Data-Driven Fault-Tolerant Tracking Control for Linear Parameter-Varying Systems

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ABSTRACT This article proposes a data-driven passive fault-tolerant tracking controller for single-input single-output (SISO) discrete-time linear systems with slowly-varying unknown parameters in the presence of actuator faults. Initially, a parameterized controller is considered. Using the internal model principle, the controller structure is determined such that the passive-fault-tolerance and tracking objectives are achieved. The obtained fixed-structure controller is utilized for both normal and faulty conditions without using any feedback from the fault information. Thus, the controller is simple to implement and responds fast to the effects of faults. Then, using a data-driven technique and based on the input/output (I/O) data, the controller parameters are adjusted online to tackle the problem of parameter variation. A data-based constraint on the controller parameters is proposed for ensuring the stability of the closed-loop system. The proposed technique is extended to multi-input multi-output (MIMO) systems using the sequential loop closing concept and the relay auto-tuning method. Simulation results of applying the proposed controller to a direct current (DC) servo motor and a three-tank system demonstrate its effectiveness.

INDEX TERMS Data-driven control, fault-tolerant control, internal model principle, linear parameter-varying systems, tracking control.

I. INTRODUCTION

LINEAR parameter-varying (LPV) systems are dynamical systems whose models depend on time-varying parameters. These systems are often used to model nonlinear systems. Indeed, the time-varying parameters interpret the location of the system’s operating points. Since the introduction of the LPV paradigm in 1988 [1], it has become a standard formalism in the systems control, and a lot of research has been done on the synthesis of controllers for LPV systems [2]–[5].

In practical engineering, faults are usually inevitable and can occur in actuators, sensors, and/or internal components of the controlled system. Faults have deteriorative effects on the closed-loop performance and may cause severe performance degradation or even lead to system instability. Therefore, it is so important to design controllers that maintain the stability and performance of the system under the undesirable effects of faults and enhance the system safety and reliability. A lot of techniques have been proposed in the area of fault-tolerant control (FTC) (e.g., [6], [7] are survey articles on the topic). Abrupt and gradual faults are common types of faults that have received significant attention in the literature. For instance, the problem of detecting and diagnosing the gradual faults for linear time-invariant systems has been investigated in [8]. In [9], based on the marginalized likelihood ratio approach, a unified framework has been proposed for the detection and isolation of abrupt-type faults in linear time-invariant systems.

The FTC methods can be primarily classified into active and passive methods. Active methods are based on fault detection and diagnosis (FDD) [10]–[12]. Passive methods are independent of FDD and use a fixed controller that is robust against faults for both of the normal and faulty cases [13]. There are several studies available in the literature to
design FTCs for LPV systems. For instance, unknown input observers have been designed to estimate the abrupt and gradual actuator faults for LPV systems in [14], [15]. In [16], an adaptive polytopic observer has been designed to estimate abrupt and time-varying actuator faults for LPV systems. Active and passive FTCs for LPV systems in the presence of actuator faults have been investigated in [17] and [18], respectively. These approaches use the system model and have the drawback of difficulty to identify the model of the system. Furthermore, the modeling error can decrease the control performance. Indirect and direct adaptive controls have also been developed for controlling LPV systems, but they also explicitly or implicitly utilize the model information and rely on the model accuracy. Online measurement data and historical data collected from plants and recorded during their operations are easily available and can be directly used to design successful controllers. It is worthy to design the controller using data and without the usage of the process model thanks to the development of computers, which can process a large amount of data.

Data-driven controls (DDCs) are a group of techniques in which the controller is directly designed using the input/output (I/O) data without utilizing any information from the mathematical model of the system. The studies on DDC techniques have attracted considerable attention in recent years. Existing methods in literature have designed DDCs for LPV systems without considering the occurrence of faults [19]–[23]. Besides, some data-driven fault-tolerant techniques based on fault detection and diagnosis have been proposed, which are considered as active fault-tolerant techniques and can be applied to LPV systems [24]–[28]. For instance, a data-driven output-feedback fault-tolerant compensation scheme has been proposed for time-varying linear systems under the digital proportional-integral-derivative (PID) control in [25]. In [26], a data-driven output-feedback fault-tolerant tracking control has been proposed and applied to a direct current servo motor. Active FTCs have the drawbacks of complexity and decreasing their performance due to time delay of online FDDs before computing and applying control signals. In comparison, passive FTCs are simple to be implemented and respond faster to the adverse effects of faults.

