Electromagnetic dipole moment and time reversal invariance violating interactions for high energy short-lived particles in bent and straight crystals at Large Hadron Collider

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The channelled particle, which moves in a crystal, alongside with electromagnetic interaction also experiences weak interaction with electrons and nuclei, as well as strong interaction with nuclei. Measurements of polarization vector and angular distribution of particles scattered by axes (planes) of unbent crystal enable to obtain limits for the EDM value and for values of constants describing P- and T-odd interactions. The same measurements also allow studying magnetic dipole moment of charged and neutral particles. Investigation of left-right asymmetry by the use of two unbent crystals makes it possible to measure EDM, MDM and other constants without studying the angular distribution of decay products of scattered particles: it is sufficient to measure the intensity of flow of particles experienced double scattering. Spin precession of channelled particles in bent crystals at the LHC gives unique possibility for measurement of constants determining $T_{odd}, P_{odd}$ (CP) violating interactions and $P_{odd}, T_{even}$ interactions of baryons with electrons and nucleus (nucleons), similarly to the possibility of measuring electric and magnetic moments of charm, beauty and strange charged baryons. Methods to separate P-noninvariant rotation from the MDM- and EDM-induced ($T_{odd}$) spin rotations are discussed.
I. INTRODUCTION

The violation of parity (P) and time reversal (T) symmetries lead to appearance of numerous processes allowing investigation of physics beyond the Standard Model (SM). Recently, it has been proposed to search for the electromagnetic dipole moments (EDM) of charged short lived heavy baryons using bent crystals at LHC [1, 2]. According to [3, 4] bent and straight crystals allow investigation of both EDM of charged and neutral baryons, and $P_{\text{odd}}T_{\text{even}}$ interactions between short lived baryons and electrons and nuclei.

This paper is devoted to consideration of influence of P- and T-odd effects, which accompany interaction of charged and neutral baryons with electrons and nuclei in crystals, on EDM measurement at the LHC. Experimental methods to distinguish different contributions to spin rotation are suggested.

II. RELATIVISTIC PARTICLES SPIN INTERACTIONS WITH CRYSTALS

Owing to quasiclassical nature of high energy particle movement in crystal, to describe evolution of its spin in electromagnetic fields inside of the crystal, Thomas–Bargmann–Michel–Telegdi (T-BMT) equations [2] are used. Let us consider a particle with spin $S$ which moves in the electromagnetic field. The term “particle spin” here means the expected value of the quantum mechanical spin operator $\hat{S}$ (hereinafter the symbol marked with “hat” means a quantum mechanical operator). Movement of high-energy particles (Lorentz-factor $\gamma \gg 1$) in non-magnetic crystals (magnetic field $\vec{B} = 0$) will be considered below.

In this case spin motion is described by the Thomas–Bargmann–Michel–Telegdi equation (T-BMT) as follows:

$$\frac{d\vec{S}}{dt} = [\vec{S} \times \vec{\Omega}],$$

(1)

where $\vec{S}$ is the spin vector, $t$ is the time in the laboratory frame, $m$ is the mass of the particle, $e$ is its charge, $\gamma$ is the Lorentz-factor, $\vec{\beta} = \vec{v}/c$, where $\vec{v}$ denotes the particle velocity, $\vec{E}$ is the electric field at the point of particle location in the laboratory frame, $g$ is the gyromagnetic ratio (by definition, the particle magnetic moment $\mu = (eg\hbar/2mc)S$, where $S$ is the particle spin).

The T-BMT equation describes the spin motion in the rest frame of the particle, wherein the spin is described by the three component vector $\vec{S}$. In practice the T-BMT equation well describes the spin precession in the external electric and magnetic fields encountered in typical present–day accelerators. Study of the T-BMT equation enables one to determine the major peculiarities of spin motion in an external electromagnetic field, to describe spin rotation effect for particles in a crystal and to apply it for measuring magnetic moments of unstable particles [2, 5–14]. However, it should be taken into account that particles in an accelerator or a bent crystal have energy spread and move along different orbits. This necessitates one to average the spin–dependent parameters of the particle over the phase space of the particle beam. That is why one must always bear in mind the distinction between the beam polarization $\vec{\xi}$ and the spin vector $\vec{S}$. A complete description of particle spin motion can be made by the use of spin density matrices equation (in more details see [3, 15]). For the case of ultra relativistic baryons T-BMT equations supplied with the term that is responsible for interaction between particle EDM and electric field can be written as follows ($\gamma \gg 1, \gamma$ is the Lorentz-factor) [1, 2, 16–17]:

$$\frac{d\vec{\xi}}{dt} = [\vec{\xi} \times \vec{\Omega}_{\text{magn}}] + [\vec{\xi} \times \vec{\Omega}_{\text{EDM}}],$$

(3)

where $\vec{\xi}$ is the particle polarization vector, $\vec{\Omega}_{\text{magn}} = -\frac{e(g-2)}{2mc} \vec{n} \times \vec{E}_\perp$, $\vec{\Omega}_{\text{EDM}} = \frac{2ed}{\hbar} \vec{E}_\perp$. $\vec{E}_\perp$ is an electric field component perpendicular to the particle velocity $\vec{V}$, $\vec{n}$ is the unit vector parallel to the velocity $\vec{V}$, $ed$ is the electric dipole moment.

It would be recalled that the particle refractive index in matter formed by different scatterers has the form:

$$n = 1 + \frac{2\pi N}{k^2} f(0),$$

(4)

where $N$ is the number of scatterers per cm$^3$ and $k$ is the wave number of the particle incident on the target, $f(0) = f_{\text{acc}}(k' - k = 0)$ is the coherent elastic zero angle scattering amplitude. In this scattering, momentum of the
scattered particle $\mathbf{p}' = \hbar \mathbf{k}'$ (where $\mathbf{k}'$ is a wave vector) equals to initial momentum $\mathbf{p} = \hbar \mathbf{k}$. Atom (nucleus) that was in quantum state before interaction with the incident particle characterized by stationary wave function $\Phi_a$ will stay in the same quantum state after interaction with the incident particle. If the energy of interaction between particle and a scatterer depends on spin of the particle, then scattering amplitude $\hat{f}(\mathbf{k}' - \mathbf{k})$ can also depend on spin. As a consequence refractive index $\hat{n}$ (symbol $\hat{}$ means that mentioned magnitude is an operator in spin space of a particle) depends on spin as well [7].

If the matter is formed by different scatterers, then

$$ n = 1 + \frac{2\pi}{k^2} \sum_j N_j f_j(0), $$

where $N_j$ is the number of j-type scatterers per cm$^3$, $f_j(0)$ is the amplitude of the particle coherent elastic zero-angle scattering by j-type scatterer.

Let us consider a relativistic particle refraction on the vacuum-medium boundary (see [7]). The wave number of the particle in the vacuum is denoted $k$. The wave number of the particle in the medium is $k' = kn$. As is evident the particle momentum in the vacuum $\mathbf{p} = \hbar \mathbf{k}$ is not equal to the particle momentum in the medium.

Therefore, the particle energy in the vacuum $E = \sqrt{\hbar^2 k^2 c^2 + m^2 c^4}$ is not equal to the particle energy in the medium $E_{med} = \sqrt{\hbar^2 k'^2 c^2 + m^2 c^4}$.

The energy conservation law immediately requires the particle in the medium to have the effective potential energy $U_{eff}$. This energy can be easily found from relation:

$$ E = E_{med} + U_{eff}, $$

i.e.

$$ U_{eff} = E - E_{med} = -\frac{2\pi \hbar^2}{m\gamma} N f(E, 0) = (2\pi)^3 NT_{aa}(\mathbf{k}' - \mathbf{k} = 0), $$

$$ f(E, 0) = -(2\pi)^2 \frac{E}{c^2 \hbar^2} T_{aa}(\mathbf{k}' - \mathbf{k} = 0) = -(2\pi)^2 \frac{m\gamma}{\hbar^2} T_{aa}(\mathbf{k}' - \mathbf{k} = 0), $$

where $T_{aa}(\mathbf{k}' - \mathbf{k} = 0)$ is the matrix element of the T-operator describing elastic coherent zero-angle scattering.

Let us remind that T-operator is associated with scattering matrix $S$ [18, 19]:

$$ S_{ba} = \delta_{ba} - 2\pi i \delta(E_b - E_a) T_{ba}, $$

where $E_a$ is the energy of scattered particles before the collision, $E_b$ - after the collision, matrix element $T_{ba}$ corresponds to states $a$ and $b$ that refer to the same energy.

For the matter formed by different scatterers effective potential energy can be written as:

$$ U_{eff} = \sum_j N_j f_j(E, 0). $$

Due to periodic arrangement of atoms in a crystal the effective potential energy is a periodic function of coordinates of a particle moving in a crystal [8]:

$$ U(\vec{r}) = \sum_{\vec{r}} U(\vec{r}) e^{i\vec{\tau}\vec{r}}, $$

where $\vec{\tau}$ is the reciprocal lattice vector of the crystal;

$$ U(\vec{r}) = \frac{1}{V} \sum_j U_j(\vec{r}) e^{i\vec{r}_j \vec{r}}, $$

here $V$ is the volume of the crystal elementary cell, $\vec{r}_j$ is the coordinate of the atom (nucleus) of type $j$ in the crystal elementary cell.
\[ U_j(\vec{\tau}) = -\frac{2\pi \hbar^2}{m\gamma} F_j(\vec{\tau}), \]  

(13)

According to (13) effective potential energy \( U(\vec{\tau}) \) is determined by amplitude \( F_j(\vec{\tau}) = F_{j \alpha\alpha}(\vec{k}^\prime - \vec{k} = \vec{\tau}) \). In contrast to the case of chaotic matter where effective potential energy is determined by the amplitude of elastic coherent scattering \( f(\vec{k}^\prime - \vec{k}) \), here it is defined by the amplitude \( F(\vec{\tau}) \) (see Annex and [2]), which can be written as:

\[
F_j(\vec{k}^\prime - \vec{k}) = f_j(\vec{k}^\prime - \vec{k}) - i \frac{k}{4\pi} \int f_j^*(\vec{k}^\prime - \vec{k}) f_j(\vec{k}^\prime - \vec{k}) d\Omega_k. \tag{14}
\]

where \( d\Omega_k \) means integration over all of the vector \( \vec{k} \) directions, \( |\vec{k}| = |\vec{k}^\prime| \).

The occurrence of the amplitude \( F(\vec{k} - \vec{k}) \) instead of the amplitude of elastic coherent scattering \( f(\vec{k}^\prime - \vec{k}) \) in (9) is specified by the fact, that unlike of an amorphous matter, the wave elastically scattered in a crystal, due to rescattering by periodically located centers is involved in formation of a coherent wave propagating through the crystal (see Appendix VIII hereinafter).

III. EFFECTIVE POTENTIAL ENERGY OF A SPIN-PARTICLE MOVING CLOSE TO CRYSTAL PLANES (AXES)

Suppose a high energy particle moves in a crystal at a small angle to the crystallographic planes (axes) close to the Lindhard angle. This motion determined by the plane (axis) potential \( U(x) \) (Figure 3), which can be determined from \( \hat{U}(\vec{k}) \) by averaging over distribution of atoms (nuclei) in a crystal plane (axis). Similar result is obtained when all the terms with \( \tau_y \neq 0, \tau_z \neq 0 \) for the case of planes or \( \tau_x \neq 0 \) for the case of axes are removed from the sum (11).

As a consequence for the potential of periodically placed axes we can write:

\[
U(\vec{\rho}) = \sum_{\vec{\tau}_x} U(\vec{\tau}_x, \vec{\tau}_z = 0) e^{i\vec{\tau}_x \cdot \vec{\rho}}, \tag{15}
\]

\( z \)-axis of the coordinate system is directed along the crystallographic axis. For the potential of a periodically placed planes we have:

\[
U(x) = \sum_{\vec{\tau}_x} U(\vec{x}, \tau_y = 0, \tau_z = 0) e^{i\tau_x x}, \tag{16}
\]

\( y, z \)-planes of the coordinate system are parallel to the chosen crystallographic planes family. Lets remind that according to (12,13) the magnitude \( U(x) \) is expressed in terms of the amplitude \( F_j(\vec{\tau}) \).

Since the amplitude \( \hat{F}(\vec{k}^\prime - \vec{k}) \) depends on spin, the effective potential energy \( \hat{U} \) depends on a spin as well [4]. The magnitude \( \hat{U} \) is the operator in spin space of a particle incident on a crystal.

