The quantum-to-classical transition: Bohr’s doctrine of classical concepts, emergent classicality, and decoherence

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Abstract
It is now widely accepted that environmental entanglement and the resulting decoherence processes play a crucial role in the quantum-to-classical transition and the emergence of “classicality” from quantum mechanics. To this extent, decoherence is often understood as signifying a break with the Copenhagen interpretation, and in particular with Bohr’s view of the indispensability of classical concepts. This paper analyzes the relationship between Bohr’s understanding of the quantum–classical divide and his doctrine of classical concepts and the decoherence-based program of emergent classicality. By drawing on Howard’s reconstruction of Bohr’s doctrine of classical concepts, and by paying careful attention to a hitherto overlooked disagreement between Heisenberg and Bohr in the 1930s about the placement of the quantum–classical “cut,” we show that Bohr’s view of the quantum–classical divide can be physically justified by appealing to decoherence. We also discuss early anticipations of the role of the environment in the quantum–classical problem in Heisenberg’s writings. Finally, we distinguish four different formulations of the doctrine of classical concepts in an effort to present a more nuanced assessment of the relationship between Bohr’s views and decoherence that challenges oversimplified statements frequently found in the literature.

Key words:
Quantum-to-classical transition, classical concepts, Niels Bohr, Copenhagen interpretation, environmental decoherence, entanglement
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1 Introduction

Bohr’s interpretation of quantum mechanics, first sketched in his Como lecture of 1927 (Bohr, 1928) and subsequently developed in his papers in the late 1920s and 1930s, is characterized by two main features. The first feature is the assumption that classical concepts are indispensable for describing the results of experiments involving quantum phenomena, in spite of their limited applicability. The second feature is the concept of complementarity, which describes the need to use mutually exclusive experimental arrangements in the use of classical concepts such as position and momentum (Camilleri, 2007). While the notion of complementarity is by far the more prominent of the two, as Howard (1994, p. 202) explains, “the doctrine of classical concepts turns out to be more fundamental to Bohr’s philosophy of physics than are better-known doctrines, like complementarity.” In perhaps his clearest expression of the doctrine, Bohr argued:

It is decisive to recognize that, however far the phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms. The argument is simply that by the word “experiment” we refer to a situation where we can tell others what we have done and what we have learned and that, therefore, the account of the experimental arrangement and of the results of the observations must be expressed in unambiguous language with suitable application of the terminology of classical physics (Bohr, 1949, p. 209).

Scholars have long pondered over precisely what Bohr meant by “classical concepts” or why he felt they should play such a primary role in quantum physics. Bohr’s doctrine was a controversial one and remains the subject of much debate and disagreement to this day. Bohr believed quantum physics to be the universally correct theory, which thus would in principle—i.e., given an appropriate experimental arrangement—have to also apply to the description of macroscopic measurement apparatuses and observers. However, as the passage above indicates, Bohr felt that the experimental setup must be described in terms of classical physics, if it is to serve as a measuring instrument at all. One is left with the impression from Bohr’s writings that the quantum–classical divide is a necessary part of the epistemological structure of quantum mechanics. This view finds expression in the works of physicists like Heisenberg and Rosenfeld, who were deeply influenced by Bohr.

We may therefore raise two questions about the views of Bohr and his contemporaries, such as Heisenberg. First, what exactly was the meaning, justification, and location of the quantum–classical divide? Second, how were transitions between the quantum and the classical realm, i.e., the “crossings” of the quantum–classical boundary, understood and explained—in particular,
in what sense and how may classicality arise from within quantum mechanics? These questions lie at the heart of many interpretations of quantum mechanics whose main goal is to solve the so-called measurement problem—a merely historically motivated term that should be more appropriately subsumed under the general heading of the problem of the quantum-to-classical transition. Remarkably, recent developments in the study of decoherence (for reviews, see Joos et al., 2003; Zurek, 2003; Schlosshauer, 2004, 2007) have shown that it is possible to address this problem (at least partially) by realizing that all realistic quantum systems are inevitably coupled to their environment. Studies of decoherence have shed new light on the emergence of classical structures from within the quantum realm and have led to enormous progress in a quantitative understanding of the quantum-to-classical transition.

The natural question is then to ask to what extent such a research program may run counter to Bohr’s assumptions of intrinsic, undervisible classical concepts and of different, mutually contradictory descriptions as embraced by the complementarity principle. Can fuzzily defined concepts such as measurement, quantum–classical dualism, and complementarity be reduced to effective concepts derivable in terms of unitarily evolving wave functions and the influence of decoherence, and can they thus shown to be but superfluous semantic or philosophical baggage? Needless to say, this is an extremely complex question pertaining to many issues of both theory and interpretation.

Questions concerning the relationship between the decoherence approach and Bohr’s interpretation of quantum mechanics have been raised by a number of physicists working on decoherence. However, there has been little serious and nuanced investigation on this matter. For example, Zeh has contrasted the dynamical approach of decoherence with the “irrationalism” of the Copenhagen school (Joos et al., 2003, p. 27). However, one may object that this characterization perpetuates the familiar myth of the Copenhagen interpretation, and in particular Bohr’s viewpoint, as entailing some kind of radical subjective idealism. It is our view that a deeper understanding of the relationship between the views of Bohr and his followers and the program of decoherence merits more careful historical and philosophical investigation.

This paper is organized as follows. Sec. 2 prepares the way by sketching the relevant components of the problem of the quantum-to-classical transition (Sec. 2.1) and by providing a brief review of the decoherence program (Sec. 2.2). In particular, we will discuss to what extent, and in what sense, decoherence may be said to explain the emergence of classicality (Sec. 2.3).

Secs. 3 and 4 then examine relationships between the decoherence account of the quantum-to-classical transition and Bohr’s doctrine of classical concepts, in the following two ways.
First, in Secs. 3.1 and 3.2, we will investigate the degree to which Bohr’s understanding of the quantum–classical divide may be recovered as emergent, and how this may challenge or support both Bohr’s interpretation of quantum mechanics and his intuition about such concepts. Inevitably, given the large degree of scholarly dispute about the exact meaning of Bohr’s writings and his views, this poses a difficult problem. Rather than take into account different possible readings of Bohr’s philosophy, we will focus specifically on his view of the quantum–classical divide in the context of the interaction between object and measuring apparatus, and the way it may differ crucially from Heisenberg’s interpretation. A close examination of Bohr’s and Heisenberg’s writings and correspondence in the 1930s reveals an underlying disagreement on how the dividing line, or “cut,” is to be understood (Sec. 3.3). This point has often gone unnoticed. We can gain a deeper insight into Bohr’s position, and why he disagreed with Heisenberg on the cut, through Howard’s reconstruction of Bohr’s doctrine of classical concepts (Sec. 3.4). This, in turn, sheds new light on the points of convergence and divergence between Bohr and the current decoherence-based program of emergent classicality. We will also show (Sec. 3.5) how certain passages of Heisenberg’s writing point out the importance of the openness of quantum systems in the problem of classicality.

Second, in Sec. 4 we look in more detail at the doctrine of classical concepts, as it was understood by Bohr and his followers, in particular Heisenberg, Weizsäcker, and Rosenfeld. While their views have often been taken as representative of what is commonly called the “Copenhagen interpretation,” we must be clear that as it is commonly used, this term refers to a range of different physical and philosophical perspectives which emerged in the decades following the establishment of quantum mechanics in the late 1920s. ¹ Indeed, a close reading reveals that there is not one single version of Bohr’s “doctrine of classical concepts” which emerges from the writings of the Copenhagen school (Sec. 4.1). Here we will draw a distinction (Sec. 4.2) between those who defended the view that we must use classical concepts in quantum mechanics (Bohr) and those who took a more pragmatic position, in arguing that it is simply the case that we do use classical concepts (Weizsäcker). A key to understanding the relationship between decoherence and the doctrine of classical concepts is the distinction which can be drawn between an epistemological and physical formulation formulation of the doctrine (Sec. 4.3). In Sec. 4.4 we shall

¹ As Jammer points out in his Philosophy of Quantum Mechanics, “the Copenhagen interpretation is not a single, clear-cut, unambiguously defined set of ideas but rather a common denominator for a variety of related viewpoints. Nor is it necessarily linked with a specific philosophical or ideological position” (Jammer, 1974, p. 87). Indeed the very idea of a unitary interpretation only seems to have emerged in the 1950s in the context of the challenge of Soviet Marxist critique of quantum mechanics, and the defense of Bohr’s views, albeit from different epistemological standpoints, by Heisenberg and Rosenfeld.
discuss to what extent decoherence can be regarded as providing a physical account of Bohr’s doctrine.

This paper avoids the tendency which has become customary in much of the literature to compare the newly emerging ideas on the foundations of quantum mechanics with some reconstructed, or worse still, some imagined, version of the “Copenhagen interpretation.” Instead we focus on what Bohr had to say, as well as on the writings of Heisenberg, Rosenfeld, and Weizsäcker. To what extent their views are in stark contrast with Bohr’s interpretation or when they represent an approach entirely consistent with his general viewpoint is particularly relevant in the context of our discussion of decoherence. Decoherence marks the most successful attempt of explaining the quantum-to-classical transition wholly within the framework of quantum mechanics. But this can be seen as part of a general approach which a number of Bohr’s followers—notably Weizsäcker and Rosenfeld—began to pursue in the 1960s. Far from seeing it as an invalidation of Bohr’s basic insight, they regarded it as providing a justification of his views. Of course, whether this was true to Bohr’s original vision remains a point of some conjecture. However, it would be just as premature and foolish to declare Bohr’s views (an interpretation of quantum mechanics) as invalidated by decoherence (a consequence of quantum theory and a physical process), as to deny that decoherence suggests a reconsideration of the status of Bohr’s concepts. It is our hope that our investigation will establish a more refined view on the complex connections between Bohr’s views and those of his followers on the one hand, and the insights gathered from decoherence and the related decoherence-inspired interpretive programs on the other hand.

