Dynamics of tachyon field with an inverse square potential in loop quantum cosmology

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The dynamical behavior of tachyon field with an inverse potential is investigated in loop quantum cosmology. It reveals that the late time behavior of tachyon field with this potential leads to a power-law expansion. In addition, an additional barotropic perfect fluid with the adiabatic index $0 < \gamma < 2$ is added, and the dynamical system is shown to be an autonomous one. The stability of this autonomous system is discussed using phase plane analysis. There exist up to five fixed points with only two of them possibly stable. The two stable node (attractor) solutions are specified, and their cosmological indications are discussed. For the tachyon dominated solution, the further discussion is stretched to the possibility of considering tachyon field as a combination of two parts which respectively behave like dark matter and dark energy.

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I. INTRODUCTION

The tachyon field with various potential has been studied a lot in cosmology. Many models have been built by treating tachyon field as inflaton field [1], candidate of dark energy [2, 3], or a dual role of the two [4]. But Ref. [4] also argued that, for the class of potentials which $V(\phi) \to 0$ as $\phi \to \infty$, radiation domination will never commence since the tachyon field energy density $\rho_\phi$ can at best scale as $a^{-3}$. The radiation energy density would always redshift faster than the tachyon field. The tachyon field with an inverse square potential is shown to be able to produce a power-law expansion [5]. Coupled with a barotropic perfect fluid, the dynamical behavior of this potential has been studied in classical cosmology [6]. However, the tracking solution, in which the energy density of the field and the barotropic fluid scales as a same power of $a$, may not be viable because its constraints on the adiabatic index $\gamma$.

Our work of tachyon field cosmology is performed under the framework of loop quantum cosmology. LQC is a canonical quantization of homogeneous spacetime using the techniques developed in loop quantum gravity (LQG). The loop quantum effects can be very well described by the effective theory of LQC. A modified Friedmann equation is proposed and two corrections are often considered: the inverse volume correction and the holonomy correction. However, the holonomy correction dominates over the inverse volume correction for a universe with a large scale factor, and thus the latter can be neglected without harm. Therefore, we only consider the holonomy correction in this paper.

Currently, tachyon matter has not been thoroughly investigated in loop quantum cosmology (LQC). A. A. Sen [4] generalized the description of tachyon matter in standard cosmology to LQC under inverse volume correction. Xiong and Zhu [1] investigated the inflation scenario of a pure tachyon field with an exponential potential under the holonomy correction. Xiao and Zhu [8] performed a phenomenological analysis of tachyon warm inflation, in which the tachyon field with an exponential potential is coupled with radiation and interaction between the two matters were considered. However, the dynamics of tachyon field in LQC has not been investigated, yet. In this paper, we discuss the dynamics of tachyon field coupled with a barotropic perfect fluid in LQC. We focus on the inverse square potential, and try to explore the possibility of a viable dark energy model.

The organization of this paper is as follows. In Sec. II, we try to derive the expansion law for a pure tachyonic matter in LQC. In Sec. III, we couple the field with a barotropic perfect fluid and analyze the dynamics of the autonomous system. The cosmological implications of the phase plane analysis are presented in Sec. IV A and IV B. The conclusions are made in Sec. V.

II. TACHYON MATTER IN LOOP QUANTUM COSMOLOGY

Based on the holonomy correction in loop quantum cosmology, the modified Friedmann equation of a flat $(k = 0)$ Friedmann-Robertson-Walker (FRW) cosmological model is

$$H^2 = \frac{1}{3} \rho \left( 1 - \frac{\rho}{\rho_c} \right),$$

(1)

where $H$ is the Hubble parameter, $\rho$ and $\rho_c$ denote the matter density and critical density, respectively. We also

