Improving sampling and calibration of GRBs as distance indicators

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We present a sample of 81 Gamma-Ray Bursts (GRBs) observed by Fermi-GBM for which we compute the distance moduli and use them to constrain effective dark energy models. To overcome the circularity problem affecting the use of GRBs as distance indicators, we calibrate the Amati relation of our sample by employing a cosmology-independent technique. Specifically, the latest observational Hubble parameter data are used to approximate the cosmic expansion through a Bezier parametric curve. We subsequently obtain the distance moduli of the GRBs and include it in a suite of recent cosmological (Planck Compressed 2018 and 2012 BOSS release of BAO data) and local (Pantheon SNIa) observations of the expansion history to compute Bayesian posterior constraints for the standard cosmological model ΛCDM, ωCDM, and for the CPL parametrization.

We highlight the advantages that our dataset and method represent over other recent GRB data.

I. INTRODUCTION

In the endeavour of characterising the cosmological expansion, standard candles are a keystone for precise distance determinations. In practice, however, the search for high precision in many of the distance indicators exposes several sources of bias that prevent astrophysical objects from reaching the status of standard candles. The luminosity of Supernovae of Type Ia (SNe Ia), for example, seems to be subject to its environment [1] and such dependence could only be accounted for through precise observations. In this sense, any contribution from alternative distance indicators which preferably cover a wide range of redshifts is key to improve cosmological distance determinations, and ultimately, characterise the Dark Energy component.

An attractive prospect is the distance modulus of Gamma Ray Bursts (GRBs) (for pioneer works see, e.g. [2,4]). Despite the well known dispersion in the luminosity correlations and other sources of uncertainty, GRBs remain good candidates for distance indicators. GRBs represent the most powerful explosions in the Universe. They are bright enough to be detected up to high redshifts[1]. Therefore, they are often proposed as complementary tools to SNe Ia observations to probe the expansion history of the Universe. The prompt emission of GRBs lies mostly in the range from 0.001 to a few MeV, and lasting from 0.01 to 1000 seconds. This property classifies naturally the set of GRBs in two categories, those with $T_{90} > 2$ seconds are classified as long and are associated to the collapse of certain types of massive stars [8]; while the short kind is associated to the merger of compact objects [8]. Despite several efforts to model the explosion mechanism of the GRBs (e.g. [9,12]), there is no single satisfactory explanation and their nature is still not fully understood. In consequence, the distance calibration of GRBs presents more difficulties as that of SNe Ia.

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[1] The highest redshifts recorded lie at $z = 8.2$ (GRB090423) [5,6] and $z = 9.4$ (GRB090429) [7].
So far, several methods have been proposed to calibrate GRBs \cite{2, 3, 4, 13, 14, 15, 16, 17, 18}. Most calibrating methods rely on empirical luminosity correlations found in long GRBs. Among the known correlations are those between spectrum lag and isotropic peak luminosity ($\tau_{\text{lag}} - L$ relation, \cite{19}), the correlation between time variability and isotropic peak luminosity ($\nu - L$ relation, \cite{20}), a tight correlation between the peak energy of $\nu F_\nu$ spectrum and isotropic equivalent energy ($E_p - E_{\text{iso}}$ relation, \cite{21}), a correlation between the peak energy and the collimation-corrected energy ($E_p - E_{\gamma}$ relation, \cite{22}), the correlation between peak energy and isotropic peak luminosity ($E_p - L$ relation, \cite{23}), and the correlation between minimum rise time of light curve and isotropic peak luminosity ($\tau_{\text{RT}} - L$ relation, \cite{4}). Recently, some other correlations have been reported in the literature \cite{24, 29} (See \cite{31} for a review and more details on these correlations).

In many of these correlations, the luminosity of GRBs appears correlated with the temporal and spectral properties. While, as already stated, these correlations are not yet fully understood from first principles, their existence naturally leads to the consideration that GRBs could be used as distance indicators, offering a possible route to probe the expansion history of the Universe up to $z \gtrsim 9$. Datasets of GRBs distance moduli are often used (either alone or in combination with other observational data such as SNe Ia), to constraint cosmological parameters \cite{2, 3, 13, 15, 32, 37}. In the calibration of the empirical correlations, two caveats stand out that prevent the improvement of GRBs distance moduli as distance indicators. First is a number of sources of uncertainty in the determination of luminosity parameters. It is known that combining databases from different telescopes may introduce an unknown selection bias due to the different thresholds and spectroscopic sensitivity \cite{38, 39}. Additionally, mixing methods for the redshift determination (photometric vs. spectral) is also a source for uncertainty \cite{40}.

On the other hand an inherent circularity problem arises in the calibrations of GRBs, since the determination of energy flux typically assumes an underlying cosmological model. Several works have attempted to tackle these circularity problems by adopting model-independent methods to estimate parameters in the calibration (see for instance \cite{3, 13, 16, 33, 35, 41, 42}).

In this paper we present a sample of 81 Fermi-GRBs, carefully selected to avoid speculation (and large errors) in the determination of luminosity parameters. After listing their spectral properties, we calibrate the set in a model-independent way by employing the Amati relation \cite{36, 43} which relates the rest frame peak energy of the spectra $E_p$ to the isotropic energy emitted $E_{\text{iso}}$. The calibration is performed following the recent work of \cite{44} where a compilation of 31 measurements of the Hubble parameter were used to fit a Bezier parametric curve in order to obtain the Hubble’s rate at arbitrary redshifts without assuming an a priori cosmological model. In that paper, the values for the $E_{\text{iso}}$ was determined for 193 GRBs taken from \cite{45} and references therein. In contrast, our sample originates from a single catalogue, thus avoiding selection biases and other instrument-associated systematics.

