Analyzing Wuhan's Strategies for Controlling Covid-19 Epidemic

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Abstract. Because of the high infectiousness of COVID-19, the paper divided infected people into two groups, the confirmed cases to be treated and isolated in hospitals and unconfirmed carriers in free environment. First, a COVID-19 transmission model was built based on the classification of infected population, and a sensitivity analysis algorithm was constructed to optimize unknown parameters in the model, such as the probabilities of transmission and the diagnosis rate. Second, the transmission model and an optimization procedure were used to simulate the COVID-19 epidemic in Wuhan and Diamond Princess Cruise, and the simulation results were compared with actual data reported by governments. Finally, Wuhan’s strategies for controlling COVID-19 epidemic at different stages were analyzed through the COVID-19 transmission model. The results showed: only isolating and treating the confirmed patients suffering severe symptoms could not effectively inhibit the rapid spread of COVID-19; isolating all the confirmed patients could reduce the infected population over 30 times; besides isolating all the confirmed patients, the city-wide lockdowns and fast test methods could dramatically contain the spread of the epidemic, including decreasing the cumulative infected population and shortening the period of epidemic; compared with Wuhan's control strategies, the protection and isolation measures of Diamond Princess Cruise could not effectively inhibit the spread of COVID-19.

1. Introduction
Throughout history, human beings have experienced various infectious epidemics, such as smallpox, the Black Death, cholera and Ebola. These epidemics damaged public health and brought heavy blows to the global economy. SARS in 2003 was a massive epidemic in China, causing about 7000 infected people and economic losses of more than 200 billion RMB [1]. By now, human beings have been in the fight against the COVID-2019 epidemic for over 1 year. How to adopt effective strategies for controlling COVID-2019 is a hot issue.

For many years, experts and scholars in the field of public health and disease prevention have been researching and evaluating epidemic control strategies. Wang Ming et al. summarized SARS control strategies in Guangzhou, and pointed out that the prevention of clustered SARS cases and iatrogenic infection was essential [2]. Wang Jie-xiu et al. emphasized that close contacts with confirmed SARS cases should be traced and isolated to cut off the SARS transmission chain [3]. Xie Zhong-lun et al. compared different strategies for controlling SARS, and concluded that expanding the treatment ranges of suspected cases could contain the SARS transmission more effectively than restricting population flow [4, 5]. Zhang Juan et al. proposed a SARS transmission pattern, and simulated the influence of control strategies implemented in China on SARS spread [6]. Wu Ling-xiao introduced the necessary measures to reduce the incidence of measles, such as active surveillance and passive testing [7]. Li Rui-ping et al. suggested that the routine and supplementary immunization should be
promoted to control rubella [8]. Zhang Cai-jun et al. analyzed the control strategies of hand-foot-and-mouth disease, and advocated health education for children under 5 years old [9]. Kimberlyn Roosa et al. predicted the spread of COVID-19 in Guangdong and Zhejiang, China, by applying phenomenological models and actual data reported by National Health Commission of China, and pointed out that the predictions could reflect the effectiveness of control strategies [10]. Fang Yaqing et al. employed the SEIR model to simulate the COVID-19 epidemics in Hubei, China, and highlighted the importance of developing medicine for COVID-19 and optimizing therapeutic plans [11].

This paper built a COVID-19 transmission model to assess Wuhan’s control strategies at different stages, such as building mobile cabin hospitals, isolating all the confirmed patients, implementing city-wide lockdowns and adopting fast COVID-19 diagnostic methods. First, due to the high infectiousness of COVID-19, the infected people were divided into the confirmed patients in hospitals and unconfirmed carriers in free environment. Second, based on the classification of infected population, a COVID-19 transmission model was built by modifying the traditional SIR model. Then, the transmission model was verified through using it to simulate the COVID-19 epidemic in Wuhan and Diamond Princess Cruise and comparing the simulation results with the actual data reported by governments. Finally, Wuhan’s strategies for controlling COVID-19 were assessed.

2. Building and verifying the COVID-19 transmission model

Compared with the past epidemics, COVID-19 is a highly contagious disease, and a COVID-19 carrier can even affect others prior to the onset of the illness. According to official statistics, the average incubation period of COVID-19 is 4.95 days. Based on the probabilities of transmission 0.3365 (1/day), an evaluated value in Table 4, in the initial epidemic, it could be estimated that an infected person could affect 5 people prior to the onset of the illness and the confirmed population would not exceed 18.91% of the infected people with no symptoms.

