Predicting the X-ray flux of evolved pulsar wind nebulae based on VHE $\gamma$-ray observations

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Context. Energetic pulsars power winds of relativistic leptons which produce photon nebulae (so-called pulsar wind nebulae, PWNe) detectable across the electromagnetic spectrum up to energies of several TeV. The spectral energy distribution has a double-humped structure: the first hump lies in the X-ray regime, the second in the $\gamma$-ray range. The X-ray emission is generally understood as synchrotron radiation by highly energetic leptons, the $\gamma$-ray emission as Inverse Compton scattering of energetic leptons with ambient photon fields. The evolution of the spectral energy distribution is influenced by the time-dependent spin-down of the pulsar and the decrease of the magnetic field strength with time. Thus, the present spectral appearance of a PWN depends on the age of the pulsar: while young PWNe are bright in X-rays and $\gamma$-rays, the X-ray emission of evolved PWNe is suppressed. Hence, evolved pulsar wind nebulae may offer an explanation of the nature of some of the unidentified VHE ($\gamma$-ray sources not yet associated with a counterpart at other wavelengths.

Aims. The purpose of this work is to develop a model which allows to calculate the expected X-ray fluxes of unidentified VHE $\gamma$-ray sources considered to be PWN candidates. Such an estimate may help to evaluate the prospects of detecting the X-ray signal in deep observations with current X-ray observatories in future studies.

Methods. We present a time-dependent leptonic model which predicts the broad-band emission of a PWN according to the characteristics of its pulsar. The values of the free parameters of the model are determined by a fit to observational VHE $\gamma$-ray data. For a sample of representative PWNe, the resulting model predictions in the X-ray and $\gamma$-ray range are compared to observations.

Results. The comparison shows that the energy flux of the X-ray emission of identified PWNe from different states of evolution can be roughly predicted by the model. This implies the possibility of an estimate of the non-thermal X-ray emission of unidentified VHE $\gamma$-ray sources in case of an evolved PWN scenario.

I. INTRODUCTION

Due to a new generation of Imaging Atmospheric Cherenkov Telescopes (IACTs), the number of detected Galactic sources emitting VHE ($\gamma$-rays with $E > 100$ GeV) has increased significantly during the last decade. Pulsar wind nebulae (PWNe) form the most abundant class among these sources. PWNe are usually associated with the non-thermal emission from a magnetized plasma of relativistic particles fed by an energetic pulsar. In current models, the plasma is thought to consist mainly of energetic leptons (see, e.g., Gaensler & Slane 2006) which emit non-thermal radiation over a wide energy range. Interacting with magnetic fields, the leptons produce synchrotron radiation up to several MeV. In addition, low-energy photons, e.g. from the cosmic microwave background (CMB), can be up-scattered by the energetic leptons to very high energies via the Inverse Compton effect. Therefore, the emission in X-rays and VHE $\gamma$-rays is tightly linked, emerging from the same lepton population (see, e.g., Gelfand et al. 2008). The second largest population of Galactic VHE $\gamma$-ray sources consists of unidentified sources without an unambiguous counterpart at other wavelengths (Aharonian et al. 2008). However, in many cases an energetic pulsar can be found in the vicinity, suggesting a possible connection between these unidentified objects and pulsar wind nebulae. Provided that the rotational period and its first time derivative can be measured, e.g. by radio observations, the spin-down energy loss of the pulsar can be estimated and hence the viability of the pulsar as an energy source of the nebula can be investigated. In addition, in the PWN scenario of broad-band emission by energetic leptons a spatial association of the VHE $\gamma$-ray source with an X-ray nebula counterpart is expected. There are mainly two issues complicating this identification scheme: In some cases, PWNe are slightly displaced from the pulsar, which may result from an interaction with the supernova remnant reverse shock (see, e.g., Blondin et al. 2001) and from a proper motion of the pulsar, gained from a kick at its birth (e.g., van der Swaluw et al. 2004). Furthermore, in particular for older systems the X-ray emission becomes fainter and hence harder to detect since the energetic leptons injected during earlier epochs have been cooled and, at the same time, the supply of fresh leptons is reduced. Moreover, the synchrotron emission by the freshly injected leptons is suppressed because the magnetic field...
strength decreases with time. Since the accumulated less energetic leptons can still produce VHE $\gamma$-rays via Inverse Compton (IC) scattering, such evolved PWNe have been proposed (de Jager & Diennani-Atal 2003) as an explanation of some of the as yet unidentified VHE $\gamma$-ray sources.

