On the Hard Gamma-Ray Spectrum of the Potential PeVatron Supernova Remnant G106.3 + 2.7

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Abstract

The Tibet ASγ experiment has measured a γ-ray flux of supernova remnant G106.3+2.7 of up to 100 TeV, suggesting it is potentially a “PeVatron.” Challenges arise when the hadronic scenario requires a hard proton spectrum (with spectral index ≈1.8), while usual observations and numerical simulations prefer a soft proton spectrum (with spectral index ≥2). In this paper, we explore an alternative scenario to explain the γ-ray spectrum of G106.3+2.7 within the current understanding of acceleration and escape processes. We consider that cosmic ray particles are scattered by turbulence driven by Bell instabilities. The resulting hadronic γ-ray spectrum is novel, dominating the contribution to the emission above 10 TeV, and can explain the bizarre broadband spectrum of G106.3+2.7 in combination with leptonic emission from the remnant.

1. Introduction

Galactic cosmic rays (CRs) are mostly charged particles (mainly protons) with energies up to the so-called “knee” (~1–10 PeV). Based on energetic arguments, supernova remnants (SNRs) are usually believed to be the accelerators of Galactic CRs. However, although a large number of SNRs have been detected in γ-rays, none of them has been confirmed to be a PeV particle accelerator (i.e., the so-called “PeVatron”; see, e.g., Bell et al. 2013). Discovered in the DRAO Galactic-plane survey (Joncas & Higgs 1990), G106.3 + 2.7 is a cometary SNR with a tail in the southwest and a compact head (containing PSR J2229+6114) in the northeast, at a distance of 800 pc away from Earth (Kothes et al. 2001). Recently, HAWC and the Tibet ASγ experiments have reported the γ-ray spectrum of SNR G106.3 + 2.7 above 40 TeV, arguing that the remnant is a promising PeVatron candidate (Albert et al. 2020; Tibet ASγ Collaboration et al. 2021). In particular, Tibet ASγ Collaboration et al. (2021), for the first time, measured the γ-ray flux up to 100 TeV, finding that the centroid of γ-ray emissions deviates from the pulsar at a confidence of 3.1σ and is well correlated with a molecular cloud (MC). The offset of the γ-ray emission centroid from PSR J2229 + 6114 is measured to be 0.44 (≈6 pc at a distance of 800 pc).

Both leptonic and hadronic models have been proposed to give a plausible explanation to the spectral energy distribution (SED) of the SNR. In the leptonic scenario, electrons are suggested to be transported to their current position from the pulsar wind nebula (PWN, Tibet ASγ Collaboration et al. 2021; Liu et al. 2020), or accelerated by the blast wave directly (Tibet ASγ Collaboration et al. 2021; Ge et al. 2021). In the hadronic scenario, protons are suggested to be accelerated by the blast wave in earlier ages, or reaccelerated by the PWN adiabatically (Ohira et al. 2018; Tibet ASγ Collaboration et al. 2021). The large offset between PSR J2229 + 6114 and the γ-ray emission centroid indicates that the γ-rays are more likely to mainly originate from the particles accelerated by the blast wave in the southwestern “tail” region (Tibet ASγ Collaboration et al. 2021). In order to figure out the origin of the γ-rays, Ge et al. (2021) separated the PWN-dominated X-ray emitting region in the northeast from the other (the “tail”) part of the SNR using XMM-Newton and Suzaku observations. They found that a pure leptonic model can hardly fit the radio, X-ray, and γ-ray spectral data of the “tail” region simultaneously. Therefore, there must be a hadronic component in the γ-ray spectrum, and the SED can only be explained with a hadronic (with a proton spectral index ≈1.8; Tibet ASγ Collaboration et al. 2021) or a leptonic–hadronic hybrid (with a proton index ≈1.5; Ge et al. 2021) model. However, challenges arise as a hard proton spectrum is required in both hadronic and leptonic–hadronic hybrid models, while numerical simulations show that diffusive shock acceleration can only give rise to a soft proton spectrum (with spectral index ≥2; see, e.g., Caprioli et al. 2020).

