The essence and origin of quantum theory

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According to De Broglie’s idea of analogy, the relation between quantum mechanics and classical mechanics is similar to that between wave optics and geometric optics, we have given the quantum equation of the gravitational field intensity $E_g(\vec{r}, t)$ for matter particles, since the gravitational field intensity $E_g(\vec{r}, t)$ relates to the particle position distribution function $\psi(\vec{r}, t)$, the quantum equation convert into the Schrödinger equation. In addition, we have further studied the essence and origin of quantum theory, and obtained some new results.

PACS: 03.65.-w, 05.70.Ce, 05.30.Rt

Keywords: photon; matter particles; electromagnetic field; gravitational field; schrodinger equation

1. Introduction

In 1900, the derivation of the black-body spectrum due to Planck is taken as the birth of quantum theory [1]. After Einstein proposed the light quantum hypothesis and successfully explained the photoelectric effect, people accepted the theory that light has wave-particle duality. In 1922, D. E. Broglie argued that all particles, like photons, have wave-particle duality [2, 3]. Broglie further thinking matter wave theory, he thought the matter waves of wave mechanics and classical mechanics is similar to the relationship between wave optics and geometrical optics, the relationship between the analogy thought for later founded the schrodinger wave mechanics to lay the important foundation of schrodinger in material Broglie wave theory, on the basis of schrodinger quantum wave equation is given [4-7].

In the development of physics in the 20th century, Einstein and Bohr are the two greatest scientist. They both created the glory of modern physics, but they had their own unique and profound views on the basic problems of modern physics, which caused a long-term debate. Bohr think quantum own existence form, can be described by probability wave function, when the quantum system interact with the outside world, the wave function will collapse to a specific value can be observed, for quantum system, it is impossible to get something other than the probability, the laws of quantum mechanics is only spontaneously, must abandon the decisive principle of cause and effect. Bohr later put forward the famous correspondence principle and complementary principle, which further caused a great shock in the physics.

In 1935, Einstein, Podorsky and Rosen proposed the criterion of the completeness of physical theoretical system and the famous EPR paradox [8], which involves how to understand the reality of the micro world and demonstrates the incompleteness of the description of physical reality by quantum mechanics. In 1950s, Bohm proposed the quantum theory of hidden parameters inspired by EPR paradox [9]. In the 1960s, John Bell derived a quantitative Bell’s inequality [10, 11], on the quantum correlation of distant particles from mathematics according to the quantum theory of hidden parameters. It was possible to design experiments to test the EPR paradox. Physicists completed experiment results are in violation of Bell’s inequality and consistent with the predictions of quantum mechanics [12-14]. The above experiments only show that quantum theory is related at a distance and non-local, but do not determine whether quantum theory is deterministic or non-deterministic, that is to say, whether the causality of the microscopic world is established has not been determined, and the debate on the basis of quantum theory needs to go on. Einstein acknowledged that the internal system of quantum mechanics was self-consistent, but he insisted that quantum mechanics was not the final description of a complete microscopic system.

Although quantum mechanics has made many achievements in developing new technologies, many fundamental questions still exist and need to be studied. In order to understand the microscopic world, whether we need to introduce new concepts and ideas to explain why we should introduce the concept of probability into quantum mechanics, thereby unifying the ideas of determinism and probability theory.

In classical electrodynamics, charged particle is treated as point charge, which leads to infinite self-energy. Therefore, it is problematic to treat particles as points. From the point of view of quantum mechanics, Compton wavelength is usually used to describe the distribution area of particle, and the concept of point particles should be abandoned.

According to De Broglie’s idea of analogy, the relation between quantum mechanics and classical mechanics is similar to that between wave optics and geometric optics, we have given the quantum equation of the gravitational field intensity $E_g(\vec{r}, t)$ for matter particles, since the gravitational field intensity $E_g(\vec{r}, t)$ relates to the particle position

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distribution function $\psi(\vec{r}, t)$, the quantum equation convert into the Schrödinger equation. The photon and matter particles such as electrons, protons, neutrons, etc all are not point particles, the photon is the electromagnetic energy distribution, the neutral matter particles are the gravitational field energy distribution, and the electric charge matter particles there are electromagnetic energy distribution besides the gravitational field energy distribution. On this basis, we have further studied the essence and origin of quantum theory, and obtained some new results.

