Design of Space-Frequency Index Modulation Waveforms for MIMO–OFDM Heterogeneous Networks

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ABSTRACT

In this paper, a novel framework to efficiently design the waveforms toward heterogeneous networks (HetNets) is proposed. The proposed scheme combines the index modulation (IM) concept with multiple-input multiple-output (MIMO) and orthogonal frequency-division multiplexing (OFDM) techniques and jointly considers the resources on both spatial and frequency domains to optimize the transmission signals. Furthermore, a generalized union bound expression is derived first to disclose the potential diversity gain for the spatial-frequency index modulation (SFIM) system and based on the developed bound different impacts of activating spatial and frequency domains resources for transmission are analyzed. Following the theoretical insights, two novel transmit schemes, termed as enhanced SFIM with non-overlap (ESFIM-NO) and enhanced SFIM with overlap (ESFIM-O), toward higher diversity gains are proposed. We show that the proposed methods can significantly improve the diversity of both index and amplitude phase modulation (APM) domains for achieving higher reliability for the entire HetNets. Finally, the simulation results are given to verify the theoretical analysis and system error performance.

INDEX TERMS
Multi-dimensional index modulation, coding gains, diversity gains.

I. INTRODUCTION

The heterogeneous networks (HetNets) have emerged as a promising solution to support high data rate and a variety of services required by future wireless communication networks by enabling the terminals be capable of processing signals transmitted from both macrocell and small cell [1]–[6]. However, the cross-tier interferences induced by the overlaid architecture pose a significant challenge for HetNets [7]–[10]. Recently, non-orthogonal multiple access (NOMA) techniques were developed to help HetNets manage the interferences [11]–[13]. Moreover, the unique feature of index modulation (IM) also makes it a candidate waveform for the signals of one tier in HetNets to control the interferences [14]. Therefore, a novel framework to design the waveforms for IM aided multiple-input multiple-output (MIMO)-orthogonal frequency-division multiplexing (OFDM) systems is proposed in this paper to further enhance the reliability of the tier signals.

The generalized IM technology differs from conventional transmission schemes, where the information bits are conveyed only by amplitude phase modulation (APM) signals chosen from a fixed constellation [15]–[18]. In contrast, besides APM signals, IM systems exploit the indices of various transmission units to convey information bits implicitly. Thanks to the researchers’ efforts, so far, a large variety of indices can be utilized for IM systems, such as the indices of transmit/receive antennas [19], subcarriers [20], time slots [21], and so on. Since the inactivated transmission units can convey information bits without energy consumption, the index modulation is regarded as a promising technology for the future wireless networks including HetNets [22].

Categorized in the family of IM technology, the spatial modulation (SM), which uses the indices of antennas to convey additional information bits, is well-known for its high spectral and energy efficiency with low complexity of
transmitter design. Interested reader are referred to [23], [24] to have a comprehensive view of the achievements with regard to SM.

Inspired by SM, the concept of IM is extended to the OFDM system recently, where the indices of subcarriers are used to convey additional information bits. Similar to SM, various schemes are introduced to improve the system performances. More specifically, the authors of [25] proposed the design of interleaving for subcarriers with better bit error rate (BER) performance by making the fadings in a block uncorrelated. Moreover, in [26], a novel transmission scheme, termed as coordinate interleaved OFDM-IM (CI-OFDM-IM) is proposed by selecting two groups of subcarriers independently to convey the I and Q branches of APM signals, respectively. Therefore, the transmit diversity gains can be obtained for CI-OFDM-IM. Also aiming to increase the transmit diversity order, in [27], the authors proposed to utilize the APM signals chosen from multiple constellations to bear the same information bits. Furthermore, in [28], the authors leveraged the repeated coding to propose a novel scheme, termed as repeated MCIIK-OFDM (ReMO), to increase the transmit diversity order. However, the transmit diversity orders of APM signals and index part are not matched in ReMO. The index part can only achieve a diversity order of two, which limits the whole system error performance.

