A study on Soft Pre-Open Sets using γ Operation

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Abstract

The concept of strong soft γ pre-open set was initiated by Biswas and Parasnan. We utilize this notion to study several characterizations and properties of this set. We investigate the relationships between this set and other types of soft open sets. Moreover, the properties of the strong soft γ pre-interior and closure are discussed. Furthermore, we define a new concept by using strong soft γ pre-closed that we denote as locally strong soft γ pre-closed, in which several results are obtained. We establish a new type of soft pre-open set, namely soft γ pre-open. Also, we continue to study pre-γ soft open set and discuss the relationships among all these sets. Some counter examples are given to show some relationships obtained in this work.

Keywords: soft γ regular space, soft γ pre open, strong soft γ pre closure, locally strong soft γ pre closed, soft open.

1. Introduction

Moldstov [1] investigated the soft set theory, as a new approach for uncertainties, and the vague set theory. Also, he presented many uses of soft sets in some directions, such as game theory, Perron integration, and probability.
Kasahara [2] initiated the notion of γ operation on topological space and explored several important properties. Later, Maji and others [3] studied several operations on soft sets in more details and proved many propositions about these operations. Jamil [4] established new open sets by using the idea of operation and provided several characterizations.

Moreover, Ogata [5] utilized the idea of operation to introduce new types of open sets, called γ open and pre-γ open sets. Also, he mentioned several properties of each set.

Recently, Biswas and Prasanna [6] introduced and discussed various types of soft open sets by using the notions \( \tau_{\gamma}, \text{int}_{\gamma} \) and \( \text{cl}_{\gamma} \). In this paper, we continue studying such many forms of soft pre-open sets involving γ operation.

Finally, Al-shami [7-11] studied a new type of soft open sets and investigated the properties of its separation axioms.

2. Preliminaries and basic results

Definition 2.1 [1]. Let \( X \) be the set of the universe and \( A \) be a set of parameters, then the pair \((F, A)\) is named a soft set over space \( X \) such that \( F \) is mapping from \( A \) to the family of all subsets of \( X \), which is denoted by \( F : A \to P(X) \).

Definition 2.2 [3]. Let \((N, A)\) and \((M, A)\) be two soft sets over a soft space \( X \) on a common parameter \( A \), then \((N, A) \subseteq (M, A)\), if \( N(e) \subseteq M(e) \) for each \( e \in A \).

Definition 2.3 [3]. Let \((N, A)\) and \((M, A)\) be two soft sets over a space \( X \) on a common parameter \( A \). \((N, A) \subseteq (M, A)\) and \((M, A) \subseteq (N, A)\), if \( N(e) \subseteq M(e) \) and \( M(e) \subseteq N(e) \) for each \( e \in A \). Hence \((N, A)\) is equal to \((M, A)\).

Definition 2.4 [3]. A soft set \((F, A)\) of \( X \) is named a null soft set, which is represented by \( \tilde{\phi} \), if for any \( e \in A \), \( F(e) = \phi \).

Definition 2.5 [3]. A soft set \((F, A)\) of \( X \) is named an absolute soft set that is represented by \( \tilde{X} \) if for any \( e \in A \), \( F(e) = X \).

Definition 2.6 [1]. Let \((N, A)\) and \((M, A)\) be two soft sets over \( X \), then
1) \((S, A) = (N, A) \cup (M, A)\) is determined by \( S(e) = N(e) \cup M(e) \) for each \( e \in A \).
2) \((T, A) = (N, A) \cap (M, A)\) is determined by \( T(e) = N(e) \cap M(e) \) for each \( e \in A \).

Definition 2.7 [10]. Relative complement for any soft set \((F, A)\) is defined by \((F, A)^c = (F^c, A)\), such that \( F^c : A \to P(X)\) is a function given by \( F^c(e) = X - F(e) \) for each \( e \in A \).

Definition 2.8 [12]. The family \( \tau \) of soft sets is in the universe set \( X \), and \( A \) is a set of parameters, then \( \tau \) is called soft topology on \( X \) if the following conditions hold
1) \( \tilde{\phi}, \tilde{X} \) belong to \( \tau \),
2) If \( \{ (N_i, A) : i \in I \} \in \tau \), then \( \bigcup_{i \in I} (N_i, A) \in \tau \),
3) If \( (N, A), (M, A) \in \tau \), then \( (N, A) \cap (M, A) \in \tau \).

That is, \((X, \tau, A)\) being a soft topological space.

The family of all soft open sets is stated by \( S^O(X) \).

Definition 2.9 [13]. The soft closure of a soft subset \((F, A)\) over a space \((X, \tau, A)\) is the intersection of each soft closed supersets of \((F, A)\) which is stated by \( cl(F, A) \).

Definition 2.10 [13]. The soft interior of a soft set of \((F, A)\) in space \((X, \tau, A)\) is the union of all soft open subsets of \((F, A)\), which is stated by \( int(F, A) \).

