One-loop counterterms in the Yang-Mills theory with
gauge invariant ghost field Lagrangian.

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Abstract

One-loop calculations of renormalization constants $Z_1$, $Z_2$ in the model, proposed in the paper [1] with gauge invariant ghost field Lagrangian are performed. It is shown that the model is asymptotically free and the renormalization constants satisfy the same equation as in the ordinary Yang-Mills theory.

1 Introduction.

In the Yang-Mills theory ghost field Lagrangian doesn’t possess gauge symmetry and together with a gauge fixing term breaks gauge invariance of the effective action. It makes problems for regularization and renormalization of the theory. The Slavnov-Taylor identities are more complicated then the Ward identities in electrodynamics. Recently a new formulation of the Yang-Mills theory was proposed [1] in which the gauge invariance is broken only by the gauge fixing term. In the paper [1] it was shown that in the framework of perturbation theory this model is equivalent to the usual Yang-Mills theory. Calculating the observables in this model one can pass to the Lorentz-type gauge in which the renormalizability is evident. Nevertheless a proof of renormalizability directly in the gauge proposed in the paper [1] is absent.

In the present paper we calculate at one-loop the gauge field and $A^3$-vertex renormalization constants. It is shown that a relationship between this constants is the same as in the ordinary Yang-Mills theory.

Let introduce notations. The Lagrangian proposed in [1] looks as follows

$$\mathcal{L} = \mathcal{L}_{YM} + (D_\mu \varphi)^* (D_\mu \varphi) - (D_\mu X)^* (D_\mu X) + i ((D_\mu e)^* (D_\mu b) - (D_\mu b)^* (D_\mu e)) \quad (1)$$

The fields $\varphi$, $X$ are commuting complex $SU(2)$-doublets and $e$, $b$ are anticommuting ones.

$$\varphi = \frac{1}{\sqrt{2}} \left( i\varphi^1 + \varphi^2 \right); \quad X = \frac{1}{\sqrt{2}} \left( iX^1 + X^2 \right);$$

$$e = \frac{1}{\sqrt{2}} \left( ie^1 + e^2 \right); \quad b = \frac{1}{\sqrt{2}} \left( ib^1 + b^2 \right);$$

$$e^0 - ie^3 \right); \quad b^0 - ib^3 \right);$$
Dφ = ∂φ + ig 2 Aμaφ is the covariant derivative; τa are Pauli matrices. The Yang-Mills Lagrangian is
\[ L_{YM} = -\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \partial_{\mu} A_{\mu}^a - \partial_{\mu} A_{\mu}^a + g^2 \epsilon^{abc} A_{\mu}^b A_{\mu}^c. \] Let introduce the fields \( B^{\pm,a} = \phi^a \pm X^a, \phi^0 = \phi^0 \pm X^0 \). In terms of these fields the gauge transformation leaving the Lagrangian \( \Pi \) invariant is

\[
\begin{align*}
\delta A_{\mu}^a(x) &= -\partial_{\mu} \phi^a(x) + g \epsilon^{abc} A_{\mu}^b(x) \alpha^c(x) \\
\delta B_{\mu}^{+,a}(x) &= -\frac{g}{2} \phi^+(x) \alpha^a(x) + \frac{g}{2} \epsilon^{abc} B^{+,b}(x) \alpha^c(x) \\
\delta \phi^+(x) &= -\frac{g}{2} B_{\mu}^{-,a}(x) \alpha^a(x) \\
\delta B_{\mu}^{-,a}(x) &= \sqrt{2} a \alpha^a(x) + \frac{g}{2} \phi^-(x) \alpha^a(x) + \frac{g}{2} \epsilon^{abc} B^{-,b}(x) \alpha^c(x) \\
\delta \phi^-(x) &= -\frac{g}{2} B_{\mu}^{-,a}(x) \alpha^a(x) \\
\delta \epsilon^a(x) &= \frac{g}{2} \epsilon^0(x) \alpha^a(x) + \frac{g}{2} \epsilon^{abc} \epsilon^b(x) \alpha^c(x) \\
\delta \epsilon^0(x) &= -\frac{g}{2} \epsilon^a(x) \alpha^a(x) \\
\delta b^a(x) &= \frac{g}{2} b^0(x) \alpha^a(x) + \frac{g}{2} \epsilon^{abc} b^b(x) \alpha^c(x) \\
\delta b^0(x) &= -\frac{g}{2} b^a(x) \alpha^a(x)
\end{align*}
\]