For exploiting the potential of IM further, some researchers are studying to combine OFDM with index modulation (OFDM-IM) with the MIMO technology to take advantages from both sides. More specifically, in [29], the concept of OFDM-IM has been extended to multiple antennas for achieving higher spectral efficiency. The proposed system is defined as MIMO-OFDM-IM and it is reported that significant SNR gains can be obtained by the MIMO-OFDM-IM system compared with the conventional MIMO-OFDM system. It is worthy to note that each antenna in the MIMO-OFDM-IM system activates the subcarrier independently, which limits the number of available index patterns. In order to solve this limitation, in [30], the authors proposed two spatial-frequency joint index modulation schemes, termed as generalized joint space-frequency index modulation (G-JSFIM) and Kronecker product-based joint space-frequency index modulation (KP-JSFIM). The simulation results showed that G-JSFIM can achieve the same error performance as MIMO-OFDM-IM with higher spectrum efficiency, while KP-JSFIM can provide the best error performance for low spectrum efficiency (SE) scenarios. Similar to KP-JSFIM, the authors of [31] also considered to jointly use spatial-frequency resource units to generate waveforms, however, antenna selection is carried out before index modulation to reduce the number of required radio frequency links. In [32] and [33], the performances of the SFBC-OFDM system are evaluated with channel estimation error and TWDP Fading Channel, respectively. Moreover, the index modulation concept has also been combined with compressed sensing technique in [34] to reduce the complexity of the receiver. For relay systems, a novel scheme was proposed in [35] to reduce the overhead while improving the error performance. A novel amplify-and-forward relay-assisted OFDM-IM system was proposed in [36] to reduce the system delay and energy consumption.

In this paper, a systematic waveform design to achieve enhanced transmit diversity for one tier of HetNets is proposed and the contributions of this paper are summarized as follows. Two novel transmit pattern generation schemes, termed as ESFIM-NO and ESFIM-O, are proposed. Specifically, ESFIM-O achieves higher order of transmit diversity at the same spectral efficiency compared to ESFIM-NO, while ESFIM-NO outperforms ESFIM-O at low SNR region due to larger minimum Euclidean distance. The difference to current SFIM systems lies in that we focus on enhancing the transmit diversity order by designing the transmit patterns. Unlike other current SFIM systems where a group of subcarriers are mapped with different IQ symbols, the proposed system activates multiple subcarriers to convey the same IQ symbol and the positions of active subcarriers are carefully designed. The theoretical and simulation results show that the proposed schemes obtain higher order of transmit diversity compared to existing SFIM systems without increasing the number of transmit antennas. Moreover, a generalized union bound expression for analyzing the transmit diversity orders of the SFIM system is derived.

The remainder of this paper is organized as follows. In Section II, the system model is presented. In Section III, two novel waveform design methods are proposed. In Section IV, the numerical results are given to verify the analysis and system performance. Finally, the conclusion is given in Section V.

Notations: \( a \) and \( A \) denote a vector and a matrix, respectively. \( \odot \) is the Hadamard product and \( | \) is the union operator for sets. \( \otimes \) is the Kronecker product. \( [ ] \) is the floor operator. \( A_{i, j} \) and \( A_{j, i} \) represent the \( i \)th column and \( j \)th row of the matrix \( A \), respectively.

II. SYSTEM MODEL

Consider a system equipped with \( N_t \) transmit and \( N_r \) receive antennas and each antenna transmits OFDM signals with an IFFT length of \( N \). Hence, there are \( N_r \times N \) space-frequency resource units available for each OFDM symbol period and the architecture of the transmitter is shown in Fig. 1. Assuming \( M \) information bits are conveyed during each OFDM symbol time, the coming bits are partitioned into \( G \) groups with \( m = M / G \) bits for each group. Accordingly, the spatial-frequency resources are divided into \( G \) groups and \( n = N_t \times n_b \) units are responsible for the transmission of the \( m \) bits, where \( n_b \) is the number of subcarriers selected from each transmit antenna. Without loss of generality, we only consider the first space-frequency group in the following parts of this paper.

