Establishment of damage estimation rule for brittle fracture after cyclic plastic prestrain in steel

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Abstract. Material damage caused by complicated prestrain like cyclic prestrain and various direction prestrain. It is so important to understand the mechanism of material damage for high-precision lifetime evaluation of steel structure. Material damage depends on not only equivalent plastic strain but the order of prestrain loaded. Also, material damage can be expressed by effective damage strain from back stress uploading and total dislocation density calculated from conventional mechanism based on strain gradient plasticity (CMSGP). These ideas can be extended to multiaxial problems and material damage can be estimated by these parameters as well as uniaxial problems.

1 Introduction

Brittle fracture of steel is suggested to occur because microcrack appearing in brittle layer at the grain boundary propagates into the matrix as being driven by a lot of piled-up dislocation [1]. Generally, dislocation is piled up when plastic deformation proceeds. Thus, loading plastic deformation, equivalent to dislocation’s increase, is directly connected to the increase of danger of brittle fracture. Because of these mechanisms, loading prestrain to material means getting close to material limit conditions and leads to deterioration of fracture toughness.

Though deterioration of fracture toughness derived from single prestrain has been investigated very well, there are not enough data about deterioration from complicated prestrain, for example cyclic prestrain and various direction prestrain, and mechanism of deterioration have not been discovered. Current main material damage rule is Coffin-Manson rule which defines material damage is equal to equivalent plastic strain [2], but this rule makes evaluation of the too safety side. For high-precision lifetime evaluation of steel structure which has the various load history, it is important to generalize the effect of various direction cyclic prestrain on the deterioration of the fracture toughness.
In this paper, in the beginning, as an easy case material toughness change when tensile and compression prestrain are loaded in the same direction of cleavages is investigated and material damage rule is discussed using Finite Element Method (FEM). Next, based on these results, FEM analysis is extended to multiaxial stress filed and material damage in the case that prestrain is given in two directions is investigated.

2 Experiment

First, uniaxial cyclic prestrain test was done. Test specimen shown in Fig. 1 (a) was used and prestrain patterns are shown in Table 1. General use carbon steel material was used and micro structure of this steel was general ferrite-pearlite. Loaded prestrain was total strain which sums up elastic strain and plastic strain. Also, some prestrain conditions, P4, P5 and P6, had almost the same equivalent plastic strain, while strain order was different. 12 specimens were tested for each prestrain condition.

Next, notch bending test specimen shown in Fig. 1 (b) was made from prestrain specimen, and general three-point bending test was employed at low temperature as the fracture evaluation [3]. Also, not only specimens with prestrain but specimens without prestrain (As) were tested. In this experiment, though fracture test was not CTOD test exactly because test specimens didn’t have fatigue crack, quasi-CTOD was calculated using WES1108. Bending test was done at 4 temperatures at each prestrain condition and two specimens were tested at each temperature, and then transition curve was calculated. Ductile brittle transition temperature was defined as the temperature at which δ quasi equals to 0.1, and the difference between transition temperature of As and that of each prestrain condition was defined as ΔT. Change of ΔT according to each prestrain condition is shown in Fig. 2. As seems in Fig. 2, fracture toughness is clearly different according to prestrain conditions. This means that material damage from prestrain is dependent on the load history. When some prestrain conditions, P4, P5 and P6, are focused, fracture toughness of P4 and P6 is much worse than that of P5 while equivalent plastic strain of these condition is not different very much. Thus the dependency of the prestrain order on material damage is indicated.

![prestrain test specimen](image1)

![quasi-CTOD test](image2)

**Fig. 1.** Configurations of specimens used

**Table 1.** Prestrain conditions

|        |                |
|--------|----------------|
| As     | No prestrain   |
| P1     | compression 3% |
| P2     | compression 3% ⇒tensile 0.5% |
| P3     | compression 3% ⇒tensile 1% |
| P4     | gradual increase: +1→-1→-2→+2→...+3→-3→+4 (%) |
| P5     | gradual decrease: +4→-3→-3→+2→-2→...-1→+1 (%) |
| P6     | random: +2→-1→+3→-2→...+1→-3→+4 (%) |
3 FEM analysis

In this section, FEM analysis about cyclic prestrain problem is discussed. Conventional macroscopic analysis and analysis based on conventional mechanism based strain gradient plasticity (CMSGP) is shown in this section.

