Which-way measurement and momentum kicks

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Abstract – The two-slit interference experiment with a which-way detector has been a topic of intense debate. The scientific community is divided on the question whether the particle receives a momentum kick because of the process of which-way measurement. It is shown here that the same experiment can be viewed in two different ways, depending on which basis of the which-way detector states one chooses to look at. In one view, the loss of interference arises due to the entanglement of the two paths of the particle with two orthogonal states of the which-way detector. In another view, the loss of interference can be interpreted as arising from random momentum kicks of magnitude $\hbar/2d$ received by the particle, $d$ being the slit separation. The same scenario is shown to hold for a three-slit interference experiment. The random momentum kicks for the three-slit case are of two kinds, of magnitude $\pm \hbar/3d$. The analysis is also generalized to the case of $n$-slit interference. The two alternate views are described by the same quantum state, and hence are completely equivalent. The concept of “local” vs. “nonlocal” kicks, much discussed in the literature, is not needed here.

Introduction. – The two-slit experiment with particles has become a cornerstone for the issue of wave-particle duality or the concept of complementarity introduced by Niels Bohr [1]. A debate was set in motion when Einstein proposed his famous recoiling-slit experiment, in an unsuccessful bid to refute Bohr’s complementarity principle [2]. This thought experiment has now been beautifully realized in different ways [3–5]. Bohr had countered Einstein by pointing out that measuring the momentum of the recoiling slit, in order to find which slit the particle went through, would produce an uncertainty in the position of the recoiling slit, which, in turn, would wash out the interference. Bohr’s specific resolution led many authors to surmise that complementarity was probably another way of stating the uncertainty principle, and that complementarity has its roots in the uncertainty principle. Scully, Englert and Walther proposed a which-way experiment using micromaser cavities which, they claimed, does not involve any position-momentum uncertainty [6]. Their conclusion was that the which-way detection process does not involve any momentum transfer to the interfering particle. Storey et al. countered this claim by proving that if an interference pattern is destroyed in a which-way experiment, a momentum of at least the magnitude $\hbar/d$ should be transferred to the particle, where $d$ is the separation between the two slits [7]. A momentum transfer of an amount smaller than that would not destroy the interference completely. This led to a shift in the focus of the debate to the question as to whether there is a momentum transfer to the particle involved in the process of which-way detection [8–12].

Later it was shown that the complementarity principle can be understood in terms of the ubiquitous entanglement between the particle and the which-way detector, and also equivalently in terms of the uncertainty between certain operators of the which-way detector, and the well-known wave-particle duality relation [13] can be derived from both [14,15]. However, the question as to whether there is a momentum transfer to the particle or not, still appears to be not settled [16,17]. Wiseman has tried to reconcile between the two views by introducing the concept of nonlocal momentum kicks [11], and has proposed that the momentum kicks could be probed through weak measurements [18]. In the following we show that the two views which say that there is, and there is not a momentum kick, are completely equivalent.

Two-slit which-way experiment. – Let $\psi(x)$ represent the state of the particle just as it emerges from the
double slit:
\[ \psi(x) = \frac{1}{\sqrt{2}} [\psi_1(x) + \psi_2(x)], \]
(1)

where \( \psi_1, \psi_2 \) are narrow states localized at \( x = 0 \) and \( x = d \), respectively. The states \( \psi_1, \psi_2 \) are orthogonal by virtue of their spatial separation. Let us now introduce a which-way detector at the double slit, as schematically shown in fig. 1. We need not assume any form of the which-way detector. According to von Neumann’s concept of measurement [19], if the which-way detector is to make a measurement on which of the two paths the particle followed, its two states should get entangled with the states of the two paths:
\[ \Psi(x) = \frac{1}{\sqrt{2}} [\psi_1(x)|d_1 \rangle + \psi_2(x)|d_2 \rangle], \]
(2)

