Effect of non-unitary neutrino mixing in Lorentz violation and dark NSI

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Neutrino flavour oscillation is one of the primary indication of the existence of new physics beyond standard model. The presence of small neutrino mass is essentially required for the neutrino flavour oscillation to occur which is one of the most primary necessity to look for physics beyond standard model (SM). While in SM neutrinos are considered to be left handed (LH) massless Weyl fermions, in several extensions of SM neutrino mass may appear due to some radiative correction [1–3] or by the inclusion of right handed (RH) chiral neutrinos [4–6]. In generally, the non zero neutrino mass is expected to appear due to breaking of some hidden symmetry [7,8]. The smallness of neutrino mass can be explained from type-I Seesaw mechanism which involves the addition of heavy neutral lepton in neutrino oscillation probability is possible to be observed within the energy interval, 2 – 2.75 GeV, which corresponds to the maximum flux observed at DUNE, in the scenario of dark NSI only, under the assumption of non-unitary neutrino mixing.

I. INTRODUCTION

Neutrino oscillation is a well established phenomenon at present in the domain of particle physics. The presence of small neutrino mass is essentially required for the neutrino flavour oscillation to occur which is one of the most primary necessity to look for physics beyond standard model (SM). While in SM neutrinos are considered to be left handed (LH) massless Weyl fermions, in several extensions of SM neutrino mass may appear due to some radiative correction [1–3] or by the inclusion of right handed (RH) chiral neutrinos [4–6]. In generally, the non zero neutrino mass is expected to appear due to breaking of some hidden symmetry [7,8]. The smallness of neutrino mass can be explained from type-I Seesaw mechanism which involves the addition of heavy neutral lepton [9].

Due to the presence of these additional leptons the unitary neutrino mixing matrix, which is a subset of a general lepton mixing matrix, acquires non-unitary properties. Such non-unitarity condition affects the neutrino oscillation and its propagation through the medium. Non unitarity (NU) of mixing matrix also gives rise to lepton flavour violation (LFV) which is another signature of beyond standard model (BSM) phenomena [10]. The Seesaw scale may exist at any point within electroweak (EW) and Planck scale. Although high scale Seesaw mechanism are unable to provide momentous deviation from unitarity, several low scale mechanisms are capable of doing so [11,12]. The effect of non-unitary mixing matrix has been discussed in several previous works in context of different aspects of neutrino oscillation [13–15]. In addition to the feature of non-unitarity of the mixing matrix, two of the other most profound BSM scenarios in the neutrino sector are the violation of Lorentz symmetry and the presence of nonstandard interaction (NSI).

Lorentz symmetry is conserved in two of the foremost theories of nature, SM of particle physics and general theory of relativity (GR). Various models have been developed allowing the spontaneous violation of Lorentz symmetry at the Planck scale (~ 10^{16} GeV) which unifies SM with gravity. Although it is impossible to test any theory at this energy scale, it is expected that such NP signals are manifested at lower energy scale which can be probed with current experimental facilities. Mathematically, SM is portrayed as the effective field theory (EFT) at lower energy scale consisted of the operators with dimension D ≤ 4, while the new physics (NP) signals are represented in terms of nonrenormalizable higher dimensional operators (D > 4) suppressed by the scale of NP. The technique of interferometry has been widely utilized to investigate the Lorentz symmetry [16–19]. Neutrino oscillation provides a natural source of interferometer [20] which is an excellent tool to search for Lorentz violation (LV). Currently ongoing oscillation experiments are able to search for the violation of such a fundamental symmetry. Several experimental facilities have constrained the strength of LV [21–23].

