DIPOLAR DARK MATTER AND COSMOLOGY

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The phenomenology of the modified Newtonian dynamics (MOND) can be recovered from a mechanism of “gravitational polarization” of some dipolar medium playing the role of dark matter. We review a relativistic model of dipolar dark matter (DDM) within standard general relativity to describe, at some effective level, a fluid polarizable in a gravitational field. At first order in cosmological perturbation theory, this model is equivalent to the concordance cosmological scenario, or Λ-cold dark matter (CDM) model. At second order, however, the internal energy of DDM modifies the curvature perturbation generated by CDM. This correction, which depends quadratically on the dipole, induces a new type of non-Gaussianity in the bispectrum of the curvature perturbation with respect to standard CDM. Recent observations by the Planck satellite impose stringent constraints on the primordial value of the dipole field.

1 Introduction

Contemporary cosmology, based on the so-called Λ-CDM cosmological model, beautifully interprets all observations at cosmological scales, but at the price of the introduction of a cosmological constant Λ in Einstein’s field equations, and of an unknown form of non-relativistic, non-baryonic matter called cold dark matter (CDM). This model brilliantly accounts for the mass discrepancy between the dynamical and luminous masses of clusters of galaxies, the precise measurements of the anisotropies of the cosmic microwave background (CMB) by the WMAP and Planck satellites, the formation and growth of large-scale structures as seen in deep redshift and weak lensing surveys, and the accelerated expansion as evidenced by the fainting of the light curves of distant supernovae. However, the model Λ-CDM as it stands faces some rather severe challenges when extrapolated, thanks to high resolution cosmological N-body simulations, down to the smaller scales of galaxies\textsuperscript{1,2}.
On the one hand, several predictions of the Λ-CDM model are not confirmed by observations: Numerous satellites of large galaxies obtained in N-body simulations remain unseen; phase-space correlations of galaxy satellites instead of an expected quasi-isotropic distribution; generic formation of dark matter cusps in the center of galaxies while rotation curves of galaxies favor a constant density profile; tidal dwarf galaxies dominated by dark matter while they are predicted to be mostly baryonic.

On the other hand, there are actual observations that are unpredicted and unexplained (or not naturally explained) by Λ-CDM: Strong correlation between the mass discrepancy (i.e., the presence of dark matter) and the typical acceleration scale; surface brightness of galaxies which is always below the Freeman limit; flat asymptotic rotation curves of galaxies; baryonic Tully & Fisher relation for spirals; Faber & Jackson law for ellipticals. All these challenges are mysteriously solved, and sometimes with incredible efficiency, by the empirical formula of MOND — Milgrom’s Modified Newtonian Dynamics. Although it is possible that some of these challenges will be solved within the Λ-CDM model, we take the view that MOND’s successes point toward a drastic modification of the standard cosmological model at small scales. Note, however, that MOND does not account for all the mass discrepancy at the intermediate scale of galaxy clusters.

A number of relativistic field theories have been proposed, recovering the MOND formula in the non-relativistic limit, and in which dark matter appears to be an apparent reflection of a fundamental modification of gravity. The Tensor-Vector-Scalar (TeVeS) theory of Bekenstein & Sanders extends general relativity with a timelike vector field and one scalar field. Einstein–æther theories, which involve a unit timelike vector field that is non-minimally coupled to the metric, provide interesting examples of relativistic MOND theories, when modified to allow for non-canonical kinetic terms. Other proposals include a bimetric theory of gravity, a variant of TeVeS using a Galileon field and a Vainstein mechanism to prevent deviations from general relativity at small distances, and a theory based on a preferred time foliation labelled by the so-called Khronon scalar field. The cosmology in several of these theories has been extensively investigated, notably in TeVeS and non-canonical Einstein–æther theories. It is however fair to say that all these theories have difficulties in reproducing the CMB spectrum, even when adding a component of hot dark matter.

In the present paper we shall be interested in an alternative to CDM, coined dipolar dark matter (DDM), which does reproduce the CMB spectrum in first approximation. The physical motivation for this model is the striking (and perhaps deep) analogy between MOND, in the non-relativistic approximation, and the electrostatics of non-linear dielectric media. In this view the dark matter appears to be a dipolar medium which can be polarized by the gravitational field of ordinary baryonic matter (galaxies). In the relativistic version the DDM has the potential of reproducing MOND at galactic scales, and is strictly equivalent to Λ-CDM at the level of first-order cosmological perturbations around a Friedman-Lemaître-Robertson-Walker (FLRW) background. Thus, the DDM behaves like ordinary CDM at early cosmological times, and furthermore the model naturally involves a cosmological constant.