To the best of our knowledge, there is a literature gap in designing data-driven passive fault-tolerant controllers for LPV systems. In this paper, a data-driven passive fault-tolerant tracking controller is proposed for single-input single-output (SISO) discrete-time linear systems with slowly-varying unknown parameters considering the occurrence of abrupt and gradual actuator faults. The proposed passive fault-tolerant technique employs the internal model principle (IMP) to design a parameterized fixed-structure controller which compensates the system for undesirable effects of faults. Furthermore, the structure of the controller is designed such that the tracking objective is also achieved. The controller parameters are adjusted online based on a data-driven technique to cope with the variation of parameters. To ensure the stability of the closed-loop system, a data-based constraint on the controller parameters is proposed. The proposed control technique is also extended to multi-input multi-output (MIMO) systems using the sequential loop closing idea. Overall, the main contribution of the paper is to design a simple yet effective passive fault-tolerant controller for LPV systems without using the process model. The main advantages of the proposed control technique are as follows:

- Compared to the works in [14]–[18], the controller is directly designed utilizing the I/O data and without any use of the process model. The proposed control scheme does not require to identify a model of the system. Therefore, the drawback of difficulty to derive the LPV model for the system is alleviated. At the same time, control performance due to modeling errors is also not affected.

- Compared to the works in [24]–[28], a fixed controller, which is robust against abrupt and gradual actuator faults, is used for both the normal and faulty conditions without using any feedback from the fault information. Hence, the proposed control technique is simple to be implemented and responds fast to the adverse effects of faults.

The rest of the paper is organized as follows. The data-driven FTC design problem is described in Section II. In Section III, the proposed data-driven fault-tolerant tracking controller is presented in detail. In Section IV, a data-based constraint is proposed for ensuring the frozen-time stability of the closed-loop system. In Section V, the results of applying the proposed data-driven FTC to a direct current (DC) servo motor and a three-tank system are presented. Finally, Section VI concludes the paper.

II. PROBLEM STATEMENT
Consider the control design architecture shown in Figure 1, where \( u(k) \), \( y(k) \), \( r(k) \), and \( f(k) \) denote the control input, system output, reference input, and additive actuator fault, respectively. The plant model is unknown SISO discrete-time linear with a set of slowly-varying parameters. For the frozen values of the parameters, the system’s transfer function is

\[
P^*(z^{-1}) = \frac{B(z^{-1})}{A(z^{-1})},
\]

where \( P^*(z^{-1}) \) is the discrete-time transfer function of the plant \( P \) whose parameters are frozen at their values at time step \( k^* \); \( z^{-1} \) is the unit-delay operator; \( A(z^{-1}) = 1 + a_1(k^*)z^{-1} + \ldots + a_m(k^*)z^{-m} \) and \( B(z^{-1}) = b_0(k^*) + \ldots + b_n(k^*)z^{-n} \).
is denoted as $\Sigma$, and the closed-loop system with the frozen-time plant $P^*$ is denoted as $\Sigma^*$. $\Sigma$ is said to be frozen-time stable if each frozen-time closed-loop system $\Sigma^*$ is stable [36]–[38].

**Theorem 1.** Suppose the closed-loop system in Figure 1 is frozen-time internally stable, and $B(z^{-1})$ does not have zeros at $z = 1$. Assume further that the additive actuator fault $f(k)$ affects the system where $F(z^{-1})$ is as (3), $\hat{n} = 1, \ldots, n$, and $n$ is a user-defined parameter which is determined according to the types of faults that should be tolerated. Then, the effects of $f(k)$ on the tracking error $e(k) = r(k) - y(k)$ converges to zero if $\phi_4(z^{-1}) = (1 - z^{-1})^n$.

**Proof.** For $\phi_4(z^{-1}) = (1 - z^{-1})^n$, the error response to the fault is
\[
E(z^{-1}) = \frac{-P^*(z^{-1})}{1 + P^*(z^{-1})C(z^{-1})}F(z^{-1}) = -B(z^{-1})(1 - z^{-1})^n G(z^{-1}) \frac{\phi(z^{-1})}{\phi_1(z^{-1})(1 - z^{-1})^n} = -B(z^{-1})G(z^{-1})(1 - z^{-1})^n \phi_1(z^{-1}) \phi_4(z^{-1}),
\]
where $\phi_4(z^{-1}) = A(z^{-1})(1 - z^{-1})^n + B(z^{-1})(K_1 \phi_1(z^{-1}) + K_2 \phi_2(z^{-1}) + K_3 \phi_3(z^{-1}))$, and $n - \hat{n} \geq 0$. Since $\Sigma$ is frozen-time internally stable, all roots of $\phi(z^{-1})$ are inside the unit circle. Hence, the effect of $f(k)$ on $e(k)$ asymptotically tends to zero.

Using Theorem 1, $C(z^{-1})$ in (4) is rewritten as follows:
\[
C(z^{-1}) = K_1 \frac{\phi_1(z^{-1})}{(1 - z^{-1})^n} + K_2 \frac{\phi_2(z^{-1})}{(1 - z^{-1})^n} + K_3 \frac{\phi_3(z^{-1})}{(1 - z^{-1})^n},
\]

As can be concluded from Theorem 1, (5) is a fault-tolerant fixed-order controller which is robust against faults $F(z^{-1}) = G(z^{-1})(1 - z^{-1})^n$ with $\hat{n} = 1, \ldots, n$. The order $n$ of $C(z^{-1})$ is determined according to the types of faults that should be tolerated. Theorem 2 achieves the other control objective for the system, i.e. output tracking.

