We study bulk-boundary correlators in topological open membranes. The basic example is the open membrane with a WZ coupling to a 3-form. We view the bulk interaction as a deformation of the boundary string theory. This boundary string has the structure of a homotopy Lie algebra, which can be viewed as a closed string field theory. We calculate the leading order perturbative expansion of this structure. For the 3-form field we find that the C-field induces a trilinear bracket, deforming the Lie algebra structure. This paper is the first step towards a formal universal quantization of general quasi-Lie bialgebroids.
1. Introduction

In this paper we study bulk-boundary correlators for topological open membrane models as discussed in [1, 2]. These are basically deformed BF type theories on a 3-manifold with boundary, with a manifest BV structure implemented. The basic gauge fields, ghosts and antifields are combined into superfields, which can be understood as maps from the superworldvolume into a super target manifold $\mathcal{M}$. These models will be referred henceforth as BV sigma models. Passing to a superworldvolume automatically describes differential form fields. The manifest BV structure will however make it rather straightforward to gauge fix the gauge theory.

A particular example, and indeed the main motivation for this work, is the open 2-brane (according to the terminology of [1]) with a topological WZ coupling to a closed 3-form. In [1] the open 2-brane model was shown to correspond to a BV sigma model of BF type. This model can be viewed as the membrane analogue of the Poisson-sigma model [7]. The latter model was studied by Cattaneo-Felder in [8] to describe deformation quantization in terms of deformed boundary correlation functions of a topological open string theory. In fact it was shown in [1], that the open membrane model can be seen as a deformation of this model. We tackle the topological open 2-brane theory in this paper in a way similar to [8]. The CF model captures the effect of a 2-form field background in string theory [9], which gives rise to noncommutative geometry [10, 11].

This model could be a toy model for string theory in a background 3-form field. Models for WZ couplings to a large 3-form field were studied in [3, 4, 5, 6], but rather from the point of view of the somewhat ill-defined boundary string theory. In [3] it was shown that in a particular decoupling limit a stack of M5-branes in a 3-form field the open membrane action of the M2-brane reduces to such a particularly simple topological membrane model with a large C-field. Our model could perhaps shed some light on the role of the mysterious generalized theta parameter that is central in decoupling limits of open membranes [3, 12, 13, 14, 15, 16]. Admittedly, our treatment is perturbative in the 3-form, and therefore not able to directly describe this situation. In [2] we argue that in the context of our model, at least in some cases, a large 3-form can be related by a canonical transformation to a model with another value for the 3-form, which can be small. However this involves the choice of an auxiliary Poisson structure, whose interpretation is not clear to us at the moment.

More general models in this class are defined in [2]. They are shown to describe deformations of so-called Courant algebroids [17, 18, 19] (which could also be called more descriptively quasi-Lie bialgebroids). These structures have a deep relation to problems of
quantization. For example, the original exact Courant algebroid was developed as an attempt
to geometrically describe quantization of phase space with constraints and gauge symmetries
[17]. It is related to a mix of the tangent space and the cotangent space of a manifold. It
can be deformed by a 3-form, which induces a deformation of the Poisson structure to a
quasi-Poisson structure [19, 20, 21]. This induces a deformed deformation quantization.

To explain some of the words above, let us start with a (quasi)-Lie bialgebra, also known
equivalently as a Manin pair. It can be described in terms of a Lie algebra \( g \) and its dual
space \( g^* \), such that the total space \( g \oplus g^* \) is also a Lie algebra. Note that the latter has a
natural inner product and \( g \) is a maximally isotropic Lie subalgebra. This is the formulation
used for a Manin pair \((g \oplus g^*, g)\). In the language of quasi-Lie bialgebras, the bracket on
the total space is formulated in terms of extra structure on the Lie algebra
\( g \): a 1-cocycle called the cocommutator \( \delta : g \to \Lambda^2 g \) dual to the bracket restricted to \( g^* \), and an element
\( \varphi \in \Lambda^3 g \), such that \( \delta \varphi = 0 \) and \( \delta^2 = -[\varphi, \cdot] \). When \( g^* \) is also a Lie subalgebra or equivalently
\( \varphi = 0 \), \((g \oplus g^*, g, g^*)\) is called a Manin triple and \( g \) a Lie bialgebra. (Quasi-)Lie bialgebras are
the infinitesimal objects corresponding to (quasi-)Hopf algebras [22]. Next, an algebroid is a
vector bundle \( A \) with a Lie bracket on the space of sections, acting as differential operators of
degree 1 in both arguments. A well known example of a Lie algebroid is the tangent bundle
\( TM \) of a manifold. When the base manifold is a point, the definition of a Lie algebroid
simply reduces to that of a Lie algebra. A (quasi-)Lie bialgebroid combines the structure
of (quasi-)Lie bialgebra and Lie algebroid. It is an algebroid \( A \) which has a Lie bracket, a
cocommutator and an element \( \Lambda^3 A \) satisfying certain integrability relations. For a precise
definition we refer to the literature, see [19] and references therein. There it was also shown
that in general a quasi-Lie bialgebra is equivalent to the structure of Courant algebroid.
Courant algebroids appeared originally as an attempt to geometrize general quantization of
constraint gauge systems [17]. The particular model mentioned above, for the coupling to
the 3-form field, is related to the exact Courant algebroid, for which \( A = T^* M \).

The correlation functions we will calculate can be understood as a deformation of a
homotopy Lie \( L_\infty \) algebra, on the boundary of the membrane induced by the bulk couplings.
The definition of this \( L_\infty \) in terms of correlation functions was explained in [23]. This \( L_\infty \)
structure can be identified with the structure of the closed string field theory of this boundary
string. The \( L_\infty \) structure of closed string field theory was demonstrated in [24]. It was
discussed in the context of topological strings in [25, 26]. It is indeed this \( L_\infty \) structure that
is naturally deformed by bulk membrane couplings [23]. The semiclassical approximation of
this \( L_\infty \) structure is equivalent to the structure of quasi-Lie bialgebra and more generally a
quasi-Lie bialgebroid, as explained in [2]. This structure is of course natural in string theory,
and plays an important role in CS and WZW models.

As the deformation to first order in the bulk couplings induce the quasi-Lie bialgebroid structure, we could expect that by taking higher order correlators into account we should find its quantization. For the “rigid” case of quasi-Lie bialgebras the quantization is a quasi-Hopf algebra [22]. The boundary theory of CS theory, which is the WZW model, indeed has the structure of a quasi-Hopf algebra. In a subsequent paper [27] we will discuss an explicit construction of this quasi-Hopf algebra for our model. The existence of a universal quantization for Lie bialgebras was proven in [28]. More generally, this could be applied to the models related to genuine Courant algebroids. The path integral of the BV sigma models studied in this paper can be used to define a formal universal quantization, extended to quasi-Lie bialgebras and even quasi-Lie bialgebroids. The explicit quantization of the model discussed here will be an important first step in this quantization program.

This paper is organized as follows. In the next section we review the basic structure of the BV sigma models for the topological open membranes. In Section 3 we will perform the gauge fixing and calculate the propagators. In Section 4 these will be used to calculate the bulk-boundary correlation functions relevant for the deformed $L_{\infty}$ structure of the boundary algebra. We conclude with some discussion.

2. BV Action For the Open Membrane

Here we shortly discuss topological open membrane models. We will only provide a sketch; more details can be found in [2, 1].

2.1. BV Quantization

We start by reviewing shortly the method of BV quantization [29, 30]. We will be very brief, and refer to the literature for more details, see e.g. [31] for a good introduction.

The models we will study in this paper will contain gauge fields. In order to properly define the path integral in this context we need to divide out the (infinite) volume of the gauge group, and we have to construct a well defined quotient measure for the path integral. The BV formalism is a convenient and general procedure to construct this measure.

One first fermionizes the gauge symmetry, by introducing anticommuting ghost fields for all the infinitesimal generators of the gauge symmetry. The charge of the corresponding fermionic symmetry is the BRST charge $Q$. It squares to zero if the gauge symmetries close.
There will be a corresponding charge \( g \), called ghost number, such that the original fields have \( g = 0 \) and the ghost fields \( g = 1 \). Hence \( Q \) has ghost number 1. The parity of the field will correspond to the parity of the ghost number. When there are relations between gauge symmetries one needs in addition also ghost-for-ghost fields with \( g = 2 \), etcetera. All these fields will be referred to simply as “fields”. In addition to these fields one needs to introduce corresponding “antifields”. They correspond to the equations of motion. Generally, if a field \( \phi \) has ghost number \( g \) then its antifield \( \phi^+ \) has ghost number \( -1 - g \). Also, in our conventions, the antifields of a \( p \)-form field will have form degree \( d - p \) in \( d \) dimensions. Regarding the “fields” and “antifields” as conjugate coordinates in an infinite dimensional phase space, one has a natural symplectic structure and a dual Poisson bracket \( (\cdot, \cdot) \). Due to the relation of the ghost numbers, the latter is an odd Poisson bracket and has ghost number 1. It is called the BV antibracket. Often we will just call it the BV bracket in this paper. Due to the odd degree, it is graded antisymmetric and it satisfies a graded Jacobi identity, 

\[
(\alpha, \beta) = (-1)^{|\alpha|+1}(-1)^{|\beta|+1}(\beta, \alpha), \tag{1}
\]

\[
(\alpha, (\beta, \gamma)) = ((\alpha, \beta), \gamma) + (-1)^{|\alpha|+1}(-1)^{|\beta|+1}(\beta, (\alpha, \gamma)), \tag{2}
\]

where \(|\alpha|\) denotes the ghost number of \( \alpha \). For BV quantization one requires in addition a BV operator \( \Delta \). This is an operator of ghost number 1 satisfying \( \Delta^2 = 0 \) and is such that the BV bracket can be given by the failure for \( \Delta \) for being a derivation of the product, 

\[
(\alpha, \beta) = (-1)^{|\alpha|}\Delta(\alpha \beta) - (-1)^{|\alpha|}\Delta(\alpha) \beta - \alpha \Delta \beta. \tag{3}
\]

At the linear level in antifields, the dependence of the action on the “antifields” is determined by the gauge symmetry. More precisely, if \( \phi^+_I \) is the antifield for the field \( \phi^I \), the terms linear in the antifields are given by \( S_1 = \int \phi^+_I Q \phi^I \). Note that this implies that the gauge transformation of the fields can be recovered in terms of the corresponding Hamiltonian vector field, \( Q \phi^I = (S_1, \phi^I) \). More generally, the BV-BRST operator \( Q \) is determined by the full BV action \( S_{BV} \) by the relation \( Q = (S_{BV}, \cdot) \). It squares to zero if the BV action satisfies the classical master equation \( (S_{BV}, S_{BV}) = 0 \). Quantum mechanically this is modified to the quantum master equation \( (S_{BV}, S_{BV}) - 2i\hbar \Delta S_{BV} = 0 \). This is equivalent to nilpotency of the quantum version of the differential, \( Q - i\hbar \Delta \). The Jacobi identity for the BV bracket implies that the BV-BRST operator is a graded derivation of the BV bracket.

