Voltage from mechanical stress in type-II superconductors: Depinning of the magnetic flux by moving dislocations

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Mechanical stress causes motion of defects in solids. We show that in a type-II superconductor a moving dislocation generates a pattern of current that exerts the depinning force on the surrounding vortex lattice. Concentration of dislocations and the mechanical stress needed to produce critical depinning currents are shown to be within practical range. When external magnetic field and transport current are present this effect generates voltage across the superconductor. Thus a superconductor can serve as an electrical sensor of the mechanical stress.

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Material defects such as dislocations can be set into motion by subjecting the sample to an external stress. When the stress becomes large the velocity of dislocations can be as high as the speed of sound. The dynamics of moving dislocations have been intensively studied in the past both theoretically [1, 2] and experimentally [3]. Within continuous linear theory of elasticity the dislocation speed is limited by the shear wave velocity $c_t$. When the anharmonicity of the crystal is taken into account the speed of dislocations has been shown to be inersonic (between $c_t$ and the speed of longitudinal sound $c_l$) and in some cases even supersonic [4, 5, 6, 7], that is above $c_l$. It has been well established that the fracture of a crystal under a large external stress is caused by the built-up of dislocations moving at velocities comparable to the speed of sound. For this reason, timely detection of fast-moving dislocations has practical importance for preventing material fracture. In a transparent material this can be achieved by optical methods. However, in metals the motion of dislocations is very difficult to detect. In this Letter we show how this goal can be achieved in a superconductor.

In type-II superconductors, dislocations have been studied in the context of vortex pinning [8, 9, 10]. The effect of moving dislocations has not received much attention. A stationary dislocation is a source of strong pinning provided it is oriented parallel to the vortex line. Point defects act as weak pinning sites that may collectively pin vortices in bundles [11]. The strength of pinning is determined from the depinning Lorenz force $\mathbf{F} = (1/c) \mathbf{j} \times \Phi_0$ produced by an externally driven critical current $\mathbf{j} = j_c$, with $\Phi_0$ being the flux quantum. At low temperature, when a large external stress is applied, dislocations accelerate to high velocities ($v \sim c_t$) while point defects remain relatively immobile. It is therefore reasonable to expect that the flux lattice will be dragged by dislocations in the direction of their motion. This situation is not generic however because it only exists when linear dislocations are parallel to the flux lines. Only in this case the normal core of a vortex line can be effectively pinned by the dislocation. If the dislocation is at an angle with the flux line then the pinning is more or less equivalent to the pinning by a point defect, which is much weaker than the pinning of the flux line by the entire length of the dislocation.

In this Letter we examine a more general situation in which dislocations are not necessarily parallel to the flux lines. The effect we are going to discuss is not due to pinning of normal cores of flux lines by the dislocations. We will show that high-speed dislocations generate superconducting currents of order $j_c$, thus exerting the depinning force on the surrounding vortex lattice. Depinning forces produced at a given point in space by an array of moving dislocations are random. However, at high speed and sufficient concentration of dislocations these local random forces will be depinning the entire flux lattice, thus resulting in a finite resistance of the superconductor. We shall now discuss the origin of the superconducting current that surrounds a moving dislocation.

It is well known that a global mechanical rotation of a superconductor at an angular velocity $\Omega$ results in a macroscopic current. According to the Larmor theorem, in the rotating reference frame, Cooper pairs feel the effective magnetic field $\mathbf{B} = (2mc/e)\Omega$, where $e$ and $m$ are the bare electron charge and mass [12]. This field causes the Meissner current which is the same in the rotating and laboratory frames due to the fact that electric current is the motion of electrons with respect to the ions. Consequently, global rotation generates the magnetic moment in a superconducting sample, which is known as the London’s effect [13, 14, 15]. Recently the authors demonstrated [16] that high-frequency transverse ultrasound can generate large superconducting currents via local rotations of the crystal that occur at an angular velocity $\Omega$

$$\Omega(r,t) = \frac{1}{2} \nabla \times \mathbf{u}(r,t),$$

where $\mathbf{u}(r,t)$ is the phonon displacement field. In this Letter we consider a similar effect produced by moving dislocations. We will show that a moving dislocation is accompanied by a pattern of the superconducting cur-
rent. Even far from the dislocation core this current can exceed \( j_e \) when the dislocation is moving at a high speed. Consequently, an array of moving dislocations can depin the entire flux lattice.

