Influence of the degree of off-design of the traction nozzle of a jet reaction turbine on its efficiency

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Abstract. At the present stage of social development, the need for rational use of energy resources of the planet is particularly acute. Most countries have already faced the problem of the energy crisis. An appropriate way to solve this problem is the utilization of secondary energy resources. With the help of turbine generators, it is possible to reduce the pressure and utilize the potential energy of excess pressure of gas or vapours to produce electricity. Such units can be created based on a jet reaction turbine. The study aims to assess the impact of the degree of off-design of the traction nozzle on the efficiency of the jet reactive turbine. The analysis of the factors influencing the efficiency of the jet reaction turbine is carried out in the work. The obtained efficiency rate for the jet reactive turbine depending on the given dimensionless velocity of \( \lambda_{\text{out}} \) at the inlet pressure of the inlet nozzle of 2, 4, 6 and 10 MPa in the design mode (at the degree of off-design of \( S = 1 \)) and in the off-design modes of operation (\( S > 1 \)). Graphical dependencies of the jet reaction turbine efficiency rate on the given circular velocity of the impeller are constructed (\( U = 0...0.7 \)). The research shows that with increasing pressure at the inlet to the inlet nozzle, the efficiency rate of the turbine increases, and the optimum of the efficiency rate is shifted towards the increase of the given circular velocity of the impeller, both in the design and off-design operating modes. It is found that the greatest value of the efficiency rate is acquired at the design mode of operation of the traction nozzle, i.e. at \( S = 1 \).

1. Problem setting and purpose of the research

With the rapid development of modern society, the need for energy is increasing every day. The issue of energy conservation and the rational use of the energy potential of our planet has always been of high priority and urgency. As far as many countries are on the verge of an energy crisis, this issue is especially acute. The problem of energy conservation is closely intertwined with the problems of energy, environment, technical re-equipment, and structural adjustment of the entire economy.

A large amount of exergy of pressure gases and vapours is irretrievably lost on gearboxes and pressure regulators at gas distribution stations and points. An effective method of solving such a problem is the utilization of secondary energy resources. The issue of the rational use of the exergy of pressure gases and vapours, the creation of recycling plants in Europe, and the USA is drawing great attention [1, 2]. Turbine-generator sets based on axial and centrifugal turbines are mainly used, but they are not widely introduced (especially for the capacities of up to 500 kW) since constructive solutions are needed to provide quick and inexpensive modernization of pressure reduction systems.
A promising direction is the creation of energy-saving low-power gas turbine expanders (up to 500 kW), which are developed based on jet reaction turbines, for utilizing the energy of pressure gases and vapours at throttling units with a payback period of no more than 2 years.

In the previously published materials, we indicated that the urgent task of the modern gas transportation industry is to create a pneumatic (gas), efficient, reliable and easy-to-use drive for the safe operation of gas pipelines (the information about the safe use and effective operation of gas turbines and gas turbine engines is given in materials [3-5]). This task is solved by creating pneumatic drives based on jet reaction machines (the term “jet reaction turbine (JRT)” is also often used in the literature).

One of the most important reasons for the insignificant use of the JRT is the incomplete study of the features of the gas flow in the flow part of a jet reaction turbine, as well as the issues of designing the units based on it. Details of the gas flow in a jet reaction turbine can be found in our previously published materials, as well as in the works of foreign authors who describe in detail the gas flow models in the flow part of various turbine constrictions [6-10], which significantly differ from the original JPT constructions and flow models.

It is worth noting that the novelty of this work is to assess the influence of various factors on the efficiency of a jet reaction turbine, in particular, on the efficiency rate of the JPT.

