Corrigendum to “Random grid-based visual secret sharing with abilities of OR and XOR decryptions” [Journal of Visual Communication and Image Representation. 24(2013) 48-62]

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Abstract

It has been observed that the contrast values for (2, 3) VSS scheme, (2, 4) VSS scheme, (3, 5) VSS scheme and (4, 5) VSS scheme claimed by Wu and Sun (2013) are incorrect. Since the same values are cited and compared by many other researchers in their works, we have calculated and presented the correct values of contrast in this note.

Keywords: Visual Secret Sharing, Random Grid, Threshold, Visual Cryptography, OR, XOR, Contrast, Shares

1. Introduction

Wu and Sun (2013) have recently introduced a (k, n) VSS scheme which has capability of OR and XOR decryption both simultaneously. For large number of shares, the reconstructed secret image by OR operation has a very low contrast. However, if a light weight computational device is available, the secret image can be reconstructed using XOR operation which has higher contrast. The scheme works very well, but the value of contrast mentioned in the paper are erroneous due to possible oversight in computation by the authors.

The rest of the note is organized as follows. In Section 2 we have computed the correct contrast values for (2, 3) VSS scheme as an example and presented the corrected contrast values for all other schemes. Finally, we conclude in section 3.

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2. Contrast Computation

Contrast values as depicted in Wu and Sun (2013) (Table 5, Table 6, Table 8, Table 9) are incorrect. In the following, we present the correct calculations and the final corrected values of contrast for (2, 3), (2, 4), (3, 5) and (4, 5) VSS schemes.

Definitions and formulae used in present note are the same as those introduced by Wu and Sun (2013) and Chen and Tsao (2011). For various VSS schemes, we have computed the values of contrast for OR and XOR decryption both in Appendix using those formulae.

So, we calculate the value of contrast by OR decryption for (2, 3) VSS scheme. Consider a secret pixel s, n be total number of pixels generated corresponding to each secret pixel, t is the number of pixels to be stacked, r is the reconstructed secret pixel. As the secret can be reconstructed by stacking 2 or 3 shares, t can be 2 or 3.

Case 1 : t = 2

Average light transmission when \( s = 0 \),

\[
T^{OR,2}(r[s = 0]) = \frac{1}{(n - k + 1)} \left[ T^{OR,2}_{(k,n)}(r[s = 0]) + T^{OR,2}_{(k+1,n)}(r[s = 0]) + \ldots + T^{OR,2}_{(n,n)}(r[s = 0]) \right]
\]

\[
= \frac{1}{(3 - 2 + 1)} \left[ T^{OR,2}_{(2,3)}(r[s = 0]) + T^{OR,2}_{(3,3)}(r[s = 0]) \right]
\]

\[
= \frac{1}{2} \times \left\{ \left( \frac{2}{3} \right) \times \left( \frac{1}{2} \right)^{2-1} + \left( 1 - \frac{2}{3} \right) \times \left( \frac{1}{2} \right)^2 + \left( \frac{1}{2} \right)^2 \right\}
\]

\[
= \frac{7}{24}.
\]

Average light transmission when \( s = 1 \),

\[
T^{OR,2}(r[s = 1]) = \frac{1}{(n - k + 1)} \left[ T^{OR,2}_{(k,n)}(r[s = 1]) + T^{OR,2}_{(k+1,n)}(r[s = 1]) + \ldots + T^{OR,2}_{(n,n)}(r[s = 1]) \right]
\]

\[
= \frac{1}{(3 - 2 + 1)} \left[ T^{OR,2}_{(2,3)}(r[s = 1]) + T^{OR,2}_{(3,3)}(r[s = 1]) \right]
\]

\[
= \frac{1}{2} \times \left\{ \left( 1 - \frac{2}{3} \right) \times \left( \frac{1}{2} \right)^2 + \left( \frac{1}{2} \right)^2 \right\}
\]

\[
= \frac{5}{24}.
\]

Contrast is calculated as

\[
\alpha = \frac{T^{OR,2}(r[s = 0]) - T^{OR,2}(r[s = 1])}{1 + T^{OR,2}(r[s = 1])} = \frac{\frac{7}{24} - \frac{5}{24}}{1 + \frac{5}{24}} = \frac{2}{29}.
\]
case 2 : \( t = 3 \)