For the first group, \( m \) information bits are further divided into two parts, containing \( m_1 \) and \( m_2 \) information bits, respectively, i.e., \( m = m_1 + m_2 \). The \( m_1 \) information bits are utilized to select a specific transmit pattern, which activates only
part of available resource units while keeping the remaining silent. Assuming the group has $k$ active resource units, $m_1 = \left\lceil \log_2(C(n, k)) \right\rceil$ bits can be mapped into the index domain. The remaining $m_2$ bits are conveyed by the conventional APM signals chosen from an $M$-ary constellation $Q$. After information mapping, $G$ groups are concatenated and distributed to $N_t$ antennas. On each antenna, normalized IFFT operations are applied after interleaving. Then, the waveform for transmission is formed by adding the cyclic prefix with the length of $h_{2p}$.

The waveforms are sent via a frequency selective Rayleigh fading MIMO channel. Denote $q^{(i)}_m \in \mathbb{C}^{L_b \times 1}$ as the $L$-tap channel fading from the $r$th transmit antenna to the $r$th receive antenna in the time domain, whose elements are i.i.d Rayleigh random variables obeying $CN(0, \frac{1}{\pi})$. The corresponding frequency domain fading coefficients from the $r$th transmit antenna to the $r$th receive antenna for the first group can be represented as $h^{(1)}_r = [h_{1,r}^{(1)}, \ldots, h_{N_r}^{(1)}]^T$, where $h_{a,r}^{(1)} \in \mathbb{C}$ is the channel fading coefficient on the $a$th subcarrier. Thus, for the first group, the coefficient matrix related to the $r$th receive antenna can be given as $H^{(1)} = [h^{(1)}_1, h^{(1)}_2, \ldots, h^{(1)}_{N_r}]$ and the entire channel coefficients in the frequency domain are given as $H = [(H^{(1)})^T, (H^{(2)})^T, \ldots, (H^{(N)})^T]^T$.

Assuming the $i$th, $i = 1, \ldots, 2^m$, legitimate space-frequency index modulation symbol is selected, the transmit signals on the $r$th transmit antenna can be presented as $s^{(i)}_r = [s^{(i)}_{1,r}, s^{(i)}_{2,r}, \ldots, s^{(i)}_{N_s,r}]^T$, where $s^{(i)}_{a,r} \in Q = \{0, Q\}$, $a = 1, 2, \ldots, N_s$. Thus, the signal matrix for entire $N_t$ antennas can be denoted as $S^{(i)} = [s^{(i)}_1, s^{(i)}_2, \ldots, s^{(i)}_{N_t}]$. Therefore, the received signal $y_r \in \mathbb{C}^{n_b \times 1}$ in the frequency domain on the $r$th receive antenna for the first group can be represented as

$$y_r = \sqrt{\rho} \sum_{i=1}^{N_t} h^{(i)}_{r,t} \otimes s^{(i)}_{r,t} + n_r,$$

where $\rho = n/k$ is the normalization factor, each element in $n_r \in \mathbb{C}^{n_b \times 1}$ is additive white Gaussian noise (AWGN) in the frequency domain and obeys $CN(0, N_0)$. Therefore, after maximum likelihood (ML) detection, the estimate of the signal can be expressed as

$$\hat{i} = \arg\min_i \sum_{t=1}^{N_t} \left\| y_r - \sqrt{\rho} \sum_{i=1}^{N_t} h^{(i)}_{r,t} \otimes s^{(i)}_{r,t} \right\|^2.$$

### III. DIVERSITY ANALYSIS AND WAVEFORM DESIGN

In this section, a novel expression for the diversity of spatial-frequency index modulation (SFIM) is derived first, which offers an insight into the impacts of using spatial and frequency units for the diversity gains of IM. Based on the insights, two novel waveform designs, termed as enhanced SFIM with non-overlap (ESFIM-N0) and enhanced SFIM with overlap (ESFIM-O), are proposed to enhance the transmit diversity gain, which can be used for the signals of one tier in HetNets to enhance the reliability of the entire system.

