Effects of Strong Langmuir Turbulence on Hydrogen Lines

I Hannachi, M Meireni, H Capes, F Guzman, M Koubiti, Y Marandet, M T Meftah, L Mouret, J Rosato, R Stamm

University of Batna, Department of Physics, Batna, Algeria
Aix-Marseille University and CNRS, PIIM, 13397 Marseille, France
University of Ouargla, Department of Physics, 30000 Ouargla, Algeria

E-mail: roland.stamm@univ-amu.fr

Abstract We consider conditions of strong Langmuir turbulence, for which a wave packet cycle is observed. The dynamics of such intense Langmuir electric fields may affect the line broadening of simple atoms. We use a simple stochastic model to evaluate the effect of such turbulent fields on the Lyman $\alpha$ line in low density plasmas.

1. Introduction
Following the pioneering work of Baranger and Mozer [1], the effect of plasma oscillations on spectral line shapes in plasmas have long been studied [2]. A problem of interest is the case of strongly non linear wave turbulence, observed in conditions where the energy of the Langmuir wave is not small compared to the plasma thermal energy. Such conditions may be reached during the interaction of the solar wind with the planet magnetospheres, or in laser plasmas. They may occur also each time an energetic electron beam interacts with plasma. In the strong Langmuir turbulence regime, Langmuir wave packets will form, collapse, dissipate and reform, thus creating a wave packet cycle [3]. With the help of a stochastic model for the Langmuir wave packet cycle [2], the effect on a line shape of non linear wave collapse may be investigated. Using the Lyman $\alpha$ line of hydrogen emitted in the center of mass as a benchmark, we have analyzed the changes in the line shape for different parameters found in the strong Langmuir turbulence regime.

2. Renewal process for the wave packet cycle in the strong Langmuir turbulence regime
We consider the case of electron-plasma or Langmuir waves, which are electrostatic waves in unmagnetized plasma. In the linear regime, the Langmuir waves oscillate at a high-frequency close to the plasma frequency $\omega_p = \sqrt{4\pi Ne^2/m}$ (with $e$ and $m$ the electron charge and mass, and $N$ the density), and coexist with transverse electromagnetic waves and low-frequency ion sound waves. The dispersion relations of high-frequency waves depend on the density through the plasma frequency, enabling the density fluctuations of the ion sound waves to affect the high-frequency waves [3]. This provides a nonlinear coupling mechanism between low and high frequency waves. Equations taking account of such a coupling where first derived by Zakharov [4]. The first Zakharov equation explains how the density fluctuations affect Langmuir waves. Weaker densities correspond to a higher refractive index, causing the high-frequency waves to refract into low density regions. A Langmuir wave packet can produce a density depression via the ponderomotive force, which is a nonlinear force proportional to the gradient of the square of the wave electric field.
In a region where the wave packet electric field is strong, this force can be shown to expel the plasma [3], thus creating the density depression and trapping the wave packet. The non-linear dynamics of such regions has been studied with the Zakharov equations [4], and particle-in-cell simulations. It has been found that the three-dimensional wave packets spatially narrow and become more intense. Such simulations also reveal that for a steadily driven system, many collapsing Langmuir wave packets are present. Since they are unstable, these packets will form, collapse, dissipate, and reform, thus creating a wave packet cycle. The characteristic times for such cycles have been studied [5], and take values much larger than one in units of the inverse electron plasma frequency. If we consider an emitting atom immersed in such turbulent plasma, its energy levels will be strongly affected by the electric field of the coherent wave packets located in the vicinity.

A stochastic model already proposed for the Langmuir fields in the linear regime [2] may also be used for strong Langmuir turbulence, for which the ratio of the Langmuir field energy and plasma energy densities \( W = E_L^2 / 8 \pi N k T \) reaches values of the order of one or larger. Our stochastic model uses probability density functions (PDFs) for the lifetime of a wave packet cycle, and for the values of the electric field in the wave packet. We assume that the Langmuir field applied on an emitter is a sequence of oscillating field with random amplitude and phase, \( \tilde{E}(t) = \sum_j \tilde{E}_j \cos(\omega_p t + \varphi_j) \), where the field \( \tilde{E}_j \) and phase \( \varphi_j \) change at times prescribed by a waiting time distribution (WTD).

