Temperature induced transition of thin superconductor-ferromagnet-normal (S/F/N) metal hybrid structure to in-plane Fulde-Ferrel-Larkin-Ovchinnikov (FFLO) state is accompanied by vanishing of effective inverse magnetic field penetration depth $\Lambda^{-1}$ (Phys. Rev. Lett. 121, 077002 (2018)). Here we show that $\Lambda^{-1}$ goes to zero only in limit of zero magnetic field $H \to 0$ and at any finite parallel $H$ or in-plane current $I$ it is finite and positive in FFLO state which implies diamagnetic response. We demonstrate that $\Lambda^{-1}$ has a nonmonotonic dependence on $H$ and $I$ not only in the parameter range corresponding to the FFLO phase domain but also in its vicinity. We find that for S/F/N/F/S structures with certain thicknesses of F layers there is temperature, current and magnetic field driven transition to and out of FFLO phase with a simultaneous jump of $\Lambda^{-1}$.

I. INTRODUCTION

In superconductor-ferromagnet (S/F) bilayers proximity induced odd-frequency spin triplet superconducting component in F layer gives negative contribution to square of inverse London penetration depth $\lambda^{-2}$ [1][5], which is a coefficient in relation between superconducting current density and vector potential: $j = -cA/4\pi \lambda^2$. At some parameters this contribution can exceed positive contribution from singlet superconducting component in S and F layers and makes effective inverse magnetic field penetration depth $\Lambda^{-1} = \int_0^d \lambda^{-2}(x)dx$ ($d$ is a thickness of the bilayer) negative which implies paramagnetic response of whole structure. In Ref.2 it is argued that the state with $\Lambda^{-1} < 0$ is unstable and authors find that the S/F bilayer transits to in-plane Fulde-Ferrel-Larkin-Ovchinnikov (FFLO) state as $\Lambda^{-1} \to +0$. In the recent work it is predicted that such an in-plane FFLO state can emerge at temperature much below the critical one, it is characterized by unusual current-phase relation and can be realized in S/F/N trilayer with realistic parameters, where N is a low resistive normal metal (Au, Ag, Cu or Al), S is a disordered superconductor with large residual resistivity in the normal state (NbN, WSi, NbTiN etc.) and F is ordinary ferromagnet (Fe, CuNi, etc.).

Motivated by these results and expected unusual electrodynamic response of FFLO state in S/F/N trilayer with rather usual parameters, easily realizable by modern experimental technique, we theoretically study effect of parallel magnetic field and in-plane current on $\Lambda^{-1}$ in S/F/N trilayer being in FFLO state or in state with large contribution of odd-frequency triplet component to $\Lambda^{-1}$. In section IV we consider different types of the $\pi \to$ FFLO transitions in S/F/N/F/S structures and their influence on the screening properties. Section V contains a brief summary.

II. MODEL

To study the superconducting properties of S/F/N and S/F/N/F/S structures we use the one-dimensional Usadel equation for normal $g$ and anomalous $f$ quasi-classical Green functions. With standard angle parametrization $g = \cos \Theta$ and $f = \sin \Theta \exp(i\phi)$ the Usadel equations in different layers can be written as

$$\frac{\hbar D_S}{2} \frac{\partial^2 \Theta_S}{\partial x^2} - \left( \hbar \omega_n + \frac{D_S}{2\hbar} q^2 \cos \Theta_S \right) \sin \Theta_S + \Delta \cos \Theta_S = 0, \quad (1)$$

$$\frac{\hbar D_F}{2} \frac{\partial^2 \Theta_F}{\partial x^2} - \left( \hbar \omega_n + i\hbar \right) + \frac{D_F}{2\hbar} q^2 \cos \Theta_F \right) \sin \Theta_F = 0, \quad (2)$$

$$\frac{\hbar D_N}{2} \frac{\partial^2 \Theta_N}{\partial x^2} - \left( \hbar \omega_n + \frac{D_N}{2\hbar} q^2 \cos \Theta_N \right) \sin \Theta_N = 0, \quad (3)$$