2 The quantum-to-classical transition and decoherence

In this section, we shall briefly review the problem of the quantum-to-classical transition and the implications of decoherence for this problem. This will allow us to clearly state the scope of decoherence as required for a careful comparison with Bohr’s views on the quantum-to-classical transition. The basic formalism of quantum measurement and decoherence will also be relevant to Howard’s reconstruction of Bohr’s doctrine of classical concepts discussed in Sec. 3.4 below.

2.1 The problem of the quantum-to-classical transition in quantum mechanics

Broadly speaking, the problem of the quantum-to-classical transition is concerned with the difficulty of how to reconcile the quantum-mechanical descrip-
tion of systems by unitarily evolving wave functions—which may, for example, describe coherent superpositions of macroscopically distinguishable states—with the fact that objects in the macroscopic world are perceived to be in well-defined, robust “classical” states and not in superpositions thereof. Specifically, we may distinguish three related but distinct problems (Schlosshauer, 2007):

1. **Preferred-basis problem.** What singles out the set of “preferred” physical quantities in nature? For example, why do we observe macroscopic systems to be in definite positions rather than in superpositions of positions?

2. **Nonobservability of interferences.** Why is it so prohibitively difficult to observe interference effects, in particular on macroscopic scales?

3. **Problem of outcomes.** How can we explain the apparent probabilistic selection of definite outcomes in measurements?

A fourth aspect is the apparent insensitivity of macroscopic systems to measurements: Measurements on closed quantum systems usually alter the state of the system, posing the question of how states become “objectified” in the sense of classical physics.

The problem of the quantum-to-classical transition is often illustrated in the context of the von Neumann measurement scheme (von Neumann, 1932), which demonstrates how microscopic (and thus “unproblematic”) superpositions are readily amplified to the macroscopic realm. Here one considers interactions of a quantum system $S$ with an apparatus $A$ which is also treated quantum-mechanically. Suppose the apparatus measures a set of states $\{|s_n\rangle\}$ of the system in the sense that the system–apparatus interaction is of the form $|s_n\rangle|a_0\rangle \rightarrow |s_n\rangle|a_n\rangle$, where $|a_0\rangle$ is the initial “ready” state of the apparatus.

The linearity of the Schrödinger equation then implies that the tensor-product state of an arbitrary initial state $\sum_n c_n |s_n\rangle$ of the system and the initial apparatus state $|a_0\rangle$ will evolve into a composite entangled state according to

$$\left( \sum_n c_n |s_n\rangle \right) |a_0\rangle \rightarrow \sum_n c_n |s_n\rangle |a_n\rangle.$$  \hspace{1cm} (1)

Evidently, measurement in this sense amounts to the creation of quantum correlations between the system and the apparatus. No individual state vector can be assigned to either the system or the apparatus at the conclusion of the interaction. All aspects of the problem of the quantum-to-classical transition listed above appear here: In general the final state on the right-hand side of Eq. (1) can be rewritten in different bases; it should be possible to measure interference between different “pointer” positions of the apparatus; and no one outcome (or pointer position “$n$”) has been singled out. Inclusion of further systems, such as a secondary apparatus or human observer, will not terminate the resulting von Neumann chain if these systems are treated in the same way as before (i.e., as interacting quantum systems with globally unitary time
evolution), as already recognized by Bohr (1928) and von Neumann (1932) themselves.

2.2 Basics of decoherence

The origins of the decoherence program can be traced back to a paper of Zeh (1970) who emphasized that realistic macroscopic quantum systems are never closed. If the Schrödinger equation is assumed to be universally valid, Zeh then showed that such systems must become strongly entangled with their environments, leading to a “dynamical decoupling” of wave-function components and the inability of describing the evolution of the system alone by the Schrödinger equation. Zeh proposed that this mechanism could help explain the observed fragility of quantum states of macroscopic systems and the emergence of effective superselection rules. Zeh’s work, although supported by Wigner, remained in relative obscurity for the better part of the next decade. Zeh (2006) recently called this period the “dark ages of decoherence.” In the 1980s, crucial progress was made through contributions by Zurek (1981, 1982), who also played an important role in popularizing decoherence through an article published in Physics Today (Zurek, 1991).

Since then decoherence has evolved into an exponentially growing field of research that has attracted massive attention from the foundational, theoretical, and experimental communities. Decoherence is a consequence of a realistic application of standard quantum mechanics and is an experimentally well-confirmed physical process. It is therefore neither an interpretation nor a modification of quantum mechanics. However, the implications of decoherence are intimately related to interpretive issues of quantum mechanics (Schlosshauer, 2004), in particular to the problem of measurement. Bub (1997) even suggested that decoherence represents the “new orthodoxy” of understanding quantum mechanics, i.e., as the working physicist’s approach to motivating the postulates of quantum mechanics from physical principles.

We shall now briefly summarize the formalism and physical mechanism of decoherence (for in-depth reviews of the field, see Joos et al., 2003; Zurek, 2003; Schlosshauer, 2004, 2007). Readers familiar with decoherence may choose to skip ahead to Sec. 3. We may define decoherence as the practically irreversible dislocalization (in Hilbert space) of superpositions due to ubiquitous entanglement with the environment. The key insight is that every realistic quantum system is open, i.e., interacts with its environment, and that such interactions lead to entanglement between the two partners. This means that there exists

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2 In fact, Wigner abandoned his views on the special role of consciousness in quantum measurement (Wigner, 1961) once he became aware of Zeh’s ideas on decoherence (Wigner, 1995, pp. 66, 75, 215–6, 334, 338, 341, 583, 606, 615).
no longer a quantum state vector that could be attributed to the system alone. In its most basic form, the process of decoherence can be described as a von Neumann measurement of the system by its environment,

$$\left( \sum_n c_n |s_n\rangle \right) |E_0\rangle \xrightarrow{t} |\Psi(t)\rangle = \sum_n c_n |s_n\rangle |E_n(t)\rangle. \quad (2)$$

The superposition initially confined to the system has now spread to the larger, composite system–environment state. Coherence between the components $|s_1\rangle$ can no longer be considered a property of the system alone. Since we have in practice rarely access to all environmental degrees of freedom, this process is irreversible for all practical purposes.\(^3\) The particular basis of the system in which the von Neumann measurement happens is determined by the relevant Hamiltonians, typically the system–environment interaction Hamiltonian ("environment-induced superselection"; Zurek, 1981, 1982).

Suppose now we inquire about the consequences of the environmental interactions for future measurements on the system. The local measurement statistics are exhaustively contained in the reduced density matrix (Landau, 1927; von Neumann, 1932; Furry, 1936) for the system, obtained by tracing over the environmental degrees of freedom,

$$\hat{\rho}_S(t) = \text{Tr}_E |\Psi(t)\rangle\langle\Psi(t)| = \sum_n c_m c_n^* |s_m\rangle\langle s_n| |E_n(t)\rangle\langle E_m(t)|. \quad (3)$$

The presence of interference terms $m \neq n$ embodies the quantum coherence between the different components $|s_n\rangle |E_n(t)\rangle$. Concrete models for the environment show that, with respect to the basis $\{|s_n\}\}$ of the system, the corresponding “relative” states (Everett, 1957) $|E_n(t)\rangle$ of the environment become rapidly orthogonal, meaning that they are able to resolve the differences between the states $|s_n\rangle$ (similar to a “pointer” on a scale).

Then the reduced density matrix (3) will become rapidly orthogonal in the preferred “pointer” basis $\{|s_n\}\}$ dynamically selected by the system–environment interaction Hamiltonian (Zurek, 1981, 1982),

$$\hat{\rho}_S(t) \longrightarrow \sum_n |c_n|^2 |s_n\rangle\langle s_n|. \quad (4)$$

An observer of the system cannot measure interference effects in this basis and therefore cannot empirically confirm the presence of the superposition. However, it is important to bear in mind that in the global system–environment state the superposition is of course still present, or as Joos and Zeh (1985, p. 224) put it, “the interference terms still exist, but they are not there.” Thus

\(^3\) In fact, such effective irreversibility is a necessary condition for Eq. (2) to count as a real, measurement-like decoherence process—otherwise it would be merely a case of “virtual” decoherence (see, e.g., Schlosshauer, 2007, Sect. 2.13).
in principle a suitable measurement (albeit in practice usually prohibitively difficult to implement) could always confirm the existence of the global superposition: No single outcome $|s_n⟩$ is selected, as required by the unitarity of the global evolution.

The environment-superselected states $\{|s_n⟩\}$ are also robust in the sense that they become least entangled with the environment, whereas superpositions of these states are rapidly decohered. In this way, the familiar observables such as position, momentum, and spin are dynamically superselected through the physical structure of the system–environment interaction. Recent related research programs based on studies of further consequences of environmental entanglement, such as quantum Darwinism and redundant environmental encoding, have shown how observers can indirectly gather information about the system without disturbing its state (see, e.g., Blume-Kohout and Zurek, 2006, and references therein).

