Generalized Master–Slave-Splitting Method and Application to Transmission–Distribution Coordinated Energy Management

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Abstract—Transmission–distribution coordinated energy management (TDCEM) is recognized as a promising solution to the challenge of high distributed energy resource (DER) penetration, but there is a lack of a distributed computational method that universally and effectively works for the TDCEM. To bridge this gap, this paper presents a generalized master–slave-splitting (G-MSS) method. This method is based on a general-purpose transmission–distribution coordination model called G-TDCM, which enables the G-MSS to be applicable to most of the central functions of the TDCEM. In this G-MSS method, a basic heterogeneous decomposition (HGD) algorithm is first derived from the heterogeneous decomposition of the coupling constraints in the optimality conditions of the G-TDCM. Its optimality and convergence properties are proved. Then, inspired by the sufficient conditions for convergence, a modified HGD algorithm that utilizes the subsystem’s response function is developed and demonstrated to converge faster. The distributed G-MSS method is then demonstrated to successfully solve a series of central functions of the TDCEM, e.g., power flow, contingency analysis, voltage stability assessment, economic dispatch, and optimal power flow. The severe issues of over-voltage and erroneous assessment of the system security that are caused by DERs are thus resolved by the G-MSS method with modest computation cost.

Index Terms—Distributed energy resource (DER), distributed optimization, distribution, energy management, transmission.

ACRONYMS

| ACRONYM | DESCRIPTION |
|---------|-------------|
| APP     | Auxiliary problem principle |
| ATC     | Analytical target cascading |
| DER     | Distributed energy resource |
| DPS     | Distribution system |
| DSO     | Distribution system operator |
| ED      | Economic dispatch |
| G-MSS   | Generalized master-slave-splitting |
| G-TDCM  | Generalized T-D coordination model |
| HGD     | Heterogeneous decomposition |
| ITD     | Integrated transmission and distribution |
| KKT     | Karush–Kuhn–Tucker |
| LMP     | Local marginal price |
| MSS     | Master-slave-splitting |
| OCD     | Optimality condition decomposition |
| OPF     | Optimal power flow |
| T-D     | Transmission-distribution |
| TDCEM   | T-D coordinated energy management |
| TDPF    | T-D power flow |
| TPS     | Transmission system |
| TSO     | Transmission system operator |
| VSA     | Voltage stability assessment |

OMENCLATURE

| SYMBOL | DESCRIPTION |
|--------|-------------|
| •      | Magnitude of a complex number/vector. |
| arg(•) | Argument of a complex number/vector. |
| A(•),a(•),a_f(•) | Auxiliary functions used in the G-MSS. |
| c, f, g | Functions that appear in the objective, equalities, and inequalities. |
| d, e   | Right-hand vectors in the ED model. |
| D      | Damping coefficient of the G-MSS. |
| D      | Damping coefficient of the G-MSS. |
| E, F   | Coefficient matrices in the ED model. |
| h_M, h_S, h_MB, h_HS, h_MB, h_HS | Auxiliary functions used in the G-MSS. |
| l_M, l_B, l_S | Mappings associated with h_M and h_S, and h_MB, respectively. |
| M, B, S | Master, boundary, and slave subsystems when they are used as subscripts. |
| P_F, P_D | Partial derivatives of Lagrangian L of the G-TDCM with regard to z_M, z_B, and z_S, respectively. |
| P_B    | Coefficients of the active power injections in the TPS and DPS buses. |
| P_T, P_D, P_B, P_B | Active power flowing from the TPS to the DPS. |
| s_MB, s_B | Subscripts that distinguish the coefficient matrices in the ED model. |
| s_MB, s_B | Vectors of the complex power flowing through the branches connecting two adjacent subsystems. |
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Relative to the MSS method [7] that is only applicable to the complex outer nodal power injections (like the reactive power from the shunts) in the boundary and slave subsystems.

$T$ Transpose when it is used as a superscript, and it denotes a TPS when it is used as a subscript.

$u$ Control over a system.

$v$ Vector of nodal complex voltages of a system.

$x$ State of a system.

$y_B$ Value of $h_{BS}$ under a given pair $(ξ_B, ξ_S)$.

$Y_{φ, ψ}$ Admittance matrix regarding the buses in the subsystems $φ$ and $ψ$.

$Y\text{eq}_{S}$ Equivalent admittance of a DPS.

$z$ Optimal variable regarding a system.

$\mu, \nu_{MB, BS}$ Auxiliary functions associated with the objective function in the G-TDCM.

$λ, ω$ Multipliers regarding the equality and inequality constraints.

$ξ$ Vector containing the primal and dual variables regarding a system.

I. INTRODUCTION

A. Backgrounds

T is well known that high penetration of distributed energy resources (DERs) challenges both distribution system (DPS) and transmission system (TPS) operation. A research group at Massachusetts Institute of Technology and a working group of IEEE have identified that for a country with high penetration of DERs, the generation in a DPS “could impact a country’s transmission system” [1] and “a closer cooperation between transmission system operators (TSOs) and distribution system operators (DSOs) is imperative” [2]. Moreover, DERs can contribute to improving power system operation in terms of congestion mitigation, voltage support, etc. [3], [4]. To realize these services provided by DERs, transmission-distribution (T-D) coordination is also essential, and even “of utmost importance” [1]. Nevertheless, as reviewed in [4], while there are some works on T-D coordination, e.g., power flow, VSA, ED, optimal power flow (OPF), etc. [28] Remarkably, developing such a method is not only of theoretical value but also of notable practical significance: it will reveal a potential TSO-DSO coordination mechanism that would be universally effective for the central functions of the TDCM, and also a possible way in which TSOs and DSOs modify their respective energy management systems and exchange necessary data to accomplish this coordination [28].

To bridge this gap, this paper presents a distributed generalized master-slave splitting (G-MSS) method that comes from the first author’s Ph.D. dissertation [28], demonstrates its optimality and convergence properties, and shows how it is applied to the central functions of the TDCM. This research expands our previous works in the following aspects:

- Relative to the MSS method [7] that is only applicable to the power flow problem, the G-MSS method is applicable to a general-purpose continuous optimization model of which the power flow, ED, OPF, etc., are all special instances. Moreover, its convergent solution is not only feasible but also satisfies at least the first-order optimality conditions. Hence, relative to the MSS method, this G-MSS method is ingenious in algorithm design and has a wider application scope.
- Relative to [17] and [25], this paper provides a rigorous proof of the local convergence property, which increases the theoretic value and the applicability of the G-MSS.

B. Literature Review

Most works on T-D coordination could be classified into the following subjects:

1) Modelling and dynamic simulation of an integrated transmission and distribution (ITD) system [6]–[11]: esp-

2) Coordinated voltage control at the boundary of the TPS and the DPS: passive control was first investigated to reduce the impact of DERs on the TPS [12]; then active control where TSOs and DSOs are coordinated was studied in [13]–[17], in which both heuristic methods [13]–[15] and mathematical decomposition methods [16], [17] were tested. Recently, [18] showed that using transformer tap stagger in a DPS improves the DPS’s voltage support capability, which provides another measure that can be taken in T-D coordination.

3) Voltage stability assessment (VSA) of the ITD system: initially, the impact of the load tap changers on the static voltage stability of an ITD system was noticed [19]; several years later, the impact of DERs on the stability was investigated in [20]–[23], and a distributed method was recently proposed in [25] to assess the critical point of an ITD system.

4) Active power dispatch: [24] studied a distributed solution to a unit commitment problem by using the so-called ATC method; [25] and [26] studied distributed solutions to an economic dispatch (ED) problem based on the heterogeneous decomposition (HGD) algorithm and parametric programming, respectively; [27] investigated a coordinated market in the context of T-D coordination.

C. Contributions

As seen above, T-D coordinated energy management (TDCM) that enables distributed cooperation between TSOs and DSOs is a promising solution to the challenge of high DER penetration. In this regard, however, there remains an important but unsolved problem in the literature: “Is there a distributed computation method that universally and effectively works for the central functions of a TDCM system, e.g., power flow, VSA, ED, optimal power flow (OPF), etc.?" [28] Remarkably, developing such a method is not only of theoretical value but also of notable practical significance: it will reveal a potential TSO-DSO coordination mechanism that would be universally effective for the central functions of the TDCM, and also a possible way in which TSOs and DSOs modify their respective energy management systems and exchange necessary data to accomplish this coordination [28].

$\sigma_{BB}$ Vector of the complex power that flows from one node to another in the boundary subsystem.

$s_{B}, s_{S}$ Vector of the complex outer nodal power injections regarding a system.

1ENTSO-E is short for European Network of Transmission System Operators for Electricity, the association which was a successor of six regional associations of electricity transmission system operators. More information is referred to http://www.entsoe-europe.eu/
method. Moreover, this proof further inspires a new algorithm called modified HGD that converges faster and has a larger domain of convergence.

