TESTS OF IN SITU FORMATION SCENARIOS FOR COMPACT MULTIPLANET SYSTEMS

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ABSTRACT

Kepler has identified over 600 multiplanet systems, many of which have several planets with orbital distances smaller than that of Mercury. Because these systems may be difficult to explain in the paradigm of core accretion and disk migration, it has been suggested that they formed in situ within protoplanetary disks with high solid surface densities. The strong connection between giant planet occurrence and stellar metallicity is thought to be linked to enhanced solid surface densities in disks around metal-rich stars, so the presence of a giant planet can be a sign of planet formation in a high solid surface density disk. I formulate quantitative predictions for the frequency of long-period giant planets in these in situ models by translating the proposed increase in disk mass into an equivalent metallicity enhancement. I rederive the scaling of giant planet occurrence with metallicity as $P_{\mathrm{gp}} = 0.05^{+0.02}_{-0.02} \times 10^{2.1 \pm 0.4}[\text{M/H}] = 0.08^{+0.02}_{-0.03} \times 10^{2.3 \pm 0.4}[\text{Fe/H}]$ and show that there is significant tension between the frequency of giant planets suggested by the minimum mass extrasolar nebula scenario and the observational upper limits. Consequently, high-mass disks alone cannot explain the observed properties of the close-in Kepler multiplanet systems and therefore migration is still important. More speculatively, I combine the metallicity scaling of giant planet occurrence with small planet occurrence rates to estimate the number of solar system analogs in the Galaxy. I find that in the Milky Way there are perhaps $4 \times 10^6$ true solar system analogs with an FGK star hosting both a terrestrial planet in the habitable zone and a long-period giant planet companion.

Key words: Galaxy: general – methods: statistical – planetary systems – planets and satellites: detection – planets and satellites: formation – stars: statistics

Online-only material: color figures

1. INTRODUCTION

Kepler has discovered many multiplanet systems with several planets with orbital periods $P < 50$ days.2 Indeed, 40% of solar-type stars in the Kepler field have at least one planet with $P < 50$ days (e.g., Fressin et al. 2013). Even though these systems differ from the solar system, their apparent ubiquity suggests that they may represent a frequent outcome of planet formation.

In the traditional minimum-mass solar nebula (MMSN) scenario, there is probably insufficient solid material in protoplanetary disks to form the Kepler multiplanet systems where they are observed today (Weidenschilling 1977; Hayashi 1981). Instead, formation further out in the parent protoplanetary disk combined with subsequent inward migration has been suggested as a possible formation channel for this class of system (e.g., Alibert et al. 2006). The apparent excess of planets just outside of mean-motion resonances may also support formation in the inward migration scenario (e.g., Lissauer et al. 2011; Fabrycky et al. 2012). However, the rate and even direction of migration is thought to sensitively depend on the unknown thermodynamic state of the disk (e.g., Paardekooper et al. 2010; Kley & Nelson 2012).

Alternative models of in situ formation in disks with solid surface densities enhanced beyond the MMSN expectation have also been suggested to explain the ubiquity of close-in multiple systems. In the minimum-mass extrasolar nebula (MMEN) scenario of Chiang & Laughlin (2013), the Kepler multiplanet systems formed in protoplanetary disks that were about six times more massive than those envisioned in the MMSN scenario. In that picture, the more massive MMEN disks describe the typical protoplanetary disk in the Galaxy, while the less massive MMSN disk is the outlier. I illustrate the MMEN in Figure 1. On the other hand, Hansen & Murray (2012) invoke the rapid inward migration of planetesimals into the inner regions of the disk. The enhanced solid surface density of the inner disk naturally leads to the in situ formation of planetary systems closely resembling those observed by Kepler (Hansen & Murray 2013).

Both models of in situ planet formation described above provide useful, fresh looks at planet formation. As I will show, both models are also amenable to quantitative tests. At face value, both models make qualitative predictions for the formation of long-period giant planets. All else being equal, a disk with higher solid surface density in the giant planet forming region has a better chance of forming a giant planet than does a disk with lower solid surface density in the giant planet forming region (e.g., Hubickyj et al. 2005). Consequently, Chiang & Laughlin (2013) extrapolated their model to larger semimajor axes and suggested that the enhanced solid surface density in the MMEN scenario should lead to the efficient formation of giant planets outside of 1 AU. In contrast, the concentration of a significant amount of a disk’s solid material in the inner disk as suggested by Hansen & Murray (2012) should lead to inefficient formation of giant planets outside of 1 AU. Unfortunately, it is not currently possible to directly assess with either the transit or radial velocity (RV) technique the frequency of long-period giant planets in the observed Kepler multiple systems themselves.

