Analysis of students’ spatial thinking in geometry: 3D object into 2D representation

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Abstract: The aim of this study is to find out the spatial thinking process of students in transforming 3-dimensional (3D) object to 2-dimensional (2D) representation. Spatial thinking is helpful in using maps, planning routes, designing floor plans, and creating art. The student can engage geometric ideas by using concrete models and drawing. Spatial thinking in this study is identified through geometrical problems of transforming a 3-dimensional object into a 2-dimensional object image. The problem was resolved by the subject and analyzed by reference to predetermined spatial thinking indicators. Two representative subjects of elementary school were chosen based on mathematical ability and visual learning style. Explorative description through qualitative approach was used in this study. The result of this study are: 1) there are different representations of spatial thinking between a boy and a girl object, 2) the subjects has their own way to invent the fastest way to draw cube net.

1. Introduction
Geometry is a branch of mathematics concerned with shape, size, a relative position of figures, and the properties of space. Geometry is an important area of mathematics which can be found in the natural world as well as in virtual areas of human creativity and ingenuity [1]. Geometric representations can help students make sense of area and fraction, histograms, and scatterplots, can give insight about data, and coordinate graph, can serve to connect geometry and algebra [2]. Through the study of geometry, students learn about space and reasoning. Spatial is building and manipulating the mental representation of two-and three-dimensional objects and perceiving an object from different perspectives [2]. Spatial visualization is an important aspect of geometrical thinking. It is important across several disciplines including engineering and the basic sciences enables an individual to visualize, edit, reorganize and generalize facts and is required in diverse workplace settings, such as mechanical engineering, pilot training, and scientific crime scene investigation [3]. It would be difficult to whom who not able to communicate about a position, relationships between objects, giving and receiving directions and imagine change taking place in position or size of shapes [4]. Mathematics curriculum places geometry as a one of the ability to learn mathematics. There are several standarts for students to learn geometry in every school stages [5].
Although the geometrical object is really close to human activity, the authors found from the pilot study that as many as 60% students from three different school in a city still have difficulty in classifying four sides object such as square, rectangular, parallelogram and rhombus. Four fifth grade of elementary students were asked to draw a net of a given cube. Although the students can do the task correctly the model of cube nets drawn by the students is the same. The students can only drew the cube nets according to what the teacher taught. The students have difficulty in constructing different cube models and saying that there is no other model of cube nets to the same cube and tend to accept the explanation of the teacher without any attempt to explore further by manipulating the cube model that has been given. Hence, the exploration of students’ spatial thinking is needed to be done. It is not only to enhance students’ knowledge in mathematics but also to play a role in the development of science and technology.

1.1. Theoretical Framework
Spatial ability is a cognitive aspect, evolving in line with the cognitive development of spatial concepts in motor sensory, spatial concepts at pre-operational stages and spatial stages in concrete operation stages and spatial stages in formal operations [6]. The children understands that perspective is an integrated system and logically interconnected ie the right to the left when seen from opposite direction. While spatial perspective includes the ability to respond to mutual coordination of separate objects in space.

1.2. Spatial Thinking
Spatial thinking is a thinking that finds meaning in the shape, size, orientation, location, direction or trajectory of objects, process or phenomena or the relative positions in space of multiple objects, process or phenomena and which uses the properties of space as a vehicle for structuring problem, for finding answers, and for expressing solutions [7]. It applies to explore and understand which used to manipulate and change the object we have created and to interpret the relationships among these objects, such as rotating objects in space, changing the scale of an image object etc. Thus spatial thinking demands its users to think divergently, where everyone can cite the reasons why to use a way or the path he or she takes. Spatial thinking is defined as the knowledge, skills, and habits of mind to use concepts of space, tools of representation like maps and graphs, and processes of reasoning to organize and solve problems [8]. There are two kinds of representation i.e. internal representation and eksternal representation. Internal representation in this study means an internal procesess including coding, abstraction and distortion to build a thinking product. Eksternal representation in this study is a a result of a thinking in visual, verbal or kinesthetics. Spatial thinking in this study is identified through geometrical problems of transforming a 3-dimensional object into a 2-dimensional object image.

