Caroll-Field-Jackiw electrodynamics in the pre-metric framework

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We analyze the Carroll-Field-Jackiw (CFJ) modification of electrodynamics reformulated as the ordinary Maxwell theory with an additional special axion field. In this form, the CFJ model appears as a special case of the pre-metric approach recently developed by Hehl and Obukhov. This embedding turns out to be non-trivial. Particularly, the pre-metric energy-momentum tensor does not depend on the axion. This is in contrast to the CFJ energy-momentum tensor which involves the axion addition explicitly. We show that the relation between these two quantities is similar to the correspondence between the Noether conserved tensor and the Hilbert symmetric tensor. As a result the CFJ energy-momentum tensor appears as the unique conserved closure of the pre-metric one. Another problem is in the description of the birefringence effect, which in the pre-metric framework does not depend on the axion. The comparison with the CFJ model shows that the corresponding wave propagation (Fresnel) equation has to be extended by a derivative term, which is non zero for the axion field. In this way, the CFJ birefringence effect is derived in the metric-free approach. Consequently the Lorentz and CPT violating models can be embedded without contradictions in the pre-metric approach to electrodynamics. This correspondence can be useful for both constructions.

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I. INTRODUCTION — THE CFJ-MODEL

The Carroll-Field-Jackiw (CFJ) modification of electrodynamics was formulated [1] with a view to examine the possibility of Lorentz and PCT violations in Maxwell’s electrodynamics. The model predicts the rotation of the plane of polarization of radiation from distance galaxies, an effect which is not observed [1]. However the original construction gives some general framework to treat Lorentz and PCT violations in field theories. Particularly, the similar ideas appear in the Lorentz violating extensions of the standard model [3], in the models with the spacetime variation of the coupling constants [4], in the Chern-Simon extension of GR [5], [6], and in various other contexts.

In the present paper we show that the original CFJ-model can be viewed as a specific case of the pre-metric framework of electrodynamics recently developed by Hehl and Obukhov, see [7], [9], [10].

The notations in [5] and [6] are slightly different from those originally used in [1]. Thus we start with a brief account of the CFJ-electrodynamics.

The first field equation for the electromagnetic field (a 2-nd order antisymmetric tensor) \(F_{ab}\) is postulated to be the same as in the ordinary electrodynamics:

\[
\partial_a F^{ab} = 0 \quad \text{and} \quad F^{ab} = \frac{1}{2} \varepsilon^{abcd} F_{cd}.
\] (1)

Consequently, the potential \(A_a\) appears in the ordinary form \(F_{ab} = \partial_a A_b - \partial_b A_a\). The equation (1) does not involve the metric of spacetime.

The second Maxwell equation

\[
\partial_a F^{ab} = J^b, \quad F^{ab} = g^{am} g^{bn} F_{mn}
\] (2)

involves the metric tensor explicitly. In the CFJ-model, it is modified to

\[
\partial_a F^{ab} + v_a \ast F^{ab} = J^b,
\] (3)

where \(v_a\) is a covector. The electromagnetic current \(J^b\) is conserved, \(\partial_b J^b = 0\), provided the covector \(v_a\) satisfies

\[
\partial_a v_b - \partial_b v_a = 0.
\] (4)

The Lagrangian associated with the modified field equation (3) involves the Chern-Simon-like term in addition to the standard Maxwell Lagrangian

\[
L = -\frac{1}{4} F_{ab} F^{ab} + \frac{1}{2} v_a A_b \ast F^{ab} - J^a A_a.
\] (5)

This expression is gauge invariant provided (4) holds.

The energy-momentum tensor for the electromagnetic field, in the CFJ theory, is given in the form [1]

\[
\Theta^{a}{}_{b} = -F^{ac} F_{bc} + \frac{1}{4} \delta^{a}_{b} F^{cd} F_{cd} + \frac{1}{2} v_b \ast F^{ac} A_c.
\] (6)

The divergence of this tensor is equal to the standard Lorentz force expression

\[
\partial_a \Theta^{a}{}_{b} = J^a F_{ab},
\] (7)

provided (4) is restricted to

\[
\partial_a v_b = 0.
\] (8)

Consequently, in the absence of sources, (6) is conserved by virtue of the field equations.

