Low-scale leptogenesis and dark matter

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An extension of the Standard Model with Majorana singlet fermions in the 1-100 GeV range can give rise to a baryon asymmetry at freeze-in via the CP-violating oscillations of these neutrinos: this is the well known ARS mechanism. In this paper we consider possible extensions of the minimal ARS scenario that can account not only for successful leptogenesis but also explain other open problems such as dark matter. We find that an extension in the form of a weakly coupled $B − L$ gauge boson, an invisible QCD axion model, and the singlet majoron model can simultaneously account for dark matter and the baryon asymmetry.

I. INTRODUCTION

The Standard Model (SM) of particle physics needs to be extended to explain neutrino masses, the missing gravitating matter (DM) and the observed matter-antimatter asymmetry in the universe.

Some of the most minimal extensions of the SM include new fermions, namely two or three sterile Majorana neutrinos (singlets under the full gauge group), which can account for the tiny neutrino masses, through the seesaw mechanism [1–4], and explain the observed matter-antimatter asymmetry through leptogenesis [5]. The simplest version of leptogenesis establishes the source of the matter-antimatter asymmetry in the CP violating the out-of-equilibrium decay of the heavy neutrinos. This scenario requires however relatively large Majorana masses $> 10^8$ GeV [6] (or $\sim 10^6$ GeV with flavour effects [7] included), which makes these models difficult to test experimentally. For Majorana neutrinos in the 1-100 GeV range, it has been shown by Akhmedov, Rubakov and Smirnov (ARS) [8] and refined by Asaka and Shaposhnikov [9] that a different mechanism of leptogenesis is at work. In this case the asymmetries are produced at freeze-in of the sterile states via their CP-violating oscillations. In both cases, the lepton asymmetry is reprocessed into a baryonic one by electroweak sphalerons [10]. The extra heavy neutrinos in this case could be produced and searched for in beam dump experiments and colliders (see [11–23] for an incomplete list of works), possibly giving rise to spectacular signals such as displaced vertices [15, 16, 18–20, 21]. Since only two sterile neutrinos are needed to generate the baryon asymmetry [9, 24–39], the lightest sterile neutrino in the keV range can be very weakly coupled and play the role of DM [40]. This is the famous $\nu$MSM [9]. However the stringent X-ray bounds imply that this scenario can only work in the presence of a leptonic asymmetry [41] significantly larger than the baryonic one, which is quite difficult to achieve. A recent update of astrophysical bounds on this scenario can be found in [42, 43].

In this paper our main goal is to consider scenarios compatible with Majorana masses in the 1-100 GeV range and study the conditions under which the models can explain DM without spoiling ARS leptogenesis [4]. In particular, we will focus on models that are minimal extensions of the type I seesaw model with three singlet neutrinos. We will first consider an extension involving a gauged $B − L$ model [45], which includes an extra gauge boson and can explain DM in the form of a non-thermal keV neutrino. We will then consider an extension which includes a CP axion [46] that can solve the strong CP problem and explain DM in the form of cold axions. Finally we consider the majoron singlet model [47, 48] which can also explain DM under certain conditions both in the form of a heavy majorana neutrino or a majoron.

The plan of the paper is as follows. We start by briefly reviewing the ARS mechanism and the essential ingredients and conditions that need to be met when the sterile neutrinos have new interactions. In section II, we discuss the gauged $B − L$ model, in section III we study the invisible axion model with sterile neutrinos and in section IV we reconsider the singlet majoron model. In section V we conclude.

II. THE ARS MECHANISM

For a recent extensive review of the ARS mechanism see [49]. The model is just the type I seesaw model with three neutrino singlets, $N_i$, $i = 1 − 3$, which interact with the SM only through their Yukawa couplings. The Lagrangian in the Majorana mass basis is

$$\mathcal{L} = \mathcal{L}_\text{SM} + i \bar{N}_i \gamma^\mu \partial_\mu N_i \Phi^\dagger - \left( Y_{\alpha i} \bar{L}_\alpha N_i \Phi + \frac{m_{N_i}}{2} N_i^c N_i + \text{h.c.} \right).$$

In the early Universe before the electroweak (EW) phase transition, the singlet neutrinos are produced through

\[ \text{some very recent work along these lines in the scotogenic model can be found in [41].} \]
their Yukawa couplings in flavour states, which are linear combinations of the mass eigenstates. Singlet neutrinos then oscillate, and since CP is not conserved, lepton number $L$ gets unevenly distributed between different flavours. At high enough temperatures $T \gg m_{N_i}$, total lepton number vanishes, in spite of which a surplus of baryons over antibaryons can be produced, because the flavoured lepton asymmetries are stored in the different species and transferred at different rates to the baryons. As long as full equilibration of the sterile states is not reached before the EW phase transition ($T_{EW} \sim 140 \text{GeV}$), when sphaleron processes freeze-out, a net baryon asymmetry survives. It is essential that at least one of the sterile neutrinos does not equilibrate by $t_{EW}$. The rate of interactions of these neutrinos at temperatures much higher than their mass can be estimated to be

$$\Gamma_\alpha \propto \kappa y_\alpha^2 T,$$

where $y_\alpha$ are the eigenvalues of the neutrino Yukawa matrix, $T$ is the temperature and $\kappa = \text{few} \ 10^{-3}$ \cite{50, 52}. The Hubble expansion rate in the radiation dominated era is

$$H(T) = \sqrt{\frac{4\pi^3 G_N g_*}{45}} T^2 \approx \frac{T^2}{M_P^2},$$

where $g_*$ is the number of relativistic degrees of freedom ($g_* \sim 100$ above the EW phase transition). The requirement that no equilibration is reached before $t_{EW}$ is:

$$\Gamma_\alpha(t_{EW}) \leq H(t_{EW}),$$

which implies yukawa couplings of order

$$y_\alpha \lesssim 10^{-7},$$

i.e. not much smaller than the electron yukawa. These yukawa couplings are compatible with the light neutrino masses for Majorana masses in the 1 GeV-100 GeV range.

Any model that extends the one described above with new fields/interactions should be such that the new interactions do not increase the equilibration rate of the sterile neutrinos for the out-of-equilibrium requirement in the ARS mechanism to be met. We will now consider the implications of this requirement on various extensions of the minimal seesaw model of eq. \ref{eq:1} that are well motivated by trying to explain also the dark matter and in this case also the strong CP problem.

