Stator flux oriented multiple sliding-mode speed control design of induction motor drives

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Abstract
Due to superior robustness characteristic of sliding-mode control techniques, this study proposes a multiple sliding-mode control (MSMC) strategy based on the stator flux oriented vector scheme for speed control of three-phase AC induction motor (IM) drives in the presence of an external disturbance and uncertainties. At first, the dynamic model of a three-phase IM drive is transformed into two-axe orthogonal model (i.e. d and q axes) in the synchronously rotating frame so that vector control can be applied. Then, based on the stator flux oriented scheme (i.e. zero stator flux at q-axis and constant at d-axis), the proposed MSMC causes mechanical angular speed and stator current at q-axis reach toward predefined sliding surfaces. Moreover, stator flux and current at d-axis are respectively indirect and direct controlled such that tracking errors approach toward designed sliding surfaces. The closed-loop stability of the proposed MSMC is proved to possess uniformly ultimately bounded (UUB) performance by Lyapunov stability criteria. Furthermore, the simulation results reveal that the proposed MSMC strategy has a high level of robustness despite addition of an external load and random uncertainties on system parameters. In the meantime, the simulations for comparing the baseline controller (i.e. conventional PI control) are also conducted to verify the superiority of the proposed control scheme.

Keywords
Multiple sliding-mode control, induction motor drives, stator flux oriented vector control, Lyapunov stability criteria, conventional PI control

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Introduction
IM drives have widely been applied to numerous kinds of fields such as industries, traffic transportation and medical equipment etc. Many researchers and engineers have been devoting to control of AC IM drives in order to improve control performance and reduce cost. Over past two decades, vector control has become a popular technique to effectively manipulate AC motor drives in terms of control performance.¹,² However, as ones know, control of motor drives with excellent performance is a challenging problem as operated at low-speed in the presence of system uncertainties. Moreover, an external load, that is, disturbance, to motor drives is another big challenge in fast response and precise tracking. Based on these concerns,
developing a speed controller with a high-level of robustness and a superior performance for AC motor drives is strongly in need.

There have been some research findings in the published literatures about the control of IM drives and some representative studies are reviewed here. Field-oriented control (FOC) and direct torque control (DTC) schemes are two most popular approaches to control of AC motor drives. Like in Wu et al., the FOC and DTC approaches are both applied to a five-phase permanent magnet motor. The results reveal that FOC scheme shows robustness and DTC scheme has fast dynamic response. Awan et al. presented an exact input-output feedback linearization structure. The performance improvement of permanent-magnet synchronous motors and reluctance motors. In addition, Lin et al. proposed an advanced deadbeat direct synchronous motor. The role of the NN controller is to substitute the two decoupled current-loop PI controllers in conventional vector control techniques. The experimental results demonstrated that the NN vector control can succeed in driving the induction motor without audible noise using relatively lower switching frequency or lower sampling rate.

One of well-known robust controllers, that is, sliding-mode control, has been proven to the most effective and feasible technique for any kinds of control systems. Here, some typical works with regards to induction motors are surveyed and reviewed. An adaptive sliding-mode control based model predictive torque control was proposed to enhance the robustness and improve the performance by adjusting the switching gain adaptively. In Chen and Yu presented a backstepping sliding mode control combining with a disturbance observer to regulate the speed of an induction motor. Experimental results verify its effectiveness and practicability. Furthermore, a speed observer design based on backstepping and sliding-mode strategies is used to perform speed operation, especially, at low-speed operation. From experimental tests, the control system is indeed robust on machine parameters and under load torque injection even operating at low-speed. Obviously, it reveals that the sliding-mode control strategy may be a suitable solution for control of induction motor drives.

Based on the aforementioned literatures review, FOC and DTC based approaches are usually used to tackle three-phase AC motor drives. To the best of our knowledge, a multiple sliding-mode control (MSMC) strategy based on stator flux oriented vector strategy can become a feasible technique for any kinds of control systems. Here, some typical works with regards to induction motors are surveyed and reviewed. An adaptive sliding-mode control based model predictive torque control was proposed to enhance the robustness and improve the performance by adjusting the switching gain adaptively. In Chen and Yu presented a backstepping sliding mode control combining with a disturbance observer to regulate the speed of an induction motor. Experimental results verify its effectiveness and practicability. Furthermore, a speed observer design based on backstepping and sliding-mode strategies is used to perform speed operation, especially, at low-speed operation. From experimental tests, the control system is indeed robust on machine parameters and under load torque injection even operating at low-speed. Obviously, it reveals that the sliding-mode control strategy may be a suitable solution for control of induction motor drives.