The Chern-Simon-type addition breaks the symmetry of the energy-momentum tensor (6) and its positive definiteness. This fact is treated in [1] as an indication of
the absence of the Lorentz invariance. Observe also that \( \Theta_a^a \) has a non zero trace \( \Theta_a^a = (1/2)v_a^* F^{ac} A_c \).

The appearance of a “fixed” (in the sense of (8)) covector \( v_a \) in the field equation (2) and in the Lagrangian (5) is treated in [1] as a breaking of the Lorentz invariance of the theory. Moreover, if \( v_a \) is considered as a covector (not a pseudo-covector), the parity symmetry is lost too. To preserve \(\text{SO}(3)\) space invariance, the covector \( v_a \) is taken in the form

\[
v_a = (\mu, 0, 0, 0),
\]

where \( \mu \) is a constant.

A physical consequence of the modified electrodynamics appears as a certain variation of the dispersion law. For a plane monochromatic wave it takes the form [1]

\[
w = \sqrt{|\mathbf{k}|^2 \pm \mu |\mathbf{k}|} \approx |\mathbf{k}| \pm \frac{\mu}{2}.
\]

This birefringence of the vacuum generates a Faraday like rotation on polarized light. Actual astronomical measurements impose an upper bound on the parameter \( \mu \).

As it was already mentioned in [1], the CFJ model can be equivalently reformulated as an ordinary electrodynamics with additional axion field. This new formulation is not complete because the corresponding axion field is non-dynamical. However, two equivalent representations of the model have to give the same physical consequences. The non-dynamical axion field naturally appears in the pre-metric electrodynamics as a certain variation of the dispersion law. For a plane monochromatic wave it takes the form [1]

\[
w = \sqrt{|\mathbf{k}|^2 \pm \mu |\mathbf{k}|} \approx |\mathbf{k}| \pm \frac{\mu}{2}.
\]

This birefringence of the vacuum (10) appear in this context.

In the pre-metric electrodynamics, the pre-metric form. The pre-metric energy-momentum tensor of the CFJ type with the fixed metric and the variable axion field, we reformulate the CFJ-model in the pre-metric form. The pre-metric energy-momentum tensor turns to be of the Hilbert type, while the CFJ expression is a Noether conserved quantity. Moreover, we derive the CFJ quantity by means of the conserved closure of the pre-metric tensor. In an opposite way, the pre-metric tensor is the symmetric part of the CFJ one.

In the fourth section, we deal with the effects of the light propagation. By comparison with the CFJ model, we show that the pre-metric Fresnel equation [9], [10] has to be supplemented with the terms involving the derivatives of the constitutive tensor. For the models of the CFJ type with the fixed metric and the variable axion part, we derive the explicit form of the additional term. The corresponding Fresnel equation appears to be of the 4-th order as in the case of a constant constitutive tensor.

II. THE PRE-METRIC APPROACH TO ELECTRODYNAMICS

In the pre-metric electrodynamics [11], [10] one starts with the spacetime considered as a 4 dimensional differential manifold without additional structures as metric or connection. The conservation of the electro-magnetic current is treated as a basic fundamental fact. When necessary constraints on the topology of the spacetime are applied, this conservation law results in the existence of the electromagnetic excitation \( H^{ab} \) — an antisymmetric 2-nd order tensor

\[
\partial_a J^b = 0 \quad \text{yields} \quad \partial_a H^{ab} = J^b.
\]

The quantities \( J^a \) and \( H^{ab} \) are considered as twisted (odd) tensors (differential forms). It means that they change their signs under a map of spacetime with a negative determinant, for instance, under \( P \) or \( T \) transformations.

The second basic fact assumed in the pre-metric framework is the Lorentz law of interaction between the charges and the electromagnetic field

\[
f_a = J^b F_{ab}.
\]

This law is viewed as an operational definition of the electromagnetic strength \( F_{ab} \). This is an untwisted (even) tensor, which does not change under the maps of spacetime with a negative determinant. The first Maxwell field equation

\[
\partial_a F^{ab} = 0
\]
is treated as an expression of the magnetic flux conservation law. In other words, the absence of magnetic monopoles is considered as a fundamental fact.

