Boosted Jet Tagging with Jet-Images and Deep Neural Networks

Michael Kagan\textsuperscript{1,a}, Luke de Oliveira\textsuperscript{2}, Lester Mackey\textsuperscript{2}, Benjamin Nachman\textsuperscript{1,2,b}, and Ariel Schwartzman\textsuperscript{1,c}

\textsuperscript{1}SLAC National Accelerator Laboratory, Menlo Park, CA, USA
\textsuperscript{2}Stanford University, Stanford, CA, USA

Abstract. Building on the jet-image based representation of high energy jets, we develop computer vision based techniques for jet tagging through the use of deep neural networks. Jet-images enabled the connection between jet substructure and tagging with the fields of computer vision and image processing. We show how applying such techniques using deep neural networks can improve the performance to identify highly boosted W bosons with respect to state-of-the-art substructure methods. In addition, we explore new ways to extract and visualize the discriminating features of different classes of jets, adding a new capability to understand the physics within jets and to design more powerful jet tagging methods.

1 Introduction

Jets, or collimated streams of particles arising from the production of high energy quarks and gluons, are crucial signatures for discovering signs of new physics at the Large Hadron Collider (LHC) \cite{1}. Quarks and gluons, and subsequently jets, are produced ubiquitously from the LHC proton-proton collisions. However, in many theories of new physics, new TeV-mass-scale particles decay into high energy $W^\pm$, $Z$, $H$ bosons, and top quarks. These high energy bosons and top quarks can subsequently decay into single jets containing all of their decay products. Identifying, or tagging, the jets from bosons and top quarks is thus vital to reconstruct and identify signs of new TeV-mass-scale particles. The field of jet-substructure (see \cite{2–4} and references therein) has emerged to overcome this challenge, whereby physics inspired features are engineered to exploit the differences in the internal structure of jets produced by bosons and top quarks from that of jets produced directly from quarks and gluons. Concurrently, modern computer vision and machine learning techniques have made great progress in recent years in deep learning \cite{5–7}; large multi-hidden layer neural networks operating on low level information (e.g. pixels in an image), without feature engineering, have been found to learn complex representations of information within images and efficiently use this information for classification.

The jet-images approach to jet tagging combines computer vision with jet-substructure. Jet measurements from LHC detectors are formatted into images, where detector energy measurements play...
the role of pixel intensities, and the jet-images are analyzed using computer vision inspired techniques. The computer vision techniques include linear methods, such as Fisher discriminant analysis, and deep learning methods, including convolutional and dense MaxOut neural network architectures. These techniques not only provide improved discrimination power, but also insight into the physics learned for discrimination.

The power of the jet-images and computer vision techniques is explored in a benchmark jet-substructure problem, discriminating high energy \( W^\pm \) bosons from quarks and gluons, as discussed in references [8] and [9]. We refer the reader to these references for further details.

2 Samples and selection

To study the jet-images technique, the benchmark task of discriminating high energy \( W^\pm \) bosons from quarks and gluons is examined. Monte Carlo (MC) simulations of proton-proton collisions at \( \sqrt{s} = 14 \text{ TeV} \) based on the \textsc{Pythia} 8.170 [10, 11] event generator are used. High energy \( W^\pm \) bosons are simulated through the decay of a hypothetical new heavy \( W' \) boson which decays to one \( W^\pm \) boson and one \( Z \) boson. Subsequently, the \( W \) is forced to decay hadronically (\( W \rightarrow q\bar{q}' \)), and the \( Z \) to decay invisibly (\( Z \rightarrow \nu\bar{\nu} \)). The background is the generic (multijet) production of quarks and gluons.

Since the classification performance can depend on the transverse momentum, \( p_T \), and jet mass, \( m \), of the jets, we focus our studies on a restricted range of \( 250 \text{ GeV}/c < p_T < 300 \text{ GeV}/c \), and a \( 65 \text{ GeV}/c^2 < m < 95 \text{ GeV}/c^2 \) mass window that contains the peak of the \( W \). The minimum angular spread of the \( W \) decay products is approximately \( \Delta R = \sqrt{\Delta\eta^2 + \Delta\phi^2} \approx 0.6 \), where \( \Delta\eta \) and \( \Delta\phi \) are the distances between \( W \) boson decay products in \((\eta,\phi)\) coordinates. Further, to remove any dependence on the jet transverse momentum, we re-weight the signal to have the same \( p_T \) distribution as the background.

The discrete size and finite acceptance of a real detector are modeled by binning the particles into cells with size \( 0.1 \times 0.1 \) in \((\eta,\phi)\) extending out to \( \eta = 5.0 \). Each cell’s 4-vector is defined by \( p_j = \sum_{\text{incident on } j} E_i (\cos\phi_j/\cosh\eta_j, \sin\phi_j/\cosh\eta_j, \sinh\eta_j/\cosh\eta_j, 1) \), where the sum runs over particles incident on the cell.

