Electromagnetic Signal from a Circumbinary Gas Disk Around Merging Binary Black Holes

Zhuoyun Huang1 *, Haoran Yu2 *
1 Westlake Girls High School, Auckland, Auckland Region, New Zealand
2 Liangjiatan International School, Xi’an, Shaanxi, China
*17713@my.westlakegirls.school.nz , alex.yu@student.xalis.com

Abstract. The detection of gravitational waves (GW) is a major achievement in physics and astrophysics. Up to now, more than 30 GW events resulting from the merger of compact celestial bodies have been observed by LIGO and VIRGO. Because electromagnetic radiation is the main method for studying astrophysical processes, whether physics can be combined with astronomy depends on whether the corresponding body of gravitational wave events can be found on electromagnetic radiation. Unfortunately, only the electromagnetic counterparts of the twin neutron star merger are currently seen. In gravitational wave astronomy, a very important subject is: Is it possible to generate electromagnetic radiation when the two black holes’ merge? In this essay, the dynamic process of the gas disk surrounding the binary black hole is studied through numerical simulation, and the long-term evolution process of these gases on the cosmological time scale is studied at the same time. The evolution process of the gas disk of the ring binary black hole can be obtained, which provides a theoretical limit on whether the electromagnetic counterpart of the merger of the binary black hole can be seen in the future.

1. Introduction
The direct detections of gravitational waves (GWs) by LIGO and VIRGO have satisfied people’s anticipations. Detections such as GW150914 and GW170104 not only provided the confirmations on Einstein’s theory of general relativity but also the direct observable evidence of black holes mergers. Unlike the detected GW waves such as GW170817 and GW190425 from the merging of binary neutron star inspiral, any electromagnetic counterparts (EM) signatures from the merging binary black holes are not expected. However, at the same time detection of GW150914 from LIGO and VIRGO on 2015 September 14, the Gamma-ray Burst Monitor (GBM) on the Fermi Gamma-ray Space Telescope was triggered for over 5000 times due to the possible impulses of photons from the merging of binary black holes. It is concluded that EM signals had a background fluctuation of non-astrophysical origin from the black holes merging. Nevertheless, the existence of EM counterparts during BBH mergers worth detailed studies.

The public consensus on the merges of typical binary black holes are “dark” is questioned in this essay. There is a possibility that the EM signal will follow the merge of binary black holes, with a time delay of detection. In order to make this mechanism work, a circumbinary disk must be a part of the binary black holes all the time during the merging process with the mass of a small fraction of that shed as the whole system evolves. The formation of a circumbinary disk must be natural, unlike the circumbinary disk from the dynamic interaction of a star cluster. The circumbinary disk is under the interaction from tidal stress and viscous. If the disk survives until the end of the merge, the mass of the disk will decrease substantially in a form of kinetic energy, and eventually radiates away from the newly
formed black hole. At that time, the disk is expected to be cool and thin, so the orbital motion of the remaining particles in the disk that are still in contact with the newly formed black hole is hypersonic and easy to prompt shock waves. These can, in principle, produce EM signals. Also, with the merge of binary black holes, the production of GWs will work as a means of reduction to “kick” some of the central mass out of the system. This course of actions from the GWs can be an illustrated way to explain the way that the disk radiates away from the merged black hole.

The EM signals from the merging binary black holes are crucial for the investigation of the strong-field general relativity and potential criteria of a merging binary black hole. Finally, there will be scientific payoff by carefully studying the EM signal emissions from a merging binary black hole.

2. Theoretical model

2.1. Background
Throughout the calculations and essay, we used the cgs unit and made several assumptions. We assume that there will be a gas disk formed around two BHs which their track (origin) in the formation of binary BHs. We assume there is no dead zone, the disc is thin, the angular velocity is equivalent to the Keplerian velocity and it is efficiently cooled. The mass of binary BHs, \( M_{\text{bin}} = 50 M_\odot \). The mass ratio \( q = \frac{M_s}{M_p} \) is 0.8 and the total heating rate per unit surface, \( Q^+ \), is equivalent to the total cooling rate per unit surface \( Q^- \). The disk-BH mass ratio, \( f \), is \( 10^{-2} \). The initial condition before the binary black hole merger is that the separation of two black holes is \( 10^{-2} \) in the unit of \( R_g \). We then investigate the evolution of two BHs over cosmic time. Firstly, we have to provide the evolution of the circumbinary disk and the final equations to be solved. Then, we need to adopt two typical timescales \( t_{gw} \) and \( t_\Lambda \) to plot the effects of different binary separation on the merger time. Lastly, we estimate the luminosity of an accreting BH.