#### A. DIVERSITY ANALYSIS FOR SFIM SYSTEMS

In order to analyze and compare all the SFIM schemes under the same framework, SFIM is regarded as selecting a block-wise signal $S^{(i)}$ from an index modulation constellation whose elements are in $Q$. Therefore, the transmit signal for the first group can be shown in Fig. 2, where $n_b \times N_t$ basic resource units are available for the first group.

With the ML detection, the conditional probability of the pairwise error in the case where $S^{(i)}$ is detected when $S^{(j)}$ is sent can be represented as (3), as shown at the bottom of the next page.

Using the Q-function, the instantaneous pairwise error probability (PEP) can be represented as

$$P(i \rightarrow j|\mathbf{H}) = Q\left(\frac{1}{2N_0} \sum_{t=1}^{N_t} \sum_{i=1}^{N_t} \sqrt{\rho} h^{(i)}_{r,t} \otimes (s^{(i)}_{t,r} - s^{(j)}_{t,r})^2\right),$$

where $Q(x) = \frac{1}{2} \int_x^\infty \exp(-\frac{t^2}{2}) dt$ is the Gaussian tail probability [28]. By using the approximation
on this subcarrier, i.e., $S_{a,i}^j \neq S_{a,j}^i$. The signals on different transmit antennas with the same subcarrier index can only determine the coding gain. Thus, the transmit diversity order corresponding to the PEP $P(i \rightarrow j)$ can be calculated as

$$
\Phi(i, j) = \sum_{a=1}^{n_r} I(S_{a,i}^j, S_{a,j}^i).
$$

Therefore, the transmit diversity order for the entire system can be represented as

$$
D = \min_{i,j} \Phi(i, j) = \min(D_I, D_M).
$$

where $D_I$ and $D_M$ are the transmit diversity order of the index and APM signal domains, respectively, and the analytic upper bounds of the symbol error rate and BER can be expressed as

$$
P_s = \frac{1}{2^m} \sum_{i} \sum_{j \neq i} P(i \rightarrow j),
$$

$$
P_b = \frac{1}{m2^m} \sum_{i} \sum_{j \neq i} \text{Ham}(i,j)P(i \rightarrow j),
$$

where $\text{Ham}(i,j)$ is the hamming distance.

### B. WAVEFORM DESIGN

Inspired by the results in Sec. III-A, we propose two novel waveform designs, termed as ESFIM-NO and ESFIM-O by binding the basic resource units into blocks and the basic resource units in a block are mapped with the same APM symbol. The indices of the constructed blocks and the APM symbols mapped to the blocks are named as virtual index and virtual APM symbols in order to distinguish from the conventional SFM systems. By sophisticatedly binding, the transmit diversity order of the both virtual index and virtual APM symbol can be increased, which leads to the improvement of the error performance for the entire system.

It should be emphasized that the proposed scheme in [28] modulated multiple active subcarriers with the same APM symbol as well. However, it can only increase the transmit diversity order of the APM symbol but the transmit diversity order of the index domain is still limited to two, leading the transmit diversity order of the entire system to be two. In contrast, our approaches can improve the transmit diversity
order of both the virtual index and virtual APM symbol simultaneously.

Denote \( u_{at} \) as the basic resource unit provided by the \( at \) subcarrier on the \( tr \)th transmit antenna. Then, the resource units in the first group can be illustrated in Fig. 3. In the following sections, two situations are considered according to whether the different virtual blocks contain the same basic resource units.