where subscripts S, F and N refer to superconducting, ferromagnetic and normal layers, respectively. Here $D$ is the diffusion coefficient for corresponding layer, $\hbar$ is the reduced Planck constant, $\hbar \omega_n = \pi k_B T(2n+1)$ are the Matsubara frequencies ($n$ is an integer number), $q = \nabla \varphi + 2\pi A/\Phi_0$ is the quantity that is proportional to
supervelocity \( v_s = \hbar q / m \) directed in z direction (see Fig. 1), \( \varphi \) is the phase of the order parameter, \( \mathbf{A} \) is the vector potential, \( \Phi_0 = \pi \hbar c / |e| \) is the magnetic flux quantum. The \( x \)-axis is oriented perpendicular to the surface of \( S \) layer accordingly to Fig. 1. \( \Delta \) is the superconducting order parameter, which satisfies to the self-consistency equation

\[
\Delta \ln \left( \frac{T}{T_c} \right) = 2\pi k_B T \sum_{\omega_n > 0} \Re \left( \sin \Theta_S - \frac{\Delta}{\hbar \omega_n} \right),
\]

where \( T_c \) is the critical temperature of single \( S \) layer (film) in the absence of magnetic field. These equations are supplemented by the Kupriyanov-Lukichev boundary conditions between layers: \( \mathbf{A} = (0, 0, -H x) \) in case of trilayer and \( \mathbf{A} = (0, 0, -H (x - d_S - d_F - d_N / 2)) \) in case of pentalayer.

To calculate the supercurrent density we use the following expression

\[
j = \frac{2\pi k_B T}{e \rho} q \sum_{\omega_n > 0} \Re (\sin^2 \Theta),
\]

where \( \rho \) is the residual resistivity of the corresponding layer. From Eq. (6) and London relation \( j = -cA / 4\pi \lambda^2 \), one can find expression for square of inverse London penetration depth

\[
\frac{1}{\lambda^2} = \frac{16\pi^2 k_B T}{\hbar c^2 \rho} \sum_{\omega_n > 0} \Re (\sin^2 \Theta),
\]

and for the inverse effective penetration depth

\[
\Lambda^{-1} = \int_0^d \frac{dx}{\lambda^2(x)},
\]

where the total thickness \( d = d_S + d_F + d_N \) for the \( S/F/N \) and \( d = 2d_S + 2d_F + d_N \) for the \( S/F/N/F/S \) structures. In case of thin \( S \) film \( \Lambda \) coincides with Pearl penetration depth.

Because we neglect variation of \( H \) due to screening we simply use the Helmholtz free energy per unit of square

\[
F_H = \pi N(0) k_B T \sum_{\omega_n > 0} \int \Re \{ \hbar D (\nabla \Theta)^2 + \sin^2 \Theta (q / \hbar)^2 \} dx - 4(\hbar \omega_n + i\hbar) (\cos \Theta - 1) - 2\Delta \sin \Theta \} dx.
\]

In numerical calculations we use the dimensionless units. The magnitude of the order parameter is normalized in units of \( k_B T_c \), length is in units of \( \xi = \sqrt{\hbar D / k_B T_c} \), the free energy per unit of square is in units of \( F_0 = N(0) (k_B T_c)^2 \xi_c \). The magnetic field is measured in units of \( H_0 = \Phi_0 / 2\pi \xi_c^2 \), the effective penetration depth is in units of \( \Lambda = \lambda_S^2 / d_S \), where \( \lambda_0 \) is the London penetration depth of the single \( S \) layer at zero temperature.

To find the effective penetration depth \( \Lambda^{-1} \), we numerically solve equations [1–4] using Kupriyanov-Lukichev boundary conditions [5]. In calculations we assume that the density of states on the Fermi level \( N(0) \) is the same for all layers and, therefore, the ratio of resistivities is inversely proportional to the ratio of corresponding diffusion coefficients. To reduce the number of free parameters we also assume that the resistivity of \( S \) layer and \( F \) layers are equal, i.e. \( \rho_S / \rho_F = 1 \), which roughly corresponds to parameters of real \( S \) and \( F \) films. Because formation of FFLO state in the \( S/F/N \) structure needs the large ratio of resistivities between \( N \) layer and \( S \) layers, we use \( \rho_S / \rho_N = 150 \) in our calculations, which is close to the parameters of real materials. For example for pair \( \mathrm{NbN}/\mathrm{Al} \) the ratio \( \rho_S / \rho_N \) could be as large as 400 (Ref. 13) while for pair \( \mathrm{NbN}/\mathrm{CuNi} \) \( \rho_S / \rho_F \sim 1.5 \) (Ref. 14). The exchange field of the ferromagnet is assumed to be of the order of the Curie temperature \( T_{Curie} \) (for example in \( \mathrm{CuN} \) \( h \sim 13 k_B T_c \)).

