A novel spike-train generator suitable for QCA implementation towards UWB-IR applications

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Abstract: In this paper, a novel spike-train generator suitable for quantum-dot cellular automaton (QCA) implementation is proposed. As an analysis tool of the proposed generator, a novel spike phase map is derived. Then, using the map, it is shown that the proposed generator can generate spike-trains with various spike patterns in terms of period, density, and correlation. Furthermore, a stochastic algorithm for parameter tuning for the proposed generator is proposed. It is shown that the parameter tuning algorithm enables the proposed generator to generate spike-trains suitable for ultra wide band impulse radio (UWB-IR) communication, ranging, and positioning systems. A QCA layout of the generator obtained by the parameter tuning algorithm is designed and its operation is verified by a QCA simulator.

Key Words: quantum-dot cellular automaton, spike-train generator, ultra wide band impulse radio

1. Introduction

Most computer devices are basically operated on the basis of the metal-oxide semiconductor field-effect transistor (MOSFET). The quantum-dot cellular automaton (QCA) is nanoscale technology, which has been expected to be alternative to the conventional MOSFET based architecture [1–4]. Figure 1(a) shows a basic element of the QCA called a QCA cell. As shown in the figure, the QCA cell has four quantum dots arranged in a square pattern, where two dots are assumed to have electrons and the other dots are assumed to have no electrons. Due to Coulomb forces, there are two stable patterns of the dots as shown in Fig. 1(a), where these patterns are said to have polarizations $Pol = +1$ and $Pol = -1$. Hence, the QCA cell can work as a binary memory, where the polarizations $Pol = +1$ and $Pol = -1$ may represent binary states $Logic = 1$ and $Logic = 0$, respectively. In Fig. 1(b), QCA cells are located adjacent to each other. As shown in the figure, there are two stable patterns of the quantum dots in the located QCA cells due to the Coulomb forces. These patterns can be switched by
appropriate low potential barriers among the dots and applying external Coulomb forces. Typically, the potential barriers are controlled by four-phase synchronized clocks, where the QCA cells driven by the same clock are said to belong to the same clock zone as shown in Fig. 1(c). By applying the four-phase synchronized clocks to the four clock zones appropriately, binary information can be transmitted through the QCA cells. It should be emphasized that the QCA cells can transmit the binary information without using current flows, whereas the conventional MOSFET based architecture cannot avoid energy consumption due to current flows. This ultra low energy consumption property is one of the most significant advantages of the QCA based architecture. Then, using the QCA cells, many memory units and memoryless units have been designed so far [21–27]. In order to realize proper functions of such QCA units, designers are recommended to obey the following design guidelines [28–31].

**G1** The number of adjacently located QCA cells in the same clock zone should be more than or equal to 2 as shown in Fig. 1(c).

**G2** The number of adjacently located QCA cells in the same clock zone should be less than or equal to 28 at a clock frequency of 1 THz as shown in Fig. 1(c).

**G3** The center-to-center distance between two adjacently located QCA cells should be as short as possible as shown in Fig. 1(c).

The cellular automaton (CA) has been investigated intensively from both fundamental scientific viewpoint and engineering application viewpoint. For example, since the CA is different from traditional dynamical systems such as ordinary and partial differential equations but can exhibit a huge variety of spatio-temporal phenomena, it is even called a “new kind of science” [5–7]. Also, the CA has been applied to many engineering systems such as traffic flow model, image classification, and music generation [8–10]. Among such applications, this paper focuses on application of the CA to spike-train generators and ultra wide band impulse radio (UWB-IR) communication, ranging, and positioning systems. The UWB-IR systems have been applied to various engineering systems such as vehicular radar, wireless sensor networks, and position estimation, where their advantages include high data rate, low power consumption, and high resistivity to noise [17–19]. (A) Recall that the QCA is a
nanoscale device and has the ultra low energy consumption property. So, if we design a UWB-IR system based on the QCA, such a system is expected to have the ultra small circuit area property and the ultra low power consumption property. An important building block of such a QCA based UWB-IR system is a QCA based spike-train generator that generates spike-trains suitable for the UWB-IR. Our group has been developing design methods of the CA to generate spike-trains suitable for the UWB-IR systems [11–13]. However, these design methods can not be applied to the QCA based architecture since these methods can not handle transmission delays among QCA cells and thus they can not satisfy the design guidelines G1 and G2. In order to overcome this difficulty, this paper aims at

- proposing a novel CA spike-train generator that can be implemented by the QCA and can generate spike-trains with various spike patterns;
- proposing a rigorous analysis method of the proposed spike-train generator; and
- proposing a stochastic algorithm for parameter tuning for the proposed spike-train generator so that it can generate spike-trains suitable for UWB-IR systems.