1) BINDING RESOURCE UNITS WITH NON-OVERLAP

In this section, the binding with non-overlapping basic resource units is considered. Without loss of generality, we still consider the first group, which contains \( n_b \times N_t \) space-frequency resource units. All the units in the first group can be represented as a matrix \( U \), where \( u_{at} \) is its element, i.e., \( U_{a,t} = u_{at} \). By binding \( d \) units without overlapping, the virtual blocks after binding can be given as \( W = \{w_{ct}\} \), where

\[
\begin{align*}
    w_{ct} & = \begin{cases} 
        U_{((c-1)\times d+1),t} \\
        U_{((c-1)\times d+2),t} \\
        \vdots \\
        U_{((c-1)\times d+d),t} 
    \end{cases} 
\end{align*}
\]

(14)

represents the \( ct \)th virtual block on the \( tr \)th transmit antenna. Therefore, the binding with non-overlap means that

\[
    w_{ct} \cap w_{c't'} = \emptyset \quad c \neq c' \text{ and } t \neq t'.
\]

Let \( n_v \) and \( K_r \) represent the number of virtual blocks and the active ones, respectively, the number of virtual index patterns is \( V = 2^{[\log_2 C(n_v,K_r)]} \).

In order to illustrate the method of binding with non-overlap more clearly, a specific example is given as follows, where \( N_t = 2, n_b = 4, d = 2 \). The binding strategy is shown in Fig. 4, where the basic resource units with the same colour belong to the same virtual block and each virtual block can be expressed as

\[
\begin{align*}
    w_{11} & = \{u_{11}, u_{21}\} \\
    w_{21} & = \{u_{31}, u_{41}\} \\
    w_{12} & = \{u_{12}, u_{22}\} \\
    w_{22} & = \{u_{32}, u_{42}\}.
\end{align*}
\]

(15)

If higher transmit diversity order is required by using the binding with non-overlap, the group with larger number of basic resource units is needed. In Fig. 5, a binding with \( d = 3 \) is presented, where each virtual block can be expressed as

\[
\begin{align*}
    w_{11} & = \{u_{11}, u_{21}, u_{31}\} \\
    w_{21} & = \{u_{41}, u_{51}, u_{61}\} \\
    w_{12} & = \{u_{12}, u_{22}, u_{32}\} \\
    w_{22} & = \{u_{42}, u_{52}, u_{62}\}.
\end{align*}
\]

(16)

Obviously, there is a SE cost for pursuing higher error performance if using the binding with non-overlap.

For evaluating the error performance of the scheme, we category the error events into two types: the virtual block index errors and virtual APM symbol errors. Moreover, denote \( \Psi^i = \{v^i_1, v^i_2 \ldots v^i_{K_r}\} \) the index set of the active virtual blocks of the transmitted signal while \( \Psi^j = \{v^j_1, v^j_2 \ldots v^j_{K_r}\} \) as the detected index set of the active virtual blocks and \( v^i_k, v^j_k \in \{1, 2, \ldots, N_t n_b/d\} \). Then, only the PEP \( P(\Psi^i \rightarrow \Psi^j) j \in \Lambda_i \) satisfying \( |\Psi^i - \Phi^i_j| = 1 \) is considered for virtual index errors, where \( \Phi^i_j = \Psi^i \cap \Psi^j \). Then, the virtual index error rate (VIER) can be approximated as

\[
P_{VI} \approx \frac{1}{V} \sum_{i=1}^{V} \sum_{j \in \Lambda_i} P(\Psi^i \rightarrow \Psi^j).
\]

(17)

Denoting \( \tilde{\psi}^i = \Psi^i - \Phi^i_j \) and \( \tilde{\psi}^j = \Psi^j - \Phi^i_j \), the basic resource units in the virtual blocks corresponding to
the indice \( \vec{v} \) and \( \vec{v}^\prime \) can be represented as \( \{\vec{u}_{at}^j\} \) and \( \{\vec{u}_{at}^{j,\prime}\} \), respectively. These units can be further divided into two sets, where

\[
\Omega_{ij}^j = \{\vec{u}_{at}^j, \vec{u}_{at}^{j,\prime}\}
\]

contains the units with the same subcarrier index but different antenna indices, i.e., \( a = a', t \neq t' \) and

\[
\Omega_{ij}^t = \{\vec{u}_{at}^j\} \cup \{\vec{u}_{at}^{j,\prime}\} - \Omega_{ij}^j
\]

contains the units with different subcarrier indice, i.e., \( a \neq a' \).

