Monopole creation operators
as confinement–deconfinement order parameters

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Abstract

We study numerically two versions of the monopole creation operators proposed by Fröhlich and Marchetti. The disadvantage of the old version of the monopole creation operator is due to visibility of the Dirac string entering the definition of the creation operator in the theories with coexisting electric and magnetic charges. This problem does not exist for the new creation operator which is rather complicated. Using the Abelian Higgs model with a compact gauge field we show that both definitions of the monopole creation operator can serve as order parameters for the confinement–deconfinement phase transition. The value of the monopole condensate for the old version depends on the length of Dirac string. However, as soon as the length is fixed the old operator certainly discriminates between the phases with condensed and non–condensed monopoles.

1 Introduction

The order parameters are useful tools for investigation of phase transitions. Up to now there is no good definition of the order parameter for the temperature phase transition in QCD with dynamical quarks. The traditional order parameters like the expectation value of the Polyakov line and the tension of the chromoelectric string work well in the quenched case (no dynamical quarks) but they fail to distinguish between the phases of the full QCD. Indeed, even at zero temperature the string spanned on a quark and an anti–quark can be broken by sea quarks making both the Polyakov line and the string tension useless candidates for the order parameter. Moreover, the value of the quark condensate $\langle \bar{\Psi}_q \Psi_q \rangle$ is strict order parameter only for massless dynamical quarks.

Below we discuss the quantities which may serve as the order parameters for full QCD if the monopole (or, ”dual superconductor”) confinement mechanism \cite{1} is valid. In this picture the monopoles defined with the help of the Abelian projection \cite{2} are supposed to be condensed...
in the confinement phase. The monopole condensate causes a dual analogue of the Abrikosov vortex to be formed between quarks and anti–quarks. As a result the quarks and anti–quarks are confined into colorless states. In the deconfinement phase the monopoles are not condensed and quarks are not confined. Thus the natural confinement–deconfinement order parameter is the value of the monopole condensate, which should be nonzero in the confinement phase and zero in the deconfinement phase.

There are two (formal) difficulties in defining of the monopole condensate. At first, the monopole condensate is defined as an expectation value of the monopole field. However, the monopoles are the topological defects in the compact Abelian gauge field and, as a result, the immediate output of the lattice simulations can not provide us with values the monopole fields. This difficulty can be overcome by noticing that the lattice output comes in a form of information about the monopole trajectories. Then one can apply a known procedure which allows us to rewrite the path integral over monopole trajectories as an integral over the monopole fields, and get the monopole condensate.

The second difficulty is that the expectation value of the \( \phi \) should always be zero regardless whether this field is condensed or not. The reason is very simple: the path integral includes the integration over all possible gauges while the charged field is a gauge dependent quantity. These two problems were solved in Ref. \( [3] \), where the gauge invariant monopole creation operator for compact QED (cQED) was explicitly constructed. The numerical calculations in the lattice cQED \( [4] \) and in the Maximal Abelian projection of the lattice \( SU(2) \) gluodynamics \( [5] \) confirm that the expectation value of this operator is an order parameter for the confinement–deconfinement phase transition. Other types of the monopole creation operators were investigated in Refs. \( [6] \).

However, it noticed recently in Ref. \( [8] \) that the “old” monopole creation operator \( [3] \) depends on the shape of the Dirac string in the models with electrically charged dynamical fields. Exactly the same situation appears in the Abelian projection of QCD: the off-diagonal gluons become electrically charged dynamical fields while the diagonal gluons turns into compact Abelian gauge fields containing monopole singularities. Below we figure out whether the dependence on the Dirac string in the “old” version of the monopole creation operator is crucial for the usage of the expectation value of this operator as an order parameter. Another (“new”) monopole creation operator was suggested in Ref. \( [8] \). This operator does not depend on the shape of the Dirac string even in the presence of the dynamical electric charges. However, the numerical calculations \( [7] \) show that the construction of this operator is rather complicated and the simulations are time consuming.

The structure of the this paper is as follows. We explicitly describe the “new” and the “old” monopole creation operators in Sections 2 and 3, respectively. The results of the numerical calculations of these creation operators in the compact Abelian Higgs model are presented in Section 4. We demonstrate that both these operators can be used to detect confinement–deconfinement phase transitions in the theories with charged matter. Our conclusions are presented in Section 5.
2 “Old” monopole creation operator.

