$B \to K^*l^+l^-$: Zeroes of angular observables as test of standard model

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We calculate the zeroes of angular observables $P_4'$ and $P_5'$ of 4-body angular distribution of $B \to K^*(\to K\pi)l^+l^-$ where LHCb, in its analysis of form factor independent angular observables, has found deviations from standard model predictions in one of the $q^2$ bins. In the large recoil region, we obtain relations between the zeroes of $P_4'$, $P_5'$ and the zero of forward-backward asymmetry of lepton pair. These relations, in the considered region, are independent of hadronic uncertainties and depend only on Wilson coefficients. We also construct a new observable, $O_L^{L,R}$, whose zero in the standard model coincides with the zero of forward-backward asymmetry but in presence of new physics contributions will show different behavior. Moreover, the profile of the new observable, even within the standard model, is very different from the forward backward asymmetry. We point out that precise measurements of these zeroes in near future would provide crucial test of the standard model and would be useful in distinguishing between different possible new physics contributions to Wilson coefficients.

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\( K^*l^+l^- \) is governed by effective Hamiltonian for \( b \to s \) transitions written as

\[
H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb}^* V_{ts} \sum C_i(\mu) O_i + C'_i(\mu) O'_i + \text{h.c.}
\]  

(1)

where contribution of the term \( \propto \frac{V_{ub}^* V_{tb}}{V_{tb}^* V_{ts}} \) is neglected. \( O_i \) are the effective local operators and their coefficients \( C_i(\mu) \) are called Wilson coefficients evaluated at scale \( \mu \). The factorization scale \( \mu \) distinguishes between short distance physics (above scale \( \mu \)) and long distance physics (below scale \( \mu \)). Wilson coefficients depend on factorization scale and are the only source of information about heavy degrees of freedom which have been integrated out while matrix elements of local operators \( O_i \) dictate the low energy dynamics (for a review see [5]).

The operators contributing significantly to the process \( B \to K^*l^+l^- \) in SM are semileptonic vector operator \( O_9 \), axial vector operator \( O_{10} \) and magnetic photon penguin operator \( O_7 \). Their explicit form is given by

\[
O_7 = \frac{e}{16 \pi^2} m_b (\hat{s}_\alpha \sigma_{\mu \nu} R \hat{b}_\alpha) F^{\mu \nu},
\]

\[
O_9 = \frac{e^2}{16 \pi^2} (\hat{s}_\alpha \gamma^\mu L \hat{b}_\alpha) (\hat{\ell}_\gamma \mu l),
\]

\[
O_{10} = \frac{e^2}{16 \pi^2} (\hat{s}_\alpha \gamma^\mu L \hat{b}_\alpha) (\hat{\ell}_5 \mu \gamma_{5l}),
\]

(2)

Here, \( \alpha, \beta \) are the color indices, \( L,R = \frac{(1 \mp \gamma_5)}{2} \) represent chiral projections, \( T^a \) are the SU(3) color charges. \( m_b \) is the b-quark mass. The primed operators have same tensorial structure as unprimed ones but with helicity flipped. Their contribution within SM is severely suppressed or vanishes.

The effective coefficient of operator \( O_9 \) is given by

\[
C_9^{\text{eff}} = C_9 + Y(\hat{s})
\]

(3)

Here \( s \) is lepton invariant mass and \( \hat{s} \) is the invariant mass (\( s \)) normalized by B-meson mass square, i.e., \( \hat{s} = s/m_B^2 \). \( Y(\hat{s}) \) is one loop function and contains contribution from one loop matrix elements of operators \( O_{1,2,3,4,5,6} \). The form of function \( Y(\hat{s}) \) can be found in [6]. Due to \( Y(\hat{s}) \), \( C_9^{\text{eff}} \) is not real but has a small imaginary part. For calculating zeroes of different observables and getting analytic relations among them, we will treat these Wilson coefficients as real and also neglect small \( Y(\hat{s}) \) but for numerical calculations we include \( Y(\hat{s}) \) in \( C_9^{\text{eff}} \). As will be evident later, this turns out to be a good working approximation.