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set $8\pi G = 1$ for convenience. The energy conservation equation is the same as the classical one,
\begin{equation}
\dot{\rho} = -3H (\rho + p),
\end{equation}
where $p$ is the pressure. Differentiate Friedmann equation with respect to time, we have
\begin{equation}
\dot{H} = -\frac{1}{2} (p + \rho) \left(1 - \frac{2\rho}{\rho_c}\right),
\end{equation}
where 'dot' denotes the derivative with respect to the cosmological time $t$. Therefore the conditions for superinflation ($\dot{H} > 0$) are
\begin{equation}
\begin{cases}
\omega < -1, & \text{if } 1 - \frac{2\rho}{\rho_c} > 0, \\
\omega > -1, & \text{if } 1 - \frac{2\rho}{\rho_c} < 0.
\end{cases}
\end{equation}
where $\omega = p/\rho$ is the equation of state. It’s easy to see the existence of superinflation in LQC is purely an effect of quantum geometry, because it originates from the time derivative of the modification term $(1 - \rho/\rho_c)$. The Raychaudhuri equation then becomes
\begin{equation}
\frac{\ddot{a}}{a} = \dot{H} + H^2 = -\frac{1}{6} \left[3p \left(1 - \frac{2\rho}{\rho_c}\right) + \rho \left(1 - \frac{4\rho}{\rho_c}\right)\right],
\end{equation}
which indicates the conditions for $\ddot{a} > 0$:
\begin{equation}
\begin{cases}
w < -\frac{1-2\rho}{3-2\rho}, & \text{if } 1 - \frac{2\rho}{\rho_c} > 0, \\
w > -\frac{1-2\rho}{3-2\rho}, & \text{if } 1 - \frac{2\rho}{\rho_c} < 0.
\end{cases}
\end{equation}
The regions for $\dot{H} > 0$ and $\ddot{a} > 0$ are portrayed explicitly in Fig. 1. Obviously, as the matter density decreases, the correction term becomes less and less important, and the Friedmann equation as well as the conditions for $\ddot{a} > 0$ and $\dot{H} > 0$ are consistent with classical cosmology in an asymptotical way.

Now we consider the model with only tachyon field. Note that the tachyon field referred in this paper is just a scalar field with an nonquadratic kinetic term. We don’t claim any identification with the tachyon in string theory. According to Sen [9, 10], the energy density and pressure of the tachyon field in a flat FRW cosmology can be expressed as
\begin{equation}
\rho_\phi = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}}, \quad p_\phi = -V\sqrt{1 - \dot{\phi}^2},
\end{equation}
where $\phi$ is the tachyon field, $V(\phi)$ denotes its potential, and the equation of state is
\begin{equation}
\omega_\phi = \frac{p_\phi}{\rho_\phi} = \dot{\phi}^2 - 1.
\end{equation}
We see $\omega_\phi$ ranges smoothly from $-1$ to 0. Using the energy conservation Eq. (2), the evolution equation of tachyon field can be written explicitly as
\begin{equation}
\ddot{\phi} + \left(1 - \dot{\phi}^2\right) \left(\frac{V'}{V} + 3H\dot{\phi}\right) = 0.
\end{equation}
FIG. 1: $\ddot{a} > 0$ in the region between the two solid curves. The region I and II correspond to $H > 0$.

where $V' = dV/d\phi$.

Here, we are interested in the inverse square potential
\begin{equation}
V = \beta \phi^{-2},
\end{equation}
with $\beta > 0$. According to [3] this potential is able to produce a power law expansion in classical cosmology. In classical cosmology, one can use the Hubble parameter instead of the field $\phi$ as a fundamental quantity by employing the Hamilton-Jacobi formulation. By using Eqs. (1) and (3) and dropping the $\rho/\rho_c$ terms, one can obtain
\begin{equation}
\ddot{\phi}^2 = -\frac{2H}{3H^2}.
\end{equation}
which indicates $\dot{H} \leq 0$, and therefore superinflation will not occur in classical cosmology. Divide both sides by $\dot{\phi}$, we have
\begin{equation}
\dot{\phi} = -\frac{2H'}{3H^2}.
\end{equation}
and thus
\begin{equation}
H(\phi)^2 - \frac{9}{4}H(\phi)^4 + \frac{1}{4}V(\phi)^2 = 0.
\end{equation}
For the inverse square potential, an exact solution $H \sim \phi^{-1}$ is found, and after some algebra one can arrive at
\begin{equation}
\begin{cases}
a(t) = t^n, \\
\phi(t) = \sqrt{\frac{2}{3n}}t,
\end{cases}
\end{equation}
where
\begin{equation}
n = \frac{1}{3} + \frac{1}{6}\sqrt{4 + 9\beta^2}.
\end{equation}
The solution is inflationary when $n > 1$, or $\beta > \sqrt{4/3}$. Although one can have arbitrarily fast expansion with
an arbitrarily big \( n \) or \( \beta \), the condition for an accelerated expansion already calls for an energy scale close to a Plank mass \([2, 3]\). Refs. \([4, 11]\) also discussed the problem of tachyon model in reheating. Therefore this tachyon model is more suitable as a dark energy model rather than an inflaton model.

