Harmonic Analysis of symmetric and anti-symmetric laminated skew plate using finite element method

Manoj Narwariya¹, Avadesh K. Sharma², Vinod Patidar³ and Premanand S. Chauhan⁴

¹Research Scholar, Sir Padampat Singhania University Bhatewar Udaipur 313 601, India
²Associate Professor, Rajkiya Engineering College, Mainpuri, India
³Professor, Sir Padampat Singhania University Bhatewar Udaipur 313 601, India
⁴Principal, IPS College of Technology and Management, Gwalior, India

manoj.narwariya@gmail.com

Abstract. In this paper, harmonic behaviour of laminated composite skew plates is presented with a symmetric and anti-symmetric arrangement of layers. The convergence and validity of the model have been checked by comparing different type of example available in published literature. From the comparison study, it can be easily observed that proposed model is being capable of solving the skew structures with adequate accuracy. The effects of various skew angles, lamination scheme and number of layers have been demonstrated. The results show the maximum variation in the non-dimensional fundamental frequency is limited to 3% for a symmetrically laminated skew plate. It is clear from the results that the maximum resonance amplitude occurs at the first fundamental mode of vibration at different skew angle. Finite element package ANSYS is employed for analysis of the plates. The SHELL281 element is chosen to mesh the model.

Keywords: Composite Plate; Harmonic behaviour; Skew angle; Frequency Response Function; FEM.

1. Introduction
Skew plate is commonly used in various applications such as in constructing the wings of aero structure, skew bridges, ship hulls etc. Laminated skew plates being parts of these structures are often subjected to various types of forces which create unwanted vibration. Careful designs can minimize these vibrations. Thus, their natural frequencies and resonance amplitude must be calculated to find safe and efficient design.

Farag and Ashour [1] developed a new method to analyze the vibration effect on the composite skew plate and found the fast converging semi-analytical method. Zeng and Bert [2] investigated the free vibrations of skew plates stiffened orthogonally and concluded that relatively higher frequencies are found with the higher skew angle and deeper stiffeners provide more stiffness. Liew et al. [3]...
studied the vibrational behaviour of a symmetric thick laminated composite square, rectangle and skew plate using FSTD and observed that more accurate results found taking high-order basis assumption. Dey and Singha [4] investigated the dynamic stability behaviour of laminated composite skew plates under the simply supported boundary condition using the FEA process. Singha and Rupesh [5] worked on plate bending element which was high précised four noded shears flexible composite skew plate and investigated the large amplitude for the plate vibration. The isotropic laminated skew plate has been studied by Das et al. [6] for the large deflection analysis. Sharma et al. [7] analyzed the free vibration behaviour of shear deformable anti-symmetric laminated plates and also studied for the cross-ply rectangular plate with rotational and translational edge constraints. Rao and Reddy [8] analyzed the displacements and natural frequencies of a propeller of composite material with metal using ANSYS software. Useche et al. [9] studied the vibration and harmonic behaviour of orthotropic plates having a crack and having a shear deformable property using the boundary element method. Jyoti Vimal et al. [10] carried out a modal analysis of functionally graded skew plates using FEM. The experimental study on free vibration behavior of isotropic and laminated skew plates has been carried out by Srinivasa et al. [11]. The dynamic analysis of laminated composite plates is studied by Maithry and Rao [12] using FEM package ANSYS. Kumari Shipra et al. [13] presented a meshless solution for linear free vibration response of laminated composite skew plates. Mandal et al. [14] carried out a numerical and experimental study on laminated skew plates with s-glass epoxy material with and without a cut-out. Pankaj Katariya et al. [15] investigated numerically, the free vibration responses of the skew sandwich shear deformable composite plate with the help of the finite element method. The free vibration and buckling analysis of a skew sandwich plate (three layered) have been carried out by Joseph and Mohanty [16].

Many researchers have been paid more attention to analysis the laminated composite structures. Different techniques have been used to carry out the numerical investigation on the vibration of skew composite plates. From a detailed study of the literature, it has been observed that the study of harmonic behaviour of the skew structure appears to have not been studied as yet. This necessitates the study of harmonic behaviour of laminated skew plates which may be helpful to construct efficient and safe structures.

