Open issues in stellar modelling
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ABSTRACT

An important goal of helio- and asteroseismology is to improve the modelling of stellar evolution. Here I provide a brief discussion of some of the uncertain issues in stellar modelling, of possible relevance to asteroseismic inferences.

Key words. solar interior – helioseismology – stellar evolution – physics of stellar interiors – asteroseismology

1. Introduction

The goal of the workshop was to investigate ways to improve our understanding of the Sun; this is obviously intimately linked to the general understanding of stellar structure and evolution, and indeed there are considerable prospects that our growing possibilities of asteroseismic sounding of other stars will inform our studies of the solar interior.

From the point of view of asteroseismology, the relevant aspects of stellar modelling include both the study of stellar structure and evolution and the modelling of stellar oscillations, in particular their frequencies, for a given model. The latter aspect provides the diagnostic link between the observations and the stellar models; although the adiabatic approximation is valid for the oscillations in most of the star, departures from adiabaticity, and other uncertainties in the modelling of the near-surface layers, give rise to substantial systematic errors that must be taken into account in the analyses.

Unlike what is perhaps a common perception, we are still far from adequate modelling of stellar interiors. Here I can only touch on a few issues, mainly in connection with the modelling of main-sequence stars showing solar-like oscillations. A more detailed discussion of these issues, and further references, was provided by Christensen-Dalsgaard & Houdek (2010). For an extensive presentation of stellar oscillations and helio- and asteroseismic techniques and results, see Aerts et al. (2009).

2. Numerical issues

A prerequisite for meaningful asteroseismic diagnostics of the physics of stellar interiors is that stellar modelling presents a faithful representation of the physical assumptions. Thus the models must be numerically sufficiently accurate. Perhaps the best test of this is to compare independently computed models with the same physical assumptions. A major effort towards such comparisons was carried out in the ESTA project initiated as part of the preparations for analysis of CoRoT data (Monteiro 2008). In general, the agreement was reasonable between the results of the evolution codes included in the comparison, although not obviously adequate for the asteroseismic analysis; also, it would be very valuable to extend the comparison to other codes commonly used for general stellar modelling. The results of adiabatic oscillation calculations for a given model agreed quite well, but the analysis highlighted the importance of adequate numerical resolution in the evolution and oscillation calculations, and of consistency in the description of the stellar physics.

An important, and probably often inadequately treated, aspect of the computations is the accurate specification, implementation and documentation of the unavoidable approximations in stellar modelling. Without appropriate care in this area it is difficult or impossible to use asteroseismic inferences to test the validity of these approximations.

3. Stellar parameters

Efficient utilization of the asteroseismic data requires the best possible information about other properties of the star. In the solar case, the mass, radius, luminosity and age are determined quite accurately (or at least precisely) from independent observations. Solar composition, characterized by the ratios of abundances of elements heavier than helium to the abundance of hydrogen, can be determined from spectroscopic observations. Recently, however, there has been a substantial revision in some of these abundances, leading to conflicting comparisons between the resulting solar models and helioseismic inferences (e.g., Asplund et al. 2009 and references therein).

Parameters of other stars are in general known far less well. Masses can be obtained in the rare cases where the star is a member of a well-observed binary system. The effective temperature and surface gravity can be obtained from spectroscopic observations, but subject to the possible limitations of modelling of stellar atmospheres and hence with substantial (and probably often underestimated) uncertainties. The stellar luminosity requires knowledge, from parallax observations, of the distance and hence is currently restricted to relatively nearby stars; also, to the observed stellar magnitude must be applied a bolometric correction which again depends on atmosphere models. Also, in a few cases the stellar radius can be determined from interferometry, again assuming that the distance is known and with some sensitivity to atmospheric structure through limb darkening. Stellar composition is obtained from spec-
troscopy; in the case of stars similar to the Sun this is most often done differentially, relative to the solar spectrum, and hence the abundances are directly affected by the uncertainty in the solar composition.

4. Microphysics

For the purpose of stellar modelling the equation of state is probably in general sufficiently well known from recent tabulations of sophisticated equations of state. On the other hand, helioseismology clearly demonstrates that these are not yet correct, at the level of the observational precision (e.g., Basu et al. 1999). This would also affect the asteroseismic determination of the helium abundance from the signatures in the frequencies of helium ionization (Houdek & Gough 2007).

The computation of stellar opacities is considerably more uncertain than the equation of state, with direct effect on the structure of the radiative part of stellar models. Since the heavy-element abundance affects stellar structure predominantly through the opacity, uncertainties in the heavy-element abundances and the opacity are closely linked. Thus an obvious way to correct solar models, given the revised abundances, is to claim substantial opacity increases (e.g., Bahcall et al. 2003, Christensen-Dalsgaard et al. 2003), although possibly beyond what is physically realistic. An independent indication of a need for opacity increases, although at somewhat lower temperatures than relevant in the Sun, comes from the lack of predicted instability of some observed modes in β Cephei stars (Dziembowski & Pamyatnykh 2008).

