Optimization of a Low-alpha Lattice for the HLS-II Storage ring

Siwei Wang\textsuperscript{1}, Wei Xu\textsuperscript{1}, Ke Xuan\textsuperscript{1}, Yigang Wang\textsuperscript{1}, Jingyi Li\textsuperscript{2}

\textsuperscript{1} NSRL, University of Science and Technology of China, Hefei, Anhui 230029, China
\textsuperscript{2} Institute of High Energy Physics, CAS, Beijing 100049, China

E-mail: wxu@ustc.edu.cn

Abstract. To generate terahertz radiation at HLS-II, a low-\(\alpha_c\) lattice scheme is proposed. The new lattice can reduce the bunch length in the storage ring, thus enhancing the coherent synchrotron radiation in the THz region. In this paper, the design and optimization of a low-\(\alpha_c\) lattice is reported. The new lattice preserves the symmetry of nominal lattice and reduces the first and second order momentum factor at the cost of increasing maximum \(\beta\) function and natural emittance. The bunch length is tracked and the result shows that the low-\(\alpha_c\) lattice can effectively compress bunches in the storage ring. The performance of this low-\(\alpha_c\) lattice can be further studied and improved.

1. Introduction

The low-alpha lattice has been developed in synchrotron light sources for THz generation [1, 2]. In this work, a low-alpha lattice is designed to generate terahertz (THz) radiation in the HLS-II storage ring. To get minimum impact on the current lattice, the adjustment of quadrupoles go along the tune free knobs which is acquired from the tune response matrix. This process preserves the lattice symmetry and has little impact on the tune of the storage ring. After the \(\alpha_c\) is adjusted to a small value, the chromaticity, dynamic aperture and the second order momentum compaction factor (\(\alpha_2\)) should also be optimized. This optimization is accomplished by adjusting the sextupoles along the chromaticity knobs and chromaticity free knobs. This process also preserves the lattice symmetry. After the optimization, the bunch length is tracked with the accelerator program ELEGANT [3]. The results with different RF cavity parameters are compared.

In this paper, the optimization of the low-\(\alpha\) lattice is reported. The bunch length tracking in the new lattice is also introduced. The result shows that the low-\(\alpha_c\) lattice can be applied in the HLS-II storage ring with its current magnets.

2. Optimization of the low-\(\alpha_c\) lattice at HLS-II

The HLS-II storage ring is a compact one with a circumference of about 66 m. Its linear lattice is shown in Fig. 1. The lattice is 2-fold periodic with 4 DBA cells. The quadrupoles are grouped into 8 families labeled from Q1 to Q8. The strengths of the quadrupoles with the same label remain the same.

When designing the tune knob for the HLS-II storage ring, the tune response matrix was studied. The tune response matrix estimates the tune change with respect to quadrupole
strength changes and explicitly expressed as

\[
\begin{pmatrix}
\Delta \nu_x \\
\Delta \nu_y
\end{pmatrix}
= M
\begin{pmatrix}
\Delta K_1 \\
\Delta K_2 \\
\vdots \\
\Delta K_8
\end{pmatrix}
= \begin{pmatrix}
a_1 \\
b_1 \\
\vdots \\
a_8 \\
b_8
\end{pmatrix}
\begin{pmatrix}
\Delta K_1 \\
\Delta K_2 \\
\vdots \\
\Delta K_8
\end{pmatrix}.
\]

(1)

A singular value decomposition can be performed to the tune response matrix \(M = U\Lambda V^T\). Matrix \(V\) is \(8 \times 8\) and the first two columns are connected with the tune knob, and the last 6 columns are the “tune free knobs”. Adjusting the quadrupole strengths along the tune free knobs does not affect the tune. This is only valid in a small range because the linearity between tune and quadrupole strength is not strict in a larger range.

The design of low-\(\alpha_c\) lattice adopts all the quadrupoles and adjust the quadrupoles along the tune free knobs in a small range, which can reduce the impact on the lattice symmetry and tune. A particle swarm optimization (PSO) program is used to find the optimized result around the nominal lattice [4]. The adjustment coefficients before the tune free knobs are set as the optimization variables. The adjustment ranges are all set to be -0.5 to 0.5. The goal value is set to be \(|\alpha_c - 10^{-5}|\) to reduce the first order momentum compaction factor to the order of \(10^{-5}\). This adjustment aims at making minimum modification to the current lattice.

