Study of the $Z$ Boson at LEP

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Abstract

The $Z$ line shape is measured at LEP with an accuracy at the per mille level. Usually it is described in the Standard Model of electroweak interactions with account of quantum corrections. Alternatively, one may attempt different model-independent approaches in order to extract quantities like mass and width of the $Z$ boson. If a fit deviates from that in the standard approach, this may give hints for New Physics contributions. I describe two model-independent approaches and compare their applications to LEP data with the Standard Model approach.

1 Introduction

From 1989 till 1995 about 18 millions of $Z$ bosons have been produced at LEP1 and about 200 000 at SLC. Due to this, and due to the lack of direct hints for the existence of a Higgs boson, the $Z$ boson and its interactions became for several years the central theme of tests of the Standard Model \cite{1,2,3}, recently accompanied by the discovery of the $t$ quark at the Tevatron \cite{4,5}.

The predictions of the electroweak Standard Model depend on the particle masses, fermion mixings, and one coupling constant. One of the best measured parameters is by now the $Z$ boson mass.

In 1983, at the $p\bar{p}$ collider SPS (CERN) the $Z$ boson was discovered \cite{6,7} and the mass could be determined at that time with an accuracy of several GeV; in 1986:

$$M_Z = 92.6 \pm 1.7 \text{ GeV}$$

At the end of 1989 LEP1 and SLC started operation and dominated the precision experiments for tests of the electroweak Standard Model for a decade. This may be exemplified by quoting the following improvements of precision from August 1989 \cite{8} till October 1997 \cite{9}:

$$M_Z = 91.120 \pm 0.160 \text{ GeV} \quad \rightarrow \quad 91.1867 \pm 0.0020 \text{ GeV}$$
$$\sin^2 \theta^\text{eff}_w = 0.23300 \pm 0.00230 \quad \rightarrow \quad 0.23152 \pm 0.00023$$
\[
m^{\text{pred}} = 130 \pm 50 \text{ GeV} \quad \rightarrow \quad m^{\text{meas}} = 175.6 \pm 5.5 \text{ GeV} \quad (4)
\]
\[
M^{\text{pred}}_H > \text{ few GeV} \quad \rightarrow \quad \geq 77 \text{ GeV} \quad (5)
\]
\[
\alpha_s(M_Z) = 0.110 \pm 0.010 \quad \rightarrow \quad 0.119 \pm 0.003 \quad (6)
\]

2 The \( Z \) line shape

The \( Z \) boson may be studied as a resonance at LEP from a measurement of the cross-section

\[
e^+e^- \rightarrow (\gamma, Z) \rightarrow \bar{f}f(\pm n\gamma) \quad (7)
\]
as a function of the beam energy; see Fig. 1. The determinations of mass \( M_Z \) and width \( \Gamma_Z \) are dominated by hadron production in a small region around the peak: \(|\sqrt{s} - M_Z| < 3 \text{ GeV}\). The \( Z \) is not a pure Breit-Wigner resonance. We want to study a \( 2 \rightarrow 2 \) process with intermediate \( Z \), but have also virtual photon exchange. In addition, there are huge \( 2 \rightarrow 3 \) contributions due to initial state radiation (ISR) of photons and due to final state radiation (FSR). Further, many virtual corrections are contributing as quantum corrections: vertex insertions, self energy insertions, box diagrams, and all their iterations.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{The muon production cross-section over a wide energy range; figure by courtesy of Frederic Teubert.}
\end{figure}
3 Real photonic corrections

The QED corrections may be taken into account in a universal way by the following convolution formula ([10, 11, 12, 13] and references therein):

\[ \sigma(s) = \int \frac{ds'}{s} \sigma_0(s') \rho \left( \frac{s'}{s} \right) + \int \frac{ds'}{s} \sigma_0^{\text{int}}(s, s') \rho^{\text{int}} \left( \frac{s'}{s} \right) \]  

- \( \rho(s'/s) \) – the radiator describes initial and final state radiation, including leading higher order effects and soft photon exponentiation;
- \( \sigma_0(s') \) – the basic scattering cross-section, which is the object of investigation.

The \( \rho^{\text{int}}(s'/s) \) takes into account the initial-final state interference effects which are comparatively small (a few per mille) near the \( Z \) resonance but are bigger off the resonance, and \( \sigma_0^{\text{int}}(s, s') \) is a function similar to \( \sigma_0(s') \), but suppressed if \( \rho^{\text{int}}(s'/s) \) is small.

