Interferometry signatures for QCD first-order phase transition in heavy ion collisions at GSI-FAIR energies

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Abstract

Using the technique of quantum transport of the interfering pair we examine the Hanbury-Brown-Twiss (HBT) interferometry signatures for the particle-emitting sources of pions and kaons produced in the heavy ion collisions at GSI-FAIR energies. The evolution of the sources is described by relativistic hydrodynamics with the system equation of state of the first-order phase transition from quark-gluon plasma (QGP) to hadronic matter. We use quantum probability amplitudes in a path-integral formalism to calculate the two-particle correlation functions, where the effects of particle decay and multiple scattering are taken into consideration. We find that the HBT radii of kaons are smaller than those of pions for the same initial conditions. Both the HBT radii of pions and kaons increase with the system initial energy density. The HBT lifetimes of the pion and kaon sources are sensitive to the initial energy density. They are significantly prolonged when the initial energy density is tuned to the phase boundary between the QGP and mixed phase. This prolongations of the HBT lifetimes of pions and kaons may likely be observed in the heavy ion collisions with an incident energy in the GSI-FAIR energy range.

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I. INTRODUCTION

One of the central goals of high energy heavy ion collisions is to find and quantify the QCD phase transition from hadronic matter to quark-gluon plasma (QGP). Under the assumption of first-order phase transition there is a mixed phase of the QGP and hadronic gas. In the absence of pressure gradient, a slow-burning fireball is expected when the initial system is at rest in the mixed phase, and this leads to a considerable time-delay of the system evolution \[1, 2, 3, 4, 5, 6\]. It is therefore of interest to investigate the time-delay signatures for the first-order phase transition.

Two-particle Hanbury-Brown-Twiss (HBT) interferometry is a useful tool for detecting the space-time structure of particle-emitting sources in high energy heavy ion collisions \[7, 8, 9, 10\]. For the first-order phase transition between the QGP and hadronic matter, the time-delay of the system evolution may prolong the emission duration of particles and lead to unusually large HBT lifetime, as compared to a crossover transition or a hadron gas without the QCD phase transition \[1, 2, 3, 4, 5, 6\]. It is known that the phase transitions occurring in the heavy ion collisions at RHIC and top SPS energies are crossover and in small baryon density regions. Moreover, at AGS energies the systems are almost in hadronic phase although with higher baryon densities. The future Facility for Antiproton and Ion Research (FAIR) at GSI with heavy ion beams from 2 – 45A GeV will provide an opportunity to explore the first-order QCD phase transition at high baryon densities \[11, 12, 13, 14, 15, 16\]. Investigating how the HBT results variate with the incident energy and what the HBT signatures the first-order phase transition is possessed of in the GSI-FAIR energy range is thereby the subject of this work.

In high energy heavy ion collisions the final particles include the contributions of direct production and excited-state particle decay. Also, the particles will subject to the multiple scattering with other particles when they propagate in the system up to the thermal freeze-out. In Ref. \[17\] a HBT analysis technique with quantum transport of the interfering pair was developed to investigate the effects of particle decay and multiple scattering on the extracted HBT radii in the heavy ion collisions at AGS and RHIC energies. This HBT analysis technique allows one to follow the trajectories of the test particles after emission up to the thermal freeze-out in model calculations. It is suitable for examining the space-time geometry of evolving sources in detail. In this study we will use relativistic hydrodynamics
with an equation of state of first-order phase transition to describe the evolution of the particle-emitting sources produced at GSI-FAIR energies. We will use the HBT analysis technique with quantum transport of the interfering pair to examine the HBT radii and lifetimes of the sources for different initial energy densities. Because the central heavy ion collisions at GSI-FAIR energies are almost full stopped, we assume that the systems are at rest initially in the center-of-mass frame and with spherical shape for simplicity.

As compared with pion HBT interferometry, kaon HBT interferometry may present more clearly the source space-time geometry at emission configuration, because kaons can escape easily from the system after hadronization and therefore seldom be affected by the multiple scattering and particle decay. By comparing the results extracted from the two-pion and two-kaon HBT interferometry, we find that the multiple scattering and particle decay lead to a larger radius of the pion source than that of kaon’s. However, both the HBT lifetimes of the pion and kaon sources have significant increase when the initial energy density approaches to the boundary between the QGP and mixed phase.

The paper is organized as follows. In section II we describe the model of equation of state (EOS) used in our calculations. The adiabatic paths for the EOS and the space-time configuration of the evolving system are also discussed in this section. In section III we give a brief description to the quantum probability amplitudes in a path-integral formalism of HBT and the two-pion and two-kaon HBT results for the evolving sources produced in the collisions at GSI-FAIR energies. Finally, the summary and discussion are presented in section IV.

