Resonances in radiative hyperon decays

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Abstract

The importance of resonances for the radiative hyperon decays is examined in the framework of chiral perturbation theory. Low lying baryon resonances are included into the effective theory and tree contributions to these decays are calculated. We find significant contributions to both the parity-conserving and parity-violating decay amplitudes and a large negative value for the asymmetry parameter in polarized $\Sigma^+ \rightarrow p\gamma$ is found, in agreement with the experimental result $\alpha_{\Sigma^+}^p = -0.76 \pm 0.08$.

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1 Introduction

The radiative hyperon decays—$\Sigma^+ \to p\gamma$, $\Lambda \to n\gamma$, etc.—have been studied both experimentally and theoretically for over three decades, but still a number of mysteries still exist.\[1\] The primary problem has been and remains to understand the size of the asymmetry parameter in polarized $\Sigma^+ \to p\gamma$ decay\[2\]

$$\alpha_\gamma = -0.76 \pm 0.08$$ (1)

The difficulty here is associated with the restrictions posed by Hara’s theorem, which requires the vanishing of this asymmetry in the SU(3) limit.\[3\] The proof of this theorem is easily given. By gauge invariance the radiative decay amplitudes must have the form

$$\text{Amp}(B \to B'\gamma) = \bar{u}_{B'}(p') \frac{-i}{2(M_B + M_{B'})} \sigma_{\mu\nu} F^{\mu\nu}(A_{B'B} + B_{B'B}\gamma_5)u_B(p)$$ (2)

where $F^{\mu\nu}$ is the electromagnetic field strength tensor, $A_{B'B}$ is the parity conserving M1 amplitude and $B_{B'B}$ is its parity violating E1 counterpart. Now under U-spin, the electromagnetic current is a singlet, while the weak $\Delta S = 1$ Hamiltonian acts like the $\Delta U_3 = -1$ component of a U-spin vector. Thus the effective current for this transition transforms like a CP-even U-spin lowering operator. This is completely analogous to the situation in nuclear beta decay involving the isospin operator, and the same arguments which require the vanishing of tensor matrix element—$-i\sigma_{\mu\nu}\gamma_5 F^{\mu\nu}$—for the axial current between isospin analog states such as $n, p$\[4\] guarantee the vanishing of the E1 radiative hyperon decay amplitude between states such as $\Sigma^+, p$ which are members of a common U-spin multiplet.\[4\] Since the asymmetry parameter is related to the decay amplitudes via

$$\alpha_\gamma = \frac{2\text{Re}A_{B'B}B_{B'B}^*}{|A_{B'B}|^2 + |B_{B'B}|^2}$$ (3)

the vanishing of the E1 amplitude also guarantees the vanishing of the asymmetry in contradiction to the near maximal number—Eq. [1]—measured experimentally.

Of course, in the real world U-spin is broken and one should not be surprised to find a nonzero value for the asymmetry—what is difficult to understand is its size. Indeed theorists have modelled this decay for nearly three decades and a definitive explanation for the $\Sigma^+ \to p\gamma$ asymmetry remains lacking. This is certainly not for lack of trying—indeed many ideas have been pursued.\[1\] Early approaches utilized simple baryon pole models with weak parity conserving $BB'$ matrix elements determined from fits to ordinary hyperon decay.\[5\] The SU(3) breaking in this case comes from the difference between experimental $\Sigma^+$ and proton magnetic moments, which leads to asymmetry estimates in the 10% range. Corresponding parity violating $BB'$ matrix elements must vanish in the SU(3) limit via the Lee-Swift theorem.\[6\] However, SU(3) breaking leads to a nonvanishing asymmetry again only at the $\sim 10\%$ level.\[6\] Recent work involving the calculation of chiral loops has also not lead to a resolution, although slightly larger asymmetries can be accommodated.\[8\] In addition, over the years there have been claims that Hara’s theorem is not to be trusted.\[9\] However, these have proven to be incorrect.\[10\]

An exception to this pattern is the work of LeYouanc et al., who have argued that inclusion of the $(70, 1^-)$ intermediate states can lead to a simultaneous resolution of the s/p-wave problem.\[3\] Note that the same argument would apply to the transition $\Xi^- \to \Sigma^-\gamma$. However, there is presently no experimental information on the asymmetry for this decay.
in ordinary hyperon decay as well as the asymmetry problem in radiative hyperon decay. In a previous work we have studied this approach within a chiral framework and have shown that it is indeed possible to find a simultaneous fit to both s- and p-wave hyperon decay amplitudes if contributions from the lowest lying $^{1/2}^{-}$ and $^{1/2}^{+}$ baryon octet resonant states are included in the formalism. In the present paper we extend this discussion to radiative hyperon decay.