Average light transmission when \( s = 0 \),

\[
T^{\text{OR},3}(r[s = 0]) = \frac{1}{(n-k+1)}[T^{\text{OR},3}_{(k,n)}(r[s = 0]) + T^{\text{OR},3}_{(k+1,n)}(r[s = 0]) + ...
\]
\[
+ T^{\text{OR},3}_{(n,n)}(r[s = 0])]
\]
\[
= \frac{1}{(3-2+1)}[T^{\text{OR},3}_{(2,3)}(r[s = 0]) + T^{\text{OR},3}_{(3,3)}(r[s = 0])]
\]
\[
= \frac{1}{2} \times \left( \frac{3}{2} \right) \times \left( \frac{1}{2} \right)^{3-1} + (1 - \frac{3}{2}) \times \left( \frac{1}{2} \right)^{3} + \frac{3}{3} \times \left( \frac{1}{2} \right)^{3-1} + (1 - \frac{3}{3}) \times \left( \frac{1}{2} \right)^{3}
\]
\[
= 1/4.
\]

Average light transmission when \( s = 1 \),

\[
T^{\text{OR},3}(r[s = 1]) = \frac{1}{(n-k+1)}[T^{\text{OR},3}_{(k,n)}(r[s = 1]) + T^{\text{OR},3}_{(k+1,n)}(r[s = 1]) + ...
\]
\[
+ T^{\text{OR},3}_{(n,n)}(r[s = 1])]
\]
\[
= \frac{1}{(3-2+1)}[T^{\text{OR},3}_{(2,3)}(r[s = 1]) + T^{\text{OR},3}_{(3,3)}(r[s = 1])]
\]
\[
= \frac{1}{2} \times \left( (1 - \frac{3}{2}) \times \left( \frac{1}{2} \right)^{3} + (1 - \frac{3}{3}) \times \left( \frac{1}{2} \right)^{3} \right]
\]
\[
= 0.
\]

Contrast is calculated as

\[
\alpha = \frac{T^{\text{OR},3}(r[s = 0]) - T^{\text{OR},3}(r[s = 1])}{1 + T^{\text{OR},3}(r[s = 1])} = \frac{1}{4} - 0 = \frac{1}{4}.
\]

Now, we calculate the value of contrast for \((2, 3)\) VSS scheme by XORed decryption

case 1 : \( t = 2 \)
Average light transmission when \( s = 0 \),

\[
T_{XOR,t}(r[s = 0]) = \frac{1}{(n - k + 1)}[T_{XOR,t}^{(k,n)}(r[s = 0]) + T_{XOR,t}^{(k+1,n)}(r[s = 0]) + \ldots + T_{XOR,t}^{(n,n)}(r[s = 0])] \\
= \frac{1}{(3 - 2 + 1)}[T_{XOR,2}^{(2,3)}(r[s = 0]) + T_{XOR,2}^{(3,3)}(r[s = 0])] \\
= \frac{1}{(3 - 2 + 1)}\left[\frac{1}{2} \times \left(1 + \frac{1}{(3)}\right) + \frac{1}{2}\right] \\
= 7/12.
\]

Average light transmission when \( s = 1 \),

\[
T_{XOR,t}(r[s = 1]) = \frac{1}{(n - k + 1)}[T_{XOR,t}^{(k,n)}(r[s = 1]) + T_{XOR,t}^{(k+1,n)}(r[s = 1]) + \ldots + T_{XOR,t}^{(n,n)}(r[s = 1])] \\
= \frac{1}{(3 - 2 + 1)}[T_{XOR,2}^{(2,3)}(r[s = 1]) + T_{XOR,2}^{(3,3)}(r[s = 1])] \\
= \frac{1}{(3 - 2 + 1)}\left[\frac{1}{2} \times \left(1 - \frac{1}{(3)}\right) + \frac{1}{2}\right] \\
= 5/12.
\]

Contrast is computed as

\[
\alpha = \frac{T_{XOR,2}^{(r[s = 0])} - T_{XOR,2}^{(r[s = 1])}}{1 + T_{XOR,2}^{(r[s = 1])}} = \frac{7/12 - 5/12}{1 + 5/12} = \frac{2}{17}.
\]

case 2 : \( t = 3 \)