The gauge invariant creation operator $\Phi$ was suggested by Dirac [9]:

$$\Phi = \phi(x) \exp \left\{ i \int E_k(\vec{x} - \vec{y}) A_k(\vec{y}) \, d^3y \right\},$$

(1)

here $\phi(x)$ and $A_k(x)$ are the electrically charged field and the gauge potential, which transform under the gauge transformations as

$$\phi(x) \rightarrow \phi(x)e^{i\alpha(x)}, \quad A_k(x) \rightarrow A_k(x) + \partial_k \alpha(x).$$

(2)

The Coulomb field, $E_k(x)$, satisfies the equation:

$$\partial_k E_k = \delta^{(3)}(x).$$

(3)

It is easy to see that the operator $\Phi$, eq. (1), is invariant under the gauge transformations (2).

The Fröhlich–Marchetti construction [3] of the monopole creation operator in cQED is based on eq. (1). At first step the partition function of cQED is transformed to a dual representation. For the general form of the cQED action it can be shown [5] that the dual theory is an Abelian Higgs model (AHM) in the limit when both the Higgs boson mass and the gauge boson mass are infinite. In this theory the Higgs field, $\phi_x$, corresponds to the monopole defect in the original cQED. The gauge field $^*B$ is dual to the original gauge field $\theta$. Thus the gauge invariant creation operator (1) for the AHM model corresponds to the monopole creation operator in the original cQED. The explicit expression for this operator on the lattice is (cf. eq.(2)):

$$\Phi_{\text{mon}, \text{old}} = \phi_x e^{i(\ ^*B, ^*H_x)},$$

(4)

where $^*H_x$ is the Coulomb field of the monopole, $^*\delta^*H_x = ^*\delta_x$, and $^*\delta_x$ is the discrete $\delta$–function defined on the dual lattice. Here and below we will use the differential form notations on the lattice: $(a, b) = \sum_c a_c b_c$ is the scalar product of the forms $a$ and $b$ defined on the $c$–sells; $(a, a) \equiv ||a||^2$ is the norm of the form $a$; $d$ is the forward derivative (an analog of the gradient); $\delta$ is the backward derivative (an analog of the divergence) and $^*$–operation transfers a form to the dual lattice. For a description of the language of the differential forms on the lattice see, e.g., review [10].

Performing the inverse duality transformation for the expectation value of the creation operator (4), we get the expectation value, $\langle \Phi^{\text{mon}} \rangle$, of this operator in cQED:

$$\langle \Phi^{\text{mon}, \text{old}} \rangle = \frac{1}{Z} \int_{-\pi}^{\pi} \mathcal{D}\theta \exp\{-S(d\theta + W)\}, \quad Z = \int_{-\pi}^{\pi} \mathcal{D}\theta \exp\{-S(d\theta)\},$$

(5)

where $d\theta$ is the plaquette angle, and $S$ is the periodic lattice action, $S(d\theta + 2\pi n) = S(d\theta)$, $n \in \mathbb{Z}$. The form $W = 2\pi \delta \Delta^{-1}(*H_x - \omega_x)$ depends on the Dirac string $^*\omega_x$ defined on the dual lattice. The Dirac strings start and end on the monopoles and anti–monopoles, $^*\delta_x = \delta_x$. The operator $\Delta = d\delta + \delta d$ is the lattice Laplacian. The numerical investigation of this
creation operator in cQED shows [4] that it can be used as the confinement–deconfinement order parameter.

The operator (4) is well defined for the theories without dynamical matter fields. However, if an electrically charged matter is added, then the creation operator (4) depends on the position of the Dirac string. To see this fact let us consider the compact Abelian Higgs model with the Villain form of the action:

\[ Z_{AHM} = \int_{-\pi}^{\pi} D\theta \int_{-\pi}^{\pi} D\varphi \sum_{n \in \mathbb{Z}(c_2)} \sum_{l \in \mathbb{Z}(c_1)} \exp \left\{ -\beta \left| \left| d\theta + 2\pi n \right| \right|^2 - \gamma \left| d\varphi + q\theta + 2\pi l \right|^2 \right\}. \] (6)

Here \( \theta \) is the compact Abelian gauge field and \( \varphi \) is the phase of the dynamical Higgs field. The integer \( q \) is the charge of the Higgs field. For the sake of simplicity we consider the London limit (the Higgs mass is infinitely large while the Higgs condensate is fixed).

