The Secular Dynamics of TNOs and Planet Nine Interactions

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Abstract

The existence of Planet Nine has been suggested to explain the pericenter clustering of extreme trans-Neptunian objects (TNOs). However, the underlying dynamics involving Planet Nine, test particles, and Neptune is rich, and it remains unclear which dynamical processes lead to the alignment and how they depend on the properties of Planet Nine. Here we investigate the secular interactions between an eccentric outer perturber and TNOs starting in a near-coplanar configuration. We find that a large number of TNOs could survive outside of mean-motion resonances at 4 Gyr, which differs from previous results obtained in the exact coplanar case with Neptune being treated as a quadrupole potential. In addition, secular dynamics leads to the orbital clustering seen in N-body simulations. We find that a near-coplanar Planet Nine can flip TNO orbital planes, and when this happens, the geometrical longitudes of pericenter of the TNOs librate around 180° during the flip. Orbital precession caused by the inner giant planets can suppress the flips while keeping the longitude of pericenter librating when 30 au \( \lesssim r_p \lesssim 80 \) au and \( \alpha \gtrsim 250 \) au. This results in the alignment of the pericenter of the low-inclination TNOs (\( i \lesssim 40° \)). We find that the anti-aligned population and flipped orbits could be produced by an eccentric (\( e_0 \gtrsim 0.4 \)) outer planet of \( \sim 10M_\oplus \) in a wide \( a_0 \gtrsim 400 \sim 800 \) au orbit. Future surveys of the high-inclination TNOs will help further constrain the properties of possible outer planets.

Key words: celestial mechanics – Kuiper belt: general – planetary systems

1. Introduction

Recent observations of the vast expanse of the outer solar system have revealed around a dozen distant (\( a \gtrsim 150 \) au) trans-Neptunian objects (TNOs) in our solar system with pericenter distances outside the orbit of Neptune (Gladman et al. 2002; Brown et al. 2004; Chen et al. 2013; Trujillo & Sheppard 2014). The orbits of such objects exhibit interesting architectures. For instance, there seems to be a clustering in the orbital orientation of the TNOs (e.g., Trujillo & Sheppard 2014).

Many studies have shown that the alignment of the orbits is not due to selection biases (de la Fuente Marcos & de la Fuente Marcos 2014; Gomes et al. 2015; Brown & Batygin 2016; Sheppard & Trujillo 2016; Brown 2017), although Shankman et al. (2017a) demonstrated that the “Outer Solar System Origins Survey” (OSSOS; Bannister et al. 2016) contains nonintuitive biases for the detection of TNOs that lead to apparent clustering of orbital angles in their data, and the angular elements of the distant TNOs are consistent with uniform distribution (Bannister et al. 2018). Recent observations have suggested additional clustering features of the TNOs (Sheppard & Trujillo 2016; Brown 2017). Ongoing observational TNO surveys will provide a better understanding of the architecture of the outer solar system and the details (if any) of the TNO clustering.

It has been suggested that the clustering of the TNO orbits can be explained by an undetected outer planet, “Planet Nine,” in our own solar system (Trujillo & Sheppard 2014; Batygin & Brown 2016a). The location of the putative Planet Nine has been constrained using dynamical simulations of TNOs orbiting under the gravitational influence of Planet Nine (Brown & Batygin 2016; de la Fuente Marcos & de la Fuente Marcos 2016) and by simulations of the tidal perturbation induced by Planet Nine on the relative distance between the Earth and the Cassini spacecraft (Holman & Payne 2016b) and Pluto and other TNOs (Holman & Payne 2016a). In addition, the formation mechanism for Planet Nine has been investigated, including scenarios for capturing Planet Nine from another star, scattered giant planets originating within the solar system (Bromley & Kenyon 2016; Li & Adams 2016; Mustill et al. 2016; Parker et al. 2017), pebble accretion in a large (250–750 au) ring of solids (Kenyon & Bromley 2016), and circularization of Planet Nine with an extended cold planetesimal disk (Eriksson et al. 2018).

The dynamics involved in the interactions between TNOs and the putative Planet Nine are rich, and the mechanism by which the clustering of TNO orbits arises due to interactions with Planet Nine and the four known giant planets is not well characterized. Batygin & Brown (2016a) suggested that the origin of the clustering is produced by mean-motion resonances (MMRs). This is supported by Malhotra et al. (2016), who noted that some of the TNOs are likely to be in MMR with an exterior planet. Indeed, dynamical simulations of the detected TNOs with hypothetical orbits of Planet Nine show that the TNOs can move between different MMRs with Planet Nine (Becker et al. 2017; Millholland & Laughlin 2017; Hadden et al. 2018).

On the other hand, secular interactions can also produce similar orbital alignment in the longitude of pericenter of the TNOs. This has been investigated in detail in the coplanar configuration, where the TNOs and Planet Nine all lie in the same plane. In particular, Beust (2016) investigated the secular interactions of the test particles with Planet Nine in the coplanar case and found that the alignment can be produced by secular effects. Recently, Batygin & Morbidelli (2017) found from a detailed study of the coplanar case that the effect of secular dynamics embedded in MMR is to regulate the clustering of the TNOs. In addition, Hadden et al. (2018)
noted that TNOs with pericenters initially aligned with that of Planet Nine follow secular trajectories but are more likely to be ejected due to overlaps of MMRs if their pericenter distances become small. Thus, the clustering of the TNO orbits is likely due to a combined effect of MMRs and secular interactions.

It is likely that Planet Nine is not in an exact coplanar configuration with TNOs: with small inclinations, many TNOs could survive outside MMRs with Planet Nine. Thus, in this article, we focus on the secular interactions between TNOs and Planet Nine in a near-coplanar configuration. We use the orbital phase-averaged Hamiltonian and generalize the dynamical analysis to higher dimensions, which allows the inclination of the TNOs to vary, and we consider the clustering of the orbits due to secular effects. Extending beyond the coplanar configurations, Saillenfent et al. (2017) recently identified the secular resonances of the TNO orbits using surface of sections. Here we focus on the near-coplanar configuration and study the how the secular interactions lead to the clustering of the TNOs.

The remainder of this article is organized as follows. In Section 2, we analyze the secular effects following the orbital phase-averaged Hamiltonian, and in Section 3, we perform full N-body simulations and compare with the secular results from Section 2. In Section 4, we discuss the inclination distribution of the TNOs and the origin of the aligned orbits. Finally, we present our conclusions in Section 5.

2. Secular Interactions between TNOs and Planet Nine

For a large range of orbital parameters, TNO orbits can cross that of Planet Nine. Although orbit-crossing (outside of commensurability) will eventually lead to close encounters between Planet Nine and TNOs, which may result in the ejection of TNOs, Gronchi & Milani (1999) and Gronchi (2002) have shown that the averaging principle is a powerful tool to study the secular evolution for a long time span, with applications to asteroids whose orbits are planet-crossing. The secular interactions also play a key role in the dynamics of TNOs. In particular, it has been shown that in a coplanar configuration, secular effects guide the overall evolution of the TNOs and cause the libration of the longitude of pericenter about an anti-aligned configuration with Planet Nine (Δω ~ 180°; Beust 2016; Batygin & Morbidelli 2017). In particular, Hadden et al. (2018) showed that any clustering that initially exists near Δω ~ 0° will be removed due to instabilities caused by overlaps of MMR, and that secular interactions shape the clustering near Δω ~ 180°.

It is likely that TNOs are misaligned in inclination with that of Planet Nine, and N-body simulations have demonstrated interesting clustering of the TNOs that start near coplanar with that of Planet Nine (Batygin & Morbidelli 2017; Khain et al. 2018). It is not clear how the secular dynamics contribute to the overall clustering of the TNOs in the misaligned configurations. Here we focus on the pure secular analysis and extend the previous secular study to higher dimensions, allowing the inclination of the TNOs to vary, and characterize the secular effects in the observed clustering of the TNO orbital orientations.