1.3. Transformation of Spatial Object Representation
Spatial object representation transformation is a tool for describing representation. The transformation process of representation of spatial objects occurs in the mind [9]. There are two classes of mental transformation: object-based spatial representations and egocentric perspective transformation. Object-based transformations are imagined rotation or translations of objects relative to reference frame of environment. Egocentric perspective transformations are imagined rotations and transformation of one’s point-of-view relative to that reference frame. Both of them are imagined rotations or translation of mental process with different aspect. Transformation of object representation, encoding operations establish mental representation of the spatial of the spatial word by mentally rotating object, mentally extrapolating, mentally extending [10]. Mental representation is the internalization of physical change, that is, there is an internal process that is done when we interact with the world or physical objects. The interaction performed by each individual is different, including to solve the same problem. Transformation of spatial object representation in this study is referred to an operation used to change
perspective (frame of reference), change orientation (rotation), change shape, move thoroughly, and reconfigure parts of an object in mind.

1.4. Spatial Reasoning Process
Spatial reasoning process is a process when an information of the object of space and information associated with various ways such as measurement, observation, and conclusion. The conclusion is used up to a valid conclusion about an object, relationship, and how to determine the completion of a particular task. The process of spatial reasoning is used in the inference of all possible spatial relationships between one group of objects using a particular subgroup. The distortion of spatial thinking and abstract spatial thinking are two things that play a role in the process of spatial reasoning.

2. Methods
Exploratif descriptive through qualitative approach was used in this study.

2.1. Subject
The subjects of this study were the 5th grade of visual learning style of both boy and girl students. The 5th-grade students were chosen because they have learned space matter in school and still in concrete operation phase, also it is important to study this subject in the earliest of cognitive development. By choosing a visual subject, students are expected to optimize spatial thinking that is closely related to visualization. Studies show that the gender influence students’ performance and concept in mathematics and that boys do better in in space [11-13] than the girls. Thus, besides exploring the spatial thinking of students, the author also distinguishes between boys and girls’ work to provide comprehensive information.

2.2. Technique
Subjects did geometry test for three days with one problem in each day. The subjects’ work in solving geometry test were explored and analyzed through an unstructured interview where the interviewer follows the interviewees’ narration and generates questions spontaneously based on his or her reflections on that narration. A deep interview conducted to the subject regarding what they thought, done, written and spoken while doing the test. The audio and video format were used to record subjects along the research process from solving geometry test until interview section.

2.3. Instrument
Two types of instruments were used in this study: main instrument and tracer instrument. The main instrument contained statements about the spatial concepts with which the students could explore the cube nets from the given cube object. The tracer instrument was used to classify student learning style. Four validity test data were used in this study: 1) credibility test (internal validity), 2) test of dependency (reliability), 3) confirmation test (objectivity) and 4) transferability test.

3. Result and Discussion
3.1. Internal Representation
Internal activity begins when the senses of the subject capture the cube model that is in front of him, the subject was looking at the model of the cube in front of him. The subject encoded the information obtained when viewing the cube model into his mind, the subject attributing information possessed to the current information that is the subject of reconstructing the cube into cubes of the cube in his mind. Representation internal shown when the subjects silence before deciding on something in speech or act. The subjects both boy and girl imagine opening the cube model until a cube net is obtained. The following text represented the explanation of students related to how they imagine the cube object to cube nets.
3.2. External Representation
Both the female and male subjects are able to represent what they have understood by using words, using a new term for ease of completion, mention the steps of drawing a cube sketch based on a given cube model. When the subject was asked whether there is another way to find out the cube nets, the subject can easily explain. The subject mentions another way of determining the sketch of the cube nets of a cube model.

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Figure 1. The explanation of students about cube nets

| Question | Can you tell me, how did you draw a sketch of cube nets based on the given dotted cube model? |
|----------|--------------------------------------------------------------------------------------------------|
| Boy      | Firstly, I imagined opening the model of this cube, like a car changes into a transformer robot until the shape of the image of the nets made, then I make the net’s sketch... |
| Girl     | ... I try to start imagining the letters or anything I've ever seen which can be made with 6 pieces of a square. Thus, if it folded then can construct a cube, ... |

Figure 2. Student’s technique to determine the sketch of the cube nets

This shows the verbal external representations that have been performed by the subject.

Boy : the model of this cube can be cut out ribs then if I opened it, it will become cube nets then I drew the sketch. But I prefer to imagine it because the cube model is only one, meaning it can only be used to make a cube net only
Girl : you just opened the model of the cube and it will form the cube nets.

