be controlled by varying the amplitude of the external field, the barrier height/width, and the frequency mismatch.

The recent advances in the fabrication of semiconductor nano-structures have generated a rebirth of one-dimensional quantum mechanics. Indeed, while one dimensional (1D) scattering problems were considered on a purely academical level in the sixties, one now has the means to fabricate and design such 1D structures using molecular beam epitaxy techniques, and test the prediction of relatively simple theoretical models. The motivation to study such artificial structures comes to a large extent from the hope that these could be ultimately used as elementary building blocks in the next generation of electronic devices and computers. Unfortunately, this research is still at its preliminary stage, and the conceptualization of even the simplest devices already represents a considerable challenge. In this letter, we do a step in this direction, and we consider a device which allows to control the switching of an electric current: the device exploits the Rabi oscillations between a bound state and quasi levels in the presence of a microwave field. We will restrict the discussion to the single electron case: electron–electron interactions will be ignored throughout the paper.

Consider a two state system, with ground state energy $E_G$ and excited state $E_1$ in the presence of a harmonic perturbation. If the frequency of the perturbation matches roughly the spacing between the two levels, the system undergoes oscillations with a frequency which is much smaller than the excitation frequency $\omega$. This Rabi frequency depends on the mismatch $\delta \omega \equiv (E_1 - E_G)/\hbar - \omega$ between the level spacing and the excitation frequency, and the amplitude, or matrix element $F_{1G}$ of the perturbation $\hat{E}$.

\[
\omega_R = \sqrt{\delta \omega^2 + \frac{|F_{1G}|^2}{\hbar^2}}
\]  

If we start with the system initially in the ground state, transitions to the excited state will occur, but the system will ultimately return in the ground state after a period $T_R = 2\pi/\omega_R$.

We propose to use these oscillations in the following device. Consider the potential landscape depicted in Fig. 1: A potential well with depth $V$ is separated from a continuum region by a thin barrier with height $W$. We adjust the well width $a$ so that there is only one bound state in the well. The continuum states with $E > 0$ extend from the well to the free region. If the barrier was infinite, we would have a discrete set of states in the well. For a finite potential barrier, these states become quasi–levels, which have a finite lifetime in the well. We choose the barrier parameters in such a way that only one quasi level exists for the interval $0 < E < W$. By applying a dipole field:

\[
\phi(x,t) = \epsilon x \cos(\omega t)
\]

with a frequency which is close to the transition between the ground state and the quasi level, the system will undergo Rabi oscillations (as long as the wave function has not leaked totally out of the well region). By adjusting the amplitude $\epsilon$ of the external field, the excitation frequency $\omega$ and the barrier parameters, we can control how much of the wave function penetrates the free region.

We perform a numerical calculation to probe this device. In a first step, we need to determine the bound and continuum states for the potential of Fig. 1. The “free” region is chosen to have a width $c$ which is much larger than the well and barrier parameters. The wave functions in all three regions are determined using the connection formulas for square potential steps, and the corresponding energies for the bound and “continuum” states are found numerically. To identify the energy around which the quasi level is centered, we calculate the integrated density of all states with $E > 0$, and select the levels for which this quantity is a maximum. Alternatively, we calculated all matrix elements of the position operator between the ground and excited states, and select the level for which the probability of transition is a maximum (these two procedures give the same result). Once the spacing between the quasi-level energy $E_Q$ and the ground state energy $E_G$ is known, we choose the driving frequency $\omega$ with the desired mismatch $\delta \omega = (E_Q - E_G)/\hbar - \omega$. 

\[
\frac{\delta \omega}{\hbar} = \frac{|F_{1G}|^2}{\hbar}
\]

1
For the time evolution, the dipole potential of Eq. (2) requires special care. Because of the presence of the position operator in Eq. (2), all terms in the Hamiltonian do not commute with each other, and the time ordering in the evolution operator (3) has to be considered. However, by performing a gauge transformation such that the scalar potential in the new gauge is zero and the driving field is fully included in the vector potential, we are able to obtain a Hamiltonian for which all terms commute (in each constant region of the static potential V(x)): the corresponding vector potential is constant in space, and therefore commutes with the momentum operator. For an elementary time step, the time evolution operator is then written in the Caley form, and a generalization of the finite difference scheme of Goldberg, Schey and Schwartz (6) for the case of a vector potential is derived. More details on the method of solution are provided elsewhere (6). At t = 0, the particle is taken to be in the ground state, and at each time step, we compute the integrated density in the free region:

$$\rho_i(t) = \int_{a+b}^{a+b+c} dx \rho(x, t),$$

as well as the overlap |< G|\psi(t) >|^2$ with the ground state.