Being quantized in the Lorentz gauge \( \partial_{\mu} A_{\mu} = 0 \) the model is described by the effective action which contains the gauge fixing term and non gauge invariant Faddeev-Popov ghost Lagrangian. However in this model the Lorentz invariant condition \( B^{-,a} = 0 \) is also an admissible gauge. In this case the gauge invariance is broken by only the gauge fixing term: \( L_{ef} = L + \lambda^a B^{-,a} \), here \( L \) is the Lagrangian \( \Pi \). The Lagrangian \( \Pi \) at the surface \( B^{-,a} = 0 \) looks as follows

\[
\mathcal{L} = L_{YM} + \frac{1}{2} \partial_{\mu} \phi^+ \partial_{\mu} \phi^- + \frac{\alpha}{\sqrt{2}} A_{\mu}^a \partial_{\mu} B^{+,a} + \frac{g}{4} A_{\mu}^a \left( \phi^- \partial_{\mu} B^{+,a} - B^{+,a} \partial_{\mu} \phi^- \right) + \\
+ \frac{ag\sqrt{2}}{4} A_{\mu}^2 \phi^+ + \frac{g^2}{8} A_{\mu}^2 \phi^+ \phi^- + i \left( \partial_{\mu} \epsilon^0 \partial_{\mu} b^0 + \partial_{\mu} \epsilon^a \partial_{\mu} b^a \right) + \\
+ ig^2 2 A_{\mu} \left\{ \epsilon^{abc} (b^b \cdot \partial_{\mu} \epsilon^c - \partial_{\mu} b^b \cdot \epsilon^c) + (b^b \cdot \partial_{\mu} \epsilon^0 - \partial_{\mu} b^b \cdot e^0) - (b^0 \cdot \partial_{\mu} \epsilon^a - \partial_{\mu} b^0 \cdot e^a) \right\} + \\
+ \frac{ig^2}{4} A_{\mu}^2 (\epsilon^0 e^0 + e^a b^a) \tag{3}
\]

The propagators and vertices are given in the appendix.

## 2 Renormalization constants.

We start with considering the gauge field renormalization constant \( Z_2 \) and \( A^3 \)-vertex renormalization constant \( Z_1 \) and show that in the one-loop approximation these constants coincide with the corresponding ones in the ordinary Yang-Mills theory. We also
consider anticommuting fields renormalization constant $\bar{Z}_2$ and $A_{\mu;eb}$-vertex renormalization constant $\bar{Z}_1$ and show that the relationship $Z_1Z_2^{-1} = \bar{Z}_1\bar{Z}_2^{-1}$ holds in one-loop approximation. The calculations can be found in the appendix. The dimensional regularization is used, hence all tadpole diagrams are equal to zero.

The constant $Z_2$ renormalizes the transversal part of one-particle irreducible diagrams with two external gauge field lines $A_\mu$. There are three types of the contributions to the divergent part of the corresponding diagrams. The first one is a contribution of gauge field $A_\mu$ loops. This contribution is the same as in the ordinary Yang-Mills theory, because the vertices $A^3$, $A^4$ and the propagators coincide with the usual ones. The second one is the contribution of loops with commuting scalar fields $B^{+a}$, $\varphi^\pm$. And the third one is the contribution of loops with anticommuting fields $e$, $b$. The sum of the second and the third contributions equals to

$$\frac{ig^2}{48\pi^2\varepsilon}\delta^{ab}(g_{\mu\nu}p^2 - p_\mu p_\nu)$$

In the ordinary Yang-Mills theory divergent part of Faddeev-Popov ghosts loop is the following [2], [3].

$$\frac{ig^2}{48\pi^2\varepsilon}\delta^{ab}(g_{\mu\nu}p^2 + 2p_\mu p_\nu)$$

One can see that transversal parts of (4) and (5) coincide. As it was mentioned above the contributions of gauge field $A_\mu$ loops in the present theory and in the Yang-Mills theory coincide and therefore constants $Z_2$ coincide too.

