Rotation as an origin of high energy particle collisions

O. B. Zaslavskii

Department of Physics and Technology,
Kharkov V.N. Karazin National University,
4 Svoboda Square, Kharkov 61022, Ukraine and
Institute of Mathematics and Mechanics, Kazan Federal University,
18 Kremlyovskaya St., Kazan 420008, Russia

We consider collision of two particles in rotating spacetimes without horizons. If the metric coefficient responsible for rotation of spacetime is big enough, the energy of collisions in the centre of mass frame can be as large as one likes. This can happen in the ergoregion only. The results are model-independent and apply both to relativistic stars and wormholes.

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I. INTRODUCTION

In recent years, interest to high energetic phenomena in the strong gravitational field arose after an important observation made in [1]. It turned out that if two particles moving towards a black hole collide near the horizon, their energy in the centre of mass frame can grow unbound. This is so-called Bañados-Silk-West (BSW) effect. It renewed an interest to more early works on high energy collisions [2] - [4] of particles that moved in opposite directions or on circular orbits. Other mechanisms of high energy collisions were also suggested. Some of them implied that the event horizon either exists or is about to form. Meanwhile, there are also quite different types of collisions not connected with the horizon at all. One of examples is collision inside the ergosphere (generally speaking, not in the vicinity of the horizon) when one of particles has a large negative angular momentum (but modest individual energy) [5], [6]. Quite recently, one more example of high energy collision where the horizon is irrelevant was found. It was shown [7] that head-on collision of particles near the throat of the Teo

*Electronic address: zaslav@ukr.net
wormhole \(^{[8]}\) leads to high \(E_{c.m.}\).

The goal of the present paper is to show that in space-times with rotation rapid enough, high \(E_{c.m.}\) can be obtained (i) both for wormholes and relativistic stars and not only for head-on collisions but also for particles moving in the same directions or orbiting a body or the wormhole throat, (ii) it is an universal phenomenon irrespective of the details of concrete models. The present work extends the list of mechanisms that give rise to high energy particle collisions.

II. BASIC FORMULAS

We consider the stationary axially-symmetric metric

\[
\begin{equation}
\text{ds}^2 = -N^2 \text{dt}^2 + g_\phi (d\phi - \omega \text{dt})^2 + \frac{dr^2}{A} + g_\theta d\theta^2, \tag{1}
\end{equation}
\]

where the coefficients do not depend on \(t\) and \(\phi\). Correspondingly, the energy \(E = -mu_t\) and the angular momentum \(L = mu_\phi\) of a particle moving in this background are conserved. Here, \(m\) is its mass, \(u^\mu\) being the four-velocity. These quantities are related by a simple formula \(^{[9]}\)

\[
\begin{equation}
X = \mathcal{E}N, \tag{2}
\end{equation}
\]

where

\[
\begin{equation}
X = E - \omega L, \tag{3}
\end{equation}
\]

\[
\begin{equation}
\mathcal{E} = \frac{m}{\sqrt{1 - V^2}}, \tag{4}
\end{equation}
\]

has the meaning of the energy measured by the zero angular momentum observer \(^{[10]}\), \(V\) is the local speed measured by this observer.

The equations of motion of a test particle read

\[
\begin{equation}
m \frac{dt}{d\tau} = \frac{X}{N^2}, \tag{5}
\end{equation}
\]

\[
\begin{equation}
m \frac{d\phi}{d\tau} = \frac{L}{g_\phi} + \frac{\omega X}{N^2}, \tag{6}
\end{equation}
\]

\[
\begin{equation}
m \frac{N}{\sqrt{A}} \frac{dr}{d\tau} = \sigma Z, \tag{7}
\end{equation}
\]
where $\sigma = \pm 1$ determines the sign of the radial momentum,

$$Z = \sqrt{X^2 - N^2\left(\frac{L^2}{g_\phi} + m^2\right)}.$$  \hspace{1cm} (8)

Let two particles 1 and 2 collide. Then, one finds for the energy in the centre of mass frame $E_{c.m.}^2 = -(m_1u_1^\mu + m_2u_2^\mu)(m_1u_1\mu + m_2u_2\mu)$ that

$$E_{c.m.}^2 = m_1^2 + m_2^2 + 2m_1m_2\gamma,$$ \hspace{1cm} (9)

where $\gamma = -u_1\mu u_2^\mu$ is the Lorentz factor of relative motion. For simplicity, we restrict ourselves by motion in the equatorial plane $\theta = \frac{\pi}{2}$. It follows from the equations of motion (5) - (7) that

$$m_1m_2\gamma = \frac{X_1X_2 - \sigma_1\sigma_2Z_1Z_2}{N^2} - \frac{L_1L_2}{g_\phi}.$$ \hspace{1cm} (10)

