Anisotropic transport for $\nu = 2/5$ FQH state at intermediate magnetic field

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The $\nu = 2/5$ state is spin-unpolarized at weak magnetic field and fully polarized at strong field. At intermediate field, a plateau of half the maximal polarization is observed. We study this phenomenon in the frame of composite fermion theory. Due to the mixing of the composite fermion Landau levels, the unidirectional charge/spin density wave state of composite fermions is lower in energy than the Wigner crystal. It means that transport anisotropy, similar to those for electrons in higher Landau levels at half fillings, may take place at this fractional quantum Hall state when the external magnetic field is in an appropriate range. When the magnetic field is tilted an angle, the easy transport direction is perpendicular to the direction of the in-plane field. Varying the partial filling factor of composite fermion Landau level from 0 to 1, we find that the energy minimum occurs in the vicinity of one-half.

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The Coulomb interaction between electrons in two-dimensions plays a dominant role when the kinetic energy is quenched into a series of degenerated Landau levels (LLs), and gives rise to a variety of unusual phenomena. The most famous example is the fractional quantum Hall (FQH) effect, where at certain filling factors the system condenses into the so-called Laughlin liquid which acquires gaps for making charge excitations. Another manifestation of Coulomb interaction is the formation of unidirectional charge density wave (UCDW) or stripe phases in high LLs at half fillings. A unified description of all the fractional quantum states in the lowest LLs $\nu = n/(2pn+1)$ was achieved by the composite fermion (CF) picture, in which each electron carries 2p statistical magnetic quanta to form a composite particle. At a mean field level, the CFs experience an effective magnetic field $B^* = B/(2pn+1)$, in which they fill n CF-LLs, and exhibit integer quantum Hall effects.

In recent years, the spin polarization of electrons for integral as well as fractional quantum Hall states have been studied extensively. Many novel and interesting spin-related phenomena have been revealed. A large amount of data have been accumulated for various fractional filling states, as well as for $\nu = 1$. In a tilted magnetic field experiment carried out by W. Pan et al., the transport becomes highly anisotropic for even-number filling factors ($\nu = 4, 6, \cdots$) when the tilted angle exceeds a large critical value. A new phase called spin density wave was proposed to explain this phenomenon.

It is now well established that for some filling factors ($\nu = 1/m$, m odd number) the ground state is fully spin polarized for all values of Zeeman splitting, while for other filling factors (for example, $\nu = 2/5, 3/7$, etc.) the ground state is fully polarized only for large values of Zeeman splitting but unpolarized for small (or zero) values of Zeeman energy. One interesting observation is that at intermediate values of Zeeman energy there appears a plateau of half the maximal spin polarization for $\nu = 2/5$. The stability of the half-polarized state implies that the ground state energy of the system as a function of spin polarization has non-monotonic behavior at half polarization. The Zeeman energy $E_Z = g_\mu B_{tot}$ favors spin-polarization, while the electron-electron interaction favors a singlet state. However, the origin of the half-polarization phenomenon remains controversial.

According to theory of composite fermion with a spin, each CF-LL is split into two subbands. As the $n = 0 \uparrow$ spin band is always fully occupied, it can be treated as a non-dynamical background. At intermediate $E_Z$, the $n = 0$ $\downarrow$-spin CF-LL and the $n = 1 \uparrow$-spin CF-LL can both be partially occupied with filling factors $\nu_1$ and $\nu_2$, respectively. $\nu_1 + \nu_2 = 1$. The half-polarized state corresponds to $\nu_1 = \nu_2 = 1/2$. The composite fermions occupying the partially filled CF-LLs can be thought as a system consisting of two types of fermions. V.M. Apalkov et al mapped the two-component fermion system onto a system of excitons and described the ground state as a liquid state of excitons with nonzero values of exciton angular momentum. Their calculation reveals that a downward cusp occurs at half the maximal spin polarization. On the other hand, G. Murthy proposed a partially polarized density wave (PPDW) state of CFs based on a Hartree-Fock theory developed earlier. He compared the ground state energies of the PPDW state, in which one set of CDW was placed directly over the other, with the Halperin-(1,1,1) liquid state and found that the PPDW state is slightly lower in energy than Halperin-(1,1,1) state. Activated by the above works, we compare the cohesive energies of the CDW and the UCDW states for CF-LL filling factor $\nu = 2$. Allowing for different stacking possibilities of the two sets of interacting CDW or UCDW states consisting of type-1 and type-2 fermions, we find that one set of the lattice is rigidly displaced with respect to the other. Since each type of fermions has opposite spin, the shifted lattices form an antiferromagnet-like structure. It is called the spin density wave (SDW). We carry out a Hartree-Fock computation for such a system, with the external magnetic field tilted a variety of angles.
The results show that the cohesive energy of the shifted UCDW is steadily lower than that of the shifted WC’s, which means that the magneto-transport anisotropy, similar to those for electrons in high LLs at half fillings, may take place in the lowest LL at \( \nu = 2/5 \) provided that the Zeeman energy is adjusted to an appropriate range. The easy transport direction is always perpendicular to the direction of the in-plane field. We also calculate the Zeeman energy is adjusted to an appropriate range.