This paper is organized as follows. In Section II, a novel spike-train generator suitable for the QCA implementation is proposed. Also, a rigorous analysis tool of the proposed generator is proposed. In Section III, a QCA layout of the proposed spike-train generator is designed and its operation is verified by a QCA simulator called QCADesigner [20]. Using the analysis tool, it is shown that the proposed generator can generate spike-trains with various spike patterns in terms of periods of spike-trains, second peaks of auto-correlation functions, and numbers of spikes. In Section 4, a parameter tuning algorithm for the proposed generator is proposed, where the analysis tool is used as its subroutine to accelerate parameter tuning speed. It is shown that the parameter tuning algorithm enables the proposed generator to generate spike-trains suitable for applications to the UWB-IR systems. Also, the resulting generator is designed as a QCA layout and its operation is verified by the QCA simulator.

2. A novel spike-train generator suitable for QCA implementation

2.1 Proposed spike-train generator

In this subsection, a novel spike-train generator suitable for quantum-dot cellular automaton (QCA) implementation is proposed. The proposed generator has the following discrete time.

\[ t = 0, 1, 2, \cdots. \]

Figure 2(a) shows a circuit diagram of the proposed generator. As shown in the figure, the generator has \( M \) cells \((M > 0)\) called \( p \)-cells, which have the following binary states.

\[ p_i(t) \in B = \{0, 1\}, \]

where \( i \in \{0, 1, \cdots, M - 1\} \) is an index of the \( p \)-cell. For simplicity, the following vector form of the binary states is introduced.

\[ P(t) = (p_0(t), \cdots, p_{M-1}(t))^T \in B^M. \]

As shown in Fig. 2(a), the \( p \)-cells are ring-coupled and thus the dynamics of the \( p \)-cells is described by

\[ P(t + 1) = (p_{M-1}(t), p_0(t), \cdots, p_{M-2}(t))^T. \quad (1) \]

In this paper, the initial conditions of the \( p \)-cells are fixed to \( P(0) = (1, 0, \cdots, 0) \) and thus the \( p \)-cells oscillate periodically with period \( M \) as shown in Fig. 2(b). As shown in Fig. 2(a), the proposed generator has reconfigurable wires from \( M \) left terminals (connected from the \( p \)-cells) to \( N \) \((N \geq M)\) right terminals (connected to the reset unit). It is assumed that each left terminal has one wire and each right terminal can accept any number of wires. Then the wires transform the binary state vector \( P(t) \in B^M \) of the \( p \)-cells into the following binary signal vector.
Fig. 2. A novel spike-train generator suitable for quantum-dot cellular automaton (QCA) implementation. (a) Circuit diagram. The generator consists of $M$ $p$-cells, $N$ $x$-cells, $M$ reconfigurable wires with delay $\rho$, and a reset unit with delay $\delta$, where $M = N = 5$ in this figure. The wires are characterized by the matrix $A$ in Eq. (2). (b) Typical time waveforms. $\rho = 2$ and $\delta = 2$. The white triangle represents $p_i(t) = 1$. The black circle represents $b_j(t) = 1$. The white circle represents $b_j(t + \delta - \rho) = 1$. The black box represents $x_j(t) = 1$ and the gray box represents $x_j(t) = \phi$. The generator outputs a spike-train $Y(t)$ with the period $T = 20$ and the number $Q = 4$ of spikes.
\[ b(t) \in B^N = (b_0(t), \cdots, b_{N-1}(t))^T. \]

In order to characterize the transformation by the wires, the following function \( a(j, i) : \{0, 1, \cdots, M-1\} \times \{0, 1, \cdots, N-1\} \rightarrow B \) and its matrix form \( A \) are introduced.

\[
a(j, i) = \begin{cases} 
1 & \text{if the } j\text{-th left terminal is wired to the } i\text{-th right terminal,} \\
0 & \text{otherwise,} 
\end{cases}
\]

\[
A = \begin{pmatrix}
a(0,0) & a(1,0) & \cdots & a(N-1,0) \\
a(0,1) & a(1,1) & \cdots & a(N-1,1) \\
\vdots & \vdots & \ddots & \vdots \\
a(0,M-1) & a(1,M-1) & \cdots & a(N-1,M-1)
\end{pmatrix}.
\]