We assume the forward in time condition $\frac{dt}{d\tau} \geq 0$. Then, (5) entails

$$X \geq 0.$$ \hspace{1cm} (11)

We discuss the situation when there is no horizon, so in the point of collision $N = O(1)$. In what follows, we assume that

$$\omega = \varepsilon\omega_0(r) > 0,$$ \hspace{1cm} (12)

where $\omega_0$ is the bounded function and the dimensionless parameter $\varepsilon \gg 1$. We are interested in the individual energies $E = O(1)$. Then, it is seen from (11) that $L \leq 0$. It is worth noting that independently of the sign of $L$, inside the ergoregion $\frac{d\phi}{d\tau} > 0$.

### III. HIGH ENERGY COLLISIONS

We consider three cases separately. It is essential that, if $L \neq 0$, $Z \approx X \approx \varepsilon\omega_0 |L|$ is big.

#### A. Case 1. Head-on collision, $\sigma_1\sigma_2 = -1$.

Hereafter, we assume that, although $g_\phi$ itself can be small (as it takes place for the Teo wormhole [8]), it satisfies the inequality

$$\frac{1}{\omega^2g_\phi} \ll 1.$$ \hspace{1cm} (13)
Then, the main contribution to (10) comes from the terms with $\omega$. If $L_1 \neq 0$, it is seen from (9), (10) that

$$E_{c.m.}^2 \approx \frac{4\omega^2 |L_1 L_2|}{N^2}$$

(14)

can be always made as large as one likes.

If $L_1 = 0$,

$$E_{c.m.}^2 \approx \frac{2\omega |L_2| (E_1 + Z_1)}{N^2}.$$ 

(15)

B. Case 2. Circular motion

Let at least one of particles move on a circular orbit, so $Z_1 = 0$. Then,

$$E_{c.m.}^2 \approx \frac{2\omega^2 |L_1 L_2|}{N^2}$$

(16)

and we arrive at the same conclusion. If $L_1 = 0$,

$$E_{c.m.}^2 \approx \frac{2\omega |L_2| E_1}{N^2}.$$ 

(17)

C. Case 3. Motion in the same direction, $\sigma_1 \sigma_2 = +1$

This case is the most interesting one since fine-tuning is mandatory here in analogy with the BSW effect [1]. We require individual energies $E_{1,2}$ to be finite. Then, one can obtain from (10) that for both nonzero angular momenta $\gamma$ is finite as well, so the effect under discussion is absent.

Let us now assume that $L_1 = 0$ (this can be thought of as a counterpart of the critical particle in the BSW effect [1]) and $L_2 < 0$. Then, it is easy to obtain from (10) that

$$E_{c.m.}^2 \approx \frac{2\omega |L_2| (X_1 - Z_1)}{N^2}$$

(18)

can be made as big as we want.

As far as the relative motion of two particles is concerned, cases 1 and 2 are analogues of particle collisions near a rotating black hole considered in [2] - [4] whereas case 3 is a rather close counterpart of the BSW effect [1]. The quantity $X_1$ is modest whereas $X_2 \sim \omega$. Therefore, it is seen from (2) - (4) that kinematically, case 3 represents collision between a slow particle 1 and rapid particle 2 according to (3) in full analogy with the kinematics of the BSW effect [9].
D. Comparison to the Teo wormhole

For the Teo wormhole, \( \omega = \frac{2a}{r^3} \), and, for motion the equatorial plane, \( g_{\phi} = r^2 \), \( N = 1 \). If collision occurs near the throat, \( r = b \), eq. (14) gives us

\[
E_{\text{c.m.}}^2 \approx \frac{16a^2 |L_1L_2|}{b^6}
\]

that corresponds to eq. (3.8) of [7]. Condition (13) gives us \( \frac{b^4}{a^2} \ll 1 \) in agreement with the assumption \( b \ll \sqrt{a} \) made in Sec III B of [7].

