Statistics of the Z–R Relationship for Strong Convective Weather over the Yangtze–Huaihe River Basin and Its Application to Radar Reflectivity Data Assimilation for a Heavy Rain Event

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ABSTRACT

The relationship between the radar reflectivity factor (Z) and the rainfall rate (R) is recalculated based on radar observations from 10 Doppler radars and hourly rainfall measurements at 6529 automatic weather stations over the Yangtze–Huaihe River basin. The data were collected by the National 973 Project from June to July 2013 for severe convective weather events. The Z–R relationship is combined with an empirical q–R relationship to obtain a new Z–q relationship, which is then used to correct the observational operator for radar reflectivity in the three-dimensional variational (3DVar) data assimilation system of the Weather Research and Forecasting (WRF) model to improve the analysis and prediction of severe convective weather over the Yangtze–Huaihe River basin. The performance of the corrected reflectivity operator used in the WRF 3DVar data assimilation system is tested with a heavy rain event that occurred over Jiangsu and Anhui provinces and the surrounding regions on 23 June 2013. It is noted that the observations for this event are not included in the calculation of the Z–R relationship. Three experiments are conducted with the WRF model and its 3DVar system, including a control run without the assimilation of reflectivity data and two assimilation experiments with the original and corrected reflectivity operators. The experimental results show that the assimilation of radar reflectivity data has a positive impact on the rainfall forecast within a few hours with either the original or corrected reflectivity operators, but the corrected reflectivity operator achieves a better performance on the rainfall forecast than the original operator. The corrected reflectivity operator extends the effective time of radar data assimilation for the prediction of strong reflectivity. The physical variables analyzed with the corrected reflectivity operator present more reasonable mesoscale structures than those obtained with the original reflectivity operator. This suggests that the new statistical Z–R relationship is more suitable for predicting severe convective weather over the Yangtze–Huaihe River basin than the Z–R relationships currently in use.

Key words: Z–R relationship, Weather Research and Forecasting (WRF) model, three-dimensional variational (3DVar) system, data assimilation, observation operator

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1. Introduction

In numerical models, radar reflectivity data are generally used for moist processes by the definition of the observation operator that converts the model control variables into the equivalent observed quantities. One way to introduce reflectivity data into numerical models is the assimilation of the precipitation rate, which is obtained from the relationship between the radar reflectivity factor (Z) and the rainfall rate (R). For example, Lopez and Bauer (2007) used the 1D + 4D-Var (one-dimensional + four-dimensional variational) method to assimilate the NCEP stage-IV analyses of hourly accumulated surface precipitation over the US mainland, and their experimental results showed that NEXt-generation RADars (NEXRAD) data could be beneficial in the analyses and
subsequent forecasts of the moisture field. Zhang et al. (2012) used the 1D + 3D-Var method to assimilate the precipitation rate to produce a better water vapor field for high-resolution models, in which the precipitation rate was calculated from the radar reflectivity based on the $Z-R$ relationship with $Z = 200R^{1.6}$.

The other way to introduce reflectivity data into numerical models is the direct assimilation of reflectivity data, which is implemented through the relationship between reflectivity and the rainwater mixing ratio ($Z-q_r$), as used in the WRF (Weather Research and Forecasting) 3DVar (Xiao et al., 2007). With the assumption of clouds in a stationary state with respect to warm microphysical processes, the other related variables—including the cloud water mixing ratio ($q_c$), the cloud water mixing ratio ($q_r$) and the vertical velocity ($\omega$)—can be obtained through the rain water mixing ratio ($q_r$) (Liu et al., 2007). Furthermore, Sun and Crook (1997) reported that the $Z-q_r$ relationship can be obtained by eliminating the rainfall rate between the Marshall–Palmer expressions from the $Z-R$ and $q_r-R$ relationships (Battan, 1973). Therefore, the appropriate $Z-R$ relationship plays an important part in the assimilation of radar reflectivity data.

