Strength calculation of risers near the supports of reinforced concrete three-hinged frames based on the concrete plasticity theory

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Abstract. The necessity of estimation of residual strength of risers sections near supports of reinforced concrete three-hinged frames at their reuse after operation in aggressive environment conditions and damages presence is indicated. The method of calculating the elements strength under the shear on the joint action of the transverse and longitudinal forces based on the plasticity theory is elaborated. The variational method, the virtual velocities principle, discontinuous solutions are applied. The kinematically possible failure schemes, the realization of which depends on the values ratio of transverse and longitudinal forces are proposed. The application boundary of these schemes is defined. The permissible level of concrete class reduction and the degree of its integrity violation were established, under which the residual strength of serial three-hinged frames near the supports was provided.

1. Introduction
Three-hinged reinforced concrete frames are widespread in the construction industry due to their well known advantages: reduced structures weight up to 30% compared to traditional beam frames, reduction of the number of mounting elements up to 2.5 times and construction duration up to 15%. At present, the number of structures reuse has increased. Thus, the three-hinged frames of agricultural objects are dismantled and subsequently used for the construction of trade pavilions, indoor swimming pools, cinemas etc. Particular attention in such structures should be paid to eaves and supporting joints. Eaves joints are discussed in details in [1, 2], with respect to the supporting joints there is some ambiguity in assessing their strength. This is due to the fact that in the field tests of the serial frames without any defects and damages the failure of the risers sections near the supports is not observed due to the considerable reserves of their strength. However, during long-term structures exploitation of livestock in the aggressive environment conditions, there is a violation of the concrete structure and its strength reduction in these areas. In addition, near the supporting joints, the cross-sections height of the frame is reduced. This leads to the risk of their failure by shear. Meanwhile, the methods of calculating the strength of areas near supports according to current [3, 4] and previous norms [5] do not take into account the joint action of longitudinal and transverse forces due to the high level of compression forces at serial schemes and the reinforcement intensity. Therefore, it is of great current interest to develop a
method of calculating reinforced concrete elements under shear. Moving along the failure surface, which characterizes the shear, is possible only in the presence of directional plastic deformation. The above noted justifies the prospect of plasticity theory application for the strength problems solution [6 – 15].

2. Theoretical calculation the strength of the risers near the supports

To calculate the strength of reinforced concrete elements under shear into the joint action of transverse \( V \) and longitudinal \( N \) forces at the Yuri Kondratyuk National Technical University (PoltNTU), a variational method in the plasticity theory is applied [6, 7, 16 – 21]. Concrete is considered as a rigid-plastic body. Plastic deformations are considered localized in thin layers on the failure surface. Other areas are considered absolutely rigid (disks I and II in fig. 1 and 2). The mathematical apparatus of the ideal plasticity theory and discontinuous solutions are used. On the failure surface AB (fig. 1 and 2), velocity jumps in the normal and tangential directions are considered.

The strength calculation of the riser sections near the supports of the three-hinged frames is based on the strength problem solution of the truncated concrete wedge loaded with tangent \( V \) and normal \( N \) forces, as a model of this section. Two possible wedges failure cases are considered: near the facets of the right angle and near the facets of the blunt angle. The proposed kinematic failure schemes of truncated concrete wedges on the joint action of \( V \) and \( N \) are shown in fig. 1.

![Figure 1](image_url)

**Figure 1.** Kinematically possible failure scheme of concrete wedge under shear near facets of right angle – type I (a) and near facets of blunt angle – type II (b).

Kinematically possible failure scheme of the risers of the three-hinged frame near the supports, which corresponds to the failure case of the wedge in fig. 1,b, and the stress distribution in concrete, transverse and longitudinal reinforcement in the serial reinforcement scheme are presented in fig. 2.

The expression for determining the ultimate value of the force \( N \) through the unknown \( k = V / N \) and \( \tan \theta \) is realized by finding its minimum value during their variation

\[
N (1 + \tan \beta k) = mbh \left[ 2B \sqrt{1 + \frac{1}{4} \left( \frac{k + \tan \theta}{k \tan \theta - 1} \right)^2} - 1 \right] \frac{k \tan \theta - 1}{1 - \tan \alpha \tan \theta} + k_c f_c A_e \cos \alpha \times \\
\left[ 1 + \left( \frac{1}{2k_c} \frac{k \tan \alpha + 1}{k - \tan \alpha} \right)^2 \right] (k - \tan \alpha) + k_c f_y A_{sw,e} \left[ 1 + \left( \frac{k}{2k_c} \right)^2 \right],
\]

where \( \tan \beta = V / N \) – the ratio of the longitudinal \( N \) and the transverse \( V \) forces applied on the riser support; \( m = f_c - f_{ct} \), here \( f_c \) and \( f_{ct} \) – compressive and tensile strength of concrete;
B = \sqrt{\left[1 + \chi \left(1 - \chi^2\right) / 3 \right]} , \text{ here } \chi = f_{cv} / f_c; \ b, h - \text{ the width and height of support area of the frame riser near the support; } \alpha - \text{ the inclination angle of the inner facet (fig. 2,a); } k_c = 1 / 3 - \text{ for heavy concrete; } f_{vy}, A_v' \text{ and } f_{yw}, A_{sw}' - \text{ strength and area of longitudinal and transverse reinforcement respectively; } A_{w,0} = A_{w0} / s \left[\left( h - c'/\cos \alpha \right) \tan \theta / \left( 1 - \tan \alpha \tan \theta \right) - c_{sw} \right]; \ s - \text{ step of transverse reinforcement, } c', c_{sw} - \text{ the parameters shown in the fig. 2,b.}

