A low-dissipation monotonicity-preserving scheme for turbulent flows in hydraulic turbines

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Abstract. The objective of this work is to improve the inherent dissipation of the numerical schemes under the framework of a Reynolds-averaged Navier-Stokes (RANS) simulation. The governing equations are solved by the finite volume method with the k-ω SST turbulence model. Instead of the van Albada limiter, a novel eddy-preserving limiter is employed in the MUSCL reconstructions to minimize the dissipation of the vortex. The eddy-preserving procedure inactivates the van Albada limiter in the swirl plane and reduces the artificial dissipation to better preserve vortical flow structures. Steady and unsteady simulations of turbulent flows in a straight channel and a straight asymmetric diffuser are demonstrated. Profiles of velocity, Reynolds shear stress and turbulent kinetic energy are presented and compared against large eddy simulation (LES) and/or experimental data. Finally, comparisons are made to demonstrate the capability of the eddy-preserving limiter scheme.

1. Introduction
At part load conditions of a Francis turbine, an undesirable vortex rope develops within the draft tube, and induces severe low frequency and large amplitude pressure fluctuations. The pressure fluctuations endanger the hydro turbine, especially when its frequency approaches the turbine natural frequency. Thus, knowledge of the dynamic load on the turbine is a major concern of the hydraulic community, and the simulation of unsteady flow in draft tubes has attracted attention during the past two decades. In order to accurately capture the flow features, efforts have been made by employing more advanced turbulence models, improving the grid quality and extending the computational domain to the complete hydro turbine. Apart from turbulence models and grid effects, discretization schemes also play a crucial role in the simulation of the vortex rope. With the vast adoption of commercial software, the role of numerical schemes have unfortunately been removed. It is known that the dissipation error is characterized by a loss of wave amplitude, while the dispersion error is characterized by a phase difference between the numerical and analytical solution. Since the frequency of the vortex rope was well predicted while the pressure pulsation amplitudes were underestimated in most previous studies [1, 2, 3, 4], the inherent dissipation of the numerical schemes requires a careful reexamination.

Pulsating flow in a straight asymmetric diffuser is a good representative model for investigating the unsteady effects of flow in draft tubes. The pulsating flow is characterized by several parameters. The parameter \( a_{uc} \) is the ratio between the oscillating and mean centerline velocity, and if \( a_{uc} < 1 \), the flow is current-dominated. Current-dominated flows are largely controlled by the dimensionless forcing frequency \( \omega^+ = \omega \nu / u_s^2 = 2/t_s^+ \), where \( \omega \), \( \nu \), \( u_s \) and \( t_s^+ \) represent the angular frequency of the oscillation, the kinematic viscosity, the friction velocity,
and the dimensionless Stokes length, respectively. The dimensionless turbulent Stokes length $l^+_t$ is given by,

$$l^+_t = l_s^+ \left( \frac{\kappa l^+_s}{2} + \sqrt{1 + \left( \frac{\kappa l^+_s}{2} \right)^2} \right),$$  

where $\kappa$ is the von Kármán constant. The turbulent Stokes length $l^+_t$ defines a scale of the depth that the vorticity waves generated at the wall penetrate into the flow, and is inversely proportional to $\omega^+$. The effect of the oscillation on the flow is confined to a layer of thickness $2l^+_t$. If the thickness $2l^+_t$ is greater than the half-height of the channel $h^+ = Hu^*/\nu$, then the entire flow is affected by the unsteadiness. On the other hand, in the regime $2l^+_t < h^+$, there exists a region near the centreline of the channel where the flow is not affected and behaves like a plug flow. As a consequence, the pulsating flow can be classified into different regimes according to its forcing frequency. For low ($\omega^+ < 0.005$), medium ($0.005 < \omega^+ < 0.02$), and high ($0.02 < \omega^+ < 0.04$) frequency, the corresponding flows are in quasi-steady, relaxation, and quasi-laminar regimes respectively.

In this paper, motivated by the potential to improve the ability of capturing unsteady flow features, efforts are made in the aspect of numerical schemes. We adopt the eddy-preserving limiter [5] to develop a low-dissipation scheme for RANS simulations in hydraulic turbines.

2. Methodology

The compressible Reynolds-averaged Navier-Stokes (RANS) equations are solved with a second-order finite-volume approach and the $k-\omega$ SST turbulence model. Here we present two numerical schemes: the Jameson-Schmidt-Turkel (JST) scheme serves as the baseline scheme for comparison against the novel eddy-preserving limiter scheme.

