Virtual Compton scattering off the nucleon (VCS) is studied in the regime of low energy of the outgoing real photon. This regime allows one to directly access the generalized polarizabilities of the nucleon in a VCS experiment. In the derivation of the low energy theorem for VCS that exists in the literature, the low energy limit taken for virtual initial photons does not match on that for real photons, when one approaches the initial photon's mass shell. While this problem has for a long time been attributed to the non-analyticity of the Compton amplitude with respect to the photon virtuality, I demonstrate that it is merely due to an ill-defined low energy limit for VCS, on one hand, and to a particular way of constructing the VCS amplitude, use in the literature, on the other.

I provide a uniform description of low energy Compton scattering with real and virtual photons by defining a Lorentz-covariant operator sub-basis for Compton scattering in that regime, that has six independent structures. Correspondingly, six new generalized polarizabilities are introduced in the Breit frame. These polarizabilities are defined as continuous functions of the photon virtuality and at the real photon point match onto the nucleon polarizabilities known from real Compton scattering.

PACS numbers:

I. INTRODUCTION

Scattering of photons has been among the basic tests used to study the structure of matter and to understand the nature of light. The experiments of Faraday and theoretical studies of Maxwell established the notion of electromagnetic waves light was identified with. Compton effect, the phenomenon of the wave length shift of light scattered off an electron that is impossible for a classical wave, established the picture of particle-wave duality, along with Einstein’s theory of the photoeffect. With the discovery of the four fundamental interactions and the continuing quest for the sub-nucleon structure, Compton scattering with real and virtual photons reemerged as a clean way to study this structure experimentally and theoretically. The merit of photons as a universal elementary probe of nucleon structure becomes especially emphasized within the framework of sum rules and dispersion relations, the connection between scattering of photons of low frequency, and absorption of photons of high frequencies [1][2][3]. These are relations between low energy coefficients, polarizabilities that describe the response of the nucleon structure to the quasi-static external electromagnetic field, and the nucleon’s photoabsorption spectrum. Analyticity, along with unitarity of the Compton amplitude play the central role in this derivation.

Introduction of polarizabilities is based on the low energy theorem (LET) [4]. Electric polarizability $\alpha$ and magnetic susceptibility $\beta$ quantify the linear response of the nucleon to the incoming photon’s electric (magnetic) field $E(B)$, respectively. The induced electric (magnetic) dipole interacts with the outgoing photon’s field $E'(B')$ leading to an effective interaction Hamiltonian $4\pi\alpha E \cdot E' + 4\pi\beta B \cdot B'$. Since at low photon energy $\omega$ both fields are $\sim \omega$, one can state that generally, the contribution of the unknown nucleon structure enters Compton observables at order $\omega^2$. Once the known, energy-independent classical Thomson term is separated out, the polarizabilities are directly measurable. Here one can already guess a potential complication: the polarizabilities arise in a Hamiltonian, rather than Lagrangean. This means that the procedure of low energy expansion is in general frame dependent, since the operator is not Lorentz-covariant. Fortunately, for real Compton scattering the frame dependence arises as corrections in powers of a small quantity $\omega/M$ and at the accuracy at which the low energy expansion is truncated, is irrelevant.

This is not generally the case when one of the photons is virtual, i.e. originating from electron scattering. The finite initial photon ”mass” $Q^2$ ensures that only outgoing photon’s energy vanishes. The incoming and outgoing photon energy vanishes simultaneously in Breit frame that treats the two photons symmetrically, but not in center-of-mass or laboratory frame. Then it is clear that expanding the virtual Compton amplitude in one frame or another can differ by terms $\sim Q^2/M^2$.

Low energy theorem for virtual Compton scattering (VCS) was introduced through low energy expansion of the VCS scattering coefficients in c.m. frame [5]. The VCS amplitude was decomposed into multipoles that correspond to dipole and quadrupole transitions in the initial and final $\gamma N$ state. At low outgoing photon energy, there are only ten multipoles that vanish linearly, and factoring out this energy dependence, the ten generalized polarizabilities (GP’s) were introduced. The term ”generalized” refers to the fact that these GP’s are not numbers, but functions of the three-momentum of the virtual photon $q$ that is kept fixed, thus they generalize the real Compton scattering (RCS) polarizabilities that
should thus be just the limit \( q \to 0 \) of the GP’s.

This correspondence is troublesome, for instance since ten GP’s should be related to six polarizabilities of RCS [9]. The procedure of [5] is clearly non-covariant. Implementing Lorentz invariance, in particular crossing that relates VCS reactions with interchanged initial and final nucleons, it was possible to eliminate four out of ten GP’s [7], equalling the number of independent GP’s that should be confronted to the RCS polarizabilities. However, only four of the six GP’s match onto their RCS counterparts, as two of them vanish at the real photon point. This mismatch is usually attributed to the non-analyticity of the Compton amplitude as function of the photon virtuality \( Q^2 \) [6, 7]. Recalling the special role the analyticity has played in the derivation of the nucleon sum rules, it is highly desirable to clarify the origin of such non-analytical behavior, if it indeed takes place. In fact, non-analytical behavior has to be related to a physical singularity, otherwise it is not acceptable in a field theory.

A similar problem was observed some time ago when performing low energy expansion of the forward doubly virtual Compton scattering [8]. The authors found a striking result that in this case, even the lowest term of the expansion does not match to the classical Thomson term. This mismatch was again attributed to the non-analyticity of the Compton amplitude with respect to \( Q^2 \). In a recent paper [9], it was shown that this mismatch is a pure artifact of an ill-defined low energy limit. In [8], the LEX is performed around a point that does not correspond to nucleon near its physical mass shell. In [9] it is shown that the LEX for VVCS cannot be performed at forward direction, and instead requires non-zero nucleon recoil for virtual photons. The new formulation of LET allowed to define the low energy coefficients in a continuous way, and to derive the generalized sum rules of the nucleon.

In this article, I aim at revising the LET for VCS in order to ensure that the Compton amplitude is continuous and analytical function of all its variables. In a paper by L’vov et al. [10], an alternative approach to analyzing low-energy VCS process was proposed. It was based on i) operating with the Compton basis written in terms of the electromagnetic field strength tensors, thus using a Lorentz covariant description to begin with, and ii) evaluating those operators in Breit frame and rewriting them in terms of electromagnetic field three-vectors. This approach showed that one does not in general need to perform a multipole decomposition of the VCS amplitude, but rather use the classical analogy and interpret the scattering coefficients that multiply the covariant structures as Breit-frame polarizabilities. Unfortunately, that work only included the spin-independent part. Another important point that is missing in [10], is that the low energy limit of Guichon et al. was used, even if implicitly. Correspondingly, neither a complete set of GP’s was introduced, nor brought in correspondence with the VCS observables that is ultimately the principal reason for performing LEX. The aim of the present work is in improving on both points. This study, as well, capitalizes to a large extent on the work by Ragusa [6] who introduced the full low energy expansion of the RCS amplitude.

The article is organized as follows. I will start with defining the VCS kinematics in Section II and rewriting the covariant VCS tensor in terms of electromagnetic strength tensors and in Lorentz-covariant form in Section III. I will then propose the new way to define the low energy limit for VCS that is explicitly Lorentz invariant, respects crossing symmetry and ensures that the path along which this limit is taken always lies inside the physical region in Section IV. I will perform the low energy expansion (LEX) of the VCS amplitude in Section V, relating the tensors to operators involving electromagnetic field three-vectors in Breit frame, and provide thus the interpretation of the scattering coefficients as polarizabilities. Finally in Section VI I investigate the relations between the new GP’s and the VCS observables.

## II. VIRTUAL COMPTON KINEMATICS

I consider the virtual Compton scattering process \( \gamma^*(q) + N(p) \to \gamma(q') + N(p') \). Its kinematics is described in terms of Lorentz scalars

\[
\begin{align*}
    s &= (p + q)^2 = (p' + q')^2 = (P + K)^2 \\
    u &= (p - q')^2 = (p' - q)^2 = (P - K)^2 \\
    t &= (q - q')^2 = (p' - p)^2 \\
    Q^2 &= -q^2 \geq 0
\end{align*}
\]

where the nucleon and photon average momenta were introduced, \( P = \frac{p + p'}{2}, \ K = \frac{q + q'}{2} \). The sum of these variables is fixed by

\[ s + u + t + Q^2 = 2M^2, \]

with \( M \) the nucleon mass. The above relation implies that only three of them are independent, and it is useful to introduce the “crossing” variable

\[ \nu = \frac{s - u}{4M^2} = \frac{PK}{M} = \frac{s - M^2 + \frac{t + Q^2}{4}}{2M} \]

