Moduli constraints on primordial black holes

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Abstract. The amount of late decaying massive particles (e.g., gravitinos, moduli) produced in the evaporation of primordial black holes (PBHs) of mass $M_{BH} \lesssim 10^9$ g is calculated. Limits imposed by big-bang nucleosynthesis on the abundance of these particles are used to constrain the initial PBH mass fraction $\beta$ (ratio of PBH energy density to critical energy density at formation), as: $\beta \lesssim 5 \times 10^{-19} (x_\phi/6 \times 10^{-3})^{-1} (M_{BH}/10^9 \text{ g})^{-1/2} (\bar{Y}_\phi/10^{-14})$; $x_\phi$ is the fraction of PBH luminosity going into gravitinos or moduli, $\bar{Y}_\phi$ is the upper bound imposed by nucleosynthesis on the number density to entropy density ratio of gravitinos or moduli. This notably implies that such PBHs should never come to dominate the cosmic energy density.

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1. Introduction – The spectrum of locally supersymmetric theories generically contain fields whose interactions are gravitational, and whose mass \( m_\phi \sim \mathcal{O}(100 \text{ GeV}) \). The Polonyi and gravitino fields of supergravity theories, or the moduli of string theories, are such examples. This leads to well-known cosmological difficulties: quite notably, such particles (hereafter generically noted \( \phi \) and termed moduli) decay on a timescale \( \tau_\phi \sim M_{\text{Pl}}^2/m_\phi^3 \sim 10^8 \text{s (}m_\phi/100 \text{ GeV})^{-3} \), i.e., after big-bang nucleosynthesis (BBN), and the decay products may drastically alter the light elements abundances \( \text{[1]} \). The success of BBN predictions provides in turn a stringent upper limit on the number density to entropy density ratio (\( Y_\phi \)) of these moduli, generically \( Y_\phi < \sim 10^{-14} \text{[2]} \) (see Sec. 3).

It is argued in this letter that these same constraints can be translated into stringent constraints on the abundance of primordial black holes (PBHs) with mass \( M_{\text{BH}} < \sim 10^9 \text{g} \). In effect, moduli are expected to be part of the Hawking radiation of an evaporating black hole as soon as the temperature of the black hole exceeds (roughly speaking) the rest-mass \( m_\phi \); and indeed, the Hawking temperature of a PBH reads \( T_{\text{BH}} \equiv m_\phi^2/M_{\text{BH}} \simeq 10^4 \text{GeV (}M_{\text{BH}}/10^9 \text{g})^{-1} \text{[3]} \).

Primordial black holes are liable to form in the early Universe at various epochs, e.g., when a density fluctuation re-enters the horizon with an overdensity of order unity \( \text{[4]} \), or when the speed of sound vanishes \( \text{[5]} \) (as may occur in phase transitions). As a consequence, constraints on the abundance of PBHs can be translated into constraints on the structure of the very early Universe \( \text{[6]} \). Until recently, the only existing constraint on PBHs of mass \( M_{\text{BH}} < \sim 10^9 \text{g} \) relied on the assumption that via evaporation, PBHs leave behind stable Planck mass relics \( \text{[7]} \). However, recent work from the perspective of string theories seems to indicate that this is not the case \( \text{[8]} \), in particular that evaporation proceeds fully. Nevertheless, Green \( \text{[9]} \) has pointed out recently that such PBHs would also produce supersymmetric particles, and consequently, cosmological constraints on the lightest supersymmetric particle (LSP) density could be turned into constraints on the initial PBH mass fraction \( \beta \) (defined as the ratio of PBH energy density to critical energy density at formation). This constraint relies on the assumption that the LSP is stable, i.e. \( R \)–parity is a valid symmetry; and, as attractive as \( R \)–parity is, it is not of a vital necessity altogether. The constraint related to the production of gravitinos or moduli, to be derived below, is thus complementary to this \( R \)–parity constraint, and it also turns out to be more stringent. Hereafter, units are \( \hbar = k_B = c = 1 \), and \( m_{\text{Pl}} \equiv M_{\text{Pl}}/(8\pi)^{1/2} \simeq 2.4 \times 10^{18} \text{GeV} \) is the reduced Planck mass.

