Design of Phase Control System for Parametric Array Loudspeakers Based on FPGA

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Abstract. In order to achieve accurate control of the audible sound of the acoustic parametric array in the air, fine phase control of the ultrasonic signal as a carrier is required. By establishing the parametric matrix mathematical model and the directivity function formula, the effects of delay accuracy on the ultrasonic carrier and the audible sound deflection angle are simulated and analyzed. The control of 16-channel phase-controlled parametric array excitation signal is realized by using the fast parallel processing speed of FPGA. The signal processing circuit is designed to D/A conversion and amplification of the parametric array excitation signal to drive the piezoelectric transducer array efficiently. The experimental results show that the system can achieve fine control of the parameter array carrier transmission delay. The control accuracy of the parametric array carrier for the 40KHz carrier is 1 ns, and the error delay is less than 1 ns.

1. Introduction

Parametric array loudspeakers, also known as audio directional loudspeakers, utilize non-linear interaction of ultrasound in the air to achieve directional propagation of sound. It is based on the acoustic parametric array theory [1]. The audible sound is modulated on an ultrasonic carrier wave, and then transmitted to the air through an ultrasonic transducer. The audible sound is demodulated by a non-linear effect in the air. Deflecting the audible sound can improve the flexibility of the parametric array loudspeakers system application, but it is required that the parametric array system has the ability to finely control the audible sound beam. Therefore, the carrier ultrasonic phase control technology is the key technology to ensure the high directivity of the parametric array signal.

Luo Ying [2] realized a phase delay control integrated phase transmission system with a delay resolution of 3.75ns based on FPGA and a delay algorithm; Li Yanzhong [3] implemented a 4-channel delay resolution of 20ns based on STM32 ultrasonic phased array; Assef [4] implemented an 8-channel phased focusing system with adjustable amplitude and delay based on FPGA; Kim [5] designed a 32-channel phased control with a frequency of 500 kHz based on FPGA Signal generator; Du Chunhui [6] tested 8-channel ultrasonic phased array excitation signal transmission with 2.5ns delay resolution based on FPGA.

The paper analyzes the influence of delay accuracy on the parametric array carrier theoretically. The deflection angle formula is derived, and then the influence of the delay accuracy on the carrier wave and the difference frequency waves are analyzed based on the simulation results. In addition, a
parametric array carrier signal control system based on FPGA and DDS is designed to optimize the accuracy of channel independent acoustic parametric array ultrasonic carrier delay.

2. Theory

2.1 Parametric Array Phase Control Principle

According to Huygens’ principle, the total sound pressure of the array is the superposition of each array element signal, as shown in equation (1) [7].

\[
p(r, \theta, t) = \sum_{i=1}^{N} p_i(r, \theta, t) = \frac{p_0}{r} \sin\left(\frac{ka \sin \theta}{2}\right) \sin\left[\frac{(\omega t - kl \sin \theta)N}{2}\right] \exp\left[-j\left(\frac{ka \sin \theta}{2}\right)\right] \\
\times \exp\left[\frac{-j(\omega t - kl \sin \theta)}{2}\right](N-1) \exp[j(\omega t - kr)]
\]

\( a \) is the array element aperture; \( l \) is the array element spacing; \( k \) is the wave number; \( N \) is the number of array elements; \( \theta \) is the radiation angle, and \( r \) is the delay between adjacent array elements. Based on the sound path difference, the deflection angle \( \theta_s \) can be written as:

\[
\theta_s = \sin^{-1}\left(\frac{cr}{l}\right)
\]

(2)

The radiation directivity function of the matrix array far-field carrier can be derived as:

\[
D_\theta(\theta) = \frac{p(r, \theta, t)}{p(r, \theta, t)} = \frac{\sin\left(\frac{ka \sin \theta}{2}\right) \sin\left[\frac{(\omega t - kl \sin \theta)N}{2}\right]}{\sin\left(\frac{ka \sin \theta}{2}\right) \sin\left(\frac{\omega t - kl \sin \theta}{2}\right)N \sin\left(\frac{\omega t - kl \sin \theta}{2}\right)}
\]

\[
= \frac{\sin\left(\frac{\pi a sin \theta}{2}\right) \sin\left(\frac{\pi l (sin \theta_i - sin \theta)}{\lambda}\right)}{\pi a \sin \theta \frac{\sin\left(\frac{\pi l (sin \theta_i - sin \theta)}{\lambda}\right)}{\lambda}}
\]