2.2. Evolution of circumbinary disk
The formation of binary blackholes can categorized into 3 ways: the classical common-envelope scenario, chemically homogeneous evolution scenario, and dynamical formation channels. In classical common-envelope scenario, which the binary blackholes in the isolated system is formed through non-conventional energy transfer; the chemically homogeneous evolution scenario, which binary blackholes are formed through two tidally distorted binary stars stay compact as they experience strong internal mixing; the dynamical formation channels, which binary black holes are formed through multiple three-body interactions in dense star clusters. During those different evolution phases, the circumbinary materials exist in the system. Circumbinary disks can be formed by mass loss from one or both stars through the outer Lagrangian points in the highly evolved system. The Circumbinary disks can also be formed when the remaining mass after the mass shedding circles due to centrifugal force. The mass shedding can be both conventional and non-conventional. The succession of close to impacts expected to create a tight binary blackholes will likely strip it of any circumbinary material. We presume that EM signals from an enduring circumbinary disk can possibly recognize arrangement situations.

Tidal torque: Within the binary BHs, if \( q \) is moderately small i.e. \( q \leq 1 \), a gap will be formed around the primary BH and exchange angular momentum with the gas. Moreover, \( g \) will be smaller and thus reduce the tidal torque. Here, as we adopt \( q = 0.8, g = 0.25 \).

\[
g(q) = \frac{q}{(1+q)^2} \tag{1}
\]

if \( q \) is relatively large, the angular momentum flux from the secondary locally dominates the viscous flux. Thus, gas is repelled from high-m resonance. The surface density drops near \( R = a \),
forming a gap, the conditions for the effective gap opening are $q \geq 3 \left(\frac{H}{R}\right)^3$, which is easily satisfied and thus prevents further accretion with an ideal tidal barrier. This forms a decretion disk and removes angular momentum of the binary to the gas disk which accelerates the merger. Currently, the binary BHs we observed have moderately large $q$ value due to the tidal torque. The Lindblad resonance is used to describe the tidal torque between the gas disc and the binary BHs. Excitation of density waves is the form of the resonance which carries energy and angular momentum in the disc and dissipates through shocks. Considering the impulse approximation, the tidal torque can be written in the following two ways, under different conditions:

$$\Lambda_\tau(R,a) = \frac{f}{2} g^2 \Omega^2 R^3 \left(\frac{a}{\Delta}\right)^4, R > a$$  \hspace{1cm} (2)

When $R > a$, the gas will gain the orbital angular momentum of the binary BHs system and then flow outwards. It is crucial that $R > a$ as the gas will not be able to fall in the black hole which allows the long-term survival of the gas.

$$\Lambda_\tau(R,a) = -\frac{f}{2} g^2 \Omega^2 R^3 \left(\frac{R}{\Delta}\right)^4, R < a$$  \hspace{1cm} (3)

When $R < a$, the gas transfers its angular momentum to the binary BHs system, then the gas will gradually fall downwards promptly and result in accretion.

**Mechanics of angular momentum transformation:** For general accretion disk, we have magneto rotational instability (MRI). According to Gammie 1996, the gas will experience a torque and lose angular momentum and the BHs acquire angular momentum. The disk flow can break down into turbulence and largely helps the transportation of angular momentum. What MRI brought us is the outward transformation of angular momentum, that is the continuous falling of gas(accretion).

**The dead zone:** A dead zone at the disc will be formed if it satisfies two following conditions: the temperature and the surface density. The temperature must be approximately lower than 800 Kelvin in order to make MRI operational. The disc will be thermally ionized and have an active MRI if the temperature is higher than 800K. The surface density must exceed the critical in order for the disk to be fully ionized externally. The inner edge of the dead zone is fixed by the radius at which the temperature of the fully turbulent steady state disc model is equal to the critical temperature required for the MRI.

