Reflection symmetries and universal four-zero texture

Masaki J. S. Yang

1Department of Physics, Saitama University, Shimo-okubo, Sakura-ku, Saitama, 338-8570, Japan

In this paper, we consider exact reflection symmetries with universal four-zero texture for quarks and leptons. The previous study of $\mu - \tau$ reflection symmetries can be translated to forms $P m_{\nu,\nu} P = m_{u,v}, m_{d,e} = m_{d,e}$ with $P = \text{diag}(-1,1,1)$ in the basis of four-zero texture. We call such a symmetry reflection because it is just a extended CP symmetry and no longer a $\mu - \tau$ reflection. These symmetries can constrain the Majorana phases and then enhance predictivity of the leptogenesis.

In this scheme, fermion mass matrices are restricted to have only four parameters. It predicts the Dirac phase $\delta_{CP} \simeq 203^\circ$ and $m_1 \simeq 3$[meV] in the case of the normal hierarchy.

Mass matrix of the right-handed neutrinos $M_R$ also exhibits a four-zero texture with the reflection symmetry because four-zero textures are type-I seesaw invariant. An $u - \nu$ unification predicts mass eigenvalues as $(M_{R1}, M_{R2}, M_{R3}) = (O(10^3), O(10^9), O(10^{14}))$[GeV].

II. FOUR-ZERO TEXTURE AND $\mu - \tau$ REFLECTION SYMMETRIES

In this section, we review the previous study which discussed exact $\mu - \tau$ reflection symmetries and four-zero texture [80]. At the beginning, the mass matrices of the Standard Model fermions $f = u, d, e$ and neutrinos $\nu_L$ are defined by

$$\mathcal{L} \ni \sum_f -f_L^j m_{fij} f_R^j - \bar{\nu}_L^{ij} m_{\nu} \nu_{Lj}^j + \text{h.c.}.$$  (1)

As textures of mass matrices, we assume hermitian four-zero textures [80] and a symmetric neutrino mass:

$$m_u = \begin{pmatrix} i & 0 & 0 \\ 0 & A_u & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & A_u & 0 \\ A_u & B_u & r_u B_u \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -i & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix},$$  (2)

$$m_d = \begin{pmatrix} 0 & A_d \\ A_d & B_d \\ 0 & r_d B_d \end{pmatrix},$$  (3)

$$m_{\nu} = \begin{pmatrix} -i & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} a_{\nu} & b_{\nu} & c_{\nu} \\ b_{\nu} & d_{\nu} & e_{\nu} \\ c_{\nu} & e_{\nu} & f_{\nu} \end{pmatrix} \begin{pmatrix} -i & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix},$$  (4)

$$m_e = \begin{pmatrix} 0 & A_e \\ A_e & B_e \\ 0 & r_e B_e \end{pmatrix},$$  (5)

with real parameters $r_j, A_j \sim C_f, A_j \ll B_j \ll C_f$ and $a_{\nu} \sim f_{\nu}$. For the later convenience, the relative phases are pressed on $m_{u,\nu}$.

Bi-maximal transformation of the basis by the following $U_{BM}$:

$$m_f^{BM} = U^\dagger_{BM} m_f U_{BM}, \quad m_{\nu}^{BM} = U^\dagger_{BM} m_{\nu} U_{BM},$$  (6)

$$U_{BM} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{2} & \sqrt{2} \\ 0 & -\sqrt{2} & \sqrt{2} \end{pmatrix},$$  (7)
These matrices (8)-(11) separately satisfy exact reflection symmetries:

\[
\begin{align*}
T_u(m_{u \nu}^{BM})^\dagger T_u &= m_{u \nu}^{BM}, & T_d(m_{d e}^{BM})^\dagger T_d &= m_{d e}^{BM},
\end{align*}
\]  

(12)

where

\[
T_u = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad T_d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.
\]  

(13)

Note that this symmetry is not imposed on \(m_\nu\) in the basis of four-zero texture [4].

