Primordial perturbations with pre-inflationary bounce

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Abstract

Based on the effective field theory (EFT) of nonsingular cosmologies, we build a stable model, without the ghost and gradient instabilities, of bounce inflation (inflation is preceded by a cosmological bounce). We perform a full simulation for the evolution of scalar perturbation, and find that the perturbation spectrum has a large-scale suppression (as expected), which is consistent with the power deficit of the cosmic microwave background (CMB) TT-spectrum at low multipoles, but unexpectedly, it also shows itself one marked lower valley, which actually provides a better fit to the dip at multipole \( l \sim 20 \). The depth of valley is relevant with the physics around the bounce scale, which is model-dependent.

PACS numbers:

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I. INTRODUCTION

Inflation [1][2][3][4] is the current paradigm of early universe. It predicts nearly scale-invariant scalar perturbation, which is consistent with the cosmic microwave background (CMB) observations [5][6], as well as the gravitational waves (GWs). However, it is not the final story of the early universe. As pointed out by Borde, Vilenkin and Guth [7][8], inflation is past-incomplete, and “inflationary models require physics other than inflation to describe the past boundary of the inflating region of spacetime.” [8].

This past-incompletion (singularity) of inflation has inspired radical alternatives to inflation, e.g., [9][10][11][12]. However, how to make the inflation happen in a past-complete scenario is also a noteworthy issue. In certain sense, this actually requires that the pre-inflationary phase should be past-complete. One possibility is that it is slow contracting, so that the infinite past is complete Minkowski spacetime. In such a scenario, a nonsingular bounce preceding inflation must occur (so-called the bounce inflation scenario) [13].

Recently, the Planck collaboration [14][15] have observed the power deficit of CMB TT-spectrum at large scale. This might be a hint of the pre-inflationary physics, which happens around $\sim 60$ efolds, e.g., [16]. The idea of bounce inflation accounted for not only the power deficit on large angular scales [13][17][18], but also a large dipole power asymmetry [17][19] in the CMB fluctuation. Thus we conjectured that the physics hinted by the CMB anomalies might be relevant with the pre-inflationary bounce, see also [20][21][22][23][24][25][26][27].

In physical time, the equation of motion of scalar perturbation $\zeta$ is

$$\ddot{\zeta}_k + \left(3H + \frac{\dot{Q}_s}{Q_s}\right)\dot{\zeta}_k + c_s^2 k^2 a^2 \zeta_k = 0. \quad (1)$$

Generally, $Q_s \sim \epsilon_{cont} = const. \gg 1$ for the contraction, while $Q_s \sim \epsilon_{inf} < 1$ for the inflation, where $\epsilon = -\dot{H}/H^2$. Thus $Q_s$ inevitably shows itself a jumping around the nonsingular bounce, even if this phase lasts shortly enough. Previous studies neglected the effect of $Q_s$ on the perturbation spectrum, since this effect is ambiguous without a fully stable (without the ghost and gradient instabilities) nonsingular bounce. Recently, with the effective field theory (EFT) of nonsingular cosmologies [28][29][30], we have been able to stably manipulate the bounce [31][32], see also [33][34]. This impels us to reconsider the relevant issue.

In this paper, inspired by [28][29][31][32], we build a fully stable model of bounce inflation, in which initially the universe is in the ekpyrotic contraction. By numerically solving Eq.
(1), we find that the pre-inflationary bounce not only brings the power deficit of the CMB TT-spectrum at low multipoles (as expected in [13][17]), but unexpectedly, also provides a better explanation to the dip at multipole \( l \sim 20 \) hinted by Planck [6].

II. THE LAGRANGIAN

Recently, it has been found that the nonsingular cosmological models usually suffer from the ghost or gradient instabilities \((c_s^2 < 0)\) [35][36], see also [37][38]. Based on the EFT of nonsingular cosmologies [28][29][30], this No-go result has been clearly illustrated. The cubic Galileon interaction \( \sim \Box \phi \) in Horndeski theory [39][40][41] only moves the period of \( c_s^2 < 0 \) to the outside of bounce phase, but cannot dispel it completely [42][43]. It has been found first in [28][29] that the operator \( R^{(3)} \delta g^{00} \) in EFT could play significant role in curing the gradient instability of scalar perturbation. Recently, we have built fully stable cosmological bounce models in Ref. [31] by applying the covariant \( L_{R^{(3)} \delta g^{00}} \).

We follow Ref. [31], and after defining \( \phi_\mu = \nabla_\mu \phi, \phi^\mu = \nabla^\mu \phi, \phi_{\mu\nu} = \nabla_\nu \nabla_\mu \phi, X = \phi_\mu \phi^\mu \) and \( \Box \phi = \phi^\mu \phi_\mu \), write the effective Lagrangian of nonsingular bounce inflation as (\( \phi \) is set dimensionless)

\[
L \sim \frac{M_p^2}{2} R - \frac{M_p^2}{2} X - V(\phi) + \frac{\tilde{P}(\phi, X)}{2} + L_{R^{(3)} \delta g^{00}} + L_{\delta K \delta g^{00}},
\]

\[(\text{Ghost free}) \text{ Bounce Removing } c_s^2 < 0\]

where

\[
L_{R^{(3)} \delta g^{00}} = \frac{f_1(\phi)}{2} \delta g^{00} R^{(3)}
\]

\[
= \frac{f_1(\phi)}{2} 
\frac{X}{2} \int f_{\phi \phi} d \ln X - \left( f_\phi + \int \frac{f_\phi}{2} d \ln X \right) \Box \phi
\]