Jets are built by clustering the cells using the anti-\( k_t \) algorithm [12] with distance parameter \( R = 1.0 \), via \textsc{FastJet} [13] 3.0.3. The large distance parameter is used in order to capture all the decay products of a boosted \( W \) boson. To reduce the impact of pileup\(^1\) and underlying event, jets are trimmed [14] by re-clustering the constituents into \( R = 0.3 \) \( k_t \) subjets and dropping those which have \( p_{T,\text{subjet}} < 0.05 \times p_T \).

In order to compare the jet-images approach with standard substructure techniques, we compare performance with known highly performant jet substructure features. These features are the jet mass defined as \( m_{T,\text{jet}}^2 = \sum_{i,j} p_i p_j \) with jet constituent four-vectors \( p_i \), \( n\)-subjettiness [15] in the form of \( \tau_{21} \) using the winner-take-all axis definition [16], and the distance in \((\eta,\phi)\) space between subjets of the trimmed jet (\( \Delta R \)).

3 Jet-images and pre-processing

Jet-images are formed by taking a \( 25 \times 25 \) cell grid around the jet axis, with the intensity of jet-image pixel \( i \) defined to be the corresponding cell transverse energy \( p_{T,j} = E_i / \cosh(\eta_j) \).

In order for machine learning algorithms to learn more efficiently, we make use of space-time symmetries to pre-process the jet-images into a standardized representation. The pre-processing steps are:

\(^1\)Pileup is generated by multiple proton-proton collisions per bunch crossing
- Translation: Jet-images are translated such that the leading $p_T$ subjet is located at $(\eta, \phi) = (0, 0)$. Note that translations in $\phi$ are effectively rotations around the detector $z$-axis, while translations in $\eta$ are Lorentz boosts along the $z$-axis. As such, translations in $\eta$ can alter the mass of a jet-image if the pixel energies are kept fixed. However, the transverse energy is invariant to such $\eta$ translations, and thus is used for the pixel intensities.

- Rotation: Jets are rotated such that the second leading $p_T$ subjet is aligned along the vertical axis of the jet-image. If the jet has only one subjet, the first principle component of the energy distribution in $(\eta, \phi)$ is rotated to align with the vertical axis of the jet-image.

- Parity Flip: After rotation, images are flipped over the vertical axis such that the right side of the image has a higher energy than the left. This helps to standardize the location of additional radiation in the jet-image.

After pre-processing, the leading subjet within the jet-image is located at the center of the image, and the second subjet (if it exists) is aligned along the vertical axis of the image. In facial recognition tasks, this is equivalent to aligning the eyes within an image of a face. With such a standardized jet-image representation, the machine learning algorithms do not need to learn about the symmetries in the jet-image, thus allowing the learning to focus more effectively on discrimination. The effect of the pre-processing on a collection of $W$ jets can be found in Figure 1.

![Figure 1](image_url)

**Figure 1.** The average jet-image for $W$ jets before (left) and after (right) pre-processing. The average is taken over jet-images with $240 \text{ GeV} / c < p_T < 260 \text{ GeV} / c$ and $65 \text{ GeV} / c^2 < m < 95 \text{ GeV} / c^2$.

4 Deep learning architectures and training

Discrimination between $W$ and quark/gluon jet-images is performed using deep neural networks (DNN), which have been found to outperform competing algorithms in computer vision tasks similar to jet tagging with jet-images. DNNs have been found to learn rich high-level representations from raw (pixel-level) image data [5–7]. We make use of the power of such networks by training them on jet-images, with the pixel level information as input. We focus our attention here on two
state-of-the-art architectures: fully connected MaxOut networks [6], and convolutional neural networks [5].

Complete descriptions of the architectures can be found in reference [9], and are only briefly described here. The MaxOut network consists of two MaxOut layers followed by two fully connected layers, followed by a classification layer. The convolutional network (ConvNet) consists of three convolution layers, followed by one fully connected layer, followed by a classification layer. Convolution kernel sizes were 11 × 11 in the first convolutional layer, and 3 in subsequent convolutions. The atypically large kernel size in the first convolution was found to help performance on the relatively sparse jet-images (only about 5–10% of pixels contain energy on average in a jet-image). Each convolution is followed by a max-pooling layer [17] and then by a dropout layer [7]. All MaxOut, dense, and convolutional layers use the rectified linear unit (ReLU) activation function [18], while the classification layer uses sigmoid activations.

