Energy Trading between Microgrids Individual Cost Minimization and Social Welfare Maximization

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Abstract—Microgrids use renewable energy to mitigate the pressure on the Macrogrid and environment. But the renewable energy generation is fluctuant and hard to be predicted. There are mainly two techniques to tackle with this problem. The first one is installing the battery to buffer the renewable energy and smooth out the fluctuation. The other one is exchanging energy between the Microgrids. This technique is benefit from averaging effect produced by diversity in renewable energy production across different areas. However there still are two fundamental questions which need to be answered: how to deal with several stochastic parameters caused by the renewable energy generation and how to maximize Microgrids own profit by serving their demands and trading energy. In this paper, we design a double-auction based mechanism which can ensure the Microgrids and Macrogrid who is served as auctioneer profits. In addition, our algorithm need less information communication and can help Microgrids get more profits than without double auction. In order to solve the cost-minimization problem combined with stochastic parameters, we introduced the Lyapunov optimization which only need to know the current values without requiring any knowledge of their statistics. Simulation results based on real world data show the reducing in total cost and energy from Macrogrids.

Index Terms—Renewable energy, Lyapunov optimization, Double auction

1 INTRODUCTION

Growing attention to the environment and demands of electricity force the power grid modernizing. Power grids install the renewable energy generator, energy storage and transform themselves to the future power grid named smart grid or Microgrid. Then the function of smart grids are widely extended. They need to sense, calculate and control the renewable energy generation in order to allocate their energy to satisfy different customers demands and minimize their own cost. However using renewable energy has its own deficits. For example, the renewable energy generation is hard to be predicted. These characteristics may sometimes force the Microgrids to ignore some demands. No matter how, the stability and reliability are the most important factors of the Microgrids. If the power produced by the renewable generators cannot satisfy all demands, Microgrids have to find other way and ensure the stability.

In order to make Microgrids to be more satisfied, there are two possible way. The first one is to install batteries into the Microgrids [6]. When there is extra energy in one day, batteries can store the extra energy in case of shortage in another day. The energy charging in the batteries can help Microgrids dealing with the shortage without purchasing energy from the Macrogrids. However, Batteries are not panacea to this problem, because the batteries are expansive and have a lot of constraints like charging and discharging rates and capacities. The other one is that Microgrids are allowed to exchange energy with each other [16]. Todays power grid is consist of hundred of Microgrids. The renewable energy produced in different geographical areas have averaging effect. So the Microgrids which locate at several place have chance to mitigate the problem caused by
the intermittent nature of the renewable energy. The mechanism which can pool Microgrids extra energy together will help Microgrids replace the fossil energy from Macrogrid with the renewable energy from other Microgrids. However, problems on energy storage and trading still need to be further studied.

One main challenge is motivating the Microgrids to participate in the trade. When the capacity of battery of one Microgrid is great, the possibility that this Microgrid will attend the trade is little. Then the benefit brought by the trade will be minimized. Another challenge is that if Microgrids decide to attend the trade, Microgrids need to make some practical decision: whether use more energy to serve their own loads or trade? How much money should be charged for per unit energy? These decisions and question are critical to the Microgrids which intend to minimize cost.

There have been several studies with similar solution or smart grid topic. [15] have a similar approach to the problem. The authors apply this mechanism to another scenario called inter-cloud trading. In addition, the double auction designed by the author is not suitable to the Microgrids scenario. [16] want to Microgrids borrow energy from the Microgrids which have extra energy. But when Microgrids have a great capacity battery, Microgrids are tend to put extra energy into battery instead of exchanging. It also doesn’t consider about the delay-tolerant loads. [17] and [24] use the double auction to solve the double auction problem. This algorithm have a good performance, but it need iterative double auction which need more information communication and time to reach the best outcome. [18] take delay-tolerant load into consideration, but it don’t have a electricity market and need a large capacity battery.