\[
+ \frac{f}{2X} \left[ \phi_\mu \phi_\nu - (\Box \phi)^2 \right] - \frac{f - 2X f_X}{X^2} \left[ \phi^\mu \phi_\mu \phi^\nu \phi_\nu - (\Box \phi) \phi^\mu \phi_\mu \phi^\nu \right],
\]

\[
L_{\delta K \delta g^{00}} = \frac{g_1(\phi)}{2} \delta K \delta g^{00}
\]

\[
= \frac{g}{2 \sqrt{-X}} \left( \frac{\phi^\mu \phi_\mu \phi^\nu \phi_\nu}{X} - \Box \phi \right) - \frac{3}{2} g H,
\]

\[
f = f_1(\phi) \left[ 1 + \frac{X}{f_2(\phi)} \right], \quad g = g_1(\phi) \left[ 1 + \frac{X}{f_2(\phi)} \right],
\]

\[(5)\]
with \( f_2 = \frac{X}{\delta g^{00}} \) = \( \dot{\phi}^2(t) \), \( R^{(3)} \delta g^{00} \) and \( \delta K \delta g^{00} \) being the EFT operators (\( R^{(3)} \) is the 3-dimensional Ricci scalars on the spacelike hypersurface). We briefly review the EFT of nonsingular cosmologies in Appendix A, see (A3) for the definition of \( \delta g^{00} \) and \( \delta K \). Though \( L_{\delta g^{00} R^{(3)}} \) has the higher order of the second order derivative of \( \phi \), it is Ostrogradski ghost-free \cite{44,45}. Additionally, \( L_{\delta g^{00} R^{(3)}} \) and \( L_{\delta K \delta g^{00}} \) do not affect the cosmological background.

III. A STABLE MODEL OF BOUNCE INFLATION

A. Background

A sketch of the bounce inflation scenario is plotted in Fig. 1. We will show how to build its stable model with the Lagrangian (2).

As a specific model, we set

\[
\tilde{P}(\phi, X) = \frac{\alpha_0}{(1 + (\phi/\lambda_1)^2)^2} M_p^2 X/2 + \frac{\beta_0}{(1 + (\phi/\lambda_1)^2)^2} X^2/4,
\]

\[
V(\phi) = -\frac{V_0}{2} e^{V/\sqrt{\phi}} \left[ 1 - \tanh \left( \frac{\phi}{\lambda_2} \right) \right] + \frac{\Lambda}{2} \left( 1 - \left( \frac{\phi}{\lambda_3} \right)^2 \right) \left[ 1 + \tanh \left( \frac{\phi}{\lambda_2} \right) \right],
\]

with the positive constants \( \lambda_{1,2,3} \) and \( q, \alpha_0, \beta_0 \) being dimensionless. We have \( \tilde{P}(\phi, X) \neq 0 \) only around \( \phi \approx 0 \) \cite{46,47,48}, while \( \tilde{P}(\phi, X) = 0 \) for \( |\phi| \gg \lambda_1 \).
Thus we have
\[ 3H^2M_p^2 = \left[ 1 - \frac{\alpha_0}{(1 + (\phi/\lambda_1)^2)^2} \right] M_p^2 \phi^2/2 + \frac{3\beta_0}{(1 + (\phi/\lambda_1)^2)^2} \phi^4/4 + V(\phi), \] (8)
\[ \dot{H}M_p^2 = -\left[ 1 - \frac{\alpha_0}{(1 + (\phi/\lambda_1)^2)^2} \right] M_p^2 \phi^2/2 - \frac{\beta_0}{(1 + (\phi/\lambda_1)^2)^2} \phi^4/2. \] (9)

In infinite past, the universe is almost Minkowski, which will experience the ekpyrotic contraction. In the ekpyrotic phase \( \phi \ll -\lambda_1 \) and \(-\lambda_2 \), we have \( \dot{P} = 0 \) and \( V_{ekpy} = -V_0 e^{\sqrt{2}q\phi} \) \( (q \ll 1) \). Thus we could write Eqs. (8) and (9) as
\[ 3H^2 = \frac{\dot{\phi}^2}{2} - \frac{V_0}{M_p^2} e^{\sqrt{2}q\phi}, \quad \dot{H} = -\frac{\dot{\phi}^2}{2}. \] (10)

By solving (10), we have
\[ a \sim (-t)^{1/\epsilon}, \quad \dot{\phi} = \sqrt{\frac{2}{\epsilon}} (-t)^{-1}, \] (11)
and
\[ \phi(t) = \sqrt{\frac{2}{\epsilon}} \ln \left[ \frac{\sqrt{\epsilon - 3}}{\epsilon \sqrt{V_0/M_p}} (-t)^{-1} \right], \] (12)
where \( \epsilon = -\dot{H}/H^2 = 1/q \gg 1 \), which suggests \( H = -\epsilon^{-1}(-t)^{-1} \).

When \( \phi \sim \lambda_1 \), we could have
\[ \dot{H} \sim \left( \frac{\alpha_0}{4} - \frac{\beta_0 \dot{\phi}^2}{4M_p^2} - 1 \right) \frac{\dot{\phi}^2}{2} > 0, \] (13)
the nonsingular bounce will occur. While after \( \phi \gg \lambda_1, \lambda_2 \), the field \( \phi \) will be canonical \((\dot{P} = 0)\) again. We have
\[ 3H^2 = \frac{\dot{\phi}^2}{2} + \frac{\Lambda}{M_p^2} \left( 1 - \left( \frac{\phi}{\lambda_3} \right)^2 \right)^2, \quad \dot{H} = -\frac{\dot{\phi}^2}{2}. \] (14)

Thus the slow-roll inflation will occur. Actually, after the nonsingular bounce, the Lagrangian (2) will reduce to \( L \sim M_p^2 R/2 - M_p^2 X/2 - V_{inf} \) with \( V_{inf} \) being the potential of slow-roll inflation.