Let us now relate this to the path integral. A BV observable \( \mathcal{O} \) is a functional of the fields and antifields satisfying \( Q \mathcal{O} - i\hbar \Delta \mathcal{O} = 0 \). The expectation value of such an observable is calculated by the path integral

\[
\langle \mathcal{O} \rangle = \int L D\phi e^{\frac{i}{\hbar}S_{BV}} \mathcal{O}, \tag{4}
\]
where the integration is performed over a Lagrangian subspace \( L \) in field space. The quantum master equation is equivalent to the condition that this expectation value does not change under continuous deformations of \( L \) for any BV observable. A choice of Lagrangian subspace \( L \) is called a gauge fixing. The Lagrangian subspace \( L \) can be given in terms of a gauge fixing fermion \( \Psi \), which is a function of the fields \( \phi^I \) of ghost number \(-1\). In terms of \( \Psi \), the subspace \( L \) is then given by fixing the antifields as \( \phi^+_I = \frac{\partial \Psi}{\partial \phi^I} \). The quantum master equation then implies that the above expectation values are independent of continuous variations of \( \Psi \). The idea is to choose \( \Psi \) such that the kinetic terms in the action become nondegenerate, so that one can define a propagator and apply perturbation theory.

### 2.2. Superfields and Action

The topological open membranes we study are of BF types, that is, the fields are differential forms \( \phi^I_{(p)} \) on the worldvolume \( V \) of the membrane. An essential role will be played by the BRST operator \( Q \) and the 1-form charge \( G_\mu \), satisfying the crucial anti-commutation relations \( \{ Q, G_\mu \} = \partial_\mu \). The existence of \( G_\mu \) is guaranteed for any topological field theory, as the energy-momentum tensor is BRST exact. The above anti-commutation relation gives rise to descent equations for the observables. We will define descendants of operators by the recursive relation as \( O^{(p+1)} = G O^{(p)} \). When the scalar operator \( O^{(0)} \) is BRST closed, they satisfy the descent equation \( Q O^{(p+1)} = d O^{(p)} \).

The theories we are interested in start from differential \( p \)-forms \( \phi^I_{(p)} \) with gauge transformations giving a BRST operator of the form \( Q \phi^I_{(p)} = d \phi^I_{(p-1)} + \cdots \), where the dots contain no derivatives. Possibly by introducing auxiliary fields, we can always choose the ghosts \( \phi^I_{(p-1)} \) such that the field \( \phi^I_{(p)} \) is its descendant. This can be extended to higher gauge symmetries. Higher descendants have negative ghost number, hence they will be antifields. We will consider all descendants found in this way as “fundamental” in our BV theory. It will be convenient to combine all these descendants—gauge fields, ghosts, and antifields—into superfields, introducing anticommuting coordinates \( \theta^\mu \) of ghost number \(-1\),

\[
\phi^I(x, \theta) = \phi^I(x) + \theta^\mu \phi^{(1)}_\mu(x) + \frac{1}{2} \theta^\mu \theta^\nu \phi^{(2)}_{\mu\nu}(x) + \frac{1}{3!} \theta^\mu \theta^\nu \theta^\rho \phi^{(3)}_{\mu\nu\rho}(x).
\]

On superfields we have \( G_\mu = \frac{\partial}{\partial \theta^\mu} \). Viewing \( (x^\mu | \theta^\mu) \) as coordinates on the supermanifold \( \mathcal{V} = \Pi TV \), where \( \Pi \) denotes the shift of (ghost) degree in the fiber by \( +1 \), these superfields can be viewed as functions on this super worldvolume. They take value in some target superspace \( \mathcal{M} \). In this way superfields are maps between these supermanifolds, and we can
formulate the model as a sigma-model with superspaces as target and base spaces. We will sometimes use the notation \( \bar{x} \) for the collection of supercoordinates \( (x^\mu|\theta^\mu) \).

For the general model, we start from a supermanifold \( \mathcal{M} \), with a symplectic structure \( \omega \) of degree 2. This will be the target space of our sigma-model. Let \( \phi^I \) denote a set of coordinates on \( \mathcal{M} \). They will induce superfields \( \phi^I \) on the super worldvolume \( \mathcal{V} \). The set of supercoordinates form a map \( \phi : \mathcal{V} \to \mathcal{M} \). The symplectic structure on \( \mathcal{M} \) induces a BV symplectic structure on superfield space given by

\[
\omega_{BV} = \int_{\mathcal{V}} \phi^* \omega = \frac{1}{2} \int_{\mathcal{V}} \omega_{IJ} \delta \phi^I \delta \phi^J ,
\]

where the variations \( \delta \phi^I \) can be understood as a basis of one-forms on superfield space. Note that this symplectic structure has ghost degree \(-1\), due to the integration over the super worldvolume. This symplectic structure induces a twisted Poisson bracket of degree 1, or BV bracket, acting on functionals of the superfields. It is given by

\[
(\alpha, \beta) = \int_{\mathcal{V}} \omega^{IJ} \frac{\partial^R \alpha}{\partial \phi^I} \frac{\partial^L \beta}{\partial \phi^J} \equiv \sum_{I,J,p} \int_{\mathcal{V}} \omega^{IJ} \frac{\partial^R \alpha}{\partial \phi^I{(p)}} \frac{\partial^L \beta}{\partial \phi^J{(3-p)}} ,
\]

where the superscripts \( R, L \) denote right and left derivatives, and \( \omega^{IJ} \) is the inverse of \( \omega_{IJ} \). It is easily seen that this bracket can be derived from a BV operator \( \Delta \).

The BV action functional will be given by

\[
S_{BV} = \int_{\mathcal{V}} \left( \frac{1}{2} \omega_{IJ} \phi^I d\phi^J + \gamma \right) ,
\]

where \( \gamma = \phi^* \gamma \) for \( \gamma \) a function on \( \mathcal{M} \). The kinetic terms in this action have the usual BF structure \( \int B dA \), with the “\( B \)” and “\( A \)” fields residing in conjugate superfields with respect to the BV structure. For \( \gamma = 0 \) the BV-BRST operator is given by \( Q_0 = d \). This satisfies the correct anticommutation relation with \( G_\mu = \frac{\partial}{\partial \theta^\mu} \). In fact the form \( S_{BV} \) of the action is essentially the only one consistent with this requirement. In order for it to satisfy the BV master equation, the function \( \gamma \) should satisfy a corresponding identity, and the superfields should satisfy appropriate boundary conditions. These conditions can be described as follows.

We denote by \( [\cdot, \cdot] \) the Poisson bracket on the space \( C^\infty(\mathcal{M}) \) dual to the symplectic structure \( \omega \). Similarly we can construct a BV-like second order differential operator \( \Delta = \frac{1}{2} \omega^{IJ} \frac{\partial^2}{\partial \phi^I \partial \phi^J} \) for this bracket. Note that the BV bracket and BV operator on superfield space can be related from these structures on \( \mathcal{M} \) by pullback. Then the BV master equation gives the condition \( [\gamma, \gamma] + 2i\hbar \Delta \gamma = 0 \). In addition there are conditions coming from the boundary terms. When restricted to the important set of operators of the form \( f = \phi^* f = f(\phi) \) for
a function $f$ on $M$, we can express the deformed operator $Q$ in terms of the deformation $\gamma$ and the poisson bracket $[,]$ as $Qf = df + \phi^* [\gamma, f]$. In other words, the deformation of the BRST operator acts on these functions essentially through the Poisson differential $Q = [\gamma, ·]$. To get good boundary conditions, we choose a Lagrangian submanifold $L \subset M$, and restrict $\phi$ to take values in $L$ on the boundary $\partial V$. The Lagrangian condition will guarantee that the action satisfies the master equation for $\gamma = 0$, and reduces the more general master equation to an algebraic equation (from the worldvolume point of view) for $\gamma$. We can also add a boundary term,

$$S_{bdy} = \int_{\partial V} \beta,$$

where $\beta = \phi^* \beta$ and $\beta$ is a function on $L$.

2.3. Courant Algebroid and 3-Form Deformations

We will only consider models with fields of non-negative ghost number. This also means that the ghost number should not exceed 2 (as otherwise its anti superfield will have negative ghost number). There will be superfields of ghost number 0, which will be denoted $X^i$, and their antifields have ghost number 2, and are denoted $F_i$. Furthermore there are superfields of ghost number one. They come in conjugate pairs $\chi^a$ and $\psi^a$, containing each others antifields. The expansions of these superfields will read

$$X^i = X^i + \theta \cdot P^+ + \frac{1}{2} \theta^2 \cdot \eta^+ + \frac{1}{3!} \theta^3 \cdot F^+, \quad F_i = F_i + \theta \cdot \eta_i + \frac{1}{2} \theta^2 \cdot P_i + \frac{1}{3!} \theta^3 \cdot X^+,$$

$$\chi^a = \chi^a + \theta \cdot A^a + \frac{1}{2} \theta^2 \cdot B^+_a + \frac{1}{3!} \theta^3 \cdot \psi^+_a, \quad \psi^a = \psi^a + \theta \cdot B^a + \frac{1}{2} \theta^2 \cdot A^+_a + \frac{1}{3!} \theta^3 \cdot \chi^+_a,$$

where we suppressed the worldvolume indices and their contractions. The ghost degree zero scalars $X^i$ are coordinate fields on some bosonic target space $M$. The ghost degree one fields $\chi^a$ form coordinates on the fiber of some odd fiber bundle $\Pi A$ over $M$, while their antifields $\psi^a$ are coordinates on the fiber of the dual fiber bundle $\Pi A^*$. The total target superspace can be identified with the twisted cotangent bundle $M = T^*[2](\Pi A)$. Note that the fiber of this cotangent bundle contains the conjugate $F_i$ to the base coordinates $X^i$ and the conjugate
$\psi^a$ to the fiber coordinates $\chi_a$. A special case arises when we take $A = T^*M$, which leads to a so-called exact Courant algebroid. It was shown in [1] that this particular model gives rise to the topological membrane coupling to a 3-form WZ term.

