Error-heralded generation and self-assisted complete analysis of two-photon hyperentangled Bell states assisted by single-sided quantum-dot-cavity systems

Yan-yan Zheng, Lei-xia Liang, and Mei Zhang
Department of Physics, Applied Optics Beijing Area Major Laboratory, Beijing Normal University, Beijing 100875, China

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Hyperentangled Bell-state analysis (HBSA) is critical for high-capacity quantum communication. Here we design two effective schemes for error-heralded deterministic generation and self-assisted complete analysis of hyperentangled Bell states for two-photon systems in both the polarization and spatial-mode degrees of freedom, assisted by single-sided quantum-dot-cavity systems. We construct an error-heralded block with a singly charged quantum dot inside a single-sided optical microcavity, two circular polarization beam splitters, one half-wave plate, and one single-photon detector, in which the errors due to imperfect interactions between photons and quantum dot systems can be heralded. With this error-heralded block, the fidelity of our two schemes for hyperentangled Bell-state generation and complete HBSA can reach unit one. What interesting is that by using the measured spatial-mode state to assist the analysis of the polarization state, our complete HBSA scheme works in a self-assisted way, which greatly simplifies the analysis process and largely relaxes the requirements on nonlinearities. These advantages make our schemes much easier to implement experimentally, and have more practical applications in long-distance high-capacity quantum communication.

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I. INTRODUCTION

Quantum entanglement has received much attention in recent years due to its important application in quantum information processing [1]. It is a key quantum resource in quantum communication, such as quantum teleportation [2], quantum key distribution [3–5], quantum dense coding [6, 7], quantum secret sharing [8], and quantum secure direct communication [9–14]. In quantum communication, information is encoded on different quantum states, for example the four Bell states of a two-photon entangled system. In order to read the quantum information, we must completely determine the different quantum states. So, Bell-state analysis (BSA) is an essential technology in quantum communication schemes, and different types of Bell state analysis schemes are proposed. For instance, in 1999, Vaidman et al. [15] and Lütkenhaus et al. [16] proposed two BSA schemes for teleportation with only linear optical elements. However, the linear BSA schemes are impossible to deterministically and completely distinguish the four Bell states [17–19]. In recent years, researchers have proposed some interesting ways for achieving complete BSA with the use of nonlinear media. In 2005, Barrett et al. [20] proposed a Bell state analyzer using weak nonlinearities. In 2015, Sheng et al. [21] proposed a more optimal proposal by using cross-Kerr nonlinearity, which can realize nearly complete logic Bell-state analysis.

Recently, hyperentanglement [22–29], which means particles are simultaneously entangled in more than one degree of freedom (DOF), has attracted much attention [30–49]. Up to now, there are some theoretical and experimental schemes for the generation of hyperentangled states proposed in optical systems [22–27], such as polarization-momentum DOFs [24], polarization-orbital-angular momentum DOFs [25], and multipath DOFs [26]. In experiment, hyperentanglement can be generated by the combination of the techniques used for creating entanglement in a single DOF [27]. For instance, Vallone et al. [28] demonstrated the generation of a two photon six-qubit hyperentangled state in three DOFs in experiment. Two-photon polarization-spatial hyperentangled state can be produced by spontaneous parametric down-conversion source with the β barium borate crystal in 2009. Compared to the entanglement in a single DOF, hyperentanglement enlarges the Hilbert space for encoding and has numerous applications in high-capacity quantum information processing. For example, it can largely improve the quantum channel capacity, beat the limit of linear photonic superdense coding [51], accomplish the deterministic entanglement purification [51, 52], quantum error-correcting [53], and construct quantum repeaters [54]. In addition, hyperentanglement is also possible to assist the analysis of Bell state [57, 61]. In 1998, Kwiat and Weinfurter [57] first introduced the method of the complete BSA assisted by hyperentanglement. In 2003, Walborn et al. [58] proposed a more simple complete BSA scheme using only linear optical elements. In 2006, Schuck et al. [59] completely distinguished four polarization states in
experiment assisted by polarization-time-bin hyperentanglement. In 2007, Barbieri et al. accomplished another BSA experiment using polarization-momentum hyperentanglement. However, with only linear optics, one cannot distinguish the 16 Bell states completely. The discrimination of the hyper-entangled Bell states has become a new task. In 2010, Sheng et al. firstly presented a way to distinguish the 16 hyperentangled Bell states completely with cross-Kerr nonlinearity, and they also discussed its application in quantum hyper teleportation and hyperentanglement swapping. Subsequently, some interesting hyperentangled Bell-state analysis (HBSA) schemes were proposed. For example, in 2012, Ren et al. proposed a complete HBSA scheme for 16 hyperentangled Bell states in both polarization and momentum DOFs by using one-sided quantum-dot (QD) cavity system, and Wang et al. proposed the schemes to generate and completely analyze the 16 hyperentangled Bell states of two-photon systems assisted by double-sided QD-cavity systems. In 2015, Liu et al. proposed two schemes for hyperentangled Bell-state generation (HBSG) and HBSA assisted by nitrogen-vacancy (NV) centers in resonators, which can realize non-destructive discrimination of 16 two-photon hyperentangled states by using four NV systems. In 2016, Wang et al. gave the way for error-detected generation and complete analysis of hyperentangled Bell states for photon systems assisted by double-sided QD-cavity systems, and Li et al. presented a simplified complete HBSA scheme, which use spatial information to assist polarization state differentiation. As the Kerr medium is difficult to achieve experimentally, solid state system with giant nonlinearity is another promising candidate for quantum information processing.

