Double-ring interference of binary diffractive axicons

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Abstract: We report on the interference between the double rings generated by the Fourier transform of a binary diffractive axicon. These two rings have the same size and correspond to the \( \pm 1 \) diffracted order beams. The interference condition between both rings can be easily changed by adding a constant phase bias, resulting in a central ring that is either dark or bright. Additionally, this interference condition can be changed along the ring and can be easily tuned, thus allowing greater flexibility. We present experimental results obtained with a binary \( \pi \)-phase liquid-crystal spatial light modulator. These patterns might find applications in optical trapping systems, where the bright or dark regions could trap particles whose refractive index is either higher or lower than the medium.

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1. Introduction

Optical trapping has become a huge field after the first discovery by Ashkin et al. [1]. In this case particles having an index of refraction larger than the medium can be trapped in three dimensions by a tightly focused laser beam. A similar trapping mechanism exists when the samples are illuminated with a circular vortex beam, which presents a dark hole in the center and orbital angular momentum (OAM) [2]. The intense region of the ring not only traps these high refractive index particles, but also transfers angular momentum to them. In the same time frame, similar trapping was reported for black or reflective particles [3], as well as with particles having an index of refraction smaller than the medium [4]. However, in these cases, the particle is trapped in the dark central area of the vortex beam.

One problem with these techniques is that the radius of the vortex beam increases as the charge increases [5]. Recently a new mechanism for producing vortex beams based on axicons, known as the “perfect vortex” beam, has been proposed [6]. These perfect vortex beams are generated either by combining an axicon with a spiral phase [7], or by Fourier transforming a higher-order Bessel beam [8]. This combination leads to a ring of light where the change in diameter with the topological charge is significantly reduced (although not completely independent, as pointed out in [9]). An optical trapping system based on perfect vortices was demonstrated in [10].

Furthermore, other related designs based on computer generated holograms can produce intensity circuits in the Fourier transform with arbitrary shapes and phase variation along the light path [11–13]. These studies show that phase gradients can create optical forces transverse to the optical axis.

All these works typically use spatial light modulators (SLM) producing continuous phase-only modulation. These are nematic liquid-crystal SLMs with rather limited frame rates. An alternative technology is based on ferroelectric liquid crystal SLMs, capable to reach rates of more than thousand frames per second [14]. However, these are binary phase-only SLMs. Therefore, when
displaying a phase mask, the binary SLM generates a series of other harmonic components where the negative first order is the most relevant, having the same weight as the positive first order. This effect is well-known, but here we analyze its implications in the design of axicons and perfect vortex masks. Because their application in optical traps is one of their major interest, we also analyze their potential implication in this field.

In this work we use diffractive axicons displayed onto a binary \( \pi \)-phase SLM. These diffractive axicons are radial diffraction gratings, thus generating a ring of light in the Fourier domain. The binary axicon creates both positive and negative orders, and therefore generates two rings of light, both having the same diameter. These two rings interfere and their interference condition can be changed by adding a constant phase bias. In this way, we can create a single intense ring, or alternatively a dark ring surrounded by two bright rings. This property has been recently identified and applied in a materials’ laser processing system to improve resolution [15].

Also, very recently, Liang et al. analyzed a double ring effect in the context of perfect vortex beams [16]. They generated these beams by Fourier transforming an azimuthally polarized Bessel beam and the phase-shift between the circular polarization components was used to determine the interference condition between the two rings.

Here we further experimentally analyze different variations of these previous recent works. We consider different variations of a binary phase axicon. First, we change the phase bias to change the interference condition. Then we combine them with a spiral phase pattern. This creates a continuous variation of the interference condition along the ring, resulting in a ring of light with spiral interference. These spiral fringes illustrate the opposite phases between positive and negative orders. Finally, we present a new situation where the phase bias can be changed in azimuthal sectors and create what we call a “conveyor belt” ring of light, where the central bright / dark condition changes along the ring.

The paper is organized as follows: after this introduction, Section 2 briefly reviews the axicon and provides a physical explanation on the formation of the two rings. The experimental system and methods are described in Section 3, which also presents experimental results of the two-ring interference generated by the binary phase axicon and how their phase condition can be changed with a phase bias. Next, Section 4 shows the effect of adding a spiral phase to the original design (as required to create a perfect vortex beam) and how the helical interference displays the interference of the two rings. In Section 5, we analyze the phase gradients of some of these designs due because of their effect in generating transversal forces in the optical trapping systems. Then, in Section 6 we present the “conveyor belt” design. Finally, we present the conclusions of the work.

