Mixed convection of single-walled carbon nanotubes in a triangular cavity containing a pentagonal impedance

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Abstract. Mixed convection of nanofluid (a homogeneous mixture of water and single-walled carbon nanotubes) is investigated numerically in a triangular cavity containing an adiabatic pentagonal obstacle close to the top surface. Analytical models are employed for effective thermophysical properties of nanofluid regarding carbon nanotubes and water. The top wall of the cavity is kept at a higher temperature, while the inclined sidewalls are kept at a lower temperature. A constant magnetic field of intensity $B_0$ is introduced in the momentum equation. The effects of radiation and viscous heating on heat transfer are neglected. The impacts of Richardson number on the dimensionless velocity and temperature are presented graphically in terms of streamlines and isothersms. The Galerkin finite element method with a penalty function is employed to obtain the solution of dimensionless partial differential equations. The numerical results are examined for governing parameters, including volume fraction of carbon nanotubes, Richardson, and Hartmann numbers. It is concluded that the Nusselt number increases with increasing the governing parameters in the presence of an adiabatic pentagonal obstacle.

Keywords: Mixed convection; Magnetic field; SWCNT-water nanofluid; Triangular cavity; pentagonal block; Finite element method.

1. Introduction

The rate of heat transfer can be increased by increasing the thermal conductivity of base fluids. Due to high thermal conductivity, CNTs are used to obtain high heat transfer rates in several engineering applications like cooling of electronic components, furnaces, thermal storage systems, high-performance building insulation, drying technologies, and heat exchangers. In such applications, the transfer of heat energy plays an important role. Several studies exist related to the heat convection in different cavity configurations filled with various fluids and imposed boundary conditions on the walls. For instance, Selimefendigil [1] and Jafari et al. [2] studied mixed convection of single- and multiple-walled carbon nanotubes in cavities and noticed a better improvement in the heat transfer than other nanoparticles. The effects of mixed convection in a wavy walled lid-driven enclosure filled with nanofluid were also investigated by Öztop et al. [3]. They discovered an increase/decrease in
Nusselt number with the increasing volume fraction of nanoparticles based on governing parameters. Selimefendigil and Öztöp [4] employed various thermal boundary conditions and studied mixed convection of a nanofluid in a lid-driven cavity. They concluded that the solid volume fraction of nanoparticles enhances the rate of heat transfer in the cavity. Later, Selimefendigil and Öztöp [5] extended their study to a moving wall and internal heat generation. Using non-uniform heating conditions on both sidewalls, Sivasankaran et al. [6] deliberated mixed convection in a lid-driven cavity. It was demonstrated that the Nusselt number increases with increasing amplitude ratio. Mekroussi et al. [7] also found higher heat transfer rates in the cavity due to mixed convection and proved that the wavy walls have a powerful impact on enhancing heat transfer.

Sivasankaran et al. [8] studied mixed convection in a lid-driven square cavity with a sinusoidal boundary condition at the base surface. Their results depend upon Richardson and Hartmann numbers. Sivakumar and Sivasankaran [9] studied mixed convection numerically in an inclined square cavity. They noticed a substantial increase in the heat transfer rate in the cavity. Using extended-Darcy model, Sivasankaran and Pan [10] examined mixed convection in a cavity with nonuniform thermal boundary conditions. It was observed that an increase in amplitude ratio increases the heat transfer rate. Bakar et al. [11] examined the behavior of mixed convection numerically with internal heat generation/absorption in a square lid-driven cavity.

They observed the reduction of heat transfer with an increase in heat generation. For pseudoplastic fluids, Manchanda and Gangawane [12] reported an insignificant effect on fluid and thermal structure during mixed convection. Gangawane and Manikandan [13] employed isothermal and isoflux thermal boundary conditions for mixed convection of Newtonian fluids in a lid-driven cavity. They noticed a significant increase in the heat transfer with increasing Reynolds number up to 200 and then decreases. Gangawane [14] investigated mixed convective flow in a cavity using isoflux boundary condition on the walls. He confirmed the numerical results of [13]. Kalteh et al. [15] used a triangular heat source in a lid-driven square cavity filled with different nanofluids. They confirmed a substantial increase in Nusselt number with the volume fraction of nanoparticles.