Since the theory of radar-based precipitation estimation was proposed by Bent (1943), many $Z-R$ relationships have been established based on different raindrop spectrum data successively (e.g., Marshall and Palmer, 1948; Marshall et al., 1955; Jones, 1956; Imai, 1960; Fujiwara, 1965; Jatila and Puhakka, 1972; Joss and Gori, 1978; List, 1988; Huggel et al., 1996; Caracciolo et al., 2008; Moumouni et al., 2008; Tapiador et al., 2010; Bamba et al., 2014). The $Z-R$ relationship varies in different regions and for different types of precipitation (Stout and Mueller, 1968; Steiner and Houze, 1997; Rao et al., 2001; Lee et al., 2002). Some relevant works have been carried out in the middle and lower reaches of the Yangtze and Huaihe River basins using radar observations (Liu et al., 1999; Chi et al., 2000; Li et al., 2005; Chen et al., 2006; Wong et al., 2006; Yao et al., 2007; Liu et al., 2010; Wang et al., 2013; Xu et al., 2013). For example, Liu et al. (1999) classified radar reflectivity according to its value and obtained a sequence of $Z-R$ relationships using one 731 weather radar and rainfall measurements from 52 automatic weather stations (AWSs) and 7 meteorological stations. Xu et al. (2013) established $Z-R$ relationships in terms of different types of precipitation along the Yangtze River in Jiangsu Province and found that the estimated accuracy of precipitation was improved by different degrees for Meiyu and typhoon precipitation compared with the single $Z-R$ relationship.

The Observation, Prediction, and Analysis of severe Convection of China (OPACC) project, a 5-yr National 973 Fundamental Research Program, emphasizes the effective utilization of high-resolution, modern operational observing networks, as well as special experimental observing facilities, including polarimetric Doppler radars, during intensive observing periods (IOPs; Xue, 2016). Large amounts of observational data have been collected for mesoscale convective systems over the lower Yangtze and Huaihe River basins during IOPs, including observations from Doppler radars, wind profile radars, rain gauges, satellites, and lightning location systems. With the favorable information, we recalculated the $Z-R$ relationship for severe convective weather over the Yangtze–Huaihe River basin based on observations from multiple Dopple radars and AWSs collected by the OPACC project. A new $Z-q_r$ relationship is then obtained by combining the recalculated $Z-R$ relationship with the $q_r-R$ relationship reported by Battan (1973), which is used to correct the observational operator for the assimilation of reflectivity data in the WRF 3DVar system. The effectiveness and performance of the corrected reflectivity operator are tested with a heavy rain event that occurred on 23 June 2013, observations for which are not used for the statistics of the $Z-R$ relationship. All experiments are conducted with WRF model and its 3DVar system.

The paper is organized as follows: Section 2 describes the data used for the statistics of the $Z-R$ relationship, in which the observational data and their quality control are introduced. Section 3 presents the calculation of the $Z-R$ relationship, and a new $Z-q_r$ relationship to correct the reflectivity operator used in the WRF 3DVar system is given in Section 4. Section 5 examines the application of the corrected reflectivity operator to a heavy rain event that occurred on 23 June 2013. A summary and discussion are presented in Section 6.

## 2. Data

All the data used in this paper are provided by the OPACC project. In June and July 2013 and 2014, about 20 IOPs focusing on the lower Yangtze and Huaihe River basins in Jiangsu and Anhui provinces were carried out (Xue, 2016). We processed the observations from 6 IOPs between 23 June and 21 July 2013 (Table 1), in which the last five IOPs were used to recalculate the $Z-R$ relationship and the IOP of 23 June 2013 was used in reflectivity data assimilation experiments.

In the lower reaches of the Yangtze and Huaihe River basins, 10 Chinese Next Generation Weather Radars
CINRAD-SA/SB are deployed in the cities of Shangqiu, Hefei, Fuyang, Nanjing, Nantong, Xuzhou, Lianyungang, Hangzhou, Ningbo, and Jiujiang (Fig. 1). Each radar provides volume scan data with nine elevation angles (0.5°, 1.45°, 2.4°, 3.35°, 4.3°, 6.0°, 9.9°, 14.6°, and 19.5°) every six minutes. Within the areas covered by these radars, there are 6529 AWSs providing hourly precipitation data.