In-plane FFLO state could be realized as FF-like state (in this case \( f(z) \sim \exp(iq_0 z) \)) or as LO-like state (in this case \( f(z) \sim \cos(q_0 z) \) near \( T^{FFLO} \)). In addition to 1D calculations we also numerically solved 2D Usadel equation (in \( x \) and \( z \) directions in Fig. 1) with following
boundary conditions along $z$ direction: $f(z = 0) = f(z = \pi/q_0) = 0$ and found that such a LO-like state has an energy (per unit of volume) larger than FF-like state at any $q_0 = \partial \varphi/\partial z$. Therefore throughout our paper under FFLO state we assume FF-like state $f(z) \sim \exp(iq_0 z)$ and solve 1D problem in $x$ direction.

III. S/F/N TRILAYER

Let us first consider the S/F/N trilayer. As it is shown in Ref. 7, in the case $\rho_N \ll \rho_S$ there is a range of parameters when in-plane FFLO phase appears below the certain critical temperature $T_{FFLO}^c < T_c$. In the FFLO phase the effective penetration depth $\Lambda^{-1} = 0$ as $H \to 0$, which signals about vanishing of magnetic response at $T \leq T_{FFLO}^c$. Here we calculate effect of finite $H$ and $I$ on $\Lambda^{-1}$.

![FIG. 2](image)

**FIG. 2.** Dependence of the free energy $F_H$ (a) and inverse effective penetration depth $\Lambda^{-1}$ (b) on $q_0$ for the S/F/N trilayer being in in-plane FFLO state at different values of the parallel magnetic field. The arrows indicate the values of $\Lambda^{-1}$ corresponding to the left minimum of the free energy. We use the following parameters of the system: $h = 5k_BT_{c,0}$, $d_S = 1.1\xi_c$, $d_F = 0.5\xi_c$, $d_N = \xi_c$ and $T = 0.3T_{c,0}$.

In Fig. 2(a) we show dependence $F_H(q_0)$ for the trilayer below the FFLO transition temperature $T_{FFLO}^c$. One can see that in the absence of the external field two states with $q_0 \neq 0$ have a minimal energy. Both states correspond to $\Lambda^{-1} = 0 = \partial F_H/\partial q_0$ (Fig. 2b). The parallel magnetic field $H$ breaks the symmetry $F_H(q_0)$ and leads to the increase and furthermore disappearance of one of the energy minimums (right one in Fig. 2a). Corresponding to this minimum state has negative value of $\Lambda^{-1}$ at $H > 0$ and according to the arguments suggested in Ref. 7 should be considered as an unstable one. Indeed, one can show that the term corresponding to contribution of kinetic energy to $F_H$ is proportional to $\Lambda^{-1}q^2$. When $\Lambda^{-1} < 0$ it is energetically favorable to have nonzero supervelocility $\sim q$ (if it was zero) or increase it (if it was finite) which makes such a state with negative $\Lambda^{-1}$ unstable. To see how this instability evolves in time and what is the finite state one should solve 3D problem in our case (taking into account $q \neq 0$ in all directions) and it is out of scope of the present research. Further we consider only the state with $\Lambda^{-1} \geq 0$, corresponding to the left minimum of $F_H(q_0)$ in Fig. 2a.

