Some Improved Correlation Coefficients for q-Rung Orthopair Fuzzy Sets and Their Applications in Cluster Analysis

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The structure of q-rung orthopair fuzzy sets (q-ROFSs) is a generalization of fuzzy sets (FSs), intuitionistic FSs (IFSs), and Pythagorean FSs (PFSs). The notion of q-ROFSs has the proficiency of coping with uncertainty without any restrictions. In addition, the structure of q-ROFSs can effectively cope with the situations involving dual opinions without any restrictions, instead of dealing with only single opinion or dual opinions under certain restrictions. In clustering problems, the correlation coefficients are worthwhile because they provide the degree of similarity or correlation between two elements or sets. The theme of this study is to formulate the correlation coefficients for q-ROFSs that are basically the generalization of correlation coefficients of IFSs and PFSs. Moreover, an application of these correlation coefficients to a clustering problem is proposed. Also, an analysis of the outcomes is carried out. Furthermore, a comparison is carried out among the correlation coefficients for q-ROFSs and the existing ones. Finally, the downsides of the existing works and benefits of the correlation coefficients for q-ROFSs are discussed.

1. Introduction

Zadeh [1] initiated the notion of fuzzy set theory and logic in 1965. The fuzzy set (FS) is characterized by a function known as membership grade that attains values from a unit interval. This innovative theory was a nice tool for handling the uncertainties in practical life. Adlassing [2] applied the FS theory in medical diagnosis, Bezdek and Douglas Harris [3] defined the fuzzy partitions and relations, and Kandel [4] proposed a fuzzy technique in pattern recognition. The downside of FSs is that they do not describe the non-membership grade, in spite of the fact that the non-membership grade can be acquired in the fuzzy environment by subtracting the membership grade from 1. Henceforth, Atanassov [5] defined the intuitionistic FS (IFS) which describes both the membership and nonmembership grades independently. The addition of IFS contributed greatly in the FS theory. Chaira [6] proposed a novel concept of the IF C means clustering algorithm and applied it to medical images, Dengfeng and Chuntian [7] discussed the similarity measures of IFSs and applied them in pattern recognitions, and Hung and Yang [8] also worked on the similarity measures of IFSs based on Hausdorff distance. Eventhough IFSs discuss both the membership and nonmembership grades, still they have restrictions in the structure that the sum of both the grades must not exceed 1. To overcome this barrier, Yager [9] developed the Pythagorean FS (PFS) which relaxes the restrictions on the membership and nonmembership grades by keeping the sum of the squares of both the grades within the unit interval. Li and Zeng [10] formulated the
distance measures of PFSs, Li and Lu [11] offered the similarity and distance measures of PFSs and their applications, and Ejegwa and Awolola [12] applied the distance measures for PFSs to pattern recognition problems. Yet again, the structure of PFSs has certain limitations that affected the decision-making abilities of the professionals. So Yager [13] gave the concept of the q-rung orthopair fuzzy set (q-ROFS) that not only discusses the membership and nonmembership grades but also provides the largest possible domain for better decision-makings. A comparison among the domains of IFSs, PFSs, and q-ROFSs is portrayed in Figures 1–4. Liu et al. [14] devised the multiple-attribute decision-making based on q-ROF power Maclaurin symmetric mean operators, Liu and Wang [15] defined some q-ROF aggregation operators and applied them in multiple-attribute decision-making, and Wang et al. [16] discussed the similarity measures of q-ROFSs with their applications.

