Kinetic Scale Structure of Low-frequency Waves and Fluctuations

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Abstract

The dissipation of solar wind turbulence at kinetic scales is believed to be important for the heating of the corona and for accelerating the wind. The linear Vlasov kinetic theory is a useful tool for identifying various wave modes, including kinetic Alfvén, fast magnetosonic/whistler, and ion-acoustic (or kinetic slow), and their possible roles in the dissipation. However, the kinetic mode structure in the vicinity of ion-cyclotron modes is not clearly understood. The present paper aims to further elucidate the structure of these low-frequency waves by introducing discrete particle effects through hybrid simulations and Klimontovich formalism of spontaneous emission theory. The theory and simulation of spontaneously emitted low-frequency fluctuations are employed to identify and distinguish the detailed mode structures associated with ion-Bernstein modes versus quasi-modes. The spontaneous emission theory and simulation also confirm the findings of the Vlasov theory in that the kinetic Alfvén waves can be defined over a wide range of frequencies, including the proton cyclotron frequency and its harmonics, especially for high-beta plasmas. This implies that these low-frequency modes may play predominant roles even in the fully kinetic description of kinetic scale turbulence and dissipation despite the fact that cyclotron harmonic and Bernstein modes may also play important roles in wave–particle interactions.

Key words: methods: analytical – methods: numerical – plasmas – solar wind – turbulence – waves

1. Introduction

The heating of the solar corona and the acceleration of the wind are mutually related outstanding problems in contemporary space and astrophysics. It is generally believed that the dissipation of the pervasive magnetohydrodynamic (MHD) turbulence at the kinetic scale plays an important role in the heating and acceleration process. However, the detailed physics of the kinetic scale dissipation have not been addressed in any conclusive manner. Based on recent observations (Bale et al. 2005; Sahraoui et al. 2009, 2010; Chen et al. 2013; Podesta 2013), most contemporary theories assume that the dissipation involves the kinetic Alfvén waves (KAWs; Howes et al. 2011; Boldyrev et al. 2013). Of course, KAWs are beyond the description of MHD. Thus, Howes (2009) and Boldyrev et al. (2013), among others, pointed out the limitations of MHD or Hall MHD by correctly describing the kinetic plasma modes. However, there are reasons to believe that the dissipation at kinetic scales not only involves KAWs, but also a slew of other plasma waves such as the magnetosonic/whistler (MSW) mode (Gary & Smith 2009; Narita et al. 2011, 2016), the kinetic slow (KS) mode (Yao et al. 2011; Howes et al. 2012; Narita & Marsch 2015), and ion-Bernstein modes (Perschke et al. 2013). In this regard, Verscharen et al. (2012) and Comišel et al. (2013) carried out hybrid simulations to show that the cascade of MHD turbulence involves waves in the ion-cyclotron frequency region, which standard MHD or gyrokinetic theories cannot handle. Podesta (2012) also pointed out the potential significance of ion-Bernstein waves (IBWs) as a dissipation channel of solar wind turbulence. Narita & Marsch (2015) emphasized the role of the KS (or, equivalently, the ion-acoustic) mode, which is distinct from the kinetic Alfvén wave. Comišel et al. (2015) employed a two-dimensional (2D) hybrid simulation in order to demonstrate that kinetic plasma wave modes involved in the kinetic scale dissipation may include kinetic Alfvén, fast MSW, ion-acoustic, and ion-Bernstein cyclotron modes that satisfy the dispersion relation even at a nonlinear saturation stage. They also reported frequency broadening around the normal modes, which they interpreted as the sideband generation. In short, there seems to emerge an awareness that the kinetic scale physics of the solar wind turbulence dissipation involves wave modes that are far more rich and complicated than of those centered around the simple kinetic Alfvén wave concept.