As the full target space is a cotangent bundle, it has a natural symplectic structure. On the space of all superfields, this induces the following odd symplectic structure,

\begin{equation}
\omega_{BV} = \int_V (\delta F_i \delta X^i + \delta \psi^a \delta \chi_a),
\end{equation}

giving a BV bracket of the form

\begin{equation}
(\cdot, \cdot) = \int_V \left( \frac{\partial}{\partial X^i} \wedge \frac{\partial}{\partial F_i} + \frac{\partial}{\partial \chi_a} \wedge \frac{\partial}{\partial \psi^a} \right).
\end{equation}

It is related to the BV operator $\Delta = \int_V \left( \frac{\partial^2}{\partial X^i \partial F_i} + \frac{\partial^2}{\partial \chi_a \partial \psi^a} \right)$.

The BV action of the deformed theory is given by

\begin{equation}
S_{BV} = \int_V (F_i dX^i + \psi^a d\chi_a + \gamma).
\end{equation}

The interactions we will consider will be of the form

\begin{equation}
\gamma = a^i a^j F_i \psi^a + b^i a^j F_i \chi_a + \frac{1}{3!} c_{abc} \psi^a \psi^b \psi^c + \frac{1}{2} c_{ab} \psi^a \psi^b \chi_a + \frac{1}{2} c_{bc} \psi^a \chi_b \chi_c.
\end{equation}

Here the coefficients $a, b, c$ can be any functions of the degree zero superfields $X^i$. The deformation $\gamma$ should of course satisfy the master equation $[\gamma, \gamma] + 2i \hbar \Delta \gamma = 0$.

The canonical example is the exact Courant algebroid, based on the cotangent bundle $A = T^*M$. Here the indices $a$ and $i$ can be identified. We then take the coefficient of the $\psi F$ term to be $a^i_j = \delta^i_j$. This will generate the Lie bracket on vector fields. The main other interaction is the cubic interaction $\frac{1}{3!} c_{ijk} \psi^i \psi^j \psi^k$. Here $c$ should be a closed 3-form. More generally we can also turn on an antisymmetric bivector $b^{ij}$. The full deformation is then given by

\begin{equation}
\gamma = b^{ij} F_i \chi_j + \frac{1}{2} (\partial_k b^{ij} + b^{il} b^{jm} c_{klm}) \psi^k \chi_i \chi_j + \frac{1}{2} b^{il} c_{ijkl} \psi^j \psi^k \chi_i + \frac{1}{6} c_{ijk} \psi^i \psi^j \psi^k.
\end{equation}

It satisfies the master equation provided that $c_{ijk}$ is a closed 3-form and $3b^{[i} \partial_j b^{k]} + b^{il} b^{jm} b^{kn} c_{lmn} = 0$. Notice that this is a deformation of the Poisson condition for the bivector $b^{ij}$.

After integrating out the linearly appearing superfield $F$ it gives the Poisson sigma-model studied by Cattaneo-Felder [8] on the boundary with a bulk membrane coupling to the 3-form $c$ (by pull-back) [1].

\begin{equation}
\frac{1}{3!} \int_V c_{ijk} dX^i dX^j dX^k + \int_{\partial V} \left( \chi_i dX^i + \frac{1}{2} b^{ij} \chi_i \chi_j \right).
\end{equation}
Hence this model is the basic example of a (topological) string deformed by the 3-form.

Another special case arises when there are only multiplets of degree one. The only coefficients in the bulk terms are the $c$'s above. The target space has the form $\mathcal{M} = \Pi \mathfrak{g} \oplus \Pi \mathfrak{g}^*$, where $\mathfrak{g}$ is the vector space associated to the $\chi$ and $\mathfrak{g}^*$ the dual vector space associated to $\psi$. As shown in [2] the master equation is equivalent to the conditions that $\mathfrak{g}$ is a quasi-Lie bialgebra—or equivalently it says that $(\mathfrak{g} \oplus \mathfrak{g}^*, \mathfrak{g})$ is a Manin pair. This is well known to be the infinitesimal structure of a quasi-Hopf algebra. We therefore expect to find this latter structure when quantizing the model.

2.4. Boundary Conditions

The boundary conditions are restricted by the following rules. First, the restriction to the boundary of $F_i dX^i + \psi^a d\chi_a$ should be zero. In general, the boundary condition for a field $\phi$ is the same as for the Hodge dual of its antifield, $* \phi^+$. Furthermore the boundary condition for a 1-form $\phi^{(1)}$ is the same as for $d \phi^{(0)}$, in order not to break the BRST invariance. What remains is to provide the boundary conditions of the scalars. We choose here for $X^i$ and $\chi_a$ Neumann boundary conditions, and for $F_i$ and $\psi^a$ Dirichlet boundary conditions. The boundary conditions are therefore

\begin{align}
    d_\perp X &= 0, & F &= 0, & d_\perp \chi &= 0, & \psi &= 0, \\
    P^+_{\perp} &= 0, & \eta_{\parallel} &= 0, & A_{\perp} &= 0, & B_{\parallel} &= 0, \\
    (* \eta^+)_\parallel &= 0, & (* P)_{\perp} &= 0, & (* B^+)_\parallel &= 0, & (* A^+)_\perp &= 0.
\end{align}

(16)

All the 3-forms are zero on the boundary. This will however not be relevant, as these will vanish after gauge fixing anyway. For the superfields $\mathbf{F}$ and $\psi$ these boundary conditions can be conveniently rephrased by saying that they vanish on the boundary $\partial V$, i.e. at $(x_\perp | \theta_\perp) = 0$. More generally, boundary conditions can be chosen by restricting the coordinate fields $\phi^I$ to map to a Lagrangian submanifold $\mathcal{L} \subset \mathcal{M} = T^*\Pi A$. Above we chose $\mathcal{L} = \Pi A$.

In the following we will call the fields $(X, \chi)$ living on the boundary the basic fields, and their anti-superfields $(F, \psi)$ conjugate fields.

2.5. Boundary Observables, Boundary Algebra, and Correlators

We can relate boundary observables to functions on the Lagrangian submanifold $\mathcal{L}$ (equal to $\Pi A$ in the situation described in the last subsection). For $f \in \mathcal{B} \equiv C^\infty(\mathcal{L})$ and a $p$-cycle $C_p \subset \partial V$, we can build the coordinate invariant integrated operators $O^{(p)}_{f,C_p} = f_{C_p} f^{(p)}$. Let
$x$ be a point on the boundary $\partial V$. Then we have an operator from the scalar component of
the superfield $f = \phi^* f$, evaluated at $x = (x|0)$,
\[ \mathcal{O}_{f,x} = \phi^* f(x) = f(\phi(x)). \] (17)

If $C$ is a 1-cycle in $\partial V$, let us denote by $C = \Pi TC$ the super extension in $\partial V$. Then we have
the first descendant operator
\[ \mathcal{O}^{(1)}_{f,C} = \oint_C f = \oint_C f^{(1)}. \] (18)

Note that the integral over $C$ includes an integration over $\theta$ in the tangent direction to $C$.
Lastly, we have the operators for 2-cycles $S \subset \partial V$,
\[ \mathcal{O}^{(2)}_{f,S} = \int_S f = \int_S f^{(2)}, \] (19)
where $S = \Pi TS$. For example, for the full boundary $S = \partial V$, this corresponds to a
deformation of the boundary interaction.

The operators $\mathcal{O}_{f,C_p} = \int_{C_p} f$, with $C_p = \Pi TC_p$, are closed with respect to the undeformed
BRST operator $Q_0 = d$, due to Stokes’ theorem and the fact that $C_p$ has no boundary.
Also note that the BV operator is manifestly zero on the boundary, due to the Lagrangian
condition on $\mathcal{L}$. More generally, the deformed BRST operator will act on $f$ for $f \in \mathcal{B}$ through the
differential $Q = [\gamma, \cdot]$ restricted to the “boundary algebra” $\mathcal{B}$. Therefore the operators
above are genuine observables as long as this differential vanishes. If $Q$ is nonzero on $\mathcal{B}$, we
will still loosely speak of the above operators as “observables”, even though they are not
necessarily closed. Genuine observables should then be constructed from the $Q$-cohomology
of $\mathcal{B}$. A more extended discussion of observables, especially relevant for nonzero $Q$, is beyond
the scope of this paper and will appear elsewhere [27].

We will be interested in the effect of the bulk terms on the string theory living on the
boundary. This topological closed string field theory has the structure of a $L_\infty$ algebra
[24, 26, 25], generating its closed string field theory. The bracket in this $L_\infty$ algebra is
defined as the current algebra bracket of the boundary string,
\[ \{ f, g \} = \frac{1}{i\hbar} \oint_C f^{(1)} g, \] (20)
where $C$ is a 1-cycle on the boundary enclosing the insertion point of $g$ on the boundary. This
bracket can more concretely be calculated using the correlation functions. More generally,
the $L_\infty$ brackets can be defined by the correlation functions [23]
\[ \langle O_{\delta \phi_0, \infty} O_{f_1, \ldots, f_n} x \rangle = \frac{(-1)^{\sum_k (n-k)!(f_k+1)}}{(i\hbar)^{n-1}} \langle O_{\delta \phi_0, \infty} O^{(1)}_{f_1,C} O^{(2)}_{f_2,\partial V} \cdots O^{(2)}_{f_n,\partial V} \rangle, \] (21)
where $C$ is a 1-cycle enclosing the point $x$. The powers of $i\hbar$ are included for convenience to cancel the leading behavior. The first insertion is a delta-function $\delta_{\phi_0}(\phi) = \delta(\phi - \phi_0)$ inserted in a point at “infinity”. This outgoing test-observable is inserted to give an expectation value $\phi_0$ to the scalar fields living on the boundary. In most of the rest of this paper we will have the insertion of this operator understood, and will not write it down explicitly.

Let us first discuss the correlation functions of boundary operators in the open membrane theory, in the presence of a nontrivial bulk term $\gamma$. As we discussed above, the basic boundary observables are determined by functions on the Lagrangian subspace $\mathcal{L} \subset \mathcal{M}$. The bulk observables are induced by elements of the bulk algebra $\mathcal{A} = C^\infty(\mathcal{M})$, while the boundary observables are induced by elements of the boundary algebra $\mathcal{B} = C^\infty(\mathcal{L})$. Furthermore we have the projection $P_L : \mathcal{A} \to \mathcal{B}$, restricting a function to $L$.

