Research on the Modeling of Optimal Allocation of Engineering Machinery Groups for Road Emergency Maintenance

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Abstract. To solve the problem of allocation to various places of engineering machinery groups for road emergency maintenance, the characteristics of emergency maintenance were analyzed and the emergency maintenance process queue network and Petri net model were built to simulate the emergency maintenance process. Then the calculation method of maintenance time was put forward, and the mathematical model of optimal allocation of engineering machinery groups for road emergency maintenance is established, which provides the basis for solving the optimal allocation scheme of the engineering machinery groups.

1. Introduction

Natural disasters often lead to road damage, and engineering machinery is becoming indispensable in road emergency rescue. When the road is damaged by a large natural disaster, it is usually repaired by a number of engineering machines of different types, and the solution to allocate the machinery groups to complete the maintenance task in the shortest time is urgently demanded. Most of the existing researches on the modelling of the allocation of maintenance resources for emergency rescue use the value coefficient method [1-2] to evaluate the efficiency of maintenance, lacking consideration of the maintenance process.

Using Petri nets to simulate the earthwork process has gradually aroused the interest of researchers [3-4]. Petri net is one of the main methods for modelling and analysing discrete event systems. In recent years, Petri nets have been widely used in the simulation of discrete event systems such as design and manufacturing, network system and engineering maintenance. However, the working face is always limited when roads are damaged, and the machines often need to work in a queue under the condition of multiple machines working at the same time. Increasing the number of machines could speed up the maintenance process, but it would cause inefficiency and waste resources due to queuing. Therefore, it is particularly important to allocate the engineering machinery group in efficient ways. According to the existing research, queuing theory is an effective method to solve the above problems. Based on the analysis of the road maintenance process, this paper uses Petri nets and queuing theory for modelling, and obtains the time calculation expression of emergency maintenance, which provides a basis for the optimal allocation of the engineering machinery groups.
2. Methods of road emergency maintenance

The main types of road damages are as follows:

- Earth and stone obstruction, such as landslides and debris flow caused by the accumulation of earth and stone, blocking the road
- Roadway damage, such as the subsidence of the roadway, tensile crack, deformation and dislocation
- Subgrade damage, such as subgrade subsidence and subgrade collapse

The purpose of road emergency maintenance is to restore passage in the shortest time. There are some commonly used road emergency maintenance engineering machines such as bulldozers, excavators, loaders. All the machinery work together to dig, load, transport and push in a certain process to complete the emergency maintenance task. The types and main functions of engineering machinery for various types of functions are shown in table 1.

| Types of damage          | Machinery | Usage                        |
|--------------------------|-----------|------------------------------|
| Earth and stone obstruction | Bulldozer | Transferring earth and stone |
|                          | Excavator | Grading                     |
|                          | Loader    |                             |
| Roadway damage           | Bulldozer | Excavating the damaged roadway |
|                          | Excavator | Backfilling                 |
|                          | Loader    | Grading                     |
| Subgrade damage          | Bulldozer | Excavating the damaged subgrade |
|                          | Excavator | Backfilling in layers       |
|                          | Loader    | Grading                     |

3. Road emergency maintenance Petri net model

The road emergency maintenance is a dynamic discrete event, and the overlapping relation between the maintenance processes can be expressed by the flow relation between transitions in Petri net [5].

3.1 The basic definition of Petri net

A timed Petri net is a 5-tuple, $PN = (P, T, F, M_0, DI)$ where:

- $P = \{p_1, p_2, ..., p_m\}$ is a finite set of places.
- $T = \{t_1, t_2, ..., t_n\}$ is a finite set of transitions.
- $F \subseteq (P \times T) \cup (T \times P)$ is a set of arcs (flow relation).
- $M_0$ is the initial marking.
- $DI$ is a function of time defined on the transition set $T$. For $t \in T$, $DI(t) = a$ represents that it takes time $a$ of occurrence of transition $t$ [6].