We show the usefulness of our dataset by comparing our results with previous samples, through the performance of a Bayesian parameter estimation for the $\Lambda$CDM, $\omega$CDM and the CPL models employing GRB data and the latest compilation of Supernovae Ia data (SNe Ia) \cite{46}, Baryon Acoustic Oscillations (BAO) \cite{47, 49} and Cosmic Microwave Background (CMB) data \cite{50}.

The paper is organized as follows. In Sec. \ref{sec:sample} we present in detail our Fermi/GBM GRBs sample; in Sec. \ref{sec:calibration} we calibrate the observables of our sample and subsequently the Amati relation. In Sec. \ref{sec:fit} we include the calibrated sample in a suite of observations to fit parameters of the three dark energy (DE) models mentioned before. We carry this analysis including SNe Ia; BAO for clusters with redshifts up approximately 2, as well as CMB data from Planck-compressed 2018. We discuss our results in Sec. \ref{sec:discussion} and we present our conclusions in Sec. \ref{sec:conclusions}. 

II. GAMMA-RAY BURSTS OBSERVATIONS

The GRBs spectrum is described in terms of an empirical spectral function, the Band function [51], which explicitly is

\[
f(E) = \begin{cases} 
N_0 \left( \frac{E}{100\text{keV}} \right)^\alpha \exp \left( -\frac{E}{E_0} \right) & E \leq E_b \\
N_0 \left( \frac{E_0 (\alpha - \beta)}{100\text{keV}} \right)^{(\alpha - \beta)} \exp(\beta - \alpha) \left( \frac{E}{100\text{keV}} \right)^\beta & E > E_b 
\end{cases}
\]

(1)

with \( E_b = (\alpha - \beta) E_0 \). This spectrum peaks at \( (E_{p,\text{obs}}, \alpha, \beta) \), which is related to the spectral parameters as \( E_{p,\text{obs}} = E_0(2 + \alpha) \).

Due to the intense radiation emitted in GRBs, it is possible to detect such explosions at high redshift \( z \) [5]. The precise determination of \( z \) is crucial to infer the distance to the object (luminosity distance), which is necessary to determine the radiated energy \( (E_{\text{iso}}) \) in gamma band. The redshift can be computed analysing spectral emission or absorption lines of the afterglow spectrum, or by its photometric analysis at lower energy bands (from X-rays to radio), from observations generally performed by auxiliary telescopes.

A. The sample

While the SWIFT satellite has provided the largest number of GRBs with redshift to the existing catalogues, the BAT instrument of this satellite is limited to energies up to 150 keV [52]. This value lies below the average \( E_{p,\text{obs}} \) of GRBs [53], which prevents the determination of most of the spectral parameters in the Band function or even the cut-off power-law. Consequently, it is impossible to obtain directly the flux and luminosity for many of the GRBs observed by the BAT-SWIFT satellite.

On the other hand, Fermi features two instruments GBM and LAT sensible to energy bands of 8 keV to 40 MeV [54], and 100 MeV to 300 GeV [55], respectively.

For our compilation, FERMI spectral data were taken from the FERMI-GBM catalogue [56, 57], and the redshifts were retrieved from the BAT-SWIFT database available at https://swift.gsfc.nasa.gov/archive/grb_table.html/ and the webpage of J. Greiner http://www.mpe.mpg.de/~jcg/grbgen.html.

It is important to mention that some of the GRBs in the GBM catalogue presented no value for the spectral parameters. Thus we reduced the raw data, employing the Gamma Ray Spectral Fitting Package (RMFIT V4.3.2). In particular, we did this for the cases of GRB120712571, GRB180728728, GRB181020792, GRB190114873, GRB190324947.

Since the determination of redshift from photometry is subject to learning-curve effect, that is, there is a drift in the mean redshift over time as a consequence of different instruments contributing to redshift acquisition (see for instance [40] for more details), we avoid further bias and discard the GRBs with redshift set through such method [58]. Thus, we limit our sample to those GRBs with redshift determined through spectroscopic methods either from the afterglow or from the host galaxy (recall that the short GRBs do not satisfy the Amati relation for long GRBs and they are also left out of our sample). Finally, we also discarded GRBs which present significant uncertainties in the spectral parameters, namely \( E_p \) and \( F_{\text{bolo}} \), because of their poor contribution during our fit procedure.

Thus, from an initial sample of 107 GRBs, we selected objects meeting the above criteria and we finally present in Table II a sample of 81 GRBs covering the redshift range \( 0.117 \leq z \leq 5.283 \). The table presents the
spectral parameters of each GRB and their associated errors.

In the following section we show the process to derive the distance moduli for these objects.