Definition 2.11 [2]. A soft space \((X, \tau, A)\). An operation \( \gamma \) is a function from soft topology \( \tau \) into \( P(X) \), that is \((K, A) \subseteq (K, A)^\gamma \) for all \((K, A) \in \tau \), in which \((K, A)^\gamma \) represents the value of \( \gamma \) at \((K, A)\).

For simplicity, we use the notation \( \hat{X}_\gamma \) to indicate the soft space \((X, \tau, A)\) in which \( \gamma \) is an operation defined on \( \tau \).

Definition 2.12 [6]. A soft set \((U, A)\) in \( \hat{X}_\gamma \) is named as a \( \gamma \) soft open set, if for any \( e_F \in (U, A) \), there is a soft open set \((H, A)\) containing \( e_F \) such that \((H, A)^\gamma \subseteq (U, A)\). The family of all \( \gamma \) soft open sets is stated by \( S^O_\gamma (\hat{X}_\gamma) \).

Definition 2.13 [13]. A space \( \hat{X}_\gamma \) is named a soft \( \gamma \) regular space if, for any \( e_F \in (X, A) \) and for all soft open sets \((K, A)\), there exists a soft open set \((H, A)\) containing \( e_F \) in which \((K, A)^\gamma \subseteq (H, A)\).
Definition 2.14 [13]. The soft γ interior of \((H, A)\) over \(\tilde{X}_\gamma\) is identified as the union of every \(\gamma\) soft open set super-set of \((F, A)\), which is stated by \(τ_\gamma \text{int}(H, A)\) .
\[
τ_\gamma \text{int}(H, A) = \cup\{(U, A)\colon (U, A) \text{ is } \gamma \text{ soft open set }, (U, A) \subseteq (H, A)\}.
\]

Definition 2.15 [13]. The soft gamma closure of \((H, A)\) over \(\tilde{X}_\gamma\) is identified as the intersection of every \(\gamma\) soft closed super-set of \((H, A)\), which is stated by \(τ_\gamma \text{cl}(H, A)\) .
\[
τ_\gamma \text{cl}(H, A) = \cap\{(U, A)\colon (U, A) is a \gamma \text{ soft closed set }, (U, A) \subseteq (H, A)\}.
\]

Definition 2.16 [14]. A soft set \((F, A)\) in soft space \((X, τ, A)\) is named pre-soft open if \((F, A) \subseteq \text{ int } cl(F, A)\).

Definition 2.17 [15]. A soft set \((H, A)\) over \(\tilde{X}_\gamma\) is called soft \(\gamma\) semi-open, if there is \(\gamma\) soft open set \((U, A)\) in which \((U, A) \subseteq (H, A) \subseteq τ_\gamma \text{cl}(U, A)\).

Definition 2.18 [13]. An operation \(\gamma\) associated with soft topology \(τ\) is named soft regular, if for any soft open \((K, A)\) and \((H, A)\) and any \(e_F \in (X, A)\), there is a soft open \((T, A)\) containing \(e_F\) in which \((T, A)^\gamma \subseteq (K, A)^\gamma \cup (H, A)^\gamma\).

Proposition 2.19 [12]. Let \((Y, τ_\gamma, A)\) be soft subspace of \((X, τ, A)\), then \((K, A)\) is soft open over \(Y\) if and only if \((K, A) = Y \cap (G, A)\) for soft open set \((G, A)\).

Proposition 2.20 [12]. Let \((Y, τ_\gamma, A)\) be a soft subspace of soft topological space \((X, τ, A)\), then \((F, A)\) is soft closed set over \(Y\) if and only if \((F, A) = Y \cap (H, A)\) for soft closed set \((H, A)\).

Proposition 2.21 [6]. Each \(\gamma\) soft open set is soft open.

Proposition 2.22 [6]. If \((N, A)\) is soft open and \((M, A)\) is soft subset of \((X, τ, A)\), then \(cl([N, A]) \cap (M, A) = cl(N, A) \cap cl(M, A)\).

Proposition 2.23 [13]. Let \((U, A)\) be a soft subset of \(\tilde{X}_\gamma\), then \((U, A)\) is a soft \(\gamma\) semi-open, if and only if \((U, A) \subseteq τ_\gamma cl \tau_\gamma int(U, A)\).

Proposition 2.24 [6]. Let \((U, A)\) be soft set in \(\tilde{X}_\gamma\), then \((U, A)\) is soft open if and only if \(τ_\gamma int(U, A) = (U, A)\).

Proposition 2.25 [6]. Let \((N, A)\) be soft set over \(\tilde{X}_\gamma\), then
1) \(cl(N, A) \subseteq τ_\gamma cl(N, A)\),
2) \(τ_\gamma int(N, A) \subseteq int(N, A)\).

