NEW DEVELOPMENTS IN PRECISION ELECTROWEAK PHYSICS

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ABSTRACT

Our picture of the electroweak interactions continues to improve, with ever more precise constraints on the masses of the Higgs boson(s) and on non-standard physics. Some recent developments include: (a) a calculation of higher-order effects on the weak mixing angle which reduces scheme-dependence, (b) a long-awaited improvement in the experimental accuracy of measurement of parity violation in atomic cesium, and (c) improvements in top quark and \( W \) mass measurements from Fermilab and \( W \) mass measurements from LEP.

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INTRODUCTION

Numerous measurements not only support the unified theory of weak and electromagnetic interactions, but provide constraints on the symmetry-breaking ("Higgs") sector of the theory and on possible extensions of standard-model physics. A streamlined language for these constraints was provided several years ago by Peskin and Takeuchi. Within the past year, enough has taken place that it is worth updating some recent analyses. For several reasons, the constraints on new heavy fermions in the electroweak sector and upper limits on the Higgs boson mass have become somewhat stronger. In the absence of new heavy fermions, one can finally begin to place a significant upper limit on the Higgs boson mass. However, our limit will be somewhat more conservative than some which are quoted in recent literature.

The developments since the summer conferences of 1996 include: (a) a calculation by Degrassi, Gambino, and Sirlin of higher-order effects on the weak mixing angle which reduces scheme-dependence, (b) a long-awaited improvement in the experimental accuracy of measurement of atomic parity violation (APV) in

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cesium [3], and (c) improvements in top quark and W mass measurements from Fermilab [7, 8, 9, 10] and LEP [11]. A consistent picture of electroweak parameters now includes determination of the weak mixing angle $\sin^2 \theta$ to about 0.3% and of the top quark mass to about 3%. Other parameters (such as the W mass and the weak charge of the cesium nucleus) are specified in terms of these quantities, and, remarkably, agree with the predictions at present levels of accuracy. The present article discusses what level of agreement is significant, and what level of disagreement would point the way to new physics.

In Section II we review notation and conventions. Section III is devoted to the effects of the calculation of Ref. [5]. The experimental inputs are described in Section IV, while Section V gives the results of a simultaneous fit to the observables. Possible future improvements in measurements and their theoretical interpretation are treated in Section VI, while Section VII summarizes.

II. NOTATION AND CONVENTIONS

The strengths of electroweak interactions are specified by two coupling constants, $g$ and $g'$, corresponding to the SU(2) and U(1) gauge groups, and a Higgs boson vacuum expectation value $v = 2^{-1/4}G_F^{-1/2} \approx 246$ GeV, where $G_F = 1.16639(2) \times 10^{-5}$ is the Fermi coupling constant. Separate measurements of $g$ and $g'$ are not available, since – in contrast to the unit of electric charge $e$ – neither is probed by a long-range interaction. However, the two couplings are related to $e$ through $e = gg'/(g^2 + g'^2)^{1/2}$. It is most convenient to express all couplings in terms of their values at some convenient mass scale. Since so many measurements have taken place at the Z peak, this scale is taken to be $M_Z$. Extrapolation of the fine-structure constant $\alpha = e^2/4\pi = 1/137.036$ to $M_Z$ then leads to the estimate \[ \alpha^{-1}(M_Z) = 128.9 \pm 0.1. \]

The mass of the Z boson itself involves another combination of the parameters $g$, $g'$, and $G_F$. To lowest order, $G_F/\sqrt{2} = (g^2 + g'^2)/8m_Z^2$. The measurement of $M_Z$ at LEP has become so precise that one must correct for distortions of the LEP ring by earth tides and changes in ground water, and effects induced by passage of the TGV train between Geneva and Paris. The result \[ M_Z = 91.1863 \pm 0.0020 \text{ GeV}/c^2. \]

