Changes in atomic populations due to tokamak edge plasma fluctuations

R Hammami¹, H Capes¹, F Catoire², L Godbert-Mouret¹, M Koubiti¹, Y Marandet¹, A Mekkaoui³, J Rosato¹ and R Stamm¹

¹ PIIM, Aix-Marseille Université and CNRS, centre Saint Jérôme, F-13397 Marseille Cedex 20, France.
² CELIA, Université de Bordeaux – CNRS - CEA, Talence, 33405, France
³ IEK-4 Plasma Physics, Forschungszentrum Jülich GmbH, Trilateral Euregio Cluster, Euratom Association, D-52425 Jülich, Germany

E-mail: ramzi.hammami@univ-amu.fr

Abstract. The population balance of atoms and ions in a tokamak edge plasma is calculated in the presence of density and temperature fluctuations. We apply a stochastic model to simplified atomic systems and examine the sensitivity of the energy level populations to the statistical properties of the fluctuations.

1. Introduction

The energy level populations of atoms and ions are important quantities for characterizing the edge plasmas of magnetic fusion devices such as tokamaks. Radiation losses, ionization equilibrium, or the intensity of spectral lines used for diagnosing the plasma are directly related to the atomic and ionic populations. Such quantities are used to understand how the plasma interacts with the plasma facing materials. Their determination relies on the solution of an atomic population kinetic model, which is usually obtained using a stationary collisional-radiative model (CRM), assuming constant plasma parameters such as the electronic density and the temperature of each species [1]. In this work we examine the case where these parameters fluctuate as a consequence of edge plasma turbulence. In edge plasmas, density and temperature fluctuations of order unity on time scales of the order of 10 μs have been measured, and characterized with statistical quantities such as the probability density function (PDF) and the time correlation function [2]. We consider a previously developed model, which assumes that the fluctuating quantity obeys a stochastic process [5]. The density or temperature are distributed according to a prescribed PDF, and jump from one constant value to another one according to a given waiting time distribution (WTD) [5]. Assuming such a process for the fluctuations, we can solve the time dependent CRM by an analytical model or a Monte Carlo type simulation. In section 2, we briefly describe how atomic populations may be calculated in the presence of the renewal stochastic process for the fluctuations reported in [7]. Our model is applied to two ideal atomic systems in section 3, a reduced hydrogen atom, and the carbon ionization balance.
2. Modeling the plasma parameters with a renewal process

The atomic or ionic populations are described by a vector $X(t) = (x_1(t), \ldots, x_N(t))$ containing the different population $x_j$ of each level $j$ with an initial value $X_0 = X(t = 0)$. The kinetics of the atomic populations is governed by a differential equation of the form [5]:

$$\frac{dX(t)}{dt} = M(Y(t))X(t),$$

where $M$ is a time dependent matrix filled with the collisional-radiative rates, and $Y(t)$ is the fluctuating plasma parameter (electronic temperature or density in the following). We now assume that $Y$ follows a renewal process with stepwise constant values separated by instantaneous jumps. The duration on each step obeys a prescribed waiting time distribution (WTD), and the values taken by $Y$ are distributed along a chosen probability density function (PDF) $P(Y)$. Equation (1) is then a linear stochastic equation, which may be solved by a numerical simulation or an analytic approach. The analytic solution takes a particularly simple form if an exponential WTD $\phi(t) = \nu \exp(-\nu t)$ is chosen where $\nu$ the typical fluctuation frequency of $Y$. It may be expressed as the Laplace transform of the Green function $G$ associated to equation 1:

$$\langle G(s) \rangle = \left[I - \nu G_{ST}(s)\right]^{-1} G_{ST}(s),$$

where the brackets $\langle \cdots \rangle$ imply that an ensemble average has been taken, and the subscript ST means that a static average over the PDF $P(Y)$ has been taken. It is possible to obtain the average $\langle X(t) \rangle$ of the population vector in the limit of long times by using:

$$\lim_{s \to 0^+} s\langle G(s) \rangle X_0 = \lim_{s \to 0^+} s\langle G(s) \rangle X_0.$$

The solution of equation (1) may also be obtained by a numerical solution coupled to a simulation of the chosen stochastic process. The fluctuating plasma variable is again assumed to follow a stepwise constant stochastic process with prescribed PDF and WTD, but we now generate variable histories on the computer with pseudorandom number algorithms, associated to numerical techniques such as transformation or rejection methods [8]. In the following, the calculations are performed assuming an exponential WTD with an average time duration of steps equal to the inverse of the typical frequency $\nu$ of the fluctuations. This entails that we have chosen a process, which loses all memory of its previous states at every time. Other stochastic processes may be considered by our approach if we expect memory effects to be significant [11].

