Measuring Dirac CP-violating phase with intermediate energy beta beam facility

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Taking the established nonzero value of $\theta_{13}$, we study the possibility of extracting the Dirac CP-violating phase by a beta beam facility with a boost factor $100 < \gamma < 450$. We compare the performance of different setups with different baselines, boost factors and detector technologies. We find that an antineutrino beam from $^6$He decay with a baseline of $L = 1300$ km has a very promising CP discovery potential using a 500 kton Water Cherenkov (WC) detector. Fortunately this baseline corresponds to the distance between FermiLAB to Sanford underground research facility in South Dakota.
I. INTRODUCTION

The developments in neutrino physics in recent 15 years have been overwhelmingly fast. Nonzero neutrino mass has been established and five out of nine neutrino mass parameters have been measured with remarkable precision. In 2012, at last the relatively small mixing angle, $\theta_{13}$ was measured. This nonzero value of $\theta_{13}$ opens up the possibility of having CP-violating effects in the neutrino oscillations; i.e., $P(\nu_\alpha \to \nu_\beta) \neq P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$. With this nonzero value of $\theta_{13}$, the quest for measuring the Dirac CP-violating phase, $\delta_D$, has been gathering momentum. A well-studied way to extract $\delta_D$ is the precision measurement and comparison of $P(\nu_\mu \to \nu_\tau)$ and $P(\bar{\nu}_\mu \to \bar{\nu}_\tau)$ by superbeam and neutrino factory facilities. However, this is not the only way. In fact by studying the energy dependence of just one appearance mode e.g., $P(\nu_\mu \to \nu_\tau)$, the value of $\delta_{CP}$ can be extracted. In Ref. [6], a novel method for extracting $\delta_D$ (or more precisely $\cos \delta_D$) was suggested that was based on reconstructing the unitary triangle in the lepton sector.

Recently, beta beam facilities producing $\nu_e$ or $\bar{\nu}_e$ beams from the decay of relativistic ions have been proposed and studied as an alternative machine to establish CP-violation in the neutrino sector. Most studies were however performed before the measurement of $\theta_{13}$, with a focus on the CP-discovery reach for values of $\theta_{13}$ much smaller than the measured one. Ref. [10] shows that using the $\nu_e$ beam from $^{18}$Ne decay with energies peaked around 1.5-2 GeV, information on $\delta_D$ can be extracted without a need for an antineutrino beam. In this setup, the boost factor of the decaying ions is $\gamma = 450$. In the present work, we shall consider a similar setup; however, with lower boost factors yielding neutrino energies below the 13-resonance energy in the mantle which is about 6.5 GeV. For a detailed analysis of the matter effects see Ref. [12]. For a neutrino beam with a given energy in the range 400 MeV $< E_\nu < 1.5$ GeV, the oscillation probability can be approximately written as

$$P_{e\mu} \simeq |s_{12}^m c_{23} e^{i\lambda_2 - i\lambda_1}| + s_{13}^m s_{23} e^{-i\delta_{CP}} (e^{i\lambda_3 - i\lambda_1})^2 \approx |s_{12}^m c_{23} e^{i\lambda_2 - i\lambda_1}| + s_{13}^m s_{23} |e^{-i\delta_{CP}}|^2 \approx |s_{12}^m c_{23} e^{i\lambda_2 - i\lambda_1}| + s_{13}^m s_{23} |e^{-i\delta_{CP}}|^2, \quad (1)$$

where $c_{12}^m \ll 1$ is the cosine of the 12-mixing angle in matter and $\lambda_i$ are the phases resulting from the propagation; i.e., for a constant density $\lambda_i = (m_i^2)_{eff} L/(2E)$. For the antineutrino mode, a similar equation holds with $s_{12}^m \ll 1$ and

$$P_{e\mu} \simeq |s_{12}^m c_{23} e^{i\lambda_2 - i\lambda_1}| + s_{13}^m s_{23} e^{i\delta_D} (e^{i\lambda_3 - i\lambda_1})^2 \approx |s_{12}^m c_{23} e^{i\lambda_2 - i\lambda_1}| + s_{13}^m s_{23} |e^{i\delta_D}|^2. \quad (2)$$

Notice that in the above formulas, the deviations of the values of $\theta_{23}$ and $\delta_D$ in matter from those in vacuum are neglected. These deviations are of order of $\Delta m^2_{21}/\Delta m^2_{32}$ [12]. As long as $|\lambda_2 - \lambda_1| \sim 1$, the two terms in Eq. (1) as well as those in Eq. (2) are of the same order so the interference terms which are sensitive to $\delta_D$ are of order of the oscillation probabilities themselves. This means that the variation in the oscillation probabilities due to the change of $\delta_D$ within (0 $\pi$) is of order of the oscillation probabilities, themselves. As a result for these energies and $|\lambda_2 - \lambda_1| \sim 1$, even a moderate precision in the measurement of the probabilities will be enough to extract the value of $\delta_D$.