First we write the action as the sum of a kinetic term and an interaction term, $S = S_0 + S_\gamma$, where we took $S_\gamma = \int \gamma$. This gives rise to the path integral representation of the correlation functions

$$\langle \prod_a O_a \rangle = \int D\phi e^{i(S_0 + S_\gamma)} \prod_a O_a,$$

(22)

which we calculate as usual in an $\hbar$ expansion by perturbation theory, treating $S_\gamma$ as a perturbation. The propagator has the form

$$\langle \phi^I(x)\phi^J(y) \rangle = -i\hbar \omega^{IJ} G(x,y),$$

(23)

where $G(x,y)$ is the integral kernel for the inverse kinetic operator $d^{-1}$ (after gauge fixing) and $\omega^{IJ}$ is the inverse of the symplectic structure $\omega_{IJ}$. We recognize in this the BV bracket structure. Because of this we will see that we can effectively describe the algebraic structure on the boundary operators in terms of the original BV bracket.

Let us consider for concreteness the bracket defined by the correlation function

$$\{f,g\}(\phi_0) = \langle O_{\delta_{\phi_0}} O_{f,x}^{(1)} O_{g,x} \rangle,$$

(24)

where all the operators are put on the boundary and $\delta_{\phi_0}$ is a delta function fixing the scalar fields to a fixed value $\phi_0$ consistent with the boundary condition. After contractions, and using the expression for the propagator above, the lowest order term can be written,

$$\pm \int_V dz \int_C dy G(z,y)G(z,x) \int_M d\phi \delta(\phi - \phi_0) \omega^{KL} \omega^{IJ} \frac{\partial^2 \gamma}{\partial \phi^K \partial \phi^L} \frac{\partial f}{\partial \phi^I} \frac{\partial g}{\partial \phi^J}.$$

(25)

This is just the Feynman integral corresponding to a 2-legged tree-level diagram. The integral is a universal factor, which no longer depends on the precise choice of operators. The
dependence on the functions $f$ and $g$, and therefore the choice of boundary observables, is expressed in terms of differential operators acting on these functions.

In terms of the the boundary algebra of functions $\mathcal{B} = C^\infty(\mathcal{L})$, the bracket can now be written
\[
\{f, g\} = (-1)^{|f|+1} P_\mathcal{L}[[\gamma, f], g] - (-1)^{|f|+1}|g| P_\mathcal{L}[[\gamma, g], f].
\] (26)

Here the $P_\mathcal{L}$ results from the integration against the outgoing state $\delta\phi_0$, or equivalently the delta-function in the integral over zero-mode $\phi$. More precisely, we should interpret the boundary operators like $f$ as embedded in the algebra $\mathcal{A}$; so we should more properly use a lift $f$ to the bulk algebra.

Similarly, the 4-point function, defined by
\[
\langle \mathcal{O}_{\delta\phi_0, \infty} \mathcal{O}^{(2)}_{h, \partial V} \mathcal{O}^{(1)}_{g, C} \mathcal{O}_{f, x} \rangle,
\] (27)
at tree level is proportional to the Feynman integral
\[
\int_\mathcal{V} du \int_{\partial \mathcal{V}} dz \int_C dy G(u, z) G(u, y) G(u, x),
\] (28)
multiplied by a 3-differential operator acting on $f, g, h$ and depending on $\gamma$. The Feynman integral calculates again the universal coefficient corresponding to this term in the expansion of the trilinear bracket. The rest can again be expressed in terms of $\gamma$ and the BV bracket $[\cdot, \cdot]$, as
\[
\{f, g, h\} = (-1)^{|g|+1} P_\mathcal{L}[[[\gamma, f], g], h] \pm \text{perms.}
\] (29)
The signs are such that the bracket is skew symmetric with respect to the ghost degree shifted by one.

We see in general that the integrals over propagators give some universal coefficients, while the rest is determined by the algebra of the bracket. The essential point is that the nontrivial operations, i.e. the brackets defined above, correspond to nonvanishing Feynman integrals.

3. Sigma Model Computations

In this section we will compute the propagators using the BV quantization of the sigma-model.
3.1. GAUGE FIXING

The BV model having form fields, will have gauge invariance. We therefore need to gauge fix. The BV language we have adopted will make this quite simple. We mainly have to choose a gauge fixing fermion $\Psi$ to gauge fix the anti-fields. Note that in order to preserve the topological nature of our model we need to choose the fields and anti-fields according to ghost number: the anti-fields are the fields with negative ghost number.

There were two types of “BV multiplets”: $X^i$ and $F_i$ having degree 0 and 2, and $\chi_a$ and $\psi^a$, both having degree 1. The fields will have different degrees in the two cases, so the gauge fixing will be slightly different. We will therefore treat them separately. We will leave out the indices, as they can be easily reinserted.

3.1.1. GHOST DEGREE 1 MULTIPLET

We start with the ghost degree 1 multiplets $(\chi, \psi)$. The gauge fields are 1-form fields $A$ and $B$. We will use a covariant Lorentz gauge. To implement this gauge fixing, we introduce antighost fields and Lagrange multiplier fields. They both are scalars and have ghost numbers $-1$ and $0$ respectively. They are of course supplemented by their antifields, which are 3-forms of ghost number 0 and $-1$ respectively. For the gauge field $A$ we have an antighost $\bar{\chi}$ and Lagrange multiplier $\chi$; for $B$ the antighost is $\bar{\psi}$ and the Lagrange multiplier $\psi$. The boundary conditions for the antighosts and Lagrange multipliers will be the same as for the scalar field in the corresponding superfield.

To fix the gauge we introduce the following antighost terms in the action,

$$S_{\text{antighost}} = \int (\bar{\chi} \chi + \bar{\psi} \psi^+). \quad (30)$$

The gauge fixing fermion will be given by

$$\Psi = \int (d\bar{\chi} * A + d\bar{\psi} * B). \quad (31)$$

This implies the following gauge fixing of the antifields

$$A^+ = * d\bar{\chi}, \quad B^+ = * d\bar{\psi}, \quad \bar{\chi}^+ = -d * A, \quad \bar{\psi}^+ = -d * B, \quad (32)$$

while all other antifields vanish. The antifields of the Lagrange multipliers all vanish. After gauge fixing, the kinetic terms in the action $S_0$ become

$$S_{\text{kin}} = \int_V (B dA + A * d\bar{\chi} + B * d\bar{\psi} + \chi d * d\bar{\chi} + \psi d * d\bar{\psi}). \quad (33)$$

13
These kinetic terms can be grouped into basically two multiplets: there are second order terms involving 2 scalars ($\chi$, $\bar{\chi}$) and ($\psi$, $\bar{\psi}$) and a set of 2 vectors and two scalars, ($A, B, \chi, \bar{\psi}$).

In the following we will denote by $d_p : \Omega^p \rightarrow \Omega^{p+1}$ the De Rham differentials acting on $p$-forms and by $\Delta_p = -d_{p-1}d_p^* - d_p^*d_{p+1}$ the corresponding Laplacians acting on $\Omega^p$. The kinetic operator for two scalars is given by $\Delta_0 = *d_2*d_0 : \Omega^0 \rightarrow \Omega^0$. This operator has finite dimensional kernel, and therefore can be inverted on the fluctuations. The kinetic operator for the vector ‘multiplet’ can be conveniently organized in a matrix form as

$$
\begin{pmatrix}
* d_1 & d_0 \\
* d_2 & 0
\end{pmatrix}
: \Omega^1 \oplus \Omega^0 \rightarrow \Omega^1 \oplus \Omega^0.
$$

(34)

This matrix operator is a Dirac operator, in the sense that it squares to (minus) the Laplacian. This allows us to write the propagators in matrix notation as

$$
\begin{pmatrix}
\langle AB \rangle & \langle A\chi \rangle \\
\langle \bar{\psi} B \rangle & \langle \bar{\psi}\chi \rangle
\end{pmatrix}
= i\hbar \begin{pmatrix}
* d_1 & d_0 \\
* d_2 & 0
\end{pmatrix}^{-1}
= i\hbar \begin{pmatrix}
* d_1 \Delta_1^{-1,D} & -d_0 \Delta_0^{-1,N} \\
* d_2 * \Delta_1^{-1,D} & 0
\end{pmatrix}.
$$

(35)

The extra subscript on the inverse Laplacians denotes the boundary condition.

### 3.1.2. Ghost Degree 0 Multiplet

Next consider the ($X, F$) multiplet. This one is slightly more complicated, as it involves a 2-form $P$ in $F$. Therefore we have to worry about an extra gauge-for-gauge symmetry. The antighost and Lagrange multiplier fields will be given by

| gauge field | antighost degree | ghost # | Lagr. mult. degree | ghost # |
|-------------|------------------|---------|--------------------|---------|
| $P$         | $\bar{\eta}$     | 1       | $\eta$             | 1       |
| $\eta$      | $\bar{\Lambda}$  | 0       | $\Lambda$          | 0       |
| $\bar{\eta}$| $\Lambda$        | 0       | $\bar{\Lambda}$    | 0       |

(36)

The antighost terms in the action will be

$$
S_{\text{antighost}} = \int (\bar{\eta} \eta^+ - \bar{FF}^+ - \bar{\Lambda} \Lambda^+).
$$

(37)

and the gauge fermion is

$$
\Psi = \int (d\bar{\eta} * P + d\bar{F} * \eta + d\bar{\Lambda} * \bar{\eta}).
$$

(38)

The antifields will be replaced by the gauge fixing according to

$$
P^+ = *d\bar{\eta}, \quad \eta^+ = *d\bar{F}, \quad \bar{\eta}^+ = d*P + *d\bar{\Lambda}, \quad \bar{F}^+ = -d*\eta, \quad \bar{\Lambda}^+ = *d\bar{\eta}.
$$

(39)
and fixes all other antifields to zero. This gives the gauge fixed kinetic action

$$S_{\text{kin}} = \int_V (Fd^*d\bar{F} + PdX + P*d\eta + \eta*d\bar{\eta} - \eta*d\bar{\lambda} + \eta*d\bar{\lambda} + \eta*d\bar{\eta} + \eta*d\bar{\eta})$$.