**Figure 2.** Kinematically possible failure scheme of risers of the three-hinged frame near the supports: normal and tangential efforts in concrete \( N_c \) and \( T_c \) (a); longitudinal \( N_s \) and \( N_{sw} \), transverse \( V'_s \) and \( V'_sw \) reinforcement efforts (b).

The calculations result analysis according to the formula (1) allows to determine the different factors influence on the strength of the risers section near the frame supports under shear. Thus, it is found that reducing the reinforcement cross-section up to 25% does not significantly reduce the strength of the section. Simplified dependencies (2) and (3) are proposed to evaluate the influence of concrete strength characteristics:

- for failure case I

\[
N / f_c bh = \frac{2 \sqrt{(1 - \chi + \chi^2)} / 3 \sqrt{(k - \tan \gamma)^2 + 0.25(1 + k \tan \gamma)^2 - (1 - \chi)(k - \tan \gamma)}}{(1 - k \tan \beta) \tan \gamma};
\]

- for failure case II

\[
N / f_c bh = \frac{2 \sqrt{(1 - \chi + \chi^2)} / 3 \sqrt{(k - \tan \gamma)^2 + 0.25(1 + k \tan \gamma)^2 - (1 - \chi)(k - \tan \gamma)}}{(1 - k \tan \beta)(\tan \gamma - \tan \alpha)}.
\]

The transverse and equilibrium forces are determined from the equations \( V = N \tan \beta \) and \( P = \sqrt{V^2 + N^2} \).

The effect of concrete compressive strength the considered areas strength is approximate linear. So efforts are perceived by the risers at the supports are directly proportional to the magnitude \( f_c + 0.08 f_{cv} \).
3. Comparative analysis of theoretical and experimental results
The received theoretical results are compared with test data of truncated wedges, which were made in Poltava National Technical University. During the experiment, the wedge angle $\alpha$ was varied from 0 to 45° every 15° and the load application angle $\beta$ was varied from $-20^\circ$ (direction to the wedge right angle) to $40^\circ$ (direction to the wedge blunt angle) every 10°, and also the samples with $\beta = -5^\circ$ i 5° were tested. The load on the truncated facet was transferred through a steel plate glued to the concrete by means of a special device (fig. 3), which provided a load centered application. 28 samples were tested. Results comparison of theoretical strength by formulas (2) and (3) and experimental one is shown in fig. 4.

![Figure 3](image)

**Figure 3.** Test scheme of wedges (a), sample in press (b): 1 – device; 2 – sample.

![Figure 4](image)

**Figure 4.** Comparative analysis of theoretical and experimental strength of truncated wedges.

The following statistical characteristics were obtained: average ratio $P^{\text{cal}}/P^{\text{test}} = 0.98$, coefficient of variation 0.0962.
The failure pattern (fig. 5) confirms the accepted kinematic schemes.

Figure 5. The failure character of the truncated wedges in the direction of the transverse force \( V \) to the right angle (a), from the right angle at \( \beta \leq \alpha / 2.5 \) – I failure case (b), at \( \beta \geq \alpha / 2.5 \) – II failure case (c).

The results of the riser calculation near the supports of three-hinged frames for series [22] are shown in table 1.

Table 1. The calculation results of the efforts perceived by the concrete risers of the frame.

| \( f_{cd} \), MPa | \( f_{ctd} \), MPa | \( f_{cd} / f_{ctd} \) | \( b \), mm | \( h \), mm | \( \alpha^o \) | \( \beta^o \) | \( k \) | \( \tan(\gamma) \) | \( N_{rd} / f_{cd}bh \) | \( V_{rd} / f_{cd}bh \) |
|-----------------|-----------------|-----------------|----------|----------|-------------|-------------|-----|-------------|-----------------|-----------------|
| 7.7             | 0.66            | 0.086           | 180      | 400      | 8.5         | 40          | 8.63| 1.43        | 0.407            | 0.342            |

The data analysis of the table 1, dependence (1) and the loads values, on which serial three-hinged reinforced concrete frames are designed with 21 m span, taking into account changes in the regulatory framework, indicate the provision of the risers strength in the areas near the supports while reducing the class of concrete to C12/15 and integrity violation of the support units to 25%.

4. Conclusions

The calculation method of elements under shear for the joint action of transverse \( V \) and longitudinal \( N \) forces was developed based on the concrete plasticity theory, which allows to estimate the strength of risers section near supports of reinforced concrete three-hinged frames.