Each value of \( Q^2 \), \( \nu \) and \( t \) can be related to the incoming and outgoing photon energy \( \omega, \omega' \), respectively, and scattering angle \( \theta \) that are frame dependent. For given \( Q^2 \), one has the three-vector of the virtual photon \( |\vec{q}| = \sqrt{\omega^2 + Q^2} \equiv q \), while \( |\vec{q}'| = \omega' \). In the c.m. frame defined by \( \vec{p} + \vec{q} = 0 \) that was used in [5], one has

\[
\begin{align*}
    \omega' &= \frac{s - M^2}{2\sqrt{s}} \\
    \omega &= \frac{s - M^2 - Q^2}{2\sqrt{s}} \\
    \cos \theta &= \frac{\omega}{q} + \frac{t + Q^2}{2q\omega'}
\end{align*}
\]
Alternatively, in this work the nucleon Breit frame will be used. This frame is defined by $\vec{P} = 0$. Breit frame is of advantage because it treats the photons in a symmetric manner,

$$\omega = \omega' = \frac{PK}{P^0} = \frac{M\nu}{\sqrt{M^2 - t/4}} = s - M^2 + \frac{t + Q^2}{2\sqrt{M^2 - t/4}}$$

$$\cos \theta_{T}\gamma = \frac{\omega}{q} + \frac{t + Q^2}{2q\omega}$$

(5)

### III. COMPTON AMPLITUDE

The Lorentz covariant and explicitly gauge invariant tensor basis for VCS was introduced long ago. It was then used to study VCS in LEX approach and in dispersion relations approach. I will use the form of the VCS tensor of Ref. 11

$$T_{VCS}^{\mu\nu} = \sum_{i=1}^{12} F_i(u, v, Q^2)\bar{u}(p')\rho_i^{\mu\nu}u(p)$$

(6)

as a starting point. Above, the $\rho_i$’s are Lorentz covariant tensors that are linearly independent and explicitly gauge-invariant by construction. The tensors are cast between the initial and final nucleon spinors $u(p)$ and $u(p')$, respectively. The numeration of the tensors can be different in different references, and for definitiveness I use the one of Refs. 7, 12. The corresponding amplitudes $F_i$ are Lorentz scalars that are functions of the kinematical variables $\nu, t$, and $Q^2$. The VCS tensor is then embedded into the full, physical amplitude $T_{VCS}$ for the scattering process $e + p \rightarrow e + p + \gamma$ as

$$T_{VCS} = \frac{e}{Q^2}\bar{u}(k', h)\gamma_i u(k, h)T_{VCS}^{\mu\nu}\zeta_{i\nu}(q', \lambda)$$

$$e \sum_{i}^{12} \Omega(\lambda, \alpha)\epsilon_\mu(q, \lambda)T_{VCS}^{\mu\nu}\zeta_{i\nu}(q', \lambda),$$

(7)

where $u(k), u(k')$ denote the initial and final electron’s spinors, $h$ the conserved helicity of massless electrons, and $\Omega(\lambda, \alpha) = \bar{u}(k', h)\gamma_5(q, \lambda)u(k, h)$. I refer the reader to the Appendix for the explicit form of the polarization vectors of the initial and final photons. Above, $e$ stands for the (positron’s) electric charge, and my conventions for the VCS amplitude differ from those used in the literature by a factor of $-e^2$ for further convenience. I will next rewrite the twelve tensors in terms of the electromagnetic field strength tensors. I define these latter as $F_{\mu\nu} = ie(q^\alpha \epsilon_\mu^\alpha - q^\mu \epsilon_\nu^\alpha)$ and $F_{\nu\lambda} = -ie(q^\beta \epsilon_\nu^\beta - q^\nu \epsilon_\lambda^\beta)$. The following expressions can be found for $\rho_i$:

$$\rho_1 = \frac{1}{2} F_{\mu\nu} F_{\nu\mu}$$

$$\rho_2 = -\frac{1}{4} \left( P^\mu F_{\mu\nu} \right) \left( P^\nu F_{\nu\alpha} \right)$$

$$\rho_3 = \frac{2}{PK} \left( q^2 g_{\alpha\beta} - q_\alpha q_\beta \right) \left( P^\mu F_{\mu\alpha} \right) \left( P^\nu F_{\nu\beta} \right)$$

(8)

for the spin-independent part, and

$$\rho_4 = 2P^\mu \left[ F_{\mu\alpha} F_{\alpha\beta} - F_{\mu\alpha} F_{\alpha\beta} \right] i\gamma^5 \gamma^\beta$$

$$\rho_5 = -\frac{1}{2} \left( q \epsilon_\nu^\gamma F_{\nu\alpha} \right) \left( q \epsilon_\alpha^\beta F_{\nu\beta} \right) i\gamma^5 \gamma^\beta$$

$$\rho_6 = -\frac{1}{2} \left( q \epsilon_\nu^\gamma F_{\nu\alpha} \right) \left( q \epsilon_\alpha^\beta F_{\nu\beta} \right) i\gamma^5 \gamma^\beta$$

$$\rho_7 = -\frac{1}{2} \left( q \epsilon_\nu^\gamma F_{\nu\alpha} \right) \left( q \epsilon_\alpha^\beta F_{\nu\beta} \right) i\gamma^5 \gamma^\beta$$

$$\rho_8 = -\frac{1}{2} \left( q \epsilon_\nu^\gamma F_{\nu\alpha} \right) \left( q \epsilon_\alpha^\beta F_{\nu\beta} \right) i\gamma^5 \gamma^\beta$$

$$\rho_9 = -\frac{1}{2} \left( q \epsilon_\nu^\gamma F_{\nu\alpha} \right) \left( q \epsilon_\alpha^\beta F_{\nu\beta} \right) i\gamma^5 \gamma^\beta$$

$$\rho_{10} = -2F_{\mu\alpha} F_{\mu\beta} i\sigma_{\alpha\beta}$$

$$\rho_{11} = \frac{1}{2} \left( q \epsilon_\nu^\gamma F_{\nu\alpha} \right) \left( q \epsilon_\alpha^\beta F_{\nu\beta} \right) i\gamma^5 \gamma^\beta$$

$$\rho_{12} = \frac{1}{2} \left( q \epsilon_\nu^\gamma F_{\nu\alpha} \right) \left( q \epsilon_\alpha^\beta F_{\nu\beta} \right) i\gamma^5 \gamma^\beta$$

for the spin-dependent part of the VCS amplitude. In the above, the notation $\Delta = q - q'$ was used. Transversality condition fixes $q_{\beta} F_{\beta\nu} = 0$, while a similar condition for the virtual photon is not required. Nevertheless, for the reasons of symmetry that will be important in the discussion of the properties of the amplitudes $F_i$, I keep terms $\sim q_{\beta} F_{\beta\nu}$ in Eq. 9. The dual tensor is defined as $F_{\alpha\beta} = \frac{1}{2} \varepsilon_{\alpha\beta\mu\nu} F_{\mu\nu}$ and similarly for $F'$, and in general for the virtual photon. One observes that all twelve tensors can be expressed through the field strength tensors in a compact way. The first three tensors do not depend on the nucleon spin, and coincide with the expressions found in 10. The remaining nine tensors are spin-dependent, and were not represented in this form in the literature. I next consider properties of the VCS amplitudes under two crossing transformations: nucleon crossing relates the original reaction $\gamma^*(q) + N(p) \rightarrow \gamma(q') + N(p')$ to $\gamma^*(q) + N(-p') \rightarrow \gamma(q') + N(p)$, while the photon crossing - to the process $\gamma(-q') + N(p) \rightarrow \gamma(-q) + N(p')$. Under these transformations, the tensors transform according to

$$P \rightarrow -P$$

$$PK \rightarrow -PK$$

$$\gamma^\mu \gamma^\nu \rightarrow C\gamma^\gamma C^\dagger = +\gamma^\gamma$$

$$\sigma^{\mu\nu} \rightarrow C\sigma^\nu\sigma^\dagger = -\sigma^{\nu\mu}$$

(10)
for nucleon crossing and

\[
F^{\alpha\beta} \leftrightarrow F'^{\alpha\beta} \\
q \leftrightarrow -q' \\
K \rightarrow -K \\
PK \rightarrow -PK \\
q^2 \leftrightarrow q'^2
\]

under photon crossing. Requiring the VCS amplitude to be invariant under these transformations one obtains following properties of the amplitudes \[\text{[7, 11]}\]:

\[
F_i(-\nu, t, q^2, q'^2) = F_i(\nu, t, q'^2, q^2), \\
i = 1, 2, 5, 6, 7, 9, 11, 12
\]

\[
F_i(-\nu, t, q^2, q'^2) = -F_i(\nu, t, q'^2, q^2), \\
i = 3, 4, 8, 10
\]

\[
F_i(-\nu, t, q'^2, q'^2) = F_i(\nu, t, q'^2, q^2), \\
i = 1, 2, 5, 6, 11, 12
\]

\[
F_i(-\nu, t, q^2, q^2) = -F_i(\nu, t, q'^2, q'^2), \\
i = 3, 4, 7, 8, 9, 10
\]

(11)

For both photons real, the tensors \(\rho_{1,5,8,12}\) vanish. Furthermore, the amplitudes \(F_{7,9}\) vanish in that limit. Correspondingly, real Compton scattering is described in terms of six amplitudes \(F_{1,2,4,6,10,11}\) and the corresponding tensors.