2. Methods of the research
The efficiency rate of the jet reaction turbine in the design mode due to the dimensionless quantities that characterize the energy loss is determined as follows:

\[ \eta_T = \frac{N_T}{N_s} = \frac{M_T \omega_T}{N_s} = \frac{(M_U - M_{rr}) \omega_T}{N_s} = \frac{N_U - N_{rr}}{N_s} = \]

\[ = \frac{G_t h_U - N_{rr}}{G_{in} h_s} = \eta_U \frac{G_t}{G_{in}} - \frac{N_{rr}}{N_s} = \eta_U (1 - \alpha_l) - \frac{N_{rr}}{N_s} \]  

(1)

where \( M_U \) is the moving moment; \( N_U \) is the power corresponding to the moment of \( M_U \); \( M_{rr} \) is the torque resistance; \( N_{rr} \) is the power corresponding to the moment of \( M_{rr} \); \( G_{in} \) is the mass flow of gas through the inlet nozzle; \( G_t \) is the mass gas flow through the traction nozzle; \( \alpha_l \) is the cost ratio; \( h_U \) is the force of 1 kg of gas passing through the traction nozzle (Euler operation):

\[ h_U = \frac{N_U}{G_t} = \frac{M_T \omega_T}{G_t} = \frac{G_t C_{out,t} U_{out,t} L}{G_t L} = C_{out,t} U_{out,t} \]

(2)

\( \eta_U \) is the circular efficiency rate:

where \( C_s \) is the velocity corresponding to the isentropic force of \( h_s \), i.e. the velocity that would 1 kg of gas have during the expansion without any loss of energy and heat exchange with the environment from the braking parameters at the inlet to the turbine to the ambient pressure:

\[ C_s = \sqrt{2 h_s} \]

(4)

\( U \) is the given circular velocity of the impeller

\[ U = \frac{U_{out,t}}{C_s} \]

(5)

\( C_{out,t} \) is the given absolute velocity of the gas on the cut of the traction nozzle

\[ U_{out,t} = \frac{C_{out,t}}{C_s} \]

(6)
The circular efficiency rate of the JPT takes into account energy losses due to the friction along the length of the gas path, wave losses (loss on jump seals), and losses with the initial velocity.

The research shows that you can use a simpler value of the velocity of \( \mathbf{C}_{d,t,s=0} \) in the calculations instead of \( W_{\text{out},t} \) (where \( W_{\text{out},t} \) is the relative flow velocity on the traction nozzle section, and \( C_{d,t,s=0} \) is the gas velocity on the traction nozzle section on the start-up mode).

If we ignore the difference between the velocities of \( W_{\text{out},t} \) and \( \mathbf{C}_{d,t,s=0} \), the formula for the JPT efficiency rate can be converted to:

\[
\eta = \frac{\left( \mathbf{C}_{d,t,s=0} - U_{\text{out},t} \right) U_{\text{out},t}}{h_s} (1-\alpha_l) - \frac{K_{r,r} \cdot U_{\text{out},t}^3}{L^3 G_{\text{in}} h_s} (1-\alpha_l) - \frac{K_{r,r} \cdot U_{\text{out},t}^3}{L^3 G_{\text{in}} h_s} = 2U \left( 1-\alpha_l \right) \left( \frac{\mathbf{C}_{d,t,s=0}}{C_s} - \frac{U}{\bar{U}} \right) - \frac{K_{r,r} \cdot U_{\text{out},t}^3}{L^3 G_{\text{in}} h_s}
\]

\[ \eta = \eta \left( \frac{1}{L^3 G_{\text{in}} h_s} \right)
\]

Figure 1 presents the dependenc (working environment - air) according to the formula (7). In comparison with the dependencies shown in Figure 2, maximum efficiency rate values are reduced for the corresponding inlet pressures in the JPT in the range of 1.2 to 5.3% and are shifted towards smaller values of the reduced circular velocity.

\[ \eta \]

**Figure 1.** Dependency of the change in the efficiency rate in the design mode on the given circular velocity at the inlet pressure of \( P_{\text{in}} = 2; 4; 6; 10 \text{ MPa} \).

One of the dimensionless coefficients characteristic of jet reaction turbines is the degree of gasdynamic off-design of the traction nozzle, which estimates the static pressure at the cut of the traction nozzle relative to the ambient pressure and is derived from the theory of rocket engines.

Taking into account this coefficient, the equation for the traction force of the traction nozzle is as follows:

\[ S = \frac{P_{\text{tn}}}{P_{\alpha,p}} \]

It is known that the traction force \( P_{\text{tn}} \) has a maximum value at \( S = 1 \). Accordingly, the moving moment of \( M_u \) at \( S = 1 \) will be greater than at other values of \( S \). Therefore, the choice of mode \( S = 1 \) is essential: it can be selected, for example, as a start-up mode (when using the JPT in ball
valve drives) or a maximum mode of the JPT efficiency rate (when using a JPT in turbine generators).

