Observation of Supercavity Modes in Subwavelength Dielectric Resonators

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Enhancement of the electromagnetic field is one of the key problems of optics, radiophysics, photonics, and related branches of science. Light trapping structures with high quality factors ($Q$ factor) are vital for photonics industry and technologies. Optical resonators and microcavities are based on materials or systems that forbid outgoing waves by different means. The common principle of operation for traditional resonators and microcavities is due to reflection from the resonator's boundaries under the condition of constructive interference of the trapped waves with themselves. For such structures, high $Q$ factors usually demand a large resonator size. The $Q$ factor can be increased by engineering the environment as for cavities in photonic bandgap structures or by exploiting the total internal reflection at glancing angles of incidence in whispering-gallery-mode resonators.

For compact geometries, the lowest electric and magnetic Mie resonances dominate the optical response explaining why the value of the $Q$ factor drops rapidly with a decrease of size. The creation of subwavelength resonators with a high $Q$ factor is still a challenge.

Recently, high-refractive-index dielectric nanoparticles were suggested as a novel platform for localization of light in subwavelength photonic structures. Very recently, a new prospective mechanism was proposed for achieving high values of $Q$ factor in high-index resonators of the subwavelength scale. The mechanism allows to achieve high-quality (high-$Q$) modes, or supercavity modes, in a single dielectric resonator via destructive interference of a pair of leaky modes with similar far-field profiles. The interference is enabled by continuous tuning of the resonator's aspect ratio, which induces strong coupling of the leaky modes when their frequencies come close to degeneracy. The physics of supercavity modes was associated with bound states in the continuum (BICs)—exotic nonradiative states proposed in quantum mechanics almost a century ago and rediscovered in photonics only in 2008. Until now, optical BICs have been predicted and observed essentially in dielectric structures, such as photonic crystals, waveguide arrays, and metasurfaces, and used for various applications including lasing, biosensing, and nonlinear frequency conversion.

The interaction between two resonances with finite lifetimes results in a mixing of the resonant states and the appearance of an avoided resonance crossing of their energies in the parameter space—the hallmark of the strong coupling regime. For closed systems with interaction, the losses of the dressed states are always between the losses of the bare states—eigenstates without interaction. For open systems, the situation is essentially different because the modes with a common radiative
channel can couple externally via constructive and destructive interference. The lifetime of each interacting mode changes dramatically in this case. Thus, the constructive interference gives rise to super-radiant modes with enhanced radiation and the destructive interference can result in the appearance of the mode with completely damped radiative losses—BIC—that was shown by Friedrich and Wintgen for quantum mechanical systems.[24] For a true BIC, the radiative losses through all the channels are completely damped making the mode dark and optically inactive. In optics, BICs can exist only in unbounded structures or in structures with epsilon-near-zero materials,[25] while for finite samples, the complete loss cancellation is impossible.[35] Nevertheless, we can substantially increase the $Q$ factor of the resonator by the cancellation of radiation through the main radiative channels. This regime can be associated with supercavity modes also known as quasi-BICs. Recently, it was shown that the change of the resonator’s size can play a role of the interaction between the modes and the radiation losses can be suppressed dramatically via continuous deformation indicating the formation of a high-$Q$ supercavity mode.[9,10,26] (Figure 1a).