Thus, knowing $G_F$, $\alpha(M_Z)$, and $M_Z$, we should be in a position to predict all electroweak observables, including the weak mixing angle $\theta$, where $\sin^2 \theta = e^2/g^2$. For example, one expects $M_W = M_Z \cos \theta$. This lowest-order relation, and most others, are affected by fermion loops in gauge boson propagators. The most important such effect \[ \frac{G_F}{\sqrt{2}} \rho = \frac{g^2 + g'^2}{8m_Z^2}, \quad \rho \approx 1 + \frac{3G_Fm_t^2}{8\pi^2\sqrt{2}}. \]
Here we have neglected all quark masses except $m_t$. The Collider Detector Facility (CDF) Collaboration at Fermilab reported $m_t = 176.8 \pm 6.5$ GeV/$c^2$ at the summer conferences in 1996, while a new value from the D0 Collaboration has recently appeared: $m_t = 173.3 \pm 5.6 \pm 6.2$ GeV/$c^2$. The average of the two measurements is $m_t = 175.5 \pm 5.5$ GeV/$c^2$, leading to about a 1% upward correction to $\rho$. Many electroweak measurements have far surpassed this accuracy and thus are sensitive to the top quark mass.

The Higgs boson mass $M_H$ also affects electroweak observables through loops, but only logarithmically. Thus one has to know both $m_t$ and $M_H$ in order to use the values of $G_F$, $\alpha(M_Z)$, and $M_Z$ to predict other quantities. The effects of $m_t$, $M_H$, and a number of other sources of new physics (“oblique corrections”) whose effects are felt mainly through loops in gauge boson propagators can be gathered into three parameters called $S$, $T$, and $U$ by Peskin and Takeuchi. Each observable can be expressed as a “nominal” value for fixed $G_F$, $\alpha(M_Z)$, $M_Z$, $m_t$, and $M_H$, plus a linear combination of $S$, $T$, and $U$. The only place $U$ appears is in the expression for the $W$ mass. A pair of parameters equivalent to $S$ and $U$ is $S_Z \equiv S$ and $S_W \equiv S + U$. We shall take as nominal values $m_t = 175$ GeV/$c^2$ and $M_H = 300$ GeV/$c^2$.

When $m_t$ and $M_H$ deviate from their nominal values, and when new heavy fermions $U$ and $D$ with $N_C$ colors and charges $Q_U$ and $Q_D$ are present, one finds the following contributions:

$$T \simeq \frac{3}{16\pi \sin^2 \theta M_W^2} \left[ m_t^2 - (175 \text{ GeV})^2 + \sum_{N_C} \left( \frac{m_U^2 + m_D^2}{m_U^2 - m_D^2} \ln \frac{m_U^2}{m_D^2} \right) \right]$$

$$- \frac{3}{8\pi \cos^2 \theta} \ln \frac{M_H}{300 \text{ GeV}} ,$$

where $\Delta \rho \simeq \alpha T$;

$$S_Z = \frac{1}{6\pi} \left[ \ln \frac{M_H}{300 \text{ GeV}/c^2} - 2 \ln \frac{m_t}{175 \text{ GeV}/c^2} + \sum_{N_C} \left( 1 - 4Q \ln \frac{m_U}{m_D} \right) \right] ,$$

$$S_W = \frac{1}{6\pi} \left[ \ln \frac{M_H}{300 \text{ GeV}/c^2} + 4 \ln \frac{m_t}{175 \text{ GeV}/c^2} + \sum_{N_C} \left( 1 - 4Q_D \ln \frac{m_U}{m_D} \right) \right] .$$

The expressions are written for doublets of fermions with $N_C$ colors and $m_U \geq m_D \gg m_Z$, while $Q \equiv (Q_U + Q_D)/2$. The sums are taken over all doublets of new fermions. The leading-logarithm expressions are of limited validity for $M_H$ and $m_t$ far from their nominal values. We do not consider multi-Higgs models here; a discussion can be found in [19].

Most electroweak observables can be written as homogeneous functions of $\rho$ (typically of degree 0, 1, or 2) times linear functions of $\sin^2 \theta$ (which itself is a linear function of $S$ and $T$). Since $\Delta \rho = \alpha T$, it is then straightforward to evaluate
the coefficients of $S$ and $T$ in such observables\,[2,17]. Examples of these coefficients will be given in Sec. V when we discuss fits to the data.