2.1. Hamiltonian Framework

We consider an outer planet (Planet Nine) and a TNO orbiting the Sun, illustrating the configuration of the system in Figure 1. The mass of the TNO is much smaller than that of Planet Nine and our Sun, and thus the TNO can be treated as a test particle.

![Figure 1](image_url). Configuration of the system. The brown circle represents a TNO, and the blue circle represents Planet Nine. The black arrows represent the position vectors of the TNO and Planet Nine. On the other hand, the blue and brown arrows represent the angular momentum direction of the orbits of Planet Nine and the TNO separately.

We allow the orbit of Planet Nine to be eccentric and misaligned with the inner orbit, different from the circular restricted case.

In the nonhierarchical configuration, when a/a_9 ≥ 0.1, the usual expansion in the semimajor axis ratio is not a good approximation. However, in the case when the outer perturber is much less massive than the center body, the perturbation from the outer companion is not strong enough to destabilize the system; i.e., the semimajor axes of the orbits are almost constant. In this case, one can consider the long-term secular evolution of the system by averaging out the fast varying orbital phase of the inner and outer orbits.

Specifically, the secular (averaged) Hamiltonian of the interaction energy can be expressed as

$$H_{sec,\phi} = \frac{Gm_9}{4\pi^2} \int_0^{2\pi} \int_0^{2\pi} \left( \frac{1}{|r - r_9|} - \frac{r - r_9}{r_9^2} \right) dl \, dl_9, \tag{1}$$

where r and r_9 are the distance to the test particle and the perturber from the central object, m_9 is the mass of the perturber as illustrated in Figure 1, and l and l_9 are the mean anomaly of the test particle and the perturber, respectively. Here $r - r_9$ can be expressed as

$$|r - r_9| = \sqrt{r^2 + r_9^2 - 2rr_9 \cos \phi}, \tag{2}$$

and $r_9$, $\phi$, and $\phi$ can be expressed by orbital elements.

The evolution of the TNO’s orbit can be well described using the averaged Hamiltonian, which converges in most configurations even for crossing orbits, as discussed in Gronchi & Milani (1999) and Gronchi (2002). For illustration, we compare the secular evolution with the N-body simulations in Figure 20 in Appendix A.

The TNOs in the outer solar system undergo perturbations from Planet Nine, as well as the known inner giant planets. The TNOs are quite far away from the inner giant planets; thus, the secular effects of the inner giant planets on the TNOs can be well approximated based on the Hamiltonian to the second order in the ratio of the TNO to giant planet semimajor axes. Combining the secular Hamiltonian for the interaction energy of the TNO with a perturber (Equation (1)), the secular Hamiltonian can be expressed as (e.g., Kaula 1964;


\begin{equation}
H_{\text{sec},1} = H_{\text{sec},0} - \frac{1}{8} GM \left( \frac{3}{2} \sum_{i=1}^{4} m_{pi} a_{pi}^2 \right),
\end{equation}

where $M$ is the mass of the Sun; $a$, $e$, and $i$ are the semimajor axis, eccentricity, and inclination of the TNO; and $m_{pi}$ and $a_{pi}$ are the masses and semimajor axes of the inner giant planets. Here we assume that the inner planets are coplanar with the orbit of Planet Nine.

The orbit of Planet Nine is also perturbed by the inner giant planets, which causes precession. The precession rate of Planet Nine’s orbit in the low-inclination regime can be expressed as

\begin{equation}
\frac{d\omega_0}{dt} = \frac{3}{2} \sqrt{GM} \left( \sum_{i=1}^{4} m_{pi} a_{pi}^2 \right),
\end{equation}

and

\begin{equation}
\frac{d\Omega_0}{dt} = -\frac{3}{4} \sqrt{GM} \left( \sum_{i=1}^{4} m_{pi} a_{pi}^2 \right),
\end{equation}

where $m_0$, $a_0$, $e_0$, $\omega_0$, and $\Omega_0$ are the mass, semimajor axis, eccentricity, argument of pericenter, and longitude of the ascending node of Planet Nine separately.

The Hamiltonian of the TNOs (Equation (3)) is implicitly expressed in terms of the canonical variables of the TNOs,

\begin{equation}
q = (M, \omega, \Omega),
\end{equation}

\begin{equation}
p = (\sqrt{GM}a, \sqrt{GM}(1-e^2), \sqrt{GM}(1-e^2)\cos(i)),
\end{equation}

where $M$ is the mean anomaly of the TNO, $J = \sqrt{GMa(1-e^2)}$, and $L = \sqrt{GMa(1-e^2)}\cos(i)$ are the angular momentum and z-component of angular momentum of the TNO, and $L = \sqrt{GMa}$ is a constant in the secular regime.

To obtain the alignment of the orbit of the TNO relative to that of Planet Nine, we transfer the coordinates of the TNOs to the difference in the argument of pericenter and the longitude of the ascending node between the TNO and Planet Nine. Using the type-III generating function, $G_3 = -2(\Delta\omega + \omega_0) J - 2(\Delta\Omega + \Omega_0) J_z$ we transform to new canonical angles $\Delta\omega = \omega - \omega_0$ and $\Delta\Omega = \Omega - \Omega_0$. Then, the new Hamiltonian becomes

\begin{equation}
H_{\text{sec}} = H_{\text{sec},1} - \omega_0 J - \Omega_0 J_z.
\end{equation}

The secular result following Equation (7) is a good approximation until close encounters occur between the TNO and Planet Nine or the inner giant planets. For illustration, Figure 2 compares the secular and N-body results, which show the evolution of a TNO starting with $a = 368.75$ au, $e = 0.867$, $\Delta\omega = 180^\circ$, and $i = 10^\circ$ relative to the ecliptic and with Planet Nine and the inner giant planets all coplanar in the ecliptic plane. The semimajor axis and eccentricity of Planet Nine are $a = 500$ au and $e = 0.6$. The blue crosses are the secular results, and the red dashed lines are the N-body results. To isolate the effects of close encounters with Planet Nine, we substitute the inner giant planets (Jupiter, Saturn, Uranus, and Neptune) with an equivalent $J_2$ term in the N-body simulation. We use the package Mercury for the N-body simulation, with the “hybrid” Wisdom–Holman/Bulirsch–Stoer integrator (Press et al. 1992; Wisdom & Holman 1992; Chambers 1999) and a time step of $dt = 3000$ days, which is roughly 5% of Neptune’s orbital period.

Figure 2 shows that the secular result is a good approximation up to $\sim 50$ Myr, at which point the TNO has a close encounter with Planet Nine, causing changes in the semimajor axis of the TNO. However, the secular effects are not immediately suppressed after the close encounter. In particular, the libration of $\Delta\omega$ around $180^\circ$ shown in the secular results can still be observed in the N-body results after the close encounter until the end of simulation. Neither $\Delta\omega$ nor $\Delta\Omega$ librates for both the secular and N-body results. The secular results agree qualitatively with the N-body results after the change in the semimajor axis. This is due to the weak dependence of the secular interactions on the semimajor axis for TNOs with large semimajor axes ($\gtrsim 300$ au), as shown in Figure 4 and Hadden et al. (2018). Note that the precession of the orbits increases the chance of close encounters between TNO and Planet Nine, which causes the secular approximation to deviate from the N-body results. Thus, the secular integration is a better approximation for the three-body interactions without $J_2$ precession, as illustrated in Appendix A. For instance, the $J_2$ precession timescale due to the inner giants in $\Omega$ for Planet Nine is 13 Gyr, and for the TNO, it is 276 Myr.

