Instrumental variable methods using dynamic interventions

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**Summary.** Recent work on dynamic interventions has greatly expanded the range of causal questions that researchers can study. Simultaneously, this work has weakened identifying assumptions, yielding effects that are more practically relevant. Most work in dynamic interventions to date has focused on settings where we directly alter some unconfounded treatment of interest. In policy analysis, decision makers rarely have this level of control over behaviours or access to experimental data. Instead, they are often faced with treatments that they can affect only indirectly and whose effects must be learned from observational data. We propose new estimands and estimators of causal effects based on dynamic interventions with instrumental variables. This method does not rely on parametric models and does not require an experiment. Instead, we estimate the effect of treatment induced by a dynamic intervention on an instrument. This robustness should reassure policy makers that these estimates can be used to inform policy effectively. We demonstrate the usefulness of this estimation strategy in a case-study examining the effect of visitation on recidivism.

**Keywords:** Causal inference; Dynamic interventions; Instrumental variables; Non-parametrics; Recidivism

1. **Introduction**

This paper is the only extension, to our knowledge, of dynamic interventions to instrumental variables (IVs). We use these developments to study the effects of inmate visitation on future criminal behaviour. Beyond helping to answer an important substantive question, our proposed method provides four important contributions: inference is fully non-parametric, the method is valid in observational settings, the method can incorporate continuous instruments and, finally, the method estimates the effect of more realistic interventions than have generally been available, relaxing positivity assumptions and providing practicable decision support.

Previous evidence from theoretical and empirical research in the criminology literature suggests that visitation reduces recidivism (Bales and Mears, 2008; Borge, 1985; Casey-Acevedo et al., 2004; Cochran, 2014; Derkzen et al., 2009; Duwe and Clark, 2013; Holt and Miller, 1972; Mears et al., 2012). For an excellent review of the literature on recidivism and desistance, see Bersani and Doherty (2018). Intuitively, the salutory effect of visitation makes sense: visitation enables prisoners to remain connected to the world outside prison and encourages them to feel...
a sense of responsibility to their family and friends. As is often the case in policy work, however, research into the effects of visitation on recidivism is associational (Durose et al., 2014), or depends on parametric assumptions (Bales and Mears, 2008; Cochran, 2014; Duwe and Clark, 2013) and/or matching (Cochran, 2014; Mears et al., 2012). These studies are often sensitive to model misspecification or violations of other strong assumptions.

Causal inference in such complex settings must be able to overcome three central challenges. First, experiments are rarely available, and ‘no-unmeasured-confounding’ assumptions may be violated. Thus researchers often resort to using IVs. A second difficulty arises when continuous rather than binary instruments are most suitable, since we are rarely interested in studying the effect of setting everyone to a specific instrument level in the non-binary case. Finally, policy settings are complex and often involve many covariates; we have little reason to believe that we know a parametric relationship between covariates, instrument, treatment and outcome. The method that we propose can satisfy all of these demands, providing robust inference in a complex setting.

In this paper, we present a case-study to illustrate these methods, looking at the effects of visitation on recidivism. In this example, we cannot intervene directly on the treatment—visitation—but we can implement policies that encourage or discourage visits. In addition, we know that visitation is confounded with recidivism rather than assigned randomly or pseudo-randomly, which forecloses many causal methods. Using our estimator, we can robustly and non-parametrically examine the effects of visitation on recidivism among those whose visitation we can affect.

The paper is organized as follows. In the remainder of this section, we describe the relevant statistical and criminological concepts. We give a primer on potential outcomes as well as IVs and describe common approaches in the causal literature to the parameterization, identification and estimation of causal parameters. In Section 2, we discuss the shift estimator proposed and its properties. We demonstrate these properties through a simulation study in Section 3 and apply the shift estimator to our case-study in Section 4. We conclude with a discussion and future work.

The programs that were used can be obtained from

https://rss.onlinelibrary.wiley.com/hub/journal/1467985x/series-a-datasets.

1.1. Language and notation of causal inference
Throughout, we shall rely on the Neyman–Rubin potential outcomes framework (Rubin, 1974). In this setting, one imagines a ‘potential outcome’: the outcome that we would have seen if we had implemented a certain treatment (or after a given exposure). We ask, for example, whether a prisoner would have committed a new crime if he had been visited and compare this with whether he would have committed a new crime if he were not visited. If there is a difference between these two potential outcomes, we say that there is a causal effect of visitation on his recidivism.

We shall also rely on IVs, which are a fundamental tool in observational causal inference and are particularly popular in econometrics (Acemoglu, 2002; Angrist et al., 1996; Heckman, 1990; Etilé and Sharma, 2015; Wooldridge, 1999). An instrument, denoted by $Z$, is in essence an un-confounded variable which affects the outcome only through the treatment. When the structure of the relationships between the instrument and other variables meets some additional criteria that are described later, the instrument acts almost as a proxy for randomization and makes causal inference possible without an experiment or in an experiment with non-compliance.
Throughout, we use the following notation. Our observed data are denoted $O = (Y, A, Z, X) \sim \text{IID } P$, where $Y$ denotes the outcome of interest (recidivism), $A$ denotes the binary treatment of interest (visitation), $Z$ denotes the instrument (distance in minutes to the inmate’s family home) and $X$ is a set of observed covariates (demographics, crime type, etc.).

In our approach, a step towards estimating the causal effect is first to estimate so-called ‘nuisance parameters’. We use the term ‘nuisance’ because, although they are important quantities that feed into our final estimate, we are not directly interested in them. The relevant nuisance quantities are denoted by

$$\eta = (\mu(Z, X), \lambda(Z, X), \pi(Z|X))$$

where $\mu(z, x) = E(Y|Z = z, X = x)$ is the outcome regression, $\lambda(z, x) = E(A|Z = z, X = x)$ is the treatment regression and $\pi(z|x) = P(Z = z|X = x)$ is the conditional instrument density (i.e. instrument propensity score function).

Finally, we follow conventions in the causal literature for potential outcomes and denote the potential treatment under $Z = z$ as $A^z$; similarly $Y^z$ is the potential outcome under $Z = z$, and $Y^A$ is the potential outcome when the treatment takes its potential value under $Z = z$.

1.2. Dynamic interventions

Causal effects are summaries of outcomes under some intervention on a treatment variable, where every intervention falls into a $2 \times 2$ classification of types: static versus dynamic, and deterministic versus stochastic. Static interventions change treatment in a way that does not depend on any other information (e.g. covariate or past treatment information); dynamic interventions are allowed to depend on such other information. Deterministic interventions change treatment in a uniform way that is not random; stochastic interventions incorporate some randomness.

Interest in dynamic interventions (both stochastic and deterministic) is growing, because these interventions are often more plausible in practice and because they allow identification under relaxed assumptions compared with deterministic intervention effects like the average treatment effect. For example, Munoz and Van Der Laan (2012) proposed a particular class of stochastic dynamic interventions for continuous treatments, based on an intervention that replaces the observed treatment with a random draw from the observed conditional treatment distribution (given covariates) shifted by some factor $\delta$. Mathematically, the observed treatment $A$ is replaced with a new $A^* = A + \delta$. Note that this intervention is stochastic since it is a random draw, and it is dynamic because it depends on the observed covariate and treatment.

Haneuse and Rotnitzky (2013) recast effects under such interventions as effects of ‘modified treatment policies’ and showed that they have a deterministic interpretation, in which the observed treatment is replaced not with a random draw but with the precise value $A^* = A + \delta$, which is completely known given the observed treatment. These deterministic ‘modified treatment policies’ ask what would happen if each unique observed treatment value was shifted by the quantity $\delta$. Haneuse and Rotnitzky (2013) also showed that effects under this deterministic interpretation can be identified under slightly weaker positivity assumptions. Young et al. (2014) studied similar effects of interventions that change treatment depending on how the observed value relates to a relevant threshold. For more examples of dynamic interventions we refer the reader to Cain et al. (2010), Dudik et al. (2014), Moore et al. (2012), Murphy (2003), Pearl (2009), Robins (2004), Robins et al. (2008), Taubman et al. (2009), Tian (2008), van der Laan and Petersen (2007) and Kennedy (2019).
Standard causal inference methods were built for estimating effects of static interventions and require each individual to have some non-zero chance of receiving each level of treatment. This is often called a ‘positivity’ or ‘experimental treatment assignment’ assumption. In reality, there are often some subjects who would never receive certain interventions; in the example that we discuss shortly, for example, it would never be possible to ensure that every prison inmate was exactly 20 min away from his or her family home. Some family homes are more than 20 min from every prison. Dynamic interventions can relax the strong positivity assumptions that are required to identify causal effects; for example in the intervention that was proposed by Haneuse and Rotnitzky (2013), one only needs each subject to have some chance of receiving the treatment level $\delta$ units away from their observed value, instead of more extreme levels that are further away from what was actually observed.

The aforementioned work on dynamic effects has expanded the field of causal inference to incorporate more feasible interventions. However, previous work has relied exclusively on a strong exchangeability or no-unmeasured-confounding assumption, i.e. that treatment is randomized given observed covariates. This may be reasonable in some (e.g. medical) settings but is often implausible in policy or other settings. In these cases, the assumptions that are required for an IVs strategy like that proposed here may be more plausible, since they do not require an unconfounded treatment. In our case, prison administrators have considerable control over the distance that prisoners will be from their next of kin. This can therefore be modified by a policy change and is more likely to be explained by measured variables, i.e. less likely to be confounded. In contrast, prison administrators have no direct control over the complex process determining whether and how prisoners receive visits, except in their ability to forbid it.

The primary goal of our paper is to fill the gap between policies of interest and the available statistical methods by incorporating dynamic interventions with IVs. This approach yields more relevant effects under weaker identifying assumptions and allows the researcher to use non-parametric models without sacrificing fast convergence rates.

