Hydrodynamic forces on tubular cylinders fitted with sacrificial anodes – numerical analysis

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Abstract. In offshore engineering, the design of effective marine platforms compatible with the offshore environmental condition is a challenging task, especially accurate estimation of hydrodynamic coefficients that affect the viscous forces. Practically, the well-known Morison’s equation is often adopted by designers to calculate wave loads on slender structures, where the values of drag and inertia coefficients are selected as constant values based on the design code of practice’s recommendations. However, the mass and drag coefficients must be determined empirically based on specific met-ocean data for the operation location. Thus, the objective of this study is to evaluate the Hydrodynamic Forces on Circular Cylinders fitted with sacrificial anodes using empirical methods, and validate the results using Computational Fluid Dynamic (CFD). It is found that the major parameters that affect the drag and inertia coefficients are the water depth, wave heights, wave frequencies, the pipe diameters, and the presence of anode fittings.

Keywords. Hydrodynamic Forces, Tubular Cylinders, Sacrificial Anodes, Hydrodynamic coefficients.

1. Introduction
Fixed offshore structures are made of tubular steel members, welded together to form a three-dimensional rigid frame. These steel platforms are subjected to corrosion, and their successful use in engineering and commercial applications depends on the corrosion protective mechanisms. Thus, impressed current cathodic protection systems, corrosion allowance and galvanic anodes techniques are commonly adopted by designers for corrosion protection of offshore structures, besides organic surface coatings [1]. Cathodic protection is a common method used to protect immersed parts of steel surfaces from corrosion. However, added hydrodynamic loading due to sacrificial anodes is not well understood [2]. Accurate estimation of wave forces on cylindrical elements are of considerable interest in the design of offshore platforms. The simplest and most efficient method used by the engineers to predict the wave forces on submerged vertical tubular cylinders with a diameter less than half of the wavelength was proposed by Morison [3]. This equation consists of two components: drag force due to the water particle velocity and inertia force due to the water particle acceleration [3]. However, the application of this equations is limited to linear wave theory. In addition, the total hydrodynamic force on circular cylinders...
is a function of drag and inertia coefficients. Read et al. [4] examined the effect of the drag and inertia coefficients used in Morison's equation as predicted by some calibration approaches on the short-term wave-induced force acting on a fixed vertical cylinder. Sarpkaya [5] has presented a comprehensive review of Morison's equation and its dependence on various factors. Boccotti et al. [6] conducted a field experimental investigation to determine the accuracy of Morison equation for estimating wave forces on circular cylinders. Accurate estimation of drag (CD) and inertia coefficient (CM) plays an important role in the numerical prediction of the wave forces on circular cylinders, thus, a series of experimental investigations have been carried out by Sarpkaya [7], Troesch and Kim [8] to estimate the values of CD and CM. The design codes of practices such as API [9], DNV [10] normally recommend specific values for CD and CM to be used by engineers during estimation of hydrodynamic forces of tubular cylinders. However, accurate values for CD and CM are not well established when the cylinders are fitted with sacrificial anodes. Fixed offshore structures are typically protected against corrosion by installing sacrificial anodes. Anodes are highly active metals that are used to prevent a less active material surface from corroding. When sacrificial anodes are used, the anodes are physically connected to the structures, and an electrochemical cell is formed between the anode and the cathode through the surrounding seawater. The review of the existing literature shows that, very limited studies have been conducted to determine the effects of sacrificial anodes on wave forces using CFD tools.

Thus, the objective of this paper is to determine the hydrodynamic forces on circular cylinders fitted with sacrificial anodes using Morison equations and the findings are validated using CFD simulations. The results provide insights into better understanding of the effects of sacrificial anodes on the hydrodynamic forces of tubulars cylinders fitted with anodes for offshore applications.