However, it is possible to indirectly infer the frequency of long-period giant planets in at least two ways. First, the frequency can be characterized by proxy, a technique in which
objects in the solar neighborhood that can be studied in detail stand in for the more distant objects in the solar neighborhood that can be studied in detail. In this case, the close-in multiple systems of low-mass planets discovered in the solar neighborhood with the RV technique are a proxy for the close-in multiple systems of low-mass planets discovered in the solar neighborhood that can be studied in detail. For that reason, I select systems orbiting solar-type stars with $0.22 < J - H < 0.62$, $0.00 < H - K < 0.10$, and $0.22 < J - K < 0.72$ roughly corresponding to spectral types in the range F5–K5 (Covey et al. 2007). From here, I refer to the systems in this sample as the Kepler multiplanet systems.

Likewise, I select systems of exoplanets discovered with the RV technique that have at least two planets with $M \sin i < 0.1 M_{Jupiter} = 31.8 M_\oplus$ from both http://exoplanets.eu and http://exoplanets.org (Schneider et al. 2011; Wright et al. 2011). For each planet host star, I obtain Hipparcos parallaxes and $B - V$ colors from van Leeuwen (2007) and apparent Tycho-2 $V$-band magnitudes from Høg et al. (2000). I transform Tycho-2 $B_T$ and $V_T$ magnitudes into approximate Johnson–Cousins $V$-band magnitudes using the relation $V = V_T - 0.09(B_T - V_T)$. I then select exoplanet systems orbiting solar-type stars with $0.44 < B - V < 1.15$ and $3.5 < M_V < 7.4$, roughly corresponding to spectral types in the range F5–K5 (Binney & Merrifield 1998). I give the exoplanet systems and host star properties that result from this selection in Tables 1 and 2. From here, I refer to the systems in this sample as the RV multiplanet systems. I plot both samples in Figure 2. I make no distinction between mini-Neptunes and super-Earths despite the possibility that their origins may be unique.

It is important to establish the utility of the RV multiplanet systems as a proxy for the Kepler multiplanet systems. Radial velocity surveys measure the stellar reflex velocity, which for a given host star mass is a function of planet mass, orbital period, eccentricity, and inclination. On the other hand, transit surveys measure transit depth, which for a given host star radius is a function of planet radius. Transit probability also biases transit surveys toward the shortest-period planets in an underlying population. Previous analyses have explored this relationship and discovered that while the two populations are broadly consistent, there may be differences (e.g., Wolfgang & Laughlin 2012). Connecting the two populations in this way requires an accurate and precise mass–radius relationship valid at all periods as well as knowledge of the orbital inclinations of the RV multiplanet systems. Despite the progress made for short-period planets, a general empirical mass–radius relation is not yet available (e.g., Marcy et al. 2014; Weiss & Marcy 2014). The orbital inclinations of the RV multiplanet systems are also unknown.

However, the similarity of the Kepler and RV multiplanet systems can be established in another way. If the same planet formation process is responsible for the formation of both the Kepler and the RV multiplanet systems, then the orbital spacing of the two populations should be similar. In a planetary system, the ratio of the separation between two planets to the sum of the Hill radii of two planets is

$$S_{\text{Hill}} = \frac{a_1 - a_2}{R_{H,1} + R_{H,2}},$$

where $a_1 > a_2$. If $P_i^2 = [(4\pi^2)/(GM_{\text{host}})]a_i^3$ and

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3 http://mastweb.stsci.edu/kplr/casjobs/guide.aspx
\[ R_{H,i} = \left( \frac{M_i}{3M_*} \right)^{1/3} a_i, \]

then

\[ S_{\text{Hill}} = \frac{a_1 - a_2}{R_{H,1} + R_{H,2}}, \]

\[ P_{1}^{2/3} - P_{2}^{2/3} = \frac{P_{1}^{2/3}(\rho_1/3 \rho_*)^{1/3}(r_1/r_*) + P_{2}^{2/3}(\rho_2/3 \rho_*)^{1/3}(r_2/r_*)}{[P_{1}^{2/3}(\rho_1/3 \rho_*)^{1/3}(r_1/r_*) + P_{2}^{2/3}(\rho_2/3 \rho_*)^{1/3}(r_2/r_*)]^2}. \]
Equations (1) and (2) and the published values of an average density. For the RV multiplanet systems, I use HD 215152 112190 215152 6.47 0.97
HD 192310 99825 192310 5.98 0.88 2
HD 136352 75181 136352 4.80 0.64 3
HD 134606 74653 134606 4.75 0.74 3
61 Vir 64924 115617 5.08 0.71 3
HD 109271 61300 109271 4.11 0.66 2
HD 96700 54400 96700 4.47 0.61 2
HD 69830 40693 69830 5.47 0.75 3
HD 51608 33229 51608 5.46 0.77 2
HD 40307 27887 40307 6.59 0.94 3
HD 39194 27080 39194 6.02 0.76 3
HD 31527 22905 31527 4.56 0.61 3
HD 21693 16085 21693 5.40 0.76 2
HD 20794 15510 20794 5.36 0.82 2
HD 20003 14530 20003 5.17 0.77 2
HD 13808 10301 13808 6.10 0.87 2
HD 10180 7599 10180 4.37 0.63 6
HD 1461 1499 1499 4.63 0.67 2

