Developing a Simplified Method to Investigate the Dynamic Behavior of Fluid Conveying Pipes under Mean Internal Pressure

Mahdi Bayrami Atashgah,1 Mehdi Iranmanesh,1 and Alireza Mojtahedi2

1Department of Maritime Engineering, Amirkabir University of Technology, Hafez St., Tehran 15914, Iran
2Department of Water Resources Engineering, Faculty of Civil Engineering, University of Tabriz, 29 Bahman Blvd., Tabriz, Iran

Correspondence should be addressed to Mehdi Iranmanesh; iranmaneshmehdi2021@gmail.com

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1. Introduction

Fluid conveying pipes are used most widely in different fields of engineering, such as marine engineering, oil exploration, refining, and chemical transport. Flow-induced vibrations in these pipes require the study of the interaction between the fluid and the pipe [1–3]. The precise prediction of the natural frequencies and critical flow velocities are two important significant factors in studying the vibrations of the fluid conveying pipes. These factors are strongly dependent on the velocity of the internal fluid. In this regard, predicting the critical fluid velocity to determine the onset of instability is a challenge in the fluid conveying pipes. Critical velocity relates to the value at which the imaginary part of the natural frequency becomes zero.

The vibration of fluid conveying pipes is a profound challenge in many areas. The hot topic started when Paidoussis [2] realized the vibration in pipes caused by the fluid flow and introduced two types of instabilities for them. The first type is the divergence instability that occurs at lower velocity, and the second type is the instability of the flutter that occurs at higher velocity. Benjamin [4] discussed the flutter instability of a cantilevered fluid conveying pipe. Gregory and Paidoussis [5] concluded that the flutter instability occurs earlier than divergence instability for a cantilevered fluid conveying pipe. Paidoussis and Li [6] described in detail the instability of cylindrical pipes due to the fluid by the linear vibration equations. Buckling instability and flutter instability were two issues addressed by them. Paidoussis and Issid [7] proposed a variety of methods to study the dynamic behavior of fluid conveying pipes. So far, different methods have been presented to solve the vibration problem of fluid conveying pipes both in linear and nonlinear dynamics, such as the Galerkin method [7–12] and finite element method (FEM) [13–16]. Housner [17] analytically determined the flow-induced vibration of pipes with pinned-pinned boundary conditions. However, the analytical solutions are problematic for other kinds of
boundary conditions. Ibrahim [18, 19] and Li et al. [1] have systematically summarized research on fluid conveying pipes. Also, Dai et al. [20] studied the vibrations of a flexible fluid conveying pipe for transmitting oscillating currents, and the main parameters were analyzed and discussed.

In this regard, some complicated methods have been developed by other researchers. A homotopy perturbation method was proposed to investigate the dynamic behavior of fluid conveying pipes. Xu et al. [21] evaluated a pinned-pinned fluid conveying pipe and validated the results using the finite element method and experimental data. Yundong and Yi-ren [22] proposed He’s variational iteration method for calculating the natural frequencies and critical velocities of fluid conveying pipes under different boundary conditions. The dynamic response of the Timoshenko beam under random stimulation using the superposition method was analyzed by Zhai et al. [23]. Kuiper and Metrikine [24] studied the instability of a free-hanging riser using power series expansion and the D-decomposition method. Lin and Qiao [25] used the differential quadrature method (DQM) under the harmonic excitation to obtain the vibrational response of fluid conveying pipes. Ni et al. [26] used the differential transformation method (DTM) to analyze the vibration of the fluid conveying pipes with several scenarios. A semianalytical method was proposed by Liang et al. [27] to evaluate the dynamic behavior of a fluid conveying pipe under transverse external fluid flow, employing the differential quadrature method and the Laplace transform. The method was validated by an exact solution. Also, a mathematical model for the free vibration of the fluid-conveying cantilevered pipe-in-pipe system considering the thermal effect and two-phase flow was proposed by Guo et al. [28].

Some researchers were focused on sensitivity analysis of fluid conveying pipe. Gregory and Paidoussis [5] examined the effects of the internal damping on stability of fluid conveying pipes. Chen [29] discussed and evaluated the forced vibrations of a fluid conveying pipe. Linear governing equations of the fluid conveying pipe and the critical velocities were discussed by Paidoussis and Issid [7] for different boundary conditions. Long Jr [30] observed the vibration characteristics of fluid conveying pipes experimentally and measured natural frequencies. He realized that as the internal fluid velocity of the fluid conveying pipes increased, the natural frequencies decreased. A mathematical model for the lateral motion of a marine riser was developed to examine the effect of the internal flow and bending rigidity of the pipe on the dynamic behavior of the riser by Wu and Lou [31]. Meng and Chen [32] investigated nonlinear-free vibrations and vortex-induced vibrations of a fluid-conveying steel catenary riser. The dynamic behavior of catenary pipelines subjected to internal slug-flow was investigated by Chatjigeorgiou [33]. He et al. [34] reported a theoretical investigation of an elastic and slender fluid-conveying pipe with a top-end excitation subjected to uniform cross flows. Oke and Khulief [35] discussed how the internal surface damage is reflected in the vibration behavior of a composite pipe conveying fluid. Gu et al. [36] proposed a stochastic dynamic model for the dynamic characteristics analysis of pipe-conveying fluid. Askarian et al. [37] investigated the gravity effects in vertical and horizontal fluid conveying pipes. An analytical solution for nonlinear vibration and postbuckling of functionally graded pipes conveying fluid considering the rotary inertia and shear deformation effects was presented by Khodabakhsh et al. [38]. Shahali et al. [39] investigated the nonlinear dynamic response of a viscoelastic pipe conveying fluid subjected to a uniform external cross flow based on the Euler–Bernoulli theory. The nonplanar vibrations and multimodal responses of pinned-pinned risers in shear cross flow were numerically studied by Jiang et al. [40]. Ma et al. [41] presented an experimental study of large-amplitude intermittent vibrations of an inclined bendable riser transporting an air-water flow with an unsteady slug-flow pattern. Liang et al. [42, 43] developed the dynamical modeling for the spinning pipe conveying fluid and analyzed its vibrations.

