CONFRONTATION OF DOUBLE-INFLATIONARY MODELS WITH OBSERVATIONS

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Abstract

We consider double-inflationary models with two noninteracting scalar fields, a light scalar field $\phi_l$ with potential $\frac{1}{2}m_l^2\phi_l^2$ and a heavy scalar field $\phi_h$ with potential $\frac{1}{n}\phi_h^n$ with $n = 2, 4$. CDM with the initial spectrum of adiabatic perturbations produced in these models is compared with observations. These models contain two more free parameters than the standard CDM model with an initial scale-invariant spectrum. We normalize our spectra to COBE DMR and compare the predictions with observations on the biasing factor, large-scale peculiar velocities, quasar and galaxy formation and the Stromlo-APM counts-in-cells analysis. The model with $n = 4$ is excluded by the data while for the $n = 2$ model, taking cosmic variance into account, a small window of parameters compatible with observations is found.

PACS Numbers:
INTRODUCTION

Inflationary models [1] can solve some of the outstanding problems in cosmology. Also, as emphasized already some time ago [2], for a given model it is possible to calculate the various spectra of fluctuations produced during the inflationary phase and to put the predictions to test using observational data. In the simplest inflationary models, the Fourier components of the gravitational potential are Gaussian stochastic quantities with an approximately flat (Harrison-Zel’dovich) r.m.s. spectrum [3–5]. For more complicated spectra, calculation of the spectrum can be implemented either analytically or numerically, (for the latter possibility, see for ex. [6]), and the spectrum can be obtained with very high accuracy. The (approximately) flat (Harrison-Zel’dovich) spectrum, together with the assumptions of the standard CDM (Cold Dark Matter) model was put to test using N-body simulations and a very good agreement was found with the observed galaxy-galaxy correlation function on scales \((0.5 - 10)h^{-1}\text{Mpc}\) when \(h \approx 0.5\) [7], see also [8] \((h \equiv H_0/100 \text{ km/s/Mpc})\). Observations seem to imply more power on scales greater than \(20h^{-1}\text{Mpc}\), strong evidence coming from the APM galaxy survey and observed peculiar velocities [9] and from large-angle \(\Delta T\) fluctuations [10–12]. There are several ways to solve this problem, either abandoning the assumptions of CDM, one may consider a mixture of hot and cold dark matter [13] or a nonvanishing cosmological constant, or trying CDM with an initial perturbation spectrum having more power on large scales. Following the second possibility, an attractive solution is to consider so called “tilted” spectra, with spectral index \(n < 1\). Such a spectrum will occur in power-law and in extended inflation. Recent studies however [14,15] have shown that no value of \(n\) seems able to reconcile the CDM model with all the observations (in accordance with the earlier “pre-COBE” discussion in [16,17]), although the value \(n \approx 0.7 - 0.8\) comes closest to it.

So, if we want to reconcile the CDM model with observations without introducing neutrinos with a restmass of a few \(eV\), we have to consider models belonging to the next level of complexity, i.e. having at least one more additional parameter characterizing the initial perturbation spectrum. Inflationary models with flat spectra need one free parameter to specify the amplitude of the spectrum of density perturbations, for example the coupling constant of the inflaton in the simplest versions of inflation, while those generating a tilted spectrum need one more free parameter, for example the Brans-Dicke parameter \(\omega\) in extended inflation models. We will consider here double inflationary models [17–25]. In the specific models considered here, the inflationary stage is driven by scalar fields without mutual interaction potential. As a general rule, such models produce a spectrum of adiabatic perturbations having more power on scales larger than some characteristic scale [22]. Three free parameters are now needed: one for the height and the form of the “step”, one for its location and finally one for the overall normalization. We study here two double inflationary models consisting of two noninteracting scalar fields, a light scalar field \(\phi_l\) with potential \(\frac{1}{2}m_l^2\phi_l^2\) and a heavy scalar field \(\phi_h\) with potential \(\frac{1}{n}\phi_h^n\). The spectra obtained numerically are then compared with observational data. Recently, an analogous study was performed for another double-inflationary model [26] and we will present the results in a way which makes the comparison easier. In section I we give some basic results for the models considered here and specify their free parameters. In section II, we give the various observational tests to which our models are submitted for different values of their parameter. In section III finally,
we give a brief discussion of our results.