The electron spin in QD is another suitable candidate for solid-state quantum information processing. The electron-spin coherence time of a charged QD can be maintained for more than 3 s and the electron spin-relaxation time can reach (ns). Moreover, it is easy to embed the QDs in the cavities and they can be manipulated fast and initialized easily. Based on these advantages, many schemes for quantum information processing have been proposed using QD-cavity systems, including QD in a double-sided cavity system, and QD in a single-sided cavity system, such as entanglement beam splitter, entanglement purification and concentration for electron spins, universal quantum gates, hyper-parallel photonic quantum computation, complete HBSA, hyperentanglement purification, and the heralded quantum repeater for a quantum communication network with QD in double-sided cavity system. The schemes based on single-sided cavity system are presented successively, such as entangler, complete HBSA, hyperentanglement concentration, hyper-parallel photonic quantum computation, and error-rejecting quantum computation. In the ideal condition, the fidelities and the efficiencies of these schemes can reach 100%. Considering the realistic condition, their fidelities and the efficiencies would be affected by the parameters of the QD-cavity systems more or less. In 2009, An et al. presented a quantum information processing protocol by using a single photon interacted with low-Q single-sided cavity system. In 2011, Kastoryno et al. proposed a scheme for the generation of a maximally entangled state using the decay of the cavity to improve the fidelity, and thus herald the error. In 2012, Li et al. proposed an improved scheme for atom-photon entangling gates, which has robust fidelity. In 2016, Li et al. and Wang et al. proposed two error-detected schemes with QD in double-sided cavity system for quantum repeater and HBSA, respectively.

In this paper, we firstly constructed an error-heralded block with QD in a single-sided cavity system and then proposed two schemes for deterministic HBSG and complete HBSA with this block. As the errors caused by the imperfect interaction between photons and QD-cavity systems can be heralded, our two schemes have a high fidelity of unit one. In our error-heralded HBSG scheme, the deterministic generation of hyperentangled two-photon systems can be performed by repeating until success. Moreover, our schemes work in both the weak coupling regime and the strong coupling regime. By using the measured spatial-mode state to assist the analysis of the polarization state, our complete HBSA scheme works in a self-assisted way and uses only two nonlinear QDs, and it can also realize the function of error avoidance, which greatly reduces the resource loss. Compared to a double-sided cavity system, a single-sided cavity system does not require the balance between the upper and lower sides, and is easier to construct experimentally. These features make our schemes more useful in high-capacity quantum communication with hyperentanglement in the future.

II. INTERACTION BETWEEN A CIRCULARLY POLARIZED LIGHT AND A SINGLY CHARGED QD IN A SINGLE-SIDED MICROCAVITY

The singly charged quantum dot (e.g., a self-assembled In(Ga)As QD or a GaAs interface QD) embedded in the center of a single-sided microcavity (the top distributed Bragg reflectors are 100% reflective and the bottom distributed Bragg reflectors are partially reflective) is shown in Fig. 1(a).
In the cold cavity case ($g = 0$), the reflection coefficient is
and an electron state with the spin projections

\[
\omega \sigma_r \mathcal{O}_r, \quad \omega \sigma_r \mathcal{O}_s \mathcal{O}_r
\]

The relative energy of $\hat{\omega}_r \mathcal{O}_r$ of $\mathcal{O}_r$ is in the state $\hat{\omega}_r \mathcal{O}_r$ and $\hat{\omega}_r \mathcal{O}_r$.

Criteria of spin-dependent transition is based on the Pauli’s exclusion principle [89]. That is, if the excess electron is in the state $| \uparrow \rangle$, the negatively charged exciton $X^-$ in the state $| \uparrow \uparrow \rangle$ is created by absorbing the left circularly polarized light $| L \rangle$. If the excess electron is in the state $| \downarrow \rangle$, the right circularly polarized light $| R \rangle$ can be absorbed to create the negatively charged exciton $X^-$ in the state $| \downarrow \downarrow \rangle$. Here, $| \uparrow \rangle$ ($| \downarrow \rangle$) and $| \uparrow \rangle$ ($| \downarrow \rangle$) represent a hole state and an electron state with the spin projections $| + \rangle$ ($| - \rangle$) and $| + \rangle$ ($| - \rangle$), respectively.

In the interaction picture [90], the Heisenberg equations of cavity field operator $\hat{a}$ and exciton $X^-$ operator $\hat{\sigma}_-$ are described as

\[
\begin{align*}
\frac{d\hat{a}}{dt} &= -i(\omega_c - \omega)\hat{a} + \frac{\kappa_s}{2}\hat{a} - g\hat{\sigma}_- - \sqrt{\kappa}\hat{a}_\text{in}, \\
\frac{d\hat{\sigma}_-}{dt} &= -i(\omega_{X^-} - \omega)\hat{\sigma}_- + \frac{\gamma}{2}\hat{\sigma}_- - g\hat{\sigma}_z\hat{a}, \\
\hat{a}_\text{out} &= \hat{a}_\text{in} + \sqrt{\kappa}\hat{a}.
\end{align*}
\]

where $\omega$, $\omega_c$ and $\omega_{X^-}$ denote the frequencies of the input photon, the columnar microcavity and the $X^-$ transition, respectively. $g$ is the coupling intensity of the quantum dot and microcavity. $\gamma/2$ and $\kappa/2$ represent the decay rates of $X^-$ and microcavity field mode. $\kappa_s/2$ is the microcavity leakage rate, $\hat{a}_\text{in}$ and $\hat{a}_\text{out}$ represent input and output operators, respectively.