The flux at the detector scales as $\gamma^2$ and the scattering cross section of neutrinos increases by increasing the energy (i.e., increasing $\gamma$). As a result, for a given baseline, the statistics increases with $\gamma$. Based on this observation, most attention in the recent years has been given to $\gamma > 300$. However, one should bear in mind that for $\gamma < 300$, there is the advantage of using very large Water Cherenkov (WC) detectors. In this energy range, the neutrino interaction will be dominantly quasi-elastic and its scattering cross section is known with high precision. In the literature, the CP-discovery potential of a beta beam setup from CERN to Frejus with $\gamma < 150$ and $L = 130$ km has been investigated [12]. Moreover, varying the values of $\gamma$ and baselines, it has been shown that for $150 < \gamma < 300$ with 500 kton WC detector or iron calorimeter, there is a very good chance of CP-discovery. Ref. [9] explores the $\theta_{13} - \delta_D$ discovery reach with a 300 kton WC and 50 kton LAr detector at Deep Underground Science and Engineering Laboratory (DUSEL), taking maximum boosts possible at Tevatron. Now that the value of $\theta_{13}$ is measured and found to be sizeable, reconsidering $\gamma < 300$ setup is imperative. In vacuum, the dependence of the oscillation probability on $L$ and $\gamma$ would be through $L/\gamma$. However, for setups under consideration, because of the matter effects, the dependence on $L$ and $\gamma$ is more sophisticated so the dependence on $E$ and $L$ has to be investigated separately. In particular, while Ref. [14] focuses on $L/\gamma = 2.6$ km, we have found that for $L/\gamma > 2.6$ km, there is a very good discovery potential. The present paper is devoted to such a study.

In section II, we describe the inputs and how we carry out the analysis. We outline the characteristics of the beam and the detector as well as the sources of the background and the systematic errors. In section III, we present our results and analyze them. In section IV, we summarize our conclusions and propose an optimal setup for the $\delta_D$ measurement.
II. THE INPUTS FOR OUR ANALYSIS

Using the GloBES software \cite{18}, we investigate the CP-discovery potential of a beta beam setup with various baselines and beam boost factors, $\gamma$. For the central values of the neutrino parameters, we have taken the latest values in Ref \cite{19}. The hierarchy can be determined by other experiments such as PINGU \cite{20, 21} or combining PINGU and DAYA Bay II results \cite{22} so we assume that hierarchy is known by the time that such a beta beam setup is ready. We study both normal and inverted hierarchies. T2K and Nova can also solve the octant degeneracy and determine whether $\theta_{23} < 45^\circ$ or $\theta_{23} > 45^\circ$ \cite{23}. The data already excludes the $\theta_{23} > 45^\circ$ solution at 1 $\sigma$ C.L. For the uncertainty of the mixing parameters, we take the values that will be achievable by forthcoming experiments. Namely, we take the following uncertainties: 0.4\% for $\theta$, 0.7\% for $\Delta m^2_{32}$, 2\% for $\theta_{23}$ \cite{24}, 0.2\% for $\Delta m^2_{31}$ \cite{24} and 0.7\% for $\Delta m^2_{31}$ \cite{24}. As predefined by GloBES, we use the matter profiles in \cite{27}. We consider 5\% error for matter density. The uncertainties are treated by the so-called pull-method \cite{18}. While the effects of uncertainty in matter density is more for larger baselines, the uncertainties of neutrino parameters affect the results from smaller baselines more.

As the source of neutrino (antineutrino) beam, we take decays of $^{18}$Ne ($^6$He):

$$^{18}\text{Ne} \rightarrow ^{18} \text{F} + e^+ + \nu_e$$

and

$$^6\text{He} \rightarrow ^6 \text{Li} + e^- + \bar{\nu}_e.$$  

The endpoint energies of these two decays are very close to each other: $E_0 = 3.4$ MeV for $^{18}$Ne and $E_0 = 3.5$ MeV for $^6$He. As a result for equal $\gamma$, the energy spectrum of $\nu_e$ and $\bar{\nu}_e$ from their decays will be approximately similar. The pair of $^8$Li and $^8$B isotopes have also been discussed in the literature as a potential source of the $\nu_e$ and $\bar{\nu}_e$ beams. In these cases, the endpoints are higher so to have neutrino beams with energies $E_0 < 1.5$ GeV, the values of $\gamma$ should be lower than in the case of $^{18}$Ne/$^6$He. On the other hand, the flux at the detector drops as $\gamma^{-2}$ so with the $^8$Li and $^8$B isotopes, the number of decays should be larger to compensate for the $\gamma^{-2}$ suppression. We will not consider the $^8$Li/$^8$B isotopes in this paper and will focus on the $^{18}$Ne/$^6$He pair. For neutrino (antineutrino) mode, we take $2.2 \times 10^{18}$ ($5.8 \times 10^{18}$) decays of $^{18}$Ne ($^6$He) per year which seems to be realistic \cite{28}. Tevatron accelerator can accelerate $^{18}$Ne and $^6$He up to boost factors 586 and 351, respectively.