The excitation and the electromagnetic strength can be related, in general, by an operator

$$H^{ab} = \chi^{ab}(F_{cd}).$$  \hspace{1cm} (13)

The restriction of this equality to a linear, homogeneous, local relation yields

$$H^{ab} = \chi^{abcd} F_{cd},$$  \hspace{1cm} (14)

where $\chi^{abcd}$ is the constitutive tensor, with the symmetries

$$\chi^{abcd} = -\chi^{bacd} = -\chi^{abdc}.$$  \hspace{1cm} (15)

The irreducibly decomposition of this tensor, under the group of linear transformations, involves three independent pieces

$$\chi^{abcd} = (1) \chi^{abcd} + (2) \chi^{abcd} + (3) \chi^{abcd}.$$  \hspace{1cm} (16)

The axion and the skewon parts are defined as $[12]

\begin{align*}
(3) \chi^{abcd} &= \chi^{[abcd]} , \\
(2) \chi^{abcd} &= \frac{1}{2}(\chi^{abcd} - \chi^{[cdab]}),
\end{align*}

while the principal part is

\begin{align*}
(1) \chi^{abcd} &= \chi^{abcd} - (2) \chi^{abcd} - (3) \chi^{abcd}.
\end{align*}

The Lagrangian, in pre-metric electrodynamics, may be taken in the form

$$\mathcal{L} = -\frac{1}{4} F_{ab} H^{ab} - J^a A_a.$$  \hspace{1cm} (19)

The variation relative to the potential field $A_a$ yields the field equation (11), see [13].

The energy-momentum tensor, in the pre-metric electrodynamics, is postulated as

$$T^a_b = - F_{bm} H^{am} + \frac{1}{4} \delta^a_b \Gamma_{mn} H^{mn}.$$  \hspace{1cm} (20)

This tensor is traceless for an arbitrary constitutive tensor. Its divergence equals to the Lorentz force plus an additional term that depends on the derivatives of $\chi^{abcd}$. The axion part $(3) \chi^{abcd}$ is also not involved in (20) [10].

The wave propagation in this pre-metric framework is managed by a 4th order equation for the wave-covector $q_a$

$$G^{abcd} q_a q_b q_c q_d = 0,$$  \hspace{1cm} (21)

where

$$G^{abcd} = \frac{1}{4!} \epsilon_{mnpqrsstu} \chi^{mnr(a} \chi^{b|ps|c} \chi^{d)stu}$$  \hspace{1cm} (22)

is the tensor density of the weight $+1$. The axion part drops out also from this tensor [10].

### III. CFJ Embedded in Metric Free Framework

#### A. The CFJ Constitutive Tensor

We start with embedding standard electrodynamics in the pre-metric framework. Comparing the equations (2) and (11) we see that the Maxwell excitation $H^{ab}$ has to be equal to $F^{ab}$. Take also into account that the current $J^a$ and the excitation $H^{ab}$ have to be treated as twisted (odd) tensors, while $F^{ab}$ is untwisted. Consequently, the Maxwell excitation is

$$(\text{Max}) H^{ab} = \sqrt{-g} g^{ac} g^{bd} F_{cd}.$$  \hspace{1cm} (23)

Hence the Maxwell constitutive tensor

$$(\text{Max}) \chi^{abcd} = \frac{1}{2} \sqrt{-g} \left(g^{ac} g^{bd} - g^{bc} g^{ad}\right),$$  \hspace{1cm} (24)

involves only the principal part.

The CFJ modified field equation (3) involves the term proportional to $F^{ab}$ itself, i.e., to the first order derivatives of the potential $A_a$. Thus the CFJ excitation can be constructed as a type of integral operator. Although the general case of a linear operator relation between $H_{ab}$ and $F_{ab}$ was mentioned above (13), it is preferable to deal with an ordinary local tensor expression. For this, we rewrite (3) in an equivalent form. The condition (4) is satisfied by a covector $v_a$ equal to the gradient of an arbitrary function

$$v_a = \partial_a \theta; \quad \theta = \theta(x^a).$$  \hspace{1cm} (25)

With this redefinition, the field equation (3) is equivalent to

$$\partial_a \left(F^{ab} + \theta \star F^{ab}\right) = J^b,$$  \hspace{1cm} (26)

provided the field equation (1). Comparing with (11) we derive the CFJ electromagnetic excitation

$$H^{ab} = \sqrt{-g} F^{ab} + \theta \star F^{ab},$$  \hspace{1cm} (27)

i.e., the constitutive tensor in CFJ electrodynamics involves an axiom in addition to the principal part

$$(\text{CFJ}) \chi^{abcd} = \frac{1}{2} \sqrt{-g} \left(g^{ac} g^{bd} - g^{bc} g^{ad}\right) + \frac{1}{2} \theta \varepsilon^{abcd}.$$  \hspace{1cm} (28)