Before moving on to discuss low energy behavior of the tensors and the amplitudes, the ground state and continuum contributions to the \(F_i\)'s should be separated. On general grounds, the VCS amplitude can have two kinds of singularities, poles corresponding to an exchange of an on-shell particle in one of the channels, and cuts along which the amplitude has a non-zero discontinuity corresponding to multi-particle exchanges. The positions of singularities and various kinematical regions for the VCS process are displayed in Fig. 1. The full amplitude can then be split into two pieces, out of which one would contain poles, and the other one can only have cuts.

\[
T^{\mu\nu}_{VCS,NB} \equiv T_{VCS} - T^{\mu\nu}_{VCS,Born}
\]

(13)

The Born (nucleon pole) contribution is due to an exchange of a single nucleon in the direct and crossed channel, as shown in Fig. 2. This amplitude is given by

\[
T^{\mu\nu}_{VCS,Born} = -e^2\bar{u}(p) \left[ \frac{\Gamma^\nu(q')(P + K + M)\Gamma^\mu(q)}{(P + K)^2 - M^2 + i\epsilon} + \frac{\Gamma^\mu(q)(P - K + M)\Gamma^\nu(q')}{(P - K)^2 - M^2 + i\epsilon} \right] u(p)
\]

(14)

where the nucleon electromagnetic vertex is given by

\[
\Gamma^\mu(q) = F_1(q^2)\gamma^\mu + F_2(q^2)i\sigma^{\mu\nu}\frac{q_\nu}{2M}, \quad \text{and} \quad \Gamma^\nu(q') = e_N\gamma^\nu + \kappa_N\sigma^{\mu\nu}\frac{q'^\nu}{2M}.
\]

In the above, \(e_N\) and \(\kappa_N\) denote the nucleon charge and anomalous magnetic moment, whereas \(F_{1,2}(q^2)\) stand for the usual Dirac and Pauli form factors. For practical purposes the form factors are taken in the phenomenological form. This choice corresponds to taking out that part of the VCS amplitude that is known from other experiments and does not contain any new information. It should be noticed that this choice is not unique: the phenomenological form factors describe the nucleon on its mass shell. Then, in Eq. (14), it is only the imaginary part that always corresponds to the exchange of an on-shell nucleon in either direct or crossed channel. The real part of the Born amplitude contains off-shell nucleons, and its form factors are in general unknown. One may argue that, since the imaginary part in the \(s\) and \(u\) channel is known and given by two \(\delta\)-functions, a dispersion relation in those channels would restore the picture of the real part being given in terms of the same on-shell form factors. However, such a dispersion representation is incomplete because it neglects the analytical structure in the \(t\)-channel. For instance, Fig. 3 shows a contribution that simultaneously contains the nucleon pole and the \(\pi N\) continuum. The contribution in Fig. 3 can be obtained from unitarity in the \(t\)-channel for any ”mass” of the off-shell nucleon, but this cannot be done in a model that uses the phenomenological parametrizations of the nucleon form factors. Then, this (model-dependent) energy running of the \(\gamma^*NN\) vertex has to accompany the model-independent part associated with the on-shell nucleons. The ambiguity arises when the Born and non-Born amplitudes are calculated in two different models. Typically, the Born part is evaluated with the phenomenological form factors that are used in the analysis of VCS experiments (see Section VI for details). While the divergent \(\sim 1/\omega\) terms are model-independent, the subleading terms \(\sim \omega^0, \omega^1\) included in this contribution, are not. They can contribute at the same order as the GP’s and should be taken with care when calculating VCS amplitude in a model, and comparing results to the experiment. Any model gives GP’s with respect to the Born contribution defined and evaluated in the same model.

In the following, I will use the Born amplitude defined in terms of the phenomenological form factors, as the most practical choice.
FIG. 1: (Color online) Mandelstam plane for VCS with $Q^2 = 0.33 \text{ GeV}^2$. On the plane $\nu,t$, the kinematical regions and the positions of the singularities of the VCS amplitude are shown. The dotted lines correspond to the nucleon pole in the $s(u)$-channel, and $\pi^0$-pole in the $t$-channel. The inelastic thresholds are shown by the dashed lines. The scattering regions in the $s(u)$-channels correspond to the regions between the upper (forward scattering) and lower (backward scattering) solid lines at positive (negative) values of $\nu$, respectively. The VCS amplitude is purely real for all variables below the corresponding inelastic threshold (within the triangle). The intersection of the scattering regions with the shaded triangle represents the area where the low energy expansion can be used.

It has been argued, as well, that the $\pi^0$-exchange contribution should be included into the Born part since it is a tree-level graph, and it has a pole at $t = m^2_{\pi}$. There is however an important difference between the nucleon pole graphs and the pion pole: the former depend on interaction of a single photon with the nucleon, whereas the latter represents a local two-photon interaction, and thus contains information complementary to the nucleon pole contributions. There is still no consensus in the community, where this contribution should be included, so I choose to follow the tradition of attributing the pion pole to the continuum contribution. For the purpose of low energy scattering in the $s$-channel, this $t$-channel pole lies far enough so that its contribution is a continuous function of all variables.

Once the Born amplitude is specified, the Born contributions to the amplitudes $F_i, F^B_i$ can be calculated. The results are known [7, 11, 12], and I will not quote them here. The residual part of the amplitude can be generically introduced as $F^{NB}_i \equiv F_i - F^B_i$, and the non-Born amplitude contains no poles. Therefore, (if one stays away from the $t$-channel), this amplitude should be a regular function of all its arguments, and the only kind of singularity that it has are the unitarity cuts in $\nu$ corresponding to nucleon excitations and continuum in the $s$ and $u$ channels. The inelastic thresholds in these channels are separated from the nucleon pole by the finite pion mass. Then, in the energy range between the nucleon pole and the threshold the non-Born amplitude is a purely real regular function (see Fig. 1) that can be Taylor expanded in powers of $\omega'/m_\pi$, $m_\pi$ being the pion mass and $\omega'$ the energy of the real photon. This gives rise to the LET and LEX approach: separate out the singular part of the amplitude that you can calculate; Taylor expand the unknown residual amplitude, thus limiting the unknowns to a (minimal) set of constants; relate these constants to the observables and interpret them as polarizabilities. In the next section, I will discuss the general procedure of taking the limit of low energy for VCS.

IV. LOW ENERGY THEOREM FOR VCS

Before proceeding with the low energy expansion, one has to specify the way the low energy limit is realized. The problem is twofold: firstly, it has to be controlled that when performing the low energy limit all the symmetries of the Compton amplitude remain intact; secondly, the Born part that is to be separated out is singular precisely at the point where the LEX has to be performed. This implies that the choice of the kinematical point for
This expansion should be made with care. For instance, for real Compton scattering, the low energy limit of the Compton amplitude is well known from classical electrodynamics and is given by the constant Thomson term. However, if putting only the nucleon in the direct-channel on-shell but not the crossed channel one, it would be impossible to obtain a constant since a pole cannot be cancelled by a regular function. Therefore, unless one goes to the point \( s = u = M^2 \) one would never find the correct low energy limit. The point where zero energy limit should be taken is analogous for VCS,

\[
\begin{align*}
  s &= u = M^2 \\
  \nu &= 0 \\
  t + Q^2 &= -2(qq') = 0
\end{align*}
\] (15)

Note that in Breit frame both initial and final photon energy vanishes at this point simultaneously. As it was stated in [5], this limit should be realized as

\[
\nu \to 0 \quad \text{at fixed } Q^2 \quad \text{and } (qq') = 0
\] (16)

in order to ensure that i) the path on the Mandelstam plane along which the limit is taken lies completely inside the physical region for \( s \) or \( u \) channel process, and ii) this limit can be approached symmetrically either from positive (\( s \)-channel) or negative (\( u \)-channel) values of \( \nu \). In Breit frame, this limit corresponds to angle \( \theta \to 90^\circ \) since \( \cos \theta((qq') = 0) = \frac{\omega}{q} \to 0 \). Since the value of energy should be small enough, \( \omega \lesssim m_\pi \), this condition is too restrictive on the values of scattering angles for which the above LEX prescription is viable. To access all the kinematics, one have to relax the condition \( (qq') = 0 \) but ensure that \( (qq') \) and its first derivative with respect to \( \nu \) vanishes at \( \nu = 0 \). Vanishing of the first derivative is required by observing that \( (qq') \) is even under crossing \( \nu \to -\nu \). Therefore, the proposed procedure to perform the low energy limit for VCS is the following. Assume that a VCS measurement is carried out at the kinematical point \( \nu_0, t_0 \) (\( \omega_0 = \frac{M\nu_0}{\sqrt{M^2 - t_0^4}} \) and \( |q_0| = \sqrt{\omega_0^2 + Q^2} \) accordingly). The limit of low energy, \( \nu = 0, t = -Q^2 \) can be approached along the path \((\omega, \omega_0, t_0)\) given by

\[
\begin{align*}
  t(\omega, \omega_0, t_0) + Q^2 &= \frac{\omega^2}{\omega_0^2}(t_0 + Q^2) \quad \text{or} \\
  \cos \theta &= \frac{\omega|q|}{\omega_0|q|} \cos \theta_0,
\end{align*}
\] (17)

shown in Fig. 4. In the limit of \( Q^2 = 0 \) this path reduces simply to \( \cos \theta = \cos \theta_0 \), fixed angle that is used in LEX for RCS. There exist more than one functional form that satisfy crossing condition and reduce to fixed angle for real Compton scattering. However, they can only differ by corrections in powers of \( \omega/M \), and are equivalent for LEX. For comparison, in [5], the low energy limit is realized at fixed scattering angle \( \theta_0 \) and fixed three-vector magnitude \( |q| \). Then, along that path the VCS amplitude is decomposed into a series of multipoles with definite orbital momentum in the initial and final channel, and this series is truncated at the dipole order for the outgoing photon.