Table 2

| Name     | HIP  | HD  | $M_V$ (mag) | $B - V$ (mag) | $N_P$ |
|----------|------|-----|-------------|--------------|-------|
| HD 1461  | 1499 | 1461| 4.63        | 0.67         | 2     |
| HD 10180 | 7599 | 10180| 4.37        | 0.63         | 6     |
| HD 13808 | 10301| 13808| 6.10        | 0.87         | 2     |
| HD 20003 | 14530| 20003| 5.17        | 0.77         | 2     |
| HD 20794 | 15510| 20794| 5.36        | 0.71         | 3     |
| HD 20781 | 15526| 20781| 5.71        | 0.82         | 2     |
| HD 21693 | 16085| 21693| 5.40        | 0.76         | 2     |
| HD 31527 | 22905| 31527| 4.56        | 0.61         | 3     |
| HD 39194 | 27080| 39194| 6.02        | 0.76         | 3     |
| HD 40307 | 27887| 40307| 6.59        | 0.94         | 3     |
| HD 51608 | 33229| 51608| 5.46        | 0.77         | 2     |
| HD 69830 | 40693| 69830| 5.47        | 0.75         | 3     |
| HD 93385 | 52676| 93385| 4.37        | 0.60         | 2     |
| HD 96700 | 54400| 96700| 4.47        | 0.61         | 2     |
| HD 109271| 61300| 109271| 4.11        | 0.66         | 2     |
| 61 Vir   | 64924| 115617| 5.08        | 0.71         | 3     |
| HD 134606| 74653| 134606| 4.75        | 0.74         | 3     |
| HD 136352| 75181| 136352| 4.80        | 0.64         | 3     |
| HD 192310| 99825| 192310| 5.98        | 0.88         | 2     |
| HD 215152|112190|215152| 6.47        | 0.97         | 2     |

Figure 2. Multiplanet system architectures. Left: validated Kepler multiplanet systems. The size of each planet is proportional to its estimated radius. The terrestrial planets in the solar system are included for scale. Right: systems of multiple Neptune-mass planets discovered with the radial velocity technique (RV multiplanet systems). The size of each planet is proportional to its estimated minimum mass. The terrestrial planets in the solar system are included for scale. The architectures of the Kepler and RV multiplanet systems are similar, indicating that the properties of RV multiplanet systems can reasonably be used as proxies for the properties of the Kepler multiplanet systems.

(A color version of this figure is available in the online journal.)

3. ANALYSIS

3.1. Giant Planet Formation

It is well established that giant planets occur more frequently around metal-rich stars (e.g., Santos et al. 2004; Fischer & Valenti 2005). The high metallicity of a star is interpreted as evidence that its parent disk was enriched in dust. A dust-enriched protoplanetary disk is thought to be more likely to form the $\sim 10 M_{\oplus}$ core necessary for giant planet formation in the few million years available before the disk disappears. Giant planets are also less frequently found around M dwarfs than solar-type stars (e.g., Endl et al. 2006; Butler et al. 2006; Bonfils et al. 2013). M dwarfs presumably formed from lower-mass disks than solar-type stars, and the reduced amount of dust present in a low-mass disk is thought to make it more difficult to assemble a core. Recent observational evidence that disk mass scales approximately linearly with host star mass below 1 $M_{\odot}$ supports this view (Andrews et al. 2013). In short, giant planet formation efficiency is proportional to the solid surface density in a protoplanetary disk $\Sigma_{\text{solid}}$, where

$$\Sigma_{\text{solid}} \propto f_{\text{dust}} M_{\text{disk}} \propto Z M_{\text{disk}}.$$
The first proportionality is true by construction. The second is true because observations of the LMC and Milky Way show that the dust-to-gas fraction $f_{\text{dust}}$ scales approximately linearly with metal mass fraction $Z$ (Gordon et al. 2003). If the MMEN disk can be extrapolated beyond 1 AU, then the six times increase in disk mass in the MMEN scenario relative to the MMSN scenario should have an equivalent effect on giant planet formation efficiency as increasing $Z$ by a factor of six, or increasing [M/H] by 0.78 dex.

