Coupled TM surface plasmon features of graphene-metal layered structure in the sub-THz frequency range

Hameda Alkorre\(^1\), Gennady Shkerdin\(^2\), Johan Stiens\(^1\) and Roger Vounckx\(^1\)

\(^1\) Laboratory for micro- and photon electronics (LAMI), Department of Electronics and Informatics (ETRO-IR), Vrije Universiteit Brussel (VUB), Pleinlaan 2, B-1050 Brussels, Belgium
\(^2\) Kotel’nikov Institute of Radio Engineering and Electronics of the Russian Academy of Science, Vvedenskogo Square 1, 141120 Fryazino (Moscow region), Russia

E-mail: hailkorr@etro.vub.ac.be

Received 10 November 2014, revised 30 January 2015
Accepted for publication 19 February 2015
Published 27 March 2015

Abstract

TM surface plasmon (SP) characteristics of a four-layer structure, consisting of air as the superstrate, a monolayer of graphene, a dielectric buffer layer and metal as the substrate are analyzed at sub-THz frequencies. TM SPs in such case are represented by metal-like and graphene-like branches. For small frequencies the metal-like plasmon splits up into two branches depending on the graphene electron concentration; one of the branches goes into cutoff at the point where the branch features Brewster-type characteristics. Graphene-like plasmon modes are converted into short-range modes for small buffer thicknesses. Brewster-type SP modes can be effectively modulated in the vicinity of their cutoff thicknesses by means of the graphene electron concentration.

Keywords: surface plasmons, modes, Brewster, multi-layer, sub-THz frequencies

(Some figures may appear in colour only in the online journal)

1. Introduction

Electronic, optical and mechanical properties of graphene were extensively studied after the appearance of the first publication devoted to experimental investigations of graphene [1]; results of these studies were widely published [2–4]. Free carriers in graphene can be created either by doping or by field effects. A graphene electron gas of high mobility forms an ideal two dimensional sheet [1]. The optical surface conductivity of a graphene monolayer was studied in a wide range of frequencies, from the terahertz to the visible range [2, 5–8]. Electromagnetic wave (EMW) propagation in structures containing graphene layers was investigated in a number of publications [9–12] and various perspective applications for graphene in photonics and optoelectronics have been discussed [13]. Special attention was paid to plasmons supported by graphene layers and which are tunable by a gate voltage. Perspective applications of graphene plasmons in nanophotonics due to their high confinement around the graphene layer have been discussed in many papers [14–23].

In spite of the fact that graphene plasmons were investigated in a number of structures containing a few graphene monolayers or graphene bilayers, the case of graphene plasmons coupled with other types of structure modes (surface metal-plasmon or waveguide modes) has not been thoroughly examined. The coupling of graphene plasmons with surface metal plasmons in a structure containing a graphene layer and metal substrate separated by an air gap was studied [24]: the solution of the coupled plasmon mode shows a linear dispersion behavior in a specific parameter range.
In this article we analyze the propagation characteristics of TM coupled modes in a structure containing a metal substrate, dielectric buffer layer, a monolayer of graphene, and air as the superstrate.

The article is divided into two parts. In the first part, the dispersion equation for TM eigenmodes and some of its approximate solutions are given on the basis of Maxwell’s equations solution. In the second part, dispersion characteristics of coupled graphene-metal plasmons are analyzed on the basis of numerical solutions of the dispersion equation for sub-THz frequencies. The results are summarized in the conclusions.

2. Dispersion equation for the coupled graphene-metal surface plasmons (MSPs) in four-layered structure

We consider the propagation of TM-polarized EMWs along the Z direction in the structure shown in figure 1; magnetic field of TM-polarized EMW is directed along X axis and electric field vector of TM-polarized EMW is parallel to (Y, Z) plane, the magnetic and electric field component respectively can be written as: \( H = (H_x, 0, 0) \) and \( E = (0, E_y, E_z) \). The structure of figure 1 consists of a metal substrate with relative dielectric permittivity \( \varepsilon_3 \), a buffer layer of thickness \( d_2 \) and relative dielectric permittivity \( \varepsilon_2 \), a graphene monolayer created on the buffer layer surface \( y = 0 \) (Y-axis is orthogonal to the multi-layer structure) and air as the lossless superstrate medium behind the graphene layer with relative dielectric permittivity \( \varepsilon_1 = 1 \).

