THE AVERAGE BINDING NUMBER OF GRAPHS AND ITS ALGORITHM

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Abstract. As a result of the interaction of rapid development and competition in information technologies, the reliability of a network and how solid it remains is important. It is called the hat vulnerability of the network to measure the endurance of the network until communication is interrupted by the deterioration of the connection lines between some centers or centers in a network. The centers of the network can be modeled such that the vertex of the network and the connecting lines are the distances of the graph, while investigating the strength of a communication network against disturbances that may occur in the centers or connecting lines. Networks can be modeled with graphs and there are several parameters to measure the vulnerability of these graphs. In this study, the average binding number was studied. For \( v \in V(G) \), the local binding number of \( v \) is \( \text{bind}_v(G) = \min_{S \subseteq F_v(G)} \left\{ \frac{|N(S)|}{|S|} \right\} \), where \( F_v(G) = \{ S \subseteq V(G) \mid v \in S, S \neq \emptyset, N(S) \neq V(G) \} \). Furthermore, the average binding number of \( G \) is defined as \( \text{bind}_{av}(G) = \frac{1}{n} \sum_{v \in V(G)} \text{bind}_v(G) \), where \( n \) is the number of vertices in graph \( G \).

In this paper, some bounds of the average binding number of some special graphs are obtained. Finally, the algorithm for calculation of average binding numbers of graphs is given. The algorithms of these parameters are developed that calculates for any graph and the algorithms are explained. The algorithms are analyzed by code metrics and their usefulness is shown.

Keywords: Graph theory; Network design; Connectivity; Vulnerability; Average binding number; Algorithm

1. Introduction

Any network, such as computer networks, is a series of centers connected by transmission routes. One of the most important problems which is solved by the help of graph theory is to design a network model whose resistance for the disruptions is more than other networks. Graphs are often used to model real world problems such as in a communication, computer, or spy network. In a network, the vulnerability parameters measure the resistance of the network to disruption of operation after the failure of certain stations or links.

These networks are modeled with graphs with their centers corresponding to the vertices and their connections to the edges. When designing networks, security vulnerabilities can be measured and cost calculated when modeled with graphs. There are some vulnerability measurement parameters that have been developed in order to detect and
prevent security weaknesses at the network design stage and also to design more economical and secure networks by taking into account the network cost. Communication problems occur when a network’s hubs or connection lines are damaged[13].

Vulnerability measurements are performed to answer the question of how long and how the network will last until communication is interrupted in a network. In short, vulnerability is a measure of the resistance of a network that it shows up until its communication ends. We can determine how durable, sensitive and dispersible the graph structure designed with these measurements. Some of these vulnerability measurement parameters are as follows; connectivity [33], scattering number [20], binding number [36], and average binding number [1]. Average parameters have been found to be more useful in some cases than the corresponding worst case based measurements [1,4].

Our aim for the paper is to create algorithm for average binding number and we considered same based vulnerability parameters that are average binding number. Therefore, we developed the algorithm of these vulnerability parameters and analyzed the average binding number algorithm with code metrics. We use Python language for developing the algorithm. That’s way we used the Radon [37] library to analyze code metrics. Radon can compute cyclomatic complexity, maintainability index, raw metrics and Halstead metrics. Cyclomatic Complexity corresponds to the number of selections a block of code incorporates plus 1. This number (additionally known as McCabe number) is identical to the number of linearly unbiased paths via the code. This number may be used as a manual while checking out conditional good judgment in blocks. Maintainability Index is a software metric which measures using some values of different metrics. Raw metrics use the line of code numbers, number of comment lines, multi-line strings and the number blank lines to compute the code metrics. Halstead’s goal is to get to know measurable properties of code and their relations. The static numbers computed from the source code are: number of distinct operands and operators, and total number of these [25, 27, 15, 17]. In section 2, Software Metriks are obtained. In section 3, the average binding number definition and basic theorems are obtained. In section 4, Special Graphs are obtained. In section 5, Algorithms are obtained. In section 6 Test results and Complexity are obtained.

