Reheating-temperature independence of cosmological baryon asymmetry in Affleck-Dine leptogenesis

Masaaki Fujii\textsuperscript{1}, K. Hamaguchi\textsuperscript{1}, and T. Yanagida\textsuperscript{1,2}

\textsuperscript{1} Department of Physics, University of Tokyo, Tokyo 113-0033, Japan
\textsuperscript{2} Research Center for the Early Universe, University of Tokyo, Tokyo 113-0033, Japan

Abstract

In this paper we point out that the cosmological baryon asymmetry in our universe is generated almost independently of the reheating temperature $T_R$ in Affleck-Dine leptogenesis and it is determined mainly by the mass of the lightest neutrino, $m_{\nu_1}$, in a wide range of the reheating temperature $T_R \simeq 10^5$–$10^{12}$ GeV. The present baryon asymmetry predicts the $m_{\nu_1}$ in a narrow region, $m_{\nu_1} \simeq (0.3$–$1) \times 10^{-9}$ eV. Such a small mass of the lightest neutrino leads to a high predictability on the mass parameter $m_{\nu_e\nu_e}$ contributing to the neutrinoless double beta decay. We also propose an explicit model in which such an ultralight neutrino can be naturally obtained.
1 Introduction

The origin of baryon (matter-antimatter) asymmetry in the present universe is one of the fundamental problems in particle physics as well as in cosmology. Recently, leptogenesis \[1\] becomes very attractive among various baryogenesis scenarios, since there are now convincing evidences of neutrino oscillations, especially the atmospheric neutrino oscillation reported by the Superkamiokande collaboration \[2\]. The small but nonzero masses of neutrinos suggested from neutrino-oscillation experiments strongly indicate the existence of lepton-number violation, which is a crucial ingredient of the leptogenesis. It is extremely interesting in the leptogenesis scenario that the baryon asymmetry of the present universe is closely related to observable low-energy physics, namely, neutrino masses and mixings.

Among various mechanisms \[3, 4, 5, 6\] for leptogenesis proposed so far, the leptogenesis of Affleck-Dine (AD) mechanism \[7\] is naturally expected to work once one introduces supersymmetry (SUSY) in the standard model together with the nonzero neutrino mass. Recently, a detailed analysis on this mechanism (AD leptogenesis \[3\]) has been performed \[8\], which has shown that the dynamics of the flat direction field is drastically changed by thermal effects, as pointed out in Refs. \[9, 10\]. Actually, it has been shown \[8\] that the resultant baryon asymmetry is indeed suppressed for relatively high reheating temperatures \(T_R\), and an ultralight neutrino is required in order to obtain the sufficient baryon asymmetry \(n_B/s\) in the present universe. (Here \(n_B\) and \(s\) are the baryon number and entropy density in the present universe, respectively.)

In this paper, we perform a reanalysis on the AD leptogenesis, and emphasize that the present baryon asymmetry \(n_B/s\) is determined mainly by the mass of the lightest neutrino \(m_{\nu_1}\) for \(T_R \simeq 10^5\)–\(10^{12}\) GeV, and its dependence on the reheating temperature \(T_R\) is rather mild. Here, we include an additional thermal effect observed recently in Ref. \[11\], which makes the dependence of the resultant baryon asymmetry on the reheating temperature even milder. Notice that in many baryogenesis scenarios the obtained baryon asymmetry depends crucially on the reheating temperature \(T_R\) of the inflation. Thus, it is very attractive that the baryon asymmetry in our universe is predicted almost independently of the reheating temperature \(T_R\) in the AD leptogenesis.

Furthermore, this reheating-temperature independence of the AD leptogenesis means that the mass of the lightest neutrino \(m_{\nu_1}\) can be determined from the present baryon asymmetry \(n_B/s\). Actually, we show that the observed baryon asymmetry \(n_B/s \simeq (0.4–1) \times 10^{-10}\) \[12\] predicts \(m_{\nu_1} \simeq (0.1–3) \times 10^{-9}\) eV. Thus, neutrinos can not be degenerate in mass, and the masses of the two heavier neutrinos are also determined from the
neutrino-oscillation experiments for the atmospheric and the solar neutrinos, that is, 
\[ m_{\nu_3} \simeq \sqrt{\delta m_{\text{atm}}^2} \simeq (3-8) \times 10^{-2} \text{ eV} \] and 
\[ m_{\nu_2} \simeq \sqrt{\delta m_{\text{sol}}^2} \sim 10^{-3} - 10^{-2} \text{ eV} \] 

Although it is hard to confirm the mass of such an ultralight neutrino \( \nu_1 \), it can be tested indirectly. A crucial observation here is that such a small mass of the lightest neutrino together with the masses and mixings of the neutrinos obtained from neutrino-oscillation experiments suggests a high predictability on the rate of neutrinoless double beta (0\( \nu\beta\beta \)) decay. We show that the \( \nu_e - \nu_e \) component of the neutrino mass matrix, \( m_{\nu_e\nu_e} \), contributing to the 0\( \nu\beta\beta \) decay, is determined with high accuracy, depending on the solution to the solar neutrino deficits and the \( e-3 \) component of the neutrino mixing matrix, \( U_{e3} \). For the case of large angle MSW solution, which is favored from the recent Superkamiokande experiments \[13\], a sizable \( m_{\nu_e\nu_e} \), say, \( |m_{\nu_e\nu_e}| \sim 10^{-2} - 10^{-3} \text{ eV} \) is predicted. It is very encouraging that such a large \( |m_{\nu_e\nu_e}| \) is accessible at future experiments for the 0\( \nu\beta\beta \) decay such as GENIUS \[14\]. On the other hand, we find that the obtained \( |m_{\nu_e\nu_e}| \) depends highly on \( U_{e3} \) for the case of small angle MSW and LOW solution. We also stress that this predictability on the 0\( \nu\beta\beta \) decay is a generic consequence of the mass hierarchy \( m_{\nu_1} \ll m_{\nu_2,\nu_3} \).

Finally, we propose an explicit model based on a Froggatt-Nielsen (FN) mechanism \[13\], in which such an ultralight neutrino is naturally predicted. Here, we impose a discrete \( Z_6 \) group as the FN symmetry.