2. The relationship between quantum equation of photon and Maxwell's equations

The Maxwell's equations are the macroscopic equation of electromagnetic field, which are description the change rule of electric and magnetic fields for a beam of light or a large number of photons. The single photon also has electric and magnetic fields, it satisfies the Maxwell's equations.

(1) The Maxwell's equations in vacuum are

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

(1)

$$\nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

(2)

$$\nabla \cdot \vec{E} = 0$$

(3)

$$\nabla \cdot \vec{B} = 0$$

(4)

In the Ref. [15], the quantum vector wave equation of photon is

$$i\hbar \frac{\partial}{\partial t} \vec{\psi} = c\hbar \nabla \times \vec{\psi} + V \vec{\psi},$$

(5)

where the $\vec{\psi}$ is the vector wave function of photon, the $V$ is the potential energy of photon in medium, it is

$$V = \hbar \omega (1 - n),$$

(6)

where the $n$ is the refractive index of photon in medium. When the photon is in the air or vacuum, the refractive index $n = 1$, the potential energy $V = 0$, i.e., it is a free photon, the Eq. (2) becomes

$$i\hbar \frac{\partial}{\partial t} \vec{\psi} = c\hbar \nabla \times \vec{\psi}.$$

(7)

In the Ref. [16], we have given the quantum spinor wave equations of free and non-free photons, they are (see Appendix A and B):

$$i\hbar \frac{\partial}{\partial t} \psi = -icz\vec{\alpha} \cdot \vec{\nabla} \psi,$$

(8)

and

$$i\hbar \frac{\partial}{\partial t} \psi = -icz\vec{\alpha} \cdot \vec{\nabla} \psi + V \psi,$$

(9)

The photon spinor wave function $\psi$ and the famous Gell-Mann matrices $\vec{\alpha}$ are:

$$\psi(\vec{r}, t) = \begin{pmatrix} \psi_1(\vec{r}, t) \\ \psi_2(\vec{r}, t) \\ \psi_3(\vec{r}, t) \end{pmatrix},$$

(10)

and

$$\alpha_x = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \alpha_y = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}, \quad \alpha_z = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

(11)
Using the method of separation variable $\psi(\vec{r}, t) = \psi(\vec{r}) f(t)$, the Eqs. (8) and (9) become
\[ -ic\hbar \vec{\alpha} \cdot \vec{\nabla} \psi(\vec{r}) = E\psi(\vec{r}), \] (12)
and
\[ [ -ic\hbar \vec{\alpha} \cdot \vec{\nabla} + V] \psi(\vec{r}) = E\psi(\vec{r}), \] (13)
where $E$ is the total energy of photon, The Eqs. (9) and (13) are the spinor wave equations of time-dependent and time-independent of photon in medium, which can be used to study the quantum property of photon in medium.

Substituting Eqs. (10) and (11) into (9), we have
\[ i\hbar \frac{\partial}{\partial t} \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} = -ic\hbar \begin{pmatrix} 0 & -i \frac{\partial}{\partial x} & i \frac{\partial}{\partial y} \\ i \frac{\partial}{\partial y} & 0 & -i \frac{\partial}{\partial z} \\ -i \frac{\partial}{\partial x} & i \frac{\partial}{\partial z} & 0 \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} + \begin{pmatrix} V & 0 & 0 \\ 0 & V & 0 \\ 0 & 0 & V \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}. \] (14)

With Eq. (14), we obtain
\[ \hbar c \left( \frac{\partial \psi_3}{\partial y} - \frac{\partial \psi_2}{\partial z} \right) = i\hbar \frac{\partial}{\partial t} \psi_1 - V\psi_1, \] (15)
\[ \hbar c \left( \frac{\partial \psi_1}{\partial z} - \frac{\partial \psi_3}{\partial x} \right) = i\hbar \frac{\partial}{\partial t} \psi_2 - V\psi_2, \] (16)
and
\[ \hbar c \left( \frac{\partial \psi_2}{\partial x} - \frac{\partial \psi_1}{\partial y} \right) = i\hbar \frac{\partial}{\partial t} \psi_3 - V\psi_3. \] (17)

If we set $\vec{\psi} = \psi_1 \vec{i} + \psi_2 \vec{j} + \psi_3 \vec{k}$, the Eqs. (15)-(17) can be written as
\[ i\hbar \frac{\partial}{\partial t} \vec{\psi} = c\hbar \vec{\nabla} \times \vec{\psi} + V \vec{\psi}. \] (18)

We can find the quantum vector wave equation (5) and the quantum spinor wave equation (9) are equivalent.

