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DISSIPATION AND EXTRA LIGHT IN GALACTIC NUCLEI: IV. EVOLUTION IN THE SCALING RELATIONS OF SPHEROIDS

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ABSTRACT

We develop a model for the physical origin and redshift evolution of spheroid scaling relations. We consider spheroid sizes, velocity dispersions, dynamical masses, profile shapes (Sersic indices), stellar and supermassive black hole masses, and their related scalings. Our approach combines advantages of prior observational constraints in halo occupation models and libraries of high-resolution hydrodynamic simulations of galaxy mergers. This allows us to separate the relative roles of dissipation, dry mergers, formation time, and evolution in progenitor properties, and identify their impact on observed scalings at each redshift. We show that, at all redshifts, dissipation is the most important factor determining spheroid sizes and fundamental plane scalings, and can account (at $z = 0$) for the observed fundamental plane tilt and differences between observed disk and spheroid scaling relations. Because disks (spheroid progenitors) at high redshift have characteristic larger gas fractions, this predicts more dissipation in mergers, yielding systematically more compact, smaller spheroids. In detail, this gives rise to a mass-dependent evolution in the sizes of spheroids of a given mass, that agrees well with observations. This relates to a subtle weakening of the tilt of the early-type fundamental plane with redshift, important for a number of studies that assume a non-evolving stellar mass fundamental plane. This also predicts evolution in the black hole-host mass relations, towards more massive black holes at higher redshifts. Dry mergers are also significant, but only for large systems which form early – they originate as compact systems, but undergo a number of dry mergers (consistent with observations) such that they have sizes at any later observed redshift similar to systems of the same mass formed more recently. Most of the observed, compact high-redshift ellipticals will become the cores of present BCGs, and we show how their sizes, velocity dispersions, and black hole masses evolve to become consistent with observations. We also predict what fraction might survive intact from early formation, and identify their characteristic $z = 0$ properties. We make predictions for residual correlations as well: e.g. the correlation of size and fundamental plane residuals with formation time of a given elliptical, that can be used as additional tests of these models.

Subject headings: galaxies: elliptical and lenticular, cD — galaxies: evolution — galaxies: formation — galaxies: nuclei — galaxies: structure — cosmology: theory

1. INTRODUCTION

Understanding the scaling relations between the photometric and kinematic properties of galaxy spheroids – their masses, sizes, velocity dispersions, and luminosities – is fundamental to explaining their origin. Any model which attempts to account for these correlations must, of course, also determine how they evolve as a function of redshift. In a $\Lambda$CDM universe, objects grow hierarchically, implying that ellipticals do not comprise a monolithic population formed at an early epoch, but have evolved and grown by mergers from $z \gtrsim 6$ to $z = 0$.

Observations are beginning to probe the populations of spheroids at these epochs, and reveal that there is indeed evolution in such scaling relations (see e.g. [Trujillo et al. 2006; McIntosh et al. 2003; Zirm et al. 2007; van Dokkum et al. 2008]), generally in the sense of massive ellipticals being more compact at high redshifts. Some of these systems, in fact, appear sufficiently compact (see e.g. Zirm et al. 2007; van Dokkum et al. 2008) that there may be no similar $z = 0$ analogs whatsoever, leading to considerable debate regarding the fate of such objects. Ultimately, the causes of such evolution and its implication for e.g. the fundamental plane of ellipticals at those redshifts (and today) are not well understood.

Moreover, observations now demonstrate that essentially all massive spheroids host a supermassive black hole at their centers (Kormendy & Richstone 1995). The mass of this black hole is tightly correlated with a variety of structural parameters: the mass ($M_{\text{BH}}$, velocity dispersion ($\sigma$), concentration/Sersic index ($\kappa$), and mass-to-light ratio ($M/L$). Recent results from gravitational lensing studies (Junor et al. 2006) demonstrate that the mass of these black holes increases with the mass of the host galaxy, consistent with the idea that black hole masses are determined by galaxy mass at some early time. It is also consistent with the notion that the mass of the black hole is tightly correlated with the total mass of the galaxy, which itself is a function of the mass of the dark matter halo.

Observations suggest that there may be evolution in the black hole-host galaxy scalings (e.g. Peng et al. 2006, but the details remain ambiguous (see e.g. Woo et al. 2006; Salviander et al. 2006; Lauer et al. 2007a). As pointed out in Hopkins et al. 2007b, at least some such evolution should
be a natural consequence of evolution in host spheroid scaling relations, provided that there is some fundamental quantity or property traced by black hole mass. Again, if true, this is of basic importance for models of black hole growth, AGN and quasar activity, star formation, and galaxy formation and evolution.

A number of important questions therefore arise: If indeed the observations are correct, why are these high-redshift spheroids different from their analogs at $z = 0$? What can this tell us about the process of spheroid formation? Can we use them to constrain and test models for spheroid formation, quenching, and black hole growth? Are such objects typical, or are they the result of complex selection effects? What happens to them by $z = 0$? Can we find the remnants or left-overs of the population today? Are evolution in spheroid structural properties related to evolution in their hosted black holes? And how is this connected to the cosmological process of quasar fueling and black hole growth?

In a series of papers, Hopkins et al. (2008a,b,c,d) (hereafter Paper I-Paper IV), we combined libraries of hydrodynamics simulations of galaxy merger remnants and observations of nearby ellipticals spanning the largest available dynamic range in e.g. spatial scale, surface brightness, and galaxy properties, and developed a methodology by which we could empirically separate spheroids into their two dominant physical components. First, a dissipationless component – i.e. an “envelope,” formed from the violent relaxation/scattering of stars that are already present in merging stellar disks that contribute to the final remnant. Because disks are very extended, with low phase-space density (and collisionless processes in a merger cannot raise this phase-space density), these stars will necessarily dominate the profile at large radii, hence the envelope, with a low central density.

Second, a dissipational component – i.e. a dense central relic of starbursts from gas that has been brought into the remnant, lost its angular momentum, and turned into stars in a compact central starburst similar to those observed in e.g. local ULIRGs (although these would represent the most extreme cases). Because gas can radiate, it can collapse to very high densities, and will dominate the profile within radii $\sim 0.5-1$ kpc, accounting for the high central densities in ellipticals. The gas will probably reflect that initially brought in from merging disks, but could in principle also stem from new cooling or stellar mass loss in the elliptical (see e.g. Ciotti & Ostriker 2007). In subsequent mergers the two components will act dissipationlessly (they are just stars and dark matter), but the segregation between the two is sufficient that they remain distinct even after multiple major dissipationless or “dry” re-mergers: i.e. one can still, in principle, distinguish the dense central stellar component that is the remnant of the combined dissipational starburst(s) from the less dense outer envelope that is the remnant of low-density disk stars.

In Paper I we developed a methodology to empirically separate these components in observed systems with data of sufficient quality, and tested this on observed samples of nearby merger remnants from Rothberg & Joseph (2004). Using e.g. comparison with stellar populations, colors, and other properties as well as direct comparison of simulations with observed surface brightness profiles, galaxy shapes, and kinematics, we confirmed that this could reliably extract the dissipational component of the galaxy (see also Kormendy et al. 2008, and references therein). In Paper II we extended this to observed ellipticals with central “cusps” and in Paper III to those with central “cores” (believed to have undergone subquent dry mergers, but as we argue, still retaining evidence for the initial dissipational/dissipationless stellar components), from Kormendy et al. (2008) and Lauer et al. (2007). In Paper IV, we used these results to argue that the relative mass fraction in the dissipational component – i.e. the total mass fraction of the elliptical built up dissipationally, as opposed to brought in as stars in stellar disks, was the most important parameter determining the sizes and densities of ellipticals at $z = 0$. We showed that this fraction depends systematically on mass: low mass ellipticals experienced more dissipation than high-mass ones (presumably reflecting the well-established observational fact that low-mass disks are more gas-rich than higher-mass disks). We then empirically demonstrated that this systematic dependence is both necessary and sufficient to explain e.g. the difference between the size-mass relation of ellipticals and that of their (ultimate) progenitor disks, and to explain the observed “tilt” in the fundamental plane relation of spheroids.

There are, however, limitations in our ability to theoretically model these processes using idealized simulations and to understand them observing only $z = 0$ ellipticals. Many galaxies are expected to have had complex merger histories (perhaps forming early in a rapid series of multiple, highly dissipational mergers, rather than a single disk-disk merger at late times), which might, in principle, “smear out” or bias some of these comparisons or systematically alter some predictions (see e.g. Kobayashi 2004, Naab et al. 2007). Moreover, the gas content in pre-merger disks (and possibly in ellipticals themselves) is both expected and observed (e.g. Erb et al. 2006) to evolve strongly as a function of redshift. The disks building up ellipticals at $z \geq 2$ are much more gas rich, so their remnant ellipticals at these times would necessarily be more dissipational, and as such might have different structural properties and obey different scaling relations from their $z = 0$ counterparts. It is therefore of particular interest to combine our empirical understanding of the role of dissipation from local samples, and our inferences from idealized simulations, with fully cosmological models that can account for the evolution in elliptical progenitors and merger histories with redshift.

Some initial attempts at studying these correlations have been made using cosmological simulations (Naab et al. 2007; Di Matteo et al. 2008; Sijacki et al. 2008; Croft et al. 2008). It is, unfortunately, prohibitively expensive to simultaneously resolve the $\sim 100$ pc scales necessary to meaningfully predict e.g. spheroid scaling lengths, black hole masses, and central velocity dispersions in a fully cosmological simulation which would also contain a volume capable of modeling a large number of galaxies up to masses $\sim 10^{12} M_\odot$. Moreover, disk formation remains an unsolved problem in such simulations, with the uncertain physics of e.g. star formation and feedback probably important to form disks with the appropriate scale lengths, thicknesses, and bulge-to-disk ratios for their masses (Robertson et al. 2004; D’Onghia et al. 2006; Governato et al. 2007; Ceverino & Klypin 2007; Zavala et al. 2007). If initial disks do not have the appropriate observed structural properties, then even a perfect model for spheroid formation will have severe systematic inaccuracies.

In order to get around some of these limitations, alternative attempts have been made to instead predict scaling resolutions from semi-analytic models of galaxy formation (Khochfar & Silk 2006; Almeida et al. 2007). Although they could not calibrate the details of their predictions with nu-
merical simulations, Khochfar & Silk (2006) used this to illustrate, with very simple assumptions, that increasing dissipational fractions in ellipticals at higher redshift leads to the expectation that spheroids should be more compact. Almeida et al. (2007) explicitly ignored dissipation, and as a result found that none of the models they considered could reproduce the observed $z = 0$ fundamental plane correlations of spheroids (nor their redshift evolution).

Such models, however, potentially suffer from the same problems regarding the nature of disks (the “initial conditions”): uncertainties related to modeling spheroid formation are difficult to disentangle from prescriptions for e.g. cooling, disk growth, star formation, and feedback. Moreover, while a few properties can easily be estimated (in a mean sense) from a priori analytic arguments, many important properties and physical factors affecting spheroid evolution – the role of dissipation in changing the shape and velocity structure of a spheroid, the detailed density profiles (especially at small radii and in extended envelopes) of ellipticals, and the dark matter to stellar or baryonic mass ratios within various radii in the galaxy – require detailed, high-resolution numerical simulations to robustly model.

One way of circumventing the problems in these cosmological models is to adopt a halo occupation distribution (HOD) approach, in which (beginning from a halo+subhalo distribution determined in numerical simulations) halos are populated with galaxies according to empirical constraints on their clustering properties. The success of such models at reproducing the observed distributions at a variety of redshifts, as a function of various galaxy properties, has now been demonstrated (see e.g. Conroy et al. 2006). This allows us to begin with the empirical knowledge of where galaxies lie, and what their gas properties are, without reference to some (potentially incomplete) a priori model of e.g. star formation and feedback (which we are not, in any case, interested in testing here). Using this approach to identify and follow galaxies and their subsequent mergers, we can then reference each merger to an idealized hydrodynamic simulation with similar properties (e.g. initial morphologies, gas fractions, sizes, masses, etc.), and use this to predict what the properties of the remnant should be. Constructing a sample of such remnant spheroids at any given redshift is straightforward, and allows us to compare with observed scaling relations in a direct manner.

In this paper, we adopt this method to combine the advantages of high-resolution simulations, semi-analytic models, and observational constraints on spheroid progenitors in order to develop robust predictions for the evolution of spheroid structure and the correlation between black hole mass and host properties. We study how e.g. the evolution in disk gas fractions, structural properties, and the interplay between gas-rich and gas-poor mergers and multiple mergers in a hierarchical cosmology relate to observed correlations from $z = 0$ – 4.

In § 2 we describe our methodology, including our cosmological models of merger histories (§ 2.2), empirical models for progenitor disks (§ 2.3), and library of simulations which we use to determine remnant properties (§ 2.4). In § 3 we consider and compare observations to the resulting predicted scalings of spheroids at $z = 0$, and specifically highlight the effects of dissipation (§ 3.1) and dry mergers (§ 3.2). In § 4 we make a number of predictions and comparisons to observations regarding how these correlations evolve with redshift, and again illustrate the effects of dissipation (§ 4.2) and dry mergers (§ 4.3). In § 5 we follow the history of individual systems forming at different redshifts to compare them at later redshifts, and discuss the fates of early-forming galaxies which (initially) reflect these evolved correlations. Finally, in § 6 we summarize our results and outline future observational tests.

Throughout, we adopt a WMAP5 ($\Omega_M, \Omega_{\Lambda}, h, \sigma_8, n_s) = (0.274, 0.726, 0.705, 0.812, 0.960$) cosmology (Komatsu et al. 2008), and normalize all observations and models shown to these parameters. Although the exact choice of cosmology may systematically shift the associated halo masses (and high-redshift evolution) at a given galaxy mass (primarily scaling with $\sigma_8$), our comparisons (in terms of stellar mass) are for the most part unchanged (many of these differences are implicitly normalized out in the halo occupation approach). Repeating our calculations for a “concordance” (0.3, 0.7, 0.7, 0.9, 1.0) cosmology or the WMAP1 (0.27, 0.73, 0.71, 0.84, 0.96) and WMAP 3 (0.268, 0.732, 0.704, 0.776, 0.947) results of Spergel et al. (2003) and Spergel et al. (2007) has little effect on our conclusions. We also adopt a Salpeter (1955) stellar initial mass function (IMF), and convert all stellar masses and mass-to-light ratios accordingly. Again, the choice of the IMF systematically shifts the normalization of stellar masses herein, but does not substantially change our comparisons. All magnitudes are in the Vega system, unless otherwise specified.

2. THE SIMULATIONS AND COSMOLOGICAL MODEL

2.1. Overview

The model we will adopt consists of the following steps, summarized here and described in detail below:

- We construct the halo+subhalo mass function, and populate this with galaxies of various masses according to a standard halo occupation model approach. We allow this population to vary within the range allowed by observational constraints, but find this to be a small source of uncertainty.

- We assign un-merged galaxies a scale length and gas fraction appropriate for their stellar mass and redshift according to observational constraints (with appropriate scatter). We also systematically vary assignments to span the range allowed by observational constraints, and find this is the dominant source of uncertainty in our predictions.

- We evolve this model forward in time (following the dark matter) and identify mergers. After a merger, the properties of the remnant galaxy are calculated as a function of the progenitor properties, according to the results from hydrodynamic simulations.

- At some later time, we construct a mock catalogue (uniformly sampling the mock population in stellar mass) and compare the predictions with observed galaxy properties.

The exact implementation of the halo occupation model and merger rate calculations are outlined in the Appendix, for readers interested in reproducing the model calculations. Here we outline the relevant physics and model approach, and highlight the calibrations from simulations used to predict the properties of merger remnants, as well as the important sources of uncertainty for those predictions.
2.2. Cosmological Model

We wish to track merger histories in a cosmologically motivated manner, in order to follow the growth of spheroids. As discussed in §1, it remains prohibitive to simulate the relevant hydrodynamic processes (with the necessary \( \leq 100 \, \text{pc resolution} \)) in full cosmological simulations (where, in order to probe the massive galaxies of interest here, we would require \( \sim \, \text{Gpc}^3 \) box sizes). In principle, we could begin with a full semi-analytic model: attempt to describe, in an a priori manner, the entire process of cooling, disk growth, star formation, and feedback. This has been done by Khochfar & Silk (2006), for example, as well as Almeida et al. (2007), and we compare with their results below. We have also experimented with this methodology applied to the approach of Somerville et al. (2008a), and find qualitatively similar results.

