Internal spin resistance of spin batteries

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For spin batteries we introduce the concept of internal spin resistance which quantifies the amount of backflow from the load to the battery. It allows to relate through a Thévenin-Norton relation, spin current sources to spin accumulation sources. The value of the internal spin resistance is derived explicitly for several spin batteries based on spin injection, ferromagnetic resonance or spin Hall effect.

I. INTRODUCTION. WHAT IS A SPIN BATTERY? INTERNAL SPIN RESISTANCE.

Spin batteries are a crucial component of spintronics. The goal of this paper is to argue that an important characteristic of spin batteries is their internal spin resistance, which is the analog for spin batteries of the internal resistance of charge batteries. Many specific realizations of spin batteries have been proposed. The simplest spin current source in spintronics is probably a ferromagnet through which a charge current flows, a spin accumulation and a spin current will appear in any adjacent paramagnetic metal: this is the mechanism of spin injection which is used in many (so-called) non-local setups where a spin current flows separately from a charge current. Spin pumping through ferromagnetic resonance is another popular way to create a spin source. The spin Hall effect, where a charge current generates a transverse spin current can also be used for spin current generation (e.g. Ref.8). A solar spin battery has been proposed as well.

To the extent of our knowledge the idea of an internal spin resistance has not been spelled out explicitly in the literature; the closest would be in Ref9, where Brataas and co-workers computed maximum spin voltage and maximum spin current generated by the ferromagnetic resonance operated spin pump. As we will show later the internal spin resistance for collinear spin batteries is just the ratio of maximum spin voltage to maximum spin current.

Introducing the concept of internal spin resistance leads to distinguish spin current batteries from spin accumulation sources; the obvious practical import of that distinction is that this can help optimize the design of a spin battery depending on whether one requires a strong spin current (as in STT (spin transfer torque) applications) or strong spin accumulations (as in GMR (giant magneto-resistance)).

We will consider both collinear and non-collinear spin batteries and in section II will explicitly compute the internal spin resistance for several spin batteries operated by spin accumulation, spin Hall effect or ferromagnetic resonance.

A. Defining internal spin resistance.

We confine ourselves in this sub-section to collinear setups so that all magnetizations have the same direction. The spin current is also assumed to be one dimensional so that a single scalar can describe it.

The simplest way to define a spin battery is as a spin current source. But a spin current induces in general a non-equilibrium spin build-up: therefore one could argue that a spin battery can also be viewed as a spin accumulation source. This is quite obvious in Johnson-Silsbee thermodynamic theory as well as magnetoelectronic circuit theory, where spin accumulation is the counterpart of charge voltage and drives both charge and spin currents.

When one turns to charge batteries, two alternative descriptions are possible: battery as current source or as voltage source. The Thévenin-Norton equivalence ensures that both points of view are interchangeable.

When comparing charge batteries with spin batteries, spin accumulation acts as the analog of (charge) voltage while spin current plays the role of charge current. To make the analogy clearer we define a spin voltage as

$$V_s = -\frac{\mu_{\uparrow} - \mu_{\downarrow}}{2e}$$  \hspace{1cm} (1)

(which is the spin accumulation cast in electrical units) and we will also measure the spin current in electrical units:

$$I_s = I_{\uparrow} - I_{\downarrow} \propto -e.$$  

If one connects a spin battery to a load (e.g. a paramagnetic metal) a spin current will flow away; concurrently a spin accumulation will build up in general in the load, inducing a backflow spin current and spin accumulation in the battery. The spin current will therefore be reduced in comparison to what it would be if there was no backflow. Within linear-response this can be described through the equation:

$$I_s = I_{s,max} - \frac{V_s}{r_{IS}}$$  \hspace{1cm} (2)

or

$$V_s = V_{s,max} - r_{IS} I_s$$  \hspace{1cm} (3)

where $V_s$ is the spin voltage of the battery ($V_{s,max}$ being its maximum value), $I_s$ its spin current (maximum
load spin resistance being the ratio to the spin resistance of the load, both being evaluated measured by a comparison of the internal spin resistance.

These relations imply a Thévenin-Norton equivalence between spin current source and spin accumulation source exactly as with charge batteries. The spin battery Thévenin-Norton relations will be derived exactly for several examples of spin batteries in Section II in confirmation of the previous simple argument.

One potential point of confusion is the fact that spin current and spin accumulation are not uniform within the spin battery: which spin currents and spin accumulations enter in the previous equations? Within a circuit theory, given the assumption of one-dimensional connections between battery and load, the equations should be understood as local relations at the point where spin current exits the battery to enter the load. Departures from one-dimensional spin current flow can be handled through an averaging process so that the relations should be quite general by replacing \( I_s \) with \( \langle I_s \rangle \) the average over the contact surface between battery and load, and a similar definition for \( V_s \).

