Quantum Informational Dark Energy: Dark energy from forgetting

Jae-Weon Lee
School of Computational Sciences, Korea Institute for Advanced Study, 207-43 Cheongnyangni 2-dong, Dongdaemun-gu, Seoul 130-012, Korea

Jungjai Lee
Department of Physics, Daejin University, Pocheon, Gyeonggi 487-711, Korea

Hyeong-Chan Kim
Center for Quantum Spacetime, Sogang University, Seoul 121-742, Republic of Korea.
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We suggest that dark energy has a quantum informational origin. Landauer’s principle associated with the erasure of quantum information at a cosmic horizon implies the non-zero vacuum energy having effective negative pressure. Assuming the holographic principle, the minimum free energy condition, and the Gibbons-Hawking temperature for the cosmic event horizon we obtain the holographic dark energy with the parameter \( d \simeq 1 \), which is consistent with the current observational data. It is also shown that both the entanglement energy and the horizon energy can be related to Landauer’s principle.

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I. INTRODUCTION

The cosmological constant problem is one of the most important unsolved puzzles in modern physics [1]. There are strong evidences from Type Ia supernova (SN Ia) [2, 3], cosmic microwave background radiation [4] and Sloan digital sky survey (SDSS) observations [5] that the current universe is under an accelerating expansion, which can be explained by dark energy (a generalization of the cosmological constant) having energy density \( \rho_\Lambda \) and negative pressure \( p_\Lambda \) satisfying \( w_\Lambda \equiv p_\Lambda/\rho_\Lambda < -1/3 \). Although, there are already various models relying on materials such as quintessence [6, 7], k-essence [8], phantom [9, 10], Chaplygin gas [11], and quintom [12] among many [13, 14, 15], the identity of the dark energy is still mysterious. There are also many attempts to derive dark energy from the quantum vacuum fluctuation (See [16, 17, 18, 19, 20]). A fundamental problem in all these models is that there is no obvious way to cancel the \( O(M_P^4) \) zero point energy from the quantum vacuum fluctuation. Here \( M_P \) is the reduced Planck’s mass. On the contrary, the holographic dark energy (HDE) model has an intrinsic advantage over other models in that it does not need fine-tuning of parameters or an \textit{ad hoc} mechanism to cancel the zero-point energy of the vacuum, because quantum fields have less degrees of freedom from the start in this model, according to the holographic principle [21]. Furthermore, in this model, the cosmic coincidence problem could be also solved if there was an inflation with the number of efoldings \( N \simeq 65 \) [22, 23]. However, HDE model has its own difficulties [24, 21, 20] and the physical origin of HDE itself has yet to be adequately justified. Recently, we suggested that there can be a relation between dark energy and entanglement (nonlocal quantum correlation) which are the two great puzzling entities in modern physics [27].

In this paper, we suggest a new idea that dark energy is originated from the erasure of the quantum information at the cosmic horizon. This idea linking dark energy to quantum informational concepts is not so strange than it may appear, because we can treat every physical process in the universe as essentially a kind of quantum information processing [28, 29]. We will show that Landauer’s principle applied to this cosmological information erasing implies the existence of non-zero vacuum energy having effective negative pressure. Being one of main concepts in quantum information science [31, 32], Landauer’s principle states that to erase one bit information of a system irreversibly at least \( k \ln 2 \) entropy should be consumed, where \( k \) is the Boltzman’s constant and \( T \) is the temperature of a thermal bath contacting with the system. (Henceforth, we set \( k = 1 \).) Since this erased information can be connected to the vacuum fluctuation, our model provides a new way to obtain dark energy from the vacuum fluctuation. Bennett [33] solved the Maxwell’s demon problem using this principle, which is also related to reversible computation [29] and quantum computation. Landauer’s principle have been also used to study thermodynamics of black holes by many authors [34, 35, 36, 37, 38]. We will also show that the entanglement energy and the horizon energy can be explained by Landauer’s principle.