Diagonalizing the mass matrices \(m_f = U_L \hat{m}_f^{\text{diag}} U_R\), one obtains the CKM and MNS mixing matrices

\[
V_{\text{CKM}} = U_L^\dagger U_{Ld}, \quad U_{\text{MNS}} = U_{Ue}^\dagger U_{Lv}.
\]  

(14)

The predicted CKM matrix has the maximal \(CP\) phase in the Fritzsch–Xing parameterization [3],

\[
V_{\text{CKM}} = \begin{pmatrix} c_u & s_u & 0 \\ -s_u & c_u & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -i & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{pmatrix} \begin{pmatrix} c_d & -s_d & 0 \\ s_d & c_d & 0 \\ 0 & 0 & 1 \end{pmatrix},
\]  

(15)

where \(c_f = \cos \theta_f, s_f = \sin \theta_f\). A choice of parameters \(r_u \simeq r_d \simeq 1.6\) gives a nice agreement between the predicted \(V_{cb}, V_{ts}\) and the observation [34].

Similarly, the MNS matrix is obtained as

\[
U = V_e^\dagger \begin{pmatrix} +i & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} V_\nu,
\]  

(16)

\[
= \begin{pmatrix} 1 & -\frac{m_e}{m_\mu} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} +i & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}.
\]  

(17)

The Majorana phases are discussed later. Since the sign of \(A_e \simeq \pm \sqrt{m_e/m_\mu}\) only affects the sign of \(CP\) phase \(\delta_{CP}\), we fix \(A_e > 0\). Moreover, the 2-3 mixing of the \(V_\nu\) is small \((\sim \sin^{-1}(\pm m_\mu/m_\nu) = \pm 3.37^\circ)\). Then, it is tentatively absorbed to \(c_{23}\) and \(s_{23}\). It leads to

\[
|U_{e3}| = |s_{13}^{PDG}| = |s_{13} - \frac{m_e}{m_\mu} c_{13} s_{23}|.
\]  

(18)

The parameter \(\sqrt{m_\mu/m_\nu} \simeq 0.07\) and mixing angles of the latest global fit [2] is

\[
\theta_{23}^{PDG} = 49.7^\circ, \quad \theta_{12}^{PDG} = 33.82^\circ, \quad \theta_{13}^{PDG} = 8.61^\circ,
\]  

(19)

determines mixing parameters as

\[
s_{13} = \pm \sqrt{(s_{13}^{PDG})^2 - \frac{m_e}{m_\mu} (c_{13}^{PDG})^2 (s_{23}^{PDG})^2} = \pm 0.140,
\]  

(20)

\[
s_{23} = 0.763 \simeq s_{23}^{PDG}, \quad s_{12} = 0.555 \simeq s_{12}^{PDG}.
\]  

(21)

The sign \(\pm\) in Eq. (20) corresponds to the sign of \(\cos \delta_{CP}\). Since the latest global fit found \(\cos \delta_{CP} < 0\) [2], minus sign \(s_{13} = -0.140\) is adopted.

The Dirac phase \(\delta_{CP}\) can be evaluated from the Jarlskog invariant [3],

\[
J^{PDG} = \sin \delta_{CP} s_{12}^{PDG} c_{13}^{PDG} (s_{13}^{PDG})^2 s_{23}^{PDG} c_{23}^{PDG}.
\]  

(22)

From Eq. (10), \(J\) is calculated as

\[
J = \text{Im} [U_{\mu 3} U_{\tau 2} U_{\mu 2}^* U_{\tau 3}^*] = c_\mu s_\tau c_{13} c_{23} [-c_{12} s_{12} s_{23}^2 + s_{13} c_{23} (c_{12}^2 - s_{12}^2) + s_{13}^2 c_{12} s_{23}^2] = -0.0130.
\]  

(23)

Since \(s_{13} = -0.14\) is small,

\[
\sin \delta_{CP} = -0.390 \simeq \sqrt{\frac{m_e}{m_\mu} c_{13} s_{23}}, \quad \delta_{CP} \simeq 203^\circ.
\]  

(25)