To observe a supercavity mode we manufacture a ceramic cylindrical resonator based on the lanthanum aluminate–calcium titanate (LaAlO$_3$–CaTiO$_3$) microwave ceramics with the permittivity $\varepsilon = 44.8$ and loss tangent of the ceramics about $10^{-4}$.[29] The values of the dielectric material properties were originally provided by the ceramic manufacturer and later specified through measurements of the resonance frequencies of the dielectric cylindrical resonator excited by the loop antenna. The resonator consists of a stack of the disks of different heights and the same radii $r = 15.7$ mm (Figure 1b). This design makes it possible to gradually change the height of the resonator and consequently its aspect ratio. The thickness of the thinnest disk defining the height change step is 0.2 mm and the thickest disk is 15 mm. The resonator was placed on a holder made of dielectric foam with the permittivity approximately $\varepsilon = 1.1$ and low material losses at GHz frequencies. For fabrication details see Section S2 in the Supporting Information. The cylindrical resonator hosts radially oscillating modes and axially oscillating modes, in which electromagnetic fields can be characterized with an azimuthal degree of freedom (orbital angular momentum). We focus only on the azimuthally symmetric modes with zero angular momentum because they possess $Q$ factors higher than for less symmetric modes. To excite the azimuthal symmetric modes selectively we use a loop antenna placed beneath the resonator concentrically with its axis. To study the scattering properties, we use the vector network analyzer (VNA) and measure the complex reflectivity via the $S_{11}$ coefficient of the scattering matrix (for details see Section S3 in the Supporting Information). The measured dependence of $1 - |S_{11}|$ on the frequency $kr$ and cylinder aspect ratio $r/L$ demonstrates the avoided resonance crossing behavior with a characteristic linewidth narrowing for the high-frequency mode (Figure 1c). The numerical simulations with identical excitation conditions show perfect coincidence with the experiment (Figure 1d) (for details see Section S1 in the Supporting Information).
With the same setup, we perform measurements of the reflectivity $S_{11}$ for a wider range of cylinder aspect ratio and observe four supercavity modes (Figure 2a). Using fitting procedures, we extract the experimental unloaded $Q$ factors of the modes and study their dependence on the cylinder aspect ratio in the vicinity of each avoided resonance crossing (Figure 2c). The supercavity mode labeled as B possesses the highest $Q$ factor of 12,500, which matches perfectly with the numerical simulations (Figure 2b,d). For details on $Q$ factor extraction see Section S6 in the Supporting Information. For the numerical simulations in Figure 2d, we apply an expansion of Maxwell’s equations into the basis of resonant states (quasi-normal modes).[30,31] This value of the $Q$ factor is limited mainly by the material loss in the ceramics (loss tangent about $10^{-4}$). Explicit finite element simulations show that the pure radiative $Q$ factor reaches a value of around $1.8 \times 10^3$. To get this value with direct measurements, we need to use a resonator made of a material with loss tangent less than $6 \times 10^{-6}$, which is not available for existing ceramic materials. The calculated near-field electric profiles for the supercavity modes show highly symmetric distributions with the increase of axial and radial order for the high-frequency modes (Figure 2e).

In order to gain deeper insight into the physics of supercavity modes in individual high-index resonators, we illustrate cancellation of supercavity mode radiative losses through the dominant scattering channel in terms of multipoles. As was shown theoretically, in the supercavity regime, the radiation through the dominant multipole channel becomes negligible.[13] We perform the experimental study to demonstrate cancellation of supercavity mode radiation losses through the dominant channel in the far-field. In the experiment, we excited the cylindrical resonator with the loop antenna placed concentrically to the axis of the cylinder (for details see Section S4 in the Supporting Information). The resonator together with the loop antenna rotated around the transverse axis as shown in Figure 3a. The scattered field in the orbital plane was

Figure 2. Near-field measurements of the $Q$ factors for supercavity modes. a,b) Measured and calculated map of the coefficient $1 - |S_{11}|$ versus frequency $kr$ and aspect ratio $r/L$. The supercavity modes labeled as A–D correspond to aspect ratios $r/L = 0.71, 0.55, 0.47, and 0.42$. c) Measured and d) calculated dependence of $Q$ factor on aspect ratio $r/L$ for supercavity modes in the vicinity of the avoided resonance crossings. e) Calculated near-field patterns of the azimuthal component of the electric field for the supercavity modes (points A–D).
measured by the distantly positioned horn antenna. We observe that the appearance of a supercavity mode "A" (Figure 2b) is accompanied by a drastic change in the far-field radiation pattern from the magnetic dipolar to the magnetic octupolar one (Figure 3b,c) that agrees with earlier theoretical predictions.[13]

