A Sun Sensor Based on Regular Pyramid Sensor Arrays

J Wang¹,², *, X Wang² and J Y Yang¹

¹School of Information and Communication Engineering, University of Electronic Science and Technology of China, Chengdu, Sichuan, 610054, China
²College of Electronic Engineering, Chengdu University of Information Technology, Chengdu, Sichuan, 610225, China

* e-mail: rivers2000@126.com

Abstract. Non-planar sun sensors determine solar position by existing photodiodes or solar panels without increasing the size or mass of aerospace equipment such as satellites. However, it suffers from the coarse orientation accuracy. A sun sensor that is formed by photodiodes or solar panels mounted on the lateral surfaces of regular pyramid was proposed to reduce the estimated error of the sun vector, which indirectly improve the accuracy of solar orientation determination. The orientation formulation of the sun sensor and its sun vector error were derived to evaluate the interference suppression for the sun sensor. Experimental data validates the maximum error of orientation is less than 1° in a clear sky by the sun sensor consisting of 16 solar panels, which are mounted on the lateral surfaces of a regular 16-pyramid with the output error less than ±5% and the assembly error less than ±1°.

1. Introduction

Sun sensors have been widely adopted to determine solar orientation for satellites and aircrafts attitude control [1-2], assisted positioning for planetary rovers [3] and visual odometry [4], ground-based navigation systems [5], and efficiency improvement of photovoltaic systems [6]. These sun sensors are divided into two categories: one calculates the sun’s direction by imaging positions on the planar array when sunlight passes through the small holes or slits on the mask, such as complementary metal-oxide semiconductors (CMOSs) [7], charge coupled devices (CCDs) [8], and micro-electro-mechanical systems (MEMS) [9]. The other calculates the sun’s direction by the least squares solution of a linear equations, in which the coefficient matrix consists of the unit normal vectors of illuminated sensor planes (photodiodes or solar panels) on the non-planar array and the constant consists of the irradiances on these planes [1-3, 5, and 10-13]. The former has highly accuracy orientation but the sensor field of view (FOV) is less than 180°. Therefore, at least three such sun sensors are required to determine the sun’s position for full FOV applications. However, this requirement leads to an increased load for small aerospace equipment such as nano-satellites, which is problematic because of their limited size, weight, and power supply. The latter is constructed by photodiodes mounted on different surfaces of satellites [1] and aircrafts [2], sometimes by the direct re-use of solar panels of the spacecraft [12], but it is susceptibility to interference. Thus, for meeting both the sizing limitations of small-scale aerospace equipment and maintaining a highly accurate orientation of the sun, improving the accuracy of orientation of the non-planar sun sensor method could be one way to achieve this balance.
The interference in solar orientation determination by non-planar sun sensors can be divided into two categories: 1) zero mean interference, such as zero-mean Gauss white noise, which may originate from misalignments and undesired characteristics of sensors and imperfect surrounding measurement circuitry [14-17]; and 2) constant interference, such as the interference coming from the uniform scattering light, which is about 8% of the incoming sunlight in a clear sky [14]. Recent works mostly centre on improving the accuracy of solar orientation determination by suppressing the zero mean Gauss white noise, such as the calibration [15] and optimal design [16-17]. Reference [14] introduces a model to eliminate the constant interference caused by the uniform scattering light in a clear sky, but the model is only applicable to the interference that can be measured by experiment. However, few works address the unmeasured constant interference and evaluate the sun vector error caused by constant interference or zero mean interference, which limits further improvement the accuracy of solar orientation determination.

For suppressing constant interference and zero mean interference in applications, a sun sensor based on regular pyramid sensor arrays is proposed. The orientation formulation of the sun sensor is introduced in Section 2 and the sun vector error of the sensor is deduced to assess the suppression of interference in Section 3, then the interference suppression of the sensor is verified by field experiment in Section 4. Finally the summary and discussions is provided in Section 5.