**Theorem 2.** Suppose the closed-loop system in Figure 1 is frozen-time internally stable. Under the data-driven controller (5), $y(k)$ tracks $r(k)$ if $\phi_i(z^{-1})$ ($i = 1, 2, 3$) are selected as follows:

\[
\phi_1(z^{-1}) = 1, \\
\phi_2(z^{-1}) = \Delta(z^{-1}) = 1 - z^{-1}, \\
\phi_3(z^{-1}) = \Delta^2(z^{-1}) = (1 - z^{-1})^2.
\]

**Proof.** To achieve the output tracking, $\phi_i(z^{-1})$ ($i = 1, 2, 3$) should be determined such that the DC gain of the closed-loop transfer function converges to 1. Using (6) and from Figure 1, the DC gain of the closed-loop transfer function $T(z^{-1})$ is computed as follows:
\[
T_{\text{ns}} = \lim_{z \to 1} T(z^{-1}) = \lim_{z \to 1} \frac{P^*(z^{-1})C(z^{-1})}{1 + P^*(z^{-1})C(z^{-1})} = \lim_{z \to 1} \frac{B(z^{-1})(K_1 + K_2 \Delta + K_3 \Delta^2)}{A(z^{-1}) \Delta^2 + B(z^{-1})(K_1 + K_2 \Delta + K_3 \Delta^2)} = 1.
\]

The proof is completed.

**III. PROPOSED DATA-DRIVEN FAULT-TOLERANT TRACKING CONTROLLER**

To solve the problem defined in the previous section, a novel data-driven fault-tolerant tracking controller is proposed. Motivated by [34], [35], the following parameterized fixed-structure controller is considered:
\[
C(z^{-1}) = K_1 \frac{\phi_1(z^{-1})}{\phi_4(z^{-1})} + K_2 \frac{\phi_2(z^{-1})}{\phi_4(z^{-1})} + K_3 \frac{\phi_3(z^{-1})}{\phi_4(z^{-1})},
\]
where $K_1$, $K_2$, and $K_3$ are the controller parameters which should be adjusted online using a DDC technique to tackle the variation of parameters, $\phi_4(z^{-1})$, $\phi_2(z^{-1})$, $\phi_3(z^{-1})$, and $\phi_4(z^{-1})$ are suitable polynomials in $z^{-1}$, which should be determined such that the tracking and fault-tolerance objectives are achieved.

**Definition 1.** The closed-loop system with the LPV plant $P$
In closing, it can be claimed that the following controller achieves both the control objectives for the system:

\[ C(z^{-1}) = K_1 \frac{1}{(1 - z^{-1})^2} + K_2 \frac{1 - z^{-1}}{(1 - z^{-1})^3} + K_3 \frac{1 - z^{-1}}{(1 - z^{-1})^4}. \]  

(7)

**Remark 2.** For \( n = 1 \), (7) is a PID controller which is not causal. In this case, a supplementary large-magnitude pole is added to the third term of (7) to cope with this problem.

A data-driven technique is also proposed to attain an online implementation of (7). Using (7) and from Figure 1, the control law is obtained as

\[ \Delta^3 u(k) = K_1(k)e(k) - K_2(k)\Delta y(k) - K_3(k)\Delta^2 y(k). \]  

(8)

The problem is to identify proper values for \( K_1, K_2, \) and \( K_3 \). To this end, let us rewrite (8) as follows:

\[ u(k) = g(\phi'(k)), \]

\[ \phi'(k) \triangleq [K(k), r(k), y(k), y(k - 1), y(k - 2), u(k - 1), \]

\[ \quad \cdots, u(k - n)]. \]  

(9)

\[ K(k) = [K_1(k), K_2(k), K_3(k)], \]

where \( g(\cdot) \) is an appropriate function of \( \phi'(k) \). Taking the inverse Z-transform of (1), the output equation is written as

\[ y(k) = P(\phi(k - 1)), \]  

(10)

where \( P(\phi(k - 1)) : \mathbb{R}^{2m_1} \rightarrow \mathbb{R} \) expresses a function whose output is determined by the information vector \( \phi(k - 1) \) defined by

\[ \phi(k - 1) \triangleq [y(k - 1), \cdots, y(k - m_1), u(k - 1), \cdots, u(k - m_1)]. \]  

(11)

By substituting \( u(k) \) from (9) into (11) and using (10), the system equation is derived as

\[ y(k + 1) = h(\tilde{\phi}(k)), \]

\[ \tilde{\phi}(k) = [y(k), \cdots, y(k - \max((m_1 - 1), 2)), K(k), \]

\[ r(k), u(k - 1), \cdots, u(k - \max((m_1 - 1), n))], \]

where \( h(\cdot) \) is a function of \( \tilde{\phi}(k) \). Therefore, \( K(k) \) is related to the information vector \( \tilde{\phi}(k) \) as follows:

\[ K(k) = \tilde{h}(\tilde{\phi}(k)), \]

\[ \tilde{\phi}(k) \triangleq [y(k + 1), y(k), \cdots, y(k - \max((m_1 - 1), 2)), \]

\[ r(k), u(k - 1), \cdots, u(k - \max((m_1 - 1), n))]. \]