Proposition 2.26 [6]. Let \((N, A)\) be any soft set over soft \(\gamma\) regular space \(\tilde{X}_\gamma\), then \(τ_\gamma int(N, A) = int(N, A)\).

Proposition 2.27. Let \(γ\) be soft regular-operation defined over soft topology \(τ\), the intersection of two soft \(\gamma\) open over \(\tilde{X}_\gamma\) is soft \(\gamma\) open set.

Proof: Consider \((U, A)\) and \((V, A)\) are \(\gamma\) soft open sets over space \(X\). Let \(e_F \in (U, A) \cap (V, A)\). Then \(e_F \in (U, A)\) and \(e_F \in (V, A)\). It follows that there exist two soft open \((H_1, A)\) and \((H_2, A)\) containing \(e_F\), in which \((H_1, A)^\gamma \subseteq (U, A)\) and \((H_2, A)^\gamma \subseteq (V, A)\). Hence, \(e_F \in (H_1, A)^\gamma \cap (H_2, A)^\gamma \subseteq (U, A) \cap (V, A)\). By definition 2.18, there is soft open \((H_3, A)\) containing \(e_F\), in which \((H_3, A)^\gamma \subseteq (U, A) \cap (V, A)\). Hence, \((U, A) \cap (V, A)\) is \(\gamma\) soft open set.

Proposition 2.28. Let \((N, A)\) and \((M, A)\) be any soft subsets over \(\tilde{X}_\gamma\), then the following statements are true
1) If \((N, A) \subseteq (M, A)\), then \(τ_\gamma cl(N, A) \subseteq τ_\gamma cl(M, A)\),
2) \(τ_\gamma cl(N, A) \cup τ_\gamma cl(M, A) \subseteq τ_\gamma cl([N, A] \cup (M, A)]\),
3) \(τ_\gamma cl([N, A] \cap (M, A)] \subseteq τ_\gamma cl(N, A) \cap τ_\gamma cl(M, A)\),
4) For soft regular-operation \(γ\), \(τ_\gamma cl(N, A) \cap τ_\gamma cl(M, A) = τ_\gamma cl([N, A] \cup (M, A)]\).

Proof: Obvious.

Proposition 2.29. If \(\tilde{Y}_\gamma\) is a soft subspace of \(\tilde{X}_\gamma\) and \((F, A) \subseteq \tilde{Y}_\gamma\), then \(τ_\gamma cl(F, A) = τ_\gamma cl(F, A) \cap \tilde{Y}_\gamma\).

Proof: Consider \(τ_\gamma cl(F, A) = \cap\{(S, A)\colon (S, A) is \gamma \text{ soft closed in } Y \text{ and } (F, A) \subseteq (S, A)\}\).

Proposition 2.30. Let \(γ\) be soft-regular operation set to \(\tilde{Y}\). If \((K, A)\) is any soft set and \((L, A)\) is \(γ\) soft open set over \(\tilde{X}_\gamma\), then \(τ_\gamma cl(K, A) \cap (L, A) \subseteq τ_\gamma cl((K, A) \cap (L, A))\).
Proof: Let $e_F \in \tau_y cl(K, A) \cap (L, A)$, then $e_F \in \tau_y cl(K, A)$ and $e_F \in (L, A)$. It follows that for every $\gamma$ soft open set $(H, A)$ containing $e_F$, $(K, A) \cap (H, A) \neq \emptyset$, and by proposition 2.27, $(K, A) \cap (H, A)$ is soft $\gamma$ open set containing $e_F$. But $e_F \in \tau_y cl(K, A)$, therefore $(K, A) \cap (L, A) \cap (H, A) \neq \emptyset$ which implies that $e_F \in \tau_y cl((K, A) \cap (L, A))$.

3. Some properties of strong soft $\gamma$ pre-open set

Definition 3.1 [6]. A soft set $(K, A)$ in $\hat{X}_y$ is named strong soft $\gamma$ pre-open set if $(K, A) \subseteq \text{int} \tau_y cl(K, A)$.

The family of all strong soft $\gamma$ pre-open sets over $\hat{X}_y$ is stated by $SS_y^*P(\hat{X}_y)$.

Example 3.2. Consider $X = \{h_1, h_2, h_3\}$ and $A = \{a_1, a_2\}$. Let $\tau = \{\phi, X, (K_1, A), (K_2, A), (K_3, A)\}$ where $K_1(a_1) = \{h_1\}, K_2(a_1) = \{h_2\}, K_3(a_1) = \{h_3\}, K_1(a_2) = \{h_2, h_3\}$, $K_2(a_2) = \{h_2\}, K_3(a_2) = \{h_3\}$.

Assign an operation $\gamma: \tau \rightarrow P(X)$ as $\gamma(U, A) = (U, A) \cap (K, A)$ for every $(U, A) \in \tau$, then $\gamma = \{\phi, X, (K_1, A)\}$ where $(K, A)$ is strong soft $\gamma$ pre-open set.

