Quarkonium in Heavy Ion Collisions

--- high-energy multiple scattering of quark pair in nuclei ---

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Quarkonium suppression in heavy ion collisions is a potential signature of the formation of the quark-gluon plasma. After a very brief review of the $J/\psi$ result at CERN, we restrict our discussion to the effects of the high-energy multiple scattering of the quark pair in the colliding nuclei.

§1. Quarkonium in heavy ion collisions

Investigation of the QCD plasma, which is a system of the strongly interacting gas of the quarks and the gluons, is one of the fundamental subjects in the physics of the strong interaction. In the experimental study with high-energy nucleus-nucleus collisions, the $J/\psi$ production is considered as a special signal sensitive to the plasma which is created in the early stage of the reactions. The color attraction between the heavy $c\bar{c}$ pair produced in the initial hard parton scatterings will be screened in the plasma, which will suppress the binding to the physical $J/\psi$. This predicted suppression was observed in the NA38 experiment. However it was seen even for the proton-nucleus ($pA$) interactions as well as the nucleus-nucleus ($AB$). Because the exponential scaling with the effective target thickness $L^{**}$ is found and the $J/\psi$ and the $\psi'$ are suppressed in the same rate, this suppression is interpreted as a normal nuclear effect or absorption of the produced pre-resonance state by the nucleons in the nucleus with $\sigma_{\text{abs}} = 6 \sim 7$ mb. The large cross section and the state independence are explained as the importance of the color-octet pre-meson state in the production of the quarkonia. This is a typical example to show the necessity of the joint studies of the quarkonium in $p\bar{p}$, $pA$ and $AB$ reactions.

The Pb-Pb collisions brought us a real surprise; much stronger suppression was reported. Threshold behavior is seen in the data, which seems very hard to explain theoretically (even with the plasma). All medium effects must be studied duly and critically before the final conclusion is reached, although the plasma is the most immediate possibility.

The absorption of the $J/\psi$ state by the comoving secondaries produced in the reaction is examined extensively. As a small color singlet object, $J/\psi$'s interactions with the light secondaries are expected to be weak. Quantitatively, however, it is

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** One should be careful that $L$ is defined by a model.

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largely unknown with little experimental information. Model calculations are done for the processes like $\psi\pi, \psi\rho \rightarrow D\bar{D}, D^*D^*$, etc. But the resulting cross sections at the threshold region are very model-dependent.\textsuperscript{14} We note that the possible modifications of the $D$ state in medium also affect the quarkonium dissociation rate.\textsuperscript{15,16}

NA50 collaboration recently reported\textsuperscript{2} the smaller cross section $\sigma_{\text{abs}} = 4.4 \pm 0.5$ mb with the improved statistics, and the $J/\psi$ yield in the peripheral region with the smaller errors, which apparently makes the threshold clearer. Any suppression model proposed so far should be subjected to this data.

Strictly speaking, the formation amplitude of the $J/\psi$ must be calculated in the $AB$ collisions, where the cross section of the $J/\psi$ or the pre-meson state with a nucleon is unlikely to have a definite meaning. In §2 we study a model in which the nuclear effect modifies the $c\bar{c}$ state before forming the $J/\psi$.

At the collider energies, the situation will change at least in two respects. One cannot think of the independent scatterings of the $J/\psi$ off the valence nucleons because they are localized in the extremely thin regions in the CM frame (see §3). Multiple $c\bar{c}$-pair production may alter the assumption of no accidental coalescence, which seems valid at the CERN-SPS energy owing to the rare $c\bar{c}$ production.\textsuperscript{17}

\section{A multiple scattering model for $J/\psi$ suppression}

Recently Qiu, Vary and Zhang (QVZ) proposed a new nuclear mechanism for the quarkonium suppression, which successfully explains the observed data at CERN\textsuperscript{18}. In their model the inclusive cross section of the $J/\psi$ production in the collision of the hadrons $A$ and $B$ is written in a factorized form:

$$
\sigma_{AB \rightarrow J/\psi X} = K_{J/\psi} \sum_{a,b} \int dq^2 \left( \frac{\hat{\sigma}_{ab \rightarrow c\bar{c}}(Q^2)}{Q^2} \right) 
\times \int dx_F \phi_a/A(x_a) \phi_b/B(x_b) \frac{x_a x_b}{x_a + x_b} F_{c\bar{c} \rightarrow J/\psi}(q^2),
$$