In this work, we introduce a time-dependent leptonic model of the non-thermal emission of PWNe (Section 2) and apply the model to PWNe of different evolutionary states (Section 3). For each individual source, the free parameters are fixed by fitting the model to the VHE $\gamma$-ray data. Subsequently, we show that the fitted model allows a rough prediction of the X-ray emission of these objects. Hence, in future studies it may serve as a means to estimate the X-ray flux of unidentified VHE $\gamma$-ray sources in a PWN scenario, allowing to evaluate the prospects of detection in deep observations with current X-ray observatories.

II. THE MODEL

In this Section, we introduce a leptonic model describing the time evolution of the non-thermal radiation from PWNe. The time dependence of the energy output $E$ of the pulsar, which derives from the slow-down of the rotation, has to be taken into account:

$$\dot{E} = -\frac{dE_{\text{rest}}}{dt}.$$  

Following Pacini & Salvati (1973), the energy output evolves with time as

$$\dot{E}(t) = \dot{E}_0 \left(1 + \frac{t}{\tau_0}\right)^{-\frac{n+1}{n-1}},$$

where $\dot{E}_0 = \dot{E}(t = 0)$, $\tau_0$ denotes the spin-down timescale of the pulsar and $n$ the braking index. The latter has been measured only for a few young pulsars (see, e.g., Magalhaes et al. 2012, and references therein). Such a measurement exists, for instance, for PSR B1509–58, which is one of the sample pulsars discussed in Section 3. Thus, for the modeling of the PWN associated with PSR B1509–58 we used the measured value of $n = 2.839$ (Livingstone et al. 2007). For the other cases we adopted $n = 3$ (Manchester & Taylor 1977), corresponding to spin-down via magnetic dipole radiation. The spin-down timescale $\tau_0$ is defined by

$$\tau_0 = \frac{2\tau_c}{n-1} \left(\frac{P_0}{P}\right)^{n-1},$$

with $P_0$ and $P$ being the initial and the current period, respectively, and $\tau_c = P/(2\dot{P})$ the characteristic age of the pulsar ($\dot{P}$ denoting the time derivative of the rotational period). For $n = 3$ and $P_0 \ll P$ the present true age $T$ of the pulsar corresponds to the characteristic age $\tau_c$, whereas for other cases it can be calculated as

$$T = \frac{P}{(n-1)\dot{P}} \left(1 - \left(\frac{P_0}{P}\right)^{n-1}\right).$$

$P$ and $\dot{P}$ can usually be derived from radio observations, while the initial period $P_0$ will be treated as a free parameter of our model.

In the following, the evolution of the non-thermal emission of a PWN is calculated in discrete time steps with an adaptive step size of $\delta t$. In each time step only a fractional amount $\Delta E_p(t)$ of the energy output of the pulsar is converted into relativistic leptons, i.e. electrons and positrons. Assuming the corresponding conversion efficiency $\eta$ to be constant over time, $\Delta E_p(t)$ is determined as

$$\Delta E_p(t) = \eta \int_t^{t+\delta t} \dot{E}(t') \, dt'$$

for a time interval $[t, t + \delta t]$ and with $\eta \in [0, 1]$. The conversion efficiency is strongly correlated with $P_0$, such that the quality of the fit does not benefit from an additional free parameter. Therefore, $\eta$ is fixed to the value of 0.3, which is in agreement with e.g. the modeling results for MSH 15–52 carried out by Schöck et al. (2010) and Zhang et al. (2008). We transferred this value to the other selected PWNe.

We assume that the differential energy spectrum of the injected leptons can be described by a simple power law

$$\frac{dN_{\text{inj}}}{dE}(E, t) = \Phi_0(t) \left(\frac{E}{1 \text{ TeV}}\right)^{-2}.$$  

Assuming that the spectral shape does not change within a time bin, $\Phi_0(t)$, denoting the normalization of the distribution at 1 TeV, can be calculated by integrating the injection spectrum over energy:

$$\Delta E_p(t) = \frac{1}{\int_{E_{\text{min}}}^{E_{\text{max}}} \frac{dN_{\text{inj}}}{dE}(E, t) \, dE}.$$  