The sound speed in the disc is given by $c_s = \sqrt{\frac{kT}{\mu m_H}}$, where $k$ is the Boltzmann constant, $T$ is the temperature, and $\mu = 2.3$, and $m_H$ is the mass of a hydrogen atom. Thus, the disc temperature is given by

$$T = 3.3 \times 10^4 \left(\frac{H}{R}\right)^2 \left(\frac{M}{60 M_\odot}\right) \left(\frac{R}{10 R_\odot}\right)^{-1} k$$  \hspace{1cm} (4)

Now, the critical temperature of the operational MRI is denoted as $T_{cri}$. Let the $T = T_{cri}$, radius of the inner edge of the dead zone is

$$R_{de} = 417 \left(\frac{H}{R}\right)^2 \left(\frac{M}{60 M_\odot}\right) \left(\frac{T_{cri}}{800 K}\right)^{-1} R_\odot$$  \hspace{1cm} (5)
Note that both the temperature and the inner radius of the dead zone depend upon the disc aspect ratio. The dead zone will not be considered in this essay.

**The mass leakage:** However, taking idealized conditions into consideration, the tidal barrier is not perfect. Gas can flow across the barrier where \( R = a \), enter the inner zone from the outer zone, the edge of the cavity, being accreted by the BHs and result in a loss of total gas mass within the accretion disk due to stream fuel to both BHs. As a consequence, once the gas leakage is massive, not enough gas will be conserved and we will not observe electromagnetic counterpart during the binary BH merger. As shown by Ragusha et al. 2016, When \( \frac{H}{R} \leq 1 \), the accretion rate of leaked mass is greatly decreased in comparison with the single-BH accretion. Note that we will omit the gas leakage in our work.

**Energy balance:** For gas disk, we assume it can effectively be cooled down, similar to the cooled standard thin accretion disk (Shakura & Sunyaev 1973; Kato et al. 2008, Frank et al. 2003) which the disks are long lived and We assume the heating of the accretion disk is mainly from viscosity.

The total heating rate per unit surface \( Q^+ \) can be expressed as:

\[
Q^+ = \frac{9}{8} v \sum \Omega_k^2
\]  

The total cooling rate per unit surface \( Q^- \) can be expressed as:

\[
Q^- = \frac{\sigma T^4}{1 + \tau}
\]  

Where \( \sigma_{sb} \) is the Stefan-Boltzmann constant, and is the optical depth of the whole system? Finally, we need to determine the temperature of the active layer and dead zone. Since there is no dead zone, \( T_a = T_e \). The optical depth is \( \tau = \frac{1}{2} \sum \tau_k \). And thus, the effective temperature of the active layer can be expressed as:

\[
T_a^4 = T_{surf}^4 (1 + \tau_a)
\]  

In the above expression, \( 1 + \tau_a \) is adopted, to agree with both optically thin and optically thick cases. Note that in our work we simply the model and only considering the ideal condition which \( Q^+ = Q^- \).

**Basic equations for the evolution of circumbinary gas disk:**

The evolution of the disc surface density around binary BHs is described by the classical hydrodynamical equations of accretion theory. We adopt a cylinder coordinate \((R, \phi, Z)\) which is centered at the mass centroid of the binary BHs. Thus, the evolution of total surface density in binary black hole mergers is:

\[
\frac{\partial \Sigma}{\partial t} = \frac{3}{R} \frac{\partial}{\partial R} \left[ R^2 \frac{\partial}{\partial R} \left( \frac{1}{R^2} \Sigma \right) \right] - \frac{2}{R} \frac{\partial}{\partial R} \left( \frac{\Lambda_{\Sigma} \Sigma_{tot}}{\Omega_k} \right)
\]  

Which is the sum of the diffusion and the tidal impact. Tidal torque restrains the inward migration of the gas disk. For disks that have sparsely surface density, the inward migration will be slowed down. The value of \( R_A \) will tell us the areas that are significantly influenced by the tidal torque.

### 2.3. Evolution of binary BHs

**The change of binary BHs separation a due to GW:** The decay of orbit of the binary system due to Gravitational-wave radiation, leads the evolution of separation \( a \), is given by:
The influence under the existence of circumbinary BHs gas disk: The tidal torque $\Lambda_T$ acts on the binary BHs and shrink binary separation $a$. It is worthy to emphasize that, with respect to time scales of binary BHs, tidal torque has a larger influence compared with GW radiation. This has never been involved in previous work. Assume the angular momentum of the whole system is conserved, the disk binary torque can be expressed as:

$$\left(\frac{da}{dt}\right)_{gw} = -\frac{64c}{5} g \left(\frac{R_g}{a}\right)^3$$

Thus, we ended up having the following equation for the evolution of binary separation in BBH mergers:

$$\frac{da}{dt} = \left(\frac{da}{dt}\right)_{\Lambda} + \left(\frac{da}{dt}\right)_{gw}$$

When the gravitational wave is produced by the merging of binary black holes, there will be a sudden drop in the total mass of the system. The mass lost does not affect the angular momentum, in which a recoil presents. However, due to the change in mass, every particle initially at radius will have a circularization radius. Thus, in accordance with a decrease in mass, the circularization radius increases by the same fractional amount, so that after the merger the particles are in elliptical orbits. This can give a rather stronger EM signal. The Crude estimation of the characteristics of the EM signals based on scaling relations calibrated against the numerical simulations. Be cautious that the numerical estimates below are subjected to large uncertainties.