2. Approach proposed

2.1. Parameterization of the estimand

The first step in the analysis of a causal effect is to decide what causal effect we actually want to learn. The local average treatment effect (LATE) with a binary $Z$ is a common example. In this case, we want to learn the effect of treatment $A$ on individuals who receive treatment if assigned $Z = 1$ but do not if $Z = 0$. When $Z$ is not binary, researchers might seek to estimate a version of the LATE among compliers who take treatment at some specific $Z = z$ but not at another $Z = z'$:

$$\psi_{\text{LATE}} = \mathbb{E}(Y^1 - Y^0 | A^z > A^{z'})$$.

This parameter presents three challenges that are not posed by the usual LATE parameter:

(a) substantively, it does not relate to an intuitive intervention—could one actually set $Z = z$ or $Z = z'$ for all subjects?,

(b) it is difficult to estimate because it is not pathwise differentiable and does not admit $\sqrt{n}$-consistent estimation non-parametrically and

(c) it requires stronger identifying assumptions than the usual LATE, because of the continuous IV.

Another approach to dealing with non-binary $Z$ is to take the classical IV approach, often a two-stage least squares or some variant, which estimates the LATE as a parameter in a regression. This model posits
This parametric approach is popular but relies on the strong assumption that the researcher can explicitly describe the relationships between possibly high dimensional covariates, instrument, treatment and outcome. We shall show that the use of influence-function-based estimation enables us to forgo these assumptions and still to estimate the effect of any number of dynamic interventions.

To define the causal effect of interest, we need to think about the intervention under study. In the ‘usual LATE’ case given in equation (1), the intervention is to set $Z = z$ for everyone and to compare that with setting $Z = z'$ for everyone. More generally, denote interventions by $h$: functions of $Z$ which may depend on $X$ and may be stochastic or deterministic. Some examples of such functions are as follows: ‘usual LATE’,

$$h_1(Z; X) = z, \quad h_2(Z; X) = z';$$

‘single shift’,

$$h_1(Z; X) = Z + \delta, \quad h_2(Z; X) = Z, \quad \delta \in \mathbb{R};$$

‘double shift’,

$$h_1(Z; X) = Z + \delta, \quad h_2(Z; X) = Z - \delta, \quad \delta \in \mathbb{R};$$

‘random shift’,

$$h_1(Z; X) = Z + \delta_1(X), \quad h_2(Z; X) = Z + \delta_2(X);$$

$$\delta_1 \sim \text{Unif}(0, |X|), \quad \delta_2 \sim \text{Unif}(-|X|, 0);$$

‘modified treatment policy’,

$$h_1(Z; X) = Z \mathbb{1}(Z \geq 30) + 30 \mathbb{1}(Z \leq 30), \quad h_2(Z; X) = Z.$$  

The effect of interest is then generally defined as

$$\psi_h = \mathbb{E}(Y^1 - Y^0 | A^{h_1(Z; X)} - A^{h_2(Z; X)} > 0).$$

Each intervention defines a complier population and will lead to estimates of treatment within that population. The single-shift intervention, for example, defines a complier population as those who receive treatment when their instrument increases by a set amount $\delta$ but do not at its current level. The double-shift complier population is those who receive treatment when their instrument is increased by $\delta$ but does not when the instrument is decreased by $\delta$. Any compliers in the single shift are therefore also included in the double-shift complier population for a given $\delta$.

The choice of $h$-functions depends on the intervention of interest. If we want to imagine a policy that encourages administrators to prioritize keeping prisoners close to home, we may want to use the double-shift estimand (5), which shifts the entire distribution of $Z$ (distance from home) by some amount $\delta$ in each direction from where we observe it currently.

In this paper, all results will be given for the treatment effect among compliers with respect to the double-shift intervention above, i.e.

$$\psi_{\text{shift}} = \mathbb{E}(Y^1 - Y^0 | A^{Z+\delta} > A^{Z-\delta})$$
when $Z$ has unbounded support on the whole real line; to the best of our knowledge, this kind of IV causal effect has not yet been proposed in the literature. In the case where the support of $Z$ is a bounded interval $[z_{\text{min}}, z_{\text{max}}]$, we consider a slightly modified version

$$
\psi^{*}_{\text{shift}} = \mathbb{E}(Y^{\text{1}} - Y^{\text{0}} | AZ + \delta_{l} > AZ - \delta_{u})
$$

(9)

for $\delta_{l} = \delta I(Z \leq z_{\text{max}} - \delta)$ and $\delta_{u} = \delta I(Z > z_{\text{min}} + \delta)$. The latter estimand simply considers compliers whose observed $Z$-values are within $\pm \delta$ of the bounds on the support, i.e. those subjects for whom a shift intervention on the IV is feasible.

Conceptually, these effects can be thought of in one of two ways. In the first, we imagine an intervention which takes each individual’s observed distance from their next of kin, $Z$, and increases or decreases it by an amount $\delta$. We then estimate the effect of this intervention on the recidivism of those whose visitation changes when the distance is changed. In the second conception, the intervention shifts the current distribution of distances and draws new values of $Z$ from that shifted distribution. We then examine the effect of visitation among those whose visitation changes between the original and shifted distributions of distance.

The second interpretation may be more attractive in a policy setting, because it roughly corresponds to a policy change that would direct administrators to prioritize keeping inmates closer to home or facilitating transportation.

It is relatively straightforward to move from the shift case to the more general $h$ case, although some subtleties arise, especially around positivity and the conditions that must be imposed on the $h$-functions. See Haneuse and Rotnitzky (2013) for an idea of what the general case will look like.

2.2. Identification

Having chosen a causal parameter to estimate, the fundamental problem of causal inference arises: the goal is to make inference about a functional of the distribution of potential outcomes, but these counterfactuals are only partially observed. The procedure to move from a functional of unknown potential outcome distributions to an estimable functional of the observational distribution $P$ is called identification.

We shall use the following base assumptions for identifying versions of the IV estimands that were mentioned in the previous subsection:

(a) consistency, $Z = z \Rightarrow A = A^{z}$ and $Z = z \Rightarrow Y = Y^{z}$;
(b) ignorability of $Z$, $Z \perp \perp (Y^{z}, A^{z})|X$;
(c) exclusion restriction, $Y^{z} = Y^{z'}$ for units with $A^{z} = A^{z'}$;
(d) monotonicity, $\mathbb{P}(A^{z} \geq A^{z'}) = 1$ for $z > z'$.

We shall also require instrumentation and positivity assumptions, which we shall describe shortly, and which depend on the particular form of the double-shift parameter. Assumptions (a)–(d) are common to the IV literature. It is important to note that, having made these assumptions and identified our parameter in the data, we have moved from a causal to a purely statistical question. The positivity assumption that is required is determined by the effect under study.

Note that the ignorability assumption is not saying that $Z$ is independent of $A$ given $X$. Clearly, that would create problems for our entire procedure because it would require that the instrument has no effect on the treatment given covariates. Instead, we assume that $Z$ is conditionally independent of the potential outcomes of $A$, which will hold if $Z$ is as good as randomized within covariate strata. It would be violated if, for example, inmates are moved closer to home for good behaviour, and good behaviour was related to the probability of recidivating.
Under assumptions (a)–(d), the LATE parameter with a binary \( Z \) can be identified as

\[
\psi_{\text{LATE}} = \mathbb{E}(Y^1 - Y^0 | A^1 > A^0) = \frac{\mathbb{E}\{\mu(1, X) - \mu(0, X)\}}{\mathbb{E}\{\lambda(1, X) - \lambda(0, X)\}}.
\]  

(10)

The positivity assumption that is needed in this case is

\[
P\{\epsilon < \pi(z|X) < 1 - \epsilon \} = 1.
\]  

(11)

This positivity assumption requires that each subject has positive probability of receiving all levels of \( z \). In the binary \( Z \) case, this means that both \( Z = 0 \) and \( Z = 1 \) must have positive probability. When \( Z \) is continuous or multivalued rather than binary, this assumption becomes stronger, because it says that each subject must have positive probability of receiving each value of \( z \). We shall refer to equation (11) as the ‘usual’ positivity requirement and we shall show that the estimator that we propose relaxes this assumption in important ways.

For the case of the double-shift interventions on \( Z \), we make the instrumentation assumption,

\[
P(A^Z + \delta > A^Z - \delta) > 0,
\]  

(12)

and the positivity assumption,

\[
P\{\epsilon < \pi(Z + \delta|X) < 1 - \epsilon \} = 1,
\]  

(13)

and

\[
P\{\epsilon < \pi(Z - \delta|X) < 1 - \epsilon \} = 1.
\]

Combined with monotonicity, this is similar to the no-zero-average causal effect of the instrument on treatment in Angrist et al. (1996) for a binary \( A \) extended to the case where \( Z \) is continuous.

This positivity assumption implies that, if we observe \( Z = z \), there must be a non-zero chance of observing both \( z \pm \delta \) (replacing \( \delta \) with \( \delta_u^* \) or \( \delta_l^* \) as needed). Note that this is weaker than assumption (11), because the instrument is not required to take on every value in its support: only values around its observed value. The indicator function ensures that the positivity requirement holds when \( z_{\max} \) and \( z_{\min} \) are known but, for convenience, we may drop the indicator function and assume that the support of \( Z \) is unbounded. For the proof of this (and all other proofs) see Appendix A.

Under assumptions (a)–(d), instrumentation (12) and positivity (13) the double-shift LATE parameters are identified as

\[
\psi_{\text{shift}} = \frac{\mathbb{E}\{\mu(Z + \delta, X) - \mu(Z - \delta, X)\}}{\mathbb{E}\{\lambda(Z + \delta, X) - \lambda(Z - \delta, X)\}},
\]  

(14)

\[
\psi_{\text{shift}}^* = \frac{\mathbb{E}\{\mu(Z + \delta_u^*, X) - \mu(Z - \delta_l^*, X)\}}{\mathbb{E}\{\lambda(Z + \delta_u^*, X) - \lambda(Z - \delta_l^*, X)\}}.
\]  

(15)

This is similar to the usual LATE, except that instead of obtaining predicted values under \( Z = 0 \) and \( Z = 1 \) we do so under shifted versions of the observed \( Z \).