It is very close to the best fit \(\delta_{CP}/^\circ = 217^{+40}_{-28}\) [2].
Including the Majorana phases
\[ U_{\text{MNS}} = U P, \quad P \equiv \text{diag}(1, e^{i \alpha_2/2}, e^{i \alpha_3/2}), \] (26)
one can reconstruct the neutrino mass matrix \( m_\nu \) as
\[ m_\nu = \begin{pmatrix} i & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} V_\nu P \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} P V_\nu \begin{pmatrix} i & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \] (27)
The \( \mu - \tau \) reflection symmetry \([12]\) restricts the Majorana phases to be \( \alpha_{2,3}/2 = n \pi/2 \) \((n = 0, 1, 2, \ldots)\) \([76, 78]\). Moreover, if universal texture \((m_f)_{11} = 0\) for \( f = u, d, \nu, e \) \([23]\) is assumed, we can determine the lightest neutrino mass \( m_1 \) from the condition of the texture
\[ m_1 = -e^{i \alpha_2} m_2 s_{13}^2 - e^{i \alpha_3} m_3 t_{13}^2 \] (28)
where \( t_{13} \equiv s_{13}/c_{13} \). The numerical value of the mass becomes
\[ |m_1| = 6.23 \, \text{[meV]} \quad \text{for} \quad (\alpha_2, \alpha_3) = (0, 0) \quad \text{or} \quad (\pi, \pi), \] (29)
\[ = 2.52 \, \text{[meV]} \quad \text{for} \quad (\alpha_2, \alpha_3) = (0, \pi) \quad \text{or} \quad (\pi, 0), \] (30)
for the normal hierarchy (NH) case. Here, we used mass differences from the global fit \([2]\)
\[ \Delta m^2_{21} = 73.9 \, \text{[meV]}^2, \quad \Delta m^2_{32} = 2525 \, \text{[meV]}^2. \] (31)

III. UNIVERSAL FOUR-ZERO TEXTURE

Since the mass matrix \( m_\nu \) is a function of \( \alpha_2, \alpha_3 \) and \( m_1 \), it can be reconstructed as
\[ m_\nu \simeq e^{i \alpha_3} \begin{pmatrix} 0 & -0.57i & -10.6i \\ -0.57i & 32.7 & 22.0 \\ -10.6i & 22.0 & 23.1 \end{pmatrix} \text{[meV]} \] (32)
for \( (\alpha_2, \alpha_3) = (0, 0) \) or \( (\pi, \pi) \),
\[ \simeq e^{i \alpha_3} \begin{pmatrix} 0 & -8.91i & -0.66i \\ -8.91i & 26.2 & 27.5 \\ -0.66i & 27.5 & 18.4 \end{pmatrix} \text{[meV]} \] (33)
for \( (\alpha_2, \alpha_3) = (0, \pi) \) or \( (\pi, 0) \).

In the case of Eq. \((32)\), the matrix \( m_\nu \) also exhibits approximate four-zero texture. In particular, a choice of parameter \( r_y B_\nu/C_\nu \sim r_y m_\mu/m_\tau \sim +0.08 \) makes \( (m_\nu)_{13} \) to be zero. Then, with these conditions, \( m_\nu \) becomes a four-zero texture such as
\[ m_{00} \equiv \begin{pmatrix} 0 & -8.91i & 0 \\ -8.91i & 26.2 & 27.5 \\ 0 & 27.5 & 18.4 \end{pmatrix} \text{[meV]} \] (34)
for \( (\alpha_2, \alpha_3) = (\pi, 0) \).

The eigenvalues of \( m_{00} \) are found to be
\[ (m_1, m_2, m_3) = (3.06, -9.39, 50.9) \text{[meV]}. \] (35)

Indeed the majorana phases \( \alpha_2 = \pi, \alpha_3 = 0 \) are realized.

The right-handed neutrino mass \( M_R \) can be reconstructed from the type-I seesaw mechanism \([34, 36]\) with some GUT relations. For example, Yukawa interaction for neutrinos is determined in a Pati–Salam GUT \([32]\)
\[ Y_\nu = \begin{pmatrix} i & 0 & 0 \\ 0 & A_\nu & 0 \\ 0 & r_y B_\nu & C_\nu \end{pmatrix} \begin{pmatrix} -i & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \] (36)
with the Georgi–Jarlskog relation \( B_\nu = -3B_\tau \) \([57]\).
\( M_R \) also displays the four-zero texture because the four-zero texture is seesaw invariant \([1]\)
\[ M_R = Y_\nu^T m_{00}^{-1} Y_\nu \]
\[ = \begin{pmatrix} 0 & 8.82 \times 10^7 & 0 \\ 8.82 \times 10^7 & 4.63 \times 10^{11} & -2.68 \times 10^{13} \\ 0 & -2.68 \times 10^{13} & 1.60 \times 10^{15} \end{pmatrix} \text{[GeV]}. \] (37)
Here, we used quark masses at the \( m_Z \) scale \( m_f(m_Z) \) \([83]\).
\[ m_u = 1.27 \text{[MeV]}, \quad m_c = 0.619 \text{[GeV]}, \quad m_t = 171.7 \text{[GeV]}, \] (39)
and \( r_y = 1.6 \). Indeed \( M_R \) also satisfies
\[ T_u M_{R, u}^B T_u = M_{R, u}^{BM}, \quad M_{R, d}^{BM} \equiv U_{BM}^\dagger M_R U_{BM}. \] (40)

Therefore, all the fermion mass to be the four-zero texture with the \( \mu - \tau \) reflection symmetries.

