Diffraction-unlimited position measurement of quantum particles

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We consider a method of high-fidelity, spatially resolved position measurement of ultracold atoms in an optical lattice. We show that the atom-number distribution can be nondestructively determined at a spatial resolution beyond the diffraction limit by tracking the progressive evolution of the many-body wavefunction collapse into a Fock state. We predict that the Pauli exclusion principle accelerates the rate of wavefunction collapse of fermions in comparison with bosons. A possible application of our principle of surpassing the diffraction limit to other imaging systems is discussed.

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The classical theory of electromagnetism predicts that two objects with a distance less than a wavelength cannot be distinguished [1]. This fundamental limit known as the diffraction limit has long imposed insurmountable constraints on optical physics. Recent achievements of the single-site resolved imaging [2,3] and addressing [4] of ultracold lattice gases are not exceptions: the diffraction limit requires a high numerical aperture lens and a large number of signals which requires use of a near-resonant probe light causing destruction of atomic states. As a result, all the experiments on single-site-resolved detection performed to date are destructive.

Quantum gases in an optical lattice offer an ideal playground to investigate strongly correlated systems and quantum information [5,6]. Recently, the single-site resolved detection and addressing have emerged as a powerful tool for those studies [8,12]. Against such a backdrop, development of a nondestructive measurement at the single-site level will have a significant impact on the field of quantum simulation [13,14], quantum information processing [15,16], and open quantum many-body systems [17,18]. Furthermore, it will also have an important application for the study of influence of measurement back-action on quantum many-body states [19-24] and open up the possibility of extending the concepts of quantum feedback control [24,25] and quantum non-demolition measurement (QND) [26,27] to quantum many-body systems. To move toward these goals, there is an obvious need to develop methods which allow us to overcome the difficulty posed by the diffraction limit.

In this Letter, we propose a method that achieves this aim. Quantum theory predicts that measurements on one (e.g., light) of a pair of entangled systems affect another (e.g., atoms). By diffracting a dispersively scattered light through a lens aperture, the spatial degrees of freedoms of atoms get entangled with the light and hence, a detection of a photon induces the wavefunction collapse of atoms. We show that this measurement back-action will localize the atom-number distribution and tracking the progressive collapse into a Fock state allows us to perform diffraction-unlimited position measurement with near-unit fidelity. Our method enables us to distinguish between different Fock states in contrast with the conventional single-atom detection schemes which are restricted to parity measurements. Previous works discussing non-destructive methods using off-resonant scattering with the angle-resolved measurement [29-34] and with the use of cavity [35,36] cannot achieve such a high spatial resolution. Furthermore, we find that the Pauli exclusion principle accelerates the rate of wavefunction collapse of fermions compared with bosons and, thus, our scheme is particularly advantageous for the single-site detection of fermionic gases, which has not yet been implemented.

Although we here focus on an optical lattice system, the principle of surpassing the diffraction limit considered in this Letter will have much broader applications to other imaging systems ranging from single trapped ions in quantum information [37,38] to fluorescent protein molecules in biology [40,41].

Main idea. — According to the classical argument [1] on the diffraction limit, two point sources whose distance is less than a wavelength scale cannot be resolved because the diffraction pattern of two sources overlaps with each other. The crucial point here is that the position of a source is inferred from the peak position and the width of an interference pattern. The main idea of our scheme surpassing this limit is as follows. Since quantum measurement theory automatically takes into account the method of Bayesian inference, we can extract—by tracking the progressive wavefunction collapse—the unbiased position information of atoms beyond the diffraction limit. In contrast to the classical argument above, the conditional probability distribution of source-positions calculated from measurement outcomes allows us to make the most likelihood estimation of positions of objects in our model. Remarkably, our method can—with a prior knowledge of the total number of point sources [46,47]—precisely distinguish multi-fluorescence objects even if the objects reside in a region narrower than the diffraction limit.

Model. — We consider two-level atoms in a lattice described by the many-body Hamiltonian
point diametrically opposite to \( \Delta \).

\( \frac{\text{do not consider measuring the polarization of a photon, where the polarization vector is averaged out since we}}{\text{to the function } F} \)

through a lens aperture and detected on the screen. The position of a detected photon is denoted by \( R \).