The standard electrodynamic boundary conditions represented by the continuity of the electric field tangential component at the interfaces \( y=0, y=-d_2 \) and by the continuity of the magnetic field tangential component at the interface \( y=-d_2 \) are used to calculate the EMW propagation in the structure of figure 1. The boundary condition for the tangential component of the magnetic field at \( y=0 \) is modified by the presence of the monolayer of graphene, which features a surface electron concentration \( n_s \) and a surface conductivity \( \sigma \). In the SI units, the boundary condition can be written down as follows:

\[
H_y(y=0) = H_y(y=-d_2) = -\sigma E_x(y=0),
\]

where \( H_y \) and \( H_y \) are the tangential components of the magnetic field in the superstrate and the buffer layer, respectively; \( E_x(y=0) \) is the amplitude of the tangential component of the electric field at \( y=0 \).

Straightforward calculations for the TM eigenmodes lead to the spatial distribution of the magnetic field amplitude given by equations (2a) and (2c) and to the dispersion equation (3):

\[
H_{yx1} = H_{yx1} e^{-Q_{y1}y+iy_{23}z -j\omega t},
\]

\[
H_{yx2} = \left[H_{yx2} + H_{yx2} e^{-Q_{y2}(y+d_2)}\right] e^{iy_{23}z -j\omega t},
\]

\[
H_{yx3} = H_{yx3} e^{-Q_{y3}(y+d_2)} e^{iy_{23}z -j\omega t},
\]

\[
1 + \frac{\varepsilon_3 Q_{y1}}{\varepsilon_2 Q_2} \left(\frac{\varepsilon_2 Q_2}{\varepsilon_1 Q_1} + \frac{\varepsilon_1}{Q_1} + j \sigma(\omega, q) \omega \varepsilon_0\right) - e^{-Q_{y2}d_2} \times \left(1 - \frac{\varepsilon_3 Q_{y1}}{\varepsilon_2 Q_2} \left(\frac{\varepsilon_2 Q_2}{\varepsilon_1 Q_1} - \frac{\varepsilon_1}{Q_1} - j \sigma(\omega, q) \omega \varepsilon_0\right)\right) = 0,
\]

where:

\[
H_{H_1} = \left(\frac{\varepsilon_1}{\varepsilon_2}Q_2 - j Q_1, \frac{\varepsilon_2 Q_2}{\varepsilon_1 Q_1} - j Q_1, \frac{\varepsilon_2}{\varepsilon_1}Q_1\right),
\]

\[
H_{H_2} = \frac{\varepsilon_2}{\varepsilon_1}Q_1\left(\frac{\varepsilon_2}{\varepsilon_1}Q_1 - j Q_1\right),
\]

\[
Q_1 = \sqrt{q^2 - k_0^2 \varepsilon_j^2} \quad \text{for} \quad j = 1, 2, 3, \quad k_0 = \omega/c \quad \text{where} \quad c \quad \text{is the EMW speed in vacuum,} \quad \varepsilon_j \quad \text{and} \quad \mu_0 \quad \text{are permittivity and permeability of vacuum.}
\]

Throughout the paper we ignore the spatial dispersion of graphene conductivity, suggesting that \( q \nu \ll \omega \), the Fermi velocity is \( \nu_F = 10^6 \text{ m s}^{-1} \) \[25\], however temporal dispersion of graphene conductivity is taken into account therefore \( \sigma(\omega, q) = \sigma(\omega) \). We use equation (4a) for graphene’s conductivity \( \sigma(\omega) \) consisting of intraband \( \sigma^{\text{intra}}(\omega) \) and interband \( \sigma^{\text{inter}}(\omega) \) contributions \[2\]:

\[
\sigma(\omega) = \sigma^{\text{intra}}(\omega) + \sigma^{\text{inter}}(\omega),
\]

where

\[
\sigma^{\text{intra}}(\omega) = \frac{2T e^2}{\pi \hbar^2 \omega \left[1 + i \omega/\nu_F\right]},
\]

\[
\sigma^{\text{inter}}(\omega) = \frac{e^2}{4\hbar} \left\{G(\omega/2) + \frac{4\hbar \omega}{\pi} a \int_{-\infty}^{0} d\epsilon G(\epsilon) - G(\omega/2)\right\} \left[\hbar \omega^2 - 4e^2\right]
\]