The aim of the present contribution, which is a summary of the detailed paper, is to present the DDM model at second order in cosmological perturbations, and to show that the non-linear dynamics of DDM starts departing from that of ordinary CDM at this order. As a result, we shall find that DDM predicts an additional contribution to the curvature perturbation, specifically given by the internal energy of the DDM fluid. This extra contribution in the evolution of the curvature perturbation will be analyzed at second order in perturbations using the formalism of Langlois & Vernizzi. We compute the bispectrum of the curvature perturbation, limiting ourselves to super-Hubble scales, and find a specific contribution to non-Gaussianity due to the DDM. Although the amplitude of the DDM non-Gaussianity signal depends on the a priori unknown value of the dipole moment at early times, recent observations of the CMB by the Planck satellite impose stringent constraints on the primordial value of the dipole moment. In
contrast with usual models of primordial non-Gaussianities, where the curvature perturbation is frozen on super-Hubble scales during the standard cosmological era, we find that the amplitude of the non-Gaussianity induced by the DDM increases with time after the radiation-matter equality on super-Hubble scales. This distinctive feature of the DDM model, as compared with standard CDM, could thus provide a specific signature in the CMB and large-scale structure probes of non-Gaussianity.

2 Model of dipolar dark matter and dark energy

The model of dipolar dark matter (DDM) and dark energy of Blanchet & Le Tiec\textsuperscript{23,24} is based on a matter action in curved spacetime, which is to be added to the standard Einstein-Hilbert action of general relativity (without a cosmological constant), and to the actions of all the other matter fields (such as baryons, photons, neutrinos, etc), all described in the standard way.

The DDM fluid in a spacetime with metric $g_{\mu\nu}$ is described by (i) a conserved current $J^\mu = \sigma u^\mu$, such that $\nabla_\mu J^\mu = 0$, where $u^\mu$ is the four-velocity normalized according to $u_\mu u^\mu = -1$ and $\sigma = (-J_\mu J^\mu)^{1/2}$ is the rest mass density; and (ii) a dipole moment vector field $\xi^\mu$, which intervenes in the dynamics only through its projection $s^\mu \equiv (\delta^\mu_\nu + u_\nu u_\nu) \xi^\nu$ orthogonal to the four-velocity $u^\mu$. From this vector we construct the polarization field $P^\mu = \sigma s^\mu$, i.e., the dipole density.\textsuperscript{a} The Lagrangian describing the DDM fluid simply reads\textsuperscript{23,24}

\[ L = -\sigma + J_\mu \dot{\xi}^\mu - V(P), \]

where the overdot stands for the covariant derivative with respect to proper time, $\dot{\xi}^\mu \equiv u^\nu \nabla_\nu \xi^\mu$. Notice that $J_\mu \dot{\xi}^\mu = J_\mu \dot{s}^\mu + \nabla_\mu (J^\nu u_\nu \xi^\nu)$, so that the Lagrangian (1) admits an equivalent form depending only on the orthogonal projection $s^\mu$: this shows that the only dynamical degrees of freedom of the dipole moment $\xi^\mu$ are those of the projection $s^\mu$, which is a spacelike vector.\textsuperscript{b} The first term in Eq. (1) is the Lagrangian of a pressureless perfect fluid, i.e., that of ordinary CDM, while the second term is analogous to the coupling of the charge current to the four-potential in electromagnetism.

The potential $V$ is a function of the norm $P = (P_\mu P^\mu)^{1/2}$ of the polarization. Its expansion in powers of $P$ is determined, in the weak-field limit $P \ll a_0$ only, by the requirement of recovering the phenomenology of MOND in the non-relativistic regime. Up to third order, it reads\textsuperscript{23}

\[ V(P) = \frac{\Lambda}{8\pi} + 2\pi P^2 + \frac{16\pi^2}{3a_0} P^3 + \mathcal{O}(P^4), \]