However, it is difficult to find an exact solution like this in LQC due to the existence of the quantum modification term. Combining equation (1) and (3), we have

\[
\dot{\phi}^2 = \frac{2\dot{H}}{3H^2} \left( 1 - \frac{\rho_0}{\rho_c} \right) \left( 1 - 2\frac{\rho_0}{\rho_c} \right)
\]

(14)

Obviously, \( \dot{H} > 0 \) when \( (1 - 2\rho_0/\rho_c) < 0 \), which means superinflation will naturally happen and it’s purely an effect of quantum geometry as we said before. Divide both sides by \( \dot{\phi} \), we obtain

\[
\dot{\phi} = -\frac{2H'}{H^2} \left( 1 - \frac{\rho_0}{\rho_c} \right) \left( 1 - 2\frac{\rho_0}{\rho_c} \right)
\]

(15)

and therefore

\[
H'^2 \left( 1 - \frac{\rho_0}{\rho_c} \right)^2 - \frac{9}{4} H^4 + \frac{1}{4} V^2 \left( 1 - \frac{\rho_0}{\rho_c} \right)^2 = 0.
\]

(16)

Fortunately, we are still able to analyze its asymptotical behavior. When \( \rho_0 \ll \rho_c \), we can neglect the derivatives of the correction terms since these terms, since \( (1-\rho_0/\rho_c) \) and \( (1-2\rho_0/\rho_c) \) will not change significantly. Then, an approximated power-law expansion

\[
a(t) \sim t^m
\]

(17)

can be obtained, with

\[
m = \frac{1}{6} \left( 1 - \frac{\rho_0}{\rho_c} \right) \left[ 2 + \sqrt{4 + \frac{9\beta^2 (1-2\rho_0/\rho_c)^2}{(1-\rho_0/\rho_c)^2}} \right].
\]

(18)

So we have

\[
\phi(t) = \sqrt{\frac{1 - \rho_0/\rho_c}{1 - 2\rho_0/\rho_c}} \frac{2}{3m} t
\]

(19)

\[
2 \sqrt{4 + \frac{9\beta^2 (1-2\rho_0/\rho_c)^2}{(1-\rho_0/\rho_c)^2}}
\]

Obviously, \( m \to n \) as \( \rho_0/\rho_c \to 0 \). The quantum geometry results in a different evolution of tachyon field and the scale factor, but the evolution will converge to the classical one at late time when the quantum effects vanishes.

**III. WITH BAROTROPIC FLUID**

In order to obtain a viable dark energy model, we add a barotropic perfect fluid in our model, for which the equation of state is \( p_\gamma = (\gamma - 1)\rho_\gamma \), where \( \gamma \) is a constant. Therefore, the equation of state for the whole is

\[
\omega = \frac{p}{\rho} = \frac{\omega_\phi \rho_\phi + \omega_\gamma \rho_\gamma}{\rho} = \omega_\phi \Omega_\phi + \omega_\gamma \Omega_\gamma = (\phi^2 - \gamma) \Omega_\phi + \gamma - 1.
\]