Average light transmission when \( s = 0 \),

\[
T_{XOR,t}(r[s = 0]) = \frac{1}{(n - k + 1)}[T_{XOR,t}^{(k,n)}(r[s = 0]) + T_{XOR,t}^{(k+1,n)}(r[s = 0]) + \ldots + T_{XOR,t}^{(n,n)}(r[s = 0])] \\
= \frac{1}{(3 - 2 + 1)}[T_{XOR,3}^{(2,3)}(r[s = 0]) + T_{XOR,3}^{(3,3)}(r[s = 0])] \\
= \frac{1}{(3 - 2 + 1)}\left[\frac{1}{2} + \frac{1}{2} \times \left(1 + \frac{1}{(3)}\right)\right] \\
= 3/4.
\]
Average light transmission when $s = 1$,

$$
T_{XOR,t}(r[s = 1]) = \frac{1}{(n-k+1)}[T_{XOR,t}^{(k,n)}(r[s = 1]) + T_{XOR,t}^{(k+1,n)}(r[s = 1]) + ..
$$

$$
... + T_{XOR,t}^{(n,n)}(r[s = 1])]
$$

$$
= \frac{1}{(3 - 2 + 1)}[T_{XOR,3}^{(2,3)}(r[s = 1]) + T_{XOR,3}^{(3,3)}(r[s = 1])]
$$

$$
= \frac{1}{(3 - 2 + 1)}\left[\frac{1}{2} + \frac{1}{2} \times (1 - \frac{1}{(3)})\right]
$$

$$
= 1/4.
$$

Contrast is calculated as

$$
\alpha = \frac{T_{XOR,3}^{(r[s = 0])} - T_{XOR,3}^{(r[s = 1])}}{1 + T_{XOR,3}^{(r[s = 1])}} = \frac{2 - \frac{1}{4}}{1 + \frac{1}{4}} = \frac{2}{5}.
$$

Similarly, contrast values for (2, 4) VSS scheme, (3, 5) VSS scheme and (4, 5) VSS scheme are also computed in Appendix. Table 1 shows the comparison between reported contrast by [Wu and Sun (2013)] and correctly computed contrast in this note.

Table 1: Comparison between previously stated contrast and our calculated contrast

| VSS scheme | $t$ | Wu and Sun (2013) Contrast values | Corrected Contrast values |
|------------|-----|---------------------------------|---------------------------|
|            |     | $\alpha_{OR}$ | $\alpha_{XOR}$ | $\alpha_{OR}$ | $\alpha_{XOR}$ |
| (2, 3)     | 2   | $\frac{10}{17}$ | $\frac{6}{5}$ | $\frac{10}{17}$ | $\frac{6}{5}$ |
|            | 3   | $\frac{11}{17}$ | $\frac{2}{5}$ | $\frac{11}{17}$ | $\frac{2}{5}$ |
| (2, 4)     | 2   | $\frac{14}{17}$ | $\frac{9}{5}$ | $\frac{14}{17}$ | $\frac{9}{5}$ |
|            | 3   | $\frac{11}{17}$ | $\frac{2}{5}$ | $\frac{11}{17}$ | $\frac{2}{5}$ |
|            | 4   | $\frac{12}{17}$ | $\frac{3}{5}$ | $\frac{12}{17}$ | $\frac{3}{5}$ |
| (3, 5)     | 3   | $\frac{129}{132}$ | $\frac{1}{4}$ | $\frac{129}{132}$ | $\frac{1}{4}$ |
|            | 4   | $\frac{269}{132}$ | $\frac{1}{4}$ | $\frac{269}{132}$ | $\frac{1}{4}$ |
|            | 5   | $\frac{1}{16}$ | $\frac{1}{4}$ | $\frac{1}{16}$ | $\frac{1}{4}$ |
| (4, 5)     | 4   | $\frac{12}{16}$ | $\frac{1}{4}$ | $\frac{12}{16}$ | $\frac{1}{4}$ |
|            | 5   | $\frac{1}{16}$ | $\frac{1}{4}$ | $\frac{1}{16}$ | $\frac{1}{4}$ |
3. Conclusion

In the corrigendum, the computation errors in Wu and Sun (2013) are corrected.

Appendix A. Appendix

Let \( I \) be the secret image and \( I' \) be its reconstructed image by stacking \( t \) shares i.e., \( I' = R_1 \otimes \ldots \otimes R_t \). The contrast of \( I' \) w.r.t \( I \) is defined by Wu and Sun (2013)

\[ \alpha = \frac{T(I_1' \otimes i_2 \otimes \ldots \otimes i_t) - T(I_1' \otimes i_2 \otimes \ldots \otimes i_t[I(1)])}{1 + T(I_1' \otimes i_2 \otimes \ldots \otimes i_t[I(1)])} \]

where \( I(0) \) and \( I(1) \) denote the region consisting of white (transparent) pixels and black (opaque) pixels respectively in \( I \). Further, \( I'[I(0)] \) and \( I'[I(1)] \) is the area of all white and black pixels in the reconstructed image \( I' \) corresponding to the regions \( I(0) \) and \( I(1) \).

