DYNAMIC APERTURE CALCULATION FOR THE DAΦNE-II PROJECT

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INTRODUCTION

The DAΦNE-II project is considered now as a possible candidate for upgrading the DAΦNE electron-positron collider and improving its luminosity. The basic idea is to use a strong RF focusing mechanism [1] to compress a bunch at the interaction point and hence to get a chance for the vertical beta reduction.

An intrinsic feature of the strong RF focusing is a large value of synchrotron tune and one can assume intuitively that just like the betatron strong focusing results in the transverse dynamic aperture limitation, the synchrotron strong focusing can provide the same but for the energy dependent dynamic aperture.

In order to check that, we were performing a 3-D simulation of the DAΦNE-II dynamic aperture under various assumptions (weak and strong RF focusing). The results of this tracking are presented in the paper.

DAΦNE-II LATTICE

The DAΦNE-II lattice proposed by C.Biscari [2] has rather large negative momentum compaction factor necessary for effective bunch length squeezing and as a promising counteraction against the microwave bunch lengthening [3]. In order to achieve it, the arc cell contains negative and positive curvature dipole magnets, a number of quadrupole magnets to focus the beam in the transverse plane and sextupole magnets to compensate for rather large natural chromaticity.

The betatron and dispersion functions of the DAΦNE-II are shown in Fig.1.1 and Fig.1.2 while the main parameters are listed in Table 1.1. Dispersion-free straight sections are discussed for detector and RF cavities accommodation.

Table 1.1: DAΦNE-2 main parameters.

| Parameter                  | Value               |
|----------------------------|---------------------|
| Beam energy, \( E \)       | 511 MeV             |
| Circumference, \( L \)     | 103.45 m            |
| Revolution frequency, \( f_0 \) | 2.898 MHz           |
| Revolution period, \( T \) | 0.345 \( \mu \)s    |
| Betatron tune, \( v_l/v_t \) | 8.792/7.893        |
| Natural chromaticity, \( \xi_x/\xi_z \) | -18.4/-37.2 |
| Momentum compaction factor, \( \alpha \) | -0.214 |
| Beta-functions at IP, \( \beta_x/\beta_z \) | 50.0/0.25 cm |
| Energy loss per turn, \( U_0 \) | 35.45 KeV           |
| Partition numbers, \( J_x/J_y/J_z \) | 1.67/1.0/1.33 |
| Damping times, \( \tau_x/\tau_y/\tau_z \) | 5.9/9.9/7.5 ms |
| Horizontal emittance, \( \varepsilon_x \) | 6.38\( \times \)10\(^{-8} \) m-rad |
| Energy spread, \( \sigma_{E/\delta} \) | 5.53\( \times \)10\(^{-4} \) |

To study the influence of synchro-betatron resonances in the case of the strong longitudinal focusing, the simulation of the non-linear beam behavior was performed with the help of the ACCELERATICUM computer code [4].

SIMULATION RESULTS

The ACCELERATICUM code is a general-purpose code to study different aspects of particle motion in a circular accelerator.

It provides a symplectic 6D tracking for the transversely and longitudinally coupled magnetic lattice according to the formalism proposed by G.Ripken in [5]. The formalism uses the canonical variables, which are commonly used in the six-dimensional linear theory

\[
\begin{pmatrix}
-x \\
p_x \\
-x \\
p_x \\
\sigma(s) = s - c \cdot t(s), \delta = \frac{E - E_a}{E_q}
\end{pmatrix},
\]

and which are also canonical in the non-linear formalism if the transformation through the nonlinear elements is performed with the help of Hamiltonian generating functions approach.

Besides the nonlinear dynamics, the 6D tracking allows us to investigate linear parameters of the machine as a
function of the beam momentum deviation (betatron functions, dispersion, etc.). For instance, Fig.2.1 shows the nonlinear part of the chromaticity when the linear part is corrected to zero by the sextupoles and Fig.2.2 shows the momentum compaction factor as a function of energy deviation.

![Fig.2.1 Residual tune chromaticity after sextupole correction.](image1)

![Fig.2.2 Momentum compaction factor vs. energy deviation.](image2)

Two families of sextupole magnets (see their integrated strength in Table 2.1) in the arc cells were set to adjust the natural chromaticity to zero.

### Table 2.1 Sextupole magnets’ integrated strength.

| Name   | (mI), m² |
|--------|----------|
| SD     | -6.06    |
| SF     | 2.69     |

To consider dynamic aperture limitation due to the strong RF focusing we have used several values of RF voltage during simulation (see Table 2.2). The case of $U_{RF}=300$ kV corresponds to the weak focusing while 3 MV and 5.8 MV provide strong synchrotron focusing of the bunch at the IP.