2.3 Emergence of classicality through decoherence

To what extent decoherence explains the emergence of classical structures and properties from within quantum mechanics depends to a significant degree on the specific axiomatic and interpretive framework of quantum mechanics that one adopts (Schlosshauer, 2004). Since we are chiefly interested in the consequences of decoherence for observers and measurements, matters are most clear-cut if we assume the usual measurement postulates of quantum mechanics (i.e., the projection postulate with Born’s rule). In this case the use of reduced density matrices poses no further interpretive difficulties, and decoherence resolves the preferred-basis problem and explains the difficulty of observing most interference phenomena. Although the problem of outcomes is absent by assumption, decoherence allows us to quantify when it is appropriate to assume that an event or outcome has happened for all practical purposes.

One may go beyond the measurement axioms and use decoherence to show how the projection postulate may be effectively derived from the influence of decoherence in local measurement-like interactions. Decoherence leads to effective ensembles of quasiclassical wave-function “trajectories”—in phase space (Paz et al., 1993) or other bases—which can be re-identified over time and may then be associated with observed trajectories. In this way, classical structures and dynamics may be understood as emergent, at the level of intrinsically local observers, from a global wave function (Zeh, 1970, 1973, 2000; Zurek, 1998, 2005; Schlosshauer, 2006, 2007). The key interpretive difficulty concerns the question of how to reconcile the fact that the global quantum state contains all possible trajectories (outcomes) while only a single trajectory is observed.  

To resolve this problem, one may have to resort to many-worlds and many-minds
We shall not further concern ourselves with these interpretive questions but rather concisely summarize, in the most interpretation-neutral manner possible, in what sense decoherence can be said to account for the emergence of “classicality.” Decoherence leads to the dynamical superselection, within the quantum formalism, of certain preferred observables that correspond to the familiar quantities of our experience (such as position and momentum). Decoherence causes coherences between the preferred states to become effectively uncontrollable and unobservable at the level of the system in the sense that phase relations are dislocalized into the composite system–environment state.\(^5\) Decoherence thus accounts for classicality in so far as it describes effective restrictions on the superposition principle for subsystems interacting with other systems described by a larger composite Hilbert space. Save for the fundamental problem of outcomes, this arguably explains how to reconcile our experience of robust, seemingly measurement-independent states characterized by a small set of definite physical quantities with the fact that the linear Hilbert-space formalism seems to theoretically admit a vast number of never-observed quantum states.

3 Bohr and Heisenberg on the quantum–classical divide

3.1 Bohr and the quantum–classical divide

Much has been made of the fact that Bohr’s epistemological approach to quantum mechanics was opposed to von Neumann’s formal measurement scheme in which these systems were treated as unitarily evolving and interacting quantum systems, i.e., on an equal footing with microscopic systems. For Bohr, quantum mechanics, as a universally valid theory, certainly could be employed to describe the interaction of the system under investigation and the measuring instrument, but in doing so one was precluded from treating the measuring instrument as a measuring instrument. To this extent Bohr’s approach was fundamentally different to that of von Neumann, taking the “classicality” of the measuring instrument as something we must assume a priori. Hence it is often asserted that there was an irreducible divide for Bohr between the quantum and classical realms. According to Jammer:

interpretations, adopt a purely epistemic interpretation of quantum states, introduce hidden variables, or consider deviations from the Schrödinger dynamics. See Schlosshauer (2004, 2007) for discussions of these different options in the context of decoherence.\(^5\) See Zurek (2005) for an approach to formalizing this local inaccessibility of phase relations without resorting to reduced density matrices.
Contrary to Planck and Einstein, Bohr did not try to bridge the gap between classical and quantum, but from the very beginning of his work, he searched for a scheme of quantum conceptions which would form a system just as coherent, on the one side of the abyss, as that of classical notions on the other (Jammer, 1966, p. 86).

Later Bohr’s conception of classical mechanics, which gave deeper significance to his previous ideas on the irreconcilable disparity between classical and quantum theory, precluded, now on epistemological grounds, the possibility to interpret the correspondence principle as asserting the inclusion of classical mechanics within quantum theory (Jammer, 1966, p. 117).

This passage captures what many physicists have understood to be Bohr’s point of view. But in the time since Jammer wrote this passage in 1966, there has been a considerable effort on the part of scholars to come to a deeper understanding about precisely what Bohr meant. In the late 1960s Paul Feyerabend (1968, 1969) complained that Bohr’s views had been systematically distorted by both his critics and his followers, more interested in pursuing their own philosophical agendas than seriously understanding what he had to say. With this in mind Feyerabend urged physicists and philosophers to go “back to Bohr.” A concerted effort to make philosophical sense of Bohr’s philosophical writings gathered momentum after the Bohr centennial in 1985 (Howard, 1994, pp. 201–2).

In spite of the attention Bohr’s writings have received over the past three decades, scholarly opinion on how we should understand his thinking remains divided. This is not the place to discuss the different interpretations of Bohr’s thought, suffice it to say that there is widespread agreement now that complementarity does not fit neatly with the views of the logical empiricists, nor should it necessarily be characterized as anti-realist. Bohr’s viewpoint, articulated in his reply to the EPR challenge, was that it is not simply the case that we cannot measure the two well-defined attributes of an object, but rather that the mutually exclusive experimental arrangements serve to define the very conditions under which we can unambiguously employ such classical concepts as position and momentum (Bohr, 1949). The key point for Bohr then is that quantum mechanics reveals to us the previously “unrecognized presuppositions for an unambiguous use of our most simple concepts” (Bohr, 1937, pp. 289–90). We cannot ascribe to a particle a “position” in space or a “momentum” independently of the specific experimental conditions under which we observe the particle.
3.2 Bohr, isolated systems, and entanglement

It has sometimes been argued that the characteristic feature of quantum mechanics crucial for decoherence—namely entanglement—plays effectively no role in Bohr’s thinking. However, a closer reading of Bohr suggests this is not so. Indeed, Bohr’s understanding of the quantum–classical divide turns out to depend on his recognition of the nonseparability between object and instrument in the act of measurement.

Early on Bohr recognized that the classical concept of an isolated “object” which has a well-defined “state” and which interacts with a measuring instrument is rendered problematic in quantum mechanics. This is grounded in the fact that, as Born observed in 1926, in quantum mechanics “one cannot, as in classical mechanics, pick out a state of one system and determine how this is influenced by a state of the other system since all states of both systems are coupled in a complicated way” (Born, 1983, pp. 52–53). As Bohr noted, this paradoxical situation in quantum mechanics has serious implications for the concept of observation. In order to observe a quantum system we must interact with it using some device serving as a measuring instrument. On the one hand, in order to observe something about an electron, say its momentum, we must assume that the electron possesses an independent dynamical state (momentum), which is in principle distinguishable from the state of the instrument with which it interacts. On the other hand, such an interaction, if treated quantum-mechanically, destroys the separability of the object and the instrument, since the resulting entanglement between the two partners means that they must be described by a single composite nonseparable quantum state. Such entangled states represent quantum correlations between the two systems that frequently embody entirely new physical properties for the composite system that are not present in any of the subsystems. In some sense, the two entangled partners have thus become a single quantum-mechanical system.

For Bohr, this lay at the heart of the epistemological paradox of quantum mechanics. Bohr regarded the condition of isolation to be a simple logical demand, because, without such a presupposition, an electron cannot be an “object” of empirical knowledge at all. “The crucial point” he explained in 1949, is that contrary to the situation in classical physics, in quantum mechanics we are confronted with the “impossibility of any sharp separation between the behavior of atomic objects and the interaction with the measuring instruments which serve to define the very conditions under which the phenomena appear” (Bohr, 1949, p. 210, emphasis in original). Bohr had earlier emphasized this point at the 1936 “Unity of Science” congress, where he had explained:

A still further revision of the problem of observation has since been made
necessary by the discovery of the universal quantum of action, which has taught us that the whole mode of description of classical physics . . . retains its adequacy only as long as all quantities of action entering into the description are large compared to Planck’s quantum . . . This circumstance, at first sight paradoxical, finds its elucidation in the recognition that in this region [where classical mechanics breaks down] it is no longer possible sharply to distinguish between the autonomous behavior of a physical object and its inevitable interaction with other bodies serving as measuring instruments, the direct consideration of which is excluded by the very nature of the concept of observation itself (Bohr, 1937, p. 290).

The “whole mode of description of classical physics” to which Bohr refers in this passage is nothing other than the condition of separability. It is not merely that the act of measurement influences or disturbs the object of observation, but that it is no longer possible to distinguish between the object and its interaction with the device serving as a measuring instrument. A quantum-mechanical treatment of the observational interaction would paradoxically make the very distinction between object and instrument ambiguous. However, such a distinction is a necessary condition for empirical inquiry. After all, an experiment is carried out precisely to reveal information about some atomic object. As Bohr was to put it, only so far as we can neglect the “interaction between the object and the measuring instrument, which unavoidably accompanies the establishment of any such connection” can we “speak of an autonomous space-time behavior of the object under observation” (Bohr, 1937, p. 291). To speak of an interaction between two separate systems—an object and measuring instrument—is to speak in terms of classical physics.

