THE USE OF QUANTUM DOTS AS HIGHLY SENSITIVE SWITCHES BASED ON SINGLET-TRIPLET TRANSITION AND SYMMETRY CONSTRAINT

Y.M. Liu, G.M. Huang, and C.G. Bao*

The State Key Laboratory of Optoelectronic Materials and Technologies, and Department of Physics, Zhongshan University, Guangzhou, 510275, P.R. China

Based on symmetry constraint that leads to the appearance of nodes in the wave functions of 3-electron systems at regular triangle configurations, it was found that, if the parameters of confinement are skillfully given and if a magnetic field is tuned around the first critical point of the singlet-triplet transition, a 2-electron quantum dot can be used as a highly sensitive switch for single-electron transport.

PACS numbers: 73.61.-r

* The corresponding author.

Modern experimental techniques, e.g., using electrostatic gates and etching, allow a certain number of electrons (from a few to a few thousand) to be confined in quantum dots. As a kind of systems different from those existing in nature, rich physics is contained. Therefore, they attract certainly the interest of academic research. On the other hand, the properties of the dots can be controlled, e.g., by changing the gate voltage or by applying an external magnetic field, etc. Therefore, these systems have a great potential in application. In particular, the dots can be used as a single-electron switch to control the electric microcurrent. Making use of the Coulomb blockade, current switches have already been designed by a number of authors (refer to ref.8 and 9, and references therein). This kind of efforts is crucial for developing microtechniques. In general, there are two factors harmful to the sensitivity of this kind of device, one is thermal fluctuation, another is quantum tunneling. The former can be effectively reduced by lowering the temperature, the latter can not. In order to eliminate the quantum tunneling, we propose an idea of a new mechanism in this paper. Related theoretical evaluation has been performed, however, technical details are not involved.

Let us review briefly the main physics of Coulomb blockade. Let a dot be connected to a source and a drain by tunnel barriers. A gate-voltage \( V_g \) is applied on the dot, while bias-voltages \( V_s \) and \( V_d \) are applied on the source and drain, respectively. Let \( N \) electrons be confined in the dot with the ground state energy \( \epsilon_N \). If an extra electron comes in from the source, the lowest energy of the \((N+1)\)-electron system is denoted as \( E_{N+1} \). The extra electron is driven by the bias-voltage \( V_{SD} = V_s - V_d \). Thus the condition of Coulomb blockade is \( \Delta_0 = -\epsilon \beta V_{SD} < E_{N+1} - \epsilon_N \equiv \Delta \) (where the constant \( \beta = 0.81 \) is introduced by Foxman et al.\(^{10} \), however the actual value of \( \beta \) is not essential to the following discussion). Whereas if \( \Delta_0 \geq \Delta \), the blockade is released and the extra electron would go through the dot to the drain. By increasing \( V_g \), \( \Delta \) can be reduced as shown schematically in Fig.1a. When \( V_g \) increases and arrives at a critical value, the condition \( \Delta_0 = \Delta \) holds, and resonant transmission occurs. It is noted that \( \Delta \) is changed continuously with \( V_g \). Therefore, even when \( \Delta > \Delta_0 \), the leak of current might occur due to quantum tunnelling. Thus, the blockade is not strict, and the sensitivity of the device as a switch is more or less spoiled.

![FIG. 1: A sketch to show the continuous variation of \( \Delta \) in the usual device of Coulomb blockade (a), and in an improved device (b). When \( \Delta = \Delta_0 \), resonant transmission occurs. X in (b) is a quantity to control the variation of \( \Delta \) (not yet specified) .](image)

On the other hand, if one can design a device so that \( \Delta \) varies abruptly as shown in Fig.1b, the quantum tunnelling would be remarkably suppressed and the sensitivity would be greatly improved. To achieve the abrupt change, we suggest a source-dot-drain device with the following three features.

1) The dot is axial symmetric and contains two electrons. The eigenstates states have the orbital angular momentum \( L \) and total spin \( S \) to be conserved. In particular, the ground state has \( L_2 \), \( S_2 \), and energy \( \epsilon_{L_2S_2} \).