II. EQUATION OF STATE AND SYSTEM EVOLUTION

For non-dissipative ideal fluid hydrodynamics is defined by the local conservations of energy-momentum and other conserved quantities (e.g. entropy, baryon number, and strangeness) [18,19]. To solve the hydrodynamical equations one needs the EOS which gives the relation among the thermodynamical quantities in the conserved equations [18,19]. In what follows we discuss our EOS model and system evolution described by hydrodynamics and the EOS.
A. EOS model

In our EOS model the QGP phase is described by an ideal gas of gluons, $u, d, s$ quarks and antiquarks, with the constant vacuum energy $B$ associated with QCD confinement \[20, 21\]. The pressure, energy density, and the conserved charge density in the QGP phase are given by

\[ p^Q = \sum_i p_i(T, \mu_i) - B, \]
\[ \varepsilon^Q = \sum_i \varepsilon_i(T, \mu_i) + B, \]
\[ n^Q_A = \sum_i A_i n_i(T, \mu_i), \]

where \( p_i(T, \mu_i), \varepsilon_i(T, \mu_i), \) and \( n_i(T, \mu_i) \) are the pressure, energy density, and number density of particle species \( i \) in the ideal gas with temperature \( T \) and chemical potential \( \{\mu_i\} \). \( A_i \) is the conserved charge number of the particle species \( i \). In our calculations we use the quark masses \( m_u = m_d = 5 \text{ MeV}, m_s = 150 \text{ MeV} \) and the bag constant \( B = (235 \text{ MeV})^4 \) \[21\].

For the hadronic phase we adopt the excluded volume model \[21, 22, 23\] and consider the particles $\pi, K, N, \Lambda, \Sigma, \Delta$, and their antiparticles. The pressure, energy density, and the conserved charge density in the hadronic phase are given by \[21, 22, 23\]

\[ p^H = \sum_i p_i(T, \tilde{\mu}_i), \]
\[ \varepsilon^H = \frac{\sum_i \varepsilon_i(T, \tilde{\mu}_i)}{1 + V_0 \sum_i n_i(T, \tilde{\mu}_i)}, \]
\[ n^H_A = \frac{\sum_i A_i n_i(T, \tilde{\mu}_i)}{1 + V_0 \sum_i n_i(T, \tilde{\mu}_i)}, \]

where

\[ \tilde{\mu}_i = \mu_i - V_0 p^H, \]
\[ V_0 = (1/2)(4\pi/3)(2a)^3 \]

is the excluded volume which is assumed to be the same for all hadrons with \( a = 0.5 \text{ fm} \) \[21\].

For the first-order phase transition, there are Gibbs relationships in the mixed phase of the QGP and hadron gas. We have \( T^Q = T^H, \mu_{N,\Delta} = 3\mu_u, \mu_{\Lambda,\Sigma} = 2\mu_u + \mu_s, \mu_{\pi^+,\pi^0,\pi^-} = 0, \mu_{K^+,\bar{K}^0} = \mu_u - \mu_s, \ldots, \) and

\[ p^M = p^Q(T, \mu_u, \mu_s) = p^H(T, \mu_u, \mu_s), \]
\[ \varepsilon^M = \alpha \varepsilon^Q(T, \mu_u, \mu_s) + (1 - \alpha) \varepsilon^H(T, \mu_u, \mu_s), \]  

(9)

\[ n^M_A = \alpha n^Q_A(T, \mu_u, \mu_s) + (1 - \alpha) n^H_A(T, \mu_u, \mu_s), \]  

(10)

where \( \mu_u \) and \( \mu_s \) are the chemical potential of \( u \) and \( s \) quarks, and \( \alpha = V_Q/V \) is the fraction of the volume occupied by the plasma phase. The boundaries of the coexistence region are found by putting \( \alpha = 0 \) (the hadron phase boundary) and \( \alpha = 1 \) (the plasma boundary).

Using the thermodynamical relations of ideal gas one can get other thermodynamical quantities, such as entropy density \( s \), in the QGP, hadronic, and mixed phases from Eqs. \([\text{11} - \text{3}], [\text{14} - \text{6}], \) and \([\text{8} - \text{10}]\), and get numerically the EOS for solving the hydrodynamical equations.