The effects of mean internal pressure on the natural frequencies of fluid conveying pipes with different typical boundary conditions have not been studied in detail. The main aim of this study is to introduce a simplified approach to determine the natural frequencies and critical velocities of fluid conveying pipes. For this purpose, the elements of the diagonal matrices of mass, damping, and stiffness are considered to calculate the natural frequencies of pipe before occurring the instability (flow velocity in a range higher than the critical velocity), instead of all ones that gained by the Galerkin discretization. At the first step, a linear equation of motion for an Euler–Bernoulli beam is considered to simulate the vibration of fluid conveying pipes with mean internal pressure. For the second step, the Galerkin method is used to solve the partial differential equation for the condition of $N = 4$ ($N$ is the number of mode shapes). At the next step, the element in the row $n$ and column $n$ of the mass, damping, and stiffness matrices is used to calculate the $n$th natural frequency by which the matrix form is converted to an algebraic form. Then, the results are validated by experimental data and four numerical methods besides two analytical solutions. Finally, the effects of mean internal pressure variations on the natural frequencies are discussed, and critical values related to internal pressures are investigated. In each vibration mode of the pipe and fluid system, there is a critical value of internal pressure for which the natural frequency of the system becomes zero. This value is named Mean Critical Pressure. This study is the first stage of our work that aims to justify the applicability of an efficient and simplified method to determine and evaluate the natural frequency of fluid conveying pipes. The advantage of the presented method is the simplicity of equations, and the disadvantage of the method is that the real part of the natural frequency is not calculated.

2. Methodology

In this section, the dynamic equation of the fluid conveying pipe is considered, and the Galerkin discretization method is formulated using first four modes to calculate the natural frequencies. The details of the proposed simplified method are also explained.
2.1. Dynamic Equation of the Fluid Conveying Pipes. For a pipe, considered as an Euler–Bernoulli beam, the linear equation of motion for the vibration of the fluid conveying pipe with an external excitation is given as follows [44]:

\[
EI \frac{\partial^4 w(x,t)}{\partial x^4} + (M_f V^2 + \bar{p} A (1 - 2\mu)) \frac{\partial^2 w(x,t)}{\partial x^2} + 2M_f V \frac{\partial^2 w(x,t)}{\partial x \partial t} + (M_f + M_p) \frac{\partial^2 w(x,t)}{\partial t^2} = f, \tag{1}
\]

where \( EI \) is the flexural rigidity, \( A \) stands for the flow area, \( M_f \) and \( M_p \) are the mass-per-unit-length of fluid and pipe, \( V \) is the fluid flow velocity, \( \bar{p} \) is the mean internal pressure, \( \mu \) is the Poisson ratio which only considered in the clamped-clamped pipe, \( w(x,t) \) is the transverse deflection of the pipe, \( f \) is external excitation load, \( x \) is the horizontal coordinate along the centerline of the pipe, and \( t \) stands for the time. In the case of \( f = 0 \), the fluid conveying pipe vibrates due to the motion of the internal fluid, and the dynamic equation is

\[
EI \frac{\partial^4 w}{\partial x^4} + (M_f V^2 + \bar{p} A (1 - 2\mu)) \frac{\partial^2 w}{\partial x^2} + 2M_f V \frac{\partial^2 w}{\partial x \partial t} + (M_f + M_p) \frac{\partial^2 w}{\partial t^2} = 0. \tag{2}
\]

The related nondimensional parameters are defined as follows:

\[
\begin{align*}
\eta &= \frac{w}{L}, \\
\tau &= \frac{t}{\sqrt{M_f + M_p} L^2}, \\
u &= \frac{M_f}{EI} V, \\
\xi &= \frac{x}{L}, \\
P &= \frac{\bar{p} A}{EI}, \\
\beta &= \frac{M_f}{M_f + M_p}, \\
\omega &= \sqrt{\frac{M_f + M_p}{EI} \omega_0 L^2},
\end{align*}
\]

where \( w \) is the dimensionless natural frequency and \( \omega_0 \) is the natural frequency (rad/s). Substituting equation (3) into (2), the dimensionless form of equation (2) can be expressed as follows:

\[
\frac{\partial^4 \eta}{\partial \xi^4} + (u^2 + P (1 - 2\mu)) \frac{\partial^2 \eta}{\partial \xi^2} + 2\sqrt{\beta} u \frac{\partial^2 \eta}{\partial \xi \partial \tau} + \frac{\partial^2 \eta}{\partial t^2} = 0. \tag{4}
\]

The framework of this study is presented in Figure 1. The framework is performed in two main directions. In the first direction, by using the Galerkin method and considering 4 to 8 mode shapes of the pipes, the dynamic matrices of the system are extracted, and the natural frequencies in different boundary conditions are calculated. In the second direction, instead of using four mode shapes of the pipes, only one mode shape of the pipes is used, and the relationships related to natural frequencies and critical velocities are explicitly obtained. The main idea of this research is to use one mode shape of the pipe and convert the matrix form of dynamic equations to algebraic form. Consequently, only the diagonal components of dynamic matrices are used to calculate the natural frequencies.

2.2. Galerkin Discretization. The main purpose of this study is to provide explicit formulas to determine the natural frequencies of the fluid conveying pipe under different boundary conditions presented in Figure 2. The Galerkin method is used to obtain the explicit formulas and reduce complexities compared to the other methods such as the finite element method and finite difference method.

Equation (4) can be discretized using the Galerkin method to solve the related partial differential equation. The dimensionless lateral deflection is represented as follows:

\[
\eta(\xi, \tau) = \sum_{i=1}^{N} \varphi_i(\xi) q_i(\tau) + \epsilon, \tag{5}
\]

where \( q_i(\xi) \) is the shape function that satisfied all considered boundary conditions for the pipe (see Table 1), and \( q_i(\tau) \) is the generalized coordinate of the discretized pipe structure, and \( \epsilon \) is the order of the Galerkin truncation error (\( N \) is the number of mode shapes).