(20)

where

\[
\omega_\phi = \frac{\rho_\phi}{\rho}, \quad \Omega_\phi = \frac{\rho_\phi}{\rho} = 1 - \Omega_\phi.
\]

(21)

If we simply assume there is no interaction between the tachyon field and barotropic fluid, then their evolution equations are

\[
\ddot{\phi} + \left( 1 - \phi^2 \right) \left( \frac{V'}{V} + 3H \phi \right) = 0,
\]

(22)

and

\[
\dot{\rho}_\gamma + 3\gamma H \rho_\gamma = 0.
\]

(23)

Eq. (24) now becomes

\[
\dot{H} = -\frac{1}{2} \left( \phi^2 \rho_\phi + \gamma \rho_\gamma \right) \left( 1 - 2\frac{\rho}{\rho_c} \right).
\]

(24)

Combining Eqs. (22)-(24) and the Friedmann equation, we can construct a 4-dementional autonomous system. To see this, we usually introduce 4 convenient variables \([6, 12]\):

\[
\begin{align*}
\phi &\equiv \phi, \quad \rho \equiv \sqrt{\frac{V}{3H}}, \\
z &\equiv \frac{\rho}{\rho_c}, \quad \lambda \equiv \frac{V'}{3HV}.
\end{align*}
\]

(25)

Moreover, we use \( N = \ln a^3 \) instead of the cosmological time \( t \) as an independent variable, therefore for any time-dependent function \( f \), we have

\[
\frac{df}{dN} = \frac{j}{3H}.
\]

(26)

Here, one should be careful that this treatment may not be practicable when \( \dot{a} = 0 \), however, at that point, one can always switch back to \( t \) without causing any problem. With the help of new variables, the Eqs. (1) and (22)-(24) can now be expressed respectively as follows:

\[
\begin{align*}
\frac{\rho_\phi}{3H^2} + \frac{y^2}{\sqrt{1-x^2}} (1-x) &= 1, \\
\dot{\phi} &= -\left( 1 - x^2 \right) \left( \frac{y'}{y} + 3Hx \right), \\
\frac{\rho_\gamma}{3H^2} &= -3\gamma \left( \frac{1}{1-z} - \frac{y^2}{\sqrt{1-x^2}} \right), \\
\frac{H}{3H^2} &= -(1-2z) \left[ \frac{y^2}{\sqrt{1-x^2}} + \gamma \left( \frac{1}{1-z} - \frac{y^2}{\sqrt{1-x^2}} \right) \right].
\end{align*}
\]

(27)
According to Lyapunov’s theory of stability, the stability of a fixed point can be determined by the property of the linearized system about it. The linearization is done by expanding Eq. (31) about the fixed points and keeping only the linear parts. After that, one can obtain a matrix

\[
\begin{pmatrix}
\frac{\partial x}{\partial \xi} & \frac{\partial x}{\partial \eta} & \frac{\partial x}{\partial \zeta} \\
\frac{\partial y}{\partial \xi} & \frac{\partial y}{\partial \eta} & \frac{\partial y}{\partial \zeta} \\
\frac{\partial z}{\partial \xi} & \frac{\partial z}{\partial \eta} & \frac{\partial z}{\partial \zeta}
\end{pmatrix}
\]

(33)

at each fixed point. If the all the eigenvalues of \( \Phi \) for a fixed point have negative real parts, the fixed point is (locally) exponentially stable. However, if there exist one or more eigenvalues which have positive real parts, then the fixed point is unstable. In both stable and unstable cases, the fixed point is a node if the eigenvalues are all real, otherwise it will be a spiral point. Furthermore, the fixed point is called saddle point if the eigenvalues have both positive and negative real parts.