### B. Adiabatic paths

In our model calculations the system evolves from a thermalized initial state to final freeze-out. In the absence of dissipation, the entropy of the system is conserved during evolution. On the other hand, the baryon number is also conserved. So the ratio of their local densities \( n_B/s \) is a constant. For our EOS model we show in Fig. 1 the adiabatic cooling paths of the systems with \( n_B/s = 0.08 \) and 0.06, which correspond to the incident energies about 10 and 30 \( A \)GeV, respectively \([23, 24]\). The dotted line in Fig. 1 is the transition curve between the QGP and hadron gas. The mixed phase is on the transition curve from the endpoint of the QGP branch (point 1 or 1') up to the beginning of the hadronic branch (point 2 or 2'). The non-trivial zigzag shape of the trajectories indicates that the system has a re-heating in the mixed phase \([23, 25]\). The reason is that at a certain point \((T, \mu)\) on the phase-transition curve, the number of degrees of freedom and hence the specific entropy in the plasma phase are larger than the corresponding values in the hadronic phase. Hence the temperature must increase during hadronization to conserve both the total entropy and baryon number simultaneously \([25]\).

In Fig. 2 we show the hydrodynamical relevant relation, \( p/\varepsilon \), for \( n_B/s = 0.06 \) and 0.08. At the boundaries between the hadronic and mixed phases (2 and 2') the ratio \( p/\varepsilon \) has maximums. The minimums of the ratio, named the “softest points”, are at the boundaries between the mixed phase and QGP (1 and 1'), corresponding to \( \varepsilon = \varepsilon^{MQ} = 1.83 \) and 1.90 GeV/fm\(^3\) for \( n_B/s = 0.06 \) and 0.08, respectively.
FIG. 1: Adiabatic paths for \( n_B/s = 0.06 \) and \( n_B/s = 0.08 \).

FIG. 2: The ratio of pressure to energy density \( p/\varepsilon \) for \( n_B/s = 0.06 \) and \( n_B/s = 0.08 \).

C. System evolution

After knowing system EOS we can obtain the solutions of hydrodynamical equations for the certain initial conditions \([4, 26, 27, 28]\), by using the HLLE scheme \([29, 30]\) and Sod’s operator splitting method \([31]\). Because the central heavy ion collisions at GSI-FAIR energies are almost full stopped, we assume that the initial system is at rest in a sphere with a constant energy density \( \varepsilon^0 \). For \( n_B/s = 0.06 \), the incident energy is about 30 AGeV \([23, 24]\). We investigate the system evolution with the initial energy densities \( \varepsilon^0 = 4.12 \text{ GeV/fm}^3 > \varepsilon^{\text{MQ}} \) and \( \varepsilon^0 = \varepsilon^{\text{MQ}} = 1.83 \text{ GeV/fm}^3 \). The corresponding initial temperatures are \( T^0 = 180 \) and 142 MeV. Meantime, the corresponding initial baryon chemical potentials are \( \mu_B^0 = 3\mu_u^0 = 990 \) and 780 MeV. For the system with \( n_B/s = 0.08 \), the corresponding incident energy about 10 AGeV enable only the initial energy density to approach the region of the QGP boundary.
FIG. 3: The space-time contours of energy density for the systems with $n_B/s = 0.06$ (a,b) and $n_B/s = 0.08$ (c,d), for the initial energy density $\epsilon^0 > \epsilon^{MQ}$, $\epsilon^0 = \epsilon^{MQ}$, and $\epsilon^0 = \epsilon^{HM}$.

In this case we calculate the system evolution with the initial energy densities $\epsilon^0 = \epsilon^{MQ} = 1.90$ GeV/fm$^3$ and $\epsilon^0 = \epsilon^{HM} = 172$ MeV/fm$^3$, where $\epsilon^{HM}$ is the energy density at the boundary between the hadronic and mixed phases. The corresponding initial temperatures are 132 and 152 MeV. The corresponding initial baryon chemical potential are 960 and 480 MeV.

Figure 3(a) and (b) show the space-time contours of the local energy densities at $\epsilon^{MQ}$, $\epsilon^{HM}$, and $\epsilon^{TFO}$ for $\epsilon^0 > \epsilon^{MQ}$ and $\epsilon^0 = \epsilon^{MQ}$ for the system with $n_B/s = 0.06$, respectively. Here $r_0$ and $\epsilon^{TFO}$ are the initial system radius and the energy density at the thermal freeze-out. One can see that the system takes more time evolving through the mixed phase (from $\epsilon^{MQ}$ to $\epsilon^{HM}$) than that through the pure QGP phase (from $\epsilon^0$ to $\epsilon^{MQ}$) or the pure hadronic phase (from $\epsilon^{HM}$ to $\epsilon^{TFO}$). The duration of the evolution through the mixed phase is larger when the initial energy density is at the soft point. Figure 3(c) and (d) show the contours at $\epsilon^{HM}$ and $\epsilon^{TFO}$ for $\epsilon^0 = \epsilon^{MQ}$ and $\epsilon^0 = \epsilon^{HM}$ for the system with $n_B/s = 0.08$, respectively. For $\epsilon^0 = \epsilon^{MQ}$, because the soft point for $n_B/s = 0.08$ is higher than that for $n_B/s = 0.06$ (see Fig. 2), the evolving time through the mixed phase for $n_B/s = 0.08$ is short than that for $n_B/s = 0.06$. For $\epsilon^0 = \epsilon^{HM}$ there is only hadronic phase. The evolution is fast because of the larger $p/\epsilon$ in the hadronic gas. In our calculations the energy density at the thermal
freeze-out is taken to be 45 MeV/fm³, which corresponds to the thermal freeze-out temperatures 110 and 100 MeV for $n_B/s = 0.06$ and 0.08. The initial system radii are taken to be 6 fm.

