The Periastron Shift of Binary Pulsar with Kozai-Lidov Oscillation

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ABSTRACT
We study a hierarchical triple system with the Kozai-Lidov mechanism, and analyse the emission effect of gravitational waves on the cumulative shift of periastron time. Time evolution of the osculating orbital elements of the triple system is calculated by directly integrating the first-order post-Newtonian equations of motion. The Kozai-Lidov mechanism will bend the evolution curve of the cumulative shift when the eccentricity becomes large. We also investigate the parameter range of mass and semi-major axis of the third object with which the above phenomena could occur within 100 years.

Key words: gravitational waves – binaries (including multiple): close – stars: kinematics and dynamics – pulsars: general – stars: black holes

1 INTRODUCTION
The PSR B1913+16 system (Hulse-Taylor binary) found in 1975, is one of the most famous pulsar binaries (Hulse & Taylor 1975). This binary pulsar has a highly eccentric close orbit: the semi-major axis is about 0.013 au and the eccentricity is about 0.617 (Taylor et al. 1976). Because of these features, the orbital energy of the system is extracted by the gravitational wave (GW) emission and its orbital period is decreasing gradually. This period shift has been detected for over 30 years as the cumulative shift of periastron time, which is explained quite well by the GW emission from the binary in general relativity (Weisberg & Taylor 2005). It is the first indirect evidence of the existence of GW.

We have so far found many binary pulsars(see e.g. Lorimer (2008)). Near some of the observed binary pulsars, there may exist a third companion. In fact, the PSR B1620-26 system (Thorsett et al. 1999) and the PSR J0337+1715 system (Ransom et al. 2014) are triplet systems. If the GW emission affects the orbital evolution of such a triple system, one may wonder what kinds of evidence could be found in the observation and how to observe them.

In a hierarchical triple system, the inner binary sometimes shows quite different orbital motion from that of an isolated binary. The Kozai-Lidov (KL) mechanism (Kozai 1962; Lidov 1962), which is one of the most important phenomena in a hierarchical triple system, is particularly interesting. KL-mechanism occurs when the inner orbit inclines enough from the outer orbit. The main feature of KL-mechanism is the secular changes of the eccentricity of the inner orbit and the relative inclination. Both values oscillate amongst each other as a seesaw, that is, when the eccentricity decreases, the inclination increases, and vice versa. When the eccentricity is large, the tidal dissipation becomes important and the GW emission is more efficient because the periastron comes closer. As a result, the KL oscillation will play important roles in various relativistic astrophysical phenomena, for example: the merger of black holes (Blaes et al. 2002; Miller & Hamilton 2002; Liu & Lai 2017), the tidal disruptions of stars by supermassive black holes (Ivanov et al. 2005; Chen et al. 2009, 2011; Wegg & Bode 2011; Li et al. 2015), and the formation of hot Jupiters (Naoz et al. 2012; Petrovich 2015; Anderson et al. 2016) or ultra-short-period planets (Oberst et al. 2017).

In this letter, we study the cumulative shift of the periastron time by the GW emission in a hierarchical triple system with the KL-mechanism and discuss what we will observe. The paper is organized as follows: We briefly summarize the important features of KL-mechanism in §2. After we explain our approach in §3, we describe our models and present the results and discussions in §4. In our result, KL-effects gives the different evolution curve of the cumulative shift from that of an isolated binary. The detectability of
such phenomenon is also discussed. The conclusion follows in §5.

2 HIERARCHICAL TRIPLE SYSTEM AND KOZAI-LIDOV MECHANISM

In this paper, we treat the so-called hierarchical triple system, whose schematic picture is given in Fig. 1. It is a three-body system characterized by the following features: The distance between the first and second bodies is much shorter than the distance to the third body. We also assume that the effect of the third body is much smaller than the gravitational interaction between the first and second bodies. As a result, we can separate the three-body motion into the two-body inner binary orbit and the outer companion’s orbit.