Proposition 3.3. A strong soft $\gamma$ is pre-open $(K, A)$ over $\hat{X}_y$ if and only if there is soft open set $(N, A)$ in which $(K, A) \subseteq (N, A) \subseteq \tau_y cl(K, A)$.

Example 3.4. Every soft open over $\hat{X}_y$ is strong soft $\gamma$ pre-open set.

Proof: assume that $(K, A)$ is soft open over $\hat{X}_y$, then $(K, A) \subseteq \text{int} (K, A) \subseteq \tau_y cl(K, A)$.

Note that the intersection of finite strong soft $\gamma$ pre-open sets may not be a strong soft $\gamma$ pre-open set, as shown in the next example.

Example 3.6. Consider $X = \{h_1, h_2, h_3\}$ and $= \{a_1, a_2\}$. Let $\tau = \{\phi, X, (K_1, A), (K_2, A), (K_3, A), (K_4, A)\}$ where $K_1(a_1) = \{h_1\}, K_2(a_1) = \{h_3\}, K_3(a_1) = \{h_1, h_2\}, K_4(a_1) = \{h_1, h_3\}$, $K_1(a_2) = \{h_1\}, K_2(a_2) = \{h_2\}, K_3(a_2) = \{h_1, h_2\}, K_4(a_2) = \{h_1, h_3\}$.

Assign an operation $\gamma: \tau \rightarrow P(X)$ as $\gamma(K, A) = \{(K, A) \cap (K, A) \cap (K, A) \cap (K, A) \subseteq \text{int} \tau_y cl(K, A)$ for every $(K, A) \in \tau$, then $\tau_y = \{\phi, X, (K_1, A), (K_2, A), (K_3, A)\}$. Clearly, $(K_1, A)$ and $(K_2, A)$, represented as $K_5(a_1) = \{h_2, h_3\}$ and $K_6(a_2) = \{h_2\}$, respectively, are strong soft $\gamma$ pre-open sets, but the intersection of $(K_3, A)$ and $(K_5, A)$ is $(F_6, A)$ such that $K_6(a_1) = \{h_2\}$ in which $K_6(a_2) = \{h_2\}$, which is not strong soft $\gamma$ pre-open set.

Proposition 3.7. If $(K, A) \in SS_y^*O(\hat{X}_y)$ and $(H, A) \in SS_y^*P(\hat{X}_y)$, then $(K, A) \cap (H, A) \in SS_y^*P(\hat{X}_y)$.

Proof: As $(H, A) \in SS_y^*P(\hat{X}_y)$, then by Proposition 3.3, there exists a soft open $(G, A) \in SS_y^*O(X)$ in which $(H, A) \subseteq (G, A) \subseteq \tau_y cl(H, A)$. And so, $(K, A) \cap (H, A) \subseteq (K, A) \cap (G, A) \subseteq (K, A) \cap \tau_y cl(H, A) \subseteq \tau_y cl((K, A) \cap (H, A))$. Also, since $(G, A) \subseteq (H, A)$, then $(K, A) \cap (H, A) \subseteq SS_y^*P(\hat{X}_y)$.

Proposition 3.8. If $\{(K_i, A): i \in I\}$ is a collection of strong soft $\gamma$ pre-open sets over $\hat{X}_y$, then $U_{i \in I}(K_i, A)$ is strong soft $\gamma$ pre-open set.

Proof: Let $U_{i \in I}(K_i, A) \subseteq \text{int} \tau_y cl((K_i, A))$, then $U_{i \in I}(K_i, A) \subseteq \text{int} \tau_y cl((K_i, A))$ and so, $U_{i \in I}(K_i, A) \subseteq \text{int} \tau_y cl((K_i, A)) \subseteq \tau_y cl(U_{i \in I}(K_i, A))$. Hence $U_{i \in I}(K_i, A)$ is strong soft $\gamma$ pre-open set.

Proposition 3.9. For $\hat{Y}$ is a soft subspace of $\hat{X}_y$, if $(K, A) \in SS_y^*P(\hat{Y}_y)$, then $(K, A) \in SS_y^*P(\hat{X}_y)$.

Proof: Let $(K, A) \in SS_y^*P(\hat{Y}_y)$, then there exists $(U, A) \in SS_y^*O(Y)$ in which $(K, A) \subseteq (U, A) \subseteq \tau_y cl(K, A)$. According to Proposition 2.19, there exists $(H, A) \in SS_y^*O(X)$ in which $(K, A) \subseteq (H, A) \cap \hat{Y} \subseteq \tau_y cl(K, A) \cap \hat{Y}$. It follows by Proposition 2.29 that $(K, A) \subseteq (H, A) \subseteq \tau_y cl(K, A)$. Hence $(K, A) \in SS_y^*P(\hat{X}_y)$.
Proposition 3.10. Let $\tilde{\mathcal{Y}}_{\gamma}$ be soft subspace of $\tilde{X}_{\gamma}$ such that $(K, A) \subseteq \tilde{\mathcal{Y}}$. If $(K, A) \in SS_{\gamma}^{\ast}P(\tilde{X}_{\gamma})$, then $(K, A) \in SS_{\gamma}^{\ast}P(\tilde{\mathcal{Y}}_{\gamma})$.