In case of interface discontinuities (for instance contact resistance) spin current and spin voltage are understood to be defined on the battery side.

Observe that the internal spin resistance depends solely on the spin battery; it is by definition independent of the loads connected to the battery. The explicit values taken by \( V_s \) and \( I_s \) however depend on the load.

### B. Ideal spin batteries.

The definition of the internal spin resistance makes obvious two subsequent definitions:

- ideal spin current source: \( r_{IS} = \infty \);
- ideal spin accumulation source: \( r_{IS} = 0 \).

In terms of applications an ideal spin accumulation source should be better suited to inducing large spin accumulations in the load, which can be detected as charge voltages through Johnson-Silsbee charge-spin coupling. Ideal spin current sources should be more interesting when one is in need of large angular momentum transfer such as with spin transfer torques.

In practice since one has always non-ideal sources, the relevance of a spin battery to a given load should be measured by a comparison of the internal spin resistance to the spin resistance of the load, both being evaluated locally at the connection between battery and load (the load spin resistance being the ratio \( V_s^{load}/I_s \)).

\[ r_{IS} = V_{s,max}/I_{s,max} \] (4)

since for \( V_s = 0 \), \( I_s = I_{s,max} \) by Eq. 2, which in turn implies by Eq. 0 \( 0 = V_{s,max} - r_{IS} I_{s,max} \).

Let us consider now a load connected to a spin battery. We will show how the internal spin resistance modifies the response to a spin battery.

**General response to a spin battery.** As noted by Brataas and coll. a spin battery is unipolar: there needs be a single terminal connecting the battery and its load (while a charge battery is of course dipolar). Through Johnson-Silsbee charge-spin coupling a charge response of the load can be measured; this requires at least two terminals through which either a voltage drop \( V_c \) or a charge current \( I_c \) is measured. The most general setup for a spin battery driven device is therefore a three terminal one (see Fig. 1). We call \( T_1 \) and \( T_2 \) the charge measurement terminals on the load and \( T_3 \) the terminal connecting the spin battery to the load. The charge response can be driven either by a spin current \( I_s \) or a spin accumulation (or spin voltage \( V_s \)). If one assumes there is no interface spin-flip, spin current is continuous at the battery-load interface.

Within linear response in the three terminal geometry of Fig. 1 this leads to linear relations:

\[ I_s = g_{ss} V_s + g_{sc} V_c, \] (5)
\[ I_c = g_{cs} V_s + g_{cc} V_c. \] (6)

These transport parameters are load-dependent. Reciprocity implies that \( g_{cs} = g_{sc} \). If furthermore the two charge measurement terminals are shorted, then \( V_c = 0 \) and one measures \( I_c \). Or conversely a voltmeter can be used and then \( I_c = 0 \).

To have an idea of the magnitude of the transport coefficients of the load, let us take a two channel model; then:

\[ g_{cc} = g_{ss} = g_{\uparrow\uparrow} + g_{\downarrow\downarrow} \] (7)
\[ g_{cs} = g_{sc} = g_{\uparrow\downarrow} - g_{\downarrow\uparrow} = g_{cc} p \] (8)

with \( p \) being a conductance polarization. More generally one expects therefore \( g_{cc} \) and \( g_{ss} \) to be of same order.
In addition to the ‘charge-spin conductance’

\[
g_{cs} = \frac{\partial I_c}{\partial V_s} = \frac{I_c}{V_s} \bigg|_{V_s=0},
\]

several additional transport coefficients can also be defined to measure the charge response to a spin drive:

\[
r_{cs} = \frac{\partial V_c}{\partial I_s} \bigg|_{I_c=0}, \quad h_{cs} = \frac{\partial I_c}{\partial V_s} \bigg|_{V_s=0}, \quad k_{cs} = \frac{\partial V_c}{\partial V_s} \bigg|_{I_c=0},
\]

which have respectively the dimensions of a resistance or are dimensionless. The voltage drop \( V_c \) and charge current \( I_c \) are related through the charge conductance \( g_{cc} = \frac{\partial I_c}{\partial V_c} \), so that for instance \( k_{cs} = -g_{cs}/g_{cc} \). While the charge conductance \( g_{cs} \) is always positive, \( g_{cs} \) may be negative.

It is against \( g_{ss} \) that the internal spin resistance should be compared as we shall now show.

