OPTIMAL RESIDENTIAL LOAD CONTROL COMPARISON USING LINEAR PROGRAMMING AND SIMULATED ANNEALING FOR ENERGY SCHEDULING

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ABSTRACT

Many tariffs are used for electricity pricing. Electricity bills can be reduced by choosing the optimal working hours of appliances and for appropriate tariffs in smart homes. In this study, Linear Programming (LP) and Simulated Annealing (SA) methods are realized and compared each other to find minimum bill. The multi-time tariffs that have different energy unit prices at different times of the day are preferred. Energy Management System (EMS) shifts time slots of some appliances to cheap energy unit prices. Optimal load control is applied to prevent high peak demand because there may be overload in system because of shifting of working hours of appliances. Mathematical model of the problem is constructed and LP method is solved by GAMS and SA technique is realized by C# program. The cost table consisting of the energy unit prices of each time slots is used as input. Optimum electricity cost, working hours of the appliances and peak to average ratio are achieved by two different solutions and the results are compared. According to result, LP gives lower cost than SA.

Keywords: Linear programming, Simulated annealing, Smart home, Energy management system, Load control

1. INTRODUCTION

Reducing energy cost is one of the main topics in energy management for smart buildings. Scheduling of the appliances can be realized by using optimization techniques in smart home energy management system (HEMS). Electricity costs can also be reduced by shifting usage time of appliances in smart homes. However, shifting of high load of appliances to nighttime tariff causes peak to average ratio (PAR) to increase. To schedule household appliances, some mathematical based and heuristic methods optimization techniques are used for optimizing electricity cost [1].

The Advanced Metering Infrastructure (AMI) devices enable to communicate between power utilities and smart home users by accepting instruction information and sending data information [2-3]. Thus, HEMS schedule working times of household appliances. Recently, diverse decision support tools have been notified to optimize and implement scheduling appliances for residential consumers in smart homes [4-12].Demand response (DR) strategies of power market are mainly named as a price based DR or incentive based DR. Emergency demand response program (EDRP), direct load control (DLC), capacity/ancillary service program, interruptible load program (ILP), and demand side bidding (DSB) are classed in incentive based DR. Time of use pricing (TOU), critical peak pricing (CPP), and real time pricing (RTP) are categorized in price based DR [13].

Classical optimization methods are mainly divided into two parts as deterministic and heuristic methods. Branch and bound, branch and cut, branch and price, constraint programming (CP), dynamic programming (DP), A’, Iterative Deeping A’ (IDA’) are the parts of deterministic methods. Heuristic algorithms may divide into metaheuristics and problem-specific heuristics [14].

The mathematical model with under certain constraints created for a specific form in the purpose of profit maximization or cost minimization by making the most efficient use of scarce resources is called
as LP. Some assumptions need to be provided to minimize the loss of information in mathematical expression of a real system. The assumption of linearity shows that the contribution of each decision variable to the objective function is proportional to the value of that decision variable unlike the values of other decision variables. The assumption of certainty is that each parameter and coefficients are known in the LP model. The assumption of summability is a natural consequence of the assumption of linearity while expressing the sum of the contributions formed according to the value of the decision variables. The assumption of severability is the ability of decision variables to take continuous values [15,16]. By using conventional mathematical optimizations, the optimal DR solution is determined in [17]. A non-linear programming (NLP) approach is used to solve DR problem. Mixed Integer Linear Programming (MILP) is an exact optimization method. It can be used for smart home energy scheduling. There are many researches about energy scheduling using MILP [2, 3, 18-21].

Tariffs may vary for each country. There are single and multi-time tariffs for residential in Turkey. Single time tariff has same price for all time slots although multi-time tariff has three different prices for daytime, peak and nighttime slots. Optimal load control is a necessity because most of appliances in smart home may lean to work the cheapest tariff, which is nighttime tariff. This tendency can cause peak load demand increasing. Reducing and shifting consumption is aim of residential load management programs [5]. Optimal residential load control is also one of the subject that is worked on by many researchers [8, 22-24].