In the next section then we introduce the effective weak and strong-electromagnetic Lagrangians including resonant states and we evaluate the pole diagram contributions of both ground state baryons and resonant states to radiative hyperon decay. Numerical results are presented in Sec. 3, while in Sec. 4 we conclude with a brief summary. In the Appendix, we determine the strong couplings of the resonances to the ground state octet by a fit to the electromagnetic decays of these resonances.

## 2 Radiative hyperon decays

There exist six weak radiative hyperon decays: $\Sigma^+ \rightarrow p\gamma$, $\Xi^- \rightarrow \Sigma^-\gamma$, $\Sigma^0 \rightarrow n\gamma$, $\Lambda \rightarrow n\gamma$, $\Xi^0 \rightarrow \Sigma^0\gamma$, $\Xi^0 \rightarrow \Lambda\gamma$ which can be studied experimentally, and each can be represented in terms of a parity-conserving and a parity violating matrix element as in Eq. (2). The aim of the present work is to calculate the amplitudes $A_{B'B}$ and $B_{B'B}$ within the framework of chiral perturbation theory and, to this end, we consider first the Lagrangian without resonances, which will be included in the following section. It can be decomposed into a strong-electromagnetic and a weak part. The former reads

$$L^s_B = i \text{tr}(\bar{B}\gamma_\mu[D^\mu, B]) - \hat{M} \text{tr}(\bar{B}B) + l_d \text{tr}(\bar{B}\sigma_{\mu\nu}\{f_{+}^{\mu\nu}, B\}) + l_f \text{tr}(\bar{B}\sigma_{\mu\nu}[f_{+}^{\mu\nu}, B]) ,$$

where $f_{+}^{\mu\nu}$ is the chiral field strength tensor of the electromagnetic field and $\hat{M}$ represents the mass of the baryon octet in the chiral limit. The coupling constants $l_d$ and $l_f$—so-called low-energy-constants (LECs)—can be determined from a fit to the baryon magnetic moments, cf. App. A, and are the only counterterms contributing to the radiative hyperon decays up to second chiral order. (Note, that at next chiral order, there is an additional term possible which is proportional to $B\gamma_\mu[D_\nu, f_{+}^{\mu\nu}]B$ but this vanishes for real photons.)

We now turn to the weak piece of the meson-baryon Lagrangian, whose lowest order form is

$$L^w_{\phi B} = d \text{tr}(\bar{B}\{h_+, B\}) + f \text{tr}(\bar{B}[h_+, B]) .$$

In our previous work the LECs $d$ and $f$ have been determined from the nonleptonic hyperon decays by two independent means. In a calculation was performed which included all terms at one-loop order. This work suffered from the fact, however, that at this order too many new unknown LECs enter the calculation so that the theory lacks predictive power. In order to proceed these parameters were estimated by means of spin-3/2 decuplet resonance exchange. The results for the $p$-waves were still in disagreement with the data and, therefore, additional counterterms that were not saturated by the decuplet had to be taken into account, leading to the possibility of an exact fit to the data. In a second approach we included lowest lying spin-$1/2^+$ and $1/2^-$ resonances in the theory and performed a tree level calculation. Integrating out the heavy degrees of freedom provides then a plausible estimate of the weak counterterms,
which have been neglected completely in [13] and [10]. A satisfactory fit for both s- and p-waves was achieved. Since we herein apply this scheme for the weak radiative hyperon decays we will use the values for $d$ and $f$ from [13].

Having introduced the Lagrangian for the ground state baryons, we can then proceed by including the low lying resonances. In [11, 12] it was argued that in a simple constituent quark model including the lowest lying spin $1/2^-$ octet from the $(70,1^−)$ multiplet leads to significant improvements in both radiative and nonleptonic hyperon decays. We confirmed in a recent calculation [13] that indeed exist significant contributions from these resonances for the nonleptonic hyperon decays in the framework of chiral perturbation theory. We begin therefore with the inclusion of the octet of spin-parity $1/2^−$ states, which include the well-established states $N(1535)$ and $Λ(1405)$. As for the rest of the predicted $1/2^−$ states there are a number of not so well-established states in the same mass range—cf. [11] and references therein. In order to include resonances one begins by writing down the most general Lagrangian at lowest order which exhibits the same symmetries as the underlying theory, i.e. Lorentz invariance and chiral symmetry. For the strong part we require invariance under $C$ and $P$ transformations separately, while the weak piece is invariant under $CP$ transformations where the transformation $S$ interchanges down and strange quarks in the Lagrangian. We will work in the $CP$-conserving limit so that all LECs are real, and denote the $1/2^−$ octet by $R$.

