Thermal wakefield oscillations of laser-induced plasma channels and their spectral signatures in luminescence

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Abstract. Starting from a general microscopic model for an interacting two-component plasma including the interaction with a quantized light field, the equations of motion in the Wigner representation are derived. In contrast to the case of strongly focused laser beams which are known to leave behind so called wakefield oscillations of the electron plasma, thermal wakefield oscillations dominate the dynamics of a femtosecond laser generated plasma rod. It is shown that the photoluminescence from the resulting electron-ion plasma bears spectral features related to the plasma frequency due to these thermal radial wakefield oscillations.

1. Introduction
In recent years, an interesting research area of nonlinear optics has evolved around so-called light strings where a relatively weakly focused laser beam ionizes air molecules via multi-photon ionization. The presence of the resulting electron gas in turn influences the propagation of the laser beam. Due to the defocusing effect of the plasma, beams with typical widths of about 100 microns do not collapse, as would be expected from the nonlinear Kerr effect alone, but can propagate over centimeters or even meters\[1, 2\].

While the presence of the plasma is important for the understanding of the beam propagation, it is also interesting in its own right. Namely, after the laser beam has passed, the electron plasma does not vanish immediately but lives on for times of the order of nanoseconds, determined by typical timescales of radiative and nonradiative recombination processes. Since the excitation of the plasma is a multiphoton process, typical diameters of the resulting plasma rod are well below the diameters of the light string. In several publications the observation of terahertz (THz) or far-infrared emission from these electron plasmas has been reported \[3, 4\]. This emission is interesting from a technological point of view since it provides an extended THz source.

In addition to the potential technological importance of femtosecond light filaments as an extended source, the mechanism of THz emission from the generated plasma raises basic physics issues. For strongly focused laser beams, plasmas with relatively small extent can be created. In this case, the ponderomotive force leads to so-called wakefield oscillations after the laser pulse has passed\[5, 6, 7\]. Such oscillating plasmas can provide a coherent source for terahertz emission\[8, 9\].
In the case of typical light strings with diameters of the order of 100 μm which can reach lengths up to several meters, however, we expect other mechanisms to be more important. In a recent paper, we investigated incoherent light-emission as one possible emission mechanism[10]. In the present paper, we study plasma oscillations driven by the thermal pressure inside the electron gas. These oscillations do not provide a transverse, coherent source for cylindrically symmetric systems. But it will be shown that the oscillations can be observed as spectral signatures in the luminescence spectrum.

In the first part of the paper, we present the outline of a general theory for an interacting two-component electron-ion plasma coupled to a quantized light field. In Sec. 2 we present the Hamiltonian and introduce our notation. Since the incoherent light emission (i.e. luminescence) will play an important role, we have to treat photons on the same quantum-mechanical level as electrons and ions. In Sec. 3 we study the electron dynamics and show that the semi-classical Vlasov equation can be obtained as limiting case and how the fluid-dynamics equations of an electron plasma can be derived from there. In the second part, we turn toward the specific geometry of light strings. In Sec. 4 we give an estimate of the different contributions to the electron dynamics for typical light-string parameters and determine the thermal pressure as the dominant mechanism for wakefield oscillations. We present an approximative solution of these oscillations before we investigate the spectral signatures in the luminescence spectrum caused by them in Sec. 5.

In our investigations, we are led by recent studies of light-strings geometries[11, 12, 13, 14] and possible emission mechanisms there [3, 4, 15]. Nevertheless, the general theory can be applied to quite different situations. For example, it is equally valid for electron-hole plasmas in semiconductors or in glasses.

2. Hamiltonian
Starting point is the Hamiltonian in second-quantized form, given by[16]

\[ H_{\text{min}} = \sum_{\lambda, s} \int d^3 r \Psi_{\lambda}^\dagger (r, s) \frac{1}{2m_{\lambda}} \left( \frac{\hbar}{i} \nabla - q_{\lambda} A(r) \right)^2 \Psi_{\lambda} (r, s), \]  

where \( \Psi_{\lambda} (r, s) \) and \( \Psi_{\lambda}^\dagger (r, s) \) are annihilation and creation operators for electrons (\( \lambda = e \)) and ions (\( \lambda = i \)), \( m_{\lambda} \) and \( q_{\lambda} \) denote the respective mass and charge, and

\[ A(r) = \sum_{q, \sigma} F_q U_{q, \sigma}(r) B_{q, \sigma} + \text{h.c.} \]

is the operator of the quantized vector potential. In this mode expansion,

\[ U_{q, \sigma}(r) = e^{i q \cdot r} \epsilon_{q, \sigma}, \]

denotes the eigemmode of the electro-magnetic field in vacuum, and the prefactor

\[ F_q = \sqrt{\frac{\hbar}{2\varepsilon_0 \omega_q V}}, \]

with the quantization volume \( V \) ensures the correct Bosonic commutation relations for the photon operators \( B_{q, \sigma}(B_{q, \sigma}^\dagger) \) annihilating (creating) a photon with wave number \( q \) and polarization direction \( \sigma \). Expressed in the photonic operators, the free-field Hamiltonian is given by

\[ H_L = \sum_{q, \sigma} \hbar \omega_q \left( B_{q, \sigma}^\dagger B_{q, \sigma} + \frac{1}{2} \right) \]
and resembles a sum over linear harmonic oscillator Hamiltonians.

Since we are working in the $\mathbf{A} \cdot \mathbf{p}$-picture, we generally set up all our equations in momentum space. The real and momentum space operators are related via

$$\Psi_e(r,s) = \frac{1}{\sqrt{V}} \sum_k e^{i k \cdot r} e_{k,s},$$

where the operator $e_{k,s}$ annihilates an electron with momentum $\hbar k$ and spin $s$. An analogous relation defines the ionic operators in momentum space $p_k$.