(40)

They split into a scalar multiplet \((F, F)\), a 1-form multiplet \((\bar{\eta}, \eta, X, \bar{\lambda})\) (note that we dualized the 2-form \(P\)), and another 1-form multiplet \((\eta, \bar{\eta}, \bar{\lambda}, F)\) which has different kinetic terms. The scalar multiplet and the first 1-form multiplet are handled in the same way as above, so we find the propagators

$$\left( \begin{array}{cc}
\langle \bar{\eta} \bar{\eta} \rangle & \langle \bar{\eta} P \rangle \\
\langle X \eta \rangle & \langle X X \rangle
\end{array} \right) = i\hbar \left( \begin{array}{cc}
-\star d_1 \Delta_{1,D}^{-1} & -d_0 \Delta_{0,N}^{-1} \\
\star d_2 \star \Delta_{1,D}^{-1} & 0
\end{array} \right)$$.

(41)

The fermionic 1-form multiplet needs extra attention. The kinetic operator of this multiplet is

$$\left( \begin{array}{cc}
-\star d_1 * d_1 & -d_0 \\
*d_2 * & 0
\end{array} \right) : \Omega^1 \oplus \Omega^0 \to \Omega^1 \oplus \Omega^0$$.

(42)

Similar to the above we find the propagator

$$\left( \begin{array}{cc}
\langle \eta \bar{\eta} \rangle & \langle \eta F \rangle \\
\langle \bar{\eta} F \rangle & \langle \bar{\eta} F \rangle
\end{array} \right) = i\hbar \left( \begin{array}{cc}
-\star d_1 * d_1 & -d_0 \\
*d_2 * & 0
\end{array} \right)^{-1} = i\hbar \left( \begin{array}{cc}
-\star d_1 * d_1 \Delta_{1,D}^{-2} & d_0 \Delta_{0,D}^{-1} \\
-\star d_2 * \Delta_{1,D}^{-1} & 0
\end{array} \right)$$.

(43)

3.2. Explicit Propagators and Superpropagators

To give explicit expressions for the propagators, we take for the membrane simply the upper half space. We choose coordinates \((x^\alpha, x_\perp)\), \(\alpha = 1, 2\), with the boundary at \(x_\perp = 0\), and the bulk at \(x_\perp > 0\). We define reflected coordinates \(\tilde{x}^\mu\) such that \(\tilde{x}^\alpha = x^\alpha\), \(\tilde{x}_\perp = -x_\perp\). We will also introduce a reflected Kronecker \(\tilde{\delta}\) such that \(\tilde{\delta}^\perp = -1\), \(\tilde{\delta}^{\alpha\alpha} = 1\).

We will denote the kernels of the inverse Laplacians \(\Delta_{p,B}^{-1}\) by \(\Pi_{p,B}\). Here \(p\) is the form degree, and \(B \in \{D, N\}\) denotes the boundary condition. The propagator of the scalar \(\chi\) and \(\bar{\chi}\), which have Neumann boundary conditions, is given by

$$\langle \chi(x) \bar{\chi}(y) \rangle = i\hbar \Pi_{0,N}^{0,N}(x, y) = \frac{i\hbar}{4\pi} \left( \frac{1}{\|x - y\|} + \frac{1}{\|x - \tilde{y}\|} \right)$$.

(44)

Similarly, there is a minus sign in between the two terms for Dirichlet boundary conditions,

$$\langle \psi(x) \bar{\psi}(y) \rangle = \langle F(x) F(y) \rangle = i\hbar \Pi_{0,D}^{0,D}(x, y) = \frac{i\hbar}{4\pi} \left( \frac{1}{\|x - y\|} - \frac{1}{\|x - \tilde{y}\|} \right)$$.

(45)

The kernel for the inverse Laplacian \(\Delta_{1,N}^{-1}\) for two vectors with Neumann boundary conditions is given by

$$\Pi_{\mu\nu}^{1,N}(x, y) = \frac{1}{4\pi} \left( \frac{\delta_{\mu\nu}}{\|x - y\|} + \frac{\tilde{\delta}_{\mu\nu}}{\|x - \tilde{y}\|} \right)$$.

(46)
while Π^{1,D}(x, y) has a minus sign in front of the reflected term. The propagator between the two vectors A and B can then be written
\[ \langle B_\mu(x)A_\nu(y) \rangle = i\hbar\epsilon_{\mu\rho\sigma} \frac{\partial}{\partial x^\sigma} \Pi^{1,N}_\rho(x, y) = i\hbar\epsilon_{\mu\rho\sigma} \frac{\partial}{\partial y^\sigma} \Pi^{1,D}_\rho(x, y). \] (47)

Notice that this indeed satisfies the boundary conditions for A and B.

The propagator between a vector and a Lagrange multiplier scalar, having always the same boundary condition, is given by the formula \( d_0\Delta_0^{-1} \), where the propagator \( \Delta_0^{-1} \) of course is chosen for the correct boundary conditions. For example,
\[ \langle *P_\mu(x)X(y) \rangle = -i\hbar d_0\Pi^0,N(x, y) = -i\hbar \left( \frac{(x - y)^\mu}{\|x - y\|^3} + \frac{(x - \tilde{y})^\mu}{\|x - \tilde{y}\|^3} \right), \] (48)
indeed satisfying Neumann boundary conditions.

In practice we will not need all the propagators. For the calculations in this paper we can ignore the Lagrange multiplier fields. In fact, all we need are the components of the gauge fixed superfields, which are
\[ X(x, \theta) = X + \theta \cdot d\eta + \frac{1}{2}\theta^2 \cdot dF, \quad F(x, \theta) = F + \theta \cdot \eta + \frac{1}{2}\theta^2 \cdot P, \]
\[ \chi(x, \theta) = \chi + \theta \cdot A + \frac{1}{2}\theta^2 \cdot d\chi, \quad \psi(x, \theta) = \psi + \theta \cdot B + \frac{1}{2}\theta^2 \cdot d\chi. \] (49)

One can combine the above propagators in terms of a superpropagator. Let us define the superpropagator more generally. We introduce the supercoordinates \( x = (x^\mu|\theta^\mu) \) and \( y = (y^\mu|\zeta^\mu) \). Next we combine the propagators of the \( p \)-forms into a single superpropagator,
\[ \sum_{p=0}^{2} \frac{1}{p!(2-p)!} \theta^{2-p} \cdot *d_p \Pi^p(x, y) \cdot \zeta^p. \] (50)
Here \( \Pi^p = *\Pi^{3-p} \). We express this in terms of a superpropagator
\[ \Pi(x, y) = \sum_{p=0}^{3} \frac{1}{p!(3-p)!} \theta^{3-p} \cdot *\Pi^p(x, y) \cdot \zeta^p, \] (51)
and the operator \( d^\dagger \) representing \( d^\dagger \), that is \( \frac{1}{p!}d^\dagger(\theta^p \cdot \alpha_p) = \frac{1}{(p-1)!}\theta^{p-1} \cdot d^\dagger(\alpha_p) \). We then find that the above superpropagator can be written in the form \( d^\dagger\Pi(x, y) \). Note that the superpropagator \( \Pi \) can be seen as the inverse of the super-Laplacian \( \Delta = -dd^\dagger - d^\dagger d \), as it satisfies
\[ \Delta\Pi(x, y) = \delta^{(3)}(x - y)(\theta - \zeta)^3. \] (52)

In the flat upper half space, we can write down simple explicit expressions for these propagators. The Laplacian is given by \( \Delta = \Delta = \partial^\mu\partial_\mu \). Furthermore we need boundary
conditions. We denote by \((\tilde{x}^\mu) = (\tilde{x}^\mu|\tilde{\theta}^\mu)\) the reflected supercoordinates. Then the boundary for the supercoordinates is at \(\tilde{x} = x\). Depending on the boundary condition, we find the explicit solution

\[
\Pi(x, y) = -\frac{1}{4\pi} \left( \frac{(\theta - \zeta)^3}{\|x - y\|} \pm \frac{(\theta - \tilde{\zeta})^3}{\|x - \tilde{y}\|} \right),
\]

with \(+\) (\(-\)) for Dirichlet (Neumann) boundary conditions. Furthermore, we have the explicit form \(d_x^\dagger = \frac{\partial}{\partial y^\mu} T_{x^\mu}\). In terms of these operations we can write the superpropagator as

\[
\langle X(x)F(y) \rangle = \langle \chi(x)\psi(y) \rangle = i\hbar d_x^\dagger \Pi^D(x, y) = i\hbar d_y^\dagger \Pi^N(x, y).
\]

(54)

4. INTERACTIONS AND BRACKETS

In this section we will calculate the boundary correlators with a single bulk insertion. Note that bulk deformations of order \(n\) in the conjugate fields (i.e. \(\psi \) and \(F\)) give rise to \(n\)-linear brackets on the boundary.

4.1. A BASIC INTERACTION

We start with a simple bulk interaction quadratic in the (conjugate) fields,

\[
\int_V \psi F = \int_V (BP + \eta \ast d\chi).
\]

(55)

Indeed this is the interaction that should already be turned on in the undeformed exact Courant algebroid (and is responsible for the Schouten-Nijenhuis bracket on multivector fields).

This will have an effect on the \(AX\) correlator on the boundary. It can be motivated formally by noting that \(\psi\) and \(F\) are the conjugate fields to \(\chi\) and \(X\) respectively. So there is a Feynman diagram with the above interaction in the bulk and \(X\) and \(\chi\) on the boundary.

In fact, as the term above is quadratic it gives a correction to the propagators. Let us first, formally, discuss this correction. The correction to the 2-point function of \(A\) and \(X\) is

\[
\langle AX \rangle' \sim \frac{i}{\hbar} \langle AB \rangle \langle PX \rangle \sim -i\hbar * d_1 \Delta_{1,D}^{-1} d_0 \Delta_{0,N}^{-1} \equiv i\hbar \Xi.
\]

(56)

Naively, this vanishes as we can pull \(d_0\) through \(\Delta_{1,D}^{-1}\) where it is annihilated by \(d_1\). There is however a catch in this argument, as the two propagators have different boundary conditions.
This makes that pulling the \( d_0 \) through is not allowed. That indeed \( \Xi \) is nonzero can be seen by acting with \( *d \),

\[
* d_1 \Xi = (\Delta^1 - d_0 * d_2 *) \Delta^{-1}_{1,D} d_0 \Delta^{-1}_{0,N} = d_0 \Delta^{-1}_{0,N} - d_0 \Delta^{-1}_{0,D} * d_2 * d_0 \Delta^{-1}_{0,N} = d_0 (\Delta^{-1}_{0,N} - \Delta^{-1}_{0,D}). \tag{57}
\]

One can show that pulling through \( *d_2 * \) is allowed because the expression is sandwiched in \( d_0 \)'s. Indeed we see that the above is nonzero due to the difference in boundary condition.