We plot the background evolution in Fig. 2 with \( \alpha_0 = 20, \beta_0 = 5 \times 10^9, \lambda_1 = 0.224, \lambda_2 = 0.0667, \lambda_3 = 12, V_0 = 5 \times 10^{-9} M_p^4 \), \( q = 0.1, \Lambda = 2.5 \times 10^{-9} M_p^4 \). The initial values are set by (11) and (12).
FIG. 2: The background evolution of our model with $\alpha_0 = 20$, $\beta_0 = 5 \times 10^9$, $\lambda_1 = 0.224$, $\lambda_2 = 0.0667$, $\lambda_3 = 12$, $V_0 = 5 \times 10^{-9} M_p^4$, $q = 0.1$, $\Lambda = 2.5 \times 10^{-9} M_p^4$.

B. Simulation for the scalar perturbation spectrum

In unitary gauge $\delta \phi = 0$, the quadratic action of scalar perturbation $\zeta$ for (2) is (see Appendix A and also our [28])

$$S^{(2)}_\zeta = \int a^3 Q_s \left( \dot{\zeta}^2 - c_s^2 \left( \frac{\partial \zeta}{\partial a} \right)^2 \right) d^4 x,$$

in which

$$Q_s = \frac{2\dot{\phi}^4 \tilde{P}_{XX} - M_p^2 \dot{H}}{\gamma^2} + 3 \left( \frac{g_1}{2\gamma M_p} \right)^2,$$

$$c_s^2 Q_s = \frac{\dot{c}_3}{a} - M_p^2,$$

$$c_3 = \frac{a M_p^2 \dot{\beta} + \gamma \left( \frac{1 + 2f_1}{M_p^2} \right)}{\gamma},$$

with $\gamma = H + \frac{\dot{\phi}^2}{2M_p^2}$.

The stabilities require $Q_s > 0$ and $c_s^2 > 0$. Generally, $Q_s > 0$ can be obtained by applying
\( \dot{P}(\phi, X) \). While around the bounce point \( H \simeq 0 \),

\[
c_s^2 \sim -\dot{\gamma} \left( 1 + \frac{2f_1}{M_p^2} \right) + \frac{2\dot{f}_1\gamma}{M_p^2} - \gamma^2.
\] (18)

We will have \( c_s^2 > 0 \) only for \( 2f_1 < -M_p^2 \), as has been clarified in Refs. [28][30]. Thus the gradient instability \( (c_s^2 < 0) \) is cured by \( L_{\delta g^0}R^{(3)} \), since if \( f_1 \equiv 0 \), we have \( c_s^2 \sim -\dot{\gamma} - \gamma^2 < 0 \) around the bounce point. Here, we always could set \( c_s^2 \sim O(1) \) with a suitable \( f_1(\phi) \) (see also [30]) satisfying

\[
2f_1(\phi) = \frac{\gamma}{a} \int a (Q_s c_s^2 + M_p^2) \, dt - M_p^2.
\] (19)

In conformal time \( \eta = \int dt/a \), the motion equation of \( \zeta \) is

\[
u'' + \left( c_s^2 k^2 - \frac{z''}{z_s} \right) \nu = 0,
\] (20)

where \( u = z_s \zeta \) and \( z_s = \sqrt{2a^2 Q_s} \). In infinite past, the universe is almost Minkowski, and will come through the ekpyrotic phase. The perturbation modes have the wavelength \( \lambda \simeq 1/k \ll \sqrt{z_s/z_s''} \) and \( c_s^2 = 1 \). Thus the initial state of the perturbation is

\[
u \simeq \frac{1}{\sqrt{2k}} e^{-ik\eta}.
\] (21)

The perturbation modes will pass through the ekpyrotic phase, the bounce phase and the inflation phase, sequentially. The resulting spectrum \( P_\zeta \) of \( \zeta \) (at \( -k\eta \ll 1 \)) is

\[
P_\zeta = \frac{k^3}{2\pi^2} |\zeta|^2.
\] (22)

In physical time, the motion equation of \( \zeta \) is (1). In the ekpyrotic phase, \( z_s \sim a \sim (-\eta)^{1/\epsilon_{ekpy}} \), since \( Q_s \sim \epsilon_{ekpy} = \text{const.} \gg 1 \). While in the inflationary phase, \( \epsilon_{inf} < 1 \). This suggests that \( Q_s \) (or \( z_s \sim a\sqrt{Q_s} \)) will show itself a jumping around the nonsingular bounce, which will inevitably affect \( P_\zeta \). Whether the jumping of \( Q_s \) is gentle or not is model-dependent. We will simulate its effect on \( P_\zeta \) by numerically solving Eq. (1), with \( c_s^2 = 1 \) set by Eq.(19).

It should be mentioned that if \( g_1 = 0 \) (\( L_{\delta K} \delta g^{00} \) is absent), we will have \( \gamma = H = 0 \) at the bounce point and \( Q_s \sim 1/\gamma^2 \) is divergent, see (17), so that Eq. (1) is singular. Here, in order to avoid it, we apply \( g_1(\phi) \), see also [30].

