Cooling-induced structure formation and evolution in collapsars

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Abstract

The collapse of massive rotating stellar cores and the associated accretion on to the newborn compact object is thought to power long gamma-ray bursts (GRBs). The physical scale and dynamics of the accretion disc are initially set by the angular momentum distribution in the progenitor, and the physical conditions make neutrino emission the main cooling agent in the flow. The formation and evolution of structure in these discs are potentially very relevant for the energy release and its time variability, which ultimately imprint on the observed GRB properties. To begin to characterize these, taking into account the three-dimensional nature of the problem, we have carried out an initial set of calculations of the collapse of rotating polytropic cores in three dimensions, making use of a pseudo-relativistic potential and a simplified cooling prescription. We focus on the effects of self-gravity and cooling on the overall morphology and evolution of the flow for a given rotation rate in the context of the collapsar model. For the typical cooling times expected in such a scenario, we observe the appearance of strong instabilities on the cooling time-scale following disc formation, which modulate the properties of the flow. Such instabilities, and the interaction they produce between the disc and the central object, lead to significant variability in the obtained mass accretion and energy-loss rates, which will likely translate into variations in the power of the relativistic outflow that ultimately results in a GRB.

Key words: accretion, accretion discs – hydrodynamics – instabilities – gamma-ray burst: general – supernovae: general.

1 INTRODUCTION

Gamma-ray bursts (GRBs) are bright flashes of radiation with a spectral energy peaking in the gamma-ray band (Fishman & Meegan 1995). Their duration ranges from fractions of a second to a few minutes and produces directed fluxes of relativistic material with kinetic luminosities exceeding \(10^{53}\) ergs \(^{-1}\). In order to produce such high luminosities, and observed millisecond variability in flux, it is common and reasonable to invoke the presence of a compact object [such as a neutron star (NS) or a black hole (BH)] as part of the GRB production mechanism at the level of the central engine. To date, despite the lack of definitive direct evidence, it is generally thought that GRBs are the result of catastrophic events involving NSs (perhaps magnetized) or BHs and violent, so-called hypercritical accretion which produces a violent episode of energy release that is subsequently transformed into electromagnetic radiation (see Piran 2004; Lee & Ramirez-Ruiz 2007; Nakar 2007; Gehrels, Ramirez-Ruiz & Fox 2009, for comprehensive reviews).

From GRB afterglow observations that cover the range from X-rays to the radio, it was possible to locate the origin of this sources at cosmological distances (see van Paradijs, Kouveliotou & Wijers 2000, for an initial review). Short GRBs (SGRBs) are typically located at \(z \lesssim 1\); meanwhile, long GRBs (LGRBs) are located at \(z \simeq 1-5\) or more, and compete with quasars for the most distant objects observed. On the whole, the hosts of SGRBs, and by extension the progenitors, are not drawn from the same parent population of LGRBs. SGRBs appear to be more diffusely positioned around galaxies, and their associated hosts contain a generally older population of stars (Lee & Ramirez-Ruiz 2007; Berger 2011). Observations also show that LGRBs can sometimes be associated with a supernova (SN) (with no H lines, i.e. Type Ib or Ic) taking place at the same time and at the same place. The observation of GRB980425 in conjunction with one of the most unusual SNe ever seen (SN1998bw, Galama et al. 1998b) was the first of this kind, but not the last. Further, the host properties, and the locations of the bursts within them, provide strong evidence that they are related to vigorous star formation, strengthening the link with the death of massive stars (Fruchter et al. 2006; Levesque 2013). The review by Woosley & Bloom (2006) shows the existing evidence for the link of LGRBs at low redshift with Type Ic SNe, and the progenitor mechanisms currently explored.

Woosley (1993) proposed the collapsar model to explain the formation of a GRB, from a pre-SN star in which the shutdown of
nuclear reactions in the core leads to collapse and the formation of a BH, rather than a neutron star. The accretion of the infalling material would form an accretion disc, releasing energy to power the burst. Two main variants on this model may occur. In the first, the Fe core is massive enough to induce a direct collapse of the core into a BH, while in the second an intermediate stage produces a protoneutron star first, which later collapses after enough matter has been accreted on to its surface (typically this would take a few seconds).

Two key ingredients, presumably associated with the progenitor, make this a relatively rare occurrence (as they must, considering that the rate of core-collapse SNe far exceeds the observed GRB rate): rotation and the lack of a hydrogen envelope. The first is necessary in order to ensure that a large fraction of the available energy is released in a disc close to the BH through accretion, rather than simply be swallowed whole by the BH (in something akin to Bondi accretion). Evolutionary models for rapidly rotating stars (Woosley & Heger 2006) show that the core is able to retain enough angular momentum to make this a possibility depending on the mass-loss history and the presence (or lack) of magnetic fields coupling the envelope to the inner regions. The second is required in order for the relativistic jet that is launched at the centre of the star to perforate it, break out and eventually lead to high energy emission far from the stellar surface, which we observe as a GRB. The envelope may either have been lost through interaction with a binary companion, or, if efficient mixing occurs throughout the star, the giant phase may be avoided altogether keeping the radius relatively small (Yoon & Langer 2005; Woosley & Heger 2006). Once a centrifugally supported disc forms, the temperature will be high enough that neutrino emission becomes the main cooling mechanism, as pointed out in the context of SNe by Chevalier (1989) and Houck & Chevalier (1991), allowing accretion to proceed at extremely high rates with the attending energy release. In principle, the burst itself may be powered by a combination of neutrinos themselves, or magnetic mechanisms that tap the rotation in the disc and/or the BH. MacFadyen & Woosley (1999) carried out the first detailed numerical study of the collapsar, and further explored jet production, propagation and breakout from the star for a variety of configurations (MacFadyen, Woosley & Heger 2001; Zhang, Woosley & MacFadyen 2003). One possibility is that the explosion eventually does launch the stellar envelope outward and produces an extremely energetic event, leading to the observed hypernovae.

A critical assessment of the outcome of collapsing cores has recently been given by Dessart, O’Connor & Ott (2012) in their one-dimensional general relativistic (GR) simulations. They found that most of the LGRB progenitors obtained by Woosley & Heger (2006) failed to form a BH directly from the collapse of the iron core, and resulted instead in rapidly rotating protoneutron stars which could experience magnetorotational instability (MRI) prior to BH formation. Nevertheless, there are other works on two-dimensional GR simulations (Mizuno et al. 2004a,b; Sekiguchi & Shibata 2007, 2011), where direct formation of a BH is readily obtained from different collapsing cores. The equation of state (EoS), and microphysics (neutrino transport included) used in each one of these works, seems to be of importance in determining the fate of the collapsing core, but the greatest uncertainty remains in the properties of the pre-collapse, such as rotation rate, mass and radius, which can only be addressed by improving stellar evolution models. An important point in the collapsar scenario is that the amount of angular momentum in the star is crucial for GRB production (Lee & Ramirez-Ruiz 2006). Too much of it results in an accretion disc that forms far from the BH. The temperatures and densities are then not high enough for efficient cooling through neutrinos, and hypercritical accretion cannot proceed. Too little of it leads to quasi-spherical accretion, where the mass accretion rate can be extremely large but with near-zero efficiency for the conversion of gravitational binding energy into thermal energy through shocks (and subsequently radiation).

Much effort has been applied to explore the behaviour of collapsars in two dimensions, assuming azimuthal symmetry. Some of them are GR magnetohydrodynamical (MHD) simulations with a simple (e.g. Mizuno et al. 2004a,b; Sekiguchi & Shibata 2007) or detailed (Shibata, Sekiguchi & Takahashi 2007; Sekiguchi & Shibata 2011) EoS and microphysics, and a number of authors have addressed the problem using Newtonian or pseudo-Newtonian approximations in MHD or hydrodynamical simulations with approximate or realistic EoSs and neutrino cooling effects (Proga et al. 2003; Fujimoto et al. 2006; Nagataki et al. 2007; López-Cámara, Lee & Ramirez-Ruiz 2009, 2010; Nagataki 2009). However, relatively little work has been done in three dimensions, thus neglecting the potentially important role of the self–gravity of the infalling gas, and generic instabilities in three dimensions. The first study we are aware of was carried out by Rockefeller, Fryer & Li (2006), and based on a 60 M⊙ rotating Population III star. They observed the formation of instabilities within the disc, mainly as spiral waves, which contribute to angular momentum transport. More recently, Taylor, Miller & Podsiadlowski (2011) considered a very rapidly rotating progenitor, formed from the merger of two He stars. They also saw the development of instabilities in the disc and based on the observed energy release concluded that some of their models are indeed capable of producing a GRB. We note that all of these models consider that the BH is motionless at the origin of coordinates, which is a good approximation if the disc mass is negligible (which may not be the case) or if the infalling layers do not show strong asymmetries in their mass distribution. A recent analysis by Korobkin et al. (2011) explores the evolution of self-gravitating relativistic discs with a simple EoS, around a non-rotating BH with disc-to-BH mass ratios of 0.24, 0.17 and 0.11. Their work focused on the effects of the initial disc configuration and perturbations in the formation of non-axisymmetric instabilities at the disc.