2. Orientation determination of sun sensors based on regular pyramid arrays

The Sun is far enough from the observation site and the sunlight reaching the observation site can be assumed parallel to each other. Thus, for determining the sun’s position, we define the sun vector to be pointing towards the sun from the observation point with a magnitude of intensity of its irradiance. Note that the sun vector in this work is defined at the opposite direction of sunlight.

The geometric relationship between the sun vector and the sun sensor that consists of sensor planes mounted on the lateral surfaces of a regular M-pyramid (\(M \geq 3\)) is shown in Figure 1. The \(x\)-\(y\)-\(z\) Cartesian coordinate system is set with the bottom of regular \(M\)-pyramid as the \(x\)-\(y\) plane, the centre of the bottom as the origin \(O\), which is the observation point. In the system, the sun vector is donated by \(s\); the illuminated sensor plane \(P_i\) (where \(i \in \{1,2,\ldots,M\}\)) is mounted at azimuth \(\alpha_i\) and zenith \(\gamma\); its unit normal vector \(n_i\) aligns with local vertical, forming an angle \(\varphi_i\) with the sun vector \(s\). Azimuth angle is the angle from the true north (if applied on the Earth), here set as the positive \(\gamma\) direction, and rotating to east to a projected vector on the \(x\)-\(y\) plane. Elevation angle is the angle between the \(x\)-\(y\) plane and a vector. From the cosine law for radiation [18] — the irradiance at any plane varies with the angle between the normal of this plane and radiation-energy propagation direction, and consequently, the solar irradiance on the plane \(P_i\) is \(|s|\cos \varphi_i\), which is just equal to the inner product of vector \(s\) and vector \(n_i\). Therefore, we have

\[
s \cdot n_i = |s|\cos \varphi_i, \tag{1}
\]

where \(|s|\) is the length of the sun vector \(s\).

For the irradiance passing through a sensor plane \(P_i\), the output measurement value \(e_i\) usually relates linearly to the input by a factor \(\xi_i\) and can be assumed to be

\[
e_i = \frac{1}{\xi_i}|s|\cos \varphi_i. \tag{2}
\]

According to (2), the irradiance passing through the sensor is \(|s|\cos \varphi_i = \xi_i e_i\). Then the following matrix equation can be obtained for the sun sensor:

\[
\begin{bmatrix}
\mathbf{n}_1^T \\
\mathbf{n}_2^T \\
\vdots \\
\mathbf{n}_M^T
\end{bmatrix}
\mathbf{s} = 
\begin{bmatrix}
\xi_1 e_1 \\
\xi_2 e_2 \\
\vdots \\
\xi_M e_M
\end{bmatrix}.
\tag{3}
\]
According to the geometric relationship shown in Figure 1, \( s \) is the sun vector and \( \mathbf{n}_i \) is the unit normal vector of the sensor plane \( P_i \), if denoted by \( B \), and then we have

\[
B = \begin{pmatrix}
\sin \alpha_i \cos \gamma & \cos \alpha_i \cos \gamma & \sin \gamma \\
\sin \alpha_2 \cos \gamma & \cos \alpha_2 \cos \gamma & \sin \gamma \\
\vdots & \vdots & \vdots \\
\sin \alpha_M \cos \gamma & \cos \alpha_M \cos \gamma & \sin \gamma
\end{pmatrix}.
\]  

So, (3) becomes

\[
Bs = \begin{pmatrix}
\xi_1 e_1 \\
\xi_2 e_2 \\
\vdots \\
\xi_M e_M
\end{pmatrix}.
\]  