III. CALIBRATION

We followed the model-independent calibration put forward by [44]. We thus apply the empirical relation $E_p - E_{\text{iso}}$ that connects $E_p = E_{p, \text{obs}}(1 + z)$ with the isotropic equivalent energy $E_{\text{iso}}$ derived by [21, 43],

$$E_{\text{iso}}(z) = 4\pi d_L^2(z)S_{\text{bolo}}(1 + z)^{-1},$$

where $S_{\text{bolo}}$ is the bolometric fluence of gamma rays in the GRB at redshift $z$, the factor $(1 + z)^{-1}$ transforms the observed GRB duration into the source cosmological rest-frame one and $d_L(z)$ is the luminosity distance of the GRB given by

$$d_L(z) = c(1 + z) \int_0^z \frac{dz'}{H(z')}.$$

Clearly, from the above equation it can be seen that the calibration of GRBs depends on the cosmological model through the expansion history $H(z)$. In fact a good fit can be obtained when a cosmological model is assumed a priori (see Fig. 1), although this is the cause of the circularity problem we want to tackle.

Making use of the cosmic chronometers approach [59, 60], which provides an independent technique to constrain $H(z)$ from the differential evolution of massive and passive early-type galaxies, [44] approximated these Hubble parameter data by employing a Bézier parametric curve of degree $n$ given by

$$H_n(z) = \sum_{d=0}^n \beta_d h_n^d(z),$$

$$h_n^d = \frac{n!(z/z_m)^d}{d!(n-d)!} \left(1 - \frac{z}{z_m}\right)^{n-d},$$

where $\beta_d$ are coefficients of the linear combination of Bernstein basis polynomials $h_n^d(z)$, positive in the range.
| GRB          | t0 [s] | t90 [s] | ∆t [s] | t90/t0 | ∆t/t0 | E_bolo [ergs] | F_bolo [ergs/cm²/s] | E_peak [keV] | E_0.01 [keV] | E_0.1 [keV] | α [eV] | n < 1 | n > 1 | n ≈ 1 | E_0.1 [keV] | F_0.1 [ergs/cm²/s] | E_0.01 [keV] | F_0.01 [ergs/cm²/s] | E_0.1 [keV] | F_0.1 [ergs/cm²/s] |
|--------------|--------|---------|--------|--------|--------|--------------|---------------------|--------------|---------------|-------------|-------|-------|-------|-------|-------------|---------------------|--------------|---------------------|-------------|---------------------|
| GRB121128212 | 2.7400 | 2.726E+01 | 1.706E-01 | 4.10E+00 | 2.80E-01 | 5.826E+00  | 5.490E-02  | 9.329E-02  | -2.416E+00 | 9.637E-02 | 1.091E-06 | 2.308E-07 |
| GRB130518580 | 2.4900 | 4.858E+01 | 2.066E-02 | 1.90E+00 | 2.00E-01 | 1.457E+00  | 1.739E-01  | 1.561E-02 | -2.181E+00 | 6.691E-02 | 1.746E-06 | 9.416E-08 |
| GRB121128212 | 2.2000 | 1.734E+01 | 6.334E-02 | 3.15E+00 | 3.73E-01 | 6.008E+01  | 3.849E+00  | 1.195E-01 | -2.424E+00 | 9.205E-02 | 5.345E-07 | 1.644E-07 |
| GRB090926181 | 2.1062 | 1.376E+01 | 6.448E-02 | 7.427E-04 | 3.338E+02 | 5.839E+00  | 8.619E-03  | -2.378E+00 | 4.558E-02 | 4.542E-06 | 1.285E-07 |
| GRB130612141 | 2.0060 | 7.424E+00 | 5.056E-01 | 2.085E+00 | 2.894E+01 | 8.235E+00  | 3.295E-01  | 5.639E-01 | -2.254E+00 | 1.824E-01 | 1.837E-07 | 7.850E-07 |
| GRB131011741 | 1.8740 | 7.706E+01 | 8.097E-03 | 8.791E-04 | 2.176E+02 | 4.088E+01  | 7.417E-02  | -2.085E+00 | 8.092E-02 | 5.037E-06 | 1.178E-07 |
| GRB120326056 | 1.7980 | 1.178E+01 | 6.022E-02 | 2.483E-02 | 4.431E+01 | 5.587E+00  | 2.344E-01  | -2.335E+00 | 1.347E-01 | 3.562E-07 | 1.898E-07 |
| GRB100906576 | 1.7270 | 1.106E+02 | 2.684E-02 | 1.039E-02 | 7.491E+01 | 2.427E+01  | 9.263E-01  | -1.861E+00 | 1.001E-01 | 7.480E-07 | 4.083E-07 |
| GRB091020900 | 1.7100 | 2.426E+01 | 8.973E-03 | 1.057E-03 | 2.283E+02 | 4.881E+01  | 1.245E-00  | -2.454E+00 | 3.775E+01 | 5.035E-07 | 2.996E-06 |
| GRB110213220 | 1.4600 | 3.430E+01 | 8.441E-03 | 7.378E-04 | 1.126E+02 | 1.204E+01  | 4.812E-02  | -4.870E+00 | 0.000E+00 | 4.099E-05 | 5.837E-08 |
| GRB100817929 | 1.3200 | 7.186E+05 | 5.106E+00 | 2.834E+05 | 4.842E+05 | 1.527E+05  | 3.846E+00  | 3.925E+01 | 1.138E+07 | 4.883E+06 | 2.323E+06 |
| GRB160804065 | 1.2900 | 2.650E+01 | 2.440E-02 | 2.832E-04 | 6.635E+02 | 1.537E+01  | 1.384E-00  | -3.534E+00 | 1.245E+00 | 3.581E-06 | 7.504E-07 |
| GRB150314205 | 1.1700 | 3.607E+05 | 1.857E+01 | 1.476E+01 | 2.396E+02 | 1.063E+00  | 4.01E+00   | -2.601E+00 | 1.021E-01 | 6.361E-07 | 2.866E-07 |
| GRB131231198 | 0.9150 | 5.632E+01 | 1.228E-02 | 2.387E-03 | 1.784E+02 | 1.928E+01  | 8.495E-01  | -2.059E+00 | 1.662E-01 | 4.170E-07 | 1.482E-07 |
| GRB160804065 | 0.7360 | 1.316E+02 | 1.165E-02 | 1.888E-03 | 7.139E+01 | 4.175E+00  | 3.731E-00  | -2.819E+00 | 9.034E-01 | 1.672E-07 | 4.966E-08 |
| GRB131231198 | 0.6420 | 3.123E+01 | 5.624E-02 | 9.278E-04 | 1.781E+02 | 3.018E+01  | 7.337E+00  | 7.285E-02 | 8.976E-01 | 2.285E-07 | 5.807E-08 |
| GRB130518580 | 0.5970 | 1.437E+02 | 6.217E-03 | 9.168E-04 | 2.099E+02 | 4.231E+01  | 9.329E-02  | -1.408E+00 | 1.212E+00 | 5.096E-07 | 1.237E-07 |