2. Software Metrics

The values that can be measured or those can be calculated from these measurements are called as software metrics. To get these metrics; tools that can calculate these metrics automatically can be used. In this way more detailed information about a software project that includes billions lines of codes can be reached. There are five primary software metrics including size, effort, duration, cost, and quality. These metrics can be used to measure the quality levels of softwares such as complexity, reusability, testability, effectivity/efficiency, durability, intelligibility. Shortly all of these metrics can be used measure quality of softwares. There are lots of metric measurement clusters to make these measurements. Among these most accentuated clusters are: Chidamber & Kemerer, Brito e Abreu, Bansiya & Davis.

McCabe Cyclomatic Complexity which is one of the functional software complexity measurement method, is used to find out non-reliable and hard to test program module. Shortly named CC, this method basically using control flow graph to try to measure complexity of a module. It makes this according to nodes and edges on the graph. Every node on the graph is generated according to a logic code such as if, while, switch, for, goto. Each logic code refers a node on the graph and they are connected with edges. Each branching in the program code determines
direction of the edges in the control graph. When the graph is ready, sum of the numbers of edges and nodes calculated and they are used in the correlation below.

\[ V(G) = E - N + 2 \times P \]  \hspace{1cm} (1)

\( E \); number of edge, \( N \); number of node, \( P \); number of module, equals to 1 as initial. Cyclomatic complexity model calculates complexity value of program code that shows how much errancy it has. Errancy shows the risk of that code.

\[ \text{Comment Rate} = \frac{\text{Comment Statements}}{\text{Total Statements}} \]  \hspace{1cm} (2)

Comment rate is calculated from division of sum of comment statements to sum of total statements like equation 2. Here, \( E \) is the number of edges of the control flow graph of the software, \( N \) is the number of nodes and \( P \) is the number of connected components in this graph. Comment Rate is calculated by dividing the number of comment statements by the total number of statements as seen in equation 2.

Halstead’s complexity metric; Halstead’s Complexity which is one of the functional software complexity measurement method, is used to find out complexity of a code block according to number of the operators and operands. In this method the basic elements are operators and operands. Operators refers symbols and key words that carries out arithmetic and logical transactions. On the other hand operands refers numbers and other variables used in these transactions. Halstead finds metrics with summing up these operators and operands like shown below

\( \mu_1 = \text{number of distinct operators} \)

\( \mu_2 = \text{number of distinct operands} \)

\( N_1 = \text{total number of operators} \)

\( N_2 = \text{total number of operands} \)

| Table 1. Calculations of Some Software Metric. |
|-----------------------------------------------|
| Program vocabulary \( \mu = \mu_1 + \mu_2 \) |
| Program length \( N_1 + N_2 \) |
| Calculated program length \( H = \mu_1 \log_2 \mu_1 + \mu_2 \log_2 \mu_2 \) |
| Halstead Program Volume \( V = N \times \log_2 \mu \) |
| Halstead Program Difficulty \( D = (\mu_1 / 2) \times (N_2 / \mu_2) \) |
| Halstead Program Effort \( E = V \times D \) |

\( N = N_1 + N_2 \) (Program Length)

\( \mu = \mu_1 + \mu_2 \) (Program Vocabulary)

Program length and program vocabulary is used to calculate other metrics of Halstead as basic entries. Other metrics shown below.

**Halstead Program Length;** \( N = N_1 + N_2 \) (Program Length) should be maximum 300. If this value reaches above 300 its suggested to redesign the code block. If its above 500 it must be redesigned.

**Halstead Program Volume;** \( V = N \times \log_2 \mu \) (Volume) this value should be between 20-1000 . If its above 1000 code must be redesigned.