All deep learning experiments were conducted in Python with the Keras [19] deep learning library, utilizing NVIDIA C2070 graphics cards. 8 million examples were used for training, with an additional 2 million validation samples for tuning the hyper-parameters, and 3 million examples were used for testing. Signal examples were weighted such that the total sum of weights was the same as the total number of background examples. These weights were used by the cost function in the training and in the ROC curve computations of the test samples. The networks were trained with the Adam [20] algorithm. The training consisted of 100 epochs, with a 10 epoch patience parameter on the increase in area under the ROC curve between 0.2 and 0.8 on a validation set. Batch sizes of 32 were used for the MaxOut network, while batch sizes of 96 were used for the convolution networks.

5 Results

The primary figure of merit used in this note to compare the performance of different classifiers is the ROC curve, which shows the quark/gluon background rejection (the inverse misclassification efficiency) as a function of the W jet signal efficiency. Efficiencies and rejection are computed by scanning thresholds on the signal-to-background likelihood ratio distribution (i.e. the optimal use of the variables).

The ROC curve showing the DNNs as well as several substructure variables can be found on the left in Figure 2. Both DNNs are seen to have significantly better rejection than the substructure variables for all efficiencies, with an improvement in rejection of about 2 at a signal efficiency of 30%. We also see that the MaxOut network tends to outperform the ConvNet, which we believe is due to difficulties for convolutional networks to learn rich discriminating information from kernels when images are sparse.

In order to understand if the DNNs have learned the information contained within the substructure variables, we combine the DNN outputs with substructure variables into a single discriminant using the likelihood ratio of the 2D distribution of the DNN and the substructure variable. The combination with the ConvNet is shown on the right in Figure 2. We can see that the combination with ΔR and τ21 adds little discrimination to the baseline performance of the ConvNet, indicating that the ConvNet has learned the discriminating information contained in these variables. However, when the jet mass is combined with the ConvNet, the performance is seen to increase, indicating that the ConvNet has not completely learned the mass information. Similar behavior is found for the MaxOut network.

To examine more deeply into the information learned by the DNNs, Figure 3 shows the 2D distribution of τ21 (left) and jet mass (right) conditioned on the ConvNet output, i.e. \( p(X|\text{DNN output}) \) where \( X \) is the substructure variable. As we can see, the ConvNet output is strongly correlated with τ21, again indicating that this information has been learned by the network. However, the correlation
with mass is not as strong, with the distributions of mass significantly spread out over various ConvNet output values. While some correlation is present, the network does not seem to have learned all information contained in the jet mass.

In order to study what is learned by the DNNs beyond known substructure variables, we examine the networks in several ways. First, we find the 500 most activating images for a given neuron,
and visualize the average of these images. This is shown in Figure 4 for several neurons in the last hidden layer of the ConvNet. Also shown above each images is the fraction of images found to be most activating for each neuron that were signal images. As we can see, the signal correlated neurons have a clear two-prong structure with information between the leading two subjets but very little information outside of the two-prong structure. In contrast, background correlated neurons have much broader energy distributions indicating the presence of additional wide-angle radiation beyond the leading two subjets.

Figure 4. Average most activating input images for several neurons in the last hidden layer of the ConvNet.

To further understand how the DNNs use the information present in the jet-images for discrimination, we compute the Pearson correlation coefficient for each jet-image pixel with the output of the DNN. These correlations with the ConvNet can then be visualized as an image, as seen in Figure 5. From this correlation image, we see that the presence of a leading subjet at the center of the image provides little discrimination power. This is not surprising, as both signal and background jets have at least one high energy subjet. However, the location of the subleading subjet is strongly correlated with signal-like images, as seen by the red region at $(\eta, \phi) \approx (-0.5, 0)$. In addition, radiation present around the the leading subjet and radiation at the periphery of the image away from either subjet is correlated with background like images. This indicates that the radiation pattern beyond the leading subjets provides important information for discrimination.

While we have focused on jets with $240 \text{GeV}/c < p_T < 260 \text{GeV}/c$ and $65 \text{GeV}/c^2 < m < 95 \text{GeV}/c^2$, we can restrict the phase space of the images in such a way as to remove discriminating information from known powerful features. In doing so, we can examine the information learned by the DNNs beyond the known substructure features. As such we restrict the phase space to a signal-like region of $240 \text{GeV}/c < p_T < 260 \text{GeV}/c$, $79 \text{GeV}/c^2 < m < 81 \text{GeV}/c^2$, and $0.19 < \tau_{21} < 0.21$. The ROC curve comparing classification performance, as well as the DNN-pixel correlation image, can be found in Figure 6. In this phase space, we can see that mass and $\tau_{21}$ have little discrimination power, as expected in such a restricted region. However, we can also see that the DNNs still provide discrimination power, further indicating that the DNNs have learned physics information beyond these known substructure variables. The ConvNet-pixel correlation image can help to elucidate what information has been learned. In this phase space, we see again that the radiation pattern provides important discrimination power. The radiation between subjets is correlated with signal-like images,
while radiation at the periphery is correlated with background like images. This could be due to the difference between radiation patterns of the decays of a color singlet $W$ boson, as apposed to the decays of a color octet gluon or color triplet quark. While features engineered to exploit such colorflow information have been studied in the past [21], the features learned here may present a new way to access this information for improved discrimination.