2. Modelling

2.1. Geometric Modelling

The Geometry of the laminate is shown in figure 1. The dimensions length (a), width (b) and thickness (t) are fixed and taken as 1 m, 1 m and 0.01 m respectively. The skew angle (α) is varied from 15° to 75°.

![Figure 1. Geometry of skew plate](image)

2.2. Boundary Condition

A skew orthotropic plate having dimensioned of \( 1 \text{ m} \times 1 \text{ m} \) is considered in this study. Clamped boundary condition (i.e. all the edges of skew plate are constrained to all the six degrees of freedom) is used to analyze the plate. A force of 1 N is applied on the plate at its node no. 415.
2.3. Material Used
The Graphite/epoxy material is used for harmonic analysis having the following properties:

\[ E_1 = 172720 \text{ MPa}, \quad E_2 = E_3 = 6909 \text{ MPa}, \quad G_{12} = G_{13} = 3450 \text{ MPa}, \quad G_{23} = 1380 \text{ MPa}, \quad \nu_{12} = \nu_{13} = \nu_{23} = 0.25 \]

and \( \rho = 1600 \text{ kg/m}^3 \) where the subscripts 1 denotes parallel and 2 denotes perpendicular directions with respect to the direction of fibres in a layer.

3. Verification of present analysis
The convergence study and comparison study has been done to assure the present method to be accurate and efficient

3.1. Convergence Study
Cross-ply symmetric (0°/90°/0°) and anti-symmetric (0°/90°) laminated skew plate under the clamped boundary condition has been analysed with following parameters:

- Length to width ratio \((a/b) = 1\)
- Thickness ratio = 0.01
- Skew angle \((\alpha) = 30°\)

Convergence study is listed in Table 1. It has been observed that the mesh size of 17 × 17 is sufficient to mesh the geometry.

Table 1: Convergence study for a skew cross-ply laminated plate under clamped Boundary condition.

| a/b | t/b | Skew angle (α) | Mesh Size | 7×7 | 9×9 | 11×11 | 13×13 | 15×15 | 17×17 | 19×19 | 21×21 |
|-----|-----|---------------|-----------|-----|-----|-------|-------|-------|-------|-------|-------|
|     | 0.01| 30°           |           | 18.049 | 18.045 | 18.044 | 18.043 | 18.043 | 18.043 | 18.043 | 18.043 |
| 1   |     |               |           | 27.203 | 27.170 | 27.159 | 27.154 | 27.152 | 27.151 | 27.151 | 27.151 |
|     |     |               |           | 32.839 | 32.805 | 32.796 | 32.790 | 32.787 | 32.787 | 32.787 | 32.787 |
|     |     |               |           | 36.726 | 36.635 | 36.605 | 36.595 | 36.586 | 36.586 | 36.586 | 36.586 |

3.2. Comparison Study
Laminated composite plate with different skew angles have been analyzed and compared with the published results of Mandal A. et al (2017) as shown in table 2. The comparison is done for non-dimensionalised fundamental frequency \((\omega = (\alpha a^2)/(\pi^2 t)\sqrt{(\rho/E_t)})\) of symmetric angle-ply [45°/-45°/45°/-45°/45°] laminated plate at different skew angle. It is clear from the table 2 that the results show reasonably good agreement.