The resulting contribution from the canonic momentum to the kinetic energy of the electrons is given by

$$H_{k}^{e_{\text{kin}}} = \sum_{k,s} \epsilon_e^{k,s} e^\dagger_{k,s} e_{k,s},$$

with the matrix element

$$\epsilon_e^k = \frac{\hbar^2 k^2}{2 m_e},$$

the light-matter interaction is given by

$$H_{A \cdot p}^e = -\sum_{k,q,s} J_k^e \mathbf{A}_q e^\dagger_{k+q/2,s} e_{k-q/2,s},$$

with the canonical current matrix element

$$J_k^e = \frac{q e}{m_e} \hbar k,$$

and the pure $A^2$-contribution is obtained as

$$H_{A^2}^e = \frac{q_e^2}{2 m_e} \sum_{k,k',q,s} \mathbf{A}_{q'}^\dagger \mathbf{A}_q e^\dagger_{k+q,s} e_k e_{k',s}. $$

In both Eqs. (9) and (11) the Fourier transformation of the vector field is denoted $A_q$ and given by

$$A_q = \sum_{\sigma} \mathcal{F}_q \left( \epsilon_{q,\sigma} B_{q,\sigma} + \epsilon^*_{-q,\sigma} B^\dagger_{-q,\sigma} \right).$$

We also include Coulomb interaction between all the charge carriers by using the Hamiltonian

$$H_C^{ee} = \frac{1}{2} \sum_{k,k',q,s,s'} V_q e^\dagger_{k,s} e^\dagger_{k',s'} e_{k'+q,s'} e_{k-q,s},$$

$$H_C^{ei} = -\sum_{k,k',q,s,s'} V_q e^\dagger_{k,s} p_{k',s'} p_{k'+q,s'} e_{k-q,s},$$

and an ion-ion interaction similar to Eq. (13). The 3D-Coulomb matrix element in Fourier space is given by

$$V_q = \frac{1}{\epsilon_0 V} \frac{|q_e|}{q^2}.$$

Starting point for all investigations is the total Hamiltonian

$$H_{\text{tot}} = \sum_{\lambda} \left( H_{k}^{\lambda_{\text{kin}}} + H_{A \cdot p}^{\lambda} + H_{A^2}^{\lambda} + H_{C}^{\lambda,\lambda} \right) + H_L + H_{ei}^{\lambda},$$

with the contributions given by Eqs. (7), (9), (11), (13), and the corresponding ionic terms, and by Eqs. (5) and (14). This Hamiltonian leads to the operator version of Maxwell’s equation such that all results known from classical optics are in principle fully contained in our approach.
3. Electron dynamics

In the past, many theoretical investigations of Coulomb interacting many-body systems have been based on a two-time Green’s functions approach\[17\]. In contrast, our present approach is formulated in terms of a single-time density-matrix formalism. We begin by studying the equation of motion for the single-electron coherences $\langle \epsilon_k^\dagger \epsilon_{k'} \rangle$. In general, we work in the Heisenberg picture where the operators carry the full time dependence, which is described by the Heisenberg equation

$$\frac{\partial}{\partial t} \langle O \rangle = \langle [O, H_{\text{tot}}] \rangle.$$ \hspace{1cm} (17)

Here, $O$ can be any operator product of interest, in our case the electron coherence. We obtain the equation of motion

$$i\hbar \frac{\partial}{\partial t} \langle \epsilon_k^\dagger \epsilon_{k'} \rangle = (\epsilon_k^\dagger - \epsilon_k^\dagger) \langle \epsilon_k^\dagger \epsilon_{k'} \rangle + \sum_{p} \left( J_k^p \cdot \langle A_p \epsilon_{k+p,s}^\dagger \epsilon_{k'} \rangle - J_{k'}^p \cdot \langle A_p \epsilon_k^\dagger \epsilon_{k'-p,s} \rangle \right)$$

$$- \frac{\hbar^2}{2m_e^2} \sum_{p,p'} \left( \langle A_p^\dagger \cdot A_{p'} \epsilon_{k+p-p'}^\dagger \epsilon_{k'} \rangle - \langle A_{p'} \cdot A_p \epsilon_k^\dagger \epsilon_{k'-p+p'} \rangle \right)$$

$$- \sum_{q, l, s'} V_q \left( \langle e_{k+q,s}^\dagger e_{l+s'}^\dagger e_{l+q,s'} \epsilon_{k'} \rangle - \langle e_{k,s}^\dagger e_{l+s'}^\dagger e_{l+q,s'} \epsilon_{k'-q,s} \rangle \right)$$

$$+ \sum_{q, l, s'} \sum_{q, l, s'} V_q \left( \langle e_{k+q,s}^\dagger p_{l+s'}^\dagger p_{l+q,s'} \epsilon_{k'} \rangle - \langle e_{k,s}^\dagger p_{l+s'}^\dagger p_{l+q,s'} \epsilon_{k'-q,s} \rangle \right). \hspace{1cm} (18)$$

We see that both the many-body Coulomb interaction and the quantized light-matter interaction lead to a hierarchy problem, i.e., on the right-hand side of Eq. (18) new operator products occur such that the system of equations is not closed. In order to systematically truncate this hierarchy of equations, we use the so-called cluster or cumulant expansion\[18, 19\] where each multi-particle or mixed expectation value is split according to

$$\Delta \langle B_{q,s}^\dagger B_{q,s} \rangle = \langle B_{q,s}^\dagger B_{q,s} \rangle - \langle B_{q,s}^\dagger \rangle \langle B_{q,s} \rangle, \hspace{1cm} (19)$$

$$\Delta \langle B_{q,s}^\dagger e_{k_1,s_1}^\dagger e_{k_2,s_2} \rangle = \langle B_{q,s}^\dagger e_{k_1,s_1}^\dagger e_{k_2,s_2} \rangle - \langle B_{q,s}^\dagger \rangle \langle e_{k_1,s_1}^\dagger e_{k_2,s_2} \rangle, \hspace{1cm} (20)$$

$$\Delta \langle e_{k_1,s_1}^\dagger p_{k_2,s_2}^\dagger p_{k_3,s_3} e_{k_4,s_4} \rangle = \langle e_{k_1,s_1}^\dagger p_{k_2,s_2}^\dagger p_{k_3,s_3} e_{k_4,s_4} \rangle - \langle e_{k_1,s_1}^\dagger e_{k_4,s_4} \rangle \langle p_{k_2,s_2}^\dagger p_{k_3,s_3} \rangle. \hspace{1cm} (21)$$

Here, we have defined the purely correlated part denoted by $\Delta \langle \ldots \rangle$ by subtracting the Hartree-Fock contribution from the full two-particle terms. Similar definitions exist for higher order correlations.

