A $\text{Sim}(2)$ invariant dimensional regularization

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Abstract

We introduce a $\text{Sim}(2)$ invariant dimensional regularization of loop integrals. Then we can compute the one loop quantum corrections to the photon self energy, electron self energy and vertex in the Electrodynamics sector of the Very Special Relativity Standard Model (VSRSM).

The Weinberg-Salam model (SM) is a very successful description of Nature, that is being verified at the LHC with a great precision. Moreover, until now, neither new particles nor new interactions have been discovered at the LHC\cite{1}. This cannot be the whole story, though. The SM assumes that the neutrino is a massless particle, whereas we know that the neutrino is massive in order to describe the observed neutrino oscillations\cite{2}.

If we assume that Lorentz's is an exact symmetry of Nature, we have to introduce new particles and interactions in order to give masses to the observed neutrinos through, for instance, the seesaw mechanism\cite{3}.

A new possibility to have a massive neutrino arises in Very Special Relativity (VSR)\cite{4}. Instead of the 6 parameter Lorentz group, a 4 parameters subgroup ($\text{Sim}(2)$) is assumed to be the symmetry of Nature. $\text{Sim}(2)$ transformations change a fixed null four vector $n_\mu$ at most by a scale factor, so ratios of scalar quantities containing the same number of $n_\mu$ in the numerator as in the denominator are $\text{Sim}(2)$ invariant, although they are not Lorentz invariant. In this way it is possible to write a VSR mass term for left handed neutrinos\cite{5}.

Recently, we have proposed the SM with VSR\cite{6} (VSRSM). It contains the same particles and interactions as the SM, but neutrinos can have a VSR mass without lepton number violation. Since the electron and the electron neutrino form a $SU(2)_L$ doublet, the VSR neutrino mass term will modify the QED of the electron.

A main obstacle in exploring the loop corrections in the VSRSM is the non-existence of a gauge invariant regulator that preserve the $\text{Sim}(2)$ symmetry of the model.

In this letter, we define an appropriate regulator, based on the calculation of integrals using the Mandelstam-Leibbrandt (ML)\cite{7} prescription introduced in
We want to emphasize that our method directly lead to the ML prescription, the only one compatible with canonical quantum field theory. The regulator preserve gauge invariance, a property inherited from the ML prescription, as well as the Sim(2) symmetry.

Then we proceed to compute one loop corrections. We find the divergent and finite part of the vacuum polarization and electron self energy. Moreover we compute the leading correction to the standard QED result for the anomalous magnetic model of the electron.

We want to emphasize that meanwhile no new particles or interactions are discovered at the LHC or elsewhere, we have to consider the VSRSM as a very strong candidate to describe weak and electromagnetic interactions. It contains all the predictions of the SM plus neutrino masses and neutrino oscillations. It is renormalizable (as we show explicitly in this letter) and unitarity of the S metric is preserved. If future experiments validates the predictions of the model, it would be the first evidence of Lorentz Symmetry violation.

1 Mandelstam-Leibbrandt (ML) prescription from a hidden symmetry

In this section we review the results of [9].

Let us compute the following simple integral:

\[ A_\mu = \int dp \frac{f(p^2)p_\mu}{(n \cdot p)} \]

where \( f \) is an arbitrary function, \( dp \) is the integration measure in \( d \) dimensional space and \( n_\mu \) is a fixed null vector \((n \cdot n) = 0\). This integral is infrared divergent when \((n \cdot p) = 0\).

The ML is:

\[ \frac{1}{(n \cdot p)} = \lim_{\varepsilon \to 0} \frac{(p \cdot \bar{n})}{(n \cdot p)(p \cdot \bar{n}) + i\varepsilon} \tag{1} \]

where \( \bar{n}_\mu \) is a new null vector with the property \((n \cdot \bar{n}) = 1\).