\section{Affleck-Dine leptogenesis}

The Affleck-Dine (AD) flat direction scalar field for leptogenesis is \[ H_u = L_i = \frac{1}{\sqrt{2}} \phi_i , \] 
and along this direction we have effective dimension-five operators in superpotential;
\[ W = \frac{1}{2M_i} (L_i H_u)^2 = \frac{m_{\nu_i}}{2 \langle H_u \rangle^2} (L_i H_u)^2 , \]
where \( \langle H_u \rangle = \sin \beta \times 174 \text{ GeV} \) and \( \tan \beta \equiv \langle H_u \rangle / \langle H_d \rangle \). Here \( H_u \) (\( H_d \)) and \( L_i \) are Higgs field which couple to up (down) type quarks and SU(2)\(_L\)-doublet lepton fields, respectively. (They denote chiral superfields or their scalar components.) Hereafter, we take \( \sin \beta \simeq 1 \). Note that the scale \( M_i \) are related to the neutrino masses \( m_{\nu_i} \) through the seesaw formula \( m_{\nu_i} = \langle H_u \rangle^2 / M_i \) \[16\]. Here we have taken a basis where the mass matrix for neutrinos \( \nu_i \) is diagonal, and for simplicity we will suppress the family index \( i \) (= 1, 2, 3) unless we denote it explicitly.
Before discussing the detailed dynamics of the $\phi$ field, we first roughly describe the evolution of $\phi$ and note the relevant epoch for the AD leptogenesis. After the end of inflation, the inflaton $\chi$ starts a coherent oscillation. (At this stage the energy density of the universe is dominated by the oscillating inflaton $\chi$ and the Hubble parameter $H$ of the expanding universe decreases with cosmic time $t$ as $H = (2/3) t^{-1}$\textsuperscript{[12].}) After that, when the Hubble parameter $H$ becomes comparable to the decay rate of the inflaton, $\Gamma_\chi \sim T_\chi^2/M_\ast$ ($M_\ast = 2.4 \times 10^{18}$ GeV is the reduced Planck mass), the energy density of the radiation starts to dominate the universe. The evolution of the $\phi$ field is as follows. During the inflation, the $\phi$ field takes a large value determined from the effective potential discussed below. After the end of inflation, the value of $\phi$ gradually decreases as the Hubble parameter $H$ decreases and then, at some time, the $\phi$ starts its coherent oscillation. As we will see, the net lepton number is fixed when the flat direction field $\phi$ starts the coherent oscillation.

Let us discuss the dynamics of the $\phi$ field. The method of the following analysis in this section is based on Ref. \textsuperscript{[8]}. First, we show the total effective potential for the flat direction field $\phi$ relevant to the leptogenesis. In addition to the usual $F$-term potential and SUSY-breaking terms, there are additional SUSY-breaking terms caused by the nonzero energy density of inflaton, which depend on the Hubble parameter $H$, and also there are thermal potential terms depending on the temperature $T$. It turns out that the total potential is given by the following form:\textsuperscript{[3]}

$$V_{\text{total}} = \left( m_\phi^2 - H^2 + \sum_{f_k |\phi| < T} c_k f_k^2 T^2 \right) |\phi|^2 + m_{3/2}/8M \left( a_m \phi^4 + \text{H.c.} \right) + H/8M \left( a_H \phi^4 + \text{H.c.} \right) + a_d \alpha_s^2 T^4 \log \left( \frac{|\phi|^2}{T^2} \right) + \frac{1}{4M^2} |\phi|^6. \quad (3)$$

We explain each terms by turns. First of all, the $F$-term potential directly comes from the superpotential Eq. (2):

$$V_F = \frac{1}{4M^2} |\phi|^6. \quad (4)$$

Next, the SUSY-breaking potential for $\phi$ is given by

$$\delta V_0 = m_\phi^2 |\phi|^2 + m_{3/2}/8M \left( a_m \phi^4 + \text{H.c.} \right), \quad (5)$$

\textsuperscript{1}The thermal potential proportional to $T^4 \log (|\phi|^2)$ in Eq. (3), which has been recently found out in Ref. \textsuperscript{[11]}, was not considered in Ref. \textsuperscript{[8]}.\textsuperscript{3}
where $m_\phi$ and $m_{3/2}a_m$ are SUSY-breaking mass parameters. They are expected to be $m_\phi \sim m_{3/2} \sim 1$ TeV and $|a_m| \sim 1$.\(^2\)

During the inflation and during the oscillation period of inflaton $\chi$ after the end of inflation, the energy density of the universe is dominated by the inflaton $\chi$. Thus, there appears additional SUSY-breaking effects caused by the nonzero energy density of $\chi$:\(^3\)

$$\delta V_{\text{inf}} = -c_H H^2 |\phi|^2 + \frac{H}{8M} (a_H \phi^4 + H.c.),$$

where $c_H$ and $a_H$ are real and complex constants, respectively. Hereafter, we assume $c_H \simeq |a_H| \approx 1$, since we find that the conclusions in the present paper do not depend much on these parameters unless $c_H \ll -1$.\(^4\)

The rests in Eq. (3) correspond to the thermal effects which we discuss now. Although the energy density is dominated by the inflaton $\chi$ during the inflaton-oscillation period, there is a dilute plasma consisting of the decay products of the inflaton $\chi$ even in this period. The temperature of this dilute plasma is given by \(^1\)

$$T = \left( \frac{T^2_H M_* H}{2} \right)^{1/4}.$$

This dilute plasma has crucial effects on the dynamics of the $\phi$ field. First, the fields $\psi_k$ which couple to $\phi$ are produced by the inflaton decay and/or by thermal scatterings if their effective masses are less than the temperature;

$$f_k |\phi| < T,$$

and hence the flat direction field $\phi$ receives a thermal mass of order $\sim f_k T$. Here, $f_k$ denote Yukawa or gauge coupling constants of $\psi_k$ to $\phi$. The induced thermal mass term is given by \(^1\):

$$\delta V_{1}^{\text{th}} = \sum_{f_k |\phi| < T} c_k f_k^2 T^2 |\phi|^2,$$

where $c_k$ are real positive constants of order unity. (Details for $c_k$ and couplings $f_k$ relevant to the flat direction $\phi$ can be seen in Ref. \[^8\].)

Moreover, it has been recently pointed out \(^1\) that there is another thermal effect. In the $\phi/\sqrt{2} = L = H_u$ flat direction in Eq. (1), the SU(3)$_C$ gauge group is not broken, and gluons, gluinos and down-type (s)quarks remain massless. These fields generate a free

\(^2\)In this paper we assume the gravity-mediation model of SUSY-breaking.