Observe that the function $\theta(x)$ is arbitrary in (27,28), thus both expressions are Lorentz invariant. As for the parity invariance, for $\theta(x)$ a scalar, parity symmetry is loosed, while, for $\theta(x)$ is a pseudo-scalar (twisted scalar), it is preserved. We will consider these two cases simultaneously. It means, that we allow the constitutive tensor to have a non-homogeneous parity although, in pre-metric electrodynamics, it is assumed to be twisted.
B. The CFJ Lagrangian

Substituting the CFJ excitation (27) into the pre-metric Lagrangian (19) we obtain
\[ \mathcal{L} = -\frac{1}{4} F_{ab} F^{ab} \sqrt{-g} - \frac{1}{4} \theta F_{ab}^* F^{ab} - J^a A_a \ . \] (29)

Observe that
\[ \frac{1}{4} \theta F_{ab}^* F^{ab} = -\frac{1}{2} v_a A_b^* F^{ab} - \frac{1}{2} \theta A_b \partial_a (\theta A^* F^{ab}) + \frac{1}{2} \theta A^* (\theta A^* F^{ab}) . \] (30)

The second term vanishes, provided the field equation (1). Thus the pre-metric Lagrangian (19) is equivalent to the CFJ one (5), up to a total derivative.

C. The CFJ energy-momentum tensor

Substituting (27) in the energy-momentum tensor of metric-free electrodynamics (20) we obtain the ordinary Maxwell energy-momentum tensor
\[ (\text{Max}) T^a_b = -F^{ac} F_{bc} + \frac{1}{4} \delta^a_b F^{cd} F_{cd} \] (31)
since the axion term does not give any addition [12]. This expression is, in fact, the Hilbert energy-momentum tensor. Indeed, since the Chern-Simon-like term is independent on the metric, \( T^{ab} = \delta L/\delta g_{ab} = \delta \mathcal{L}/\delta g_{ab} \).

Certainly, (31) is traceless, symmetric and positive defined. Its divergence, however, is not equal to the Lorentz force. In order to compare this expression to the CFJ tensor (6), we consider the divergence of the general expression (20). It takes the form [10]
\[ \partial_a T^a_b = f_b + X_b , \] (32)
where
\[ X_b = \frac{1}{4} \left( H^{ij} \partial_b F_{ij} - F_{ij} \partial_b H^{ij} \right) . \] (33)

Substituting here the constitutive relation (14), we obtain the reduction to two terms of different origin \( X_b = Y_b + Z_b \), with
\[ Y_b = -\frac{1}{4} F_{ij} F_{j\rho} \partial_b \left( \chi^{ijkl} + \chi^{ijkl} \right) , \] (34)
and
\[ Z_b = \frac{1}{4} \chi^{ijkl} F_{ij} \partial_b F_{kl} . \] (35)

In the second term, \( Z_b \), the derivatives of the electromagnetic field are involved, so it is of the same fashion as the Lorentz force itself. Thus \( Z_b \) gives a type of a “hard violation” of the conservation law. This fact is rather natural, since the skewon term is not involved in the Lagrangian. In the first term, \( Y_b \), only the derivatives of the constitutive tensor are involved. This term is not zero for a spacetime with a variable metric even without the axion term modification. This is a type of a “soft violation” of the conservation law. It is usually treated by the change of the partial derivative to the covariant one.

In the CFJ model we deal with a constant principal part and a variable axion part (28). Moreover, the axion part does not involve the energy-momentum tensor (20). Consequently the relation (32) takes the form
\[ \partial_a (\text{Max}) T^a_b = f_b - \frac{1}{4} F_{mn}^* F^{mn} \partial_b \theta . \] (36)
This relation is equivalent to
\[ \partial_b \left( (\text{Max}) T^a_b + \frac{1}{2} F^{ac} A_c \partial_b \theta \right) = f_b \] (37)
provided the field equation (1) and the condition (8). The expression in brackets coincides with the CFJ energy-momentum tensor (6). Thus this tensor can be viewed as a conserved closure of the pre-metric energy-momentum tensor. This extension is unique up to a total derivative.