Epidemic transmission models mainly include SI, SIR, SIRS and SEIR on the basis of different population classification methods. In SIR model, the whole population includes susceptible people, infected patients and recovered individuals with lifelong immunity [12, 13]. In this section, a COVID-19 transmission model is built by modifying the traditional SIR model.

Due to the high infectiousness of COVID-19, the whole population is divided into three groups, the susceptible people $N_S$, the confirmed patients $N_{I1}$, who are treated in isolated environment, and the unconfirmed carriers $N_{I2}$, who are in free environment and transmit the virus to the susceptible people. $N_{I1}$ and $N_{I2}$ are functions of time $t$ and satisfy

$$N_{I1}(t) = N_{I1}(0) + N_{I2}(t)$$  \hspace{1cm} (1)

where, $N_{I1}(t)$ is the whole infected population. $S(t)$ is used to stand for the ratio of the susceptible people $N_S$ to the whole population $N_0$, satisfying

$$S(t) = 1 - \frac{N_{I1}(t)}{N_0}$$  \hspace{1cm} (2)

where, $N_0 = N_S(t) + N_I(t)$. If $\beta(t)$ is used to denote the probabilities of transmission between a carrier of COVID-19 and a susceptible individual, $\gamma(t)$ the diagnosis rate describing a percentage of newly confirmed cases per unit time relative to the unconfirmed infected population, the COVID-19 transmission model can be expressed as

$$\begin{align}
\frac{dN_{I1}(t)}{dt} &= \gamma(t) \cdot N_{I2}(t) \\
\frac{dN_{I2}(t)}{dt} &= \beta(t) \cdot S(t) \cdot [N_{I2}(t) + aN_{I1}(t)] - \gamma(t) \cdot N_{I1}(t) \\
\frac{dN_S(t)}{dt} &= -\beta(t) \cdot S(t) \cdot [N_{I2}(t) + aN_{I1}(t)]
\end{align}$$  \hspace{1cm} (3)

Here, the parameter $a$ is set to 0 or 1 to evaluate the influence of control strategies on the COVID-19 epidemic. If the confirmed cases are isolated, $a = 0$, otherwise $a = 1$.

The COVID-19 transmission model (3) is the first-order ordinary differential equations, and can be solved by means of Runge-Kutta method. However, some undetermined parameters in equation (3),
such as the probabilities of transmission $\beta(t)$, the diagnosis rate $\gamma(t)$, initial values of the confirmed and the unconfirmed infected population, $N_{I1}(t_0)$ and $N_{I2}(t_0)$, should be given first. In order to solve the undetermined parameters, an objective function is constructed

$$F[\beta(t), \gamma(t), N_{I1}(t_0), N_{I2}(t_0)] = \frac{1}{2} \int_{t_0}^{t_f} \rho(t) \cdot [N_{I1}(t) - N_{I1}^*(t)]^2 \, dt$$

(4)

where, $N_{I1}^*(t)$ is the actually cumulative confirmed population reported by governments; $t_0$ and $t_f$ are the initial and end times among the actual data; $\rho(t)$ is the weight function and can be set as required. In the following numerical examples, set $\rho(t) = 1$. The probabilities of transmission $\beta(t)$, the diagnosis rate $\gamma(t)$, initial values $N_{I1}(t_0)$ and $N_{I2}(t_0)$ can be obtained by minimizing the objective function (4).

Some efficient optimization algorithms like BFGS can be adopted to minimize the objective function (4). During the solving procedure, the derivative or sensitivity of the objective function with respect to its undetermined parameters should be known. To obtain the derivative or sensitivity information, an iterative algorithm is constructed as followed.