The different levels of approximation have a clear physical meaning and determine, for example, whether one treats the light field classically or quantum mechanically, whether one treats charge carriers on an averaged Hartree-Fock level or whether one includes multi-particle correlations. In the simplest case of a classical light field coupled to electrons and ions not assumed to be correlated, the dynamics is fully described by the single-particle expectation values of the form $\langle \epsilon_k^\dagger \epsilon_k \rangle$ and the classical vector potential $\langle A \rangle$. Restricting ourselves for a moment to this semiclassical coherent regime, we can simplify Eq. (18) to

$$i\hbar \frac{\partial}{\partial t} \langle \epsilon_k^\dagger \epsilon_{k'} \rangle = (\epsilon_k^\dagger - \epsilon_k^\dagger) \langle \epsilon_k^\dagger \epsilon_{k'} \rangle + \sum_p A_p \cdot \left( J_k^p \langle \epsilon_k^\dagger \epsilon_{k+p} \epsilon_{k'} \rangle - J_{k'}^p \langle \epsilon_k^\dagger \epsilon_{k'} \epsilon_{k-p} \rangle \right)$$
simplicity and introduced the Fourier component of the electron and ion densities via

\[
- \frac{q^2}{2m_e} \sum_{p,p'} A_p^* \cdot A_p \left( \langle e_{k+p}^+ e_{k'-p'}^+ \rangle - \langle e_{k,s}^+ e_{k'-p}^+ \rangle \right) \\
+ \sum_{q,s} V_q \left( n_{q,s}' - n_{q,s}^e \right) \left( \langle e_{k+q,s}^+ e_{k,s}^+ \rangle - \langle e_{k,s}^+ e_{k-q,s}^e \rangle \right) \\
+ \sum_{q,l,s'} V_q \left( \langle e_{k+q,s}^l e_{l,s'}^+ \rangle - \langle e_{k,s}^l e_{l,s'}^e \rangle \right),
\]

(22)

where we have dropped the expectation values for the classical vector potential for notational simplicity and introduced the Fourier component of the electron and ion densities via

\[
n_{q,s}' = \sum_1^e \langle e_{l,s}^l e_{l+q,s}^e \rangle
\]

and the analogous counterpart for ions. In Eq. (22) the first term with the kinetic energies describes the free ballistic electron motion, the second line contains the light-matter interaction, and the third the correction due to the \(A^2\)-term. The fourth line gives the Hartree contribution of the Coulomb interaction while the last line results from the exchange term of the electron-electron correlation. Whereas for homogeneous density distributions the Hartree term vanishes, it is clearly dominant for any slight deviation from local neutrality.

In order to derive the hydrodynamic plasma equations as a limiting case of Eq. (22) we first introduce the Wigner distribution[20]

\[
f^e(r, \mathbf{k}) = \sum_{\mathbf{q}} \text{e}^{i\mathbf{q} \cdot \mathbf{r}} \langle \epsilon_{\mathbf{k} - \mathbf{q}/2}^+ \epsilon_{\mathbf{k} + \mathbf{q}/2}^e \rangle.
\]

(24)

Taking the time derivative of this equation, inserting the different terms of Eq. (22), and using a gradient expansion in first order in \(\mathbf{q}\), we obtain

\[
\frac{\partial}{\partial t} f^e(r, \mathbf{k}) \bigg|_{\text{kin}} = -\nabla_r \cdot \left( \frac{\hbar \mathbf{k}}{m_e} f^e(r, \mathbf{k}) \right),
\]

(25)

\[
\frac{\partial}{\partial t} f^e(r, \mathbf{k}) \bigg|_{A^p} = \frac{q_e}{m_e} \left[ \nabla_r (\mathbf{A}(r) f^e(r, \mathbf{k})) - \langle \nabla_r (\mathbf{A}(r) \cdot \mathbf{k}) \rangle \cdot (\nabla_k f^e(r, \mathbf{k})) \right],
\]

(26)

\[
\frac{\partial}{\partial t} f^e(r, \mathbf{k}) \bigg|_{A^2} = \frac{q_e^2}{2\hbar m_e} \left( \nabla_r |\mathbf{A}(r)|^2 \right) \cdot (\nabla_k f^e(r, \mathbf{k})),
\]

(27)

\[
\frac{\partial}{\partial t} f^e(r, \mathbf{k}) \bigg|_{\text{Coul}} = \frac{q_e}{\hbar} \langle \nabla_r \Phi(r) \rangle \cdot (\nabla_k f^e(r, \mathbf{k}))
\]

(28)

\[
+ \frac{q_e}{\hbar} \left[ \langle \nabla_r \Delta \varepsilon_k(r) \rangle \cdot (\nabla_k f^e(r, \mathbf{k})) - (\nabla_k \Delta \varepsilon_k(r)) \cdot (\nabla_r f^e(r, \mathbf{k})) \right],
\]

(29)

where we have defined the energy renormalization

\[
\Delta \varepsilon_k(r) = \sum_{k'} V_{k-k'} f^e(r, \mathbf{k}')
\]

(30)

and the average Coulomb potential

\[
\Phi(r) = \int V(r - r') \left( n^+(r') - n^e(r') \right) d^3 r'
\]

(31)

with the usual Coulomb matrix element

\[
V(r) = \frac{1}{4\pi\varepsilon_0} \frac{q_i}{r},
\]

(32)
in real-space representation. Here, the ionic charge \( q_i \) is the absolute value of the electronic charge \( q_e = -|q_e| \). In Eq. (31), the macroscopic densities are defined via

\[
n^e(\mathbf{r}) = \frac{1}{V} \sum_k f^e(\mathbf{r}, \mathbf{k}),
\]

for electrons and analogously for ions.

Since we have not included the coupling to two-particle correlations, the combined Eqs. (25)–(29) provide the type of a collisionless kinetic equation. By defining the kinetic velocity matrix element

\[
u^e_k(\mathbf{r}) = \frac{\hbar \mathbf{k} - q_e \mathbf{A}(\mathbf{r})}{m_e}
\]

and making use of the Coulomb gauge condition \( \nabla \cdot \mathbf{A}(\mathbf{r}) = 0 \), we can rewrite the equation of motion as

\[
\frac{\partial}{\partial t} f^e(\mathbf{r}, \mathbf{k}) = -\nabla_r (\nu^e_k(\mathbf{r}) f^e(\mathbf{r}, \mathbf{k})) - \frac{q_e}{\hbar} \left[ \nu^e_k(\mathbf{r}) \times \mathbf{B}(\mathbf{r}) + (\nu^e_k(\mathbf{r}) \cdot \nabla_r) \mathbf{A}(\mathbf{r}) \right] \cdot (\nabla_k f^e(\mathbf{r}, \mathbf{k})) + \frac{q_e}{\hbar} \left[ (\nabla_r \Delta \varepsilon_k(\mathbf{r})) \cdot (\nabla_k f^e(\mathbf{r}, \mathbf{k})) - (\nabla_k \Delta \varepsilon_k(\mathbf{r})) \cdot (\nabla_r f^e(\mathbf{r}, \mathbf{k})) \right],
\]

where we have introduced the magnetic field \( \mathbf{B} = \nabla \times \mathbf{A} \). This equation is in close analogy to the dynamics of classical distribution function studied by many authors (see e.g. [20, 21, 22]). In cases where the exchange contribution in the last line can be neglected, we recover the classical Liouville equation when we rewrite Eq. (35) for the phase space density \( \tilde{f}^e(\mathbf{r}, \mathbf{v}) = f^e(\mathbf{r}, (m_e \mathbf{v} + q_e \mathbf{A})/\hbar) \).