Let us perform the Berezinsky-Kosterlitz-Thouless (BKT) transformation [11] with respect to the compact gauge field \( \theta \):

\[ d\theta + 2\pi n = dA + 2\pi \delta \Delta^{-1} j, \quad \text{with} \quad A = \theta + 2\pi \delta \Delta^{-1} m[j] + 2\pi k. \] (7)

Here \( A \) is the non–compact gauge field, \(*m[j]\) is a surface on the dual lattice spanned on the monopole current \(*j\) (\( \delta^*m[j] = *j \)) and \( k \) is the integer–valued vector form\(^1\). We substitute eqs.(7) in eq.(6) and shift of the integer variable, \( l \rightarrow l + qk \).

Next we perform the BKT transformation with respect to the compact scalar field \( \varphi \):

\[ d\varphi + 2\pi l = d\vartheta + 2\pi \delta \Delta^{-1} \sigma, \quad \text{with} \quad \vartheta = \varphi + 2\pi \delta \Delta^{-1} s[\sigma] + 2\pi p. \] (8)

Here \( \vartheta \) is the non–compact scalar field, \(*s[\sigma]\) is a 3D hyper–surface on the dual lattice spanned on the closed surface \(*\sigma\) (\( \delta^*s[\sigma] = *\sigma \)) and \( p \) is the integer–valued scalar form.

Substituting eqs. (7,8) into the partition function (6) and integrating the fields \( A \) and \( \varphi \) we get the representation of the compact AHM in terms of the monopole currents \(*j\) and world sheets of Abrikosov strings \(*\sigma\ (“\text{the BKT–representation}”):\n
\[ Z_{AHM} \propto Z_{BKT} = \sum_{*s[j] \in \mathbb{Z}(c_3)} \sum_{\delta^*s[j] = *j} \exp \left\{ -4\pi^2/\beta \left( j, \frac{1}{\Delta + m^2} j \right) - 4\pi^2 \gamma \left( \sigma_j, \frac{1}{\Delta + m^2} \sigma_j \right) \right\}, \] (9)

the dual surface variable \(*\sigma_j = \sigma + q^*m[j]\) is spanned \( q \)–times on the monopole current \( j \), \( \delta^*\sigma_j = q^*j \), since the flux of the magnetic monopole having an unit magnetic charge can be taken out by \( q \) strings carrying the elementary flux \( 2\pi/q \). The mass of the gauge boson \( \theta \) is \( m = q \sqrt{\gamma/\beta} \).

The BKT–representation (9) of the AHM partition function (6) can also be transformed into the dual representation using simple Gaussian integrations. We use two dual compact fields \(*B\) (vector field) and \(*\xi\) (scalar field) in order to represent the closeness properties of the

\(^1\)A detailed description of the duality and BKT transformations in terms of the differential forms on the lattice can be found, e.g., in Ref. [10].
currents $\ast \sigma_j$ and $\ast j$, respectively. We also introduce two dual non–compact fields, $\ast F$ (vector field) and $\ast G$ (rank-2 tensor field), in order to get a linear dependence on, correspondingly, the currents $\ast \sigma_j$ and $\ast j$ under the exponential function:

$$Z_{BKT} = \text{const.} \int_{-\infty}^{\infty} D \ast F \int_{-\infty}^{\infty} D \ast G \int_{-\pi}^{\pi} D \ast B \int_{-\pi}^{\pi} D \ast \zeta \sum_{j \in \mathbb{Z}(\ast c_3)} \sum_{\ast \sigma_j \in \mathbb{Z}(\ast c_2)} \exp \left\{ -\ast \beta (\ast G, (\Delta + m^2) \ast G) - \ast \gamma (\ast F, (\Delta + m^2) \ast F) + i(\ast F, \ast \sigma_j) + i(\ast G, \ast j) + i(\ast B, \delta \ast \sigma_j - q \ast j) - i(\ast \zeta, \delta \ast j) \right\} ,$$

(10)

where

$$\ast \beta = \frac{1}{16 \pi^2 \gamma}, \quad \ast \gamma = \frac{1}{16 \pi^2 \beta} .$$

