Polarized $\Lambda_b \to X_c \tau \nu$ in the SM and THDM

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Abstract

The inclusive rate and $\tau$ spectrum for a polarized $\Lambda_b$-baryon to decay to charm hadronic final states and leptons $\tau \nu$ in the SM and a two-Higgs doublet model are computed. The $O(\alpha_s)$ QCD corrections to $\tau$ spectrum in the two-Higgs model are also given.

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I. Introduction

The semileptonic decay $B \rightarrow X_c l \nu$ has been extensively studied in both the standard model (SM) and a two-Higgs-doublet model (THDM) [1,2]. Compared with the B decay, in addition to the spectrum of the lepton arising from the decay, the various spin correlation quantities are of interest for the decay of a polarized $\Lambda_b$. It’s well-known [3,4] that the heavy quarks produced in $Z^0$ decay are polarized and only charmed and beautiful $\Lambda$ baryons seem to offer a practical method to measure the polarization of the corresponding heavy quark. The polarization transferred from a heavy quark $Q$ to the corresponding $\Lambda$ spin analysers for the decays of heavy quarks. Some aspects of the inclusive rate and $l$ spectrum of polarized $\Lambda_b \rightarrow X_c l \nu$ have been studied in the SM [3,4,5].

The $\Lambda_b \rightarrow X_c \tau \nu$ (and $B \rightarrow X_c \tau \nu$) decay is sensitive to new physics, in particular, models with charged Higgs bosons. Because the charged Higgs bosons contribute at tree level, its contribution can not be cancelled by other new particles in the models. Therefore, the calculations of charged Higgs contributions with high accuracy will provide strong bound on parameters of the models when experimental measurement of the decay is available.

In this paper we investigate the polarized inclusive decay $\Lambda_b \rightarrow X_c \tau \nu$ in both the SM and THDM. In the SM we extend the results of Manohar and Wise [3] to the case of non-zero mass of the final state lepton (tau). We calculate the spin dependent form factors in the hadronic tensor to the $1/m_b^2$ order in THDM which do not contribute to the decay rate when the mass of the final state lepton is neglected. In the THDM we compute the inclusive rate and $\tau$ spectrum for a polarized $\Lambda \rightarrow X_c \tau \nu$ included the $\Lambda_b^2 QCD/m_b^2$ nonperturbative corrections and the $O(\alpha_s)$ perturbative corrections to $\tau$ spectrum.

The paper is organized as follows. In section II we calculate $\tau$ spectrum from a polarized $\Lambda_b$ decay to the order $1/m_b^2$ in the $1/m_b$ expansion in the SM. Section III is devoted to calculations in the THDM. The $O(\alpha_s)$ QCD corrections to the decay are included. In section IV numerical results are given. Finally, a summary and discussions are presented in section V.

II. $\tau$ spectrum of $\Lambda_b \rightarrow X_c \tau \nu$ in the SM

We consider the inclusive semileptonic decay $\Lambda_b \rightarrow X_c \tau \nu$ in the SM. At the tree level, for unpolarized leptons the partial decay width can be written as

$$d\Gamma = \frac{G_F^2 |V_{cb}|^2}{(2\pi)^3 E_H} \mathcal{L}_{\mu\nu} W_{\mu\nu} \frac{d^3 p_\tau}{2E_\tau} \frac{d^3 p_\nu}{2E_\nu} \frac{1}{2},$$

where $E_H$ is the energy of $\Lambda_b$, $L_{\mu\nu}$ is the leptonic tensor

$$L^{\mu\nu} = 2(p_\tau^\mu p_\tau^\nu + p_\nu^\mu p_\tau^\nu - g^{\mu\nu} p_\tau^2) + i\epsilon^{\mu\nu\alpha\beta}(p_\tau^\alpha p_\nu^\beta),$$

and $W_{\mu\nu}$ is the hadronic tensor

$$W_{\mu\nu} = (2\pi)^3 \sum_X \delta^4(p_{\Lambda_b} - q - p_X) \langle \Lambda_b(v, s)| J_\mu |X \rangle \langle X| J_\nu| \Lambda_b(v, s) \rangle$$

with $p_{\Lambda_b} = m_{\Lambda_b} v, q = p_\tau + p_\nu$ and $s$ being the spin of $\Lambda_b$. $J_\mu = \bar{c} \gamma_\mu (1 - \gamma_5) b$ in eq.(3) is the hadronic current. The expansion of $W_{\mu\nu}$ in terms of Lorentz invariant structure functions $W_i$ is defined by

$$W_{\mu\nu} = -g_{\mu\nu} W_1 + v_\mu v_\nu W_2 - i\epsilon_{\mu\nu\alpha\beta} v^\alpha q^\beta W_3 + q_\mu q_\nu W_4 + (q_\mu v_\nu + q_\nu v_\mu) W_5 + \left\{ -g_{\mu\nu} W_1 + \epsilon_{\mu\nu\alpha\beta} v^\alpha q^\beta W_3 + q_\mu q_\nu W_4 + (q_\mu v_\nu + q_\nu v_\mu) W_5 \right\}$$

$$+ (s_\mu v_\nu + s_\nu v_\mu) W_6 + (s_\mu q_\nu + s_\nu q_\mu) W_7 + i\epsilon_{\mu\nu\alpha\beta} v^\alpha s^\beta W_8 + i\epsilon_{\mu\nu\alpha\beta} q^\alpha s^\beta W_9 \right\}.$$ (4)