For example, the proposed generator in Fig. 2(a) is characterized by the following matrix \( A \).

\[
A = \begin{pmatrix}
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0
\end{pmatrix}. \tag{2}
\]

Then the binary state vector \( P(t) \) is transformed into the binary signal vector \( b(t) \) as follows.

\[
b(t) = AP(t). \tag{3}
\]

In Fig. 2(b), black circles represent a time waveform of the binary signal vector \( b(t) \), which corresponds to the pattern of the wires in Fig. 2(a). As noted in Fig. 2(a), the signal \( b(t) = (b_0(t), \cdots, b_{N-1}(t))^T \) is transmitted to the reset unit with delay \( \rho \). Then the reset unit accepts the following signal

\[
b(t - \rho) = (b_0(t - \rho), \cdots, b_{N-1}(t - \rho))^T.
\]

As shown in Fig. 2(a), the proposed generator has \( N \) cells called \( x\)-cells, which have the following tri-state states.

\[ x_j(t) \in T = \{0, 1, \phi\}, \]

where \( j \in \{0, 1, \cdots, N-1\} \) is an index of the \( x\)-cell. Also, “0” and “1” correspond to those in the binary set \( B \), while “\( \phi \)” is used to represent a delay in the reset unit. For simplicity, the following vector form of the tri-state states is introduced.

\[ X(t) = (x_0(t), \cdots, x_{N-1}(t))^T \in T^N. \]

As shown in Fig. 2(a), the \( x\)-cells are basically ring-coupled and then obey the following dynamics.

If \( X(t) \neq (0, \cdots, 0, 1) \), then

\[ X(t + 1) = S(X(t)), \tag{4} \]

where

\[ S((x_0, \cdots, x_{N-1})^T) = (0, x_0, \cdots, x_{N-2})^T \]

is a shift operator. As shown in Fig. 2(a), the \( x\)-cells accept the binary signal vector \( b(t - \rho) \) via the reset unit and then additionally obey the following dynamics.

If \( X(t) = (0, \cdots, 0, 1) \), then

\[ X(t + 1 + d) = \begin{cases} 
(\phi, \cdots, \phi) & \text{for } d \in \{0, 1, \cdots, \delta - 1\}, \\
 b(t + \delta - \rho) & \text{for } d = \delta,
\end{cases} \tag{5} \]

where \( \delta \in \{1, 2, \cdots, \rho\} \) is a parameter characterizing the delay of the reset unit, and is called a reset delay. In Fig. 2(b), black and gray boxes represent a time waveform of the tri-state state vector \( X(t) \), which corresponds to the pattern of the wires in Fig. 2(a). When the black box is below the highest position \( N - 1 \), the black box is shifted upward due to Eq. (4). When the black box reaches
the highest position \(N - 1\), the black box is reset to the white circle with the reset delay \(\delta\) due to Eq. (5). Repeating such dynamics, the state vector \(X(t)\) oscillates as shown in Fig. 2(b). In addition, depending on the discrete state \(X(t)\), the proposed generator outputs the following spike-train \(Y(t)\) as shown in Fig. 2(b).

\[
Y(t) = \begin{cases} 
1 & \text{if } X(t) = (0, \cdots, 0, 1), \\
0 & \text{otherwise.}
\end{cases}
\]

As a result, the proposed generator can be summarized as follows.

- **Time:** Discrete time \(t\)
- **States:** Binary states \(P(t)\) and Tri-state states \(X(t)\)
- **Dynamics:** Eqs. (1), (3), (4), and (5)
- **Output:** Spike-train \(Y(t)\) in Eq. (6)
- **Parameters:** Numbers \(M\) and \(N\) of the cells, Matrix \(A\) characterizing the wires, Transmission delay \(\rho\), and Reset delay \(\delta\)

**Remark 1 (Novelty):** It should be emphasized that our previous spike-train generators [11–13] cannot be implemented by the QCA since they cannot handle transmission delays among QCA cells and thus they cannot satisfy the design guidelines G1 and G2. In order to overcome this difficulty, in this paper, the delays \(\rho\) and \(\delta\) are introduced in the reconfigurable wires and the reset unit, respectively. Due to the delays \(\rho\) and \(\delta\), the proposed spike-train generators cannot be analyzed by analysis methods in [11–13]. In order to overcome this difficulty, in the next subsection, a novel rigorous analysis tool of the proposed generator is proposed.