The eigenvalues of \( M_R \) are found to be
\[ (M_{R, 1}, M_{R, 2}, M_{R, 3}) \]
\[ = (6.63 \times 10^5, 1.17 \times 10^9, 1.60 \times 10^{15}) \text{[GeV]}, \] (41)
\[ \simeq (\sqrt{m_u m_c m_t}, m_c, m_3^2/m_2). \] (42)

Relations in the third line are obtained from perturbative treatments.

On the other hand, quark masses at the GUT scale \( \Lambda_{\text{GUT}} = 2 \times 10^{16} \text{GeV} \) \([83]\)
\[ m_u = 0.48 \text{[MeV]}, \quad m_c = 0.235 \text{[GeV]}, \quad m_t = 74 \text{[GeV]}, \] (43)
lead to smaller eigenvalues
\[ (M_{R, 1}, M_{R, 2}, M_{R, 3}) \]
\[ = (9.52 \times 10^4, 1.68 \times 10^9, 2.97 \times 10^{14}) \text{[GeV]}. \] (44)
The true values are supposed to be between Eq. \((41)\) and Eq. \((43)\).

Since the mass matrix \( M_R \) have strong hierarchy \( M_R \sim Y_\nu^T Y_\nu \), the lightest mass eigenvalue \( M_{R, 1} \) is too small \([33, 34]\) for the naive thermal leptogenesis \([2]\). However, leptogenesis may be achieved by the decay of the second lightest neutrino \( \nu_{R, 2} \) \([2]\) with the maximal Majorana phase \( \alpha_2/2 = \pi/2 \). Besides, the \( O(100) \) TeV neutrino \( \nu_{R, 1} \) might have some relation to the IceCube gap \([33, 34]\).
IV. REFLECTION SYMMETRIES

Let us consider mathematical and physical meaning of the $\mu - \tau$ reflection symmetries:

$$T_{u,d} m^B_{u,d} T_{u,d} = m^B_{u,d}, \quad m^T_f = U^T_B m_f U_B.$$  (45)

In the basis of the four-zero texture, these conditions lead to

$$U_B T_{u,d} U^T_B m^*_{u,d} U^*_{u,d} = m_{u,d}.$$  (46)

Surprisingly,

$$-U^T_B T_u U^T_B = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = P_1,$$  (47)

$$U^T_B T_d U^T_B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = 1_3.$$  (48)

Then, the $\mu - \tau$ reflection symmetries in the four-zero basis are found to be

$$P_1 m^*_{u,\nu,R} P_1 = m_{u,\nu,R}, \quad m^*_{d,e} = m_{d,e}.$$  (49)

Hermitian or symmetric mass matrices which satisfy Eq. (49) are given by

$$m_u = \begin{pmatrix} a_u & i b_u & i c_u \\ -i b_u & d_u & e_u \\ -i c_u & e_u & f_u \end{pmatrix}, \quad m_{\nu,R} = \begin{pmatrix} a_{\nu,R} & i b_{\nu,R} & i c_{\nu,R} \\ i b_{\nu,R} & d_{\nu,R} & e_{\nu,R} \\ i c_{\nu,R} & e_{\nu,R} & f_{\nu,R} \end{pmatrix},$$  (50)

$$m_{d,e} = \begin{pmatrix} a_{d,e} & b_{d,e} & c_{d,e} \\ b_{d,e} & d_{d,e} & e_{d,e} \\ c_{d,e} & e_{d,e} & f_{d,e} \end{pmatrix},$$  (51)

with real parameters $a_f \sim f_f$. The mass matrices [3]-[7], [34] and [98] satisfy these conditions. We call such a symmetry reflection because it is just a extended CP symmetry and no longer a $\mu - \tau$ reflection.