where $\Lambda$ is the cosmological constant and $a_0$ is the constant MOND acceleration scale, measured to the value\textsuperscript{1,2} $a_0 \simeq 1.2 \times 10^{-10} \text{ m} \cdot \text{s}^{-2}$. Appearing both in the expansion (2), the constants $\Lambda$ and $a_0$ should naturally be related numerically in this model, $\Lambda \sim a_0^2$, which happens to be in good agreement with observations. Indeed, if we define the natural acceleration scale associated with the cosmological constant, $a_\Lambda \equiv 1/\Lambda \sqrt{\Lambda/3}$, then the current astrophysical measurements yield $a_0 \simeq 1.3 a_\Lambda$. The related numerical coincidence $a_0 \sim H_0$ was pointed out very early on by Milgrom\textsuperscript{3,4,5}. The near agreement between $a_0$ and $a_\Lambda$ has a natural explanation in this model, although the exact numerical coefficient between the two acceleration scales cannot be determined. In the strong-field (but still non-relativistic) regime $P \gg a_0$, the potential $V$ can be adjusted so as to recover the ordinary Newtonian limit\textsuperscript{23}.

Varying the Lagrangian (1) with respect to the dipole moment $\xi^\mu$ and the mass current $J^\mu$, one obtains a non-geodesic equation of motion for the fluid and an equation of evolution for the

\textsuperscript{a}Hereafter, we set $G = c = 1$ and use a metric signature $+2$. Greek indices $\mu, \nu, \ldots$ are used for spacetime coordinate components and Latin indices $i,j, \ldots$ for the corresponding spatial indices.

\textsuperscript{b}This is in sharp contrast with modified gravity theories such as TeVeS and non-canonical Einstein-æther theories where the fundamental vector field is timelike.
At second order, however, directly at the level of the Lagrangian (1), we introduced $\Omega_\mu \equiv \delta_\mu + u_\mu (1 + 2sV')$, as well as the notations $V' \equiv dV/dP$ and $\delta^\mu \equiv s^\mu/s$. The first term in the right-hand side of (3b) is akin to a pressure gradient, while the second term, which involves the Riemann tensor $R_{\mu\nu\rho\sigma}$, is analogous to the standard coupling to curvature for the motion of a particle with spin. The stress-energy tensor of the dipolar fluid can be derived by varying the Lagrangian (1) with respect to the metric, and reads\(^\text{24}\)

\[
T_{\mu\nu} = \Omega_{(\mu} J_{\nu)} - \nabla_\rho \left( P^\rho u_{\mu} u_{\nu} - u^\rho P_{(\mu} u_{\nu)} \right) - (V - PV') g_{\mu\nu},
\]

where parenthesis around indices denote symmetrisation. The first term is monopolar, the second one is dipolar, and the third represents a dynamical dark energy contribution. The stress-energy tensor (4) is conserved, $\nabla_\nu T^{\mu\nu} = 0$, as a consequence of the equations of motion (3a) and evolution (3b).

Up to an hypothesis of “weak clusterisation” of DDM during the cosmological evolution, which has been justified in spherical symmetry but not in the general case, the model was shown to reproduce the phenomenology of MOND in the non-relativistic limit\(^\text{23}\), in the sense that the Bekenstein & Milgrom\(^\text{25}\) modification of the Poisson equation is recovered. The Euclidean norm of the ordinary gravitational field is then given by the derivative of the potential, $g = V'(P)$, and the MOND interpolating function is related to the potential by $\mu = 1 - 4\pi \Phi(g)/g$, where $\Phi(g)$ is the inverse function of $V'(P)$, i.e., is such that $P = \Phi(g)$.

### 3 Evolution of the curvature perturbation in cosmology

In the cosmological context, we now consider perturbations around an homogeneous and isotropic FLRW background. Since the dipole vector would break the isotropy of the background (because it is spacelike), it must belong to the perturbation; we thus write $s^\mu = \mathcal{O}(1)$. To second order in cosmological perturbations, the dipolar fluid can be described using an energy density $\rho$ and a four-velocity $v^\mu$ that obey $v_\mu v^\mu = -1 + \mathcal{O}(3)$ and $\nabla_\mu (\rho v^\mu) = \mathcal{O}(3)$, and such that the stress-energy tensor (4) reduces to [using the expansion (2)]

\[
T_{\mu\nu} = -\frac{\Lambda}{8\pi} g_{\mu\nu} + (\varepsilon + p) v_\mu v_\nu + \rho g_{\mu\nu} + \pi_{\mu\nu} + \mathcal{O}(3),
\]

where $\varepsilon$ and $p$ are the energy density and pressure of the matter fluid, as measured in a frame comoving with $v^\mu$. We have $\varepsilon = \rho(1 + W)$, with $W$ the specific internal energy. The anisotropic stress tensor $\pi_{\mu\nu}$ is orthogonal to the velocity and traceless: $v^\mu \pi_{\mu\nu} = \mathcal{O}(3)$ and $\rho g^{\mu\nu} \pi_{\mu\nu} = \mathcal{O}(3)$.

