Improvement of Sandcastle Stability Model Based on Analytic Hierarchy Process

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Abstract. The strength of a sandcastle is closely related to its shape and material. We developed a sand castle model of optimal sand-to-water mixture proportion. The collected data were used to fit the regression linear equation and the statistical regression model was used to analyze the optimal water-sand ratio. Based on the sensitivity analysis, the consistency between the model and the reality is verified, and the robustness of the model is also verified. After the general shape of the sand castle is determined, the sandcastle shape is optimized and adjusted taking the influence of rainfall into consideration. Based on the relationship between soil slope and infiltration rate and erosion rate, we divided the rainwater erosion sand castle into two stages: infiltration effect of different slopes on water-sand ratio and slope effect on erosion rate after water content saturation. After obtaining the optimal slope angles of two stages, the optimal precise 3d geometry was evaluated by AHP with the stability parameters and two slope angles.

Keywords: Curve fitting, Analytic hierarchy process, Fuzzy evaluation method

1. Introduction
Building a sandcastle is a necessary childhood game for almost every child, but it’s worth investigating how to make them last the longest on the beach. Based on a large amount of literature and practical experience, the shape of a sand castle should be roughly cylindrical because of the large deformation caused by the ridged shape under the impact of waves and tides. However, the material of the sandcastle also has a great influence on its stability. Finding the best materials is also worth studying and analyzing. How to consider the influence of rainfall and how much slope to choose are also the dimensions of further optimizing the shape of sand castles. Such research also provides guidance on how to build buildings at risk of tsunamis.

2. Problem hypothesis
   • Considering that sandcastles are usually built on relatively flat sandy beaches, it is not necessary to consider large waves, which can be devastating to sandcastles.
   • Because we use the same type of sand, roughly the same amount of sand and water to sand ratio, so we can assume that all sandcastles are 1 cubic meter in volume and 1 meter in height.
   • The supporting effect between sand grains is not considered, that is, the large-scale collapse of
sand grains is ignored.

3. Model establishment and solution

3.1. Problem analysis

In the current analysis, there is little research on the influence of sand castle materials on its stability. Here, we changed the sand and water mixing ratio, changing the stability of the sand castle, to study which material is the best material to build the sand castle.

In the finite element model used to solve the optimal 3D shape of sand castle, the difference of water and sand ratio can affect the elastic modulus and poisson's ratio of the material, thus resulting in the change of sand castle stability. However, we found that there was no strong data support among water and sand ratio, elastic modulus and poisson's ratio, and it was difficult to find the exact functional relationship. We used the statistical regression model to analyze the relationship among sand particle size, water content, erosion velocity and erosion time with the collected experimental data, so as to master the internal mechanism. The following is the experimental data obtained from a large number of literatures.

Table 1. Experimental data on sand particle size, water content, erosion velocity and erosion time

| Particle size(um) | Water content(%) | Erosion velocity(cm/s) | Erosion time(s) |
|-------------------|------------------|------------------------|-----------------|
| 29.81             | 33.11            | 44.63                  | 759             |
| 29.81             | 33.11            | 47.78                  | 445             |
| 29.81             | 33.11            | 39.61                  | 1849            |
| 29.81             | 33.11            | 43.8                   | 1049            |
| 29.81             | 33.11            | 53.25                  | 222             |
| 34.69             | 27.75            | 44.63                  | 820             |
| 34.69             | 27.75            | 47.78                  | 324             |
| 38.16             | 29.68            | 50.51                  | 182             |
| 38.16             | 29.68            | 39.61                  | 1011            |
| 38.16             | 29.68            | 43.8                   | 369             |
| 33.71             | 27.29            | 50.02                  | 126             |
| 33.71             | 27.29            | 41.53                  | 1310            |
| 33.71             | 27.29            | 48.56                  | 381             |
| 39.51             | 26.27            | 46.27                  | 602             |
| 39.51             | 26.27            | 41.53                  | 539             |
| 39.51             | 26.27            | 37.55                  | 976             |
| 28.96             | 24.62            | 51.02                  | 1344            |
| 28.96             | 24.62            | 46.27                  | 3051            |
| 28.96             | 24.62            | 48.56                  | 1559            |
| 28.96             | 24.62            | 56.51                  | 782             |
| 28.96             | 24.62            | 60.28                  | 668             |
| 7.47              | 36.51            | 90.2                   | 7200            |
| 7.47              | 36.51            | 106.49                 | 7200            |
| 7.89              | 36.06            | 90.2                   | 7200            |
| 7.89              | 36.06            | 100.66                 | 7200            |