In Wu and Sun (2013) according to Lemma 1 the average light transmission of stacked result when \( s = j \), where \( j = 0 \) or \( j = 1 \) is given as

\[ T_{OR,t}^{(k,n)}(r[s = j]) = \frac{1}{(n-k+1)}[T_{OR,t}^{(k,n)}(r[s = j]) + T_{OR,t}^{(k+1,n)}(r[s = j]) + \ldots + T_{OR,t}^{(n,n)}(r[s = j])] \]

\( T_{OR,t}^{(k,n)}(r[s = 0]) \) is average light transmission of stacked results by \( t \) number of pixel values by Chen and Tsao (2011) threshold RG based VSS scheme

\[ T_{OR,t}^{(k,n)}(r[s = 0]) = \begin{cases} \left(\frac{1}{2}\right)^t \times \left(\frac{1}{2}\right)^{t-1} + \left(1 - \left(\frac{1}{2}\right)^t\right) \times \left(\frac{1}{2}\right)^t & \text{for } t \geq k \text{ from Chen and Tsao (2011)} \\ \left(\frac{1}{2}\right)^t & \text{for } t < k \text{ from Chen and Tsao (2011)} \end{cases} \]

(1202, Lemma 7)

(1200, Lemma 5)

In Wu and Sun (2013) according to Lemma 6, the average light transmission of XORed result when \( s = j \) is given as

\[ T_{OR,t}^{XOR}(r[s = j]) = \frac{1}{(n-k+1)}[T_{OR,t}^{XOR}(r[s = j]) + T_{OR,t}^{XOR}(r[s = j]) + \ldots + T_{OR,t}^{XOR}(r[s = j])] \]
$T_{XOR,t}^{(k,n)}$ is average light transmission of XORed results by t number of pixel values by Chen and Tsao [Chen and Tsao (2011)] threshold RG based VSS scheme.

$$T_{(k,n)}^{XOR,t}(r[s = 0]) = \begin{cases} 
\frac{1}{2} \times \left(1 + \frac{1}{(k)}\right) & \text{for } t = k \ldots \text{from Wu and Sun (2013) (p.52, Lemma 4)} \\
\frac{1}{2} & \text{for } t > k \ldots \text{from Wu and Sun (2013) (p.52, Lemma 4)} \\
\frac{1}{2} & \text{for } t < k \ldots \text{from Wu and Sun (2013) (p.52, Lemma 3)}
\end{cases}$$

$$T_{(k,n)}^{XOR,t}(r[s = 1]) = \begin{cases} 
\frac{1}{2} \times \left(1 - \frac{1}{(k)}\right) & \text{for } t = k \ldots \text{from Wu and Sun (2013) (p.52, Lemma 4)} \\
\frac{1}{2} & \text{for } t > k \ldots \text{from Wu and Sun (2013) (p.52, Lemma 4)} \\
\frac{1}{2} & \text{for } t < k \ldots \text{from Wu and Sun (2013) (p.52, Lemma 3)}
\end{cases}$$

In [Wu and Sun (2013)](Table 6) contrast for (2,4) VSS scheme are depicted incorrectly. So, using the above stated formulae, we calculate the value of contrast by OR decryption.

case 1: $t = 2$

Average light transmission when $s = 0$,

$$T_{(k,n)}^{OR,2}(r[s = 0]) = \frac{1}{(n - k + 1)} \left[ T_{(k,n)}^{OR,2}(r[s = 0]) + T_{(k+1,n)}^{OR,2}(r[s = 0]) + \ldots + T_{(n,n)}^{OR,2}(r[s = 0]) \right]$$

$$= \frac{1}{(4 - 2 + 1)} \left[ T_{(2,4)}^{OR,2}(r[s = 0]) + T_{(3,4)}^{OR,2}(r[s = 0]) + T_{(4,4)}^{OR,2}(r[s = 0]) \right]$$