(b) A measurement back-action caused by the detection of a photon at position \( X \). The many-body wavefunction shrinks according to the function \( F \) which peaks at \( x_X \), where \( x_X \) is the lattice point diametrically opposite to \( X \) with respect to the center of the lens aperture.

\[
\hat{H} = \int d^3r \left[ \hat{H}_a(r) + \hat{H}_{af}(r) \right] + \hat{H}_f, \tag{1}
\]

where \( \hat{H}_a(r) = \sum_{i=g,e} \hbar \omega_i \hat{\Psi}_{i}^{\dagger}(r) \hat{\Psi}_{i}(r) \) is the Hamiltonian of atoms and \( \hbar \omega_{i,g,e} \) are their ground (\( g \)) and excited (\( e \)) state energies, and \( \hat{\Psi}_{i,g,e}(r) \) are the corresponding field operators; \( \hat{H}_{af}(r) = -\langle d \cdot \hat{E}(r) \rangle \hat{\Psi}_{f}^{\dagger}(r) \hat{\Psi}_{f}(r) + \text{H. c.} \) is the electric-dipole interaction and \( \hat{d} \) is the electric dipole-moment, and \( \hat{E}(r) \) is the electric field operator; \( \hat{H}_f = \sum_{k' \sigma} \hbar \omega_{k'} \hat{a}_{k',\sigma}^{\dagger} \hat{a}_{k',\sigma} \) is the free-field Hamiltonian and \( \hat{a}_{k',\sigma} \) is the annihilation operator of a photon with wave vector \( k' \) and polarization \( \sigma \). Atoms are illuminated by an off-resonant probe light whose positive frequency component is \( E_p^{(+)}(r) = e_p \hat{E}_0 e^{i k r} / 2 \). Each scattered photon is diffracted through a lens aperture and detected on a screen (see Fig. 1(a)). We first focus on a 1D lattice and then discuss the generalization to a 2D lattice. In this Letter, we ignore tunneling of atoms through lattice potentials during imaging and focus on light scattering.

The scattered field can be calculated \([48]\) by integrating out the Heisenberg equation of motion under the geometry shown in Fig. 1. After performing the adiabatic elimination of the excited state and employing the tight-binding approximation, we arrive at the following expression of the scattered field at position \( X \) on the screen:

\[
\hat{E}_{\text{sc}}^{(+)}(X) = \gamma \sum_{m} e^{-i \Delta k \cdot m} \frac{J}{\sigma} \int \frac{d^3r}{\sigma} \left[ x_X - m d \right] \hat{b}_{m}^{\dagger} \hat{b}_{m}, \tag{2}
\]

where the polarization vector is averaged out since we do not consider measuring the polarization of a photon, \( \Delta k \) is the wave-vector difference between incident and scattered photons, the operator \( \hat{b}_{m} \) annihilates an atom at site \( m \), \( x_X \) is the diagonal coordinate of the detected position (Fig. 1(b)), and \( \mathcal{F}[y] \equiv J_1(y)/y \) which vanishes rapidly for \( y \gg 1 \). Here we introduce the parameter \( \sigma \) characterizing the resolution of the classical imaging method which is defined by the numerical aperture of the lens \( N_A \) as \( \sigma = 1 / k N_A \). The diffraction limit is usually characterized by the first zero of the Airy disk, \( d_{\text{diff}} = 0.61 \lambda / N_A \), which can be related to \( \sigma \) as \( d_{\text{diff}} = 3.8 \sigma \).

Note that the classical imaging method can achieve the single-site resolved measurement only when the diffraction limit reaches the scale of the lattice constant \( d_{\text{diff}} \lesssim d \).

Let us now consider the physical content of Eq. (2). The measured observable can be continuously varied as we control the parameter \( \sigma \) by, for example, changing the distance between a lens and a lattice. When the lens is positioned at a far-field region and the numerical aperture is so low that \( \sigma \) is much larger than the lattice constant, the phase factor in Eq. (2) can induce a Bragg diffraction pattern rather than the space-resolved imaging \([40]\).

We here develop a continuous quantum measurement theory and show that each observation of a dispersively scattered photon will localize the atom-number distribution and a sequence of these observations causes progressive collapse of the atomic state into a Fock state which, in turn, allows us to surpass the diffraction limit. We first consider an ideal situation in which the collection efficiency of scattered photons is unity and later discuss the effect of uncollected photons. Since the effective Hamiltonian commutes with the atom-number operator at each lattice site, the ideal photodetection of a dispersive scattered photon constitutes a QND measurement of the atom-number statistics in the sense of Ref. \([50]\). Hence, our model presents a dual approach compared with a QND model \([26–28]\) of the photon number based on two-level systems. 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For the sake of concreteness, let us consider \( N \) atoms trapped in a 1D optical lattice with \( N_L \) sites. The state of a quantum gas is represented in terms of Fock states \( \{ n_m \} \equiv \{ n_1, \ldots, n_{N_L} \} \) satisfying \( \sum_{m=1}^{N_L} n_m = N \). Let \( \rho_0 \) be the density matrix of the initial motional state of atoms and \( P_{0} \{ n_m \} \) be the corresponding initial atom-number distribution. When we detect a photon at the screen position \( X \), the change of the conditional state can be described by the measurement operator \([23]\) as

\[
\rho_0 \rightarrow \mathcal{E}_{\text{sc}}^{(+)}(X) \rho_0 \mathcal{E}_{\text{sc}}^{(+)}(X) / \text{Tr}[\mathcal{E}_{\text{sc}}^{(+)}(X) \rho_0 \mathcal{E}_{\text{sc}}^{(+)}(X)].
\]