The reflection symmetries [99] has freedom of unitary transformation. For example, the down-type mass $m_{d,e}$ also can have phases by transformation of a phase matrix

$$P = \text{diag}(e^{i\phi}, 1, 1):$$

$$\tilde{m}_u = P_1^T m_u P = \begin{pmatrix} a_u & i e^{-i\phi} b_u & i e^{-i\phi} c_u \\ -i e^{i\phi} b_u & d_u & e_u \\ -i e^{i\phi} c_u & e_u & f_u \end{pmatrix},$$  (52)

$$\tilde{m}_d = P_1^T m_d P = \begin{pmatrix} a_{d,e} & e^{-i\phi} b_{d,e} & e^{-i\phi} c_{d,e} \\ e^{i\phi} b_{d,e} & d_{d,e} & e_{d,e} \\ e^{i\phi} c_{d,e} & e_{d,e} & f_{d,e} \end{pmatrix}. $$  (53)

In this case, by the following equivalent transformation

$$P_u = PP_1 P = \begin{pmatrix} -e^{2i\phi} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$  (54)

$$P_d = P1_3 P = \begin{pmatrix} +e^{2i\phi} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. $$ (55)

deforms the reflection symmetries as

$$P_u^T \tilde{m}_{u,d} P_u = \tilde{m}_{u,d}, \quad P_d^T \tilde{m}_{d,e} P_d = \tilde{m}_{d,e}.$$  (56)

Note that the reflection symmetries control only phases and zero texture conditions are not imposed. As a justification of the four-zero texture, a chiral symmetry breaking $S_{2L} \times S_{2R} \rightarrow 0$ have been studied in the context of democratic textures [33-40]. In this scheme, $S_{2L,R}$ is a permutation symmetry between the first and second generation of left-(right-)handed fermions. These chiral symmetries retain the lightest fermions to be massless.

A permutation-symmetric texture can be transformed to a hierarchical texture by a bi-maximal mixing between 1-2 generations, as shown in Table 1. In the hierarchical basis, the up-type Yukawa matrix respects a “$\mu - e$” reflection symmetry:

$$T_{12} Y^*_{u,\nu} T_{12} = Y_{u,\nu}, \quad Y^*_{d,e} = Y_{d,e}, \quad T_{12} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 2a & \sqrt{2b} \\ 0 & \sqrt{2c} & d \end{pmatrix},$$  (57)

and they restrict $\delta_{u,\nu} = \epsilon_{d,e} = 0$. This idea suggests that CP violation of flavor mixing only comes from a chiral symmetry breaking of the first generations. Large CP violation in $m_{\nu}$ and $m_{u}$ is desirable from the viewpoint of the leptogenesis.

V. CONCLUSIONS AND DISCUSSIONS

In this paper, we consider exact reflection symmetries with universal four-zero texture for quarks and leptons. The previous study of $\mu - \tau$ reflection symmetries can be translated to forms $P m^*_{u,\nu} P = m_{u,\nu}$, $m^*_{d,e} = m_{d,e}$ with $P = \text{diag} (-1, 1, 1)$ in the basis of four-zero texture. We call such a symmetry reflection because it is just a extended CP symmetry and no longer a $\mu - \tau$ reflection. These symmetries can constrain the Majorana phases and then enhance predictivity of the leptogenesis.

In this scheme, fermion mass matrices are restricted to have only four parameters. It predicts the Dirac phase $\delta_{C P} \approx 203^\circ$ and $m_1 \approx 3 \text{[meV]}$ in the case of the normal hierarchy.

Mass matrix of the right-handed neutrinos $M_R$ also exhibits a four-zero texture with the reflection symmetries because four-zero textures are type-I seesaw invariant. An $\alpha - \nu$ unification predicts mass eigenvalues as

$$P^T \tilde{m}_{u,a} P = \tilde{m}_{u,a}, \quad P^T \tilde{m}_{e,e} P = \tilde{m}_{e,e}.$$  (56)
\[ (M_{R1}, M_{R2}, M_{R3}) = (O(10^5), O(10^7), O(10^{14})) \text{ [GeV]} \]

Since the mass matrix \( M_R \) have strong hierarchy \( M_R \sim Y_{eR}^2 Y_e \), the lightest mass eigenvalue \( M_{R1} \) is too small for the naive thermal leptogenesis. However, leptogenesis may be achieved by the decay of second lightest neutrino \( \nu_{R2} \) with the maximal majorana phase \( \alpha_2/2 = \pi/2 \). Besides, the \( O(100) \text{ TeV} \) neutrino \( \nu_{R1} \) might have some relation to the IceCube gap.

