Fourth-generation effects on the rare $B \rightarrow K^* \nu \bar{\nu}$ decay

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Abstract. The rare $B \rightarrow K^* \nu \bar{\nu}$ decay is analysed with a new up-like quark $\hat{t}$ in a sequential fourth-generation model. Two possible solutions for the fourth-generation CKM (Cabibbo–Kabayashi–Maskawa) factor $V^*_{ts}V^{\pm}_{tb}$ obtained as a function of the new $\hat{t}$-quark mass are used. The branching ratio (BR) and missing-energy spectrum of this decay in the two cases are estimated. In one case, it is shown that for $m_{\hat{t}} \geq 200$ GeV a significant enhancement to the BR and the missing-energy spectrum of this decay over the SM (standard model) is recorded, while the results are almost same in the other case. If a fourth generation should exist in nature and nature chooses this case, this B-meson decay mode could be a good probe for the existence of the fourth generation, or perhaps a signal for a new physics.

1. Introduction

The SM has been widely discussed in the literature, and serves as an explicit model for studying all low-energy experimental data. But there is no doubt that the SM is an incomplete theory. Among the unsolved problems within the SM is the CP violation, and the number of generations. In the SM there are three generations, and yet there is no theoretical argument to explain why there are three and only three generations of fermions in the SM. From the LEP result of the invisible partial decay width of the $Z$ boson it follows that the mass of an extra-generation neutrino $N$ should be larger than 45 GeV [1]. Having this experimental result in mind, and if we believe that the fourth-generation fermions really exist in nature, we should be able to find their mass spectrum, and take into account their physical effects in low-energy physics.

One of the promising areas in the experimental search for a fourth generation via its indirect loop effects is that of the rare B-meson decays. The experimental observation of the inclusive $b \rightarrow X_s \gamma$ [2], and exclusive $B \rightarrow K^* \gamma$ [3] decays, together with the recent Babar
Collaboration [4] upper limit on the exclusive decay \( BR(B \rightarrow K^*\ell^+\ell^-) < 2.9 \times 10^{-6} \), and the Belle Collaboration [5] result \( BR(B \rightarrow K\ell^+\ell^-) = (0.75^{+0.25}_{-0.21} \pm 0.9) \times 10^{-6} \), at SLAC and KEK laboratories, respectively, will allow us to set up a complete programme to test the SM properties at the loop level and constrain various new physics scenarios.

On this basis, serious attempts to study the effects of the fourth-generation fermions on the rare B-meson have been made by many authors. For example, the effect of the fourth-generation quarks on the process \( b \rightarrow s \) have been previously investigated in [8, 9], and it is shown that in the fourth-generation model the \( b \rightarrow s\gamma \) branching ratio (BR) is essentially within the range allowed by CLEO [9]. On the other hand, the \( B \rightarrow X_u\ell^+\ell^- \), and the \( B \rightarrow X_u\gamma \) decays with the fourth-generation fermions are analysed in [10]. Recently, the fourth-generation effects on the rare decays \( B \rightarrow K^*\ell^+\ell^- \) [11], \( B \rightarrow \ell^+\ell^- \), \( B \rightarrow \ell^+\ell^-\gamma \) [12], and \( B_s \rightarrow \nu\bar{\nu}\gamma \) [13] were studied.

The exclusive decay \( B \rightarrow K^*\nu\bar{\nu} \) provokes special interest in the SM and beyond [14]–[18]. In particular, it is shown that the MSSM remains perturbative up to the unification scale \( M_U \) of the Yukawa couplings. The implications of a fourth generation of quarks on the process \( b \rightarrow s \) have been previously investigated in [8, 9], and it is shown that in the fourth-generation model the \( b \rightarrow s\gamma \) branching ratio (BR) is essentially within the range allowed by CLEO [9]. On the other hand, the \( B \rightarrow X_u\ell^+\ell^- \), and the \( B \rightarrow X_u\gamma \) decays with the fourth-generation fermions are analysed in [10]. Recently, the fourth-generation effects on the rare decays \( B \rightarrow K^*\ell^+\ell^- \) [11], \( B \rightarrow \ell^+\ell^- \), \( B \rightarrow \ell^+\ell^-\gamma \) [12], and \( B_s \rightarrow \nu\bar{\nu}\gamma \) [13] were studied.

