COMPOUND PROPOSITIONAL LAW FOR LOGICAL EQUIVALENCE, TAUTOLOGY AND CONTRADICTION

Umair Khalid Qureshi¹, Parivish Sami Lander², Shahzad Ali Khaskheli³, Manzar Bashir Arain⁴, Zubair Ahmed Kalhoro⁵, Syed Hasnain Ali Shah⁶, Amir Khan Mari⁷, Saifullah Bhatti⁸

¹Department of Business Administration, Shaheed Benazir Bhutto University, Sanghar, Sindh, Pakistan
²,³,⁴,⁶,⁷,⁸Department of Information Technology, Shaheed Benazir Bhutto University, Sanghar, Sindh, Pakistan
⁵Institute of Mathematics and Computer Science, University of Sindh, Jamshoro, Sindh, Pakistan

Corresponding Author: Umair Khalid Qureshi
umair.khalid_sng@sbbusba.edu.pk

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Abstract
This paper presents a Compound Propositional Law for Logical Equivalence, Tautology and Contradiction. The proposed Law is developed with the help of negation, disjunction, conjunction, exclusive or, conditional statement and bi-conditional statement. The idea of research is taken from de-Morgan law. This proposed law is important and useful for Logical Equivalence, Tautology and Contradiction for the research purpose because these are the rare cases in the field of research. This article aims to help readers understand the compound proposition and proposition equivalence in conducting research. This article discusses propositions that are relevant for proposition equivalence. Six main compound propositions are distinguished and an overview is given in the article. Hence, it is observed from the result and discussion that the compound proposition law is a good achievement in discrete structure for the logical Equivalence, Tautology and Contradiction purpose.

Keywords: Proposition Equivalence, Compound Proposition, Truth Table, Result Analysis, Logical Symbols.

I. Introduction
Discrete mathematics is a branch of applied mathematics that deals with arrangements of discrete objects which are separated from each other, such as sets, functions, relations, matrix algebra, combinatorial and finite probability, graph theory, finite differences and recurrence relations, logic, mathematical induction, and
algorithmic thinking [II], [X]. It has many applications in computer science and software engineering, such as how to describe a static structure and dynamic behavior of the software system, and how to verify a software specification by logic statements, etc. In computer science, it can help people understand and construct a computer system to solve a given problem. Discrete mathematics has applications to all fields of computer science, it is used extensively in telecommunications and information processing. We learn concepts associated with them, properties and relationships among them. Because of this variety of topics, it is perhaps preferable to study all these contents of discrete mathematics. They concluded that the Discrete mathematics material should be taught with examples and applications from computer science because the applications would enhance the understanding of Discrete mathematics [IV], [VI], [V], [III]. Similarly, in this research, we focused on discrete mathematics' main topic of Logic. Logic is a language for reasoning for some assertion. It is a collection of rules which can be used when doing logical reasoning. Human reasoning has been observed over centuries from at least the times of Greeks, and patterns appearing in reasoning have been extracted and abstracted. The foundation of the logic was laid down by a British mathematician Boole in the middle of the 19th century. Logic is interested in the true or false of statements, and how the truth/false of a statement can be determined from other statements. We use symbols to represent arbitrary statements so that the results can be used in many similar but different situations, so logic can promote clarity of thought and eliminate ambiguity and mistakes. There are various topics in logic such as a proposition, tautology, contradiction, consistency, logical equivalence predicate, quantifiers, etc. In this research, we are concern with Logical Equivalence, Tautology and Contradiction, and which we are discussed one by one. Firstly tautology, a tautology is a compound statement that is true for every value of the individual statements [IX], [I]. The word tautology is derived from the Greek word ‘tauto’ means ‘same’ and ‘logy’ means ‘logic’. A compound statement is made with two more simple statements by using some conditional words such as ‘and’, ‘or’, ‘not’, ‘if’, ‘then’, and ‘if and only if’. Now contradiction, contradiction is just the opposite of tautology or you can it contradict the tautology statement. When a compound statement formed by two simple given statements by performing some logical operations on them, gives the false value only is called a contradiction or in different terms, it is called a fallacy [VII]. Logical equivalence, compound propositions are said to be logical equivalence if they have the same truth value [VII]. Correspondingly, in this research, we have proposed a compound propositional law for Logical Equivalence, Tautology and Contradiction. The compound propositional law depends on negation, disjunction, conjunction, exclusive or, conditional statement and bi-conditional statement. The proposed compound propositional law validity shows from methodology and result.