Lets express the amplitude \( \hat{F}(q) \) as Fourier transformation of function \( \hat{F}(\vec{\tau}) \):

\[
\hat{F}(q) = \int \hat{F}(\vec{\tau}) e^{-i\vec{q} \cdot \vec{\tau}} \, d^3\tau' \tag{17}
\]

Considering mentioned above we can conduct summation of \( \tau_x \) and \( \tau_x \) in (16) using following expression:

\[
\sum_{\tau_x} e^{i\tau x} = d_x \sum_l \delta(x - X_l), \tag{18}
\]

where \( d_x \) is the lattice period along axis \( x \); \( X_l \) are coordinates of \( l \) plane.

\[
\sum_{\tau_x, \tau_y} e^{i\tau_x \cdot \vec{\rho}} = d_x d_y \sum_l \delta(\vec{\rho} - \vec{\rho}_l), \tag{19}
\]

where \( \vec{\rho}_l \) is a coordinate of an axis, located in point \( \vec{\rho}_l \); \( d_x, d_y \) are lattice periods along axes \( x \) and \( y \).
As a result we obtain following expression for the effective potential energy of interaction between an incident particle and a plane (axis) (the lattice is assumed to consist of atoms of one kind):

\[
\hat{U}(x) = - \sum_{\tau_x} \frac{2\pi \hbar^2}{m\gamma V} \hat{F}(q_x = \tau_x, q_y = q_z = 0) e^{i\tau_x x} = - \frac{2\pi \hbar^2}{m\gamma V d_y d_z} \hat{F}(x, q_y = q_z = 0), \tag{20}
\]

\[
\hat{U}(\vec{\rho}) = - \frac{2\pi \hbar^2}{m\gamma V} \sum_{\tau_x, \tau_y} \hat{F}(q_x = \tau_x, q_y = \tau_y, q_z = 0) e^{i\tau_x \cdot \vec{\rho}} = - \frac{2\pi \hbar^2}{m\gamma d_z} \hat{F}(\vec{\rho}, q_z = 0), \tag{21}
\]

\(d_z\) is the lattice period along the axis \(z\).

IV. P- AND T-ODD SPIN INTERACTIONS IN CRYSTALS

Let consider baryon scattering in thin crystal, when channeling effects can be neglected. General expression for amplitude of elastic coherent scattering of a spin 1/2 particle by a spinless (unpolarized) nuclei in presence of electromagnetic, strong and \(P\)-, \(T\)-odd weak interactions can be written as:

\[
\hat{F}(\vec{q}) = A(\vec{q}) + B(\vec{q}) \sigma \tilde{N} + B_w(\vec{q}) \sigma \tilde{N}_w + B_T \sigma \tilde{N}_T, \tag{22}
\]

where \(A(\vec{q})\) is spin-independent part of scattering amplitude, which is caused by electromagnetic, strong and weak interactions. \(h\vec{q} = \vec{h}\vec{k} - \vec{h}\vec{k}'\) is the transmitted momentum, \(\vec{h}\vec{k}\) is the momentum of the scattered particle, \(\vec{h}\vec{k}'\) is the momentum of the incident baryon, \(\vec{k}', \vec{k}\) are the wave vectors, \(\tilde{N} = \frac{|\vec{k} \times \vec{k}|}{|\vec{k}' \times \vec{k}|}, \tilde{N}_w = \frac{\vec{k}' + \vec{k}}{|\vec{k}' + \vec{k}|}, \tilde{N}_T = \frac{\vec{k}' - \vec{k}}{|\vec{k}' - \vec{k}|}, \sigma = (\sigma_x, \sigma_y, \sigma_z)\) are the Pauli matrices.

The term, which is proportional to \(\sigma \tilde{N}\), is responsible for spin-orbit interaction contribution to scattering process. For electromagnetic interaction this contribution is determined by the particle magnetic moment. \(P_{\text{odd}} T_{\text{even}}\) part of scattering amplitude, which is proportional to \(\sigma \tilde{N}_w\) is determined by \(P_{\text{odd}} T_{\text{even}}\) interactions of baryon with electron and nucleus. \(T_{\text{odd}}\) part of scattering amplitude, which is proportional to \(\sigma \tilde{N}_T\) is determined by electric dipole moment and short range baryon-electron and baryon-nucleus \(T_{\text{odd}}\) interactions.

With amplitude \(\hat{F}(\vec{q})\) one can find the cross-section of particle scattering by a crystal and polarization vector of the scattered particle. The scattering cross-section for a thin crystal can be written as [8]:

\[
\frac{d\sigma_{\text{cr}}}{d\Omega} = \frac{d\sigma}{d\Omega} \left\{ (1 - e^{-\bar{u}^2 \bar{q}^2}) + \frac{1}{N} \left| \sum_n e^{i\bar{p} \vec{r}_n} \right|^2 e^{-\bar{u}^2 \bar{q}^2} \right\}, \tag{23}
\]

where \(\bar{r}_n^0\) is the coordinate of the center of gravity of the crystal nucleus, \(\bar{u}^2\) is the mean square of thermal oscillations of nuclei in the crystal. The first term describes incoherent scattering and the second one describes the coherent due to periodic arrangement of crystal nuclei (atoms). This contribution leads to the increase in the cross section \(\sigma_{\text{cr}}\).

The value \(\frac{d\sigma}{d\Omega}\) describes baryon scattering cross section by atoms of crystal:

\[
\frac{d\sigma}{d\Omega} = tr \hat{\rho} \hat{f}^+(\vec{q}) \hat{f}(\vec{q}), \tag{24}
\]

where \(\hat{\rho}\) is the spin density matrix of the incident particle.

The polarization vector of a particle that has undergone a single scattering event can be found using the following expression:

\[
\xi = \frac{tr \hat{\rho} \hat{f}^+ \sigma \hat{f}}{tr \hat{\rho} \hat{f}^+ \hat{f}} = \frac{tr \hat{\rho} \hat{f}^+ \sigma \hat{f}}{\frac{d\sigma}{d\Omega}}. \tag{25}
\]
Using (22) and (92) one can obtain the following expressions for polarization vector of the scattered particle and differential cross-section:

$$\tilde{\xi} = \tilde{\xi}_{so} + \tilde{\xi}_{w} + \tilde{\xi}_{T},$$

where $\tilde{\xi}_{so}$ is the change of polarization vector due to spin-orbit interaction, $\tilde{\xi}_{w}$ is the change of polarization vector caused by weak parity violating interaction, $\tilde{\xi}_{T}$ is the change of polarization vector caused by $T$-odd interaction.

$$\tilde{\xi}_{so} = \left\{ (|A|^2 - |B|^2)\tilde{\xi}_0 + 2|B|^2\tilde{N}(\tilde{N}\tilde{\xi}_0) + 2\Im(AB^*)[\tilde{N}\tilde{\xi}_0] + 2\Re(AB^*) \right\} \cdot \left( \frac{d\sigma}{d\Omega} \right)^{-1},$$

$$\tilde{\xi}_{w} = \left\{ (|A|^2 - |B_w|^2)\tilde{\xi}_0 + 2|B_w|^2\tilde{N}_w(\tilde{N}_w\tilde{\xi}_0) + 2\Im(AB_{w}^*)[\tilde{N}_w\tilde{\xi}_0] + 2\Re(AB_{w}^*) \right\} \cdot \left( \frac{d\sigma}{d\Omega} \right)^{-1},$$

$$\tilde{\xi}_{T} = \left\{ (|A|^2 - |B_T|^2)\tilde{\xi}_0 + 2|B_T|^2\tilde{N}_T(\tilde{N}_T\tilde{\xi}_0) + 2\Im(AB_{T}^*)[\tilde{N}_T\tilde{\xi}_0] + 2\Re(AB_{T}^*) \right\} \cdot \left( \frac{d\sigma}{d\Omega} \right)^{-1}.$$  

(26)

The differential cross-section reads as follows:

$$\frac{d\sigma}{d\Omega} = \text{tr} \rho f^+ f =$$

$$|A|^2 + |B|^2 + |B_w|^2 + |B_T|^2 + 2\Re(AB^*)\tilde{N}\tilde{\xi}_0 + 2\Re(AB_{w}^*)\tilde{N}_w\tilde{\xi}_0 + 2\Re(AB_{T}^*)\tilde{N}_T\tilde{\xi}_0.$$  

(27)

While deriving expressions (27) and (28) small terms containing production of $B_w$ and $B_T$, which are much smaller comparing to other terms, were omitted. According to (27) the angle of polarization vector rotation for a baryon scattered in a crystal is determined by rotations around three mutually orthogonal directions (see terms proportional to $N$, $N_w$, $N_T$). The indicated rotations are determined by electromagnetic, strong and weak $P,T_{odd}$ interactions. It should also be noted that initially unpolarized particle beam ($\xi_0 = 0$) in a crystal acquires polarization directed along one of three vectors $\tilde{N}$, $\tilde{N}_w$, $\tilde{N}_T$, which carries information about all types of interaction too. According to (28) amplitudes interference results in asymmetry in scattering caused by orientation of vectors $\tilde{N}_T$, $\tilde{N}$, $\tilde{N}_w$ with respect to $\xi_0$, $\vec{k}'$ and $\vec{k}$. Therefore, the angular distribution of scattered particles intensity is anisotropic. Thus, measurements of the rotation angle and of the angular distribution of intensity for a particle beam scattered by crystal axes enables to study $T_{odd}$ interactions of positive(negative) charged and neutral short-lived baryons. In particular, such measurements allow one to obtain restrictions on electric dipole moment of short-lived baryons and other constants describing these interactions. The same makes also possible to study $P_{odd}$ and spin-orbit interactions. Let us remind that in case of electromagnetic interaction spin-orbit interaction is determined by magnetic moment of the particle. Important notice follows from (27) and (28): measurement of the angular distribution (which depends on the spin orientation) of products of the weak decay of the baryon is not a single method to measure spin rotation angle for the short-lived baryons. The same can be made by studying left-right asymmetry of the baryon scattered by two crystals. In this case it is sufficient to measure the angular distribution of intensity of flow of double scattered particles. The first crystal provides spin rotation, while the second enables to study angular distribution of particles scattered by the first crystal i.e. left-right asymmetry. This can be realized, for example, by using set of crystals with axes directed at small angle with respect to momentum of the scattered particles and detection of either nuclear reaction or ionizing losses inside crystal detector. Computer modelling is essential for further analysis. The angle of spin rotation $\theta_p$ and scattering anisotropy increases significantly for a baryon in an unblank crystal, when particles move at a small angle with respect to a crystal axes, since in this case the scatterers density grows. As a result, even for short-lived beauty (bottom) baryons with negative and neutral charge the $P_{odd}$ and $T_{odd}$ amplitude can be measured. Study of the spin rotation of a particle, which moves at a small angle with respect to a crystal axes, is hampered by depolarization effect [7, 8, 20]. Let us note that trajectories of the scattered particles, which azimuth angles are in the vicinity $\varphi$ and $\varphi + \pi$ (z axes is directed along the crystal axes) contributes to EDM (Todd interaction) spin rotation with opposite signs due to different electric field signs. At the same time, the $P$-odd rotation occurs around the momentum direction and is not affected by the electric field direction. As a result the $T_{odd}$ spin rotation can be...
observed in unbent crystal if we use subtraction of the measurements results for angle ranges $\varphi$ and $\varphi + \pi$ from each other. Such procedure leads to summation of contributions from $T$-odd rotation. Simultaneous measurement of spin orientation for all $\varphi$ values (as well as for all polar angles for scattering by the axes) provides intensity increase. For unbent crystals the same measuring procedure enables to use crystal with higher nucleus charge that also contributes to effect increase. The similar reasoning is valid for measuring the anomalous magnetic moment by means of axial scattering in unbent crystals. Subtraction procedure can be applied for measuring both anomalous magnetic moment and EDM ($T_{\text{odd}}$ interactions) of neutral charm and beauty short-lived baryons.

V. EFFECTIVE POTENTIAL ENERGY OF A SPIN-PARTICLE MOVING CLOSE TO THE CRYSTAL PLANES (AXES)

According to $[20]$ and $[21]$ to obtain expression for potential energy of baryon in crystal one should derive the expressions for contributions to $\hat{U}$ caused by different types of interactions.