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This paper is organized as follows. In section 2, the relevant effective Hamiltonian for the decay \( B \rightarrow K^*\nu\bar{\nu} \) and the existence of a new up-like quark \( t \) in a sequential fourth-generation model SM, which we shall call SM4 hereafter for the sake of simplicity. This model is considered as a natural extension of the SM, where the fourth-generation model is introduced in the same way as the three generations are introduced in the SM [19], so no new operators appear, and clearly the full operator set is exactly the same as in the SM. Hence, the fourth generation will change only the values of the Wilson coefficients via virtual exchange of the fourth-generation up-like quark \( t \).

This paper is organized as follows. In section 2, the relevant effective Hamiltonian for the decay \( B \rightarrow K^*\nu\bar{\nu} \) and the existence of a new up-like quark \( t \) in a sequential fourth-generation model SM4 is presented; therein the dependence of the BR and the missing-energy spectrum on the fourth-generation model parameters for the decay of interest are investigated using the results of the light-cone QCD sum rules for estimating form factors. Section 3 is devoted to the numerical analysis and concluding remarks.

2. Effective Hamiltonian

In the SM, the process \( B \rightarrow K^*\nu\bar{\nu} \) is described at quark level by the \( b \rightarrow s\nu\bar{\nu} \) transition, and receives contributions from Z-penguin and box diagrams, where dominant contributions come from intermediate top quarks. The effective Hamiltonian responsible for \( b \rightarrow s\nu\bar{\nu} \) decay is described by only one Wilson coefficient, namely \( C_{11}^{(SM)} \), and its explicit form is [20]

\[
H_{\text{eff}} = \frac{G_F \alpha}{2\pi \sqrt{2} \sin^2 \theta_W} C_{11}^{(SM)} V_{ts}^* V_{tb} \bar{s} \gamma_\mu (1 - \gamma_5) b \nu \bar{\nu} (1 - \gamma_5) \nu,
\]

New Journal of Physics 4 (2002) 25.1–25.12 (http://www.njp.org/)
where \( G_F \) is the Fermi coupling constant, \( \alpha \) is the fine-structure constant (at the Z mass scale), and \( V_{ts}^*V_{tb} \) are products of Cabibbo–Kobayashi–Maskawa matrix elements. In equation (1), the Wilson coefficient \( C_{11}^{SM} \) in the context of the SM has the following form including \( O(\alpha_s) \) corrections [21]:

\[
C_{11}^{SM} = \left[ X_0(x_t) + \frac{\alpha_s}{4\pi} X_1(x_t) \right],
\]

with

\[
X_0(x_t) = \frac{x_t}{8} \left[ x_t + 2 + \frac{3(x_t - 2)}{(x_t - 1)^2} \ln(x_t) \right],
\]

where \( x_t = m_t^2/m_W^2 \), and

\[
X_1(x_t) = \frac{4x_t^3 - 5x_t^2 - 23x_t}{3(x_t - 1)^2} - \frac{x_t^4 + x_t^3 - 11x_t^2 + x_t}{(x_t - 1)^3} \ln(x_t) + \frac{x_t^4 - x_t^3 - 4x_t^2 - 8x_t}{2(x_t - 1)^3} \ln^2(x_t)
\]

\[
+ \frac{x_t^3 - 4x_t}{(x_t - 1)^2} \text{Li}_2(1-x_t) + 8x_t \frac{\partial X_0(x_t)}{\partial x} \ln(x_\mu).
\]

Here \( \text{Li}_2(1-x_t) = \int_{x_t}^{1} \left( \ln t \right) / (1-t) \, dt \) is a specific function, and \( x_\mu = \mu^2/m_W^2 \) with \( \mu = O(m_t) \).

At \( \mu = m_t \), the QCD correction for the \( X_1(x_t) \) term is very small (around \( \sim 3\% \)).

From the theoretical point of view, the transition \( b \rightarrow s\nu\bar{\nu} \) is a very clean process, since it is practically free from scale dependence, and free from any long-distance effects. In addition, the presence of a single operator governing the inclusive \( b \rightarrow s\nu\bar{\nu} \) transition is an appealing property. Therefore, the theoretical uncertainty within the SM is just related to the value of the Wilson coefficient \( C_{11}^{SM} \) due to the uncertainty in the top-quark mass. In this work, we have considered possible new physics in \( b \rightarrow s\nu\bar{\nu} \) only through the value of the Wilson coefficient.