The convergence can be obtained when the condition of \( N \geq 4 \) is satisfied [9]. In this study, it is considered \( N = 4 \). Therefore, we have

\[
\eta(\xi, \tau) \equiv \sum_{i=1}^{4} \varphi_i(\xi) q_i(\tau), \tag{6}
\]

By considering the \( \phi = \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \varphi_4 \end{bmatrix} \) and \( Q = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} \) and introducing the parameters presented in (7), (8), and (5) is changed into the matrix form as shown in (9),

\[
\eta(\xi, \tau) = \phi^T Q = \phi^T \left[ \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \varphi_4 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} \right] = \phi^T \left[ \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} \right], \tag{7}
\]

where

\[
(\beta \omega_0 L^4) = \begin{bmatrix} M_f & M_f & M_f & M_f \\ M_f & M_f & M_f & M_f \\ M_f & M_f & M_f & M_f \\ M_f & M_f & M_f & M_f \end{bmatrix}, \tag{8}
\]

\[
(\beta \omega_0 L^4) = \begin{bmatrix} M_f & M_f & M_f & M_f \\ M_f & M_f & M_f & M_f \\ M_f & M_f & M_f & M_f \\ M_f & M_f & M_f & M_f \end{bmatrix}, \tag{8}
\]

\[
(\beta \omega_0 L^4) = \begin{bmatrix} M_f & M_f & M_f & M_f \\ M_f & M_f & M_f & M_f \\ M_f & M_f & M_f & M_f \\ M_f & M_f & M_f & M_f \end{bmatrix}, \tag{8}
\]

\[
(\beta \omega_0 L^4) = \begin{bmatrix} M_f & M_f & M_f & M_f \\ M_f & M_f & M_f & M_f \\ M_f & M_f & M_f & M_f \\ M_f & M_f & M_f & M_f \end{bmatrix}, \tag{8}
\]

\[
(\beta \omega_0 L^4) = \begin{bmatrix} M_f & M_f & M_f & M_f \\ M_f & M_f & M_f & M_f \\ M_f & M_f & M_f & M_f \\ M_f & M_f & M_f & M_f \end{bmatrix}, \tag{8}
\]

\[
(\beta \omega_0 L^4) = \begin{bmatrix} M_f & M_f & M_f & M_f \\ M_f & M_f & M_f & M_f \\ M_f & M_f & M_f & M_f \\ M_f & M_f & M_f & M_f \end{bmatrix}, \tag{8}
\]
Figure 1: The framework of this study.

Figure 2: Fluid conveying pipes with different boundary conditions.
2.3. Simplified Method Based on Galerkin Discretization (SMGD). In this method, the $n^{th}$ mode shape of the Galerkin discretization is used to determine the $n^{th}$ natural frequency of the fluid conveying pipe. The components of the diagonal matrices $M$, $C$, and $K$, which are calculated based on the Galerkin method, are used to calculate the natural frequencies, orders of the matrices of mass, damping, and stiffness being equal to 1. Hence, the matrix form is converted to an algebraic form. The characteristic equation for the $n^{th}$ dimensionless natural frequency can be expressed as follows:

$$K_n - M_n \omega_n^2 + i C_n \omega_n = 0,$$  \hspace{1cm} (13)

where $M_n$, $C_n$, and $K_n$ are the $n^{th}$ components of the mass, damping, and stiffness matrices, respectively, as follows:

$$M_n = M_{cof}(n),$$  \hspace{1cm} (14)

$$C_n = C_{cof}(n),$$  \hspace{1cm} (15)

$$K_n = K_{cof}(n) + \Pi K_{cof}(n),$$  \hspace{1cm} (16)

where $M_{cof}(n)$, $C_{cof}(n)$, and $K_{cof}(n)$ are the diagonal matrices of $M_{cof}$, $C_{cof}$, and $K_{cof}$, respectively.

By substituting (8) into equation (13) and using the components of the diagonal matrices, we get the dimensionless form of $n^{th}$ natural frequency, as a function of $u$, $P$, and $\beta$, and the dimensionless critical velocities for different types of boundary conditions (pinned-pinned, clamped-pinned, cantilevered and clamped-clamped), can be obtained as shown in Table 2 and 3. We explicitly derive the dynamic matrices of the system, including the mass matrix, the damping matrix, and the stiffness matrix. Considering only one mode shape to determine the natural frequency is the main idea of the research. Next, by considering only one mode shape, the characteristic equation is obtained only to determine the natural frequency. The relevant relationships were listed in Table 2. According to the relationships of the mode shapes described in Table 1, the integral multiplication of the mode shape with its derivative, which is described in equation (12), becomes zero for pinned-pinned, clamped-pinned and clamped-clamped conditions. Therefore, the damping term does not appear in the dynamic equations. Furthermore, the relationships presented in Tables 2 and 3 show that increasing the mean internal pressure reduces the natural frequencies and critical flow velocities.

The simplified method, as shown in Figure 1, is fundamentally different from the Galerkin method. In the
Galerein method, the dynamic components of the system are matrices. But in the simplified method, the dynamic components of the system are algebraic.

3. Validations

3.1. Numerical and Analytical Validation. In this section, the equations presented in Tables 2 and 3 are validated using several numerical and analytical methods and experimental data. The structure of the pipe can be considered as a straight beam when the fluid velocity is equal to zero, and internal pressure does not exist. Hence, the natural frequencies can be obtained analytically. At the first step, the case of \( \mu = 0 \) and \( P = 0 \) is considered to compare the results of the explicit formulas with those of the DTM, DQM, and the exact solutions. Data in Table 4 show that the results of the explicit formulas are in good agreement with the exact solutions for the different boundary conditions. More importantly, the results indicate the higher accuracy of explicit formulas in comparison with the DTM and DQM. To investigate the precision of the presented method, the natural frequencies of fluid conveying pipes with four considered boundary conditions are calculated using the equations defined in Table 2. The observed results imply that the parameter of the fluid velocity plays a vital role in the natural frequencies. The results of the explicit formulas are compared for the case of a cantilevered pipe, \( \beta = 0.1 \) and the case of clamped-clamped, \( \mu = 0.3 \) with those predicted by DTM, DQM, and exact solutions. A good agreement in the results is observed, and the relative errors are less than 0.01 percent for all boundary conditions. Moreover, the explicit formulas would be beneficially easier and computational cost lower than the other methods.