In this paper, we assume all Microgrids are rational and selfishness which just want to minimize their cost and maximize their profit. We intend to design a double-auction based mechanism which can ensure the Microgrids maximize their own benefit, meanwhile the mechanism should be efficient, have potential to deal with stochastic parameters and need little calculation. Double auction means both potential buyers and sellers can submit their bids which are according to their real-time situation to the auctioneer. Auctioneer will choose the selling and purchasing price which can ensure the profits. In addition, the mechanism can let the rational participants put true price into auction rather than the price deviating Microgrids needs. The mechanism only need one-shot double auction instead of iterative double auction which is efficient and reliable. Then a Lyapunov-optimization based algorithm will help each Microgrid to deal with the stochastic parameters and decide the auction parameters. In particular, our objectives will show as below:

- Propose a mechanism which can deal with stochastic parameters to help Microgrids which have distributed renewable energy generation, energy storage and delay-tolerant and delay-intolerant loads decide the price and amount of energy submit to the double auction, meanwhile it can solve the cost minimization problem.
- Develop a double auction which can let both of the participants and auctioneers be benefit from the auction. Our non-iterative double auction is efficient and allow distributed Microgrids attend the auction. The mechanism also will make Microgrids reveal the true needs.
- Through theoretical analysis, we show our algorithm can help individual Microgrid can be benefit when they attend auction.

In the remainder of the paper, we discuss the system model and double auction framework in Section 2. Then, in Section 3, we show the solution to former problem based on the time average cost minimization problem using the method of Lyapunov optimization, and prove the theoretical performance of this method. Numerical results are in Section 4 and conclusion is in the Section 5.

2 SYSTEM MODEL AND DOUBLE AUCTION FRAMEWORK

As described in the previous section, to handle stochastic and trading problem, we design a time average cost minimization plus double auction mechanism to help the Microgrids to be participate in the electricity market. The
meanwhile discharging leaving the battery queue. The energy in the battery can be considered as a queue and can store extra energy. The energy in the battery device.

2 Broker: The Macrogrid is served as Broker which can get selling or buying energy demands from Microgrids. The Macrogrid needs to figure out the best solution which can maximize the social welfare.

2.1 Cost Minimization Problem

To individual Microgrids, they need to minimize their cost. But both renewable energy supply and users’ demands are stochastic. So Microgrids need an algorithm to deal with these stochastic variables.

2.1.1 Battery

Each Microgrid has its own battery which can store extra energy. The energy in the battery can be considered as a queue \( B_i(t) \). Charging \( C_i(t) \) is new energy enter the battery queue, meanwhile discharging \( D_i(t) \) means the energy leaving the battery queue.

\[
B_i(t + 1) = B_i(t) - D_i(t) + C_i(t)
\]

To be specific, the batteries have a lot of constraints. Firstly, we don’t allow charging and discharging happening simultaneously and batteries have their own capacities.

\[
1_C_{i(t)>0} + 1_D_{i(t)>0} \leq 1
\]

\[
0 \leq B_i(t) \leq B_i^{\max}
\]

Secondly, denoting \( C_i^{\max} \) is the maximum amount energy that battery can be charged safely at a single time, and \( D_i^{\max} \) is the maximum amount energy that battery can be discharged safely at a on slot. In addition, the energy charged into battery should be energy from Macrogrid \( G_i^C(t) \) and renewable generator \( R_i^C(t) \).

\[
0 \leq C_i(t) \leq \min[B_i^{\max} - B_i(t), C_i^{\max}(t)]
\]

\[
0 \leq D_i(t) \leq \min[B_i(t), D_i^{\max}]
\]

\[
C_i(t) = G_i^C(t) + R_i^C(t)
\]

\[
D_i(t) = D_i^I(t) + D_i^I(t)
\]

2.1.2 Load Service

The users have both delay-intolerant(DI) and delay-tolerant(DT) load demands. DI load demands need to be served when they come. DT load demands should be served before a certain deadline. In addition, we assume the \( T_i(t) \) is an i.i.d non-negative stochastic process, and \( 0 \leq T_i(t) \leq T_i^{\max} \), now we define the \( Q_i(t) \) as the length of queue buffering of delay-tolerant jobs at Microgrid \( i \) on time \( t \) and \( J_i(t) \) as the energy allocated to serve the DT loads demands. Similar to \( C_i(t) \), \( J_i(t) \) and \( I_i(t) \) are came from different sources named as \( G_i^I(t), D_i^I(t), X_i^I(t), R_i^I(t), G_i^C(t) \) and \( D_i^I(t) + R_i^I(t) \) which will be specific in Table I.