While the disappearance probabilities (i.e., $P(\nu_e \rightarrow \nu_e)$ or $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$) are not sensitive to $\delta_D$, appearance probabilities (i.e., $P(\nu_e \rightarrow \nu_\mu)$ or $P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu)$) contain information on $\delta_D$. In our analysis, we however employ both appearance and disappearance modes. In principle, the disappearance mode can help to reduce the effect of uncertainty in other parameters but we have found that the effect of turning off the disappearance mode on the $\delta_D$ measurement is less than 1\%. To derive the value of $\delta_D$, the detector has to distinguish $\nu_\mu$ from $\nu_e$. We focus on a 500 kton Water Cherenkov (WC) detector and compare its performance with a 50 kton Totaly Active Scintillator Detector (TASD).

In the energies of our interest with $\gamma < 300$, the main interaction mode is Charged Current (CC) quasi-elastic mode with a non-negligible contribution from inelastic charged current interaction which produce one or more pions along with the charged lepton. In principle, the quasi-elastic CC events can be distinguished from the inelastic CC ones by counting the number of Cherenkov rings. However with a WC detector distinguishing the two interactions will be challenging. We take the signal to be composed of both quasi-elastic and inelastic charged current events and conservatively assume WC detector cannot distinguish between the two.

In the case of QE interaction by measuring the energy and the direction of the final charged lepton, the energy of the initial neutrino can be reconstructed up to an uncertainty of 0.085 GeV caused by the Fermi motion of nucleons inside the nucleus. However, in the inelastic interaction, a fraction of the initial energy is carried by pions so the energy of the initial neutrino cannot be reconstructed by measuring the energy and the direction of the final lepton alone. A WC detector cannot measure the energy deposited in hadronic showers so with this technique, the reconstruction of the energy spectrum will be possible only for the QE interactions. Following the technique in \cite{14, 29} we take an unknown normalization for QE events and use its spectrum as a basis for energy reconstruction. Of course with this method, energy reconstruction cannot be carried out on an event by event basis and information on the spectrum will be only statistical. TASD can measure the energy deposited in hadronic showers, too. As a result, energy reconstruction by TASD can be possible on an event by event basis.

As shown in \cite{30}, the background from atmospheric neutrinos can be neglected and the main source of background for both TASD and WC detectors are neutral current interactions of the beam neutrinos. In our analysis, the cross sections of the quasi-elastic, inelastic and neutral current interactions we employ the results of \cite{31}. Recently the MiniBooNE collaboration has measured the antineutrino cross section in the energy range of our interest \cite{32} with remarkable precision. In the near future, the measurement of cross section will become even more precise. Unless otherwise stated, we assume four years of data taking.
For the treatment of the efficiencies and backgrounds we implement the same methods used in [14, 29]. While for the purpose of this paper, the methods used in [14, 29] are adequate, we would like to note that a more complete discussion of reconstruction of events in large WC detectors can be found in [30]. To be more specific similarly to [14], we assume the following characteristics for the WC detector performance. We take a signal efficiency of 55% for neutrinos and that of 75% for antineutrinos. We take the uncertainty in normalization of the total signal to be 2.5% but as we mentioned above, we take a free normalization for QE events. We assume a background rejection of 0.3% for neutrinos and 0.25% for antineutrinos. The normalization uncertainty of the background is taken to be 5%. For both background and signal, the calibration error is 0.0001. For the energy reconstruction of the background, we use the migration matrices tabulated for the GLoBES package [33]. We consider the energy range between 0.2 and 3 GeV and divide it into 28 bins. The energy resolution for QE CC interactions is assumed to be of form \( \frac{E}{GeV} \). For both background and signal, the calibration error is 0.0001. For the energy reconstruction of the background, we have studied the dependence of our results on the number of the bins. It seems that the results do not change by increasing the number of the bins to 30.