As noted earlier, the positivity assumptions that are required for identifying \( \psi_{\text{shift}} \) are substantially weaker than those required for typical LATEs. For every set of covariates \( X \) with positive probability, the usual positivity assumption says that we must be able to observe any value \( z \in \text{Supp}(Z) \). This would imply, for example, that every prisoner must have a positive probability of being at every distance from home within the support of \( Z \).
In contrast, identifying the shift estimand requires an assumption about shifts around the observed values of $Z$, instead of the likelihood we could attain an arbitrary $z$ for all subjects. So each subject need not have a positive probability across a wide range of values $z$, but only for their own observed value (which is automatic), and for a value that is shifted away from their observed value.

We can illustrate this with a simple simulation. Imagine that our data-generating process is a mixture of truncated normal distributions, with means dependent on a single binary covariate $X$:

$$X \sim \text{Bern}(\frac{1}{2})$$

$$Z \sim \text{TruncNorm}\{\mu = 2X - 1, \sigma = 0.5, a = -X - 3(1 - X), b = 3X + 1(1 - X)\}.$$  

In our example, we could imagine that rural ($X = 0$) inmates are on average slightly further from home than urban ($X = 1$) inmates. Distance is continuous in each case, but there are hard maximum and minimum values for each population at the nearest and furthest prisons. It may be that we do not know these edges or how they depend on covariates, and therefore we cannot set up our estimates to account for this. Although in this setting we have focused on $\psi_{\text{shift}}$ these results hold for $\psi_{\text{shift}}^*$ by the same logic as presented here.

In the usual positivity setting, we require all $z$ within the support $[-3, 3]$ to have positive probability. However, we know that, for any subject with covariates $X = 1$, all $z < -1$ have zero probability. Likewise, for all subjects with $X = 0$, any value of $z > 1$ has zero probability.

In the shift case, however, the positivity assumption will be violated only if we draw observations where a shift up or down pushes $Z$ outside its support conditional on $X$. For example, if $X = 0$ and we draw a $Z$-value such that $Z < -3 + \delta$, we know that $Z - \delta < -3$ will have zero probability and will be a positivity violation. If we set $\delta = 0.1$, for example, the chances of drawing such an observation are extremely small.

We illustrate this in Fig. 1. The light boxes outline positivity violations in the usual setting. The narrow dark boxes represent an area where observations would violate positivity. In our simulations ($n = 5000; \delta = 0.1$), there were no violations of the shift parameter’s positivity assumption.

2.3. Estimation

The next step is to estimate the identified parameter (in our case, equations (14) or (15)). This is often accomplished in one of three ways: through regression-based estimators, weighting-based estimators or doubly robust estimators. Estimation has historically relied on parametric models like linear or logistic regression (Wooldridge, 1999). The two-stage least squares approach in equation (2) is one example of this approach. In the parametric setting, the relationship between covariates, instrument, treatment and outcome is assumed known (for example, it is assumed to be linear). This assumption may be too strong in many cases, especially in contexts with a large number of covariates or a complex system.

Regression-based estimators take parameters like that in equation (14) and simply plug in estimates of the regression functions. For this reason, we sometimes refer to them as ‘plug-in’ estimators. The regression-based estimator of the shift parameter is therefore

$$\hat{\psi}_{\text{reg}} = \frac{\hat{E}\{\hat{\mu}(Z + \delta, X) - \hat{\mu}(Z - \delta, X)\}}{\hat{E}\{\hat{\lambda}(Z + \delta, X) - \hat{\lambda}(Z - \delta, X)\}}.$$  

To estimate this, one might assume that the regression function $\mu(Z + \delta, X) = E(Y|Z + \delta, X)$ is a linear function of the covariates. $\hat{\mu}(Z + \delta, X)$ is then predicted by taking a linear regression of $Y$ on the covariates. Averaging the predictions from these regression functions gives the estimate
Fig. 1. Values of $Z$ or $z$ which violate positivity (low overlap on a single binary covariate $X$): dark narrow rectangles show the range of observed draws of $Z = z$ that would violate our positivity assumption; lighter long rectangles show the range of $Z$ that is a violation of usual positivity requirements.

of $\psi_{\text{shift}}$. Alternatively one could use non-parametric estimators of these regression functions, e.g. based on kernel smoothing or random forests.

A weighting-based approach is similar but, instead of modelling the conditional means of the outcome and treatment variables, the propensity scores—the chances of receiving treatment conditional on covariates—are modelled. This yields estimators of the form

$$\hat{\psi}_{\text{IPW}} = \frac{\mathbb{E}\left\{ \frac{\hat{\pi}(Z - \delta|X) - \hat{\pi}(Z + \delta|X)}{\hat{\pi}(Z|X)} Y \right\}}{\mathbb{E}\left\{ \frac{\hat{\pi}(Z - \delta|X) - \hat{\pi}(Z + \delta|X)}{\hat{\pi}(Z|X)} A \right\}}.$$ (19)

These estimators have good properties when the propensity scores (the $\pi$-terms) are known, as in an experiment. When they must be estimated, however, these models become vulnerable to model misspecification and, if estimated non-parametrically, will not attain $\sqrt{n}$-rates except under specific circumstances (e.g. for particular and undersmoothed nuisance estimators (Hirano et al., 2003; Hahn, 1998)).

Modern methods have shed some light on how to overcome some of these concerns. Notably, influence-function-based estimation frees researchers from many of the untenable parametric assumptions of earlier methods. We say that an estimator has a particular influence function when the estimator (minus its target) is equivalent to a sample average of the influence function, up to $o_P(1/\sqrt{n})$ error. A somewhat deeper and more nuanced notion of influence function concerns the influence function of a parameter; a parameter has a particular influence function if the parameter admits a von Mises or distributional Taylor series expansion, with the influence function acting as the derivative term in the expansion. For more details on influence functions, see Tsiatis (2006), Bickel et al. (1975), van der Laan and Robins (2003), van der Vaart (1998) or Kennedy (2017a).

In practice, influence-function-based estimators will often be of a form that blends the regression and weighting approaches. Our next result gives the influence functions for the two double-shift LATE parameters that we consider, which motivates appropriate estimators that
can attain fast convergence rates even when the nuisance estimators converge at slower non-parametric rates. The variance of the efficient influence function also acts as a benchmark for the smallest variance of any regular asymptotically linear estimator, as well as a local minimax lower bound on the mean-squared error (Bickel et al., 1975).

**Theorem 1.** Under the causal conditions and the positivity condition given in equation (13), the efficient influence function of the functional $\psi_{\text{shift}}$ defined in equation (9) is given by

$$\varphi(z; \eta, \psi) = \frac{\Xi(Y; \delta_u^*, \delta_l^*) - \psi_\Xi(A; \delta_u^*, -\delta_l^*)}{\mathbb{E}\{\lambda(Z + \delta_u^*, X) - \lambda(Z - \delta_l^*, X)\}}$$

(20)

where $\Xi(T; a, b) \equiv \xi(T; a) - \xi(T; b)$ for

$$\xi(T; \delta) = \xi_\mathbb{P}(T; X, Z, \delta) \equiv \frac{\pi(Z - \delta | X)}{\pi(Z | X)} \{T - \mathbb{E}(T | X, Z)\} + \mathbb{E}(T | X, Z + \delta)$$

the (uncentred) efficient influence function for the mean of arbitrary $T$ (here replace $T$ with either $A$ or $Y$) under a stochastic or dynamic intervention that sets $Z$ to $Z + \delta$.

Recall that $\delta_u^* = \delta l (Z \leq z_{\text{max}} - \delta)$, $\delta_l^* = \delta l (Z \geq z_{\text{min}} + \delta)$. When the support of $Z$ is unbounded, the efficient influence function for $\psi_{\text{shift}}$ is the same as above except with $\delta_u^*$ and $\delta_l^*$ replaced with simply $\delta$.

For the proof of theorem 1, see Appendix A.4.3.

A standard approach for constructing efficient estimators based on influence functions is to solve an estimating equation by using the estimated influence function as an estimating function, i.e. we solve for $\hat{\psi}$ in

$$\mathbb{P}_n \{\varphi(z; \hat{\eta}, \hat{\psi})\} = 0.$$  

We use the notation $\mathbb{P}_n$ to denote the empirical measure $\mathbb{P}_n f(X) = (1/n) \sum_{i=1}^n f(X_i)$.

On the basis of the formulation of $\varphi$ given in theorem 1, we propose the influence-function-based estimator:

$$\hat{\psi}_{\text{IF}} = \frac{\mathbb{P}_n \{\hat{\Xi}(Y; \delta_u^*, -\delta_l^*)\}}{\mathbb{P}_n \{\hat{\Xi}(A; \delta_u^*, -\delta_l^*)\}}$$  

(21)

for $\hat{\Xi}$ an estimate of $\Xi$. Similarly, for our double-shift parameter with unbounded support, the corresponding influence-function-based estimator is given by

$$\hat{\psi}_{\text{IF}} = \frac{\mathbb{P}_n \{\hat{\Xi}(Y; \delta, -\delta)\}}{\mathbb{P}_n \{\hat{\Xi}(A; \delta, -\delta)\}}.$$  

(22)

The remaining task is simply to pick a non-parametric method (or set of methods) to estimate the nuisance parameters. We shall show in the following section that non-parametric rates on the nuisance parameters are sufficient to achieve parametric rates on the causal estimand overall. In practice, we use SuperLearner, which is an ensemble learner, to estimate the $\mu$- and $\lambda$-parameters as well as possible. (We use the following algorithms: ‘SL.gam’, ‘SL.glm’, ‘SL.glm.interaction’, ‘SL.ranger’ and ‘SL.mean’.) To estimate the conditional density, we rely on random forests and a Gaussian kernel. We chose these tools for maximum flexibility, but the user is relatively free to pick different tools if they prefer. Appendix A.3 gives more detail on the strategy that we used and the conditions that need to be met in nuisance parameter estimation.