2. Reviewing the axicon and its Fourier transform

Usually axicons are most known for producing non-diffracting Bessel beams [17]. However, they are also useful to generate in the far field (Fourier transform) a ring of light with a diameter that depends on the period of the axicon. In fact, Durning [18] created the Bessel beam by Fourier transforming an annular slit aperture.

After the pioneering works [19,20], diffractive axicons are nowadays routinely encoded onto SLMs, typically as continuous phase patterns reproducing either the axicon phase function [21] or a related computer-generated hologram [22]. The polarization properties of liquid-crystal SLMs have been exploited also to encode birefringent axicons that generate Bessel beams where the polarization state changes along propagation [23].

A diffractive axicon consists of a radial diffraction grating having a period \( d \), i.e., its transmission function is given by

\[
\hat{r}(r) = \exp \left( \frac{i2\pi}{d} \frac{r}{d} \right) \exp(i\phi_0)
\]

(1)
where \( r \) denotes the radial coordinate, and \( \phi_0 \) is a constant phase bias that defines the phase at the origin. Usually these axicon elements are encoded so that the phase varies linearly across each period, as shown in Fig. 1(a). Alternatively, binary-phase diffractive axicons are also of interest since binary diffractive structures are easier to fabricate [24]. In addition, they can be encoded on binary ferroelectric liquid-crystal SLMs, which are much faster than standard continuous phase-only SLMs [14]. In some cases, low-cost twisted-nematic liquid-crystal SLMs provide limited ranges of phase modulation, not reaching \( 2\pi \) radians (especially for long wavelengths) making them more useful as binary \( \pi \)-phase modulators. This is the case in this work. A binary \( \pi \)-phase version of the axicon in Fig. 1(a) is shown in Fig. 1(b).

**Fig. 1.** (a) Regular continuous-phase diffractive axicon. (b) Binary-phase version of the axicon. (c) Ray diagrams of the double-ring formation in the Fourier plane with the positive axicon (red rays) and negative axicon (yellow rays). Rays 1-4 correspond to the converging axicon (+1 order). Rays 5-8 correspond to the diverging axicon (-1 order).

Here, we do not concentrate on the non-diffracting beam that is created after the axicon. Instead, we examine its Fourier transform formed in the focal plane of a converging lens as shown in Fig. 1(c). Now, considering the paraxial approximation and the diffraction grating’s law, the radius of the ring depends on the period of the grating as

\[
R = \frac{f\lambda}{d}
\]

where \( f \) is the lens focal length and \( \lambda \) is the wavelength.

In the case of the conventional continuous-phase axicon, only one ring will be obtained in the Fourier plane. However, the binary version of Eq. (1) satisfies all of the conditions to be expanded in a Fourier series which is written as the following transmission function [25]:

\[
t(r) = \sum_{n=-\infty}^{\infty} c_n \exp\left(in2\pi \frac{r}{d}\right) \exp(in\phi_0)
\]
The Fourier series coefficients $c_n$ are determined by integrating the transmission function over a period as:

$$c_n = \frac{1}{d} \int_0^d t(r) \exp \left( i2\pi n \frac{r}{d} \right) dr$$ (4)

In this work, we encode binary gratings where each period has two phase steps of equal width and a $\pi$ phase difference. In this case, the main diffracted orders are the $n=\pm 1$ orders, each having a diffraction efficiency of $|c_{\pm 1}|^2 = (2/\pi)^2 = 40.5\%$. The remaining 19% energy is diffracted onto higher diffraction orders. As a result, we can simplify Eq. (3) to consider only the $+1$ and $-1$ orders as follows:

$$t(r) = c_1 \left\{ \exp \left( +i2\pi \frac{r}{d} \right) \exp(i\phi_0) + \exp \left( -i2\pi \frac{r}{d} \right) \exp(-i\phi_0) \right\}$$ (5)

Therefore, the binary $\pi$-phase axicon produces, in the first diffraction order, the converging conical beam characteristic of the regular axicon [red rays in Fig. 1(c)]. But it also generates a diverging conical beam [yellow rays in Fig. 1(c)], corresponding to the negative first order. This beam is diffracted with the same angle but diverging from the axis [15,26]. Now, when we look at the Fourier transform plane, the negative component produces a ring of equal diameter as the positive component, both given by Eq. (2). Therefore, a superposition of these two rings is obtained and their interference condition depends on their relative phase. The purpose of this paper is to describe this interference and outline possible use in optical trapping.

