Effects of mass models on dynamical mass estimate: the case of ultra diffuse galaxy NGC1052-DF2

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ABSTRACT

NGC1052-DF2 was recently discovered as the dark-matter deficient galaxy claimed by van Dokkum et al. (2018a, vD18). However, large uncertainties on its dynamical mass estimate have been pointed out, concerning the paucity of sample, statistical methods and distance measurements. In this work, we discuss the effects of the difference in modeling of the tracer profile of this galaxy on the dynamical mass estimate. To do this, we assume that the tracer densities are modeled with power-law and Sérsic profiles, and then we solve the spherical Jeans equation to estimate the dynamical mass. Applying these models to kinematic data of globular clusters in NGC1052-DF2, we compare 90 per cent upper limits of dynamical mass-to-light ratios estimated between from this analysis and from vD18. We find that the upper limit obtained by the power-law is virtually the same as the result from vD18, whilst this limit estimated by the Sérsic is significantly greater than that from vD18, thereby suggesting that NGC1052-DF2 can still be a dark-matter dominated system. Consequently, we propose that dynamical mass estimate of a galaxy is largely affected by not only small kinematic sample but the choice of tracer distributions, and thus the estimated mass still remains quite uncertain.

Key words: galaxies: kinematics and dynamics - galaxies: structure - galaxies: individual: NGC1052-DF2

1 INTRODUCTION

Owing to recent deep photometric observations, ultra diffuse galaxies (UDGs) have been discovered in clusters and groups of galaxies (e.g., van Dokkum et al. 2015a,b; Koda et al. 2015; Yagi et al. 2016; van der Burg et al. 2017; Trujillo et al. 2017). These galaxies have commonly the typical luminosity of a dwarf galaxy, but they are similar to Milky-Way-sized galaxies in physical size. Therefore, UDGs are characterized as an extremely low surface brightness galaxies. From dynamical analysis for kinematic data of globular clusters (GCs) within UDGs, they are, in general, thought to be largely dominated by dark matter as well as the the Galactic dwarf spheroidal galaxies (e.g., van Dokkum, et al. 2016), but how these diffuse galaxies are formed and evolved in their dark matter halo is still ongoing debate (e.g., van Dokkum et al. 2015a,b; Amorisco & Loeb 2016; Di Cintio et al. 2017).

Interestingly enough, however, van Dokkum et al. (2018a, hereafter vD18) have recently discovered a dark matter deficient the UDG that is deficient in dark matter, NGC1052-DF2, which is a satellite of NGC1052 elliptical galaxy. They adopted mass tracer estimator (MTE) constructed by Watkins et al. (2010) to estimate the dynamical mass within a given radius, and applied it to the kinematic data of the 10 GCs of the galaxy. Then, they estimated the dynamical mass to be only $< 3.4 \times 10^8 M_\odot$ (at 90% confidence) within 7.6 kpc from its centre, even though the stellar mass of this galaxy is estimated to be $2 \times 10^8 M_\odot$.

If their mass estimation is correct, this UDG is a certainly exciting galaxy in terms of the deficit of dark matter and understanding its formation (e.g., Ogiya 2018). However, previous studies have pointed out uncertainties of this mass estimation due to the paucity of kinematic sample, statistical methods and distance measurements (Laporte et al. 2018; Martin et al. 2018; Trujillo et al. 2018). All of them argued that the mass estimate of NGC1052-DF2 still remains largely uncertain, hence it is difficult to conclude that the UFD is a galaxy lacking dark matter.