### III. DEFINITIONS OF WEAK MIXING ANGLE

Several definitions of the weak mixing angle have appeared in the literature. The on-shell scheme\,[20] defines a value of $\theta$ valid in the presence of loop corrections via the tree-level relation $M_W = M_Z \cos \theta$. The $\overline{\text{MS}}$ (modified-minimal-subtraction) scheme is more directly related to the ratio of coupling constants: $\hat{s}^2 \equiv \sin^2 \theta_{\text{MS}} \equiv (e^2/g^2)_{\text{MS}}$ with a particular prescription for removing divergences in loop corrections. Finally, $\sin^2 \theta_{\text{eff}}^{\text{lept}}$ is the effective value of $\sin^2 \theta$ measured in leptonic asymmetries at the $Z$, which probe the ratio of the vector and axial vector couplings of leptons to the $Z$:

\[ g_V = -(1/4) + \sin^2 \theta_{\text{eff}}^{\text{lept}}, \quad g_A = 1/4. \]

In previous analyses\,[3,4] we made use of a connection between the nominal value of $\hat{s}^2$ as quoted by DeGrassi, Kniehl, and Sirlin\,[21] for the difference $\sin^2 \theta_{\text{eff}}^{\text{lept}} - \hat{s}^2 = 0.0003$, to quote $\hat{s}^2 = 0.2315$ for $m_t = 175$ GeV/$c^2$, $M_H = 300$ GeV/$c^2$. This difference has now diminished to 0.0001\,[3], and the nominal value of $\hat{s}^2$ is now quoted as 0.2320. The net effect of these changes is to raise the nominal value of $\sin^2 \theta_{\text{eff}}^{\text{lept}}$ from 0.2318 to 0.23211. As we will see in Sec. V, the experimental values of $\sin^2 \theta_{\text{eff}}^{\text{lept}}$ measured at LEP and SLC then imply a more negative value of $S$ when other constraints are taken into account. This shift favors a lower Higgs boson mass for a fixed value of $m_t$.

### IV. EXPERIMENTAL INPUTS

#### A. Improved measurement of parity violation in atomic cesium

An early prediction of the electroweak theory was the violation of parity in atomic transitions through the mixing of opposite-parity levels induced by the weak neutral current. Within the past 10 years increasingly precise experiments\,[23,24,25,26,27] have been performed in various systems, and all (so far) have agreed with electroweak predictions. At the time of a recent review\,[3], the most precise information was provided by a measurement in cesium\,[23]. The result can be quoted in terms of a weak charge $Q_W$, which measures the strength of the (coherent) vector coupling of the $Z$ to the nucleus:

\[ Q_W \simeq \rho(N - Z - 4Z\hat{s}^2). \]

It was found that $Q_W\,(\text{Cs}) = -71.04 \pm 1.58 \pm 0.88$. The first error is experimental, while the second is theoretical\,[28,29].

When the measurements of Ref.\,[23] were first reported, theoretical calculations hadn’t been done well enough yet for the result to have an impact, and the Peskin-Takeuchi $S - T$ language\,[2] was not yet currently in use (though there were some intimations of it in the literature)\,[30].

The atomic physics calculations by the Novosibirsk group in 1989\,[28] and the Notre Dame group in 1990\,[29], and the $S - T$ description by Peskin and Takeuchi
changed the situation. It had been noticed \[31, 32\] that the weak charge was insensitive to the electroweak radiative corrections due to the top quark mass once the Z mass was specified. The impact of this result in the $S-T$ language \[17\] was realized at a workshop at Snowmass in June of 1990, where it was found that the weak charge for cesium (and atoms in that range of A and Z) was almost independent of $T$. Thus it took a couple of years between the reporting of the experimental results and the full realization of their significance for particle physics.

The prediction \[17\] $Q_W(\text{Cs}) = -73.20 \pm 0.13$ is insensitive to standard-model parameters \[17, 31\]; discrepancies are good indications of new physics (such as exchange of an extra Z boson). The 1988 result in cesium could be interpreted as implying $S = -2.6 \pm 2.3$.

