Smooth Hybrid Inflation and Non-Thermal Type II Leptogenesis

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We consider a smooth hybrid inflation scenario based on a supersymmetric \(SU(2)_L \times SU(2)_R \times U(1)_{B-L}\) model. The Higgs triplets involved in the model play a key role in inflation as well as in explaining the observed baryon asymmetry of the universe. We show that the baryon asymmetry can originate via non-thermal triplet leptogenesis from the decay of \(SU(2)_L\) triplets, whose tiny vacuum expectation values also provide masses for the light neutrinos.

I. INTRODUCTION

There exists an attractive class of supersymmetric models in which inflation is closely linked to the supersymmetric grand unification scale \([1,4]\). Among these models, supersymmetric hybrid inflation (with minimal Kähler potential) predicts a scalar spectral index close to 0.985 \([1]\), to be compared with \(n_s = 0.968 \pm 0.014\) presented by WMAP7 \([5]\). Smooth hybrid inflation, a variant of supersymmetric hybrid inflation, yields a spectral index of 0.97 if supergravity effects are ignored. However, inclusion of supergravity corrections with minimal Kähler potential leads to higher values of the spectral index even in this case \([6]\). It has been shown in \([7,8]\) that the predicted scalar spectral index in smooth hybrid inflation model is affected if the non-minimal terms in the Kähler potential are switched on, and \(n_s\) close to the WMAP prediction is easily realized. For supersymmetric hybrid inflation with soft terms, it is also possible to reduce \(n_s\) to 0.968 \([9]\).

Inflation in these models is naturally followed by leptogenesis \([10]\). Type I leptogenesis from the decay of right handed neutrinos has been discussed in some details in recent papers \([11]\), where the light neutrino masses are obtained from type I seesaw. Care has to be exercised to ensure that leptogenesis is consistent with constraints that may arise from the observed solar and atmospheric neutrino oscillations \([12]\). Light neutrino masses can also arise from the so-called type II seesaw mechanism \([13]\) in which heavy scalar \(SU(2)_L\) triplets acquire tiny vacuum expectation values (vevs) that can contribute to the masses of the observed neutrinos.

An interplay between type I and type II seesaw in the generation of light neutrino masses \([14]\) is also a possibility (for example, while considering a left-right symmetric model). If the right handed neutrinos all have superheavy masses comparable to \(M_{\text{GUT}} = O(10^{16} \text{ GeV})\) or close to it, the type I seesaw contribution to neutrino masses alone would be too much small to be compatible with the neutrino oscillation data. A situation similar to this is adopted in this paper where the triplet vev is the main source of light neutrino masses. It is well known that these triplet scalars can play an additional important role by producing the desired lepton asymmetry \([15,16]\). They could be present in the early universe from the decay of the inflaton, and their own subsequent decay can lead to leptogenesis.

We implement this scenario (type II leptogenesis with smooth hybrid inflation) within a supersymmetric version of the well known gauge symmetry \(SU(2)_L \times SU(2)_R \times U(1)_{B-L}\) \([17]\). (Generalizations to other (possibly larger) gauge symmetries seems quite plausible.) We restrict our attention to non-thermal leptogenesis which is quite natural within an inflationary setting. (For type II thermal leptogenesis see...
II. HIGGS TRIPLETs IN LEFT-RIGHT MODEL

The quark and lepton superfields have the following transformation properties under the gauge group $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ \cite{17}:

\[ Q = (3, 2, 1, \frac{1}{3}), \quad Q^c = (3^*, 1, 2, -\frac{1}{3}), \quad L = (1, 2, 1, -1), \quad L^c = (1, 1, 2, 1). \]

The Higgs sector consists of

\[ H = (1, 2, 2, 0), \quad \Delta_0^a_L = (1, 3, 1, 2), \quad \Delta_0^a_L = (1, 3, 1, -2), \quad a = 1, 2, \]
\[ \Delta_R = (1, 1, 3, -2), \quad \Delta_R = (1, 1, 3, 2). \]

Our primary goal, as stated earlier, is the implementation of non-thermal type II leptogenesis, and to realize it we consider two pairs of triplets $\Delta_L, \Delta_R$ (indicated by index $a = 1, 2$) which, through mixing, can produce the CP violation necessary for generating an initial lepton asymmetry \cite{27}. The model also possesses a gauge singlet superfield $S$ which plays a vital role in inflation.