\[
Q_i(t + 1) = \max[Q_i(t) - J_i(t), 0] + T_i(t)
\]

\[
J_i(t) = G_i^I(t) + D_i^I(t) + X_i^I(t) + R_i^I(t)
\]

\[
I_i(t) = G_i^I(t) + D_i^I(t) + R_i^I(t)
\]

Then in order to make the equation become concise, we denote \( R_i(t) \) as total renewable energy generated in time slot \( t \), \( G_i(t) \) as total energy purchase from Macrogrid, \( X_i(t) \) as total
energy bought by Microgrid $i$ and $Y_i(t)$ as total energy sold from Microgrid $i$.

$$R_i(t) = R_i^I(t) + R_i^J(t) + R_i^Y(t) + R_i^C(t) \quad (11)$$

$$G_i(t) = G_i^I(t) + G_i^J(t) + G_i^C(t) \quad (12)$$

$$X_i^J(t) = \sum_{l=1, l \neq i}^{N} \hat{x}_{il}(t) \quad (13)$$

$$R_i^Y(t) = \sum_{l=1, l \neq i}^{N} \hat{y}_{il}(t) \quad (14)$$

However, the DT load demands can be delayed, but users still feel uncomfortable when these demands cannot be served. So we denote $Z_i(t)$ as delay aware queue.

$$Z_i(t+1) = \max[Z_i(t) - J_i(t), 0] + \varepsilon_i 1_{Q_i(t) > 0} \quad (15)$$

In the extreme condition, both queue still can be stable. We have constraint:

$$G_i^{\max} \geq J_i^{\max}(t) \geq G_i^{I,max} \geq \max[\varepsilon_i, T_i^{\max}] \quad (16)$$

So that

$$0 \leq Q_i(t) \leq Q_i^{\max} \quad (17)$$

$$0 \leq Z_i(t) \leq Z_i^{\max} \quad (18)$$

No matter how, the energy get into the Microgrids should be excess the energy get out of the Microgrids

$$I_i(t) + J_i(t) + \sum_{l=1, l \neq i}^{N} \hat{y}_{il}(t) + C_i(t) \leq R_i(t) + G_i(t) + D_i(t) + \sum_{l=1, l \neq i}^{N} \hat{x}_{il}(t) \quad (19)$$

And we want have a better control of the battery, so we formulate a virtual queue.

$$X_i(t) = B_i(t) - \Theta_i - D_i^{\max} \quad (20)$$

$\Theta_i$ will be specified later.
\[ Z_i(t) \] Length of virtual queue at Microgrid \( i \), slot \( t \)
\[ \varepsilon_i \] Set by users denote as users’ delay aware coefficient

TABLE II
THE VARIABLES ABOUT DOUBLE AUCTION

| Variable | Description |
|----------|-------------|
| \( \beta_i(t) \) | The price of buying per unit energy by Microgrid \( i \), slot \( t \) |
| \( \alpha_i(t) \) | The price of selling per unit energy from Microgrid \( i \), slot \( t \) |
| \( \hat{\beta}(t) \) | Actual price of buying per unit energy, slot \( t \) |
| \( \hat{\alpha}(t) \) | Actual price of selling per unit energy, slot \( t \) |
| \( x_{ij}(t) \) | The amount of energy bought by the Microgrid \( i \) to Microgrid \( l \), slot \( t \) |
| \( y_{ij}(t) \) | The amount of energy sold from Microgrid \( i \) to Microgrid \( l \), slot \( t \) |
| \( \hat{x}_{ij}(t) \) | The actual amount of energy bought by the Microgrid \( i \) to Microgrid \( l \), slot \( t \) |
| \( \hat{y}_{ij}(t) \) | The actual amount of energy sold from Microgrid \( i \) to Microgrid \( l \), slot \( t \) |
| \( \beta^\text{min}_i \) | The minimum price that Microgrid \( i \) can get a unit of energy. It should cover all necessary infrastructure costs |
| \( \rho_1, \rho_2 \) | Set by Auctioneer measure the maximum energy can trade in the auction |