In this paper, we focus on the dynamical effects in three dimensions, coming essentially from the self-gravity of the infalling gas and cooling, which occur during the initial stages of the collapse of a rotating polytropic envelope on to a central BH in the context of the collapsar model. Rather than make use of a very complex EoS, we consider generic solutions with simplified cooling (assumed to be through neutrinos). In particular, we pay special attention to the comparison between the cooling and dynamical time-scales in order to gauge their effect on the formation of structure and the evolution of the mass accretion and energy losses. Section 2 deals with our setup and input physics, Section 3 presents our results, and our conclusions are given in Section 4.

2 INITIAL CONDITIONS AND INPUT PHYSICS

In the context of the collapsar model, we studied the collapse and accretion of 2.5 M⊙ rotating polytropic envelopes with adiabatic index γ = 5/3 on to a 2 M⊙ BH fixed at the centre of the mass distribution. In this section, we will present the relevant details of the initial conditions and physical processes taken into account in our simulations which were carried out with GADGET-2 (Springel, Yoshida & White 2001; Springel 2005), which we have modified in

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order to include the necessary physics to account for accretion and cooling in this context.

2.1 The polytropic envelope and its characteristics

The polytropic envelope was constructed by solving the Lane-Emden equation for hydrostatic equilibrium (see Shapiro & Teukolsky 1983), from which we obtained radial profiles for the density and internal energy of $4.5 \, M_\odot$ polytropic stars with central density $\rho_c = 2.53 \times 10^9 \, \text{g cm}^{-3}$ and $\gamma = 5/3$. From the density profile we mapped a three-dimensional particle distribution with an accept/reject Monte Carlo procedure. To account for the BH, we removed the innermost $2 \, M_\odot$ from this three-dimensional polytropic star and concentrated it in a $2 \, M_\odot$ sink particle placed at the centre of the distribution, leaving the remaining mass unaffected, now with inner radius $r_{\text{in}}$. In Table 1, we show the total mass of the system $M_s$, as well as the initial outer and inner radii of the polytropic envelope ($R_s$ and $r_{\text{in}}$, respectively). From these quantities, we can obtain a characteristic time-scale $t_{\text{dyn}} = \sqrt{R_s^3/GM_s}$, from which all physical units are scaled in the code.

For the distribution of angular momentum in the star, of great relevance for the collapsar model, we have assumed rigid body rotation, and assigned a constant angular velocity $\Omega_0$, defined in terms of the circularization radius $r_c$, the infalling gas would have in a Keplerian orbit around the BH. This circularization radius is estimated assuming a Newtonian gravitational potential (a pseudo-Newtonian potential like Paczynski & Wiita’s would result in a smaller $r_c$, but differing in less than $\sim 3$ per cent from the Newtonian one at the values involved here), a negligible contribution from gas pressure and conservation of angular momentum for the envelope material at its initial radius $r_i = (R_i^2 + z^2)^{1/2}$, where $R_i$ is the cylindrical radius. Thus $r_c$ can be expressed as

$$r_c = \Omega_0^2 R_i^3 / G M(r_i),$$

where $M(r_i)$ is the mass contained inside $r_i$. We have assumed spherical symmetry in the mass distribution, so the gravitational pull only depends on the radius $r_i$. The former consideration implies that $r_c$ would be underestimated. Fig. 1 shows $r_c$ as a function of the initial cylindrical radius $R_i$ for our $2.5 \, M_\odot$ polytropic envelopes at the equatorial plane. Each colour represents a different constant angular velocity $\Omega_0$. The dotted lines only consider the BH mass and the solid lines consider the BH mass and the spherical envelope mass contained at $r_c$.

![Figure 1. Circularization radius $r_c$ in the equatorial plane for our $2.5 \, M_\odot$ polytropic envelope as a function of the cylindrical distance $R/R_s$. Each colour represents a different angular velocity $\Omega_0$. The dotted lines only consider the BH mass and the solid lines consider the BH mass and the spherical envelope mass contained at $r_c$.](image1)

![Figure 2. Circularization radius $r_c$ for our $4.5 \, M_\odot$ polytropic star as a function of the cylindrical distance $R/R_s$ (x-axis) and $Z/R_s$ (y-axis) for three different angular velocities ($\Omega_0 = 8.32, 6.0$ and $3.5 \, \text{s}^{-1}$ from the bottom to top). This map assumes a spherical mass distribution $M(r_i)$.](image2)

Table 1. Envelope and total system masses, along with the characteristic time-scale $t_{\text{dyn}}$, and the outer and inner radius of the envelope.

| System parameter | Value       |
|------------------|-------------|
| $M_s$            | $4.5 \, M_\odot$ |
| $M_{\text{env}}$ | $2.5 \, M_\odot$ |
| $R_s$            | $1715.7 \, \text{km}$ |
| $r_{\text{in}}$  | $844.69 \, \text{km}$ |
| $t_{\text{dyn}}$ | $0.0919 \, \text{s}$ |

$r_{\text{in}}$, and so $r_c$ will give an approximate position for the inner edge of the disc with respect to the BH. Fig. 2 shows the colour-coded circularization radius in a meridional slice. Clearly, material close to the equator has the greatest rotation rate and will circularize at a larger radius. Close to the rotation axis, angular momentum goes to zero and matter will essentially free fall into the BH.

With the purpose of choosing an adequate value for $\Omega_0$, we first obtained from the initial spherical profiles, an estimation of the breakup rotational velocity for such polytropic envelope. Then, assuming rigid body rotation, we compared the rotational velocities obtained for an angular velocity $\Omega_0$, with the breakup rotational velocity for the polytrope. Fig. 3 shows the calculated breakup rotational velocity for such a polytropic star (red line) and rotational velocities in the equatorial plane for different values of the circularization radius $r_c$ ($r_c \propto \Omega_0^2$). Rotational velocities below the breakup velocity curve ensure formation of a disc around the BH at $r_c$ ($r_c$ for the innermost material). This estimation does not take into account...
asymmetric effects on the mass distribution due to rotation, given that the envelope should be rotating from the beginning and therefore it could depart from spherical symmetry. Fig. 3 shows that in order to obtain rotating velocities below breakup, we have to consider a circulation radius $r_c$ smaller than $\approx 12.8 r_{\text{acc}}$, where $r_{\text{acc}}$ is the accretion radius (its value is given in the BH physics which follows). All simulations were made using $r_c = 7.49 r_{\text{acc}}$ and thus correspond to maximal discs, placing as much matter as possible in the disc while avoiding centrifugal mass-loss. The estimates for energy release which we obtain should be considered accordingly.

The previous setup ensures the formation of an accretion disc outside the region of the innermost circular stable orbit around a Schwarzschild BH given by $r_{\text{acc}} = 3 r_g = 6 G M_{\text{BH}}/c^2$. Gas orbiting the BH at $r < r_{\text{acc}}$ would fall inevitably on to it, irrespective of its rotational velocity.

In order to have a handle on the accuracy of the simulations, we carried out convergence tests of the collapse and accretion of such rotating polytropic envelopes on to an accreting sink particle, using 50 000, 500 000 and 5000 000 smoothed particle hydrodynamics (SPH) particles of equal mass. After substantial testing, we have found no difference in the accretion rates observed, or in the general properties of the accretion disc, when using more than 500 000 SPH particles, and so all runs described in this paper have this initial resolution. Moreover, given that the snapshots obtained for such resolution had a reasonable size ($\lesssim 30$ MB), we were able to achieve a very good time resolution between snapshots $\sim 1/20 t_{\text{dyn}}$, which proved to be helpful when looking for structure formation at the accretion disc.