In this paper, we point out uncertainties of tracer models assumed in dynamical mass estimations. In particular, we show that the dynamical mass of NGC1052-DF2 might
be affected by tracer distribution models assumed in analysis. As mentioned above, vD18 utilized MTE to determine the dynamical mass. This mass estimator is based on the projected virial theorem and a spherical Jeans equation. Moreover, this estimator assumes that the density profile of the tracers is modeled with single power-law form because of requirement from their analytic treatment in the MTE modelling. However, a power-law profile is only acceptable to have a diverged profile at the centre of system without any apparent physical motivation or evidence. Furthermore, since vD18 reported that the stellar system of NGC1052-DF2 is fitted with a Sérisc profile, which has cored profile in inner parts, it may be natural to expect that GC tracers might follow a similar profile. Therefore, in order to investigate the effects of model differences on mass estimate, especially tracer distribution, we calculate dynamical masses of NGC1052-DF2 with two models: Sérisc and power-law tracer density profiles. In addition, to estimate dynamical mass, we do not utilize MTE but use line-of-sight velocity dispersion derived from the spherical Jeans equation. Thus, we set constraints on dark halo parameters from the information of positions and line-of-sight velocities of tracers, and then we estimate the dynamical mass using these best-fitting parameters. However, in principle, both methods should result in virtually similar results if there is not significant statistical uncertainty in the tracer distribution.

This Letter is organized as follows. In Section 2 we introduce the method of dynamical mass estimation based on our analysis. In Section 3, we show the results of mass estimation and then comparison with vD18’s estimation. Summary and conclusion are shown in Section 4.

2 DYNAMICAL MASS ESTIMATIONS

In this section, we briefly introduce the methods of dynamical mass estimates for NGC1052-DF2 based on spherical Jeans equations. Since NGC1052-DF2 is far from the Sun ($D_\odot \sim 20$ Mpc estimated from vD18), the available observed information of its GCs are their projected distributions, line-of-sight velocities and velocity dispersions. Thus, Jeans equations should be integrated along the line of sights. In assumptions of spherically symmetric mass distribution and no net-streaming motions for the tracers, the line-of-sight velocity dispersion is straightforwardly written as

$$
\sigma_{l.o.s}^2(R) = \frac{2}{\Sigma_\star(R)} \int_R^\infty dr \left( 1 - \beta_{ani} \frac{R^2}{r^2} \right) \frac{\nu_\star(r) \sigma^2_{ani}(r)}{\sqrt{1 - R^2/r^2}} \tag{1}
$$

where $R$ denotes the projected radius from the centre of the galaxy, and $\Sigma_\star(R)$ is the projected tracer distribution integrated by $\nu_\star(r)$ along the line-of-sight direction. The three dimensional velocity dispersions of the tracers in the system are represented with $\sigma_r$, $\sigma_\theta$, and $\sigma_\phi$ which denote components along radial, polar and azimuthal directions, respectively. For spherical symmetry, we take $\sigma_\phi = \sigma_\theta$. The anisotropy parameter $\beta_{ani} = 0$ indicates an isotropic velocity ellipsoid of the tracers, while positive and negative $\beta_{ani}$ are radially- and tangentially-biased velocity dispersions, respectively. Then, the anisotropy parameter, $\beta_{ani}$, is defined as $\beta_{ani} = 1 - \sigma_\phi/\sigma_r$. Radial dispersion $\sigma_r$ is obtained by the spherical Jeans equation under assumptions of steady-state and dark matter dominated system (Binney & Tremaine 2008), which is expressed as

$$
\sigma^2_{l.o.s}(r) = \frac{1}{\nu_\star(r)} \int_r^\infty r' \nu_\star(r') \frac{2\beta_{ani} GM(r')}{r'^2} \frac{\rho_{DM}(r') dr'}{r^2} , \tag{2}
$$

where $G$ is the gravitational constant, and $M(r)$ is the enclosed mass of the spherical dark matter halo: $M(r) = \int_0^r 4\pi r'^2 \rho_{DM}(r') dr'$. From Eq. (1) and Eq. (2), we can estimate the DM profile $\rho_{DM}$ by adopting the line-of-sight velocity dispersion $\sigma_{l.o.s}(R)$ to the observational kinematic data.