Note that another increasingly popular approach to non-parametric causal inference involves the use of targeted maximum likelihood estimation. This method similarly uses machine learning
to estimate nuisance parameters and relies on the influence function. It differs from estimating equation approaches by using an additional targeting step to create a plug-in estimator, ensuring that the estimator respects the bounds of the parameter space. The ratio structure of our parameter probably makes such a targeted maximum likelihood estimation slightly more complicated than that for an unconfounded average treatment effect; developing such an approach would be valuable future work.

In the next subsection we study the large sample properties of our proposed estimators and show that they can attain fast $\sqrt{n}$-rates of convergence, even when built from flexible non-parametric machine learning tools.

2.4. Asymptotic properties

In this section we highlight some of the notable properties of our proposed influence-function-based estimator of the shift parameter $\psi_{\text{shift}}$, namely that they are doubly robust and can converge at faster rates than the nuisance estimators on which they rely. This makes them less sensitive to the curse of dimensionality than simple plug-in inverse probability weighting or regression estimators. Although we focus on estimating $\psi_{\text{shift}}$, the same properties also hold for the LATE $\psi_{\text{LATE}}$ based on feasible IV interventions. Throughout, we let $\|f\| = \int f(z)^2 dP(z)$ denote the squared $L_2(P)$ norm.

Theorem 2. Dropping the distinction between the bounded and unbounded cases for ease of notation, suppose that the following conditions hold.

(a) The nuisance function and their estimators belong to a Donsker class.
(b) Strengthened positivity holds: $P\{\epsilon < \pi(Z \pm \delta|X)/\pi(Z|X) < C\} = 1$ for some $\epsilon > 0$ and $C < \infty$.

Then,

$$\hat{\psi}_{\text{IF}} - \psi_{\text{shift}} = O_p \left[ \frac{1}{\sqrt{n}} + \left\| \hat{\pi}(Z - \delta|X) - \frac{\pi(Z - \delta|X)}{\pi(Z|X)} \right\| \right] \times \left\{ \| \mu(Z + \delta, X) - \hat{\mu}(Z + \delta, X) \| + \| \lambda(Z + \delta, X) - \hat{\lambda}(Z + \delta, X) \| \right\}$$

$$+ \left\| \frac{\hat{\pi}(Z + \delta|X)}{\pi(Z|X)} - \frac{\pi(Z + \delta|X)}{\pi(Z|X)} \right\| \times \left\{ \| \mu(Z - \delta, X) - \hat{\mu}(Z - \delta, X) \| + \| \lambda(Z - \delta, X) - \hat{\lambda}(Z - \delta, X) \| \right\} .$$

(23)

If further

$$\left\| \frac{\hat{\pi}(Z - \delta|X)}{\pi(Z|X)} - \frac{\pi(Z - \delta|X)}{\pi(Z|X)} \right\| \left\{ \| \mu(Z + \delta, X) - \hat{\mu}(Z + \delta, X) \| + \| \lambda(Z + \delta, X) - \hat{\lambda}(Z + \delta, X) \| \right\}$$

$$+ \left\| \frac{\hat{\pi}(Z + \delta|X)}{\pi(Z|X)} - \frac{\pi(Z + \delta|X)}{\pi(Z|X)} \right\| \left\{ \| \mu(Z - \delta, X) - \hat{\mu}(Z - \delta, X) \| + \| \lambda(Z - \delta, X) - \hat{\lambda}(Z - \delta, X) \| \right\}$$

$$= o_p \left( \frac{1}{\sqrt{n}} \right)$$

(24)

then we have

$$\sqrt{n}(\hat{\psi}_{\text{IF}} - \psi_{\text{shift}}) \rightsquigarrow N\{0, \mathbb{E}(\varphi \varphi^T)\} .$$

(27)
This shows that the asymptotic variance of $\psi_{\text{shift}}$ is given by the variance of $\varphi$ and is therefore easy to estimate, allowing simple closed form asymptotically valid confidence intervals. Specifically we estimate the asymptotic variance simply by computing the empirical variance of the estimated influence function.

In addition, the estimator $\hat{\psi}_{\text{IF}}$ will converge at a $\sqrt{n}$-rate and will be optimally efficient if the product of the nuisance error rates converges to 0 faster than $n^{1/2}$, e.g. if they each converge at faster than $n^{1/4}$. This $n^{1/4}$-rate requirement does not require parametric model assumptions and in fact can hold under general smoothness, structural (e.g. generalized additive model) or sparsity assumptions, allowing a wide variety of modern machine learning tools to be used, while still providing classical inferential guarantees.

Finally, theorem 2 shows that the estimator $\hat{\psi}_{\text{IF}}$ is doubly robust, since, if either the $\hat{\pi}$-estimator or if the regression estimators $\hat{\lambda}, \hat{\mu}$ are estimated consistently (not necessarily both), then the estimator $\hat{\psi}_{\text{IF}}$ is consistent. Importantly, if we estimate the nuisance functions $\pi, \lambda, \mu$ on a separate independent sample, the Donsker assumption can be avoided entirely, and we need only consistency of the nuisance functions at any rate for expression (27) to hold.

Although theorem 2 is useful for pointwise inference at particular $\delta$-values, in many cases one may prefer confidence bands across a continuous range of shifts, i.e. we want a 95% confidence band that holds across the full range of $\delta$s under investigation simultaneously, rather than for a single $\delta$. These sorts of confidence bands are termed ‘uniform’, because they cover the true parameter uniformly across a range of measurements. For such function-valued parameters, the following theorem provides uniform confidence bands.

**Theorem 3.** Assume the conditions in theorem 2 and that

(a) $\varphi$ is Lipschitz in $\delta$,
(b) $\|\delta(\delta)/\sigma(\delta)\|_D - 1 = o_p(1),$
(c) $\sup_{\delta \in D} \left\| \frac{\hat{\pi}(Z - \delta |X) - \pi(Z - \delta |X)}{\hat{\pi}(Z |X) - \pi(Z |X)} \right\| \times \{ \|\mu(Z + \delta, X) - \hat{\mu}(Z + \delta, X)\| + \|\lambda(Z + \delta, X) - \hat{\lambda}(Z + \delta, X)\| \} = o_p(1/\sqrt{n})$

and
(d) $\sup_{\delta \in D} \left\| \frac{\hat{\pi}(Z + \delta |X) - \pi(Z + \delta |X)}{\hat{\pi}(Z |X) - \pi(Z |X)} \right\| \times \{ \|\mu(Z - \delta, X) - \hat{\mu}(Z - \delta, X)\| + \|\lambda(Z - \delta, X) - \hat{\lambda}(Z - \delta, X)\| \} = o_p(1/\sqrt{n}).$

Then,

$$\sup_{\delta \in D} \left[ \frac{\sqrt{n}\{\hat{\psi}_{\text{IF}}(\delta) - \psi_{\text{shift}}(\delta)\}}{\hat{\sigma}(\delta)} - \sqrt{n(\bar{P}_n - \bar{P})} \frac{\varphi(z; \eta, \delta)}{\sigma(\delta)} \right] = o_p(1/\sqrt{n}).$$  \hspace{1cm} (28)

This result shows that uniform confidence bands for our estimators across a continuous range of $\delta$-values are possible under assumptions that are not much stronger than the pointwise requirements. In particular, under these assumptions, the multiplier bootstrap can be used to construct uniform confidence bands (Kennedy, 2019).

### 3. Simulations

We run the following simulation, similar to that used by Kennedy (2019). Note that for these
estimators, however, it is preferred that we have an unbounded support for $Z$, although as we have seen, setting a $z_{\text{max}}$- and $z_{\text{min}}$-value within the functional enables us to adapt easily to the bounded support setting:

\[
(Y^0, X) \sim N(0, I_5), \\
Z|X, Y^0 \sim N(\alpha^T X, 2), \\
A = \mathbb{I}(Z \geq Y^0), \\
Y = Y^0 + \psi A. \tag{29}
\]

For these simulations, we set $\alpha = (1, 1, -1, -1)$ and our causal effect $\psi = 2$. This obeys all the identifying assumptions that we have made and does not involve particularly complex models or high dimensions.

To examine convergence rates, we use the true forms of our nuisance parameters and add noise of the form $\epsilon(Z)/N^{1/k}$ where we add error of the form $\epsilon(z) \sim N(z, 1)$ to the $\mu(z)$ and $\lambda(z)$ terms and $\epsilon \sim N(1, 1)$ to the $\pi$-terms. (This error form may not be obvious compared with something like $\epsilon \sim N(1, 1)$. This type of additive error will cancel out in the double-shift estimator and both the plug-in and influence-function-based estimators will perform as if there were no error. In contrast, taking asymptotic error like $\mu(Z) - \hat{\mu}(Z) \sim N(1, 1)$ and $\mu(Z + \delta) - \hat{\mu}(Z + \delta) \sim N(2, 1)$ is incoherent across all $Z$-values.) $k$ is the rate at which we estimate our parameters. So when $k = 2$, for example, we have $\hat{\mu} = \mu + \epsilon/N^{1/2}$, meaning that we are estimating $\mu$ at roughly a $\sqrt{n}$-rate.

We compare influence-function-based estimators and a plug-in estimator by using these perturbed nuisance parameters. The plug-in estimator is roughly equivalent to a two-stage least-squares-type estimator, adapted to estimate the same parameter as the shift estimator. This type of simulation has two fundamental advantages:

(a) it saves a large amount of computation and, more importantly,
(b) it enables us to control the error rates on the nuisance parameters exactly.

If we were to run a simulation based on existing two-stage least squares and influence-function-based estimation algorithms, we would not know for sure the error rates on the nuisance parameters.