First of all, we select the radar reflectivity and rainfall data from the five IOPs at times when both reflectivity and rainfall data are available. The rainfall measurements are subjected to quality control processes, such as inspection revisions (Ren et al., 2010). The reflectivity data are processed by using the quality control method in the Weather Surveillance Radar-1988 Doppler precipitation estimation algorithm (Fulton et al., 1998), which includes the removal of super-refraction echoes and isolated points, and the detection of singular echoes. A scanning composite plane is then generated in which the reflectivity data are extracted from the fourth tilt (3.35°) within 20 km, from the third tilt (2.4°) between 20 and 35 km, from the second tilt (1.5°) between 35 and 50 km, and from the first tilt (0.5°) between 50 and 230 km.

Through the quality control procedure and the generation of the scanning composite plane, a total of 1386 samples are obtained. Each sample consists of scanning composite planes and the available hourly precipitation data that are valid at the same time. Considering that the 1386 samples cover multiple stages of severe precipitation events—such as occurrence, development, and disappearance—we reselected samples for the statistics of the $Z-R$ relationship to exclude interference from weaker phases during severe weather processes. The proportion of reflectivity $\geq 30$ dBZ (dBZ30) of a single sample is simply used as the screening standard. For all 1386 samples, the value of dBZ30 ranges from 0.004 to 0.373 and the sample size varies with different values of dBZ30. For example, the number of samples is 969 when dBZ30 = 0.1, but only 396 samples for dBZ30 = 0.2 and 28 samples for dBZ30 = 0.3. We select 891 samples (64.3% of the total number of samples, when dBZ30 = 0.12) for the statistics of the $Z-R$ relationship.

### 3. Establishment of $Z-R$ relationship

The radar reflectivity factor ($Z; \text{mm}^6 \text{m}^{-3}$) is commonly converted to an estimated precipitation rate ($R; \text{mm h}^{-1}$) through a power law relationship:

$$Z = AR^b,$$

where $A$ and $b$ are parameters related to the raindrop spectrum, which are in the ranges of 31–500 and 1.1–1.9, respectively (Chumchean, 2004).

The $Z-R$ relationship is calculated with a widely used optimization method, in which the discriminant function is given as

$$C_{TF} = \min \left\{ \sum_{i=1}^{n} \left[ (R_i - I_i)^2 + (R_i - I_i) \right] \right\},$$

where $R_i$ is the rainfall estimated from radar reflectivity and $I_i$ is the hourly rainfall measured by the AWSs. The optimum values of $A$ and $b$ in the $Z-R$ relationship are determined by adjusting the values of $A$ and $b$ to find a minimum value for $C_{TF}$. Based on data from the 891 samples described in Section 2, we obtained new statist-
ics for the $Z–R$ relationship for severe convective weather over the Yangtze–Huaihe River basin:

$$Z = 109R^{1.74}.$$  \hfill (3)

To preliminarily evaluate the applicability of this new $Z–R$ relationship over the Yangtze–Huaihe River basin, two $Z–R$ relationships are used as a comparison. One is the classic Marshall–Palmer relationship, $Z = 200R^{1.6}$ (Marshall and Palmer, 1948; Marshall et al., 1955), and the other is $Z = 300R^{1.4}$, which is used by the National Weather Service for convective rainfall (Fulton et al., 1998) and the new generation of weather radar precipitation series in China (Yao et al., 2007). Table 2 shows the values of $C_{TF}$ from the three $Z–R$ relationships. It can be seen that the $C_{TF}$ value calculated from the new $Z–R$ relationship decreases by 18.3% and 16.4%, compared with the existing $Z–R$ relationships of $Z = 300R^{1.4}$ and $Z = 200R^{1.6}$, respectively. This implies that the new $Z–R$ relationship gives an improved estimation of precipitation from radar observations.

To further illustrate the role of new $Z–R$ relationship, Fig. 2 shows these three $Z–R$ relationships and the scattered pairs of $Z$ and $R$. Compared with the existing relationship of $Z = 300R^{1.4}$ for convective rainfall (Fig. 2, dotted line), the new $Z–R$ relationship (Fig. 2, dashed line) produces a larger value of $R$ for $Z < 42.9$ dBZ, but a smaller $R$ for $Z > 42.9$ dBZ when this difference increases significantly with increasing $Z$. The rainfall rate derived from the new $Z–R$ relationship is larger for values of $Z$ between 25 and 50 dBZ relative to the relationship of $Z = 200R^{1.6}$. For example, the rainfall rate calculated from the new $Z–R$ relationship increases by 16.43% when $Z = 40$ dBZ. From the value of $C_{TF}$ in Table 2 and the distribution of scattered data shown in Fig. 2, the new $Z–R$ relationship should be more reasonable for severe convective weather over the Yangtze–Huaihe River basin than the other two $Z–R$ relationships.