As was stated above elastic coherent scattering of a particle by an atom is caused by electromagnetic interaction of the particle with the atom electrons and nucleus as well as weak and strong nuclear interaction with electrons and nucleus. The general expression for the amplitude of elastic scattering of a particle with spin $\frac{1}{2}$ by a spinless or unpolarized nuclei $[22]$ can be written as:

$$\hat{F}(\vec{q}) = A_{\text{coul}}(\vec{q}) + A_{s}(\vec{q}) + (B_{\text{magn}}(\vec{q}) + B_{S}(\vec{q}))\vec{n} \times \vec{q} + \vec{F}_{\text{orb}}(\vec{q}) + (B_{\text{we}}(\vec{q}) + B_{\text{urnuc}}(\vec{q}))\vec{N}_{w} + (B_{\text{EDM}}(\vec{q}) + B_{T_{e}}(\vec{q}) + B_{T_{\text{nuc}}}(\vec{q}))\vec{q},$$

(29)

where $\vec{q} = \vec{k}' - \vec{k}, \vec{n} = \frac{\vec{q}}{|\vec{k}' + \vec{k}|}, A_{\text{coul}}(\vec{q})$ is the spin-independent part of the amplitude of elastic Coulomb scattering of a particle by an atom; $A_{s}(\vec{q})$ is the spin-independent part of the scattering amplitude, which is caused by strong interaction (the similar contribution caused by weak interaction it is negligibly small and hereinafter is omitted).

The spin-dependent amplitude, which is proportional to $B_{\text{magn}}(\vec{q})$, is determined by electromagnetic spin-orbit interaction. The term proportional to $B_{s}(\vec{q})$ is responsible for the contribution of the spin-orbit strong interaction to a scattering process of a baryon by a nucleus.

The term proportional to the parity odd pseudo scalar $\vec{N}_{w}$ (unit vector $\vec{N}_{w} = \frac{\vec{F} + \vec{k}}{|\vec{k}' + \vec{k}|}$) includes two contributions:

a). Contribution to the amplitude proportional to $B_{\text{we}}(\vec{q})$, which describes elastic scattering caused by the parity violating weak interaction between the baryon and electrons.

b). Contribution to the amplitude proportional to $B_{\text{urnuc}}(\vec{q})$, which describes elastic scattering caused by the parity violating weak interaction between the baryon and the nucleus.

The term proportional to the time ($T$) violating (CP non-invariant) pseudo scalar $\vec{q}$ includes three contributions:

a). Contribution proportional to $B_{\text{EDM}}(\vec{q})$ describes elastic scattering of the baryon with EDM by the atom’s Coulomb field.

b). Contribution proportional to $B_{T_{e}}(\vec{q})$ describes possible short-range $T$-non-invariant interaction between the baryon and electrons.

c). Contribution to the amplitude, which is proportional to $B_{T_{\text{nuc}}}(\vec{q})$, describes scattering caused by $T$-non-invariant interaction between the baryon and nucleons.

Using the amplitude $\hat{F}(\vec{q})$ potential energy $\hat{U}(\vec{r})$ can be expressed as a sum of terms that describe contribution of different interactions to $\hat{U}(\vec{r})$:

$$\hat{U}(\vec{r}) = U_{\text{coul}}(\vec{r}) + U_{S}(\vec{r}) + \hat{U}_{\text{magn}}(\vec{r}) + \hat{U}_{s-p-\text{orb}}(\vec{r}) + \hat{U}_{W}(\vec{r}) + \hat{U}_{T}(\vec{r}),$$

(30)

where $U_{\text{coul}}(\vec{r})$ is Coulomb potential energy of interaction between baryon and crystal, which is details investigated during description of the effect of particles channeling in a crystal; $U_{S}(\vec{r})$ describes spin independent contribution of nuclear interactions to the potential energy of interaction with crystal; $\hat{U}_{\text{magn}}(\vec{r})$ describes contribution to $\hat{U}(\vec{r})$ caused by interaction between baryons magnetic moment and atoms electric field; $\hat{U}_{s-p-\text{orb}}(\vec{r})$ is contribution caused by spin-orbital nuclear interactions; $\hat{U}_{W}(\vec{r})$ describes the contribution caused by parity violating weak interactions; $\hat{U}_{T}(\vec{r})$ describes the contribution caused by $T$-violation interactions between baryon and crystal.

According to the analysis given in appendices of the paper following expressions can be given for potential energy $\hat{U}(\vec{r})$:
a) Potential energy for the plane \( U_S(x) \):
\[
U_S(x) = -\frac{2\pi \hbar^2}{m\gamma d_y d_z} N_{\text{nuc}}(x) A_S(0) ,
\]
where \( N_{\text{nuc}}(x) = \iint N_{\text{nuc}}(x, y', z') dy' dz' \) is the probability density for vibrating nuclei detection in point \( x \) (in direction orthogonal to the chosen crystallographic plane).

Similarly, for the axis:
\[
U(\vec{\rho}) = -\frac{2\pi \hbar^2}{m\gamma d_z} N_{\text{nuc}}(\vec{\rho}) A_S(0) ,
\]
where \( N_{\text{nuc}}(\vec{\rho}) = \int N_{\text{nuc}}(\vec{\rho}, z') dz' \), \( \vec{\rho} = (x, y) \) is a vector laying in a plane orthogonal to the chosen crystallographic axis.

b) Effective potential energy determined by the anomalous magnetic moment
\[
\hat{U}_{\text{magn}}(x) = -\frac{e\hbar}{2mc} \frac{g - \frac{2}{3}}{2} \hat{\sigma}[\hat{E}_{\text{plane}} \times \vec{n}] - i \frac{1}{4d_y d_z mc^2} \left(\frac{g - \frac{2}{3}}{2}\right) \frac{\partial}{\partial x} \delta V^2(x) \hat{\sigma} \vec{N} = -(\alpha_m + i\delta_m)\vec{\sigma}\vec{N} ,
\]
where
\[
\vec{N} = [\vec{n}^\perp \times \vec{n}],
\]
\[
\alpha_m = \frac{e\hbar}{2mc} \frac{g - \frac{2}{3}}{2} E_x ,
\]
\[
\delta_m = \frac{1}{4d_y d_z mc^2} \left(\frac{g - \frac{2}{3}}{2}\right) \frac{\partial}{\partial x} \delta V^2(x) ,
\]
where \( \delta V^2(x) = \int \left\{ \left[ \int V_{\text{coul}}(x, y, z) dz \right]^2 - \left[ \int V_{\text{coul}}(x, y, z) dz \right]^2 \right\} dy \) is the mean-square fluctuation of energy of Coulomb interaction between baryon and atom.

Similarly for the case of axis it can be obtained:
\[
\hat{U}_{\text{magn}}(\vec{\rho}) = -\frac{e\hbar}{2mc} \frac{g - \frac{2}{3}}{2} \hat{\sigma}[\hat{E}_{\text{axis}} \times \vec{n}] + \hat{U}_{\text{magn}}^{(2)}(\vec{\rho}) ,
\]
where
\[
\hat{U}_{\text{magn}}^{(2)}(\vec{\rho}) = -i \frac{1}{d_z mc^2} \left(\frac{g - \frac{2}{3}}{2}\right) \hat{\sigma} [\nabla_\rho \delta V^2(\vec{\rho}) \times \vec{n}] .
\]

For the axisymmetric case:
\[
\hat{U}_{\text{magn}}^{(2)}(\rho) = -i \frac{1}{4d_z mc^2} \left(\frac{g - \frac{2}{3}}{2}\right) \frac{\partial}{\partial \rho} \delta V^2(\rho) \hat{\sigma} [\vec{n}_\rho \times \vec{n}] ,
\]
where \( \delta V^2(\rho) = \left[ \int V_{\text{coul}}(\vec{\rho}, z) dz \right]^2 - \left[ \int V_{\text{coul}}(\vec{\rho}, z) dz \right]^2 \) is the mean-square fluctuation of energy of Coulomb interaction between baryon and atom, \( \vec{n}_\rho = \frac{\vec{\rho}}{\rho} \) is the unit vector, \( \vec{n}_\rho \perp \vec{n} \).

c) Effective potential energy \( \hat{U} \) determined by spin-orbit interaction between baryon and nucleus:
\[
\hat{U}_{\text{ssp-orb}} = -(\alpha_s + i\delta_s)\vec{\sigma}\vec{N} ,
\]
where in case of plane:
\[
\vec{N} = [\vec{n}^\perp \times \vec{n}] ,
\]
\[
\alpha_s = -\frac{2\pi \hbar^2}{m\gamma d_y d_z} \frac{\partial N_{\text{nuc}}}{\partial x} B'' ,
\]
\[
\delta_s = \frac{2\pi \hbar^2}{m\gamma d_y d_z} B' \frac{\partial N_{\text{nuc}}}{\partial x} .
\]
In case of axis:

\[
\hat{U}_{ssp-orb}(\rho) = \frac{2\pi \hbar^2}{m\gamma d_z} (B'' - iB') \vec{\nabla}_{\rho} [\nabla_{\rho} N_{nuc}(\rho) \times \vec{n}].
\]  

(38)

d) Effective potential energy \( \hat{U} \) determined by \( P_{odd} \) and \( T_{even} \) interactions in the case of plane:

\[
\hat{U}_w(x) = \hat{U}_{we}(x) + \hat{U}_{wnuc}(x) = -(\alpha_w(x) + i\delta_w(x)) \vec{n},
\]

where

\[
\alpha_w(x) = \frac{2\pi \hbar^2}{m\gamma d_y d_z} (\vec{B}_{we}(0)\n_N(x) + \vec{B}_{wnuc}(0)\n_{nuc}(x)),
\]

\[
\delta_w(x) = \frac{2\pi \hbar^2}{m\gamma d_y d_z} (\vec{B}_{we}(0)\n_N(x) + \vec{B}_{wnuc}(0)\n_{nuc}(x)).
\]

(40)

In the case of axis:

\[
\hat{U}_{we}(\rho) = \frac{2\pi \hbar^2}{m\gamma d_z} (\vec{B}_{we}(0)\n_N(\rho) + \vec{B}_{wnuc}(0)\n_{nuc}(\rho)) \vec{n}_w,
\]

\[
\n_{nuc}(\rho) = \int \n_{nuc}(\rho, z) dz,
\]

(41)

where \( \vec{B}' \) and \( \vec{B}'' \) are the real and imaginary parts of \( \vec{B} \), respectively.

e) \( T \)-violation interactions leads to the following contribution to potential energy:

\[
\hat{U}_T(x) = \hat{U}_{EDM} + \hat{U}_{Te} + \hat{U}_{Tnuc} = -(\alpha_T(x) + i\delta_T(x)) \vec{n}_T,
\]

where \( \alpha_T = \alpha_{EDM} + \alpha_{Te} + \alpha_{Tnuc}, \delta_T = \delta_{EDM} + \delta_{Te} + \delta_{Tnuc} \).

Energy of interaction between electric dipole moment with atoms electric field:

\[
\hat{U}_{EDM} = -e d E_{pl}(x) \vec{N}_T - i \frac{d}{2d_y d_z \hbar c} \frac{\partial}{\partial x} \vec{N}_T \vec{N}_T,
\]

where unit vector \( \vec{N}_T \) is orthogonal to the plane, \( E_{pl}(x) = E_x \vec{N}_T \).

Evidently, \( \hat{U}_{EDM} \) can be expressed as:

\[
\hat{U}_{EDM} = -(\alpha_{EDM} + i\delta_{EDM}) \vec{n}_T.
\]

(44)

Similar to \( \hat{U}_{magn} \) energy \( \hat{U}_{EDM} \) has non-zero both real and imaginary parts. The expression for \( \hat{U}_{magn} \) converts to \( \hat{U}_{EDM} \) when using replacement \( \frac{\alpha}{2} \rightarrow 2\frac{\alpha}{\lambda_c} \) (\( \lambda_c = \frac{\hbar}{mc} \) is the Compton wave-length of the particle) and \( \vec{N} \rightarrow \vec{N}_T \).