We found up to 5 fixed points for Eq. (31), their properties are listed in Table I, where

\[
y_1 = \sqrt{\frac{\sqrt{\alpha^2 + 4} - \alpha^2}{2}}, 0 < \alpha y_1 < 1.
\]

(34)

The physical requirements mentioned before also set restrictions on the existence and stability of these fixed points. By considering \( \gamma/\alpha^2/\sqrt{1 - \gamma} \leq 1 \), we found

\[
0 \leq \frac{\gamma(\gamma - 2)^2}{\sqrt{1 - \gamma}} \leq f(\gamma) \leq \alpha^2(2 - \gamma)^2,
\]

(35)

where \( f(\gamma) = 4\alpha^2 - 20\gamma\alpha^2 + 17\gamma^2\alpha^2 + 16\gamma^2(1 - \gamma) \), which indicates the two eigenvalues of \( P_3 \) with seemingly complicated square root part are definitely real and nonpositive. Note that, when \( \gamma > \alpha^2 y_1^2 \), \( P_3 \) doesn’t exist. There are only four fixed points in the system, and \( P_4 \), as a stable node, is the only attractor in the system. When \( \gamma = \alpha^2 y_1^2 \), a bifurcation occurs as the fifth fixed point, \( P_5 \), emerges and coincides with \( P_4 \). As \( \gamma \) decreases from \( \alpha^2 y_1^2 \) to 0, \( P_4 \) turns into an unstable saddle and \( P_3 \) becomes a new attractor. The position of \( P_3 \) is located on a straight line on the x-y plane described by \( (x, y) = (\sqrt{\gamma}, \sqrt{\gamma/\alpha}) \) and in the limit case \( \gamma = 0 \), it will coincide with \( P_1 \).

IV. LATE TIME EVOLUTION

From TABLE I we can see that all the fixed points locate at \( z = 0 \), where the energy density vanishes and the LQC modification term \( \rho/\rho_c \) becomes unimportant. We have proved in last section that the physically possible phase plane projection on the \( x - y \) plane shrinks as \( z \) decreases from 1 to 0. In fact, as \( z \to 0 \) the phase plane projection becomes \( y^4 + x^2 \leq 1 \) asymptotically.

The equilibrium points or fixed points \( (x_e, y_e, z_e, \lambda_e) \) are solutions acquired by setting

\[
\frac{dx}{dN} = \frac{dy}{dN} = \frac{dz}{dN} = 0.
\]

(32)

According to Lyapunov’s theory of stability, the stability of a fixed point can be determined by the property of the linearized system about it. The linearization is done
investigate the properties of late time evolution near the attractor, regardless of the history before it. The fine-tuning problem can thus be waived since the same ending occurs for a wide range of initial conditions. Here we analyze two different solutions, the tachyon dominated solutions and tracker solutions.

A. Tracker solutions

When \( \gamma < \alpha^2 y_1^2 \), \( P_4 \) becomes an unstable saddle while \( P_3(\sqrt{\gamma}, \sqrt{\gamma}/\alpha, 0) \) comes out as the only attractor in our system. The fractional density is a constant at \( \rho = \gamma/\alpha^2 \), \( \phi(t) = \sqrt{\gamma} t + \phi_0 \), which means both matter scale as the same power of \( \alpha \). Therefore, we have

\[
\Omega_\phi \approx \frac{\gamma/\alpha^2}{\sqrt{1-\gamma}}, \quad \omega_\phi \approx \omega_\gamma = \gamma - 1, \tag{36}
\]

\[
\phi(t) \approx \sqrt{\gamma} t + \phi_0, \tag{37}
\]

\[
\rho_\gamma \propto \rho_\phi \propto a^{-3\gamma}, \quad a(t) \propto t^{\frac{2}{3}}, \tag{38}
\]

B. Tachyon dominated solutions

In the case \( \gamma > \alpha^2 y_1^2 \), \( P_4 \) does not exist. As the solutions converge to the only attractor \( P_3(\alpha y_1, y_1, 0) \) in late time evolution, tachyon will become dominant while both matters are decreasing. In fact, near the attractor