### 4. Correction of reflectivity operator

In the WRF 3DVar system, radar reflectivity data are assimilated through the relationship between the radar reflectivity ($Z$) and the rainwater mixing ratio ($q_r$), which is expressed by

$$Z = 43.1 + 17.5\log(\rho q_r),$$  \hfill (4)

where $\rho$ (kg m$^{-3}$) is the density of the atmosphere and $q_r$ (g kg$^{-1}$) is the rainwater mixing ratio. It should be noted that Eq. (4) is derived analytically by assuming the Marshall–Palmer distribution for raindrop size (Sun and Crook, 1997).

The $Z–q_r$ relationship can be obtained by eliminating the rainfall rate between the Marshall–Palmer expressions for the $Z–R$ and $q_r–R$ relationships (Battan, 1973), as described in Sun and Crook (1997). Therefore, we obtain a new $Z–q_r$ relationship according to the recounted $Z–R$ relationship in Section 2.2:

$$Z = 43.0 + 19.77\log(\rho q_r).$$  \hfill (5)

The $Z–q_r$ relation in Eq. (5) is used to update Eq. (4) as a correction of the observational operator for the reflectivity data in the WRF 3DVar system. The two $Z–q_r$ relationships are shown in Fig. 3. In addition, the $Z–q_r$ relationship derived from $Z = 200R^{1.6}$ is plotted in Fig. 3 as a reference, i.e., $Z = 43.78 + 18.2\log(\rho q_r)$. It can be found that the variations of $q_r$ with $Z$ are consistent with

| $Z–R$ relationship  | $C_{TF}$  |
|---------------------|-----------|
| $Z = 109R^{1.74}$   | 690,578   |
| $Z = 300R^{1.4}$    | 845,217   |
| $Z = 200R^{1.6}$    | 825,638   |

**Fig. 2.** Scattered pairs of radar reflectivity $Z$ (dBZ) and corresponding rainfall rate $R$ (mm h$^{-1}$) over the Yangtze–Huaihe River basin from five IOPs in 2013 (gray crosses), as well as the $Z–R$ relationships from Marshall–Palmer (solid line), Fulton et al. (1998) (dotted line), and new statistics in this study (dashed line).

**Fig. 3.** Three $Z–q_r$ plots, from the WRF 3DVar system (solid line), the Marshall–Palmer (dotted line), and the recounted $Z–R$ relationship in this study (dashed line). The inset graph shows an enlargement of the $Z–q_r$ curves between 30 and 50 dBZ.
5. Case study for the corrected reflectivity operator

5.1 Case overview

A heavy rainfall event, that occurred in Jiangsu and Anhui provinces and surrounding regions on 23 June 2013, was captured by seven S-band Doppler radars located in the cities Fuyang, Hefei, Lianyungang, Nantong, Shangqiu, Xuzhou, and Yancheng in China. At 0000 UTC 23 June 2013, a convective cell was located at the junction of Henan and Anhui provinces and another convective cell was located near Nanjing City. The system then developed and moved southward and formed an echo band over Hefei, Nanjing, Fuyang, and the surrounding region at about 0230 UTC, with a maximum echo intensity of 55 dBZ around Hefei and Nanjing. The strong radar echo band had moved out of Hefei and Nanjing by 0600 UTC. The maximum hourly precipitation reached 43.5 mm h\(^{-1}\) at 0300 UTC 23 June 2013 in Feixi County, Hefei City, Anhui Province.