II. Proposed Compound Propositional Law

In this section, we have presented the proposed compound proposition law for logical equivalence, tautology and contradiction with the help of negation, disjunction, conjunction, exclusive or, conditional statement and bi-conditional statement. The proposed compound propositional laws can be defined as:

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i. \( p \lor (p \rightarrow q) \)

ii. \( p \lor (\neg p \lor q) \)

iii. \( p \lor (\neg q \lor q) \)

iv. \( \neg(p \land q) \rightarrow (\neg p \oplus q) \)

v. \( \neg(p \lor q) \rightarrow (\neg p \leftrightarrow q) \)

vi. \( \neg(p \oplus \neg q) \rightarrow [\neg(p \land q)] \)

vii. \( \neg(p \lor q) \rightarrow (\neg p \leftrightarrow \neg q) \)

viii. \( \neg(p \land q) \lor (q \oplus \neg q) \)

ix. \( \neg(p \lor q) \rightarrow (\neg p \leftrightarrow q) \)

x. \( \neg(p \lor q) \rightarrow (\neg p \leftrightarrow \neg q) \)

xi. \( \neg(p \lor q) \lor (\neg q \rightarrow q) \)

xii. \( \neg(p \rightarrow q) \lor (p \lor \neg q) \)

xiii. \( \neg(p \lor q) \lor (\neg p \lor q) \)

xiv. \( \neg(p \lor q) \lor (\neg q \lor q) \)

xv. \( \neg(p \lor q) \lor (p \lor \neg q) \)

xvi. \( \neg(p \lor q) \lor (p \lor \neg q) \)

xvii. \( \neg(p \lor q) \lor (\neg p \lor q) \)

Hence these are the compound propositional law for logical equivalence, tautology and contradiction. Where i to ix are the tautology and contradiction and x to xviii are the logical equivalence. Proposed laws are used all the symbol the same as de-morgan law. From these laws, it can be observed that this research is a good achievement in discrete Mathematics, and it is also used to further research purposes. In the below segment, we have produced the results of the proposed compound propositional law.

### III. Results and Discussions

This segment is producing the results of the proposed compound propositional law. The results were created by truth table to better understand the proposed laws, but before that let us learn about the logical operations performed on given statements. Logical operations are used to connect the two simple statements in the form of compound statements and this process is called logical operations. There are 5 major logical operations performed based on respective symbols, such as AND,
OR, NOT, if and then, if and only if. Let us perform one by one with their logical operation.

i. $p \lor (p \rightarrow q)$

| $p$ | $q$ | $p \rightarrow q$ | $p \lor (p \rightarrow q)$ |
|-----|-----|-------------------|--------------------------|
| T   | T   | T                 | T                        |
| T   | F   | F                 | T                        |
| F   | T   | T                 | T                        |
| F   | F   | T                 | T                        |

ii. $\neg(p \lor q) \land (p \oplus q)$

| $p$ | $q$ | $p \lor q$ | $p \oplus q$ | $\neg(p \lor q)$ | $\neg(p \lor q) \land (p \oplus q)$ |
|-----|-----|-----------|-------------|-----------------|----------------------------------|
| T   | T   | T         | T           | F               | F                                |
| T   | F   | T         | T           | F               | F                                |
| F   | T   | T         | T           | F               | F                                |
| F   | F   | T         | T           | T               | F                                |