Step ①: in the $i^{th}$ iteration, solve equation (3) in the internal $[t_0, t_f]$ by means of the current values of $\beta(t)$, $\gamma(t)$, $N_{I1}(t_0)$ and $N_{I2}(t_0)$ to obtain the current values $N_{I1}(t)$ and $N_{I2}(t)$, and calculate the value of the objective function (4) through the numerical integration;

Step ②: based on the results in Step ①, solve the conjugate equation (5) in the internal $[t_0, t_f]$ by means of the boundary condition $M_1(t_f) = 0$ and $M_2(t_f) = 0$, and calculate the current values $M_1(t)$ and $M_2(t)$;

$$\begin{align*}
\frac{dM_1(t)}{dt} &= \left(\frac{N_{I1}(t) + \alpha N_{I1}(t)}{N_0} - \alpha \cdot S(t)\right) \beta(t) \cdot M_2(t) + \rho(t) \cdot \left(N_{I1}(t) - N_{I1}^*(t)\right) \\
\frac{dM_2(t)}{dt} &= \left(\frac{N_{I2}(t) + \alpha N_{I1}(t)}{N_0} - S(t)\right) \beta(t) \cdot M_2(t) + \gamma(t) \cdot \left(M_2(t) - M_1(t)\right)
\end{align*}$$

(5)

Step ③: calculate the sensitivity of the objective function (4) with respect to $\beta(t)$, $\gamma(t)$, $N_{I1}(t_0)$ and $N_{I2}(t_0)$

$$\begin{align*}
\frac{\delta F}{\delta N_{I1}(t_0)} &= -M_1(t_0) \\
\frac{\delta F}{\delta N_{I2}(t_0)} &= -M_2(t_0) \\
\frac{\delta F}{\delta \gamma(t)} &= N_{I2}(t) \cdot \left(M_2(t) - M_1(t)\right) \\
\frac{\delta F}{\delta \beta(t)} &= -S(t) \cdot M_2(t) \cdot \left(N_{I2}(t) + \alpha \cdot N_{I1}(t)\right)
\end{align*}$$

(6)

Assuming $\beta(t)$ and $\gamma(t)$ are piecewise functions in the internal $[t_0, t_f]$, $\beta_k$ and $\gamma_k$ are the values of the two functions in the $k^{th}$ internal $[t_k, t_{k+1}]$, the last two expressions in equation (6) can be rewritten as

$$\begin{align*}
\frac{\delta F}{\delta \beta_k} &= \int_{t_k}^{t_{k+1}} \left(M_2(t) - M_1(t)\right) \cdot N_{I1}(t) \, dt \\
\frac{\delta F}{\delta \gamma_k} &= -\int_{t_k}^{t_{k+1}} \left(N_{I2}(t) + \alpha \cdot N_{I1}(t)\right) \cdot S(t) \cdot M_2(t) \, dt
\end{align*}$$

(7)

Please refer to Appendix for the derivation procedure of the formulas.

2.1 Simulating the COVID-19 epidemic on Diamond Princess Cruise

The COVID-19 transmission model (3) was utilized to simulate the COVID-19 epidemic on Diamond Princess Cruise. The total number of crew members and tourists on Diamond Princess Cruise was 3,711, and the cumulative confirmed population reported by Japanese government were shown in table 1. Based on the COVID-19 transmission model (3) and the official data in table 1, the undermined parameters in equation (3) were obtained, as shown in table 2. The protection measure of Diamond Princess Cruise was that the confirmed cases disembarked from the ship and were sent to the hospitals while the others stayed on the ship and were isolated. In table 2, $\alpha = 0$, indicating the confirmed cases were isolated; $N_{I1}(3)$ and $N_{I2}(3)$ were predicted initial values, where $N_{I1}(3)$ was the confirmed cases.
on Feb. 03, 2020 and \( N_{I2}(3) \) was the unconfirmed infected population on Feb. 03, 2020. Based on the model parameters in Table 2 and equation (3), the COVID-19 epidemic on Diamond Princess Cruise was simulated, as shown in Figure 1. The horizontal and vertical axes were date and cumulative confirmed population, respectively. The blue curve stood for official data, and the red one represented simulation results of the transmission model (3).

Table 1. Cumulative confirmed population reported by Japanese government.

| Date       | Number |
|------------|--------|
| 02-03      | 10     |
| 02-04      | 20     |
| 02-05      | 61     |
| 02-06      | 64     |
| 02-07      | 61     |
| 02-08      | 64     |

| Date       | Number |
|------------|--------|
| 02-09      | 70     |
| 02-10      | 130    |
| 02-11      | 135    |
| 02-12      | 174    |
| 02-13      | 218    |
| 02-14      | 218    |

| Date       | Number |
|------------|--------|
| 02-15      | 285    |
| 02-16      | 355    |
| 02-17      | 454    |
| 02-18      | 542    |
| 02-19      | 634    |
| 02-20      | 634    |

Table 2. Parameters in COVID-19 transmission model.

| Parameter      | Unit  | Value |
|----------------|-------|-------|
| \( \alpha \)   |       | 0     |
| \( \beta(t) \) | 1/day | 1.1430|
| \( \gamma(t) \)| 1/day | 0.9126|
| \( N_{I1}(3) \)|       | 7.8374|
| \( N_{I2}(3) \)|       | 6.5776|

Figure 1. The COVID-19 epidemic on Diamond Princess Cruise.