Suppose now that \( n \) photons were detected at the positions \( X \equiv \{ X_1, \ldots, X_n \} \). Then the atom-number distribution of the quantum state becomes

\[
P_n \{ n_m \} |X \rangle = \frac{P_{0} \{ n_m \} \prod_{k=1}^{n} P \{ X_k | n_m \}}{\sum_{n_m}^{n} P_{0} \{ n_m \} \prod_{k=1}^{n} P \{ X_k | n_m \}}.
\]
Here \( P[X|\{n_m\}] \) is the conditional probability of detecting a photon at \( X \), given that the atomic state is the Fock state \(|\{n_m\}\rangle\):

\[
P[X|\{n_m\}] = \frac{\sum_{m=1}^{N_L} n_m F\left[\frac{|x-X_m|}{\sigma}\right]^2}{\int dX' \sum_{m=1}^{N_L} n_m F\left[\frac{|x-X_m|}{\sigma}\right]^2},
\]

where we neglect the contribution of the phase factor \( e^{i\phi} \) in Eq. (2). We can track the progressive dynamics of the wavefunction collapse by applying Eq. (3) iteratively. After detecting a sufficiently large number of photons, the state will collapse into a Fock state \(|\{n_m\}\rangle\) and, hence, the occupied atom number at each lattice will be precisely determined. Note that the order of the measurement outcomes is irrelevant to the final collapsed Fock state because all measurement operators commute with each other. Therefore, only the accumulated histogram of the positional information of photodetections is needed to determine the final atom-number distribution.

Let us here discuss our principle of surpassing the diffraction limit. The crucial point is that since Eq. (3) derived from the quantum measurement theory automatically takes into account the Bayesian inference, we can extract more detailed information about positions of point sources than the classical imaging method, which only utilizes the peak and the width of photodetection histogram to deduce the positions of atoms and fails to fully exploit a wealth of information hided in the histogram.

**Numerical simulations.**—To illustrate this point and clarify the principle of surpassing the diffraction limit, we here discuss numerical simulations of our model. Figures 2(a) and (b) show the collapse of a bosonic state and that of a fermionic state into Fock states, respectively, where the initial state is chosen as a superposition of all possible Fock states. For comparison, we also show the histograms of detected positions of photons (Figs. 2(c) and (d)). Our method enables us to distinguish between different Fock states by tracking the wavefunction collapse and hence, we can determine the atom number at each lattice site with near-unit fidelity in contrast to the conventional parity measurement. On the other hand, the classical diffraction-limited images (Figs. 2(c) and (d)) that utilize a peak and the width of the histogram to obtain position information cannot resolve atoms placed at neighboring. The distribution of the collapsed states obtained by many realizations reproduces the initial state distribution. Furthermore, we note that the required number of photocounts is several orders of magnitude smaller than that required by the present imaging method.

To investigate how quantum statistics of atoms affects the evolution of the wavefunction collapse, we plot the rate of wavefunction collapse against the resolution parameter \( \sigma \) (Fig. 3). We find that the rate of convergence is faster for fermions than bosons. This can be attributed to the Pauli exclusion principle which greatly reduces the number of possible configurations for fermions. Hence, our method is particularly advantageous for the single-site resolved detections of fermionic gases. Another interesting feature is that the average number of photodetections needed to cause the many-body wavefunction collapse grows almost exponentially with the resolution parameter \( \sigma \) in a large-\( \sigma \) region. Because the rate of convergence can be related to the relative entropy of each measurement, this finding might have some information-theoretic background.