The structure functions $W_i$ can be calculated in heavy quark effective theory (HQET) [17] and results to the $1/m_b^2$ order are

$$W_1 = \delta(z) \left( \frac{m_b}{2} + \frac{K_b m_b}{6} - \frac{q.v}{2} \right) - \delta'(z) \left( \frac{2K_b m_b q^2}{3} + K_b m_b^2 q.v - \frac{5K_b m_b q.v^2}{3} \right) +$$

$$\delta''(z) \left( -\frac{2K_b m_b^3}{3} (q^2 - q.v^2) \right) + 2K_b m_b^2 q.v \left( \frac{q^2 - q.v^2}{3} \right),$$
\[ W_2 = \delta(z) \left( m_b + \frac{5 K_b m_b}{3} \right) - \frac{14 \delta'(z) K_b m_b^2 q \cdot v}{3} + \frac{\delta''(z) - 4 K_b m_b^3 (q^2 - q \cdot v^2)}{3}, \]
\[ W_3 = \frac{\delta(z) - 5 \delta'(z) K_b m_b q \cdot v}{3} + \frac{\delta''(z) - 2 K_b m_b^2 (q^2 - q \cdot v^2)}{3}, \]
\[ W_4 = -\frac{4 \delta'(z) K_b m_b}{3}, \]
\[ W_5 = \frac{-\delta(z) - \delta'(z) \left( -4 K_b m_b^2 - 5 K_b m_b q \cdot v \right)}{3} + \frac{2 \delta''(z) K_b m_b^2 (q^2 - q \cdot v^2)}{3}, \]
\[ W_6^* = \frac{2 \delta'(z) K_b m_b}{3}, \]
\[ W_7^* = 0, \]
\[ W_8^* = \frac{2 \delta'(z) K_b m_b}{3}, \]
\[ W_9^* = \frac{2 \delta'(z) K_b m_b}{3}, \]
\[ W_6^* = \frac{-\delta(z) (1 + \epsilon_b)}{2} - \frac{5 K_b m_b}{3} - \frac{\delta'(z) \left( -5 K_b m_b^2 q \cdot v \right)}{3} + \frac{2 \delta''(z) K_b m_b^3 (q^2 - q \cdot v^2)}{3}, \]
\[ W_7^* = \frac{2 \delta''(z) K_b m_b^2 (q^2 - q \cdot v^2)}{3}, \]
\[ W_8^* = \frac{-\delta(z) (1 + \epsilon_b)}{2} - \frac{K_b m_b}{6} - \frac{\delta'(z) \left( 5 K_b m_b^2 q \cdot v \right)}{3} - \frac{2 \delta''(z) K_b m_b^3 (q^2 - q \cdot v^2)}{3}, \]
\[ W_9^* = \frac{2 \delta''(z) K_b m_b^2 (q^2 - q \cdot v^2)}{3}. \]  

(5)

where
\[ K_b = -\langle \Lambda_b(v, s) | \bar{b} \gamma^\lambda \gamma_5 b | \Lambda_b(v, s) \rangle, z = (m_b v - q^2)^2 - m_c^2, \]  

(6)

and \( \epsilon_b \) is defined by
\[ \langle \Lambda_b(v, s) | \bar{b} \gamma^\lambda \gamma_5 b | \Lambda_b(v, s) \rangle = (1 + \epsilon_b) u(v, s) \gamma^\lambda \gamma_5 u(v, s). \]  

(7)

\( K_b \) and \( \epsilon_b \) are only unknown two parameters at \( O(m_b^{-2}) \) for the \( \Lambda_b \) decay which parametrize the nonperturbative phenomena and are expected to be of order \( (\Lambda_{QCD}/m_b)^2 \). We will discuss them in section IV. The other parameter at \( O(m_b^{-2}) \)
\[ G_b = Z_b |H_b(v, s) | \bar{b} \gamma^\lambda \gamma_5 b |H_b(v, s) \rangle \]  

(8)

is equal to zero for \( H_b = \Lambda_b \) due to the zero spin of the light degrees of freedom inside \( \Lambda_b \). \( W_i(i=1,2,3) \) and \( W_i^*(i=4,5,6,7,8,9) \) have already been given by Manohar and Wise \[ \text{[3]} \] and \( W_i(i=4,5) \) by Balk et al. \[ \text{[3]} \]. We list them here only for completeness. From eqs(1),(2),(4) and (5) we get the differential decay rate
\[ \frac{d \Gamma_W}{\Gamma_b dt dx dy d\cos \theta} = \hat{A}(x, t, y, \eta, \epsilon) + \hat{B}(x, t, y, \eta, \epsilon) \cos \theta, \]  