**Time scales:** First, the orbit of the disk decays as the gravitational wave radiates away from the center. The time scale of the specific process is given by:

$$t_{GW} = \frac{1}{4} \times \left(\frac{da}{dt}\right)_{GW}$$

The merger timescale due to the tidal torque driven by the circumbinary disc can be expressed as:

$$t_{\Lambda} = \frac{a}{\left(\frac{da}{dt}\right)_{\Lambda}}$$

Heating of the disk by shocks occurs on a characteristic timescale of the dynamical time

$$t_{dyn} = \frac{GM}{v^3_{rec}}$$

$v_{rec}$ is the recoil velocity imparted on the center mass. The light-travel time required to tell the disk about the GW event is much shorter than Therefore, $t_{dyn}$ describes the delay with respect to the GW event and energy deposition in the disk. This means that there will be a time delay between the merger and the EM signal, if photons are radiated away promptly. The time delay becomes longer and can be years if a significant fraction of the radiation is trapped.

**Disk dimension:** Defining a recoil radius $R_{rec}$ such that the Kepler velocity $v_k$ at this radius is equal to the recoil velocity $v_{rec}$.
\[ R_{\text{rec}} = \frac{GM}{v_{\text{rec}}^2} \]  

(16)

Usually, \( v_{\text{rec}} \) is bigger than \( v_k \), thus the inner radius \( R_{\text{in}} \) of the disk falls within this radius. The outer disk radius is much larger than when the disk was formed, as it must absorb the angular momentum of the gas which has spiraled inward. Therefore, expecting \( R_{\text{out}} \) to be significantly larger than \( R_{\text{rec}} \).

**Luminosity:** After coalescence, the luminosity occurs due to the perturbed gas that suffers shocks and dissipation, and thus it is sensitive to the disk mass distribution, BH kick velocity, and BH moving direction. As the accretion disk formed post-merger, weak EM signals will also be released. The time delay of the appearance of the radiation determines the peak EM luminosity. Here, we provide the estimation of Luminosity of an accreting BH,

\[ L = \int 2\pi R \sigma T^4 dR \]  

(17)

We only consider the region where there is gas, i.e. \( \sum R > 0 \). Moreover, we define a luminosity weighted temperature with the adaptation of a blackbody assumption, i.e. \( h \nu \approx 3kT_{\text{weighted}} \) weighted to help us define which energy band to observe such signal:

\[ T = \frac{\int RT^5 dR}{\int RT^4 dR} \]  

(18)

The temperature has an influence on the spectral energy distribution post-merger. Concerning spectral energy distribution, the characteristic temperature related with the burst, \( T \), should satisfy the following condition:

\[ T_b < T < T_s \]

\( T_b \) the blackbody temperature can be expressed as:

\[ T_b = \left( \frac{L}{2\pi\sigma R^2} \right)^{\frac{1}{4}} \times 3 \times 10^6 \nu_3^3 M_\odot \left( \frac{q_d}{10^{-7}} \right)^{\frac{1}{2}} \left( \frac{f}{0.1} \right)^{\frac{1}{2}} K \]  

(19)

Where \( v \) is the kick velocity, is the Stefan-Boltzmann constant at \( 5.67032 \times 10^{-8} \), and \( q_d \) is the disk-to-black-hole mass ratio. In blackbody radiation, higher temperatures will cause higher photon energy and therefore a higher frequency. \( T_s \), the shock temperature is:

\[ T_s = \frac{3\mu m_H \nu^2}{16k} \times 2 \times 10^7 \nu_3^2 K \]  

(20)

Where \( m_H \) is the mass of hydrogen at \( 1.6735575 \times 10^{-27} \text{kg} \). Under the idealized conditions, in which \( \nu \) and prompt emission, the EM signal will appear mostly in X-rays around hours later after the merger. With the estimation of luminosity and timescales given above, the detection of stellar-mass events signals may have already existed.