Another important issue for the problems of the fluid conveying pipe is the determination of dimensionless critical velocities obtained by the equations presented in Table 3. As reported by Paidoussis [2], the fluid conveying pipe system is stable for dimensionless velocities in the range less than dimensionless critical velocities and loses stability at dimensionless velocities equal to dimensionless critical velocities. For a pipe with positively supported ends, the system will lose stability at dimensionless critical velocities through divergence. However, the flutter is the expected form of instability for a cantilevered system because the cantilever system is nonconservative. In this study, the dimensionless critical velocities of fluid conveying pipe are obtained using the explicit formulas (SMGD) for different boundary conditions. They are listed in Table 5 to compare with the results of the DTM, DQM, and given by Paidoussis [2]. As can be seen, the relative errors are less than 2.08 percent for all boundary conditions. Also, Table 5 shows that the internal pressure leads to a reduction in the dimensionless critical velocities, except for the first mode of cantilevered cases.

In Table 6, the specifications of a pinned-pinned fluid conveying pipe are given to compare the results further. To verify the accuracy of the results, they are compared with those given by Xu et al. [21] and Housner [17].

A homotopy perturbation method (HPM) was used to determine the dimensionless natural frequency of a pinned-pinned fluid conveying pipe [21]. Dimensionless natural frequencies of a simply supported pipe are obtained as follows:

\[
\left( \frac{\omega_n}{\omega_N} \right)^2 = n^2 \left( \frac{V_c}{V} \right)^2 + 16n^5 \beta \left( \frac{V}{V_c} \right)^2 \sum_{k=1}^{\infty} \frac{k^2((-1)^n-k-1)^2}{(n^2+k^2)(n^2-k^2)}, \quad n = 1, 2, \ldots
\]

(17)

where \( \omega_N \) is the fundamental natural frequency of the pipe in the absence of the flow and can be expressed as follows:

\[
\omega_N = \frac{\pi^2}{L^2} \sqrt{\frac{EI}{M_f}}.
\]

(18)

and \( V_c \) is the critical velocity of flow for static buckling of the pipe:

\[
V_c = \frac{\pi}{L} \sqrt{\frac{EI}{M_f}}.
\]

(19)

Housner [17] suggested that the first two dimensionless natural frequencies of a pinned-pinned fluid conveying pipe can be computed by

\[
\left( \frac{\omega_n}{\omega_N} \right)^2 = \alpha \pm \sqrt{\alpha^2 - 4 \left[ 1 - \left( \frac{V}{V_c} \right)^2 \right] \left[ 4 - \left( \frac{V}{V_c} \right)^2 \right]}, \quad n = 1, 2,
\]

(20)

where

\[
\alpha = 8.5 - \left( \frac{V}{V_c} \right)^2 \left[ 2.5 + 128\beta \right].
\]

(21)

The SMGD shows that the dimensionless natural frequency of a simply supported pipe for the case of \( P = 0 \) is as follows:
Table 3: Formulas of SMGD for the first four dimensionless critical velocities of the fluid conveying pipes with internal pressure for the different boundary conditions.

| Boundary conditions          | Expression                                                                 |
|-----------------------------|-----------------------------------------------------------------------------|
| Pinned-pinned               | $u_1 = \sqrt{8.8696 - P}$                                                   |
|                             | $u_2 = \sqrt{39.4784 - P}$                                                  |
|                             | $u_3 = \sqrt{88.8264 - P}$                                                  |
|                             | $u_4 = \sqrt{157.9137 - P}$                                                 |
| Clamped-pinned              | $u_1 = \sqrt{20.649 - P}$                                                   |
|                             | $u_2 = \sqrt{58.1984 - P}$                                                  |
|                             | $u_3 = \sqrt{115.571 - P}$                                                  |
|                             | $u_4 = \sqrt{192.701 - P}$                                                  |
| Cantilever                  | $u_1 = \sqrt{\left(\sqrt{4}\beta - 4 \left(0.8583 \left(12.3622 + 0.8583P \right) / 2 \left(0.8583 \right)\right)\right)^2}$ |
|                             | $u_2 = \sqrt{\left(\sqrt{4}\beta - 4 \left(-13.2943 \left(485.5182 - 13.2943P \right) / 2 \left(-13.2943 \right)\right)\right)^2}$ |
|                             | $u_3 = \sqrt{\left(\sqrt{4}\beta - 4 \left(-45.9043 \left(3806.6 - 45.9043P \right) / 2 \left(-45.9043 \right)\right)\right)^2}$ |
|                             | $u_4 = \sqrt{\left(\sqrt{4}\beta - 4 \left(-98.9169 \left(14617 - 98.9169P \right) / 2 \left(-98.9169 \right)\right)\right)^2}$ |
| Clamped-clamped             | $u_1 = \sqrt{20.649 - P \left(1 - 2\mu\right)}$                           |
|                             | $u_2 = \sqrt{58.1984 - P \left(1 - 2\mu\right)}$                          |
|                             | $u_3 = \sqrt{115.571 - P \left(1 - 2\mu\right)}$                          |
|                             | $u_4 = \sqrt{192.701 - P \left(1 - 2\mu\right)}$                          |

Table 4: The first four dimensionless natural frequencies of the fluid conveying pipes for the value of $u = 0$ and $P = 0$ for the different boundary conditions.