Because the unit normal vectors of the lateral surfaces of a regular \( M \)-pyramid are non-coplanar, the rank of \( B \) is equal to the number of its columns, i.e., \( \text{rank}(B) = 3 \). Then, there is a unique solution for the matrix equation:

\[
s = (B^T B)^{-1} B^T \begin{pmatrix}
\xi_1 e_1 \\
\xi_2 e_2 \\
\vdots \\
\xi_M e_M
\end{pmatrix}.
\]
For a non-planar sensor array using similar sensors, the measurement coefficients $\xi_i$ may be reasonably assumed to be equal to a constant $\xi$ ($\xi > 0$). If we denote the measurement vector as $e = (e_1, e_2, \ldots, e_M)^T$, the sun vector solution can be simplified as

$$s = \xi (B^T B)^{-1} B^T e.$$  \hspace{1cm} (7)

Letting $B = (b_x, b_y, b_z)^T$, we have

$$\begin{align*}
  b_x &= (\sin \alpha_1 \cos \gamma \quad \cdots \quad \sin \alpha_M \cos \gamma)^T \\
  b_y &= (\cos \alpha_1 \cos \gamma \quad \cdots \quad \cos \alpha_M \cos \gamma)^T \\
  b_z &= (\sin \gamma \quad \cdots \quad \sin \gamma)^T.
\end{align*}$$  \hspace{1cm} (8)

According to the geometric relationships shown in Figure 1, $\alpha_i = 2\pi (i - 1)/M + \alpha_1$. Thus we can deduce that $b_x$, $b_y$, and $b_z$ are orthogonal each other. Then we further obtain the following equation for calculating the sun vector.

$$s = \xi \left( \frac{b_x^T}{b_x^T b_x} b_x \right) e$$  \hspace{1cm} (9)

3. Suppression of interference

As mentioned earlier, the irradiance measured in practical applications always contains influence of interferences. Assume that an interference vector $\varepsilon$ is added to the measurement vector $e$. According to (9), the error of the sun vector, denoted by $\Delta s$, can be derived as:

$$\begin{align*}
\Delta s = \begin{pmatrix}
\Delta s_x \\
\Delta s_y \\
\Delta s_z
\end{pmatrix} = \begin{pmatrix}
1/b_x^2 b_x & 0 & 0 \\
0 & 1/b_y^2 b_y & 0 \\
0 & 0 & 1/b_z^2 b_z
\end{pmatrix} \begin{pmatrix}
b_x^T \varepsilon \\
b_y^T \varepsilon \\
b_z^T \varepsilon
\end{pmatrix}.
\end{align*}$$  \hspace{1cm} (10)

where $\Delta s_x$, $\Delta s_y$, and $\Delta s_z$ are the estimated errors of the projection component on $x$, $y$ and $z$ axis of sun vector $s$ in the Cartesian coordinate system, respectively.

As $\sum_{i=1}^{M} \sin \alpha_i = \sum_{i=1}^{M} \sin(2\pi (i - 1)/M + \alpha_1) = 0$, we can see that the sum of elements of $b_x^2$ is equal to 0. Thus $\Delta s_x$ is independence of constant interference. Similarly, $\sum_{i=1}^{M} \cos \alpha_i = 0$, we can see that $\Delta s_y$ is also independence of constant interference because the sum of elements of $b_y^2$ is equal to 0. Because $\Delta s_x$ and $\Delta s_y$ are independence of constant interference, we can conclude that the estimation of solar azimuth angle is independence of zero mean interference. Unlike $b_x^2$ and $b_y^2$, $b_z^2$ is a constant, which makes $\Delta s_z$ is independence of zero mean interference but proportional to constant interference. Considering the greater of $|\Delta s_x|$, $|\Delta s_y|$, and $|\Delta s_z|$, the greater of $|\Delta s|$, and then the greater of the maximum error angle between the sun vector and its estimation vector. Therefore, we can conclude that the sun sensor based on regular pyramid sensor arrays will improve the accuracy of solar orientation determination in case of constant interference and zero mean interference.