In the weak excitation approximation ($\langle \hat{\sigma}_z \rangle = -1$ and $\hat{\sigma}_z \hat{a} = -\hat{a}$), the solution of the reflection coefficient [81] of the single-sided quantum-dot-cavity system can be obtained as

\[
r(\omega) = 1 - \frac{\kappa [i(\omega_{X^-} - \omega) + \frac{\gamma}{2}]}{[i(\omega_{X^-} - \omega) + \frac{\gamma}{2}] [i(\omega_c - \omega) + \frac{\kappa_s}{2} + \frac{\gamma}{2}]} + g^2.
\]

In the cold cavity case ($g = 0$), the reflection coefficient is

\[
r_0(\omega) = \frac{i(\omega_c - \omega) - \frac{\kappa_s}{2} + \frac{\gamma}{2}}{i(\omega_c - \omega) + \frac{\kappa_s}{2} + \frac{\gamma}{2}}.
\]

Then the interaction of the photon and the single-sided quantum-dot-cavity system can be described by the reflection operator $\hat{r}(\omega)$. Here

\[
\hat{r}(\omega) = |r_0(\omega)| e^{i\phi_0} \langle | R \rangle \langle \downarrow \rangle \rangle | \uparrow \rangle + |r_h(\omega)| e^{i\phi_h} \langle | L \rangle \langle \uparrow \rangle \rangle | \downarrow \rangle + |r_l(\omega)| e^{i\phi_l} \langle | R \rangle \langle \uparrow \rangle \rangle | \downarrow \rangle,
\]

where $\phi_0 = \arg[r_0(\omega)]$ and $\phi_h = \arg[r_h(\omega)]$ denote the phase shifts of reflection light in the cold cavity and hot cavity, respectively. If the state of the injected electron spin is $| \uparrow \rangle$, the left circular light $| L \rangle$ gets a reflection phase shift $\phi_h$, and the right circular light $| R \rangle$ has a reflection phase shift $\phi_0$. If the injected electron spin is in the state $| \downarrow \rangle$, the right circular light $| R \rangle$ has a reflection phase shift $\phi_h$, and the left circular light $| L \rangle$ gets a reflection phase shift $\phi_0$. By adjusting $\omega$ and $\omega_c$ ($\omega - \omega_c = 0$), $\phi_h - \phi_0 = \pi$ can be obtained. In the condition $\omega = \omega_{X^-} = \omega_c$ and $\kappa \gg \kappa_s$, the

FIG. 1: (a) Schematic diagram of a singly charged QD inside a single-sided optical micropillar cavity. (b) The relative energy levels and the optical transitions of a QD.
reflection coefficients of cold and hot cavities can achieve \(|r_o(\omega)| \simeq 1\) and \(|r_h(\omega)| \simeq 1\). Then, the reflection operator \(\hat{\tau}(\omega)\) can be expressed as

\[
\hat{\tau}(\omega) = e^{i\varphi_o}[(|R\rangle \langle R| \otimes |\uparrow\rangle \langle \uparrow| + |L\rangle \langle L| \otimes |\downarrow\rangle \langle \downarrow|) + e^{i\varphi_h}(|L\rangle \langle L| \otimes |\uparrow\rangle \langle \uparrow| + |R\rangle \langle R| \otimes |\downarrow\rangle \langle \downarrow|)].
\]  

After performing reflection operator \(\hat{\tau}(\omega)\) on the states of the photon and the electron spin, the input-output relation of the single-sided QD-cavity system can be obtained as

\[
\begin{align*}
|R, \uparrow\rangle &\to |r_o||R, \uparrow\rangle, & |L, \uparrow\rangle &\to -|r_h||L, \uparrow\rangle, \\
|R, \downarrow\rangle &\to -|r_h||R, \downarrow\rangle, & |L, \downarrow\rangle &\to |r_o||L, \downarrow\rangle.
\end{align*}
\]  

III. ERROR-HERALDED BLOCK FOR THE IMPERFECT INTERACTION BETWEEN PHOTONS AND QD-CAVITY SYSTEM

In order to avoid the errors caused by the imperfect interactions between photons and QD-cavity systems, we construct an error-heralded block with an QD in single-sided cavity system, two circular polarization beam splitters (CPBS), one half-wave plate (Hp) and one single-photon detector (D). The schematic diagram of our error-heralded block is shown in Fig. 2. The QD in the single-sided cavity is initially prepared in the state \(|\varphi^+\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)\).

If the input photon is in the left-circularly polarized state \(|L\rangle\), the state of the whole system composed of the photon and the QD in the cavity is

\[
|\Phi\rangle_0 = |L\rangle \otimes \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle).
\]  

The photon will be transmitted by the CPBS on the left and then pass through the half-wave plate(Hp). The state of the whole system will change to \(|\Phi\rangle_1\), which can be expressed as

\[
|\Phi\rangle_1 = \frac{1}{\sqrt{2}}(|R\rangle - |L\rangle) \otimes \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle).
\]  