In Sec. II we show that the minimum free energy condition and the holographic principle give a rise to HDE with \( d \simeq 1 \) as observed. In Sec. III, we review Landauer’s principle and suggest that this minimum free energy con-
dion is related to the quantum information erasing at the cosmic horizon and dark energy comes from this principle. In Sec. IV, we consider more general situations by relaxing the restrictions for dark entropy (the entropy related to dark energy) $S_A$ and the horizon temperature $T$. We show that even in this case the quantum informational dark energy has a form of HDE generally. Section V contains discussions.

II. HOLOGRAPHIC DARK ENERGY AND MINIMUM FREE ENERGY CONDITION

In this section we show that by assuming the holographic principle, the minimum free energy condition and Gibbons-Hawking temperature for the cosmic event horizon, we can easily obtain the HDE with the parameter $d \approx 1$ as observed. In section II and III we choose the event horizon as an IR-cutoff. Before going further we need to shortly review the entanglement dark energy model [22]. The vacuum entanglement entropy [39] associated with the entanglement entropy $S_{Ent}$ [40] between inside and outside parts of the cosmic event horizon could give dark energy density in the form of HDE [22]

$$\rho_A = \frac{3d^2 M_p^2}{R_h^2},$$

where $R_h$ is the radius of the future event horizon and $d$ is a parameter. Obtaining the exact value of $d$ is important, because it determines the properties of HDE and its approximate value was derived recently only in [27].

More precisely, we suggested that (entanglement) dark energy satisfies

$$dE_A = TdS_A,$$

where the temperature $T$ is the Gibbons-Hawking temperature of the event horizon [41] [42] [43] and the dark entropy $S_A$ is given by the entanglement entropy of the vacuum fluctuation defined as

$$S_{Ent}(\rho_A) \equiv -Tr(\rho_A \log \rho_A) = h(\rho_A),$$

where $h$ is the von Neumann entropy. This energy can be also interpreted as the ‘horizon energy’ [14] representing the vacuum energy inside the horizon according to the holographic principle. It is natural to divide the universe $(AB)$ into the inside $(A)$ and the outside $(B)$ of the event horizon to calculate the entanglement entropy, because the event horizon represents the global causal structure. (See Fig. 2.) The reduced density matrix $\rho_A \equiv Tr_B \rho_{AB}$ represents an effective subsystem $A$ of the whole system $AB$ described by a density matrix $\rho_{AB}$ [29] [45]. This entanglement dark energy can be also related to the cosmic Hawking radiation (See the comments in [46, 47]).

Using $M_p$ as a UV cut-off and the spin degrees of freedom ($N_{dof} = 118$) in the standard model, we obtained the parameter (Eq. (12) in Ref. [27])

$$d = \sqrt{\frac{0.3 N_{dof}}{2\pi}} \approx 0.95,$$

which is well consistent with current observational data $d = 0.91^{+0.26}_{-0.18}$ [15]. Although the chosen values for these input parameters seems to be reasonable, it is desirable to get a more plausible and general model having less free parameters, which we pursue in this paper.

To remove the ambiguity of input parameters, in this section, we assume that the quantum informational dark entropy $S_A$ is given by the holographic principle [41], i.e., $S_A = S_{hod} \equiv \frac{A m_p^2}{4} = \pi R_h^2 m_p^2$, where $A \equiv 4\pi R_h^2$ is the surface area of the cosmic event horizon and $m_p = \sqrt{8\pi M_p}$. For the temperature $T$ we use the Gibbons-Hawking temperature [41] $T = 1/2\pi R_h$. This is a plausible choice because our universe is under an accelerating expansion and going to a dark energy dominated universe, which can be a quasi-de Sitter universe. (This kind of space-time attracts much interest recently in relation with dS/CFT correspondence [49].) Note that although this temperature is very low, due to the huge entropy proportional to the horizon area, dark energy $E_A \sim T S_A$ could be comparable to the observed value. By inserting $S_A$ and $T$ into Eq. (2) and integrating it one can obtain