Since \( y(k + 1) \) should tend to \( r(k + 1) \), \( \tilde{\phi}(k) \) can be considered as

\[ \tilde{\phi}(k) = [r(k + 1), r(k), y(k), \cdots, y(k - \max((m_1 - 1), 2)), u(k - 1), \cdots, u(k - \max((m_1 - 1), n))]. \]

Now, it is possible to adjust the parameters of the controller in an online manner. Motivated by the developments in [23], \( K_1, K_2, \) and \( K_3 \) for each time step can be determined by utilizing the I/O database. The procedure is performed in the following five steps.

**Step A** A simple PID controller is designed to obtain an initial set of the controller parameters, close the feedback loop, and generate an initial database. The structure of the PID controller is as follows:

\[ u(k) = K_c \left( e(k) + \frac{T_s}{T_1} \sum_{i=1}^{k} e(i) + \frac{T_D}{T_s} [e(k) - e(k - 1)] \right), \]

where \( K_c, T_s, T_1, \) and \( T_D \) are the proportional gain, sampling interval, integral time constant, and derivative time constant, respectively. These parameters can be adjusted using the well-known Ziegler-Nichols tuning methods [39]. Then, the initial parameters of the proposed data-driven fault-tolerant tracking controller are calculated as

\[ \begin{cases} K_1 = \frac{K_c}{T_1}, & K_2 = K_c, \quad K_3 = \frac{K_c T_D}{T_s} \quad \text{for } n = 1, \\ K_1 \leq \frac{K_c T_D}{T_s}, & K_2 = \frac{K_c T_D}{T_s}, \quad K_3 = K_c, \quad \text{for } n \geq 2. \end{cases} \]  

(12)

The format of the dataset is defined as \( \Phi(j) \triangleq [\tilde{\phi}(j), K(j)], \) \( j = 1, 2, \ldots, N(k), \) where \( j \) is an index of the database, and \( N(k) \) is the number of information vectors at the step \( k \). Note that, \( N(0) \) denotes the size of the initial database. At each time step, \( \hat{N} + 1 \) pairs of the latest input-output data are used to construct the information vector \( \Psi(k) = [y(k), u(k), y(k - 1), \cdots, y(k - \hat{N}), u(k - \hat{N})] \).

**Step B** It is necessary to determine the distances between \( \tilde{\phi}(k) \) (which indicates the current system state) and \( \tilde{\phi}(j) \) \( (j \neq k) \) stored in the database. To this end, the mentioned distance is calculated as follows:

\[ D(\tilde{\phi}(k), \tilde{\phi}(j)) = \sum_{i=1}^{r_d} \left| \frac{\tilde{\phi}_i(k) - \tilde{\phi}_i(j)}{\max_m (\tilde{\phi}_i(m)) - \min_m (\tilde{\phi}_i(m))} \right|, \]

(13)

where \( r_d = \max((m_1 - 1), 2) + \max((m_1 - 1), n) + 3; \)

\( \tilde{\phi}_i(k) \) and \( \tilde{\phi}_i(j) \) denote the \( i \)th element of the vector \( \tilde{\phi}(k) \) and the \( i \)th element of the \( j \)th information vector, respectively; \( \max_m (\tilde{\phi}_i(m)) \) and \( \min_m (\tilde{\phi}_i(m)) \) are the maximum and minimum elements among the \( i \)th element of all vectors stored in the database \( (\tilde{\phi}(m), \quad m = 1, \ldots, N(k)), \) respectively. Then, \( p \)-pieces datasets with the smallest distances are selected as the neighbor datasets.

**Step C** The controller parameters are computed using the \( p \)-neighbors selected in Step B as follows:

\[ \hat{K}(k) = \sum_{i=1}^{p} w_i K(i), \quad w_i = \frac{1}{\sum_{i=1}^{p} \frac{1}{D_i}}, \]

(14)

where \( \hat{K}(k) \) is the computed parameters and \( w_i \) \( (i = 1, \ldots, p) \) are weights corresponding to the information vectors selected in Step B. Now, using the stability constraint (19) proposed in Section IV, it should be determined whether or not the controller (7) with \( \hat{K}(k) \) stabilizes the frozen-time closed-loop system \( \Sigma^* \). If (19) holds for \( \hat{K}(k) \) computed by (14), \( u(k) \) is computed and applied to the system; and then, \( y(k + 1) \) is measured. Otherwise, \( \hat{K}(k) \) is selected among the control parameters stored in the database and is considered as one that fulfills (19) and minimizes (13); and then, \( u(k) \) is computed and applied to the system.
**Step D** To enhance the tracking performance, the control parameters computed in **Step C** must be updated. Then, the updated parameters are stored in the database. These parameters should be adjusted such that the tracking error is decreased. This aim can be achieved using the following steepest descent strategy:

\[
K(k) = \hat{K}(k) - \eta \frac{\partial J(k+1)}{\partial K(k)}, J(k+1) \doteq \frac{1}{2} \varepsilon (k+1)^2, \tag{15}
\]

where \(\eta = \text{diag}\{\eta_1, \eta_2, \eta_3\}\) is the learning rate; \(J(k)\) is the error criterion; and \(\varepsilon(k) = y_t(k) - y(k)\) is the tracking error between the system output and the following reference model:

\[
y_t(k) = \frac{z^{-1}T(1)}{T(z^{-1})} r(k). \tag{16}
\]