We run 500 simulations with a sample size of 100, 1000, 5000 and 10000 for $\delta \in [0.5:4]$ and $k \in \{2, 3, 4, 6\}$. The results at 5000 are shown in Fig. 2; the others are given in Appendix A. As theory would dictate, both the plug-in and the influence-function-based estimators do well when our regression and propensity scores are estimated at $n^{1/2}$. However, we see that, when the rate is slower than that, the plug-in starts to experience major bias. The influence-function-based estimator, in contrast, maintains its performance until the rate is slower than $n^{1/4}$.

In Fig. 2, we show the empirical standard deviation over the simulations. In practice, we can construct confidence bands by using the multiplier bootstrap.

4. Application

We now apply the double-shift estimator to the question of whether visitation can affect recidivism risk. This intervention imagines shifting prisoners both nearer and further from their home, and estimates to what degree the change in their visitation due to this shift changes the chances of their being arrested for a new crime within 3 years of release. Summary statistics are given in Table 1. In general, prisoners who are visited are slightly less likely to recidivate (45% compared with 50%). They are also nearer to home on average (155 min compared with 195 min). However, they score more favourably on a number of other covariates which
are likely to be associated with reduced recidivism—the level-of-service inventory (revised) score, marital status, number of prior incarcerations, etc.—suggesting that there are unmeasured confounders contributing to the observed association between visitation and recidivism. (The level-of-service inventory (revised) score is a measure of the estimated risk of recidivism. See, for example, https://www.uscourts.gov/sites/default/files/713_4_0.pdf.) This reinforces the need for a robust causal estimation strategy.
Instrumental Variable Methods

The data set has been supplied by the Pennsylvania Department of Corrections. The data include information on all male prisoners who were released from the Pennsylvania Department of Corrections in 2008. They contain a unique identifier for each of 5749 prisoners, the facility in which they were last imprisoned before being released and the distance (in minutes) from each of the 25 prisons in the system to their next of kin.

Each of the prisons has on average 230 of the prisoners in the data set. The largest is the Chester prison, with 503 prisoners. The smallest is Greene with 101. On average, prisoners would move 139 min closer to home if they moved to their nearest prison.

We use the distance from next of kin to the prison as an instrument, because previous research has established that when prisoners are closer to home they are more likely to be visited (Jackson et al., 1997). This approach assumes that distance to home has no other effect on recidivism that is not accounted for by observed covariates—notably the effects of a particular prison’s characteristics (prison fixed effects). Both of these are strong assumptions that ought to be examined. That being said, they are also both assumptions that would be made in a classic IV setting.

The intervention is then to shift prisoners both nearer and further from home and to estimate the effects on recidivism among those for whom this shift changes their chance of being visited. This is a more plausible intervention than moving all prisoners to within a certain distance from home. Note, for example, that some prisoners’ next of kin are a certain minimum distance from all Pennsylvania prisons. This means that any hypothesized intervention which moves them closer than that minimum distance is impossible. We measure distance in estimated travel time by car. The shift estimator therefore alleviates some of the issues with the positivity assumption, which requires that all subjects have a positive probability of the theorized instrument value. The positivity assumption is not required by parametric models and is strong.

In our simulations, we used an unbounded $Z$. In this case, that no longer holds. No one can be a negative distance from prison, for example. We can easily incorporate this in our analysis, setting values for $z_{\text{max}}$ and $z_{\text{min}}$ such that $Z \in [z_{\text{min}}, z_{\text{max}}]$. In this data set, we know the maximum and minimum distances that prisoners can be from prison, and we set $z_{\text{max}}$ and $z_{\text{min}}$ accordingly.

The results of shifts between 20 and 120 min nearer or further from home are illustrated in Fig. 3. We run over five folds. To estimate the nuisance parameters, we use SuperLearner for the outcome regressions and ranger combined with a kernel for the conditional densities. See Appendix A.3 for details.

The estimated effect is an increase between 6.4% and 9.4% in the chance of recidivating when visitation is lost. In this case, the complier population at some $\delta$ are those who are currently being visited but would lose visitation if moved $\delta$ min further away. It also includes those who are not being visited but would be if moved $\delta$ min closer. At a 120-min shift level, the 95% pointwise and uniform confidence bands no longer include zero, implying that this is a statistically significant effect. The confidence bands narrow as the shifts grow larger, probably because of the increased complier population.

Another useful feature of the multiplier bootstrap is that we can test a hypothesis of effect homogeneity. To do so, we check whether we can draw a horizontal line across Fig. 3 which is fully contained within the confidence bounds. Here, we can do so at 0.25, for example, meaning that we cannot reject the hypothesis of effect homogeneity.

The complier population ranges from about 6% of prisoners at a 20-min increase, to 25% of prisoners at a 120-min increase.

These are the results for any visitation at all at the last location. We can similarly examine the effects of visitation among specific types of visitors. For example, we can specifically look at...
Fig. 3. Estimated effects of visitation induced by shifting driving distance on recidivism (confidence bands for the shift estimator): ○, estimate; ■, pointwise band type; □, uniform band type.

Fig. 4. Estimated effects for child visitation induced by a positive shift in driving distance on recidivism (confidence bands for the shift estimator): ○, estimate; ■, pointwise band type; □, uniform band type.
Table 2. Regression results

| Results for the following δ-values: | 20     | 40     | 60     | 90     | 120    |
|------------------------------------|--------|--------|--------|--------|--------|
| All visits                         | 0.064  | 0.094  | 0.08   | 0.074  | 0.094  |
| Uniform confidence interval        | (−0.118, 0.246) | (−0.097, 0.285) | (−0.11, 0.27) | (−0.044, 0.193) | (0.034, 0.154) |
| Compliers                          | 0.057  | 0.105  | 0.143  | 0.21   | 0.247  |
| Child visits                       | 0.248  | 0.269  | 0.25   | 0.192  | 0.269  |
| Uniform confidence interval        | (−0.251, 0.746) | (−0.238, 0.775) | (−0.264, 0.763) | (−0.095, 0.478) | (0.132, 0.406) |
| Compliers                          | 0.023  | 0.045  | 0.06   | 0.087  | 0.096  |

the effects of being visited by one’s children. Note that, although we do not restrict explicitly to inmates with children, the complier population should exclude anyone without children, since we necessarily cannot induce or end child visitation among the childless. The results of this analysis are given in Fig. 4.

These effects, again, are not significant at the 95% level until a 120-min shift. The point estimates are much higher than for visitation by anyone; they rise almost to 30%. The complier population, in contrast, is understandably much smaller. At the smallest shift level it is 2% of the total population and rises to just 10%. We again cannot reject homogeneity, since for example a 25% effect is within the confidence bands at all δ-levels.

These results are summarized in Table 2.

5. Discussion and conclusion

Our analyses provide evidence that increased visitation may reduce recidivism. The effects are potentially quite large and, given how difficult it is to reduce recidivism, should be encouraging to policy makers who have undertaken to keep prisoners near their families. Although the hypothesized intervention in this case was moving prisoners nearer to or further from home, the results suggest that any intervention which increases visitation is likely to reduce recidivism.

Preliminary results also suggest that there may be ways to target those inmates who are particularly sensitive to visitation. Especially in the analyses of child visitation, we saw that there may be effect heterogeneity. We hypothesize that those most affected are those with the strongest ties, but more analysis is needed to test that hypothesis.

There are some caveats to keep in mind. First, in our analysis, recidivism is measured as being rearrested for a new crime within 3 years. This is not a perfect measure of the act of recidivating. We seek to measure the effect of visitation on the overall probability of ever committing a new crime. Our measure, however, clearly overlooks anyone who is rearrested after 3 years. It also overlooks anyone who has committed a new crime, but has not been caught. Similarly, anyone who is falsely accused and arrested will be considered to have recidivated. Finally, those with a parole violation are not considered to have recidivated, but are also not free to recidivate after their rearrest (they have been excluded from the analysis).

Second, we use distance as an instrument. This distance figure was reported by the prisoners on entry into the prison system and may not be entirely accurate. Worse, it may be biased in a way that is associated with recidivism, e.g. if prisoners who are likely to commit more crimes tend to record a ‘home’ location far from their prison. This would be an ignorability restriction,
where there is some unobserved variable which links the distance metric and the probability of recidivating.

Third, it may be that prisoners are moved between prisons in a way that is associated with their chances of committing a new crime. If prisoners are moved closer to home because of good behaviour, those who are less likely to commit new crimes are also likely to be closer to home. This is another violation of the ignorability assumption.

There may additionally be a causal path between distance and recidivism that does not pass through the treatment or visitation. Perhaps prisoners feel more comfortable when closer to home and suffer less stress. Conversely, it may be that their criminal networks are more easily maintained close to home, making them more likely to recidivate. These are both examples of exclusion restriction violations.

Depending on the way that we hypothesize a change in distance, there may also be positivity violations. These occur when a prisoner cannot receive the instrument value that we assign them. Monotonicity, instrumentation and consistency are probably unlikely to be violated in this example.

The need for robust causal inference tools for observational data is pressing. In this paper, we have presented one such tool for new estimands based on dynamic interventions on IVs. These estimators enable researchers to investigate interventions that consider the effect of each individual having their observed instrument value perturbed, rather than setting the instrument to a specific level, which can require strong and impractical assumptions. Our tools also enable researchers to make use of non-parametric machine learning tools, while still allowing fast parametric rate inference.

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Appendix A

A.1. Comparison with two-stage least squares

Two-stage least squares assumes a parametric model and constant effects, among other things. Thus, by assumption, the estimate of the causal effect of visitation on recidivism is the average treatment effect. Here, we estimate that losing visitation causes the rate of recidivism to rise by about 10% for everyone in our population (Table 3). This is not significantly different from 0 according to this model. The 10% figure is not directly comparable with the estimates that we have found, since our estimates are local effects, and this is a global effect. That being said, at our largest complier population, we find about an 11% effect of losing visitation, so the two are not wildly different.