The field dependence of $\Lambda^{-1} \geq 0$ is present in Fig. 2a. One can see that $\Lambda^{-1}$ nonmonotonically changes with a field. Increase of $\Lambda^{-1}$ at relatively weak magnetic field is connected with two effects. The first one is the suppression of the superconducting correlations (including triplet one) in N layer by magnetic field and we find that it gives main contribution to increase of $\Lambda^{-1}$. Besides that there is slight enhancement of of singlet superconductivity in S layer, because weak magnetic field decreases supervelocility $\sim q = q_0 + 2\pi A/\Phi_0$ in S layer, and it also provides enhancement of $\Lambda^{-1}$. The second effect is responsible for enhancement of $T_{FFLO}^c$ (see Fig. 3) by applied field - earlier this effect was predicted for S/F bilayer being in FFLO state in Ref. 15. Note that the found enhancement of $T_c$ is rather small for S/F/N trilayer with realistic parameters.

Sufficiently large magnetic field destroys proximity-induced superconductivity in F/N layers and $\Lambda^{-1}$ reaches the maximum value - see Fig 2a. The following decrease of $\Lambda^{-1}$ is explained by gradual increase of $q \sim A$ in S layer and gradual suppression of $|\Delta|$ as in usual S film. These results show that $\Lambda^{-1}$ is finite and positive at any finite $H$ for S/F/N trilayer being in FFLO state. One may also conclude that the magnetic response is diamagnetic and nonlinear even at $H \to 0$ because $\Lambda^{-1}$ changes from zero up to the finite value.

![FIG. 3](image)

**FIG. 3.** Dependence of the inverse effective penetration depth of the magnetic field $\Lambda^{-1}$ in the S/F/N trilayer on the parallel magnetic field $H$ at different thicknesses of F layer $d_F$: (a) $0.5\xi_c$ (FFLO state); (b) $0.8\xi_c$; (c) $\xi_c$; (d) $1.2\xi_c$. The other parameters of the trilayer are following: $h = 5k_BT_{c,0}$, $d_S = 1.1\xi_c$, $d_N = \xi_c$ and $T = 0.2T_{c,0}$.
FIG. 4. Dependence of the critical temperature of S/F/N trilayer on the parallel magnetic field $H$. The parameters of the system are following: $h = 25k_BT_{c0}$, $d_S = 1.2\xi_c$, $d_F = 0.16\xi_c$, $d_N = \xi_c$. Using smaller thickness of S layer one can obtain larger relative change of $T_c$ but $T_c$ itself goes to lower temperatures. The similar result could be obtained for $h = 5k_BT_{c0}$ too.

Even if $\Lambda^{-1}$ is positive at $H = 0$ and trilayer is not in the FFLO state the dependence $\Lambda^{-1}(H)$ may be non-monotonic due to contribution of triplet component to $\Lambda^{-1}$. In Fig. 3(b,c,d) we demonstrate it by varying the thickness of F layer and keeping over parameters of trilayer constant. With increasing $d_F$ contribution of triplet component to $\Lambda^{-1}$ decreases, but it stays finite. Small increase of $d_F$ (see Fig. 3(b)) drives the system from FFLO state but due to considerable contribution of triplet component to $\Lambda^{-1}$ it cannot be used to study states which are not in the FFLO phase domain and ii) it is difficult to relate coefficients in Ginzburg-Landau functional with microscopic parameters in F layer.

At larger $d_F$, i.e. with getting further from the FFLO domain, $\Lambda^{-1}(H = 0)$ increases and starting from some value of $d_F$ ($\approx 2\sqrt{hD_F}/h$) the inverse penetration depth $\Lambda^{-1}$ decreases in a weak magnetic field (see Fig. 3(c)). Our calculations show that the effect is connected with faster decay of singlet component than the triplet one in N layer at weak magnetic field. At the field larger some value (it roughly corresponds to the minimum in dependence $\Lambda^{-1}(H)$ shown in Fig. 3(c)) the proximity induced superconductivity is getting suppressed stronger and $\Lambda^{-1}$ increases as in Fig. 3(a,b) and the dependence $\Lambda^{-1}(H)$ has both minimum and maximum. Further increase of $d_F$ (see Fig. 3(d)) leads to the monotonic decrease of $\Lambda^{-1}$ in magnetic field (triplet component gives small contribution to $\Lambda^{-1}$) and the influence of N layers manifests itself in rapid vanishing of $\Lambda^{-1}$ at relatively weak fields when superconductivity is terminated there.