(9)

here
\[ \hat{A}(x, t, y, \eta, \epsilon) = \left[ -12 ty + 12 ty - 12 y \eta + K_b (16 t + 20 xy + 16 \eta) \right] \delta(z) - \\
4 K_b (4 t^2 - 4 t y - 4 t y - 5 t x y + 7 x^2 y - 5 t y^2 + 7 x y^2 + 4 x \eta \\
-4 y \eta - 5 x y \eta - 5 y^2 \eta - 4 \eta^2) \delta'(z) + 4 K_b (4 t - x^2 - 2 x y \\
- y^2) (t - x + \eta) \delta''(z), \]

\[ \hat{B}(x, t, y, \eta, \epsilon) = \frac{1}{\sqrt{x^2 - 4 \eta}} \left( 24 t^2 - 24 t x y + 12 x^2 y + 24 x \eta - 12 x y \eta - \\
24 \eta^2 + K_b (-24 t x + 20 x^2 y + 24 x \eta - 32 y \eta) + \epsilon_b (24 t^2 - 24 t x \\
-12 t x y + 12 x^2 y + 24 x \eta - 12 x y \eta - 24 \eta^2) \right) \delta(z) - 4 K_b (8 t^2 + \\
6 t^2 x - 10 t x^2 + 10 t^2 y - 18 t x y - 5 t x^2 y + 7 x^3 y - 5 t x y^2 + \\
7 x^2 y^2 + 10 x^2 \eta + 8 t y \eta + 2 x y \eta - 5 x^2 y \eta - 5 x y^2 \eta - 8 \eta^2 - \\
6 x \eta^2 - 2 y \eta^2) \delta'(z) + 4 K_b (4 t - x^2 - 2 x y - y^2) (t - x + \eta) ( \\
-2 t + x y + 2 \eta) \delta''(z) \right) \]

(10)

where

\[ \Gamma_b = \frac{G_F^2 |V_{cb}|^2 m_b^5}{192 \pi^3}, x = \frac{2E_x}{m_b}, y = \frac{2E_y}{m_b}, t = \frac{q^2}{m_b^2}, \eta = \frac{m_z^2}{m_b^2}, \epsilon = \frac{m_c}{m_b} \]

and \( \theta \) is the angle between the \( \tau \) direction and the \( \Lambda_b \) spin in the rest frame of the \( \Lambda_b \). After integrating over \( t \) and \( y \), one obtains the \( \tau \) energy spectrum

\[ \frac{d\Gamma_W}{d\epsilon d\epsilon \cos \theta} = A_W(x, \eta, \epsilon) + B_W(x, \eta, \epsilon) \cos \theta, \]

(11)

where \( A_W(x, \eta, \epsilon) \) and \( B_W(x, \eta, \epsilon) \) are given in Appendix. The \( \eta \to 0 \) limits of \( A_W \) and \( B_W \) agree with the results of Manohar and Wise [3]. The total inclusive decay width of \( \Lambda_b \to X_\tau \nu \) can be obtained by integrating the spectrum formula over range

\[ 2 \sqrt{\eta} \leq x \leq 1 - \rho + \eta, \]

the result is not present here because one can easily obtain it from ref. [3] by taking \( G_b = 0 \). Perturbative \( O(\alpha_s) \) QCD corrections to the double differential distribution of the \( \tau \) energy and the invariant mass of the lepton system for \( b \to c \tau \nu \) have been studied by M. Ježabek et al. [18,19]. We will use eq.(30) in ref. [19] in our numerical analysis for the nonpolarized distribution of the \( \tau \) energy.

### III. \( \tau \) spectrum of \( \Lambda_b \to X_\tau \nu \) in THDM

We consider the THDM [22] in which the up-type quarks get masses from Yukawa couplings to the one Higgs doublet \( H_2 \) (with the vacuum expectation value \( v_2 \)) and down-type quarks and leptons get masses from Yukawa couplings to the another Higgs doublet \( H_1 \) (with the vacuum expectation value \( v_1 \)). Such a model occurs as a natural feature in supersymmetric theories. For the sake of simplicity we shall use the Feynman rules of the THDM in MSSM [23]. In a THDM there are three diagrams contributing to the decay \( \tau \) spectrum of \( \Lambda_b \to X_\tau \nu \) which correspond to W-exchange, Goldstone boson-exchange, and charged Higgs boson-exchange respectively if one uses a non-physical gauge. It is shown in ref. [3] that in the Landau gauge the rate can be decomposed into the sum of two incoherent decays:

\[ M = M_W + M_S \Rightarrow |M|^2 = |M_W|^2 + |M_S|^2 \]

where \( M_S = M_G + M_M, M_W, M_G \) and \( M_M \) are the W-mediated, Goldstone boson-mediated and Higgs boson-mediated decay amplitudes respectively. This decomposition has an advantage to simplify calculations, in particular, the calculations of QCD corrections. We assume the Landau gauge hereafter. Then the new thing needed to do is to calculate \( M_S \).

For the purpose of calculating \( M_S \) the hadronic current \( J_\mu \) in eq.(3) is replaced by

\[ J_i = \bar{c}(a_i + b_i \gamma_5)b \quad (i = H, G) \]

(12)

with
\[ a_H = m_0 \tan \beta + m_c \cot \beta, \quad b_H = m_b \tan \beta - m_c \cot \beta, \quad a_G = -m_b + m_c, \quad b_G = -m_b - m_c. \]

Following the same steps as those in section II, a straightforward calculation leads to

\[
\frac{d\Gamma_H}{d\theta} = A_H(x, \eta, \epsilon, \xi, \tan \beta) + B_H(x, \eta, \epsilon, \xi, \tan \beta) \cos \theta, \tag{13}
\]

\[
\frac{d\Gamma_I}{d\theta} = A_I(x, \eta, \epsilon, \xi, \tan \beta) + B_I(x, \eta, \epsilon, \xi, \tan \beta) \cos \theta, \tag{14}
\]

with \( A_i(x, \eta, \epsilon, \xi, \tan \beta) \) and \( B_i(x, \eta, \epsilon, \xi, \tan \beta)(i = H, I) \) given in Appendix. Here \( d\Gamma_i/d\theta(i = H, I) \) denotes the contributions to the \( \tau \) spectrum from Higgs-mediated and the interference term between Higgs-mediated and Goldstone boson-mediated respectively, and \( \xi = m_H/m_b \).