The pioneering work of Fischer & Valenti (2005) showed that the probability of giant planet occurrence $P_{\text{gp}}$ is

$$P_{\text{gp}} = 0.03 \times 10^{2.00\text{[Fe/H]}}. \quad (8)$$

They argued that the dependence of $P_{\text{gp}}$ on the square of the number of iron atoms present was the natural expectation from the collisional agglomeration of dust grains into planetesimals. However, their result lacked an uncertainty estimate and may have been affected by the need to bin their data. Moreover, many new, longer-period giant planets have been discovered in the interim, so a new calculation is timely. For those reasons, I update the interpretation of the coefficient $\beta_i$ changes the log odds ratio of the probability of the event by a factor of $\beta_i$, or the probability of the event itself by $e^{\beta_i}$. The coefficient $\beta_0$ is the log odds of the event when all predictors $x_i = 0$. I use the glm function in R to compute a logistic regression model (R Core Team 2013). I build models predicting the effect of both

4 For a pedagogical introduction to logistic regression, see Chapter 12 of Chatterjee et al. (2000).

5 I arrive at quantitatively similar results if I instead use the Valenti & Fischer (2005) SPOCS sample.

6 http://www.star.bristol.ac.uk/~mbt/topcat/

7 http://www.R-project.org/
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$[\text{M/H}]$ and $[\text{Fe/H}]$ on the probability of giant planet occurrence. I compute $[\text{M/H}]$ assuming the solar abundances from Asplund et al. (2005). I find that

$$P_{\text{gp}}([\text{M/H}]) \propto 10^{(2.1 \pm 0.4)\alpha [\text{M/H}]},$$
(11)

$$P_{\text{gp}}([\text{Fe/H}]) \propto 10^{(2.3 \pm 0.4)\alpha [\text{Fe/H}]},$$
(12)

To determine the absolute probability of giant planet occurrence at $[\text{M/H}] = [\text{Fe/H}] = 0$, I calculate the fraction of stars with giant planets in the sample in the ranges $-0.05 < [\text{M/H}] < 0.05$ and $-0.05 < [\text{Fe/H}] < 0.05$. I use bootstrap resampling to determine confidence intervals, and I find that

$$P_{\text{gp}} = 0.08^{+0.02}_{-0.03} \times 10^{2.3 \pm 0.4} [\text{Fe/H}],$$
(13)

$$P_{\text{gp}} = 0.05^{+0.02}_{-0.02} \times 10^{2.1 \pm 0.4} [\text{M/H}].$$
(14)

I plot the result in Figure 5.

These results suggest that if the MMEN disk can be extrapolated past 1 AU, then the probability of giant planet occurrence in the Kepler multiplanet systems should be $P_{\text{gp}}(0.78) \approx 1$. Given that these giant planets would be found at $a \gtrsim 1$ AU, they would only rarely be observed to transit and therefore be nearly invisible to the transit technique. However, these planets would likely be detectable with the RV technique.

3.2. Stability and Completeness Constraints

The analysis in Section 3.1 indicates that practically every Kepler multiplanet system formed in the extrapolated MMEN scenario should have a long-period giant planet. However, it is currently impractical to search for long-period giant planets in the Kepler multiplanet systems themselves. Instead, I use the RV multiplanet systems in Table 2 as a proxy and use the published RV completeness estimates for those systems to derive a constraint on the frequency of long-period giant planets in the Kepler multiplanet systems.

The existence and stability of the Kepler multiplanet systems with orbital periods $P < 50$ days precludes the existence of giant planets with similar orbital periods. On the other hand, longer-period giant planets are not prohibited by stability arguments. A multiplanet system is likely to be stable if its planets are separated by 10 or more mutual Hill radii $R_H = [M_p/(3M_\star)]^{1/3} a$ (e.g., Chambers et al. 1996; Smith & Lissauer 2009). For a system of multiple planets orbiting a 1 $M_\odot$ star with its outermost Neptune-mass planet $M_1 = 17.147 M_\oplus$ at an

![Figure 4. Logistic function.](A color version of this figure is available in the online journal.)

![Figure 5. Effect of host star composition on giant planet occurrence. I use logistic regression to estimate the effect of composition on giant planet occurrence, so the arbitrary binning reflected in the plots does not affect the derived scaling. Left: the effect of iron metallicity on giant planet occurrence. There is a hint that giant planet occurrence levels out in the low-metallicity tail of the thin disk metallicity distribution (e.g., Santos et al. 2004). Right: the effect of total metallicity on giant planet occurrence. The hint of a plateau at low metallicity disappears when considering total metallicity. This occurs because non-solar abundance patterns (e.g., Z-enhancement) begin to appear in stars with $[\text{Fe/H}] \approx -0.4$ as the thick disk stellar population starts to become considerable relative to the thin disk (e.g., Adibekyan et al. 2012a).](A color version of this figure is available in the online journal.)
that describes the probability of a number of successes prior for \( \theta \)

\[
a_1 = \left( \frac{P_1}{365} \right)^{2/3},
\]

(15)

\[ A = a_1 + 10 \left( \frac{M_1}{3 M_\odot} \right)^{1/3} a_1, \]

(16)

the smallest semimajor axis \( a_2 \) at which a \( M_2 = 1 M_{\text{up}} \) giant planet would not make the system obviously unstable is

\[ a_2 = A \left[ 1 - 10 \left( \frac{M_2}{3 M_\odot} \right) \right]^{-1}. \]

(17)

I find that \( a_2 \approx 1 \) AU, or \( P_2 \approx 365 \) days. About 75% of the giant planets identified around solar-type stars with the RV technique have \( P \geq 365 \) days (Cumming et al. 2008), and therefore would not render a Kepler multiplanet system obviously unstable. For that reason, the fraction of the observed giant planet systems permitted by stability considerations is \( \eta_{\text{mill}} = 0.75 \). Schmitt et al. (2013), Cabrera et al. (2014), and Lissauer et al. (2014) recently identified KOI-351/Kepler-90 as a presumably stable system with multiple small, close-in planets with giant planets at longer orbital periods.