- Relative to [29], the modified HGD algorithm proposed in this paper no longer limits to an ED problem; instead, it is demonstrated to be generally workable for a series of functions of the TDCEM, as a general formula of the response function is derived from a general-purpose model. The physical interpretation of this general response function is explained, and it is also analyzed why introducing this response function accelerates the convergence of the algorithm for general cases.

A detailed comparison between our previous works and this paper is listed in Table I.

In short, in comparison with the above previous works as well as the first author’s Ph.D. Thesis [28], we believe that the G-MSS method in this paper has the following advantages: (i) theoretically sounder with a proven new convergence theorem that has a weaker assumption, (ii) broader application scope as it is proposed for general problems rather than a specific application, and (iii) converges faster for general cases if a response function is adopted. Moreover, this paper also presents new contents on the computational performances of the G-MSS method under various loading conditions and its positive effect when it is applied to VSA and OPF problems under different penetration rates of DERS, and a field demonstration project.

The remainders are as follows. In Section II, a generalized transmission-distribution coordination model (G-TDCM) will be established. In Section III, two distributed algorithms in the G-MSS method are presented with proved optimality and convergence properties. In Section IV, the application of the G-MSS method to the power flow calculation, contingency analysis, VSA, ED, and OPF is demonstrated. Finally, conclusions are presented.

II. GENERALIZED TRANSMISSION-DISTRIBUTION COORDINATION MODEL

A G-TDCM means that it is applicable to most of the central functions of the TDCEM. Hence, it allows one to develop a universally applicable distributed solution to a TDCEM system instead of designing algorithms for every specific function thereof, so it will save effort in establishing a TDCEM system and help to reveal basic TSO-DSO coordination rules.

To commence with this model, an ITD system is divided into master, boundary, and slave subsystems. The boundary refers to the interface of the TPS and the DPS, typically a high-voltage or low-voltage bus of a distribution substation. The master and slave subsystems consist of the other components (e.g., buses, lines, generator, loads, etc.) of the TPS and the DPS, respectively, which are separately supervised by the TSO and the DSO. In addition, the practice in power system operations reveals the following facts:

Fact 1: The control over the boundary subsystem is decided by either a TSO or a DSO.

Fact 2: The master and slave subsystems are coupled by the state of the boundary subsystem. In other words, any power flow path that connects the master and slave subsystems must pass through the boundary subsystem, as is illustrated in Fig. 1.2

As for this master-slave-structured ITD system, let \( z \) be the optimal variable that contains both \( x \), the state of the system, and \( u \), the control, and let \( f \) and \( g \) be the equality and inequality functions, respectively. Then, the G-TDCM is formulated as a general-purpose continuous optimization problem below [28]:

\[
\min_{z_M, x_B, z_S} \left\{ c_M (z_M, x_B) + c_S (x_B, z_S) \right\} \\
\text{s.t.} \quad f_M (z_M, x_B) = 0, \quad f_B (z_M, x_B, z_S) = 0, \quad f_S (x_B, z_S) = 0, \quad g_M (z_M, x_B) \geq 0, \quad g_S (x_B, z_S) \geq 0
\]

where the subscripts \( M, B, \) and \( S \) denote the master, boundary, and slave subsystem variables and functions, respectively. Notice that, due to Fact 1, the control in the boundary subsystem, \( u_B \), is contained by either \( z_M \) or \( z_S \).

In (1), \( c_M \) should be a function of \( z_M \) and \( x_B \), and \( c_B \) is a function of \( x_B \) and \( z_S \), because of the field management boundary between a TSO and a DSO. Moreover, \( c_M \) and \( c_S \) here are formulated in a general form to make the G-TDCM as general and flexible as possible. For example, as for the power flow problem, let \( c_M = 0 \) and \( c_S = 0 \); as for the ED problem, let \( c_M \) and \( c_S \) be the cost functions of the generators; as for the OPF problem, \( c_M \) and \( c_S \) can be either the generational cost functions or other common operational targets. Remarkably, although the objective in (1) seems to limit to a family of functions that do not contain a master-slave-coupling term like \( c_B (z_M, x_B, z_S) \), a similar term still can be involved in the objective function if it can be reformulated as \( \mu (\nu_M (z_M, x_B, z_S) + \nu_S (x_B, z_S)) \), where \( \mu, \nu_M \) and \( \nu_S \) are functions.\(^3\)

In addition, the equality constraints in (1) are restricted to power flow equations, the very ones that couple the TPS and the DPS. The other possible equality constraints, e.g., those regarding the operating of a device \( \alpha \), are modeled by \( g_M (\alpha) \geq 0 \) and \( -g_S (\alpha) \geq 0 \), and thus represented by the inequality constraints in (1). In addition, the inequality functions \( g_M, g_B, \) and \( g_S \) include common operational constraints, like power capacity of lines and transformers, active and reactive power limits of generators, maximum ramp rate of generators, nodal voltage magnitude, and phase angle constraints. Notice that as per the classical steady-state power system model, those functions in (1) are typically smooth functions of \( z \) and/or \( u \).

As can be seen from Fact 2 and Fig. 1, the master-slave-structure of an ITD system enables one to reformulate the power flow equations \( f_M (z_M, x_B, z_S) \) as the difference of a function \( f_{MB} (z_M, x_B) \) and a function \( f_{BS} (x_B, z_S) \), namely \( f = f_{MB} - f_{BS} \), which will be formally proved in Appendix A.

\(^2\)In Fig.1, there are loops interconnecting different nodes in the boundary and master subsystems, but there is a tree-like structure in the slave subsystem, because (a) a DUS is usually radial; and (b) two DPs that are connected to different TPS’s buses are seldom allowed to be interconnected in field operations.

\(^3\)The proof is omitted here to save space. The details are referred to [28].
This reformulation will facilitate developing the G-MSS method below.

III. GENERALIZED MASTER-SLAVE-SPLITTING METHOD [28]

A. Heterogeneous Decomposition of Optimality Conditions

Below, let $L$ denote the Lagrangian of the G-TDCM:

$$L = c_M + c_S - \lambda^T_M f_M - \lambda^T_B (f_{MB} - f_{BS}) - \lambda^T_S f_S - \omega^T_M g_M - \omega^T_S g_S$$

(2)

where $\lambda$ and $\omega$ are the multipliers regarding the equality and inequality constraints, respectively, and $\omega \geq 0$; superscript $T$ stands for transpose. Note that $f_B = f_{MB} - f_{BS}$ is used in (2).

With certain constraint qualifications, the Karush–Kuhn–Tucker (KKT) conditions of the G-TDCM hold and are formulated in the following canonical form:

$$\left\{ \begin{array}{l}
l_M (\xi_M, \xi_B) = 0, \quad l_B (\xi_M, \xi_B, \xi_S) = 0, \quad l_S (\xi_B, \xi_S) = 0,

f_M (z_M, x_B) = 0, \quad f_B (z_M, x_B, z_S) = 0, \quad f_S (x_B, z_S) = 0,
g_M (z_M, x_B) \geq 0, \quad g_S (x_B, z_S) \geq 0, \quad \omega_M \geq 0, \quad \omega_S \geq 0,

\omega^T_M g_M (z_M, x_B) = 0, \quad \omega^T_S g_S (x_B, z_S) = 0
\end{array} \right. $$

(3)

where the variables $\xi_M := [z_M; \lambda_M; \omega_M]$, $\xi_B := [x_B; \lambda_B]$ and $\xi_S := [z_S; \lambda_S; \omega_S]$ contain the primal and dual variables of every subsystem, respectively; $l_M, l_B$, and $l_S$ are the partial derivatives of $L$ with regard to $z_M, x_B$, and $z_S$, respectively.

In (3), both $f_B$ and $l_B$ couple the variables of the master and slave subsystems. Given $f_B = f_{MB} - f_{BS}$, $l_B$ can also be reformulated as $l_B (\xi_M, \xi_B, \xi_S) = l_{MB} (\xi_M, \xi_B, \xi_S)$, where

$$l_{MB} = \frac{\partial l_M}{\partial x_B} (\xi_M, \xi_B, \xi_S) + (\frac{\partial l_M}{\partial z_M} (\xi_M, \xi_B, \xi_S) - \omega^T_M) \lambda_M - \omega^T_S \lambda_B + \frac{\partial l_S}{\partial x_B} (\xi_B, \xi_S) \lambda_S + (\frac{\partial g_M}{\partial x_B} (\xi_M, \xi_B, \xi_S) - \omega^T_M) \lambda_B$$

Based on these difference-type formulae of $f_B$ and $l_B$, the KKT conditions in (3) are decomposed into two parts, i.e., the (KKT-M) in (4) and the (KKT-S) in (5):

$$\text{KKT - M : } \begin{cases} h_M (\xi_M, \xi_B) = 0, \\
\omega^T_M g_M (\xi_M, x_B) = 0, \quad g_M (z_M, x_B) \geq 0, \quad \omega_M \geq 0 \end{cases}$$

(4)

$$\text{KKT - S : } \begin{cases} h_S (\xi_B, \xi_S) = 0, \quad \omega^T_S g_S (x_B, z_S) = 0, \\
g_S (x_B, z_S) \geq 0, \quad \omega_S \geq 0 \end{cases}$$

(5)

where $h_M := [f_M; l_M]$, $h_S := [f_S; l_S]$, $h_{MB} := [f_{MB}; l_{MB}]$, and $y_B$ is the value of the function $h_{BS} := [f_{BS}; l_{BS}]$ of a given pair $(\xi_B, \xi_S)$. Because of the asymmetric formulae of the (KKT-M) and the (KKT-S), we call this decomposition heterogeneous decomposition. Indeed, this decomposition format corresponds to the practice that a TSO (or a DSO) only supervises the variables regarding its TPS (or the DPS), which mathematically requires that the above (KKT-M) and (KKT-S) should be only concerned with $(\xi_M, \xi_B)$ and $(\xi_B, \xi_S)$, respectively.