Tibet ASγ Collaboration et al. (2021) suggest that a very hard proton spectrum can be formed under very efficient acceleration (which seems to be an extreme case) and after very slow particle diffusion. We here explore an alternative self-consistent scenario, in which the γ-ray spectrum can be explained more naturally. The magnetic field in the upstream of the blast wave is believed to be amplified via the nonresonant Bell instability (Bell 2004; Amato & Blasi 2009); protons escaping from the upstream of the shock can drive nonresonant turbulence, whose scale is smaller than the Larmor radius of the escaped particles, and the relatively low-energy particles are thus diffused by the magnetic field, which is amplified by the nonresonant turbulence (for reviews see, e.g., Blasi 2013). Based on numerical simulations (Bell 2004; Bell et al. 2013), we study the escape process based on the first principle instead of a model with a priori phenomenological assumptions. The resulting proton spectrum is novel and can explain the hard γ-ray spectrum well in combination with leptonic emission from the SNR. The model is described in Section 2, and the SED of the remnant is fitted in Section 3. A summary is presented in Section 4.
2. Model Description

For simplicity, we follow the approximation that the global maximum energy $E_{\text{max, global}}$ is reached at the beginning of the Sedov phase, in which $R_{sh} \propto t^{2/5}$ (see, e.g., Ohira et al. 2012). The maximum proton energy is given by Bell et al. (2013),

$$E_{\text{esc}}(t) = 230 n_e^{1/2} \left( \frac{\eta}{0.03} \right) \left( \frac{v_{sh}}{10^4 \text{ km s}^{-1}} \right)^2 \left( \frac{R_{sh}}{\text{ pc}} \right) \text{TeV},$$

where $\eta$ is the acceleration efficiency, $n_e$ is the electron number density of the interstellar medium (ISM); we assume that the ISM near the SNR is fully ionized, $v_{sh}$ is the velocity of the shock, and $R_{sh}$ is the radius of the shock.

Following Cardillo et al. (2015), we calculate the spectrum of escaped CR protons in the Sedov phase. In the upstream, the protons are scattered by the wave generated by the escape of the higher-energy protons, and therefore there is no wave upstream to scatter the protons with the highest energy $E_{\text{esc}}(t)$ (Bell 2004; Bell et al. 2013). Hence, the latter escape the system quasi-ballistically at a speed $\sim c$, inducing a current $j_{CR} = n_{CR} e v_{sh}$ at the shock (Bell 2004, where $n_{CR}$ represents the number density of the CRs). The differential number of the escaped protons with energy $E_{\text{esc}}(t)$ can thus be evaluated via

$$e \frac{dN_{\text{esc}}(E_{\text{esc}})}{dE_{\text{esc}}} \, dE_{\text{esc}} = 4\pi R_{sh}^2 j_{CR} \, dt.$$  

(2)

We further assume that the CR pressure at the shock is a fixed fraction $\xi$ of the ram pressure.

$$j_{CR} = n_{CR} e v_{sh} = \frac{e \xi n_{e} v_{sh}^3}{E_0 \Psi(E_{\text{esc}})},$$  

(3)

where

$$\Psi = \begin{cases} \left( \frac{E_{\text{esc}}}{E_0} \right) \ln \left( \frac{E_{\text{esc}}}{E_0} \right) & \alpha = 2 \\ \alpha - 1 \left( \frac{E_{\text{esc}}}{E_0} \right)^{\alpha - 1} - \left( \frac{E_0}{E_{\text{esc}}} \right)^{\alpha - 2} & \alpha > 2, \end{cases}$$  

(4)

where $\alpha$ is the power-law index of the parent CR spectrum at the shock, and $E_0$ is the minimum energy of protons.

Finally, $N_{\text{esc}}$ can be expressed as (see Cardillo et al. 2015 for the derivation)

$$\frac{dN_{\text{esc}}(E_{\text{esc}})}{dE_{\text{esc}}} = 4 \pi R_{sh}^2 j_{CR} \frac{dE_{\text{esc}}}{dt}$$

$$= 4 \pi \xi n_{e} v_{sh}^3 \frac{dR_{sh}}{dt} \frac{d\Psi}{dE_{\text{esc}}}$$

$$\propto \frac{n_{e} v_{sh}^3 R_{sh}^3}{E_0 \Psi} \left[ 1 + \ln \left( \frac{E_{\text{esc}}}{E_0} \right) \frac{\ln \left( \frac{E_{\text{esc}}}{E_0} \right)^2}{\ln \left( \frac{E_{\text{esc}}}{E_0} \right)^2} \right]^\alpha \alpha = 2 \left( \frac{1}{\alpha - 2} \right) \alpha > 2.$$  