Under $CP$ transformations the fields behave as

\[ B \rightarrow \gamma_0 C B^T, \quad B \rightarrow B^T C \gamma_0, \quad f^{\mu\nu}_+ \rightarrow -f^{\mu\nu+}_T, \]

\[ h_+ \rightarrow h^T_+, \quad D^\mu \rightarrow -D^T_\mu, \]

\[ R \rightarrow -\gamma_0 C R^T, \quad \bar{R} \rightarrow -R^T C \gamma_0, \]

where $C$ is the usual charge conjugation matrix. The kinetic term of the $1/2^−$ Lagrangian is straightforward

\[ \mathcal{L}^\text{kin}_R = i \text{tr}(\bar{R} \gamma_\mu [D^\mu, R]) - M_R \text{tr}(\bar{R} R) \quad (7) \]

with $M_R$ being the mass of the resonance octet in the chiral limit. The resonances $R$ couple electromagnetically to the $1/2^+$ baryon octet $B$ via the Lagrangian

\[ \mathcal{L}^*_{RB} = ir_d \left[ \text{tr}(\bar{R} \sigma_{\mu\nu} \gamma_5 \{f^{\mu\nu}_+, B\}) + \text{tr}(\bar{B} \sigma_{\mu\nu} \gamma_5 \{f^{\mu\nu}_+, R\}) \right]
+ ir_f \left[ \text{tr}(\bar{R} \sigma_{\mu\nu} \gamma_5 \{f^{\mu\nu}_+, B\}) + \text{tr}(\bar{B} \sigma_{\mu\nu} \gamma_5 \{f^{\mu\nu}_+, R\}) \right], \quad (8) \]

and the couplings $r_d$ and $r_f$ can be determined from a fit to the electromagnetic decays of the resonances—cf. App. A. The corresponding weak Lagrangian is

\[ \mathcal{L}^W_{RB} = i w_d \left[ \text{tr}(\bar{R} h_+, B) - \text{tr}(\bar{B} h_+, R) \right] + i w_f \left[ \text{tr}(\bar{R} h_+, B) - \text{tr}(\bar{B} h_+, R) \right]. \quad (9) \]

with two couplings $w_d$ and $w_f$ which have been determined from a fit to the nonleptonic hyperon decays in [13].

We will not include additional resonances from the $(70,1^−)$ states, which were the only resonances considered in [11, 12]. But in many applications the spin-$3/2^+$ decuplet and the Roper-like spin-$1/2^+$ octet states play an important role, cf. e.g. [17]. The decuplet is only $231$ MeV higher in average than the ground state octet and the Roper octet masses are comparable to the $1/2^−$ states $R$. One should therefore also account for these resonances.
Considering first the decuplet, due to angular momentum conservation the spin-3/2 decuplet states can couple to the spin-1/2 baryon octet only together with Goldstone bosons or photons. Therefore, intermediate decuplet states can contribute only through loop diagrams to radiative hyperon decay. Such loop diagrams saturate contact terms of the same chiral order as the loop corrections with the baryon octet. Since in this work we restrict ourselves to lower chiral orders we can disregard such decuplet contributions. Their effect begins only at higher chiral orders, which we have neglected from the beginning. In addition, the calculation of relativistic loop diagrams in the resonance saturation scheme leads to some complications. The integrals are in general divergent and have to be renormalized which introduces new unknown parameters. The absence of a strict chiral counting scheme in the relativistic formulation leads to contributions from higher loop diagrams which are usually neglected in such calculations, cf. [17].

