On the Dirac monopole’s concept

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Abstract

The Dirac monopole is discussed in view of the gauge invariance in Quantum Electrodynamics. It is shown the monopole existence implies the violation of the gauge invariance principle. The monopole field is essentially a longitudinal field and so a mass is naturally associated to it. Interpretation for the case the monopole charge is different from zero is addressed at the conclusion.

1 The topological monopole

Any derivation of the Dirac Quantization condition \([1]\), \(eg/c = n\hbar/2\) (where \(e\) is the electric charge, \(g\) the magnetic flux and \(n = 0, \pm 1, \pm 2,...\)), is based on two steps: First, one has to calculate the angular momentum due to the fields; Second, to quantize it through Schröedinger Quantum Mechanics (for instance).

The first step was first performed in 1904 when J. J. Thomson [2] derived the term \(ev \times B\) known as the magnetic part of the Lorentz force upon an electric charge (\(B\) is the magnetic field and \(v\) the charge’s velocity, both relative to the inertial laboratory system). The angular momentum of the field resulting from an electric charge and a magnetic monopole (or simply, monopole) can be written as the volume integral of the angular momentum density \(r \times (E \times B)/4\pi c\), with \(r = (x, y, z)\) (the position vector) and \(E\) the electric field. It results the total angular momentum stored in the field for this system is \(eg/c\). Considering the angular momentum part due to the particles has to compensate the one of the fields, Thomson derived the magnetic
Lorentz term which leads to a variation of the angular momentum of $2eg/c$
for the particles in order to preserve the conservation of the total angular
momentum of the system.

The second step towards the definition of the Dirac Quantization Con-
dition is to consider Wave Mechanichs as valid for particles, [1]. Any gauge
change on the electromagnetic potentials leaves the Schroedinger equation in-
variant if the wave functions are altered as $\psi \rightarrow \psi' = \psi \exp\left[i\phi/hc\right]$, where $\phi$
is the gauge function. It is clear that, once the phase $e\phi/hc$ has no dimension,
$\phi$ is directly connected to some magnetic flux. In the system of one charge
and one monopole this magnetic flux is $4\pi g$, and requiring the phase must
be unaltered by the presence of the monopole, i.e., that the phase change
due to monopole’s presence must be a multiple of $2\pi$, one gets the Dirac
Quantization Condition, [1]. It has been considered this condition makes the
gauge due to the monopole just an artifact. Accordingly, it was shown by
other approaches (Wu and Yang’s, [3]) that if such condition is observed the
monopole is a topological set-up of the gauge fields, being the monopole’s
gauge not observable.

It is interesting to quote this condition was derived by Dirac [1] and
Wu-Yang [3] in order to the charges never cross a singularity on the vector
potential gauge associated to the monopoles. In Brandt and Primack’s work,
[4], it was shown also that the two approaches are equivalent. In Dirac’s
approach the wave functions of the charges are set to be exactly null on
the singular part of the vector field, while in Wu-Yang’s a Fiber-Bundle
description is given as the traduction of this condition. In fact, it is impossible
to define a monopole through a vector potential (from which the magnetic
field is to be derived) free of singularities in $\mathbb{R}^3$ (the three-dimensional flat
world), [3], and Dirac Quantization Condition makes the singularity non-
observable.

As the approaches of Dirac, [1] and Wu-Yang, [3], are equivalent, [4],
we proceed considering the consequences in assuming the existence of a
monopole $g$ and the condition $eg/c = nh/2$. He, Qiu and Tze, [5], considered
this situation and showed it is impossible to get $g \neq 0$ in pure Quantum
Electrodynamics (QED). It can be demonstrated that the longitudinal photon field does not alter Physics in a gauge-invariant theory as QED, [5], so the unphysical gauge coupling (which has an arbitrary value) associated to this field is not observable. It was shown however, [5], that in deriving the Dirac Quantization Condition, this unphysical gauge coupling appears related to the monopole flux in the same way it appears for the physical electric charge
in the usual relation \( e g/c = n\hbar/2 \). This is due to the fact the non-integrable phase for the particle with charge \( e \) in the presence of the fields \( A_\mu \) (physical) and \( [A_0]_\mu \) (unphysical longitudinal field) can be written as:

\[
P(x_2, x_1; C) = \exp \left[ i \frac{e}{\hbar c} \int_{x_1}^{x_2} A_\mu(x) dx^\mu + i \frac{e_0}{\hbar c} \int_{x_1}^{x_2} [A_0]_\mu(x) dx^\mu \right]
\]  

(1)

where the line-integral is along the path \( C \) from \( x_1 \) to \( x_2 \). For a closed loop \( C \), equation (1) gives:

\[
P(x, x; C) = \exp \left[ i \frac{e}{\hbar c} \int a F_{\mu \nu} da^{\mu \nu} \right]
\]  

(2)

where \( F_{\mu \nu} \) is the electromagnetic field strength and \( a \) is the area bounded by \( C \). Now \( P \) is \( U(1) \) gauge-invariant as the unphysical field \( [A_0]_\mu \) makes no contribution, [5].