### III. B-L GAUGE SYMMETRY

The SM is invariant under an accidental global $U(1)_{B-L}$ symmetry, that couples to baryon minus lepton number. If one promotes this symmetry to a local one \cite{45}, the model needs to be extended with three additional right handed neutrinos to avoid anomalies, which interestingly makes the type I seesaw model the minimal particle content compatible with this gauge symmetry.

![Fig. 1. Summary of present (shaded regions) and future (unshaded) constraints on $g_{B-L}$ and $m_\nu$ for the $B-L$ model, adapted from ref. \cite{62}. The dashed region labeled SHIP is the sensitivity of bremsstrahlung searches in SHIP \cite{67, 68}. The dashed lines labeled ARS indicate the upper limits on $g_{B-L}$ below which the ARS mechanism should be insensitive to the new gauge interaction for $m_\nu$ below or above the two heavy neutrino threshold. The solid, dotted and dashed black lines correspond to the correct DM relic abundance in the form of sterile neutrinos of mass 1, 10 and 100 keV respectively.](image)

In this case, we have interactions between SM lepton and quark fields with the new gauge boson, $V_\mu$, as well as an additional term involving sterile neutrinos

$$\mathcal{L} \sim g_{B-L} \left( \sum_f Q_{f B-L} V_\mu \gamma_\mu f - \sum_a V_\mu N_a \gamma_\mu N_a \right),$$

where $Q_{f B-L} = 1/3, -1$ for quarks and leptons respectively. Consequently we obtain new channels for production and decay of the $N$, and if we want to maintain successful ARS leptogenesis, we have to ensure that the new interactions do not result in the thermalization of the $N$’s before $t_{EW}$, which can only be met for sufficiently small $g_{B-L}$.

Existing constraints on this model come from direct searches for $V$ in elastic neutrino-electron scattering, $V$ gauge boson production at colliders, Drell-Yan processes and new flavour changing meson decays \cite{53–61, 63–65}. The status of these searches is summarized in Fig. 1 adapted from \cite{62} (see also \cite{63–65}). For masses, 1 GeV $\leq m_\nu \leq 10$ GeV, $g_{B-L}$ is bounded to be smaller than $\sim 10^{-4}$, while the limit is weaker for larger masses. The improved prospects to search for right-handed neutrinos exploiting the $U(1)_{B-L}$ interaction have been recently studied in \cite{92}, where the authors consider the displaced decay of the $N$ at the LHC and the proposed SHIP beam dump experiment \cite{66}.

For $m_\nu \leq 1$ GeV the strongest constraints come from supernova cooling \cite{69, 72, 74}, beam dump searches \cite{71}.
For $T$ues of ($\gamma$-decays are assumed and the kinematical limits of Figs. 4 and 5 are included. The curves indicate the regions that the light particles can account for. At high enough temperatures, the thermal corrections to the masses of the form $\delta m_i^{-2}$ are included. In the relevant region of parameter space, they are seen to depend on the lifetime of the decaying particle and its abundance per baryon prior to decay. The corresponding region in Fig. 2 is a sketch of the excluded region in the latter analysis, which is approximately bounded by the lines corresponding to the lifetimes $\tau_V \in [0.1-100]$s. The threshold for $V$ decays to electrons, $m_V \geq m_e$, is determined by the fraction of the $V$ decay to charged particles per baryon, $\nu_e$.

Considering now the ARS mechanism, let us assume that at least two of the sterile neutrinos have masses in the 1-100 GeV range, while the $V$ boson is lighter, $m_V \leq 2m_N$. To ensure that processes like the one depicted in Fig. 3 do not affect leptogenesis, we should have

$$\langle \Gamma(f \rightarrow N N) \rangle_{T \rightarrow T} \ll H(T_{EW}) \quad (7)$$

For $T \gg m_V, m_N$, the thermal average of the total rate (including all fermions) is

$$\langle \Gamma \rangle_T \approx \frac{\pi}{648\xi(3)} g_{B-L}^4 T \sim 4 \times 10^{-3} g_{B-L}^4 T, \quad (8)$$

and eq. (7) implies

$$g_{B-L} \lesssim 4 \times 10^{-4}. \quad (9)$$

Therefore $2 \leftrightarrow 2$ processes lead to constraints from successful leptogenesis that are roughly of the same order of the ones from experimental limits.

For $m_V \geq 2m_N$, the dominant production goes via the decay of the gauge boson into two sterile neutrinos $V \rightarrow NN$, which, if kinematically allowed, scales with $g_{B-L}^2$. In the case where the decay rate is

$$\Gamma(V \rightarrow NN) = \frac{g_{B-L}^2 m_V^2}{24\pi} \left(1 - \frac{4m_N^2}{m_V^2}\right)^{3/2}. \quad (10)$$

Requiring that it is smaller than $H(T_{EW})$ implies for $m_N \ll m_V$

$$g_{B-L} \lesssim 10^{-5} \left(\frac{100 \text{GeV}}{m_V}\right). \quad (11)$$

Therefore, to have a successful low scale leptogenesis within the range to be probed by future accelerator experiments we must have $m_V \leq 2m_N$, at least for two of the three heavy neutrinos, forbidding the $V \rightarrow NN$ decay. One may wonder whether this is enough when thermal corrections are included. At high enough temperatures both sterile neutrinos and the gauge boson acquire thermal corrections to the masses of the form

$$m(T) \sim g_{B-L} T. \quad (12)$$

The thermal mass of the gauge boson is larger than that of the sterile neutrino, because all fermions charged under $B-L$ will contribute to the former and only the gauge boson loop contributes to the latter:

$$m_V^T = m_V + \sqrt{\frac{4}{3}} g_{B-L} T, \quad m_N^T = m_N + \frac{1}{\sqrt{8}} g_{B-L} T. \quad (13)$$

We substitute the temperature dependent mass in eq. (10) and we show in Fig. 4 the ratio $\Gamma(V \rightarrow NN)/H$ close to the minimum threshold temperature (where $m_V^T \geq 2m_N^T$), for $m_N = 1$ and 100 GeV as a function of $g_{B-L}$. The upper limit for $g_{B-L}$ is in the same ballpark as that derived in eq. (9) from $2 \leftrightarrow 2$ processes.