2.1.3 Problem 1 Formulation

The Microgrids want to cost less, meanwhile they can serve more demands. Firstly, we define the \( U_i(t) \)

\[ U_i(t) = P(t)G_i(t) + \sum_{l=1, l \neq i}^{N} \hat{\beta}(t)\hat{x}_{il}(t) - \sum_{l=1, l \neq i}^{N} \hat{\alpha}(t)\hat{y}_{il}(t) \]

Then The average time cost minimization problem is shown below:

\[ P1 : \min \lim_{T \to \infty} \frac{1}{T} \mathbb{E}\{U_i(t)\} \]  \( (21) \)

subject to:

\[ \text{(1)-(20)} \]

2.2 Auctioneer Allocation Problem

We want to use double auction to motivate the Microgrids to take part in the electricity market. The proposed reference structure is shown in Fig.2. We show the Social Welfare Maximization Problem \((P2)\)

\[ P2 : \max \sum_{i=1}^{\text{i}'} \sum_{l=1}^{\text{l}'} (\rho_1 \beta_i(t) \log(x_{il}(t)) - \rho_2 \alpha_l(t) \frac{y_{li}(t)}{2}) \]  \( (22) \)

subject to:

\[ x_{il}(t) = y_{li}(t) \] \( \forall i \in \{1, 2, \cdots, \text{i}'\}, \forall l \in \{1, 2, \cdots, \text{l}'\} \]  \( (23) \)

\[ P_i(t) \geq \beta_{\text{V}}(t) > \alpha_{\text{V}}(t) \]  \( (24) \)

We denote \( \beta_{\text{V}}(t) \) and \( \alpha_{\text{V}}(t) \) as the sell-bid and buy-bid which may get accepted at time \( t \). \( (23) \) make sure the balance between the supply and demands of energy in the electricity market and \( (24) \) promise that both Microgrids and Macrogrid attend the auction can get profits. In addition, \( (24) \) ensure the Microgrids will not offer energy to and get energy from the same Microgrids, because \( \beta_{\text{V}}(t) > \alpha_{\text{V}}(t) \)

2.3 Discussion of the System Model

We now illustrate some notice on the system models.

- We assume the energy bought by Microgrids from auction only can serve their
loads demands. In other words, the energy got from auction cannot be charged into the battery and Microgrids cannot use energy bought from auction for further trading. Because in the real world, we need consider about the efficiency of charging and discharging energy. If we put the charging and discharging efficiency into the system model, we will find that the possibility of purchasing energy for further trading is little. This more complicated model with physical model will be considered in the further work.

- Transferring energy between the Microgrids will cost some of energy. However, the cost of energy can be considered in our model which can be interpreted as a higher price which can cover the energy loss. Moreover Microgrids may use DC power line connection in the future, it will cost less energy than the traditional methods. So temporarily we ignore the energy loss caused by energy exchanging.

- All Microgrids in our system model are selfish and rational. It means they only want to maximize their own profit and minimize cost. They also want to fulfill customers’ demands as much as possible.

3 **Algorithm**

It is obvious that P1 is a time-coupling problem. And there are a lot of stochastic variables in this problem. So we reformulate this problem with Lyapunov optimization theory and convert P1 into P3.

### 3.1 Lyapunov optimization based Problem1

Firstly, we need to define the Lyapunov function as

\[ L_i(t) = \frac{1}{2}(Q_i^2(t) + X_i^2(t) + Z_i^2(t)) \]

The one-slot conditional Lyapunov drift can be defined as

\[ \triangle (L_i(t)) = \mathbb{E}\{L_i(t+1) - L_i(t)|\overrightarrow{K}_i(t)\} \] (25)

And devote a vector \( \overrightarrow{K}_i(t) = (Q_i(t), X_i(t), Z_i(t)) \)

At last we can formulate P3

\[ \text{P3} : \min \quad \triangle (L_i(t)) + V_i\mathbb{E}\{U_i(t)|\overrightarrow{K}_i(t)\} \]

subject to:

\[ (1)-(20) \]

Then we can have following Lemma.