3.3 The Heisenberg cut and the Bohr–Heisenberg disagreement about its “shiftiness”

Bohr’s view made an immediate and lasting impression on Heisenberg. As Heisenberg observed, it follows then that from an epistemological point of view “a peculiar schism in our investigations of atomic processes is inevitable” (Heisenberg, 1952b, p. 15). In the discussions at the Como conference in September 1927, Heisenberg explained that in “quantum mechanics, as Professor Bohr has displayed, observation plays a quite peculiar role.” In order to observe a quantum-mechanical object, “one must therefore cut out a partial system somewhere from the world, and one must make ‘statements’ or ‘observations’ just about this partial system” (Bohr, 1985, p. 141). The existence of “the cut between the observed system on the one hand and the observer and his apparatus on the other hand” is a necessary condition for the possibility of empirical knowledge. Without the assumption of such a divide we could not speak of the “object” of empirical knowledge in quantum mechan-
ics. Heisenberg emphasized the significance of the cut \([\text{Schnitt}]\) throughout the 1930s. In his lecture on “Questions of Principle in Modern Physics” delivered in November 1935 in Vienna, Heisenberg explained:

In this situation it follows automatically that, in a mathematical treatment of the process, a dividing line must be drawn between, on the one hand, the apparatus which we use as an aid in putting the question and thus, in a way, treat as part of ourselves, and on the other hand, the physical systems we wish to investigate. The latter we represent mathematically as a wave function. This function, according to quantum theory, consists of a differential equation which determines any future state from the present state of the function . . . The dividing line between the system to be observed and the measuring apparatus is immediately defined by the nature of the problem but it obviously signifies no discontinuity of the physical process. For this reason there must, within certain limits, exist complete freedom in choosing the position of the dividing line (Heisenberg, 1952a, p. 49, emphasis added).

This point had been emphasized in his lecture the previous year, in which Heisenberg argued that “there arises the necessity to draw a clear dividing line in the description of atomic processes, between the measuring apparatus of the observer which is described in classical concepts, and the object under observation, whose behavior is represented by a wave function” (Heisenberg, 1952b, p. 15). This was the central theme in an unpublished paper, written in 1935 entitled \textit{Ist eine deterministische Ergänzung der Quantenmechanik möglich?}, in which Heisenberg outlined his own response to the criticisms of quantum mechanics which had emerged from such physicists as von Laue, Schrödinger and Einstein in the 1930s. A draft of the paper is contained in his letter to Pauli on 2 July 1935 (Pauli, 1985, pp. 409–18 [item 414]). The paper, which was written at Pauli’s urging, argued that a deterministic completion of quantum mechanics is in principle impossible. This is because in quantum mechanics we must draw a cut between the quantum-mechanical system to be investigated, represented by a wave function in configuration space, and the measuring instrument described by means of classical concepts. The critical point for Heisenberg is that “this cut can be shifted arbitrarily far in the direction of the observer in the region that can otherwise be described according to the laws of classical physics,” but of course, “the cut cannot be shifted arbitrarily in the direction of the atomic system” (Heisenberg, 1985, p. 414). No matter where we chose to place the cut, classical physics remains valid on the side of the measuring device, and quantum mechanics remains valid on the side of the atomic system.

There were, however, key differences, which have often been overlooked, between Bohr and Heisenberg in their respective analyses of the quantum–classical divide. The crucial point seems to have been that for Heisenberg
the location of the cut “cannot be established physically”—it represents no physical discontinuity—and moreover it is precisely the arbitrariness in the choice of the location of the cut that is decisive for the application of quantum mechanics” (Heisenberg, 1985, p. 416). We know from an exchange of correspondence in the 1930s that Bohr objected to Heisenberg’s view that the cut could be shifted arbitrarily far in the direction of the apparatus (Archives for the History of Quantum Physics: Heisenberg to Bohr 10 August 1935, Bohr to Heisenberg 10 September 1935, Bohr to Heisenberg 15 September 1935, Heisenberg to Bohr 29 September 1935). Heisenberg explained to Bohr that without such a presupposition one would have to conclude that there exist “two categories of physical systems—classical and quantum-mechanical ones” (Archives for the History of Quantum Physics, Heisenberg to Bohr 29 September 1935). Heisenberg acknowledged that strictly speaking, however, the laws of quantum mechanics are applicable to all systems (including the measuring instrument). As Heisenberg pointed out in the discussions that followed Bohr’s Como paper in September 1927: “One may treat the whole world as one mechanical system, but then only a mathematical problem remains while access to observation is closed off” (Bohr, 1985, p. 141). 6 It was therefore an epistemological condition that one had to introduce the cut into the quantum-mechanical description.

The view outlined by Heisenberg leaves it ambiguous under what circumstances we are entitled to consider the apparatus “classically,” and under what circumstances it should be treated as a “quantum-mechanical” system. Certain physicists were also troubled by how we are to understand the interaction between a “quantum-mechanical” system and a “classical” measuring apparatus. Such an interaction does not appear to be subsumed under either classical or quantum theory. Moreover the division between the quantum and the classical realms seems to coincide exactly with the object–instrument and microscopic–macroscopic distinctions. It appears odd that there should be a fundamental difference between microscopic and macroscopic systems.

Heisenberg’s treatment of the cut between the “quantum” object and the “classical” measuring device has often been taken as a faithful representa-

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6 Interestingly, Everett’s relative-state interpretation (Everett, 1957)—and its subsequent development into many-worlds (DeWitt, 1970, 1971; Deutsch, 1985) and many-minds interpretations (Lockwood, 1996; Zeh, 2000)—have taken precisely the route of considering a single closed quantum-mechanical system, containing, among other things, the observers themselves, and then try to account for observers and their observations from within this formalism. Heisenberg’s charge that in this case “only a mathematical problem remains” would then be transformed into a merit by Everett’s idea of letting the quantum-mechanical formalism provide its own interpretation—a program that Bohr would have likely disagreed with, since for him the formalism by itself would become meaningless without the prior assumption of classical concepts.
tion of Bohr’s most carefully considered view of the problem of measurement. However, Heisenberg frequently alluded to his disagreement with Bohr on this matter in the 1930s. In *Physics and Philosophy*, he explained that “Bohr has emphasized that it is more realistic to state that the division into the object and rest of the world is not arbitrary” and the object is determined by the very nature of the experiment (Heisenberg, 1989, p. 24). Writing to Heelan in 1975, Heisenberg explained that he and Bohr had never really resolved their disagreement about “whether the cut between that part of the experiment which should be described in classical terms and the other quantum-theoretical part had a well defined position or not.” In his letter Heisenberg stated that he had “argued that a cut could be moved around to some extent while Bohr preferred to think that the position is uniquely defined in every experiment” (Heelan, 1975, p. 137). Weizsäcker also later recalled that Heisenberg had disagreed with Bohr over the cut in the 1930s (Weizsäcker, 1987, p. 283). The Heisenberg–Bohr exchange would seem to suggest that for Bohr, the quantum–classical distinction corresponds to something “objective,” and is not merely an arbitrary division.

3.4 Howard’s reconstruction of Bohr’s doctrine of classical concepts and improper mixtures in decoherence

So how are we to understand Bohr’s own view of the cut argument, and what does this have to do with the modern decoherence approach? An important clue can be found in a paper in which Howard (1994) suggested a new interpretation (termed a “reconstruction”) of Bohr’s classical concepts. In particular, Howard addressed the question of what Bohr meant when he insisted on the use of a classical description, and the question of precisely where such a classical description is to be employed. He argued that a careful reconstruction of Bohr’s views shows that for him “the classical/quantum distinction” did not exactly coincide with the “instrument/object distinction” whereas this seems to have been precisely the case for Heisenberg. Bohr, it seems, was keen to avoid the mistaken impression that the “classical” instrument somehow interacts with the “quantum” object. According to Howard,

> it is widely assumed that Bohr’s intention was that a classical description be given to the measuring apparatus in its entirety, a quantum description being given presumably, to the observed object in its entirety. On this view, the classical/quantum distinction would coincide with the instrument/object distinction; hence, its designation in what follows as the “coincidence interpretation” of the doctrine of classical concepts. I will argue instead that the two distinctions cut across one another, that Bohr required a classical description of *some*, but not necessarily *all*, features of the instrument and more surprisingly, perhaps, a classical description of some features of the
observed object as well. More specifically I will argue that Bohr demanded a classical description only of those properties of the measuring instrument that are correlated, in the measurement interaction, with the properties of the observed object that we seek to measure; and that this implies, as well, a classical description of the associated measured properties of the observed object itself (Howard, 1994, p. 203).

Howard’s reconstruction provides a new reading of many crucial passages from Bohr’s writings, in which “the classical/quantum distinction corresponds to an objective feature of the world.” Furthermore, this reading of Bohr makes it clear that there is no explicit or implicit appeal to the vague notion that the measuring instrument is a “macroscopic” object, having certain dimensions (Howard, 1994, p. 211). Howard argues that Bohr’s classical descriptions may be interpreted—in agreement with, but not forced out by, Bohr’s own writings—as the appropriate use of proper mixtures for the measured system in place of the global pure state describing the entangled system–apparatus state which is produced as the result of a von Neumann measurement. Accordingly, this transition is motivated by Bohr’s insistence on an unambiguous and objective description of quantum phenomena, which requires the classical concept of separability between the observed (the system) and the observer (the apparatus). However, the feature of quantum entanglement shows—as evidenced by examples such as EPR (Einstein et al., 1935) and von Neumann’s measurement scheme with its famous application to Schrödinger’s cat (Schrödinger, 1935a)—that such separability no longer holds. As discussed in Sec. 2.1, in a von Neumann measurement,

\[ \Psi_0 = \left( \sum_n c_n |s_n\rangle \right) |a_0\rangle \rightarrow \Psi = \sum_n c_n |s_n\rangle |a_n\rangle, \]  

the final joint system–apparatus state does not factor into a tensor-product state \(|s'\rangle|a'\rangle\), and thus no individual quantum state can be attributed to either the system or the apparatus. Evidently, the classical concept of independence and separability between the system and the apparatus no longer holds.

