HOLOMORPHIC MAPS ONTO VARIETIES OF NON-NEGATIVE KODAIRA
DIMENSION

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1. INTRODUCTION AND STATEMENT OF RESULTS

A classical result in complex geometry says that the automorphism group of a manifold of general type is discrete [Mat63]. It is more generally true that there are only finitely many surjective morphisms between two fixed projective manifolds $X$ and $Y$ of general type [KO75].

Rigidity of surjective morphisms, and the failure of a morphism to be rigid have been studied by a number of authors, the most general results being those of Borel and Narasimhan [BN67]. For target manifolds $Y$ with Chern numbers $c_1(Y) = 0$ and $c_{dim Y}(Y) \neq 0$, rigidity has been shown by Kalka, Shiffman and Wong [KSW81]. These results have recently been generalized by Hwang [Hwa03] to the case where $Y$ is a compact Kähler manifold with $c_1(Y) = 0$. Although in Hwang’s setup deformations need not be rigid, he is able to give a good description of the space of surjective morphisms.

In this paper we give a complete description of the space of surjective morphisms in the general setup where $Y$ is a normal projective variety that is not covered by rational curves. Our main result, Theorem 1.2, states that surjective morphisms are rigid, unless there is a clear geometric reason for it.

- Deformations of surjective morphisms between normal projective varieties are unobstructed unless the target variety is covered by rational curves.
- If the target is not covered by rational curves, then surjective morphisms are infinitesimally rigid, except for those morphisms that factor via a variety with positive-dimensional automorphism group.

Notation 1.1. If $X$ and $Y$ are normal compact complex varieties, $\text{Hom}(X,Y)$ denotes the space of holomorphic maps $X \to Y$ and $\text{Hom}_s(X,Y)$ the space of surjective holomorphic maps. Given a morphism $f \in \text{Hom}(X,Y)$, let $\text{Hom}_f(X,Y)$ be the connected component of $\text{Hom}(X,Y)$ that contains $f$.

Theorem 1.2. Let $X$ be a normal compact complex variety and $Y$ be a projective normal variety which is not covered by rational curves. If $f : X \to Y$ is a surjective morphism,
then there exists a factorization

\[ X \xrightarrow{\alpha} Z \xrightarrow{\beta} Y \]

where

1. \( \beta \) is a finite morphism which is étale outside of the singular set of \( Y \)
2. if \( \text{Aut}^0(Z) \) is the maximal connected subgroup of the automorphism group of \( Z \), then \( \text{Aut}^0(Z) \) is an Abelian variety, and the natural morphism

\[ \text{Aut}^0(Z)/\text{Aut}(Z/Y) \cap \text{Aut}^0(Z) \to \text{Hom}_f(X, Y) \]

is isomorphic.

In particular, all deformations of surjective morphisms \( X \to Y \) are unobstructed, and the associated components of \( \text{Hom}(X, Y) \) are smooth Abelian varieties.

**Corollary 1.3.** In the setup of Theorem 1.2, if \( Y \) is smooth, then \( Y \) has a finite étale cover of the form \( T \times W \), where \( T \) is an Abelian variety of dimension \( h^0(X, f^*(T_Y)) \) and

\[ \dim \text{Hom}_f(X, Y) \leq \dim Y - \kappa(Y), \]

where \( \kappa(Y) \) is the Kodaira dimension.

**Remark 1.4.** We conjecture that Theorem 1.2 and Corollary 1.3 are true when \( Y \) is a compact Kähler manifold of nonnegative Kodaira dimension. Our proof needs the projectivity assumption because it employs Miyaoka’s characterization of uniruledness.

The following corollaries are immediate consequences of Corollary 1.3.

**Corollary 1.5.** Let \( Y \) be a projective manifold which is not uniruled. If \( \pi_1(Y) \) is finite, then for each connected normal compact complex variety \( X \) the space \( \text{Hom}_s(X, Y) \) is discrete.