In Eq. (7), if we set
\[ \vec{\psi} = \frac{1}{\sqrt{2}} (\sqrt{\varepsilon_0} \vec{E} + i \frac{1}{\sqrt{\mu_0}} \vec{B}), \] (19)
substituting Eq. (19) into (7), we obtain
\[ i\hbar \sqrt{\varepsilon_0} \frac{\partial}{\partial t} \vec{E} - h \frac{1}{\sqrt{\mu_0}} \frac{\partial}{\partial t} \vec{B} = c\hbar \sqrt{\varepsilon_0} \vec{\nabla} \times \vec{E} + ic\hbar \frac{1}{\sqrt{\mu_0}} \vec{\nabla} \times \vec{B}, \] (20)
comparing the real and imaginary parts of the both sides of equation (20), we get
\[ \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \] (21)
\[ \vec{\nabla} \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}, \] (22)
if we let $\vec{\nabla} \cdot \vec{\psi} = 0$, there is
\[ \vec{\nabla} \cdot \vec{E} = 0, \] (23)
\[ \nabla \cdot \vec{B} = 0. \]  

(24)

By the following quantum wave equation of photon and gauge condition

\[ i\hbar \frac{\partial}{\partial t} \psi = c\hbar \nabla \times \psi, \]  

(25)

\[ \psi = \frac{1}{\sqrt{2}}(\sqrt{\varepsilon_0} \vec{E} + i \frac{1}{\sqrt{\mu_0}} \vec{B}), \]  

(26)

\[ \nabla \cdot \psi = 0. \]  

(27)

We can obtain the Maxwell's wave equations (1)-(4) in vacuum.

(2) The Maxwell's equations in medium are

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \]  

(28)

\[ \nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \]  

(29)

\[ \nabla \cdot \vec{E} = 0 \]  

(30)

\[ \nabla \cdot \vec{B} = 0. \]  

(31)

With Eqs. (5) and (6), we can obtain the quantum wave equation of photon in medium, it is

\[ i\hbar \frac{\partial}{\partial t} \tilde{\psi}(\vec{r}, t) = c\hbar \nabla \times \tilde{\psi}(\vec{r}, t) + V\tilde{\psi}(\vec{r}, t) \]

\[ = c\hbar \nabla \times \tilde{\psi}(\vec{r}, t) + \hbar \omega (1 - n)\tilde{\psi}(\vec{r}, t). \]  

(32)

By the separation of variables

\[ \tilde{\psi}(\vec{r}, t) = \psi(\vec{r}) f(t), \]  

(33)

substituting Eq. (33) into (32), we have

\[ c\nabla \times \psi(\vec{r}) = n\omega \psi(\vec{r}) \]  

(34)

if we let

\[ \psi = \frac{1}{\sqrt{2}}(\sqrt{\varepsilon \vec{E}} + i \frac{1}{\sqrt{\mu}} \vec{B}), \]  

(35)

with Eqs. (34) and (35), we get

\[ \nabla \times \vec{E} = i\omega \mu \vec{H}, \]  

(36)

\[ \nabla \times \vec{H} = -i\omega \varepsilon \vec{E}, \]  

(37)

by the gauge condition \( \nabla \cdot \vec{\psi} = 0 \), we have

\[ \nabla \times \vec{E} = 0, \]  

(38)
The Eqs. (36)-(39) are the Maxwell’s wave equations for the monochromatic light in the medium. By the following quantum wave equation of photon and gauge condition

\[ c \nabla \times \vec{\psi}(\vec{r}) = n\omega \vec{\psi}(\vec{r}) \]  
\[ \vec{\psi} = \frac{1}{\sqrt{2}} (\sqrt{\varepsilon} \vec{E} + i \sqrt{\frac{1}{\mu}} \vec{B}) , \]  
\[ \nabla \cdot \vec{\psi} = 0. \]  