Places set and transition set are the basic components of directed net from which flow relation is constructed. Each place represents a resource whose flow is defined by the flow relationship. A Petri net simulation model is built by taking the engineering machines as the element of the places and the state change process of the machines and the earthwork as the transitions, which can describe the dynamic change of the emergency maintenance system.
3.2 The basic principle of Queuing theory

When machines of the same kind carries out the same work, we can regard the earthwork to be constructed as the server, the maintenance machines as the customers waiting for the service, and the maintenance process as receiving service. Assuming that the arrival rate of engineering machines conforms to the Poisson distribution, a M/M/1/m queuing system is formed between the machines and the working face, that is, the number of server is 1, the customer source is quota M, the arrival time interval is independent of each other and conforms to the negative exponential distribution, so do the service time interval of the server [7]. The queuing principle of loaders is shown in figure 1 as an example.

![Figure 1. Queuing principle of loaders](image)

The main indexes of the system are as follows:

- Average arrival rate $\lambda$, Average arrival time interval $1/\lambda$.
- Average service rate $\mu$, Average service time $1/\mu$.
- Probability that there are zero unit in the system $P_0 = \sum_{n=0}^{m} \frac{m!}{(m-n)!} \left(\frac{\lambda}{\mu}\right)^n$.

3.3 Petri net model

Take the emergency maintenance of earth and stone obstruction as an example to establish a Petri net model. When there are earth and stone obstructed in the road, a single excavator is usually allocated to cooperate with loaders to transfer earth and stone. Next, bulldozers follow to grade. There is no queue when loaders and bulldozers unload soil. When the earthwork amount of loading, unloading and grading reached the task limit, the simulation can be regarded as the end. Its Petri net model is shown in figure 2, and the descriptions of the model's places and transitions are shown in table 2.
Figure 2. Petri net model of emergency maintenance of earth and stone obstruction

| Places                  | Transitions       |
|-------------------------|-------------------|
| P1: soil to be loaded   | T1: soil excavation |
| P2: excavator           | T2: soil unloading |
| P3: Loaders in queue    | T3: loaders queuing |
| P4: laden               | T4: end of loading |
| P5: unstow              | T5: bulldozing    |
| P6: Unloaded soil       | T6: soil unloading |
| P7: soil to be graded   | T7: bulldozers queuing |
| P8: laden               |                   |
| P9: unstow              |                   |
| P10: bulldozers in queue|                   |
| P11: graded soil        |                   |

Table 2. Definitions of places and transitions

4. Allocation model of engineering machinery groups

4.1 Posing problem
A sudden disaster results in earth and stone obstruction, roadway damage and subgrade damage respectively, whose earthwork volume is $Q_1, Q_2, Q_3$ respectively. There are engineering machinery groups formed by bulldozers, excavators and loaders, whose amount is $T, W, Z$ respectively. The cycle time of a single operation is respectively $1/\lambda_T, 1/\lambda_W, 1/\lambda_Z$ and the lading time is respectively $1/\mu_T, 1/\mu_W, 1/\mu_Z$. The groups allocation scheme for each task can be expressed as:

$$X = \begin{bmatrix} T_1 & W_1 & Z_1 \\ T_2 & W_2 & Z_2 \\ T_3 & W_3 & Z_3 \end{bmatrix}$$

Consider how to allocate the groups in each task to minimize the time to complete the overall task of emergency maintenance. Since the total task completion time is equal to the maximum completion time of each task. The objective function of the optimal allocation model of engineering machinery group in emergency maintenance is:

$$\min t = \max \{t_1, t_2, t_3\}$$

4.2 Calculation method of maintenance time
It is known that the time interval of same kind of machines' arrival in the queuing system conforms to negative exponential distribution, and the lading time of machines also conforms to negative exponential
distribution. According to the principle of queuing theory and Road emergency maintenance Petri net model, the time of emergency maintenance of earth and stone obstruction is:

\[
    t_1 = \frac{Q_{IZ}}{\mu_z} \left\{ 1 - \left[ \sum_{n=0}^{Z-1} \frac{Z!}{(Z-n)!} \left( \frac{\lambda_z}{\mu_z} \right)^n \right]^{-1} \right\} + \frac{Q_{IT}}{\mu_r} \left\{ 1 - \left[ \sum_{n=0}^{T-1} \frac{T!}{(T-n)!} \left( \frac{\lambda_r}{\mu_r} \right)^n \right]^{-1} \right\}
\]  

(1)

Where, \( Q_{IZ} \) and \( Q_{IT} \) are respectively the loading and bulldozing earthwork volume, subjected to \( Q_{IT} + Q_{IT} = Q_1 \).