Given the nature of our work, it is of great importance to ensure that the accretion disc has enough particle resolution to observe the formation of structures such as clumps or spiral arms. As shown by Bate & Burkert (1997), in order to properly represent fragmentation in an SPH simulation, the Jeans mass in the disc must be greater than the minimum resolvable mass $M_{\text{min}}$ given by

$$M_{\text{min}} = 2 M_{\text{tot}} (N_{\text{neigh}}/N_{\text{tot}}),$$

where $M_{\text{tot}}, N_{\text{tot}}$ and $N_{\text{neigh}}$ are the total gas mass, the total number of particles and the SPH number of neighbours, respectively. For all of our simulations, we found that, at all times, Jean’s mass $M_1$ at radii $R < 0.9 R_p$ was at least one order of magnitude above the resolution mass of the simulation. Meanwhile, for $R > 0.9 R_p$, this condition was not always satisfied. It is therefore safe to say that for $R \lesssim 0.9 R_p$, where more than 90 per cent of the disc’s mass is contained, the formation of structure is properly resolved.

### 2.2 BH physics

We consider the formation of a Schwarzschild BH at the centre of the distribution from the innermost 2 M$_\odot$ of the initial 4.5 M$_\odot$ polytropic star. In general relativity (GR), the Schwarzschild solution for a non-rotating, neutral spherical BH of mass $M_{\text{BH}}$ implies the definition of the gravitational radius $r_g = 2 G M_{\text{BH}}/c^2$. This radius represents the event horizon, and therefore there cannot be static observers within $r < r_g$. When studying the motion of a test particle in the Schwarzschild metric, and considering a circular orbit at radius $r$ around the BH, a relationship emerges between the orbital angular momentum $L$ and the radius $r$, from which the innermost stable circular orbit can be obtained as $r_{\text{acc}} = 3 r_g$. At $r > 3 r_g$ there can be stable circular orbits, and at $r < 3 r_g$ they are all unstable. Given that at distances $r < r_{\text{acc}}$ all circular orbits will fall inevitably on to the singularity, we will consider $r_{\text{acc}}$ as our accretion radius ($r_{\text{acc}} = 3 r_g$), and all material at $r < r_{\text{acc}}$ will be considered to enter the event horizon (at $r_g$), and all of its properties (such as mass, angular and linear momentum, etc.) will be transferred to the BH.

As our study is mainly focused on studying the effects of cooling and self-gravity in the formation of instabilities at the accretion disc, we will consider a pseudo-Newtonian potential to account for the most important GR effects determining the motion of matter near a non-rotating BH. This together with an adequate equation of state (EoS) and cooling mechanisms will give important information about the accretion flow without solving the problem in GR. Therefore, we will consider the BH as a 2 M$_\odot$ particle, artificially fixed at the origin (by cancelling the forces acting on it). Nevertheless, all linear and angular momentum accreted from the gas is stored, and would be taken into account for future simulations where a freely moving BH will be considered.

We considered a Paczynski–Wiita (PW) potential $\Phi_{\text{PW}}$ (Paczynski & Wiita 1980), which reproduces exactly the location of the marginally stable and the innermost stable circular orbit ($r_{\text{mb}} = 2 r_g$ and $r_{\text{acc}} = 3 r_g$, respectively) for a Schwarzschild BH.

$$\Phi_{\text{PW}} = -\frac{G M_{\text{BH}}}{r - r_g}. \quad (3)$$

This pseudo-Newtonian potential reproduces quite accurately the form of the Keplerian angular momentum distribution $L(r) \equiv (r^2 d\Phi/dr)^{1/2}$ obtained for a test particle orbiting a Schwarzschild BH (see Fig. 4). The deviation from the Schwarzschild distribution translates into a slightly different accretion rate from that expected for GR, given that it would have a direct effect on the angular momentum transported, which in turn would affect how material is transported at the disc. Nevertheless, considering a PW potential results in a more realistic agreement with GR than considering a Newtonian potential.

When accretion of a gas particle occurs, its mass is removed from the gas and transferred to the BH. This changes the BH properties after an integration time $d\tau$ on which the BH mass will have accreted a mass $M_{\text{acc}}$ at a rate $M_{\text{acc}}/d\tau$. This modifies $r_g$ and therefore the accretion radius $r_{\text{acc}}$ and the PW potential at $r$. We carried out a series of tests of simple models of accretion, with and without rotation, and with and without hydrodynamical effects playing a role (by effectively reducing drastically the internal energy in the gas) to ensure that the modifications in the GADGET code accurately
conserved mass, and linear and angular momentum during the accretion process.

2.3 Thermodynamics and cooling schemes

The code GADGET-2 (Springel 2005) includes an ideal gas EoS, in terms of an entropic function $A(s) = P/\rho s^\gamma$, where $\gamma = 5/3$ is the adiabatic index. Therefore, if we study the collapse of our 2.5 $M_\odot$ polytropic envelopes without the inclusion of any cooling mechanism on the code, the collapse would be an adiabatic one. The gas would only lose energy (internal, potential and kinetic) when it gets accreted by the BH. Although this scenario is not the one we expect in a collapsar, given that there would be an important neutrino cooling at some regions of the disc (where $T$ and $\rho$ are high enough to achieve pair creation and annihilation), it is important to study it to determine the importance of cooling mechanisms in the accretion flow and the overall morphology of the accretion disc formed around the BH.

The adiabatic collapse will give us information about the collapse and formation of instabilities in an envelope whose cooling mechanisms are highly inefficient. On the other hand, an isothermal collapse accounts for an undetermined cooling process which ‘radiates’ energy in order to maintain a constant temperature throughout the envelope as it collapses and orbits the BH. In the former scenario, the energy losses are immediate, and the envelope would have already lost a great amount of energy prior to disc formation. Therefore, the isothermal scheme accounts as a collapse with a highly efficient cooling mechanism. This isothermal collapse is obtained by setting $\gamma \simeq 1$ in the EoS.

In order to explore the implications of different efficiencies in the cooling mechanism, we explored the transition from an adiabatic collapse ($\gamma = 5/3$) without cooling to an isothermal one ($\gamma \simeq 1$), in a 2.5 $M_\odot$ polytropic envelope with a fixed circularization radius $r_c = 7.5 r_{\text{acc}}$. For $\gamma = 5/3$, we adopted a simplified cooling prescription based on a fixed cooling time $t_{\text{cool}}$ dependent on the dynamical time-scale of the accretion disc (orbital period), $t_{\text{disc}}$, formed around the BH:

$$t_{\text{cool}} = \beta t_{\text{disc}}, \quad \frac{dt}{dr} = \frac{u_t}{t_{\text{cool}}}.$$  (4)

This cooling prescription is determined by the internal energy $u_t$ and the efficiency parameter $\beta$, which determines how many times

the gas must orbit the BH before it gets significantly cooled. If $\beta > 1$ then the gas goes around the BH many times before losing a significant fraction of its internal energy and the infall will resemble an adiabatic one. Meanwhile, if $\beta \ll 1$, then the envelope cools down before forming the disc and the cold gas would fall freely (except for rotation) on to the BH. Without compression or expansion of the envelope, the only change in the internal energy would be given by equation (4) and the cooling would be exponential.

The dynamical time-scale for the disc can be estimated from the initial angular velocity $\Omega_0 = 8.33 \times 10^{-3}$ s$^{-1}$ obtained for a circularization radius $r_c = 7.5 r_{\text{acc}}$, such that $t_{\text{disc}} = 1/\Omega_0 = 0.12$ s. After disc formation, the accretion disc would have a nearly Keplerian angular momentum distribution. Therefore, $\Omega_0$ will not represent anymore the angular velocity of the whole disc, and this $t_{\text{disc}}$ will not correspond to the local dynamical time at every radius. Nevertheless, it will still give us an estimate of the time it takes the innermost material to orbit around the BH.

2.3.1 Neutrino cooling time

In order to better understand and characterize the formation and evolution of instabilities within the disc with the presence of an effective cooling mechanism, such as neutrino emission, we explored the case where $t_{\text{cool}}$ is close to the physical neutrino cooling time-scale, $t_\nu$. A first approximation to $t_\nu$ can be obtained from the work by Narayan, Piran & Kumar (2001), where they study different accretion flow scenarios on to a compact object (BH) in the context of GRB production. For a given temperature, $T$, and density, $\rho$, the cooling rate per unit volume due to neutrinos is given by (Narayan et al. 2001)

$$q_{\nu} \simeq 5 \times 10^{33} T_{11}^9 + 9.0 \times 10^{32} \rho T_{11}^3 \text{erg cm}^{-3} \text{s}^{-1},$$

(5)

where the first term on the right-hand side comes from pair annihilation and the second term from pair capture on to free nucleons (estimated for a fully photodissociated gas where the mass fraction of free nucleons is $X_{\text{nuc}} = 1$). Both terms depend sensitively on temperature, $T_{11} = T/10^{11}$ K, so high temperatures, $T \gtrsim 10^{10}$ K, are required for this to become relevant.