Recalling the master-slave structure of an ITD system, one can see that the (KKT-M) with a given $y_B$ is only concerned with the variables of the TPS, and that the (KKT-S) with a given $\xi_B$ is only concerned with the variables of the DPS. Hence, if $y_B$ and $\xi_B$ are given, the primal and dual variables of the TPS and the DPS can be independently solved from the (KKT-M) and the (KKT-S), respectively. Furthermore, these solved variables satisfy the KKT conditions in (3) if $y_B$ and $\xi_B$ are consistent, i.e., $y_B$ equals the counterpart that is produced by (5) with this $\xi_B$, and $\xi_B$ equals the counterpart that is produced by (4) with this $y_B$.

To arrive at this “consistency” point, two iterative algorithms are designed below. We call these algorithms the basic HGD and modified HGD algorithms, respectively, because they are based on the above heterogeneous decomposition of the KKT conditions.

B. Basic HGD Algorithm

1) Computation Procedures: The basic HGD algorithm is an iterative algorithm. In every iteration, a TSO and a DSO solve a

*The proof of this assertion is straightforward; please see [28].
transmission subproblem (6) and a distribution subproblem (7), respectively:

\[
\min_{z_M, x_B} c_M (z_M, x_B) - (p_{sp}^B)^T x_B \\
\text{s.t.} \quad \begin{cases} 
   f_M (z_M, x_B) = 0, \\
   g_M (z_M, x_B) \geq 0 
\end{cases} \\
\min c_S (x_{sp}^B, z_S) + (\lambda_{sp}^B)^T f_{BS} (x_{sp}^B, z_S) \\
\text{s.t.} \quad f_S (x_{sp}^B, z_S) = 0, \quad g_S (x_{sp}^B, z_S) \geq 0
\]

where the superscript \(sp\) denotes the specified variables in each iteration, and \(\lambda_{MB}\) is the multiplier with regard to the equality \(f_{MB}(z_M, x_B) = f_{BS}\).

Given the above subproblems, the procedures of this basic HGD algorithm are presented below:

**HGD Algorithm Procedures (Starting from the Distribution Subproblem).**

1. **Step 1**
   a) Set the maximum iteration number \(K\) and the tolerance \(\varepsilon\).
   b) Initialize \(x_{sp}^B\) as \(x_{sp}^B, 0 = [x_{sp}^B, 0 ; \lambda_{MB}, 0]\).
   c) Let the iteration counter \(k = 1\).

2. **Step 2**
   a) For iteration \(k\), the DSO solves (7) with a given \(x_{sp}^B, k-1\), and the solution is denoted by \(x_{sp}^B, k = [x_{sp}^B, k ; \lambda_{sp}^B, k ; \omega_{sp}, k]\).
   b) The DSO computes \(y_{sp}^B, k = [y_{sp}^B, k ; p_{sp}^B, k]\) where \(f_{BS, k} = f_{BS} (x_{sp}^B, k-1 ; z_S, k)\) and \(p_{sp}^B, k = l_{BS} (x_{sp}^B, k-1, \xi_{sp}, k)\).

3. **Step 3**
   The TSO solves (6) with a given \(y_{sp}^B, k\) and obtains the primal and dual variables \(\xi_{sp}^B, k = [\xi_{sp}^B, k ; \lambda_{MB}, k]\).

4. **Step 4**
   If \(\|\xi_{sp}^B - \xi_{sp}^B, k-1\| < \varepsilon\), the HGD algorithm is deemed to converge. Otherwise, return to Step 2 and let \(k = k + 1\) unless \(k = K\).

This HGD algorithm can also start from the transmission subproblem, and the procedures are similar to the above and thus omitted. The initialization step for online operation can be conducted via online measurements and/or the latest forecast at hand. As will be shown in Section IV, this basic HGD algorithm was successfully applied to the T-D coordinated ED and OPF problems in [17] and [25].

2) **Optimality and Convergence Properties:** As for the optimality, recalling that \(y_B = h_{BS} = [f_B; l_{BS}]\), one can prove via direct comparison that the KKT conditions of the subproblems in (6) and (7) are exactly those in (4) and (5). Hence, when this HGD algorithm converges, which implies that \(y_{sp}^B\) and \(\xi_{sp}^B\) are consistent, this convergent solution will satisfy the KKT conditions in (3). Therefore, it is a candidate local optimum of the G-TDCM, and it is indeed a local optimum if the second-order sufficient optimality conditions are satisfied or if the G-TDCM is convex. In the latter case, this convergent solution is also globally optimal.

As for the convergence, we will show that this basic HGD algorithm linearly converges in the neighborhood of a local optimum of (1), which is denoted by \((\xi_{sp}^B, *; y_{sp}^B, *)\). Per the sensitivity theorem (Theorem 5.1 in [30]), if the local primal and dual solutions to the distribution subproblem with \(\xi_{sp}^B, *\) satisfy the following conditions: (i) the second-order sufficient optimality conditions; (ii) the strict complementarity slackness condition; and (iii) the primal solution is a regular point, then for every \(\xi_{sp}^B\) located in a certain neighborhood of \(\xi_{sp}^B, *\), there is a unique once continuously differentiable function \(H_{BS}(\xi_{sp}^B)\) representing the unique local optimum \(\xi_{BS}\) of (7), i.e., \(\xi_{BS} = H_{BS}(\xi_{sp}^B)\). Moreover, since the functions in (1) are smooth, it follows that \(y_B = h_{BS}\) and \(h_{BS}\) is continuously differentiable; therefore, there exists a unique once continuously differentiable function \(h_{MB}\) such that \(y_{sp}^B = h_{BS}(\xi_{sp}^B)\). Similarly, if the local primal and dual solutions to the transmission subproblem with \(y_{sp}^B, *\) satisfy the similar conditions, then for every \(y_{sp}^B\) in the neighborhood of \(y_{sp}^B, *\), there also exists a continuously differentiable function \(h_{MB}\) representing the associated unique local optimum \(\xi_{MB}\) of (6). Hence, \(\xi_{sp}^B, * = h_{MB}^{-1}(h_{BS}(\xi_{sp}^B))\), which indicates that \(\xi_{sp}^B, *\) should be the fixed point of this composite function. Thus, the fixed-point theorem can be used to analyze the local convergence property.

The above analysis is formally stated below:

**Lemma 1:** If the conditions (i)–(iii) required by the sensitivity theorem hold for \((\xi_{sp}^B, *, y_{sp}^B, *)\), it follows that:

1) There exist domains \(D_M, D_B\) and \(D_S\) such that for any \(\xi_{MB} \in D_B\), the distribution subproblem and the transmission subproblem have unique local optima \(\xi_{BS}\) and \(\xi_{MB}\) in \(D_M\) and \(D_S\) respectively.

2) In the domain \(D_B\), define composite mappings \(h_{BS}(\cdot) = h_{BS}(\xi_{BS}, \xi_{MB})\) and \(h_{MB}(\cdot) = h_{MB}(\xi_{MB}, \xi_{BS})\). Then the convergence of the basic HGD algorithm is equivalent to that the mapping \(\Phi: D_B \subset \mathbb{R}^{n_B} \rightarrow D_B\) defined in (8) converges to its fixed point, if \(h_{MB}\) has an inverse mapping in the domain \(D_B\) and \(h_{BS}(D_B) \subset h_{MB}(D_B)\).

\[
\Phi = h_{MB}^{-1} \circ h_{BS}
\]

where the notation \(\circ\) represents the composite of mappings.

Lemma 1 allows one to derive the conditions guaranteeing the local convergence of the basic HGD algorithm from the fixed-point theorem, which is formally stated below:

**Convergence Theorem [28]:** Suppose Lemma 1 holds. The basic HGD algorithm converges linearly in the neighborhood of \(\xi_{sp}^B, *\), if either of the conditions in (9) and (10) is satisfied:

\[
\rho \left( \left| \left| \frac{\partial h_{BS}}{\partial \xi_{MB}} \right|^{-1} \frac{\partial h_{BS}}{\partial \xi_{MB}} \right| \right) < 1
\]

\[
\left| \left| \frac{\partial h_{BS}}{\partial \xi_{BS}} \right|^{-1} \frac{\partial h_{BS}}{\partial \xi_{BS}} \right| < 1
\]

where \(\rho(\cdot)\) stands for the spectral radius of a matrix.