**Response to a Spin accumulation source.** The two relevant transport coefficients are \( g_{cs} \) and \( k_{cs} \).

Suppose one measures \( I_c \) by shorting terminals \( T1 \) and \( T2 \) with an ammeter. Eq. 5-6 apply with \( V_c = 0 \). We look for the transport coefficient:

\[
g'_{cs} = \frac{\partial I_c}{\partial V_s} \bigg|_{V_s=0}.
\]

and want to show it will be reduced to the internal spin resistance. For an ideal spin accumulation source the response would be \( g'_{cs} = g_{cs} \) since \( V_s = V_{s,max} \).

The relation \( V_s = V_{s,max} − r_{IS} I_s \) (eq. 2) implies \( V_s = V_{s,max} − r_{IS} g_{ss} I_s \). Thus

\[
V_s = V_{s,max} / (1 + r_{IS} g_{ss}) \cdot
\]

Therefore:

\[
g'_{cs} = \frac{g_{cs}}{(1 + r_{IS} g_{ss})}
\]

which means that the charge current \( I_c \) measured will be reduced by the factor

\[
1 + r_{IS} g_{ss}.
\]

If we consider spin injection by a ferromagnet (see for instance [1] for an N-F-N trilayer acting as a spin battery) \( r_{IS} \sim r_N \) is in the 1 Ω range (if \( r_N = \rho_N l_N /\Delta \) is the spin resistance of the N layer, \( l_N \) the spin relaxation length in N, \( \Delta \sim 10^{-15} m^2 \) the cross-section for a nanopillar for example). This means a load with \( g_{ss}^{-1} \gg 1 \Omega \) is better suited to that kind of battery. If the load is a paramagnetic metal, the usual range is \( g_{ss}^{-1} \sim 1 \Omega \).

The previous description assumes that there is no interface resistance between the spin battery and the load. This is easy to incorporate using:

\[
V_s - V_s^{\text{load}} = r_c I_s
\]

which results in a further decrease of the response

\[
g'_{cs} = \frac{g_{cs}}{[1 + (r_c + r_{IS}) g_{ss}]}.
\]

Inclusion of interfacial spin flip is straightforward and left to the reader.

If one prefers to measure a voltage between terminals \( T1 \) and \( T2 \), similar calculations lead to:

\[
k'_{cs} = \frac{k_{cs}}{1 + r_{IS} g_{ss} g_{cc} / g_{cc}}
\]

where

\[
k'_{cs} = \frac{\partial V_c}{\partial V_s}.
\]

In conclusion if \( r_{IS} \ll g_{ss}^{-1} \) (typically this would mean \( r_{IS} \gg 1 \Omega \)) the spin battery can be considered to be an ideal spin accumulation source.

**Response to a Spin current source.** The relevant transport coefficients are now \( r_{cs} \) and \( h_{cs} \).

Suppose first \( V_c = 0 \) (terminals \( T1 \) and \( T2 \) shorted). The relation \( I_s = I_{s,max} − V_s / r_{IS} \) (eq. 2) combined with Eq. 5 implies \( I_s = I_{s,max} − (r_{IS} g_{ss})^{-1} I_s \). Thus the incoming spin current is reduced

\[
I_s = I_{s,max} / (1 + (r_{IS} g_{ss})^{-1}).
\]

Therefore:

\[
h'_{cs} = \frac{I_c}{I_{s,max}} = \frac{h_{cs}}{(1 + (r_{IS} g_{ss})^{-1})}
\]

which means that the charge current \( I_c \) measured will be reduced by the factor

\[
1 + (r_{IS} g_{ss})^{-1}.
\]

If one rather measures a voltage between \( T1 \) and \( T2 \), similar calculations lead to:

\[
r'_{cs} = \frac{V_c}{I_{s,max}} = \frac{r_{cs}}{(1 + [r_{IS} (g_{ss} g_{cc} − g_{cs} g_{cc}) / g_{cc}]^{-1})}.
\]

If \( r_{IS} \gg g_{ss}^{-1} \) there is no renormalization and the spin battery can then be considered to be an ideal spin current source.

**D. Non-collinear internal spin resistance: internal spin conductance tensor.**

We keep the hypothesis of one-dimensional flow so that the spin current is a vector instead of a rank-2 tensor and spin voltage is now a vector instead of a scalar (see Eq. 1). For non-collinear setups the obvious generalization of Eq. 2 is:

\[
I_s = I_{s,max} − g_{IS} V_s
\]
or

\[ V_s = V_{s,\text{max}} - r_{IS} I_s \]  \hspace{1cm} (21)

where \( g_{IS} = r_{IS}^{-1} \) is an internal spin conductance tensor (see IIC-IID for proofs of that relation for two examples of non-collinear spin batteries). Eq. (22) and (23) are replaced by:

\[ I_s = g_{ss} V_s + g_{ss} V_c, \]  \hspace{1cm} (22)

\[ I_c = g_{cs} \cdot V_s + g_{cs} V_c. \]  \hspace{1cm} (23)

where \( g_{ss} \) is a rank 2 tensor and the vectors \( g_{cs} \) are equal by reciprocity. The charge current response to a spin accumulation source is renormalized when one takes into account internal spin resistance; instead of

\[ I_c = g_{cs} \cdot V_{s,\text{max}} \]

one has

\[ I_c = \left\{ \left[ (1 + r_{IS} g_{ss})^{-1} \right]^T g_{cs} \right\} \cdot V_{s,\text{max}}. \]  \hspace{1cm} (24)