### 2.2 Novel spike maps for the proposed generator

As shown in Fig. 2(b), let \(t_n\) denote the \(n\)-th spike position of the output \(Y(t)\) of the generator. Then the dynamics of the spike position \(t_n\) can be described by the following spike position map \(g\).

\[
t_{n+1} = g(t_n) = t_n + M - \beta(t_n + \delta - \rho(\text{mod} M)) + \delta,
\]

where

\[
\beta(j) = i \quad \text{if the } j\text{-th left terminal is wired to the } i\text{-th right terminal,}
\]

\[
\beta : \{0, 1, \cdots, M - 1\} \rightarrow \{0, 1, \cdots, N - 1\}.
\]

Figure 3(a) shows an example of the spike position map \(g\). Note that, by iterating the spike position map \(g\), the spike position \(t_n\) ever increases and thus \(g\) is not suitable for analysis of the proposed generator. So, the following spike phase \(\theta_n\) is introduced.

\[
\theta_n = t_n(\text{mod} M).
\]

The dynamics of the spike phase \(\theta_n\) can be described by the following spike phase map \(G\) from a finite set into itself.

\[
\theta_{n+1} = G(\theta_n) = g(\theta_n)(\text{mod} M),
\]

\[
G : \{0, 1, \cdots, M - 1\} \rightarrow \{0, 1, \cdots, M - 1\}.
\]

Figure 3(b) shows an example of the spike phase map \(G\). Using the spike phase map \(G\), fundamental characteristics (e.g., cycle \(Q\) of a periodic orbit and number of co-existing periodic orbits) of the spike phase \(\theta_n\) can be analyzed rigorously. For example, it can be shown that the spike phase map \(G\) in Fig. 3(b) has the following periodic orbit with cycle \(Q = 4\).

\[
(\theta_1, \theta_2, \theta_3, \theta_4) = (1, 0, 3, 4).
\]

Then, using the periodic orbit \((\theta_1, \cdots, \theta_Q)\), the corresponding sequence of the spike position \(t_n\) can be derived as follows.
Fig. 3. Spike maps corresponding to the proposed spike-train generator in Fig. 2(a), where the wires are characterized by the matrix $A$ in Eq. (2). (a) Spike position map $g$. (b) Spike phase map $G$.

\[ t_n = \theta_1 + \sum_{k=1}^{n-1} (M - \beta(\theta_k - \rho \text{mod } M)) + \delta. \]  

For example, using the periodic orbit $(\theta_1, \theta_2, \theta_3, \theta_4) = (1, 0, 3, 4)$ of the spike phase map $G$ in Fig. 3(b), the following sequence of the spike position $t_n$ can be derived.

\[ (t_1, t_2, t_3, t_4) = (1, 5, 8, 14). \]

It can be confirmed that these spike positions are identical with those in Fig. 2(b). In addition, the period $T$ of the spike train $Y(t)$ can be derived as follows.

\[ T = t_{Q+1} - t_1 = \sum_{k=1}^{Q} (M - \beta(\theta_k - \rho \text{mod } M)) + \delta - \theta_1. \]

For example, using the above mentioned periodic orbit $(\theta_1, \theta_2, \theta_3, \theta_4) = (1, 0, 3, 4)$, the period $T = 20$ can be obtained and it can be confirmed that so obtained period $T = 20$ is identical with that in Fig. 2(b).

Remark 2 (Significance of the spike phase map): As explained in this subsection, the spike phase map $G$ can be used as a rigorous analysis tool for the proposed spike-train generator. It should be emphasized that the spike phase map $G$ will be also utilized as a kind of subroutine to accelerate execution speed of a parameter tuning algorithm in Section 4. As a result, it can be said that the spike phase map $G$ is useful not only for analysis but also for design of the proposed spike-train generator.

3. QCA implementation of the proposed spike-train generator

3.1 QCA basics

In this section, a brief of QCA basics is introduced. Figure 4(a) shows a QCA layout of a majority gate consisting of five QCA cells, where $A$, $B$, and $C$ are inputs and $M = AB + BC + CA$ is an output. If $C$ is fixed to 1 (i.e. polarization of the cell corresponding to $C$ is fixed to 1), the majority gate works as an OR gate as shown in Fig. 4(b). If $C$ is fixed to 0 (i.e. polarization of the cell corresponding to $C$ is fixed to $-1$), the majority gate works as an AND gate as shown in Fig. 4(c). Furthermore, by arranging QCA cells appropriately, other logic gates and complicated logic functions
Fig. 4. (a) QCA layout of a majority gate. The majority gate has three inputs $A$, $B$, and $C$, and an output $M = AB + BC + CA [1–4]$. (b) and (c) show simulation results of the majority gate obtained by the QCADesigner. (b) The majority gate works as an OR gate if the input $C$ is fixed to 0. (c) The majority gate works as an AND gate if the input $C$ is fixed to 1.