Another important multiplet of excited states is the octet of Roper-like spin-1/2 fields. While it was argued in [18] that these play no role, a more recent study seems to indicate that one cannot neglect contributions from such states to, e.g., decuplet magnetic moments. It is thus important to investigate also the possible contribution of these baryon resonances to the LECs. The Roper octet, which we denote by \( B^* \), consists of the \( N^*(1440) \), the \( \Sigma^*(1660) \), the \( \Lambda^*(1600) \) and the \( \Xi^*(1620?) \). The transformation properties of \( B^* \) under \( CP \) are the same as for the ground state baryons \( B \), and the effective Lagrangian of the \( B^* \) octet coupled to the ground state baryons takes the form

\[
\mathcal{L}_{B^*B} = \mathcal{L}_{B^*B}^{\text{kin}} + \mathcal{L}_{B^*B}^{S} + \mathcal{L}_{B^*B}^{W} \tag{10}
\]

with the kinetic term

\[
\mathcal{L}_{B^*B}^{\text{kin}} = i \text{tr} \left( \bar{B}^* \gamma_{\mu} [D^\mu, B^*] \right) - M_{B^*} \text{tr} \left( \bar{B}^* B^* \right) \tag{11}
\]

(with \( M_{B^*} \) being the resonance mass in the chiral limit), an electromagnetic interaction part

\[
\mathcal{L}_{RB}^S = \sum_{i} \left[ \text{tr} \left( \bar{R} \sigma_{\mu \nu} [f_{\mu \nu}^i, B^*] \right) + \text{tr} \left( \bar{B} \sigma_{\mu \nu} [f_{\mu \nu}^i, R^*] \right) \right]
\]

\[
+ \sum_{i} \left[ \text{tr} \left( \bar{R} \sigma_{\mu \nu} [f_{\mu \nu}^i, B] \right) + \text{tr} \left( \bar{B} \sigma_{\mu \nu} [f_{\mu \nu}^i, R] \right) \right] \tag{12}
\]

and a weak piece

\[
\mathcal{L}_{B^*B}^W = \sum_{i} \left[ \text{tr} \left( \bar{B}^* [h^+_i, B^*] \right) + \text{tr} \left( \bar{B} [h^+_i, B^*] \right) \right] + \sum_{i} \left[ \text{tr} \left( \bar{B}^* [h^+_i, B] \right) + \text{tr} \left( \bar{B} [h^+_i, B^*] \right) \right] \tag{13}
\]

As in the case of their 1/2− counterparts, the coupling constants \( f^*, d^* \) have been determined from nonleptonic hyperon decay in [13], while the electromagnetic couplings \( l^*, l_{\gamma}^* \) are found from radiative decays of the resonances—(cf. App. A). There exist no additional unknown parameters in this approach once the weak couplings are fixed from the nonleptonic decays. Study of radiative hyperon decay provides, therefore, a nontrivial check on whether the results from the simple quark model are consistent with chiral perturbation theory.

### 2.1 Ground state contributions

We begin by considering the diagrams which include only ground state baryons, as depicted in Fig. 1. The relevant Lagrangians are given in Eqs. (4) and (5). The photon couples not only via the field strength tensor but also through the covariant derivative \( D_\mu \)

\[
[D_\mu, B] = \partial_\mu B + [\Gamma_\mu, B] \tag{14}
\]
Here the chiral connection
\[
\Gamma_\mu = -i v_\mu + \ldots = -i e Q A_\mu + \ldots
\]  
contains the external photon field \( A_\mu \) and the ellipses denote terms that do not contribute in our calculation, while the field strength tensor reads
\[
f_+^{\mu\nu} = 2(\partial^\mu v^\nu - \partial^\nu v^\mu) + \ldots
\]  
The explicit calculation reveals that the contributions to the decays from the chiral connection cancel if one uses the physical mass of the internal baryon for the propagator, which is consistent to the order we are working. Consequently, the only contribution to the radiative hyperon decays stems from the terms with the couplings magnetic \( l_d \) and \( l_f \). Such pole diagrams with intermediate ground state baryons contribute only to the parity conserving amplitudes \( A^{ji} \), yielding\(^4\)

\[
\begin{align*}
A^{n\Sigma^0} &= e \frac{8\sqrt{2}}{M_\Sigma - M_N} \frac{8\sqrt{2}}{3} \, d \ell_d \\
A^{n\Lambda} &= e \frac{8\sqrt{2}}{M_\Lambda - M_N} \frac{8\sqrt{2}}{3\sqrt{3}} \, d \ell_d \\
A^{\Sigma^0\Xi^0} &= e \frac{8\sqrt{2}}{M_\Sigma - M_\Xi} \frac{8\sqrt{2}}{3} \, d \ell_d \\
A^{\Lambda\Xi^0} &= e \frac{8\sqrt{2}}{M_\Lambda - M_\Xi} \frac{8\sqrt{2}}{3\sqrt{3}} \, d \ell_d \\
A^{p\Sigma^+} &= A^{n\Xi^-} = 0
\end{align*}
\]  

where we work in the limit of identical \( u \)- and \( d \)-quark masses and employ the physical masses for the internal baryons. If one were to include only ground state baryons in the effective theory and neglect resonances then, there would be no additional contributions to the amplitudes at tree level. This would lead to a vanishing asymmetry parameter for any of the radiative hyperon decays and in particular for the decay \( \Sigma^+ \to p\gamma \). The inclusion of meson loops leads to a small contribution for such asymmetry parameters\(^8, 20\) in clear contradiction to the experimental result Eq. (1).