$$= \frac{1}{3} \times \left(\frac{2}{4}\right)^2 - 1 + \left(1 - \frac{2}{4}\right) \times \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2$$

$$= 19/72.$$

Average light transmission when $s = 1$,

$$T_{(k,n)}^{OR,2}(r[s = 1]) = \frac{1}{(n - k + 1)} \left[ T_{(k,n)}^{OR,2}(r[s = 1]) + T_{(k+1,n)}^{OR,2}(r[s = 1]) + \ldots + T_{(n,n)}^{OR,2}(r[s = 1]) \right]$$

$$= \frac{1}{(4 - 2 + 1)} \left[ T_{(2,4)}^{OR,2}(r[s = 1]) + T_{(3,4)}^{OR,2}(r[s = 1]) + T_{(4,4)}^{OR,2}(r[s = 1]) \right]$$

$$= 17/72.$$
Contrast is calculated as

\[ \alpha = \frac{T^{OR,2}(r[s = 0]) - T^{OR,2}(r[s = 1])}{1 + T^{OR,2}(r[s = 1])} = \frac{\frac{19}{72} - \frac{17}{72}}{1 + \frac{17}{72}} = \frac{2}{89}. \]

case 2 : \( t = 3 \)

Average light transmission when \( s = 0 \),

\[ T^{OR,3}(r[s = 0]) = \frac{1}{(n - k + 1)} \left[ T^{OR,3}_{(k,n)}(r[s = 0]) + T^{OR,3}_{(k+1,n)}(r[s = 0]) + \ldots + T^{OR,3}_{(n,n)}(r[s = 0]) \right] \]

\[ = \frac{1}{(4 - 2 + 1)} \left[ T^{OR,3}_{(2,4)}(r[s = 0]) + T^{OR,3}_{(3,4)}(r[s = 0]) + \ldots + T^{OR,3}_{(4,4)}(r[s = 0]) \right] \]

\[ = \frac{1}{3} \times \left( \frac{3}{4} \right) \times \left( \frac{1}{2} \right)^{3-1} + (1 - \frac{3}{4}) \times \left( \frac{1}{2} \right)^3 + \left( \frac{3}{4} \right) \times \left( \frac{1}{2} \right)^{3-1} + \]

\[ (1 - \frac{3}{4}) \times \left( \frac{1}{2} \right)^3 + \left( \frac{1}{2} \right)^3 \]

\[ = 15/96. \]

Average light transmission when \( s = 1 \),

\[ T^{OR,3}(r[s = 1]) = \frac{1}{(n - k + 1)} \left[ T^{OR,3}_{(k,n)}(r[s = 1]) + T^{OR,3}_{(k+1,n)}(r[s = 1]) + \ldots + T^{OR,3}_{(n,n)}(r[s = 1]) \right] \]

\[ = \frac{1}{(4 - 2 + 1)} \left[ T^{OR,3}_{(2,4)}(r[s = 1]) + T^{OR,3}_{(3,4)}(r[s = 1]) + \ldots + T^{OR,3}_{(4,4)}(r[s = 1]) \right] \]

\[ = \frac{1}{3} \times \left( \frac{3}{4} \right) \times \left( \frac{1}{2} \right)^3 + (1 - \frac{3}{4}) \times \left( \frac{1}{2} \right)^3 + \left( \frac{3}{4} \right) \times \left( \frac{1}{2} \right)^3 + \left( \frac{1}{2} \right)^3 \]

\[ = 9/96. \]

Contrast is calculated as

\[ \alpha = \frac{T^{OR,3}(r[s = 0]) - T^{OR,3}(r[s = 1])}{1 + T^{OR,3}(r[s = 1])} = \frac{\frac{15}{96} - \frac{9}{96}}{1 + \frac{9}{96}} = \frac{6}{105}. \]
case 3: \( t = 4 \)
Average light transmission when \( s = 0 \),