3.1 Macroscopic material damage analysis

As macroscopic analysis, analysis based on back stress is done using Abaqus. Back stress is parameter which expresses amount of kinematic hardening and have some components as well as normal stress. Back stress is defined as

$$\alpha_{ij} = \sigma_{ij} - \sigma_0,$$

where $\sigma_0$ is yield stress when plastic strain equals 0. Combined hardening rule is defined as

$$\sigma^0 = \sigma_0 + Q_\infty (1 - e^{-b\dot{\varepsilon}^{pl}}),$$

$$\dot{\alpha}_{ij} = C \frac{1}{\sigma^0} (\sigma_{ij} - \alpha_{ij}) \dot{\varepsilon}^{pl} - \gamma \alpha_{ij} \dot{\varepsilon}^{pl},$$

where $\sigma^0$ is equivalent stress deciding the size of yield surface, and $Q_\infty$ and $b$ are intrinsic material parameters, and $C$ is initial kinematic hardening coefficient and $\gamma$ is the parameter which expresses damping factor of kinematic hardening coefficient. Isotropic hardening is expressed in (2) and kinematic hardening is expressed in (3). In this analysis, each parameter is defined as shown in Table 2.

| Table 2. Parameter of Combined hardening |
|-----------------------------------------|
| $\sigma_0$ (MPa) | 332.778 |
| $Q_\infty$ (MPa) | 100 |
| $b$ | 5 |
| $C$ (MPa) | 10000 |
| $\gamma$ | 81 |
In this analysis, based on the method suggested by Ohata [4], it is assumed that the plastic strain necessary for updating untracked back stress field contributes to material damage. This assumption is based on the idea that piled-up dislocation is opened while bauschinger effect occurs and material damage doesn’t occur. When back stress is updating, the amount of piled-up dislocation is thought to increase. Thus, the total equivalent strain in the field where back stress updates is defined as effective damage strain ($\bar{\varepsilon}_p^d$). Also, as scalar value of back stress, equivalent back stress ($\alpha$) is defined as

$$\alpha = \sqrt{\left(\frac{a_{xx} - a_{yy}}{2}\right)^2 + \left(\frac{a_{yy} - a_{zz}}{2}\right)^2 + \left(\frac{a_{zz} - a_{xx}}{2}\right)^2 + 6\left(\frac{a_{xy}^2 + a_{yz}^2 + a_{zx}^2}{2}\right),}$$  \hspace{1cm} (4)

and $\bar{\varepsilon}_p^d$ is defined as

$$\bar{\varepsilon}_p^d = \int \{\varepsilon_p | \alpha > \alpha_{\text{max}}\} d\varepsilon_p,$$  \hspace{1cm} (5)

where $\varepsilon_p$ is equivalent plastic strain and $\alpha_{\text{max}}$ is maximum value of equivalent back stress at each point. From these conditions, $\bar{\varepsilon}_p^d$ is calculated at each prestrain condition shown in Table 1. Comparison between $\bar{\varepsilon}_p^d$ and $\Delta T$ is shown in Fig. 3. As seen in Fig. 3, good correlation between $\bar{\varepsilon}_p^d$ and $\Delta T$ is observed. In the next subsection, more detailed analysis by CMSGP will be done in order to make high-precision evaluation.

![Fig. 3. Comparison between $\bar{\varepsilon}_p^d$ and $\Delta T$](image)

### 3.2 Analysis of conventional mechanism based strain gradient plasticity

In this subsection, analysis based on CMSGP is done and detailed evaluation of material damage of cyclic prestrain is made. CMSGP is the theory suggested by Bassani [5] and deals with plastic strain gradient. Calculation method of stress is not very different from that of conventional macroscopic problem, but flow stress ($\sigma_{\text{flow}}$) is defined as

$$\sigma_{\text{flow}} = \sigma_{\text{ref}} \sqrt{f^2(\varepsilon^p) + l\eta^p},$$  \hspace{1cm} (6)

where $\sigma_{\text{ref}}$ is reference stress and $f(\varepsilon^p)$ expresses strain hardening. Also, $\eta^p$ is plastic strain gradient and $l$ is intrinsic material scale length. Using CMSGP, dislocation density can be calculated based on plastic strain gradient and equivalent plastic strain. Martínez [6] suggests that dislocation density is defined as
\[ \rho_{\text{GND}} = \frac{\eta b p}{b} \quad (7) \]

\[ \rho_{\text{SSD}} = \left[ \sigma_{\text{ref}}(e^p)/(M\alpha b) \right]^2 \quad (8) \]

where \( b \) is burgers vector and \( M , \alpha \) is intrinsic material parameter and \( \mu \) is modulus of transverse elasticity. Also, \( \rho_{\text{GND}} \) is the density of geometrically necessary dislocation (GND) and \( \rho_{\text{SSD}} \) is the density of statistically stored dislocation (SSD). As brittle fracture occurs because of piled-up dislocation, total dislocation density, sum of \( \rho_{\text{GND}} \) and \( \rho_{\text{SSD}} \), can be defined as parameter which express the material damage in this analysis.