The notion of correlation coefficient is often used in the statistical problem. Bonizzoni et al. [17] worked on the correlation clustering and consensus clustering, Cheung and Li [18] proposed a quantitative correlation coefficient method for business intelligence, and Kumar et al. [19] conceived the method for ranking of L-R type generalized fuzzy numbers. Actually, the intention of correlation coefficient is to determine the strength of correlation or similarity between two objects or sets. These correlation coefficients have momentous use and applications in the theories of FSs, IFSs, and PFSs. Yang and Lin [20] proposed the similarity and inclusion measures for type-2 FSs and used these measures in the clustering, Chen et al. [21] defined the correlation coefficients of hesitant FSs and used these notions for analysis of clustering, Xu et al. [22] presented the clustering algorithm for IFSs, Hox et al. [23] discussed the techniques and applications of multilevel analysis, Garg [24] came up with a new correlation coefficient among PFSs and applied them in decision-making, Park et al. [25] devised the correlation coefficient of interval-valued IFSs and illustrated their application by using them in the problems of multiple-attribute group decision-making, Nguyen [26] concocted the similarity or dissimilarity measure for IFSs with its applications in pattern recognition, and Du [27] developed the correlation and correlation coefficients of q-ROFSs. Garg and Kumar [28, 29] studied the similarity measures of IFS and thought up of the aggregation operators for linguistic IFS with their applications in decision-making processes. In 2017, Garg [30] proposed a new method for IF decision-making founded on the improved operation laws with applications. Singh and Garg [31] gave distance measures for type-2 IFSs with their application to multicriterion decision-making. Garg [32] formulated the distance and similarity measures for intuitionistic multiplicative preference relation and its applications, and Jamkhan and Garg [33] perceived some new operations over the generalized IFSs and applied them in the decision-making process.

The progression in the theory as well as the applications of correlation coefficients to practical problems drove us to study these notions. Hereafter, this study presents the correlation coefficients for q-ROFSs and their clustering algorithm. Unlike fuzzy sets, the q-ROFSs involve both the membership grade and nonmembership grade. The IFSs and PFSs also talk about the membership and nonmembership
2. Preliminaries

This section defines some of the fundamental concepts such as FS, IFS, PFS, q-ROFS, information energy, correlation, and the correlation coefficients for IFSs.

Definition 1 (see [1]). For a nonempty set \( U \), a fuzzy set (FS) is defined as \( F = \{x, m(x) : x \in U\} \), where the membership grade \( m(x) \) maps each \( x \in U \) into \([0, 1]\). Additionally, \( 1 - m(x) \) represents the uncertainty of \( x \in U \) and \( m \) is known as the fuzzy number (FN).

Definition 2 (see [5]). For a nonempty set \( U \), an intuitionistic FS (IFS) is defined as \( F = \{x, m(x), n(x) : x \in U \land 0 \leq m(x) + n(x) \leq 1\} \), where the membership grade \( m(x) \) and the nonmembership grade \( n(x) \) map each \( x \in U \) into \([0, 1]\). Additionally, \( 1 - (m(x) + n(x)) \) represents the uncertainty of \( x \in U \) and \( (m, n) \) is known as the intuitionistic FN (IFN).

Definition 3 (see [9]). For a nonempty set \( U \), a Pythagorean FS (PFS) is defined as \( F = \{x, m(x), n(x) : x \in U \land 0 \leq (m(x))^2 + (n(x))^2 \leq 1\} \), where the membership grade \( m(x) \) and the nonmembership grade \( n(x) \) map each \( x \in U \) into \([0, 1]\). Additionally, \( \sqrt{1 - ((m(x))^2 + (n(x))^2)} \) represents the uncertainty of \( x \in U \) and \( (m, n) \) is known as the Pythagorean FN (PFN).

Definition 4 (see [13]). For a nonempty set \( U \), a q-rung orthopair FS (q-ROFS) is defined as \( F = \{x, m(x), n(x) : x \in U \land 0 \leq (m(x))^q + (n(x))^q \leq 1\} \), where \( q \) is a non-negative integer, and the membership grade \( m(x) \) and the nonmembership grade \( n(x) \) map each \( x \in U \) into \([0, 1]\). Additionally, \( \sqrt[1 - ((m(x))^q + (n(x))^q)} \) represents the uncertainty of \( x \in U \) and \( (m, n) \) is known as the q-rung orthopair FN (q-ROFN).

In the light of above definitions, a summary of generalizations of FSs is given in Figure 5.