2.1 Tracer number density models

In order to solve the above Jeans equation, a three-dimensional profile of tracer number densities is assumed. vD18 assumed that their number density falls off according to a power-law, $\nu_\star(r) \propto r^{-\gamma}$, following Watkins et al. (2010), and they found $\gamma_\star = 0.9 \pm 0.3$. However, there is no physical justification for whether this cusped profile is the most likely model, and thus there is a possibility that a cored profile is also acceptable.

In this work, we therefore assume that the member tracers in the NGC1052-DF2 are distributed according to a Sérisc profile (Sérisc 1968) as well as a power-law profile. This is motivated by the facts that the stellar distribution...
of this galaxy is expressed by a two-dimensional Sérisc profile (vdD18), and the GC tracers might follow a distribution similar to the stars. Sérisc profile on the sky plane is written by $\Sigma(R) \propto \exp[-R^{1/m}]$, where $m$ is the Sérisc index, which measures the curvature of the profile, and $m = 1$ corresponds to the exponential profile, and $R$ is the projected radius from the centre of the galaxy. The three-dimensional density $n_\star(r)$ is obtained from the surface density $\Sigma(R)$ by deprojection through the Abel transform derived by Lima Neto et al. (1999).

To obtain structural parameters of power-law and Sérisc profiles, we fit them to the cumulative profile of the projected number density of the tracers (GCs). Since, as was done in vD18, we employ the power-law model that is fitted to the three-dimensional density profile of the tracers, we use Abel integral to calculate the projected power-law profile, and then we cumulate the projected number density with $R$ from inside to outside. The distribution and the best-fitting profiles are shown in Figure 1. We use data of the 10 GCs including the positions and line-of-sight velocity distributions by van Dokkum et al. (2018b), and we employ a simple $\chi^2$ fitting. From the fitting results, we find $\chi^2 = 3.16$ for the power-law model with $\chi^2_{\text{Ser}} = 2.16$, respectively. In comparison with these models, there is no significant difference in the goodness of fit, due to the paucity of the sample tracers. In what follows, we calculate dynamical mass of the galaxy using these two tracer number density models. When we solve the Jeans equation, we adopt two kinds of scale radii of the stellar distributions. It is found from this result that all masses estimated with the different tracer models (e.g., power-law v.s. Sérisc profiles, and isotropic v.s. anisotropic velocity ellipsoids) and our adopted scale radii of the stellar distributions. It is found from this result that all masses estimated with the Sérisc profile are systematically more massive than those with the power-law profile. Also, understandably, considering velocity anisotropy of tracers makes their uncertainties larger. The reason why using Sérisc profile makes dynamical mass more massive than power-law may be explained as follows: It is suggested that the shape of tracer density profile have large impact on determination of halo density profile (e.g., Evans, An & Walker 2009).

### 2.2 Dark matter halo model

For the dark matter halo, we adopt a generalized Hernquist profile given by Hernquist (1990) and Zhao (1996),

$$\rho_{\text{dm}}(r) = \rho_0 \left(\frac{r}{r_s}\right)^{-\gamma} \left[1 + \left(\frac{r}{r_s}\right)^{\alpha}\right]^{\beta-\gamma},$$

where $\rho_0$ and $r_s$ are the scale density and radius, $\alpha$ is the sharpness parameter of the transition from the inner slope $-\gamma$ to the outer slope $-\beta$. For $(\alpha, \beta, \gamma) = (1, 3, 1)$, we recover the NFW profile (Navarro et al. 1996, 1997) motivated by cosmological pure dark matter simulations, while $(\alpha, \beta, \gamma) = (1.5, 3, 0)$ corresponds to the Burkert cored profile (Burkert 1995). Therefore, this dark matter halo model enables us to explore a wide range of physically plausible dark matter profiles.