In the formula (7), the degree of off-design of the traction nozzle is \( S = 1 \).

The formula for the efficiency rate of a jet reaction turbine for the off-design mode and without taking into account the difference between the velocities of \( W_{\text{out},t} = C_{\text{d,t,s}=0} \):

\[
\eta_T = 2\bar{U} \left[ (1 - \alpha_l) \left( \frac{\lambda_{\text{out},t} \cdot a_{kr}}{C_s} - \bar{U} \right) + \frac{p_{\text{in}}^{*} f_{kr, t} (1 - \alpha_l) (S - 1)}{C_s G_{in} y (\lambda_{\text{out},t}) S} - \frac{\bar{U}^2 C_s K_{r,r}}{L^2 G_{in}} \right] \quad (9)
\]

Figure 2 presents the dependencies according to the formula (9).

![Figure 2](image_url)

**Figure 2.** The dependency of the change in the efficiency rate in the off-design mode on the given circular velocity at the inlet pressure of \( P_{\text{in}} = 2; 4; 6; 10 \) MPa.

Comparing Figures 1 and 2, we can find out how the degree of off-design of the traction nozzle affects the efficiency rate of the JPT at \( W_{\text{out},t} = C_{\text{d,t,s}=0} \).

The formula showing the relationship between the relative flow velocity at the cut of the traction nozzle and the gas velocity at the cut of the traction nozzle in the start-up mode is as follows:

\[
W_{\text{out},t} = \frac{\lambda_{\text{d,t,s}=0} a_{kr}^2 + \frac{k - 1}{k + 1} U_{\text{out},t}^2}{\sqrt{\lambda_{\text{d,t,s}=0}^2 (1 + \frac{k - 1}{2kRT_{\text{in}}} U_{\text{out},t}^2)}} = C_{\text{d,t,s}=0} \quad (10)
\]

The formula (10) shows that the greater the circular velocity \( U \), the greater the difference between the relative flow velocity at the cut of the traction nozzle and the gas velocity at the cut of the traction nozzle in the start-up mode is.

The formula for the efficiency rate of the jet reaction turbine at the degree of off-design of \( S = 1 \) taking into account the difference between the velocities of \( W_{\text{out},t} \) and \( C_{\text{d,t,s}=0} \):

\[
\eta_T = 2\bar{U} \left[ (1 - \alpha_l) \left( \frac{\lambda_{\text{out},t} \cdot a_{kr}}{C_s^2} + \frac{k - 1}{k + 1} \bar{U}^2 - \bar{U} \right) \right] - \frac{2\bar{U}^3 C_s K_{r,r}}{L^2 G_{in}} \quad (11)
\]

Figure 3 presents dependencies according to the formula (11).
Figure 3. The dependency of the change in the efficiency rate in the design mode on the reduced circular velocity of the impeller at different pressures at the inlet to the JPT for the degree of off-design of $S = 1$.

The formula for the jet reaction turbine efficiency rate for the off-design mode (the degree of off-design is $S > 1$) and taking into account the difference between the velocities of $W_{\text{out},t} > C_{d,t=0}$

$$
\eta_T = 2\bar{U} \left[ (1 - \alpha_l) \left( \frac{a_{kr}^2}{\sqrt{C_s^2 + \frac{k-1}{k+1} \bar{U}^2 - \bar{U}}} \right) + \frac{p_n f_{kr, t}(1 - \alpha_l)(S - 1)}{C_s G_{in} y(\lambda_{out,t})S} - \frac{K_{rr} \bar{U}^2 C_s}{L^2 G_{in}} \right] \quad (12)
$$

Figure 4 shows the dependency of the change in the efficiency rate in the off-design mode on the given circular velocity.

Figure 4. The dependency of the change in the efficiency rate in the off-design mode on the given circular velocity at the inlet pressure of $P_{in} = 2; 4; 6; 10$ MPa.