**Corollary 1.6.** Let \( Y \) be a projective \( n \)-dimensional manifold which is not uniruled. If \( c_n(Y) \neq 0 \), then for each connected normal compact complex variety \( X \) the space \( \text{Hom}_s(X, Y) \) is discrete.

### 2. Proof of Theorem 1.2

**2.1. Step 1: Setup.** Let \( X \) be a normal variety. Then the tangent sheaf \( T_X \) is by definition the dual of the sheaf \( \Omega_X^1 \) of differentials. If \( f : X \to Y \) is holomorphic, we consider \( \text{Hom}_f(X, Y) \), the connected component of \( \text{Hom}(X, Y) \) that contains \( f \). If \( f \) is additively surjective, since \( X \) is reduced, it is then well-known that

\[ T_{\text{Hom}_f(X, Y)}|_f \cong \text{Hom}(f^*(\Omega_Y^1), \mathcal{O}_X). \]

See e.g. [Kol96, I, Thm. 2.16] for a proof in the algebraic case. We note that if \( Y \) is smooth, then \( \text{Hom}(f^*(\Omega_Y^1), \mathcal{O}_X) \cong H^0(X, f^*(T_Y)) \).

If in the set-up of Theorem 1.2 there are no infinitesimal deformations of the morphism \( f \), i.e. if \( \text{Hom}(f^*(\Omega_Y^1), \mathcal{O}_X) = \{0\} \), there is nothing to prove. We will therefore assume throughout that \( \text{Hom}(f^*(\Omega_Y^1), \mathcal{O}_X) \neq \{0\} \).

**2.2. Step 2: Reduction to a finite morphism.** In this section we reduce the proof of Theorem 1.2 to the case that the morphism \( f \) is finite. To this end, we will consider the Stein factorization of \( f \).

\[ X \xrightarrow{\beta, \text{conn. fibers}} W \xrightarrow{h, \text{finite}} Y, \]
assume that Theorem 1.2 holds for the finite morphism \( h \), and show that \( \text{Hom}_f(X, Y) \) and \( \text{Hom}_h(W, Y) \) are then naturally isomorphic. The argumentation is based on the following elementary observation whose proof we leave to the reader.

**Fact 2.1.** Let \( h : S \to B \) be a morphism of complex spaces. Assume that \( S \) is smooth and compact, \( B \) is connected and that the associated morphism between the Zariski tangent spaces is everywhere isomorphic. Then \( h \) is surjective and étale.

In particular, \( h \) is an isomorphism if it is injective.

In order to apply Fact 2.1, observe that the Stein factorization (2.1) yields a canonical morphism of complex spaces

\[
A : \text{Hom}_h(W, Y) \to \text{Hom}_f(X, Y)
\]

which is injective because \( g \) is surjective. If \( \gamma \in \text{Hom}_h(W, Y) \) is any morphism, it is known that associated morphism between the Zariski tangentspaces at \( \gamma \) and \( \gamma \circ g \)

\[
T A : \frac{T_{\text{Hom}(W, Y)}|_\gamma}{\text{Hom}(\gamma^*(\Omega^1_Y), O_W)} \to \frac{T_{\text{Hom}(X, Y)}|_{\gamma \circ g}}{\text{Hom}(g^*\gamma^*(\Omega^1_Y), O_X)}
\]

is the pull-back via \( g \). Since \( g \) has connected fibers, \( g_*(O_X) = O_W \), and since \( g_* \) and \( g^* \) are adjoint functors, [Har77, p. 110], this map is isomorphic.

If Theorem 1.2 holds for the finite morphism \( h \), \( \text{Hom}_h(W, Y) \) will be a projective manifold. By Fact 2.1, the morphism \( A \) will then be isomorphic, and Theorem 1.2 will hold for \( f \), too. We are therefore reduced to showing Theorem 1.2 under the additional assumption that \( f \) is finite. We maintain this assumption throughout the rest of the proof.

**Remark 2.2.** If \( f \) is finite and \( H \in \text{Pic}(Y) \) ample, then \( f^*(H) \) will again be ample. Thus, the assumption that \( f \) is finite implies that \( X \) is projective. We can therefore argue in the algebraic category for the remainder of the proof.

