Walking from 750 GeV to 950 GeV in the Technipion Zoo

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If the 750 GeV diphoton excess is identified with the color-singlet isosinglet-technipion, $P^0 (750)$, in the one-family walking technicolor, as in our previous paper, then there should exist another color-singlet technipion, isorotplet one, $P^{\pm,3}$, definitely predicted at around 950 GeV independently of the dynamical details. The $P^{\pm,3}(950)$ are produced at the LHC via vector boson and photon fusion processes, predominantly decaying to $W\gamma$, and $\gamma\gamma$, respectively. Those walking technicolor signals can be explored at the Run 2, or 3, which would further open a way to a plethora of yet other (colored) technipions.

The ATLAS and CMS groups\textsuperscript{[1,2]} have reported a diphoton excess with the global significance of about 3 standard deviations at around 750 GeV. It would provide a clue for new physics beyond the standard model.

In the previous work\textsuperscript{3} the authors gave an interpretation for the diphoton excess by identifying the 750 GeV resonance as a color-singlet isosinglet technipion, $P^0$, of the one-family model\textsuperscript{4}, which was shown\textsuperscript{5,6} to have mass of this large in the walking technicolor having the large anomalous dimension $\gamma_m = 1$. In this paper, we present another implication following the 750 GeV resonance: that is the presence of the technipion with the mass of 950 GeV, which is color-singlet isorotplet (denoted as $P^{\pm,3}$), enrolled in the technipion “zoo” with sixty entries in total.

The one-family walking technicolor is a scale-invariant (walking) version of the original one-family technicolor model\textsuperscript{4} a naive-scale up of QCD. The theory possesses eight technifermion flavors, $F = (Q^c, L)$, which consists of six techniquarks ($Q^c = (U, D)^c$) having the QCD charge ($c = r, g, b$) and two technileptons ($L = (N, E)$), singlet under the QCD. The chiral symmetry in the theory is thus enlarged from the $SU(2)_L \times SU(2)_R$ in the standard model to the $SU(8)_L \times SU(8)_R$. The technifermions develop the chiral condensate $\langle FF \rangle$ by the strong dynamics to break the chiral $SU(8)_L \times SU(8)_R$ symmetry down to the vectorial $SU(8)_V$. The sixty-three Nambu-Goldstone (NG) bosons then emerge, among which three are eaten by the $W$ and $Z$ bosons once the electroweak gauge is turned on, while other sixty become pseudo-NG bosons due to the explicit breaking effects supplied outside the walking technicolor dynamics. Thus the low-lying spectra consist of those sixty technipions, as well as the characteristic composite Higgs ( "technidilaton", a pseudo-NG boson of the scale symmetry, predicted in the walking technicolor\textsuperscript{4,8}), identified as the 125 GeV LHC Higgs. (Several discussions on the lightness of the technidilaton and the consistency of its coupling property with the LHC Higgs have been given in recent works. See Refs.\textsuperscript{9,10}.)

The technipions are classified on the basis of the standard model charges: the color-singlet technipions $P^0$ and $P^i$ (with $i = 1, 2, 3$ being the isospin charges) are constructed from technifermions as $P^0 \sim \frac{1}{\sqrt{3}}(\bar{Q}\gamma_5 Q - 3\bar{L}\gamma_5 L)$, $P^i \sim \frac{1}{\sqrt{3}}(\bar{Q}\gamma_5 \sigma^i Q - 3\bar{L}\gamma_5 \sigma^i L)$, where $\sigma^i$ stands for the Pauli matrices. As was discussed in Refs.\textsuperscript{5,6} in the context of the walking technicolor, they get the masses due to a four-fermion interaction induced by an extended technicolor which explicitly breaks the associated chiral symmetry (but keeps the standard-model symmetry),

$$
\frac{1}{\Lambda_{ETC}}(\bar{Q}QL - \bar{Q}\gamma_5 \sigma^i Q\bar{L}\gamma_5 \sigma^i L) .
$$

The masses are calculated by using the standard current algebra. Then one gets the formula\textsuperscript{5,6},

$$
m_{P^i}^2 = \frac{8}{5}m_{P^0}^2 .
$$

Remarkable to note is that this formula is fixed without any detail of the walking dynamics and modeling of the extended technicolor: the prefactor $(8/5)$ has merely come from the difference in the associated chiral charges for $P^0$ and $P^{\pm,3}$. As was shown in Ref.\textsuperscript{3}, the $P^0$ can be interpreted as the 750 GeV diphoton resonance, so we take $m_{P^0} = 750$ GeV in Eq.\textsuperscript{2} to get the $P^i$ mass:

$$
m_{P^i} = \sqrt{\frac{8}{5}}m_{P^0}\bigg|_{P^0 \equiv P^0(750)} \simeq 950 \text{ GeV} .
$$

Thus the presence of the 750 GeV resonance simultaneously predicts the 950 GeV isorotplet technipion, $P^i \equiv P^i(950)$.