We can estimate the temperature that the infalling gas would acquire upon arrival to the disc by calculating its free-fall velocity $v_{\text{ff}}$ at the centrifugal barrier ($r = r_c$), where material would be shocked and heated. Assuming that at the shock ($r = r_c$) all kinetic energy is transformed into thermal energy, we can estimate an upper limit $T_{\text{up}}$ to the temperature $T$ the infalling gas would acquire as $T_{\text{up}} = m v_{\text{ff}}^2/3k$. To estimate $v_{\text{ff}}$, we will assume the hydrogen gas has negligible pressure support and no rotation. This results in an overestimation of $v_{\text{ff}}$ since the velocity will not be completely radial when rotation is included but is a fair approximation since the rotation rate is actually quite low. So, we may plot the temperature $T_{\text{up}}$ acquired by the infalling gas at $r_c$ as a function of its initial radius $r_i$ (Fig. 5). For the circularization radius used in our simulations, $r_c = 7.5 r_{\text{acc}}$, $T_{\text{up}}$ falls in the range $10^{10} \lesssim T_{\text{up}} \lesssim 10^{11}$ K. Therefore, assuming that the infalling gas will acquire temperatures in the range $10^{10} \lesssim T \lesssim 10^{11}$ K at the innermost part of the disc is seen to be a good approximation.

On the other hand, the mean density of the pre-collapse polytropic envelope, $\bar{\rho}$, can be obtained from the data in Table 1, as $\bar{\rho} = 1.7 \times 10^8$ g cm$^{-3}$. This implies that after collapsing on to the BH, with a change in spatial scale of a factor of a few to 10 at the least, material will be further compressed in some regions and can easily reach densities from 10 to 1000 times larger, depending on the local pressure and the cooling mechanism implemented. Thus, densities
comparable to those considered by Narayan et al. (2001), \(10^{10} \lesssim \rho \lesssim 10^{12} \, \text{g cm}^{-3}\) will also occur.

The internal energy of the gas will have three main contributions. First, from an ideal gas composed of \(\alpha\) particles and free nucleons. Secondly, a contribution from radiation, which would be effectively trapped due to the high densities reached at neutrino cooling regions (the photon mean free path is many orders of magnitude smaller than the typical length-scale of the problem). Finally, a relativistic electron–positron component, which can have arbitrary degeneracy. To obtain an approximation of the expected internal energy under such conditions, we will consider that the relativistic electron–positron pairs are hot and fully non-degenerate. Thus, \(u_{\text{gas}} = \frac{3}{2}kT\rho/(\mu m_p)\), and photons and pairs will contribute together a radiation energy density \(u_{\text{rad}} + u_{\text{pair}} = (11/4)aT^4\). Therefore, the full internal energy of the gas (per unit volume) can be estimated by

\[
u = \frac{3}{2}kT\rho + \frac{11}{4}aT^4.
\]

From equations (5) and (6) and considering densities in the range \(\rho \simeq 10^{10} - 10^{12} \, \text{g cm}^{-3}\) and temperatures of the order of \(T \simeq 10^8 - 10^{11} \, \text{K}\), we can estimate a neutrino cooling time-scale \(t_\nu = u/q_\nu\) as shown in Fig. 6. We limited the plot to \(10^{-4} \leq t_\nu \leq 10\) s, which contains the cooling times explored here. \(t_\nu\) depends strongly on the temperature of the system when reaching \(T \sim 10^{11} \, \text{K}\), when neutrinos from pair annihilation dominate \(q_\nu\). Table 2 shows the efficiency parameter \(\beta\) used in our simulations, and the corresponding cooling time, \(t_{cool}\). Our models will be labelled after their \(t_{cool}/t_{dyn}\) ratio, as shown in Table 2.

### 3 RESULTS AND DISCUSSION

We will first discuss the general features of the accretion process, such as the BH mass accretion rate \(\dot{M} = dM/dt\), the evolution of the BH mass \(M_{\text{BH}}\) and the energy-loss rate \(L_e = d\dot{E}/dt\), defined by the cooling times from Table 2. These quantities will give us information on the general changes in the behaviour of the system when implementing different cooling schemes. Then, we will study in detail some specific models, in order to look for a relation between these quantities’ behaviour and the disc’s specific properties, or the cooling time-scale itself.

#### 3.1 Accretion rates and BH mass

Given that the polytropic envelope is intrinsically located at a distance \(\sim R_s/2\) from the BH at \(t = 0\), the accretion will not begin until the innermost material reaches the centre of the distribution. The time the envelope takes to reach the BH is affected by the gas pressure which, in the case of being negligible, would translate into a free-fall time of the envelope \(t_{\text{fall}} = \tau_{\text{dyn}}/2\). In reality, gas pressure will make the envelope reach the BH at a slightly earlier time, \(t_{\text{fall}} \lesssim \tau_{\text{dyn}}/2\).

Fig. 7 shows the accreted mass as a function of time for all different models. In every case, accretion begins at a time \(0.02 \lesssim t \lesssim 0.05\) s depending on the cooling scheme. The more efficient the cooling, the more \(t_{\text{fall}}\) and the accreted mass increase. At the bottom panel of Fig. 7 we can see the slowly cooled envelopes (\(\beta = 1.34\) and \(\beta = 2.68\) and \(\beta = 1.34\)) which resemble the adiabatic (red line) the most. All these models show a smooth increase in the accreted mass for at least the first \(\sim 0.9\) s. Models \(\beta = 2.68\) and \(\beta = 1.34\) have an abrupt increase in the accreted mass after \(0.9\) s and \(1.9\) s, respectively. In the top panel, we can see the cooled envelopes with higher cooling efficiency (\(\beta = 0.67\), \(\beta = 0.134\) and \(\beta = 0.067\)) which resemble the isothermal envelope (red line) the most. Efficiently cooled models show abrupt changes in the accreted mass and considerably higher accreted mass than the slowly cooled ones. Models with high cooling efficiency (as well as the isothermal one) are terminated at earlier times because of the computational demands imposed by an increasingly shorter time-step in the final stages. Special care...
was taken in order to keep the gas from cooling below $u = 0$, by imposing a threshold in the cooling subroutine so that the energy was lost only for gas with $T > 1000$ K.

Fig. 8 shows the evolution of the accretion mass rate for all models, whose first peak at $t \lesssim 0.1$ s is due to the accretion of infalling material with low angular momentum. As material reaches the BH, a shock is formed and propagates outwards. If no cooling is enabled, the evolution is adiabatic, which will slow down the infalling material and prevent some of it from being accreted. Meanwhile, with finite cooling, the shocked material is able to lose energy, and the shock is slowed down faster than in the adiabatic envelope. This translates into a higher accretion rate shown as a peak at $t \sim 0.1$ s in Fig. 8 (more distinguishable in the bottom panel).

If the shock is strong enough to slow down the infalling material, the accretion rate will be diminished considerably. This, together with the exhaustion of low angular momentum material, translates into a decrease in the accretion rate, shown at times $0.1 \lesssim t \lesssim 0.25$ s in Fig. 8. This does not seem to hold for models $\beta = 0.134$, $\beta = 0.067$ and $\beta = 0.134$ which show strong variations in $\dot{M}$ after $t \simeq 0.1$ s. Once the shock has passed through the whole envelope, material will settle around the BH and form an accretion disc, whose thickness decreases with increasing cooling efficiency, as expected. From this point on, material from the disc will be able to fall on to the BH if angular momentum transport occurs. This can be seen in the bottom panel of Fig. 8 around $t \approx 0.3$ s, when the accretion rate has decreased by nearly one order of magnitude with respect to the initial peak.

As we can see from Fig. 8, the slowly cooled models show smaller variations in the accretion rate $\dot{M}$ than the more efficiently cooled ones. Such variations must be due to a change in the way material is transported within the disc. Particularly, model $\beta = 0.134$ (blue line in the top panel of Fig. 8) shows several peaks in $\dot{M}$ at times $t \sim 0.2, 0.3$ and $0.4$ s. Some quasi-periodic pattern may be present in the accretion rate of model $\beta = 0.134$, and it will be studied carefully below.