**Lemma 1**: Given \( \triangle (L_i(t)) \) shown in (16) We can have:

\[ \triangle (L_i(t)) + V_i\mathbb{E}\{U_i(t)|\overrightarrow{K}_i(t)\} \]

\[ \leq A_i + \mathbb{E}\{X_i(t)(C_i(t) - D_i(t))|\overrightarrow{K}_i(t)\} \]

\[ + \mathbb{E}\{Q_i(t)(T_i(t) - J_i(t))|\overrightarrow{K}_i(t)\} \]

\[ + \mathbb{E}\{Z_i(t)(\epsilon_i(t) - J_i(t))|\overrightarrow{K}_i(t)\} \]

\[ + V_i\mathbb{E}\{U_i(t)|\overrightarrow{K}_i(t)\} \] (27)

where \( A_i \) is the constant i.e.,

\[ A_i = \frac{\varepsilon_i^2}{(e_i^{\text{max}})^2 + (j_i^{\text{max}})^2} + \frac{\max(C_i^{\text{max}})^2(D_i^{\text{max}})^2}{2} \]

**Proof**: At first, we can get following equation according (16)

\[ 1) \quad \frac{Q_i^2(t+1) - Q_i^2(t)}{2} \leq \frac{J_i^2(t) + T_i^2(t)}{2} + Q_i(t)(T_i(t) - J_i(t)) \]

\[ \leq \frac{(j_i^{\text{max}})^2 + (T_i^{\text{max}})^2}{2} + Q_i(t)(T_i(t) - J_i(t)) \]

\[ 2) \quad \frac{X_i^2(t+1) - X_i^2(t)}{2} \leq \frac{(C_i(t) - D_i(t))^2}{2} + X_i(t)(C_i(t) - D_i(t)) \]

\[ \leq \frac{\max(C_i^{\text{max}})^2(D_i^{\text{max}})^2}{2} + X_i(t)(C_i(t) - D_i(t)) \]

\[ 3) \quad \frac{Z_i^2(t+1) - Z_i^2(t)}{2} \leq \frac{\varepsilon_i^2(t) + J_i^2(t)}{2} + Z_i(t)(\epsilon_i(t) - J_i(t)) \]

\[ \leq \frac{\varepsilon_i^2}{(e_i^{\text{max}})^2 + (j_i^{\text{max}})^2} + Z_i(t)(\epsilon_i(t) - J_i(t)) \]

Next we can get:

\[ \triangle (L_i(t)) + V_i\mathbb{E}\{U_i(t)|\overrightarrow{K}_i(t)\} \]

\[ \leq \frac{\varepsilon_i^2}{(e_i^{\text{max}})^2 + (j_i^{\text{max}})^2} + \frac{\max(C_i^{\text{max}})^2(D_i^{\text{max}})^2}{2} \]

\[ + \mathbb{E}\{X_i(t)(C_i(t) - D_i(t))|\overrightarrow{K}_i(t)\} \]

\[ + \mathbb{E}\{Q_i(t)(T_i(t) - J_i(t))|\overrightarrow{K}_i(t)\} \]

\[ + \mathbb{E}\{Z_i(t)(\epsilon_i(t) - J_i(t))|\overrightarrow{K}_i(t)\} \]

Then the (27) directly follow. Because \( A_i + \mathbb{E}\{Z_i(t)\epsilon_i(t) + Q_i(t)T_i(t)\} \) cannot change when the slot t begin and we need to minimize the right-hand of (27) and .Then we reformulate the P3,we can get P4

\[ \text{P4} : \min \quad X_i(t)(C_i(t) - D_i(t)) \]

\[ - J_i(t)(Q_i(t) + Z_i(t)) \] (28)

\[ + V_iU_i(t) \]
Put (6), (7), (9)-(14) into P4 and consider about the feature of DI loads demands, we can get P5.

\[
\text{P5 : min } \quad (X_i(t) + V_i P(t)) G_i^C(t) \\
+ (X_i(t) + V_i \alpha_i(t)) R_i^C(t) \\
- (X_i(t) + Q_i(t) + Z_i(t)) D_i^J(t) \\
- (Q_i(t) + Z_i(t) - V_i P(t)) G_i^J(t) \\
- (Q_i(t) + Z_i(t) - V_i \beta_i(t)) X_i^J(t) \\
- (Q_i(t) + Z_i(t) - V_i \alpha_i(t) R_i^J(t) \\
\tag{29}
\]

subject to

\[(1)-(20)\]