After the interaction between the photon and the QD-cavity system, the state becomes

\[
|\Phi\rangle_2 = \frac{1}{2}(r_h |R\rangle |\uparrow\rangle + r_o |R\rangle |\downarrow\rangle - r_o |L\rangle |\uparrow\rangle - r_h |L\rangle |\downarrow\rangle).
\]
Then the photon passes through the half-wave plate (Hp) again, the state will change to

\[ |\Phi\rangle_3 = \frac{1}{2\sqrt{2}}[(r_h - r_o) |R\rangle |\uparrow\rangle + (r_h + r_o) |L\rangle |\downarrow\rangle - r_h (|R\rangle - |L\rangle) |\downarrow\rangle - r_o (|R\rangle - |L\rangle) |\downarrow\rangle] \]

\[ = \frac{1}{2\sqrt{2}}[(r_h - r_o) |R\rangle |\uparrow\rangle + (r_h + r_o) |L\rangle |\downarrow\rangle - (r_h - r_o) |R\rangle |\downarrow\rangle - (r_h + r_o) |L\rangle |\downarrow\rangle] \]

\[ = \frac{1}{2\sqrt{2}}[(r_h - r_o) |R\rangle (|\uparrow\rangle - |\downarrow\rangle) + (r_h + r_o) |L\rangle (|\uparrow\rangle + |\downarrow\rangle))] \]

\[ = \frac{1}{2}[(r_h - r_o) |R\rangle |\varphi^-\rangle + (r_h + r_o) |L\rangle |\varphi^+\rangle]. \quad (10) \]

At last, the photon passes through the CPBS on the right and the reflected \(|L\rangle\) component will be detected by the single photon detector (D), and the final state can be described as

\[ |\Phi\rangle_4 = \frac{1}{2}(r_h - r_o) |R\rangle |\varphi^-\rangle. \quad (11) \]

We can see that if the photon is reflected by the error-heralded block, the polarization of the photon and the state of the QD would not change, the photon will be detected by the detector and the click of detector represents the error-herald process. If there is no click of the detector, the photon will transmitted from the error-heralded block and means there are no errors in the process.

**IV. ERROR-HERALDED GENERATION OF HYPERENTANGLED BELL STATES FOR PHOTONS WITH SINGLE-SIDED QUANTUM-DOT-CAVITY SYSTEMS**

The hyperentangled Bell states in both polarization and spatial-mode DOFs can be written as

\[ |\Phi\rangle_{AB} = |\varphi_P\rangle_{AB} \otimes |\varphi_S\rangle_{AB}. \quad (12) \]

Here \(AB\) denotes two photons. \(|\varphi_P\rangle_{AB}\) is one of the four polarization Bell state:

\[ |\varphi_P^\pm\rangle_{AB} = \frac{1}{\sqrt{2}}(|RR\rangle \pm |LL\rangle), \quad |\psi_S^\pm\rangle_{AB} = \frac{1}{\sqrt{2}}(|RL\rangle \pm |LR\rangle). \quad (13) \]

\(|\varphi_S\rangle_{AB}\) is one of the four spatial-mode Bell states:

\[ |\varphi_S^\pm\rangle_{AB} = \frac{1}{\sqrt{2}}(|a_1 b_1\rangle \pm |a_2 b_2\rangle), \quad |\psi_S^\pm\rangle_{AB} = \frac{1}{\sqrt{2}}(|a_1 b_2\rangle \pm |a_2 b_1\rangle). \quad (14) \]

Here \(a_1 (b_1)\) and \(a_2 (b_2)\) denote the two spatial modes of photon \(A (B)\). The states \(|\varphi_P^\pm\rangle_{AB}\) and \(|\varphi_S^\pm\rangle_{AB}\) are in even-parity mode, and the states \(|\psi_P^\pm\rangle_{AB}\) and \(|\psi_S^\pm\rangle_{AB}\) are in odd-parity mode. The principle for our hyperentangled Bell states generation (HBSG) is shown in Fig. 3. The initial states of the two electron spins \(e_1\) and \(e_2\) in two QD-cavity systems QD\(_1\) and QD\(_2\) are \(|\varphi^+\rangle_1\) and \(|\varphi^+\rangle_2\), respectively. Here \(|\varphi^\pm\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle \pm |\downarrow\rangle)\), and the two QD-cavity systems QD\(_1\) and QD\(_2\) are in the ideal condition. The two photons A and B are prepared in the same initial state \(|\varphi^+\rangle = |\varphi^+\rangle_B = \frac{1}{\sqrt{2}}(|R\rangle + |L\rangle)\), the process for HBSG can be described in detail as follows.
states through some appropriate single-bit operations.

states by measuring the state of two QDs, and then generate the other 12 polarization-spatial hyperentangled Bell state. 

\[ |\psi\rangle = \frac{1}{\sqrt{2}} (|\varphi^+\rangle_R |\varphi^+\rangle_L + |\varphi^-\rangle_R |\varphi^-\rangle_L) \]

corresponds to hyperentangled Bell state \[ |\psi\rangle = \frac{1}{\sqrt{2}} (|\varphi^+\rangle_R |\varphi^+\rangle_L - |\varphi^-\rangle_R |\varphi^-\rangle_L) \].

FIG. 3: Schematic diagram of our error-heralded hyperentangled Bell state generation (HBSG) scheme for two-photon polarization-spatial. BS is a 50 : 50 beam splitter which performs the spatial-mode Hadamard operation \[ |x\rangle \rightarrow \frac{1}{\sqrt{2}}(|x\rangle + |x\rangle), |x\rangle \rightarrow \frac{1}{\sqrt{2}}(|x\rangle - |x\rangle) \], \( x = a, b \). \( Z \) is a half-wave plate which performs a polarization bit-flip operation \[ Z = |R\rangle \langle L| + |L\rangle \langle R| \]. WFC is waveform corrector mapping \[ |i\rangle_2 \rightarrow \frac{1}{2}(r_h - r_o)|i\rangle_2 \], \[ |i\rangle_1 \] and \[ |i\rangle_2 \] denote two spatial modes of photon \( i \) (i=a,b).