III. RESULTS AND THE INTERPRETATION

In Figs. 1 and 2, the vertical axis shows the precision with which \( \delta_D = 90^\circ \) can be determined at 1 \( \sigma \) % C.L. We take \( \delta_D = 90^\circ \) and define \( \Delta \delta_D \) the range for which \( \Delta \chi^2 < 1 \). More precisely, \( \Delta \delta_D \) is defined as the difference between maximum and minimum values of \( \delta_D \) around \( \delta_D = 90^\circ \) for which \( \Delta \chi^2 = 1 \). From Fig. 1, we observe that low energy set-up with \( \gamma = 300 \) and WC detector can outperform the setup with \( \gamma = 450 \) and TASD detector for both normal and inverted hierarchies. The oscillatory behavior of the curves is driven by the 13-splitting and has a frequency given by \( \sim \Delta m^2_{31}/(2E) \). Such a behavior can be understood by the following consideration on Eqs. (12). While \( \lambda_2 - \lambda_1 \) is driven by \( \Delta m^2_{31} \) and slowly varies with \( L \), \( \lambda_3 - \lambda_2 \) is driven by \( \Delta m^2_{21} \) and varies rapidly. For the values of \( L \) that \( \lambda_3 - \lambda_2 = 2\pi \), the sensitivity is lost. This consideration explains the oscillatory behavior of Figs. 1. Notice, however that this consideration holds for a given \( E_\nu \). If the energy spectrum is wide, the effect will smear out. In other words, if the number of energy bins from which information on \( \delta_D \) can be deduced (i.e., the bins for which the number of events without oscillation is sizeable and the quasi-elastic interactions dominate) is relatively large, missing information in few of these bins for which \( \lambda_3 - \lambda_2 \rightarrow \lambda_2 \) will not affect much the precision in the determination of \( \delta_D \). In the opposite case when at all such bins \( \lambda_3 - \lambda_2 \rightarrow \lambda_3 \), the precision in \( \delta_D \) will be dramatically deteriorated. Increasing the boost factor increases both the peak energy and the energy width. Thus, we expect for higher \( \gamma \), the oscillatory behavior to be smeared out. Fig. 1 confirm this expectation. In case of antineutrinos, the information on \( \delta_D \) can be deduced from a larger range of spectrum mainly because of the shape of the spectrum at the source and the fact that for antineutrinos, the QE interactions dominate over inelastic interaction for a wider energy range compared to the case of neutrino [31]. As a result, the modulation driven by \( \Delta m^2_{31} \) is less severe for antineutrinos. As seen in the lower panels of Fig 1, the antineutrino beam with \( \gamma = 300 \) and a WC detector can achieve an impressive precision of better than 20° for baselines over 500 km.

Figs. 2 show the fraction of the parameter \( \delta_D \) for which CP-violation can be established. From these figures, we also observe that sets with \( \gamma < 300 \) and 500 kton WC detector can outperform the setup with 50 kton TASSD detector and \( \gamma = 450 \) for \( L < 2500 \) km.

Figs. 3 and 4 compare the CP-discovery potential of a \( \nu \) run with an \( \nu \) run and a mixed balanced run. For the antineutrino run the decay rate is taken to be about 2.6 times that of neutrinos to compensate for the low cross sections of antineutrinos. For 200 km < \( L < 5000 \) km, the antineutrino run seems to outperform both the neutrino run and the mixed run in the precision measurement of \( \delta_D = 90^\circ \). This result is at odds with the results of [14]. However, we should remember that Ref. [14] focuses on a specific value of \( L/\gamma \). In this energy and baseline range, the sensitivity of average \( P(\nu_e \rightarrow \nu_\mu) \) to \( \delta \) is higher than that of average \( P(\nu_\mu \rightarrow \nu_e) \).

For \( L = 1300 \) km, (corresponding to the baseline of the LBNE setup from the FermiLAB to Sanford underground research facility in south Dakota [34]), we also observe that \( \gamma = 300 \) with WC detector is promising and can outperform the \( \gamma = 450 \) setup with TASSD detector. Fig. 5 shows \( \Delta \delta_D \) versus \( \gamma \) for \( L = 1300 \) km and \( L = 2300 \) km. The latter corresponds to the baseline for the LBNO setup from CERN to Finland [35]. Plots show that the setup with \( \gamma = 200 - 300 \) and \( L = 1300 \) km can measure \( \Delta \delta_D = 90^\circ \) with a remarkable precision and also have an outstanding coverage of the \( \delta_D \) range. At this baseline, increasing \( \gamma \) from 200 to 300 does not much improve the sensitivity to \( \delta_D \).
For relatively short baselines $L \sim 100 \text{ km}$, $\sin(\lambda_2 - \lambda_1)/2 \ll 1$ so the contributions of the first terms in Eqs. (12) are subdominant relative to the second terms. As a result, the interference between the first and second terms which is the only contribution sensitive to $\delta_D$ will be suppressed; i.e., when $\delta_D$ varies between 0 and $\pi$ the variation of $P_{\mu\mu}$ and $P_{\bar{\nu}\mu}$ will be of order of $\sin(\lambda_2 - \lambda_1)/2 \sim 0.05L/(130 \text{ km})$. On the other hand, for $L > 1000 \text{ km}$, $\sin(\lambda_2 - \lambda_1)/2 \sim 1$ and the two terms in Eqs. (12) are of the same order, making the variation of the oscillation probabilities due to the variation of $\delta_D$ of order of the oscillation probabilities themselves. As a result, deriving $\delta_D$ from a 130 km setup such as CERN to Frejus requires a different strategy than that of a very long baseline setup with $L > 1000 \text{ km}$. This is demonstrated in Figs. 5 and 6. From Fig. 5 we observe that the pure $\nu_e$ run in the case of the setup with $L = 1300 \text{ km}$ has a better prospect but as seen in Fig. 6, in case of $L = 130 \text{ km}$ km baseline a mixed run of neutrino and antineutrino can perform better than pure $\nu_e$ or $\bar{\nu}_e$ runs. For $\sin(\lambda_2 - \lambda_1)/2 \ll 1$, the uncertainties in neutrino parameters (especially the uncertainties of $\theta_{13}$ and $\theta_{31}$) induce a significant uncertainty in the derivation of $\delta_D$. If we turn off the error in these parameters, the performance of the CERN to Frejus setup will be competitive with that of the LBNE setup but considering the realistic uncertainty in these parameters as outlined in the previous section, the sensitivity of the LBNE setup to $\delta_D$ is much better than the $L = 130 \text{ km}$ setup. This can be confirmed by comparing Figs. 5 and 6.