Another way to see this equation is to write down the full quadratic action for the two coupled vector multiplets when including the \( \psi F \) term. In matrix notation the relevant part of the Lagrangian density can be written as

\[
\begin{pmatrix}
B \\
\chi \\
\eta \\
X
\end{pmatrix}^t \begin{pmatrix}
* d_1 & d_0 & 1 & \cdots \\
- * d_2 & \ddots & \ddots & \ddots \\
\cdots & \ddots & * d_1 & d_0 \\
\cdots & \cdots & - * d_2 & \ddots
\end{pmatrix} \begin{pmatrix}
A \\
\psi \\
* P \\
\Lambda
\end{pmatrix}.
\]

The deformed propagator is the inverse of the kinetic matrix appearing above,

\[
\begin{pmatrix}
- * d_1 \Delta^{-1}_{1,D} & -d_0 \Delta^{-1}_{0,N} & - * d_1 * d_1 \Delta^{-2}_{1,D} & \Xi \\
* d_2 * \Delta^{-1}_{1,D} & \ddots & \cdots & \Xi \\
\cdots & \ddots & - * d_1 \Delta^{-1}_{1,D} & - d_0 \Delta^{-1}_{0,N} \\
\cdots & \cdots & * d_2 * \Delta^{-1}_{1,D}
\end{pmatrix} \begin{pmatrix}
\Delta^{-1}_{0,D} \\
\Delta^{-1}_{0,N}
\end{pmatrix}, \tag{59}
\]

with the operator \( \Xi \) appearing in the top right corner. The relation (57) is required to get the zero in the top right corner of the product of the kinetic matrix with the propagator matrix. The other equation \( \Xi \) has to satisfy, needed to get a zero in the second row, is \( d_2 * \Xi = 0 \). This is indeed trivially satisfied. We note that in the explicit coordinates the combination of scalar propagators appearing in the equation (57) for \( \Xi \), \( \Pi^{0,N}(x,y) - \Pi^{0,D}(x,y) = -\frac{1}{2\pi} \frac{1}{\|x-y\|} \), only depends on \( x - \tilde{y} \). As a result also the kernel for \( \Xi \) should depend only on this combination of its arguments. This observation will be useful in the explicit calculation of this kernel.

Let us now be more precise and do the actual calculation of \( \Xi \), using the explicit form for the propagators given before. As these propagators have singularities at coincident points we will need some kind of regularization. We will use a point-splitting regularization. As the above subtleties suggest the regularization might be important for the result. We will comment on this below. The above 2-point function becomes

\[
\langle A(y)X(x) \rangle' = \frac{i}{\hbar} \int_V d^3z \langle A(y)B(z) \rangle \langle P(z)X(x) \rangle
= -i\hbar \int_V d^3z d_1 \Pi^{1,N}(z,y)d_0 \Pi^{0,N}(z,x)
= -i\hbar \int_{\partial V} d^2z d_1 \Pi^{1,N}(z,y)\Pi^{0,N}(z,x). \tag{60}
\]

18
Figure 1: The point splitting regularization, where \( x \) and the contour over \( y \) are taken on the boundary, and the bulk integral is performed over a region a distance \( \epsilon \) away from the boundary (shaded region).

The fact that naively this vanishes is reflected by the fact that the integrand is a total derivative. However as \( V \) has a boundary, there remains a boundary term. Inserting the propagators we found above, we obtain

\[
\langle A_\mu(y)X(x) \rangle' = \frac{-i\hbar}{4\pi^2} \int_{\partial V} d^2z \frac{\epsilon^{(2)}_{\mu\beta}(y-z)^\beta}{\|y-z\|^3 \|z-x\|} \frac{1}{2\pi \|(x-y)\|^2} \left(1 - \frac{x_\perp + y_\perp}{\|x - \tilde{y}\|}\right).
\]

The explicit calculation of the integral can be found in Appendix A. This result is nonsingular when either \( x \) or \( y \) are in the bulk. When both are on the boundary, it reduces to a simple \( 1/r \) behavior. This could of course have been inferred from the scaling behavior.

Notice that in the integral above a nonzero \( x_\perp \) and \( y_\perp \) regularize the integral. This can be related to the point-splitting regularization. We will be mainly interested in the deformed boundary correlators, so we will now take \( x \) and \( y \) on the boundary. The bulk integral over \( z \) has to be regularized, cutting out small balls around \( x \) and \( y \). We can alternatively move the bulk integration slightly away from the boundary, taking it over \( z_\perp > \epsilon \) for some \( \epsilon > 0 \). This is depicted in Figure 1. We can then safely use Stokes’s theorem and reduce to the integral over the boundary. Instead of taking \( z \) off the boundary we can equivalently take \( y_\perp \) and \( x_\perp \) different from zero of order \( \epsilon \). We can safely take \( x_\perp = 0 \) as the singularity at \( z = x \) is harmless. We then end up with the above integral, with \( y_\perp \) of order \( \epsilon \).

This \( 1/r \) behavior of the boundary 2-point function implies that in the presence of a bulk deformation \( \gamma = a^i_\alpha \psi^\alpha F_i \) there is a nonvanishing boundary correlator

\[
\{\chi_\alpha, X^i\}' \equiv \frac{1}{i\hbar} \langle O^{(1)}_{\chi_\alpha, C} O_{X^i, x} \rangle' = a^i_\alpha.
\]

giving the nontrivial bracket. As expected this correlator is independent of the contour \( C \) (as long as it encloses the point \( x \)). The same calculation is valid for \( \langle *d\bar{\eta}(y)\chi(x) \rangle \), which gives
the other term in the bracket, i.e. $\frac{1}{\hbar}\langle \mathcal{O}_{\chi_a,x}^{(1)} \rangle = a_i^i$ is also nonzero. In the case of the undeformed exact Courant algebroid $A = T^*M$ we had the coupling $\gamma = \psi^a F_i$. To find the bilinear bracket on general functions $f, g \in \mathcal{B}$, we substitute observables $\mathcal{O}_{f,C}^{(1)}$ and $\mathcal{O}_{g,x}$. As the $\mathcal{O}_{f,C}^{(1)}$ contains just a single field $A$ or $\overline{\eta}$, there will only be a single deformed contraction (factorizing in two undeformed contractions as above). With no other contractions, the remaining fields $X$ and $\chi$ in the observables will only contribute through their zero modes. Therefore, the calculation of the bracket reduces to the calculation of the simpler correlator above, see also the discussion around (28). The bracket (20) therefore is given by

$$\{f, g\}' = (-1)^{|f|+1} \frac{\partial f}{\partial X^i} \frac{\partial g}{\partial X^i} - \frac{\partial f}{\partial \chi_i} \frac{\partial g}{\partial \chi_i},$$

(63)

where the signs come from the explicit sign in the definition of the bracket and commuting $\overline{\eta}$ through $\frac{df}{dX^i}$. This is precisely the Schouten-Nijenhuis bracket on the boundary algebra of multivector fields, $\mathcal{B} = \Gamma(\Lambda T^*M)$.

Let us make a last remark about the above computation. At first sight the result seems to be non-topological. If we would calculate the correlator above, but with the cycle $C$ in the bulk rather than on boundary, we would get a nonzero result even though $C$ is now contractable. This correlator however is not topological, as the operator $\oint_C A$ is not BRST invariant when $C$ lies inside the bulk, as $Q \oint_C A = \oint_C d\psi + \oint_C \eta = \oint_C \eta$. Note that on the boundary we do have a BRST invariant operator due to the boundary condition on $\eta$. More generally as $Q P = d\eta$, we could make the observable closed by adding a term $\int_D P$, where $D$ is a disc with boundary $C$. The contribution of this extra term will however cancel completely the contribution of the original term, making it trivially invariant. We could have added the same contribution to the boundary observable. Now the regularization becomes relevant. If we regularize by moving the disc slightly in the bulk, the above correlator vanishes. However with a point splitting regularization we should cut a small hole in the disc. Furthermore, as $P$ vanishes on the boundary, the extra contribution is zero and we find the same result as above.

4.2. Interactions Quadratic in Conjugate Fields

We will now generalize the above calculation to other interactions still quadratic in the conjugate superfields $F$ and $\psi$, but which might include extra $X$ and $\chi$ fields. As these are quadratic in conjugate superfields, they give contributions to the boundary bracket. We will only do the calculation for this bracket. So we consider an interaction of the form $\gamma = \varphi_{ab} \psi^a \psi^b$ term in the action, where the coefficients $\varphi$ are functions of the fields $X$ and $\chi$. 

20
To see the effect on the bracket, we insert two boundary operators (apart from the outgoing delta-function). We will show that

$$\frac{i}{\hbar} \langle \int_V (\phi \psi \psi)(z) \mathcal{O}^{(1)}_{x,C} \mathcal{O}_{x,x} \rangle = i\hbar \varphi(x).$$

(64)

Here $C$ is a 1-cycle on the boundary enclosing the point $x$. We do not include the indices, as these are obvious. The only important thing will be the propagators and the integrals. Explicitly the only contributing term is proportional to

$$\langle \int_V (\varphi B * d\chi)(z) \oint_C A(y) \chi(x) \rangle = \int_V d^3z \int_C dy \varphi(z) \langle B(z) A(y) \rangle \langle * d\chi(z) \chi(x) \rangle,$$

(65)

where of course $x$ is on the boundary and $C$ is a cycle on the boundary enclosing $x$. At the right hand side we have worked out the contractions that occur. Because the $* d\chi$ propagator is the same as the $PX$ propagator, this is actually almost the same as the calculation of the bracket we did above. The only difference is the presence of $\varphi$. As we saw that the calculation basically reduced to local interactions, we should expect that in fact the $z$-dependence of this term does not matter, and therefore it can be replaced by $\varphi(x)$. This gives exactly the result stated above. Let us now confirm that this expectation is correct.