The weak charge has also been measured in atomic thallium: $Q_W(\text{Tl}) = -114.2 \pm 1.3 \pm 3.4$ \[20\] and $-120.5 \pm 3.5 \pm 4.0$ \[27\] (this number is deduced \[8\] from the published result), to be compared with the theoretical estimate \[33, 34\] $Q_W = -116.8$. Here the first error refers to the total experimental error, while the second refers to the error associated with the atomic physics calculations. Averaging the two values, we find $Q_W(\text{Tl}) = -115.0 \pm 2.1 \pm 4.0 = -115.0 \pm 4.5$, implying $S = -4.5 \pm 3.8$. As in the case of cesium, the central value is negative, but consistent with zero. An accurate measurement in lead \[25\] awaits comparable progress in the theoretical calculation \[35\].

A new experimental value for cesium has now appeared \[4\]: $Q_W(\text{Cs}) = -72.11 \pm 0.27$. The theoretical error from the atomic physics calculations is still $\pm 0.88$, leading to a total error of $\pm 0.93$. This result implies $S = -1.3 \pm 0.3 \pm 1.1$, a value still consistent with zero, but with a reduced error.

**B. New W mass measurements**

1. **Direct production of $W^+W^-$ pairs at LEP.** All four LEP experiments reported W mass measurements at the 1996 Warsaw Conference \[36, 37\], with an average value of $M_W = 80.3 \pm 0.4 \pm 0.1$ GeV/$c^2$. These results were based on the reaction $e^+e^- \rightarrow W^+W^-$ just above threshold, at a center-of-mass energy of 161 GeV. For this energy, the W mass measurement is based mainly on comparison of the experimentally measured cross section with the theoretical prediction.

   The LEP Collider has now been operated at c.m. energies up to 172 GeV, where the W mass must be measured by reconstructing its decay. This entails a larger systematic error, though the higher cross section farther above threshold makes for a lower statistical error for a given integrated luminosity. The combined result of all four LEP experiments \[11\] is now $M_W = 80.38 \pm 0.14$ GeV/$c^2$.

2. **Production in the Tevatron.** The CDF and D0 collaborations at Fermilab have reported new $M_W$ values based on data obtained during Run 1B, which ended in 1996. The new CDF value \[8\], based on $W \rightarrow \mu \nu$, is $M_W = 80.430 \pm 0.100 \pm 0.040 \pm 0.115$ GeV/$c^2$, where the first error is statistical, the second is due to uncertainty in the momentum scale, and the third is the remaining systematic
error. This value of $M_W = 80.430 \pm 0.155 \text{ GeV/c}^2$ may be combined with previous CDF $e\nu$ and $\mu\nu$ data to yield $M_W = 80.375 \pm 0.120 \text{ GeV/c}^2$. Analysis of the $W \to e\nu$ decays from Run 1B is in progress.

The new D0 $W \to e\nu$ value, based on a value recently reported \cite{10} and one published last year \cite{9}, is $80.44 \pm 0.11 \text{ GeV/c}^2$. When the earlier UA2 \cite{38} result of $80.36 \pm 0.37 \text{ GeV/c}^2$ is combined with the CDF and D0 results, taking account of common errors, the average is $M_W = 80.41 \pm 0.09 \text{ GeV/c}^2$.

3. World average of direct measurements. The LEP II and hadron collider measurements may be combined to give $M_W = 80.401 \pm 0.076 \text{ GeV/c}^2$.

4. Indirect measurement using deep inelastic neutrino scattering. The ratio $r_\nu \equiv \sigma(\nu_\mu N \to \nu_\mu + \ldots)/\sigma(\nu_\mu N \to \mu^- + \ldots)$ is very sensitive to $\sin^2 \theta$ and provides a good measure of it. It turns out that the $\sin^2 \theta$ and $\rho$ dependences combine in such a way that this ratio actually depends on $m_t$ and $M_H$ in very much the same way as does $M_W$. As a result, measurements of $R_\nu$ can be quoted as effective measurements of $M_W$.

A new measurement by the CCFR Collaboration at Fermilab \cite{39} implies $M_W = 80.35 \pm 0.21 \text{ GeV/c}^2$. This analysis was performed for $m_t = 175 \text{ GeV/c}^2$ and $M_H = 150 \text{ GeV/c}^2$; the variation of $M_W$ with $m_t$ and $M_H$ within the region of interest may be ignored. Previous determinations \cite{40, 41} imply slightly lower values of $M_W$ but are hard to combine with the CCFR value without updated analyses. Consequently, we do not include them in our averages.