First, photon A is injected into the quantum circuit through the left input port, then enters photon B. It must be ensured that the time interval between the two input photons is less than the decoherence time of the electrons in QD. CPBS\(_1\) will transmit the photons in state \( |R\rangle \) to path \( a_2(b_2) \) and reflect the \( |L\rangle \) photons to path \( a_1(b_1) \). The photons in path \( a_1(b_1) \) will then pass through the error-detected block QD\(_1\) and HP\(_1\). The photons in path \( a_2(b_2) \) will then pass through the WFC (waveform corrector) \[ S \], which changes \[ |i\rangle_2 \rightarrow \frac{1}{2}(r_h - r_o)|i\rangle_2 \]. This slightly decreases the overall success probability, but leaves the fidelity intact. The state of the entire system changes from \( \Psi_0 = |\varphi^+\rangle_A |\varphi^+\rangle_B |\varphi^+\rangle_1 |\varphi^+\rangle_2 \) to \( \Psi_1 \), before the photons reach the BS. Here \( \Psi_1 \) is

\[
\Psi_1 = \frac{1}{2}(|LLa_1(b_1)|\varphi^+\rangle_1 + |LRa_1(b_2)|\varphi^-\rangle_1
+ |RLa_2(b_1)|\varphi^-\rangle_1 + |RLa_2(b_2)|\varphi^+\rangle_1)|\varphi^+\rangle_2).
\]

Then the two wave packets splitted by CPBS\(_1\) will interfere at BS, which complete the spatial-mode Hadamard operation \[ |a_1(b_1)\rangle \rightarrow \frac{1}{\sqrt{2}}(|c_1(d_1)\rangle + |c_2(d_2)\rangle), |a_2(b_2)\rangle \rightarrow \frac{1}{\sqrt{2}}(|c_1(d_1)\rangle - |c_2(d_2)\rangle) \], and lead photons to path \( c_1(d_1) \) or \( c_2(d_2) \). Photons in path \( c_1(d_1) \) will then pass through the second error-heralded block consisting of the two CPBSs, QD\(_2\) and HP\(_2\). The whole system will change to the following state

\[
\Psi_2 = \frac{1}{2}(|\varphi^+\rangle_1 |\varphi^+\rangle_S |\varphi^+\rangle_2 + |\varphi^-\rangle_1 |\varphi^-\rangle_S |\varphi^-\rangle_2 + |\varphi^+\rangle_1 |\varphi^-\rangle_S |\varphi^-\rangle_2
\]

\[
+ |\varphi^-\rangle_1 |\varphi^+\rangle_S |\varphi^+\rangle_2 + |\varphi^-\rangle_1 |\varphi^-\rangle_S |\varphi^-\rangle_2) \]

Eq.(16) shows the relationship between the polarization-spatial hyperentangled Bell states of the two photons system and the measurement results of the two QD-cavity systems. If QD\(_1\) and QD\(_2\) are in the states \( |\varphi^+\rangle_1 \) and \( |\varphi^+\rangle_2 \), respectively, the two photon is in the hyperentangled Bell state \( |\psi^+\rangle_1 |\psi^+\rangle_S \). When QD\(_1\) and QD\(_2\) are in the states \( |\varphi^-\rangle_1 \) and \( |\varphi^-\rangle_2 \), respectively, the two photon is in the hyperentangled Bell state \( |\psi^-\rangle_1 |\psi^-\rangle_S \). Similarly, QD states \( |\varphi^-\rangle_1 |\varphi^+\rangle_2 \) corresponds to hyperentangled Bell state \( |\psi^+\rangle_1 |\psi^-\rangle_S \), QD states \( |\varphi^+\rangle_1 |\varphi^-\rangle_2 \) corresponds to hyperentangled Bell state \( |\psi^-\rangle_1 |\psi^+\rangle_S \). In this way, we can deterministically generate 4 two-photon polarization-spatial hyperentangled Bell states by measuring the state of two QDs, and then generate the other 12 polarization-spatial hyperentangled Bell states through some appropriate single-bit operations.
V. SELF-ASSISTED COMPLETE POLARIZATION-SPATIAL HYPERENTANGLED BELL STATES ANALYSIS WITH ERROR-HERALDED BLOCK

The principle for distinguishing the 16 hyperentangled Bell states is shown in Fig. 4. Which contains two steps: spatial-mode Bell states analysis and polarization Bell states analysis. The two error-heralded block are used to record the space-state information. The Z represents a half-wave plate which is used to perform a bit-flip operation \( Z = |R\rangle\langle L| + |L\rangle\langle R| \) in the polarization DOF, and the beam splitters(BSs) guide the photons from each input port to those two output ports with equal probabilities. The waveform correctors(WFCs) in path \( a_2(b_2) \) is used to eliminate the infidelity caused by the cavity-QD-system. The circular polarization beam splitter(CPBS) transmits the input right-circularly polarized photon and reflects the left-circularly polarized photon, the polarization beam splitter(PBS) transmit horizontal polarized states while reflecting vertical polarized states. The CPBS and two PBSs form a single-photon Bell state measurement device (SPBSM), which discriminates four single-photon Bell states completely, assisted by the spatial information recorded in the two QDs, we can completely distinguish the 16 two-photon hyperentangled Bell states. We describe our process for HBSA as follows.