(3)

Under free field conditions, the directivity function of the difference frequency wave \( D_{\text{diff}} \) is determined by the product of the directivity function of the two columns of carriers [8-9], which can be expressed as

\[
D_{\text{diff}}(\theta) = D_{\text{ia}}(\theta)D_{\text{ib}}(\theta)
\]

(4)

\( D_{\text{ia}}(\theta) \) and \( D_{\text{ib}}(\theta) \) are the directivity functions of \( \omega_a \) and \( \omega_b \) respectively. From equation (4), we can know that by controlling the directivity of the carrier to achieve beam deflection of differential frequency waves. Therefore, the deflection accuracy of the carrier wave is directly proportional to the deflection accuracy of the audible sound. Figure 1 shows the arrangement of rectangular array elements.
2.2 Effect of delay accuracy on carrier deflection angle

In order to calculate the influence of the delay accuracy on the deflection angle, we can derive from equation (5):

\[ \sin \theta_s = \frac{c}{l} \tau \]  

(5)

Differentiate the variables of formula (6) simultaneously:

\[ \Delta \theta = \frac{c}{l \cos \theta_s} d \Delta \tau \]  

(6)

It can be seen from equation (6) that the magnitude of the deflection angle error \( \Delta \theta \) is not only related to the delay accuracy \( \Delta \tau \), but also to the deflection angle \( \theta_s \). The speed of sound \( c \) is 340 m/s, and the distance between array elements \( l \) is 8 mm. Table 1 shows the influence of delay accuracy on the deflection angle error. It can be known from Table 1 that with the improvement of the delay accuracy, the effect of the delay error on the carrier deflection angle becomes smaller and smaller.

| \( \theta_s \) (°) | \( \Delta \tau \) (ns) | \( \Delta \theta \) (°) | Error ratio (%) |
|------------------|-----------------|-----------------|-----------------|
| 10               | 1000            | 2.4573          | 13.07           |
| 10               | 100             | 0.2457          | 1.31            |
| 10               | 10              | 0.0246          | 0.13            |
| 10               | 1               | 0.0025          | 0.01            |

When the speed of sound and the spacing of the array elements are not changed, the delay accuracy is set to 1 us and 1 ns, respectively. The corresponding error angles and are obtained. The relationship between these is shown in Table 2.

Table 2. Effect of deflection angle \( \theta_s \) on error angle \( \Delta \theta \) at different delay accuracy

| \( \theta_s \) (°) | \( \Delta \theta_i \) (°) | \( \Delta \theta_i \) (°) |
|------------------|-----------------|-----------------|
| 15               | 2.5210          | 0.0025          |
| 25               | 2.6868          | 0.0027          |
| 45               | 3.4437          | 0.0034          |
| 65               | 5.7619          | 0.0058          |
| 85               | 27.9393         | 0.0279          |

In combination with Table 2 and Figure 3, it can be seen that the influence on the error is within 5° when the delay accuracy is less than 55°, which is at a low level. With the \( \theta_s \) increase to 75°, the
effect on the error is already close to 10°. When $\theta_s$ is increased to 85°, the angle error is 28°, which is close to one-third. In actual applications, the validity of the pointing cannot be guaranteed. At the same time, it can be seen from Figure 3 that as the delay accuracy is improved to 1ns, the error angle is always less than 0.03° over the entire 0 ~ 85° deflection range, and the effect on the target deflection angle can be ignored. Therefore, it can be seen that the delay accuracy can reduce the carrier beam deflection angle error.

2.3 Impact of Delay Accuracy on Difference Frequency Sound

The sound field simulation of $M \times N$ uniform piston matrix sound source is shown in Figure 1. Limited to the actual element diameter of 16mm, the simulation parameters are selected as the array element spacing $l = 8$mm; the number of array elements $M$ is 8; the number of channels $N$ is 16. And the ultrasonic carrier frequency $f_1 = 39$kHz, $f_2 = 40$kHz.