2.2. Secular Clustering of Anti-aligned TNOs

2.2.1. Alignment in Pericenter Orientation

It has previously been found (e.g., Hadden et al. 2018 and our Section 3) that particles starting with anti-aligned pericenters are more likely to survive and play an important role in sculpting the overall orbital architecture of the TNOs. Thus, we start the analysis by focusing on the secular evolution of an (initially) anti-aligned population. We consider a set of 500 test particles initialized with $150$ au $< a < 550$ au, $30$ au $< r_p < 50$ au, $i = 10^\circ$, $\omega = 180^\circ$, and $\omega$ uniformly distributed between $0^\circ$ and $360^\circ$. As in the illustrative example in Figure 2, Planet Nine lies in the ecliptic plane, together with the inner four giant planets, and we substitute the inner giant planets with the equivalent $J_2$ potential. The semimajor axis and eccentricity of Planet Nine are set to be $a_9 = 500$ au and
$e_0 = 0.6$, and the longitude of pericenter of Planet Nine is $\varpi_0 = 0^\circ$. Thus, the pericenters of the test particles are initially anti-aligned with that of Planet Nine. We follow the particles' secular evolution by integrating equations of motion generated from the secular Hamiltonian, Equation (7). Details of our numerical method are given in Appendix A.

Figure 3 shows the secular evolution of the TNOs. Dividing the particles into semimajor axis regions, we notice that $\Delta \varpi$ circulates when TNO semimajor axes are low: $a \lesssim 200$ au (upper left panel), where we illustrate the trajectories of a few representative particles. This is the region where the $J_2$ precession dominates. For instance, the precession caused by the $J_2$ term is roughly $\varpi = 4200^\circ$/Gyr, calculated following Equation (4). On the other hand, the change rate of $\varpi$ caused by Planet Nine is $\varpi = -360^\circ$/Gyr, following the averaged Hamiltonian (Equation (1)) without the $J_2$ term in a coplanar configuration. Therefore, the $J_2$ precession leads to the circulation in $\varpi$ in the positive direction.

When $a$ increases, the dynamical influence due to the giant planets becomes weaker and the perturbation due to the outer planet becomes stronger. Thus, the $J_2$ term no longer dominates the evolution. As shown in the upper middle and left panels of Figure 3, when $a$ increases to $\sim 210 - 300$ au, $\Delta \varpi$ stops circulating and starts to librate around $180^\circ$. In the upper panels, where $a < 250$ au, the inclinations of the TNOs stay low for the entire 4 Gyr simulation.

As $a$ further increases ($a \gtrsim 250$ au), the initial eccentricities approach closer to unity because we chose a fixed range of initial pericenter distances ($30$ au $< r_p < 50$ au). This is shown in the lower panels in Figure 3: as the eccentricity increases, the $J_2$ precession increases, while the precession due to Planet Nine decreases, leading to the circulation in $\Delta \varpi$ at high eccentricity.

Thus far, the secular dynamics of anti-aligned particles we have described are qualitatively the same as those seen in the strictly coplanar case previously studied by Beust (2016), Batygin & Morbidelli (2017), and Hadden et al. (2018). In particular, an island of librating trajectories appears at a critical semimajor axis where the apsidal precession induced by Planet Nine is able to balance that from the solar system giants.

Figure 3 shows that Planet Nine can also excite the inclinations of some TNOs, a dynamical effect that, by construction, is absent from previous coplanar studies. When TNOs reach very high eccentricity orbits, Planet Nine is able to excite extreme inclinations, often leading to orbital flips. We will study the dynamics of these orbital flips in more detail in Section 4.

During the flips, the pericenter orientation of the TNO in the ecliptic plane ($\theta$, the geometrical longitude of pericenter) librates as illustrated in Section 4. This is similar to the coplanar flips shown in the hierarchical octopole limit (Li et al. 2014b). In contrast to the nearly coplanar cluster, for high-inclination orbits ($i > 140^\circ$), $\Delta \varpi$ circulates, as can be seen in the lower panels of Figure 3. This differs from Figure 10 in Batygin & Morbidelli (2017), where $\Delta \varpi$ remains confined as the inclination becomes larger. This is because the objects selected in Batygin & Morbidelli (2017) only stay briefly in the high-inclination region, before circulation is completed. We can see both types of objects in our secular and $N$-body simulations (e.g., some trajectories in

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Footnote:

8 This was referred to as the “pericenter longitude” by Brown & Batygin (2016).
Dynamical evolution in the color represents the pericenter distance of the TNOs. For the high-inclination objects with larger semimajor axes, which are more likely to allow high-inclination excitations of TNOs. Figures 3 and 26 present trajectories in the plane of geometrical longitude of pericenter (Δθe) vs. pericenter distance for different semimajor axes. We only plot the TNOs with larger semimajor axes, which are more likely to allow high-inclination excitations of TNOs (lower panels in Figure 3). When inclination is low, θe ∼ ϖ and the dynamical evolution in Δθe vs. rp looks similar to that in Δϖ vs. rp. Different from Δϖ, libration of Δθe can also be seen when the TNO inclination is high, i ≥ 140°.

In addition, we note that in the upper right panel of Figure 3, where 250 au < a < 280 au, there are two teal objects displaying an interesting evolution, where their eccentricities vary with large amplitudes. There are only two teal objects in this regime; one of them has ϖ centered in and oscillating around 0°.

To illustrate the libration of the geometrical longitude of pericenter in the ecliptic plane for some of the flipped orbits, we show in Figure 4 the evolution of TNO pericenter distances versus Δθe in the same semimajor axis ranges as those in the lower panels of Figure 3, where the inclination of the TNOs can be excited. At high inclinations (e.g., i ≥ 150°), Δθe librates, while Δϖ rapidly circulates. We note that θe ∼ ϖ when inclinations are low (i ∼ 0°), and ϖ ∼ 2Ω−ϖ when TNOs counterorbit with regard to Planet Nine (i ∼ 180°). The libration of θe is consistent with the libration of 2Ω−ϖ noted by Batygin & Morbidelli (2017) for high-inclination orbits.

Out of the 500 particles, 105 have their inclinations excited to retrograde configurations during the 4 Gyr simulation. It is more likely for the inclination to flip if the particles start with large semimajor axes and small pericenter distances, but the detailed dependencies are more complicated, as we demonstrate in Section 4.

To illustrate the clustering of high-inclination TNO orbits, Figure 5 presents trajectories in the plane of (Δθe, i), color-coded in the pericenter distance. There is a moderate clustering of trajectories around Δθe ∼ 90° and ∼270° when the inclinations are high, i ≥ 60°−120°, and the pericenter distances are low, ≤80 au. Around i ≥ 140°, Δθe is clustered around 150°−210°.

Combining all the particles in the secular simulations with different semimajor axes, we show the scatter in semimajor axis...
starting with TNOs in the near-coplanar configuration. We note that the clustering in \( \Delta \theta_e \) is stronger than that in \( \Delta \varpi \) because \( \Delta \varpi \) no longer librates for the high-inclination population. This is consistent with what we find in Figures 3–5. The clustering in both \( \Delta \varpi \) and \( \Delta \Omega \) is very weak and cannot be seen in the scatter plots (as shown in the histogram plots in Figure 8). Thus, we do not include them here.

2.2.2. Alignment in Argument of Pericenter \( \omega \) and Longitude of Node \( \Omega \)

We now consider the alignment of the orbital plane in argument of pericenter and longitude of ascending node. We focus on TNOs with \( a \gtrsim 300 \) au, which allows the inclination of the TNOs to be excited to large values. Figure 7 presents the evolution of the TNOs in the plane of \((\Delta \varpi, i)\) and \((\Delta \Omega, i)\), color-coded in the pericenter distance \( r_p \). The figure illustrates “looping” trajectories followed by TNOs that reach high inclination, resulting in the clustering of \( i \) near \( \Delta \varpi \sim 0^\circ \) and \( 180^\circ \) and \( \Omega \) near \( \Delta \varpi \sim 90^\circ \) and \( 270^\circ \) among the high-inclination TNOs when the pericenter distance is low, \( \lesssim 80 \) au.