A. HBSA protocol for Bell states in spatial mode

The first step is used to distinguish the 4 spatial-mode Bell states assisted by the single-sided QD-cavity systems. The initial states of the two electron spins \( e_1 \) and \( e_2 \) in two QD-cavity systems QD\(_1\) and QD\(_2\) are \( |\psi^+\rangle_1 = |\psi^+\rangle_2 \), and the two QD-cavity systems QD\(_1\) and QD\(_2\) are in the ideal condition. The state of photon pair \( AB \) is one of the 16 hyperentangled Bell states. Then photons A and B enter the state analysis device shown in Fig. 4, one after another, the interval time between the two photons should be less than the spin coherence time of the QDs. After the two photons interacting with QD-cavity systems QD\(_1\), before the two wave packets in mode \( a(b)_1 \) and \( a(b)_2 \) interfere.
Electron in the cavity is reflecting four different two-photon entanglement spatial states. We assume that the initial states of the excess with the QDs, Hps, BSs, the whole system returns to the initial state. Different combinations of two QDs status, after passing through the two QD systems, the two photons A and B can be entangled with the electron spins into parity information. Therefore we can similarly use QD1 to see that the even-parity Bell states can be distinguished from the odd-parity Bell states assisted by the QD-cavity bit-flip operation. The two QDs are used to record spatial information and do not destroy the polarization information. QD1 performs a Hadamard operation on the spatial-mode state while transforming the phase information, without affecting the state of the photon pair. The spin state of excess electron in QD1 is in the even-parity mode. If the spin e1 is in the state |ψ+⟩, the spatial-mode state of photon pair AB is in the odd-parity mode. Now, we can see that the even-parity Bell states can be distinguished from the odd-parity Bell states assisted by the QD-cavity systems QD1, without affecting the state of the photon pair AB. Then the photons in two spatial-mode will interfere at BS1, which performs a Hadamard operation on the spatial-mode state while transforming the phase information into parity information. Therefore we can similarly use QD2 to record the phase information of the spatial-mode state. The corresponding transformations on the states before the c-PBS can be described as follows:

\[
|\phi^+_P⟩_A|\phi^+_S⟩_B \xrightarrow{QD_1, Z_1} |\phi^+_P⟩_A|\phi^+_S⟩_B|\psi^+⟩_1|\varphi^+⟩_2,
\]

\[
|\psi^+_P⟩_A|\phi^+_S⟩_B \xrightarrow{QD_1, Z_1} |\psi^+_P⟩_A|\phi^+_S⟩_B|\psi^+⟩_1|\varphi^+⟩_2,
\]

\[
|\phi^+_P⟩_A|\phi^-_S⟩_B \xrightarrow{QD_1, Z_1} |\phi^+_P⟩_A|\phi^-_S⟩_B|\psi^+⟩_1|\varphi^-⟩_2,
\]

\[
|\psi^+_P⟩_A|\phi^-_S⟩_B \xrightarrow{QD_1, Z_1} |\psi^+_P⟩_A|\phi^-_S⟩_B|\psi^+⟩_1|\varphi^-⟩_2,
\]

\[
|\phi^+_P⟩_A|\psi^+_S⟩_B \xrightarrow{QD_1, Z_1} |\phi^+_P⟩_A|\psi^+_S⟩_B|\psi^+⟩_1|\varphi^+⟩_2,
\]

\[
|\psi^+_P⟩_A|\psi^-_S⟩_B \xrightarrow{QD_1, Z_1} |\psi^+_P⟩_A|\psi^-_S⟩_B|\psi^+⟩_1|\varphi^+⟩_2,
\]

\[
|\phi^+_P⟩_A|\psi^-_S⟩_B \xrightarrow{QD_1, Z_1} |\phi^+_P⟩_A|\psi^-_S⟩_B|\psi^+⟩_1|\varphi^-⟩_2,
\]

\[
|\psi^+_P⟩_A|\psi^-_S⟩_B \xrightarrow{QD_1, Z_1} |\psi^+_P⟩_A|\psi^-_S⟩_B|\psi^+⟩_1|\varphi^-⟩_2.
\]

After passing through the two QD systems, the two photons A and B can be entangled with the electron spins in the two cavities. It can be seen that such a two-photon hyperentangled state does not change after interacting with the QDs, Hps, BSs, the whole system returns to the initial state. Different combinations of two QDs status, reflecting four different two-photon entanglement spatial states. We assume that the initial states of the excess electron in the cavity is |φ^+⟩ = \frac{1}{\sqrt{2}}(|↑⟩ + |↓⟩). The Z represents a half-wave plate which is used to perform a bit-flip operation Z = |R⟩⟨L| + |L⟩⟨R| in the polarization DOF, Hp represents a half-wave plate, which can perform the polarization Hadamard operation |R⟩ \rightarrow \frac{1}{2}(|R⟩ + |L⟩), |L⟩ \rightarrow \frac{1}{2}(|R⟩ - |L⟩) on the photon, and the beam splitters (BSs) can accomplish the Hadamard operation on the spatial-mode DOF. The circular polarization beam splitter (c-PBS) transmits the input right-circularly polarized photon and reflects the left-circularly polarized photon, the polarization beam splitter (PBS) transmit horizontal polarized states while reflecting vertical polarized states. The two QDs are used to record spatial information and do not destroy the polarization information. QD1 is used to record the parity information, and QD2 to record the phase information. The spin state of excess electron in QD1 is
changed for odd-parity states and unchanged for even-parity. The spin state of excess electron in QD2 is changed for negative phase states and unchanged for positive phase states. The relation between the outcomes of the two QDs and the initial space states is shown in Table I.

We can detect whether or not there is a photon interacting with the QD-cavity system by measuring the spin state of the excess electron. If the excess electron is in the initial states, there is no photon (or two photons) interacting with the QD-cavity system. If the excess electron is in the state \(|\varphi^-\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle)\) states, there is a photon interacting with the QD cavity system (with a bit-flip operation \(Z = |R\rangle\langle L| + |L\rangle\langle R|\) on the photon, its original polarization state is recovered). At this point, we can fully distinguish the four space Bell states and do not affect it.