2) \( V_g \) is fixed at a skillfully prescribed value (see below), while an adjustable external magnetic field \( B_g \) is applied and tuned around a critical point \( B_C \) so that the ground state undergoes singlet-triplet transitions\(^{11-17} \). Accordingly, \( (L_2, S_2) \) jumps to \( (L_2', S_2') \), or reversely, while the corresponding ground state energies \( \epsilon_{L_2S_2} \) and
\[ H = \sum_{j=1}^{N} \left[ -\frac{\hbar^2}{2m} \nabla_j^2 + U(r_j) + \frac{1}{2} m^* (\frac{\omega_0}{\Omega})^2 r_j^2 \right] - g \mu_B \frac{\hat{S}}{2} + \frac{e^2}{4\pi \varepsilon_0} \sum_{j<k} N_{jk} \]  

where \( \omega_c = eB/(m^*c) \), \( N \) is the number of electrons (equal to 2 or 3), the Zeeman term is included. \( m^* = 0.067m_e \), the dielectric constant \( \varepsilon = 12.4 \) (for a GaAs dot), and the units meV, nm, and Tesla are used throughout this paper. The Hamiltonian is diagonalized to obtain the eigenenergies by using the methods as outlined in [18, 19].

\[ H = \sum_{j=1}^{N} \left[ -\frac{\hbar^2}{2m} \nabla_j^2 + U(r_j) + \frac{1}{2} m^* (\frac{\omega_0}{\Omega})^2 r_j^2 \right] - g \mu_B \frac{\hat{S}}{2} + \frac{e^2}{4\pi \varepsilon_0} \sum_{j<k} N_{jk} \]  

The ground state energies \( \epsilon_{L,S} \) calculated with \( U_o = 9 \) and \( r_o = 60 \) (just as an example) are plotted in Fig.2 as a function of \( B_o \). The qualitative feature of Fig.2 would remain unchanged if \( U_o \) and \( r_o \) vary inside a large reasonable region. One can see that the increase of \( B_o \) causes a number of singlet-triplet transitions and totally three critical points appear. It was found that the vicinity of the first critical point, denoted as \( B_C \) (equal to 0.463 in Fig.2) is suitable for our purpose. We shall concentrate in the vicinity of \( B_C \). In region I (\( B_o < B_C \)) and II, (\( L_2, S_2 = 0, 0 \) and \( 1, 1 \)), respectively. It was found that, when \( r_o \) is given, the value of \( B_C \) depends on \( U_o \) nearly linearly. E.g., when \( r_o = 35 \), \( B_C \approx 1.185 + 0.0255U_o \); when \( r_o = 60 \), \( B_C \approx 0.325 + 0.0153U_o \). The larger the \( U_o \), the larger the \( B_C \), while the larger the \( r_o \), the smaller the \( B_C \).

When \( B_o \) is given in region I, if the third electron with S-wave can come in, \( L_3, S_3 \) should be \( 0, 1/2 \). Selected values of \( \Delta_1 = E_{0,1/2} - \epsilon_{0,0} \) are calculated and plotted as functions of \( B_o \) as shown in Fig.3. On the other hand, from dynamical consideration, if the three electrons form a regular triangle (RT), the total potential energy can be minimized. Thus the RT is a favorable configuration. However, from the study of symmetry constraint \( 20^{22} \), it was known that the spatial wave function of a 3-electron state \( \Psi_{LS}(123) \) would be zero at the RTs if \( L = 3j \) and \( S = 1/2 \), or if \( L \neq 3j \) and \( S = 3/2 \), where \( j = 0, 1, 2, \ldots \). This is called a prohibition of regular triangle (PRT), and will definitely cause serious effect.

\[ H = \sum_{j=1}^{N} \left[ -\frac{\hbar^2}{2m} \nabla_j^2 + U(r_j) + \frac{1}{2} m^* (\frac{\omega_0}{\Omega})^2 r_j^2 \right] - g \mu_B \frac{\hat{S}}{2} + \frac{e^2}{4\pi \varepsilon_0} \sum_{j<k} N_{jk} \]  