2.2 Simulating the COVID-19 epidemic in Wuhan

The COVID-19 transmission model (3) was utilized to simulate the COVID-19 epidemic in Wuhan, and the undetermined parameters in the model were obtained by means of actual data reported by Chinese National Health Commission. Wuhan has a population of 14 million. In order to stop COVID-19 virus from spreading across China, Wuhan locked down the city on Jan. 23, 2020. In the locked-down Wuhan, there were about 9 million residents. So set \( N_0 = 9,000,000 \). Table 3 gave the cumulative confirmed population in Wuhan reported by Chinese National Health Commission. Based on the COVID-19 transmission model (3) and the official data in table 3, the undermined parameters in equation (3) were obtained. \( \beta(t) \) and \( \gamma(t) \) were piecewise functions, as shown in table 4; the initial values of \( N_{I1}(t) \) and \( N_{I2}(t) \) on Jan. 20, 2020 were predicted as 433.3830 and 531.4037. Besides, set \( \alpha = 0 \) to indicate that the confirmed cases were isolated in Wuhan. Based on equation (3) and the model parameters \( \beta(t) \), \( \gamma(t) \), \( \alpha = 0 \) and the initial values of \( N_{I1}(t) \) and \( N_{I2}(t) \), the COVID-19 epidemic in Wuhan was simulated, as shown in figure 2, where the blue and red curves stood for the official data and simulation results, respectively. Compared with the official data in table 3, the standard deviation of the simulation results was 333.7, about 0.86% of the median of the official data. Figure 2 showed that the results simulated by the COVID-19 transmission model (3) coincided with the official data, verifying the transmission model (3).
Table 3. Cumulative confirmed population reported by Wuhan.

| Date  | 01-20  | 01-21  | 01-22  | 01-23  | 01-24  | 01-25  | 01-26  | 01-27  |
|-------|--------|--------|--------|--------|--------|--------|--------|--------|
| Number| 258    | 363    | 425    | 495    | 572    | 618    | 698    | 1590   |

| Date  | 01-28  | 01-29  | 01-30  | 01-31  | 02-01  | 02-02  | 02-03  | 02-04  |
|-------|--------|--------|--------|--------|--------|--------|--------|--------|
| Number| 1905   | 2261   | 2639   | 3215   | 4109   | 5142   | 6384   | 8351   |

| Date  | 02-05  | 02-06  | 02-07  | 02-08  | 02-09  | 02-10  | 02-11  | 02-12  |
|-------|--------|--------|--------|--------|--------|--------|--------|--------|
| Number| 10117  | 11618  | 13603  | 14982  | 16902  | 18454  | 19558  | 32994  |

| Date  | 02-13  | 02-14  | 02-15  | 02-16  | 02-17  | 02-18  | 02-19  | 02-20  |
|-------|--------|--------|--------|--------|--------|--------|--------|--------|
| Number| 35991  | 37914  | 39462  | 41152  | 42752  | 44412  | 45027  | 45346  |

| Date  | 02-21  | 02-22  | 02-23  | 02-24  | 02-25  | 02-26  | 02-27  | 02-28  |
|-------|--------|--------|--------|--------|--------|--------|--------|--------|
| Number| 45860  | 46201  | 46607  | 47071  | 47441  | 47824  | 48137  | 48557  |

| Date  | 02-29  | 03-01  | 03-02  | 03-03  | 03-04  | 03-05  | 03-06  | 03-07  |
|-------|--------|--------|--------|--------|--------|--------|--------|--------|
| Number| 49122  | 49426  | 49540  | 49671  | 49797  | 49871  | 49871  | 49912  |

| Date  | 03-08  | 03-09  | 03-10  | 03-11  | 03-12  | 03-13  | 03-14  | 03-15  |
|-------|--------|--------|--------|--------|--------|--------|--------|--------|
| Number| 49948  | 49965  | 49978  | 49986  | 50007  | 50026  | 50046  | 50066  |