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**TABLE I: Properties of fixed points**

| Points | Coordinates | Existence | Eigenvalue | \( \Omega_\phi \) | Stability |
|--------|-------------|-----------|------------|----------------|-----------|
| \( P_1 \) | (0, 0, 0) | for all \( \gamma \) | \((-1, -\gamma, \frac{3}{2})\) | 0 | Unstable saddle |
| \( P_{2\pm} \) | \((\pm 1, 0, 0)\) | for all \( \gamma \) | \((2, -\gamma, \frac{3}{2})\) | 1 | Unstable saddle |
| \( P_3 \) | \((\sqrt{\gamma}, \sqrt{\gamma}/\alpha, 0)\) | \(0 < \gamma \leq \alpha^2 y_1^2\) | \(-\gamma, -\frac{(2-\gamma)\alpha - \sqrt{f(\gamma)}}{4\alpha}, -\frac{(2-\gamma)\alpha + \sqrt{f(\gamma)}}{4\alpha}\) | \(\frac{2/\alpha^2}{\sqrt{1-\gamma}}\) | Stable node, if \(0 < \gamma < \alpha^2 y_1^2\) |
| \( P_4 \) | \((\alpha y_1, y_1, 0)\) | for all \( \gamma \) | \((-\alpha^2 y_1^2, -\frac{1}{2}(2\alpha^2 y_1^2), -(\gamma - \alpha^2 y_1^2))\) | 1 | Stable node, if \(\gamma > \alpha^2 y_1^2\); Unstable saddle, if \(\gamma < \alpha^2 y_1^2\) |

---

**FIG. 2**: Phase plane for \( \gamma = 0.3, \alpha = 1 \). The outer contour corresponds to \( y^4 + x^2 = 1 \). Solutions start at \( z=0.01 \). Nearly all solutions end up at \( P_3 \).

**FIG. 3**: Phase plane for \( \gamma = 4/3, \alpha = 1 \). The outer contour corresponds to \( y^4 + x^2 = 1 \). Solutions start at \( z=0.01 \). Almost all solutions end up at \( P_4 \).
we will have
\[ \Omega_\phi \approx 1, \quad \rho_\phi \approx \rho < \rho_c, \quad \omega \approx \omega_\phi \approx \alpha^2 y_1^2 - 1, \]
\[ \dot{\phi}(t) \approx \alpha y_1 t, \]  
(39)
\[ \rho_\phi \propto a^{-3(1+\omega_\phi)} = a^{-3\alpha^2 y_1^2}, \quad a(t) \propto t^{2/3(1+\omega_\phi)} = t^{2/3\alpha^2 y_1^2}. \]

This result is consistent with the result we derived in the second section. It is easy to see that if \( \alpha^2 y_1^2 < 2/3 \), the solutions depict an eternal acceleration. On the other hand, the universe will end up in deceleration if \( \alpha^2 y_1^2 > 2/3 \).

An interesting possibility is to see tachyon field as a combination of two parts \([12, 14]\), which behave like a pressureless dust (dark matter, denoted by lower case \( DM \)) and a cosmological constant (dark energy, denoted by lower case \( \Lambda \)), respectively:
\[ \rho_\phi = \rho_{DM} + \rho_\Lambda, \quad p_\phi = p_{DM} + p_\Lambda, \]  
(40)

where
\[ \rho_{DM} = \frac{V(\phi)\dot{\phi}^2}{\sqrt{1 - \dot{\phi}^2}}, \quad p_{DM} = 0, \]  
(41)
and
\[ \rho_\Lambda = V(\phi)\sqrt{1 - \dot{\phi}^2}, \quad p_\Lambda = -\rho_\Lambda. \]  
(42)

In this way, dark matter and dark energy originate from a same scalar field, and the dynamics of tachyon field becomes a description of their dynamics and interaction. The ratio between the two parts is
\[ \frac{\rho_{DM}}{\rho_\Lambda} = \frac{\dot{\phi}^2}{1 - \dot{\phi}^2}. \]  
(43)