The remaining parts of the paper are structured as follows: In the following section; objective function and assumptions of problem, constraints of household appliances, optimal load control, and algorithm of SA are described. The third part of this paper consists from simulation and results of electricity cost and PAR values. The last section explains the results and the contribution of the study.

2. METHODOLOGY

Objective function, the constraints for scheduling appliances, algorithm of SA, and optimal load control system are presented in this section. In addition, formulations of LP method and assumptions are described.

2.1. Objective Function of the Problem

The appliance can work during 24 hours within some constraints, j represents the appliances and i represents the time slots. Minimum cost function is shown as:

$$\min \sum_{i \in I} \sum_{j \in J} C_{ij} \cdot x_{ij}$$  \quad (1)

Let us assume binary variable x indicate turning “On”

\(x_{ij} = 1; \) jth appliance is used at ith time slot

\(= 0; \) else

\(P_j = \) power of jth appliance

\(t = \) working hours of appliances

\(E_j = \) the energy consumption of jth appliance consumes at a single time slot (kWh)

$$E_j = P_j \cdot t$$  \quad (2)
and \( a_i \) is the price of \( i \)th time slot (kr/kWh).

\( C_{ij} = \text{the cost (kr) of } j \text{th appliance consumes at } i \text{th time slot} \)

\[ C_{ij} = E_j \cdot a_i \]  

(3)

Multi-time tariff prices without taxes are given in Table 1 for Turkey. All prices have been shown without taxes through paper. Power values of appliances and how long will they work are shown in Table 2.

| Tariffs   | Time Slots | Price(kr/kWh) |
|-----------|------------|---------------|
| Daytime   | 06-17      | 43.8686       |
| Peak      | 17-22      | 63.8935       |
| Nighttime | 22-06      | 27.8547       |

Table 1. Residential multi-time tariff prices without taxes in Turkey since 1/1/2019[25]

| Appliances   | j | Power(kW) | Hours of work |
|--------------|---|-----------|---------------|
| Washing Machine | 1 | 0.8       | 3             |
| Dishwasher   | 2 | 1.8       | 2             |
| Air Conditioner | 3 | 2         | 3             |
| TV           | 4 | 0.2       | 2             |
| Iron         | 5 | 2.5       | 1             |
| Oven         | 6 | 2         | 2             |
| Refrigerator | 7 | 0.9       | 24            |

Table 2. Power information of household appliances

2.2. Constraints of scheduling appliances using LP

Appliances are represented by \( j \) and appliances are washing machine, dishwasher, air conditioner, television, iron, oven, refrigerator respectively. Equation 4 means every appliance has to work at least one time in a day. In our model, working hours of some appliances can be any hour, others’ potential working hours are determined by user. In example, washing machine, dishwasher, refrigerator can be work anytime in a day. Air conditioner has to work 3 hours between 12:00 - 18:00. TV works 2 hours between 20:00 - 24:00. Iron works one hour between 09:00 - 24:00 while oven works 2 hours between 09:00 - 21:00 according to equations 9 and 10. Equation 11 means refrigerator works 24 hours.

\[ \sum_{i=1}^{24} x_{ij} \geq 1, \forall j \]  

(4)

\[ \sum_{i=1}^{24} x_{ij} = 3, \forall j = 1 \]  

(5)

\[ \sum_{i=1}^{24} x_{ij} = 2, \forall j = 2 \]  

(6)
\begin{align}
\sum_{i=12}^{17} x_{ij} &= 3, \forall j = 3 \\
\sum_{i=20}^{23} x_{ij} &= 2, \forall j = 4 \\
\sum_{i=9}^{23} x_{ij} &= 1, \forall j = 5 \\
\sum_{i=9}^{20} x_{ij} &= 2, \forall j = 6 \\
\sum_{i=1}^{24} x_{ij} &= 24, \forall j = 7
\end{align}

2.3. Algorithm of SA

The objective of algorithm is to find best hours to turn household appliances. SA algorithm relates an analogy between the optimization problem and the physical annealing process: X corresponds solutions, C corresponds the cost of energy, T represents temperature and it simulates cooling process. Algorithm compares time slots. If compared one is smaller, it is considered a new solution. If it is bigger and the possibility of acceptance is realized, then it is considered as new solution. Initial temperature is considered as \( T_0 = 30 \), cooling ratio \( P_r = 0.85 \), temperature length \( T_L = 24 \) and algorithm is as follows [26]:

Select an initial state \( X_i \) and temperature \( T_0 \);

Repeat

for \( i = 1 \) to \( T_L \) do

Generate a new state \( X_n \) by applying a small randomly generated perturbation;

Calculate the change in energy cost \( \Delta C = C(X_n) - C(X_i) \);

if \( \Delta C \leq 0 \) then

The new state is accepted as the starting point for the next move: \( X = X_n \);

else if \( \Delta C > 0 \) then

The new state is accepted with probability

\[ P_r = \exp(-\Delta C / T) \]

end if

end for
Decrease the temperature monotonically \((T = Pr \times T)\); until stopping criterion is reached.

### 2.4. Optimal Load Control

Most of appliances are supposed to work at cheaper tariff to minimize electricity cost when multi-time tariff is used in optimization model. This may lead to an increasing peak to average ratio (PAR). Let assume \(L\) load at hour of \(h \in H = \{1, \ldots, H\}\) where \(H=24\). The daily peak load is:

\[
L_{\text{peak}} = \max_{h \in H} L_h
\]

and the average load is:

\[
L_{\text{avg}} = \frac{1}{H} \sum_{h \in H} L_h
\]

Also, PAR is calculated by [5]:

\[
\text{PAR} = \frac{L_{\text{peak}}}{L_{\text{avg}}} = \frac{H \max_{h \in H} L_h}{\sum_{h \in H} L_h}
\]

If hourly cost is greater than 0.9 TL, up to two appliances will work to reduce PAR for proposed study. Let us assume \(a_i\) is hourly cost value, this operation is shown like:

\[
\text{if } (a_i) > 0.9, \text{ then } \sum_{j=1}^{j} x_{ij} \leq 2, \forall i
\]

### 3. SIMULATION AND RESULTS

In this section, electricity costs and PAR values are presented with some related figures and tables. The simulation part of this study consists of two parts. The first part explains scheduling appliances by both optimization methods. The second part explains hourly load values of system and peak values for both scenarios.

#### 3.1. Scheduling Appliances

Use of household appliances during day is shown in Figure 1 and Figure 2 for Scenario 1 and 2.
The applied scenarios are represented from which scenario number is shown in Table 3[27].

**Table 3. Optimization scenarios**

| Scenario Number | Applied Mode                                      |
|------------------|---------------------------------------------------|
| Scenario 0       | Single time tariff                                |
| Scenario 1       | LP optimized and optimal load control is not applied |
| Scenario 2       | LP optimized and optimal load control is applied  |
| Scenario 3       | SA optimized and optimal load control is not applied |
| Scenario 4       | SA optimized and optimal load control is applied  |

Use of household appliances during day is shown in Figure 3 and 4 for Scenario 3 and Scenario 4.
3.2. Peak Load Control

Peak load for optimal load control is not applied (Scenario 1) and it is obtained as 4.4 kW while for load control is applied (Scenario 2) and it is obtained as 3.4 kW by LP optimization. Peak load for optimal load control is not applied (Scenario 3) and it is obtained as 6.2 kW and for load control is applied (Scenario 4) and it is obtained as 3.5 kW by SA algorithm. Load of appliances for all scenarios is given in Figure 5.
3.3. Electricity Costs Using LP

Single time tariff price is 43.3187 kr/kWh and energy consumption is 40.5 kWh for the proposed system. Therefore, electricity cost for single time tariff is obtained as 17.544 TL for one day. LP optimizes the system as electricity cost is 15.838 TL for optimal load control is not applied (Scenario 1) for one day. Multi-time tariff with LP optimization and optimal load control is not applied provides 9.72% saving than single time tariff. LP optimizes the system as electricity cost is 15.911 TL for optimal load control is applied (Scenario 2) for one day. Multi-time tariff with LP optimization and optimal load control is applied scenario provides 9.3 % saving than single time tariff.