I. THE MODEL AND ITS FLUCTUATION SPECTRUM

Let us start with a short description of the homogeneous background. We consider the following Lagrangian density describing matter and gravity

\[ L = \frac{R}{16\pi G} + \frac{1}{2} \phi_h,\mu \phi_h^\mu - \frac{\lambda}{n} \phi_h^n + \frac{1}{2} (\phi_h,\mu \phi_l^\mu - m_h^2 \phi_l^2) \] (1)

where \( \mu = 0, \ldots, 3, c = \hbar = 1 \) and the Landau-Lifshitz sign conventions are used. When \( n = 2, \lambda \equiv m_h^2 \), whereas for \( n = 4, \lambda \) is a dimensionless coupling constant. The space-time metric has the form

\[ ds^2 = dt^2 - a(t)^2 \delta_{ij} dx^i dx^j, \quad i, j = 1, 2, 3. \] (2)

Spatial curvature may always be neglected because it becomes vanishingly small during the first period of inflation driven by the heavy scalar field. The homogeneous background is treated classically, it is determined by the scale factor \( a(t) \) and the two scalar fields \( \phi_h, \phi_l \). Their equations of motion are given by

\[ \dot{a} = aH, \quad H^2 = \frac{4\pi G}{3} (\dot{\phi}_h^2 + \dot{\phi}_l^2 + 2\frac{\lambda}{n} \phi_h^n + m_h^2 \phi_l^2), \]
\[ \ddot{\phi}_h + 3H \dot{\phi}_h + \lambda \phi_h^{n-1} = 0, \quad \ddot{\phi}_l + 3H \dot{\phi}_l + m_l^2 \phi_l = 0, \]

where a dot denotes a derivative with respect to \( t \). We have the useful equation

\[ \dot{H} = -4\pi G (\dot{\phi}_h^2 + \dot{\phi}_l^2) \] (3)

which shows that \( H \) always decreases with time in this model.

Let us consider now the inhomogeneous perturbations. We consider a perturbed FRW background whose metric, in the longitudinal gauge, reads

\[ ds^2 = (1 + 2\Phi)dt^2 - a^2(t) \delta_{ij} dx^i dx^j \] (4)

(in Bardeen’s notations [27], \( \Phi = \Phi_A, \Psi = -\Phi_H \)). We get from the perturbed Einstein equations (exp(\( ikr \)) spatial dependence is assumed and the Fourier transform convention is \( \Phi_k \equiv \frac{1}{(2\pi)^{3/2}} \int \Phi(r) e^{-ikr} d^3k \))

\[ \Phi = \Psi, \]
\[ \dot{\Phi} + H \Phi = 4\pi G (\dot{\phi}_h \delta \phi_h + \dot{\phi}_l \delta \phi_l), \]
\[ \delta \ddot{\phi}_h + 3H \delta \dot{\phi}_h + \left( \frac{k^2}{a^2} + (n-1)\lambda \phi_h^{n-2} \right) \delta \phi_h = 4\dot{\phi}_h \Phi - 2\lambda \phi_h^{n-1} \Phi, \]
\[ \delta \ddot{\phi}_l + 3H \delta \dot{\phi}_l + \left( \frac{k^2}{a^2} + m_l^2 \right) \delta \phi_l = 4\dot{\phi}_l \Phi - 2m_l^2 \phi_l \Phi \] (5)

We see that, contrary to the case when only one scalar field is involved, when we have more than one scalar field, the dynamics of the perturbed system cannot be described by
just one equation for the master quantity $\Phi$ \cite{28} or else for the gauge-invariant quantity $\zeta = \delta \phi + (\dot{\phi}/H)\Phi$ in terms of which the action for the fluctuations can be written \cite{29}. Analogously to \cite{17} one can now compute the spectrum of growing adiabatic perturbations. These perturbations arise from the vacuum fluctuations of the scalar fields $\phi_h$ and $\phi_l$. The fluctuations are Gaussian and the power spectrum $\Phi^2(k)$ of the gravitational potential, defined through $\langle \Phi^* \Phi \rangle = \Phi^2(k) \delta(k-k')$ characterizes them completely. For scales crossing the Hubble radius when both scalar fields are in the slow rolling regime, the spectrum of growing adiabatic perturbations, when those scales are outside the Hubble radius during the matter-dominated stage (assuming $a(t) \propto t^{\frac{4}{3}}$ at the present time), is given by

$$k^\frac{3}{2} \Phi(k) = \frac{6}{5} \sqrt{2\pi G H} \sqrt{\left(\frac{2\phi_h}{n}\right)^2 + \phi_l^2}$$

$$\simeq \frac{4}{5} \sqrt{6\pi^3 G^3 \lambda} \frac{2}{n} \phi_h^2 \phi_l$$

$$= \frac{\sqrt{24\pi G \lambda}}{5} \left(\frac{4\pi G}{n}\right)^{2-n} \sqrt{s \ln \frac{n}{k}} \frac{k_f}{k}$$

where the r.h.s. has to be taken at $t = t_k$, the time at which a perturbation with wavenumber $k$ comes outside the Hubble radius during inflation, $k = a(t_k) H(t_k)$. The wavenumber $k_f$ corresponds to the characteristic scale appearing in the spectrum and it is close to the scale crossing the Hubble radius near the end of the first inflation. Hence, there exists a very broad interval of scales for which the dynamics of inflation at the time of the first Hubble radius crossing is determined by the field $\phi_h$ while the main contribution to $\phi(k)$ is made by $\phi_l$. We see from (7) that the upper part of the spectrum, corresponding to small $k'$s or very large scales, is not flat but has a logarithmic dependence $\propto \ln \frac{n}{k}$. This gives, as we will see, a crucial difference between the $n = 2$ and $n = 4$ spectra. The quantity $s(t)$ is the number of e-folds from time $t$ till the end of the second inflation and it is given by