\( e, \nu_F \) are the electron energy and Fermi level energy, respectively, \( T = k_B t, k_B \) is Boltzmann constant, \( t \) is temperature in Kelvin, \( e \) is the electron charge and \( \tau \) is the electron relaxation time; \( G(\epsilon) = f(-\epsilon) - f(\epsilon) \) and \( f(\epsilon) \) is the electron distribution function.
Dependence of Fermi level energy \( E_F \) (from now on this energy is denoted as Fermi level) versus graphene surface electron concentration \( n_s \) and temperature \( t \) is defined by the following integral equation:

\[
n_s = \frac{2}{\pi [\hbar v_F]} \int_{+\infty}^{0} \varepsilon_0 [f(\varepsilon - E_F) - f(\varepsilon + E_F)]. \tag{5}
\]

Interband transitions become more important for higher optical frequencies and lower Fermi levels. At room temperature, and low terahertz frequencies \( f \lesssim 30 \) THz, intraband transitions give the largest contribution to the optical conductivity of the graphene monolayer for \( E_F > 0.05 \) eV (EMW frequency is denoted as \( f \), in contrast to electron distribution function which is denoted as \( f(\varepsilon) \)).

Equation (3) was derived in [26] although the coupling of MSPs and surface carrier plasmons created by a two dimensional surface charge \( q \) embedded in the interface \( y = 0 \) had not been studied. For a very wide buffer layer thickness when \( d_2 \to \infty \), equation (3) reduces to the dispersion equations of both decoupled TM MSPs (equation (6)) and TM graphene plasmons (equation (7), see also [7, 12, 26]):

\[
1 + \frac{\varepsilon_3 Q_k}{\varepsilon_2 Q_k} = 0, \tag{6}
\]

\[
\frac{\varepsilon_2}{Q_k} + \frac{\varepsilon_1}{Q_k} + i\sigma(\omega, q)/\omega \varepsilon_0 = 0. \tag{7}
\]

Rewriting equation (6) leads to the well-known dispersion relation for MSPs [27] given by equation (8):

\[
q_{3, 2} = k_0 \sqrt{\varepsilon_3 \varepsilon_2 f(\varepsilon_3 + \varepsilon_2)} . \tag{8}
\]

Equation (7) is described by the 4th power equation for the \( q^2 \) value and its general solutions are too cumbersome to write them down. However, the solution is considerably simplified for the case \( q \gg k_0 \sqrt{\varepsilon_1, 2} \). When the dispersion relation for graphene plasmons can be written down as follows [7]:

\[
q_0 \approx \frac{(\varepsilon_1 + \varepsilon_2) i \varepsilon_0 \omega}{\sigma(\omega)} . \tag{9}
\]

Taking into account only the intraband contribution to graphene’s optical conductivity given by equation (4b) for \( \tau \to \infty \), equation (3) is reduced to equation (10) for \( q \gg \sqrt{k_0 \varepsilon_1} \) (\( j = 1, 2, 3 \), \( \varepsilon_1 = \varepsilon_2 = 1 \), \( \varepsilon_3 = 1 - \omega^2/\omega_p^2 \) (\( \omega_p \) is the metal electron plasma frequency):

\[
\left( \frac{2\omega^2}{\omega_p^2} - 1 \right) \left( \frac{\omega^2}{\omega_p^2(q)} - 1 \right) = e^{-2q\varepsilon_1}, \tag{10}
\]

where \( q_0(q) = \sqrt{4\varepsilon_0 \ln \left(2 \cosh(E_F/2T)\right)} \) is the graphene plasmon frequency defined by equation (9). Equation (10) is coincident with equation (22) derived in [24]; the equation describes frequencies of coupled metal-graphene plasmons for \( \varepsilon_1 = \varepsilon_2 = 1 \) when the plasmon absorption is totally neglected. However, this equation is valid only for large propagation constant values when the conditions \( q \gg k_0 \sqrt{\varepsilon_1} \) are fulfilled and these inequalities are not fulfilled for long-range modes of low terahertz frequencies propagating in the structure of figure 1 with material parameters under study (refer to the graphs plotted in figure 2). In this case it is necessary to solve equation (3) to find out the dispersion of the coupled modes. The terms long/short-range modes are used to define the amount of mode damping: for long-range or short-range modes the propagation length is respectively larger or smaller than the mode’s wavelength.

