The analyzation of 2D complicated regular polygon photonic lattice

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Abstract. We have numerically simulated the light intensity distribution, phase distribution, far-field diffraction of the two dimensional (2D) regular octagon and regular dodecagon lattices in detail. In addition, using the plane wave expansion (PWE) method, we numerically calculate the energy band of the two lattices. Both of the photonic lattices have the band gap. And the regular octagon lattice possesses the wide complete band gap while the regular dodecagon lattice has the incomplete gap. Moreover, we simulated the preliminary transmission image of photonic lattices. It may inspire the academic research both in light control and soliton.

1. Introduction
In these decades, photonic lattices have attracted substantial research interests. More and more research achievements are reported about new lattice fabrication, light control [1], localized transmission and energy band gap. Especially the band gaps, which opens a new door for studying the optical phenomenon of period photonic lattices. It is easier to find the localized transmission of the period lattice whose frequency is close to a bang gap.

As we know, a great number of studies report on the band structure of two dimensional (2D) simple period photonic lattices such as hexagon, honeycomb [2] and kagome [3] lattice by now [4]. Hexagon lattice has a range of application in nonlinear optic fiber and optical soliton transmission due to their wide band gaps [5]. The lattice solitons transmission, localization and interband transitions have been studied in honeycomb lattices [6]. Moreover, the kagome lattice has been generated by the technique of optical induction [7,8].

In this paper, we mainly discussed a new kind of structure lattices, the 2D complicated period regular polygon photonic lattice. We analysed the light field of photonic lattice by simulating the light intensity distribution, phase distribution as well as far-field diffraction pattern. For further studies, we calculated the energy band gap of it via the plane wave expansion (PWE) method in TE mode and TM mode. The periodic dielectric equation of photonic crystal and Maxwell equations are the major components. Also we need make a Fourier series expansion for the periodic function. And, the band gap has been found in the lattices whose shapes are regular octagon and regular dodecagon via calculation. In order to analysis the band structure briefly, we have simulated the image of Gauss source passing through the photonic crystal by means of the Finite-Difference Time-Domain(FDTD).
2. Analyzation of the light field
We analysed two types of 2D period photonic lattices, one is regular octagon lattice, the other is regular dodecagon lattice. The lattice is a superposition of simple lattices with a certain angle and different phases [9]. And we simulated the light intensity distribution, phase distribution and far-field diffraction of them.

![Simulation results](image1)

Figure 1. The simulation results. (a)-(d)Intensity pattern, Phase distribution, far-field pattern and three dimensional (3D) intensity pattern of regular octagon photonic lattice. (e)-(h)Intensity pattern, Phase distribution, far-field pattern and 3D intensity pattern of regular dodecagon photonic lattice.

Based on the Bessel beams and Fourier transformation [10], a series of results on the light field has been simulated. As is shown in Figure 1(a), the light field contains periodic distribution of a large number of regular octagons [11]. Also, the phase distribution in Figure 1(b) are arranged periodically. Comparing (a) and (b), there is a π-phase difference between two lattice sites [12]. From the far-field pattern (c), we can see that the distribution of far-field is a superposition of two equal squares. Also, the distribution in (g) is a superposition of two same hexagon. As the 3D intensity pattern shows, the intensity of regular octagon lattice is like ridges arranged periodically. It suggests the periodicity of regular octagon lattice and the non-diffraction of Bessel beams. According to the figure from (e) to (h), we can conclude that the phenomenon of regular dodecagon lattice is similar to that of regular octagon lattice.

3. Analyzation of the energy band structure
For further analyses, we calculated the band structure [13] of the two complicated photonic lattice. The photonic lattice has the great application value and the huge development potential due to the existence of energy band gap. Essentially, we need the master equations of 2D photonic lattice first:

\[
\frac{1}{\varepsilon(r')} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) E_z(r') + \frac{w^2}{c^2} E_z(r') = 0 \quad (TE \text{ mode}) \tag{1}
\]
In the TE mode, the electric field distribution is in XY plane. And we mainly studies band gap in this mode. In the TM mode, the magnetic field distribution is in XY plane. Next, we calculate the band structure by the PWE.

The Fourier series expansion of the periodic dielectric function in reciprocal space:

\[
\frac{1}{\varepsilon(r')} = \sum_G k(G)e^{iG \cdot r'}
\]  

And, \( G = l_1b_1 + l_2b_2 \), which is basic vectors in reciprocal space. The \( k(G) \) is Fourier coefficients of \( \frac{1}{\varepsilon(r')} \).

Expanding the \( E_z(r') \):

\[
E_z(r') = \sum_G E_z(G)e^{i(k + G) \cdot r'}
\]  

The eigenvalue equation in TE mode is deduced via substituting (3) and (4) into the master equation of 2D photonic lattice (1):

\[
\sum_G k(G - G')|k + G + G'|E_z(G') = \frac{w^2}{c^2} E_z(G)
\]  

The expansion of \( H_z(r') \):

\[
H_z(r') = \sum_G H_z(G)e^{i(k + G) \cdot r'}
\]  

Similarly, the eigenvalue equation in TM mode:

\[
\sum_G k(G - G')(k + G) \cdot (k + G')H_z(G') = \frac{w^2}{c^2} H_z(G)
\]  

The energy band structure can be calculated by using the eigenvalue equations above [14,15].
Figure 2. The simulation results of band structure. (a) and (b) The band structure of regular octagon lattice in TE mode and TM mode. (c) and (d) The band structure of regular dodecagon lattice in TE mode and TM mode.