To select \(T(z^{-1})\), it is possible to use the desired rise-time \((\sigma)\) and the desired damping coefficient \((\mu)\) as follows [23]:

\[
T(z^{-1}) = 1 + t_1 z^{-1} + t_2 z^{-2},
\]

\[
t_1 = -2 \exp\left(-\frac{\rho}{2\mu}\right) \cos\left(\sqrt{2\mu-1}\right), \quad t_2 = \exp\left(-\frac{\rho}{\mu}\right).
\]

\[\rho = \frac{T_s}{\sigma}, \quad \mu = 0.25(1 - \delta) + 0.51\delta.\]

\(\mu\) is adjusted by changing \(\delta\), which can be \(\sigma = 0\) or \(\delta = 1\) representing the step response of the reference model per the Binominal model response and the Butterworth model response, respectively [23]. The elements of the Jacobian \(\partial J(k+1)/\partial K(k)\) are written as

\[
\begin{align*}
\partial J(k+1) \quad \partial K_1(k) &= \partial J(k+1) \partial z(k+1) \partial y(k+1) \partial u(k) \\
&\quad \partial u(k) \partial \partial K_1(k) \\
&= -\varepsilon(k+1) \varepsilon(k+1) - \partial y(k+1) \\
\partial K_2(k) &= \partial J(k+1) \partial z(k+1) \partial y(k+1) \partial u(k) \\
&\quad \partial u(k) \partial \partial K_2(k) \\
&= \varepsilon(k+1) \varepsilon(k+1) (y(k) - y(k-1)) - \partial y(k+1) \\
\partial K_3(k) &= \partial J(k+1) \partial z(k+1) \partial y(k+1) \partial u(k) \\
&\quad \partial u(k) \partial \partial K_3(k) \\
&= \varepsilon(k+1) (y(k) - 2y(k-1) + y(k-2)) - \partial y(k+1) \\
&\quad \partial u(k).
\end{align*}
\]

The system Jacobian is defined as \(\tilde{y}(k+1)/\partial u(k) = \tilde{y}(k+1)/\partial u(k) \text{ sign}(\tilde{y}(k+1)/\partial u(k))\) where \(\text{sign}(x)\) is the signum function, i.e., \(\text{sign}(x) = 1\) for \(x > 0\), while \(\text{sign}(x) = -1\) for \(x < 0\). In this paper, it is supposed that the sign of the system Jacobian is known. Moreover, \(\tilde{y}(k+1)/\partial u(k)\) is included in \(\eta\).

**Step E** This step is taken to avoid excessive stored data. Toward this end, except for the \(p\)-neighbors selected in **Step B**, the vectors satisfying both of the following conditions are deleted from the database.

\[
\begin{align*}
D(\tilde{\phi}(k), \tilde{\phi}(j)) &\leq \alpha_1, \\
\sum_{i=1}^{3} \left( \frac{K_i(k) - \hat{K}_i(k)}{\hat{K}_i(k)} \right)^2 &\leq \alpha_2, \quad j = 1, 2, \ldots, \hat{N}(k) - p \quad \text{(18)}
\end{align*}
\]

where \(\alpha_1\) and \(\alpha_2\) are small user-defined coefficients that should be set between 0.1 and 1.

**IV. FROZEN-TIME STABILITY CONSTRAINT**

To ensure the stability of the closed-loop system, the constraint (2) is imposed on the system parameters to confirm the slow variation of them, and the controller parameters are selected such that the closed-loop system with the LPV plant is frozen-time stable. In this section, a data-based constraint on the controller parameters is proposed to ensure the frozen-time stability of the closed-loop system. At each time step \(k\), given the data \(\Psi(k) = [y(k), u(k), \ldots, y(k-N), u(k-N)]\) which are demonstrative of the current plant behavior, the task is to determine whether or not (7) with \(K(k)\) stabilizes \(\Sigma^*\). The frozen-time stability constraint is considered as a bound on a cost function as follows:

\[
\max_{k \in [k-N, k]} \left\| \frac{\varepsilon_t}{\hat{K}_k} \right\|_k \leq \gamma, \tag{19}
\]

where \(0 < \gamma < 1\) is defined by the user; \(\varepsilon_t(k)\) is a fictitious desired signal which would have exactly reproduced the data \(\Psi(k)\) if (7) with \(\hat{K}(k)\) had been in the loop and is obtained as follows; and

\[
\varepsilon(t) = \hat{y}_t(t) - y(t) = \frac{z^{-1}T(1)}{T(z^{-1})} r(t) - y(t),
\]

\[
\varepsilon_t = C^{-1}(z^{-1}) u(t) + y(t), \quad t \in [k-N, k],
\]

\[
\max_{k \in [k-N, k]} \left\| \frac{\varepsilon_t}{\hat{K}_k} \right\|_k = \left\| \frac{\varepsilon_t}{\hat{K}_k} \right\|_k = \max_{k \in [k-N, k]} \sqrt{\sum_{k-N}^{k} \varepsilon_t^2}. \tag{19}
\]