For the spin independent terms \( A_H \) and \( A_I \), our results agree with those obtained by Y. Grossman et al. \[24\]. Note that \( B_I = 0 \) at the leading order of \( 1/m_b \) expansion as \( m_c \to 0 \). This is due to the chiral difference of the vertices of \( W \) and Higgs. As we can see in numerical analysis that this makes \( B_I \) much smaller than \( B_W \).

Combining eq.(11), eq.(13) and eq.(14), one obtains the \( \tau \) spectrum of \( A_b \to X_c \tau \nu \) in the THDM

\[
\frac{d\Gamma_{THDM}}{d\theta} = (A_W + A_I + A_H) + (B_W + B_I + B_H) \cos \theta. \tag{15}
\]

We now come to the position to calculate \( O(\alpha_s) \) QCD corrections. Making use of the results obtained by A.Czarnecki et al. \[24\] and Y. Grossman et al. got the \( O(\alpha_s) \) corrections of the total width of \( b \to c \tau \nu \) mediated by Higgs \[8\]. To get the \( O(\alpha_s) \) corrections of \( \tau \) spectrum one can also use their results. We find that the relation between \( d\Gamma_{\alpha_s}^{H(I)}/dxdt \) and \( d\Gamma_{\alpha_s}^{H(I)}/dt \) is very simple, as expected. \( d\Gamma_{\alpha_s}^{H(I)}/dxdt \) is independent of \( x \) because Goldstone boson and Higgs are both scalar particles. Therefore, \( d\Gamma_{\alpha_s}^{H(I)}/dxdt \) can be simply obtained by dividing \( d\Gamma_{\alpha_s}^{H(I)}/dt \) by \( x_{\text{max}} - x_{\text{min}} \), where \( x_{\text{max}}, x_{\text{min}} \) denote \( x \)'s kinematical upper and lower limits respectively. \( d\Gamma_{\alpha_s}^{H(I)}/dt \) can be easily obtained from eq.(8) of ref. \[24\] by multiplying the lepton part. The results are

\[
\frac{d\Gamma_{\alpha_s}^{H}}{dxdt} = \frac{\sqrt{3}m_b^2\eta(t-\eta)\tan^2 \beta}{16\pi^2 \xi^3 p_3} \Gamma(c_1^H, c_2^H, c_3^H), \]

\[
\frac{d\Gamma_{\alpha_s}^{I}}{dxdt} = \frac{\sqrt{2}m_b^2\eta(t-\eta)\tan \beta}{8\pi^2 \xi^3 p_3} \Gamma(c_1^I, c_2^I, c_3^I), \tag{16}
\]

here

\[
c_1^H = 2 \tan^2 \beta + 2 \epsilon^2 \cot^2 \beta, \quad c_2^H = 4 \epsilon, \quad c_3^H = \tan^2 \beta - \epsilon^2 \cot^2 \beta,
\]

\[
c_1^I = -2 \tan^2 \beta + 2 \epsilon^2 \cot^2 \beta, \quad c_2^I = 2 \epsilon (\tan \beta - \cot \beta), \quad c_3^I = -\tan \beta - \epsilon^2 \cot \beta, \tag{17}
\]

and \( \Gamma(c_1, c_2, c_3) \) is \[23\]

\[
\Gamma(c_1, c_2, c_3) = \frac{\alpha_s}{6\pi^2} \frac{G_F m_b^3 |V_{cb}|^2}{\sqrt{2}} [c_1 G_+ + c_2 G_- + c_3 G_0] \tag{18}
\]

with

\[
G_+ = p_0 H + p_0 |L_{i2}(p_+)| - 2L_{i2}(1 - \frac{1}{p_+}) - 2L_{i2}(1 - \frac{1}{p_+}) + \frac{1}{2} Y_p \ln \left( \frac{16\rho_p^2}{\epsilon^2 t} \right) - Y_w \ln |\epsilon| + 2Y_w(1 - \epsilon^4) + \frac{2}{t} p_3 \ln (\epsilon + t - \epsilon^2),
\]

\[
G_- = H + p_3 (6 - 2 \ln \left( \frac{16\rho_p^2}{\epsilon^2 t} \right) + \frac{1}{t} Y_p (1 - t - \epsilon^2 + 4 - 3\epsilon^2),
\]

\[
G_0 = -6 p_0 p_3 \ln \epsilon,
\]

where

\[
H = 4 p_0 |L_{i2}(p_+)| - L_{i2}(p_-) - 2L_{i2}(1 - \frac{1}{p_+}) + \frac{1}{2} Y_p \ln \left( \frac{16\rho_p^2}{\epsilon^2 t} \right) - Y_w \ln |\epsilon| + 2Y_w(1 - \epsilon^4) + \frac{2}{t} p_3 \ln (\epsilon + t - \epsilon^2),
\]

\[
p_0 \equiv \frac{1}{2} (1 - t + \epsilon^2), \quad p_3 \equiv \frac{1}{2} \sqrt{1 + t^2 + \epsilon^4 - 2(t + \epsilon^2 + \epsilon^2)}, \quad p_\pm \equiv p_0 \pm p_3,
\]

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\[ Y_p \equiv \frac{1}{2} \ln \frac{p_+}{p_-}, W_0 \equiv \frac{1}{2} (1 + t - e^2), W_\pm \equiv W_0 \pm p_3, Y_w \equiv \frac{1}{2} \ln \frac{W_+}{W_-.} \]

After integrating (16) over \( t \), we get \( \tau \) spectrum numerically with only parameters \( \tan \beta \) and \( \xi \). As a check, we find that our numerical results of \( \Gamma^H_{\alpha_x} \) and \( \Gamma^I_{\alpha_x} \) agree with those obtained by Y. Grossman et al. 