### 3.2 Cooling efficiency and heat losses

Models implemented with our cooling prescription will be losing energy each time-step at a rate $\frac{dF}{dt} = \frac{u}{\dot{u}}/\dot{e}_{\text{cool}}$. This cooling rate depends on the internal energy $u$, the position $(R, \phi, z)$. Therefore, energy losses will not be uniform throughout time and/or space. Considering the contribution from each SPH particle, we can obtain the total energy-loss rate $L_c = \sum \left( \frac{\dot{u}}{\dot{e}_{\text{cool}}} \right)$ at a given time $t$.

Energy-loss rates $L_c$ (i.e. neutrino luminosities) for different cooling efficiencies are shown in the bottom panel of Fig. 9. The maximum $L_c$ is obtained for the model with higher cooling efficiency, reaching up to $L_c \sim 10^{54}$ erg s$^{-1} = 1000$ foe s$^{-1}$. The total energy lost $u_{\text{lost}} = \int L_c dt$ is shown in the top panel of Fig. 9, with a maximum value $u_{\text{lost}} \sim 10^{53}$ erg that is also reached by the most efficiently cooled model, in only $t \lesssim 0.7$ s.

As the cooling efficiency increases, the changes in the energy-loss rates $L_c$ become more drastic. By comparing Figs 8 and 9, we see that such abrupt changes coincide in time with intense increases in the accretion rates. Given that a rapid increase in the energy-loss rate $L_c$ is only possible if there is an increase in the internal energy ($\dot{u}/\dot{u} = u/\dot{u}_{\text{cool}}$), some mechanism must be causing the increase in the local internal energy $u$. As these intensity variations on both $\dot{M}$ and $L_c$ appear as the cooling efficiency is improved, it seems logical to assume that the formation of structure within the accretion disc is triggering these variations. Therefore, we study in detail the formation of structures, such as spiral arms, in order to look for a coincidence of structure formation events with the intense increases in both the accretion rate and the energy-loss rate.
Based on the obtained energy-loss rates $L_c$ from our different models, and the assumption that the neutrino cooling would have a cooling time-scale $t_c$, ranging within the cooling times $t_{cool}$ explored here, we can expect to obtain neutrino luminosities $L_n$ in the range $1 \lesssim L_n \lesssim 2000$ foe s$^{-1}$ (depending on the local temperature and density conditions) when a realistic neutrino prescription is used. An efficient neutrino cooling will be restricted to regions with really high temperatures ($T \gtrsim 10^{11}$ K) and densities ($\rho \gtrsim 10^{10}$ g cm$^{-3}$), and this cooling mechanism will not be efficient during the initial collapse of the envelope, when the bulk of the gas has not yet reached the BH. This may affect the energy-loss rates significantly at earlier times. Moreover, here we do not consider the possibility of energy deposition from the cooling mechanism itself, which will be important for neutrino cooling at regions of high density ($\rho \gtrsim 10^{10}$ g cm$^{-3}$) when the opacity due to neutrons and $\alpha$ particles becomes important (Shapiro & Teukolsky 1983). This may diminish the neutrino luminosity and could also prevent the material from cooling down so efficiently.

### 3.3 Characteristic time-scales in the disc

In order to look for characteristic time-scales for the gas in the accretion disc, we performed Fourier transforms of the time series for $M$, $L_c$ and the cylindrical radial component of the momentum $P_r$. Any characteristic time-scale occurring in them will appear as a peak in the Fourier transform amplitude $|F(\nu)|^2$. Figs 10 and 11 show the Fourier transforms for $L_c$ (red line), $M$ (blue line) and $P_r$ (black line) for all models (except $L_c$ for the adiabatic and isothermal models) as a function of inverse frequency $t = 1/\nu$ (in seconds).

Fig. 10 shows that there is a transition in the peaks in $|F(\nu)|^2$ for $M$ appearing at $t \approx 0.2$ and 0.05 s when the cooling efficiency $\beta$ increases. These two peaks in $M$ also appear in $L_c$ for models $\beta 1.34$ and $\beta 2.68$, but the peak at $t \approx 0.05$ s is not that clear for model $\beta 1.34$. The transform for $L_c$ also shows less intense peaks at $t \approx 0.2$ and 0.05 s that also appear in the spectrum of $L_c$. Increasing the cooling efficiency alters the position and intensity of the peak time-scales and in model $\beta 1.34$ an additional peak is present at $t \approx 0.02$ s.

Comparing Figs 11 and 10 we can see that increasing the cooling efficiency increases the number of peaks appearing at higher frequencies (smaller characteristic time-scales) in the Fourier transforms. This coincides with the increase of variations in $M$ and $L_c$ seen when increasing the cooling efficiency. We note that only the
more efficiently cooled models show significant variations on time-scales shorter than 10 ms, and as stated in the previous sections, these variations seem to be produced by the formation of instabilities in the disc.

### 3.4 Morphological features

If the formation of patterns in the disc is responsible for the rapid variations in both $M$ and $L_\phi$, we must start by looking for instabilities that may give rise to the formation of such structures. This can be done by looking for regions where local gravitational instability arises because of the high density and/or low pressure (internal energy). This may be observable in density and/or internal energy maps in the disc plane. Moreover, the appearance of any asymmetric structures in the disc could alter the angular momentum distribution and transport. Spiral arms and high-density gas clumps can alter the gravitational interaction between the disc and the BH and lead to the transport of angular momentum across the disc, independently of the presence of any viscous mechanism (such as the MRI).

In order to characterize non-axisymmetric instabilities, we performed a one-dimensional Fourier transform of the azimuthal distribution of mass $\Phi_M = \int \rho(\phi, r, z) d\phi d\ell$ as in the work of Zurek & Benz (1986), defining the amplitude of the $n$th mode by

$$C_m = \frac{1}{2\pi} \int_0^{2\pi} e^{i m \phi} \Phi_M d\phi.$$  

The power in each mode, $|C_m|^2$, will give us information about the presence of overdense structures with $2\pi/m$ azimuthal symmetry in the disc. Therefore, the relative power $|C_m|^2 = |C_0|^2$ will give us information on the intensity of $m$ spiral arms compared to the integrated disc mass $C_0$ at time $t$. By plotting the evolution in time of such relative powers $|C_m|^2$, we can study the evolution of the disc and look for the formation and disruption of spiral arms forming at the disc.

Such spiral structures should also be observable in density maps at the $z = 0$ plane or, more appropriately, in the surface density, $\Sigma(r, \phi) = \int \rho(r, \phi, z) dz$, maps as overdense regions. Nevertheless, in order to obtain quantitative information about the formation of structure in our simulations, we have found it more useful to study the evolution of the Toomre parameter in the disc (Toomre 1964), which provides insight into the conditions under which the accretion disc around the BH becomes unstable. The Toomre parameter is given by

$${Q_T} = \frac{\kappa_c}{\pi G \Sigma},$$  

where $\kappa = (\partial W/\partial r)^{1/2}$ is the epicyclic frequency of motion for material in the disc, obtained from first-order perturbations and subject to the effective potential $W(r) = \phi(r) + \partial^2/2r^2$, $\Sigma(r, \phi)$ is the surface density of the disc and $c_s$ is the local sound speed. By evaluating the Toomre parameter $Q_T$ throughout the disc, we should be able to notice the formation of any spiral structures, as a region with $Q_T$ lower than the average value, given the higher surface density caused by its collapse. Considering that we are only using $Q_T$ as a parameter to visualize collapsing regions with significantly higher density and/or lower temperature, we will consider an approximation to the epicyclic frequency which neglects the contribution of the disc to the gravitational potential. The epicyclic frequency of a gas particle orbiting a BH with a PW potential is given by

$$\kappa = \left(\frac{\partial W}{\partial r}\right)^{1/2} = \left(\frac{GM_B (r - 3r_g)}{r(r - r_g)^3}\right)^{1/2}.$$  

This approximation will translate in values for $\kappa$ that are $1.5$–$3.5$ times smaller than those obtained directly from the simulation data, because of the self-gravity of the disc, but it will be significantly simpler to evaluate them uniformly, thus. In the next section, we will study the morphology of some models individually, in order to obtain information on the importance of structure formation for the variability in the mass accretion and energy-loss rates. We will focus on the models with $t_{cool} \lesssim 0.2$ s, which show the most significant variability.