This theorem can be straightforward proved via the fixed-point theorem, and the conditions in (9) and (10) can also be verified thereby. The proof of Lemma 1 and Convergence Theorem is provided in Appendix B. Moreover, notice that the Theorem 1 in [7] is a special case of this convergence theorem. Finally, notice that the domain \(D_B\) in Lemma 1 is in a certain neighborhood of \(\xi_{sp}^B, *\), which means that this Convergence Theorem only guarantees a local convergence property. Fortunately, however, online supervisory control and data acquisition systems usually provide a relatively good initial point, so a method that has a local convergence still often provides a satisfying result in field operations.
C. Modified HGD Algorithm

1) Basic Idea: Recalling \( h_{MB} = [f_{MB}; l_{MB}] \), \( h_{BS} = [f_{BS}; l_{BS}] \) and the difference-type formulae of \( f_B \) and \( l_B \), one can see that the basic HGD algorithm essentially decomposes \( h_B \) as \( h_B = h_{MB} - h_{BS} \) from which two subproblems in (6) and (7) are constructed. If \( h_B \) is decomposed in an alternative way, e.g., \( h_B = h_M^B - h_{BS} \), with the following property

\[
\left( \frac{\partial h_M^B}{\partial \xi_{MB}} \right)^{-1} \left( \frac{\partial h_{BS}}{\partial \xi_{BS}} \right) \leq \beta \left( \frac{\partial h_M^B}{\partial \xi_{MB}} \right)^{-1} \left( \frac{\partial h_{BS}}{\partial \xi_{BS}} \right)
\]

where \( 0 < \beta < 1 \), and \( h_M^B \) are derived from \( h_{MB} \) and \( h_{BS} \) as \( h_M^B \) as \( h_{MB} \) and \( h_{BS} \), then this new decomposition leads to a modified HGD algorithm that converges faster. Moreover, this modified HGD algorithm may also have a larger domain of convergence, because the points outside the domain of convergence associated with the basic HGD algorithm enter the domain of convergence associated with \( h_M^B \) and \( h_{BS} \). Following this idea, we will show a typical way of constructing \( h_M^B \) and \( h_{BS} \), which exploits the response function of the distribution subproblem (or the transmission subproblem) with regard to \( \xi^B \) (or \( y^B \)).

2) Construction of the New Decomposition: Suppose there exist mappings \( h_{MB} \) and \( h_{BS} \), as are defined in Lemma 1, associated with \( h_{MB} \) and \( h_{BS} \) such that \( h_B = h_{MB} - h_{BS} \). Let

\[
\hat{h}_M^B(\xi_M, \xi_B) := h_{MB}(\xi_M, \xi_B) - a(\xi_B) \quad \text{and} \quad \hat{h}_S^B(\xi_B) := h_{BS}(\xi_B) - a(\xi_B),
\]

where \( a(\xi_B) \) is a continuously differentiable function. Thus, \( h_B = h_M^B - h_S^B \) holds. Further, define

\[
\hat{h}_M^B = h_B - a, \quad \hat{h}_S^B = h_S - a, \quad \text{and} \quad \hat{f} = \hat{h}_M^B - 1 \circ \hat{h}_S^B. \]

Then we have

\[
\left( \frac{\partial \hat{h}_M^B}{\partial \xi_{MB}} \right)^{-1} \left( \frac{\partial \hat{h}_S^B}{\partial \xi_{BS}} \right) = \left( \frac{\partial h_M^B}{\partial \xi_{MB}} - \frac{\partial a}{\partial \xi_{MB}} \right)^{-1} \left( \frac{\partial h_S}{\partial \xi_{BS}} - \frac{\partial a}{\partial \xi_{BS}} \right).
\]

It follows from (12) that if \( \left( \frac{\partial \hat{h}_M^B}{\partial \xi_{MB}} - \frac{\partial a}{\partial \xi_{MB}} \right) \) is bounded above and if \( \left( \frac{\partial \hat{h}_S^B}{\partial \xi_{BS}} - \frac{\partial a}{\partial \xi_{BS}} \right) \) is bounded above by a small number (ideally zero), then \( \left( \frac{\partial \hat{h}_M^B}{\partial \xi_{MB}} \right)^{-1} \left( \frac{\partial \hat{h}_S^B}{\partial \xi_{BS}} \right) \) is likely to satisfy the property in (11), which ensures that faster convergence is achieved via this new decomposition. Therefore, to accelerate the convergence, \( \frac{\partial a}{\partial \xi_{BS}} \) should be close to \( \frac{\partial h_S}{\partial \xi_{BS}} \), which is simply the sensitivity, or “response”, of the output of the distribution subproblem with regard to the input \( \xi^B \). Thus, \( \frac{\partial a}{\partial \xi_{BS}} \) is called a distribution-response function.

The above observation yields a way of constructing \( \frac{\partial a}{\partial \xi_{BS}} \) or equivalently \( a(\xi_B) \): let \( \frac{\partial a}{\partial \xi_{BS}} \) be equal to the total derivative of \( h_{BS} \) with regard to \( \xi_B \), namely \( \frac{\partial h_{BS}}{\partial \xi_{BS}} + \frac{\partial h_M^B}{\partial \xi_{MB}} \frac{\partial \xi_{MB}}{\partial \xi_B} \), which can be obtained by solving the sensitivity equations derived from the (KKT)-S. Furthermore, in view of the structure of \( h_{BS} \), \( a(\xi_B) \) is further structured as \( a_I(\xi_B) : a_I(\xi_B) \) where \( a_I \) and \( a_I \) correspond to \( f_{BS} \) and \( l_{BS} \), respectively. Since \( f_{BS} \) is the power from a TPS to a DAPS (cf. Fig. 1), \( a_I \) can be physically understood as an equivalent of the (negative) DPS power injection at the boundary bus.

Notice that this response function \( \frac{\partial a}{\partial \xi_{BS}} \) is generally workable for the central functions of the TDCEM, as it is derived from the G-TDCM, which is a general-purpose model. Moreover, the formula \( \frac{\partial a}{\partial \xi_{BS}} = \frac{\partial h_{BS}}{\partial \xi_{BS}} + \frac{\partial h_M^B}{\partial \xi_{MB}} \frac{\partial \xi_{MB}}{\partial \xi_B} \) implies that this distribution-response function can be physically interpreted as:

1) how much the boundary power injection \( f_{BS} \) will change with an increase in the boundary voltage \( v_B \);
2) how much the boundary power injection \( f_{BS} \) will change with an increase in the boundary price \( \lambda_B \) (notice that \( \lambda_B \) is the locational marginal price when the OPF is to minimize the generational cost);
3) how much the optimum of the distribution subproblem will change with an increase in the boundary voltage \( v_B \);
4) how much the optimum of the distribution subproblem will change with an increase in the boundary price \( \lambda_B \).

Based on the \( h_M^B \) and \( h_S^B \) as constructed above, one can then construct new transmission and distribution subproblems and obtain the following computation procedures.

3) Computation Procedures: The computation procedures of this modified HGD algorithm are similar to those of the basic HGD algorithm except for Steps 2.b and 3:

- Step 2.b: In every iteration \( k \), after solving the distribution subproblem in (7), the DSO computes \( y^p_{MB,k} = [r^p_{MB,k} \times y^B_{MB,k}] \) and sends \( y^p_{MB,k} \) to the TSO, where

\[
\begin{align}
\begin{cases}
\min_{z_{MB}, x_{MB}} & c_M(z_{MB}, x_{MB}) - (l_{BS,sp} \times T_{MB})^T x_B \\
\text{s.t.} & \begin{cases}
J_M(z_{MB}, x_{MB}) = 0, \\
J_{MB}(z_{MB}, x_{MB}) - a_I(x_B) = f_{BS,sp, k}, \quad \lambda_{MB}
\end{cases}
\end{cases}
\end{align}
\]

The convergent solution of this modified HGD algorithm must be a candidate local optimum of the G-TDCM, which can be proven via direct comparison between the KKT conditions of (7), (15) at the convergent point and those listed in (3). The convergence of this algorithm will be faster than the basic HGD algorithm if the property in (11) holds.

Alternatively, one can introduce a transmission-response function representing the response of the output of the transmission subproblem (6) with regard to the parameter \( y^p_{MB} \), and construct another version of the modified HGD algorithm. This construction is in general similar to the above except that in every iteration \( k \), the TSO solves the transmission subproblem (6) and the DSO solves a new distribution subproblem (16) with a given \( \lambda_{MB, k-1} \). The optimality and convergence properties of this algorithm are same as those of the algorithm with a distribution-response function.

\[
\begin{align}
\min_{z_{SP}} & \begin{cases}
\left( z_{SP} \right)_{MB, k-1} + \left( \lambda_{MB, k-1} - b_{SP}f_{BS, k-1} \right) \cdot T_{SP} \\
J_{SP}(z_{SP}) = 0,
\end{cases} \\
y_{BS} \left( z_{SP} \right)_{MB, k-1} \geq 0
\end{align}
\]
where $B(x_B, z_S)$ and $b(y_B)$ are differentiable functions such that $\frac{\partial b}{\partial y_B}$ is the transmission-response function and $\frac{\partial y_B}{\partial x_B} = (\frac{\partial y_B}{\partial x_B})^T b_x$, $\frac{\partial \lambda}{\partial z_S} = (\frac{\partial \lambda}{\partial z_S})^T b_x$.