**Halstead Program Difficulty;** \( D = (\mu_1 / 2) \times (N_2 / \mu_2) \) this value shows the fault tolerance of program and it should be maximum 50. If its above cod must be redesigned.

**Halstead Program Effort;** \( E = V \times D \) this value should be less then 500.000, if its above that its suggested to redesign the code. And this code is estimated as low quality.
**Halstead Program Time Effort:** $T = E / 18$; this value should be less than 5.000. Programs that have more than that value are estimated low quality and it is suggested to redesign the code.

**Halstead Program Bug Number:** $B = \left( E^{(2/3)} \right) / 3000$, it shows the estimated bug number.

These metrics calculated were statically computed from the source code. The Halstead program length (H) is calculated according to the formula below.

$$H = n_1 \log_2 n_1 + n_1 \log_2 n_2$$  \hspace{1cm} (3)

The meaning of $n_1$ is the number of distinct operators. The meaning of $n_2$ is the number of distinct operands. The meaning of $N_1$ is the total number of operators. The meaning of $N_2$ is the total number of operands. The validity of the Halstead software metrics has been confirmed statistically so many times over a wide range of software languages. From these numbers, several software measures can be calculated.

In conclusion, Halstead complexity gives numerical knowledge about a program with measuring complexity. In this method, operator and operands used as basic entries. He claims to measure complexity of a program with summing up these numbers.

### 3. Average Binding Number and Main Theorems

We give the definition of average binding number below.

**Definition 3.1.** The average binding number was studied. For $v \in V(G)$, the local binding number of $v$ is $bind_v(G) = \min_{S \subseteq F_v(G)} \left\{ \frac{|N(S)|}{|S|} \right\}$, where $F_v(G) = \{S \subseteq V(G) | v \in S, S \neq \emptyset, N(S) \neq V(G)\}$.

Furthermore, the average binding number of $G$ is defined as $bind_{av}(G) = \frac{1}{n} \sum_{v \in V(G)} bind_v(G)$, where $n$ is the number of vertices in graph $G$. We determine the average binding number in networks.

The related theorems of the average binding number and other graph parameters are provided as the followings.

**Theorem 3.1.** If $G$ is a graph of order $n$, then

$$bind_{av}(G) \leq n - 1.$$  

**Proof.** Let $v \in V(G)$ and $S_v$ be a local binding set at $v$. Certainly $|S_v| \geq 1$ and $|N(S_v)| \leq n - 1$. Thus $S^*_v$. Hence

$$bind_{av}(G) = \frac{1}{n} \sum_{v \in V(G)} bind_v(G) \leq n - 1.$$  

**Theorem 3.2.** If $G$ is a graph of order $n$, then

$$bind_{av}(G) \leq bind_{av}(G + e).$$

**Proof.** Let $v \in V(G)$ and $S_v$ and $S^*_v$ be a local binding set of $V(G)$ and $V(G+e)$, respectively. Clearly
$|S_v| = |S^*|_v$ and $|N(S_v)| \leq |N(S^*|_v)$. Hence,

$$bind_v(G) = \frac{|N(S_v)|}{|S_v|} \leq \frac{|N(S^*_v)|}{|S^*_v|} = bind_v(G + e)$$

Thus by the definition,

$$bind_{av}(G) \leq bind_{av}(G + e).$$

**Definition 3.2.** [?????????????] The connectivity number $\kappa(G)$ is defined as the minimum number of vertices whose removal from $G$ results in a disconnected graph or in the trivial graph (=a single vertex).

**Definition 3.3.** [jung ???????] The scattering number of a graph $G$, denoted $sc(G)$, is defined by $sc(G) = \max\{ \omega(G-S) - |S| : S \subseteq V(G) \text{ and } \omega(G-S) > 1 \}$ where $\omega(G-S)$ denotes the number of components in $G-S$.