Besides the color-singlet technipions, the theory predicts the color-octet and -triplet ones. The masses of the colored technipions are originated from a different source: those are generated by the QCD interactions, just like the
photon exchange contribution to the charged pion mass in QCD. The explicit breaking effect of all the technipions is actually amplified by the large anomalous dimension $\gamma_m \simeq 1$ characteristic to the walking technicolor to lift the mass up to $O(\text{TeV})$. The precise size of the mass is, however, subject to the nonperturbative calculation of the vector current correlator in the walking dynamics, in sharp contrast to the case of the color-singlet technipions, particularly the ratio $m_{\mu \nu}/m_{\rho \sigma}$ which is free from the dynamical details as mentioned above.

The couplings of $P^i(950)$ to the standard-model gauge bosons are only given by the Wess-Zumino-Witten term for the non-Abelian anomaly of the underlying walking technicolor, since the three-NG boson vertex is forbidden by the low-energy theorem of the spontaneously broken chiral symmetry in the non-anomalous part (See, e.g., Sec. 2.2. of Ref. [12]), which is in sharp contrast to the coupling of the (charged) non-Abelian NG boson-heavy Higgs boson in extended Higgs models. The Wess-Zumino-Witten construction for the chiral $SU(8)_L \times SU(8)_R$ symmetry reads

$$S_{WZW} = -\frac{N_C}{12\pi^2 F_\pi} \int_{M^4} \text{tr}[(3dVdV + dAdA) \pi],$$

which breaks the intrinsic parity $^1$ where $N_C$ denotes the number of the technicolor and $F_\pi$ is the technipion decay constant, fixed by the electroweak scale $v_{EW} = 246$ GeV as

$$F_\pi = v_{EW}/\sqrt{N_D} \bigg|_{N_D=N_F/2=4} = 123\text{ GeV},$$

for the one-family model with the eight techni-flavors, forming the four electroweak doublets ($N_D = N_F/8 = 4$). Equation (4) has been written in terms of differential form. The $P^i(950)$ are parametrized in the $\pi$ matrix, $\pi \equiv P^i X^i_\mu$, with the corresponding $SU(8)$ generator,

$$X^i_\mu = \frac{1}{4\sqrt{3}} \begin{pmatrix} \sigma^i \otimes 1 & 0 \\ 0 & -3 \cdot \sigma^i \end{pmatrix} \otimes 8 \times 8. \tag{6}$$

The standard-model gauge boson fields $(W^\pm_\mu, Z_\mu, A_\mu)$ are embedded in the chiral-external gauge fields $V_\mu$ and $A_\mu$ as follows [3]:

$$V_\mu = e Q_{em} A_\mu + \frac{e}{2sc} \left( I_3 - 2s^2 Q_{em} \right) Z_\mu 
+ \frac{e}{2\sqrt{2} s} \left( W^\pm_\mu I^+ + W^-_\mu I^- \right),$$

$$A_\mu = -\frac{e}{2sc} I_3 Z_\mu - \frac{e}{2\sqrt{2} s} \left( W^+_\mu I^+ + W^-_\mu I^- \right), \tag{7}$$

where $e$ is the electromagnetic coupling, $s$ ($c^2 = 1 - s^2$) denotes the weak mixing angle, and

$$Q_{em} = I_3 + Y,$$

$$I_3 = \frac{1}{2} \begin{pmatrix} \sigma^3 \otimes 1 & 0 \\ 0 & \sigma^3 \end{pmatrix},$$

$$Y = \frac{1}{6} \begin{pmatrix} 1_{2x2} \otimes 1 & 0 \\ 0 & -3 \cdot 1_{2x2} \end{pmatrix},$$