However, a full semi-analytic model necessitates an attempt to predict e.g. the star formation histories, gas fractions, disk galaxy populations, and clustering of galaxies in an a priori manner. Although these are certainly worthy of investigation, they are not the quantities we are interested in here — therefore attempting to model e.g. disk formation, star formation, and stellar winds in this manner will introduce additional uncertainties and dependences.

For example, Khochfar & Silk (2006) find that their most massive galaxies are predicted to evolve very rapidly. However, their analysis results in more rapid gas fraction evolution than observed, a problem common to semi-analytic models without strong feedback (see e.g. Somerville et al. 2001). In fact, over the redshift range \( z \sim 0 - 2 \), progenitor disk galaxy properties (at least the broad properties of greatest interest to us here, namely their sizes (or the Tully-Fisher relation) and gas fractions) are reasonably well constrained empirically, but theory has had mixed success at best in reproducing them from a fully a priori framework. If current a priori models are incorrectly describing the cooling of gas and disk formation (in particular the formation of massive galaxies), then the predicted properties of ellipticals formed in mergers of those progenitors are suspect.

We therefore adopt a halo occupation approach — essentially beginning with the observational constraints on e.g. disk gas fractions and the Tully-Fisher relation, and predicting the properties of spheroids from these empirically constrained progenitors. For our purposes, we are not concerned with why a given disk population is e.g. gas-rich or gas-poor, or has a particular effective radius — we need those numbers to compute the properties of the remnant of a major merger of such disks. This allows us to be as conservative as possible, and to identify the robust prediction for spheric scaling laws without reference to uncertainties in e.g. disk formation models. We can (and do) include the uncertainties in the observational constraints in our modeling, but we find that these are unimportant.

The details of our scheme are described in Hopkins et al. (2008b) and Hopkins et al. (2008c), but we briefly review them here. For a particular cosmology, we identify the halo and subhalo populations, and associate each main halo and subhalo with a galaxy in a Monte Carlo fashion. Specifically: at a given redshift, we calculate the halo mass function \( n(M_{\text{halo}}) \) for our adopted cosmology following Sheth et al. (2001). For each halo, we calculate the (weakly mass and redshift dependent) subhalo mass function (or distribution of subhalos, \( P(N_{\text{subhalo}} | M_{\text{subhalo}}, M_{\text{halo}}) \)) following Zentner et al. (2005) and Kravtsov et al. (2004). Alternatively, we have adopted it directly from Gao et al. (2004); Nurmi et al. (2004) or calculated it following van den Bosch et al. (2005); Vale & Ostriker (2006), and obtain similar results: the uncertainties at this stage are negligible. Note that the subhalo masses are defined as the masses upon accretion by the parent halo, which makes them a good proxy for the hosted galaxy mass (Conroy et al. 2006) and removes the uncertainties owing to tidal stripping.

We then populate the central galaxies in each halo and subhalo according to an empirical halo occupation model. These empirical models determine both the mean stellar mass and dispersion in stellar masses of galaxies hosted by a given halo/subhalo mass \( P(M_{\text{gal}} | M_{\text{subhalo}}) \), from fitting the observed galaxy correlation functions as a function of e.g. scale and stellar mass or luminosity at a given redshift. Other properties, such as e.g. the sizes and gas fractions of galaxies, are determined as a function of their stellar mass (and, according to our adopted model, merger history) as described in §2.3.2 below.

Although such models are constrained, by definition, to reproduce the mean properties of the halos occupied by galaxies of a given mass/luminosity, there are known degeneracies between parameterizations that give rise to some (usually small) differences between models. We therefore repeat all our calculations for our “default” model (for which we follow the empirical constraints in Conroy et al. 2006) (see also Vale & Ostriker 2006) and an alternate halo occupation model (Yang et al. 2003) (see also Yan et al. 2005; Zheng et al. 2005), which bracket the range of a number of calculations (e.g., Cooray 2005, 2006; Zheng et al. 2005; van den Bosch et al. 2007; Brown et al. 2008) and direct observations of groups (Wang et al. 2006).

We have also compared a variety of prescriptions for the redshift evolution of various components in the halo occupation model: we have adopted that directly fitted by the authors above at various redshifts, we have considered a complete re-derivation of the HOD models of Conroy et al. (2006) and Vale & Ostriker (2006) at different redshifts from each of the the observed mass functions of Fontana et al. (2006); Bundy et al. (2005); Borch et al. (2006); Blanton (2006) (see Hopkins et al. 2008f), and have also found similar results assuming no evolution in \( P(M_{\text{gal}} | M_{\text{subhalo}}) \) (for star forming galaxies). Indeed, a number of recent studies suggest that there is very little evolution in halo occupation parameters (in terms of mass, or relative to \( L_\star \)) with redshift (Yan et al. 2003; Cooray 2005; Conroy et al. 2006; Brown et al. 2008), or equivalently that the masses of galaxies hosted in a halo of a given mass are primarily a function of that halo mass, not of redshift (Heymans et al. 2006; Conroy et al. 2007). This appears to be especially true for star-forming galaxies (Conroy et al. 2007), unsurprising given that quenching is not strongly operating in those systems to change their mass-to-light ratios, but it also appears to be true for red galaxies at least at moderate redshifts (Brown et al. 2008).

Consequently, the differences between any of these choices are small (at least at \( z \leq 3 \), and negligible compared to the uncertainties in our predictions from e.g. the uncertainties in disk sizes and gas fractions as a function of their stellar mass and redshift. Systems which have not undergone a major merger in the model are assumed to be disk-dominated, and we can populate them either according to a halo occupation model that treats all galaxies in an identical fashion, or as blue/star forming galaxies in a model where the constraints are derived for blue and red galaxies separately (we find it...
makes little or no difference).

Finally, having populated a given halo and its subhalos with galaxies, we follow the evolution of those halos forward and identify major mergers. The details of the treatment of subhalo–subhalo and subhalo–halo mergers are described in Hopkins et al. (2008b) and Hopkins et al. (2008a), but in short the halo merger timescale is straightforward and well defined. When two subhalos are fully merged, we can either assume the galaxies have merged, or allow for some additional merger timescale within the merged halo. We have experimented with a variety of models for this, including e.g. the dynamical friction timescale and alternative timescales calibrated from simulations or calculated based on group capture or collisional cross section estimates and angular momentum (orbital cross section) capture estimates. But because these merger times within a halo are always much less than the Hubble time at the relevant redshifts, and we are interested only in statistical properties across populations evolving over a Hubble time, it makes no difference to our calculations what choice we adopt.

When galaxies merge, we estimate the properties of the remnant based on those of the progenitor galaxies, according to our numerical simulations, as described in §2.4. This approach yields reasonable agreement with global quantities such as the spheroid mass function, the mass density of ellipticals as a function of mass, and the fraction of early-type galaxies as a function of mass (Hopkins et al. 2008a). Again, despite the assumptions involved thus far, we stress that the detailed choice of halo occupation model yields negligible differences in progenitor galaxy properties and in our predictions at all the masses and redshifts of interest, where the clustering and abundances of massive galaxies are reasonably well-constrained.

2.3. Progenitors: Laying Galaxies Down

Given a host halo mass and disk stellar mass, both determined from the halo occupation model, the two important parameters we must assign to each disk are a size (effective radius $R_e$) and gas fraction ($f_{\text{gas}} = M_{\text{gas}}/(M_* + M_{\text{gas}})$). This is where the dominant uncertainties in our modeling arise – we will show that e.g. the gas fraction in pre-merger disks is an important parameter determining the characteristics of the remnant, so we can only predict those properties to within the uncertainties in disk gas fractions at the appropriate redshifts.

To minimize the uncertainties in modeling this, we therefore begin with an empirical estimate of the gas fraction distribution in spiral galaxies as a function of mass and redshift. At $z = 0$, there is no significant uncertainty: various measurements (e.g. [Kennicutt 1998; Bell & de Jong 2000; Kannappan 2004; McGaugh et al. 2008; McGaugh 2005]) and indirect constraints from star formation and the baryonic Tully-Fisher relation (Bell et al. 2003; Noeske et al. 2007; Calura et al. 2007) give consistent estimates of the median gas fraction in spirals as a function of their stellar mass, and the scatter at each mass. We obtain identical results if we use any of these constraints (or the data points themselves), but note for convenience that all of these observations can be well-fitted by the trend

$$f_{\text{gas}}(z = 0) \approx 1/(1 + (M_*/10^{15}M_\odot)^{0.1})$$

with a constant $\sim 0.2 - 0.25$ dex scatter at each mass. Similar, albeit less robust, estimates exist at $z = 1$ and $z = 2$ (see e.g. Shapley et al. 2005; Erb et al. 2006; Trujillo et al. 2006). We therefore adopt a simple functional form that interpolates between these measurements. Motivated by the power-law form of the observed Kennicutt-Schmidt star formation relation ($M_* \propto \Sigma_{\text{gas}}^{1.4}$), we adopt the following interpolation formula:

$$f_{\text{gas}}(z) = f_{\text{gas}}(z = 0) \left[1 + \tau_{\text{LB}}(1 - f_{\text{gas}}(z = 0)^{\beta})\right]^{-2/3}$$

where $\beta \approx 3/2$ and $\tau_{\text{LB}}(z)$ is the fractional lookback time to redshift $z$. This can be derived, for example, by assuming a simple model where the star formation rate scales according to the observed relation, the disk scale length varies as a power of the baryonic mass, and the net mass accretion rate (inflows minus outflows) is also a power-law function of the mass of the system; given the requirement that the system begin at an initial time $t = 0$ with $f_{\text{gas}} = 1$ and have the observed $f_{\text{gas}}$ at $z = 0$.

In any case, the values above provide a good fit to the observations at $z = 0 - 2$ (Bell & de Jong 2000; Shapley et al. 2005; Erb et al. 2006) and a reasonable approximation to results of cosmological simulations (Keres et al. 2005; Keres et al. 2007) and semi-analytic models (Somerville et al. 2008b), and interpolate smoothly between $z = 0$ and high redshift. Lowering $\beta$ systematically weakens the implied redshift evolution in $f_{\text{gas}}$ – in order to represent the uncertainty in the observations, we consider a range $\beta \sim 0.5 - 2.0$ (bracketing the observational errors). Similar prescriptions for $f_{\text{gas}}$ evolution can be obtained from various toy model star formation histories including e.g. commonly adopted exponential $\tau$-models (see e.g. Bell & de Jong 2000; Noeske et al. 2007) and integration of specific tau models enforcing proportionality between inflow, star formation, and outflow rates (Erb 2008); these are all discussed in more detail in Hopkins et al. (2007b, 2008a), but they generally yield similar results and lie within the uncertainties we consider (differences being much less than the typical observed scatter in $f_{\text{gas}}$ at each mass and redshift).

The other important property we must estimate for progenitor disks is their effective radius. At $z = 0$, this is well-measured for all disk masses of interest. We employ the fits from Shen et al. (2003) to late-type galaxy sizes as a function of stellar mass (from $\lesssim 10^9 M_\odot$ to $\gtrsim 10^{12} M_\odot$), and the scatter in those sizes. Adopting different local estimates or even completely ignoring this scatter makes little difference. Observations are somewhat more ambiguous regarding the redshift evolution of sizes, but fortunately the observational constraints (e.g. Trujillo et al. 2006; Barden et al. 2005, Ravindranath et al. 2004; Rix et al. 2004; Ferguson et al. 2004; Akivama et al. 2008) and state-of-the-art models (Somerville et al. 2008b) suggest that any evolu-
tions suggest that at least a significant fraction of this popula-

We therefore consider two extremes. In the first case, disk sizes and the baryonic Tully-Fisher relation are completely fixed with redshift, according to their $z = 0$ values. In the second, disk sizes evolve according to observational estimates from the various measurements above. A simple power law

$$R_{\text{e, disk}}[M_\bullet, z] = (1 + z)^{\beta_d} R_{\text{e, disk}}[M_\bullet, z = 0]$$

(3)

where $\beta_d \approx 0.4$ provides a good fit to the observations and is consistent with all of the measurements, spanning the redshift range $z = 0 - 3$. We adopt this estimate, but obtain a similar result if we use the more detailed cosmologically motivated model in Somerville et al. (2008b), which allows disks to form conserving specific angular momentum from halo gas (see Mo et al. (1998)), yielding (approximately) $R_{\text{e}} \propto R_{\text{vir}} / c$, where $c \approx 1/(1 + z)$ is the halo concentration – although $R_{\text{vir}}$ is smaller for halos of a given mass at high redshift, they are less concentrated, yielding a weak size evolution similar to that observed (Bullock et al. 2001; Wechsler et al. 2002; Neto et al. 2007; Comerford & Natarajan 2007; Buote et al. 2007).

Together this gives us an estimate of our input disk parameters, and we allow for the described range in both the estimated gas fraction and disk size evolution. At low redshift, this introduces little uncertainty (most spheroids at $z = 0$ have assembled relatively recently, so their structural properties reflect those of low-redshift disk progenitors, where the uncertainties are minimal and the relevant sizes, etc. can be directly measured from observations). However, these uncertainties begin to grow rapidly at $z \gtrsim 2 - 3$, where there are no more direct observational constraints. Moreover, at these redshifts, it is not clear that “un-merged” galaxies will necessarily be disks analogous to those well-understood at $z < 2$ (see e.g. the clumpy morphologies and increased dispersions observed in Reddy et al. 2006; Flores et al. 2006; Bournaud et al. 2008), although higher-resolution observations suggest that at least a significant fraction of this population does exhibit regular rotation and smoother morphologies (albeit most likely still puffier and more dispersion-supported than low-redshift disks) in the rest-frame optical (Puech et al. 2007; Akiyama et al. 2008; Genzel et al. 2008; Shapiro et al. 2008).

In either case, the uncertainties as this limit is approached can be considered part of the uncertainty in e.g. progenitor sizes and structural properties, as observational estimates of this quantity often do not make strict morphological cuts at these redshifts but rather assign gas-rich star-forming systems to a single category (and this is how such uncertainties would enter into our model). In this aspect, the uncertainties from assigning them implicit “disk” properties may actually be less than one might expect, because at the point where a galaxy is very gas-rich (increasingly common at these high redshifts), it does not matter what initial configuration this gas is in (whether e.g. clumpy, filamentary, or infalling in direct collapse), as it is dissipational and in any case will (in mergers) lose angular momentum and form a dense central starburst. In any case, these uncertainties lead us to limit our predictions to $z < 4$. Fortunately, because at any lower redshift most spheroids have assembled relatively recently, with decreasing mass fractions contributed by galaxies assembled at very early times, the uncertainties associated with the lack of constraints at these high redshifts rapidly become less important to our predictions.

2.4. The Simulations

In order to determine the effects that a given merger will have on galaxy properties, we require some estimates of how e.g. the gas content of initial galaxies (and other properties) translates into properties of the remnant. We derive these prescriptions from a large library of hydrodynamic simulations of galaxy encounters and mergers, described in detail in Robertson et al. (2006); Cox et al. (2008); Younger et al. (2008) and Paper I. These amount to several hundred unique simulations, spanning a wide range in progenitor galaxy masses, gas fractions, orbital parameters, progenitor structural properties (sizes, concentrations, bulge-to-disk ratios), and redshift.

Most of the simulations are major (mass ratios $\sim 1 : 3 - 1 : 1$), binary encounters between stellar disks (which provide an idealized scenario useful for calibrating how details of the remnant depend on specific progenitor properties), but they include a series of minor mergers (from mass ratios $\sim 1 : 20$ to $\sim 1 : 3$), as well as spheroid-spheroid “re-mergers” or “dry mergers” (i.e. mergers of the elliptical remnants of previous merger simulations), mixed-morphology (spiral-elliptical) mergers (see also Burkert et al. 2007; Johansson et al. 2008, for a detailed study of these mergers), multiple mergers, and rapid series of hierarchical mergers. Our adopted prescriptions are robust to these choices. The simulations usually include accretion and feedback from supermassive black holes, as well as feedback from supernovae and stellar winds. However, we have performed parameter studies in these feedback prescriptions, and find that the structural properties of interest here are relatively insensitive to these effects (Cox et al. 2008; Hopkins et al. 2007b, 2008a).