3. Experimental results

In our experimental system a He-Ne laser beam of 632.8 nm wavelength is spatially filtered and collimated. The beam illuminates a twisted-nematic transmissive liquid-crystal display from the company CRL, model XGA-3 TN-LCD, with 1074×768 pixels, and pixel spacing $\Delta = 18$ $\mu$m. We configured the device with the elliptically polarized eigenvector configuration [27] to produce phase-only modulation. However, this device does not reach $2\pi$ phase modulation for this wavelength. Thus, we configured the contrast and gain such that gray levels of 0 and 255 correspond to phases of 0 and $\pi$ radians, resulting in a binary $\pi$-phase modulation. A converging lens of focal length $f = 50$ cm is placed behind the SLM [Fig. 1(c)], and the beam is focused onto a WinCam detector located at the back focal plane of the lens.

Figure 2 shows the experimental results obtained with an axicon of period $d = 10$ pixels. According to Eq. (3), a relative phase $2\phi_0$ between the $\pm 1$ diffraction orders occurs when adding a constant phase $\phi_0$ to the axicon phase before binarizing it. This phase-shift can be used to control the two-ring interference condition. For instance, in Fig. 2(a) such phase was adjusted to provide a maximum intense bright narrow ring of light. In this situation the two rings are in phase and can be considered as the phase bias origin, $\phi_0 = 0$. However, note that there are two weak radial sidelobes, which are visible because the ring of light has a non-negligible width. This inevitably happens in any experimental physical realization [9], as opposed to the ideal case that would generate a radial delta function. To better visualize these radial sidelobes, Fig. 2 includes a profile of the radial intensity in the region of the ring for each case.

In Fig. 2(c) this constant phase is changed to be $\phi_0 = \pi/2$ in order to generate a $2\phi_0 = \pi$ relative phase-shift between the two overlapping rings, which are now out of phase. Note that now the center of the ring is dark with two equally bright radial sidelobes. The intensity profile shows indeed a narrow dark ring surrounded by two equally intense bright rings. In Figs. 2(b) and 2(d) two intermediate situations are shown, when phases $\phi_0 = \pi/4$ and $\phi_0 = 3\pi/4$ are applied, so the corresponding phase shifts between the $\pm 1$ orders are $\pi/2$ and $3\pi/2$ respectively. In both cases we observe the dark central ring but now the two sidelobes show asymmetry in the intensity. In the first case, the outer ring is brighter, while in the second case, the inner ring is brighter.
These results are similar to those recently reported for laser processing applications [15] and for generating perfect vortices in [16].

Figure 3 shows additional results where here we change the period $d$ of the axicon grating to 15, 20 and 30 pixels respectively. In all cases the phase bias $\phi_0$ is the same, and produces the dark interference always in the center of the ring, independently of the period as in Fig. 2(c). Figure 3(c) shows the case with the larger period, thus showing the ring in the Fourier transform with smaller diameter, but also a weak third order ring is visible. By changing the phase shift between the two components in Eq. (5), these would all become the bright rings as in Fig. 2(a).

**Fig. 2.** Experimental results of the two-ring interference obtained by Fourier transform of a binary-phase diffractive axicon with a period of $d = 10$ pixels and (a) in phase, (b) with a relative phase of $\pi/2$, (c) with a relative phase of $\pi$ and (d) with a relative phase of $3\pi/2$. The corresponding intensity profile is shown on the right for each case.

**Fig. 3.** Experimental results of the two-ring interference obtained by Fourier transform of a binary-phase diffractive axicon with period (a) $d = 15$ pixels, (b) $d = 20$ pixels, (c) $d = 30$ pixels.

**4. Interference of perfect vortex beams**

Let us now combine the axicon phase function in Eq. (1) with a spiral phase term $\exp(i\ell \theta)$, where $\theta$ is the azimuthal coordinate and $\ell$ denotes the topological charge of the encoded vortex. Such a combined design has been described as generating the “perfect” vortex beam [6], since the circular shape of the beam does not exhibit the diameter variation with $\ell$ of standard vortex
beams. In most cases of optical trapping using such circular beams, the beam possesses orbital angular momentum which is transmitted to the trapped particles, causing them to rotate [10].