III. HBT INTERFEROMETRY WITH QUANTUM TRANSPORT OF THE INTERFERING PAIR

The two-particle Bose-Einstein correlation function $C(k_1,k_2)$ is defined as the ratio of the two-particle momentum distribution $P(k_1,k_2)$ to the the product of the single-particle momentum distribution $P(k_1)P(k_2)$. For an evolving source, using quantum probability amplitudes in a path-integral formalism, $P(k_i)$ ($i = 1, 2$) and $P(k_1,k_2)$ can be expressed as

$$P(k_i) = \int d^4x \rho(x)e^{-2i\text{Im}\tilde{\phi}_s(x)}A^2(\kappa x),$$

$$P(k_1,k_2) = \int d^4x_1d^4x_2 \ e^{-2i\text{Im}\tilde{\phi}_s(x_1)}e^{-2i\text{Im}\tilde{\phi}_s(x_2)} \times \rho(x_1)\rho(x_2)|\Phi(x_1x_2;k_1k_2)|^2,$$

where \( \rho(x) \) is the four-dimension density of the particle-emitting source, \( A(\kappa x) \) is the magnitude of the amplitude for producing a particle with momentum \( \kappa \) at \( x \), \( e^{-2i\text{Im}\tilde{\phi}_s(x)} \) is the absorption factor due to multiple scattering, and \( \Phi(x_1x_2;k_1k_2) \) is the wave function for the two identical bosons produced at \( x_1 \) and \( x_2 \) with momenta \( \kappa_1 \) and \( \kappa_2 \), and detected at \( x_{d1} \) or \( x_{d2} \) with momenta \( k_1 \) and \( k_2 \), respectively.

In our HBT calculations final identical kaons are considered to be emitted thermally from the space-time hypersurface at \( \varepsilon^{\text{HM}} \) and keep freeze-out after emission. However, final identical pions (for example \( \pi^+ \)) include the primary pions emitted from the hypersurface at \( \varepsilon^{\text{HM}} \) and the secondary pions from the “excited-state” particle decays during the system evolving in hadronic phase until to the thermal freeze-out. The four-dimension density of the pion source can be expressed as

$$\rho(x) = n_\pi(x)\delta(t - \tau^0) + \sum_{j\neq \pi} D_{j\rightarrow\pi}n_j(x),$$

where \( n_i(x) \) and \( \tau^0 \) are the particle number density and the hadronization time in local frame, \( D_{j\rightarrow\pi} \) is the product of the decay rate in time and the fraction of the decay \( \tilde{d}_{j\rightarrow\pi} \).
For example, \( D_{\Delta \rightarrow \pi} = \Gamma_{\Delta} \times \frac{1}{3} \) and \( D_{\pi^0 \pi^0 \rightarrow \pi^+ \pi^-} = v_r n_\pi \sigma(\pi^0 \pi^0 \rightarrow \pi^+ \pi^-) \times 1 \), where \( v_r \) is the relative velocity of the two colliding pions and the cross section \( \sigma(\pi^0 \pi^0 \rightarrow \pi^+ \pi^-) \) is equal to the absorption cross section of \( \pi^+ \pi^- \rightarrow \pi^0 \pi^0 \) \([17]\). In calculations we neglect the contributions of \( \Lambda \) and \( \Sigma \) decays to the final pions, because most of them exist until the thermal freeze-out.

When a test pion propagating in the source it will subject to multiple scattering with the medium particles in the source. Based on Glauber multiple scattering theory \([36]\), the absorption factor due to the multiple scattering in Eqs. (11) and (12) can be written as \([17, 26, 33, 34, 35]\)

\[
e^{-2 \text{Im} \phi_s(x)} = \exp \left[-\int_x^{x_f} \left( \sum_i' \sigma_{\text{abs}}(\pi i) n_i(x') \right) d\ell(x') \right],
\]

where \( \sum_i' \) means the summation for all medium particles except for the test pion along the propagating path \( d\ell(x') \), \( \sigma_{\text{abs}}(\pi i) \) is the absorption cross section of the test pion with the particle species \( i \) in the medium, and \( x_f \) is the freeze-out coordinate. In calculations we only consider the dominant absorption processes for the identical pions, for example the reactions of \( \pi^+ \pi^- \rightarrow \pi^0 \pi^0 \) and \( \pi^+ N \rightarrow \Delta \) for \( \pi^+ \), as we did in Ref. \([17]\).

