Numerical modeling of waves in the port of Manzanillo, Mexico by cnoidal theory

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Abstract

This paper presents the application of one computer program developed to calculate the physical characteristics of the progressive water waves using cnoidal theory of Keulegan and Patterson: profile of water surface, wave length, horizontal and vertical orbital velocities, horizontal and vertical orbital accelerations, period, and under water surface pressure. The Turbo-Pascal programming language was used. CNOIDAL program requires the following data to be introduced: depth, height and phase angle of waves, as the depth ‘z’ of the particle to be analyzed. In the first part of the program, the Newton-Raphson method is used to determine the wave length. In its second part, the nine physical characteristics of waves, providing of equations with elliptic Jacobeans functions, are calculated: profile of water surface, celerity, wave length, horizontal and vertical orbital velocity, horizontal and vertical orbital acceleration, under water surface pressure, and period. A measurement of some characteristics of the waves in the port of Manzanillo was realized by twelve months in the year of 2013 and 2014 using the dates obtained by the Transportation Mexican Institute (TMI) in this port. Results after application this computer program validate the application region proposal by Littman and Struik and the experimental works of Lé Mehauté, Divoky and Lyn in the Tetra Tech wave tank, of 32 meters of length, square across section of 1.219 meters and waves generator with a speed control in a rank from 1 to 12 seconds of period and with waves ranks $0.244 \text{ m/s}^2 \geq \frac{d}{T^2} \geq 0.015 \text{ m/s}^2$. These results were compared with the calculus of the program proposed in this work for the free surface profile, speeds and accelerations of the particles and it was observed a very good accurate by the Keulegan and Patterson cnoidal theory.

Keywords: Cnoidal theory; full elliptical integrals; wave characteristics; Manzanillo
1. Introduction

Since 1877, Boussinesq organized the first previews wave’s studies in shallow waters and with finite fullness. However, Korteweg and Devries, in 1895, were who first developed the cnoidals wave’s theory. The term “cnoidal” is applied due to the free surface of waves is expressed through the elliptic jacobi’s functions.

This theory proposes that the shape of the free surface of waves and the other characteristics are given by the curves of the three elliptic jacobi’s functions. There is no just one cnoidal theory; the bibliography presents several alike which are not identical. Like stokesians theories, since all the cnoidals representations are unfinished series, the order of nearness is important because certain factors equal zero in the low order theory.

There are two kinds of cnoidal theories; the oldest are intuitive in nature while the newest theories are integers and more severe, all being non-rotational. The intuitive elementary theory is by Korteweg and de Vries; the first and second terms of the series are deducted but one scheme for the extension to higher order terms is not presented, Cardoso-Landa (1999).

2. Description of the cnoidal theory selected and the program and equation developed

2.1. Equation developed

The severe cnoidal theories are by Keulegan and Patterson and others; all of them are based upon a disturbance expansion developed by Friedrichs. The work of Keller confirms the results of Korteweg and de Vries, while Laitone and Chappelear studies give a higher order term, Cardoso-Landa (1999).

Unfortunately, even when the rigor predominates, the actual theories are different after the third term. The theories by Keulegan and Patterson are not mathematically consistent, since several terms of second order are unvalued while the third order ones are included, regardless it can be the most physically attractive.

Using the following equations system, proposed by Keulegan and Patterson, Wiegel (1960).

\[ m = k^2 = \frac{H/d}{\left(2L+1 - \frac{Z_t}{d}\right)} \]  \hspace{1cm} (1)

\[ \left(2L+1 - \frac{Z_t}{d}\right)E(k) = \left(2L+2 - \frac{H}{d}\right)K(k) \]  \hspace{1cm} (2)

\[ \frac{Z_t}{d} = \frac{H}{d[kK(k)]^2} \left(K(k)\left[K(k) - E(k)\right]\right) + 1 - \frac{H}{d} \]  \hspace{1cm} (3)

It was obtained the following equation, who related directly the full elliptical integrals of first K (k) and second E (k) class with their modulus k, Cardoso-Landa (1999).

\[ k^2 = m = \frac{E(k)H/d}{\left(\left[H/d\left[\frac{K(k)\left[K(k) - E(k)\right]}{[kK(k)]^2} + 1 - \frac{1}{m}\right]\right] + 2 - \frac{H}{d}\right)K(k)} \]  \hspace{1cm} (4)
2.2. Description of the program developed

The Turbo-Pascal, version 6.0 programming language was used (it was necessary to deduct and use the above equation). This program requires the following data to be introduced: depth, height and phase angle of waves, as the depth “z” of the particle to be analyzed. In the first part of the program, the Newton-Raphson method is used to determine the wave length. In its second part, the nine physical characteristics of waves, providing of equations with elliptic jacobi functions, are calculated: profile of water surface, celerity, wave length, horizontal and vertical orbital velocity, horizontal and vertical orbital acceleration, under water surface pressure and period, Cardoso-Landa (1999). In the next table, shown the nine used equations of the cnoidal theory of Keulegan and Patterson.