2. Morison’s equation

2.1. Morison’s equation

To estimate wave forces on slender structures, designers normally use the well-known Morison Equation. For a stationary cylinder in a plane flow field, with flow stream, with a free-stream velocity, \( U=U(t) \) the total hydrodynamic force per unit length on the cylinder can be determined by superimposing the drag and inertia forces using the equation (1).

\[
\bar{q} = C_D \rho \frac{D}{2} |u| u + C_M \rho \pi \frac{D^2}{4} \dot{u}
\]  

(1)

This equation assumes that the total in-line hydrodynamic force on a structure consists of semi-empirical values of inertia and drag forces supplemental linearly. The inertia force is developed because the water taking possession a wave carries a momentum with it, whereas the drag force part is caused because of the presence of wake region on the stream side of the cylinder. The total hydrodynamic force on the jacket member will be calculable from equation (2).

\[
F = \int_0^d f ds = \int_0^d \left[ C_m \rho \pi D^2 \frac{dy}{dt} + C_d \rho \frac{D^2}{2} |U + U_c|(U + U_c) \right] ds
\]  

(2)

where, \( d = \) water depth, \( C_m = \) hydrodynamic coefficient of mass, \( C_d = \) hydrodynamic coefficient of drag, \( ds = \) integration is done over the full wetted length of the cylinder, \( \rho = \) density of water \((1000 \text{kg/m}^3)\), \( D = \) pipe diameter, \( U = \) horizontal water particle velocity, \( dy/dt = \) water particle acceleration, and \( U_c = \) current velocity.

As reported by Wilson [11], the Morison’s equation is deduced by applying the principle of conservation of linear fluid momentum. This principle is based on Newton’s second law applied to fluid of fixed control volume \( V \) at any time \( t \). Here, \( V \) is a rectangular box surrounding a disk of a submerged circular cylinders, where \( V \) can be determined as \( D2\Delta z \), where \( D \) is the disk diameter, and \( \Delta z \) is the
height of the disk. Therefore, for water flow along the x-directions, with horizontal velocity \( u \), the net horizontal shear force on the disc can be determined as equation (3).

\[
\sum F_x = \frac{\partial}{\partial t} \int_{V} \rho u dV + \int_{A_0} \rho u. udA_0
\]  

(3)

Where \( dV \) is a small element of the control volume, and \( dA_0 \) is the element of area on the surface of the control volume, perpendicular to the flow direction. General derivation of equation (3) is discussed by Munson et al. [12].

2.2. Linear wave theory

Linear wave theory is a first order, small amplitude theory developed by Airy [13]. This theory forms the basis for the probabilistic spectral description of waves. The governing equations and the solutions for the waves are essentials for the prediction of wave forces on offshore structures [11].

**Figure 1.** Definition of simple harmonic waves past a cylinder fitted with sacrificial anodes.

Assuming the simple harmonic plane wave defined in figure 1, is propagating along the positive x direction. The origin of the coordinate is located at still water level (SWL), the vertical coordinate is \( z \), directed upward as the positive direction. The density of water \( \rho \) is taken as 1025 kg m\(^{-3}\). The differential equations and the boundary conditions for the water particles velocity and pressure can be expressed as follows:

\[
\frac{\partial u}{\partial z} - \frac{\partial \omega}{\partial x} = 0
\]  

(4)

\[
\frac{\partial u}{\partial x} + \frac{\partial \omega}{\partial z} = 0
\]  

(5)

\[
\frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{\partial \rho}{\partial x}
\]  

(6)

\[
\frac{\partial \omega}{\partial t} = -\frac{\partial \rho}{\partial x} - g
\]  

(7)

\[
\omega = -\frac{\partial \eta}{\partial t} \text{ at } z = 0
\]  

(8)
\[ p = p_a \quad \text{at} \quad z = 0 \]  \hspace{1cm} (9)

\[ \omega = 0 \quad \text{at} \quad z = -d \]  \hspace{1cm} (10)