Figure 5. Pearson correlation coefficient of each input pixel with the ConvNet output.

Figure 6. ROC curve comparing DNN and substructure variable performance (left), and Pearson correlation coefficient of each input pixel with the ConvNet output (right). Both figures are computed in the restricted phase space of $240 \text{ GeV}/c < p_T < 260 \text{ GeV}/c$, $79 \text{ GeV}/c^2 < m < 81 \text{ GeV}/c^2$, and $0.19 < \tau_{21} < 0.21$. 

DOI: 10.1051/epjconf/2016000000EPJ Web of Conferences epjconf/20160000  (2016) 201
6 Conclusions

In this paper, we have used the paradigm of jet-images to interpret LHC data based on tools inspired by image processing, computer vision, and deep learning. Deep learning proved to be a powerful paradigm for jet tagging when combined with jet-images, significantly improving performance over state-of-the-art jet substructure features. The performance of deep learning techniques was aided by domain specific inputs, whereby physics inspired pre-processing allowed deep neural networks to learn powerful discriminating information without having to simultaneously learn the symmetries of space-time. Finally, we showed several techniques to extract information about what the deep neural networks are learning. A key aspect to this was being able to visualize the learned information in several ways.

Acknowledgments

This work was supported by the Stanford Data Science Initiative and by the US Department of Energy (DOE) grant DE-AC02-76SF00515.

References

[1] L. Evans, P. Bryant, JINST 3, S08001 (2008)
[2] A. Altheimer et al., J. Phys. G39, 063001 (2012), arXiv:1201.0008
[3] A. Altheimer et al., Eur. Phys. J. C74, 2792 (2014), arXiv:1311.2708
[4] D. Adams et al., Eur. Phys. J. C75, 409 (2015), arXiv:1504.00679
[5] K. Simonyan, A. Zisserman (2014), arXiv:1409.1556
[6] I.J. Goodfellow, D. Warde-Farley, M. Mirza, A. Courville, Y. Bengio (2013), arXiv:1302.4389
[7] G.E. Hinton, N. Srivastava, A. Krizhevsky, I. Sutskever, R. Salakhutdinov (2012), arXiv:1207.0580
[8] J. Cogan, M. Kagan, E. Strauss, A. Schwartzman, JHEP 02, 118 (2015), arXiv:1407.5675
[9] L. de Oliveira, M. Kagan, L. Mackey, B. Nachman, A. Schwartzman (2015), arXiv:1511.05190
[10] T. Sjostrand, S. Mrenna, P.Z. Skands, Comput. Phys. Commun. 178, 852 (2008), arXiv:0710.3820
[11] T. Sjostrand, S. Mrenna, P.Z. Skands, JHEP 0605, 026 (2006), arXiv:0603175
[12] M. Cacciari, G.P. Salam, G. Soyez, JHEP 0804, 063 (2008), arXiv:0802.1189
[13] M. Cacciari, G.P. Salam, G. Soyez, Eur. Phys. J. C72, 1896 (2012), arXiv:1111.6097
[14] D. Krohn, J. Thaler, L.T. Wang, JHEP 1002, 084 (2010), arXiv:0912.1342
[15] J. Thaler, K. Van Tilburg, JHEP 1103, 015 (2011), arXiv:1011.2268
[16] A.J. Larkoski, D. Neill, J. Thaler, JHEP 04, 017 (2014), arXiv:1401.2158
[17] D. Scherer, A. Müller, S. Behnke, Evaluation of pooling operations in convolutional architectures for object recognition, in Artificial Neural Networks - ICANN 2010 (Springer, New York, 2010)
[18] X. Glorot, A. Bordes, Y. Bengio, Journal of Machine Learning Research 15, 315 (2011)
[19] F. Chollet, Keras, https://github.com/fchollet/keras (2015)
[20] D.P. Kingma, J. Ba, Proceedings of the 3rd International Conference for Learning Representations (2014), arXiv:1412.6980
[21] J. Gallicchio, M.D. Schwartz, Phys. Rev. Lett. 105, 022001 (2010), arXiv:1001.5027