$$s \simeq 4\pi G \left(\frac{\phi_h^2}{n} + \frac{\phi_l^2}{2}\right)$$

In order to have a sufficiently long second inflationary stage that will put the characteristic length scale of the spectrum on a scale in agreement with observations, we have that $\phi_l \simeq 3M_p$ near the end of the first inflation so $s(t_f) \simeq 60$. A very little change in this initial value will be enough to shift the spectrum in $k$-space while leaving the form of the spectrum practically unaltered. We define the parameter $p \equiv \sqrt{\lambda M_p^2/m_i}$, $p^2$ gives the order of magnitude of the ratio of the energy of the heavy scalar field to the energy of the light scalar field near the end of the first inflation; it specifies the form of the spectrum, namely its “step” with more power on large scales, and the width of the transition region in $k$-space. We will now estimate $\Delta_k$, the height of the “step” in the spectrum between a scale which is on the upper plateau (but of course still inside the cosmological horizon) and a scale at the beginning of the lower plateau. For scales crossing the Hubble radius at the beginning of the second inflationary stage (neglecting possible small oscillations), we have the standard result of inflation driven by one scalar field in slow-rolling regime

$$k^\frac{3}{2} \Phi(k) = \frac{\sqrt{24\pi G m_i^2}}{5} s_0$$

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$$k^\frac{3}{2} \Phi(k) = \frac{\sqrt{24\pi G m_i^2}}{5} s_0$$
\[ s_0 = 2\pi G \phi_0^2. \]

We therefore get for \( \Delta_k \)

\[ \Delta_k \approx \sqrt{\lambda} \frac{2 \phi_h^n(t_k)}{m_i \phi_0} \]

\[ \approx \sqrt{\lambda} \frac{4\pi G}{n} \left( \frac{2 + \ln h}{\sqrt{s_0}} \right)^{\frac{2n}{n}} \frac{k_f}{k} \]

\( k \ll k_f. \) \hspace{1cm} (11)

For \( n = 2 \), this is

\[ \Delta_k \approx 0.13p \ln \frac{k_f}{k} \]

\[ \approx 0.33p \phi_h(t_k), \] \hspace{1cm} (13)

whereas \( n = 4 \) yields

\[ \Delta_k \approx 0.073p \ln \frac{k_f}{k}, \]

\[ \approx 0.24p \phi_h^2(t_k). \] \hspace{1cm} (15)

where \( \phi_h \equiv \phi_h^M \). We will investigate models with \( 3 \leq p \leq 16 \) for \( n = 4 \) and \( 6 \leq p \leq 28 \) for \( n = 2 \). How we choose the scales of our spectra is very important when we compare them with the observations. For this purpose, we need a precise definition and we adopt here the convention of [26], and define it with the help of \( k_b \), the scale where the extrapolated upper part intersects the lower plateau. One shows numerically that the evolution of the background also during the transition between the two main inflationary stages is inflationary, in the sense that \( \ddot{a} > 0 \), for \( p < 25 \) when \( n = 2 \) [30] and for \( p < 50 \) when \( n = 4 \). Also, the initial perturbation spectrum will have no oscillations for \( p < 15 \) when \( n = 2 \) and also for all \( p \)'s considered here when \( n = 4 \). A last comment concerns the power spectrum \( P(k) \) defined through \( \langle \delta_k \delta_k^* \rangle = P(k) \delta(k - k') \). Linear perturbations grow at different rates depending on the relation between their wavelengths, the Jeans length and the Hubble radius and this is specified by the transfer function \( T(k) \) [more accurately, one should write \( T(k, t_0) \)]

\[ P(k, t_0) = \frac{4}{9} \frac{k^4}{H_0^4} \Phi^2(k) T^2(k). \]

where by definition, \( T(k \to 0) = 1 \). \( T(k) \) is computed numerically making assumptions about the matter content of the universe, and depends on parameters like \( \Omega_0 \) and \( h \). We use here the transfer function for the standard CDM model given by [31]

\[ T(q) = \frac{\ln(1 + 2.34q)}{2.34q} \left[ 1 + 3.89q + (16.1q)^2 + (5.46q)^3 + (6.72q)^4 \right]^{-\frac{1}{4}} \]

\( q \equiv \frac{k}{H_0 Mpc^{-1}}, \Omega_0 = 1, h = 0.5 \). We will assume tacitly in all the formulas that the density parameter \( \Omega_0 = 1 \), in accordance with inflation. The power spectra \( P(k) \) for different values of the parameters are displayed for the \( n = 2 \), resp. \( n = 4 \) model in Fig.1, resp. Fig.2.
II. CONFRONTATION WITH OBSERVATIONS