Without loss of generality, we set

\[
Q_s = A_Q \left[ B - \tanh \left( \frac{t}{t_s} \right) \right],
\] (23)

\[
7
\]
which requires

\[ g_1(\phi(t)) = -\frac{2HM_p^2Q_s - 2\sqrt{3H^2M_p^6Q_s + M_p^4(3M_p^2 - Q_s)\left(\dot{H}M_p^2 - 2\dot{\phi}^2P_{XX}\right)}}{Q_s - 3M_p^2} \tag{24} \]

in Lagrangian (2), see (17). We plot the spectrum \( P_\zeta \) of scalar perturbation in Fig. 3 for the background in Fig. 2 and the different values of \( B \) and \( t_s \), where \( P^{inf}_\zeta = \frac{H_{inf}^2}{8Q_{inf}^2\pi^2M_p^2}\left(\frac{k}{\mathcal{H}_{inf}}\right)^{n_s-1} \)

is that of the inflation, with \( Q_{inf}^s \) being the value of \( Q_s \) during inflation, \( n_s - 1 \simeq 0 \) (but is slightly red). The evolutions of \( Q_s \), \( g_1 \) and \(|\zeta|\) with respect to \( t \), respectively, are plotted in Figs. 7 and 8 of Appendix B.

As expected in [13], \( P_\zeta \) shows itself a large-scale cutoff, but is flat (with a damped oscillation) at small scale. However, due to the step-like evolution of \( Q_s \), the peaks and valleys of the oscillations are obviously pulled lower. Actually, after the nonsingular bounce, with Eq. (1), we shortly have the effective Hubble parameter

\[ H_{inf}^{eff} = H_{inf} + \frac{\dot{Q}_s}{3Q_s} < H_{inf}, \tag{25} \]

since \( \dot{Q}_s < 0 \), see Figs. 7(b) and 8(b) in Appendix B. Thus \( P_\zeta \) is pulled lower at the corresponding scale, since \( P_\zeta \sim (H_{inf}^{eff})^2 \). The change rate of \( Q_s \) is relevant to the physics of nonsingular bounce, as showed in Eq. (23), so the depth of valley pulled lower is actually model-dependent.

In Sec. IV B, we will show that such a marked lower valley at corresponding scale helps to explain the dip around \( l \simeq 20 \) hinted by Planck [6].

IV. MORE ON THE SPECTRUM

A. Analytical estimation

We will attempt to analytically estimate \( P_\zeta \). The equation of motion for \( \zeta \) is (20). In [26], the spectrum of primordial GWs has been calculated. Here, if the effect of \( Q_s \) is neglected, the calculation will be similar.

The bounce phase is the evolution with \( \dot{H} > 0 \). We define that it begins and ends at \( \eta_{B-} \)

and \( \eta_{B+} \), respectively, at which \( \dot{H} = 0 \). We set that \( H = 0 \) at \( \eta_B \), which corresponds to the bounce point. Generally, \( \Delta \eta_B = \eta_{B+} - \eta_{B-} \lesssim 1/\mathcal{H}_{B+} \).
In our model (Sec. III), the contracting phase ($\eta < \eta_{B-}$) is ekpyrotic-like, $a$ is almost constant for $\epsilon_{ekpy} \gg 1$. Considering the continuities of $a$ and $H$ at $\eta_{B-}$, we have

$$a(\eta) = a_{B-} \left[ \frac{x}{(\epsilon_{ekpy} - 1)^{-1}H_{B-}^{-1}} \right]^{\frac{1}{\epsilon_{ekpy} - 1}},$$

(26)

see [26] for the details, where $H_{B-}$ is the comoving Hubble parameter at $\eta_{B-}$ and $x = \eta - \eta_{B-} + (\epsilon_{ekpy} - 1)^{-1}H_{B-}^{-1}$. We have $z_s''/z_s = a''/a$, since $Q_s$ is constant. Thus the solution of (20) is

$$u_k = \frac{\sqrt{\pi|x|}}{2} c_{1,1} H_{\nu_1}^{(1)}(-kx)$$

(27)

where $\nu_1 = 1/2$ for $\epsilon_{ekpy} \gg 1$, and the initial condition (21) has been used.

In the nonsingular bounce phase ($\eta_{B-} < \eta < \eta_{B+}$), $H$ should cross 0. We parameterize it as $H = \alpha(t - t_B)$ [49] with $\alpha M_P^2 \ll 1$. We have

$$a \simeq a_B e^{\frac{1}{2} \alpha (t-t_B)^2} \simeq a_B \left[ 1 + \frac{\alpha}{2} (t-t_B)^2 \right],$$

(28)

where $a = a_B$ at the bouncing point $t = t_B$. The continuities of $a$ and $H$ at $\eta_{B-}$ and $\eta_{B+}$ suggest $H_{B+} = H_{B-} + \alpha a_B^2 (\eta_{B+} - \eta_{B-})$. In our models, $|H_{B-}| \lesssim H_{B+}/4$, see Figs. 7 and 8 in Appendix B, so that we approximately have

$$H_{B+} \simeq \alpha a_B^2 \Delta \eta_B.$$  \hspace{1cm} (29)
Thus in this phase the equation (20) is

\[ u''_k + (k^2 - \alpha a_B^2)u_k = 0. \tag{30} \]

Its solution is

\[ u_k(\eta) = c_{2,1}e^{l(\eta-\eta_B)} + c_{2,2}e^{-l(\eta-\eta_B)}, \tag{31} \]

where \( l = \sqrt{\alpha a_B^2 - k^2} \). Here, we have neglected the effect of \( Q_s \), or it is difficult to solve Eq. (20).

In inflationary phase \((\eta \geq \eta_B)\), \( Q_s^{inf} \) is almost constant. Considering the continuities of \( a \) and \( H \) at \( \eta_B \), we have

\[ a_{inf}(\eta) = a_{B+}(-yH_{B+})^{\frac{1}{\epsilon_{inf}}}, \tag{32} \]

where \( y = \eta - \eta_B + 1/H_{B+} \), and \( H_{B+} = H_{B+}/a \), \( H_{inf} \lesssim H_{B+} \). The solution of (20) is

\[ u_k = \frac{\sqrt{\pi} |y|}{2} \left[ c_{3,1}H_{\nu_2}^{(1)}(-ky) + c_{3,2}H_{\nu_2}^{(2)}(-ky) \right] \tag{33} \]

where \( \nu_2 = \frac{\epsilon_{inf} - 3}{2(\epsilon_{inf} - 1)} \).