We now evaluate in detail the effect on ARS leptogenesis induced by the new scatterings of Fig. 3.
A. Leptogenesis

The sterile neutrinos relevant for leptogenesis are the heavier ones, $m_{N_{2,3}}$, with masses in the 1-100 GeV, and we focus on the scenario where $m_V \leq 2m_{N_{2,3}}$. We have included the terms involving the $B-L$ gauge interactions, i.e. $ff \leftrightarrow NN$, in the quantum kinetic equations for ARS leptogenesis derived in [31]. Following Raffelt-Sigl approach [84], we consider a density matrix, describing the expectation value of number densities of new collision terms in the equation for the evolution of the heavy sterile neutrinos that could be observable in SHIP, and furthermore this measurement, in combination with input from neutrinoless double beta decay and CP violation in neutrino oscillations, could provide a quantitative prediction of the baryon asymmetry. Adding the $B-L$ terms to the equations for $\rho$ and $\bar{\rho}$, that have the same flavour structure as the neutral current interactions is the addition of new collision terms in the equations for the electron neutrino equilibrium distribution function of the particle with momentum $p_i$ with $p_4 \equiv k$; $\{\}$ is the anticommutator, and the normalized matrices are:

$$r(k) \equiv \frac{\rho(k)}{f_F(k)}, \quad \bar{r}(p_3) \equiv \frac{\bar{\rho}(p_3)}{f_F(p_3)}.$$ (15)

The additional collision terms for $\bar{\rho}$ have the same form with the substitution $k \leftrightarrow p_3$.

As usual we are interested in the evolution in an expanding universe, where the density matrices depend on momentum, $y \equiv p/T$ and the scale factor or inverse temperature $x \propto T^{-1}$. We consider the averaged momentum approximation, which assumes that all the momentum dependence factorizes in the Fermi-Dirac distribution and the density $r$ is just a function of the scale factor, i.e. $\rho(x,y) = f_F(y)r(x)$. In this approximation we can do the integration over momentum and the $B-L$ terms in the equation for $r$ and $\bar{r}$ become:

$$\left( xH \frac{dr}{dx} \right)_{B-L} = \left( xH \frac{d\bar{r}}{dx} \right)_{B-L} = \frac{\langle \gamma^{(0)}_V \rangle}{2} (2 - \{r,\bar{r}\})$$

$$- \frac{\langle \gamma^{(1)}_V \rangle}{2} (r + \bar{r} - \{r,\bar{r}\}),$$ (16)

where $H$ is the Hubble expansion parameter.

The averaged rates are computed in the appendix with the result:

$$\langle \gamma^{(0)}_V \rangle = 3.2(3) \times 10^{-3} g_{B-L}^4 T,$$

$$\langle \gamma^{(1)}_V \rangle = 3.4(1) \times 10^{-4} g_{B-L}^4 T.$$ (17)

The new interactions do not modify the chemical potential dependent terms, nor the evolution equation for $\mu_{B/3-L_\alpha}$. The equations are therefore those in [31] with the additional $B-L$ terms in eq. (16).

To illustrate the effect of the $B-L$ gauge interaction, we have considered the test point of ref. [31] with masses for the heavy steriles $m_{N_{2,3}} \sim 0.8$ GeV. Within the parameter space of successful leptogenesis, this point was chosen because it leads to charmed meson decays to heavy sterile neutrinos that could be observable in SHIP, and furthermore this measurement, in combination with input from neutrinoless double beta decay and CP violation in neutrino oscillations, could provide a quantitative prediction of the baryon asymmetry. Adding the $B-L$ terms to the equations for $r$ and $\bar{r}$ of [31], and solving them numerically (for details on the method see [31]) we obtain the curves in Fig. [3]. The evolution of the baryon asymmetry as a function of $T_{EW}/T$ is shown by the solid line of Fig. [3] in the absence of $B-L$ interactions or for a sufficiently small value of $g_{B-L} \leq 10^{-4}$. The suppression of the asymmetry is visible for larger values of $g_{B-L}$, as shown by the dashed and dashed-dotted lines. The naive expectations are therefore confirmed and we do not expect a significant modification of the baryon asymmetry of the minimal model, as long as $g_{B-L} \lesssim \text{few} \times 10^{-4}$.
DM has been studied in [68]. We now quantify the neutrino mass is small enough). A similar scenario for be suppressed as much as needed (provided the lightest bounds, is controlled also by mixing, which can therefore

\[ \text{FIG. 5.} \ Y_B \text{ as a function of } T_{EW}/T \text{ for the } Y \text{ and } m_N \text{ parameters corresponding to the test point in [31]} \text{ and } g_{B-L} \leq 10^{-4} \text{ (solid), } g_{B-L} = 5 \times 10^{-3} \text{ (dotted), } g_{B-L} = 10^{-2} \text{ (dashed). The horizontal line is the observed value.} \]

B. Dark Matter

Now we want to discuss possible dark matter candidates in the \( B-L \) scenario without spoiling the ARS mechanism, which as we have seen imposes a stringent upper bound on the gauge coupling, \( g_{B-L} \). We will be interested in the region where the \( V \) boson can decay to the lightest neutrino, ie \( m_V \geq 2m_{N_1} \). The small value needed for \( g_{B-L} \) suggests to consider the possibility of a freeze-in scenario [85, 86], where the gauge boson does not reach thermalization, and neither does the lightest sterile neutrino, \( N_1 \). The status of dark matter in a higher mass range through freeze-out has been recently updated in [64].

As it is well known, \( N_1 \) in the keV mass range is sufficiently long lived to provide a viable warm DM candidate [40, 41]. The \( B-L \) model is as we will see a simple extension of the \( \nu \text{MSM} \) [9], which avoids the need of huge lepton asymmetries to evade X-ray bounds. In our scenario the keV state is produced from the decay \( V \rightarrow N_1N_1 \), while the lifetime of \( N_1 \), relevant in X-ray bounds, is controlled also by mixing, which can therefore be suppressed as much as needed (provided the lightest neutrino mass is small enough). A similar scenario for DM has been studied in [68]. We now quantify the parameter space for successful DM and leptogenesis in this scenario.