In this work, we adopt six parameters $(r_s, \rho_0, \beta_{\text{dm}}, \alpha, \beta, \gamma)$ to be determined by fitting to the observed line-of-sight velocity distribution for the GCs in NGC1052-DF2. In order to set constraints on these parameters and to determine their uncertainties, we utilize Markov Chain Monte Carlo (MCMC) techniques, based on Bayesian parameter inference, with the standard Metropolis-Hasting algorithm (Metropolis et al. 1953; Hastings 1970). For likelihood function, we assume that the line-of-sight velocity distribution is Gaussian and centered on the systemic velocity of the galaxy $(\mu)$. Given that the total number of tracers is $N$, and the $i$th tracer has the measured line-of-sight velocity and its observational error $\sigma_i$, at the sky plane coordinates $(x_i, y_i)$, the likelihood function is constructed as

$$L = \prod_{i=1}^{N} \frac{1}{(2\pi)^{1/2}[(\sigma_i)^2 + (\sigma_\mu)^2]^{1/2}} \exp\left[-\frac{1}{2} \left(\frac{(x_i - \mu_j)}{\sigma_i}\right)^2\right],$$

where $\sigma_\mu$ is the theoretical line-of-sight velocity dispersion at $(x_i, y_i)$ specified by model parameters (as described above) and derived from the Jeans equations.

Using posterior distribution of each dark halo parameter, we calculate a marginalized dynamical mass at a given radius. To compare with vD18’s results, we estimate the mass within 7.6 kpc, which is the radius of the outermost GC tracer.

### 3 COMPARISON WITH ESTIMATED DYNAMICAL MASSES

Using the results of the MCMC fitting analysis for the kinematic data of the GC tracers in NGC1052-DF2, we estimate the dynamical mass within 7.6 kpc, with marginalizing all dark halo parameters $(r_s, \rho_0, \alpha, \beta, \gamma)$. Table 1 lists the dynamical masses estimated with the different tracer models (e.g., power-law v.s. Sérisc profiles, and isotropic v.s. anisotropic velocity ellipsoids) and our adopted scale radii of the stellar distributions. It is found from this result that all masses estimated with the Sérisc profile are systematically more massive than those with the power-law profile. Also, understandably, considering velocity anisotropy of tracers makes their uncertainties larger. The reason why using Sérisc profile makes dynamical mass more massive than power-law may be explained as follows: It is suggested that the shape of tracer density profile have large impact on determination of halo density profile (e.g., Evans, An & Walker 2009;
Strigari et al. 2010; Hayashi & Chiba 2012). If a tracer distribution has a flat profile in the central region, the Jeans equation can predict a relatively low \( \sigma_{\text{tr},r,s} \) in the inner region, which can be significantly lower than an observed value. In order to match the prediction with the observation in the inner region, therefore, the halo modeling method prefers dark matter profiles that have relatively large masses in the central region, i.e. it can lead to cuspy density profiles with high \( \gamma \), large scale radii and/or high scale densities. On the other hand, for a steeper tracer density profile in an inner region, it comes out in the opposite sense. Namely, a relatively steeper tracer distribution can prefer a less massive halo than a flatter tracer distributions. Thus, the halo mass estimation is quite sensitive to the assumption of inner tracer profiles, as shown in our result.3

Figure 2 shows posterior distribution functions (PDFs) of the marginalized dynamical mass within 7.6 kpc when assuming isotropic (top panel) and anisotropic velocity distributions (bottom panel). Firstly, to compare with the results from vD18, we estimate the mass under the assumption in which the GC tracers have an isotropic velocity ellipsoid, i.e., \( \beta_{\text{ani}} = 0 \). From the top panel, we find that in the case of power-law profile of tracer distribution (magenta PDF and dashed line), the 90 per cent upper limit of the estimated mass is almost the same as the result from vD18 (red solid line) which estimates dynamical mass with the same power-law profile. Thus, we can confirm that our results using Jeans analysis are consistent with those with the MTE if the power-law profile is assumed for the tracers, and thus our method can reproduce the result of vD18. On the other hand, comparing between the upper limits estimated by vD18 and our Sérsic tracer density model (blue dashed line), the latter is about one order of magnitude grater than the former. In addition, the bottom panel shows comparison between mass estimations in the cases where the isotropic assumption is relaxed for the tracer velocities, i.e., \( \beta_{\text{ani}} \neq 0 \). The difference of the upper limits between vD18 and our Sérsic model still remains in the anisotropic cases.