Due to the complicated structure of regular octagon lattice, we regard it as the compound lattice. The Fourier coefficients $k(G)$ of regular octagon lattice:
\[ k(G) = \frac{1}{a_c a_c} \int dr' \frac{1}{\varepsilon(r')} e^{-i G r'} \]
\[ = \left\{ \frac{1}{\varepsilon_b} + \frac{1}{\varepsilon_a} - \frac{1}{\varepsilon_b} \right\} \frac{1}{\varepsilon_b} \sum_{n=1}^{6} 2 \cos(G \cdot P_n) \]

The \( f \) means the filling ratio of regular octagon lattice. \( \varepsilon_a \) and \( \varepsilon_b \) are the dielectric constants of lattice point and background medium respectively [11]. And the \( r_n \) is the radius of the lattice point. After the calculation, we got the \( P_n \) that are the coordinates of the lattice points in compound lattice: \( \text{P}_1 = (\frac{1}{2}, \frac{1+\sqrt{2}}{2}) = \text{P}_{1,1}, \text{P}_2 = (\frac{1}{2}, \frac{1+\sqrt{2}}{2}) = \text{P}_{1,2}, \text{P}_3 = (\frac{1}{2}, \frac{1+\sqrt{2}}{2}) = \text{P}_{1,3} \) and \( \text{P}_4 = (\frac{1}{2}, \frac{1+\sqrt{2}}{2}) = \text{P}_{1,4} \). And the unit is micron. Similarly, we got the Fourier coefficients \( k(G) \) of regular dodecagon lattice:

\[ k(G) = \frac{1}{a_c a_c} \int dr' \frac{1}{\varepsilon(r')} e^{-i G r'} \]
\[ = \left\{ \frac{1}{\varepsilon_b} + \frac{1}{\varepsilon_a} - \frac{1}{\varepsilon_b} \right\} \frac{1}{\varepsilon_b} \sum_{n=1}^{6} 2 \cos(G \cdot P_n) \]

The \( P_n \) in regular dodecagon lattice: \( \text{P}_1 = (\frac{1}{2}, \sqrt[4]{\frac{7}{4} + \frac{\sqrt{3}}{2}}) = \text{P}_{1,1}, \text{P}_2 = (\sqrt[4]{\frac{7}{4} + \frac{\sqrt{3}}{2}}, \sqrt[4]{\frac{7}{4} + \frac{\sqrt{3}}{2}}) = \text{P}_{1,2}, \text{P}_3 = (\sqrt[4]{\frac{7}{4} + \frac{\sqrt{3}}{2}}, -\sqrt[4]{\frac{7}{4} + \frac{\sqrt{3}}{2}}) = \text{P}_{1,3} \), \( \text{P}_4 = (\sqrt[4]{\frac{7}{4} + \frac{\sqrt{3}}{2}}, \sqrt[4]{\frac{7}{4} + \frac{\sqrt{3}}{2}}) = \text{P}_{1,4} \), \( \text{P}_5 = (\sqrt[4]{\frac{7}{4} + \frac{\sqrt{3}}{2}}, -\sqrt[4]{\frac{7}{4} + \frac{\sqrt{3}}{2}}) = \text{P}_{1,5} \) and \( \text{P}_6 = (\sqrt[4]{\frac{7}{4} + \frac{\sqrt{3}}{2}}, \sqrt[4]{\frac{7}{4} + \frac{\sqrt{3}}{2}}) = \text{P}_{1,6} \). As is shown in Figure 2(a) and (b), there are three band gaps in TE mode and two gaps in TM mode [15]. The band gap in regular octagon lattice is wide. More importantly, there is a large gap in the range of (0.402–0.445), whatever the mode is TE or TM, which means a complete band gap in 2D regular octagon lattice. The complete band gap [16] can achieve the light induction at any direction in range of two dimensional space. It portends that there is great application value for regular octagon photonic medium. From Figure 2(c) and (d), there are also band gaps of regular dodecagon lattice in TE and TM mode. But, the gaps of it are not complete. Comparing the two lattices, the regular octagon lattice is better in application.
Figure 3. Transmission image. (a) and (b) The simulation results of output image on the back of the regular octagon lattice and the regular dodecagon lattice.

As Figure 3 shows, the transmission image was simulated by the method of FDTD. We can see the solitons arranged in the regular octagon or the regular dodecagon on the back when the Gauss beams irradiate on a cell lattice of the front surface in Figure 3(a) and (b) [17]. The localization wavepacket transition [18] results from band gaps of the complicated lattice [8].

4. Conclusion
The research mainly studies on the 2D complicated regular polygon photonic lattices, regular octagon lattice and regular dodecagon lattice. The lattice is a superposition of two equal simple lattices with a certain angle and phase. And the light field of the complicated lattice has been derived in this paper. The regular octagon lattice and regular octagon lattice possess the huge application value due to the energy band gaps. The regular octagon lattice which has the wide complete band gap. And we use the method of PWE to calculate the band structure. Moreover, the solitons can be observed in the photonic lattice we design [19]. This paper only analyses the complicated lattice preliminarily. Thus, many problems are still to be studied and solved.

Acknowledgments
This work is supported by the National Natural Science Foundation of China (Grant nos. 11304187, 11404196, 11574185, 11404196 and 11604183). A Project of Shandong Province Higher Educational Science and Technology Program (Grant no. J14LA55) and China postdoctoral science foundation projects (Grant no.2015M582126).

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