Strategies to link tiny neutrino masses with huge missing mass of the Universe

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With the start of the LHC, interest in electroweak scale models for the neutrino mass has grown. In this letter, we review two specific models that simultaneously explain neutrino masses and provide a suitable DM candidate. We discuss the implications of these models for various observations and experiments including the LHC, Lepton Flavor Violating (LFV) rare decays, direct and indirect dark matter searches and Kaon decay.

I. INTRODUCTION

Although the Standard Model (SM) of particle physics has been triumphant in explaining the observations in high energy accelerators, it has failed to accommodate the Lepton Flavor Violation (LFV) in the neutrino oscillation. Moreover, there is no suitable candidate for dark matter in the SM. Extensive literature exists on models explaining each of these phenomena. For example, the conventional type I seesaw mechanism with very heavy right-handed neutrinos explains the small neutrino masses. However, this model does not provide a candidate for Dark Matter (DM). Moreover, since the scale of new physics in this scenario (i.e., the masses of the right-handed neutrinos) is much higher than the reach of the LHC or any other man-made accelerator in the foreseeable future, directly testing this scenario is a dream that may not ever come true.

Testable models that can simultaneously explain neutrino masses and dark matter have been in the center of attention in recent years. To a great extent, this attention owes to the start of the LHC and a host of direct and indirect DM search experiments that are either in the process of data taking or planned to do so in the near future. To explain neutrino oscillation, the model should include sources for LFV. If the mass scale of the new particles is low, we in general expect rather large LFV decay rates, $\text{Br}(\mu \rightarrow e\gamma)$, $\text{Br}(\tau \rightarrow e\gamma)$ and $\text{Br}(\tau \rightarrow \mu\gamma)$, that can be in principle observed at the experiments searching for these decays. With the MEG experiment taking data [1] and the superflavor project under consideration, this possibility is becoming more exciting.

In this article, we review two specific models that can simultaneously explain neutrino masses and dark matter. Both models are based on a $Z_2$-symmetry under which all SM particles are even and all new particles are odd. As a result, the lightest new particle is stable against decay and provides a suitable DM candidate. In both these models, right-handed neutrinos are added to the SM; however, the $Z_2$ symmetry prevents a Dirac mass term for neutrinos. Neutrinos acquire Majorana mass terms at one-loop level.

One of these models, the so-called SLIM model, contains a low energy sector with masses less than $O(10)$ MeV [2, 3]. The low energy sector can show up at various observable phenomena like rare Kaon decay or supernovae. As we shall see, the upper bounds on the masses of these low energy sector particles and lower bounds on the couplings provide a way to test the SLIM model. The second model, the so-called AMEND model [4], does not contain such a low energy sector; however, it can lead to interesting phenomenology at direct searches for DM and at the LHC.

The paper is organized as follows: In section II we review the structure of the low energy SLIM scenario and its implication for low energy observations. In section III we explain how the low energy effective SLIM scenario can be embedded within an electroweak symmetric model. In section IV we review the prediction of this model for various observations such as DM annihilation into electron-positron or photon pairs, self-interaction of DM and LFV rare decays. In section V we discuss the possibility of discovering the model at the LHC. In section VI we first describe the structure of AMEND. We then review the neutrino mass matrix, LFV rare decay, DM annihilation and direct and indirect DM searches, electroweak precision test and signatures at colliders within the context of this model. Results are summarized in section VII.

II. THE REAL SLIM SCENARIO AND ITS IMPLICATIONS

In this section, we review the so-called SLIM scenario which has been first introduced in [2]. SLIM is the abbreviation of Scalar as LIght as Mev which describes the characteristics of the DM candidate within this scenario. The scenario is

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quite minimalistic and adds only a scalar, $\delta$ and two (or more) right-handed Majorana neutrinos, $N_i$. A $Z_2$ symmetry is imposed on the scenario under which $\delta$ and $N_i$ are odd but the SM particles are all even. As discussed in [2], the scenario can be realized both for real and complex $\delta$. We will however concentrate on the real SLIM scenario. $\delta$, which is called SLIM, is lighter than the other $Z_2$ odd particles and as a result is stable and can play the role of the DM candidate.

The scenario is based on the following Lagrangian:

$$\mathcal{L} = m_\delta^2 \delta^2 \left( g_{i\alpha} \bar{N}_i \nu_\alpha + \frac{m_{N_i}}{2} \bar{N}_i^c N_i + \text{H.c.} \right).$$  \hspace{1cm} (1)

Of course, if we assume neither $N_i$ nor $\delta$ have electroweak interactions, the $g_{i\alpha}$ coupling in Eq. (1) will not be symmetric under SU(2)×U(1). As a result, it can be only effective. That is, the scenario should be augmented to become symmetric under SU(2)×U(1) at higher energies. This step will be taken in the next section. In this section, we are concerned about the low energy implications of the scenario. Notice that the Majorana mass term for $N_i$ violate lepton number and the coupling $g_{i\alpha}$ is a source of LFV. As shown [2], this Lagrangian through the one-loop diagram shown in Fig. 1 leads to the Majorana mass for active neutrinos

$$m_\nu = U \cdot \text{Diag}[m_1, m_2 e^{2i\gamma_2}, m_3 e^{2i\gamma_3}] U^T.$$  \hspace{1cm} (3)

Combining (2,3), we find

$$g = \text{Diag}(X_1, ..., X_n) \cdot O \cdot \text{Diag}(\sqrt{m_1}, \sqrt{m_2 e^{i\gamma_2}}, \sqrt{m_3 e^{i\gamma_3}}) U^T,$$

where $n$ is the number of Majorana neutrinos and $O$ is an arbitrary $n \times 3$ matrix that satisfies $O^T \cdot O = \text{Diag}(1, 1, 1)$. Finally,

$$X_i = 4\pi \left( \frac{1}{m_{N_i}} \right)^{1/2} \left( \log \frac{\Lambda^2}{m_{N_i}^2} - \frac{m_2^2}{m_{N_i}^2 - m_\delta^2} \log \frac{m_{N_i}^2}{m_\delta^2} \right)^{-1/2}.$$  \hspace{1cm} (5)

As is well-known, within thermal production scenario, the dark matter energy budget of the universe is almost independent of mass and is determined by the DM annihilation cross section. From this observation, the DM annihilation cross section is determined: $\langle \sigma v \rangle \sim 10^{-26}$ cm$^3$sec$^{-1}$. Within the SLIM scenario the main annihilation modes are $\delta\delta \rightarrow \nu_\alpha \nu_\beta, \bar{\nu}_\alpha \bar{\nu}_\beta$. Notice that these processes violate lepton number by two units. Indeed, these processes proceed through $t$-channel diagrams with a lepton number violating right-handed neutrino propagator proportional to $m_{N_i}$:

$$\langle \sigma(\delta\delta \rightarrow \nu_\alpha \nu_\beta)v \rangle = \langle \sigma(\delta\delta \rightarrow \bar{\nu}_\alpha \bar{\nu}_\beta)v \rangle = \frac{1}{4\pi} \sum_i g_{i\alpha} g_{i\beta} m_{N_i}^2.$$

Taking $\Lambda \sim 200$ GeV, 0.01 eV $< m_\nu < 1$ eV and $\langle \sigma(\delta\delta \rightarrow \text{everything})v \rangle \sim 10^{-26}$ cm$^3$sec$^{-1}$, from Eqs. (2,6), we obtain

$$O(1) \text{ MeV} \lesssim m_{N_i} \lesssim 10 \text{ MeV} \quad \text{and} \quad 3 \times 10^{-4} \lesssim g_{1\alpha} \lesssim 10^{-3},$$  \hspace{1cm} (7)
where \( N_1 \) is the lightest right-handed neutrino whose propagator dominates the DM annihilation. Remember that \( \delta \) is taken to be lighter than \( N_1 \) so the DM candidate within this scenario is lighter than 10 MeV. This argument does not set a lower bound on \( m_\delta \); however from primordial nucleosynthesis a lower bound of \( O(1) \) MeV is derived [9]. That is within this scenario, the DM mass is in the MeV range. This is the rationale for naming \( \delta \) as Scalar as Light as Mev, SLIM.

Although in this scenario DM pairs mainly annihilate to neutrino or antineutrino pairs, the energy of the produced neutrinos (\( E_\nu \approx m_\delta < 10 \) MeV) will be too low to be detectable at neutrino telescopes such as ICECUBE. In principle, as shown in [10], neutrinos produced by annihilation of 20-30 MeV DM candidates in dark halo can be observed by future large neutrino detectors such as LENA [11]. However, for the mass range relevant for real SLIM, we do not expect a sizeable signal [12].