### Table I: The relation between the outcomes of the two QDs and the initial space states.

| Initial space states | QD1 | QD2 |
|----------------------|-----|-----|
| \(|\phi^+_S\rangle_{AB}\) | \(|\varphi^+_1\rangle\) | \(|\varphi^+_2\rangle\) |
| \(|\phi^-_S\rangle_{AB}\) | \(|\varphi^-_1\rangle\) | \(|\varphi^-_2\rangle\) |
| \(|\psi^+_S\rangle_{AB}\) | \(|\varphi^-_1\rangle\) | \(|\varphi^+_2\rangle\) |
| \(|\psi^-_S\rangle_{AB}\) | \(|\varphi^-_1\rangle\) | \(|\varphi^-_2\rangle\) |

### B. HBSA protocol for Bell states in polarization mode

The second step is discrimination of 4 polarization Bell states assisted by the spatial-mode entanglement. The C-PBS and two PBSs form a single-photon Bell state measurement device (SPBSM), which can be used to discriminate 4 single-photon polarization Bell states completely. The 8 single-photon Bell states of two photons \(A, B\) that composed of the polarization and spatial-mode DOFs can be expressed as

\[
|\phi^\pm\rangle_X = \frac{1}{\sqrt{2}}(|Rx_2\rangle \pm |Lx_1\rangle), \quad |\psi^\pm\rangle_X = \frac{1}{\sqrt{2}}(|Rx_1\rangle \pm |Lx_2\rangle).
\]  

Here \(X(x)\) can be either \(A(a)\) or \(B(b)\). For example, after the photon \(A\) passes through c-PBS, the four single-photon Bell states will change to

\[
|\phi^+_A\rangle \rightarrow \frac{1}{\sqrt{2}}(|R\rangle + |L\rangle)|a_1\rangle = |H\rangle|a_1\rangle, \quad |\phi^-_A\rangle \rightarrow \frac{1}{\sqrt{2}}(|R\rangle - |L\rangle)|a_1\rangle = |V\rangle|a_1\rangle, \\
|\psi^+_A\rangle \rightarrow \frac{1}{\sqrt{2}}(|R\rangle + |L\rangle)|a_2\rangle = |H\rangle|a_2\rangle, \quad |\psi^-_A\rangle \rightarrow \frac{1}{\sqrt{2}}(|R\rangle - |L\rangle)|a_2\rangle = |V\rangle|a_2\rangle.
\]

(20)

Here \(|R\rangle = \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)\) and \(|L\rangle = \frac{1}{\sqrt{2}}(|H\rangle - |V\rangle)\). After the photon \(A\) passing through two PBSs, the photon in state \(|H\rangle|a_1\rangle, |V\rangle|a_1\rangle, |H\rangle|a_2\rangle\) or \(|V\rangle|a_2\rangle\) will be detected by the photon detector \(a_1^+, a_2^+, a_1^-\) or \(a_2^-\), respectively. The four single-photon Bell states of photon \(B\) can be detected in the same way. The relationship of the 8 single-photon Bell states and the corresponding response of photon detectors is summarized as follows:

\[
|\phi^\pm\rangle_X \leftrightarrow x_1^\pm, \quad |\psi^\pm\rangle_X \leftrightarrow x_2^\pm.
\]

(21)

Here \(X(x)\) can be either \(A(a)\) or \(B(b)\). Before performing the single-photon Bell state measurements (SPBSMs), the spatial-mode state of the two photons is known, and can assist the analysis of the four polarization Bell states. If the spatial-mode state after the first step is \(|\phi^+_S\rangle_{AB}\), the four possible combinations of SPBSMs will be

\[
|\phi^+_T\rangle_{AB}|\phi^+_S\rangle_{AB} = \frac{1}{2}(|\phi^+_A\phi^+_B\rangle + |\phi^+_A\phi^-_B\rangle + |\phi^-_A\phi^+_B\rangle + |\phi^-_A\phi^-_B\rangle), \\
|\psi^+_T\rangle_{AB}|\phi^+_S\rangle_{AB} = \frac{1}{2}(|\phi^+_A\psi^+_B\rangle - |\phi^-_A\psi^-_B\rangle + |\psi^+_A\phi^+_B\rangle - |\psi^-_A\phi^-_B\rangle).
\]

(22)

There are 16 possible measurement combinations, which can be divided into four different groups, each group consists of 4 different polarization and space Bell states. The relationship between the initial hyperentangled states and the possible detections is shown in Table II. We can determine which group the probe result belongs to, assisted by the
known spatial information, it is possible to completely distinguish the 4 polarization states. And finally determine the original hyper-entangled Bell states of the two-photon.

For example if the two QDs are both unchanged, their states are $|\varphi^{+}\rangle_1|\varphi^{+}\rangle_2$, it means that the space state is $|\phi_2^+\rangle$, and if the detector $a_1^+$ and $b_1^+$ have a response, it means that the single photon Bell state is $|\phi_2^+\rangle_A|\varphi^{+}\rangle_B$, from table II we can see it belongs to the first group , so the polarization state is $|\phi_1^+\rangle$, the original hyperentangled Bell states of the two photons is $|\phi_2^+\rangle_A|\phi_2^+\rangle_B$. If the detector $a_1^-$ and $b_1^-$ clicked, it means that the single photon Bell state is $|\phi^-\rangle_A|\varphi^{+}\rangle_B$, from table II we can see it belongs to the fourth group , and we also know the space state is $|\phi_2^-\rangle$, so the polarization state is $|\psi^-\rangle$, the original hyper-entangled Bell states of the two-photon is $|\psi^-\rangle_A|\phi_2^+\rangle_B$. Similarly, we can completely distinguish 16 two-photon hyperentangled Bell states with the state of two QDs and the response of eight single photon detectors.