**TABLE I:** Spectral parameters for the employed GRBs taken from the GBM-FERMI catalog. The (α) represents the GRBs that we processed. Columns are: name, redshift, t0, spectral normalization, the standard deviation for the spectral normalization, peak energy, standard deviation for the peak energy, spectral index of low energy, standard deviation for the spectral index of low energy, spectral index of high energy, standard deviation for the spectral index of high energy, bolometric fluence and the standard deviation for the bolometric fluence.
0 \leq z/z_m \leq 1, with z_m the maximum z of the Hubble parameter dataset which consists of 31 measurements of Hubble parameter taken from [61]. In particular, [44] considered a Bézier curve of degree n = 2 in order to obtain a monotonic growing function in such way that with d = 0 and z = 0 it can be identify \( \beta_0 = H_0 \).

By employing the dataset of Hubble parameter reported in [61], we performed a non-linear least-squares minimization by using the Python software package lmfit [62]. The best-fit parameters we obtained for the Bézier curve with \( n = 2 \) are

\[
H_2(z) = \beta_0 h_0^2(z) + \beta_1 h_1^1(z) + \beta_3 h_2^1(z),
\]

with \( \beta_0 = H_0 = 67.7652 \pm 3.6864 \), \( \beta_1 = 102.9455 \pm 10.8574 \) and \( \beta_2 = 208.7820 \pm 14.1192 \) all in units of km s\(^{-1}\)Mpc\(^{-1}\). In addition, the correlations between these parameters were also obtained: \( C(\beta_0, \beta_1) = -0.839 \), \( C(\beta_1, \beta_2) = -0.702 \) and \( C(\beta_0, \beta_2) = 0.507 \). The best-fit with its 1\( \sigma \) confidence region are shown in Fig. 2. It is worth to mention these values are in agreement with the previous estimation made by [44].

The next step consists of extrapolating the function \( H_2(z) \) to redshift \( z > z_m \) and construct the luminosity distance \( d_L^{cal}(z) \), i.e.,

\[
d_L^{cal}(z) = c(1 + z) \int_0^z \frac{dz'}{H_2(z')},
\]

and therefore the isotropic energy \( E_{iso}^{cal} = 4\pi (d_L^{cal}(z))^2 S_{bolo}(1 + z)^{-1} \). In order to obtain the corresponding errors \( \sigma E_{iso}^{cal} \), the \( \sigma d_L \) is calculated by taking into account the correlations between the parameters \( \beta \)'s besides the GRBs systematics on the observables. The corresponding \( E_p - E_{iso}^{cal} \) distribution is shown in Fig. 3.

In order to calibrate the Amati relation for our sample of 81 Fermi-GRBs, we employed a Python module for performing robust linear regression on data points where both variables have measurement errors. The fitting method is the bivariate correlated errors and intrinsic scatter (BCES) that follows [63]. In particular, this method is useful when it is not clear which variable should be treated as the independent variable and
which is the dependent one. Following this method, it was obtained the best fit for the Amati relation

$$\log \left( \frac{E_{\text{iso}}^{\text{cal}}}{\text{erg}} \right) = A \log \left( \frac{E_p}{300\text{keV}} \right) + B,$$

where $A = 0.3537 \pm 0.3764$, $B = 53.2168 \pm 0.1420$ and the respective covariance matrix is given by

$$\text{cov} = \begin{bmatrix} 0.1417 & 0.0435 \\ 0.0435 & 0.0202 \end{bmatrix}.$$  

Finally, the GRBs distance moduli, from the definition $\mu_{\text{GRB}} = 5 \log(d_{L}^{\text{cal}}/\text{Mpc}) + 25$, can be calculated by using all the fitting parameters obtained from the Amati relation for the sample of 81 GRBs with $0.117 \leq z \leq 5.283$. The variance of $\mu$ is computed by using error propagation method and is given by

$$\sigma_{\mu_{\text{GRB}}}^2 = \left( \frac{\partial \mu_{\text{GRB}}}{\partial A} \right)^2 \sigma_A^2 + \left( \frac{\partial \mu_{\text{GRB}}}{\partial B} \right)^2 \sigma_B^2 +$$

$$2 \left( \frac{\partial \mu_{\text{GRB}}}{\partial A} \right) \left( \frac{\partial \mu_{\text{GRB}}}{\partial B} \right) \sigma_{AB} + \left( \frac{\partial \mu_{\text{GRB}}}{\partial E_p} \right)^2 \sigma_{E_p}^2 +$$

$$+ \left( \frac{\partial \mu_{\text{GRB}}}{\partial S_{\text{bolo}}} \right)^2 \sigma_{S_{\text{bolo}}}^2,$$

where it is not included the covariances for $E_p$ and $S_{\text{bolo}}$ since they are not correlated.