In HBT analyses the variables usually used are the Pratt-Bertsch variables, \( q_{\text{out}} \), \( q_{\text{side}} \), and \( q_{\text{long}} \) \([37, 38]\). Here \( q_{\text{out}} \) and \( q_{\text{side}} \) are the components of the relative momentum of identical particle pair in the directions parallel and perpendicular to the total transverse momentum of the pair, and \( q_{\text{long}} \) is the relative momentum along the beam direction of collision. Using the Pratt-Bertsch variables the HBT radius in the side-direction, \( R_{\text{side}} \), reflects the transverse size of the source. However, the HBT radius in the out-direction, \( R_{\text{out}} \), is related to not only the source size, but also the source expanding velocity and lifetime \([37, 38]\). So a detailed joint analysis of \( R_{\text{out}} \) and \( R_{\text{side}} \) as a function of transverse momentum of the pair may also provides the information of source dynamics \([8, 9, 37, 38]\). Motivated by investigating the source lifetime directly and clearly, we use the variables \( q = |k_1 - k_2| \) and \( q_0 = |E_1 - E_2| \) and the simple Gaussian fitting formula

\[
C(q, q_0) = 1 + \lambda e^{-q^2 R^2 - q_0^2 \tau^2},
\]

where \( R \), \( \tau \), and \( \lambda \) are the source HBT radius, lifetime, and chaotic parameter.

From Eqs. (11) – (14) we can construct numerically the two-particle HBT correlation function for each \((q, q_0)\) bin \([17, 26, 27, 39, 40]\). Figure 4 shows the two-particle correlation
functions $C(q, q_0 < 15 \text{ MeV}/c)$ for the evolving sources with $n_B/s = 0.06$ and 0.08, and for $\varepsilon^0 > \varepsilon^{MQ}$, $\varepsilon^0 = \varepsilon^{MQ}$, and $\varepsilon^0 = \varepsilon^{HM}$. The symbols $\triangle$ and $\bullet$ are the two-kaon and two-pion correlation function results. For comparison, the symbols $\square$ present the two-pion correlation functions calculated with the pions emitted from the thermal freeze-out (TFO) configuration. Table I gives our HBT fitted results. One can see that for the same initial conditions, the two-kaon HBT radius is smaller than that of the two-pion’s. It is because that the kaons are emitted earlier and from smaller system configuration. The multiple scattering and particle decays during the source evolving in hadronic phase increase the HBT radii of the pion sources. Because the pion sources have largest configuration at TFO, the corresponding HBT radius is the largest for the certain initial energy density.

For the system with $n_B/s = 0.06$ we can see that for the initial energy density $\varepsilon^0 > \varepsilon^{MQ}$ the two-pion HBT lifetimes are larger than that of two-kaon’s. However, when $\varepsilon^0$ drops to the soft point $\varepsilon^{MQ}$ the HBT lifetimes for the pions and kaons increase significantly and almost are the same, while the corresponding HBT radii decrease. The reasons are related to their different source space-time geometries [see Fig. 3(a) and (b)] and expanding velocities for the two kinds of initial conditions. When the initial energy density is at the soft point, the source has the smallest expansion and correspondingly the smallest average spatial size.
### TABLE I: The HBT fitted results.