A. Normalization of the spectrum to COBE DMR

Let us start with the normalization of the spectrum. Since the discovery of large angular scale fluctuations in the Cosmic Microwave Background Radiation by COBE DMR [10], one can normalize the spectrum of fluctuations using these COBE DMR observations. These observations probe the spectrum of fluctuations on very large scales, from several hundreds Mpc up to the cosmological horizon and allow for a normalization based on first principles. More precisely, one has

\[ \sigma_T^2(10^0) = \sum_{l \geq 2} \frac{2l + 1}{4\pi} \langle |a_{lm}|^2 \rangle \exp[-l(l+1)\theta_0^2] \equiv \sum_{l \geq 2} a_l^2 \exp[-l(l+1)\theta_0^2]; \tag{19} \]

where \( \theta_0 = 0.425\theta_{FWHM} = 4.25^0 \) is the Gaussian angle corresponding to the antenna beam and additional smearing of the raw data. It is to be noted that due to the exponential factor, only the multipoles up to \( l \leq 20 \) contribute significantly to the observed anisotropy. The r.m.s. coefficients \( \sqrt{\langle |a_{lm}|^2 \rangle} \), with \( a_{lm} \) defined by

\[ \Delta T \equiv \sum_{l,m} a_{lm} Y_{lm} \tag{20} \]

are actually independent of \( m \). Based on the latest results of COBE DMR [11,12], we take \( \sigma_T^2(10^0) = (1.25 \pm 0.2) \times 10^{-5} \), with error bars at the 1\( \sigma \)–level. This allows us to normalize the fluctuation spectrum and constitutes a great step forward. For the CMBR anisotropy, on large angular scales the dominant effect is the Sachs-Wolfe effect and, for adiabatic perturbations, we have the fundamental relation

\[ \langle |a_{lm}|^2 \rangle = \frac{H_0^4}{2\pi} \int_0^\infty dk k^{-2} P(k) j_l^2(kr_{\text{rec}}), \tag{21} \]

where \( j_l \) is a spherical Bessel function and \( r_{\text{rec}} \) is the comoving distance between us and the surface of recombination, we have in very good approximation \( r_{\text{rec}} = \frac{2}{H_0} \). In the models we are considering here, normalization of the power spectrum \( P(k) \), which is obtained for given \( p \) and location of the “step”, through eq. (21) will fix the value of the remaining free parameter \( \lambda \) or equivalently \( m_h \). The parameter \( m_h \) will have the order of magnitude given thereafter when we vary \( p \) and \( k_b \) earlier [30]. As a result, the energy density at the beginning of the second inflation is also fixed: it is \( \sim \frac{2}{p^2} \times 10^{-11}M_p^4 \) for \( n = 2 \). For the parameter \( \lambda \), we get the following result, when we vary the parameters: \( \sqrt{\Lambda} \sim 2 \times 10^{-6} \) and the corresponding energy density is then \( \sim \frac{2}{p^2} \times 10^{-11}M_p^4 \) for \( n = 4 \). Another remark concerns the contribution to the CMBR anisotropy on large scales \( (l \leq 40) \) which comes from gravitational waves (tensor metric perturbations). One can show that this contribution is equal for both models and rather small, namely \( \sqrt{\langle |a_{lm}|^2 \rangle_{\text{tot}}} \equiv \sqrt{\langle 1 + \frac{4}{5} \rangle \langle |a_{lm}|^2 \rangle_{\text{AP}}} \approx 1.05 \sqrt{\langle |a_{lm}|^2 \rangle_{\text{AP}}} \) where the subscript \( \text{AP} \) refers to that part of the fluctuations due to adiabatic perturbations, i.e., the quantity calculated in (21). Although this effect is not large, it is important here for the \( n = 2 \) model, which for some choices of the parameters will turn out to be in marginal agreement.
with observations. Also, one has to take into account “cosmic variance” which is connected to the fact that we try to estimate r.m.s. values of physical quantities in the universe from a limited sample. Therefore, independently on how good the COBE measurements may be, the quantities \[ \sum_{l=\nu}^{m} \frac{|a_{lm}|^2}{\langle |a_{lm}|^2 \rangle} \] obey a \( \chi^2 \)-distribution for \( 2l + 1 \) d.o.f.

In connection with recent interest in relations between \( T_S \equiv \langle |a_{lm}|^2 \rangle_{GW} \langle |a_{lm}|^2 \rangle_{AP} \) (for \( 1 \leq l \leq 40 \)) and the slopes of power spectra of adiabatic perturbations, \( n_S = 1 + \frac{d\log(k^3\Phi^2(k))}{d\log k} \), and of gravitational waves, \( n_T = \frac{d\log(k^3\Phi^2(k))}{d\log k} \), it is interesting to note that the relation \( n_T \approx n_S - 1 \) (proposed, e.g., in [32,33]) is valid in our models for \( k \ll k_f \) (and not too small) though it is not valid for \( k > k_f \) and also not valid for single chaotic inflation. On the other hand, the relation \( T_S \approx 6.2n_T \) which is valid for single slow-rolling inflation (see e.g. [34] for chaotic inflation) is strongly violated if \( k_{hor} \ll k_f \) (this issue will be addressed in a separate publication [35]).