The most interesting feature of this scenario is that there is a lower bound on the coupling of the new sector to neutrinos, this model can be eventually tested. This feature and its phenomenological implications have been elaborated on in [12]. Consider the decay \( A \rightarrow B + \nu \) where \( A \) and \( B \) can be detected but \( \nu \) appears as missing energy. If the mass difference between \( A \) and \( B \) is less than the sum of the masses of \( \delta \) and \( N_i \), there will be another contribution to the missing energy signal:

\[
A \rightarrow BN_i \delta
\]

where both \( \delta \) and \( N_i \), like \( \nu \), escape detection. \( N_i \) eventually decay into \( \nu \delta \). Thus, by studying the decay mode \( A \rightarrow B + \text{missing energy} \), information on the parameters of the scenario can be deduced. Similar analysis has extensively been carried out (see Refs. [9,12]) in the case of the Majoron couplings to neutrinos. In particular, consider the decay of \( K^+ \) to the charged leptons. By comparing \( \Gamma(K^+ \rightarrow e^+ + \text{missing energy})/\Gamma(K^+ \rightarrow \mu^+ + \text{missing energy}) \) with the SM prediction, the coupling \( g_{i\alpha} \) can be constrained:

\[
\frac{\Gamma(K^+ \rightarrow e^+ + \text{missing energy})}{\Gamma(K^+ \rightarrow \mu^+ + \text{missing energy})} = \frac{\Gamma_{SM}(K^+ \rightarrow e^+\nu_e) + \sum_i \Gamma(K^+ \rightarrow e^+N_i\delta)}{\Gamma_{SM}(K^+ \rightarrow \mu^+\nu_e) + \sum_i \Gamma(K^+ \rightarrow \mu^+N_i\delta)} \approx \frac{\Gamma_{SM}(K^+ \rightarrow e^+\nu_e) + \sum_i \Gamma(K^+ \rightarrow e^+N_i\delta)}{\Gamma_{SM}(K^+ \rightarrow \mu^+\nu_e)} .
\]

In the last line, we have taken

\[
\Gamma(K^+ \rightarrow e^+N_i\delta) \ll \Gamma(K^+ \rightarrow \mu^+N_i\delta), \quad \frac{\Gamma(K^+ \rightarrow e^+\nu_e)}{\Gamma(K^+ \rightarrow \mu^+\nu_e)} .
\]

Considering the fact that \( \Gamma(K^+ \rightarrow e^+\nu)/\Gamma(K^+ \rightarrow \mu^+\nu) \sim (m_e/m_\mu)^2 \ll 1 \), this assumption is justified. The recent bounds on the ratio \( \Gamma(K^+ \rightarrow e^+ + \text{missing energy})/\Gamma(K^+ \rightarrow \mu^+ + \text{missing energy}) \) from KLOE [13] yields

\[
\sum_i |g_{ie}|^2 < 10^{-5}
\]

where the sum runs over \( N_i \) lighter than \( K^+ \).

The spectrum of the charged lepton in the two-body decay \( K^+ \rightarrow \ell^+ + \nu \) will be of course different from that in the three body decay \( K^+ \rightarrow \ell^+ + N_i + \delta \). Thus, by studying the spectrum of \( \mu^+ \) in \( K^+ \rightarrow \mu^+\text{missing energy} \), information can be derived on \( \sum_i |g_{i\mu}|^2 \). The analysis based on 1973 LBL Bevatron data [14] gives [8]

\[
\sum_i |g_{i\mu}|^2 < 9 \times 10^{-5} .
\]

A more thorough investigation of \( K^+ \rightarrow \mu^+N_i\delta \) can be performed with the present KLOE data as well as with the upcoming NA62 results [15].

Another situation where the SLIM scenario can show up is the supernova explosion during which a neutrino gas of temperature of \( T \sim \) few 10 MeV is formed inside the supernova core. Since the SLIM particles are relatively light and are coupled to neutrinos, they can be produced at the supernova explosion. The produced \( \delta \) can interact with neutrinos in that environment with a cross section given by [8]

\[
\sigma(\delta N_i \rightarrow \delta N_i) \sim \frac{g^4T^2}{4\pi(T^2 + m_{N_i}^2)^2} .
\]

Taking \( T \sim \) few \( \times 10 \) MeV, we find that the mean free path of the SLIM particles is \( (\sigma n_\nu)^{-1} \sim 10 \) cm which is far shorter than the supernova core. As a result, the SLIM will be trapped inside the core. The energy transferred by diffuse out of the SLIM particles can be tolerated within the present uncertainties of supernova model [2]. In case of supernova explosions in the future, the scenario might be tested by studying the neutrino energy spectra [2].
III. ULTRAVIOLET COMPLETION OF THE SLIM SCENARIO

In the previous section, we introduced the low energy SLIM scenario based on the effective Lagrangian in Eq. (1) and briefly discussed its implications on the relevant low energy phenomena. The Lagrangian in Eq. (1) has to be embedded within a SU(2) × U(1) symmetric model. In [2], several ideas for the ultraviolet completion of the SLIM scenario have been suggested. In [3] a minimalistic model have been introduced that embeds the SLIM scenario. In this section, we review this model and in the next section we discuss its implications.

The model presented in [3] is quite minimalistic and is composed of (1) an electroweak singlet, \( \eta \); (2) two Majorana right-handed neutrinos, \( N_i \) and (3) an electroweak doublet with nonzero hypercharge, \( \Phi^T = [\phi^0 \phi^-] \) where \( \phi^0 \equiv (\phi_1 + i\phi_2)/\sqrt{2} \) with real \( \phi_1 \) and \( \phi_2 \). As emphasized before, all these new particles are odd under the \( Z_2 \) symmetry. Imposing the \( Z_2 \) symmetry, the most general \( Z_2 \) even renormalizable Lagrangian involving only the scalars will be of form

\[
\mathcal{L} = - m^2_\Phi \Phi^\dagger \Phi - m^2_\eta \eta^2 - m_{\eta\Phi}\eta(H^T(i\sigma_2)\Phi) + \text{H.c.}
\]

\[
- \lambda_1 |H^T(i\sigma_2)\Phi|^2 - \text{Re}[\lambda_2 (H^T(i\sigma_2)\Phi)^2] - \lambda_3 \eta^2 H^\dagger H - \lambda_4 \Phi^\dagger \Phi H^\dagger \cdot H
\]

\[
- \lambda_5 \frac{\Phi^\dagger \Phi}{2} - \frac{\lambda^2}{4} \eta^4 - \lambda_3 \eta^2 \Phi^\dagger \Phi - m^2_H H^\dagger \cdot H - \frac{\lambda}{2} (H^\dagger \cdot H)^2 .
\] (9)

Positivity of the potential at infinity puts constraints on the couplings [16],

\[
\lambda_1, \lambda_2 > 0, \lambda_3 > -(\lambda_1 \lambda_2)^{1/2}, \lambda_4 > -(\lambda_1 \lambda_2)^{1/2}
\]

and

\[
\lambda_1 - |\lambda_2| + \lambda_4 > -(\lambda_1 \lambda_2)^{1/2} .
\]

Phases of \( \lambda_2 \) and \( m_{\eta\Phi} \) are sources of CP-violation. For simplicity, we impose CP-symmetry on the Lagrangian in Eq. (1) which makes all the parameters in Eq. (9) real.

Setting \( H^T = (0 v_H/\sqrt{2}) \), the mass terms will be of form

\[
\mathcal{L}_m = - m^2_\phi^- |\phi^-|^2 - \frac{m^2_\eta}{2} \eta^2 - \frac{m^2_{\eta\phi}}{2} \phi_1 - m_{\eta\Phi} v_H \phi_1 \eta
\] (10)

where

\[
m^2_{\phi^-} = m^2_\Phi + \lambda_4 \frac{v^2_H}{2} \quad (11)
\]

\[
m^2_\eta = m^2_\phi + \lambda_3 \frac{v^2_H}{2} \quad (12)
\]

\[
m^2_{\phi_1} = m^2_\phi + \lambda_1 \frac{v^2_H}{2} + \lambda_2 \frac{v^2_H}{2} \quad (13)
\]

\[
m^2_{\phi_2} = m^2_\phi + \lambda_1 \frac{v^2_H}{2} - \lambda_2 \frac{v^2_H}{2} . \quad (14)
\]

The parameters are taken in a range that neither of the scalars, except the SM Higgs, develops a vacuum expectation value. \( \phi^- \), being a charged particle should be heavier than \( \sim 80 \) GeV to avoid the direct search bounds [17]. Notice that while \( \phi_1 \) mixes with \( \eta \) through the \( m_{\eta\phi} \) term, there is no such a mixing between \( \phi_2 \) and \( \eta \). Had we taken \( m_{\eta\phi} \) complex, the mixing term would be

\[
\Re[m_{\eta\phi}] v_H \phi_1 \eta + \Im[m_{\eta\phi}] v_H \phi_2 \eta .
\]

However as we discussed above, we take the Lagrangian to be CP-symmetric so \( \phi_2 \) is a mass eigenstate itself. The other neutral mass eigenstates are \( \delta_1 \) and \( \delta_2 \) defined as follows:

\[
\begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} = \begin{bmatrix} \cos \alpha & - \sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \eta \\ \phi_1 \end{bmatrix} \quad (15)
\]
Through a one-loop diagram active neutrinos acquire the following mass

\[
\text{tan} 2\alpha = \frac{2 v_H m_{\nu\Phi}}{m_{\phi_1}^2 - m_{\eta}^2}
\]

(16)

\[
m_{\delta_1}^2 \simeq m_{\eta}^2 - \frac{(m_{\nu\Phi} v_H)^2}{m_{\phi_1}^2 - m_{\eta}^2}
\]

(17)

\[
m_{\delta_2}^2 \simeq m_{\phi_1}^2 + \frac{(m_{\nu\Phi} v_H)^2}{m_{\phi_1}^2 - m_{\eta}^2}
\]

(18)

where in the last two equations we have taken \((m_{\nu\Phi} v_H)^2/(m_{\phi_1}^2 - m_{\eta}^2)^2 \ll 1\). In other words, the mixing, \(\alpha\) is small and the interactions of lightest scalar \(\delta_1\) with the \(W\) and \(Z\)-bosons are suppressed by \(\sin \alpha\) but \(\delta_2\) approximately corresponds to the real component of the electroweak doublet \(\Phi\). Direct searches [18] restrict \(\delta_2\) and \(\phi_2\) to be heavier than ~90 GeV. On the other hand, \(\delta_1\) can be light and play the role of the SLIM described in the previous section. To see this more clearly, let us add the couplings with fermions:

\[
\mathcal{L} = -g_{i\alpha} N_i \Phi^\dagger \cdot \nabla_\alpha \Phi
\]

(19)

where \(L_\alpha\) is the lepton doublet of flavor \(\alpha\): \(L_\alpha^T = (\nu_{L\alpha}, \ell_{L\alpha})\). We focus on the following range of parameters:

\[
m_{\delta_0}^2 < m_{N_1}^2 \ll m_{\phi_2}^2 \simeq m_{\phi_3}^2 \simeq m_{\phi_1} - \sim m_{\text{electroweak}}^2
\]