\(^3\)See, for example, Ref. \[^{17}\].
energy which is proportional to $g_s(T)^2 T^4$ at two loop order\[4\] where $g_s$ is the coupling constant of the SU(3)$_C$ gauge field. On the other hand, the evolution of the running coupling $g_s$ is given by;

$$\frac{d}{d \log \mu} g_s(\mu) = \frac{g_s^3}{16\pi^2} \left(-3C_2 + \sum_{f_i|\phi|<\mu} T(R_i)\right), \quad (10)$$

where $C_2 = 3$ and $T(R_i) = 1/2$ for fundamental representations. Notice that the evolution changes when the scale $\mu$ passes through an effective mass of a field, $f_i|\phi|$, as shown in Fig. [1]. In our case $f_i$ denote Yukawa couplings of up-type quarks. Thus, the coupling constant of SU(3)$_C$ depends on $|\phi|$ as follows;

$$g_s(T)|_{y_u|\phi|>T} = g_s(T)|_{\phi=0} + \frac{g_s(M_G)^3}{32\pi^2} \sum_{y_u|\phi|>T} T(R_u) \log \left(\frac{y_u^2|\phi|^2}{T^2}\right), \quad (11)$$

where $y_u$ are Yukawa coupling constants of up-type quarks and $M_G$ is the ultraviolet scale where $g_s$ is fixed. Then, there is an additional potential through the modification

---

\[4\] In this flat direction, the down type (s)quarks also generate a free energy of order $O(y_b^2)$, where $y_b$ is the Yukawa coupling of the bottom quark. This gives an analogous free energy, which does not, however, give a dominant effect as long as $y_b \lesssim 1$. 

Figure 1: A schematic behavior of the SU(3)$_C$ gauge coupling $\alpha_s$. The dashed line represents the running coupling when the $\phi$ field does not have a vacuum-expectation value.
of the gauge coupling constant in Eq. (11);
\[
\delta V_{1}^{th} = a_{g}^{2} T^{4} \left( \sum_{y_{u} |\phi| > T} \frac{2}{3} T(R_{u}) \right) \log \left( \frac{|\phi|^{2}}{T^{2}} \right),
\]
where \( a_{g} \) is a constant which is a bit larger than unity \([11]\) and \( \alpha_{s} \equiv g_{s}^{2}/(4\pi) \). Hereafter, we take the factor \( \sum (2/3) T(R) \) to be unity since it does not change the result much. \[\text{5}\]

Now we obtain the total effective potential Eq. (3), i.e., \( V_{\text{total}} = V_{F} + \delta V_{0} + \delta V_{\text{inf}} + \delta V_{1}^{th} + \delta V_{2}^{th} \). The evolution of \( \phi \) is described by the equation of motion with this \( V_{\text{total}} \) as
\[
\ddot{\phi} + 3H \dot{\phi} + \frac{\partial V_{\text{total}}}{\partial \phi^{*}} = 0,
\]
where the dot denotes a derivative with time.

During the inflation, there is no plasma and hence no thermal effects. In this stage the Hubble parameter \( H \) takes a constant value which is much larger than the soft SUSY-breaking mass \( m_{\phi} \). Thus, the potential is dominated by the Hubble-induced terms \( \delta V_{\text{inf}} \) in Eq. (3) and \( |\phi|^{6} \) term in Eq. (1), and hence the flat direction \( \phi \) rolls down to the minimum of the potential,
\[
|\phi| \simeq \sqrt{MH} \quad \text{arg}(\phi) \simeq -\frac{\text{arg}(a_{H}) + (2n + 1)\pi}{4}, \quad n = 0 - 3.
\]
Note that we have assumed the Hubble-induced mass squared is negative \((c_{H} \simeq +1 > 0)\). \[\text{6}\]

After the inflation ends, the inflaton \( \chi \) starts to oscillate and its decay produces a dilute plasma. However, the potential is still dominated by Hubble-induced terms and \( |\phi|^{6} \) term at the first stage of the oscillation. Thus, the flat direction field \( \phi \) is trapped for a while in the above minimum given in Eq. (3). \[\text{4}\]

Then, as the Hubble parameter decreases, the negative Hubble-induced mass term is eventually exceeded by another term in the potential;
\[
H^{2} \lesssim m_{\phi}^{2} + \sum_{f_{k} |\phi| < T} c_{k} f_{k}^{2} T^{2} + a_{g} \alpha_{s}^{2}(T) \frac{T^{4}}{|\phi|^{2}}.
\]
As we shall see below, it is this time when the oscillation of \( \phi \) starts. Let us denote the Hubble parameter at this time by \( H_{\text{osc}} \). Using the relations \( |\phi| \simeq \sqrt{MH} \) and \( T = \)

\[\text{5 At least the top Yukawa coupling } f_{t} \text{ always satisfies } f_{t} |\phi| > T \text{ before the oscillation of } \phi. \text{ Thus, the resultant baryon asymmetry changes by only a factor of } \sqrt{3} \text{ at most. See Eq. (14).}\]
The evolution of $\phi$ after $H \simeq H_{ osc}$ depends on which term in Eq. (3) dominates the effective potential. There are basically three cases; the potential is dominated by (i) $m^2|\phi|^2$ term, (ii) $T^2|\phi|^2$ term, or (iii) $T^4 \log(|\phi|^2)$ term. First, if the potential is dominated by the $m^2|\phi|^2$ term, the equation of motion Eq. (13) is given by

$$\ddot{\phi} + 3H \dot{\phi} + m^2 \phi = 0.$$  

(18)

It is clear that the field $\phi$ oscillates around the origin ($\phi = 0$) and the amplitude of the oscillation dumps as $|\phi| \propto H \propto t^{-1}$. Second, when the potential is dominated by the thermal mass term $c_k f_k^2 T^2 |\phi|^2$, the $\phi$ field oscillates around $\phi = 0$ and the amplitude dumps as $|\phi| \propto H^{7/8} \propto t^{-7/8}$.

The third case is given when the $T^4 \log(|\phi|^2/T^2)$ term dominates the potential. If we neglect the time dependence of $T^4$, the damping rate of the oscillation amplitude due to such a flat potential, $V \sim \log(|\phi|^2)$, can be estimated by using the virial theorem, and it is given by $|\phi| \propto H^2 \propto t^{-2}$. In the actual case, however, the potential itself gradually decreases with time as $T^4 \propto t^{-1}$. We have numerically checked that the amplitude dumps as $|\phi| \propto H^{\alpha} \propto t^{-\alpha}$ with $\alpha \simeq 1.5$. Notice that, in all above cases, the dumping rate is faster than the rate before the beginning of the $\phi$'s oscillation ($|\phi| \propto H^{1/2} \propto t^{-1/2}$).