The purpose of the present paper is to elucidate the fundamental structure associated with plasma eigenmodes as well as the spontaneously generated electromagnetic fluctuations in the vicinity of the normal modes. The spontaneous emission is particularly useful in identifying the most relevant normal modes and for properly interpreting certain computer simulations. Navarro et al. (2014) showed that spontaneous emission of electromagnetic fluctuations, due to the thermal random motion of particles, significantly contributes to the observed magnetic fluctuations in the solar wind at 1 au (Bale et al. 2009). Wu et al. (2013a, 2013b) showed that Alfvén waves can be generated by a spontaneous process due to the presence of heavy ions, and Yoon et al. (2017) calculate the steady-state spectra of low-frequency electromagnetic fluctuations of the Alfvénic type for thermal equilibrium plasma. The existing theories of low-frequency wave structures based upon Vlasov kinetic theory (Gary & Borovsky 2008; Gary & Smith 2009; Howes 2009; Podesta 2012; Sahraoui et al. 2012; Boldyrev et al. 2013; Comišel et al. 2015; Narita & Marsch 2015) do not take into account the discrete particle effects. We thus rely on the Klimontovich formalism in order to describe the spontaneous emission. In general there exists an infinite array of normal modes, especially near multiple harmonic ion-cyclotron modes. However, a quiescent plasma naturally “selects” the
most relevant modes, so that one may locate the most important roots guided by the spontaneous emission calculation. Indeed, some confusion exists, at least in terms of terminology, regarding the multiple-harmonic cyclotron modes, or equivalently, ion-Bernstein modes, and highly damped quasi-modes. For instance, the closely packed multiple-harmonic cyclotron modes that Podesta (2012) identifies as the IBWs may actually be higher-order quasi-modes. The broadened frequency, which Comisiel et al. (2015) interpret as the sideband generation, may be alternatively understood in terms of the presence of non-collective quasi-modes. The present paper will show that the emission in the quasi-mode bands is a natural consequence of the spontaneously emitted fluctuations.

The organization of the present paper is as follows. In Section 2 we outline the theory and one-dimensional hybrid simulation of a spontaneously emitted low-frequency (that is, frequency lower than electron plasma or cyclotron frequencies) regime. Guided by the intensity of spontaneous emission pattern in the angular frequency versus the wavenumber spectrum, we also carry out the linear Vlasov analysis in order to locate the most relevant complex roots depicting the plasma dispersion relation. Section 3 concludes the present paper and we discuss the most salient findings.

2. Theory and Simulation of Spontaneous Emission in the Low-frequency Regime

In the present section we discuss the simulation and theory of spontaneous emission as well as the linear Vlasov analysis. We perform a 1.5D (one spatial dimension and three-dimensional velocity) hybrid simulation. We treat the ions (protons) as particles and the electrons are approximately treated as a neutralizing background massless fluid. We set the constant background magnetic field to be at an angle to the simulation direction,

\[ B_0 = B_0 (\sin \theta \, e_1 + \cos \theta \, e_3) , \]

where \( e_1 \) represents the simulation dimension. The system size is \( L = 502.6 \, v_A / \Omega_p \), with \( n_p = 4096 \) grid cells and 4000 particles per cell. Here,

\[ v_A = \frac{B_0}{\sqrt{4 \pi n_p m_p}} \]

is the Alfvén speed, where \( n_p \) and \( m_p \) are the proton density and mass. On the other hand,

\[ \Omega_p = \frac{e B_0}{m_p c} \]

is the proton cyclotron frequency, where \( c \) is the speed of light. We ran the simulation until \( t_{max} = 655.36 / \Omega_p \) using a time step of \( \Delta t = 0.01 / \Omega_p \). Initially the protons are distributed in the velocity space according to their thermal equilibrium (Maxwellian) form. Both the protons and the massless fluid electrons have the same plasma beta,

\[ \beta = \frac{8 \pi n_a T_a}{B_0^2} = \beta , \]

where \( n_a \) is the density and \( T_a \) is the temperature of species \( a \).

As the system evolves the random thermal motions of the protons spontaneously emit electromagnetic field fluctuations, which are reabsorbed by the protons until a balance between the emission and absorption (or fluctuation-dissipation) is achieved in a steady state.