From Eq. (35), equations for macroscopic quantities like the carrier density, Eq. (33), and the velocity density

\[
v^e_{\text{kin}}(\mathbf{r}) = \frac{1}{V} \sum_k \nu^e_k(\mathbf{r}) f^e(\mathbf{r}, \mathbf{k}).
\]

can be computed. These two equations are

\[
\frac{\partial}{\partial t} n^e(\mathbf{r}) = -\nabla_r \cdot v^e_{\text{kin}}(\mathbf{r}),
\]

\[
\frac{\partial}{\partial t} v^e_{\text{kin}}(\mathbf{r}) = -\partial_{r_i} \left( v^e_{\text{kin},j}(\mathbf{r}) \frac{v^e_{\text{kin}}(\mathbf{r})}{n^e(\mathbf{r})} \right) - \partial_{r_j} P^e_{ij}(\mathbf{r}) \mathbf{e}_i + \frac{q_e}{m_e} \left[ n^e(\mathbf{r}) \mathbf{E}(\mathbf{r}) + v^e_{\text{kin}}(\mathbf{r}) \times \mathbf{B}(\mathbf{r}) \right] + \frac{q_e^2}{2m_e\varepsilon_0 V^2} \sum_{k,k',K} \left( \frac{1 - \mathbf{k} \cdot \mathbf{k}'}{k'^2} \right) \nabla_{r_i} \left( f^e(\mathbf{r}, \mathbf{K} + k/2) f^e(\mathbf{r}, \mathbf{K} - k/2) \right),
\]

where we have introduced the total electric field

\[
\mathbf{E}(\mathbf{r}) = \mathbf{E}_r(\mathbf{r}) + \mathbf{E}_i(\mathbf{r}) = \mathbf{E}_r(\mathbf{r}) - \nabla_r \Phi(\mathbf{r})
\]

with the average Coulomb potential given by Eq. (31), and the pressure tensor

\[
P^e_{ij}(\mathbf{r}) = \frac{1}{V} \sum_k \delta u^e_{k,j}(\mathbf{r}) \delta u^e_{k,i}(\mathbf{r}) f^e(\mathbf{r}, \mathbf{k}),
\]
with the deviation from mean velocity given by
\[ \delta u_k^e(r) = u_k^e(r) - \frac{v_{\text{kin}}^e(r)}{n^e(r)}. \] (41)

For a Boltzmann distribution in \( k \), the pressure tensor is proportional to the identity matrix with the classic pressure
\[ P_{ij}^e(r) = \delta_{i,j} p_{\text{class}}^e(r) = \delta_{i,j} \frac{k_B T}{m_e} n^e(r). \] (42)

Equation (37) and (38) constitute the “warm plasma wave equations”[22].

4. Plasma oscillations
Different mechanisms for plasma oscillations after an incoming optical laser pulse have been considered in the past. While two slightly detuned pulses can produce a beat-wave[5], the dominant mechanism typically associated with plasmas generated by strongly focused laser beams is that of the wake field[6, 7] relying on ponderomotive forces. Yet another mechanism proportional to the density gradient is discussed in Refs. [8, 9]. All of these publications have modelled the electron gas as cold by neglecting the thermal pressure term, Eq. (42). This was justified because of the very strong field intensities of the order of \( 10^{17} - 10^{18} \text{ W/cm}^2 \) and the strong focus of the incoming radiation with a typical diameter of only a few microns in all of these experiments. In the case of a light-string, however, the focus is of the order of \( 100 \mu\text{m} \) and the peak intensities are in the range between \( 10^{13} \) and \( 10^{14} \text{ W/cm}^2 \). Thus, the typical ponderomotive forces are expected to be much weaker and the importance of the pressure term might be drastically increased.

From the macroscopic equations of the previous section, we can calculate both the second-order ponderomotive response of an inhomogeneous electron gas to incoming radiation and the effect of the pressure term. First neglecting the pressure term, we obtain
\[
\frac{\partial}{\partial t} v_2^e \approx \left\{ \frac{q_e^2 n_{bg}^e(r)}{8 m_e^2 \omega^2} \left[ \nabla \left( E^e(r) \cdot E^e(r) \right) \right] + \frac{q_e^2}{2 m_e^2} \frac{E^e(r)}{\omega^2 - \omega_{\text{pl}}^2} \nabla n_{bg}^e(r) \right\} + \text{c.c.}
\]
\[
- \frac{q_e^2 n_{bg}^e(r)}{4 m_e^2 \omega^4} \nabla \left( E^e(r) \cdot E^e(r) \right)
\] (43)
for the second order velocity field. Here, \( E^e(+) \) (\( E^e(-) \)) denotes the positive (negative) frequency component of the electric field. The first two lines provide the second-harmonic response in agreement with Ref. [21] while the third line gives the ponderomotive force as derived in Ref. [22].

