NUCLEON MAGNETIC MOMENTS, THEIR QUARK MASS DEPENDENCE AND LATTICE QCD EXTRAPOLATIONS

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1 Introduction

The chiral symmetry of QCD is spontaneously broken at low energies, leading to the appearance of Goldstone Bosons. For 2-flavor QCD we identify the resulting 3 Goldstone Bosons with the 3 physical pion states, the lowest lying modes in the hadron spectrum. In addition to being spontaneously broken, chiral symmetry is broken explicitly via the non-zero quark mass $\hat{m}$ in the QCD lagrangian. This explicit breaking is responsible for the non-zero masses $m_\pi$ of the pions. In the now experimentally established large-condensate scenario with parameter $B_0$ one obtains

$$m_\pi^2 = 2 \hat{m} B_0 \{1 + O(\hat{m} B_0)\}$$

for this connection. At low energies QCD is represented by a chiral effective field theory ($\chi$EFT) with the dynamics governed by the Goldstone Bosons, coupling to matter fields and external sources. The important aspect for this work is the fact that this $\chi$EFT incorporates both the information on the spontaneous and on the explicit breaking of chiral symmetry. We report on recent work utilizing $\chi$EFT to study the quark mass (pion mass) dependence of the magnetic moments of the nucleon.

2 The Calculation

We use $\chi$EFT with pions, nucleons and deltas as explicit degrees of freedom. When including matter fields with differing masses—as is the case between $\Delta (1232)$ and the nucleon—one has to make a decision on the power counting one employs. Throughout this work we follow the so called Small Scale Expansion (SSE) of refs. However, in one crucial aspect we differ from refs. For the leading order $N\Delta$ transition lagrangian we employ

$$\mathcal{L}^{(1)}_{N\Delta} = \tilde{T}_i^{\mu} \left[ c_A w_\mu^i + c_V i f_\mu^i S^\nu \right] N^\nu + h.c.,$$

(2)
which treats vector and axial-vector couplings $c_V$, $c_A$ to this transition on a symmetric footing. Usually the (leading order) $N\Delta$ vector coupling is relegated to subleading order based on standard (“naive”) power counting arguments. Nevertheless, we find it necessary to resort to Eq. (2) to capture essential quark-mass dependent effects in the anomalous magnetic moments already at leading one-loop order, resulting in a better behaved perturbative expansion. Our goal is to study the quark (pion) mass dependence of the magnetic moments of the nucleon. Treating the electromagnetic field as an external vector source, to leading one-loop order—according to SSE—one has to evaluate 11 diagrams, displayed in Fig. 1.

3 Isovector Anomalous Magnetic Moment

For the isovector anomalous magnetic moment one obtains

$$
\kappa_v = \kappa_v^0 - \frac{g_A^2 m_\pi M}{4\pi F_\pi^2} + \frac{2c_A^2 \Delta M}{9\pi^2 F_\pi^2} \left\{ \sqrt{1 - \frac{m_\pi^2}{\Delta^2}} \log[R(m_\pi)] + \log \left[ \frac{m_\pi^2}{2\Delta} \right] \right\} 
$$

$$
-8E_1(\lambda) M m_\pi^2 + \frac{4c_A c_V g_A M m_\pi^2}{9\pi^2 F_\pi^2} \log \left[ \frac{2\Delta}{\lambda} \right] + \frac{4c_A c_V g_A M m_\pi^3}{27\pi F_\pi^2 \Delta}
$$

Figure 1.
\[-\frac{8c_{AV}gA\Delta^2M}{27\pi^2F^2}\left\{ \left(1-\frac{m^2_\pi}{\Delta^2}\right)^{3/2}\log[R(m_\pi)] + \left(1-\frac{3m^2_\pi}{2\Delta^2}\right)\log\left[\frac{m_\pi}{2\Delta}\right]\right\}\right\] 

+ N^2\text{LO} \right. 

where \(\Delta\) is the nucleon-delta mass difference. Most of the parameters in this expression are known and specified in ref.\(1\), except for \(\kappa_0^s, c_V, E_1\). At a chosen regularization scale \(\lambda\) we fit these 3 parameters to reproduce quenched lattice results for \(\kappa_s\) reported in ref.\(1\). Note that these lattice data correspond to lattice pions heavier than 600 MeV. With the parameters now fixed one obtains the full curve in Fig.2, which at \(m_\pi = 140\text{ MeV}\) comes very close to the physical isovector anomalous magnetic moment, indicated by the full circle. A priori it is not guaranteed that this extrapolation curve over such a wide range of quark (pion) masses would come anywhere near to the physical value, but remarkably it does so, albeit with a large error band (dashed curves). We also note that our approach rests on the assumption that for lattice data with effective pion masses larger than 600 MeV the differences between quenched and fully dynamical lattice simulations are small\(1\), allowing us to utilize “Standard” instead of “Quenched” \(\chi\)EFT methods.

4 Isoscalar Anomalous Magnetic Moment

To the same leading-one loop order in SSE one only obtains analytic quark mass dependence for the isoscalar anomalous magnetic moment \(\kappa_s\):

\[\kappa_s = \kappa_0^s - 8E_2m^2_\pi + N^2\text{LO} \right. \]

The 2 unknown couplings \(\kappa_0^s, E_2\)—parameterizing short-distance physics beyond the realm of \(\chi\)EFT—can again be fitted to lattice data\(1\).

5 Comparison with Pade-Formula

Combining isovector and isoscalar results one obtains the quark (pion) mass dependence of the magnetic moments of proton and neutron, as shown in the full line of Fig.3. Surprisingly our result is rather close—at least within the present error band—to the Pade-fit extrapolation formula of the Adelaide group\(3\), shown as the dashed curve.

Acknowledgments

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