Note that in this representation the integer variables $\ast \sigma_j$ and $\ast j$ are no more restricted by the closeness relations. Therefore we can use the Poisson summation formula with respect to these variables and integrate out the fields $\ast F$ and $\ast G$. Finally, we obtain the dual field representation of the partition function (6):

$$Z_{BKT} \propto Z_{\text{dual field}} = \int_{-\pi}^{\pi} D \ast B \int_{-\pi}^{\pi} D \ast \zeta \sum_{u \in \mathbb{Z}(\ast c_3)} \sum_{v \in \mathbb{Z}(\ast c_2)} \exp \left\{ -\ast \beta \left( d \ast B + 2 \pi \ast u, (\Delta + m^2) (d \ast B + 2 \pi \ast u) \right) - \ast \gamma \left( d \ast \zeta + q \ast B + 2 \pi \ast v, (\Delta + m^2) (d \ast \zeta + q \ast B + 2 \pi \ast v) \right) \right\} ,$$

(12)

where $\ast u$ and $\ast v$ are the integer valued forms defined on the plaquettes and links of the dual lattice, respectively. Clearly, this is the dual Abelian Higgs model with the modified action.

The gauge field $\ast B$ is compact and the radial variable of the Higgs field is frozen. The model is in the London limit and the dynamical scalar variable is the phase of the Higgs field $\ast \zeta$.

Thus in the presence of the dynamical matter the dual gauge field $\ast B$ becomes compact.

The compactness of the dual gauge field implies that it transforms under the gauge transformations as:

$$\ast B \rightarrow \ast B + d \ast \alpha + 2 \pi \ast k ,$$

(13)

where the integer valued field $k$ is chosen in such a way that $\ast B \in (-\pi, \pi]$.

One can easily check that the operator (4) is not invariant under these gauge transformations:

$$\Phi_{x, \text{mon}}^{\text{old}}(H) \rightarrow \Phi_{x, \text{mon}}^{\text{old}}(H) e^{2 \pi i (\ast k, \ast H_x)} .$$

(14)

Another way to establish this fact is to realize that the pure compact gauge model is dual to the non-compact $U(1)$ with matter fields (referred above as the (dual) Abelian Higgs model). Reading this relation backwards one can conclude that the presence of the matter field leads to the compactification of the dual gauge field $\ast B$. 

5
3 “New” monopole creation operator.

The invariance of the operator \( \Phi \) under the gauge transformations \( \delta \) can be achieved if and only if the function \( *H_x \) is an integer–valued form. Thus, if we take into account the Maxwell equation, \( \delta *H_x = *\delta_x \), we find that \( *H_x \) should be a string attached to the monopole ("the Mandelstam string"): \( *H_x \rightarrow *j_x, *j_x \in \mathbb{Z}, \delta *j_x = *\delta_x \). The string must belong to the three–dimensional time–slice. However, one can show \( \cite{8} \) that for a fixed string position the operator \( \Phi \) creates a state with an infinite energy. This difficulty may be bypassed \( \cite{8} \) by summation over all possible positions of the Mandelstam strings with a measure \( \mu(*j) \):

\[
\Phi^{\text{mon,new}}_x = \phi_x \sum_{\delta^*j_x \in \mathbb{Z}} \mu(*j_x) e^{i(\ast B, \ast j_x)}.
\]

The summation over the strings provides an entropy factor which cancels the energy suppression. An example of a “reasonable” measure \( \mu(j) \) is \( \cite{8} \):

\[
\mu(*j_x) = \exp\left\{ -\frac{1}{2\kappa} ||*j_x||^2 \right\}.
\]

If the Higgs field \( \phi \) is \( q \)–charged \( (q \in \mathbb{Z}) \), the summation in eq.(15) should be taken over \( q \) different strings with the unit flux. Measure \( \cite{16} \) corresponds to the dual formulation of the \( 3D \) XY–model with the Villain action:

\[
S_{XY}(\chi, r) = \frac{\kappa}{2} ||d\chi - 2\pi B + 2\pi r||^2.
\]

The two point correlation function in the XY-model \( \cite{17} \) is given by

\[
\langle e^{ix_x} e^{-ix_y} \rangle = \sum_{j_{xy} \in \mathbb{Z}} \mu(*j_{xy}) e^{i(\ast B, \ast j_{xy})},
\]

where \( \delta *j_{xy} = *\delta_x - *\delta_y \). For sufficiently large coupling \( \kappa \) and sufficiently small field \( B \) we get

\[
\langle e^{ix_x} e^{-ix_y} \rangle \rightarrow \text{const},
\]

as \( |x - y| \rightarrow \infty \). So that the correlation function \( \cite{18} \) might yield an appropriate measure of the Mandelstam strings.