To compute \( A_\mu \) we must know what \( f \) is, provide an specific form of \( n_\mu \) and \( \bar{n}_\mu \), and evaluate the residues of all poles of \( \frac{f(p^2)}{(n \cdot p)} \) in the \( p_0 \) complex plane, a difficult task for an arbitrary \( f \).

Instead we want to point out the following symmetry:

\[ n_\mu \to \lambda n_\mu, \bar{n}_\mu \to \lambda^{-1}\bar{n}_\mu, \lambda \neq 0, \lambda \in \mathbb{R} \tag{2} \]

It preserves the definitions of \( n_\mu \) and \( \bar{n}_\mu \):

\[ 0 = (n \cdot n) \to \lambda^2 (n \cdot n) = 0 \]
\[ 0 = (\bar{n} \cdot \bar{n}) \to \lambda^{-2}(\bar{n} \cdot \bar{n}) = 0 \]
\[ 1 = (n \cdot \bar{n}) \to (n \cdot \bar{n}) = 1 \]
We see from (1) that:
\[
\frac{1}{(n \cdot p)} \rightarrow \frac{1}{(n \cdot p)^{\lambda - 1}}
\]
Now we compute \( A_\mu \), based on its symmetries. It is a Lorentz vector which scales under (2) as \( \lambda^{-1} \). The only Lorentz vectors we have available in this case are \( n_\mu \) and \( \bar{n}_\mu \). But (2) forbids \( n_\mu \). That is:
\[
A_\mu = a\bar{n}_\mu
\]
Multiply by \( n_\mu \) to find \((A \cdot n) = a\). Thus \( a = \int dp f(p^2) \). Finally:
\[
\int dp \frac{f(p^2)p_\mu}{(n \cdot p)} = \bar{n}_\mu \int dp f(p^2)
\]
By the same arguments, we can compute the generic integral:
\[
A = \int dp \frac{F(p^2, p \cdot q)}{(n \cdot p)} = (\bar{n} \cdot q)f(q^2, (n \cdot q)(\bar{n} \cdot q)) \tag{3}
\]
\( q_\mu \) is an external momentum, a Lorentz vector. \( F \) is an arbitrary function. The last relation follows from (2), for a certain \( f \) we will find in the following.
Taking the partial derivative respect to \( q_\mu \) in both sides of (3), we obtain that
\[
\frac{\partial A}{\partial q^\mu} n_\mu = \int dp F_{,u} = g(x) =
\]
\[
f(x, y) + 2y \frac{\partial}{\partial x} f(x, y) + y \frac{\partial}{\partial y} f(x, y) \tag{4}
\]
We defined \( u = p \cdot q, x = q^2, y = (n \cdot q)(\bar{n} \cdot q) \). \( (),_u \) means derivative respects to \( u \).
Assuming that the solution and its partial derivatives are finite in the neighborhood of \( y = 0 \), it follows from the equation that \( f(x, 0) = g(x) \). That is the partial differential equation has a unique regular solution.
Now we apply this result to compute integrals that appear in gauge theory loops:
\[
\int dp \frac{1}{[p^2 + 2p \cdot q - m^2]^a} \frac{1}{(n \cdot p)} = (\bar{n} \cdot q)f(x, y)
\]
In this case
\[
g(x) = -2a \int dp \frac{1}{[p^2 - x - m^2]^{a + 1}}
\]
The unique regular solution of (4) is:
\[
f(x, y) = -\frac{1}{y} \left\{ \int dp [p^2 - x - m^2]^{-a} - \int dp [p^2 - x + 2y - m^2]^{-a} \right\}
\]
We can check that \( f(x, 0) = -2a \int dp [p^2 - x - m^2]^{-a - 1} = g(x) \).
In the same way we can compute the whole family of loop integrals:

\[ \int dp \frac{1}{|p^2 + 2p \cdot q - m^2|^a} \frac{1}{((n \cdot p))^b} = (\tilde{n} \cdot q)^b (-2)^b \frac{\Gamma(a + b)}{\Gamma(a) \Gamma(b)} \int_0^1 dt t^{b-1} \int dp [p^2 - q^2 + 2n \cdot q \tilde{n} \cdot t - m^2]^{-a-b} \]