I use the completeness contours in Figure 6 of Mayor et al. (2011) to determine the expected number of long-period giant planets that would have been discovered around the RV multiplanet systems if every system had a long-period giant planet with mass and period drawn from the observed distributions of those quantities. Since all 20 RV multiplanet systems are in the catalog of Mayor et al. (2011), the completeness contours given in the paper apply for each system. For all \( p \) planets in the exoplanets.org catalog with \( M \sin i > 100 M_{\oplus} \) and \( P > 365 \) days, I check whether each is above the (100%, 95%, 80%, 60%, 40%) completeness contours. If so, I increment the expected number of detections \( q \) by (1, 0.95, 0.8, 0.6, 0.4). If the planet is below the 40% contour, I assume that it would be undetectable. I find that the fraction of giant planet systems that would have been recovered by Mayor et al. (2011) is \( q/p = \eta_c = 0.71 \). I illustrate this calculation in Figure 6.

3.3. Inference

The goal is to determine both the posterior distribution for \( P_{\text{sp}} \) inferred from the non-detection of giant planets in the RV multiplanet systems and the posterior of \( P_{\text{sp}} \) expected under the extrapolated MMEN scenario after taking into account stability and completeness. If there is tension between the two posteriors, then the MMEN scenario may not be an accurate description of Kepler multiplanet system formation.

Bayes’ theorem guarantees

\[
f(\theta|y) = \frac{f(y|\theta)f(\theta)}{\int f(y|\theta)f(\theta)d\theta},
\]

(18)

where \( f(y|\theta) \) is the posterior distribution of the model parameter \( \theta \), \( f(y|\theta) \) is the likelihood of the data \( y \) given \( \theta \), and \( f(\theta) \) is the prior for \( \theta \). In this case, the likelihood is the binomial likelihood that describes the probability of a number of successes \( y \) in \( n \) Bernoulli trials each with probability \( \theta \) of success:

\[
f(y|\theta) = \binom{n}{y} \theta^y (1 - \theta)^{n-y}.
\]

(19)

The orbital period of \( P_1 = 50 \) days

\[
a_1 = \left( \frac{P_1}{365} \right)^{2/3},
\]

\[ A = a_1 + 10 \left( \frac{M_1}{3 M_\odot} \right)^{1/3} a_1, \]

\[ a_2 = A \left[ 1 - 10 \left( \frac{M_2}{3 M_\odot} \right) \right]^{-1}. \]

\[ \eta_{\text{mill}} = 0.75 \]

\[ \eta_c = 0.71 \]

\[ q/p = \eta_c \]

\[ q \]

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equally likely. However, if \( i \) is large then there is imagined to be a lot of prior information and the posterior distribution will mostly reflect the prior when \( n < i \). On the other hand, if \( n \gg i \), then the posterior will be dominated by the data. For that reason, I take \( \alpha = \beta = 1 \).

In the exoplanet context, \( \theta \) is the unknown probability of giant planet occurrence \( P_{\text{gp}} \), \( n = 20 \) is the number of RV multiplanet systems in Table 2, and \( y \) is the number of detected long-period giant planets. No giant planets have been detected in the RV multiplanet systems, so \( y = 0 \). After accounting for completeness and stability, the equivalent number of systems searched at 100% completeness would be \( n' = \eta_c \eta_{\text{Hill}} n \approx (0.71)(0.75)(20) \approx 10.6 \). The posterior distribution of \( P_{\text{gp}} \) inferred from the non-detection of long-period giant planets in the RV multiplanet systems is

\[
P_{\text{gp,obs}} = \text{Beta}(\alpha + y, \beta + n' - y),
\]

\[
= \text{Beta}(1 + 0, 1 + \eta_c \eta_{\text{Hill}} n - 0),
\]

\[
= \text{Beta}(1, 11.6).
\]  
(24)

(25)

(26)

In the extrapolated MMEN scenario, every RV multiplanet system should have a long-period giant planet. Consequently, the expected number of successful planet discoveries after examining \( n \) systems should be \( y' = \eta_c \eta_{\text{Hill}} n \). The posterior distribution of \( P_{\text{gp}} \) in the extrapolated MMEN scenario should be

\[
P_{\text{gp,MMEN}} = \text{Beta}(\alpha + y', \beta + n - y'),
\]

\[
= \text{Beta}(1 + \eta_c \eta_{\text{Hill}} n, 1 + n - \eta_c \eta_{\text{Hill}} n),
\]

\[
= \text{Beta}(11.6, 10.4).
\]  
(27)