Non standard interaction (NSI) is another elegant way to incorporate NP in the neutrino sector. Introduced by Wolfenstein in 1978 in terms of flavour changing neutral current (FCNC) when the neutrino undergoes the coherent forward elastic scattering in the matter [24], NSI has been widely studied in many literatures. It should be mentioned that the discussions in these works involve neutrino interaction mediated by a vector boson, which directly contributes to the matter potential, while the neutrinos may also interact via a scalar mediator, giving rise to scalar NSI. Since the universe contains a large abundance of dark matter (DM), it is also possible that the scalar particle may belong to the dark sector, generating dark NSI [31]. Unlike the case of vector NSI, the neutrino-scalar interaction induces correction to the neutrino mass term which is possible to be represented in terms of dimension-6 four fermion operator. The bounds on the NSI parameters corresponding to the scalar mediator is much weaker, as compared to the case of vector NSI. Borexino collaboration has been able to put constraints on the scalar NSI parameters [22,23].

In this work, the effect of non-unitary mixing matrix is analyzed in the sector of Lorentz violation and dark NSI within...
the framework of neutrino oscillation in context of DUNE experimental setup. It is found that the parameter space varies significantly in the different scenario of LV and dark NSI under the consideration of unitary and nonunitary neutrino mixing matrix. In section II the formalism of this work is described including the different aspects of NP scenarios in the framework of neutrino oscillation. In section III the result of this analysis is discussed and finally the conclusion is mention in section IV.

II. FORMALISM

In this section the formalism of this analysis is presented. In section IIA the facet of non-unitarity of neutrino mixing matrix is discussed, while the scenarios of LV and dark NSI are described in section IIB and IIC respectively. In section IID the analytical framework to estimate the neutrino oscillation probability is presented.

A. Non unitarity of mixing matrix

Recent measurement by KATRIN collaboration have estimated the neutrino mass to be $m_e < 0.8$ eV/c$^2$ [34]. Neutrino mass can be either of Dirac or Majorana in nature. However, it is not favourable for the neutrino to acquire pure Dirac mass term, as the corresponding Yukawa couplings are extremely small ($\sim 10^{-11}$) compared to the other fermions. Such problem is possible to be resolved if the additional RH neutrino is of Majorana nature which is the origin of the well known Seesaw mechanism [35]. The small neutrino mass can be generated significantly in the different scenario of LV and dark NSI [36]. The small neutrino mass can be generated through non-unitary property of the PMNS mixing matrix. In the presence of $n$ number of neutrinos, the generic mixing matrix takes the following form

$$U_{\alpha\alpha'} = \left( T_{3\times3} Q_{3\times(n-3)} V_{(n-3)\times(n-3)} \right)$$

where $T_{3\times3}$ and $U_{(n-3)\times(n-3)}$ represent the mixing in the sector of LH and RH neutrinos respectively, provided there are three active LH neutrinos and $(n-3)$ number of RH neutrinos. $Q_{3\times(n-3)}$ and $V_{(n-3)\times3}$ contain the coupling parameters in the RH sector. In case of unitary mixing, $T_{3\times3}$ is represented as the PMNS mixing matrix, given by

$$U_{PMNS} = \begin{pmatrix}
  c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\
  -s_{12} c_{23} + c_{12} s_{13} s_{23} e^{i\delta} & c_{12} c_{23} - s_{12} s_{13} s_{23} e^{i\delta} & c_{13} s_{23} \\
  s_{12} s_{23} - c_{12} s_{13} c_{23} e^{i\delta} & -c_{12} s_{23} - s_{12} s_{13} c_{23} e^{i\delta} & c_{13} c_{23}
\end{pmatrix}$$

Here $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$ and $\delta$ is the CP violating phase. In presence of NU, $T_{3\times3}$ can be parametrized in a model independent way, assuming that the additional neutral heavy leptons do not contribute to neutrino oscillation [12]. One of the most convenient parametrization of $T_{3\times3}$ is expressed as follows [37][38]

$$T_{3\times3} = \begin{pmatrix}
  a_{00} & 0 & 0 \\
  a_{10} & a_{11} & 0 \\
  a_{20} & a_{21} & a_{22}
\end{pmatrix} U_{PMNS},$$

where $a_{ij}$’s are the parameters that invoke NU. In generally, the diagonal parameters are real, while the off-diagonal ones are complex ($a_{ij} = |a_{ij}|e^{i\phi_{ij}}$, $i \neq j$) that can provide an additional source of CP violation. The constraints on NU parameters in eqn. 3 are obtained from several decay processes [37][39][40] and oscillation experiments [15][38].