The design procedure of the proposed data-driven fault-tolerant tracking controller is summarized below.

**Algorithm 1.**

- **Step 1** A PID controller is firstly designed using the Ziegler-Nichols tuning methods [39] to generate an initial set of controller parameters using (12).
- **Step 2** Select the order of the controller (7) according to the types of faults that should be tolerated. Generate the control law (8).
- **Step 3** Calculate the distances between \(\tilde{\phi}(k)\) and \(\tilde{\phi}(j)\) \((j \neq k)\) using (13); then, \(p\)-neighbors data are selected.
- **Step 4** Stabilizing controller parameters \(K(k)\) are computed using Step C and the stability constraint (19); then, they are applied to the system.
- **Step 5** The fictitious reference signal and the correction term \(\partial J(k+1)/\partial K(k)\) are calculated using (16) and (17), respectively. Then, the controller parameters are modified using (15), and the database is learned using the modified parameters \(K(k)\).
- **Step 6** Redundant data are removed using (18).
- **Step 7** Repeat Steps 3-6 at each time step \(k\).

Figure 2 shows the structure of the closed-loop system using the proposed data-driven fault-tolerant tracking control.

**Remark 3.** The proposed method can be extended to design data-driven fault-tolerant tracking controls for MIMO systems.
systems. The only further steps which must be taken are pairing the input and output variables and designing a multi-loop PID controller. The pairing mode can be determined via the relative gain array (RGA) method or the singular value analysis (SVA) [40]. Then, it is possible to design a multi-loop PID controller for the MIMO process using the sequential loop closing idea and the relay auto-tuning method [41]–[43]. The tuned PID parameters are used to generate initial databases, and the data-driven fault-tolerant tracking controllers are independently designed for the SISO loops using Algorithm 1. The main idea of the sequential loop closing with a relay auto-tuning method is to control the MIMO system loop by loop. For instance, for a process with two inputs and two outputs, a SISO controller is tuned for the first loop using a relay tuner, while the other loop is on manual. Then, the first loop is closed and another SISO controller is designed for the second loop by placing another relay tuner on it. This procedure is repeated until converging of the controller parameters is achieved.

V. SIMULATION RESULTS

A. DC SERVO MOTOR CONTROL SYSTEM

The state-space model of a DC servo motor is as follows [32]:

\[
\begin{bmatrix}
\dot{\theta}(t) \\
\dot{\omega}(t) \\
\dot{i}_a(t)
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 \\
0 & -\frac{B}{K_e} & \frac{K_i}{L_a} \\
0 & -\frac{1}{L_a} & 0
\end{bmatrix}
\begin{bmatrix}
\theta(t) \\
\omega(t) \\
i_a(t)
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
\frac{1}{L_a}
\end{bmatrix} V_a(t),
\]

\[
y(t) =
\begin{bmatrix}
0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
\theta(t) \\
\omega(t) \\
i_a(t)
\end{bmatrix},
\]

where \(\theta(t)\) is the angular position; \(\omega(t)\) is the angular velocity; \(i_a(t)\) is the armature current; \(V_a(t)\) is the control input and represents the armature voltage. The parameters of the system are \(B = 0.1\) N-m/(rad/s), \(J = 0.055\) kg-m\(^2\), \(K_i = 55\) N-m/A, \(K_e = 0.022\) V s/rad, and \(L_a = 0.033\) H. \(R_a(t)\) is the armature resistance, and it is assumed that its value changes slowly due to the change of temperature as

\[
R_a(t) = \begin{cases} 6 \Omega, & 0 \leq t < 2, \\ 6 + 1.5(t-2) \Omega, & 2 \leq t < 5. 
\end{cases}
\]

By using the Euler approximation [44] with the sampling time \(T_s = 0.001\) s, the discrete-time equivalent of the system is obtained as follows:

\[
\begin{align*}
\theta(k+1) &= \theta(k) + T_s \omega(k), \\
\omega(k+1) &= \omega(k) + T_s \left(-\frac{B}{J} \omega(k) + \frac{K_i}{J} i_a(k)\right), \\
i_a(k+1) &= i_a(k) + T_s \left(-\frac{K_e}{L_a} \omega(k) - \frac{R_a(t)}{L_a} i_a(k) + \frac{1}{L_a} V_a(k)\right), \\
y(k) &= \omega(k).
\end{align*}
\]

