Abstract. We present recent measurements of time-dependent CP asymmetries in two neutral B decay modes: $B^0 \to \eta' K^0$ and $B^0 \to \omega K_S^0$. The measured values of mixing-induced ($\sin 2\phi_1^B$) and direct CP violation parameters are: $S_{\eta'K^0} = +0.68 \pm 0.07_{\text{stat}} \pm 0.03_{\text{syst}}, A_{\eta'K^0} = +0.03 \pm 0.05_{\text{stat}} \pm 0.04_{\text{syst}}$ and $S_{\omega K_S^0} = +0.91 \pm 0.32_{\text{stat}} \pm 0.05_{\text{syst}}, A_{\omega K_S^0} = -0.36 \pm 0.19_{\text{stat}} \pm 0.05_{\text{syst}}$, respectively. These results are fully consistent with the current world average of $\sin 2\phi_1$ measured in $b \to c\bar{c}s$ transitions, and represent the most accurate measurements in these modes up-to-date. In addition, we present a new preliminary update on branching ratio measurement in $B^0 \to \pi^0 \pi^0$ decay mode, the result of which: $\mathcal{B}(B^0 \to \pi^0 \pi^0) = (9.90 \pm 0.12_{\text{stat}} \pm 0.10_{\text{syst}}) \times 10^{-6}$ represents an important ingredient in a $\pi$ $\pi$ SU(2) isospin analysis, used to experimentally extract $\phi_2$ angle from $B \to \pi \pi$ measurements. To obtain $CP$ violation parameters ($\mathcal{B}(B^0 \to \pi^0 \pi^0)$ the final data set of $772 \times 10^6$ ($751.5 \times 10^6$) $BB$ pairs collected at the $\Upsilon(4S)$ resonance with the Belle detector at the KEKB asymmetric $e^+e^-$ collider in Japan ($3.5 \times 8.0$ GeV) is used.

1. Introduction
Within the Standard Model (SM) the $CP$ violation is established via charged weak currents and three generation Cabbibo-Kobayashi-Maskawa quark mixing matrix (CKM) [1], namely its single irreducible complex phase. Requirement on CKM unitarity gives rise to six triangles in the complex plane, one of which naturally describes the $b$-quark transitions and is characterized by three measurable quantities (angles): $\phi_1$, $\phi_2$ and $\phi_3$. The $B$-factories with their experimental apparatuses, Belle [2] and BaBar, were designed to test the CKM mechanism. Particularly, one of the main tasks was an experimental determination of the angles together with the sides of the unitarity triangle. In the past decade, both Belle and BaBar clearly observed the $CP$ violation in all statistically significant $b \to c\bar{c}s$ induced decays [3, 4]. The experimental outcome is a precise determination of $\sin 2\phi_1 = 0.667 \pm 0.023_{\text{stat}} \pm 0.012_{\text{syst}}$ (Belle result) [3] via time-dependent $CP$ asymmetries in $B^0$ decays. Despite the fact that these measurements proved CKM predictions to be extremely successful, the matter-antimatter asymmetry (as incorporated in SM via $CP$ violation) is well below what Universe observations require to solve the baryon asymmetry puzzle [5]. To search for new sources of $CP$ violation, experimental effort has gradually focused on other quark transitions, e.g. $b \to q\bar{q}s$. They dominantly proceed via loop Feynman diagram, and thus can be regarded highly sensitive to a possible new heavy particle(s) contribution.

2. Recent $\phi_1$ related Belle measurements
In $B$ factories the $B$ mesons are produced coherently from a $\Upsilon(4S)$ resonance. The $\Upsilon(4S)$ decays almost exclusively to a $BB$ pair, where one $B^0_{\text{rec}}$ may be reconstructed in a studied decay mode while the other $B^0_{\text{tag}}$ may reveal its flavour. Using a technique called time-dependent $CP$
analysis, one can extract \( CP \) violation parameters by measuring a \( CP \) asymmetry evolving in time. For a \( B \) decaying into a given \( CP \) final state \( f \) the \( CP \) asymmetry follows as:

\[
a_{CP}(\Delta t) = A_t \cos(\Delta m_d \Delta t) + S_t \sin(\Delta m_d \Delta t),
\]

where \( \Delta t \) stands for the proper time interval between \( B^0_{\text{rec}} \) and \( B^0_{\text{tag}} \), \( \Delta m_d \) (\( \Delta \Gamma_d \)) is the difference between the masses (decay rates) of heavy and light \( B^0 \) mass eigenstates, and \( A_t \) and \( S_t \) represent direct and mixing-induced \( CP \) violation parameters, respectively. This relation assumes \( CPT \) invariance, perfect tagging and that \( \Delta \Gamma_d \sim 0 \). For \( b \rightarrow c\bar{s}s \) transitions, the SM predicts that \( S_t = -\xi \sin 2\phi_1 \) (with a very small theoretical uncertainty), where \( \xi = -1(+1) \) for \( CP \)-odd (\( CP \)-even) final state. The \( \phi_1 \) angle is related to the CKM elements as: \( \phi_1 \equiv \arg[\bar{V}_{cd} V_{cb}^* / V_{td} V_{tb}^*] \).