Heavy rainfall occurs in a favorable large-scale environmental field. At 1200 UTC 22 June 2013, the center of the South Asian high was located over the Qinghai–Tibetan Plateau and its eastern ridge point was located near 128°E at 200 hPa. There was an upper-level jet (39°–41°N, 70°–150°E) on the north side of the South Asian high with a maximum wind speed of up to 53 m s\(^{-1}\) (Fig. 4a). The area covered by the jet increased over time, and Jiangsu and Anhui provinces were located on the right-hand side of the jet at 0000 UTC 23 June 2013 (Fig. 4b). At 500 hPa, the mid- and upper-latitude areas of East Asia presented a circulation scenario referred to as “two troughs and one ridge”. In this case, a shortwave trough moved eastward in the midlatitude areas and the 588-ridge line of the steady western Pacific subtropical high stretched westward to 120°E, providing abundant water vapor for this heavy rainfall event (Figs. 4c, d). At 700 hPa, Jiangsu and Anhui provinces and their surrounding areas were located to the front of the trough and the low-level jet was located in the north of these areas (Fig. 4e). At 0000 UTC 23 June 2013, the low-level jet moved into Jiangsu and Anhui provinces and surrounding areas and the wind speed at the center of the low-level jet reached 18 m s\(^{-1}\) (Fig. 4f).

The above analysis shows that heavy rainfall occurred under a favorable large-scale circulation that included the upper-level divergence caused by the South Asian high and the upper-level jet at 200 hPa, the eastward motion of a shortwave trough at 500 hPa, and abundant water vapor provided by the steady western Pacific subtropical high. The coupling between the westerly upper-level jet and the low-level jet is conducive to the development of vertical movement over Anhui and Jiangsu provinces. The trough line at 700 hPa and the low-level jet are mesoscale systems that caused this heavy rainfall event.

5.2 Experimental design

The WRF model (version 3.5) and its 3DVar system are used to perform reflectivity data assimilation experiments. The model is configured with two one-way nested domains with grid spacings of 15 and 5 km, respectively. The outer domain covers mainland China and the inner domain covers Jiangsu and Anhui provinces and their surrounding regions. There are 30 vertical layers with the model top at 50 hPa. The physical parameterization schemes used in all domains include the Rapid Radiative Transfer Model longwave radiation (Mlawer et al., 1997), the Dudhia shortwave radiation (Dudhia, 1989), the Mellor–Yamada–Janjić planetary boundary layer scheme (Janjić, 1994), the WRF Single-Moment six-class microphysics scheme (Hong et al., 2004), and the Noah land surface scheme (Chen and Dudhia, 2001). The Grell–Dévényi cumulus scheme (Grell and Dévényi, 2002) is only used for the outer domain. The WRF simulation without data assimilation starts at 2300 UTC 22 June 2013 and ends at 1200 UTC 23 June 2013. The initial and boundary fields for the outer domain are obtained by interpolating the NCEP-FNL analysis dataset. The initial and boundary fields for the inner domain are derived from the model output in the outer domain.

The 30-min update cycle is performed in the inner domain with reflectivity data assimilation during 0000–0100 UTC 23 June 2013 (Fig. 5). At the start of the analysis (0000 UTC 23 June 2013), the 1-h forecast from the WRF model run for the inner domain initiated at 2300 UTC 22 June 2013 provides the first guess for the initial 3DVar analysis. Then the 30-min WRF forecast started
from the current analysis provides the first guess for the next analysis. Subsequently, an 11-h forecast is made starting from the last analysis at 0100 UTC 23 June 2013. During the assimilation process, reflectivity data from seven S-band Doppler radars, which have been passed through quality-control check described in Section 2, are mapped into the gridded WRF model coordinates using the 88D2ARPS module in the Advanced Regional Prediction System (Xue et al., 1995).

To test the performance of the corrected $Z-q_r$ relationship used in the assimilation of the reflectivity data, two assimilation experiments are conducted with the WRF 3DVar system: experiments REF and REF_NEW. The REF experiment is a radar data assimilation process with the WRF 3DVar system in which only the reflectivity data are assimilated. The REF_NEW experiment is similar to the REF experiment, except the use of the new $Z-q_r$ relationship in the WRF 3DVar system. The experiment
without the assimilation of reflectivity data (CON) is used to examine the impact of the assimilation of reflectivity data on the analysis and forecast of this heavy rain event.

In the assimilation experiments, the background error covariance is generated by using the gen_be module in the WRF 3DVar system, according to the National Meteorological Center method (Parrish and Derber, 1992). A set of forecasts over the inner domain are initiated from the NCEP-FNL analysis at 0000 and 1200 UTC every day in June 2013. The differences between the 24- and 12-h forecasts valid at the same time during the month are used to calculate the domain-averaged background error statistics.

