Interactions of $\eta$-meson in asymmetric nuclear matter

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Abstract. The interactions between $\eta$-meson and nucleons are studied by the unification of the chiral SU(3) model and chiral perturbation theory. The $\eta$-meson-nucleon interactions for the next-to-leading order terms are derived by expanding the $\eta N$ interaction Lagrangian within the chiral perturbation theory. Using the chiral SU(3) model, we calculate the in-medium scalar density of protons $\rho_p$ and neutrons $\rho_n$ for different values of temperature $T$, isospin asymmetry $I$ and nucleonic density $\rho_N$. Within the chiral SU(3) model the in-medium properties of nucleons are evaluated in terms of scalar-isoscalar fields $\sigma$, and $\zeta$ and the scalar-isovector field $\delta$. The scalar densities of nucleons calculated using chiral SU(3) model are used as input in the equation of motion of $\eta$ mesons obtained from chiral perturbation theory. We solve the equation of motion for sigma term $\Sigma_{\eta N}=280\pm130$ MeV and scattering length $a_{\eta N}=1.025$ fm for different values of asymmetry of the medium. We find an attractive mass-shift of the $\eta$ meson in nuclear medium and the magnitude of this shift increases with increasing density. Finite isospin asymmetry of the medium causes less drop in the in-medium mass of $\eta$ mesons. The negative mass-shift indicates the possibility of the formation of $\eta$-mesic nuclei.

1. Introduction
The study of the baryon-meson interactions is a very interesting topic of research in the strong interaction physics, both theoretically and experimentally [1,2]. The interactions of kaon/pion/quarkonia with baryons have been extensively studied in the literature [2–4], but $\eta$-baryons interactions still require more attention. The $\eta N$ interactions were studied by Q. Haider and L. Liu and it was anticipated that eta-nucleon interactions are attractive and there is a possibility of formation of eta-mesic nuclei [4,5]. Since then the bound states of $\eta$-nucleons has been one of the interesting research topics in the hadron physics [1,2].

In a work [1], for a correct description of the $\eta$-nuclear interactions the off-shell terms, i.e. next-to-leading order terms, were added in the Lagrangian using heavy baryon chiral perturbation theory [2] and in-medium masses of $\eta$ mesons were evaluated. The scalar densities appearing in the equation of motion of $\eta$ mesons were calculated using relativistic mean-field model at zero temperature. In current study, we follow the work [1] for the interaction of $\eta$ mesons with nucleons. However, the values of scalar densities of nucleons are evaluated using chiral SU(3) model for finite temperature and isospin asymmetry of the medium. Within chiral SU(3) model, coupled equations of motion for the scalar fields $\sigma$ and $\zeta$, the scalar-isovector field $\delta$, the scalar dilaton field $\chi$ and the vector fields $\omega$ and $\rho$ are solved to obtain their in-medium values under mean-field approximation. The scalar fields $\sigma$, $\zeta$ and $\delta$ are used to calculate the scalar densities of nucleons. The chiral model incorporates the QCD trace anomaly and nonlinear realization property of the chiral symmetry [3] and it has been used in literature for study
of in-medium meson properties and finite nuclei\cite{3,6}.

The layout of the paper is follows: In section 2, the equation of motion of $\eta N$ interactions is derived from the unified approach of the chPT and chiral model. Afterward, in section 3, the results are discussed considering effects of finite isospin asymmetry and temperature. Finally, we have concluded our paper in section 4.