\subsection*{2.2 Resonance contributions}

Any explanation for the large asymmetry then must come from inclusion of additional intermediate states. The diagrams including resonances are shown in Fig. 2. The spin-1/2\(^-\) resonances contribute to the parity violating amplitudes \( B^{ji} \)

\[
\begin{align*}
B^{p\Sigma^+} &= e \frac{4(M_\Sigma - M_N)}{(M_\Sigma - M_R)(M_N - M_R)} \left( \frac{1}{3} r_d + r_f \right) (w_d - w_f) \\
B^{\Sigma^-\Xi^-} &= e \frac{4(M_\Xi - M_\Sigma)}{(M_\Sigma - M_R)(M_\Xi - M_R)} \left( \frac{1}{3} r_d - r_f \right) (w_d + w_f)
\end{align*}
\]  

\(^4\)Corresponding parity violating couplings vanish due to the Lee-Swift theorem.\(^6\)
\[ B^{n}\Sigma^0 = e \frac{4}{(M_R - M_\Sigma)} \frac{\sqrt{2}}{3} r_d(w_d - w_f) + e \frac{4}{(M_R - M_N)} \frac{\sqrt{2}}{3} r_d(w_d + w_f) \ ; \]

\[ B^{n}\Lambda = e \frac{4}{(M_R - M_\Lambda)} \frac{\sqrt{2}}{3} r_d(w_d + 3w_f) + e \frac{4}{(M_R - M_N)} \frac{\sqrt{2}}{3} r_d(w_d - 3w_f) \ ; \]

\[ B^{\Sigma^0 \Xi^0} = e \frac{4}{(M_\Xi - M_R)} \frac{\sqrt{2}}{3} r_d(w_d - w_f) + e \frac{4}{(M_\Sigma - M_R)} \frac{\sqrt{2}}{3} r_d(w_d + w_f) \ ; \]

\[ B^{\Xi^0 \Xi^0} = e \frac{4}{(M_\Xi - M_R)} \frac{\sqrt{2}}{3} r_d(w_d + 3w_f) + e \frac{4}{(M_\Sigma - M_R)} \frac{\sqrt{2}}{3} r_d(w_d - 3w_f) \ . \] (18)

while the octet of spin-1/2\(^+\) resonances contributes to the parity conserving amplitudes \( A^{ji} \)

\[ A^{p\Sigma^+} = e \frac{4}{M_\Sigma - M_{B^*}} (\frac{1}{3} l_d^* l_f^*) (d^* - f^*) + e \frac{4}{M_N - M_{B^*}} (\frac{1}{3} l_d^* l_f^*) (d^* - f^*) \ ; \]

\[ A^{\Sigma^- \Xi^-} = e \frac{4}{M_\Xi - M_{B^*}} (\frac{1}{3} l_d^* l_f^*) (d^* + f^*) + e \frac{4}{M_\Sigma - M_{B^*}} (\frac{1}{3} l_d^* l_f^*) (d^* + f^*) \ ; \]

\[ A^{n\Sigma^0} = e \frac{4}{M_\Sigma - M_{B^*}} \frac{\sqrt{2}}{3} l_d^* (d^* - f^*) + e \frac{4}{M_{B^*} - M_N} \frac{\sqrt{2}}{3} l_d^* (d^* + f^*) \ ; \]

\[ A^{n\Lambda} = e \frac{4}{M_\Lambda - M_{B^*}} \frac{\sqrt{2}}{3} l_d^* (d^* + 3f^*) + e \frac{4}{M_{B^*} - M_N} \frac{\sqrt{2}}{3} l_d^* (d^* - 3f^*) \ ; \]

\[ A^{\Sigma^0 \Xi^0} = e \frac{4}{M_{B^*} - M_\Xi} \frac{\sqrt{2}}{3} l_d^* (d^* - f^*) + e \frac{4}{M_\Sigma - M_{B^*}} \frac{\sqrt{2}}{3} l_d^* (d^* + f^*) \ ; \]

\[ A^{\Xi^0 \Xi^0} = e \frac{4}{M_{B^*} - M_\Xi} \frac{\sqrt{2}}{3} l_d^* (d^* + 3f^*) + e \frac{4}{M_\Lambda - M_{B^*}} \frac{\sqrt{2}}{3} l_d^* (d^* - 3f^*) \ . \] (19)