We can obtain the the Maxwell’s equations (36)-(39) in medium. The probability density of photon in space \( \vec{r} \) is

\[ \rho_H(\vec{r}) = |\vec{\psi}(\vec{r})|^2 = \frac{1}{2} (\varepsilon \vec{E}^2 + \frac{1}{\mu} \vec{B}^2) = \varepsilon \vec{E}^2 = \rho_{EB}(\vec{r}). \]  

From Eq. (43), we find the probability density \( \rho(\vec{r}) \) of photon is equal to the energy density \( \rho_{EB}(\vec{r}) \) of the electromagnetic field, we can obtain the following results:

1. For a lot of photons, the electromagnetic field energy density \( \rho_{EB}(\vec{r}) \) is in direct proportion to the photon numbers \( N(\vec{r}) \) and the single photon probability density \( \rho_H(\vec{r}) \), it is

\[ \rho_{EB}(\vec{r}) \propto N(\vec{r}) \propto \rho_H(\vec{r}). \]  

2. For a single photon, it is not a point particle, instead, it has a very small distribution area \( \Omega \) of electromagnetic fields, the whole distribution area represents a photon.

We define a concept of partial photon, which is described by the occupancy \( P(\vec{r}) \), it is

\[ P(\vec{r}) = \frac{\varepsilon \vec{E}^2(\vec{r})}{\int_{\Omega} \varepsilon \vec{E}^2(\vec{r}) \, d^3\vec{r}}. \]  

At space \( \vec{r} \), the bigger the occupancy \( P(\vec{r}) \), the bigger the photon component, there is

\[ \int_{\Omega} P(\vec{r}) \, d^3\vec{r} = 1. \]  

Since the photon itself is a very small distribution area of electromagnetic fields, the each point in the region represents the partial photon, the photon is not positioned. The wave-particle duality of photon can be understood as: The entire electromagnetic field distribution area of photon represents a photon, which manifests as the particle nature of photon. The electromagnetic field energy density distribution of photon manifests as the wave nature of photon. At space \( \vec{r} \), the probability density \( |\psi(\vec{r})|^2 \) of photon is in direct proportion to its electromagnetic fields energy density, it is

\[ |\psi(\vec{r})|^2 \propto \varepsilon \vec{E}^2(\vec{r}) , \]  

with Eqs. (45)-(47), we have

\[ P(\vec{r}) = |\psi(\vec{r})|^2. \]  

Photon is not a point particle, it exists in the distribution area of electromagnetic field, where the energy density of electromagnetic field is large, it means that the photon appears to be of great weight. So, the every point of the electromagnetic field distribution region, all are a part of the photon, such as the A and B points in electromagnetic field distribution region, they can be represented as part of photon. The photon can be expressed as the superposition of the every point of the electromagnetic field distribution region, that is, the photon can appear at both point A and point B. This structure of the photon is the reason for existing the quantum superposition. When the photon interacts with the outside world, such as photon through the slit or colliding with particle, the electromagnetic field distribution of photon should be changed, and form the probability distribution \( |\psi(\vec{r})|^2 \) at space \( \vec{r} \), the wave nature of photon is
from its own electromagnetic field energy distribution change. The photon produces interferes through the double slit, it is because the electromagnetic field of photon is redistributed by double slit, thus forming interference fringe. Therefore, Using the electromagnetic field distribution instead of point photon, which can better explain the quantum phenomena of photon shown in the experiment.

3. The gravitational field of particle

(1) The non-relativistic gravitational theory

The gravitational potential for a continuous mass distribution is

$$ \Phi(\vec{x}) = -\int \frac{G \rho(\vec{x}')}{|\vec{x} - \vec{x}'|} d^3 \vec{x}' $$

where $\rho(\vec{x}')$ is the mass density. The Eq. (49) satisfies the Poisson equation

$$ \nabla^2 \Phi(\vec{x}) = 4\pi G \rho(\vec{x}), $$

the self gravitational energy is

$$ w_g = \frac{1}{2} \int \rho(\vec{x}) \Phi(\vec{x}) d^3 \vec{x} $$

$$ = -\frac{1}{2} \int G \rho(\vec{x}) \rho(\vec{x}') \frac{d^3 \vec{x} d^3 \vec{x}'}{|\vec{x} - \vec{x}'|} $$

$$ = \int \frac{1}{8\pi G} (\nabla \Phi(\vec{x}))^2 d^3 \vec{x} + \int \rho(\vec{x}) \Phi(\vec{x}) d^3 \vec{x}, $$

where

$$ \rho_g = \frac{(\nabla \Phi(\vec{x}))^2}{8\pi G}, $$

is the energy density of gravitational field, and $\rho(\vec{x}) \Phi(\vec{x})$ is the energy density of gravitational field interacts with matter.