### DYNAMIC APERTURE SIMULATION

All plots of the dynamic aperture are presented for the interaction point. We use the 4D (without synchrotron motion) dynamic aperture shown in Fig.3.1 as a reference. The picture is typical for the coupling resonance limitation of a stable area: large 1D aperture (along the x-axis) is reduced if the vertical motion with arbitrary small amplitude is switched on.

![Fig.3.1 4D dynamic aperture of DAΦNE-II (1000 turns).](image3)

Fig.3.2 shows the DAΦNE-II off-energy dynamic aperture for the constant energy deviation (no synchrotron oscillation is turned on) for the weak (left plot) and strong (right plot) RF focusing. In this case, the limitation of the particle stable area can be explained by different on- and off-energy particle trajectories in the magnetic field.

A rather different situation can be seen in Fig.3.3 and Fig.3.4, where the synchrotron oscillation is taken into account. While for the weak RF focusing the dynamic aperture does not differ much from that with a constant energy deviation (Fig.3.2, left), for the strong RF focusing the dynamic aperture became very small even for $\Delta p/p=0$.

The following schematic mechanism can be proposed. For the general tune-amplitude dependence expressions

$$\nu_x + \nu_{x0} + C_{xx} A_x^2 = n,$$
$$\nu_z + \nu_{z0} + C_{zz} A_z^2 = n,$$

and in the presence of synchrotron oscillation, the resonant condition has the form

$$\nu_x + \nu_{x0} + C_{xx} A_x^2 + C_{zz} A_z^2 + k \nu_s = n,$$  \hspace{1cm} (3.1)

where $1 < k < N$ is the number of oscillation modes. Now suppose that at every point of the dynamic aperture curve we have some particular betatron resonance that limits the stable area in this point. Then the synchrotron motion generates a set of satellite resonances, which are represented by lines at the amplitude plane $A_x(A_z)$. The resonance line equation can be defined from (3.2). For the sake of simplicity, consider only the horizontal resonance and main (strongest $k=1$) satellite $m_x (\nu_{x0} + C_{xx} A_x^2) + \nu_s = n$, the following expression for the horizontal position of the satellite resonance line can be deduced:

![Fig.3.3 Synchrotron oscillation and dynamic aperture.](image4)

![Fig.3.4 Synchrotron oscillation and dynamic aperture.](image5)
Fig. 3.2 Off-energy DA with constant energy deviation (no synchrotron oscillation). $U_{\text{RF}} = 300$ kV (left), $= 3$ MV (right).

Fig. 3.3 Off-energy DA with synchrotron oscillation. $U_{\text{RF}} = 300$ kV (left plot), $= 3$ MV (right plot), $= 5.8$ MV (below).

Fig. 3.4 The same as in Fig. 3.3. High resolution survival plot.
\[ A_x = A_{x0} \sqrt{1 - \frac{\nu_s}{m_s \delta}}, \]  

(3.3)

where \( \delta = \nu_{x0} - n/m \) is a distance from the resonance and \( A_{x0} = \delta / C_{xx} \) is the position of the original (\( \nu_s = 0 \)) betatron resonance on the amplitude plane. The strong satellite resonance inside the initial dynamic aperture provides an additional reduction of the stable area as is clearly seen in the right of Fig.3.3 and Fig.3.4.

In the case of weak RF focusing, the situation is not so serious for two possible reasons:

1. The distance (in the amplitude space) between the main and satellite resonances is small and only slightly distorts later the edge of the dynamic aperture. Some evidence of this fact one can see in the left-hand side of Fig.3.3 (the blue and green curve).

2. The amplitude of the satellite resonance depends on the synchrotron tune and drops down with the satellite number \( k \) (most probably like the Bessel function).

CONCLUSIONS

The 6D tracking with synchrotron oscillation shows that in the case of strong RF focusing the dynamic aperture of the DAΦNE-II is reduced as compared to the week focusing case or constant energy deviation. A possible mechanism of this reduction is that the synchrotron motion produces satellites of the strong sextupole resonances, which limit the dynamic aperture in the 4D case. The satellite resonances locate inside the initially stable area and additionally reduce it. The following plan for the further study and recover of this phenomenon can be proposed:

1. More detailed investigation of the satellite behaviour for the weak, strong and intermediate RF focusing, including the satellites amplitude values.

2. In the case of the strong RF focusing dependence of dynamic aperture on the tune point is to be explored (in other words, more accurate choosing of the betatron and synchrotron tunes). It seems that all the three tunes are important now.

3. As the satellites resonances location depends on the detuning coefficients, it is necessary to check if it possible to control it by octupole magnets.

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