Simple technique for generating the partially coherent and partially polarized electromagnetic source

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Abstract. The technique for generating the partially coherent and partially polarized source starting from the completely coherent and completely polarized laser source is proposed and analyzed. This technique differs from the known ones by the simplicity of its physical realization. An original technique for measuring the cross-spectral density matrix is employed. Experimental results of the characterization the coherence and polarization properties of the generated source are shown.

1. Introduction

Many recent investigations have been devoted toward the problem of generating the partially coherent and partially polarized electromagnetic source in the vector coherence theory [1]. Very important advances have been reported specially in connection with the use of spatial light modulators (SLM) [2,3,4]. In the present paper we propose a very simple technique for modulating the coherence and polarization of laser radiation which does not need any LC SLM. This technique bears some resemblance with the one reported in [5], but differs from it as by another action principle so by the simplicity.

In practice, together with theoretical results, a very strong urge of experimental investigations is present in the electromagnetic coherence theory. So, once the desired secondary partially coherent and partially polarized source has been created, it must be subject to experimental characterization. The idea of such a determination is well known [6-8], but its physical realization has not been yet reported. Here we propose also a rather simple technique for characterizing the coherence of electromagnetic source, which was used in our experiments.

2. Background

As has been shown by Wolf [9], the second-order statistical properties of a random planar (primary or secondary) electromagnetic source can be completely described by the cross-spectral density matrix (for brevity we omit the explicit dependence of the considered quantities on frequency $\nu$)

$$W(x_i, x_j) = \begin{bmatrix}
\langle E'_r(x_i) E'_r(x_j) \rangle & \langle E'_r(x_i) E'_l(x_j) \rangle \\
\langle E'_l(x_i) E'_r(x_j) \rangle & \langle E'_l(x_i) E'_l(x_j) \rangle
\end{bmatrix},$$  

(1)

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where \( E_x(x) \) and \( E_y(x) \) are the orthogonal components of the electric field vector \( \mathbf{E}(x) \), asterisk denotes the complex conjugate, and the angle brackets denote the average over the statistical ensemble. Using this matrix, Wolf defines the following three fundamental statistical characteristics of the source:

the power spectrum

\[
S(x) = \text{Tr} \mathbf{W}(x, x),
\]

the spectral degree of coherence

\[
\mu(x_1, x_2) = \frac{\text{Tr} \mathbf{W}(x_1, x_2)}{[\text{Tr} \mathbf{W}(x_1, x_1) \text{Tr} \mathbf{W}(x_2, x_2)]^{1/2}},
\]

and the spectral degree of polarization

\[
P(x) = \left[ 1 - \frac{4 \text{Det} \mathbf{W}(x, x)}{[\text{Tr} \mathbf{W}(x, x)]^2} \right]^{1/2}.
\]

In equations (2) – (4) \( \text{Tr} \) stands for the trace and \( \text{Det} \) denotes the determinant.

It is not out of place to mention here that, equally with Wolf’s definition of the degree of coherence, there are known the other definitions [10-12]. But, for our following analysis, the definition given by equation (3) proves to be quite sufficient.

3. Technique for generation

Let the incident laser radiation the primary source, linearly polarized in some plane normal to the direction of propagation, which can be characterized by the cross-spectral density matrix

\[
\mathbf{W}_{ps}(x_1, x_2) = S_0 \exp \left( - \frac{x_1^2 + x_2^2}{4 \epsilon^2} \right) \begin{bmatrix} \cos^2 \theta & \sin \theta \cos \theta \\ \sin \theta \cos \theta & \sin^2 \theta \end{bmatrix},
\]

where \( S_0 \) is the value of power spectrum at the source centre, \( \epsilon \) is the effective (rms) size of the source, and \( \theta \) is the angle that the direction of the linear polarized electric field makes with the \( x \) axis. It can be shown that in this case \( |\mu_{ps}(x_1, x_2)| = 1 \) and \( P_{ps}(x) = 1 \), i.e. that such a source is completely coherent and completely polarized.

It is well known that we can change the coherence of the source by means of a dynamic random diffuser [13,14]. However due to the need of measuring all the matrix components we need to ensure that the interfering beams related to the diagonal components must come from the same point, i.e., from the same diffuser. For the off diagonal components, this requirement is quite different. The interfering beams involved must come from different diffusers. These facts because of the parallel and orthogonal components are completely dependent and completely independent from each other respectively.