Howard suggests that Bohr’s classical concepts may be identified with the selection of *subensembles that are appropriate to the measurement context* (i.e., in Bohr’s terminology, that are appropriate to the particular “experimental arrangement”), in the following sense. Consider the density matrix corresponding to the final state on the right-hand side of Eq. (5),

\[ \hat{\rho}_{SA} = |\Psi\rangle\langle\Psi| = \sum_{nm} c_n c_m^* |s_n\rangle \langle s_m| \otimes |a_n\rangle \langle a_m| \neq \hat{\rho}_S \otimes \hat{\rho}_A, \]  

and suppose that the measurement of interest is described by the apparatus observable \( \hat{O} = \sum_m o_m |a_m\rangle \langle a_m|. \) Then the statistics of such a measurement
will be exhaustively described by the subensemble

\[ \hat{\rho}_{SA|\hat{O}} = \sum_n |c_n|^2 |s_n\rangle\langle s_n| \otimes |a_n\rangle\langle a_n| \equiv \sum_n |c_n|^2 \hat{\rho}^{(n)}_{SA|\hat{O}}. \]  

(7)

Here the interference terms \( n \neq m \) have been \textit{a priori} neglected, since they cannot be measured by the chosen observable \( \hat{O} \). For all purposes of this particular measurement, an \textit{ignorance interpretation} (d’Espagnat, 1966, 1976, 1995) is then attached to the conditional density matrix (7). That is, \( \hat{\rho}_{SA|\hat{O}} \) is interpreted as describing a situation in which the system–apparatus combination is in \textit{one} of the pure states \( |s_n\rangle|a_n\rangle \) but we simply do not know in which. In this way, the density matrix (7) is by assumption (associated with Bohr’s assumption of “classical concepts”) taken to represent a \textit{proper} (classical) mixture of the “outcome states” \( |s_n\rangle|a_n\rangle \) for a measurement (“properization” of the full density matrix conditioned on the particular measurement).

According to Howard, Bohr’s mutually exclusive experimental arrangements may then be identified with the choice of such effective mixtures conditioned on the particular measurement setup. Thus we may say that the different decompositions of the global density matrix into proper subensembles correspond to the different observables measured by each of the arrangements. No single “properized” mixture of the form (7) will give the correct statistics for all possible observables but will suffice for the measurement of observables co-diagonal in the basis used in expanding the mixture. This ties in with Bohr’s notion of the existence of classical measurement apparatuses: In Howard’s reading, the assumption of classicality for such apparatuses would then correspond to the existence of preferred apparatus observables whose eigenbases determine the particular “properized” mixtures, which in turn exhaustively describe the statistics of these particular measurements.

Howard’s reconstruction actually fits rather nicely with what we know of the disagreement between Bohr and Heisenberg concerning the cut between “object” and the instrument. It also allows us to draw a particularly interesting connection to decoherence, and in fact gives Bohr’s “classical concepts” (interpreted in the sense of Howard’s reconstruction) a more precise meaning. In Howard’s picture, for Bohr the choice between different “properized” mixtures was simply a result of knowing which observable was measured by a particular experimental arrangement. However, first, what determines this observable on physical grounds? And second, in many cases the formalism allows us to rewrite such a mixture in many different bases (e.g., for Bell states) and thus does not uniquely fix the basis which supposedly should correspond to a particular experimental arrangement. How can one circumvent this problem of basis ambiguity as posed by the formalism of quantum mechanics? Third, what precisely justifies neglecting the interference terms in the global density matrix?
To all questions decoherence provides an answer. In any realistic account of measurement, we ought to include further interactions with the environment. As a consequence of decoherence, there will be at least one preferred basis in which the interference terms between different one-to-one quantum-correlated system–apparatus states in the reduced system–apparatus density matrix will be sufficiently small in order to be neglected in practice. We thus arrive at a system–apparatus density matrix that is formally identical to (7). The relevant observable is determined by the structure of the apparatus–environment interaction Hamiltonian (Zurek, 1981, 1982), while in other bases system–apparatus correlations would be rapidly destroyed by the environment and therefore the apparatus could not function reliably (this is the “stability criterion” introduced by Zurek, 1982). The preferred (environment-superselected) basis may then be regarded as corresponding to Bohr’s notion of a particular physical arrangement that is used to measure the system. This allows us to explain why measurement devices appear to be designed to measure certain physical quantities but not others, while in the absence of decoherence we would in general face the preferred-basis problem (Zurek, 1982; Schlosshauer, 2004). Decoherence thus supplies a physical criterion for the choice of the particular “properized” ensemble in Howard’s reconstruction.

This dynamical picture also allows us to precisely quantify the location of the Heisenberg cut, or, in the terminology of Howard’s reconstruction, of \textit{when} and \textit{where} we may make the replacement of the density matrix by a “properized” mixture. In any given physical situation, we can (at least in principle) model the relevant interactions between the subsystems and thus, for each chosen subsystem, precisely determine the degree to which certain interference effects may be observable in a local measurement performed on this subsystem. This allows us to quantify the degree to which the subsystem is rendered effectively classical with respect to different local observables. This ability to precisely pinpoint the location of the quantum–classical divide by taking into account the relevant decoherence effects represents an enormous progress over the Heisenberg picture, where it was simply left to the judgment of the observer where to place the cut.

One important point, however, remains. Reduced density matrices obtained by tracing out the environment are not ignorance-interpretable since, as Pessoa Jr. (1998, p. 432) put it, “taking a partial trace amounts to the statistical version of the projection postulate.” By contrast, the system–apparatus density matrices in the Bohr–Howard picture are derived from simply neglecting the interference terms and then assuming that the resulting mixture is ignorance-interpretable. This involves a conceptual leap just as severe as that of choosing to interpret reduced density matrix as ignorance interpretable. In both cases, quantum mechanics forbids us to attach such an interpretation. Scholars working on decoherence have been (at least lately) very careful in making this point. Howard emphasizes this issue, too, but his reconstruction
is precisely all about associating Bohr’s assumption of classical concepts with a deliberately ignorance of this point.

We see that Howard’s reconstruction has the merit of providing a specific formalization of Bohr’s notion of classical concepts, and decoherence shows how this formalization can be physically motivated, justified, and quantified. If we follow Howard and take Bohr’s classical concepts as amounting to a replacement of quantum-mechanical ensembles by classical mixtures that depend on the measurement context, then decoherence indeed allows us to derive these concepts (modulo the fundamental problem of the transition to ignorance-interpretable mixtures). Or, put differently, decoherence allows us to tell a dynamical, physical story of these concepts.

3.5 Heisenberg, entanglement, and the external world

Looking back over the history of the foundations of quantum mechanics, we can now see the crucial obstacle to an understanding of the quantum-to-classical transition was the erroneous assumption that we can treat quantum systems as isolated from the environment. It is true that Bohr had earlier understood very well that the properties exhibited by quantum systems cannot be separated from the experimental conditions under which they are observed. But he does not appear to have extended this argument to the measuring apparatus and its environment: Bohr was simply content to assume the classicality of the experimental apparatus. The recognition that it is precisely the openness of quantum systems and the resulting environmental entanglement that may explain how these systems become effectively classical was the crucial insight in the decoherence account.

However, there do appear to be anticipations of the relevance of the environment in certain passages in Heisenberg’s writings from the 1950s. In his contribution to the volume commemorating Bohr’s seventieth birthday, Heisenberg gave his most systematic defense of what he referred to as the “Copenhagen interpretation of quantum mechanics.” In the paper Heisenberg remarked that many authors had pointed out that the so-called “reduction of wave-packets cannot be deduced from the Schrödinger’s equation” and that to this extent a number of physicists had drawn the conclusion “that there is an inconsistency in the ‘orthodox’ interpretation” (Heisenberg, 1955, p. 23). Yet, as Heisenberg emphasized, the reason that the quantum-mechanical treatment of the “interaction of the system with the measuring apparatus” does not of itself “as a rule lead to a definite result (e.g. the blackening of a photographic plate)” is that in such a treatment “the apparatus and the system are regarded as cut off from the rest of the world and treated as a whole according to quantum mechanics” (Heisenberg, 1955, p. 22). However, as Heisenberg explained: “If
the measuring device would be isolated from the rest of the world, it would be
neither a measuring device nor could it be described in the terms of classical
physics at all” (Heisenberg, 1989, p. 24). Somewhat surprisingly Heisenberg
attributed this view to Bohr:

Bohr has rightly pointed out on many occasions that the connection with the
external world is one of the necessary conditions for the measuring apparatus
to perform its function, since the behavior of the measuring apparatus must
be capable of being … described in terms of simple [classical] concepts,
if the apparatus is to be used as a measuring instrument at all. And the
connection with the external world is therefore necessary … We see that a
system cut off from the external world … cannot be described in terms of
classical concepts (Heisenberg, 1955, pp. 26–7, emphasis added).