Therefore:

\[
\frac{U_{EDM}}{U_{magn}} = \frac{4d}{\lambda_c(g - 2)} = \frac{e d}{\mu_A} = \frac{D}{\mu_A}.
\]

(45)

Short range \( T_{odd} \) interactions give contributions to the effective potential energy that can be written as:

\[
\hat{U}_{Te(nuc)}(x) = -(\alpha_{Te(nuc)} + i\delta_{Te(nuc)}) \vec{n}_T,
\]

where

\[
\alpha_{Te(nuc)} = \frac{2\pi \hbar^2}{m\gamma d_y d_z} \vec{B}_{Te(nuc)} \frac{dN_{e(nuc)}(x)}{dx},
\]

\[
\delta_{Te(nuc)} = \frac{2\pi \hbar^2}{m\gamma d_y d_z} \vec{B}_{Te(nuc)} \frac{dN_{e(nuc)}(x)}{dx}.
\]

In the case of axis:

\[
\hat{U}_{Te(nuc)}(\rho) = \frac{2\pi \hbar^2}{m\gamma d_z} (\vec{B}''_{Te(nuc)} + i\vec{B}''_{Te(nuc)}) \vec{n}_N N_{Te(nuc)}(\rho).
\]

(47)
Thus in the experiment aimed to obtain the limit for the EDM value, the limits for the scattering amplitude, which is determined by short-range T(CP)-noninvariant interactions between baryons and electrons, and nuclei, will be obtained as well. The obtained values of these amplitudes for different interaction types allows one to restore the values of corresponding constants, too. The simplest model for such a potential is the Yukawa potential [21].

Using it all the equations for $\alpha_{Te(nuc)}$ can be obtained by replacement in (27) of $V_{coul} + V_{EDM}$ by $V_{coul} + V_T$, $V_T = -d_T \sigma r \xi_T$. Here $d_T$ is the interaction constant, $\xi_T = 1/M_T$, where $M_T$ is the mass of heavy particles, exchange of which leads to interaction $V_T$ [21]. It should be noted that constant $d_T$ for interaction between the heavy baryon and the nucleon can be greater than that for nucleon-nucleon interaction. This effect can be explained by the reasoning similar to that explaining expected EDM growth for the heavy baryon. $T_{odd}$ interaction mixes heavy baryon stationary states with different parity more effectively than for light baryons due to probably smaller spacing between energy levels corresponding to these states.

Let’s note that as scattering amplitude $\hat{F}$ is a complex value potential energy $\hat{U}$ is also a complex value. The real part of this energy describes changes in particles energy as a result of interaction with the matter, the imaginary part describes absorption.

Every spin dependent contribution to $\hat{U}$ is of following structure:

$$\hat{A} = -(\alpha + i\beta)\sigma \hat{N}. \quad (48)$$

Let’s compare this expression with the one for energy of interaction between magnetic moment $\vec{\mu}$ and magnetic moment $\vec{B}$:

$$\hat{U}_{magn} = -\vec{\mu} \sigma \vec{B}. \quad (49)$$

It can be seen that terms proportional $\alpha$ in $\hat{A}$ causes spin rotation around $\hat{N}$. The imaginary part shows that absorption in matter depends on spin orientation regarding $\hat{N}$. As a result spin component directed along $\hat{N}$ can appear (spin dichroism occurs [3]).

The analogy between (48) and (49) leads us to the conclusion that particle in the matter is affected by pseudomagnetic fields caused by strong and weak interactions (in neutron low energy area effects determined by such fields are discovered and have been investigated for many years, see [7]). Let now baryons with the polarization vector oriented at a certain angle to the direction of $\vec{n}$ be incident on medium. This baryon state can be considered as a superposition of two states with polarizations along and opposite to the momentum direction defined by unit vector $\vec{n}$. The wave function of a particle before entering the target has the form

$$\psi(\vec{r}) = e^{i\vec{k}\vec{r}} \chi_{n, n} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}, \quad (50)$$

or

$$\psi(\vec{r}) = c_1 e^{i\vec{k}\vec{r}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 e^{i\vec{k}\vec{r}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad (51)$$

Choose the direction $\vec{n}$ as the $z$-axis. The state of the type $\left( \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right)$ has the refractive index $n_+$, while the state of the type $\left( \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right)$ has the refractive index $n_-$. If a baryon with spin parallel to vector $\vec{n}$ (spin state $\left( \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right)$) were incident on the target, its motion in matter would be described by the wave function $\psi_+(\vec{r}) = e^{ikn_+z} \left( \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right)$. If a baryon with spin antiparallel to $\vec{n}$ (spin state $\left( \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right)$) is incident on the target, then in matter it is described by the wave function $\psi_-(\vec{r}) = e^{ikn_-z} \left( \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right)$. If a baryon with an arbitrary spin direction falls on the target, its wave function (see (51)) is the superposition of states $\left( \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right)$ and $\left( \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right)$. As a consequence, the wave function of baryons in matter is also the superposition of these states, and can be written as

$$\psi(\vec{r}) = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \psi_+ (\vec{r}) = c_1 e^{ikn_+z} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 e^{ikn_-z} \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \quad (52)$$

Now let us consider how the polarization of baryons changes as they penetrate into the interior of the target (with the growth of the target thickness). Suppose we have a detector that transmits the particles with spin polarized along a certain direction in the detector (the axis of the detector) and absorbs the particles with the opposite spin direction. Such a detector is the analog of the Nicol prism [22] used in optics for analyzing the polarization of light. When polarized light is incident on the Nicol prism, one component of the light polarization passes through it, while the component orthogonal to the axis of the Nicol prism is absorbed. In the case of baryons, a target with polarized nuclei may act as a detector. As the scattering cross section of a polarized baryon depends on whether the baryon spin is oriented along the direction of the polarization vector of the nucleus or opposite to it, baryon absorption in
the detector exhibits the same dependence. Suppose that the axis of the detector is parallel to the $z$-axis along which the detector nuclei are polarized. In this case the detector analyzes those components of the baryon spin, which are directed along the $z$-axis and opposite to it. From (52) follows that the probability amplitude $A^{(+)}$ of finding the baryon with spin state $(\frac{1}{2})$, i.e., of finding the baryon polarized parallel to the $z$-axis, is given by the expression:

$$A^{(+)} = (1 \ 0)\psi = c_1 e^{i kn + z}.$$  

The probability

$$P_z^{(+)} = |(1 \ 0)\psi|^2 = |c_1|^2 e^{-2i mn + z} = |c_1|^2 e^{-\rho \sigma + z}.$$  

Similarly, the probability $P_z^{(-)}$ of finding the baryon polarized opposite to the $z$-axis is

$$P_z^{(-)} = |(0 \ 1)\psi|^2 = |c_2|^2 e^{-2i mn - z} = |c_2|^2 e^{-\rho \sigma - z},$$  

where $\sigma_{\pm}$ is the total cross section of scattering by the nucleus of the baryon polarized parallel (antiparallel) to the baryon momentum. Since in matter $\text{Im} n \neq \text{Im} - n$ ($\sigma_+ \neq \sigma_-)$ owing to P-violation, one of the components of the baryon spin wave function decays faster and at some depth the rapidly damped component may be neglected. The beam will appear polarized along the $z$-axis (along the direction of the particle momentum $n$). Let us now rotate the detector so that its polarization axis becomes perpendicular to the direction of $n$. Choose the direction of the polarization axis of the detector as the $x$-axis. Now the detector analyzes those components of the baryon spin, which are directed along and opposite to the $x$-axis. To determine the probability $P_x^{(\pm)}$ of finding the component of the baryon spin parallel (antiparallel) to the direction of the $x$-axis, one should expand the wave function (52) in terms of the spin wave functions $\chi_x^{\pm}$, which are the eigenfunctions of operator $S_x$ of the spin projection onto the $x$-axis. They have the form

$$\chi_{x}^{\pm} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ \pm 1 \end{pmatrix}.$$  

As a result, we find that the probabilities $P_x^{(\pm)}$ of baryon spin polarization along and opposite to the $x$-axis change with $z$ as:

$$P_x^{(+)} = \frac{1}{2} \left\{ |c_2|^2 e^{-2i kn + z} + |c_2|^2 e^{-2i mn + z} + 2|c_1| c_2 |e^{-i kn + z} \cos (k \text{Re}(n_+ - n_-)z + \delta) \right\},$$

$$P_x^{(-)} = \frac{1}{2} \left\{ |c_1|^2 e^{-2i mn + z} + |c_2|^2 e^{-2i mn - z} - 2|c_1| c_2 |e^{-i kn + z} \cos (k \text{Re}(n_+ - n_-)z + \delta) \right\},$$

where $\delta = \delta_1 - \delta_2$ is the difference of the initial phases of states with baryon spin polarization along and opposite to the $z$-axis ($c_1 = |c_1| e^{i \delta_1}; c_2 = |c_2| e^{i \delta_2}$). If at $z = 0$, the baryon is polarized along $x$, i.e.,

$$c_1 = c_2 = \frac{1}{\sqrt{2}}, \quad \delta = 0,$$

then with growing $z$ the polarization opposite to $x$ appears and further change of the polarization acquires the character of oscillations. As the baryons pass through the target, one of the components decays more strongly and the baryon beam eventually becomes polarized along or opposite to the $z$-axis. When a beam polarized along the $z$-axis is incident onto the target, no oscillations emerge: only damping occurs. Using (52), one can find the baryon polarization vector

$$\vec{p}_n = \frac{\langle \psi | \vec{S}_x | \psi \rangle}{\langle \psi | \psi \rangle}.$$  

As a result,

$$p_{nx} = 2\text{Re} \delta^* c_2 \psi^* \langle 0 | \psi \rangle, \quad p_{ny} = 2\text{Im} \psi^* c_2 \psi^* \langle 0 | \psi \rangle, \quad p_{nz} = (|c_1|^2 - |c_2|^2) \langle 0 | \psi \rangle.$$  

(57)
Suppose that baryon spin in a vacuum is directed perpendicular to the polarization vector of the nuclei. Choose this direction as the $x$-axis. In this case

$$c_1 = c_2 = 1/\sqrt{2}.$$  

Using relations\(57\) we obtain

$$p_{nx} = \cos[k \text{Re}(n_+ - n_-)z]e^{-k\text{Im}(n_+ + n_-)z}(\psi|\psi)^{-1},$$

$$p_{ny} = -\sin[k \text{Re}(n_+ - n_-)z]e^{-k\text{Im}(n_+ + n_-)z}(\psi|\psi)^{-1},$$

$$p_{nz} = \frac{1}{2}(e^{-2k\text{Im}n_+z} - e^{2k\text{Im}n_-z})(\psi|\psi)^{-1}$$

$$p_x^2 + p_y^2 + p_z^2 = 1.$$  \(58\)

According to \(58\) as the baryon penetrates into the interior of the target, its polarization vector rotates about the particle’s momentum direction $\vec{n}$ through the angle

$$\theta = k \text{Re}(n_+ - n_-)z = \frac{2\pi\rho}{k} \text{Re}(f_+ - f_-)z = \frac{Re(U^+_w - U^-_w) z}{\hbar} c.$$  \(59\)

At the same time, as the baryons pass through matter, the transverse components $p_{nx}$ and $p_{ny}$ of the polarization vector decay because baryon absorption depends on spin orientation, and finally the beam appears to be polarized along or opposite to the $z$-axis. Thus, the dependence of the absorption of baryons in the target on the orientation of their spin results in the fact that the polarization vector $\vec{p}_n$ (recall that $|\vec{p}_n| = 1$) not only rotates about the $z$-axis (about the direction of momentum) but also undergoes additional rotation in the direction of the $z$-axis (the end point of the polarization vector moves along the unit sphere). If the dependence of absorption on spin orientation can be neglected, the polarization vector rotates about the direction of particle momentum $\vec{n}$ only in the $(x, y)$ plane. In terms of kinematics, this phenomenon is analogous to the light polarization plane rotation in a magnetic field (the Faraday effect), while spin oscillations along and opposite to the direction of the $x$-axis are analogous to the transitions $K^0 \leftrightarrow K^0$ occurring in regeneration of neutral $K$-mesons (see e. g. \[23\]).

Let us now consider particle moving in straight (non bent) crystal. The expression for $U$ contains group of terms proportional to either electric field projection on to $x$-axis or derivative of electrons and nuclei density \(dN_e/(dx)\). As a result, particle moving between the planes experience influence of pseudomagnetic fields that reverse sign due to transverse oscillations of a channeled particle. This leads to the fact that total spin rotation in such fields is suppressed (although suppression fades with the growth of energy of the particle). Mentioned above does not concern to spin rotation effect and spin dichroism caused by weak P-violation $T_{even}$ interaction. This effect increases with the growth of crystal thickness (the effect also occurs in amorphous medium) \[7\].