We also compare our estimated dynamical mass to those estimated by the other papers (Laporte et al. 2018; Martin et al. 2018), which pointed out that there are large uncertainties on dynamical mass estimates of NGC1052-DF2 due to lack of tracer sample and contamination by field GCs. To this end, applying the mass estimator derived by Walker et al. (2009) to the intrinsic velocity dispersions estimated by these studies, we calculate the dynamical masses within 7.6 kpc. For Laporte et al. (2018) and Martin et al. (2018), the intrinsic velocity dispersions are inferred \( \sigma_{\text{int}} = 11.4^{+5.8}_{-5.0} \) \( \text{km s}^{-1} \) and \( 10.0^{+10.5}_{-3.0} \) \( \text{km s}^{-1} \), and then the dynamical masses are \( M(< 7.6 \text{kpc}) = 9.85^{+12.55}_{-6.24} \times 10^8 M_\odot \) and \( 7.58^{+22.42}_{-3.97} \times 10^8 M_\odot \) respectively. Comparing them with our estimates shown in Table 1, the masses with their velocity dispersions are as large as those from our analysis with Sérsic profile, even though their mass estimates are similar to vD18’s method. This indicates that small kinematic tracers and the estimates of the intrinsic velocity dispersions of NGC1052-DF2 have a large impact on dynamical mass measurements, thereby implying that in combination with these effects and our dynamical analysis, accuracy of mass estimate of the galaxy could be even worse. Furthermore, we compare the results of our Jeans models to the mass estimated using the method by Errani et al. (2018). They adopted the mass estimator, \( M_{\text{est}}(< 1.8 R_{\text{half}}) = 3.5 \times 10^8 M_\odot \) which was introduced by Amorisco & Evans (2012) and Campbell et al. (2017), and showed that this estimator would have the minimum uncertainty compared to other estimators. Using this estimator, we calculate the dynamical masses within the radius of half number of GC tracers (i.e. \( R_{\text{half}} = 3.1 \text{kpc} \)) using the intrinsic velocity dispersions, \( \sigma_{\text{int}} = 3.2^{+5.5}_{-3.2} \text{ km s}^{-1} \) estimated by vD18 and \( 11.4^{+5.8}_{-4.3} \text{ km s}^{-1} \) estimated by Martin et al. (2018). As a result, we obtain \( M_{\text{est}} = 0.46^{+3.98}_{-0.46} \times 10^9 M_\odot \) for vD18 and \( 5.90^{+7.50}_{-4.32} \times 10^8 M_\odot \) for Martin et al. (2018), and find that there is a little difference between our Jeans analysis and any mass estimators. This result can confirm that even if we employ the different mass estimators, the estimated mass depends largely on the inferred velocity dispersions.

Finally, we estimate dynamical mass-to-light ratios \( (M/L) \) assuming the stellar mass of NGC1052-DF2, \( M_* = 2.2 \times 10^9 M_\odot \), derived by vD18 to be fixed. Table 2 shows 90 per cent confidential upper limits of \( M/L \) when we use the dynamical masses within 7.6 kpc. It is clear that assuming the Sérsic profile for tracers makes \( M/L \) significantly larger than those in the cases assuming the power-law profile. Moreover, our estimated \( M/L \sim 7.1 \) is similar to those of Fornax and Sculptor classical dwarf spheroidal galaxies \( (M/L \sim 4.5 \text{ for Fornax and } M/L \sim 9.7 \text{ for Sculptor, taken from McConnachie 2012}) \). Therefore, if the tracer distribution is assumed to follow the Sérsic profile, NGC1052-DF2 can be considered to be a DM dominant system like the Galactic dwarf galaxies.

