Evolution of elliptic and triangular flow as a function of beam energy in a hybrid model

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Abstract. Elliptic flow has been one of the key observables for establishing the finding of the quark-gluon plasma (QGP) at the highest energies of Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC). As a sign of collectively behaving matter, one would expect the elliptic flow to decrease at lower beam energies, where the QGP is not produced. However, in the recent RHIC beam energy scan, it has been found that the inclusive charged hadron elliptic flow changes relatively little in magnitude in the energies between 7.7 and 39 GeV per nucleon-nucleon collision. We study the collision energy dependence of the elliptic and triangular flow utilizing a Boltzmann + hydrodynamics hybrid model. Such a hybrid model provides a natural framework for the transition from high collision energies, where the hydrodynamical description is essential, to smaller energies, where the hadron transport dominates. This approach is thus suitable to investigate the relative importance of these two mechanisms for the production of the collective flow at different values of beam energy. Extending the examined range down to 5 GeV per nucleon-nucleon collision allows also making predictions for the CBM experiment at FAIR.

1. Introduction
In 2010, the RHIC beam energy scan program was launched to study the features of the QCD phase diagram and to search for signs of the possible first-order phase transition between the confined and deconfined matter [1]. The existence of a critical point marking the boundary of cross-over and the aforementioned first-order phase transition in the plane of baryochemical potential $\mu_B$ and temperature $T$ was predicted by lattice calculations [2, 3, 4]; it has, however, been put to question by the continuum extrapolated results [5, 6] which suggest the phase transition remaining cross-over also at large values of $\mu_B$.

Elliptic flow $v_2$ is one of the key observables that supports the formation of a strongly coupled quark-gluon plasma at the highest energies of RHIC and the Large Hadron Collider (LHC). Thus the naive expectation for $v_2$ in the beam energy scan would be a decrease at lower beam energies
where the hydrodynamic phase is short or the QGP is not created at all. However, the measured inclusive charged hadron elliptic flow $v_2$ demonstrates relatively little dependence on the collision energy $\sqrt{s_{NN}}$ between 7.7 and 39 GeV [7].

One possible method for investigating the importance of the hydrodynamical evolution for the flow production is the hybrid approach, where one uses a transport model for the non-equilibrium phases at the beginning and in the end of a heavy-ion collision event, and hydrodynamics for the intermediate hot and dense stage and the phase transition between the quark-gluon plasma and hadronic matter. This approach should be applicable for a wide range of heavy ion collision energies and thus optimal for studying the beam energy dependence of the flow observables down to $\sqrt{s_{NN}} = 5$ GeV, an energy reachable also at the future heavy ion collisions at FAIR.

2. Hybrid model

In this study, a transport + hydrodynamics hybrid model by Petersen et al. [8] is utilized. The initial state in this model is produced by the Ultrarelativistic Quantum Molecular Dynamics (UrQMD) string / hadronic cascade [9, 10]. The transition to hydrodynamics is done when the two colliding nuclei have passed through each other: $t_{\text{start}} = \max\{2R(\gamma_{CM}^2 - 1)^{-1/2}, 0.5 \, \text{fm}\}$, where $R$ represents the nuclear radius and $\gamma_{CM} = (1 - v_{CM}^2)^{-1/2}$ is the Lorentz factor. The minimum time of 0.5 fm has been determined by the model results at the collision energy $\sqrt{s_{NN}} = 200 \, \text{GeV}$ [11]. At $t_{\text{start}}$, the energy-, momentum- and baryon number densities of the particles, represented by Lorentz-contracted 3D Gaussian distributions with the width $\sigma = 1.0 \, \text{fm}$, are mapped onto the hydro grid. Spectator particles are excluded from this procedure and propagated separately in the cascade.

The SHASTA algorithm [12, 13] is used to solve the (3+1)-D ideal hydrodynamics evolution equations. The equation of state (EoS) is based on a hadronic chiral parity doublet model with quark degrees of freedom, coupled to Polyakov loop to include the deconfinement phase transition [14]. It possesses the important feature of being applicable also at finite baryon densities. At the end of the hydrodynamical evolution, the active EoS is changed to the hadron gas EoS, so the active degrees of freedom on both sides of the transition hypersurface match exactly [15].

The particlization, i.e. the transition from hydro to transport, is done when the energy density $\rho$ reaches the critical value $2\epsilon_0$, where $\epsilon_0 = 146 \, \text{MeV/fm}^3$ is the nuclear ground state energy density. The particle distributions are generated according to the Cooper-Frye formula from the iso-energy density hypersurface, which is constructed using the Cornelius hypersurface finder [16]. The rescatterings and final decays of these particles are then computed in the UrQMD. The final distribution of particles can then be directly compared against the experimental data. It has been tested that the hybrid model has a reasonable agreement with the experimental data for particle $m_T$ spectra at midrapidity $|y| < 0.5$ for energies ranging from $E_{\text{lab}} = 40 \, \text{AGeV}$ to $\sqrt{s_{NN}} = 200 \, \text{GeV}$ [17, 18].

3. Results

3.1. Elliptic flow

In this study, the flow coefficients $v_n$ are computed from the particle momentum distributions using the event plane method [19, 20]. This, together with the new implementation of the Cooper-Frye hypersurface finder and particlization, forms the core difference compared to previous studies of elliptic flow in the same hybrid approach [21, 22]. The primary interest in the following is to see, if the experimentally observed weak sensitivity of the elliptic flow on the collision energy is manifested also in the hybrid model results.