| Boundary conditions          | Result          | $\omega_1$ | $\omega_2$ | $\omega_3$ | $\omega_4$ |
|-----------------------------|-----------------|------------|------------|------------|------------|
| SMGD                        | 9.8700          | 39.4800    | 88.8300    | 157.9000   |
| DTM [26]                    | 9.8669          | 39.4784    | 88.8264    | 157.9137   |
| Relative error (%)          | 0.0314          | 0.0041     | 0.0041     | 0.0087     |
| DQM [26]                    | 9.8716          | 39.4863    | 88.8442    | 157.9454   |
| Relative error (%)          | -0.0162         | -0.0160    | -0.0160    | -0.0287    |
| Exact [46]                  | 9.8696          | 39.4784    | 88.8264    | 157.9137   |
| Relative error (%)          | 0.0041          | 0.0041     | 0.0041     | 0.0087     |
| Clamped-pinned              | 15.4200         | 49.9600    | 104.2000   | 178.3000   |
| DTM [26]                    | 15.4182         | 49.9649    | 104.2477   | 178.2697   |
| Relative error (%)          | 0.0117          | -0.0098    | -0.0458    | 0.0170     |
| DQM [26]                    | 15.4228         | 49.9799    | 104.2790   | 178.3234   |
| Relative error (%)          | -0.0182         | -0.0398    | -0.0758    | -0.0131    |
| Exact [46]                  | 15.4182         | 49.9649    | 104.2477   | 178.2697   |
| Relative error (%)          | 0.0117          | -0.0098    | -0.0458    | 0.0170     |
| Cantilever                  | 3.5160          | 22.0300    | 61.7000    | 120.9000   |
| DTM [26]                    | 3.5160          | 22.0345    | 61.6972    | 120.9019   |
| Relative error (%)          | 0.0000          | -0.0204    | 0.0045     | -0.0016    |
| DQM [26]                    | 3.5167          | 22.0389    | 61.7096    | 120.9260   |
| Relative error (%)          | -0.0199         | -0.0404    | -0.0156    | -0.0215    |
| Exact [46]                  | 3.5160          | 22.0345    | 61.6972    | 120.9019   |
| Relative error (%)          | 0.0000          | -0.0204    | 0.0045     | -0.0016    |
| Clamped-clamped             | 22.3730         | 61.6700    | 120.9000   | 199.9000   |
| DTM [26]                    | 22.3733         | 61.6728    | 120.9034   | 199.8594   |
| Relative error (%)          | -0.0147         | -0.0045    | -0.0028    | 0.0203     |
| DQM [26]                    | 22.3778         | 61.6852    | 120.9276   | 199.8995   |
| Relative error (%)          | -0.0349         | -0.0246    | -0.0228    | 0.0003     |
| Exact [46]                  | 22.3733         | 61.6728    | 120.9034   | 199.8594   |
| Relative error (%)          | -0.0147         | -0.0045    | -0.0028    | 0.0203     |

$\omega_n = \sqrt{(nn)^4 - (nn)^2 (u^2)}$, $n = 1, 2, \ldots$ (22)

Equation (22) is simpler than (17) and (20). The first four dimensionless natural frequencies of a pinned-pinned fluid conveying pipe are calculated by equations (18), (14), and (17) and listed in Table 7. The results of the explicit formulas (SMGD) are in good agreement with the results of the FEM, HPM, and Housner’s, and the relative errors are less than 1.83 percent for all cases.
To compare the results with available experimental data, a pinned-pinned fluid conveying pipe is considered based on the specifications in Table 8 [48]. The changes of natural frequencies with increasing dimensionless flow velocity are shown in Figure 3. The results of the present study are compared to those of the experimental data [48], FEM [21], HPM (17), and Housner (20). Figure 3 confirms that the results of the presented study are significantly consistent with the experimental data and the results of the finite element method.

### 4. Results and Discussions

For the sake of the argument, the material properties of the studied cases are chosen based on the literature as follows: $E$ (the elastic modulus of the pipes), $\mu$ (the Poisson ratio), $\rho_p$ (the density of the pipe), $D$ (the outer diameter of the pipes), $d$ (the internal diameter of the pipes), $L$ (length of the pipes), $\rho_f$ (the density of the fluid), and $\overline{P}$ (the mean internal pressure) are considered as $68 \text{ GPa}$, $0.3$, $2700 \text{ kg.m}^{-3}$, $46 \text{ mm}$, $40 \text{ mm}$, $2 \text{ m}$, $870 \text{ kg.m}^{-3}$, and $2 \text{ MPa}$, respectively. At the first step, three cases including $u = 0$, 1, and 2, and $P = 1.57$ are considered to compare the results with the Galerkin method ($N = 4$, for all cases $\beta = 0.5$). Table 9 shows that the results of the explicit formulas (SMGD) are in good agreement with the Galerkin method for the different boundary conditions. Furthermore, the acceptable accuracy of the results is obtained by the SMGD. For all cases, relative errors are less than 2.60 percent (except for the first frequency of the cantilever boundary condition, with $u = 2$). At high velocities, the system practically fails, and there is no pipe. Since the behavior of the system tends to be nonlinear at higher critical velocities and there are complex issues related to instability, the accuracy of relationships in these areas is reduced.

### Table 5: The first four dimensionless critical velocities of the fluid conveying pipes for the different values of $P$ in the different boundary conditions.