**Theorem 3.3.** [20??????????] Let $G$ be a graph of order $n$. Then

$$2\alpha(G) - n \leq sc(G) \leq \alpha(G) - K(G).$$

**Theorem 3.4.** If $G$ is a graph of order $n$ with scattering number $sc(G)$ and independence number $\beta(G)$, then

$$bind_{av}(G) \leq sc(G) + 2\beta(G) - 1.$$

**Proof.** By Theorem 1, we have

$$bind_{av}(G) \leq n - 1.$$

We know that $n = \alpha(G) + \beta(G)$. Hence,

$$bind_{av}(G) \leq \alpha(G) + \beta(G) - 1.$$

$$bind_{av}(G) - 2\beta(G) + 1 \leq \alpha(G) - \beta(G)$$

By Theorem 3, we get

$$bind_{av}(G) - 2\beta(G) + 1 \leq \alpha(G) - \beta(G) \leq sc(G)$$

$$bind_{av}(G) \leq sc(G) + 2\beta(G) - 1.$$

**Theorem 3.5.** If $G$ is a graph of order $n$ with connectivity $K(G)$, covering number $\alpha(G)$ and independence number $\beta(G)$, then

$$bind_{av}(G) \leq \alpha(G) - K(G) + 2\beta(G) - 1.$$

**Proof.** By Theorems 3.3 and 3.4, we have

$$bind_{av}(G) \leq \alpha(G) - K(G) + 2\beta(G) - 1.$$
4. Special Graphs

In this section, the average binding numbers of graphs, Banana Tree, Comet Graph and Firecracker Graph are calculated.

**Definition 4.1.** The Banana Tree $B_{r,m}$ is a graph obtained by connecting one leaf of each of $r$ copies of an $m$-star graph with a single root vertex that is distinct from all the stars.

![Fig.1. Banana Tree ($B_{5,4}$)](image)

**Theorem 4.1.** If $B_{r,m}$ is a Banana Tree with $r \geq 2$ and $m \geq 2$, then

$$\text{bind}_{av}(B_{r,m}) = \left(\frac{r \cdot m + (r + 1)^2}{(r \cdot m + 1) \cdot (r \cdot m - r + 1)}\right) \cdot \frac{1}{(r \cdot m) - 1}$$

**Proof.** Let $v \in V(B_{r,m})$, $S_v \in F_v(B_{r,m})$. There is a local binding set $S_v$ of $V(B_{r,m})$ such that $|S_v| = r \cdot m - r$, when $S_v$ contains all isolated vertices in set of $B_{r,m}$ then $|N(S_v)| = r \cdot m + 1$. Thus $\text{bind}_v(B_{r,m}) = \frac{r \cdot m + 1}{r \cdot m - r}$ for $r$ vertices of $B_{r,m}$.

If $\deg(v) = r$ then there is a local binding set $S_v$ of $V(B_{r,m})$ such that $|S_v| = r \cdot m - r + 1$ and $|N(S_v)| = r + m$. Thus $\text{bind}_v(K_{r,m}) = \frac{r + m}{r \cdot m - r + 1}$.

If $\deg(v) = m$ then there is a local binding set $S_v$ of $V(B_{r,m})$ such that $|S_v| = r \cdot m + r + 1$ and $|N(S_v)| = 2r + 1$. Thus $\text{bind}_v(K_{r,m}) = \frac{2r + 1}{r \cdot m + r + 1}$.

By the definition

$$\text{bind}_{av}(B_{r,m}) = \frac{1}{r \cdot m + 1} \sum_{v \in V(B_{r,m})} \text{bind}_v(B_{r,m}) = \frac{r \cdot m + 1}{r \cdot m - r} \cdot \frac{r + m}{r \cdot m - r + 1} \cdot \frac{2r + 1}{r \cdot m + r + 1}$$

$$\text{bind}_{av}(B_{r,m}) = \left(\frac{r \cdot m + (r + 1)^2}{(r \cdot m + 1) \cdot (r \cdot m - r + 1)}\right) \cdot \frac{1}{(r \cdot m) - 1}$$