Here Heisenberg appears to connect the fact that the measuring instrument
cannot be isolated from the rest of the world with the need to use classi-
cal concepts in the description of the experiment. In Physics and Philosophy
Heisenberg makes more explicit this connection between the inseparability of
the “system” and the “external world” and the quantum-to-classical transi-
tion:

Again the obvious starting point for the physical interpretation of the for-
malism seems to be the fact that mathematical scheme of quantum me-
chanics approaches that of classical mechanics in dimensions which are large
compared to the size of atoms. But even this statement must be made with
some reservations. Even in large dimensions there are many solutions of
the quantum-mechanical equations to which no analogous solutions can be
found in classical physics. In these solutions the phenomenon of the “inter-
ference of probabilities” would show up … [which] does not exist in classi-
cal physics. Therefore, even in the limit of large dimensions the correlation
between the mathematical symbols, the measurements, and the ordinary
concepts [i.e., the quantum-to-classical transition] is by no means trivial.
In order to get at such an unambiguous correlation one must take another
feature of the problem into account. It must be observed that the system
which is treated by the methods of quantum mechanics is in fact a part of
a much bigger system (eventually the whole world); it is interacting with
this bigger system; and one must add that the microscopic properties of the
bigger system are (at least to a large extent) unknown. This statement is
undoubtedly a correct description of the actual situation … The interac-
tion with the bigger system with its undefined microscopic properties then
introduces a new statistical element into the description … of the system
under consideration. In the limiting case of the large dimensions this sta-
tistical element destroys the effects of the “interference of probabilities” in
such a manner that the quantum-mechanical scheme really approaches the
classical one in the limit (Heisenberg, 1989, pp. 121–2).
In this intriguing passage Heisenberg recognizes that it cannot simply be the macroscopic dimensions of the measuring apparatus that ensure that the apparatus becomes effectively classical. In the reconstruction of Bohr’s view of the quantum–classical divide presented earlier, attention is restricted to the interaction between the system and the apparatus. There the description of the measured system by a proper mixture is justified by referring to the need for an objective account of experimental results. In the passages quoted above, however, Heisenberg enlarges the system–apparatus composite to include couplings to further degrees of freedom in the environment (the “external world”). This is a most interesting point. Of course, for practical purposes, Heisenberg admits that we often treat the quantum system and the measuring apparatus as isolated from the rest of the world. But the quantum-to-classical transition depends on “the underlying assumption,” implicit in the Copenhagen interpretation, “that the interference terms are in the actual experiment removed by the partly undefined interactions of the measuring apparatus, with the system and with the rest of the world (in the formalism, the interaction produces a ‘mixture’)” (Heisenberg, 1955, p. 23).

Heisenberg’s account may be read as anticipating results of the decoherence program in two ways. First, in the formal description of decoherence, the environment is traced out, arriving at an improper mixtures, which is then, for all practical purposes of statistical prediction, interpreted as a proper mixture. Second, because the Quantum Darwinism program (Blume-Kohout and Zurek, 2006) precisely shows how the environment plays the role of an information channel that “objectifies” (in an effective sense) the information represented by the system–apparatus quantum correlations through redundant encoding. The account also shows some interesting parallels to the distinction between decoherence and classical noise. Noise describes a situation in which the system is perturbed by the environment. However, decoherence corresponds to a measurement-like process in which the system perturbs the environment, in the sense that the superposition initially confined to the system spreads to the system–environment combination. The nonlocal nature of quantum states then implies that this “distortion” of the environment by the system in turn influences the observable properties at the level of the system (as formally described by the reduced density matrix).

However, while Heisenberg emphasizes the importance of environmental interactions, nowhere does he explicate the role of entanglement between the system and the environment as the crucial point in the emergence of classicality in the system. In discussing the extent to which quantum mechanics gives an “objective” description of the world, Heisenberg draws attention to the fact that “classical physics is just that idealization in which we speak about the parts of the world without any reference to ourselves” (Heisenberg, 1989, p. 22). Here Heisenberg is careful to avoid the impression that quantum mechanics contains any “genuine subjective features”—he categorically
denies that the mind of the observer plays any crucial role in the measurement process. But he does suggest that “quantum theory only corresponds to this ideal [of the separability of ‘objects’] as far as possible.” Heisenberg maintains that the somewhat arbitrary “division of the world into the ‘object’ and the rest of the world” is the starting point of quantum mechanics. To this extent, he seems to have been of the view that such a division between “object” and the “rest of the world” was indispensable for physics, in spite of the fact that quantum systems exhibit radical nonseparability (Heisenberg, 1989, p. 23). It is the hallmark of decoherence that it begins from the assumption that the quantum system and apparatus cannot be isolated from the surrounding environment, and moreover that it is precisely this feature of quantum mechanics which results in the emergence of classicality.

Physicists such as Bohr, Heisenberg, and Schrödinger had recognized the nonseparability of quantum systems—i.e., entanglement—as a characteristic feature of quantum mechanics. For example, Schrödinger, who had coined the term “entanglement” (Verschränkung in German) in 1935 (Schrödinger, 1935a,b, 1936), referred to this nonseparability not as “one but rather the characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought” (Schrödinger, 1935b, p. 555, emphasis in the original). But the feeling prevailed that entanglement was something unusual and a peculiarly microscopic phenomenon that would have to be carefully created in the laboratory (such as in an EPR-type experiment). Entanglement was regarded as an essential quantum feature that would necessarily have to be irreconcilable with classicality. These long-held beliefs likely contributed to the comparably late “discovery” of decoherence (Schlosshauer, 2007). It is indeed a particular irony that entanglement would turn out to be not something that had to be tamed in some way to ensure classicality but would instead assume a key role in the emergence of classicality.

4 The doctrine of classical concepts

4.1 The doctrine of classical concepts revisited

It remains here to comment on the extent to which we can say that the decoherence account of measurement runs counter to Bohr’s original view of classical concepts. In order to do this, it seems necessary to get a clearer picture of precisely why Bohr thought we must use classical concepts. This is a question that has puzzled many physicists and philosophers, but unfortunately Bohr never really clarified his views on this issue. Howard’s reconstruction of Bohr’s doctrine of classical concepts is instructive, but in the end it simply begs the question: Why are we are entitled to replace the quantum-mechanical density
matrix with a particular “classical” mixture? Or, within the decoherence-based
account of Howard’s reconstruction, how can we justify the use of improper
mixtures without presuming the usual axioms of measurement, which underlie
the formalism and interpretation of improper mixtures as the formal entities
that completely encapsulate all local measurement statistics (Schlosshauer,
2004)?

Indeed much of what we know of Bohr’s position comes from scattered re-
marks, often interpreted through his contemporaries, many of whom took
up the role of Bohr’s self-appointed spokesmen. The situation becomes more
difficult when we realize that Bohr’s views were appropriated by a number
of different philosophical schools of thought such as positivism, Kantianism,
critical realism, linguistic idealism, dialectical materialism, and even pragma-
tism. One reason that Bohr’s writings were so readily adapted to different
philosophical positions is that many of his contemporaries saw it as their task
to clarify Bohr’s views, which were often not expressed quite as clearly as
they might have been. To this extent, many different versions of Bohr’s views
had emerged by the 1960s, some of which were diametrically opposed to one
another.

In spite of the difficulty of this situation, we can make sense of the differ-
ent versions of Bohr’s doctrine of classical concepts by carefully drawing two
useful distinctions. The first is what might be termed the pragmatic versus
categorical version of the doctrine of classical concepts, and the second is the
distinction between an epistemological and physical formulation of the doc-
trine. We will address both in turn here.

4.2 Pragmatic versus categorical formulations

In probing the doctrine of classical concepts, it is important to note something
which Beller (1999, pp. 197–9) has highlighted in her recent work. Bohr’s edict
that we must use classical concepts was sometimes given a more pragmatic
interpretation by Bohr and his followers, namely, that we simply do use classi-
cal concepts in describing the results of measurements in quantum mechanics,
and that this is the situation we find ourselves in. One certainly finds this
“weaker” version of the doctrine of classical concepts in the writings of Rosen-
feld, Heisenberg, and Weizsäcker. Thus the doctrine is transformed from a
categorical imperative to a pragmatic statement of the fact. In his book on
the Worldview of Modern Physics, Weizsäcker makes explicit this subtle, but
important, shift of emphasis:

We ought not to say, “Every experiment that is even possible must be clas-
sically described,” but “Every actual experiment known to us is classically
described, and we do not know how to proceed otherwise.” This statement is not sufficient to prove that the proposition is *a priori* true for all, merely possible future knowledge; not is this demanded by the concrete scientific situation. It is enough for us to know that it is a priori valid for quantum mechanics. We have resolved not to say, “Every experiment *must* be classically described” but simply, “Every experiment *is* classically described.” Thus the factual, we might almost say historical situation of physics is made basic to our propositions (Weizsäcker, 1952, pp. 128, 130).