We assume that the abundance of \( V \) and \( N_1 \) is zero at a temperature below the EW phase transition where all the remaining particles in the model are in thermal equilibrium. All fermions in the model couple to the \( V \) and therefore its production is dominated by the inverse decay process: \( f\bar{f} \rightarrow V \). The kinetic equation describing the production of \( V \) is the following:

\[ \dot{n}_V + 3Hn_V = \sum_f \int \frac{d^3p_f}{(2\pi)^32E_f} \frac{d^3p_f}{(2\pi)^32E_f} \frac{d^3p_V}{(2\pi)^32E_V} \]

\[ (\pi^4\delta^4(p_V - p_f - p_{\bar{f}}))[|M_{f\bar{f} \rightarrow V}|^2f_f\bar{f}_f(1 + f_V) + -|M_{V \rightarrow f\bar{f}}|^2f_f(1 - f_f)], \]

where \( f_i(p) \) are the distribution function of the particle, \( i \), with momentum \( p \), and

\[ n_i = g_i \int \frac{d^3p}{(2\pi)^3}f_i(p), \]

is the number density, with \( g_i \) the number of spin degrees of freedom. \( g_f = 4 \) for a Dirac fermion, \( g_N = 2 \) for a Majorana fermion and \( g_V = 3 \) for a massive gauge boson. \( M \) is the amplitude for the decay \( V \rightarrow f\bar{f} \) at tree level.

The sum over \( f \) is over all fermions, but we can safely neglect the contribution of the \( N_1 \) and also those that are non-relativistic. We can also neglect the Pauli-blocking/stimulated emission effects (\( f_i \pm 1 \sim \pm 1 \)) and approximate the distribution function in equilibrium for fermions and bosons by the Maxwell-Boltzmann, \( f_i(p_i) = e^{p_i/T}/T \). Taking into account the relation

\[ |M_{V \rightarrow f\bar{f}}|^2 = |M_{f\bar{f} \rightarrow V}|^2, \]

and the principle of detailed balance

\[ f_f^q f_{\bar{f}}^q = f_f^q, \]

the equation can be simplified to

\[ \dot{n}_V + 3Hn_V = -\sum_f \int \frac{d^3p_f}{(2\pi)^32E_f} \frac{d^3p_f}{(2\pi)^32E_f} \frac{d^3p_V}{(2\pi)^32E_V} \]

\[ (\pi^4\delta^4(p_V - p_f - p_{\bar{f}}))[|M_{V \rightarrow f\bar{f}}|^2f_f(1 - f_f)], \]

As long as \( f_V \ll f_f^q \), the first term on the right-hand side can be neglected and the equation simplifies further to:

\[ \dot{n}_V + 3Hn_V \simeq 3 \sum_f \frac{m_f^2}{2\pi^2} \Gamma_{V \rightarrow f\bar{f}} TK_1 \left( \frac{m_V}{T} \right), \]

where \( K_1 \) is the first modified Bessel Function of the 2nd kind. The decay width in the \( V \) rest frame is given by

\[ \Gamma(V \rightarrow f\bar{f}) = \frac{g_{B-L}^2N_C Q_f^2 m_V}{12\pi} \left( 1 + \frac{2m_f^2}{m_V^2} \right) \left( 1 - \frac{4m_f^2}{m_V^2} \right)^{1/2}, \]

where \( N_C = 3(1) \) and \( Q_f = 1/3(1) \) for quarks/leptons.

As usual we define the yield of particle \( i \) as

\[ Y_i = \frac{n_i}{s}, \]
where $s$ is the entropy density

$$ s = \frac{2\pi^2}{45} g_* T^3, $$

(26)

and we can assume $g_*^n \simeq g_*^n$. We also consider the averaged momentum approximation which amounts to assuming that $f_V$ has the same momentum dependence as $f^n$. Changing variable from time to temperature, the final evolution equation for $Y$ reads:

$$ \frac{dY}{dT} = -3 \sum_{\gamma} \frac{m^2 V - f_j}{2\pi^2 H s} \left[ 1 + \frac{1}{3} \frac{d g_*}{dT} \right] K_1 \left( \frac{m_V}{T} \right) \left( 1 - \frac{s Y}{n_{\nu V}} \right), $$

and the production of $N$ is

$$ \dot{n}_1 + 3Hn_1 = 2 \frac{K_1(x)}{K_2(x)} \Gamma(V \rightarrow N_1 N_1) n_V, $$

(28)

and in terms of the yield

$$ \frac{dY_{N_1}}{dT} = - \frac{2}{HT} \frac{K_1(x)}{K_2(x)} \Gamma(V \rightarrow N_1 N_1) Y_V, $$

(29)

where

$$ \Gamma(V \rightarrow N_1 N_1) = \frac{g^2_{B-L} m_V}{24\pi} \left( 1 - \frac{4m^2_{N_1}}{m^2_V} \right)^{3/2}. $$

(30)

It is straightforward to solve these equations. In Fig. 6 we show the yields of $V$ and $N$ as function of the inverse temperature for $m_V = 10\text{ MeV}$, $m_{N_1} = 10\text{ keV}$ and $g_{B-L} = 10^{-11.4}$. The resulting abundance of $N_1$ is

$$ \Omega_{N_1} h^2 \equiv \frac{8\pi m_{N_1}}{3 \rho_c h^2} Y_{N_1} \approx 2.7 \times 10^2 Y_{N_1} \frac{m_{N_1}}{\text{keV}}, $$

(31)

where $s_0 = 2889.2 \text{ cm}^{-3}$ is the entropy today and $\rho_c = 1.05110^{-5} h^2 \text{ GeV cm}^{-3}$ is the critical density. The evolution of $\Omega_{N_1} h^2$ is shown in Fig. 6 for two values of $m_V$ and a fixed value of $g_{B-L}$. Requiring that $\Omega_{N_1} h^2$ equals the full DM contribution of $\Omega_{\text{DM}} h^2 \simeq 0.12$ implies a relation between $m_V$ and $g_{B-L}$ as shown in the curves of Fig. 6. Interestingly some region of the parameter space is excluded from supernova and BBN observations.

A final comment concerns the comparison of our calculation of the DM abundance and that in ref. [68]. In this reference only the evolution of the $N_1$ is considered, and the collision term corresponds to the scattering process $f \bar{f} \rightarrow N_1 N_1$, where the narrow width approximation is assumed. We believe this method is only equivalent to ours when all $f, \bar{f}$ and $V$ distributions are the equilibrium ones, but this is not the case here. Nevertheless the results we obtain are similar to those in [68] in the region they can be compared.