In contrast to \( c\bar{s}s \) transitions, \( B^0 \rightarrow \eta' K^0 \) and \( B^0 \rightarrow \omega K^0_S \) represent physics decays induced by \( b \rightarrow u\bar{s}u \) and \( b \rightarrow d\bar{s}d \) transitions, where a penguin graph plays a dominant role. Nevertheless, a small pollution by CKM and colour-suppressed tree processes occurs and the \( CP \) violation parameters \( S_t \) and \( A_t \) may slightly deviate from the nominal values measured in \( c\bar{s}s \) transitions. We expect \( S_t \simeq -\xi \sin 2\phi_1, A_t \sim 0 \). Therefore, observing a large deviation (further denoted as \( \Delta S_t \)) would be a clear sign for New Physics (NP). For the \( B^0 \rightarrow \eta' K^0 \) the SM predicts \( \Delta S_t \) to be in the range: \([-0.05;+0.09] \) (\( SU(3)_F [6] \)) or \([-0.03;+0.03] \) (QCD factorization scheme [7]). For the \( B^0 \rightarrow \omega K^0_S \) the SM predicts \( \Delta S_t \) to be within \([+0.1;+0.2] \) (pQCD, QCDF, \ldots [8]).

### 2.1. Measurement of \( CP \) violation parameters in \( B^0 \rightarrow \eta' K^0 \)

The overall reconstruction is performed separately in two \( CP \) eigenstates: \( B^0 \rightarrow \eta' K^0_S \) (\( CP = -1 \)) and \( B^0 \rightarrow \eta' K^0_L \) (\( CP = +1 \)) [9]. After a signal/background shape is determined, both channels are combined together and a common \( CP \) fit is performed. In order to increase the signal yield, several \( \eta' \) decay chains are used to reconstruct \( B^0 \) candidates: \( \eta' \rightarrow \eta \pi^+ \pi^- \) (\( \eta \rightarrow \gamma \gamma \) or \( \eta \rightarrow \pi^+ \pi^- \pi^0 \)) and \( \eta' \rightarrow \rho^0 \gamma \) (\( \rho^0 \rightarrow \pi^+ \pi^- \)). The last decay chain is not used in \( B^0 \rightarrow \eta' K^0_L \) reconstruction due to its low signal-to-background ratio.

![Figure 1](image.png)

**Figure 1.** From left to right: Signal-enhanced \( M_{bc} \), \( \Delta E \) and \( R_{S/B} \) distributions of \( B^0 \rightarrow \eta' K^0_S \) candidates (red - total PDF, yellow - \( q\bar{q} \) background, blue - \( B\bar{B} \) background); signal-enhanced \( p_B^* \) and \( R_{S/B} \) distributions of \( B^0 \rightarrow \eta' K^0_L \) candidates (red - total PDF, blue, green and yellow - combinatorial background with fake \( \eta' \), real \( \eta' \) with fake \( K^0_L \), and both real \( \eta' \) with real \( K^0_L \), respectively). All analysed \( \eta' \) decay modes are combined in the plots.

In order to find the signal/background fraction, several event-distributions are simultaneously fitted (see Fig. 1). In \( \eta' K^0_S \) analysis the following variables are used: a beam-constraint mass, \( M_{bc} = \sqrt{(E^{CMS}_{\text{beam}})^2 - (y_B^{CMS})^2} \), difference between the \( B \) meson energy and beam energy, \( \Delta E = E^{CMS}_B - E^{CMS}_{\text{beam}} \), and \( q\bar{q} \) (continuum) suppressing variable \( (q = u, d, s, c) \) - likelihood ratio, \( R_{S/B} \). Similarly, in \( \eta' K^0_L \) decay the yield is extracted by \( R_{S/B} \) and the \( B^0 \) momentum in the CMS, \( p_B^* \), which is calculated using \( \eta' \) momentum and the \( K^0_L \) flight direction, assuming \( \Delta E = 0 \). In total, \( 3541 \pm 90_{\text{stat}} \) signal candidates are reconstructed. Afterwards, the values of \( \sin 2\phi_1^{\text{eff}} \) and...
$A_{\eta'K^0}$ are obtained by performing an unbinned, maximum-likelihood (uML) fit to the $(\Delta t; q)$ distributions of the signal region events (see Fig. 2). In the plots $q = +1$ and $q = -1$ stand for the $B$ flavour on the tag side $B^0$ and $\bar{B}^0$, respectively. The new result, superseding previous Belle measurements [11], is: $S_{\eta'K^0} = +0.68 \pm 0.07_{\text{stat}} \pm 0.03_{\text{syst}}$ and $A_{\eta'K^0} = +0.03 \pm 0.05_{\text{stat}} \pm 0.04_{\text{syst}}$, which is fully consistent with the older Belle measurement [11], BaBar result [12] and the SM predictions, see the HFAG plot in Fig. 2.