3. Results of numerical calculations and discussion

Numerical calculations were performed for the structure of figure 1 with the following material parameters for the different layers: the relative dielectric permittivity of air \( \varepsilon_1 = 1 \), for the buffer layer we consider \( \varepsilon_2 = 4 + 0.04i \), valid for EMW frequency \( f \leq 3 \) THz [28], and finally for the metal we used Au. Frequency dependent dielectric relative permittivity \( \varepsilon_3 \) of Au is described by Drude model with plasma and damping frequencies taken from [29]. The optical conductivity of graphene depends on the electron relaxation time \( \tau \), whereby the \( \tau \) value is evaluated from the DC mobility \( \mu \) according to the relation \( \tau = \mu \hbar / \sqrt{\pi \varepsilon_0 e^2} \). As the mobility values measured in [1] varied between \((0.3-1) \times 10^4 \) cm\(^2\) V\(^{-1}\) s\(^{-1}\) at room temperature we suggest \( \mu = 10^4 \) cm\(^2\) V\(^{-1}\) s\(^{-1}\) in our calculations.

Mathematically speaking, equation (3) can yield a large set of complex valued wave vector solutions, leading to many branches in the dispersion spectrum. This solution set can be divided into waveguide mode and surface plasmon (SP) like solutions. The analyses of the latter type of solutions will be the topic of this paper. From these SP mode solutions, we discuss only the bound modes which are the most relevant in the perspective of applications. Such modes feature positive
values of the real and imaginary parts of the propagation constant with small (short range) or large (long range) propagation lengths. For further classification of the SP mode branches we first assume an infinite thick buffer, such that one can define purely decoupled but Fermi level dependent graphene surface plasmons (GSP) and MSP solutions. The MSP solution exist over the full considered Fermi level range (0–0.5 eV), however the GSP do not.

The dependences of the real parts of the propagation constant of the decoupled MSP and GSP modes on the plasmon frequencies for \( E_F = 0.5 \) eV are depicted in figure 2 versus.

The pure MSP results from equation (8), which even approximates to \( q = k_0 \sqrt{\varepsilon_2} \) for low THz frequencies. The pure GSP exists in the three-layer system air/graphene/buffer: equation (7) yields the exact solution and equation (9) for \( \varepsilon \gg k_0 \sqrt{\varepsilon_2} \) yields an approximate solution.

The approximate solution of equation (9) leads to considerable errors over the whole considered frequency band. It is also seen that the difference of decoupled MSP and GSP propagation constant vanishes for low THz and GHz frequencies. Hence we may expect stronger coupling effects in this frequency range. All subsequent numerical calculations are mainly performed in the strong coupling regime for \( f = 0.3 \) THz.

The dispersion dependence of decoupled GSP on graphene’s electron Fermi energy level is shown in figure 3 for the EMW frequency \( f = 0.3 \) THz. From figure 3 one can observe that the decoupled GSP is represented by a long-range mode only for \( E_F \geq 0.32 \) eV, when \( \text{Im}(q) < \text{Re}(q)/2\pi \) and the GSP propagates at least one wavelength.

It is obvious that for a thinner buffer layer decoupled plasmon dispersion dependences will modify. In the extreme case of a very thin buffer layer when \( d_2 \rightarrow 0 \), the only long-range solution of equation (3) exists and this is represented by the MSP for the two-layer superstrate/metal structure. This solution is very close to that given by equation (8) with the substitution \( \varepsilon_2 \rightarrow \varepsilon_1 \left( \varepsilon_{1,2} \rightarrow \varepsilon_{1,3} \right) \), although slightly modified due to the presence of the monolayer graphene. The long-range SP tends to become the surface metal plasmon of the three-layer structure superstrate/buffer/metal in the case of small electron concentrations in graphene. Increasing the graphene electron concentration leads to a considerable modification of the coupled SP spectrum.

We classify coupled SPs as metal-like surface plasmons (MLSP) and graphene-like surface (GLSP) plasmons depending on their behavior for \( \varepsilon \rightarrow \infty \).

At this juncture the dependences of coupled plasmon modes on the buffer thickness and the Fermi level \( E_F \) in the graphene will be elaborated.