We only consider leptons injected into the pulsar wind in the range between $E_{\text{min}} = 0.1$ TeV and $E_{\text{max}} = 1000$ TeV, well suited to accommodate the VHE $\gamma$-rays as well as the X-ray emission from PWNe. Leptons with energies outside this range do not significantly contribute to the emission in the considered photon wavebands. Given a differential yield of leptons $dN(E, t - \delta t)/dE$ with energy $E$ at a time $t - \delta t$ the number of leptons remaining after cooling at the time $t$ can be calculated. Following Zhang et al. (2008), the cooling of the lepton population during a time step $\delta t$ is implemented in the model by means of an exponential function:

$$\frac{dN_{\text{cooled}}}{dE}(E, t) = \frac{dN}{dE}(E, t - \delta t) \cdot \exp\left(-\frac{\delta t}{\tau_{\text{eff}}(E, t)}\right).$$

This approach uses an effective cooling timescale $\tau_{\text{eff}}^{-1} = \tau_{\text{syn}}^{-1} + \tau_{\text{esc}}^{-1} + \tau_{\text{ad}}^{-1}$ taking into account synchrotron,
escape and adiabatic energy losses. In general, the cooling timescale of a particle with energy $E$ and a current energy loss rate $\dot{E}_p$ is defined by $\tau = -\frac{\dot{E}_p}{\dot{E}_p}$. The synchrotron and escape cooling time scales $\tau_{\text{syn}}$ and $\tau_{\text{esc}}$ are likewise adopted from Zhang et al. (2008):

$$\tau_{\text{syn}}(E, t) = 12.5 \cdot \left[ \frac{B(t)}{10 \, \mu G} \right]^{-2} \cdot \left[ \frac{E}{10 \, \text{TeV}} \right]^{-1} \text{kyr}$$

(9)

$$\tau_{\text{esc}}(E, t) = 34 \cdot \left[ \frac{B(t)}{10 \, \mu G} \right] \cdot \left[ \frac{E}{10 \, \text{TeV}} \right]^{-1} \cdot \left[ \frac{R(t)}{1 \, \text{pc}} \right]^2 \text{kyr},$$

(10)

where $R(t)$ and $B(t)$ describe the time evolution of the PWN radius and the magnetic field strength inside the PWN, respectively. For evolved PWNe, $R(t)$ is given by (see Gaensler & Slane 2006, and references therein):

$$R(t) = \begin{cases} a \cdot t^{11/15} & \text{for } t < \tau_0 \\ b \cdot t^{7/10} & \text{for } t \geq \tau_0 \end{cases},$$

(11)

where the coefficients $a$ and $b$ can be calculated using the present-day size of the PWN. Based on the radius evolution, we can calculate the timescale for adiabatic energy losses following de Jager & Harding (1992):

$$\frac{dE_{\text{ad}}}{dt} = -\frac{E}{3} \nabla \vec{v}_\perp(R) = \dot{E}_p$$

(12)

with $\vec{v}_\perp(R)$ being the radial component of the particle velocity. In general, its divergence can be calculated to

$$\nabla \vec{v}_\perp(R) = \frac{1}{R^2} \cdot \frac{\partial (R^2 \vec{v}_\perp)}{\partial R}$$

(13)

$$= \frac{1}{R(t)^2} \cdot \frac{\partial (R(t)^2 \vec{v}_\perp(t))}{\partial t} \cdot \frac{\partial t}{\partial R}$$

(14)

$$\Rightarrow \tau_{\text{ad}} = -\frac{E}{\dot{E}_p} = \begin{cases} 45/18 \cdot t & \text{for } t < \tau_0 \\ -30 \cdot t & \text{for } t \geq \tau_0 \end{cases}$$

(15)

This results in an adiabatic energy gain of particles in older PWNe. However, this energy gain is negligible due to the significantly larger time scale compared to the other processes. The magnetic field strength $B(t)$ is adapted from Zhang et al. (2008):

$$B(t) = \frac{B_0}{1 + (t/\tau_0)\alpha} + B_{\text{ISM}}.$$  

(16)

$B_{\text{ISM}}$ represents a time-independent component of $3 \, \mu G$ to account for the magnetic field strength of the ambient medium. Assuming the conservation of magnetic flux density for large time scales ($t \gg \tau_0$) implies $\alpha = 0.6$. Finally, $B_0$, the initial magnetic field strength inside the PWN, is a free parameter. All in all, the model has two free parameters ($P_0$ and $B_0$) defining the starting conditions of the PWN evolution.

