Supplemental document accompanying submission to Optics Express

Title: Enhanced on-chip frequency measurement using weak value amplification

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Enhanced on-chip frequency measurement using weak value amplification: supplemental document

This supplemental document details the numerical methods used to create the Bragg grating simulations in Section III of the main paper. The grating is treated as a lumped element, and its effects are calculated using a transfer matrix method. Two variants will be discussed here: the fundamental transfer matrix approach and the thin layer approach.

1. FUNDAMENTAL MATRIX APPROACH

The fundamental matrix approach is computationally inexpensive and easy to implement, but only works for a single-frequency grating. It consists of finding a matrix $F$ that relates the forward- and backward-traveling waves $A_f$ and $A_b$ at either side of the grating. This can be written as a matrix equation,

$$
\begin{bmatrix}
A_f(0) \\
A_b(0)
\end{bmatrix} =
\begin{bmatrix}
F_{11} & F_{12} \\
F_{21} & F_{22}
\end{bmatrix}
\begin{bmatrix}
A_f(L)e^{-i\beta L} \\
A_b(L)e^{i\beta L}
\end{bmatrix},
$$

(S1)

where we set $A_b(L)$ to zero. The matrix elements of $F$, which can be calculated using coupled-mode theory [1, 2], are

$F_{11} = F_{22} = (\cos qL + \frac{i\delta}{q} \sin qL) e^{i\beta L}$

$F_{21} = F_{12} = -\frac{i\kappa}{q} \sin qL e^{i(\beta L - \pi/2)}$.

(S2)

The reflection and transmission coefficients are then

$$r_g = \frac{F_{21}}{F_{11}},$$

$$t_g = \frac{1}{F_{11}},$$

(S3)

and can be used to calculate the transmission and group delay of the grating using

$$\frac{L}{V_g} = \frac{\partial \text{arg}(r_g)}{\partial \omega}.$$  

(S4)

2. THIN LAYER APPROACH

To simulate a grating with two band gaps, or any other arbitrary $n(z)$, we can use the thin layer method [2, 3]. We model the grating as $N$ thin segments of length $l \ll L$, each with an
approximately constant index of refraction, as shown in Fig. S1. The index of segment \( p \) is \( n_p \equiv n(pl) \). We can relate the forward- and backward-traveling waves across an interface using

\[
\begin{bmatrix}
A^f_{p-1} \\
A^b_{p-1}
\end{bmatrix}
= \frac{1}{2n_p} M_p T_p \begin{bmatrix}
A^f_p \\
A^b_p
\end{bmatrix},
\]

(S5)

where

\[
\frac{1}{2n_p} M_p = \frac{1}{2n_p} \begin{bmatrix}
n_{p-1} + n_p & n_{p-1} - n_p \\
n_{p-1} - n_p & n_{p-1} + n_p
\end{bmatrix}
\]

(S6)

is derived [2] using the Fresnel equations to relate the forward- and backward-traveling waves at each interface, and

\[
T_p = \begin{bmatrix}
e^{-i\frac{2\pi}{\lambda} n_p l} & 0 \\
0 & e^{i\frac{2\pi}{\lambda} n_p l}
\end{bmatrix}
\]

(S7)

implements the phase resulting from propagation by a distance \( l \) through a segment with index of refraction \( n_p \). The full effect of the grating is the product of the matrices representing each layer,

\[
F = \prod_{p=1}^{N} \frac{1}{2n_p} M_p T_p.
\]

(S8)

The transmission and reflection coefficients can then be calculated in the same way as before, using Eq. (S3). The thin layer approach is more computationally expensive than the fundamental matrix approach since it must include many segments within each grating period in order to be accurate, but it allows for any arbitrary periodic index variation \( n(z) \).

REFERENCES

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