|       | $2K$               | $2\pi$             | $2\pi$(TFO)         |
|-------|--------------------|--------------------|--------------------|
| (a)   | $R = 3.25 \pm 0.12$ fm | $R = 4.90 \pm 0.20$ fm | $R = 6.95 \pm 0.30$ fm |
| $\varepsilon^0 > \varepsilon^{MQ}$ | $\tau = 5.35 \pm 0.36$ fm/c | $\tau = 7.31 \pm 0.54$ fm/c | $\tau = 7.84 \pm 0.74$ fm/c |
| $n_B/s = 0.06$ | $\lambda = 0.81 \pm 0.05$ | $\lambda = 0.83 \pm 0.05$ | $\lambda = 0.90 \pm 0.06$ |
| (b)   | $R = 2.61 \pm 0.09$ fm | $R = 3.70 \pm 0.14$ fm | $R = 5.20 \pm 0.19$ fm |
| $\varepsilon^0 = \varepsilon^{MQ}$ | $\tau = 12.50 \pm 0.68$ fm/c | $\tau = 12.53 \pm 0.58$ fm/c | $\tau = 12.43 \pm 0.65$ fm/c |
| $n_B/s = 0.06$ | $\lambda = 0.91 \pm 0.05$ | $\lambda = 1.04 \pm 0.05$ | $\lambda = 1.03 \pm 0.05$ |
| (c)   | $R = 2.35 \pm 0.07$ fm | $R = 3.70 \pm 0.10$ fm | $R = 4.98 \pm 0.16$ fm |
| $\varepsilon^0 = \varepsilon^{MQ}$ | $\tau = 9.58 \pm 0.36$ fm/c | $\tau = 9.52 \pm 0.31$ fm/c | $\tau = 9.13 \pm 0.35$ fm/c |
| $n_B/s = 0.08$ | $\lambda = 0.98 \pm 0.05$ | $\lambda = 1.04 \pm 0.04$ | $\lambda = 0.95 \pm 0.04$ |
| (d)   | $R = 1.66 \pm 0.03$ fm | $R = 2.38 \pm 0.06$ fm | $R = 3.54 \pm 0.04$ fm |
| $\varepsilon^0 = \varepsilon^{HM}$ | $\tau = 2.33 \pm 0.08$ fm/c | $\tau = 2.43 \pm 0.10$ fm/c | $\tau = 1.73 \pm 0.14$ fm/c |
| $n_B/s = 0.08$ | $\lambda = 0.93 \pm 0.01$ | $\lambda = 1.02 \pm 0.02$ | $\lambda = 1.00 \pm 0.01$ |

and largest evolution time from the initial state to the hadronization. Because there is no the influence of the system evolution on kaons after hadronization, the increase of the HBT lifetime of the kaons reflects the prolongation of the system evolution in the mixed phase. In Table I, the results of chaotic parameter $\lambda$ for $\varepsilon^0 > \varepsilon^{MQ}$ are obviously smaller than unit. This is mainly because that the particle-emitting sources in this case are much different from Gaussian distribution.

For the system with $n_B/s = 0.08$, the two-pion and two-kaon HBT lifetimes for $\varepsilon^0 = \varepsilon^{MQ}$ are much larger than those for $\varepsilon^0 = \varepsilon^{HM}$. Also, the HBT radii for $\varepsilon^0 = \varepsilon^{MQ}$ are larger than the corresponding results for $\varepsilon^0 = \varepsilon^{HM}$. The main reason for these results is that the space-time configuration for $\varepsilon^0 = \varepsilon^{HM}$ is much smaller than that for $\varepsilon^0 = \varepsilon^{MQ}$ [see Fig. 3 (c) and (d)]. Our HBT investigations indicate that both for $n_B/s = 0.06$ and 0.08, the maximums of the HBT lifetimes appear when the initial energy density reaching at the soft points. Because the ratio $p/\varepsilon$ at the soft point $\varepsilon^{MQ}$ for $n_B/s = 0.08$ is higher than that for
For the systems with the $n_B/s$ values between 0.06 and 0.08, we also find that their HBT lifetimes increase significantly when $\varepsilon^0$ approaches at the corresponding soft points (between 1 and 1' in figure [2]). The maximums of the HBT lifetime are about 10 fm/c much larger than the results for $\varepsilon^0 > \varepsilon^{MQ}$ and $\varepsilon^0 = \varepsilon^{HM}$ [in Table I (a) and (d)]. Based on the evolving trajectories calculated by hydrodynamics [24], the events with the initial thermalized states staying in the mixed phase of the QGP and hadronic gas will happen when the incident energies are between 10 and 30$A$ GeV (see the figure 19 of [24], the beginnings of the bold parts of the trajectories for the 10 and 30$A$ GeV are at the two sides of the transition region, respectively). Correspondingly, the ratio of $n_B/s$ is about between 0.08 and 0.06 (see the figure 18 of [24]). From our model calculations, the maximums of the two-pion and two-kaon HBT lifetimes will be observed simultaneously in GSI-FAIR energy range when the initial energy density is tuned to the soft point.

IV. SUMMARY AND DISCUSSION

Recently the heavy ion collisions at the energies between AGS and the top-energy SPS attract special attention, for example the SPS and RHIC low energy programs [41, 42, 43, 44] and the project of GSI-FAIR [11, 12, 13, 14, 15, 16]. In this energy range it is expected that the heavy ion collisions may produce the QGP with high baryon density, which is different from that have been observed in RHIC and top-energy SPS experiments. Correspondingly, the phase transition from the high-baryon-density QGP to hadronic matter is the first-order transition, which will lead to a system evolution much different from the crossover transition happened in the low baryon density region at RHIC and top SPS energies.