This beam satisfies the Nyquist criterion during the two columns of carrier synthesis:\(^{[12]}\)

$$\sin \theta_{\text{max}} \leq \frac{\lambda}{2l}$$

(7)

$\theta_{\text{max}}$ is the maximum value of the set beam deflection angle; $\lambda$ is the carrier wavelength, and $l$ is the transducer element spacing. Therefore, in order to avoid the occurrence of the sub-maximum value, the maximum deflection angle $\theta_{\text{max}}$ is set as 30° under the above conditions. Different frequency sounds have different directivity at different deflection angles and different delay accuracy. The simulation results are shown in Figures 3 and 4.

![Figure 3](image_url)

Figure 3. Difference frequency $f_1 = 1000$Hz beam contrast with delay accuracy of 1us and 1ns
It can be seen from figure 3 and 4 that with the increase of deflection angle, the higher the delay precision, the more accurate the deflection angle will be. In order to analyze the above beam pattern more specifically, the maximum sound pressure of the beam and the 3 dB attenuation point data are taken. As shown in Figure 5, this beam is a directivity diagram of time-frequency frequency sound, where point A is the peak, which $\theta_s = 10^\circ$ is the maximum point of sound pressure, and the corresponding angles are that points L and R represent the left and right sound pressure intensity attenuated by 3dB from point A.

| $\theta_s$ ($^\circ$) | $\Delta \tau$ | $\theta_\Delta$ ($^\circ$) | $\theta_{\Delta L}$ | $\theta_{\Delta R}$ | $L - R$ ($^\circ$) |
|-----------------------|--------------|-----------------|------------------|------------------|-----------------|
| 10                    | lus          | 9.729           | -12.94           | -6.635           | 6.305           |
|                       | Ins          | 9.958           | -13.17           | -6.921           | 6.249           |
| 20                    | lus          | 17.12           | -20.56           | -14.08           | 6.48            |
|                       | Ins          | 19.87           | -23.25           | -16.85           | 6.4             |
| 30                    | lus          | 24.85           | -28.46           | -21.73           | 6.73            |
|                       | Ins          | 29.61           | -33.33           | -26.69           | 6.64            |
Table 4. Difference frequency \( f_d = 2000\text{Hz} \), audible beam directivity and beam width

| \( \theta_1 (\degree) \) | \( \Delta \tau \) | \( \theta_s (\degree) \) | \( \theta_{3,\beta}^L \) | \( \theta_{3,\beta}^R \) | \( L - R (\degree) \) |
|----------------|--------|----------------|----------------|----------------|----------------|
| 10             | 1\mu s | 9.7            | -12.94         | -6.635         | 6.305          |
|                | 1ns    | 10.07          | -13.22         | -7.036         | 6.184          |
| 20             | 1\mu s | 17.58          | -20.5          | -14.08         | 6.42           |
|                | 1ns    | 19.76          | -23.02         | -16.55         | 6.47           |
| 30             | 1\mu s | 24.85          | -28.46         | -21.76         | 6.7            |
|                | 1ns    | 30.01          | -33.51         | -26.97         | 6.54           |

From the above chart we can draw the following conclusions. The main beam positions are 9.729° and 9.958° at different delay accuracy when the deflection angle is 10°, and the difference frequency waves are 1000Hz; and the main beam positions are 9.7° and 10.07°, respectively when the difference frequency waves are 2000Hz. The actual deflection angle and the target angle are within 10° of 0.5°, and the beam directivity is basically the same. It shows that when the deflection angle is small, different delay accuracy has little effect on the angle of the difference frequency sound. It can be seen that when the deflection angle is greater than 20°, the directivity of the difference frequency sound under different delay accuracy has been significantly different. It can be seen that when the deflection angle is gradually increased, the angular error caused by the delay accuracy is also increasing.