4 Method (I): A model-independent ansatz

QED corrections are treated by the convolution formula introduced in section 3. For a careful discussion of their influence on height and location of the \( Z \) peak see [14].

The following ansatz for \( \sigma_0(s) \) is a good choice without explicit reference to the Standard Model [13, 10, 17, 18]:

\[ \sigma_0(s) = \frac{4}{3} \pi \alpha_{\text{em}}^2 \left[ \frac{r^{\gamma}}{s} + \frac{s \cdot R + (s - M_Z^2) \cdot J}{s - M_Z^2 + is \Gamma_Z/M_Z^2} \right] \]  

The line shape is described by five parameters:
- \( r^{\gamma} \sim \alpha_{\text{em}}^2 (M_Z^2) \) – may be assumed to be known
- \( M_Z, \Gamma_Z \)
- \( R \) – measure of the \( Z \) peak height; related to \( \sigma_0^{\text{had}}, \sigma_0^{\text{lept}} \)
- \( J \) – measure of the \( \gamma Z \) interference; often fixed to standard model value

4.1 Z line shape fit (I)

With the model-independent ansatz, the following nearly uncorrelated observables may be determined from the \( Z \) peak data [14, 20]:

\[ M_Z = 91.1867 \pm 0.0020 \ \text{GeV} \quad (\delta = 0.0025 \%) \]  
\[ \Gamma_Z = 2.4948 \pm 0.0025 \ \text{GeV} \quad (\delta = 1.3 \%) \]
\[ \sigma_{0}^{\text{had}} = 41.486 \pm 0.053 \text{ nb } \quad (\delta = 1.9 \%) \]  
(12)

\[ R_l = \frac{\sigma_{0}^{\text{had}}}{\sigma_{0}^{\text{lept}}} = 20.775 \pm 0.027 \quad (\delta = 1.5 \%) \]  
(13)

\[ A_{FB,0}^{\text{lept}} = 0.0171 \pm 0.0010 \]  
(14)

Here, \( M_Z, \Gamma_Z, \sigma_{0}^{\text{had}} \) are from \( \sigma^{\text{had}}(s) \), while \( R_l \) and \( A_{FB,0}^{\text{lept}} \) from \( \sigma^{\text{lept}}(s) \): with \( \sigma_{0}^{\text{had(lep)}} \) as hadronic (leptonic) peak cross-section, and \( A_{FB,0}^{\text{lept}} \) as forward-backward asymmetry at the peak. These parameters are considered to be primary parameters in contrast to derived ones, e.g. the effective leptonic weak neutral current couplings of leptons or the effective weak mixing angle [19, 20]:

\[ v_l = -0.03793 \pm 0.00058 \]  
(15)

\[ a_l = -0.50103 \pm 0.00031 \]  
(16)

\[ \sin^2 \theta_{w}^{\text{eff}} \equiv \frac{1}{4} \left( 1 - \frac{v_l}{a_l} \right) = 0.23152 \pm 0.00023 \]  
(17)

5 Method (II): Virtual corrections in the Standard Model

All virtual corrections may be written in some theory, e.g. the Standard Model, for massless particle production in the following way (see e.g. [21, 22] and [23, 24, 25, 26, 27] and references therein):

\[
\mathcal{M}_{\text{net}} \sim \frac{\alpha_{em}(s)}{s} \left\{ \frac{\alpha_{em}(s)}{\alpha_{em}} |Q_e Q_f| \gamma_{\beta} \otimes \gamma_{\beta} + \chi(s) \varrho_{ef} \left[ L_{\beta} \otimes L_{\beta} - 4s^2_w |Q_e| \kappa_{e\gamma} L_{\beta} \otimes \gamma_{\beta} + 16s^4_w |Q_e Q_f| \kappa_{e\gamma} \otimes \gamma_{\beta} \right] \right\} \]  
(18)

We use short notations: \( L_{\beta} = \gamma_{\beta}(1 + \gamma_{5}) \), \( A_{\beta} \otimes B_{\beta} = [\bar{v}_e A_{\beta} u_e] \cdot [\bar{u}_b B_{\beta} v_b] \), and

\[ \chi = \chi(s) = \frac{G_{\mu} M^2_Z}{\sqrt{2} \cdot 8 \pi \alpha_{em} s} \frac{s}{s - M^2_Z + iM_Z \Gamma_Z(s)} \quad \Gamma_Z(s) = \frac{s}{M^2_Z} \Gamma_Z \]  
(19)