Quarter model of regular octagon is used. Outline of this model and boundary condition are shown in Fig. 4. Regular octagon is regarded as one grain and how the dislocation gathers when prestrain is loaded is calculated. The maximum amount of dislocation density at the last step of prestrain is defined as a parameter of material damage.

Comparison between dislocation density and \( \Delta T \) is shown in Fig. 5. As seen in Fig. 5, very strong correlation between dislocation density and \( \Delta T \) is indicated. In addition, comparison between dislocation density and \( \varepsilon_p^d \) is shown in Fig. 6. As seen in Fig. 6, strong correlation between dislocation density and \( \varepsilon_p^d \) is observed and these values are thought to be reasonable as parameters which express material damage.

Fig. 4. Outline of model and boundary conditions

Fig. 5. Comparison between dislocation density and \( \Delta T \)
3.3 Summary of material damage from cyclic prestrain

From subsection 3.1 and 3.2, it is suggested that material damage of uniaxial cyclic prestrain problem can be expressed by effective damage strain from back stress updating and dislocation density calculated from CMSGP. This idea is different from conventional idea of material damage using Coffin-Manson rule, and these parameter can make very high-precision evaluation.

4 Extension of material damage rule to multiaxial stress field

As mentioned above, actual steel structures are loaded various direction prestrain. Thus, it is desirable that material damage rule for various direction prestrain is generalized. In this section, prestrain from two directions is simulated using FEM and material damage rule is discussed utilizing effective damage strain and dislocation density expressed in section 3.

As is shown in Fig. 7, the case that y-axis direction prestrain is loaded after x-axis prestrain is loaded is discussed. As with section 3, $\bar{\varepsilon}_p^d$ and dislocation density are calculated, and the relationship of these parameter is discussed. In this analysis, prestrain conditions shown in Table 3 is loaded.
Table 3. Prestrain conditions of two axial problem

|   |   |
|---|---|
| V1 | 1(x-axis) → 1(y-axis) |
| V2 | 1(x-axis) → 2(y-axis) |
| V3 | 2(x-axis) → 1(y-axis) |
| V4 | 2(x-axis) → 3(y-axis) |
| V5 | 2(x-axis) → 4(y-axis) |
| V6 | 3(x-axis) → 2(y-axis) |
| V7 | 1(x-axis) → 1(y-axis) → 1(x-axis) |
| V8 | 1(x-axis) → 2(y-axis) → 3(x-axis) |
| V9 | 3(x-axis) → 2(y-axis) → 1(x-axis) |
| V10 | 2(x-axis) → 3(y-axis) → 1(x-axis) |

Comparison between $\bar{\varepsilon}_p^d$ and dislocation density is shown in Fig. 8. As seen in Fig. 8, these parameters can be organized in the same level as uniaxial cyclic prestrain problem, and positive correlation between them is observed. The fact that these parameters strongly depend on $\Delta T$ at the uniaxial problem indicates that material damage of the multiaxial problem can be estimated by these parameter as well as uniaxial problem.

5 Conclusion

In this paper, material damage from complicated prestrain like cyclic prestrain and various direction prestrain is investigated. As a result, material damage strongly depends on the load history and the amount of damage is different according to the order of prestrain even if the amount of equivalent plastic strain is the same. This result is different from conventional idea, Coffin-Manson rule, which defines material damage as equivalent plastic strain.

Also, material damage can be expressed by effective damage strain from back stress updating and dislocation density calculated from CMSGP. Especially, the tendency of dislocation density matches with $\Delta T$ in high-precision. Thus material damage can be estimated accurately.

In addition, material damage about not only uniaxial problem but also multiaxial problem can be estimated by effective damage strain and dislocation density. This is very interesting because actual steel structures are loaded in various prestrain directions.
In the future, validity of these parameters in the multiaxial problem should be investigated by experiment of fracture toughness deterioration from the various direction prestrain.

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