Similar expressions can be derived for the voltage \( V_c \):

\[ V_c = \left\{ \left[ (1 + r_{IS} g_{ss}')^{-1} \right]^T k_{cs} \right\} \cdot V_{s,\text{max}} \]  \hspace{1cm} (25)

where \( V_c = k_{cs} \cdot V_s \) and

\[ g_{ss,ij} = g_{ss,ij} - \frac{g_{cs,i} g_{cs,j}}{g_{cc}}. \]  \hspace{1cm} (26)

E. Power dissipation.

For collinear spin batteries there is complete isomorphism of internal spin resistance to its charge counterpart since Eq. (2) has the same structure. We suppose spin current is exiting the battery at a single point.

The spin battery is unipolar with \( I_c = 0 \) but \( I_t = -I_s \), so the spin battery is equivalent to a charge battery with the two poles merged and with charge current \( I_t \).

The power is:

\[ P = \sum_{\alpha} -I_{\alpha} \frac{V_{\alpha}}{R_{\alpha}} \]  \hspace{1cm} for current counted positive when leaving the device. Therefore:

\[ P = -V_s I_s \]

One ends up with:

\[ P = -V_{s,\text{max}} I_s + r_{IS} I_s^2 \]  \hspace{1cm} (27)

for a spin current in electrical units and counted positive when leaving the battery. This expression shows that the internal spin resistance is always positive.

For non-collinear spin batteries the power dissipated by the battery is likewise:

\[ P = -V_s \cdot I_s = -V_{s,\text{max}} \cdot I_s + I_s \cdot r_{IS} I_s \]  \hspace{1cm} (28)

which implies that \( r_{IS} + r_{IS}^T \) is a positive semi-definite matrix to ensure correct sign for power dissipation.

II. ILLUSTRATIONS.

We explicitly derive the internal spin resistance for five spin batteries: (i) a single ferromagnetic layer which induces spin accumulation in neighbouring paramagnetic metals through spin injection, (ii) a F - N - F trilayer, (iii) bulk spin pumping, (iv) the ferromagnetic resonance operated spin battery of Brataas and coworkers, (v) spin Hall effect.

A. Single ferromagnetic layer .

The prototypal spin battery is a single ferromagnetic layer through which a charge current flows. This is a spin accumulation and spin current source for paramagnets thanks to spin injection. We now compute its characteristic spin battery parameters, namely its internal spin resistance and its maximum spin voltage.

We consider the following non-local geometry: a paramagnetic N electrode brings current to a ferromagnetic F rod connected to a second paramagnetic N’ electrode which collects the current. For maximum spin voltage, the load must be connected close to an interface. In our model we have chosen the load connection at the interface F-N’ on the N’ side but it could have been on the F side (see Fig. 2).

![Image](image.png)

Figure 2. N - F - N trilayer used as a spin emf battery. The load should be connected close to an interface for maximum spin voltage. The charge current \( I_c \) crosses the ferromagnet F and induces through spin injection a spin current \( I_s \) in the load as well as a spin accumulation \( V_s \).

We rely on a one-dimensional modellization within the standard drift-diffusion formalism. The ferromagnet has length \( d \) and is sandwiched between two paramagnetic metals N and N’ which are assumed to be infinite. The origin \( O \) is in the middle of F and the F-N interfaces are located at \( O_L (z = -d/2) \) and \( O_R (z = d/2) \). The spin current at \( O_L \) is continuous \( I_s^L = I_s (O_L) \) if one neglects interfacial spin-flip; at the other interface \( O_R \) however:

\[ I_s^R = I_s^F (O_R) = I_s^N (O_R) + I_s^{load} \]  \hspace{1cm} (29)

which expresses the spin current continuity. All the spin currents are counted positive in the direction of axis \( z \) (or away from the battery for \( I_s^{load} \)).
The paramagnets are supposed to be infinite. After tedious but straightforward calculations one eventually gets

\[ r_{IS} = r_N - \frac{r_N^2}{r_F} \frac{1 + X_1 \tanh \frac{d}{r_F}}{r_F (X_1 + X_2) + (1 + X_1 X_2) \tanh \frac{d}{r_F}} \]

(\(r_F\) is the ferromagnet spin relaxation length, \(d\) is the ferromagnet length, \(r_N\) is the paramagnet spin relaxation length, \(r_{N1} = \rho_N A\) is the paramagnet spin resistance with \(A\) its cross-section, \(r_{ci}\) \((i = 1, 2)\) are the contact resistances at the interfaces \(O_L\) and \(O_R\).