In the upper row of Figure 7, the dynamical regions where the trajectories librate around \( \Delta \varpi \sim 90^\circ \) and \( 270^\circ \) for \( 40^\circ < i < 90^\circ \) and \( 90^\circ < i < 140^\circ \) are analogous to the quadrupole-order Kozai–Lidov resonant regions, which are seen for both interior and exterior test particles (e.g., Kozai 1962; Lidov 1962; Naoz et al. 2017; Vinson & Chiang 2018). These resonant regions are also shown in the secular study by Saillenfest et al. (2017) on TNOs perturbed by Planet Nine using surface of sections. However, due to the close separation between the TNOs and Planet Nine, some of the hierarchical approximation breaks down, as shown in Batygin & Brown (2016a). In the lower row of Figure 7, in the \((\Delta \Omega, i)\) plane, the regions where the trajectories librate around \( \Delta \Omega \sim 180^\circ \) are analogous to the octopole Kozai–Lidov resonances, where \( i \) can be excited from near zero inclination. The cat-eye-shaped regions centered around inclination \( i \sim 90^\circ \) and \( \Delta \Omega \sim 0^\circ \) and \( 180^\circ \) are also analogous to the octopole Kozai–Lidov resonances when the interaction energies are higher (e.g., Figure 4 of Li et al. 2014a, panel \( H = -1 \) and \( H = -0.5 \)).

We summarize the orbital alignment in Figure 8, which shows histograms of \( \Delta \varpi, \Delta \omega, \) and \( \Delta \Omega \). We choose only particles with \( t > 3 \) Gyr and \( 30 \text{ au} < r_p < 80 \text{ au} \) since these objects are more likely to be detectable. We break up the particles into three inclination bins in Figure 8: the upper panel selects particles with \( a > 250 \text{ au} \) and \( i < 40^\circ \), the middle panel focuses on the high-inclination (\( 60^\circ < i < 120^\circ \)) particles, and the lower panel focuses on the retrograde particles with high inclination (\( i > 140^\circ \)). For the low-inclination population (\( i < 40^\circ \)), there is a strong clustering around \( \Delta \varpi \sim 180^\circ \) and the clusterings in \( \Delta \omega \) and \( \Delta \Omega \) are weak. The high-inclination population, shown in the middle panel, exhibits a clear deficit of particles around \( \Delta \varpi \sim 180^\circ \) and peaks near \( \Delta \varpi = 90^\circ \) and \( 270^\circ \). We note that the clustering in \( \Delta \varpi \sim 180^\circ \) for the high-inclination TNOs (middle panel of Figure 8) is missing due to the detectability cut, but clustering in \( \Delta \varpi \sim 180^\circ \) can be seen for high-pericenter objects in Figure 5. In addition, there is an excess of objects with \( \Delta \Omega \sim 90^\circ \) and \( 270^\circ \) and \( \Delta \omega \sim 0^\circ \) and \( 180^\circ \), which illustrate clustering of high-inclination orbits for the low-pericenter TNOs.

Figure 6. Results of secular simulations in which the particles are initially anti-aligned from the pericenter of Planet Nine (\( \Delta \varpi = 180^\circ \)). The top panel is color-coded in pericenter distances, and the lower panel is color-coded in inclination. Particles with \( t > 3 \) Gyr and \( 30 \text{ au} < r_p < 80 \text{ au} \) are plotted (with a time step of 1 Myr). We find that there is a strong clustering at \( \Delta \theta_e \sim 180^\circ \) when \( 30 \text{ au} < r_p < 80 \text{ au} \) (top). In addition, there is a clustering in \( \Delta \varpi \sim 180^\circ \) when the inclination is low, \( i \lesssim 40^\circ \), and there is a wider clustering in \( \varpi \sim 180^\circ \) when the inclination is high: \( i \gtrsim 140^\circ \) (bottom). It shows that the pure secular effects can lead to the observed TNO orbital clusterings. There is no apparent clustering in \( \Delta \omega \) or \( \Delta \Omega \) (not plotted), except very mild ones, which can be seen in the histogram in Figure 8.

versus geometrical longitude of pericenter in Figure 6. The top panel is color-coded by the pericenter distance, and the bottom panel is color-coded by the inclination of the TNOs. To approximate detectable objects, we select only particles with \( t > 3 \) Gyr. In addition, we select only particles with \( 30 \text{ au} < r_p < 80 \text{ au} \) to focus on the closer-in objects, which are more likely to be detectable. All 500 particles are started with anti-aligned pericenters (\( \Delta \varpi = 180^\circ \)) and inclinations of \( 10^\circ \) from the ecliptic. Each point corresponds to a test particle, with snapshots taken at 1 Myr time steps. There is a clear alignment in \( \Delta \theta_e \) that begins when the semi-major axes of the TNOs are around \( \gtrsim 200–300 \text{ au} \), corresponding to the libration region in Figure 3. In addition, when we consider only the lower-inclination objects (\( \lesssim 40^\circ \)), the alignment near \( \sim 180^\circ \) is stronger, particularly when \( a \gtrsim 300 \text{ au} \). This shows that the observed clustering can be produced by pure secular effects alone when
We note that we make the detectability cut at $r_p < 80$ au to facilitate comparison with observational results, in order to clarify the role of secular interactions in producing the observed clustering of the TNO orbits. Many dynamical effects, e.g., scatterings with Neptune, Planet Nine, and MMR, are neglected in the secular approach. Thus, we do not intend to reproduce the full dynamical interactions between the TNOs and Planet Nine using the secular methods. In addition, we note that when including randomly initialized TNOs with $\omega$ and $\Omega$ uniformly distributed, the secular dynamics is similar to that of the exact coplanar case, where there are two clusterings of $\varpi$ around $0^\circ$ and $180^\circ$. The $0^\circ$ clustering is unstable if one considers the full dynamics, including close encounters with Neptune and MMR (e.g., Saillenfest et al. 2017) using surface of sections that are analogous to Kozai resonances.

### 3. N-body Results

In this section, we apply the secular results of Section 2 to interpret N-body simulations of the evolution of TNOs orbiting under perturbations from Planet Nine, as well as to study the role of secular dynamics in sculpting the orbits of TNOs. We include a central 1 $M_\odot$ star, $J_2$ moment corresponding to the inner three giant planets but model both Neptune and Planet Nine as fully interacting standard massive particles.

Similar to the near-coplanar configuration included in Hadden et al. (2018), we set the the semimajor axis of Planet Nine $a_9 = 500$ au, and the eccentricity of Planet Nine $e_9 = 0.6$. We set the inclination of Planet Nine to be $3^\circ$ for a near-coplanar configuration, and we set the initial condition $\omega_9 = \Omega_9 = 0$. The initial condition of Neptune is set to agree with its configuration at the J2000 epoch: $a_N = 30.07$ au, $e_N = 0.0086$, $i_N = 1.77^\circ$, $\omega_N = -86.75^\circ$, $\Omega_N = 131.72^\circ$, and $\lambda_N = 304.88^\circ$ (Murray & Dermott 1999). We include 5000 test particles in our simulation, where the pericenters of the test particles are uniformly distributed within 30–50 au, and we draw the semimajor axes uniformly between 150 and 550 au. The inclinations of the test particles are initially zero ($i = 0^\circ$), while the non-coplanar Neptune torques the orbits of the TNOs and Planet Nine further excites their inclinations. The argument of pericenter, longitude of ascending node, and mean anomaly are all uniformly distributed between $0^\circ$ and $360^\circ$ for the TNOs. We use the Mercury package (Chambers 1999) for the N-body simulation. We adopt the “hybrid” Wisdom–Holman/ Bulirsch–Stoer integrator (Press et al. 1992; Wisdom & Holman 1992) with a time step of $dt = 3000$ days, which is roughly 5% of Neptune’s orbital period.

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4 We do not use $i = 0$ because we wish to have well-defined $\Omega_9$ and $\omega_9$ for Planet Nine in our simulations, relative to which we can measure the TNO orientations.
After 3 Gyr, ∼12.5% of particles survived the simulation (bound to the Sun with a < 3000 au), and at t = 4 Gyr, 8.7% of the TNOs survived. The histogram of the TNO period ratios with regard to Planet Nine is shown in Figure 9. In contrast to the coplanar case studied by Batygin & Morbidelli (2017), who treated Neptune as a J2 term, many particles survive outside of the lower-order MMR with Planet Nine, and some of them spend only a small fraction of time in high-order MMR, as illustrated in Figure 21 in Appendix B1. Analyzing 100 surviving TNOs, we find that more than ∼80% of them spend more than ∼80% of their time outside of MMRs. This is because scattering with the point-mass Neptune removes TNOs from MMR with Planet Nine (Hadden et al. 2018). Moreover, the misalignment between the TNO and Planet Nine’s orbit helps to avoid orbit intersections and decreases the likelihood.
of collisions or close encounters with Planet Nine that eject TNOs. The TNOs that survive outside of MMRs support the importance of secular dynamics in the evolution of TNOs, in addition to the effects of the MMR highlighted by Batygin & Morbidelli (2017).