The effective Born cross-section now is uniquely determined once the net matrix element \( \mathcal{M}_{\text{net}} \) is known:

\[ \sigma_{0}(s) = N_c^{f} \sqrt{1 - 4m^2_f/s} \frac{4\pi \alpha_{em}^2}{3s} \times \]  
(20)

\[
\left\{ \left( 1 + \frac{2m^2_f}{s} \right) |Q_e Q_f|^2 \frac{\alpha_{em}(s)}{\alpha_{em}^2} + 2 |Q_e Q_f| \text{Re} \left( \chi \frac{\alpha_{em}(s)}{\alpha_{em}} \varrho_{ef} v_{ef} \right) + |\varrho_{ef}|^2 (1 + |v_e|^2 + |v_f|^2 + |v_{ef}|^2) \right\} - \frac{6m^2_f}{s} |\varrho_{ef}|^2 (1 + |v_e|^2) \]
with

\[ v_i = 1 - 4s_w^2|Q_i|\kappa_i, \quad i = e, f \]  
\[ v_{ef} = 1 - 4s_w^2|Q_e|\kappa_e - 4s_w^2|Q_f|\kappa_f + 16s_w^4|Q_eQ_f|\kappa_{ef} \]  

Further, \( N_c^f = 1, 3 \) is the colour factor and QCD corrections also have to be taken into account.

The virtual corrections with higher order parts for the form factors may be found in [23, 27, 28] and references therein. Further, we need expressions for \( M_W, \Gamma_Z, \alpha_{em} \) and a reasonable treatment of QCD corrections and explicit expressions for the structures shown above.

The \( W \) boson mass is:

\[ M_W = M_Z \sqrt{1 - \left(1 - \frac{4\pi\alpha_{em}}{\sqrt{2}G_\mu M_Z^2[1 - \Delta r]}\right)} \]  

For \( \Delta r \), the \( Z \) width [29, 30, 31, 32], \( \alpha_{em} [33, 34] \), as well as QCD corrections [35], and all the other expressions left out here I have to refer to literature quoted above and to references therein.

### 5.1 \( Z \) line shape fit (II)

I quoted all the above formulae in order to demonstrate explicitly how involved a Standard Model fit ansatz is. The input quantities are: \( \alpha_{em}, G_\mu \) (for \( M_W \), \( M_Z, m_f, M_H, \alpha_s \). Some of them are precisely known (e.g. \( G_\mu \)), others are subject of determination at LEP (e.g. \( M_Z \)), others are completely unknown (\( M_H \)). The \( t \) quark mass may be determined from weak loop corrections at LEP or directly from \( t \) quark production at Fermilab.

Quantities like the \( Z \) width or the weak mixing angle are not a subject of fits since they are considered to be secondary quantities. In this respect, there is a basic difference to the approach of the foregoing section.

The most recent Standard Model fit is [13, 20]. The \( t \) quark mass from Fermilab is:

\[ m_t = 175.6 \pm 5.5 \text{ GeV} \]  

The global fit to all data yields:

\[ M_Z = 91.1867 \pm 0.0020 \text{ GeV} \]  
\[ m_t = 173.1 \pm 5.4 \text{ GeV} \]  
\[ M_H = 115^{+16}_{-66} \text{ GeV} \]  
\[ \alpha_s(M_Z) = 0.120 \pm 0.003 \]

In a next step, one may calculate the other quantities like the \( Z \) width and relate to values from model-independent fits. Whatever one does, there is no unique hint to New Physics. For a detailed discussion of this see [9].
6  Method (III): The S-matrix approach

When the $Z$ boson is treated as a resonance, the S-matrix approach may be used for its description. This was proposed in the context of LEP physics in [17], where the perturbation expansion in the Standard Model was studied. In [18] it was proposed to use this approach for a direct fit to LEP data and the first S-matrix fit was performed therein. The first fit by a LEP collaboration was due to L3 [36, 37]. The treatment of asymmetries near the peak was discussed in [18]. For the role of QED corrections to asymmetries see [38].

A recent survey on the definition of $Z$ mass and width and their treatment in fermion pair production is [39]. Here, I give a short introduction to the technical essentials.