Finally, we derive the resultant lepton asymmetry generated by the AD leptogenesis mechanism. Since the $\phi$ field carries lepton charge, its number density is related to the lepton number density $n_L$ as

$$n_L = \frac{1}{2} i \left( \dot{\phi}^* \phi - \phi^* \dot{\phi} \right).$$  

(19)

From Eqs. (3), (13) and (19), the evolution of $n_L$ is given by

$$\dot{n}_L + 3H n_L = \frac{m_{3/2}}{2M} \text{Im} \left( a_m \phi^4 \right) + \frac{H}{2M} \text{Im} \left( a_H \phi^4 \right).$$  

(20)

A nontrivial motion of $\phi$ along the phase direction can generate a net lepton asymmetry. Although the $\phi$ field almost traces one of the valleys in Eq. (14), the phase of $\phi$ is slightly
kicked by the relative phase between $a_m$ and $a_H$ in the total potential in Eq. (3) during its rolling towards the origin. Thus, the first term in Eq. (20) plays a role of the source of the lepton asymmetry.\(^6\) By integrating Eq. (20), we obtain the resultant lepton asymmetry at the time $t$,

$$
\left[ R^3 n_L \right] (t) \approx \frac{m_{3/2}}{2M} \int_0^t dt' R^3 \text{Im} \left( a_m \phi^4 \right),
$$

where $R$ denotes the scale factor of the expanding universe, which scales as $R^3 \propto H^{-2} \propto t^2$ in the universe dominated by the oscillation energy of the inflaton $\chi$. We can see that the total lepton number increases with time as $R^3 n_L \propto t$ until the oscillation of $\phi$ starts ($H > H_{osc}$), since $\phi^4 \propto H^2$ and hence $R^3 \phi^4 \sim \text{const}$ in this stage. On the other hand, after the $\phi$ starts its oscillation, the production of lepton number is strongly suppressed. This is because $\text{Im} (a_m \phi^4)$ changes its sign rapidly due to the oscillation of $\phi$, and also because the amplitude of $\phi$’s oscillation damped as $R^3 \phi^4 \sim t^{-n}$ with $n > 1$. \([\text{See discussion below Eq. (18).}]\) Therefore, as mentioned in the beginning of this section, the net lepton asymmetry is fixed when the oscillation of $\phi$ starts. The generated lepton number at this epoch is given approximately by

$$
n_L = \left. \frac{m_{3/2}}{2M} \text{Im} \left( a_m \phi^4 \right) t \right|_{H=H_{osc}} = \frac{1}{3} m_{3/2} M H_{osc} \delta_{\text{eff}},
$$

where $\delta_{\text{eff}} \approx \sin(4 \arg \phi + \arg a_m)$ represents an effective $CP$ violating phase. From Eq. (22), the lepton-to-entropy ratio is estimated as\(^7\)

$$
\frac{n_L}{s} = \frac{MT_R}{12M_*^2} \left( \frac{m_{3/2}}{H_{osc}} \right) \delta_{\text{eff}},
$$

when the reheating process of inflation completes. This lepton asymmetry is partially converted\(^8\) into the baryon asymmetry through the “sphaleron” effects\([19]\), since it is produced before the electroweak phase transition at $T \simeq 10^2$ GeV. The present baryon asymmetry is given by\([20]\)

$$
\frac{n_B}{s} = \frac{8}{23} \frac{n_L}{s}.
$$

\(^6\)The contribution to the lepton asymmetry from the second term in Eq. (20) is always less than or comparable to that from the first term, since $\text{Im}(a_H \phi^4)$ is suppressed. See Eq. (14).

\(^7\)In deriving Eq. (23) we have assumed that the lepton number is fixed before the reheating process of the inflation completes, namely, $H_{osc} \gtrsim T_R^2 / M_*$, which is satisfied as long as $T_R \lesssim 10^{17}$ GeV for $m_\nu \lesssim 10^{-6}$ eV.

\(^8\)In the present analysis we neglect the relative sign between the produced lepton and baryon asymmetries.
Figure 2: The contour plot of the baryon asymmetries $n_B/s$ in the $m_{\nu_1}$-$T_R$ plane. The lines represent the contour plots for $n_B/s = 10^{-9}$, $10^{-10}$, $10^{-11}$, and $10^{-12}$ from the left to the right. The short-dashed lines represent the baryon asymmetry when one neglects the thermal effects. The long-dashed lines represent the ones including only the thermal mass $\propto T^2|\phi|^2$. The solid lines represent the baryon asymmetry including both thermal mass and $T^4\log(|\phi|^2)$ terms. The shaded region corresponds to the present baryon asymmetry, $n_B/s \simeq (0.4-1) \times 10^{-10}$. We have taken $\delta_{\text{eff}} = 1$ in this figure. (See discussion in the text.)

Thus, after all, the present baryon asymmetry is given by

$$\frac{n_B}{s} = \frac{2}{69} \frac{M_{T_R}}{M_*^2} \frac{M_{3/2}}{H_{\text{osc}}} \delta_{\text{eff}}.$$  \hspace{1cm} (25)

We see that the produced baryon asymmetry becomes larger as the scale $M$ increases, i.e., as the neutrino mass $m_\nu$ decreases. Therefore, the relevant flat direction for the AD leptogenesis corresponds to the first family field, i.e., $\phi/\sqrt{2} = L_1 = H_u$.