Denoting \( \mathbf{H}'(\vec{u}_{at}^j) \) as the channel coefficient on the \( r \)th subcarrier from the \( r \)th transmit to the \( r \)th receive antenna, the instantaneous PEP with \( j \in \Lambda_j \) can be given as

\[
P(\Psi^j \rightarrow \Psi^j|\mathbf{H}) = O(\sqrt{\frac{T_i^d + T_j^d + T_{ij}^d}{2N_0}})
\]

where

\[
T_i^d = \sum_{r=1}^{N_r} \sum_{\vec{u}_{at}^j \in \Omega_{ij}^j} \rho \left\| \mathbf{H}'(\vec{u}_{at}^j) \right\|^2,
\]

\[
T_j^d = \sum_{r=1}^{N_r} \sum_{\vec{u}_{at}^{j,\prime} \in \Omega_{ij}^j} \rho \left\| \mathbf{H}'(\vec{u}_{at}^{j,\prime}) \right\|^2,
\]

\[
T_{ij}^d = \sum_{a=1}^{N_a} \rho \left\| \mathbf{H}'(\vec{u}_{at}^j) - \mathbf{H}'(\vec{u}_{at}^{j,\prime}) \right\|^2,
\]

where \( \vec{u}_{at}^j, \vec{u}_{at}^{j,\prime} \in \Omega_{ij}^j \).

By using the \( Q \) function approximation, we can obtain

\[
P(\Psi^j \rightarrow \Psi^j|\mathbf{H}) \approx \frac{1}{12} C_1 + \frac{1}{4} C_2.
\]

where

\[
C_1 = e^{-\frac{T_i^d + T_j^d + T_{ij}^d}{2N_0}},
\]

\[
C_2 = e^{-\frac{T_i^d + T_j^d + T_{ij}^d}{2N_0}}.
\]

Then, the unconditional PEP can be expressed as

\[
P(\Psi^j \rightarrow \Psi^j) \approx \frac{1}{12} E[C_1] + \frac{1}{4} E[C_2].
\]

Then, we can obtain the following results by using the MGF:

\[
E\left[e^{\frac{T_i^d}{2N_0}}\right] = E\left[e^{\frac{-\sum_{r=1}^{N_r} \sum_{\vec{u}_{at}^j \in \Omega_{ij}^j} \rho \left\| \mathbf{H}'(\vec{u}_{at}^j) \right\|^2}{2N_0}}\right],
\]

\[
= \left(1 + \frac{\rho}{4N_0}\right)^{-N_r |\Omega_{ij}^j|/2}
\]

Similarly, \( E[e^{\frac{T_j^d}{2N_0}}] \) can be attained as

\[
E[e^{\frac{T_j^d}{2N_0}}] = (1 + \frac{\rho}{4N_0})^{-N_r |\Omega_{ij}^j|/2}.
\]

Then, the \( E[e^{\frac{T_{ij}^d}{2N_0}}] \) with \( \vec{u}_{at}^j, \vec{u}_{at}^{j,\prime} \in \Omega_{ij}^j \) can be represented as

\[
E\left[\exp \left(\frac{\sum_{r=1}^{N_r} \sum_{\vec{u}_{at}^j \in \Omega_{ij}^j} \rho \left\| \mathbf{H}'(\vec{u}_{at}^j) - \mathbf{H}'(\vec{u}_{at}^{j,\prime}) \right\|^2}{2N_0}\right)\right] = \left(1 + \frac{\rho}{2N_0}\right)^{-N_r |\Omega_{ij}^j|/2}.
\]

The components in (26) can be obtained similarly. Hence, the virtual index PEP with \( M \)-PSK and \( j \in \Lambda_j \) is shown in (33), as shown at the bottom of the next page.