C. Progress on the top quark

As of the 1996 Warsaw Conference, CDF was reporting $m_t = 176.8 \pm 6.5 \text{ GeV/c}^2$, while D0 reported $m_t = 169 \pm 11 \text{ GeV/c}^2$. A new D0 result has now appeared \cite{7}: $m_t = 173.3 \pm 5.6 \pm 6.2 \text{ GeV/c}^2$, leading to a world average of $m_t = 175.5 \pm 5.5 \text{ GeV/c}^2$. The result is unlikely to change much in the next few years, as both experiments have analyzed their full data set.

D. Summary of inputs

We summarize in Table 1 the experimental inputs for a simultaneous fit to electroweak observables, performed in the spirit of Refs. \cite{2} and \cite{17}.

We do not use the following quantities in the fit: (1) The total width of the $Z$, $\Gamma_Z = 2.4946 \pm 0.0027 \text{ GeV}$, has been used here to extract the leptonic width \cite{13}, and is not an independent quantity. To include it in the fit one should take account of correlations, as in the treatments of Refs. \cite{42, 43}. The independent information which it provides is a confirmation of the currently accepted value of $\alpha_s = 0.118 \pm 0.006$ \cite{14, 44} and/or the absence of significant additional decay channels of the $Z$ into new particles. (2) The partial width $\Gamma(Z \to b\bar{b})$ can receive a contribution from a virtual top quark loop and thus is expected to differ from $\Gamma(Z \to d\bar{d})$ or $\Gamma(Z \to s\bar{s})$. Until recently the effects of the top quark loop, which
Table 1: Electroweak observables described in fit

| Quantity | Experimental value | Theoretical value |
|----------|--------------------|-------------------|
| \(Q_W\) (Cs) | \(-72.11 \pm 0.93\) \(^a\) | \(-73.20 \) \(^b\) - 0.80S - 0.005T |
| \(Q_W\) (Tl) | \(-115.0 \pm 4.5\) \(^c\) | \(-116.8 \) \(^d\) - 1.17S - 0.06T |
| \(M_W\) (GeV/c\(^2\)) | 80.41 \pm 0.10 \(^e\) | 80.308 \(^f\) - 0.29S + 0.45T |
| \(M_W\) (GeV/c\(^2\)) | 80.38 \pm 0.14 \(^g\) | 80.308 \(^f\) - 0.29S + 0.45T |
| \(M_W\) (GeV/c\(^2\)) | 80.35 \pm 0.21 \(^h\) | 80.308 \(^f\) - 0.29S + 0.45T |
| \(\Gamma_{\ell\ell}(Z)\) (MeV) | 83.91 \pm 0.11 \(^i\) | 83.90 - 0.18S + 0.78T |
| \(\sin^2 \theta_{\text{eff}}\) | 0.23200 \pm 0.00027 \(^j\) | 0.23211 \(^k\) + 0.0036S - 0.0026T |
| \(\sin^2 \theta_{\text{eff}}\) | 0.23061 \pm 0.00047 \(^l\) | 0.23211 \(^j\) + 0.0036S - 0.0026T |
| \(m_t\) (GeV/c\(^2\)) | 175.5 \pm 5.5 | 175 + 241S + 82T |

\(^a\) Weak charge in cesium \[6\]
\(^b\) Calculation \[17\] incorporating atomic physics corrections \[28, 29\]
\(^c\) Weak charge in thallium \[26, 27\] (see text)
\(^d\) Calculation \[34\] incorporating atomic physics corrections \[33\]
\(^e\) Average of direct hadron collider measurements
\(^f\) Including perturbative QCD corrections \[3\]
\(^g\) LEP II value as of March, 1997
\(^h\) Value from deep inelastic neutrino scattering \[38\]
\(^i\) LEP average as of November, 1996 \[13\]
\(^j\) From asymmetries at LEP \[13\]
\(^k\) As calculated \[4\] with correction for relation between \(\sin^2 \theta_{\text{eff}}\) and \(s^2\)
\(^l\) From left-right asymmetry in annihilations at SLC \[46\]

should act to suppress the partial width somewhat, had not been detected. The ALEPH Collaboration \[47\], after a silence of several years, presented results at the 1996 Warsaw Conference much more in accord with this expectation, and other LEP experiments now appear to agree with this finding. Nonetheless, since \(\Gamma(Z \rightarrow \bar{b}b)\) is still in a state of flux, we do not use it in our fit.