Fig. 8 shows the contour plot of the produced baryon asymmetry in the $m_\nu$-$T_R$ plane. (Here we have used the relation $m_\nu = \langle H_u \rangle^2 / M$.) As shown in the figure, the present baryon asymmetry $n_B/s$ is determined almost independently of the reheating
temperature for a wide range of $10^5 \lesssim T_R \lesssim 10^{12}$ GeV. In particular, for a relatively high reheating temperature $10^8 \lesssim T_R \lesssim 10^{12}$ GeV, the baryon asymmetry derived from the Eqs. (16) and (25) is given by the following simple from:

$$n_B/s \simeq 10^{-11}\delta_{\text{eff}} \times \left(\frac{m_{\nu_1}}{10^{-8}\text{eV}}\right)^{-3/2} \left(\frac{m_{3/2}|a_m|}{1\text{TeV}}\right). \tag{26}$$

The reason why it is independent of the reheating temperature $T_R$ is that the oscillation time $H_{\text{osc}}$ is determined by the thermal potential $T^4 \log(|\phi|^2)$ in the higher temperature regime, and $T_R$ dependence is canceled out in Eq. (25). Even in the lower reheating temperature region $10^5 \lesssim T_R \lesssim 10^8$ GeV, where $H_{\text{osc}}$ is determined by the thermal-mass term potential $T^2|\phi|^2$, $T_R$ dependence is still mild, i.e., $n_B/s \propto T_R^{1/3}$. The reheating-temperature independence discussed here is a very attractive and remarkable feature of the present mechanism since the produced baryon asymmetry crucially depends on $T_R$ in many other baryogenesis scenarios.

In Fig. 8, we have taken $\delta_{\text{eff}} = 1$. It is expected that $\delta_{\text{eff}} \sim \mathcal{O}(1)$, say, $\delta_{\text{eff}} \simeq 0.1–1$, unless there is an unnatural cancellation between $\text{arg}(a_m)$ and $\text{arg}(a_H)$. Thus, the present baryon asymmetry in our universe $n_B/s \simeq (0.4–1) \times 10^{-10}$ suggests an ultralight neutrino of a mass $m_{\nu_1} \simeq (0.1–3) \times 10^{-9}$ eV for $T_R \simeq 10^5–10^{12}$ GeV and $\delta_{\text{eff}} \simeq 0.1–1$. We consider the region of $10^5 \lesssim T_R \lesssim 10^{12}$ GeV throughout this paper, since it is the case for a large class of the inflation models proposed in the framework of supergravity [21, 22]. In Sec. 4 we will propose a model in which such an ultralight neutrino can be naturally obtained.

### 3 Prediction on the rate of neutrinoless double beta decay

The neutrinoless double beta ($0\nu\beta\beta$) decay, if observed, is the strongest evidence for the lepton number violation. The crucial parameter to determine the $0\nu\beta\beta$ decay rate is $|m_{\nu_e\nu_e}| \equiv |\sum_{\alpha} U_{e\alpha}^2 m_{\nu_\alpha}|$, where $U_{\alpha e}$ is the mixing matrix which diagonalize the neutrino mass matrix. If the mass of the lightest neutrino is actually so small, $m_{\nu_1} \sim 10^{-9}$ eV, as discussed in the previous sections, the contribution from $m_{\nu_1}$ to $m_{\nu_e\nu_e}$ can be completely neglected and hence the parameter $|m_{\nu_e\nu_e}|$ is written in terms of masses and mixings of two other neutrinos as

$$|m_{\nu_e\nu_e}| = |U_{e2}^2 m_{\nu_2} + U_{e3}^2 m_{\nu_3}|. \tag{27}$$

As for general studies of the $|m_{\nu_e\nu_e}|$ using the neutrino masses and mixings, see, for example, Ref. [23] and references therein.
Therefore, the $|m_{\nu_{e}}\nu_{e}|$ is determined by $U_{e3}$ and the parameters of atmospheric and solar neutrino oscillations; $\delta m_{\text{atm}}^{2} \simeq m_{\nu_{3}}^{2}$, $\delta m_{\text{sol}}^{2} \simeq m_{\nu_{2}}^{2}$ and $\tan^{2}\theta_{\text{sol}} \equiv |U_{e2}/U_{e1}|^{2}$. Namely, it is given by

$$|m_{\nu_{e}}\nu_{e}| \simeq \left|1 - |U_{e3}|^{2}\right| \sin^{2}\theta_{\text{sol}} \sqrt{\delta m_{\text{sol}}^{2}} + |U_{e3}|^{2} e^{i\alpha} \sqrt{\delta m_{\text{atm}}^{2}} \right|,$$

where $\alpha$ denotes the relative phase between the two terms.

We calculate the predicted value of $|m_{\nu_{e}}\nu_{e}|$ for the large angle MSW, the small angle MSW and the LOW solutions, taking the parameters allowed for atmospheric and solar neutrino oscillations that are shown in Fig. 3 [2] and Fig 4 [24]. In the case of large angle MSW solution, $|m_{\nu_{e}}\nu_{e}|$ is sensitive mainly to the parameter of the solar neutrino oscillation, $\sin^{2}\theta_{\text{sol}} \sqrt{\delta m_{\text{sol}}^{2}}$. The predicted value of $|m_{\nu_{e}}\nu_{e}|$ for the large angle MSW solution is shown in Fig. 5. Here, we have required $|U_{e3}| < 0.15$ from the CHOOZ experiment [25]. For a comparison, we also show the possible values of $|m_{\nu_{e}}\nu_{e}|$ when we allow $m_{\nu_{1}}$ to be relatively large as $m_{\nu_{1}} \leq (1/\sqrt{2})m_{\nu_{2}}$. We see from Fig. 5 that the $|m_{\nu_{e}}\nu_{e}|$ is predicted in a narrow range. It is very encouraging that the predicted $|m_{\nu_{e}}\nu_{e}|$ can be large enough to be accessible at future experiments such as GENIUS [14]. Furthermore, if the $U_{e3}$ becomes more constrained by future experiments [26], $|m_{\nu_{e}}\nu_{e}|$ is predicted in a much narrower range as shown in Fig. 6, where we have required $|U_{e3}| < 0.10$.

On the other hand, the $|m_{\nu_{e}}\nu_{e}|$ is sensitive to $|U_{e3}|$ in the case of the small angle MSW and the LOW solutions. The results are shown in Fig. 7 and 8. Because $|U_{e3}|$ is highly constrained by the CHOOZ experiment, the predicted value of $|m_{\nu_{e}}\nu_{e}|$ is so small. Even in these cases the contribution from $m_{\nu_{1}}$ can not enhance $|m_{\nu_{e}}\nu_{e}|$ because it is too small.

In all cases, the $|m_{\nu_{e}}\nu_{e}|$ is predicted with high accuracy, depending on the solar and atmospheric neutrino oscillation parameters and $U_{e3}$. Therefore, the presence of such an ultralight neutrino indicated from the present baryon asymmetry can be tested at near future experiments. However, notice that the results shown in this section is a generic consequence of the mass hierarchy $m_{\nu_{1}} \ll m_{\nu_{2,3}}$. Thus, we consider that the $0\nu\beta\beta$ decay provides only a consistency test for our hypothesis $m_{\nu_{1}} \sim 10^{-9}$ eV.