**Model $\beta = 1.34$.** As with model $\beta = 2.68$, this model, with $t_{cool} = 0.123$ 19 s, shows an abrupt change in both the accretion rate and the energy-loss rate, but at an earlier time, $t \approx 0.9$ s (Fig. 12). Therefore, it also suggests there should be a structure formation event producing such variations. In Fig. 13, we see the evolution in time of the relative power $|C_m|^2$ for the modes $m = 1, 2, 3$ and 4, and there is an intense structure formation event starting at $t \approx 0.8$ s, shown in every Fourier mode in Fig. 13. Nevertheless, the $m = 2$ mode seems to be the most intense of all, and therefore we expect to observe two spiral arms on the Toomre parameter plots at $t \approx 0.9$ s when it reaches its maximum.

*Figure 11.* Logarithm of the renormalized Fourier transform power $|F(\nu)|^2$ of the mass accretion rate $M$ (blue line), the energy-loss rate $L_\phi$ (red line) and the cylindrical radial component of momentum $P_r$ (black line) versus $1/\nu$ (time), for models $\beta = 0.067$, $\beta = 0.134$, $\beta = 0.067$ and the isothermal case. More efficient cooling seems to increase the number of intense peaks appearing also at shorter time-scales (higher frequencies).
Cooling-induced structure in collapsars

Figure 12. Mass accretion and energy-loss rates for model $\beta 1.34$ (top and bottom panels, respectively). There is an intense increase in both $L_c$ and $\dot{M}$ at $t \simeq 0.9$ s.

Figure 13. Relative power $|c_m|^2 = |C_m|^2 / |C_0|^2$ ($m = 1, 2, 3, 4$) for the azimuthal mass distribution $\Phi_M$ Fourier transform in model $\beta 1.34$. At $t \gtrsim 0.75$ s mode $m = 2$ begins to rise and peaks at $\sim 0.9$ s.

Figs 14, 15 and 16 show the evolution in time of the Toomre parameter $Q_T$ for times $0.78 \lesssim t \lesssim 0.89$ s. The initial and final times are indicated at the top left-hand panel and the bottom right-hand panel, respectively, and the evolution of time goes from the top to bottom and from the left-hand to right-hand side. The last panels show the formation of two spiral arms at $t \simeq 0.89$ s.

Figure 14. Evolution of the Toomre parameter $Q_T$ for model $\beta 1.34$ at times $0.78 \lesssim t \lesssim 0.89$ s. The initial and final times are indicated at the top left-hand panel and the bottom right-hand panel, respectively, and the evolution of time goes from the top to bottom and from the left-hand to right-hand side. The last panels show the formation of two spiral arms at $t \simeq 0.89$ s.

Figure 15. Evolution of the Toomre parameter $Q_T$ for model $\beta 1.34$ at times $0.91 \lesssim t \lesssim 1.03$ s. The spiral arms remain intense for $\sim 0.1$ s and break into several structures in the last panel.

Figs 14, 15 and 16 show the evolution in time of the Toomre parameter $Q_T$ for times $0.78 \lesssim t \lesssim 1.2$ s where the mode $m = 2$ reaches its maximum. Last panel from Fig. 14 shows the appearance of two spiral arms at $t \simeq 0.9$ s. These spiral arms are disrupted at $t \simeq 1$ s, but they are followed by the appearance of another two spiral arms on the following Toomre parameter maps. This is also visible on the Fourier mode $m = 2$ showing strong variations at such times. Fig. 17 shows the Toomre parameter $Q_T$ at times $1.74 \lesssim t \lesssim 1.86$ s, where an increase of both $\dot{M}$ and $L_c$ can be seen in Fig. 12. This
Figure 16. Evolution of the Toomre parameter $Q_T$ for model $\beta = 1.34$ at times $1.05 \lesssim t \lesssim 1.17$ s. The two spiral arms are now barely noticeable.

Figure 17. Evolution of the Toomre parameter $Q_T$ for model $\beta = 1.34$ at times $1.74 \lesssim t \lesssim 1.86$ s. There are several spiral structures at the disc which are shown as intense variations in modes $m = 2, 3$.

increase is also shown in the Fourier modes in Fig. 13 as strong variations in modes $m = 2, 3$.

There seems to be a relationship between the time of the first structure formation event and the cooling time, but the formation of structures should also depend on the amount of mass contained in the disc. As increasing the cooling efficiency allows more material to be accreted during the initial collapse, the disc’s mass can be considerably reduced and therefore the first structure formation event could be delayed due to the fact that the disc does not have enough material to become unstable until enough cooling has taken place.

As in Korobkin et al. (2011), we calculated the phase and velocity of the Fourier mode $m = 2$ according to Woodward, Tohline & Hachisu (1994) (based on Williams & Tohline 1987) at a time $t \approx 0.09$ s, when two spiral arms are clearly visible in density maps. Then, in order to obtain the corotation radius we calculated the time derivative of the mode’s phase $\frac{d\phi}{dt}$ as a function of the radius $r$. Thus, by comparing $\frac{d\phi}{dt}(r)$ and the disc’s integrated angular velocity distribution $\Omega(r)$, it was straightforward to obtain the corotation radius of both components. We found that at times where the spiral structure is clearly visible in density maps, the phase, and intensity of such modes, coincides very well with the overdense regions (see Fig. 18). For radii where there is no visible spiral structure in the density map, the phase of the mode becomes quite noisy, and there are large phase jumps until the Fourier approximation finds a structure identifiable with a $2\pi/m$ mode.

Model $\beta = 0.67$. As can be seen in Fig. 19, this model, with $t_{\text{cool}} = 0.0616$ s shows at least two events where both $\dot{M}$ and $L_c$ have intense and rapid increases. Comparing with Fig. 20 where we plot the evolution in time of the relative power $|c_m|^2$ for the modes $m = 1, 2, 3$ and 4, we see that the first event can be related to an increase in all Fourier modes beginning at $t \approx 0.45$ s. At that time, all Fourier modes seem to have comparable power, but modes $m = 1$ and $2$, peaking at $t \approx 0.5$ s, reach the highest values and have the longest duration. We therefore expect to see a prominent presence of one or two spiral arms that will remain strong for a time $\lesssim 0.05$ s. Comparing models $\beta = 1.34$ and $\beta = 0.67$ we see that changing the cooling efficiency alters the width of the peaks in the Fourier power, which seems to be smaller (i.e. more sharply defined in time) with increasing cooling efficiency.

Figs 21, 22 and 23 show the evolution in time of the Toomre parameter $Q_T$ in the range $0.45 \lesssim t \lesssim 0.98$ s. We see in Fig. 21 that at about $t = 0.5$ s one and two spiral arms are formed and disrupted in less than 0.1 s. Remarkably, these spiral structures then break up into small and dense gas clumps, noticeable in Fig. 23. These then interact with the rest of the spiral pattern, disrupting it and further breaking it up.
Cooling-induced structure in collapsars

Figure 19. Mass accretion and energy-loss rates for model $\beta 0.67$ (top and bottom panels, respectively). There are intense variations in both $L_c$ and $\dot{M}$ at times $\approx 0.5$ and $1.6$ s.

Figure 20. Relative power $|c_m|^2 = |C_m|^2/|C_0|^2$ ($m = 1, 2, 3, 4$) for the azimuthal mass distribution $\Phi_M$ Fourier transform of model $\beta 0.67$. All modes show an important increase at $t \approx 0.5$ s. Modes $m = 2, 1$ reach the highest values at times $0.55 \lesssim t \lesssim 0.7$ s.

As can be seen in Fig. 19, there is another important variation in $\dot{M}$ and $L_c$ at times $1.6 \lesssim t \lesssim 1.8$ s, when all modes reach at some point values $|c_m|^2 \gtrsim 10^{-3}$. In this event, all peaks are extremely narrow, because the pattern is rapidly broken up, possibly due to presence of gas clumps formed at earlier times. Fig. 24 shows the evolution of $Q_T$ for $1.58 \lesssim t \lesssim 1.70$ s, also showing clumps and spiral patterns in the disc.