Definition 5 (see [22]). For an IFS \( F \) on \( U \), the information energy \( E_{IFS} \) is given by

\[
E_{IFS}(F) = \sum_{i=1}^{n} [(m_F(x_i))^2 + (n_F(x_i))^2].
\]  (1)

Definition 6 (see [22]). The correlation \( C_{IFS} \) of two IFSs \( F \) and \( G \) is given by

\[
C_{IFS}(F, G) = \sum_{i=1}^{n} [m_F(x_i) \cdot m_G(x_i) + n_F(x_i) \cdot n_G(x_i)].
\]  (2)
Definition 7 (see [22]). The correlation coefficient \( \mathcal{K}_{\text{IFS}} \) of two IFSs \( F \) and \( G \) is given by

\[
\mathcal{K}_{\text{IFS}}(F, G) = \frac{C_{\text{IFS}}(F, G)}{\sqrt{E_{\text{IFS}}(F) \cdot E_{\text{IFS}}(G)}}
\]

The notions of information energy, correlation, and correlation coefficients in Definitions 5–7 are suitable for intuitionistic fuzzy environment [22]. But these notions flop when the information is of q-rung orthopair fuzzy type. Hence, in the following section, the generalization of existing correlation coefficients is presented.

3. Correlation Coefficient for q-Rung Orthopair Fuzzy Sets

This section generalizes the correlation coefficients of IFSs and PFSs in order to formulate the correlation coefficients for q-ROFSs.

Definition 8 (see [27]). For a q-ROFS \( F \) on \( U \), the information energy \( E_{\text{qROFS}} \) is given by

\[
E_{\text{qROFS}}(F) = \sum_{i=1}^{n} \left[ (m_F(x_i))^q + (n_F(x_i))^q \right] \cdot \sum_{i=1}^{n} \left[ (m_G(x_i))^q + (n_G(x_i))^q \right]
\]

Remark 1. For \( q = 2 \), equation (4) gives the information energy of a PFS.

Definition 9 (see [27]). The correlation \( C_{\text{qROFS}} \) of two q-ROFSs \( F \) and \( G \) is given by

\[
C_{\text{qROFS}}(F, G) = \sum_{i=1}^{n} \left[ (m_F(x_i))^q \cdot (m_G(x_i))^q \right] + \left[ (n_F(x_i))^q \cdot (n_G(x_i))^q \right]
\]

Definition 10 (see [27]). The correlation coefficient \( \mathcal{K}_{\text{qROFS}} \) of two q-ROFSs \( F \) and \( G \) is given by

\[
\mathcal{K}_{\text{qROFS}}(F, G) = \frac{C_{\text{qROFS}}(F, G)}{\sqrt{E_{\text{qROFS}}(F) \cdot E_{\text{qROFS}}(G)}}
\]
Remark 2. For \( q = 2 \), equation (7) gives the correlation coefficient of a PFS.

The correlation coefficients of PFSs and q-ROFSs are established through the membership grade and nonmembership grade.

Theorem 1. A \( \mathcal{K}_{q\text{ROFS}}(F, G) \) fulfils the following:

(i) \( \mathcal{K}_{q\text{ROFS}}(F, G) = \mathcal{K}_{q\text{ROFS}}(G, F) \)

(ii) \( 0 \leq \mathcal{K}_{q\text{ROFS}}(F, G) \leq 1 \)

(iii) The Cauchy–Schwarz inequality states that for \( (x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n) \in \mathbb{R}^n \),

\[
\sqrt{(x_1y_1 + x_2y_2 + x_3y_3 + \cdots + x_ny_n)} \leq \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2} \cdot \sqrt{y_1^2 + y_2^2 + \cdots + y_n^2}.
\]