Since \( CP \) phases are restricted in the first row and column, this idea suggests that \( CP \) violation of flavor mixing only comes from a chiral symmetry breaking of the first generation.

[1] Y. Fukuda et al. (Super-Kamiokande), Phys. Rev. Lett. 81, 1562 (1998), hep-ex/9807003.
[2] Q. R. Ahmad et al. (SNO), Phys. Rev. Lett. 87, 071301 (2001), nucl-ex/0106015.
[3] Q. R. Ahmad et al. (SNO), Phys. Rev. Lett. 89, 011301 (2002), nucl-ex/0204008.
[4] K. Eguchi et al. (KamLAND), Phys. Rev. Lett. 90, 021802 (2003), hep-ex/0212021.
[5] H. Fritzsch and Z.-z. Xing, Phys. Lett. B353, 114 (1995), hep-ph/9502297.
[6] H. Nishiura, K. Matsuda, and T. Fukuyama, Phys. Rev. D60, 013006 (2009), hep-ph/0902385.
[7] K. Matsuda, T. Fukuyama, and H. Nishiura, Phys. Rev. D61, 053001 (2004), hep-ph/0906433.
[8] Z.-z. Xing and H. Zhang, Phys. Lett. B569, 30 (2003), hep-ph/0304234.
[9] M. Bando, S. Kaneko, M. Obara, and M. Tanimoto, Prog. Theor. Phys. 112, 533 (2004), hep-ph/0405071.
[10] K. Matsuda and H. Nishiura, Phys. Rev. D74, 033014 (2006), hep-ph/0606112.
[11] S. Dev, Phys. Rev. D76, 013006 (2007), hep-ph/0703005.
[12] Y. Koide, Phys. Rev. D89, 015011 (2014), hep-ph/0702253.
[13] C. S. Lam, Phys. Lett. B507, 214 (2001), hep-ph/0104116.
[14] E. Ma and M. Raidal, Phys. Rev. Lett. 87, 011802 (2001), [Erratum: Phys. Rev. Lett.87,159901(2001)], hep-ph/0102255.
[15] R. K. S. Balaji, W. Grimus, and T. Schwetz, Phys. Lett. B508, 301 (2001), hep-ph/0104035.
[16] Y. Koide, H. Nishiura, K. Matsuda, T. Kikuchi, and T. Fukuyama, Phys. Rev. D66, 093006 (2002), hep-ph/0203333.
[17] T. Kitabayashi and M. Yasue, Phys. Rev. D67, 015006 (2003), hep-ph/0209294.
[18] Y. Koide, Phys. Rev. D69, 093001 (2004), hep-ph/0312207.
[19] A. Ghosal (2003), hep-ph/0304090.
[20] I. Aizawa, M. Ishiguro, T. Kitabayashi, and M. Yasue, Phys. Rev. D70, 015011 (2004), hep-ph/0405201.
[21] A. Ghosal, Mod. Phys. Lett. A19, 2579 (2004).
[22] R. N. Mohapatra and W. Rodejohann, Phys. Rev. D72, 053001 (2005), hep-ph/0507312.
[23] Y. Koide, Phys. Lett. B607, 123 (2005), hep-ph/0411280.
[24] T. Kitabayashi and M. Yasue, Phys. Lett. B621, 133 (2005), hep-ph/0504212.
[25] N. Habu and W. Rodejohann, Phys. Rev. D74, 017701 (2006), hep-ph/0603206.
[26] Z.-z. Xing, H. Zhang, and S. Zhou, Phys. Lett. B641, 189 (2006), hep-ph/0607091.
[27] Y. H. Ahn, S. K. Kang, C. S. Kim, and J. Lee, Phys. Rev. D73, 093005 (2006), hep-ph/0602160.
[28] A. S. Joshipura, Eur. Phys. J. C53, 77 (2008), hep-
[58] J. C. Gomez-Izquierdo and A. Perez-Lorenzana, Phys. Rev. D82, 033008 (2010), 0912.5210.
[59] T. Fukuyama, PTEP 2017, 033B11 (2017), 1701.04985.
[60] P. F. Harrison and W. G. Scott, Phys. Lett. B547, 219 (2002), hep-ph/0210197.