C. Masses

Until now we assumed non-zero masses for the $V$ and the sterile neutrinos, which should always be possible via the Stuckelberg mechanism, but masses can also be generated dynamically from the spontaneous breaking of
\( B - L \) by the VEV of a singlet scalar \( \phi \) with charge 2:

\[
\mathcal{L} \supset Q^\phi_{B-L}g_{B-L}\phi^\dagger V^\mu \partial_\mu \phi - \frac{h_N}{2} N\phi + h.c. \tag{32}
\]

In this scenario, the masses of the gauge boson and the heavy neutrinos are given by

\[
m_V \sim g_{B-L}\langle \phi \rangle, \quad m_{N_i} = h_N\langle \phi \rangle. \tag{33}
\]

Thus, in order to obtain, for example, a mass \( m_V \sim 1 \) MeV, with the gauge coupling needed to generate DM

\[
g_{B-L} \sim 10^{-11.8}, \tag{34}
\]

the VEV should be

\[
\langle \phi \rangle \sim 6.3 \cdot 10^8 \text{ GeV}. \tag{35}
\]

After spontaneous symmetry breaking a massive higgs from the \( B - L \) breaking, \( \sigma \), remains in the spectrum with a mass that we can assume to be \( M_\sigma \sim \langle \phi \rangle \).

In order to get the \( N_1 \) and \( N_{2,3} \) in the target range of keV and 1-100 GeV respectively, small and hierarchical \( h_N \)'s couplings are needed:

\[
h_{N_2} \simeq h_{N_3} \sim 10^{-6} - 10^{-8}, \tag{36}
\]

for the heavy sterile neutrinos involved in ARS leptogenesis and

\[
h_{N_1} \sim 10^{-13}, \tag{37}
\]

for the dark matter candidate. Note that the required \( h_N \)'s couplings are in the same ballpark as the yukawa couplings.

The question is whether the interactions with \( \sigma \) can modify ARS leptogenesis. The leading order process we have to consider is the decay of the scalar into two sterile neutrinos \( \sigma \rightarrow NN \). Since \( M_\sigma \gg m_N \), the requirement that this process does not thermalize the sterile neutrinos before the EW temperature, implies that the decay rate, \( \Gamma_\sigma \), is slower than the Hubble rate at \( T \geq M_\sigma \):

\[
\Gamma_D(M_\sigma) = \frac{h_N^2 M_\sigma}{16\pi} \lesssim H(M_\sigma). \tag{38}
\]

For \( m_N = 1 - 100 \) GeV this implies

\[
M_\sigma \sim \langle \phi \rangle \geq 2 \times 10^3 - 5 \times 10^6 \text{ GeV}, \tag{39}
\]

which is orders of magnitude below the target region.

The gauged \( B - L \) model works nicely to explain neutrino masses, the baryon asymmetry and dark matter. Unfortunately it also requires a very small \( g_{B-L} \) which will be very hard to test experimentally. An alternative might be to consider a flavoured \( U(1) \), for example \( L_\mu - L_\tau \), that might be compatible with a larger \( g_{B-L} \), provided the assignment of charges to the singlet states ensures that not all of them reach thermalization via the flavoured gauge interaction before \( t_{EW} \).

\section{IV. AXION AND NEUTRINOS}

As a second example we consider an extension of \( \text{eq. (1)} \) with a scalar doublet and a scalar singlet. This model is also an extension of the invisible axion model \cite{87} with sterile neutrinos, that was first considered in \cite{46}, providing a connection between the Peccei-Quinn (PQ) symmetry breaking scale and the seesaw scale of the neutrino masses.

The model contains two scalar doublets, \( \Phi_i \), and one singlet, \( \phi \). A \( U(1)_{PQ} \) global symmetry exists if the two Higgs doublets couple separately to the up and down quarks and leptons so that the Yukawa Lagrangian takes the form:

\[
\mathcal{L} \supset -Y_u\overline{Q}_L \Phi_1 u_R - Y_d\overline{Q}_L \Phi_2 d_R
\]

\[
- \frac{Y_i\overline{L}_L \Phi_i N - Y_i\overline{L}_L \Phi_i l_R - \frac{h_N}{\sqrt{2}} \overline{\phi} N + h.c.}{\langle \phi \rangle}. \tag{40}
\]

leading naturally to type II two-Higgs-doublet models without FCNC \cite{88,89}.

The most general scalar potential of the model compatible with a global \( U(1)_{PQ} \) is the following

\[
V = m_1^2 |\Phi_1|^2 + m_2^2 |\Phi_2|^2 + m_3^2 |\phi|^2 + \frac{\lambda_1}{2} |\Phi_1|^4 + \frac{\lambda_2}{2} |\Phi_2|^4 + \frac{\lambda_3}{2} |\phi|^4
\]

\[
+ \lambda_4 |\Phi_1|^2 |\Phi_2|^2 + \lambda_5 |\Phi_1|^2 |\Phi_2|^2 + \lambda_6 |\phi|^2 |\phi|^2
\]

\[
+ k |\Phi_1|^2 |\phi|^2 + h.c. \tag{41}
\]

The couplings in this potential can be chosen such that \( \phi \) gets an expectation value,

\[
\langle \phi \rangle = \frac{1}{\sqrt{2}} f_a. \tag{42}
\]

\( U(1)_{PQ} \) is then spontaneously broken and a Nambu-Goldstone boson appears, the QCD axion. Furthermore the Majorana singlets \( N \) get a mass. Expanding around the right vacuum, the field can be written as

\[
\phi = \frac{1}{\sqrt{2}} (f_a + \sigma + ia), \tag{43}
\]

where \( \sigma \) is a massive field, while \( a \) is the axion. Therefore after symmetry breaking we obtain an interaction term between sterile neutrinos and axions

\[
\mathcal{L} \supset - \frac{ih_N}{2} \overline{\phi} N + h.c. \tag{44}
\]

The breaking scale \( f_a \) must be much larger than the vacuum expectation values of the doublets, \( \gg v_1,2 \), so that the axion can evade the stringent bounds from rare meson decays and supernova cooling, which sets a stringent lower bound \( f_a \gtrsim 4 \cdot 10^9 \text{ GeV} \).