4 A model for the ultralight neutrino

In Sec. 2 we have shown that the baryon asymmetry in the present universe predicts the mass for the lightest neutrino in a narrow region, $m_{\nu_{1}} \simeq (0.1-3) \times 10^{-9}$ eV. Together with neutrino masses required to solve the solar and atmospheric neutrino anomalies, this suggests a very large mass hierarchy between the lightest and the heavier two neu-
trinos. In this section we show an explicit model based on a Foggatt-Nielsen (FN) mechanism [17], in which such an large mass hierarchy is naturally obtained.

We adopt a discrete $Z_6$ as the FN symmetry instead of a continuous $U(1)_{\text{FN}}$. We see that the discrete symmetry is crucial to produce the required large mass hierarchy in the neutrino sector. Before describing our model, we briefly review the FN model [15], which explains the observed hierarchies in quark and lepton mass matrices. This model is based on a $U(1)_{\text{FN}}$ symmetry that is broken by the vacuum-expectation value of $\Phi$, $\langle \Phi \rangle \neq 0$. Here $\Phi$ is a gauge singlet FN field carrying the FN charge $Q_\Phi = -1$. Then, all Yukawa couplings arise from nonrenormalizable interactions of $\Phi$ and are given by the following form;

$$W = h_{ij} \frac{\langle \Phi \rangle}{M_*} \Psi_i \Psi_j H_{u(d)} \epsilon^{Q_i + Q_j},$$

(29)

where $h_{ij}$ are $O(1)$ coupling constants, $Q_i$ are the FN charges of various chiral superfields $\Psi_i$ and $\epsilon \equiv \langle \Phi \rangle / M_*$. The observed mass hierarchies for quarks and charged leptons are well explained by taking suitable FN charges for them. For instance, we assign FN charges $(a, a, a + 1)$ for lepton doublets $L_i$, while giving charges $(0, 1, 2)$ to the right-handed charged leptons $E_i$, with $\epsilon \simeq 0.05-0.1$ [27, 28]. Here, we take $a = 0$ or 1 [27]. Charges for the quarks are determined if one assumes the SU(5) grand unified theory [27].

The mass matrix for the neutrinos in this model is determined by the FN charges for the lepton doublets $L_i$ [27]. To see this we first discuss the mass matrix for the heavy right-handed neutrinos $N_i$, which is given by;

$$M_{Rij} = g_{ij} \epsilon^{Q_i + Q_j} M_0,$$

(30)

where $M_0$ represents some right-handed neutrino mass scale and $g_{ij}$ are coupling constants of $O(1)$ like $h_{ij}$. Hereafter, we will take a basis where the mass matrix for the charged leptons is diagonal.\[10\] The charges for the lepton doublets $L_i$ and right-handed neutrinos $N_i$ are listed in Table. []. Then, the neutrino Dirac mass matrix $m_D$ and the right-handed neutrino mass matrix $M_R$ are given by the following forms;

$$m_D = \langle H_u \rangle \begin{pmatrix} e^{a+1} & 0 & 0 \\ 0 & e^a & 0 \\ 0 & 0 & e^a \end{pmatrix} \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix} \begin{pmatrix} e^d & 0 & 0 \\ 0 & e^c & 0 \\ 0 & 0 & e^b \end{pmatrix},$$

\[10\]One might wonder if the mixing matrix from the charged lepton sector would change the discussion above, since the mass matrix for the charged leptons has off-diagonal elements in the FN mechanism. However, the correction from this effect yields higher order terms in $\epsilon$, and hence we can safely neglect it.
Table 1: The FN charges of lepton doublets and right-handed neutrinos. $a = 0$ or $1$. We assume, for simplicity, $0 \leq b \leq c < d$.

\[
M_R = M_0 \begin{pmatrix}
\epsilon^d & 0 & 0 \\
0 & \epsilon^c & 0 \\
0 & 0 & \epsilon^b
\end{pmatrix}
\begin{pmatrix}
g_{11} & g_{12} & g_{13} \\
g_{12} & g_{22} & g_{23} \\
g_{13} & g_{23} & g_{33}
\end{pmatrix}
\begin{pmatrix}
\epsilon^d & 0 & 0 \\
0 & \epsilon^c & 0 \\
0 & 0 & \epsilon^b
\end{pmatrix}.
\] (31)

We obtain the neutrino mass matrix as

\[
m_\nu = m_D \frac{1}{M_R} m_D^T
\]

\[
= \frac{\epsilon^{2a} \langle H_u \rangle}{M_0} \begin{pmatrix}
\epsilon & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
\{h_{ij}\} & \{g_{ij}\} & \{h_{ij}\} \\
\{h_{ij}\} & \{g_{ij}\} & \{h_{ij}\}
\end{pmatrix}^{-1}
\begin{pmatrix}
\epsilon & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

\[
\approx \frac{\epsilon^{2a} \langle H_u \rangle}{M_0} \begin{pmatrix}
\epsilon^2 & \epsilon & \epsilon \\
\epsilon & 1 & 1 \\
\epsilon & 1 & 1
\end{pmatrix}.
\] (32)

As shown in Ref. [27], this mass matrix can naturally lead to a large $\nu_\mu - \nu_\tau$ mixing angle, which is suggested from the atmospheric neutrino oscillation [2]. It is very crucial that the FN charges of the right-handed neutrinos are completely canceled out in the neutrino mass matrix in Eq. (32) and hence the masses of the neutrinos are determined only by the charges of the lepton doublets, $(a, a, a + 1)$. This gives a mild mass hierarchy,

\[
m_{\nu_3} : m_{\nu_2} : m_{\nu_1} \simeq 1 : 1 : \epsilon^2 \simeq \mathcal{O}(1) : \mathcal{O}(1) : \mathcal{O}(10^{-2}).
\] (33)

Now let us turn to our FN model. To change the above point, we suppose that the broken FN symmetry is not a $\text{U}(1)_\text{FN}$ but a discrete symmetry $\text{Z}_n$ with $n = 2d$. Then, the mass matrix for the right-handed neutrino $M_R$ changes into the following form:

\[
M_R = M_0 \begin{pmatrix}
g_{11} & g_{12} \epsilon^{c+d} & g_{13} \epsilon^{b+d} \\
g_{12} \epsilon^{-c-d} & g_{22} \epsilon^{2c} & g_{23} \epsilon^{b+c} \\
g_{13} \epsilon^{-b-d} & g_{23} \epsilon^{b+c} & g_{33} \epsilon^{2b}
\end{pmatrix}.
\] (34)