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Dynamic model of an IM drive, decoupled d-q axes transformation and problem formulation

Dynamic model of the IM drive

First, an AC IM drive is three-phase balanced, hence, its dynamic model of the stator, rotor electrical circuits
and electromechanical part in space vector form can be expressed as:

\begin{align}
R_i \dot{i}_s + L_m \dot{i}_s + L_{m}i_s e^{j\theta_s} + j\omega_m L_m i_s e^{j\theta_m} &= v_s \\
R_r \dot{i}_r + L_r \dot{i}_r + L_{m}i_r e^{j\theta_s} - j\omega_m L_m i_r e^{j\theta_m} &= 0 \\
J_m \dot{\omega}_m + B_m \omega_m + T_L &= T_e
\end{align}

(1a) (1b) (1c)

where \( i_s = i_{ds} + j i_{qs} \); \( i_r = i_{ds} + j i_{qs} \); \( v_s = v_{ds} + j v_{qs} \); \( v_{qs} \) is the superscript * indicates complex conjugate; \( R_r = R_r + \Delta R_r \) denotes the stator winding resistor of each phase; \( L_m = L_m + \Delta L_m \) is the mutual inductance; \( R_r = R_r + \Delta R_r \) represents the rotor winding resistor of each phase; \( L_r = L_r + \Delta L_r \) is the rotor inductance of each phase; \( \omega_m \) is mechanical angular speed. If the pole pairs of the IM is \( P \), it has relationship \( \omega_m = \frac{2}{P} \omega_r \), where \( \omega_r \) denotes rotor angular speed, \( J_m = J_m + \Delta J_m \); \( B_m = B_m + \Delta B_m \); \( \theta_r \), \( T_L \) and \( T_e \) respectively represent as moment of inertia, friction coefficient, rotor angle, external load and electromagnetic torque. To reflect real situation, the above system parameters include nominal denoted \( \bar{X} \) and uncertain terms indicated \( \Delta \bar{X} \); \( \text{Im} [\bullet] \) represents as an imaginary part. The above equation (1) denotes the complete dynamic model of a three-phase IM drive with 7 state variables (i.e. \( i_s \), \( i_r \), and \( \omega_m \)). In terms of position control, one more state variable (i.e. \( \theta_r \)) is added. The dynamic model is based on a three-phase system, that is, a three-axis coordinate system respectively with 120° phase difference in two-dimensional space. The mathematical expression is quite complicated compared with conventional orthogonal coordinate system. Therefore, the dynamic model (1) is not suitable to be directly used for controller design. The suitable coordinate transformation for decoupling is required. In the next subsection, the often used d-q axes transformation is addressed for controller design.

**Decoupled d-q axes transformation**

Consider a coordinate transformation from an original three-phase coordinate system to an orthogonal coordinate frame rotating with an angular speed of power source \( \omega_r \), the original dynamic model of the IM drive (1) expressed in the synchronously rotating frame is given as Kocabas et al.\textsuperscript{14} and Comanescu:\textsuperscript{15}

\begin{align}
(R_r + j \omega_r L_r + L_r \nu) \bar{v}_s &= (R_r + j \omega_r L_r + L_r \nu) \bar{v}_s \\
(R_r + j \omega_r L_r + L_r \nu) \bar{v}_s &= (R_r + j \omega_r L_r + L_r \nu) \bar{v}_s \\
J_m \nu \omega_m + B_m \omega_m + T_L &= T_e
\end{align}

(2a) (2b) (2c)

where \( \nu = \frac{d}{dt} \) (\( \cdot \)) is a differential operator; \( \bar{v}_s = \bar{v}_{ds} + j \bar{v}_{qs} \); \( \bar{v}_s = \bar{v}_{ds} + j \bar{v}_{qs} \) is denoted as stator voltage; \( \omega_s \) is also called as synchronous angular speed and \( \omega_s \) represents slip angular speed with the relation \( \omega_s = \omega_r - \omega_r \). In this expression, there are only five state variables, namely \( \bar{v}_s \), \( \bar{v}_r \) and \( \bar{\omega}_m \). For the stator flux oriented control, the rotor current \( \bar{i}_r \) must be replaced with the stator flux and stator current as

\[
\bar{i}_r = \frac{1}{L_m} (\Phi_s - L_i \bar{v}_s)
\]