Equation (4) represents the irrotational condition, which is also known as the zero velocity. Equation (5) is the well-known continuity equation. Equations (11) and (12) are the Eulerian equations of motions. Similarly, the boundary conditions at the water surface as presented in equations (8) and (9), while equation (10) represents the boundary condition at seabed. The solution that satisfy the linear equation is the Laplace Equations, for surface elevation \( \eta = \eta(x,t) \) presented in equation (11) which describes a progressive harmonic waves moving in the positive x direction.

\[ \eta(x,t) = A \sin(kx - \omega t) \]  \hspace{1cm} (11)

Using equation (11), together with equations (4) until (10), the water particles velocities, \( u \) and \( \omega \), and the corresponding accelerations \( \frac{\partial u}{\partial t} = \dot{u} \) and \( \frac{\partial \omega}{\partial t} = \dot{\omega} \) are determined.

3. Methodology

3.1. Empirical methods

In this phase of the research, the total hydrodynamic forces on tubular members fitted with sacrificial anodes are evaluated using the well-known Morison’s equation. The wave kinematics were estimated using Air Wave theory, then the total forces are determined considering the appropriate drag and inertia coefficients recommended by design code of practices. The met-ocean data such as water depth, wave heights, wave periods used in the theoretical analysis represents the operation criteria for selected operation locations in the Malaysian water. A MATLAB code was developed for estimating all the necessary parameters influencing the hydrodynamic coefficients of circular cylinders fitted with anodes.

3.2. Computational fluid dynamic (CFD) simulation

CFD is a numerical technique used for the solution of the complex equations governing fluid flow and heat transfer problems inside a defined flow geometry [14]. In this study, CFD was adopted for comparison of the theoretical results. Wave forces acting on the circular cylinder were determined, using ANSYS Fluent software, which is used to solve conservation equations. The incompressible equation Navier-Stokes Equations is set as follows:

\[ \nabla \cdot \mathbf{u} = 0 \]

\[ \frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{u} \otimes \mathbf{u}) = -\frac{1}{\rho} \nabla p + \frac{\mu}{\rho} \Delta \mathbf{u} + \mathbf{g} + \mathbf{f}, \]  \hspace{1cm} (12)

where \( \mathbf{u} \) is the fluid velocity, \( \rho \) is the density of the fluid, \( p \) is the pressure, \( \mu \) is the molecular velocity, \( \mathbf{g} \) is the gravitational force, and \( \mathbf{f} \) denotes additional momentum sources. The general overview of CFD setup is shown in figure 2. The CFD simulation consists of Geometry, Engineering data, Model setup, meshing, specifications of the component names, boundary setup, solution of complex equations describing the interaction of fluid with the cylinder, and results presentation.
Although the theoretical analysis addressed several values for the outer diameters of the cylinders, effects of numerical simulation was limited to one cylinder only. The numerical simulation was limited to a circular cylinder with outer diameter, \( D = 0.8 \) m, and the wave height was varied from 5.0 to 14.0 m while the wave frequency was varied from 3 to 21.6 seconds. The model geometry, meshing and the model setup implemented for the CFD simulation is depicted in figure 3.

To ensure consistency of the boundary conditions, and it is strongly reflecting the application conditions that is being simulated, the boundary conditions are specified as following. A uniform flow velocity \( U \) is specified in the streamwise direction at the inlet boundary. At the outlet, the Neumann boundary condition was adopted for the velocity, and the pressure is specified as a reference value of zero. Non-slip boundary condition was applied on the perpendicular walls, the floor bed and the vertical cylinder [15, 16].