(iv) Applying Cauchy–Schwarz inequality to \( \mathcal{K}_{q\text{ROFS}}(F, G) \) implies

\[
(C_{q\text{ROFS}}(F, G))^2 \leq \left( \sum_{i=1}^{n} \left( (m_F(x_i))^q + (n_F(x_i))^q \right) \cdot (m_G(x_i))^q \right) \times \left( \sum_{i=1}^{n} \left( (m_G(x_i))^q + (n_G(x_i))^q \right) \cdot (m_F(x_i))^q \right)
\]

\[
= \mathcal{E}_{q\text{ROFS}}(F) \cdot \mathcal{E}_{q\text{ROFS}}(G).
\]

(v) Therefore, \( (C_{q\text{ROFS}}(F, G))^2 \leq \mathcal{E}_{q\text{ROFS}}(F) \cdot \mathcal{E}_{q\text{ROFS}}(G) \). Thus, \( 0 \leq \mathcal{K}_{q\text{ROFS}}(F, G) \leq 1 \).

(vi) Since, \( F = G \cdot (m_F(x_i))^q = (m_G(x_i))^q \) and \( (n_F(x_i))^q = (n_G(x_i))^q \), where \( x_i \in U \). Therefore, \( \mathcal{K}_{q\text{ROFS}}(F, G) = 1 \).

Remark 3. For \( q = 2 \), the above theorem reduces to the correlation coefficient of PFS.

Example 1. Consider two q-ROFSs \( F \) and \( G \) on \( U = \{x_1, x_2, x_3\} \) with \( q = 7 \), such that

\[
F = \{ (x_1, 0.7, 0.9), (x_2, 0.7, 0.4), (x_3, 0.5, 0.5) \},
\]

\[
G = \{ (x_1, 0.7, 0.5), (x_2, 0.9, 0.9), (x_3, 0.4, 0.5) \}.
\]

The information energies of \( F \) and \( G \) are calculated as

\[
\mathcal{E}_{q\text{ROFS}}(F) = 0.24,
\]

\[
\mathcal{E}_{q\text{ROFS}}(G) = 0.46.
\]

The correlations of \( F \) and \( G \) are calculated as

\[
\mathcal{K}_{q\text{ROFS}}(F, G) = 0.05.
\]
\[ \mathcal{C}_{q\text{-ROFS}}(F, G) = 0.15. \] (14)

Another definition of correlation coefficient for \( q \)-ROFSs is stated.

\[
\mathcal{C}'_{q\text{-ROFS}}(F, G) = \frac{C_{q\text{-ROFS}}(F, G)}{\max\{C_{q\text{-ROFS}}(F, F), C_{q\text{-ROFS}}(G, G)\}}
\]
\[
= \frac{\sum_{i=1}^{n}\left[ (m_F(x_i))^q \cdot (m_G(x_i))^q + (n_F(x_i))^q \cdot (n_G(x_i))^q \right]}{\max\left\{ \sum_{i=1}^{n}(m_F(x_i))^q + (n_F(x_i))^q, \sum_{i=1}^{n}(m_G(x_i))^q + (n_G(x_i))^q \right\}}.
\] (15)

**Theorem 2.** A \( \mathcal{C}'_{q\text{-ROFS}}(F, G) \) fulfills the following:

(i) \( \mathcal{C}'_{q\text{-ROFS}}(F, G) = \mathcal{C}'_{q\text{-ROFS}}(G, F) \)
(ii) \( 0 \leq \mathcal{C}'_{q\text{-ROFS}}(F, G) \leq 1 \)
(iii) \( \mathcal{C}'_{q\text{-ROFS}}(F, G) = 1 \iff F = G \)

**Proof**

(i) The proof is straight forward
(ii) Clearly, \( 0 \leq \mathcal{C}'_{q\text{-ROFS}}(F, G) \). Theorem 1 implies the following:
\[
C_{q\text{-ROFS}}(F, G) \leq \sqrt{C_{q\text{-ROFS}}(F, F) \cdot C_{q\text{-ROFS}}(G, G)}.
\] (16)
(iii) Hence,
\[
\mathcal{C}_{q\text{-ROFS}}(F, G) \leq \max\{C_{q\text{-ROFS}}(F, F), C_{q\text{-ROFS}}(G, G)\}.
\] (17)