Using the technique of quantum transport of the interfering pair we examine the two-pion and two-kaon HBT interferometry for the particle-emitting sources produced in the heavy ion collisions at GSI-FAIR energies. We use relativistic hydrodynamics with the EOS of first-order phase transition between the QGP and hadronic gas to describe the system evolution. The two-particle HBT correlation functions are calculated with the quantum probability amplitudes in a path-integral formalism, where the effects of particle decay and multiple scattering are taken into consideration. We find that both the HBT radii of pions
and kaons increase with the system initial energy density. The particle decay and multiple scattering lead to the larger HBT radii of pions than the corresponding HBT radii of kaons. The HBT lifetimes of the pion and kaon sources are sensitive to the initial energy density. They are significantly prolonged when the initial energy density is tuned to the soft point. Our model calculations indicate that this significant prolongation of the HBT lifetimes of pions and kaons will be observed in the heavy ion collisions at GSI-FAIR energies.

As a useful space-time probe HBT interferometry has been extensively used in high energy heavy ion collisions. However, there are still some open problems on HBT analysis technique and the understanding of HBT results. The HBT measurements at RHIC indicate that the values of the ratio of the transverse HBT radii \( R_{\text{out}} \) to \( R_{\text{side}} \) are smaller than those from the hydrodynamical calculations [45, 46, 47, 48]. Various models and techniques have been put forth to explain the RHIC HBT puzzle [27, 40, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61]. At GSI-FAIR energies, the heavy ion collisions are almost full stopped. We used an approximation of spherical evolving sources and assumed that the initial states are static and uniform in our calculations. It would be interesting to consider more reasonable evolving sources and study the effect of initial conditions on the HBT results in future investigations. Also, a systematical investigation to HBT interferometry in different energy ranges will be of great interest.

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[1] Pratt S 1986 \textit{Phys. Rev.} D \textbf{33} 1314
[2] Bertsch G and Brown G E 1989 \textit{Phys. Rev.} C \textbf{40} 1830
[3] Hung C M and Shuryak E 1995 \textit{Phys. Rev. Lett.} \textbf{75} 4003
[4] Rischke D H and Gyulassy M 1996 \textit{Nucl. Phys.} A \textbf{608} 479
[5] Soff S, Bass S A, Hardtke D H and Panitkin S Y 2002 \textit{Phys. Rev. Lett.} \textbf{88} 072301
[6] Zschiesche D, Stöcker H, Greiner W and Schramm S 2002 \textit{Phys. Rev.} C \textbf{65} 064902
[7] Wong C Y *Introduction to High-Energy Heavy-Ion Collisions* (World Scientific, Singapore, 1994), Chap. 17.

[8] Wiedemann U A and Heinz U 1999 *Phys. Rept.* 319 145

[9] Weiner R M 2000 *Phys. Rept.* 327 249

[10] Lisa M A, Pratt S, Soltz R and Wiedemann U 2005 *Ann. Rev. Nucl. Part. Sci.* 2005 55 357

[11] Höhne C 2005 *Nucl. Phys. A* 749 141c

[12] Friesa V 2006 *J. Phys. G: Nucl. Part. Phys.* 32 S439

[13] Peters K 2006 *Nucl. Phys. B (Proc. Suppl.)* 154 35

[14] Arsene I C, Bravina L V, Cassing W, Ivanov Yu B, Larionov A, Randrup J, Russkikh V N, Toneev V D, Zeeb G, and Zschiesche D 2007 *Phys. Rev. C* 75 034902

[15] Rosner G 2007 *Nucl. Phys. B (Proc. Suppl.)* 167 77

[16] Henning W F 2008 *Nucl. Phys. A* 805 502c

[17] Yu L L, Zhang W N and Wong C Y 2008 *Phys. Rev. C* 78 014908

[18] Rischke D H *Proceedings of the 11th Chris Engelbrecht Summer School in Theoretical Physics, Cape Town, February 4-13, 1998* Preprint [nucl-th/9809044](http://arxiv.org/abs/nucl-th/9809044)

[19] Kolb P F and Heinz U *Invited review for 'Quark Gluon Plasma 3', World Scientific, Singapore 2003* Preprint [nucl-th/0305084](http://arxiv.org/abs/nucl-th/0305084)

[20] Zhang W N, Tang G X, Chen X J, Huo L, Liu Y M, and Zhang S 2000 *Phys. Rev. C* 62 044903

[21] Toneev V D, Nikonov E G, Friman B, Nörenberg W, and Redlich K 2003 *Eur. Phys. J. C* 32 399

[22] Rischke D H, Gorenstein M I, Stöcker H, and Greiner W 1991 *Z. Phys. C* 51 485