In order to further analyze the relationship between the delay accuracy and the error of the difference frequency sound angle, we take the delay accuracy to 10ns. And the deflection angle is set to 30°. The simulations of difference frequency sound waveforms are performed separately. The data are as follows:

Table 5. Simulation results of audible beam directivity and beamwidth at different frequencies under delay accuracy \( \Delta \tau = 10\text{ns} \)

| \( \theta_1 (\degree) \) | \( f \) (Hz) | \( \theta_s (\degree) \) | \( \theta_{3,\beta}^L \) | \( \theta_{3,\beta}^R \) | \( L - R (\degree) \) |
|----------------|--------|----------------|----------------|----------------|----------------|
| 30             | 1000   | 29.32          | -33.12         | -26.27         | 6.85           |
|                | 2000   | 29.32          | -33.13         | -26.29         | 6.84           |

It can be seen from Table 5 that the deviation between the actual deviation and the target angle is still greater than 0.5° at the delay accuracy of 10ns and the maximum deflection angle of 30°. This cannot meet the target requirements. Combining the data in Tables 3 and 4, the actual deflection angle is 0.39° when the delay accuracy is 1ns, and the target deflection is 30°. The target angle deviation is 0.01 correspondingly. Based on the above analysis, it can be seen that the delay accuracy needs controlled within 1ns and the audible sound beam error is below 0.5°. Only in this way, deflection angle can be in the range of -30° ~ 30°. At this time the array element spacing \( l \) is equal to the carrier wavelength \( \lambda \).

3. System design

The delay of this system is a combination of coarse delay control and fine delay control. This method can obtain a delay with an accuracy of 1ns.

3.1 Coarse delay control

The system uses the advantages of DDS technology to discretize the continuous waveform data and store it in the waveform ROM. By adjusting the starting position of the transmission channel data, the digital phase difference of the digital waveforms of different channels is generated. After D/A conversion, the phase difference of the analog signal is converted. The ultrasound generated by the system in this paper is a narrowband sinusoidal signal of 40kHz. The number of bits of the DDS phase adder used is 10 bits, and the corresponding number of quantization points is 1024.

In the system, the phase resolution of a periodic waveform is given by:

\[
\Delta \phi = \frac{2\pi}{1024}
\]  

(8)
According to the phase time relationship, the delay resolution can be obtained:
\[ \Delta \phi = \omega \Delta t \]  \hspace{1cm} (9)
\[ \omega = 2\pi f \] \hspace{1cm} (10)
\[ \Delta t = \frac{1}{1024 f} \] \hspace{1cm} (11)

Therefore, the delayed accuracy is \( \Delta t = 24.41 \text{ns} \).

3.2 Fine delay control

The fine delay diagram is shown in Figure 7. Generally, the delay is mainly controlled by the system clock, which is an integer multiple of the system clock period. In this paper, the input clock of a 50MHz crystal system is used, and then it is multiplied to 200MHz as the system clock. The clock cycle is 5ns. Based on the coarse phase-controlled delay, the phase-shifted 200MHz output is phase-shifted to 0°, 72°, 144°, 216°, and 288° with 5ns five-way clocks by using the phase-locked loop phase-shifting feature of FPGA. Each phase is phase-shifted by 72° relative to the previous one. In this way, a fine phase control delay of 1 ns is achieved.

Combining the DDS coarse delay and the system fine delay, the phase-controlled delay is:
\[ T_{\text{delay}} = a \times T_a + b \times T_b \] \hspace{1cm} (12)

Where, \( a \) and \( b \) are the number of coarse delays and the number of fine delays, respectively.

4. Experimental results

In order to verify whether the system meets the requirements of phased array emission, an experimental platform is built to test and analyze the phased delay of the phased ultrasonic transmission signal. The 16-channel phase-controlled ultrasound system is designed. FPGA is used as the system's data generation and control center. The main function is to generate a digital sine signal at 40 kHz and implement the on-chip delay algorithm. An external 16-channel signal conditioning circuit is used to complete the analog-to-digital conversion of the signal, low-pass filtering and power amplification, and the ultrasonic excitation signal whose output power amplitude meets the design requirements. The transducer used in the laboratory is a small air ultrasonic transducer. The array element diameter is 16mm, the array channel number is 16, the array element spacing is 8mm, and the center frequency is 40kHz.