**Definition 12.** The weighted correlation coefficient \( \mathcal{W} \mathcal{C}_{q\text{-ROFS}} \) of two \( q \)-ROFSs \( F \) and \( G \) is given by
\[
\mathcal{W} \mathcal{C}_{q\text{-ROFS}}(F, G) = \frac{\sum_{i=1}^{n}(w_i)(m_F(x_i))^q \cdot (m_G(x_i))^q + (n_F(x_i))^q \cdot (n_G(x_i))^q}{\sqrt{\sum_{i=1}^{n}(w_i)[(m_F(x_i))^q + (n_F(x_i))^q] \cdot \sum_{i=1}^{n}(w_i)[(m_G(x_i))^q + (n_G(x_i))^q]}}.
\] (18)

**Definition 13.** The weighted correlation coefficient \( \mathcal{W}' \mathcal{C}_{q\text{-ROFS}} \) of two \( q \)-ROFSs \( F \) and \( G \) is given by
\[
\mathcal{W}' \mathcal{C}_{q\text{-ROFS}}(F, G) = \frac{\sum_{i=1}^{n}(w_i)(m_F(x_i))^q \cdot (m_G(x_i))^q + (n_F(x_i))^q \cdot (n_G(x_i))^q}{\max\{\sum_{i=1}^{n}(w_i)[(m_F(x_i))^q + (n_F(x_i))^q], \sum_{i=1}^{n}(w_i)[(m_G(x_i))^q + (n_G(x_i))^q]\}}.
\] (19)

**Remark 4.** For \( q = 2 \), equations (18) and (19) give the weighted correlation coefficient of a PFS.

**Remark 5.** For \( w = ((1/n), (1/n), (1/n), \ldots, (1/n))^T \), the weighted correlation coefficient reduces to correlation coefficient, i.e., \( \mathcal{W} \mathcal{C}_{q\text{-ROFS}}(F, G) = \mathcal{C}_{q\text{-ROFS}}(F, G) \) and \( \mathcal{W}' \mathcal{C}_{q\text{-ROFS}}(F, G) = \mathcal{C}'_{q\text{-ROFS}}(F, G) \).

**Theorem 3.** A \( \mathcal{W} \mathcal{C}_{q\text{-ROFS}}(F, G) \) fulfills the following:

(i) \( \mathcal{C}_{q\text{-ROFS}}(F, G) = \mathcal{C}_{q\text{-ROFS}}(G, F) \)
(ii) \( 0 \leq \mathcal{C}_{q\text{-ROFS}}(F, G) \leq 1 \)
(iii) \( \mathcal{C}_{q\text{-ROFS}}(F, G) = 1 \iff F = G \)

**Proof:** The proofs are straight forward.

**Theorem 4.** \( \mathcal{W}' \mathcal{C}_{q\text{-ROFS}}(F, G) \) fulfills the following:

(i) \( \mathcal{C}'_{q\text{-ROFS}}(F, G) = \mathcal{C}'_{q\text{-ROFS}}(G, F) \)
(ii) \( 0 \leq \mathcal{C}'_{q\text{-ROFS}}(F, G) \leq 1 \)
(iii) \( \mathcal{C}'_{q\text{-ROFS}}(F, G) = 1 \iff F = G \)
4. Clustering Algorithm for q-Rung Orthopair Fuzzy Numbers

This section extends the clustering algorithms proposed for IFS in [22] to the environment of q-ROFSs. Additionally, a solution to a clustering problem involving the q-rung orthopair fuzzy information is given.