For a function that is regular and analytical in the vicinity of the point \( \nu = 0 \), \( (qq') = 0 \) (as the non-Born VCS amplitude is by construction), that point can be approached along any path, and the result should be path-independent. However, if the function is decomposed into a series in powers of energy and truncated at a given order, the path independence can become hard to control. The above discussion implies that performing the low energy limit at fixed angle for VCS can lead to complicated correlations between different terms in the low energy expansion. Just because that path on Mandelstam plane is asymmetric, the crossing symmetry of the VCS amplitude enforces constraints onto the strength.
of different multipoles, as shown in [7]. The multipoles are designed to form a basis, order by order in \( \omega' \), and existence of such correlations indicates the breakdown of the formalism. 1 Then, the prescription of Eq. [7] can be seen as a convenient choice to incorporate the crossing symmetry of the VCS amplitude to its expansion in powers of energy, order by order. But having the scattering angle depend on the energy immediately invalidates the multipole expansion approach to LEX since the multipoles and the corresponding harmonics have now to be evaluated at \( \cos \theta = 0 \) and higher orders can contribute at the same order as the lowest ones.

V. EXPANSION OF THE VCS AMPLITUDE IN BREIT FRAME AND THE NEW SET OF GP’S

In this Section, low energy expansion of the non-Born part of the VCS amplitude will be performed. As it was mentioned before, the polarizabilities are in general frame dependent quantities since they parametrize Hamiltonian, rather than Lagrangean, and they multiply various combinations of the electromagnetic field three-vectors. This can be easily seen if one considers, for instance the structure \( (P^\nu P'_{\nu 0}) \) appearing in Eq. [8]. The electric and magnetic fields can be read off the tensors \( F = F^{0i} = \vec{E}^i \), \( F^{ij} = \epsilon^{ijk} \vec{B}^k \). In Breit frame, this structure is purely electric, \( P^\nu \vec{E}' \). In the c.m., it contains electric and magnetic fields since \( P = (P + K) - K = (\sqrt{s}, 0) - (\langle \omega + \omega' \rangle, 2, (q^0 + q')/2) \), so there will be terms \( \sim \vec{q} \times \vec{B}' \) and such. Such terms are higher order in \( \omega \) for RCS, but are not suppressed for VCS.

I will approach this problem from a slightly different perspective, that of effective field theory (EFT). Low energy expansion corresponds to pionless EFT, integrating the pion-mediated interactions out of the Lagrangean and replacing them with a number of contact interactions that are then organized hierarchically in powers of \( \omega/m_\pi \). These interactions are characterized by the corresponding number of constants that can be related to the observables. The operator basis of the Lagrangean was already introduced in Eqs. (8). The corresponding amplitudes can be expanded into a series in powers of \( \nu \) (rather than \( \omega \) to keep Lorentz invariance) and only the leading order coefficients of this expansion can be kept.

According to the properties of the amplitudes \( F_i \) under crossing, we can therefore introduce twelve low energy coefficient functions \( f_i(Q^2) \) as

\[
\begin{align*}
 f_i(Q^2) &= F_i(\nu = 0, t = -Q^2, Q^2), \quad i = 1, 2, 5, 6, 11, 12 \\
 f_i(Q^2) &= \frac{1}{\nu} F_i(\nu = 0, t = -Q^2, Q^2), \quad i = 3, 4, 8, 10 \\
 f_i(Q^2) &= \frac{1}{Q^2} F_i(\nu = 0, t = -Q^2, Q^2), \quad i = 7, 9
\end{align*}
\]

Pulling out the explicit factors of \( \nu \) and \( Q^2 \) has to be accompanied by a redefinition of tensors, \( \nu \rho_{3,4,8,10} \) and \( Q^2 \rho_{7,9} \). These low energy coefficient functions can now be related to polarizabilities when specifying a particular reference frame. I will choose the nucleon Breit frame and evaluate the basis tensors in that frame to represent them in terms of electric and magnetic field three-vectors.

In RCS, the low energy expansion of the non-Born amplitude starts from \( \omega^2 \) for the spin-independent part, and \( \omega^3 \) for the spin-dependent one. Instead, in VCS this expansion starts at order \( \omega \) for both. Therefore, one can readily eliminate some of the structures from the tensor of Eqs. (8) by noticing that any structure \( \sim Q^2 \omega^2 \) or \( \omega^4 \) will contribute at higher order.

For power counting, one has in Breit frame: \( \vec{E}_L \sim \sqrt{Q^2}, \vec{B} \sim |q| \), and \( \vec{E}_T, \vec{B}' \) \( \sim \omega \). The structures to eliminate are \( \rho_3, 4\rho_7 \sim \rho_4, 2\rho_8 \sim \rho_6 \) that contribute at order \( Q^2 \omega^2 \) (see Appendix B for details) and cannot enter the LEX neither for RCS nor for VCS. This, in fact was already observed 2 [7] [12].

As a result, one is left with eight structures that can contribute to LEX at lowest order, \( \rho_{1,2,4,5,6,10,11,12} \) times the corresponding amplitudes \( F_{1,2,4,5,6,10,11,12} \). Six of them are relevant for RCS, \( \rho_{1,2,4,6,10,11} \) while \( \rho_{5,12} \) vanish for real photons. In turn, for VCS it is \( \rho_{1,10} \) that do not contribute to the LEX due to the crossing behavior of the respective amplitudes, as found in [7]. This is the formal origin of the mismatch between the low energy expansions of the RCS and VCS.

The main idea of this work is to assume that a low-energy reduction of the VCS amplitude can be found, that would consist of only six basis structures that should be the same for RCS and VCS. This amounts in building four linear combinations out of \( \rho_{4,5,10,12} \) such that two of them give the right limit at low energies, whereas the other two should be subleading in LEX. If building such linear combinations can be realised in a covariant way and without introducing any spurious kinematical singularities, this would be the solution to the problem. After a little algebra reported in Appendix B, such combina-

1 Apart from the fact that in [2] the low energy limit is realized in a crossing-asymmetric way, further problems appear: by keeping \( |q| \) fixed and varying \( \omega' \), one actually varies the \( Q^2 \) which is a Lorentz invariant. Starting at \( \omega' = |q| \) and letting \( \omega' \rightarrow 0 \), results in \( Q^2 \rightarrow Q^2 \) with \( Q^2 \approx Q^2/\left(1+\omega'/M\right) \). So one in practice relates observables at one value of \( Q^2 \) to polarizabilities at a different \( Q^2 \), as shown in Fig. [4] The kinematical point of zero energy for \( Q^2 \) lies outside the physical region for the original VCS process with \( Q^2 \), then it is no surprise that analyticity problems might come up.

2 Redefining \( \rho_{7,9} \) requires redefining the amplitudes \( F_{6,11} \) as \( F_6 = F_6 + \frac{1}{2} F_{11}, F_{11} = F_{11} + \frac{1}{2} F_6 \). It is these combinations that enter the LEX for VCS, and not \( F_{6,11} \) [22].
and this low energy reduction of the VCS amplitude is continuous in the limit \( Q^2 \to 0 \). The original numer-
tion of Ref. [7] of the amplitudes is kept to avoid any confusion. The six low energy constants (for fixed $Q^2$) completely describe the effects of the proton structure on Compton scattering with real and virtual photons and at leading order in low energy expansion. The form of the above tensor suggests that the fact that the low energy limits for RCS and VCS were found in previous studies to not match with one another, could be attributed to the particular way that was used in the literature to construct the VCS basis starting from the RCS one. It was done by adding structures that explicitly vanish for real photon, and it resulted in only partial overlap of the low energy reduction of the VCS basis with the RCS one.

Next, I will introduce the (generalized) Breit frame polarizabilities following the approach of Ragusa [6] who complemented the LEX of the spin-independent part of Low [4] by introducing the four spin-dependent polarizabilities in Breit frame, as well. While in Ref. [6], the Compton amplitude was written in terms of polarization vectors of the photons in order to have an explicit power counting in photon energy, I will rather write it in terms of electromagnetic fields. In this way, the power counting and polarization content (longitudinal or transverse) is implicit, but the generality of this description (i.e., RCS and VCS) and the analogy with the classical electromagnetic polarizabilities become more transparent.