A.2. Additional simulations

We can see how the convergence behaves as we change the size of simulated data set (Fig. 5). At 100, there is a large amount of noise and the influence-function-based estimator is unstable. By 1000, however, it seems to be stabilized. At 10000 the estimates are very exact.

We can also look at these figures taking the standard deviation that we would use in practice, derived from the influence function (Fig. 6). These are generally though not necessarily, slightly narrower.

A.3. Estimation

Our influence-function-based estimator leaves the user a choice of how to estimate the nuisance parameters. The trade-off is generally between flexibility and precision. If the user is willing to assume that the nuisance functions take parametric forms, estimates may appear more precise (have narrower confidence bands). However, if the user has misspecified the parametric form, these confidence bands may be misleading.
Table 3. Two-stage least squares estimate of the effect of visitation on recidivism

|                      | Estimate | Standard error | t-value | Pr(>|t|) |
|----------------------|----------|----------------|---------|----------|
| Intercept            | 0.32     | 0.08           | 4.22    | 0.00     |
| No visits at last location | 0.10     | 0.06           | 1.53    | 0.13     |
| White                | -0.01    | 0.02           | -0.67   | 0.50     |
| Length of stay last location | 0.00     | 0.00           | 1.18    | 0.24     |
| County 3             | -0.05    | 0.02           | -3.39   | 0.00     |
| County 4             | 0.02     | 0.05           | 0.39    | 0.70     |
| County 5             | -0.00    | 0.05           | -0.10   | 0.92     |
| County 6             | -0.01    | 0.05           | -0.23   | 0.82     |
| County 7             | 0.01     | 0.08           | 0.08    | 0.94     |
| County 8             | -0.08    | 0.10           | -0.88   | 0.38     |
| Age                  | -0.01    | 0.00           | -10.66  | 0.00     |
| Urban                | 0.05     | 0.05           | 1.09    | 0.28     |
| Prior arrests        | 0.01     | 0.00           | 10.91   | 0.00     |
| Married              | 0.01     | 0.02           | 0.31    | 0.75     |
| Violent              | 0.01     | 0.02           | 0.92    | 0.36     |
| Level-of-service inventory (revised) score | 0.00 | 0.00 | 4.18 | 0.00 |
| High school graduate | 0.01     | 0.01           | 1.08    | 0.28     |
| Custody level        | 0.06     | 0.01           | 7.87    | 0.00     |
| Number of prior incarcerations | 0.04 | 0.01 | 5.63 | 0.00 |
| Number of total misconducts | 0.00 | 0.00 | 1.46 | 0.15 |

Fig. 5. Simulations at 100, 1000 and 10000 sample sizes: estimates by estimator type (●, influence function; ▲, plug-in), error rate and shift amount

In our results, we use SuperLearner, an ensemble learner, to estimate the regression functions. Our library includes the following learners: SL.gam, SL.glm, SL.glm.interaction, SL.ranger and SL.mean.

To estimate the conditional densities, we adopt a kernel approach. First, we model the mean $\mu_z = E(Z|X)$ by using ranger (a version of random forests). We find the variance as

$$\sigma^2_z = E\{Z - E(Z|X)\}^2.$$  (30)

We then estimate the conditional density as

$$f(z|x) = K\left(\frac{z - \hat{\mu}_z}{\hat{\sigma}_z^2}\right)$$  (31)

where $K$ is a Gaussian kernel.
This gives flexible but relatively imprecise estimates of the conditional density. If we replace ranger as the estimator of the mean function with glm, our final estimates of the confidence intervals narrow considerably. We repeat the analysis by using a glm estimate of the mean of the propensity score function and see in Fig. 7 that the point estimates are largely unchanged, but the confidence bands narrow.

The results of the analysis of a single-shift estimate by using glm are given in Fig. 8. In this case, the
uniform confidence bands do not always include 0, so the choice of nuisance parameter estimator can make a considerable difference to the conclusions that we draw.

Future work on these continuous IV estimators should incorporate refinement of the conditional density estimation.

In fact, work in the area of non-parametric conditional density estimation is advancing rapidly. Izbicki, Lee and colleagues (e.g. Izbicki et al. (2014) and Izbicki and Lee (2017)) propose converting conditional density estimation into a problem of many non-parametric regression functions. Their package, FlexCode, is an implementation of this work. Likewise, Ivan Díaz Muñoz and Mark van der Laan have developed methods for non-parametric conditional density estimation (Díaz Muñoz and van der Laan, 2011). This is implemented in their condensier package. In our case, these approaches resulted in unstable values for the conditional density estimates, but we believe that they hold great promise.

Another promising line of research is to target the ratio of density estimates directly. The laboratory of Masashi Sugiyama at the University of Tokyo has made considerable inroads in this area. Unfortunately, these are not implemented in R yet. Again, we believe that this line of research holds enormous potential.

A.4. Proofs

A.4.1. Proof of identification
We shall show that

\[
\psi_{\text{shift}} = \mathbb{E}(Y^{A=1} - Y^{A=0} | A^{Z+\delta} > A^{Z-\delta})
\]

\[
= \frac{\mathbb{E}\{\mathbb{E}(Y | Z + \delta, X) - \mathbb{E}(Y | Z - \delta, X)\}}{\mathbb{E}\{\mathbb{E}(A | Z + \delta, X) - \mathbb{E}(A | Z - \delta, X)\}}.
\]

This follows from (slightly modified forms of) the usual assumptions made in IV cases:

(a) consistency, \(Z = z \Rightarrow A = A^z\) and \(Z = z \Rightarrow Y = Y^z\);
A.4.2. General derivation of the influence function that takes the form given

Recall that we are using $Y^*$ to denote the potential outcome $Y_u^{Z*}$. The above assumptions enable us to make the following statements:

\[
\begin{align*}
\mathbb{E}(Y - Y^0 | A^{Z+\delta} > A^{Z-\delta}) &= \mathbb{E}\{Y_1 - Y_0 \mathbb{I}(A^{Z+\delta} > A^{Z-\delta})\}/\mathbb{P}(A^{Z+\delta} > A^{Z-\delta}), \\
\mathbb{P}(A^{Z+\delta} > A^{Z-\delta}) &= \mathbb{P}(A^{Z+\delta} = 1, A^{Z-\delta} = 0) \\
&= \mathbb{E}\{\mathbb{I}(A^{Z+\delta} = 1, A^{Z-\delta} = 0)\} \\
&= \mathbb{E}\{\mathbb{E}\{\mathbb{I}(A^{Z+\delta} = 1, A^{Z-\delta} = 0)|X\}\} \\
&= \mathbb{E}\{\mathbb{E}(A^{Z+\delta} - A^{Z-\delta}|X)\} \\
&= \mathbb{E}\{\mathbb{E}(A|Z + \delta, X) - \mathbb{E}(A|Z - \delta, X)\} \\
\mathbb{E}\{(Y - Y^0) \mathbb{I}(A^{Z+\delta} > A^{Z-\delta})\} &= \mathbb{E}\{\mathbb{E}(Y - Y^0) \mathbb{I}(A^{Z+\delta} > A^{Z-\delta})|X\} \\
&= \mathbb{E}\{\mathbb{E}(Y|Z + \delta, X) - \mathbb{E}(Y|Z - \delta, X)\}
\end{align*}
\]

where the final line follows because $Y_{Z=\delta} = Y_{Z=\delta, A} = Y_{A'}$. This gives the desired result

\[
\psi_{\text{shift}} = \frac{\mathbb{E}\{\mathbb{E}(Y|Z + \delta, X) - \mathbb{E}(Y|Z - \delta, X)\}}{\mathbb{E}\{\mathbb{E}(A|Z + \delta, X) - \mathbb{E}(A|Z - \delta, X)\}}
\]

A.4.2. General derivation of the influence function that takes the form given

Recall that our parameter of interest is

\[
\psi_{\text{shift}} = \frac{\mathbb{E}\{\mathbb{E}(Y|Z + \delta, X) - \mathbb{E}(Y|Z - \delta, X)\}}{\mathbb{E}\{\mathbb{E}(A|Z + \delta, X) - \mathbb{E}(A|Z - \delta, X)\}}
\]

\[
= \frac{\mathbb{E}\{\mathbb{E}(Y|Z + \delta, X) - \mathbb{E}(Y|Z - \delta, X)\}}{\mathbb{E}\{\mathbb{E}(A|Z + \delta, X) - \mathbb{E}(A|Z - \delta, X)\}}
\]

To find the efficient influence function of this quantity, first we find the influence function of each piece:

(a) \( \mathbb{E}(Y|Z + \delta, X) = \mu(Z + \delta, X) \);
(b) \( \mathbb{E}(Y|Z, X) = \mu(Z - \delta, X) \);
(c) \( \mathbb{E}(A|Z + \delta, X) = \lambda(Z + \delta, X) \);
(d) \( \mathbb{E}(A|Z, X) = \lambda(Z - \delta, X) \).