### III. ANGULAR OBSERVABLES OF \( B \to K^*l^+l^- \) IN LARGE RECOIL LIMIT

To calculate observables for \( B \to K^* \) process, one needs to calculate matrix elements of the local operators \( O_i \). These matrix elements are usually expressed in terms of seven form factors \( V, A_1, A_2, T_1, T_2, T_3 \) which are functions of momentum transfer between B and \( K^* \). These form factors are calculated via non-perturbative methods like QCD sum rules on the light cone (LCSRs) [7] when daughter mesons energies are large. Working in QCD factorization (QCDF) framework, and within heavy quark and large recoil limit, all seven form factors can be written in terms of only two independent universal factors, namely, \( \xi_\perp \) and \( \xi_\parallel \) [8]. The decay amplitude can be represented as \( \sim C_a \xi_a + \Phi_B \otimes T_a \otimes \Phi_{K^*} \), with ‘a’ corresponding to polarization of \( K^*, \perp, \parallel \). \( C_a \) contains factorizable and non factorizable correction which are calculated in perturbation theory with the help of renormalization group (RG) techniques.

The two set of form factors are related to each other through following identities (see for example [9])

\[
\xi_\perp = \frac{m_B}{m_B + m_K^*} V(q^2),
\]

\[
\xi_\parallel = \frac{m_B + m_K^*}{2E} A_1(q^2) - \frac{m_B - m_K^*}{m_B} A_2(q^2)
\]

(4)

The 4-body angular distribution of \( b \to K^*(\to K\pi)l^+l^- \) offers experimentally accessible observables which are independent of form factors and hence theoretically cleaner. The fully differential decay distribution can be described by four kinematical variables, given by

\[
\frac{d^4 \Gamma(b \to K^*(\to K\pi)l^+l^-)}{dq^2 d\cos \theta_K d\cos \theta_l d\phi} = \frac{9}{32 \pi} J(q^2, \theta_l, \theta_K, \phi)
\]

(5)
where kinematical variables dilepton invariant mass \( q^2 \), \( \theta_l \), \( \theta_K \) and \( \phi \) are defined in [10] and
\[
J(q^2, \theta_l, \theta_K, \phi) = \sum_i J_i(q^2)f(\theta_l, \theta_K, \phi)
\] (6)

The angular coefficients \( J_i(q^2) \) are generally expressed in terms of transversity amplitude. There are in total seven transversity amplitudes. There will be an additional amplitudes once scalar interactions are also taken into account which we do not consider in this work. At the leading order in \( 1/m_B \) and \( \alpha_s \) the transversity amplitudes render to following simple expression:
\[
A_{1}^{L,R} = \sqrt{2}N_{m_B}(1-\hat{s}) \left[ (C_{9}^{eff} + C_{9}^{ieff}) \mp (C_{10} + C_{10}') + 2\frac{\tilde{m}_b}{\hat{s}}(C_{7}^{eff} + C_{7}^{ieff}) \right] \xi_{\perp}(E_{K^*}),
\]
(7)
\[
A_{||}^{L,R} = -\sqrt{2}N_{m_B}(1-\hat{s}) \left[ (C_{9}^{eff} - C_{9}^{ieff}) \mp (C_{10} - C_{10}') + 2\frac{\tilde{m}_b}{\hat{s}}(C_{7}^{eff} - C_{7}^{ieff}) \right] \xi_{\perp}(E_{K^*}),
\]
(8)
\[
A_{0}^{L,R} = -\frac{N_{m_b}}{2\tilde{m}_b\sqrt{\hat{s}}}(1-\hat{s})^2 \left[ (C_{9}^{eff} - C_{9}^{ieff}) \mp (C_{10} - C_{10}') + 2\tilde{m}_b(C_{7}^{eff} - C_{7}^{ieff}) \right] \xi_{\parallel}(E_{K^*}),
\]
(9)
\[
A_{l} = \frac{N_{m_b}}{\tilde{m}_b \sqrt{\hat{s}}}(1-\hat{s})^2 \left[ C_{10} - C_{10}' \right] \xi_{\parallel}(E_{K^*})
\]
(10)

Here, \( \tilde{m}_b = m_b/m_B \) and \( E_{K^*} \) is the energy of \( K^* \) meson. Terms of \( \mathcal{O}(\tilde{m}_K^2) \) have been neglected. However it is worth mentioning that these relations holds only in the kinematical region \( 1 < q^2 < 6 \). This is precisely the region of interest.