Following the standard conventions for the Compton amplitude, and correcting for Breit kinematics with virtual initial photon, I obtain the natural generalization of the Ragusa’s LEX:

$$\frac{1}{2\rho_0} T_{LEX}^{NB} = 4\pi \alpha(Q^2) \vec{E} \cdot \vec{E}' \chi + 4\pi \beta(Q^2) \vec{B} \cdot \vec{B}' \chi$$

$$+ 4\pi \gamma_1(Q^2) - \gamma_2(Q^2) - 2\gamma_3(Q^2) \frac{1}{2} \left[ \vec{E} \times [\vec{q}' \times \vec{B}'] - \vec{E}' \times [\vec{q} \times \vec{B}] \right] \chi i \sigma \chi$$

$$+ 4\pi \gamma_2(Q^2) \frac{1}{2} \left[ \vec{q}' \times [\vec{E}' \times \vec{B}] - \vec{q} \times [\vec{E} \times \vec{B}'] \right] \chi \chi i \sigma \chi$$

$$+ 4\pi \left( \gamma_1(Q^2) + \gamma_2(Q^2) + \gamma_3(Q^2) + \gamma_4(Q^2) \right) \left[ \vec{B} (\vec{q} \vec{E}') - \vec{B}' (\vec{q}' \vec{E}) \right] \chi \chi i \sigma \chi$$

$$- 4\pi \left( \gamma_1(Q^2) + \gamma_4(Q^2) \right) \chi \chi i \sigma \Delta \chi (\vec{E}' \cdot \vec{B} + \vec{E} \cdot \vec{B}'),$$

(27)

where $\alpha(Q^2), \beta(Q^2),$ and $\gamma_i(Q^2),$ $i = 1, 2, 3, 4$ are the generalized polarizabilities for VCS, and the notation is used since they reduce to the polarizabilities of real Compton scattering at $Q^2 = 0$. Note that the operators that multiply the polarizabilities are more general than those of Ragusa [6] since they should also incorporate the virtual incoming photon. In particular, it can be noticed that the original structures of Ref. [6] that multiply $\gamma_2$ and $\gamma_4$ are not linearly-independent for VCS. I choose an appropriate linear combination of the two to accompany $\gamma_2$, and the remaining structure is chosen to coincide with that arising due to the physical contribution of the $\pi^0$ exchange in the $t$-channel. One can easily derive the relations between the basis structure listed above and the original ones of Ragusa, by diagonalizing Eq. (27) with respect to $\gamma_i$’s.

An important feature of the basis of Eq. (27) is that no distinction is made for transverse or longitudinal polarizability of the virtual photon, as for instance in [10]. In that reference, two different electric polarizabilities were introduced, $\alpha_L$ and $\alpha_T$, and it was then shown in a model that $\alpha_L$ is dominant. A similar result is obtain here, with the only difference that while in [10] this dominance of $\alpha_L$ over $\alpha_T$ is realized as function of $Q^2$, in the present work the dominance is in $\omega$, i.e. due to the neglect of terms $\sim \omega^2 Q^2$ that go beyond the LEX precision. Phenomenologically, it is important to realize that even if two distinct electric polarizabilities may be introduced in a special frame, there is no practical way to determine them both in LEX formalism.

In the remainder of this section, I list the relations between the generalized polarizabilities and the Lorentz invariants $f_i(Q^2)$ listed earlier, and obtain the two missing relations between $\gamma_i$’s and the GP’s of Guichon et al.
\[4\pi \beta(Q^2) = -f_1(Q^2)\]
\[4\pi \alpha(Q^2) = f_1(Q^2) + 4P^2 f_2(Q^2) + Q^2 \left[2f_6(Q^2) + Q^2 f_9(Q^2) - f_{12}(Q^2)\right]\]
\[4\pi \gamma_1(Q^2) = \frac{4M}{F_0} \left[\frac{Q^2}{4} f_7(Q^2) + f_{11}(Q^2)\right] - \left[f_5(Q^2) + Q^2 f_7(Q^2) + 4f_{11}(Q^2)\right]\]
\[4\pi \gamma_2(Q^2) = \left[f_5(Q^2) + Q^2 f_7(Q^2) + 4f_{11}(Q^2) + 4M f_{12}(Q^2)\right]\]
\[4\pi \gamma_3(Q^2) = -\frac{2M}{F_0} \left[\frac{Q^2}{4} f_7(Q^2) + f_{11}(Q^2)\right] - 2M \left[2f_6(Q^2) + Q^2 f_9(Q^2)\right]\]
\[4\pi (\gamma_2(Q^2) + \gamma_4(Q^2)) = \frac{2M}{F_0} \left[\frac{Q^2}{4} f_7(Q^2) + f_{11}(Q^2)\right]\]  

These relations represent the definition of the GP’s for finite \(Q^2\), and the correct limit at \(Q^2 = 0\) is ensured by the relations of Eqs. (23,25). While some of the relations only contain Lorentz scalars, the presence of a factor \(\frac{M}{F_0}\) in the others means that there is a fundamental frame dependence in expanding VCS observables in powers of energy times polarizabilities, and this dependence is expressed in terms of recoil corrections \(\sim Q^2/M^2\). In the above relations, the new GP’s are given in terms of \(f_5, f_{12}\) unlike the old GP’s \(P^{(01,01)}, P^{(11,11)} \sim Q^2 f_5, Q^2 f_{12}\) that vanished for real photons. Then, it is the slope of these two GP’s at \(Q^2 = 0\) that should match the RCS polarizabilities, and not their values at \(Q^2 = 0\),

\[\frac{4\pi}{\epsilon^2} \gamma_1(0) = -6M \frac{d}{dQ^2} P^{(11,11)}(0)\]
\[\frac{4\pi}{\epsilon^2} \gamma_2(0) = 6M \frac{d}{dQ^2} P^{(01,01)}(0),\]

where relations between \(f_5\)’s and GP’s [12] were used along with the results of Eq. (28). Eqs. (23,25,29) state model-independent relations that are only based on general properties of the VCS amplitude, plus the assumption of its analyticity at \(Q^2 = 0\). This assumption is worth checking in models, and chiral perturbation theory seems to be the perfect tool to study VCS at very low energy and \(Q^2\). Calculations of the low energy VCS amplitude exist in the linear \(\sigma\)-model [13], heavy-baryon ChPT [14,15], effective Lagrangian model [16], non-relativistic constituent quark model [17] (for a recent review, see [18]). Unfortunately, none of the above references include both real and virtual Compton scattering within the same formalism, and moreover frame dependence may be crucial since it introduces corrections \(\sim Q^2/M^2\) that alter the slope of the GP’s that have to be computed. Therefore, the two relations either should be checked on the level of the invariant amplitudes, or both sides of the two equalities should be evaluated within the same model. The author leaves this for an upcoming work.

VI. VCS OBSERVABLES

Since the set of GP’s introduced in the present work is different from those of Guichon et al., I will consider the effect of the new GP’s on the observables, and since LEX was performed in Breit frame, the same frame will be used in this section, too. I will repeat the main steps that were done in [19] for c.m. kinematics. In the process \(e + p \to e + p + \gamma\), the real photon can be emitted from one of the electron legs (Bethe-Heitler, shown in Fig. [5,b]), from a local coupling to one of the nucleon legs (Born, Fig. [5,a]) or can originate from a non-local two-photon interaction (non-Born, Fig. [5,c]),

\[T_{ep\to ep\gamma} = T_{BH} + T_{FVCS} + T_{NB}^{FVCS}\]

Bethe-Heitler amplitude is given by

\[T_{BH} = -\frac{e^3}{c} \bar{N}(p') \Gamma^\mu(\Delta) N(p) \bar{u}(k') \left[\frac{\gamma^\nu(k' + q' + m_e) \gamma^\mu}{(k' + q')^2 - m_e^2} + \frac{\gamma^\mu(k' - q' + m_e) \gamma^\nu}{(k - q')^2 - m_e^2}\right] u(k) \epsilon'^\nu.\]
\[
T_{FVCS}^B = -\frac{e^3}{Q^2} \bar{u}(k') \gamma_\mu u(k) \bar{N}(p') \left[ \frac{\Gamma'(q')(p' + q' + M)\Gamma'(q)}{(p' + q')^2 - M^2} + \frac{\Gamma'(q)(p' - q' + M)\Gamma'(q')}{(p' - q')^2 - M^2} \right] N(p) \varepsilon^*_{\nu} \tag{32}
\]

These two amplitudes can also be expanded into a series in powers of the outgoing photon energy. The expansion starts with \( \nu^{-1} \) since both amplitudes diverge at zero energy. The regular part of the VCS amplitude is embedded into the full amplitude in a similar manner,

\[
T_{FVCS} = \frac{e}{Q^2} \bar{u}(k') \gamma_\mu u(k) \sum_{i=1}^{12} \bar{N}(p') \rho_i^{\mu\nu} F_i(\nu, t, Q^2) N(p) \varepsilon^*_{\nu} \tag{33}
\]

and it allows for an expansion in powers of \( \omega \),

\[
|T_{BH} + T_{FVCS}^B|^2 = \frac{a_{BH+B}}{\omega^2} + \frac{a_{BH+B}^2}{\omega} + a_{BH+B}^0 + O(\omega)
\]

\[
[(T_{BH} + T_{FVCS}^B)^* T_{FVCS}^B + (T_{BH} + T_{FVCS}^B) T_{FVCS}^B] = a_{0}^{GP} + O(\omega)
\]

and

\[
|T_{FVCS}^{NB}|^2 = O(\omega^2). \tag{35}
\]