We average several quantities before fitting them. By combining \(W\) mass measurements from hadron and \(e^+e^-\) colliders and from deep inelastic neutrino scattering we find \(M_W = 80.395 \pm 0.071\) GeV/c\(^2\). The average of \(\sin^2 \theta_{\text{eff}}\) from LEP and SLC gives \(\sin^2 \theta_{\text{eff}} = 0.23166 \pm 0.00058\) when we multiply the error by a scale factor \[18\] of \([\chi^2/(N-1)]^{1/2} = 2.55\) to account for the discrepancy between the two values. [We cannot think of a plausible fundamental reason for this discrepancy. One could suspect systematic errors, e.g., in polarization measurements at SLC or in the interpretation of data at LEP to extract such quantities as \(A_{FB}^\ell\).] Adding a theoretical error of \(\Delta \sin^2 \theta_{\text{eff}} = \pm 0.00026\) associated with the assumed error of
Figure 1: Allowed ranges of $S$ and $T$ at 68% (inner ellipses) and 90% (outer ellipses) confidence levels, corresponding to $\chi^2 = 2.3$ and 4.6 above the minima (crosses at center of ellipses). Dotted, dashed, and solid lines correspond to standard model predictions for $M_H = 100, 300, 1000$ GeV/$c^2$. Symbols $\times$, from bottom to top, denote predictions for $m_t = 100, 140, 180, 220, and 260$ GeV/$c^2$. (a) Fit including APV experiments with present errors; (b) errors on APV experiments reduced from $\Delta Q_W = \pm 0.93$ (present value) to $\pm 0.3$ (corresponding to a hypothetical improvement in theoretical error), with present central values of $Q_W$ retained.

$\pm 0.1$ in $\alpha^{-1}(M_Z)$, we employ the value $\sin^2 \theta_{\text{eff}}^{\text{lept}} = 0.23166 \pm 0.00064$ in our fits.

V. RESULTS OF FIT TO OBSERVABLES

The results can be displayed in two different ways. In Figure 1 we show error ellipses corresponding to the allowed ranges of the parameters $S$ and $T$ discussed in Section II for the data summarized in Table 1, but not including the constraint due to the top quark mass. Figure 2(a) shows the fit for this same data set, with the additional stipulation that $m_t = 175.5 \pm 5.5$ GeV/$c^2$.

The present data [Fig. 1(a)] are fully consistent with electroweak predictions. $S$ and $T$ can be viewed as free parameters pointing the way to new physics. Fig. 1(b), to be discussed below, shows that improved errors on the theoretical prediction for parity violation in atomic cesium can have a notable effect on the error ellipses in $S$ and $T$, given present accuracy of experiments.

One can also view $S$ and $T$ simply as a way of parametrizing the electroweak theory. In this case one can include information on the top quark mass by linearizing Eqs. (2) and (3) in $m_t - 175$ GeV/$c^2$ and eliminating the dependence on $\ln(M_H/300$ GeV/$c^2$), resulting in the relation $m_t = 175$ GeV/$c^2 + 241S + 82T$.  


Figure 2: Magnified view of Figure 1. Dotted, dashed, and solid lines correspond to standard model predictions for $M_H = 100, 300, 1000 \text{ GeV}/c^2$. Symbols $\times$ denote predictions for $m_t = 140$ (bottom) and 180 (top) $\text{GeV}/c^2$ on $M_H = 100 \text{ GeV}/c^2$ curve, and for $m_t = 180 \text{ GeV}/c^2$ on $M_H = 300, 1000 \text{ GeV}/c^2$ curves. (a) The constraint $m_t = 175.5 \pm 5.5 \text{ GeV}/c^2$ has been imposed. (b) Dashed ellipses: same as (a) but with $\Delta Q_W(C_s) = \pm 0.3$. Solid ellipses: same as (a) but with $\Delta M_W = \pm 30 \text{ MeV}/c^2$.

