Asymmetric Neutrino Production in Strongly Magnetized Proto-Neutron Stars

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Abstract

We calculate the neutrino production cross-section in proto-neutron star matter in the presence of a strong magnetic field. We assume isentropic conditions and introduce a new equation of state parameter-set in the relativistic mean-field approach that can reproduce neutron stars with $M > 1.96 M_\odot$ as required by observations. We find that the production process increases the flux of emitted neutrinos along the direction parallel to the magnetic field and decreases the flux in the opposite direction. This means that the neutrino flux asymmetry due to the neutrino absorption and scattering processes in a magnetic field becomes larger by the inclusion of the neutrino production process.

Keywords: neutrino production, strong magnetic field, relativistic mean field approach

The magnetic field in neutron stars plays an important role in the interpretation of many observed phenomena. Indeed, strongly magnetized neutron stars (dubbed magnetars\textsuperscript{[2, 1, 3]}) hold the key to understanding the asymmetry in supernova (SN) remnants, pulsar kicks\textsuperscript{[4]}, and the still unresolved mechanism of non-spherical SN explosions. Such strong magnetic fields are also closely related to the unknown origin of the kick velocity\textsuperscript{[5]} that proto-neutron stars (PNSs) receive at birth. Although several post-collapse instabilities have been studied as a possible source to trigger a non-spherical explosion leading eventually to a pulsar kick, there remain uncertainties in the global initial asymmetric perturbations and the numerical simulations\textsuperscript{[6, 7]}.

In strongly magnetized PNSs, asymmetric neutrino emission emerges from parity violation in the weak interaction\textsuperscript{[8, 9]} and/or an asymmetric distribution of the magnetic field\textsuperscript{[10]}.

Indeed, recent theoretical calculations\textsuperscript{[11, 12]} have suggested that even a $\sim 1\%$ asymmetry in the neutrino emission out of total neutrino luminosity $\sim 10^{53}$ ergs might be enough to explain the observed pulsar kick velocities.

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In our previous work \[13, 14\], we calculated neutrino scattering and absorption cross sections in hot, dense magnetized neutron-star matter including hyperons in a fully relativistic mean field (RMF) theory \[15\]. We evaluated both the associated pulsar kick velocities \[14\] and the spin deceleration \[16\] for PNSs. The magnetic field was shown to enhance the scattering cross-section in a direction parallel to the magnetic field for the final neutrino momentum and to reduce the absorption cross-section along the same direction for the initial neutrino momentum. When the neutrino momentum is anti-parallel to the magnetic field, the opposite effect occurs. For a magnetic field strength of $B = 2 \times 10^{17}$ G and densities in excess of nuclear matter $\rho_B \approx 1 - 5 \rho_0$, the enhancement in the scattering cross-section was calculated to be about 1\% \[14\], while the reduction in the neutrino absorption was 2 - 4\%. This enhancement and reduction were conjectured to increases the neutrino momentum flux emitted along the north magnetic pole, while decreasing the flux along the south pole when the magnetic field has a poloidal distribution. By exploiting a one-dimensional Boltzmann equation in the attenuation approximation and including only neutrino absorption, we estimated that the pulsar kick velocity is about 520 km/s for a star with baryon mass $M_{NS} = 1.68 M_\odot$, $B = 2 \times 10^{17}$ G, $T = 20$ MeV, and $E_T \approx 3 \times 10^{53}$ erg.

In those calculations, however, we did not consider the neutrino production process through the direct URCA (DU) and modified URCA (MU) processes. A strong magnetic field may lead to an angular-dependence of the neutrino production in the URCA process because of the spin polarization of electrons and positrons in matter \[17, 18\]. It has also been reported \[19, 20\] that the Landau levels due to a magnetic field can cause an asymmetry in the neutrino emission which causes a pulsar kick velocity. Furthermore, an angular dependence of the neutrino production caused by a magnetic field has even been reported \[22, 21\] to occur in a pion condensation phase or in a quark-matter color-super conducting phase \[23\], etc.

Therefore, the neutrino production process in the presence of a magnetic field may also lead to asymmetric neutrino emission from PNSs. In this report, we take this production process into account in our model by calculating the cross-sections using the PM1L1 parameter-set \[24\] with an isothermal neutron-star model. However, including Λ particles in this parameter-set cannot reproduce the observed neutron star mass of 1.97 $M_\odot$ for PSR J1614-2230 \[25\]. In addition, an isothermal model gives a temperature that is too high in the low density region. In this work, therefore, we first improve the RMF parameter-set to allow a more massive neutron star. We then study the neutrino absorption and production through the DU process using a relativistic mean-field (RMF) approach in an isentropic neutron star.