### 2.2. Measurement of $CP$ violation parameters in $B^0 \rightarrow \omega K^0_S$

In $B^0 \rightarrow \omega K^0_S$, the $CP$ parameters are directly obtained from a simultaneous seven-dimensional uML fit to signal/background distributions: $M_{bc}$, $\Delta E$, Fisher ($F_{B\bar{B}/q\bar{q}}$), $\omega$ invariant mass ($M_{3\pi}$), $\omega$ helicity ($H_{3\pi}$) and $(\Delta t; q)$. The black curve corresponds to the total PDF, green to signal PDF, dashed red to $q\bar{q}$ background, blue dotted to $B\bar{B}$ background.

In $B^0 \rightarrow \omega K^0_S$, the $CP$ parameters are directly obtained from a simultaneous seven-dimensional uML fit to signal/background distributions: $M_{bc}$, $\Delta E$, invariant mass of $\omega \rightarrow \pi^+\pi^-\pi^0$, $\omega$ helicity angle, $H_{3\pi}$, Fisher discriminant, $F_{B\bar{B}/q\bar{q}}$ (used to suppress continuum), and fit to $(\Delta t; q)$ distribution (Fig. 3). In total, $234 \pm 22_{\text{stat}}$ signal candidates are reconstructed [13].
Figure 4. Signal-enhanced fit projections: $\Delta E$ (left), $M_{bc}$ (middle), $T_{\text{cont}}$ (right). Total PDF is displayed in black, signal in red, $q\bar{q}$ in green, rare $B$ decays in brown and $B \to \rho\pi$ in blue.

The final result can be summarized as: $S_{\omega K^0} = +0.91 \pm 0.32_{\text{stat}} \pm 0.05_{\text{syst}}$ and $A_{\omega K^0} = -0.36 \pm 0.19_{\text{stat}} \pm 0.05_{\text{syst}}$. By performing a likelihood scan, a $S_{\omega K^0}$ deviation of 3.1 $\sigma$ from the CP conservation hypothesis is found. That makes this result the first evidence of CP violation in this mode. In addition, the values are in a good agreement with previous Belle [14] and BaBar [12] results, so no evidence of NP is observed at the current level of exp. sensitivity.

3. New preliminary update on branching fraction measurement in $B^0 \to \pi^0\pi^0$

This measurement is a preliminary update on a previous Belle result: $\mathcal{B} = (2.3^{+0.4+0.2}_{-0.5-0.3}) \times 10^{-6}$ [15], and represents an important ingredient in a $\pi\pi$ SU(2) isospin analysis [16]. This analysis is applied to disentangle contribution of $\Delta\phi_2$ in $S_t \sim \sin(2\phi_2 + 2\Delta\phi_2)$ relation, and thus to experimentally extract $\phi_2$ angle ($\phi_2 \equiv \arg[-V_{td}V_{tb}^*/V_{ud}V_{ub}^*]$) from $B \to \pi\pi$ measurements [17]. The theoretical prediction based on QCD factorization scheme is $\mathcal{B} < 1.0 \times 10^{-6}$ [18].

The analysis strategy consisted in a sequential reconstruction of four gammas, two neutral pions and $B^0$ candidates. The event-distributions used in a final uML fit are: $M_{bc}$, $\Delta E$ and a $q\bar{q}$ suppression variable, $T_{\text{cont}}$. $T_{\text{cont}}$ is the output of a Fisher discriminant which combines event shape variables: $\cos\theta$ of the $\pi^0\pi^0$ system to the $B^0$-flight direction and $\cos\vartheta_{\text{thrust}}$ of the reconstructed $B^0$ compared to the thrust of the rest of the event. In the distributions (see Fig. 4) the continuum component forms the largest background contribution. The Belle new result is: $\mathcal{B}(B^0 \to \pi^0\pi^0) = (0.90 \pm 0.12_{\text{stat}} \pm 0.10_{\text{syst}}) \times 10^{-6}$, measured with $6.7\sigma$ significance.

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