In this section we examine whether this could be applied to the binary axicon case. Figure 4(a) shows the same regular binary phase axicon as in Fig. 1(b) but with a phase bias \( \phi_0 = \pi/2 \) (note the difference in the center), while Fig. 4(b) shows the same axicon with a charge of \( \ell = 2 \) which adopts the form of a spiral axicon.

![Fig. 4. Binary masks for displaying a binary axicon with parameters (a) \( \ell = 0 \) and \( \phi_0 = \pi/2 \), (b) \( \ell = 2 \) and \( \phi_0 = 0 \).](image)

Note that such a binary spiral axicon can be regarded as the previous regular binary axicon, but with a phase bias that depends azimuthally as \( \ell \theta \). Therefore, a continuous variation along the ring of the above described interference condition is expected.

The transmission function of the binary spiral axicon can be written as:

\[
t(r) = \sum_{n=-\infty}^{\infty} c_n \exp\left(\text{i}n2\pi\frac{r}{d}\right) \exp(\text{i}n\phi_0) \exp(\text{i}n\ell\theta) \\
\cong c_1 \left\{ \exp\left(\text{i}2\pi\frac{r}{d}\right) \exp(\text{i}\phi_0) \exp(\text{i}\ell\theta) + \exp\left(-\text{i}2\pi\frac{r}{d}\right) \exp(-\text{i}\phi_0) \exp(-\text{i}\ell\theta) \right\},
\]

where again \( \phi_0 \) is a constant phase and the transmission function is again approximated to consider only the most intense \( \pm 1 \) diffraction orders, which have amplitude factors \( c_1 = 2/\pi \). Equation (6) shows that the addition of the spiral phase results in the superposition of two perfect vortex beams corresponding to the \( \pm 1 \) diffraction orders having opposite charge values. They generate two rings in the Fourier plane, each one having opposite sense azimuthal phase. However, the phase along the ring does not show the azimuthal continuous variation usually required to induce rotation in optical traps. Instead, it shows an interesting spiral interference caused by the opposite charge and the opposite convergence of the two diffraction orders.

The corresponding experimental results are shown in Fig. 5. In the left column [Figs. 5(a) and 5(b)] the case with \( \ell = 1 \) is shown for \( \phi_0 = 0 \) and \( \phi_0 = \pi/2 \), respectively. Compared to the case without spiral phase [Fig. 2(a)], it is observed that now the radial interference condition transits azimuthally from the situation to be bright in the center to be dark in the center. This is marked in the images by yellow and red arrows, respectively. The azimuthal phase difference is \( 2\ell\theta \) and therefore two transitions from constructive to destructive interference are observed. When the phase shift \( \phi_0 = \pi/2 \) is added, the pattern rotates by 90 degrees. Because the central constructive interference has an intensity maximum greater than the two lateral sidelobes, a 3D plot of the intensity (plots under each capture in Fig. 5) reveals the locations of these radial constructive/destructive interference by the maxima/minima. In the 3D plots for Fig. 5(a) and 5(b) the rotation is clearly shown.

Figures 5(c) and 5(d) correspond to the results obtained for a charge of \( \ell = 2 \). Now the topological charge difference between the \( \pm 1 \) diffraction orders is four, and the ring pattern exhibits four angles with bright interference in the center of the ring, and another four angles
Fig. 5. Experimental results of the Fourier transform of the binary-phase diffractive axicon with added spiral phase of charge $\ell = 1$ and constant phase $\phi_0$. (a) $\ell = 1$ and $\phi_0 = 0$, (b) $\ell = 1$ and $\phi_0 = \pi/2$, (c) $\ell = 2$ and $\phi_0 = 0$, (d) $\ell = 2$ and $\phi_0 = \pi/2$, (e) $\ell = 3$ and $\phi_0 = 0$ and (f) $\ell = 8$ and $\phi_0 = 0$. Red and yellow arrows indicate angular locations where the interference pattern shows darkness or brightness in the center of the ring, respectively.

where there is a dark central interference (these are again marked in the figure by yellow and red arrows). Once again, the interference pattern is reversed by adding a phase shift $\phi_0 = \pi/2$, as shown in Fig. 5(d). The corresponding 3D plot clearly visualizes this rotation.