Performing the inverse duality transformation of the creation operator \( \cite{15} \) in the dual representation of the compact AHM \( \cite{12} \) we get the expectation value of this operator in terms of the compact gauge field \( \theta \) in the original representation \( \cite{8} \):

\[
\langle \Phi^{\text{mon,new}} \rangle = \frac{1}{Z} \sum_{*j \in \mathbb{Z}(*\delta_x)} \int_{-\pi}^{\pi} D\theta \exp\left\{ -\frac{1}{2\kappa} ||d^*j||^2 - S(d\theta + \frac{2\pi}{q} \tilde{j}) \right\}.
\]

The current \( \tilde{j} \equiv *\tilde{\delta} \) is defined via a double duality operation which is first applied in the 3D time slice and then in the full 4D space.

We thus defined “old” \( \cite{8} \) and “new” \( \cite{20} \) monopole creation operators.
4 Numerical results

Below we present results of the numerical simulation of the old and the new monopole creation operators. We study them in the Abelian Higgs model with compact gauge field and infinitely deep potential for the Higgs field corresponding to the London limit. This is the simplest model containing both the monopoles and the electrically charged fields. We used the Wilson form of the action which is more suitable for the numerical simulations than the Villain action used for the analytical calculations above:

\[
Z^{\text{AHM}}_{\text{W}} = \int_{-\pi}^{\pi} D\theta \int_{-\pi}^{\pi} D\varphi \, e^{\beta \cos d(\theta) + \gamma \cos (d\varphi + q\theta)}.
\]  

(21)

The expectation values of the old and new monopole creation operators in the Wilson form are given by

\[
\langle \Phi_{\text{mon, old}} \rangle = \frac{1}{Z} \int_{-\pi}^{\pi} D\theta \exp \left\{ \beta \cos(d\theta + 2\pi \delta^{-1}(H_x - \omega_x)) + \gamma \cos(q\theta) \right\},
\]

(22)

\[
\langle \Phi_{\text{mon, new}} \rangle = \frac{1}{Z} \sum_{\delta \in \mathbb{Z}} \int_{-\pi}^{\pi} D\theta \exp \left\{ -\frac{1}{2\kappa} ||d^* j||^2 + \beta \cos \left( d\theta + \frac{2\pi}{q} j \right) + \gamma \cos(q\theta) \right\},
\]

(23)

respectively. We have fixed the unitary gauge, therefore the phase of the Higgs field is eaten up by the corresponding gauge transformation.

The value of the monopole order parameter, \( \langle \phi \rangle \), corresponds to the minimum of the (effective constraint) potential on the monopole field. This potential can be estimated as follows:

\[
V_{\text{eff}}(\Phi) = -\ln \left[ \delta \left( \Phi - \Phi_{\text{mon, new/old}} \right) \right].
\]

(24)

We are studying the model with \( q = 7 \). The compact Abelian Higgs model with multiple charged Higgs fields is known to have a nontrivial phase structure containing the Coulomb, Higgs and Confinement phases \([12]\). In this paper we concentrate on the phase transition between the confinement and the Coulomb phases.

We simulated the 4D Abelian Higgs model with anti-periodic boundary conditions in the 3D space (the single monopole charge can not exist in the finite volume with periodic boundary conditions). For the new creation operator we perform our calculations on the \( 4^4, 6^4, 8^4 \) lattices and the coupling constant \( \gamma = 0.3 \). For each configuration of 4D fields we simulated 3D model to get the ensemble of the Mandelstam strings with the weight \( \mu(j_x) \). We generated 60 statistically independent 4D field configurations, and for each of these configurations we generated 40 configurations of 3D Mandelstam strings.

According to eq. (17) the weight function (16) corresponds to the 3D XY–model with the Villain action. The XY model has the phase transition at \( \kappa_c(B = 0) \approx 0.32 \) \([13]\). Our numerical observations show that in presence of the external field \( B \) the critical coupling constant gets shifted, \( \kappa_c(B) \approx 0.42 \).