So, we can make use of a Mach-Zehnder interferometer starting with a polarizing beam splitter PBS that separates the orthogonal field components \( E_x(x) \) and \( E_y(x) \) so that each of them can be independently changed by means of two rotating ground glass plates GGP1 and GGP2 placed at the opposite arms of the interferometer. At the exit of the interferometer is placed a 50/50 beam splitter cube as is shown in figure1.

The considering system can be considered as a thin phase screen with an amplitude transmittance given by the Jones matrix

\[
\mathbf{T}(x) = \begin{bmatrix} \exp[i \varphi_1(x)] & 0 \\ 0 & \exp[i \varphi_2(x)] \end{bmatrix}
\]

where \( \varphi_1(x) \) and \( \varphi_2(x) \) are the position-dependent random variables associated with the roughness of the ground glass plates. Following Ref. 2, we assume that the random variables \( \varphi_1(x) \) and \( \varphi_2(x) \) possess the same Gaussian probability density and are Gauss-correlated for two different points \( x \) and \( x_2 \), separated by a distance \( \xi \).
Figure 1. Schematic illustration of the technique for generating the partially coherent and partially polarized source: BS, beam splitter; PBS, polarizing beam splitter; M, mirror; GGP$_1$, GGP$_2$, rotating ground glass plates.

For the off diagonal components, both variables $\varphi_1(x)$ and $\varphi_2(x)$ are originated by two different ground glass plates, so we can assume that they are statistically independent, i.e.

$$ p[\varphi_1(x), \varphi_2(x)] = p[\varphi_1(x)]p[\varphi_2(x)]. $$

(7)

The cross-spectral density matrix of the secondary source (SS) created just behind the interferometer can be calculated as follows:

$$ W_{ss}(x_1, x_2) = \langle T^*(x_1)W_{ss}(x_1, x_2)T(x_2) \rangle, $$

(8)

where the dagger denotes the Hermitian conjugation. On substituting from equations (5) and (6) into equation (8), we find

$$ W_{ss}(x_1, x_2) = S_x \exp\left( -\frac{x_1^2 + x_2^2}{4\sigma^2} \right) \times \exp\left(-\sigma^2 \left[ 1 - \exp\left(-\frac{x_1^2}{2\gamma^2}\right) \right] \right) \cos^2 \theta \exp\left(-\sigma^2 \sin \theta \cos \theta \right) \exp\left(-\sigma^2 \sin \theta \cos \theta \right) \exp\left(-\sigma^2 \left[ 1 - \exp\left(-\frac{x_2^2}{2\gamma^2}\right) \right] \right) \sin^2 \theta. $$

(9)

Then, substituting this result into definitions given by equations (3) and (4), we obtain, respectively,

$$ \mu_{ss}(x_1, x_2) = \exp\left(-\sigma^2 \left[ 1 - \exp\left(-\frac{x_1^2}{2\gamma^2}\right) \right] \right), $$

(10)

$$ P_{ss}(x) = \sqrt{1 - 4\sin^2 \theta \cos^2 \theta \left[ 1 - \exp\left(-2\sigma^2\right) \right]} $$

(11)

Equations (10) and (11) shows that the generated source is partially coherent and partially polarized. The output degree of coherence and polarization depends on the parameters $\sigma$ and $\gamma$, and the degree of polarization reaches its minimum value of $P_{ss}(x) = \exp[-2\sigma^2]$ with the choice of $\theta = \pi/4$.

Also we can see that for different values of the root mean square phase retardation of the GGP, particularly for $\pi \leq \sigma$ the degree of coherence and polarization changes in the full range from 1 to 0. The behavior of equations (10) and (11) for different values of $\sigma$ is illustrated in figure 2.
4. Technique for characterization

Once the desired secondary partially coherent and partially polarized source has been created, it must be subject to experimental characterization, i.e. the elements $W_{ss}$ of the matrix $W_{ss}$ have to be experimentally determined. Taking into account the symmetry of the cross-spectral density given by equation (9), for this purpose the technique sketched schematically in figure 3 can be used.

This technique represents a modified version of well known two-pinhole Young’s experiment. The Mach-Zehnder interferometer is employed here to extend the effective distance between the pinholes so that the radiation emerged from each pinhole can be processed independently by finite size optical components. The polarizers $P_1$ and $P_2$ serve to cut off only one of the orthogonal field components. The removable rotators $R_1$ and $R_2$ serve to produce the rotation of one of the transmitted field component through 90°. For such a purpose a suitably oriented half-wave birefringent plate can be used.