Finally, Figs. 5(e) and 5(f) contain the result for charges $\ell = 3$ and $\ell = 8$ respectively, showing how the interference pattern exhibits $2\ell$ transitions from bright to dark interference.

5. Discussion for optical trapping systems

As mentioned before, given the number of works using axicons in optical trapping, we believe these interference effects caused by binarization might have interest for applications in this field. Most of the works in this area use a SLM to encode a continuous phase function. With the aid of a relay optical system, the phase function is projected onto the back aperture of a microscope objective, that provides the optical Fourier transform (FT) in the optical trap. The requirements
for optical trapping can be obtained using high power laser sources as well as the high numerical objective (typically 100X) lenses [5].

In this section we include some analysis of the use of the binary version of the axicon for its potential application in optical trapping. The first aspect to note is that the total optical diffraction efficiency of the ring generated by the binary axicon is of 81%, close to the 100% efficiency that can be obtained using the conventional continuous phase versions.

In optical trapping, the high intensity gradient forces provide the particle confinement while the scattering forces associated with the beam’s transverse phase gradients allow rotating the particle along the path [11]. Specifically, these transverse forces are proportional to the optical current [28], defined as $j = IV \phi$ where $I$ and $\phi$ are the intensity and the phase distributions in the Fourier transform plane. In these situations, a path of constant high intensity is typically desired and the motion of trapped particles is due to $\nabla \phi$, the gradient of the phase function. These patterns can be produced with a vortex mask [5], with a perfect vortex mask [9], or with computer generated holograms designed to generate specific intensity patterns and phase gradients aligned with the intensity path [11–13].

In order to gain some insight of these optical currents, in Fig. 6 we compare computer simulations of the performance that different designs produce in the focal plane of the Fourier transform lens. We consider four cases – the spiral phase plate, a continuous phase axicon combined with a spiral phase, the binary phase axicon, and the binary version of the axicon - spiral phase combination. In these simulations, we considered a SLM pixel spacing of 18 µm on a 512 × 512 screen and a Fourier transform lens with a focal length of 500 mm, equivalent to the parameters used in the experimental system. In all cases, we also multiplied the patterns by a circular binary mask having a diameter of 512 pixels to remove the square aperture of the SLM.

We start with the original case of the vortex beam trap [5] generated with a spiral phase plate, in this case having a charge of $\ell = 10$. In this case, the detector plane is considered to have a pixel size of 1.5 µm. Figure 6(a) shows three parts. The left [Fig. 6(a1)] shows the circular harmonic phase function multiplied by a circular mask to remove the square aperture of the SLM. The middle [Fig. 6(a2)] shows the well-known doughnut of light intensity originated by the vortex phase singularity. This circle of light is the optical trap circuit. In order to examine the phase along this intensity circuit, we multiplied the phase pattern in the FT by a binary mask corresponding to the maximum of the circular intensity pattern. This phase variation along the high intensity ring of light is shown on the right panel [Fig. 6(a3)] and shows a blazed angular grating and the phase gradient that causes the rotation of the trapped particles.

Next we consider a continuous phase axicon in Fig. 6(b). The left panel [Fig. 6(b1)] shows the continuous phase axicon with a period of $d = 10$ pixels multiplied by a spiral phase with topological charge of again $\ell = 10$. The output intensity ring of light is shown in Fig. 6(b2) and corresponds to a perfect vortex output. In this case, we increased the pixel size in the detector plane to 10 µm because this ring is larger than the one generated by the vortex beam in Fig. 6(a1). Therefore the relay optics mentioned above in the optical trapping system must be modified. The diameter of the ring remains roughly constant as the charge changes as it is characteristic of the perfect vortex beam generated in the FT domain [6]. Again the right panel shows the phase variation along the high intensity ring of light. This phase variation is spiral along the ring. Again, the phase gradient generated by this azimuthal variation is the cause of a tangential force that is able to induce rotation along the ring, as demonstrated with such perfect vortices in [10]. Note that this phase gradient is not perfectly well aligned with the intensity circuit, especially for low values of $\ell$.