3.2 Algorithm Properties

In this section, we summarize the properties of our algorithm as follows:

**Theorem 1.** All \( V_i \) in (26) should meet the constraints that \( 0 < V_i < V_i^{\max} \) for all \( t \in \{0, 1, 2, \cdots \} \)

\[
V_i^{\max} = \frac{B_i^{\max} - T_i^{\max} + \varepsilon_i^{\max}}{P_i^{\max} - P_i^{\min}} \tag{30}
\]

our Lyapunov optimization based Problem1 has the following properties:

1) Both \( Q_i(t) \) and \( Z_i(t) \) are upper bounded by \( Q_i^{\max} \) and \( Z_i^{\max} \) at all slot \( t \), where

\[
Q_i^{\max} = V_i P_i^{\max} + T_i^{\max} \tag{31}
\]

\[
Z_i^{\max} = V_i P_i^{\max} + \varepsilon_i^{\max} \tag{32}
\]

Further here we denote \( Q_i(t) + Z_i(t) \) upper bound as \( \Theta_i \)

\[
\Theta_i = V_i P_i^{\max} + T_i^{\max} + \varepsilon_i^{\max} \tag{33}
\]

2) we denote the worst-case delay as \( \delta_i^{\max} \), where

\[
\delta_i^{\max} = \frac{Q_i^{\max} + Z_i^{\max}}{\varepsilon_i} = \frac{2 V_i P_i^{\max} + T_i^{\max} + \varepsilon_i^{\max}}{\varepsilon_i} \tag{34}
\]

3) According the definition of virtual queue \( X_i(t) \)

\[
- \Theta_i - D_i^{\max} \leq X_i(t) \leq D_i^{\max} - \Theta_i - D_i^{\max} \tag{35}
\]

4) If \( \forall i, R_i(t), I_i(t) \) and \( T_i(t) \) are i.i.d, then time-average-cost is shown below

\[
\lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E} \{ P(t) G_i(t) + \sum_{t=1, t \neq i}^{N} \beta(t) \hat{y}_{il}(t) \\
- \sum_{t=1, t \neq i}^{N} \hat{a}(t) \hat{y}_{il}(t) \} \leq P_i^* + \frac{A_i}{V_i} \tag{36}
\]

**Proof.** We set \( Q_i(0), Z_i(0) \) and \( X_i(0) \) satisfying all these constraints, then we use induction to prove equation above.

1) If \( 0 < Q_i(t) \leq V_i P_i^{\max} \), then \( Q_i(t) \leq V_i P_i^{\max} + T_i^{\max} \).\( Q_i(t) \geq V_i P_i^{\max} \), because \( Z_i(t) \geq 0, \forall t \in \{0, 1, 2, \cdots \} \) and our target is to minimize the P5. We can find \( -(Q_i(t) + Z_i(t)) + V_i P_i^{\max} \leq 0 \). It means no matter how, increasing \( G_i^J(t) \) can still minimize P5. So we choose \( G_i^J(t) = G_i^{J, \max} \), then P5 can be minimized. According (8) and (16), \( Q_i(t+1) \leq V_i P_i^{\max} + T_i^{\max} - J_i^{\max} \leq V_i P_i^{\max} + T_i^{\max} \)

To sum up, \( Q_i^{\max} = V_i P_i^{\max} + T_i^{\max}, Z_i^{\max} \) and \( \Theta_i \) can be proven similarly.