(28)

(29)

I plot these posterior distributions and 95% credible intervals in Figure 7. I find the probability that these posterior distributions overlap to be \( p \approx 1 \times 10^{-3} \). This implies a 3\( \sigma \) (one-sided) difference between the two distributions: there are too few long-period giant planets observed in RV multiplanet systems for them to have formed in the extrapolated MMEN scenario. The concentration of solid material in the inner region of the disk hypothesized by Hansen & Murray (2012) should make long-period giant planets less common in the RV multiplanet systems than in the field planet host population. About 7% of field FGK stars have a giant planet with \( P \) in the range 1 yr \( \leq P \leq 10 \) yr (e.g., Cumming et al. 2008). Unfortunately, in RV multiplanet systems the current 1\( \sigma \) upper limit on long-period giant planet occurrence is 15%, so the Hansen & Murray (2012) scenario is unconstrained. In the future, a search for long-period giant planets in \( n \approx 50 \) RV multiplanet systems would be able to resolve the issue.

3.4. Metallicity Constraints

The frequency of giant planets can also be constrained by examining the metallicity distribution of a sample of stars. Giant planets at all orbital periods preferentially occur around metal-rich stars, so Kepler multiplanet systems that have a long-period giant planet should preferentially orbit metal-enriched stars. No strong metallicity effect has been measured for low-mass or small-radius planets orbiting solar-type stars, at least for the single systems that dominate the Kepler sample (e.g., Schlaufman & Laughlin 2011; Buchhave et al. 2012).

To quantify the maximum fraction of Kepler multiplanet systems that could host a long-period giant planet yet not cause a noticeable metallicity enhancement in the Kepler multiplanet sample, I use a Monte Carlo simulation. I create many random mixed control samples composed of the metallicities of both multiple small-planet and large-planet host stars from Buchhave et al. (2012). I vary the fraction of large planet-host stars and then compare the metallicity distribution of the pure multiple small-planet host sample to the resultant mixed control sample distributions using the Anderson–Darling test. I plot the results in Figure 8 and find that control samples including less than about 50% of the multiple small-planet host stars in the Kepler field also possess an unobserved long-period giant planet based on metallicity alone. This is in contrast to the extrapolated MMEN expectation, where nearly all systems of small planets should also have successfully formed a long-period giant planet companion. In fact, a giant planet host fraction of 1 suggested by the extrapolated MMEN scenario is rejected at the \( p = 1 \times 10^{-3} \) level, or about 3\( \sigma \) (one-sided).

4. DISCUSSION

The MMEN scenario failed two independent tests of its apparent prediction that long-period giant planets should be ubiquitous in close-in multiplanet systems, each at 3\( \sigma \). Failing these two independent tests at 3\( \sigma \) is approximately equivalent to failing a single test at 4\( \sigma \). This tension indicates that massive disks alone cannot fully explain the properties of the Kepler multiplanet systems. If in situ formation and migration are the only two relevant processes, then migration must still have played a role. Raymond & Cossou (2014) reached a...
similar conclusion after examining disk surface density profiles. Therefore, the properties of the Kepler multiplanet systems can in principle be used to help determine the rate, direction, and stopping mechanism for migration in a gaseous disk as well as their dependencies on planet mass, stellar magnetic field, disk thermodynamics, turbulence, accretion rate, and dissipation mechanism. It is possible that increasing the solid surface density of a disk by increasing metallicity does not have the same effect on giant planet formation as increasing the solid surface density through overall disk mass. It may be that the increased dust mass in a metal-rich protoplanetary disk efficiently removes ions and therefore affects the disk structure. The MRI-inactive regions of a metal-rich disk may therefore be larger than a solar-metallicity disk, and consequently promote especially efficient giant planet formation. In that case, the increased incidence of giant planets around metal-rich stars may not be a useful guide to giant planet formation in massive disks. It is also possible that the MMEN disk asymptotes to the MMSN disk beyond 1 AU, though in that case migration is still likely an important process in planet formation (e.g., Hansen & Murray 2012).

This study of in situ planet formation has produced many insights that are useful in the estimation of the frequency of exoplanet systems with “solar-system-like” architectures. Though this was not the aim of this effort, the topic is of considerably current interest and worth examining in detail. Indeed, the presence of long-period giant planets in systems of terrestrial planets may play an important role in the habitability of those planets. Wetherill (1994) argued that the formation of Jupiter prevented the formation of many comets and that Jupiter’s subsequent presence ejected many more comets from the solar system. In systems with no long-period giant planets, the cometary impact flux in the terrestrial planet region is expected to be 1000 times the observed value in the solar system. Frequent comet impacts may sterilize a planet, and will definitely inhibit the evolution of intelligent life. As a result, the presence of a long-period giant planet in an exoplanet system may play an important role in its habitability.