2.3. **Step 3: Further setup.** In the sequel we will use the following notation:

- **ample bundle:** \( H \) . . . an ample line bundle on \( Y \)
- **exceptional sets:** \( X_s \ldots \) singular locus of \( X \)
  
  \[ Y_s := f(X_s) \cup \{ \text{singular locus of } Y \} \]
- **open sets:** \( Y_0 := Y \setminus Y_s \)
  
  \[ X_0 := f^{-1}(Y_0) \]
  
  \[ f_0 := f|_{X_0} : X_0 \to Y_0 \]

It is well-known that the finite morphism \( f_0 \) defines a vector bundle on the quasi-projective target manifold \( Y_0 \).

**Fact 2.3.** The trace map gives a splitting

\[
(f_0)_*O_{X_0} \cong O_{Y_0} \oplus E^*_0
\]

where \( E^*_0 \) is a vector bundle on \( Y_0 \). In particular, the projection formula gives

\[
(f_0)_*(f_0)^*T_{Y_0} \cong T_{Y_0} \oplus (E^*_0 \otimes T_{Y_0}).
\]

**Remark 2.4.** The exceptional set \( Y_s \) is of codimension \( \geq 2 \). Thus, if \( m \in \mathbb{N} \) is sufficiently large and \( H_1, \ldots, H_{\dim Y - 1} \in |mH| \) are general members, then the general complete intersection curve

\[ C := H_1 \cap \ldots \cap H_{\dim Y - 1} \]

does not intersect \( Y_s \). In particular, the vector bundle \( E^*_0 \) is defined all along \( C \).
2.4. **Step 4: Construction of the étale cover.** In this section we construct a factorization of the morphism $f$, which we assume to be finite, via an étale cover of $Y$. The important properties of the construction are summarized in the following proposition.

**Proposition 2.5.** In the setup of Theorem 1.2 there exists a canonical factorization of $f$ via a finite morphism $\beta$ that is étale outside of the singular set of $Y$,

$$
\begin{array}{cccc}
X & & & Y \\
\alpha & \overset{f}{\longrightarrow} & Z & \overset{\beta}{\longrightarrow} \\
\end{array}
$$

such that all infinitesimal deformations of $f$ come from pull-backs of vector fields on $Z$, i.e. that the natural injective morphism

$$
\text{Hom}(\Omega^1_Z, \mathcal{O}_Z) \to \text{Hom}(\Omega^1_Z, \alpha^*(\mathcal{O}_X)) \cong \text{Hom}(\alpha^*(\Omega^1_Z), \mathcal{O}_X) \cong \text{Hom}(f^*(\Omega^1_Y), \mathcal{O}_X)
$$

is isomorphic.

**Remark 2.6.** In the formulation of Proposition 2.5 we have identified $\text{Hom}(\alpha^*(\Omega^1_Z), \mathcal{O}_X)$ and $\text{Hom}(f^*(\Omega^1_Y), \mathcal{O}_X)$. For this, we use the assumptions that $f$ is finite and that $\beta$ is étale outside of a set of codimension 2: the (reflexive) sheaves $(\alpha^*(\Omega^1_Z))^\vee$ and $(f^*(\Omega^1_Y))^\vee$ agree in codimension 1. Since $X$ is normal, they must be isomorphic.

If $Y$ is smooth, then $Z$ must also be smooth and the natural morphism discussed in Proposition 2.5 is simply the pull-back map

$$
\alpha^*: H^0(Z, T_Z) \to H^0(X, \alpha^*(T_Z)) \cong H^0(X, f^*(T_Y)).
$$

We start the proof of Proposition 2.5 with the following lemma which links the existence of elements in $\text{Hom}(f^*(\Omega^1_Y), \mathcal{O}_X)$ that do not come from vector fields to the structure of the bundle $\mathcal{E}_0$.