Model $\beta 0.134$. The cooling time used in this model was so short that the envelope was significantly cooled before $t \approx t_{\text{dyn}}$. It is difficult
to associate the intense and rapid increase shown in $M$ and $L_c$ at $t \simeq 0.28$ s in Fig. 25 with the spiral structure formation event seen in Fig. 26 given that all modes oscillate between $10^{-3} \lesssim |c_m|^2 \lesssim 10^{-2}$ right before $t \simeq 0.3$ s. The same happens with the increase shown in Fig. 25 at $t \simeq 0.35$ s, and which lasts for $\sim 0.1$ s. Nevertheless, the increase shown on Fig. 25 at $t \gtrsim 0.5$ s can be associated with an intense increase on all Fourier modes at the same time from Fig. 26.
Cooling-induced structure in collapsars

Figure 27. Evolution of the Toomre parameter $Q_T$ for model $\beta 0.134$ at times $0.13 \lesssim t \lesssim 0.25$ s. The spiral structures formed at very early times are quickly disrupted into clumps.

At $t \approx 0.525$ s all Fourier modes reach a maximum value $\gtrsim 10^{-1}$, which is not attained by less efficiently cooled models. This means that the mass ratio between the spiral structures and the disc’s mass is greater than in previous models, which can be due to the fact that the BH has accreted a significantly greater amount of mass from the disc (more than twice the mass accreted with respect to less efficiently cooled models).

By looking at the Fourier modes amplitude $|c_m|^2$ in Fig. 26, we can see that modes $m = 1, 2$ are quite intense ($|c_m|^2 > 10^{-2}$) before $t \approx 0.2$ s when the accretion disc has just been formed. This suggests that we should see spiral structure at times as early as 0.15 s. This can be seen in Fig. 27 where the Toomre parameter $Q_T$ evolution is plotted for times $0.13 \lesssim t \lesssim 0.25$ s, where we can observe the presence of spiral arms that form and rapidly dissipate. Before $t \approx 0.2$ s there is an increase in all Fourier modes which seems to be related to the intense variation seen in Fig. 25 in both $M$ and $L_c$ at those times.

Figs 28 and 29 show the evolution of the Toomre parameter $Q_T$ at times $0.27 \lesssim t \lesssim 0.38$ s and $0.50 \lesssim t \lesssim 0.61$ s, respectively. As can be seen from both figures, $Q_T$ reaches higher values within the disc, probably because the surface density is considerably lower after the rapid BH mass accretion. Nevertheless, all regions ranging from green to blue have $Q_T \lesssim 1$ and it is clear that there are intense spiral structures and clumps forming in the disc. These clumps can be observed as early as $t \approx 0.2$ s. In order to better appreciate the presence of instabilities in the disc (regions with $Q_T \lesssim 1$), we fixed a lower limit on the Toomre parameter at 0.02 so that all regions with $Q_T \lesssim 1$ will have a colour between blue and green ($Q_T$ reaches values as low as $\sim 0.002$).

This is the model with one of the highest cooling efficiencies that we have applied to our simplified system, and its results should be taken with care when relating them to an actual collapsar model due to the fact that the envelope was significantly cooled down before reaching the BH. Nevertheless, the intense structure formation observed is something we would expect to see in the innermost part of the disc of a collapsar, given that the neutrino cooling efficiency is a very steep function of temperature.
4 SUMMARY AND CONCLUSIONS

We have presented a thorough three-dimensional numerical study of the accretion of infalling envelopes on to BHs, using a simplified EoS for the gas, considering that the ideal gas contribution from free nuclei dominates over radiation and a hot $e^\pm$ gas. We have further included a simplified prescription for cooling based on realistic emission rates expected at the given densities and temperatures within a collapsar, with which we can study scenarios ranging from adiabatic to isothermal, with the aim of identifying and characterizing the morphology and the variations in structure induced within the disc as a result of the energy losses to neutrinos. With these caveats, which we discuss further below, we find global features which are likely to be present in real collapsing stellar cores and are relevant for the dynamics and energy release leading to the production of cosmological GRBs from massive stars.

Our main results are as listed below:

(i) The most important parameter governing the energy release in the collapse of a stellar core in the context of a collapsar is the rotation rate. If it is too low, the gas will flow essentially in a radial fashion, and accrete on to the central BH, releasing very little of its energy (akin to a Bondi flow). If it is too high, the disc will form at a radius that may be too large for efficient cooling to kick in (recall the sensitive dependence on neutrino emissivity on temperature), and the accretion efficiency may be too low to provide sufficient energy. It is the combination of placing shocked matter in centrifugal support as close to the BH as possible (and hence deep in the potential well), but not allowing it to fall in, which is critical for a successful event. In two dimensions, this has been characterized before (Lee & Ramirez-Ruiz 2006; López-Cámara et al. 2009, 2010), and it is plainly important in three dimensions as well. In this sense, modelling the BH through the pseudo-potential of Paczynski & Wiita (1980) is the most important ingredient as far as gravitational effects are concerned, which is why we have adopted it in the present work. Of course, considering a rotating BH may change the quantitative results somewhat, but not the qualitative nature of this conclusion.

(ii) Increasing the cooling efficiency induces more profuse and intense structure formation which in turn produces strong variations both in the mass accretion rate and the energy losses. The duration and intensity of these variations is related to the intensity and lifetime of the collapsed structures, whether they are spiral arms or clumps. Structures with a Toomre parameter $Q \ll 1$ and high relative power $|c_{in}|^2$ contain considerable amounts of gas within compact regions, and can therefore be accreted quickly, leading to large variations in $M$ and $L_c$. The frequency of these variations (and structure formation) increases with rising cooling efficiency, as observed in the Fourier power spectra of $M$ and $L_c$.

(iii) The spiral structures that form are clearly transient in nature, forming and disappearing within a few orbital periods. Their relative power is particularly strong in the lowest azimuthal modes, $m = 1, 2$, at modest cooling, when $f_{cool} \simeq f_{disc}$ and rapidly spreads to higher modes, $m = 3, 4$ for increasing cooling efficiency, when $f_{cool} \ll f_{disc}$. The development of such instabilities is due to the inhomogeneities obtained when generating the three-dimensional SPH particle distribution from our spherically symmetric polytropic stars, done by a Monte Carlo sampling on the density profile $\rho(r)$. Therefore, other than changing the number of SPH particles, or the random number generator, we exert no control over the intensity and direction of the initial perturbation given to the system. Having said that, all models have the same initial perturbation, yet they can develop different non-axisymmetric modes (spiral structure) for different cooling time-scales. Therefore, given an initial random perturbation on the particle distribution, the main driver of structure formation in our system is the cooling efficiency, given that it determines which mode will develop to be the strongest and the time it will take to become important. We follow the evolution of the instabilities through their power in the corresponding Fourier modes both in radius and in angle, and show that they are associated with episodes of enhanced mass accretion.

(iv) The spiral structure and gas clumps in the disc not only induce intense variations (with different durations) in both $M$ and $L_c$, but are also significant enough to break the symmetry in gravitational interaction between the disc and the BH. This can be seen in Figs 10 and 11 where the Fourier transforms of the radial momentum component $P_r$ of the disc are shown. As cooling becomes more efficient, greater power is seen at higher frequencies (shorter time-scales) in $P_r$. Such characteristic frequencies are easily noticeable just by looking at the time series of the components of momentum $P_r$ and $P$, separately, where one can identify a ‘periodic’ oscillation of the accretion disc. By forcing the BH to remain fixed at the origin during the simulations, we cannot here give a full account of this effect, but clearly these oscillations could have important consequences for the overall dynamics of the flow and are further discussed below.

(v) The integrated energy losses obtained range from 1 to −2000 foe ($10^{51}$ to $10^{55}$ erg) for adiabatic to isothermal models. Now, this is only what would count as ‘neutrino’ energy release. The integrated accretion energy $E_{acc} = \int M c^2 \, dr$ ranges from $10^{54}$ to $3 \times 10^{54}$ erg. Both of these indicate that as expected, the available power to drive a GRB is present in the system. How it eventually manages to do so is a different matter, but, more to the point here, it is clear that the time variations imprinted on the outflow are dependent on the initial mechanism driving it. This is in a way akin to the argument presented in Carballido & Lee (2011), where temporal variability in local shearing box simulations for different neutrino cooling prescriptions, when integrated over the entire disc on a large scale, produces a different and characteristic power spectrum which is the underlying shape upon which further processes are superimposed, each adding its typical signature layer of behaviour.