\[
T^{OR,A}(r[s = 0]) = \frac{1}{(n - k + 1)}[T^{OR,A}_{(k,n)}(r[s = 0]) + T^{OR,A}_{(k+1,n)}(r[s = 0]) + \ldots + T^{OR,A}_{(n,n)}(r[s = 0])]
\]
\[
= \frac{1}{(4 - 2 + 1)}[T^{OR,A}_{(2,4)}(r[s = 0]) + T^{OR,A}_{(3,4)}(r[s = 0]) + T^{OR,A}_{(4,4)}(r[s = 0])]
\]
\[
= \frac{1}{3} \times \left( \frac{4}{3} \right) \times \left( \frac{1}{2} \right)^{4-1} + (1 - \left( \frac{4}{3} \right)) \times \left( \frac{1}{2} \right)^4 + \left( \frac{4}{3} \right) \times \left( \frac{1}{2} \right)^{4-1} + (1 - \left( \frac{4}{3} \right)) \times \left( \frac{1}{2} \right)^4
\]
\[
= \frac{1}{3} \times \left( \frac{4}{3} \right) \times \left( \frac{1}{2} \right)^{4-1} + (1 - \left( \frac{4}{3} \right)) \times \left( \frac{1}{2} \right)^4 + \left( \frac{4}{3} \right) \times \left( \frac{1}{2} \right)^{4-1} + (1 - \left( \frac{4}{3} \right)) \times \left( \frac{1}{2} \right)^4
\]
\[
= \frac{1}{8}.
\]

Average light transmission when \( s = 1 \),

\[
T^{OR,A}(r[s = 1]) = \frac{1}{(n - k + 1)}[T^{OR,A}_{(k,n)}(r[s = 1]) + T^{OR,A}_{(k+1,n)}(r[s = 1]) + \ldots + T^{OR,A}_{(n,n)}(r[s = 1])]
\]
\[
= \frac{1}{(4 - 2 + 1)}[T^{OR,A}_{(2,4)}(r[s = 1]) + T^{OR,A}_{(3,4)}(r[s = 1]) + T^{OR,A}_{(4,4)}(r[s = 1])]
\]
\[
= \frac{1}{3} \times \left( \frac{4}{3} \right) \times \left( \frac{1}{2} \right)^{4-1} + (1 - \left( \frac{4}{3} \right)) \times \left( \frac{1}{2} \right)^4 + \left( \frac{4}{3} \right) \times \left( \frac{1}{2} \right)^{4-1} + (1 - \left( \frac{4}{3} \right)) \times \left( \frac{1}{2} \right)^4
\]
\[
= \frac{1}{3} \times \left( \frac{4}{3} \right) \times \left( \frac{1}{2} \right)^{4-1} + (1 - \left( \frac{4}{3} \right)) \times \left( \frac{1}{2} \right)^4 + \left( \frac{4}{3} \right) \times \left( \frac{1}{2} \right)^{4-1} + (1 - \left( \frac{4}{3} \right)) \times \left( \frac{1}{2} \right)^4
\]
\[
= 0.
\]
Contrast is calculated as

\[
\alpha = \frac{T^{OR,A}(r[s = 0]) - T^{OR,A}(r[s = 1])}{1 + T^{OR,A}(r[s = 1])} = \frac{\frac{1}{8} - 0}{1 + \frac{1}{8}} = \frac{1}{8}.
\]

Now, we calculate the value of contrast for (2, 4) VSS scheme by XORed decryption

case 1: \( t = 2 \)
Average light transmission when $s = 0$,

$$T_{XOR,t}^{(r[s = 0])} = \frac{1}{(n - k + 1)} [T_{XOR,t}^{(k,n)}(r[s = 0]) + T_{XOR,t}^{(k+1,n)}(r[s = 0]) + \ldots + T_{XOR,t}^{(n,n)}(r[s = 0])]$$

$$= \frac{1}{(4 - 2 + 1)} [T_{XOR,t}^{(2,4)}(r[s = 0]) + T_{XOR,t}^{(3,4)}(r[s = 0]) + T_{XOR,t}^{(4,4)}(r[s = 0])$$

$$= \frac{1}{(4 - 2 + 1)} \left[ \frac{1}{2} \times (1 + \frac{1}{16}) + \frac{1}{2} + \frac{1}{2} \right]$$

$$= 19/36.$$

Average light transmission when $s = 1$,

$$T_{XOR,t}^{(r[s = 1])} = \frac{1}{(n - k + 1)} [T_{XOR,t}^{(k,n)}(r[s = 1]) + T_{XOR,t}^{(k+1,n)}(r[s = 1]) + \ldots + T_{XOR,t}^{(n,n)}(r[s = 1])]$$

$$= \frac{1}{(4 - 2 + 1)} [T_{XOR,t}^{(2,4)}(r[s = 1]) + T_{XOR,t}^{(3,4)}(r[s = 1]) + T_{XOR,t}^{(4,4)}(r[s = 1])$$

$$= \frac{1}{(4 - 2 + 1)} \left[ \frac{1}{2} \times (1 - \frac{1}{16}) + \frac{1}{2} + \frac{1}{2} \right]$$

$$= 17/36.$$

Contrast is calculated as

$$\alpha = \frac{T_{XOR,t}^{(r[s = 0])} - T_{XOR,t}^{(r[s = 1])}}{1 + T_{XOR,t}^{(r[s = 1])}} = \frac{19/36 - 17/36}{1 + 17/36} = \frac{2}{53}.$$
case 2: \( t = 3 \)