The predicted simulation scalings are discussed in detail in Paper I-Paper III, where we take advantage of high-resolution observations of local merger remnants (from Rothberg & Joseph 2004, as well as both nuclear cusp and core ellipticals (from Lauer et al. 2007b; Kormendy et al. 2008) to test them observationally. We find good agreement between the predicted and observed scalings with e.g. stellar mass and gas content (of the progenitor galaxies) at the time of the merger. If, as is commonly believed, core ellipticals are the product of spheroid-spheroid re-mergers, then our comparisons in Paper III also confirm the scalings derived from simulated re-mergers.

2.4.1. Mergers without Gas

In mergers, the stars and dark matter form a collisionless system, and hence gas free (dissipationless or dry) mergers are particularly easy to handle. Furthermore, direct comparison with our simulations suggests that, to the desired accuracy, it is a good approximation to treat the dissipational and dissipationless components of a merger separately, allowing us to apply simple rules to the stellar remnant in mergers of both gas-rich disks and gas-poor spheroids.

It is well established in numerical simulations that, to lowest order, the effect of any merger of dissipationless components is to “puff them up” by a uniform factor, while roughly conserving profile shape (at least for certain “equilibrium” profile shapes such as the Navarro et al.
merger remnant randomly from that fitted to our simulated mergers of the same mass ratio, and so on – we obtain similar predictions.

Finally, having the profile of the dissipationless stellar and halo components, we can calculate the velocity dispersion $\sigma$ of stars within some radius (here we take the mass-weighted velocity dispersion of stars within $R_e$, and assume isotropic orbits, although this is not a dominant uncertainty in our predictions). We can instead (without any reference to the isotropy assumption) decide that the dynamical mass estimator, $\sigma^2 R_e$, is a good tracer of the true enclosed total mass within the stellar effective radius $R_e$ (modulo a normalization constant that is the same for most ellipticals) – a fact seen in our simulations (Paper IV) and observations (Cappellari et al. 2006; Bolton et al. 2007, 2008) – and use the known total mass ($M(< R_e)$) to predict $\sigma$. The results are similar.

We note that halos are tracked with stars in these processes, but between mergers, the halos still grow. We assume at all times that the dark matter halo follows a Hernquist (1990) profile (essentially identical to an Navarro et al. (1996) profile at the radii that matter for the baryonic galaxy, but with finite mass and analytically tractable properties, a more useful approximation since we are considering halos as if in isolation), with a concentration from the mean concentration-halo mass-redshift relation in simulations and observations (see § 2.3 and Springel et al. (2005)). We then calculate the halo contribution to quantities such as e.g. the dynamical mass within $R_e$ and $\sigma$ based on this profile. However, we could also assume the dark matter within the stellar effective radius “freezes out” with each major merger (new dark matter within $R_e$ only being what is brought in by subsequent mergers, within the $R_e$ of those galaxies at the time of their first merger) – it makes relatively little difference, since the mass densities in the cores of halos do not strongly evolve with redshift (Bullock et al. 2001).

2.4.2. Mergers with Gas

In a gas-rich merger, the gas loses angular momentum and, being dissipative, radiates its energy and collapses to the center of the galaxy. The collapse will, in general, continue until the gas becomes self-gravitating, at which point it rapidly turns into stars as it further contracts (for details, see Hopkins et al. 2008d). The remnant will exhibit a “dissipational component” – the dense central stellar concentration which is the relic of these dissipational merger-induced starbursts.

Covington et al. (2008) consider the dissipation of energy in mergers and use it to estimate the effect this will have on the size of the merger remnant (see also Ciotti et al. 2007), which we test in Paper II and Paper III and show is a good approximation to our simulations and observed systems, and can be reduced to the approximation

$$R_e \approx \frac{R_e(\text{dissipationless})}{1 + (f_{\text{gas}}/f_0)},$$

where $f_0 \approx 0.25 - 0.30$, $R_e(\text{dissipationless})$ is the effective stellar radius of the final merger remnant if it were entirely dissipationless, and $f_{\text{dissipational}}$ is the baryonic mass fraction from the starburst. For most major mergers, gas in the disks at the time of the merger will rapidly lose energy, and star formation is very efficient, so $f_{\text{dissipational}}$ reflects the gas fractions of the progenitor disks immediately before the merger $f_{\text{dissipational}}(\text{merger}) = f_{\text{gas}} = M_{\text{gas}}(\text{disks})/M_e(\text{remnant})$.\footnote{Hernquist (1990), or de Vaucouleurs (1948) profiles; see e.g. Barnes 1992; Boylan-Kolchin et al. 2005, 2006; Hopkins et al. 2008h. The conservation of both energy and phase space density means that, modulo a small normalization offset owing to possibly different profile shapes the remnant of e.g. two initial stellar disk or spheroid mergers will have a similar final effective radius. In detail, if we temporarily ignore the halos of the galaxies (a good approximation in most simulations, since the specific binding energy of the galaxy baryonic mass is much larger than that of the halo) and consider isotropic systems, we obtain for the parabolic merger of systems of mass $M_1$ and $M_2 = f M_1$ ($f \leq 1$) the energy conservation equation

$$E_f = k_f (M_1 + M_2) \sigma_f^2 = E_i = k_i M_1 \sigma_i^2 + k_2 M_2 \sigma_2^2$$

(4)

where $k$ is a constant that depends weakly on profile shape (for greater detail, see Ciotti et al. 2007). If profile shape is roughly preserved and $R \propto M/\sigma^2$, then we expect

$$R_f = R_i \left(1 + \frac{f^2}{1 + f^2 \frac{\sigma_i^2}{\sigma_f^2}} \right).$$

A merger of two perfectly identical spheroids will double $R_e$ and conserve $\sigma$. It is straightforward to solve the appropriate energy conservation equation numerically, allowing for arbitrary profile shapes and for halo components (assuming the halo profile follows Navarro et al. (1996) or Hernquist (1990) profiles), but in practice we find that these subtleties make no almost no difference compared to using Equation (5).

In a merger of two stellar disks, violent relaxation will transform the stellar distribution from initially exponential disks into de Vaucouleurs (1948)-like profiles, more generally Sersic (1968) profiles (of the form $I \propto \exp[-(r/r_0)^{1/n_r}]$ with $n_r \sim 2.5 - 3.5$ after a single major disk-disk merger (see Paper I-Paper II and Naab & Trujillo 2006), while the halo profile shape is approximately preserved (Paper IV, Boylan-Kolchin et al. 2005). These trends in simulations can both be roughly explained by allowing the merger to scatter stars and dark matter (in addition to uniformly puffing up the profile) in their final three-dimensional radii by some lognormal broadening factor $\sigma_r \sim 0.3 - 0.4$ dex (in a 1 : 1 merger). In other words, while the median final (post-merger) effective radius of stars at some initial radius $r_i$ is given by the uniform puffing up or stretching of the profile in Equation (5), there is a lognormal scatter with dispersion $\sigma_r$ in the final radii of stars from that initial radius. Over the relevant dynamic range for most practical observational purposes (from $\sim 0.01 R_e$ to $\gtrsim 10 R_e$, or as much as 15 magnitudes in surface brightness), this will effectively transform an initial exponential profile into an $n_r \approx 3$ ($\sigma_r = 0.3$) or $n_r \approx 4$ ($\sigma_r = 0.4$) profile.

Subsequent spheroid-spheroid or spheroid-disk mergers continue to scatter stars out to larger radii, building an extended envelope of material, and raising the best-fit Sersic index of the dissipationless component of the remnant. We study this in Paper III, and find to a good approximation that a re-merger of mass ratio $f \leq 1$ will raise the $n_r$ of the dissipationless component by $\Delta n_r \sim f$, or equivalently “broaden” the three-dimensional stellar distribution by a lognormal factor $\sim 0.3 f$ (the prescription we adopt). At $z = 0$ we can then determine an “observed” Sersic index by projecting this mock profile (with that of the dissipational component) and fitting to the system. If we consider the Sersic indices directly fitted to our simulations – i.e. draw a Sersic index for a disk-disk
(gas blown out by feedback from black holes and stellar winds, even with extreme feedback prescriptions, makes little difference here: it predominantly effects \( f_{gas} \) of the pre-merger disks over much longer timescales, rather than the consumption in mergers). We show in [Hopkins et al. (2008d)] that this is not strictly true in extremely gas-rich mergers \((f_{gas} > 0.5)\), where torques are inefficient at driving dissipation, and ellipticals may not be formed from even major mergers: these extreme cases however are seen only at the lowest observed masses, and do not affect most of our predictions (though we note where this may be important).

Given the typical exponential-like \((n_e \sim 1 - 2)\) character of these dissipational components when they first form (owing to their formation from infalling gas), we can invert this to infer what the effective radius of the dissipational component specifically must be (given that effective radius and profile, we can then construct a mock profile of the entire galaxy). Based on our study of re-merger simulations in Paper III, we find that the two components (those which were originally dissipational at their formation, and those which were not – i.e. which originally came from stellar disks) tend to be separately conserved even after several dry or gas-poor spheroid-spheroid re-mergers. In a series of mergers, then, it is a good approximation to treat the two separately: dissipation-less components grow and add as described above. Dissipational components form with a mass given by the gas content of merging disks with sizes appropriate for the above relation, and then evolve either by gas poor (spheroid-spheroid) merging (where they will add with other stellar dissipational components formed in earlier mergers, following the rules for dissipationless merging above since both components are stellar), or by more gas-rich (spheroid-disk) merging (in which case new dissipational mass is added given the prescriptions above).

Dissipation will also fuel growth of the central supermassive black hole. Based on our theoretical modeling of self-regulated black hole growth in simulations ([Di Matteo et al. 2005; Hopkins et al. 2006a; Hopkins & Hernquist 2006; Hopkins et al. 2007]), and that of others (e.g. [Silk & Rees 1998; Murray et al. 2005; Ciotti & Ostriker 2007]), essentially any model where the black hole regulates its growth by halft accretion (allowing some fraction of the energy or momentum of accretion to couple to the surrounding media) will give rise to a fundamental correlation between black hole mass and spheroid binding energy ([Hopkins et al. 2007b]), more specifically with the binding energy of the gas in the central regions whose inflow must be suppressed to regulate black hole growth.

There is direct evidence for this in the observations as well, with Hopkins et al. (2007b) finding \( \sim 3 \sigma \) preference in the data for a correlation of the form \( M_{BH} \propto \sigma_b^{0.5-0.7} \sigma_b^{1.5-2.0} \), similar to and consistent with the “fundamental” correlation being one with bulge binding energy, which is roughly traced by \( E_b \propto M_{bh}\sigma_b^2 \). \( M_{BH} \propto \sigma_b^{1.71} \) (see also Aller & Richstone 2007). indirectly, this can then explain the observed \( M_{BH} \propto \sigma_b^{1.71} \) (Ferrarese & Merritt 2000, Gebhardt et al. 2000) and \( M_{BH} \propto M_* \) (Magorrian et al. 1998) correlations.

Motivated by our simulations and these observational results, we model black hole growth as follows: in each merger, we allow the black hole to grow by an amount proportional to the binding energy of the new dissipational components of the merger (i.e. the starburst gas), in addition to summing the masses of the two progenitor black holes (we do not model any gravitational kicks that might expel black holes, but theoretical estimates suggest that by \( z = 0 \) in the massive galaxies we are interested in here, such effects are small if they are present at all; Volonteri et al. 2006; Volonteri 2007). We include an intrinsic scatter \( \sim 0.2 \sigma \), again motivated by simulations (although there will be some scatter in the resulting correlations regardless, reflecting different gas content and potential depth at the time of merger, with subsequent evolution to \( z = 0 \)). The exact proportionality constant is a reflection of the physics of feedback efficiencies and coupling, but we adopt a constant calibrated in simulations to match the normalization of the observed \( z = 0 M_{BH} - \sigma \) relation ([Di Matteo et al. 2005; Hopkins et al. 2005a, 2007b]). Any other results (e.g. the slope of these correlations, residual correlations or correlations with other structural parameters, and/or redshift evolution in the correlations) are genuine predictions of the model, not this calibration.

3. Predicted Scaling Laws at \( z = 0 \)

Figures 1 & 2 plot the fundamental plane correlations of ellipticals at \( z = 0 \), produced by this simple model. Here (and
in subsequent figures) we draw systems randomly from the halo occupation distribution with the restriction that we draw equal numbers of simulated systems per logarithmic interval in halo mass (in order to represent the full dynamic range of interest in the plots). This differs slightly from selecting e.g. a volume-limited sample, which would be dominated by low-mass systems (less useful for studying the distribution in a parameter at a given mass, which is our intention here).

We compare with a large sample of observed ellipticals: specifically the combination of the Kormendy et al. (2008) compilation of Virgo ellipticals and the bright elliptical sample of Lau et al. (2007b) (which extends this dynamic range at the cost of slightly higher uncertainties in the data), with nuclear HST observations and ground-based data at large radii (allowing accurate surface brightness profile measurements from \( \sim 50 \) pc to \( \sim 50 \) kpc\(^5\)). The correlations traced by these samples throughout this paper are statistically indistinguishable from the best-fit correlations (over the range observed) determined from volume-limited samples of (primarily field) early-type galaxies in the SDSS, in e.g. Shen et al. (2003), Bernardi et al. (2003, 2007); Gallazzi et al. (2006); von der Linden et al. (2007). There may be a slight (not highly significant) offset between the data-sets, in the sense that the objects in the Virgo sample may have slightly larger \( R_e \) and smaller \( \sigma \) than the Lau et al. (2007b) sample at fixed mass; if real, this may relate to the former being a cluster sample, or to the likely inclusion of a non-negligible S0 population in the latter. But in any case, the possible offset between the two is smaller than the uncertainties in our modeling (in terms of normalizing e.g. absolute sizes of systems, requiring accurate priors on their progenitor disk and halo sizes), and within the predicted and observed scatter, and so reasonably brackets the range of different observations. (For more details of the observational samples and their analysis, we refer to those papers and Paper II.)

The correlations are accurately reproduced at \( z = 0 \). Specifically, the size-mass relation has a steep logarithmic slope \( R_e \sim M_{\star}^{0.16} \), compared to \( R_e \sim M_{\star}^{0.3} \) for the progenitor disks – the sense of this is specifically that low-mass ellipticals are more compact, relative to their progenitor disks. The velocity dispersion-mass (Faber-Jackson) relation is also reproduced, although this correlation is less dramatically different from that obeyed by disks (the baryonic Tully-Fisher relation). The resulting fundamental plane, expressed as either \( M_{\text{dyn}}(M_{\star}) \) or \( R_e(\sigma, M_{\star}) \), is also reproduced; i.e. the predicted ratios of enclosed dark matter to stellar mass within the stellar \( R_e \) agree with observations as a function of mass (note that with the adopted definition of \( M_{\text{dyn}} \propto \sigma^2 R_e, M_{\text{dyn}} \) can be less than \( M_{\star} \)). Specifically, the tilt of the fundamental plane is reproduced – rather than \( M_{\text{dyn}} \propto M_{\star} \), the expectation if systems were perfectly homologous, we predict \( M_{\text{dyn}} \propto M_{\star}^{1+\alpha} \) (where \( \alpha \approx 0.2 \) is the FP tilt), namely that lower-mass systems (as a consequence of their being more compact) are more baryondominated within their stellar effective radii. (The nature of the FP scalings are discussed in much greater detail in Paper IV; however, we outline the dominant physical effects in § 3.1 & 3.2 below.)

We plot the simulated systems, with colors denoting the redshift of their last gas-rich merger – i.e. approximately the time at which they should have stopped forming stars. It is clear that, at fixed stellar mass (or velocity dispersion, given that it more tightly correlates with stellar mass than e.g. effective radius), systems with larger radii (or larger \( M_{\text{dyn}}/M_{\star} \)) tend to be older – they are often systems that merged at early times and have grown dissipationlessly since then. This appears to be borne out in recent observational estimates based on stellar population constraints in local ellipticals (Gallazzi et al. 2006, Graves et al. 2008). (We discuss this further below.)

Figure 3 plots the Sersic index of the galaxy light profiles predicted at \( z = 0 \). We stress that these are rough estimates at best – individual galaxy profiles show a degree of diversity representative of many details in their formation and merger history that only high-resolution numerical simulations can capture. We therefore refer to Paper II and Paper III for a more detailed study and comparison of Sersic indices in simulated and observed systems; however, comparison of our estimates with numerical simulations suggests that the adopted prescriptions capture at least the median behavior in simulations.