Very similar results we obtain for the larger value of exchange field ($h = 25k_BT_{c0}$) with the only difference that they occur in much narrower range of $d_F$ with respect to $\xi_c$, reflecting smaller value of characteristic decay length of superconducting correlations in F layer $\xi_F \approx 1/\sqrt{h}$ (results are not shown here). Qualitatively the same dependencies $\Lambda^{-1}(H)$ (except the one with two extremum shown in Fig. 3(c)) could be found at fixed $d_F$ when one increases temperature from $T < T_{FFLO}$ up to $T_{FFLO} < T < T_c$ when the trilayer is driven from in-plane FFLO to uniform state but with still noticeable contribution of triplet superconductivity to $\Lambda^{-1}$.

FFLO state in S/F/N trilayer could be tuned not only by parallel magnetic field but in-plane current too. As in the case of parallel magnetic field applied current breaks the symmetry $q_0 \rightarrow -q_0$ and in Fig. 5 we show dependence of $\Lambda^{-1}$ on the in-plane current for the same parameters as in Fig. 3. External current (supervelocity) suppresses stronger proximity induced superconductivity in N layer than the superconductivity in S layer (like in S/N bilayer and) and $\Lambda^{-1}$ increases with the current for some $d_F$ (see Figs. 3(a,b)). Qualitatively, the results shown in Fig. 5(a) could be found using modified Ginzburg-Landau equation as it was done in Ref. where current-carrying FFLO state was studied. This approach is much simpler than the used here Usadel equations and allows to obtain analytical solution for current states but it has two disadvantages: i) it cannot be used to study states which are not in the FFLO phase domain and ii) it is difficult to relate coefficients in Ginzburg-Landau functional with microscopic parameters of S/F/N structure.

FIG. 5. Dependence of the inverse effective penetration depth $\Lambda^{-1}$ in the S/F/N trilayer on the in-plane $I$ at different thicknesses of F layer $d_F$: (a) $0.5\xi_c$ (FFLO state); (b) $0.8\xi_c$; (c) $\xi_c$; (d) $1.2\xi_c$. Current is expressed in units of the critical current of the FFLO-state $I_c^{FFLO}$. The rest of parameters is the same as in Fig. 3.
IV. $\pi \to$ FFLO TRANSITION IN S/F/N/F/S STRUCTURES

Let us discuss now symmetric S/F/N/F/S pentalayer (it could be imagined as doubled trilayer). Our interest to this system is mainly connected with existence of $\pi$ state, corresponding to the phase difference $\pi$ between outer S layers, together with 0 state (uniform or FFLO one) considered in previous section. We restrict ourself by consideration of the uniform $\pi$-state, because in the chosen parameter range modulated (FFLO) $\pi$-state is not realized (note that in the recent work\textsuperscript{25} such a state is predicted for the S/F/S structure in certain range of parameters).

![Graph](image)

**FIG. 6.** (a) Dependence of the critical temperature of 0 ($T_0^\pi$), $\pi$ ($T_\pi^\pi$) and FFLO ($T_{FFLO}$) states on the thickness of F layer for the S/F/N/F/S pentalayer. Below temperature $T^*$ the $\pi$-state is energetically more favorable than the 0-state. (b) Temperature dependence of $\Lambda^{-1}$ for the pentalayer with $d_F = 0.4\xi_c$ being in 0 or $\pi$-states. The arrow indicates temperature of the 0-$\pi$ transition. We use the following parameters: $h = 5k_B T_c$, $d_S = 1.4\xi_c$, $d_N = 2\xi_c$.

The response of trilayer and pentalayer on the parallel magnetic field is somewhat different due to different orbital effect produced by $H$. In trilayer magnetic field induced supervelocity is maximal in N layer while in pentalayer it is maximal in S layers. It is the reason why for pentalayer we did not find enhancement of $T_c$ by parallel magnetic field (shown in Fig. 4 for trilayer) and dependence $\Lambda^{-1}(H)$ like one shown in Fig. 3(c). Besides, due to symmetry of considered pentalayer the parallel magnetic field does not remove degeneracy of FFLO state with respect to sign of $g_0$ as it does for trilayer. Despite these differences we find that in FFLO state and at parameters close to FFLO phase domain $\Lambda^{-1}$ increases in weak magnetic field and decreases in large field which leads to maximum in dependence $\Lambda^{-1}(H)$ like in trilayer (see Fig. 3(a,b)). The only quantitative difference is that in pentalayer the FFLO state exists in narrower range of $d_F$ than in trilayer (with the same parameters) because of its competition with $\pi$ state. Further in this section we mainly focus on temperature and current/magnetic field driven $\pi \to$ FFLO transition in symmetric pentalayer.