Dependences of MLSP propagation constant \( q \) versus buffer layer thickness for \( f = 0.3 \) THz at different Fermi level values \( E_F \) (a) real part, (b) imaginary part.
The MLSP splits up into two mode branches. One of them exists within the whole range of buffer thicknesses and this solution is called the continuous MLSP branch, the other MLSP solution exists only for buffer thicknesses smaller than the cutoff thickness $d_{2c}$ and this is called the cutoff MLSP mode. For $d_2 \to \infty$, the propagation constant solution converges to $q_{3,2}$, and it converts from long-range solution to a short-range solution for buffer layer thinner than $d_{2c}$. On the other hand the cutoff MLSP branch exists only for buffer layer thicknesses thinner than $d_{2c}$ ($\mu \leq d_{2c} \leq 250.15 \mu m$ for 0.179 eV $\leq E_F \leq 0.5$ eV), and for $d_2 \to 0$ the propagation constant solution tends towards $q_{3,1} = (62.88 + 1.3 \times 10^{-5}) \text{ cm}^{-1}$, also the imaginary part of the propagation constant vanishes at the cutoff thickness $d_{2c}$. Furthermore, the cutoff MLSP branch converts from the bound type mode for $d_2 > d_{2c}$, to the growing type mode for $d_2 > d_{2c}$, and this branch is represented by an incident plane wave without any reflection for $d_2 > d_{2c}$. These kinds of modes are called Brewster-type modes [30]. The whole incident wave is actually absorbed inside the structure [31, 32]. The propagation constant values real parts $q_r$ at the cutoff conditions for TM modes can be found from the solution of equation (11):

$$\text{Im} \left[ \varphi_{TM}(q) \right] = \text{Im} \left\{ -\frac{1}{2Q_2} \right\} \times \left\{ \begin{array}{c} \left[ \begin{array}{c} 1 + \frac{\varepsilon_3 Q_3}{\varepsilon_2 Q_2} \right] \frac{\varepsilon_2}{Q_2} + \frac{\varepsilon_1}{Q_1} + i \frac{\sigma(\omega, q)}{\omega \varepsilon_0} \\
1 - \frac{\varepsilon_3 Q_3}{\varepsilon_2 Q_2} \end{array} \right] \\
\left[ \begin{array}{c} \frac{\varepsilon_2}{Q_2} - \frac{\varepsilon_1}{Q_1} - i \frac{\sigma(\omega, q)}{\omega \varepsilon_0} \\
2\pi i N \right] \end{array} \right\} = 0, \tag{11}$$

where $N = 0, \pm 1, \pm 2, \ldots$.

Buffer layer thicknesses at the cutoff conditions are found using the relation: $d_{2c,N} = \varphi_{TM}(q_{d_{2c,N}})$. The splitting of the MLSP is due to its interaction with an additional cutoff mode solution of dispersion equation (3), which behaves as a short-range mode for small buffer thickness $d_2$. As illustration, the real and imaginary parts of the propagation constant values of the following modes have been plotted in figure 6 versus buffer thickness for $f=0.3$ THz: for $E_F=0.178$ eV the propagation constant of the additional cutoff mode, MLSP, and at $E_F=0.179$ eV the propagation constant of the continuous and cutoff MLSP branches, (a) real part, (b) imaginary part.

![Figure 5](image1.png)

![Figure 6](image2.png)

Figure 5. Dependences of the propagation constant values $q$ for continuous MLSP (solid line) and cutoff MLSP (dot line) branches versus buffer layer thickness for $f=0.3$ THz at different Fermi level values $E_F$ (a) real part, (b) imaginary part, of the propagation constant.

Figure 6. Dependences of the modes propagation constant $q$ versus buffer layer thickness for $f=0.3$ THz: for $E_F=0.178$ eV the propagation constant of the additional cutoff mode, MLSP, and at $E_F=0.179$ eV the propagation constant of the continuous and cutoff MLSP branches, (a) real part, (b) imaginary part.
considers the MLSP mode and the additional cutoff mode; for \( E_F = 0.179 \text{ eV} \), one considers the two branches of the MLSP mode, the continuous one and the cutoff one. The additional cutoff mode appears at a graphene Fermi level \( E_F \approx 0.155 \text{ eV} \) for \( f = 0.3 \text{ THz} \) when its propagation constant values strongly differ from those of the MLSP modes. As such splitting is out of question. However, for increasing \( E_F \) values the propagation constant solutions of both modes approach each other. For \( E_F = 0.178 \text{ eV} \) and \( d_2 \approx 126.5 \mu \text{m} \) both propagation constant values almost coincide: add propagation constant value cutoff \( q_{ad} = (61.1 + 30.9i) \text{ cm}^{-1} \), and the MLSP \( q_m = (66.4 + 29i) \text{ cm}^{-1} \). For larger thicknesses the add cutoff mode will go into cutoff \( d_{2c,ad} \) which equals at 148.71 \( \mu \text{m} \) for \( E_F = 0.155 \text{ eV} \).