(vi) By obtaining the cooling efficiency $L_c / L_{acc}$ as a function of the mass accretion rates, we are able to compare our results with previous work. The cooling efficiency obtained in our simulations is consistent with the ones found by Chen & Beloborodov (2007) and Di Matteo, Perna & Narayan (2002) for stationary accretion discs. As our configuration is not stationary, the accretion rate is not the only factor determining the cooling efficiency. As shown in Fig. 30, there are intense increases in $dM/dr$ shown as a loop where a maximum accretion rate and a minimum cooling efficiency are reached, followed by an increase in the energy-loss rate due to material heating from approaching the BH and a decrease in the accretion rate. All models show a loop due to the initial collapse of the envelope, and models showing structure formation events exhibit similar events further on. This plot clearly shows the strong transient behaviour at early times, followed by a near stabilization of the flow, with intermittent episodes of intense accretion when structure forms (red and blue lines). As our approach does not consider neutrino transport and scattering, not negligible within the regions of highest density ($\rho \gtrsim 10^{11}$ g cm$^{-3}$), the cooling efficiency could reach smaller values in principle. This indicates that in some cases, our assumption of a totally transparent fluid (to neutrinos) may need revising when including a more detailed EoS and neutrino emission formalism.
The formation and destruction of accretion structures, in particular arms and clumps, is not only transient, but recurrent in an orderly fashion. Inspection of Figs 8 and 9, where the mass accretion rates and energy release are plotted, shows that for greater efficiency, the interval between enhancements in $M$ and $L$ is smaller. The disc is draining of matter in the process as well, of course, and so each subsequent episode is of lesser intensity. However, the trend appears to be clear: as structure forms, the disruption of azimuthal symmetry allows for a degree of angular momentum transport through the dense spiral arms (or even clumps), leading to an accretion episode. Having suppressed this structure, a second cooling interval must elapse before new condensations form and allow the process to repeat, indicating a correlation between the time elapsed between accretion (or luminosity) spikes and the cooling time itself. For decreasing cooling times, the disc is being depleted ever faster and so the trend becomes increasingly difficult to see (note the accretion rate and luminosity are plotted in a logarithmic scale).

(viii) Given our cooling implementation, equation (4), the internal energy per unit mass of the accretion disc $u_M = U/M_{\text{disc}}$ is directly related to the energy-loss rate $L$ by

$$u_M = \frac{U}{M_{\text{disc}}} = \frac{1}{M_{\text{disc}}} \left( \sum_j u_{j,t} \right) = \frac{t_{\text{cool}}L}{M_{\text{disc}}},$$

where $u_{j,t}$ is the internal energy of the particle $j$ and $M_{\text{disc}}$ is the mass of the accretion disc at time $t$. Thus, $u_M$ contains information on both the energy-loss rate and the accretion rate (determining the disc’s remaining mass). The evolution of $u_M$ as a function of the normalized time $t/t_{\text{cool}}$ shows some intrinsic properties of this particular cooling scenario.

Fig. 31 shows the normalized internal energy per unit mass $u_M/\mu_0$ ($u_{M,0} = u_M(t = 0)$) as a function of $t/t_{\text{cool}}$ for all simulations with our cooling prescription, equation (4). All models show the initial decrease in $u_M$ due to the cooling and collapse of the envelope, followed by an intense increase from the outward shock produced by the heated material reaching its centrifugal barrier near the BH. Both of these events take place at times $t \leq t_{\text{dyn}}$, and therefore models with $t_{\text{cool}} \geq t_{\text{dyn}}$ will start accreting and will have formed an accretion disc at times $t \leq t_{\text{cool}}$. Meanwhile, models with $t_{\text{cool}} \ll t_{\text{dyn}}$ will form an accretion disc only after several cooling times have elapsed. Models displaying such scenarios are intrinsically different, and hence we do not expect to observe similar behaviour when comparing them. Models with $t_{\text{cool}} \geq t_{\text{dyn}}$ display an increase in $u_M$ at a time $t > t_{\text{cool}} > 8.2$, which corresponds to the structure formation event previously noted in models β1.34 and β0.67. The exception to this trend is model β13.4, which shows no significant increase in $u_M$ and no structure formation event at such times.

The exception can be explained by considering the minimum azimuthally averaged sound speed in the disc, $c_{s,\text{min}} = \min[c_s(R), (0.05 < R/R_s < 1)]$, from which we can estimate an upper limit for the time, $t_{\text{pert}} \equiv R_s/c_{s,\text{min}}$: it would take a pressure perturbation to transfer its information throughout a disc of size $R_s$. In particular, this perturbation could be induced precisely by cooling. Thus, if the cooling time-scale is smaller than this perturbation time-scale, there is a region within the disc where any significant drop in pressure (brought about by a reduction in the internal energy) could not be immediately compensated by hydrodynamical processes and it could experience a collapse if the pressure drop is strong enough. On the other hand, if the cooling time-scale is significantly higher than $t_{\text{pert}}$, any drop in pressure will be quickly accounted for, and softened, by hydrodynamical processes before the gas gets cool enough to undergo collapse. This is the case in model β13.4 which, at all times, satisfies $t_{\text{cool}} > 2t_{\text{pert}}$: as seen in Fig. 32 where the evolution of $t_{\text{cool}}/t_{\text{pert}}$ versus $t/t_{\text{cool}}$ is plotted. Hence, we do not expect to see any structure formation event from this model as long as the condition $t_{\text{cool}}/t_{\text{pert}} \lesssim 1$ is not satisfied.

Clearly there is also room for improvement in what we have presented here, and we detail some of these issues below:

(ix) First, a more detailed implementation of neutrino cooling will lead to more realistic results. Particularly, this refers to the fact that we have assumed a uniform cooling efficiency throughout the flow, $\beta$. As the temperature dependence of cooling is significant, the outer regions will not emit as copiously as the inner disc, leading to different behaviours. We believe this is one of the two most important points which would require addressing. The inclusion of a more accurate EoS could also be of importance, given that different components will react differently to the cooling. In particular, the pressure changes induced by neutrino cooling could be significantly different from what we observed.
(x) We have alluded to the second improvement already, namely that the gravitational interaction between the disc and the BH does not always allow one to assume that the later is always lying at the origin. Rather, it will oscillate with the disc, as in a binary. This has two important consequences: first, the accretion rate will be modified somewhat, as clumps and spiral arms will not be disrupted/accreted in the same way, and secondly, the site of accretion, and potential jet driving through the envelope, will be in continuous motion within the disc and stellar envelope. A quantitative analysis of this in effect is clearly necessary, and it may have far-reaching consequences, as the common assumption has been that the energy deposition driving a relativistic outflow leading to breakout sits motionless at the centre of the star.

(xi) Although the PW pseudo-potential, as mentioned above, captures the most essential features of GR for accretion purposes, a full relativistic treatment of the problem is desirable. In particular, not only the effects of a spinning (Kerr) BH, but also of the field produced by the flow itself, which, as we have seen, can in some cases produce a significant pull on the central object.

(xii) By considering a constant cooling time we neglected some important effects of neutrino transport and energy deposition. Our approach did not consider that, at high-density regions ($\rho \gtrsim 10^{11} \text{g cm}^{-3}$), the emitted neutrinos could be able to deposit a significant amount of energy in the gas due to inelastic scattering off free nucleons and $\alpha$ particles, and because of neutrino opacity effects even in the elastic scattering regime, some of the structure formation we see may be inhibited, or limited, preventing the formation of the densest features. Thus, our energy-loss rates should be treated as a maximum limit for the expected neutrino luminosities.

(xiii) Finally, another energy deposition mechanism we are not including in the code is viscous heating. By applying an $\alpha$ disc prescription to our data, we were able to quantify the rate of viscous heating expected against the rate of cooling from our prescription. We found that at all times the viscous heating is between one and three orders of magnitude smaller than the cooling, even when the spiral arms are formed. Nevertheless, it could be worth to add an $\alpha$-like viscosity term to the SPH code, which should translate not only in an energy deposition term, but also in a slightly different dynamical evolution of the disc and spiral structures.

The rich behaviour seen here, and characterized for the first time in three dimensions, clearly shows that using only two-dimensional studies of the collapse of stellar cores is insufficient to explain all of the behaviour and variability, even qualitatively, that is likely to occur in such systems. As we already know that only a small fraction of collapsing massive stars will produce a GRB, it is relevant in the sense of identifying precisely which conditions will lead to one in the evolution of the star. Preliminary work on the first two topics, in particular the disc–BH interaction, leads us to believe that these may be significant for the behaviour and evolution of collapsar discs, and they will be the subject of future work.

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