IV. Numerical Results

In order to do numerical calculations we need to discuss values of parameters which are \( K_b, \epsilon_b, m_c, m_b \) and the parameters in THDM.

1. Constraints on the parameters of THDM

In ref. \[25,26\], constraints on \( \tan \beta \) from \( K - \bar{K} \) and \( B - \bar{B} \) mixing, \( \Gamma(b \to s\gamma), \Gamma(b \to c\tau\nu) \) and \( R_b \) have been given

\[ 0.7 < \tan \beta < 0.6 \frac{m_{H_+}}{1 \text{ GeV}} \]

and also the lower limit \( m_{H_+} > 200 \text{ GeV} \) has been given there. Taking the radiative correction and the \( 1/m_Q^2 \) correction into account in the B meson decay, Y. Grossman et al. have a improved bound of \( R \) (defined by \( R = \tan \beta / m_{H_\perp} \)) which is

\[ R < 0.49 \text{ GeV}^{-1} \]

We will predict the \( \tau \) spectrum and total width of \( \Lambda_b \) decay under these constraints.

2. About the parameters \( K_b, \epsilon_b \) and \( m_b, m_c \)

\( K_b \) and \( \epsilon_b \) characterize the \( 1/m_Q^2 \) corrections to the decay distribution for \( \Lambda_b \to X_c\tau\nu \) and are nonperturbative quantities independent of \( m_Q \). Since quarks are not free physical particles, \( m_b \) and \( m_c \) can not be determined directly by experiment. However, we can estimate them by the phenomenological analysis of the heavy hadron spectra to the order \( 1/m_Q \). From the effective Lagrangian in HQET, the mass of a heavy hadron can be written as \[30,32\]

\[ m_{b_Q} = m_Q + \bar{\Lambda} (j^P_{I,S}) + \frac{a(j^P_{I,S})}{m_Q} + \frac{b(j^P_{I,S})}{m_Q} (\vec{S}_Q \cdot \vec{j}_l) + ... \]

(19)

where \( \langle \vec{S}_Q \cdot \vec{j}_l \rangle = \frac{1}{2}[J(J+1) - j_l(j_l + 1) - \frac{3}{2}] \) with J and \( j_l \) being the spins of the hadron and the light degrees of freedom inside the hadron respectively. The parameter \( \bar{\Lambda} \) represents contributions come from the effective Lagrangian in the \( m_Q \to \infty \) limit, and \( a, b \) are respectively associated with the kinetic energy and the color magnetic energy of the heavy quark inside the hadron. In present case, \( a(0^+, 0, 0) = m_b^2 K_b \) and \( b(0^+, 0, 0) = 0 \). It is shown that \( \epsilon_b \leq -2/3 K_b \). Furthermore, one can take \( \epsilon_b = -2/3 K_b \) if one omits the contributions of terms arising from double insertions of chromomagnetic operator \[27,29\]. Starting from eq.(19), it is shown \[32\] that one can obtain \( a(1^-/2, 1^+, 0), m_c \) (and \( \bar{\Lambda}(1^-/2, 1^+, 0, b(1^-/2, 1^+, 0) \) ) by using the observed masses of the doublets \( (B^*, B) \) and \( (D^*, D) \) if choosing \( m_b \) as input. Furthermore, \( a(0^+ , 0, 0) \) which is the parameter we need is determined by \[32,38\]

\[ a(0^+, 0, 0) = \frac{h m_b}{1 - h} [(m_{\Lambda_b} - \bar{m}_D) - (m_{\Lambda_b} - \bar{m}_B)] + a(1^-/2, 1^+, 0), \]

(20)

where

\[ h = \frac{m_{B^*} - m_B}{m_{D^*} - m_D}, \]

(21)

and \( \bar{m}_H = \frac{1}{4}(m_H + 3m_{H^\ast}) \), \( \Lambda = B, D \). Therefore, if we choose \( m_b = 5.1 \text{ GeV} \), the other parameters will be†:

\[ m_c = h m_b = 1.65 \text{ GeV} \]

† the experimental data used in ref. \[32\] have been improved since then. Our data are from ref. \[33\].
\[ K_b = \frac{0.142 m_b GeV}{m_b^2} \approx 0.009, \alpha_b \approx -0.006 \]

If we choose \( m_b = 5.044 \) which is a "critical" value based on eq.(19) \[32\], other parameters will be
\[ m_c = 1.63 GeV, K_b \approx 0.006, \alpha_b \approx -0.004 \]

We will use these two sets of values and discriminate them by the first(\( m_b = 5.044 GeV \)) and second(\( m_b = 5.1 GeV \)) set respectively in the numerical computations.