where \( |d_1 \rangle, |d_2 \rangle \) are certain normalized states of the which-way detector. The particle travels a distance \( D \) to the screen in a time \( t \), and the state is given by
\[ \Psi(x, t) = \frac{1}{\sqrt{2}} [\psi_1(x, t)|d_1 \rangle + \psi_2(x, t)|d_2 \rangle], \]
(3)

where \( \psi_1(x, t), \psi_2(x, t) \) remain orthogonal. Now it is easy to see that when one calculates the probability density of the particle falling on the screen at a position \( x \), namely \( |\Psi(x, t)|^2 \) the cross-terms in the above, which represent interference, have a factor proportional to \( |\langle d_1 | d_2 \rangle| \):
\[ |\Psi(x, t)|^2 = \frac{1}{2} [ \langle \psi_1(t) \rangle^2 + \langle \psi_2(t) \rangle^2 + \psi_1^*(t)\psi_2^*(t)|d_1 \rangle\langle d_2 | + \psi_2^*(t)\psi_1^*(t)|d_2 \rangle\langle d_1 | ], \]
(4)

where we have suppressed the \( x \)-dependence of the states for brevity. If \( |d_1 \rangle, |d_2 \rangle \) are orthogonal, the last two terms in the above equation, which represent interference, drop off. It needs to be stressed that \( \psi_1(x), \psi_2(x) \) in (22) are the same as those in (1), and the which-way detection does not change the individual states of the particle emerging from the slits, hence there is no question of any additional momentum kick due to the which-way detection. This was the point of view of Scully, Englert and Walther [6].

However, the loss of interference described here may also be viewed in a slightly different fashion. If \( |d_1 \rangle, |d_2 \rangle \) are orthonormal, one can introduce another set of orthonormal states:
\[ |d_\pm \rangle = (|d_1 \rangle \pm |d_2 \rangle)/\sqrt{2}. \]

The state of the particle emerging from the double slit and the which-way detector being combined, eq. (22) can then be written as
\[ \Psi(x) = \frac{1}{2} [\psi_1(x) + \psi_2(x)]|d_+ \rangle \]
\[ + \frac{1}{2} [\psi_1(x) - \psi_2(x)]|d_- \rangle. \]
(5)

The state of the particle at the screen and the which-way detector being combined, eq. (3) can then be written as
\[ \Psi(x, t) = \frac{1}{2} [\psi_1(x, t) + \psi_2(x, t)]|d_+ \rangle \]
\[ + \frac{1}{2} [\psi_1(x, t) - \psi_2(x, t)]|d_- \rangle. \]
(6)

Although (6) shows no interference, if the particle is detected in coincidence with the which-way state \( |d_+ \rangle \), it shows an interference which is exactly the same as that shown by (1). Alternatively, if the particle is detected in coincidence with the which-way state \( |d_- \rangle \), it shows an interference which is slightly shifted. The two interferences may be represented as
\[ |\Psi_+ (x, t)|^2 = \frac{1}{4} [ \langle \psi_1(t) \rangle^2 + \langle \psi_2(t) \rangle^2 + \psi_1^*(t)\psi_2^*(t) + \psi_2^*(t)\psi_1^*(t) ], \]
\[ |\Psi_- (x, t)|^2 = \frac{1}{4} [ \langle \psi_1(t) \rangle^2 + \langle \psi_2(t) \rangle^2 - \psi_1^*(t)\psi_2^*(t) - \psi_2^*(t)\psi_1^*(t) ], \]
(7)

where \( \psi_\pm (x, t) \) is \( \langle d_\pm | \Psi(x, t) \rangle \). A temporary mixing of the Dirac notation may be excused here. The fact that coincident detection of the particle with \( |d_\pm \rangle \) states brings back interference is called quantum erasure [20,21]. Looking at (6), one can understand the loss of interference as arising due to the spinor \( |d_- \rangle \) flipping the relative phase between the two paths by \( \pi \). This has been recognized earlier [22,23].