(2.1)

where $\sum_{a,b}$ runs over all parton flavors, $Q^2 = q^2 + 4m_c^2$, $\phi_a/A(x_a)$ is the distribution function of parton $a$ in hadron $A$, and $x_F = x_a - x_b$ and $x_a x_b = Q^2/s$. $\hat{\sigma}$ is the parton cross section and $K_{J/\psi}$ is a phenomenological constant. The factor $F_{c\bar{c} \rightarrow J/\psi}(q^2)$, which describes the transition probability for the $c\bar{c}$ state of the relative momentum $q^2$ to evolve into a physical $J/\psi$, is parametrized as

$$
F_{c\bar{c} \rightarrow J/\psi}(q^2) = N_{J/\psi}(q^2) \theta(4m^2 - 4m_c^2 - q^2) \left( 1 - \frac{q^2}{4m_c^2 - 4m_c^2} \right)^{\alpha_F}.
$$

(2.2)

This form includes the effect of the open charm threshold at $4m_c^2$ and simulates the gluon radiation effect with the parameter $\alpha_F > 0$ which puts the larger weight to
the smaller $q^2$.

The nuclear effect is taken into account as the coherent multiple scattering of the pair, which at the leading of the nuclear enhancement may result in shifting of the relative momentum in the transition probability,\(^{(18),(19)}\)

\[
F_{c\bar{c} \rightarrow J/\psi}(q^2) = F_{c\bar{c} \rightarrow J/\psi}(q^2 + \varepsilon^2 L),
\]

where $L$ is the effective length of the nuclear medium in the $AB$ collisions. The model assumption here is the separation of the multiple scattering regime from the later formation stage of the physical resonance. We note here that for a large enough $L$ such that $\bar{q}^2 > 4m^2_c - 4m^2_c$ the transition probability essentially vanishes due to the existence of the open charm threshold $(2.2)$. This apparently gives a stronger suppression than the exponential one which follows in the Glauber model.

The vanishing of the probability for the large $L$ may be altered due to the subleading nuclear effects, by which we expect a diffusion of the momentum distribution.\(^{(20)}\) We model the momentum diffusion here by modifying the transition probability $F_{c\bar{c} \rightarrow J/\psi}$. The $c\bar{c}$ pair with relative momentum $q$ produced in a hard parton collision, will change its momentum to $q'$ after the random multiple scattering, and then transforms into the $J/\psi$ with the probability $F_{c\bar{c} \rightarrow J/\psi}(q^2 + \varepsilon^2 L)$.

[After many scatterings, this classical, elementary diffusion process of the momentum results in the Gaussian distribution around the initial value $q$ with the variance $\varepsilon^2 L$. For demonstration, we replace the transition probability by $\bar{F}_{c\bar{c} \rightarrow J/\psi}(q^2) \equiv \left( \frac{1}{2\pi \varepsilon^2 L} \right)^{3/2} \int d^3 q' e^{-\frac{(q' - q)^2}{2\varepsilon^2 L}} F_{c\bar{c} \rightarrow J/\psi}(q^2)$.]

Note that $\bar{F}_{c\bar{c} \rightarrow J/\psi}(q^2)$ never vanishes for any $q$ although the average momentum of the pair increases as $\langle q'^2 \rangle = q^2 + 3\varepsilon^2 L$ after the multiple scattering. Importantly, the transition probability behaves more moderately as $\bar{F}_{c\bar{c} \rightarrow J/\psi}(q^2) \propto L^{-3/2}$ for the asymptotically large $L$, which stems from the depletion of the normalization factor.

In Fig. 1 we show our result on the $J/\psi$ suppression calculated using the formula $(2.1)$ with the smeared probability $(2.4)$ with $\varepsilon^2 = 0.185$ GeV$^2$/fm. Other parameters are fixed to the same as 18): $\alpha_F = 1$ and $f_{J/\psi} \equiv K_{J/\psi}N_{J/\psi} = 0.485$. Our model reasonably fits the data in the $pA$ and $AB$ collisions taken from 11) except the Pb-Pb point. The curve bends upward slightly in the semi-log plot. The original QVZ model $(2.3)$ with $\varepsilon^2 = 0.25$ GeV$^2$/fm (dashed line) can explain all the data points in Fig. 1. The downward bending of QVZ model is the result of the existence of the open charm threshold in $(2.2)$ and the uniform momentum shift $(2.3)$.

This simple analysis indicates the importance of the subleading effect or the momentum diffusion to the $L$-dependence of the $J/\psi$ formation.