| Boundary conditions | Pressure | Result | $u_{c1}$ | $u_{c2}$ | $u_{c3}$ | $u_{c4}$ |
|---------------------|----------|--------|----------|----------|----------|----------|
| Pinned-pinned       | 0        | SMGD   | 3.1416   | 6.2832   | 9.4248   | 12.5664  |
|                     |          | DTM [26] | 3.1416   | 6.2832   | —        | —        |
|                     |          | Relative error (%) | -0.0002 | -0.0002 | —        | —        |
|                     | 1.57     | SMGD   | 3.0400   | 6.2330   | 9.3914   | 12.5414  |
|                     |          | DTM [26] | 3.1416   | 6.2832   | —        | —        |
|                     |          | Relative error (%) | -0.0129 | -0.0098 | —        | —        |
|                     |          | Paidoussis [2] | 3.1400   | 6.2800   | —        | —        |
|                     |          | Relative error (%) | 0.0507  | 0.0507   | —        | —        |
| Clamped-pinned      | 0        | SMGD   | 4.5441   | 7.6288   | 10.7502  | 13.8818  |
|                     |          | DTM [26] | 4.4934   | —        | —        | —        |
|                     |          | Relative error (%) | 1.1288  | —        | —        | —        |
|                     | 1.57     | SMGD   | 4.4745   | 7.5875   | 10.7210  | 13.8591  |
|                     |          | DTM [26] | 4.4937   | —        | —        | —        |
|                     |          | Relative error (%) | 1.1220  | —        | —        | —        |
|                     |          | Paidoussis [2] | 4.4900   | —        | —        | —        |
|                     |          | Relative error (%) | 1.2054  | —        | —        | —        |
| Cantilevered        | 0        | SMGD   | 3.7951   | 6.0432   | 9.1062   | 12.1561  |
|                     |          | DTM [26] | —        | —        | 9.3224   | —        |
|                     |          | Relative error (%) | —        | —        | -2.3187  | —        |
|                     | 1.57     | SMGD   | 3.9966   | 5.9119   | 9.0196   | 12.0913  |
|                     |          | DTM [26] | —        | —        | 9.3233   | —        |
|                     |          | Relative error (%) | —        | —        | -2.3281  | —        |
|                     |          | Paidoussis [2] | —        | —        | 9.3000   | —        |
|                     |          | Relative error (%) | —        | —        | -2.0834  | —        |
| Clamped-clamped     | 0        | SMGD   | 6.3787   | 9.0882   | 12.1571  | 15.2576  |
|                     |          | DTM [26] | 6.2832   | —        | —        | —        |
|                     |          | Relative error (%) | 1.5196  | —        | —        | —        |
|                     | 1.57     | SMGD   | 6.3293   | 9.0536   | 12.1312  | 15.2370  |
|                     |          | DTM [26] | 6.2838   | —        | —        | —        |
|                     |          | Relative error (%) | 1.5099  | —        | —        | —        |
|                     |          | Paidoussis [2] | 6.2800   | —        | —        | —        |
|                     |          | Relative error (%) | 1.5714  | —        | —        | —        |
|                     |          | DTM [26] | —        | —        | 9.3233   | —        |
|                     |          | Relative error (%) | —        | —        | -2.3281  | —        |
|                     |          | Paidoussis [2] | —        | —        | 9.3000   | —        |
|                     |          | Relative error (%) | —        | —        | -2.0834  | —        |

### Table 6: Specifications considered for a pinned-pinned fluid conveying pipe [47].

| Elasticity modulus | Outer diameter | Wall thickness | Pipe length | Density of the pipe | Density of the internal flow |
|--------------------|----------------|----------------|-------------|---------------------|----------------------------|
| 210 GPa            | 324 mm         | 16 mm          | 32 m        | 8200 kg.m\(^{-3}\) | 908.2 kg.m\(^{-3}\)       |
without internal pressure.

Figure 4 depicts how the first four fluid conveying pipe.

4.1. μT_he Effects of Mean Internal Pressure on Pinned-Pinned

Table 7: The first four dimensionless natural frequencies (rad/s) of a pinned-pinned fluid conveying pipe for the different values of μ and without internal pressure.

| Flow velocity (m/s) | 0   | 15  | 25  | 35  | 45  | 55  |
|--------------------|-----|-----|-----|-----|-----|-----|
| Dimensionless velocity | 0   | 0.6015 | 1.0025 | 1.4035 | 1.8045 | 2.2055 |
| 1st mode | SMGD | 4.3733 | 4.2925 | 4.1449 | 3.9126 | 3.5799 | 3.1144 |
| FEM [21] | 4.3732 | 4.2921 | 4.1441 | 3.9116 | 3.5781 | 3.1115 |
| Relative error (%) | 0.0017 | 0.0094 | 0.0186 | 0.0254 | 0.0604 | 0.0934 |
| HPM [21] | 4.3732 | 4.2870 | 4.1293 | 3.8800 | 3.5222 | 3.0145 |
| Relative error (%) | 0.0017 | 0.1284 | 0.3771 | 0.8400 | 1.6377 | 3.3142 |
| Housner [17] | 4.3732 | 4.2971 | 4.1576 | 3.9372 | 3.6183 | 3.1660 |
| Relative error (%) | 0.0017 | -0.1070 | -0.3062 | -0.6250 | -1.0618 | -1.6296 |
| 2nd mode | SMGD | 17.4931 | 17.4129 | 17.2692 | 17.0511 | 16.7561 | 16.3800 |
| FEM [21] | 17.4928 | 17.4123 | 17.2682 | 17.0499 | 16.7544 | 16.3775 |
| Relative error (%) | 0.0017 | 0.0034 | 0.0059 | 0.0070 | 0.0104 | 0.0153 |
| HPM [21] | 17.4928 | 17.4171 | 17.2682 | 17.0765 | 16.7991 | 16.4457 |
| Relative error (%) | 0.0017 | -0.0242 | 0.0062 | -0.1488 | -0.2557 | -0.3995 |
| Housner [17] | 17.4928 | 17.3922 | 17.2122 | 16.9330 | 16.5686 | 16.0585 |
| Relative error (%) | 0.0017 | 0.1189 | 0.3313 | 0.6974 | 1.1320 | 1.8508 |
| 3rd mode | SMGD | 39.3595 | 39.2794 | 39.1364 | 38.9206 | 38.6313 | 38.2666 |
| FEM [21] | 39.3587 | 39.2783 | 39.1350 | 38.9190 | 38.6292 | 38.2638 |
| Relative error (%) | 0.0019 | 0.0027 | 0.0036 | 0.0041 | 0.0054 | 0.0073 |
| HPM [21] | 39.3587 | 39.2858 | 39.1559 | 38.9602 | 38.6978 | 38.3672 |
| Relative error (%) | 0.0019 | -0.0164 | -0.0498 | -0.1016 | -0.1718 | -0.2622 |
| 4th mode | SMGD | 69.9724 | 69.8923 | 69.7496 | 69.5346 | 69.2472 | 68.8863 |

Table 8: Specifications considered for a pinned-pinned fluid conveying pipe [48].

| Elasticity modulus | Outer diameter | Wall thickness | Pipe length | Mass-per-unit-length of fluid | Mass-per-unit-length of fluid and pipe |
|--------------------|----------------|----------------|-------------|-------------------------------|----------------------------------------|
| 68.948 Gpa | 2.54 cm | 0.165 cm | 3.2 m | 0.383 kg.m⁻¹ | 0.7134 kg.m⁻¹ |

4.1. The Effects of Mean Internal Pressure on Pinned-Pinned Fluid Conveying Pipe. Figure 4 depicts how the first four dimensionless natural frequencies vary with dimensionless internal pressure for the three dimensionless flow velocity values. The results are obtained using the equations listed in Table 2. It can be seen that the mean internal pressure has more effect on the first frequency for the pinned-pinned boundary condition. Significantly, the value of the critical mean internal pressure (the case of \( u_c = 0 \)) is equal to the dimensionless natural frequency of the pipe with \( u = 0 \) and \( P = 0 \).