Table 2: Comparison of non-dimensional fundamental frequencies \((\omega = (\alpha a^2)/(\pi^2 t)\sqrt{(\rho/E_t)})\) for clamped symmetric angle-ply [45°/-45°/45°/-45°/45°] laminated plate for different skew angles.

| Skew angle(α) | Present | Mandal et. al (2017) | Liew et. al. (2003) |
|---------------|---------|----------------------|---------------------|
| 15°           | 2.30487 | 2.649                | 2.3502              |
| 30°           | 2.613086| 3.287294             | 2.661               |
| 45°           | 3.287294| 4.74623              | 3.358               |
| 60°           | 4.74623 |                      | 4.857               |

%Error

| Present | Mandal et. al (2017) | Liew et. al. (2003) |
|---------|----------------------|---------------------|
| 1.355773| 1.341721             | 1.461629            |
| 1.928758| 1.800617             | 2.105603            | 2.313721           |
4. Numerical results and discussion

Harmonic analysis of the plate is studied by finite element package ANSYS using the full method without uncertainty. The plates are excited by external force and the responses of plates with respect to different natural frequencies are plotted on the Frequency Response Function (FRF) graph. This graph is used to obtain the frequencies of resonant amplitude, damping and mode shapes of any physical substance. The analysis is done to compare the results for increasing layers of symmetric and anti-symmetric skew plate. The effect of skew angles and damping factor are also obtained.

4.1. Effect of number of layers.

The laminated skew plates with symmetric and anti-symmetric arrangement of layers are analyzed to study the effect of number of layers on natural frequency and resonance amplitude. The results are obtained for two-layered, four-layered and six-layered plate for anti-symmetric plate and three-layered, five-layered and seven-layered for a symmetric plate. The thickness ratio (t/b) as 0.01 and skew angle as 15° has been taken. From the table 3, it has been observed that the natural frequency increases and resonance amplitude decreases as the number of layer increases for both symmetric and anti-symmetric arrangement. It can also be seen that resonance amplitude found in a lesser amount for a symmetric plate.

Table 3: Effect of number of layers for on the natural frequency and resonance amplitude for anti-symmetric and symmetric skew plate.

| Lamination scheme | No. of layers | Natural frequency (Hz) | Resonance amplitude (mm) |
|-------------------|---------------|------------------------|-------------------------|
| Anti-symmetric    | (0°/90°) 1    | 71.22                  | 0.167393                |
|                   | (0°/90°) 2    | 105.7                  | 0.0761053               |
|                   | (0°/90°) 3    | 111.05                 | 0.0689831               |
| Symmetric         | (0°/90°) 1,0° | 109.91                 | 0.0685977               |
|                   | (0°/90°) 2,0° | 112.35                 | 0.0663408               |
|                   | (0°/90°) 3,0° | 113.29                 | 0.0657142               |

Harmonic response for the same plates has been plotted on Frequency Response Function (FRF) graph as shown in figures 2(a-f). Figure 2(a) shows FRF for anti-symmetric two layered plate for frequency range between 60 Hz and 200 Hz. It has been found that high resonance (0.167393 mm) occurred at first fundamental mode of frequency i.e. 71.22 Hz and figure 2(b) shows for symmetric three-layered plate for frequency range of 80 Hz to 400 Hz. Maximum resonance 0.0685977 mm found at natural frequency of 111.05 Hz.

Figure 2(c) shows FRF for anti-symmetric four layered plate for frequency range between 75 Hz and 300 Hz. High resonance 0.0761053 mm found at first mode of frequency i.e. 105.7 Hz and figure 2(d) shows an FRF for symmetric five layered plate for frequency range between 75 Hz and 300 Hz. Maximum resonance 0.0663408 mm has been observed at first natural frequency of 112.35 Hz.

Figure 2(e) shows FRF for anti-symmetric six layered plate for frequency range between 80 Hz and 280 Hz. Resonance 0.0689831 mm observed at natural frequency of 111.05 Hz and figure 2(f) shows FRF for symmetric seven layered plate for frequency range between 90 Hz and 250 Hz. Resonance 0.0657142 mm occurred at first frequency of 113.29 Hz.
Figure 2(a). FRF for anti-symmetric two layered plate.

Figure 2(b). FRF for symmetric three-layered plate

Figure 2(c). FRF for anti-symmetric four layered plate.

Figure 2(d). FRF for symmetric five layered plate

Figure 2(e). FRF for anti-symmetric six layered plate.