**Lemma 2.7.** Assume that there exists an infinitesimal deformation $\sigma \in \text{Hom}(f^*(\Omega^1_Y), \mathcal{O}_X)$ which does not come from the pull-back of a vector field on $Y$. Then, if $C$ is a general complete intersection curve and $\mathcal{E}_0$ the dual of $\mathcal{E}_0^*$, the restriction $\mathcal{E}_0|_C$ is nef, but not ample.

**Proof.** Since $C$ is not contained in the branch locus, the fact that $\mathcal{E}_0|_C$ is nef is shown in [PS00] Thm. A of the appendix by R. Lazarsfeld] —as we need only the nefness on a general curve, we could also use the general semi-positivity theorem of Viehweg for images of relative dualizing sheaves.

Recall that $\text{codim}_X X \setminus X_0 \geq 2$. Sections in a reflexive sheaf which are defined on $X_0$ therefore extend uniquely to all of $X$. This yields identifications

$$
\begin{align*}
\text{Hom}(f^*(\Omega^1_Y), \mathcal{O}_X) &= H^0(X_0, (f_0)^*(T_{Y_0})) \\
&= H^0(Y_0, T_{Y_0} \oplus (\mathcal{E}_0^* \otimes T_{Y_0})) \\
&= H^0(Y, T_Y) \oplus H^0(Y_0, \mathcal{E}_0^* \otimes T_{Y_0}).
\end{align*}
$$

Since we assume that the infinitesimal deformation $\sigma$ does not come from the pull-back of a vector field, we obtain a section $\check{\sigma} \in H^0(Y_0, \mathcal{E}_0^* \otimes T_{Y_0})$, i.e., a morphism of vector bundles

$$
\check{\sigma}: \mathcal{E}_0 \to T_{Y_0}.
$$

After removing further sets of codimension 2, if necessary, we may assume without loss of generality that

$$
\mathcal{F} := \text{Image}(\check{\sigma}) \subset T_{Y_0}
$$

is a locally free subsheaf of $T_{Y_0}$. The restriction of its dual to a general complete intersection curve, $\mathcal{F}^*|_{C}$, is then a torsion-free quotient of $\Omega^1_{Y_0}|_{C}$, which, by Miyaoka’s celebrated theorem [Miy87] cor. 6.4] (see also Theorem 9.0.1 of Shepherd-Barron’s article in [Kol92]).
has non-negative degree. Equivalently, we can say that \( F|_C \) has non-positive degree. But \( F|_C \) is a quotient of \( E_0|_C \) and should therefore have positive degree if \( E_0|_C \) was ample. We conclude that \( E_0|_C \) is not ample. \( \square \)

The existence of a factorization of \( f \) via a cover of \( Y_0 \) now follows from the argumentation of \cite{PS05} proof of Prop. 3.8. For the reader’s convenience, we reproduce the proof here.

**Lemma 2.8.** In the setup of lemma \ref{2.7} after perhaps removing further subsets of codimension two, if necessary, the morphism \( f_0 \) factors via an étale cover \( Y_0^{(1)} \to Y_0 \), which is not an isomorphism.

**Proof.** To factorize the morphism \( f_0 \), it suffices to find a coherent subsheaf \( F \subset E_0^* \) such that \( \mathcal{O}_Y \oplus F \subset \mathcal{O}_Y \oplus E_0^* \cong (f_0)_* \mathcal{O}_X \) is a sheaf of \( \mathcal{O}_Y \)-algebras, i.e. closed under the multiplication map

\[
\mu : (\mathcal{O}_Y \oplus E_0^*) \otimes (\mathcal{O}_Y \oplus E_0^*) \to \mathcal{O}_Y \oplus E_0^*
\]

We can then set \( Y_0^{(1)} := \text{Spec} \mathcal{F} \). If \( F \subset E_0^* \) is a sub-vectorbundle that has degree zero on the general complete intersection curve, then it follows that the natural morphism \( Y_0^{(1)} \to Y_0 \) is étale.