First, we consider the Sersic index fitted to the entire galaxy profile – this is of course a useful observational parameter, but it reflects a mix of different physical components of the galaxy. In order to transform a galaxy from an exponential disk profile (a Sersic index \( n_s = 1 \)) to a typical massive elliptical with a high Sersic index \( \geq 4 \), both dissipation and violent relaxation are necessary. Violent relaxation will scatter stars at large radii, building up an extended envelope (characteristic of high-\( n_s \) systems), but high-\( n_s \) profiles rise steeply at small radii, so dissipation is needed to build up a dense central stellar concentration. The effects of repeated mergers on the profile at large radii are relatively straightforward, and (more or less) monotonically increase \( n_s \), but the consequences of dissipation are more complex (related not just to the dissipational mass fraction but to the relative concentration of the dissipa-
tive component and its profile shape) and non-monotonic.

We construct the best-fit Sersic index fitting a dynamic range comparable to the observations plotted (representative of local galaxies for which the highest-quality data is available), from \( \sim 50 \text{pc} \) to \( \sim 30 \text{kpc} \). Because the galaxy profiles are multi-component, the resulting \( n_s \) reflects a complex combination of the observed dynamic range, the relative mass, size, and slope of the dissipational component, and the merger history. Together, these effects yield an apparent steep dependence of Sersic index on mass, in agreement with the observations, but with large scatter (whereas if the profiles were entirely dissipationless, the lack of dense stellar concentrations at small radii would restrict them to smaller Sersic indices and lead to a weaker dependence of \( n_s \) on merger history and mass). Most of the strength of the dependence here is ultimately a coincidence reflecting particular combinations of parameters moving into or out of the dynamic range for which the fits are sensitive.

If we consider the Sersic indices of just the dissipationless component (i.e. the sum of the stellar components of pre-merger disks that were violently relaxed in the initial gas-rich mergers that formed each spheroid progenitor), the dependence on mass is weaker, but it more directly reflects the merger history of the systems (and depends much less on e.g. the dynamic range of the fit). We compare this with multi-component decompositions of the same observed systems, based on a more detailed, physically motivated approach to studying galaxy profiles which we discuss in detail in Paper I-Paper III, where we demonstrate that the “dissipationless” component of observed ellipticals can, in fact, be recovered. By definition in our simple model, the dependence we predict now is entirely driven by the mean dependence of merger history on mass – systems which have experienced more mergers (and more violent mergers) will have had more stars scattered to large radii in those mergers, raising the \( n_s \) of the outer component. At low masses, most systems formed in \( \sim 1 \) major merger at relatively low redshifts \( z < 1 \sim 2 \), so a typical \( n_s \sim 2 \sim 3 \) is obtained. At high masses, systems often have their first merger at very high redshifts, involving a large degree of dissipation, forming a dense core, and then experience a number of dry mergers, building up a more extended dissipationless outer component and envelope and raising \( n_s \).

The apparent trends of Sersic index with mass are much stronger if dwarf spheroidals are included, as in many observational samples (e.g. Graham & Driver 2007, Ferrarese et al. 2006). These systems have most likely not experienced any significant mergers – as a structural family, they are clearly related to disks, not ellipticals, and their Sersic indices (typical \( n_s \sim 1 \)) demonstrate this; that they dominate at low masses reflects e.g. the dominance of disks and ellipticals, and is again driven by the cosmological dependence of merger history on mass.

Figure 4 compares the predicted and observed correlations between nuclear black hole mass and host bulge/spheroid properties (effective radius, velocity dispersion, stellar mass, and binding energy). Recall, motivated by observational and theoretical expectations of a black hole “fundamental plane” (Hopkins et al. 2007c), the fundamental “assumed” correlation here is between black hole mass and binding energy of the dissipational component of the bulge, determined at the time of that dissipation (time of gas-rich merger). Despite the complex interplay between this and the total bulge mass and size, dry mergers, and evolution in halo bulge properties, the \( z = 0 \) correlations (including the residual correlations observed

\[ M_{\bullet} \propto M^\alpha \]
Dissipation is one of the most important factors controlling the properties of remnant ellipticals. We demonstrate this robustly with detailed observations and simulations in Paper II and Paper III. Here, we discuss how its effects are manifest in a cosmologically representative population, including systems which have experienced a number of dry mergers. Figure 6 shows the integrated fraction of our $z = 0$ ellipticals which formed in dissipative starbursts (from gas in disks at the time of each merger), as opposed to dissipationlessly scattered stellar disks. Recall, dissipation allows gas to collapse to small scales, building up dense central components in the stellar mass distribution and changing the phase-space distribution of the remnant. We compare with the observationally estimated dissipational fractions in local ellipticals, determined in Paper II and Paper III. The agreement is good at each mass - in other words, the amount of dissipation observed in ellipticals can be explained by our simple model.

What drives the trend of dissipation with mass? We find that it reflects the typical gas fractions of disks as a function of mass. That trend (in disks) may, of course, be a complex consequence of poorly understood processes such as star formation, cold gas accretion, and stellar feedback; but for our purposes here we are not interested in how this property arises but for our purposes here we are not interested in how this property arises, as we are interested in how this property arises, as we are interested in how this property arises. However, these systems will also be in the most biased environments, and so (in a $\Lambda$CDM context) undergo relatively more gas-exhausted and/or quenched) and correspondingly lower dissipational fractions. At fixed mass, systems with earlier formation times tend to have slightly lower dissipational fractions; we emphasize that this is true when the systems are observed at $z = 0$. At their time of formation, early-forming systems may have had very large dissipational fractions, reflecting the characteristically higher gas fractions of disk progenitors at these redshifts (we show this in greater detail in § 5 below). However, these systems will also be in the most biased environments, and so (in a $\Lambda$CDM context) undergo relatively more gas-exhausted and/or quenched) and correspondingly lower dissipational fractions.

This simple correspondence is expected, because most of the mass in ellipticals is assembled by a relatively small number of mergers at relatively low redshifts $z \sim 0$–2. As a result, dissipational fractions roughly reflect those of disks in this redshift interval.
have had just one major merger, with even massive ellipticals at \( \sim a \) few \( L_\star \) still having only a small number of such mergers.

In [Hopkins et al. 2008c], we show that the fraction of systems with e.g. only one major merger in their history versus multiple major mergers, or for which the last major merger was gas-rich, agrees well with the observed trend in the fraction of ellipticals with central cusps (power-law nuclear slopes), rapid rotation, or disky isophotal shapes as opposed to central cores, slow rotation, or boxy isophotal shapes (indicators of wet versus dry mergers, discussed in detail in e.g. Paper II and Faber et al. 1997, Naab et al. 2006a, Cox et al. 2006). The most recent major mergers, where the final stellar mass (at least \( \sim 1/2 \) of it) is assembled, occurs at relatively low redshifts, with a median \( z \sim 1 \) and spanning the range \( z \sim 0 - 2 \). Of course, the most massive BCGs form a tail in this distribution with much more complex histories, and for massive galaxies undergoing \( \gtrsim 1 - 2 \) major mergers, the most recent mergers are dry, but in any case the broad point remains: most of the \( z = 0 \) mass in ellipticals is assembled in a relatively small number of major mergers at \( z \lesssim 2 \), and therefore their dissipational fractions reflect those of disks over the same interval.

This general conclusion (that major mergers with mass ratios \( \lesssim 1:3 \), as opposed to more numerous minor mergers, dominate the mass assembly in mergers once gas is exhausted and star formation "quenched") is supported by calculations from other halo occupation models (Zheng et al. 2007) and cosmological simulations (Maller et al. 2006). However, these calculations, as well as clustering estimates (Masjedi et al. 2007) and hydrodynamic simulations (Naab et al. 2006) suggest that as galaxies approach the most massive \( M_\star \gtrsim 10^{12} M_\odot \) BCG mass regimes, growth by a large series of minor mergers becomes more important than growth by major mergers.

Roughly speaking, these calculations suggest that, since most of the mass is in \( \sim L_\star \) systems, mergers with such systems dominate mass growth: so when galaxies are at masses \( \lesssim a \) few \( L_\star \), their growth is dominated by major mergers, and at higher masses, their growth is dominated by progressively more minor mergers. Our predictions in this limit should therefore be treated with some degree of caution, in contrast with the bulk of the spheroid population at \( \lesssim L_\star \) (typically observed in field or low-density environments, where the expectation for growth by a small number of major mergers since \( z \sim 2 - 4 \) is reasonable; see Blanton et al. 2005, Wang et al. 2006, Masjedi et al. 2006). On the other hand, it is still unclear whether such minor mergers actually contribute significantly to mass growth even in the high-mass regime, or whether satellites are disrupted in this regime or kicked into orbits where the merger time is longer than the Hubble time (see e.g. Hashimoto et al. 2003, Tormen et al. 2004, Benson et al. 2004, Zheng et al. 2007, Kazantzidis et al. 2007, Brown et al. 2008).

We, in any case, have experimented with extending our modeling to include minor mergers, and find that this yields qualitatively similar conclusions (with moderate quantitative but no qualitative differences in the predicted properties of the most massive galaxies, and no significant differences otherwise). However, our predictions (and those of other halo-occupation based approaches) are less certain for minor mergers owing to ambiguities in e.g. satellite disruption and weaker constraints on the satellite mass function at small mass ratios.

### 3.1.2. What Effects Does this Dissipation Have?

How does this amount of dissipation change the properties of the \( z = 0 \) ellipticals? We consider this question in detail in idealized simulations and observed ellipticals in Paper IV: in short, increasing the amount of dissipation yields remnants with more of their stellar mass in a compact central starburst remnant. This yields a stellar remnant with a smaller effective radius \( R_e \), and therefore overall a more baryon-dominated object (lower \( M_{\text{dyn}}/M_\star \)). We demonstrate that these general conclusions remain both qualitatively and quantitatively valid in a more general scenario here. In order to compare the effects with and without dissipation, we re-run our model, but without including the effects of dissipation: this effectively amounts to treating gas identically to stars in mergers, or equivalently treating initial disks as if they are all gas-free.

Figures 8 & 9 plot the fundamental plane correlations of ellipticals at \( z = 0 \), yielded from this alternative model. Low-mass ellipticals, without dissipation, now have sizes similar to those of their progenitor disks, and much larger than the sizes...
observed. The slope of the size-mass relation, $R_e \propto M_0^{0.3}$, reflects that in progenitor disks and halos, and disagrees dramatically with the observed slope for ellipticals $R_e \propto M_0^{0.6}$. As we show in Paper II, the trend of increasing dissipation at low masses, owing to increasing gas fractions in lower-mass disks, drives smaller mass ellipticals to relatively lower masses, steepening the size-mass relation and giving rise to the trend in Figure 1. Other trends, such as e.g. the relation of dry mergers to mass, are insufficient to explain the observed size-mass relation (this is expected: dissipationless and dry mergers can only lower the density of objects; they can increase radii at large mass but cannot make low-mass ellipticals smaller or more dense as needed). Invoking evolution in disk sizes and high-redshift mergers can make ellipticals more compact overall, but affects the size-mass slope in the opposite sense required (since the highest-mass ellipticals will have their first mergers at the earliest times, they would be the most compact relative to disks today; the opposite is observed).

This difference in our predicted size-mass relation is also reflected in the velocity dispersion-mass and fundamental plane relations, albeit more weakly, since the effective radius of the stars enters only indirectly: much of $\sigma$ is set by the potential of the halo, which is not altered much by dissipation. Nevertheless, the Faber-Jackson relation disagrees with that observed (again, in the sense that ellipticals are not dense enough), and the fundamental plane tilt is erased (we obtain $M_{\text{dyn}} \propto M_0^{0.9-1.0}$, essentially identical to that observed in disks).

These discrepancies are also apparent from the predictions in Almeida et al. (2007), who use the semi-analytic models from both Baugh et al. (2005) and Bower et al. (2006) to predict galaxy properties in a similar manner as we have done, but treating gas identically to stars (in fact, their prescriptions for galaxy sizes and dark matter scalings are almost identical to ours in this “no dissipation” case). As a direct result, at $z = 0$, their predicted size-mass relation is much too flat ($R_e \propto M_0^{0.5}$) relative to that observed for spheroids, and the predicted fundamental plane has no tilt (in fact it is slightly tilted in the incorrect sense $M_{\text{dyn}} \propto M_0^{0.8-0.9}$). They consider the differences between the two semi-analytic models, as well as the inclusion or removal of adiabatic contraction in the galaxy halos, supernova and AGN feedback, self gravity in the baryons, and systematically higher or lower-energy orbits, and find that none of these significantly change the predicted fundamental plane scalings. In short, at $z = 0$, dissipation (making galaxy stellar mass distributions more compact) is much more important and more significant to the fundamental plane correlations than any differences between various models for galaxy formation and evolution – only dissipation works as a viable explanation of the observed scalings.

The Sersic profiles in the absence of dissipation will be those of the outer/dissipationless components, already shown in Figure 5 (in this case, both the Sersic index of the entire profile and the Sersic index of just the dissipationless component will be the same, and be identical to that of just the dissipationless component in our full model). We directly compare this with the observed Sersic indices in Figure 10. Although this matches the outer profiles of observed systems, it clearly is not the same as the Sersic indices obtained fitting the observed galaxies to a single Sersic law over the entire dynamic range – the high Sersic values (for the entire profile) $n_s \sim 5-10$ observed can only be reproduced with some dissipation to make a dense central concentration of light (matching the steep rise at small radii of large-$n_s$ Sersic profiles). This ultimately points to the same issue as the size-mass relations: dissipation is needed to raise the central stellar densities in order to match observed correlations.

Without dissipation, there is of course no way to grow the central black hole; there is no correlation between black hole mass and host galaxy properties for us to predict. We could in principle assume that black hole mass is dissipationlessly conserved in mergers, but initially set by disk masses – however, this would predict that it is disk, not bulge properties that set the black hole mass, in direct contradiction with observations. We could assume that $M_{\text{BH}}$ strictly traces the stellar mass or velocity dispersion in the bulge (or in the dissipationless component of the bulge), but as demonstrated in Hopkins et al. (2007c), this fails to explain residual correlations between black hole mass and bulge binding energy at fixed $M_*$ or $\sigma$.

3.2. The Effects of Dry Mergers

Having analyzed the effects of dissipation, we now turn to the effects of dissipationless or dry mergers. Since essentially all disk-dominated galaxies do, in fact, have significant gas fractions, this in practice refers to subsequent spheroid-spheroid re-mergers. What effect do such mergers have on the $z = 0$ scaling relations of ellipticals? In order to study this, we re-run our model, including dissipation but excluding such dry mergers (we have also considered allowing them to occur in the sense that they add stellar mass, but not allowing them to alter the structural properties of the galaxy; the results are similar).

Figures 11 & 12 plot the fundamental plane correlations of ellipticals at $z = 0$ from this alternative model. At intermediate and low masses, the number of mergers in our full model is small (see Figure 7), and dissipation is dominant, so the correlations predicted here are essentially identical to those in Figures 11 & 2. At high masses or luminosities ($> a$ few $L_*$), however, there is an appreciable effect on the projections of the FP. The highest mass systems have their first mergers at early times, when their progenitors are more gas-rich, and are therefore compact when first formed. Without dry mergers to puff them up at later times, they do not have effective radii as large as observed. The difference in the scaling laws over
especially at (Hausman & Ostriker 1978; Hernquist et al. 1993). However, 2007) therefore demands some dry mergers in their history. The observations that e.g. older BCGs tend to be the most els are seriously incorrect) had their first mergers at early masses or velocity dispersion. Without dry mergers, then (weak) change along projected correlations (e.g. $M_{BH} - \sigma$) reflects the small structural changes of the remnants in Figure 11.

The dynamic range observed in Figure 11 is subtle, becoming dramatic only at the highest masses.

There is, however, a significant effect in the residual correlations with e.g. size and merger history at fixed stellar mass or velocity dispersion. Without dry mergers, then (because of the same effect), the oldest systems at fixed mass would be the smallest. This is the opposite of observed trends (Gallazzi et al. 2006; Graves et al. 2008): older BCGs are extremely clustered, and so (unless CDM models are seriously incorrect) had their first mergers at early times, where spheroids are observed to be compact (see § 4).

The observations that e.g. older BCGs tend to be the most extended systems (von der Linden et al. 2007; Bernardi et al. 2007), therefore demands some dry mergers in their history (Hausman & Ostriker 1978; Hernquist et al. 1993). However, as shown in Figure 7 the number of such dry mergers (especially at $z \lesssim 1 - 2$) is still relatively small, so this is consistent with the $\sim 1$ major dry merger since $z \sim 1$ which direct observations of dry merging pairs (van Dokkum 2005; Bell et al. 2006; Lin et al. 2008) and constraints from mass function evolution (Bundy et al. 2005; Borch et al. 2006; Pérez-González et al. 2008; Brown et al. 2007; Hopkins et al. 2007a; Zheng et al. 2007) suggest.

