STATISTICAL ASPECTS OF NEVEU AND SCHWARZ DUAL MODEL

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Abstract

Statistical method is applied to Neveu and Schwartz model for obtaining the average characteristics of the heavy resonances: spin distribution, decay widths etc. The properties of the dual model spectrum of states constructed of both commuting and anticommuting operators are considered. In this case the spin characteristics coincide with the results of the statistical bootstrap model.


1 Introduction

In this paper we investigate the average characteristics of the Neveu and Schwarz dual model from the statistical viewpoint [1—4]. The exponential rise of the density of hadron states at high energies in the dual resonance model (DRM) creates the preconditions for the efficiency of the statistical approach. Besides, there may exist an essential connection [3] between the properties of the resonance spectrum in the DRM and in the statistical bootstrap model [5, 6].

For some observable $A$ the average in the resonances with fixed mass $M$ (i.e. microcanonical average) $\alpha(M^2) \equiv \alpha(0) + \alpha'M^2 = N$ is defined in dual resonance model to be [3]

$$
\langle A \rangle_N = \frac{\sum_{R \in N} \langle R | A | R \rangle}{\sum_{R \in N} \langle R | R \rangle} = \frac{1}{2\pi i} \oint dz z^{-N-1} \text{Sp}(z^H A),
$$

where $H$ is the Hamiltonian operator in DRM, and the integration contour in the complex $z$-plane envelopes the point $z = 0$.

In the paper [4] a new type of averaging has been proposed

$$
\langle \langle A \rangle \rangle_N = \frac{\sum_{R \in N} |\langle 0 | V(p) | R \rangle|^2 \langle R | A | R \rangle}{\sum_{R \in N} |\langle 0 | V(p) | R \rangle|^2} = \frac{1}{(2\pi)^2} \oint dx \oint dy \oint dx' y^{-N-1} \langle 0 | V(p) x^H A y^H V+(p) | 0 \rangle
$$

where $V(p)$ is the vertex operator in DRM. Every resonance state is contained in the average (2) with the weight proportional to the square of the coupling constant with the basic states of the model (conditionally $\pi$-mesons) $g_{\pi\pi R} = \langle 0 | V(p) | R \rangle$. This provides an effective accounting of the dynamical features of the reactions and allows us to call (2) the dynamical average.

In the papers [3, 4] the averages such as (1) and (2) were considered in the framework of the Veneziano dual model. It is clear that the statistical approach can be applied to the other possible formulations of the dual model. We shall consider here the dual model of Neveu and Schwarz which possesses some important theoretical advantages comparing with the usual Veneziano

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model. In this model the resonance states are generated by the infinite sequence of operators of the usual Veneziano model

\[ [a_n^\mu, a_{n'}^{\nu+}] = -g^{\mu\nu} \delta_{nn'} \]  
\[ (n, n' = 1, 2, ...) \]  

and the new infinite set of anticommuting operators

\[ \{b_m^\mu, b_{m'}^{\nu+} \} = -g^{\mu\nu} \delta_{mm'} \]  
\[ (m, m' = 1/2, 3/2, ...) \]

satisfying conditions \[ [a_n^\mu, b_{m'}^{\nu+}] = 0 \]. The role of Hamiltonian is played by the operator

\[ H = H_a + H_b = -\sum_{n=1}^{\infty} na_n^{+}a_n - \sum_{m=1/2}^{\infty} mb_m^{+}b_m \]  

and the vertex operator is defined as

\[ V(p) = V_a(p)V_b(p) = \exp \left[ -\sqrt{2p} \sum_n \frac{a_n^{+}}{\sqrt{n}} \right] \exp \left[ \sqrt{2p} \sum_n \frac{a_n}{\sqrt{n}} \right] p \sum_m (b_m^{+} + b_m) \].