Using dimensional regularization, we obtain:

\[ \int dp \frac{1}{|p^2 + 2p \cdot q - m^2|^a} \frac{1}{((n \cdot p))^b} = (-1)^{a+b} i(\pi)^{\omega} (-2)^b \frac{\Gamma(a + b - \omega)}{\Gamma(a) \Gamma(b)} (\tilde{n} \cdot q)^b \int_0^1 dt t^{b-1} \frac{1}{(m^2 + q^2 - 2(n \cdot q)(\tilde{n} \cdot t))^a + b - \omega}, \quad \omega = d/2 \quad (5) \]

### 2 \( \text{Sim}(2) \) invariant regulator

The prescription to regularize the infrared divergences that we have reviewed in chapter 1, always produces finite results depending on two fixed null vectors \( \bar{n}_\mu, n_\mu \). Moreover it preserves gauge invariance because it respects the shift symmetry of the loop integral \( \int dp f(p_\mu) = \int dp f(p_\mu + q_\mu) \) for arbitrary \( q_\mu \). However ML does not respect \( \text{Sim}(2) \) symmetry of VSRS. Below we show how to remedy this.

We start from the ML result for the integral \((5)\).

We trade \( \bar{n}_\mu \) by \( q_\mu \), i.e. \( \bar{n}_\mu = an_\mu + bq_\mu \). From the conditions: \( \bar{n} \cdot \bar{n} = 0, \bar{n} \cdot n = 1 \) we get \( \bar{n}_\mu = -\frac{q^2}{2(n \cdot q)^2} n_\mu + \frac{q_\mu}{n \cdot q} \). Moreover, we see that \( \bar{n}_\mu \) satisfies the scaling \((2)\) and is real for any value of \( q^2 \) in Minkowsky space. So, all the conditions to apply the procedure reviewed in section 1 are satisfied. Therefore,

\[ \int dp \frac{1}{|p^2 + 2p \cdot q - m^2|^a} \frac{1}{((n \cdot p))^b} = (-1)^{a+b} i(\pi)^{\omega} (-2)^b \frac{\Gamma(a + b - \omega)}{\Gamma(a) \Gamma(b)} \left( \frac{q^2}{2n \cdot q} \right)^b \int_0^1 dt t^{b-1} \frac{1}{(m^2 + q^2 - 2(n \cdot q)(\tilde{n} \cdot t))^a + b - \omega}, \quad \omega = d/2 \quad (6) \]

Notice that now \((6)\) respects the \( \text{Sim}(2) \) invariance of the original integral. The same procedure can be applied to other integrals found in \((9)\). Notice that first we keep \( \bar{n} \) fixed, derive \((5)\) with respect to \( q_\mu \) as many times as necessary and then replace \( \bar{n}_\mu = -\frac{q^2}{2(n \cdot q)^2} n_\mu + \frac{q_\mu}{n \cdot q} \). The rationale for this prescription derives from the observation that we could compute the integral with whatever power of \( p_\mu \) in the numerator using Cauchy theorem of residues in \( p_\mu \) complex plane. In this way it doesn’t matter whether \( \bar{n}_\mu \) depends on \( q_\mu \) or not.