Similarly, the emergency maintenance of roadway damage can be regarded as three queuing systems in series, which takes time:

\[
    t_2 = \frac{Q_z}{\mu_r} \left\{ 1 - \left[ \sum_{n=0}^{T_z} \frac{T_z!}{(T_z-n)!} \left( \frac{\lambda_z}{\mu_r} \right)^n \right]^{-1} \right\} + \frac{Q_w}{\mu_w} \left\{ 1 - \left[ \sum_{n=0}^{W_z} \frac{W_z!}{(W_z-n)!} \left( \frac{\lambda_w}{\mu_w} \right)^n \right]^{-1} \right\} + \frac{Q_t}{\mu_z} \left\{ 1 - \left[ \sum_{n=0}^{Z_t} \frac{Z_t!}{(Z_t-n)!} \left( \frac{\lambda_z}{\mu_z} \right)^n \right]^{-1} \right\}
\]

(2)

When the subgrade is damaged, first the excavators excavate the damaged subgrade, and others excavate the nearest soil for backfilling. After the damaged subgrade is removed, the loaders will transport the soil to the damaged place for backfilling, and then cooperate with the bulldozers to compact. A two-stage cyclic queuing system is formed by bulldozers compacting soil in layers loaders unloading [8], as shown in figure 3.

![Figure 3. Two-stage cyclic queuing system](image)

The average service time of server II is equal to bulldozers’ lading time \( 1/\mu_r \). The probability that there are zero unit in server I is:

\[
    P_0 = \frac{1 - \frac{\mu_r}{\mu_z}}{1 - \left( \frac{\mu_r}{\mu_z} \right)^{Z_z + 1}}
\]

(3)

the time of emergency maintenance of subgrade damage is:

\[
    t_3 = \frac{Q_z}{\mu_w} \left\{ 1 - \left[ \sum_{n=0}^{W_z} \frac{W_z!}{(W_z-n)!} \left( \frac{\lambda_w}{\mu_w} \right)^n \right]^{-1} \right\} + \frac{Q_t}{\mu_z} \left\{ 1 - \left[ \sum_{n=0}^{Z_t} \frac{Z_t!}{(Z_t-n)!} \left( \frac{\lambda_z}{\mu_z} \right)^n \right]^{-1} \right\}
\]

(4)

4.3 The mathematical model of optimal allocation of engineering machinery groups

The purpose of optimal allocation of machinery groups for emergency maintenance is to solve the optimal allocation plan \( X \). Through the analysis of the problem, the following mathematical model can be established:

\[
    \min t = \max \{ t_1, t_2, t_3 \}
\]
5. Conclusion

This paper analyzes the method for engineering machinery to undertake road repairing tasks. A model based on Petri net and queuing theory is established to simulate the construction process and to calculate the construction time. The mathematical model of optimal allocation of engineering machinery groups is put forward. This model aims at solving the problem of multi-point allocation of engineering machinery group for road emergency maintenance and provides a basis for solving the optimal allocation scheme of the machinery groups.

References

[1] Liang Q. (2016) Dispatching Method and Model of the Construction Machinery for Traffic Emergency Rescue. Construction Machinery Technology & Management, 29:82-85.

[2] Zhang C, Hu Q, Wang J, et al. (2019). Modeling method and solving algorithm of personnel, machinery and task distribution model suitable for emergency rescue and disaster relief. http://pss-system.cnipa.gov.cn/sipopublicsearch/patentsearch/showViewList-jumpToView.shtml

[3] Cheng F, Wang Y, Ling X, et al. (2011) A Petri net simulation model for virtual construction of earthmoving operations. Automation in construction, 20:181-188.

[4] ZHANG Z, AN J. (2015) Whole Life Cycle Process Modeling of Underground Coal Mine Project Based on Petri Net. Coal Engineering, 47:142-144,148.

[5] Wu Y, Yu L, Li Z, at al. (2005) Study on simulation method for traffic and transportation system of underground structure group based on Petri Network. Water Resources and Hydropower Engineering, 36:90-92,102.

[6] Wu Z. (2006) An introduction to Petri net. China Machine Press, Beijing.

[7] Cao H, Peng L, Du K, et al. (2019) Optimal allocation of long tunnel construction machinery groups in middle hard surrounding rock based on queuing theory. Journal of Railway Science and Engineering, 16:535-541.

[8] Meng Y. (1989) The basis and application of queuing theory. Tongji University Press, Shanghai.