Now we wish to point out that the phase flip in (6) can also be interpreted as a momentum kick. We write (5) as
\[ \Psi(x) = \frac{1}{2} [\psi_1(x) + \psi_2(x)]|d_+ \rangle \]
\[ + e^{i p_0 x/\hbar} \frac{1}{2} [\psi_1(x) + \psi_2(x)]|d_- \rangle, \]
(8)

where \( p_0 = h \pi/d \) is a momentum kick the particle receives whenever the which-way detector state is \( |d_- \rangle \). To see if
writing (6) as (8) is valid or not, we simplify (8) as

\[
\Psi(x) = \frac{1}{2} (\psi_1(x) + \psi_2(x))|d_+\rangle \\
+ \frac{i}{2} (\psi_1(x) + \psi_2(x))|d_-\rangle \\
= \frac{1}{2} (\psi_1(x) + \psi_2(x))|d_+\rangle \\
+ \frac{1}{2} (\psi_1(x, t) + e^{ip_0d/\hbar} \psi_2(x))|d_-\rangle \\
= \frac{1}{2} (\psi_1(x) + \psi_2(x))|d_+\rangle \\
+ \frac{i}{2} (\psi_1(x) - \psi_2(x))|d_-\rangle,
\]

where we have used the fact that \(\psi_1(x)\) is a narrow state localized at \(x = 0\) and \(\psi_2(x)\) is a narrow state localized at \(x = d\). This simple analysis shows that (8) is the same as (5), and the phase flip can also be seen as momentum kick of magnitude \(p_0 = h/2d\) which the particle receives randomly, fifty percent of the time, \(i.e.,\) whenever the which-way detector state is \(|d_-\rangle\).

Now (22) and (8) represent the same state. If one viewed the which-way detector in the basis described by \(|d_1\rangle, |d_2\rangle\), one would say that the two paths of the particle are correlated with two orthogonal states, and so there is no interference. Alternately if one viewed the which-way detector in the basis described by \(|d_+\rangle, |d_-\rangle\), one would say that the particles receive a momentum kick equal to \(p_0 = h/2d\), fifty percent of the time, and the two interference patterns corresponding to the particles which receive or do not receive a kick, are mutually shifted, washing out the result.

**Choice of basis.** One may wonder what happens if one uses another basis. The answer is that only one basis whose states are unbiased with respect to \(|d_1\rangle, |d_2\rangle\), will lead to the concept of momentum kicks. We demonstrate that in the following analysis. Suppose we choose another basis \(|\alpha\rangle, |\beta\rangle\) for the path detector, such that

\[
|d_1\rangle = \frac{1}{\sqrt{2}} (e^{i\theta_1}|\alpha\rangle + e^{i\theta_2}|\beta\rangle), \\
|d_2\rangle = \frac{1}{\sqrt{2}} (e^{i\theta_1}|\alpha\rangle + e^{i\theta_3}|\beta\rangle).
\]

The above represent the most general basis which is unbiased with respect to \(|d_1\rangle, |d_2\rangle\). Orthogonality of \(|d_1\rangle, |d_2\rangle\) demands that \(\theta_1 - \theta_2 = \theta_3 - \theta_4 + \pi\). The state of the particle at the screen and the which-way detector being combined, eq. (22) can then be written as

\[
\Psi(x) = \frac{1}{2} (e^{i\theta_1} \psi_1 + e^{i\theta_2} \psi_2)|\alpha\rangle \\
+ \frac{i}{2} (e^{i\theta_1} \psi_1 + e^{i\theta_3} \psi_2)|\beta\rangle, \\
= \frac{1}{2} (e^{i\theta_1} \psi_1 + e^{i\theta_3} \psi_2)|\alpha\rangle \\
+ \frac{i}{2} e^{ip_0d/\hbar} (e^{i\theta_1} \psi_1 + e^{i\theta_3} \psi_2)|\beta\rangle,
\]