Further increase of the Fermi level in graphene leads to the modification of the MLSP spectrum: the MLSP mode splits up and the continuous and cutoff MLSP branches are created for \( E_F = 0.179 \text{ eV} \) instead of the MLSP and additional cutoff mode exists for \( E_F = 0.178 \text{ eV} \).

The continuous MLSP branch for \( E_F = 0.179 \text{ eV} \) turns out to be close to the additional cutoff mode at \( E_F = 0.178 \text{ eV} \) for buffer thickness \( d_2 < 126.5 \mu \text{m} \) and to the MLSP at \( E_F = 0.178 \text{ eV} \) for buffer thickness \( d_2 > 126.5 \mu \text{m} \). On the other hand the cutoff MLSP branch for \( E_F = 0.179 \text{ eV} \) turns out to be close to the additional cutoff mode at \( E_F = 0.178 \text{ eV} \) for buffer thickness \( d_2 > 126.5 \mu \text{m} \) and to the MLSP at \( E_F = 0.178 \text{ eV} \) for buffer thickness \( d_2 < 126.5 \mu \text{m} \).

It is seen that there is no crossing points for the MLSP branches dependences when \( E_F = 0.179 \text{ eV} \) unlike for MLSP mode and additional cutoff mode dependences at \( E_F = 0.178 \text{ eV} \) when these crossing points exist both for real and imaginary parts of the propagation constant. This is clearly seen in figure 6(a) and (b) graphs. Dependences of the GLSP propagation constant values on buffer thickness are shown in figure 7 for different Fermi levels \( E_F \).

One can notice that for a thinner buffer layer, these modes become more damped and finally they are converted into short-range ones for small buffer thickness due to the stronger mode localization around the graphene layer. This effect is more pronounced for smaller Fermi level energies. GLSP for \( E_F = 0.2 \text{ eV} \) and \( E_F = 0.25 \text{ eV} \) are represented by short-range modes in the whole range of buffer thicknesses up to a very wide buffer thickness, when \( q_2 = (80.06 + 144.03i) \text{ cm}^{-1} \) for \( E_F = 0.2 \text{ eV} \) and \( q_2 = (83.27 + 65.11i) \text{ cm}^{-1} \) for \( E_F = 0.25 \text{ eV} \). GLSP modes are represented by long-range modes for \( d_2 > 440 \mu \text{m} \) when \( E_F = 0.35 \text{ eV} \) \( q_2 = (117.26 + 13.26i) \text{ cm}^{-1} \) for a very wide buffer thickness and for \( d_2 > 38.65 \mu \text{m} \) when \( E_F = 0.5 \text{ eV} \) \( q_2 = (125.05 + 4.88i) \text{ cm}^{-1} \) for a very wide buffer thickness.

It is interesting to mention that GLSP dispersion dependences for \( E_F = 0.35 \text{ eV} \) and \( E_F = 0.5 \text{ eV} \) are markedly different (the same is valid for continuous MLSP branches shown in figure 5).

This is explained by mode modification spectrum analogously to that resulting in the MLSP splitting. The point is that dispersion dependences for GLSP mode and continuous MLSP branch become very close for Fermi level about \( E_F = 0.48 \text{ eV} \) and buffer layer thickness about 750 \( \mu \text{m} \) (see figure 8). This leads to a modified mode spectrum. As a result, the dependences for the real parts of the mode propagation constant values are repulsed in this region of buffer thickness for \( E_F = 0.5 \text{ eV} \) and GLSP mode (continuous MLSP branch) propagation constant dependence for \( E_F = 0.5 \text{ eV} \) turns out to be close to continuous MLSP branch (GLSP mode) propagation constant dependence for \( E_F = 0.48 \text{ eV} \) in the region of buffer thickness \( d_2 < 700 \mu \text{m} \).

If we replace GLSP dispersion dependence for \( E_F = 0.5 \text{ eV} \) in figure 7 by continuous MLSP branch dispersion dependences for \( E_F = 0.5 \text{ eV} \) taken from figure 5 we see that continuous MLSP branch dispersion dependences fits much better, the set of graphs plotted in figure 7 (the same is valid for the analogous replacement of graphs in figure 5). The modes are defined as graphene (metal)-like ones according to their behavior for very large buffer layer when \( d_2 \) tends to infinity, therefore GLSP branches can be replaced by the MLSP ones due to the mode modification. At higher frequencies, one can show that the difference between propagation constant values of decoupled metal and graphene plasmons (see figure 2) becomes larger as the graphene surface conductivity \( \sigma(\omega) \) decreases due to the decreased}

![Figure 7](image-url)

**Figure 7.** Dependences of TM GLSP propagation constant \( q \) versus buffer layer thickness for \( f = 0.3 \text{ THz} \), at different Fermi level values \( E_F \) (a) real part, (b) imaginary part (logarithmic scale).
intraband surface conductivity contribution $\sigma^{\text{intra}}(\omega)$ (see equation (4b)).