2) Then we will prove the worst-case delay \( \delta_i^{\max} \) by contradiction

\[
Z_i(t+1) \geq Z_i(t) - J_i(t) + \varepsilon_i I_{Q_i(t)}>0 \\
\]

So \( Z_i(t_0 + 1 + \delta_i^{\max}) - Z_i(t_0) \geq - \sum_{t=t_0}^{t_0+\delta_i^{\max}} J_i(t) + \varepsilon_i I_{Q_i(t)}>0 \\
\]

Due to \( Z_i(t_0 + 1 + \delta_i^{\max}) \leq Z_i^{\max} \) and \( Z_i(t_0) > 0 \)

Rearrange the terms

\[
\sum_{t=t_0}^{t_0+\delta_i^{\max}} J_i \geq \varepsilon_i I_{Q_i^{\max}} - Z_i^{\max} \\
\]

We assume the DT load demands came in at slot \( t_0 \) and cannot be served at slot \( t_0 + 1 + \delta_i^{\max} \), So \( \sum_{t=t_0}^{t_0+\delta_i^{\max}} J_i \leq Q_i^{\max} \)

Then \( Q_i^{\max} \geq \varepsilon_i I_{Q_i^{\max}} - Z_i^{\max} \), rearrange the equation and use contradiction. We can get:

\[
\delta_i^{\max} = \frac{Q_i^{\max} + Z_i^{\max}}{\varepsilon_i} = \frac{2 V_i P_i^{\max} + T_i^{\max} + \varepsilon_i^{\max}}{\varepsilon_i} \\
\]

3) To be specify that \( \beta_i^{\min} \) as the minimum price that Microgrid \( i \) can get a unit of energy.

When \( t=0, X_i(0) = B_i(0) - \Theta_i - D_i^{\max} < B_i^{\max} - \Theta_i - D_i^{\max} \)

If \( B_i^{\max} - \Theta_i - D_i^{\max} \geq X_i(t) > 0 \), according to the P5, \( D_i^J(t) > 0, C_i(t) = 0 \). Then \( B_i^{\max} - \Theta_i - D_i^{\max} \geq X_i(t) > X_i(t+1) \)

If \( 0 \geq X_i(t) > -V_i P_i^{\min} \), then
Theorem 2: based on the cost-minimization problem. Double Auction Mechanism has two step:

3.3 Double Auction

3.3.1 Microgrids Decide Auction Parameters

The Microgrids will decide different cases

Proof. We prove the Theorem2 depending on different cases

Case 1. The buy-bid win, but sell-bid doesn’t. Then P4 transform into

3.3.2 Auctioneer Decide Auction Parameters

In Section 2.2, we illustrate P2. We show the double auction mechanism in this section. The auctioneer sorts all received buy-bids from Microgrids in descending order and sell-bids from Microgrids are sorted in ascending order in the sell prices. $\overline{\beta}(t) \geq \overline{\beta}_2(t) \geq \cdots \geq \overline{\beta}_n(t)$ and $\overline{\sigma}_1(t) \leq \overline{\sigma}_2(t) \leq \cdots \leq \overline{\sigma}_n(t)$

Next we reformulate the Social Welfare Maximization Problem(P2) based on new conditions.

subject to:
\( \rho_1, \rho_2 \) are set by auctioneer which can estimate \( \bar{x}_i(t) = \bar{y}_i(t) \) is close to \( \sqrt{\frac{\rho_1 \beta_i(t)}{\rho_2 \alpha_i(t)}} \) which can maximize \( P_5 \).

We assume \( \beta_{i^*}(t) \) and \( \alpha_{i^*}(t) \) as the solution to the \( P_5 \), then \( \beta_{i^*-1}(t) \) and \( \alpha_{i^*-1}(t) \) will be accepted by the auctioneer. Further more, auctioneer will announce the \( \text{cox}_{il} \) and \( \text{coy}_{il} \) as the coefficients which can ensure (40).

The result will announced by auctioneer:

The price charge to each buyer Microgrid \( i \) of per unit of energy is:

\[
\hat{\beta}_i(t) = \begin{cases} 
\beta_{i^*}(t) & \text{if } \beta_i(t) \text{ wins} \\
0 & \text{otherwise}
\end{cases} \tag{41}
\]

The price pay to each seller Microgrid \( i \) of per unit of energy is:

\[
\hat{\alpha}_i(t) = \begin{cases} 
\alpha_{i^*}(t) & \text{if } \alpha_i(t) \text{ wins} \\
0 & \text{otherwise}
\end{cases} \tag{42}
\]

The amount of energy Microgrid \( i \) bought from Microgrid \( l \)

\[
\hat{x}_{il}(t) = \begin{cases} 
\text{cox}_{il} \cdot x_{il}(t) & \text{if bid } \beta_i(t) \text{ win} \\
0 & \text{otherwise}
\end{cases} \tag{43}
\]