iii. $\neg p \iff q$

| $p$ | $q$ | $\neg p$ | $\neg p \iff q$ |
|-----|-----|---------|-----------------|
| T   | T   | T       | F               |
| T   | F   | T       | F               |
| F   | T   | F       | F               |
| F   | F   | T       | F               |

iv. $\neg(p \lor q) \leftrightarrow (\neg p \rightarrow q)$

| $p$ | $q$ | $p \lor q$ | $\neg(p \lor q)$ | $\neg p$ | $\neg p \rightarrow q$ | $\neg(p \lor q) \leftrightarrow (\neg p \rightarrow q)$ |
|-----|-----|-----------|-----------------|---------|------------------|----------------------------------|
| T   | T   | T         | F               | T       | T                | F                                |
| T   | F   | T         | F               | F       | F                | T                                |
| F   | T   | T         | F               | T       | T                | F                                |
| F   | F   | T         | T               | F       | T                | F                                |

v. $\neg(p \oplus \neg q) \lor [\neg(p \land q)]$

| $p$ | $q$ | $\neg q$ | $p \oplus \neg q$ | $\neg(p \lor q)$ | $\neg p \land q$ | $\neg(p \land q)$ | $\neg(p \oplus \neg q) \lor [\neg(p \land q)]$ |
|-----|-----|---------|-------------------|-----------------|-----------------|-----------------|----------------------------------|
| T   | T   | F       | F                 | T               | T               | F               | T                                |
| T   | F   | F       | F                 | T               | F               | T               | T                                |
| F   | T   | F       | F                 | F               | T               | T               | T                                |
| F   | F   | T       | F                 | T               | T               | T               | T                                |

vi. $p \lor q \rightarrow (\neg p \iff \neg q)$

| $p$ | $q$ | $p \lor q$ | $\neg(p \lor q)$ | $\neg p$ | $\neg q$ | $\neg p \iff \neg q$ | $p \lor q \rightarrow (\neg p \iff \neg q)$ |
|-----|-----|-----------|-----------------|---------|---------|----------------------|----------------------------------|
| T   | T   | T         | F               | F       | F       | T                    | T                                |
| T   | F   | T         | F               | F       | T       | T                    | T                                |
| F   | T   | T         | F               | T       | F       | F                    | T                                |
| F   | F   | T         | T               | T       | T       | T                    | T                                |

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vii. \( (\neg p \rightarrow q) \lor (q \oplus \neg q) \)

| p   | q   | \neg p | \neg q | \neg p \rightarrow q | q \oplus \neg q | (\neg p \rightarrow q) \lor (q \oplus \neg q) |
|-----|-----|--------|--------|-------------------|----------------|----------------------------------|
| T   | T   | F      | F      | T                 | T              | T                                |
| T   | F   | F      | T      | T                 | T              | T                                |
| F   | T   | T      | F      | T                 | T              | T                                |
| F   | F   | T      | F      | T                 | T              | T                                |

viii. \( (p \oplus q) \leftrightarrow (\neg p \oplus \neg q) \)

| p   | q   | p \oplus q | \neg p | \neg q | \neg p \oplus \neg q | (p \oplus q) \leftrightarrow (\neg p \oplus \neg q) |
|-----|-----|-------------|--------|--------|---------------------|-----------------------------------------------|
| T   | T   | F           | F      | F      | F                   | T                                             |
| T   | F   | F           | T      | T      | T                   | T                                             |
| F   | T   | T           | F      | T      | T                   | T                                             |
| F   | F   | T           | F      | T      | T                   | T                                             |

ix. \( \neg (p \leftrightarrow q) \equiv \neg p \oplus \neg q \)