2. Moduli production – Although one is generally interested in \( Y_\phi \) itself, and not in its momentum dependence, it will prove necessary in a first approach to keep track of \( dY_\phi/dk \) (where \( k \) is the momentum) integrated over the black hole lifetime. In effect, during their evaporation, PBHs produce moduli over a whole spectrum of momenta, with high Lorentz factors, and the existing constraints on \( Y_\phi \) depend strongly on the (cosmic) time at which moduli decay (\( \tau_\phi \) is the decay timescale in the modulus rest frame), hence on whether they are relativistic or not.

More quantitatively, the mass and temperature of a PBH evolve with time \( t \) during evaporation as: \( M(t) = M_{\text{BH}}[1 - (t - t_i)/\tau_{\text{BH}}]^{1/3} \) and \( T(t) = T_{\text{BH}}[1 - (t - t_i)/\tau_{\text{BH}}]^{-1/3} \text{[3]} \). Here, \( t_i \) denotes the time of formation, \( t_i \ll \tau_{\text{BH}} \), with \( \tau_{\text{BH}} \) the PBH lifetime: \( \tau_{\text{BH}} \simeq \ldots \text{[3]} \).
0.14 s (MBH/10^9 g)^3 \[\text{[1]}\]. Toward the end of the evaporation process, the temperature increases without apparent bound, although the standard analysis breaks down at T \sim m_{Pl} \text{ (see Ref. [3] for a discussion of the end point of evaporation). Once the black hole temperature } T \gg m_{\phi}, \text{ moduli can be considered as massless. Then the number of moduli emitted per PBH with momentum } k \text{ between } k \text{ and } k + dk; \text{ and per unit of time, is, for a Schwarzschild black hole [3]} q_{\phi}(k, t) = (2\pi)^{-1} \Gamma_{\phi}(M(t), k)/[\exp(k/T(t)) - (1)^{2s}]. \text{ The absorption coefficient } \Gamma_{\phi} \text{ is a non-trivial function of } M, \text{ and } s \text{ which has to be calculated numerically [10], and } s \text{ is the spin of } \phi. \text{ As announced any PBH will thus produce moduli at some point, and, moreover, these moduli will be produced over a whole range in momentum. To give an example of the sensitivity of the constraints on } Y_{\phi} \text{ on the time of decay: if } \phi \text{ decays into photons, pair creation on the cosmic background (of temperature } T_{\gamma} \text{) suppresses cascade photons whose energy } E \gtrsim \frac{m_{\phi}^2}{22T_{\gamma}} \text{; since } T_{\gamma} \simeq 1 \text{ MeV } (t/1 \text{ s})^{-1/2}, \text{ at early times } t \lesssim 10^4 \text{ s, the cut-off lies below the threshold of deuterium photo-dissociation (} \sim 2 \text{ MeV)}, \text{ and the constraints on } Y_{\phi} \text{ are evaded, while at later times, the cut-off is pushed above this threshold, and photo-dissociation becomes highly effective. Finally, since a modulus carrying momentum } k \text{ at cosmic time } \tau_{\phi} \text{ will decay at time } t \sim \tau_{\phi} \max[(k/m_{\phi})^{3/2}, 1], \text{ it is necessary to follow } dY_{\phi}/dk \text{ as a function of time. As an aside, this will permit the calculation of } Y_{\phi} \text{ produced by PBHs such that } T_{BH} < m_{\phi}.\text{ }