Comparing Figures 3 and 4, we can find out how the degree of off-design of the traction nozzle affects the efficiency rate of the JPT at $W_{\text{out},t} > C_{d,t=0}$. 
The analysis of the graphs shown in Figures 4 and 3 allows us to assess the effect of the degree of off-design on the efficiency rate of the JPT: at \( S = 1 \) in the studied pressure range at the inlet to JPT, maximum efficiency values are 3-13% higher than for the dependencies presented in Figure 4 and the maximum of the efficiency rate is shifted towards larger values of the given circular velocity.

The analysis of the graphs shown in Figures 4 and 2 allows us to assess the effect of the difference between the velocities of \( W_{\text{out}, t} \) and \( C_{d,t=0} \), which occurs when the impeller rotates due to the compressor effect, on the efficiency rate of the JPT. This effect is less significant than the effect of the degree of off-design of the traction nozzle, a failure of taking into account the compressor effect leads to a decrease in the efficiency rate in the range of 0.7-2.2% and a slight shift towards smaller values of the circular velocity.

The formula that correlates the given velocity in the initial section of the traction nozzle and the degree of off-design of the traction nozzle is given below:

\[
\lambda_{tp,s} = \sqrt{\frac{k+1}{k-1}} \left[ 1 - \left( \frac{P_{\text{a.p}} \cdot S}{P_{\text{in}} \cdot \sigma} \right)^{\frac{k-1}{k}} \right]^{(13)}
\]

where \( k \) is the coefficient of gas isentropy; \( P_{\text{a.p}} \) is the ambient pressure; \( \sigma \) is the coefficient of the recovery of total pressure in the flow part of the JPT from the inlet to the inlet nozzle to the outlet of the traction nozzle.

Graph 5 shows the dependency of the change of the dimensionless gas flow rate on the section of the vehicle in the off-design mode on the degree of off-design of the traction nozzle, according to formula 13, the graph shows that at \( \lambda_{\text{out}, t} = 1 \), the degree of off-design for each pressure will be maximum and with an increase of lambda, the degree of off-design decreases.

![Graph 5](image)

**Figure 5.** The dependency of the change in the dimensionless gas flow rate at the cut of the vehicle in the off-design mode on the degree of off-design of the traction nozzle, at the inlet pressure of \( P_{\text{in}} = 2; 4; 6; 10 \) MPa.

Figure 6 presents the dependencies of the change in the maximum values of the efficiency rate in the design and off-design mode on the degree of off-design of the traction nozzle, at the inlet pressure...
for the entire range of parameters. The graph shows that at the degree of off-design of $S = 1$ (i.e. at the design mode of operation), the efficiency rate indicator has the greatest values.

![Graph showing the dependency of the change of the maximum values of the efficiency rate in the off-design mode on the degree of off-design of the traction nozzle, at the inlet pressure of $P_{in} = 2; 4; 6; 10$ MPa.]

**Figure 6.** The dependency of the change of the maximum values of the efficiency rate in the off-design mode on the degree of off-design of the traction nozzle, at the inlet pressure of $P_{in} = 2; 4; 6; 10$ MPa.

3. Conclusions
Formulas for the efficiency rate of the jet reaction turbine have been obtained for:
- the off-design mode (the degree of off-design is $S > 1$) and taking into account the difference between the velocities of $W_{out,t} > C_{d,t,s=0}$;
- the design mode of operation ($S = 1$) and at $W_{out,t} = C_{d,t,s=0}$;
- the design mode and taking into account the difference between the velocities of $W_{out,t}$ and $C_{d,t,s=0}$.
- the off-design mode and without taking into account the difference between the velocities of $W_{out,t}$ and $C_{d,t,s=0}$.

The graphs are constructed and analyzed according to the above--mentioned dependencies.

The influence of the degree of off-design of the traction nozzle thrust is estimated. It has been found that with an increase of pressure at the inlet to the inlet nozzle, the turbine efficiency rate increases, and the optimum of the efficiency rate shifts towards the increase of the impeller velocity, both in the design and in off-design modes. It has been concluded that the greatest value of the efficiency rate is acquired at the design mode of operation of the traction nozzle, i.e. at $S = 1$.

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