In the framework of chiral perturbation theory at tree level one must include the spin-1/2\(^+\) resonances in order to ensure a nonvanishing asymmetry parameter for the decays \( \Sigma^+ \rightarrow p\gamma \) and \( \Xi^- \rightarrow n\gamma \) since the parity conserving component vanishes if only the baryon octet is retained. In [12, 20] the coupling of the photons to the ground state baryons was expressed directly in terms of the experimental baryon magnetic moments which implicitly includes higher order chiral contributions and leads to nonvanishing parity conserving amplitudes for the decays \( \Sigma^+ \rightarrow p\gamma \) and \( \Xi^- \rightarrow \Sigma^-\gamma \). As in the case of the nonleptonic hyperon decays one must to include the spin-1/2\(^+\) resonances to account for such higher order effects.[13]

### 3 Numerical results and discussion

In this section we present the numerical results for the decay amplitudes and the decay parameters. For the electromagnetic couplings we use the values which can be obtained from the magnetic moments of the ground state baryons and the radiative decays of the resonances, cf. App. A, while the weak parameters are fixed from the nonleptonic hyperon decays.[13] For the parity conserving amplitudes we obtain, in units of 10\(^{-7}\) GeV\(^{-1}\),

\[ A^{p\Sigma^+} = 0 - 1.81 = -1.81 \quad A^{\Sigma^- \Xi^-} = 0 + 0.08 = 0.08 \]

\[ A^{n\Sigma^0} = 0.50 - 0.52 = -0.02 \quad A^{n\Lambda} = 0.41 + 0.11 = 0.52 \]

\[ A^{\Sigma^0 \Xi^0} = -1.01 + 1.06 = 0.05 \quad A^{\Xi^0 \Xi^0} = -0.36 + 0.02 = -0.34 \ , \] (20)
where the first and second numbers denote the contributions from the ground state octet and the spin-1/2+ resonances, respectively. The parity violating amplitudes read, in units of $10^{-7}$ GeV$^{-1}$,

\[
\begin{align*}
B^{\rho\Sigma^+} &= 0.47 & B^{\Sigma^-\Xi^-} &= 0.15 \\
B^{n\Sigma^0} &= -0.45 & B^{n\Lambda} &= -0.05 \\
B^{\Sigma^0\Xi^0} &= 0.70 & B^{\Lambda\Xi^0} &= -0.08
\end{align*}
\]

(21)

The decay rates are given by

\[
\Gamma^{ij} = \frac{1}{\pi} \left( \frac{M_i^2 - M_j^2}{2M_i} \right)^3 \left( |A^{ij}|^2 + |B^{ij}|^2 \right)
\]

(22)

and one obtains, in units of GeV

\[
\begin{align*}
\Gamma^{\rho\Sigma^+} &= 1.3 \times 10^{-16} \quad (1.0 \times 10^{-17}) \\
\Gamma^{n\Sigma^0} &= 7.5 \times 10^{-18} \\
\Gamma^{\Sigma^0\Xi^0} &= 2.7 \times 10^{-18} \quad (7.9 \times 10^{-18})
\end{align*}
\]

\[
\begin{align*}
\Gamma^{\Sigma^-\Xi^-} &= 1.6 \times 10^{-19} \quad (5.3 \times 10^{-19}) \\
\Gamma^{n\Lambda} &= 3.8 \times 10^{-18} \quad (4.6 \times 10^{-18}) \\
\Gamma^{\Lambda\Xi^0} &= 2.5 \times 10^{-18} \quad (2.5 \times 10^{-18})
\end{align*}
\]