5.3 Analyzed field

The analyzed horizontal wind, water vapor, equivalent temperature, and vertical velocity fields from the three experiments (CON, REF, and REF_NEW) are compared in an attempt to understand the effect of the corrected reflectivity operator on the analysis and forecast results. Figure 6 shows the background and analyzed fields from the two assimilation experiments (REF and REF_NEW) for the 850-hPa horizontal wind at 0100 UTC 23 June 2013. Without the assimilation of reflectivity data (Fig. 6a), the wind fields converge in an area of strong echo near Liu’an City, Anhui Province (A in Fig. 6a), where wind shear is formed by the convergence of northerly and westerly winds. At the same time, a southwesterly wind dominates in other areas of strong echo (B in Fig. 6a), covering the cities of Chuzhou, Nanjing, Yangzhou, and Taizhou and their surrounding regions. With the assimilation of reflectivity data, the northwesterly and southwesterly winds converge in the area of strong echo marked by B in Fig. 6b and the convergence strengthens in area A, which is conducive to the generation and development of convective activity. The analyzed wind field obtained from the REF_NEW exper-

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**Fig. 5.** Schematic diagram of the experimental design illustrating the data assimilation (DA) cycle and forecast.

**Fig. 6.** (a) 850-hPa background wind field, the analyzed field (arrows), and increments of analysis (shaded) from the two assimilation experiments (b) REF and (c) REF_NEW at 0100 UTC 23 June 2013.
iment (Fig. 6c) is mostly consistent with that from the REF experiment, but the cyclonic convergence in the analyzed wind field obtained from the REF_NEW experiment is stronger than that from the REF experiment in the areas of strong echo (A and B), which is clearly seen in the shaded areas in Figs. 6b, c.

The analyzed horizontal winds at 500 hPa indicated (figure omitted) that the wind is modified from westerly to southwesterly in the area of strong echo covering Chuzhou, Jiangsu, and the surrounding areas (B) when the reflectivity data are assimilated, which results in deepening of the trough. At the same time, the divergence at 200 hPa increases in these areas by modification of the wind from westerly to northwesterly. This configuration of upper-level divergence and lower-level convergence could provide dynamic lifting conditions for the generation and development of precipitation. The results from experiments REF and REF_NEW are similar, but the convergence of winds in REF_NEW is more intense than that in REF.

Figure 7 shows the 850-hPa water vapor mixing ratio $q_v$ at 0100 UTC 23 June 2013 from the three experiments (CON, REF, and REF_NEW). Compared with the results from experiment CON (Fig. 7a), the value of the water vapor mixing ratio from the two assimilation experiments (Figs. 7b, c) is increased in the areas of strong echo (regions B, C, and D in Fig. 7). The location of high values of $q_v$ in region D moves to the north when the reflectivity data are assimilated. This indicates that the water vapor mixing ratio is adjusted by the assimilation of radar reflectivity data in radar echo regions. The adjusted pattern of high water vapor mixing ratios is more consistent with the distribution of the areas of strong echo (Fig. 7). It should be noted the areas of reflectivity denoted by regions C and D in Fig. 7 are the same as those in region A in Fig. 6, but regions C and D are located to the western and eastern edges, respectively, of region A.

The pattern of the analyzed field for water vapor from experiment REF_NEW (Fig. 7c) is similar to that from experiment REF (Fig. 7b). However, the coverage of the water vapor mixing ratio exceeding 0.016 kg kg$^{-1}$ is larger than that from experiment REF in the areas of strong echo (regions B, C, and D). The increase in the water vapor mixing ratio from experiment REF_NEW can reach 0.002 kg kg$^{-1}$ relative to experiment REF. A reduction with a maximum of 0.0008 kg kg$^{-1}$ from experiment REF_NEW occurs in the center of region B (32.5° N, 119° E) where the radar echo is greater than 43.87 dBZ (Fig. 7b, c). This adjustment of the water vapor mixing ratio is consistent with the change in the reflectivity operator from the $Z$–$q_r$ relationship used in WRF 3DVar to the new $Z$–$q_r$ relationship.