The eigenstates of the operator \( H \) are divided into two groups with different quantities of \( G \)-parity [7] \((G = (-1)^{\sum m b_m^{+} b_m})\):

1) the states with eigenvalues \( N = \frac{1}{2}, \frac{3}{2}, ... \) composed of the odd number of \( "b" \)-operators and an arbitrary number of \( "a" \)-operators, \( G = -1 \);

2) the states with eigenvalues \( N = 1, 2, ... \) composed of the even number of \( "b" \)-operators and an arbitrary number of \( "a" \)-operators, \( G = 1 \) (the state with \( N = 0 \) is spurious [7]).

In the particle interactions the value of \( G \)-parity is conserved. The \( \pi \)-meson is associated with the lowest physical state of the model with the mass \( m_\pi^2 = -\frac{1}{2} \).

The low energy part of the spectrum in the Neveu-Schwarz model is similar to the experimental data.
2 Statistical Averages

We shall start with considering the degeneracy of the resonance level \( N \), i.e. the number \( d(N) \) of the solutions of

\[
H | R_i \rangle = N | R_i \rangle, \quad (i = 1, 2, \ldots d(N)).
\]

(7)

It is obvious that the degeneracy \( d(N) \) can be presented in the following form

\[
d(N) = \frac{1}{2\pi i} \int dz \, z^{-2N-1} \text{Sp} (z^{2H}) = \frac{1}{2\pi i} \int dz \, z^{-2N-1} \times
\]

\[
\times \prod_{n=1}^{\infty} (1 - z^{2n})^{-4} (1 + z^{2n-1})^{4} = \frac{1}{2\pi i} \int dz \, z^{-2N-1} \theta_2^{-2}(0|\tau) \theta_4^{-2}(0|\tau) 4z^{1/2},
\]

(8)

where \( \tau = -i \frac{\ln z}{\pi} \), and \( \theta_2, \theta_4 \) are the Jacobi \( \theta \)-functions [8]. The behaviour of (8) as \( N \) goes to infinity is determined by the quantity of the integrand at \( z \approx 1 \), which is given by means of the imaginary Jacobi transformation [8]. The integral (8) is calculated by employing the saddle-point technique, and

\[
T = \frac{1}{\pi} \sqrt{N}
\]

(9)

determines the saddle point \( Z_0 = e^{-\frac{\pi}{T}} \). We get

\[
d(N) \approx \frac{1}{2i} N^{-7/4} \exp[2\pi \sqrt{N}].
\]

(10)

Taking into consideration that \( N \approx \alpha' M^2 \), we get the expression for the hadron state density \( \rho(M) \) at \( M \to \infty \)

\[
\rho(M) \approx \text{const} \cdot M^{-5/2} \exp \left[ \frac{M}{T_0} \right],
\]

(11)

where \( T_0 = \frac{1}{2\pi \sqrt{\alpha'}} \) plays the role of Hagedorn limiting temperature [5]. For \( \alpha' \approx 1 \text{ GeV}^{-2} \) we get \( T_0 \approx 160 \text{ MeV} \), and this result remarkably coincides with the prediction of the thermodynamic model [5] based on the comparison with the experiment.

Let us calculate now the average number of the oscillator excitations. According to (1), we have
\[ \langle -a_n^+ a_n \rangle_N = \frac{1}{d(N) 2\pi i} \oint dz \, z^{-2N-1} \text{Sp} \left( -a_n^+ a_n z^2 H \right) = \frac{1}{d(N) 2\pi i} \oint dz \, z^{-2N-1} \frac{4z^{2n}}{1-z^{2n}} \prod_{k=1}^{\infty} (1-z^{2^k})^{-4} (1+z^{2^k-1})^4. \quad (12) \]

At fixed \( n \) and \( N \to \infty \) we get

\[ \langle -a_n^+ a_n \rangle_N \approx \frac{4}{e^{n/T} - 1}, \quad (13) \]

where \( T \) is given by (9).