Here Weizsäcker is clear that the doctrine of classical concepts has “not logical but factual necessity.” In other words, it is simply the case that quantum mechanics is founded upon experimental results that *are* described by means of classical concepts. Physicists defending Bohr’s doctrine of classical concepts often resorted to this pragmatic formulation when challenged. Heisenberg provides another case in point. Responding to the suggestion that it might be possible to “depart from classical concepts” in providing a genuinely *quantum-mechanical description* of experiments, Heisenberg explained that classical concepts are an essential part of the language which forms the basis of all natural science. Our actual situation in science is such that we *do* use classical concepts for the description of experiments, and it was the problem of quantum theory to find a theoretical interpretation of the experiments on this basis (Heisenberg, 1989, p. 23, emphasis added).

Here the historical and pragmatic dependence of quantum mechanics on classical concepts is once again emphasized.

Of course simply appealing to the fact that we happen to use classical concepts does not of itself prove that we cannot conceptualize the world of experience in any other way. Weizsäcker and Heisenberg certainly recognized this, but the point they stressed was that our current theory of quantum mechanics only corresponds to what can be observed if we assume that the results of measurements are described in terms of the basic concepts of classical physics. For Bohr, on the other hand, the doctrine of classical concepts was expressed more categorically: “It lies in the nature of physical observation . . . that all experience *must* ultimately be expressed in terms of classical concepts” (Bohr, 1987, p. 94, emphasis added); “the unambiguous interpretation of any measurement *must* be essentially framed in terms of classical physical theories, and we may say that in the sense the language of Newton and Maxwell will remain the language of physics for all time” (Bohr, 1931, p. 692, emphasis added). This brings us back to the question posed at the beginning of this section: Why must we interpret observations classically? In a letter to Bohr written in October 1935, Schrödinger asked Bohr why this remained one of his deepest convictions (Bohr, 1996, pp. 508–9). Bohr’s reply unfortunately does
not shed much light on the matter, as he simply restated what he saw as “the seemingly obvious fact that the functioning of the measuring apparatus must be described in space and time” (Bohr, 1996, pp. 511–2).

The task of answering the question was left largely to Bohr’s followers such as Heisenberg, Weizsäcker, Rosenfeld, and Petersen, many of whom were in essential disagreement about the finer points of Bohr’s interpretation. Yet, in spite of the diversity of viewpoints, we can distinguish two fundamentally different approaches to answering the question left by Bohr. These may be labeled as the epistemological and physical formulations of the doctrine of classical concepts. A deeper understanding of these two approaches provides an important clue to understanding what is really meant by Bohr’s doctrine of classical concepts and how it relates to decoherence. It is to this that we now turn our attention.

4.3 Epistemological versus physical versions

Much of the discussion over the extent to which decoherence marks a break from the “Copenhagen” viewpoint suffers from the failure to fully appreciate the different ways in which the question of why we must use classical concepts in the description of experiments was interpreted by Bohr’s followers. Here we need to distinguish two fundamental approaches.

In the first instance those who defended Bohr tended to frame the question in epistemological terms. This amounts to asking why our conceptual framework is so wedded to our classical intuitions about the world. Physicists who approached the doctrine of classical concepts from this perspective attempted to give the doctrine a decidedly Kantian, linguistic, or pragmatic reading. Weizsäcker and Heisenberg, for example, were inclined to interpret Bohr as having “pragmatized” or “relativized” Kant’s philosophy (Camilleri, 2005). In this context Heisenberg would pronounce in 1934 that “modern physics has more accurately defined the limits of the idea of the a priori in the exact sciences, than was possible in the time of Kant” (Heisenberg, 1952b, p. 21). Some authors such as Faye (1991) have argued that Bohr himself was influenced by Kantian tradition through his association with the Danish philosopher Høffding. Petersen argued that “Bohr’s remarks” on the indispensability of classical concepts “are based on his general attitude to the epistemological status of language and to the meaning of unambiguous conceptual communication, and they should be interpreted in that background” (Petersen, 1968, p. 179). Much has been made of Petersen’s remarks on Bohr’s view of the primacy of language over reality in quantum mechanics. Indeed on many occasions Bohr emphasized that an “objective” description amounts to the possibility of “unambiguous communication,” and to this extent it must be
expressed through concepts of classical physics. Petersen saw Bohr’s writings as having provided a deep insight “into the epistemological role of the conceptual framework” of classical physics (p. 185).

While approaching the meaning of doctrine of classical concepts from an altogether different philosophical perspective, Rosenfeld agreed that Bohr’s primary concerns arose from “general epistemological considerations about the function of language as a means of communicating experience” (Rosenfeld, 1979b, p. 526). Rosenfeld argued that “we must make use of the concept of classical physics” in describing phenomena, simply because we must attempt to make ourselves understood to other human beings, and here the concepts of classical physics provide the means by which we can unambiguously communicate the results of our observations. The concepts of classical physics are for Rosenfeld not to be understood as somehow part of the a priori structure of the human mind, but have adapted to our experience of the world. Given that all experimental knowledge of the atomic world involves the amplification of effects such that they can be perceived by human beings at the macrolevel, it should therefore come as no surprise that the classical concepts form the basis of our description of experience even in quantum mechanics.

Yet, the epistemological perspective invariably leads to another question, which amounts to a reformulation of the doctrine of classical concepts in physical terms. We may state this as follows: Why is the world such that the concepts of classical physics can be employed, at least to a very good approximation, in certain situations? Or, to put it another way: Why are classical concepts applicable at all to the quantum world? This is a salient question, given that, strictly speaking, the world is nonclassical. While Bohr himself did not attempt to provide such a physical explanation for the doctrine of classical concepts, in the 1950s and 1960s a number of physicists turned their attention to accounting for the emergence of classicality wholly within the framework of quantum mechanics. This approach, which is closer in spirit to the decoherence program, was pursued by those who saw themselves as working within the “Copenhagen” tradition. The passage quoted earlier from Heisenberg, in which he attempts to explain the quantum-to-classical transition, may be taken as one example.

In the 1960s Weizsäcker and Rosenfeld both attempted to defend this kind of approach to the physics of the quantum-to-classical transition as entirely in keeping with the spirit in which Bohr had intended his doctrine of classical concepts. As Weizsäcker put it at a colloquium in 1968, “the crucial point in the Copenhagen interpretation” is captured, “but not very luckily expressed, in Bohr’s famous statement that all experiments are to be described in classical terms” (Weizsäcker, 1971, p. 25). As a devotee of Bohr, this was a view that Weizsäcker endorsed wholeheartedly, but which he now wished to justify. “My proposed answer is that Bohr was essentially right” in arguing that the results
of all measurements must be classically describable, i.e. localized in space-time, “but that he did not know why” (Weizsäcker, 1971, p. 28). The paradox at the heart of the Copenhagen interpretation for Weizsäcker is therefore to be stated: “Having thus accepted the falsity of classical physics, taken literally, we must ask how it can be explained as an essentially good approximation” when describing objects at the macrolevel. He spells this out:

This amounts to asking what physical condition must be imposed on a quantum-theoretical system in order that it should show the features which we describe as “classical.” My hypothesis is that this is precisely the condition that it should be suitable as a measuring instrument. If we ask what that presupposes, a minimum condition seems to be that irreversible processes should take place in the system. For every measurement must produce a trace of what has happened; an event that goes completely unregistered is not a measurement. Irreversibility implies a description of the system in which some of the information that we may think of as being present in the system is not actually used. Hence the system is certainly not in a “pure state”; we will describe it as a “mixture.” I am unable to prove mathematically that the condition of irreversibility would suffice to define a classical approximation, but I feel confident it is a necessary condition (Weizsäcker, 1971, p. 29, emphasis in original).

Already in the 1960s a number of physicists had devoted themselves to investigating the thermodynamic conditions of irreversibility that would need to hold in order for a measurement to be registered macroscopically as “classical.” In 1965 Rosenfeld conceded that “it is understandable that in order to exhibit more directly the link between the physical concepts and their mathematical representation, a more formal rendering of Bohr’s argument should be attempted” (Rosenfeld, 1979a, p. 536). In fact, Rosenfeld felt that this had been carried out by Daneri, Loinger, and Prosperi (1962) in their thermodynamic analysis of the irreversible amplification process triggered by the interaction between the quantum system and the measuring device. For Rosenfeld this work had clarified many of the misunderstandings, which had arisen through “the deficiencies in von Neumann’s axiomatic treatment” (Rosenfeld, 1979a, p. 537).

However, this would prove to be a controversial claim, which generated much debate and discussion well into the 1970s. As Jammer puts it, “Rosenfeld’s unqualified endorsement of the Daneri, Loinger, and Prosperi measurement theory raises the question whether this is really congenial, or at least not incompatible, with the basic tenets of the Copenhagen interpretation” (Jammer, 1974, p. 493). Whereas for Rosenfeld, this theory of measurement was “in complete harmony with Bohr’s ideas” (Rosenfeld, 1979a, p. 539), for Bub it represented an approach to quantum theory fundamentally at odds with Bohr’s (Bub, 1971, p. 65). While a closer historical analysis is beyond the
scope of this paper, the episode is indicative of the attempts to reformulate Bohr’s doctrine of classical concepts in the 1960s. It was nevertheless clear to Wigner that “the transition to a classical description of the apparatus” in this kind of approach was “an arbitrary step” which only served to “postulate the miracle which disturbs us” (Wigner, 1995, p. 65). It was not until studies of decoherence were conducted that physicists were able to give an adequate account of the elusive quantum-to-classical transition.