Therefore, the graphene influence on the MLSP generally weakens and it is expected that modifications of the MLSP spectrum like that calculated for $f=0.3$ THz (see figures 4 and 5) appear for a higher Fermi levels in graphene if the SP frequency is increased. Numerical calculations confirm this statement.

The behavior of MLSP for $f=0.6$ THz is similar to that calculated for $f=0.3$ THz, but the splitting of the MLSP takes now place for a larger Fermi energy $E_F = 0.22$ eV. The MLSP modes for this frequency are characterized by the larger damping of the modified spectrum and by smaller values of buffer thickness $d_2$, at the cutoff conditions compared to the case when $f=0.3$ THz. Further increase of the SP frequency leads to the disappearance of the splitting effect for MLSP e.g. for a SP frequency $f>0.75$ THz, the MLSP propagation constant dependences are represented by continuous finite functions within the whole range of buffer thicknesses.

Dependences of MLSP propagation constant on Fermi level values are shown in figure 9 for $f=0.3$ THz and different buffer thicknesses. One observes the propagation constant modulation is the strongest for buffer thickness $d_2=150 \mu m$ when the MLSP splits up into two branches for $E_F \approx 0.179$ eV. Both real and imaginary parts of the MLSP propagation constant can be effectively modulated by changing the Fermi level in graphene; modulation of the imaginary part of the propagation constant, in particular, can lead to a large increase of the MLSP damping and to the SP propagation blockage.

These observations are in agreement with what was found in figures 4 and 5 where the MLSP propagation constant was considerably modulated by changing the graphene electron concentration. This effect is especially strong near the regions when SP splits up into two branches.

4. Conclusions

The dispersion dependences of TM graphene-metal coupled SPs were analyzed versus the buffer layer thickness and Fermi level in graphene. Calculations were mainly performed for sub-THz frequencies.
It is shown that TM SPs are represented by metal-like and graphene-like branches depending on their behavior for very thick buffer layers. For frequencies smaller than 0.75 THz (for the concrete structure under study), the MLSP split up into two branches depending on the graphene electron concentration: one of the branches (continuous MLSP branch) exists in the whole range of the buffer thickness and being a short-range mode for small thicknesses, another one (cutoff MLSP branch) undergoes cutoff and exists only within a limited range of buffer thicknesses smaller than the cutoff thickness. It is found that cutoff MLSP branches are represented by Brewster-type modes at the cutoff thicknesses and cutoff MLSP branches are converted into unphysical growing modes for larger buffer thicknesses. The graphene-like plasmons are converted into short-range modes for small buffer thicknesses due to strong mode confinement around graphene.

It is demonstrated that propagation constant values of SP mode near cutoff can be effectively modulated by changing the graphene electron concentration in the vicinity of the modes cutoff thicknesses.

Acknowledgments

The authors acknowledge Vrije Universiteit Brussel (VUB) through the SRP-project M3D2, the COST action MP1204 TERA-MIR, and the Libyan Ministry of Higher Education for technical and financial support.