In this spirit, the transition \( b \rightarrow s\nu\bar{\nu} \) in equation (1) can only include extra contribution due to the fourth-generation fermion \( m_t \); hence, the fourth-generation fermion contribution modifies only the value of the Wilson coefficient \( C_{11}^{SM} \), and it does not induce any new operators:

\[
C_{11}^{SM4} = C_{11}^{SM} + \frac{V_{ts}^*V_{tb}}{V_{tb}V_{ts}} C^{(new)},
\]

where \( C^{(new)} \) can be obtained from \( C_{11}^{SM} \) by making the substitution \( m_t \rightarrow m_t \).

As a result, we obtain a modified effective Hamiltonian, which represents \( b \rightarrow s\nu\bar{\nu} \) decay in the presence of the fourth-generation fermion:

\[
H_{eff} = \frac{G\alpha}{2\pi \sqrt{2} \sin^2 \theta_w} V_{ts}^*V_{tb} [C_{11}^{SM4}] \bar{s}\gamma_\mu(1-\gamma_5)b\bar{\nu}\gamma_\mu(1-\gamma_5)\nu.
\]

However, in spite of such theoretical advantages, it would be a very difficult task to detect the inclusive \( b \rightarrow s\nu\bar{\nu} \) decay experimentally, because the final state contains two missing neutrinos and many hadrons. Therefore, only the exclusive channels, namely \( B \rightarrow K^*\nu\bar{\nu} \), are well suited for consideration in the search for possible ‘new physics’ effects and constraints.

In order to compute \( B \rightarrow K^*\nu\bar{\nu} \) decay, we need the matrix elements of the effective Hamiltonian, equation (6), connecting the final and initial meson states. This problem is related to the non-perturbative sector of QCD and can be solved only by using non-perturbative methods. The matrix element \( \langle K^* | H_{eff} | B \rangle \) has been investigated in a framework of different approaches.
such as chiral perturbation theory [22], three-point QCD sum rules [23], the relativistic quark model with the light front formalism [24], effective heavy-quark theory [25], and light-cone QCD sum rules [26, 27]. To begin with, let us denote by \( P_B \) and \( P_{K^*} \) the four-momentum of the initial and final mesons, and define \( q = P_B - P_{K^*} \) as the four-momentum of the \( \nu \bar{\nu} \) pair, and \( x \equiv E_{\text{miss}}/M_B \) as the missing-energy fraction, which is related to the squared four-momentum transfer \( q^2 \) by \( q^2 = M_B^2[2x - 1 + r_{K^*}] \), where \( r_{K^*} \equiv M_{K^*}^2/M_B^2 \), with \( M_B \) and \( M_{K^*} \) being the initial and final meson masses. Then, the hadronic matrix element for \( B \to K^* \nu \bar{\nu} \) can be parametrized in terms of five form factors:

\[
\langle K_h^* | \bar{s}_\mu (1 - \gamma_5) b | B \rangle = \frac{2V(q^2)}{M_B + M_{K^*}} \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu}(h) P_B^\alpha P_{K^*}^\beta \\
\quad - i \left[ \epsilon_\mu(h)(M_B + M_{K^*})A_1(q^2) - [\epsilon^*(h) \cdot q](P_B + P_{K^*})_\mu \frac{A_2(q^2)}{M_B + M_{K^*}} \right] \\
\quad - q_\mu [\epsilon^*(h) \cdot q] \frac{2M_{K^*}}{q^2} [A_3(q^2) - A_0(q^2)],
\]

(7)

where \( \epsilon(h) \) is the polarization four-vector of the \( K^* \)-meson. The form factor \( A_3(q^2) \) can be written as a linear combination of the form factors \( A_1 \) and \( A_2 \):

\[
A_3(q^2) = \frac{1}{2M_{K^*}} \left[ (M_B + M_{K^*})A_1(q^2) - (M_B - M_{K^*})A_2(q^2) \right].
\]

(8)