We can simplify the expressions by assuming identical interface parameters at \(O_L\) and \(O_R\), \(P_{c1} = P_{c2} = P_c\) (interface conductance polarizations, noted usually \(\gamma\) in Valet-Fert notation\(^\text{10}\)) and \(r_{c1} = r_{c2} = r_c\), then with \(X = (r_N + r_c)/r_F\):

\[ r_{IS} = r_N - \frac{r_N^2}{r_F} \frac{1 + X \tanh \frac{d}{r_F}}{2X + (1 + X^2) \tanh \frac{d}{r_F}}, \]

and the maximum spin voltage is found as:

\[ V_{s,max} = r_N I_c \frac{P_c r_c + P_F r_F \tanh \frac{d}{2r_F}}{r_N + r_c + r_F \tanh \frac{d}{2r_F}} \]

(where \(P_F\) is the usual conductivity polarization of the ferromagnet, noted \(\beta\) in Valet-Fert notation\(^\text{10}\)). The term in brackets is actually the current polarization \((I_s/I_c)\); it is bounded by \([P_c r_c + P_F r_F \tanh \frac{d}{2r_F}] / [r_c + r_F \tanh \frac{d}{2r_F}]\) which is a weighted average of \(P_c\) and \(P_F\). Therefore the larger the spin injection, the larger the spin voltage.

Let us examine limiting cases:

- \(d \to 0\) (thin ferromagnet): then the spin voltage is dominated by interfaces and one gets:

\[ V_{s,max} \to r_N I_c \frac{P_c r_c}{r_N + r_c} I_c; \]

\[ r_{IS} \to \frac{r_N r_c}{r_N + r_c}. \]

This implies the spin voltage is set by the smallest of either the contact or normal layer resistance. For Co-Cu layers for instance, using \(P_c \sim 0.75\), \(P_F \sim 0.5\), \(r_F \sim 10\Omega\), \(r_N \sim 1\Omega\), \(r_c \sim 1\Omega\) (ranges extracted from Ref\(^\text{10}\) and\(^\text{12}\) with an area \(A = 1 \text{fm}^2\) and \(I_c \sim 0.1 \text{mA}\) yielding \(V_{s,max} \sim 0.01 \text{mV} = 10 \mu\text{V}\) and \(r_{IS} \sim 0.5\Omega\).

- \(d \to \infty\) (thick ferromagnet): then the spin voltage becomes

\[ V_{s,max} \to r_N I_c \frac{P_c r_c + P_F r_F \tanh \frac{d}{2r_F}}{r_N + r_c + r_F \tanh \frac{d}{2r_F}}; \]

\[ r_{IS} \to \frac{r_N (r_c + r_F)}{r_N + r_c + r_F}. \]

This is also in the 10 \(\mu\text{V}\) range for \(V_{s,max}\) while \(r_{IS} \sim 1\Omega\). More generally, the scale of \(V_{s,max}\) is set by \(r_N I_c \sim 10\mu\text{V}\) so we don't expect to go much beyond the 10 \(\mu\text{V}\) range with this kind of battery.

The internal spin resistance scale is given by \(r_N\) which is its maximum value. It is therefore in the 1\(\Omega\) range.

It is interesting to compute a conversion gain \(C_{cs} = V_s/V_c = V_s/R I_c\) which is a measure of the efficiency to convert a charge voltage into a spin voltage. One finds:

\[ C_{cs} = \frac{r_N}{r} \frac{P_c r_c + P_F r_F \tanh \frac{d}{2r_F}}{r_N + r_c + r_F \tanh \frac{d}{2r_F}}. \]

For metallic multilayers, the resistance of the trilayer \(N-F-N'\) \(R\) is in the 10\(\Omega\) range; plugging in the figures we used for a Co-Cu interface, this yields \(C_{cs} \sim 1\%\) which is small and consistent with the figures \(V_{s,max} \sim 10\mu\text{V}\) (for \(V_c \sim 1\text{mV}\)).

### B. F-N-F trilayer

![Figure 4](image-url) (color online) F - N - F nanopillar used as spin battery. The load is connected to the spin battery through the N paramagnetic layer. A charge current \(I_c\) crossing the nanopillar ensures spin injection occurs into the load (as a spin current \(I_s\) or a spin voltage \(V_s\)).