We start from the RMF Lagrangian comprised of nucleons, Λ fields, sigma and omega meson fields, and the iso-scalar and Lorentz vector interaction between nucleons. We parameterize the nucleon mean-fields to reproduce the consensus nuclear-matter properties, i.e. a binding energy of 16 MeV, a nucleon effective mass of $M^*/M = 0.65$, and an incompressibility coefficient of $K = 250$ MeV in symmetric nuclear matter at a saturation density of $\rho_0 = 0.17$fm$^{-3}$. In the analysis of heavy-ion experiments \[21\] an EOS with $M^*/M = 0.65$ and $K = 200 - 400$MeV simultaneously reproduces the results of the transverse-flow and sub-threshold $K^+$-production experiments. In the analysis of such data one needs to consider the momentum-dependence of the RMF approach. However, such momentum dependence only affects observables at an energy above a few hundred MeV per nucleon and does not significantly affect the properties of infinite nuclear matter at the temperatures experienced by proto-neutron stars.

The $\sigma - \Lambda$ and $\omega - \Lambda$ couplings are taken to be $2/3$ that of the nucleon, i.e. $g_{\sigma,\omega}^\Lambda = 2/3g_{\sigma,\omega}$ by taking account of the quark degrees of freedom. For the $\Lambda-\Lambda$ interaction we use $h_s = 0.3467g_s$ and $h_v = 0.3467g_v$.

In order to stiffen the EOS when including the lambda (Λ) particles, we introduce an addi-
tional Λ-Λ interaction term with a Lagrangian written as

\[ L_{\Lambda\Lambda} = \frac{h_s^2}{2m_s^2} \{ \bar{\psi}_\Lambda \psi_\Lambda \}^2 + \frac{h_v^2}{2m_v^2} \{ \bar{\psi}_\Lambda \gamma_\mu \psi_\Lambda \} \{ \bar{\psi}_\Lambda \gamma^\mu \psi_\Lambda \} , \]

where \( h_s \) and \( h_v \) are the scalar and vector couplings between the two \( \Lambda \)s, respectively. The scalar and vector meson masses are taken to be \( m_s = 550\text{MeV} \) and \( m_v = 783\text{MeV} \), as in previous calculations \[13, 14, 16\]. Of course the new EOS reproduces massive neutron stars with 2.0 solar masses even if the nuclear medium is composed of \( \Lambda \) particles.

![Figure 1](image)

Figure 1: (Color online) Upper panels (a) and (d): Density dependence of the total energy per baryon \( E_T/A \) of neutron-star matter for entropies \( S/A = 1 \) (a) and 2 (d). Middle panels (b) and (e): Temperature profiles for entropies \( S/A = 1 \) (b) and 2 (e). Solid and dashed lines represent results with and without \( \Lambda \) particles, respectively, in the EOS. Lower panels (c) and (f): Number fractions of protons \( x_p \) and of \( \Lambda \) particles \( x_\Lambda \) for entropies \( S/A = 1 \) (c) and 2 (f). Solid and dashed lines stand for \( x_p \) with and without \( \Lambda \) particles, respectively. Dot-dashed lines show the \( x_\Lambda \) fraction.

Figure 1 shows total energies per baryon \( E_T/A \), temperature profiles, and number fractions for various constituent particles in an isentropic system with entropy per baryon \( S/A = 1 \) or 2. The proton fraction is \( x_p \approx 0.3 \) in all density region. When one includes \( \Lambda \)s in the system, they appear at a density \( \rho_B \gtrsim 2\rho_0 \) and the number fraction \( x_\Lambda \) increases with increasing density. In these isentropical models, the proton fraction slightly decreases even when the \( \Lambda \)s appear, while in an isothermal model \( x_p \) decreases more rapidly.