After performing summation over \( K^* \)-meson polarization and taking into account the number of light neutrinos \( N_\nu = 3 \) for the differential of the decay width, one can get [20]

\[
\frac{d\Gamma(B \to K^* \nu \bar{\nu})}{ds} = \frac{G_F^2 \alpha^2 |V_{ub}V_{ts}^*|^2}{2^{10} \pi^3 \sin^3 \theta_w} \lambda^1/2(1, r_{K^*}, s) M_B^2 |C_{11}^{SM}|^2 \\
\times \left\{ 8\lambda s \frac{V^2(q^2)}{(1 + \sqrt{r_{K^*}})^2} + \frac{1}{r_{K^*}} \left[ \frac{\lambda^2}{(1 + \sqrt{r_{K^*}})^2} \\
\quad + (1 + \sqrt{r_{K^*}})^2(\lambda + 12r_{K^*} s) A_1^2(q^2) - 2\lambda(1 - r_{K^*} - s) \text{Re}(A_1(q^2)A_2(q^2)) \right] \right\},
\]

(9)

where \( \lambda(1, r_{K^*}, s) = 1 + r_{K^*}^2 + s^2 - 2r_{K^*} s - 2r_{K^*} - 2s \) is the usual triangle function, and \( s = q^2/M_B^2 \).

From equation (7), it is easy to derive the missing-energy distribution corresponding to the helicity \( h = 0, \pm 1 \) of the \( K^* \)-meson:

\[
\frac{d\Gamma(B \to K^*_{h=0} \nu \bar{\nu})}{dx} = \frac{G_F^2 \alpha^2 M_B^2 |V_{ub}V_{ts}^*|^2}{64\pi^3 \sin^4 \theta_w} |C_{11}^{SM}|^2 \sqrt{(1 - x)^2 - r_{K^*}} \\
\times \left\{ (1 + \sqrt{r_{K^*}})^2 (1 - x - r_{K^*}) A_1(q^2) - 2[(1 - x)^2 - r_{K^*}] A_2(q^2) \right\}^2,
\]

(10)

\[
\frac{d\Gamma(B \to K^*_{h=\pm 1} \nu \bar{\nu})}{dx} = \frac{G_F^2 \alpha^2 M_B^2 |V_{ub}V_{ts}^*|^2}{64\pi^3 \sin^4 \theta_w} |C_{11}^{SM}|^2 \sqrt{(1 - x)^2 - r_{K^*}} \\
\times \frac{2x - 1 + r_{K^*}}{(1 + \sqrt{r_{K^*}})^2} [2\sqrt{(1 - x)^2 - r_{K^*}} V(q^2) \mp (1 + \sqrt{r_{K^*}})^2 A_1(q^2)]^2.
\]

(11)

From equations (9)–(11), one can see that the decay width for \( B \to K^* \nu \bar{\nu} \) contains three form factors: \( V \), \( A_1 \), and \( A_2 \). These form factors were calculated in the framework of QCD sum
Figure 1. The dependence of the BR of the decay $B \to K^*\nu\bar{\nu}$ on the fourth up-like-quark mass $m_t$. 

rules in [26]–[28]. However, in this work, in estimating the total decay width, we have used the results of [28], where these form factors were calculated by including one-loop radiative corrections to the leading twist-two contribution:

$$F(q^2) = \frac{F(0)}{1 - a_F(q^2/M_B^2) + b_F(q^2/M_B^2)^2},$$

and the relevant values of the form factors at $q^2 = 0$ are

$$A_1^{B\to K^*}(q^2 = 0) = 0.34 \pm 0.05,$$

with $a_F = 0.6$, and $b_F = -0.023$,

$$A_2^{B\to K^*}(q^2 = 0) = 0.28 \pm 0.04,$$

with $a_F = 1.18$, and $b_F = 0.281$, 

and

$$V_{B\to K^*}(q^2 = 0) = 0.46 \pm 0.07,$$

with $a_F = 1.55$, and $b_F = 0.575$. 

Note that all errors which come out are due to the uncertainties of the b-quark mass; the Borel parameter variation, wavefunctions, and radiative corrections are added in quadrature. Finally, to obtain quantitative results one needs the values of the fourth-generation CKM matrix elements $V_{ts}^*V_{tb}$. Following [6], the values of the fourth-generation CKM factor $V_{ts}^*V_{tb}^\pm$ due to the masses of $t$ are listed in table 1.