Based on Galerkin Weighted Residual technique, Ali et al. [16] studied mixed convection in a hexagonal cavity in the presence of a magnetic field. They assumed isothermal boundary conditions and recovered the considerable impacts of governing parameters on the flow and the dimensionless heat transfer. Louaraychi et al. [17] investigated the effects of major parameters on the behavior of mixed convection in lid-driven cavities. They observed the zones showing the dominance of natural convection. A complete review of the mixed convection of nanofluids in several enclosures was presented by Izadi et al. [18]. It was concluded that the Nusselt number and pressure drop depends upon Reynolds and Richardson numbers as well as the volume fraction of nanoparticles. The impacts of pertinent parameters on the flow and heat transfer were examined by Ezzaraa et al. [19]. They found that the thermal performance of the cavity depends upon the emissivity of the walls, the velocity of the external flow, and the position of the outlet port.

Using water–Al$_2$O$_3$ nanofluid, Ghasemi and Aminossadati [20] conducted a numerical study on the mixed convection and compared two different scenarios of moving walls. They found a higher heat transfer rate for the downward sliding wall motion. Nayak et al. [21] considered a non-homogeneous model to examine the influence of governing parameters on the flow and heat transfer of a nanofluid in a partially heated enclosure. They noticed the effects of the governing parameters on both the flow and heat transfer of a nanofluid. Using Buongiorno’s model and internal heat generation, Yu et al. [22] examined the behavior of nanofluid in an inclined lid-driven cavity. They observed the dependence of governing parameters on the streamlines, isotherms, and the local Nusselt number in the cavity. In the presence of a magnetic field, Javed and Siddiqui [23] investigated mixed convection in a square enclosure containing micropolar fluid. They noticed the dependence of mixed convective parameters in the convection regime. Kapil et al. [24] considered an enclosed cavity filled with nanofluid and inferred that the solid volume fraction of nanoparticles has a substantial impact on the heat transfer rate. Using numerical simulations in a cubical cavity, Al-Rashed et al. [25] studied mixed convection of nanofluids. They noted that the volume fraction of nanoparticles has a weak impact on heat transfer.
Using a square enclosure, Wang et al. [26] examined mixed convection and verified the dependence of heat transfer on the pulsating temperature of the inner cylinder. They concluded that the unsteady mixed convection enhances the heat transfer rate in the enclosure. Other studies related to mixed convection in lid-driven cavities filled in with Newtonian/no-Newtonian fluids under different thermal boundary conditions on the cavity walls can be found in [27-37]. Their findings confirm the results of the published work.

The above literature indicates the shortage of studies related to mixed convection in triangular cavities filled with CNTs and containing a pentagonal block. The main objective of this study is to investigate the effects of mixed convection parameter on streamlines, isotherms and heat transfer from a nanofluid comprising single-walled carbon nanotubes with water in a triangular cavity with an adiabatic pentagonal block.

2. Model Formulation

A 2-D triangular cavity, containing a heated pentagonal obstacle close to the top wall, is considered for the numerical analysis of mixed convection of the selected nanofluid. The top wall of the cavity is parallel to the x-axis of the Cartesian Coordinate system and is moving with velocity $u_0$ towards the right. The top wall is maintained at higher temperature $T_h$, whereas the inclined sidewalls are kept cold at lower temperature $T_c$, as shown in Figure 1. Consequently, the nanofluid motion is subjected to a buoyancy force owing to the differential heating and temperature difference. The gravitational force is acting vertically downward. A constant magnetic field of strength $B_0$ is enforced on the cavity walls. The magnetic Reynolds number is assumed to be negligible.

\[ u = u_0, v = 0, T = T_h \]

![Figure 1. Geometry of triangular cavity with inner pentagonal obstacle.](image)

Under these assumptions, the dimensionless governing equations can be written as [20]:

\[ \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \quad (1) \]

\[ U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\rho_f}{\rho_{nf}} \frac{\partial P}{\partial X} + \frac{v_{nf}}{\nu_f} \frac{1}{\text{Re}} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right), \quad (2) \]

\[ U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\rho_f}{\rho_{nf}} \frac{\partial P}{\partial Y} + \frac{v_{nf}}{\nu_f} \frac{1}{\text{Re}} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) - \frac{\sigma_{nf}}{\sigma_f} \frac{Ha^2}{\text{Re}} V + \frac{(\rho\beta)^{nf}}{(\rho\beta)_f} \text{Ri} \theta, \quad (3) \]
The above dimensionless governing equations are obtained using the following dimensionless variables and parameters:

\[ X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad U = \frac{u}{u_0}, \quad V = \frac{v}{u_0}, \quad P = \frac{P}{\rho_j u_0^2}, \quad \theta = \frac{T - T_c}{T_h - T_c}, \quad \text{Re} = \frac{u_0 L}{\nu_f} \]

\[ \text{Pr} = \frac{\nu_f}{\alpha_f}, \quad \text{Gr} = \frac{\beta_f g (T_h - T_c) L^3}{\nu_f \alpha_f}, \quad \text{Ha} = \frac{\sigma_f}{\rho_f \nu_f}, \quad \text{Ri} = \frac{\text{Gr}}{\text{Re}^2} \]

where \( \text{Pr}, \ \text{Gr}, \ \text{Ha}, \ \text{Ri} \) are the Prandtl number (\( \text{Pr} \approx 6.2 \) for water), Rayleigh number, Hartman number, and Richardson number, respectively. The corresponding boundary conditions in the non-dimensional form are given by

1. Top wall: \( U = 1, \quad \theta = 1, \quad 0 \leq X \leq 1, Y = 0.85 \) \( V = 0 \) \( (5) \)
2. Right inclined wall: \( U = V = \theta = 0 \) \( (6) \)
3. Left inclined wall: \( U = V = \theta = 0 \) \( (7) \)
4. For Inner pentagonal shaped obstacle temperature \( \frac{\partial \theta}{\partial Y} = 0 \) \( (8) \)
5. Velocities at all walls of the obstacle are zero, i.e., \( U = V = 0 \). \( (9) \)

The local and average Nusselt numbers along the domain of enclosure which is partially heated are described as:

\[ Nu = -\frac{k_{nf}}{k_f} \frac{\partial \theta}{\partial Y}\bigg|_{Y=0.85}, \quad Nu_{avg} = \int_{L} Nu dX. \] \( (10) \)

The empirical relations for the effective physical properties for nanofluid concerning base fluid and nanoparticles are given as follows [38]:

\[ \rho_{nf} = (1 - \phi) \rho_f + \phi \rho_s, \quad (\rho C_p)_{nf} = (1 - \phi)(\rho C_p)_f + \phi(\rho C_p)_s, \]

\[ \mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}}, \quad \frac{k_{nf}}{k_f} = \frac{1 - \phi + 2\phi}{1 - \phi + 2\phi} \ln \left( \frac{k_{CNT} + k_f}{k_{CNT} - k_f} \right) \frac{2k_f}{2k_f}, \]

\[ (\rho \beta)_n = (1 - \phi)(\rho \beta)_f + \phi(\rho \beta)_s, \quad r = \frac{\sigma_s}{\sigma_f}, \quad \frac{\sigma_{nf}}{\sigma_f} = 1 + \frac{3(r - 1)\phi}{(r + 2) - (r - 1)\phi}, \]

where \( \rho_f, k_f, \mu_f, C_{pf} \), and \( \beta_f \) is the density, thermal conductivity, dynamic viscosity, the specific heat of the water, and thermal expansion coefficient, respectively, \( \rho_s, \beta_s, k_s, \mu_s, \) and \( C_{ps} \) are the respective properties of single-walled carbon nanotubes and \( \phi \) is the solid volume fraction of carbon nanotubes.

3. Numerical Procedure
The governing momentum and energy equations (2-4) with specified boundary conditions have been solved numerically, employing the Galerkin method. To determine the solution via penalty function [39-40], we use the penalty parameter "\( \gamma \)" to remove the pressure \( P \) as:

\[
P = -\gamma \left( \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} \right),
\]

(11)

The penalty parameter \( \gamma \) automatically satisfies the continuity equation. It is noted that for a significant value of the penalty parameter \( \gamma \), it is satisfied automatically. Using Eq. (4), the momentum equation becomes:

\[
U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = \frac{\rho_f}{\rho_{nf}} \frac{\partial}{\partial X} \left( \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} \right) + \frac{\nu_{nf}}{\nu_f} \frac{1}{Re} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right),
\]