In a two-body problem in Newtonian dynamics, the elliptical orbit is described by six orbital elements; the semi-major axis $a$, the eccentricity $e$, the argument of periastron $\omega$, the longitude of ascending node $\Omega$, and the mean anomaly $M$. Although these elements are constant in the isolated two-body system, in the hierarchical three-body system, the perturbations from the tertiary companion affect the binary motion and modify the trajectory from that of the isolated one. Such a trajectory is not closed in general, but we can define it as an osculating orbit at each time, whose trajectory is approximated by the elliptic orbit with the above six orbital elements determined by the instantaneous position and velocity (Murray & Dermott 2000). As for the outer orbit, we pursue the centre of mass of the inner binary rotating around the tertiary companion (see Fig. 1). It can also be described as another osculating orbit. Hence, we introduce two osculating orbits, which are called as inner and outer orbits: The masses of the inner binary are $m_1$ and $m_2$, while the tertiary companion has the mass $m_3$. We use subscriptions 'in' and 'out' to show the elements of inner and outer orbits, respectively.

In the hierarchical three-body system with large relative inclination, an orbital resonance known as Kozai-Lidov (KL) oscillation may occur. This mechanism, which was discovered by Kozai (1962) and Lidov (1962), is characterized by the oscillation of the inner eccentricity $e_{\text{in}}$ and the relative inclination $I$. The relative inclination $I$ is defined as the argument between the inner and outer orbital planes. It is given by two orbital elements as

$$\cos I = \cos i_{\text{in}} \cos i_{\text{out}} + \sin i_{\text{in}} \sin i_{\text{out}} \cos (\Omega_{\text{in}} - \Omega_{\text{out}}).$$

(1)

This oscillation occurs in secular time-scale under the conservation of the energy and angular momentum. In Newtonian quadrupole approximation method (see e.g. Shevchenko (2017)), it results in the secular exchange of $e_{\text{in}}$ and $I$ with the conserved value of $\Theta$, which is defined by

$$\Theta = (1 - e_{\text{in}}^2)^{\cos 2 I}. $$

(2)

This approximation also gives the criterion of KL-oscillation as

$$0 \leq |\cos I| \leq \frac{\sqrt{3}}{5}. $$

(3)

which is equivalent to $39.2315^\circ \leq I \leq 140.7685^\circ$. The KL-oscillation time-scale is evaluated as

$$T_{\text{KL}} \sim P_{\text{in}} \frac{m_1 + m_2}{m_3} \left(\frac{a_{\text{out}}}{a_{\text{in}}}\right)^3 \left(1 - e_{\text{out}}^2\right)^{\frac{3}{2}}, $$

(4)

where $P_{\text{in}}$ is the orbital period of the inner orbit.

The effect of general relativity (GR) changes these KL-criterion as (See e.g. Migaszewski & Goździewski (2011))

$$0 \leq 1 - \cos I \leq \frac{3}{5} \left(1 - \frac{4}{9} \langle H_{\text{GR}} \rangle \right). $$

(5)

where the GR correction term $\langle H_{\text{GR}} \rangle$ is derived from the double-averaged post-Newtonian Hamiltonian of two-body relative motion (Richardson & Kelly 1988) as

$$\langle H_{\text{GR}} \rangle = -\frac{3G^2(m_1 + m_2)^2}{c^2d^2\sqrt{1 - e^2}} \left(1 - \frac{m_1m_2}{3(m_1 + m_2)^2}\right) + \text{constant}. $$

(6)

The KL-oscillation time-scale is also modified when the GR effect is taken into account.

3 PERIASTRON SHIFT AND GRAVITATIONAL WAVES

We study the GW emission effects on the hierarchical triple system and analyse the cumulative shift of the periastron time of the inner binary. In particular, we focus on the systems whose inner orbit is initially inclined enough, such that the KL-oscillation will occur.

In order to solve the three-body system, we employ the first-order post-Newtonian equations of motion, called the Einstein-Infeld-Hoffmann (EIH) equations (Einstein, Infeld,
In order to see this period shift, it is convenient to observe the cumulative shift of periastron time $\Delta P$ defined by

$$\Delta P(T_N) = T_N - P_{m,0} N,$$

where $T_N$ is the $N$-th periastron passage time and $P_{m,0}$ is the initial orbital period of the inner binary. Using the definition

$$N = \int_0^{T_N} \frac{1}{P_{m}(t)} dt,$$

where $P_{m}(t)$ is the binary period at time $t$, which changes in time by the GW emission as

$$P_{m}(t) = P_{m,0} + \int_0^t \frac{\dot{P}_m(t')}{P_{m,0}} dt',$$

we obtain the cumulative shift of periastron time $\Delta P$ as

$$\Delta P(T_N) = T_N - \int_0^{T_N} \frac{\dot{P}_m(t')}{P_{m,0} + \int_0^t \frac{\dot{P}_m(t')}{P_{m,0}} dt'} dt'$$

$$= \int_0^{T_N} \frac{\dot{P}_m(t')}{P_{m,0} + \int_0^t \frac{\dot{P}_m(t')}{P_{m,0}} dt'} dt'.$$