In this section, we are going to estimate whether for our configuration any of these mechanisms are comparable to the thermal pressure. The important parameters as obtained from the experiment are the maximum power per unit area \( I_{\text{max}} \), the pulse length \( \tau \), the excitation frequency \( \omega \), the width of the light string, i.e., the transverse profile of the electric field \( R_L \), and the width of the electron plasma channel \( R_e \). The latter may differ from the width of the light string because the plasma creation as a multi-photon process is most effective in the center of the light string. Exact parameters have to be obtained from the experiment. Typical values used in the subsequent estimates are
\[
I_{\text{max}} = 10^{14} \text{ W/cm}^2.
\] (44)
\( \tau = 10 \text{ fs}, \quad (45) \)
\( \omega = \frac{2\pi c}{800 \text{ nm}} = 2.4 \times 10^{15} / \text{s}, \quad (46) \)
\( R_L = 50 \mu \text{m}, \quad (47) \)
\( R_e = 10 \mu \text{m}. \quad (48) \)

The average electric field inside the plasma channel can be calculated from the energy balance inside a small volume,

\[ \varepsilon_0 |E_{\text{max}}|^2 \Delta V = I_{\text{max}} \tau \pi R_L^2, \quad (49) \]

where the characteristic volume is given by \( \Delta V = \pi R_L^2 c \tau \) such that

\[ |E|^2 = \frac{I_{\text{max}}}{c \varepsilon_0}. \quad (50) \]

The characteristic velocity change due to the ponderomotive force according to Eq. (43) is given by

\[ \frac{q_e^2 n_{\text{bg}}^e(r)}{4 m_e^2 \omega^2} \nabla \left| \tilde{\mathbf{E}}(\mathbf{r}) \right|^2, \quad (51) \]

where \( \tilde{\mathbf{E}} \) denotes the slowly varying envelope of the electric field. If we assume its functional form as Gaussian,

\[ \tilde{\mathbf{E}}(\mathbf{r}) = e_{\text{pol}} E_{\text{max}} e^{-\left( \frac{r^2}{R_L^2} + \frac{z^2}{c^2 \tau} \right)}, \quad (52) \]

we obtain a characteristic velocity change

\[ \left| \frac{1}{n_{\text{bg}}(\mathbf{r})} \frac{\partial}{\partial t} \mathbf{v}(\mathbf{r}) \right| \approx \frac{q_e^2}{4 m_e^2 \omega^2} \left( \frac{2}{R_L R_L} + \frac{2 e_z z}{c \tau c \tau} \right) e^{-\left( \frac{r^2}{R_L^2} + \frac{z^2}{c^2 \tau} \right)} \frac{I_{\text{max}}}{c \varepsilon_0}, \quad (53) \]

where the prefactors

\[ \frac{q_e^2 I_{\text{max}}}{2 m_e^2 \omega^2 c \varepsilon_0 R_L} \approx 2 \times 10^{-3} \text{ nm ps}^{-2}, \quad \frac{q_e^2 I_{\text{max}}}{2 m_e^2 \omega^2 c^2 \varepsilon_0} \approx 3.5 \times 10^{-2} \text{ nm ps}^{-2}, \quad (54) \]

respectively give a measure of the typical acceleration acting on an electron.

In a similar manner, we can obtain a characteristic velocity change from the thermal spread of the initial Gaussian. Using the classical pressure, Eq. (42), the velocity change is given by

\[ \left| \frac{1}{n_{\text{bg}}(\mathbf{r})} \frac{\partial}{\partial t} \mathbf{v}(\mathbf{r}) \right| \approx \frac{k_B T}{m_e R_L} \nabla n_{\text{bg}}(\mathbf{r}) \approx \frac{k_B T}{m_e} \frac{2}{R_e}, \quad (55) \]

Since typical carrier temperatures are determined by the photon energies used for multi-photon ionization, we use an electronic temperature of \( T = 10000 \text{ K} \) which gives an acceleration of

\[ \frac{2 k_B T}{m_e R_e} \approx 30 \text{ nm ps}^{-2}. \quad (56) \]

This is higher by orders of magnitude such that we conclude that the dominant mechanism for the spread of the electrons is the thermal pressure.
4.1. Channel geometry

Following the philosophy usually adopted for the description of laser-induced electron plasmas, we assume that we can separate the time scales of the creation of the plasma channel via multi-photon ionization and the subsequent dynamics. This should be even more justified in our case where the ponderomotive forces do not play any major role. Therefore, we assume as an initial condition directly after the creation of the electron channel an electron distribution of the form

\[
\langle \Psi_e^\dagger (R - r/2) \Psi_e (R + r/2) \rangle = \frac{N_e}{L \pi R_e^2} e^{-\frac{r^2}{4\lambda_{th}^2}}. 
\]

(57)

Here, \( N_e \) denotes the total number of ionized electrons such that \( n_0 = \frac{N_e}{L \pi R_e^2} \) is the peak density at the center of the plasma channel. The average kinetic energy which can be estimated by the excess energy of one photon is assumed to lead to a thermal distribution with the characteristic thermal wavelength

\[
\lambda_{th} = \sqrt{\frac{2\hbar^2}{m_e k_B T}}. 
\]

(58)

An identical spatial distribution with radius \( R_i = R_e \) is assumed for the ions. Because of their large mass, however, we neglect the ionic motion due to thermal pressure and assume the ion distribution as static.

Once the electron channel starts spreading, there is a strong restoring Coulomb force which tends to attract the electrons back to the ions. Intuitively, we expect oscillations of the electron channel, i.e., an oscillatory breathing. Since for a Gaussian density profile a broader electron distribution is equivalent to a smaller peak value at the center of the channel, we approximate the Coulomb potential

\[
V_{\text{eff}}(R_\parallel) = \frac{e^2}{\epsilon_0} \int_0^{R_\parallel} \frac{1}{\rho} \left( \int_0^{\rho'} \Delta n(\rho'') \rho'' d\rho'' \right) d\rho' 
\]

(59)

by

\[
V_{\text{eff}}(R_\parallel) \approx \frac{e^2}{4\epsilon_0} R_\parallel^2 \left[ n^i - n^e \right]_{R=0}, 
\]

(60)

where we have replaced the full \( \Delta n(\rho'') \) in Eq. (59) by its value at the origin. Figure 1 shows a comparison between the full solution of the integral in Eq. (59) and the harmonic approximation. Up to the width of the string, the harmonic force provides a reasonable approximation. For the current paper, we neglect deviations from the parabolic potential. The resulting equation of motion for the single-electron correlation without the exchange contribution is thus given by

\[
\frac{i\hbar}{\partial t} \langle \Psi_e^\dagger (r) \Psi_e (r') \rangle = \left[ \hat{H}_{\text{LHO}}(r') - \hat{H}_{\text{LHO}}(r) \right] \langle \Psi_e^\dagger (r) \Psi_e (r') \rangle 
\]

(61)

with the linear-harmonic oscillator Hamiltonian

\[
\hat{H}_{\text{LHO}}(r) = -\frac{\hbar^2 \nabla_r^2}{2m_e} + \frac{e^2}{4\epsilon_0} \Delta n(r = 0)|r_\parallel|^2. 
\]

(62)