(2) The relativistic gravitational theory

In the presence of gravitational field, the dynamic problem of particle can be equivalently transformed into the geometric problem of Riemann space. That is, the motion profile of particle in the gravitational field is the geodesic line of Riemann space. Einstein not only geometrized the particle dynamics in the gravitational field, but also geometrized the gravitational field itself. That is, the metric field $g_{\mu\nu}$ of Riemann space represents the gravitational field. In this way, it is always controversial. The gravitational field, like electromagnetic field, is an objective material field, the geometrization of gravitational field is only an equivalent theory. The relationship between curved space-time metric $g_{\mu\nu}$, flat space-time metric $\eta_{\mu\nu}$ and gravitational field $h_{\mu\nu}$ is as follows

$$ g_{\mu\nu} = \eta_{\mu\nu} + kh_{\mu\nu}. $$

From Eq. (53), we can find if there is the gravitational field $h_{\mu\nu}$ then there is the curved space-time metric $g_{\mu\nu}$, if there is no the gravitational field, the space-time is flat, and the metric is $\eta_{\mu\nu}$. Therefore, it is the gravitational field causes the space-time bending, and can not be considered gravitational field as the curved space-time, The Gravitational field and curved space-time are causal relation. It is only an equivalent theoretical method to study gravitational field with the curved space-time.

The electromagnetic field is from electromagnetic current

$$ J^\mu = (\rho, \vec{J}), $$

where $\rho$ is the electric density, $\vec{J}$ is the electric current density, the electromagnetic field vector $A^\mu = (\varphi, \vec{A})$ satisfy an equation

$$ \Box A^\mu = -\mu_0 J^\mu, $$
and Lorentz condition
\[ \partial_\mu A^\mu = 0. \] (56)

The source of gravitational field is energy-momentum tensor \( T^{\mu\nu} \), and the gravitational field tensor \( h^{\mu\nu} \) equation is
\[ \Box (h^{\mu\nu} - \frac{1}{2} \eta^{\mu\nu} h) = -kT^{\mu\nu}, \] (57)

and gauge condition is
\[ \partial_\mu (h^{\mu\nu} - \frac{1}{2} \eta^{\mu\nu} h) = 0, \] (58)

where \( k \) is a constant, \( h = h_\mu \) is the trace of \( h^{\mu\nu} \), and \( \eta^{\mu\nu} \) is the metric of flat space-time.

Definition a new gravitational field
\[ \phi^{\mu\nu} = h^{\mu\nu} - \frac{1}{2} \eta^{\mu\nu} h, \] (59)

the Eqs. (57) and (58) can be written as
\[ \Box \phi^{\mu\nu} = -kT^{\mu\nu}, \] (60)
\[ \partial_\mu \phi^{\mu\nu} = 0, \] (61)

the energy-momentum tensor of field \( \phi^{\mu\nu} \) is
\[ t^{\mu\nu} = \frac{1}{4} [2\phi^{\alpha\beta,\mu} \phi^{\alpha\beta}_{\nu} - \phi^{\mu}_{\nu} \phi^{\alpha,\nu} - \eta^{\mu\nu} (\phi^{\alpha\beta,\sigma} \phi^{\alpha\beta}_{\sigma} - \frac{1}{2} \phi^{\alpha}_{\sigma} \phi^{\alpha}_{\sigma})], \] (62)

where \( t^{00} \) is the energy density.