(3)

Through the coordinate transformation, equations (2) can be rewritten as

\begin{align}
R_s \bar{v}_s + (j \omega_r + \nu) \Phi_s &= \bar{v}_s \\
\left(\frac{R_r}{L_m} + j \omega_r L_r + \frac{L_r}{L_m} \nu\right) \Phi_s &= \bar{v}_s \\
- \left(\frac{L_r R_r}{L_m} + j \omega_r \sigma L_r L_r + \frac{\sigma L_r L_r}{L_m} \nu\right) \bar{v}_s &= 0 \\
J_m \nu \omega_m + B_m \omega_m + T_L &= T_e
\end{align}

(4a) (4b) (4c)

where complex vectors \( \Phi_s = \Phi_{ds} + j \Phi_{qs} \) and \( \nu = \nu_{ds} + j \nu_{qs} \); \( \sigma = 1 - \frac{j}{L_m \nu} = \bar{\sigma} + \Delta \sigma \).

**Problem formulation**

After the above coordinate transformation, the two-axle dynamic model with orthogonal features along with the electro-mechanical model is obtained in the equations (4) so that the individual axis can be controlled via a feedback decoupled approach. To achieve the stator flux oriented vector control, the stator flux must be controlled to zero at q-axis, that is, all stator flux is located at d-axis. In the meantime, the stator flux at d-axis must be maintained as a constant value. In this way, the electromagnetic torque \( T_e \) generated by the IM drives is proportional to the stator current at q-axis (i.e. \( \bar{i}_{qs} \)). In other words, the decoupled stator currents (i.e. \( \bar{i}_{ds} \) and \( \bar{i}_{qs} \)) at d and q-axes can be separately controlled. The control problem is to design \( \bar{v}_s \) (i.e. \( \nu_{ds} \) and \( \nu_{qs} \)) using the proposed SMC strategies such that the reference stator current \( \bar{i}_{ds} \) can be obtained where the reference stator current \( \bar{i}_{ds} \) at d-axis and \( \bar{i}_{qs} \) at q-axis are respectively generated using the proposed SMC strategies. Subsequently, the reference mechanical angular speed \( \omega_{sm} \) and reference stator flux \( \Phi_s \) can be achieved using the proposed SMC strategies as well. The overall control block diagram called MSMC based on stator flux oriented vector control scheme is illustrated in the following Figure 1.

**Stator flux oriented multiple sliding-mode control design**

As the foregoing, the stator current needs to be controlled where the reference stator current is generated via the desired stator flux and its dynamics. In addition,
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Figure 1. Overall control block diagram of the IM drive system.

where \( k_{Ps} \) is the selected proportional and integral gains such that the first sliding surface is stable and determines its convergent rate. To make the mechanical angular speed error converge into the designed sliding surface and then asymptotically approach to zero, the reference electromagnetic torque \( T^*_{e,eq} \) designed and contains two parts: ones is equivalent control \( T^*_{e,sw} \) that deals with a nominal system and the other one is switching control \( T^*_{e,sw} \) which is used to cope with external disturbances or uncertainties as

\[
T^*_{e,eq} = B_m \omega_m + \frac{J_m}{k_{Ps}} (k_{Pm} \dot{\omega}_m + k_{Ie} \omega_m) \quad (7a)
\]

\[
T^*_{e,sw} = \frac{J_m}{k_{Ps}} \left[ \alpha_1 s_1 + \alpha_2 s_1 (|s_1| + e_1) \right] / (1 - \lambda_1) \quad (7b)
\]

\[
T^* = T^*_{e,eq} + T^*_{e,sw} \quad (7c)
\]

where \( \alpha_1 > \delta_1 / 2 > 0 \) and \( \delta_1 \) represents the exponential convergent rate to the sliding surface; \( \alpha_2 > 0 \) is the switching gain; \( e_1 \geq 0 \) is a boundary layer of the first sliding surface; \( \lambda_1 > 0 \) denotes the upper bound of the uncertain control gain and satisfies the following inequality

\[
|\tilde{J}_m \Delta_j| \leq \lambda_1 < 1 \quad (8a)
\]

where \( \Delta_j = -(\tilde{J}_m + \Delta J_m)^{-1} \tilde{J}_m^{-1} \Delta J_m \). In addition, it is assumed that the uncertainty function has an upper bound and satisfies the following inequality

\[
\left| -k_{Ps} T^*_{e,eq} \Delta_j + k_{Ps} (1 + \Delta_j) \tilde{J}_m^{-1} (T_L + \Delta B_m \omega_m) \right| \leq \mu_{10} + \mu_{11} \beta_1 \quad (8b)
\]

where \( \beta_1 \) is an known positive function and \( \mu_{10}, \mu_{11} \geq 0 \) are constants constrained by the stability of the closed-loop function.