Typical configurations of the Mandelstam strings corresponding to the phase with the condensate of these strings (disordered phase, large \( \kappa \)) and to the phase without the string condensate (ordered phase, small \( \kappa \)) are shown in Figures 1(a,b).
Figure 1: The typical Mandelstam strings in the auxiliary 3D model in (a) the ordered phase (no string condensate, $\kappa = 0.3$) and (b) disordered phase (nonzero string condensate, $\kappa = 0.5$).

One can expect that the operator (15) plays the role of the order parameter in the phase, where the Mandelstam strings are condensed ($\kappa > \kappa_c$). In Figures 2 we present the effective potential (24) in the confinement ($\beta = 0.85$) and deconfinement ($\beta = 1.05$) phases. The potential is shown for two values of the 3D coupling constants $\kappa > \kappa_c$ corresponding to high densities of the Mandelstam strings. In the confinement phase, Figure 2(a), the potential $V(\Phi)$ has a Higgs form signaling the monopole condensation. According to our numerical observations this statement does not depend on the lattice volume. In the deconfinement phase, Figure 2(b), the potential has minimum at $\Phi = 0$ which indicates the absence of the monopole condensate.

For small values of the 3D coupling constant $\kappa$ (in the phase where Mandelstam strings $j_x$ are not condensed), we found (Figure 3) that the potential $V(\Phi)$ has the same behavior for the both phases of 4D model. Thus the operator (15) serves as the order parameter for
the deconfinement phase transition, if Mandelstam strings are condensed, \textit{i.e.} the coupling constant \( \kappa \) should be larger than the critical value \( \kappa_c(B) \).

Now let us show that the expectation value of the old monopole creation operator (1) behaves as an order parameter for the Dirac string with a fixed length. We generate the compact Abelian Higgs model for \( \gamma = 0.3, q = 1 \) and couplings \( \beta = 0.6 \) (confinement phase) and \( \beta = 1.2 \) (deconfinement phase).

We use Dirac strings of the given length in proportion 1 : 1.5 on the \( 4^4, 6^4, 8^4, 10^4, 12^4, 16^4 \) lattices. The form of the Dirac strings is shown in Figure 4. For each value of \( \beta \) and for

Figure 3: The effective monopole potential \((24)\) in the low–\( \kappa \) region of the 3\( D \) model.

Figure 4: Two configurations of the Dirac strings used in our simulations on the lattice with anti–periodic boundary conditions.
the fixed form of the Dirac string we took average over 3000 values of the monopole creation operator. The dependence of the minimum of the effective potential, \( \min V(\Phi) \), on lattice of the size \( L \) for the fixed length of Dirac string \( \ell \) is shown in Figure 5. We fitted the data for

\[
\Phi_{\text{min}} = aL^b + \Phi_{\text{inf}}^{\text{min}},
\]

where \( a, b \) are fitting parameters. It occurs that \( b = -1 \) within statistical errors. The resulting values of \( \Phi_{\text{inf}}^{\text{min}} \) are collected in Table 1. We obtain that

| \( \beta \) | \( \ell \) | \( \Phi_{\text{inf}}^{\text{min}} \) |
|---|---|---|
| 0.6 | \( L \) | 0.45(7) |
| 0.6 | \( 1.5L \) | 0.36(5) |
| 1.2 | \( L \) | -0.03(6) |
| 1.2 | \( 1.5L \) | -0.07(5) |

Table 1: The minimum of the monopole potential, \( \Phi_{\text{inf}}^{\text{min}} \), based on the old operator vs. the gauge coupling \( \beta \) and the Mandelstam string length \( \ell \).

\( \Phi_{\text{inf}}^{\text{min}} \) vanishes at the point of the phase transition. Thus the operator (15) serves as the order parameter for the deconfinement phase transition provided the length of Dirac string is fixed.
5 Conclusions

The new monopole creation operator proposed in Ref. 8 can be used as a test of the monopole condensation in the theories with electrically charged matter fields. Our calculations indicate that the operator should be defined in the phase where the Mandelstam strings are condensed. The minimum of the effective potential corresponds to the value of the monopole field which is zero in deconfinement phase and nonzero in the confinement phase.

In the infinite volume limit the potential corresponding to the old version of the monopole creation operator shows the same features as the potential calculated with the use of the new operator. The shape of the old effective potential depends on the length of the Dirac string. This fact indicates that the Dirac strings with different shapes provide different monopole creation operators and all of them can serve as order parameters for the confinement–deconfinement phase transition.

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