VI. RELATIVISTIC PARTICLE SPIN ROTATION AND INTERACTIONS IN BENT CRYSTALS

Let us now consider particle moving in bent crystal. Expressions for energy of interaction between a baryon and a crystal plane (axis), which are obtained above, allow us to find the equation describing evolution of the particle polarization vector in a bent crystal. The mentioned equations differ from those, which describe spin evolution in external electromagnetic fields in vacuum, by presence of terms, which define contributions from P and T(CP) noninvariant interactions between electrons and nuclei to the spin rotation. Moreover, a new effect, which is caused by nonelastic processes, arises: along with the spin precession around vectors $\vec{N}_m$, $\vec{N}_T$, $\vec{n}$, the spin components directed along vectors $\vec{N}_m$, ($\vec{N}_T$, $\vec{n}$) appear and, thus, spin dichroism occurs. Equations, which describe spin rotation in this case, can be obtained by the following approach \[7\]. Spin wave function $|\Psi(t)\rangle >$ meets the equation as follows:

$$i\hbar \frac{\partial |\Psi(t)\rangle}{\partial t} = \hat{U}_{eff} |\Psi(t)\rangle >.$$  \(60\)

Baryon polarization vector $\vec{\xi}$ can be found via $|\Psi(t)\rangle >$:

$$\vec{\xi} = \frac{<\Psi(t)|\vec{\sigma}|\Psi(t)>}{<\Psi(t)|\Psi(t)>},$$  \(61\)
Thus the equation for spin rotation of a particle ($\gamma \gg 1$), which moves in a bent crystal, reads as follows:

$$\frac{d\vec{\xi}}{dt} = [\vec{\xi} \times \Omega_{mso}] - \frac{2}{\hbar}(\delta_m(x) + \delta_{s0}(x))\{\vec{N}_m - \vec{\xi}(\vec{N}_m \vec{\xi})\} +$$

$$+ [\vec{\xi} \times \Omega_{T}] + \frac{2}{\hbar}(\delta_{EDM}(x) + \delta_{E}(x) + \delta_{Tnuc}(x))\{\vec{N}_T - \vec{\xi}(\vec{N}_T \vec{\xi})\} +$$

$$+ [\vec{\xi} \times \Omega_{W}] - \frac{2}{\hbar}\delta_w \{\vec{n} - \vec{\xi}(\vec{n} \vec{\xi})\} \quad (62)$$

where $\Omega_{mso} = \Omega_{MDM} + \Omega_{so} = \frac{\alpha (x - 1)}{2 \hbar \gamma m c} E(x) + \frac{\alpha_{s0}(x)}{\hbar} N_m$, $\Omega_{T} = \Omega_{EDM} + \Omega_{Tnuc} = \frac{2}{\hbar}(dE(x) + \alpha_T + \alpha_{Tnuc}(x))N_T$, $\Omega_{W} = \frac{2}{\hbar}\delta_w \vec{n}$. Let us note that vector $[\vec{n} \times \vec{E}]$ is parallel to vector $N_m = [\vec{n} \times \vec{n}_x]$ and $\vec{N}_m = -\vec{N}$ (see 36), vector $\vec{E}$ is parallel to $N_T = \vec{n}_x$, $\vec{n} = \frac{\vec{E}}{E}$ is the unit vector parallel to the direction of the particles momentum.

Lets compare equations (63) and (62):

$$\frac{d\vec{\xi}}{dt} = [\vec{\xi} \times \Omega_{magn}] + [\vec{\xi} \times \Omega_{EDM}], \quad (63)$$

According to 62 baryon spin rotates around three axes [3]: effective magnetic field direction $\vec{N}_m||[\vec{n} \times \vec{E}]$, electric field direction $\vec{N}_T||\vec{E}$ and momentum direction $\vec{n}$. Nonelastic processes in crystals result in the new effect: terms proportional to $\delta$ lead to appearance of the polarization vector component in directions of vectors $\vec{N}_m$, $\vec{N}_T$ and $\vec{n}$. Let’s pay attention to the fact that appearance of the spin component directed along effective magnetic field $B^*$ ($\vec{N}_m$ direction) is caused by both spin rotation around direction of the electric field $\vec{E}$ ($\vec{N}_T$ direction) due to T-noninvariant violation and spin dichroism due to nonelastic processes at interaction between the magnetic moment and the bent crystal. It can be seen that appearance of such spin component imitates the result of the T-noninvariant rotation (Figures 1, 2). Contributions to equation 62, which are caused by the interaction between baryon and nuclei, depend on distribution of nuclei density $N_{nuc}(x)$ (see terms proportional to $\alpha_{s0}(x), \delta_{s0}(x), \alpha_{Tnuc}(x), \delta_{Tnuc}(x)$). As a result, for positively charged particles, moving in the channel along the trajectories located in the center of the channel, such contributions are suppressed. Thus, according to 62, when one conducts and interprets experiments aimed for measuring EDM, one should consider the fact that measuring spin rotation provides information about the sum of contributions to T-noninvariant rotation. The mentioned rotation is determined by both EDM and short-range CP-noninvariant interactions. Nonelastic T-noninvariant processes lead to spin dichroism in direction of $\vec{N}_T$ as well, which gives additional opportunities for EDM measurement. Let’s evaluate the most important new effects described by the equation 62 and consider the contribution to spin rotation caused by spin dichroism in direction of $\vec{N}_m$. 

FIG. 1. Spin rotation caused by magnetic moment and T-reversal violation interactions (including EDM). Black arrows represent spin rotation about effective magnetic field (about bent axis, direction $\vec{N}_m$), red arrows represent spin component caused by EDM (direction $\vec{N}_T$), purple arrows represent the new effect - appearance of the spin component directed along $\vec{N}_m$ owing to the spin dichroism (spin rotation and dichroism in direction $\vec{N}_T$ owing to T-reversal violation and P-violating interactions, is not shown here for simplicity).
According to (33) coefficient \( \delta_m \) reads as follows:

\[
\delta_m = \frac{1}{4d_yd_zmc^2} \left( g - \frac{2}{2} \right) \frac{\partial}{\partial x} \frac{3V^2(x)}{\partial x} = \frac{1}{4d_yd_zmc^2} \left( g - \frac{2}{2} \right) \int \left\{ \left[ \int V_{coul}(x, y, z) dz \right]^2 - \left[ \int V_{nuc}(x, y, z) dz \right]^2 \right\} dy,
\]

(64)

where \( V_{coul}(x, y, z) = \sum_i V_e(x - x_i, y - y_i, z - z_i) - V_{nuc}(x - \eta_{fx}, y - \eta_{fy}, z - \eta_{fz}) \), \( x_i, y_i, z_i \) are the coordinates of the \( i \)-th electron in atom, \( \eta_{fx}, \eta_{fy}, \eta_{fz} \) are the coordinates of the atom nucleus. Let us choose the position of equilibrium point for the oscillating nucleus as the origin of coordinates. The overline denotes averaging of electrons’ and nuclei’ positions over electron density distribution and nuclei oscillations; in other words, averaging with wave-functions of atoms in crystal. By means of these functions, the density distribution can be expressed as follows:

\[
N(\vec{r}_1, \vec{r}_2, ..., \vec{r}_z, \vec{\eta}) = N_e(\vec{r}_1, \vec{r}_2, ..., \vec{r}_z, \vec{\eta}) N_{nuc}(\vec{\eta}),
\]

(65)

where \( N_e \) is the density distribution of electrons in atom, \( N_{nuc}(\vec{\eta}) \) is the density distribution of nucleus oscillations. Let’s introduce the function \( W(x, y) = \int V(x, y, z) dz \). From (64) we have:

\[
W(x, y) = \sum_i \int V_e(x - x_i, y - y_i, \xi) d\xi - \int V_{nuc}(x - \eta_x, y - \eta_y, \xi) d\xi = \sum_i W_e(x - x_i, y - y_i) - W_{nuc}(x - \eta_x, y - \eta_y)
\]

(66)

\[
W^2(x, y) = \left[ \sum_i W_e(\vec{\rho} - \vec{\rho}_i) - W_{nuc}(\vec{\rho} - \vec{\eta}_\perp) \right]^2 \times \\
N_e(\vec{\rho}_1 - \vec{\eta}_1, ..., \vec{\rho}_z - \vec{\eta}_\perp) N_{nuc}(\vec{\eta}_\perp) d^2\rho_1 d^2\rho_2 d^2\eta_\perp,
\]

(67)
where $\vec{\rho} = (x, y), \vec{\eta}_\perp = (\eta_x, \eta_y), Z$ is the number of electrons in atom. In other words:

$$W^2(\vec{\rho}) = \int \{ \sum_i W_e(\vec{\rho} - \vec{\rho}_i) \}^2 -$$

$$- 2 \sum_i W_e(\vec{\rho} - \vec{\rho}_i)W_{nuc}(\vec{\rho} - \vec{\eta}_\perp) +$$

$$+ W_{nuc}^2(\vec{\rho} - \vec{\eta}_\perp) \} N_e(\rho_1 - \eta_1, \ldots, \rho_z - \eta_1)$$

$$N_{nuc}(\vec{\eta}_\perp) d^2 \rho_1 d^2 \rho_2 d^2 \eta_1.$$  

(68)

The result of averaging $W^2(\vec{\rho})$ includes two contributions: that for density distribution of a single electron in atom and one dependent on coordinates of two electrons in the atom, which describes pair correlations in electrons positions in the atom. However, the influence of pair correlations will be ignored during the estimations. As a result the expression (68) can be represented as follows:

$$W^2(\vec{\rho}) = \int d^2 \eta_\perp \{ Z[(W_e^2(\vec{\rho}, \vec{\eta}_\perp))_{\vec{\rho}} -$$

$$- (W_e(\vec{\rho}, \eta_\perp))_{\vec{\rho}}^2] + Z^2 < W_e(\vec{\rho}, \vec{\eta}_\perp) > -$$

$$- 2Z < W_e(\vec{\rho}, \vec{\eta}_\perp) > + W_{nuc}^2(\vec{\rho} - \vec{\eta}_\perp) \} N_{nuc}(\vec{\eta}_\perp),$$

(69)

where the function

$$< W_e(\vec{\rho}, \vec{\eta}_\perp) >_e = \int W_e(\vec{\rho} - \vec{\rho}', \vec{\eta}_\perp) d\rho'$$

$$< W^2_e(\vec{\rho}, \vec{\eta}_\perp) >_e = \int W_e^2(\vec{\rho} - \vec{\rho}', \vec{\eta}_\perp) d\rho'',$$

that means

$$W^2(\vec{\rho}) = \int d^2 \eta_\perp \{ Z[< W_e^2(\vec{\rho}, \vec{\eta}_\perp) > - < W_e(\vec{\rho}, \eta_\perp) >^2] +$$

$$+(Z < W_e(\vec{\rho}, \vec{\eta}_\perp) >_e - W_{nuc}(\vec{\rho} - \vec{\eta}_\perp))^2 \} N_{nuc}(\vec{\eta}_\perp).$$

(70)

According to (64), the function $\int [W^2(\vec{\rho}) - \overline{W(\vec{\rho})}^2] dy$ determines the expression for $\delta_m$. It should be noted that when fluctuations caused by nuclei oscillations are neglected, only fluctuations, which are determined by distribution of electrons’ coordinates in the atom, are left. As a result, the following equation for $\delta_m$ can be obtained:

$$\delta_m = \frac{1}{4d_y d_z} mc^2 \left( \frac{g - 2}{2} \right) \frac{\partial}{\partial x} \int \left\{ W^2(x, y) - \overline{W(x, y)} \right\} dy,$$

(71)

where

$$\overline{W(\vec{\rho})} = \int \left\{ ZW_e(\vec{\rho}, \eta_\perp) \right\} N_{nuc}(\eta_\perp) d^2 \eta_\perp,$$

$$\overline{W_e(\vec{\rho}, \eta_\perp)} = \int W_e(\vec{\rho} - \vec{\rho}')N_e(\rho' - \eta_\perp) d^2 \rho'.$$

(72)