The fraction of solar-type stars that host a stable exoplanet system with at least one low-mass planet and at least one long-period giant planet is

\[ \eta_{\text{ss}} = \eta_{\text{single}} \eta_{\text{sp}} \eta_{\text{gp}} \eta_{\text{Hill}}, \]

where \( \eta_{\text{single}} \) is the fraction of solar-type stars in single star systems, \( \eta_{\text{gp}} \) is the fraction of solar-type stars that host small planets, \( \eta_{\text{gp}} \) is the fraction of solar-type stars that host a giant planet, and \( \eta_{\text{Hill}} \) is the fraction of systems that are Hill stable. The factor \( \eta_{\text{Hill}} \) is necessary because I assume that \( \eta_{\text{gp}} \) and \( \eta_{\text{gp}} \) are independent of each other and planet period. This assumption would lead to an overestimate of \( \eta_{\text{ss}} \), as some of the hypothetical systems would be unstable. I correct after-the-fact using the factor \( \eta_{\text{Hill}} \) to ensure that I only count systems that would not be obviously unstable.

About 1/3 of the FGK stars in the solar neighborhood are in single systems, so \( \eta_{\text{single}} = 0.33 \) (e.g., Duquennoy & Mayor 1991). Fressin et al. (2013) report that 40% of solar-type stars have at least one planet with \( P < 50 \) days, so \( \eta_{\text{sp}} = 0.4 \). The factor \( \eta_{\text{gp}} \) depends sensitively on metallicity, so it is a function of the metallicity distribution of a stellar population. Because giant planet occurrence is such a strong function of metallicity, Galactic giant planets are only likely to be found in the Milky Way’s disk or bulge. For the disk metallicity distribution, I use the distribution of \([\text{M/H}]\) from Casagrande et al. (2011). For the bulge metallicity distribution, I use the distribution of \([\text{M/H}]\) from Bensby et al. (2013), assuming that the abundances inferred from microlensed dwarf stars studied in that survey are representative of the bulge metallicity distribution. Also for the bulge sample, non-solar abundance patterns are important to consider. For that reason, I compute \([\text{M/H}]\) from the measured values of \([\text{Fe/H}], [\text{O/Fe}], [\text{Mg/Fe}], \) and \([\text{Si/Fe}]\) assuming the solar abundances from Asplund et al. (2005). These four elements contribute 99% of the total stellar metallicity of the available abundances in the Bensby et al. (2013) catalog, and I therefore neglect the other elements. To calculate \( \eta_{\text{gp}} \) for each population, I use

\[ \eta_{\text{gp}} = \frac{1}{m} \sum_{i=1}^{m} P_{\text{gp}}([\text{M/H}]_i), \]

where \( m \) is the number of measured metallicities in a catalog and \( P_{\text{gp}}([\text{M/H}]_i) \) is Equation (13). I find that \( \eta_{\text{gp}} = 0.06 \) for the disk and \( \eta_{\text{gp}} = 0.17 \) for the bulge.

Plugging in the numbers, I find that \( \eta_{\text{ss}} = 0.0067 \) in the disk and \( \eta_{\text{ss}} = 0.017 \) in the bulge. Recently, Petigura et al. (2013) published a tentative extrapolation of the frequency of Earth-size planets with orbital periods in the range 200 days \( \leq P \leq 400 \) days; they found that \( \eta_{\text{gp}} = \eta_{\text{gp}} \approx 0.06 \). An Earth-mass planet orbiting at \( a = 1 \) AU implies that any giant planets in the system must be beyond \( a \approx 3.5 \) AU to avoid Hill instability. Using the giant planet orbital distribution from Cumming et al. (2008), that implies that \( \eta_{\text{Hill}} = 0.44 \). As a result, the fraction of solar-type stars hosting true solar system analogs including both an Earth-size planet near 1 AU and a long-period Jupiter-mass planet is \( \eta_{\text{ss}} = 0.00059 \) in the disk and \( \eta_{\text{ss}} = 0.0015 \) in the bulge.

The total number of such systems in the Galaxy is

\[ N_{\text{ss}} = \eta_{\text{FGK}} (\eta_{\text{ss}, \text{disk}} M_{\text{disk}} + \eta_{\text{ss}, \text{bulge}} M_{\text{bulge}}). \]
where \( \eta_{\text{FGK}} \) is the FGK mass fraction in a stellar population, \( M_{\text{disk}} \) is the stellar mass of the Milky Way’s disk, and \( M_{\text{bulge}} \) is the stellar mass of the Milky Way’s bulge. The stellar mass of the Milky Way’s disk is \( M_{\text{disk}} \approx 4.6 \times 10^{10} \, M_\odot \), while the stellar mass of the bulge is \( M_{\text{bulge}} \approx 1 \times 10^{10} \, M_\odot \) (Bovy & Rix 2013; Widrow et al. 2008). I assume a Chabrier (2003) initial mass function (IMF) truncated at \( 0.08 \, M_\odot \) and \( 120 \, M_\odot \) for both the disk and bulge population, as there is no conclusive evidence that the IMF varies between the two populations (e.g., Bastian et al. 2010). Given the Chabrier (2003) IMF, I use the algorithm from the SLUGS code\(^8\) to randomly sample the IMF and compute the fraction of stellar mass in FGK stars (\( 0.8 \, M_\odot \leq M_* \leq 1.25 \, M_\odot \)). I find that 10% of a stellar population’s stellar mass is in FGK stars, and because each star has \( M_* \approx 1 \, M_\odot \), the number of stars is equivalent to the stellar mass in FGK stars. As a result, \( \eta_{\text{FGK}} = 0.1 \).