2.1. Stiffened gas equation of state

The stiffened gas law is employed as the equation of state (EOS) to close the system, which is given by $p = (\gamma - 1)\rho e - \gamma p_\infty$, and the speed of sound is computed by $c = \sqrt{\gamma \frac{p + p_\infty}{\rho}}$, where $p, \rho, e$ and $c$ are respectively the pressure, density, specific internal energy, and speed of sound. The specific heat ratio, $\gamma$ and $p_\infty$ are two parameters used to define liquids with different properties.

The stiffened gas equation of state can provide a reasonable approximation for liquids under high pressure conditions. For water, values for $\gamma$ and $p_\infty$ used by various authors are listed in Table 1. In this paper, to respect the physical speed of sound in water at $20^\circ C$, $\gamma$ and $p_\infty$ are chosen to be 7.15 and $3.1 \times 10^8$ Pa respectively. As a consequence of the large speed of sound, the Mach number is typically very low which makes the system difficult to converge. To alleviate this problem, a low speed preconditioner [6] is implemented in this work.

2.2. Jameson-Schmidt-Turkel (JST) scheme

In the JST scheme, the numerical flux across the interface between cells $i$ and $i+1$ is evaluated as,

$$F_{i+\frac{1}{2}} = \frac{1}{2}(F_i + F_{i+1}) - D_{i+\frac{1}{2}},$$  

where $F_i = F(V_i)$, and $V = \{\rho, \rho u, \rho v, \rho w, \rho E\}^T$ is the state variable vector. The artificial dissipation is defined as,

$$D_{i+\frac{1}{2}} = c^{(2)}_i r_{i+\frac{1}{2}} \Delta V_{i+\frac{1}{2}} - c^{(4)}_i r_{i+\frac{1}{2}} (\Delta V_{i+\frac{1}{2}} - \Delta V_{i-\frac{1}{2}} + \Delta V_{i+\frac{1}{2}} - \Delta V_{i-\frac{1}{2}}),$$  

where $c^{(2)}_i$ and $c^{(4)}_i$ are the second and fourth order artificial dissipation coefficients, respectively.
Table 1: Parameters for stiffened gas equation of state used by various authors.

| Authors                     | $\gamma$ | $p_\infty$ ($\times 10^8$ Pa) | Sound Speed (m/s) |
|-----------------------------|----------|-------------------------------|-------------------|
| Goncalves & Patella [7]     | 1.01     | 0.1211                        | 110.7 (artificial)|
| Chang & Liou [8]            | 1.932    | 11.645                        | 1533              |
| Le Métayer et al. [9]       | 2.35     | 10.0                          | 1543              |
| Paillere et al. [10]        | 2.8      | 8.5                           | 1543              |
| Barberon & Helluy [11]      | 3.0      | 8.533                         | 1600              |
| Saurel & Abgrall [12]       | 4.4      | 6.0                           | 1626              |
| Shyue [13]                  | 7.0      | 3.0                           | 1449              |
| Luo et al. [14]             | 7.0      | 3.03975                       | 1459              |
| Gallouët et al. [15]        | 7.15     | 3.0                           | 1465              |
| Present paper               | 7.15     | 3.1                           | 1489              |

where $r_{i+\frac{1}{2}}$ is the spectral radius of the Jacobian matrix. The dissipative coefficients are given by $\epsilon^{(2)}_{i+\frac{1}{2}} = \kappa_2 \max(s_i, s_{i+1})$ and $\epsilon^{(4)}_{i+\frac{1}{2}} = \max(0, \kappa_4 - \epsilon^{(2)}_{i+\frac{1}{2}})$, where the pressure sensor is defined as $s_i = \frac{p_{i+1} - 2p_i + p_{i-1}}{p_{i+1} + 2p_i + p_{i-1}}$.

2.3. Eddy-preserving limiter scheme

The state variables are reconstructed at the interfaces for the MUSCL scheme using,

$$V_i^+ = V_i + \frac{\Phi_i}{4}[(1 - \kappa)\Delta_i^u + (1 + \kappa)\Delta_i^c], \quad \text{(4)}$$

$$V_j^- = V_j + \frac{\Phi_j}{4}[(1 - \kappa)\Delta_j^u + (1 + \kappa)\Delta_j^c], \quad \text{(5)}$$

where $\Phi_i$ and $\Phi_j$ are slope limiters, and $-1 \leq \kappa \leq 1$. The upwind and central increments are computed by,

$$\Delta_i^c = V_j - V_i, \quad \Delta_j^u = 2 \nabla V_i \cdot s_{ij} - (V_j - V_i), \quad \text{(6)}$$

$$\Delta_j^u = 2 \nabla V_j \cdot s_{ij} - (V_j - V_i), \quad \text{(7)}$$

where $s_{ij} = x_j - x_i$. In our baseline variable reconstruction scheme, the van Albada limiter is employed and $\kappa = \frac{1}{3}$. When $\kappa$ is further increased, the artificial dissipation decreases. In the limit, where $\kappa = 1$, a purely central but unstable convective flux calculation scheme is obtained.