### B. Large-scale peculiar velocities

A lot of information can be gained from the observation of peculiar velocities, velocities in addition to the Hubble flow. As all matter contributes gravitationally, the peculiar velocities sample all the mass and not just the galaxies. Hence, knowing the peculiar velocity field would give us an information on the primordial spectrum of the same interest as the CMBR anisotropy. It should however be stressed that large-scale peculiar velocities have rather large uncertainties. In the linear regime, gravitational instability produces a velocity field that is irrotational at sufficiently late times, the velocity field \( \mathbf{v} \) then derives from a velocity potential \( \Psi \) with \( \mathbf{v} = -\nabla \Psi \). Measurements of redshifts \( z \) and of galaxy distances \( r \) (actually of \( H_0r \)) provide the radial component of the peculiar velocity field: \( v_r = cz - H_0r \), for a galaxy at small redshift. For a potential flow, the radial velocity field when integrated along radial paths gives the velocity potential out of which the other velocity components can be derived. The difficulty with this method is to construct a smooth radial velocity field and to eliminate the statistical uncertainties in \( v_r \). This is done with interpolation and smoothing of the raw data and we finally have the following equation

\[
\langle v^2 \rangle_R = \frac{H_0^2}{2\pi^2} \int_0^\infty dk P(k)W_{TH}^2(kR) \exp(-kR_s) \tag{22}
\]

where \( W_{TH}(kR) \) stands for the Fourier transform, up to a constant, of the “Top Hat” window function

\[
W(kR) = \frac{3}{(kR)^3} (\sin kR - kR \cos kR). \tag{23}
\]

We will compare our predictions with the Potent data. In these observations the raw data are smoothed with a Gaussian smoothing radius \( R_s = 12h^{-1}\text{Mpc} \) while spheres of radii \( R = 40h^{-1}\text{Mpc} \) and \( R = 60h^{-1}\text{Mpc} \) were considered. The data are \( v_{40} = 405 \pm 60 \text{ km/s} \) and \( v_{60} = 340 \pm 50 \text{ km/s} \) [36], error bars at the 1\( \sigma \) level. Analogously to what was found in [26], this test is crucial for our models too.

Velocities are generally too low and all the models are excluded if one doesn’t invoke cosmic variance for the measured peculiar velocities. For the \( n = 2 \) models, the best velocities
are systematically obtained for $\frac{2\pi}{k_b} \sim 6h^{-1}\text{Mpc}$. This is also the case for $n = 4$, except when $p < 6$, but the velocities do not grow significantly for smaller $k_b$. Also, it turns out that the $n = 4$ model can be excluded: for $p < 5$ one gets the best velocities, but other tests exclude these models while for $p \geq 5$, the velocities become too small.

C. The biasing factor $b$

Before the COBE DMR observations of the CMBR anisotropy, one way to normalize the fluctuation spectrum was through the quantity $\sigma_8 \equiv \langle \left( \frac{\delta M}{M} \right)^2 \rangle_{R=8h^{-1}\text{Mpc}}$. This quantity measures the variance of the total mass fluctuations in a sphere of radius $R = 8h^{-1}\text{Mpc}$. The reason for considering spheres of radii $R = 8h^{-1}\text{Mpc}$ is that for bright galaxies $\sigma_8$ is equal to one [37]. However, one doesn’t expect the total matter to be as clustered as bright galaxies and one tries therefore the simplest assumption, namely that there is a scale independent bias, given by the biasing factor $b$

$$\sigma^2_{R,g} = b^2 \sigma^2_R \quad \xi_{gg} = b^2 \xi$$  \hspace{1cm} (24)

where the subscript $g$ refers to galaxies while $\xi$ is the two-point correlation function. $\sigma^2_R$ can be computed from the power spectrum $P(k)$

$$\sigma^2_R = \frac{1}{2\pi} \int_0^\infty dk k^2 W^2_{TH}(kR) P(k).$$  \hspace{1cm} (25)

Early numerical simulations of CDM models [7] were able to reproduce the correct two-point correlation function $\xi$ which is certainly a great success for CDM. It required however, if one imposes $\Omega_0 = 1$, that $h$ is as low as $h \sim 0.25$. However for $h \approx 0.5$ one gets the right amplitude for $\xi$ assuming that $b \approx 2$. Another reason for considering a bias comes from the observed r.m.s. peculiar velocities between galaxy pairs. If $\Omega_0 = 1$ and there is no bias then numerical simulations give velocities $\sim 1000\text{km/s}$, as expected also on theoretical grounds, much larger than the observed ones $\sim 300 \pm 50\text{km/s}$ (see however [38] for a possible velocity bias). Here also, an $\Omega_0 = 1$ model will be compatible with the observed peculiar velocity fields if we introduce a biasing parameter $b \approx 2.5$. Finally, $b = 2.0 \pm 0.2$ is needed in order to get the correct amount of clusters of galaxies [39].