$$E_A = \int dE_A = \int TdS_A = 8\pi R_h M_p^2$$

and HDE density $\rho_A = E_A/(4\pi R_h^3/3) = 6M_p^2/R_h^2$ with $d = \sqrt{2}$, which, however, somewhat deviates from the observed value $d = 0.91^{+0.26}_{-0.18}$. This problem can be alleviated by considering the minimum free energy condition $dF = d(E_A - T S_A) = 0$, or,

$$dE_A = d(T S_A) = TdS_A + S_A dT,$$

instead of Eq. (2). The second term may represent the contribution from the change of the temperature in addition to the typical thermal energy term $T dS_A$. (In section III we will consider another interpretation of this minimum free energy condition.) In this case

$$dE_A = d \left( \frac{\pi R_h^2 m_p^2}{2\pi R_h} \right) = \frac{m_p^2 dR_h}{2},$$

from which one can obtain $d = 1$ by repeating the integration in Eq. (5). For $d = 1$ the equations (25) and (26) below give the equation of state $w_A = -0.903$ and its derivative $w_1 = 0.208$ at the present. (See Eq. (28) for the exact definition of $w_1$. ) These results are comparable to the recent observational data; $w_A = -1.03 \pm 0.15$ and $w_1 = 0.409^{+0.582}_{-0.567}$ [29] [31]. (For $d = \sqrt{2}$, $w_A = -0.736$ and $w_1 = 0.12$. ) This is our first main result, which suggests that the correct equation for the dark energy could be Eq. (6) rather than Eq. (2). This seems to be reasonable, because we need to consider the effect of the variation of $T$ on $E_A$.

Since our universe is not exactly equal to the de Sitter universe, $T$ can be slightly different from that of the de Sitter universe. Thus, it is also expected that $d$ is approximately 1 as observed. Recall that we get this result
without introducing ambiguous parameters such as the UV-cutoff or $N_{\text{dof}}$ in our calculation. With the simple and reasonable assumptions, HDE with $d \simeq 1$ can be easily derived.

It was shown in [22] that if the event horizon of our universe is that of a black hole, using the black hole Hawking temperature $T = 1/4\pi R_h$ and $dE_\Lambda = TdS_\Lambda$ one can obtain $E_\Lambda = 4\pi R_h M^2_\Lambda$ and $\rho_\Lambda = 3M^2_\Lambda/(8\pi)^2$, that is, $d = 1$. However, it is obvious that our universe is not exactly like a black-hole.

It is tempting to interpret the minimum free energy condition in Eq. (6) as the condition for the vacuum in a canonical ensemble, however, it is unclear whether we can treat the horizon as a thermal system in a canonical ensemble. In the next section we consider an alternative possibility that this condition comes from Landauer’s principle and dark energy has a quantum informational origin.

III. LANDAUER’S PRINCIPLE AND DARK ENERGY

First of all, it is important to make a clear distinction between two terminologies, the information ‘loss’ in black hole physics and the information ‘erasing’ in quantum information science. By information ‘loss’ we mean, for example, a non-unitary transformation from a pure state to a mixed thermal state at a black hole. On the other hand the information ‘erasing’ in quantum information science is a transformation from unknown states to a specific state by compulsion, a process usually reducing the entropy of the state [32]. It is also different from ‘measurements’ where the unknown states randomly collapse to one of measurement basis states. For example, consider a one-bit memory which consists of cylinder and an atom in it as shown in Fig. 1. A thermal bath with the temperature $T$ keeps in contact with the cylinder. Let us denote ‘0’ (‘1’) by locating the atom in the left (right) partition. To erase this memory we use the piston to push the atom into the left partition regardless of its initial position by investing a work $W_{\text{sys}}$. Then, the initial information of the atom is erased irreversibly, which is similar to what happens during a formatting of a computer hard disk. According to Landauer’s principle the entropy of the cylinder decreases by ln2 at the cost of at least $T\ln2$ free energy consumption, which means $W_{\text{sys}} \geq T\ln2$. (See [32, 53, 54] for details.) This energy is eventually converted to thermal energy of the bath increasing the total entropy of the whole system (the bath+the cylinder) and saving the second law of thermodynamics. Note that, for general systems, $W_{\text{sys}}$ needs not to be a mechanical work. Any free energy that can be used to erase the information and to reduce free energy of a system can play a similar role.