(20)

and

\[
\frac{|m_{\phi_2}^2 - m_{\delta_2}^2|}{m_{\phi_2}^2 + m_{\delta_2}^2} \simeq -\frac{\lambda_2 v_H^2}{2} \simeq \frac{\sin^2 \alpha}{2} \ll 1.
\]

(21)

\(\delta_1\), which is called SLIM, is the dark matter candidate. The main annihilation mode of DM is to neutrino (antineutrino) pair:

\[
\langle \sigma(\delta_1 \delta_1 \rightarrow \nu_{L\alpha} \nu_{L\beta}) \rangle = \frac{\sin^4 \alpha}{8\pi} \sum_i \frac{g_{i\alpha} g_{i\beta} m_{N_i}}{m_{\delta_1}^2 + m_{N_i}^2}^2.
\]

(22)

Considering that \(m_{\delta_1} \ll m_{N_i}\), from this formula we expect the lightest right-handed neutrino to be one of the main contributors to the annihilation cross section so we find

\[
\text{Max}[g_{i\beta}] \sin \alpha \sim 5 \times 10^{-4} \left( \frac{m_{N_1}}{\text{MeV}} \right)^{1/2} \left( \frac{\langle \sigma v_r \rangle}{3 \times 10^{-26}\text{cm}^3\text{sec}^{-1}} \right)^{1/4} \left( 1 + \frac{m_{\delta_1}^2}{m_{N_1}^2} \right)^{1/2}.
\]

(23)

Through a one-loop diagram active neutrinos acquire the following mass

\[
(m_{\nu})_{\alpha\beta} = \sum_i \frac{g_{i\alpha} g_{i\beta}}{32\pi} m_{N_i} \left[ \sin^2 \alpha \left( \frac{m_{\delta_1}^2}{m_{N_i}^2 - m_{\delta_2}^2} \log \frac{m_{N_i}^2}{m_{\delta_2}^2} - \frac{m_{\delta_1}^2}{m_{N_i}^2 - m_{\delta_1}^2} \log \frac{m_{N_i}^2}{m_{\delta_1}^2} \right) + \frac{m_{\phi_2}^2}{m_{N_i}^2 - m_{\delta_2}^2} \log \frac{m_{N_i}^2}{m_{\delta_2}^2} - \frac{m_{\phi_2}^2}{m_{N_i}^2 - m_{\delta_1}^2} \log \frac{m_{N_i}^2}{m_{\delta_1}^2} \right].
\]

(24)

This formula resembles the mass formula in Eq. [2] with the difference that after UV completion, the UV cutoff has disappeared and instead the masses of the heavy particles, \(m_{\phi_2}\) and \(m_{\delta_2}\), show up in the formulas for the active neutrinos. For \(m_{\delta_1}^2 < m_{N_i}^2 \ll m_{\phi_2}^2\), we find

\[
(m_{\nu})_{\alpha\beta} \simeq \sum_i \frac{g_{i\alpha} g_{i\beta}}{32\pi} m_{N_i} \left[ \sin^2 \alpha \left( \frac{m_{\delta_1}^2}{m_{N_i}^2 - m_{\delta_2}^2} \log \frac{m_{N_i}^2}{m_{\delta_2}^2} - \frac{m_{\delta_1}^2}{m_{N_i}^2 - m_{\delta_1}^2} \log \frac{m_{N_i}^2}{m_{\delta_1}^2} - 1 \right) - \lambda_2 v_H^2 \right].
\]

(25)

We can divide the parameter space to the following two separate regimes: (1) \(\lambda_2 v_H^2/(m_{\phi_2}^2) \gg \sin^2 \alpha \log(m_{\phi_2}^2/m_{N_i}^2)\); (2) \(\lambda_2 v_H^2/(m_{\phi_2}^2) \sim \sin^2 \alpha \log(m_{\phi_2}^2/m_{N_i}^2)\) or \(\lambda_2 v_H^2/(m_{\phi_2}^2) \ll \sin^2 \alpha \log(m_{\phi_2}^2/m_{N_i}^2)\). In the first case, Eq. [25] combined with
Eq. (25) implies $m_{N_1} \ll 1$ which is disfavored by big bang nucleosynthesis [5]. For $\lambda_2 v_H^2/(m_{\phi_2}^2) \gtrsim \sin^2 \alpha \log(m_{\phi_2}^2/m_{N_1}^2)$, we find

$$m_{\delta_1} \ll m_{N_1} \sim \text{few MeV}$$

which is the same condition as we found for the low energy SLIM scenario in the previous section.

Notice that within this model no upper bound on the masses of $\phi^-$, $\delta_2$ or $\phi_2$ is found. However, as we will see below, for relatively light $\Phi$, the model is more natural. From Eq. (23) and the perturbativity of $g_{1\beta}$, we find that $\sin \alpha$ cannot be smaller than $\sim 10^{-4}$. For $\sin \alpha \ll 1$ and $m_{\phi}^2 \gg m_{\eta}^2$, we can write (see Eqs. 16,17)

$$m_{\delta_1}^2 \simeq m_{\eta}^2 - \sin^2 \alpha m_{\phi}^2.$$ (26)

Thus, for $m_{\phi} \gg 100$ GeV, a fine tuned cancelation between the two terms in Eq. (26) is required to maintain $m_{\delta_1}$ below 10 MeV. In other words, based on naturalness of the model we expect $\phi^-$, $\delta_2$ or $\phi_2$ to be within the reach of the LHC. We shall discuss this point in section IV.

Notice that this model has some features in common with the so-called inert model [19] but with the difference that here we have an extra singlet scalar and the main annihilation mode of dark matter pair is into neutrinos. The contribution to the oblique parameters in our model is however similar to that in the inert model. Similarly to the inert model, within this model the SM Higgs can be as heavy as a few 100 GeV without violating bounds from the electroweak precision data. That is because the contribution from the new doublet to the oblique parameters can cancel the one from a SM Higgs.

IV. IMPLICATIONS OF THE SLIM MODEL

The impact of this model on the low energy phenomena such as the decay of light mesons and supernovae is similar to the low energy scenario discussed in section II except that the coupling $g_{1\beta}$ has to be replaced by $g_{1\beta} \sin \alpha/\sqrt{2}$. However, since this model also contains new heavy states, its phenomenology is richer. In particular, the heavy states can be produced at the LHC. We will discuss about this in more detail in section IV. Here, we discuss the impact that this model can have on other phenomena: (1) Annihilation modes of DM to an electron positron pair or photon pair; (2) Dark matter self-interaction; (3) Magnetic dipole moment of the muon; (4) LFV rare decay of charged lepton. As we shall see, bounds on rare decay already constrains a part of the parameter space. Let us discuss them one by one.

A. Annihilation into electron positron pair and photon pair

The annihilation to the $e^-e^+$ pair is loop suppressed by a factor of $e^4/(16\pi^2 \sin^2 \theta_W)^4$ [3]. Because of this suppression, the rate of $DM + DM \rightarrow e^-e^+$ is too low to account for the disputed 511 keV signal. Moreover the flux of radiation from the $e^-e^+$ pair would be too low to be detectable. Of course, in this model the DM is too light to annihilate to $\mu^-\mu^+$ pair.

At one-loop level a pair of $\delta_1$ can also annihilate into a photon pair with cross section

$$\sigma(\delta_1\delta_1 \rightarrow \gamma\gamma) \sim \frac{e^8 \sin^4 \alpha}{8\pi (16\pi^2)^2 \cos^4 \theta_W} \frac{m_{\delta_1}^2}{m_W^2} \sim \text{few} \times 10^{-41} \left( \frac{M_{\delta_1}}{\text{MeV}} \right)^2 \sin^4 \alpha \text{ cm}^3/\text{sec}.$$ (27)

Because of the loop suppression, the flux of photons would be too small to be detectable at Fermi telescope (see, e.g., Fig 4 of [20]).

B. Self-interaction of Dark Matter

The $\lambda'$ couplings in Eq. (9) can lead to the self-interaction of the DM pairs with the following cross section

$$\langle \sigma(\delta_1\delta_1 \rightarrow \delta_1\delta_1)v^2 \rangle \sim \text{Max}[|\lambda_1|^2 \sin^4 \alpha / 8\pi m_{\delta_1}^2, |\lambda_2|^2 \cos^4 \alpha / 8\pi m_{\delta_1}^2, |\lambda_3|^2 \sin^2 \alpha \cos^2 \alpha / 8\pi m_{\delta_1}^2].$$

The self-interaction of DM is constrained by merging galaxy clusters [21]: $\sigma/m_{DM} \lesssim 1 \text{ cm}^2/\text{g}$ which translates into $|\lambda_1|^2 \sin^4 \alpha, |\lambda_2|^2 \cos^4 \alpha, |\lambda_3|^2 \sin^2 \alpha \cos^2 \alpha \lesssim 10^{-4}$. 

C. $(g - 2)_\mu$

Via coupling $g_{i\mu}\bar{N}_i\mu_L(\phi^-)^{\dagger}$, muons receive a magnetic dipole moment at one-loop level:

$$\delta \frac{g - 2}{2} = \sum_i \frac{|g_{i\mu}|^2 m_{\mu}^2}{16\pi^2 m_{\phi^-}^2} K(t_i),$$

where

$$K(t_i) = \frac{2t_i^2 + 5t_i - 1}{12(t_i - 1)^3} - \frac{t_i^2 \log t_i}{2(t_i - 1)^4},$$

in which $t_i = m_{N_i}^2 / m_{\phi^-}^2$. For $t_i \ll 1$,

$$\delta \frac{g - 2}{2} = 5 \times 10^{-12} \sum_i \frac{|g_{i\mu}|^2}{10^{-2}} \left(\frac{100 \text{ GeV}}{m_{\phi^-}}\right)^2,$$

which is two orders of magnitude below the present bound.