The N-body results on the clustering of the TNO orbits in the near-coplanar configuration have previously been shown in the literature (e.g., Batygin & Brown 2016b), and we include the clustering of the TNO orbits in the Appendix B2 (Figures 22 and 23) to allow detailed comparison with our secular results.

To illustrate the dependence of the surviving particles on their initial longitude of pericenter in more detail, we show the histogram of the initial longitude of pericenter for the surviving particles ($t_{\text{final}} > 3$ Gyr) in Figure 10. There is a strong peak near $\Delta\pi_0 \sim 180^\circ$. The initially anti-aligned population is more likely to survive due to the phase protection of the orbits in the libration region around $\Delta\theta_e \sim 180^\circ$. This keeps the TNO orbits away from being tangential with the orbit of Planet Nine, where the overlap of the MMRs leads to chaotic evolution of TNOs, as discussed in Hadden et al. (2018) for the coplanar case. Then, they are carried into the low pericenter distance Neptune-scattering region and are often ejected (Khain et al. 2018). The dominance of surviving particles with $\Delta\pi_0 \sim 180^\circ$ also shows that secular investigations can reproduce the dynamical features of N-body simulations, as discussed in Appendix B.

Figure 11. Evolution of particles for $t > 3$ Gyr, $a > 300$ au, $10$ au $< r_p < 1000$ au in the $r_p-\theta_e$ plane for the N-body simulations seen in Figure 22. To highlight the high-inclination evolution, we increase the transparency of the low-inclination part of the trajectories ($i < 40^\circ$). The libration of the particles in $\theta_e$ is consistent with that caused by the secular resonances, as shown in Figure 4. We show a wide range of pericenter distances here in order to illustrate the overall dynamics.

Moreover, we can see the libration of the geometrical longitude of pericenter during the evolution of the TNO orbits in the plane of $(\Delta\theta_e, r_p)$ (Figure 11). The libration in $\Delta\theta_e \sim 180^\circ$ when the inclination is low is similar to the coplanar case (Hadden et al. 2018). This leads to the clustering in $\Delta\theta_e \sim 180^\circ$. When the inclination is higher, the trajectories can sometimes also librate in $\Delta\theta_e \sim 180^\circ$. This feature can also be seen in the secular results, as shown in Figure 3.

To summarize the clustering features of the TNO orbits, we combine all the long-lived ($t > 3$ Gyr) and small-pericenter particles (30 au $< r_p < 80$ au) in Figure 12. The upper panel shows the histogram for particles with low inclination, $i < 40^\circ$. We select those with $a > 250$ au, since there is no clustering at smaller semimajor axes (due to the fast $J_2$ precession, as shown in Figure 22). We plot histograms in the geometrical longitude of pericenter $\Delta\theta_e$, argument of pericenter $\Delta\omega$, and longitude of ascending node $\Delta\Omega$. There is a significant peak around $\Delta\theta_e \sim 180^\circ$, indicating strong clustering in the geometrical longitude of pericenter. However, the clustering in $\Delta\omega$ and $\Delta\Omega$ is weak.

The middle panel shows the high-inclination population, $60^\circ < i < 120^\circ$. Similar to the secular results, we see clustering in $\Delta\omega \sim 0^\circ$ and $180^\circ$, $\Delta\Omega \sim 90^\circ$ and $270^\circ$, and $\Delta\theta_e \sim 90^\circ$ and $270^\circ$ within the detection limit for the low-pericenter TNOs. Only a small fraction of TNOs reached the counterorbiting configuration with $i > 140^\circ$. Similar to the nearly anti-aligned population illustrated in Figure 8, there are two peaks around $\Delta\theta_e \sim 135^\circ$ and $225^\circ$. The clustering in $\Delta\omega$ and $\Delta\Omega$ is weak for the near counterorbiting population. These N-body clusterings are similar to the secular results in Figure 8. This indicates that the clusterings in the N-body results can be produced by pure secular interactions for the anti-aligned population.
4. Orbital Flips and Dependence on Planet Nine

The $N$-body simulations have shown that TNO orbits can flip with large amplitude and cross 90° (e.g., Batygin & Brown 2016b; Lawler et al. 2017; Shankman et al. 2017b). This might explain the origin of the detected retrograde TNOs (e.g., Chen et al. 2016). The flip of the orbits is very similar to the near-coplanar flip of the hierarchical ($a_{\text{TNO}} \ll a_{\text{pert}}$) three-body interactions discovered in Li et al. (2014b), where a test particle’s orbit can be flipped by nearly 180° by a near-coplanar perturber. Here we investigate the flip in the much less hierarchical configurations ($a_{\text{TNO}} \sim a_{\text{pert}}$) in this section and characterize the dependence of the flip on the properties of the perturber, Planet Nine.

4.1. Secular Investigation

For simplicity, we start with the secular approximation. To characterize the range of parameter space where the TNOs could flip, we investigate the flips in the plane of TNO semimajor axes and initial eccentricities using the secular integration in Figure 13. Similar to the secular simulations discussed above, we set $a_0 = 500$ au, $e_0 = 0.6$, and $i_0 = \omega_0 = \Omega_0 = 0$. We set the initial longitude of pericenter of the TNO to be $\omega_0 = 180°$ ($\omega_0 = \pi$, $\Omega_0 = 0$) here, since the surviving TNOs mostly have $\Delta \omega_0 \sim 180°$ (as shown in Figure 10). The inclinations of the TNOs are all set to be $5°$, slightly misaligned from the ecliptic plane. The parameter space that allows the TNOs to flip for the anti-aligned TNOs does not depend sensitively on the initial mutual inclination, as long as it is nonzero and near-coplanar.

To understand the flips better, we break down the problem into pieces and consider the three-body interactions first, without the $J_2$ precession. Including only the central star, a TNO, and the perturbing Planet Nine, the orbit of the TNO can be easily flipped both when the TNO is inside the orbit of Planet Nine and when it is farther. For illustration, we represent the runs that could not flip using plus signs and then use crosses to represent the runs that can flip when we ignore $J_2$ precession.

Next, we marked with circles the runs that can flip in the presence of the $J_2$ precession. Including $J_2$ precession, the flip of the orbits can be suppressed. This is determined by the libration and $J_2$ precession timescales of the TNOs. To compare the timescales, we color-coded the crosses using the ratio of the libration timescale of $\Delta \theta_e$ and the $J_2$ precession timescale. To obtain the timescales, we numerically calculated the libration timescale of the geometrical longitude of pericenter following the secular integration of the three-body interaction, and we calculated the $J_2$ precession due to the inner four giant planets analytically based on Equation (4).

When the libration timescale exceeds 4 Gyr, we color the crosses red. As expected, the runs can still flip when the libration timescale is shorter than the $J_2$ precession timescales in general. Most of the TNOs can still flip when they are farther from the inner giant planets with $a \gtrsim 300$ au and $r_p \gtrsim 100$ au. Note that we only focus on the anti-aligned configurations here, since these TNOs are more likely to survive. The TNO orbits are less likely to flip if their pericenters are aligned with that of Planet Nine, analogous to the hierarchical limit (Li et al. 2014b). The general flip condition in the nonhierarchical limit is quite complicated and beyond the scope of our paper.

As illustrated in Figure 13, the overall dynamics can be divided in the following three regions.

(1) Small semimajor axis region ($a \lesssim 150$ au), where the precession timescale is much shorter than that of the $\Delta \theta_e$ libration timescale. Both the flip and the libration in $\Delta \theta_e$ can be suppressed.