Let’s neglect nuclei oscillation to estimate the value $\delta_m$. In this case contribution to meansquare fluctuation of the energy of Coulomb interaction between baryon and atom is caused by fluctuations of positions of electrons in atom. As a result the expression for $\delta_m$ takes the following form:

$$\delta_m = \frac{1}{4d_y d_z} mc^2 \left( \frac{g - 2}{2} \right) \int$$

$$\times \frac{\partial}{\partial x} \int \left\{ z[< W_e^2(x, y) > - < W_e(x, y) >^2] \right\} dy$$

where function $W_e(x, y) = \int V(x - x', y - y', z) dz$. In expression (72) averaging of electrons distribution in atom (see explanations to the expression (69) ) is conducted over variables $x', y'$. To conduct estimations, for Coulomb energy
of interactions between baryon and electrons shielded Coulomb potential is used. Let's suppose that electrons are distributed uniformly in area about the same size as the radius of the shielding. In this case, the following estimations for \( \delta_m \) can be obtained: \( \delta_m \sim 10^8 \div 10^9 \text{sec}^{-1} \) depending on the position of the baryon trajectory in the planar channel. According to \([1, 2]\) estimated experimental sensitivity for EDM is \( cd \sim 10^{-17} \text{e cm} \). Spin rotation frequency \( \Omega_{EDM} = \frac{2 \pi dE}{m_R} \). The field \( E \) affecting baryons in bent crystal can be obtained from the expression \( E = \frac{m \gamma c^2}{e R} \), where \( R \) is the radius of crystal curvature. Therefore \( \Omega_{EDM} = 2 \frac{d}{\sqrt{3}} \frac{W}{m} \), where \( W \) is the energy of baryon. For \( R = 30 \text{m} \), \( d \sim 10^{-17} \text{cm} \) and \( W = 1 \text{TeV} \) we have \( \Omega_{EDM} \sim 10^7 \text{sec}^{-1} \). As a result, the nonelastic processes, which are caused by magnetic moment scattering, can imitate the EDM contribution. Surely, more detailed computer simulation is needed. The contributions of \( \sigma_{odd} \) and \( \sigma_{even} \) rotation effect to the general spin rotation can be evaluated by the following way. Precession frequency \( \Omega_w \) is determined by the real part of the amplitude of baryon weak scattering by an electron (nucleus). This amplitude can be evaluated by Fermi theory \([23, 24]\) for the energies, which are necessary for \( W \) and \( Z \) bosons production or smaller:

\[
ReB \sim G_F k = 10^{-5} \left( \frac{1}{m_p} \right)^2 k = 10^{-5} \frac{\hbar}{m_p c} \frac{m}{m_p \gamma} = 10^{-5} \lambda_{cp} \frac{m}{m_p \gamma},
\]

where \( G_F \) is the Fermi constant, \( m_p \) is the proton mass, \( \lambda_{cp} \) is the proton Compton wavelength. For particles with energy from hundreds of GeV to TeV \( ReB \sim G_F k = 10^{-10} \text{cm} \). For different particle trajectories in a bent crystal the value of precession frequency \( \Omega_w \) could vary in the range \( \Omega_w \sim 10^4 \div 10^4 \text{sec}^{-1} \). Therefore, when a particle passes 10 cm in a crystal, its spin undergoes additional rotation around momentum direction at angle \( \theta_p \sim 10^{-6} \div 10^{-7} \text{rad} \). The effect grows for a heavy baryon as a result of the mechanism similar to that of its EDM growth (see the explanation for the growth of constant \( \rho \)). A consequence of this absorption is that either the root-mean-square scattering angle appears changed and depends on spin orientation with respect to vectors \( \vec{N}_m \) and \( \vec{N}_T \). Particularly, due to interference of magnetic, weak and coulomb interactions, the imaginary part of weak scattering amplitude and is proportional to the difference of total scattering cross-sections by the following way (see Fig.3).

\[
\delta \ll \sqrt{\frac{G_F}{m}} \chi \frac{e}{m_p c} \frac{m}{m_p \gamma},
\]

When baryon trajectory passes in the area, where collisions with nuclei are important (this occurs in the vicinity of potential barrier for positively charged particles), the value \( \Omega_w \sim 10^8 \div 10^8 \text{sec}^{-1} \). Similar to the real part \( ReB \) for the case of heavy baryons the difference in cross-sections grows. Multiple scattering also contributes to spin rotation and depolarization \([3, 7, 8, 15]\). Particularly, due to interference of magnetic, weak and coulomb interactions, the root-mean-square scattering angle appears changed and depends on spin orientation with respect to vectors \( \vec{N}_m \), \( \vec{N}_T \) and \( \vec{n} \). When measuring MDM and \( T_{odd} \) spin rotation in a bent crystal, one can eliminate parity violating rotation by the following way (see Fig.3).

By turning the crystal 180° around the direction of incident baryon momentum one could observe that \( \sigma_{odd} \) spin rotation does not change, while the sign of MDM and \( T_{odd} \) spin rotations does due to change of the electric field direction. Subtracting results of measurements for two opposite crystal positions one could obtain the angle of rotation, which does not depend on \( P_{odd} \) effect. Such measurement also can be made using the idea presented in \([2]\), according to which to control systematic uncertainties two crystals (up and down bending) should be used to induce opposite spin precession to channelled baryons. Separation of the contributions caused by MDM and \( T_{odd} \) spin rotation is possible when comparing experimental results for two initial orientations of polarization vector \( \vec{\xi} \). Namely: \( \vec{\xi} \parallel \vec{N}_m \) and \( \vec{\xi} \parallel \vec{N}_T \), i.e. the initial \( \vec{\xi} \) is parallel to the bending axis of the crystal or \( \vec{E} \) (see Fig.4).

VII. CONCLUSION

Besides electromagnetic interaction the channelled particle, which moves in a crystal, experiences weak interaction with electrons and nuclei, as well as strong interaction with nuclei. Measurements of polarization vector and angular distribution of particles scattered by axes (planes) of unbent crystal enable to obtain limits for the EDM value and for values of constants describing P- and T-odd interactions. The same measurements also allow studying magnetic dipole.
FIG. 3. By turning the crystal 180° around the direction of incident baryon momentum one could observe that $P_{\text{odd}}$ spin rotation does not change, while the sign of MDM and $T_{\text{odd}}$ spin rotations does due to change of the electric field direction. Subtracting results of measurements for two opposite crystal positions one could obtain the angle of rotation, which does not depend on $P_{\text{odd}}$ effect.

FIG. 4. Separation of the contributions caused by MDM and $T_{\text{odd}}$ spin rotation is possible when comparing experimental results for two initial orientations of polarization vector $\xi$. Namely $\xi \parallel N_m$ and $\xi \parallel \tilde{N}_t$, i.e. the initial $\xi$ is parallel to the bending axis of the crystal or $\tilde{E}$. In real situation rotating the crystal by 90° so that direction of $S_0$ is parallel to $B^*$ can be more convenient.
moment of charged and neutral particles. Investigation of left-right asymmetry by the use of two unbent crystals makes it possible to measure EDM, MDM and other constants without studying the angular distribution of decay products of scattered particles: it is sufficient to measure the angular distribution of intensity of flow of particles experienced double scattering. When analyzing particle’s spin rotation, which is caused by electric dipole moment interaction with electric field, one should consider non-invariant spin rotations both \( P_{\text{odd}}, T_{\text{even}} \) and \( P_{\text{odd}}, T_{\text{odd}} \), resulting from weak interaction with electrons and nuclei. As demonstrated hereinafore, spin precession of channelled particles in bent crystals at the LHC gives unique possibility for measurement of constants determining \( T_{\text{odd}}, P_{\text{odd}} \) (CP) violating interactions and \( P_{\text{odd}}, T_{\text{even}} \) interactions of baryons with electrons and nucleus (nucleons), similarly to the possibility of measuring electric and magnetic moments of charm, beauty and strange charged baryons. For a particle moving in a bent crystal a new effect, which is caused by nonelastic processes, arises: in addition to the spin precession around three directions \( \vec{N}_m, \vec{N}_T, \vec{n} \) the spin dichroism effect causes the appearance of the spin components in directions of \( \vec{N}_m, \vec{N}_T, \vec{n} \). To separate P-noninvariant rotation from the MDM- and EDM-induced \( (T_{\text{odd}}) \) spin rotations both the method of turning crystal by \( 180^\circ \) and the method of using two crystals suggested in [2] can be used. To separate contributions caused by MDM and \( T_{\text{odd}} \) interactions, two bent crystals placed perpendicular to each other can be used.

VIII. APPENDICES

A. Scattering amplitude

As follows from (14):

\[
F_j(0) = f_j(0) - i \frac{k}{4\pi} \int f_j^*(\vec{k}' - \vec{k}) f_j(\vec{k}' - \vec{k}) d\Omega_{k''}. \tag{76}
\]

The integral in (76) is equal to the total cross-section of the elastic coherent scattering by a nucleus (atom). According to the optical theorem:

\[
\text{Im} f_j(0) = \frac{k}{4\pi} \sigma_{\text{tot}} = \frac{k}{4\pi} \sigma_{\text{elast}} + \frac{k}{4\pi} \sigma_{\text{nonelast}}. \tag{77}
\]

In contrast to the matter with chaotically distributed scatterers the amplitude \( F_j(0) \) in crystal is expressed as follows:

\[
F_j(0) = \bar{f}_j(0), \bar{f}_j(0) = f_j(0) - \frac{k}{4\pi} \sigma_{\text{elast}}. \tag{78}
\]

In other words, the cross-section of elastic coherent scattering in crystal does not contribute to the imaginary part of the amplitude \( F_j(0) \). This imaginary part is solely determined by the cross-section of nonelastic processes (cross-section of reactions):

\[
F_j(0) = \text{Re} F_j(0) + i \text{Im} F_j(0) = \text{Re} F_j(0) + i \frac{k}{4\pi} \sigma_{\text{nonelast}}. \tag{79}
\]

The nonzero-angle scattering possesses the similar features. This fact becomes clear when one uses the equality, which is correct for elastic scattering [5]:

\[
\text{Im} f_{\text{elast}}(\vec{k}' - \vec{k}) = \frac{k}{4\pi} \int f_{\text{elast}}^*(\vec{k}' - \vec{k}) f_{\text{elast}}(\vec{k}' - \vec{k}) d\Omega_{k''}. \tag{80}
\]

and subtracts the elastic scattering contribution from the imaginary part of \( f(\vec{k}' - \vec{k}) \) using (14).

The same result can be obtained by considering the interaction with the scatterer in terms of the perturbation theory. In case when the first Born approximation is used, scattering amplitude \( f^{(1)}(\vec{k}' - \vec{k}) \) has zero imaginary part:

\[
\text{Im} f^{(1)}_{aa}(\vec{k}' - \vec{k}) = 0. \tag{81}
\]

The non-zero imaginary part arises, when one uses the second order Born approximation.

\[^{1}\text{Let us remind that T-operator, which determines the scattering amplitude (see [5]), satisfies the following equation}\]

\[^{18, 19}\]
\[ T = V + V \frac{1}{E - H_0 + i\eta} T. \]  

(82)

where \( V \) is the interaction energy, \( H_0 \) is the Hamilton operator of colliding systems located at large distance from each other.

As a result for the elastic coherent scattering amplitude \( f_{aa} \) with the accuracy up to the second order terms over the interaction energy, one gets:

\[ f_{aa}(\vec{k}' - \vec{k}) = -(2\pi)^2 \frac{m\gamma}{\hbar^2} (< \Phi_{k'}a|V|\Phi_{ka}> + < \Phi_{k'a}|V|\Phi_{ka}>), \]

(83)

where \( \Phi_{ka} \) is an eigenfunction of Hamilton operator \( H_0 \),

\[ \Phi_{ka} = \frac{1}{(2\pi)^{3/2}} e^{i\vec{k}\vec{r}} \Phi_a, \]

(84)

\( \Phi_a \) is the wave function of scatterer stationary states and \( H_0 \Phi_{ka} = E_a(\vec{k})\Phi_{ka}. \)

Using the completeness of function \( \Phi_{k'a} \) and replacing "1" in \( (83) \) by \( \sum_{\vec{k}''} |\Phi_{k''a}><\Phi_{k''a}| = 1 \) one obtains the sum over the intermediate states \( b \), which includes states with \( b = a \). This term contains the following expression:

\[ \frac{1}{E_a(\vec{k}) - E_a(\vec{k}'')} + i\eta = \frac{1}{E_a(\vec{k}) - E_a(\vec{k}'')} - i\pi \delta(E_a(\vec{k}) - E_a(\vec{k}'')). \]

(85)

The "\( P'\)" symbol in the real part in \( (83) \) means, that all integrals containing the "\( P'\)" symbol in \( (83) \) are the principal-value integrals.

This real part contribution is small as compared to the first Born approximation, therefore, it will not be further considered. The imaginary unit in the second term of \( (85) \), which is proportional to the \( \delta \) function, leads to occurrence of an imaginary part in amplitude \( f(\vec{k}' - \vec{k}). \)

Substitution of the expression with \( \delta \)-function into \( (83) \) makes it obvious that the term in the sum, in which \( b = a \), is equivalent to the term subtracted from the amplitude \( f_{aa}(\vec{k}' - \vec{k}) \) in \( (14) \).