4 SUMMARY AND CONCLUSIONS

NGC1052-DF2 was recently discovered as the dark-matter deficient UDG claimed by vD18. However, large uncertainties of its dynamical mass estimate have been pointed out in terms of the paucity of kinematic sample, statistical methods and distance measurements.

In this work, we argue the effects of the difference in mass models on dynamical mass estimate. In particular, we focus on the modeling of a tracer density profile and challenge the single power-law profile assumed in vD18. Their MTE modelling based on Watkins et al. (2010) posits on a power-law profile for a tracer distribution due to require-

\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
\textbf{Models} & \textbf{\( M/L \)} \\
\hline
\textbf{Spherical, power-law, isotropy} & \( \leq 1.28 \) \\
\textbf{Spherical, Sérsic, isotropy} & \( \leq 7.14 \) \\
\textbf{vD18 (Spherical, power-law, isotropy)} & \( \leq 1.55 \) \\
\hline
\end{tabular}
\caption{The values of 90 per cent confidential upper limits of \( M/L \) ratios within 7.6 kpc. The stellar mass of NGC1052-DF2 is fixed to \( 2.2 \times 10^9 M_\odot \).}
\end{table}

\footnote{For the case of isotropic velocity ellipsoid, the best-fit values of dark halo parameters, especially \( r_s, \rho_0, \gamma \), are \( (\log r_s) = 4.14^{+0.58}_{-0.72} \), \( \log (\rho_0) = -3.52^{+1.29}_{-1.79} \), \( \gamma = 0.57^{+0.49}_{-0.34} \) with the Sérsic and \( (\log r_s) = 3.62^{+0.52}_{-1.07} \), \( \log (\rho_0) = -3.60^{+1.01}_{-0.74} \), \( \gamma = 0.55^{+0.41}_{-0.37} \) with the power-law profile, respectively. The values of \( r_s \) and \( \rho_0 \) in the units of pc and \( M_\odot \text{ pc}^{-3} \).}
A possible reason for this result might be explained as that if a tracer density has a cored profile such as Sérsic model, the predicted line-of-sight velocity dispersions can decrease within the cored region and be lower than an observed value. In order to match the predicted value to the observation, the corresponding dark halo parameters such as a scale density and radius tend to become large values that can lead the prediction to a relatively large dynamical mass. By contrast, in the case for the cusped tracer density, the dark halo parameters tend to become small values that prefer a relatively low dynamical mass. Consequently, the dynamical mass derived with the cored tracer density tends to be larger than that in the case of the cusped distribution.

We compare our estimated dynamical masses with those calculated by the other studies that pointed out large uncertainties on dynamical mass estimates of NGC1052-DF2 and inferred higher intrinsic velocity dispersions than those from vD18. Comparing them to our results, their estimated dynamical masses accord roughly with those from our analysis with Sérsic profiles, and thereby confirming that small tracer sample and the intrinsic velocity dispersion estimation of NGC1052-DF2 have a large impact on dynamical mass measurements. Also, we confirm that a dynamical mass estimate depends largely on the inferred intrinsic velocity dispersion, irrespective of mass estimator modellings.

Finally, by comparing between the 90 per cent confidence upper limits of the dynamical masses estimated in this work and vD18, we find two main results. When assuming the power-law cusped density profile of the tracers, the upper limit of the mass estimated in this case is nearly the same as the result from vD18. Thus, our results using the Jeans analysis are consistent with those obtained from the MTE modelling used in vD18. On the other hand, when we adopt the Sérsic cored density profile for the tracers, the value of upper limit is about one order of magnitude greater than that from vD18. Correspondingly, the upper limit of dynamical mass-to-light ratio determined with the Sérsic density profile is significantly higher than that obtained with the power-law distribution. Also, this mass-to-light ratio is compatible with those of luminous dwarf galaxies in the Local Group, thereby suggesting that NGC1052-DF2 can still be a dark-matter dominated system, and this galaxy may not be deficient in dark matter.