B. Violation of Lorentz symmetry

It is widely known that SM is invariant under Lorentz transformation, which leads to the conservation of CPT symmetry in the universe. It is assumed in several extensions of SM that the Lorentz symmetry is possible to be violated at some higher energy scale, possibly at Planck scale. The origin of Lorentz violation (LV) is discussed extensively in various theoretical models of string theory [41] and quantum gravity [42][44]. In string theory, the violation of Lorentz symmetry can occur spontaneously in presence of a non-perturbative vacuum generating tensor fields acquiring non-zero vacuum expectation value (vev) [45][46]. In non-commuting field theory, the phenomenon of Lorentz violation occurs naturally. In quantum theory of gravity, the Lorentz violation may appear due to an anisotropic scaling of the space and time [47].

The observation of Lorentz violation is only possible at Planck scale ($E_{Pl} \sim 10^{19}$ GeV) which is not accessible by any current terrestrial experimental facility. At electroweak (EW) scale ($E_{EW} \sim 100$ GeV) the effect of Lorentz violation is suppressed by a factor of $E_{EW}/E_{Pl} \sim 10^{-17}$ However, several experimental techniques have been developed over the years to test the existence of LV. The system of three flavour neutrino oscillation is one of the most commendatory sector to study such effects, as it appears under the category of the interferometry effect. The analysis of LV in the framework of neutrino oscillation has been done previously in several works, see refs [22][48][54]. The Lagrangian density corresponding to LV considering the effect of CPT violation is expressed as [55]

$$\mathcal{L}_{LV} < a_{\alpha\beta}^L (\bar{\nu}_\alpha \gamma_\lambda P_L \nu_\beta),$$

where $P_L = (1 - \gamma_5)/2$ and $a_{\alpha\beta}^L$ is the parameter exhibiting the strength of LV. Here $\lambda$ is the spacetime index ($\lambda = 0, 1, 2, 3$) and $\alpha, \beta$ are the flavour indices, $\alpha, \beta \in \{e, \mu, \tau\}$. In this analysis, only the isotropic component of LV parameters ($\lambda = 0$) are taken into account. The interaction Hamiltonian correspond-
The phenomenon of neutrino oscillation is based on the fact that neutrino flavour eigen states are represented as a superposition of the mass eigen states. Considering an initial neutrino beam $\nu_\alpha (\alpha = e, \mu, \tau)$, the relation between the flavour and the mass eigen states is represented as

$$|\nu_\alpha (t = 0)\rangle = \sum_{i=1}^{3} U_{\alpha i}^{3\times 3} |\nu_i (t = 0)\rangle$$

(11)

The matrix $U^{3\times 3}$ may be unitary ($U_{PMNS}$) or non-unitary ($T_{3\times 3}$), represented by eqn. (3) and (2) respectively. Now, Time evolution of mass eigen state is expressed as, $|\nu_i (t)\rangle = e^{-iH_m t} |\nu_i (t = 0)\rangle = e^{-iE_\nu t}$, where $H_m$ is the interaction matrix.

C. Dark Non standard interaction

In neutrino sector another way to introduce the BSM physics is via the inclusion of the non standard interaction (NSI) which appears in several mass models. Considering SM as a lower energy approximation of a full theory existing at a much higher energy scale ($\Lambda$), the effective Lagrangian is possible to be represented in terms of a number of higher dimensional Lorentz structures. Each of the operator is non-renormalizable, in general, and is restricted by the NP energy scale, $\Lambda$ at which the phenomenon of LV might also occur.

