Perpendicular to steel reinforcing bars axis displacements of rebars for determination the torsional stiffness of reinforced concrete elements with normal cracks

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Abstract. The article provides a technology for determining the displacement of the end of the reinforcing bar embedded in concrete for measuring the shifting force in the longitudinal reinforcement. Dependences of the displacement of the end of the reinforcing bar on different protective layers are shown. A formula to define the displacement of steel reinforcing bars due to the shifting force is given. The formula was obtained by the numerical computation approximation using three-dimensional finite elements. A method for determining the torsional stiffness of reinforced concrete element with normal cracks is presented. In this method the pliability of the reinforcement perpendicular to its axis is also considered.

1. Research analysis and problem-setting

The redistribution of forces between separate items of coverings, bridges and other complex statically indeterminate systems in spatial behavior depends on their torsional and bending stiffness relation [2, 14, 21]. Spatial, normal and inclined cracks have a significant impact on the stiffness in reinforced concrete structures [8].

Bending stiffness of reinforced concrete elements with cracks have been researched wide enough. Most of the work related to torsional deformations involves a spatial crack. However, such techniques are not suitable for calculation of rotational movements of elements with normal cracks resulting from bending moments.

The torsional stiffness of reinforced concrete elements with normal cracks has been researched by the authors of this article [1, 2, 18, 19]. According to these researches, the torsional stiffness should be determined by cutting the longitudinal reinforcement first, then determining mutual displacement of the faces. Then the shifting force and stiffness of the element with normal cracks are determined. The forepart of the problem is approximate when solving analytically and rather time-consuming when solving by modeling three-dimensional finite elements. In this connection, in [2, 18, 19] the empirical formula for the displacement of the reinforcing bar by the force perpendicular to its axis is used to determine the shifting force. However, this formula is derived from tests of embedded details clamped in a mass concrete. It does not take into account the influence of protective layer depth of the reinforcing bar.

Based on these issues method development for the determination of the value of shifting force in longitudinal reinforcing bar for calculation the torsional stiffness of concrete elements with normal cracks becomes relevant.

In relation to the above-mentioned target, the object of this article is to develop the method for the determination of shifting force in longitudinal reinforcement according to its location in the cross-
section of the concrete element and method for the determination of torsional stiffness of normally cracked concrete element.

2. Presentation of the basic material
Consider a concrete element with normal cracks (Figure 1).

![Figure 1](image1.png)

**Figure 1.** Model of reinforced concrete element with normal cracks subjected to torsion.

Figure 1 shows the distance between normal cracks via \( l_{oc} \) (which can be determined by any of well-known method, including the method in regulations). From block \( A \) to block \( B \) the torque is transmitted through the uncracked part of concrete with \( z_{oc} \) height and through the longitudinal reinforcing bars due to the shifting force arising in the reinforcing bars. The torsional stiffness of the element with a normal crack is calculated without any difficulty after determination of shifting force in longitudinal reinforcing bars. Scheme of the shifting forces action in longitudinal reinforcement in normal crack is in Figure 2 [1].

![Figure 2](image2.png)

**Figure 2.** Scheme of forces (a) and rotation (b) in a section with a crack.

Consider separately the movements caused by local deformation. At first glance, it seems most accurate to solve this problem by modelling three-dimensional finite elements using software complexes like «Ansys», «Abacus», «Lira», etc. However, there are major obstacles in the detailed analysis. The most important of these is the correct modelling of the bond between the concrete and the anchor under the action of a perpendicular to its axis load. The reason is that when the anchor works under the transverse load, one part of concrete under the metal rod is crumpled, and the opposite part «secedes» from concrete practically without any resistance (Figure 3).
Figure 3. Deformation model of a reinforcing rod under the action of a transverse load.