| p   | q   | \neg p | \neg q | p \leftrightarrow q | \neg (p \leftrightarrow q) | \neg p \oplus \neg q |
|-----|-----|--------|--------|-------------------|---------------------------|---------------------|
| T   | T   | F      | F      | T                 | F                         | F                   |
| T   | F   | F      | T      | F                 | F                         | F                   |
| F   | T   | T      | F      | T                 | T                         | T                   |
| F   | F   | T      | F      | T                 | T                         | T                   |

x. \( \neg (p \land q) \equiv \neg (p \land q) \lor (\neg p \oplus \neg q) \)

| p   | q   | \neg p | \neg q | p \land q | \neg (p \land q) | \neg p \oplus \neg q | \neg (p \land q) \lor (\neg p \oplus \neg q) |
|-----|-----|--------|--------|-----------|-----------------|---------------------|-----------------------------------------------|
| T   | T   | F      | F      | T         | F               | F                   | F                                             |
| T   | F   | F      | T      | F         | T               | T                   | T                                             |
| F   | T   | T      | F      | T         | T               | T                   | T                                             |
| F   | F   | T      | T      | F         | T               | T                   | T                                             |

xi. \( \neg p \leftrightarrow \neg q \equiv \neg (p \land q) \rightarrow (\neg p \leftrightarrow \neg q) \)

| p   | q   | \neg p | \neg q | p \land q | \neg (p \land q) | \neg p \leftrightarrow \neg q | \neg (p \land q) \rightarrow (\neg p \leftrightarrow \neg q) |
|-----|-----|--------|--------|-----------|-----------------|-----------------------------|-----------------------------------------------|
| T   | T   | F      | F      | T         | F               | T                           | T                                             |
| T   | F   | F      | T      | F         | T               | T                           | T                                             |
| F   | T   | T      | F      | F         | T               | F                           | F                                             |
| F   | F   | T      | T      | F         | T               | T                           | T                                             |
xi. \( p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q \)

| \( p \) | \( q \) | \( \neg p \) | \( \neg q \) | \( p \leftrightarrow q \) | \( \neg p \leftrightarrow \neg q \) |
|-----|-----|-----|-----|-----|-----|
| T   | T   | F   | F   | T   | T   |
| T   | F   | F   | T   | F   | F   |
| F   | T   | T   | F   | F   | F   |
| F   | F   | T   | T   | T   | T   |

xii. \( p \oplus q \equiv \neg p \oplus \neg q \)

| \( p \) | \( q \) | \( \neg p \) | \( \neg q \) | \( p \oplus q \) | \( \neg p \oplus \neg q \) |
|-----|-----|-----|-----|-----|-----|
| T   | T   | F   | F   | F   | F   |
| T   | F   | F   | T   | T   | T   |
| F   | T   | T   | F   | T   | T   |
| F   | F   | T   | T   | F   | F   |

xiii. \( (p \oplus q) \equiv (p \oplus q) \lor (\neg p \oplus \neg q) \)

| \( p \) | \( q \) | \( \neg p \) | \( \neg q \) | \( p \oplus q \) | \( \neg p \oplus \neg q \) | \( p \oplus q \lor \neg p \oplus \neg q \) |
|-----|-----|-----|-----|-----|-----|-----|
| T   | T   | F   | F   | F   | F   | F   |
| T   | F   | F   | T   | T   | T   | T   |
| F   | T   | T   | F   | T   | T   | T   |
| F   | F   | T   | T   | F   | F   | F   |

xiv. \( p \leftrightarrow q \equiv (p \leftrightarrow q) \land (\neg p \leftrightarrow \neg q) \)

| \( p \) | \( q \) | \( \neg p \) | \( \neg q \) | \( p \leftrightarrow q \) | \( \neg p \leftrightarrow \neg q \) | \( (p \leftrightarrow q) \land (\neg p \leftrightarrow \neg q) \) |
|-----|-----|-----|-----|-----|-----|-----|
| T   | T   | F   | F   | T   | F   | F   |
| T   | F   | F   | T   | F   | F   | F   |
| F   | T   | T   | F   | T   | F   | F   |
| F   | F   | T   | T   | T   | T   | T   |