The electric current is given by

\[
j = en_s (v_s - \mathbf{u}) + en_n (v_n - \mathbf{u}),
\]

where \( n_s \) and \( n_n \) are concentrations of superconducting and normal electrons, while \( v_s \) and \( v_n \) are their drift velocities respectively. In what follows we will neglect the contribution of the normal electrons to the total current because their motion is impeded by viscous forces and is negligible as compared to the motion of the charged superfluid. For certainty we will consider deformation \( \mathbf{u}(r, t) \) produced by a moving screw dislocation. The effect from the edge dislocations is similar and will be reported elsewhere. Within the continuous theory of elasticity a screw dislocation along the z-axis, moving in the x-direction with velocity \( v \), is described by the displacement field

\[
\mathbf{u}(r, t) = \frac{bp}{2\pi} \arctan\left(\frac{\gamma y}{x - vt}\right) \mathbf{e}_z,
\]

where \( b \) is the Burgers vector, \( p = \pm 1 \) is the chirality of the dislocation, and \( \gamma \) is the effective Lorentz factor:

\[
\gamma = \left(1 - \frac{v^2}{c^2}\right)^{1/2}.
\]

To simplify mathematics, in what follows we will consider the case of \( v^2 \ll c^2 \). Estimates based upon this approximation will be valid up to \( v \sim 0.3c \). We shall see that in fact even much smaller velocities of dislocations may be sufficient to depin the flux lattice. In this case

\[
\mathbf{u}(r, t = 0) = \frac{bp}{2\pi} \frac{\sin \theta}{r} \mathbf{e}_z,
\]

where \( \theta \) is the angle in cylindrical coordinates.

It is convenient to work with the gauge invariant quantity

\[
Q = \mathbf{A} - (\hbar c/2e) \nabla \varphi,
\]

where \( \mathbf{A}(r, t) \) is the electromagnetic vector potential and \( \varphi \) is the phase of the superfluid wave function. For the superconducting current one has

\[
j = -\frac{n_s e^2}{mc} \left( Q + \frac{mc}{e} \mathbf{u} \right).
\]

The term in Eq. (7) proportional to \( Q \) is the standard one and the term proportional to \( \mathbf{u} \) comes from the motion of the underlying crystal lattice, Eq. (2). The current \( j \) and the magnetic field \( \mathbf{B} = \nabla \times \mathbf{A} \) satisfy the Maxwell equation:

\[
\nabla \times \mathbf{B} = \frac{4\pi}{ce} j + \frac{1}{c} \mathbf{E}.
\]

Since \( \mathbf{A} \) produced by a moving dislocation is a function of \( r - vt \) and \( \mathbf{E} = -\mathbf{A}/c \), the last term in Eq. (8) is proportional to \( (v/c)^2 \) and it can be safely omitted. In terms of \( Q \) Eq. (8) then becomes

\[
\lambda^2 \nabla \times (\nabla \times Q) + Q = -\frac{mc}{e} \mathbf{u},
\]

where \( \lambda = (mc^2/4\pi n_s e^2)^{1/2} \) is the London penetration length.

It is convenient to introduce the dimensionless parameter \( \beta = (bp/2\pi \lambda) \) and dimensionless distance from the dislocation \( \rho = r/\lambda \). Substituting Eq. (3) into Eq. (4) we obtain

\[
\rho^2 \frac{\partial^2 Q_z}{\partial \rho^2} + \frac{\partial^2 Q_z}{\partial \rho^2} + \rho \frac{\partial Q_z}{\partial \rho} - \rho^2 Q_z = \frac{mcv}{e} \beta \rho \sin \theta.
\]

If we choose solution in the form \( Q_z(\rho, \theta) = (mcv/e) f(\rho) \sin \theta \) then Eq. (10) becomes an ordinary differential equation for \( f(\rho) \):

\[
\rho^2 f''(\rho) + \rho f'(\rho) - (1 + \rho^2) f(\rho) = \beta \rho.
\]