This calculation is carried out below in the following limits. As a first approximation, it is sufficient to assume that all } \phi \text{ particles are emitted at the same average energy, parametrized as } \alpha T(t); \alpha \text{ is a constant which depends on } s, \text{ with } \alpha \approx 2.8 \text{ for } s = 0, \alpha \sim 4 \text{ for } s = 1/2, \text{ and } \alpha \sim 7 - 8 \text{ for } s = 3/2 \text{ [11]. This approximation suffices as the energy at peak flux corresponds to the average energy to within } \approx 10\% \text{ [11], and since the injection spectrum cuts-off exponentially for } k > \alpha T, \text{ and as a power-law for } k < \alpha T. \text{ The initial mass fraction of PBHs is approximated to a delta function centered on } M_{BH}. \text{ Although recent considerations tend to indicate otherwise [12], this remains a standard and simple approximation; moreover, the extension of the results to a more evolved mass fraction is easy to carry out. Finally, it is also assumed that the Universe is radiation dominated all throughout the evaporation process, which implicitly implies that black holes never dominate the energy density. This latter assumption will be justified in Section 3.}

Then the distribution } f_{\phi}(k, t) \equiv s^{-1}dn_{\phi}/dk = dY_{\phi}/dk, \text{ where } s \text{ denotes the radiation entropy density, at times } \tau_{BH} < t < \tau_{\phi} \text{ reads:}

\[ f_{\phi}(k, t) = Y_{BH} \int_{t_{i}}^{\tau_{BH}} q_{\phi}(k', t') \frac{dk'}{dk} dt'. \tag{1} \]

\[ \text{1 The lifetime of a black hole depends on the number of degrees of freedom } g_x \text{ in each spin } s \text{ in the radiation [11], i.e. } \tau_{BH} = 6.2s f(M_{BH})^{-1}(M_{BH}/10^9 g)^3, \text{ with } f(M_{BH}) \sim 0.267g_0 + 0.147g_{1/2} + 0.06g_1 + 0.02g_{3/2} + 0.007g_2. \text{ Here the particle content of the minimal supersymmetric standard model (MSSM) with unbroken supersymmetry has been used, } g_0 = 98, g_{1/2} = 122, g_1 = 24, g_{3/2} = 2, g_2 = 2.\]

\[ \text{2 The value of } \alpha \text{ for } s = 3/2 \text{ is based on extrapolation of the results of Ref. [11] for other spins, while the fraction of luminosity emitted in spin } s = 3/2 \text{ (noted } x_{s} \text{ in the following) is given in Ref. [11]. It does not seem that a detailed study of Hawking radiation of gravitinos has ever been performed. Here it is assumed that the helicity states } \pm 1/2 \text{ and } \pm 3/2 \text{ of the gravitino are produced with values of } \alpha \text{ and } x_{\phi} \text{ as quoted for generic spin } s = 1/2 \text{ and } s = 3/2 \text{ respectively.}\]
In this expression, \( q_\phi(k', t') \) is the injection spectrum per black hole as above, \( Y_{\text{BH}} \equiv n_{\text{BH}}/s \), where \( n_{\text{BH}} \) represents the PBH number density, and \( k' \equiv k a(t)/a(t') \), where \( a \) is the scale factor. The factor \( d k'/d k \) results from redshifting of \( k' \) at injection time \( t' \) down to \( k \) at time \( t \). Equation (1) can be derived as the solution of the transport equation:

\[
\partial_t f_\phi = H \partial_k (k f_\phi) + Y_{\text{BH}} q(k, t),
\]

where \( H \) is the Hubble scale at time \( t \), and the first term on the r.h.s accounts for redshift losses. This equation and its solution Eq. (1) are valid for \( t \ll \tau_\phi \), when the decay of \( \phi \) particles can be neglected. It should be recalled that in the range of masses \( m_\phi \) and \( M_{\text{BH}} \) considered, indeed \( \tau_{\text{BH}} \ll \tau_\phi \). Equation (1) also neglects the entropy injected in the plasma by PBH evaporation, which remains a good approximation as long as PBHs carry only a small fraction of the total energy density at all times.