**Definition 14.** For a set of q-ROFNs \( F_i \), define a matrix of correlation coefficients \( M = (\mathcal{K}_{ij})_{m \times m} \), where \( \mathcal{K}_{ij} = K(F_i, F_j) \) is a correlation coefficient among \( (F_i, F_j) \), such that

(i) \( 0 \leq \mathcal{K}(F_i, F_j) \leq 1 \)
(ii) \( \mathcal{K}(F_i, F_j) = 1 \)
(iii) \( \mathcal{K}(F_i, F_j) = \mathcal{K}(F_j, F_i) \)

**Theorem 5.** If \( M_1 = M \circ M = (\mathcal{K}_{ij})_{m \times m} \) where \( M = (\mathcal{K}_{ij})_{m \times m} \) is a matrix of correlation coefficients, then such \( M_1 \) is known as composite matrix that is symbolized by \( (\mathcal{K}_{ij})_{m \times m} \) and defined as

\[
(\mathcal{K}_{ij})_{m \times m} = \max_k \{ \min(\mathcal{K}_{ik}, \mathcal{K}_{kj}) \}. \tag{20}
\]

**Definition 15.** If \( M^2 = M \circ M = (\mathcal{K}_{ij})_{m \times m} \) where \( M = (\mathcal{K}_{ij})_{m \times m} \) is a matrix of correlation coefficients, then such \( M^2 \) is known as composite matrix that is symbolized by \( (\mathcal{K}_{ij})_{m \times m} \) and defined as

\[
(\mathcal{K}_{ij})_{m \times m} = \max_k \{ \min(\mathcal{K}_{ik}, \mathcal{K}_{kj}) \}. \tag{21}
\]

**Theorem 6.** For a correlation matrix \( M = (\mathcal{K}_{ij})_{m \times m} \) \( \forall h \in \mathbb{Z} \), that the finite repeated compositions of \( M \), i.e., \( M^h \), is also a correlation matrix, i.e., \( M^h_1 \circ M^h_2 = M^h \circ M^h_2 \) is a correlation matrix.

**Definition 17.** The \( \alpha \)-cutting matrix of a matrix of correlation coefficients \( M = (\mathcal{K}_{ij})_{m \times m} \) is symbolized and defined as

\[
M_\alpha = (\alpha \mathcal{K}_{ij})_{m \times m}, \tag{22}
\]

where \( 0 \leq \alpha \leq 1 \) and \( \alpha \mathcal{K}(F_i, F_j) = \begin{cases} 0, & \text{if } \mathcal{K}_{ij} \leq \alpha, \\ 1, & \text{if } \mathcal{K}_{ij} \geq \alpha. \end{cases} \)

The comprehensive clustering algorithm for q-ROFNs is explained as follows (Algorithm 1)

**Algorithm 1:**

Step 1. The initial step is to construct a matrix of correlation coefficients, i.e., \( M = (\mathcal{K}_{ij})_{m \times m} \) for the set of q-ROFNs \( F_i \).

Step 2. If the matrix of correlation \( M \) is not an equivalent matrix, then an equivalent matrix \( M^{2h} \) is constructed through the finite repeated compositions till \( M^{2h} = M^{2(h+1)} \).

Step 3. In the final step, the classification of q-ROFNs is carried out by forming the \( \alpha \)-cutting matrix. The following principle is used to classify each q-ROFN:

“The q-ROFNs are said to be of the same type if every element of \( i \)-th line and the corresponding element of \( j \)-th line belonging to \( M_i \) are same.”

A visual representation of the algorithm is portrayed in Figure 6 through a flowchart.

5. Illustrated Example

This section demonstrates the application of the correlation coefficients defined for q-rung orthopair fuzzy information through a numerical example.

**Example 2.** Consider a situation in which an automobile company wants to classify their vehicles on the basis of some features. This example is set to classify four vehicles on the basis of three features. Suppose that the set \( F = \{ f_1, f_2, f_3, f_4 \} \) be the representative of the collection of four vehicles and the set \( G = \{ g_1, g_2, g_3 \} \) be the representative of the collection of three features, such that \( g_1, g_2, \) and \( g_3 \) symbolize the fuel efficiency, the safety, and the selling cost of the vehicle, respectively. Assume that the weighted vector is \( w = [0.5, 0.3, 0.2]^T \). Furthermore, the assessments of the professionals on each vehicle are listed in Table 1. Each value provided in the table is absolutely q-ROFN for \( q = 3 \).