The general solution of this equation that goes to zero at \( \rho \to \infty \) is

\[
f(\rho) = \beta \left[ CK_1(\rho) - \frac{1}{\rho}\right],
\]

where \( K_1(\rho) \) is a modified Bessel function and \( C \) is a constant of integration that can be obtained from the requirement that \( Q \) is finite everywhere. Since \( K_1(\rho) \to 1/\rho \) as \( \rho \to 0 \), this gives \( C = 1 \). Eq. (7) then gives

\[
j = -\frac{c}{4\pi \lambda^2} \left[ \frac{mc}{e} \mathbf{u} + Q \right] = -\frac{mc^2v}{4\pi e \lambda^2} \beta K_1(\rho) \sin \theta \mathbf{e}_z.
\]

Because the angle \( \theta \) is defined with respect to the x-axis, \( j \) vanishes in the plane spanned by the Burgers vector \( \mathbf{b} \) and the dislocation velocity \( \mathbf{v} \). In the yz-plane the current flows along a closed loop. It generates a dipole-like magnetic field,

\[
\mathbf{B} = \nabla \times Q = \frac{mcv}{e \lambda^2} [f(\rho) \cos \theta \mathbf{e}_r - \rho f'(\rho) \sin \theta \mathbf{e}_\theta].
\]

The equicurrent lines from an array of moving parallel dislocations are shown in Fig. 1. As the velocity of dislocations increases the equicurrent loops in Fig. 1 expand. In the presence of the transport current, \( j_e \), normal to a flux line, the line becomes locally mobile if the combined force exerted on it by \( j_e \) and the current \( j_d \) due to moving dislocations exceeds the depinning threshold. To compute this effect one should notice that the direction and amplitude of \( j_d \) fluctuates in space and time due to random distribution of dislocations. For an ensemble of parallel dislocations moving at the same speed \( v \) the depinning threshold should be roughly determined by the condition

\[
\langle j_d \rangle^{1/2} \sin \theta = j_e - j_t,
\]

where \( \langle j_d \rangle \) is the mean current density and \( j_t \) is the threshold current density.
where $j_c > j_t$ is the critical current in the absence of moving dislocations and $\vartheta$ is the angle that dislocations make with the flux lines. The latter enters Eq. (15) because only the component of $j_d$ normal to the flux line exerts a force on the line.

The amplitude of the fluctuating current that appears in Eq. (15) can be computed as

$$\langle j_d^2 \rangle = n_d \int d^2r j_d^2(r),$$

where $n_d$ is a 2D concentration of dislocations and $j_d(r)$ is given by Eq. (13). At the lower limit this integral should be cutoff by the size of the dislocation core, $r \sim b$. This gives

$$\langle j_d^2 \rangle = n_d \left( \frac{mc^2v}{8\pi^2eA^2} \right)^2 \pi \ln \frac{\lambda}{b}.$$  

Substituting this result in Eq. (15), one obtains the critical (depinning) concentration of dislocations as function of their velocity:

$$\sqrt{n_d} = \frac{j_c}{\lambda} \left( 1 - \frac{j_t}{j_c} \right) \frac{ct}{v},$$

where we have introduced a dimensionless critical current

$$\bar{j}_c = \left[ \frac{64\pi^3}{\ln(\lambda/b)} \right]^{1/2} \frac{e\lambda^3}{mc^2c_t b \sin \vartheta} j_c.$$ 

For typical values of the parameters: $\lambda \sim 10^{-5}$ cm, $b \sim 2 \times 10^{-8}$ cm, $c_t \sim 2 \times 10^5$ cm/s, and $\vartheta = 90^\circ$, the parameter $\bar{j}_c$ is of order unity at $j_c \sim 10^5$ A/cm$^2$. According to Ref. 2 the speed of a screw dislocation very rapidly approaches the speed of sound on increasing the elastic stress. Taking $v \sim 0.1c_t$ and $j_t \sim 0.9j_c$, we obtain a reasonable value of the critical concentration of dislocations: $n_d \sim 1/\lambda^2$. Even smaller concentration of dislocations will be required if the transport current is brought closer to $j_c$. In experiment this effect will manifest itself as a rapid shift of the critical current towards lower values in the presence of plastic deformation of the material. The resulting depinning of flux lines will generate voltage across the superconductor. Since this voltage originates from the elastic stress, this would be a remarkable example of a strong non-equilibrium piezoelectric effect in a conducting material.

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