For mono-energetic injection at \( k' = \alpha T(t') \):

\[
q_\phi(k', t') = \frac{x_\phi}{\alpha T(t')} \left| \frac{dM}{dt'} \right| \delta[k' - \alpha T(t')].
\]

Here \( x_\phi \) denotes the fraction of PBH luminosity \( |dM/dt'| \) carried away by moduli; for the MSSM content, \( x_\phi \simeq 6 \times 10^{-3} \) for \( s = 0 \) with one degree of freedom (e.g., a modulus field), \( x_\phi \simeq 6 \times 10^{-3} \) for \( s = 1/2 \) with two degrees of freedom (e.g., helicity \( \pm 1/2 \) states of the gravitino), and \( x_\phi \simeq 9 \times 10^{-4} \) for \( s = 3/2 \) with 2 degrees of freedom (e.g., helicity \( \pm 3/2 \) states of the gravitino) \([11]\) (see also previous footnote). The \( \delta \) distribution can be rewritten as a function of \( t \), using the identity: \( \delta[f(t)] = |df/dt|^{-1} \delta(t - t_s) \), where \( t_s \) is such that \( f(t_s) = 0 \) (here \( t_s \) is uniquely and implicitly defined in terms of \( k, k' \)). Equation (1) can be integrated in the limits \( k \ll k_0 \) and \( k \gg k_0 \), where \( k_0 = \alpha T_{\text{BH}} (t/\tau_{\text{BH}})^{-1/2} \) is the momentum at time \( t \) of a particle injected at time \( \tau_{\text{BH}} \) with momentum \( \alpha T_{\text{BH}} \). In particular, modes with \( k \ll k_0 \) were injected with energy \( \simeq \alpha T_{\text{BH}} \) at time \( t' \simeq t(k/\alpha T_{\text{BH}})^2 \ll \tau_{\text{BH}} \), while modes with \( k \gg k_0 \) were produced in the final stages at time \( t' \simeq \tau_{\text{BH}} \) with momentum \( k' \simeq \alpha T(t') \gg \alpha T_{\text{BH}} \). One obtains:

\[
f_\phi(k, t) \simeq \frac{2}{3} x_\phi \frac{M_{\text{BH}}}{(\alpha T_{\text{BH}})^2} \frac{k}{\alpha T_{\text{BH}}} \frac{t}{\tau_{\text{BH}}} Y_{\text{BH}} \quad (k \ll k_0),
\]

(3)

\[
f_\phi(k, t) \simeq x_\phi \frac{M_{\text{BH}}}{(\alpha T_{\text{BH}})^2} \left( \frac{k}{\alpha T_{\text{BH}}} \right)^{-3} \left( \frac{t}{\tau_{\text{BH}}} \right)^{-1} Y_{\text{BH}} \quad (k \gg k_0),
\]

(4)

and both expressions agree to within a factor 3/2 at \( k = k_0 \).

If initially \( T_{\text{BH}} < m_\phi \), moduli are produced only in the final stages for \( k' \gg m_\phi \) at injection. Hence the above spectrum should remain valid if a low-momentum cut-off \( k_c \sim m_\phi (t/\tau_{\text{BH}})^{-1/2} \gg k_0 \) is introduced. The total number of \( \phi \) particles produced (hence the constraint on \( \beta \)) is thus suppressed (weakened) by a factor \( \sim (m_\phi/T_{\text{BH}})^2 \), after integration of \( f_\phi(k, t) \) over \( k > k_c \), if \( T_{\text{BH}} < m_\phi \), i.e., if \( M_{\text{BH}} > 10^9 \text{g} (m_\phi/10 \text{TeV})^{-1} \). Since this mass range \( M_{\text{BH}} \gtrsim 10^9 \text{g} \) is moreover strongly constrained by the effects on BBN of quarks directly produced in the evaporation \([13]\), it will be ignored in the following.