Consider four independent helicity amplitudes in the case of massless fermions:

$$M_{fi}^s = \frac{R_f^f}{s} + \frac{R_Z^{fi}}{s-s_Z} + \sum_{n=0}^{\infty} \frac{F_n^{fi}}{m_Z^n} \left( \frac{s-s_Z}{m_Z} \right)^n, \quad i = 1, \ldots, 4. \quad (29)$$

The position of the $Z$ pole in the complex $s$ plane is given by

$$s_Z = \frac{m_Z^2}{2} - i m_Z \Gamma_Z. \quad (30)$$

The $R_f^f$ and $R_Z^{fi}$ are complex residua of the photon and the $Z$ boson, respectively. One may approximate (29) by setting $F_n^{fi} \to 0$. There are four residua $R_Z^{fi}$ for $e_L^e_L e_R^R e_R^e_L \to f_R^+ f_L^-, e_R^e_R e_R^e_L \to f_R^+ f_L^-, e_R^e_L e_R^e_L \to f_R^+ f_L^-$, and $e_R^e_R e_R^e_L \to f_R^+ f_L^-$. The amplitudes $M_{fi}^s$ give rise to four cross-sections $\sigma_i$: $\sigma_T^0(s), \sigma_{FB}^0(s), \sigma_{pol}^0(s), \sigma_{lr}^0(s)$. Here, the $\sigma_T^0$ – the total cross-section, $\sigma_{FB}^0$ – numerator of the forward-backward asymmetry, $\sigma_{pol}^0$ – that of the final state polarization etc. All these cross-sections may be parameterized by the following master formula ($A = T, F B, \ldots$):

$$\sigma_A^0(s) = \frac{4}{3} \pi \alpha_{em} \frac{r^A}{s} + \frac{sr_A}{(s-m_Z^2)^2 + m_Z^2 \Gamma_Z^2} + \ldots \quad (31)$$

Without QED corrections, asymmetries are:

$$A_A^0(s) = \frac{\sigma_A^0(s)}{\sigma_T^0(s)} = A_T^A + A_1^A \left( \frac{s}{m_Z^2} - 1 \right) + \ldots, \quad A \neq T \quad (32)$$

They take the above extremely simple approximate form around the $Z$ resonance. At LEP1, the higher order terms in the Taylor expansion may be neglected since $(s/m_Z^2 - 1)^2 < 2 \times 10^{-4}$. The coefficients have a quite simple form:

$$A_T^0 = \frac{r_T}{r_T^A}, \quad A_1^0 = \left[ \frac{j_T}{r_A} - \frac{j_T^A}{r_T^A} \right] A_T^0 \quad (33)$$

A comment is necessary concerning the definition of mass and width of the $Z$ boson. The so-called pole definition with a constant width [39] as a natural
consequence of the S-matrix ansatz leads to different numerical values compared to the usual Standard Model approach ([19] [20, 21, 22]. A very precise approximation is:

\[
\overline{m}_Z = \left[1 + \left(\frac{\Gamma_Z}{M_Z}\right)^2\right]^{-\frac{1}{2}} M_Z \approx M_Z - \frac{1}{2} \frac{\Gamma_Z^2}{M_Z} = M_Z - 34 \text{ MeV} \tag{34}
\]

### 6.1 Z line shape fit (III)

The interest of the community in an S-matrix based fit to the LEP data has several origins. One is the wish for a model-independent description of the resonance. Closely related is the question on the number of independent parameters needed to describe the peak: four (per channel) suffice to describe a cross-section: \(M_Z, \Gamma_Z, r_T, j_T\), provided we assume QED interactions to be understood. Among these parameters, \(Z\) mass and width are universal for all channels. Any asymmetry introduces two additional degrees of freedom (per channel): \(r_A, j_A\).

There are practical aspects of all this. If the number of different energy points needed for a scan of the \(Z\) peak is asked for, the answer is at least five (four plus one) for cross-sections, at least three (two plus one) for asymmetries. Further, the \(\gamma Z\) interferences \(j_A\) form separate degrees of freedom. The \(j_T\) and \(M_Z\) are highly correlated. This became more important recently when the highest statistics were taken, and also with the data collected at energies farther away from the peak. There the interference becomes more influential.