Fig. 5. (a) QCA layout of a shift register consisting of D-type flip-flops $DFF_0$, $DFF_1$, and $DFF_2$. (b) Clocks $C_0(t)$, $C_1(t)$, $C_2(t)$, and $C_3(t)$. (c) Typical time waveforms obtained by the QCADesigner. It can be seen that the QCA layout works as a shift register.

can be designed [21–24]. In order to design a D-type flip-flop by the QCA cells, we introduce the following definition.

**Definition:** Suppose $R$ QCA cells are arranged in serial and is said to form a QCA subset as shown in Fig. 5(a), where $2 \leq R \leq 28$. Suppose four QCA subsets are arranged in serial as shown in Fig. 5(a). Suppose a QCA cell $D_i$ is arranged next to an end point of the four QCA subsets and a QCA cell $D_{i+1}$ is arranged next to the other end point of the QCA subsets as shown in Fig. 5(a). Suppose the clock $C_0(t)$ is applied to the QCA subset next to the QCA cell $D_i$ as shown in Fig. 5(a). Suppose the clocks $C_1(t)$, $C_2(t)$ and $C_3(t)$ are applied to the other QCA subsets in serial order as shown in Fig. 5(a). The set of $4R + 2$ QCA cells arranged in the above manner is said to form a set $DFF_i$.

The set $DFF_i$ works as the D-type flop-flop. Figure 5(a) shows a QCA layout of a shift register consisting of $DFF_0$, $DFF_1$, and $DFF_2$, where $D_0$, $D_1$, $D_2$ are inputs of $DFF_0$, $DFF_1$, $DFF_2$, respectively, and $D_1$, $D_2$ and $D_3$ are outputs of $DFF_0$, $DFF_1$, $DFF_2$, respectively. Figure 5(b) explains how to provide clocks to the flip-flop. When a tunneling barrier between quantum-dots is high, the electrons in the dots are tightly confined and thus the polarization is locked. On the other hand, when the tunneling barrier is low, the electrons can be rearranged in the dots and thus the polarization can be changed.
are introduced in order to adjust the phases of the clocks. Figure 5(b) shows examples of such clocks \( C_0(t), C_1(t), C_2(t), \) and \( C_3(t) \). Each clock \( C_k(t) \) has four phases called Switch, Hold, Release, and Relax. (B) If a clock \( C_k(t) \) is in the Hold and the Relax phases, the tunneling barrier is high and low, respectively. As shown in Fig. 5(b), the four clocks \( C_0(t), C_1(t), C_2(t), \) and \( C_3(t) \) are assumed to be synchronized with different phases. Also, as shown in Fig. 5(a), these four clocks are applied to the ten QCA cells in each set \( DFF_i \) of QCA cells. This clocking method realizes that each set \( DFF_i \) works as a D-type flip-flop. As shown in Fig. 5(a), the D-type flip-flops are connected in serial and then they work as a shift register. Figure 5(c) shows a simulation result of the shift register, which is obtained by the QCA simulator QCADesigner [20]. It can be seen that the QCA layout in Fig. 5(a) works as a shift register, where \( D_0 \) is its input. Other types of sequential logics can be also designed in a similar fashion [25–27].

### 3.2 QCA implementation of the proposed spike-train generator

Figure 6(a) shows a QCA layout of the proposed generator in Fig. 2(a). In this layout, the upper loop and the lower loop correspond to the \( p \)-cells and the \( x \)-cells in Fig. 2(a), respectively. Also, crossover cells are introduced in order to realize crosses of unconnected wires [4, 20], and delay cells (i) and (ii) are introduced in order to adjust the phases of the clocks \( C_0(t), C_2(t), C_3(t), \) and \( C_4(t) \) appropriately. The delays caused by the delay cells (i) correspond to the transmission delay \( \rho \) of the reconfigurable wires in Fig. 2(a). The delays caused by the delay cells (ii) and the crossover cells correspond to the reset delay \( \delta \) of the reset unit in Fig. 2(a). Figure 6(b) shows typical time waveforms of the QCA layout in Fig. 6(a) obtained by the QCADesigner. Comparing Fig. 6(b) with Fig. 2(b), it can be confirmed that the spike-train \( Y(t) \) generated by the QCA layout in Fig. 6(a) is equivalent to the spike-train \( Y(t) \) generated by the proposed generator in Fig. 2(a). It should be emphasized that the proposed generator with other values of the parameters \((M, N, A, \rho, \delta)\) can be also implemented by the QCA layout in the same fashion.