Figure 2(f). FRF for symmetric seven layered plate.
4.2. Effect of skew angle

3-layered symmetric and 2-layered anti-symmetric cross-ply skew plates are used for presenting the effect of skew angle on natural frequency and resonance amplitude for thickness ratio as 0.01. From table 4, it has been found that as the skew angle increases, natural frequency also increases but resonance amplitude decreases for both symmetric and anti-symmetric plate. It can also be seen that resonance amplitude is suddenly decreased and frequency is reached in higher value for skew angle of 75°.

Table 4: Effect of skew angle on the natural frequency and resonance amplitude for anti-symmetric and symmetric skew plate.

| Skew Angle | Symmetric (0°/90°/0°) | Anti-Symmetric (0°/90°) |
|------------|-----------------------|-------------------------|
|            | Natural Frequency in Hz | Resonance Amplitude in mm | Natural Frequency in Hz | Resonance Amplitude in mm |
| 15°        | 109.91                | 0.0685977               | 71.22                   | 0.167393               |
| 30°        | 115.98                | 0.0686759               | 85.108                  | 0.135868               |
| 45°        | 134.51                | 0.0635678               | 121.15                  | 0.088471               |
| 60°        | 199.33                | 0.0442216               | 227.62                  | 0.040236               |
| 75°        | 560.78                | 0.0132627               | 761.38                  | 0.00732799             |

Harmonic response on symmetric and anti-symmetric cross-ply laminated skew plate for different skew angles for thickness ratios 0.01 are plotted as shown in figures 3(a-j).

FRF shown in figure 3(a) is for symmetric three-layered plate with a 15° skew angle for the frequency range of 80 Hz to 400 Hz. It is found that the maximum resonance (i.e. 0.0685977 mm) occurred at natural frequency of 109.91 Hz and figure 3(b) shows FRF for anti-symmetric two layered skew plate with 15° skew angle for the frequency range of 60 Hz to 200 Hz. Resonance 0.167393 mm occurred at natural frequency of 71.22 Hz.

Figure 3(c) shows FRF for symmetric three-layered plate with a 30° skew angle for the frequency range of 90 Hz to 300 Hz. In this arrangement, maximum resonance (i.e. 0.0686759 mm) occurred at natural frequency of 115.98 Hz. Figure 3(d) shows FRF for anti-symmetric two layered skew plate with 30° skew angle for the frequency range of 70 Hz to 200 Hz and Resonance found maximum of amount of 0.135868 mm at natural frequency of 85.108 Hz.

Figure 3(e) shows FRF for the symmetric three-layered plate with a 45° skew angle for the frequency range of 100 Hz to 300 Hz. Resonance 0.0635678 mm found at natural frequency of 134.51 Hz. Figure 3(f) shows FRF for anti-symmetric two layered plate with a 45° skew angle for the frequency range of 90 Hz to 280 Hz. Resonance 0.088471 mm found at natural frequency of 121.15 Hz.

Figure 3(g) shows FRF for the symmetric three-layered plate with a 60° skew angle for frequency range of 120 Hz to 480 Hz. Resonance 0.0442216 mm found at natural frequency of 199.33 Hz. Figure 3(h) shows FRF for anti-symmetric two layered plate with a 60° skew angle for the frequency range of 160 Hz to 450 Hz. Resonance 0.0402336 mm found at natural frequency of 227.62 Hz.

Figure 3(i) shows FRF for the symmetric three-layered plate with a 75° skew angle for the frequency range of 400 Hz to 1200 Hz. Resonance 0.0132627 mm found at natural frequency of 560.78 Hz. Figure 3(j) shows FRF for anti-symmetric two layered plate with a 75° skew angle for the frequency range of 600 Hz to 1300 Hz. Resonance 0.00732799 mm found at natural frequency of 761.38 Hz.
Figure 3(a). FRF for symmetric three-layered plate with a 15° skew angle.

Figure 3(b). FRF for anti-symmetric two layered plate with a 15° skew angle.

Figure 3(c). FRF for symmetric three-layered plate with a 30° skew angle.

Figure 3(d). FRF for anti-symmetric two layered plate with a 30° skew angle.