Table 4. Parameters in COVID-19 transmission model

| Parameter | $\beta(t)$ | $\gamma(t)$ | $\beta(t)$ | $\gamma(t)$ | $\beta(t)$ | $\gamma(t)$ | $\beta(t)$ | $\gamma(t)$ |
|-----------|------------|-------------|------------|-------------|------------|-------------|------------|-------------|
| Date      | Jan.20----Feb.05 | Feb.05----Feb.11 | Feb.11----Feb.12 | Feb.12----Mar.11 |
| Value     | 0.336       | 0.1109      | 0.0838      | 0.0899      | 0.1073     | 0.1016      | 0.2079     | 0.3693      |

Figure 2. The COVID-19 epidemic in Wuhan.

3. Analyzing Wuhan's strategies for controlling COVID-19 at different stages

From Jan. 20 to Mar. 11, 2020, Wuhan adjusted control strategies 2 times, which could be reflected by the parameter variation in table 4. In the early stage of the epidemic, i.e. before Feb. 05, 2020, Wuhan mainly isolated and treated the confirmed patients suffering severe symptoms in designated hospitals, and the probabilities of transmission and the diagnosis rate were $\beta(t) = 0.3365$ (1/day) and $\gamma(t) = 0.1109$ (1/day), respectively. $\gamma(t)$ was very low and $\beta(t)$ was much higher than $\gamma(t)$, demonstrating that the epidemic was spreading rapidly and the control strategies in this stage could not effectively inhibit the spread of COVID-19. On Feb. 05, 2020, Wuhan adjusted control strategies, including building mobile cabin hospitals, isolating all the confirmed patients but not only isolating the confirmed cases with severe symptoms, treating critical patients in the designated hospitals and non-critical patients in the mobile cabin hospitals. The newly adjusted control strategies made $\beta(t)$ and $\gamma(t)$ reduce dramatically from 0.3365 (1/day) to 0.0899 (1/day). From Feb. 05 to Feb. 11, 2020, $\beta(t)$ and $\gamma(t)$ were basically equal, meaning that the COVID-19 outbreak had been contained to a certain extent.
However, because the COVID-19 diagnosis was a time consuming process and the accuracy of the test method was not high, the diagnosis rate remained much low, i.e. \( \gamma(t) = 0.0899 \) (1/day). On Feb. 11, 2020, Wuhan adjusted the control strategies again, including implementing the city-wide lockdowns and adopting the fast test methods like CT imaging to confirm COVID-19 cases for the purpose of the goal that all the infected people in Wuhan could be treated and isolated. These new and strict control strategies greatly increased the confirmed population and effectively controlled the epidemic spread. On Feb. 12, 2020, the increased confirmed cases was 13,436 and after this date, the diagnosis rate \( \gamma(t) \) was much higher than the probabilities of transmission \( \beta(t) \).

Based on the COVID-19 transmission model (3) and the parameters in table 4, the COVID-19 epidemic in Wuhan was simulated as shown in figure 3, where the green and red curves stood for the cumulative infected cases and the cumulative confirmed population, respectively. It could be found that almost all the infected people were isolated and treated by the end of March, and the cumulative confirmed population was about 50,000. The simulation showed that the COVID-19 epidemic would be finally over in late March, 2020.

Figure 3. Simulating the COVID-19 epidemic in Wuhan.

In order to assess the influence of Wuhan’s control strategies at different stages on the COVID-19 epidemic, the COVID-19 transmission model (3) was used to simulate the epidemic under the different strategies.

**Case 1:** Assuming that Wuhan had been implementing the control strategies --- mainly isolating and treating confirmed cases suffering severe symptoms in hospitals. The COVID-19 epidemic was shown in figure 4. It could be found that the cumulative infected population would reach up to 8,483,400 by the mid-May, 2020, about 170 times of the population in figure 3.

**Case 2:** Assuming that Wuhan had been implementing the control strategies --- building mobile cabin hospitals, isolating and treating all the confirmed patients in designated hospitals and mobile cabin hospitals. The COVID-19 epidemic was shown in figure 5. Compared with figure 4, the cumulative infected population dropped dramatically to 247,620 although the period of epidemic was longer, over 2 years.