The mass of the axion is induced by the QCD anomaly in the sub-eV range:

\[
m_a \simeq \frac{z^{1/2} m_{\pi} f_\pi}{1 + z (\langle \phi \rangle)} \tag{45}
\]
where \( z = m_a / m_d \). For \( f_a \geq 4 \cdot 10^8 \) GeV, we have

\[
m_a \leq O \left( 10^{-2} \right) \text{eV}. \tag{50}\]

It is well known that the invisible axion is a viable cold DM candidate, through the misalignment mechanism \[57, 91, 92\] (for recent reviews see \[93, 94\]). The DM energy density is given by

\[
\Omega_a h^2 \sim 2 \cdot 10^4 \left( \frac{f_a}{10^{16} \text{GeV}} \right)^{7/6} (\theta_0)^2, \tag{51}\]

where \( \theta_0 \) is the misalignment angle. Assuming an order one initial misalignment angle, the requirement to not exceed the observed DM density implies an upper bound on the scale of PQ symmetry breaking. The constraints on \( f_a \) depend also on whether the breaking of the PQ symmetry happens before or after inflation; in the latter case the misalignment angle is the average value taken over many patches

\[
\langle \theta_0 \rangle^2 \sim \frac{\pi^2}{3} \tag{52}\]

so \( \Omega_a \leq \Omega_{DM} \) implies

\[
f_a \lesssim 1.2 \cdot 10^{11} \text{GeV}, \tag{53}\]

with the equality reproducing the observed cold dark matter energy density \( \Omega_{CDM} h^2 \approx 0.12 \).

The axion can also manifest itself as dark radiation \[95\], given that it is also thermally produced \[96\]. This population of hot axions contributes to the effective number of relativistic species, but the size of this contribution is currently well within the observational bounds \[97\].

In this model the VEV of the scalar singlet gives a Majorana mass to the sterile neutrinos:

\[
m_N \approx h_N f_a. \tag{54}\]

So, if we want a mass in the electroweak range, \( O(1-10^2) \) GeV and \( f_a \in [10^8, 10^{11}] \) GeV, we need the coupling \( h_N \) to be in the range:

\[
h_N \in [10^{-11}, 10^{-6}]. \tag{55}\]

The hierarchy between \( f_a \) and the electroweak scale requires that some couplings in the scalar potential in eq. (\[\ref{eq:scalar_potential}\] \( k, \lambda_{1}, \lambda_{20} \)) are very small. Even if not very appealing theoretically, these small numbers are technically natural as already pointed out in \[98\], where the authors studied the same model with very heavy sterile neutrinos.

A relevant question is that of naturalness or fine-tuning of the Higgs mass in this model. In \[98\], this issue was studied in the context of high-scale thermal leptogenesis, and it was concluded that stability imposes relevant constraints. In particular, a relatively small \( v_2 \lesssim 30 \) GeV is necessary to ensure viable leptogenesis for lower \( m_N \approx 10^5 - 10^6 \) GeV so that yukawa’s are small enough, \( y \lesssim 10^{-4} \), and do not induce unnaturally large corrections to the Higgs mass. In our case, the yukawa couplings, eq. (\[\ref{eq:yukawa}\]), are too small to give large corrections to the Higgs mass, so no additional constraint needs to be imposed on \( v_2 \). As a consequence other invisible axion models, such as the KSVZ \[99, 100\], would also work in the context of low-scale \( m_N \), but leads to tension with stability bounds in the high-scale version \[101, 102\].

**A. Baryon Asymmetry**

The possibility to generate the baryon asymmetry in this model a la Fukugita-Yanagida for very heavy neutrinos \( m_N \sim f_a \) was recognized in the original proposal \[10\] and further elaborated in \[98\]. We want to point out here that for much smaller values of \( h_N \), the ARS mechanism could also work successfully.

As explained above the crucial point is whether the new interactions of the sterile states in this model are fast enough to equilibrate all the sterile neutrinos before EW phase transition. The leading order process we have to consider is the decay of the scalar into two sterile neutrinos \( \sigma \rightarrow NN \), exactly as we considered in the previous section. The limit of \( M_{\sigma} \sim 2 \times 10^5 - 5 \times 10^6 \) GeV derived in eq. (\[\ref{eq:limit}\]) also applies here, which is safely satisfied given the supernova cooling bounds.

At second order, we must also consider the new annihilation process of sterile neutrinos to axions \( NN \leftrightarrow aa \) as shown in Fig. \[\ref{fig:annihilation}\]. The rate of this process at high tem-
peratures, $T \gg m_N$, is given by
\[ \Gamma_{N_a} = \frac{T^3 m_N^2}{192\pi f_a^4}. \] (56)

The condition $\Gamma_{N_a}(T) < H(T)$ is satisfied for $T \leq f_a$ if
\[ f_a \geq 1.2 \cdot 10^5 \left( \frac{m_N}{1\text{GeV}} \right)^{2/3} \text{GeV} \] (57)

(for $m_N \in [1, 10^2]$ GeV), safely within the targeted range. Fig. 8 shows the region on the ($f_a, h_N$) plane for which successful baryogenesis through the ARS mechanism and DM can work in this model.

Even if the necessary condition for ARS leptogenesis is met for $f_a \geq 10^8$ GeV, the presence of the extra degrees of freedom, the axion, the heavy scalar and the second doublet could modify quantitatively the baryon asymmetry. For example, the presence of two scalar doublets could modify the scattering rates of the sterile neutrinos considered in the ARS scenario, where the main contributions are:

- $2 \leftrightarrow 2$ scatterings on top quarks via higgs exchange
- $2 \leftrightarrow 2$ scatterings on gauge bosons
- $1 \leftrightarrow 2$ decays or inverse decays including resumed soft-gauge interactions

Sterile neutrinos are coupled to the same Higgs doublet that also couples to the top quarks; in this case nothing changes with respect to the usual calculation, in which the reactions with top quarks are mediated by $\Phi_1$. However, we could have coupled sterile neutrinos with the doublet interacting with down quark, like in [18]. In this case top quark scattering does not contribute to sterile neutrino production at tree level. The baryon asymmetry is not expected to change within an order of magnitude, since the scattering rate on gauge bosons and the $1 \leftrightarrow 2$ processes are equally important [40] [52]. The process $\sigma \rightarrow NN$ is not foreseen to be relevant for $M_\sigma \sim f_a$, since the scalar is long decoupled when the generation of the asymmetry starts, while the new process $NN \leftrightarrow aa$ is expected to be very small according to the above estimates. It could nevertheless be interesting to look for possible corners of parameter space where the differences with respect to the minimal model is not negligible since this could provide a testing ground for the axion sector of the model.

\[ V. \ MAJORON \ MODEL \]