There are 12 angular coefficients \( J_i(q^2) \) and considering \( \tilde{J}_i(q^2) \) (corresponding to CP conjugate mode of \( B \rightarrow K^*(\rightarrow K\pi)l^+l^- \) ), there are in total 24 angular coefficients. The CP conjugated coefficients \( \tilde{J}_i \) are given by \( J_i \) with weak phases conjugated. One can construct, taking certain ratios and combinations such that form factors and hadronic uncertainties pertaining to such observables more or less cancel and we are left with observables which are cleaner and have high sensitivity to NP effect. One can define such CP -averaged and CP violating observables as in [10], [11].

IV. ZEROES OF ANGULAR OBSERVABLES

It is well known that the zero of the forward backward asymmetry of the lepton pair in the decay \( B \rightarrow K^*l^+l^- \) is highly insensitive to form factors and precise measurement of this quantity can reveal new physics [12]. The zero of forward backward asymmetry is known to depend on ratios of form factors, value of b quark mass and Wilson coefficients \( C_{7}^{eff} \) and \( C_{9}^{eff} \). However, in the heavy quark limit and in the region where energy of \( K^* \) (\( \sim m_B/2 \)) is comparable with B-meson mass, the hadronic uncertainties cancel in ratios of form factors and, to a good approximation, zero of the forward backward asymmetry of the lepton pair is essentially independent of form factor uncertainties. The position of the zero is thus heralded as a test of SM since the position shifts significantly for most models beyond SM. The zero of the forward backward asymmetry is given by a clean relation:
\[
\text{Re}(C_{9}^{eff}(\hat{s}_0)) = -2\frac{\tilde{m}_b}{\hat{s}_0} C_{7}^{eff} \frac{1 - \hat{s}_0}{1 + \tilde{m}_b^2 - \hat{s}_0} - 2\frac{\tilde{m}_b}{\hat{s}_0} C_{7}^{eff}
\]
(11)

Here, \( \hat{s}_0 \) is position of the zero of the forward-backward asymmetry. Taking it as cue, we investigate other observables, particularly \( P_3^\perp \) and \( P_4^\parallel \) as there has been a tension between SM prediction and experimental data for these observables in one of the bins.

To bring out the power to differentiate various NP scenarios, we calculate the zeroes of some of the angular observables including the primed operators as well. The associated Wilson coefficients are assumed to be real for simplicity though it is straight forward to generalise the relations below to complex coefficients. This is not necessary for the present as our main motivation is to study the situation within SM, where as we show below, there are tight correlations of the zeroes of the angular observables considered and the zero of the forward backward asymmetry. We
also propose a new observable, $\mathcal{O}^{L,R}_T$, defined below. This new observable, and its zero, carries quite a complimentary information compared to the observables already studied in literature. To the best of our knowledge, such correlations and their impact as in providing a litmus test for SM has not been studied before. We would again like to emphasize that the analytic expressions below have been obtained by neglecting the $Y(\hat{s})$ contribution from $C_9^{eff}$ and treating the leptons as massless. However, in the numerical evaluations, we have retained the $Y(\hat{s})$ contribution and have massive leptons.

To calculate zeros of any of the observables, one needs to look solution of numerator only. We set them to zero and obtain the solution.