D. LFV rare decay

The coupling $g_{i\alpha}\bar{N}_i\ell_L\phi^- \ell'$ leads to the Lepton Flavor Violating rare decays, $\mu \rightarrow e\gamma$, $\tau \rightarrow \mu\gamma$ and $\tau \rightarrow e\gamma$. From the formulas in [22], we find

$$\Gamma(\ell_\alpha \rightarrow \ell_\beta \gamma) = \frac{m_\alpha^3}{16\pi} |\sigma_R|^2,$$

where

$$\sigma_R = \sum_i g_{i\alpha}g_{i\beta}^* \frac{ie m_\alpha}{16\pi^2 m_{\phi^-}^2} K(t_i),$$

where $t_i = m_{N_i}^2 / m_{\phi^-}^2$ and $K(t_i)$ is defined in Eq. (28). Within this model $t_1 \ll 1$ however $t_2$ can be either small or larger than 1. In case that $t_2$ is also small we can write

$$\text{Br}(\mu \rightarrow e\gamma) \sim 2 \times 10^{-4} \sum_i |g_{i\mu}g_{e1}^*|^2 \left(\frac{100 \text{ GeV}}{m_{\phi^-}}\right)^4$$

$$\text{Br}(\tau \rightarrow \ell_\alpha \gamma) \sim 5 \times 10^{-5} \sum_i |g_{i\tau}g_{e\alpha}^*|^2 \left(\frac{100 \text{ GeV}}{m_{\phi^-}}\right)^4.$$

The latest bound [?] on these branching ratios are

$$\text{Br}(\mu \rightarrow e\gamma) < 1.2 \times 10^{-11}$$

$$\text{Br}(\tau \rightarrow e\gamma) < 3.3 \times 10^{-8}$$

$$\text{Br}(\tau \rightarrow \mu\gamma) < 4.4 \times 10^{-8}.$$

These bounds already excludes large values of the couplings. However, the following range is consistent with bounds and is particularly interesting from phenomenological point of view as in this range the forthcoming LFV searches have a good chance of observing a signal:

$$m_{\phi^-} \sim 100 \text{ GeV} \quad g_{i\mu}, g_{i\tau} \sim \text{few} \times 10^{-2} \quad \text{and} \quad g_{i\epsilon} \sim \text{few} \times 10^{-3}.$$
V. SIGNATURE OF THE SLIM MODEL AT THE LHC

As we discussed in the previous section, naturalness implies that the new particles $\phi^-$, $\delta_2$ and $\phi_2$ have masses not much higher than $O(100$ GeV). As a result, these particles are expected to be produced in pairs at the LHC. The produced $\phi^-$ can decay via its coupling in Eq. (19) to charged leptons:

$$\Gamma(\phi^- \to l_\beta N_i) = \frac{|g_{i\beta}|^2 (m_{\phi^-}^2 - m_{N_i}^2)^2}{16\pi m_{\phi^-}^3} \quad \text{for} \quad m_{N_i} < m_{\phi^-} - m_{l_\beta}. \quad (36)$$

Notice that the same couplings that determine the flavor structure of the neutrino mass matrix as well as branching ratios of the LFV rare decays of the charged leptons also determine these decays. This fact provides a means for the cross check of the model. In [24], the feasibility of determining the coupling at the LHC has been studied considering the various sources of background and employing state-of-the-art techniques to enhance the signal to background ratio. In this section, we review the results of this analysis. The details of the analysis and the software that has been used to perform this analysis can be found in [24].

As we saw in section III, at least the lightest $N_1$, which we call $N_1$ has to be light which means the decay modes $\phi^- \to N_1 e^-$, $N_1 \mu^-$ and $N_1 \tau^-$ are all kinematically allowed. However, the second right-handed neutrino can be heavier. The following three situations are possible:

- $m_{N_2} > m_{\phi^-}$. In this case, only $\phi^- \to N_1 l_\beta^-$ will be possible.
- $m_{N_2} < m_{\phi^-}$ and $m_{N_2} \sim m_{\phi^-}$. In this case, both $\phi^- \to N_1 l_\beta^-$ and $\phi^- \to N_2 l_\beta^-$ are kinematically possible and by studying the energy spectrum of the charged lepton, these two decay modes can in principle be distinguished. However, considering that the contribution of $N_2$ to the neutrino mass cannot be larger than about $\sim \sqrt{\Delta m_{\text{atm}}/\Delta m_{\sun}} \sim 10$ times the contribution of $N_1$, Eq. (24) implies that

$$|g_{1\beta}|^2 \ll |g_{2\beta}|^2.$$ 

This means for this situation, $\phi^- \to N_1 l_\beta^-$ will dominate over $\phi^- \to N_2 l_\beta^-$. As a result, the signal for $\phi^- \to l_\beta^- + \text{missing energy}$ will be mostly composed of $\phi^- \to N_1 l_\beta^-$. 

- $m_{N_2} \ll m_{\phi^-}$. In this case, the masses of both $N_1$ and $N_2$ can be neglected and the energy of $l_\beta^-$ in both cases will be approximately equal to $m_{\phi^-}/2$ in the rest frame of $\phi^-$. Thus, the signals for $\phi^- \to N_1 l_\beta^-$ and $\phi^- \to N_2 l_\beta^-$ cannot be distinguished. The signal for $\phi^- \to l_\beta^- + \text{missing energy}$ is determined by the following sum

$$|g_{1\beta}|^2 + |g_{2\beta}|^2.$$ 

In [24] only the case $m_{N_2} > m_{\phi^-}$ is studied but from the above discussion, we conclude that the analysis in [24] also applies for the case $m_{N_2} \ll m_{\phi^-}$ because $\phi^-$ practically only decays to $N_1$ just like the case $m_{N_2} > m_{\phi^-}$. For the case $m_{N_2} \sim m_{N_1}$, the analysis of [24] also applies but one has to replace $|g_{1\beta}|^2$ with $|g_{1\beta}|^2 + |g_{2\beta}|^2$.

To make the analysis simpler it has been assumed in [24] that $m_{\phi_2} \sim m_{\phi^-}$, $m_{\delta_2} \sim m_{\phi^-}$ and $|m_{\delta_2} - m_{\phi_2}|$ do not exceed 80 GeV to forbid two body decays $\delta_2, \phi_2 \to W^+ \phi^-$ or $\phi_2 \to \delta_2 Z$ (or $\delta_2 \to \phi_2 Z$). Moreover, $\phi^-$ can have other decay modes such as $\phi^- \to W^- \delta_1$, $\phi^- \to \delta_1 l^- \nu$ and $\phi^- \to \delta_1 W^- \gamma$ but these modes can be neglected for $g_{1\alpha} \approx 0.01$ and $\sin \alpha \approx 0.01$.

[24] studies the pair production of $\phi^+ \phi^-$, $\phi^+ \phi_2$ and $\phi^+ \delta_2$ at the LHC and the subsequent decay of $\phi^\pm$ to charged leptons. To perform the analysis, the two benchmark points with parameters shown in Table I have been studied. At point A, $\text{BR}(\phi^\pm \to \pm N_1) = 0$ and $\text{BR}(\phi^\pm \to \pm \mu N_1) \approx \text{BR}(\phi^\pm \to \tau N_1) \approx 0.5$. At point B, $\text{BR}(\phi^\pm \to \pm N_1) \approx \text{BR}(\phi^\pm \to \pm \mu N_1) \approx \text{BR}(\phi^\pm \to \tau N_1) \approx 1/3$. The main focus in [24] is on point A.

The cross section of the $\phi^+ \phi^-$ production at the 14 TeV run of the LHC for $m_{\phi^\pm}$ between 80 GeV to 130 GeV varies between 800 fb to 200 fb. At the benchmark point A, the subsequent decay of $\phi^\pm$ will lead to four types of signal $\mu^+ \mu^- + \text{missing energy}$, $\tau^+ \mu^- + \text{missing energy}$, $\mu^+ \tau^- + \text{missing energy}$ and $\tau^+ \tau^- + \text{missing energy}$. The $\tau\tau$ final state is contaminated by large hadronic backgrounds such as $W + \text{jets}$ followed by the decay of $W$ to jets or $\tau$. Light jets fake $\tau$ even after applying the cuts this mode has not been discussed in [24]. In [24], the sum of signals $\tau^+ \mu^- + \text{missing energy}$ and $\mu^+ \tau^- + \text{missing energy}$ has been collectively studied.

The main sources of background are $W^+W^-, t\bar{t}, W+\text{jets}$ and $Z+\text{jets}$. Their cross sections at 14 TeV center of mass energy are shown in Table [11]. Like the $\phi^+ \phi^-$ signal, the $W^+W^-$ pair can lead to $l_\beta^0 l_\gamma^- + \text{missing energy}$. Notice that the cross section of the $W^+W^-$ production is about two orders of magnitude higher than that of the $\phi^+ \phi^-$ production.
|                  | Point A | Point B |
|------------------|---------|---------|
| $m_{N_1}$ (MeV)  | 1       | 1       |
| $m_{N_2}$        | > $m_{\phi^-}$ | > $m_{\phi^-}$ |
| $\alpha$        | 0.01    | 0.01    |
| $\lambda_2$     | 0       | 0       |
| $m_{\phi_1}$ (GeV) | 90      | 90      |
| $\theta$        | $\pi/2$ | 0       |
| $g_{1\alpha}$    | (0, 0.03, -0.01) | (0.01)   |

TABLE I: Model parameters.