The internal energy $W$, pressure $p$, and anisotropic stress tensor $\pi_{\mu\nu}$ are second-order quantities. Therefore, to first order in cosmological perturbations, Eq. (5) coincides with the stress-energy tensor of a cosmological constant and a pressureless perfect fluid, so that the model is indistinguishable from the concordance cosmological model\(^\text{23,24}\). At second order, however, deviations from $\Lambda$-CDM appear because of the non-zero internal energy, pressure and anisotropic stresses of DDM. Hereafter, we will only need the expression of the specific internal energy, which reads\(^\text{25}\)

\[
W = 2\pi \sigma s^2 - \frac{1}{2} (\mathcal{L}_v s)^2 + \mathcal{O}(3),
\]

which involves the norm squared of $\mathcal{L}_v s^\mu$, the Lie derivative of $s^\mu$ along $v^\mu$.

The imprint of these quadratic deviations from cold dark matter on the evolution of the curvature perturbation can be investigated using the geometric approach of Langlois & Vernizzi\(^\text{26,27}\),

\(^{\text{24}}\)This fact can also be proven\(^\text{25}\) directly at the level of the Lagrangian (1).
which provides a non-linear generalization of the usual gauge-invariant quantity $\zeta$ used in linear perturbation theory. We introduce the one-form

$$\zeta_\mu \equiv D_\mu N - \frac{N}{\varepsilon} D_\mu \varepsilon,$$

where the local number of e-folds $N$ obeys $\dot{N} = \frac{1}{3} \nabla_\mu v^\mu$, and $D_\mu \equiv (\delta_\mu^\nu + v_\nu v^\mu) \nabla_\mu$ is the projected derivative orthogonal to the four-velocity. From the projected conservation law $v_\mu \nabla_\nu T^{\mu\nu} = 0$, one can derive an evolution equation for $\zeta_\mu$ that is valid non-linearly and on all scales.

Applying this result to the model of DDM at second order in the perturbations yields

$$\mathcal{L}_v \zeta_\mu = \frac{1}{3} D_\mu \dot{W} + \mathcal{O}(3).$$

(8)

For a FLRW background, the spatial part of the curvature one-form $\zeta_\mu$ reduces to the ordinary gradient of the second-order curvature scalar $\zeta$, and the evolution equation (8) can be readily integrated on super-Hubble scales, yielding the remarkably simple result

$$\zeta = \zeta_{\text{CDM}} + \frac{1}{3} W + \mathcal{O}(3).$$

(9)

That is to say, the additional contribution to the curvature perturbation scalar due to the non-conservation of the curvature one-form at second order is given, on large scales, by a fraction of the specific internal energy of the dipolar fluid.

Introducing the line element $ds^2 = a^2 (\text{d}t^2 + \gamma_{ij} \text{d}x^i \text{d}x^j)$ of the FLRW background, where $\eta$ is the conformal time, as well as the spatial component of the dipole $s^i = (0, \lambda^i)$, with $\lambda^i = \mathcal{O}(1)$, the covariant expression (6) becomes

$$W = 2\pi \bar{\sigma} a^2 \lambda^i \lambda_i - \frac{1}{2} \lambda^i \lambda'_i + \mathcal{O}(3),$$

(10)

where $\bar{\sigma}$ is the background energy density, $\lambda_i \equiv \gamma_{ij} \lambda^j$ and $' \equiv \partial/\partial \eta$. The specific internal energy $W$ is quadratic in the dipole moment, which obeys the first-order evolution equation

$$\lambda^{''} + \mathcal{H} \lambda^{'} - 4\pi \bar{\sigma} a^2 \lambda^i = \mathcal{O}(2),$$

(11)

where $\mathcal{H} = a'/a$ is the conformal Hubble parameter. Considering that the background Universe is a mixture of a radiation and a pressureless matter fluid, this equation can be solved analytically.