Once we have obtained \((6)\), we notice that it provides a unique analytic continuation of the integral from \( b < 0 \) to \( b > 0 \). Since for \( b < 0 \) we do not need an infrared regulator, we can compute the integral using standard dimensional

\footnote{This is the more general form for \( \bar{n}_\mu \) compatible with reality, right scaling under \((2)\) and \( \bar{n}_\mu \) dimensionless.}

For instance in \( d = 3 \) we must have \( \bar{n}_\mu = a \frac{q^2 n_\mu}{(n \cdot q)^2} + b \frac{n_\mu}{(n \cdot q)} + c \epsilon_{\mu \nu \lambda} \frac{n_\nu}{(n \cdot q)} q_{\lambda} \sqrt{q^2} \) with \( a, b, c \) pure numbers. This fails to be real for \( q^2 < 0 \).
regularization. By integration by parts in the integral over \( t \), we can check that \([6]\) gives the right answer for \( b < 0 \).

3 The model

The leptonic sector of VSRSM consists of three \( SU(2) \) doublets \( L_a = \left( \nu^0_aL, e^0_aL \right) \), where \( \nu^0_aL = \frac{1}{2}(1 - \gamma_5)\nu^0_a \) and \( e^0_aL = \frac{1}{2}(1 - \gamma_5)e^0_a \), and three \( SU(2) \) singlet \( R_a = \nu^0_aR = \frac{1}{2}(1 + \gamma_5)n^0_a \). We assume that there is no right-handed neutrino. The index \( a \) represents the different families and the index 0 say that the fermionic fields are the physical fields before breaking the symmetry of the vacuum.

In this letter we restrict ourselves to the electron family. \( m \) is the VSR mass of both electron and neutrino.

After spontaneous symmetry breaking (SSB), the electron acquires a mass term \( M = \frac{G_e}{\sqrt{2}}v \), where \( G_e \) is the electron Yukawa coupling and \( v \) is the VEV of the Higgs. Please see equation (52) of [6]. The neutrino mass is not affected by SSB: \( M_{\nu_e} = m \).

Restricting the VSRSM after SSB to the interactions between photon and electron alone, we get the VSR QED action. \( \psi \) is the electron field. \( A_\mu \) is the photon field. We use the Feynman gauge.

\[
\mathcal{L} = \overline{\psi} \left( i \left( D + \frac{1}{2} hm^2(n \cdot D)^{-1} \right) - M \right) \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{(\partial_\mu A_\mu)^2}{4}
\]

\[
D_\mu = \partial_\mu - ieA_\mu, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu
\]

We see that the electron mass is \( M_e = \sqrt{M^2 + m^2} \), where \( m \) is the electron neutrino mass.

3.1 Feynman rules

To draw the Feynman graphs we used [11] In the following sections, we have used extensively the program FORM [12].

Figure 1: Feynman rules for one loop computations: electron propagator, photon propagator, \( A_\mu ee \) and \( A_\mu A_\nu ee \) vertex.
4 Photon Self Energy in VSRSM

In this section we present the computation of the photon self-energy. In VSRSM it is given by two graphs:

Applying the $\text{Sim}(2)$ invariant regulator to the addition of the graphs of Figure 2, and after a long calculation, we get:

\[ i\Pi_{\mu\nu} = A(\eta_{\mu\nu}q^2 - q_{\mu}q_{\nu}) + B\left(-q^2 \frac{n_{\mu}n_{\nu}}{(n.q)^2} + \frac{n_{\mu}q_{\nu} + n_{\nu}q_{\mu}}{n.q} - \eta_{\mu\nu}\right) \]  

(7)

with

\[
A = (-ie)^2 \frac{i}{(4\pi)^\omega} \int_0^1 dx \Gamma(2 - \omega) \frac{8x(1 - x)}{(M_e^2 - (1 - x)q^2)^{2-\omega}}
\]

\[
B = -m^2 i \frac{e^2}{4\pi^2} \int_0^1 \frac{dx}{(1 - x)} \log \left[1 - \frac{q^2(1 - x)^2}{M_e^2 - q^2(1 - x)x}\right]
\]

(8)

Here $-e$ is the electron electric charge, $m$ the electron neutrino mass and $M_e$ is the electron mass. $q_{\mu}$ is the virtual photon momentum.