4. Experiments and analysis

Our sun sensor is designed such that 16 solar panels are mounted on the lateral surfaces of a regular 16-pyramid. For increasing the detectable FOV, the angle between the lateral and bottom faces of the pyramid is designed to be 26.4°, which makes 127.2° of FOV. The solar panels used are monocrystalline silicon solar batteries with 5V open circuit voltage and 160mA short circuit current. As short circuit current of solar batteries varied with light-intensity change approximately linearly, the irradiances on the pyramidal surfaces are replaced by the output current of the batteries in the measurements. With the same irradiance, the output deviation of solar panels is $<\pm 5\%$. The assembly error of solar panels on the pyramid is $<\pm 1^\circ$, which makes an additional output error on the solar panels $<\pm 1.8\%$. In addition, five solar panels mounted on the top of the pyramid in a horizontal plane parallel to the ground surface are used to measure the total solar irradiance on the ground by the average of their short-circuit current outputs.
For determining the sun’s position, a Cartesian coordinate system is set with the centre of the pyramid base as the origin, the ground surface as the $x$-$y$ coordinate plane with $y$-axis pointing toward true north and $x$-axis toward east. In the coordinate system, the reference of solar azimuth angle and solar elevation angle are calculated using the astronomical formulas, which use the local latitude, solar hour, and solar declination angle [18, 19].

The field experiment was conducted on August 5, 2015 in the suburb of Chengdu, China (longitude 103°59', latitude 30°35'). The measurement platform was installed inside the meteorological observation site with a broad sight span. The measurement was carried out from 8:50 to 17:17, with sunny weather in the morning, thin clouds around the sun after 15:25 BJT (Beijing Time), and thin clouds covering the sun at 15:45–16:30 BJT and at approximately 17:07 BJT.

![Figure 2](image)

Figure 2. (a) Solar azimuth angle, (b) solar elevation angle, (c) the total solar irradiance measured by the average output current (mA) of the five solar panels mounted on top of the pyramid, and (d) the error of solar orientation determination.

All solar panels were illuminated by sunlight within the solar azimuth angle ranged from 86.8° to 271.7° (Figure 2a) during the period of observation, while the elevation angle ranged from 29.9° to 76.7° and then returned to 32.6° (Figure 2b). The total solar irradiance changed in a cosine waveform with respect to solar elevation angle and dropped sharply in the cloudy periods (16:00–16:30 and around 17:07 BJT) as shown in Figure 2c, which was consistent with the actual weather conditions. According to Ref. [20], about 10% of radiation comes from scattered radiation in a clear sky, which
makes a constant interference on each solar panel. In addition, the total output error caused by undesired characteristics and misalignments of solar panels is $\pm 6.8\%$, which usually can be considered as a zero mean Gaussian noise [18]. The solar orientation was determined every 10 seconds and the orientation errors are shown in Figure 2d. The maximum error is less than $1^\circ$ in clear periods (8:50–15:30) BJT in case of greater constant (approximately $10\%$ of the total solar irradiance) interference and zero mean interference caused by solar panels with the maximum output error of $\pm 6.8\%$. The result validates the suppression of the sun sensor to constant interference and zero mean interference.

5. Conclusion
We designed a sun sensor by regular pyramid sensor arrays. With the geometric relationship of regular pyramid, the error of the sun vector is deduced to evaluate the interference suppression of the sun sensor in this study. For any sun sensor formed by all lateral surfaces of a regular pyramid, the accuracy of solar orientation determination is improve in case of constant interference or zero mean interference. The estimation of the projection component on $x$ and $y$ axis of the sun vector in a Cartesian coordinate system are unbiased if the interference keeps constant, which makes the estimation of solar azimuth angle to be independence of constant interference. The estimation of the projection component on $z$ axis of the sun vector is unbiased if the mean of the interference is zero, but its estimation error is proportional to constant interference. Our field experiment shows that the maximum error of solar orientation determination is less than $1^\circ$ by the sun sensor, which consists of 16 solar panels mounted on the lateral surfaces of a regular 16-pyramid in a clear sky.

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