After the PSO program finds a set of optimum solution for the quadrupole strengths, the linear low-\(\alpha_c\) lattice is acquired. But the performance of such a lattice is not so good because of the affected chromaticity and dynamic aperture. When the momentum compaction factor is reduced to a low level, the second order momentum compaction factor becomes important and it may affect longitudinal dynamics. Luckily both the dynamic aperture and \(\alpha_2\) can be adjusted.

Figure 1. Schematic of the HLS-II linear lattice. There are 32 quadrupoles in the storage ring and they are grouped into 8 families according to lattice symmetry.
Figure 2. Result of the MOGA optimization of dynamic aperture and second order $\alpha_c$. The horizontal axis denotes minus DA area and the vertical axis denotes the second order $\alpha_c$.

by adjusting the sextupoles. There are 32 sextupoles in the HLS-II storage ring and they are grouped into 4 families with the strengths in each family remain the same.

Similarly the chromaticity response matrix can be calculated by

$$\begin{pmatrix} \Delta C_x \\ \Delta C_y \end{pmatrix} = \begin{pmatrix} d_{11} & d_{12} & d_{13} & d_{14} \\ d_{21} & d_{22} & d_{23} & d_{24} \end{pmatrix} \begin{pmatrix} \Delta S_1 \\ \Delta S_2 \\ \Delta S_3 \\ \Delta S_4 \end{pmatrix} = R \begin{pmatrix} \Delta S_1 \\ \Delta S_2 \\ \Delta S_3 \\ \Delta S_4 \end{pmatrix}. \quad (2)$$

A SVD can be performed to the matrix $R$ and obtain two sets of chromaticity free knobs $\vec{f}_1$, $\vec{f}_2$. Unlike the tune free knobs, the chromaticity free knobs are strict, adjusting the sextupole strengths along the chromaticity free knob does not affect the chromaticity. Also the pseudo-inverse of the matrix $R$ can be computed to obtain two sets of chromaticity knobs $\vec{\xi}_1$, $\vec{\xi}_2$.

When optimizing the nonlinear performance of the low-$\alpha_c$ lattice, a multi-objective genetic algorithm (MOGA) program is adopted [5]. The dynamic aperture area and the $\alpha_2$ are the optimization goal values. They are explicitly:

$$f(1) = -DA \text{ area}, \quad f(2) = |\alpha_2 - 10^{-5}|. \quad (3)$$

These two goals aim at increasing the dynamic aperture and adjusting $\alpha_2$ to the order of $10^{-5}$. To increase the degree of freedom for adjustment, the two sets of chromaticity knobs and two sets of chromaticity free knobs are all used and the sextupoles are expressed by

$$\vec{S} = \vec{S}_0 + x_1 \vec{\xi}_1 + x_2 \vec{\xi}_2 + x_3 \vec{f}_1 + x_4 \vec{f}_2. \quad (4)$$

The four coefficients $x_1$, $x_2$, $x_3$ and $x_4$ are the optimization variables. The coefficients $x_3$ and $x_4$ are set to vary in the range $[-50,50]$ while $x_1$ and $x_2$ are set in a narrower range to ensure
the chromaticities remain in the positive range of $[0,10]$. This kind of adjustment can deal with the optimization while constraining the chromaticity change. 200 populations are iterated for 500 generations. After the MOGA program converges, a set of solution can be obtained. The converged solutions are plotted in Fig. 2. One of the results has the largest concentration, which will be estimated in the following parts.

3. Parameters of the low-$\alpha_c$ lattice

![Graphs of various functions](image)

**Figure 3.** Optical functions for low-$\alpha_c$ lattices and nominal lattice. (a) The horizontal and vertical $\beta$ functions. (b) The horizontal $\eta$ function.