Since the emission energy of gravitational waves is very small, we usually expect

$$\int_0^{T_N} \frac{\dot{P}_m(t')}{P_{m,0}} dt' \ll P_{m,0}.$$

In fact, for the Hulse-Taylor binary pulsar (Weisberg & Taylor 2005), since we have

$$P_o = 0.32299 \text{ [day]},$$

$$P_o = -2.4184 \times 10^{-12} \text{ [s]},$$

the condition (14) is true if $t \ll 3.656 \times 10^8 \text{ [yr]}$. Hence, when we are interested in the time-scale such that $T_N \ll 10^8 \text{ [yr]}$, we can approximate $\Delta P$ as

$$\Delta P(T_N) \approx \frac{1}{P_{m,0}} \int_0^{T_N} dt \int_0^t \frac{\dot{P}_m(t')} {P_{m,0}} dt'.$$

If we assume $P_{m}(t) = P_{m,0} = \text{constant}$, we obtain

$$\Delta P(T_N) \approx \frac{P_{m,0}}{2P_{m,0}} T_N^2,$$

which was used in Weisberg & Taylor (2005). However, in a hierarchical triple system with the KL oscillation, $P_{m}(t)$ depends on time. Hence, we evaluate $\Delta P$ by Eq. (17) by use of the averaged time-dependent orbital elements.

We remark that the back reaction of GW emission on the orbital evolution corresponds to the 2.5 order post-Newtonian terms, which are not included in our calculation. It is because the effect of back reaction is quite small.

4 RESULTS AND DISCUSSIONS

To show our numerical results about the periastron shift of a binary in a hierarchical triple system, we shall choose the PSR J1840-0643 as an example\(^3\). This binary pulsar was

\(^3\) The most famous Hulse-Taylor binary pulsar shows the cumulative shift of the periastron time given by Eq. (18). However, the presence of a third stellar-mass object within $a_{out} < 100[au]$ was ruled out (Smarr & Blandford 1976).
discovered in Einstein@Home project, whose detail and the orbital parameters of the binaries are given in Knispel et al. (2013). The Doppler shift effect caused by the acceleration due to third body gives a constraint for the parameters of an outer orbit, if it exists. The Doppler time-scale $T_D = c a_1^{-1} m_1^{-1}$ should be longer than the characteristic age of pulsar $T_\nu$ defined by $T_\nu = \nu/(2|\dot{\nu}|)$, where $\nu(t)$ is the spin frequency of the pulsar. It gives the upper limit for the mass of a third body and its distance. The PSR J1840-0643 system has the characteristic age $T_\nu = 2.56 \times 10^6 [\text{yrs}]$, so it seems that this system also has a strict constraint on the presence of a third body like Hulse-Taylor binary. This characteristic age was, however, evaluated in the topocentric frame with the ansatz of an isolated binary. In the barycentric frame, the spin period is increasing, which looks unphysical (Knispel et al. 2013). We therefore assess that this system has not yet had a strict constraint for the presence of a third body.

Assuming that this system has a third body, we set up initial values of the inner binary of our model by using the observed parameters of this binary system, and analyse the cumulative shift of the periastron time $\Delta P$. Table 1 shows the initial condition of our model.

The results of our triple-system model are shown in Figs. 2 and 3. Fig. 2 shows the time evolution of the averaged inner eccentricity $\bar{e}_{\text{in}}$, averaged relative inclination $\bar{I}$ and averaged orbital elements of our three-body system with $m_1 = 1.4 M_\odot, m_2 = 0.16 M_\odot$ and $m_3 = 30 M_\odot$. $a$, $e$, $i$, $\Omega$, $\omega$, and $M$ are the semi-major axis, eccentricity, inclination, longitude of ascending node, argument of periastron, and mean anomaly, respectively. Those are fixed by the observational data for the inner orbit, while assumed for the outer orbit. The argument of periastron $\omega$ can be arbitrary because the eccentricity is zero. We have used this freedom to fix the axis of the reference frame.