By following the steps for the clustering discussed in the previous section, the stepwise calculations are carried out.

Step 1. The matrix of correlation coefficients is constructed by computing the correlation coefficients from Table 1:

\[
M = \begin{pmatrix}
1 & 0.5284 & 0.8062 & 0.5423 \\
0.5284 & 1 & 0.4465 & 0.8373 \\
0.8062 & 0.4465 & 1 & 0.4375 \\
0.5423 & 0.8373 & 0.4375 & 1
\end{pmatrix}.
\]

Step 2. The equivalent correlation matrix is constructed by finite repeated compositions.
Since \( M^8 = M^4 \), therefore, \( M^4 \) is an equivalent correlation matrix.

Step 3. The classifications are worked out by forming the \( \alpha \)-cutting matrix as

1. If \( 0 \leq \alpha \leq 0.5384 \), then all of \( f_1, f_2, f_3, \) and \( f_4 \) are of the same type, \( \{ f_1, f_2, f_3, f_4 \} \).
2. If \( 0.5384 \leq \alpha \leq 0.8062 \), then the vehicles are classified into two types as \( \{ f_2 \} \) and \( \{ f_1, f_3, f_4 \} \).
3. If \( 0.8062 \leq \alpha \leq 0.8373 \), then the vehicles are classified into three types as \( \{ f_1, f_3 \}, \{ f_2 \}, \) and \( \{ f_4 \} \).
4. If \( 0.8373 \leq \alpha \leq 1 \), then the vehicles are classified into four types as \( \{ f_1 \}, \{ f_2 \}, \{ f_3 \}, \) and \( \{ f_4 \} \).

Clearly, the outcomes achieved specify the efficiency of correlation of q-ROFSs, since every vehicle is classified into a different type, which is infrequent in clustering analysis.

### 6. Comparative Analysis

This section studies the comparison among applications of the proposed correlation coefficients in the environment of IFSs, PFSs, and q-ROFSs. The data provided in the Example 2 cannot be modeled by using the correlations of IFSs and PFSs.

**Remark 7.** For \( q = 1 \), equation (7) gives the correlation coefficient for an IFS.

**Remark 6.** Verify the generalization of correlation of q-ROFSs. The following example is presented in order to compare the proposed correlation coefficients with the existing ones.

**Example 3.** This example solves the problem discussed in the previous example in the environment of intuitionistic fuzzy information. Table 2 contains the similar information to Table 1. Step by step calculation is carried out. Moreover, for \( q = 1 \), an IFS is a special case of q-ROFS. Therefore, in this example, we consider \( q = 1 \) in the solution.

By following the steps for the clustering, the stepwise calculations are carried out.
Table 1: q-rung orthopair fuzzy information.

|     | $g_1$          | $g_2$          | $g_3$          |
|-----|----------------|----------------|----------------|
| $f_1$| (0.67, 0.46)   | (0.93, 0.46)   | (0.46, 0.71)   |
| $f_2$| (0.74, 0.50)   | (0.10, 0.46)   | (0.67, 0.74)   |
| $f_3$| (0.59, 0.46)   | (0.90, 0.10)   | (0.89, 0.00)   |
| $f_4$| (0.90, 0.20)   | (0.50, 0.40)   | (0.60, 0.40)   |

(i) Step 1. The matrix of correlation coefficients is constructed by computing the correlation coefficients from Table 2:

$$M = \begin{pmatrix} 1 & 0.8536 & 0.9244 & 0.9079 \\ 0.8536 & 1 & 0.7316 & 0.7521 \\ 0.9244 & 0.7316 & 1 & 0.8463 \\ 0.9079 & 0.7521 & 0.5463 & 1 \end{pmatrix}.$$  \( (24) \)

(ii) Step 2. The equivalent correlation matrix is constructed by finite repeated compositions.

$$M^2 = MMM = \begin{pmatrix} 1 & 0.8536 & 0.9244 & 0.8042 \\ 0.8536 & 1 & 0.9244 & 0.9079 \\ 0.9244 & 0.9244 & 1 & 0.9079 \\ 0.8042 & 0.9079 & 0.9079 & 1 \end{pmatrix}.$$  \( (25) \)

(iii) Since $M^8 = M^4$, therefore, $M^4$ is an equivalent correlation matrix.