Now consider we are going to measure the magnetic flux over the area \( a \), \( \int \mathbf{B} \cdot da \), knowing that \( \mathbf{B} = \nabla \times \mathbf{A} \), and that \( \mathbf{A} = \mathbf{A} + \nabla \phi \), where \( \phi \) represents the gauge-freedom for the vector potential. The measure of the magnetic flux can be written as:

\[
\int \nabla \times \mathbf{A} \cdot da
\]  

(3)

According to Wu and Yang, [3], for each point over the surface \( a \) we must choose \( \mathbf{A} \) such that no singularity is visited on going over the surface, i.e., if some point which will be considered is the place occupied by the singular string on this surface, the gauge changing (from one previous point to this) must respect the condition \( e g/c = n\hbar/2 \) (a \( 2\pi \) change of phase), in a way that the string is moved to another place in \( \mathbb{R}^3 \). This way no singularity is seen, the term associated to the longitudinal field makes no contribution and the Dirac Quantization Condition is respected, [3].

According to reference [5], this is wrong. It is clear that the transformation from equation (1) to (2) is only possible if the Stokes theorem can be applied, and unfortunately, \( \mathbf{A} \) is not a continuous function to support such theorem. It changes abruptly from one point to the next in space due to the gauge changing necessary to hide the singularity associated to the monopole’s definition. The Stokes theorem cannot be applied, therefore the unphysical gauge coupling \( e_0 \) cannot be disregarded and will appear connect to the magnetic flux \( 4\pi g \) as \( e_0 g/c = n\hbar/2 \), [5]. It is easy to see that if primary the gauge is \( \mathbf{A}_1 = \phi g(1 - \cos \theta)/r \sin \theta \), for \( \theta < \pi - \varepsilon \), and changes to
\[ A_2 = -\phi g(1 + \cos \theta)/r \sin \theta, \] for \( \theta > \varepsilon \), for some electric charge’s path in the region of the overlap of the two gauges (\( \varepsilon \) is some arbitrary angle), one goes suddenly from \( G(1 - Y)\phi / G(-1 - Y)\phi \) (if \( G \equiv g/r \sin \theta \), and \( Y \equiv \cos \theta \)). The only possible solution for this situation is to set \( g = 0 \) as in reference [5], otherwise a physical interpretation must be assumed for \( e_0 \) in pure QED since we get both \( eg/c = nh/2 \) and \( e_0 g/c = nh/2 \).

In the next section an alternative approach to such problem is addressed. It is shown the gauge invariance is broken because longitudinal fields are related to the monopole’s definition. We then proceed considering the case \( g \neq 0 \) and an interpretation is addressed. If the string of singularities is not to be seen, some physical reason (strange to pure QED) must be given for this to happen.

### 2  The definition of a Dirac monopole

In the following steps we will start from the Maxwell equations in the vacuum and consider the necessary conditions for the existence of a monopole with \( g \neq 0 \). We do so because the definition \( B = \nabla \times A \) was first derived from the condition \( \nabla \cdot B = 0 \). The departure equations are those which describe photons in the usual theory (gauge invariant). At the end, after defining the composition of fields and conditions which define the monopole, one desires part of the flux of its field (the returning flux) to be associated to the arbitrary gauge of the system in a way the definition of the monopole is free of singularities (the returning flux) in a topological manner, [3]. In the next steps it will become clear that in order to define a monopole, the gauge invariance must be broken and as a consequence this violation is not only connected to the magnetic charge but with the monopole mass.