(23)

where the number in the brackets denotes the experimental value. (The decay $\Sigma^0 \rightarrow n\gamma$ is dominated by the electromagnetic decay $\Sigma^0 \rightarrow \Lambda\gamma$ and, therefore, no experimental value can be given in this case.) Finally, the corresponding asymmetry parameters are found to be, cf. Eq. (3),

\[
\begin{align*}
\alpha^{\rho\Sigma^+} &= -0.49 & \alpha^{\Sigma^-\Xi^-} &= 0.84 \\
\alpha^{n\Sigma^0} &= 0.12 & \alpha^{n\Lambda} &= -0.19 \\
\alpha^{\Sigma^0\Xi^0} &= 0.15 & \alpha^{\Lambda\Xi^0} &= 0.46
\end{align*}
\]

(24)

Our results are only indicative, of course. A full discussion would have to include both the effects of chiral loops as well as contributions from additional resonant states. In this regard, we do not anticipate that our predictions should be able to precisely reproduce the experimental values for the decay widths, but it should be noted that we obtain in our approach radiative hyperon decay widths which are in reasonable agreement with experiment except in the case of $\Sigma^+ \rightarrow p\gamma$, which is about an order of magnitude larger than the experimental value. We could, of course, by use of average resonant mass rather than non-strange mass or by twiddling parameters, bring this number into better agreement with experiment. However, our purpose herein is not a precise fit to data, but rather to ask whether the resonance saturation is able to represent the basic phenomenology of these decay rates. In this regard, the answer is then yes—our results suggest that the spin-1/2− resonances play an essential role for the radiative decays as was found in the constituent quark model. ([12]) (Indeed at lowest chiral order without such resonant contributions both parity conserving and parity violating amplitudes would vanish!) But in addition, in the chiral approach one must include the octet of spin-1/2+ resonances in order to account for SU(3) breaking effects, which are higher order in the chiral expansion. A similar conclusion was reached in the case of the nonleptonic hyperon decays. ([13])

What is perhaps more important here is that with the inclusion of resonant contributions, the origin of the “large” negative asymmetry in the radiative $\Sigma^+$ hyperon decay is no longer a
mystery. Indeed it becomes almost natural. The resonant contribution to the parity conserving and parity violating amplitudes are comparable in size, leading to significant asymmetries for a number of the radiative modes, including $\Sigma^+ \rightarrow p\gamma$. It should also be noted that there is no conflict with Hara’s theorem. Examination of Eq. (18) clearly shows that the parity violating amplitudes for the decays $\Sigma^+ \rightarrow p\gamma$ and $\Xi^- \rightarrow \Sigma^-\gamma$ vanish in the SU(3) limit.

4 Summary

In this work we examined the significance of low lying baryon resonant contributions to radiative hyperon decay. To this end, we included the spin-1/2$^-$ octet from the (70,1$^-$) states and the octet of Roper-like 1/2$^+$ fields in the effective theory. The most general Lagrangian incorporating these resonances coupled to the ground state baryons introduces twelve new parameters, four of which can be determined from the electromagnetic decays of the resonances, two can be fitted from the ground state baryon magnetic moments and the remaining six weak couplings have already been determined from nonleptonic hyperon decays within the framework of chiral perturbation theory.[13] Thus, the inclusion of the spin-1/2 resonant states leads to no additional unknown parameters. (It should be noted that an alternative approach—inclusion of the spin-3/2$^+$ decuplet, as performed in [14] for nonleptonic hyperon decay, generates terms at the same chiral order as the loop corrections—$O(p^2)$, which is beyond the accuracy of this calculation and therefore can be neglected.) In [11, 12] it was argued that within the quark model the inclusion of the spin-1/2$^-$ octet is sufficient to obtain a satisfactory fit for both nonleptonic and radiative hyperon decays. We have shown that in the framework of chiral perturbation theory the structure of the contributions from such resonances agrees with the results in the quark model to the order we are working. In [12] the pole terms of the ground state baryons to the parity conserving amplitudes were expressed in terms of experimental magnetic moments and thereby a significant nonzero value for the asymmetry parameter in polarized $\Sigma^+ \rightarrow p\gamma$ was obtained. However, this approach includes the anomalous magnetic moment components which are of higher chiral order and, therefore, do not appear to the order we are working. On the other hand, in our tree level chiral perturbative calculation the improvement of experimental agreement is brought about by the inclusion of the Roper-octet, which is in the same mass range as the 1/2$^-$ octet. We found that by using the electromagnetic couplings determined from a fit to the magnetic moments of the ground state baryons and from the electromagnetic decays of the resonances, and the weak couplings from the nonleptonic hyperon decays we obtained reasonable predictions for the decay amplitudes and significant negative values for the $\Sigma^+ \rightarrow p\gamma$ asymmetry as a very natural result of this picture, even though Hara’s theorem is satisfied.