Compared with figure 3, the cumulative infected population was 4.95 times of the population in figure 3, and the period of epidemic was 13 times of that in figure 3. The differences between figure 3 and figure 5 demonstrated that the control strategies adjusted by Wuhan on Feb. 11, 2020, namely the city-wide lockdowns and fast test methods like CT imaging to confirm COVID-19 cases, could dramatically contain the spread of the epidemic, including decreasing the cumulative infected population and shortening the period of epidemic.

**Case 3:** Assuming that Wuhan had been implementing the control strategies --- building mobile cabin hospitals, isolating all the confirmed patients, adopting the fast test methods like CT imaging to confirm COVID-19 cases, and implementing the city-wide lockdowns, such as suspending public transportation, temporarily closing factories and schools, and asking people to stay at home. The COVID-19 epidemic was simulated under these strict control strategies, as shown in figure 6. It could
be found that the COVID-19 epidemic would be over in 2 months and the cumulative confirmed population was not more than 1,700, about 3% of the population in figure 3.

**Case 4:** Assuming that Wuhan hadn't implemented any protection and control strategies. The COVID-19 epidemic was shown in figure 7. It could be found that the COVID-19 virus would affect the whole Wuhan in 3 months. Case 3 and Case 4 considered two extremes, and the simulation results in figure 6 and figure 7 illustrated the importance of the timely response to the COVID-19 epidemic and effective control strategies.

**Case 5:** In order to compare the control strategies of Wuhan and Diamond Princess Cruise, Diamond Princess Cruise's model parameters in table 2 were used to predict the COVID-19 epidemic in Wuhan. Figure 8 showed the prediction for the COVID-19 epidemic in Wuhan under the Diamond Princess Cruise's control strategy. It could be found that the COVID-19 virus would affect 3,367,800 people in 3 months, about 67 times of the population in figure 3. Obviously, the protection and isolation measures of Diamond Princess Cruise could not effectively inhibit the rapid spread of COVID-19.

![Figure 4. Simulation results of Case 1.](image1)

![Figure 5. Simulation results of Case 2.](image2)

![Figure 6. Simulation results of Case 3.](image3)
4. Conclusions
This paper built a COVID-19 transmission model to analyze the control strategies of Wuhan and Diamond Princess Cruise. Some valuable conclusions were drawn:

1) the COVID-19 transmission model could be used to predict the spread of COVID-19 in various regions by means of actual data reported by governments;

2) Wuhan's control strategies prior to Feb. 05, 2020, i.e. mainly isolating and treating the confirmed patients suffering severe symptoms in hospitals, could not effectively inhibit the rapid spread of COVID-19;

3) Wuhan's control strategies implemented from Feb. 05, 2020, i.e. building mobile cabin hospitals and isolating all the confirmed patients in the designated hospitals and the mobile cabin hospitals, could reduce the infected population over 30 times;

4) Wuhan's control strategies implemented from Feb. 11, 2020, i.e. city-wide lockdowns and fast test methods like CT imaging to confirm COVID-19 cases for the purpose of the goal that all the infected people could be treated and isolated, could dramatically contain the spread of the epidemic, including decreasing the cumulative infected population and shortening the period of epidemic;

5) the protection and isolation measures of Diamond Princess Cruise could not effectively inhibit the spread of COVID-19;

6) assuming that Wuhan had been implementing the control strategies announced on Feb. 05 and Feb. 11, 2020, i.e. building mobile cabin hospitals, isolating all the confirmed patients, adopting the fast test methods like CT imaging to confirm COVID-19 cases, and implementing the city-wide lockdowns, the COVID-19 epidemic would be over in 2 months and the cumulative confirmed population was not more than 1,700.
5. Appendices
The optimization problem (4) can be rewritten as
\[
\begin{align*}
\min & \quad F(\vec{T}(t), \vec{N}(t_0)) = \frac{1}{2} \int_{t_0}^{t_f} \rho(t) \cdot (N_{i1}(t) - N_{i1}^*(t))^2 \, dt \\
\text{s.t.} & \quad \frac{d\vec{N}(t)}{dt} = \vec{f}(\vec{N}, \vec{T}, t)
\end{align*}
\]
(A.1)