#### 2.1  Basic construction of a monopole

The electromagnetic fields, solution of the wave equation \( [\nabla^2 - \partial^2/\partial(ct)^2]F = 0 \), can be described by a multipole expansion with:

\[
B_{lm} = \Omega e^{-i\omega t} f_l(kr)LY_{lm} \\
E_{lm} = \frac{i}{k} \nabla \times B_{lm}
\]

where \( \Omega \) is a constant, \( L = \frac{1}{r} \times \nabla \), \( Y_{lm} \) is the spherical harmonic of the order \( l, m \) and \( f_l \) a Hankel plus Bessel type solution. The parameter \( k \) is \( \omega/c \) and
the spherical coordinate of distance and time in the laboratory frame. Taking the Hankel solutions, the fields \( E_{10}, B_{10} \) can be approximated in the \( kr << 1, r \neq 0 \) region by:

\[
B_{1,0} = e^{-i\omega t}(-\Omega)kLY_{10}/r^2 \\
E_{1,0} = e^{-i\omega t}(-\Omega)\nabla\left(Y_{10}/r^2\right)
\]

The \( E_{1,0} \) solution is exactly the field of a dipole in this approximation and it is also written as:

\[
e^{-i\omega t}(-\Omega)\nabla\left(Y_{10}/r^2\right) = (3n(p \cdot n) - p)/r^3
\]

with \( p \equiv \sqrt{3/4\pi}e^{-i\omega t}\Omega z \) (\( |z| = 1 \)) and \( n \) is the unitary vector directioned from the origin to \( r \).

It is possible to construct a sum of solutions \( E_{1,0} \) (with the corresponding magnetic field associated to it) composed over a line path over the space. The \( E_{1,0} \) may also be written as \( \nabla \times [p \times r/r^3] \) and it is possible to introduce an elementary contribution like:

\[
\nabla \times \left[ dp \times r/r^3 \right] \\
dp \equiv \sqrt{3/4\pi}e^{-i\omega t}\Omega \lambda dl
\]

where \( \lambda \) is a density and \( dl \) the elementary oriented path of some line in space. Defining the line path along the \( z \) axis from zero to \( -L \) (\( L > 0 \), \( L << 1/k \)), the resulting electric field \( E_{\text{line}} \) in the region \( kr << 1 \) will be:

\[
q \left( \frac{r}{4\pi r^3} - \frac{r + Lz}{4\pi |r + Lz|^3} + \delta(x)\delta(y)[\Theta(-z) - \Theta(-z - L)]z \right)
\]

where \( \Theta(x) \) is the Heaviside function, \( \delta(x) \) is the Dirac Delta function and \( q = \sqrt{3/4\pi}e^{-i\omega t}\Omega \lambda \). From this expression it is possible to calculate the magnetic field as \( \nabla \times B = -i\omega E_{\text{line}}/c \) for \( kr << 1 \). There are of course other components for the fields (other terms besides the first of the Hankel function for \( x = kr << 1, -(2l - 1)!!(1 - x^2/(2 - 4l) + ...) / x^{l+1} \), which are small as the difference \( r << 1/k \) is pronounced.

Consider now an infinite set of elementary \( E_{\text{line}} \) solutions in a way the lines are isotropically distributed on the radial direction with the ends \( dq \) at the origin of the coordinate system and the \(-dq\) ends at the far infinite,
away from the origin. This construction defines what we could call as a radial system of currents with spherical symmetry. In this particular situation the first order contribution to the magnetic field is null due to the symmetry of the construction.

Louis de Broglie proposed the similar oscillating electric charge situation, [6], calculating the electric field in space out of the places where there is an inward flux. Consider $A^\mu = (V, A)$ such that (in spherical coordinates):

$$V \sim -\frac{|k|^2 e^{i(\kappa ct - |k|r)}}{k_0^2 r}$$  \hspace{1cm} (9)

$$A_r \sim \frac{i\kappa \partial V}{|k|^2 \partial r}$$

$$A_\theta = A_\phi = 0$$

with $\kappa^2 = |k|^2 + k_0^2$, where $k_0$ is a parameter related to the mass $m$ of the longitudinal field by $k_0 = mc/\hbar$. In guided wave problems the parameter $k_0$ is related to the constraint given by the walls which guide the wave. The electric field derived from $A^\mu$ has only radial component different from zero and will be:

$$E_r \sim \left[ \frac{1}{r^2} + \frac{i|k|}{r} \right] e^{i(\kappa ct - |k|r)}$$  \hspace{1cm} (10)

$$E_\theta = E_\phi = 0$$

and the magnetic field is null. If $k$ approaches to zero, $\kappa$ approaches to $k_0$, so that in this limit:

$$E_r \sim \frac{1}{r^2} e^{ik_0 ct}$$  \hspace{1cm} (11)

which is the same result developed above for the electric field in places outside the inward flux and with $r << 1/k$.