The presence of the extra factor of $\varphi(z)$ first of all gives an extra factor from the partial integration, when the derivative acts on $\varphi$. Furthermore, it gives the extra insertion of $\varphi(z)$ in the boundary term. We find, leaving out a factor of $\frac{i\hbar}{4\pi^2}$,

$$\int_{\partial V} d^2z \int_C dy \alpha \epsilon_{\alpha\beta}(y - z) \frac{\epsilon_{\alpha\beta}(y - z, z' - y)}{\|z - y\| \|z - x\|} - \int_{\partial V} d^2z \int_C dy \epsilon_{\alpha\beta} \frac{z^\gamma \partial_\gamma \varphi(z) + (z - y)^\beta \partial_\beta \varphi(z)}{\|z - y\| \|z - x\|}.$$

In the following we will take $x = 0$ for simplicity. Then we will rescale $z' = z/\|y\|$. We start with the second term, which is written

$$\int_{\partial V} d^2z' \int_C dy' \epsilon_{\alpha\beta} \left( \frac{z'^\gamma \partial_\gamma \varphi(\|y\| z')}{\|z' - y\| \|z' - y\|^2 \|z'\|^2} + \frac{(\|y\| z' - y)^\beta \partial_\beta \varphi(\|y\| z')}{\|y\| \|z' - y\| \|z'\|^3 \|z'\|^2} \right).$$

(66)

Both terms have no pole in $y$, and therefore the contour integral gives zero.

For the first term we use a Taylor expansion of $\varphi(\|y\| z')$, which becomes an expansion in powers of $\|y\|$. We find

$$\int_{\partial V} d^2z' \int_C dy' \epsilon_{\alpha\beta} \frac{z'^\gamma \partial_\gamma \varphi(\|y\| z')}{\|y\| \|z' - y\| \|z'\|^2 \|z'\|^3 \|z'\|^2} \varphi(0) + \|y\| (z')^\gamma \partial_\gamma \varphi(0) + \mathcal{O}(\|y\|^2).$$

(67)

The term of order $\|y\|^2$ has no pole in $y$, and therefore the contour integral vanishes. The second term in the expansion gives zero because of antisymmetry in the integral under
simultaneous reflection of \( z \) and \( y \). The first term in the expansion is the term we are interested in. It is proportional to the original integral, and therefore gives \( i\hbar\varphi(0) \).

Substituting back \( x \), we conclude therefore that the complete integral equals \( i\hbar\varphi(x) \), as expected. This calculation shows that a bulk term of the form \( \frac{1}{2} \int_V \varphi_{ab} \psi^a \psi^b \) —with the coefficients \( \varphi_{ab} \) functions of the fields \( X \) and \( \chi \)—induces in the deformed boundary theory a contribution to the bracket of the form

\[
\{ f, g \}' = \varphi_{ab} \frac{\partial f}{\partial \chi^a} \frac{\partial g}{\partial \chi^b} + \cdots,
\]

where the ellipses denote contribution from other terms.

The calculation above can also be done directly in the superfield notation. This has the advantage that several calculations, for different degree forms, are done at once.

There is a generalization of the above, which in superfield notation can be written

\[
\frac{i}{\hbar} \left\langle \int_V \varphi \psi(z) \int_C \chi(y) \chi(x) \right\rangle = i\hbar\varphi(x).
\]

The difference with the above is that we did not restrict the dependence of the last insertion on \( \theta \) (to the zeroth descendant). What we see from this is that the combination of the boundary operators behaves like a delta-function on the boundary.

There is a quick way to see this \( \delta \)-function behavior of the integral. Let us shift the integration variables \( z \) and \( y \) by \( x \) and scale by \( R \), i.e. \( z \rightarrow R(z - x) + x \). The scaling of the propagators will compensate for the scaling of the density. Furthermore, the integral is independent of the size of the contour \( C \). Therefore the only change is to replace \( \varphi(z) \) by \( \varphi(R(z - x) + x) \). Taking \( R \rightarrow 0 \) we find that the full \( \varphi \) dependence is replaced by \( \varphi(x) \). For this argument to work we have to be careful that the limit \( R \rightarrow 0 \) is continuous. Otherwise, there might be extra terms involving derivatives of \( \varphi \). Luckily, these terms turn out to vanish. Above we showed this was correct for derivatives with respect to \( z \). For derivatives with respect to \( \xi \) a similar calculation will give the more general result. In this paper we will actually not need this more general result, so we do not give the full derivation.

### 4.3. Interactions Cubic in Conjugate Fields

Next we consider the interactions that are cubic in conjugate fields, i.e. of the form \( \gamma = \frac{i}{3!} c_{abc} (X, \chi) \psi^a \psi^b \psi^c \). Having three conjugate fields \( \psi \) we need to insert three boundary observables involving \( \chi \). This leads to a correlator of the form

\[
\frac{i}{6\hbar} \left\langle \int_V (c\psi \psi \psi)(z) \mathcal{O}^{(1)}_{x, C} \mathcal{O}^{(2)}_{x, \partial V} \right\rangle.
\]

22
In fact, we can use the result above for the quadratic interactions to calculate this seemingly more complex correlator. After the contractions we can write this correlator as

$$i \bar{h} \int_V d^3z \int_{\partial V} d^2u \oint_C d\sigma \, c(z) \langle \psi(z) \ast \bar{\psi}(u) \rangle \langle B(z) A(y) \rangle \langle \ast d\chi(z) \chi(x) \rangle.$$  \hspace{1cm} (71)

This has indeed the form of the correlator in (64) with $\phi(z)$ replaced by

$$\phi(z) = i \bar{h} \int_{\partial V} d^2u \, c(z) \langle \psi(z) \ast \bar{\psi}(u) \rangle.$$ \hspace{1cm} (72)

The result (64) then gives for the above correlator

$$i \hbar \phi(x) = -\frac{\hbar^2 c(x)}{2\pi} \int_{\partial V} d^2u \frac{u_\perp}{\| u - x \|^3} = -\hbar^2 c(x) \int_0^\infty dr \frac{r}{(1 + r^2)^{3/2}} = -\hbar^2 c(x),$$ \hspace{1cm} (73)

where $r = \| u_\perp - x \| / |u_\perp|$. Note that here we need $u_\perp > 0$, which is satisfied because of the point-splitting regularization. It follows that the correlator (70) is equal to $-\hbar^2 c(x)$. Here the coefficient $c$ can again be any function of the fields $X$ and $\chi$. The correlator (70) calculates the trilinear bracket in (21). As for the bilinear bracket, for general arguments $f, g, h \in \mathcal{B}$ of this bracket, the calculation reduces essentially to the above correlator, with some extra signs coming from the definition (21) and from straightforward ordering of the factors. We conclude that the interaction term $\frac{1}{3!} \int_V c_{abc} \psi^a \psi^b \psi^c$ gives a contribution to the trilinear bracket of the form

$$\{ f, g, h \}' = c_{abc} \frac{\partial f}{\partial \chi_a} \frac{\partial g}{\partial \chi_b} \frac{\partial h}{\partial \chi_c} + \cdots,$$ \hspace{1cm} (74)

with the ellipses again denoting contributions from other terms.

### 4.4. Boundary Closed String Field Theory

The $L_\infty$ brackets we calculated through the correlation functions generate the closed string field theory action of the boundary string. Indeed, the $L_\infty$ algebra of the bosonic closed string field theory of [24] will be the same as the $L_\infty$ algebra discussed in this paper for the present topological situation. The structure constants of this $L_\infty$ algebra, together with the natural pairing defined by the 2-point functions, can be interpreted as the coefficients of an action functional for the closed string field theory [24]. Therefore, we have basically calculated the string field theory action to lowest order for the boundary string theory of the open membrane.

For the models discussed in this paper, the string field of the boundary closed string field theory is an element $\Phi$ living in the boundary algebra $\mathcal{B} = \mathcal{C}^{\infty}(\mathcal{L})$. The inner product is
defined in terms of the 2-point function. This can be reduced to an integral over the zero modes, which are the coordinates \((X^i, \chi_a)\) on the supermanifold \(L\). The string field theory action is given in terms of the brackets by

\[
S = \int_L \left( \frac{1}{2} \Phi Q \Phi + \frac{1}{3} \Phi \{ \Phi, \Phi \} + \frac{1}{4} \Phi \{ \Phi, \Phi, \Phi \} + \cdots \right).
\] (75)

Let us summarize the results for the case of the 3-form model, based on the target superspace \(\mathcal{M} = T^*[2](\Pi T^*M)\). In this case the boundary algebra \(\mathcal{B} = C^\infty(\Pi T^*M) = \Gamma(M, \Lambda TM)\) can be identified with the algebra of polyvector fields. The basic string field is related to a bivector \(\Phi = \frac{1}{2} b_{ij}(X)\chi_i \chi_j\), the other components correspond to ghosts and antifields in the closed string field theory. The deformations \(\{\cdot, \cdot\}\) were based on a closed 3-form \(c_{ijk}\) and a quasi-Poisson bivector \(b^{ij}\). We conclude that the induced \(L_\infty\) structure is given to first order in \(c\) by

\[
Q = b^{ij} \chi_j \frac{\partial}{\partial X^i} + \frac{1}{2} (\partial_k b^{ij} + c_{klm} b^{ji} b^{mj}) \chi_i \chi_j \frac{\partial}{\partial \chi_k} + O(c^2),
\]

\[
\{\cdot, \cdot\} = \frac{\partial}{\partial X^i} \wedge \frac{\partial}{\partial \chi_i} + \frac{1}{2} c_{ijk} b^{ki} \chi_i \frac{\partial}{\partial X_i} \wedge \frac{\partial}{\partial \chi_j} + O(c^2),
\]

\[
\{\cdot, \cdot, \cdot\} = \frac{1}{6} c_{ijk} \frac{\partial}{\partial \chi_i} \wedge \frac{\partial}{\partial \chi_j} \wedge \frac{\partial}{\partial \chi_k} + O(c^2).
\] (76)

We recognize in the undeformed bracket the Schouten-Nijenhuis bracket on polyvector fields. Furthermore, for \(c = 0\) the BRST operator \(Q\) is the standard Poisson differential for the Poisson structure \(b^{ij}\). We also note that the essential deformation due to the 3-form \(c\) is the trilinear bracket, corresponding to a cubic interaction in the string field theory action.

### 5. Conclusions and Discussion

We calculated the propagators and correlators for the topological open membrane to leading order. This leads to a confirmation that the \(L_\infty\) algebra of the boundary string theory indeed is given by the algebraic expressions related to the bulk couplings parameters.