[61] W. Grimus and L. Lavoura, Phys. Lett. B579, 113 (2004), hep-ph/0305309.
[62] W. Grimus, S. Kaneko, L. Lavoura, H. Sawanaka, and M. Tanimoto, JHEP 01, 110 (2006), hep-ph/0510326.
[63] W. Grimus, S. Kaneko, L. Lavoura, H. Sawanaka, and M. Tanimoto, JHEP 01, 059 (2007), hep-ph/0610337.
[64] A. S. Joshipura and B. P. Kodrani, Phys. Lett. B670, 369 (2009), 0706.0953.
[65] B. Adhikary, A. Ghosal, and P. Roy, JHEP 10, 040 (2009), 0908.2686.
[66] A. S. Joshipura, B. P. Kodrani, and K. M. Patel, Phys. Rev. D79, 115017 (2009), 0903.2161.
[67] Z.-z. Xing and Y.-L. Zhou, Phys. Lett. B693, 584 (2010), 1008.4906.
[68] S.-F. Ge, H.-J. He, and F.-R. Yin, JCAP 1005, 017 (2010), 1001.0940.
[69] S. Gupta, A. S. Joshipura, and K. M. Patel, Phys. Rev. D85, 031903 (2012), 1112.6113.
[70] W. Grimus and L. Lavoura, Fortsch. Phys. 61, 535 (2013), 1207.1675.
[71] A. S. Joshipura and K. M. Patel, Phys. Lett. B749, 159 (2015), 1507.01235.
[72] Z.-z. Xing and Z.-h. Zhao, Rept. Prog. Phys. 79, 076201 (2016), 1512.04207.
[73] X.-G. He, Chin. J. Phys. 53, 100101 (2015), 1504.01560.
[74] P. Chen, G.-J. Ding, F. Gonzalez-Canales, and J. W. F. Valle, Phys. Lett. B753, 644 (2016), 1512.01551.
[75] R. Samanta, P. Roy, and A. Ghosal, JHEP 06, 085 (2018), 1712.06555.
[76] Z.-z. Xing and J.-y. Zhu, Chin. Phys. C41, 123103 (2017), 1707.03676.
[77] C. C. Nishi, B. L. Sánchez-Vega, and G. Souza Silva, JHEP 09, 042 (2018), 1806.07412.
[78] N. Nath, Z.-z. Xing, and J. Zhang, Eur. Phys. J. C78, 289 (2018), 1801.09931.
[79] R. Sinha, P. Roy, and A. Ghosal, Phys. Rev. D99, 033009 (2019), 1809.06615.
[80] M. J. S. Yang (2020), 2002.09152.
[81] H. Fritzsch and Z.-Z. Xing, Phys. Lett. B413, 396 (1997), hep-ph/9707215.
[82] I. Esteban, M. C. Gonzalez-Garcia, A. Hernandez-Cabezudo, M. Maltoni, and T. Schwetz, JHEP 01, 106 (2019), 1811.05487.
[83] C. Jarlskog, Phys. Rev. Lett. 55, 1039 (1985).
[84] P. Minkowski, Phys. Lett. 67B, 421 (1977).
[85] M. Gell-Mann, P. Ramond, and R. Slansky, Conf. Proc. C790927, 315 (1979).
[86] T. Yanagida, Conf. Proc. C7902131, 95 (1979).
[87] H. Georgi and C. Jarlskog, Phys. Lett. B86, 297 (1979).
[88] Z.-z. Xing, H. Zhang, and S. Zhou, Phys. Rev. D77, 113016 (2008), 0712.1419.
[89] S. Davidson and A. Ibarra, Phys. Lett. B535, 25 (2002), hep-ph/0202239.
[90] K. Hamaguchi, H. Murayama, and T. Yanagida, Phys. Rev. D65, 043512 (2002), hep-ph/0109030.
[91] M. Fukugita and T. Yanagida, Phys. Lett. B174, 45 (1986).
[92] O. Vives, Phys. Rev. D73, 073006 (2006), hep-ph/0512160.
[93] M. G. Aartsen et al. (IceCube), Phys. Rev. Lett. 111, 021103 (2013), 1304.5356.
[94] M. G. Aartsen et al. (IceCube), Science 342, 1242856 (2013), 1311.5238.