A Markovian choice for the WTD is a simple Poisson law \( \nu \exp(-\nu t) \), with \( \nu \) the jumping frequency, but memory effects may be retained with other choices for this distribution. In this work we will take a Poisson law with the jumping frequency equal to the inverse of the lifetime of a wave packet cycle [3]. Whereas the phase is uniformly distributed in the interval \((0, 2\pi)\), the Langmuir field values are chosen to be distributed with a log-normal PDF. Such PDFs of fluctuating plasma parameters have been found in solar wind plasmas [6] or in tokamak plasmas [7]. We have used a log-normal PDF with a standard deviation \( \sigma = 1/4 \) for the underlying normal distribution. For a ratio \( W=1 \), the average Langmuir field \( E_L \) is generally much larger than the Holtsmark field \( E_0 = e r_0^2 \), with \( r_0 \) the mean interparticle distance. At a density \( N = 10^{12} \, \text{cm}^{-3} \) and a temperature \( T = 10^4 \, \text{K} \), \( E_L \) is almost 50 times larger than \( E_0 \).

3. Results: Lyman \( \alpha \) line shape in the presence of strong Langmuir turbulence
We have calculated Lyman \( \alpha \) with a numerical integration of the Schrödinger equation, and an average over about 100 histories of the field. It is possible to compute pure Langmuir profiles, or profiles resulting from a convolution with a thermal Stark effect. On Fig.1(a) pure Langmuir profiles for \( \nu = \alpha_p / 50 \) are calculated for four values of the average Langmuir field expressed in units of \( E_0 \). A first doubling of \( E_L \) results in an increase of the width by a factor 2.6, whereas a second doubling increases the width only by 20%. For \( E_L = 100 \, E_0 \), one can see on Fig.1(b) a linear behaviour of the width as a function of \( \nu \), with a doubling of the width following a doubling of \( \nu \). Compared to pure Stark profiles obtained with an approximated impact, or a simulation, the action of Langmuir wave packets may result in a strong broadening, as can be seen on Fig.1(c) where the full profile is almost three times larger than the pure Stark for a density \( N = 10^{12} \, \text{cm}^{-3} \), \( E_L = 100 \, E_0 \) and \( \nu = \alpha_p / 50 \). Keeping \( W \) values of the order of 1, this effect is much reduced for a higher density case \( N = 10^{15} \, \text{cm}^{-3} \). Even for an unrealistic value \( \nu = \alpha_p / 5 \), the broadening effect of strong turbulence Langmuir waves is limited to 30%.

4. Conclusion
We have described the main features of Strong Langmuir turbulence, a regime which is expected as the energy of the Langmuir wave becomes comparable to the plasma thermal energy. The effect of wave packet cycles may be modelled by a stochastic process using PDFs for the lifetime of the cycle, and the values of the turbulent electric field. This field can reach values up to two orders of
magnitude larger than the Holtsmark field in such conditions. For plasmas with a density $N=10^{12}$ cm$^{-3}$ and a temperature $T=10^4$ K, an additional broadening by a factor of three has been calculated. The broadening effect increases linearly with the jumping frequency $\nu$, but has saturation behaviour as the average Langmuir field is increased. For the higher density $N = 10^{15}$ cm$^{-3}$, the broadening effect is weaker if one uses realistic $\nu$ values. Further work will concern other lines than Lyman $\alpha$, other PDFs for the electric field, the role of memory in the stochastic process, and applications to various plasmas affected by strong Langmuir turbulence.

**Acknowledgments**

This work was carried out within the framework of the European Fusion Development Agreement and the French Research Federation for Fusion Studies. We also acknowledge the support of the French National Research Agency (contract ANR-11- BS09-023, SEDIBA).

**References**

[1] Baranger M and Mozer B 1961 *Phys. Rev.* **123** 25
[2] Oks E 1995 *Plasma Spectroscopy, The influence of Microwave and Laser Fields* (Berlin: Springer Ser. on Atoms and Plasmas)
[3] Robinson P 1997 *Rev. Mod. Phys.* **69** 507
[4] Zakharov V E 1972 *Sov. Phys. JETP* **35** 908
[5] DuBois D F, Rose H and Russel D 1988 *Phys. Rev. Lett.* **61** 2209
[6] Bakshi P and Kalman G 1976 *J. Geophys Res* **47** 307
[7] Sattin F et al 2004 *Phys. Plasmas* **11** 5032