4. Results and Discussion

4.1. The effect of wave heights and wave periods on total hydrodynamic forces

In this section, the effects of wave heights and wave periods on the hydrodynamic forces are discussed. A MATLAB code was developed to estimate the hydrodynamic forces based on the well-known Morison Equations. Figure 4 shows the variation of total hydrodynamic forces with the wave height, \( H_{\text{max}} \). The value of \( H_{\text{max}} \) was varied from 3 to 21.6 seconds. From the graph, one can observe that increasing the wave height has significantly influenced the corresponding forces. The graph also shows that the wave forces decreased with the increasing wave periods.
4.2. The effect of water depth on hydrodynamic forces
Figure 5 shows the variation of wave forces as a function of water depths. In this study, the water depth was varied as $d = 30\,\text{m}, 50\,\text{m}$ and $75\,\text{m}$. These water depths values represent typical offshore operation locations within the Malaysian water. The figure shows that for a given water depth, the values of hydrodynamic forces increased with the increasing wave height, $H_{\text{max}}$. Further, the graphs show that the maximum force was determined as $1210\,\text{kN}$, corresponding to water depth of $75\,\text{m}$, and $T_{\text{ass}} = 14$ seconds. Generally, the trend of the graphs suggests a consistent increase in the wave forces as the wave heights increase. Similarly, one can observe that the values of wave forces increased exponentially as the water depth increases.

4.3. The effect of pipe diameter on hydrodynamic forces
Figure 6 shows the variation of wave forces with respect to the outer diameter of the pipe. The pipe diameter was varied as $D = 0.8\,\text{m}, 1.5\,\text{m}$ and $2\,\text{m}$. It can be observed that as the pipe diameter increases, the total hydrodynamic forces increased as well. In addition, the graph also shows that the maximum wave forces are recorded at low wave periods, and the forces decreased gradually to reach negligible values at $T = 22$ seconds.
4.4. Comparison between clean and sacrificial anode fitted members

In this section a comparison of the total hydrodynamic forces on smooth cylinders and cylinders fitted with anodes was conducted. The waves forces are estimated using the specified values of drag and inertia coefficients in PTS [17].

Figures 7 - 9 show the time series records for drag, inertia and the total forces, for smooth and anode fitted cylinders, together with the corrected force determined using the $C_D$ and $C_M$ values determined using CFD simulations. The total force for the smooth cylinder and the one fitted with sacrificial anodes are determined as 1670 kN and 3199 kN respectively, while the total forces on the cylinder fitted with sacrificial anodes determined using $C_D$ and $C_M$ values determined from CFD was 2719 kN. The percentage different for the total force between the corrected values of hydrodynamic coefficient and the ones determined from PTS is 8.12%. This shows that the values of hydrodynamic coefficients recommended by the code of practices are more conservatives. Thus, cost effective platforms can be designed if more accurate values of hydrodynamic coefficients are used to estimated wave forces on anode fitted circular members.

**Figure 6.** Variation of wave forces with outer diameter, D = 0.8 m, 1.5 m, 2 m, for different wave periods.

**Figure 7.** Time series records for drag forces on smooth and anode fitted cylinders.
Figure 8. Time series records for inertia forces on smooth and anode fitted cylinders.

Figure 9. Time series records for the total hydrodynamic forces on smooth and anode fitted cylinders.

5. Conclusion
Wave forces acting on smooth and anode fitted circular cylinders are determined by employing the Morison equation and CFD simulations. The effects of sacrificial anodes on the hydrodynamic forces was initially determined using the well-known Morison equation and the findings are compared using CFD simulation techniques. The effect of wave height, wave period, water depth and the outer diameters on the hydrodynamic forces are investigated. The general conclusions drawn from this parametric study are as follows:

- The numerical calculations show that increasing the wave height has significantly increased the total hydrodynamic, while increasing the wave periods resulted in a comparatively smaller hydrodynamic forces.
- The study shows that the values of hydrodynamic coefficient determined using the numerical simulations are generally smaller that the values specified in the code of practice. A decrease
of 8.15% was observed on $C_D$ and $C_M$ values.

- The results show that water depth, wave heights, wave frequencies, and the pipe diameters have major influences on the hydrodynamic forces of circular cylinders fitted with sacrificial anodes.

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