Proof: Let $(K, A) \in SS_{\gamma}^{\ast}P(\tilde{X}_{\gamma})$, there is $(V, A) \in S^\ast O(X)$ in which $(K, A) \subseteq (V, A) \subseteq \tau_{\gamma}cl_{\gamma}(K, A)$, and since $(K, A) \subseteq \tilde{\mathcal{Y}}$, then $(K, A) \subseteq (H, A) \subseteq \tau_{\gamma}cl_{\gamma}(K, A) \cap Y$ and $(H, A) \subseteq S^\ast O(Y)$. By Proposition 2.29, we get $(K, A) \subseteq (H, A) \subseteq \tau_{\gamma}cl_{\gamma}(K, A)$. Hence $(K, A) \in SS_{\gamma}^{\ast}P(\tilde{\mathcal{Y}}_{\gamma})$.

Definition 3.11. A soft set $(K, A)$ in $\tilde{X}_{\gamma}$ is called strong soft $\gamma$ pre-closed, if $\tilde{X} - (K, A)$ is strong soft $\gamma$ pre-open set, equivalently, $cl_{\gamma}int_{\gamma}(K, A) \subseteq (K, A)$. The family of all strong soft $\gamma$ pre-closed is stated by $SS_{\gamma}^{\ast}PC(\tilde{X}_{\gamma})$.

Proposition 3.12. If $\{ (K_i, A_i) : i \in I \}$ is a family of strong soft $\gamma$ pre-closed sets over $\tilde{X}_{\gamma}$, then $\bigcap_{i \in I}(K_i, A_i)$ is strong soft $\gamma$ pre-closed.

Proof: Follows from Proposition 3.8.

Proposition 3.13. If $(K, A) \in SS_{\gamma}^{\ast}C(\tilde{X}_{\gamma})$ and $(H, A) \in SS_{\gamma}^{\ast}PC(\tilde{X}_{\gamma})$ are closed sets over $\tilde{X}_{\gamma}$, then $(K, A) \cap (H, A) \in SS_{\gamma}^{\ast}PC(\tilde{X}_{\gamma})$.

Proposition 3.14. Let $\tilde{\mathcal{Y}}_{\gamma}$ be a soft subspace of $\tilde{X}_{\gamma}$ and let $(K, A) \subseteq \tilde{\mathcal{Y}}$. If $(K, A) \in SS_{\gamma}^{\ast}PC(\tilde{\mathcal{Y}}_{\gamma})$, then $(K, A) \in SS_{\gamma}^{\ast}PC(\tilde{X}_{\gamma})$.

Proposition 3.15. Let $\tilde{\mathcal{Y}}_{\gamma}$ be a soft subspace of $\tilde{X}_{\gamma}$ such that $(K, A) \subseteq \tilde{\mathcal{Y}}$. If $(K, A) \in SS_{\gamma}^{\ast}PC(\tilde{\mathcal{Y}}_{\gamma})$, then $(K, A) \in SS_{\gamma}^{\ast}PC(\tilde{X}_{\gamma})$.

Proposition 3.16. A soft subset $(K, A)$ of space $\tilde{X}_{\gamma}$, then $(K, A)$ is strong soft $\gamma$-pre-closed if and only if $cl_{\gamma}(K, A) - (K, A) \subseteq \text{int}\left(\tilde{X} - \tau_{\gamma}int_{\gamma}(K, A)\right) - \left(\tilde{X} - cl_{\gamma}(K, A)\right)$.

Proof: since $cl_{\gamma}(K, A) - (K, A) \subseteq \text{int}\left(\tilde{X} - \tau_{\gamma}int_{\gamma}(K, A)\right) - \left(\tilde{X} - cl_{\gamma}(K, A)\right)$.$\Leftrightarrow cl_{\gamma}(K, A) - (K, A) \subseteq \left(\tilde{X} - cl_{\gamma}(K, A)\right)$.

Definition 3.17. A soft element $e_{\gamma} \in P(X)$ is said to be strong soft $\gamma$ pre-interior element of $(K, A) \subseteq \tilde{X}_{\gamma}$ if there is strong soft $\gamma$ pre-open set $(G, A) \subseteq (K, A)$. The set of each strong soft $\gamma$ pre-interior element of $(K, A)$ is named strong soft $\gamma$ pre-interior set of $(K, A)$ which is stated by $spint_{\gamma}(K, A)$.