Correspondingly, it was proposed in [5] to extract the GP’s from the discrepancy of the measured cross section, on one hand, and the Bethe-Heitler plus Born cross section that can be calculated, on the other hand. This amounts to calculating the coefficient

\[
a_0^{GP} = (T_{BH}^{-1} + T_{FVCS}^{B-1})^{*} T_{FVCS,1}^{NB}
\]

\[
+ (T_{BH}^{-1} + T_{FVCS}^{B-1}) T_{FVCS,1}^{NB*} \tag{36}
\]

of the interference between the leading \( \sim 1/\omega \) terms of the BH+B part and the \( \sim \omega \) term of the non-Born amplitude that is parametrized in terms of GP’s. The (model-independent) divergent parts are given by

\[
T_{BH}^{-1} = -\frac{e^3}{t} \bar{u}(p') \Gamma_\mu (\Delta) u(p) \bar{u}(k') \gamma_\mu u(k)
\]

\[
\times \left[ \frac{k'^\nu}{(k'q')} - \frac{k^\nu}{(kq')} \right] \varepsilon^*_{\nu}
\]

\[
T_{FVCS}^{B-1} = -\frac{e^3}{Q^2} \bar{u}(p') \Gamma_\mu (q) u(p) \bar{u}(k') \gamma_\mu u(k)
\]

\[
\times \left[ \frac{p'^\nu}{(p'q')} - \frac{p^\nu}{(pq')} \right] \varepsilon^*_{\nu}. \tag{37}
\]

and its expansion in energy starts at \( \nu^3 \). The differential \( (e, e'\gamma) \) cross section is related to the squared amplitude,

\[
d^6\sigma \sim |T_{BH} + T_{FVCS}^B + T_{FVCS}^{NB}|^2 = |T_{BH} + T_{FVCS}^B|^2 + [(T_{BH} + T_{FVCS}^B)^* T_{FVCS}^{NB} + (T_{BH} + T_{FVCS}^B) T_{FVCS}^{NB*}]
\]

\[
+ |T_{FVCS}^{NB}|^2 \tag{34}
\]

and the leading term of the non-Born part is given by

\[
T_{FVCS}^{NB,1} = \frac{e}{Q^2} \bar{u}(k') \gamma_\mu u(k) \bar{u}(p') \sum_i \rho_i^{\mu\nu} f_i(Q^2) u(p) \varepsilon^*_{\nu} \tag{38}
\]

with \( f_i(Q^2) \) related to the GP’s as in Eq.(28). Note that for the leading term, \( t \) can be substituted with \(-Q^2 \) since \( t + Q^2 = -2(qq') \sim \omega \).

I will only deal here with the unpolarized case. After some algebra, details of which can be found in the Appendix, the coefficient \( a_0^{GP} \) can be represented in the familiar form (cf. [5])

\[
a_0^{GP} = \frac{8M e^4}{1 - \epsilon} [v_1(\epsilon P_{TL} - P_{TT}) + v_2 P_{LT}], \tag{39}
\]

in terms of three structure functions that are related to the GP’s as

\[
P_{LL} = G_E \left[ 4\pi \alpha + \frac{Q^2}{2M} 4\pi (\gamma_1 + \gamma_2 + 2\gamma_3) \right]
\]

\[
P_{TT} = \frac{Q^2}{4M} G_M 4\pi (\gamma_1 + 2\gamma_3)
\]

\[
P_{LT} = -G_E 4\pi \beta + \frac{Q^2}{4M} G_M 4\pi \gamma_2, \tag{40}
\]

with the dependence of the GP’s and form factors on \( Q^2 \) suppressed for shortness. \( v_1 \) and \( v_2 \) are shorthands for somewhat lengthy kinematical factors that are listed in the Appendix, and \( \epsilon \) denotes the usual virtual photon polarization parameter, and is also given in the Appendix. As already known in the literature, three
FIG. 5: The different contributions to the scattering process $e+p \to e+p+\gamma$: FVCS Born (case a), Bethe-Heitler (case b), and non-Born FVCS (case c). For a) and b), the blob denotes the elastic nucleon form factors.

independent structure functions can be measured in the unpolarized VCS experiment at low energies. Now, they all are expressed through the polarizabilities that are a direct generalization of those of RCS. Comparing Eq. (40) to the original results of [5], one notices the presence of the spin-dependent GP’s in the structure function $P_{LL}$. The origin for that contribution is in terms $\sim F'_0 \sigma_0 \nu \chi^\dagger \chi$. This difference may be attributed to the use of the Breit kinematics instead of the c.m. kinematics used in [5]. These new terms in $P_{LL}$ are expected to be less important for low and moderate values of $Q^2$, as in the kinematics of Mainz [19] and MIT-Bates [20] experiments, but might affect the extracted values of $\alpha(Q^2)$ and $\beta(Q^2)$ for the kinematics of the JLab experiments [21].

VII. SUMMARY

To summarize, I considered the virtual Compton scattering process at low energy of the outgoing real photon. I formulated the low energy theorem for that reaction, and the present formulation is realized in an explicitly crossing-symmetric way that is an improvement with respect to the previous formalism [5]. I demonstrated that for virtual photons, the requirement of crossing symmetry of the VCS amplitude makes the LEX in terms of multipole expansion too complicated, if at all viable since it introduces relations upon different multipole transitions that are supposed to form a basis at leading order in that expansion. Instead, I proposed a different approach based on a Lorentz covariant EFT description of VCS at low energies. Within this approach, it was possible to define the low energy limit in a continuous way, with respect to the virtuality of the initial photon, and the same six structures and associated with them low energy coefficient functions fully describe Compton scattering at low energies with real and virtual photons. These six low energy constants (for fixed $Q^2$) can be interpreted as polarizabilities only when going to a specific reference frame. I chose Breit frame since it treats the initial and final photons in a symmetric way, and crossing symmetry and power counting are realized in a simple manner in that frame. Using the classical notion of the polarizability and working in a framework closely related to that of Ref. 9 where the complete set of nucleon polarizabilities was introduced for RCS, I obtained the new set of the six generalized polarizabilities. These new GP’s are defined such as to reduce to the polarizabilities of RCS for real initial photon, the feature that was missing in the formalism of [5]. The continuous limit at $Q^2 = 0$ imposes two relations between the values of four invariant amplitudes for VCS at the real photon point, leading to two relations of $\gamma_{1,2}$ to the slope of two GP’s of [5] in that kinematical point. These two relations should be checked in models, most notably within chiral perturbation theory in its relativistic or heavy baryon form. I also computed the contribution of the GP’s to the unpolarized VCS cross section in Breit frame. While confirming the general structure of this contribution as reported in [5, 7], I found that the structure function $P_{LL}$ has a contribution from spin-dependent GP’s that is not present in the analysis of [5].

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APPENDIX A: VCS KINEMATICS, POLARIZATION VECTORS AND NUCLEON SPINORS IN BREIT FRAME

I use the standard definition of the nucleon Breit frame,

$$P^\mu = (P^0, \vec{0})$$

$$\Delta^\mu = (0, 0, 0, |\vec{\Delta}|)$$

with $t = -\Delta^2$, and the energy of the initial (final) nucleon $E(E')$ are equal to $E = E' = P^0 = \sqrt{M^2 - \frac{t}{4}}$. I use $\Delta = |\Delta|$ and $q = |\vec{q}|$ in the following. The photons’
momenta in this frame are given by
\[ q^\mu = (\omega, q \cos \alpha, 0, q \sin \alpha) \]
\[ q'^\mu = \omega(1, \cos \beta, 0, -\sin \beta) \]
\[ \sin \beta = \frac{t + Q^2}{2\omega \Delta} \]
\[ \cos \alpha = \frac{\omega}{q} \cos \beta \] (A2)

In the following, I will also use the photon kinematics in the limit of very small photon energy,
\[ q^\mu \approx \Delta^\mu = (0, 0, 0, \Delta) \]
\[ q'^\mu = \omega(1, \sin \theta, 0, \cos \theta) \] (A3)

with \( \theta = \frac{\pi}{2} + \beta \). The polarization vectors for transverse photons (also notation \( \tilde{\varepsilon}_T \) is used in the text) are
\[ \varepsilon^\mu_{\lambda=\pm}(\vec{q}) = -\frac{\lambda}{\sqrt{2}}(0, \sin \alpha, i\lambda, -\cos \alpha) \]
\[ \tilde{\varepsilon}^\mu_{\lambda=\pm}(\vec{q}') = -\frac{\lambda'}{\sqrt{2}}(0, -\sin \beta, i\lambda', -\cos \beta). \] (A4)

For the longitudinal polarization of the virtual photon, one has
\[ \varepsilon^\mu_{\lambda=0}(\vec{q}) = \frac{1}{\sqrt{Q^2}}(q, \omega \vec{q}) \] (A5)

with \( \vec{q} \) the unit vector in the direction of the virtual photon’s three-momentum. One can identically rewrite it as
\[ \varepsilon^\mu_{\lambda=0}(\vec{q}) = \frac{q}{\omega \sqrt{Q^2}} q^\mu - \frac{\sqrt{Q^2}}{\omega}(0, \vec{q}) \] (A6)

and using gauge invariance of the VCS tensor, \( q_\mu T^{\mu\nu} = 0 \), one can verify that the first term does not contribute. The longitudinal polarization vector (denoted as \( \varepsilon_L \) in the body of the article) of the initial photon is used in the form
\[ \varepsilon^\mu_{\lambda=0}(\vec{q}) = -\frac{\sqrt{Q^2}}{\omega}(0, \vec{q}). \] (A7)

For nucleon spinors describe a Dirac particle with the three-vector \( \vec{p} \), mass \( M \) and energy \( E = \sqrt{\vec{p}^2 + M^2} \), one has
\[ u(\vec{p}) = \sqrt{E + M} \begin{bmatrix} \chi \\ \bar{\sigma} \tilde{p} \end{bmatrix}, \]
\[ u(\vec{p}') = \sqrt{E' + M} \begin{bmatrix} \chi \\ \bar{\sigma} \tilde{p}' \end{bmatrix}, \] (A8)

and due to Breit kinematics one has \( \bar{\sigma} \tilde{p}' = -\bar{\sigma} \tilde{p} = \frac{1}{2} \bar{\sigma} \Delta \).