Suppose the two-dimensional electron system is confined by a harmonic potential in the \( z \)-direction with the characteristic frequency \( \Omega \). The external magnetic field is tilted an angle \( \theta \) with \( \mathbf{B} = (B \tan \theta, 0, B) \). According to G. Murthy \textit{et al.}, the Hartree-Fock theory in terms of CF variables gives a reasonably good account of physical properties. The wave functions for the \( n \)th CF-LL are:

\[
\phi_{n,X}(r) = \frac{1}{\sqrt{l_y}} e^{-i\frac{x}{\omega} x} \Phi_0^\omega ((x - X) \sin \hat{z} + z \cos \hat{z})
\]

where \( l \) is the magnetic length in the effective field and \( X \) is an integer multiple of \( 2\pi l^2/L \). \( \Phi_0^\omega \) is the harmonic oscillator wave function corresponding to the frequency \( \omega \) and \( \tan \tilde{\theta} = \frac{\omega}{\tilde{\omega}_x} \tan \theta \). The frequency \( \omega_{\pm} \) are given by

\[
\omega_{\pm} = \frac{1}{2} (\Omega^2 + \frac{\omega^2}{\cos^2 \theta}) \pm \sqrt{\frac{1}{4} (\Omega^2 - \frac{\omega^2}{\cos^2 \theta})^2 + \Omega^2 \omega_{\pm}^2 \tan^2 \theta}.
\]

(2)

The electron density operator is expressed in the momentum space as

\[
\hat{\rho}(\mathbf{q}) = \sum_{n,n',X} e^{-i\mathbf{q} \cdot \mathbf{r}} \rho_{n,n',X}^\dagger \rho_{n,n',X}^\dagger (\mathbf{q}),
\]

where \( X = X_\pm = X \pm \frac{q_y l^2}{2} \). \( \rho_{n,n',X} \) destroys a CF in the state \( \phi_{n,X}(r) \). The matrix element \( \rho_{n,n',X} \) can be computed with the one-particle states Eq. (3). The Hamiltonian can now be written as:

\[
H = \frac{1}{2L_x L_y} \sum_{\mathbf{q},X} v(q)e^{-i\mathbf{q} \cdot \mathbf{X}} \rho_{n_1 n_2}(\mathbf{q})
\]

\[
\times \rho_{n_3 n_4}(\mathbf{q})^\dagger \rho_{n_5 n_6}^\dagger (\mathbf{q})^\dagger \rho_{n_7 n_8} (\mathbf{q}),
\]

(4)

where \( v(q) = 2\pi \alpha e^2 / q \). Equation (4) is the correct form for the CF Hamiltonian, and the energy coming from normal ordering represents the Hartree interaction of an electron with its own correlation hole.

By using the standard manipulation, we can write explicitly the effective potential \( U_{nn',\mathbf{q}} \) as a sum of a Hartree term (in units of \( e^2 / 4kd \))

\[
H_{nn',\mathbf{q}} = \int \frac{dq_y}{\pi} \frac{1}{q_y^2 + q_x^2} |F_{nn',\mathbf{q}}^\theta|^2
\]

and a Fock term

\[
X_{nn',\mathbf{q}} = -2\pi i^2 \int \frac{d\mathbf{p}}{(2\pi)^3} H_{nn',\mathbf{p}} e^{i\mathbf{p} \cdot \mathbf{q}^2},
\]

(6)

where \( F_{nn',\mathbf{q}}^\theta \) is given by

\[
F_{nn',\mathbf{q}}^\theta = (n'/n)^2 (a^2 / 2)^{(n-n')/2} e^{-\gamma^2/4 - a^2/4}
\]

\[
\times L_{nn'}^{n-n'} (a^2 / 2)
\]

(7)

with

\[
\alpha^2 = (q_x \cos \theta - q_z \sin \theta)^2 l_2^2 + q_y^2 l_4^2 / l_2^2 \cos^2 \theta
\]

\[
\gamma^2 = (q_x \cos \theta + q_z \sin \theta)^2 l_2^2 + q_y^2 l_4^2 / l_2^2 \sin^2 \theta.
\]

(8)

Here \( l_2^2 = h / m\omega_\pm \) and \( L_{nn'}^m (x) \) is the Laguerre polynomial.