The superpotential is given by:

\begin{equation}
W = S \left[ \frac{(\Delta_R \Delta_R^c)^2}{M_S^2} - M_X^2 \right] + \frac{\alpha_{ab}}{M_S} \Delta_0^a_L \Delta_0^b_L \Delta_R + \frac{\gamma}{M_S} H H \Delta_0^c_L \Delta_R + f_{L_1 L_1} \Delta_R + Y^i L^c i L^c \Delta_0^a_L + Y^a Q Q L \Delta_0^a_L, \end{equation}

where $a, b = 1, 2$, and the $SU(2)_c$ generation and color indices are suppressed. $M_X$ is a superheavy mass scale and $M_S$ is the cutoff scale which controls the non-renormalizable terms in the superpotential. We take the matrix $\alpha_{ab}$ to be real and diagonal ($\alpha_{ab} = \delta_{ab} \alpha_a$) in our calculation for simplicity. The first two terms (in the square bracket) are responsible for inflation. The importance of the remaining terms will be discussed later in connection with the inflaton decay, reheating, leptogenesis and neutrino mass generation. A $Z_2$ symmetry along with $U(1)_R$ global symmetry is imposed in order to realize the above superpotential. The charges of all the superfields are listed in Table I. The inclusion of the $Z_2$ symmetry forbids terms like $\Delta_0^a L \Delta_0^b L$ in the superpotential, but allows the term $\Delta_0^a L \Delta_0^b L \Delta_R$. This ensures that the $SU(2)_L$ triplets are lighter than the superheavy right handed neutrinos. Apart from its importance in realizing inflation (would be discussed in the next section), the global $R$-symmetry plays another important role in our analysis. Its unbroken $Z_2$ subgroup acts as `matter parity', which implies a stable LSP, thereby making it a plausible candidate for dark matter. We see from Table I that baryon number violating superpotential couplings $QQQ, Q_e Q_e Q_e$ and $QQQL$ are forbidden by the $U(1)_R$ symmetry. This also holds for the higher dimensional operators, so that the proton is essentially stable \cite{22}.

| Charges | $S$ | $\Delta_0^a_L$ | $\Delta_0^b_L$ | $\Delta_R$ | $H$ | $L_1$ | $Q$ | $Q_e$ |
|---------|----|----------------|----------------|------------|-----|-------|-----|-------|
| $R$     | 1  | 2              | 2              | 0          | 0   | 1     | 0   | 1     |
| $Z_2$   | 1  | 1              | -1             | 1          | -1  | 1     | 1   | 1     |

**Table I:** $R$ and $Z_2$ charges of superfields.
III. SMOOTH HYBRID INFLATION

The superpotential term responsible for inflation is given by

$$W_{inf} = S \left[ \frac{(\Delta R \Delta_R)^2}{M_S^2} - M_X^2 \right].$$  \hfill (2)

Note that under $U(1)_R$, $S$ carries the same charge as $W$ and therefore guarantees the linearity of the superpotential in $S$ to all orders (thus excluding terms like $S^2$ which could ruin inflation \cite{1}). The scalar potential derived from $W_{inf}$ is

$$V_{inf} = \left\{ \frac{(\Delta R \Delta_R)^2}{M_S^2} - M_X^2 \right\}^2 + 4 |S|^2 \frac{|\Delta_R|^2 |\Delta_R|^2}{M_S^4} (|\Delta_R|^2 + |\Delta_R|^2) + D \text{ terms.}$$  \hfill (3)