Starting with an initial population $X_0$ at $t = 0$, we first apply during a time interval $\Delta t_1$ a constant value $Y_1$ of the fluctuating parameter. The population $X_1$ at time $\Delta t_1$ may be written as:

$$X_1 = e^{-M(Y_1)\Delta t_1} X_0$$

For a given time history of the variable $Y$, we may then iterate this expression until we reach a time for which the value of $X$ is constant and a stationary result has been obtained. We have averaged the values of $X(t)$ over a set of such time histories for a fixed value of $\nu$. The number of histories required for obtaining a stable result depends on the number of time jumps required before we reach
the stationary result. According to the values of the fluctuation and atomic frequencies, about 10 to 10^4 histories may be required for obtaining results being reproducible to within 1 percent. Our simulation technique, although much less computer efficient than our analytic model, is flexible since it allows to rapidly switch between different PDF and WTD already available in numerical libraries. The two approaches have been used to crosscheck several of our results.

3. Results and discussion

3.1 The gamma PDF
The three level system 1s, 2s, 2p of hydrogen has been recently analysed for the case of temperature fluctuations. Changes in the population of the 2s level depend on the typical frequency and the rate $r = \frac{\Delta T}{\langle T \rangle}$ of the electron temperature fluctuations, where $\Delta T = \langle T^2 \rangle - \langle T \rangle^2$, and the average electron temperature is given by $\langle T \rangle = \int_0^\infty T P(T) dT$ with $P(T)$ the PDF of the fluctuating temperature. Assuming a gamma function for the PDF $P(T)$ is written:

$$P(T) = \frac{\alpha^\beta}{\Gamma(\beta)} T^{\beta-1} \exp(-\alpha T). \quad (5)$$

In reference [5], it has been shown that significant changes in the 2s population are observed for a fluctuating plasma temperature. Such changes depend on the typical frequency $\nu$ of the fluctuations, and reach two limits as $\nu$ is small (static case) or large (diabatic case) compared to the typical atomic frequencies given by the collisional and radiative processes. If the analysis of a specific plasma device is performed, the average frequency $\nu$ (the inverse of the turbulence correlation time) is in principle known and determined from the measure of the correlation function of the fluctuating variable.

3.2 Three level hydrogen atom
In the following we first use the same statistical properties of the fluctuating temperature, and study a three level hydrogen atom with the principal quantum levels $n=2$, 3 and 4. The collisional excitation rate employs the simple formula for allowed transitions found in the work of Van Regemorter [11]. That is, for the collisional frequency $C_{ij}$ from a state $i$ to a state $j$:

$$C_{ij} = N_e \times 10^{-11} \frac{1}{E_{ij}} \sqrt{T} e^{-\frac{E_{ij}}{T}} f_{ij}, \quad (6)$$

where $E_{ij}$ is the energy separation of the states $i$ and $j$, $f_{ij}$ is the absorption oscillator strength, and $T$ the electron temperature in eV. A principle of detailed balance is used for obtaining de-excitation frequencies.

Our first result concerns temperature fluctuations obeying a gamma distribution with an average temperature of 2 eV, and $r=0.9$. The average populations (Eq. (4)) of the levels $n=3, 4$ plotted as a function of the fluctuation frequency $\nu$ are shown in figure 2 for a density of $N_e=10^{19}$ m^{-3}.  

3
Figure 2. Populations of the n=3 and n=4 levels of a three level hydrogen atom, as a function of the fluctuation frequency of the temperature, and for a fixed electron density of $N_e=10^{19}$ m$^{-3}$.

This couple of density and temperature sets us in a regime for which both radiative and collisional processes contribute to the population kinetics. For frequencies $\nu$ lower than about $10^6$ s$^{-1}$, we are in the static regime, for which $\nu<<\nu_{at}$. One enters the opposite diabatic limit where $\nu>>\nu_{at}$ for frequencies $\nu$ higher than about $10^{10}$ s$^{-1}$. In between those two asymptotic regimes, it can be seen in figure 2 that the population of the n=3 level increases with increasing frequencies by 11%, while in the same range the population of n=4 decreases by 31%. In our closed system, the population of the n=2 level changes in order to maintain a constant total population of atoms. If we consider the ratio of level populations n=3 to n=4 (the Balmer $\alpha$ to Balmer $\beta$ intensity ratio), we find that it increases by about 70% as we vary the frequency from the static to the diabatic regime.

We now examine the effect of electronic density fluctuations which are assumed to follow a gamma distribution given by equation (6), with $r = 0.9$.

Figure 3. Populations of the n=3,4 levels of a three level hydrogen atom, as a function of the fluctuation frequency of the density, and for a fixed electron temperature of 2 eV.
In figure 3, we have plotted the stationary populations of levels n=3, 4 for an average density of $10^{19}$ m$^{-3}$ and a temperature of 2eV. A change of population is also observed between the static and diabatic limits, with a simultaneous increase of the two levels as the frequency is increased, by 19% and 22% for respectively levels n=3 and 4. Moreover, we note from figure 3, that the diabatic limit tends to the regime without plasma electron density fluctuations. Our simulation of the stochastic process is in a perfect agreement with the analytical results. Indeed, we have shown [5] that solving equation (1) for the diabatic regime, is equivalent to solve the following equation:

$$\langle M \rangle_{s \to 0} \lim s \langle \tilde{G}(s) \rangle X_0 = \langle M \rangle X_{DB} = 0,$$

where

$$X_{DB} = \lim s \langle \tilde{G}(s) \rangle X_0,$$

is the population vector for the diabatic regime. From the two cases studied, it appears difficult to predict in a general case the behaviour of a set of populations submitted to a fluctuating plasma parameter. A specific study with our model is required for each plasma condition analysed.