As a first step towards the construction of \( \mathcal{F} \), we fix a complete intersection curve \( C \subset Y_0 \) and construct \( Y_0^{(1)} \) only over \( C \). Since the restriction \( E_0|_C \) is nef, but not ample, it follows from \cite{PS00} Lem. 2.3 that there exists a unique maximal ample subbundle \( V_C \subset E_0|_C \) such that the quotient \( E_0|_C / V_C \) has degree zero. Let \( \mathcal{F}_C \subset E_0^* \) be the kernel of the associated map \( E_0^*|_C \to V_C^* \) which is a sub-vectorbundle of degree zero. It is then clear that \( \mathcal{O}_C \oplus \mathcal{F}_C \subset \mathcal{O}_C \oplus E_0^*|_C \) is closed under multiplication, as the map

\[
\mu' : (\mathcal{O}_C \oplus \mathcal{F}_C) \otimes (\mathcal{O}_C \oplus \mathcal{F}_C) \to \mathcal{O}_C \oplus \mathcal{E}_0^*|_C / \mathcal{O}_C \oplus \mathcal{F}_C
\]

is necessarily zero.

To end the proof of Lemma \ref{2.8} we need to extend the sub-vectorbundle \( V_C \subset E_0|_C \) to all of \( Y_0 \), i.e. we need to find a sub-vectorbundle \( T \subset E_0 \) such that for a general complete intersection curve \( C' \subset Y_0 \), the restriction \( T|_{C'} \subset E_0|_{C'} \) is the unique maximal ample subbundle. For this, consider the Harder-Narasimhan filtration of \( E_0|_{C'} \),

\[
0 = E_0|_{C'}^{(0)} \subset E_0|_{C'}^{(1)} \subset \cdots \subset E_0|_{C'}^{(f)} = E_0|_{C'}.
\]

It is an elementary computation to see that there exists a number \( k \) such that \( V_C = E_0|_{C'}^{(k)} \). In this setup, after removing further subsets of codimension two, if necessary, the theorem of Mehta-Ramanathan \cite{Kol92} Thm. 9.1.1.7 (see also \cite{MR82}) guarantees that \( V_C \) extends to all of \( Y_0 \), as required. \( \square \)

**Remark 2.9.** János Kollár pointed out to us that the proof of Lemma \ref{2.8} really shows that if an antinef vector bundle on a curve has a section after pull back, then it has a section after an étale pull back

It is a classical result that the cover \( Y_0^{(1)} \to Y_0 \) can be extended to \( Y \).

**Corollary 2.10.** In the setup of Lemma \ref{2.7} the morphism \( f \) factors via a normal variety \( Y^{(1)} \),

\[
X \xleftarrow{a} Y^{(1)} \xrightarrow{b} Y
\]

where \( b \) is a finite morphism of degree \( >1 \), étale outside of the singular locus of \( Y \).
Proof. The factorization for $f_0 : X_0 \to Y_0$ via an unbranched cover $b_0 : Y_0^{(1)} \to Y_0$ is shown in Lemma 2.8. Since $Y$ is normal, [DG24 Thm. 3.5] says that there exists a unique normal compactification $Y^{(1)} \supset Y_0^{(1)}$ with a finite morphism $b : Y^{(1)} \to Y$ that extends $b_0$ and is étale outside of the singular set.

The proof is finished if we show that the associated rational map $a : X \dasharrow Y^{(1)}$ is a morphism. That, however, follows from the fact that $b$ is a morphism. Thus, the following two prerequisites are verified. Note that the proof of Theorem 1.2 is finished if we show that $f = b \circ a$ is a morphism and that $b$ is finite. □

Proof of Proposition 2.5. If all infinitesimal deformations of $f$ come from pull-backs of vector fields on $Z$, i.e. if $\text{Hom}(f^*(\Omega^1_Y), \mathcal{O}_X) \cong H^0(Z, T_Z)$, there is nothing to prove: set $Z = Y$.