Using the above EOS we calculated the neutrino absorption and production cross-sections. In this work we assume a uniform dipole magnetic field along the \( z \)-direction, i.e. \( B = B\hat{z} \). Since even for an astronomically strong magnetic field the associated energy scale is still much weaker than the strong interactions, \( \sqrt{eB} \ll \mu_a \), where \( \mu_a \) is the chemical potential of the particle \( a \), we can treat the magnetic field perturbatively. Hence, we ignore the contribution from the convection current and only consider the spin-interaction. The wave function for a baryon \( b \) in a strong magnetic field is obtained by solving the following Dirac equation

\[ [\gamma_\mu p^\mu - M_b^0 - U_0(b)\gamma_0 - \mu_b B\sigma_z] u_b(p, s) = 0, \]
with $M_b^* = M_b - U_b(b)$. The quantities, $U_b(b)$ and $U_0(b)$, are the scalar mean-field and the time-component of the vector mean-field, respectively. These scalar and vector fields are calculated within RMF theory. We set $B = 10^{17}$ G as a representative maximum field strength inside a neutron star. This value corresponds to $\mu N B = 0.32$ MeV which satisfies $|\mu_b B| \ll \varepsilon _\nu \ll E^*_b(p) \equiv \sqrt{p^2 + M^*_b}$. The initial momentum here is taken to be the chemical potential $|k| = \varepsilon _\nu$.

We calculated the absorption and scattering neutrino cross-sections perturbatively, and separated the cross section into the two parts: $\sigma_{S,A} = \sigma_{S,A}^0 + \Delta \sigma_{S,A}$, where $\sigma_{S,A}^0$ is independent of $B$, and $\Delta \sigma_{S,A}$ is proportional to $B$. The subscripts $S$ and $A$ indicate scattering ($\nu_e \rightarrow \nu_e$) or absorption ($\nu_e \rightarrow e^-$), in obvious notation. Related weak couplings are taken from Ref. [27].

![Figure 2](image)

Figure 2: (Color online) Ratio of the magnetic part of the absorption cross-section $\Delta \sigma_A$ to the cross-section without a magnetic-field $\sigma_A^0$. Lines are drawn for matter without $\Lambda$s (a) and with $\Lambda$s (b) at $T = 20$ MeV. Solid, dot-dashed and dashed lines represent the results at $\rho_B = \rho_0$, $3\rho_0$ and $5\rho_0$, respectively. Neutrino incident energies are taken to be equal to the neutrino chemical potentials.

In Fig. [2] we show the magnetic part of the absorption cross-section (a and b) as a function of the initial neutrino angle for entropy $S/A = 1$ without (a) and with (b) $\Lambda$ particles in the EOS, and for $S/A = 2$ without (c) and with (d) $\Lambda$ particles.

At $\rho_B = \rho_0$, the magnetic field suppresses the absorption cross-sections in a direction parallel to the magnetic field $\mathbf{B}$ by about 8.3% for an entropy of $S/A = 1$ and by about 4% for $S/A = 2$. Hence, the magnetic field increases the emitted neutrino flux in the direction of the north magnetic pole and decreases the flux along the south magnetic pole. The suppression of $\sigma_A$ for $S/A = 1$ at $\rho_B = \rho_0$ turns out to be much larger than in an isothermal model with $T = 20$ MeV, because the temperature at nuclear matter density in the isoentropic model is only about $T = 7$ MeV. At lower temperature the magnetic contribution becomes larger. However, at higher densities and temperatures the suppression is comparable in the two models.

As discussed in Ref. [14], the normal parts of the cross-sections, $\sigma_0$, decrease as the temperature and the density become lower. In contrast, the magnetic parts, $\Delta \sigma$, increase as the
temperature becomes lower. Also, as the density decreases the magnetic part decreases more slowly than the normal part. This is because $\Delta \sigma$ is approximately proportional to the fractional area of the distorted Fermi surface caused by the magnetic field. Because of these two effects, the relative strength $\Delta \sigma_A/\sigma_A^0$ becomes significantly larger when the density and entropy are small.

Now considering the neutrino production process, we define the integrated cross-section for neutrino production as follows,

$$\sigma_{pr}(\nu_e) = \int \frac{d^3 k_i}{(2\pi)^3} n_e(e_i(k_i)) \frac{d^3}{dk_i^3} \sigma_{pr}(\nu_e, k_i),$$

(3)

where $e_i(k_i)$ is the single particle energy of electrons with momentum $k_i$. The cross-section and the electron momentum distribution function in the presence of a magnetic field are separated in a perturbative way into the two parts

$$\frac{d^3 \sigma_{pr}}{dk_i^3} = \frac{d^3 \sigma_{pr}^0}{dk_i^3} + \frac{d^3 \Delta \sigma_{pr}}{dk_i^3},$$