The distance moduli of the 81 GRBs, $\mu_{\text{GRB}}$, and their $1\sigma$ uncertainties calibrated through the Amati relation are listen in Table II. The corresponding distribution of $\mu_{\text{GRB}}$ versus $z$ is shown in Fig. 4 together the most recent compilation of SNe Ia, the Pantheon dataset [46].

Before we proceed, we consider important to point out that although there is a debate as to whether the Amati relation is an intrinsic effect or the result of detection biases or even a combination of these two [38, 64–71]. There are also works that claimed the instrumental selection biases, even if they may affect the sample,
| Name         | z_{\text{GRB}} | \mu_{\text{GRB}} | \sigma_{\text{GRB}} |
|--------------|----------------|-------------------|----------------------|
| GRB180728728 | 0.117          | 42.8925           | 0.3202               |
| GRB150727793 | 0.313          | 44.8198           | 0.2120               |
| GRB171010792 | 0.3285         | 45.0890           | 0.2084               |
| GRB130427324 | 0.34           | 40.7755           | 0.7320               |
| GRB130925173 | 0.347          | 41.9251           | 0.6303               |
| GRB140606133 | 0.384          | 45.8540           | 0.5696               |
| GRB190114873 | 0.425          | 41.5673           | 0.7661               |
| GRB091127976 | 0.493          | 44.5669           | 0.6197               |
| GRB090618353 | 0.546          | 41.9673           | 0.2065               |
| GRB101219686 | 0.5519         | 45.5073           | 0.4163               |
| GRB170607971 | 0.557          | 45.2022           | 0.2349               |
| GRB180720598 | 0.654          | 43.8625           | 0.6303               |
| GRB080914060 | 0.689          | 44.3602           | 0.2588               |
| GRB111228657 | 0.7163         | 43.7251           | 0.7325               |
| GRB140512814 | 0.725          | 43.1577           | 0.6938               |
| GRB130215063 | 0.736          | 44.0437           | 0.3068               |
| GRB131223198 | 0.738          | 44.5001           | 0.5444               |
| GRB140602660 | 1.1175         | 44.2628           | 0.2436               |
| GRB160509374 | 1.17           | 41.7320           | 0.4133               |
| GRB190324947 | 1.17           | 40.6119           | 0.2267               |
| GRB140213807 | 1.24           | 45.3785           | 0.3163               |
| GRB130420313 | 1.297          | 44.4991           | 0.4700               |
| GRB140801792 | 1.32           | 46.3119           | 0.2246               |
| GRB160509374 | 1.368          | 44.1446           | 0.6472               |
| GRB160625945 | 1.406          | 41.3129           | 0.5167               |
| GRB180205184 | 1.409          | 46.5709           | 0.6687               |
| GRB180141610 | 1.44           | 44.2451           | 0.2165               |
| GRB180314030 | 1.445          | 45.4831           | 0.2547               |
| GRB190213220 | 1.46           | 45.5781           | 0.2381               |
| GRB161117066 | 1.549          | 44.3270           | 0.3228               |
| GRB190728095 | 1.567          | 43.3861           | 0.3023               |
| GRB090120900 | 1.71           | 46.1073           | 0.3894               |
| GRB100906576 | 1.727          | 43.6092           | 0.3680               |
| GRB12019170  | 1.728          | 44.9427           | 0.2248               |
| GRB150314205 | 1.758          | 44.4233           | 0.4052               |
| GRB120326056 | 1.798          | 46.6725           | 0.5366               |
| GRB131011741 | 1.874          | 45.0715           | 0.2895               |
| GRB190612141 | 2.006          | 47.8070           | 0.9432               |
| GRB170705115 | 2.01           | 45.7002           | 0.2684               |
| GRB161017745 | 2.0127         | 45.7348           | 0.2917               |
| GRB140620219 | 2.04           | 45.2549           | 0.3761               |
| GRB150409313 | 2.06           | 44.8412           | 0.4816               |
| GRB090926181 | 2.1062         | 44.6289           | 0.3914               |
| GRB120624933 | 2.1974         | 43.0422           | 0.6318               |
| GRB121128212 | 2.2            | 46.0748           | 0.4233               |
| GRB081221681 | 2.26           | 44.9163           | 0.2998               |
| GRB141028455 | 2.33           | 45.0981           | 0.3481               |
| GRB080905705 | 2.374          | 45.3735           | 0.2518               |
| GRB130518580 | 2.49           | 44.4747           | 0.4165               |
| GRB081121858 | 2.512          | 44.8830           | 0.2120               |
| GRB170214649 | 2.53           | 43.4524           | 0.5245               |
| GRB120811649 | 2.671          | 46.9384           | 0.4537               |
| GRB140206304 | 2.74           | 45.2477           | 0.2227               |
| GRB081222204 | 2.77           | 46.0675           | 0.2076               |
| GRB110731465 | 2.83           | 46.0880           | 0.3799               |
| GRB181020792 | 2.938          | 45.1314           | 0.3299               |
| GRB140703026 | 3.114          | 46.0270           | 0.2579               |
| GRB140233556 | 3.26           | 45.2132           | 0.2346               |
| GRB140808038 | 3.29           | 47.6998           | 0.2397               |
| GRB110818860 | 3.36           | 45.9832           | 0.2544               |
| GRB170405777 | 3.51           | 44.5288           | 0.3177               |
| GRB090323002 | 3.57           | 44.3246           | 0.5026               |
| GRB120909007 | 3.93           | 45.5415           | 0.2444               |
| GRB090516353 | 4.109          | 45.5758           | 0.2189               |
| GRB120712571 | 4.1745         | 47.3048           | 0.2222               |
| GRB140304557 | 5.283          | 47.2699           | 0.2422               |