3.4. Electricity Costs Using SA

The system which is optimized by SA method and optimal load control is not applied (Scenario 3) it shows same performance as LP optimized and optimal load control is not applied(Scenario 1).Scenario 3 performs as 9.72% saving than single time tariff. On the other hand, SA is optimized and load control is applied scenario costs higher than LP optimized and load control is applied to system. Electricity cost for Scenario 4 is 16.31 TL for one day. Even so, it saves 7.03% than single time tariff.

Electricity costs are same in LP and SA if the scenario which the systems do not use optimal load control is realized. On the other hand, Figure 6 shows electricity costs are significantly higher in SA algorithm than LP when the systems utilize optimal load control. Electricity cost for all scenarios applied by LP model and SA algorithm are less than the cost when user uses electricity with single time tariff.
3.5. Comparison of PAR Values

Multiple scenarios are tested considering optimal load control, tariff sorts and optimization techniques. PAR values are nearly same for both optimization methods but SA optimized system has a bit more PAR than LP when both systems use optimal load control.

The load control system reduces PAR in Scenario 2 as 22.7 % than Scenario 1 for LP optimization. Therefore, PAR for Scenario 1 and Scenario 2 are respectively,

$$\text{PAR} = \frac{L_{\text{peak}}}{L_{\text{avg}}} = \frac{4.4}{1.687} = 2.607$$ \hspace{1cm} (16)

$$\text{PAR} = \frac{L_{\text{peak}}}{L_{\text{avg}}} = \frac{3.4}{1.687} = 2.015$$ \hspace{1cm} (17)

The load control system reduces PAR in Scenario 4 as 43.5 % than Scenario 3 for SA optimization. Therefore, PAR for Scenario 3 and Scenario 4 are respectively,

$$\text{PAR} = \frac{L_{\text{peak}}}{L_{\text{avg}}} = \frac{6.2}{1.687} = 3.675$$ \hspace{1cm} (18)

$$\text{PAR} = \frac{L_{\text{peak}}}{L_{\text{avg}}} = \frac{3.5}{1.687} = 2.074$$ \hspace{1cm} (19)

However, when the systems do not use optimal load control, SA has higher PAR values 1.41 times than LP. Graphically comparison of PAR is shown in Figure 7.

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**Figure 6.** Comparison of electricity costs of LP and SA methods
Scenario 2 gives the best PAR value and Scenario 1 and 3 gives optimum electricity cost as it is seen in Table 4.

### Table 4. Comparison electricity costs and PAR for all scenarios

| Scenario Number | Applied Mode                               | Electricity Cost (TL) | PAR    |
|-----------------|--------------------------------------------|-----------------------|--------|
| Scenario 0      | Single time tariff                         | 17.544                | -      |
| Scenario 1      | LP optimized and optimal load control is not applied | 15.838                | 2.607  |
| Scenario 2      | LP optimized and optimal load control is applied | 15.911                | 2.015  |
| Scenario 3      | SA optimized and optimal load control is not applied | 15.838                | 3.672  |
| Scenario 4      | SA optimized and optimal load control is applied | 16.310                | 2.704  |

### 4. Conclusions

Minimizing electricity costs in smart homes requires some optimization techniques to utilize in HEMS. In this study, the optimization methods SA and LP are applied to the system to shift the load of the devices from high cost time slots to low cost time slots. This can cause overload during low cost time slots. In order to avoid overloading, optimal load control is implemented for both solutions in scenario 2 and 4. In the case of load control is applied (Scenario 2 and 4), the electricity cost of LP is significantly lower than that of SA. PAR values of them are nearly same.
The electricity costs are same in scenario 1 and 3 when load control is not performed. On the other hand, PAR value is considerably higher in scenario 3 than scenario 1. All optimized scenarios (using multi-time tariff) gives better result than Scenario 0 (using single time tariff). Scenario 2 gives the best result if low PAR value is determined as the most important criterion and Scenario 1 when low electricity cost is the most important criterion. In any case, the results of LP are better than SA. In scheduling and load control problem, LP method might have more parameters and variables. In this case, mathematical based methods could not be enough. For this reason, a metaheuristic algorithm is used the problem of this study. For the future work, parameters that are more complicated, variables, and decision variables can be handled.

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