Using the $D$-flatness condition $|\langle \Delta_R \rangle| = |\langle \Delta_R \rangle|$, we see that the supersymmetric vacuum lies at $M = |\langle \Delta_R \rangle| = |\langle \Delta_R \rangle| = \sqrt{MS} M_S$ and $\langle S \rangle = 0$. Defining $\zeta/2 = |\Delta_R^0| = |\Delta_R^0|$ and $\sigma/\sqrt{2} = |S|$, one can rewrite the scalar potential as \cite{20, 21}

$$V_{inf} = \left\{ \frac{\zeta^4}{16 M_S^2} - M_X^2 \right\}^2 + \frac{\sigma^2 \zeta^6}{16 M_S^2}. \hfill (4)$$

The importance of this potential in the context of inflation is discussed in \cite{21}. Here we can briefly summarize it. Although $\zeta = 0$ is a flat direction, it is actually a point of inflection with respect to any value of $\sigma$. It also possesses two (symmetric) valleys of local minima (containing the supersymmetric vacua) which are suitable for inflation. Unlike ‘regular’ supersymmetric hybrid inflation, the inclination of these valleys is already non-zero at the classical level and the end of inflation is smooth.

If we set $M = M_{GUT} = 2.86 \times 10^{16} \text{ GeV}$, and substitute in the expression for the quadrupole anisotropy, $(\delta T/T)_Q$, we find $M_X \simeq 1.8 \times 10^{15} \text{ GeV}$ and $M_S \simeq 4.6 \times 10^{17} \text{ GeV}$ \cite{6}. Here we have employed WMAP7 \cite{5}, measurement of the amplitude of curvature perturbation $(\Delta_R)$ and set the number of e-foldings $N_Q \simeq 57$. The value of $\sigma$ is $1.3 \times 10^{17} \text{ GeV}$ at the end of inflation (corresponding to the slow roll violating parameter, $\eta = \frac{M_S^2 V''}{8 e V} = -1$), and it is $2.7 \times 10^{17} \text{ GeV} \langle \sigma_Q \rangle$ at the horizon exit. The spectral index is estimated to be $n_s \simeq 0.97$ (without supergravity corrections), close to the value of $n_s$ from WMAP.

Note that the supergravity corrections are important and this is studied in \cite{4}. Once these are included (with minimal Kähler potential), $n_s$ approaches unity (for $M \gtrsim 1.5 \times 10^{16} \text{ GeV}$) \cite{6}. By lowering the scale $M$ compared to the $M_{GUT}$, one can achieve $n_s$ in the acceptable range. However, in this case the inflaton field-value $\sigma_Q$ would be larger than the cutoff scale $M_S$ providing a threat to the effective field theory concept.

If we employ a non-minimal Kähler potential

$$K = |S|^2 + |\Delta_R|^2 + |\Delta_R|^2 + \frac{\kappa_S |S|^4}{4 M_p^2},$$  \hfill (5)

then along the D-flat direction $|\Delta_R| = |\Delta_R|$, the inflationary potential for $\sigma^2 \gg M^2$ is given by,

$$V = M_X^4 \left[ 1 - \kappa_S \frac{\sigma^2}{2 M_p^2} + \left( 1 - \frac{7}{2} \kappa_S + 2 \kappa_S^2 \right) \frac{\sigma^4}{8 M_p^4} - \frac{2}{27} M^4 \right].$$  \hfill (6)

The spectral index calculated from this potential is in the desired range $(0.968 \pm 0.014)$ for different choices of $\kappa_S$. An analysis of this case is extensively studied in \cite{8}. We have tabulated sets of values of $M, M_S, \sigma_Q$ in Table II corresponding to different choices of $\kappa_S$ with different predictions for the spectral index (for more examples, see Figs. 7 and 8 of \cite{8}). With non minimal Kähler terms included, there arises the possibility of having observable tensor to scalar ratio $r$, a canonical measure of gravity waves produced during inflation \cite{22}. 