For the 130 km setup, the oscillation probabilities can be approximately written as $P_{\bar{\nu}\mu} \simeq |i s_{13}^m c_{23} \sin(\lambda_2 - \lambda_1) + s_{12}^m s_{23} e^{i \delta_D} (e^{i \lambda_3} - 1)|^2$ and $P_{\mu\mu} \simeq |i c_{12}^m c_{23} \sin(\lambda_2 - \lambda_1) + s_{12}^m s_{23} e^{i \delta_D} (e^{i \lambda_3} - 1)|^2$. Since we are far from the 31-resonance, $s_{13}^m$ is not much different from $s_{13}$ and is approximately the same for normal and inverted hierarchies. As a result, replacing $\delta_D \rightarrow \pi - \delta_D$ and $\Delta m_{32}^2 \rightarrow - \Delta m_{31}^2$ (i.e., $\lambda_3 \rightarrow - \lambda_3$), the oscillation probability does not change. That is why in Fig. 6 the $\Delta \delta_D$ plots for normal and inverted hierarchies are practically the same. If we take a value other than 90° as the true value of $\delta_D$, we will not have such a symmetry. However, as seen in Figs. 3 and 6 the general behavior for normal and inverted hierarchies are similar. With the present SPS setup, CERN cannot enhance $\gamma$ over 150 for the $^6\text{He}$ ions. On the other hand, from Fig. 6 we observe that with $L = 130 \text{ km}$ km, there is no point in seeking higher values of $\gamma$. In fact, at $\gamma = 150$, the fraction of CP-violating $\delta_D$ parameter for which CP-violation can be established is slightly higher than that for $\gamma > 250$.

From comparing Figs. 5 and 6 we observe that the best performance can be achieved by $L = 1300 \text{ km}$ setup and antineutrino run. For example, while with CERN to Frejus setup, $\delta_D = 90^\circ$ can be measured with only uncertainty of $\Delta \delta_D = 35^\circ$; with a 1300 km setup, the uncertainty can be lowered down to $\Delta \delta_D = 15^\circ$. Notice that for these setups, the same detector (500 kton WC) is assumed. Although with longer baselines, the flux decreases but instead we observe that for $200 < \gamma < 300$, the curves corresponding to the $\bar{\nu}_e$ run are almost flat.

As expected for $L > 100 \text{ km}$, the results are highly sensitive to the central values of $\Delta m_{31}^2$. The oscillatory behavior in Figs. 7 that we discussed before implies such a sensitivity. In fact, the setup that we are proposing can simultaneously extract $\delta_D$ and $\Delta m_{31}^2$. Figs. 7 show 68% and 95% C.L. contours for $\gamma = 300$ and $L = 1300 \text{ km}$ (LBNE). In drawing these plots, the hierarchy is assumed to be known; however, the measured value of $\Delta m_{31}^2$ is not used. The precision in $\Delta m_{31}^2$ can drastically be improved by forthcoming experiments. In 21, it is shown that combining the T2K and PINGU results, 0.7% precision in $\Delta m_{31}^2$ is achievable. In Fig 7 the vertical lines show the 0.7% uncertainty in $\Delta m_{31}^2$ around the “true” value of $\Delta m_{31}^2$. As seen from the figure, for the case of the antineutrino beam the uncertainty in $\Delta m_{31}^2$ will not significantly increase the uncertainty in the $\delta_D$ determination.

IV. CONCLUSIONS

Measuring the CP-violating phase by a beta beam facility has been extensively studied in the literature. Most of the recent studies have focused on relatively high energy beams with $\gamma > 300$. The reason is that for a given baseline, the number of detected neutrinos increases approximately as $\gamma^3$. However, for lower energy beta beam, large volume WC detectors can be employed that can compensate for the decrease of flux and cross section. Moreover with the relatively large value of $\theta_{13}$ chosen by the nature, having enough statistics will not be the most serious challenge for measuring the CP-violating phase. Considering these facts, we have explored the CP-discovery reach of an intermediate energy beta beam for various baselines and different neutrino vs antineutrino combinations using the GLoBES software. We have discussed the precision with which $\delta_D$ can be measured, assuming that by the time that the required facilities are ready the hierarchy is also determined. Our results do not depend much on which mass ordering is chosen.