It interesting to make the change from the Transverse-Magnetic solutions at the begining to the Transverse-Electric ones, $E_{TM} \rightarrow -B_{TE}$, $B_{TM} \rightarrow E_{TE}$, from which we obtain the same physics for $g = -\sqrt{3/4\pi} e^{-i\omega t} \Omega \lambda$, as we got for $q$. Now we are close to define a monopole. It is first necessary to make $\omega \rightarrow 0$ in order the outward (inward) magnetic flux be as constant as desidered ($g \rightarrow -\sqrt{3/4\pi} \Omega \lambda$). It is now necessary to create conditions for the returning flux not to be seen. It is important to remember that the geometry of the construction eliminates the leading contribution for the
resulting electric field. It is clear that in the given conditions the measure 
\(|B|^2 - |E|^2|\) is no longer zero. In the de Broglie’s example, [6], 
\(|E|^2 - |B|^2 \neq 0\), then the field associated to the construction is longitudinal. If one defines 
the vector potential [6], 
\(A_x = A_0 e^{i(\kappa ct - |k|z)}\), \(A_y = A_z = 0\), the electric and magnetic fields result: 
\(E_x = -i\kappa A_0 e^{i(\kappa ct - |k|z)}\), \(B_y = -i|k|A_0 e^{i(\kappa ct - |k|z)}\), and 
so \(|E|^2 - |B|^2 = k_0^2|A|^2\), with \(\kappa^2 = |k|^2 + k_0^2\), i.e., if \(k_0 \neq 0\). The mass related to this gauge dependent object is 
\(k_0 \bar{h}/c\). In the magnetic case, \(|B|^2 - |E|^2 \neq 0\)
and we arrive at the same conclusion about the mass associated to the field 
since by the present approach the charges (electric or magnetic) are defined 
starting from the Maxwell equations in the vacuum, and so vector potentials 
can be assigned to each field.

Now it is clear the conclusions in the pioneer work of He, Qiu and Tze, [5], follows. In defining a monopole from the construction above, the inward 
flux lines must be non-observable. The magnetic field is purely longitudinal, 
so the relation (now with \(e\) as the electron’s electric charge) \(eg/c = n\bar{h}/2\) for 
g \(\neq 0\) (\(g\) well approximated as constant in time for Physics once we define 
\(1/\omega \rightarrow \infty\)) violates the gauge invariance principle, a fundamental principle 
in Quantum Electrodynamics. If this principle is broken by the presence of a 
monopole, attempts (like in reference [3]) to make the Dirac string an artifact 
(non-observable) are wrong.

2.2 One-string description and quantization of the mass

Now consider the line path which defines equation (8) to be defined as a 
round loop - it is made when one joins the ends of the line path defined for the 
 elementary solution (7). In this new situation the magnetic field results 
to be (for a round loop centered at the origin on the \(x - y\) plane with some 
radius \(a\)):

\[
B_{loop}^r = \frac{Ia^2 \cos \theta}{2} \frac{1}{r^3}
\]

\[
B_{loop}^\theta = \frac{Ia^2 \sin \theta}{4} \frac{1}{r^3}
\]

where \(I \equiv -i\sqrt{3/4\pi} e^{-i\omega t} \lambda \Omega \omega/c\) and it is considered the \(a << r << 1/k\) 
region.

Consider now that a solenoid made up of such loops is defined over space 
(which symmetry axis follows a line over the \(z\) axis) from the origin to \(z = \)
−\mathcal{L} with a density \sigma of solenoids per unity distance along its axis. Taking \mathcal{L} \ll 1/k and a \to 0 the magnetic field is:

\begin{equation}
\mathbf{B}_{\text{solenoid}} = g \left( \frac{\mathbf{r}}{4\pi r^3} - \frac{\mathbf{r} + \mathcal{L}\mathbf{z}}{4\pi|\mathbf{r} + \mathcal{L}\mathbf{z}|^3} + \delta(x)\delta(y)\left[\Theta(-z) - \Theta(-z - \mathcal{L})\right] \mathbf{z} \right) \tag{13}
\end{equation}

with

\begin{equation}
g = \Upsilon \frac{\omega}{c} \sin \omega t \tag{14}
\end{equation}

or

\begin{equation}
g = \Upsilon \frac{\omega}{c} \cos \omega t
\end{equation}

where \Upsilon is defined to be constant and equal to \sqrt{3/4\pi a^2 \sigma \Omega \lambda / 4} when the parameters \sigma and \lambda are set to compensate variations on the parameter \(a\). The electric field is null except at the solenoid (where it is infinite, i.e., not defined) in this approximation (\(r \ll 1/k\)).