TABLE II: For a given value of $\kappa_S$, the predicted values of the spectral index ($n_s$), the gauge symmetry breaking scale ($M$), the cutoff scale ($M_S$), and the inflaton field at the time of horizon exit ($\sigma_Q$) are presented.

| Set | $\kappa_S$ | $n_s$ | $M$ (GeV) | $M_S$ (GeV) | $\sigma_Q$ (GeV) |
|-----|------------|-------|-----------|-------------|-----------------|
| I   | 0.005      | 0.005 | $2.2 \times 10^{16}$ | $5.5 \times 10^{17}$ | $2.1 \times 10^{17}$ |
| II  | 0.01       | 0.968 | $4 \times 10^{15}$   | $1.5 \times 10^{15}$ | $3 \times 10^{17}$ |

IV. REHEATING

Let us now discuss inflaton decay and reheating. The inflaton field(s) smoothly enter an era of damped oscillation about the supersymmetric vacuum. The oscillating system consists of two scalar fields $S$ and $\theta = (\delta \theta + i \tilde{\theta})/\sqrt{2}$ with $\theta = \Delta_R - M$ and $\delta \theta = \Delta_L - M$ with a common mass $m_{inf} = 2\sqrt{2}M_{MS}/M_S$, which decay into a pair of left triplets ($\Delta_L^a, \tilde{\Delta}_L^a$) and their fermionic partners ($\tilde{\Delta}_R^a, \Delta_R^a$) respectively through the Lagrangian [see Eq.(2)]

$$L^s = \sqrt{2}a \frac{M}{M_S} m_{inf} S^* \Delta_L^a + h.c., \quad L^\theta = \sqrt{2}a \frac{M}{M_S} \theta \tilde{\Delta}_L^a + h.c..$$ (7)

The decay widths of both $S$ and $\theta$ turn out to be

$$\Gamma_{inf} = \frac{3}{4\pi} a^2 \left( \frac{M}{M_S} \right)^2 m_{inf} = \frac{3}{4\pi} \left( \frac{M_a}{M} \right)^2 m_{inf},$$ (8)

where $M_a$ is the mass of the $SU(2)_L$ triplet given by $M_a = a \frac{M^2}{M_S}$ (generated via the non-renormalizable superpotential coupling $\tilde{\Delta}_L^a \Delta_L^a \Delta_R^a$, after $\Delta_R, \tilde{\Delta}_R$ acquire vevs). For this decay to be kinematically allowed, $a \lesssim \sqrt{\frac{2M}{M_S}}$. The splitting between $M_1$ and $M_2$ (i.e. between $\alpha_1$ and $\alpha_2$) will be important in estimating the lepton asymmetry. The decay of inflaton into right-handed neutrinos is kinematically forbidden since the latter have superheavy mass acquired from the renormalizable coupling $f_2 L_L L_R \Delta_R$, with $f_2$ of order unity.

The reheating temperature from the decay of the inflaton is $T_R \approx \frac{1}{4} (\sqrt{\Gamma M_P}$, where $\Gamma$ represents the total decay width of the inflaton (here it is $\Gamma_{inf}$), where $M_P = 2.4 \times 10^{18}$ GeV is the reduced Planck scale. Using the first set of values for $M, M_S$ specified in Table II, one finds

$$T_R \approx 0.12 \times \alpha \left( \frac{M}{M_S} \right)^2 \sqrt{M M_P} \text{ GeV},$$ (9)

where $\alpha = \sqrt{\alpha_1^2 + \alpha_2^2}$. With the parameters involved in Table II (set II and III), we find $M/M_S \sim O(10^{-2})$. Hence the reheating temperature is $T_R \sim O(10^{10-11})$ GeV, where the constraint on $\alpha_a$ is taken into account ($\alpha \sim O(10^{-3})$). Note that such a reheating temperature does not pose any threat if the graviton is sufficiently heavy [24]. Therefore we conclude from the above discussion that at the end of inflation, the inflaton system has decayed away into $SU(2)_L$ triplets. We will show in the next section that the subsequent decay of these $SU(2)_L$ triplets creates a lepton asymmetry, which is partially converted into the observed baryon asymmetry via the electroweak sphaleron effects [25].