### D. Discussions

First, the G-MSS is universally applicable to the central functions of a TDCEM system, because it is designed for the G-TDCM that is a general-purpose coordination model. Moreover, it points out a universal coordination mechanism to realize this distributed TDCEM.

Second, although what we presented above is for a one-TSO-to-one-DSO case, the G-MSS is indeed applicable to a one-TSO-to-multiple-DSO case as long as the DSOs are not directly connected with each other, which is typically the case in practice. To see this, just let the boundary subsystem involves all the boundary buses in an ITD system, and then the above derivation and assertions still hold.

Third, when TPSs and DPSs have to be separately modeled as single- and three-phase models, the G-MSS can be applied in the way that the single-phase transmission subproblem and the three-phase distribution subproblem are solved by the TSO and the DSO, respectively, and then the obtained single-phase $\xi_{a,b,c}^s$ or the three-phase $y_{B,a,b,c}^s$ is converted to the three- or single-phase counterpart that will be used in the next iteration. This conversion is based on the following assumptions that usually hold in practice: (i) the three-phase nodal voltages are nearly symmetric in the T-D interface; and (ii) a change in the boundary power injection of any phase identically affects the solution to the single-phase transmission subproblem. The conversion can be conducted as follows:

- Convert a single-phase $\xi_{a,b,c}^s$ into a three-phase $\xi_{a,b,c}^s$: Let the phase-a voltage $x_A^s$ equal $x_{a}^s$, and obtain the phase-b and phase-c voltages $x_B^s$ and $x_C^s$ by shifting the phase angle of $x_B^s$ by 120° and 240°, respectively. Because of the above assumption (ii), the third three-phase multiplier $\lambda_B^s = \lambda_C^s = \lambda_{B,C}^s$, the latter which is the single-phase multiplier.

- Convert a three-phase $y_{B,a,b,c}^s$ into a single-phase $y_{B,a,b,c}^s$: Sum up the three-phase power $y_{B,a,b,c}^s$ to produce the single-phase value $y_{B,a}^s$. Compute the mean of the three-phase sensitivity $y_{B,a,b,c}^s$ to produce the single-phase $y_{B,a}^s$, which is used to approximate the impact of $x_{B,a}^s$ (note that $x_{B,a}^s$ is generated from $x_{B,a}^s$) on the optimum of the distribution subproblem.

Fourth, in the G-MSS method, the transmission and distribution subproblems are usually solvable by existing tools. For example, as for the power flow problem that will be shown in Section IV-A, the corresponding transmission and distribution subproblems can be directly solved by the existing power flow tools and the system operators only need to invoke a data transfer program to exchange $f_{BS}^{sp}$ and $x_{BS}^{sp}$. Hence, the G-MSS method allows TSOs and DSOs to continue using the existing tools that they are familiar with, which would be an advantage in field applications.

Lastly, if a transmission or distribution subproblem becomes infeasible, slack variables accompanied by a penalty can be introduced into the subproblems to ensure the algorithm to proceed smoothly 0.

### IV. APPLICATION TO TDCEM

In an energy management system, the central functions regarding steady-state power system operations generally include power flow calculation and the related contingency analysis and static VSA, state estimation, ED and OPF. Distributed solutions to these functions of a TDCEM system can be designed via the G-MSS method, because the models of these functions are all special instances of the G-TDCM. Below we will present the application of the G-MSS method to these functions.

#### A. PF & Contingency Analysis & VSA

The T-D power flow (TDPF) model is a special case of (1) with a zero-valued objective and only equality constraints. Moreover, only the state of the system needs to be solved from the equality constraints. Thus, in every iteration, the subproblems (6) and (7) in the basic HGD algorithm turn out to be (17) and (18), and the exchange data $y_{BS}^{sp}$ and $\xi_{B}^{sp}$ turn out to be $f_{BS}^{sp}$ and $x_{BS}^{sp}$, respectively, which are exchanged between the TSO and the DSO until the change in $x_{BS}^{sp}$ is smaller than $\varepsilon$. This coincides with the MSS method [7], which implies that it is only a special case of the G-MSS method.

$$\begin{align*}
&\left\{ f_M (x_M, x_B) = 0 \\
&f_{MB} (x_M, x_B) = f_{BS}^{sp} \\
&f_S (x_{BS}^{sp}, x_S) = 0
\end{align*}$$

Although this basic HGD algorithm is simple and intuitive, the modified HGD algorithm with a distribution-response function is typically preferable for it converges faster. Notice that only $a_f(x_B)$ needs to be constructed in this case. Apart from using the $\frac{\partial y_{BS}^{sp}}{\partial x_B}$, one can also construct $a_f$ via a static network equivalencing approach below [28], which will be more convenient in field operations.

Provided that the equivalent admittance of a DPS denoted by $y_{BS,eq}$ is given, the complex power $s_{BS}$ in Fig. 1 is formulated as follows:

$$s_{BS} = \text{diag} \{ v_B \} \text{Y}_{B,S} (\text{Y}_{S,S})^{-1} \text{diag} \{ v_S \}^{-1} s_S + \text{diag} \{ v_B \} \text{Y}_{S,eq} v_B$$

In (19), in addition to the notations in Fig. 1, $y_{BS,eq}$ is the admittance matrix regarding the buses in the subsystems $\phi$ and $\varphi$ ($= M, B$ or $S$); $v_S$ and $s_S$ are respectively the complex voltages and power injections regarding the slave-subsystems; $\text{diag}$ stands for conjugate. Informally, in a distribution power flow solution, the magnitude of $v_S$ typically increases with that of $v_B$, so it can be expected that the term in the second line of (19) dominates the response of $s_{BS}$ with regard to $v_B$, namely the response of $f_{BS} = \text{Re}(s_{BS})$ with regard to $x_B = \left[ \text{Re}(v_B) ; \text{arg}(v_B) \right]$. Thus, $a_f$ is constructed as follows:

$$a_f := \left( \text{Re}(\text{diag} \{ v_B \} \text{Y}_{S,eq} v_B) ; \text{Im}(\text{diag} \{ v_B \} \text{Y}_{S,eq} v_B) \right)$$

$\uparrow$Comparison between the G-MSS and common distributed optimization algorithms is provided in Appendix C.

$\uparrow$This assumption is taken as an item of network operation regulations in many places.

$\uparrow$Since the TPS is usually modeled in single-phase (more specifically, the positive sequence) in the power flow calculation, a single-phase $S_{eq}$ that corresponds to the positive sequence equivalent of the DPS should be generated via the method in [6] if the DPS is has a three-phase model.
A distributed TDPF algorithm based on the above modified HGD algorithm is then outlined below:

**Distributed TDPF algorithm based on the modified HGD algorithm.**

**Step 1**
- a) Add the distribution network equivalent \( Y_{S,eq} \) to the admittance matrix of the TPS to establish \( f_{MB} = a_f \) in (15).
- b) Initialize \( x_{B,0} \).

**Step 2**
- a) For iteration \( k \), the DSO solves (18) with a given \( x_{B,k-1}^{sp} \), and then computes \( f_{B,k}^{sp} (x_{B,k-1}^{sp}, x_{S,k}) \) and \( a_f(x_{B,k-1}^{sp}) \) via (20).
- b) The DSO computes \( f_{BS,k}^{sp} = f_{BS,k} - a_f(x_{B,k}^{sp}) \) to be sent to the TSO.

**Step 3**
The TSO solves \( \{ f_M(x_M, x_B) = 0, f_{MB}(z_M, x_B) - a_f(x_B) = f_{BS,k}^{sp} \} \) and obtains \( x_{B,k}^{sp} \).

**Step 4**
If \( \| x_{B,k}^{sp} - x_{B,k-1}^{sp} \| < \varepsilon \), this algorithm is deemed to converge. Otherwise, return to Step 2 and let \( k = k + 1 \) unless \( k = K \).

To test the computational performances of this TDPF algorithm under different loading conditions, the ITD system called T14D5, which was introduced in [23], was used as a test system. As was described in [23], five DPSs are connected to a modified IEEE 14-bus transmission system, and several distributed generators that have a local voltage maintaining capability and will also trip off at a low voltage are integrated into the five DPSs. Let \( \lambda \) denote the increased system loading in the per-unit value, so \( \lambda = 0 \) corresponds to the base case; for the critical point, \( \lambda = 2.24 \) and all distributed generators tripped off.