Analogous calculation for the number of "b"-excitations gives

\[ \langle -b_m^+ b_m \rangle_N \approx \frac{4}{e^{m/T} + 1}. \quad (14) \]

Expressions (13) and (14) are the equilibrium distributions of Bose-Einstein and Fermi-Dirac, respectively. Thus, we have come to the canonical distribution with the temperature \( T \) given by (9). The use of the saddle point technique while calculating the integrals such as (1) enables us to fulfill the normal in statistical mechanics transition from the microcanonical to the canonical (Gibbs) ensemble. The canonical averages of the physical values are defined as

\[ \langle A \rangle_T = \frac{\text{Sp} \left( e^{-H/T} A \right)}{\text{Sp} e^{-H/T}}. \quad (15) \]

Let us continue considering the average characteristics of the resonance spectrum using (15). The total mean number of the oscillator excitations of "a" and "b" type are equal, respectively, to

\[ \langle - \sum_{n=1,2,...} a_n^+ a_n \rangle_T = \sum_{n} \frac{4}{e^{n/T} - 1} \approx 4T \ln T, \quad (16) \]

\[ \langle - \sum_{m=1/2,3/2,...} b_m^+ b_m \rangle_T = \sum_{m} \frac{4}{e^{m/T} + 1} \approx 4T \ln 2. \quad (17) \]

The canonical average of energy has to coincide with the energy \( N \) of the microcanonical ensemble and is given by the expression:
\[ \langle H \rangle_T = \langle H_a \rangle_T + \langle H_b \rangle_T \simeq \sum_n \frac{4n}{e^{n/T} - 1} + \sum_m \frac{4m}{e^{m/T} + 1} = \frac{2}{3} \pi^2 T^2 + \frac{1}{3} \pi^2 T^2. \quad (18) \]

In (16), (17), (18) we substitute the summation for the integration which is reasonable for large \( T \).

Let us proceed to the question of spin distribution in the Neveu and Schwarz dual model. In the case of Veneziano model this task has been solved in the papers [2, 9]. The operator of the \( z \)-component of spin is [10]

\[ L_z = L^a_z + L^b_z = i \sum_{n=1}^{\infty} (a_n^x a_n^y - a_n^y a_n^x) + i \sum_{m=1/2}^{\infty} (b_m^x b_m^y - b_m^y b_m^x). \quad (19) \]

Choosing the basis of states to be diagonal in \( L_z \) [2], we find the characteristic function of the operator \( L_z \) according to (15)

\[ \langle e^{i \varphi L_z} \rangle_T = \prod_{n=1}^{\infty} \left[ \frac{(1 - e^{-n/T})^2}{1 - 2 \cos \varphi e^{-n/T} + e^{-2n/T}} \right] \left[ \frac{1 + 2 \cos \varphi e^{-n/2} + e^{-2n/2}}{(1 + e^{-n/2})^2} \right] = \theta^{-1}_1(v|\tau)\theta_2(0|\tau)\theta_3(v|\tau)\theta_4(0|\tau) \sin \pi v, \quad (20) \]

where \( v = \frac{\varphi}{2\pi}, \tau = \frac{i}{2\pi T} \) and \( \theta_1, \theta_2, \theta_3, \theta_4 \) are the Jacobi \( \theta \)-functions. For \( T \to \infty \) we apply the imaginary Jacobi transformation for the evaluation of (20), and get

\[ \langle e^{i \varphi L_z} \rangle_T \simeq \frac{2\pi T \sin \frac{\varphi}{2}}{\sinh \pi \varphi T}. \quad (21) \]

By differentiating (21) we find the average value

\[ \langle L_z^2 \rangle_T = -\frac{d^2}{d\varphi^2} \langle e^{i \varphi L_z} \rangle_T |_{\varphi=0} \simeq \frac{\pi^2 T^2}{3} = \frac{N}{3}. \quad (22) \]