The fundamental plane itself is less affected by dry mergers. This is not surprising; numerical simulations demon-
mergers (and the projected correlation is roughly $M_{\text{BH}} \propto M_*$) there is almost no change in that correlation with or without dry mergers. Total spheroid binding energy is also conserved (which leads to velocity dispersion being nearly constant) in dry mergers, but the correlation here is not linear (although it is the more fundamental correlation in our models). If we assume the “initial” correlation is the power-law $M_{\text{BH}} \propto E_{\text{halo}}^{0.7}$ (or $M_{\text{BH}} \propto \sigma^4$ for the $M_{\text{BH}} - \sigma$ relation), then (assuming $M_* \propto \sigma^4$, which gives $E_{\text{halo}} \propto M_*^{3/2}$) after a merger of mass ratio $f$ ($f \leq 1$) two systems initially on the correlation will move a factor $(1 + f)/(1 + f^{3/2})^{0.7}$ off the correlation (likewise for the $M_{\text{BH}} - \sigma$ relation). This amounts to a $\sim 20\%$ ($\sim 0.1$ dex) effect in major mergers – so we expect (given the $\sim 0.3$ dex intrinsic scatter in the correlations) it will only be noticeable at the most massive end where systems have undergone a large number of dry mergers. We do see this, but over the dynamic range of the present data, there is little distinction that can be clearly drawn – the scatter in various properties (and the fact that e.g. neither $M_{\text{BH}} - \sigma$ nor $M_{\text{BH}} - M_*$ is the most fundamental correlation in this model) mostly washes it out. In terms of the correlations with effective radius $R_e$, there are more noticeable changes, directly reflecting the different size-mass relation in Figure[11].

4. EVOLUTION OF SCALING LAWS AS A FUNCTION OF REDSHIFT

We have shown how dissipation and dry mergers alter the correlations of ellipticals at $z = 0$. Of course, gas fractions of their progenitors evolve with redshift, and dry mergers preferentially occur at low redshift. In addition, other properties of the progenitors, such as disk and halo sizes, will evolve. We therefore expect there could be evolution in these scaling laws, and make predictions for this here. We adopt our standard model, but construct samples as they would be observed at a given non-zero redshift.

4.1. Predicted Evolution In Our Standard Model

4.1.1. Spheroid Sizes

Figure[15] shows how the median effective radii of spheroids at fixed stellar mass are predicted to evolve with redshift. We vary the assumptions described in §2, most importantly (other uncertainties being relatively negligible at the redshifts of interest here) whether (and how strongly, within the observational limits) disk sizes gas fractions evolve with redshift, and show the resulting allowed range. We compare with observations of the spheroid-size mass relation from [Trujillo et al. 2006, McIntosh et al. 2005, van Dokkum et al. 2008] and [Zirm et al. 2007] (see also [Franx et al. 2008, Damjanov et al. 2008]). Although the observational constraints are weak, they agree with our predictions at redshifts $z \sim 0 - 3$.

Evolution in galaxy gas fraction, which directly translates to evolution in the amount of dissipation in mergers, drives this behavior. This is expected – in §2.1, and in greater detail in Paper II, we show that the degree of dissipation is the dominant factor determining the stellar effective radius. Disk gas fractions are larger at high redshift, so in mergers, a larger mass fraction will be formed in a compact dissipational starburst, yielding a smaller stellar remnant. Dry mergers have also had less time to act and puff up remnants at high redshift. If disks are more compact at high redshift, this drives further evolution; however, observational constraints suggest that this is relatively weak (see §2.3), and so it is less important than the evolution in gas fractions.

![FIG. 15 — Predicted evolution in spheroid sizes with redshift. We show the predicted median size of spheroids at a given stellar mass, relative to the median size of objects of that mass at $z = 0$. Shaded range is the uncertainty owing to both cosmology and how rapidly progenitor (disk and halo) properties evolve. Solid lines are simple power-law fits (Equation 7). Dashed line shows the power-law fit corrected (approximately) to the $R_e$ that would be measured in optical/UV bands (as opposed to the half-mass radius: stellar population effects tend to make very young ellipticals with recent central starbursts more concentrated in short-wavelength light). Dotted line is the prediction from the semi-analytic model of [Khochfar & Silk 2006]. Observational estimates for galaxies in the appropriate stellar mass intervals are shown from McIntosh et al. (2005, circles), Trujillo et al. (2006, stars), Zirm et al. (2007, square), and van Dokkum et al. (2008, diamond). Progenitors are more gas-rich at high-$z$, yielding mergers with more dissipation and therefore smaller remnants. At high masses, low-$z$ disks are gas poor (and dry mergers have been more important), so the relative difference is larger; at low masses, there is little room for gas fractions to increase, hence weak evolution.

This also predicts that the relative size evolution should be stronger in high-mass systems. Fitting these predictions to simple scaling laws of the form

$$R_e(M_* | z) = (1 + z)^{-\beta} R_e(M_* | z = 0)$$

we obtain $\beta = (0.0 - 0.2, 0.24, 0.48, 0.64) \pm 0.05$ or so for each) for $M_* \sim (10^9, 10^{10}, 10^{11}, 10^{12}) M_\odot$ (roughly $\beta \approx \text{MAX}[0.23 \log (M_* / 10^9 M_\odot), 0.2]$). Low mass disks are still gas-rich at $z = 0$ (and have generally not experienced many dry mergers), so there is limited room for evolution (the evolution we do predict is almost entirely driven by whatever evolution in progenitor disk sizes is assumed, hence the quoted range $0.0 - 0.2$ in the lowest-mass bin). At high mass, however, dry mergers have been increasing spheroid sizes for a significant redshift interval. Furthermore, disks at high mass at low redshift are relatively gas-poor; by $z \sim 3$, they have doubled or tripled their gas fractions ([Erb et al. 2008]. The relative size difference is therefore much more pronounced.

These predictions are similar to those made by [Khochfar & Silk 2006], who adopted a full semi-analytic model to estimate the history of disk galaxies, accretion, and star formation, and then modeled disk sizes by assuming effective radius is proportional to the mass fraction in the dissipationless (non-starburst) component: $R_e(M_* | z) = R_e(M_* | 0) \times (f_{\text{dissipationless}}(z) / f_{\text{dissipationless}}(0))$. For comparison, we show their predictions in Figure[15] at intermediate and low masses, they are similar to ours. This is because their simple model captures the key qualitative aspect which drives our predicted evolution, namely the evolution in disk gas fractions with redshift, making high-redshift ellipticals more dissipational than their low-redshift counterparts. Ultimately, their determination of the size evolution with dissipational fraction is not a bad approximation over intermediate gas fractions (to the accuracy here). However, there is a significant difference in the predictions at the
highest masses – Khochfar & Silk (2006) predicted much stronger evolution than we do. This is the regime where our attempts to improve on their estimates are of particular importance.

First, their predictions are tied to some assumptions and modeling of disk galaxies in their semi-analytic model; this is precisely why we have tried to be as empirical as possible in constructing our progenitor galaxies. They note that their strong evolution in this mass bin is the result of a high-redshift overcooling problem in describing massive galaxies in their semi-analytic model (a well-known difficulty without strong feedback). As a consequence, their disk gas fraction evolution to \( z \sim 2 \) is much steeper than observed – i.e., “progenitor” disks at these masses and redshifts are demonstrably too gas-rich and in halos of too low a mass. Erb et al. (2006) see typical gas fractions for \( M_\ast \geq 10^{11} M_\odot \) galaxies at \( z \sim 2 \) of \( \sim 20\% \) (whereas Khochfar & Silk (2006) predict \( \sim 70-80\% \)) – even if the ellipticals observed at \( z = 2 \) formed from high-redshift mergers with gas fractions say, double this value, this predicts factor \( \lesssim 2 \) size evolution at \( z = 2 \), similar to our estimates. By using a halo occupation approach, we avoid this problem.

Second, our size estimates are calibrated directly to numerical simulations: we track the evolution of multiple galaxy components in merger simulations and use this to design our prescriptions. Of particular importance, we include the effects of dry mergers appropriately and extrapolate our prescriptions to arbitrary dissipational fractions: at high \( f_{\text{dissipational}} \) the simple size estimator from Khochfar & Silk (2006) clearly must break down, as galaxies will still have finite sizes. This direct calibration with simulations avoids ambiguity in e.g. degeneracies between cosmological effects and assumptions about how spheroid sizes scale, and allows for more robust predictions.

Nevertheless the reason for our predicted evolution, and its sense, is essentially the same as that identified in Khochfar & Silk (2006). Ellipticals at high redshift have, on average, formed a larger fraction of their mass in dissipational starbursts (owing to their progenitors being, on average, more gas-rich) at these redshifts. Dissipation allows this component to be significantly more compact than the dissipationlessly violently relaxed stars from the progenitor disks, giving rise to a more compact remnant.

At the highest masses (\( \gg 10^{11} M_\odot \)), the evolution we predict is somewhat weaker than that observationally inferred by Zirm et al. (2007), van Dokkum et al. (2008), Franx et al. (2008). However, as demonstrated in Paper I, this can be explained by an additional bias – if systems at high \( z \) are very young (ages \( \lesssim 0.5 \) – 1 Gyr) remnants of gas-rich mergers (as we predict precisely these systems are, and as observations of their stellar populations suggest, see Kriek et al. (2006) and observed in even rest-frame optical wavelengths (let along rest-frame UV), the gradients in \( M_\ast/L \) introduce a bias towards smaller \( R_e \) (there may also be some related, but weak, biases in these stellar population parameters and stellar masses, see e.g. Wuyts et al. (2007)). The younger stellar population from the starburst – i.e., the dissipational component – is significantly brighter in \( B \)-band at ages \( \lesssim 1 \) Gyr, so this dominates the fit, yielding a smaller \( R_e \) (equivalent to artificially further increasing the dissipational mass fraction – this is why it actually looks as if the Khochfar & Silk (2006) prediction provides an acceptable match to these objects).

In line with this expectation, the sizes we obtain if we ignore the dissipationless components of our predicted \( z > 2 - 3 \) systems are consistent with the observations from Zirm et al. (2007), van Dokkum et al. (2008), Franx et al. (2008); Damianov et al. (2008). Essentially, this effect can yield a bias of an additional factor \( \sim 2 \) (see Figure 20 in Paper I) for gas-rich systems. It will not be a problem at lower redshifts, both because systems are older (the effect is negligible at starburst ages \( > 1 \) Gyr, or even earlier, as induced metallicity gradients from the starburst can partially cancel it out), and because gas fractions are lower (we expect significant bias only for \( f_{\text{dissipational}} \gtrsim 0.3 \) – 0.4). Furthermore, there is no bias within the starburst light (there is a strong gradient within the starburst itself), so for low-mass systems (which are largely dissipational at every redshift), there is no significant effect. It is precisely for large mass systems at high redshift that we expect this to be an important concern, and measurements that could ultimately estimate e.g. color gradients and stellar population gradients in these systems will be necessary for more detailed constraints.

4.1.2. Velocity Dispersions

Figure 16 shows the corresponding evolution in velocity dispersion with redshift. The evolution is much weaker than might be intuitively expected: if \( \sigma^2 \sim GM/R_e \), then the factor \( \sim 3 \) evolution in size in massive systems in Figure 15 would translate to factor \( \sim \sqrt{3} \) evolution in \( \sigma \). Instead, we predict such systems have roughly the same or at most a factor \( \sim 1.3 \) larger \( \sigma \). This is because, at \( z = 0 \), dark matter plays an important role in setting the velocity dispersion – the potential at \( r = 0 \) has a contribution \( \sim GM_\text{halo}/R_\text{halo} \) (here \( R_\text{halo} \) is the effective radius of the halo, \( \text{not} \) the virial radius; \( R_\text{halo} \sim R_\text{vir}/c \), where \( c \) is the halo concentration), which is comparable to or larger than \( \sim GM_\ast/R_e \) in most systems (although \( R_\text{halo} \gg R_e \), \( M_\text{halo} \gg M_\ast \) by about the same factor). This is especially true for the high-mass ellipticals (for which the evolution in \( R_e \) is strongest) which, at \( z = 0 \), are the most dark-matter dominated within their stellar \( R_e \). At a given halo mass, \( R_\text{halo} \) evolves weakly – the evolution in halo concentrations almost completely offsets the evolution in virial radii (this reflects the fact seen in most simulations that massive halos build inside out – the central potential is set first and then the outer halo builds up; see Bullock et al. 2001, Wechsler et al. 2002), so assuming the ratio \( M_\ast/M_\text{halo} \) does not evolve much with redshift (as inferred in most observations and halo occupation models; see § 2.2), then the contribution of the halo to \( \sigma (\sigma_\text{halo}) \) FIG. 16. — Predicted evolution in spheroid velocity dispersions with redshift (style as Figure 15). Despite significant evolution in \( R_e \) at fixed mass, the role of dark matter halos (which evolve more weakly) in setting \( \sigma \) yields much less evolution in \( \sigma \) for the same systems. Lines are the simple prediction given the fitted scaling of \( R_e \) with \( z \) (Figure 15) and Equation (8).
we obtain linearly, we obtain a galaxy potential and halo potential which add to reflect a galaxy potential and halo potential which add inside their stellar potential. Again, low-mass systems are already highly dissipational and baryon-dominated inside $R_e$ at $z = 0$, so there is little room for evolution.

evolves weakly at fixed $M_\ast$. If anything, it can decrease — if the baryonic component is more compact at high-$z$, then the enclosed dark matter mass in the stellar $R_e$ is smaller.

Roughly, then, if we consider the observed velocity dispersion to reflect a galaxy potential and halo potential which add linearly, we obtain $\sigma^2 \propto (M_\ast/R_e + M_{\text{halo}}/R_{\text{halo}})$. If we consider systems of fixed $M_\ast$, and $M_{\text{halo}}(M_\ast)$ and $R_{\text{halo}}(M_{\text{halo}})$ evolve weakly, then we obtain

$$\langle \sigma(z| M_\ast) \rangle = \frac{1}{\sqrt{1 + \gamma}} \sqrt{\gamma + \langle \sigma_0(0) \rangle / \langle R_e(z) \rangle},$$

where $\gamma \equiv (M_{\text{halo}}/R_{\text{halo}})/(M_\ast/R_e) \sim 1 - 2$ is the relative fraction of the central potential contributed by the dark matter at $z = 0$. For the values of $\gamma$ we estimate ($\sim 1$ at $M_\ast \sim 10^{11} M_\odot$ and $\sim 2$ at $M_\ast \sim 10^{12} M_\odot$), this simple expectation fits the observed trends well.

4.1.3. The Fundamental Plane

Combining these trends, we anticipate the evolution in $M_{\text{dyn}}/M_\ast$, shown in Figure 17. Since $M_{\text{dyn}} \equiv k \sigma^2 R_e / G$, the stronger evolution in $R_e$ largely drives the evolution in $M_{\text{dyn}}$. We emphasize that this reflects a real difference in the enclosed matter within the stellar $R_e$ (as we have defined it, there are no significant structural or kinematic non-homology effects). The decrease in $M_{\text{dyn}}$ at fixed $M_\ast$ in the highest-mass systems is a consequence of the fact that at low-$z$, they have small dissipational content and have larger stellar $R_e$, so have larger dark matter masses enclosed in $R_e$ (and thus higher enclosed mass in $R_e$). At high-$z$, their progenitors are gas-rich, so they are formed in highly dissipational mergers and have compact stellar distributions, which enclose less dark matter in the stellar $R_e$.

Figure 18 shows the implications of this evolution for the FP as a function of redshift. At low redshift, the observed tilt ($\alpha \approx 0.2$) is recovered. At high redshift, however, all progenitors are gas-rich, so there is not much difference in the dissipational content at low and high masses — meaning their compactness (relative to e.g. their dark matter halos or progenitor disks) is no longer a strong function of mass. The high-mass systems, being nearly as dissipational as low-mass systems, are similarly baryon-dominated in their stellar $R_e$, and there is less tilt.

At the level predicted here, it may be difficult to observe the predicted FP evolution. Robust velocity dispersions and effective radii in the same rest-frame band (avoiding a bias towards smaller $R_e$ as observations probe closer to the rest-frame UV) may be obtainable in large samples at $z \lesssim 1$, but the predicted evolution in that interval is weak — within the systematic uncertainties at $z \sim 0$.