Before discussing the result of the proposed controller, the following property about “uniformly ultimately bounded (UUB)” is given.

Definition 1: Wu and Karkoub\(^6\) The solutions of a dynamic system are said to be UUB if there exist positive constants \( \upsilon \) and \( \theta \), and for every \( \delta \in (0, \theta) \), there is a positive constant \( T = T(\delta) \), such that \( |x(t_0)| < \delta \Rightarrow |x(t)| \leq \upsilon, \forall t \geq t_0 + T \).

Theorem 1: Consider the transformed two-axes dynamic model of an IM drive with the known upper bound of an external load in (4), applying the proposed sliding-mode control results in finite time to reach the predefined sliding surface and satisfy the following convex set \( D_\sigma \)

\[
D_\sigma = \{ \sigma_n \in \mathbb{R} | |\sigma_n| \leq s_\sigma \} \quad (9a)
\]
where
\[
s_{ar} \equiv \left(-\nu_{ar1} + \sqrt{\nu_{ar1}^2 + 4\nu_{ar2}v_{m}}\right)/2\nu_{ar2} \tag{9b}
\]

\[
v_{ar2} = |\alpha_1 - \delta_1/2|
\]

\[
v_{ar1} = \varepsilon_1\alpha_1 + \varepsilon_2 - (\mu_{10} + \mu_{11}p_1) - \varepsilon_1\delta_1/2 \tag{9c}
\]

\[
v_{m} = -\varepsilon_1(\mu_{10} + \mu_{11}p_1)
\]

Then, \(e_{oum}\) is UUB as \(t \to \infty\).

**Proof:** See Appendix A.

Moreover, as the mechanical angular speed is equal or less than rated speed, the stator flux is a constant value under constant power operation. Hence, based on the result of stator flux oriented control, the reference \(i_{qs}^*\) can be given as

\[
i_{qs}^* = \frac{4T_{ie}}{3P\phi_s} \tag{10}
\]

To achieve the tracking of the reference \(i_{qs}^*\), the second sliding-mode control is proposed and then the second sliding surface is defined as follows:

\[
s_2 = k_{pq}i_{qs} + k_{iq}\int i_{qs}dt \tag{11}
\]

where \(k_{pq}\) and \(k_{iq}\) are the chosen proportional and integral gains such that the second sliding surface is stable. Based on the dynamic model of IM drives, the equivalent and switching control of \(i_{qs}\) are designed as

\[
\begin{align*}
    v_{qs,cq}^e &= \bar{L}_a \left[\frac{R_s}{L_a} + \frac{1}{\sigma_T}\right]i_{qs} + \omega_d i_{d}^e + \frac{\omega_r}{L_a}\phi_s^e \\
    + &\frac{k_{iq}}{k_{pq}}i_{qs} \\
    v_{qs,sw}^e &= \bar{L}_a \left[\beta_1 s_2 + \beta_2 s_2/(|s_2| + e_2)\right]/(1 - \lambda_2) \tag{12b}
\end{align*}
\]

where \(L_a = \sigma L_s = \bar{L}_a + \Delta L_a\); \(\beta_1 > \delta_2/2 > 0\) and \(\delta_2\) is an exponentially convergent rate to the sliding surface; \(\beta_2 > 0\) is the switching gain; \(e_2 \geqslant 0\) is the boundary layer of the second sliding surface; \(\lambda_2 > 0\) is the upper bound of the uncertain control gain and satisfies the following inequality

\[
|\Delta L_a/\bar{L}_a| \leq \lambda_2 < 1 \tag{13}
\]

**Remark:** In equation (12a), \(i_{qs}^e\) is substituted with

\[
i_{qs}^e = \left[i_{qs}^e(kT) - i_{qs}^e((k - 1)T)\right]/T \text{ where } T \text{ denotes the sampling time of the control cycle. In general, the smaller the sampling time } T \text{ is, the more accurate the time derivatives is. The approximation error between them is regarded as part of uncertainties or disturbances. The proof is analogous to the proof of Theorem 1. For brevity, it is omitted.}