Recent experimental studies are summarized in [43]. The data of table 1 are obtained from the \(Z\) line shape scans at LEP which were performed mainly in 1993 and 1995 (from table 7 of [43]). The biggest error correlations are shown in table 2 (from table 8 of [43]). Including into the analysis cross-sections measured at TRISTAN energies does not improve substantially e.g. the resolution of \(M_Z\) and \(j_T\) [43].

| Parameter | S-matrix fit | SM Prediction |
|-----------|--------------|---------------|
| \(\overline{m}_Z\) [GeV] | 91.153 \(\pm\) 0.0034 | 91.1534 \(\pm\) 0.0033 |
| \(\Gamma_Z\) [GeV] | 2.4924 \(\pm\) 0.0026 | 2.4924 |
| \(j_T^\text{rad}\) | 2.9623 \(\pm\) 0.0067 | 2.9623 |
| \(j_T^\text{had}\) | 0.15 \(\pm\) 0.15 | 0.22 |
| \(j_T^\text{lept}\) | 0.14239 \(\pm\) 0.00034 | 0.14253 |
| \(j_T^\text{lept}\) | 0.009 \(\pm\) 0.012 | 0.004 |
| \(j_T^\text{FB}\) | 0.00304 \(\pm\) 0.00018 | 0.00266 |
| \(j_T^\text{FB}\) | 0.789 \(\pm\) 0.013 | 0.799 |

Table 1: Results from a combined LEP1 line shape fit

\(^1\)Note that the table shows values of the complex pole mass \(\overline{m}_Z\). The Standard Model fits use the on shell mass \(M_Z\); the relation of both is given in [44].
| Correlation     | Value |
|-----------------|-------|
| $M_Z - j_T^{\text{had}}$ | -0.77 |
| $M_Z - j_T^{\text{lept}}$ | -0.47 |
| $\Gamma_Z - \Gamma_T^{\text{had}}$ | 0.80 |
| $\Gamma_Z - \Gamma_T^{\text{lept}}$ | 0.62 |
| $\Gamma_T^{\text{had}} - \Gamma_T^{\text{lept}}$ | 0.78 |
| $j_T^{\text{had}} - j_T^{\text{lept}}$ | 0.49 |

Table 2: Biggest correlations in the S-matrix fit

To summarize, from both the strong experimental correlations in the S-matrix fit and the excellent agreement of the central values of fitted parameters in all fit scenarios one may conclude that the different scenarios are highly compatible with each other.

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References

[1] S. L. Glashow, *Nucl. Phys.* 22 (1961) 579.
[2] S. Weinberg, *Phys. Rev. Lett.* 19 (1967) 1264.
[3] A. Salam, “Weak and Electromagnetic Interactions”, in N. Svartholm (ed.), *Elementary Particle Theory*, Proceedings of the Nobel Symposium held 1968 at Lerum, Sweden, (Almqvist and Wiksell, Stockholm, 1968), pp. 367-377.
[4] CDF Collaboration, F. Abe *et. al.*, *Phys. Rev. Lett.* 74 (1995) 2626–2631.
[5] D0 Collaboration, S. Abachi *et. al.*, *Phys. Rev. Lett.* 74 (1995) 2632–2637.
[6] UA1 Collaboration, G. Arnison *et. al.*, *Phys. Lett.* 126B (1983) 398–410.
[7] UA2 Collaboration, P. Bagnaia *et. al.*, *Phys. Lett.* 129B (1983) 130–140.
[8] G. Altarelli, “Theory of precision electroweak experiments”, in *Proc. of the 14th Int. Symp. on Lepton and Photon Interactions, 7-12 Aug 1989, Stanford* (M. Riordan, ed.), pp. 286–304, World Scientific, Teaneck, N.J., 1990.

[9] G. Altarelli, “The status of the standard model”, preprint CERN-TH. 97-278 (1997), rapporteur's talk at LP97, Hamburg, hep-ph/9710434.

[10] D. Bardin, M. Bilenky, A. Chizhov, A. Sazonov, Y. Sedykh, T. Riemann, and M. Sachwitz, *Phys. Lett.* **B229** (1989) 405.

[11] D. Bardin, M. Bilenky, A. Sazonov, Y. Sedykh, T. Riemann, and M. Sachwitz, *Phys. Lett.* **B255** (1991) 290–296.

[12] D. Bardin, G. Passarino, and W. Hollik (eds.), “Reports of the working group on precision calculations for the Z resonance”, report CERN 95-03 (1995).

[13] P. Christova, M. Jack, and T. Riemann, in preparation.

[14] W. Beenakker, F. A. Berends, and S. C. van der Marck, *Z. Phys.* **C46** (1990) 687–692.

[15] A. Borrelli, M. Consoli, L. Maiani, and R. Sisto, *Nucl. Phys.* **B333** (1990) 357.

[16] F. Jegerlehner, *Prog. Part. Nucl. Phys.* **27** (1991) 1–76.

[17] R. G. Stuart, *Phys. Lett.* **B262** (1991) 113–119.

[18] A. Leike, T. Riemann, and J. Rose, *Phys. Lett.* **B273** (1991) 513–518.