In Eq. (61) for microscopic expectation values, the pressure term is not directly visible anymore. But the initial spread of the electronic channel is fully contained in the kinetic contribution of Eq. (62).
The density dynamics of Eq. (61) in absence of any external fields can be solved analytically by means of the propagator of the linear-harmonic oscillator. Alternatively, we use the insight from this analytical solution to write the most general expression

$$\langle \Psi_e^*(\mathbf{R} - \mathbf{r}/2)\Psi_e(\mathbf{R} + \mathbf{r}/2) \rangle = n_0 N e^{-\left(\frac{R_i}{\lambda_{th}}\right)^2} e^{-\left(\frac{R_i}{\lambda_{th}}\right)^2} e^{-iC\Re(r_1 e^{-\left(\frac{x}{\lambda_{th}}\right)^2}}. \quad (63)$$

for the solution and derive equations of motion for the four coefficients

$$\frac{\partial}{\partial t} N(t) = \frac{2\hbar}{m_e} C(t) N(t), \quad (64)$$
$$\frac{\partial}{\partial t} A(t) = -\frac{\hbar}{m_e} C(t) A(t), \quad (65)$$
$$\frac{\partial}{\partial t} B(t) = -\frac{\hbar}{m_e} C(t) B(t), \quad (66)$$
$$\frac{\partial}{\partial t} C(t) = -\frac{\hbar}{m_e} \left( \left(\frac{2}{A(t)B(t)}\right)^2 - (C(t))^2 - \frac{m_e^2}{2\hbar^2} \omega_{pl}^2 (1 - N(t)) \right). \quad (67)$$

Their initial values are related to our initial distribution, Eq. (57), and given by

$$N(t = 0) = 1, \quad (68)$$
$$A(t = 0) = R_i, \quad (69)$$
$$B(t = 0) = \lambda_{th}, \quad (70)$$
$$C(t = 0) = 0. \quad (71)$$

It is interesting to note that there is no need for the exciting optical pulse to be resonant with the plasma frequency. In contrast to other mechanisms, the only effect of the pulse is to create an incoherent density distribution with electrons with a certain excess energy. Then their thermal spread leads to small density fluctuations which in turn result in a strong restoring Coulomb force and subsequent density oscillations. In analogy to the traditional wakefield oscillations we term them *thermal wakefield oscillations*.

In order to gain some insight into the oscillatory motion, we derive a second-order oscillator equation for $\Delta N = 1 - N$. Taking the time derivative of Eq. (64) leads to

$$\frac{\partial^2}{\partial t^2} \Delta N = - \left[ N \omega_{pl}^2 + \frac{2\hbar^2}{m_e^2} \left( -\frac{4}{A^2B^2} - 3C^2 \right) \right] \Delta N + \frac{2\hbar^2}{m_e^2} \left( \frac{4}{A^2B^2} - 3C^2 \right). \quad (72)$$
where we have introduced the plasma frequency

$$\omega_{pl} = \sqrt{\frac{q^2 n_0}{\varepsilon_0 m_e}}.$$  \hspace{1cm} (73)

If we assume that

$$\frac{1}{A^2 B^2} \gg C^2,$$  \hspace{1cm} (74)

which we have confirmed from the numerical solution, and if we furthermore assume that the functions $A$, $B$, and $N$ can be approximated by their initial values $R_i$, $\lambda_{th}$ and 1, respectively, Eq. (72) is a simple oscillator equation which can be solved by

$$\Delta N(t) = \frac{8h^2}{m_e^2 R_i^2 \lambda_{th}^2} \tilde{\omega}_{pl}^2 [1 - \cos(\tilde{\omega}_{pl})],$$  \hspace{1cm} (75)

where we have defined the new effective plasma frequency

$$\tilde{\omega}_{pl}^2 = \omega_{pl}^2 + \frac{8h^2}{m_e^2 R_i^2 \lambda_{th}^2}.$$  \hspace{1cm} (76)

This solution drives Eq. (67) for $C(t)$ which in turn influences the dynamics of $A(t)$ and $B(t)$.

Figure 2 presents the numerical solution of the set of Eqs. (64)–(67) for a carrier temperature of $T = 20000$ K and for varying plasma radius. The peak density is chosen to be $1.242 \times 10^{16}$ cm$^{-3}$ corresponding to a plasma frequency of $\omega_{pl} = 2\pi$ ps$^{-1}$. We have chosen a relatively high temperature and small values for the radius in order to clearly demonstrate the varying oscillation period for different values of $R_i$ in accordance with Eq. (76). We notice that the density oscillations lie in the percent regime consistent with Fig. 1.

From the general form of the solution, Eq. (63), we can derive the kinetic current density

$$\mathbf{j}_e(R) = -\frac{\hbar q_e n_0}{m_e} C(t) N(t) R_{||} e^{-\left(\frac{R_{||}}{\lambda_{th}}\right)^2}.$$  \hspace{1cm} (77)

This current represents the electron motion around the ionic plasma channel. For the azimuthal symmetry assumed here, this current is purely longitudinal and does not provide a source to the classical wave equation. This can be easily verified by calculating the curl of Eq. (77). Under different circumstances, such as an anisotropic temperature[23], an asymmetric initial condition, or an additional $z$-dependence due to the delayed excitation dynamics, it is probable that the electronic oscillation provides a source to coherent THz emission.
5. Incoherent emission

We leave the coherent regime and rather investigate the effect of the electronic breathing on incoherent light emission, i.e., luminescence coming from the electron plasma. It is known that under steady state conditions, the emission rate of energy into a certain mode gives a good measure of the incoherent emission. Since the breathing plasma channel provides a time dependent source, however, we rather choose

\[ I_{q,\sigma}^{\text{PL}} = \hbar \omega_q \Delta \langle B_{q,\sigma}^\dagger B_{q,\sigma} \rangle, \tag{78} \]

i.e., the total energy per mode. Which quantity is more closely related to an experimentally measurable spectrum is a question which depends on the detailed detection geometry and will not be addressed here.

The equation of motion for the diagonal incoherent photon number is given by

\[ \frac{\partial}{\partial t} \Delta \langle B_{q,\sigma}^\dagger B_{q,\sigma} \rangle \bigg|_{\text{SE}} = -\frac{2}{\hbar} \text{Im} \left[ \sum_{k,s} \mathbf{J}_k \cdot e_{q,\sigma}^s \mathcal{F}_q \Delta \langle \mathcal{B}_{q,\sigma}^\dagger e_{k-q/2,s}^s e_{k+q/2,s} \rangle \right], \tag{79} \]

where we have neglected the contribution from the \( A^2 \)-term. We know that the main effect of the \( A^2 \)-term can be incorporated by using adapted mode functions of an electromagnetic field propagating in the presence of the plasma channel\[10\] instead of the plane wave expansion, Eq. (3), suited for propagation in vacuum. Thus the main consequence is a new frequency dependent coupling strength inside the plasma channel. Due to the large extension of the light string compared to characteristic electronic length scales, we can calculate the 3D emission spectrum without the \( A^2 \)-term and then multiply our result by the frequency dependent coupling strength.