Thereby, the authors of the article [1, 2] have used the empirical formula to define the movement of the anchor under the transverse load used to determine the pliability of embedded steel details [20]. This formula was obtained from experimental data processing and is used to determine cross movement of a reinforcing bar subjected to a force perpendicular to its axis:

$$\Delta_{loc} = 1000 \frac{Q^2}{d_s^3 E_c} + \frac{Q}{d_s E_c},$$  \hspace{1cm} (1)$$

where $d_s$ and $E_c$ – respectively reinforcing bars diameter and modulus of deformation of concrete; $Q$ – force applied to the reinforcing bar in a direction perpendicular to its axis.

In the correlation formula (1) all the details of complex stress-strain state of the system reinforcement-concrete have been integrally taken into consideration. However, the proximity of the reinforcement to the lateral and bottom surface of the cross-section of the element has not been taken into account (see Figure 2).

From the point of view of the authors, it is correct to modeling the transverse movement of the reinforcing bar as it is shown in Figure 4.

Figure 4. Model for determining the displacement of the reinforcing bar subjected to a load perpendicular to the reinforcing bar axis.

In Figure 4 the gap between the finite elements of the reinforcing bar and the finite elements of concrete is labeled «2». This is to eliminate the resistance of finite elements on the upper half of the bar diameter of the displacement of the reinforcing bar vertically under the load $Q$.

Vertical movement at point 1 below the reinforcing bar (along the Z axis in figure 4) depends on the thickness of the lateral protective layers $t_1$ and $t_2$, the lower protective layer $a$ and the upper protective layer $c$.

It is obvious that a dependence can be obtained from a series of calculations using three-dimensional finite elements:
\[ \Delta = Q \cdot f(a, t, c), \]  

(2)

where \( \Delta \) – vertical movement under the reinforcing bar.

To reduce the number of calculations it can be assumed that \( t_1 \) and \( t_2 \) are equal. This is appropriate while taking into account that basically the reinforcing bars in the cross section are disposed symmetrically (see Figure 2).

To obtain the dependency (2) you can first obtain separately dependencies of the form:

\[ \Delta = f_1(a) \text{ at } t = \text{const}; \ c = \text{const}; \]  
\[ \Delta = f_2(a) \text{ at } a = \text{const}; \ c = \text{const}; \]  
\[ \Delta = f_3(a) \text{ at } t = \text{const}; \ c = \text{const}. \]  

Then, using these dependencies, obtain a complete dependence (2).

To obtain dependencies (3), a numerical experiment has been carried out with a modelling according to scheme in figure 4. This modeling in a well-known program «Lira» uses three-dimensional finite elements. Thus the modulus of elasticity of finite elements of concrete takes the value \( E_c = 25 \ 000 \text{ MPa}. \) For any dependence (3), it is easy to switch from the basic modulus of elasticity to the modulus of elasticity of the element under considering by its multiplying by a factor equal to these elastic modules ratio.

Figure 5 shows the displacement relationship \( \Delta = f(a) \) at \( C = 10 \text{ mm} = \text{const} \), and in figure 6 at \( C = 49 \text{ mm} \) and fixed values \( t = \text{const} \).

![Figure 5. Displacement dependence \( \Delta = f(a) \) at \( C = 10 \text{ mm} = \text{const} \) and fixed values of \( t = \text{const} \).](image-url)
Figure 6. Displacement dependence $\Delta = f(a)$ at $C = 40 \text{ mm} = \text{const}$ and fixed value $s = \text{const}$.

A comparison of figures 5 and 6 shows that $\Delta = f(a)$ is similar. However, the offset value of the end of the reinforcing bar at $C=40 \text{ mm}$ is much bigger than the similar displacement at $C=10 \text{ mm}$ which confirms the above-mentioned assumption about the dependence of displacement of the end of the reinforcing bar on all three sizes $a, c, t$ (Figure 4).