We conclude that the inclusion of spin-1/2 resonances in nonleptonic hyperon decays provides a reasonable explanation of the importance of higher order counterterms and gives a satisfactory picture of both radiative and nonradiative nonleptonic hyperon decay. In order to make a more definite statement one should, of course, go to higher orders and include meson loops as well as the contributions from additional resonances. However, this is clearly beyond the scope of the present investigation.
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A Determination of the strong couplings

In this appendix we present the determination of the photon baryon couplings used in the effective Lagrangian. We start with the ground state baryons. In this case the two appearing LECs $l_d$ and $l_f$ can be fit to the baryon magnetic moments which are defined by

$$\mu^{ji} = \frac{e}{M_i + M_j} \left[ F_1^{ji}(0) + F_2^{ji}(0) \right] = \frac{e}{M_i + M_j} F_1^{ji}(0) + A^{ji}$$  \hspace{1cm} (A.1)

The form factor $F_1^{ji}(0)$ is related to the electromagnetic charges of the baryons

$$F_1^{ji}(0) = q_i \delta_{ji} , \quad q_i = \{-1, 0, 1\} .$$  \hspace{1cm} (A.2)

A fit for $l_d$ to the magnetic moments of the ground state baryons delivers

$$l_d = 0.25 \text{ GeV}^{-1}$$  \hspace{1cm} (A.3)

and we neglected the uncertainties in our fit since we are only interested in the order of magnitude for this parameter. (Note, that $l_f$ does not contribute to the amplitudes at tree level.)

We now turn to the determination of the couplings $r_d, r_f$ and $l_d^*, l_f^*$ appearing in the electromagnetic part of the effective resonance-ground state Lagrangian. The decays listed in the particle data book, which determine the coupling constants $r_d$ and $r_f$, are $N(1535) \rightarrow N\gamma$. The width is given by

$$\Gamma^{ji} = \frac{1}{8\pi M^2_R} |k_\gamma||T^{ji}|^2$$  \hspace{1cm} (A.4)

with

$$|k_\gamma| = E_\gamma = \frac{1}{2M_R} (M_R^2 - M_B^2)$$  \hspace{1cm} (A.5)

being the three-momentum of the photon in the rest frame of the resonance and $M_R$ and $M_B$ being the masses of the resonance and the ground state baryon, respectively. For the resonance mass $M_R$ we use the mass of $N(1535)$. The mistake we make in the case the other resonance octet states is of higher chiral order and, therefore, beyond the accuracy of our calculation. For the transition matrix one obtains

$$|T^{ji}|^2 = 128 e^2 (p_i \cdot k)^2 (C^{ji})^2$$  \hspace{1cm} (A.6)

with $p_i$ the momentum of the decaying baryon and the coefficients

$$C^{pp(1535)} = \frac{1}{3} r_d + r_f , \quad C^{pn(1535)} = \frac{2}{3} r_d .$$  \hspace{1cm} (A.7)

Using the experimental values for the decay widths we arrive at the central values

$$e r_d = 0.033 \text{ GeV}^{-1} , \quad e r_f = -0.046 \text{ GeV}^{-1} .$$  \hspace{1cm} (A.8)
We do not present the uncertainties in these parameters here, since for the purpose of our considerations a rough estimate of these constants is sufficient.

For the determination of $l_d^*$ and $l_f^*$ we use the decays $N(1440) \rightarrow N\gamma$. One has to replace the resonance mass by $M_{B^*} \simeq 1440$ MeV in Eq. (A.4,A.5) and the coefficients read

$$C_{pp}^{(1440)} = \frac{1}{3} l_d^* + l_f^*, \quad C_{nn}^{(1440)} = -\frac{2}{3} l_d^*.$$

(A.9)

The fit to the decay widths delivers

$$e l_d^* = -0.024 \text{ GeV}^{-1}, \quad e l_f^* = -0.009 \text{ GeV}^{-1}.$$

(A.10)

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Figure captions

Fig.1 Diagrams that contribute to radiative hyperon decays. Solid and wavy lines denote ground state baryons and photons, respectively. Solid squares and circles are vertices of the weak and electromagnetic interactions, respectively.

Fig.2 Diagrams including resonances that contribute to radiative hyperon decays. Solid and wavy lines denote ground state baryons and photons, respectively. The double line represents a resonance. Solid squares and circles are vertices of the weak and electromagnetic interactions, respectively.
Figure 1

Figure 2