Moreover, misidentification of some of the jets or other misidentifications can lead to mimicking the signal. To reduce the background and therefore enhance the signal significance (i.e., signal/\sqrt{background}), several cuts are suggested in [24]. Using these cuts, explicit computation of the signal significance is carried out for benchmark point A at 14 TeV run of the LHC and for 30 fb$^{-1}$. The results are displayed for the $\tau\mu +$ missing energy and $\mu\mu +$ missing energy signals respectively in tables III and IV. As seen from table IV for the values of $m_{\phi^-}$ lower than 130 GeV, the discovery can be made by 30 fb$^{-1}$ of data. Considering that the background is almost the same, the signal significance of the $e^-e^+ +$ missing energy signal can be obtained by scaling that of $\mu^-\mu^+ +$ missing energy by a factor of

$$\frac{\text{Br}(\phi^- \rightarrow N_1 e^-)\text{Br}(\phi^- \rightarrow N_1 e^-)}{\text{Br}(\phi^+ \rightarrow N_1 \mu^+)\text{Br}(\phi^- \rightarrow N_1 \mu^-)} |_{\text{At Point B}} \approx \frac{4}{9},$$

That is at $m_{\phi^\pm} = 80$ GeV and the benchmark point B, the signal significance of $e^-e^+ +$ missing energy for 30 fb$^{-1}$ of data will be 4.1$\sigma$ C.L. As indicated in [24], this is a simplified estimation as there might be some difference between muon and electron reconstruction and selection efficiencies in the detector. A detailed study of these features needs a full simulation of the detector.

A crucial question is whether $g_{i\beta}$ can be derived from the data. As discussed before, deriving the flavor structure of $g_{i\beta}$ helps us to cross-check the model as the same couplings determine the neutrino mass matrix and LFV branching ratios. To derive $g_{i\beta}$, one should extract the signal number, $N_S$ which is generally given by

$$N_S = \frac{N_{\text{obs}} - N_B}{\epsilon_S}$$

(37)

where $\epsilon_S$ is the selection efficiency of the signal, $N_{\text{obs}}$ is the observed number of events and $N_B$ is the contamination due to the background. $N_B$ is calculated by simulation using input such as Parton Distribution Functions (PDF) or total luminosity. The uncertainty in these inputs induce an uncertainty in extracting $N_S$ and therefore the couplings. The main source of uncertainty is the uncertainty in PDFs which at present is about 10 % uncertainty. These uncertainties induce an uncertainty of about 60 % in extracting $N_S$. In [24], it was shown that $\Delta N_S/N_S$ due to these uncertainties does not improve by increasing the luminosity. However, increasing the center of mass energy will enhance signal to background ratio and therefore improve $\Delta N_S/N_S$. Of course, if by using the data of LHC or some other machine, the uncertainty in PDFs are reduced, the precision of extracting $g_{i\alpha}$ can be improved.

Other modes that have been discussed in [24] are $pp \to \phi^+\phi^-$ and $pp \to \phi^+\phi^-$. As discussed earlier in this section, because of the simplifying assumptions on the mass spectrum of the components of $\Phi$, decay modes such as $\phi^- \to W^-\delta_1$ are forbidden so $\delta_2$ and $\phi_2$ can have only invisible decay modes $\delta_2, \phi_2 \to N_1\nu$. As a result, the signal will be composed of a charged lepton from the decay of $\phi^\pm$ plus missing energy which is composed of $N_1$ from $\phi^\pm$ decay and the decay products of $\phi_2$ or $\delta_2$. At benchmark point A and for $m_{\phi^\pm} = 80$ GeV, the significance of the $\mu +$ missing energy and $\tau +$ missing energy signals can reach as high as 9$\sigma$ and 4.6$\sigma$ at 14 TeV energy and 30 fb$^{-1}$ integrated luminosity [24]. Again under simplifying assumption that the selection efficiency of detecting muon and electron is not much different the significance of the $e +$ missing energy signal at the B point is equal to that of $\mu +$ missing energy signal at the A point rescaled by a factor of $\text{Br}(\phi^+ \rightarrow N_1 e^+) |_{\text{At point B}}/\text{Br}(\phi^+ \rightarrow N_1 \mu^+) |_{\text{At point A}} \approx 2/3$.

The above results are for the 14 TeV run of the LHC. In [24], an estimation of the signal significance for the 7 TeV is made by rescaling the cross sections of both background and signal to their values at 7 TeV run. The results for 30 fb$^{-1}$ are displayed at table V.
| Process   | $W^+W^-$ | $t\bar{t}$ | $W$+jets | Z+jets |
|-----------|----------|----------|----------|--------|
| Cross Section | 115.5±0.4 pb | 878.7±0.5 pb | 187.1±0.1 nb | 258.9±0.7 nb |

TABLE II: Background cross sections

| $m(\phi^\pm)$ | 80 GeV | 90 GeV | 110 GeV | 130 GeV |
|---------------|--------|--------|--------|---------|
| Signal significance | 2.8 | 2.2 | 1.4 | 1 |

TABLE III: Signal significance in the $\tau\mu E_T^{miss}$ final state for different $m(\phi^\pm)$ hypotheses at 30 $fb^{-1}$.

VI. AMEND

In this section, we review the AMEND model which was introduced in [4]. AMEND stands for A Model Explaining Neutrino masses and Dark matter. Like the SLIM model, there is a $Z_2$ symmetry that protects DM from decay. The $Z_2$ symmetry also forbids Dirac mass at one-loop level. The $Z_2$ symmetry in this model is the remnant of a global $U(1)_X$ symmetry which is explicitly broken by small parameters. In the limit of exact $U(1)_X$, neutrino masses vanish. Neutrino masses are suppressed both by a loop factor and the small $U(1)_X$ breaking term (i.e., ‘t Hooft criterion). The particle content of the model includes two fermionic doublets $R$ and $R'$ with opposite hypercharges, an electroweak triplet $\Delta$ and a complex singlet, $\phi$. These particles can in principle be produced at colliders. In particular, one of the components of $\Delta$ is doubly charged and can lead to almost background free signal of same sign charged lepton pair plus missing energy via $\Delta^{++} \rightarrow l_1^+l_2^+\delta_{1,2}$. The LFV couplings of these particles lead to LFV rare decays such as $\mu \rightarrow e\gamma$, $\tau \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ at loop level.

Within this model, the DM production in the early universe is thermal. In [4], various possible annihilation modes have been discussed. It was found that the dominant annihilation mode was the DM annihilation through $s$-channel Higgs exchange which can account for the observed density of DM. DM in this model can be counted as a Weakly Interacting Massive Particle (WIMP) and can show up in direct DM search experiment based on measuring recoil energy from scattering DM off nuclei in a background free environment. Various experiments are designed for this purpose. Their results are contradictory. On one hand, experiments such as XENON do not find any signal and on the other hand, the DAMA experiment [25] reports a positive signal at more than 8 $\sigma$ C.L. To reconcile these conflicting results, several attempts have been made. Among these solutions, inelastic DM solution [27] and light DM (< 10 GeV) [28] have received more attention. In [4], null results interpreted as an upper bound on the cross-section as well as the two solutions accommodating the positive signal from direct DM searches have been studied and shown that they can be embedded within the AMEND model by going to the proper regions of the parameter space. However, more recent data from XENON100 [29] and a re-analysis of XENON10 data [26] respectively disfavor the inelastic DM and light DM solutions. We therefore focus on the constraint from DM searches.

In this section, we first describe AMEND and then discuss its implications for various observations.

A. Description of AMEND

The particle content of this model is listed in table VII. As seen from this table, the new particles are the following.

- A complex scalar field which can be decomposed in terms of real fields as $\phi \equiv (\phi_1 + i\phi_2)/\sqrt{2}$;
- A scalar triplet with the following components

$$\Delta = \begin{bmatrix} \Delta^+ \\ \Delta^0 \\ \Delta^{++} \end{bmatrix}/\sqrt{2}. \tag{38}$$


| $m(\phi^\pm)$ | 80 GeV | 90 GeV | 110 GeV | 130 GeV |
|---------------|--------|--------|--------|---------|
| Signal significance | 9.2 | 8.4 | 6.6 | 4.9 |

TABLE IV: Signal significance in $\mu\mu E_T^{miss}$ final state for different $m(\phi^\pm)$ hypotheses.
TABLE V: Signal significance in different final states for the 7 TeV run, provided that 30 fb⁻¹ data is collected at this energy before any switch to higher machine energies.

| Channel | Mass Point | Signal significance |
|---------|------------|---------------------|
| $\phi^+\phi^- \to \tau\mu E_T^{miss}$ | $m_{(\phi^\pm)} = 80$ GeV | 1.6 |
|        | $m_{(\phi^\pm)} = 90$ GeV | 1.2 |
|        | $m_{(\phi^\pm)} = 110$ GeV | 0.7 |
|        | $m_{(\phi^\pm)} = 130$ GeV | 0.5 |
| $\phi^+\phi^- \to \mu\mu E_T^{miss}$ | $m_{(\phi^\pm)} = 80$ GeV | 6.4 |
|        | $m_{(\phi^\pm)} = 90$ GeV | 5.7 |
|        | $m_{(\phi^\pm)} = 110$ GeV | 4.2 |
|        | $m_{(\phi^\pm)} = 130$ GeV | 3.0 |
| $\phi^\pm\phi_2 \to \tau E_T^{miss}$ | $m_{(\phi^\pm)} = 80$ GeV | 2.6 |
|        | $m_{(\phi^\pm)} = 90$ GeV | 5.0 |

TABLE VI: Particle content and gauge quantum numbers.

| particle | SU(3)c | SU(2)L | U(1)Y |
|----------|--------|--------|-------|
| $Q_L$    | 3      | 2      | 1/6   |
| $u_R$    | 3      | 1      | 2/3   |
| $d_R$    | 3      | 1      | -1/3  |
| $\ell_L$ | 1      | 2      | -1/2  |
| $e_R$    | 1      | 1      | -1    |
| $R = R_R$| 1      | 2      | -1/2  |
| $R' = R'_R$| 1  | 2      | 1/2   |
| $H$      | 1      | 2      | 1/2   |
| $\Delta$| 1      | 3      | 1     |
| $\phi$  | 1      | 1      | 0     |

We can write $\Delta^0 = (\Delta_1 + i\Delta_2)/\sqrt{2}$ where $\Delta_1$ and $\Delta_2$ are real scalar fields.