We note that adopting the point-splitting regularization is essential in getting precisely this result. It is similar to what happens in the case of strings coupling to a 2-form \(\Pi\). For the string theory, the bulk interaction induces a 2-point function with a step-function behavior. This results in the noncommutativity of the product in the boundary open string theory \(\Pi\). In our case we find a deformed 2-point function that has a \(1/r\) behavior on the boundary. This shows that rather the bracket is deformed.
In fact as the deformation has degree 3, the generic deformation will generically contain a term of order 3 in the conjugate fields. As our calculations show, this gives rise to the trilinear bracket. In the quantized algebra, this integrates to a Drinfeld associator. This is most clearly seen in the simplest situation where we have only degree 1 multiplets \((\chi_a, \psi^a)\). The semiclassical trilinear bracket we have calculated is induced from the coefficients \(c_{abc}\) of the deformation. As shown in [2] this situation corresponds to a quasi-Lie bialgebra, with these coefficients identified with the structure constants of the coassociator. This indeed is the infinitesimal structure of the Drinfeld associator [22] of a quasi-Hopf algebra.

The other special situation is the exact Courant algebroid, for which \(M = T^*[2](T^*M)\). It is the toy model of a string in a 3-form background. We found that it gives rise to a closed string field theory with a quartic coupling proportional to the 3-form. This coupling is equivalent to the trilinear bracket in the \(L_\infty\) algebra. The open string version gives rise to a deformation of the problem of deformation quantization, as discussed in [2, 20].

The quantization performed in this paper is only the first step towards a quantization of the open membrane. Indeed, the calculations we did here only reproduced the semi-classical quasi-Lie bialgebroid structure. The full quantum correlators, involving higher orders in the bulk coupling, will give a quantization of this quasi-Lie bialgebroid structure, what could be called a quasi-Hopf algebroid. The \(L_\infty\) structure we found will integrate the product structure in this object. This idea is more easily tested in the much better understood case of quasi-Lie bialgebras, when only the degree 1 multiplets are present. The full quantum result in this case should reflect the quasi-Hopf algebra structure. As these models are of a Chern-Simons type, this relation can be viewed as a generalized Chern-Simons/WZW correspondence [32]. We will leave the study for further quantization of these open membrane models and the emergence of the quasi-Hopf algebra structure for a later paper [27].

Acknowledgements

We are happy to thank Hong Liu, Jeremy Michelson, Sangmin Lee, David Berman and Jan-Pieter van der Schaar for interesting discussions. The research of C.H. was partly supported by DOE grant #DE-FG02-96ER40959; J.-S.P. was supported in part by NSF grant #PHY-0098527 and by the Korea Research Foundation.


APPENDIX A. CALCULATION OF THE INTEGRAL

In this appendix we calculate explicitly the integral in (61). We shift $z$ along $x_\parallel$ (parallel to the boundary), rescale $z$ by $\| (x - y) \| \| \parallel x \|$ and change to polar coordinates. We see that the only surviving component is the one perpendicular to the direction of $(x - y)\parallel$, which is given by

$$\frac{-i\hbar}{4\pi^2 \| (x - y) \| \| x \|} \int_0^\infty dr \int_0^{2\pi} d\phi \frac{r^2 \cos \phi - r}{(r^2 - 2r \cos \phi + 1 + \eta^2)^{3/2}(r^2 + \xi^2)^{1/2}},$$

(77)

where $\eta = \frac{y_\perp}{\| (x - y_\parallel) \|}$ and $\xi = \frac{x_\perp}{\| (x - y_\parallel) \|}$. In general this integral can not be calculated exactly. However, when we take $x$ on the boundary, we can explicitly perform the integral. The integral, without the factor $\frac{-i\hbar}{4\pi^2 \| (x - y) \| \| x \|}$ in front, reduces to

$$\int_0^{2\pi} d\phi \int_0^\infty dr \frac{r \cos \phi - 1}{(r^2 - 2r \cos \phi + 1 + \eta^2)^{3/2}} = \int_0^{2\pi} d\phi \frac{-r \sin^2 \phi - \eta^2 \cos \phi}{(\sin^2 \phi + \eta^2)\sqrt{r^2 - 2r \cos \phi + 1 + \eta^2}} \bigg|_{r=0}^{\infty}$$

$$= \int_0^{2\pi} d\phi \frac{-1}{\sin^2 \phi + \eta^2} \left( \frac{\sin^2 \phi - \eta^2 \cos \phi}{\sqrt{\eta^2 + 1}} \right) = -2\pi \left( 1 - \frac{\eta}{\sqrt{\eta^2 + 1}} \right) = -2\pi \left( 1 - \frac{y_\perp}{\| x - y \|} \right).$$

(78)

We now use the fact that the correlator only depends on the combination $x - \tilde{y}$, or equivalently $y - \tilde{x}$. This also means that it depends on the normal coordinates only through $x_\perp + y_\perp$. This allows us to find the full answer for nonzero $x_\perp$, simply by replacing $y - x$ by $y - \tilde{x}$ and $y_\perp$ by $x_\perp + y_\perp$.

REFERENCES

[1] J.-S. Park, Topological Open $p$-Branes, hep-th/0012141.

[2] C. Hofman and J.-S. Park, Topological Open Membranes, hep-th/0209148.

[3] E. Bergshoeff, D.S. Berman, J.P. van der Schaar, and P. Sundell, A Noncommutative $M$-Theory Five-Brane, Phys. Lett. B492 (2000) 193, hep-th/0005026.

[4] S. Kawamoto and N. Sasakura, Open Membranes in a Constant C-field Background and Noncommutative Boundary Strings, JHEP 0007 (2000) 014, hep-th/0005123.

[5] Y. Matsuo and Y. Shibusa, Volume Preserving Diffeomorphism and Noncommutative Branes, JHEP 0102 (2001) 006, hep-th/0010040.
[6] B. Pioline, *Comments on the Topological Open Membrane*, Phys. Rev. **D66** (2002) 025010, hep-th/0201257.

[7] P. Schaller and T. Strobl, *Poisson Structure Induced (Topological) Field Theories*, Mod. Phys. Lett. **A9** (1994) 3129.

[8] A.S. Cattaneo and G. Felder, *A Path Integral Approach to the Kontsevich Quantisation Formula*, math.QA/9902090.

[9] V. Schomerus, *D-branes and Deformation Quantisation*, JHEP **9906** (1999) 030, hep-th/9903205.

[10] A. Connes, M.R. Douglas and A. Schwarz, *Noncommutative Geometry and Matrix Theory: Compactification on Tori*, JHEP **9802** (1998) 003, hep-th/9711162.

[11] N. Seiberg and E. Witten, *String Theory and Noncommutative Geometry*, JHEP **9909** (1999) 032, hep-th/9908142.

[12] R. Gopakumar, S. Minwalla, N. Seiberg and A. Strominger, *OM Theory in Diverse Dimensions*, hep-th/0006062.

[13] E. Bergshoeff, D.S. Berman, J.P. van der Schaar, and P. Sundell, *Critical Fields on the M5-Brane and Noncommutative Open Strings*, Phys. Lett. **B492** (2000) 193, hep-th/0006112.

[14] D.S. Berman, M. Cederwall, U. Gran, H. Larsson, M. Nielsen, B.E.W. Nilsson, and P. Sundell, *Deformation Independent Open Brane Metrics and Generalized Theta Parameters*, JHEP **0202** (2002) 012, hep-th/0109107.

[15] J.P. Van der Schaar *The Reduced Open Membrane Metric*, JHEP **0108** (2001) 048, hep-th/0106046.

[16] Bergshoeff and J.P. Van der Schaar *Reduction of Open Membrane Moduli*, JHEP **0202** (2002) 019, hep-th/0111061.

[17] T. Courant, *Dirac Manifolds*, Trans. Amer. Math. Soc. **319** (1990) 631, funct-an/9702004.

[18] Z.-J. Liu, A. Weinstein, and P. Xu, *Manin Triples for Lie Bialgebroids*, J. Diff. Geom. **45** (1997) 547, dg-ga/9508013.
[19] D. Roytenberg, *Courant Algebroids, Derived Brackets and Even Symplectic Supermanifolds*, Ph.D. thesis, University of California at Berkeley (1999) [math.DG/9910078](http://arxiv.org/abs/math.DG/9910078).

[20] P. Severa and A. Weinstein, *Poisson Geometry With a 3-Form Background*, [math.SG/0107133](http://arxiv.org/abs/math.SG/0107133).

[21] P. Severa, *Quantization of Poisson Families and of Twisted Poisson Structures*, [math.QA/0205294](http://arxiv.org/abs/math.QA/0205294).

[22] V.G. Drinfeld, *Quasi-Hopf Algebras*, Leningrad Math. J. 1 (1990) 1419.

[23] C. Hofman and W.K. Ma, *Deformations of Closed Strings and Topological Open Membranes*, JHEP 0106 (2001) 033, [hep-th/0102201](http://arxiv.org/abs/hep-th/0102201).

[24] B. Zwiebach, *Closed String Field Theory: Quantum Action and the BV Master Equation*, Nucl. Phys. B390 (1993) 33, [hep-th/9206084](http://arxiv.org/abs/hep-th/9206084).

[25] T. Kimura, A.A. Voronov, G.J. Zuckerman, *Homotopy Gerstenhaber Algebras and Topological Field Theory*, q-alg/9602009.

[26] T. Kimura, J. Stasheff, A.A. Voronov, *On Operad Structures of Moduli Spaces and String Theory*, hep-th/9307114.

[27] C. Hofman and J.-S. Park, In preperation.

[28] P. Etingof and D. Kazhdan, *Quantization of Lie Bialgebras, I–VI*, q-alg/9506005, q-alg/9701038, q-alg/9610030, [math.QA/9801043](http://arxiv.org/abs/math.QA/9801043), [math.QA/9808121](http://arxiv.org/abs/math.QA/9808121), math.QA/0004042.

[29] I. Batalin and G. Vilkovisky, *Gauge Algebra and Quantization*, Phys. Lett. B102 (1981) 27.

[30] I. Batalin and G. Vilkovisky, *Quantization of Gauge Theories with Linearly Dependent Generators*, Phys. Rev. D29 (1983) 2567.

[31] M. Henneaux and C. Teitelboim, *Quantization of Gauge Systems*, Princeton University Press, Princeton, 1992.

[32] E. Witten, *Quantum Field Theory and the Jones Polynomial*, Comm. Math. Phys. 121 (1989) 351.