A more meaningful description of (6) can be given in the framework of the Noether procedure. Indeed, this expression is derivable from the Lagrangian (5) by the standard formula for the canonical energy-momentum tensor. Thus (6) can be interpreted as an energy-momentum tensor of a system of two interacted fields — the electromagnetic field \( F_{ab} \) and the non-dynamical field \( v_a \), see [13]. The additional term can be viewed as the energy of interaction of the electromagnetic field with an “infinite sea” of the covector field \( v_a \). It is rather natural that the total expression is not positive defined and non-symmetric. These features of the energy-momentum tensor cannot be seen, however, as an indication of the Lorentz or parity symmetry breaking. They appear even in the case of the covector field restricted by a Lorentz invariant condition (8).

The quantity (6) shares the well known properties of canonical energy-momentum tensors. Particularly, it is gauge invariant only up to a total derivative. It is a general property of the Noether tensors derived from the Lagrangians which are gauge invariant only up to a total derivative [13]. Although the total derivative is not important in most situations, it prevents (6) from being used as a source in Einstein’s gravity equation.

IV. LIGHT PROPAGATION

In the framework of the pre-metric approach, the light propagation is managed by the 4th order equation (21). The axion part of the constitutive tensor does not alter this equation [10]. Consequently, for the CFJ constitutive tensor (28), the equation (21) does not give any birefringence effect at all. This is in a contradiction with the CFJ dispersion law (10). In fact, the equation (21)
is derived in the geometrical optics approximation. Thus the derivatives of the constitutive tensor are neglected. However, the CFJ birefringence effect is proportional to $\mu^2$, i.e., to the square of the first order derivative of $\chi^{abcd}$. The CFJ dispersion law uses the exact plane wave solution, while (21) is based on the geometrical optics limit. Thus, in order to have a correspondence between two formulas, the expression (22) has to be supplied with a certain correction term.

In fact we have here two different types of the birefringence effects: (i) The pre-metric birefringence is generated by an algebraic structure of the constitutive tensor. (ii) The CFJ birefringence effect is generated by derivatives of the constitutive tensor.

For description of the light propagation, we consider the wave-type ansatz

$$F_{ab} = f_{ab} e^{i\varphi}, \quad (38)$$

where $\varphi = \varphi(x^a)$ while $f_{ab}$ is a constant tensor. We denote the wave covector as $q_a = \partial_a \varphi$. For the ansatz (38), the tensor of excitation (13) is

$$H^{ab} = \chi^{abcd} f_{cd} e^{i\varphi}. \quad (39)$$

Observe, that, in general, the amplitude of $H_{ab}$ is not a constant tensor, even if $f_{ab}$, the amplitude of $F_{ab}$, is a constant.

Substituting (38, 39) into the field equations (1) and (11) and putting to zero the current vector, we obtain a system of 8 linear equations

$$\varepsilon^{abcd} q_b f_{cd} = 0 \quad (40)$$

$$\left( \chi^{abcd} q_b - i\chi^{abcd, a} \right) f_{cd} = 0 \quad (41)$$

for 6 independent variables $f_{ab}$. For a constant constitutive tensor, this system coincides with the corresponding system of [9], which was derived by means of the Hadamard method. The equations (40) and (41) are linearly dependent. Indeed, the contraction of (40) with the covector $q_a$ is identically zero. The solution of (40) may be written as

$$f_{ab} = q_a a_b - q_b a_a, \quad (42)$$

where $a_a$ is an arbitrary covector. It is defined only up to an arbitrary shift $a_a \rightarrow a_a + \lambda q_a$, which corresponds to the gauge transformations of the potential. In order to fix this “gauge” freedom, we use the covector in the form of Tamm’s ansatz [9]

$$a_a = \frac{a_0}{q_0} q_a + l_a. \quad (43)$$

The new covector $l_a$ has only 3 independent components $l_a = (0, l_a), \alpha = 1, 2, 3$. Substituting (43) into (41) we obtain 4 linear equations

$$\left( \chi^{abc\delta} q_b q_c + i\chi^{abc\delta, \gamma} q_c \right) l_\delta = 0 \quad (44)$$

for 3 independent components $l_a$. Following [9], [10] we introduce a specific frame such that the wave vector is directed in the positive time direction, i.e., $q_a = (1, 0, 0, 0)$. In $1 + 3$ decomposition, (44) reads

$$\chi^{\alpha 000} \delta l_\delta = 0, \quad (45)$$

$$\left( \chi^{\alpha \alpha 00} + i\chi^{\alpha 000} \delta + i\chi^{\alpha 000, 0} \right) l_\delta = 0. \quad (46)$$

We still deal with an over-determined system of 4 linear equations.