Pauli spinors \( \chi \) are taken to correspond to a definite z-projection of the nucleon spin both for the initial and final nucleons,
\[ \chi_{+1/2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \chi_{-1/2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \] (A9)

With these definitions, one finds the following useful relations:
\[ \bar{u}(\vec{p}')u(\vec{p}) = 2E\chi\chi \]
\[ \bar{u}(\vec{p}')\gamma_5 u(\vec{p}) = 2E\chi\left[ \sigma - \frac{E - M}{E} \bar{\sigma} \right] \chi \]
\[ \bar{u}(\vec{p}')\gamma_5 u(\vec{p}) = 0 \]
\[ \bar{u}(\vec{p}')i\sigma^{ij}u(\vec{p}) = i\epsilon^{ijk}2M\chi\left[ \bar{\sigma}^k + \frac{E - M}{M} \bar{\sigma}^k \right] \chi \]
\[ \bar{u}(\vec{p}')i\sigma^{ij}u(\vec{p}) = \bar{\Delta}^i \chi \chi \] (A10)

I use conventions \( \gamma_5 = i\gamma_0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \), and \( \epsilon_{0123} = +1 \).

### APPENDIX B: POWER COUNTING IN BREIT FRAME AND LOW ENERGY REDUCTION OF THE VCS INVARIANT BASIS

For power counting, one has in Breit frame
\[ E' = \tilde{E}_T = i\epsilon \omega \varepsilon \sim \omega \]
\[ \tilde{B}' = i\epsilon \vec{q} \times \varepsilon \sim \omega \]
\[ \tilde{E}_T = i\epsilon \omega \varepsilon_T \sim \omega \]
\[ \tilde{E}_L = i\epsilon \omega \varepsilon_L \sim \sqrt{Q^2} \]
\[ \tilde{B} = i\epsilon \vec{q} \times \varepsilon_T \sim |\vec{q}|, \] (B1)

and I refer the reader to the Appendix A for explicit expressions of the polarization vectors. The structures to eliminate are
\[ \nu\rho_3 = \frac{2}{M} \left[ -Q^2 g_{\alpha\beta} - q_\nu q_\beta \right] (P_\mu F^{\mu\alpha}) (P_\nu F^{\nu\beta}) \]
\[ Q^2 (\rho_7 - \frac{1}{4} \rho_11) = Q^2 (q_\mu F^{\mu\alpha} F^{\nu\beta} i\gamma_5 \gamma_\alpha \gamma_\beta) \]
\[ \nu\rho_8 = \frac{1}{2} \left( q_\nu q_\beta + q_\mu q_\mu' \right) F^{\alpha\mu} F^{\nu\beta} i\sigma_\alpha \beta \]
\[ -\frac{1}{2} q_\nu q_\beta \left( F^{\alpha\beta} F^{\mu\nu} + F^{\nu\beta} F^{\mu\alpha} \right) i\sigma_{\mu\nu} \]
\[ Q^2 (\rho_9 - \frac{1}{4} \rho_6) = 2Q^2 (P_\alpha q_\beta F^{\alpha\beta}) F^{\mu\nu} i\sigma_{\mu\nu} \]
\[ +Q^2 \left( \frac{q q'}{4M} (\rho_{11} - 4\rho_3) \right) \] (B2)

All the structures listed above contribute at order \( Q^2 \omega^2 \) and cannot enter the LEX neither for RCS nor
for VCS. 3

As a result, one is left with eight structures that can contribute to LEX at lowest order, $\rho_{1,2,4,5,6,10,11,12}$ times the corresponding amplitudes $F_{1,2,4,5,6,10,11,12}$. Six of them are relevant for RCS, $\rho_{1,2,4,6,10,11}$ while $\rho_{5,12}$ vanish for real photons. In turn, for VCS it is $\rho_{4,10}$ that do not contribute to the LEX due to the crossing behavior of the respective amplitudes, as found in [2]. This is the formal origin of the mismatch between the low energy expansions of the RCS and VCS. I will review the situation in detail by rewriting the eight tensors in terms of electromagnetic fields in Breit frame following in grand line Refs. [6][10]. Understanding in the following the tensors $\rho_i$ cast between the initial and final nucleon (Pauli) spinors, these expressions read

$$
\begin{align*}
\rho_1 &= 2P^0(\langle \vec{E}\vec{E} \rangle - (\vec{B}\vec{B}')) \\
\rho_2 &= 8(P^0)^3(\vec{E}\vec{E}') \\
\nu\rho_1 &= 4 \left( \frac{P^0}{M} \right)^3 \delta \bar{\sigma} \cdot \left( [\vec{E}_T \times [\vec{q} \times \vec{B}']] - \frac{\omega^2}{q^2} [\vec{E}' \times [\vec{q} \times \vec{B}']] \right) \\
\rho_5 &= 8M P^0 \left( \delta \bar{\sigma} \cdot ([\vec{q} \times \vec{B}] + (\vec{E}_T \times [\vec{q} \times \vec{B}'])) \right) \\
\rho_6 &= 8M P^0 \left( \delta \bar{\sigma} \cdot (q\vec{E}) \right) - 2P^0 q^2 (\vec{E}_L \vec{E}') \\
\rho_11 &= 4\rho_5 = -4M i\bar{\sigma} \cdot (\vec{E}\vec{E}' + \vec{E}'\vec{B}) \\
\rho_{12} &= 4MP^0 \left[ i(\delta \bar{\sigma} \cdot (q\vec{E})) - 2P^0 q^2 (\vec{E}_L \vec{E}') \right] \\
&+ \frac{Q^2}{8M} (\rho_{11} - 4\rho_5)
\end{align*}
$$

where the higher order terms in $\omega$ were omitted, and the use was made of the relations $\vec{B} = [\vec{q} \times \vec{E}_T], \vec{E}_T = -\frac{\omega^2}{q^2} [\vec{q} \times \vec{B}], \vec{E}_L = \bar{q} \vec{q}$, with the unit vector along the direction of the virtual photon $\bar{q}$. 4 Appendix A enlists relations with the nucleon spinors that were used to derive the above results.

First, consider the similar structures \( i\bar{\sigma} \cdot [\vec{E}' \times [\vec{q} \times \vec{B}']] \), \( i\bar{\sigma} \cdot (\vec{E}_T \times [\vec{q} \times \vec{B}']) \) that enter $\nu\rho_4, \rho_5$. For both photons real, they reduce to $\omega\bar{\sigma} \cdot (\vec{E}' \times \vec{E})$ and come with the opposite sign, so they exactly cancel in $\rho_5$, and double in $\rho_4$. For VCS, it is only the first of the two that is leading order, thus $\rho_5$ obtains a contribution at leading order but in $\rho_4$ it is multiplied by $\omega^2$, and this amplitude is subleading in VCS. The possible way out is to build a third structure with the needed limit for RCS and for VCS, that would thus interpolate between the two low energy limits. This structure is

$$
\begin{align*}
o_1 &= i\bar{\sigma} \cdot (\vec{E}' \times [\vec{q} \times \vec{B}'] - [\vec{E}' \times [\vec{q} \times \vec{B}]] \quad (B4)
\end{align*}
$$

The other two structures that do not match in low energy RCS and VCS are $\nu\bar{\sigma} \cdot (\vec{B}' \times \vec{B})$ and $i(\delta \bar{\sigma} \cdot (q\vec{E}))$, the first being part of $\rho_{10}$, and the second of $\rho_{12}$. The first tensor is purely transverse, being magnetic, whereas the second one is purely longitudinal with respect to the virtual photon. Once again, I will be looking for an interpolating tensor. The tensor of interest is

$$
\begin{align*}
o_2 &= i\bar{\sigma} \cdot (\vec{q} \times [\vec{E} \times \vec{B}] - [\vec{q} \times \vec{E}' \times \vec{B}]) \quad (B5)
\end{align*}
$$