For $n = 4$, we get an unacceptably low $b$ ($b < 1.5$) for $p < 5$, for $p = 5$, $b \approx 1.5$. For $n = 2$, we get too low $b$ for $p < 8$ which are just the values that produce the best velocities.

D. Formation of galaxies and quasars

The very existence of compact objects observed at high redshifts constrains any proposed model for the formation of galaxies as these objects must already have formed at these high $z$. Quasars have now been observed up to redshifts near $z = 5$. They are believed to be powered by massive black holes located at the center of galaxies. From luminosity bounds one can estimate the mass of the black holes ($M \approx 10^9M_\odot$), while for the host galaxy the estimates give $M \approx 10^{11} - 10^{12}M_\odot$. Although the formation of gravitationally bound objects is a complicated non-linear process one can, using rather simple assumptions, make a connection with the linear theory [8][10][11]. The fraction $F(> M)$ of bound objects with
mass greater than some given mass \( M \) can be expressed as a function of \( \sigma_R \) [12], where \( R \) is the radius of the sphere containing an amount of mass \( M \) today, and \( \sigma_R \) is calculated assuming a linear evolution (in this subsection we will adopt the notation \( \sigma(M) \):

\[
F(> M) = \text{erfc}(\sqrt{2} \frac{\nu}{\sqrt{2}})
\]

(26)

where \( \text{erfc} \) is the complementary error function, \( \nu = \delta_c(1+z)\sigma^{-1}(M) \). The spherical collapse model gives for the collapse threshold value \( \delta_c = 1.686 \), the value used here, though estimates from numerical simulations suggest other acceptable values \((1.33 \leq \delta_c \leq 2)\). The most recent estimates of the mass fraction \( F(10^{11}M_\odot) \) in host galaxies of quasars at \( z = 4 \) [13] yields the following lower bound [20], using (26):

\[
\sigma(10^{11}M_\odot) \approx 2.2 \pm 0.5
\]

(27)

When comparing our predictions with the data, we should keep in mind that equation (27) is only a lower bound as quasars do not necessarily form in all potential host galaxies and the real number of quasars at \( z = 4 \) might be larger than the observed one. Also important is the fact that many large galaxies seem to have formed already at \( z = 1 \). From this observation we get the lower bound

\[
\sigma(10^{12}M_\odot) \approx 2.0 \pm 0.4
\]

(28)

Approximately the same estimate follows from Ref. [13]

For \( n = 2, \ p \geq 25 \), considering the best velocities obtained for \( \frac{2\pi}{k_b} \approx 6h^{-1}\text{Mpc} \), the values obtained for this test are too low. For these models, it is interesting to point out that if the overall normalization goes up, hence improving these numbers, the biasing factor of these models will become too low so that these models must be rejected.

E. Counts-in-cells analysis

We finally compare our models with the counts-in-cells analysis of large-scale clustering of the Stromlo-APM redshift survey [14]. Values for the counts-in-cells variance \( \sigma_l^2 \), where \( l \) is the cell size expressed in \( h^{-1}\text{Mpc} \), obtained with our spectra normalized according to \( \sigma_8 = 1 \), corresponding to optical galaxies in redshift space, are compared with the Stromlo-APM data. In order to decide whether our model fits the data well, we apply a \( \chi^2 \) analysis. Considering the 9 data points (for 9 different cell sizes) as independent and the error bars quoted in [14] as 2\( \sigma \) ones, while we test here a theory with 2 parameters \( p \) and \( k_b \) (we still have the possibility to change the normalization, this is just changing \( b \)), we have a \( \chi^2 \) distribution with 7 d.o.f. The variance \( \sigma_l^2 \) can be written as

\[
\sigma_l^2 = \frac{1}{2\pi^2} \int_0^\infty dk k^2 W_l^2(k)P(k).
\]

(29)

where \( W_l^2 \) is the analogous of \( W_{TH}^2 \), but now for a cell of size \( l \)

\[
W_l^2(kl) = 8 \int_0^1 dx \int_0^1 dy \int_0^1 dz (1-x)(1-y)(1-z) \frac{\sin(klr)}{klr} \quad r \equiv |\vec{x}|.
\]

(30)
A $\chi^2 < 7$ will be considered good while $\chi^2 > 18$ will be considered bad.

The $n = 4$ model gives very bad numbers, $\chi^2 > 30$ for $p \leq 5$, $\chi^2 > 20$ for $p < 8$, for $p = 8$ the test is still not too good, $\chi^2 > 11$. All the models with $n = 2$, $p > 10$ will yield very good results, $2 < \chi^2 < 3$ for $\sqrt{n} \approx (6 - 10)h^{-1}\text{Mpc}$.