For simplicity, the model dot is assumed to be 2-dimensional. The potential of confinement is an axial-symmetric square well \( U(r) = -U_o \) if \( r = \sqrt{x^2 + y^2} \leq r_o \), or \( U(r) = 0 \) elsewhere, where \( U_o \) is positive. Furthermore, a uniform magnetic field \( B_o \) is applied perpendicular to the X-Y plane. The Hamiltonian reads
Let us prove the PRT when \( S = 3/2 \). In this case \( \Psi_{L,S}(123) \) is totally antisymmetric. When the three electrons form an RT, a rotation of the system by \( \frac{2\pi}{3} \) is equivalent to a cyclic permutation of particles. The rotation leads to an extra factor \( e^{-i2\pi L/3} \), while the permutation is equivalent to two interchanges and thus causes nothing. Thus, the equivalence leads to
\[
(e^{-i2\pi L/3} - 1)\Psi_{L,S}(123) = 0
\]
(2)

This equation holds only at the RT configurations. Evidently, when \( L \neq 3j \), the first factor of (2) is nonzero, therefore \( \Psi_{L,S} \) must be zero at the RT. It implies the appearance of an inherent node at the RT, and the PRT holds. The case \( S = 1/2 \) can be similarly proved\(^{20}\).

![Figure 4: The evolution of \( \Delta_I \) and \( \Delta_{II} \) with respect to \( B_o \) in the vicinity of \( B_C \). \( r_o \) is fixed at 60 nm, and \( U_o \) is given at a number of values (in meV).](image)

When PRT holds, the inherent node at the RT will remarkably increase the kinetic energy, and at the same time push the wave function away from the area of lower potential energy. Therefore, the associated energy \( E_{L_3,S_3} \) is considerably higher than those free from the PRT. For this reason, \( E_{0,1/2} \) is in general high. By adjusting \( U_o \) or \( r_o \), \( E_{0,1/2} \) can be easily much higher than \( \epsilon_{0,0} \) resulting in a large \( \Delta_I \) as confirmed by Fig.3.

In region II, the S-wave limitation leads to two choices \((L_3,S_3) = (1,1/2) \) and \((1,3/2)\). Due to the PRT, \( E_{1,3/2} \) is considerably higher and therefore can be neglected. Whereas, due to being free from the PRT, \( E_{1,1/2} \) is remarkably lower, this leads to a small \( \Delta_{II} = E_{1,1/2} - \epsilon_{1,1} \) as plotted in Fig.3. Thus the PRT assures \( \Delta_I \geq \Delta_{II} \), which leads to the great jump shown in Fig.3.

The second condition \( \Delta_{II} = \Delta_o \) can also be satisfied by adjusting the parameters of confinement. In what follows \( \Delta_o = 0 \) is assumed due to the very low bias-voltage. When \( r_o = 60 \), we know from Fig.3 that \( \Delta_{II} = \Delta_o = 0 \) holds if \( U_o = 5.97 \). In general, for a given \( r_o \), there is a corresponding \( U_o = (U_o)_a \) so that the pair of parameters \((U_o)_a \) and \( r_o \) gives \( \Delta_{II} = 0 \) at \( B_o = B_C \). Based on our numerical results \((30 \leq r_o \leq 70)\), a nearly linear relation \((U_o)_a \approx 20.253 - 0.238 \) was found. The larger the \( r_o \), the smaller the \((U_o)_a \).

For a further discussion, a typical low-lying spectrum of the 2- and 3-electron dots in the vicinity of \( B_C \) is given in Fig.4 (where \( U_o \) is a little larger than \((U_o)_a\)). For these systems the levels not appearing in the figure are much higher. If \( U_o \) is given considerably larger than \((U_o)_a \), \( E_{1,1/2} \) would become considerably lower and lead to \( \Delta_{II} = 0 \). In this case the intruding electron may find some way to release its energy (e.g., by emitting a photon or a phonon) and falls into the level \( \Delta_{II} = 0 \). This occurs, the intruding electron can not go out again but remain inside, thereby the previous mechanism of transport would be spoiled. Therefore \( U_o \) is better given at \((U_o)_a \), this leads to \( \Delta_{II} = \Delta_o \). In this case a resonant transmission takes place and the intruding electron has no chance to remain inside.