The Fourier transformation in both the space and the time of the total magnetic field fluctuations obtained from the simulation is displayed in Figure 1, for \( \beta = 0.1 \), and for several different magnetic field inclination angles, \( \theta \), which is equivalent to the angle between the wave vector \( k \) and the ambient magnetic field. Figure 1(a) corresponds to \( \theta = 10^\circ \), which is a quasi-parallel case, and is consistent with previous works for \( \theta = 0^\circ \) (Araneda et al. 2011; Navarro et al. 2014).

The two quasi-parallel normal modes, Alfvén/ion-cyclotron (AIC) and MSW branches, can be identified by the enhanced emission pattern in the vicinities of these modes. In addition, a broad swath of spontaneously emitted fluctuations starting from \( \omega = \Omega_p \) for the near \( k \sim 0 \) region can be observed and as \( k \) increases, the frequency progressively widens. Such a feature corresponds to the non-collective modes spontaneously emitted by the thermal plasma, as first identified by Araneda et al. (2011).

The situation of most interest in the context of solar wind turbulence research is that of quasi-perpendicular angles. For \( \theta = 70^\circ \), Figure 1(b), the emission pattern begins to make the transition from the AIC/MSW dual mode structure to a more complicated form possessing multiple-harmonic ion-cyclotron (ion-Bernstein) bands. For \( \theta = 80^\circ \) and \( 85^\circ \), Figures 1(c) and (d), respectively, it can be seen that the fast MSW branch splits up and smoothly transitions to the IBW mode at each harmonic. However, it is interesting to note that while the MSW mode is interspersed by the IBW mode, the mode nevertheless retains a continuity beyond the strict limit of low frequency. Note that MSWs and KAWs are defined under the assumption of \( \omega \ll \Omega_p \), whereas Figure 1 shows that these modes are well defined beyond the limit of low frequency, \( \omega \ll \Omega_p \), and extend to proton cyclotron frequency and beyond. While this is well known within the context of the linear Vlasov analysis of the wave dispersion relation, Figure 1 demonstrates this feature within the context of the spontaneous emission spectrum. For oblique angles, the AIC mode now takes on the characteristics of KAWs, which merge into the higher-order mode (HOM) at the high \( k \)-region, beyond which the mode loses its characteristics. Again, however, it should be noted that KAWs are defined up to the proton cyclotron frequency range, which is beyond the strict limit of the definition of a KAW. The KS (ion-acoustic) mode is a quasi-electrostatic mode for small \( \theta \), so it does not show up in the magnetic field spectrum for \( \theta = 10^\circ \). For quasi-perpendicular \( \theta \), however, the KS mode contains a compressional magnetic field perturbation, so it does appear in the \( B \)-field spectrum, but for the present low beta case the mode has a low frequency, and as such, it is not evident.

In a recent paper Yoon & López (2017a, 2017b) formulated a fully general expression for the spontaneously emitted electromagnetic fluctuations in magnetized and thermal plasmas. According to their paper, the magnetic field fluctuation spectrum
is given by

\[
\langle |dB|^2 \rangle = \frac{1}{2\pi \omega |\Lambda|^2} \omega^2 
\left[ |a_{12}|^2 \sigma_{11} + |a_{23}|^2 \sigma_{22} + |a_{23}|^2 \sigma_{33}
\right.
\left. - (a_{12}a_{22}^* - a_{12}a_{23}^*)\sigma_{12} - (a_{12}a_{33}^* + a_{12}a_{23}^*)\sigma_{13}
\right.
\left. - (a_{23}a_{23}^* - a_{23}a_{23}^*)\sigma_{23} + \cos \theta |a_{11}|^2 \sigma_{11}
\right. \\
+ \left. |a_{12}|^2 \sigma_{22} + |a_{13}|^2 \sigma_{33} + (a_{14}a_{12}^* - a_{14}a_{12}^*)\sigma_{12}
\right.
\left. + (a_{14}a_{13}^* + a_{14}a_{13}^*)\sigma_{13} - (a_{12}a_{13}^* - a_{12}a_{13}^*)\sigma_{23}
\right]
\left. + \sin^2 \theta |a_{13}|^2 \sigma_{13} + |a_{23}|^2 \sigma_{22} + |a_{33}|^2 \sigma_{33}
\right.
\left. + (-a_{13}a_{23}^* + a_{13}a_{23}^*)\sigma_{12} + (a_{13}a_{33}^* + a_{13}a_{33}^*)\sigma_{13}
\right.
\left. + (-a_{23}a_{33}^* + a_{23}a_{33}^*)\sigma_{23}
\right]
\left. - \cos \theta \sin \theta |a_{14}a_{12}^* + a_{14}a_{12}^*\sigma_{11}
\right]
\left. - (a_{13}a_{23}^* + a_{13}a_{23}^*)\sigma_{23} + (a_{13}a_{33}^* + a_{13}a_{33}^*)\sigma_{33}
\right.
\left. + (a_{12}a_{12}^* + a_{12}a_{12}^*)\sigma_{13} + a_{13}a_{13}^* + a_{13}a_{13}^*\sigma_{13}
\right.
\left. - (a_{12}a_{33}^* + a_{12}a_{33}^* - a_{13}a_{23}^* - a_{13}a_{23}^*)\sigma_{23} \right].
\]