Introducing the variable $y \equiv a/a_{\text{eq}}$, with the subscript “eq” referring to the time of the matter-radiation equality, when $\bar{\rho}_{\text{rad}} = \bar{\sigma} \equiv \bar{\rho}_{\text{eq}}$, we find

$$\lambda'(y, x) = \left(1 + \frac{3}{2} y \right) \lambda_i^*(x),$$

(12)

where we discarded the decaying mode in the solution. We introduced a function of space, $\lambda_i^*(x)$, corresponding to the primordial value of the dipole, early in the radiation-dominated era. Now, Eq. (10) simplifies into

$$W(y, x) = F(y) \lambda_i^2(x), \quad \text{with} \quad F = \frac{9}{16} \Omega^2 \left(y + 2 + \frac{4}{3y}\right),$$

(13)

where $\lambda_i^2 \equiv \gamma_{ij} \lambda_j^* \lambda_i^*$ and $\Omega/a_{\text{eq}} \equiv (8\pi \bar{\rho}_{\text{eq}}/3)^{1/2}$ is the inverse characteristic timescale for the collapse of a medium of uniform density $\bar{\rho}_{\text{eq}}$. At late times, we see that the correction $W$ to the conserved CDM curvature perturbation grows linearly with the scale factor.
4 Statistical description of vector perturbations and non-Gaussianity

Next, we turn to the statistical description of the vector perturbation $\lambda^i$. The additional contribution to the curvature perturbation, given by (13), depends quadratically on the dipole moment. Our aim is to analyze the effect of this contribution on the curvature spectrum and bispectrum. Borrowing from previous works on the statistical description of a vector-type perturbation, we decompose the Fourier modes of $\lambda^i(x)$ into left $(L)$, right $(R)$, and longitudinal $(\ell)$ polarizations according to $\lambda^i(\mathbf{k}) = \sum_\alpha \lambda^\alpha_i(\mathbf{k}) e^\alpha(\mathbf{k})$, with the polarization vectors $e^L = (1,1,0)/\sqrt{2}$, $e^R = (1,-1,0)/\sqrt{2}$, and $e^\ell = (0,0,1)$, with the $\hat{z}$ direction aligned with that of the wavevector $\mathbf{k}$. In absence of a fundamental description for the dipole field, we simply describe each polarization $\lambda^\alpha_i$ as a Gaussian and statistically isotropic random field. We further assume that the polarizations are not correlated among themselves or to the other quantities entering the problem, and in particular to $\zeta_{CDM}$. The power spectra $P_{\alpha}(k)$ are then defined by the relations

$$\langle \lambda_i^\alpha(\mathbf{k}) \lambda_i^{\beta}(\mathbf{k}') \rangle \equiv - (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}') P_{\alpha}(k) \delta_{\alpha\beta},$$

where $\delta$ is the three-dimensional Dirac delta distribution. Similarly, for the two-point function $\langle \lambda_i^\alpha(\mathbf{k}) \lambda_i^{\beta}(\mathbf{k}') \rangle$, the general form of the vector power spectrum is

$$P_{ij}(k) = T_{ij}^{even}(k) P_x(k) + i T_{ij}^{odd}(k) P_{-}(k) + T_{ij}^{long}(k) P_L(k),$$

where we introduced $T_{ij}^{even} = \delta_{ij} - \hat{k}_i \hat{k}_j$, $T_{ij}^{odd} = \varepsilon_{ijk} \hat{k}_k$, $T_{ij}^{long} = \hat{k}_i \hat{k}_j$, and $P_{\pm} = \frac{1}{2}(P_R \pm P_L)$. Notice that if the parity invariance that is manifest in the phenomenological Lagrangian (1) was satisfied at a more fundamental level, then we would have $P_{-} = 0$. We will also use the standard definitions for the spectrum $P_\zeta$ and bispectrum $B_\zeta$ of the curvature perturbation, namely

$$\langle \zeta(k) \zeta(k') \rangle \equiv (2\pi)^3 \delta(k + k') P_\zeta(k, k'),$$

$$\langle \zeta(k) \zeta(k') \zeta(k'') \rangle \equiv (2\pi)^3 \delta(k + k' + k'') B_\zeta(k, k', k'').$$