Using the parameters given above, we obtain the total width in terms of \( \tan\beta \) and \( m_H \) as follows:
\[
\begin{align*}
\Gamma^1_W & = C^1_W + D^1_W \alpha_s, \\
\Gamma^1_H + \Gamma^1_L & = C^1_H + D^1_H \alpha_s, \\
\Gamma^2_W & = C^2_W + D^2_W \alpha_s, \\
\Gamma^2_H + \Gamma^2_L & = C^2_H + D^2_H \alpha_s, \\
\end{align*}
\]

where
\[
\begin{align*}
C^1_W & = 0.109, \\
C^1_H & = \frac{0.0141 \left( 0.253 + 1.16 \tan^2 \beta \right)}{\xi^2} + \frac{0.0141 \left( 0.025 + 0.112 \tan^2 \beta + 0.239 \tan^4 \beta \right)}{\xi^4}, \\
D^1_W & = -0.0476, \\
D^1_H & = \frac{0.00894 \left( -0.165 - 0.577 \xi^2 - 0.374 \tan^2 \beta - \xi^2 \tan^2 \beta - 0.331 \tan^4 \beta \right)}{\xi^4}, \\
C^2_W & = 0.112, \\
C^2_H & = \frac{0.0139 \left( 0.262 + 1.19 \tan^2 \beta \right)}{\xi^2} + \frac{0.0139 \left( 0.0256 + 0.107 \tan^2 \beta + 0.245 \tan^4 \beta \right)}{\xi^4}, \\
D^2_W & = -0.0493, \\
D^2_H & = \frac{0.0082 \left( -0.163 - 0.574 \xi^2 - 0.37 \tan^2 \beta - \xi^2 \tan^2 \beta - 0.328 \tan^4 \beta \right)}{\xi^4}. \\
\end{align*}
\]

The superscript \( i \) (\( i=1, 2 \)) in eqs.\((22)\) and \((23)\) denotes that the \( i \)th set of values of \( K_c, \alpha_c, m_c \) and \( m_b \) is used.

The explicit dependence of total width \( \Gamma^\tau_{THDM} = \Gamma_W + \Gamma_H + \Gamma_L \) (normalized to the electron channel) on \( \tan\beta \) and \( m_H \) is plotted in fig.1. It should be noted that the absolute value of \( \Gamma^\tau_{THDM} \) is very sensitive to \( m_b \). Using the ratio between the width \( \Gamma^\tau_{THDM} \) of \( H^\tau \) and \( \Gamma^\tau_{THDM} \) for the second set of parameters is three percent larger than that for the first set. (2)

In the range of \( \tan\beta \) which is interesting physically, say, \( \tan\beta \leq 60 \), the normalized width changes roughly five to fifteen percent when \( \tan\beta \) changes from twenty to sixty and \( m_{H^\pm} \) is fixed. The normalized width changes the same order of magnitude for fixed \( \tan\beta \) and changing \( m_{H^\pm} \) from 200 to 400 Gev. From eqs. \((22)\) and \((23)\) it follows that the \( \alpha_s \) corrections decrease the total width roughly 20 percent which is larger than that in SM. Because when \( \tan\beta \gg 1 \), \( C^\tau_H \) is proportional to \( r^4 \left( -1 + 0.2 r^2 \right) \) and \( D^\tau_H \) is proportional to \( -r^2 \left( 1 + 0.3 r^2 \right) \) where \( r=R m_b \), one can obtain constraints on \( r \) from the measurement of the total width.

\( \tau \) spectrum for some typical values of \( \tan\beta \) and \( m_H \) is calculated and the result is plotted in figs.2-5. The predictions in the SM are also plotted in the figs. Here the \( \alpha_s \) corrections for the spin-dependent term are not considered. We can see from figs.4-5 that the spin-dependent spectrum is quite different for \( r \leq 1 \) and \( r > 2 \) (Note that we have the constraints \( r < 0.49 Gev^{-1} m_b \approx 2.5 \) and \( m_{H^\pm} \geq 200 Gev \) from experiments, as mentioned before). The reason is as follows. We know from Appendix that \( B_H = -\frac{r^2 \eta}{8 \xi^2} B_W \approx -r^4 \eta B_W / 4 \)

That is, it depends \( r^4 \). For \( r \leq 1 \), \( B_H \ll B_W \). As pointed in section III, \( B_I \) is negligibly small comparing with \( B_W \) due to the chiral difference of b quark couplings to W and \( H^\pm \) which is deduced from Model II of
THDM. Therefore, the spectrum is almost the same as that in the SM, as can be seen from figs.4-5. For \( r \geq 2 \), \( B_H \) is as the same order of magnitude as \( B_W \) so that \( B_H \) and \( B_W \) tend to cancel each other, which makes the spin-dependent distribution of \( \tau \) energy very small and a little dependent of the \( \tau \) energy, as shown in figs.4-5. Thus one can say that if the \( \tau \) spectrum is somewhat more isotropic than what the SM predicts, the THDM with large \( \tan \beta \) (> 80) and \( m_H \geq 200 \text{ Gev} \) is preferred in describing the nature. For the nonpolarized term, as can be seen from figs.4-5, the spectrum is very similar to that of B decay since the difference between the \( \Lambda_b \) decay and B decay comes from the \( 1/m_b^2 \) corrections.