Putting all of the factors together, I find that the number of “solar systems” in the Galaxy with at least one Neptune-size planet with orbital period \( P < 50 \) days and a long-period giant planet is \( N_{\text{ss}} \approx 5 \times 10^7 \). The number of true solar system analogs in the Galaxy with an Earth–size planet in the habitable zone and a long-period Jupiter-mass giant planet is \( N_{\text{ss}} \approx 4 \times 10^6 \).

In this analysis, I have not accounted for the possibility of metallicity gradients in the disk of the Milky Way. The inner disk has a higher stellar density than the solar neighborhood, and it is likely more metal-rich too (e.g., Frinchaboy et al. 2013). Similarly, the outer disk has a lower stellar density than the solar neighborhood and may be metal-poor as well. This is a very active area of a research and future observations may resolve the issue.

5. CONCLUSIONS

Kepler multiple planet systems with several planets orbiting with periods \( P < 50 \) days may be difficult to explain in the traditional core accretion and Type I migration paradigm. In response, two in situ models of plant formation were proposed: the MMSN scenario of Chiang & Laughlin (2013) and the planetesimal migration scenario of Hansen & Murray (2012). Both models make predictions for the occurrence rate of long-period giant planets. In the extrapolated MMSN scenario, they are ubiquitous. In the planetesimal migration scenario, they are less common than in the field population. I find that the prediction from the extrapolated MMSN scenario for the occurrence of long-period giant planets in multiple low-mass systems discovered with the RV technique fails at 3\( \sigma \). The lack of metallicity enhancement in the hosts of multiple small-planet systems discovered by Kepler provides an independent test of the extrapolated MMSN scenario, which it also fails at 3\( \sigma \). I am unable to constrain the planetesimal migration scenario with the current sample. As a result, migration is still a necessary step in the formation of systems of close-in low-mass planets.

I also rederived the scaling of giant planet occurrence with metallicity, and I find that \( P_{\text{gp}} = 0.05^{+0.02}_{-0.02} \times 10^{2.1 \pm 0.4} \text{[M/H]} = 0.08^{+0.02}_{-0.02} \times 10^{2.3 \pm 0.4} \text{[Fe/H]} \). I used these relations to calculate the frequency of “solar systems” in the Galaxy, where a solar system is defined as a single FGK star orbited by a planetary system with at least one small planet interior to the orbit of a giant planet. The presence of a giant planet exterior to an Earth-size planet may be necessary to prevent frequent comet impacts from inhibiting the evolution of life on an otherwise habitable planet. I find that in the solar neighborhood, about 0.7% of solar-type stars have a “solar system” consisting of both a Neptune-size planet with \( P < 50 \) days and a protective long-period companion. Intriguingly, I find that true solar system analogs with both a terrestrial planet in the habitable zone and a long-period giant planet companion to protect it occur around only 0.06% of solar-type stars. There are perhaps \( 4 \times 10^6 \) such systems in the Galaxy.

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APPENDIX
KEPLER MULTIPLANET SYSTEM QUERY

The following SQL query can be used in the Kepler CasJobs database available from MAST to reproduce my sample of Kepler multiple small-planet candidate systems. It is first necessary to set “Context” to “kepler.”

SELECT a.kepid, a.kepoi_name, a.koi_prad, a.koi_period, a.koi_srad, c.kepler_name FROM kepler_koi a INNER JOIN ( SELECT kpid FROM kepler_koi WHERE koi_disposition != 'FALSE POSITIVE' AND koi_prad < 5 GROUP BY kpid HAVING COUNT(kepoi_name) > 1 ) b ON a.kepid = b.kpid LEFT OUTER JOIN published_planets c ON a.kepid = c.kepid AND a.kepoi_name = c.kepoi_name INNER JOIN keplerObjectSearchWithColors d ON a.kepid = d.kic_kepler_id WHERE a.koi_disposition != 'FALSE POSITIVE' AND d.jh BETWEEN 0.22 AND 0.62 AND d.hk BETWEEN 0.00 AND 0.10 AND d.jk BETWEEN 0.22 AND 0.72 ORDER BY a.kepoi_name, a.koi_period;

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