Some preference is shown for a particular range of Higgs boson masses when information on the top quark mass is included, as shown in Fig. 2(a). We find that $M_H \simeq 166 \times 2^{\pm 1} \text{ GeV}/c^2$ leads to a variation of $\chi^2$ of one unit above the minimum (2.13 for 4 degrees of freedom – the 6 data points in Table 1 minus the two parameters $S$ and $T$). For $\Delta \chi^2 = 2.3$, the corresponding uncertainty in $M_H$ is about $(2.8)^{\pm 1}$. The higher central value of our $M_H$ in comparison with the values $127^{+143}_{-71} \text{ GeV}/c^2$ [3] or $124^{+125}_{-71} \text{ GeV}/c^2$ [13] is due primarily to our use of a scale factor in quoting the error on $\sin^2 \theta^{\text{lept}}_{\text{eff}}$. Until the discrepancy between the LEP and SLC values is resolved, we regard this as prudent. Another point of view [37, 43] is to explore the effect of ignoring one or more mildly discrepant values of $\sin^2 \theta^{\text{lept}}_{\text{eff}}$, such as the SLC value or the value $\sin^2 \theta^{\text{lept}}_{\text{eff}} = 0.23246 \pm 0.00041$ (entering into the LEP average) due to the forward-backward asymmetry in $e^+e^- \rightarrow Z \rightarrow b\bar{b}$. The corresponding uncertainty in the Higgs boson mass is similar to that implied by Fig. 2(a).

In any case, our analysis now disfavors Higgs bosons heavier than 500 $\text{ GeV}/c^2$, unless new contributions to the parameter $T$ (beyond the top quark) exist. That this is still a possibility can be seen in Fig. 1(a) from the solid curve corresponding to $M_H = 1000 \text{ GeV}/c^2$, which passes within the 68% c.l. ellipse for a range of top
quark masses above 200 GeV/c².

VI. POSSIBLE IMPROVEMENTS AND THEIR IMPACT

A. Atomic parity violation

The present measurement can be placed in the context of present constraints on $S$ and $T$ from high energy experiments. As in 1988, not much can be said until the theoretical errors shrink to match the experimental ones. The 90% error ellipses in $S$ and $T$ specified by high energy experiments (primarily those at LEP, SLC, and Fermilab) allow a range of only ±0.4 in $S$ from the central value ($S$ close to 0). When the total error on $S$ (experiment plus theory) from atomic physics experiments is ±0.4, this experiment will again provide a significant constraint in the $S−T$ context. An example is shown in Fig. 1(b). Here, it is assumed that the theoretical errors in extracting $Q_W(\text{Cs})$ from experiment have been decreased so that the total error on $Q_W$ is only ±0.3. The central value has been assumed to be the present one. A significant constraint on the electroweak theory results.

Viewed simply as alternative ways to measure electroweak parameters, atomic parity violation experiments have less impact than the accelerator experiments. In Fig. 2(b) we show via the dashed curves the $S−T$ contours for an improved determination of $Q_W(\text{Cs})$ when the top quark mass is taken into account. The curves are indistinguishable from those in Fig. 2(a). Another illustration of this point is provided by the discussion of the impact of measurements of isotope ratios of $Q_W$. These turn out to be primarily sensitive to $\sin^2 \theta_{\text{eff}}^\text{lept}$ and, as such, do not compete in accuracy with LEP and SLC measurements.

We reiterate that a crucial comparison involves Figs. 1(a) and 1(b), where the possibility of new physics is kept open. The key measurement is the absolute scale of the parity-violating transition, as recently improved so notably for cesium. A corresponding improvement in the atomic theory is of high priority in order to extract a value of $Q_W$ with suitably small errors.