Having established the framework for cooling and injection processes, we can calculate the number of leptons with energy $E$ present in the nebula at a time $t + \delta t$. This number comprises leptons injected and cooled until time $t$ as well as freshly injected leptons between $t$ and $t + \delta t$:

$$\frac{dN}{dE}(E, t + \delta t) = \frac{dN_{\text{cooled}}}{dE}(E, t) + \frac{dN_{\text{inj}}}{dE}(E, t + \delta t).$$

(17)

By iteratively evaluating Eq. (17), it is possible to determine the energy distribution of the leptons inside the PWN at an arbitrary time. Based on this distribution, the corresponding photon population can be calculated, with synchrotron radiation and IC scattering as the most relevant emission processes in the considered energy range. A detailed account of these mechanisms can be found in Blumenthal & Gould (1970). We neglect a synchrotron self-Compton (SSC) scattering component in the VHE $\gamma$-ray spectrum, since this work is focused on evolved pulsars, whereas SSC is relevant mostly for the highly magnetized PWNe of very young and energetic pulsars, e.g. the Crab Nebula (Meyer et al. 2010). The target photon fields considered for IC scattering – CMB, starlight and infrared photons – are adopted from the GALPROP code (Porter & Strong 2003).

As a first step, we can use the model to study the development of the spectral energy distribution (SED) with progressing age for a generic PWN system. The SED shown in Fig. 1 is based on the characteristics of the pulsar PSR J1826–1334 and its nebula (see Table III), representing an example of an evolved PWN. The free parameters are exemplarily set to $P_0 = 30 \, \text{ms}$ and $B_0 = 50 \, \mu G$.

Since the magnetic field strength decays strongly with time, the X-ray emission is suppressed for high PWN ages. At the same time, energy-dependent cooling effects become visible in the $\gamma$-ray band, reducing the emissions

![FIG. 1. Typical evolution of the modeled spectral energy distribution of a PWN with time. The color scale represents the age of the PWN, starting with a young system (500 years, yellow) and proceeding in equidistant steps on a logarithmic time scale to an old system (150 kyr, dark red).](image-url)
The comparison shows that the \( \gamma \) -ray data which are in the order of 0.5%. The modeled SEDs are confronted with observational X-ray, \( \gamma \) -ray (where available) and VHE \( \gamma \) -ray data in Fig. 3. In the VHE \( \gamma \) -ray range spectral points were available, while in X-rays and \( \gamma \) -rays the shown lines and areas correspond to the published power-law fits.

The comparison shows that the \( \gamma \) -ray and VHE \( \gamma \) -ray data are reasonably well described by the model, whereas the X-ray data are strongly overestimated. However, this is expected since the extraction region for the spectrum determination is usually much smaller in X-rays than in VHE \( \gamma \) -rays. The resulting mismatch between the lepton population used for the modeling and the one observed in X-rays has to be taken into account. In the following, we assume an outflow velocity of the leptons for the innermost part of the PWN of \( v/3 \) (see, e.g., Kennel & Coroniti 1984). Starting from the given size of the X-ray spectrum extraction region, we determine the corresponding maximum age \( \tau_{\text{lept., max}} \) of the leptons producing the emission. In the next step, we re-calculate the amount and energy distribution of the leptons contained within this region. The time scale of adiabatic energy losses for these freshly-injected leptons was adjusted accordingly.