Table 1. Initial orbital elements of our three-body system with $m_1 = 1.4 M_\odot, m_2 = 0.16 M_\odot$ and $m_3 = 30 M_\odot$. $a$, $e$, $i$, $\Omega$, $\omega$, and $M$ are the semi-major axis, eccentricity, inclination, longitude of ascending node, argument of periastron, and mean anomaly, respectively. Those are fixed by the observational data for the inner orbit, while assumed for the outer orbit. The argument of periastron $\omega$ can be arbitrary because the eccentricity is zero. We have used this freedom to fix the axis of the reference frame.

| orbit | $a$ [au] | $e$ | $i$ [deg] | $\Omega$ [deg] | $\omega$ [deg] | $M$ [deg] |
|-------|----------|-----|-----------|----------------|----------------|-----------|
| inner | 2.17373  | 0   | 60        | 0              | -              | 0         |
| outer | 20.0     | 0   | 0         | 0              | -              | 20        |

Figure 2. The time evolution of the osculating orbital elements of our model; the averaged eccentricity of the inner binary $\bar{e}_{\text{in}}$, averaged relative inclination $\bar{I}$ and averaged KL-conserved value $\bar{\Theta}$ for 100 yrs. The red, green and blue lines show the evolution of $\bar{e}_{\text{in}}$, $\bar{I}$ and $\bar{\Theta}$ respectively. Many small oscillations are caused by the outer companion’s motion, while the KL-oscillation with time-scale of about 100 yrs is seen.

Figure 3. The cumulative shift of periastron time $\Delta P$ for 100 yrs. The blue line shows the result calculated for inner binary of our model, which shows the deviation from the red line named 'isolated binary' calculated for a single binary system. In our model, the eccentricity is initially small, which $\Delta P$ is fitted by the quadratic function $\Delta P(T_N)$ given in the text, but increases by the KL oscillation, which induces the deviation from the red dotted line, and eventually decreases again to small value, which makes the curve to another quadratic function $\Delta P_{\text{KL}}(T_N)$ defined in the text.

and averaged KL-conserved value $\bar{\Theta}$ for 100 yrs. $\bar{\Theta}$ is given by Eq. (2) by use of $\bar{e}_{\text{in}}$ and $\bar{I}$. As for the evolutions of $\bar{e}_{\text{in}}$ and $\bar{I}$, we find two kinds of oscillations with different time-scales: One is the period of outer orbit $P_{\text{out}} = 15.92$ [yrs], and the other is the secular oscillation time-scale $\sim 100$ [yrs], in which the eccentricity increases from 0 to about 0.6 while the inclination decreases from 60° to about 48°. This secular oscillation of $\bar{e}_{\text{in}}$ and $\bar{I}$ corresponds to the KL-oscillation. Actually, the KL time-scale calculated by Eq. (4), $T_{\text{KL}} \sim 103.91$ [yrs], is consistent with the result of our simulation. We remark that KL-conserved value $\Theta$ is approximately conserved but shows small oscillation with the period $P_{\text{out}}$ around $\bar{\Theta} = 0.3$. This is because our model is not an ideal hierarchical triple system, that is, the perturbation from outer body is not small enough to satisfy the condition for quadrupole approximation. However, one important point is that the stable KL-oscillation is observed even in such a non-ideal hierarchical triple system.