More generally, according to the principle to erase the information of a system (for example, the cylinder in Fig. 1) represented by $dS_{\text{sys}} < 0$, the work $W_{\text{sys}} \geq T|dS_{\text{sys}}|$ should be invested, if $T$ is fixed. This eventually increases the entropy of the bath by $dS_{\text{bath}}$ and the thermal energy of the bath at least by $T|dS_{\text{sys}}|$ such that the total entropy increases by $dS_{\text{tot}} = dS_{\text{bath}} + dS_{\text{sys}} \geq 0$. Alternatively, Landauer’s principle can be restated as the claim that the net free energy gain of the whole system during the erasure can not be larger than the work invested, i.e.,

$$dF_{\text{tot}} \equiv dF_{\text{bath}} + dF_{\text{sys}} = d(E_{\text{bath}} - TS_{\text{bath}}) + d(E_{\text{sys}} - TS_{\text{sys}}) \leq W_{\text{sys}}.$$  (8)

Otherwise, one can use the increased net free energy $dF_{\text{tot}}$ to run a perpetual motion machine of the second kind, because the free energy is a measure of the amount of work extractable from a system. This clearly violates the second law of thermodynamics. One can say that the erasure is done most efficiently when the above equality holds. In this case the work $W_{\text{sys}}$ is used solely to erase the information and the net increase of the total entropy is zero. For this optimal case and $T$ fixed,

$$dS_{\text{bath}} = |dS_{\text{sys}}|, \quad dE_{\text{sys}} = 0, \quad W_{\text{sys}} = T|dS_{\text{sys}}| = dF_{\text{bath}},$$  (9)

and the condition in Eq. (8) reduces to $dF_{\text{bath}} = 0$, the minimum free energy condition, which leads to $dE_{\text{bath}} = TdS_{\text{bath}}$. This suggests that the the horizon energy [44] or the ‘first-law’ often given in this form has a quantum informational origin. In Ref. [27], by applying a similar condition to the black hole horizon, we derived the first law of black hole physics and the discrete black hole mass. For more general cases where $T$ can vary, we can expect that $W_{\text{sys}} = dF_{\text{sys}}$ and the minimum free energy condition ($dF_{\text{bath}} = 0$) still holds for an optimal erasing process, where information is erased most efficiently.

In this paper, we suggest that the cosmic horizon plays the role of the bath and is one of the most efficient information eraser saturating the bound in Eq. (8) and

![FIG. 1: According to Landauer's principle, to erase one bit information (the position of an atom, the gray points) in the cylinder at least $T\ln2$ work $W_{\text{sys}}$ should be invested. Note that the local entropy within the cylinder decreases while the entropy of whole system (cylinder+bath) should not decrease.](image-url)
dark energy is this marginal horizon (vacuum) energy $E_{\Lambda}$ originated from the quantum information erasing at the cosmic horizon with the Gibbons-Hawking temperature $T$. That is, $dF_{\text{bath}} = dF_{\Lambda} = 0$. Thus, the change of dark energy $E_{\Lambda}$ is related to the increase of dark entropy by $dS_{\Lambda}$ at the horizon as $dE_{\Lambda} = d(TS_{\Lambda})$.