![Figure 5: The evolution of \((U_o)_a \) versus \( r_o \). The parameters \((U_o)_a \) and the associated \( r_o \) together assure the condition of resonant transmission, i.e., \( \Delta_{II} = \Delta_o \) (here \( \Delta_o = 0 \) is assumed).](image)

If the confinement potential contains nonsymmetric component \( U_{non}(r)_L \), \( L \) is no more strictly conserved. In region I (in the status "off") if the 2-body ground state contains a small \( L_2 = 1 \) component, this component can absorb the incoming S-wave electron and goes to the \( E_{1,1/2} \) level, thereby a leak may occur. Alternatively, if the 2-body ground state contains a small \( L_2 = 2 \) component, this component can not absorb the extra S-wave electron because the level \( E_{2,1/2} \) is
quite high (cf. Fig.4). This fact implies that only the kind of $U_{\text{non}}(\mathbf{r})$ with odd parity would cause a leak, this kind should be diminished as far as possible (e.g., in the technical aspect, the two pipes connected to the dot are placed by the two sides as symmetric as possible). A representative $U_{\text{non}}$ with odd parity can be written as $U_{\text{non}}(r, \theta) = U(r)(\lambda_\alpha \cos \theta + \lambda_\beta \sin \theta)$, where $U(r)$ is the original one in eq.(1). By using the perturbation theory, for two representative cases, the ratio of the weights of the $L = 1$ and $L = 0$ components in the 2-body ground state is $\tau = 1.15(\lambda_\alpha^2 + \lambda_\beta^2)$ if $r_o = 60$, $U_o = (U_o)_a = 5.971$ and $B_o = B_C = 0.416$; and $\tau = 0.131(\lambda_\alpha^2 + \lambda_\beta^2)$ if $r_o = 35$, $U_o = (U_o)_a = 11.922$, and $B_o = B_C = 1.525$. From this evaluation, if $\lambda_\alpha$ and $\lambda_\beta$ are in the order of $10^{-1}$, $U_{\text{non}}$ would not cause a serious problem. Besides, a smaller $r_o$ would lead to a larger level-spacing $\epsilon_{1,0} - \epsilon_{0,0}$, and therefore a smaller $\tau$.

The Kondo effect would do no harm in this device. This effect ceases to exist in the status "off" because the spin of the dot is meanwhile zero, while this effect is promotive in "on". However, the thermal fluctuation might spoil the sensitivity of the device. In the status "off", if the 2-electron state has been raised up from the ground state to the level $\epsilon_{1,1}$ by thermal fluctuation, the third electron may come in and the system may fall to the level $E_{1,1/2}$, and thus the "off" is no more strict. Let us define $\gamma = \exp[-(\epsilon_{1,1} - \epsilon_{0,0})/kBT]$, which is the ratio of the probabilities of staying in the levels $\epsilon_{1,1}$ and $\epsilon_{0,0}$, respectively, and is plotted in Fig.5 as a function of $B_o$, where $T$ is given at a number of values. From this figure, we know that, if $T \leq 0.01K$, when $|B_o - B_C| > 0.01$, the thermal fluctuation is negligible. Therefore, in this temperature, a change of $B_o$ in the order of 0.01 around $B_C$ is sufficient to initiate the "on" or "off" of the switch. When $T$ is much smaller than 0.01K, a very small change of $B_o$ is sufficient to initiate the "on" or "off", thus the switch becomes highly sensitive.