NSI can be exchanged by a vector or scalar mediator. The vector mediator can be both charged and neutral. The charged current (CC) NSI is tightly constrained, while the constraints on the neutral current (NC) NSI is much weaker. The importance of NC vector NSI has been discussed in several literatures [25][26][28]. The interaction Hamiltonian corresponding to NC vector NSI, which contributes a subleading effect to the neutrino oscillation, is given by

$$H_{NC-NSI}^{rec} = A \begin{pmatrix} \epsilon_{ee} & \epsilon_{e\mu} & \epsilon_{e\tau} \\ \epsilon_{e\mu} & \epsilon_{\mu\mu} & \epsilon_{\mu\tau} \\ \epsilon_{e\tau} & \epsilon_{\mu\tau} & \epsilon_{\tau\tau} \end{pmatrix}$$

(6)

The parameters, $\epsilon_{\alpha\beta}$ determine the strength of NSI which corresponds to the dimension-6 four fermion lepton number conserving operator. The off-diagonal elements ($\alpha \neq \beta$) are complex, in general. $A = \pm G_F n_e$, with $G_F$ and $n_e$ representing Fermi constant and the number density of electron in the medium respectively. The ‘+’ and ‘−’ signs correspond to neutrino and anti-neutrino respectively. In a recent analysis, the degeneracy existing between the scenario of LV and vector NSI has been discussed with the role of atmospheric neutrino to lift up this degeneracy [59].

While the vector NSI affects the matter potential during neutrino propagation through a medium, the NSI mediated by a scalar has completely different consequences. Instead of modifying the SM matter potential, the scalar NSI directly influences the neutrino mass term. The effect of scalar mediator is added up to the mass term as a perturbation, which is represented in terms of Hamiltonian as follows

$$H_{NSI}^{scalar} = \frac{1}{2E_v} (M + \delta M)(M + \delta M)^\dagger$$

(7)

According to modern cosmological model, the universe is filled with dark matter (DM) particles ($\sim 27\%$). From several astrophysical constraints the local dark matter density is estimated to be $\rho_e \sim 0.47 \text{ GeV/cm}^3$. Since the number density of the DM particles is inversely proportional to its mass, there can be a large abundance of DM present in the universe if their mass is sufficiently small. These light DM particles ($m_\chi \leq 100 \text{ eV}$) can be either scalar or vector, and they may also interact with neutrinos. Such interaction generates another kind of NSI, named as dark NSI. If the DM mediator is a scalar particle ($\phi$) of mass $m_\phi$, the neutrino-DM interaction Lagrangian is given by [51][60]

$$-\mathcal{L}_{NSI}^{dark} \sim \frac{1}{2} M_{\alpha\beta} \bar{\nu}_\alpha \gamma_5 \nu_\beta + \frac{1}{2} m_\phi^2 \phi^2$$

(8)

Here $\gamma_{\alpha\beta}$ is the Yukawa coupling corresponding to neutrino-DM interaction and $M_{\alpha\beta}$ is the neutrino mass matrix. The corresponding interaction Hamiltonian is given by [51][60]

$$H_{NSI}^{dark} = \frac{1}{2E_v} (MM^\dagger + \delta M^2)$$

(9)

Here the correction to the mass term is represented as, $\delta M_{\alpha\beta}^2 = 2m_\phi^2 \gamma_{33} \gamma_{3j}$. Such coupling of neutrino with DM is shown to violate the CPT symmetry [60]. The correction to the mass term is parametrized as follows [29][30]

$$\delta M \equiv \sqrt{\Delta m_{31}^2} \begin{pmatrix} \eta_{ee} & \eta_{e\mu} & \eta_{e\tau} \\ \eta_{e\mu} & \eta_{\mu\mu} & \eta_{\mu\tau} \\ \eta_{e\tau} & \eta_{\mu\tau} & \eta_{\tau\tau} \end{pmatrix}$$

(10)

Here $\eta_{\alpha\beta}$ determines the strength of dark NSI. From the recent data obtained from Borexino 2017, the constraint on only one diagonal NSI parameter is estimated as, $\eta_{ee} = -0.16$ [29]. In the next section, the analytical description to obtain neutrino oscillation probability is discussed in the scenario of LV and dark NSI with the consideration of unitary and non-unitary neutrino mixing matrix.