The Newton gravitational theory is the non-relativistic limit of relativistic gravitational theory, there are
\[ \Phi = \frac{1}{2} kh_{00}, \] (63)
\[ \nabla^2 h_{00} = \frac{1}{2} \rho, \] (64)

with Eqs. (51) and (52), we can obtain the equation of the Newton gravitational field
\[ \nabla^2 \Phi = 4\pi G \rho, \] (65)

where \( k = 4\sqrt{\pi G} \), and \( \rho \) is mass density. In the Newtonian approximation, the gravitational field energy density is
\[ \rho_g = t^{00} = \frac{(\nabla \Phi)^2}{8\pi G}, \] (66)

defining the strength of the gravitational field \( \vec{E}_g \) as
\[ \vec{E}_g = -\nabla \Phi, \] (67)

the gravitational field energy density \( \rho_g \) becomes
\[ \rho_g = \frac{\vec{E}_g^2}{8\pi G}, \] (68)

the energy density of electromagnetic field is
\[ \rho_{(EB)} = \varepsilon \vec{E}_g^2. \] (69)
From Eqs. (68) and (69), we find the gravitational field energy density is in direct proportion to the square of the gravitational field strength \( \vec{E}_g \), and the electromagnetic field energy density is in direct proportion to the square of the electromagnetic field strength \( \vec{E} \).

All particles such as electron, protons and neutrons cannot be regarded as point particles, they have a wide energy distribution area of gravitational field.

We define a concept of partial electron, which is described by the occupancy \( P(\vec{r}) \), it is

\[
P(\vec{r}) = \frac{\int_0^{t_0} \rho_0(\vec{r}) d\vec{r}}{\int_V E_2^2(\vec{r}) d^3\vec{r}}.
\]

(70)

At space \( \vec{r} \), the bigger the occupancy \( P(\vec{r}) \), the bigger the electron component, At the whole gravitational field distribution area \( V \) of the electron, there is

\[
\int_V P(\vec{r}) d^3\vec{r} = 1.
\]

(71)

Where the volume \( V \to \infty \). The bigger the occupancy \( P(\vec{r}) \), the bigger the electron component. For the microscopic particles, such as electron, proton, neutron and so on, their gravitational field distribution can be divided into two areas, one area is the spherical area that its radius is about the Compton wavelength of microscopic particle, which is called Compton area, the other one is the outside Compton area. In the two areas, their gravitational field energy are almost equal. The every point in the gravitational field distribution areas, it is a part of the microscopic particle, such as the \( A \) and \( B \) points in the gravitational field areas, they can be represented as part of microscopic particle. The microscopic particle can be expressed as the superposition of the every point occupancy \( P(\vec{r}) \). This structure of the microscopic particle is the reason for existing the quantum superposition. When the microscopic particle interacts with the outside world, the gravitational field distribution of microscopic particle should be changed, and form the probability distribution \( |\psi(\vec{r})|^2 \) at space \( \vec{r} \), the wave nature of microscopic particle is from its own gravitational field energy distribution change. The microscopic particle produces interferes through the double slit, it is because the gravitational field of microscopic particle is redistributed by double slit, thus forming interference fringe. Therefore, Using the gravitational field distribution instead of point particle, which can better explain the quantum phenomena of microscopic particle shown in the experiments.

4. The quantum wave equation

In 1923, DE Broglie had extended the wave-particle duality of photon to physical particle [2], like electrons, protons and so on. Later, he had perfected the theory of matter waves [3], and by analogy Fermat and Morperto principle, he believed the relation between the new wave theory and classical mechanics is similar to the relationship between wave optics and geometric optics, this analogy inspired Schrodinger when he founded wave mechanics. In the following, we should give the quantum wave equation of particle with the analogy method. In addition, we define wave function and give the new physical meaning of wave function.

(1) The time-independent wave equation of particle

The particle nature of photon is described by the Fermat principle, it is

\[
\delta \int n ds = 0,
\]

(72)

the motion of physical particles is described by Morperto principle, it is

\[
\delta \int \sqrt{2m(E-V)} ds = 0,
\]

(73)

the time-independent photon wave equation is

\[
\nabla^2 \vec{E} + \frac{\omega^2 n^2}{c^2} \vec{E} = 0,
\]

(74)

where \( \vec{E} \) is electric field intensity of photon, \( \omega \) is photon frequency, \( n \) is refractive index of medium and \( c \) is velocity of light.
comparing equations (72) and (73), we find
\[ n \propto \sqrt{2m(E - V)}, \]  
(75)
with Eqs. (74) and (75), the particle wave equation can be written as
\[ \nabla^2 \vec{E}_g + A \cdot 2m(E - V) \vec{E}_g = 0, \]  
(76)
the Eq. (76) is applicable to both charged particles and neutral particles, the \( \vec{E}_g \) is the gravitational field intensity of particle.