We first notice that $q^\mu\Pi^\mu = 0$ as required by $U(1)$ gauge invariance of the photon field. It is obtained by a straightforward application of the regularized integrals of . Moreover $B(q^2 = 0) = 0$, therefore the photon remains massless. Also the photon wave function divergence is the same as in QED.

5 Electron Self Energy in VSRSM

Here we calculate the electron self-energy. Again we have two graphs contributing to the 2-proper vertex. See Figure 3.

Figure 3: Electron self energy one loop graphs. The second graph vanishes in Feynman gauge.
with:

\[ C = (-ie)^2m^2\frac{i}{16\pi^2} \int_0^1 dx (1-x)^{1-\omega} \ln \left( 1 + \frac{q^2(1-x)}{(M^2 - q^2 - i\varepsilon)} \right) + \frac{2i(4\pi)^{-\omega}}{\Gamma(2-\omega)} \int_0^1 dx \frac{\Gamma(2-\omega)}{\mu^2 x - x(1-x)q^2 + (M^2 + m^2)(1-x) - i\varepsilon}^2 \]

\[ D = -2(-ie)^2(\omega - 1)i(4\pi)^{-\omega} \int_0^1 dx \frac{\Gamma(2-\omega)}{\mu^2 x - x(1-x)q^2 + (M^2 + m^2)(1-x) - i\varepsilon}^2 \]

\[ E = (-ie)^22\omega Mi(4\pi)^{-\omega} \int_0^1 dx \frac{\Gamma(2-\omega)}{\mu^2 x - x(1-x)q^2 + (M^2 + m^2)(1-x) - i\varepsilon}^2 \]

6 Electron-Electron-Photon Proper vertex \( \Gamma^\mu(p + q, p) \)

In this subsection we discuss the 3 points proper vertex and verify the Ward-Takahashi identity. This is an important test of the gauge invariance of the regulator. The one loop contribution to \( \Gamma^\mu(p' = p + q, p) \) consists of the addition of 3 graphs (Figure 4):

Figure 4: One loop contribution to the 3 points proper vertex

As a result of the shift symmetry which is respected by the regulator, \( \int dp \psi(p_{\mu}) = \int dp \psi(p_{\mu} + q_{\mu}) \) for arbitrary \( q_{\mu} \), we can prove the Ward-Takahashi identity:

\[ -iq_{\mu} \Gamma^\mu(p + q, p) = S^{-1}(p + q) - S^{-1}(p) \]  

(11)

Here \( S(p) = \frac{1}{p - M - \Sigma(p)} \) is the full electron propagator and \( \Gamma^\mu(p + q, p) \) is the three proper vertex.

Below we explicitly verified that the pole at \( d = 4 \) satisfies \[ 11 \] \[ ^2 \]

Pole contribution:

\[ \mathcal{P}\Sigma(q) = -(-ie)^2 \frac{1}{16\pi^2} \left\{ 2m^2 \frac{\not{p}}{n.q} - \not{q} + 4M \right\} \frac{1}{2 - \omega} \]  

(12)

\[ ^2 \text{The finite part of the Ward-Takahashi identity is true also, but the computation is too long to show it in this letter.} \]
The divergent piece satisfies the Ward identity:

\[ q_\mu \mathcal{P} \Gamma^\mu(p + q, p) = \mathcal{P} \Sigma(p) - \mathcal{P} \Sigma(p + q) = -(ie)^2 \frac{1}{16\pi^2} \frac{1}{2 - \omega} \left( \gamma_\mu + 2m^2 \frac{n_\mu}{n.p.(p + q)} \right) \] (13)

The on-shell proper vertex can be written as follows:

\[ \bar{u}(p + q)^{\mu} \{ G_2 [-i\sigma_{\mu}q_\nu, \lambda i] + G_3 \lambda \mu q_\nu + F_3 \lambda \mu q_\nu \lambda + \lambda \mu F_1 + F_2 \lambda \mu q_\nu \} u(p) \] (14)

where:

\[ \lambda \mu = \gamma_\mu + \frac{m^2}{2} \frac{\lambda q_\mu}{n.p(n.p + n.q)}, \quad Q_\mu = q_\mu - q^2 \frac{n_\mu}{n.q} \]

\( F_1, F_2, F_3, G_2, G_3 \) are form factors (Lorentz scalar combinations of \( n_\mu, p_\mu, q_\mu \)).