### III. DISCUSSION AND CONCLUSION

The obtained large scale peculiar velocities are low for all values of $p$ and $k_b$. A possible solution to this problem is to assume that, due to “cosmic variance”, the velocities observed around us are higher than the real average. If we assume that the peculiar velocities are measured from just one independent volume, $v^2$ (actually $v_{10}^2$ or $v_{60}^2$) itself obeys a $\chi^2$-distribution (with one d.o.f. and variance $\sigma^2(v^2) = 2$), for example there is a 20% probability to have $v^2 \geq 1.6\langle v^2 \rangle$ and a 30% probability to have $v^2 \geq 1.1\langle v^2 \rangle$. But one has to be cautious, when invoking cosmic variance since a $\chi^2$-distribution with one d.o.f. for $v^2$ might be too crude.

When $n = 4$, we get the highest, though still rather low, velocities for the lowest values of $p$. But for $p < 5$, we get unacceptably low $b$ and unacceptably high values for $\sigma(10^{11}M_\odot)$ and $\sigma(10^{12}M_\odot)$. Comparison with the counts-in-cells analysis gives very bad results for $p < 8$. For $p = 6, 8$ we get acceptable $\sigma(10^{11}M_\odot)$ and $\sigma(10^{12}M_\odot)$, however the velocities become unacceptably low and this situation becomes only worse with growing $p$. For example $p = 14$ gives unacceptably high $b$ and unacceptably low $\sigma(10^{11}M_\odot)$, $\sigma(10^{12}M_\odot)$ and $v$. Hence, the $n = 4$ model is very unlikely and can be confidently excluded.

An improved situation is obtained when $n = 2$. We obtain a window of allowed parameters for $\frac{\Delta v}{k_b} \sim (6 - 10)h^{-1}\text{Mpc}$ and $10 < p < 15$ (see fig.3). However, also for this window, the velocities are too low compared to the POTENT data and one has to invoke “cosmic variance” and higher observed velocities around us than the real average one. In order to avoid that the magnitude of this effect is too unprobably high, we can also push the COBE data to their upper (1 - 1.5)$\sigma$ error bar, an increase of about (20-25)% resulting in the same increase in the power spectrum $P(k)$ and a corresponding decrease of the biasing factor $b$ ($\chi^2$ is unaffected). We would still get acceptable numbers, though $b$ would become rather low ($\approx 1.5$), in particular $\sigma(10^{11}M_\odot)$, $\sigma(10^{12}M_\odot)$ which are otherwise a little bit low get better. Note however, that even for the mean COBE normalization, the values of the large-scale bulk velocities in the $n = 2$ model for the allowed range of parameters are slightly higher than those in the CDM model with a cosmological constant and flat initial perturbation spectrum (see e.g. [47]) which are in turn higher than those in the tilted CDM model with $n_S \leq 0.9$. Thus, this problem is less severe for the double inflationary $n = 2$ model than for the latter models.

In conclusion, we have considered here two double-inflationary models with two noninteracting scalar fields: a light scalar field $\phi_l$ with potential $\frac{1}{2}m_l^2\phi_l^2$ and a heavy scalar field $\phi_h$ with potential $\frac{1}{2}m_h^2\phi_h^2$. We have analyzed numerically the cases $n = 2$ and $n = 4$. Trying CDM with an initial spectrum produced by these models, one has three free parameters, that is two more than the standard CDM model with a (approximately) scale-invariant initial spectrum. For given $k_b$ (location of the “step”) and given $p \equiv \frac{\sqrt{M^{n-2}_{pl}}}{m_l}$, normalization to COBE DMR constrain the remaining free parameter of the model: for $n = 2$, $m_h \sim 3 \times 10^{-6}M_p$. 
a result already reported earlier [30], for $n = 4$, $\sqrt{\lambda} \sim 2 \times 10^{-6}$. In this way also, the energy scale of the inflationary phase is determined, scales $\sim \frac{2\pi}{b_0}$ cross the horizon (for the first time) close to the beginning of the second inflation corresponding to an energy density $\sim \frac{2}{p^2} \times 10^{-11}M_p^4$ for $n = 2$ and $\sim \frac{2}{p^2} \times 10^{-11}M_p^4$ for $n = 4$. As was recently emphasized in the study of another double-inflationary model [26], here too the peculiar velocities obtained are very small for all values of the parameters and this poses a severe constraint on the model. The $n = 4$ model is shown to be excluded while the $n = 2$ model is marginally admissible for the range of parameters $\frac{2\pi}{b_0} \sim (6 - 10) h^{-1} \text{Mpc}$ and $10 < p < 15$. In the latter case, the remaining difficulty is still with low large-scale bulk velocities though it is less severe than in the CDM+$\Lambda$ model or the tilted CDM model.

If one assumes, invoking cosmic variance, that the average values of the measured peculiar velocities are higher than their actual r.m.s. values, and taking the upper $(1 - 1.5)\sigma$ limit of the COBE DMR data for the overall normalization, a window of parameters mentioned above for the $n = 2$ model remains compatible with observations, however with velocities about 30% less than the lower 1$\sigma$ measured ones. If the COBE DMR measurements go a little bit up as they did recently, while the measured peculiar velocities on which there are still rather large uncertainties go down, then these models will do better. Another potential difficulty which might also be cured by an overall increase of amplitude is the rather small total density fluctuations at galaxy scales.