(2) Large semimajor axis region ($a \gtrsim 150$ au) within the two black lines inside $30$ au $\lesssim r_p < \lesssim 100$ au in Figure 13, where the two timescales are comparable. The flips are suppressed while the libration in $\Delta \theta_e$ still persists.

(3) Large semimajor axis region ($a \gtrsim 150$ au), where the libration timescale is much shorter. Both the libration and the flip remain.

Figure 13. Flip condition in the secular approximation. The colors represent the ratio of the libration timescale of $\theta_e$ due to Planet Nine to the timescale of $J_2$ precession. The red plus signs indicate the TNOs that do not flip in 4 Gyr even without the $J_2$ potential, and the circles denote the runs that still flip in the presence of the $J_2$ potential. The initial inclinations of the TNOs are set to be $5°$, and the pericenters of the TNOs are anti-aligned with that of Planet Nine ($\omega = \pi$, $\Omega = 0$) for illustration. The orbits are more likely to be flipped when the libration timescale of the longitude of pericenter vector is shorter than the $J_2$ precession timescale, with $a \lesssim 300$ au. Flips in the region within $30$ au $\lesssim r_p \lesssim 100$ au are suppressed due to $J_2$ precession.
To illustrate the flips in more detail, we show examples of arbitrarily selected trajectories in these three regions below.

Figure 14 illustrates the evolution in region 1, where the semimajor axis of the TNO is small, and thus the $J_2$ term from the inner four giant planets dominates over the perturbation from the outer Planet Nine. The left panel shows the case with $J_2$ precession, and the right panel shows the case without $J_2$ precession. Without $J_2$ precession, the flip is analogous to that in the hierarchical limit (Li et al. 2014b). The precession timescale is much shorter compared with the $\Delta \theta_e$ libration timescale (as shown in Figure 13), and thus the libration of $\Delta \theta_e$ is suppressed when the $J_2$ term is included. The flip of the orbit is also suppressed in this case.

Figure 15 illustrates the evolution in region 2, where the semimajor axis of the TNO is larger, and thus the $J_2$ term from the inner four giant planets becomes comparable to the perturbation from the outer Planet Nine. Similar to Figure 14, the left panel shows the case with the $J_2$ term, and the right panel shows the case without the $J_2$ term. The $J_2$ precession timescale is similar to the $\Delta \theta_e$ libration timescale (as shown in Figure 13). The libration of $\Delta \theta_e$ is not suppressed when the $J_2$ term is included; however, the flip of the orbit is suppressed in this case. Orbits lying in this region are detectable with $r_p \lesssim 100$ au, and they contribute to the alignment of the orbits around $\Delta \theta_e \sim 180^\circ$.

Finally, Figure 16 illustrates the evolution in region 3, where the eccentricity of the TNO is lower and the perturbation from the outer Planet Nine is more dominant. Again, the left panel shows the case with the $J_2$ term, and the right panel shows the case without the $J_2$ term. The $J_2$ precession timescale is lower than the $\Delta \theta_e$ libration timescale (as shown in Figure 13), and neither the libration of $\Delta \theta_e$ nor the flip of the orbit are suppressed when the $J_2$ term is included. Orbits lying in this region contribute to the flipped orbits.

In addition, the TNO could stay in the high-inclination regime for many cycles of low-amplitude $2\Omega - \omega$ libration, and $\omega$ circulates during this high-inclination phase. This is similar to the $N$-body results presented in Figure 23, but this is different from the particles selected in Figure 10 of Batygin & Morbidelli (2017), where the particles fall back to the low-inclination regime quickly and $\omega$ is still confined. We note that there are also particles that only spend one libration cycle at high inclination in our simulation, similar to Figure 10 of Batygin & Morbidelli (2017).

Interestingly, we notice that orbits within $30$ au $\lesssim r_p \lesssim 100$ au and $a \gtrsim 200$ au inside the solid and dashed black lines in Figure 13 do not flip, and the geometrical longitude of pericenter $\Delta \theta_e$ still librates in the presence of the $J_2$ term, since the two timescales are comparable to each other. These TNOs lie within our selection criteria based on observational limits, and they lead to the alignment of the low-inclination TNO orbits with $\Delta \theta_e \sim 180^\circ$, as shown in the N-body and secular simulations discussed in the previous sections.

The parameter space of clustered low-inclination TNOs corresponds to the central libration region in the plane of $(\Delta \varpi, r_p)$ (e.g., Figure 3), near $\Delta \varpi = 180^\circ$. Thus, characterizing the dependence of the libration region on the properties of Planet Nine can help constrain the possible outer planet based on the detected clustering of low-inclination orbits.

In Figure 17, we show the corresponding pericenter distances of the fixed points as a function of semimajor axis for different Planet Nine orbital parameters. The fixed point of the libration region is located at $\Delta \varpi = 180^\circ$, and we numerically calculate the pericenter distances of the fixed point at different semimajor axes by searching for eccentricity corresponding to the maximum energy when $\Delta \varpi = 180^\circ$.

The libration regions exist around the fixed points. The secular resonances around $\Delta \varpi = 180^\circ$ disappear for low TNO semimajor axes (e.g., upper left panel of Figure 3).
marked the minimum semimajor axes when the libration regions appear with black circles. There are no fixed points when the semimajor axis is small, in particular for wider and more circular Planet Nine’s orbit. In other words, when the orbit of Planet Nine is more circular, the anti-aligned libration appears only for TNOs with larger semimajor axes. In addition, when Planet Nine is more distant, it also requires a more eccentric Planet Nine orbit to produce the alignment for the closer-in TNOs. This is consistent with the N-body results shown in Brown & Batygin (2016), where Planet Nine favors a more eccentric orbit around $a \sim 500$ au to produce the clustering.

4.2. Inclination Distribution Based on N-body Results

As shown in the previous section, the flip of the orbits depends on the existence of the libration region of $\Delta \theta_e \sim 180^\circ$, and this itself depends on the properties of Planet Nine. Thus, the retrograde TNO orbits and the signatures of the inclination distribution can provide valuable constraints on the properties of any possible outer planet. In this section, we illustrate the dependence of the inclination distribution using N-body simulations. First, we present the inclination distribution of the TNOs under perturbations from a Planet Nine with $m_9 = 10\, M_\odot$, $a_9 = 500$ au, $e_9 = 0.6$, and $i_9 = 3^\circ$, similar to that included in Section 3. We find that some high-inclination
uniform when \( a = 150 \) near \( \sim \) circular and farther.

Figure 17. Fixed-point pericenter distance vs. TNO semimajor axis for different Planet Nine properties. The solid lines represent the fixed points due to an \( a = 500 \) au Planet Nine, and the dashed lines represent those due to an \( a = 800 \) au Planet Nine. The black circles represent the critical minimum semimajor axes when the libration regions appear. The libration region around \( \Delta \omega = 180^\circ \) arises in pericenter distance for closer TNOs when Planet Nine’s orbit is more eccentric, and this libration region disappears when Planet Nine is circular and farther.

The inclination distribution depends on the properties of Planet Nine. For instance, if Planet Nine is less massive (\(~M_{\oplus}\)), the perturbations on the TNOs are weaker, and the \( J_2 \) precession due to the inner planets dominates. Thus, it is difficult to flip the TNO orbits, and none of the TNO orbits are flipped over 90°.

Since the flips are associated with the libration of the geometrical longitude of pericenter (\( \Delta \theta_p \)) around 180°, and since this only occurs for Planet Nine orbits that have high eccentricity and are wide (\( a \sim 400–800 \) au; see Figure 17), we expect that the TNO orbits can flip only for such Planet Nine orbits. To test this, we performed more N-body simulations to calculate the fraction of TNOs that can have their inclinations excited using different Planet Nine properties. We included a 10\( M_{\oplus} \) Planet Nine and varied the Planet Nine orbital parameters in these simulations.