The emission of photons is thus determined by photon-assisted densities of the form \( \Delta \langle \mathcal{B}_{q,\sigma}^\dagger e_{k-q/2,s}^s e_{k+q/2,s} \rangle \). These expectation values correspond to processes where an electron changes its momentum, while emitting a photon with the corresponding energy. The full Heisenberg equation of motion for the photon-assisted terms as obtained from Eq. (17) using the Hamiltonian from Eq. (16) is

\[ i\hbar \frac{\partial}{\partial t} \Delta \langle \mathcal{B}_{q,\sigma}^\dagger e_{k-q/2,s}^s e_{k+q/2,s} \rangle = \left( \varepsilon^s_{k+q/2} - \varepsilon^s_{k-q/2} - \hbar \omega_q \right) \Delta \langle \mathcal{B}_{q,\sigma}^\dagger e_{k-q/2,s}^s e_{k+q/2,s} \rangle + \Omega_{\text{SE}}(q,k) + \Omega_{\text{ST}}(q,k) \]

\[ + \text{i} \hbar \frac{\partial}{\partial t} \Delta \langle \mathcal{B}_{q,\sigma}^\dagger e_{k-q/2,s}^s e_{k+q/2,s} \rangle \bigg|_{\text{A}^2} \]

\[ + \text{i} \hbar \frac{\partial}{\partial t} \Delta \langle \mathcal{B}_{q,\sigma}^\dagger e_{k-q/2,s}^s e_{k+q/2,s} \rangle \bigg|_{\text{Coul}} \]

\[ + \text{i} \hbar \frac{\partial}{\partial t} \Delta \langle \mathcal{B}_{q,\sigma}^\dagger e_{k-q/2,s}^s e_{k+q/2,s} \rangle \bigg|_{\text{scat}} \tag{80} \]

with the source of spontaneous emission,

\[ \Omega_{\text{SE}}(q,k) = \mathcal{F}_q e_{q,\sigma} \cdot \left( \mathbf{J}_k^e \langle e_{k+q/2,s}^s e_{k+q/2,s} \rangle - \sum_{l,s'} \mathbf{J}_l^e \langle e_{k-q/2,s'}^e e_{k+q/2,s} \rangle \langle e_{k-q/2,s'}^s \rangle \right) \]

\[ + \mathcal{F}_q e_{q,\sigma} \cdot \sum_{l,s'} \left( \mathbf{J}_l^e \Delta \langle e_{k-q/2,s}^s e_{l+q,1,s'}^s e_{k+q/2,s} \rangle + \mathbf{J}_l^e \Delta \langle e_{k-q/2,s}^s p_{l+q,1,s'} e_{k+q/2,s} \rangle \right), \tag{81} \]

consisting of contributions from single-particle and higher order correlations, a stimulated term,

\[ \Omega_{\text{ST}}(q,k) = \sum_p \Delta \langle \mathcal{B}_{q,\sigma}^\dagger \mathbf{A}_p \rangle \cdot \left( \mathbf{J}_k^e \langle e_{k+q/2,p,s}^s e_{k+q/2,s} \rangle - \mathbf{J}_k^e \langle e_{k-q/2,p,s}^s e_{k+q/2,p,s} \rangle \right), \tag{82} \]
and all the contributions resulting from the $A^2$-term and the Coulomb interaction summarized in the third and fourth line of Eq. (80).

It is known that the Coulomb scattering of densities is provided by the coupling to two-particle correlations in the last line of Eq. (18). In a similar way, the scattering for photon-assisted densities is provided by coupling to photon-assisted two-particle correlations of the form $\Delta \langle B^\dagger e^\dagger p^\dagger pe \rangle$ which is schematically summarized as the scattering contribution in the last line of Eq. (80). While a non-interacting single-component plasma clearly cannot emit or absorb radiation due to the kinetic energy term

$$
\epsilon^{\sigma}_{k|q/2} - \epsilon^{\sigma}_{k-q/2} = \hbar^2 k \cdot q - \hbar \omega_q = \left( \frac{\hbar^2}{m_e} k \cdot q - c \right) \hbar q
$$

and the fact that the electronic velocity $\frac{\hbar k}{m_e}$ is always smaller than the velocity of light $c$, the inclusion of Coulomb scattering of electrons among each other and with the positively charged ions releases the energy conservation and allows for light emission[10].

In our present numerical solution, only the dominant source term of Eq. (80) is included and the Coulomb scattering is modelled by a constant scattering rate $\gamma_{\text{eff}}$. In the low-density and long-wavelength limit, the resulting equation is thus given by

$$
i \hbar \frac{\partial}{\partial t} \Delta \langle B^\dagger_{k,s} e^\dagger_{k-q/2} e_{k+q/2} \rangle = - \left( \hbar \omega_q + i \hbar \gamma_{\text{eff}} \right) \Delta \langle B^\dagger_{k,s} e^\dagger_{k-q/2} e_{k+q/2} \rangle + \mathcal{F}_q \epsilon_{q,s} \cdot J^e_k f^e_k.
$$

(84)

Combined with Eq. (79), it still gives a partial insight on the polarization dependence of the emitted light.

We study the influence of the breathing electron channel by solving Eq. (84) both for constant carrier distributions and with the time dependent input as obtained from the spatial Fourier transform of Eq. (63).