- Two Weyl fermion SU(2)L doublets, $R^T = (\nu_R E_R^-)$ and $(R')^T = (E_R^+ \nu'_R)$.

A symmetry, called G symmetry, is defined under which each of the new particles are charged under a separate U(1). The G symmetry is defined as follows.

$$G \equiv U(1)_R \times U(1)_\phi \times U(1)_{\Delta} \times U(1)_{\ell}$$ (39)

where $U(1)_\phi$ and $U(1)_{\Delta}$ are symmetries under which only $\phi$ and $\Delta$ are respectively charged and $U(1)_R$ is the symmetry under which $R$ and $R'$ have opposite quantum numbers. $U(1)_{\ell}$ is the familiar lepton number U(1) symmetry associated with lepton number. The model is constructed such that the main part of its Lagrangian preserves the G symmetry. In addition to the kinetic and the gauge interaction terms, the most general G-preserving Lagrangian is composed of the following scalar potential

$$V = -\mu_H^2 H^\dagger H + \mu_\Delta^2 Tr (\Delta^\dagger \Delta) + \mu_\phi^2 \phi^\dagger \phi$$

$$+ \frac{\lambda}{4}(H^\dagger H)^2 + \frac{\lambda_\phi}{4}(\phi^\dagger \phi)^2 + \frac{\lambda_{\Delta_1}}{2} Tr(\Delta^\dagger \Delta)^2 + \frac{\lambda_{\Delta_2}}{2} Tr(\Delta^\dagger |\Delta, \Delta| \Delta)$$

$$+ \lambda_{H\Delta_1} H^\dagger H Tr(\Delta^\dagger \Delta) + \lambda_{H\Delta_2} H^\dagger |\Delta, \Delta| H + \lambda_\Delta \phi^\dagger \phi Tr (\Delta^\dagger \Delta) + \lambda_{H\phi} \phi^\dagger \phi H^\dagger H$$ (40)

and the fermionic part which is a Dirac mass term for the fermionic doublet

$$- \mathcal{L}_R = m_{RR}(R^C)^\dagger \cdot R + \text{h.c.}$$ (41)

where $(R^C)^T = (\nu_R^C - (E_R^+)^C)$. $m_{RR}$ should be heavier than $\sim 100$ GeV to avoid the bounds from direct searches. As a reference point, we shall take $m_{RR} = 300$ GeV. At a very high energy scale, the G symmetry breaks to a smaller
U(1)$_X$ symmetry under which the SM particles are all neutral and the quantum numbers of the new particles are as follows.

\[ R \xrightarrow{U(1)_X} +1 \, , \, R' \xrightarrow{U(1)_X} -1 \, , \, \Delta \xrightarrow{U(1)_X} +1 \, \text{and} \, \phi \xrightarrow{U(1)_X} -1. \]

Notice that U(1)$_X$ is anomaly-free and can in principle be fixed. The terms that break G to its U(1)$_X$ subgroup are the following

\[ \mathcal{V}_{H \Delta \phi} = \lambda_{H \Delta \phi} H^T i \sigma_2 \Delta \phi^\dagger H \phi + \text{h.c.} \]

\[ \mathcal{V}_{t_{\ell \phi}} = g_\alpha \phi^\dagger R^\dagger \ell L_\alpha + \text{h.c.} . \]

At a lower energy scale, U(1)$_X$ breaks to a Z$_2$ symmetry under which the SM particles are even but the new particles are all odd. After U(1)$_X \rightarrow Z_2$, the Lagrangian includes the following terms for the scalars

\[ \bar{\mathcal{V}}_{\text{scalar}} = \lambda_{H \Delta \phi} H^T i \sigma_2 \Delta \phi^\dagger H \phi + \tilde{\mu}_\phi^2 \phi^2 + \tilde{\lambda}_\phi \phi^4 + \lambda_{H \Delta \phi} H^T i \sigma_2 \Delta \phi^\dagger H \phi + \tilde{\lambda}_{H \Delta \phi} \Delta^\dagger \Delta \phi^2 + \text{h.c.} . \]

and the following terms for the fermions

\[ - \bar{\mathcal{E}}_{\ell \phi} = \tilde{g}_\alpha \phi^\dagger R^\dagger \ell L_\alpha + \text{h.c.} \, \text{and} \, - \bar{\mathcal{E}}_{\ell \Delta} = (\tilde{g}_\Delta) \alpha R^\dagger \cdot \Delta \cdot \ell L_\alpha + \text{h.c.} \]

The pattern of symmetry breaking implies \( g \gg \tilde{g}_\Delta \) and \( \lambda_{H \Delta \phi} \gg \tilde{\lambda}_{H \Delta \phi} \).

**B. The scalar sector**

Within this model, only the SM Higgs obtains a vacuum expectation value and

\[ \langle \phi_1 \rangle = \langle \phi_2 \rangle = \langle \Delta_1 \rangle = \langle \Delta_2 \rangle = 0 . \]

After electroweak symmetry breaking, \( \phi \) and \( \Delta^0 \) mix with each other. For the CP-symmetric case, CP even scalars \( \phi_1 \) and \( \Delta_1 \) mix only with each other and the CP-odd scalars \( \phi_2 \) and \( \Delta_2 \) mix among each other. The neutral scalar mass eigenstates, \( \delta_1 \), \( \delta_2 \), \( \delta_3 \) and \( \delta_4 \) can be written in terms of the components of \( \phi \) and \( \Delta^0 \) as follows

\[ \begin{pmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \end{pmatrix} = \begin{pmatrix} \cos \alpha_1 & 0 & \sin \alpha_1 & 0 \\ 0 & \cos \alpha_2 & 0 & \sin \alpha_2 \\ -\sin \alpha_1 & 0 & \cos \alpha_1 & 0 \\ 0 & -\sin \alpha_2 & 0 & \cos \alpha_2 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \Delta_1 \\ \Delta_2 \end{pmatrix} , \]

where \( |\tan 2\alpha_1| \approx 2\tan \theta_W \approx 2m_{\phi\Delta}^2/(m_{\Delta}^2 - m_{\phi}^2) \). In the following the masses of \( \delta_i \) are denoted by \( M_i \). The formula for \( M_i \) can be found in [4]. The difference \( ||\alpha_1|| - |\alpha_2|| \) as well as the mass splittings \( |M_2 - M_1| \) and \( |M_4 - M_3| \) are suppressed by the U(1)$_X$-breaking terms. We take \( \delta_1 \) to be the lightest new particle and therefore a DM candidate. As discussed in [2], the CP-odd scalar \( \delta_2 \) could also play the role of DM.

The coupling of \( \delta_1 \) to the Z-boson is of form

\[ \frac{i g_{SU(2)} \sin \alpha_1 \sin \alpha_2}{\cos \theta_W} [\delta_2 \partial_\mu \delta_1 - \delta_1 \partial_\mu \delta_2] Z^\mu , \]

where \( g_{SU(2)} \) is the SM weak gauge coupling and \( \theta_W \) is the Weinberg angle. If \( M_1 + M_2 < m_Z \),

\[ \Gamma(Z \rightarrow \delta_1 \delta_2) = \frac{G_F \sin^2 \alpha_1 \sin^2 \alpha_2}{6\sqrt{2} \pi} m_Z^3 . \]

\( \delta_2 \) will eventually decay to \( \delta_1 \) and a neutrino pair so this decay mode will count as an extra contribution to invisible decay mode of the Z boson. For \( M_1 + M_2 < m_Z \), the upper bound on the extra invisible decay modes of the Z boson [29] implies

\[ \sin \alpha_1 \sin \alpha_2 < 0.07 \]
C. Neutrino masses and LFV rare decays

Within this model, there are one-loop contributions to the neutrino mass matrix of form [3]

\[ (m_\nu)_{\alpha\beta} = |g_\alpha(\hat{g}_{\Delta}\beta + g_\beta(\hat{g}_{\Delta})_{\alpha})\tilde{\eta} + [\hat{g}_\alpha(\hat{g}_{\Delta})_{\beta} + \tilde{g}_\beta(\hat{g}_{\Delta})_{\alpha}]\eta |, \]  

\[ (50a) \]

\[ \eta = \frac{m_{RR}}{64\pi^2} \left( \frac{M_3^2}{m_{RR}^2 - M_3^2} \ln \frac{m_{RR}^2}{M_3^2} - \frac{M_1^2}{m_{RR}^2 - M_1^2} \ln \frac{m_{RR}^2}{M_1^2} \right) \sin 2\alpha_1 - \left[ (\alpha_1, M_1^2, M_2^2) \rightarrow (\alpha_2, M_2^2, M_3^2) \right], \]

\[ (50b) \]

The parameters denoted by tilde are all suppressed by U(1) \( X \) breaking terms. Notice that for \( \hat{g}_{\Delta} = 0 \), the neutrino masses vanish. This is expected as for \( \hat{g}_{\Delta} = 0 \), by assigning lepton number equal to +1 and −1 respectively to \( R \) and \( R' \), lepton number will be conserved so the neutrinos cannot have a Majorana mass term. It is straightforward to confirm that regardless of the flavor structure of the couplings, the determinant of \( m_\nu \) vanishes which means one of the neutrino mass eigenvalues is zero and the neutrino mass scheme is hierarchical. This structure is due to the fact that only two right-handed neutrinos are incorporated within this model. In order to make the neutrino mass scheme non-hierarchical (i.e., \( \text{Det}[m_\nu] \neq 0 \)), another pair of \( R \) and \( R' \) should be added.