where

\[
\Lambda = \frac{\Lambda_{11} \Lambda_{22} \Lambda_{33} + \Lambda_{11} \Lambda_{23}^2 + \Lambda_{12}^2 \Lambda_{33}}{\Lambda_{22} \Lambda_{13}^2 + 2 \Lambda_{12} \Lambda_{13} \Lambda_{33}},
\]

is the determinant of the dielectric response (or “dispersion”) tensor, \( \Lambda \) (Stix 1992; López & Yoon 2017; Yoon & López 2017a, 2017b), given by

\[
\Lambda = \begin{pmatrix} \Lambda_{11} & \Lambda_{12} & \Lambda_{13} \\ -\Lambda_{12} & \Lambda_{22} & \Lambda_{23} \\ \Lambda_{13} & -\Lambda_{23} & \Lambda_{33} \end{pmatrix}.
\]

Figure 1. Magnetic fluctuation spectra obtained from the hybrid simulation for \( \beta = 0.1 \) and various \( \theta \).
Here, \( \omega_p \) is the plasma frequency of species \( a \), \( e_a \) and \( m_a \) are the charge and mass of species \( a \),

\[
\Lambda_{23} = i \sum_a \frac{\omega_p^2}{\omega^2} \sum_{n=-\infty}^{\infty} \frac{k_{\parallel} v_{Ta}}{2 \Omega_a} \Lambda'_n(\lambda_a) \xi_a Z'(\zeta_a^n),
\]

\[
\Lambda_{33} = 1 - \frac{c^2 k_{\parallel}^2}{\omega^2} - \sum_a \frac{\omega_p^2}{\omega^2} \sum_{n=-\infty}^{\infty} \Lambda_n(\lambda_a) \xi_a^n Z'(\zeta_a^n).
\]

Figure 2. Magnetic fluctuation spectra obtained from theory, Equation (1), for \( \beta = 0.1 \), \( v_\parallel/c = 10^{-3} \), and various \( \theta \).

The Astrophysical Journal, 845:60 (7pp), 2017 August 10 López et al.
must employ the more general Klimontovich formalism a statistically averaged distribution function, and therefore the since the Vlasov equation describes the spacetime evolution of
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Figure 3. The Astrophysical Journal, 845:60 (7pp), 2017 August 10

\[
\begin{align*}
\sigma_{12} &= \pi^{1/2} \sum a \frac{\omega_{pe}^2}{\omega^2} \sum_{n=-\infty}^{\infty} n \Lambda_n(\lambda_a) \xi_n e^{-\langle \xi_n \rangle^2}, \\
\sigma_{11} &= \pi^{1/2} \sum a \frac{\omega_{pe}^2}{\omega^2} \sum_{n=-\infty}^{\infty} k_1 \gamma_a \frac{n \Lambda_n(\lambda_a)}{\lambda_a} \xi_n \xi_n^* e^{-\langle \xi_n \rangle^2}, \\
\sigma_{23} &= \pi^{1/2} \sum a \frac{\omega_{pe}^2}{\omega^2} \sum_{n=-\infty}^{\infty} k_1 \gamma_a \Lambda_n(\lambda_a) \xi_n \xi_n^* e^{-\langle \xi_n \rangle^2}, \\
\sigma_{33} &= \pi^{1/2} \sum a \frac{\omega_{pe}^2}{\omega^2} \sum_{n=-\infty}^{\infty} 2 \lambda_n(\lambda_a) \xi_n \xi_n^* e^{-\langle \xi_n \rangle^2}.
\end{align*}
\]