We have \( P_\zeta \) as

\[ P_\zeta(k, H_{B+}, H_{B-}, \Delta \eta) \approx \frac{H_{inf}^2}{8\pi^2 Q_s^{inf} M_p^2} |c_{31} - c_{32}|^2 = P_\zeta^{inf}|c_{31} - c_{32}|^2, \tag{34} \]

where \( P_\zeta^{inf} = \frac{H_{inf}^2}{8\pi^2 Q_s^{inf} M_p^2} \) is that of the slow-roll inflation. Requiring the continuities of \( \zeta \) and \( \dot{\zeta} \), we could write the coefficients as

\[ \begin{pmatrix} c_{3,1} \\ c_{3,2} \end{pmatrix} = M^{(3,2)} \times M^{(2,1)} \times \begin{pmatrix} c_{1,1} \\ c_{1,2} \end{pmatrix}, \tag{35} \]

see Appendix C for the matrices \( M^{(2,1)} \) and \( M^{(3,2)} \).

The effects of bounce has been encoded in \( M^{(3,2)} \) and \( M^{(2,1)} \) (or \( |c_{3,1} - c_{3,2}|^2 \)). We approximately have

\[ |c_{3,1} - c_{3,2}|^2 \approx 1 - A \sin \left( \frac{2k}{H_{B+}} \right) - A \sin \left( \frac{2k}{H_{B+}} + 2k\Delta \eta_B \right) \tag{36} \]

for \( k \gg H_{B+} \), where

\[ A = \frac{H_{B+}}{k} \left( 1 - \frac{\alpha a_B^2}{2H_{B+}} \Delta \eta_B \right) \approx \frac{H_{B+}}{2k} \tag{37} \]
and (29) is used, which suggests that on small scale \( k \gg \mathcal{H}_{B+} \), \( P_\zeta \) is flat with a rapidly damped oscillation, its maximal oscillating amplitude is around \( k \simeq \mathcal{H}_{B+} \). However, if the bounce phase lasts shortly enough, \( \Delta \eta_B \ll 1/\mathcal{H}_{B+} \), (36) will be

\[
|c_{3,1} - c_{3,2}|^2 \approx 1 - \frac{\mathcal{H}_{B+}}{k} \sin \left( \frac{2k}{\mathcal{H}_{B+}} \right).
\]  

(38)

While on large scale \( k \ll \mathcal{H}_{B+} \), \( P_\zeta \sim k^2 \) will have a strongly blue tilt, since

\[
|c_{3,1} - c_{3,2}|^2 \approx w(\Delta \eta_B) \left( \frac{k}{\mathcal{H}_{B+}} \right)^2
\]

(39)

where

\[
w(\Delta \eta_B) = \left[ (1 - \frac{l^2}{2\mathcal{H}_{B+}}) \cosh(l \Delta \eta_B) + \frac{l}{2} \mathcal{H}_{B+} - \Delta \eta_B + \frac{l^2}{4\mathcal{H}_{B+}} \Delta \eta_B^2 \sinh(l \Delta \eta_B) \right]^2
\]

(40)

which is \( w(\Delta \eta_B) \approx 1 \) for \( \Delta \eta_B \approx 0 \).

We plot \( P_\zeta \) for (34) in Figs. 4 for the different values of \( \Delta \eta \) and \( \mathcal{H}_{B-} \). We see that for \( k > \mathcal{H}_{B+} \), \( P_\zeta \sim k^0 \) but has a damped oscillation, while for \( k < \mathcal{H}_{B+} \), \( P_\zeta \sim k^2 \) shows itself a large-scale cutoff. Thus (34) is consistent with our simulation result (see Fig. 7 in Sec. III) well at large and small scales, respectively.

However, since we have neglected the step-like evolution of \( Q_s \), the pull-lower around \( k \simeq \mathcal{H}_{B+} \) in Fig. 7(d) cannot be reflected in (34).

![Fig. 4: The power spectrum with different \( \Delta \eta \) and different \( \mathcal{H}_{B-}/\mathcal{H}_{B+} \).](image)

**B. Template**

To conveniently fit the observation data, a simple “Template” capturing the essential shape of \( P_\zeta \) is indispensable. Based on the simulation in Sec. III and the analytical estimate
in Sec. IV A, we write it as

\[ P_{\zeta} = F(k, \mathcal{H}_{B+}, A_d, \omega_d) \cdot P_{\zeta}^{inf}, \]

where \( P_{\zeta}^{inf} = A_{inf}(\frac{k}{k_*})^{n_{inf}-1} \) is the spectrum predicted by slow-roll inflation, and \( A_{inf} \) is the amplitude at the pivot scale \( k_* \), \( n_{inf} \) is its tilt, and

\[
F(k, \mathcal{H}_{B+}, A_d, \omega_d) = \left\{ 1 + e^{-\left(\frac{k}{\mathcal{H}_{B+}}\right)^2} \left( \frac{k}{\mathcal{H}_{B+}} \right)^2 \right. \\
+ e^{-\left(\frac{k}{\mathcal{H}_{B+}}\right)^2} \left( \frac{\sin(2k/\mathcal{H}_{B+})}{k/\mathcal{H}_{B+}} \right) \left[ 1 - A_d \cdot e^{-O(1)\omega_d} \right] \right\}.
\]