(iv) Step 3. The classifications are worked out by forming the $\alpha$-cutting matrix as

(5) If $0 \leq \alpha \leq 0.7521$, then all of $f_1, f_2, f_3$, and $f_4$ are of the same type, i.e., $\{f_1, f_2, f_3, f_4\}

(6) If $0.7521 \leq \alpha \leq 0.8536$, then the vehicles are classified into two types as $\{f_2\}$ and $\{f_1, f_3, f_4\}

(7) If $0.8536 \leq \alpha \leq 0.9244$, then the vehicles are classified into three types as $\{f_1, f_3\}, \{f_3\}$, and $\{f_4\}

(8) If $0.9244 \leq \alpha \leq 1$, then the vehicles are classified into four types as $\{f_1\}, \{f_2\}, \{f_3\}$, and $\{f_4\}$

The outcomes achieved specify that the correlation of q-ROFSs generalizes the correlation of IFSs. Because these outcomes achieved by substituting $q=1$ in the proposed method are identical to the outcomes through the correlation of IFS. Thus, the proposed correlation coefficient for q-ROFSs can be applied to the problems of an intuitionistic fuzzy nature. These examples verify the superiority and dominance of the correlations of q-ROFSs.

7. Downsides of Current Structures and Benefits of the Proposed Methods

This section discusses the shortcomings of the current structures, and the benefits of the proposed methods over the existing ones are also talked over.

7.1. Downsides

(1) When speaking of the problems with dual opinions, i.e., the membership and the nonmembership grades, the concept of FSs fails to model them

(2) Despite the fact that IFSs can model problems with dual opinions, they also flop because of the strong constraints on its characteristic functions. These restrictions bound the decision makers to a limited set of choices.

(3) Likewise, PFSs also have constraints on making the choice of membership and nonmembership grades which bound the decision makers to a certain domain.

(4) Because of these restrictions in the structures of FSs, IFSs, and PFSs, their correlation coefficients are inoperable at dealing with the information in q-ROFSs.

7.2. Benefits

(1) The notion of q-ROFSs generalizes the structures of FSs, IFSs, and PFSs which implies that the structure of q-ROFSs is capable of dealing with the information provided in the article.

(2) The proposed correlation coefficients of q-ROFSs also generalize the correlation coefficients of IFSs and PFSs. Therefore, these correlation coefficients of q-ROFSs are capable of coping with the intuitionistic fuzzy information and Pythagorean fuzzy information, as discussed in Example 3 in the previous reaction.

(3) With some modifications, the correlation coefficients of q-ROFSs can be applied to the intuitionistic fuzzy information and Pythagorean fuzzy information.
8. Conclusion
This research work discusses some fundamental concepts such as fuzzy sets (FSs), intuitionistic FSs (IFSs), and Pythagorean FSs (PFSs). Furthermore, the structure without any restrictions called the q-rung orthopair FSs (q-ROFSs) is discussed, that is, the generalization of aforementioned structures. In addition, the correlation coefficients for IFSs and PFSs are discussed. Furthermore, the shortcomings of these correlations are also identified. Moreover, some innovative correlation coefficients for q-ROFSs are introduced, and their generalizations are proved through examples and remarks. Also, the properties and results of the proposed correlation coefficients are presented. Furthermore, an algorithm for clustering via the proposed correlation coefficients is given along with an application to the practical clustering problem. Finally, a comparative analysis is carried out among the proposed correlation coefficients and the existing conceptions. The benefits of the proposed generalization and the downsides to the other available theories are argued. In future, these concepts can be introduced for other generalizations of fuzzy theory, which will develop many interesting structures, results, and applications.

Data Availability
No data were used to support this study.

Conflicts of Interest
The authors declare that they have no conflicts of interest.

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