Note that contrary to the standard formulation of the Yang-Mills theory the radiative corrections generate also renormalization of the longitudinal part of the polarization operator, that is the counterterm proportional to $(\partial_\mu A_\mu)^2$. Such a counterterm does not break the gauge invariance, because it originates from the admissible gauge invariant counterterm $(D^2\phi)^* (D^2\phi)$ after the shift $\phi^0 \rightarrow \phi^0 + \sqrt{2}g^{-1}a$.

The constant $Z_1$ performs renormalization of $A^3$-vertex. Again there are three types of contributions. Contribution of the gauge field $A_\mu$ loops coincide with the corresponding one in the ordinary Yang-Mills theory. In one-loop approximation the total contribution of commuting scalar fields $B^{+a}$, $\varphi^\pm$ and anticommuting fields $e$, $b$ is equal to

$$\frac{g^3}{96\pi^2\varepsilon}\varepsilon^{abc}\left(g_{\mu\nu}(p - q)_\lambda - g_{\nu\lambda}(2p + q)_\mu + g_{\mu\lambda}(p + 2q)_\nu\right),$$

It coincides with the contribution of Faddeev-Popov ghosts in the ordinary Yang-Mills theory and therefore the constant $Z_1$ equals Yang-Mills constant $Z_1$.

Thus in one-loop approximation the constants $Z_2$, $Z_1$ are the same as in the Yang-Mills theory and therefore the expression $Z_1Z_2^{-1}$ is the same as in the Yang-Mills theory.

$$Z_1Z_2^{-1} = 1 - \frac{3g^2}{16\pi^2\varepsilon}$$

which means that the model is asymptotically free.
To find the constants $\bar{Z}_2$, $\bar{Z}_1$, it is necessary to calculate the divergent part of the diagrams represented on fig. 1, 2. Performing the calculations we obtain

$$\bar{Z}_2 = 1 + \frac{9g^2}{32\pi^2\varepsilon}$$
$$\bar{Z}_1 = 1 + \frac{3g^2}{32\pi^2\varepsilon}$$

(8)

$$\bar{Z}_1\bar{Z}_2^{-1} = \left(1 + \frac{3g^2}{32\pi^2\varepsilon}\right)\left(1 - \frac{9g^2}{32\pi^2\varepsilon}\right) = 1 + \left(\frac{3}{32} - \frac{9}{32}\right)\frac{g^2}{\pi^2\varepsilon} = 1 - \frac{3g^2}{16\pi^2\varepsilon}$$

Thus $Z_1Z_2^{-1} = \bar{Z}_1\bar{Z}_2^{-1}$.

Figure 1: Diagram for calculation of anticommuting fields renormalization constant

Figure 2: Diagrams for calculation of renormalization constant $\bar{Z}_1$

3 Discussion.

In the present paper it was shown that the theory with the gauge invariant ghost Lagrangian agrees with the standard Yang-Mills theory in one-loop approximation. The relationship $Z_1Z_2^{-1} = \bar{Z}_1\bar{Z}_2^{-1}$ between renormalization constants which is one of the necessary conditions of gauge invariance of renormalized Lagrangian holds. A complete renormalization of the theory requires further investigation.

Acknowledgements
I am grateful to A.A. Slavnov for useful discussion and remarks.
References

[1] A.A. Slavnov, A Lorentz invariant formulation of the Yang-Mills theory with gauge invariant ghost field Lagrangian, JHEP 08(2008)047

[2] L.D. Faddeev, A.A. Slavnov Gauge Fields. Introduction to quantum theory. Second edition. Addison-Wesley Publishing Company 1991. Chapter 4.