As a result only terms caused by nonelastic processes and reactions with \( b \neq a \) make contributions to the imaginary part of the amplitude in crystal. To simplify consideration below, when expressing \( F \), the contribution from the elastic coherent scattering to the imaginary part of the amplitude \( f_{aa} \) will not be considered and the second term in \( (14) \) will not be also written explicitly.

**B. Effective potential energy of a spin-particle moving close to the crystal planes (axes)**

Let’s consider the expression for effective potential energy in detail. According to \( (15,20,21) \) the contributions to the effective potential energy are caused by interactions of different types including short-range and long-range interactions. In the presence of several types of interaction, to describe their different contributions to the scattering amplitude, it is convenient to separate scattering caused only by long-range interactions and present amplitude in following form:

\[ f(\vec{q}) = f_{long}(\vec{q}) + f_{shortlong}(\vec{q}), \]

(86)

where \( f_{long}(\vec{q}) \) is a scattering amplitude determined by long-range coulomb and magnetic interactions (assuming that short-range interactions are absent), \( f_{shortlong}(\vec{q}) \) is a scattering amplitude determined by short-range interactions (calculating this amplitude waves scattered by long-range interactions were used as an incident waves). For general scattering theory in the presence of several interactions see, for example, \( (18,19) \).

When several types of interactions influence on the scattering amplitude, it can be easily studied with the help of perturbation theory. Let interaction energy \( V \) be a sum of several interactions: \( V = \sum_i V_i \). Then at the first Born approximation scattering amplitude is a sum of scattering amplitudes caused by every interaction separately: \( f = \sum_i f_i(V_i) \). But in the second Born approximation additional term \( f_2 \), which is determined by the following expression, appears in the scattering amplitude (see \( (3,18,19) \))

\[ f_2 = V \frac{1}{E - H_0 - i\eta} V = \sum_p V_p \frac{1}{E - H_0 - i\eta} \sum_i V_i, \]

(87)
As one can see, equality \([87]\) contains interference contributions to \(f\) proportional to \(V_pV_j\).

Let us now consider how different terms, which amplitude \([20]\) includes, contribute to the effective potential energy of particle interaction with the crystal.

The Coulomb amplitude, described by the first term in \([20]\), leads to conventional expression for potential energy of interaction between a charged particle and a plane (axis).

The second term \(A_s(q)\) is caused by the short-range interaction. Amplitude \(A_s(q)\) can be written as:

\[
A_s(q) = A_{nuc}(q)\Phi_{osc}(q),
\]

where \(A_{nuc}(q)\) is the spin independent part of the amplitude of elastic scattering by the resting nucleus, \(\Phi_{osc}(q)\) is the form-factor caused by nucleus oscillations in crystal.

Owing to the short-range kind of strong interactions amplitude, \(A_{nuc}(q)\) is equal to zero-angle scattering amplitude \(A(0)\) within the range of scattering angles \(\theta \leq \frac{1}{kR_{osc}} \ll 1\).

Form-factor \(\Phi_{osc}(q)\) has the form \([2]\):

\[
\Phi_{osc}(q) = \sum_n \rho_n <\varphi_n(r)|e^{-i\vec{q}\cdot \vec{r}}|\varphi_n(r)> = \int e^{-i\vec{q}\cdot \vec{r}}N_{nuc}(\vec{r})d^3r,
\]

where \(\varphi_n(r)\) is the wave function describing vibrational state of nuclei in crystal, summation \(\sum_n \rho_n\) means statistical averaging with Gibbs distribution over vibrational states of nucleus in crystal. Let’s remind, that squared form-factor \(\Phi_{osc}(q)\) is equal to Debye-Waller factor, \(N_{nuc}(\vec{r})\) is a probability density of vibrating nuclei detection in point \(\vec{r}\), \(\int N_{nuc}(\vec{r})d^3r = 1\). As a result, equations \([31], [32]\) can be obtained.

### C. Effective potential energy determined by the anomalous magnetic moment

According to \([29]\) the scattering amplitude, which is determined by baryon’s anomalous magnetic moment, has the form:

\[
\hat{F}_{magn}(q) = B_{magn}(q)\vec{\sigma}[\vec{n} \times \vec{q}].
\]

Defining the scattering amplitude at the first step one could solely consider magnetic scattering and its interference with Coulomb scattering (see \([85]\)), and at the second step add the term caused by interference between magnetic and nuclear interactions.

For the first step perturbation theory can be used. In the first order of perturbation theory the interference of the magnetic moment scattering by the Coulomb field with the Coulomb scattering of baryon electric charge by the Coulomb field is absent. The amplitude \(\hat{F}^{(1)}_{magn}(q)\) reads as:

\[
\hat{F}^{(1)}_{magn}(q) = if_{coul}(q)\frac{\hbar}{mc}\left(\frac{g-2}{2}\right)\frac{1}{2}\vec{\sigma}[\vec{n} \times \vec{q}],
\]

where \(f_{coul}(q)\) is the amplitude of coulomb scattering of a baryon by an atom in the first Born approximation; \(\vec{n} = \frac{\vec{E}}{E}\), \(m\) is the baryon mass.

It should be noted that the coefficient, by which \(\vec{\sigma}\) is multiplied, in the expression for amplitude \(\hat{F}^{(1)}_{magn}(q)\) is purely imaginary. After substitution of \([91]\) into \([20]\) and summation over \(\tau_x\) one obtains the expression for effective interaction energy as follows:

\[
\hat{U}_{magn}(x) = -\frac{e\hbar}{2mc}\left(\frac{g-2}{2}\right)\vec{\sigma}[\vec{E}_{plane}(x) \times \vec{n}],
\]

where \(\vec{E}_{plane}(x)\) denotes the electric field, produced by the crystallographic plane in point \(x\). In axis case \(U_{magn}(\vec{p})\) can be obtained by replacement of \(x\) by \(\vec{p}\) in \([92]\) and \(\vec{E}_{plane}(x)\) by \(\vec{E}_{axis}(\vec{p})\), respectively.

Using \([92]\) and Heisenberg equations for spin operator, the motion equation for polarization vector \([1,2]\) for the case of \(B = 0\) and \(\gamma \gg 1\) can be obtained.

Effective interaction energy \([92]\) can be rewritten as follows:

\[
\hat{U}_{magn} = -\frac{e\hbar}{2mc}\left(\frac{g-2}{2}\right)\vec{E}_{plane}(x)\hat{\sigma}\vec{N},
\]

\([93]\)
where $\vec{N} = [\vec{n}_x \times \vec{n}]$ is the unit vector, $\vec{n}_x \perp \vec{n}$, unit vector $\vec{n}$ is parallel to the crystallographic plane.

Expression (93) for the effective potential energy comprises factor, which is purely real. However, the coefficient in the expression for scattering amplitude $\tilde{F}(\vec{q})$, by which $\vec{q}$ is multiplied, has non-zero both real and imaginary parts. Due to this fact, the effective potential energy $\tilde{U}$ also has non-zero both real and imaginary parts.

In the second order of perturbation theory this coefficient in amplitude $\tilde{F}(\vec{q})$ is not purely imaginary as well – it has a non-zero real part. By means of (14), (83)-(85) the following expression for the contribution $\tilde{F}(2)(q)$ to the amplitude $\tilde{F}(\vec{q})$ can be obtained:

$$\tilde{F}^{(2)}(q = \vec{r}) = i \frac{k}{4\pi \hbar^2} \times$$

$$\times \left\{ \Phi_d \int \int e^{-i\vec{r} \cdot \vec{r}} \left[ \int \tilde{V}(\vec{r}, z) dz \right]^2 d\vec{r} | \Phi_a > - \right.$$  

$$- \int \int e^{-i\vec{r} \cdot \vec{r}} \left[ \int < \Phi_d | \tilde{V}(\vec{r}, z) | \Phi_a > d\vec{r}^2 \right] =$$

$$= i \frac{k}{4\pi \hbar^2} \int \int e^{-i\vec{r} \cdot \vec{r}} \left[ \int \tilde{V}(\vec{r}, z) dz \right]^2 | \Phi_a > -$$

$$- \left[ \int < \Phi_d | \tilde{V}(\vec{r}, z) | \Phi_a > d\vec{r}^2 \right] =$$

$$= \left\{ \int \tilde{V}(\vec{r}, z) dz \right\}^2 d\vec{r}^2,$$

where $\tilde{V}(\vec{r}, z) = \tilde{V}_{coul}(\vec{r}, z) + \tilde{V}_{mag}(\vec{r}, z)$, $\tilde{V}_{mag}(\vec{r}, z) = -\mu_a \tilde{E}(\vec{r}, z) \times \vec{n}$, $z$ axis of the coordinate system is directed along the unit vector $\vec{n}$, $\vec{n}$ is the unit vector directed along the particle momentum before scattering $\hbar \vec{k}$, $\mu_a$ is the anomalous magnetic moment of the particle $\mu_a = \frac{e\hbar}{2mc} (\frac{g}{2} - 2)$.

When deriving (94), it was considered that the particle energy is much greater than the electrons’ binding energy in atoms and the atoms’ binding energy in crystal. As a result it is possible at first to examine scattering by electrons and nuclei, which rest in points $r_i$, and then to average the result over the electrons and nuclei positions with wave functions $|\Phi_a >$ (impulse approximation, for example see [18]). The overline in (94) and hereinafter denotes such kind of averaging. The contribution caused by interference between magnetic and nuclear scattering, and the contributions determined by the particle squared magnetic moment should complete the expression mentioned above. For positively charged particles, moving far from the top of the potential barrier, the contribution caused by interactions with nuclei is suppressed and will be omitted in consideration hereinafter. Contributions proportional to the particle squared magnetic moment are smaller then those caused by interference between magnetic and Coulomb scattering and will, thus, be also omitted. After substitution of (94) into (20) and summation over $\tau_x$ the following expression for the contribution to the effective potential energy caused by the amplitude $\tilde{F}_{mag}(\vec{r})$ can be obtained:

$$\tilde{U}_{mag}^{(2)}(x) = -i \frac{1}{4d_\rho d_x m \epsilon^2} \left( \frac{g}{2} - 2 \right) \frac{\partial}{\partial x} \delta V^2(x) \vec{r} \cdot \vec{N},$$

where $\vec{N} = [\vec{n}_x \times \vec{n}]$, $\vec{n}_x \perp \vec{n}$, $\vec{n}_x$ is the unit vector along axis $x$,

$$\overline{\delta V^2(x)} = \int \left\{ \left[ \int V_{coul}(x, y, z) dz \right]^2 - \left[ \int V_{coul}(x, y, z) dz \right]^2 \right\} dy$$

Similarly for the case of axis it can be obtained:

$$\tilde{U}_{mag}^{(2)}(\vec{r}) = -i \frac{1}{d_\rho m \epsilon^2} \left( \frac{g}{2} - 2 \right) \vec{r} \cdot \nabla \overline{\delta V^2(\vec{r}) \times \vec{n}}.$$
For the axisymmetric case:

$$
\hat{U}^{(2)}_{\text{magn}}(\rho) = -i \frac{1}{4d_{\text{2mc}}^2} \left( \frac{g - 2}{2} \right) \frac{\partial}{\partial \rho} \delta V^2(\rho) \hat{\rho} \times \hat{\rho},
$$

(97)

where $\delta V^2(\rho) = \left[ \int V_{\text{coul}}(\rho, z) dz \right]^2 - \left[ \int V_{\text{coul}}(\rho, z) dz \right]^2$, $\hat{\rho} = \frac{\vec{r}}{r}$ is the unit vector, $\hat{n}_\rho \perp \hat{n}$.

In the planar channeling case $\hat{U}_n$ is determined by the expression (83).

### D. Effective potential energy $\hat{U}$ determined by spin-orbit interaction

According to (29) the part of the scattering amplitude caused by strong spin-orbit interaction has the form:

$$
\hat{F}_{\text{ssp-orb}}(\vec{q}) = B_s(\vec{q}) \vec{\sigma} \hat{N} \times \hat{\rho},
$$

(98)

The coefficient $B_s(\vec{q})$ can be expressed similar to (88) as follows:

$$
B_s(\vec{q}) = B_{nuc}(\vec{q}) \Phi_{\text{osc}}(\vec{q}),
$$

(99)

where $B_{nuc}(\vec{q})$ describes scattering by a resting nucleus, $\Phi_{\text{osc}}(\vec{q})$ is the form-factor determined by nucleus oscillations in crystal.