(42)

Here, the parameters set \((\mathcal{H}_{B+}, A_d, \omega_d)\) reflects the effect of pre-inflationary bounce on the spectrum. Around \( k \gtrsim \mathcal{H}_{B+} \), we have

\[
F(k, \mathcal{H}_{B+}, A_d, \omega_d) \simeq 1 - A_d e^{-O(1)\omega_d},
\]

(43)

so \( A_d \) and \( \omega_d \) (related with the parameter \( \Delta \eta < 1/\mathcal{H}_{B+} \) in Sec. IV A) depict the width and depth of valley around \( k \gtrsim H_{B+} \), respectively. Here, \( A_d \) is related with the change rate of \( Q_s \) (neglected in Sec. IV A). With Eq. (25), we have approximately

\[
A_d \simeq \frac{2 \left| \dot{Q}_s \right|_{max}}{3H_{inf}Q_s}
\]

(44)

noting \( \dot{Q}_s < 0 \). In (42), we have

\[
F(k, \mathcal{H}_{B+}, A_d, \omega_d) \sim 1 - \frac{\sin(2k/\mathcal{H}_{B+})}{k/\mathcal{H}_{B+}}
\]

(45)

for \( k \gg \mathcal{H}_{B+} \), which equals to (38), while for \( k \ll \mathcal{H}_{B+} \), we approximately have

\[
F(k, \mathcal{H}_{B+}, A_d, \omega_d) \simeq \left( \frac{k}{\mathcal{H}_{B+}} \right)^2,
\]

which is consistent with (39). \( P_{\zeta} \) for the “Template” (42) is plotted in Fig. 5. We see that (42) has effectively captured the essential shape of \( P_{\zeta} \) showed in Fig. 3.

C. Data fitting

We modified the CAMB and CosmoMC code package and perform a global fitting with Planck2015 data. The parameter set of the lensed-ΛCDM model is \( \{ \Omega_b h^2, \Omega_c h^2, 100\theta_{MC}, \tau, \ln(10^{10} A_{inf}), n_{inf} \} \), with \( \Omega_b h^2 \) the baryon density, \( \Omega_c h^2 \) the cold dark
matter density, $\theta_{\text{MC}}$ the angular size of the sound horizon at decoupling, and $\tau$ the reionization optical depth. We also include the parameters set $\{H_B, A_d, \omega_d\}$ (so-called the bounce 3-parameters) defined in (42), which captures the physics of pre-inflationary bounce, as has been argued. We set the pivot scale $k_* = 0.05\text{Mpc}^{-1}$, roughly in the middle of the logarithmic range of scales probed by Planck.

With (42), we plot the CMB TT-spectrum $D_l^{TT} \equiv l(l + 1)C_l^{TT}/2\pi$ and $\Delta D_l^{TT}$ in Fig. 6 with the best-fit parameters set $\{\Omega_\text{b}h^2, \Omega_ch^2, 100\theta_{\text{MC}}, \tau, \ln(10^{10}A_{inf}), n_{inf}, H_B, A_d, \omega_d\}$. Since WMAP and Planck, some models attempting to explain the anomalies of CMB at large scale (but not solving the initial singularity) have been proposed [50][51][52][53][54][55].

We see that the spectrum (42) of scalar perturbation predicted by our model could fit better not only the power deficit of the CMB TT-spectrum at low multipoles, but also the dip at $l \sim 20$. Actually, after we add the bounce 3-parameters $\{H_B, A_d, \omega_d\}$ into the parameter set of the $\Lambda$CDM model, the corresponding $\Delta\chi^2$ value can be greatly improved. The details will be presented in upcoming work.

V. CONCLUSION

In bounce inflation scenario, the inflation is singularity-free (past-complete). However, its pathology-free model has been still lacking. Here, we showed such a model. The nonsingular bounce is implemented by applying $\tilde{P}(\phi, X)$, see (6), which is ghost-free, while $\epsilon_s^2 < 0$ is dispelled by $L_{\delta h_{\text{de}} R^{(3)}}$ [31].

We perform a full simulation for the evolution of scalar perturbation, and find that the spectrum $P_\zeta$ has a suppression at large scale $k \ll H_B$ but is flat (with a damped oscillation) at small scale $k \gg H_B$, which confirms the earlier results showed in [13][17] and is consistent with the power deficit of the CMB TT-spectrum at low multipoles $l \lesssim 30$; but unexpectedly, $P_\zeta$ also shows itself one marked lower valley at $k \gtrsim H_B$, though the depth is model-dependent. We show that this lower valley actually provides a better fit to the dip at $l \sim 20$ hinted by Planck [6]. Based on the simulation and the analytical estimation for the perturbation spectrum, we also offer a “Template” of $P_\zeta$ (effectively capturing the physics of bounce) to fit data.
The equation of motion of GWs mode $\gamma_{ij}$ for (2) is

$$\ddot{\gamma}_k + \left(3H + \frac{Q_T}{Q_T}\right)\dot{\gamma}_k + c_s^2\frac{k^2}{a^2}\gamma_k = 0 ,$$

(46)

which is unaffected by the operators $R^{(3)}\delta g^{00}$ and $\delta K\delta g^{00}$, where $Q_T = M_p^2$. We plot the primordial GWs spectrum $P_T$ in Fig. 5 (the black dot curve) with $P_T^{inj} = \frac{2H^{2}_{inf}}{\pi^2 M_p^4}$, see also [26]. It should be mentioned that if $Q_T \neq M_p^2$ around the nonsingular bounce (the gravity is modified completely), $P_T$ will be different. It is also possible that the corresponding gravity has a large parity violation [56], which might be imprinted in CMB.