### 4. Generation of various spike-trains and parameter tuning for UWB application

Figure 7 shows the proposed generator with different patterns of reconfigurable wires (i.e., different values of the parameter \( A \)) and corresponding spike-trains \( Y(t) \). It can be seen that the proposed generator can generate spike-trains with various spike patterns by adjusting the pattern of the reconfigurable wires. In this section, a stochastic algorithm for parameter tuning for the proposed generator so that it can generate spike-trains \( Y(t) \) suitable for ultra wide band impulse radio (UWB-IR) communication, ranging, and positioning systems is proposed. In order to characterize the spike-train \( Y(t) \) of the proposed generator, the following quantities and function are introduced.

- The period \( T \) of the spike-train \( Y(t) \), which can be obtained by using the spike phase map \( G \) as explained in Section 2.2.

- The number \( Q \) of spikes of the spike-train \( Y(t) \) during the period \( 0 \leq t < T \), which is identical with the cycle of the corresponding periodic orbit \((\theta_1, \ldots, \theta_Q)\) of the spike phase and thus can be obtained by using the spike phase map \( G \).

- The autocorrelation function \( C(\tau) \) of the spike-train \( Y(t) \) and its second peak \( S \), which are defined by

\[
C(\tau) = \frac{1}{T} \sum_{t=0}^{T-1} Y(t)Y(t+\tau),
\]

\[
S = \begin{cases} 
\max_{0 \leq \tau < T} C(\tau) & \text{for } Q > 1, \\
C(0) & \text{otherwise.}
\end{cases}
\]
Fig. 6. (a) QCA layout of the proposed generator in Fig. 2(a). The upper loop and lower loop correspond to the \( p \)-cells and the \( x \)-cells, respectively. The crossover cells realize crosses of unconnected wires [4, 20]. The delay cells are introduced in order to adjust the phases of the clocks. The delays caused by the delay cells (i) correspond to the transmission delay \( \rho \). The delays caused by the delay cells (ii) and the crossover cells correspond to the reset delay \( \delta \). (b) Typical time waveforms obtained by the QCADesigner. By comparing with Fig. 2(b), it can be confirmed that the QCA layout in (a) can realize the dynamics of the proposed generator in Fig. 2(a).

For example, the spike-trains \( Y(t) \) in Figs. 7(a), (b), and (c) are characterized by \((T, Q, S) = (3, 1, 0.33), (25, 4, 0.12), \) and \((20, 4, 0.095)\), respectively. Hence it can be confirmed that, by adjusting the pattern of the reconfigurable wires (i.e., the values of the parameter \( A \)), the proposed generator can generate spike-trains with various spike patterns in terms of the period \( T \), the number \( Q \) of spikes, and the auto-correlation function \( C(\tau) \) and its second peak \( S \).
Fig. 7. Generation of various spike-trains by the proposed generator. The numbers of the $p$-cells and the $x$-cells are $M = N = 5$. The transmission delay and the reset delay are $\rho = \delta = 2$. (a) The reconfigurable wires are characterized by $a(0, 4) = a(1, 4) = a(2, 4) = a(3, 4) = a(4, 4) = 1$. The spike-train $Y(t)$ is characterized by the period $T = 3$, the number $Q = 1$ of spikes during the period, and the second peak $S = 0.33$ of the auto-correlation function $C(\tau)$. (b) $a(0, 0) = a(1, 0) = a(2, 0) = a(3, 0) = a(4, 3) = 1$. $Y(t)$ is characterized by $(T, Q, S) = (25, 4, 0.12)$. (c) $a(0, 4) = a(1, 3) = a(2, 2) = a(3, 1) = a(4, 0) = 1$. $Y(t)$ is characterized by $(T, Q, S) = (20, 4, 0.095)$. 

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Now, a parameter tuning algorithm for the proposed generator to generate spike-trains $Y(t)$ suitable for the UWB-IR systems is proposed. In the UWB-IR systems, the following characteristics of the spike-train $Y(t)$ are desired \([14, 15]\).

\begin{itemize}
    \item[(C1)] The second peak $S$ of the autocorrelation function $C(\tau)$ of the spike-train $Y(t)$ should be lower in order to realize higher resistivity against noise.
    \item[(C2)] The period $T$ of the spike-train $Y(t)$ should be longer in order to realize lower second peak $S$ of the autocorrelation function $C(\tau)$.
    \item[(C3)] The number $Q$ of spikes should be larger in order to realize higher resistivity against noise.
\end{itemize}

Then, the following objective function $F$ of the spike-train $Y(t)$ is introduced.

$$F = \alpha T + \beta Q - \gamma S,$$

where $\alpha > 0$, $\beta > 0$, and $\gamma > 0$ are parameters of the objective function $F$. Using the objective function $F$, the following parameter tuning algorithm is proposed.