If the structures $o_{1,2}$ can be represented in a covariant form without introducing any spurious singularity, the low energy limit of the VCS amplitude will be related to the new, universal set of polarizabilities that are defined for real and virtual photons. This amounts in building four linear combinations out of $\rho_{4,5,10,12}$ such that two of them give the right limit at low energies, whereas the other two should be subleading in LEX. Such combinations are

$$
\begin{align*}
\phi_1 &= 2\rho_5 - \nu\rho_10 \\
\phi_2 &= 2\nu\rho_5 + Q^2 \rho_{10} \\
\phi_3 &= 4M \rho_5 - \rho_{12} - \nu\rho_4 \\
\phi_4 &= -\nu\rho_{12} + Q^2 \rho_{14}.
\end{align*}
$$

APPENDIX C: VCS OBSERVABLES

To recollect, the leading contribution of the GP’s to the VCS cross section arises as an interference between the divergent \( \sim 1/\omega \) parts of the Bethe-Heitler and FVCS Born amplitudes, and the first, \( \sim \omega \) term in energy expansion of FVCS non-Born amplitude,

$$
\begin{align*}
\begin{multlined}[t][0.8\textwidth]
d_{0}^{GP} = \sum_{spins} \left[ (T_{BH}^{-1} + T_{FVCS}^{B^{-1}})^{\dagger} T_{FVCS,1}^{NB} \right. \\
\left. + (T_{BH}^{-1} + T_{FVCS}^{B^{-1}}) T_{FVCS,1}^{NB*} \right]
\end{multlined}
\end{align*}
$$

The divergent parts are given by

$$
\begin{align*}
T_{BH}^{-1} &= -\frac{e^3}{t} \bar{u}(p') \Gamma_{\mu}(\Delta) u(p) \bar{u}(k') \gamma^{\mu} u(k) \\
&\times \left[ \frac{k^{\nu}}{(k'q')} - \frac{k^{\nu}}{(k'q')} \right] \bar{\epsilon}^{*}_{\nu} \\
T_{FVCS}^{-1} &= -\frac{e^3}{Q^2} \bar{u}(p') \Gamma_{\mu}(q) u(p) \bar{u}(k') \gamma^{\mu} u(k) \\
&\times \left[ \frac{p^{\nu}'}{(p'q')} - \frac{p^{\nu}'}{(p'q')} \right] \bar{\epsilon}'^{*}_{\nu},
\end{align*}
$$

\[3\] The only tensor for which it is not obvious right away is $\rho_3$. Explicit evaluation in terms of $\vec{E}$ fields gives $\nu\rho_3 \sim Q^2 (\vec{E} \vec{E}') - (q\vec{E}) (\vec{q} \vec{E}) - \omega^2 (\vec{E}_L \vec{E}_L')$.\[4\] To simplify the above expressions, recoil corrections were neglected in tensors $f_{5,12}$ as i.e. $1 - \frac{\Delta^2}{8M^2} \approx 1$, but not as $0 - \frac{\Delta^2}{8M^2} \approx 0$.
and the leading term of the non-Born part is given by
\[ T_{FVCS}^{NB,1} = \frac{e}{Q^2} \bar{u}(k')\gamma_\mu u(k)\sum_i \rho_i^{\mu\nu} f_i(Q^2)u(p)\varepsilon^*_\nu \]
with \( f_i(Q^2) \) related to the GP’s as in Eq. (C8). Note that for the leading terms \( t \) can be substituted by \(-Q^2\) since \( t + Q^2 = -2(qq') \sim \omega \). Unlike in [5], I perform the sum over spins in Eq. (C1) in the covariant form. For the unpolarized case that is studied here, the calculation involves the trace
\[ \left\langle \frac{1}{4} \sum_{\text{spins}} \left[ (T_{BH} + T_{FVCS}^B)^\dagger T_{FVCS,1}^{NB} + (T_{BH} + T_{FVCS}^B) - T_{FVCS,1}^{NB} \right] \right\rangle 
\]
where the shorthands were introduced, \( \hat{f}_5 = f_5 - \frac{Q^2}{2M} f_{12} \), \( 2\hat{f}_6 = 2f_6 + Q^2 f_9 \), and \( \hat{f}_{11} = Q^2 f_7 + 4f_{11} + \frac{Q^2}{2M} f_{12} \). The three traces that have to be computed, are
\[ \begin{align*}
\frac{1}{2} Tr[(q' + M)\Gamma^{\mu'}(q)(q' + M)] &= 4MP^{\mu'} G_E(Q^2) \\
\frac{1}{2} Tr[(q' + M)\Gamma^{\mu'}(q)(q' + M)i\gamma_5 \gamma_3] &= -2G_M(Q^2)\epsilon^{\mu'
u\lambda\beta} P_\sigma \Delta_\lambda \\
\frac{1}{2} Tr[(q' + M)\Gamma^{\mu'}(q)(q' + M)i\sigma^{\alpha\beta}] &= \frac{2}{M} F_2(Q^2) P^{\mu'} \left[ P^\alpha \Delta^\beta - P^\beta \Delta^\alpha \right] + 2MG_M(Q^2) [\Delta^\alpha g^{\mu'\beta} - \Delta^\beta g^{\mu'\alpha}] 
\end{align*} \]
Performing now Lorentz contraction, the hadronic tensor takes the following form
\[ H^{\mu\nu} = \frac{1}{2} Tr[(q' + M)\Gamma^{\mu'}(q)(q' + M)\sum_i \rho_i^{\mu\nu}] \]
and the hadronic tensor is given by
\[ T_{FVCS,7}^{NB} = 2 \bar{u}(k')\gamma_\mu u(k)\sum_i \rho_i^{\mu\nu} f_i(Q^2)u(p)\varepsilon^*_\nu \]
\[ H^{\mu \nu} = ((Pq')q' - (qq')P') \left\{ AMP^{\mu} P^{\nu} \left[ 4G_{E}f_{2} + \frac{Q^{2}}{M^{2}} F_{2}(2\hat{f}_{6} - f_{12}) \right] - Q^{2}g^{\mu \nu} G_{M}(\hat{f}_{5} + \hat{f}_{11} + 8M\hat{f}_{6}) \right\} + P^{\mu}(-(qq')g^{\mu \nu} + q'^{m} q^{n}) \left[ 4M_{E}f_{1} + Q^{2}G_{M}(\hat{f}_{5} + \hat{f}_{11} + 4Mf_{12}) \right] \]

(C10)

To proceed, the electron kinematics needs to be defined. It is, in the case of the massless electron,

\[
k^{\mu} = E_{e}(1, \sin \theta' \cos \phi, \sin \theta' \sin \phi, \cos \theta'),
\]

\[
k'^{\mu} = k^{\mu} - q^{\mu}.
\]

(C11)

with \( E_{e} \) the energy of the initial electron, and \( E'_{e} = E_{e} - \omega \) that of the final one. For the purpose of calculating the contribution of the GP’s to the VCS observable to leading order, it is enough to take the electron kinematics in the limit of zero energy of the final photon, that is the kinematics of the elastic electron-proton scattering, \( q \approx \Delta \). Then, one finds

\[
E_{e} = E'_{e},
\]

\[
\cos \theta' = \sqrt{\frac{1 - \epsilon}{1 + \epsilon}}
\]

\[
\sin \theta' = \sqrt{\frac{2\epsilon}{1 + \epsilon}}
\]

(C12)

Above, I introduced the photon’s longitudinal polarization parameter \( \epsilon \),

\[
\epsilon = \frac{(E_{e} + E'_{e})^{2} - (\vec{k} - \vec{k}')^{2}}{(E_{e} + E'_{e})^{2} + (\vec{k} - \vec{k}')^{2}}.
\]

(C13)

The kinematical factors introduced in Section [VI] can be cast in the following form, after a straightforward but somewhat lengthy algebra:

\[
v_{1} = \frac{1}{Q^{2}}(Pq'q' - qq'P') \left[ \frac{k'_{\nu}}{(k'q')} - \frac{k_{\nu}}{(kq')} - \frac{p'_{\nu}}{(p'q')} + \frac{p_{\nu}}{(pq')} \right]
\]

\[
= \sin \theta \left[ \frac{4P^{2} \sin \theta}{4P^{2} - Q^{2} \cos^{2} \theta} + \frac{p^{0}}{E_{e}} \left( \sin \theta - \sqrt{\frac{2\epsilon}{1 + \epsilon}} \cos \phi \right) \right] + \frac{\epsilon}{\sin \theta} \left[ \frac{\cos \phi}{\sin \theta} \right] - 2 \frac{\epsilon}{E_{e} + \epsilon} \left[ \frac{\cos \phi}{\sin \theta} \right] - \epsilon v_{1}
\]

\[
v_{2} = \frac{1 - \epsilon}{2Q^{2}} (q'^{\mu} q'^{\nu} - qq'g^{\mu \nu}) P^{\nu} l^{\mu}
\]

\[
= - \frac{\cos \phi}{\sin \theta} \left[ \frac{\cos \phi}{\sin \theta} \right] - \frac{2\epsilon}{E_{e} + \epsilon} \left[ \frac{\cos \phi}{\sin \theta} \right] - \epsilon v_{1}
\]

(C14)

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