The S-wave limitation is a basic requirement. To evaluate in a semi-classical way, let us assume that a classical electron comes in with a velocity $\nu$ along a straight pipe with a radius $d_v$ aimed at the center of the dot. Then the angular momentum is $\hbar/2$, or $d_v \nu < 0.0028$ G, where $d_v$ is in nm, and $C$ is the velocity of light in vacuum. On the other hand, the thermal velocity $\nu_{\text{ther}} = \sqrt{2kBT/m^*} = 0.0000710 \nu TC$, where $T$ is the temperature in K. Thus, the requirement can be rewritten as $d_v < 40.56(\nu_{\text{ther}}/\nu)/\sqrt{C}$. If $T = 0.01$, even $\nu$ as large as $10\nu_{\text{ther}}$, the requirement, namely $d_v < 40.56$, can be realized with the present technics of fabrication.

As final remarks, two distinguished features of the suggested device as a sensitive switch are noticeable. Firstly, the quantum tunneling has been greatly suppressed. Secondly, the device is highly sensitive to the variation of $B_o$ at low temperature, this is the superiority. However, since very low temperature together with the S-wave limitation and good axial symmetry of the dot are required, the effectiveness of the switch depends on the technical aspect, and remains to be checked. Nonetheless, it is believed that the idea proposed in this paper, i.e., making use of the effect of symmetry constraint and phase transition, is in general useful in the design of various microdevice.

Acknowledgment

This paper is supported by the NSFC of China under the grant No.90103028, 90306016, and No.10174098.

1. L. Jacak , P. Hawrylak, A. Wójs, Quantum Dots (Springer, Berlin, 1998)
2. T. Chakraborty, Quantum Dots (Elsevier, Amsterdam, 1999)
3. M.S. Kushwaha, Surface Science Reports, 41, 1 (2001)
4. S.M Reimann and M. Manninen, Rev. Mod. Phys. 74, 1283 (2002)
5. G.W. Bryant, Phys. Rev. Lett. 59, 1140 (1987)
6. P.A. Maksym, H. Imamura, G.P. Mallon, and H. Aoki, J. Phys.: Condens. Matter 12, R299 (2000)
7. L.P. Kouwenhoven and P.L. McEuen, Nanotechnology, edited by G. Timp (Springer, New York, 1999), 471
8. F. Nakajima, Y. Ogasawara, J. Motohisa, T. Fukui, Physica, E 13, 703 (2002)
9. I.H. Chan, P. Fallahi, R.M. Westervelt, K.D. Maranowski, A.C. Gossard, Physica E 17, 584 (2003)
10. E.B. Foxman, et al., Phys. Rev. B 47, 10020 (1993)
11. M. Wagner, U. Merkt, and A.V. Chaplik, Phys. Rev. B 45, 151 (1992)
12. D. Piaffkuche, V. Gudmundsson, and P.A. Maksym, Phys. Rev. B 47, 2244 (1993)
13. P.Hawrylak, Phys. Rev. Lett. 71, 3347 (1993)
14. M.Fusilnik, I.I. Glazman, and W. Hofstetter, Plts, Phys. Rev. B 68, 161303R (2003)
15. A. Fuhrer, T.Ibn, K. Euslin, W.Wegscheider, M. Bichler, Phys. Rev. Lett. 91, 206802 (2003)
16. A. Kogan, G. Granger, M.A. Kastner, D. Goldhaber-Gordon, and H. Shtrikman, Phys. Rev. B 67, 113309 (2003)
17. J. Kyriakidis, M. Pioro-Ladriere, M. Ciorga, A.S. Sachrajda, and P. Hawrylak, Phys. Rev. B 66, 035320 (2002)
18. C.G. Bao, J. Phys.: Conden. Matter. 14, 8549 (2002)
19. G.M. Huang, Y.M. Liu, C.G. Bao, Phys. Rev. B 68, 165334 (2003)
20. C.G. Bao, W.F. Xie, and C.D. Lin, J. Phys. B: At. Mol. Opt. 27, L193 (1994)
21. W.Y. Ruan, Y.Y. Liu, C.G. Bao and Z.Q. Zhang, Phys. Rev. B51, 7942 (1995).
22. C.G. Bao, Phys. Rev. Lett. 79, 3475 (1997).