3.3 Application to the carbon ionisation equilibrium

Neglecting particle transport, we have studied the ionization balance of carbon in the presence of temperature fluctuations. The equation of ionization equilibrium without transport can be written for the average population of the ion with charge $Z$:

$$\frac{d\langle n_Z \rangle}{dt} = -\langle n_Z N_e \alpha_{Z \to Z+1} \rangle - \langle n_{Z-1} N_e \beta_{Z \to Z-1} \rangle + \langle n_{Z+1} N_e \alpha_{Z-1 \to Z} \rangle + \langle n_{Z+1} N_e \beta_{Z+1} \rangle$$

where $\alpha_{Z \to Z'}$ and $\beta_{Z \to Z'}$ are respectively the ionization and recombination coefficients.

Figure 4. Different carbon ions populations (from $C^{+1}$ to $C^{+4}$) as a function of the averaged temperature are plotted with (solid lines) and without (dotted lines) temperature fluctuations.
These data are taken from the ADAS database [12]. The fluctuation frequency used in our calculation is equal to $10\mu s^{-1}$ which is a typical fluctuation frequency measured in edge plasma of current magnetic fusion devices. The PDF of temperature is assumed to be a gamma distribution with a fluctuation rate $r = 0.5$. We show in Fig.4 the abundances of several carbon ions (from $C^{+1}$ to $C^{+4}$) as a function of the average temperature in the range 1 eV to 20 eV, i.e. conditions which can be found in divertor plasmas. Changes in the ion populations are clearly seen, with shifts in the position of the maximum abundance modifying the ratio of the different ion species for a given temperature. Fluctuations have therefore to be retained in analysis aimed at measuring particle transport from the abundances curves, since the latter are sensitive to both of them, provided the fluctuation rate is large enough.

4. Conclusion

The effects of temperature and density fluctuations on the population of ideal atomic or ionic systems have been investigated. We have shown that significant changes occur in the populations of a three level system modeling hydrogen Balmer emission as we scan the typical frequency fluctuation of temperature or density. The ionization balance of carbon has also been studied in the presence of temperature fluctuations. The abundances of the different ions are modified in the presence of such temperature fluctuations. All these effects are significant only for fluctuation rates $r$ larger than several tens of percent, values which can be found in tokamak edge plasmas. With the aim of achieving a fluctuating edge plasma diagnostic, further studies will concern more realistic atomic system, which will require retaining more atomic processes and a more detailed atomic structure.

Acknowledgments

This work was carried out within the framework of the European Fusion Development Agreement and the French Research Federation for Fusion Studies. It is supported by the European Communities under the contract of Association between Euratom and CEA. The views and opinions expressed herein do not necessarily reflect those of the European Commission. We also acknowledge the support of the French National Research Agency (contract ANR-11- BS09-023, SEDIBA).

References

[1] Fujimoto T 2004 Plasma Spectroscopy, (Oxford: Clarendon Press)
[2] Graves J P, Horacek J, Pitts R A and Hopcraft K I 2005 Plasma Phys. Control. Fusion 47 L1
[3] Dudson D et al. 2008 Plasma Phys. Control. Fusion 50 124012
[4] Rosato J, Capes H, Catoire F, Kadomtsev M B, Levashova M G, Lisitsa V S, Marandet Y, Rosmej F B and Stamm R 2011 J. Nucl. Mater. 415 S617
[5] Catoire F, Capes H, Marandet Y, Mekkaoui A, Rosato J, Koubiti M and Stamm R 2011 Phys. Rev. A. 83 012518
[6] Catoire F, Capes H, Rosato J, Marandet Y, Mekkaoui A, Koubiti M and Stamm R 2011 Eur. Phys. J. D. 65 481
[7] Lax M and Scher H 1977 Phys. Rev. Lett. 39 781
[8] Vesely F J 1994 Computational Physics, an Introduction (New York: Plenum Press)
[9] IMSL 2012, http://www.roguewave.com
[10] Rosato J, Catoire F, Marandet Y, Mekkaoui A, Capes H, Koubiti M, Stamm R, Kadomtsev M, Levashova M G, Lisitsa V S and Rosmej F B 2011 Phys. Lett. A. 375 4187
[11] Van Regemorter H 1962 Ap. J. 136 906
[12] Summers H P 2004 The Atomic Data and Analysis Structure manual, version 2.6, http://www.adas.ac.uk/