Figure 19. Fraction of TNOs with inclination over 60° (left panel) and over 90° (right panel) at 4 Gyr for different Planet Nine orbital parameters. It is more likely to flip the TNO orbits for a more eccentric Planet Nine orbit with semimajor axis \(~400–800\) au for a 10\( M_{\oplus} \) Planet Nine.
Figure 19 shows the fraction of TNOs that have inclinations over 60° and 90° at 4 Gyr. Specifically, it is more likely to flip the TNO orbits for an eccentric Planet Nine (\( e \gtrsim 0.4 \)) with a large semimajor axis of \( \sim 400-800 \) au. This agrees with our expectation based on the appearance of the libration region around \( \Delta \theta_p \sim 180° \). The recent discovery and dynamical analysis of high-inclination objects, e.g., 2015 BP519 (Becker et al. 2018), supports the existence of possible outer planets. Future surveys of the inclination distribution of Centaurs can help further constrain the parameters of possible outer planets (e.g., Petit et al. 2017; Lawler et al. 2018).

5. Conclusion

A deeper understanding of the dynamics involved in interactions between TNOs and the hypothesized Planet Nine can help characterize the orbits of possible planets in the outer solar system and facilitate the observational detection of such planets. The secular interactions with Planet Nine play an important role in shaping the orbits of the TNOs. We obtained the averaged Hamiltonian for a Planet Nine in the plane of the ecliptic but allowing TNOs small initial inclinations, and we compared our secular results with \( N \)-body simulations extending beyond the secular effects in the exact coplanar configurations studied in the literature (Beust 2016; Batygin & Morbidelli 2017; Hadden et al. 2018). Our findings are listed below.

1. For systems in which Neptune is a fully interacting point mass and Planet Nine is slightly inclined (\( \sim 3° \)), many TNOs survive outside of MMR for 4 Gyr.
2. The clustering of TNO orbits near \( \Delta \varpi (\Delta \theta_p) \sim 180° \) can be produced by secular interactions, similar to the coplanar results (Beust 2016; Batygin & Morbidelli 2017; Hadden et al. 2018). In addition, the clustering near \( \Delta \varpi \sim 180° \) is stronger if one selects only the low-inclination (\( \sim <40° \)) TNO population. The clustering in \( \Delta \varpi \) and \( \Delta \Omega \) for the low-inclination population is weak.
3. For the high-inclination population (\( 60° < i < 120° \)), there are clusters for the TNOs within the detection limit (\( r_p < 80 \) au) near \( \Delta \theta_p \sim 90° \) and 270°, while the clusters in \( \Delta \varpi \) are weaker. In addition, we can see clusters in \( \Delta \varpi \) and \( \Delta \Omega \), around \( \Delta \varpi \sim 0° \) and 180° and around \( \Delta \Omega \sim 90° \) and 270° for the low-pericenter TNOs. The orbital alignment of the high-inclination population can also be produced by secular effects.
4. Eccentric (\( e \gtrsim 0.4 \)) Planet Nine beyond \( a \gtrsim 400 \) au can facilitate flips of TNO orbits over 90° due to secular interactions, analogous to the near-coplanar flips in the hierarchical configurations (Li et al. 2014b). During the flips, the geometrical longitude of pericenter (\( \Delta \theta_p \)) of TNOs librates.
5. The \( J_2 \) precession caused by the inner giant planets can suppress the aforementioned flips of TNO planets and lead to the clustering of low-inclination TNOs around \( \Delta \varpi (\Delta \theta_p) \sim 180° \) with 30 au < \( r_p < 80 \) au.

Configurations involving higher-inclination Planet Nine may explain the clustering in \( \Delta \varpi \). This is consistent with the results of Batygin & Brown (2016a) and Brown & Batygin (2016). A detailed analysis of the dynamics of a highly inclined Planet Nine is a promising future direction that is beyond the scope of this paper.

Future observations with more detailed orbital features of TNOs and, in particular, of the high-inclination population will provide valuable information to constrain the properties of any outer planets in our solar system. In addition to the orbital distribution of TNOs, it is likely that ecliptic comets can also help predict and/or constrain the properties of Planet Nine, for which Nesvorný et al. (2017) suggested that the inclination distribution of ecliptic comets would be wider than the observed one for a range of Planet Nine configurations.

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Appendix A

Secular Nonhierarchical Three-body Interactions

To verify that the secular approach serves as a good approximation, we illustrate in Figure 20 results that use both direct secular averaging (dashed red lines) and \( N \)-body integrations (solid blue lines). We use the high-accuracy adaptive time-stepping integrator “ias15” (Rein & Spiegel 2015) from the REBOUND package to obtain the \( N \)-body trajectories, since the eccentricities of the particles approach close to unity in the three-body interactions without the \( J_2 \) terms. We also include results using the Hamiltonian at the octopole level of expansion in the semimajor axis ratio (dot-dashed yellow lines), which characterizes the Kozai–Lidov oscillation (Kozai 1962; Lidov 1962) and is a good approximation when the hierarchical parameter \( \epsilon = (a/a_0) e_0 / (1 - e_0^2) < 0.1 \) (for a review, see Naoz 2016).

As shown in Figure 20, the hierarchical limit is still a good approximation when \( a = 30 \) au (\( a_0 = 500 \) au, \( e_0 = 0.6, \epsilon = 0.056 \)), and the octopole results deviate from the \( N \)-body results when \( a > 70 \) au (\( \epsilon > 0.13 \)). The secular results all agree very well with the \( N \)-body results over the 4 billion yr evolution. When \( a > 350 \) au, the trajectories are chaotic.
Appendix B
Some Details of N-body Results

B.1. Presence of MMRs

In this section, we present the detailed TNO orbital evolution starting in the near-coplanar configuration as mentioned in Section 3 based on the N-body results. First, we illustrate the presence of high-order MMRs during the evolution of TNOs. As mentioned in the main text, Figure 9 shows that the TNOs could survive outside of the lower-order MMRs. To illustrate any presence of higher-order MMRs, we plot the mean anomaly of the TNOs when that of Planet Nine is zero in Figure 21.

The evenly spaced blank regions in Figure 21 illustrate the presence of MMRs due to commensurability between the TNO and Planet Nine. Two out of the three randomly selected TNOs spend only a negligible fraction of time in MMR, while the TNO in the middle panel spends around 20%–30% of the time in MMR. All of them stay out of MMR for a large fraction of time, indicating the importance of pure secular interactions in their orbital evolution. Performing a similar analysis for 100 surviving TNOs, we find that more than 80% of them remain outside all MMRs for more than 80% of the time.

In addition, we adopt a more systematic approach, where we cut the time series of $M_{\text{TNO}}$ at $M_9 = 0$ to 1 and 10 Myr segments and perform Kolmogorov–Smirnov (ks) tests in each segment. If the ks test shows that the $M_{\text{TNO}}$ agrees with a uniform distribution, we mark the TNO to be out of MMR for the time segment. Using time segments of 1 Myr, 95% of the TNOs spend more than 80% of the time outside of MMR, and using time segments of 10 Myr, 85% of the TNOs spend more than 80% of the time outside of MMR. We note that this approach only gives a rough estimate and is very sensitive on the segment widths.

B.2. Orbital Clustering

Next, we illustrate the clusterings of the TNO orbits in more detail and compare with the secular results. The alignment of the test particles in pericenter ($\Delta \varpi$ and $\Delta \theta_e$) as a function of particle semimajor axis is shown in Figure 22. We record the longitude of pericenter of the test particles every 1 Myr for $t > 3$ Gyr. To illustrate the dependence of the alignment on different parameters of the system, we include three different color codes. The upper left panel of Figure 22 color-codes the TNOs in the pericenter distance, and it shows the orbital distribution of the TNOs in the plane of $(a, \Delta \varpi)$. It illustrates that there is a clear alignment of the test particles around the geometrical longitude of pericenter, $\Delta \theta_e \sim 180^\circ$, for small-pericenter objects ($30 \text{ au} < r_p < 80 \text{ au}$), consistent with Brown & Batygin (2016) and the observational results of Trujillo & Sheppard (2014). Particles with smaller pericenter distances, $r_p \lesssim 30 \text{ au}$, exhibit faster precession in the longitude of pericenter, suppressing any clustering, and the clustering is weak for longer pericenter distance TNOs, $\gtrsim 80$. In addition, the anti-aligned particles exhibit drifts in their semimajor axes at approximately constant pericenters. This is due to close encounters with Neptune when the particles are close to their pericenter, as shown in the coplanar case investigation by Hadden et al. (2018).