Average light transmission when \( s = 0 \),

\[
T^{\text{XOR},t}(r[s = 0]) = \frac{1}{(n-k+1)} \left[ T^{\text{XOR},t}_{(k,n)}(r[s = 0]) + T^{\text{XOR},t}_{(k+1,n)}(r[s = 0]) + \ldots + T^{\text{XOR},t}_{(n,n)}(r[s = 0]) \right]
\]

\[
= \frac{1}{(4-2+1)} \left[ T^{\text{XOR},3}_{(2,4)}(r[s = 0]) + T^{\text{XOR},3}_{(3,4)}(r[s = 0]) + T^{\text{XOR},3}_{(4,4)}(r[s = 0]) \right]
\]

\[
= \frac{1}{(4-2+1)} \left[ \frac{1}{2} + \frac{1}{2} \times \left( 1 + \frac{1}{\binom{4}{3}} + \frac{1}{2} \right) \right]
\]

\[
= \frac{13}{24}
\]

Average light transmission when \( s = 1 \),

\[
T^{\text{XOR},t}(r[s = 1]) = \frac{1}{(n-k+1)} \left[ T^{\text{XOR},t}_{(k,n)}(r[s = 1]) + T^{\text{XOR},t}_{(k+1,n)}(r[s = 1]) + \ldots + T^{\text{XOR},t}_{(n,n)}(r[s = 1]) \right]
\]

\[
= \frac{1}{(4-2+1)} \left[ T^{\text{XOR},3}_{(2,4)}(r[s = 1]) + T^{\text{XOR},3}_{(3,4)}(r[s = 1]) + T^{\text{XOR},3}_{(4,4)}(r[s = 1]) \right]
\]

\[
= \frac{1}{(4-2+1)} \left[ \frac{1}{2} + \frac{1}{2} \times \left( 1 - \frac{1}{\binom{4}{3}} + \frac{1}{2} \right) \right]
\]

\[
= \frac{11}{24}
\]

Contrast is calculated as

\[
\alpha = \frac{T^{\text{XOR},3}(r[s = 0]) - T^{\text{XOR},3}(r[s = 1])}{1 + T^{\text{XOR},3}(r[s = 1])} = \frac{13}{24} - \frac{11}{24} = \frac{2}{35}
\]
case 3 : $t = 4$

Average light transmission when $s = 0$,

$$T_{XOR,t}^t(r[s = 0]) = \frac{1}{(n - k + 1)}[T_{(k,n)}^{XOR,t}(r[s = 0]) + \ldots + T_{(n,n)}^{XOR,t}(r[s = 0])]
\nonumber$$

$$= \frac{1}{(4 - 2 + 1)}[T_{(2,4)}^{XOR,t}(r[s = 0]) + T_{(3,4)}^{XOR,t}(r[s = 0]) + T_{(4,4)}^{XOR,t}(r[s = 0])]
\nonumber$$

$$= \frac{1}{(4 - 2 + 1)}\left[\frac{1}{2} + \frac{1}{2} + \frac{1}{2} \times (1 + \frac{1}{4})\right]
\nonumber$$

$$= \frac{2}{3}.$$

Average light transmission when $s = 1$,

$$T_{XOR,t}^t(r[s = 1]) = \frac{1}{(n - k + 1)}[T_{(k,n)}^{XOR,t}(r[s = 1]) + \ldots + T_{(n,n)}^{XOR,t}(r[s = 1])]
\nonumber$$

$$= \frac{1}{(4 - 2 + 1)}[T_{(2,4)}^{XOR,t}(r[s = 1]) + T_{(3,4)}^{XOR,t}(r[s = 1]) + T_{(4,4)}^{XOR,t}(r[s = 1])]
\nonumber$$

$$= \frac{1}{(4 - 2 + 1)}\left[\frac{1}{2} + \frac{1}{2} + \frac{1}{2} \times (1 - \frac{1}{4})\right]
\nonumber$$

$$= \frac{1}{3}.$$

Contrast is calculated as

$$\alpha = \frac{T_{XOR,t}^t(r[s = 0]) - T_{XOR,t}^t(r[s = 1])}{1 + T_{XOR,t}^t(r[s = 1])} = \frac{\frac{2}{3} - \frac{1}{3}}{1 + \frac{1}{3}} = \frac{1}{4}.$$