As in the last subsection, in order to describe an oscillating magnetic charge at the origin of the coordinate system it is necessary to define at least a line of returning flux of magnetic field as well as to take \(r \ll 1/k\). In the present construction \(\nabla \cdot \mathbf{B}_{\text{solenoid}} = 0\) everywhere.

Now, following the initial proposal of this section, one wants to associate the returning magnetic flux to the gauge (arbitrary) in a way it is possible to define a topological monopole. The first step towards this is to make \(\omega \to 0\), and to make \(\mathcal{L} \to \infty\). The same problem raised in the last section happens in the one-string description too: The measure \(|\mathbf{B}|^2 - |E|^2\) is different from zero and so the resulting field related to the monopole is purely longitudinal. If the string is not seen due to some mechanism, the quantization condition is related to the longitudinal field only, and so, taking \(g \neq 0\) one is assuming the gauge has some physical interpretation.

It is important to consider the case such mechanism exists. It is then natural to consider the relation \(eg/c = n\hbar/2\) (\(e\) the electron’s charge - this means a gauge is chosen) when \(\omega \to 0\) in order \(g\) variation in time is less than our precision detection:

\begin{equation}
g = \Upsilon \omega/c = \Upsilon mc/h = n\hbar c/2e \tag{15}
\end{equation}

which leads to:

\begin{equation}
m = n\hbar^2/2\Upsilon e \tag{16}
\end{equation}

i.e., the monopole mass must be quantized if \(\Upsilon\) is constant and \(m \to 0\). This does not mean the resulting monopole mass to be necessarily vanishing.
as we will see with the help of the next section. For the sake of generality, we will just quote there are approaches pointing towards a vanishing monopole mass anyway, [7]. As it was shown magnetic monopoles as well as electric charges can be described by the present procedure, this quantization is defined for both types of particle’s charges (with \( g \) instead of \( e \) in the last expression for the case of electric charge’s particles). There is a good reason to believe \( \Upsilon \) is a constant in Nature, it is probably related to the other constants: Another interesting consideration about the quantization condition is that, if the minimum frequency \( \omega \) possible is \( 2\pi c/R_u \), where \( R_u \) is the physical observable Universe’s radius, and so:

\[
\frac{\hbar c}{e} \sim \frac{\Upsilon}{R_u} \tag{17}
\]

i.e., there must be a relation between the constants of Nature (wavelength of a particle) and the age of the Universe (or the magnetic charge varies with the Universe’s time scale).

### 2.3 Arbitrariness in the monopole description

Other results follow from oscillating charges as in the constructions above once a charge (magnetic or electric) at \( r' \) can be defined through the density \( \rho \) by \( [\rho + \rho_0(t) - \rho_0(t)]\delta(r-r') \), where \( \rho_0 \) is function of the time and arbitrary in magnitude (\( \delta \) is the Dirac-delta function). It can be shown this arbitrariness means also no topological mechanism is possible in case of monopoles in pure Quantum Electrodynamics.

A null magnetic charge can be defined as the composition \( A = A_1 - A_2 \) (as in the first section):

\[
A = [\phi g_0 (1 - \cos \theta)/r \sin \theta] + [\phi g_0 (1 + \cos \theta)/r \sin \theta] = \phi g_0 2/r \sin \theta \tag{18}
\]

i.e., two monopoles of opposite charge at the same position (at the origin of the coordinate system), with \( \varepsilon < \theta < \pi - \varepsilon \) (\( \varepsilon \) is some arbitrary vanishing angle). The magnetic flux over a spherical surface centered at the origin is calculated as \( \oint A \cdot dl \) over a closed curve around the singular string for some \( \varepsilon < \theta < \pi/2 \) (\( \theta \to \varepsilon \)), plus \( -\oint A \cdot dl \) for a closed curve around the singular string for \( \pi - \theta \), that give us zero for the net flux. This is quite obvious to see since the surface is open by inside and outside (the first closed curve must be integrated on the clockwise direction while the second in the anticlockwise direction - or vice-versa) while \( A \) remains the same (\( \sin \theta = \sin \pi - \theta \)).
The vector potential representing a non-null magnetic charge \( g \) can be defined as the sum:

\[
\phi g_0 \left( \frac{1 - \cos \theta}{r \sin \theta} \right) + \phi g_0 \left( \frac{1 + \cos \theta}{r \sin \theta} \right) + \phi g \left( \frac{1 - \cos \theta}{r \sin \theta} \right)
\] (19)

with \( \varepsilon < \theta < \pi - \varepsilon \) (\( \varepsilon \) is some arbitrary vanishing angle). The net magnetic charge is \( g \).