Figure 22 shows that there is no clustering around $\Delta \theta_e \sim 0^\circ$, which differs from the secular case initialized with a uniform distribution of $\varpi$. This is because the TNOs with $\Delta \varpi_9 \sim 0$ are ejected due to instability caused by the overlap of MMRs, as discussed in Hadden et al. (2018). This is illustrated in the upper right panel in Figure 22, which is color-coded in $\Delta \varpi_9$. It shows that most of the surviving TNOs that are clustered around $\Delta \theta_e \sim 180^\circ$ started with $\Delta \varpi_9 \sim 180^\circ$. Meanwhile, there is a population of particles that exhibit drifts in pericenter...
Figure 21. Mean anomaly of TNOs when that of Planet Nine is zero vs. time for three arbitrarily selected TNOs. Regular blank regions over time indicate the existence of MMRs. For the three illustrated examples, the TNOs in the left and right panels only stay in MMR for a negligible fraction of time, while the TNO in the middle panel stays in MMR for $\sim 20\% - 30\%$ of the time. All of the TNOs show that pure secular interactions play an important role in their orbital dynamical evolution.

Figure 22. Alignment in the pericenter orientation of the TNOs under the perturbation of Planet Nine, Neptune, and inner $J_2$ momentum caused by the inner three giant planets. Planet Nine is nearly coplanar with the ecliptic, and the initial test particle $\varpi$ values are randomly distributed. Particles with $t > 3$ Gyr are selected and plotted with time steps of 1 Myr. Upper left panel: alignment in the geometrical longitude of pericenter $\Delta \theta_e$ color-coded in pericenter. We include all surviving TNOs with a wide range of pericenters to illustrate the overall dynamics. Upper right panel: alignment in $\Delta \theta_e$ color-coded in the initial longitude of pericenter, $\Delta \varpi_0$. Lower left panel: alignment in $\Delta \varpi$ color-coded in inclination. Lower right panel: alignment in $\Delta \theta_e$ color-coded in inclination. The upper right, lower left, and lower right panels only select particles with $30 \text{ au} < r_p < 80 \text{ au}$. There is a strong clustering in $\Delta \varpi \sim 180^\circ$ and $\Delta \theta_e \sim 180^\circ$ for the low-inclination TNOs with initial $\Delta \varpi_0 \sim 180^\circ$. 
distances at constant semimajor axes (most clearly seen as the multicolored vertical paths in the top left panel of Figure 22), which arise due to secular effects.

The lower panels of Figure 22 color-code the TNOs in inclination. The lower left one shows the alignment in $\Delta \omega$. Here $\Delta \omega$ clusters around $\sim 180^\circ$, and the clustering is strongest when the test particle inclinations are low, $\leq 40^\circ$. The majority ($\sim 90\%$) of the anti-aligned ($120^\circ < \omega < 240^\circ$) small-pericenter ($30 \text{ au} < r_p < 80 \text{ au}$) particles maintain a low inclination throughout their 4 Gyr evolution. On the other hand, the lower right panel shows the alignment in the geometrical longitude of pericenter ($\Delta \theta_c$), color-coded in inclination. In contrast to the result for $\Delta \omega$, $\Delta \theta_c$ shows tight clustering, even for high-inclination TNOs. This is because $\Delta \theta_c$ librates during the flips of the orbits, similar to the flips of the inner orbit in the hierarchical three-body interactions (Li et al. 2014b; see more details in Section 4).

As shown in the secular results of Section 2, the high-inclination population also shows interesting clustering in the orbital orientation. To illustrate this, we plot in Figure 23 the alignment in different orbital orientations as a function of semimajor axis. All of the long-lived ($t > 3 \text{ Gyr}$), high-inclination ($60^\circ < i < 120^\circ$) particles are selected; there are 303 long-lived TNOs that reached above $60^\circ$, and 255 of them reached retrograde configurations. The clustering in longitude of node, $\Delta \Omega$, is shown in the upper left panel; argument of pericenter, $\Delta \omega$, in the upper right panel; longitude of pericenter, $\Delta \varpi$, in the lower left panel; and geometrical longitude of pericenter, $\Delta \theta_c$, in the lower right panel.

Consistent with the secular results, there is a strong deficit of particles with $\Delta \varpi \sim 180^\circ$ and $\Delta \theta_c \sim 180^\circ$. In addition, there are clusters around $\Delta \varpi \sim 90^\circ$ and $270^\circ$, and the clusters around the geometrical longitude of pericenter are slightly tighter ($\Delta \theta_c \sim 90^\circ$ and $270^\circ$) for the low-pericenter TNOs. This is very different from the low-inclination population. The clusterings in $\Delta \omega$ around $0^\circ$ and $180^\circ$ around $90^\circ$ and $270^\circ$ are very similar to the secular results. The existence of such clustering in future TNO detection can help constrain the orbital parameters of any outer planet.

To illustrate the evolution of the particles and Planet Nine, we plot Planet Nine and selected representative TNO orbital elements as a function of time in Figures 24–26. Figure 24 shows the evolution of Planet Nine, Figure 25 shows the evolution of some examples of low-inclination objects, and Figure 26 shows the evolution of some high-inclination objects.

The semimajor axis and eccentricity of Planet Nine show little variation, as illustrated in Figure 24. The inclination of Planet Nine has low-amplitude, long-timescale variations due to interactions with Neptune. The red dashed lines in the lower
three panels represent the precession due to the $J_2$ component by the inner giant planets. The $J_2$ precession agrees well with the changes of $\omega$, $\Omega$, and $\varpi$ as a function of time.

Figure 25 shows the trajectories of low-inclination TNOs. The semimajor axes of the particles are perturbed by Planet Nine and Neptune, which cause large period-ratio variations. The eccentricity of the particles also varies throughout the 4 Gyr of integration, which allows the pericenter distance to oscillate. The inclinations stay low ($\leq 30^\circ$) within the 4 Gyr integration. Here $\Delta \varpi$ librates for the particles with large semimajor axes, as illustrated by the particle represented by the yellow line. At smaller semimajor axes, $\Delta \varpi$ starts to circulate, and $\Delta \omega$ and $\Delta \Omega$ generally circulate within 4 Gyr.

Figure 26 shows the trajectories of some representative high-inclination TNOs. The semimajor axes of the particles are perturbed by Planet Nine and Neptune, which cause large period-ratio variations, similar to the low-inclination case. The eccentricity of the particles also varies throughout the 4 Gyr of integration, while the inclination can be excited to high values. Only $\sim 20\%$ of the trajectories flip back to low inclination, $\leq 20^\circ$, within the 4 Gyr integration. Different from the low-inclination particles, the $\Delta \varpi$ of the particles does not always librate around $180^\circ$, even when the semimajor axes are large. Decreases in $\Delta \varpi$ can be seen when $\Delta \varpi$ does not librate, which explains the overdensity for $\Delta \varpi \sim 90^\circ$ compared with $\Delta \varpi \sim 270^\circ$, since the long-lived particles starting near $\Delta \varpi \sim 180^\circ$ tend to reach $\sim 90^\circ$ earlier and slightly more often than $\sim 270^\circ$. There are libration regions in $\Delta \omega$ and $\Delta \Omega$ as their inclinations increase that lead to clusterings in $\Delta \omega$ and $\Delta \Omega$ for high-inclination objects ($60^\circ < i < 120^\circ$).
Figure 25. Evolution of randomly selected particles with low inclination from the $N$-body simulation discussed in Section 3.
Figure 26. Evolution of randomly selected particles with high inclination from the N-body simulation discussed in Section 3.

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