where \(p_0 = h/2d\). Again, we have used the fact that \(\psi_1, \psi_2\) are narrow states localized at \(x = 0\) and \(x = d\), respectively. In this general case, too, the state of the particle, corresponding to the two path detector states, is the same except for the term \(e^{ip_0d/\hbar}\), when the path detector state is \(|\beta\rangle\). Thus, the magnitude of the momentum kick is the same whatever basis states one chooses. The phase factor \(e^{i(\theta_2 - \theta_1)}\) is unimportant because it does not affect the position of the conditional interference pattern corresponding to the path detector state \(|\beta\rangle\). If one assumes that the two slits are located at \(x = \pm d/2\), instead of being at \(x = 0, d\), one gets an additional phase factor of \(e^{i\pi/2}\). If one chooses to use a path detector basis which is not unbiased with respect to \(|d_1\rangle, |d_2\rangle\), the experiment cannot be interpreted in terms of momentum kicks.

**Three-slit which-way experiment.** – To convince the reader that the above view is not contrived, but very natural, we extend it to a three-slit interference experiment in the presence of a which-way detector (see fig. 2). The state of the particle plus the which-way detector, as it emerges from the triple-slit, can be written as

\[
\Psi(x) = \frac{1}{\sqrt{3}} (\psi_1(x)|d_1\rangle + \psi_2(x)|d_2\rangle + \psi_3(x)|d_3\rangle),
\]

where \(\psi_1, \psi_2, \psi_3\) are the states corresponding to the three paths of the particle and \(|d_1\rangle, |d_2\rangle, |d_3\rangle\) are the orthonormal states of the which-way detector corresponding to those paths. It is obvious that (12) will not show any interference because of the orthogonality of \(|d_1\rangle, |d_2\rangle, |d_3\rangle\).

Proceeding along the lines of the preceding discussion, one may consider a scheme of quantum erasure of three-slit interference [24] and define three mutually orthogonal states of the which-way detector,

\[
|d_α\rangle = \frac{1}{\sqrt{3}} (|d_1\rangle + |d_2\rangle + |d_3\rangle), \\
|d_β\rangle = \frac{1}{\sqrt{3}} (e^{-i2\pi/3}|d_1\rangle + |d_2\rangle + e^{i2\pi/3}|d_3\rangle), \\
|d_γ\rangle = \frac{1}{\sqrt{3}} (e^{i2\pi/3}|d_1\rangle + |d_2\rangle + e^{-i2\pi/3}|d_3\rangle).
\]
In terms of these states, (12) can be written as

\[
\Psi(x) = \frac{1}{3} \left[ |\psi_1(x) + \psi_2(x) + \psi_3(x)\rangle |d_{\alpha}\rangle 
+ \frac{1}{3} \left[ e^{-i2\pi/3} |\psi_2(x) + e^{i2\pi/3} \psi_3(x)\rangle |d_{\beta}\rangle 
+ \frac{1}{3} \left[ e^{i2\pi/3} |\psi_1(x) + e^{-i2\pi/3} \psi_3(x)\rangle |d_{\gamma}\rangle \right] \right].
\]

(14)

We assume that $|\psi_1(x), \psi_2(x), \psi_3(x)\rangle$ are narrow states localized at $x = -d, x = 0, x = d$, respectively. We claim that (14) can also be written in the following form:

\[
\Psi(x) = \frac{1}{3} \left[ |\psi_1(x) + \psi_2(x) + \psi_3(x)\rangle |d_{\alpha}\rangle 
+ \frac{1}{3} \left[ e^{ip_k x/\hbar} |\psi_1(x) + \psi_2(x) + \psi_3(x)\rangle |d_{\beta}\rangle 
+ \frac{1}{3} \left[ e^{-ip_k x/\hbar} |\psi_1(x) + \psi_2(x) + \psi_3(x)\rangle |d_{\gamma}\rangle \right] \right],
\]

(15)

where $p_k = 2\pi \hbar/3d$ is a momentum kick. It is straightforward to see that at $x = \pm d$, $e^{ip_k x/\hbar} = e^{\pm i2\pi/3}$, and (15) reduces to (14).