[23] Hung C M and Shuryak E 1998 *Phys. Rev. C* 57 1891

[24] Ivanov Y B, Russkikh V N and Toneev V D 2006 *Phys.Rev. C* 73 044904

[25] Subramanian P R, Stocker H and Greiner W 1986 *Phys. Lett. B* 173 468

[26] Zhang W N, Efaaf M J, Wong C Y and Khaliliasr M 2004 *Chin. Phys. Lett.* 21 1918

[27] Zhang W N, Efaaf M J and Wong C Y 2004 *Phys. Rev. C* 70 024903

[28] Efaaf M J, Zhang W N, Khaliliasr M et al 2005 *HEP & NP* 29(1) 46

[29] Schneider V et al 1993 *J. Comput. Phys.* 105 92

[30] Rischke D H, Bernard S and Maruhn J A 1995 *Nucl. Phys. A* 595 346

[31] Sod G A 1977 *J. Fluid Mech.* 83 785
[32] Cleymans J and Redlich K 1999 Phys. Rev. C 60 054908
[33] Wong C Y 2003 J. Phys. G: Nucl. Part. Phys. 29 2151
[34] Wong C Y 2004 J. Phys. G: Nucl. Part. Phys. 30 S1053
[35] Wong C Y 2006 AIP Conference Proc. 828 617 Preprint hep-ph/0510258
[36] Glauber R J Lectures in Theoretical Physics, (Interscience, N.Y., 1959) 1 315
[37] Pratt S 1986 Phys. Rev. D 33 72; Pratt S, Cs¨ orgo T and Zim´ anyi J 1990 Phys. Rev. C 42 2646
[38] Bertsch G, Gong M and Tohyama M 1988 Phys. Rev. C 37 1896; Bertsch G 1989 Nucl. Phys. A 498 173c
[39] Zhang W N, Li S X, Wong C Y and Efaff M J 2005 Phys. Rev. C 71 064908
[40] Zhang W N, Ren Y Y and Wong C Y 2006 Phys. Rev. C 74 024908
[41] Mitrovski M K 2006 J. Phys. G: Nucl. Part. Phys. 32 S43
[42] Š´ andor L 2006 J. Phys. G: Nucl. Part. Phys. 32 S127
[43] Stephans G S F 2006 J. Phys. G: Nucl. Part. Phys. 32 S447
[44] Stephans G S F 2008 J. Phys. G: Nucl. Part. Phys. 35 044050
[45] Adler C et al (STAR Collaboration) 2001 Phys. Rev. Lett. 87 082301
[46] Adcox K et al (PHENIX Collaboration) 2002 Phys. Rev. Lett. 88 192302
[47] Adler S S et al (PHENIX Collaboration) 2004 Phys. Rev. Lett. 93 152302
[48] Adams J et al (STAR Collaboration) 2005 Phys. Rev. C 71 044906
[49] Soff S, Bass S A, Hardtke D H and Panitkin S Y 2002 J. Phys. G: Nucl. Part. Phys. 28 1885
[50] Heinz U and Kolb P 2002 Nucl. Phys. A 702 269
[51] Lin Z W, Ko C M and Pal S 2002 Phys. Rev. Lett. 89 152301
[52] Teaney D 2003 Nucl. Phys. A 715 817
[53] Cs¨ org¨ o T and Zim´ anyi 2003 Acta Phys. Hung. New Series, Heavy-ion Phys. 17 281 nucl-th/0206051
[54] Moln´ ar D and Gyulassy M 2004 Phys. Rev. Lett. 92 052301
[55] Socolowski Jr O, Grassi F, Hama Y and Kodama T 2004 Phys. Rev. Lett. 93 182301
[56] Kapusta J and Li Y 2004 J. Phys. G: Nucl. Part. Phys. 30 S1069; Kapusta J and Li Y 2005 Phys. Rev. C 72 064902
[57] Gramer J G, Miller G A, Wu J M S and Yoon J H 2005 Phys. Rev. Lett. 94 102302
[58] Pratt. S and Schindel 2005 nucl-th/0511010 Pratt S 2007 arXiv:0710.5733 Pratt S 2008
[59] Frodermann E, Heinz U, and Lisa M A 2006 \textit{Phys. Rev.} C \textbf{73} 044908; Frodermann E, Chatterjee and Heinz U 2007 \textit{J. Phys. G: Nucl. Part. Phys.} \textbf{34} 2249

[60] Li Q, Bleicher M and Stöcker H 2007 \textit{Phys. Lett.} B \textbf{659} 525; Li Q and Bleicher M 2008 \textit{arXiv:0808.3457}

[61] Broniowski W, Chojnacki M, Florkowski W and Kisiel A 2008 \textit{Phys. Rev. Lett.} \textbf{101} 022301; Chojnacki M, Florkowski W, Broniowski W and Kisiel A 2008 \textit{Phys. Rev. C} \textbf{78} 014905