An influence function in the discrete case is a particular Gateaux derivative in the direction of a point mass. In the continuous case, it is an approximation of this derivative. Because of this, we can use the usual derivative rules when dealing with influence functions, notably

\[
\text{IF}\left(\frac{f}{g}\right) = \frac{\text{IF}(f)g - \text{IF}(g)f}{g^2}.
\]

Below we give the derivation of the influence function for quantity (1). The notation $Z_{\text{obs}}$ denotes the observed value of $Z$, and likewise for $X$:

\[
\text{IF}\{\mu(Z + \delta, X)\} = \text{IF}\left\{ \int \mathbb{E}(Y|X = x, Z = z + \delta) d\mathbb{P}(Z = z|X = x) d\mathbb{P}(X = x) \right\}
\]

\[
= \sum_x \sum_{z} \mathbb{I}(X_{\text{obs}} = x) \mathbb{I}(Z_{\text{obs}} = z + \delta) \frac{\mathbb{P}(Z = z + \delta|X = x) \mathbb{P}(X = x)}{\mathbb{P}(Z = z + \delta|X = x) \mathbb{P}(X = x)} \{Y - \mathbb{E}(Y|X = x, Z = z + \delta)\} \mathbb{P}(Z = z|X = x) \mathbb{P}(X = x)
\]
look like intervention would move them off the support. For example, if throughout is to define interventions which leave the instrument at observed levels if the specified inter-
or if we define our intervention functions in such a way to guarantee this. The approach that we take
The form above shows the necessity of the strengthened positivity condition

\[
\phi(z; \eta, \psi) = \frac{g \{ \Xi(Y; \delta, -\delta) - f \} - f \{ \Xi(A; \delta, -\delta) - g \}}{g^2} \\
= \frac{\Xi(Y; \delta, -\delta) - \psi \Xi(A; \delta, -\delta)}{g} \\
= \frac{\Xi(Y; \delta, -\delta) - \psi \Xi(A; \delta, -\delta)}{\mathbb{E}\{\lambda(Z + \delta, X) - \lambda(Z - \delta, X)\}}.
\]
Now to estimate the parameter we take
\[
\mathbb{P}_n \{ \psi(z; \hat{\delta}, \hat{\psi}) \} = 0
\]  
\[\Rightarrow \mathbb{P}_n \left[ \hat{\lambda}(Y; \delta, -\delta) - \hat{\psi} \hat{\lambda}(A; \delta, -\delta) \right] = 0\]  
\[\Rightarrow \hat{\psi}_W = \frac{\mathbb{P}_n \{ \hat{\lambda}(Y; \delta, -\delta) \}}{\mathbb{P}_n \{ \hat{\psi} \hat{\lambda}(A; \delta, -\delta) \}}.
\]  

We can now define the conditions under which our estimator is consistent and give its asymptotic standard errors. Note that, in this case, we are using \( \delta \) and \(-\delta\); below we give the proof in the bounded support case.

A.4.3. Derivation of the efficient influence function with bounded support

We define our intervention where we shift the instrument level \( Z \) up by some fixed amount \( \delta \) as long as \( Z + \delta \leq z_{\text{max}} \) and down by \( \delta \) as long as \( Z - \delta \geq z_{\text{min}} \). This gives a parameter:

\[
\psi^{*}_{\text{shift}} = \frac{\mathbb{E}[Y | Z + \delta(I(Z \leq z_{\text{max}} + \delta), X) - \mathbb{E}[Y | Z - \delta|I(Z \geq z_{\text{min}} - \delta), X]]}{\mathbb{E}[A | Z + \delta(I(Z \leq z_{\text{max}} + \delta), X) - \mathbb{E}[A | Z - \delta|I(Z \geq z_{\text{min}} - \delta), X]]}
\]  
\[= \frac{\mathbb{E}[\mu(Z + \delta | I(Z \leq M_1), X) - \mu(Z - \delta | I(Z \geq M_2), X)]}{\mathbb{E}[\lambda(Z + \delta | I(Z \leq M_1), X) - \lambda(Z - \delta | I(Z \geq M_2), X)]}
\]  
\[= \frac{f}{g}.
\]

Starting with the \( f \)-term, we can break the intervention down depending on where \( Z \) lies:

\[
f = \mathbb{E}[\{ \mu(Z + \delta) - \mu(Z - \delta) \} I(Z \leq M_1)] + \{ \mu(Z + \delta) - \mu(Z) \} I(Z \leq M_1, Z \leq M_2)
\]  
\[+ \{ \mu(Z) - \mu(Z - \delta) \} I(Z \geq M_1, Z \geq M_2)]
\[\Rightarrow \text{IF}(f) = \left[ \frac{\pi(Z - \delta)}{\pi(Z)} \{ Y - \mu(Z, X) \} + \mu(Z + \delta, X) - \mathbb{E}[\mu(Z + \delta, X)] - \frac{\pi(Z + \delta)}{\pi(Z)} \{ Y - \mu(Z, X) \} - \mu(Z - \delta, X) + \mathbb{E}[\mu(Z - \delta, X)] \right] I(Z \leq M_1)
\]  
\[+ \left[ \frac{\pi(Z - \delta)}{\pi(Z)} \{ Y - \mu(Z, X) \} + \mu(Z + \delta, X) - \mathbb{E}[\mu(Z + \delta, X)] - \{ Y - \mathbb{E}[\mu(Z)] \} \right] I(Z \leq M_1, Z \leq M_2)
\]  
\[+ \left[ Y - \mathbb{E}[\mu(Z, X)] - \frac{\pi(Z + \delta)}{\pi(Z)} \{ Y - \mu(Z, X) \} - \mu(Z - \delta, X) + \mathbb{E}[\mu(Z - \delta, X)] \right] I(Z \geq M_1, Z \geq M_2)
\]  
\[= [\xi(Y; \delta) - \xi(Y; -\delta) - \mathbb{E}[\mu(Z + \delta) - \mu(Z - \delta)]] I(Z \leq M_1, Z \geq M_2)
\]  
\[+ [\xi(Y; -\delta) - \xi(Y; 0) - \mathbb{E}[\mu(Z + \delta) - \mu(Z)]] I(Z \leq M_1, Z \leq M_2)
\]  
\[+ [\xi(Y; 0) - \xi(Y; -\delta) - \mathbb{E}[\mu(Z + \delta, X) - \mu(Z, X)]] I(Z \geq M_1, Z \geq M_2)
\]
\[ nents \text{ into smaller pieces, for example:} \]

We can treat the first ratio as a constant under the causal assumptions and break down the other components:

\[ \Xi \{ Y; \delta \| (Z \leq M_1), -\delta \| (Z \geq M_2) \} - f. \]

In the single-shift case, this reduces very considerably as follows:

\[ \xi \{ Y; \delta \| (Z \leq M_1) \} - \xi (Y; 0) = \xi (Y; \delta \| (Z \leq M_1)). \]  

(69)

The same holds for the \( g \)-term, replacing \( Y \) with \( A \). Now we can find the influence function of the parameter itself:

\[ \text{IF}(\psi) = ([\Xi \{ Y; \delta \| (Z \leq M_1), -\delta \| (Z \geq M_2) \} - f]g - [\Xi \{ A; \delta \| (Z \leq M_1), -\delta \| (Z \geq M_2) \} - g]f)/g^2 \]

(70)

This gives the influence-function-based estimator of

\[ \psi^*_{1\text{F}} = \frac{P_n \{ \Xi \{ Y; \delta \| (Z \leq M_1), -\delta \| (Z \geq M_2) \} \}}{P_n \{ \Xi \{ A; \delta \| (Z \leq M_1), -\delta \| (Z \geq M_2) \} \}}. \]

(72)

A.4.4. Proof of convergence (theorem 2)

In an abuse of notation, rewrite the estimator as

\[ \hat{\psi}_{1\text{F}} = \frac{P_n f(Y; \hat{\eta})}{P_n g(A; \hat{\eta})}. \]

(73)

And likewise the true parameter is written as

\[ \psi_{\text{shift}} = \frac{P f(Y; \eta)}{P g(A; \eta)}. \]

(74)

Now we take

\[ \psi_{1\text{F}} - \psi_{\text{shift}} = \frac{P_n f(Y; \hat{\eta})}{P_n g(A; \hat{\eta})} - \frac{P f(Y; \eta)}{P g(A; \eta)} \]

\[ = \frac{P g(A; \eta) P_n f(Y; \hat{\eta}) - P f(Y; \eta) P_n g(A; \hat{\eta})}{P_n g(A; \hat{\eta}) P g(A; \eta)} \]

\[ = \frac{P g(A; \eta) \{ P_n f(Y; \hat{\eta}) - P f(Y; \eta) \} - P f(Y; \eta) \{ P_n g(A; \hat{\eta}) - P g(A; \eta) \}}{P_n g(A; \hat{\eta}) P g(A; \eta)} \]

\[ = \frac{1}{P_n g(A; \hat{\eta})} \{ P_n f(Y; \hat{\eta}) - P f(Y; \eta) - \psi_{\text{shift}} \{ P_n g(A; \hat{\eta}) - P g(A; \hat{\eta}) \} \}. \]

(75)

We can treat the first ratio as a constant under the causal assumptions and break down the other components into smaller pieces, for example:

\[ P_n f(Y; \hat{\eta}) - P f(Y; \eta) = (P_n - P) \{ f(Y; \hat{\eta}) - f(Y; \eta) \} + (P_n - P) f(Y; \eta) - P \{ f(Y; \hat{\eta}) - f(Y; \eta) \}. \]

(76)

The first term on the right-hand side is \( o_p(1/\sqrt{n}) \) by van der Vaart (2000) as long as we assume Donsker classes or by estimating the nuisance parameters on a separate sample. The second term is normal by the central limit theorem. The third term needs to be bounded by iterated expectation as follows. First we take the expectation over \( X \) and \( Z \):

\[ P \{ f(Y; \hat{\eta}) - f(Y; \eta) \} = \mathbb{E} \left[ \frac{\hat{\pi}(Z - \delta|X)}{\hat{\pi}(Z|X)} \{ \mu(Z, X) - \hat{\mu}(Z, X) \} + \hat{\mu}(Z + \delta, X) \right] \]

(77)

\[ + \mathbb{E} \left[ \frac{\hat{\pi}(Z + \delta|X)}{\hat{\pi}(Z|X)} \{ \mu(Z, X) - \hat{\mu}(Z, X) \} + \hat{\mu}(Z - \delta, X) \right] \]

\[ - \mathbb{E} \{ \mu(Z + \delta, X) - \mu(Z - \delta, X) \}. \]
In this, we assume that either \( Z \) has infinite support if the intervention functions have been properly specified such that the terms where \( Z \pm \delta \) is outside the support of \( Z \) reduce to 0. Looking at the first line:

\[
\mathbb{E} \left[ \frac{\hat{\pi}(Z - \delta|X)}{\hat{\pi}(Z|X)} \{\mu(Z, X) - \hat{\mu}(Z, X)\} + \mathbb{E}\{\hat{\mu}(Z + \delta, X) - \mu(Z + \delta, X)\} \right].
\]