(12)

\[
U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = \frac{\rho_f}{\rho_{nf}} \frac{\partial}{\partial Y} \left( \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} \right) + \frac{1}{Re \nu_f} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) - \frac{\sigma_{nf}}{\sigma_f} \frac{Ha^2}{V} + \frac{(\rho \beta_{nf})}{(\rho \beta_f)} \cdot Ri \theta.
\]

(13)

Equations (12-13) and the related boundary conditions are solved using the Galerkin method. Consider the following solutions for velocities \((U, V)\) and the dimensionless temperature \(\theta\) using the basis set \(\{Y_n\}_{n=1}^{N}\)

\[
U \approx \sum_{n=1}^{N} U_n Y_n(X, Y), V \approx \sum_{n=1}^{N} V_n Y_n(X, Y), \theta \approx \sum_{n=1}^{N} \theta_n Y_n(X, Y).
\]

(14)

After incorporating these solutions into Eqns. (14), we obtain the following nonlinear residual equations at all nodes of the selected cavity:

\[
R_k^U = \sum_{n=1}^{N} \left[ \sum_{n=1}^{N} U_n Y_n \frac{\partial Y_n}{\partial X} + \left( \sum_{n=1}^{N} V_n Y_n \right) \frac{\partial Y_n}{\partial Y} \right] Y_k dX dY + \gamma \sum_{n=1}^{N} U_n \int \frac{\partial Y_k}{\partial X} \frac{\partial Y_n}{\partial Y} dX dY
\]

\[
+ \gamma \sum_{n=1}^{N} V_n \int \frac{\partial Y_k}{\partial Y} \frac{\partial Y_n}{\partial X} dX dY + \frac{1}{Re} \sum_{n=1}^{N} U_n \int \frac{\partial Y_k}{\partial X} \frac{\partial Y_n}{\partial X} + \frac{\partial Y_k}{\partial Y} \frac{\partial Y_n}{\partial Y} dX dY,
\]

(15)

\[
R_k^V = \sum_{n=1}^{N} \left[ \sum_{n=1}^{N} U_n Y_n \frac{\partial Y_n}{\partial X} + \left( \sum_{n=1}^{N} V_n Y_n \right) \frac{\partial Y_n}{\partial Y} \right] Y_k dX dY + \gamma \sum_{n=1}^{N} V_n \int \frac{\partial Y_k}{\partial Y} \frac{\partial Y_n}{\partial Y} dX dY
\]

\[
+ \gamma \sum_{n=1}^{N} U_n \int \frac{\partial Y_k}{\partial X} \frac{\partial Y_n}{\partial X} + \frac{\partial Y_k}{\partial Y} \frac{\partial Y_n}{\partial Y} dX dY + \frac{\sigma_{nf}}{\sigma_f} \frac{Ha^2}{V} \sum_{n=1}^{N} V_n Y_n Y_k dX dY
\]

\[
+ \frac{1}{Re} \sum_{n=1}^{N} U_n \int \frac{\partial Y_k}{\partial X} \frac{\partial Y_n}{\partial X} + \frac{\partial Y_k}{\partial Y} \frac{\partial Y_n}{\partial Y} dX dY + \frac{(\rho \beta_{nf})}{(\rho \beta_f)} \sum_{n=1}^{N} \theta_n Y_n Y_k dX dY,
\]

(16)

\[
R_k^\theta = \sum_{n=1}^{N} \left[ \sum_{n=1}^{N} U_n Y_n \frac{\partial Y_n}{\partial X} + \left( \sum_{n=1}^{N} V_n Y_n \right) \frac{\partial Y_n}{\partial Y} \right] \theta_k dX dY + \frac{1}{Pr Re} \sum_{n=1}^{N} \theta_n \int \frac{\partial Y_k}{\partial X} \frac{\partial Y_n}{\partial X} + \frac{\partial Y_k}{\partial Y} \frac{\partial Y_n}{\partial Y} dX dY,
\]

(17)
The solution set of nonlinear algebraic equations (15-17) is obtained by a reduced integration method [41] and the Newton Raphson approach [42]. We get $3N \times 3N$ a system of equations at each iteration.

$$J(a^n)[a^n - a^{n+1}] = R(a^n),$$

where $J(a^n)$ is the Jacobian matrix and $R(a^n)$ is the residual vector.