In Fig. 6(c1), we examine the binary phase axicon that we experimentally demonstrated in Fig. 2. In this case, there is no spiral phase. Again, the phase difference between the two rings is selected to be zero so the two rings interfere in phase and they provide a narrower ring than the continuous phase axicon, as described and exploited in [15] for microfabrication. The calculated
Fig. 6. Computer simulations comparing the phase patterns, the intensity at the Fourier transform plane, and the phase along the maximum of the output for (a) a spiral phase having a charge of 10 and corresponding to a vortex lens, (b) a continuous phase axicon multiplied by a spiral phase having a charge of 10, (c) a binary phase axicon without a spiral phase and (d) a binary phase axicon with a spiral phase having a charge of 8.
intensity distribution agrees very well with the experiment in Fig. 2(a). However, in this case there is no azimuthal variation of the phase and therefore there is no angular phase gradient and trapped particles would not suffer any azimuthal force that may induce rotation along the ring.

Finally, in Fig. 6(d1) we examine the case of the binary version of the axicon plus the spiral phase, corresponding to the experiments shown in Fig. 5. In this case both the intensity and the phase show distributions that differ from the standard desired situation in optical trapping that seek for constant intensity along the ring and a linear phase variation along the circle.

These results show that binary axicons where the spiral phase is included produce interference effects that might limit their application in trapping systems. While the binary axicon does create a narrow ring of light, it does not produce any phase gradient. Next, we discuss a potential way to overcome these limitations for the binary phase axicon by using a “conveyor belt” approach.

6. Discussion for a conveyor belt

A new approach is shown in Fig. 7. In Fig. 2, we showed that the condition for constructive / destructive interference in the ring can be adjusted by the additional phase bias $\phi_0$. Now changing its value along the azimuthal coordinate results in a variation along the ring. In Fig. 7(a), the phase bias alternates from zero to $\pi/2$ in 12 sectors distributed azimuthally. As a consequence, the ring of light alternates between constructive / destructive central interference, forming a kind of conveyor belt along the circle of light. Note that a longitudinal conveyor belt was shown in a trapping system in [29].

![Fig. 7](image-url)

**Fig. 7.** Experimental results of the Fourier transform of the binary phase diffractive axicon, showing a circular optical conveyor belt.

Figure 7(b) shows experimental results of the output on the CCD detector, while Fig. 7(c) is a 3D representation of the intensity profile to better show this conveyor-belt effect. This conveyor belt could provide trapping of particles with refractive indices both higher and lower than the medium in regions of the ring where the center shows brightness and darkness, respectively. It might be necessary to restrict the azimuthal width of the trapping region to achieve the required trapping intensity.

In this case there is no azimuthal variation of the phase nor a phase gradient along the ring. However, note that this pattern can be rotated simply by rotating the phase mask. Angular motion of the trapped particles could be induced around the ring by rotating the entire pattern in Fig. 7(a). Frame rates up to 4000 frames/sec are advertised [30]. Assuming a 2 degree rotation/frame, a complete rotation would require about 200 frames and a rotation rate of about 20 rotations/sec.

7. Conclusions

In conclusion, we have experimentally analyzed the FT of a binary-phase axicon. The binary version of the axicon phase function generates a number of additional harmonic diffraction components, where the $\pm 1$ diffraction orders are the most relevant, each one carrying 40.5% of
the total energy. The FT of this binary axicon produces two rings of light with the same radius whose interference can be controlled and manipulated.

We showed that adding a constant phase bias changes the interference condition so that the ring of light can be switched from constructive interference with an intense bright central ring surrounded by weak radial sidelobes to destructive interference with a dark central ring surrounded by two brighter sidelobes. This situation has been recently described in [15] for the application in laser materials’ processing. Since some particles get trapped with light [1] while others get trapped in darkness surrounded by light [3], this could be also interesting in optical trapping systems.

Next, we discussed the situation where a spiral phase pattern is added in order to generate perfect vortex beams. In this case the two-ring interference results in a spiral interferogram along the ring which increases as $2\ell$ with the topological charge encoded together with the axicon. Because these perfect vortex beams have been often used in optical trapping systems, we included computer simulations of the FT field. Intensity patterns agree very well with our experiments, while the phase patterns show the phase gradients that would be expected.

Finally, we considered a case where the interference was modified azimuthally, creating a conveyor belt effect where the ring shows simultaneously angular sectors with central brightness and other with central darkness. This effect could be used to trap particles with refractive indices both higher and lower than the medium, thus generating two different trap conditions along the ring. Since it is generated with a binary phase mask, it can be displayed with a fast ferroelectric liquid crystal SLM and rotated at frame rates in the order of kHz.

We believe these results could be interesting to the community related to axicons, Bessel beams and optical trapping. Hopefully, this last application can be explored experimentally.

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