[3] M.E. Peskin, D.V. Schroeder An Introduction to Quantum Field Theory. Addison-Wesley Publishing Company 1995. Chapter 16.
A  Propagators and vertices.

\[
\begin{align*}
A_\mu^a & A_\nu^b \quad \frac{\delta^{ab} g_{\mu\nu} - k_\mu k_\nu / k^2}{i} \\
A_\mu^a & B^{+b} \quad \frac{\delta^{ab} \sqrt{2} (-i k_\mu)}{i} \\
\varphi^+ & \varphi^- \quad \frac{1}{i} \frac{(-2)}{k^2} \\
B^{+b}_\mu & a \quad \frac{-ig}{4} \delta^{ab} i(p - q)_\mu \quad \frac{ig^{2a}}{4} \gamma_{\mu\nu} \delta^{ab} \\
\varphi^+ & n \quad \frac{-ig}{2} \delta^{ab} (p + q)_\mu \quad \frac{-ig}{2} \gamma_{\mu\nu} \delta^{ab} \\
e^c & \nu \quad \frac{-g^2}{2} \gamma_{\mu\nu} \delta^{ab} \delta_{\nu\mu} \\
ee^c & \nu \quad \frac{-g^2}{2} \gamma_{\mu\nu} \delta^{ab} \delta_{\nu\mu} \\

\end{align*}
\]

B  Calculation of \( Z_2 \).

Here we present the calculations of the commuting scalar fields \( B^{+a} \), \( \varphi \) and anticommuting fields \( e, b \) loops contributions to the divergent part of the gauge field two-point Green function.

For calculation of commuting scalar fields contribution it is necessary to find divergent part of two diagrams which are represented at the fig. 3. Left diagram (fig. 3)
Figure 3: 1PI diagrams with loops of commuting fields $B^+, \varphi^\pm$

gives the following integral.

$$
\int \frac{d^4 k}{(2\pi)^4} \left( \frac{g^2}{4} \delta^{ac}(2k - p)_\mu \right) \frac{1}{i (p + k)^2} \left( \frac{ig \sqrt{2}}{2} g_{\lambda\nu} \delta_{ab} \right) \delta^{cd} \frac{\sqrt{2} (-k_\lambda)}{a/k^2} =
$$

$$
= \frac{g^2}{2} \delta^{ab} \int \frac{d^4 k}{(2\pi)^4} \frac{(2k + p)_\mu k_\nu}{k^2} \delta^{mn} \frac{\delta^{ln}}{(p + k)^2} = \frac{g^2}{2} \delta^{ab} \left( \frac{-i}{48\pi^2 \varepsilon} (g_{\mu\nu} p^2 - p_\mu p_\nu) \right)
$$

It is easy to see that the right diagram at fig. 3 coincides with the left diagram after the change $\mu \leftrightarrow \nu$, $a \leftrightarrow b$, $p \to -p$ and consequently it gives the same result. Therefore the total commuting scalar fields contribution is

$$
- \frac{ig^2}{48\pi^2 \varepsilon} \delta^{ab} (g_{\mu\nu} p^2 - p_\mu p_\nu)
$$

(9)

Now let us find the anticommuting fields contribution. We calculate divergent part of diagrams shown at fig. 4. For the first diagram (fig. 4) we have

$$
(-1) \int \frac{d^4 k}{(2\pi)^4} \left( - \frac{ig}{2} \varepsilon^{amc} (2k - p)_\mu \right) \frac{\delta^{mn}}{k^2} \left( - \frac{ig}{2} \varepsilon^{bln} (2k - p)_\nu \right) \frac{\delta^{lc}}{(k - p)^2} =
$$

$$
= \frac{g^2}{4} \varepsilon^{amn} \varepsilon^{bln} \int \frac{d^4 k}{(2\pi)^4} \frac{(2k - p)_\mu (2k - p)_\nu}{k^2 (k - p)^2} \delta^{mn} \frac{\delta^{ln}}{48\pi^2 \varepsilon} (g_{\mu\nu} p^2 - p_\mu p_\nu)
$$

Figure 4: 1PI diagrams with loops of anticommuting fields
One can see that the remaining diagrams have identical divergent parts. Furthermore, the sum of these divergences is equal to the divergence of the first diagram. Therefore, the total anticommuting fields contribution is

\[ \frac{ig^2}{24\pi^2\varepsilon} \delta^{ab}(g_{\mu\nu}p^2 - p_\mu p_\nu) \]  

(10)

Summarizing (9) and (10) we find

\[ \frac{ig^2}{48\pi^2\varepsilon} \delta^{ab}(g_{\mu\nu}p^2 - p_\mu p_\nu) \]  

(11)

C Calculation of $Z_1$.