Under the Sim(2) scaling \( n_\mu \to \lambda n_\mu, F_1, F_2 \) are invariants, \( F_3 \to \lambda^{-2} F_3, G_3 \to \lambda^{-1} G_3, G_2 \to \lambda^{-1} G_2 \).

In the Non-Relativistic (NR) limit we get Table 1, keeping terms that are at most linear in \( q_\mu \).

| NR limit | Form factor |
|----------|-------------|
| \( 2M_c \phi_j^a \phi^b A_0 \) | \( F_1(0) \) |
| \( \frac{3m^2}{4M_c} \epsilon_{ijk} \phi_i^a \phi^b \phi^c \phi_j^a q_k A_0 \) | \( F_1(0) \) |
| \( \frac{3m^2}{4M_c} \epsilon_{ijk} \phi_i^a \phi^b \phi^c q_k A_0 \) | \( F_1(0) \) |
| \( \frac{m^2}{2M} (\epsilon_{ijk} \phi_i^a \phi^b \phi^c) q_{ij} A_i \) | \( G_2(0) \) |
| \( -2m_0 \epsilon_{ijk} \phi_i^a \phi^b \phi^c q_{ij} A_i \) | \( F_2(0) \) |
| \( -i \epsilon_{ijk} n_k \frac{m^2}{2} \phi_j^a \phi^b \phi^c q_{ij} A_i \) | \( G_2(0) \) |
| \( i(2M_c \epsilon_{ijk} q_k \phi_i^a \phi^b \phi^c + 2M_c \epsilon_{ijk} \phi_i^a \phi^b \phi^c) A_0 q_i \) | \( G_2(0) \) |
| \( 2M_c \epsilon_{ijk} q_k \phi_i^a \phi^b \phi^c Q_\mu A^\mu \) | \( G_3(0) \) |
| \( \epsilon_{ijk} n_k \phi_j^a \phi^b \phi^c q_{ij} A_i \) | \( F_2(0) \) |
| \( 4M_c \epsilon_{ijk} n_k \phi_j^a \phi^b \phi^c q_{ij} A_i \) | \( G_2(0) \) |
| \( 2M_c \epsilon_{ijk} q_k \phi_i^a \phi^b \phi^c A_0 q_i \) | \( F_3(0) \) |
| \( i \epsilon_{ijk} \phi_i^a \phi^b \phi^c A_i q_j \) | \( F_2(0) \) |
| \( -i \frac{m^2}{2M} \epsilon_{ijk} \phi^a \phi^b \phi^c \phi_j^a A_0 q_j \) | \( F_1(0) \) |

Table 1: In the right column we list the form factor. In the left column we have the NR limit of the matrix element accompanying the form factor in (14). All form factors are evaluated at \( q_\mu = 0 \). Here \( A_0 \) is the electric potential and \( A_i \) is the vector potential. \( \phi_{\mu} \) is a two dimensional constant vector that corresponds to the NR limit of the Dirac spinors.
To show the power of the Sim(2) invariant regularization prescription presented in this letter, we will compute the one loop contribution to the (isotropic)anomalous magnetic moment of the electron. It is given by $F_2(0) - 2n_0MG_2(0) - 4F_3(0)M_en_0^2i$ (See rows 11, 5 and 9 of Table 1).