Our work highlight the conjecture again that the physics hinted by the large-scale anomalies of CMB is related with the pre-inflationary bounce. The nonsingular cosmological bounce also has been implemented in some models of modified gravity [57–68], see also [69][70] for reviews. Confronting the corresponding models with the CMB data will be interesting.

*FIG. 5: The black dotted curve is the spectrum $P_T/P_{T,inf}$ of the primordial GWs in bounce inflation scenario, see [26], while the {green dotdashed, red dashed, brown solid} curves are those of the primordial scalar perturbation based on the results of “Template” (42) with $A_d = \{0.25, 0.8, 0.8\}$, $d = \{\pi, \pi, \pi\}$ and $\omega_d = \{0.25, 0.25, 0.1\}$, which are consistent with those in Fig. 3.*

**Acknowledgments**

YC would like to thank Youping Wan and Yi-Fu Cai for discussions and hospitalities during his visit at University of Science and Technology of China. YSP thanks Mingzhe Li for helpful suggestions in USTC-ICTS seminar. We acknowledge the use of CAMB and
FIG. 6: The green points show the Planck2015 data with $1\sigma$ errors. The best-fit values of parameters are $\ln(10^{10}A_{inf}) = 3.091$, $n_{inf} = 0.966$, $\ln(H_{B+}) = -7.51$, $A_d = 0.87$, $\omega_d = 5.47$.

CosmoMC. This work is supported by NSFC, No. 11575188, 11690021, and also supported by the Strategic Priority Research Program of CAS, No. XDA04075000, XDB23010100.

**Appendix A: The EFT of nonsingular cosmologies**

In this Appendix, we briefly review the EFT of nonsingular cosmologies, see [28] for the details.

With the ADM $3+1$ decomposition, we have

\[
g_{\mu\nu} = \begin{pmatrix} N_k N^k - N^2 N_j \\ N_i \\ N^j \end{pmatrix}, \quad g^{\mu\nu} = \begin{pmatrix} -N^{-2} N^j \\ N^i \\ h^{ij} - \frac{N^i N^j}{N^2} \end{pmatrix},
\]

and $\sqrt{-g} = N \sqrt{h}$, where $N_i = h_{ij} N^j$. The induced metric on 3-dimensional hypersurface is $h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$, where $n_\mu = n_0 (dt/dx^\mu) = (-N, 0, 0, 0)$, $n^\nu = g^{\mu\nu} n_\mu = (1/N, -N^i / N)$ is orthogonal to the spacelike hypersurface, and $n_\mu n^\mu = -1$. Thus

\[
h_{\mu\nu} = \begin{pmatrix} N_k N^k \\ N_i \\ h_{ij} \end{pmatrix}, \quad h^{\mu\nu} = \begin{pmatrix} 0 & 0 \\ 0 & h^{ij} \end{pmatrix}.
\]
The EFT is [28]

\[
S = \int d^4x \sqrt{-g} \left[ \frac{M_p^2}{2} f(t) R - \Lambda(t) - c(t) g^{00} \right. \\
+ \frac{M_4^2}{2}(\delta g^{00})^2 - \frac{m_3^2(t)}{2} \delta K \delta g^{00} - m_4^2(t) (\delta K^2 - \delta K_{\mu\nu} \delta K^{\mu\nu}) + \frac{\tilde{m}_4^2(t)}{2} R^{(3)} \delta g^{00} \\
- \tilde{m}_4^2(t) \delta K^2 + \frac{\tilde{m}_5(t)}{2} R^{(3)} \delta K + \frac{\tilde{x}(t)}{2} (R^{(3)})^2 + \ldots \\
- \frac{\tilde{x}(t)}{M_p^2} \nabla_i R^{(3)} \nabla^i R^{(3)} + \ldots \right],
\]

(A3)

where \( \delta g^{00} = g^{00} + 1, \) \( R^{(3)} \) is the 3-dimensional Ricci scalar, \( K_{\mu\nu} = h^{\mu}_\sigma \nabla_\sigma n_\nu \) is the extrinsic curvature, \( \delta K_{\mu\nu} = K_{\mu\nu} - h_{\mu\nu} H \).

Here, we focus on building a stable model of bounce inflation. We only consider the coefficients set \( (f, c, \Lambda, M_2, m_3, \tilde{m}_4) \), and set other coefficients in (A3) equal to 0. We always could set \( f = 1 \), which suggests \( c(t) = -M_p^2 \dot{H} \) and \( c(t) + \Lambda(t) = 3M_p^2 H^2 \).

As pointed out in Ref. [33], the \( R^{(3)} \delta K \) operator in EFT could play similar role as \( R^{(3)} \delta g^{00} \), which we will consider elsewhere. Mapping (2) into the EFT (A3), we have \( M_4^2(t) = X^2 \dot{P}_{XX}, m_3^2(t) = -g_1(\phi) \) and \( \tilde{m}_4 = f_1(\phi) \). Only with \( (M_2, m_3, \tilde{m}_4) \neq 0 \), the quadratic action of scalar perturbation \( \zeta \) is (see, e.g., our [28])

\[
S^{(2)}_\zeta = \int d^4x a^3 Q_s \left( \dot{\zeta}^2 - c_s^2 \frac{(\partial \zeta)^2}{a^2} \right),
\]

(A4)

where

\[
Q_s = \frac{2M_4^2}{\gamma} + \frac{3m_3^6}{4M_p^2 \gamma^2} - \frac{\dot{H}M_p^2}{\gamma^2},
\]

(A5)

\[
c_s^2 Q_s = \frac{\dot{c}_3}{a} - M_p^2
\]