xv. \( \neg (p \oplus q) \equiv (p \leftrightarrow q) \lor (\neg p \leftrightarrow \neg q) \)

| \( p \) | \( q \) | \( \neg p \) | \( \neg q \) | \( p \leftrightarrow q \) | \( \neg p \leftrightarrow \neg q \) | \( (p \leftrightarrow q) \lor (\neg p \leftrightarrow \neg q) \) |
|-----|-----|-----|-----|-----|-----|-----|
| T   | T   | F   | F   | T   | F   | F   |
| T   | F   | F   | T   | F   | F   | F   |
| F   | T   | T   | F   | T   | F   | F   |
| F   | F   | T   | T   | T   | T   | T   |

xvi. \( \neg (p \oplus q) \equiv p \leftrightarrow q \)

| \( p \) | \( q \) | \( p \oplus q \) | \( \neg (p \oplus q) \) | \( p \leftrightarrow q \) |
|-----|-----|-----|-----|-----|
| T   | T   | F   | T   | T   |
| T   | F   | F   | T   | T   |
| F   | T   | T   | F   | F   |
| F   | F   | F   | T   | T   |

xvii. \( \neg (p \land q) \equiv \neg p \oplus \neg q \)

| \( p \) | \( q \) | \( \neg p \) | \( \neg q \) | \( p \land q \) | \( \neg (p \land q) \) | \( \neg p \oplus \neg q \) |
|-----|-----|-----|-----|-----|-----|-----|
| T   | T   | F   | F   | T   | F   | F   |
| T   | F   | F   | T   | F   | T   | T   |
| F   | T   | T   | F   | T   | T   | T   |
| F   | F   | T   | T   | F   | T   | T   |
xviii. \( \neg(p \leftrightarrow q) \equiv \neg p \oplus \neg q \)

| p | q | \neg p | \neg q | p \leftrightarrow q | \neg(p \leftrightarrow q) | \neg p \oplus \neg q |
|---|---|-------|-------|-----------------|-----------------|-----------------|
| T | T | F     | F     | T               | F               | F               |
| T | F | F     | T     | F               | T               | T               |
| F | T | T     | F     | F               | T               | T               |
| F | F | T     | T     | T               | F               | F               |

xix. \( \neg p \leftrightarrow \neg q \equiv \neg(p \land q) \rightarrow (\neg p \leftrightarrow \neg q) \)

| p | q | \neg p | \neg q | \neg p \leftrightarrow \neg q | \neg(p \land q) | \neg(p \land q) \rightarrow (\neg p \leftrightarrow \neg q) |
|---|---|-------|-------|-----------------|----------------|-----------------------------------------------|
| T | T | F     | F     | T               | F               | T                                             |
| T | F | F     | T     | F               | F               | F                                             |
| F | T | T     | F     | F               | T               | F                                             |
| F | F | T     | T     | T               | F               | T                                             |

From the results shown in the truth table it has been clarified that the proposed compound propositional law is useful and very important for logical equivalence, tautology and contradiction. Throughout the results and discussions, it can be detected that the compound proposition law is good achievement for logical equivalence, tautology and contradiction and it is also good for further research purposes in discrete structure.

IV. Conclusion

This research has developed a compound proposition law estimated for logical equivalence, tautology and contradiction. De-Morgan law is an insight that helped to guide this study. The results and discussion appear to show support for compound proposition law and directions for further research in this area. Furthermore, the study suggests judging the validity of logical equivalence, tautology and contradiction. This research has been discussed some useful topics —i.e. logical equivalence, tautology, contradiction, negation, disjunction, conjunction, exclusive or, conditional statement and bi-conditional statement. Throughout the article, it can be observed that the compound propositional law is more effectively and efficiently for logical equivalence, tautology and contradiction.

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Conflict of Interest:

There is no conflict of interest regarding this article.

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