Table 1. Cnoidal equations.

| Equation name         | Equation                                                                                                                                 |
|-----------------------|-------------------------------------------------------------------------------------------------------------------------------------------|
| Profile of water surface | $\eta = Zt + Hcn\left[ \frac{2K(k)}{3L} \right] \left[ \frac{x}{L} - \frac{t}{T} \right]$                                  |
| Celerity              | $C = \sqrt{\frac{2d}{1 + \frac{16d^2}{3L} - k^2(k) \left[ \frac{1}{2} E(k) \right]}}$                                                                                      |
| Wave length           | $L = \sqrt{\frac{16d}{3H} - k^2(k)}$                                                                                                      |
| Orbital velocity      | $u = \sqrt{\frac{g}{Ld}} \left[ \frac{4}{4} \frac{2Z}{2d} \right] + \left[ \frac{3H - ZH}{2d} \frac{H}{3} \right] \frac{1}{E(k)} \left[ \frac{1}{2} e^{-\frac{1}{2} e^{-2}} \right]$ |
| Orbital velocity      | $w = \sqrt{\frac{g}{Ld}} \left[ \frac{4HK(k)}{3L} \left[ \frac{1}{2} \frac{d}{H} \frac{Z}{2d} \right] + \frac{16K^2(k)}{L} \left[ \frac{d}{3} - Z \right] k^2 \left[ \frac{1}{2} e^{-\frac{1}{2} e^{-2}} \right] \right]$ |
| Orbital acceleration  | $a_x = \sqrt{\frac{g}{Ld}} \left[ \frac{4HK(k)}{3L} \left[ \frac{1}{2} \frac{d}{H} \frac{Z}{2d} \right] + \frac{16K^2(k)}{L} \left[ \frac{d}{3} - Z \right] k^2 \left[ \frac{1}{2} e^{-\frac{1}{2} e^{-2}} \right] \right]$ |
| Orbital acceleration  | $a_y = \sqrt{\frac{g}{Ld}} \left[ \frac{4HK(k)}{3L} \left[ \frac{1}{2} \frac{d}{H} \frac{Z}{2d} \right] + \frac{16K^2(k)}{L} \left[ \frac{d}{3} - Z \right] k^2 \left[ \frac{1}{2} e^{-\frac{1}{2} e^{-2}} \right] \right]$ |
| Under water surface pressure | $P = \gamma (\eta - Z)$                                                                                                                        |
| Period                | $T = \sqrt{\frac{g}{d}} \left[ \frac{16d}{3H} \left[ \frac{kL}{1 + \frac{1}{2} E(k)} \right] \right]$                                              |

3. Measurements of water wave characteristics in the Port of Manzanillo

Measurement of some characteristics of the waves in the port of Manzanillo were realized by twelve months between August 2013 and August 2014 using the dates obtained by the Transportation Mexican Institute (TMI) in this Mexican port. For example, Fig. 1 shown the measurements by November 21, 2013 to November 23, 2013.
4. Conclusions

Results after application this computer program validate the application region proposal by Littman and Struik and the experimental works of Lé Mehauté, Divoky and Lyn in the Tetra Tech wave tank, of 32 meters of length, square across section of 1.219 meters and waves generator with a speed control in a rank from 1 to 12 seconds of period and with waves ranks $0.244 \, \text{m/s}^2 \geq \frac{d}{T^2} \geq 0.015 \, \text{m/s}^2$. 
These results were compared with the calculus of the program proposed in this work for the free surface profile, speeds and accelerations of the particles and it was observed a very good accurate by the Keulegan and Patterson cnoidal theory, Laitone (1962) and Dean (1970). For example, it shown some calculus using the cnoidal theory by 5 points simultaneously (maximum 20 points).

Table 2. Application of the CNOIDAL program

| Dates | Analyzed points | Calculated points |
|-------|----------------|-------------------|
| T     | 4.3            | 4.3               | 4.3             | 4.3             | 4.3             |
| D     | 11.4           | 11.4              | 11.4            | 11.4            | 11.4            |
| H     | 2.15           | 2.15              | 2.15            | 2.15            | 2.15            |
| θ     | 60             | 60                | 60              | 60              | 60              |
| Z     | -4             | -3.5              | -3              | -2.5            | -2              |
| L     | 6.482          | 6.482             | 6.482           | 6.482           | 6.482           |
| C     | -204.418       | -204.418          | -204.418        | -204.418        | -204.418        |
| η     | 11.404         | 11.404            | 11.404          | 11.404          | 11.404          |
| U     | -16.285        | -17.150           | -17.900         | -18.534         | -19.053         |
| W     | 121.067        | 107.606           | 93.477          | 78.774          | 63.593          |
| A_c   | -30.251        | -34.574           | -38.321         | -41.492         | -44.086         |
| A_a   | -99.633        | -88.554           | -76.925         | -64.824         | -52.331         |
| P     | 15835.489      | 15321.489         | 14807.489       | 14293.489       | 13779.489       |

Importance of the computer program establish oneself not necessary use graphical solutions, and it is possible to compare results between different theoretical water wave models and direct measurements, for example with the port of Manzanillo in the country of México, Cardoso-Landa (2010). Those results indicate the possibility to use the CNOIDAL program to more Mexican ports in the Pacific Ocean, for example, Lázaro Cárdenas port, Salina Cruz port, Zihuatanejo port, Acapulco port, Ensenada port, and Puerto Vallarta port. These works are in development at this moment.
Fig. 4. Comparison of the free surface profile of the waves on the port of Manzanillo, México.

Fig. 5. Comparison of the speed of the particles of the waves on the port of Manzanillo, México.

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