The considered kinematically possible failure schemes of concrete wedges as models of the frames supporting sections are confirmed by experimental studies. The implementation boundary of the proposed kinematic schemes is determined by the ratio of the values of \( V \) and \( N \) and corresponds to the inclination angle of their equivalent \( \beta = \alpha / 2.5 \).

The possible reduction of the reinforcement intensity of the risers near frame supports as a result of reinforcement corrosion damages during their operation does not significantly affect their shear strength. The relationship between the concrete compressive strength \( f_{cd} \) and the strength of the riser near the supports for engineering calculations can be assumed linear.

The strength of risers of serial three-hinged frames with 21 m span on the supporting sections while reducing the concrete strength characteristics and reducing the cross-sectional area of concrete and reinforcement of these sections up to 25% is secured. In the case of an increase of the damage degree, strengthening measures must be taken.

The use of the obtained results gives the opportunity to improve the structural solutions of reinforced concrete three-hinged frames.
References

[1] Pershakov V M 2007 Frame buildings made of three-hinged reinforced concrete frames (Kiev: Knyzhkove vydavnyttso NAU)

[2] Pershakov V M 2011 Experimental studies of three-hinged reinforced concrete frames. Construction of Ukraine 1 17-22

[3] DBN B.2.6-98: 2009 2011 Construction of buildings and structures. Concrete and reinforced concrete structures. Substantive provisions (Kyiv: Minregionstroy of Ukraine)

[4] DSTU B V.2.6-156: 2010 2011 Concrete and reinforced structures of heavy concrete. Design rules (Kyiv: Minregionstroy of Ukraine, SE "Ukrbudinform")

[5] SNiP 2.03.01-84* 1989 Concrete and reinforced concrete structures (Moscow)

[6] Kolmogorov VL 1986 Metal forming mechanics (Moscow)

[7] Mitrofanov VP 2006 The theory of perfect plasticity as the elementary mechanic pseudo-plastic ultimate state of concrete: bases, imitations, practical aspects Proc. of the 2nd fib. Congress (Naples) 7-6

[8] Mineola LJ 2008 Plasticity theory (Dover)

[9] Nielsen MP and Hoang LC 2011 Limit Analysis and Concrete Plasticity (CRC Press. Taylor & Francis Group)

[10] Ivlev DD 2001 Mechanics of Plastic Media. T. 1. Theory of Ideal Plasticity (Moscow: Fizmatlit)

[11] Ebobisse F and Reddy BD 2004 Some mathematical problems in perfect plasticity Comput. Methods Appl. Mech. Engrg. 193 5071-5094

[12] Sorensen JH, Hoang LC, Olesen JF and Fischer G 2017 Test and analysis of a new ductile shear connection design for RC shear walls Structural Concrete 18 189-204

[13] Jorgensen HB, Hoang LC 2015 Load carrying capacity of keyed joints reinforced with high strength wire rope loops 2015 Proc. of fib symposium: Concrete - Innovation and Design (Copenhagen) 13

[14] Braestrup Mikael W 2019 Concrete plasticity – a historical perspective. Proc. of the fib Symposium: Concrete – Innovations in Materials, Design and Structures (Poland: Krakow) 29-48

[15] Jorgensen H B, Hoang L C and Hagsten L G 2017 Strength of precast concrete shear joints reinforced with high-strength wire ropes Institution of Civil Engineers Proceedings Structures and Buildings 170(3) 168-179

[16] Pohribnyi V, Dovzhenko O and Karabash L 2017 The design of concrete elements strength under local compression based on the variational method in the plasticity theory MATEC Web of Conferences 116 02026

[17] Dovzhenko OA, Pohribnyi VV and Karabash LV 2018 Effective keyed connections of hollow-core floor slabs with walls in modern large-panel house building Science & technique. 17(2) 146-156

[18] Dovzhenko O, Pohribnyi V, Pents V and Mariukha D 2018 Bearing capacity calculation of reinforced concrete corbels under the shear action MATEC Web Conferences 230 02005

[19] Dovzhenko OO, Pohribnyi VV and Yurko IA Concrete and Reinforced Concrete Strength under Action of Shear, Crushing and Punching Shear 2018 IOP Conf. Ser.: Mater. Sci. Eng. 463 (1) 022026

[20] Pohribnyi V, Dovzhenko O, Kuznietsova I and Usenko D 2018 The improved technique for calculating the concrete elements strength under local compression Web of Conferences 230 02025

[21] Dovzhenko O, Pogrebyni V and Yurko I 2018 Shear Failure Form Realization in concrete News NAS RK. Series of geology and technical science 2 (428) 212-219

[22] Serie 1.822.1-2/82 1984 Reinforced concrete frames for single span agricultural buildings with a slope of the roof 1:4. Issue 1. Frames of rectangular cross-section with span 12, 18 and 21 m. (Moscow: GiproNIselkhoz. NIIZhB. TsNIEPselstroy)