Although there remain substantial uncertainties in important nuclear parameters the effect on stellar modelling is in general relatively modest, since even large changes in the parameters can be compensated by modest changes in the temperature, owing to the high temperature sensitivity of the reactions. An important exception concerns the balance between contributions to the PP chains and the CNO cycle in hydrogen burning, which has a substantial effect on the presence and extent of convective cores. This includes the relatively recent large reduction in the rate of proton capture by $^{14}$N (Angulo et al. 2005). An interesting, and so far not resolved, issue concerns electron screening of nuclear reactions (Shaviv 2004, Mao et al. 2009).

There is no doubt that diffusion and settling take place in those parts of a star where there is no macroscopic motion. These processes, in various approximations, are now universally included in ‘standard’ solar modelling, leading to an increase of a few per cent in the surface hydrogen abundance during evolution to the present solar age, and a decrease of around 10 per cent in the heavy-element abundances. In somewhat more massive stars with thinner outer convection zones the settling rate at the base of the convection zone is much higher, leading to an almost complete elimination at the stellar surface of helium and heavier elements, on a timescale short compared with the evolution timescale (Vauclair et al. 1974). To account for the ‘normal’ abundances observed in most such stars one must therefore invoke mixing processes or possibly mass loss to compensate for the settling. A possible explanation is mixing caused by rotationally induced meridional circulation (see below).

An additional complication, particularly in stars a little more massive than the Sun, is the selective effects of radiation pressure on different elements, leading to gravitational levitation counteracting settling and strong local variations in the heavy-element composition. To be taken properly into account, this requires opacity calculations for the local composition, depending on location and time, as the star evolves (e.g., Richer et al. 2000). This has so far only been consistently implemented in very few evolution calculations.

5. Properties of convective cores

Convective cores play an important role in the main-sequence evolution of stars of masses just slightly higher than the Sun and above. This is caused by the increasing dominance in hydrogen burning of the much more temperature sensitive CNO cycle over the PP chains. For stars of masses less than around 2 solar masses this involves a phase where the mass of the convective core increases, owing to the gradual conversion, on a timescale comparable to the evolution timescale, of $^{16}$O to $^{14}$N; if diffusion is neglected the growth of the core leads to a discontinuity in the hydrogen abundance at the edge of the core, and hence in the density and sound speed. Asteroseismic diagnostics of this discontinuity, and other aspects of convective cores, may be possible with sufficiently accurate data (Popielski & Dziembowski 2003, Mazumdar et al. 2006, Cunha & Metcalfe 2007).

The uncertainties in the microphysics, particularly as it affects the importance of the CNO cycle, influence the size of the convective core (see Christensen-Dalsgaard & Houdek 2010 for details). Thus the reduction in the $^{14}$N reaction rate shifts the onset of convective cores higher in stellar mass by about 0.06 $M_{\odot}$. An interesting case is the effect of the revision of solar abundances which, as discussed above, is reflected in the assumed stellar abundances (VandenBerg et al. 2007) showed that this led to a significant change in the isochrones computed for the open cluster M67; with the old composition models near the end of the central hydrogen burning had a convective core, as also suggested by the observed colour-magnitude diagram, while models with the revised composition lacked the convective core.

A probably more important uncertainty concerns the extent of convective overshoot. There is little doubt that motion continues beyond the convectively unstable region, but the extent of that motion, and its effects on stellar structure, are highly uncertain. Presumably the motion is sufficiently vigorous to cause homogenization of the composition, but it is less clear whether it leads to full mixing of entropy and hence an adiabatic stratification. The extent is typically parameterized as a fraction $\alpha_{ov}$ of the pressure scale height at the edge of the core, with a correction for very small cores, but no a priori estimate of $\alpha_{ov}$ is available. Analyses of open clusters and binary stars lead to values of $\alpha_{ov}$ of typically around 0.1 – 0.2, a value confirmed by asteroseismic analyses of β Cephei stars (e.g., Aerts et al. 2003).

An additional complication in models with growing convective cores is the presence of what has been called semi-convection. Convective instability is typically defined in terms of the temperature gradient $\nabla = d\ln T / d\ln p$, where $T$ is temperature and $p$ is pressure; convective instability sets in where the value $\nabla_{rad}$ of $\nabla$ required to transport energy by radiation exceeds the adiabatic value $\nabla_{ad}$. Since
The treatment of convective envelopes also involves substantial uncertainties. Overshoot below the convective envelope has a relatively modest effect on stellar evolution although it can affect the properties of the red bump on the red-giant branch. A more serious concern are the properties of the near-surface layer where the density is low and consequently a substantial superadiabatic gradient is required to transport the energy. Together with the structure of the stellar atmosphere this determines the specific entropy in the predominantly adiabatic bulk of the convective envelope and hence its structure, including its depth. In the solar case the treatment of this layer, e.g., using the Bohm-Vitense (1958) mixing-length formulation, is calibrated to obtain the correct radius; this calibration is typically, with little justification, used in modelling other stars. Hydrodynamical simulations of near-surface convection (Nordlund et al. 2009) provide a reasonably realistic modelling of these layers; unlike other parts of the star the relevant dynamical and thermal timescales are sufficiently similar that the relevant effects can be taken into account, although obviously still with an approximate treatment of scales smaller than the numerical resolution. The results of the simulations can then be used to calibrate the simpler formulations (e.g., Trampedach 2007); this offers a promising procedure for more realistic modelling of this part of the star, although it has so far not seen much use.