2. Methodology

In the chiral perturbation theory, the Lagrangian density of the meson-baryon interactions up to second order can be written as follows\cite{1}:

$$L_{\eta N} = \frac{1}{2} \partial^\mu \eta \partial_\mu \eta - \frac{1}{2} \left( m_\eta^2 - \frac{\Sigma_{\eta N}}{f_\pi^2} \bar{\Psi} \Psi \right) \eta^2 + \frac{1}{2} \frac{\kappa}{f_\pi^2} \bar{\Psi} \partial^\mu \eta \partial_\mu \eta. \quad (1)$$

In the above expression, the next-to-leading order contributions are included from the heavy baryon chiral perturbation theory\cite{2}. The second term in equation (1) is called the sigma correction term and we consider the value of $\Sigma_{\eta N}$ parameter as $\Sigma_{\eta N} = 280 \pm 130$ MeV\cite{1}. The last term in equation (1) is known as kappa term and the parameter $\kappa$ is evaluated from the $\eta N$ scattering length using the following expression:

$$\kappa = 4\pi f_\pi^2 \left( \frac{1}{m_\eta^2 + \frac{\Sigma_{\eta N}}{2f_\pi^2} \langle \bar{\Psi} \Psi \rangle} + \frac{\kappa}{2f_\pi^2} \langle \bar{\Psi} \Psi \rangle \partial_\mu \partial^\mu \right) \eta. \quad (2)$$

where $m_\eta$ (547 MeV) and $m_N$ (939 MeV) are the vacuum mass of $\eta$-meson and nucleon, respectively\cite{1}. In the present work, for $\eta N$ meson-nucleon scattering length we consider $a_{\eta N} = 1.02$ fm\cite{1,7–11}. Under the mean-field approach, the equation of motion is calculated by putting the $\eta N$ Lagrangian in the Euler Lagrange equations and is given as:

$$\left( \partial_\mu \partial^\mu + \frac{m_\eta^2}{2f_\pi^2} \langle \bar{\Psi} \Psi \rangle + \frac{\kappa}{2f_\pi^2} \langle \bar{\Psi} \Psi \rangle \partial_\mu \partial^\mu \right) \eta = 0. \quad (3)$$

In the above expression, $\langle \bar{\Psi} \Psi \rangle \equiv \left( \rho^*_n + \rho^*_p \right)$ represents the net scalar density of the nucleons, $\rho^*_N$, calculated in the chiral SU(3) model. The vector and scalar densities of nucleons at finite temperature are defined as:

$$\rho^*_i = \gamma_i \int \frac{d^3k}{(2\pi)^3} \left( \frac{1}{1 + \exp \left( \frac{\beta(E^*_i(k) - \mu^*_i)}{T} \right)} - \frac{1}{1 + \exp \left( \frac{\beta(E^*_i(k) + \mu^*_i)}{T} \right)} \right), \quad (4)$$

and

$$\rho^s_i = \gamma_i \int \frac{d^3k}{(2\pi)^3} \frac{m^*_i}{E^*_i(k)} \left( \frac{1}{1 + \exp \left( \frac{\beta(E^*_i(k) - \mu^*_i)}{T} \right)} + \frac{1}{1 + \exp \left( \frac{\beta(E^*_i(k) + \mu^*_i)}{T} \right)} \right), \quad (5)$$

respectively, where $\beta = \frac{1}{kT}$ and $\gamma_i$ represents the degeneracy of nucleons ($i = n, p$)\cite{6}. Also, $\mu^*_i$ and $E^*_i = \sqrt{m^*_i^2 + k^2}$ define the chemical potential and single particle energy of the nucleons, respectively. Here, $m^*_i = -g_{\eta i} \sigma - g_{\zeta i} \zeta + g_{\delta i} \delta$ define the in-medium mass of nucleons. The parameters $g_{\eta i}$, $g_{\zeta i}$ and $g_{\delta i}$ define the couplings of scalar fields with nucleons. The impact of asymmetry is incorporated by the formula $I = \frac{\Sigma_{\tau \eta i} \rho^s_i}{2\rho_N}$\cite{6}.
The Fourier transform of the equation (3) gives
\[
-\omega^2_{\eta} + \vec{k}^2 + m^2_{\eta} - \frac{\Sigma_{\eta N}}{2f^2_{\pi}} (\rho^a_p + \rho^a_n) + \frac{K}{2f^2_{\pi}} (\rho^a_p + \rho^a_n) \left( -\omega^2_{\eta} + \vec{k}^2 \right) = 0,
\]
which is solved to obtain the energy of $\eta$ meson in the nuclear matter.