**TABLE II:** Distance moduli of 81 GRBs calibrated through the Amati relation.
FIG. 4: Distance moduli $\mu_{\text{GRB}}$ for our 81 GRB sample together with the SNe Ia distance moduli compared to the flat $\Lambda$CDM model with $\Omega_m = 0.299$, $\Omega_\Lambda = 0.701$ and $h = 0.6875$.

cannot be responsible for the existence of the spectral-energy correlations [65, 72]. See also [45, 73, 74] for more detail discussion supporting the reliability of Amati relation.

IV. PARAMETER ESTIMATION OF DARK ENERGY

We have used the public Boltzmann code CLASS [75] to run the background evolution for all the dark energy models studied here: the $\Lambda$CDM, $\omega$CDM and CPL models. Then we use the cosmological parameter estimator MONTE PYTHON [76], which is linked to CLASS and adopts the Markov Chain Monte Carlo (MCMC) method to constrain the parameters of each DE model by fitting the cosmological data. The code employs the Metropolis-Hastings algorithm [77, 78] for sampling, and computes the Bayesian parameter inference of the posteriors with the convergence test given by the Gelman-Rubin criterion $R$ [79], where we require $R - 1 < 10^{-3}$ for all our chains.

The suite of datasets considered for our analysis includes those related to the expansion history of the universe, i.e., the ones describing the distance-redshift relations. More precisely, we use Type Ia Supernovae, BAO data and CMB data in the condensed form of shift parameters (also known as distance priors) as well as the calibrated samples of Gamma Ray Bursts listed above.

A. Observational data

1. Type Ia Supernovae (SNe Ia)

One of the latest SNe Ia data compilation is the Pantheon sample [46] which consists of 1048 SNe with the redshift spanning $0.01 < z < 2.3$. This sample is a combination of 365 spectroscopically confirmed SNe Ia discovered by the Pan-STARRS1 (PS1) Medium Deep Survey together with the subset of 279 PS1 SNe Ia ($0.03 < z < 0.68$) with distance estimates from SDSS, SNLS and several low-z and Hubble Space Telescope
In order to perform our analysis, we choose this dataset and use it in the usual manner to define

$$\chi^2_{SN} = \Delta \mu \cdot C^{-1} \cdot \Delta \mu,$$

(10)

where $C$ is the full systematic covariance matrix and $\Delta \mu = \mu_{\text{theo}} - \mu_{\text{obs}}$ is the vector of the differences between the observed and theoretical value of the observable quantity for SNe Ia, the distance modulus, $\mu$. It is worth mentioning that in our analysis the absolute magnitude is taken as nuisance parameter.

### 2. Baryon Acoustic Oscillations (BAO)

We used the low redshift galaxy BAO data listed in Table III. The data provide measurements of three types of ratios of comoving distance: the angular scale of the BAO ($D_A(z)/r_s$), the redshift-space BAO scale ($D_H(z)/r_s$) [47], and the spherically-averaged BAO scale ($D_V(z)/r_s$) [48, 49] being $r_s$ the comoving sound horizon at the end of the baryon drag epoch given by

$$r_s = \int_{z_d}^{\infty} \frac{c_s(z)}{H(z)} \, dz,$$

(11)

where $c_s$ denotes the sound speed in the primordial photon-baryon plasma given by $c_s = 3^{-1/2}c[1 + \frac{3}{4}\rho_b(z)/\rho_\gamma(z)]^{-1/2}$. $D_H(z) = c/H(z)$, $D_A(z)$ is the comoving angular diameter distance

$$D_A(z) = c \int_0^z \frac{dz'}{H(z')},$$

(12)

and $D_V(z)$ is the spherically averaged combination of transverse and radial BAO modes,

$$D_V(z) = \left[ zD_H(z)D_A^2(z) \right]^{1/3}.$$

(13)

Thus, the corresponding $\chi^2_{BAO}$ for BAO data is given by

$$\chi^2_{BAO} = \Delta F^{BAO} \cdot C_{BAO}^{-1} \cdot \Delta F^{BAO},$$

(14)

where $\Delta F^{BAO} = F_{\text{theo}} - F_{\text{obs}}$ is the difference between the observed and theoretical value of the observable quantity for BAO which can be different depending on the considered survey and $C_{BAO}^{-1}$ is the respective inverse covariance matrix.
3. Cosmic Microwave Background (CMB)

Instead of the full data of the CMB anisotropies, we used CMB data in the condensed form of shift parameters reported in [50], which were derived from the last release of the Planck results [80]. Evidently, the analysis proceeds much faster in this way than by performing an analysis involving the full CMB likelihood.