Now, we come to the evaluation of the contributions of the dipole moment to the spectrum and bispectrum of the curvature perturbation (9). Since the internal energy $W$ is quadratic in $\lambda^i$, there would be no tree-level contribution if the dipole field was to be treated as a statistical perturbation. Therefore, in a first calculation the dipole is assumed to have a nonzero background homogeneous value $\bar{\lambda}^i$, with an additional fluctuation $\delta \lambda^i$ described as above, such that

$$\lambda^i(x) = \bar{\lambda}^i + \delta \lambda^i(x).$$

The homogeneous part of the dipole has to be small to be consistent with the isotropy assumption for the background; it can be interpreted as the averaged field over our observable Universe. The presence of $\bar{\lambda}^i$ then allows a tree-level computation, with the result

$$P_\zeta^{tree}(k) = P_\zeta^{iso}(k) \left[ 1 + g \left( \hat{k} \cdot \hat{k}_i \right)^2 \right],$$

$$B_\zeta^{tree}(k, k', k'') = \frac{8}{27} \mathcal{F}^3 \bar{\lambda}^i \bar{\lambda}^j \left[ P^{ik}(k) P^{jk}(k') + 2 \text{ perm.} \right],$$

with $\hat{\lambda}^i$ the direction of the background dipole. The spectrum is dominated by its CDM component, as $P_{\zeta_{CDM}} \ll P_{\zeta_{CDM}}$, but acquires a small anisotropy in the direction of $\bar{\lambda}^i$, parametrized by $g(k)$. In general, the shape of the bispectrum is anisotropic and quite complicated.

In a second calculation, we treated the dipole as a perturbation only, setting $\bar{\lambda}^i = 0$, assumed scale invariance, $P \propto k^{-3}$, and performed a one-loop calculation, where logarithmic divergent integrals are regularized by introducing a cutoff $L^{-1}$, with $L$ a length scale that can be chosen to be the Hubble radius today. We found that the result obtained for the spectrum and bispectrum are consistent with the expressions above, at leading order in the cutoff, if we evaluate the tensor
\( \bar{\lambda}_i \bar{\lambda}_j \) appearing in Eq. (18b) by an average over the length scales smaller than \( L \). This shows the consistency of the two calculations in the sense of cosmic variance.

Next, to derive some quantitative estimates, the simplifying assumption \( P_\ell = P_+ \) and \( P_- = 0 \) can be made, such that the bispectrum (18b) takes a local form. Assuming scale invariance, the \( f_{\text{NL}} \) parameter, defined as

\[
 f_{\text{NL}}(k, k', k'') \equiv \frac{5}{6} B_\zeta(k, k', k'') / [P_\zeta(k)P_\zeta(k') + 2 \text{perm.}],
\]

reads

\[
 f_{\text{NL}} \simeq \frac{20}{81} F^3 \bar{\lambda}_2^2 \frac{P_+^2}{P_\zeta^2} = \frac{45}{8192} \left( y + 2 + \frac{4}{3y} \right)^3 \frac{H_{\text{eq}}^2 \alpha_{\text{eq}}^6 \bar{\lambda}_2^2 P_+^2}{P_\zeta^2}, \tag{19}
\]

where \( H \) is the Hubble parameter and \( P \equiv P k^3/(2\pi^2) \) is constant for a scale-invariant spectrum. Introducing the parameter \( x \equiv (a_{\text{eq}} \bar{\lambda}_s)H_{\text{eq}} \) comparing the dipole proper length to the Hubble radius at the matter-radiation equality, and assuming the existence of a number \( \alpha \) of order unity such that \( P_+ \simeq \alpha \bar{\lambda}_s^2 \), we obtain at the last scattering surface

\[
 f_{\text{NL}} \simeq 1.5 \times 10^{17} \alpha^2 x^6 \sim 150 \left( \frac{W}{10^{-5}} \right)^3, \tag{20}
\]

where the second estimate relies on \( \alpha \sim 1 \) and \( W \sim x^2 \). Since a value \( W \ll 10^{-5} \) is necessary for a consistent treatment of the isotropic background, we find (with our simplifying assumptions) that the obtained non-Gaussianity is compatible, although marginally, with an observable value. Recently, the Planck collaboration reported \( f_{\text{NL}} = 2.7 \pm 5.8 \) (68\% C.L. statistical) for primordial non-Gaussianities of the local type from the CMB temperature map. This implies the constraint \( s_{\text{eq}} \lsim 1.5 \times 10^{-3} H_{\text{eq}}^{-1} \) on the physical size of the dipole at the matter-radiation equality.

An important feature of the \( f_{\text{NL}} \) parameter obtained here is that its amplitude grows like the cube of the scale factor \( a \). The growing character of this additional type of non-Gaussianity would make it a distinctive effect of DDM (with respect to CDM) to look for by comparing measurements at different cosmological epochs, for instance in the temperature anisotropies of the CMB and in large-scale structures. However, to fully justify such comparison and having a meaningful test, the present calculation should be extended to sub-Hubble scales.

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