Consider now the special case appearing in the CFJ model. The corresponding constitutive tensor involves the constant principal part $(\chi^{abcd})$ and the variable axion part $(\chi^{abcd}) = \theta(x^\mu) e^{abcd}$. Consequently, the equation (45) is satisfied identically and we remain with the system (46) of 3 independent equations ($v_\beta = \partial_\beta \theta$)

$$M^{\alpha \delta} a_\delta = 0, \quad \text{with} \quad M^{\alpha \delta} = (\chi^{\alpha 000} + i\chi^{\alpha 000} \beta), \quad (47)$$

A nontrivial solution to this homogeneous linear system appears only for $\det M^{\alpha \delta} = 0$, i.e.,

$$\frac{1}{3!} \varepsilon_{\alpha \beta \gamma} \varepsilon_{\mu \nu \rho} M^{\alpha \mu} M^{\beta \nu} M^{\gamma \rho} = 0. \quad (48)$$

We substitute here (47) and observe that the imaginary part vanishes because of symmetries (the skewon part is absent in the CFJ model). Consequently we come to the equation

$$\frac{1}{3!} \varepsilon_{\alpha \beta \gamma} \varepsilon_{\mu \nu \rho} \chi^{\alpha 000} \chi^{\beta 000} \chi^{\gamma 000} - v_\alpha v_\beta v_\gamma \chi^{\alpha 000} = 0. \quad (49)$$

Following the procedure given in [9], [10] we rewrite this equation in the covariant form

$$G^{abcd} q_a q_b q_c q_d - \chi^{abcd} v_a v_b v_c q_d q_e = 0. \quad (50)$$

Substituting here the CFJ constitutive tensor (28) we obtain

$$(q_a q^a)^2 - (v_a q^a)^2 + (v_a v^a)(q_b q^b) = 0, \quad (51)$$

which coincides with [2]. Observe that for a derivation of this relation we only need the covariant condition (4) which allows to embed the CFJ model in the pre-metric setting.

Finally, in the rest frame $v_a = (m, 0, 0, 0)$, we substitute $q_a = (w, k)$ to derive

$$(w^2 - k^2)^2 - m^2 k^2 = 0, \quad (52)$$

which coincides with the CFJ dispersion law (10).

V. DISCUSSION

In the case the violations of the Lorentz and the CPT symmetries exist as electromagnetic phenomena, it is very possible that they would appear in the form of the...
CFJ modification. The reason is that this model preserves the basic features of ordinary electrodynamics. In particular, (i) the electromagnetic charge is conserved, (ii) the model is gauge invariant, (iii) the divergence of the energy-momentum is equal to the standard Lorentz force, (iv) the electromagnetic flux is conserved (absence of monopoles), and (v) the model is derivable from a Lagrangian.

The birefringence effect is a measurable result of this model. So, in the case the birefringence is not observable, one can deduce that there is no violation of Lorentz and CPT, as in [1]. The inverse deduction, in general, is not true. Even if the birefringence effect was observable, it could be originated also in a corresponding Lorentz and CPT invariant model.

The full set of Lorentz violation terms in electrodynamics is considered in [14], [15]. The corresponding addition in the Lagrangian is

$$\Delta L = -\frac{1}{4} (k_F)_{abcd} F^{ab} F^{cd}. \quad (53)$$

For $(k_F)_{abcd}$ treated as a set of coupling constants (not a tensor), this is a CPT even Lorentz violating term. Only 10 of the 19 possible terms of the form (53) generate the birefringence effect. The coefficient matrix $(k_F)_{abcd}$ is very similar to the constitutive tensor $\chi_{abcd}$. Thus the modification (53) is in a straightforward way embedded in the pre-metric framework.

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