Figure 3(e). FRF for symmetric three-layered plate with a 45° skew angle.

Figure 3(f). FRF for anti-symmetric two layered plate with a 45° skew angle.
Figure 3(g). FRF for symmetric three-layered plate with a 60° skew angle.

Figure 3(h). FRF for anti-symmetric two layered plate with a 60° skew angle.

Figure 3(i). FRF for symmetric three-layered plate with a 75° skew angle.

Figure 3(j). FRF for anti-symmetric two layered plate with a 75° skew angle.

4.3. Effect of damping ratio

The effect of damping ratios on natural frequency and resonance amplitude are presented for two layered (0°/90°) anti-symmetric and three layered (0°/90°/0°) Symmetric skew laminated composite plate for thickness ratios 0.01. From table 5, it has been observed that rasonance amplitude decreases as the damping ratio increases.

Table 5: Effect of Damping Ratios on resonance amplitude for anti-symmetric and symmetric laminated composite plate.

| Lamination scheme | Damping Ratio | Natural frequency (Hz) | Resonance amplitude (mm) |
|-------------------|---------------|------------------------|--------------------------|
| Two layered anti-symmetric | 0.01 | 85.108 | 0.135868 |
| | 0.05 | 85.108 | 0.0256293 |
| | 0.1 | 85.108 | 0.0127985 |
| Three layered symmetric | 0.01 | 115.98 | 0.0686759 |
| | 0.05 | 115.98 | 0.0135532 |
| | 0.1 | 115.98 | 0.00677883 |
Harmonic response on symmetric and anti-symmetric cross-ply laminated skew plate for different damping ratios for thickness ratios 0.01 are shown on FRF graph. It can be seen that a large damping ratio has a fairly flat resonance curve as in shown in figures 4(a-f).

Figure 4(a) demonstrates an FRF for the symmetric three-layered skew plate for a damping ratio of 0.01. The frequency range is taken between 90 Hz and 300 Hz. Resonance 0.0686759 mm observed at a natural frequency of 115.98 Hz. Figure 4(b) demonstrates an FRF for anti-symmetric two layered skew plate for a damping ratio of 0.01. The frequency range is taken between 70 Hz to 200 Hz. Resonance 0.135868 mm observed at a natural frequency of 85.108 Hz.

Figure 4(c) demonstrates an FRF for symmetric three-layered skew plate for a damping ratio of 0.05 for the frequency range of 80 Hz to 400 Hz. Resonance 0.0135532 mm observed at a natural frequency of 115.98 Hz. Figure 4(d) demonstrates an FRF for anti-symmetric two layered skew plate for a damping ratio of 0.05 for the frequency range of 60 Hz to 200 Hz. Resonance 0.0256239 mm observed at a natural frequency of 85.108 Hz.

Figure 4(e) demonstrates an FRF for the symmetric three-layered skew plate for damping ratio of 0.1 for frequency range of 80 Hz to 400 Hz. Resonance 0.0067783 mm observed at a natural frequency of 115.98 Hz. Figure 4(f) demonstrates an FRF for anti-symmetric two layered skew plate for a damping ratio of 0.1 for frequency range of 60 Hz to 200 Hz. Resonance 0.0127985 mm observed at a natural frequency of 85.108 Hz.
5. Conclusions
This study deals with the vibration and harmonic response of skew laminated composite plate. The analyses are conducted for symmetric and anti-symmetric angle ply and cross ply laminates to examine the effect of skew angle, fibre orientation, and the number of layers on the resonance amplitude and natural frequency of composite skew plates. From the convergence study, it has been found that a (17×17) mesh size is sufficient enough to compute the responses and the responses obtained are also validated with the available published literature.

It has been observed that the non-dimensional fundamental frequency increases as the number of layers increases but the variation is negligible beyond four layers. As the skew angle increases, the resonance amplitude decreases with an increase in natural frequencies. The resonance amplitude found the lowest with the highest natural frequency for the plate with a skew angle of 75°. It has also been found that as the damping ratio increases, the value of resonance amplitude decreases for all skew plates.

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