The distribution in \( l_z \) is the following

\[ \sigma_N(l_z) = \frac{1}{2\pi} \int_{-\pi}^{\pi} d\varphi e^{-i l_z \varphi} \langle e^{i \varphi L_z} \rangle_T \simeq \frac{1}{4T} \frac{1}{\cosh^2 \left( \frac{l_z}{2T} \right)}. \quad (23) \]
The integral in (23) is calculated by contour integration enclosing the contour in the lower half-plane and explicitly performing the sum over the pole residues. Using (23) one can easily obtain the spin distribution of the resonances

\[ \rho_N(l) \simeq -(2l + 1) \frac{d}{dl_z} \sigma_N(l_z)|_{l_z = l} = \frac{1}{4T^2} \frac{(2l + 1) \sinh \left( \frac{l}{2T} \right)}{\cosh^3 \left( \frac{l}{2T} \right)}, \quad (24) \]

which can give further information about the spin structure of the resonance spectrum. Let us write out the spin characteristics of the resonances constructed of ”b”-operator only. The characteristic function of the operator \( L_z^b \) is

\[ \langle e^{i\varphi L_z^b} \rangle_T = \prod_{n=1}^{\infty} \frac{1 + 2 \cos \varphi e^{-\frac{(n-1/2)}{T}} + e^{-\frac{(2n-1)}{T}}}{(1 + e^{-\frac{(n-1/2)}{T}})^2} = \frac{\theta_3(v|\tau)}{\theta_3(0|\tau)} \sim e^{-\frac{l_z^2}{2T}}. \quad (25) \]

From (25) we get

\[ \langle (L_z^b)^2 \rangle_T = -\frac{d^2}{d\varphi^2} \langle e^{i\varphi L_z^b} \rangle_T \big|_{\varphi = 0} \simeq T = \frac{\sqrt{N}}{\pi}. \quad (26) \]

We see that for \( N \to \infty \) the “b”-oscillator contribution into spin becomes asymptotically negligible, whereas their contribution into the energy makes up \( \frac{N}{3} \) according to (18). The distribution in \( l_z \) for the “b” oscillators has the Gaussian form

\[ \sigma_N^b(l_z) = \frac{1}{2\pi} \int_{-\pi}^{\pi} d\varphi e^{-i\varphi l_z} e^{-\frac{\varphi^2}{2T}} \simeq \frac{1}{\sqrt{2\pi T}} e^{-\frac{l_z^2}{2T}}. \quad (27) \]

### 3 Dynamical Averages

The dynamical averaging (2) in Neveu-Schwarz model is performed over the states with integer values of \( N \) (it is necessary for the \( G \)-parity conservation). The characteristic function of the operator \( L_z \) is given by

\[ \langle e^{i\varphi L_z} \rangle_N = \frac{1}{2\pi} \int dxx^{-N-1} \langle \pi|V(p)x^H e^{i\varphi L_z} V+(p)\pi \rangle, \quad (28) \]
where $|\pi\rangle = k\bar{b}_{1/2}^+|0\rangle$ is pion state [7]. Using kinematical relations in the c.m.s. ($\vec{p} + \vec{k} = 0$) and the mass-shell conditions ($p^2 = k^2 = m^2_\pi = -\frac{1}{2}$), we obtain

$$\langle\pi|V(p)x^HV^+(p)|\pi\rangle = [(1 - x)^{-1/2} + 2(p_x^2 + p_y^2)(1 - \cos \varphi)] \times$$

$$\times \left[\frac{1}{4}\frac{x}{1 - x} + \frac{N^2}{4}(1 - x) + (p_x^2 + p_y^2)(N x - \frac{x}{1 - x})(1 - \cos \varphi)\right].$$

(29)

From (28) and (29) we find

$$\langle\langle L_z^2 \rangle\rangle_N = -\frac{d^2}{d\varphi^2}\langle\langle e^{i\varphi L_z} \rangle\rangle_N|_{\varphi=0} \simeq 2(p_x^2 + p_y^2)\ln(N - 1) + C + O\left(\frac{1}{N}\right),$$

(30)

where $C = 0, 577\ldots$ is the Euler constant.