Here we assume $0 \leq b \leq c < d$. Notice that the Majorana mass for $N_1$ is no longer suppressed by the power of $\epsilon$, which is a basic point to yield an extremely small neutrino mass $m_{\nu_1}$. However, the structure of the neutrino mass matrix looks similar to the
original one;

\[ m_\nu = \frac{\epsilon^2 a (H_u)}{M_0} \left( \begin{array}{ccc} \epsilon & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right) \left( \begin{array}{ccc} g_{11} & \epsilon^{-2d} & g_{12} \\ g_{12} & g_{22} & g_{23} \\ g_{13} & g_{23} & g_{33} \end{array} \right)^{-1} \left( \begin{array}{ccc} \epsilon & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right) \]

\[ \sim \frac{\epsilon^2 a (H_u)}{M_0} \left( \begin{array}{ccc} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{array} \right). \tag{35} \]

We see that one of the mass eigenvalue of this mass matrix strongly suppressed as \( \sim \epsilon^{2(1+d)} \). This suppression is also understood directly by taking the determinant of the above mass matrix. For the mass hierarchy required from the successful AD leptogenesis, it is suitable to take \( d = 3 \) (\( Z_6 \)).

To demonstrate our point, we randomly generate \( \mathcal{O}(1) \) couplings \( h_{ij} \) and \( g_{ij} \). Namely we calculate the mass matrix for neutrinos, taking the magnitudes of the couplings \( h_{ij} \) and \( g_{ij} \) to be in a range \( 0.5 - 1.5 \) and their phases to be \( 0 - 2\pi \). We also take \( \epsilon = 0.05 - 0.1 \) randomly. Here, we have required the parameters \( r \equiv \delta m^2_{\nu_3}/\delta m^2_{\mu} = (m^2_{\nu_3} - m^2_{\nu_2})/(m^2_{\nu_2} - m^2_{\nu_1}) \), \( \sin^2 2\theta_{\text{atm}} \equiv 4|U_{\mu 3}|^2 (1 - |U_{\mu 3}|^2) \) and \( \tan^2 \theta_{\text{sol}} \equiv |U_{e2}/U_{e1}|^2 \) to be consistent with the parameter regions shown in Figs. 8 and 4. We have here required \( |U_{e3}| < 0.15 \) to satisfy CHOOZ limit [24]. Fig. 8 shows the obtained mass of the lightest neutrino, \( m_{\nu_1} \). We can see that, an ultralight neutrino of mass \( m_{\nu_1} \simeq (0.1-3) \times 10^{-9} \) eV is naturally obtained.

5 Discussion and conclusions

In this paper we have performed a reanalysis on Affleck-Dine (AD) leptogenesis taking into account of all the relevant thermal effects. Then, we have pointed out that the baryon asymmetry is produced almost independently of the reheating temperature \( T_R \) and it is determined mainly by the mass of the lightest neutrino \( m_{\nu_1} \) in a wide range of the reheating temperature \( T_R \sim 10^5-10^{12} \) GeV. Notice that such reheating temperatures, \( T_R \sim 10^5-10^{12} \) GeV, are naturally realized in a large class of SUSY inflation models [21, 22]. This reheating-temperature independence is a very attractive feature of the AD leptogenesis.

Furthermore, we have shown that the present baryon asymmetry predicts the mass for the lightest neutrino in a very narrow region, \( m_{\nu_1} \simeq (0.3-1) \times 10^{-9} \) eV. We have also proposed an explicit model based on a Froggatt-Nielsen mechanism [15] with a discrete

\[ \text{[11] A similar calculation was done in Ref. [28], where they adopted the U(1)_{FN} model.} \]
symmetry $Z_6$, where such an ultralight neutrino indicated from the AD leptogenesis is naturally obtained.

Such a small mass of the lightest neutrino means that the mass parameter $m_{\nu_e \nu_e}$ contributing to the $0\nu\beta\beta$ decay is determined by the masses and mixings of two other neutrinos. Actually, we have shown that $|m_{\nu_e \nu_e}|$ can be predicted with high accuracy, by observable neutrino oscillation parameters such as $\delta m_{\text{atm}}^2$, $\delta m_{\text{sol}}^2$, $\sin^2 \theta_{\text{sol}}$ and $U_{e3}$. In particular, when the large angle MSW solution is the case and $\sin^2 \theta_{\text{sol}} \sqrt{\delta m_{\text{sol}}^2}$ is relatively large, the value of $|m_{\nu_e \nu_e}|$ is predicted as $|m_{\nu_e \nu_e}| \approx 10^{-3} - 10^{-2}$ eV, which may be testable at future $0\nu\beta\beta$ decay experiments such as GENIUS [14].

Acknowledgements

The work of K.H. was supported by the Japanese Society for the Promotion of Science. T.Y. acknowledges partial support from the Grant-in-Aid for Scientific Research from the Ministry of Education, Sports, and Culture of Japan, on Priority Aerea # 707: “Supersymmetry and Unified Theory of Elementary Particles”.

References

[1] M. Fukugita and T. Yanagida, Phys. Lett. B174 (1986) 45.
[2] Y. Fukuda et al. [Super-Kamiokande Collaboration],
    Phys. Lett. B433 (1998) 9 [hep-ex/9803006];
    Phys. Lett. B436 (1998) 33 [hep-ex/9805006];
    Phys. Rev. Lett. 81 (1998) 1562 [hep-ex/9807003].
See also recent datas, C. McGrew [Super-Kamiokande Collaboration], talk presented at The 2nd International Workshop on Neutrino Oscillations and their Origin ("NOON2000"), Tokyo, Japan, December 6–8, 2000.
[3] See, for a recent review,
    W. Buchmuller and M. Plumacher, Phys. Rept. 320 (1999) 329 [hep-ph/9904310];
    M. Plumacher, Nucl. Phys. B530 (1998) 207 [hep-ph/9704231].
[4] K. Kumekawa, T. Moroi and T. Yanagida, Prog. Theor. Phys. 92 (1994) 437 [hep-ph/9405337];
    T. Asaka, K. Hamaguchi, M. Kawasaki and T. Yanagida, Phys. Lett. B464 (1999) 12 [hep-ph/9906366], Phys. Rev. D 61 (2000) 083512 [hep-ph/9907559].
G. F. Giudice, M. Peloso, A. Riotto and I. Tkachev, JHEP 9908 (1999) 014 [hep-ph/9905242];
G. Lazarides, Springer Tracts Mod. Phys. 163 (2000) 227 [hep-ph/9904428] and reference therein;
M. Kawasaki, M. Yamaguchi and T. Yanagida, hep-ph/0011104.