Table II lists the iterations numbers the TDPF algorithm requires for convergence. This indicates that although these numbers may change with the loading conditions, they are generally acceptable. In fact, except for the cases either with \( \lambda \leq 0.4 \) (where most nodes of this ITD system are in a high-voltage condition) or with \( \lambda \geq 2.0 \) (where the system is in a notably low-voltage condition), the TDPF algorithm converges in less than 6 iterations. This phenomenon can be understood by noting that a flat-start strategy is used in the algorithm, so it will converge with fewer iterations if the convergent boundary voltage is near 1.0 p.u. In fact, as a boundary voltage is typically controlled to be around 1.0 p.u. in field operations, the TDPF can be regarded as a proper computation method practically.

Furthermore, although the Newton-Raphson method can be used to solve the ITD power flow equations centrally, it requires people to establish an extremely large-size power flow model, which may cause difficulties in updating and maintaining the model. Moreover, because of the difference in the parameters of TPSs and DPSs, the condition number regarding the ITD power flow equations can be so large that the commonly used methods, like the Newton-Raphson or fast decoupled load flow, may fail to converge in a limited number of iterations. To see this, the 30E system that was introduced in [7] was used to compare the performances of the centralized Newton-Raphson method, a fast decoupled load flow method, and the distributed G-MSS method. The results are shown in Table III. For this 30E system, the condition number is 1.2 \( \times 10^8 \). The fast decoupled load flow method fails to converge; the Newton-Raphson method requires over 40 iterations for convergence, and for each iteration the Jacobian of the entire ITD system needs to be solved. As for the G-MSS method, it converges in 3 iterations, which means the TSO and the DSO need to update the boundary data only for three times. Moreover, during these three iterations, there are a total of 6 and 11 iterations performed to solve the power flow subproblems (17) and (18), respectively. This again indicates that the G-MSS method is a proper choice to solve the TDPF problem.

As power flow calculation is the core of static contingency analysis, the TDPF algorithm is also applied thereto [31]. For the test system called 30Dl therein, it is found that i) the TDPF successfully detects one dangerous contingency that is missed by the conventional contingency analysis; ii) the TDPF avoids one false alarm that is yielded by the conventional contingency analysis; and iii) the post-containment-contingency security of DPSs is successfully checked by the TDPF. The improved accuracy is because the post-contingency state of the ITD system is evaluated as a whole in the TDPF. Besides, owing to the fast convergence property of the above distributed algorithm, the average number of the iterations between a TSO and a DSO for the TDPF is only 3.8, which again demonstrates the power of the G-MSS method in solving TDPF problems.

Similarly, the static voltage stability of an ITD system can also be accurately assessed by a distributed continuation TDPF model where the above distributed TDPF algorithm is embedded. An interesting finding is that in the context of DER penetration, the true critical point and the loading margin of the ITD system are not so different from what the conventional transmission or distribution VSA method computes. To show this, we set different DER penetration rates in the T14D5 system and then computed the associated static voltage stability margin which is the maximal increase in the system loading under which the ITD power flow equations have a solution. Here, the penetration is defined as the ratio of distributed generators’ generation to the total loads of the DPSs, and the distributed generators are assumed to have a low-voltage-ride-through capability. The ITD system’s stability margins computed by T-D VSA and the conventional VSA method are listed in Table IV, where margin = 2.24 p.u., for example, means that when the ITD system’s load is increased by 224 MW, the system will reach the critical point. As the transmission VSA method neither accurately considers change in the losses of the DPSs nor that in the operating status of DERs as the load increases, this method

---

**Table II**

| \( \lambda \) | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 | 2.2 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Iterations | 17 | 13 | 3 | 3 | 4 | 4 | 5 | 6 | 10 | 14 |

**Table III**

| Computational Performances of Different Methods for 30E System |
|---|---|---|
| Methods | Newton-Raphson | Fast Decoupled Load Flow | G-MSS |
| Iterations | 46 | Did not converge | 3 |
| Results of T-D VSA (P.U.) | 2.24 | 2.43 | 2.48 | 2.71 | 3.19 |
| Results of Transmission VSA (P.U.) | 3.10 | 3.25 | 3.26 | 3.30 | 3.09 |

**Table IV**

| Penetration | 0% | 5% | 10% | 20% | 40% |
|---|---|---|---|---|---|
| Results of T-D VSA (P.U.) | 2.24 | 2.43 | 2.48 | 2.71 | 3.19 |
| Results of Transmission VSA (P.U.) | 3.10 | 3.25 | 3.26 | 3.30 | 3.09 |
TABLE V

| Penetration | 0% | 5% | 10% | 20% | 40% |
|-------------|----|----|-----|-----|-----|
| Results of T-D VSA (P.U.) | 0.109 | 0.119 | 0.121 | 0.132 | 0.156 |
| Distribution VSA (P.U.) | 1.123 | 1.145 | 1.152 | 1.162 | 1.210 |

may either overestimate the margin when the distribution losses are wrongly estimated (e.g., the 2nd to the 5th column in Table IV), or underestimate the margin when the reactive power support of the DERs is inaccurately evaluated (the last column in Table IV). For the distribution side, the computed stability margins of the DPS connected at #10 buses of the 14-bus TPS are listed in Table V. For example, margin = 0.109 p.u. means that when the this DPS-1 system’s load is increased by 10.9 MW, this DPS will reach the critical point. Table V demonstrates that the conventional distribution VSA method always significantly exaggerates the stability margin, because the boundary voltage is assumed to be unaffected by the loading process, which is never the case in field operations. The T-D VSA, by contrast, yields accurate results as it considers the whole ITD system.

B. OPF

A continuous T-D OPF problem can be directly solved by the basic HGD algorithm shown in Section 0. Here, the exchange data \( y_B^{sp} \) include \( J_{BS}^{sp} \) and \( \rho_{BS}^{sp} \), and \( \xi_B^{sp} \) include \( x_B^{sp} \) and \( \lambda_{MB}^{sp} \). Notice that \( x_B \) and \( J_{BS} \) represent the boundary voltage and the power injection from the TPS to the DPSs, respectively, so the exchange of \( y_B^{sp} \) means driving the solutions to the transmission and distribution subproblems to satisfy the boundary power flow equations in (1). Moreover, \( \lambda_{MB}^{sp} \) and \( J_{BS}^{sp} \) represent the response of the transmission subproblem and distribution subproblems with regard to the change in the boundary power and voltages, respectively, so the exchange of \( \xi_B^{sp} \) means leading the solutions to the subproblems to a candidate local optimum of the centralized T-D OPF model. Thus, the basic HGD algorithm guarantees feasibility and often local optimality of ITD system operations. More details of the algorithm were reported in [17].

This T-D OPF mode is compared with the conventional uncoordinated mode where transmission and distribution OPF are separately conducted. Some initial results have been reported in [17], where the tests on different scales of ITD systems confirmed that i) the T-D OPF solves the distribution over-voltage issue with fewer curtailments of the DERs’ generation; and ii) the T-D OPF mitigates the transmission congestion with lower redispatch costs by controlling DERs. This is because when there is a change in the state of a certain system, e.g., the overvoltage issue taking place in a DPS, the boundary variables such as the voltages or power injection, will be coordinated by the TSO as the voltages or power injection, will be coordinated by the TSO and DSOs. This coordinated dispatch will determine the phasor angles and active power, exemplifies the G-TDCM. However, an ED model can also be established via shift factors, and this version saves the phase angles of every bus and thus typically computationally cheaper. We will show below that the T-D ED problem set up via shift factor is also a special case of

TABLE VI

| DER Capacity: MW | 47.5 | 57 | 76 |
|-----------------|------|----|----|
| (Penetration: %) | (18%) | (22%) | (30%) |
| T-D OPF | 0 | 0 | 9.8 |
| Without Coordination | 2.02 | 6.14 | 16.46 |

TABLE VII

| Boundary Voltage Change in the Coordination |
|--------------------------------------------|
| VB (p.u.) | DPS-1 | DPS-2 | DPS-3 | DPS-4 |
|--------------|-------|-------|-------|-------|
| Before Coordination | 1.0385 | 1.0425 | 1.0456 | 1.0399 |
| After Coordination (18% pen.) | 1.0294 | 1.0305 | 1.0292 | 1.0268 |
| After Coordination (30% pen.) | 1.0149 | 1.0187 | 1.0206 | 1.0183 |

TABLE VIII

| Computational Performances of Different Methods for T-D OPF |
|-------------------------------------------|
| Test Systems | T14D4 | T57D10 | T57D14 |
|---------------|------|------|-------|
| Iterations | G-MSS | APP | Biskas’s |
|---------------|------|------|-------|
| 22 | 99 | 57 |
| 21 | 31 | N/A |
| 37 | 46 | N/A |
| Objective Value | G-MSS | APP | Biskas’s |
|---------------|------|------|-------|
| 7564.23 | 7564.21 | 7564.94 |
| 39831.33 | 39830.57 | N/A |
| 39815.65 | 39814.87 | N/A |
| Global Optimum | G-MSS | APP | Biskas’s |
|---------------|------|------|-------|
| 7564.23 | 7564.21 | 7564.94 |
| 39831.33 | 39830.57 | N/A |
| 39815.65 | 39814.87 | N/A |
that the G-TDCM [28], so the G-MSS method is applicable and the resultant distributed algorithms are preferable.