(5)

3. Application to SNR G106.3 + 2.7

3.1. $\gamma$-Ray Spectroscopic Luminosity

Since the kinematic/physical signature of direct MC-SNR contact remains inconclusive (Q.-C. Liu & Y. Chen, 2021 in preparation), the MC may only be illuminated by the escaped protons. In such a scenario, the MC is sometimes approximated by a truncated cone (see, e.g., Li & Chen 2012; Celli et al. 2019). Here, the MC is assumed to subtend a solid angle $\Omega$ at the SNR center with an inner radius $R_1$ and an outer radius $R_2$.

The diffusion coefficient near the SNR is adopted to be $D(E) = D_{100 \text{ TeV}} (E/100 \text{ TeV})^\delta$, where $D_{100 \text{ TeV}}$ is the diffusion coefficient for particles at 100 TeV, and $\delta$ is the energy dependence index.

The diffusion equation for the protons with energy $E_{\text{esc}}$ escaping the shock when the shock radius is $R_{sh}$ at time $T_{\text{esc}}$ writes

$$\frac{\partial}{\partial t} f(E_{\text{esc}}, r, t) = \frac{D(E_{\text{esc}})}{r^2} \frac{\partial}{\partial r} \left[ r^2 \frac{\partial}{\partial r} f(E_{\text{esc}}, r, t) \right] + Q,$$  

(6)

where

$$Q = \frac{1}{4 \pi R_{sh}^2} \frac{dN_{\text{esc}}(E_{\text{esc}})}{dE_{\text{esc}}}$$

$$\times \delta(r - R_{sh}) \delta(t - T_{\text{esc}})$$

$$= C_Q \rho_{sh}^2 R_{sh}^2 \left( \frac{E_{\text{esc}}}{E_0} \right)^{-2}$$

$$\times \delta(r - R_{sh}) \delta(t - T_{\text{esc}})$$

$$\times \left[ 1 + \ln \left( \frac{E_{\text{esc}}}{E_0} \right) \ln \left( \frac{E_{\text{esc}}}{E_0} \right)^2 \right]$$

$$\left( \alpha - 2 \right) \left( \frac{E_{\text{esc}}}{E_0} \right)^{-\alpha + 2} \alpha > 2.$$  

(7)

is the injection term, $f$ is the proton distribution function, and $C_Q$ is a free parameter that is proportional to the total proton energy.

As is shown in the Appendix, the solution of Equation (6) is (see also Atoyan et al. 1995; Celli et al. 2019)

$$f(E_{\text{esc}}, r, t) = \frac{dN_{\text{esc}}(E_{\text{esc}})}{dE_{\text{esc}}} \frac{4 \pi R_{sh}^2}{4r^3 \Omega}$$

$$\times \left[ \exp \left[ \frac{-1}{2} \left( \frac{r - R_{sh}}{R_{\text{diff}}} \right)^2 \right] - \exp \left[ \frac{-1}{2} \left( \frac{r + R_{sh}}{R_{\text{diff}}} \right)^2 \right] \right],$$  

(8)

where $R_{\text{diff}} = 2 \sqrt{D(E_{\text{esc}})(t - T_{\text{esc}})}$ is the diffusion length scale.

At time $T_{\text{age}}$, the differential number of protons with energy $E_{\text{esc}}$ lying inside the conic MC shell can thus be calculated to be

$$N_{\text{CR,MC}} = \int \rho_{sh}^2 (E_{\text{esc}}, r, t) \Omega r^2 dr.$$  

(9)

Finally, the hadronic $\gamma$-ray spectroscopic luminosity $\Phi_{\gamma}(E_{\gamma})$ is evaluated via the cross section presented in Kafexhiu et al. (2014).

$$\Phi_{\gamma}(E_{\gamma}) = \sigma n_{\text{MC}} \int d\sigma \frac{dE_{\gamma}}{dE_{\gamma}} N_{\text{CR,MC}}(E_{\gamma}) dE_{\gamma},$$  

(10)

where $d\sigma/dE_{\gamma}$ is the $\gamma$-ray differential cross section.