In order to obtain the approximation dependence, a calculation has been made in the software complex Lira using the three-dimensional finite elements of the model according to Figure 4. A total of 125 analytical models have been counted. Values $t$ ranged from 10 to 30 mm; values $a$ from 10 to 40 mm and values $C$ from 20 to 150 mm. The formula has been obtained as a result of the numerical experiment:

$$\Delta_b = (\alpha_1 C^2 + \alpha_2 C + \alpha_3) \beta_1 C^3 + \beta_2 \gamma_1 C^2 + \gamma_2 \delta_1 + \delta_2 + \delta_3,$$

where the values of approximation coefficients are:

$$\alpha_1 = 1.34850E - 02; \quad \alpha_2 = -2.61701; \quad \alpha_3 = 230.5999993$$

$$\beta_1 = -3.14300E - 05; \quad \beta_2 = 0.00620; \quad \beta_3 = -0.42800$$

$$\gamma_1 = 7.94800E - 07; \quad \gamma_2 = -0.00018; \quad \gamma_3 = 0.00724$$

$$\delta_1 = -3.41300E - 05; \quad \delta_2 = 0.00826; \quad \delta_3 = -0.69060.$$

The maximum error of $\Delta_b$ values obtained by formula (4) compared to the values obtained by calculations using three-dimensional finite elements is 7%, thus enabling usage of formula (4) in practical calculations.

Formula (4) is obtained for base values: modulus of elasticity of concrete $E_{cb,b} = 25000 \text{ MPa}$; modulus of elasticity of steel reinforcement $E_s = 200000 \text{ MPa}$; shear force value $Q_b = 10 \text{ kN}$.

$\Delta$ value for the case is obtained on the base values using the formula:

$$\Delta = \Delta_b \frac{E_{cb,b} Q_b}{E_s Q},$$

Knowing the displacement of the end of the reinforcing bar by the shifting force, we can determine the shear modulus of the reinforcement. This shear modulus of the reinforcement is equivalented to the shear modulus of concrete and the pliability from crumpling is taken into account. This pliability will be taken into account using the coefficient $K_{\text{aug}} < 1$. 
Consider the method of determining the $K_{nag}$ coefficient. For this purpose, consider the deformation of the cantilevered nog (reinforcing bar subjected to shear by a transverse force $Q$ acting perpendicularly to its axis (Figure 7).

\[ \Delta = \frac{Q \cdot l}{G \cdot A}, \]  

(6)

where $A$ – cantilever bar sectional area.

However, the reinforcing bar shifts not only under the action of shear, but also under the action of the crumpling of concrete beneath its surface at the embedment point. The displacement caused by the crumpling of concrete is marked $\Delta_0$ in Figure 7. This displacement previously determined by the empirical formula (1), is determined by the approximation formula (4) based on the research conducted in this article.

The total displacement of the end of the bar is equal to (see Figure 7):

\[ \Delta_{tot} = \Delta + \Delta_0. \]  

(7)

As we are considering the single width cross-section (the relative angle rotation in a normal crack is considered below) then the cantilever length $l$ in Figure 7 should be taken equal to one.

Define the shear modulus of conditional equivalent bar in length $l=1$ as $G_{ekv}$. The displacement of this bar is equal to $\Delta_{tot}$ (i.e. taking into account not only the shift $\Delta$ but also the shift of support $\Delta_0$). The displacement of the equivalent bar would then be equal to:

\[ \Delta_{tot} = \frac{Q \cdot (l = 1)}{G_{ekv} \cdot A}. \]  

(8)

From this it is easy to find $G_{ekv}$.

As we know the shear modulus of the equivalent bar, we determine the $K_{nag}<1$ coefficient:

\[ K_{nag} = \frac{G_{ekv}}{G_s}. \]  

(9)

This coefficient takes into account the displacement of the reinforcing bar due to the crumpling of concrete under its surface. As we know $K_{nag}, G_{ekv}, G_s, G_c$ values it is easy to find the torsion center of the cross-section with a crack:

\[ Z_c = \frac{Z_{crc} b(h - a' - Z_{css} / 2) + 2A_y \alpha_x}{Z_{crc} b + 2A_y \alpha}; \]  

(10)

\[ \alpha = \frac{G_s}{G_b} K_{nag}, \]  

(11)
where $d'$ – the concrete cover (to reinforcement) (see Figure 2,a); $\alpha$ – the ratio of the shear modulus of the reinforcement $G_s$ and the concrete $G_b$, taking into account the compliance of reinforcement in the shifting force direction.