Since the stator flux \(\phi_s\) and stator current at d-axis \(i_{qs}\) have coupling relationship with \(v_{qs}\), direct control \(i_{qs}\) and indirect control \(\phi_s\) via \(v_{qs}\) are designed. Hence, the reference \(i_{qs}^e\) can be given via SMC. The third sliding surface is then defined as

\[
s_3 = k_{pf}e_{ph} + k_{if}\int e_{ph}dt \tag{14}
\]

where the selected gains, \(k_{pf}\) and \(k_{if}\), make the sliding surface stable. Then, the reference \(i_{qs}^e\) is designed as

\[
\begin{align*}
    r_{ds,cq}^e &= (k_{pf}R_s)^{-1}\left[k_{pf}v_{ds} - k_{if}e_{ds}\right] \tag{15a}
    \\
    r_{ds,sw}^e &= (k_{pf}R_s)^{-1}\left[\gamma_1 s_3 + \gamma_2 s_3/(|s_3| + e_3)/(1 - \lambda_3)\right] \tag{15b}
    \\
    i_{ds}^e &= \bar{i}_{ds,cq} + \bar{i}_{ds,sw} \tag{15c}
\end{align*}
\]

where \(\gamma_1 > \delta_2/2 > 0\) and \(\delta_2\) is an exponentially convergent rate to the sliding surface; \(\gamma_2 > 0\) is the switching gain; \(e_3 \geqslant 0\) is the boundary layer of the third sliding surface; \(\lambda_3 > 0\) is the upper bound of uncertain control gain and satisfies the following inequality. The proof is analogous to the proof of Theorem 1 and omitted for brevity.

Then, the fourth sliding surface is defined as

\[
s_4 = k_{pd}i_{d} + k_{ld}\int i_{d}dt \tag{16}
\]

where \(k_{pd}\) and \(k_{ld}\) are the selected proportional and integral gains such that the fourth sliding surface is stable. The equivalent and switching control for the \(i_{d}^e\) can be derived as follows:

\[
\begin{align*}
    v_{ds,cq}^e &= (k_{pd}k_{pf} + k_{pd}/\bar{L}_a)^{-1} \left\{k_{pd}k_{pf}\bar{\phi}_s + k_{pd}k_{pf}\bar{R}_s i_{d}^e + k_{pd}k_{if}e_{ph} + k_{ld}e_{d}\right\} \\
    - &k_{pd}\left[\left(L_a/\bar{L}_a + 1/\sigma_T\right)i_{d}^e + \omega_d i_{d}^e + \left(1/(\lambda_3 - 1)\right)\bar{\phi}_s^e\right] \tag{17a}
    \\
    v_{ds,sw}^e &= (k_{pd}k_{pf} + k_{pd}/\bar{L}_a)^{-1} \left[\mu_1 s_4 + \mu_2 s_4/(|s_4| + e_4)/(1 - \lambda_4)\right] \tag{17b}
    \\
    v_{d}^e &= \bar{v}_{ds,cq} + \bar{v}_{ds,sw} \tag{17c}
\end{align*}
\]

where \(k_{pd}k_{pf} + k_{pd}/\bar{L}_a\) is nonsingular; \(\sigma_T\) is the nominal term of \(\sigma_T = \omega_d - \Delta \tau_r\) where \(\Delta \tau_r\) is uncertain terms; \(\mu_1 > \delta_4/2 > 0\) where \(\delta_4\) is the exponentially convergent rate to the sliding surface; \(\mu_2 > 0\) is the switching gain; \(e_4 \geqslant 0\) is the boundary layer of the fourth sliding surface; \(\lambda_4 > 0\) is the upper bound of the uncertain control gain and satisfies the following inequality.
Finally, the proposed MSMC in equations (12) and (17) generate three-phase voltage commands via coordinate transformation fed into an inverter to achieve the speed control. The proof is analogous to the proof of Theorem 1 and omitted for brevity.

\[
\left|\left(k_{p_d}k_{p_y} + k_{p_d}/\Delta L_{\sigma}\right)/\left(k_{p_d}k_{p_y} + k_{p_d}/L_{\sigma}\right)\right| \leq \lambda_4 < 1 \quad (18)
\]