Now we calculate the contribution of the commuting scalar fields $B^{+a}$, $\varphi$ and the anticommuting fields $e, b$ into divergent part of the gauge field three-point Green function.

The contribution of commuting fields $B^{+a}$ and $\varphi^\pm$ loops is given by the sum of six diagrams one of which is represented at fig. 5 and other five diagrams differ from this one by permutation of vertices. The diagram at fig. 5 gives

\[ \int \frac{d^4k}{(2\pi)^4} \left(-g_{e^amn}(k - 2q)\beta g_{a\mu} + (q - 2k)\mu g_{\alpha\beta} + (k + q)\alpha g_{\beta\mu}\right) \delta^{nl} \frac{g_{\beta\gamma} - k_\beta k_\gamma/k^2}{k^2} \]

\[ \frac{1}{i} \frac{(-2)}{k - q - p} \left(-\frac{g}{4} \delta^{s}(2k - 2q - p)_{\nu}\right) \delta^{ms} \frac{\sqrt{2}(k - q)\alpha_{\alpha}}{a (k - q)^2} \]

\[ \text{div} = \frac{g^3}{16\pi^2\varepsilon} \delta^{abc} \left(-\frac{11}{96}g_{\mu\nu}(p + 2q)_{\lambda} - \frac{11}{96}g_{\nu\lambda}(p + 2q)_{\mu} + \frac{13}{96}g_{\mu\lambda}(p + 2q)_{\nu} \right) \]  

(12)

Contributions of five remaining diagrams can be obtained from formula (12). For example, if we replace $q \rightarrow (-q - p), p \rightarrow p, a \leftrightarrow c, \mu \leftrightarrow \lambda$ in this formula, we obtain the divergent part of the diagram which differs by permutation of two bottom vertices. Summarizing all six we obtain

\[ \frac{-g^3}{32\pi^2\varepsilon} \delta^{abc} \left(g_{\mu\nu}(p - q)_{\lambda} - g_{\nu\lambda}(2p + q)_{\mu} + g_{\mu\lambda}(p + 2q)_{\nu} \right) \]  

(13)

\[ \text{Figure 5: Diagram with commuting fields loop} \]
Now let us consider the loops of the anticommuting fields \( e \) and \( b \). Corresponding diagrams are presented at fig. 6. Vertices \( A^a_{\mu}e^m b^n, A^a_{\mu}e^0 b^n, A^a_{\mu}e^m b^0 \) differ from each other only by tensors \( \varepsilon^{amn}, \delta^m_n, -\delta^m_n \) correspondingly. For the top and bottom diagrams we obtain correspondingly

\[
\delta^f g^r \delta^f b^m f^m n (-\delta^m c) = -\varepsilon^{bca} = -\varepsilon^{abc}
\]

So these diagrams give the same results. In addition one can see that each remaining diagrams give the same result too. So we have to calculate divergent part of the first diagram at fig. 6 and multiply it by eight.

\[
-\varepsilon^{abc} \int \frac{d^4k}{(2\pi)^4} \left( \frac{ig}{2} (2k + q)_\nu \right) \frac{1}{k^2} \left( -\frac{ig}{2} (2k + p + q)_\lambda \right) \frac{1}{(k + p + q)^2} \div^{\text{div}}
\]

\[
= -\frac{ig^3}{8} \varepsilon^{abc} \frac{i}{24\pi^2 \varepsilon} \left( g_{\nu \nu} (p - q)_\lambda - g_{\nu \lambda} (2p + q)_\mu + g_{\mu \lambda} (p + 2q)_\nu \right)
\]

The total contribution of the anticommuting fields loops and loops of the fields \( B^{+,a} \),

\[
+ \text{five diagrams differing by permutation of vertices}
\]
\[ \varphi^\pm = \frac{g^3}{96\pi^2\varepsilon} \epsilon^{abc} \left[ g_{\mu\nu}(p - q)\lambda - g_{\nu\lambda}(2p + q)\mu + g_{\mu\lambda}(p + 2q)\nu \right]. \quad (14) \]

## D Anticommuting fields \( e \) and \( b \).

Now we calculate the constants \( \bar{Z}_2, \bar{Z}_1 \). Let us consider the one-loop diagram at fig. 1.