(A6)

\[
c_3 = \frac{aM_p^2}{\gamma} \left( 1 + \frac{2\tilde{m}_4^2}{M_p^2} \right),
\]

(A7)

where \( \gamma = H - m_3^2/(2M_p^2) \). Only if \( Q_s > 0 \) and \( c_s^2 > 0 \), the nonsingular cosmological model is healthy. In models with the operator \( (\delta g^{00})^2 \), \( Q_s > 0 \) always can be obtained, since \( (\delta g^{00})^2 \) contributes \( \dot{\zeta}^2 \). While \( c_s^2 > 0 \) requires \( \dot{c}_3 > aM_p^2 \), which is

\[
c_3|_{t_f} - c_3|_{t_i} > M_p^2 \int_{t_i}^{t_f} a dt.
\]

(A8)

The inequality (A8) suggests that \( c_3 \) must cross 0 (\( \tilde{m}_4^2 = M_p^2/2 \) or \( \gamma \) is divergent), since the integral \( \int a dt \) is infinite. Thus if the \( R^{(3)} \delta g^{00} \) operator is absent, \( c_s^2 > 0 \) throughout is
impossible. We can set \( c_s^2 \simeq 1 \) by

\[
2m_4^2 = \frac{\gamma}{a} \int a \left( Q_s c_s^2 + M_p^2 \right) dt - M_p^2.
\]  
(A9)

**Appendix B: More on the simulation**

We plot the evolutions of \( Q_s, g_1, |\zeta| \) with respect to \( t \), and also \( P_\zeta(k) \) for the background in Fig. 2, with different values of \( B \) and \( t_* \) in this Appendix.

We see how \( |\zeta| \) evolves with \( a \) in different phases. Theoretically, \( \zeta \sim 1/a \) for the perturbation modes with \( k \gg \sqrt{z_s''/z_s} \), while \( \zeta \sim \text{const.} \) for the perturbation modes with \( k \ll \sqrt{z_s''/z_s} \), which is consistent with our Figs. 7(c) and 8(c).

![Graphs](image)

(a) \( Q_s \)  
(b) \( 10^5 (3H + Q_s/Q_s) \)

(c) \(|\zeta| \) for \( k = \{10^{-5}, 3 \times 10^{-5}, 10^{-3}, 10^{-2}\} \) from top to bottom

(d) \( P_\zeta/P_\zeta^{inf} \)

**FIG. 7:** We set \( A_Q = 3, B = 2, t_* = 4 \times 10^4 \) and the background is given by Fig. 2.
FIG. 8: We set \( A_Q = 3, \ B = 1.6, \ t_* = 3 \times 10^4 \) and the background is given by Fig. 2.

Appendix C: The matrices elements of \( \mathcal{M}^{(2,1)} \) and \( \mathcal{M}^{(3,2)} \)

We define \( l = \sqrt{\alpha a_B^2 - k^2}, \ x_1 = 1/|\mathcal{H}_{B-}|, \ x_2 = \mathcal{H}_{B+}, \ y_{1,2} = (\eta_{B+} - \eta_B) \), and have

\[
\mathcal{M}_{11}^{(2,1)} = \frac{\sqrt{\pi x_1}}{4l} \left[ (l + \alpha a_B^2 y_1) H_{\nu_1}^{(1)}(k x_1) - k H_{\nu_1-1}^{(1)}(k x_1) \right] e^{-ly_1}, \quad (C1)
\]
\[
\mathcal{M}_{12}^{(2,1)} = \frac{\sqrt{\pi x_1}}{4l} \left[ (l + \alpha a_B^2 y_1) H_{\nu_1}^{(2)}(k x_1) - k H_{\nu_1-1}^{(2)}(k x_1) \right] e^{-ly_1}, \quad (C2)
\]
\[
\mathcal{M}_{21}^{(2,1)} = \frac{\sqrt{\pi x_1}}{4l} \left[ (l - \alpha a_B^2 y_1) H_{\nu_1}^{(1)}(k x_1) - k H_{\nu_1-1}^{(1)}(k x_1) \right] e^{ly_1}, \quad (C3)
\]
\[
\mathcal{M}_{22}^{(2,1)} = \frac{\sqrt{\pi x_1}}{4l} \left[ (l - \alpha a_B^2 y_1) H_{\nu_1}^{(2)}(k x_1) - k H_{\nu_1-1}^{(2)}(k x_1) \right] e^{ly_1}, \quad (C4)
\]
\[ M^{(3,2)}_{11} = \frac{i \sqrt{\pi x_2}}{2} \left[ (l - \alpha a_B^2 y_2) H^{(2)}_{\nu_2} (k x_2) + k H^{(2)}_{\nu_2 - 1} (k x_2) \right] e^{ly_2}, \tag{C5} \]

\[ M^{(3,2)}_{12} = \frac{i \sqrt{\pi x_2}}{2} \left[ -(l + \alpha a_B^2 y_2) H^{(2)}_{\nu_2} (k x_2) + k H^{(2)}_{\nu_2 - 1} (k x_2) \right] e^{-ly_2}, \tag{C6} \]

\[ -M^{(3,2)}_{21} = \frac{i \sqrt{\pi x_2}}{2} \left[ (l - \alpha a_B^2 y_2) H^{(1)}_{\nu_2} (k x_2) + k H^{(1)}_{\nu_2 - 1} (k x_2) \right] e^{ly_2}, \tag{C7} \]

\[ -M^{(3,2)}_{22} = \frac{i \sqrt{\pi x_2}}{2} \left[ -(l + \alpha a_B^2 y_2) H^{(1)}_{\nu_2} (k x_2) + k H^{(1)}_{\nu_2 - 1} (k x_2) \right] e^{-ly_2}. \tag{C8} \]

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