**List 1. Stochastic Algorithm for Parameter Tuning.**

\textbf{Step 1: Initialization.} The numbers $M$ and $N$ of the cells and the delays $\rho$ and $\delta$ are given. Prepare $L$ matrices ($A_1, \ldots, A_L$) as follows: \(a_l(j, i) = 1\) for \(j = 0, \ldots, M - 1\) and \(a_l(j, i) = 0\) for \(j \neq i\), where \(a_l(j, i)\) is an element of the matrix $A_l$. Also, initialize an iteration counter $k$ to 0.

\textbf{Step 2: Evaluation.} A spike-train generated by the matrix $A_l$ is denoted by $Y_l(t)$. The values $(F_1, \ldots, F_L)$ of the objective function $F$ of the spike-trains $(Y_1(t), \ldots, Y_L(t))$ are calculated.

\textbf{Step 3: Selection and Mutation.} The matrix $A_{sel}$ corresponding to the maximum value $F_{\text{max}} = \text{max}_1\{F_l\}$ of the objective function $F$ is selected. A column of the selected matrix $A_{sel}$ is randomly selected and the position of “1” in the selected column is randomly changed within the column.

\textbf{Step 4: Termination.} Let $K$ be a given maximum iteration number. If $k < K$, then increment $k$ by 1 and go to Step 2. If $k = K$, then terminate this algorithm.

Note that calculation speed of the objective functions $(F_1, \ldots, F_L)$ can be accelerate by the spike phase map $G$ (the calculation speed of $T$ and $Q$ becomes $M$ times higher). Figure 8 shows an example of parameter tuning result. Figure 8(a) shows the proposed generator just after the initialization in Step 1. In this case, the period of the spike-train $Y_l(t)$ is $T_l = 5$, the second peak of the auto-correlation function $C_l(\tau)$ is $S_l = 0.167$, the number of spikes during the period is $Q_l = 1$, and the value of the objective function is $F_l = 4.08$, where $\alpha = 0.8$, $\beta = 0.1$ and $\gamma = 0.1$. Figure 8(b) shows the proposed generator after the parameter tuning. In this case, the period of the spike-train $Y_l(t)$ is $T_l = 30$, the second peak of the auto-correlation function $C_l(\tau)$ is $S_l = 0.066$, the number of spikes during the period is $Q_l = 5$, and the value of the objective function is $F_l = 24.49$. It can be seen that, after the parameter tuning, the proposed generator generates the spike-train $Y_l(t)$ with better characteristics, i.e., longer period $T_l$, larger number $Q_l$ of spikes, and lower second peak $S_l$ of the auto-correlation function. Figure 9 shows the characteristics of the parameter tuning algorithm, where the dots indicate the characteristics of the proposed generator after the parameter tuning shown in Fig. 8(b). It can be seen that, as the iterations proceed, the parameter tuning algorithm can find patterns of the reconfigurable wires (i.e., values of the parameter $A_l$), which lead to better characteristics of the spike-trains $Y_l(t)$. It can be also seen that the average characteristics of the spike-trains almost converge after the parameter tuning. Figure 10(a) shows a QCA layout of the proposed generator in Fig. 8(b) whose pattern of the reconfigurable wires is obtained by the parameter tuning algorithm. Also, Fig. 10(b) shows time waveforms of the QCA layout obtained by the QCA simulator QCADesigner. It can be confirmed that the QCA layout generates a spike-train $Y(t)$, which is equivalent to the spike-train $Y_l(t)$ in Fig. 8(b).
Fig. 8. An example of parameter tuning result. The numbers of the cells are \( M = N = 5 \) and the delays are \( \rho = \delta = 2 \). The number of the prepared matrices is \( L = 5 \) and the maximum iteration number is \( K = 5000 \). The parameters of the objective function \( F \) are \( \alpha = 0.8, \beta = 0.1 \) and \( \gamma = 0.1 \). (a) After initialization in Step 1. The spike-train \( Y_i(t) \) is characterized by \( (T_1, Q_1, S_1, F_1) = (5, 1, 0.167, 4.08) \). (b) After tuning. The spike-train \( Y_i(t) \) is characterized by \( (T_1, Q_1, S_1, F_1) = (30, 5, 0.066, 24.49) \).