For PBHs such that \( T_{\text{BH}} \gtrsim m_\phi \), it is a very good approximation to consider that emitted moduli carry at time \( t \) a momentum \( k_0 \), since \( k f_\phi(k, t) = dY_\phi/d \ln(k) \) behaves as \( k^2 \) for \( k \ll k_0 \), and as \( k^{-2} \) for \( k \gg k_0 \). Moreover at time \( t = \tau_\phi \):

\[
d f_\phi(k, t) \sim \frac{2 x_\phi}{\alpha T_{\text{BH}}} \frac{M_{\text{BH}}}{(\alpha T_{\text{BH}})^2} \left( \frac{k}{\alpha T_{\text{BH}}} \right)^{-3} \left( \frac{t}{\tau_{\text{BH}}} \right)^{-1} Y_{\text{BH}} \quad (k \gg k_0),
\]
and therefore the $\phi$ particles decay at rest (in the plasma rest frame), at time $\tau_\phi \sim M_P^2/m_\phi^3$, in the range of masses considered, $m_\phi \lesssim 10$ TeV and $M_{BH} \lesssim 10^9$ g. One then seeks the total number of moduli present at that time, which is given by:

$$Y_\phi \simeq \frac{x_\phi M_{BH}}{2 \alpha T_{BH}} Y_{BH}.$$  

This result can be obtained as a solution of the transport equation $\partial_t Y_\phi = x_\phi Y_{BH} |dM(t)/dt|/\alpha T(t)$, or by integrating $f_\phi(k,t)$ over $k$ in Eqs. (3), (4) above; all three results agree to within a factor $3/2$. Equation (5) has a simple interpretation: within a factor 2 it corresponds to the instantaneous evaporation of black holes at time $\tau_{BH}$, with total conversion of their mass $M_{BH}$ in particles of energy $\alpha T_{BH}$, a fraction $x_\phi$ of which is moduli. This result can be rewritten in terms of more conventional parameters. The mass $M_{BH}$ is taken to be a fraction $\delta$ of the mass within the horizon at the time of formation $t_i$: $M_{BH} \approx 4 \pi \delta m_P^2/H_i$, where $H_i$ denotes the Hubble scale at time $t_i$, and $\delta \sim O(1)$ is expected [12]. Furthermore, instead of $Y_{BH}$, one generally uses the mass fraction $\beta \equiv n_{BH} M_{BH}/\rho_c$ defined at the time of PBH formation $t_i$, with $\rho_c = 3H_i^2 m_P^2$ the critical energy density at that time. Using $s = (2\pi^2/45)g_* T_\gamma^3$, with $T_\gamma$ the cosmic background temperature, $T_\gamma \approx 0.5 g_{200}^{1/4} H_i^{1/2} m_P^{1/2}$, and $g_{200} = g_*/200$ ($g_*$ number of degrees of freedom), one obtains:

$$\beta \simeq 3 \times 10^{21} g_{200}^{1/4} \delta^{-1/2} \left( \frac{M_{BH}}{10^9 \text{ g}} \right)^{3/2} Y_{BH},$$  

and therefore:

$$Y_\phi \simeq 2 \times 10^4 \delta^{1/2} g_{200}^{-1/4} \left( \frac{x_\phi}{6 \times 10^{-3}} \right) \left( \frac{\alpha}{3} \right)^{-1} \left( \frac{M_{BH}}{10^9 \text{ g}} \right)^{1/2} \beta,$$

which constitutes the main result of this section. If the Universe went through a matter dominated era between times $t_a$ and $t_b$, with $t_i < t_a < t_b \ll \tau_{BH}$, then the r.h.s. of Eq. (5) must be multiplied by the factor $(H_b/H_a)^{1/2}$, where $H_{a,b}$ is the Hubble scale at time $t_{a,b}$, and the constraint on $\beta$ Eq. (8) below is weakened consequently.

3. Discussion – As mentioned previously, the most stringent constraints on $Y_\phi$ result from the effect of the decay products of $\phi$ on BBN [2]. These studies assume monoenergetic injection at energy $m_\phi$ at time $\tau_\phi$, and their results can be used safely, since the moduli emitted by PBHs decay when non-relativistic. One usually considers production of hadrons or photons in $\phi$ decay. The constraint due to hadron injection is in principle very significant for $m_\phi \gtrsim 1$ TeV, but it is not obvious that $\phi$ can decay hadronically, and moreover it relies on assumptions on the cosmic evolution of helium-3 (see, e.g., Ref. [2]), which have now been proven uncertain (see, e.g., Ref. [14] and references therein). Therefore, in the following,
only constraints on photon injection are used; the bounds presented will thus be slightly conservative. Holtmann et al. [2] obtain in this case:

\[
Y_\phi \lesssim 10^{-15} \quad \text{for} \quad m_\phi \simeq 100 \text{ GeV},
\]

\[
Y_\phi \lesssim 10^{-14} \quad \text{for} \quad m_\phi \simeq 300 \text{ GeV},
\]

\[
Y_\phi \lesssim 5 \times 10^{-13} \quad \text{for} \quad m_\phi \simeq 1 \text{ TeV}, \text{ and}
\]

\[
Y_\phi \lesssim 5 \times 10^{-10} \quad \text{for} \quad m_\phi \simeq 3 \text{ TeV}.
\]

The error on the upper limit is a factor \(\approx 4\). It results from the uncertainty in the fudge factors that enter the \(\tau_\phi(m_\phi)\) relationship when the constraints of Holtmann et al., given in the plane \(m_\phi Y_\phi - \tau_\phi\), are translated into the plane \(Y_\phi - m_\phi\). Note that these constraints assume that \(\phi\) decays into photons with a branching ratio unity, and should be scaled consequently. However these limits should also be strengthened by a factor as high as \(\sim 50\) to avoid \(^6\)Li overproduction, if one assumes that \(^6\)Li has not been destroyed in stars in which it has been observed [17]. This constraint is ignored in what follows, as it relies on yet unproven assumptions on stellar evolution; this but makes the above constraints more conservative. Finally, the observational upper limit on the amount of \(\mu\)–distortion in the cosmic microwave background implies [5, 2]: \(Y_\phi \lesssim 10^{-13}(m_\phi/100 \text{ GeV})^{1/2}\) for \(20 \text{ GeV} \lesssim m_\phi \lesssim 500 \text{ GeV} \).

Note that the most stringent constraint on \(\beta\) results from the production of the lightest of all moduli–like particles in the theory, whose mass would likely be \(\lesssim \text{few} \times 100 \text{ GeV}\). Overall it seems that \(Y_\phi \lesssim 10^{-14}\) represents a reasonable generic upper limit from BBN. Using Eq. (8), this can be rewritten as a limit on \(\beta\):

\[
\beta \lesssim 5 \times 10^{-19} \delta^{-1/2} g_{200}^{1/4} \left(\frac{x_\phi}{6 \times 10^{-3}}\right)^{-1} \left(\frac{\alpha}{3}\right) \left(\frac{M_{\text{BH}}}{10^9 \text{ g}}\right)^{-1/2} \left(\frac{\overline{Y}_\phi}{10^{-14}}\right), \tag{9}
\]

and \(\overline{Y}_\phi\) denotes the upper limit on \(Y_\phi\). This result does not depend on whether PBHs are shrouded in a photosphere, as suggested by Heckler [16], since moduli are not expected to interact with it due to their gravitational interaction cross-section. On the contrary, other astrophysical constraints on \(\beta\) for \(M_{\text{BH}} \gtrsim 10^9 \text{ g}\) are in principle sensitive to the presence of a photosphere, as they rely on the direct emission of photons and quarks [3].

If \(R\)–parity holds, the constraint on the LSP mass density \(\Omega_{\text{LSP}} < 1\) today implies:

\[
\beta \lesssim 2 \times 10^{-17} \delta^{-1/2} g_{200}^{1/4} (\alpha/3)(x_{\text{LSP}}/0.6)^{-1}(M_{\text{BH}}/10^9 \text{ GeV})^{-1/2}(m_{\text{LSP}}/100 \text{ GeV})^{-1}.
\]

This constraint has been adapted from the study of Ref. [2] and Eq. (8) above. The fraction of luminosity carried away by the LSP is \(x_{\text{LSP}} \simeq 0.6\), since each spartner produced by a PBH will produce at least one LSP in its decay [3]. This LSP constraint on \(\beta\) is thus less stringent than the moduli constraint, provided at least one modulus of the theory has mass \(\lesssim 1 \text{ TeV}\).