Note that the above fluctuation spectrum, Equation (1), could not be discussed on the basis of the collisionless Vlasov theory, since the Vlasov equation describes the spacetime evolution of a statistically averaged distribution function, and therefore the information of individual particles is not retained. Thus, one must employ the more general Klimontovich formalism (Ichimaru 1992). For a detailed derivation, see Yoon & López (2017a, 2017b). In Figure 2 we show the theoretical magnetic field fluctuation spectrum computed from Equation (1), for the same parameters as in Figure 1. Note the excellent agreement between the theory and the simulation. We thus have solid theoretical ground to interpret the simulation result.

In order to further understand the spontaneous emission spectrum, next we solve the full electromagnetic linear dispersion relation, \( \Lambda = 0 \), in Equation (2) (Stix 1992). It is well known that for a given value of \( k \) and \( \theta \), there are an infinite number of roots, although most of them are highly damped (Araneda et al. 2011; Podesta 2012). Thermal plasma naturally selects the most relevant solutions, as the spontaneous emission spectrum displays. These happen to be the least damped roots in most cases. Guided by this, we solved for a group of the most relevant roots. In Figure 3 we plotted the numerical roots for \( \beta = 0.1, \theta = 88^\circ \), and \( v_A/c = 10^{-3} \). Figures 3(a) and (b) display the real frequency, \( \omega \), and imaginary part, \( \gamma \), as a function of the wavenumber, respectively. Figures 3(a) and (b) are in many ways similar to the theoretical analysis that is already available in the literature; see, e.g., the kinetic dispersion relation analysis carried out by Podesta (2012). The purpose of the linear dispersion relation analysis presented here is to aid in the identification and interpretation of the enhanced fluctuation spectra shown in Figure 1 or 2. In order to facilitate the visual identification we have color coded each root; the red curve is for the KS mode (Narita & Marsch 2015; or ion-acoustic mode; Gary 1993), the blue curve is for the KAW mode, and green represents part of the MSW mode and part of the first harmonic IBW mode below \( \Omega_p \). Although it is not shown, we have checked the polarization of each mode (Gary 1986). The brown and orange curves are part MSW and part higher-harmonic IBW modes (Podesta 2012; Sahraoui et al. 2012). Finally, the black dotted lines are the HOMs. These are highly damped quasi-modes, and there are an infinite number of them all closely packed together. The dotted lines represent the inner envelope (the least damped ones). Podesta (2012) numerically found a few of them, which he identifies as IBW modes, but we prefer the term HOMs for their analogy with the parallel case (Araneda et al. 2011; Navarro et al. 2014; Viñas et al. 2014). In our designation the genuine IBWs are actually the \((n + 1/2)/\Omega_p\) modes. In the magnetosphere, the electron Bernstein waves (EBWs) are sometimes called the \((n + 1/2)/\Omega_p\) modes, as EBWs exist between the two adjacent harmonics. Note the similar structure for our IBWs. The salient feature in the present linear Vlasov analysis is again, the quasi-continuous feature associated with the MSW mode beyond its strict dispersion relation on top of the simulated magnetic field fluctuation spectrum. These happen to be the least damped modes. For higher plasma betas, as shown in Figure 4, spontaneously emitted B-field fluctuation spectrum becomes progressively broadened and more intense. For higher frequencies, the gap between IBWs increases as the beta value is raised. Also, the KAWs and IBWs merge into a continuous broadened
spectrum, thus justifying the treatment of the KAW as a single continuous branch regardless of the IBWs for the high-beta plasmas. The spectral power in the KAW branch as displayed in Figure 4(d) is very intense, showing the dominance of KAW in high-beta plasma, such as the solar wind near 1 au.