**Definition 4.2.** A comet $C_{t,r}$ is a graph obtained by identifying one end of a path $P_t \geq 2$, with the center of a star $K_{1,r}$ ($t \geq 2$). Consider the graph $G$ in Fig. 2 where $|V(G)| = 16$ and $|E(G)| = 15$. 
Theorem 4.2. If $C_{t,r}$ is a comet graph $r \geq 2$ and $t \geq 2$, then

$$\text{bind}_{av}(C_{t,r}) = \frac{2t + 2r + 1}{(r + t)(r + 1)}$$

Proof. Let the vertices of $P_t$ be $p_1, p_2, ..., p_t$ in order along the path, $\deg(p_i) = 1$ and $\deg(p_t) = r+1$. Let $v \in V(C_{t,r})$, $S_v \in F_v(C_{t,r})$. We distinguish two cases.

Case 1: If $v \in V(K_{1,r})$ then there is a local binding set $S_v$ of $V(C_{t,r})$ such that $|S_v| = r$, when $S_v$ contains all isolated vertices in set of $C_{t,r}$ then $|N(S_v)| = 1$.

Case 2: Assume $v \in V(P_t)$.
- If $v = \{p_1\}$ or $v = \{p_t\}$ then we have $|S_v| = r + 1$ and $|N(S_v)| = 2$. Thus $\text{bind}_v(C_{t,r}) = \frac{2}{r + 1}$ for two vertices.
- If $v = \{p_t\}$ then we get $|S_v| = r + 1$ and $|N(S_v)| = r + 2$. Thus $\text{bind}_v(C_{t,r}) = \frac{r + 2}{r + 1}$.
- If $v \in \{p_2, p_3, ..., p_{t-2}\}$ then there is a local binding set $S_v$ of $V(C_{t,r})$ such that $|S_v| = r + 1$ and $|N(S_v)| = 3$. Thus $\text{bind}_v(C_{t,r}) = \frac{3}{r + 1}$.

By the definition

$$\text{bind}_{av}(C_{t,r}) = \frac{1}{r + t} \sum_{v \in V(C_{t,r})} \text{bind}_v(C_{t,r}) = \frac{1}{r} \frac{r + 2}{r + 1} \left(\frac{2}{r + 1} + \frac{r + 2}{r + 1} + \frac{2}{r + 1} \right) \frac{(t - 3)}{r + t}$$

$$\text{bind}_{av}(C_{t,r}) = \frac{2t + 2r + 1}{(r + t)(r + 1)}$$

Definition 4.3. The double star graph $K(m,n)$ is a graph that is formed by two stars $K(m)$ and $K(n)$ via joining their centers by an edge (see Fig. 3). We give the anti-magic labeling of the Cartesian product of the double star graphs $K(m, n)$ with regular graphs.

Theorem 4.3. If $K_{m,n}$ is a double star graph with $m \geq 2$ and $n \geq 1$, then

$$\text{bind}_{av}(K_{m,n}) = \frac{3}{m + n - 1}$$
Proof. Let $v \in V(K_{m,n})$, $S_v \subseteq V(K_{m,n})$. There is a local binding set $S_v$ of $V(K_{m,n})$ such that $|S_v| = m+n-2$, when $S_v$ contains all isolated vertices in set of $K_{m,n}$, then $|N(S_v)| = 2$. Thus $\text{bind}_v(K_{m,n}) = \frac{2}{m+n-2}$ for $m+n-2$ vertices of $K_{m,n}$.

If $\deg(v) = m$ then there is a local binding set $S_v$ of $V(K_{m,n})$ such that $|S_v| = m+n-1$ and $|N(S_v)| = m+1$. Thus $\text{bind}_v(K_{m,n}) = \frac{m+1}{m+n-1}$.