Figure 8 shows a longitude–pressure cross-section of equivalent temperature and vertical velocity along 32.3°N at 0100 UTC 23 June 2013 from three experiments. It can be seen that the isentropes from two assimilation experiments (Figs. 8b, c) are denser than those from experiment CON (Fig. 8a) in the areas of strong radar echoes (116°, 119°, and 120°E, respectively). At the same time, the results from two assimilation experiments (Figs. 8b, c) show that a warm center occurs over the areas of strong echo, where warm air is lifted from the lower troposphere to the middle and upper troposphere. This suggests that the atmosphere is seen as more unstable when assimilating radar reflectivity data.

Compared with experiment REF (Fig. 8b), the isentropes obtained from experiment REF_NEW are denser in the regions of strong echo, and the uplifted warm air obtained by experiment REF_NEW is more in coverage.
and height (Fig. 8c). For example, the coverage of the equivalent temperature line exceeding 356 K from experiment REF_NEW is larger than that from experiment REF in the areas of strong echo at 116°, 119°, and 120°E. The warm center at 360 K occurs at 400 hPa in experiment REF_NEW, but at 680 hPa in experiment REF (Figs. 8b, c). This suggests that the use of the new reflectivity operator gives a convectively unstable layer with a larger range and greater intensity, which is more conducive to the occurrence and development of convection.

A strong upward motion is a necessary condition for the occurrence of heavy rainfall. Through the assimilation of reflectivity data in experiment REF_NEW, two updraft regions are seen near the areas of radar strong echo at about 118.5° and 120°E (Fig. 8c). The maximum vertical velocity reaches 6 m s\(^{-1}\) at about 400 hPa. While experiment REF shows one updraft flow at about 120°E, but does not detect the other updraft at about 118.5°E (Fig. 8b). Furthermore, the magnitude of the updraft at 120°E from experiment REF is less than that from experiment REF_NEW, which is consistent with the analysis of the horizontal wind.

The characteristic of lower-level convergence and upper level divergence can also be seen in the longitude–pressure cross-section of divergence along 32.3°N from experiments REF and REF_NEW at 0100 UTC 23 June 2013 (Figs. 9b, c). Compared with experiment REF (Fig. 9b), the range and intensity of the lower-level convergence and upper-level divergence are larger in experiment REF_NEW (Fig. 9c), especially near 118.5°E. It can also be seen from Figs. 8c and 9c that the pumping action caused by the upper-level divergence and lower-level convergence produces significant upward motion in areas of strong echo, accompanied by a downdraft on both sides of the upward motion, which results in the formation of the secondary circulation. The vertical structure of the divergence field therefore explains the distribution of the vertical velocity.

There is a correspondence between the vorticity and divergence fields (Fig. 9). As shown in Fig. 9f, the cyclonic vorticity added over the regions of radar strong echo (near 118.5° and 120°E) through the assimilation of reflectivity data in experiment REF_NEW. The vorticity reaches maximum values of \(4.0 \times 10^{-4}\) and \(6.5 \times 10^{-4}\) s\(^{-1}\) at about 850 hPa near 118.5° and 120°E, respectively. The magnitude of the vorticity and the height of the positive vorticity from experiment REF_NEW are larger than in experiment REF (Fig. 9e). This analysis indicates that the corrected reflectivity operator has a better ability to adjust the physical variables and produce a more reasonable analysis than the original reflectivity observation operator used in the WRF 3DVar system.

### 5.4 Forecasts of composite reflectivity

The impact of the corrected radar reflectivity operator on the rainfall forecast was assessed by comparing the observed composite reflectivity and its forecasts from three experiments (CON, REF, and REF_NEW). Here, the composite reflectivity is defined as the maximum reflectivity in the vertical column (Xue and Martin, 2006) and the predicted composite reflectivity is derived from the predicted hydrometeors, including the mixing ratio for rainwater, snow, and hail.

The equitable threat scores (ETS; Rogers et al., 1996) of the predicted composite reflectivity for 10-, 20-, and
30-dBZ thresholds from three experiments are shown in Fig. 10. For the 10- and 20-dBZ thresholds, the ETSs of composite reflectivity from the radar data assimilation experiments are higher than those from experiment CON, especially within the 7-h forecasts. By contrast, the ETSs from the radar data assimilation experiments for the 30-dBZ threshold are only obviously higher than those from experiment CON within the first few hours, and the results are poorer after the 6-h forecast. This means that the assimilation of radar data has a positive effect on the forecast of the radar composite reflectivity in the first few hours of the forecast, and that experiment REF_NEW performs better than experiment REF.