V. TYPE II NON-THERMAL LEPTOGENESIS AND NEUTRINO MASSES

In general both the right-handed neutrinos as well as the left-handed triplets can yield a lepton asymmetry in left-right models [18]. However, in our case with superheavy ($M \sim 10^{10}$ GeV) right handed
neutrinos, the Leptogenesis would come mainly from the $SU(2)_L$ triplets. Note that we have considered two pairs of $SU(2)_L$ superfields, so that the CP asymmetry would be nonzero.

The experimental value of the baryon to photon ratio is given by [5]

$$\frac{n_B}{n_\gamma} \simeq (6.5 \pm 0.4) \times 10^{-10}. \quad (10)$$

In this respect, the required lepton asymmetry is estimated to be [7]

$$\left| \frac{n_L}{s} \right| \simeq (2.67 - 3.02) \times 10^{-10}. \quad (11)$$

To estimate the lepton asymmetry we follow the analysis of ref [16]. The Higgs triplet $\Delta_L^a$ decays into $LL$ and $HH$ (see Fig. 1(a)), while $\bar{\Delta}_L^a$ decays into $\tilde{L}\tilde{L}$ and $\tilde{H}\tilde{H}$. The amount of CP violation in these decays is controlled by the interference of the tree level process with one-loop diagram (see Fig. 1(b)) as described in [16].

The effective mass-squared matrix of the scalar triplets $\Delta_L^a$ and $\bar{\Delta}_L^a$ is [16], $\Delta_L^{a\dagger} M^2 \Delta_L^a + \bar{\Delta}_L^{a\dagger} (M'^2)_{ab} \bar{\Delta}_L^b$, where

$$M^2 = \begin{pmatrix} M_1^2 - i\Gamma_{11} M_1 & -i\Gamma_{12} M_2 \\ -i\Gamma_{21} M_1 & M_2^2 - i\Gamma_{22} M_2 \end{pmatrix},$$

and $M'^2$ has a similar pattern with $\Gamma_{ab}$ replaced by $\Gamma'_{ab}$. The contributions to $\Gamma_{ab}$ ($\Gamma'_{ab}$) come from the absorptive part of the one loop self-energy diagrams for $\Delta_L^a \to \Delta_L^b$ ($\bar{\Delta}_L^a \to \bar{\Delta}_L^b$),

$$\Gamma_{ab} M_b = \frac{1}{8\pi} \Sigma_{ij} (f_{iij}^a f_{j11}^b) \rho_{\Delta_L} + M_a M_b g^a g^b,$$

$$\Gamma'_{ab} M_b = \frac{1}{8\pi} \Sigma_{ij} (f_{iij}^a f_{j11}^b) M_a M_b + p_{\Delta_L}^2 g^a g^b,$$

where $i, j$ are generation indices, $g^a = \gamma^a \left( \frac{M_1}{M_2} \right)$ and $p_{\Delta_L}^2$ is the momentum squared of the incoming or outgoing particle. The physical states $\chi_{1,2}^\pm$, $\xi_{1,2}^\pm$ (with masses $\sim M_{1,2}$) can be obtained [28] by diagonalizing $M^2$, $M'^2$. Here we neglect terms of order $[\Gamma_{12} M_1 M_2].$

The CP asymmetries are then defined by

$$\epsilon^a = \frac{\Delta L \Gamma(\chi^a \to l\ell) - \Gamma(\chi^{a+} \to l^{-}\ell^c)}{\Gamma_{\chi^a} + \Gamma_{\chi^{a+}}},$$

$$\epsilon = \frac{M_1 M_2}{2\pi (M_1^2 - M_2^2)} \sum_{ij} \text{Im} f_{iij} f_{i11}^2 g^1 g^{2*} \sum_{ij} |f_{i11}^2|^2 + |g^a|^2,$$
The main contribution to the neutrino mass matrix, namely