In addition to the hadronic component, we add a leptonic component contributed by the electrons accelerated by the blast
wave. We approximate the spectrum of electrons in the SNR to be a broken power law.

\[
\frac{dN_e}{dE} \propto \begin{cases} 
E^{-\alpha_1} & E \leq E_b \\
E^{-\alpha_2} & E_b < E < \min(E_{\text{max,global}}, E_{\text{loss}}), 
\end{cases}
\]  

where \(dN_e/dE\) is the differential number of electrons, and \(E_{\text{loss}}\) is the maximum energy determined by the energy loss; \(\alpha_1\) and \(\alpha_2\) are the power-law indices in the low-energy and high-energy bands, respectively. The break energy \(E_b\) of the electron spectrum can be constrained well (~9 TeV), assuming a magnetic field of 6 \(\mu\)G (Ge et al. 2021) based on the radio (Pineault & Joncas 2000) and X-ray (Fujita et al. 2021; Ge et al. 2021) emissions, which are both dominated by the SNR.

The electron spectrum above the break is quite soft, and the leptonic \(\gamma\)-ray emissions are dominated over by the hadronic emissions above 10 TeV. Hence, the total \(\gamma\)-ray spectrum is insensitive to the electron spectrum above \(E_b\).

The fitted SED is plotted in Figure 1, using the parameters listed in Table 1. We find that the parameters are insensitive to the X-ray flux, since the different sets of X-ray flux data from Ge et al. (2021) and Fujita et al. (2021) can be fitted with very similar parameters.

In Figure 2, we plot the proton spectrum inside the MC. As can be seen, the protons with energy <75 TeV are still trapped by the turbulence driven by the escaping protons via the Bell instability, which is absent in the MC. Hence, only protons with energies ranging from 75 TeV to 280 TeV can reach the MC within the relatively short lifetime of the SNR and contribute to a novel hadronic \(\gamma\)-ray spectrum. In Figure 3, we explore the dependence of the hadronic \(\gamma\)-ray spectrum on \(\alpha\) and \(\delta\) and find that the choice of \(\alpha\) (in the range 2.0–2.6) or \(\delta\) (in the range 0–1) does not impact the results significantly. The reason is that the protons that can hit the MC within the

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**Figure 1.** SED of the emission from SNR G106.3 + 2.7. The CGPS data are taken from Pineault & Joncas (2000), XMM-Newton data from Ge et al. (2021) (for Model A), Suzaku data from Fujita et al. (2021) (for Model B; see also Table 1), Fermi-LAT data from Xin et al. (2019), VERITAS data from Albert et al. (2020), HAWC data from Albert et al. (2020), and Tibet AS\(\gamma\) data from Tibet AS\(\gamma\) Collaboration et al. (2021). The hadronic \(\gamma\)-ray spectrum below 500 GeV is contributed by 75–280 TeV protons via a small cross section.

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**Table 1.** Fitting Parameters

| Parameter | Model A | Model B |
|-----------|---------|---------|
| \(T_{\text{age}}\) (yr) | 1000 | ... |
| \(R_{\text{ejecta}}\) (pc) | 5.5 | ... |
| \(E_{\text{max,global}}\) (TeV) | 2.8 \times 10^2 | ... |
| \(E_0\) (GeV) | 1.0 | ... |
| \(\alpha\) | 2.3 | ... |
| \(D_{100\,\text{TeV}}\) (cm\(^2\) s\(^{-1}\)) | 6.6 \times 10^{22} | ... |
| \(\delta\) | 1/3 | ... |
| \(R_1\) (pc) | 7 | ... |
| \(R_3\) (pc) | 10 | ... |
| \(E_{\text{SN}}\) (erg) | 1.0 \times 10^{31} | ... |
| \(d\) (pc) | 800 | ... |
| \(M_0\) (M\(_\odot\)) | 1.0 | ... |
| \(\eta\) | 0.6 | ... |
| \(\eta_{\text{MC}}C_0(\Omega/4\pi)\) (erg\(^{-2}\) cm\(^{-3}\)) | 320 | ... |
| \(E_0\) (TeV) | 12 | 10 |
| \(\alpha_1\) | 2.3 | ... |
| \(\alpha_2\) | 3.8 | 3.45 |
| \(B\) (\(\mu\)G) | 4.6 | ... |
| \(W_{\text{erg}}\) | 6.8 \times 10^{37} | ... |
| \(T_{\text{CMB}}\) (K) | 2.73 | ... |
| \(u_{\text{CMB}}\) (eV cm\(^{-3}\)) | 0.25 | ... |
| \(T_{\text{FIR}}\) (K) | 25 | ... |
| \(u_{\text{FIR}}\) (eV cm\(^{-3}\)) | 0.2 | ... |
| \(T_{\text{OPT}}\) (K) | 3000 | ... |
| \(u_{\text{OPT}}\) (eV cm\(^{-3}\)) | 0.3 | ... |