The state (15) implies that the particle passing through the triple slit, passes undisturbed one-third of the time (when the which-way detector state is $|d_{\alpha}\rangle$), experiences a momentum kick of magnitude $p_k = \hbar/3d$ one-third of the time (when the which-way detector state is $|d_{\beta}\rangle$), and experiences a momentum kick of magnitude $-\hbar/3d$ one-third of the time (when the which-way detector state is $|d_{\gamma}\rangle$). Thus, the loss of interference in a three-slit interference experiment, too, can be interpreted either as arising due to entanglement of the three paths with three orthogonal which-way states, or as arising due to the two kinds of momentum kicks the particle receives at random.

### $n$-slit which-way experiment.

To complete the picture, we look at a $n$-slit interference experiment with which-way detection, and enquire if the concept of momentum kicks is general enough to be applicable to this case. The state of the particle plus the which-way detector, as it emerges from the $n$-slit, can be written as

\[
\Psi(x) = \frac{1}{\sqrt{n}} \sum_{k=1}^{n} \psi_k(x) |d_k\rangle,
\]

(16)

where $\{|d_k\rangle\}$ are the $n$ mutually orthogonal states of the path detector and $|\psi_k\rangle$ is the state of the particle corresponding to it passing through the $k$-th slit. The probability density of the particle, at a position $x$ on the screen, is given by

\[
\Psi^* \Psi = \frac{1}{n} \sum_{k=1}^{n} |\psi_k|^2 + \frac{1}{n} \sum_{j,k} \psi_k^* \psi_k \langle d_j | d_k \rangle + \psi_k^* \psi_j \langle d_k | d_j \rangle.
\]

(17)

The interference, represented by the second summation, dies out because of the orthogonality of $\{|d_k\rangle\}$.

Although the issue of wave-particle duality in $n$-slit interference has recently been studied [25,26], a quantum eraser has not been theoretically formulated for $n$-slit interference. So, one has to look for alternate basis states of the path detector which are unbiased with respect to $\{|d_j\rangle\}$. Such a “Fourier basis” can be constructed using the $n$-th roots of unity. Let the basis states be given by $\{|\alpha_j\rangle\}$. They are related to $\{|d_j\rangle\}$ as follows:

\[
|d_1\rangle = \frac{1}{\sqrt{n}} (|\alpha_1\rangle + |\alpha_2\rangle + |\alpha_3\rangle + \cdots + |\alpha_n\rangle),
\]

\[
|d_2\rangle = \frac{1}{\sqrt{n}} (|\alpha_1\rangle + e^{\frac{i\pi}{n}} |\alpha_2\rangle + e^{\frac{2i\pi}{n}} |\alpha_3\rangle + \cdots + e^{\frac{(n-1)i\pi}{n}} |\alpha_n\rangle),
\]

\[
|d_3\rangle = \frac{1}{\sqrt{n}} (|\alpha_1\rangle + e^{\frac{2i\pi}{n}} |\alpha_2\rangle + e^{\frac{3i\pi}{n}} |\alpha_3\rangle + \cdots + e^{\frac{(n-1)i\pi}{n}} |\alpha_n\rangle).
\]

(18)

All the other states of $\{|d_j\rangle\}$ can be similarly represented. Elementary properties of $n$-th roots of unity ensure the orthonormality of states. The entangled state (16) can be represented in this new basis as

\[
\Psi(x) = \frac{1}{n} \left( \psi_1 + \psi_2 + \psi_3 + \cdots + \psi_n \right) |\alpha_1\rangle
+ \frac{1}{n} \left( \psi_1 + e^{\frac{i\pi}{n}} \psi_2 + e^{\frac{2i\pi}{n}} \psi_3 + e^{\frac{3i\pi}{n}} \psi_4 + \cdots + e^{\frac{(n-1)i\pi}{n}} \psi_n \right) |\alpha_2\rangle
+ \frac{1}{n} \left( \psi_1 + e^{\frac{2i\pi}{n}} \psi_2 + e^{\frac{3i\pi}{n}} \psi_3 + e^{\frac{4i\pi}{n}} \psi_4 + \cdots + e^{\frac{(n-1)i\pi}{n}} \psi_n \right) |\alpha_3\rangle
+ \cdots + \frac{1}{n} \left( \psi_1 + \psi_2 + \psi_3 + \cdots + \psi_n \right) |\alpha_n\rangle.
\]