We have found that a setup with only antineutrino run with $200 < \gamma < 300$ and a baseline of $L = 1300 \text{ km}$ has an excellent discovery potential. Four years run of such a setup with $5.8 \times 10^{18}$ $^6\text{He}$ decays per year can establish...
CP-violation at 95% C.L. for more than 85% of the $\delta_D$ parameter range. If $\delta_D = 90^\circ$, this setup can determine it with impressive precision $\delta_D = 90^\circ \pm 8^\circ$ for inverted hierarchy and $\delta_D = 90^\circ \pm 7^\circ$ for normal hierarchy at 1$\sigma$ C.L. Such a baseline corresponds to the distance between FermiLAB and Sanford underground research facility in South Dakota. A baseline of $L = 1300$ km seems to be close to the optimal distance to measure the Dirac CP-violating phase. We have found that for this baseline a setup with intermediate values of $\gamma$ in the range $200 < \gamma < 300$ with a 500 kton WC detector can outperform that with $\gamma = 450$ and 50 kton TASS.

For the very long baselines with $L > 1000$ km, a pure antineutrino source from $^6$He enjoys a better performance than a mixed neutrino antineutrino run. On the other hand for shorter baselines, a balanced neutrino antineutrino mode gives better results. We have specifically discussed the CERN to Frejus setup with $L = 130$ km baseline. We have found that with two years of neutrino mode from $2.2 \times 10^{18}$ decays of $^{13}$Ne per year combined with two years of antineutrino mode from $5.8 \times 10^{18}$ decays of $^8$He per year both with $\gamma = 150$ (the largest boost that can be obtained for $^8$He with the present SPS accelerator at CERN [13]), the CP-violation can be established for about 80% of the $\delta_D$ parameter range. With such setup and runtime, if the true value of $\delta_D$ is equal to 90°, it can be measured as $\delta_D = 90^\circ \pm 18^\circ$ at 1$\sigma$ C.L. By increasing $\gamma$ to higher values, the precision in the $\delta_D$ measurements slightly improves however still with a similar detector and antineutrino run, the performance of $L = 1300$ km can be better.

In sum, our conclusion is that a beta beam facility with $200 < \gamma < 300$, baseline of $L \approx 1300$ km and 500 kton WC running in the antineutrino mode from $^8$He decay is an optimal option for establishing the CP-violation in the lepton sector and the measurement of $\delta_D$. The location of source and detector can be respectively FermiLAB and Sanford underground laboratory in south Dakota.