[5] H. Murayama and T. Yanagida, Phys. Lett. B322 (1994) 349 [hep-ph/9310297].
[6] B. A. Campbell, S. Davidson and K. A. Olive, Nucl. Phys. B399 (1993) 111 [hep-ph/9302223];
H. Murayama, H. Suzuki, T. Yanagida and J. Yokoyama, Phys. Rev. Lett. 70 (1993) 1912; Phys. Rev. D 50 (1994) 2356 [hep-ph/9311326].

[7] I. Affleck and M. Dine, Nucl. Phys. B249 (1985) 361.
[8] T. Asaka, M. Fujii, K. Hamaguchi and T. Yanagida, Phys. Rev. D 62 (2000) 123514 [hep-ph/0008041].
[9] M. Dine, L. Randall and S. Thomas, Nucl. Phys. B458 (1996) 291 [hep-ph/9507452].
[10] R. Allahverdi, B. A. Campbell and J. Ellis, Nucl. Phys. B579 (2000) 355 [hep-ph/0001122].
[11] A. Anisimov and M. Dine, hep-ph/0008058.
[12] See, for example, E. Kolb and M. Turner, The Early Universe (Addison-Wisley, 1990).
[13] See, for example, Y. Suzuki [Super-Kamiokande Collaboration], talk at the “Neutrino 2000” Conference, Sudbury, Canada, June 16–21, 2000;
M. Smy [Super-Kamiokande Collaboration], talk presented at The 2nd International Workshop on Neutrino Oscillations and their Origin (“NOON2000”), Tokyo, Japan, December 6–8, 2000.
[14] J. Hellmig and H. V. Klapdor-Kleingrothaus, Z. Phys. A359 (1997) 351 [nucl-ex/9801004];
H. V. Klapdor-Kleingrothaus and M. Hirsch, Z. Phys. A359 (1997) 361.
[15] C. D. Froggatt and H. B. Nielsen, Nucl. Phys. B147 (1979) 277.
[16] T. Yanagida, “Horizontal Symmetry And Masses Of Neutrinos”, Prog. Theor. Phys. 64 (1980) 1103, and in Proceedings of the “Workshop on the Unified Theory and the Baryon Number in the Universe”, Tsukuba, Japan, Feb 13–14, 1979, Eds. O. Sawada and A. Sugamoto, KEK report KEK-79-18, p. 95;
M. Gell-Mann, P. Ramond and R. Slansky, in “Supergravity” (North-Holland,
Amsterdam, 1979) *eds.* D.Z. Freedman and P. van Nieuwenhuizen, Print-80-0576 (CERN).

[17] T. Moroi and H. Murayama, JHEP0007 (2000) 009 [hep-ph/9908223].

[18] A. de Gouvea, T. Moroi and H. Murayama, Phys. Rev. D 56 (1997) 1281 [hep-ph/9701244].

[19] V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B155 (1985) 36.

[20] S. Y. Khlebnikov and M. E. Shaposhnikov, Nucl. Phys. B308 (1988) 885; J. A. Harvey and M. S. Turner, Phys. Rev. D 42 (1990) 3344.

[21] T. Asaka, K. Hamaguchi, M. Kawasaki and T. Yanagida, in Ref. [4], and references therin.

[22] M. Kawasaki, M. Yamaguchi and T. Yanagida, Phys. Rev. Lett. 85 (2000) 3572 [hep-ph/0004243].

[23] F. Vissani, JHEP9906 (1999) 022 [hep-ph/9906525]; H. V. Klapdor-Kleingrothaus, H. Pas and A. Y. Smirnov, [hep-ph/0003219].

[24] G. L. Fogli, E. Lisi, D. Montanino and A. Palazzo, Phys. Rev. D 62 (2000) 013002 [hep-ph/9912231].

[25] M. Apollonio *et al.*, Phys. Lett. B466 (1999) 415 [hep-ex/9907037].

[26] See, for example, Y. Obayashi, talk given at the Joint U.S./Japan workshop on New Initiatives in Lepton Flavor Violation and Neutrino Oscillations with Very Intense Muon and Neutrino Sources, Honolulu, Hawaii, October 2–6, 2000.

[27] J. Sato and T. Yanagida, Nucl. Phys. Proc. Suppl. 77 (1999) 293 [hep-ph/9809307]; P. Ramond, Nucl. Phys. Proc. Suppl. 77 (1999) 3 [hep-ph/9809401].

[28] J. Sato and T. Yanagida, Phys. Lett. B493 (2000) 356 [hep-ph/0009205].
Figure 3: The parameter range we have taken for the atmospheric neutrino oscillation.

Figure 4: The parameter ranges we have taken for the large angle MSW, the small angle MSW and the LOW solutions to the solar neutrino problem.
Figure 5: The predicted value of $|m_{\nu_e\nu_e}|$ for the large angle MSW solution. The solid lines represent the upper and lower values of $|m_{\nu_e\nu_e}|$ for $m_{\nu_1} \simeq 0$. The plots represent the values for the case when the $m_{\nu_1}$ is allowed to be $m_{\nu_1} \leq (1/\sqrt{2}) m_{\nu_2}$. Here, we have required $|U_{e3}| < 0.15$, to satisfy the CHOOZ limit.

Figure 6: Same as Fig. 5, but for $|U_{e3}| < 0.10$. 
Figure 7: The predicted value of $|m_{\nu_e\nu_e}|$ for the small angle MSW solution. The solid lines represent the upper and lower values of $|m_{\nu_e\nu_e}|$ for $m_{\nu_1} \simeq 0$. The plots represent the values for the case when the $m_{\nu_1}$ is allowed to be $m_{\nu_1} \leq (1/\sqrt{2})m_{\nu_2}$.

Figure 8: Same as Fig. 7, but for the LOW solution.
Figure 9: The plot for $r = \frac{\delta m^2_{\text{sol}}}{\delta m^2_{\text{atm}}}$ and $m_{\nu_1}$ in a Froggatt-Nielsen model with a discrete $Z_6$ symmetry.