§3. High-energy $Q\bar{Q}$ state passing through random gauge fields

The coherent multiple scattering of the pair becomes more important at the collider energies, where two colliding nuclei are Lorentz-contracted to thin slabs and therefore the interaction of the $c\bar{c}$ pair with the valence nucleons should be treated as a simultaneous action.
Let us study a simpler problem of the high-energy heavy $Q\bar{Q}$ pair passing through a nucleus, which is modeled as a random filed source, and demonstrate that the penetration probability of the singlet bound state decays non-exponentially.\textsuperscript{21} Traveling through the target nucleus, the quark-anti-quark pair acquires the eikonal phase,

$$U(x, y; A) = W(x; A)W^\dagger(y; A),$$

accumulated along the path with the transverse positions, $x$ and $y$, with $W(x; A) = \mathcal{P} \exp[ig \int_0^\infty dx^+ A^a(x, x^+)\gamma^a]$. This is a matrix in the color space and $\mathcal{P}$ indicates the path ordering of the product. In the definite color basis they are written as

$$U_{ss}(x, y) = \frac{1}{N}\text{Tr}(W(x)W^\dagger(y)),
$$

$$U_{sa}(x, y) = \sqrt{\frac{2}{N}}\text{Tr}(W(x)W^\dagger(y)\gamma^a),
$$

$$U_{ba}(x, y) = 2\text{Tr}(\gamma^a W(x)\gamma^a W^\dagger(y)).$$

(3.2)

Here the color transparency is manifest as $U_{ss}(x, x) = 1$ and $U_{as}(x, x) = 0$. The survival probability of the color singlet $Q\bar{Q}$ state $\varphi(r, z)$ ($r = x - y$ and $z$ is the longitudinal momentum fraction) is written as $S \sim |\int d^2rdzd\varphi(r, z)U_{ss}\varphi(r, z)|^2$.

As we are interested only in the $Q\bar{Q}$ state, we take a closure of the final target states and average the probability over the initial target state. This is performed by dividing the path into small zones and averaging over the random gauge configurations in Eq. (3.1),\textsuperscript{21} which results in a kernel $\bar{U}(x, y)\bar{U}^\dagger(\bar{x}, \bar{y}) \equiv K(r, \bar{r}, R - \bar{R})$ with $R = \frac{1}{2}(x + y)$ and $\bar{x}$ the coordinate in the conjugate amplitude. The $r (R)$ dependence of $K$ gives rise to the relative (total) momentum diffusion of the $Q\bar{Q}$ state caused by this interaction with the random gauge fields.

In the thin and thick target limits, one finds the asymptotic behavior of the survival probability, $S = \int d^2r d^2\bar{r}\rho_0(r)\rho_0(\bar{r})K(r, \bar{r}, 0)$ with $\rho_0(r) = \int dz\varphi^2(r, z)$, as

$$S \sim 1 - \frac{L}{L_{\text{in}}} (L \to 0), \quad S \propto \frac{1}{N^2} \frac{L_{\text{in}}}{L} (L \to \infty).$$

(3.3)

The first relation defines $L_{\text{in}}$. The power-law suppression $L^{-1}$ is the result of the 2-dimensional diffusion of the relative momentum, which corresponds to $L^{-3/2}$ in §2 where we assumed the 3-dimensional random walk instead. The $1/N^2$ is the result of the color diffusion.

For concreteness we calculated the survival probability of a small Gaussian state as shown in Fig. 3. Furthermore we speculate an analytic

![Fig. 2. High-energy $Q\bar{Q}$ pair passing through a nuclear target.](image)

![Fig. 3. Survival probability of the singlet Gaussian state traversing SU(3) random fields [cross] is compared with Eq. (3.4) [solid] and the exponential form [dashed]. The first term of (3.4) is shown by a dotted line.](image)
approximation which is compatible with the asymptotic behavior (3.3):

\[ S(L/L_{in}) = \frac{2}{N^2} \frac{1}{1 + \frac{L}{L_{in}}} + \frac{N^2 - 2}{N^2} \frac{1}{(1 + \frac{1}{2} L_{in})^2}. \] (3.4)

This formula is found to fit the numerical result quite well, and clearly demonstrates the non-exponential behavior of the survival probability.

§4. Concluding Remarks

It is a theoretical challenge to predict what will happen at BNL-RHIC before the new data show up.\(^2\)\(^4\) Our elementary demonstration is an example which indicates that the J/ψ production at BNL-RHIC energy will be different from those observed at CERN-SPS energy. At the same time one should think over again the CERN-SPS data and the available pA data to construct a unified picture of the quarkonium production in the nucleus-nucleus collisions. Such steady efforts will make the study with BNL-RHIC and CERN-LHC successful, together with other various observables.

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