We adopt the Hartree-Fock (HF) approximation in a similar form introduced by Coté and MacDonald for double-layer systems only. The layer-index is replaced by the type-index in our case.

Allowing the charge density wave by making ansatz

\[
< c_{nnX}^\dagger c_{n'X} c_{nX} c_{n'X} > = e^{iQ\cdot X} \Delta_{nn'}(Q),
\]

(9)

we carry out a HF computation on the UCDW and the triangular lattice. We assume that \( \Delta_{nn} \) are nonzero only for \( n = 0, 1 \). The cohesive energy can be calculated in the same way as it has been done for the WC.

\[
E_{coh} = \frac{1}{2} \sum_{Q \neq 0} \{ U_{00}(Q) + U_{11}(Q) \}
\]

\[
+ 2 \cos\mathbf{Q} \cdot \mathbf{a} |H_{01}(Q)| |\Delta_{00}(Q)|^2,
\]

(10)

where \( \mathbf{a} \) is the relative shift of the two sets of lattice. Here we have not counted in the Zeeman energy and the uniform direct interactions. The latter are cancelled with the neutralizing positive backgrounds. The interactions of the type-1 and type-2 fermions with the CFs in the fully filled lowest \( n = 0 \) \( \uparrow \) spin CF-LL subband are also omitted.

In our computations, the relative shift of the two sets of CDW reduces the energy significantly. For the triangular lattice in the perpendicular magnetic field, the cohesive energy of the unshifted lattices is -0.0961 (in units of \( e^2 / 2kd \)). The relative displacement \( \mathbf{a} = \frac{1}{2} \mathbf{A}_0 \) reduces the energy to -0.1242. For the UCDW state, the energy reduction for shifted lattice is even larger. This reduction of energy for shifted lattice is attributed mainly to the lack of exchange symmetry between the type-1 and type-2 fermions. Fig.1 shows that the shifted UCDW state is always preferable to the shifted WC state for all of the tilted angles.

As is in the case of high LLs, a stable UCDW ground state or stripes, leads to the anisotropy in magneto-transport experiment. As each set of the UCDW has opposite spin polarization, the shifted lattice forms a pattern of antiferromagnet. We call this new phase the unidirectional spin density wave (USDW). The space period of the USDW.
is $\sim 5.2l$. For completeness, we have also considered the triangular lattice of "bubbles", with each bubble containing in general several electrons by using the scheme developed by M.M. Fogler et al. Our result shows that the lattice of one electron per "bubble" has the lowest cohesive energy.

In Fig.2 we calculate the cohesive energy of shifted UCDW by varying the partial filling factor $\nu_1$ from 0 to 1 for zero in-plane magnetic field. We assume that the space periods of the CDWs are the same for both CF-LLs bands. It can be seen that the energy minimum appears at half filling, which means $\nu_1 = \nu_2$. This figure reflects the typical characteristics of Fig.1 in Ref.10. It indicates that the state of half to half occupation for $n = 0 \downarrow$-spin CF-LL and the $n = 1 \uparrow$-spin CF-LL is energetically more stable than the full occupation of only one sub-band of the CF-LLs. Taking account of the fully occupied $n = 0 \uparrow$-spin band, the total polarization is one-half.

Because of the small $g$ factor of electrons in GaAs, spins may not be fully polarized in FQH states. Transitions between singlet, partially polarized, and fully polarized states for a number of fractional fillings can be understood in terms of CFs with a spin$^{6,9,12}$. For $\nu = 2/5$, the transition takes place when the unoccupied $n = 1 \uparrow$-spin CF-LL subband crosses the occupied $n = 0 \downarrow$-spin CF-LL subband as the Zeeman energy increases. If the cross is trivial, nothing interesting will happen. However, the competition between exchange and direct Coulomb interaction of CFs results in the spontaneously breakdown of translational symmetry.