**Experimental situations.**—Let us here make a couple of practical considerations which should be taken into account when one considers an experimental implementation of our method. First, we note that, in practice, only a portion of scattered photons can be collected and the back-action caused by destructions of uncollected photons in the far-field region also affects the dynamics of the wavefunction collapse. In this case, Eq. (4) does not reconstruct an exact progressive dynamics of the wavefunction collapse. Nevertheless, the actual final collapsed Fock state, which is of primary interest, coincides with the one identified from Eq. (3) after a sufficient number of photodetections. This is because the back-
action of uncollected photons does not alter the atomic state once the state collapses into a some Fock state owing to the commutativity of the effective Hamiltonian with atom-number operators at each site. As we pointed out earlier, the order of photodetection does not matter for the eventual wavefunction collapse; this implies that the presence of uncollected photons only delay the speed of the collapse but not alter the eventual atomic state. Hence, our method can determine the atom-number at each lattice even in the presence of uncollected photons by tracking the evolution of the conditional probability distribution with Eq. (3) and identifying the final collapsed Fock state. In this respect, our principle of surpassing the diffraction limit is also applicable to classical objects [40, 41, 45] where positions of fluorescence objects take some continuous predetermined values and there is no measurement back-action. In such case, the variable of the conditional probability distribution should be replaced by a set of position parameters of fluorescence objects in Eq. (3) instead of the label \{n_m\} of Fock states.

Second, although the required number of scattering events is orders of magnitude less than that of the present imaging method, successive recoils may induce heating and it will be beneficial to estimate a typical scale of expected total recoil energy and show an appropriate experimental parameters for our scheme. To this end, we consider the setup of Ref. [3]: \(d = 532\)nm, \(\lambda = 780\)nm and \(N_A = 0.68\) (corresponding to \(\sigma = 0.343\)).

Combining our numerical results of the required number of detections with the experimental collection efficiency \(\sim 10\%\) results in an estimation of total recoil energy per atom for the wavefunction collapse as \(\sim 93E_r\) where \(E_r \equiv \hbar^2\pi^2/2md^2\). Hence, the contribution of heating effects would be made negligible by, for example, the ramp-up of the lattice potential to several thousands of \(E_r\) during an imaging process as performed in [2, 4, 8, 12].

Conclusion.— We have demonstrated that tracking the progressive evolution of wavefunction collapse of a quantum many-body state provides a way to surpass the classical resolution limit, which—in contrast to the conventional diffraction-limited parity measurement—enables a nondestructive measurement of the atom-number distribution at the single-site level. Moreover, our principle of surpassing the diffraction limit has a much broader range of applications other than an optical lattice system and gives a powerful means to extract position information in different varieties of challenging situations.

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[16] T. Byrnes, K. Wen, and Y. Yamamoto, Phys. Rev. A 85, 040306 (2012).
[17] A. J. Daley, Adv. Phys. 63, 77 (2014).
[18] Y. S. Patil, S. Chakram, L. M. Aycock, and M. Vengalattore, Phys. Rev. A 90, 033422 (2014).
[19] S. Keffler, A. Holzner, I. P. McCulloch, J. von Delft, and F. Marquardt, Phys. Rev. A 85, 011605 (2012).
[20] A. J. Daley, H. Pichler, J. Schachenmayer, and P. Zoller, Phys. Rev. Lett. 109, 020505 (2012).
[21] J. S. Douglas and K. Burnett, Phys. Rev. A 86, 052120 (2012).
[22] S. Keffler, I. P. McCulloch, and F. Marquardt, New J. Phys. 15, 053043 (2013).
[23] Y. S. Patil, S. Chakram, and M. Vengalattore, ArXiv e-prints (2014), arXiv:1411.2678 [cond-mat.quant-gas].
[24] H. Wiseman and G. Milburn, Quantum Measurement and Control (Cambridge University Press, 2010).
Supplemental Materials

Derivation of the electric field from the Heisenberg equation of motion

The Hamiltonian under consideration is given by Eq. (1). Then, the time evolution of the scattered field obeys the following Heisenberg equation:

\[ \dot{\hat{a}}_{k',\sigma} = -i\omega_{k'}\hat{a}_{k',\sigma} + \frac{i}{\hbar}\sqrt{\hbar\omega_{k'}}\mathbf{e}_{k'} \cdot \int d^3r'e^{-ik'r'} \left[ \mathbf{d} \cdot \hat{\Psi}_g^\dagger(r') \hat{\Psi}_e(r') + \text{H.c.} \right]. \]  

(S5)

Let \( R_L \) (\( R_S \)) be the distance between the lens and the lattice (screen). We integrate out the Heisenberg equation of motion for the geometry shown in Fig. 1 and under the conditions \( kR_L, kR_S \gg 1 \). Since the probe light is far-detuned, the excited state can be eliminated adiabatically, giving \( \hat{\Psi}_e \simeq -d^* \cdot E_{\text{p}}^{(+)}(+) \hat{\Psi}_g / \hbar \Delta \), where \( \Delta \) is the detuning of the probe light. Then, after averaging out the polarization vector, we obtain the positive frequency component of the scattered field as follows:

\[ \hat{E}_{\text{sca}}^{(+)}(R) = \gamma \int d^3r'e^{-i\Delta k \cdot r} \frac{J_e(ka \xi')}{ka} \hat{\Psi}_g^\dagger(r') \hat{\Psi}_g(r'), \]  