In between the two models described in [11], there is the possibility of having a global $U(1)$ spontaneously broken, which is not related to the strong CP problem and we call it lepton number. This is of course the well-known singlet majoron model [37] [38] [104]. We assume the sterile neutrinos carry lepton number, $L_N = 1$, but Majorana masses are forbidden and replaced by a yukawa interaction as in the $B - L$ model:
\[ \mathcal{L} \supset - \left( \bar{h}_N \Phi + \frac{1}{\sqrt{2}} h_N N \phi \phi + h.c. \right) \] (58)

where $\Phi$ is the standard model Higgs doublet, while $\phi$ is a complex scalar which carries lepton number $L_\phi = -2$. Then, the complex scalar acquires a VEV
\[ \phi = f + \sigma + i\eta \sqrt{2} \] (59)

and the $U(1)_L$ is spontaneously broken giving rise to the right-handed Majorana mass matrix and leading to a Goldstone boson $\eta$, the majoron. Consequently the Lagrangian will induce the new scattering processes for neutrinos depicted in Fig. 9.

As usual we have to ensure that at least one sterile neutrino does not equilibrate before $T_{EW}$ (see [105] [106] for a recent discussion in the standard high-scale or resonant leptogenesis). As in the previous cases we have to consider the decay $\sigma \rightarrow N_i N_i$ and the annihilation into majorons, Fig. 9. The former gives the strongest constraint, as in eq. (39):
\[ M_\sigma \sim f \geq 2 \times 10^5 - 5 \times 10^6 \text{GeV}. \] (60)

These lower bounds for $m_N = 1$ and 100 GeV are shown by the horizontal lines in Fig. 10.

\[ A. \ Dark \ Matter \]

There are two candidates in this model for dark matter that we consider in turn: the Majoron and the lightest sterile neutrino, $N_1$.

\[ 1. \ Majoron \]

In this model a natural candidate for dark matter is the majoron itself, but it has to acquire a mass, therefore becoming a pseudo Nambu-Goldstone boson (pNGB). One possibility is to appeal to gravitational effects [107] [108]. However, the contribution to the mass from gravitational instantons is estimated to be [109] [111]
\[ m_\eta \sim M_P e^{-M_P f}, \] (61)

and therefore extremely tiny, unless $f$ is close to the Planck scale.
Another alternative is to consider a flavoured $U(1)_X$ and soft symmetry breaking terms in the form of yukawa couplings $^{[112]}$ $^{[113]}$. This possibility has been studied in detail in ref. $^{[113]}$. It has been shown that the majoron can be the main component of dark matter for sterile neutrino masses $m_N \geq 10^5$ GeV, while for masses in the range we are interested in ($m_N \sim 1 - 100$ GeV) neither thermal production via freeze-out nor via freeze-in works.

The possibility to produce it via vacuum misalignment, analogous to the one which produces the axion relic density has also been discussed in $^{[113]}$. It was shown to give a negligible contribution compared to the thermal one, because the majoron gets a temperature dependent mass at early times. Even if the mass of the majoron is significantly smaller in our situation, with lighter $m_N$, we find the same result, i.e. that only a small fraction of the DM can be produced via misalignment.

No matter what the production mechanism is if the majoron constitutes the dark matter, there are constraints from the requirement that the majoron be stable on a cosmological timescale and its decay to the light neutrinos

$$\Gamma(\eta \rightarrow \nu \nu) = \frac{1}{64\pi} \sum_i \frac{m_i^2}{f^2} m_\eta$$

should not spoil the CMB anisotropy spectrum $^{[114]}$ $^{[115]}$. This gives constraints on the mass $m_\eta$ and the symmetry breaking scale $f$, as showed in Fig. 10.

As in the axion case there are additional constraints from supernova cooling $^{[116]}$, but they are much weaker and give an upper bound much lower than the range shown in Fig. 10.

In the unconstrained region in Fig. 10 ARS leptogenesis and majoron DM could in principle work provided the mechanism to generate the majoron mass does not involve further interactions of the sterile neutrinos.

2. Sterile Neutrino

We want now to consider the sterile neutrino as a dark matter candidate, a possibility already explored in $^{[117]}$ $^{[118]}$. In this case the presence of the Majoron could make the sterile neutrino unstable, given that it would decay through the channel

$$\Gamma(N \rightarrow \nu \eta) = \frac{1}{32\pi} \left(\frac{m_N}{f}\right)^2 m_\nu$$

Therefore one has to consider either $\Phi$ as a real scalar (as in $^{[118]}$) or assume that the Majoron has acquired a larger mass so that this decay is forbidden.

As in the $B - L$ case, ARS leptogenesis is driven by the other two heavier neutrino states $N_{2,3}$, while $N_1$ can be produced through freeze-in from $\sigma \rightarrow N_1 N_1$ decay. Assuming $\sigma$ is in thermal equilibrium with the bath the Boltzmann equation describing the evolution of the $N_1$

$$\dot{n}_1 + 3Hn_1 = 2 \frac{M^2}{2\pi^2} \frac{\Gamma_{\sigma \rightarrow N_1 N_1}}{T} K_1 \left(\frac{M_\sigma}{T}\right),$$

where we have neglected Pauli blocking, and the inverse processes.

Following the standard procedure we end with the contribution to the abundance:

$$\Omega_{N_1} h^2 \sim \frac{10^{27}}{g_\star^{1/2}} \frac{m_{N_1} \Gamma_{\sigma \rightarrow N_1 N_1}}{M_\sigma^2},$$

Using

$$\Gamma_{\sigma \rightarrow N_1 N_1} = \frac{h_{N_1}^2 M_\sigma}{16\pi} \left(1 - \frac{4 m_{N_1}^2}{M_\sigma^2}\right) \sim \frac{h_{N_1}^2 M_\sigma}{16\pi},$$

and requiring that $\Omega_{N_1} h^2$ matches the observed DM we find

$$h_{N_1} \sim 4.3 \cdot 10^{-13} \sqrt{\frac{m_{N_1}}{M_\sigma}} \left(\frac{g_\star}{10}\right)^{3/4}.\frac{m_{N_1}}{M_\sigma}$$

The mass of the DM candidate is related to the coupling which regulates the freeze-in process through the VEV of $\phi$

$$m_{N_1} = h_{N_1} \langle \phi \rangle.$$
VI. CONCLUSION

The extension of the Standard Model with three heavy majorana singlets at the weak scale can explain neutrino masses and also account for the baryon asymmetry in the Universe via the ARS mechanism \[8\]. This scenario could be testable in future experiments. Unfortunately the simplest model cannot easily accommodate dark matter. In the νMSM \[9\], one of the three heavy states is in the keV range and provides a candidate for dark matter, but it requires huge lepton asymmetries that cannot be naturally achieved in the minimal setup.