This is consistent with observational constraints in this redshift interval from weak lensing (Heymans et al. 2006 and optical studies (di Serego Alighieri et al. 2005, Treu et al. 2005, van der Wel et al. 2005, van Dokkum & van der Marel 2007, Brown et al. 2008). These observations do see evolution in optical bands, (in the opposite sense to that predicted here, namely an increasing apparent tilt), but find that this owes to stellar population effects (di Serego Alighieri et al. 2005) – the evolution observed in terms of $M_{\text{dyn}}$ and $M_\ast$ is either negligible or slightly negative (as predicted here). Any model where the stellar populations of lower-mass ellipticals are systematically younger (as is predicted here; see Hopkins et al. 2008c) will predict the observed trend in optical bands: if, say, a low-mass system at $z = 0$ has a stellar population age of $\sim 7$ Gyr whereas a high-mass system has an age of $\sim 12$ Gyr, as observed (see e.g. Trager et al. 2000, Nelan et al. 2005, Thomas et al. 2005, Gallazzi et al. 2006), then inverting passive evolution to $z = 0.7$, the low-mass (younger) system would have a lower $M_\ast/L_B$ by a fac-
tor $\gtrsim 10$, whereas the high-mass system would have increased $M_*/L_B$ by a factor of just $\approx 2$. It is therefore challenging to observe the weak evolution here ($\sim 0.1$ dex), given expected stellar population effects of magnitude $\sim 0.5 - 1$ dex.

However, this degree of evolution is important for a number of subtle effects: for example, van Dokkum (2008) uses the evolution of the optical FP with redshift to infer evolution in elliptical mass-to-light ratios (under the assumption that the physical – i.e. stellar mass – FP is invariant). This is broadly fine, as the optical $M/L$ for any reasonable stellar population age evolves much more rapidly than the physical FP evolution in Figure 13. But the authors then compare this change in $M/L$ with time to the mean colors of ellipticals as a function of time, and argue that the relation between the two suggests evolution in the stellar initial mass function. At this level (as the authors acknowledge), the comparison is sensitive to evolution in the physical (stellar mass) FP at the $\sim 0.1$ dex level – if massive galaxies at $z \sim 1$ have lower $M_{dyn}/M_*$ by $\sim 0.1$ dex (similar to what we predict in Figure 17) this will bias the inferred $M/L$ ratios (under the assumption of no physical FP evolution), but not, obviously the galaxy colors, by an amount comparable to the significance of the effect seen (in other words, allowing for this evolution, the evidence for any evolution in the stellar IMF may be less strong).

At this level of detail, physical evolution in the FP in the manner predicted here may change this comparison, and should be considered in future, more detailed comparisons with observations. However, the effects here may not be important so long as the observed sample is appropriately selected: van Dokkum (2008) focus on satellites in rich clusters, which are not likely to undergo significant subsequent merging (especially as compared to the central galaxies modeled here, particularly those in moderately dense field and group environments that contribute much of the “lever arm” to the FP tilt evolution predicted). This emphasizes the importance of appropriate sample selection when comparing these quantities, and the need for further study to determine exactly how our predictions generalize to populations with more complex or distinct histories (such as satellites) and therefore pertain to the observations.

4.1.4. Black Hole Masses

Figure 19 & 20 show the evolution in BH masses at fixed stellar mass and velocity dispersion, respectively. Because systems are more dissipational at high redshifts, the binding energy of the baryonic material which must be supported or expelled to halt accretion is higher (at fixed stellar mass). As a consequence, the typical BH masses are larger. We discuss this evolution, its physical causes, and consequences, in greater detail in Hopkins et al. (2007b). Here, we note that the same conclusions hold, adopting a more complex cosmological model than was considered in that paper.

The evolution is similar to that suggested by a number of recent observations (e.g. Shields et al. 2006; Walter et al. 2004; Peng et al. 2006; Woo et al. 2006; Salvatier et al. 2006; and indirect estimations (Hopkins et al. 2006c; Merloni et al. 2004; Adelberger & Steidel 2005; Wuytis & Loeb 2005; Hopkins et al. 2007c; Lidz et al. 2006) but we emphasize that it is not strong (it is in fact on the lower $\sim 1$-$\sigma$ end of the estimated evolution) – typical BHs are a factor $\lesssim 2$ larger at $z \sim 2-3$. Larger differences from the local relations (in particular in very luminous quasar populations) may result from selection biases (Lauer et al. 2007a), where the most luminous quasars (most massive BHs) are likely to represent the upper end of the scatter in the BH-host correlations. The combination of such a selection effect with our predicted evolution is a very good match to the evolution in Peng et al. (2006).

We also compare with the constraints in Hopkins et al. (2006c), derived from observations of the evolution in the spheroid mass density with redshift (since the black hole mass density cannot, in any reasonable model, decrease, the maximum evolution in the typical ratio $M_{BH}/M_*$ is limited by the fact that the resulting black hole mass density, predicted from the observed spheroid mass functions, must not be higher than that observed at $z = 0$). This constraint is most directly applicable to $\sim L_*$ galaxies, since this is where most of the mass density of the universe resides. These constraints limit evolution to a factor $\lesssim 2$ at $z = 2$, consistent with our predictions. Comparing to similar estimates from Merloni et al. (2004), who adopt a similar approach but model the evolution in the black hole mass density from quasar luminosity functions, yields a similar constraint and expectation of weak evolution, $M_{BH}/M_* \sim (1+z)^{0.4-0.6}$, consistent with our predic-
The evolution in $M_{\text{BH}}/\sigma$ is somewhat more complex, reflecting the full interplay between evolution in $M_{\text{BH}}$ with spheroid binding energy and evolution in $\sigma$ with redshift (discussed above). Because deeper potentials will, in general, be reflected in higher $\sigma$ values, we expect weaker (or even inverse) evolution here, relative to $M_{\text{BH}}/M_*$ (where $M_*$ was not a measure of the central potential depth, so deeper potentials at fixed $M_*$ are translated to higher $M_{\text{BH}}$). However, because $\sigma$ reflects a combination of the halo and galaxy over a significant dynamic range, the results are non-trivial and can be non-monotonic in redshift. For example, in our most massive bin, the predicted evolution in black hole masses is sufficiently strong (and evolution in $\sigma$ relatively weak as a reflection of the tradeoff between dark matter and baryonic potential) that evolution to larger $M_{\text{BH}}/\sigma$ (albeit at the factor $\lesssim 2-3$ level) may be expected, consistent with recent observations (Shields et al. 2006; Salviander et al. 2006), although again selection effects are probably important and may explain cases where the inferred evolution appears to be much larger.

4.2. Effects of Dissipation

We have argued that these effects are largely driven by varying degrees of dissipation as a function of redshift. We briefly illustrate this here by repeating our predictions for the size evolution of ellipticals (which, as we showed in § 4, is the most pronounced evolution) in the version of our model where we ignore dissipation or treat gas identically to stars (as in §§ 3.1, 2). Figure 21 shows the results. Without evolution in gas fractions driving evolution in the degree of dissipation, only the (weaker) evolution in progenitor disk sizes has a noticeable effect, and the evolution is substantially suppressed. In fact, the predicted evolution can, in this case, be even weaker than that in the progenitor disks, because ellipticals with a given final mass tend to undergo their mergers over a similar redshift range – so their final sizes will reflect whatever the sizes of disks were at that time, whether or not those disk sizes evolved later.

A similar (relative) effect is seen for the evolution in $M_{\text{dyn}}$ and the FP, as well as BH masses. We show the evolution in black hole mass at fixed stellar mass predicted without dissipation in Figure 22. As expected based on the evolution in $R_c$, the evolution here is essentially eliminated without dissipation.

4.3. Effects of Dry Mergers

At the high-mass end, there is also some evolution because systems merging early will have dry mergers puff up their sizes at later times. Figure 23 shows the evolution in the size-mass relation, with dry mergers suppressed (as in § 3.2). The difference here is real, but small. Dry mergers – while important for the size evolution of individual objects – do not have a dramatic effect on the overall evolution of the size-mass relation. If the “pre dry-merger” size-mass relation is a power-law, $R_c \propto M_*^{\alpha}$, then by the energy conservation argument in Equation 4, a merger with mass ratio $f (f \leq 1)$ of two systems initially on this correlation will have a final effective radius a factor $(1 + f)^{\alpha}/(1 + f^{2-\alpha})$ larger than the primary. Relative to the size-mass relation, the system has moved above the relation by a factor $(1 + f)^{2-\alpha}/(1 + f^{2-\alpha})$.

For observed values of $\alpha$ in disks or ellipticals ($\sim 0.3$ and $\sim 0.6$, respectively) this amounts to only $\sim 0.1$ dex evolution per major ($f \geq 1/3$) merger, compared to e.g. $\sim 0.3$ dex scatter in the correlation. This is why, in Figure 11 the absence of dry mergers makes a difference only at the highest end of the mass function, where the number of major dry mergers might be large. Most of the population did not experience substantial dry merging, and what did, experienced it mainly at late times. As a result, the inferred redshift evolution without dry mergers is largely the same, with only an offset in the rate of
size evolution at $z \lesssim 1-2$ for the most massive systems (leading to an offset in the relative size prediction at $z \sim 3$).

It should be emphasized that the difference seen at the highest masses and redshifts in Figure 23 pertains by definition to those ellipticals which are already that massive at those redshifts – i.e. systems which have formed and assembled $\sim 10^{12} M_\odot$ by $z \gtrsim 2-3$. These systems obviously live in the most dense environments, and are very rare – their evolution is strong because they are expected to undergo a large number of dry mergers, as well as mergers with later-forming gas-poor disks and spheroids and minor mergers (see §5 below) that will build up an extended envelope and rapidly increase their sizes. The typical system with the same mass observed at lower redshifts was not fully assembled at these early times, but forms in less overdense regions and (although star formation in progenitors may have ceased at early times) assembled more recently, in a smaller number of dry mergers. So the evolution seen should not be taken to imply a large number of dry mergers for the average massive galaxy, but rather to imply a large number of subsequent mergers (“large” being $\sim$ a few; again see §5 for discussion of how this can efficiently increase the spheroid size) for, in particular, galaxies in extreme environments that are already massive at early times.

5. EVOLUTION OF INDIVIDUAL SYSTEMS: WHAT HAPPENS TO THE SYSTEMS FORMED AT HIGH REDSHIFT?

We have predicted that high-$z$ massive ellipticals should be substantially more compact than low-$z$ ellipticals of the same mass, in apparent agreement with observations. If this is true, then what happens to these early-forming, compact ellipticals as they evolve to $z = 0$?

We are now interested in the evolution of individual systems, from their formation redshift to $z = 0$, rather than the evolution of the population at the same mass (recall, the systems with $M_\ast \sim 10^{11} M_\odot$ at $z \sim 2-3$ are not, generally, the same galaxies that have $M_\ast \sim 10^{11} M_\odot$ at $z = 0$). Figure 24 shows such a case study, in fact a set of them, tracking the median (averaged over many Monte Carlo realizations) evolution in sizes of individual systems that are formed at a given redshift and will have a specific stellar mass at $z = 0$. For two $z = 0$ stellar masses where evolution is significant, we show the evolution in effective radii, velocity dispersions, and stellar masses. Figure 25 shows the same, but projects a few of the representative median tracks (for early-forming systems) into the size-stellar mass plane, to show how the systems evolve as a function of time and merger history in this space relative to e.g. the observed $z = 0$ correlation.

The systems which have their first major merger at some early time are indeed much more compact than their $z = 0$ descendants. Partly, this is because they begin life as lower-mass systems, but even for their (instantaneous) stellar mass they are more compact than $z = 0$ objects of that mass, reflecting the higher degree of dissipation (the evolution in Figure 15). However, by $z = 0$, systems of the same stellar mass from different formation epochs all have effective radii within a narrow range (a factor of $\sim 2$). The systems which form late ($z \lesssim 1$) evolve weakly – they rarely experience any dry mergers, so their $z = 0$ sizes largely reflect their sizes at formation. However, the systems which form early ($z \gtrsim 3$) evolve strongly – they experience a number of both mergers with lower-redshift, larger and much less gas-rich disks, as well as other spheroids (true spheroid-spheroid dry mergers). Such mergers increase the sizes of the ellipticals both in absolute terms and in relative terms (moving them above the size-mass relation representative of the time when they formed). Although the number of major mergers for most systems is relatively small, it is a function of their formation time. The systems which first merge at earliest times do so because they live in highly biased environments – they are precisely those expected to undergo the most dry merging. As a result, the effects of evolving dissipational content and the effects of dry mergers appear to conspire such that, despite a significant evolution in the size-mass relation with redshift, by $z = 0$ ellipticals formed at early times will have broadly similar sizes to those forming around $z \sim 0$.

If we examine Figures 24-25 we can plot out the likely course of evolution for the compact, massive passive galaxies seen at $z \sim 3$ in Kriek et al. (2006); Zirm et al. (2007); Labbé et al. (2005); Wuyts et al. (2007); Daddi et al. (2005); van Dokkum et al. (2008); Franx et al. (2008); Damjanov et al. (2008). At the redshifts observed, these systems which have formed recently (systems which form earlier will be even more rare) in a gas-rich merger (in agreement with their young observed ages, relative to $z = 0$ spheroids of the same mass; Kriek et al. 2006), very high gas fractions $\gtrsim 50-40\%$ characteristic of disks at these and higher redshifts. The large dissipational fraction yields a massive galaxy ($M_\ast \gtrsim 10^{11} M_\odot$) with a very small effective radius $\sim 1$ kpc. As discussed in §4 above, however, the velocity dispersion is not extremely high (if we took $\sigma(z) = \sigma(M_\ast = 0) \times R_e(z)/R_e(z_0)$, we would estimate $\sigma \approx 600-800$ km s$^{-1}$; however, the predicted velocity dispersion is in fact $\sim 250-300$ km s$^{-1}$). Because the system is so much more compact, the dark matter fraction within $R_e$ is much smaller than in a typical $z = 0$ spheroid of the same mass, so its contribution to the velocity dispersion is much smaller.

The system, having formed in a high density environment (consistent with their observed clustering and number densities; e.g. Quadri et al. 2007; van Dokkum et al. 2006), will undergo a significant number of mergers between this time and $z \sim 1-2$, where the merger rate begins to level off. These mergers are with a mix of disks and ellipticals. At times shortly after formation, successive mergers may be with other gas-rich disks, yielding similar amounts of dissipation and keeping the system relatively compact. This is illustrated in Figure 25 as a “similar gas-rich merger”; which will move the system parallel to the size-mass relation ($R_e \propto M_\ast^{0.6}$). Soon, nearby systems will (given the biased environment) be largely elliptical – but they may be similar, compact ellipticals. Spheroid-spheroid dry mergers will puff up the system, moving it somewhat relative to the initial size-mass relation at formation. This is shown as a “dry merger with similar (compact) elliptical” in Figure 25 – here merging two identical (equally compact) ellipticals doubles both $R_e$ and $M_\ast$ (see §4.1), moving the system “up” relative to the size-mass relation along the steeper axis $R_e \propto M_\ast^{1.0}$.

As time goes by (in a hierarchical scenario), the galaxies merging will have formed in a more extended physical region over longer periods of time (being recently accreted into the parent halo); merging disks at later times will be progressively more gas-poor (characteristic of low-$z$ disks, which have consumed their gas in star formation), and thus contribute dissipationless stars with very large effective radii, building up the stellar component (an extended, violently relaxed envelope) at large radii. Likewise, pure spheroid-spheroid mergers will include spheroids that formed later from such less gas-rich disks (and are therefore less compact), and these will serve to puff
The effective radius therefore rapidly grows, and by $z = 0$ the system lies on (or even somewhat above) the median size-mass relation for systems of the same $z = 0$ stellar mass. The velocity dispersion grows slightly as mass is accumulated, but since most of this mass is contributed from dissipation-less components to the extended envelope (recall, a merger of two identical spheroids will leave $\sigma$ unchanged), it does not contribute much, and the $z = 0$ galaxy has a peak (central) velocity dispersion $\approx 400\text{km s}^{-1}$, large but only 30% larger than its initial central velocity dispersion and completely consistent with those observed in the most massive galaxies today (Bernardi et al. 2006). In detail, in fact, the predicted descendants of early compact systems here have similar abundances, velocity dispersions, and remarkably similar predicted locations in e.g. the $z = 0$ size-mass, fundamental plane, and Faber-Jackson relations to the sample in Bernardi et al. (2006), suggesting that many of those systems may be the products of this process. For the most part, then, these extreme systems are completely consistent in all the properties we can predict here (effective radius, mass, velocity dispersion, black hole mass, and profile shape/Sersic index) with their constituting the central $\sim 10-30\%$ of the mass in a significant fraction of the brightest group or cluster galaxies today.