It reveals that the fluid velocity affects the internal critical pressure besides the natural frequencies. The fluid velocity has more effect on the mean internal pressure for the first and second frequencies. Furthermore, the values of the mean internal pressure (which leads to \( u_c = 0 \)) for the first four mode shapes of the pinned-pinned fluid conveying pipe are equal to 9.8696, 39.4784, 88.8264, and 157.9137, respectively. The higher the average internal pressure of the pipe, the lower the natural frequency of the system. At high velocities, the safety expected for the pipe decreases, and the critical velocity, which is the limit of instability, decreases.

Figures 5–7 (and equations in Table 3) illustrate that natural frequencies and critical flow velocity gradually decrease as the mean internal pressure is increased (the inverse relationship, except at the first mode of the cantilever boundary condition).

4.2. The Effects of the Flow Velocity on the Boundary Conditions. According to equations of clamped-pinned condition, presented in Table 3, it shows that the values of the critical mean internal pressure (which leads to \( u_c = 0 \)) for the first four mode shapes are equal to 15.42, 49.96, 104.2, and 178.3, respectively. The related values of the critical mean internal pressure are equal to dimensionless natural frequencies of the clamped-pinned pipe with \( u = 0 \) and \( P = 0 \). It can be comprehended that the mean internal pressure has more effect on the first mode natural frequency due to higher values of the critical mean internal pressures for the second, third, and fourth modes.

Figure 5 shows the first four dimensionless natural frequencies of a clamped-pinned pipe conveying fluid for the different values of \( u \). The velocity values according to the Im (\( \omega \)) are equal to critical velocity values. Also, the critical velocity decreases as the mean internal pressure increases.

The values of the critical mean internal pressure for the first four mode shapes of the cantilevered fluid conveying pipe (\( \beta = 0.1 \)) are equal to 3.51, 22.03, 61.7, and 120.9, respectively, which are equal to dimensionless natural frequencies for the condition of \( u = 0 \) and \( P = 0 \). In Figure 6, it can be observed that the mean internal pressure has less effect on the natural frequencies for the third and fourth frequencies and leads to a slight increase in the first frequency. Also, the values of the mean internal pressure for the first four mode shapes of the clamped-clamped fluid
Figure 3: The first third natural frequencies of a pinned-pinned fluid conveying pipe for the different values of $(V/V_c)$ and without internal pressure based on experimental data; (a) the first frequency, (b) the second frequency, and (c) the third frequency.

Table 9: The first four dimensionless natural frequencies of the pipes conveying fluid for the different values of $u$ and with internal pressure $p = 1.57$ in the different boundary conditions.

| Boundary conditions   | Dimensionless velocity | Result | $\omega_1$ | $\omega_2$ | $\omega_3$ | $\omega_4$ |
|-----------------------|------------------------|--------|------------|------------|------------|------------|
| Clamped-pinned        | 0                      | SMGD   | 14.8200    | 49.2900    | 103.5000   | 177.5000   |
|                       |                        | $N = 4$| 14.8200    | 49.2900    | 103.5000   | 177.5000   |
|                       |                        | Relative error (%) | 0.0000 | 0.0000 | -0.0386 | -0.0282 |
|                       |                        | SMGD   | 14.4300    | 48.8500    | 103.1000   | 177.1000   |
|                       |                        | $N = 4$| 14.3600    | 48.8900    | 103.1500   | 177.1600   |
|                       |                        | Relative error (%) | 0.4875 | -0.0818 | -0.0485 | -0.0339 |
|                       |                        | SMGD   | 13.1800    | 47.5000    | 101.7000   | 175.7000   |
|                       |                        | $N = 4$| 12.9400    | 47.6900    | 101.9800   | 176.0000   |
|                       |                        | Relative error (%) | 1.8547 | -0.3984 | -0.2746 | -0.1705 |
Table 9: Continued.

| Boundary conditions          | Dimensionless velocity | Result          | $\omega_1$ | $\omega_2$ | $\omega_3$ | $\omega_4$ |
|-----------------------------|------------------------|-----------------|------------|------------|------------|------------|
| Cantilevered                |                        | SMGD            | 3.703      | 21.56      | 61.11      | 120.3      |
|                             |                        | $N = 4$         | 3.716      | 21.5527    | 61.1105    | 120.2569   |
|                             |                        | Relative error (%) | -0.3498   | 0.0339     | -0.0008    | 0.0358     |
|                             |                        | SMGD            | 3.42       | 21.18      | 60.71      | 119.8      |
|                             |                        | $N = 4$         | 3.9575     | 21.3007    | 60.786     | 120.3519   |
|                             |                        | Relative error (%) | 0.7156    | -0.5666    | -0.1250    | -0.4586    |
|                             |                        | SMGD            | 2.41       | 20         | 59.49      | 118.6      |
|                             |                        | $N = 4$         | 2.28       | 20.5352    | 59.8251    | 120.6216   |
|                             |                        | Relative error (%) | 5.7018    | -2.6063    | -0.5610    | -1.6760    |
| Clamped-clamped ($\mu = 0.3$) |                        | SMGD            | 22.2000    | 61.4400    | 120.6000   | 199.6000   |
|                             |                        | $N = 4$         | 22.2300    | 61.4700    | 120.6900   | 199.6300   |
|                             |                        | Relative error (%) | -0.1350   | -0.0488    | -0.0746    | -0.0150    |
|                             |                        | SMGD            | 21.9200    | 61.0600    | 120.2000   | 199.2000   |
|                             |                        | $N = 4$         | 21.8700    | 61.1300    | 120.3300   | 199.2700   |
|                             |                        | Relative error (%) | 0.2286    | -0.1145    | -0.1080    | -0.0351    |
|                             |                        | SMGD            | 21.0600    | 59.9200    | 119.0000   | 197.9000   |
|                             |                        | $N = 4$         | 20.7900    | 60.1000    | 119.2700   | 198.2000   |
|                             |                        | Relative error (%) | 1.2987    | -0.2995    | -0.2264    | -0.1514    |

Figure 4: The first four dimensionless natural frequencies of a pinned-pinned pipe conveying fluid for the different values of internal pressure; (a) the first frequency, (b) the second frequency, (c) the third frequency, and (d) the fourth frequency.
Figure 5: The first four dimensionless natural frequencies of a clamped-pinned pipe conveying fluid for the different values of $u$.