In [Wu and Sun 2013] (Table 8) contrast for (3,5) VSS scheme for $t = 4$ by XOR decryption are depicted incorrectly. So, we calculate the average light
transmission when \( s = 0 \)

\[
T_{XOR,t}(r[s = 0]) = \frac{1}{(n - k + 1)} [T_{XOR,t}\(_{(k,n)}(r[s = 0]) + T_{XOR,t}\(_{(k+1,n)}(r[s = 0]) + \ldots
\]

\[
= \frac{1}{(5 - 3 + 1)} [T_{XOR,t}\(_{(3,5)}(r[s = 0]) + T_{XOR,t}\(_{(4,5)}(r[s = 0]) + T_{XOR,t}\(_{(5,5)}(r[s = 0])]
\]

\[
= \frac{1}{(5 - 3 + 1)} \left[ \frac{1}{2} + \frac{1}{2} \times \left( 1 + \frac{1}{\binom{5}{4}} \right) + \frac{1}{2} \right]
\]

\[
= \frac{8}{15}.
\]

Average light transmission when \( s = 1 \),

\[
T_{XOR,t}(r[s = 1]) = \frac{1}{(n - k + 1)} [T_{XOR,t}\(_{(k,n)}(r[s = 1]) + T_{XOR,t}\(_{(k+1,n)}(r[s = 1]) + \ldots
\]

\[
= \frac{1}{(5 - 3 + 1)} [T_{XOR,t}\(_{(3,5)}(r[s = 0]) + T_{XOR,t}\(_{(4,5)}(r[s = 0]) + T_{XOR,t}\(_{(5,5)}(r[s = 0])]
\]

\[
= \frac{1}{(5 - 3 + 1)} \left[ \frac{1}{2} + \frac{1}{2} \times \left( 1 - \frac{1}{\binom{5}{4}} \right) + \frac{1}{2} \right]
\]

\[
= \frac{7}{15}.
\]

Contrast is calculated as

\[
\alpha = \frac{T_{XOR,t}(r[s = 0]) - T_{XOR,t}(r[s = 1])}{T_{XOR,t}(r[s = 1])} = \frac{\frac{8}{15} - \frac{7}{15}}{\frac{7}{15}} = \frac{1}{22}
\]

In Wu and Sun (2013) (Table 9) contrast for \((4, 5)\) VSS scheme for \( t = 4 \) by XOR decryption are depicted incorrectly. So, we calculate the average light transmission when \( s = 0 \)

\[
T_{XOR,t}(r[s = 0]) = \frac{1}{(n - k + 1)} [T_{XOR,t}\(_{(k,n)}(r[s = 0]) + T_{XOR,t}\(_{(k+1,n)}(r[s = 0]) + \ldots
\]

\[
= \frac{1}{(5 - 4 + 1)} [T_{XOR,t}\(_{(4,5)}(r[s = 0]) + T_{XOR,t}\(_{(5,5)}(r[s = 0])]
\]

\[
= \frac{1}{(5 - 4 + 1)} \left[ \frac{1}{2} \times \left( 1 + \frac{1}{\binom{5}{4}} \right) + \frac{1}{2} \right]
\]

\[
= \frac{11}{20}.
\]
Average light transmission when $s = 1$,

$$T_{XOR,t}(r[s = 1]) = \frac{1}{(n - k + 1)} [T_{XOR,t}(r[s = 1]) + T_{XOR,t}((k+1,n))r[s = 1]) + \ldots + T_{XOR,t}(n(n))r[s = 1])]$$

$$= \frac{1}{(5 - 4 + 1)} [T_{XOR,4}(r[s = 1]) + T_{XOR,4}(r[s = 1])]$$

$$= \frac{1}{(5 - 4 + 1)} \left[ \frac{\frac{1}{2} \times (1 - \frac{1}{(5)}) + \frac{1}{2}}{1} \right]$$

$$= \frac{9}{20}.$$