(a) $A_{FB}$: In terms of the angular coefficients, forward-backward asymmetry is proportional to $(J_{6s} + \bar{J}_{6s})$ which in turn is $\propto [\Re(A^L_{\parallel} A^L_{\perp}) - (L \leftrightarrow R)]$. Setting $(J_{6s} + \bar{J}_{6s}) = 0$ gives following solution

$$\hat{s}_0 = -2 \left( \frac{C_{10} C_7 - C'_{10} C_7}{C_{10} C_9 - C'_{10} C_9} \right) \hat{m}_b$$  \hspace{1cm} (12)

(b) $P'_5$: $P'_5$ is proportional to $(J_5 + \bar{J}_5)$ which in massless lepton limit is $\propto [\Re(A^L_{0} A^L_{4}) + (L \leftrightarrow R)]$. The zero of $P'_5$ is given by

$$\hat{s}_0^{P_5} = \left( \frac{C_7 + C'_7}{C_{10} C_9 - C'_{10} C_9 - (C_7 - C'_7)(C_{10} + C'_{10})\hat{m}_b} \right) \hat{m}_b$$  \hspace{1cm} (13)

(c) $P'_4$: The numerator of $P'_4$ expression is $(J_4 + \bar{J}_4) \propto [\Re(A^L_{0} A^L_{4}) + (L \leftrightarrow R)]$. The zero of $P'_4$ is given by

$$\hat{s}_0^{P_4} = -2 \left( \frac{C_7 + C'_7}{(C_9 - C'_9)^2 + (C_{10} - C'_{10})^2 + 2(C_7 - C'_7)(C_9 - C'_9)\hat{m}_b} \right) \hat{m}_b$$  \hspace{1cm} (14)

(d) $\mathcal{O}^{L,R}_T$: Apart from these observables, we also construct a new observable, $\mathcal{O}^{L,R}_T$, which has following form

$$\mathcal{O}^{L,R}_T = \frac{|A^L_4|^2 + |A^L_5|^2 - (L \leftrightarrow R)}{\sqrt{-(J_{2s} + \bar{J}_{2s})(J_{2s} + \bar{J}_{2s})}}$$  \hspace{1cm} (15)

The zero of the observable $\mathcal{O}^{L,R}_T$ is given by

$$\hat{s}_0^{\mathcal{O}^{L,R}_T} = -2 \left( \frac{C_{10} C_7 + C'_{10} C'_7}{C_{10} C_9 + C'_{10} C'_9} \right) \hat{m}_b$$  \hspace{1cm} (16)

Relations in Standard Model

It is very interesting to consider these relations in the limit of SM: set $C'_i = 0$. Further, we exploit the fact that within SM, numerically, $C_9 \approx -C_{10}$. Employing these and simplifying, we obtain:

$$\hat{s}_0^{SM} = -2 \frac{C_7}{C_9} \hat{m}_b$$  \hspace{1cm} (17)

which matches with the relation (11) within large recoil limit. The LHCb collaboration has measured the point of zero crossing of the forward backward asymmetry zero: $q_0^2 = (4.9 \pm 0.9) \text{ GeV}^2$ [13]

In the case of $P'_5$, setting primed Wilson coefficients equal to zero in equation (12), relation reduces to

$$\hat{s}_0^{P'_5,SM} = -2 \frac{C_7}{C_9 + C_7} \hat{m}_b \frac{\hat{s}_0/2}{1 - \hat{s}_0/2} \approx \frac{\hat{s}_0}{2}$$  \hspace{1cm} (18)

So the relations predicts value of zero of $P'_5$ to be approximately half of forward-backward asymmetry zero.
TABLE I: Values of input parameters used for numerical calculations of zeroes of observables

| Parameter | Value       |
|-----------|-------------|
| $m_{\text{pole}}$ | 4.80 GeV   |
| $G_F$     | $1.166 \times 10^{-5}$ |
| $m_B$     | 5.280 GeV  |
| $m_{K^*}$ | 0.895 Gev  |
| $m_\mu$   | 0.106 GeV  |
| $\alpha$  | 1/129      |
| $\alpha_s$| 0.21       |

In the SM limit the zero of $P_4'$ reads

$$s_0^{P_4, SM} = -2 C_7 C_9 + 2 C_7^2 \hat{m}_b \hat{s}_0 \frac{(1 - \hat{s}_0)}{(2 - \hat{s}_0)} \approx \frac{\hat{s}_0}{2}$$  \hspace{1cm} (19)

where in the last step, we have again used the fact that $\hat{s}_0$ is very small compared to 1. So, the prediction for the zero of $P_4'$ is also about half of $\hat{s}_0$.