The expression (11) differs from the generally accepted definition of the position of the center of rigidity only in the fact that the ratio of shear modulus $\alpha$ is multiplied by the coefficient $K_{nag}<1$. This coefficient takes into account the crumpling of concrete under the reinforcing bar when a force is applied to it, perpendicular to its axis.

When turning the cross-section about the center of rigidity its external rotation torque $M_t$ is perceived due to the pure torsion resistance $M_{s,b}$ and to the shear resistance when the whole cross-section $M_n$ is turned. The moment perceived by pure torsion is determined by the formula:

$$M_{s,b} = \theta (GJ_b + 2GJ_s),$$

(12)

where $GJ_b$ – torsional stiffness of concrete rectangle with sides $Z_{rec}$ and $b$ about the centre of mass; $GJ_s$ – torsional stiffness of one reinforcing bar.

Moment from shear due to turning will be determined by the formula (see Figure 2):

$$M_s = Q_b Z_b + 2Q_s Z_s + 2Q_a a_s.$$  

(13)

At the same time, the value of the shear force $Q_b$, is determined by the well-known shear formula:

$$Q_b = \Delta_b G_i A_b,$$

(14)

where $\Delta_b$ – the shift of a rectangle with area $A_b=Z_{rec}b$ from the $Q_b$ force $\Delta b$ value is determined according to the scheme in Figure 2,b from the turn to $\theta$ angle:

$$\Delta_b = \theta \cdot Z_b.$$  

(15)

Then $Q_b$ is calculated according to the formula:

$$Q_b = \theta \cdot Z_b G_i A_b.$$  

(16)

Forces $Q_s$ and $Q_a$ are defined similarly:

$$Q_s = \theta \cdot Z_s G_i A_s; \quad Q_a = \theta \cdot a_s G_i A_s.$$  

(17)

It should be noted that in formulas (17) the shear modulus of the reinforcement $G_i$ must be multiplied by the described above $K_{nag}$ coefficient.

We finally obtain the formula for the external torque by substitution values $Q_b, Q_s, Q_a$ by (16) and (17) in the equation (13) taking into account the external moment $M_t$ equal to the sum of moments according to (12) and (13):  

$$M_t = \theta [GJ_b + 2 \cdot GJ_s + Z^2_s G_s A_s + 2 \cdot G_s A_s (Z^2_s + a^2_s)].$$

(18)

All the values in square brackets in formula (18) are known. Therefore, if we know the angular turning $\theta$ it is easy to determine $M_t$ – the part of torque perceived by the concrete part or reinforcing bar. With a known part of torque per the reinforcing bar, it is not difficult to determine the value of the shifting force $Q_s$ and $Q_a$.

The problem with a different amount of longitudinal reinforcement is solved similarly.

After determining the shifting forces, the mutual displacement of the faces of a normal crack is determined by formula (4), but with the known $Q_s$ and $Q_a$ values. In this case $K_{nag}$ for determining $Q_s$ differs from $K_{nag}$ for determining $Q_a$ because values $c, t$ and $a$ for different directions of shifting force differ. Thus, the displacements $\Delta_{loc,s}$ in the direction of the $X$ axis and displacements $\Delta_{loc,a}$ in the direction of the $Z$ axis will be obtained.

The mutual displacement in the crack is determined by the expression:

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The right of equation (19) is multiplied by 2 because concrete is crumpled on both sides by shifting forces.

After determining the full movement by (19) it is not difficult to determine the torsional stiffness $B_{rc}$ of the element with normal cracks located at a distance $l_{rc}$ from each other:

$$B_{rc} = \frac{\Delta_{lrc}}{\Delta_{lrc} + \Delta_{loc}} B_{t,0},$$

(20)

where $\Delta_{lrc}$ – displacement from the torsion of entire block with $l_{rc}$ in length and overall depth of section; $B_{t,0}$ – torsional stiffness of an element without cracks (initial torsional stiffness).