If $\deg(v) = m$ then there is a local binding set $S_v$ of $V(K_{m,n})$ such that $|S_v| = m+n-1$ and $|N(S_v)| = n+1$. Thus $\text{bind}_v(K_{m,n}) = \frac{n+1}{m+n-1}$.

By the definition

$$\text{bind}_{av}(K_{m,n}) = \frac{1}{m+n} \sum_{v \in V(K_{m,n})} \text{bind}_v(K_{m,n}) = \frac{2}{m+n-2} \frac{(m+n-2)}{m+n-1} + \frac{m+1}{m+n-1} \frac{n+1}{m+n-1}$$

$$\text{bind}_{av}(K_{m,n}) = \frac{3}{m+n-1}$$

5. Algorithms

We give the algorithms for the average binding number. As seen, some methods were used which are append_node, nodes and neighbors in neighbor average binding number algorithm. They are methods and interfaces in used graph structure which is in the NetworkX [30] for Python. Also, we used helper methods in Python like copy, clear. The Algorithm can compute the average binding number for graphs.

We give an algorithm is developed in order to calculate the average binding number for any connected undirected graph.

The Average Binding Number (Bind_{av})

Method: Bind_{av}(G)

Input: Graph G

Output: The average binding number of any graph

Begin

// Set the total Bind_{local vertex binding number} as 0
total = 0

// Take the vertices list of graph G
vertices = G.nodes

foreach $v$ in vertices

// Set the maximum bind for vertex $v$ as minimum integer
minBind$v$ = MaxInt

// Take the copy of graph G as temporary graph
tempS = copy.deepcopy(S)

// append the vertex $v$
tempS.append_node(v)

// Take the number of components of temporary graph
nbs = nx.number_connected_components(tempS)

// If number of components of temporary graph is not equal vertices,
// calculate the Bindv
if $v$ not in tempS
    tempS.append(v)
end if

// Calculate the Bind with vertex from temporary graph

tempBindv1 = len(nbs)
tempBindv2 = Bind(graph, tempS.nbs, tempS)

// Compare the Bindv with the minimum Bindv
if tempBindv1 < minBindv minBindv = tempBindv1 end if
if tempBindv2 < minBindv minBindv = tempBindv2 end if

tempS.clear()
// Take the total of Bindv
total = total + minBindv
end foreach
// Return the calculated total Bindv
return total/(len(verticless))
end

The algorithm first selects the peak number and finds the neighborhood number of the selected peaks. Then each peak is the binding number is found. Finally, it returns the average binding number of the graph.

6. Test results and Complexity

Binding number of a graph is an important characteristic quantity of a graph, which can be used to describe the graph characteristic to further understand the structure of a graph. It was an attempt to measure how “well-distributed” the edges of a graph are. The binding number of a graph can be computed in polynomial time\[39\]. The average binding number of G is defined as \( \text{bind}_{av}(G) = \frac{1}{n} \sum_{v \in V(G)} \text{bind}_v(G) \), where average binding number is bounded by a polynomial in \( n \).

The average binding number of some special graphs are obtained and some of these results are shown below.

| Graph | bindav | Elapsed Time (sec) |
|-------|--------|-------------------|
| P5    | 0.799  | 0.018             |
| P7    | 0.857  | 0.567             |
| K(3,3) | 0.428 | 11.357            |
| B(2,4) | 0.603 | 67.800            |
| B(3,4) | 0.548 | 96.687            |
| B(2,5) | 0.399 | 82.350            |
| B(3,5) | 0.441 | 106.594           |

In this note we show that \( \text{bind}_{av}(G) \) can be computed in polynomial time. Because of average binding number being polynomial complexity is identified with a ‘n’, where \( n \) is the number of vertices in graph G.

In this section, the comparative analysis of Radon code metrics of algorithms are given below. Some results can be obtained using Tables 3 to 6.