Meanwhile, in order to calculate the sensitivity of the objective function with respect to its variables in the optimization problem (A.1), the Lagrangian function is constructed
\[
L(\vec{T}(t), \vec{N}(t_0)) = F(\vec{T}(t), \vec{N}(t_0)) + \int_{t_0}^{t_f} \vec{M}(t)^T \cdot \left( \frac{d\vec{N}(t)}{dt} - \vec{f}(\vec{N}, \vec{T}, t) \right) dt
\]
(A.2)

where,
\[
\begin{align*}
\vec{N}(t) &= [N_{i1}(t), N_{i2}(t)]^T \\
\vec{T}(t) &= [\beta(t), \gamma(t)]^T \\
\vec{M}(t) &= [M_1(t), M_2(t)]^T \\
\vec{f}(\vec{N}, \vec{T}, t) &= \begin{cases} 
\begin{bmatrix} f_1(\vec{N}, \vec{T}, t), f_2(\vec{N}, \vec{T}, t) \end{bmatrix}^T & \\
\beta(t) \cdot S(t) \cdot (N_{i2}(t) + \alpha N_{i1}(t)) - \gamma(t) \cdot N_{i2}(t) 
\end{cases} \\
f_1(\vec{N}, \vec{T}, t) &= \gamma(t) \cdot N_{i2}(t) \\
f_2(\vec{N}, \vec{T}, t) &= \beta(t) \cdot S(t) \cdot (N_{i2}(t) + \alpha N_{i1}(t)) - \gamma(t) \cdot N_{i2}(t)
\end{align*}
\]
(A.3)

\(M_1(t)\) and \(M_2(t)\) belong to \(C^1(\mathbb{R})\), and are continuously differentiable functions. Substituting equation (A.1) into equation (A.2) and calculating the integral by parts, the following formula is obtained
\[
L(\vec{T}(t), \vec{N}(t_0)) = \int_{t_0}^{t_f} \left( \frac{\rho(t)}{2} \cdot (N_{i1}(t) - N_{i1}^*(t))^2 - \vec{N}(t)^T \cdot \frac{d\vec{M}(t)}{dt} \right) dt
\]
(A.4)

Calculating the variation of the Lagrangian function (A.4) with respect to \(\vec{T}(t)\) and \(\vec{N}(t)\) yields
\[
\begin{align*}
\delta L(\vec{T}(t), \vec{N}(t_0)) &= \int_{t_0}^{t_f} \delta\vec{N}(t)^T \cdot \left( \vec{B}(t) - \frac{d\vec{M}(t)}{dt} \right) dt \\
&+ \int_{t_0}^{t_f} \delta\vec{f}(t)^T \cdot \left( \frac{\partial f_1(\vec{N}, \vec{T}, t)}{\partial \vec{N}} \right)^T \cdot \vec{M}(t) dt \\
&+ \int_{t_0}^{t_f} \delta\vec{f}(t)^T \cdot \left( \frac{\partial f_2(\vec{N}, \vec{T}, t)}{\partial \vec{N}} \right)^T \cdot \vec{M}(t) dt
\end{align*}
\]
(A.5)

where
\[
\begin{align*}
\left\{ \begin{array}{l}
\frac{\partial f_1(\vec{N}, \vec{T}, t)}{\partial \vec{N}} = 0 \\
\frac{\partial f_2(\vec{N}, \vec{T}, t)}{\partial \vec{N}} = \begin{bmatrix} 0 & \alpha \cdot S(t) - N_{i2}(t) + \alpha N_{i1}(t) \end{bmatrix}^T \\
\frac{\partial \vec{f}(\vec{N}, \vec{T}, t)}{\partial \vec{T}} = \begin{bmatrix} \beta(t) \cdot S(t) - N_{i2}(t) + \alpha N_{i1}(t) \end{bmatrix}^T \\
\end{array} \right.
\end{align*}
\]
(A.6)

Letting \(\vec{M}(t)\) satisfy
\[
\begin{align*}
\left\{ \begin{array}{l}
\frac{d\vec{M}(t)}{dt} = \vec{B}(t) - \left( \frac{\partial \vec{f}(\vec{N}, \vec{T}, t)}{\partial \vec{N}} \right)^T \cdot \vec{M}(t) \\
\vec{M}(t_0) = 0
\end{array} \right.
\end{align*}
\]
(A.7)

the conjugate equation (5) can be obtained by substituting equation (A.6) into equation (A.7). Meanwhile, equation (5) can be simplified to
\[
\begin{align*}
\delta L(\vec{T}(t), \vec{N}(t_0)) &= \int_{t_0}^{t_f} \left( \vec{M}(t) \cdot (N_{i2}(t) + \alpha N_{i1}(t)) - \beta(t) \right) dt \\
&+ \int_{t_0}^{t_f} \left( \vec{M}(t) \cdot \delta\vec{f}(t) - \alpha \cdot S(t) \cdot \vec{M}(t) \cdot \delta\vec{N}(t)^T \right) dt
\end{align*}
\]
(A.8)