It is well-known that the scattering spectra of a finite size obstacle can be described by a cascade of Fano resonances.[32] Fano parameters are used widely to approximate the experimental scattering spectra and they are considered as independent fitting parameters. It was predicted that in compact structures, the supercavity modes are fundamentally linked to optical Fano resonances.[13] In particular, it was shown theoretically that the Fano asymmetry parameter diverges in the vicinity of the supercavity regime, when the $Q$ factor as a function of the aspect ratio reaches the maximum. This prediction has a very powerful practical meaning—a Fano resonance in the scattering spectrum as function an external parameter reaches the maximal $Q$ factor when it turns into the symmetric Lorentzian shape. This prediction was not verified experimentally.

To confirm it, we investigate experimentally the scattering properties of the cylindrical resonator in a radio-frequency range. The resonator was positioned between two horn antennas connected to the ports of the vector network analyzer (Figure 4a). The first antenna illuminated the quasi-plane wave in the

Figure 3. Radiation pattern measurements. a) Experimental setup for the far-field radiation pattern measurements. Azimuthally symmetric modes of the resonator are excited by the loop antenna connected to the port "1" of the vector network analyzer (VNA). The resonator and loop antenna can be rotated by 360° around their axis. The scattered field is collected by the horn antenna connected to the port "2" of the VNA. b) Contribution of the magnetic dipole and magnetic octupole to the radiated power of "A" supercavity mode (see Figure 2b,d,e) depending on the aspect ratio $r/L$. c) Normalized measured and calculated far-field radiation patterns for three values of aspect ratio $r/L = 0.66$, $0.7025$, and $0.74$. The amplitude of the scattered field increases drastically in the supercavity-regime (the excitation frequency is 2.768 GHz).

Figure 4. Characterization of the Fano asymmetry parameter in far-field measurements. a) Experimental setup for forward scattering measurements. b,c) Measured and calculated spectra of the normalized total scattering cross-section as a function of the resonator aspect ratio in the region of the avoided resonance crossing "A." d) Extracted and simulated Fano asymmetry parameter depending on $r/L$. e,f) Experimental and numerical dependencies of the total scattering cross-section on the aspect ratio of the cylinder and frequency for TE-polarized incident wave. The cross-section is normalized to the projected cross-section $2rL$. 
direction of the cylinder. The field is polarized orthogonally to the axis of the cylinder. The second horn antenna collected the field scattered in the front direction (for details see Section S5 in the Supporting Information). The result amplitude of the transmission coefficient was used to calculate the total extinction cross-section by means of the optical theorem. The measured and calculated maps of the extinction cross-section by means of the optical theorem.[33] The measured and calculated maps of the extinction cross-section by means of the optical theorem.

| Spectral band | Material          | Re(ε) | λ  | r0 | Q_{mat} | Q_{nd} | Q0 | Q_{mat}^{(d)} | Refs. |
|---------------|-------------------|-------|----|----|--------|--------|----|--------------|-------|
| Visible       | LiNbO3            | 6.5   | 400 nm | 165 nm | >10^4 | 70 | 70 | 5 | [35] |
| Visible       | GaP               | 12.9  | 500 nm | 145 nm | 320   | 1000 | 240 | 12 | [36] |
| Visible       | Halide perovskite | 5.5   | 550 nm | 235 nm | 30 | 55 | 20 | 5 | [28] |
| Near-IR       | Diamond           | 5.8   | 800 nm | 335 nm | >10^2 | 65 | 65 | 5 | [37] |
| Near-IR       | Bulk TMD (W5_{3}) | 16.7  | 800 nm | 210 nm | 50 | 2400 | 50 | 15 | [38] |
| Near-IR       | AlGaAs (Al 20%)   | 11.5  | 850 nm | 260 nm | >10^4 | 660 | 660 | 10 | [39] |
| Near-IR       | c-Si              | 12.3  | 1350 nm | 400 nm | >10^5 | 850 | 850 | 11 | [40] |
| Near-IR       | Ge                | 17.8  | 1550 nm | 380 nm | 7800 | 3700 | 2500 | 17 | [41] |
| Mid-IR        | SiC               | 9.1   | 3 μm  | 1 μm | 185 | 280 | 110 | 8 | [42] |
| Mid-IR        | a-Si:H            | 10.3  | 4 μm  | 1.3 μm | >10^4 | 440 | 440 | 9 | [22] |
| THz           | SiO             | 11.7  | 300 μm | 90 μm | >10^4 | 700 | 700 | 10 | [43] |
| Microwaves    | CaTiO3–LaAlO3    | 44.8  | 10 cm | 1.6 cm | 15 000 | 1.8 × 10^5 | 12 500 | 55 | [29] |
| Microwaves    | BaTiO3           | 72    | 5.5 cm | 0.7 cm | 5300 | 5.7 × 10^6 | 5500 | 105 | [44] |