The control problem is to adjust \(V_a(k)\) such that \(\omega(k)\) tracks a desired trajectory and the step/ramp-type faults (which represent electronic-component failures, temperature drift in components, and aging problems in components) in the actuator (a pre-amplifier and a servo amplifier) are tolerated. According to the types of faults, the order of the controller (7) is considered as \(n = 2\); and \(T(z^{-1}) = 1 - 1.021z^{-1} + 0.6702z^{-2}\). The other user-specified parameters are selected as \(m_1 = 3\), \(p = 6\), \(\eta_1 = \eta_2 = \eta_3 = 0.01\), \(\alpha_1 = 0.2\), \(\alpha_2 = 0.2\), \(N(0) = 50\), and \(\hat{N} = 50\). The initial controller parameters are considered as \(K_1 = 0.001\), \(K_2 = 0.01\), and \(K_3 = 2\). A ramp-type fault with slope \(m = 1\) is examined at the actuator at \(t = 2.5\) s.

Towards evaluating the effect of the actuator faults and demonstrating the performance of the proposed data-driven fault-tolerant control technique, another simulation study is done to control the angular velocity of the considered DC motor by applying the data-driven PID controller proposed for systems with time-variant parameters or nonlinear properties in [23]. The initial parameters of the data-driven PID controller are considered as \(K_P = 2\), \(K_I = 0.01\), and \(K_D = 0\). The angular velocity based on both literature [23] and the control scheme of this paper is depicted in Figure 3. The proposed controller tolerates the occurred fault and contributes towards getting the motor speed to successfully

![FIGURE 3. Obtained results for the angular velocity of the DC motor.](image-url)
track its set goal trajectory. For the data-driven PID controller proposed in [23], however, the angular velocity diverges from the desired-set value during fault occurrence.

B. THREE-TANK CONTROL SYSTEM

The schematic diagram of a three-tank process is presented in Figure 4. The model of the system is as follows [45]:

\[ \begin{align*}
A \dot{h}_1(t) &= q_1(t) - q_{13}(t), \\
A \dot{h}_2(t) &= q_2(t) + q_{32}(t) - q_{20}(t), \\
A \dot{h}_3(t) &= q_{13}(t) - q_{32}(t), \\
y_1(t) &= h_1(t), \\
y_2(t) &= h_2(t),
\end{align*} \]

where \( h_i(t) \) (\( i = 1, 2, 3 \)) in m is the liquid level in the \( i \)th tank at time \( t \); \( q_1(t) \) and \( q_2(t) \) are input flow rates in m³/sec; \( q_{ij}(t) \) (\( i, j \in \{1, 2, 3\} \)) represents the rate of liquid flow from the \( i \)th tank into the \( j \)th tank in m³/sec; and \( A = 0.0149 \) m² denotes the cross-sectional area of any of the identical tanks. Using the generalized Torricelli’s rule, the flow-rate equations are obtained as:

\[
\begin{align*}
q_{13}(t) &= a_1 S \sqrt{2g(h_1(t) - h_3(t))}, \\
q_{32}(t) &= a_3 S \sqrt{2g(h_3(t) - h_2(t))}, \\
q_{20}(t) &= a_2 S \sqrt{2gh_2(t)};
\end{align*}
\]

where \( g = 9.81 \) m/sec² is the acceleration of gravity; \( a_1 = 0.418 \), \( a_2 = 0.789 \), and \( a_3 = 0.435 \) are outflow coefficients; \( S = 5 \times 10^{-5} \) m² is the cross-sectional area of connection pipes. The linearized dynamic equations of the system are as follows:

\[
\begin{align*}
A \dot{\tilde{h}}_1(t) &= \tilde{q}_1(t) - C_{13} \tilde{h}_1(t) + C_{13} \tilde{h}_3(t), \\
A \dot{\tilde{h}}_2(t) &= \tilde{q}_2(t) + C_{32} \tilde{h}_3(t) - C_{32} \tilde{h}_2(t) - C_{20} \tilde{h}_2(t), \\
A \dot{\tilde{h}}_3(t) &= C_{13} \tilde{h}_1(t) - C_{13} \tilde{h}_3(t) - C_{32} \tilde{h}_3(t) + C_{32} \tilde{h}_2(t),
\end{align*}
\]

where \( C_{ij} = a_i S \sqrt{g/(2(h_i - \hat{h}))} \) with \( C_{20} = a_2 S \sqrt{g/(2h_2)}, \hat{h}_1(t) = h_1(t) - \hat{h}_1 (i = 1, 2, 3) \), and \( \hat{h}_1 \) is the level of the \( i \)th tank at the operating point. Moreover, \( \tilde{q}_1(t) = \hat{q}_1(t) - q_1(i = 1, 2); \tilde{q}_2 \) denote the supplying flow rates at the operating point. The parameters \( C_{ij} \)