5.1. Constant densities

For the case of constant $f^e_k$, we can formally solve Eq. (84) in the steady state limit,

$$
0 = - \left( \hbar \omega_q + i \hbar \gamma_{\text{eff}} \right) \Delta \langle B^\dagger_{k,s} e^\dagger_{k-q/2} e_{k+q/2} \rangle + \mathcal{F}_q \epsilon_{q,s} \cdot J^e_k f^e_k.
$$

(85)

and obtain

$$
\Delta \langle B^\dagger_{k,s} e^\dagger_{k-q/2} e_{k+q/2} \rangle = \frac{\mathcal{F}_q \epsilon_{q,s} \cdot J^e_k f^e_k}{\hbar \omega_q + i \hbar \gamma_{\text{eff}}}.
$$

(86)

If we insert this solution into Eq. (79), we obtain the luminescence spectrum

$$
\frac{\partial}{\partial t} I^\text{PL}_{q,s} = \hbar \omega \frac{\partial}{\partial t} \Delta \langle B^\dagger_{k,s} B_{k,s} \rangle = \frac{1}{\epsilon_0} \left( \frac{1}{V} \sum_k |J^e_k|^2 f^e_k \right) \frac{\gamma_{\text{eff}}}{\omega^2 + \gamma_{\text{eff}}^2}.
$$

(87)

For a Boltzmann distribution, this result can be evaluated analytically to give

$$
\frac{\partial}{\partial t} I^\text{PL}_{q,s} = \omega^2_{\text{pl}} kT \frac{\gamma_{\text{eff}}}{\omega^2 + \gamma_{\text{eff}}^2},
$$

(88)

i.e., a spectrum growing linearly with time. Since typical values for the electronic thermal wavelength are much smaller than the diameter of the light string, the electrons locally see a three-dimensional environment. Therefore, the $k$-sum in Eq. (87) is three-dimensional and independent of the polarization vector of the detected field. As an example, we have assumed
x-polarized detection. The previous result is also independent of the angle under which the
detection is performed. This independence on the polarization direction and emission angle
is a result of neglecting the $A^2$-contribution. If this contribution is properly taken into account,
the coupling strength for different modes varies and a sensitivity on the detector is expected.
In the present publication, however, we want to focus on the polarization effects induced by the
breathing dynamics.

5.2. Breathing dynamics

In this section, we combine the numerical solution of the breathing dynamics, Eqs. (64)–(67),
with the numerical solution of Eqs. (79) and (84). The time dependent source term $f_k(t)$ is
obtained by spatial Fourier transformation from Eq. (63),

$$f_k(t) = \frac{N_e}{S^2} \frac{4\pi B(t)^2}{4 + A(t)^2 B(t)^2 C(t)^2} \exp \left[ -\frac{B(t)^2 |k_\perp|^2}{4 + A^2 B^2 C^2} \right] f_{kz},$$

$$= n_0 \frac{\pi R_i^2}{S^2} \frac{4\sqrt{\pi} \lambda_{th} B(t)^2}{4 + A(t)^2 B(t)^2 C(t)^2} \exp \left[ -\frac{k^2}{4} \left( B(t)^2 \sin^2(\theta_k) + \lambda_{th}^2 \cos^2(\theta_k) \right) \right],$$

where $\theta_k$ is the angle between the vector $k$ and the direction of the plasma channel.

Intuitively, we expect some signatures at the plasma frequency due to the time dependence
of the source term. Fourier transformation of Eq. (84) shows that for a source oscillating with
$\omega_{pl}$ the energy denominator, Eq. (83), has to be replaced by

$$\frac{\hbar^2}{m_e} \mathbf{k} \cdot \mathbf{q} - \hbar \omega_q + \hbar \omega_{pl} - i\hbar \gamma_{eff}.$$  

Even in the limit of vanishing scattering, $\gamma_{eff} \to 0$, this energy denominator causes a resonance
at $\omega_q \approx \omega_{pl}$.

Figure 3 shows the rate of the total number of emitted photons per unit string length

$$\frac{1}{L} \frac{\partial}{\partial t} N_{tot} = \frac{1}{L} \frac{\partial}{\partial t} \sum_q \Delta \langle B_{q\sigma}^\dagger B_{q\sigma} \rangle$$

with $\sigma$ corresponding to the polarization perpendicular to the electron string as function of
time for the same parameters as used in Fig. 2. One can clearly see that the dynamics of the
breathing densities influences the emission dynamics of the incoherent light. The period of the
plasma oscillations is visible in the rate of change of the incoherent emission. While the average emission rate is determined by the effective scattering $\gamma_{\text{eff}}$, the fluctuations are determined by the oscillations of the electron density.

To investigate the effect of the breathing dynamics in more detail, we compare various spectra for light emitted perpendicular to the plasma string in Fig. 4. In particular, we compare spectra for different polarization directions. In order to see the features around the plasma frequency more clearly, we plot $\omega^2 I_{q, \sigma}^{\text{PL}}$, a quantity related to the power density. If the polarization direction points along the string as depicted in the right figure, we recover the constant emission rate discussed in the previous section. Here, spectra taken at equal time intervals show identical offsets between them. For the polarization direction perpendicular to the light-string, however, clear signatures are observed at the plasma frequency $\omega_{\text{pl}} = 2\pi \text{ ps}^{-1}$ and at two times the plasma frequency. The thick lines mark three times which are approximately half a plasma period apart. We note that the spectral features at $2\omega_{\text{pl}}$ repeat themselves after half a plasma period. As expected, the features at $\omega_{\text{pl}}$ in contrast show a period of roughly one picosecond, i.e., one plasma period. In addition, we also note that the plasma oscillations lead to an overall slower emission since the spectra on the right-hand side after identical computation time exhibit slightly higher values than their counterparts in the left part of the figure.

For emission directions other than 90 degrees from the plasma channel, the polarization vector for both polarizations has a component perpendicular to the channel. Therefore, in this case we expect spectral signatures for both polarization directions. Even then, however, the spectral features will be stronger for one mode than for the other, the difference being dependent on the precise emission angle.

6. Conclusion and Outlook

In this paper, we have presented a microscopic theory for the investigation of light-matter interaction in partially ionized plasmas. The formalism is very general and can be applied to a multitude of situations. Here, we have focused on the thermal spreading and the subsequent density oscillations after the creation of an electron-ion plasma via multi-photon ionization. In this scenario, the mobile electrons start to spread due to their kinetic energies and strong
Coulomb forces attract them back to the basically immobile ion distribution. While these thermal wakefield oscillations are purely longitudinal in nature and thus do not provide a source for propagating electro-magnetic fields, they are predicted to produce characteristic signatures in the luminescence spectrum. The luminescence spectrum itself is caused mainly by scattering processes which relax the strict energy and momentum conservation prohibiting luminescence of a noninteracting single-component plasma.

We have noted many places where future research is possible; for example, a deviation from the cylindrical symmetry or an anisotropic velocity distribution of the electrons after the ionization might lead to a coherent THz source; the inclusion of the $A^2$-term might lead to a dependence of the detected luminescence on the emission angle. But even under the simplified situation studied here, an interesting polarization dependence of the signatures of the oscillations in the luminescence spectrum has been predicted which should be most easily detected in a differential measurement.

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