**Figures 2.** Responses of the IM drive with a constant angular speed under addition of disturbance after 1 s. and a maximum of 10% error of random variance on the system parameters: (a) tracking of real and reference mechanical angular speeds with addition of disturbance after 1 s, (b) stator flux at d-axis, \(\phi_{ds}\), (c) stator current at d-axis, \(i_{ds}\), (d) stator current at q-axis, \(i_{qs}\), and (e) electromagnetic torque, \(T_e\).
Simulations and discussions

To evaluate the performance and validate feasibility of the proposed control, two cases of simulations are performed for the tracking of reference speeds that are respectively constant and sinusoidal speeds. The chosen system and control parameters in the simulation are shown in Table B1 of Appendix B. First, the simulations of the IM drive operated with the constant speed 2000 $r/min$ under the external disturbance $T_L = 2N \cdot m$ and a maximum of 10% error of random variance on the system parameters are conducted. The tracking of reference mechanical angular speed shown in Figure 2(a) is excellent despite suffering the external disturbance.

Figures 3. Responses of the IM drive operated with a sinusoidal angular speed under addition of disturbance after 1 s. and a maximum of 10% error of random variance on the system parameters: (a) low-speed operating, (b) stator flux at d-axis, $\phi_d$, (c) stator current at d-axis, $i_{ds}$, (d) stator current at q-axis, $i_{qs}$, (e) electromagnetic torque, $T_e$.
disturbance $T_L = 2N - m$ after 1 sec and random uncertainties. Figure 2(b) represents the response of the stator flux at d-axis and achieves the constant value 0.3019 as desired. In addition, the responses of the stator currents at d and q-axes are respectively shown in Figure 2(c) and (d). Obviously, the stator current at q-axis becomes larger after addition of an external load since 1 s and the stator current at d-axis is almost constant. Simultaneously, the value of electromagnetic torque $T_e$ becomes bigger shown as Figure 2(e) since addition of the external load. It is quite reasonable that the IM increases $T_e$ as it tackles the external load $T_L$ under the steady state of mechanical angular speed $\omega_{rm}$ (obtained from equation (2c)). Further, to demonstrate the superiority of the proposed control scheme, the tracking of the IM drive operated with a low reference sinusoidal mechanical angular speed $\omega^*_{rm} = 200\sin(\pi t/3)$ is investigated under addition of the external load. Figure 3(a) demonstrated the tracking response of the reference $\omega^*_{rm} = 200\sin(\pi t/3)$ is quite satisfactory in spite of suffering the external load and random uncertainties on system parameters. Other responses consisting of the stator flux at d-axis $\phi_d$, stator currents at d and q-axes, and the electromagnetic torque $T_e$ are exhibited in Figure 3(b) to (e), respectively. Furthermore, the control performance of the proposed MSMC is comparative with a baseline controller (i.e. conventional PI controller) under the same conditions is shown in Figure 4 where the proportional and integral gains are the identical to the gains of multiple sliding surfaces. Clearly, it can be seen that tracking performance of the mechanical angular speed applying the proposed MSMC is superior to that of using PI controllers (cf. Figures 3(a) and 4) while the IM drive is subject to an external load. The above simulation results specifically indicate that the proposed MSMC scheme possess effectiveness and a high level of robustness.

Conclusions

The dynamic model of a three-phase IM is decoupled and represented via the d-q axes transformation such that the corresponding controller can be designed with ease. Then, the speed control of an IM drive based on the stator flux oriented MSMC strategy was proposed to resolve the typical and potential problems for example, low-speed tracking performance and a high-level of robustness. The proposed MSMC not only had outstanding tracking performance but also possessed a high level of robustness in the presence of an external load and random uncertainties on system parameters. The overall system stability with the UUB performance is assured via Lyapunov stability criteria. Finally, the simulation results also validate the effectiveness and feasibility of the proposed MSMC scheme. Additionally, the comparative simulations with a baseline controller (i.e. conventional PI) exhibit the superior performance of the proposed MSMC under different operating conditions. Due to the necessity of feedback states measured by sensors for the proposed controller, it will cause the increase of cost. Thus, the direction of next research will be sensorless speed control of IM drives without sensors using our proposed control scheme.