\[ \epsilon' = \Delta L \frac{\Gamma(\xi^a \rightarrow ll) - \Gamma(\xi^a \rightarrow l\bar{l}c)}{\Gamma_{\xi^a} + \Gamma_{\xi^a}}, \]

\[ = \frac{M_1 M_2 \sum_{ij} |f_{1ij}|^2 g^1 g^2}{2\pi (M_1^2 - M_2^2)} \frac{\sum_{ij} |f_{1ij}|^2 + |g|^2}{\sum_{ij} |f_{1ij}|^2}, \quad (15) \]

where the lepton number violation $\Delta L$ changes by 2 units. We note that $\epsilon^a = \epsilon'^a$.

The lepton asymmetry is given by

\[ \frac{n_L}{s} \approx \frac{3}{2} \frac{T_R}{m_{\text{inf}}} \sum a [\epsilon^a + \epsilon'^a], \]

\[ = \frac{3}{2} \frac{T_R}{m_{\text{inf}}} \frac{3 M_1 M_2}{\pi (M_1^2 - M_2^2)} \frac{\sum_{ij} |f_{1ij}|^2 g^2}{\sum_{ij} |f_{1ij}|^2 + |g|^2}, \quad (16) \]

where the ratio of the number density of the SU(2)$_L$ triplets $(n_\Delta)$ to the entropy density $s$ is expressed as $\frac{3}{2} \frac{T_R}{m_{\text{inf}}}$. Once this asymmetry is created, one should ensure that it is not erased by the lepton-number non-conserving interactions (for example $HH \leftrightarrow \Delta_L \rightarrow LL, \bar{H}\bar{H} \leftrightarrow \Delta_L \rightarrow \bar{L}\bar{L}$). As long as the SU(2)$_L$ triplet masses ($M_a$) are sufficiently larger than $T_R$ (here $\frac{T_R}{M_a} \approx 0.12 \frac{\sqrt{M_P M_1}}{M_S}$) with the specific choice of $M_1, M_2$ as given in Table II (set II and III), there will be no significant wash-out factor, unlike thermal leptogenesis.

To estimate $n_L/s$, we need to fix some parameters appearing in Eq. (16) which are also involved in the light neutrino mass matrix. The neutrino mass matrix is represented by the type II see-saw relation

\[ m_{\nu} = 2 f_{1ij} v^a_{\Delta_L} - m_D^{-1} M_R^{-1} M_D \equiv m_{\nu_{11}} - m_{\nu_{22}}, \quad (17) \]

where $v^a_{\Delta_L}$ are the SU(2)$_L$ triplet Higgs’s vevs. With the masses of all right handed neutrinos comparable to $M_1, m_{\nu_{11}}$ are too small to account for the solar and atmospheric neutrino data. Hence $m_{\nu_{11}}$ provides the main contribution to the neutrino mass matrix, namely

\[ (m_{\nu})_{ij} \approx 2 f_{1ij} \frac{g^a}{M_a} v^2, \quad (18) \]

where $v \approx 174$ GeV. In order to estimate both the lepton asymmetry (Eq. (16)) and neutrino masses (through Eq. (15)), we first simplify by assuming $|g^1| \approx |g^2| = g, |f_{11}^1| \approx |f_{11}^2| = f_1$ (thus $|\Sigma_{ij} f_{1ij} f_{1ij}^2| = |\Sigma_{ij} f_{1ij}|^2$). Then Eqs. (16) and (15) can be expressed as

\[ \frac{n_L}{s} \approx \frac{9}{\pi} \frac{T_R}{m_{\text{inf}}} \frac{M_1 M_2}{M_1^2 - M_2^2} \frac{\sum_{ij} |f_{1ij}|^2 g^2}{\sum_{ij} |f_{1ij}|^2 + g^2}, \]

\[ \approx \frac{0.374}{\pi} \frac{\sqrt{M_P}}{M} \frac{M_1 M_2}{M_1^2 - M_2^2} \frac{\sum_{ij} |f_{1ij}|^2 g^2}{\sum_{ij} |f_{1ij}|^2 + g^2}, \quad (19) \]

\[ (m_{\nu})_{ij} \approx 2 f_{1ij} g v^2 \left( \frac{1}{M_1} + \frac{1}{M_2} \right), \quad (20) \]

where we have substituted for $T_R$ and $M_a$ and assumed the CP violating phase to be maximal.