D. Neutrino oscillation in matter

The parameters, $\alpha_{\mu\nu}$ determine the strength of NSI which corresponds to the dimension-6 four fermion lepton number conserving operator. The off-diagonal elements ($\alpha \neq \beta$) are complex, in general. $A = \pm G_F n_e$, with $G_F$ and $n_e$ representing Fermi constant and the number density of electron in the medium respectively. The ‘+’ and ‘−’ signs correspond to neutrino and anti-neutrino respectively. In a recent analysis, the degeneracy existing between the scenario of LV and vector NSI has been discussed with the role of atmospheric neutrino to lift up this degeneracy [59].

While the vector NSI affects the matter potential during neutrino propagation through a medium, the NSI mediated by a scalar has completely different consequences. Instead of modifying the SM matter potential, the scalar NSI directly influences the neutrino mass term. The effect of scalar mediator is added up to the mass term as a perturbation, which is represented in terms of Hamiltonian as follows

$$H_{NSI}^{scalar} = \frac{1}{2E_v} (M + \delta M)(M + \delta M)^\dagger$$

(7)

Here $a_{i\beta}^{d\phi}$ is simply denoted by $a_{\alpha\beta}$. As $H_{LV}$ is Hermitian, $a_{i\beta}^{d\phi} = a_{\beta\alpha}^d$. Due to the consideration of CPT-violating scenario, $a_{i\beta}^{d\phi} \overset{\text{CPT}}{=} -a_{\beta\alpha}^d$. The off-diagonal elements are generally complex, $a_{\alpha\beta} = |a_{\alpha\beta}| e^{i\phi_{\alpha\beta}}$. The constraints on these parameters are obtained from various experiments such as IceCube [56], MINOS [57], Double Chooz [58].
Hamiltonian in mass basis and $E_i$ is the corresponding energy eigen value. In mass basis the effective Hamiltonian in presence of Lorentz violation (LV) is represented as

$$H_m = \frac{1}{2E_{\nu}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2 & 0 \\ 0 & 0 & \Delta m_{31}^2 \end{pmatrix} + U^{3\times3} A \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} U^{3\times3\dagger} + U^{3\times3} \begin{pmatrix} a_{ee} & a_{e\mu} & a_{e\tau} \\ a_{\mu e} & a_{\mu\mu} & a_{\mu\tau} \\ a_{\tau e} & a_{\tau\mu} & a_{\tau\tau} \end{pmatrix} U^{3\times3\dagger}$$ (12)

while considering the effect of dark NSI, the effective Hamiltonian is modified as

$$H_m = \frac{1}{2E_{\nu}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2 & 0 \\ 0 & 0 & \Delta m_{31}^2 \end{pmatrix} + U^{3\times3} A \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} U^{3\times3\dagger} + \frac{1}{2E_{\nu}} (MM^\dagger + \delta M^2)$$ (13)

The unperturbed part of the mass matrix $M$ is given by, $M \equiv U^{3\times3} \text{Diag}(m_1, m_2, m_3) U^{3\times3\dagger}$. The time evolution of flavour state is expressed as, $|\nu_\alpha(t)\rangle = e^{-iH_{\text{flavour}}t} |\nu_\alpha(t = 0)\rangle$, $H_{\text{flavour}}$ is the Hamiltonian in the flavour basis, $H_{\text{flavour}} = U^{3\times3} H_m U^{3\times3\dagger}$. To calculate the probability of neutrino oscillation, the framework of [61] is taken into consideration, according to which the
III. RESULTS AND DISCUSSION