For analyzing the error events related to the virtual APM symbol, we model the ML receiver as an \( L \) branch MRC receiver as in [37]. Then, the error probability of a single virtual APM symbol conditioned on the instantaneous channel coefficient matrix \( \mathbf{H} \) can be expressed as

\[
P_{M|\mathbf{H}} \approx 2Q(\sqrt{\frac{2}{N_0} \sum_{r=1}^{N_r} \sum_{\vec{u}_{at}^j} \rho \left\| \mathbf{H}'(\vec{u}_{at}^j) \right\|^2 \sin(\pi/M))}.
\]

By considering \( M \)-PSK and the MGF method, the unconditional error probability can be given as

\[
P_M \approx \frac{1}{6} \left(1 + \frac{\rho}{N_0} \sin^2\left(\frac{\pi}{M}\right)\right)^{-dN_r} + \frac{1}{2} \left(1 + \frac{4}{3N_0} \sin^2\left(\frac{\pi}{M}\right)\right)^{-dN_r}.
\]

Thus, when the indices of the virtual blocks are detected correctly, the error rate of the APM symbol is \( (1 - P_V)P_M \). When the indices of the virtual blocks are detected incorrectly, we only consider the case where a single virtual block is detected by mistake. Therefore, \( K_v - 1 \) APM symbols have an error rate of \( P_M \) and the APM symbol error probability for the virtual block with index detection error is \( (M - 1)/M \). Then, the total error probability of all the virtual APM symbols in the first group can be given as

\[
P_T^M \approx (1 - P_V)P_M + P_V \left(\frac{K_v - 1}{K_v}P_M + (M - 1)/M\right).
\]

Therefore, the block error rate (BLER), defined as the ratio of the number of groups in error to the total transmitted groups, can be given as

\[
P_G \approx 1 - (1 - P_V)(1 - P_T^M).
\]

2) BINDING RESOURCE UNITS WITH OVERLAP

To address the challenge mentioned in Sec. III-B1 and improve the transmit diversity order without sacrificing SE, the scheme of binding with overlapping basic resource units and constellation rotation, termed as ESFIM-O, is proposed in this section. Investigating the first group with \( n_b \times N_t \)
Therefore, the instantaneous PEP can be given as

$$P(\Psi^i \rightarrow \Psi^j | \mathbf{H}) = \mathcal{Q}(\sqrt{\frac{T_d^d + T_j^d + T_j^d + T_{ij}^s}{2N_0}}).$$

(43)

The additional component related to $T_{ij}^s$ can be given in (44), as shown on the bottom of next page, where $d_{\min}(Q, Q')$ is the minimum Euclidean distance of arbitrary symbol pairs in $Q$ and $Q'$. And the remaining calculation is identical to the Sec. III-B1.

IV. SIMULATIONS

In this section, the numerical results are presented to verify the correctness of the theoretical analysis and system performances. For the followings, the simulation parameters are presented in Table 1.

![FIGURE 6. BLER comparison among MIMO-OFDM-IM, G-JSFIM, KP-JSFIM, ESFIM-NO, and ESFIM-O with SE = 1.5 bits/s/Hz.](image)

TABLE 1. Simulation parameters.

| Parameter                        | Value |
|----------------------------------|-------|
| Number of transmit antennas      | $N_t = 2$ |
| Number of receive antennas       | $N_r = 2$ |
| Number of subcarriers            | $N = 120$ |
| Number of groups                 | $G = 30$ |
| Number of paths of channel       | $L_h = 8$ |
| Length of CP                     | $L_{cp} = 8$ |

In Fig. 6, the BLER of the proposed ESFIM-NO and ESFIM-O are compared with MIMO-OFDM-IM, G-JSFIM, and KP-JSFIM. For the proposed scheme, two out of four virtual blocks are activated for each group consisting of two transmit antennas and four subcarriers and each virtual block is mapped with a QPSK symbol, resulting in a spectral efficiency of 1.5 bits/s/Hz. From the simulation results, the significant performance gains are achieved for ESFIM-NO and ESFIM-O. More specifically, at BLER = $10^{-3}$, the ESFIM-NO and ESFIM-O have SNR gains of approximately 1 dB and 2 dB over MIMO-OFDM-IM, respectively. The theoretical upper bounds are very tight to the simulation results and the diversities attained by ESFIM-NO and ESFIM-O are clearly shown.