**Note.** \(d\) is the distance to the SNR, \(R_{\text{ejecta}}\) is the radius of the SNR at present, \(E_{\text{SN}}\) is the explosion energy of the SNR, \(M_0\) is the ejecta mass, \(B\) is the magnetic field strength, and \(W_{\text{erg}}\) is the total electron energy. \(T_{\text{CMB}}\), \(T_{\text{FIR}}\), and \(T_{\text{OPT}}\) are the temperatures of the CMB, far-infrared, and near-infrared photons, respectively; \(u_{\text{CMB}}, u_{\text{FIR}},\) and \(u_{\text{OPT}}\) are the energy densities of the CMB, far-infrared, and near-infrared photons, respectively.
relatively short lifetime of the SNR at a narrow energy range (75–280 TeV) are almost monoenergetic (as shown in Figure 2). Hence, the energy dependence index δ and the power-law index α can hardly affect the proton spectrum in the MC. Celli et al. (2019) have also proposed a similar phenomenological escaping scenario, which can lead to a hard hadronic γ-ray spectrum in middle-aged SNRs, while in our case, we model a younger SNR based on the first principle.

Since the hadronic γ-ray luminosity $\Phi_\gamma \propto C_{Q\text{MC}}\Omega$ (where $n_{\text{MC}}$ is the number density of the gas in the MC, a free parameter), we only list $C_{Q\text{MC}}\Omega/(4\pi)$ as a single parameter in Table 1. Once $C_{Q\text{MC}}\Omega$ is obtained from data fitting, the total proton energy $E_{p,\text{tot}}$ (including that in the GeV protons trapped near the shock) can be calculated via

$$E_{p,\text{tot}} \approx \int_{E_0}^{E_{\text{max,global}}} E \, dE \cdot C_{Q\text{MC}} R_{sh}^3 \Omega \left( \frac{E}{E_0} \right)^{-2} \times \left( \frac{1 + \ln(E_{\text{esc}}/E_0)}{\ln(E_{\text{esc}}/E_0)^2} \right) \quad \alpha = 2$$

If we adopt $n_{\text{MC}} = 100 \text{ cm}^{-3}$, $E_{p,\text{tot}}$ will be $1.6(4\pi/\Omega) \times 10^{49}$ erg, which is quite reasonable.

As shown in Figure 1, the leptonic component accounts for the radio (Pineault & Joncas 2000) and X-ray (Ge et al. 2021 for Model A; Fujita et al. 2021 for Model B) emission and dominates the γ-ray flux below 500 GeV. Meanwhile, the hadronic γ-rays (yellow lines) have a very hard spectrum ($dN/dE \propto E^{-\alpha}$) below 500 GeV, which stems from the proton cutoff at 75 TeV and the low-energy tail of the pp interaction cross section. The spectrum above 10 TeV is dominated by the hadronic component, and the whole γ-ray spectrum can be explained by the hybrid model naturally.