In this paper we have explored three extensions of the minimal scenario that can accommodate dark matter without spoiling baryogenesis. This is non trivial because new interactions of the heavy singlets can disrupt the necessary out-of-equilibrium condition which is mandatory to generate a lepton asymmetry. We have shown that a extension of the minimal model with a $U(1)_{B-L}$ gauge interaction can achieve this goal. The two heavier ma- jorana fermions take part in the generation of the baryon asymmetry, while the lightest one in the keV range, $N_1$, is the dark matter. In contrast with the νMSM the production of the dark matter is not via mixing, but it is dominated by the $B-L$ gauge boson decay to $N_1 N_1$. The mixing is however what controls the decay of the $N_1$ and can be made sufficiently small to avoid the stringent X-ray constraints. The correct DM abundance is achieved for very small $B-L$ gauge couplings, $g_{B-L} \lesssim 10^{-8}$, which are safely small not to disturb the baryon asymmetry, which is essentially the same as in the minimal model. Such tiny couplings will however be difficult to test. Supernova and BBN provide the most stringent constraints in the relevant region of parameter space, while future searches in SHIP might have a chance to touch on it.

We have also considered an extension involving an invisible axion sector with an extra scalar doublet and a complex singlet. The heavy majorana singlets get their mass from the PQ breaking scale \[46\]. DM is in the form of cold axions, from the misalignment mechanism and as is well known, the right relic abundance can be achieved for a large value of the PQ breaking scale, $f_a \simeq 10^{11}$ GeV. We have shown that such large scale is compatible with having the heavy neutrinos in the 1-100 GeV scale, and ARS leptogenesis. Finally we have considered the singlet majoron extension of the minimal model, with a global $U(1)_{B-L}$, that contains two potential DM candidates, the majoron or the lightest heavy neutrino, $N_1$. Unper- turbated ARS baryogenesis requires a relatively high $B-L$ breaking scale, $f \gtrsim 10^6$ GeV. Majoron DM requires exotic production scenarios, while neutrino DM works for masses around MeV, which requires extremely small mixings to make it sufficiently long-lived, or alternatively a less theoretically appealing possiblity, where the scalar couples to only one sterile neutrino, while the other two have tree level masses or couple to a different scalar with a larger VEV.

As a general rule, adding new interactions that affect
the heavy Majorana singlets modifies the ARS leptogenesis in the minimal model and viable extensions that can explain DM are likely to involve the freeze-in mechanism as in the examples above.

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Appendix

In this appendix we give some details on the computation of the momentum averaged rates in eq. (17). The amplitude for $ff \rightarrow NN$ for vanishing masses is given by

$$\sum_{\text{spins}} |\mathcal{M}|^2 = 4g_{B-L}^4 \sum_f Q_f^2 N_c \left[ \frac{\ell^2 + u^2}{s^2} \right] \equiv A \left[ \frac{\ell^2 + u^2}{s^2} \right].$$

(A.1)

Defining the Bose-Einstein and Fermi-Dirac distributions

$$f_B(x) = \frac{1}{e^{x-1} + 1}, \quad f_F(x) = \frac{1}{e^{x+1} + 1},$$

(A.2)

and the variables

$$q = \frac{1}{2} \left( q_0 \pm |q| \right),$$

(A.3)

where $q = p_1 + p_2$. We express all momenta in units of temperature $T$.

Following the procedure of ref. [50] the rate can be written as

$$R(k) = \frac{A}{4(2\pi)^3 k_0} \left( r_1(k) + r_2(k) + r_3(k) \right),$$

(A.4)

with

$$r_1(k) \equiv \int_{k_0}^{\infty} dq_+ \int_0^{k_0} dq_- f_B(q_+ + q_-) I_1(q_+ - q_-),$$

(A.5)

and

$$r_2(k) \equiv 2 \int_{k_0}^{\infty} dq_+ \int_0^{k_0} dq_- f_B(q_+ + q_-) \sum_{i=1,2} I_i(q_+, q_-) a_i[q_+ - q_-, k_0]$$

(A.6)

$$r_3(k) \equiv 2 \int_{k_0}^{\infty} dq_+ \int_0^{k_0} dq_- f_B(q_+ + q_-) \sum_{i=1,3} I_i(q_+, q_-) b_i[q_+ - q_-, k_0]$$

(A.7)

with

$$I_n(q_+, q_-) \equiv \int_{q_-}^{q_+} x^{n-1} \left[ 1 - 2 f_F(x) \right] dx,$$

(A.8)

and

$$a_1[q_+, q_-, k_0] \equiv -1 + \frac{q_+_0 (k_0 - q_+) - q_- (k_0 - q_-)}{(q_+ - q_-)^2},$$

$$a_2[q_+, q_-, k_0] \equiv \frac{q_+ + q_- - 2k_0}{(q_+ - q_-)^2},$$

(A.9)

$$b_1[q_+, q_-, k_0] \equiv a_1^2 + 2q_+ q_- (q_+ - k_0)(q_- - k_0),$$

$$b_2[q_+, q_-, k_0] \equiv 2a_1[q_+, q_-, k_0] a_2[q_+, q_-, k_0]$$

$$- 2(q_+ + q_-) (q_+ - k_0)(q_- - k_0),$$

$$b_3[q_+, q_-, k_0] \equiv a_2^2 + 2 \frac{(q_+ - k_0)(q_- - k_0)}{(q_+ - q_-)^4}$$

(A.10)

The averaged rates $\gamma_V^{(0)}$ and $\gamma_V^{(1)}$ are then:

$$\langle \gamma_V^{(0)} \rangle = \int \frac{d^4k}{(2\pi)^3 2k_0} R[k] F_k = 3.2(2) \times 10^{-3} g_{B-L}^4 T,$$

(A.11)

$$\langle \gamma_V^{(1)} \rangle = \int \frac{d^4k}{(2\pi)^3 2k_0} R[k] f_F(k) = 3.4(1) \times 10^{-4} g_{B-L}^4 T.$$