Let $P_\text{T}$ denote the power injection in the TPS buses, $P_\text{D}$ the injection in the DPS buses, and $P_\text{f}$ the power flowing from the TPS to the DPS, which is “load” to the TSO and “generation” to the DSO. Then a T-D ED model minimizing the generation cost of the ITD system is formulated in (21):

$$
\min_{P_\text{T}, P_\text{D}, P_\text{f}} c_T (P_\text{T}) + c_D (P_\text{D})
$$

s.t.

$$
\begin{align*}
E_{P_\text{T}} P_\text{T} + F_{P_\text{f}} P_\text{f} & = d_T, \\
F_{P_\text{T}} P_\text{T} + F_{P_\text{f}} P_\text{f} & = e_T \\
E_{P_\text{D}} P_\text{D} + F_{P_\text{f}} P_\text{f} & = d_D, \\
F_{P_\text{D}} P_\text{D} + F_{P_\text{f}} P_\text{f} & = e_D
\end{align*}
$$

(21)

where $E$ and $F$ denote the coefficient matrices and their subscripts denote the right-hand vectors, and the subscripts $T$ and $D$ denote the systems the $d$ and $e$ are associated with. By introducing into (21) two auxiliary variables $P_{\text{BT}}$ and $P_{\text{BD}}$ such that $P_{\text{BT}} - P_{\text{BD}} = 0$ to replace $P_\text{f}$, and by defining $z_M = [P_\text{T}, P_{\text{BT}}], z_S = [P_\text{D}, P_{\text{BD}}]$ and a dummy boundary state $x_B = 0$, one can convert (21) into the equivalent in (22) that is a special case of the G-TDCM:

$$
\min_{z_M, x_B, z_S} c_T (z_M) + c_D (z_S)
$$

$$
\begin{align*}
& f_M (z_M, x_B) = [E_{P_\text{T}} P_\text{T} + E_{P_\text{f}} P_{\text{BT}} - d_T; x_B] = 0 \\
& f_{g_M} (z_M, x_B) = [F_{P_\text{T}} P_\text{T} + F_{P_\text{f}} P_{\text{BT}} - e_T; x_B] = 0 \\
& g_M (z_M) = F_{P_\text{D}} P_\text{D} + F_{P_\text{f}} P_{\text{BD}} - d_D \\
& g_S (z_S) = F_{P_\text{D}} P_\text{D} + F_{P_\text{f}} P_{\text{BD}} - e_D
\end{align*}
$$

(22)

With the G-MSS method being applied to (22), the exchange data $y_B^{\text{SP}}$ turns out to be $y_B^{\text{TB}}$ (i.e., $y_B^{\text{TD}}$) as $I_{BS} = 0$; the data $x_B^{\text{SP}}$ turns out to be $x_B^{\text{TB}}$ as $x_B = 0$. Hence, in the basic HGD algorithm, the transmission subproblem in (6) turns out to be an ordinary transmission ED problem with a given $P_{\text{BD}}^{\text{SP}}$ that is iteratively updated by the distribution subproblem (7) that becomes (23) in this case:

$$
\min_{P_\text{D}, P_{\text{BD}}^{\text{SP}}} c_D (P_\text{D}) + (\lambda_{\text{MB}}^{\text{SP}})^T P_{\text{BD}}
$$

s.t.

$$
E_{P_\text{D}} P_\text{D} + E_{P_{\text{BD}}^{\text{SP}}} P_{\text{BD}} = d_D, F_{P_\text{D}} P_\text{D} + F_{P_{\text{BD}}^{\text{SP}}} P_{\text{BD}} \geq e_D
$$

(23)

where $\lambda_{\text{MB}}^{\text{SP}}$ iteratively updated by the transmission subproblem is the locational marginal price (LMP) at the boundary bus [cf. (6)] in this case. Thus, with regard to this shift-factor based model (21), the basic HGD algorithm only requires to iteratively update and exchange $\lambda_{\text{MB}}^{\text{SP}}$ and $P_{\text{BD}}^{\text{SP}}$, which, in comparison with the case of using direct current power flow, saves half of the exchange data.

Moreover, this modelling also enables a simpler formulation of the preceding modified HGD algorithm. In this case, the transmission-response function should be equal to

$$
\frac{\partial b}{\partial x_B}
$$

(24)

The $\eta_{\text{k}}$ can be either solved from the sensitivity equations of the transmission subproblem or evaluated via a fitting approach based on preceding iteration data (cf. [29], and a comparison between these two approaches is also presented there).

Typically, for the T-D ED problem, this modified HGD algorithm converges faster and has a larger domain of convergence than the basic HGD algorithm does.

To validate the computational performance of the G-MSS method for this ED problem, the method is compared with two commonly used methods: the Biskas’s method [34], which is selected as the representative of the OCD methods, and the ATC method [24], which is a representative augmented Lagrangian relaxation method. Moreover, in order to test the robustness against the initialization, two versions of the basic HGD algorithm, named HGD1 and HGD2 were implemented. In HGD1, the initial $\lambda_{\text{MB}}^{\text{SP}}$ was set to 0, whereas it was arbitrarily set to be 10 in HGD2. The modified HGD algorithm was tested with the initial $\lambda_{\text{MB}}^{\text{SP}}$ set to be 0. The T24D9 and the T118D30 systems in [25] were taken as test systems. The TPS of the T24D9 and that of the T118D30 are the IEEE 24-bus and 118-bus test systems, respectively, and the DPSs are constructed as described in [25]. Thus, there are 10 subsystems, 97 buses and 35 generators in the T24D9, and 31 subsystems, 362 buses and 138 generators in the T118D30. A 24-hour ED problem was solved, and the time resolution was set to be 1 hour.

Table IX lists the results, which indicate that the G-MSS method enjoys the fewest iterations among the three methods. As the ED model (22) is convex, the convergent solution of the G-MSS is the global optimum of (22). Moreover, this table also shows that its optimality and the feature of requiring very few iterations for convergence are also robust against the different initial values for this T-D ED problem.

This G-MSS based TD-ED mode has been applied to a field demonstration project in a city in South China. This power system contains a TPS where there are two conventional coal-fired power plants and multiple DPSs where there are solar and wind distributed generators as well as 200 electric vehicles that can
respond to the system operation commands. The daily maximum load of this ITD system is 160 MW, and the penetration of the distributed generators is about 20% of the peak load. We selected two similar days: for one day there is no coordination between the transmission and distribution generation resources and for the other day the generation resources of the entire ITD system are coordinated by using the T-D ED mode. Table X shows that via this coordination, the total generation cost and carbon emissions of the ITD system are significantly reduced. Monthly statistics also show that the T-D ED mode helps this city to reduce the carbon emissions per unit of GDP by 32.94% on average, as the clean DERs are fully utilized. Moreover, since the generation resources of the entire ITD system and the potential line congestion are considered in the T-D ED problem, the LMPs of this ITD system are more accurately evaluated. Fig. 2 shows a daily LMP in a T-D interface. As the T-D ED mode utilizes all available generation resources, especially the cheaper and cleaner DERs, the LMPs are lower than their counterpart in the uncoordinated mode. Apparently, the LMPs in the T-D ED mode are more rational in terms of economics.

V. CONCLUSIONS

In this paper, a G-MSS method is suggested as a distributed solution to TDCEM. Based on the heterogeneous decomposition of the KKT conditions of the G-TDCM, two versions of HGD algorithms are presented. The basic HGD is simple and intuitive, while the modified HGD, though more complex in the formulation, typically converges faster, because it introduces a subsystem’s response function to reduce the derivative of the composite mapping of the boundary variables. Both algorithms are proved to have sure optimality and convergence properties. The distributed G-MSS method is demonstrated to successfully solve a series of central functions of the TDCEM, e.g., the power flow, contingency analysis, VSA, ED, and OPF, enabling distributed and effective cooperation between TSOs and DSOs to overcome the challenges arising from the DERs.

Future research directions include, for example, combing this G-MSS method with model predictive control to handle the uncertainties in an ITD system, and developing an effective and distributed heuristic approach to a mixed-integer G-TDCM from this G-MSS method. In addition, this G-MSS method can be used as a potential tool to establish an electricity market regarding an ITD system 0 as well as to coordinate different energy sectors of an integrated energy system 0.

APPENDIX A

PROOF OF $f_B = f_{MB} - f_{BS}$

Below we will show why $f_B(z_M, x_B, z_S)$ can be written as $f_{MB}(z_M, x_B) - f_{BS}(x_B, z_S)$, which requires no additional assumptions other than the boundary power flow equations.