We consider now the following spin battery which will prove more efficient than the previous one: a F - N -
F trilayer through which flows a charge current. A spin accumulation is generated in the N layer which is wired to a load. In the latter flows a spin current. Fig.[□] shows a nanopillar geometry while Fig.[□] is a non-local variant.

The spin currents at the interfaces located at d-layer is battery to the load). On the left and the right paramagnetic N layer (a spin current Figure 6. F-N-F spin battery. The load is connected to the spin battery through the N paramagnetic layer.

We model the spin battery in one dimension using the standard drift-diffusion equations.[□][□][□][□][□][□] The origin is in the middle of the N layer. It is convenient to use opposite axes to the left (z) and to the right (z') of O. Charge current I_c flows from left to right. The width of the N layer is d and the interfaces with the ferromagnets are located at O_L (z = d/2) and O_R (z' = d/2) (see Fig.[□]). The spin currents at the interfaces I_x = I_x (O_L) and I_x' = I_x (O_R) are continuous if one neglects interfacial spin-flip; they are counted positive in the direction of axes z and z' respectively.

Figure 5. Non-local geometry for F-N-F spin battery. The load is connected to the spin battery through the N paramagnetic layer.

The ferromagnets are supposed to be infinite. We use the same notations as in previous section. After tedious but straightforward calculations one can show that:

\[
\begin{align*}
    r_{IS} &= \frac{2 r_N \left[ \sinh \frac{d}{2 l_N} + X_i \cosh \frac{d}{2 l_N} \right]}{\Delta}, \\
    X_i &= \frac{r_{F1} + r_{c1}}{r_N}, \\
    \Delta &= \exp \frac{d}{l_N} \prod_{i=1,2} (1 + X_i) - \exp - \frac{d}{l_N} \prod_{i=1,2} (1 - X_i)
\end{align*}
\]

\(l_N\) is the paramagnet spin relaxation length and the maximum spin voltage is:

\[
V_{s,max} = \frac{2 I_c}{\Delta} \left\{ \left[ P_{c2} r_{c2} + P_{F2} r_{F2} \right] \left[ \sinh \frac{d}{2 l_N} + X_1 \cosh \frac{d}{2 l_N} \right] \right. \\
- \left. \left[ P_{c1} r_{c1} + P_{F1} r_{F1} \right] \left[ \sinh \frac{d}{2 l_N} + X_2 \cosh \frac{d}{2 l_N} \right] \right\}.
\]

This implies that the spin voltage is larger for antiparallel ferromagnetic electrodes: \(P_{F1} = -P_{F2}\) and \(P_{c1} = -P_{c2}\) while it vanishes for parallel magnetizations.

In the limit of thin width for the N layer (d \(\ll l_N\)),

\[
r_{IS} \rightarrow \frac{(r_{F1} + r_{c1}) (r_{F2} + r_{c2})}{r_{F1} + r_{c1} + r_{F2} + r_{c2}}
\]

For identical ferromagnets the internal resistance scales as \(r_{IS} = (r_F + r_c)/2\).

In the same limit (d \(\ll l_N\)), the spin voltage for identical antiparallel ferromagnet becomes:

\[
V_{s,max} = I_c \left[ P_c r_c + P_F r_F \right].
\]

Quantitatively for a Co - Cu - Co trilayer with antiparallel ferromagnets, using \(P_c \sim 0.75\), \(P_F \sim 0.5\), \(r_F \sim 10 \Omega\), \(r_c \sim 1 \Omega\) with an area \(A = 1 \text{ mm}^2\) and \(I_c = 0.1 \text{ mA}\) yields, \(V_{s,max} \sim 1 \text{ mV}\) for a conversion gain \(C_{ss} = V_s/V_c\) which will be close to unity \(C_{ss} \sim 1\) which is much better than the 1% we found for the simple spin battery of Section [□][□]. The spin voltage is therefore potentially much larger than for the spin battery made out of a single ferromagnet. The physical reason behind that is simple: it stems from the much smaller spin relaxation volume within the paramagnet.

For the same parameters one gets \(r_{IS} \sim 10 \Omega\) while typically \(g_{ss}^{-1} \sim g_c^{-1} \sim 1 \Omega\); this F-N-F trilayer is better as a spin current source for a paramagnetic metallic load. If one looks for a better spin accumulation source, loads with smaller \(g_{ss}\) need to be picked.