For \( m_{RR} = 300 \text{ GeV} \) and \( m_\nu = 0.05 \text{ eV} \), it has been found [3] that

\[ g\hat{g}_\Delta \simeq 3.4 \times 10^{-6} \frac{m_\nu}{0.05 \text{ eV}} \frac{70 \text{ GeV}}{50 \text{ MeV}} \frac{500 \text{ GeV}}{m_{RR}} \frac{m_{RR}}{300 \text{ GeV}} \alpha \beta \delta \sin \alpha_1 \left( \frac{m_{RR}^2}{-m_{RR}^2} \ln \frac{m_{RR}^2}{m_{RR}^2} + 1 - \frac{m_{RR}^2}{M_1^2} \right)^{-1} \]

\[ (51a) \]

\[ \text{for } 2\hat{m}_\phi^2 m_{\phi \Delta}^2 / m_{\Delta}^2 \simeq 2M_1 \delta |\sin \alpha_1| \gg \hat{m}_\phi^2 \, , \]

\[ g\hat{g}_\Delta \simeq 3.3 \times 10^{-6} \frac{m_\nu}{0.05 \text{ eV}} \frac{300 \text{ GeV}}{m_{RR}} \frac{1 \text{ GeV}^2}{m_{\phi \Delta}^2} \left( \frac{m_{\Delta}}{500 \text{ GeV}} \right)^2 \frac{m_{RR}^2 - m_{\Delta}^2}{m_{\Delta}^2} \left( \frac{m_{RR}^2}{m_{\Delta}^2} \right)^{-1} \]

\[ (51b) \]

\[ \text{for } 2\hat{m}_\phi^2 m_{\phi \Delta}^2 / m_{\Delta}^2 \simeq 2M_1 \delta |\sin \alpha_1| \ll \hat{m}_\phi^2 \, , \]

\[ (52) \]

The \( g_\beta \), \( \tilde{g}_\beta \) and \( (\hat{g}_{\Delta})_{\beta} \) couplings will lead to LFV rare decays such as \( l_\alpha \rightarrow l_\beta \gamma \) [3]. Since \( (\hat{g}_{\Delta})_{\beta} \rtimes \tilde{g}_\beta \ll g_\beta \), the dominant contribution is from the \( g \) coupling:

\[ \text{Br}(\mu \rightarrow e\gamma) \approx 2.5 \times 10^{-9} \left( \frac{300 \text{ GeV}}{m_{RR}} \right)^4 \left( \frac{g_\mu^e}{0.1} \frac{g_\mu}{0.1} \right)^2 \]

\[ (53a) \]

\[ \text{Br}(\tau \rightarrow l_\alpha \gamma) \approx 4.5 \times 10^{-10} \left( \frac{300 \text{ GeV}}{m_{RR}} \right)^4 \left( \frac{g_\tau^e}{0.1} \frac{g_\alpha}{0.1} \right)^2 \, . \]

\[ (53b) \]

Let us now discuss the constraints on the parameters from bounds \( l_\alpha \rightarrow l_\beta \gamma \) [? ]. The bounds on \( \text{Br}(\tau \rightarrow e\gamma) \) and \( \text{Br}(\tau \rightarrow \mu\gamma) \) allow even values of \( m_{RR} \) as small as 100 GeV and \( g_{\mu,\tau} \) as large as 0.2. For \( g_\tau \sim 0.2 \), the bound on \( \text{Br}(\mu \rightarrow e\gamma) \) requires relatively large values of \( m_{RR}, m_{RR} \gg 1.1 \text{ TeV} \). However, for \( g_\mu \sim 0.02 \) and \( g_\tau \sim 0.01, m_{RR} \) as small as 100 GeV can still be accommodated. An alternative solution is \( g_e \ll g_\tau \) or \( g_e \gg g_\mu \). In the case \( g_e \ll g_\mu \), the \( g\tilde{g}_{\Delta} \eta \) contribution dominates \( (m_\nu)_{e\alpha} \); i.e., \( (m_\nu)_{e\alpha} = [\tilde{g}_e(\hat{g}_{\Delta})_{\alpha} + \tilde{g}_\alpha(\hat{g}_{\Delta})_e]\eta \). Similarly for the case \( g_e \gg g_\mu \), \( (m_\nu)_{\mu\alpha} = [\tilde{g}_\mu(\hat{g}_{\Delta})_{\alpha} + \hat{g}_\alpha(\hat{g}_{\Delta})_\mu]\eta \).

D. DM annihilation and searches for DM

As discussed before, within this model the DM production is thermal. To obtain the observed amount of the DM density, the annihilation cross section should be

\[ <(\sigma(\delta_1 \delta_1 \rightarrow \text{anything})\nu) > \simeq 3 \times 10^{-26} \text{ cm}^3/\text{sec} \, . \]

\[ (54) \]
where \( v \) is the relative velocity. In fact, since the mass splitting between \( \delta_1 \) and \( \delta_2 \) might be small, the effect of \( \delta_2 \) at the freeze-out epoch has to be taken into account.

In [4], different possible annihilation modes were investigated. Depending on the regions of the parameter space, different annihilation modes dominate. These modes are list as follows: (1) Higgs mediated decay into \( f \bar{f} \): \( \delta_1 \delta_1 \rightarrow h^* \rightarrow f \bar{f} \); (2) Higgs mediated decay into an on-shell \( W \) plus and an off-shell \( W \): \( \delta_1 \delta_1 \rightarrow WW^* \rightarrow Wf\bar{f} \); (3) Annihilation into \( WW \) or \( WW^* \) via gauge interaction; (4) Annihilation into a Higgs pair. Of course annihilation into \( W \) or \( H \) pair can be possible only for heavy \( M_1 \). The annihilation of these particles in the sun center can lead to a hard neutrino flux [30] which is disfavored by the present indirect dark matter searches. Annihilation modes to \( \tau \bar{\tau} \) and \( e\bar{e} \) are also disfavored by indirect DM searches at the neutrino telescopes. We shall focus in the range for which \( m_b < M_1 < 70 \text{ GeV} < m_W \). In this range, the dominant annihilation mode is \( \delta_1 \delta_1 \rightarrow h^* \rightarrow b\bar{b} \) so the emerging neutrino flux from DM will be rather soft and can be tolerated within the present bounds.

The coupling of \( \delta_2 \) and \( \delta_1 \) to the SM Higgs is given by

\[
\lambda_L v_H h \delta_1^2 = \frac{v_H^2}{2} \left( (\lambda_{H\Delta_1} - \lambda_{H\Delta_2}) \sin^2 \alpha_1 + \lambda_{H\phi} \cos^2 \alpha_1 - 2\lambda_{H\Delta\phi} \sin \alpha_1 \cos \alpha_1 \right) h \delta_1^2
\]

\[
\approx \frac{(M_1^2 - \mu^2_{\Delta \phi} \cos^2 \alpha_1 - \mu^2_{\Delta \phi} \sin^2 \alpha_1)}{v_H} h \delta_1^2
\]

where \( i = 1, 2 \) and the sub-dominant \( U(1)_X \) violating terms are neglected. Notice that the couplings of \( U(1)_X \) preserving part of the Lagrangian can be made real by redefining the fields. In particular, \( \lambda_{H\Delta\phi} \) can be made real so there will be no coupling of type \( h \delta_1 \delta_2 \). The coupling of form \( h \delta_1 \delta_2 \) appears when both CP and \( U(1)_X \) symmetries are broken by e.g., \( \delta \lambda_{H \phi} \).

The \( \lambda_L \) coupling leads to

\[
\langle \sigma(\delta_1 \delta_1 \rightarrow f \bar{f})_H v \rangle = N_c \frac{|\lambda_L|^2}{\pi} \frac{m_f^2}{(4 M_1^2 - m_h^2)^2} \frac{(M_1^2 - m_f^2)^{3/2}}{M_1^3} \left( \frac{m_f}{M_1} \right)^4 \left( \frac{f}{0.1} \right)^4 \text{sec}^{-1}.
\]

which is the relative velocity. In fact, since the mass splitting between \( \delta_1 \) and \( \delta_2 \) might be small, the effect of \( \delta_2 \) at the freeze-out epoch has to be taken into account.

Eventually \( \delta_2 \) decays via coupling to a virtual \( Z \) boson exchange [4]:

\[
\Gamma(\delta_2 \rightarrow \delta_1 \nu\bar{\nu}) \approx 15 \left( \frac{M_2 - M_1}{50 \text{ MeV}} \right)^5 \left( \frac{\sin \alpha_1}{0.1} \right)^4 \text{sec}^{-1}.
\]

We therefore generally expect the decay to take place before Big Bang Nucleosynthesis (BBN) so it should not affect the BBN predictions.

The coupling in Eq. (55) leads to the interaction of DM with nuclei via \( t \)-channel Higgs boson exchange so it can be constrained by direct DM searches at underground experiments sensitive to the recoil energy of the nuclei interacting with DM particles. The cross section of the DM with nuclei can be written as [31]

\[
\sigma_n \approx \sigma_p = \frac{|\lambda_L|^2}{\pi} \frac{\mu_{\Delta n}^2 m_n^2 f^2}{M_1^2 m_h^2} \approx 6.5 \times 10^{-45} \left( \frac{\lambda_L}{0.04} \right)^2 \left( \frac{65 \text{ GeV}}{M_1} \right)^2 \left( \frac{120 \text{ GeV}}{m_h} \right)^4 \left( \frac{f}{0.2} \right)^2 \text{cm}^2,
\]

where \( \mu_{\Delta n} \) is the reduced mass of the dark matter-neutron system, \( m_n \) is the nucleon mass and \( f \) is the nuclear matrix element parameter which can vary within the present uncertainties, \( 0.14 < f < 0.66 \) [31]. The same coupling also determines annihilation of DM so we can write

\[
\sigma_n \approx \sigma_p = \frac{f^2 m_n^2 (4M_1^2 - m_h^2)^2}{\pi M_1 m_h^2 v_H^2} \frac{\langle \sigma(\delta_1 \delta_1 \rightarrow h^* \rightarrow \text{SM final states}) \rangle}{4 \Gamma(h \rightarrow \text{SM final states}) |_{m_h \rightarrow 2M_1}}.
\]

The strong bound on [32] already constrains a considerable part of the parameter space. We generally expect a signal at future DM search experiments unless \( M_1 \rightarrow m_h/2 \). The case of \( M_1 \rightarrow m_h/2 \) seems to be unnatural. If the future searches for DM do not find a signal and no neutral stable scalar with mass equal to \( m_h/2 \) is found by colliders, this model will be disfavored and will eventually be ruled out.
E. Electroweak precision tests

The new particles added to SM participate in the electroweak interactions so they can lead to corrections to the electroweak precision parameters. The contributions to the oblique parameters $S, T, W, Y$ were explicitly calculated in [3].