Fig 5(a) demonstrates the dependence of the critical temperatures of 0 (uniform and FFLO) and $\pi$ (uniform) states on thickness of F layers. The temperature dependence of $\Lambda^{-1}$ for the pentalayer being in the FFLO state resembles that dependence the S/F/N structure being in FFLO state (compare with Fig. 3(a) from 5). In the $\pi$-state the same pentalayer shows monotonic increase of $\Lambda^{-1}$ with lowering temperature which is typical for single layer of hybrid S/F or S/N/F structures with no or negligible contribution of odd frequency triplet superconductivity to $\Lambda^{-1}$. Similarly to the temperature-driven 0-$\pi$ transition in the S/F/S structures\textsuperscript{19,20} there is such a transition in our pentalayer at $T = T^*$ in some range of $d_F$. In contrast to S/F/S structure considered in Ref. 21 in our pentalayer $\Lambda^{-1}$ increases at $0 \to \pi$ transition since in the $\pi$-state there is practically no negative contribution from the triplet component to $\Lambda^{-1}$. This difference becomes even more dramatic at transition from 0 FFLO state to uniform $\pi$ state when $\Lambda^{-1}$ changes from zero up to finite value as temperature decreases - see Fig. 6(b).

We also find, at fixed temperature $T < T^* < T_{FFLO}^\pi$, current or magnetic driven transition to FFLO state. Let us first consider current-driven transition. In Fig. 7 we show dependence of Gibbs energy $G = F_H - (h/2|e|)I_0q_0$, which should be used for current driven state instead of Helmholz free energy\textsuperscript{20} on current for $\pi$ and FFLO states. One can see that at $I > I_c$ FFLO state becomes more energetically favorable. Like as for temperature-driven transition there is a jump in $\Lambda^{-1}$ (see inset in Fig. 7) and in $q$, because transition current $I_t \sim q\Lambda^{-1}$ is the same in both states. It implies that $\pi \to$ FFLO transition at $I > I_t$ should be accompanied by appearance of transitional electric field, which accelerate superconducting condensate, and the voltage pulse.

Our calculations show that near $T^*$ the transition occurs at sufficiently small currents $I_t \ll I_{FFLO}^\pi$, where $I_{FFLO}^\pi$ is the critical current of FFLO state (it corresponds to maximal possible superconducting current flowing along pentalayer in FFLO state), which is close to $I_c^\pi$ of $\pi$ state. With decreasing temperature $I_t$ increases but stays smaller than $I_c$ for both FFLO and $\pi$ states which makes current-driven transition possible at all temperatures $0 < T < T^*$. Note that there is also transition from $\pi$ to 0 uniform state when $T^* > T_{FFLO}^\pi$ but it requires larger currents and exists in narrow temperature interval below $T^*$.

The parallel magnetic field differently affects the superconductivity in the FFLO and $\pi$-states, which at temperatures $T < T^* < T_{FFLO}^\pi$ can result to the field-driven $\pi \to$ FFLO transition (Fig. 8). Similar to current-driven transition here we also have jump in $\Lambda^{-1}$ – see inset in Fig. 8 while dependence $\Lambda^{-1}(H)$ in FFLO state resembles one for the S/F/N trilayer (compare with Fig. 3(a)). Because energies of $\pi$ and FFLO states are rather close one needs relatively large magnetic field to make FFLO
FIG. 7. Current dependence of the Gibbs energy for 0- and π-states in the S/F/N/F/S pentalayer at temperature $T = 0.2T_c$. Current is expressed in units of the critical current of the FFLO-state $I_{c1}^{FFLO}$. The temperature of the 0-π transition $T^\ast = 0.26T_c$ at chosen parameters ($h = 5k_BT_c$, $d_S = 1.2\xi_c$, $d_F = 0.4\xi_c$, $d_N = 2\xi_c$). The arrow indicates the transition current $I_t$ when Gibbs energies of π and FFLO states becomes equal. In the inset we show dependence of $\Lambda^{-1}$ on supercurrent in both states.