3.2. Age of SNR G106.3 + 2.7

There remains some uncertainty about the age of SNR G106.3 + 2.7. Although the characteristic age of PSR J2229 + 6114 ($\approx 10^4 \text{ yr}$) is usually adopted to be the age of the remnant, sometimes the remnant is suggested to be very young (~1 kyr; see, e.g., Albert et al. 2020). Assuming adiabatic expansion ($R_{sh} \approx 1.2(E_{\text{SN}}/\rho_{\text{ISM}})^{1/2}$, where $\rho_{\text{ISM}}$ is the density of the ISM, the estimated $E_{\text{SN}} \approx 7 \times 10^{59}$ erg (Kothes et al. 2001) is unusually low if the age of the remnant is $\approx 10$ kyr. However, as is estimated in Cardillo et al. (2015), Equation (1) indicates that the global maximum energy is $E_{\text{max,global}} \approx E_{\text{SN}}$, and SNRs with a low $E_{\text{SN}}$ can hardly accelerate CRs to $\approx 10^5$ TeV. In order to reconcile the dilemmas, in this paper we consider a spherically symmetric scenario, in which $E_{\text{SN}} = 10^{51}$ erg (the canonical value) and $T_{\text{age}} = 1000 \text{ yr}$. Such a remnant age is plausible for a hosted pulsar with a characteristic age of $\approx 10^5 \text{ yr}$. Assuming a canonical braking index $n = \nu / \nu^2 = 3$ (where $\nu$ is the spin frequency of the pulsar), the real age of the pulsar becomes $T_{\text{age}} = \tau_c [1 - (P_0/P_{\text{now}})^2]$, where $\tau_c$ is the characteristic age of the pulsar, $P_0$ is the initial spin period of the pulsar, and $P_{\text{now}}$ is the spin period of the pulsar at present. For PSR J2229 + 6114, $P_{\text{now}} = 50 \text{ ms}$ (Halpern et al. 2001), and $T_{\text{age}}$ can be as small as $\sim 1 \text{ kyr}$ if $P_0$ is sufficiently close to $P_{\text{now}}$. Such a value of $P_0$ is allowed, since in pulsar evolution models $P_0$ is usually suggested to be in a wide range from $\sim 4 \text{ ms}$ to $\sim 400 \text{ ms}$ (see, e.g., Arzoumanian et al. 2002; Faucher-Giguère & Kaspi 2006; Popov et al. 2010). There are instances in which $P_0 \approx P_{\text{now}}$ and $T_{\text{age}} \ll \tau_c$ can also be found in the catalog: (a) CCO 1E 1207.4–5209 inside SNR G296.5 + 10.0 has $P_0 \approx P_{\text{now}} = 424 \text{ ms}$, leading to $\tau_c > 27 \text{ Myr}$, which exceeds the age of the SNR by 3 orders of magnitude (Gotthelf & Halpern 2007); and (b) PSR J1852–0040 is suggested to have $P_0 \approx P_{\text{now}} \approx 10^4 \text{ ms}$, corresponding to $\tau_c \approx 2 \times 10^8 \text{ yr}$, which is 4 orders of magnitude larger than $T_{\text{age}} \approx 5 \times 10^4 \text{ yr}$ measured from the observation of the associated SNR (Gotthelf et al. 2005). On the other hand, as can be seen from Equation (1), $E_{\text{esc}}$ is determined by $R_{sh}$ and $v_{sh}$, which can be constrained by observations directly, irrespective of $T_{\text{age}}$. Ge et al. (2021) showed that $v_{sh}$ is at least 3000 km s$^{-1}$ in the southwestern tail region based on the nonthermal X-ray spectrum up to 7 keV without a clear spectral cutoff, while $R_{sh} \approx 6 \text{ pc}$ can be estimated via radio observations (see, e.g., Tibet AS$\gamma$ Collaboration et al. 2021). Although $T_{\text{age}}$ does affect $R_{\text{sh inf}}$, its impact can be compensated by the free parameter $D_{100 \text{ TeV}}$. Hence, for simplicity we only consider a spherically symmetric remnant evolving in a uniform medium following the well-known Truelove & McKee (1999) model.$^3$

3.3. Other Issues

A sharp cutoff in the new proton spectrum is an interesting characteristic in our particle-escaping scenario with a spherically symmetric morphology applied, which can fit well the Tibet AS$\gamma$ data in this SNR. Meanwhile, instead of a sharp cutoff, a smoother break can be predicted if an asymmetric morphology is considered. In that case, the shock radius $R_{sh}$ and velocity $v_{sh}$ both vary in different directions; thus $E_{\text{esc}} \propto v_{sh}^2 R_{sh}$ varies in different directions accordingly, and the spatial variation of $E_{\text{esc}}$ may turn the sharp cutoff into a softer break. Since the data available at present cannot distinguish the asymmetric model, here we adopt a symmetric model to explain the γ-ray spectrum in the zeroth order. Further observations carried out by LHAASO may help distinguish these models. During the lifetime of SNRs, the shock may break, and the CRs can thus escape in a continuous power-law

$^3$ See Leahy & Williams (2017) for a fast Python calculator on SNR evolution.
4. Summary