The stream function $\psi$ is defined as:

$$\frac{\partial \psi}{\partial Y} = U; \frac{\partial \psi}{\partial X} = -V,$$

which takes the following form

$$\frac{\partial^2 \psi}{\partial X^2} + \frac{\partial^2 \psi}{\partial Y^2} = \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y},$$

which gives

$$R_{k}^\psi = \sum_{n=1}^{N} U_n \int_{\Omega} Y_k \frac{\partial Y_n}{\partial Y} dX dY - \sum_{n=1}^{N} V_n \int_{\Omega} Y_k \frac{\partial Y_n}{\partial X} dX dY + \sum_{n=1}^{N} \int_{\Omega} \left[ \frac{\partial Y_k}{\partial X} \frac{\partial Y_n}{\partial X} + \frac{\partial Y_k}{\partial Y} \frac{\partial Y_n}{\partial Y} \right] dX dY,$$

The local Nusselt number along the heating element is given by:

$$Nu = -\sum_{k=1}^{q} \theta_k \frac{\partial Y_k}{\partial Y}.$$

4. Results and Discussion

The mixed convection of nanofluid is investigated numerically in a triangular cavity including an adiabatic pentagonal obstacle. Different thermal boundary conditions are imposed on the cavity walls. The Richardson number is increased from 0.01 to 10 for different domains of convection. The solid volume fraction of carbon nanotubes is varied from 0 to 0.2, whereas the Hartmann number is increased from 0 to 50. The thermophysical properties of water and single-walled carbon nanotubes are reported in Table 1.

| Property | Water | SWCNT |
|----------|-------|-------|
| $\rho$   | 997.1 | 2600  |
| $c_p$    | 4179  | 425   |
| $k$      | 6.2   | 6600  |
| $\beta$  | $2.1 \times 10^4$ | $3.3 \times 10^4$ |
| $\sigma$ | 0.05  | 3x10^4 |

The impacts of different domains of convection on streamlines, isotherms, horizontal, and vertical components of velocity are presented in Figure 2(a)-(d) for the fixed values of $Ha$ and $\phi$. Depending upon the values of $Ri$, Figure 2(a)-(d) present the results for the three cases. As the top wall moves with constant velocity parallel to the x-axis, the flow is generated from the right inclined wall to the left inclined wall. This flow behavior can be observed in Figure 2(a) (i)-(iii) for forced, mixed, and free convection. Due to different thermal boundary conditions, two cells form near the top surface of the triangle. The intensity of these cells increases and the size decreases with increasing Richardson number, as shown in Figure 2(a) (ii)-(iii). Finally, these cells weaken and spread at the top of the obstacle. At the same moment, another cell of high strength appears in the right bottom part of the triangle. This cell moves towards the left part with an increase in the Richardson number. The
isotherms, for the same values, are presented in Figure 2(b) (i)-(iii) to show the thermal behavior of nanofluid in the cavity. Due to the higher temperature at the top surface, the heat is transferred in a downward direction. The buoyancy force decreases, and the heat transfer increases from forced to free convection, as evident from Figure 2(b) (i)-(iii). The contours of the horizontal and vertical velocity components are depicted in Figure 2(c) and 2(d) for each case. Due to the motion of the top wall, the horizontal velocity is found to be higher in the upper region of the cavity. Consequently, it pushes the fluid from the left side to the right side. This behavior can be observed in Figure 2(c) (i) for forced convection and in Figure 2 (c) (ii) for mixed convection. Due to lower velocities, in the case of free, the horizontal velocity contours are found to be weaker. These contours get stronger with an increase in the Richardson number. As the Richardson number increase, the velocity contours become stronger and move from the right side to the left side, as shown in Figure 2(c)(i)-(iii). Consequently, the upper vortices divide into more small parts. Similar behavior can be observed for the vertical velocity contour in each case. The strength of these contours increases with increasing Ri, as shown in Figure 2 (d) (i)-(iii).
The local Nusselt number increases along the heated top surface and around the adiabatic obstacle is shown in Figure 3(d)-(e), respectively, for the three selected cases. The local Nusselt number decreases along the horizontal surface. At both corners of the top surface, there is a sudden increase or decrease in the local Nusselt number. The maximum values of Nusselt numbers are obtained at the corners of the heater due to singularity at those points. However, along
with the adiabatic pentagonal obstacle, the variation in the local Nusselt number is strange, as shown in Figure 3(e).