state more energetically favorable – see Fig. 5. Unlike the trilayer, in S/F/N/F/S pentalayer at certain field $H_{c1}$ vortices can emerge. Using the expression, valid for the S film with thickness $d_S$, $H_{c1} \sim \Phi_0/d_S^2$ [22] and replacing thickness $d_S$ by total thickness of the pentalayer, we obtain that $H_{c1} \approx 0.2H_0$ for the used in Fig. 8 parameters. This estimation explains our choice of maximal magnetic field in Fig. 8. To study effect of vortices one needs solution of 3D problem and it is out of scope of our paper.

V. SUMMARY

We have studied effect of parallel magnetic field and in-plane current on screening properties of thin S/F/N and S/F/N/F/S structures being in or close to FFLO state. In the parameter region corresponding to the formation of in-plane FFLO-phase, the effective inverse magnetic field penetration depth $\Lambda^{-1}$ is positive at any finite magnetic field/current and $\Lambda^{-1} \rightarrow 0$ as $H, I \rightarrow 0$ which implies diamagnetic response of such structures. Due to suppression of triplet superconductivity in F/N layers by magnetic field/current the dependence $\Lambda^{-1}(H)/(I)$ has unusual field/current dependence not only in FFLO state but also at parameters close to FFLO phase domain – $\Lambda^{-1}$ increases in weak fields/currents and reaches maximal value at finite $H/I$. We also find that the parallel magnetic field not only control screening properties of FFLO state but it also can drive S/F/N/F/S pentalayer from uniform π state to in-plane FFLO state which is accompanied by giant change of $\Lambda^{-1}$. The same transition could be induced by in-plane current or by changing the temperature.

Experimentally, predicted effects could be verified, for example by two-coil technique [13,23,25] which allows to measure $\Lambda^{-1}$ of thin superconducting structures directly. Potentially found results could be used in the magnetic field sensors (due to strong magnetic field dependence of $\Lambda^{-1}$) or in kinetic inductance detectors of electromagnetic radiation or particles [25] when local heating of heterostructure due to absorbed energy may considerably change $\Lambda^{-1}$ (see for example Fig. 6), which determines the kinetic inductance of the sample.