The amount of energy sell from Microgrid \( i \) bought to Microgrid \( l \)

\[
\hat{y}_{il}(t) = \begin{cases} 
\text{coy}_{il} \cdot y_{il}(t) & \text{if bid } \alpha_i(t) \text{ win} \\
0 & \text{otherwise}
\end{cases} \tag{44}
\]

**Theorem 3.** Using the mechanism presented above, all Microgrids will submit the sell-bids and buy-bids truthfully, or they will get lower profit by deviating from the true value of the buy and sell bids in (37) and (38)

**Proof.** This proof is similar to that in [15] Theorem 2, which is omitted here for brevity.

### 4 Numerical Results

In this section, we present numerical results based on the data from the real world to examine our analysis in the previous sections.

#### 4.1 Experimental Setup

We consider a a network of five Microgrids, namely MG1, MG2, MG3, MG4, MG5 and MG6 that have renewable energy generation, batteries, interconnections, delay-intolerant loads and delay-tolerant loads. There are two types Microgrids. First type of Microgrids(indexed by \( i=1,2,3 \), both DT and DI load arrive the Microgrids during one slot are i.i.d and take value from \([100,200]\) kWh uniformly at random. For the second type of Microgirds(indexed by \( i=4,5,6 \), both DT and DI load arrive the Microgrids during one slot are i.i.d and take value from \([300,600]\) kWh uniformly at random. For the renewable energy generation, we use hourly average wind speed data provided by the Alternative Energy Institute (AEI) [25]. Specifically, we choose the scaling factors such that the average wind-driven energy production during one slot is about 200 kWh for type1 Microgrids and 600 kWh for type2 Microgrids. Detailed data are shown in Fig.3(a). For the price of purchasing energy from Macrogrids, we use hourly price of energy provided by the Power Smart Pricing administered for Ameren Illinois data [26] are shown in Fig.3(b). The total length of data is 120 hours. The maximum battery capacity of both types Microgrids is 3MW.

In addition, the max charging and discharging rate of all Microgrids are 1.5MWh. Let \( V_i = \)
$V_{max}^i$ and $\varepsilon_i = T_{min}^i$. At last, both type1 and type2 Microgrids’ $\beta_{min}^i = 1$.

The auctioneer set $\rho_1 = 1000$ and $\rho_2 = 0.0001$ which can make sure the most of bid can accepted in the auction.

### 4.2 Results

We compare the cost of all Microgrids on each slot with our algorithm and without double auction mechanism. Fig.4 shows our algorithm can help Microgrids spending less money on the energy. Although some slots indicate the double auction mechanism will make Microgrids spend more, the total costs of all Microgrids are less than the situation without double auction and reduces about 5.7%. In addition, our algorithm can ensure less delay of DT loads demands.

Using double auction allow the Microgrids exchange their extra renewable energy and purchase less energy from Macrogrids. Fig.5 illustrates the energy purchased by all Microgrids from the Macrogrids. In the most of slots, Microgrids using our algorithm need less energy from Macrogrids and it outperforms the methods without double auction about 4.96%.

At last, if Microgrids can make some profits in the auction, they intend to store their extra energy into their batteries. When $B_{i, max} \to \infty$, Microgrids will seldom exchange extra energy. The double auction can motivate the Microgrids put their energy into the markets. In Fig.6, the total energy exchanged in the market will decrease when the Microgrids have a big battery, but Microgrids will not stop offering their energy. On the contrary, they still are willing to attend the auction and get energy with a reasonable price.

### 5 CONCLUSION

This paper investigate both time average cost minimization problem and auctioneer allocation problem. The rational and selfish Microgrids which are geographical diversity can attend the electricity market to minimize their costs. We solve the time average cost minimization problem using Lyapunov optimization theory. The algorithm require less calculation and can still figure out a promising solution. We design a double auction based mechanism which can maximize the social welfare and motivate the Microgrids take part in the auction. In addition, this auction can prevent Microgrids deviating from the true value of sell-bid and buy-bid. At last the mechanism only need one-shot double auction which can reduce the frequency of communication.

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