The shift parameters, \((R, l_A, \Omega_b h^2, n_s)\) provide an efficient summary of CMB data as far as DE constraints are concerned (as it has been argued in several works [81–84]) which can be used to study models with either non-zero curvature or a smooth DE component, as in our case, but not for modifications of gravity [83, 84].

The first two quantities in the vector \((R, l_A, \Omega_b h^2, n_s)\) are defined as

\[
R \equiv \sqrt{\Omega_m H_0^2 r(z_*)/c},
\]

\[
l_A \equiv \pi r(z_*)/r_s(z_*),
\]

where \(r(z)\) is the comoving distance and \(r_s(z)\) is the comoving sound horizon, both evaluated at photon-decoupling epoch \(z_*\).

The corresponding \(\chi^2\) for the CMB is

\[
\chi^2_{CMB} = \Delta F_{CMB} \cdot C_{CMB}^{-1} \cdot \Delta F_{CMB},
\]

where \(F_{CMB} = (R, l_A, \Omega_b h^2, n_s)\) is the vector of the shift parameters and \(C_{CMB}^{-1}\) is the respective inverse covariance matrix. The mean values for these shift parameters as well as their standard deviations and normalized covariance matrix are taken from Table 1 of [50].

4. Gamma-Ray Bursts (GRBs)

We used two samples. The first one consists of 193 GRBs calibrated in [44] which cover the redshift range \(0.03351 \leq z \leq 8.1\). The second sample is our set of 81 Fermi-GRBs listed above, with a redshift range \(0.117 \leq z \leq 5.283\) calibrated in this work in a model-independent way.

The \(\chi^2\) function for the GRBs data is defined similarly to the SNe Ia data, Eq. 10 as

\[
\chi^2_{GRBs} = \Delta \mu \cdot C^{-1} \cdot \Delta \mu,
\]

where \(C\) is a diagonal matrix containing \(\sigma^2_\mu\) and \(\Delta \mu = \mu_{\text{theo}} - \mu_{\text{estimated}}\) is the vector of the differences between the theoretical and estimated value of the distance moduli for the GRBs.

V. RESULTS AND DISCUSSION

We have obtained the constraints for the \(\Lambda\)CDM, \(\omega\)CDM and CPL models from the latest observational data of SNe Ia, BAO, CMB distance priors inferred from the final Planck 2018 data, and including either the 193 GRBs calibrated by [44] or the 81 GRBs calibrated in this work. For comparison, SNe Ia + BAO + CMB without GRBs have been also analysed in order to highlight the contribution of GRBs to the joint
FIG. 5: Constraints at the 68% and 95% C.L. on the \((\Omega_m, \Omega_\Lambda)\) plane using different combinations of datasets. 1) The joint analysis of the SNe Ia, BAO and CMB distance priors, 2) SNe Ia + BAO + CMB + the sample of 193 GRBs calibrated by [44], labeled as GRBs(1) and 3) SNe Ia + BAO + CMB + the sample of 81 GRBs from Fermi-GBM catalog calibrated in this work, labeled as GRBs(2).

|                  | SNIa+BAO+CMB | SNIa+BAO+CMB + GRBs(1) | SNIa+BAO+CMB + GRBs(2) |
|------------------|--------------|------------------------|------------------------|
| Best-fit         | \(0.3179\)   | \(0.3180\)             | \(0.3181\)             |
| \(\Omega_m\)    | \(0.6820\)   | \(0.6820\)             | \(0.6819\)             |
| Mean±\(\sigma\) | \(0.3179\pm0.00057\) | \(0.3180\pm0.00057\) | \(0.3181\pm0.00057\) |
|                  | \(0.6820\pm0.00058\) | \(0.6820\pm0.00058\) | \(0.6819\pm0.00058\) |

TABLE IV: Constraints at 68% C.L. on the cosmological parameters \((\Omega_m, \Omega_\Lambda)\) in case of the \(\Lambda\)CDM model using different combinations of datasets. 1) The joint analysis of the SNe Ia, BAO and CMB distance priors, 2) SNe Ia + BAO + CMB + the sample of 193 GRBs calibrated by [44], labeled as GRBs(1), and 3) SNe Ia + BAO + CMB + the sample of 81 GRBs from Fermi-GBM catalog calibrated in this work, labeled as GRBs(2).

The results of a similar analysis for the \(\omega\)CDM model are displayed in Figure 6. In this case our calibrated sample yields a value for the \(\omega_\Lambda\) parameter lower than that obtained with the other two datasets. We find, however, consistency with the results of [44] at 2\(\sigma\) confidence level as is evident from the datasets SNe Ia...
FIG. 6: Constraints at the 68% and 95% C.L. on the \((\omega_0, \Omega_m)\) plane from the combinations of datasets mentioned in Fig. 5.

The datasets tested show most tension when analysing the CPL model. In Figure 7 we show 1σ (dark colours) and 2σ (light colours) error contours in the \((\omega_0, \omega_a)\) parameter space resulting from the combination SNe Ia + BAO + CMB + GRBs(1) and SNe Ia + BAO + CMB + GRBs(2) in that same figure. The respective best-fits of the analysis are listed in Table V. Note that the values for \(\sigma\) reported in that table indicate that our sample yields similar errors to those of [44].