A few comments about the numerical values of $(V_{ts}^*V_{tb})^\pm$ are in order. From the unitarity condition for the CKM matrix we have

$$V_{us}^*V_{ub} + V_{cs}^*V_{cb} + V_{ts}^*V_{tb} + V_{ts}^*V_{tb} = 0.$$

New Journal of Physics 4 (2002) 25.1–25.12 (http://www.njp.org/)
Figure 2. The dependence of the differential of the BR of the decay $B \to K^* \nu \bar{\nu}$ on $s$ at a fixed value of $m_t$.

Figure 3. The dependence of the ratio $R$ on the fourth up-like-quark mass $m_t$. 

New Journal of Physics 4 (2002) 25.1–25.12 (http://www.njp.org/)
Figure 4. The dependence of the ratio $R_1$ on the fourth up-like-quark mass $m_t$.

Table 1. The numerical values of $V^*_{ts}V_{tb}$ for different values of $m_t$ for $BR(B \rightarrow X_s \gamma) = 2.66 \times 10^{-4}$.

| $m_t$ (GeV) | 50 | 100 | 150 | 200 | 250 | 300 | 350 |
|------------|----|-----|-----|-----|-----|-----|-----|
| $(V^*_{ts}V_{tb})^+/10^{-2}$ | -11.591 | -9.259 | -8.126 | -7.501 | -7.116 | -6.861 | -6.580 |
| $(V^*_{ts}V_{tb})^-/10^{-3}$ | 3.564 | 2.850 | 2.502 | 2.309 | 2.191 | 2.113 | 2.205 |

| $m_t$ (GeV) | 400 | 500 | 600 | 700 | 800 | 900 | 1000 |
|------------|-----|-----|-----|-----|-----|-----|-----|
| $(V^*_{ts}V_{tb})^+/10^{-2}$ | -6.548 | -6.369 | -6.255 | -6.178 | -6.123 | -6.082 | -6.051 |
| $(V^*_{ts}V_{tb})^-/10^{-3}$ | 2.016 | 1.961 | 1.926 | 1.902 | 1.885 | 1.872 | 1.863 |

When the values of the CKM matrix elements in the SM are used [29], the sum of the first three terms in equation (16) is about $7.6 \times 10^{-2}$. Substituting in the values of $(V^*_{ts}V_{tb})^+$ from table 1, it is observed that the sum of the four terms on the left-hand side of equation (16) in this case is much better, and much closer to zero than that in the SM, because the value of $(V^*_{ts}V_{tb})^+$ is very close to the sum of the first three terms, but has the opposite sign. This holds true, and is clear when $m_t \geq 200$ GeV, and this may be the most direct lower bound on the mass of $t$. If one considers $(V^*_{ts}V_{tb})^-$, whose value is about $10^{-3}$, which is one order of magnitude smaller compared to the previous case, it is easy to see then that the values of $(V^*_{ts}V_{tb})^-$ also satisfy the unitarity condition of CKM, and the error in the sum of the four terms in equation (16)
Figure 5. The dependence of the differential of the BR of the decay $B \rightarrow K^*\nu\bar{\nu}$ on the missing-energy fraction $x$ at a fixed value of $m_t$ when $K^*$ is polarized longitudinally.

is within the SM error range. Therefore, the $(V^*_{ts} V^*_{tb})^-$ contribution to the physical quantities should be practically indistinguishable from SM results, and our numerical analysis will confirm this expectation. On the other hand, there are no available upper bounds on the mass of the $\tilde{t}$, but one generally expects to have $m_{\tilde{t}} \leq 1$ TeV in order for the perturbation theory to remain valid. The large $m_{\tilde{t}}$-values will lead on to a detailed discussion of the $\rho_0$-parameter [30].