In the considered case, similar to the approach used when deriving (31), the short-range character of the nuclear forces and small (as compared to the amplitude of nucleus oscillations) nucleus radius enables assumption $B_{nuc}(\vec{q}) = B_{nuc}(0)$. It is important that the coefficient $B_{nuc}(0)$ has non-zero both real and imaginary parts:

$$
B_{nuc}(0) = B'_{nuc} + iB''_{nuc}.
$$

(100)

This is similar to the case of amplitude, which describes scattering of the magnetic moment by the atom (nucleus). To obtain the expression for the effective potential energy the summation over $\tau_z$ should be conducted in (20). The resulted expression is similar to that for $\hat{U}_{\text{magn}}$. For example, for the crystal plane case see expression (36) for $\hat{U}_{\text{ssp-orb}}$. Let us remind that the contribution determined by elastic scattering, which is described by the second term in (76), is negligibly small in comparison with nonelastic contributions to the amplitude and, therefore, can be omitted.

### E. Effective potential energy $\hat{U}$ determined by $P_{\text{odd}}$ and $T_{\text{even}}$ interactions

The next group of terms, which are proportional to $B_{\text{en}}$, is determined by weak $P_{\text{odd}}$ and $T_{\text{even}}$ interactions. According to (29) the corresponding terms in the scattering amplitude can be written as:

$$
\hat{F}_w(\vec{q}) = (B_{\text{we}}(\vec{q}) + B_{\text{wnuc}}(\vec{q})) \vec{\sigma} \hat{N}_w.
$$

(101)

Contribution $B_{\text{we}}(\vec{q})$ caused by parity violating weak interaction between the baryon and electrons can be expressed as follows:

$$
B_{\text{we}}(\vec{q}) = \tilde{B}_{\text{we}}(\vec{q}) \Phi_{\text{e}}(\vec{q}),
$$

(102)

where $\tilde{B}_{\text{we}}$ is the coefficient defining baryon elastic scattering amplitude by resting electron $\hat{f}_{\text{we}}(\vec{q}) = \tilde{B}_{\text{we}} \vec{\sigma} \hat{N}_w$, $\Phi_{\text{e}}(\vec{q}) = \int e^{-i \vec{q} \cdot \vec{r}} F_{\text{e}}(\vec{r}) d^3r$, $\int F_{\text{e}}(\vec{r}) d^3r = Z$, $Z$ is the nucleus charge. Minor corrections caused by thermal oscillations of atoms’ centers of gravity will not be considered below. To take them into consideration one should multiply $\Phi_{\text{e}}(\vec{q})$ by $\Phi_{\text{osc}}(\vec{q})$, which is the form-factor defined by oscillations of atoms nucleus.

Term $B_{\text{wnuc}}(\vec{q})$ (see (101)), which is caused by parity violating weak interaction between the baryon and nucleus, reads as follows:

$$
B_{\text{wnuc}}(\vec{q}) = \tilde{B}_{\text{wnuc}}(\vec{q}) \Phi_{\text{nuc}}(\vec{q}),
$$

(103)

where $\tilde{B}_{\text{wnuc}}$ is the coefficient defining the amplitude of a baryon elastic scattering by a resting nucleus $\hat{f}_{\text{wnuc}} = \tilde{B}_{\text{wnuc}} \vec{\sigma} \hat{N}_w$.

Due to the short-range character of P-violating interactions, when angle $\theta \simeq \frac{\pi}{2} \ll 1$, coefficients $\tilde{B}_{\text{we}}(\vec{q}) \simeq \tilde{B}_{\text{we}}(0)$ and $\tilde{B}_{\text{wnuc}}(\vec{q}) \simeq \tilde{B}_{\text{wnuc}}(0)$. As a result expressions for the effective potential energy $\hat{U}_w$ of P-violating interaction of a baryon with a crystal plane (axis) were obtained (see (39), (41)).
F. Effective potential energy $\hat{U}$ determined by the electric dipole moment and other T-nonvariant interactions

Let us consider now the electric dipole moment and other T-nonvariant contributions to the spin rotation. According to (20) the corresponding terms in the scattering amplitude can be written as:

$$\hat{F}(q) = (B_{\text{EDM}}(q) + B_{Te}(q) + B_{T \text{nuc}}(q))\sigma \tilde{q}.$$  

(104)

Let’s consider the term $\hat{F}_{\text{EDM}}(q) = B_{\text{EDM}}(q)\sigma \tilde{q}$. The coefficient $B_{\text{EDM}}(q)$ has non-zero both real and imaginary parts $B_{\text{EDM}}(q) = B'_{\text{EDM}} + iB''_{\text{EDM}}$. By the approach used for deriving $F_{\text{magn}}(q)$, for $\hat{F}_{\text{EDM}}(q)$ one can obtain:

$$\hat{F}_{\text{EDM}}(q) = -i\frac{m\gamma d}{2\pi\hbar^2}V_{\text{coul}}(q)\sigma \tilde{q} +$$

$$+ \frac{k}{4\pi\hbar^2\epsilon^2} \times$$

$$\times \left[ \int e^{-i\tilde{q} \cdot \vec{r}} \left( \left[ \int V(\vec{r}_\perp, z)dz \right]^2 - \left[ \int \tilde{V}(\vec{r}_\perp, z)dz \right]^2 \right) d^2r_\perp, \right]$$

where $\tilde{V}(\vec{r}) = V_{\text{coul}}(\vec{r}) + V_{\text{EDM}}(\vec{r})$, $V_{\text{coul}}(\vec{r})$, $V_{\text{EDM}} = -D\sigma \tilde{E}$ is the energy of interaction between the electric dipole moment $D$ and the electric field $\tilde{E}$, $D = ed$, $e$ is the electric charge of the particle. Using (20), expression (43) for the potential energy of interaction between the particle and the crystal plane can be obtained. Let’s remind that amplitude $\hat{F}(q)$ contains terms both caused by EDM and determined by short-range T-noninvariant interactions between the baryon and electrons and nuclei $B_{Te}(q)$ and $B_{T \text{nuc}}(q)$. Contributions caused by these terms should also be added to the effective potential energy of the interaction between the baryon and nuclei of the crystal $\hat{U}(x)$:

$$\hat{U}(x) = \hat{U}_{\text{EDM}} + \hat{U}_{Te} + \hat{U}_{T \text{nuc}} = -(\alpha_T(x) + i\delta_T(x))\sigma \tilde{N}_T,$$

(106)

where $\alpha_T = \alpha_{\text{EDM}} + \alpha_{Te} + \alpha_{T \text{nuc}}$, $\delta_T = \delta_{\text{EDM}} + \delta_{Te} + \delta_{T \text{nuc}}$.

Expressions for coefficients $\alpha_{Te(nuc)}$ and $\delta_{Te(nuc)}$ can be evaluated in terms of scattering amplitude by the following way. Let’s define the form-factor determined by electrons distribution in atom and nucleus oscillations:

$$B_{Te}(q) = \tilde{B}_{Te}(q)\Phi_e(q), B_{T \text{nuc}}(q) = \tilde{B}_{T \text{nuc}}(q)\Phi_{osc}(q),$$

(107)

where $\Phi_e(q) = \int e^{-i\tilde{q} \cdot \vec{r}} N_e(\vec{r})d^4r$, $N_e(\vec{r})$ is electrons distribution density in atom, $\int N_e(\vec{r})d^4r = Z$, $Z$ is the nucleus charge, $\Phi_{osc}(q)$ is determined by (103), $\tilde{B}_{Te}$ is the coefficient defining amplitude of baryon scattering by resting electron $\tilde{f}_{Te} = \tilde{B}_{Te}(q)\sigma \tilde{q}$, $\tilde{B}_{T \text{nuc}}(q)$ is the coefficient defining amplitude of baryon scattering by resting nucleus $\tilde{f}_{T \text{nuc}} = \tilde{B}_{T \text{nuc}}(q)\sigma \tilde{q}$. Let’s remind that in compliance with (13) the contribution caused by elastic coherent scattering should be subtracted from the amplitude $\tilde{B}_{Te}$. However, at high energies this contribution is negligibly small in comparison with nonelastic contributions to the amplitude and, therefore, can be omitted.

Due to the short-range character of T-noninvariant interactions at angle $\theta \approx \frac{\pi}{2} \ll 1$ coefficients $\tilde{B}_{Te}(q) \approx \tilde{B}_{Te}(0)$ and $\tilde{B}_{T \text{nuc}}(q) \approx \tilde{B}_{T \text{nuc}}(0)$.

Due to the short-range character of T-noninvariant interactions at angle $\theta \approx \frac{\pi}{2} \ll 1$ coefficients $\tilde{B}_{Te}(q) \approx \tilde{B}_{Te}(0)$ and $\tilde{B}_{T \text{nuc}}(q) \approx \tilde{B}_{T \text{nuc}}(0)$. As a result, the following expressions can be obtained:

$$\hat{U}_{Te}(x) = i\frac{2\pi\hbar^2}{m\gamma d_y d_z} \tilde{B}_{Te}(0) \frac{dN_e(x)}{dx} \sigma \tilde{N}_T,$$

$$\hat{U}_{T \text{nuc}}(x) = i\frac{2\pi\hbar^2}{m\gamma d_y d_z} \tilde{B}_{T \text{nuc}}(0) \frac{dN_{T \text{nuc}}(x)}{dx} \sigma \tilde{N}_T,$$

$$N_{e(nuc)}(x) = \int N_{e(nuc)}(x, y, z)dydz,$$

(108)

Coefficients $\tilde{B}_{Te}(0)$ and $\tilde{B}_{T \text{nuc}}(0)$ are complex values:

$$\tilde{B}_{Te(nuc)}(0) = \tilde{B}'_{Te(nuc)} + i\tilde{B}''_{Te(nuc)}.$$

As a result expression (43) can be obtained.
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[1] Bagli E., Bandiera L., Cavoto G., et all, Eur. Phys J.C. 77 (828) (2017) 1–19.
[2] Botella F. J., Garcia Martin L. M., Marangotto D., et all, Eur. Phys J.C. 77 (181) (2017).
[3] Baryshevsky V. G., [arXiv:1708.09799 [hep-ph]], (2017).
[4] Baryshevsky V. G., [arXiv:1803.05770v1], (2018).
[5] Berestetskii, V. B., Lifshitz, E. M., Pitaevskii, L. P. Quantum Electrodynamics, in Landau, L. D. and Lifshitz, E. M. Course of Theoretical Physics, Vol. 4., 2nd edn., (Butterworth-Heinemann 1982).
[6] Baryshevsky V. G., Pis’ma Zh. Tekh. Fiz. 5 (3) (1979) 182–184.
[7] Baryshevsky V. G., High-Energy Nuclear Optics of Polarized Particles, World Scientific Publishing Company, 2012, 640 p.
[8] Baryshevsky V. G., Nucl. Instr. Methods B, 402 (2017) 5–10.
[9] Baryshevsky V. G., Phys. Lett. B, 757 (2016) 426–429.
[10] Baublis V. V., et al., LNPI Research Report, 1990–1991 E 761 Collaboration St. Petersburg, (1992) 24–26.
[11] Chen D., Albuquerque I. F. and Baublis V. V., et al., Phys. Rev. Lett. 69 (23) (1992) 3286–3289.
[12] Khanzadeev A. V., Samsonov V. M., Carrigan R. A., Chen D., Nucl. Instr. Methods B, 119 (1-2) (1996) 266–270.
[13] Baryshevsky V. G., [arXiv:1504.06702], (2015).
[14] Bezshyko O. A., Burmistrov L., Fomin A. S., et all, JHEP, 8 (107) (2017).
[15] Baryshevsky V. G., Bartkevich A. R., J. Phys. G, 39 (2012).
[16] Ford G. W., Hirt C. W, Report unpublished, Michigan Univ., 1961.
[17] Mane S. R., Shatunov Yu. M., Yokoya K., Rep. Prog. Phys. 68 (9) (2005) 1997–2266.
[18] Goldberger M. L., Watson R. M. Collision Theory, Wiley, New York, 1984.
[19] Davydov, A. S. Quantum Mechanics, BHV-Petersburg, 2010.
[20] Baryshevsky V. G., [arXiv:1608.06815], (2016).
[21] Gudkov V. P., Xiao-Gang He, McKeller B. H. J., Phys. Rev. C, 47 (5) (1993) 2365–2368.
[22] Born, M. and Wolf, E. (1965). Principles of optics: Electromagnetic Theory of Propagation, Interference and Diffraction of Light (Pergamon Press).
[23] Commins, E. D. and Bucksbaum, P. H. . Weak Interactions of Leptons and Quarks, Cambridge University Press, 1983.
[24] Leader E., Predazzi E., An introduction to Gauge Theories and the "New Physics", Cambridge University press, 1982.