ACKNOWLEDGMENTS

D.P. would like to thank L. Kofman for stimulating conversations. A.S. and D.P. are grateful to Profs Y. Nagaoka and J. Yokoyama for their hospitality at the Yukawa Institute for Theoretical Physics, Kyoto University. The financial support for research work of A.S. in Russia was provided by the Russian Foundation for Basic Research, Project Code 93-02-3631, and by the Russian Research Project “Cosmomicrophysics”. P.P is supported by SERC grant # 15091-AOZ-L9.
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FIGURES

FIG. 1. Power spectrum for $n = 2$ and, from top to bottom on the left, $p = 12, 10, 15$ and 28 (full lines), compared to a scale invariant spectrum (dashed line) while all spectra are normalized to COBE.

FIG. 2. Power spectrum for $n = 4$ and, from top to bottom on the left, $p = 3, 8$ and 16 (full line), again with a scale invariant spectrum (dashed) and all spectra normalized to COBE.

FIG. 3. Schematic representation of the behaviour of the $n = 2$ model in the $p - k_b$ plane with normalization according to the mean COBE data. For $p < 8$, $b < 1.5$ while for $p > 14$, $\sigma(10^{12}M_\odot) < 1.5$. Above the dashed line, $v_{60} < 215\text{km/s}$. Inside the upper, resp. lower curve, $\chi^2 < 7$, resp. 3.
TABLES

| p  | 8   | 10  | 12  | 14  | 15  | 28  |
|----|-----|-----|-----|-----|-----|-----|
| b  | 1.53| 1.78| 1.85| 1.94| 1.89| 1.63|
| \(v_{40}\) | 269 | 260 | 263 | 263 | 266 | 282 |
| \(v_{60}\) | 221 | 215 | 218 | 218 | 220 | 231 |
| \(\sigma(10^{11}M_\odot)\) | 3.64| 2.85| 2.32| 1.99| 1.88| 1.4 |
| \(\sigma(10^{12}M_\odot)\) | 2.69| 2.1 | 1.75| 1.50| 1.44| 1.26|
| \(\chi^2\) | 17.23| 8.98| 4.35| 2.08| 2.13| 4.47|

TABLE I. Values of the various tests for the model with \(n = 2\). All these numbers have been calculated with \(k_b^{-1} = 1h^{-1}\) Mpc.

| p  | 3   | 5   | 6   | 8   | 16  |
|----|-----|-----|-----|-----|-----|
| b  | 1.16| 1.56| 1.78| 2.2 | 3.09|
| \(v_{40}\) | 298 | 250 | 235 | 217 | 208 |
| \(v_{60}\) | 241 | 206 | 195 | 182 | 176 |
| \(\sigma(10^{11}M_\odot)\) | 5.49| 3.83| 3.22| 2.38| 2.9 |
| \(\sigma(10^{12}M_\odot)\) | 4   | 2.82| 2.38| 1.78| 0.87|
| \(\chi^2\) | 41.88| 27.39| 23.24| 11.39| 3.85|

TABLE II. Values of the various tests for the model with \(n = 4\). All these numbers have been calculated with \(k_b^{-1} = 1h^{-1}\) Mpc.

| \(k_b\) | 1   | 1.2 | 1.5 | 3   |
|--------|-----|-----|-----|-----|
| b      | 1.97| 2.07| 2.21| 2.64|
| \(v_{40}\) | 260 | 255 | 250 | 228 |
| \(v_{60}\) | 215 | 212 | 208 | 193 |
| \(\sigma(10^{11}M_\odot)\) | 2.14| 2.15| 2.16| 2.25|
| \(\sigma(10^{12}M_\odot)\) | 1.6 | 1.58| 1.58| 1.68|
| \(\chi^2\) | 2.51| 1.85| 1.97| 3.4 |

TABLE III. \(n = 2\) model for \(p = 13\) and values of \(k_b^{-1}\), expressed in \(h^{-1}\) Mpc, around the window of best parameters.

| l   | 10  | 15  | 20  | 25  | 30  | 40  | 50  | 60  | 75  |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| \(\sigma^2(l)\) | 1.346| 0.773| 0.488| 0.327| 0.230| 0.126| 0.0738| 0.0486| 0.0272|
| \(\sigma^2_{\text{obs}}(l)\) | 1.24| 0.74| 0.49| 0.37| 0.24| 0.14| 0.080| 0.048| 0.025|

TABLE IV. Comparison between predicted \(\sigma^2(l)\), where \(l\) is the cell size expressed in \(h^{-1}\)Mpc and \(\sigma^2_{\text{obs}}(l)\), the values inferred from the Stromlo-APM redshift survey, for \(n = 2\), \(p = 13\) and \(k_b^{-1} = 1h^{-1}\) Mpc. For the values displayed, the \(\chi^2\) test gives \(\chi^2 = 2.51\).
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