(S6)

where \( \gamma \) is a coefficient determined by the angular momenta of atomic states and the polarization of the probe light. As a typical case, if we assume that the angular momenta of atomic states satisfy \( m_e = m_g + 1 \) and the probe light is \( \sigma^+ \)-polarized, the coefficient is given by

\[ \gamma = \frac{-iE_0k^3e^{ik(R_L+R_S)}|d|^2}{8\pi\epsilon_0R_LR_SH\Delta} \langle J_e m_e | J_g m_g; 11 \rangle, \]  

(S7)

where \( \langle J_e m_e | J_g m_g; 11 \rangle \) is the Clebsh-Gordan coefficient.

1D-case

We can now derive the expression of the positive frequency component of the electric field on the screen, i.e., the measurement operator acting on the atomic state associated with photodetections on the screen. Let us first consider atoms trapped in a one-dimensional lattice and adopt the following tight-binding approximation:

\[ \hat{\Psi}_g(r) = \Phi(y, z) \sum_m w(x - md) \hat{b}_m, \]  

(S8)

where \( \Phi(y, z) \) is the wave function confined in the transverse direction, \( w(x) \) is the Wannier function centered at \( x = 0 \). We assume that a length-scale of our geometry is much larger than the lattice constant: \( R_L, R_S \gg d \). Under this assumption, the geometrical factor \( \xi' \) takes a simple form,

\[ \xi' \simeq \frac{|x' - x_X|}{R_L}, \]  

(S9)

where we introduce a diagonal position \( x_X \equiv -R_LX/R_S \). After substituting these expressions into the scattered field and integrating the resultant expression with respect to \( r' \), we arrive at the expression of the operator associated with a photodetection at screen position \( X \):

\[ \hat{E}_{\text{sca}}^{(+)}(X) = \gamma \sum_m e^{-i\Delta k \cdot m\sigma} J_e(\frac{|x_X - md|}{\sigma}) \hat{b}_m^\dagger \hat{b}_m, \]  

(S10)

which is Eq. (2).
For a two-dimensional lattice, we can show, under $R_L, R_S \gg d$, that the geometrical factor $\xi'$ takes the form,

$$\xi' \approx \sqrt{\left(\frac{|x'| - x_X}{R_L}\right)^2 + \left(\frac{|y'| - y_Y}{R_L}\right)^2},$$  

(S11)

where $y_Y \equiv -R_L Y/R_S$. Furthermore, we perform the tight-binding approximation and integrate the resultant scattered field with respect to $r'$ in the same manner as in the 1D lattice. Then we obtain the following expression for the 2D lattice:

$$\hat{E}_{sc}^{(+)}(X, Y) = \gamma \sum_{(m_x, m_y)} e^{-i\Delta k \cdot m} \mathcal{F}[\rho] \hat{b}_{m_x, m_y} \hat{b}_{m_x, m_y},$$  

(S12)

where $\rho$ is defined by

$$\rho \equiv \sqrt{\left(\frac{|x_X - m_x d|}{\sigma}\right)^2 + \left(\frac{|y_Y - m_y d|}{\sigma}\right)^2},$$  

(S13)

and $(m_x, m_y)$ is a label of lattice sites, and $m \equiv m_x d e_x + m_y d e_y$. This is the operator associated with a photodetection at position $(X, Y)$ on the screen.

Derivation of the conditional probability distribution

We here mention the approximations leading to Eq. (4). We consider the probe light incident from a transverse direction. Then, the projected wave-vector difference between the incident and scattered photons in the direction of the lattice is by a factor of $\sim d/(R_L + R_S)$ smaller than $k$ and the phase difference between light scattered from neighboring sites is negligibly small. Hence, the phase factor in Eq. (2) can be disregarded which results in a simple expression of the conditional probability distribution

$$P[X|\{n_m\}] = \frac{\sum_{m=1}^{N_L} n_m \mathcal{F}\left[ \frac{|x_X - m d|}{\sigma} \right]^2}{\int dX' \sum_{m=1}^{N_L} n_m \mathcal{F}\left[ \frac{|x_X' - m d|}{\sigma} \right]^2},$$  

(S14)

which is Eq. (4).