### 3. Basic results

#### 3.1. The comparison between the merger time and the time scale of gravitational waves.

Under the assumption made above, when the BBH separation is \( 10^4 R_g \) and the disk mass ratio is \( 1 \times 10^{-2} \), The merger time of torque is around 10 times larger than the time scale of gravitational waves. When the BBH separation is \( 10^5 R_g \) and the disk mass ratio is \( 1 \times 10^{-2} \).
With different binary separation $a$, the different merger timescales. The merger time due to gravitational-wave radiation $t_{gw}$ is represented in blue solid line. The red dot-dashed line is the merger timescale due to binary-disk interactions, which denoted as $t_\lambda$ with the $M_{\text{disk,eff}}/M_{\text{bin}}$ ratio of $10^{-7}$. The green dashed line is also the merger timescale due to binary-disk interactions, which denoted as $t_\lambda$ with the $M_{\text{disk,eff}}/M_{\text{bin}}$ ratio of $10^{-6}$.

The merger time and the time scale of gravitational waves are equivalent. We find that as the separation is larger than $10^5 R_g$, the effect of GW radiation will be larger than the effect of tidal torque on the merger time.

Merger timescale on different binary separation. Here $M_{\text{bin}}$ is 50 times of $M_{\text{sun}}$. $R_g$ is $7.383 \times 10^6$. The blue solid curve is the merger time due to gravitational-wave radiation $t_{gw}$. The merger timescale due to binary-disk interactions $t_\lambda$ are shown by green dashed and red dot-dashed curves, where $M_{\text{disk,eff}}$ has different values. Assuming that the torque-weighted disk mass $M_{\text{disk,eff}}$ remains constant throughout the process of disk evolution, and $t_\lambda \propto a^{7/3}$, and $t_{gw} \propto a^4$. The tidal torque will help to move angular momentum at large separation. Consequently, the actual merger time will be significantly reduced. Moreover, a larger disk mass ratio will give a shorter merger time. After we have the evolution of the circumbinary disk around the BBH, we can evaluate both luminosity and waveband.

3.2. Luminosity and waveband

The luminosity of the merging binary blackholes is inseparable from the dissipation of kinetic energy, where the expected rate of dissipation of kinetic energy is given by:

$$L = f \frac{M_{\text{bin}} v_{\text{esc}}^2}{t_{\text{dyn}}} \tag{21}$$

where $f$ is a scaling factor. We adjust the factor $f = 0.1$ against the mathematical reenactments, which accept an edge of $\theta = 15^\circ$ between the backlash bearing and the orbital plane.
The graph shows how the luminosity evolves before and after the merge of the blackholes, with different magnitudes of recoil masses and recoil velocities.

The graph shows how the luminosity evolves before and after the merge of the blackholes. Since the scattering endures a few dynamical occasions. The pinnacle EM signals relies upon how expeditiously the radiation shows up. The iridescence is profoundly super-Eddington for any heavenly mass merger. In the event that the stunned circle responds by growing homologous it would hold a high optical profundity until essentially extended. This implies it can take a long time for the photons to show up, which would emphatically reduce the radiance conceivably, delivering the sign imperceptibly. On the off chance that rather the photons are delivered on a timescale that is practically identical to the few-hour energy creation time. The pinnacle EM signals likewise depend delicately on the kick speed v and its bearing, and on the circle-to-blackhole mass proportion. Note anyway that it is generally autonomous of the blackhole mass. Amazingly, this implies the sign of heavenly mass occasions can possibly be as radiant concerning mergers of supermassive blackholes. The temperature tells us the observational window. The characteristic temperature $T_T$ associated with the burst satisfies

$$T_b < T < T_s$$

$$T_b = \left( \frac{L}{2 \pi \sigma R^2} \right)^{\frac{1}{2}} \sqrt{3 \times 10^6 \nu_3 \frac{1}{M_{\odot}}} \left( q_{\Delta} \frac{1}{10^3} \right)^{\frac{1}{2}} \left( \frac{f}{0.1} \right)^{\frac{1}{2}} K,$$  \hfill (22)

$$T_s = \frac{3 \mu m_e v^2}{16k} \sqrt{2 \times 10^7 \nu_3^2} K$$ \hfill (23)

we see that for $v_3 = 1$ and accepting brief emanation the EM signal is probably going to top in medium-energy X-rays. Discovery may likewise be conceivable with lower frequencies with longer wavelength. This might be significant if the emanation is intensely blushed. Given the luminosity estimate and timescale it seems possible that events like these may already have been observed. The EM signals can be incredibly upgraded for kicks in the orbital plane and the material falls profound into the capability of the blended dark gap. This gives redshift or blueshift of a similar request (say, $\Delta 1000kms^{-1}$ ) for the EM discharge, which can on a fundamental level be utilized to test the post-merger pull back. Further, the synthesis of the material of the circle might be improved if the material outcomes from ejecta that were handled by atomic consumption in the progenitor stars.

