Groups of fast homeomorphisms of the interval and the ping-pong argument. (English)
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Summary: We adapt the Ping-Pong lemma, which historically was used to study free products of groups, to the setting of the homeomorphism group of the unit interval. As a consequence, we isolate a large class of generating sets for subgroups of Homeo⁺(I) for which certain finite dynamical data can be used to determine the marked isomorphism type of the groups which they generate. As a corollary, we will obtain a criterion for embedding subgroups of Homeo⁺(I) into Richard Thompson’s group F. In particular, every member of our class of generating sets generates a group which embeds into F and in particular is not a free product. An analogous abstract theory is also developed for groups of permutations of an infinite set.

MSC:
20B07 General theory for infinite permutation groups
20B10 Characterization theorems for permutation groups
20E07 Subgroup theorems; subgroup growth
20E34 General structure theorems for groups
20F65 Geometric group theory

Keywords:
algebraically fast; dynamical diagram; free group; geometrically fast; geometrically proper; homeomorphism group; piecewise linear; ping-pong lemma; symbol space; symbolic dynamics; Thompson’s group; transition chain

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