These results have several implications. First of all, the approximation made in Sec. 2, namely \(\Omega_{\text{BH}} \ll 1\) at all times is justified. In effect, \(\Omega_{\text{BH}} = \beta(t/t_i)^{1/2}\) at time \(t\) in a radiation-dominated Universe, since PBHs behave as non-relativistic matter, and therefore at time \(\tau_{\text{BH}}\), \(\Omega_{\text{BH}} \simeq 2.3 \times 10^{14} \delta^{1/2}(M_{\text{BH}}/10^9 \text{ g})\beta\). Consequently, if \(\beta\) verifies the above upper limits, indeed \(\Omega_{\text{BH}} \ll 1\) at all times. However, since Eq. (8) is not valid if \(\Omega_{\text{BH}} = 1\) at some time \(t_\ast < \tau_{\text{BH}}\), one needs to consider this case as well.

An order of magnitude of \(Y_\phi\) in this case can be obtained as follows. If \(t_\ast < \tau_{\text{BH}}\), the radiation present subsequent to PBH evaporation has been produced in the evaporation
process itself. Assuming total conversion of the PBH mass $M_{\text{BH}}$ at time $\tau_{\text{BH}}$ into particles (moduli and radiation) of energy $\alpha T_{\text{BH}}$, one finds $n_\phi \approx x_\phi \rho_{\text{BH}}/\alpha T_{\text{BH}}$, and $s \approx (4/3)\rho_{\text{BH}}/T_{\text{RH}}$, where $\rho_{\text{BH}}$ is the PBH energy density at evaporation, $T_{\text{RH}} \approx 2 \text{ MeV} g_2^{1/4}(M_{\text{BH}}/10^9 \text{ g})^{-3/2}$ is the reheating temperature, and $g_{10} = g_*/10$. Therefore $Y_\phi \approx 3 \times 10^{-10} g_{10}^{-1/4}(x_\phi/6 \times 10^{-3})(\alpha/3)^{-1}(M_{\text{BH}}/10^9 \text{ g})^{-5/2}$, well above the previous limits. Note that one would naively expect $Y_\phi \sim x_\phi$ since $x_\phi$ is the fraction of PBH luminosity carried away by $\phi$ particles. However the photons emitted by PBHs carry high energy $\simeq \alpha T_{\text{BH}}$ and small number density $\sim \rho_{\text{BH}}/\alpha T_{\text{BH}}$, and their thermalization leads to many soft photons carrying high entropy. Nevertheless, this discussion shows that PBHs of any mass should never come to dominate the energy density; if this were to happen, PBHs with $M_{\text{BH}} \lesssim 10^9 \text{ g}$ would produce too many moduli, while the evaporation of PBHs with $M_{\text{BH}} \gtrsim 10^9 \text{ g}$ would lead to too low a reheating temperature. In particular, scenarios of reheating of the post-inflationary Universe by black hole evaporation, as put forward, e.g., in Ref. [18], are forbidden. This result was also envisaged in Ref. [19].

Finally, the present constraints on $\beta$ exclude the possibility of generating the baryon asymmetry of the Universe through PBH evaporation. Indeed Barrow et al. [20] have performed a detailed computation of the baryon number to entropy density ratio $n_b/s$ produced in PBH evaporation, and find: $n_b/s \simeq 7 \times 10^4 \epsilon x_H/0.01 g_{200}^{-1/4}(M_{\text{BH}}/10^9 \text{ g})^{1/2} \beta^{1/2}$, where $\epsilon$ is the baryon violation parameter, defined as the net baryon number created in each baryon-violating boson decay, $x_H$ is the fraction of PBH luminosity carried away by such bosons, and other notations are as above. Unless all moduli–like particles are heavier than $\sim 3 \text{ TeV}$, and $R$–parity does not hold, the above constraints on $\beta$ imply $n_b/s < 10^{-12} \epsilon$, which does not suffice since BBN indicates $n_b/s \sim 4 - 7 \times 10^{-11}$.

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