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[1] F. P. An et al. [DAYA-BAY Collaboration], Phys. Rev. Lett. 108, 171803 (2012) [arXiv:1203.1699 [hep-ex]].
[2] J. K. Ahn et al. [RENO Collaboration], Phys. Rev. Lett. 108, 191802 (2012) [arXiv:1204.0626 [hep-ex]].
[3] Y. Abe et al. [DOUBLE-CHOOZ Collaboration], Phys. Rev. Lett. 108, 131801 (2012) [arXiv:1112.6353 [hep-ex]].
[4] A. Blondel, I. Efthymiopoulos and G. Prior, J. Phys. Conf. Ser 408 (2013).
[5] M. V. Diwan, D. Beavis, M.-C. Chen, J. Gallardo, S. Kahn, H. Kirk, W. Marciano and W. Morse et al., Phys. Rev. D 68, 012002 (2003) [hep-ph/0303081].
[6] Y. Farzan and A. Y. Smirnov, Phys. Rev. D 65, 113001 (2002) [hep-ph/0201105].
[7] H. Zhang and Z.-z. Xing, Eur. Phys. J. C 41, 143 (2005) [hep-ph/0411183]; A. Bandyopadhyay et al. [ISS Physics Working Group Collaboration], Rept. Prog. Phys. 72, 106201 (2009) [arXiv:0710.4947 [hep-ph]]; G. Ahuja and M. Gupta, Phys. Rev. D 77, 057301 (2008) [hep-ph/0702129 [HEP-PHI]]; Z.-z. Xing and H. Zhang, Phys. Lett. B 618, 131 (2005) [hep-ph/0503118]; S. Antusch, S. F. King, C. Luhn and M. Spinrath, Nucl. Phys. B 850, 477 (2011) [arXiv:1103.5930 [hep-ph]]; A. Dueck, S. Petcov and W. Rodejohann, Phys. Rev. D 82, 013005 (2010) [arXiv:1006.0227 [hep-ph]]; Z.-z. Xing and S. Zhou, Phys. Lett. B 666, 166 (2008) [arXiv:0804.3512 [hep-ph]].
[8] P. Zacchelli, Phys. Lett. B 532, 166 (2002); C. Volpe, [arXiv:0802.3352 [hep-ph]]; C. Volpe, Prog. Part. Nucl. Phys. 64, 325 (2010) [arXiv:0911.4314 [hep-ph]]; C. Orme, JHEP 1007, 049 (2010) [arXiv:0912.2676 [hep-ph]]; P. Coloma, A. Donini, P. Migliozzi, L. Scotto Lavina and F. Terranova, Eur. Phys. J. C 71, 1674 (2011) [arXiv:1004.3773 [hep-ph]]; J. Bernabeu, C. Espinoza, C. Orme, S. Palomares-Ruiz and S. Pascoli, AIP Conf. Proc. 1222, 174 (2010); S. K. Agarwalla, A. Raychaudhuri and A. Samanta, Phys. Lett. B 629, 33 (2005) [hep-ph/0505015]; S. K. Agarwalla, S. Choubey and A. Raychaudhuri, Nucl. Phys. B 798, 124 (2008) [arXiv:0711.1459 [hep-ph]].
[9] S. K. Agarwalla and P. Huber, Phys. Lett. B 693, 114 (2010) [arXiv:0909.2257 [hep-ph]].
[10] D. Meloni, O. Mena, C. Orme, S. Palomares-Ruiz and S. Pascoli, JHEP 0807, 115 (2008) [arXiv:0802.0255 [hep-ph]]; D. Meloni, O. Mena, C. Orme, S. Pascoli and S. Palomares-Ruiz, PoS NUFACT 08, 133 (2008).
[11] S. K. Agarwalla, Y. Kao and T. Takeuchi, [arXiv:1302.6773 [hep-ph]].
[12] M. Blemmel and A. Y. Smirnov, Adv. High Energy Phys. 2013 (2013) 972485 [arXiv:1306.2903 [hep-ph]].
[13] T. R. Edgecock, O. Caretta, T. Davenne, C. Densham, M. Fitton, D. Kellihier, P. Loveridge and S. Machida et al., Phys. Rev. ST Accel. Beams 16 (2013) 021002 [arXiv:1305.4067 [physics.acc-ph]].
[14] P. Huber, M. Lindner, M. Rolince and W. Winter, Phys. Rev. D 73, 053002 (2006) [hep-ph/0506237].
[15] M. Mezzetto, J. Phys. G 29 (2003) 1771 [hep-ex/0302007].
[16] W. Winter, Phys. Rev. D 78, 037101 (2008) [arXiv:0804.4000] [hep-ph].
[17] S. K. Agarwalla, S. Choubey, A. Raychaudhuri and W. Winter, JHEP 0806, 090 (2008) [arXiv:0802.3621] [hep-ex].
[18] P. Huber, M. Lindner and W. Winter, Comput. Phys. Commun. 167, 195 (2005) [hep-ph/0407333]; P. Huber, J. Kopp, M. Lindner, M. Rolinec and W. Winter, Comput. Phys. Commun. 177, 432 (2007) [hep-ph/0701187]; http://www.mpi-hd.mpg.de/personalhomes/globes
[19] M. C. Gonzalez-Garcia, M. Maltoni, J. Salvado and T. Schwetz, JHEP 1212 (2012) 123 [arXiv:1209.3023] [hep-ph]; see also, v1.1 results in www.nu-fit.org.
[20] E. K. Akhmedov, S. Razzaque and A. Yu. Smirnov, JHEP 02 (2013) 082 [arXiv:1205.7071] [hep-ph]; S. K. Agarwalla, T. Li, O. Mena and S. Palomares-Ruiz, [arXiv:1212.2238] [hep-ph]; D. Franco et al., JHEP 1304 (2013) 008 [arXiv:1301.4332] [hep-ex]; T. Ohlsson, H. Zhang and S. Zhou, Phys. Rev. D 88 (2013) 013001 [arXiv:1303.6130] [hep-ph]; A. Esmaili and A. Yu. Smirnov, JHEP 1306 (2013) 026 [arXiv:1304.1042] [hep-ph]; M. Ribordy and A. Y. Smirnov, Phys. Rev. D 87 (2013) 113007 [arXiv:1303.0758] [hep-ph].
[21] W. Winter, Phys. Rev. D 88 (2013) 013013 [arXiv:1305.5533] [hep-ph].
[22] M. Blennow and T. Schwetz, JHEP 1309 (2013) 089 [arXiv:1306.3988] [hep-ph].
[23] S. K. Agarwalla, S. Prakash and S. U. Sankar, JHEP 1307 (2013) 131 [arXiv:1301.2574] [hep-ph].
[24] F. Capozzi, E. Lisi and A. Marrone, [arXiv:1309.1638] [hep-ph].
[25] D. V. Forero, M. Tortola and J. W. F. Valle, Phys. Rev. D 86 (2012) 073012 [arXiv:1205.4018] [hep-ph].
[26] S. K. Raut, Mod. Phys. Lett. A 28 (2013) 1350093 [arXiv:1209.5658] [hep-ph].
[27] A. M. Dziewonski and D. L. Anderson, Preliminary reference earth model, Phys. Earth Planet Interiors 25 (1981) 297; F. F. Stacey, Physics of the Earth, 2nd ed., Wiley, 1977.