![Diagram](image)

**Figure 3.** Effects of Richardson number on dimensionless velocities, temperature, and local Nusselt numbers with $Re = 500$, $Ha = 10$ and $\phi = 0.1$.

The variation of isotherms of nanofluid in the cavity with the volume fraction of carbon nanotubes is demonstrated in Figure 4(a)-(c) for the three types of convection. In the case of forced convection, for pure water $\phi=0$, the heat transfers slowly from the heated top surface to the right side, Figure 4(a)(i). The isotherms are equally spaced around the adiabatic obstacle. The heat transfer rate increases with an increase in the volume fraction of carbon nanotubes, Figure 4(a) (ii)-(iii). In the case of mixed and free convection, an increase in heat transfer can be observed with an increase in the volume fraction of carbon nanotubes, see Figure 4(b)-(c) (i)-(iii). The variation of dimensionless temperature and local Nusselt numbers with volume fraction is also presented in Figure 5(a)-(c) for different values of the Richardson number. The dimensionless temperature increases vertically in the case of forced and mixed convection, Figure 5(a)-(b) respectively. In the case of forced convection, the negligible effect of the solid volume fraction of nanotubes on the dimensionless temperature can be noticed in the left region of the cavity. However, in the right-side space, a considerable effect of the solid volume fraction of carbon nanotubes can be observed. The variation of the local Nusselt number along the horizontal heated surface is depicted in Figure 5(b) (i)-(iii). In each case, a sudden change in the local Nusselt number is noticed due to singularity at the corners. As expected, the local Nusselt numbers are lower for the pure water and increase with increasing solid volume fraction of nanotubes.
Figure 4. Effects of volume fraction of carbon nanotubes on isotherms for different Richardson numbers with $Re = 300$ and $Ha = 10$. 
Figure 5. Effects of volume fraction of carbon nanotubes on (a) vertical temperature and (b) Nusselt number for different Richardson numbers with $Ha = 10$ and $Re=200$.

The influences of $Ha$ on the isotherms for the three domains of convection are depicted in Figure 6(a)-(c) keeping other parameters fixed. When $Ha=0$, the isotherms are shown in Figure 6(a) (i) for forced convection. As the top surface moves towards the right, the heat is transported slowly from the heated surface to the colder sides and the isotherms are equally spaced. The heat transfer increases with an increase in the magnetic field. Since the magnetic field produces Lorentz forces, which resists the motion of the nanofluid, this fact is explained in Figure 6(a) (ii)-(iii). The same behavior can be observed in Figure 6(b) and (c) for mixed and free convection. As expected, the inertia forces are dominant in case of forced convection, which increases the Nusselt number.
Figure 6. Effects of Hartmann number on isotherms for different Richardson numbers with $\varphi = 0.1$ and $Re=500$.

The effects of the volume fraction of carbon nanotubes on the average Nusselt number are presented in Figure 7(a) for the three selected domains of convection. In this case, the magnetic field is kept constant. As expected, the heat transfer rate increases with the increasing volume fraction of nanotubes. This is because carbon nanotubes show higher thermal conductivity (Table 1).
Consequently, it increases the thermal conductivity of the nanofluid and the heat transfer rate increases. The effects of smaller values of the Richardson number show the negligible effect on the Nusselt number. However, forced convection shows a significant impact on the average Nusselt number. The variation of average Nusselt number with Hartmann number is depicted in Figure 7(b) for three selected Richardson number. In this case, the volume fraction of carbon nanotubes is kept constant. It is observed that, for each type of convection, the average Nusselt number practically remains constant. For free convection, the average Nusselt number is found to be the lowest.

5. Conclusion
The steady mixed convection of carbon nanotubes mixed with water is investigated numerically in a triangular cavity in the presence of an adiabatic pentagonal obstacle. The effects of the volume fraction of carbon nanotubes and the magnetic field are examined for the three domains of convection. It is concluded that

- The streamlines and isotherms explain the flow and heat transfer in the cavity.
- The heat transfer enhances due to the pentagonal obstacle in the cavity.
- The heat transfer rate is highest in case of forced convection.
- The Hartmann number reduces the heat transfer rate, whereas the volume fraction of nanotubes helps in improving the heat transfer rate in the cavity.
- The Richardson number reduces the heat transfer rate in the lid-driven cavity with a pentagonal obstacle.

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