Within the first 7-h forecasts, the ETSs for the 10- and 20-dBZ thresholds from experiment REF_NEW are higher than those from experiment REF (Figs. 10a, b). For the 30-dBZ threshold, the ETSs from experiment REF_NEW are higher than those from experiments REF and CON within the first 5-h forecasts, but the ETSs from experiment REF are lower than those from experiment CON after the 3-h forecasts (Fig. 10c). This implies that the new reflectivity operator increases the effective time for the assimilation of radar data in the prediction of strong reflectivity. This effective time can be extended from 3 to 5 h when the original reflectivity operator is updated by the new operator (Fig. 10c). This suggests that a suitable $Z-q_r$ relationship plays an important part in the assimilation of radar data, which has a significant impact on the forecast skill of composite reflectivity.

To display the performance of the corrected reflectivity operator more clearly, Fig. 11 shows the observed composite reflectivity and forecast results with experiments CON, REF, and REF_NEW at 0600 UTC 23 June 2013. At this time, the ETS for the 30-dBZ threshold from experiment CON is lower than that from experiment REF_NEW, but higher than that from experiment REF. Experiment CON captures this heavy rain in the pattern of the radar composite reflectivity, but the location of the radar echo is slightly to the south in Anhui province (Fig. 11b). The assimilation of the reflectivity data with REF (Fig. 11c) improves the location of the forecast composite reflectivity in Anhui Province, but the strong composite reflectivity observed in the west of Jiangsu Province cannot be predicted. The corrected reflectivity operator in experiment REF_NEW (Fig. 11d)
does, however, predict the reflectivity in the west of Ji-
angsu Province, and the pattern and position of the fore-
casted composite reflectivity are more consistent with the
observations than in experiments CON and REF. This
suggests that the corrected reflectivity operator improves
the assimilation effect of radar reflectivity.

6. Conclusions

Observations from multiple Doppler radars and AWSs
in June and July 2013 provided by the OPACC project
are used to investigate the Z–R relationship for strong
convective weather over the Yangtze–Huaihe River
basin. A new relationship for $Z - q_r$ is established by elim-
inating the rainfall rate between the $Z - R$ and $q_r - R$
relationships, and is then used to correct the observation op-
erator for radar reflectivity data in WRF 3DVar. Three
experiments are designed for a heavy rain event to test
the impact of the corrected reflectivity operator on the
analysis and forecast of the precipitation process. The re-
lated observational data for the test case are not used in
the statistics of the $Z - R$ relationship. The analysis of the
experimental results produced the following conclusions:

1) Compared with the existing $Z - R$ relationships ($Z =
300R^{1.4}$ and $Z = 200R^{1.6}$), the $Z - R$ relationship ($Z =
109R^{1.74}$) recalculated in this paper is more suitable for
strong convective weather over the Yangtze–Huaihe
River basin, as shown by the $C_{TF}$ value.

2) The assimilation of radar reflectivity data can im-
prove the quality of the initial analyzed field and sub-
sequently the prediction of radar composite reflectivity
within the first few hours, either with the original re-
flexivity operator or with the corrected operator. By con-
trast, the corrected observation operator based on the re-
calculated $Z - R$ relationship performed better than the ori-
ignal operator used in WRF 3DVar.

3) The use of the corrected reflectivity operator ex-
tends the effective time of radar data assimilation for the
prediction of strong reflectivity. For this heavy rain
event, the effective time was extended from 3 to 5 h for
the prediction of the radar composite reflectivity for the
30-dBZ threshold, which suggests that the $Z - R$ relation-
ship plays an important part in the assimilation of radar
data.

4) The recalculated $Z - R$ relationship is limited by the
observational samples. The corrected observational op-
erator for radar reflectivity has only been tested for a heavy
rain event. Although the preliminary experimental results show that it is more suitable for severe convective
weather over the Yangtze–Huaihe River basin, more ob-
Observational data are required to confirm the validation of the new $Z-R$ relationship, and more assimilation and forecast experiments with more weather events are needed to test the applicability of the new reflectivity operator to severe convective weather events.

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