a)Radius of the cylinder with fixed aspect ratio r/L = 0.55, corresponding to the supercavity mode B (as in Figure 2b,d); b) Q_{mat} = Re(ε)/Im(ε). For LiNbO3, diamond, AlGaAs, and Si, the value shows the lower estimate; c)Total Q factor, Q = Q_{mat} + Q_{nd}; d) Q factor of the fundamental magnetic dipole mode Q_{mat} for the same wavelength and r/L = 0.55; e)References for material parameters; f)Anisotropic, in-plane, and out-of-plane component, respectively; g)High-resistivity silicon; h)Measured data (Figure 2c).
measuring the transmission between two horn antennas placed in line with noise reduction. The background signal was subtracted by means of an anechoic chamber with subsequent time gating in order to perform measurements with the vector network analyzer E8362C Vector Network Analyzer. Measurements were performed in the operational range 1–18 GHz connected to the ports of an Agilent anechoic chamber by the linearly polarized wideband horn antenna dielectric resonator excited by the loop antenna was measured in an anechoic chamber. Practical devices for applications in nanophotonics, THz, and design supercavities, which paves the way to future compact ranges. Our work suggests the universal scalable approach and determined optimal material selection for various spectral ranges from optics to THz and microwaves through a dramatic change in the far-field radiation pattern that supercavity modes can be unambiguously recognized and they are manifested in peculiarities in the scattering cross-section spectra. We have observed in the experiment that supercavity modes can be unambiguously recognized through a dramatic change in the far-field radiation pattern of the electromagnetic fields. We compared geometrical parameters and optical properties of supercavities for a diversified range of materials from optics to THz and microwaves and determined optimal material selection for various spectral ranges. Our work suggests the universal scalable approach to design supercavities, which paves the way to future compact practical devices for applications in nanophotonics, THz, and radiophysics.

**Experimental Section**

**Sample Fabrication:** The dielectric cylindrical resonator was composed of the ceramic disks of different heights and the same diameters. Each disk was manufactured by the sintering of the ceramic powder of the calcium titanate–lanthanum aluminate (LaAlO$_3$–CaTiO$_3$) solid solution in a cylindrical form. Each disk was then polished in order to get the required height (from 0.25 to 15 mm). The dielectric foil holder for the resonator was fabricated by the computer numerical control (CNC) machine drilling of the People foam material.

**Microwave Measurements:** The reflection scattering of the dielectric resonator was obtained by measuring the reflection coefficient of the loop antenna placed below the basis of the cylindrical resonator. The vector network analyzer Rohde & Schwarz ZVB-40 was used to measure the reflection coefficient. The far-field radiation pattern of the cylindrical dielectric resonator excited by the loop antenna was measured in an anechoic chamber by the linearly polarized wideband horn antenna (operational range 1–18 GHz) connected to the ports of an Agilent EB362C Vector Network Analyzer. Measurements were performed in an anechoic chamber with subsequent time gating in order to perform noise reduction. The background signal was subtracted by means of free space measurement. The scattering cross-section was obtained by measuring the transmission between two horn antennas placed in line with the cylindrical resonator. The total scattering cross-section was calculated by means of the optical theorem.

**Supporting Information**

Supporting Information is available from the Wiley Online Library or from the author.

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