If we keep the effect of factor $\hat{s}_0$ in expressions of zeroes of $P_5'$ and $P_4'$, we see that actual value of the zero of $P_5$ is a little larger than half of $\hat{s}_0$ while the zero of $P_4'$ lies a bit below half of $\hat{s}_0$, but as clearly evident from Table 2, the effect is rather small and can be safely neglected.

The case of the proposed new observable $O_{T, L, R}$ is special one. The expressions of zero of forward backward asymmetry (12) and $O_{T, L, R}^{L, R}$ (15) have interesting features. In the SM limit, the position of the two zeroes coincides while this degeneracy is lifted in a simple but complimentary manner when the helicity flipped operators are present.

The expressions of zeroes of these observables depend only on Wilson coefficients, practically independent of form factors, thereby leading to theoretically clean predictions. To calculate these zeroes, we use $C_9 = 4.2297$, $C_{10} = -4.2068$, $C_7^{eff} = -0.2974$ [14] at scale $m_b$. In our numerical analysis, we use the values of input parameters given in Table I.

![FIG. 1: Different angular observables as a function of $\hat{s}$](image-url)
from the perspective of new physics, the new physics has a destructive contribution to $C_9$. The magnitude of $C_7^{eff}$ is very accurately known from B decay $B \to K^*\gamma$. Assuming real coefficients, this then means that $C_7^{eff}$ is known up to a sign ambiguity. This ambiguity in the sign of $C_7$ is resolved by the zero of the forward backward asymmetry of the lepton pair, and therefore precise deduction of Wilson coefficient $C_9$ can be done. We would be able to identify distinctions among different NP scenarios more accurately once these zeroes are precisely measured. We would also like to draw attention to the fact that not just the position of the zero of an angular observable but also the complete profile as a function of $\hat{s}$ is a powerful tool at hand. This is illustrated in Figure 1 where one can clearly see that though the zero of forward backward asymmetry coincides with that of the new observable $O_T^{L,R}$, the two profiles are quite different.

V. SUMMARY AND CONCLUSIONS

With the latest experimental results on angular observables of $B \to K^* (\to K \pi) l^+ l^-$ showing discrepancies with respect to SM, one would like to hope to have found first evidence of new physics. But due to uncertainties inherent in the theoretical calculations of such processes, it is difficult to infer the same in affirmation. Precise measurements of theoretically clean observables holds the best chance of unambiguously revealing the presence of physics beyond SM, if any. The zero of the forward backward asymmetry is known to fall under this category of observables. The current measurement is not precise enough to say anything definitive and is totally consistent with SM. It may be used to have more such observables measured with precision. With this picture in mind, we point out that zeroes of forward backward asymmetry, $P_3$, $P_4$, and $O_T^{L,R}$ (a new angular observable proposed in this work), can be crucial test of SM. It has been pointed out that within SM, the position of all the zeroes is essentially fixed by the zero of the forward backward asymmetry, up to small corrections. To the best of our knowledge, this is the first attempt to use such correlations as a stringent test of SM itself. The relations are quite rich and general as they include Wilson coefficients of helicity flipped operators also. The relations are obtained in large recoil region in large energy limit where estimates of theoretical uncertainties are supposed to be minimalistic. A simultaneous accurate determination of these zeroes will surely provide a conclusive evidence of any NP present. Moreover, in a general setting, the zeroes by themselves carry complimentary information about the Wilson coefficients and their measurement together with the existing data can be used to pin point the class of NP scenarios which can give rise to such predictions. This is clearly evident from the position of the zero of the proposed observable $O_T^{L,R}$ which in the standard model limit yields the same value as the zero of forward backward asymmetry but when the helicity flipped operators are included, leads to complimentary information on the Wilson coefficients compared to the zero of forward backward asymmetry.

We also hope that with more data, not just the position of various zeroes, but also the complete profiles of angular observables will be known with high precision, which can be used further as a crucial test of the standard model.

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