B. $W$ mass

The analysis in Ref. [5] entailed a range of Higgs boson masses from which one could infer an allowed range $M_W = 80.367 \pm 0.048$ GeV/c². We extract a range of $M_W$ in our fit by studying the variation of $\chi^2$ in a fit to all observables except $M_W$ for a range of fixed values of $M_W$. We find $M_W = 80.338 \pm 0.057$ GeV/c² for $\Delta \chi^2 = 1$ above the minimum. Our slightly lower central value and larger uncertainty are correlated with our slightly higher central value and larger uncertainty for $M_H$. We may combine our value of $M_W$ in this fit with the experimental average to obtain an overall average $M_W = 80.360 \pm 0.044$ GeV/c². An earlier result, $M_W = 80.352 \pm 0.033$, does not incorporate all the data considered here. Our uncertainty is larger primarily because of the scale factor by which we have multiplied the uncertainty in $\sin^2 \theta_{\text{eff}}^\text{lept}$.
Figure 3: (a) Solid curves: same as Figure 2(a), but with $m_t = 175.5 \pm 2 \text{ GeV}/c^2$ assumed. Dashed curves: same as Figure 2(a), but with $\sin^2 \theta_{\text{lept}}^{\text{eff}} = 0.23166 \pm 0.00023$ assumed. (b) Same as Figure 2(a), but with improved errors mentioned above for $M_W$, $m_t$, and $\sin^2 \theta_{\text{lept}}^{\text{eff}}$.

We thus conclude that a reduction of uncertainty in direct measurements of $M_W$, e.g., to $\pm 30 \text{ MeV}/c^2$, could match or surpass the accuracy provided by fits to the LEP and SLD data, as long as the discrepancy persists between the $\sin^2 \theta_{\text{lept}}^{\text{eff}}$ values measured at LEP and SLC. To illustrate this point, we show in Fig. 2(b) via the solid curves the effect of reducing the errors on $M_W$ to $\pm 30 \text{ MeV}/c^2$.

In all of the present discussion, we have been assuming $S_W - S_Z \equiv U = 0$, which follows if there are no weak isospin-violating effects due to new heavy particles. If such effects are present, one may spot them first through a discrepancy between direct measurements of $M_W$ and the predictions based on fits to LEP and SLC data. Present fits \cite{43} to electroweak data suggest $U \approx 0.07 \pm 0.42$.

C. Top quark mass and $\sin^2 \theta_{\text{lept}}^{\text{eff}}$

The shape of the contours in Fig. 2(a) implies that further improvement in measurement of the top quark mass is unlikely to restrict the parameters of the electroweak theory unless it is combined with an improved $W$ mass measurement or a resolution of the discrepancy in measuring $\sin^2 \theta_{\text{lept}}^{\text{eff}}$. The latter observable is closest of all those we have considered to one which would restricts the semi-major axes of the ellipses in Fig. 2(a). We show in Fig. 3(a) the separate effects of reducing the top quark mass errors to $\Delta m_t = \pm 2 \text{ GeV}/c^2$ (solid curves) and the errors on $\sin^2 \theta_{\text{lept}}^{\text{eff}}$ to $\pm 0.00023$ (dashed curves) [which would be the error if we neglected the scale factor and the error on $\alpha(M_Z)$ mentioned earlier]. Note the similar shapes of
the dashed ellipses in Fig. 3(a) and the solid ellipses in Fig. 2(b). Improvements in measurements of $\sin^2 \theta_{\text{eff}}^\text{lept}$ would have roughly the same impact as improvements in measurements of $M_W$. Consistency between the two determinations would provide further confirmation of our assumption that $S_W \simeq S_Z$.

As an example of what global fits could provide in the not-too-distant future, we show in Fig. 3(b) the $S-T$ ellipses that would result from the aforementioned improvements in errors in all of $M_W$, $m_t$, and $\sin^2 \theta_{\text{eff}}^\text{lept}$. Only with a combination of these improvements can one hope to shrink the error ellipses in such a way as to achieve meaningful limits on the Higgs boson mass.

**VII. SUMMARY**

Electroweak measurements continue to be made with ever greater precision, affording potential challenges to theory. Nonetheless, the electroweak theory has continued to be remarkably resilient. Indications for new physics have come and gone over the years, but now it appears that measurements are beginning to exclude either a very heavy Higgs boson or point to some other physics (such as further quark or lepton doublets) that would allow such a boson to exist. It appears that the indirect route to such a boson provided by these measurements is an arduous one. Fortunately, instruments for direct searches (such as LEP II, the future Large Hadron Collider at CERN, and possibly an upgraded Tevatron at Fermilab) will be available in coming years. Meanwhile, significant contributions continue to emerge from non-accelerator experiments, and for this we particle physicists are profoundly grateful.

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