The element phase delay difference is obtained when deflected by 10° according to equation (12), and MedelSim SE is used to perform phase delay simulation between 16 channels. The results are shown in Figure 8 and Figure 9. Tektronix TDS3044B oscilloscope is used for measurement. The oscilloscope has a bandwidth of 400MHz and a sampling frequency of 5GS/s. The measurement data are shown in Table 6. According to the combination of phase delay and system delay, the maximum delay error between the measured delay and the set delay is 1ns. The output signal is stable, which can achieve multi-channel phase control. The oscilloscope is used to measure the delay between channel signals. Due to space limitations, only the actual output waveforms of some channels are given here.
Figure 10 and 11 show the delays of channel 1 and channel 2 (adjacent channels), channel 1 and channel 3, respectively. It can be seen that the measured data are consistent with the theoretical values.

Table 6. Signal Delay Measurement

| Channel | Theoretical delay(us) | Measured delay(us) | Absolute error(ns) |
|---------|-----------------------|--------------------|--------------------|
| 1       | 0                     | 0                  | 0                  |
| 2       | 4.0858                | 4.085              | -0.8               |
| 3       | 8.1717                | 8.171              | -0.7               |
| 4       | 12.2575               | 12.258             | 0.5                |
| 5       | 16.3434               | 16.344             | 0.6                |
| 6       | 20.4292               | 20.429             | -0.2               |
| 7       | 24.515                | 24.515             | 0                  |
| 8       | 28.6                  | 28.601             | 1                  |
| 9       | 32.6867               | 32.687             | 0.3                |
| 10      | 36.7726               | 36.773             | 0.4                |
| 11      | 40.8584               | 40.858             | -0.4               |
| 12      | 44.944                | 44.945             | 1                  |
| 13      | 49.03                 | 49.029             | -1                 |
| 14      | 53.1159               | 53.116             | 0.1                |
| 15      | 57.2017               | 57.202             | 0.3                |
| 16      | 61.2876               | 61.287             | -0.6               |

5. Conclusion
In this paper, the problem of carrier control accuracy of parametric array loudspeakers at 40kHz is studied. The effects of delay accuracy on the ultrasonic carrier and the beam directivity of audible sound were analyzed through theoretical derivation and simulation. It is concluded that the delay accuracy has a small effect on the beam deflection angle at a small deflection angle; when the deflection angle is large, the impact of the delay accuracy error on the target angle cannot be ignored. In order to solve the problem, a parametric array phased control system is designed. The delay resolution of the system is 1ns, and the delay between channels can be obtained by calculating the phase delay and the system delay. Compared with the traditional parametric array with a single carrier and low delay accuracy, the phased transmission system designed in this paper has the characteristics of independent channel phase control, accurate delay, stable transmission signal, and small distortion and interference.
References

[1] Westervelt P J. (1963) Parametric acoustic array[J]. J. Acoust. Soc. Am., 35(4): 535–537.

[2] Luo Ying, Wang Wei, Wang Ziping, Xu Jia. (2010) Research on FPGA Phased Array Phased Transmission System Based on FPGA [J]. Instrument Technology and Sensor, (10): 51-53 + 71.

[3] Li Yanzhong, Yang Jimin, Li Wenwen, Yang Juan, Li Dapeng. (2011) Research on STM32-based ultrasonic phased array blind guide system [J]. Modern Electronic Technology, 34 (16): 14-16.

[4] Assef A A, Maia J M, Schneider F K, et al. (2013) A reconfigurable arbitrary waveform generator using PWM modulation for ultrasound research [J]. BioMedical Engineering OnLine, 12 (1): 24.

[5] Kim Y, Hall T L, Xu Z, et al. (2014) Transcranial Histotripsy Therapy: A Feasibility Study [J]. IEEE Transactions on Ultrasonics Ferroelectrics and Frequency Control, 61 (4): 582-593.

[6] Du Chunhui. (2017) Design of Ultrasonic Phased Array Transmission System Based on FPGA (English) [J]. Electronic Devices, 40 (02): 321-325.

[7] Yijun Shi. (1998) Modeling of acoustic waves for linear phased arrays. Master of science dissertation. Massachusetts Institute of Technology.

[8] Berktay H O. (1974) Farfield performance of parametric transmitters[J]. J. Acoust. Soc. Am., 55(3):539-546.

[9] Yang J , Tan K S , Gan W S , et al. (2005) Beamwidth Control in Parametric Acoustic Array[J]. Japanese Journal of Applied Physics, 44(9A):6817-6819.

[10] Hansen R C. (2009) Phased array antennas[M]. New York: Wi-ley.