3.2. Data processing

Through the curve fitting with MATLAB software, the following regression equation can be obtained, which characterizes the functional relationship between sand particle size, water content, erosion velocity and erosion time.
\[ T = 286.5227 - 5.7874x_1x_2 + 486.6467x_2 - 6.1584x_2^2 - 1.6880x_1x_3 \] (1)

Where, \( T \) represents erosion time, \( x_1 \) represents particle size, \( x_2 \) represents water content, and \( x_3 \) represents erosion velocity. Then we observed \( R^2 \), that is, the goodness of fit of the regression curve, and got its result \( R^2 = 0.94 \), very close to 1, indicating that the curve has a high degree of fitting and a strong reliability.

Next, for the \( x_1 \) in the equation, the average value of 28 in the experimental data and the average value of \( x_3 \) was 2100, and the relationship between the erosion duration and water content was obtained. It can be seen that the optimal sand-water ratio should be 0.2639. So, the ratio of sand and water is the most suitable at 2.7:1.

![Figure 1. Diagram of relationship between erosion duration and water content](image)

After, the sand-water ratio can be used to further deduce the density of the mixture, so we chose clay to replace the mixture of water and sand among the materials with similar properties and density. By introducing its modulus of elasticity and poisson's ratio, we can verify that the ultimate sandcastle displacement degree is indeed more stable than other ratios under this sand-water ratio.

3.3. Sensitivity analysis
Although we get the fitting curve of water content and scouring time, we still need to verify its reliability by further tests. That is, if the size of the data is slightly changed within a certain range, the observation results will change significantly.

By means of functional analysis, the data of flow rate is stable for a long time and will not affect the water content, so we do not consider it. The particle size of sand is the main factor affecting water content. In the scour time model, let the particle size be 28, and then we test it with a step size of 0.5 from 26 to 30.
Figure 2. The effect of particle size change on the optimal sand-water ratio

It can be seen from the figure that the relation between them is very small, that is to say, the influence of the change of sand particle size on the optimal sand-water ratio can be ignored, so the conclusion of 2.7:1 we just got has strong stability.

3.4. The Effect of Rain on Shape

In an idealized analysis, we simply assume that the effects of seawater and tides are single. That is to ignore many of the changes between the two, such as slightly different effects on sandcastles at high and low tide. If it rains when we build a sandcastle on the beach, the external forces on the sandcastle cannot be considered as a whole. Otherwise, it is difficult to get the true deformation of the sandcastle. Therefore, we should modify and supplement based on the idealized model.

According to the existing slope stability test results, when rainfall on the slope, some form slope runoff, some infiltration slope surface, slope water content increased. Therefore, we can divide the rainfall process into two stages. In the first stage, when the water content of the sand-water mixture is relatively small, the rainwater mainly penetrates into the sandcastle and continuously increases its water content. In the second stage, when the sand castle continuously absorbs the rain water and makes the water content reach the saturation degree, the rain water mainly changes the shape of the sand castle by forming slope runoff. As can be seen from the figure below, when the water content reaches 0.4, it is close to saturation, and the action mode of rainwater is mainly runoff.

Figure 3. The characteristic curve
These processes are closely related to the slope of sand castle shape, which indicates that the change of slope will affect the infiltration and surface runoff of rainwater, thus affecting its stability and durability.

3.5. Optimization of the Best 3D Shape

Through the above analysis, in order to further determine the slope, we selected the circular platform with slope angles of 90 degrees, 60 degrees and 30 degrees as the research object. Among them, the round table with slope Angle of 90 degrees is a cylinder.