Figure 6: The first four dimensionless natural frequencies of a cantilevered pipe conveying fluid for the different values of $u$ and $\beta = 0.1$.

Figure 7: The first four dimensionless natural frequencies of a clamped-clamped pipe conveying fluid for the different values of $u$ and $\mu = 0.3$. 
The first four dimensionless natural frequencies of a pinned-pinned pipe conveying fluid for the different values of $u$. The values of the critical mean internal pressure ($\mu = 0.3$) are equal to 22.37, 61.67, 120.9, and 199.9, respectively. They are equal to dimensionless natural frequencies for the condition of $u = 0$ and $P = 0$. Like the cantilevered condition, the mean internal pressure has less effect on the natural frequencies for the third and fourth frequencies.

The values of the critical mean internal pressure (which leads to $u_c = 0$) for the first four mode shapes of the pinned-pinned fluid conveying pipe are equal to 9.8696, 39.4784, 88.8284, and 157.9137, respectively. It can be concluded that mean internal pressure has more effect on the first mode natural frequency due to higher values of the critical mean internal pressures for the second, third, and fourth modes. The velocity values based on the $\text{Im}(\omega) = 0$ are equal to critical velocity values, as shown in Figure 8.

5. Conclusion

This paper presents a simplified approach to determine the dynamic behavior of fluid conveying pipes with different typical boundary conditions under the mean internal pressure. For this purpose, the method is suggested by reducing the order of problem, and the $n^{th}$ mode shape of the Galerkin discretization is used to determine the $n^{th}$ dimensionless natural frequency of the considered pipes. The presented method is validated by experimental data, FEM, HPM, DTM, DQM, and the exact solution methods. The following conclusions can be drawn:

The results showed that the explicit formulas (SMGD) for the different boundary conditions are in good agreement with the exact solutions and have higher accuracy in comparison with the DTM and DQM. More importantly, the SMGD is more efficient and easier than the other methods. The results implied that the parameter of the fluid velocity plays a vital role in natural frequencies.

The results are compared for the case of a cantilevered pipe, $\beta = 0.1$ and the case of clamped-clamped, $\mu = 0.3$ with those predicted by DTM, DQM, and exact solutions. A good agreement is observed. The relative errors are less than 0.01 percent for all boundary conditions.

The changes of natural frequencies with increasing dimensionless flow velocity are compared for the case of a pinned-pinned pipe. It was implied that the results of the presented method are significantly consistent with the experimental data and the results of the finite element method. For the cases of the pinned-pinned pipe and the clamped-pinned pipe, the mean internal pressure has more effect on the first frequency.

Values of the critical mean internal pressure for the first four mode shapes of the cantilevered fluid conveying pipe are equal to dimensionless natural frequencies for the condition of $u = 0$ and $P = 0$. Values of the mean internal pressure for the first four mode shapes of the clamped-clamped fluid conveying pipe are equal to 22.3700, 61.67, 120.9, and 199.9, respectively. Furthermore, it was observed that the critical mean internal pressure values of pipes are equal to dimensionless natural frequencies for the condition of $u = 0$ and $P = 0$ for all considered cases.

Summarily, the simplified method analytically predicts the natural frequencies and critical velocities of the fluid-conveying pipes in different boundary conditions. It is less complex with a lower computational cost and higher accuracy than other methods.

Nomenclature

- $f$: External excitation load
- $EI$: Flexural rigidity
- $A$: Stands for flow area
- $M_f$: Mass-per-unit-length of fluid
- $M_p$: Mass-per-unit-length of pipe
- $V$: Fluid flow velocity
- $\bar{P}$: Mean internal pressure
- $\mu$: Poisson ratio
- $w(x, t)$: Transverse deflection of the pipe
- $x$: Horizontal coordinate along the centerline of the pipe
- $t$: Stands for the time
- $\omega$: Dimensionless natural frequency
- $\omega_0$: Natural frequency (rad/s).
- $\varphi_i(\xi)$: Shape functions
- $N$: Number of mode shapes in the Galerkin method
- $M$: Matrices of mass
- $C$: Matrices of damping
- $K$: Matrices of stiffness
- $Q$: Generalized acceleration
- $\dot{Q}$: Generalized velocity
- $Q$: Generalized displacement.
Appendix

\[ M = \int_{0}^{1} \phi \phi^T d\xi \]

\[ M = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix} \]

\[ C = \Lambda, \int_{0}^{1} \phi \phi^T d\xi \]

\[ C = \Lambda, C_{cof}, \]

\[ K^{(1)} = \int_{0}^{1} \phi \phi^{(1)T} d\xi \]

\[ K^{(1)} = \begin{bmatrix}
\int_{0}^{1} \phi_{1} \phi_{1}^{(1)T} d\xi \\
\int_{0}^{1} \phi_{2} \phi_{2}^{(1)T} d\xi \\
\int_{0}^{1} \phi_{3} \phi_{3}^{(1)T} d\xi \\
\int_{0}^{1} \phi_{4} \phi_{4}^{(1)T} d\xi \\
\end{bmatrix} \]

\[ K^{(1)} = K_{cof}, \]

\[ K^{(2)} = \Pi \int_{0}^{1} \phi \phi^{(2)T} d\xi \]

\[ K^{(2)} = \Pi K_{cof}, \]

\[ K = K^{(1)} + K^{(2)}. \]
Data Availability

The calculated data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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