[28] A. Jansson, O. Mena, S. J. Parke and N. Saoulidou, Phys. Rev. D 78 (2008) 053002 [arXiv:0711.1075] [hep-ph].
[29] P. Huber, M. Lindner and W. Winter, Nucl. Phys. B 645 (2002) 3 [hep-ph/0204352].
[30] L. Agostino et al. [MEMPYS Collaboration], JCAP 1301 (2013) 024 [arXiv:1206.6665] [hep-ex].
[31] M. D. Messier, UMI-99-23965; E. A. Paschos and J. Y. Yu, Phys. Rev. D 65 (2002) 033002 [hep-ph/0107261].
[32] A. A. Aguilar-Arevalo et al. [MiniBooNE Collaboration], Phys. Rev. D 88 (2013) 032001 [arXiv:1301.7067] [hep-ex].
[33] P. Huber, M. Lindner and W. Winter, Comput. Phys. Commun. 167 (2005) 195 [hep-ph/0407333]; P. Huber, J. Kopp, M. Lindner, M. Rolinec and W. Winter, Comput. Phys. Commun. 177 (2007) 432 [hep-ph/0701187].
[34] http://lbne.fnal.gov/
[35] A. Stahl, C. Wiebusch, A. M. Guler, M. Kamiscioglu, R. Sever, A. U. Yilmazer, C. Gunes and D. Yilmaz et al., CERN-SPSC-2012-021.
FIG. 1. Uncertainty within which $\delta_D = 90^\circ$ can be measured at 1 $\sigma$ C.L. after four years of data taking versus baseline for different values of the boost factor. For $\gamma = 450$, a 50 kton TASSD detector and for lower $\gamma$, a 500 kton WC detector are assumed. In upper (lower) panels, neutrino (antineutrino) beam with $2.2 \times 10^{18}$ ($5.8 \times 10^{18}$) decays per year is assumed. In left (right) panels, the hierarchy is taken to be normal (inverted). We have taken the true values for the left panels as $\Delta m^2_{31} = 2.421 \times 10^{-3} \text{ eV}^2$ (normal hierarchy) and $\theta_{23} = 41.4^\circ$ (first octant). For the right panels we have taken $\Delta m^2_{31} = -2.35 \times 10^{-3} \text{ eV}^2$ (Inverted Hierarchy) and the same mixing angles.
FIG. 2. The fraction of the $\delta_D$ parameter for which $CP$ can be established at higher than 95% C.L. after four years of data taking versus baseline. The rest of the description is as in Fig. 1.
FIG. 3. Uncertainty within which $\delta_D = 90^\circ$ can be measured versus baseline at 1$\sigma$ C.L. The neutrino parameters are as in Fig. 1. The hierarchy is taken to be normal. For the neutrino and antineutrino beams, $2.2 \times 10^{18}$ and $5.8 \times 10^{18}$ decays per year are assumed, respectively. The curves shown with dashed and solid lines respectively show the results of four years run on neutrino mode from the $^{18}$Ne decay and four years run on antineutrino mode from the $^6$He decay. The curve shown by dotted line displays the results of two years of neutrino run combined with two years of antineutrino run. The boost factors of the beams are taken to be 300.
FIG. 4. The fraction of the $\delta_D$ parameter for which $CP$ can be established at higher than 95 % C.L. versus baseline. The rest of description is as that of Fig. 3.
FIG. 5. The CP-discovery potential by the setups with baselines equal to 2300 km and 1300 km after four years of data taking versus the boost factors of the neutrino and antineutrino beams. In the left (right) panels, the hierarchy is taken to be normal (inverted). Upper panels: Uncertainty within which $\delta_D = 90^\circ$ can be measured at 1$\sigma$ C.L. versus the boost factors of $\nu_e$ and $\bar{\nu}_e$ beams from the $^{18}\text{Ne}$ and $^6\text{He}$ decay. Lower panels: The fraction of the $\delta_D$ parameter for which CP can be established at higher than 95 % C.L.
FIG. 6. The CP-discovery potential of a 130 km experiment (distance between CERN to Frejus) after four years of data taking versus the boost factors of neutrino and antineutrino beams. In the left (right) panels, the hierarchy is taken to be normal (inverted). Upper panels: Uncertainty within which $\delta_D = 90^\circ$ can be measured at 1σ C.L. versus the boost factors of the $\nu_e$ and $\bar{\nu}_e$ beams from the $^{18}$Ne and $^6$He decay. Lower panels: The fraction of the $\delta_D$ parameter for which $CP$ can be established at higher than 95 % C.L.
FIG. 7. Determination of $\delta_D$ and $\Delta m^2_{31}$, with $\gamma = 300$ and $L = 1300$ km after four years of data taking. The contours show 68 % C.L. and 95 % C.L. In upper (lower) panels, neutrino (antineutrino) beam with $2.2 \times 10^{18}$ ($5.8 \times 10^{18}$) decays per year is assumed. In left (right) panels, the hierarchy is taken to be normal (inverted). The true value of $\delta_D$ is taken to be 90$^\circ$. For normal (inverted) hierarchy, we take $\Delta m^2_{31} = 2.421 \times 10^{-3}$ eV$^2$ ($\Delta m^2_{31} = -2.35 \times 10^{-3}$ eV$^2$). The vertical lines show a 0.7% uncertainty in $\Delta m^2_{31}$ (e.g., for normal hierarchy $\Delta m^2_{31} = 2.421 \times 10^{-3}$ eV$^2 \times (1 \pm 0.7\%)$ eV$^2$).