Figure 3. Boy’s [left] and girl’s [right] cube net drawing

The subjects state what they thought in a sketch of cube nets based on the cube nets comes up in their mind. This indicated the subject expressing what is thought in an external representation of images. The subjects sketched the cube object as a reference to determine the dots on each square in the cube net. They marked dots on every square on cube nets corresponding to many points on each side of the cube object. This shows the visual external representation performed by the subject.

3.3. Transformation of Spatial Object Representation
In transformation representation of this spatial object happened dimension transformation that is a change of way of view/orientation of the subject in transforming 3-dimensional object to 2-dimensional object image.
Figure 4. The transformation from 3-dimensional to 2-dimensional object.

The process of changing the shadow cubes shows the occurrence of the cube shadow webs indicating a change of frame of reference from viewing 3-dimensional abstract objects to 2-dimensional abstract objects. Then, the subject changes the shadow cubes with a particular orientation/ viewpoint. Changes in the way of the view of 3-dimensional objects are shown by using the benchmark and the opponent as a reference transforming the dimension into a shadow cube, from the shadow cube to the shadow cube webs and then declaring it into a sketch of the image of the cube nets.

Figure 5. The transformation the representation of spatial objects

The subject shifts the square on the cube nets to obtain a model of different cube nets than before. The subject changed the shape of the old cube webs into new cube webs of how to "shift" a square on the cube nets. Subjects imagine "shifting" square on the cube nets to acquire the shape of new cube webs. This shows the subject of transforming the representation of spatial objects.

3.4. Spatial Reasoning Process
Manipulation processes that occur in the subject both boy and girl included several things: 1) trying to make a model of cube nets by imagining familiar objects and can to be made into cube nets. 2) plotting location side, 3) placing the dots on the three-dimensional object to the corresponding side of the cube net. In constructing a cube net, both boy and girl are alike. This manipulation is done by observing the cube forms elements and imagining familiar objects to be made into cube nets. After manipulation of the 3-dimensional object into a 2-dimensional object, the subjects started to determine the side pair i.e. determining which side is in front, back, left or right side.

Figure 6. The Spatial Reasoning Process
This process requires consideration and leads to the decision that is obtained by the side pair. This indicates the subject is doing the reasoning process. When marking the cube nets corresponding to the number of nodes on the side of the cube whose position corresponds to each square in the cube net. There is a different way between boys and girls about how they placed the dots to the cube net. In this activity, the subject uses a scanning technique to check the square in the cube nets whose position corresponds to the sides of the dotted cube. The process of sketching the cube nets requires manipulation of matching such that each square in the cube net is marked with dots as many as dots on the cube object. This process requires consideration of some information i.e. using side pairs and placing dots. This shows that there is a reasoning process in this activity. When they asked to make different cub net, subject solved a similar problem by splitting the cube, predicting the direction of movement, cutting the cube and arranging parts. This creates a consistent attitude in using the sketching steps of the cube webs.

The subject explained the reason for choosing the method they used. The process of choosing involves grouping things according to the criteria, taking into consideration some of the ways it knows, comparing, interpreting and deciding which way to choose and deciding in a conclusion. This indicates that the subjects have made the process of reasoning while expressed the reasons why choose it. Drawing different net is a manifestation of reasoning process because there is a processing information, comparing manipulations, considering making decisions before draw cube net until understanding the way. Revealing a different way to obtain different cube net is one of the reasoning processes. This indicates the subject has done the reasoning process. The subjects are able to make a conclusion of what they have done. The subjects revealed what need to be done in solving the problem and in making a conclusion. This shows that the subjects have done the reasoning process.

4. Conclusion
In this research we have obtained research results that reveal the spatial thinking profile of elementary students in solving geometric problems based on gender: 1) male students generally use mental activities such as imagining and using subjects experience to solve problems. The imagining process 3D to 2D representation is similar to the process of of transforming robot transformers into cars. This shows the subject using an analogy in solving geometrical problems, 2) Female students in general use visual activities such as drawing sketch and creating schemes in solving geometric problems, 3) Male students tend to use mental activity by imagining and imagining spatial objects, 4) Female Students find a fast way to complete the marking cubes using pair formulation points 2-5, 3-4 and 1-6, 5) Female students use more stages of dimensional transformation. Finally, we do hope that this study can contributing the theories about spatial thinking profiles of elementary students in solving geometric problems as a foundation for the development of learning models, particularly in related to the improvement of spatial thinking ability in geometric problem solving. Also, as a reference for teachers to identify the profile spatial thinking of elementary students in geometry-based problem solving.

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