Although the standard theory of nonlinear diffusive shock acceleration can predict a hard proton spectrum with $\alpha < 2$ (Caprioli et al. 2020), it is noted that observations and recent numerical simulations seem to prefer a softer hadron spectrum (Caprioli et al. 2020). In this paper, we explore an alternative plausible scenario to explain the bizarrely hard hadronic spectrum of SNR G106.3 + 2.7 within the current acceleration theory, which predicts soft ($\alpha \geq 2$) proton spectra. Apart from the common leptonic component, which can be constrained by radio and X-ray observations, we invoke a new hadronic component. This component arises from the escape scenario proposed by Bell (2004) and Cardillo et al. (2015), in which the CRs at the shock are diffused by the turbulence that is generated by the escaping CR particles with higher energies. Consequently, at a given time, only CRs with the highest energy can escape upstream, while CRs with lower energies are confined. Therefore, only protons with energies between 75 and 280 TeV can reach the MCs at an age 1 kyr. The cutoff of the proton spectrum at 75 TeV then gives rise to a very hard hadronic $\gamma$-ray spectrum below 10 TeV, and the cutoff of the $\gamma$-ray spectrum at 280 TeV can reach the MCs at an age 1 kyr. The cutoff of the hadronic spectrum of SNR G106.3 + 2.7 well.

In our model treatment, the $\gamma$-ray spectrum is calculated in the zeroth order, assuming that a spherically symmetric SNR expands in a uniform medium. There are seven free parameters in the hadronic component, namely, $n_{\text{SMS}}, D_{100 \text{ TeV}}, R_1, R_2, M_{\text{ej}}, \eta,$ and $n_{\text{MC}}CQ(\Omega/4\pi)$. Neither parameter $\alpha$ nor $\delta$ has strong correlation to the hadronic $\gamma$-ray spectrum, because the protons that can hit the MC within the relatively short lifetime of the SNR in the narrow energy range (75–280 TeV) are approximately monoenergetic.

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Appendix

Analytical Solution of Diffusion Equation

In order to solve Equation (6) analytically, we define a new function $F = f(E_{\text{esc}}, r, t)$, and then Equation (6) reads

$$\frac{\partial}{\partial t} F = D(E_{\text{esc}}) \frac{\partial^2 F}{\partial r^2} + Q(E_{\text{esc}}, r, t), \quad (A1)$$

and the boundary condition is $\partial f/\partial r = 0$ and $F|_{r=0} = 0$. Defining the differential operator

$$L \equiv \frac{\partial}{\partial r} - D \frac{\partial^2}{\partial r^2},$$

Equation (A1) can be further written as $LF = Qr$. We first discuss a simpler equation $LG(r, \zeta) = \delta(r - \zeta)$ with the boundary condition $G|_{r=0} = 0$. The solution is apparently

$$G(r, \zeta) = \frac{1}{\sqrt{\pi} R_{\text{dif}}} \left\{ \exp\left[ - \frac{(r - \zeta)^2}{R_{\text{dif}}^2} \right] - \exp\left[ - \frac{(r + \zeta)^2}{R_{\text{dif}}^2} \right] \right\}. \quad (A2)$$

Then, we multiply Equation (A2) by $h(\zeta) = Q(\zeta)\zeta$ on both sides, integrate over $\zeta$, and then apply the operator $L$. Since $L$ is linear, one may take $L$ inside the integration, i.e.,

$$L\left( \int_0^\infty G(r, \zeta)Q(\zeta)\zeta \, d\zeta \right) = \int_0^\infty LG(r, \zeta)Q(\zeta)\zeta \, d\zeta = \int_0^\infty \delta(r - \zeta)Q(\zeta)\zeta \, d\zeta = Qr = LF. \quad (A3)$$
Hence, \( \int_0^\infty G(r, \zeta)Q(\zeta)\zeta\,d\zeta = F \), and we thus have

\[
f = \frac{1}{r} \int_0^\infty Q(\zeta)G(\varepsilon_{\text{esc}}, t, T_{\text{esc}}; r, \zeta)\,d\zeta
\]

\[
= \frac{dN_{\text{esc}}(E_{\text{esc}})/dE_{\text{esc}}}{4\pi^{3/2}R_{\text{sh}}R_{\text{dif}}} \exp \left[-\left(\frac{r - R_{\text{sh}}}{R_{\text{dif}}}\right)^2\right]
\]

\[
- \exp \left[-\left(\frac{r + R_{\text{sh}}}{R_{\text{dif}}}\right)^2\right] \quad (A4)
\]

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