The datasets tested show most tension when analysing the CPL model. In Figure 7 we show 1σ (dark colours) and 2σ (light colours) error contours in the \((\omega_0, \omega_a)\) parameter space resulting from the combination SNe Ia + BAO + CMB + GRBs(1) and SNe Ia + BAO + CMB + GRBs(2) in that same figure. The respective best-fits of the analysis are listed in Table V. Note that the values for \(\sigma\) reported in that table indicate that our sample yields similar errors to those of [44].

### TABLE V: Constraints at 68% C.L. on the cosmological parameters of the \(wCDM\) model using the combinations of datasets mentioned in Table IV

|               | SNe Ia + BAO + CMB | SNe Ia + BAO + CMB + GRBs(1) | SNe Ia + BAO + CMB + GRBs(2) |
|---------------|--------------------|-------------------------------|-------------------------------|
| \(\omega_0\)  | -0.9776            | -0.9782^{+0.01}_{-0.0096}    | -0.9755                       |
| \(\Omega_m\)  | 0.3141             | 0.3142^{+0.0018}_{-0.0018}   | 0.3138                        |
| \(\Omega_{DE}\)| 0.6858             | 0.6857^{+0.0018}_{-0.0018}   | 0.6861                        |

### TABLE VI: Constraints at 68% C.L. on the cosmological parameters of the CPL model using the combinations of datasets mentioned in Table IV

|               | SNe Ia + BAO + CMB | SNe Ia + BAO + CMB + GRBs(1) | SNe Ia + BAO + CMB + GRBs(2) |
|---------------|--------------------|-------------------------------|-------------------------------|
| \(w_0\)      | -0.9631            | -0.9515^{+0.074}_{-0.078}    | -0.8628                       |
| \(w_a\)      | -0.05507           | -0.1079^{+0.3}_{-0.27}       | -0.4361                       |
| \(\Omega_m\) | 0.3142             | 0.3144^{+0.0018}_{-0.0019}   | 0.3144                        |
of samples indicated above. The best-fit values from our calibrated sample intersect only at 2σ with those from SNe Ia + BAO + CMB + 193 GRBs, whilst consistency lies at 1σ with the data excluding GRBs. We note that the values of the parameter ω₀ in the CPL model from the three datasets employed in this work, intersect the range of values resulting from the Planck 2018 data at 2σ [80], and the recent results of [86] where a sample of GRBs covering a redshift range of 0.033 ≤ z < 9 is employed, together with direct \( H(z) \) measurements [87] and the past compilation of SNe Ia Union 2.1 [88]. A list of the results of this analysis is provided in Table VI. Note again that our adequate handling of errors yields an even smaller confidence region for the parameters of this model than the posterior of [11].

VI. SUMMARY AND CONCLUSIONS

Through the criteria described in Sec. [11] we have carefully selected a sample of 81 GRBs. The Amati relation for these GRBs is calibrated in a model-independent way. Such calibration, described in Section III, relies on \( H(z) \) data as our calibration source at low redshifts. We have computed and incorporated our GRB distance moduli to a suite of observations complemented by the latest CMB, BAO and SNe Ia data in order to fit parameters of Dark Energy and test for the usefulness of our sample. We find consistency with previous works for \( \Lambda \)CDM and \( \omega \)CDM models at 2σ in the posterior contours of the relevant parameters. An evident difference with previous results lies on the values of the parameters of the CPL model. Our data prefers a dynamic dark energy which transits from a quintessence-like equation of state (\( \omega_{DE} > -1 \)) at early times to a phantom-like component (\( \omega_{DE} < -1 \)) today. The discrepancies with results of previous works employing GRB samples may be due to one or more of the following factors. Our debugging technique for the GRBs compilation discards objects with large uncertainty and those for which the spectral parameters are undetermined. This is partially the reason why we have not included GRBs from the Swift satellite, where some of the spectrum features lie outside the range of wavelengths detected (see discussion in Sec. [IIA]). Furthermore, we have taken special
care in handling the errors of our dataset. Additionally, in the estimation of DE parameters we employ the GRBs sample combined with the latest cosmological data (Planck 2018 for CMB and DR12 for BAO) and the latest compilation of SNe Ia (Pantheon) in contrast with previous works which avoid cosmological distance estimators and take supernovae only from the JLA compilation or Union 2.1 compilation. In any case, it is clear from Table VI that the errors associated are tighter for this model than those of the Amati sample. It is worth mentioning that the difference between our posterior values and those of previous works may also reflect the increasing tension between the local and cosmological estimations of the Hubble parameter. The apparent DE equation of state evolution may just represent a different interpretation of a known problem: the tension standing between the local and cosmological determination of the expansion parameter (see e.g. [89]). Our sample covers a range of redshift up to $z = 5.283$, yet the calibration of GRBs is carried through direct measurements of $H(z)$ thus relying purely on local measurements of the expansion. As a consequence the extra weight of the local measurements may serve as a lever arm for the phantom behaviour of the DE equation of state. The above issues may be elucidated with the arrival of new and more precise data. In the meantime we are confident to have obtained a robust sample of GRBs for cosmic distance estimation and with the associated errors adequately accounted for. Our estimation of DE parameters show that GRBs are competent as a complementary probe to the other well-established cosmological observations.

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