Let $P_i$ and $Q_i$ denote the aggregate active and reactive power injections into bus $i$, respectively; let $V_i = |V_i|$ and $\theta_i = \text{arg}(V_i)$ denote the voltage magnitude and phase angle of bus $i$, respectively; let $G_{ij}$ and $B_{ij}$ denote the $(i, j)$ component of the susceptance and conductance matrices, respectively. Then, the active and reactive power flow equations for any boundary $i$ can be written as follows (25) and (26) shown at the bottom of this page.

\[
0 = P_{B,i} - V_{Bi} \sum_{j \in M} V_{M,j} (G_{ij} \cos(\theta_{B,i} - \theta_{M,j}) + B_{ij} \sin(\theta_{B,i} - \theta_{M,j})) 
\]

\[
0 = Q_{B,i} - V_{Bi} \sum_{j \in B} V_{B,j} (G_{ij} \cos(\theta_{B,i} - \theta_{B,j}) + B_{ij} \sin(\theta_{B,i} - \theta_{B,j})) 
\]

\[
0 = Q_{B,i} - V_{Bi} \sum_{j \in S} V_{S,j} (G_{ij} \cos(\theta_{B,i} - \theta_{S,j}) + B_{ij} \sin(\theta_{B,i} - \theta_{S,j})) 
\]

\[
0 = P_{B,i} - V_{Bi} \sum_{j \in S} V_{S,j} (G_{ij} \sin(\theta_{B,i} - \theta_{M,j}) - B_{ij} \cos(\theta_{B,i} - \theta_{M,j})) 
\]

\[
0 = Q_{B,i} - V_{Bi} \sum_{j \in B} V_{B,j} (G_{ij} \sin(\theta_{B,i} - \theta_{B,j}) - B_{ij} \cos(\theta_{B,i} - \theta_{B,j})) 
\]

\[
0 = Q_{B,i} - V_{Bi} \sum_{j \in S} V_{S,j} (G_{ij} \sin(\theta_{B,i} - \theta_{S,j}) - B_{ij} \cos(\theta_{B,i} - \theta_{S,j})) 
\]

Recalling that $x_M = [\|v_M\|; \text{arg}(v_M)]$, $x_B = [\|v_B\|; \text{arg}(v_B)]$, $x_S = [\|v_S\|; \text{arg}(v_S)]$, let $u_B$, (the $i$-th element of $u_B$) include the outer power injection into the boundary $i$, i.e., $P_{B,i}$ and $Q_{B,i}$, so for all the boundary buses, we can rewrite (25) and (26) as

\[
\sum_{j \in M} V_{M,j} G_{ij} \cos(\theta_{B,i} - \theta_{M,j}) + \sum_{j \in B} V_{B,j} G_{ij} \cos(\theta_{B,i} - \theta_{B,j}) + \sum_{j \in S} V_{S,j} G_{ij} \cos(\theta_{B,i} - \theta_{S,j}) = P_{B,i} 
\]

\[
\sum_{j \in M} V_{M,j} B_{ij} \sin(\theta_{B,i} - \theta_{M,j}) + \sum_{j \in B} V_{B,j} B_{ij} \sin(\theta_{B,i} - \theta_{B,j}) + \sum_{j \in S} V_{S,j} B_{ij} \sin(\theta_{B,i} - \theta_{S,j}) = Q_{B,i} 
\]
So far, we have found the domains $D_M$, $D_B$ and $D_S$ such that for any $\xi_S \in D_B$, the distribution subproblem and the transmission subproblem have unique local optima $\xi_S = H_S(\xi_B) \in D_S$ and $\xi_M = H_M(\xi_B) \in D_M$, respectively. This completes the proof.

2) In the domain $D_B$, define two mappings $\hat{y}_{BS}(\cdot) = h_{BS}(\cdot; H_S(\cdot))$, $\hat{y}_{MB}(\cdot) = h_{MB}(H_M(\cdot; \cdot))$, and $\lambda_{MB}$ is assumed to be invertible in the domain $D_B$. As shown in the basic HGD algorithm, in iteration $k$, with the input $\xi_{MB}^{p,k-1} \in D_B$, the local optimum of the distribution subproblem (7) is $\xi_S,k = [\xi_S,k; \xi_{MB,k}; \xi_{BS,k}]$, from which we have $y_{BS,k}^{p} = [y_{BS,k}^{p}; y_{BS,k}^{p}] = \hat{h}_{BS}(\xi_{BS,k}^{p})$. Next, with this $y_{BS,k}^{p}$, the local optimum of the transmission subproblem (6) is $\xi_{MB}^{p} = [\xi_{MB}^{p}; \xi_{MB}^{p}]$, and this can be equivalently written as $\xi_{MB}^{p} = \hat{h}_{MB}(y_{BS,k}^{p})$, so we have $\xi_{MB}^{p} = \hat{h}_{MB}(\xi_{BS,k}^{p})$. This means that the convergence of the HGD algorithm is equivalent to the composite mapping $\Phi = \hat{h}_{MB} \circ \hat{h}_{BS}$ converges to its fixed point in the domain $D_B$. This completes the proof.

**Proof of Convergence Theorem**

As $\Phi = \hat{h}_{MB} \circ \hat{h}_{BS}$ and $\hat{h}_{MB}, \hat{h}_{BS}$ are continuously differentiable as per the sensitivity theorem, it follows from the chain rule that in the neighborhood of $\xi_{MB}^{p}$, we have

$$\frac{\partial \Phi}{\partial \xi_B} = \frac{\partial \hat{h}_{MB} \circ \hat{h}_{BS}}{\partial \xi_B} = \frac{\partial \hat{h}_{MB}}{\partial \hat{h}_{BS}} \frac{\partial \hat{h}_{BS}}{\partial \xi_B}, \quad \frac{\partial \Phi}{\partial y_B} = \frac{\partial \hat{h}_{MB}}{\partial y_B} \frac{\partial \hat{h}_{BS}}{\partial \xi_B}$$

Hence, as long as the condition stated in (9) or (10) holds, $\Phi$ is a contraction mapping in the neighborhood of $\xi_{MB}^{p}$. As per the famous fixed-point theorem 0, this contraction mapping $\Phi$ converges linearly in the neighborhood of $\xi_{MB}^{p}$. This completes the proof.

**APPENDIX C**

**COMPARISON BETWEEN G-MSS AND OTHER COMMON DISTRIBUTED OPTIMIZATION ALGORITHMS**

To further clarify the advantages of the G-MSS method, Table XI is provided below to compare this method with other common distributed optimization algorithms. From this table, it can be clearly seen that compared with these algorithms, the G-MSS method has the following advantages: (i) in algorithm design, it neither has parameter tuning issue nor requires a central coordinator, and it is also compatible with current EMS software; (ii) its proven optimality and convergence properties do not limit to only convex problems; and (iii) computationally efficient for TDCEM applications, as was verified in Tables VIII and IX, because HGD is employed in the G-MSS method.
### TABLE XI
**COMPARISON BETWEEN G-MSS AND COMMON DISTRIBUTED OPTIMIZATION ALGORITHMS**

| Categories | Algorithms | Features | G-MSS Method |
|------------|------------|----------|--------------|
| Lagrangian Relaxation (LR) and other modified LR algorithms | LR (e.g. [37]) | Simple to implement | No need to introduce a penalty to improve convergence, so there is no parameter-tuning issue |
| | Predictor-Corrector Proximal Multiplier Method (e.g. [38]) | Usually has better convergence than the LR | A proven local first-order convergence for a convex or nonconvex optimization problem |
| | APP (e.g. [32]) | A penalty is introduced for better convergence, so there comes a parameter tuning issue that is inconvenient and tricky [33]; the convergence may be slow when the penalty is not well tuned |
| | Alternating Direction Method of Multipliers (e.g. [39]) | Two-level algorithm | Also has the parameter tuning issue |
| | ATC (e.g. [24]) | Local first-order convergence if the approximate Newton matrix is close to the accurate one | |
| Optimal Condition Decomposition | Newton Direction (AND) (e.g. [40]-[42]) | There is no parameter tuning issue | |
| | Decomposition-Cooperation Interior Point Method (e.g. [43]) | The subproblem is to solve linearized KKT equations rather than a local optimization problem, so the solution to a subproblem can be infeasible for the associated subsystem | |
| | Biskas’s Method (e.g., [31],[34]) | The convergence is said to be similar to the AND, but not formally proved | |
| | Boundary Coordinator Algorithm (e.g. [44]) | There is no parameter tuning issue | |
| Equivalencing-Based Algorithms | Network Equivalencing (e.g. [45]) | Easy to implement | Proven optimality with global optimality for a convex problem |
| | Marginal Equivalencing [46] | Optimality is not guaranteed | Proven local first-order convergence |
| | | Convergence is not guaranteed | |
| | | Proven convergence and optimality | Proven optimality with global optimality for a convex problem |
| | | Must be sequentially implemented | Proven local first-order convergence, and converges faster with response functions used |
| | | Designed for a convex ED problem and not tested for nonconvex problems, e.g. OPF | The optimality and convergence properties hold for a nonconvex problem |
| | | The dimension of a local subproblem keeps increasing, and in the worst case, its dimension can be close to that of the centralized ED model | The dimension of a subproblem maintains the same in the iteration |

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