| Loop 1 | Loop 2 |
|--------|--------|
| \( y_1 \leftrightarrow q_1 \) | \( K_{11} = 0.003, K_{12} = 0.112, K_{13} = 2.54 \) |
| \( y_2 \leftrightarrow q_2 \) | \( K_{21} = 0.003, K_{22} = 0.111, K_{23} = 2.55 \) |

can be considered as slowly-varying unknown parameters. Therefore, the three-tank process can be treated as a linear system with slowly-varying unknown parameters. Using the Euler approximation with the sampling time \( T_s = 0.1 \) s, the discrete-time equivalent of (20) is given by:

\[
\begin{align*}
h_1(k+1) &= h_1(k) + T_s \left( q_1(k) - a_1 S \sqrt{2g(h_1(k) - h_3(k))} \right), \\
h_2(k+1) &= h_2(k) + T_s \left( q_2(k) + a_3 S \sqrt{2g(h_3(k) - h_2(k))} \right) - a_2 S \sqrt{2gh_2(k)}, \\
h_3(k+1) &= h_3(k) + T_s \left( a_1 S \sqrt{2g(h_1(k) - h_3(k))} \right) - a_3 S \sqrt{2g(h_3(k) - h_2(k))}, \\
y_1(k) &= h_1(k), \\
y_2(k) &= h_2(k).
\end{align*}
\]

The control problem is to manipulate \( q_1(k) \) and \( q_2(k) \) such that the levels of the first two tanks are set to their desired trajectories and the step/ramp-type faults in pumps (which can be DC motors, for instance, and the step/ramp-type faults can occur in them due to electronic-component failures, temperature drift in components, and aging problems) are tolerated. It is assumed that \( q_1(k) \) and \( q_2(k) \) are limited to 100 cm³/s. The system equations (20) have four state regions in which they are differentiable. In this paper, the region \( h_1(k) > h_3(k) > h_2(k) \) is considered. The reference signals are considered as

\[
\begin{align*}
\begin{cases}
r_1(t) = 0.15 \text{ (m)}, & r_2(t) = 0.05 \text{ (m)}, & 0 \leq t < 50, \\
r_1(t) = 0.17 \text{ (m)}, & r_2(t) = 0.07 \text{ (m)}, & 50 \leq t < 110.
\end{cases}
\end{align*}
\]

The three-tank process is a MIMO system. Therefore, the first step is to pair the input and output variables. The pairing mode is considered as \( (y_1 \leftrightarrow q_1) \) and \( (y_2 \leftrightarrow q_2) \) [46]. Hence, two independent data-driven fault-tolerant tracking controllers of form (8) with \( n = 2 \) are designed. Table 1 represents the initial controller parameters. According to the proposed method, these parameters are used to generate two initial databases. The polynomial \( T(z^{-1}) \) is considered as \( T(z^{-1}) = 1 - 1.621 z^{-1} + 0.6702 z^{-2} \). Note that this polynomial is obtained by setting \( \sigma = 1 \) s and \( \delta = 0 \). Other user-specified parameters are selected as \( m_1 = 3, p = 6, \eta_1 = \eta_2 = 1, \eta_3 = 10, \alpha_1 = 0.5, \alpha_2 = 0.1, N(0) = 8, \) and \( N = 50 \). A fault of ramp-type with slope \( m = 0.005 \) at \( \bar{t} = 30 \) s and a step-type fault with magnitude \( m = 0.006 \) at \( \bar{t} = 80 \) s are considered in the first actuator and the second actuator, respectively. The obtained closed-loop responses for the liquid levels are shown in Figure 5. The corresponding trajectories of the controller parameters are depicted in Figure 6. From Figure 5, it can be observed that by changing the operating points at \( \bar{t} = 50 \) s and thereby changing the values of unknown parameters \( C_{ij} \), the proposed data-driven
technique. The proposed method has also been extended to adequately update based on the I/O data via a data-driven parameter variation, has been developed. The proposed algorithm for designing a controller has been proposed for linear systems with slowly-varying unknown parameters. An algorithm for designing a controller for MIMO processes and has been evaluated via simulations on the three-tank process and a DC servo motor system. The obtained simulation results demonstrate the effectiveness of the control technique. For future work, we plan to investigate and design data-driven passive fault-tolerant tracking control schemes against other types of actuator faults or sensor/component failures.

VI. CONCLUSION

In this study, a data-driven passive fault-tolerant tracking controller has been proposed for linear systems with slowly-varying unknown parameters. An algorithm for designing a parameterized fixed-structure controller, which achieves the tracking objective and is robust against actuator faults and parameter variation, has been developed. The proposed algorithm is flexible in dealing with abrupt and gradual faults such as step and ramp-type faults. The controller parameters are adequately updated based on the I/O data via a data-driven technique. The proposed method has also been extended to MIMO processes and has been evaluated via simulations on the three-tank process and a DC servo motor system. The obtained simulation results demonstrate the effectiveness of the control technique. For future work, we plan to investigate and design data-driven passive fault-tolerant tracking control schemes against other types of actuator faults or sensor/component failures.

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