To determine the torsional stiffness using the above mentioned procedure, one has to set several values of the angular turning $\theta$ and to diagram «$M_t-\theta$» by (18). With its help it is easy to determine external torque value $M_t$ corresponding to the present angular turning of cross section $\theta$.

Knowing the $M_t$ value which corresponds to the given angle $\theta$, with formula (18) the part of torque perceived by the shifting force $Q_x$ or $Q_z$ is determined, then the torsional stiffness of the element with a crack is determined by the above mentioned method.

The analysis of numerical calculations and comparison with the results by formula (1) allows us to make the following conclusion for proposing an approximate calculation methodology (technique). Change of displacement $\Delta$ from $C$ parameter becomes linear after the certain value of the concrete top layer $C$.

It is evident in the graph shown in Figure 8.

Figure 8. Dependence of reinforcing bar displacement $\Delta$ on the concrete top layer $C$ at $t=25$ mm; $a=25$ mm: 1 – data obtained from finite element analysis (FEA); 2 – data from formula (1).

In Figure 8 the horizontal line shows the movement value determined by the formula (1) in accordance with [7]. Considering that after the $C$ value of 70 mm the dependence in Figure 8 becomes almost linear and the also the fact that that the depth of cross section of actual beams is unlikely to be less than 100 mm, we take on a linear dependence $\Delta=f(C)$.

Mark the displacement at value $C=0$ by $\Delta_0$, and the displacement equal to the displacement determined by (1) by $\Delta_\infty$. For practical calculations we can take approximately $\Delta_0=\Delta_\infty/2$ taking into account the graph in Figure 8. This assumption is quite reasonable as the experimental formula (1) is derived from the test in the mass concrete (theoretically speaking at a high $C$ value). Having the $\Delta$ values at $C=70$ and at $C=150$ and considering the linear distribution, it is not difficult to determine the value of the $C$ value at which the $\Delta$ value is equal to the $\Delta_\infty$ value. Denote it through $C$. So, for the graph in Figure 8 $C_{\infty}=385$ mm is obtained.

The displacement value at the given value of the concrete top layer $C$ is determined from the obvious expression:
\[
\Delta = \Delta_0 + \frac{\Delta_s - \Delta_0}{C_{\infty}} C .
\]  

(21)

Similar \( \Delta \) values are easily obtained when the shifting force is directed along the upward vertical and in the horizontal direction. Thus, when the shifting force acts in the upward vertical direction, the dependence \( \Delta = f(C) \) has the form shown in Figure 9.

![Figure 9. Dependence of reinforcing bar displacement \( \Delta \) on the value of the concrete top layer \( C \) at \( t=25\,mm; a=25\,mm \). Shifting force is directed along the upward vertical.](image)

When the shifting force acts horizontally, the displacement \( \Delta \) is essentially independent of the size of the upper protective layer and is about 0.01 mm.

Knowing the reinforcement displacement under the action of the shifting force upward vertically, downward vertically and horizontally, it is not difficult to determine \( K_{\text{nag}} \) with formulas (8) and (9) for each of these directions. Knowing the \( K_{\text{nag}} \), it is not difficult to determine the torsional stiffness of an element with a normal crack according to the above method.

3. Conclusions and research perspectives

A new technique has been developed for determining of shifting forces in longitudinal reinforcement and torsional stiffness of reinforced concrete elements with normal cracks. An approximation formula for determining the reinforcement displacement subjected to the shifting force perpendicular to the axis of the reinforcement has been given. To determine the mutual displacement of the normal crack a flat turn of the cross-section about the torsion center has been considered. The external torque is perceived through pure torsion and shear in the longitudinal reinforcement and concrete.

The parts of the external torque applied to the uncracked concrete, the horizontal and vertical components of the shifting forces in the longitudinal reinforcement are proportional to their shear and torsional stiffnesses. After determining the shifting forces in the longitudinal reinforcement, the total displacement in the crack and then the torsional stiffness of the element are determined.

Experimental testing of the proposed method is expected in the future.

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