(19)

If the particle were to be detected in coincidence with the results of measurement of an observable of which-way detector whose $n$ eigenstates are $\{|\alpha_j\rangle\}$, each subset would yield a $n$-slit interference. This would be a quantum eraser for $n$-slit interference. However, all the $n$ interference patterns, would be mutually shifted, and their sum would wash out the interference. Assuming that the slits are located at $x = 0, d, 2d, 3d, \ldots, (n-1)d$, it can be shown that the above state can be written as

\[
\Psi(x) = \frac{1}{n} \left( \psi_1 + \psi_2 + \psi_3 + \cdots + \psi_n \right) |\alpha_1\rangle
+ \frac{1}{n} e^{\frac{i\pi}{n}} \left( \psi_1 + \psi_2 + \psi_3 + \cdots + \psi_n \right) |\alpha_2\rangle
+ \frac{1}{n} e^{\frac{2i\pi}{n}} \left( \psi_1 + \psi_2 + \psi_3 + \cdots + \psi_n \right) |\alpha_3\rangle
+ \cdots
+ \frac{1}{n} e^{\frac{(n-1)i\pi}{n}} \left( \psi_1 + \psi_2 + \psi_3 + \cdots + \psi_n \right) |\alpha_n\rangle.
\]

(20)
where \( p_j = j\hbar/nd \). It should be asserted that in arriving at the above result, one has used the fact that a state \( \psi_j(x) \) is a narrow state, virtually within the confines of a single slit, localized at \( x = (j - 1)d \). If the particle were to be detected in coincidence with the states \( \{|\alpha_j\rangle\} \), the particle state corresponding to \( |\alpha_j\rangle \) will be \( \sum_{\alpha} |\psi_1 + \psi_2 + \psi_3 + \cdots + \psi_n\rangle \), which is the original state of the incoming particle, without any effect of the path detector. The particle state corresponding to another path detector state (say) \( |\alpha_j\rangle \) will be the original state of the incoming particle, but with an additional term \( e^{ip_jx/h} \), which can be interpreted as a momentum kick of magnitude \( p_j = j\hbar/nd \). So, the effect of introducing the which-way detection on the particle can be interpreted as the particle either receiving no momentum kick, or randomly receiving a kick of one of the \( n - 1 \) magnitudes, \( p_j = j\hbar/nd, j = 1, 2, \ldots, n - 1 \). The largest kick is of magnitude \( \hbar/2d \) (for even \( n \)) and \( \pm \frac{\hbar}{2}\sqrt{h/2d} \) (for odd \( n \)), and the smallest momentum kick (other than 0) is of magnitude \( \pm \hbar/nd \).

**Momentum space interference and position kicks.** Recently Ivanov et al. [27] proposed an interesting “double-slit” experiment in momentum space. The basic idea is to consider particles in superposition of two distinct momentum states. These distinct momentum states play the role of distinct positions of the two slits in a conventional two-slit experiment. They also proposed a which-way variant of their proposed experiment. Let us see if the ideas in the preceding discussion are applicable to this momentum space experiment. The state of a particle in such a state can be represented as

\[
\Psi(p) = \frac{1}{\sqrt{2}}[\psi_1(p) + \psi_2(p)],
\]

(21)

where \( \psi_1(p) \) and \( \psi_2(p) \) represent two states with distinct momenta \( p_1 \) and \( p_2 \), respectively. It was proposed that if the particle undergoes elastic scattering, it will lead to interference [27]. However, we recognize that the source of interference is the superposition state (21). If now one introduces a two-state which-way device, the combined state will look like the following:

\[
\Psi(p) = \frac{1}{\sqrt{2}}[\psi_1(p)|d_1\rangle + \psi_2(p)|d_2\rangle],
\]

(22)

where \( |d_1\rangle \) and \( |d_2\rangle \) are two orthogonal states of the which-way detector. The entanglement with the which-way detector will lead to loss of interference.