Fig. 9. Parameter tuning characteristics. The numbers of the cells are \( M = N = 5 \) and the delays are \( \rho = \delta = 2 \). The number of the prepared matrices is \( L = 5 \) and the maximum iteration number is \( K = 5000 \). The parameters of the objective function \( F \) are \( \alpha = 0.8, \beta = 0.1 \) and \( \gamma = 0.1 \). An execution of the parameter tuning for \( K \) iterations is called a trial. The graphs in (a)–(d) are averages for 1000 trials, where the dots indicate the characteristics of the proposed generator after the parameter tuning shown in Fig. 8(b). (a) Average of the maximum value \( F_{\text{sel}} \) of the objective function \( F \). (b) Average of the period \( T_{\text{sel}} \) leading to the maximum value \( F_{\text{sel}} \) of the objective function \( F \). (c) Average of the number \( Q_{\text{sel}} \) of spikes leading to the maximum value \( F_{\text{sel}} \) of the objective function \( F \). (d) Average of the second peak \( S_{\text{sel}} \) of the auto-correlation function leading to the maximum value \( F_{\text{sel}} \) of the objective function \( F \). The influence of the parameters \( \alpha, \beta \) and \( \gamma \) is explained in Appendix.
Remark 3 (Application): The above result suggests that the QCA layout in Fig. 10(a) can generate a spike-train suitable for the UWB-IR systems in terms of the period $T$, the number $Q$ of spikes, and the second peak $S$ of the auto-correlation function. Potential applications of such QCA based UWB-IR systems include: intra and inter QCA chip wired communications, ultra low power wireless QCA based communications, and ultra low power QCA based ranging and positioning.
5. Conclusions
Recall that the purposes of this paper were to propose a QCA based spike-train generator that can generate spike-trains with various spike patterns, to propose an analysis tool of the generator, and to propose a parameter tuning algorithm for the generator so that it can generate spike-trains suitable for the UWB-IR systems. These purposes were achieved as the followings. The novel spike-train generator the dynamics of which is described by the cellular automaton with the delays $\rho$ and $\delta$ was proposed. Also, as its analysis tool, the spike phase map $G$ was derived. Using the spike phase map $G$, it was shown that the proposed generator can generate various spike-trains in terms of the period, the number of spikes, and the auto-correlation function. Also, the parameter tuning algorithm for the proposed generator was proposed, where the spike phase map $G$ is used as its subroutine, where the spike phase map $G$ accelerates execution speed of the parameter tuning algorithm. It was shown that the parameter tuning algorithm enables the proposed generator to generate spike-trains suitable for the UWB-IR systems, i.e., spike-trains with longer period $T$, larger number $Q$ of spikes, and lower second peak $S$ of the auto-correlation function. Furthermore, the proposed generator after the parameter tuning was implemented as the QCA layout and its operation was validated by the QCA simulator. Future problems are including (a) calculation of bit error rates under specific pulse modulations, (b) more detailed analysis of the spike-train of the proposed generator, (c) development of more efficient parameter tuning algorithm, and (d) design of QCA based synchronizer and inverse spreader for UWB-IR communication applications.

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Appendix
A. Influence of the parameters in the tuning algorithm
Let us clarify influence of the parameters $(\alpha, \beta, \gamma)$ of the objective function $F$ in Eq. (11) used in the parameter tuning algorithm in List 1. It has been shown that lower auto-correlation (lower second peak of the auto-correlation function) of a spike-train leads to lower bit error probability in a UWB-IR communication system [32]. Therefore, the proposed parameter tuning algorithm searches values of the parameters $A$, which lead to a lower second peak $S$ of the auto-correlation function of the spike-train $Y(t)$. Figure A-1 shows the influence of the parameters $(\alpha, \beta, \gamma)$ in the characteristics of the second peak $S$. It can be seen in the figure that the parameter values $(\alpha, \beta, \gamma) = (0.8, 0.1, 0.1)$ lead to a better characteristics of the second peak $S$ (see the green dashed curve). So, the parameter values $(\alpha, \beta, \gamma) = (0.8, 0.1, 0.1)$ are used in the paper, e.g., Figs. 9, 10, A-1 are obtained by the parameter tuning with these parameter values.

![Fig. A-1. Comparison of parameter tuning characteristics for several parameters values.](image-url)
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