The proportion of dark matter rises as \( \dot{\phi}^2 \) increases. If the barotropic fluid is radiation (\( \gamma = 4/3 \)), then, by the virtue of the attractor solution, it is possible to have a trajectory that goes from the radiation dominated era to the matter dominated era, and then to the dark energy dominated regime. Radiation should dominate first, that is \( \Omega_\gamma \approx 1 \). Then, to have the universe dominated by matters described by tachyon field after the era dominated by radiation, that is \( \Omega_\phi \approx 1 \), we need the trajectory to stay close to the boundary of the phase plane, i.e., \( (1 - z)^2 y_1^4 + x_1^2 \approx 1 \), or \( y_1^4 + x_1^2 \approx 1 \) because \( z \) is very small. To have a sufficient long matter dominated era before dark energy take over, we just need the trajectory to stay close to the saddle point \( P_{2+} \) after radiation’s domination. Therefore we will rule out the trajectories which are far away from the boundary of the phase plane in following discussion.

The negative/positive branch, which starts near \( P_{2-} / P_{2+} \), will show a quite different evolution, because the attractor only lies in the right half of the phase plane and therefore the negative branch will have to get across to arrive at the attractor. The cosmological consequence will also be quite different. For the positive branch, the ratio of dark energy will increase as \( \dot{\phi} \) decreases. If the attractor is inflationary (\( \alpha^2 y_1^2 < 2/3 \)), the allowed trajectories can go naturally into the dark energy dominated regime that leads to acceleration. While if the attractor is not inflationary (\( \alpha^2 y_1^2 > 2/3 \)), acceleration will not take place because \( \dot{\phi} \) decreases almost monotonously for the feasible trajectories which are close to the boundary. On the other hand, for the negative branch, the universe will definitely enter a dark energy dominated regime that leads to acceleration when \( \dot{\phi}^2 < 2/3 \), or \(-2/3 < \dot{\phi} < 2/3 \), since the trajectories will get across from the left half to the right half. Note that, for the negative branch, the universe is not expanding in the power-law way we described before when it first gets into the acceleration regime \(-2/3 < \dot{\phi} < 0 \). When \( \dot{\phi} = 0 \), dark energy dominates completely. After that, the ratio of matter increases again, and the final state of universe will (or will not) be inflationary if \( \alpha^2 y_1^2 < 2/3 \) (or \( \alpha^2 y_1^2 > 2/3 \)). So there is possibility that we are just currently living in a transitory accelerating period and the acceleration rate can change according to the dynamics of the tachyon field.

V. CONCLUSION

Previous works on tachyon cosmology in spatially flat FRW universe showed a purely tachyonic matter with an inverse square potential \( V = \beta \phi^{-2} \) leads to a power-law expansion \([\beta]\). In LQC scenario, although it is hard to find a exact solution, the expansion of universe is nearly a power-law one when \( \rho/\rho_c \) is small. When the tachyon field is coupled with a barotropic perfect fluid with \( 0 < \gamma < 2 \), we are able to find two kinds of stable nodes which exist exclusively to each other and represent different cosmological situations. The tracker solution exists for \( \gamma < \alpha^2 y_1^2 \), while the tachyon dominated solution exists for \( \gamma > \alpha^2 y_1^2 \). Refs.\([2, 3]\) argued that the tracker solution cannot be a viable one, since its existence requires \( \gamma < \alpha^2 y_1^2 < 1 \). Therefore we focused on the tachyon dominated solution. We considered the tachyon field as a combination of two parts which respectively behave like dark matter and dark energy. Because of the existence of stable node (attractor), we found when \( \gamma = 4/3 \), it’s possible to have a trajectory in which the universe evolves naturally from radiation dominated regime to matter dominated regime, and then into the current dark energy dominated regime through qualitative discussion. The negative and positive branches, which respectively start from close to \( \phi < 0 \) and \( \phi > 0 \), can be interpreted into different cosmological evolutions and it is possible that we are just living in a transitory accelerating period, while the final stage of universe will be identical for the same \( \alpha \) or \( \beta \). However, this perspective to see tachyon field as a combination of two parts revives the need of fine tuning. We also avoided the discussion of
the period before radiation dominated era. Further work can be done to bridge the gap.

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