[19] D. R. Ward, “Tests of the Standard Model W mass and WWZ couplings”, plenary talk at the Int. Europhysics Conf. on High Energy Physics, 19-26 Aug 1997, Jerusalem, transparencies available from http://www.hep97.ac.il/pl15.htm.

[20] LEP Electroweak Working Group and SLD Heavy Flavor Group, R. Clare et. al., “A combination of preliminary electroweak measurements and constraints on the Standard Model”, CERN Internal Note LEPEWWG/97-02 (Aug 1997).

[21] D. Bardin, P. Christova, and O. Fedorenko, *Nucl. Phys.* **B175** (1980) 435.

[22] D. Y. Bardin, P. C. Christova, and O. M. Fedorenko, *Nucl. Phys.* **B197** (1982) 1.

[23] D. Y. Bardin, M. S. Bilenky, G. Mitselmakher, T. Riemann, and M. Sachwitz, *Z. Phys.* **C44** (1989) 493.

[24] D. Bardin, M. Bilenky, P. Christova, T. Riemann, M. Sachwitz, and H. Vogt, *Comput. Phys. Commun.* **59** (1990) 303.

[25] D. Bardin, M. Bilenky, A. Chizhov, P. Christova, O. Fedorenko, M. Jack, L. Kalinovskaya, A. Olshevsky, S. Riemann, T. Riemann, M. Sachwitz, A. Sazonov, Y. Sedykh, I. Sheer, and L. Vertogradov, Fortran program ZFITTER; obtainable from http://www.ifh.de/~riemann/zf5_00.uu.
[26] D. Bardin et. al., “ZFITTER v. 4.5: An analytical program for fermion pair production in $e^+e^-$ annihilation”, CERN preprint CERN-TH. 6443/92 (1992), hep-ph/9412201.

[27] D. Bardin et. al., “Electroweak working group report”, in Reports of the Working Group on Precision Calculations for the Z Resonance, CERN 95–03 (1995) (D. Bardin, W. Hollik, and G. Passarino, eds.), pp. 7–162, hep-ph/9709229.

[28] D. Bardin et. al., in preparation.

[29] A. Akhundov, D. Bardin, and T. Riemann, Nucl. Phys. B276 (1986) 1.

[30] F. Jegerlehner, Z. Phys. C32 (1986) 425. E: ibid., C38 (1988) 51.

[31] W. Beenakker and W. Hollik, Z. Phys. C40 (1988) 141.

[32] J. Bernabéu, A. Pich, and A. Santamaria, Phys. Lett. 200B (1988) 569.

[33] S. Eidelman and F. Jegerlehner, Z. Phys. C67 (1995) 585–602.

[34] F. Jegerlehner, Nucl. Phys. Proc. Suppl. 51C (1996) 131–141.

[35] K. Chetyrkin, J. Kühn, and A. Kwiatkowski, “QCD corrections to the $e^+e^-$ cross-section and the Z boson decay rate”, in Reports of the Working Group on Precision Calculations for the Z Resonance, CERN 95–03 (1995) (D. Bardin, W. Hollik, and G. Passarino, eds.), pp. 175–263, hep-ph/9503396.

[36] L3 Collaboration, O. Adriani et. al., Phys. Rept. 236 (1993) 1–146.

[37] L3 Collaboration, O. Adriani et. al., Phys. Lett. B306 (1993) 187.

[38] T. Riemann, Phys. Lett. B293 (1992) 451–456.

[39] T. Riemann, “The Z boson resonance parameters”, preprint DESY 97-001 (1997), to appear in the Proc. of the 21st Int. Colloquium on Group Theoretical Methods in Physics, Goslar, Germany, 15-20 July 1996, hep-ph/9709208.

[40] D. Y. Bardin, A. Leike, T. Riemann, and M. Sachwitz, Phys. Lett. B206 (1988) 539–542.

[41] F. A. Berends, G. Burgers, W. Hollik, and W. L. van Neerven, Phys. Lett. 203B (1988) 177.

[42] E. N. Argyres, W. Beenakker, G. J. van Oldenborgh, A. Denner, S. Dittmaier, J. Hoogland, R. Kleiss, C. G. Papadopoulos, and G. Passarino, Phys. Lett. B358 (1995) 339–346.

[43] The S-Matrix Subgroup of the LEP Electroweak Working Group, R. Clare et. al., “An investigation of the interference between photon and Z-boson exchange”, CERN Internal Note LEPEWWG/LS/97-02 (Sept 1997).