**III. APPLICATIONS**

Based on the considerations presented above, evolved PWNe offer a potential explanation for a significant fraction of TeV sources which have remained unidentified up to now. In this scenario, an old pulsar may be surrounded by a relic TeV PWN detectable with current IACTs. However, due to the low present energy output of the pulsar the nebula does not contain enough high-energy leptons to produce a strong X-ray counterpart to the TeV PWN (compare Figs. 1 and 2). This view is in accordance with detailed studies presented, e.g., by de Jager & Djannati-Ataï (2009) and Mattana et al. (2009). For some of the unidentified VHE \( \gamma \) -ray sources where an evolved PWN scenario appears likely, deep observations performed with current satellite observatories may yet reveal X-ray counterparts despite the relative weakness of the expected X-ray emission. The model presented in this work allows to select suitable candidates based on an estimate of the required exposure for a detection in the X-ray regime. In order to investigate the reliability of the model we applied it to four selected PWNe, which are listed in Table I. The motivation for this selection was to sample PWNe from different states of evolution for which both VHE \( \gamma \) -ray and X-ray spectra of sufficient quality are available. Since the model is radially symmetric, it was necessary to define a circular source area as an approximation to the asymmetrical Gaussian morphology fits of the published VHE \( \gamma \) -ray data. Accounting for most of the emission, the radius was chosen such that the circle covers an area equivalent to the \( \sqrt{2} \sigma_{\text{ellipse}} \) extent of the ellipse obtained from the VHE morphology fits (\( \sigma_{\text{ellipse}} = \sqrt{\sigma_{x} \sigma_{y}} \)). The values of the used equivalent radii are included in Table I. Having calculated the equivalent circular extent of the VHE \( \gamma \) -ray source, the free parameters of the model were fixed by a \( \chi^{2} \) fit to the VHE \( \gamma \) -ray data using the \textsc{minuit} minimization package (James & Roos 1975). Note that only statistical errors (1 \( \sigma \)) of the VHE \( \gamma \) -ray data are taken into account. The optimized parameters with their uncertainties and the predictions of the VHE \( \gamma \) -ray fluxes are presented in Tables II and III, respectively. We used the calculated errors on the modeled parameters as well as their correlation to propagate the errors on the SEDs. Hence, it is possible to estimate the model uncertainties on the multiwavelength emission derived from the VHE \( \gamma \) -ray data which are in the order of 0.5%.

In the VHE \( \gamma \) -ray range spectral points were available, while in X-rays and \( \gamma \) -rays the shown lines and areas correspond to the published power-law fits.
FIG. 3. Spectral energy distributions for the four sources listed in Table I. The black lines show the modeled SEDs resulting from a fit to the VHE $\gamma$-ray data while the cyan lines denote the model prediction of the X-ray emission calculated for the published analysis regions (compare Table IV). H.E.S.S. data (blue filled circles) are presented along with their 1 $\sigma$ statistical errors. Red lines and areas show the corresponding X-ray data with the respective error band assuming uncorrelated errors of the parameters (see also Table IV). If available, we included $\gamma$-ray data (orange) from the Fermi Large Area Telescope (Fermi-LAT), as well. The $\gamma$-ray data are also shown along with their statistical errors. References for the VHE $\gamma$-ray and X-ray data can be found in Tables I and IV, respectively. Fermi data for MSH 15−52 and for HESS J1825−137 are adopted from Abdo et al. (2010) and Grondin et al. (2011), respectively. The uncertainties on the modeled SEDs are very small and hence not visible in these plots.

$\tau_{\text{ad}} = 3/2 \cdot t$, see Eq. (12). In order to consider projection effects in the circular region of the X-ray analysis, we added the integrated emission along the line of sight following Holler et al. (2012). In each case we used the smallest available X-ray analysis region since the validity of the assumed outflow velocity is spatially limited to a region close to the pulsar. The resulting modified SEDs, shown in Fig. 3 are clearly in better agreement with the observational data. A quantitative comparison between modeled and measured values of the X-ray flux is given in Table IV.

**IV. CONCLUSIONS**

Motivated by the large number of yet unidentified VHE $\gamma$-ray sources suspected to be evolved PWNe, we developed a time-dependent leptonic model suitable to calculate the non-thermal emission from PWNe of different ages. The presented model allows to study the expected photon SEDs evolving with the age of the PWN. Our study yields additional support for the conception that evolved PWNe are still bright in VHE $\gamma$-rays, while their X-ray emission is largely suppressed and hence difficult to detect.

Moreover, in this work we investigate the contribution of leptons of different epochs to the current photon SED in
TABLE I. Overview of the selected PWNe and their associated pulsars. The list is sorted by increasing characteristic age, representing different evolutionary states. The properties of the pulsars (characteristic age $\tau_c$, current period $P$, current spin-down luminosity $\dot{E}$ and distance $d$) are taken from the ATNF pulsar database\cite{Manchester et al. 2005}. References for the H.E.S.S. sources: \cite{Aharonian et al. 2005}, \cite{Aharonian et al. 2006b}, \cite{Aharonian et al. 2006a}.