This can explain why such objects are not ubiquitous in the local Universe. However, we could still ask what fraction of such systems might be expected to survive to $z = 0$, without disappearing into a larger $z = 0$ galaxy. In short, should any of this massive and compact population exist at $z = 0$, which might represent the direct remnants of high-redshift, compact ellipticals?

Figure 25 shows the fraction (at a given $z = 0$ stellar mass) of objects which had their last major merger of any kind (which will largely set their observed size) at (or above) a given redshift. At masses corresponding to those observed in high-redshift compact galaxy populations ($\sim 10^{11} M_\odot$), the fraction of the population today which has survived relatively intact since $z \gtrsim 2$ is expected to be small, $\sim 1-10\%$. This translates to a $z = 0$ space density of such systems $\sim 10^{-4}\text{Mpc}^{-3}$: not

**Fig. 24**.— Median size (left), velocity dispersion (middle), and instantaneous stellar mass (right) as a function of redshift, for various galaxies that have similar $z = 0$ stellar mass (labeled). These median tracks are shown for systems that have their first major merger (that first form a spheroid) within narrow intervals in $z$ (shown where each line begins: e.g. the line beginning at $z \sim 3.5$ shows the median track of all systems which have their first major merger at $z = 3.5 \pm 0.25$ and will have the same labeled stellar mass at $z = 0$). Different colors show different variations in our model, and reflect the uncertainties in redshift evolution (as the shaded range in Figure 15). Systems which form early are relatively compact, but grow in mass owing to subsequent mergers with gas-poor disks and other spheroids.
large, but not completely negligible. This is an upper limit to the number density of such systems: if they have experienced a sufficiently large number of minor mergers, they may be significantly altered by $z = 0$ (although many of the same reasons that allow some systems to avoid major mergers will allow them to avoid a large number of minor mergers). At any redshift, however, there is a scatter in elliptical sizes owing to a scatter in e.g. the dissipational fractions at the time of their formation (reflecting scatter in disk gas fractions). Since disk gas fractions have significant (factor $\sim 1.5 - 2$) scatter, that is actually expected to dominate the scatter in e.g. the size-mass relation at $z = 0$ (rather than scatter in the formation times of ellipticals). That is to say, there could be a similar fraction of ellipticals that are too compact present at $z = 0$ which formed recently (not at $z > 2$), from the most anomalously gas-rich disks at lower redshifts (say the $\sim 2\sigma$ outliers in the disk gas fraction-mass relations). This probably will not explain systems as small as $\sim 1\text{kpc}$ at these stellar masses, but it illustrates that some additional indicator is needed to distinguish those possibilities. If stellar population ages could be measured, this is straightforward: the system which formed at low redshift will have significant recent star formation (in fact, quite a lot, since we are interested in compact systems with large dissipational fractions). So it should be possible, in principle, to identify the descendents of the early-forming ellipticals.

One important caveat to this discussion is that we have focused here on the role of major mergers with mass ratios below 1:3. Our calculation and other halo occupation models (Zheng et al. 2007), clustering measurements (Masjedi et al. 2007), and cosmological simulations (Maller et al. 2008) suggest that as galaxies approach the extreme $M_* \gg 10^{12} M_\odot$ BCG populations, growth by a large series of minor mergers becomes progressively more important than growth by major mergers. Our predictions in this limit should therefore be treated with some degree of caution, in contrast with the bulks of the elliptical population at masses $\lesssim$ a few $L_*$, the vast majority of which still live in less extreme environments (e.g. Blanton et al. 2005, Wang et al. 2006, Masjedi et al. 2006) and are expected to have experienced a small number (a couple) of major re-mergers since redshifts $z \sim 2 - 4$. Our modeling can, in principle, be extended to include minor mergers, and doing so we find qualitatively similar conclusions (because merger histories are roughly self-similar, the relative growth and courses of evolution seen in Figure 24 are conserved). However, as demonstrated in Hopkins et al. (2008c), our predictions (and those of other halo occupation-based approaches) become considerably less robust in the regime of minor mergers (mass ratios $\sim 1:10$ or so), owing to greater uncertainty in the satellite mass function at these mass ratios and weaker observational constraints on the halo occupation of small galaxies.

In any case, because early-forming, compact systems were
massive and experienced mergers at such early times, many will represent the primary galaxies in the highest density peaks, being BCGs today. Indeed, this is true for most of the massive high-$z$ spheroid population. However, in order to survive to $z = 0$ from this population, mergers must be avoided (by our definition) – something very hard for BCGs. We would still expect to find such objects around high-density peaks (there is no getting around the fact that they will only form so early and so massive in high-density peaks), but those that survive, in many cases, would be expected to do so precisely because they were not the central galaxy or BCG. (In detail, they probably were the central galaxy or BCG at their time of formation, but were displaced from this position in subsequent halo-halo mergers, becoming a satellite in a still larger system, supressing their merger history).

There may be some interesting candidates for such objects among the so-called “compact elliptical” family. For example (although we intend this only as an illustrative case), NGC 4486B is a satellite of NGC 4486 in Virgo. It has a stellar mass $\gtrsim 10^{10}M_{\odot}$ (Paper II) and velocity dispersion $\sigma = 200\text{km}\text{s}^{-1}$, but an effective radius of only 200 pc (see e.g. Kormendy et al. [2008]). This is somewhat smaller than the $\sim 10^{11}M_{\odot}$ which we just described, but the scenario can be generalized to masses about this low. The stellar population age is quite old, 10.5 Gyr (perhaps even up to $\sim 13$ Gyr, depending on the indicator used; Caldwell et al. [2003], implying a time of last significant star formation $z \gtrsim 2$. This is further supported by a corresponding $\alpha$-enrichment and metallicities. The system has high rotation and ellipticity suggestive of a very gas-rich formation (Bender 1988; Bender et al. [1992]).

The estimated dynamical friction time is sufficiently long that the system could have become a satellite in the Virgo progenitor as early as $z \sim 2$ and still not merged with the central galaxy. All of these lines of evidence suggest it may have survived from an early formation time as a very compact system. Unfortunately, except for the direct stellar population age measurements, these arguments are mostly circumstantial, and those stellar population estimates are still quite uncertain. Better determinations of the star formation history in this object would be an important test of this hypothesis.

6. CONCLUSIONS

We have combined the results from a large library of numerical simulations with simple, observationally constrained models for the halo occupation (and corresponding merger history) of galaxies in order to predict how spheroid scaling laws evolve. We demonstrate that, despite a variety of progenitor properties, complex merger histories, and evolution in e.g. progenitor sizes and gas content with redshift, the most important property driving the structural scaling relations of spheroids at each redshift (and, in particular, making these scaling relations different from those obeyed by disk galaxies) is the amount of dissipation involved in the formation of the spheroid. At the highest masses, dry mergers at relatively late times are also important, but are few, consistent with observational constraints. Together, these predict a significant mass-dependent evolution in spheroid scaling laws, which has important consequences for the fundamental plane, spheroid densities, and the formation histories of massive galaxies.

At $z = 0$, a simple model accounting for the effects of dissipation in mergers is able to explain the size-mass, velocity dispersion-mass, and fundamental plane (dynamical-stellar mass) correlations, the relation between galaxy profile shape (Sersic index) and mass, the correlation between black hole mass and various host properties (bulge binding energy, mass, velocity dispersion, and profile shape), the relative mass fractions in compact dissipational components in ellipticals, and the abundance of cuspy/disky/rapidly rotating versus cored/boxy/slowly rotating ellipticals (see Hopkins et al. [2008]). Dissipation in the initial disk-disk mergers that form the spheroid progenitors is the most important parameter controlling these relations.

In particular, at high masses, where disks are gas-poor, the sizes, densities, and dynamical mass to stellar mass ratios of ellipticals reflect those of their progenitor disks. At lower masses, where disks are gas-rich, mergers involve a significant amount of new star formation in dissipational nuclear starbursts triggered by loss of energy and angular momentum in the gas in the merger, building up a compact central spheroid distribution (the “dissipational” component) that yields a smaller, more dense remnant. This mass-dependence in the dissipational content of progenitor disks makes the size-mass relation of ellipticals steeper than that of disks and gives the observed tilt in the fundamental plane (essentially the statement that low-mass disks, being more dominated by this nuclear dissipational component, have higher baryon-to-dark matter fractions within their stellar effective radii). Black holes are formed with a mass that, in a simple feedback-regulated model, corresponds to the binding energy of the dissipational component of bulge at the time of this dissipation, which the black hole must work against in order to self-regulate its growth. This matches the detailed observations of a black hole fundamental plane in Hopkins et al. [2007] and gives rise to the secondary (indirect) correlations between black hole mass and spheroid velocity dispersion, mass, size, and Sersic index (all of which agree with those observed).

To lowest order, the dissipational fractions that give rise to these trends at $z = 0$ reflect the observed gas fractions of disk galaxies at moderate redshifts $z \sim 0-2$. This is because most ellipticals are formed in a relatively small number of major mergers at low redshifts in this range. It is only when one considers the most massive populations of e.g. BCGs and massive cored, boxy, slowly rotating ellipticals that dry mergers become important. Their effect is to puff up these systems, producing extended envelopes and raising their outer Sersic indices. This is sufficient to explain the observed weak dependence of outer Sersic index on mass in elliptical galaxies, following the detailed multi-component decompositions of observed systems in Paper II and Paper III. When systems are fitted to a single Sersic index, the result is a steeper dependence that reflects a complex combination of e.g. the outer Sersic index (primarily driven by merger history), the mass fraction in a dissipational component (yielding rising densities at small radius), the shape of the two components where they are comparable, and the dynamic range of the fit (since real ellipticals are not perfect Sersic profiles, there is some dependence, and in particular when fitting the entire profile to a single Sersic law, the dependence can be significant).

The detailed interplay of dissipation and dry mergers also gives rise to important residual trends that can be used as tests of these models. For example, at a given stellar mass, the oldest systems will have the largest effective radii and dynamical masses. This is driven by two effects: first, disks which have high star formation efficiencies (earlier star formation times) will be more gas-poor at the time of their mergers – i.e. will have less mass in a dissipational component. Second, systems which may be gas-rich but form very early (i.e. have a high redshift of their last gas-rich merger) tend to do so because
they are in dense environments, and will therefore experience subsequent mergers with both more gas-poor disks and other spheroids, increasing their size and dynamical-to-stellar mass ratios.

At higher redshifts, disk gas fractions are expected and observed to be systematically larger. As a result, spheroids observed at these redshifts are expected to have formed in more gas-rich mergers and to have a larger mass fraction in their dissipational component, making them more compact. We predict that elliptical sizes evolve in a mass-dependent fashion: since low-mass disks are still quite gas-rich at $z = 0$, there is little room for them to become more so at high redshift, so they cannot evolve much. High mass disks, on the other hand, could in principle be much more gas rich than their observed $z = 0 \lesssim 10\%$ values. As well, at high masses, dry mergers become important in the predicted size evolution. Consequently, the highest mass ellipticals are expected to evolve to become a factor $\sim 2$–3 smaller at $z \gtrsim 2$. In terms of the mean effective radius of spheroids of a given stellar mass, relative to that at $z = 0$, we find evolution of the form $(1 + z)^{\beta}$ where $\beta \approx 0.23 \log (M_*/10^{11} M_\odot)$ (of course enforcing $\beta > 0$ or whatever minimum evolution is set by observed evolution in progenitor disk sizes at these masses).

This is consistent with observations from e.g. [Trujillo et al. (2006); McIntosh et al. (2005); Zirm et al. (2007); van Dokkum et al. (2008)], although we highlight important biases arising from stellar population gradients which are relevant when attempting to estimate the sizes of the most massive galaxies at $z \gtrsim 2$. We demonstrate that this size evolution owes almost entirely to the change in dissipation with redshift, although dry mergers are of some importance for the most massive systems. Whether we explicitly include it or not, the evolution cannot be explained by simple scaling of disk sizes with redshift (which, in any case, is observed to be weak; see e.g. [Somerville et al. (2008b); and references therein]).

This evolution has important implications for the other fundamental plane correlations. Because the fundamental plane tilt (at $z = 0$) arises owing to systematically higher dissipational fractions in low-mass systems (making them more compact and yielding lower dark matter fractions within the stellar component) at high redshift (where even high-mass systems must be more dissipational), we expect the tilt to be weaker. Note that we refer here to the stellar mass fundamental plane, i.e. $M_\text{dyn}$ versus $M_*$ - there will of course be stellar population effects introducing normalization and tilt changes in various observed bands. The predicted change is within the $z = 0$ uncertainties at $z \lesssim 1$, but by $z \gtrsim 3$, the tilt is predicted to largely disappear. This is important both as a test of these models, but also as a caution to observational attempts to use the fundamental plane as a detailed stellar population probe. It is certainly much less significant than the expected evolution in mass-to-light ratios in optical bands, but could be important for detailed stellar population probes such as that in [van Dokkum (2008)] which assume a fixed stellar mass fundamental plane (although selection of e.g. satellite galaxies in clusters, which are less likely to undergo significant future merging relative to central galaxies in the field or groups, may be sufficient to mitigate these evolutionary effects).

We argue that velocity dispersions will change weakly with redshift, despite the evolution in effective radii, because systems with higher dissipational fractions (smaller $R_e$) are more baryon-dominated in their stellar effective radii (i.e. the velocity dispersion includes a weaker contribution from the halo). This is important, because velocity dispersion is in general preserved or increases in subsequent re-mergers. If high-redshift massive galaxies had extreme velocity dispersions $\gtrsim 600\kms$ (corresponding to the simple assumption that $\sigma$ is inversely proportional to just the stellar effective radius), then the appropriate number of systems with equal or higher velocity dispersions would have to exist at $z = 0$ (and they do not, even in survey volumes of the SDSS; [Bernardi et al. (2006)]). However, accounting for the interplay between dark matter and baryonic mass, our predicted evolution in velocity dispersions is completely consistent with observational constraints.

Furthermore, because of the increasing dissipational components and deepening potential wells at high redshift, we expect that, in any feedback-regulated model of black hole growth, black holes must become at least somewhat more massive (relative to their host spheroid stellar mass) at high redshift. We predict a similar mass-dependent evolution in $M_\text{BH}/M_*$, with little or no significant evolution at $M_* \lesssim 10^{10} M_\odot$ (corresponding to black holes with masses $\lesssim 10^8 M_\odot$), and factor $\sim 2$ increase to $z \sim 2$–3 in the masses of the black holes in the most massive systems ($M_* \gg 10^{11} M_\odot$), corresponding to $M_\text{BH} \gtrsim 10^9 M_\odot$). Likewise, we predict the evolution in black hole mass relative to host velocity dispersion, which exhibits a more complex behavior. The physical details of driving these trends are discussed in more detail in [Hopkins et al. (2007b)], but here we embed those models in a more fully cosmological context. The evolution predicted is consistent with recent observations ([Peng et al. (2006)]) and with indirect constraints based on the relative evolution of black hole mass density and spheroid mass density ([Hopkins et al. (2006c); Merloni et al. (2004)]), however it is still relatively weak. Selection effects ([Lauer et al. (2007a)]) would be expected to explain the difference between some observational estimates that infer very strong (e.g. [Walter et al. (2004)]) or rapid low-redshift ([Woo et al. (2006)]) evolution and our predictions.

We outline the history of these massive systems which are different when first formed at high redshift (where they are very dissipational) and observed at low redshift. Most of these compact, early forming ellipticals will undergo a significant number of dry mergers, or mergers with lower-redshift, more gas-depleted and larger disks, which will serve to puff up the remnant, building up some stellar mass and an extended stellar envelope (although changing the central velocity dispersion by a relatively small amount). This raises their effective radii more rapidly than their stellar mass, in the sense of moving them up in the size-mass relation. Interestingly, the evolution in disk gas fractions and the effects of such dry mergers almost exactly offset one another in massive systems. In other words, a $\sim 10^{12} M_\odot$ galaxy which first formed from a merger at $z = 3$ (which had a high gas content, leading to a very compact $\gtrsim 10^{11} M_\odot$ initial galaxy with $R_e \sim 1\kpc$) will have almost the same size at $z = 0$ as a galaxy of the same observed stellar mass which formed at much later time, say $z = 1$, from a less dissipational merger (because disk gas fractions have decreased in that time interval). The system which formed earlier was initially more compact, but experienced more mergers in the intervening time, so for a given stellar mass, the scatter in $R_e$ is not large. Likewise, velocities dispersions and other fundamental plane properties have small
scatter at a given stellar mass at any observed redshift because of this cancellation effect.