C. Bulk spin pumping.

Let us illustrate the computation of the internal spin resistance in a non-collinear setting. The calculations are quite simple in a spin pump model proposed by Watts and co-workers[□]. We are not aware of any experimental implementation so we use this spin pump as a toy model and refer the reader to the original publication for a discussion of the physical relevance. The spin pumping relies on an rf rotating magnetic field which generates a dc spin accumulation within non-magnetic materials; this is a bulk mechanism in contrast with Brataas and co-workers spin battery operated by ferromagnetic resonance[□] and governed by interface effects between the ferromagnet and a paramagnetic load (i.e. the spin mixing conductance at the interface). The bulk spin pump model is in principle applicable to metals and semiconductors.
The basis of this model is a semi-classical equation for the non-equilibrium spin accumulation vector $\vec{f}$:

$$-\partial_t \vec{f} - g \mu_B \vec{B} = \frac{\vec{f}}{\tau} - D \nabla^2 \vec{f} + \frac{g \mu_B}{\hbar} \vec{B} \times \vec{f} \quad (39)$$

The novel feature in this equation is the magnetic field derivative which is described as a Zeeman shift of the chemical potential for each spin species. The space is divided into two metallic paramagnetic regions: for $x < 0$ region (I) where there is a rotating magnetic field

$$\vec{B} = (B_{xy} \cos \omega t, B_{xy} \sin \omega t, B_z) \quad (40)$$

and for $x > 0$ region (II) where $\vec{B} = 0$. A solution which is stationary in the rotating frame is readily found in region $I$:

$$\vec{f}(x, t) = \vec{f}_{uni} + \alpha (\vec{\omega} - \vec{\omega}_B) \tau \exp(x/l_{sf}) \quad (41)$$

where $\vec{\omega} = \omega \hat{z}$, $\vec{\omega}_B = \frac{g \mu_B}{\hbar} \vec{B}$, $l_{sf}$ is the spin diffusion length and $\vec{f}_{uni}$ is the uniform solution found when the rotating field is applied in the whole space. $\alpha$ is a constant determined by boundary conditions.

The spin current density in electrical units is:

$$\vec{j}_S(x) = \frac{1}{\rho_N^*} \partial_x \vec{f}(x) \quad (42)$$

Therefore the spin current $I_s = \vec{j}_S \cdot A$ (where $A$ is the cross-section) is:

$$I_s(0) = \frac{1}{r_N} \left( \vec{f}(0) - \vec{f}_{uni} \right) \quad (43)$$

where the spin resistance is $r_N = \rho_N^* l_{sf}/A$. This implies the following relation between the spin current $I_s$ and the spin voltage $V_s = -\vec{f}(0)$:

$$I_s = I_{s,0} - g_{IS} V_s$$

with a diagonal internal spin conductance tensor:

$$g_{IS}^{ij} = \delta_{ij} r_N \quad (44)$$

The internal spin resistance is therefore in the $1 \Omega$ range for paramagnetic metals in this kind of spin battery.

### D. Interface spin pumping

The ferromagnetic resonance operated spin battery requires a non-vanishing spin mixing conductance at a F-N interface. It is an interface driven spin pump. The defining equation for the internal spin resistance eq. (2) must therefore be understood to be evaluated on the N side of the F-N interface (in other words it is crucial to include the interface within the spin battery).

Two regimes for the ferromagnetic operated spin pump can be distinguished depending on whether the load is a good or a bad spin sink (i.e. whether the spin relaxation time in the load is small or large); the good spin sink regime corresponds to a regime where Gilbert damping is enhanced while in the bad spin sink case the spin pump is said to be in the spin battery regime. As will be shown shortly the bad spin sink regime corresponds to the spin pump being a good spin accumulation source while the good spin sink regime corresponds to a good spin current source.

It can be shown that in the limit of vanishing spin flip in the F layer, the longitudinal spin current vanished. The total spin current (in electrical units) is then reduced to:

$$\vec{I}^s_s = \vec{I}^s_{pump} - \frac{g_R}{4\pi} \vec{m} \times \vec{V}_N(0) \times \vec{m} - \frac{g_I}{4\pi} \vec{V}_N(0) \times \vec{m} \quad (45)$$

where $g_R$ and $g_I$ are the real and imaginary spin mixing conductances, $\vec{V}_N(0)$ is the spin voltage vector at the interface on the N side, $\vec{m}(t)$ is the magnetization unit vector in the ferromagnet and the pumping spin current is:

$$\vec{I}^s_{pump} = \frac{g_R}{4\pi} \vec{m} \times \vec{m} + \frac{g_I}{4\pi} \vec{m}. \quad (46)$$

The internal spin conductance tensor can be directly read off Eq. (45):

$$g_{IS}^{ij} = \frac{g_R}{4\pi} (\delta_{ij} - m_i m_j) + \frac{g_I}{4\pi} \epsilon_{ijk} m_k \quad (47)$$