| VHE Source          | equiv. VHE source radius [arcmin] | Pulsar       | $\tau_c$ [kyr] | $P$ [ms] | $\dot{E}$ [erg s$^{-1}$] | $d$ [kpc] |
|---------------------|-----------------------------------|--------------|----------------|----------|--------------------------|-----------|
| MSH 15−52$^1$       | 5.4                               | PSR J1509−58 | 1.55           | 151      | 1.8 $\times 10^{37}$    | 5.81      |
| HESS J1420−607$^2$  | 4.7                               | PSR J1420−6048 | 13.0         | 68       | 1.0 $\times 10^{37}$    | 7.65      |
| HESS J1825−137$^3$  | 20.7                              | PSR J1826−1334 | 21.4         | 101      | 2.8 $\times 10^{36}$    | 4.12      |
| HESS J1837−069$^4$  | 6.6                               | PSR J1838−0655 | 22.7         | 70       | 5.5 $\times 10^{36}$    | 6.6$^b$   |

$^a$ URL: \url{http://www.atnf.csiro.au/research/pulsar/psrcat/}

$^b$ No distance estimate provided, we use the value proposed in Gotthelf & Halpern (2008) instead.

TABLE II. Application of the model to the selected PWNe. The model was fit to the VHE $\gamma$-ray data and the fit results of the free parameters $P_0$ and $B_0$ are listed. The interval in parentheses denotes the parameter boundaries during the optimization procedure.

| Source                  | $P_0$ [ms] (5−200) | $B_0$ [$\mu$G] (5−200) | $\chi^2$/n.d.f |
|-------------------------|--------------------|------------------------|----------------|
| MSH 15−52               | 38.9 ± 0.5         | 104.3 ± 2.7            | 14.5/12        |
| HESS J1420−607          | 33.2 ± 0.7         | 23.6 ± 1.3             | 3.5/8          |
| HESS J1825−137          | 26.0 ± 0.1         | 33.7 ± 0.4             | 27.9/9         |
| HESS J1837−069          | 30.7 ± 0.8         | 24.9 ± 1.6             | 9.7/10         |

TABLE III. Comparison of predicted and measured VHE $\gamma$-ray flux for the modeled PWNe. References can be found in the caption of Table I.

| Source                  | Energy threshold [TeV] | Flux above threshold $^a$ |
|-------------------------|-----------------------|---------------------------|
|                         | Model Prediction      | Measured                  |
| MSH 15−52               | 0.28                  | 20.0                      |
| HESS J1420−607          | 1.0                   | 2.9                       |
| HESS J1825−137          | 0.27                  | 77.8                      |
| HESS J1837−069          | 0.2                   | 26.7                      |

$^a$ In units of $10^{-12}$ cm$^{-2}$ s$^{-1}$.

$^b$ No statistical errors provided.

TABLE IV. Comparison of predicted and measured X-ray emission for the modeled PWNe. For each X-ray analysis the published energy range and the radius of the used analysis region are listed. References to the X-ray data: \cite{Schöck et al. 2010}, \cite{Ng et al. 2005}, \cite{Uchiyama et al. 2000}, \cite{Gotthelf & Halpern 2008}. The uncertainties of the predicted values are in the order of 0.5%.

| Source                  | Analysis region | Energy range [keV] | $F_X^a$ | Index $^b$ |
|-------------------------|-----------------|--------------------|---------|------------|
|                         | Model Prediction | Measured           |         | Model Prediction | Measured           |
| MSH 15−52               | 30′′ − 57″      | 0.5 − 9            | 13.6    | 12.0 ± 0.3 | 2.1               |
| HESS J1420−607          | 13.5″            | 2 − 10             | 0.35    | 0.13 ± 0.03 | 1.77              |
| HESS J1825−137          | 1.5″             | 0.8 − 10           | 1.20    | 1.71 ± 0.47 | 1.82              |
| HESS J1837−069          | 1.0°            | 2 − 10             | 1.55    | 1.0$^c$    | 1.87              |

$^a$ Energy fluxes are given in units of $10^{-12}$ erg cm$^{-2}$ s$^{-1}$.

$^b$ Photon index of a power-law model.

$^c$ No statistical errors provided.
tion and cooling effects in order to explain the observed VHE $\gamma$-ray emission, whereas the X-ray emission (especially in the vicinity of the pulsar) is dominated by the young lepton population.

Finally, we tested whether our model can be used to predict the X-ray flux of an unidentified source based on the VHE $\gamma$-ray detection. We selected four representative PWNe of different evolutionary states and fixed the free parameters of the model by a fit to observational VHE $\gamma$-ray data. The comparison of modeling results and observational data shows that it is possible to roughly predict the order of magnitude of the energy flux of the X-ray emission. Thus the model may facilitate the identification of evolved PWNe.

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