Proposition 3.18. For any soft subsets $(K, A)$ and $(H, A)$ of $\tilde{X}_{\gamma}$, then the following hold

1) $spint_{\gamma}(\emptyset) = \emptyset$
2) $spint_{\gamma}(\tilde{X}) = \tilde{X}$
3) If $(K, A) \subseteq (H, A)$, then $spint_{\gamma}(K, A) \subseteq spint_{\gamma}(H, A)$
4) $spint_{\gamma}(K, A) \cup spint_{\gamma}(H, A) \subseteq spint_{\gamma}((K, A) \cup (H, A))$
5) $spint_{\gamma}((K, A) \cap (H, A)) \subseteq spint_{\gamma}(K, A) \cap spint_{\gamma}(H, A)$
6) $\tau_{\gamma}int_{\gamma}(K, A) \cap spint_{\gamma}(H, A) \subseteq spint_{\gamma}((K, A) \cap (H, A))$

Proposition 3.19. Let $(K, A)$ be a soft subset of $\tilde{X}_{\gamma}$, then $\tau_{\gamma}int_{\gamma}(K, A) \subseteq \text{int}_{\gamma}(K, A) \subseteq spint_{\gamma}(K, A)$.

Definition 3.20. The strong soft $\gamma$ pre-closure of soft subset $(K, A)$ over $\tilde{X}_{\gamma}$ is the intersection of all strong soft $\gamma$ pre-closed sets containing $(K, A)$ which is stated by $spcl_{\gamma}(K, A)$.

Proposition 3.21. Let $(K, A)$ be soft set in $\tilde{X}_{\gamma}$, then $e_{\gamma} \in spcl_{\gamma}(K, A)$ if and only if, for any $(H, A) \in SS_{\gamma}^{\ast}PC(\tilde{X}_{\gamma})$, $(H, A) \cap (K, A) \neq \emptyset$.

Proposition 3.22. Let $(K, A)$ be a soft subset of $\tilde{X}_{\gamma}$, then $spcl_{\gamma}(K, A) \subseteq cl_{\gamma}(K, A) \subseteq \tau_{\gamma}cl_{\gamma}(K, A)$.

Proposition 3.23. For any $(K, A)$ and $(H, A)$ soft sets in $\tilde{X}_{\gamma}$, then the following hold

1) $spcl_{\gamma}(\emptyset) = \emptyset$
2) $spcl_{\gamma}(\tilde{X}) = \tilde{X}$
3) If $(K, A) \subseteq (H, A)$, then $spcl_{\gamma}(K, A) \subseteq spcl_{\gamma}(H, A)$
Therefore
Proof:
Proposition named
Proposition $\gamma$
which
Proposition closed.

Definition 3.24. A soft set $(K, A)$ over $\tilde{X}_f$ is called locally strong soft pre-closed set $\gamma$, if $(K, A) = (U,A) \cap (S,A)$ where $(S,A) \in SS_f^+P(\tilde{X}_f)$. That is $K,A \subseteq spcl_f(K,A) \subseteq spcl_f(S,A) = (S,A)$. It follows that $(K,A) \subseteq (U,A) \cap spcl_f(K,A) \subseteq (U,A) \cap (S,A) = (K,A)$, thus $(K,A) = (U,A) \cap spcl_f(K,A)$.

Conversely, assume that $(K,A) = (U,A) \cap spcl_f(K,A)$. Note that $(U,A) \in S^*O(\tilde{X}_f)$ and since $spcl_f(K,A)$ is strong soft $\gamma$ pre-closed, so $(K,A)$ is locally strong soft $\gamma$ pre-closed.

Proposition 3.26. Every soft open set is locally strong soft $\gamma$ pre-closed.

Proof: given that $(K,A) = (K,A) \cap X$ in which $(K,A) \in S^*O(X)$ and since $\tilde{X}_f$ is strong soft $\gamma$ pre-closed, then $(K,A)$ is locally strong soft $\gamma$ pre-closed.

Remark 3.27. Every strong soft $\gamma$ pre-closed is locally strong soft $\gamma$ pre-closed.

Proposition 3.28. Let $(K,A)$ be a soft subset of $\tilde{X}_f$. If $(K,A)$ is locally strong soft $\gamma$ pre-closed over $\tilde{X}_f$, then $(K,A) \cup (\tilde{X} - spcl_f(K,A))$ is strong soft $\gamma$ pre-closed.

Proof: Given that $(K,A)$ is locally strong soft $\gamma$ pre-closed, there exists $(U,A) \in S^*O(\tilde{X}_f)$ in which $(K,A) = (U,A) \cap spcl_f(K,A)$. It follows that $(K,A) \cup (\tilde{X} - spcl_f(K,A)) = (U,A) \cap spcl_f(K,A) \cup (\tilde{X} - spcl_f(K,A)) = (U,A) \cup (\tilde{X} - spcl_f(K,A))$ is strong soft $\gamma$ pre-closed set.