If there exists an infinitesimal deformation $\sigma_1 \in \text{Hom}(f^*(\Omega^1_Y), \mathcal{O}_X)$ that is not a pull-back of a vector field on $Y$, apply Corollary 2.10 and obtain a factorization of $f$ via a cover $b : Y^{(1)} \to Y$. If there is a section $\sigma_2 \in \text{Hom}(f^*(\Omega^1_Y), \mathcal{O}_X) = \text{Hom}(a^*(\Omega^1_{Y^{(1)}_0}), \mathcal{O}_X)$, which is not the pull-back of a vector field on $Y^{(1)}$, apply Corollary 2.10 again to the morphism $a : X \to Y^{(1)}$. Proceed inductively, creating a sequence of covers

$$X \stackrel{f}{\longrightarrow} Y^{(d)} \longrightarrow Y^{(d-1)} \longrightarrow \cdots \longrightarrow Y^{(1)} \longrightarrow Y$$

The process terminates because the degree of $f$ is finite. Let $Z := Y^{(d)}$ be the terminal variety. □

2.5. Step 5: end of proof. The factorization of $f$ given in Proposition 2.5 yields a natural morphism

$$\iota : \text{Aut}^0(Z) \to \text{Hom}_f(X, Y)$$

$$g \mapsto \beta \circ g \circ \alpha$$

Note that the proof of Theorem 1.2 is finished if we show that $\iota$ is étale. This will be guaranteed by Fact 2.1 as soon as the following two prerequisites are verified.

**Aut$^0(Z)$ is proper:** By assumption, the variety $Y$ is not uniruled. Thus, the variety $Z$ is also not uniruled, and it follows from [Ros56] that the automorphism group $\text{Aut}^0(Z)$ does not contain an algebraic subgroup which is isomorphic to $\mathbb{C}$ or to $\mathbb{C}^*$. The group $\text{Aut}^0(Z)$ must therefore be an Abelian variety, $\text{Aut}^0(Z) \cong \mathbb{C}^m/\Gamma$. In particular, $\text{Aut}^0(Z)$ is proper.

**The tangent morphism is everywhere isomorphic:** It is known that for any automorphism $g \in \text{Aut}^0(Z)$, the morphism $T_\iota$ between Zariski tangent spaces,

$$T_{\iota} : T_{\text{Aut}^0(Z)}|_g \to T_{\text{Hom}(X, Y)}|_{\beta \circ g \circ \alpha}$$

$$H^0(Z, g^*(T_Z)) \to \text{Hom}(\beta \circ g \circ \alpha)^*(\Omega^1_Y), \mathcal{O}_X)$$

is given by the natural injective morphism of sheaves

$$\text{Hom}(g^*(\Omega^1_Z), \mathcal{O}_Z) \to \text{Hom}(g^*(\Omega^1_Y), \alpha^*(\mathcal{O}_X)) \cong \text{Hom}(\alpha^*g^*(\Omega^1_Z), \mathcal{O}_X)$$

$$H^0(Z, g^*(T_Z)) \to \text{Hom}(\beta \circ g \circ \alpha)^*(\Omega^1_Y), \mathcal{O}_X)$$

Since $g$ is an automorphism, $g^*(\Omega^1_Z) \cong \Omega^1_Z$, and Proposition 2.5asserst that this morphism is indeed isomorphic.

This ends the proof of Theorem 1.2. □

3. Proof of Corollary 1.3

In the setup of Corollary 1.3, the varieties $Y$ and $Z$ are smooth. If $\text{Aut}^0(Z)$ is trivial, i.e. $H^0(Z, T_Z) = 0$, there is nothing to prove. Otherwise, since $Z$ is not uniruled, the fact
that $Z$ is a Torus-Seifert fibration follows from [Lie78, Thm. 4.9]. By [Lie78, Thm. 4.10], we have that
\[
\dim H^0(Z, T_Z) + \kappa(Z) \leq \dim Z
\]
and therefore
\[
\dim \text{Hom}_f(X, Y) \leq \dim Y - \kappa(Z) \leq \dim Y - \kappa(Y).
\]

\[\square\]

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