Then, sensitivity formulas (6) and (7) can be obtained directly through equation (A.8).
6. References

[1] Yong-ren Yu, Ying Yang, Ming Zhu, Shuang Wang, Xiao-lin Bi, Zhao-feng Liu, Fang-fang Yu, The SARS propagation models and influence on economy, *Journal of Liaoning University: Natural Sciences Edition*, 2005, Vol.32(1), pp 48-49

[2] Ming Wang, Lin Du, Duan-hua Zhou, Biao Di, Yu-fei Liu, Peng-zhe Qin, Xin-wei Wu, Xiaoshuang Chen, Ji-chun Qiu, Ze-rong Li, Study on the epidemiology and measures for control on severe acute respiratory syndrome in Guangzhou City, *Chinese Journal of Epidemiology*, 2003, Vol.24(5), pp 353-57

[3] Jie-xiu Wang, Hong-you Feng, Dong Liu, Zhi-lun Zhang, Ai-lan Shan, Xiang-jun Zhu, Zhi-gang Gao, Xu-dong Wang, Ying-yi Xia, Qian Chen, Epidemiological characteristics of severe acute respiratory syndrome in Tianjin and the assessment of effectiveness on measures of control, *Chinese Journal of Epidemiology*, 2003, Vol.24(7), pp 565-69

[4] Zhong-Lun Xie, Nan Yang, Bao-Xu Huang, Xiao-Bo Guo, Chao-Jian Shen, Xin-Jie Wei, Simulation research of sars control strategy in china: prevention and control measure's effect comparison, *Journal of System Simulation*, 2004, Vol.16(12), pp 2667-72

[5] Nan Yang, Simulation of SARS epidemiology and research on SARS control strategies, Shandong University, 2008, Master's Thesis

[6] Juan Zhang, Jie Lou, Zhien Ma, Jianhong Wu, A compartmental model for the analysis of SARS transmission patterns and outbreak control measures in China, *Applied Mathematics and Computation*, 2005, Vol.162(2), pp 909-24

[7] Ling-xiao Wu, Epidemiological characteristics of measles and its control strategies in Lechang City, *Journal of Qiqihar University of Medicine*, 2016, Vol.37(10), pp 1337-38

[8] Ruiping Li, Cai-hong Gao, Ke-hua Yi, Hui-hong Yu, Qin-ying Shen, Mei-fang Cao, Fang Wu, Epidemiological characteristics analysis of rubella and discussion of its control strategies in Fengxian District of Shanghai, 2003-2015, *Shanghai Journal of Preventive Medicine*, 2017, Vol.29(9), pp 698-701, 06

[9] Cai-jun Zhang, Shui-ou Li, Epidemiological characteristics analysis of HFMD and its control strategies from 2014-2018, *China Practical Medicine*, 2019, Vol.14(34), pp 179-81

[10] Kimberlyn Roosa, Yiseul Lee, Ruiyan Luo, Alexander Kirpich, Richard Rothenberg, James M. Hyman, Ping Yan, Gerardo Chowell, Short-term forecasts of the COVID-19 epidemic in Guangdong and Zhejiang, China: February13–23, 2020, *Journal of Clinical Medicine*, 2020, Vol.9(2), pp 1-9

[11] Yaqing Fang, Yiting Nie, Penny Marshare, Transmission dynamics of the COVID-19 outbreak and effectiveness of government interventions: a data-driven analysis, *Journal of Medical Virology*, 2020, https://doi.org/10.1002/jmv.25750

[12] Tian Xu, Pei-pei Zhang, Yu-mei Jiang, Bei-bei Su, Da-ren He, Models epidemic spreading and SARS, *Ziran Zazhi*, 2004, Vol.26(1), pp 20-25

[13] Qizhi Chen, Application of SIR model in forecasting and analyzing for SARS, *Journal of Peking University (Health Sciences)*, 2003, Vol.35 Supplement, pp 75-80