(4)

$$n_e(e_i(k_i)) = n_e(|k_i|) + \Delta n_e(k_i).$$

(5)

The first terms are independent of the $B$ field, and the second terms are proportional to $B$. Then, the neutrino phase-space distribution for the DU process in the presence of a magnetic field also separates into the two parts

$$\sigma_{pr}(\nu_e) \approx \sigma_{pr}^0(\nu_e) + \Delta \sigma_{pr}(\nu_e)$$

$$\approx \int \frac{d^3 k_i}{(2\pi)^3} n_e^0(k_i) \frac{d^3 \sigma_{pr}^0}{dk_i^3} + \int \frac{d^3 k}{(2\pi)^3} \left[ n_e^0(k) \frac{d^3 \Delta \sigma_{pr}}{dk_i^3} + \Delta n_e(k) \frac{d^3 \Delta \sigma_{pr}^0}{dk_i^3} \right].$$

(6)

Detailed expressions for $e_i$ and $\Delta n_e$ are given in Ref. [16]. As shown in Ref. [16], we can obtain the cross-section for $e^- + B_i \rightarrow B_f + \nu_e$ by exchanging the lepton chemical potentials for the neutrinos and electrons in the cross-section for $\nu_e + B_i \rightarrow B_f + e^-$. Fig. 3 shows the magnetic part of the integrated production cross-section $\Delta \sigma_{pr}$ normalized to $\sigma_{pr}^0$ as a function of $\theta_f$. Calculations were made for densities in the range $\rho_0 \leq \rho_B \leq 5\rho_0$ as indicated. Final neutrino energies were taken to be equal to the neutrino chemical potential, $|k_{\nu_e}| = \varepsilon_{\nu_e}$.

At $\rho_B = \rho_0$ with entropy $S/A = 1$, the magnetic part is enhanced by about 10% for $\theta_\nu = 0^\circ$ and suppressed by about 6% for $\theta_\nu = 180^\circ$. This is true for both systems with and without $\Lambda$ particles. Hence, the magnetic-field gives rise to an about 8% asymmetry in the production process. As the density increases, the magnetic contribution becomes smaller, particularly in the system with $\Lambda$ particles. For $S/A = 2$, the asymmetry is about 6% at $\theta_\nu = 0^\circ$ and 4% for $\theta_\nu = 180^\circ$ at $\rho_N = \rho_0$, so that the asymmetry is slightly smaller than for $S/A = 1$. At higher density the asymmetry also becomes smaller, particularly when $S/A = 2$.

In any condition the neutrino production becomes larger in a direction parallel to the magnetic field $B$, and smaller in the opposite direction. The net result is that the magnetic field increases the momentum flux of neutrinos emitted along the north magnetic polar direction while decreasing the flux in the south polar direction. This magnetic contribution effect on the production process turns out to be of the same sign and magnitude as the absorption process. Hence, the total asymmetry induced by the magnetic field from both processes should be about twice that from absorption alone [14].
Figure 3: Normalized magnetic part of the total production cross section, $\Delta \sigma_{pr}/\sigma_{pr}^0$, as a function of final neutrino angle $\theta_f$ in a system without hyperons for the entropy $S/A = 1$ (a) and $S/A = 2$ (b), and with hyperons at $S/A = 1$ (c) and $S/A = 2$ (d). In each panel the solid, dash-dotted and dashed lines represent the results for $\rho_B = \rho_0$, $3\rho_0$ and $5\rho_0$, respectively. A magnetic field strength of $B = 10^{17}$G was used in this calculation.

In summary, we have calculated the magnetic contribution to the neutrino production through the direct URCA process and the absorption during transport. We have utilized an isoentropic model for the proto-neutron star and employed RMF theory (with and without $\Lambda$ particles) for the EOS and to compute the production cross-section. The asymmetry in the absorption becomes larger at $\rho_B = \rho_0$ than that in our previous calculation based upon an isothermal model. Furthermore, the asymmetry in the production cross-section is found to be also enhanced by the magnetic-field with the same magnitude and sign as in the absorption process.

Since the scattering process also enhances the neutrino asymmetry [14], we can conclude that the magnetic-field effect causes asymmetric neutrino emission from a PNS through the combination of the production process as well as the absorption and the scattering. Therefore, the neutrino emission asymmetry from the neutrino sphere should be significantly larger than previously estimated. In future work we will consider all magnetic effects from the above three processes in a calculation of pulsar kick velocities [14] and spin-down [16].

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