$$I^\pm = \frac{1}{2} \begin{pmatrix} \sigma^\pm \otimes 1 & 0 \\ 0 & \sigma^\pm \end{pmatrix}, \tag{8}$$

with $\sigma^\pm = (\sigma^1 \pm i\sigma^2)$. In evaluating Eq. (10) we have omitted the gluon field to which the $P^i(950)$ does not couple because of the isospin symmetry. From these, we extract the $P^i(950)$ couplings to find

$$\mathcal{L}_{P^3AA} = -\frac{e^2 N_C}{4\sqrt{3} \pi^2 F_\pi} P^3 dAdA,$$

$$\mathcal{L}_{P^3ZZ} = \frac{e^2 (c^2 - s^2) N_C}{8\sqrt{3} \pi^2 c^2 F_\pi} P^3 dZdZ,$$

$$\mathcal{L}_{P^3AZ} = -\frac{e^2 (1 - 4s^2) N_C}{8\sqrt{3} \pi^2 c F_\pi} P^3 dAdZ + \text{h.c.},$$

$$\mathcal{L}_{P^3AW} = -\frac{e^2 N_C}{8\sqrt{3} \pi^2 c^2 F_\pi} P^3 dAdW^+ + \text{h.c.},$$

$$\mathcal{L}_{P^3ZW} = \frac{e^2 N_C}{8\sqrt{3} \pi^2 c^2 F_\pi} P^3 dZdW^- + \text{h.c.}, \tag{9}$$

where $P^\pm \equiv (P^1 + i P^2)/\sqrt{2}$ and $dV_1 dV_2 \equiv e^{\mu\nu\sigma\rho} \partial_\mu V_1 \partial_\nu \partial_\rho V_2 \sigma$ for arbitrary vector fields $V_1, V_2$. Note the absence of the $P^3$ coupling to $WW$ due to the one-family $SU(8)$ symmetry, in a way that $\text{tr}[X^3_\mu \{ I^+, I^- \}] = 0$, where the contribution from techniquarks are canceled by that from technileptons, as in the case of the $P^3(750)$ [3, 4, 5]. Thus, no coupling of $P^3(950)$ to $WW$ as well as $P^0(750)$ is the characteristic feature: if the 750 GeV resonance is in the future confirmed not only in the diphoton channel, but also in the WW channel, the present one-family model will definitely be ruled out.

From Eq. (10) we thus compute the partial decay rates of the $P^i(950)$ to get

$$\Gamma(P^3 \to \gamma\gamma) = \frac{\alpha_{em} N_C}{\sqrt{3} \pi F_\pi} \frac{m_{P^3(950)}}{16\pi},$$

$$\Gamma(P^3 \to ZZ) = \frac{\alpha_{em} (c^2 - s^2) N_C}{2\sqrt{3} \pi c^2 F_\pi} \frac{m_{P^3(950)}}{16\pi} \left( 1 - \frac{4m_{Z}^2}{m_{P^3(950)}} \right)^{3/2},$$

$$\Gamma(P^3 \to Z\gamma) = \frac{\alpha_{em} (1 - 4s^2) N_C}{2\sqrt{3} \pi sc F_\pi} \frac{m_{P^3(950)}}{32\pi} \left( 1 - \frac{m_{Z}^2}{m_{P^3(950)}} \right)^{3}. \tag{10}$$

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#1 The intrinsic parity is defined to be even when a particle has the parity $(-1)^{m_{\text{part}}}$, otherwise odd
and

$$\Gamma(P^{\pm} \rightarrow W^{\pm} \gamma) = \left( \frac{\alpha_{em} N_C}{2\sqrt{3\pi} \cos F_\pi} \right)^2 \frac{m^3_{P(950)}}{32\pi} \left( 1 - \frac{m^2_{W}}{m^2_{P(950)}} \right)^3,$$

$$\Gamma(P^{\pm} \rightarrow W^{\pm} Z) = \left( \frac{\alpha_{em} N_C}{2\sqrt{3\pi} \cos F_\pi} \right)^2 \frac{m^3_{P(950)}}{32\pi} \left( 1 - \frac{(m_W + m_Z)}{m_{P(950)}} \right)^{2} \left( 1 - \frac{m_W - m_Z}{m_{P(950)}} \right)^{2} \frac{1}{2} \left( 1 - \frac{m_W - m_Z}{m_{P(950)}} \right)^{2},$$