Figure 2. Modulation range of the degree of coherence and the degree of polarization for different values of the root mean square phase retardation of the GGP and $\gamma=2$.

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Figure 2. Modulation range of the degree of coherence and the degree of polarization for different values of the root mean square phase retardation of the GGP and $\gamma=2$. 
The power spectrum of the field observed at the output of the interference system in each experiment can be described by the spectral interference law [8]

\[
S_i(x') = S_{ii}^{\text{ss}} \left( \frac{\xi}{2} \right) + S_{jj}^{\text{ss}} \left( \frac{\xi}{2} \right) + 2 W_{ij}^{\text{ss}} \left( \frac{\xi}{2}, -\frac{\xi}{2} \right) \cos \left( \frac{k \xi}{2} x' + \alpha_{ij} \left( \frac{\xi}{2}, -\frac{\xi}{2} \right) \right) (i, j = x, y),
\]

where \( S_{ii}^{\text{ss}} \) and \( S_{jj}^{\text{ss}} \) are the power spectra of the field components in the pinhole position \( \xi/2 \), \( k \) is the wave number, \( z_0 \) is the geometrical path between the pinhole plane and the observation plane, and \( \alpha_{ij} = \arg W_{ij}^{\text{ss}} \). As well known, the measure of the contrast of the interference fringes is the so-called visibility coefficient defined as

\[
V_i(\xi) = \frac{[S_i(x')]_{\text{max}} - [S_i(x')]_{\text{min}}}{[S_i(x')]_{\text{max}} + [S_i(x')]_{\text{min}}}. \tag{13}
\]

On substituting from equation (12) with \( \cos(.) = \pm 1 \) into equation (13), we find that

\[
|W_{ij}^{\text{ss}}(\xi)| = \frac{1}{2} \left( S_{ii}^{\text{ss}} \left( \frac{\xi}{2} \right) + S_{jj}^{\text{ss}} \left( \frac{\xi}{2} \right) \right) V_i(\xi). \tag{14}
\]

The spectra \( S_{ii}^{\text{ss}} \) and \( S_{jj}^{\text{ss}} \) can be easily measured when one of the pinholes is covered by an opaque screen. Hence, measuring in each experiment the visibility \( V_i \), power spectra \( S_{ii}^{\text{ss}} \), and phase \( \alpha_{ij} \), one can determine all the elements \( W_{ij}^{\text{ss}} \) of the matrix \( W_{ss} \). The degree of coherence and the degree of polarization of the generated source can be then calculated using definitions given by equations (3) and (4).

5. Experiments and results
The principal part of the experimental setup was composed of two interferometers shown in figures 1 and 3. As the primary source we used the expanded well collimated beam generated by the He-Ne laser, so that the direction of its (linear) polarization made tilt of 45˚ with respect to the laboratory axes. To generate the secondary source with two different values of the transversal length of coherence, we used two pairs of ground glass plates with the diffusion angles of 10˚ and 30˚. The interference pattern at the first output of the second interferometer was registered by the CCD camera connected to the computer and the corresponding power spectra were measured by the photodiode with optical power meter located at the second output.

The results of the experiments for different pairs of ground glass plates are plotted in figure 4 together with their theoretical interpolations in accordance with equation (10). The calculated spectral degree of polarization was about zero for both experiments. As can be seen, the obtained experimental results are in a good correspondence with the theoretical predictions.
6. Conclusions
We have proposed a rather simple technique for generating the partially coherent and partially polarized electromagnetic source using two rotating ground glass plates placed at the opposite arms of a Mach-Zehnder interferometer. The efficiency of the proposed technique has been confirmed with the physical experiment. We would like to stress particularly that such a source represents a special case of the well known Gaussian Schell-model uniformly polarized electromagnetic source [15], which appears here not as some handy mathematical construction but as a true product of physical experiment. Besides, the diagonal form of the cross-spectral density matrix given by equation (9) permits to find in a closed form the coherent-mode structure of the source [16], a fact that results in considerable simplification when analyzing optical systems with such an illumination [17].

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Figure 4. Experimental results for different pairs of GGP in dotted lines and its theoretical interpolations in solid lines.
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