Introduce the following integrals:

$$\int dk(n.k + n.q)^{a_1}(n.k)^{a_2}((k - p)^2)^{a_3}(k^2 - M_e^2)^{a_4}((k + q)^2 - M_e^2)^{a_5} = I(a_1, a_2, a_3, a_4, a_5)$$

$$\int dk(n.k + n.q)^{a_1}(n.k)^{a_2}((k - p)^2)^{a_3}(k^2 - M_e^2)^{a_4}((k + q)^2 - M_e^2)^{a_5}k_\mu = I_{11}(a_1, a_2, a_3, a_4, a_5)p_\mu + I_{12}(a_1, a_2, a_3, a_4, a_5)q_\mu + I_{13}(a_1, a_2, a_3, a_4, a_5)n_\mu$$

$$\int dk(n.k + n.q)^{a_1}(n.k)^{a_2}((k - p)^2)^{a_3}(k^2 - M_e^2)^{a_4}((k + q)^2 - M_e^2)^{a_5}k_\mu k_\nu = I_{21}(a_1, a_2, a_3, a_4, a_5)\eta_{\mu\nu} + I_{22}(a_1, a_2, a_3, a_4, a_5)p_\mu p_\nu + I_{23}(a_1, a_2, a_3, a_4, a_5)q_\mu q_\nu +$$

$$I_{24}(a_1, a_2, a_3, a_4, a_5)n_\mu n_\nu + I_{25}(a_1, a_2, a_3, a_4, a_5)(p_\mu q_\nu + p_\nu q_\mu) +$$

$$I_{26}(a_1, a_2, a_3, a_4, a_5)(p_\mu n_\nu + p_\nu n_\mu) + I_{27}(a_1, a_2, a_3, a_4, a_5)(n_\mu q_\nu + n_\nu q_\mu)$$

We get:

$$F_2(0) - 2n_0MG_2(0) - 4F_3(0)M_en_0^2i =$$

$$-4ie^2M^2\{I_{22}(0, 0, -1, -1, -1) - I_{11}(0, 0, -1, -1, -1)\} -$$

$$2ie^2m^2\{ -I(0, 0, -1, -1, -1) - 2I_{22}(0, -1, -1, -1, -1)Mn_0 + 3I_{11}(0, 0, -1, -1, -1)\}$$

Evaluating the integrals according to the Sim(2) invariant prescription to $o(m^2)$, we get:

$$F_2 - 4F_3M_en_0^2i - 2G_2n_0M = \frac{\alpha}{2\pi}$$

where $\alpha$ is the fine structure constant. Therefore to this order the QED result holds.

Notice that already at tree level, the model predicts the existence of an anisotropic electric moment of the electron, corresponding to the second line of the list and an anisotropic magnetic moment of the electron, corresponding to the fourth row of the list, both of the order of $\frac{e^2}{M_e^2}$. The electric dipole moment is:

$$|\vec{p}| = \frac{3e}{4M_e}m^2|\vec{s} \times \vec{h}| \leq \frac{3}{8}\lambda e m^2 M_e^2$$

where $\lambda = 2.4 \times 10^{-12}m$ is the Compton wave length of the electron.

Using the best bound on the electric dipole moment of the electron [13], $|\vec{p}| < 8.7 \times 10^{-29}e\text{cm}$, we get:

$$\frac{m^2}{M_e^2} < 9.7 \times 10^{-19}$$
For the muon $\lambda = 1.17 \times 10^{-14} m$. Using the best bound on the muon electric dipole moment\[14\], $|\vec{p}_\mu| < 1.8 \times 10^{-19} e\cdot cm.$, we get:

$$ \frac{m_\mu^2}{M_\mu^2} < 4 \times 10^{-7} $$

Bounds using the experimental values of the magnetic moments are much weaker\[15\].

The $\text{Sim}(2)$ invariant regularization opens the way to explore the full quantum possibilities of VSR. They should be systematically studied, in Particle Physics models as well as in Quantum Gravity models.

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