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1 F. S. Bergeret, A. F. Volkov, and K. B. Efetov, Josephson
current in superconductor-ferromagnet structures with a nonhomogeneous magnetization, Phys. Rev. B 64, 134506 (2001).
2. Y. Asano, A. A. Golubov, V.Y. Fominov, and Y. Tanaka, Unconventional Surface Impedance of a Normal-Metal Film Covering a Spin-Triplet Superconductor Due to Odd-Frequency Cooper Pairs, Phys. Rev. Lett. 107, 087001 (2011).
3. T. Yokoyama, Y. Tanaka, and N. Nagaosa, Anomalous Meissner Effect in a Normal-Metal Superconductor Junction with a Spin-Active Interface, Phys. Rev. Lett. 106, 246601 (2011).
4. Y. Asano, Y.V. Fominov, and Y. Tanaka, Consequences of bulk odd-frequency superconducting states for the classification of Cooper pairs, Phys. Rev. B 90, 094512 (2014).
5. M. Alidoust, K. Halterman, and J. Linder, Meissner effect probing of odd-frequency triplet pairing in superconducting spin valves, Phys. Rev. B 89, 054508 (2014).
6. Y. Fominov, Y. Tanaka, Y. Asano, and M. Eschrig, Odd-frequency superconducting states with different types of Meissner response: Problem of coexistence, Phys. Rev. B 91, 144514 (2015).
7. S. Mironov, A. Melnikov, and A. Buzdin, Vanishing Meissner effect as a Hallmark of in-Plane Fulde-Ferrell-Larkin-Ovchinnikov Instability in Superconductor-Ferromagnet Layered Systems, Phys. Rev. Lett. 109, 237002 (2012).
8. S. V. Mironov, D. Vodolazov, Yu. Yerin, A. V. Samokhvalov, A. S. Mel'nikov and A. Buzdin, Temperature controlled Fulde-Ferrrell-Larkin-Ovchinnikov instability in superconductor-ferromagnet hybrids, Phys. Rev. Lett. 121, 077002 (2018).
9. A. I. Buzdin, Proximity effects in superconductor-ferromagnet heterostructures, Rev. Mod. Phys. 77 935 (2005).
10. K. D. Usadel, Generalized diffusion equation for superconducting alloys, Phys. Rev. Lett. 25, 507 (1970).
11. M. Yu. Kuprianov and V. F. Lukichev, Influence of boundary transparency on the critical current of "dirty" SS'S structures, Sov. Phys. JETP 67, 1163 (1988).
12. J. Pearl, Current distribution in superconducting films carrying quantized fluxoids, Appl. Phys. Lett. 5 65 (1964).
13. D. Yu. Vodolazov, A. Yu. Aladyshkin, E. E. Pestov, S. N. Vdovichev, S. S. Ustavshikov, M. Yu. Levichev, A. V. Putilov, P. A. Yunin, A. I. El'kina, N. N. Bukharov and A. M. Klushin, Peculiar superconducting properties of a thin film superconductor-normal metal bilayer with large ratio of resistivities, Supercond. Sci. Technol. 31, 115004 (2018).
14. T. Yamashita, A. Kawakami, and H. Terai, NbN-Based Ferromagnetic 0 and π Josephson Junctions, Phys. Rev. Appl. 8, 054028 (2017).
15. A.M. Bobkov and I.V. Bobkova, Enhancing of the Critical Temperature of an In-Plane FFLO State in Heterostructures by the Orbital Effect of the Magnetic Field, JETP Letters, 99, 333 (2014).
16. A. I. Buzdin and H. Kachkachi, Generalized Ginzburg-Landau theory for nonuniform FFLO superconductors, Phys. Lett. A 225, 341 (1997).
17. K. V. Samokhin and B. P. Truong, Current-carrying states in Fulde-Ferrell-Larkin-Ovchinnikov superconductors, Phys. Rev. B 96, 214501 (2017).
18. A. V. Samokhvalov, Phase Transitions in Hybrid SFS Structures with Thin Superconducting Layers, Phys. Solid State, 59, 2143 (2017).
19. V. V. Ryazanov, V. A. Oboznov, A. Yu. Rusanov, A. V. Veretennikov, A. A. Golubov, and J. Aarts, Coupling of Two Superconductors through a Ferromagnet: Evidence for a π Junction, Phys. Rev. Lett. 86, 2427 (2001).
20. N. Pompeo, K. Torokhtii, C. Cirillo, A. V. Samokhvalov, E. A. Ilyina, C. Attanasio, A. I. Buzdin, and E. Silva, Thermodynamic nature of the 0-π quantum transition in superconductor/ferromagnet/superconductor trilayers, Phys. Rev. B 90, 064510 (2014).
21. D. E. McCumber, Intrinsic Resistive Transition in Thin Superconducting Wires Driven from Current Sources, Phys. Rev. 172, 427 (1968).
22. V.V. Schmidt, Critical current in superconducting films, Zh. Eksp. Teor. Fiz. 57, 2095 (1969) [Sov. Phys.-JETP 30, 1137 (1970)].
23. S. J. Turneaure, E. R. Ulm, T. R. Lemberger, Numerical modeling of a two-coil apparatus for measuring the magnetic penetration depth in superconducting films and arrays, J. Appl. Phys. 79, 4221 (1996).
24. J. H. Claassen, J. M. Byers and S. Adrian, Optimizing the two-coil mutual inductance measurement of the superconducting penetration depth in thin films, J. Appl. Phys. 82, 3028 (1997).
25. T. R. Lemberger, I. Hetel, A. J. Hauser, and F. Y. Yang, Superfluid density of superconductor-ferromagnet bilayers, J. of Appl. Phys. 103, 07C701 (2008).
26. P. K. Day, H. G. LeDuc, B. A. Mazin, A. Vayonakis, J. Zmuidzinas, A broadband superconducting detector suitable for use in large arrays, Nature 425, 817 (2003).