The mechanism of the USDW in this work is to some extend similar to the isospin stripes at integer filling factors for the double-layer systems at $\nu = 4n + \frac{1}{2}$, in which the real spin-index is replaced by the isospin-index. The origin of such isospin stripe order is a competition between the exchange and the direct Coulomb interaction. Exchange favors accumulating all the electrons in one layer(in order to maximize the isospin exchange field), whereas direct Coulomb energy is lower when the electron density is distributed uniformly between the layers. It is also interesting to compare our prediction of USDW to the anisotropic transport for even-number filling factors ($\nu = 4, 6, \cdots$) when the magnetic is tilted to a large angle$^{10}$. It can easily be seen that the unidirectional spin density wave in this work is just a CF-version of the unidirectional spin density wave of electrons suggested in Refs.$^{10,17}$.

Hitherto, transport anisotropy was experimentally observed only in high LLs$^{22}$. Our results predict that in the lowest LL, there may also exist stripe phases at some fractional filling factors ($\nu = 2/5, 3/7, \cdots$), provided that the Zeeman energy $E_Z$ is set to an appropriate value. Since the space period of the USDW is large enough, the USDW is stable against the quantum fluctuations in the lowest LL. We note that S.Y. Lee et al. proposed another kind of spontaneous stripe order at certain even-denominator fractions in the lowest LL. They argued that for Landau level filling factor of the form $\nu = (2n + 1)/(4n + 4)$, which corresponds to CF filling factor $\nu^* = n + 1/2$, the system phase separates into stripes of $n$ and $n + 1$ filled CF-LLs. Our picture of USDW essentially differs to theirs.

In summary, we have computed the ground-state energies of USDW and WC consisting of composite fermions. We find that the relative shift of the two sets of interacting CDW lattices reduces remarkably the cohesive energy of the partially polarized density wave state of CFs proposed by G. Murthy$^{21}$. As the external magnetic field is tilted an angle, the USDW is always energetically preferable, which means that anisotropic transport may be observed in the lowest LL at $\nu = 2/5$ if the Zeeman energy is properly adjusted. We also found that half to half occupation of the $n = 0 \downarrow$-spin CF-LL and the $n = 1 \uparrow$-spin CF-LL is more stable than the full one-band occupation. Experimental observation of the stripes in the shallowest LL will support the concept of such antiferromagnet-like charge/spin density wave of composite fermions as we have described in the context.

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1 Perspectives in Quantum Hall Effects, edited by Das Sarma S and Pinczuk A 1996 John Wiley
2 Lilly M P, Cooper K B, Eisenstein J P, Pfeiffer L N and West K W 1999 Phys. Rev. Lett. 83 824
3 Du R R, Tsui D C, Stormer H L, Pfeiffer L N and West K W 1999, Solid Stat. Commun. 109 389
4 Stancu T, Martin I and Phillips P 2000 Phys. Rev. Lett. 84 1288
5 Jungwirth T, MacDonald A H, Smrcka L and S.M. Girvin 1999 Phys. Rev. B 60 15574
6 Jain J K 1989 Phys. Rev. Lett. 63 199 1990 Phys. Rev. B 41 7653
7 Composite Fermions, edited by Olle Heinonen 1998 World Scientific
8 Eisenstien J P, Stormer H L, Pfeiffer L N and West K W 1989 Phys. Rev. Lett. 62 1540 Furneaux J E, Sypers D A and Swanson A G 1989 Phys. Rev. Lett. 63 1098
9 Du R R, Yeh A S, Stormer H L, Tsui D C, Pfeiffer L N and West K W 1995 Phys. Rev. Lett. 75 3926 Du R R, Yeh A S, Stormer H L, Tsui D C, Pfeiffer L N and West K W 1997 Phys. Rev. B 55 R7351
10 Chakraborty T and Pietiläinen P 1996 Phys. Rev. Lett. 76 4018
11 Leadley D R, Nicholas R J, Maude D K, Utjuh A N, Portai J C, Harris J J and Foxon C T 1997 Phys. Rev. Lett. 79 4246
12 Khandelwal P, Kuzma N N, Barrett S E, Pfeiffer L N and West K W 1998 Phys. Rev. Lett. 81 673
FIGURES

Figure 1 The cohesive energies (in units of $e^2/\kappa_0 l$) of shifted USDW and shifted WC versus tilted angle $\theta$ for $\Omega/\omega_c = 3.0$. The inset is the rescaled energy for USDW. It can be seen that the cohesive energy of the USDW state decreases as the tilted angle increases.

Figure 2 The cohesive energy $E_{coh}$ of USDW versus partial filling factor $\nu_1$. The minimum of energy at $\nu_1 = 1/2$ indicates that the half to half occupation of CF spin-subbands is energetically preferable to the full occupations of only one CF spin-subband.