![Three-dimensional shapes with different slopes](image)

Figure 4. Three-dimensional shapes with different slopes

The stability of the sandcastles can be characterized by M. The larger M, the slower the infiltration rate. First, make some definitions to list the functional relationship. \( c \) represents cohesion, \( l \) represents length of slope, \( P \) represents influence of rain on slope, \( Q \) represents total gravity of slope body, \( \delta \) represents angle of internal friction, \( \theta \) represents angle of slope and \( \beta \) represents angle between \( P \) and slope. Based on relevant experiments, it can be obtained from the ratio of anti-sliding force and sliding force, and the specific calculation formula is as follows:

\[
M = \frac{cl + (Q \cos \theta + P \sin \beta) \tan \delta}{Q \sin \theta + P \cos \beta} \quad (2)
\]

By calculation, we can get that the sand castle is the most stable when the slope is kept at 1:0.85 in the case of rain infiltration. So we know that in the first phase, the optimal Angle is 30 degrees.

| Slope grade | Rain rate | Total rainfall on slopes | Depth of water infiltration |
|-------------|-----------|--------------------------|----------------------------|
| 1:1.5       | 0.75      | 17.47                    | 10.2                       |
| 1:1         | 0.75      | 20.15                    | 8.3                        |
| 1:0.85      | 0.75      | 22.52                    | 7.1                        |
| 1:0.5       | 0.75      | 20.43                    | 8.7                        |

The runoff velocity measurement can be found in the literature a large number of data, here directly cited:

Table 3. The relationship between slope and runoff velocity

| Slope grade | Runoff velocity |
|-------------|-----------------|
| 1:1.5       | 0.60            |
| 1:1         | 0.62            |
| 1:0.85      | 0.53            |
| 1:0.5       | 0.51            |

The relation between runoff velocity and slope of sand-water mixture in saturated state is presented. From this we can conclude that in the second stage of rain, the optimum slope should be 1:0.5, which translates to an Angle of 60 degrees.
In the original study, we have obtained the displacement of sand castles of different shapes under the action of waves and tides, so the basic form can also be used as a criterion for judging. Preliminary models show that the best basic form is cylinder. Therefore, the basic form, the slope in the unsaturated state and the slope in the saturated state constitute the criterion. Therefore, the hierarchy division of AHP is shown in the following figure.

![Figure 5. The hierarchy division of AHP](image)

Based on the above discussion, with the optimal state of the three criteria, the judgment matrix can be written more accurately, as shown in the following table:

### Table 4. Judgment matrix 1

|       | Shape1 | Shape2 | Shape3 |
|-------|--------|--------|--------|
| Shape1| 1      | 0.5    | 0.25   |
| Shape2| 2      | 1      | 0.5    |
| Shape3| 4      | 2      | 1      |

### Table 5. Judgment matrix 2

|       | Shape1 | Shape2 | Shape3 |
|-------|--------|--------|--------|
| Shape1| 1      | 3      | 6      |
| Shape2| 0.33   | 1      | 2      |
| Shape3| 0.17   | 0.5    | 1      |

### Table 6. Judgment matrix 3

|       | Shape1 | Shape2 | Shape3 |
|-------|--------|--------|--------|
| Shape1| 1      | 6      | 3      |
| Shape2| 0.17   | 1      | 0.5    |
| Shape3| 0.33   | 2      | 1      |

The maximum eigenvalue is calculated and the corresponding eigenvectors are normalized to obtain the weight of each shape, that is, the degree of our bias towards the three shapes under our given criteria.

### Table 7. Our bias towards the three shapes

|       | Shape1 | Shape2 | Shape3 |
|-------|--------|--------|--------|
| Shape1| 0.3524 | 0.2492 | 0.3984 |
In order to check the consistency of the matrix, we calculate the consistency ratio (CR), namely it is defined as the ratio of the consistency index (CI) to the average random consistency index (RI). We get CR is equal to 0.0103 < 0.1. The consistency of the matrix is determined and the model is robust.

4. Conclusion

Therefore, we finally determined the slope of sandcastle based on the basic shape and the influence of rainfall on sandcastle. That is, the platform is the most stable when the slope is 30 degrees. This can be explained in practice. Unlike waves and tides, rainfall essentially increases downward pressure. Compared with the other two sandcastles, the sandcastle with a circular slope Angle of 30 degrees is significantly less prone to collapse and deformation.

References

[1] T. Meng and T. C. Wang, Analysis of tsunami’s effect on structure and study of engineering prevention and control methods, CNKI, 2015.
[2] Z. S. Liu and Y. P. Yao, Finite element method for stability analysis of loess slop, CNKI, 2006.
[3] Y. Liu and B. Shi, Study on sediment transport characteristics under wave action, IEEE Trans, 2012.
[4] N. Fraysse, H. Thome and L. Petit. Humidity effects on the stability of a sandpile. Springer-Verla, 1999.
[5] David M. Heyes. Modelling and measuring of cohesion in wet granular materials, ELSEVIER, 2003.