where $\alpha_{em} \equiv e^2/(4\pi)$. Note that the branching ratios are estimated independently of $N_C$ and $F_\pi$ to be

$$\text{Br}[P^3 \rightarrow \gamma\gamma] \simeq 89.5\%,$$

$$\text{Br}[P^3 \rightarrow ZZ] \simeq 10.2\%,$$

$$\text{Br}[P^3 \rightarrow Z\gamma] \simeq 0.30\%,$$  \hspace{1cm} (12)

and

$$\text{Br}[P^{\pm} \rightarrow W^{\pm}\gamma] \simeq 77\%,$$

$$\text{Br}[P^{\pm} \rightarrow W^{\pm} Z] \simeq 23\%.$$  \hspace{1cm} (13)

The total widths are estimated by using the value of $F_\pi$ in Eq.\(\text{[5]}\) and taking typical numbers for $N_C$, say, $N_C = 3,4$:

| $N_C = 3$ | $N_C = 4$ |
|------------|------------|
| $\Gamma_{tot}(950)$ [MeV] | $23$ | $42$ |
| $\Gamma_{tot}(950)$ [MeV] | $14$ | $25$ |

which shows that the $P^{\pm,3}(950)$ are quite narrow resonances.

Note that the $P^{\pm,3}(950)$ are basically NG bosons, so they do not couple to longitudinal modes of weak gauge bosons, which are essentially the NG bosons, and hence the coupling would be the forbidden three-NG-boson vertex as mentioned before, as far as the non-anomalous part with the intrinsic-parity even is concerned. The couplings to $WZ$ and $ZZ$, corresponding to the transverse modes, then arise from the loop-induced anomalous term, the Wess-Zumino-Witten term with the intrinsic-parity odd as in Eq.\(\text{[9]}\). (Note again that the $SU(8)$ symmetry forbids the coupling to $WW$.) Thus all the $P^{\pm,3}(950)$ couplings are necessarily loop-suppressed, hence the total widths are very small as in Eq.\(\text{[14]}\). Thus the $P^{\pm,3}(950)$ have small couplings to weak gauge bosons, yielding the small $P^{\pm,3}$ production cross sections to easily escape from the current LHC limits, as will be seen later.

Of interest is that the charged $P^{\pm}(950)$ mainly decay to $W^{\pm}\gamma$ rather than $W^{\pm}Z$ (See Eq.\(\text{[16]}\)). This is simply due to the supression by the weak mixing angle for the coupling to $Z$ compared to that to photon (See Eq.\(\text{[9]}\)). This feature is in sharp contrast to other model isorotplet heavy Higgses which hardly decay to $W\gamma$ as addressed above. Hence the $P^{\pm}(950) \rightarrow W^{\pm}\gamma$ channel will give the characteristic signature at the LHC, a smoking gun of the one-family walking technicolor, although the production cross section is somewhat small, as will be discussed below.

Now we discuss the $P^{\pm,3}(950)$ signatures at the LHC. First of all, we look into the neutral $P^3(950)$. Because of the large coupling to diphoton as in Eq.\(\text{[12]}\), the $P^3(950)$ can dominantly be produced by the photon photon fusion ($\gamma\gamma F$). Using the effective photon approximation \hspace{0.1cm} \cite{13} as in the literature \hspace{0.1cm} \cite{14}, we may calculate the production cross section of $P^3(950)$ at $\sqrt{s} = 13$ TeV via the elastic photon photon fusion process to get

$$\sigma^{13\text{TeV}}_{\gamma\gamma F}(pp \rightarrow P^3(950)) \simeq 0.018(0.034) \text{ fb},$$  \hspace{1cm} (15)

for $N_C = 3(4)$. Including the inelastic scattering contributions would largely enhance the cross section as discussed in several works listed in Refs.\hspace{0.1cm} \cite{15,16}. According to those literatures, the enhancement factor will be $O(20)$, or more, normalized to the elastic scattering process at the resonance mass of 750 GeV. Quoting the result in Ref.\hspace{0.1cm} \cite{16} and scaling the resonance mass ($m_R$) from 750 GeV up to 950 GeV, one finds $\sigma^{13\text{TeV}}_{\gamma\gamma F}(m_R = 950 \text{ GeV})/\sigma^{13\text{TeV}}_{\gamma\gamma F}(m_R = 750 \text{ GeV}) \sim 0.76$. Taking into account this factor together with the enhancement factor as above, we may roughly estimate the production cross section,

$$\sigma^{13\text{TeV}}_{\gamma\gamma F}(pp \rightarrow P^3(950)) \simeq 0.27(0.52) \text{ fb}.$$  \hspace{1cm} (16)