Since $R$ and $R'$ have equal masses, their contributions to $S$ and $T$ parameter exactly cancel each other. Their contribution to $W$ and $Y$ will be non-zero but tiny:

$$W = \frac{g^2_{SU(2)}}{120\pi^2} \frac{m_{W}^2}{m_{RR}^2} \quad \text{and} \quad Y = \frac{g^2_{U(1)}}{120\pi^2} \frac{m_{W}^2}{m_{RR}^2}. $$

The contribution of $\Delta$ to the oblique parameters can be written as

$$\hat{S} = \frac{g^2_{SU(2)}}{24\pi^2} \xi, \quad \hat{T} = \frac{25}{576\pi^2} g^2_{SU(2)} \frac{m_{\Delta}^2}{m_{W}^2} \xi^2, \quad W = \frac{7}{720\pi^2} g^2_{SU(2)} \frac{m_{W}^2}{m_{\Delta}^2}, \quad Y = -\frac{7}{480\pi^2} g^2_{U(1)} \frac{m_{W}^2}{m_{\Delta}^2},$$

(59)

where the relation $2 m_{\Delta}^2 = m_{\Delta}^2 + m_{\Delta}^2$ has been used and the results have been expanded in

$$\xi = \frac{m_{\Delta}^2 - m_{\Delta}^2}{m_{\Delta}^2} = \lambda_{H \Delta^2} \frac{v_H^2}{m_{\Delta}^2}. $$

(60)

The contribution of $\phi$ to the electroweak precision parameters is suppressed by a factor of $|\sin \alpha_1 \sin \alpha_2|$ relative to that of $\Delta$ so it can be neglected. Within this model, Higgs can be heavier than in the SM model because the new contributions can cancel out the contributions from the Higgs to the oblique parameters and the upper bounds from electroweak precision tests on the Higgs mass can be relaxed. Without cancelation (i.e., for a light Higgs mass), the $\hat{T}$ parameter constrains $\xi \lesssim 0.1$ which translates into a bound on the splitting of the components of the triplet. This results in a mild bound on $\lambda_{H \Delta^2}$, e.g., for $m_{\Delta} \approx 500$ GeV, the bound is $\lambda_{H \Delta^2} \lesssim 0.5$.

F. Signatures at colliders

We expect a rich phenomenology for the LHC within this model. For $M_1, M_2 < m_h/2$, the SM Higgs boson can decay into a pair of $\delta_1$ or a pair of $\delta_2$. These new decay modes can dominate over the decay to the $b\bar{b}$ pair when $\lambda_{\Delta} \sim m_h/v_H \sim 0.02$. Decay mode to $\delta_1$ pair will appear as missing energy. If $M_2 - M_1$ is larger than $2m_\alpha$, $\delta_2$ can also decay into $\delta_1 e^- e^+$ which appears as a distinct displaced vertex. In order for the decay to take place within the detector, the following condition is necessary: $d\Gamma_{\delta_2}/2\gamma \gtrsim v$ where $v$ is the velocity of $\delta_2$ and $\gamma = (1 - v^2)^{-1/2}$. $d$ characterizes the size of the detector. As shown in [3] this condition requires $|M_2 - M_1| > 500$ MeV. Remember that $M_2 - M_1$ is suppressed by the U(1)$_{X}$ violating parameters. For smaller values of the splitting the decay mode $H \rightarrow \delta_2 \delta_2$ will also appear as missing energy signal regardless of if the $\delta_2 \rightarrow \delta_1 e^- e^+$ mode is kinematically accessible or not.

If the new particles are not too heavy, the charged particles $\Delta^{++}, \Delta^+, E^-_R$ and $E^{+}_R$ as well as the neutral particles $\delta_1, \delta_2, \nu_R$ and $\nu'_R$ can be produced through electroweak interactions. They will eventually decay into the SM particles plus $\delta_1$ or $\delta_2$. In particular $\Delta^{++}$ can decay to a pair of same-sign charged leptons:

$$\Delta^{++} \rightarrow \ell^{+}_\alpha \ell^{+}_\beta \delta_{1,2}. $$

The background from the SM to the same sign charged lepton pair signal is not very high which makes the discovery of $\Delta^{++}$ easier. $\Gamma(\Delta^{++} \rightarrow \ell^{+}_\alpha \ell^{+}_\beta \delta_{1,2})$ is proportional to $|\langle \bar{g}_\Delta \rangle \alpha_g \beta_{\bar{g}_\Delta} + \langle \bar{g}_\Delta \rangle \beta_g |^2$. The decays of the charged components of $R$ and $R'$ are given by the $g_\alpha$ couplings: $\text{Br}(E^-_R \rightarrow \ell^-_\alpha \delta_{1,2}) \propto |g_\alpha|^2$. In principle, $g_\alpha$ and $\langle \bar{g}_\Delta \rangle_\alpha$ can be directly extracted by studying the flavor pattern of the decay modes of $\Delta^{++}$ and $E_R$. Notice that the coupling determining $\text{Br}(E^+_R \rightarrow \ell^+_\alpha \delta_{1,2})$ also determines $\text{Br}(\ell^{+}_\alpha \rightarrow \ell^{+}_\beta \gamma)$. Moreover the combination $(\langle \bar{g}_\Delta \rangle_\alpha g_\beta + \langle \bar{g}_\Delta \rangle \beta_{\bar{g}_\Delta})$ determining $\Gamma(\Delta^{++} \rightarrow \ell^{+}_\alpha \ell^{+}_\beta \delta_{1,2})$ is exactly the combination appearing in the neutrino mass matrix in Eq. (19).

VII. CONCLUDING REMARKS

In this letter, we have first reviewed the SLIM scenario [2] and the model embedding it [2]. We have then reviewed AMEND, which stands for A Model Explaining Neutrino masses and Dark matter [4]. Both these models are constructed to explain the tiny neutrino mass and provide us with a DM candidate. We have reviewed the implications
of these models in various experiments and observations. We have updated the results in \cite{2,4}, taking into account the most recent data release from experiments such as direct DM searches XENON10 \cite{20} and XENON100 \cite{29}.

In both models, there is a $Z_2$ symmetry that makes DM stable and forbids a Dirac mass term for neutrinos. The neutrinos acquire Majorana masses at one loop level. In the framework of the both models, we generally expect the value of $\text{Br}(\mu \to e\gamma)$ to be within the reach of MEG. The present bound already rules out a part of the parameter space.

The SLIM model has a light sector (mass $< 10$ MeV) and a heavy sector (mass $> m_W$). The light sector consists of the scalar DM candidate, $\phi_1$ and at least one right-handed neutrino, $N_1$. The heavy sector consists of the components of a scalar electroweak doublet: (i) a CP-odd neutral scalar ($\delta_2$); (ii) a CP-even neutral scalar ($\delta_2$); (iii) a charged scalar ($\delta^\pm$). The light sector can show up in supernova explosion and the decay of light mesons. The present bounds from meson decay as well as supernova explosion are too weak to constrain the model. However, the on-going KLOE experiment \cite{13} and future NA62 experiment \cite{15} can test the model. In case of future observation of supernova explosion, invaluable information on this bound can be derived. Considering the fact that there is a lower bound on the coupling of the light sector to active neutrinos and an upper bound on the masses of the light sector, this model is falsifiable by low energy experiments with enough luminosity.

The heavy sector can be produced at the LHC via the electroweak interactions. The present lower bounds are $m_{\phi^-} > 80$ GeV \cite{17} and $m_{\phi_1}, m_{\phi_2} > 90$ GeV \cite{18}. For relatively light new particles ($m_{\phi^-} < 130$ GeV), LHC with $30\text{ fb}^{-1}$ of integrated luminosity and at 14 TeV center of mass energy can make discovery via the $pp \to \phi^+\phi^- \to \mu^-\mu^+$ missing energy mode. In principle, by studying $\phi^- \to l_\alpha^+ + \text{missing energy}$, the coupling $|g_{1\alpha}|^2$ (see Eq. (1)) or for the case that $m_{N_2} \ll m_{\phi^-}$, $|g_{1\alpha}|^2 + |g_{2\alpha}|^2$ can be extracted. These are the same couplings that determine the neutrino mass matrix and the pattern of LFV rare decays. However, as discussed in \cite{3}, to make this possible the large uncertainty induced by the uncertainties in the parton distribution functions should be reduced or the energy of center of mass should be increased to enhance the signal to background ratio.

The other model that we discussed (AMEND) does not necessarily contain a low energy sector. Within this model, a scalar electroweak triplet ($\Delta$) and two fermionic doublets with opposite hypercharges exist. $\Delta$ contains a doubly charged component, $\Delta^{++}$, which can lead to signals consisting of a pair of same sign charged leptons plus missing energy. The background to this signal from the SM is not very high so the discovery chance of the signal should be high provided that $\Delta^{++}$ is not too heavy.

These two models have quite different predictions for direct DM searches. Within the SLIM model, we do not expect a signal in the future DM searches. However, within AMEND, we generally expect a signal in the future DM searches. Only in some very specific parts of the parameter space such as when the DM mass approaches half the Higgs mass, the DM nucleon cross section is suppressed. The present bounds from direct DM searches already rule out a part of the parameter space.

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