Figures 1a and 1b show the $p_T$-integrated elliptic flow $v_2$ produced in Au+Au -collisions for the $p_T$ range 0.2 - 2 GeV, compared with the STAR data for the (0-5)% and (30-40)% centrality classes. In the model these are respectively represented by the impact parameter intervals $b = 0 - 3.4 \, \text{fm}$ and $b = 8.2 - 9.4 \, \text{fm}$. Figure 1c shows the differential $v_2(p_T)$ for $b = 6.7 - 8.2 \, \text{fm},$
which roughly corresponds to (20-30)% centrality class. Figures 1a and 1b also demonstrate the magnitude of $v_2$ at three different times: just before the hydrodynamics phase begins, right after the hydrodynamics phase has ended and particlization has been done, and after the hadronic rescatterings have been performed in the UrQMD (in other words, after the full evolution).

In the most central collisions the effect of the hadronic rescatterings is negligible; in the impact parameter range $b = 8.2 - 9.4$ fm the rescatterings contribute about 10% on the final result. The hydrodynamics also produce very little elliptic flow at $\sqrt{s_{NN}} \leq 7.7$ GeV; for the mid-central collisions, $v_2$ is in practice completely produced by the transport dynamics, which include resonance formation and decay and string excitation and fragmentation processes. These initial dynamics, which are often neglected in other hybrid approaches, gain importance at lower energies. On the other hand, above $\sqrt{s_{NN}} = 19.6$ GeV the hydrodynamic phase is clearly the dominant source of $v_2$.

The simulation results overshoot the experimental data for all collision energies. This suggests that either the viscous corrections should be included, or the energy density value chosen for particlization should be higher. In the most central collisions below $\sqrt{s_{NN}} = 11.5$ GeV, the model appears to produce too much flow already at transport phase; here having agreement with the data would require modifications in how the pre-equilibrium phase is handled. However, for the purposes of this study, the most important feature is the good qualitative agreement in the midcentral collisions, as here the flow effects are at their largest. Also $v_3(p_T)$ (Fig. 1c) has relatively weak dependence on $\sqrt{s_{NN}}$, which is in accordance with the STAR results.

### 3.2. Triangular flow

Based on the above results, it appears that the hydrodynamically produced elliptic flow indeed vanishes, as was the naive expectation, but this is partially compensated by the increased flow production in the transport phase and so the observed $v_2$ has only weak collision energy dependence. To study this phenomenon further, we do the same analysis for another flow observable: the triangular flow $v_3$, which originates purely from the event-by-event variations in the initial spatial configuration of the colliding nucleons, and is thus largely independent of the collision geometry.

As illustrated by Figure 2a, the $p_T$-integrated $v_3$ increases from $\approx 0.01$ to above 0.015 with increasing collision energy in the most central collisions, whereas in midcentrality $b = 6.7 - 8.2$ fm there is a rapid rise from $\approx 0$ at $\sqrt{s_{NN}} = 5$ GeV to the value of $\approx 0.02$ for $\sqrt{s_{NN}} \geq 27$ GeV. The collision energy dependence is seen also for midcentral $v_3(p_T)$ in Fig. 2b. The energy dependence of $v_3$ in midcentral collisions qualitatively resembles the hydrodynamically produced
v_2$ in Figure 1b. Thus for the higher flow coefficients, which are more sensitive to viscosity, the transport part of the model is unable to compensate for the diminished hydro phase.

3.3. Effect of initial geometry

Let us then investigate in more detail the effect of initial collision geometry on the flow coefficients. Figure 3a illustrates the collision energy and centrality dependencies of the average initial state spatial eccentricity $\langle \epsilon_2 \rangle$ and triangularity $\langle \epsilon_3 \rangle$, where the eccentricity and triangularity are defined as in [25] and calculated at the beginning of hydrodynamical evolution $t_{\text{start}}$.

The average eccentricity and triangularity are of the same magnitude in the most central collisions, where the nuclear overlap region is nearly isotropic. The situation changes in mid-central collisions, where, due to the collision geometry, $\langle \epsilon_2 \rangle$ is clearly larger than $\langle \epsilon_3 \rangle$. The observed dependence on collision energy is largely explained by $t_{\text{start}}$, which changes rapidly at low energies, from 5.19 fm at $\sqrt{s_{NN}} = 5$ GeV to 1.23 fm at $\sqrt{s_{NN}} = 19.6$ GeV. At low energies there is thus enough time for the pre-equilibrium transport to decrease the initial spatial anisotropies.

Figure 3b shows the coefficients $v_2$ and $v_3$ scaled with $\langle \epsilon_2 \rangle$ and $\langle \epsilon_3 \rangle$, respectively. The relation of the elliptic flow to the initial eccentricity remains largely unchanged for the whole collision energy range, while the $v_3$ response to the triangularity of the initial state saturates only after 19.6 GeV. This suggests that the hadronic medium is too viscous to convert initial state fluctuations into triangular flow, and a sufficiently long-living intermediate phase with a low-viscosity fluid is needed for the $v_3$ production.

4. Summary

We have demonstrated that the experimentally observed behavior of $v_2$ as a function of collision energy $\sqrt{s_{NN}}$ can be qualitatively reproduced utilizing a hybrid transport + hydrodynamics approach. The diminished hydrodynamical evolution for $v_2$ production at lower collision energies is compensated by the pre-equilibrium transport dynamics. This compensation does not apply to triangular flow $v_3$, which decreases considerably faster, reaching zero in mid-central collisions at $\sqrt{s_{NN}} = 5$ GeV. This makes $v_3$ the better signal for the formation of quark-gluon plasma in heavy ion collisions. However, according to the preliminary STAR data, $v_3$ remains constant in central collisions at $\sqrt{s_{NN}} = 7.7 - 27$ GeV [26], which suggests that the low-viscous state of nuclear matter is manifested at the lower collision energies in greater extent than expected. The flow coefficients thus remain interesting observables also for the heavy ion collisions at FAIR energies.
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