Let us now consider this state in the basis described by \( |d_+\rangle \) and \( |d_-\rangle \), introduced earlier. The state looks like the following:

\[
\Psi(p) = \frac{1}{2}[\psi_1(p) + \psi_2(p)]|d_+\rangle + \frac{1}{2}[\psi_1(p) - \psi_2(p)]|d_-\rangle.
\]

(23)

The two states of the particle, corresponding to \( |d_+\rangle \) and \( |d_-\rangle \), differ by a “phase flip”. Now an interesting observation is that this state can also be written as

\[
\Psi(p) = \frac{1}{2}[\psi_1(p) + \psi_2(p)]|d_+\rangle + e^{-ip_0x_0/\hbar}e^{ip_0x_0/\hbar}\frac{1}{2}[\psi_1(p) + \psi_2(p)]|d_-\rangle,
\]

(24)

where \( x_0 = \frac{\hbar}{2(p_2 - p_1)} \). Notice that the first exponential factor \( e^{-ip_0x_0/\hbar} \) is just a constant phase factor. The second exponential term \( e^{ip_0x_0/\hbar} \) is such that it will just shift the position of a momentum eigenfunction by an amount \( x_0 \). Thus, this factor can treated as a position kick the particle randomly receives, whenever the which-way detector state is \( |d_-\rangle \). Thus, the loss of interference in momentum space interference, due to the introduction of which-way detection, can be attributed to random position kicks of magnitude \( \frac{\hbar}{2(p_2 - p_1)} \). It is interesting analogous to the momentum kicks of magnitude \( \hbar/2d \) for position space interference.

**Conclusion.** In conclusion, we have looked at the controvertial issue of momentum kick that a particle might receive when passing through a double slit in an interference experiment involving which-way measurement. We have shown that there are two completely equivalent ways of looking at the experiment. These two ways correspond to two different basis sets of the which-way detector. On the one hand, the loss of interference is due to the entanglement of the two paths of the particle to the two orthogonal states of the which-way detector. On the other hand, the particle passing through the double slit randomly receives a momentum kick of magnitude \( \hbar/2d \), \( d \) being the slit separation. The particles which receive a momentum kick, and those which do not receive a kick separately form two mutually shifted interference patterns, which cancel each other when added. The same scenario holds for a three-slit interference pattern, except that the particle either receives no kick or receives one of the two kinds of momentum kicks, of magnitude \( \pm \hbar/3d \). The analysis has been generalized to the case of \( n \)-slit interference, and it has been shown that the loss of interference can be interpreted as the particle receiving either no kick or one of the \( n - 1 \) kinds of kicks. It is shown that the analyses presented here applies equally well to momentum space interference of Ivanov et al. [27], and the loss of interference in a which-way measurement can be interpreted in terms random position kicks.

In the analysis presented here, the momentum kick is experienced by the particle passing through the slits, as it interacts with the which-way detector, and it is not meaningful to ask if the momentum kick is received at the location of the two slits or between the two slits, a language introduced by Wiseman [18]. As the two different views are based on exactly the same quantum state, the two views are completely equivalent. However, it needs to be stressed that what is common to both views is the entanglement between the particle paths and the states of
the which-way detector. This entanglement is at the root of complementarity as, according to von Neumann’s first process of any quantum measurement [19], any detector trying to determine which path the particle followed, will necessarily get entangled with the particle paths [14]. We believe this analysis should resolve any controversy regarding the momentum kicks in which-way experiments.

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