4.4 Decoherence and the doctrine of classical concepts

As we have seen, decoherence provides a dynamical account of key components of the quantum-to-classical transition, and in doing so, clarifies many of the issues that had troubled an earlier generation of physicists. However, the extent to which this marks a departure from Bohr’s doctrine of classical concepts depends in large part on the way in which we interpret the doctrine. In attempting to find a physical foundation for the doctrine, Heisenberg, Rosenfeld, and Weizsäcker were asking questions to which decoherence now supplies a ready-made answer. They saw this effort as an extension of the general line of thought initiated by Bohr, and entirely keeping with the spirit in which he approached quantum mechanics. We may recall that for Bohr, the experimental arrangements serve to define the very conditions under which we can unambiguously employ such classical concepts as position and momentum. Decoherence certainly goes beyond anything that Bohr had to say in identifying ubiquitous and practically irreversible entanglement with a large number of environmental degrees of freedom as the crucial process which leads to the emergence of classicality in quantum systems (at least in an effective “relative-state” sense). Decoherence shows the conditions under which classicality arises in quantum mechanics, and to this extent it may be regarded as providing a physical justification for the pragmatic use of classical concepts in a given experimental situation.

Does the irreducibility of classical concepts in the quantum description hold once we recognize that they are, so to speak, “produced” out of the quantum formalism, which may thus be considered more fundamental? One may suggest that this undermines the Copenhagen school’s insistence on the epistemological primacy of classical concepts. But since decoherence is simply a consequence of a realistic application of the standard quantum formalism, it cannot by itself give an interpretation or explanation of this formalism itself. So the question remains, as it did before, whether it is possible to render quantum mechanics as a meaningful theory about the world without the use of classical concepts such as position and momentum (see Howard, 1994): In so far as decoherence depends on the use of the quantum formalism, which must itself be given a physical interpretation, some may suggest that the use
For example, part of the quantum formalism is usually derived through the “quantization” of the classical position and momentum variables of single particles, which then define configuration space as the preferred arena for the wave function. In a similar manner, classical Hamiltonians (e.g., for the harmonically bound particle) are directly “translated” into the operator-based quantum picture. However, as Zeh (2003) has pointed out, quantum field theory indicates that the only fundamental quantization required is at the levels of (postulated) spatial fields, while the concept of particles can then be derived in terms of decoherence-induced (improper) ensembles of narrow position-space wave packets (Zeh, 1993). Thus, while it is clear that at some stage of the theory we have to identify the physical entities to which the mathematical formalism refers, these entities will not need to take the form of familiar classical concepts such as particles and their positions.

Therefore any assessment of the extent to which decoherence allows us to “derive classical concepts” must inevitably depend on their definition and the level at which one demands an explanation of such concepts. Bohr and his followers in the 1930s understood this as an epistemological, not a physical, condition imposed on our description of nature, while (as we have seen) later followers of Bohr sought to give the doctrine a more pragmatic, physical underpinning.

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7 Somewhat misleadingly, this procedure is commonly referred to as the “second quantization,” while the less fundamental quantization of particle positions and momenta is denoted as the “first quantization.”

8 Early on, Heisenberg had contemplated the derivation of quantum mechanics and particle-like structures from the quantization of fields in three-dimensional space. This insight turned out to be particularly important for Heisenberg’s own understanding of wave–particle duality (Heisenberg, 1989, pp. 86, 93–4; Camilleri, 2006). Heisenberg concluded that in this approach to quantum mechanics, the classical concept of the field, not the particle, would constitute the fundamental starting point.

9 More recently, physicists have attempted to redefine Bohr’s notion of complementarity in terms of entanglement (Bertet et al., 2001), while others now claim to derive “the” Copenhagen interpretation from more fundamental principles (Ulfbeck and Bohr, 2001). A Copenhagen-esque spirit may also be evident in the recent interpretive stance that “quantum mechanics is about information” (see, e.g., Fuchs, 2002; Zeilinger, 2002; Bub, 2005). For example, Zeilinger suggests that “information is the most basic notion of quantum mechanics, and it is information about possible measurement results that is represented in the quantum states. Measurement results are nothing more than states of the classical apparatus used by the experimentalist” (Zeilinger, 2002, p. 252, emphasis in the original). The epistemological constraints underlying Bohr’s doctrine of classical concepts are in information-based interpretations often identified with limitations on the amount of information that nature if willing to proliferate, thus motivating the view that quantum mechanics is, at least in part, a theory about information. In this way, the quantum formalism may
5 Conclusions

Decoherence allows us to analyze, in precise formal and quantitative terms and wholly from within the quantum-mechanical formalism, when and how the quantum-to-classical transition happens. It unambiguously specifies the location of the Heisenberg cut and the conditions under which certain superposition states—such as those of the Schrödinger-cat type—can be prepared and observed, and what the lifetimes of such states will be in a given experimental situation. It therefore explains, for example, why superpositions of macroscopically distinct positions of a large object are so prohibitively difficult to prepare and maintain in practice. To our knowledge, there are no experimental observations of quantum-to-classical processes that could not be accounted for, at least in principle, by decoherence.\(^\text{10}\)

Decoherence thus provides a physical, quantitative underpinning of the quantum–classical divide and the dynamics at this boundary. In doing so, it clarifies the notion of the quantum–classical cut, which was at the center of the disagreement between Bohr and Heisenberg in the 1930s. As we have seen, Howard’s reconstruction of Bohr’s view of the classical-quantum divide not only can help us make sense of the historical disagreement between Bohr and Heisenberg, but also provides an important step in reconstructing the link between Bohr’s views and decoherence. It is rather clear that the application of quantum mechanics along the entire chain of interacting systems (Einstein’s “ganzer langer Weg,” as recalled by Heisenberg, 1971; see also Zeh, 2000), including measurement devices and their environments, turned out to be a key point in an understanding of the quantum-to-classical transition, ironically prominently involving the most distinctly quantum-mechanical features of entanglement and unitary evolution. Although Bohr did not deny that in principle such a fully quantum-mechanical treatment was possible, he considered it meaningless. He thereby closed himself, on grounds of a particular philosophical stance, to the approach of subjecting the measurement process to further quantum-mechanical analysis, in spite of the fact that he arguably believed quantum theory to be universal.

As the insights brought about the study of decoherence show, Bohr’s attitude to be reconstructed (rather than interpreted) from fundamental assumptions about restrictions on the flow of information, expressed as information-theoretic principles (see, e.g., the Clifton–Bub–Halvorson theorem; Clifton et al., 2003). For critical assessments of such information-based interpretations, see, e.g., Hagar (2003); Hagar and Hemmo (2006); Daumer et al. (2006); Shafiee et al. (2006).\(^\text{10}\)

We emphasize that this statement is independent of any assessment of whether and how decoherence may help solve the measurement problem, especially in the sense of the “macro-objectification” problem (Jammer, 1974; Bassi and Ghirardi, 2000; Adler, 2003; Schlosshauer, 2004; Zurek, 2007).
tude in this matter may be considered premature. It is worth noting that Heisenberg’s writings in the 1950s appear to acknowledge that the quantum-to-classical transition can be understood only if we bear in mind that the system comprising the quantum object and the measuring apparatus is in reality part of a bigger system, which ultimately encompasses the whole world. Here it seems that Heisenberg went some way toward anticipating the results of decoherence, although he never seems to have pursued the idea in any systematic fashion, and he never makes explicit the importance of entanglement in the process.

If classical concepts are understood in the pragmatic sense—as something we simply do use—decoherence suggests why this so. Decoherence shows that for macroscopic systems, and thus any system that can legitimately count as a measuring instrument capable of sufficiently amplifying measurement results as to make them accessible to our experience, decoherence will be so strong as to dynamically preclude most quantum states, save for those that turn out to be precisely the (approximate) eigenstates of “classical” observables such as position. Furthermore, information about such classical observables becomes amplified through redundant encoding in the environment, thus meeting, at least in an effective and relative-state state sense, Bohr’s criteria of “objectification” and “unambiguous communication” that seem so inherently wedded to classical physics. In this sense, decoherence indeed allows us to formulate classical concepts in physical terms: It not only tells us why the concepts of classical physics are applicable in the macroscopic situations relevant to our experience despite the underlying quantum-mechanical description of the world, but also when and where these concepts can be applied.

On the other hand, it is much more difficult to provide a reasonably conclusive answer to the question of whether decoherence suggests that Bohr’s assumption of irreducible classical concepts as an epistemological, metatheoretical—and thus ultimately philosophical—construct may be redundant. If classical concepts are understood in Bohr’s imperative sense—as something we must use—we still may invoke decoherence to justify this philosophical stance in a practical sense. While decoherence allows us to identify dynamically created classical structures and properties within the quantum formalism, of course it does not—and cannot—in and of itself provide us with an answer to the question of how to interpret this formalism, although it may lend additional support to, or disqualify, certain interpretations (Schlosshauer, 2004). Bohr’s fundamental point was that any interpretation of quantum mechanics must in the end fall back on the use of classical concepts. In this sense one may suggest that decoherence provides a physical justification for Bohr’s intuition. In fact, Zurek (2003, 1998, 1993, 2005, 2007) locates his decoherence-based “existential interpretation” between Bohr and Everett and relates it to a “neo-Copenhagen strategy.” Such trends may further indicate that the possibility of a peaceful coexistence between Bohr’s philosophy and decoherence could be considered
more viable than it has previously often been claimed.

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