In spherical coordinate, the Eq. (76) becomes
\[ \frac{\partial^2}{\partial r^2}(r \vec{E}_g) + A \cdot 2m(E - V)(r \vec{E}_g) = 0, \]  
(77)
From the free particle, the potential energy \( V = 0 \), the Eq. (77) has a spherical wave solution, it is
\[ \vec{E}_g = \vec{E}_{g0} \frac{e^{ikr}}{r}, \]  
(78)
with Eqs. (77) and (78), there is
\[ \frac{\partial^2}{\partial r^2}(\vec{E}_{g0}e^{ikr}) + A \cdot 2mE\vec{E}_{g0}e^{ikr} = 0, \]  
(79)
there is
\[ -k^2 + A \cdot 2mE = 0, \]  
(80)
where \( E = \frac{p^2}{2m} \), and quantum hypothesis
\[ k = \frac{p}{\hbar}, \]  
(81)
with Eqs. (80) and (81), we get
\[ A = \frac{1}{\hbar^2}, \]  
(82)
substituting Eq. (82) into (76), we obtain the time-independent gravitational field equation of particle, it is
\[ \left[-\frac{\hbar^2}{2m} \nabla^2 + V(r)\right]\vec{E}_g = E\vec{E}_g, \]  
(83)
the scalar form of Eq. (83) is
\[ \left[-\frac{\hbar^2}{2m} \nabla^2 + V(r)\right]E_g = EE_g, \]  
(84)
In Eq. (66), the energy density \( t^{00}(\vec{r}) \) of gravitational field is in direct proportion to \( \vec{E}_g^2 \), the probability density \( \rho(\vec{r}) \) of particle in space is proportional to the gravitational field energy density \( t^{00}(\vec{r}) \), we define particle spatial position distribution function \( \psi(\vec{r}) \), it satisfies
\[ \rho(\vec{r}) = |\psi(\vec{r})|^2, \]  
(85)
then we have
\[ |\psi(\vec{r})|^2 \propto E_g^2, \]  
(86)
the Eq. (84) becomes
\[ \left[-\frac{\hbar^2}{2m} \nabla^2 + V(r)\right]\psi(\vec{r}) = E\psi(\vec{r}). \]  
(87)
The spatial position distribution function $\psi(\vec{r})$ is the wave function of quantum mechanics, the Eq. (87) is the time-independent Schrödinger equation. Here, we give out the physics significance of wave function $\psi(\vec{r})$, it relates to the gravitational field intensity distribution $\vec{E}_g^2$ of particle.

(2) The time-dependent field equation of particle
the time-dependent photon wave equation is

$$\nabla^2 \vec{E} - \frac{n^2}{c^2} \frac{\partial^2}{\partial t^2} \vec{E} = 0,$$

substituting Eq. (75) into (88), we can obtain the time-dependent particle wave equation, it is

$$\nabla^2 \vec{E}_g - B \cdot 2m(E - V) \frac{\partial^2}{\partial t^2} \vec{E}_g = 0.$$  

For the free particle, the potential energy $V = 0$, the equation (88) has the spherical wave solution, it is

$$\vec{E}_g^2 = \frac{\vec{E}_g^0}{r} e^{i(pr - Et)/\hbar},$$

with Eqs. (89) and (90), there is

$$B = \frac{1}{E^2},$$

the Eq. (89) becomes

$$\nabla^2 \vec{E}_g - \frac{1}{E^2} \cdot 2m(E - V) \frac{\partial^2}{\partial t^2} \vec{E}_g = 0.$$  

By separation of variable

$$\vec{E}_g(\vec{r}, t) = \vec{E}_g(\vec{r}) f(t),$$

substituting Eq. (93) into (92), there are

$$\nabla^2 \vec{E}_g(\vec{r}) - D \cdot 2m(E - V) \vec{E}_g(\vec{r}) = 0,$$

$$f''(t) - D \cdot E^2 f(t) = 0.$$  

comparing Eq. (83) with (94), we have

$$D = - \frac{1}{\hbar^2},$$

$$f(t) = e^{-\frac{iE\hbar}{\hbar^2}},$$

and

$$\vec{E}_g(\vec{r}, t) = \vec{E}_g(\vec{r}) e^{-\frac{iE\hbar}{\hbar^2}},$$

taking the derivative of both sides of the equation (98), we get

$$i\hbar \frac{\partial}{\partial t} \vec{E}_g(\vec{r}, t) = \left[ - \frac{\hbar^2}{2m} \nabla^2 + V(r) \right] \vec{E}_g(\vec{r}, t).$$  