Let us discuss in detail how one can apply Landauer’s principle to the cosmic horizon. (See Fig. 2.) Here, the horizon plays a role of both the piston and the bath in Fig. 1 at the same time. For an observer $O_A$ the larger his/her cosmic horizon expands, the more information about outer region is erased at the horizon, because $O_A$ sees an expanding spherical horizon eating up the outer space as it expands. What exactly is the information erased here? Let us consider the quantum field $\phi$ with a Lagrangian $L(\phi)$ in the universe. Following Ref. [40] we can discretize the space and think the field as a collection of linked quantum oscillators $\phi_i$ located on the lattice with a Planck length $(l_p \approx \sqrt{\hbar G})$ spacing. Then, the vacuum is the ground state of the oscillators. As the horizon expands, the inside region $(A)$ is extended by engulfing the outer region (the gray region in the figure). Since the cosmological variation of the gravitational constant $G$ seems to be negligible $\left( \frac{dG}{dt} / G \right) \approx 10^{-12} \text{ yr}^{-1}$, we expect the enlargement of the region $A$ is not by the extension of the lattice spacing $l_p$ but by the appearance of new lattice sites and oscillators on them in the ground state at the horizon. The information erased is that of the new oscillators appeared at the region having been once outside the horizon (the gray region). For the observer $O_A$, regardless of their initial quantum states, the oscillators at the erased region are forced to go to a specific state (the ground state) of the region $A$ by propulsion as the horizon expands. During this process the local entropy of the gray region decreases, while that of the horizon increases. (Note that this does not mean that normal particles outside the horizon can enter the region $A$.) This is a kind of information erasure. Since we are interested in the energy of the vacuum, for simplicity, we ignore the erasure of the information of matter at the horizon.

For a toy example, imagine the black dot in the figure as a ‘vacuum’ qubit in the gray region. For the observer this vacuum qubit is initially out of the horizon and can be described as the maximally mixed state, $\rho = I/2$ having $\ln 2$ entropy. As the horizon expands, the region is reachable and the qubit becomes a part of the visible vacuum of the observer, which has zero entropy. Thus, $\Delta S_{\text{sys}} = -\ln 2$. This ‘resetting’ requires increase of the entropy of the thermal bath (horizon) by $\ln 2$.

Consider a more realistic situation, where the horizon changes from $R_h$ to $R_h + \Delta R_h$ during the time interval $[t, t+\Delta t]$. The density matrix of the erased system which is between $R_h$ and $R_h + \Delta R_h$ changes from $\rho_{\text{sys}}(t)$ to $\rho_{\text{sys}}(t+\Delta t)$, and the total system changes from $\rho_{\text{AB}}(t)$ to $\rho_{\text{AB}}(t+\Delta t)$, while the dark entropy of the horizon $S_{\Lambda}$ changes as

$$\Delta S_{\Lambda} = S_{\Lambda}(\rho_{AB}(t+\Delta t)) - S_{\Lambda}(\rho_{AB}(t)).$$

The horizon at $t$ plays a role of the heat bath for this time interval. For the optimal erasing, it should satisfy the minimum free energy condition; $\Delta F_{\Lambda} = 0$. Thus, the increase of the horizon energy is $\Delta E_{\Lambda} = \Delta(TS_{\Lambda})$. If this energy $E_{\Lambda}$ increases as the universe expands, it can play a role of dark energy (See Eq. (21)). This is a cosmological embodiment of the famous slogan in quantum information science “information is physical”.

Let us consider an infinitesimal erasing process. In general, it is not easy to derive $dS_{\text{sys}}$ directly. Fortunately, what we need to obtain $\rho_A$ is $dS_{\Lambda}$. We already know two methods to calculate $dS_{\Lambda}$. First, if this $S_{\Lambda}$ is the entanglement entropy $S_{\text{Ent}}(\rho_A)$ in Eq. (3), the model reduces to the entanglement dark energy model in Ref. [27] (with $S_{\Lambda}dT$ ignored), where $dS_{\Lambda}(\rho_{AB}) = dh(\rho_A) = d(-Tr(\rho_A n \rho_A))$. The information erased here is the entanglement information of the vacuum fluctuation and calculated in Ref. [27, 40]. In this case the dark entropy is from the entanglement of the vacuum fluctuation (virtual particles) created near the horizon. This explains the physical origin of the entanglement energy [39], which has many different definitions. (Peacock had conjectured there is energy in entangled systems related to Landauer’s principle [50].) If we use Eq. (6) instead of Eq. (2) for the entanglement dark energy in Ref. [24], with the same UV-cutoff, one can obtain

$$d = \frac{\sqrt{0.3N_{\text{def}}}}{2\sqrt{2\pi}} \approx 0.67$$

and $w_{\Lambda} \approx -1.18$ for the standard model and ($d \approx 0.96$,
$u_{\Lambda}^{0} \simeq -0.925$ for the minimal supersymmetric standard model ($N_{\text{dof}} = 244$). That means the definition of dark energy in Eq. (6) seems to favor supersymmetry when applied to the entanglement dark energy. (However, there are also arguments that the observations favor $u_{\Lambda}^{0}$ smaller than $-1$ [12].)