The corresponding integral is equal to

\[ \int \frac{d^4k}{(2\pi)^4} \left( \frac{-ig}{2} \delta^{am}(2p + k) \mu \left( \frac{ig}{2} \delta^{bn}(2p + k) \nu \right) \frac{\delta^{ab} g_{\mu\nu} - k_{\mu} k_{\nu}/k^2}{i} \right) = \]

\[ = -3ig^2 \rho_\mu \rho_\nu \int \frac{d^4k}{(2\pi)^4} \frac{g_{\mu\nu} k^2 - k_{\mu} k_{\nu} \text{div}}{k^4(k + p)^2} = \frac{9g^2}{32\pi^2\varepsilon} p^2 \]

This divergence is canceled by the following counterterm:

\[ i \int dx(\bar{Z}_2 - 1)(i\partial_\mu e^0\partial_\mu b^0) = \int dp(\bar{Z}_2 - 1)(-p^2) e^0(p) b^0(-p) \]

\[ \text{thus} \quad -(\bar{Z}_2 - 1)p^2 = \frac{9g^2}{32\pi^2\varepsilon} p^2 \]

\[ \text{and therefore} \quad \bar{Z}_2 = 1 + \frac{9g^2}{32\pi^2\varepsilon} \quad (15) \]

To find the constant \( \bar{Z}_1 \) one has to calculate the sum of divergent parts of diagrams which are presented at fig. 2. Besides the presented diagrams there are the diagrams with three vertices, but their divergent parts are equal to zero.

For the left diagram at fig. 2 we obtain the following integral.

\[ \int \frac{d^4k}{(2\pi)^4} \left( \frac{-g^2}{2} g_{\mu\nu} \delta^{ae} \right) \frac{1}{(k + p + q)^2} \left( \frac{-ig}{2} \delta^{bd}(k + 2p + 2q) \lambda \right) \frac{\delta^{cd} g_{\nu\lambda} - k_{\mu} k_{\lambda}/k^2}{i} = \]

\[ = \frac{g^3}{2} \delta^{ab}(p + q)\lambda \int \frac{d^4k}{(2\pi)^4} \frac{g_{\mu\nu} k^2 - k_{\mu} k_{\nu} \text{div}}{k^4(k + p + q)^2} = \frac{g^3}{2} \delta^{ab}(p + q)\mu, \quad \frac{3i}{32\pi^2\varepsilon} \quad (16) \]

For the right diagram on fig. 2 we obtain

\[ \int \frac{d^4k}{(2\pi)^4} \left( \frac{-g^2}{2} \delta^{cn}(k + 2q) \nu \right) \frac{\delta^{mn}}{(k + q)^2} \left( \frac{-g^2}{2} g_{\mu\lambda} \delta^{ad} \delta^{mb} \right) \frac{\delta^{cd} g_{\nu\lambda} - k_{\mu} k_{\lambda}/k^2}{i} = \]

\[ = \frac{g^3}{2} \delta^{ab} q_\nu \int \frac{d^4k}{(2\pi)^4} \frac{g_{\mu\nu} k^2 - k_{\mu} k_{\nu} \text{div}}{k^4(k + q)^2} = \frac{g^3}{2} \delta^{ab} q_\mu, \quad \frac{3i}{32\pi^2\varepsilon} \quad (17) \]

Summarizing the results (16) and (17) one gets

\[ -\frac{ig^3}{2} \delta^{ab}(p + 2q)\mu \left( \frac{-3}{32\pi^2\varepsilon} \right) \quad (18) \]
This divergencies are canceled by counterterm \((\bar{Z}_1 - 1)^{(-i\delta/2)}\delta^{ab}(p + 2q)_\mu\). So

\[
\bar{Z}_1 = 1 + \frac{3g^2}{32\pi^2\varepsilon}
\]  

(19)