IV. BEHAVIOR OF GEOMETRY

If \( \omega \rightarrow \infty \), the curvature invariants, generally speaking, diverge. The rate of their growth is model-dependent since the parameter \( \varepsilon \) can enter different metric coefficients. For the Teo wormhole, the scalar curvature (see eq. (B1) of [7]) \( R = O(b^{-3}) \) if \( r = b \) and \( R = O(b^{-6}) \) if \( r \neq b \) but has the order \( b \). One has also \( \omega(b) = O(b^{-3}) = O(\varepsilon) \). According to (15), (17), for high energy collision on the throat \( r = b \) the energy in the centre of mass frame behaves according to \( E_{\text{c.m.}}^2 = O(R) \) in case \( L_1 = 0 \) and \( E_{\text{c.m.}}^2 = O(R^2) \) in other cases. Nonetheless, as the growth of curvature is pure classic and does not contain quantum parameters, it is possible to achieve intermediate large energies within the classical region, without entering the Planck scale. Say, one can have simultaneously \( E_{\text{c.m.}}^2 \gg 1 \) and \( R \ll \frac{1}{L^2} \), where \( L \) is some prescribed scale since both inequalities contain different parameters. We do not discuss this issue further since it is strongly model-dependent, meanwhile we would like to make emphasis on model-independent features.

V. DISCUSSION

It is seen from eq. (10) that in general, roughly speaking, there are three main sources of high \( E_{\text{c.m.}} \): (i) small \( N \), (ii) high negative \( L \), (iii) high \( \omega \). Option (i) is related to collisions near black holes [1] - [4]. It applies also to the collisions in the absence of the horizon, provided the parameters of the system correspond to the threshold of its formation and collision occurs just near such a would-be horizon [11], [12]. Option (ii) was suggested in [5] and generalized in [6]. In the present work option (iii) as a generic mechanism was considered that closes the list of possibilities.
In the derivation of our formulas, we assumed that in (3), the quantity $E$ is negligible, provided $L \neq 0$, so

$$\omega \gg \frac{|E|}{|L|}. \quad (20)$$

It is worth stressing that in (20) we require that $\omega$ be large as compared to characteristics of a particle. However, we do not impose here direct restrictions on the parameters of a relativistic object as such, say on the ratio $\frac{J}{M^2}$, where $J$ is its angular momentum, $M$ being its mass. Such parameters can enter (20) indirectly, through the quantity $\omega$ but, anyway, inequality (20) relies on the particle’s energy and angular momentum.

Also, it follows from derivation that in (8) we consider terms $N^2$ as a small corrections. Thus in addition to (20),

$$\omega^2 \gg \frac{N^2}{g_\phi}. \quad (21)$$

Therefore, the metric component $g_{00} = -N^2 + g_\phi \omega^2 > 0$, so the effect takes place in the ergoregion only.

\[ \text{VI. CONCLUSIONS AND OUTLOOK} \]

Among possible types of scenarios leading to high energy collisions, we filled an important gap. We showed that fast rotation by itself leads to the possibility to gain large energy of collision and, in this sense, it represents an universal phenomenon. As far as the issue of large $E_{c.m.}$ is concerned, there is no need in detailed investigation of equations of motion in some particular metrics, the results are obtained in a model-independent way. They apply equally to rotating wormholes (thus generalizing observations made in [7] for the Teo wormhole) and to rotating stars. The latter fact is a completely new venue for such collisions. One can hope that this can be useful for astrophysics since, in contrast to wormhole case, no exotic matter is needed. Also, in the situation under discussion, there are no subtleties connected with the relativistic time dilation and small fluxes [13], [14] since now in the point of collision $N = O(1)$. The rate of growth of $E_{c.m.}$ ($\omega$ or $\omega^2$) depends on the type of collision and the fact whether or not $L_1 = 0$. In any case, $L_2 < 0$. This is valid in the entire region, where $\omega$ is large.

In contrast to the BSW effect near rotating black holes where $\omega$ is close to the angular velocity of a black hole, now $\omega$ is formally as big as one likes, so it is the dragging effect of
the space-time itself that leads to high energy of collisions in an universal manner. The price paid for this is the appearance of large curvatures. However, one can choose the situation when the curvature scale is far from, say the Planck value but, nonetheless, the energy of collision $E_{c.m}$ is high enough. The mechanism under discussion works in the ergoregion only.

Astrophysical applications are beyond the scope of the present paper. However, as a separate issue, it would be of interest to trace the influence of this mechanism on possible instabilities of ergoregion in relativistic stars [15], [16]. Also, it would be interesting to consider the phenomenon under discussion using a more physical example than the Teo wormhole [8] that was written by hand, without solving Einstein equations. In particular, one can take the rotating wormhole obtained as a solution of field equations with the phantom scalar field [17].

The next step should consist in considering scenarios of the collisional Penrose process. For wormholes, this is expected to generalize the results found for the Teo wormhole in [18]. For relativistic stars, this will be a completely new issue.

All this needs separate treatment.

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