In this section the effects of unitary and non-unitary mixing matrix in the neutrino flavour oscillation probability in the domain of LV and dark NSI are presented in the context of DUNE experimental set up, having the baseline length \( L = 1300 \text{ km} \), which operates within the energy range of \( E_\nu \approx 1 - 10 \text{ GeV} \). The oscillation channel mainly observed in DUNE is \( \nu_\mu \to \nu_e \) and the maximum flux is observed around the energy, \( E_\nu \approx 2.5 - 3 \text{ GeV} \). The Earth matter density considered in this work is, \( \rho = 2.8 \text{ gm/cc} \), which corresponds to the matter potential, \( A = 1.01 \times 10^{-13} \text{ eV} \). The mass eigen values of the three neutrinos are considered as, \( m_1 = 10^{-5} \text{ eV} \), \( m_2 = \sqrt{\Delta m_{21}^2 + m_1^2} \), \( m_3 = \sqrt{\Delta m_{31}^2 + m_2^2} \). In this analysis only one diagonal parameter is considered to be non zero, \( \eta_{ee} = -0.3 \) [30]. The LV parameter is chosen as, \( \alpha_{\mu \mu} = 10^{-24} \text{ GeV} \) with the corresponding complex phase, \( \phi_{e\mu} = 0 \), while the rest of the parameters are taken to be zero. The standard neutrino oscillation parameters in case of normal (NO) and inverted mass ordering (IO) are illustrated in table II [32]. The bounds on NU parameters (\( \alpha_{ij} \)) are presented in table III [33].

In fig. 1 the variation of the transition probability in the \( \nu_\mu \to \nu_e \) channel is depicted in the plane of \( (E_\nu, \delta) \), for normal mass ordering (NO), in case of LV scenario in the context of DUNE experimental set up. It can be seen from the left panel that for unitary mixing the maximum value of \( P_{\mu e} \geq 0.25 \) is confined within the energy interval, \( E_\nu \approx 1.5 - 2.5 \text{ GeV} \) with the corresponding range of the CP violating phase, \( 180^\circ \leq \delta \leq 215^\circ \), while in case of non-unitary mixing, \( P_{\mu e, \text{max}} \) corresponds to, \( E_\nu \approx 2 - 2.5 \text{ GeV} \) and \( 180^\circ \leq \delta \leq 200^\circ \).

The result for dark NSI is depicted in the two panels of fig. 2. From the left panel of the figure which corresponds to the unitary mixing, it can be pointed out that \( P_{\mu e, \text{max}} \geq 0.25 \) corresponds to the energy range \( E_\nu \approx 2 - 2.5 \text{ GeV} \) for the higher values of \( \delta \) (230° ≤ \( \delta \) ≤ 257°), while in case of nonunitary mixing which is shown in the right panel of the figure, the region of \( P_{\mu e, \text{max}} \) is shifted towards lower \( \delta \) region i.e. 189° ≤ \( \delta \) ≤ 230°, spread within the energy range, \( E_\nu \approx 2 - 2.75 \text{ GeV} \). The results for fig. 1 and 2 are illustrated in table III.

\[
\begin{array}{c|c|c}
(E_\nu, \delta) & \text{Unitary (U)} & \text{Non-unitary (NU)} \\
\hline
E_\nu \text{ (GeV)} & LV & NSI & LV & NSI \\
\hline
1.5 - 2.5 & 2 - 2.5 & 2 - 2.5 & 2 - 2.75 \\
2 - 2.5 & 189 - 215 & 189 - 200 & 230 - 257 & 189 - 230 \\
\end{array}
\]

\( \Delta m^2 \text{ (eV}^2 \rangle \text{ )} \)

In fig. 3 and 4 similar results are presented for inverted mass ordering (IO) in case of LV and dark NSI respectively. Here the maximum value of \( P_{\mu e} \) is reduced, as compared to the case of NO. From fig. 3 it is observed that \( P_{\mu e, \text{max}} \geq 0.0913 \) is confined in very tiny regions in the \( (E_\nu, \delta) \) plane. While in case of unitary mixing matrix, \( P_{\mu e, \text{max}} \) corresponds to the energy range, \( E_\nu \approx 1 \text{ GeV} \) and \( E_\nu \approx 3.5 - 4 \text{ GeV} \) around \( \delta \approx 252^\circ \), for unitarity the region corresponding to \( P_{\mu e, \text{max}} \) is even further diminished. For dark NSI, it can be shown from fig. 4 that \( P_{\mu e} \) is very small (≤ 0.008) in most of the allowed \( (E_\nu, \delta) \) plane. The results corresponding to IO are illustrated in table IV.