The concept of the eddy-preserving limiter is to prevent the slope limiter to be activated (or to use a less dissipative limiter) during the reconstruction of the velocity component along the tangential direction of a vortex. To identify vortical flow regions, the enhanced swirling strength criterion of Chakraborty et al. [16] is employed, whereby in a vortical region, the velocity gradient tensor $\nabla \mathbf{v}$ possesses a conjugate pair of complex eigenvalues,

$$\sigma(\nabla \mathbf{v}) = \{\lambda_r, \lambda_{cr} + i\lambda_{ci}, \lambda_{cr} - i\lambda_{ci}\}, |\lambda_{ci}| > \epsilon, \quad \text{(8)}$$

where $\epsilon$ is a small positive real number. A local measure for the compactness of the vortical motion is added to further limit the vortical regions to areas where the following condition is satisfied,

$$-\zeta \leq \frac{\lambda_{cr}}{\lambda_{ci}} \leq \delta, \quad \text{(9)}$$
where \( \zeta \) and \( \delta \) are positive thresholds for verifying the compactness of the vortex. The velocity gradient tensor can be decomposed into the following form,

\[
\nabla v = [\hat{u}_r \hat{u}_{cr} \hat{u}_{ci}] \begin{bmatrix} \lambda_r & 0 & 0 \\ 0 & \lambda_{cr} & \frac{|\hat{u}_{cr}|}{|\hat{u}_{ci}|} \\ 0 & -\lambda_{ci} & \lambda_{cr} \end{bmatrix} [\hat{u}_r \hat{u}_{cr} \hat{u}_{ci}]^{-1},
\]  

(10)

where \( \hat{u}_r, \hat{u}_{cr}, \) and \( \hat{u}_{ci} \) are normalized eigenvectors of the velocity gradient tensor. The mapping and transformation matrix from the original Cartesian system \( S_0 \) to the local vortex system \( S_\omega \) spanned by \( \hat{u}_r, \hat{u}_{cr}, \) and \( \hat{u}_{ci}, \) are given by,

\[
[M] : S_0 \mapsto S_\omega \quad [\hat{M}] = [\hat{u}_r \hat{u}_{cr} \hat{u}_{ci}]^{-1}.
\]  

(11)

The algorithm for the eddy-preserving limiting procedure is described below:

1. Calculate eigenvalues of the velocity gradient tensor \( \nabla v \). If the velocity gradient tensor has only real eigenvalues, then exit and employ the conventional van Albada limiter.
2. Verify the compactness of the vortical motion using Eqn. (9). If the flow lacks vortical compactness, then exit and employ the conventional van Albada limiter.
3. Calculate eigenvectors of the velocity gradient tensor \( \nabla v \), and define the transformation matrix \([\hat{M}]\).
4. Transform velocity components into the vortex system \( S_\omega \).
5. In the axial direction, reconstruct variables using the conventional van Albada limiter with \( \kappa = \frac{1}{3} \).
6. In the swirl plane, reconstruct variables using a higher \( \kappa \) which leads to less artificial dissipation and inactivate the van Albada limiter by setting \( \Phi_i = \Phi_j = 1 \).
7. Transform interpolated velocity components back to the original system \( S_0 \) and evaluate the fluxes.

3. Numerical results
The eddy-preserving limiter has been implemented in the simulations of flow past a NACA0015 wing under a static stall condition [5]. As shown in Fig. 2, the simulation with the eddy-preserving limiter presents more detailed structures of the shedding vortices, an accurate stall
separation point, and a precise representation of the tip-vortex due to its low-dissipative nature, which demonstrates the superiority of the eddy-preserving limiter. Numerical results presented for the pulsating turbulent flow in this paper are obtained using the JST scheme. The simulations based on the eddy-preserving limiter are yet to be completed, and the results will be added in the final version of the paper.