V. Summary

In summary, we have calculated the rate and \( \tau \) spectrum of the inclusive semileptonic decay for a polarized \( \Lambda_b \rightarrow X_c\tau\nu \) to the \( 1/m_b^2 \) order in the \( 1/m_b \) expansion in the SM. The \( \alpha_s \) corrections are included in the numerical computations for the spin independent terms of \( \tau \) spectrum. Our results show that the spin dependent \( \tau \) spectrum is significant enough to be seen. We have also calculated the same quantities in a THDM. For the spin independent terms of \( \tau \) spectrum arising from Higgs-mediated and the interference term, we have calculated the \( O(\alpha_s) \) QCD corrections to the double differential distribution. Together with the \( \alpha_s \) corrections in the SM given in ref. [18], we obtained all the \( \alpha_s \) corrections to the non-polarized double differential distribution (and so the total width) in the THDM. The numerical results show that the branching ratio of \( \Lambda_b \rightarrow X_c\tau\nu \) in THDM is of approximately 25 percent of that in the electron channel and the spin dependent \( \tau \) spectrum can be used to estimate the size of \( \tan \beta \) and \( m_{H^\pm} \). The spectrum depends dominantly on \( R \) if \( \tan \beta \gg 1 \) so that from the measurement of the angular distribution of a polarized \( \Lambda_b \rightarrow X_u\tau\nu \) in B-factories within the coming years one can obtain constraints on \( R \).

It is obvious that substituting the u quark mass \( m_u = 0 \) one immediately obtains the decay rate and \( \tau \) spectrum for a polarized \( \Lambda_b \rightarrow X_u\tau\nu \). And with minor changes one can extend the results in the paper to the inclusive semileptonic decay of a polarized \( \Lambda_c \rightarrow X_{s,d}\tau\nu \). It is interesting to calculate the \( \alpha_s \) corrections to the spin dependent term of \( \tau \) spectrum.

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Appendix: Expressions of $A_i$ and $B_i (i = W, H, I)$

$$A_W = \frac{\sqrt{x^2-4\eta}}{(1-x+\eta)^3} \left( -1 + \epsilon^2 + x - \eta \right)^2 \left( 3x + 3\epsilon^2 x - 5x^2 - \epsilon^2 x^2 + 2x^3 - \right. $$
\begin{align*}
& 4\eta - 8\epsilon^2 \eta + 10x\eta + 3\epsilon^2\eta x - 5x^2 \eta - 4\eta^2 + 3x\eta^2 \right) + \\
& 2K_h \frac{\sqrt{x^2-4\eta}}{3(1-x+\eta)^5} \left( -5x^2 - 15\epsilon^4 x^2 + 20\epsilon^6 x^2 + 25x^3 + 21\epsilon^4 x^3 - \\
& 10\epsilon^6 x^3 - 50x^4 - 6\epsilon^4 x^4 + 2\epsilon^6 x^4 + 50x^5 - 25x^6 + 5x^7 + 14\eta + 6\epsilon^4 \eta \\
& -20\epsilon^6 \eta - 70x\eta + 78\epsilon^6 x\eta - 80\epsilon^6 x^2 \eta + 115x^2 \eta - 147\epsilon^4 x^2 \eta + \\
& 44\epsilon^6 x^2 \eta - 40x^3 \eta - 60\epsilon^4 x^3 \eta - 10\epsilon^6 x^3 \eta - 80x^4 \eta - 6\epsilon^4 x^4 \eta + 86x^5 \eta \\
& -25\epsilon^6 \eta + 70\eta^2 - 126\epsilon^4 x^2 \eta + 152\epsilon^6 \eta^2 - 280x^2 \eta^2 + 300\epsilon^4 x \eta^2 - 80\epsilon^6 x \eta^2 \\
& +370x^2 \eta^2 - 147\epsilon^4 x^2 \eta^2 + 20\epsilon^6 x^2 \eta^2 - 130x^3 \eta^2 + 21\epsilon^4 x^3 \eta^2 - 80x^4 \eta^2 \\
& +50\epsilon^5 \eta^2 + 140\eta^3 - 126\epsilon^4 \eta^3 - 20\epsilon^6 \eta^3 - 420x^2 \eta^3 + 78x^3 \eta^3 + 370\epsilon^4 x^3 \eta^3 \\
& -15\epsilon^5 x^3 \eta^3 - 40x^4 \eta^3 - 50x^4 \eta^3 + 140\eta^4 + 6\epsilon^4 \eta^4 - 280x^2 \eta^4 + 115x^2 \eta^4 \\
& +25x^3 \eta^4 + 70\eta^5 - 70x^2 \eta^5 - 5x^2 \eta^5 + 14\eta^6 \right),
\end{align*}

(24)

$$B_W = \frac{(x^2 - 4\eta)}{(1-x+\eta)^3} \left( -1 + \epsilon^2 + x - \eta \right)^2 \left( 1 - \epsilon^2 - 3x - \epsilon^2 x - 2x^2 + 4\eta + \\
3\epsilon^2 \eta - 5x\eta + 3\eta^2 \right) + \epsilon_\omega \frac{(x^2 - 4\eta)}{(1-x+\eta)^3} \left( -1 + \epsilon^2 + x - \eta \right)^2 \left( 1 - \epsilon^2 - 3x - \right. $$
\begin{align*}
& \epsilon^2 x + 2x^2 + 4\eta + 3\epsilon^2 \eta - 5x\eta + 3\eta^2 \right) + 2K_h \frac{(x^2 - 4\eta)}{3(1-x+\eta)^5} \left( 5x - \\
& 15\epsilon^4 x + 10\epsilon^6 x - 25x^2 + 21\epsilon^4 x^2 + 4\epsilon^6 x^2 + 50x^3 - 6\epsilon^4 x^3 - 2\epsilon^6 x^3 \\
& -50x^4 + 25x^5 - 5\epsilon^6 x^5 + 36\epsilon^4 \eta - 36\epsilon^6 \eta + 25x \eta - 51\epsilon^4 x \eta - 22\epsilon^6 x \eta \\
& -100x^2 \eta + 10\epsilon^6 x^2 \eta + 150x^3 \eta + 6\epsilon^4 x^3 \eta - 100x^4 \eta + 25x^5 \eta + 60\epsilon^6 \eta^2 \\
& +50x^2 \eta^2 + 51\epsilon^4 x \eta^2 - 20\epsilon^6 x \eta^2 - 150x^2 \eta^2 + 21\epsilon^4 x^2 \eta^2 + 150x^3 \eta^2 \\
& -50x^4 \eta^2 - 36\epsilon^4 \eta^3 + 50x^3 \eta^3 + 15\epsilon^4 x^3 \eta^3 - 100x^2 \eta^3 + 50x^3 \eta^3 + 25x \eta^4 \\
& -25x^2 \eta^4 + 5x \eta^5 \right) \right),
\end{align*}