3. Numerical analysis

In studying the influence of the fourth-generation model parameters on the BR $BR(B \rightarrow K^*\nu\bar{\nu})$, the missing-energy spectra, and BRs of rare $B \rightarrow K^*_L\nu\bar{\nu}$ and $B \rightarrow K^*_T\nu\bar{\nu}$ decays (where $K^*_L$ and $K^*_T$ stand for longitudinally and transversely polarized $K^*$-mesons, respectively), the following values have been used as input parameters: $G_F = 1.17 \times 10^{-5}$ GeV$^{-2}$, $\alpha = 1/137$, $\mu_b = m_b = 5.0$ GeV, $M_R = 5.28$ GeV, $|V^*_{ts} V^*_{tb}| = 0.045$, $M_{K^*} = 0.892$ GeV, and the lifetime is taken as $\tau(B_d) = 1.56 \times 10^{-12}$ s [30]; also we have run calculations of equations (9)–(11) adopting the two sets of values of $(V^*_{ts} V^*_{tb})^\pm$ from table 1. The numerical results for the BR and the missing-energy spectra are presented in series of graphs. The BR for $B \rightarrow K^*\nu\bar{\nu}$ decay as a function of $m_{\tilde{t}}$, with the different values of $(V^*_{ts} V^*_{tb})^\pm$, is shown in figure 1. It can be seen that when $V^*_{ts} V^*_{tb}$ takes positive values, i.e. for $(V^*_{ts} V^*_{tb})^-$, the BR almost matches that of the SM. That is, the results in SM4 are the same as those in the SM, except a peak in the curve when $m_{\tilde{t}}$ takes values $m_{\tilde{t}} \geq 210$ GeV. The reason for this is not that there is new predicted deviation from SM,
Figure 6. The dependence of the differential of the BR of the decay $B \rightarrow K^* \nu \bar{\nu}$ on the missing-energy fraction $x$ at a fixed value of $m_t$ when $K^*$ is polarized transversely.

but just the second term of equation (5). In this case, it does not show the new effects of $m_t$. Also, we cannot establish the existence of the fourth generation from the BR for $B \rightarrow K^* \nu \bar{\nu}$, although we cannot exclude the possibility of existence either. This is because, as seen from table 1, the values of $(V_{ts}^* V_{tb})^-$ are positive. But they are of order $10^{-3}$, i.e. very small. The values of $V_{ts}^* V_{tb}$ are about ten times larger than $V_{ts}^* = 0.045$, $V_{tb} = 0.9995$ (see [30]). But in the second case, when the values of $V_{ts}^* V_{tb}$ are negative, i.e. for $(V_{ts}^* V_{tb})^+$, the curve of BR for $B \rightarrow K^* \nu \bar{\nu}$ is quite different from that of the SM. This can be clearly seen from figure 1. The BR increases rapidly with increase of $m_t$. In this case, the fourth-generation effects are shown clearly. The reason is that $(V_{ts}^* V_{tb})^+$ is 2–3 times larger than $V_{ts}^* V_{tb}$, so the last term in equation (5) becomes important, and it depends on the $t$ mass strongly. Thus the effect of the fourth generation is significant. In figure 2, we show the differential of the BR, $dBR(B \rightarrow K^* \nu \bar{\nu})/ds$, as functions of $s$ ($0 \leq s \leq (1 + \sqrt{r_{K^*}})^2$) when $m_t = 300$ GeV. The curve of the differential decay width is quite different from that of the SM when one considers $(V_{ts}^* V_{tb})^+$. This can be clearly seen from figure 2. The differential decay width increases rapidly, and the energy spectrum of the $K^*$-meson is almost symmetrical. In figure 3, the ratio $R = BR^{SM4}(B \rightarrow K^* \nu \bar{\nu})/BR^{SM}(B \rightarrow K^* \nu \bar{\nu})$ is depicted as a function of $(V_{ts}^* V_{tb})^+$ for various values of $m_t$. Figure 3 shows that for all values of $m_t \geq 210$ GeV, the value of $R$ becomes $>1$, meaning that the value of $R = 1$ is shifted. In other words, by defining the position for which $R = 1$, information can be obtained about $m_t$, the mass of the fourth-generation fermion. For completeness, we also consider the ratio $R1 = BR^{SM4}(B \rightarrow K^* \nu \bar{\nu})/BR^{SM}(B \rightarrow X_s \nu \bar{\nu})$. This ratio is plotted as a function of $(V_{ts}^* V_{tb})^+$.
for various values of $m_t$ in figure 4. It is well known that the inclusive decay width in the SM corresponding to $B \to X_s \nu \bar{\nu}$ is given as (see [20])

$$BR(B \to X_s \nu \bar{\nu}) = \frac{3 \alpha^2}{(2\pi)^2 \sin^4 \theta_W} \left| V_{tb} V^*_{ts} \right|^2 \frac{\left| C_{11}^{SM} \right|^2}{\eta_0 f(m_c/m_b)} \eta BR(B \to X_c \ell \nu),$$