This is not to say that most systems will undergo many dry mergers – the overall density of dry mergers predicted here is low, consistent with observational constraints from e.g. Bell et al. (2006); van Dokkum (2005); Lin et al. (2008), and as we noted above most (especially $\lesssim L_e$) ellipticals today formed in a few (or just one) gas-rich mergers at relatively low redshifts. However, the systems which form earliest do so because they live in the most dense environments, which evolve the most rapidly, and are expected to undergo the most subsequent merging activity. We predict, as a result, that most of these compact systems observed at high redshifts will end up constituting some of the central dense stellar mass in observed brightest cluster or group galaxies at $z = 0$; this is consistent with the observed sizes (von der Linden et al. 2007), surface brightness profile shapes (enveloped envelopes built up through their subsequent dry merging) (see e.g. Paper III and Gallagher & Ostriker 1972; Naab et al. 2006b), and masses and velocity dispersions (Bernardi et al. 2007) of these $z = 0$ systems, and also expected given the clustering properties of high redshift massive systems (Quadrini et al. 2007).

We estimate what fraction, as a function of stellar mass, of the massive galaxy population today may have survived from these early times un-remerged and thus reflect the smaller sizes and higher dissipational content characteristic of systems formed at that time, finding that e.g. $\sim 1 - 5\%$ of $z = 0 \sim 10^{11} M_\odot$ systems may have survived since $z \sim 2$. This is small, but not negligible, and by identifying compact ellipticals with old population ages, it should be possible to locate the surviving remnants of this population. We predict that these will be massive, compact (with other indicators of gas-rich origin and high inferred dissipational fractions from their stellar profiles), with old stellar population ages. Because they still generally would initially form in high-density environments, but have avoided merging since then, we would roughly expect them to be in dense environments (rich groups and clusters) today, but not to be the central galaxy (again, most early-forming systems will be the central galaxy – but if they are all the way until $z = 0$, it is unlikely that they will have avoided further mergers, and thus would not be un-remerged). There may be some observed candidate members of this population in the so-called “compact elliptical” class, galaxies similar to NGC 4486B in Virgo which appears to satisfy these criteria. Further observations to test this hypothesis could prove extremely valuable as nearby probes of high-redshift galaxy evolution.

Future work should extend these models to include the (at present more uncertain) role of particularly minor mergers (where e.g. satellite disruption may be important to explain the extended halos in massive BCGs) and other sources of dissipation, such as stellar mass loss (see e.g. Ciotti & Ostriker 2007). However, we show that other effects, e.g. evolution in the sizes of disk galaxies, dry mergers alone, and evolution in galaxy host halo properties, are insufficient to explain effects like those predicted here, in particular to account for e.g. the tilt of the fundamental plane, the relative sizes of ellipticals (versus those of disks) as a function of mass, the scaling of elliptical galaxy profile shapes with mass, and the redshift evolution in the fundamental plane correlations. Therefore, these predictions are important tests of any model in which dissipational star formation is a significant influence on galaxy formation.

Because dissipation is necessary in a basic sense to reconcile the densities of ellipticals and spirals, this represents an important test of the merger hypothesis itself, as well as a test of our understanding of galaxy merger histories and the evolution in different galaxy components with redshift. In addition, we provide a unified theoretical lens through which to interpret a number of observations of spheroid and black hole-host scalings at both $z = 0$ and high redshift, in which spheroids are fundamentally multi-component objects (with dissipational components originally formed in central starbursts triggered in the initial gas-rich interactions that formed the system, and dissipationless components representing the scattered, violently relaxed stars from the pre-initial merger stellar disks). As multi-component objects, it is primarily the relative mass fraction in the dissipational component, reflecting the gas content of spheroid progenitors, that drives their structural properties and the evolution of those structural properties with redshift.

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APPENDIX

IMPLEMENTATION OF THE HALO OCCUPATION MODEL

Here, we present a condensed outline of the simplest implementation of the model used to make the predictions in this paper. As described in the text (§ 2), we have experimented with a wide variety of modifications to the model elements and methodology – but our intention here is to summarize the basic framework upon which these modifications represent increasing layers of complexity. We therefore leave the description of these experiments to § 2 and Hopkins et al. (2008f).

As outlined in § 2.1 our methodology consists of a few key steps:

We construct a Monte Carlo sample of halos at some observed redshift \( z_{\text{obs}} \), sampling according to the halo mass function at that redshift (constructed in standard fashion for the adopted cosmology following Sheth et al. 2001).

For each halo, we determine a mock growth history, i.e. \( M_{\text{halo}}(z) \) for all \( z > z_{\text{obs}} \) up to some initial (maximum) redshift \( z_{\text{init}} \).

There are several ways to do this: using the extended Press-Schechter formalism, adopting fits to individual halo growth histories in simulations (taken from e.g. De Lucia et al. 2006; Stewart et al. 2007) or directly tracking the main-branch progenitor halo mass of each \( z = 0 \) halo in the simulations, integrating over analytic fits to mean merger histories from initial seed popula-
Wechsler et al. (2002, 2006); Neistein et al. (2006). Because we are considering the population in a statistical fashion, it makes no difference which of these approaches we adopt so long as they yield a similar median (and the scatter in galaxy properties in a given halo is, in any case, larger than the scatter at different times between these methodologies). The simplest approach, then, is to consider each halo in the Monte Carlo sample to have a mass at each $z > z_{\text{obs}}$ given by the mean growth history (adopting the analytic fits in Neistein et al. (2006), in their Equation 11 and Appendices).

Now we have an average $M_{\text{halo}}(z)$ for each halo in our representative Monte Carlo sample. We then begin at some initial redshift $z_{\text{initial}}$ and evolve the system forward in timesteps of some $\Delta z$.$^6$ We ignore halos until they reach a mass (our effective resolution limit, corresponding to resolution limits in many of the simulations on which the HOD calculations are based) of $M_{\text{halo}} = 10^{10} M_\odot$. This corresponds, given a typical HOD, to an extremely small stellar mass $M_* \lesssim 10^7 M_\odot$, so is irrelevant for the final mass of all the galaxies considered here, and is a source of little uncertainty.

At each timestep, all galaxies that have not yet experienced a major merger above the resolution limit are initialized according to the halo occupation model (specifically, we initialize the galaxy stellar mass $M_*(M_{\text{halo}})$ and then determine $R_e(M_*)$ and $f_{\text{gas}}(M_*)$).

As described in the text, we have experimented with a variety of observational constraints regarding the implementation of the HOD (see Hopkins et al. 2008 for a more detailed comparison). It is possible, for example, to use the quoted fits from the halo occupation model (specifically, we initialize the galaxy stellar mass $M_*$ by observations – showing the difference between the two in the range of uncertainties in our model). The gas fractions (where we again consider a range to allow for observational uncertainties as described in the text, specifically Equation 2).

These assignments are re-initialized at each redshift, if the halo remains un-merged. For this reason (and because at any redshift, most galaxies of a given mass have assembled in the last redshift interval $\Delta z \sim 1 - 2$), the large uncertainties at high redshift are quickly suppressed at any lower redshift. For example, even though the uncertainty in $R_e$ of a typical disk at high redshift (e.g. $z = 4$) is large (say, 0.5dex), and this will enter as such in the uncertainty in predicted size of an elliptical forming at this redshift (although the uncertainty is somewhat suppressed since the system is likely to be gas-rich; see the discussion in §2), by $z = 0$, the original uncertainty will be suppressed by a typical factor $\sim 16 - 32$ (depending on how much the galaxy has grown by low-redshift mergers), introducing only $\sim 0.02 - 0.03$ dex uncertainty in the low-redshift prediction (in other words, at lower redshifts, the uncertainties are dominated by the uncertainties in the HOD around those redshifts, rather than by the propagation of uncertainties from higher redshifts).

The critical step at each redshift is then determining whether or not the galaxy experiences a merger. Because we are interested in galaxy-galaxy mergers, rather than e.g. a subhalo merging into the parent halo (and the two can be very different, as many subhalos, especially those which are small relative to the parent halo, may not merge for a Hubble time), we do not wish to adopt an extended Press-Schechter or simulation-based merger tree, but opt for a more sophisticated approach. At each timestep, we consider the halo to have a subhalo population according to that fitted in simulations (Kravtsov et al. 2004; alternatively, following the fits in van den Bosch et al. 2005, who adopt a semi-analytic methodology but reach similar conclusions) – this subhalo mass function is, in units of $M_{\text{subhalo}}/M_{\text{halo}}$, only weakly halo mass or redshift-dependent, and it makes no difference whether we adopt the fits from different authors (we have also compared the subhalo mass functions in Springel et al. 2001; Tormen et al. 2004; De Lucia et al. 2004; Gao et al. 2004; Zentner et al. 2005; Nurmi et al. 2006, and reach similar conclusions).

$^6$ Here, we choose $z_{\text{initial}} = 6$ and $\Delta z = 0.01$, but in general we find that our results converge with respect to $\Delta z$ for values $\Delta z \lesssim 0.1$, and the predictions at any given $z_{\text{initial}}$ converge rapidly once $z_{\text{initial}}$ is larger by some difference $z_{\text{initial}} - z_{\text{obs}} \gtrsim 1 - 2$. The reasons for this are discussed in the text (§2 and below).

$^7$ The “turnover” in $M_*(M_{\text{halo}})$ in Equation (A1) mainly corresponds to quenched, spheroid galaxies, and so one could argue should not be applied to a sample of strictly un-merged galaxies. We have experimented with not including this turnover, i.e. adopting the low-mass slope of the HOD with $M_* \propto M_{\text{halo}}^{1.90}$, with a maximum at a stellar mass equal to the halo mass times the universal baryon fraction. Because very few systems survive to such large masses without merging, however, this makes no difference to our predictions.
We then calculate the time for each such subhalo and its contained galaxy to merge with the central galaxy, according to one of several methodologies: either assuming a simple dynamical friction timescale, employing a more sophisticated timescale based on the cross section for resonant galaxy-galaxy interaction and orbital capture, or a timescale calculated based on a similar calculation in angular momentum space; the probability of a merger with each subsystem is then taken as the ratio of the timestep to the merger timescale. We compare these methodologies in detail in Hopkins et al. (2008f) and Hopkins et al. (2008c) and discuss the details of the derivation for each. For our purposes here, as demonstrated in those papers, they give similar results, and we note that the results are also similar to what is commonly adopted in many semi-analytic models based on N-body simulations, which follow subhalos until they can no longer be resolved and then approximate the remaining merger timescale with a dynamical friction timescale estimate.

In Hopkins et al. (2008f) we show the detailed PDFs for the merger rate as a function of halo mass, galaxy mass, and redshift (see specifically Figures 2-4 therein), that arise from various combinations of these assumptions. We show there that the different approaches agree reasonably well, and agree with independent estimates such as sufficiently high resolution N-body experiments (e.g. Maller et al. 2006; Somerville et al. 2008a). For our purposes here, the important thing is that Hopkins et al. (2008f) show that the model predictions agree well with the observed mass function of galaxy mergers at all redshifts observed; as well as the observed galaxy merger rate as a function of environmental density, halo mass (estimated from clustering or group dynamics), and redshift; and the clustering (both large-scale and small-scale) and therefore halo occupation of mergers and recent merger remnants (see their Figures 6-12). This gives us some confidence that the calculation yields a reasonable approximation to the merger rate. For the details and summary of the adopted calculation, we refer to § 2.1 of Hopkins et al. (2008f). For our fiducial model in this paper, we employ their “default” model: we construct the subhalo mass function for each halo adopting the analytic fits in Kravtsov et al. (2004). We populate each subhalo as if it were a random member of our halo population (of the same mass) – specifically, we draw a random halo of the same mass from our Monte Carlo sample and assign the subhalo the same properties8. We then estimate the merger timescale using the “group capture” gravitational cross-section timescales, specifically using the fitting formulae from Krivitsky & Kontorovich (1997) calibrated to a large set of numerical simulations of different encounters in group environments (see also White 1978; Makino & Hut 1997; Makino & Hohl 1999). This ratio of the timestep to this timescale gives the probability of each merger, and we determine whether each occurs in Monte Carlo fashion.

When a merger occurs above the cutoff mass ratio threshold (a “major” merger), the properties of the remnant are determined as a function of the properties of both progenitors, as described in § 2.4. The final stellar mass is the sum of the two progenitor baryonic masses; the two constituent components, the dissipational and dissipationless stellar mass, of the remnant are the sum of the dissipational and dissipationless masses of the progenitors. For un-merged progenitors, the dissipational and dissipationless mass are the pre-merger gas and stellar mass, respectively. The effective radii of the remnants are calculated as described in the text, for the dissipationless component as a function of the mass ratio and scale radii of the progenitors (Equation 4), and then with the appropriate correction applied for the dissipational mass fraction (Equation 5). Other properties follow as described in § 2.4. Properties such as the dark matter fraction within a given radius (for dynamical masses, etc) are calculated assuming a Hernquist (1990) profile for the dark matter halo, with a redshift-dependent concentration from Bullock et al. (2001), and projecting the profile along with that of the galaxy.

The stellar mass and stellar mass profile of major merger remnants are taken to be those calculated at the previous merger, although the halo grows continuously with time. In other words, major merger remnants are effectively “quenched” in some sense (although new gas can and does come into the galaxy through subsequent mergers with gas-rich galaxies) – we ignore details of the re-growth of disks around early-forming ellipticals through direct cosmological accretion of cold gas. Although this may be important for especially the most high-redshift systems, a proper treatment requires a much more complete cosmological model (ideally highly resolved cosmological simulations that can form proper low-redshift disks, still a major theoretical challenge). Moreover, it has been argued that feedback from quasar and starburst activity in major mergers may actually be an important physical agent of “quenching.” (Scannapieco & Oh 2004; Granato et al. 2004; Monaco & Fontanot 2005; Hopkins et al. 2006b; Naab et al. 2006b; Bundy et al. 2008), in which case this is a more accurate assumption than allowing for new cooling.

In any event, we show in Hopkins et al. (2008c) that such a prescription, with a major merger mass ratio threshold set to the value used in this paper (1:3), yields very good agreement with the observed mass function and mass density of quenched galaxies as a function of redshift, as well as quenched central galaxy fraction as a bivariate function of stellar and halo mass, the clustering of red galaxies as a function of mass and redshift, the distribution of stellar population ages (and implicitly, “quenching times”9) as a function of elliptical stellar mass, and the distribution of elliptical structural properties related to cooling. This argues that the current prescription is at least a good approximation to the actual cooling histories of the galaxies of interest here. We have also experimented with alternative prescriptions such as those found in semi-analytic models (with halo cooling suppressed by low-level AGN feedback, or truncated above a certain halo or stellar mass threshold, see e.g. Croton et al. 2006; Bower et al. 2006; Cattaneo et al. 2004; de Lucia & Blaizot 2007; Somerville et al. 2008a), and find similar results (this is because, it turns out, the halo mass threshold for the transition to “hot mode” accretion and effective quenching in these models, and empirically constrained stellar/halo mass ranges for quenching, correspond quite closely with the regime where most galaxies experience their first major merger; see e.g. Cattaneo et al. 2006).

8 So long as the Monte Carlo sample is sufficiently large, this is identical to tracing the growth and merger history of the subhalo itself. As discussed in the text, we have explicitly chosen to ignore effects unique to satellites such as ram-pressure stripping. Although these are important for low-mass satellites which are long-lived in e.g. massive groups and clusters, they do not generally apply to the cases of interest: namely major mergers, for which the merger times are short (too short for long-timescale effects such as stripping and harassment) and the halos are of comparable mass (making e.g. ram-pressure relatively unimportant). Moreover, for the typical \( \lesssim 1 \) \( L_e \) galaxies of interest here, the “subhalo” phase is very short in major mergers – it represents a brief intermediate merging stage between field or small/loose group galaxies on their way to merging. For more massive cluster systems, however, the caveat should apply, along with the others discussed in § 2.
This process is repeated until the desired $z_{\text{obs}}$ is reached. We then discard the systems that remain un-merged (as these will not, predominantly, be elliptical galaxies) and construct the mock observed sample of interest from our Monte Carlo population.