By the equation (86), there is

$$\psi(\vec{r}, t) \propto E_g(\vec{r}, t),$$
the equation (99) becomes
\[ i\hbar \frac{\partial}{\partial t} \psi(\vec{r}, t) = \left[ -\frac{\hbar^2}{2m} \nabla^2 + V(r) \right] \psi(\vec{r}, t). \] (101)
The equation (101) is the time-dependent quantum wave equation of particle in space location distribution, i.e., the Schrodinger equation. In the above, we can find that the Schrodinger equation comes from the gravitational field equation of particle. By equation (100), transformed equation (99) into (101). When particle interacts with the outside world, the distribution of particle gravitational field intensity \( E_g(\vec{r}, t) \) should be changed, which leads to the change of particle position distribution function \( \psi(\vec{r}, t) \). The gravitational field intensity \( E_g(\vec{r}, t) \) is a hidden variable in quantum theory, it is an objective and measurable physical quantity. In quantum theory, the reason why there is the probability is because microscopic particle is not point particle but gravitational field distributions, and the each point in the gravitational field distribution areas is a part of the microscopic particle.

For a macroscopic object, its gravitational field distribution is mainly concentrated on the range of the object volume, which is called internal gravitational field areas, and the surrounding gravitational field distribution of object is called external gravitational field areas. Therefore, changing the external gravitational field distribution of a macroscopic object has little influence on its motion state. By applying external force to a macroscopic object, the internal gravitational field distribution should be changed, and then the motion state of the macroscopic object should be changed, the motion law of macroscopic object follows the Newton’s law. For a microscopic particle, the mass is very small, its internal gravitational field energy is close to the outside gravitational field energy, by changing the distribution of outside or internal gravitational field of microscopic particle, they can all change the position distribution function of microscopic particle. When external force is applied to microscopic particle, it mainly causes internal gravitational field distribution changes. For example, in external electrostatic field, the charge particle takes the circular motion and follows Newton’s laws, it is from the internal gravitational field distribution of charge particle change. When a microscopic particle passes through narrow slits, the internal and outside gravitational field distribution are all have been changed, the microscopic particle behaves like a wave, and then produce interference fringes, it should be described by quantum theory. We can obtain the following results: (1) For a macroscopic object, the internal gravitational field energy is much larger than the external gravitational field energy, the motion of macroscopic object can be changed when it is subjected to external forces to change its internal gravitational field distribution, the motion law of macroscopic object follows the Newton’s law. (2) For a microscopic particle, the internal gravitational field energy is considerable to the external gravitational field energy. If microscopic particle is subjected to external forces to change its internal gravitational field distribution, the motion law of microscopic particle follows the Newton’s law. In this case, the microscopic particle has particle nature. (3) For a microscopic particle, when internal gravitational field distribution and external gravitational field distribution all are changed, The position and state of microscopic particles are uncertain. In this case, the microscopic particle has wave nature, it should be described by quantum theory.
5. Conclusions

According to De Broglie’s idea of analogy, the relation between quantum mechanics and classical mechanics is similar to that between wave optics and geometric optics, we have given the quantum equation of the gravitational field intensity $E_g(\vec{r}, t)$ for matter particles, since the gravitational field intensity $E_g(\vec{r}, t)$ relates to the particle position distribution function $\psi(\vec{r}, t)$, the quantum equation convert into the Schrödinger equation. The photon and matter particles such as electrons, protons, neutrons, etc all are not point particles, the photon is the electromagnetic energy distribution, the neutral matter particles are the gravitational field energy distribution, and the electric charge matter particles there are electromagnetic energy distribution besides the gravitational field energy distribution. On this basis, we have further studied the essence and origin of quantum theory, and obtained some new results.

6. Acknowledgment

This work was supported by the Scientific and Technological Development Foundation of Jilin Province (no.20190101031JC).

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