Second, if we think $S_{A}$ as the entropy given by the holographic principle $S_{\text{hol}} = \pi R_{h}^{2}m_{P}^{2}$, then we can easily obtain HDE with $d = 1$ using Eq. (7) as shown in the section II. The information erased in this case can be interpreted as the all degrees of freedom in the vacuum in the gray region. This interpretation could explain the physical origin of the horizon energy of the universe with the horizon.

Note that for both cases we get $d$ values and dark energy density explicitly. It is an important open question whether $S_{\text{Ent}}$ is exactly equal to $S_{\text{hol}}$ or not [57].

Contrary to the black hole case, the location of event horizons in the Friedmann universe depends on a choice of observer [53]. However, due to the cosmological principle every point in the universe has the same amount of dark energy at the same comoving time.

## IV. DARK ENERGY FOR MORE GENERAL HORIZONS

In this section, we even further reduce the assumptions about the dark entropy, the horizon and the horizon temperature. From information science viewpoint, the event horizon is the most natural candidate for our purpose, because it plays a role of a natural information barrier. However, we need to check whether other horizons, such as Hubble horizon, particle horizon and apparent horizon can replace the event horizon or not. We adopt the approach of Hsu and Li [22] in this paper. Since the Hubble horizon and the apparent horizon can be related to the particle horizon asymptotically [60], we can concentrate only on the future event horizon and the particle horizon. Let us denote the radius of these horizons as $r$.

The particle horizon is defined as

\[
R_{p} \equiv R(t) \int_{0}^{t} \frac{dt'}{R(t')} = R(t) \int_{0}^{R} \frac{dR(t')}{H(t')R(t')^{2}}, \tag{12}
\]

and the future event horizon is

\[
R_{h} \equiv R(t) \int_{t}^{\infty} \frac{dt'}{R(t')} = R(t) \int_{R}^{\infty} \frac{dR(t')}{H(t')R(t')^{2}}, \tag{13}
\]

where $R(t)$ is the scale factor of the universe and $H$ is the Hubble parameter as usual. Here we consider the flat ($k = 0$) Friedmann universe which is favored by observations and inflationary theory and is described by the metric

\[
ds^{2} = -dt^{2} + R^{2}(t)d\Omega^{2}. \tag{14}
\]

We investigate the behavior of dark energy during the dark energy dominated era only. In cosmology, physical quantities are often expressed as polynomial functions of some length scale. For example, for the flat Friedmann universe energy density, scale factors, and comoving time can be written as power-law functions of the Hubble horizon size. Let us assume, in this paper, that the dark entropy and the temperature are power-law functions of some horizon radius ($r$), that is, $S_{A} \propto r^{\tau_{s}}$ and $T \propto r^{\tau_{T}}$. For the entanglement dark energy $\tau_{s} = 2$ and $\tau_{T} = -1$. If this temperature is the Hawking temperature our observable boundary of the universe is something like a black hole horizon. By integrating $dE_{\Lambda} = d(TS_{A})$ one can obtain

\[
E_{\Lambda} = \epsilon M_{P}^{n+1} r^{n}, \tag{15}
\]

where $n = \tau_{s} + \tau_{T}$ and $\epsilon$ is a dimensionless proportional constant which depends on the exact definitions of $T$ and $S_{A}$. Using $dE_{\Lambda} = TdS_{A}$ will give the same results up to a proportional constant. (We will see later that this constant $\epsilon$ determines properties of dark energy.) Thus, the dark energy density is given by

\[
\rho_{\Lambda} = \frac{3E_{\Lambda}}{4\pi r^{3}} = \frac{3\epsilon M_{P}^{n+1} r^{n-3}}{4\pi} = 3M_{P}^{2}H^{2}, \tag{16}
\]

where the last equality comes from the Friedmann equation. From this we obtain

\[
r = \eta H^{-\frac{2}{\tau_{s}}}, \tag{17}
\]

where $\eta = (4\pi M_{P}^{-n+1}/\epsilon)^{-1/(n-3)}$.