Fig. 3 exhibits the time evolution of the cumulative shift of periastron time $\Delta P$ for 100 yrs. The result of our model is shown by the blue line. As a reference, we also show the result of an isolated binary with the same initial data by the red dashed line, which is described by the quadratic function

$$\Delta P_{\text{KL}}(T_N) = -3.185 \times 10^{-13} T_N^2 [\text{yrs}^2].$$

The blue line of our model coincides with the red line initially (Period A in Fig. 3), but it deviates at $t \sim 30$ [yrs] and the discrepancy between these two lines becomes larger until $t \sim 60$ [yrs] (Period B in Fig. 3). This deviation of the blue line comes from the large amount of emission of gravitational waves, which is caused by the excitation of the eccentricity via the KL-mechanism. After $t \sim 70$ [yrs], the eccentricity decreases again, and then $\Delta P$ is approximated...
by another quadratic function

$$\Delta P_2[s] = -3.185 \times 10^{-13} \left[ T_N [\text{yrs}] + 1.501 \times 10^2 \right]^2 + 1.206 \times 10^{-8}, \quad (20)$$

whose curve is also given in the figure (Period c in Fig. 3). As a result, the curve of the cumulative shift will bend when the eccentricity becomes large by the KL oscillation.

In real observations, since a pulsar signal is beamed in some direction, when the inclination will change substantially, we may not be able to observe the pulsar. As a result, we will see only the parts of $\Delta P_1$ (Period A) and $\Delta P_2$ (Period C) in the cumulative shift of the periastron time. The pulsar will disappear for the transition period B (30 $\leq t \leq$ 60). Or, if we do not see any pulsar now, but we might find a new pulsar appeared only for this transition period B.

Finally, we shall discuss the possible parameter range of the third body to be observed. In addition to the model given in Table 1, we have performed our calculation for 19200 models by changing the outer semi-major axis (10 au $\leq a_{\text{out}} \leq$ 40 au) and the mass of the third body (10 $M_\odot \leq m_3 \leq$ 90 $M_\odot$) and analysed in which parameter ranges we can observe the above phenomenon (the bending of the cumulative-shift curve) within 100 years. The result is given in Fig. 3. To observe such phenomenon within 100 years, the KL timescale should be less than 100 years. So we investigate how many years we require to detect the deviation from $\Delta P_1$. For 19200 models with different $m_3$ and $a_{\text{out}}$, we calculate $\Delta P$ and judge the detectability simply by the difference between $\Delta P$ and $\Delta P_1$, i.e.,

$$\frac{\Delta P - \Delta P_1}{\Delta P_1} > 1. \quad (21)$$

Fig. 4 shows the color contour map of the time taken for detection. Black region means that KL-oscillation is not detected in 100 yrs and the white dashed line corresponds to the theoretical boundary, $T_{KL} = 100$ [yrs]. In white region, the system became unstable; the yellow solid line is the empirical criterion given in Blaes et al. (2002). Detail explanation is in the text.

Figure 4. The detectability of the effect of KL-oscillation on $\Delta P$ in 100 yrs. The horizontal and vertical axes are the outer semi-major axis and the mass of the tertiary companion, respectively. Color shows the time taken for detection. Black region means that KL-oscillation is not detected in 100 yrs and the white dashed line corresponds to the theoretical boundary, $T_{KL} = 100$ [yrs]. In white region, the system became unstable; the yellow solid line is the empirical criterion given in Blaes et al. (2002). Detail explanation is in the text.

third object with which the above phenomenon could occur within 100 years.

For the inner binary of our triple-system model, we have employed the parameters of the binary pulsar PSR J1840-0643 with zero eccentricity. Assuming the existence of tertiary companion with the mass 30 $M_\odot$ and large relative inclination $I = 60^\circ$, we find that the KOZAI-Lidov mechanism will bend the evolution curve of the cumulative shift when the eccentricity becomes large. The pulsar will disappear suddenly and then reappear again with the different cumulative-shift curve. We have also investigated the parameter ranges of the outer companion around the binary, in which the bending of the cumulative-shift curve due to KL-mechanism is observed within 100 yrs. We find that we may observe such phenomenon if a stellar or an intermediate mass black hole ($m_3 = 10 - 100 M_\odot$) exists at the distance within 40 au from the binary.

We are now performing our analysis for more general models because we will find many more triple systems in observations in near future. Those results will be published elsewhere. We are also interested in the GW waveforms from a triple system, for which study is in progress.

5 CONCLUSIONS

We have studied a hierarchical triple system with the KOZAI-Lidov mechanism and analysed the emission effect of gravitational waves on the cumulative shift of periastron time. Time evolution of the oscillating orbital elements of the triple system is calculated by directly integrating the first-order post-Newtonian equations of motion. We also investigate the parameter range of mass and semi-major axis of the

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