Finally, in Figure 5 we show the linear dispersion relation for $\beta = 1.0$, $q = 88^\circ$, and $v_A/c = 10^{-3}$. It is seen that IBWs are the least damped modes, especially for high $k$-values. It is noteworthy that the KAWs suffer a drastic increase in the damping rate for $k v_A/\Omega_p \approx 5.5$ (Figure 4(d)), but despite the high damping rate the simulation (and theoretical calculation based upon Equation (1), which is essentially the same as Figure 4—not shown) shows an intense spectral intensity in the vicinity of the KAW mode. In the spontaneous emission (or fluctuation-dissipation) formalism, the emission intensity is balanced by the absorption (damping) in the steady state such that higher damping rate can simply be compensated for by higher emission rate (Viñas et al. 2014). Without the aid of a spontaneous emission calculation one could easily be misled to dismiss the importance of the KAWs for the high $k$-domain by simply judging from its high damping rate.

3. Conclusions

In conclusion, in the present paper we have elucidated the intricate structure associated with the low-frequency electromagnetic waves based upon the linear Vlasov analysis and the theory/simulation of spontaneous emission of electromagnetic fluctuations. The significance of the present work is that it lays a firm theoretical foundation for the study of plasma turbulence in the context of solar wind research. The general consensus is that the dissipation of solar wind turbulence at the kinetic scale is essential for understanding how the solar corona is heated and how the upper atmosphere of the Sun is organized in the form of solar wind. Despite such a general picture, the detailed physics of the kinetic dissipation process is poorly understood. Recent studies (Podesta 2012; Verscharen et al. 2012; Comišel et al. 2013, 2015; Narita & Marsch 2015) suggest that the actual physics of the kinetic scale dissipation process may involve a host of plasma eigenmodes and mutual interactions among them as well as with the particles. Before one can even attempt to develop a sophisticated model of the kinetic scale turbulence involving these modes, it is imperative that we understand the basic mode structure first. The aim of the present paper had been to accomplish such a task.

It should be emphasized, however, that the actual solar wind is strongly turbulent. In such a system the spontaneously emitted fluctuations may not play a direct dynamic role. However, the purpose of the present paper had been to elucidate the fundamental mode structure aided by the spontaneous emission theory. It was shown in the present paper that the spontaneous emission calculation is particularly
useful in identifying the most relevant normal modes. An interesting finding is that the low-frequency magnetosonic and kinetic Alfvén waves retain their characteristic wave properties beyond the strict frequency and wavenumber domains that define these modes in the sense of MHD, namely, these modes must be sufficiently lower than that of the proton cyclotron frequency, and that the wavelengths must be sufficiently longer than that of proton gyro radius or the proton inertial length. It is already known, on the basis of the linear Vlasov theory, that these MHD modes extend toward a wider range of frequencies while still maintaining their essential dispersive characteristics. The wider range of frequencies include the proton cyclotron frequency and its harmonics, and the wavelengths extend to a shorter regime than that of the proton inertial length. However, the extension of the MHD modes to wider frequencies and wavelengths is meaningful only if one ignores the ion-Bernstein and higher-order quasi-modes, which dissect the MHD modes at each harmonic. The present investigation provides the justification for such a treatment in that the spontaneously emitted fluctuations for the high-beta regime smoothes out the detailed cyclotron mode structures so that it is legitimate to ignore the gaps associated with multiple-harmonic cyclotron mode structures. While such a finding provides the justification for the focus of MHD modes as playing a predominant role in the kinetic description dissipation process, the roles of the cyclotron harmonic and the Bernstein modes in the kinetic dissipation process may nevertheless be important, especially in view of the fact that these modes are associated with high damping rates.

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**Figure 5.** Linear dispersion relation, $\Lambda = 0$, for $\theta = 88^\circ$, $\beta = 1.0$, and $v_\perp/c = 10^{-3}$.  

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![Figure 5](https://example.com/figure5.png)