Figure 2: $\lambda_2$ isosurfaces for flow around NACA0015 at 18° AOA under static stalls condition. (a) without the eddy-preserving limiter. (b) with the eddy-preserving limiter [5].

3.1. Pulsating turbulent flow in a straight channel
Simulations of steady and unsteady turbulent flows in a straight channel [17, 18] were performed. The flow is forced by a given pressure gradient. The geometry of the channel is illustrated in Fig. 3, where the height is $L_z = 2H$, and the length in the streamwise and width in the spanwise directions are $L_x = 3\pi H$ and $L_y = \pi H$ respectively. The grid size is $65 \times 65 \times 65$, which is the same as that employed for the LES reference test case [17]. Periodic boundary conditions are used in both streamwise and spanwise directions, and no-slip conditions at the upper and lower boundaries.

Figure 3: Computational domain of the channel. Figure 4: Comparison of $u^+$ profile against DNS data. Figure 5: Time series of the phase-averaged centreline velocity.
First, given a steady pressure gradient $P_f(x) = \Delta P_0 \cdot x/L_x$, a steady channel test case was performed at $Re_\tau = 395$. Fig. 4 shows a good agreement of the $u^+$ profile against the DNS data [19]. Then, by setting an unsteady pressure gradient $P_f(x,t) = \Delta P_0[1 + \alpha \cos(\omega t + \pi/2)]x/L_x$ with a medium forcing frequency $\omega^+ = 0.01$, a pulsating channel flow was simulated. The flow begins the acceleration phase at $t = 0$, and decelerates at $t = T/2$. The results were compared against the LES data, and the URANS results of Scotti and Piomelli [18]. Three different PSTEP values were tested, where PSTEP corresponds to the number of physical time steps within a period. It was found that, there was an improvement from PSTEP = 16 to PSTEP = 32, while the difference of results obtained by PSTEP = 32 and PSTEP = 64 was negligible.

The phase-averaged and time-averaged quantities are defined as,

$$<u^+(z,t) = \frac{1}{NL_xL_y} \sum_{n=1}^{N} \int_0^{L_y} \int_0^{L_x} u^+(x,y,z,t+nT) dx dy, \quad (12)$$

and

$$u_0^+ = \frac{1}{T} \int_0^{T} <u^+> dt. \quad (13)$$

Fig. 5 demonstrates a good agreement of the time series of the phase-averaged centreline velocity between the present work and the LES results [17]. Fig. 6 shows the phase-averaged streamwise velocity profiles at intervals of $T/4$. The predictions are in reasonable agreement with the LES data; however, the discrepancy is more evident during the deceleration phase. A significant contribution to the differences between the URANS and the LES is caused by the error in the prediction of the time-averaged velocity $u_0^+$. The time-averaged velocity is underpredicted by as much as 20% near the centreline, as shown in Fig. 7, however, from Fig. 7(a), we can see that all turbulence models behave similarly. Consider the phase-averaged Reynolds shear stress in Fig. 8 where good agreement is shown during the acceleration phase; however, a severe overestimation is observed at the onset of the deceleration phase. This can be explained by the early increase of the maximum phase-averaged Reynolds shear stress in Fig. 9. The phase-averaged turbulent kinetic energy are compared in Fig. 10, where good agreement is shown at $t = T/2$, while the present results are generally lower than the LES data at three other time instances. In Fig. 11, the maximum phase-averaged turbulent kinetic energy is underpredicted,
Figure 8: Profiles of the phase-averaged Reynolds shear stress. Profiles are plotted every $T/4$ and offset in the vertical direction; the bottom plot corresponds to $t/T = 0$. (a) Scotti and Piomelli [18]. (b) Present Work.

Figure 9: Time series of the maximum phase-averaged Reynolds shear stress $<u'w'>$. (a) Scotti and Piomelli [18]. (b) Present Work.

Figure 10: Profiles of the phase-averaged turbulent kinetic energy $k$. Profiles are plotted every $T/4$ and offset in the vertical direction; the bottom plot corresponds to $t/T = 0$. (a) Scotti and Piomelli [18]. (b) Present Work.

Figure 11: Time series of the phase-averaged maximum of the turbulent kinetic energy $k$.

and the peak value is significantly underestimated during the deceleration phase. However, the position of the turning point, and the slope of the increase of the turbulent kinetic energy is well predicted. In summary, the present results are comparable to other RANS turbulence models and very similar to the predictions of the $k$-$\omega$ model in [18], and the agreement between RANS and LES is generally better in the region $z^+ > 2l_t^+$, where the flow is not affected by the unsteadiness.