Now consider in the construction above \( g \) is considered independent of time but \( g_0 = g_0(t) \). The vector potential will be:

\[
A = \phi \left[ g_0(t)^2 + g(1 - \cos \theta) \right]/r \sin \theta
\] (20)

and so the measure \( \oint A \cdot dl \) around each string will no longer be constant.

The variation with time is arbitrary and so this measure for each round loop around each string of the problem. In the case one desires to move some of the strings off the way of particle’s path, the quantization condition cannot be applied for this particular string, and so, it will become observable according to the picture in reference [3]. There is an arbitrary gauge transformation instead of the one related to the Dirac Quantization Condition. Once this condition cannot be associated to each string but only to the net magnetic charge, conclusions in reference [3] about the non observability of the Dirac string do not follows (the string is not a gauge artifact which can be treated topologically). The arbitrariness is related to the longitudinal fields as first pointed by He, Qiu and Tze, [5].

### 3 Conclusions

The fundamental problem now arises: Suppose there is a Dirac Monopole in Nature. It will break gauge invariance and the Dirac string will be observable. In what conditions has the gauge a physical interpretation and at the same time gives a mechanism to hide the string? If the string of singularities is not to be seen, some physical reason (strange to pure Quantum Electrodynamics) must be given for this to happen once now the mechanism of Dirac or Wu-Yang, [3], fails. Is this physical reason accounting for the gauge interpretation?

The answer relies on the longitudinal field interpretation we arrived at the end of our attempt to built a monopole. To the longitudinal field there is associated a mass \( \omega \hbar/c^2 \). The gauge has the physical interpretation to be
related to the mass and the charge of the monopole. Can the mass produce
the mechanism to make the returning flux non-observable? In a recent work
we attempt to show that in fact a mass can hide the Dirac string, [8]. The
gravitational effect of the mass makes not all the flat three-dimensional world
available due to the distortion of the spacetime. In the construction of fields
of the last section the Hankel type of solutions make the inward flux lines
regions not defined for fields (in the construction of fields of the one-string
section the Hankel type of solutions make the solenoid region not defined
for fields and the string has naturally some internal volume). In the referred
work, [8], we propose the string, which is a place where the fields are not
defined in the three-dimensional flat world, is the same place the spacetime
is also not defined due to the gravitation provoked by the monopole mass.

In another recent work, [9], we propose there are Dirac monopoles in
Nature (in fact everywhere) i.e., in the electrons. It is discussed the spin
one-half electron can be defined as composed by an electric charge and two
monopoles of opposite magnetic charge performing an object with electric
charge and a magnetic dipole. If such construction is possible it is very
important to consider monopoles as possible entities in Nature, and so, some
interpretations about the gauge and its connection to other forces in Nature
are to be addressed.

The relation we obtain between the electric charge and the Universe’s
radius is a question to be investigated in more detail, but as Feynman used to
say, once we include gravitation to the problems, it is natural to have Mach’s
principle associated. The mass quantization relation is derived directly from
Dirac Quantization Condition in the case of one-string representation. This
particular case is characterized by the fact the forbidden region (the returning
flux region) has naturally some nonzero volume in space. It is important to
observe the arbitrariness in the monopole definition as defined in section
2.3 means that the monopole mass can be non-vanishing (in fact as big as
desired) due to the arbitrary mass of the pair $+g_0, -g_0$. In this case it
is not necessary to enforce $m \to 0$ in order to get a a large time scale for
the magnetic charge oscillation (it is enough that only $g$ varies very slowly in
time, but in this case the quantization associated to the mass is lost).

It is interesting to quote the intuition Faraday had about the connection
between Gravity and Electromagnetism, [10]. He conjectured two masses in
gravitational attraction must have a solenoidal current associated to each
body. His experiments found no result: He left solenoids of various materials
in a free fall and measured the induced current due to gravity. It is interest-
ing the mechanism performed to describe monopoles and then electrons are related to gravity in our exploration: Solenoidal currents associated to the charges in the bodies have direct relation to their gravity.

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