Uncertainties in the modelling of the near-surface layers have a substantial effect on the oscillation frequencies and hence on their use as asteroseismic diagnostics. In addition to the structure of the superadiabatic layer, these uncertainties include the dynamical effects, usually ignored, of convection on stellar structure in the form of ‘turbulent pressure’, nonadiabatic effects on the oscillations, and the coupling between convection and pulsations, in terms of the perturbation to the convective flux and the turbulent pressure, as well as the stochastic excitation of the modes, for solar-like oscillations. These effects dominate the difference between the observed and modelled frequencies of solar oscillations; they can be suppressed, however, in helioseismic analyses because of the broad range of degrees of the observed modes. In the stellar case this is not possible, in general. It was pointed out by Roxburgh & Vorontsov (2003) that combinations of frequency separations can be constructed which are insensitive to the superficial layers and retain their sensitivity to the properties of the core (see also Otó Floranes et al. 2005). For more general use of the frequencies, including calibrations of the overall properties of the star, one can attempt to estimate the near-surface effects on the frequencies, by assuming a functional form similar to the known effect in the solar case (Kjeldsen et al. 2008). This, however, remains a serious issue in asteroseismic analyses.

7. Rotation

There is no doubt that most, or indeed all, stars rotate, yet rotation is usually ignored in modelling of stellar evolution. A detailed discussion of the effects of rotation on stars was recently provided by Maeder (2004). The dynamical effects of rotation on stellar structure are relatively straightforward to incorporate, at least as long as they can be treated as perturbations around a non-rotating, spherically symmetric structure. In a slowly rotating star such as the Sun these effects are very small. However, many stars rotate so rapidly that the perturbative approach is inadequate; here two-dimensional modelling of stellar structure is required (e.g., Roxburgh 2004, MacGregor et al. 2007). For greater complications are as-
associated with the effects on stellar evolution, including the
evolution of the internal rotation rate. A naive local ap-
plication of the conservation of angular momentum would
predict that the angular velocity of the central parts of stars
will increase with age as these regions contract, while ro-
tation in the outer parts would be expected to slow down as
they expand. This is certainly too simple. As already noted
by von Zeipel and Eddington, rotation causes a thermal im-
balance which leads to circulation and hence redistribution
of angular momentum and mixing of the stellar composi-
tion. Indeed, it is likely that in many stars this mixing coun-
teracts the rapid settling discussed above. To these pro-
cesses must also be added mass loss, possibly magnetically
linked to the stellar convective envelope, which removes an-
gular momentum from the star. It seems likely that most
stars start their life with rapid rotation; stars with masses
up to somewhat higher than the Sun apparently lose an-
gular momentum to a magnetized stellar wind, leading to
a strong decrease in rotation with age (e.g., [Barnes 2003]).

A treatment of these processes was proposed by Zahn
(1992) and further developed by Maeder & Zahn (1998).
This assumes an angular momentum that depends only on
the distance to the centre of the star, as a result of strong
horizontal turbulence. Mixing of composition is a diffusive
process while transport of angular momentum in addition
includes advective terms. This formulation has been seen fairly
extensive use and has had some success in accounting for
the observed composition of massive stars.

A serious problem is to account for the helioseismically
inferred solar internal rotation rate (e.g., [Howe 2009]);
in particular, the Zahn model is unable to explain the
present slow rotation of the radiative interior. This re-
quires additional mechanisms transporting angular mo-
mmentum from the interior to the convection zone. It has
been proposed that this coupling could be mediated by
gravity waves excited at the base of the convection zone
[Mathis et al. 2008]; alternatively, it may be of magnetic
nature (Garaud & Guervilly 2009). It is obvious that a-
steroseismic information about the internal rotation of other
stars, although unavoidably quite limited in the foreseeable
future, can be extremely valuable in distinguishing between
these mechanisms.

8. Concluding remarks

It is evident that there are many serious open issues in stellar
modelling. An important task is the evaluation of the
asteroseismic signatures of these effects, including the de-
design of diagnostics that may best investigate them and a
determination of the resulting requirements on the obser-
vations. The asteroseismic observations that are currently
been obtained by the CoRoT and Kepler space missions
certainly provide excellent prospects for addressing these
issues, although the experience from CoRoT has shown that
the analysis of the data also involves serious challenges.

There is clearly a need for very substantial development
of the techniques of stellar modelling. This can be inspired,
but certainly not replaced, by further detailed hydrodynam-
ical simulations of specific aspects of stellar interior dynam-
ics. Except for the near-surface layers a serious constraint
is the huge mismatch between the relevant dynamical and
thermal timescales, implying that the simulations cannot
be run under realistic stellar conditions. A great deal of

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