Proposition 3.29. If $(K,A)$ is locally strong soft $\gamma$ pre-closed and $(H,A)$ is soft open set over $\tilde{X}_f$, then $(K,A) \cap (H,A)$ is locally strong soft $\gamma$ pre-closed.

Proof: Given that $(K,A)$ is a locally strong soft $\gamma$ pre-closed set, then there exists $(U,A) \in S^*O(\tilde{X}_f)$ in which $(K,A) = (U,A) \cap spcl_f(K,A)$. It follows that $(K,A) \cap (H,A) = (U,A) \cap (S,A) \cap (H,A)$ where $(U,A) \cap (H,A) \in S^*O(\tilde{X}_f)$, and so $(K,A) \cap (H,A)$ is locally strong soft $\gamma$ pre-closed.

Proposition 3.30. A soft set $(K,A)$ over $\tilde{X}_f$ is locally strong soft $\gamma$ pre-closed set if and only if $\tilde{X} - (K,A)$ is the union of the soft closed set and strong soft $\gamma$ pre-open.

Definition 3.31. A soft set $(K,A)$ over $\tilde{X}_f$ is named strong soft $\gamma$ pre-dense whenever $spcl_f(K,A) = \tilde{X}$.

Proposition 3.32. A soft space $(X,\tau,A)$ such that $\gamma$ be an operation defined on soft topology $\tau$ is named strong soft $\gamma$ pre-submaximal, if every strong soft $\gamma$ pre-dense is strong soft $\gamma$ pre-open.

Proposition 3.33. If any soft set over $\tilde{X}_f$ is locally strong soft $\gamma$ pre-closed, then $\tilde{X}_f$ is strong soft $\gamma$ pre-submaximal.

Proof: Assume that $(K,A)$ is locally strong soft $\gamma$ pre-closed over $X$. It follows $(K,A) \cup (\tilde{X} - spcl_f(K,A)) = (K,A) \cup \phi = (K,A)$. But $(K,A) \cup (\tilde{X} - spcl_f(K,A))$ is strong soft $\gamma$ pre-open, therefore $(K,A)$ is strong soft $\gamma$ pre-open.
4- Pre γ soft open sets and soft γ preopen sets

Definition 4.1. A soft set \((K, A)\) is named soft γ pre-open over \(\bar{X}_γ\), if \((K, A) \subseteq \tau_γ \mathit{int} \tau_γ \mathit{cl}(K, A)\).

Definition 4.2 [9]. A soft set \((K, A)\) is named soft γ pre-open over \(\bar{X}_γ\) and is called pre γ soft open, if \((K, A) \subseteq \tau_γ \mathit{int} \mathit{cl}(K, A)\).

Proposition 4.3. Let \((K, A)\) be a soft subset of \(\bar{X}_γ\). Then \((K, A)\) is soft γ pre-open (respectively pre γ soft open) if and only if there is \((V, A) \in \mathcal{S}_γ O(\bar{X}_γ)\) in which \((K, A) \subseteq (V, A) \subseteq \tau_γ \mathit{cl}(K, A)\) (respectively \((K, A) \subseteq (V, A) \subseteq \mathit{cl}(K, A)\)).

Proposition 4.4. Every pre γ soft open is soft p-open set.

Remark 4.5. The reverse of the above proposition may not be true.

Recall Example 3.6. We have \(X = \{h_1, h_2, h_3\}\) and \(= \{a_1, a_2\}\). Let \(\tau = \{\emptyset, \bar{X}, (K_1, A), (K_2, A), (K_3, A), (K_4, A)\}\) where \(K_1(a_1) = \{h_1\}\), \(K_2(a_1) = \{h_3\}\), \(K_3(a_1) = \{h_1, h_2\}\), \(K_4(a_1) = \{h_1, h_3\}\). \(K_1(a_2) = \{h_2, h_3\}\), \(K_2(a_2) = \{h_1, h_2\}\), \(K_3(a_2) = \{h_1, h_2, h_3\}\), \(K_4(a_2) = \{h_1, h_3\}\). Assign the operation \(γ: \tau \rightarrow P(X)\) as \(γ(K, A) = \{(K, A)\} \quad \text{if} \quad h_2 \in (K, A)\) for any \((K, A) \in \tau\), then \((K_3, A)\) is soft pre-open but it is not pre γ soft open set.

Proposition 4.6. Each pre γ soft open set is soft γ pre-open.

Remark 4.7. The reverse of the previous proposition needs not to be true, as shown in the next example.