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Appendices

Appendix A. (Proof of theorem 1)

Without ambiguity, the arguments of variable are omitted. First, the Lyapunov function $V_1 = s^2_1 s_1/2$ is defined. To ensure the exponentially convergent rate to $V_1 = 0$, taking time derivative of $V_1$ with the addition of $\delta_1 V_1$ where $\delta_1 > 0$ and substitution of Eqs. (4c), 5(a), (6), (7) and (8) gives the result as

$$
\dot{V}_1 + \delta_1 V_1 = s^T_1 (k_p \dot{\omega}_{w_m} + k_t e_{w_m}) + \delta_1 s^T_1 s_1/2
$$

$$
= s^T_1 \left\{ k_p \left[ \omega^*_m - (T_e - T_L - B_m \omega_m)/J_m \right] + k_t e_{w_m} \right\} + \delta_1 s^T_1 s_1/2
$$

$$
= s^T_1 \left\{ k_p \left[ \omega^*_m - T^*_e e_{w_m} - T_L - (B_m + \Delta B_m) \omega_m \right]/(J_m + \Delta J_m) \right\} + k_t e_{w_m} \right\} + \delta_1 s^T_1 s_1/2
$$

$$
= s^T_1 \left\{ k_p \left[ -T^*_e e_{w_m} - (1 + J_m \Delta J_m) \right] \right\} + \delta_1 s_1/2
$$

$$
\leq s_1^T \left\{ \left( \mu_{10} + \mu_{11} p_1 \right) s_1 - \left( (1 + J_m \Delta J_m) \right) \right\}/(1 + \alpha_1 s_1/|s_1| + e_1)/|s_1| + e_1) + \delta_1 s_1/2
$$

$$
\leq -\left( |s_1|/\left( |s_1| + e_1 \right) \right) \left\{ |\alpha_1 - \delta_1/2|/|s_1|^2 + |\hat{e}_1 \alpha_1 + e_1 - (\mu_{10} + \mu_{11} p_1) - \hat{e}_1 \delta_1/2|/|s_1| + e_1) \right\}
$$

$$
= -\left( |s_1|/\left( |s_1| + e_1 \right) \right) v(|s_1|)
$$
where $v(\|s_1\|) = v_{s_2} \|s_1\|^2 + v_{s_1} |s_1| - v_{s_0}$ with $v_{s_2}$, $v_{s_1}$, and $v_{s_0}$ described in (9c). As $\|s_1\| > s_0$, the inequality $v(\|s_1\|) \geq 0$ is satisfied. Therefore, outside of the domain $D_0$ in (9a) making $\dot{V}_1 \leq 0$ is obtained. The sliding surface $s_1$ exponentially converges to the domain $D_0$ in finite time. Then, based on Definition 1, $e_{\text{thru}}$ is UUB as $t \to \infty$.

### Appendix B

**Table B1.** System and control parameters in the simulation.

| Parameters | Description                                    | Value  |
|------------|-----------------------------------------------|--------|
| $R_s$      | Stator winding resistor of each phase         | 1.1771Ω|
| $R_r$      | Rotor winding resistor of each phase          | 1.38195Ω|
| $L_s$      | Stator inductance of each phase               | 119.0902mH|
| $L_r$      | Rotor inductance of each phase                | 118.54mH|
| $L_m$      | Mutual inductance                             | 113.0449mH|
| $p$        | Pole pairs                                    | 2      |
| $J_m$      | Moment of inertia                             | 0.676e-3kgm²|
| $B_m$      | Friction coefficient                          | 0.515e-3Nmsec/ rad|
| $k_p, k_i, a_1, a_2$ | Control and switching gains          | 10, 1, 10, 1 |
| $e_1, e_2, e_3, e_4$ | Boundary layers          | 0.01, 0.01, 0.01, 0.01 |
| $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ | Upper bound of uncertain control gains     | 0.1, 0.1, 0.1, 0.1 |
| $k_p, k_i, \beta_1, \beta_2$ | Control and switching gains          | 10, 1, 10, 1 |
| $k_f, k_{if}, \gamma_1, \gamma_2$ | Control and switching gains          | 10, 1, 50, 5 |
| $k_{p_d}, k_{i_d}, \mu_1, \mu_2$ | Control and switching gains          | 10, 1, 10, 1 |
| $\phi_s^*$ | Reference stator flux                        | 0.3019wb |

Above, $\|s_1\|$ is the sliding surface $s_1$ exponentially converges to the domain $D_0$ in finite time. Then, based on Definition 1, $e_{\text{thru}}$ is UUB as $t \to \infty$. 

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