First, let us examine the case where the horizon is the particle horizon, i.e., $r = R_{p}$. From Eq. (12) and the above equation one can obtain

\[
\int_{0}^{R} \frac{dR'}{HR'^{2}} = \frac{\eta H^{-\frac{2}{\tau_{s}}}}{R}. \tag{18}
\]

Differentiating the above equation with $R$ we obtain

\[
\frac{1}{HR^{2}} = \frac{d}{dR} \left(\frac{H^{-\frac{2}{\tau_{s}}}}{R}\right). \tag{19}
\]

Setting $H^{-1} \equiv \alpha R^{x}$ we rewrite the above equation as

\[
\alpha R^{x-2} = \eta \frac{d}{dR} \left(\left(\alpha R^{x}\right)^{-\frac{2}{\tau_{s}}}\right) = \eta \alpha^{-\frac{2}{x-3}} \left(\frac{-2x}{n-3} - 1\right) R^{\frac{2x}{3}-2}. \tag{20}
\]

Comparing both sides, one can find that $n = 1$. Thus, $\eta$ turns out to be dimensionless in this case. One can also note that $x = 1 + 1/\eta$. Thus $t^{-2} \propto H^{2} \propto R^{-2x} \propto R^{-2(1+\frac{1}{\eta})}$ and, hence, $R \propto t^{\frac{1}{1+\frac{1}{\eta}}}$. Therefore, the particle horizon does not give rise to an accelerating expansion and there is no dark energy in this case.

Now we move on the case with the future event horizon. Repeating the same procedure using $R_{h}$ for $r$ instead of $R_{p}$ (i.e., $r = R_{h}$) one can obtain $n = 1$, $x = 1 - 1/\eta$ and $R \propto t^{\frac{1}{1-\frac{1}{\eta}}}$ which shows accelerating expansion as in HDE.
model. This difference came from the position of $R$ in the integral limit of Eq. (12) and Eq. (13). Therefore, we confirm that the future event horizon is a natural horizon for dark energy in our model. Since $n = 1$, our dark energy has the mathematical form of HDE, i.e., $\rho \propto R_0^{-2}$ in general. Comparing Eq. (16) with the holographic dark energy:

$$\rho_A = \frac{3\epsilon M_p^2}{4\pi R_h^2} = \frac{3\epsilon^2 M_p^2}{R_h^2},$$  \tag{21}

one can find that

$$d = \sqrt{\frac{\epsilon}{4\pi}} = \eta.$$  \tag{22}

According to observations $[48]$ $d \approx 1$, thus $\epsilon \approx 4\pi$.

How can we verify our model by current observations? Once we obtain $\rho_A$, the negative pressure $p_A$ can be derived from the cosmological energy-momentum conservation equation as usually done in holographic dark energy models $[61]$. From the Friedmann equation with perfect fluid having stress-energy tensor

$$T_{\mu\nu} = (\rho_A + p_A)U_\mu U_\nu - p_A g_{\mu\nu},$$  \tag{23}

where $U^\mu U_\mu = 1$, one can derive the cosmological energy-momentum conservation equation;

$$p_A = \frac{d(R^3 \rho_A)}{-3R^2dR}.$$  \tag{24}

From this, one can notice that a perfect fluid with total energy increasing as the universe expands has effective negative pressure. From Eq. (24), one can find that the present equation of state for HDE in Eq. (11) is $[22, 62]$ $w_A^0 = -\frac{1}{3} \left(1 + \frac{\sqrt{\Omega_A}}{d} \right)^{-1} = -\frac{1}{3} \left(1 + 2\frac{4\pi \Omega_A}{\epsilon^2} \right),$  \tag{25}