For circular precession of the ferromagnet around a direction $z$ with angle $\theta$, $m_z = \cos \theta$ and if $g_I$ is negligible (which is often the case),

$$g_{IS}^{zz} = \frac{g_R}{4\pi} \sin^2 \theta \quad (48)$$

will be the dominant element of the tensor and one can use the internal spin resistance relation for collinear batteries. The internal spin resistance is then:

$$r_{IS} = \frac{4\pi}{g_R \sin^2 \theta} \quad (49)$$

while the maximum spin current is:

$$I_{s,max} = \frac{g_R}{4\pi} \frac{\hbar}{e} \omega \sin^2 \theta \quad (50)$$

and the maximum spin voltage is just:

$$V_{s,max} = \hbar \omega/e \quad (51)$$

(Our result differs from the maximum spin voltage quoted in Ref. as

$$V_{s,max} = \hbar \omega \sin^2 \theta/e \left( \sin^2 \theta + \eta \right) \quad (52)$$
where \( \eta \) is a positive load-dependent parameter. The discrepancy is reconciled by observing that in our internal spin resistance definition, the maximum voltage should be derived by finding the maximum against any load while \( \eta \) is a load dependent parameter. The max voltage corresponds obviously to \( \eta = 0 \) which recovers our value.)

The internal spin resistance is therefore in the range of \( g_R^{-1} \) and can be tuned by varying the precession angle \( \theta \); the spin mixing conductance per unit area is typically in the 1 \( f\Omega^{-1}/m^2 \) (see for instance Ref[12]) so for an area \( A = 10^{-15} m^2 \) this means \( r_{IS} \) in the 10 - 100 \( \Omega \) range for precession angles between 0.1 - 1 rad. This implies that with loads having spin resistance in the 1 \( \Omega \) range, such spin batteries will be good spin current source rather than good spin accumulation sources. The load is a sufficiently 'bad spin sink' whenever its spin resistance is larger than \( r_{IS} \) (this is the 'spin battery' regime discussed in Ref[13]). Observe also that the smaller the area \( A \), the larger \( r_{IS} \) will be. This implies that one will get better spin current sources, closer to ideality.

**E. Spin Hall effect**

The spin Hall effect consists in the generation of a transverse spin current from a charge current[4][5]. We model the spin battery as a bar in direction \( y \) of width \( d \); the load is perpendicular to the bar (parallel to direction \( x \)) in order to collect the spin current at \( x = 0 \) (see Fig. 7). A spin accumulation is generated in the load which spills back into the bar and decays with a relaxation spin length \( l_N \). As a result the spin current in the bar will have two components (in direction \( x \)):

\[
j_s = j_s^{SHE} + j_s^{SD}
\]

where \( j_s^{SD} \) obeys the spin diffusion equation while \( j_s^{SHE} \) is the spin Hall spin current which does not depend on \( x \).

The boundary condition for the spin current is \( j_s (x = -d) = 0 \). Using the diffusion equation

\[
j_s^{SD} = + \frac{\sigma_N}{e} \partial_x \Delta \mu
\]

which relates spin accumulation and spin current one finds after straightforward calculations:

\[
r_{IS} = r_N \coth(d/l_N), \quad V_{s,max} = \frac{\cosh(d/l_N) - 1}{\sinh(d/l_N)} r_N A j_s^{SHE}
\]

where \( r_N \) is the bar spin resistance \( (r_N = \rho_N l_N / A) \) and \( A \) is the cross-section. In the limit of infinite width \( (d \to \infty) \), the internal spin resistance is equal to \( r_N \) the spin resistance of the bar; in the opposite limit \( (d \ll l_N) \), the internal spin resistance is infinite and one has an ideal spin current source. An interesting consequence is that the internal spin resistance can be tuned from the 1 \( \Omega \) range to much larger values by varying the width \( d \).

In the limit of infinite width \( (d \to \infty) \) one finds

\[
V_{s,max} = r_N A j_s^{SHE};
\]

in the opposite limit,

\[
V_{s,max} = 0
\]

So one needs a wide bar, that is \( d \gg l_N \) where \( r_{IS} = r_N \). The spin voltage can be detected through the ISHE. The magnitude is up to several hundreds of microVolts.

**III. CONCLUSION.**

We have introduced the concept of internal spin resistance which allows to quantify whether a spin battery will be a good spin current source (internal spin resistance much larger than the load spin resistance) or a good spin accumulation source (opposite limit). This gives a criterion for the design of spin batteries according to the main usage targetted: angular momentum transfer or spin accumulation. The internal spin resistance has been shown to be a positive number to ensure dissipation (or more generally the internal spin tensor is positive semi-definite). We have also considered several spin batteries relying on spin injection, spin pumping or spin Hall effect, in collinear as well as non-collinear settings, deriving explicit expressions for the internal spin resistance.
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