Using the numbers listed in Eq.\(\text{[12]}\) we thus estimate the $P^3(950)$ signal strengths:

$$\sigma^{13\text{TeV}}_{\gamma\gamma F}(P^3) \times \text{Br}[fb]$$

| $\gamma\gamma$ | $ZZ$ | $Z\gamma$ |
|----------------|-------|---------|
| $N_C = 3$ | $0.24$ | $0.46$ |
| $N_C = 4$ | $0.028$ | $0.052$ |
| $N_C = 5$ | $0.0091$ | $0.0015$ |

The most stringent signal is seen in the diphoton channel, which is compared with the ATLAS and CMS 13 TeV limits at around 950 GeV, $\sigma^{\text{ATLAS13}} \lesssim 1.6 \text{ fb}$ ($\mathcal{L} = 3.2 \text{ fb}^{-1}$) and $\sigma^{\text{CMS13}} \lesssim 5 \text{ fb}$ ($\mathcal{L} = 2.6 \text{ fb}^{-1}$), so it is far below the present bound, to be excluded, or detected in the future experiments with higher statistics.

We next turn to the charged $P^{\pm}(950)$ production at the LHC. Looking at Eq.\(\text{[13]}\) we find that the $P^{\pm}(950)$ couple to the diboson $WZ$, so they can be singly produced by the vector boson fusion (VBF). Applying the effective vector boson approximation \hspace{0.1cm} \cite{17} with the parton distribution function CTEQ6L1 \hspace{0.1cm} \cite{18}, we may estimate the 13 TeV production cross section of the $P^{\pm}(950)$ to get

$$\sigma^{13\text{TeV}}_{\text{VBF}}(pp \rightarrow WZ \rightarrow P^{\pm} + jj) \simeq 0.18(0.31) \text{ fb},$$  \hspace{1cm} (18)
for \( N_C = 3(4) \), where \( j \) denotes quarks and anti-quarks. Using the numbers displayed in Eq. (13) we thus calculate the signal strengths of the \( P^\pm(950) \):

\[
\begin{array}{c|cc}
\text{process} & N_C = 3 & N_C = 4 \\
\hline
W\gamma + jj & 0.14 & 0.24 \\
WZ + jj & 0.041 & 0.073 \\
\end{array}
\] (19)

As to the WZ channel, the ATLAS Collaboration has placed the 95% C.L. upper limit at 8 TeV \( (L = 20.3 \text{fb}^{-1}) \) on charged scalar resonances produced via the VBF, which is \( \sigma_{\text{VBF}}(WZ) \lesssim 70 \text{ fb} \) at around 950 GeV \( [19] \). On the other hand, the \( P^\pm(950) \) predicts \( \sigma_{\text{VBF}}(pp \to P^\pm \to WZ) \approx 0.0088(0.016) \) for \( N_C = 3(4) \), so it is far below the presently available upper bound.

As noted above, the \( W\gamma \) cross section is much larger than the WZ cross section, in contrast to other charged heavy scalars like in models with the extended Higgs sector. This \( W\gamma \) signal is the salient phenomenological feature of the \( P^\pm(950) \), to be tested in the future LHC experiments.

Actually, the \( P^{\pm,3}(950) \) can be produced also through the decay of the technirho (denoted as \( \rho_T \)), which might be responsible for the 8 TeV diboson excess at around 2 TeV \( [20] \); the \( \rho_T \) couplings to the \( P^{\pm,3} \) can be read off from the third reference of Ref. \( [20] \). As done in the 8 TeV analysis in the references, we may set the overall strength of the diboson coupling \( (g_{\rho\pi\pi}) \) to 4 so as to control the total width of the \( \rho_T \) to be less than 100 GeV, which is fitted to the ATLAS diboson excess data \( [21] \). As to the Drell-Yan coupling of the \( \rho_T \) \( (F_\rho) \), however, it is now more severely constrained by the 13 TeV diboson data, most stringently on \( WZ \to jj\nu\bar{\nu} \) \( [22] \), updated from the previous publication \( [21] \), to be \( F_\rho \lesssim 350 \text{ GeV} \). (The \( \rho_T \) diboson cross section with the Drell-Yan coupling \( F_\rho \lesssim 350 \text{ GeV} \) cannot account for the 8 TeV excess, which is due to the current tension between the 8 TeV and the 13 TeV results on the diboson data.) Taking account of these, we find that the branching ratio for \( \rho_T \to PP \) is about 3%. By scaling the result in Ref. \( [21] \), we thus estimate the \( P^{\pm,3}(950) \) pair production cross section at 13 TeV:

\[
\begin{align*}
\sigma_{\text{DY}}^{13\text{TeV}}(pp \to \rho_T^3 \to P^+P^-) & \approx 0.30 \text{ fb} , \\
\sigma_{\text{DY}}^{13\text{TeV}}(pp \to \rho_T^+ \to P^\pm P^0) & \approx 0.59 \text{ fb} , \\
\end{align*}
\] (20)

for \( F_\rho = 350 \text{ GeV} \) and \( g_{\rho\pi\pi} = 4 \). In this production process the final state topology will be like multiphoton plus jets through the dominant decay modes \( P^3 \to \gamma\gamma \) and \( P^\pm \to W\gamma \), in which two of the multiphoton are to be detected with the invariant mass around 950 GeV and all the final states can fully be reconstructed to be the 2 TeV resonance. This is an exotic topology, so would be a clean signal to be tested at the future LHC experiments.

In conclusion, the LHC 750 GeV diphoton excess implies the presence of yet another resonance at 950 GeV, that is the color-singlet isorotplet-technipion, \( P^{\pm,3} \), in the one-family model of walking technicolor. The \( P^{\pm,3} \) mass is completely fixed at 950 GeV, which is free from any detail of the walking dynamics, once the 750 GeV resonance is identified with the color-singlet isosinglet-technipion, \( P^0(750) \). The \( P^{\pm,3}(950) \) are singly produced at the LHC via vector boson and photon fusion processes, and doubly produced by the (2 TeV) technirho decay. Those technipions predominantly decay to \( W\gamma \) (for the charged \( P^\pm(950) \) ) and \( \gamma\gamma \) (for the neutral \( P^3(950) \) ). In particular, the charged \( P^\pm(950) \) signal is quite intrinsic for the \( W\gamma \) channel, which yields sizable cross section, leading to an intriguing topology such as dijet plus mono-photon (along with forward jets). This is the rare signal for other charged heavy scalars as in models with the extended Higgs sector, so it will be characteristic only for the \( P^\pm(950) \), to be accessible at the Run 2, or 3.

In addition to the color-singlet technipions, there are colored ones in the technipation “zoo” in the one-family walking technicolor. As noted in the early stage of the present paper, colored technipion masses are predicted to be around TeV, though they are subject to details of the walking dynamics. The colored technipions would also show up in the LHC experiments, through the large signals in the dijet channel, or monojet and single photon, as was analyzed in the literature \( [5, 6] \). Thus, a number of technipions are standing by behind the 750 GeV one in the one-family walking technicolor.

More precise estimation of the walking signals in the technipation “zoo” and comparison with the standard model background will be pursued in another publication.

In closing, in the present analysis we have so far been restricted to discuss the technipion couplings to the standard-model gauge bosons. Besides those, actually the technipions may be allowed to couple to the standard-model fermions, through extended technicolor interactions, though those couplings are formally generated at higher loops involving physics well outside of the walking technicolor dynamics. Among the standard model fermions, the Yukawa couplings to top quark and bottom quark pairs would be most influential to give significant corrections to the branching fraction of the technipions, as explicitly discussed in Ref. \( [6] \). The strength of such Yukawa couplings are actually highly dependent on the details of the extended-technicolor model-building, such as the variants of strong extended technicolor \( [24] \) having anomalous dimension, \( 1 < \gamma_m < 2 \), even larger than the walking technicolor. Hence we have disregarded those Yukawa couplings in the present analysis, in order to estimate effects of purely the walking technicolor dynamics as a starting point of the future analyses. The detailed study on the phenomenologically allowed size of the Yukawa couplings, and the related flavor physics predicted from the walking technicolor will be done elsewhere.
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