The parameters of the modified lattice are calculated. The momentum compaction factor has been reduced to $5.4 \times 10^{-5}$, which is rather a low value. The $\beta$ functions $\beta_x$, $\beta_y$ and dispersion functions $\eta_z$ for the low-$\alpha_c$ modes are plotted in Fig. 3. They are compared with the optical functions of the nominal lattice. The low-$\alpha_c$ mode is realized by lowering the dispersion functions at the bending magnets. As a consequence, the beta functions of low-$\alpha_c$ lattices become larger compared with the nominal lattice. Other main parameters of the low-$\alpha_c$ lattice are estimated and the result is listed in Table 1.

From the figure and the table it can be seen that the low $\alpha_c$ lattice is acquired at the cost of increasing $\beta$ function and emittance. The natural bunch length is reduced, but a tracking should be performed to gain a deep study of the bunch length with different RF cavity parameters.
Table 1. Main Parameters of the Low-\(\alpha_c\) Lattice.

| Parameters     | Value          |
|----------------|----------------|
| Horizontal tune| 4.4499         |
| Vertical tune  | 2.3002         |
| Natural emittance | 162 nm-rad  |
| Natural bunch length | 4.1 ps  |
| Horizontal chromaticity | 1.3    |
| Vertical chromaticity  | 3.0       |
| 1st order \(\alpha_c\) | \(5.4 \times 10^{-5}\) |
| 2nd order \(\alpha_2\) | \(-2.0 \times 10^{-6}\) |
| Maximum \(\beta_x\) | 28.7 m     |
| Maximum \(\beta_y\) | 26.5 m     |

Figure 4. Dynamic aperture of the low-\(\alpha_c\) lattice and the nominal lattice.

The dynamic aperture is tracked for the optimized lattice. The horizontal size is over 10 mm, but the vertical size can only reach 3 mm. The dynamic aperture area is about \(4 \times 10^{-5}\). The small dynamic aperture may be due to the compact property of the HLS-II storage ring and the low momentum compaction factor. Although the dynamic aperture is optimized by MOGA program, it still can’t compare with that of the standard lattice, which is shown in Fig. 4. Such a lattice will challenge the traditional off-axis injection techniques, which is also a problem in a fourth generation synchrotron light source.

4. Bunch length tracking
To get a deeper look into the bunch length in the low-\(\alpha_c\) lattice, a tracking program is performed for the new lattice with the software ELEGANT. The RF cavity parameters in the ring are set the same as in the nominal lattice. To simplify the calculation process, a Gaussian bunch
with 1000 electrons is generated with the natural emittance. This electron bunch is tracked for 1,000,000 turns in the storage ring. The average bunch length is calculated and recorded after each turn. The tracking result is shown in Fig. 5. The result shows that the low-\(\alpha_c\) lattice can effectively reduce the bunch length.

![Figure 5](image)

**Figure 5.** Bunch length tracking of the low-\(\alpha_c\) lattice. A bunch of 1000 electrons is tracked for 1,000,000 turns and the bunch length is calculated and recorded.

5. Summary
In this paper, a low-\(\alpha_c\) lattice for the HLS-II storage ring is reported. The design and optimization process are introduced in detail. The basic performance of the low-\(\alpha_c\) lattice is estimated and the bunch length is tracked. The result shows the low-\(\alpha_c\) lattice can be applied in the HLS-II storage ring and can effectively compress the bunch length. This low-\(\alpha_c\) lattice can be further studied and optimized to improve its general performance.

References
[1] Tordeux, M. A. *et al.*, “Low-alpha Operation for the SOLEIL Storage Ring”, in *Proc. IPAC’12*, New Orleans, LA, USA, May 2012, paper TUPPP004, pp. 1608–1610.
[2] Chauwattana, T. *et al.*, “THz Coherent Synchrotron Radiation from Ultra-low Alpha Operating Mode at Diamond Light Source”, in *Proc. IPAC’16*, Busan, Korea, May 2016, pp. 1682–1684.
[3] M. Borland, “Elegant: A flexible SDDS-compliant code for accelerator simulation”, Argonne National Lab., IL, 2000.
[4] Yuhui Shi, “Particle swarm optimization: developments, applications and resources”, (IEEE Cat. No. 01TH8546), vol. 1, pp. 81-86., IEEE, 2001.
[5] K. Deb *et al.*, “A fast and elitist multiobjective genetic algorithm: NSGA-II”, *IEEE transactions on evolutionary computation*, vol. 6, no. 2, pp. 182-197, 2002.