and the change of the equation of state is given by $[62]$

$$\frac{dw_A(z)}{dz} = \frac{\sqrt{\Omega_A} (1 - \Omega_A)}{3d(1 + z)} \left(1 + 2\frac{\sqrt{\Omega_A}}{d} \right) = \frac{\sqrt{4\pi \Omega_A} (1 - \Omega_A)}{3\sqrt{\epsilon}(1 + z)} \left(1 + 2\frac{\sqrt{4\pi \Omega_A}}{\sqrt{\epsilon}} \right),$$  \tag{26}

where $z \equiv 1/R - 1$ is the red shift parameter and $\Omega_A^0 \approx 0.73$ is the observed present value of the density parameter $\Omega_A$ of the dark energy. (Here we set the current scale factor $R_0 = 1$.) To compare the results with observations it is useful to represent the above equation as a function of $1 - R$ using the relation:

$$\frac{dw_A}{dz} = \frac{d(1 - R)}{dz} \frac{dw_A}{d(1 - R)} = \frac{R^2}{d(1 - R)} \frac{1}{(1 + z)^2} \frac{dw_A}{dz}. $$  \tag{27}

Then, for $z \ll 1$, $w_A(z) \simeq w_A^0 + w_1 (1 - R)$, where

$$w_1 = \frac{dw_A}{d(1 - R)} |_{R=1} = \frac{dw_A(z)}{dz} |_{z=0}. \tag{28}$$

Now, we can compare predictions of our theory with recent observations $[51, 61]$. Inserting the observed $w_A^0 = -1.03 \pm 0.15$ into Eq. (25), one can obtain $\epsilon = 8.4^{+5.2}_{-2.8}$. From this we also obtain $w_1 = 0.29^{+0.11}_{-0.10}$ using Eq. (26) and Eq. (27), which are comparable to the observed value, although the observational uncertainty is still large. The near future observations will give stronger constraints on $w_A^0$ and $w_1$, and hence, on $\epsilon$. (Note that the specific model considered in section II and III already predicts $d \approx 1$ and hence $\epsilon \approx 4\pi$.)

V. DISCUSSION

The cosmic coincidence problem can be also easily solved in our model using the result of $[22, 23]$, if there was an ordinary inflation with the number of e-folds $N \simeq 60$ at the very early universe driven by some inflation fields, because our model gives dark energy in the form of HDE.

Where does the energy erasing information come from? A similar issue exists in the inflation theory regarding the expansive reproduction of the vacuum energy during the inflation. Since the information is erased as the horizon expands, for an inside observer, the horizon plays a role of the cylinder in Fig.1. And this horizon expands due to the cosmic expansion after the inflation. Thus, one can imagine that the energy comes from ‘kinetic’ energy of the universe originated from the inflationary expansion. Alternatively, one can also imagine the energy is borrowed from negative gravitational energy as in the idea of the zero-energy universe $[63, 65]$. In the both scenarios dark energy and inflation have a deep connection. However, note that the energy alone is usually not conserved in general relativity. What we can safely rely on is the energy momentum conservation in Eq. (24).

It is still possible that at the earlier universe the dark entropy and the temperature are not simple power-law functions of $R_h$. Furthermore, another horizon besides the event horizon could give an accelerating universe when we include the interaction between dark energy and ordinary matter. In these cases our theory may give a dark energy model deviated from the simple holographic dark energy model and be free from the known difficulties of HDE model.

It is also interesting that the cosmic Hawking radiation or the energy from Landauer’s principle may be simulated on the acoustic horizons $[64, 65]$ or optical black holes $[66, 67]$ in the future.

Our theory require neither exotic matter, fine-tuning of potential nor modification of gravity. Assuming the holographic principle, Landauer’s principle and Gibbons-Hawking temperature, one can well describe the observed
dark energy. Our work indicates that the solution of the dark energy problem may not rely on exotic materials or radical new physics but the holographic principle and new aspects of familiar quantum physics such as entanglement and Landauer’s principle.

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