Choosing the momenta in the c.m.s. along the $x$-axis, for (30) we get

$$\langle\langle L_z^2 \rangle\rangle_N \simeq \frac{N}{2}\ln N.$$  

(31)

Also, it is easy to find the contribution of "$b$"-oscillators

$$\langle\langle (L_z^b)^2 \rangle\rangle_N \simeq 4(p_x^2 + p_y^2) \simeq N.$$  

(32)

The average numbers of the excitations are

$$\langle\langle -a_n^+ a_n \rangle\rangle_N = \frac{1}{n}\left(1 - \frac{n}{N}\right); \text{ for } n < N,$$

(33)

$$\langle\langle -b_m^+ b_m \rangle\rangle_N = \frac{1}{N}; \text{ for } \frac{1}{2} < m < N.$$  

(34)

4 Resonance Decay Widths

Let us take up now a heavy resonance decay within the framework of the Neveu and Schwarz model. According to [3], the average decay width of the resonance from $N_1$-level into the resonance from $N_2$- level and $\pi$-meson is given by
\[
\langle \Gamma \rangle = \frac{1}{d(N_1)(2\pi i)^2} \oint dx \oint dy \, x^{-2N_1 - 1} y^{-2N_2 - 1} \text{Sp} \left( x^{2H} V(p) y^{2H} V^+(p) \right). \tag{35}
\]

The trace in (35) has the following form

\[
\text{Sp} \left( x^{2H} V(p) y^{2H} V^+(p) \right) = \left[ (1 - x^2)^{-1} \prod_{n=1}^{\infty} (1 - (xy)^{2n}) \right] \times \frac{(1 - (xy)^{2n})^2}{(1 - x^2(xy)^{2n})(1 - \frac{1}{x}(xy)^{2n})} \left[ \frac{1}{2} \prod_{n=1}^{\infty} (1 + (xy)^{2n})^4 \sum_{k=1}^{\infty} \frac{x^{2k-1} + y^{2k-1}}{1 + (xy)^{2k-1}} \right]. \tag{36}
\]

To calculate the integral (35), we introduce the variables \( \omega = xy \) and \( x \). The main contribution to the integral (35) with large \( N_2, N_1 - N_2 \) is given by the region \( \omega \simeq 1, x \simeq 1 \). Expressing the integrand in terms of the Jacobi \( \theta \)-functions, we obtain for \( \omega \simeq 1 \)

\[
\prod_{n=1}^{\infty} \frac{(1 - \omega^{2n})^2}{(1 - x^2 \omega^{2n})(1 - \frac{1}{x^2} \omega^{2n})} \simeq \frac{\pi \sinh(\ln x)}{\ln \omega \sin(\frac{\pi \ln x}{\ln \omega})} \exp \left[ \frac{\ln^2 x}{\ln \omega} \right], \tag{37}
\]

\[
\sum_{k=1}^{\infty} \frac{x^{2k-1} + (\omega)^{2k-1}}{1 + \omega^{2k-1}} \simeq -\frac{\pi}{2 \ln \omega \sin(\frac{\pi \ln x}{\ln \omega})}. \tag{38}
\]

The integration over \( \omega \) is performed by employing the saddle-point technique with the result

\[
\langle \Gamma \rangle = \frac{d(N_2)}{d(N_1)} \frac{1}{2\pi i} \oint dx \, x^{-2N_1 + 2N_2 - 1} (1 - x^2)^{-1} f(x, \omega_0); \]

\[
f(x, \omega_0) \simeq \frac{\pi}{2 \ln \omega_0} \left( \frac{\pi \ln x}{\ln \omega_0} \right)^2 \exp \left[ \frac{\ln^2 x}{\ln \omega_0} \right], \tag{39}
\]

where \( \omega_0 = \exp \left[ -\frac{x}{2\sqrt{N_2}} \right] \) is the saddle point. The remaining integral is easily calculated at \( N_1 - N_2 >> \sqrt{N_2} \) which corresponds to the large energy of the emitted \( \pi \)-meson. In this case \( \frac{\ln x}{\ln \omega_0} \simeq \frac{\sqrt{N_2}}{N_1 - N_2} \simeq 0 \), and so we get

\[
\langle \Gamma \rangle = \frac{d(N_2)}{d(N_1)} \frac{N_1 - N_2}{2}. \tag{40}
\]
Expression (40) may be written in terms of the decaying resonance mass $M_1$ and the $\pi$-meson energy $E$

$$\langle \Gamma \rangle \simeq (1 - \frac{2E}{M_1})^{-7/4}E M_1 \exp \left[ - \frac{E}{T_0} \sqrt{1 - \frac{2E}{M_1}} \right].$$  \hspace{1cm} (41)$$