The neutrino mass matrix $m_{\nu}$ can be diagonalized by

\[ m_{\nu} = U^*_\nu m_{\nu}^{\text{diag}} U^\dagger_{\nu}, \quad (21) \]

where $m_{\nu}^{\text{diag}} = \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})$. In the basis where the charged lepton matrix is diagonal, $U_{\nu}$ coincides with the lepton mixing matrix. Using Eqs. (20), we get

\[ \frac{n_L}{s} \approx \frac{0.374}{\pi} \frac{p}{1 - p^2} \sqrt{\frac{M_P}{M_1 M_2 M_3}} \frac{1}{M_1 M_2 M_3} \sum_{ij} |m_{\nu_{ij}}|^2 Fg^2 \frac{M_1 M_2 M_3}{\sum_{ij} |m_{\nu_{ij}}|^2 F + g^2}, \quad (22) \]
where \( F = \frac{M_1^2 M_2^2}{4\alpha^2 (M_1 + M_2)^2} = \frac{p^2}{(1+p)^4} \times \frac{M_2^2}{26}. \) Here \( p \) determines the degree of degeneracy between \( M_1 \) and \( M_2 \), defined by \( M_2 = pM_1 \). Since the parameter \( g \) is defined as \( g^a = \gamma^a \frac{M}{M_S} \), its maximum value is of order \( \frac{M}{M_S} \). Finally, using the current experimental limits for neutrino masses \([26]\), one finds that \( \Sigma_{ij} |m_{\nu_{ij}}|^2 \) is given by \( \Sigma_{ij} |m_{\nu_{ij}}|^2 \simeq 0.0025 \text{ eV}^2 \), where we have used the best fitted values of the neutrino mixing angles \( \theta_{12}, \theta_{23}, \theta_{13} \) and mass squared differences \([26]\). We have taken the lightest neutrino mass eigenvalue to be zero. In Fig. 2 we present the lepton asymmetry as a function of \( p \) and \( g \) with \( \alpha_1 = 10^{-3} \). We see that \( n_L/s \) can be of order the desired value \((2-3) \times 10^{-10} \) for \( 0.2 \lesssim p \lesssim 0.8 \) and \( g \gtrsim 2.5 \times 10^{-4} \), which means \( \gamma^a \simeq O(0.01) \). It is worth mentioning that with these values one finds that \( M_1 \) and \( M_2 \) are given by \( M_1 \simeq 10^{12} \text{ GeV} \) and \( M_2 \simeq (2-8) \times 10^{11} \text{ GeV} \). Therefore, \( M_{1,2}/T_R > 10 \), which indicates that no washout should happen.

VI. CONCLUSIONS

We have considered type II non-thermal leptogenesis in the context of smooth hybrid inflation. The scheme is consistent with the observed solar and atmospheric neutrino oscillations. Although our discussion is based on the gauge symmetry \( SU(2)_L \times SU(2)_R \times U(1)_{B-L} \), it is clear that it could be extended to other models which contain suitable \( SU(2)_L \) triplet scalars with tiny vevs responsible for the observed neutrino masses. The stability of the proton will depend on the underlying gauge symmetry.

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[27] We do not insist on a completely left-right symmetric Higgs sector. Thus, only one pair of $\Delta_R, \bar{\Delta}_R$ is considered.
[28] The physical states $\chi^{1,2}_\pm (\xi^{1,2}_\pm)$ are also similarly obtained by diagonalizing the matrix in Eq. (12) with $\Gamma_{12}$ replaced by $\Gamma^*_{12}$ and vice versa.