### VI. DISCUSSION AND CONCLUSION

Since the deterministic HBSG and the complete HBSA of the two-photon hyperentangled state are very important in quantum information process, a considerable number of theoretical and experimental programs have been proposed, and they all have their own limitations. Schemes using only linear optical element cannot deterministically generate hyperentangled state and can’t completely distinguish hyperentangled states. Schemes using nonlinear material such as QDs in microcavity can accomplish the deterministic HBSG and complete HBSA, but due to the imperfect interactions of photons and quantum dot systems, the fidelities and the efficiencies can’t reach unit one. Based on the optical transitions in a QD-cavity system, we construct an error-heralded block. With this error-detected block, we propose two self-assisted schemes for the deterministic HBSG and complete HBSA of two-photon polarization-spatial hyperentangled system. The errors due to imperfect interactions between photons and quantum dot systems can be heralded by detectors, which make the fidelities of our schemes can reach unit one.

In an ideal condition, $|\Phi\rangle_f = \frac{1}{\sqrt{2}}(r_h - r_o)|F\rangle|\varphi^-\rangle$ can be obtained after the left-polarized photon $|L\rangle$ interacting with the error-detected block and the fidelities and the efficiencies can be 100%. However, in a realistic condition, the outcomes of the interaction between the photon and the error-heralded block, which are described as Eqs. (10), are affected by the coupling strength $g$ of the quantum dot and microcavity, the decay rates of $X^-$ and microcavity field mode $\gamma/2$ and $\kappa/2$, the microcavity leakage rate $\kappa_a/2$, which would affect the fidelities and the efficiencies as well.

The fidelity of the process for deterministic HBSG and complete HBSA is defined as $F = |\langle \psi_f | \psi \rangle|^2$, where $|\psi_f\rangle$ is the final state of the total system under actual circumstances and $|\psi\rangle$ is the final state with an ideal condition. Because our schemes add WFC(wave form corrector) to the path that has no nonlinear interaction, the fidelity can be kept in unit one. The efficiency is defined as the ratio of the number of the output photons to the number of the input photons. Since in the two schemes photons passed through the same number of nonlinear materials and the fidelity is one, the efficiency of our state generation scheme and the state analysis scheme are the same $\eta_{HBSG} = \eta_{HBSA} = \frac{1}{8}(r_h - r_o)^8$. In Fig. 5 we numerically simulate the efficiency of our protocols.

| Initial states possible detections |
|-----------------------------------|
| $|\phi_1^+\rangle_A|\phi_1^+\rangle_B$ | $a_1^+ b_1^+, a_2^+ b_2^+$ |
| $|\phi_1^+\rangle_A|\phi_1^\rangle_B$ | $a_1^+ b_1^-, a_2^+ b_2^-$ |
| $|\phi_1^\rangle_A|\phi_1^+\rangle_B$ | $a_1^+ b_1^+, a_2^+ b_2^+$ |
| $|\phi_1^\rangle_A|\phi_1^\rangle_B$ | $a_1^- b_1^-, a_2^- b_2^+$ |
| $|\phi_1^+\rangle_A|\phi_2^+\rangle_B$ | $a_1^+ b_1^+, a_2^+ b_2^+$ |
| $|\phi_1^+\rangle_A|\phi_2^\rangle_B$ | $a_1^+ b_1^-, a_2^+ b_2^-$ |
| $|\phi_1^\rangle_A|\phi_2^+\rangle_B$ | $a_1^- b_1^+, a_2^- b_2^+$ |
| $|\phi_1^\rangle_A|\phi_2^\rangle_B$ | $a_1^- b_1^-, a_2^- b_2^+$ |

TABLE II: Relationship between the initial hyperentangled states and the possible detections.
Besides the parameters mentioned above, the exciton dephasing, including the optical dephasing and the spin dephasing of $X^-$ will also affect the fidelities. Exciton dephasing reduces the fidelity by the amount of $(1 - \exp(-\tau/\Gamma))$, where $\tau$ and $\Gamma$ are the cavity photon lifetime and the trion coherence time, respectively. The optical dephasing reduces the fidelity less than 10%, because the time scale of the excitons can reach hundreds of picoseconds, while the cavity photon lifetime is in the tens of picoseconds range for a self-assembled In(Ga)As-based QD with a cavity Q factor in the strong coupling regime. The effect of the spin dephasing can be neglected because the spin decoherence time is several orders of magnitude longer than the cavity photon lifetime.

In summary, we have proposed two self-assisted schemes for deterministic HBSG and complete HBSA of the polarization-spatial hyperentangled two-photon system with error-heralded blocks. With the help of our error-detected block, the errors can be heralded by detectors, which makes the fidelity of our protocols be unit one. In our proposal, the four spatial-mode Bell states are completely distinguished by using two QD-cavity systems without affecting the hyperentangled state of the two-photon system, so the four polarization Bell states can be completely distinguished only by linear optical elements assisted by the polarization-spatial hyperentanglement. Our protocols greatly reduced the difficulty of experimental realization. We only need one QD for one photon, and the device is much simpler than previous. These good features make our schemes more useful in long-distance quantum communication in the future.

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