(25)

$$A_H = \frac{\eta \tan^2 \beta \sqrt{x^2-4\eta}}{8\xi^4(1-x+\eta)^3} \left( 1 - \epsilon^2 - x - \eta \right)^2 \left( L \left( 6\epsilon x - 6\epsilon x^2 - 12\epsilon \eta + \\
\end{align*}

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[The natural text of the mathematical equations is complex and requires careful reading to understand. It involves multiple variables and operations such as addition, subtraction, multiplication, division, and trigonometric functions. The equations are presented in a form that requires attention to detail and understanding of mathematical notation.]
\[-120 \eta^2 + 66 \epsilon^2 \eta^2 + 76 \eta^3 - 16 \epsilon^2 \eta^3 + 2 \epsilon^4 \eta^3 - 8 \eta^4 - 6 \epsilon^2 \eta^4 - 6 \epsilon^2 \eta^4 + 6 \epsilon^4 \eta^4 + 54 \epsilon^2 \eta^4 + 4 \epsilon^2 \eta^4 + 6 \epsilon^2 \eta^4 + 4 \epsilon^2 \eta^4 - 96 \eta^4 + 48 \epsilon^4 \eta^4 + 212 \eta^4 - 52 \epsilon^2 \eta^4 - 2 \epsilon^2 \eta^4 + 6 \epsilon^2 \eta^4 + 4 \epsilon^2 \eta^4 + 4 \epsilon^2 \eta^4 - 64 \eta^4 + 16 \epsilon^2 \eta^4 + 74 \eta^4 - 6 \epsilon^2 \eta^4 - 16 \eta^6 + 2 \eta^6 \] \quad (29)

where

\[ F = 2 \tan^2 \beta + 2 \epsilon^2 \cot^2 \beta, \quad J = 2 \tan^2 \beta - 2 \epsilon^2 \cot^2 \beta, \quad L = 4 \epsilon. \]

When \( \tan \beta \gg 1 \), eqs. (3), (4) and (5) reduce to

\[
A_H = \frac{r^4}{4} A_W + O(r^4 \tan^2 \beta^2), \quad (30)
\]

\[
B_H = -\frac{r^4}{4} B_W + O(r^4 \tan^2 \beta^2), \quad (31)
\]

\[
A_I = -\frac{6 \eta r^2 \sqrt{x^2 - 4 \eta}}{(1 - x + \eta)} (1 - x + \eta - \epsilon^2)^2 + O(r^2 \tan^2 \beta^2). \quad (32)
\]

FIG. 1. Total width (normalized to the electron channel) in terms of \( \tan \beta \) and \( m_H \), using the (a) first \( (m_b = 5.044 \text{ GeV}) \), (b) second \( (m_b = 5.1 \text{ GeV}) \) set of value. The curves terminating at \( \tan \beta = 100, 150, 200 \) correspond to \( m_H = 200 \text{ GeV}, 300 \text{ GeV}, 400 \text{ GeV} \) respectively.

FIG. 2. \( \tau \) spectrum for different \( \alpha_s \) and \( \tan \beta \), \( m_H = 200 \text{ GeV} \). The first set of parameter value \( (m_b = 5.044 \text{ GeV}) \) is used.

FIG. 3. \( \tau \) spectrum for different \( \alpha_s \) and \( \tan \beta \), \( m_H = 200 \text{ GeV} \). The second set of parameter value \( (m_b = 5.1 \text{ GeV}) \) is used.

FIG. 4. \( \tau \) spectrum for different \( \tan \beta \), \( m_H = 200 \text{ GeV} \). The first set of parameter value \( (m_b = 5.044 \text{ GeV}) \) is used. \( O(\alpha_s) \) corrections are not considered here.

FIG. 5. \( \tau \) spectrum for different \( \tan \beta \), \( m_H = 200 \text{ GeV} \). The second set of parameter value \( (m_b = 5.1 \text{ GeV}) \) is used. \( O(\alpha_s) \) corrections are not considered here.
$A_W + A_H + A_I = 0.3$

$tan\beta = 90$

$tan\beta = 50$

$tan\beta = 20$

$tan\beta = 1$

$x = 2 E_\gamma / m_b$
SM result
$tan \beta = 90$
$tan \beta = 50$
$tan \beta = 20$
$tan \beta = 1$

$B_{W} + B_{H} + B_{I}$

$x = 2 \frac{E_{\gamma}}{m_{b}}$