In Fig. 3a, we plot $\rho_i(t)$ for several values of the driving field amplitude. The barrier width $b = 2$ and height $W = 3$ are chosen to be large enough that the “escape time” of the wave function is large compared to other characteristic times of the problem. Moreover, we have chosen the driving frequency to be close to the resonance condition ($\delta \omega/\omega = 0.0001$), which is smaller than the level spacing for “continuum” energies $E > 0$, in order to check agreement with the two level approximation. For the above parameters, the external frequency is roughly $\omega = 1.5$ in dimensionless units. The infinitesimal time step $\delta t$ for the numerical evolution has to be chosen to be small compared to the period of the external field: here we choose $\delta t = 0.0125 \times (2\pi/\omega)$. The integrated density exhibits steps or plateaux, which allow to identify the Rabi frequency $\omega_R$. Superposed to the plateaux structure are small amplitude oscillations which period corresponds to the driving frequency $\omega$. As we start the simulation, transitions to the quasi-levels and neighboring levels start occurring, but after a period $T_R = 2\pi/\omega_R$, the contribution of the wave function which remained trapped in the well has returned for the most part in the ground state. This is illustrated in Fig. 3b: indeed, aside from a slow decay associated with the transparency of the barrier, |< E_G|\psi(t) >|^2$ oscillates with period which matches exactly the plateaux structure. After the first plateau, $\rho_i(t)$ picks up again as the next plateau returns the trapped wave function to the excited states. The wave function in therefore emitted by “bursts” from the well region every time a Rabi oscillation has been completed.

We note that upon doubling the coupling strength, the period of the oscillations is halved, confirming the fact that the Rabi oscillation frequency scales linearly with the field amplitude if the resonance condition is met. We compare the measured Rabi frequency with the prediction for the two level case $h\omega_R = |< G|x >|^2$, on resonance. To estimate the magnitude of the matrix element, we choose the ground and first excited states of an infinite well, which yields $< G|x > = 8\alpha/9\pi^2$. With $\alpha = 4$, the predicted Rabi frequency is therefore $\omega_R^P \approx 0.04$ (dimensionless units). The measured Rabi frequency yields $\omega_R^M \approx 0.1$, which is in reasonable agreement with the two level result: the discrepancy between the two arises from the matrix element estimate. In addition to these effects, we notice in Fig. 3a that after a Rabi oscillation, the integrated density is, as expected, larger for higher values of the driving field amplitude.

Next, in Fig. 3a, we compare the evolution of this system for several values of the barrier widths, keeping the other parameters fixed. By changing this parameter, we effectively vary the escape time of the particle trapped in the well. For small widths ($b < 1$), the escape time of the excited wave function is so small that the Rabi oscillations are hardly noticeable: within a Rabi period, most of the wave function has tunneled in the free region. The overlap plot of Fig. 3a confirms these statements, showing a strong decay over a few Rabi oscillations. Fig. 3a illustrates how the switching of the current in the free region can be tuned by varying the width of the barrier. If we were to further reduce the width of the barrier (say, $b = 0.5$), the Rabi oscillations would cease to be noticeable: the width of the quasi level becomes large and the (on resonance) matrix element is strongly reduced in magnitude. Consequently, the activation mechanism provided by the dipole potential ceases to be efficient.

We note that it is also possible to regulate the magnitude of the leakage current by varying the off resonance mismatch $\delta \omega$. According to Eq. (1), changing the mismatch also changes the Rabi frequency. This is illustrated in Fig. 3a for the same coupling strength, we compare the integrated density for mismatches $\delta \omega/\omega = 0.01%$ (practically on resonance), $\delta \omega/\omega = 5\%$, and $\delta \omega/\omega = 10\%$. Away from resonance, the ground state couples with continuum states neighboring the quasi level position, which have smaller matrix elements. Consequently, the leakage current is smaller than on resonance. Moreover, as $\delta \omega$ is increased, the first term in the square root of Eq. (1) becomes important, and the period of the Rabi oscillations is decreased.

We conclude with a brief discussion of the feasibility of this device. Our main concern is whether phonon relaxation processes could induce transitions from the quasi level to the ground state before the excited states can tunnel out of the barrier. We proceed to convert the parameters of the problem into dimension full units. If we
choose the height of the barrier to be of the order of 1eV, and the effective mass \( m^* = 0.067m_e \) for GaAs, the width of the well corresponds to a length \( a \approx 4.8 \times 10^{-7} \text{cm} \). The excitation frequency is \( \omega \approx 0.8 \times 10^{15} \text{s}^{-1} \), and the constant electric field associated with the dipole potential is \( E \approx 3 \times 10^3 \text{V/cm} \). For GaAs, typical optical phonon relaxation rates range from \( \tau_{\text{ph}} \sim 10^{-10} \text{s} \) to \( 10^{-7} \text{s} \) which is very large compared to all other characteristic length scales of the problem. We therefore do not expect any significant changes in the qualitative behavior of the device from these processes.

In summary, we have proposed a switching device based on Rabi oscillations. The device is composed of a well region with a bound state and a quasi-level. Upon activation by a microwave field, a particle in the ground state can be activated to the quasi level, and subsequently tunnel the free region adjacent to the well. The leakage current can be controlled in several ways: by varying the intensity of the driving field, by adjusting the width or height of the tunneling barrier, or by tuning the resonance mismatch to a desired value. We also note that this device could in principle also be used “in reverse” to measure the energy of a particle incident on the barrier and well region, from the free region. The incident particle tunnels in the well, and in the event that its energy corresponds to the quasi-level energy, it will get absorbed in the ground state. If the energy of the incident does not meet this resonance condition, the wave function should tunnel back in the free region without noticeable absorption in the ground state.

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FIG. 1. Potential landscape: a well region is separated by a potential barrier from the “free region”. The well depth and barrier height are adjusted so that there is only one bound state in the well, and there is only one quasi-level below the barrier.