The proposed algorithms was implemented in Python. We analyzed the algorithms via Radon code metrics. The tables given below contain result of code metrics of neighbor average binding number algorithm.
According to Table 3, this shows that the program is explained well with comment lines. The algorithm consists of 75 lines. The number of logical lines is 46 lines. It is seen that the rate of comment line count is over 20% compared to the logical line count rate. This shows that the program is explained well with comment lines.

| Raw Metrics | bind av |
|-------------|---------|
| LOC         | 75      |
| Li. LOC     | 46      |
| SLOC        | 46      |
| Comments    | 27      |
| Single Comments | 27 |
| Multi       | 0       |
| Blank       | 2       |
| Comment Stats |        |
| (C/S)       | 36%     |
| (C%)        | 59%     |
| (C+M+S)     | 36%     |

According to Table 4, For bindav algorithm; when the program is analyzed with Halstead metrics, which are the most used metrics to measure the functional software complexity, it is seen that the program length of N1 + N2 is 42. The program does not need to be re-designed since the length of the program is below the limit number 300. Looking at the volume value, which is the program volume value; this value appears to be 29. Since this value does not have a number of 1000 and above, it is seen that the program does not have a complex structure. Effort value, which is a quality standard, is also lower than the limit value of 5000. This appears to be another plus regarding the program's metrics. Similarly, having a time value below 90 is another value that shows the quality of the program. The bugs value indicating the estimated number of errors is also very close to 0, indicating that the probability of a code related error in the program will be very low.

| Halstead Metrics | bind av |
|------------------|---------|
| h_1              | 6       |
| h_2              | 23      |
| N_1              | 14      |
| N_2              | 28      |
| Vocabulary       | 29      |
| Length           | 42      |
| Calculated Length| 119.55  |
| Volume           | 204.035 |
| Difficulty       | 3.652   |
| Effort           | 745.172 |
| Time             | 41.398  |
| Bugs             | 0.068   |

According to Table 5, Maintainability Index of bind av.

| Maintainability Index of | bind av |
|--------------------------|---------|
| A                        |         |
According to Table 5, the algorithm; Maintainability Index shows the updatability feature of the program. The developed program received the highest value, A. This shows that the program can easily meet new demands in the future, within its own structure.

According to Table 6, in the developed algorithms, Binding Number got the value A from their metrics. Bindav has taken the B value because their values are above 10. Since this value is not a value of 50 and above, it is seen that the program does not generate cyclic complexity.

The proposed algorithms was implemented in Python. We tested the algorithms on several special classes of graphs such as path, cycle, wheel and complete graphs. The tables given contain values of average binding number.

7. Conclusion

In this study, the average binding number has been worked on to measure vulnerability of a network as the average of the local binding number of every vertex of a graph. Additionally, the average binding numbers have been calculated by code and the stability of popular interconnection networks has been studied. If bindav\((G)\) is large it means that, in that graph vertices of \(G\) are well bound together in the sense that \(G\) has a lot of fairly well distributed edges.

Design of networks has important criterias which are reliability and efficiency. Reliability and efficiency are important criteria in the design of networks. When we want to design a network, we wish that it is as stable as possible. Any communication network can be modelled as a connected graph. In this study, a new graph theoretical parameter namely the average binding number has been presented for the network vulnerability.

It has been demonstrated how algorithms can be used in order to measure vulnerability in real-life networks and how this given vulnerability parameter can be useful.

The purpose of the article; defining average binding number as a vulnerability measurement parameter where vulnerability is more distinctive than average binding number, and creating a polynomial time algorithm for average binding number of interval graphs.

In this paper, we considered the average binding number is usually thought to measure the vulnerability of graphs. We proposed the algorithms to calculate these parameters. We analyzed the algorithm and compared via Radon software metrics that has shown in Tables 2 to 6. The results show that the proposed algorithms correctly computes the given vulnerability parameters. As can be seen from the results obtained from the Radon software metrics, the values of the algorithm are within the ideal limits.

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