$$P(\Psi^i \rightarrow \Psi^j) \approx \frac{1}{12} \left(1 + \frac{\rho}{4N_0}\right) - N_r |\Omega_{ij}^s| \left(1 + \frac{\rho}{2N_0}\right) - N_r |\Omega_{ij}^d|/2 + \frac{1}{4} \left(1 + \frac{\rho}{3N_0}\right) - N_r |\Omega_{ij}^s| \left(1 + \frac{2\rho}{3N_0}\right) - N_r |\Omega_{ij}^d|/2$$

(33)
In Fig. 7, the BER performance of ESFIM-NO and ESFIM-O are compared with MIMO-OFDM-IM, G-JSFIM and KP-JSFIM at SE=1.5 bits/s/Hz. We show that the BER performance of ESFIM-NO with $d=2$ and MIMO-OFDM-IM are close. Meanwhile, ESFIM-O shows its superior performance due to high transmit diversity order of three. At the BER of $10^{-4}$, ESFIM-O can harvest an SNR gain of approximately 3 dB compared to MIMO-OFDM-IM.

The similar comparisons are carried out for SE=2 bits/s/Hz, where three virtual blocks are selected from a group with four virtual blocks and each block is mapped with a QPSK symbol. The BLER and BER performance results are shown in Fig. 8 and Fig. 9. As shown in Fig. 8, the performance gain for ESFIM-NO and ESFIM-O is significant. More specifically, at BLER of $10^{-3}$, ESFIM-NO and ESFIM-O can achieve approximately 1.5 dB and 2 dB SNR gains over conventional MIMO-OFDM-IM.

$$\mathbb{E} \left[ \exp \left( - \frac{T_{ij}}{4N_0} \right) \right] = \mathbb{E} \left[ \exp \left( - \frac{1}{4N_0} \sum_{r=1}^{N_r} \sum_{\tilde{u}_{ij} \in \Omega_{ij}^r} \rho \| \mathbf{H}' \left( \tilde{u}_{ij} \right) \|^2 \left\| S' \left( \tilde{u}_{ij} \right) - S' \left( \tilde{u}_{ij} \right) \right\|^2 \right) \right]$$

$$< \left( 1 + \frac{\rho}{4N_0} \right) \left( d_{\min}(Q, Q') \right)^2 \mathbb{E} \left[ \exp \left( - \frac{T_{ij}}{4N_0} \right) \right] \quad (44)$$
concluded that space-frequency index modulation provides more flexibility for system designs.

In order to evaluate the diversity of virtual index and APM symbol domains, respectively, the simulation results of VIER and virtual symbol error rate (VSER) are depicted in Fig. 11 and Fig. 12 along with the derived theoretical bounds. It is obvious that ESFIM-O can achieve better diversity than ESFIM-NO in terms of both VIER and VSER. However, ESFIM-NO has slightly better performance than ESFIM-O in the low SNR region. This is because of the usage of different constellations in ESFIM-O, which reduces the minimum Euclidean distance of the combined signal on the overlapping resource units. Again, the spatial-frequency index modulation system provides us with more design freedom to meet the target requirements of HetNets.

V. CONCLUSION

In this paper, a novel framework to design waveforms for one tier of HetNets is investigated. By appropriate binding resources in both spatial and frequency domains to form IM symbols, two novel schemes, termed as ESFIM-NO and ESFIM-O, are proposed. The theoretical and simulation results verify that the proposed two methods can outperform the existing index modulation using spatial and frequency resources in terms of BLER and BER due to its enhanced diversity. More importantly, the analysis and simulation results also show that the space-frequency index modulation system can provide more flexibility to make a balanced tradeoff between system throughput and robustness. However, the complexity of the ML detector is still extremely high, especially for the applications in high date rate transmission. Therefore, developing novel detectors with low complexity is our future research work.

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