4. Sections, subsections and subsubsections

Regarding the detection of electromagnetic signals from a circumbinary gas disk around merging black holes, our model relies significantly on the existence of circumbinary disks. The evolution of the
circumbinary disk is influenced by many factors. The tidal torque and a larger disk mass ratio will both reduce the merger time. The tidal torque has a larger impact than the GW radiation on the diminishment of binary separation a. Both isolated binary and chemically-homogeneous evolution have surrounding gas when binary BHs formed. The chemically-homogeneous stellar evolution model, first mentioned by Maeder (1987) is crucial to our work. The hydrogen-rich envelope transfers material to the middle. This means that the envelopes ended up increasingly helium wealthy, advancing to become hotter and more luminous and thus the stars will gradually narrow.

One situation that we did not consider is that the dead zone and the mass leakage respectively leaves gas and leaks gas. Thus, there is contention between these sections. After we have the evolution of the circumbinary disk around the BBH, we can evaluate both luminosity and waveband of different BBH masses. There will be EM radiation both pre-merger and post-merger. Under idealized conditions, EM signals will appear mostly in X-rays around hours later after merger. They will be observed in the space telescope but not on Earth. The signal of a more massive merger will have a longer duration. This provides theoretical limitations for the observations of BBHs EM counterpart.

In future work, we will continue focusing on the observation of BBHs EM counterpart since it has never been witnessed. We will attempt to develop more precise and reliable Mathematical simulations and models for the dynamics of its circumbinary disk and the long-term survival of gas within the disk.

Acknowledgments
We would like to express our very great appreciation to Fu-Guo Xie, Research Associate in Shanghai Astronomical Observatory, our research supervisor, for his patient guidance and useful critiques of this research work. Our grateful thanks are also extended to Yan-Bei Chen, Professor of Physics in Caltech, for his valuable support.

References
[1] Armitage, P. J., & Natarajan, P. (2002). Accretion during the Merger of Supermassive Black Holes. The Astrophysical Journal, 567(1). https://doi.org/10.1086/339770.
[2] Gammie, C. F. (1996). Layered Accretion in T Tauri Disks. The Astrophysical Journal, 457, 355. https://doi.org/10.1086/176735.
[3] Mandel, I., & De Mink, S. E. (2016). Merging binary black holes formed through chemically homogeneous evolution in short-period stellar binaries. Monthly Notices of the Royal Astronomical Society, 458(3), 2634–2647. https://doi.org/10.1093/mnras/stw379.
[4] Martin, R. G., Lubow, S. H., Livio, M., & Pringle, J. E. (2012). Dead zones around young stellar objects: dependence on physical parameters. Monthly Notices of the Royal Astronomical Society, 420(4), 3139–3146. https://doi.org/10.1111/j.1365-2966.2011.20241.x.
[5] Martin, R. G., Nixon, C., Xie, F.-G., & King, A. (2018). Circumbinary discs around merging stellar-mass black holes. Monthly Notices of the Royal Astronomical Society, 480(4), 4732–4737. https://doi.org/10.1093/mnras/sty2178.
[6] Milosavljević, M., & Phinney, E. S. (2005). The Afterglow of Massive Black Hole Coalescence. The Astrophysical Journal, 622(2). https://doi.org/10.1086/429618.
[7] Mink, S. E. D., & Mandel, I. (2016). The chemically homogeneous evolutionary channel for binary black hole mergers: rates and properties of gravitational-wave events detectable by advanced LIGO. Monthly Notices of the Royal Astronomical Society, 460(4), 3545–3553. https://doi.org/10.1093/mnras/stw1219.
[8] Mink, S. E. D., & King, A. (2017). Electromagnetic Signals Following Stellar-mass Black Hole Mergers. The Astrophysical Journal, 839(1). https://doi.org/10.3847/2041-8213/aa67f3.
[9] Ragusa, E., Lodato, G., & Price, D. J. (2016). Suppression of the accretion rate in thin discs around binary black holes. Monthly Notices of the Royal Astronomical Society, 460(2), 1243–1253. https://doi.org/10.1093/mnras/stw1081.