The optical potential $U^*_{\eta}$ can be written as:
\[
U^*_{\eta} = \frac{m^*_{\eta} - m^2_{\eta}}{2m_{\eta}} \simeq m^*_{\eta} - m_{\eta},
\]
where $m^*_{\eta}$ denotes the in-medium mass of $\eta$ mesons at $|\vec{k}| = 0$.

### 3. Results and Discussions

In this section, we discuss how medium parameters such as density, temperature and asymmetry affect the mass of the $\eta$-meson. In figure 1 we show the results for isospin asymmetric parameter $I=0$ and $0.5$ and temperature $T=0$ (c) and $100$ MeV (d). For nuclear saturation density we consider $\rho_0=0.15$ fm$^{-3}$. The values of medium modified mass of $\eta$-meson for different permutation of medium attributes are listed in table 1. We observe that the mass of $\eta$-meson decreases with an increase in the nuclear density. If we change the value of sigma term $\Sigma_{\eta N}$ from 150 to 410 MeV, we observe more decrement in the in-medium mass of $\eta$ meson. This is because the $\Sigma_{\eta N}$ term in the meson-nucleon interaction equation of motion (see equation (3)) gives attractive contribution to the $\eta$ mass. Further, when we increase the number density of the neutrons i.e. $I=0.5$ in the nuclear medium, the trend of effective mass remains the same with density but varies with slightly less rate. Moreover, we find the temperature effects on the effective mass to be less appreciable too.

In symmetric nuclear matter, the values of mass-shift obtained using chiral SU(3) model along with chiral perturbation theory are higher in magnitude than obtained in work [1]. It should be noted that for $\eta N$ Lagrangian, only the next-to-leading order terms contribute to the $\eta$ mass. The leading-order terms do not affect the $\eta$-meson in-medium properties. Whereas in the coupled channel approach, the obtained effective mass for $\eta$-mesons is much larger than we predict. In this coupled approach, the $\eta N$ contributions are taken from the leading-order terms only [12].

Table 1. In-medium mass $m^*_{\eta}$ of $\eta$-meson at different values of medium parameters.

| $\rho_N$ | $T=0$ MeV $\eta=0$ | $T=0$ MeV $\eta=0.5$ | $T=100$ MeV $\eta=0$ | $T=100$ MeV $\eta=0.5$ | $\Sigma_{\eta N}$ (MeV) |
|----------|----------------------|----------------------|----------------------|----------------------|------------------------|
| $\rho_0$ | 150                  | 431                  | 433                  | 445                  | 445                    |
| $4\rho_0$| 280                  | 429                  | 431                  | 437                  | 438                    |
| $4\rho_0$| 410                  | 426                  | 429                  | 441                  | 442                    |

The optical potential of the $\eta$-mesons is also obtained in the present work from the equation (7). In cold symmetric nuclear matter, at $\rho_N=\rho_0$ and $\Sigma_{\eta N}=150$ MeV, we observe an attractive mass-shift of -115 MeV. For $\Sigma_{\eta N}=410$ MeV and the same medium parameters the negative mass-shift increases and become -120 MeV. The observed attractive mass-shift also predicts the formation of eta-mesic nuclei [1].
4. Conclusions
As a conclusion, we observe that the influence of the nuclear density on the mass of the $\eta$-meson in the medium is significant, while the effects of asymmetry and temperature make a smaller contribution. The attractive mass-shift predicts the possibility of $\eta$-mesic nuclei formation and can be explored in the future.

5. Acknowledgments
One of the authors, Rajesh Kumar sincerely acknowledges the support towards this work from the Ministry of Science and Human Resources Development (MHRD), Government of India.
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