(17)

where the theoretical uncertainties related to the b-quark mass dependence disappear. In equation (17) the factor 3 corresponds to the number of light neutrinos. The phase space factor $f(m_c/m_b) \approx 0.44$, the QCD correction factors $\eta_0 \approx 0.87$, $\tilde{\eta} = 1 + (2\alpha_s(m_b)/3\pi)(\frac{25}{4} - \pi^2) \approx 0.83$ [21], and the experimental measured value $BR(B \to X_c \ell \nu) = 10.14\%$. In this figure, the SM4 prediction on R1 for all the values of $m_t$ is greater than the SM prediction. This means that the SM4 contributes constructively to the decay width. In figures 5 and 6, we show the missing-energy distribution for the decays $dBR(B \to K^*_L \nu \bar{\nu})/dx$ and $dBR(B \to K^*_T \nu \bar{\nu})/dx$ as functions of $x$; $(1 - r_{K^*})/2 \leq x \leq 1 - \sqrt{r_{K^*}}$ for $m_t = 250$ GeV. It can be seen there that, when $V^*_{ts} V_{tb}$ takes positive values, i.e. for $(V^*_{ts} V_{tb})^+$, the missing-energy spectrum almost matches that of the SM. That is, the results in SM4 are the same as those in the SM. But in the second case, when the values of $V^*_{ts} V_{tb}$ are negative, i.e. for $(V^*_{ts} V_{tb})^-$, the curve of the missing-energy spectrum is quite different from that of the SM. This can be clearly seen from figures 5 and 6. The enhancement of the missing-energy spectrum is rapid, and the missing-energy spectrum of the $K^*$-meson is almost symmetrical. In figures 7 and 8, the BRs $BR(B \to K^*_L \nu \bar{\nu})$ and $BR(B \to K^*_T \nu \bar{\nu})$ are depicted as a function of $m_t$. Figures 7 and 8 show that for all values of $m_t \geq 210$ GeV the values of the BRs become greater than in the SM. The BR increases rapidly with the increase of

**Figure 7.** The dependence of the BR of the decay $B \to K^* \nu \bar{\nu}$ on the fourth up-like-quark mass $m_t$ when $K^*$ is polarized longitudinally.
In this case, the fourth-generation effects are shown clearly, whereas in our approach, the predictions for the ratio \( \frac{BR(B \to K^*\nu\bar{\nu})}{BR(B \to K^+\nu\bar{\nu})} \), as well as the transverse asymmetry \( A_T \),

\[
A_T \equiv \frac{BR(B \to K^0_{h=-1}\nu\bar{\nu}) - BR(B \to K^0_{h=+1}\nu\bar{\nu})}{BR(B \to K^0_{h=-1}\nu\bar{\nu}) + BR(B \to K^0_{h=+1}\nu\bar{\nu})},
\]

are model independent.

In conclusion, the BR and the missing-energy spectra of the rare exclusive semileptonic \( B \to K^*\nu\bar{\nu} \) decay have been investigated in a sequential fourth-generation model. The effects of a possible fourth-generation fermion \( t \)-quark mass have been considered, and sensitivities of the BR and the missing-energy spectra to the \( t \)-quark mass are observed.

Finally, note that the results for \( B \to \rho\nu\bar{\nu} \) decay can be easily obtained from \( B \to K^*\nu\bar{\nu} \) if the following replacement is made in all equations: \( V_{tb}V_{ts}^{\dagger} \) is replaced by \( V_{tb}V_{td}^{\dagger} \) and \( M_{K^*} \) is replaced by \( M_{\rho} \). In viewing these results, one must keep in mind that the values of the form factors for the \( B \to \rho \) transition generally differ from those for the \( B \to K^* \) transition. However, these differences must be in the range of \( SU(3) \) violation, namely of the order 15–20%.

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