Example 4.8. Given that \(X = \{h_1, h_2, h_3, h_4\}\) and \(= \{a_1, a_2\}\), consider \(\tau = \{\emptyset, \bar{X}, (K_1, A), (K_2, A), (K_3, A)\}\) such that \(K_1(a_1) = \{h_1\}\), \(K_2(a_1) = \{h_2, h_3\}\), \(K_3(a_1) = \{h_1, h_2, h_3\}\), \(K_4(a_1) = \{h_1, h_2, h_3\}\). \(K_1(a_2) = \{h_1\}\), \(K_2(a_2) = \{h_2, h_3\}\), \(K_3(a_2) = \{h_1, h_2, h_3\}\). Assign \(γ(K, A) = (\mathit{int} \mathit{cl}(K, A)) \quad \text{if} \quad (K, A) \in \{h_1\}\) \(\quad \text{if} \quad (K, A) \neq \{h_1\}\).

That is, \(\tau_γ = \{\emptyset, \bar{X}, (K_2, A), (K_4, A)\}\) where \(K_4(a_1) = \{h_1, h_4\}\) and \(K_4(a_2) = \{h_1, h_4\}\) is soft γ pre-open set. However, it is not pre γ soft open set.

Proposition 4.9. Every soft γ pre-open is strong soft γ pre-open.

Remark 4.10. The reverse of Proposition 4.9 may not be true, as shown in the following example.

In Example 3.6, we have \(X = \{h_1, h_2, h_3\}\) and \(A = \{a_1, a_2\}\). Let \(\tau = \{\emptyset, \bar{X}, (K_1, A), (K_2, A), (K_3, A), (K_4, A)\}\) where \(K_1(a_1) = \{h_1\}\), \(K_2(a_1) = \{h_3\}\), \(K_3(a_1) = \{h_1, h_2\}\), \(K_4(a_1) = \{h_1, h_3\}\), \(K_1(a_2) = \{h_1\}\), \(K_2(a_2) = \{h_3\}\), \(K_3(a_2) = \{h_1, h_2\}\), \(K_4(a_2) = \{h_1, h_3\}\). Assign an operation \(γ: \tau \rightarrow P(X)\) as \(γ(K, A) = \{(K, A)\} \quad \text{if} \quad h_2 \in (K, A)\) \(\quad \text{if} \quad h_2 \notin (K, A)\) for every \((K, A) \in \tau\).

\((K_3, A)\) is strong soft γ pre-open but it is not soft γ pre-open.

Proposition 4.11. Let \(γ\) be an operation defined on soft topology \(\tau\) and let \(\bar{X}\) be a soft γ regular topological space, then the followings are equivalents

1) Soft pre open
2) Pre γ soft open
3) Strong soft γ pre open
4) Soft γ pre-open

Proof: Straightforward from Proposition 2.20 and Proposition 3.2.

Proposition 4.12. If \((K, A)\) is γ soft open set over \(\bar{X}_γ\), then \((K, A)\) is soft γ pre-open set.

Proposition 4.13. If \((K, A)\) is γ soft open set in \(\bar{X}_γ\), then \((K, A)\) is pre γ soft open.

Definition 4.14. A space \(\bar{X}_γ\) is extremally γ soft disconnected if soft γ closure of every soft γ open set is soft γ open.

Proposition 4.15. A space \(\bar{X}_γ\) is extremally γ soft disconnected if and only if each soft γ semi-open is soft γ pre-open set.
Proof: Let $\tilde{X}_\gamma$ be extremally $\gamma$ soft disconnected, then for every soft $\gamma$ semi-open $(K,A)$, we get $(K,A) \subseteq \tau_\gamma cl(\tau_\gamma int(K,A)) = \tau_\gamma int(\tau_\gamma cl(K,A)) \subseteq \tau_\gamma int(\tau_\gamma cl(K,A))$. Hence $(K,A)$ is soft $\gamma$ pre-open set.

Conversely, let $(K,A)$ be soft $\gamma$ open, so $\tau_\gamma cl(K,A) = \tau_\gamma cl(\tau_\gamma int(K,A))$. Since $\tau_\gamma cl(\tau_\gamma int(K,A))$ is soft $\gamma$ semi-open, hence by hypothesis, $\tau_\gamma cl(\tau_\gamma int(K,A))$ is soft $\gamma$ pre-open. It follows that $\tau_\gamma cl(K,A) \subseteq \tau_\gamma int(\tau_\gamma cl(K,A))$, therefore $\tau_\gamma cl(K,A)$ is soft $\gamma$ open set.

5. Conclusions

Soft sets were initiated by Molodtstove in 1999 and, since then, many researchers defined and investigated several types of soft sets. Some of these studies have real applications such as solving problems for medical diagnosis, determining educational obstacles, and taking right decisions about them. In this paper, we defined and studied new soft sets and named them soft $\gamma$ pre-open sets. Also, we provided several properties and characterizations about pre $\gamma$ soft open and strong soft $\gamma$ pre-open sets. Also, the relationships among these soft sets were discussed.

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