Integrate over \( z \)

\[
= \mathbb{E} \left[ \int \frac{\hat{\pi}(Z - \delta|X)}{\hat{\pi}(Z|X)} \{\mu(Z, X) - \hat{\mu}(Z, X)\} \pi(Z|X)dz + \int \{\hat{\mu}(Z + \delta, X) - \mu(Z + \delta, X)\} \pi(Z|X)dz \right].
\]

Change variables:

\[
= \mathbb{E} \left[ \int \frac{\hat{\pi}(Z|X)}{\hat{\pi}(Z + \delta|X)} \{\mu(Z + \delta, X) - \hat{\mu}(Z + \delta, X)\} \pi(Z + \delta|X)dz + \int \{\hat{\mu}(Z + \delta, X) - \mu(Z + \delta, X)\} \pi(Z + \delta|X)dz \right]
\]

\[
= \mathbb{E} \left[ \int \frac{\hat{\pi}(Z|X)}{\hat{\pi}(Z + \delta|X)} \{\mu(Z + \delta, X) - \hat{\mu}(Z + \delta, X)\} \pi(Z|X)dz + \int \{\hat{\mu}(Z + \delta, X) - \mu(Z + \delta, X)\} \pi(Z|X)dz \right]
\]

\[
= \mathbb{E} \left[ \frac{\hat{\pi}(Z|X)}{\hat{\pi}(Z + \delta|X)} \frac{\pi(Z + \delta|X) - \pi(Z|X)}{\pi(Z|X)} \{\mu(Z + \delta, X) - \hat{\mu}(Z + \delta, X)\} \pi(Z|X)dz \right]
\]

Note that we can choose how to do the change-of-variables step. We could equally say that

\[
\mathbb{E} \left[ \int \frac{\hat{\pi}(Z|X)}{\hat{\pi}(Z + \delta|X)} \{\mu(Z, X) - \hat{\mu}(Z, X)\} \pi(Z|X)dz + \int \{\hat{\mu}(Z + \delta, X) - \mu(Z + \delta, X)\} \pi(Z|X)dz \right]
\]

This will give the final result

\[
\mathbb{E} \left[ \frac{\hat{\pi}(Z - \delta|X) - \pi(Z - \delta|X)}{\hat{\pi}(Z|X) - \pi(Z|X)} \{\mu(Z, X) - \hat{\mu}(Z, X)\} \right].
\]

The first formulation requires \( \pi(Z|X)/\pi(Z + \delta|X) \) and its empirical estimate to be bounded away from zero and the second requires that the same condition holds for \( \pi(Z - \delta|X)/\pi(Z|X) \). Assuming these, we can bound this quantity by using the Cauchy–Schwartz inequality:

\[
\leq \left\| \frac{\hat{\pi}(Z|X) - \pi(Z|X)}{\hat{\pi}(Z + \delta|X) - \pi(Z + \delta|X)} \right\| \left\| \mu(Z + \delta, X) - \hat{\mu}(Z + \delta, X) \right\|,
\]

or, equivalently,

\[
\leq \left\| \frac{\hat{\pi}(Z - \delta|X) - \pi(Z - \delta|X)}{\hat{\pi}(Z|X) - \pi(Z|X)} \right\| \left\| \mu(Z, X) - \hat{\mu}(Z, X) \right\|.
\]

We can repeat this process with the \( (Z - \delta) \)-terms, and the same proof holds for the \( \lambda \)-terms, with the same requirement of positivity in the propensity score ratios.

To achieve consistent estimation of \( \hat{\psi} \), we therefore need

\[
\left\| \frac{\hat{\pi}(Z - \delta|X) - \pi(Z - \delta|X)}{\hat{\pi}(Z|X) - \pi(Z|X)} \right\| \left\| \mu(Z + \delta, X) - \hat{\mu}(Z + \delta, X) \right\| + \left\| \mu(Z - \delta, X) - \hat{\mu}(Z - \delta, X) \right\| \leq \sigma_p(1/\sqrt{n}).
\]

This is satisfied if all terms are estimated at faster than \( n^{1/4} \) or if the propensity score ratios (or both sets of regression functions) are estimated at \( n^{1/2} \).
A.4.5. Proof of uniform convergence (theorem 3)

We closely follow the proof in Kennedy (2017a); refer there for details. The goal is to provide a bound for the quantity

$$\sup_{\delta \in \mathcal{D}} \sqrt{n} \left\{ \frac{\hat{\psi}_{IF}(\delta) - \psi_{\text{shift}}(\delta)}{\hat{\sigma}(\delta)} \right\} - \sqrt{n(\mathbb{P}_n - \mathbb{P})} \frac{\varphi(z; \eta, \delta)}{\sigma(\delta)}. \tag{84}$$

Let the norm \(\|f\|_\mathcal{D}\) signify \(\sup_{f \in \mathcal{D}}\). First, if \(\varphi\) is Lipschitz and \(\|\hat{\sigma}/\sigma - 1\|_\mathcal{D} = o_p(1)\), we can rewrite the quantity above as

$$\left\| \frac{\sqrt{n} \{\hat{\psi}_{IF}(\delta) - \psi_{\text{shift}}(\delta)\}}{\sigma(\delta)} - \sqrt{n(\mathbb{P}_n - \mathbb{P})} \frac{\varphi(z; \eta, \delta)}{\sigma(\delta)} \right\|_\mathcal{D} + o_p(1). \tag{85}$$

Now we split the above quantity into one piece which accounts for estimation error, and one which captures the error in nuisance function estimation. We do this by splitting the data over \(K\) folds:

$$\frac{\sqrt{n} \{\hat{\psi}_{IF}(\delta) - \psi_{\text{shift}}(\delta)\}}{\sigma(\delta)} - \sqrt{n(\mathbb{P}_n - \mathbb{P})} \frac{\varphi(z; \eta, \delta)}{\sigma(\delta)} = \frac{\sqrt{n}}{K\sigma(\delta)} \sum_{k=1}^{K} \left\{ \frac{K}{n} \right\} \left( \mathbb{P}_n^k - \mathbb{P} \right) \left\{ \varphi(z; \hat{\eta}_k - \varphi(z; \eta, \delta)) - \mathbb{P} \{ \varphi(z; \hat{\eta}_k, \delta) - \varphi(z; \eta, \delta) \} \right\} \tag{86}$$

$$= B_{n,1}(\delta) + B_{n,2}(\delta).$$

The proof that is given in Kennedy (2017b) to bound \(\|B_{n,1}(\delta)\|_\mathcal{D}\) is completely general, so we do not give it here. It shows that

$$\|B_{n,1}(\delta)\|_\mathcal{D} = o_p(1). \tag{87}$$

The second task is to bound \(\|B_{n,2}(\delta)\|_\mathcal{D}\). We have already shown that

$$\mathbb{P}\{\Xi(Y, \delta, -\delta) - \hat{\Xi}(Y, \delta, -\delta)\} \lesssim \left\| \frac{\hat{\pi}(Z - \delta|X)}{\pi(Z|X)} - \frac{\pi(Z - \delta)}{\pi(Z|X)} \right\| \|\mu(Z + \delta, X) - \hat{\mu}(Z + \delta, X)\|$$

$$+ \left\| \frac{\hat{\pi}(Z + \delta|X)}{\pi(Z|X)} - \frac{\pi(Z + \delta)}{\pi(Z|X)} \right\| \|\mu(Z - \delta, X) - \hat{\mu}(Z - \delta, X)\| \tag{88}$$

in \(L_2(P)\) norm. The same holds replacing \(Y\) with \(A\) and \(\mu\)-terms with \(\lambda\)-terms.

Recall that our influence function \(\varphi\) is given by

$$\varphi = \Xi(Y; \delta, -\delta) - \psi_{\text{shift}}\Xi(A; \delta, -\delta). \tag{90}$$

Thus,

$$\mathbb{P}\{\varphi(z; \hat{\eta}, \delta) - \varphi(z; \eta, \delta)\} = \mathbb{P}\{\Xi(Y; \delta, -\delta) - \hat{\Xi}(Y; \delta, -\delta) - \psi_{\text{shift}}\Xi(A; \delta, -\delta) + \hat{\psi}_{IF}\hat{\Xi}(Y; \delta, -\delta)\} \tag{91}$$

$$\lesssim \left\| \frac{\hat{\pi}(Z - \delta|X)}{\pi(Z|X)} - \frac{\pi(Z - \delta)}{\pi(Z|X)} \right\| \left\{ \|\mu(Z + \delta, X) - \hat{\mu}(Z + \delta, X)\| + \|\lambda(Z + \delta, X) - \hat{\lambda}(Z + \delta, X)\| \right\}$$

$$+ \left\| \frac{\hat{\pi}(Z + \delta|X)}{\pi(Z|X)} - \frac{\pi(Z + \delta)}{\pi(Z|X)} \right\| \left\{ \|\mu(Z - \delta, X) - \hat{\mu}(Z - \delta, X)\| + \|\lambda(Z - \delta, X) - \hat{\lambda}(Z - \delta, X)\| \right\} \tag{92}$$

where the \(\psi_{\text{shift}}\)-terms and the denominator are constant and therefore drop out. If we make the assumption that the supremum of the quantity above across all \(\delta\) in some \(\mathcal{D}\) is \(o_p(1/\sqrt{n})\), the bound for \(B_{n,2}(\delta)\) is complete. Thus the entire quantity is bounded uniformly across \(\delta\).

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Supporting information
Additional ‘supporting information’ may be found in the on-line version of this article:
On-line appendix