We see that the meson energy spectrum has the Boltzmann distribution form with the effective temperature $T_{eff} = \frac{T_0}{2}(1 + \sqrt{1 - \frac{2E}{M_1}})$. Such a dependence coincides exactly with the results of the statistical bootstrap model [12] (note that we do not take into account the whole chain of the successive decays, the consideration of which changes the value of $T_{eff}$ [12]).

A considerable difference of (41) from the analogous result obtained in the Veneziano model with $\alpha(0) \neq 1$ [3] consists in the linear growth of the decay width with $M_1$ increasing (such a behaviour would take place also in the Veneziano model with $\alpha(0) = 1$).

5 Discussion

In conclusion we would like to discuss some interesting, as it seems to us, aspects of our consideration:

1. The average resonance spectrum characteristics in the Neveu-Schwarz model for $N \to \infty$ are similar, as a whole, to those obtained in the Veneziano model [2, 3, 4]. This allows us to hope that statistical approach grasps the main features of the dual dynamics independent of the choice of the specific model.

2. The average decay width has the same dependence on the $\pi$-meson energy as in the statistical bootstrap model and grows linearly as $M \to \infty$. Concerning the total width dependence on the hadron mass, the comparison is meaningless, since no decay widths but the relative probabilities are considered in the statistical bootstrap and the total probability is normalized to be equal unit.

3. In the Neveu-Schwarz model there exists an infinite sequence of the gauge relations which allows one to remove the unphysical states. In order to obtain the number of physical states, we must add the factor $(1 - z) \prod_{n=1}^{\infty} (1 - z^{2n})(1 + z^{2n-1})^{-1}$ to the integrand (8) (the same factor as for calculating the planar loop) [13, 11]. The number of physical states is given by [13]
\[ d^{D \text{phys}}(N) = d^{D-1}(N) - d^{D-1}(N - \frac{1}{2}), \quad (42) \]

where \( D \) is the dimension of the oscillator operators (in our case \( D = 4 \)). For large \( M \) the density \( \rho^{(M)}_{\text{phys}} \) equals

\[ \rho^{(M)}_{\text{phys}}(M) \simeq \text{const} M^{-3} \exp[\pi \sqrt{3\alpha' M}]. \quad (43) \]

The removal of unphysical states can be easily taken into account in our calculations. For example, one must substitute (42) for \( d(N_1) \) and \( d(N_2) \) in the expression (40), which results in the alteration of the quantity of \( T_0 \) in (41). So, the partial widths of the decay with \( \pi \)-meson emission, which have the form \( \langle \Gamma \rangle \sim \frac{d(N_2)}{d(N_1)} \), do not essentially change as the removal of unphysical states is performed. However, the situation may be quite different while considering the general case of decay \( R_1 \rightarrow R_2 + R_3 \) where one expects the results such as \( \langle \Gamma \rangle \sim \frac{d(N_2)d(N_3)}{d(N_1)} \). Here the decay characteristic will be crucially dependent of the value of power of \( M \) in the expressions (11), (43). Note that the case of the power value equal to \(-3\) (regarding the physical states only) coincides with the results of the investigations [14, 15] and makes the decay into one heavy and one light (\( \pi \)-meson) particles dominant, while the case of the value equal to \(-5/2\) (all states are taken into account) corresponds to the primary formulation of Hagedorn [5] which predicts the decay into particles of approximately equal masses.

4. Our calculations allow us to obtain the average characteristics of the resonance spectrum constructed of "b" operators only.

The question of building the dual theory with the same hadron state density as in the statistical bootstrap has been considered in the recent paper [16]. The author concludes that the construction of resonance states with the only Fermi operators represents one of the most preferable possibilities. In this case using calculations (26) and (27) we find that also the average spin and the spin distribution coincide with that of the statistical bootstrap theory with spin [17].

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