References

[1] Novoselov K S, Geim A K, Morozov S V, Jian D, Zhang Y, Dubonos S V, Grigorieva I V and Firsov A A 2004 Electric field effect in atomically thin carbon films Science 306 666–9
[2] Falkovsky L A 2008 Optical properties of graphene and IV–VI semiconductors Phys.-Usp. 51 887–97
[3] Castro N A H, Guinea F, Peres N M R, Novoselov K S and Geim A K 2009 The electronic properties of graphene Rev. Mod. Phys. 81 109–62
[4] Wu Y H, Yu T and Shen Z X 2010 Two-dimensional carbon nanostuctures: fundamental properties, synthesis, characterization, and potential applications J. Appl. Phys. 108 071301
[5] Gusynin V P, Sharapov S G and Carbotte J P 2007 Sum rules for the optical and Hall conductivity in graphene Phys. Rev. B 75 165407
[6] Stauber T, Peres N M R and Geim A K 2008 Optical conductivity of graphene in the visible region of the spectrum Phys. Rev. B 78 085432
[7] Jablan M, Buljan H and Soljačić M 2009 Plasmonics in graphene at infrared frequencies Phys. Rev. B 80 245435
[8] Mak K F, Sfeir M Y, Wu Y, Lui C H, Miseiwich J A and Heinz T F 2008 Measurement of the optical conductivity of graphene Phys. Rev. Lett. 101 196405
[9] Blake P, Novoselov K S, Castro Neto A H, Jiang D, Yang R, Booth T J, Geim A K and Hill E W 2007 Making graphene visible Appl. Phys. Lett. 91 063124
[10] Abergel D S L, Russell A and Fal’ko V I 2007 Visibility of graphene flakes on a dielectric substrate Appl. Phys. Lett. 91 063125
[11] Hansen G W 2008 Quasi-transverse electromagnetic modes supported by a graphene parallel-plate waveguide J. Appl. Phys. 104 084314
[12] Bludov Y V, Ferreira A, Peres N M R and Vasilevskiy M I 2013 A primer on surface plasmon-polaritons in graphene Int. J. Mod. Phys. B 27 1341001
[13] Bonaccorso F, Sun Z, Hasan T and Ferrari A C 2010 Graphene photonics and optoelectronics Nat. Photonics 4 611–22
[14] Hwang E H and Das S S 2007 Dielectric function, screening, and plasmons in two-dimensional graphene Phys. Rev. B 75 205418
[15] Mikhailov S A and Ziegler K 2007 New electromagnetic mode in graphene Phys. Rev. Lett. 99 016803
[16] Geim A K and Novoselov K S 2007 The rise of graphene Nat. Mater. 6 183
[17] Jablan M, Buljan H and Soljačić M 2011 Transverse electric modes in bilayer graphene Opt. Express 19 11239
[18] Koppens F H L, Chang D E and García de Abajo F J 2011 Graphene plasmonics: a platform for strong light–matter interactions Nano Lett. 11 3370–7
[19] Fei Z et al 2011 Infrared nanoimaging of Dirac plasmons at the graphene–SiO2 interface Nano Lett. 11 4701–5
[20] Gan C H, Chu H S and Ping Li E 2012 Synthesis of highly confined surface plasmon modes with doped graphene sheets in the midinfrared and terahertz frequencies Phys. Rev. B 85 125431
[21] Svintsint D, Yyurkov V, Ryzhii V and Otsuji T 2013 Voltage-controlled surface plasmon-polaritons in double graphene layer structures J. Appl. Phys. 113 053701
[22] Bludov Y V, Vasilevskiy M I and Peres N M R 2010 Mechanism for graphene-based optoelectronic switches by tuning surface plasmon-polaritons in monolayer graphene Europhys. Lett. 92 68001
[23] Chen J et al 2012 Optical nano-imaging of gate-tunable graphene plasmons Nature 487 77–81
[24] Horing N J M 2009 Coupling of graphene and surface plasmons Phys. Rev. B 80 193401
[25] Falkovsky L A and Pershoguba S S 2007 Optical far-infrared properties of a graphene monolayer and multilayer Phys. Rev. B 76 153410
[26] Nakayama M 1974 Theory of surface waves coupled to surface carriers J. Phys. Soc. Japan 36 393
[27] Landau L D and Lifshitz E M 1960 Electrodynamics of Continuous Media (London: Pergamon)
[28] Kitamura R, Pilon L and Miroslaw J 2007 Optical constants of silica glass from extreme ultraviolet to far infrared at near room temperature Appl. Opt. 46 8118–33
[29] Ondal M A, Bell R J, Alexander R W Jr, Long L L and Querry M R 1985 Optical properties of fourteen metals in the infrared and far infrared: Al, Co, Cu, Au, Fe, Pb, Mo, Ni, Pd, Pt, Ag, Ti, V, and W Appl. Opt. 24 4493–9
[30] Burke J J, Stegeman G I and Tamir T 1986 Surface-polariton–photon waves guided by thin lossy metal films Phys. Rev. B 33 5186–201
[31] Zervas M N 1991 Surface plasmon–polariton waves guided by thin metal films Opt. Lett. 16 720–2
[32] Skherdin G, Stiens J and Vounckx R 2003 The relationship between reflectivity minima and eigenmodes in multi-layer structures J. Opt. A: Pure Appl. Opt. 5 386–96