As our conclusion, dynamical mass estimate of a galaxy with paucity of data is dependent largely on the choice of dynamical models, especially tracer distributions, and thus the estimated mass of NGC1052-DF2 is still considered to be highly uncertain. Therefore, in an attempt to determine robustly dynamical masses of NGC1052-DF2, it is required for dynamical analysis to use kinematic data of stars in the galaxy, and it needs spectroscopic observations; however it is challenging because of the faintness of the galaxy at the moment.

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REFERENCES

Amorisco, N. C., & Evans, N. W. 2012, MNRAS, 419, 184
Amorisco, N. C., & Loeb, A. 2016, MNRAS, 459, L51
Binney, J., & Tremaine, S. 2008, Galactic Dynamics (2nd ed.; Princeton, NJ: Princeton Univ. Press)
Bürkert, A. 1995, ApJ, 447, L23
Campbell, D. J. R., Frenk, C. S., Jenkins, A., et al. 2017, MNRAS, 469, 2335
Di Cintio, A., Brook, C. B., Dutton, A. A., et al. 2017, MNRAS, 466, L1
Errani, R., Peñarrubia, J., & Walker, M. G. 2018, arXiv:1805.00484
Evans, N. W., An, J., & Walker, M. G. 2009, MNRAS, 393, L50
Hastings, W. K. 1970, Biometrika, 57, 97
Hayashi, K., & Chiba, M. 2012, ApJ, 755, 145
Hernquist, L. 1990, ApJ, 356, 359
Irwin, M., & Hatzidimitriou, D. 1995, MNRAS, 277, 1354
Koda, J., Yagi, M., Yamanoi, H., & Komiyama, Y. 2015, ApJ, 807, L2
Laporte, C. F. P., Agnello, A., & Navarro, J. F. 2018, arXiv:1804.04139
Lima Neto, G. B., Gerbal, D., & Márquez, I. 1999, MNRAS, 309, 481
Martin, N. F., Collins, M. L. M., Longeard, N., & Tollerud, E. 2018, ApJ, 859, L5
McConnachie, A. W. 2012, AJ, 144, 4
Metropolis, A. W., Rosenbluth, M. N., Teller, A. H., & Teller, E. 1953, J. Chem. Phys., 21, 1087
Navarro, J. F., Frenk, C. S., & White, S. D. M. 1996, ApJ, 462, 563
Navarro, J. F., Frenk, C. S., & White, S. D. M. 1997, ApJ, 490, 493
Ogiya, G. 2018, arXiv:1804.06421
Sérsic, J. L. 1968, Cordoba, Argentina: Observatorio Astronomico, 1968,
Strigari, L. E., Frenk, C. S. & White, S. D. M. 2010, MNRAS, 408, 2364.
Trujillo, I., Roman, J., Filho, M., & Sánchez Almeida, J. 2017, ApJ, 836, 191
Trujillo, I., Beasley, M. A., Borlaff, A., et al. 2018, arXiv:1806.10141
van der Burg, R. F. J., Hoekstra, H., Muzzin, A., et al. 2017, A&A, 607, A79
van Dokkum, P. G., Abraham, R., Merritt, A., et al. 2015, ApJ, 798, L45
van Dokkum, P. G., Romanowsky, A. J., Abraham, R., et al. 2015, ApJ, 804, L26
van Dokkum, P., Abraham, R., Brodie, J., et al. 2016, ApJ, 828, L6.
van Dokkum, P., Danieli, S., Cohen, Y., et al. 2018, Nature, 555, 629 (vD18)
van Dokkum, P., Cohen, Y., Cohen, Y., et al. 2018, ApJ, 856, L30.
Walker, M. G., Mateo, M., Olszewski, E. W., et al. 2009, ApJ, 704, 1274
Watkins, L. L., Evans, N. W. & An, J. H. 2010, MNRAS, 406, 264.
Yagi, M., Koda, J., Komiyama, Y., & Yamanoi, H. 2016, ApJS, 225, 11
Zhao, H. 1996, MNRAS, 278, 488

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