3.2. Pulsating turbulent flow in an straight asymmetric diffuser

Cervantes and Engström [20] carried out an experimental investigation of flow unsteadiness in a straight asymmetric diffuser. The geometry of the closed-loop system is shown in Fig. 12, and it was driven by a pump at an average volumetric flow rate of $2.47 \times 10^{-3}$ m$^3$/s. Measurements were made at two sections: $x = 2082$ and 2632 mm. The straight duct has a length of 2102 mm and a rectangular cross section of 100 $\times$ 150 mm. Experiments were conducted at steady flow and three pulsating flows with frequencies 0.03, 0.10 and 0.35 Hz. The oscillation amplitude was taken to be 15% of the mean centreline velocity.
Figure 12: Schematic view of the test rig with dimensions in mm [20].

Figure 13: Grid for the straight diffuser.

The computational grid for numerical simulations is shown in Fig. 13, and the number of grid points is approximately 5 million. The friction Reynolds number $Re_\tau = Hu_*/\nu$ is 470, based on the half duct height and friction velocity at section 2082. The hydraulic diameter is $D_h = 4 \times \text{area}/\text{perimeter} = 0.12$ m. The flow entering the diffuser is a developing duct flow at Reynolds number 20000, based on the mean streamwise velocity and hydraulic diameter. For boundary conditions, a fully-developed laminar velocity profile which corresponds to the same volumetric flow rate as the experiment was imposed at the inlet and a constant reference pressure was fixed at the outlet. The turbulent parameters were determined using $\kappa = \frac{3}{2}(u_{avg}I)^2$ and $\omega = \frac{\kappa^{1/2}}{C_\mu l}$, where the turbulence intensity and turbulence length scale are estimated by $I = \frac{u'}{u_{avg}} = 0.16(Re_{D_h})^{-1/8}$ and $l = 0.07D_h$ respectively, and $C_\mu = 0.09$ is an empirical constant.

Steady and unsteady simulations at three different frequencies, i.e. 0.03, 0.10 and 0.35 Hz were performed.

The time averaged streamwise velocity at section 2082 in Fig. 14(b) agrees very well against the law of the wall. In the viscous sublayer region, the prediction collapses on the curve $u^+ = y^+$, and in the log-law region, the velocity profile has the same slope as the log-law curve $u^+ = \frac{1}{k} \ln y^+ + B$ ($k = 0.418$ and $B = 5.45$), but at a slightly lower magnitude. At section 2632, the streamwise velocity (Fig. 15(b)) was reasonably predicted before the log-law region. Although the streamwise velocity is supposed to be elevated above the log-law, the increase began earlier in the simulation than the experiment, and the mean centreline velocity was about 10% overpredicted.

In Fig. 16(b), the predicted time averaged vertical velocities at section 2082 are slightly greater than zero, while all the experimental vertical velocities are negative. In the experiment, the flow was tripped by the plates at the exit of the contraction. Since the contraction and trip plates were not included in the computational domain in the current computation, the turbulence generated by the trip plates were eliminated. It is also worth noting that the absolute values of vertical velocities are very small quantities close to zero, and less than 3% of the maximum streamwise velocity. In addition, the uncertainty in the experiment on $v^+$ is $5 \times 10^{-2}$ [20], which is a relatively large number in comparison with $v^+$ itself. The situation is similar in section 2632 (Fig. 17 (b)), while the maximum value is predicted much higher.

The vertical turbulent kinetic energy is shown in Fig. 18 and Fig. 19. At both sections, the vertical turbulent kinetic energy has higher levels than those in the experiment, especially in the region near the centreline. The time averaged Reynolds stress profiles at section 2082 (Fig. 20) and 2632 (Fig. 21) are similar to the experimental data, primarily due to the good prediction of the streamwise velocity.
4. Conclusion
A low-dissipation scheme was developed for turbulent flows in hydraulic turbines. The methodology is based on employing the novel eddy-preserving limiter within a MUSCL reconstruction. The scheme was implemented to simulate both steady and unsteady flows in a channel and an asymmetric diffuser. Preliminary numerical results agree well against experimental data, and the final results will compare the baseline to the proposed low-dissipation
Figure 20: Time averaged Reynolds stress at section 2082. (a) Experiment [20]. (b) Present Work.

Figure 21: Time averaged Reynolds stress at section 2632. (a) Experiment [20]. (b) Present Work.

scheme.

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