In fig. 4 that \( P_{\mu e} \) is very small (≤ 0.008) in most of the allowed \( (E_\nu, \delta) \) plane. The results corresponding to IO are illustrated in table IV.

One significant feature to observe from all the plots is that around \( E_\nu \approx 2 - 2.75 \text{ GeV} \), which corresponds to the energy for the maximum flux at DUNE (2.5 – 3 GeV), the scenario of dark NSI is only able to provide large value of \( P_{\mu e} \), with the assumption of NU of the mixing matrix, in case of NO.

\[
\begin{array}{c|c|c}
\alpha_{ij} & \text{Best fit with \( 1\sigma \) interval} & \text{Best fit with \( 3\sigma \) interval} \\
\hline
\alpha_{10} & 3.82^{+0.78}_{-0.76} & > 0.93 \\
\alpha_{11} & 49.7^{+2.0}_{-1.1} & > 0.95 \\
\alpha_{20} & 8.65^{+0.12}_{-0.13} & > 0.95 \\
\alpha_{21} & 217^{+40}_{-28} & > 3.6 \times 10^{-2} \\
\alpha_{30} & 280^{+25}_{-29} & > 0.13 \\
\alpha_{31} & 65^{+2.0}_{-0.31} & > 0.021 \\
\end{array}
\]

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
Parameters & Best fit & Best fit \\
\hline
\( \theta_{12}^2 \) & 3.82^{+0.78}_{-0.76} & 3.82^{+0.78}_{-0.75} \\
\hline
\( \theta_{23}^2 \) & 49.7^{+2.0}_{-1.1} & 49.7^{+2.0}_{-1.0} \\
\hline
\( \theta_{13}^2 \) & 8.65^{+0.12}_{-0.13} & 8.65^{+0.12}_{-0.13} \\
\hline
\( \delta \) & 217^{+40}_{-28} & 280^{+25}_{-29} \\
\hline
\( \Delta m_{21}^2 \) (eV) & 7.39^{+0.21}_{-0.20} \times 10^{-5} & 7.39^{+0.21}_{-0.20} \times 10^{-5} \\
\hline
\( \Delta m_{31}^2 \) (eV) & 2.52^{+0.03}_{-0.031} \times 10^{-3} & -2.512^{+0.034}_{-0.031} \times 10^{-3} \\
\hline
\end{tabular}
\caption{Values of neutrino oscillation parameters with \( 1\sigma \) interval}
\end{table}
Further, it can be pointed out from the figures that the nature of $P_{\mu e}$ in case of LV and dark NSI are quite different in case of both unitary and non-unitary neutrino mixing scenario for DUNE setup, which implies that these two BSM effects are completely non-degenerate, unlike the case of vector NSI [59].

IV. CONCLUSION

In this work the effect of unitary and non-unitary mixing matrix on neutrino flavour oscillation probability are analyzed considering the scenario of Lorentz violation and dark NSI, in context of DUNE experimental setup. The oscillation probability, $P_{\mu e}$ is higher for NO as compared to IO, in both scenario of LV and dark NSI. In case of IO, the value of $P_{\mu e}$ is very small in the major portion of the $(E_\nu, \delta)$ plane for both the unitary and non-unitary mixing matrix, in the scenario of dark NSI. An interesting feature of this analysis is that if large $P_{\mu e} (> 0.25)$ is observed around the energy, $E_\nu \approx 2.5$ GeV,
which corresponds to the energy of maximum flux at DUNE, it is only possible in the sector of dark NSI under the consideration of non-unitary mixing of the neutrinos. Furthermore, it is also identified from the analysis that the Lorentz violation and dark NSI are two non-degenerate BSM phenomena, unlike the scenario of vector NSI.

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