The Higgs boson $ZZ$ couplings in the Higgs-strahlung at the ILC

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Abstract

We derive the fully differential cross section of the Higgs-strahlung process $f\bar{f} \rightarrow Z \rightarrow Z(\rightarrow f_Z\bar{f}_Z)X(\rightarrow f_X\bar{f}_X)$, where $f$, $f_Z$, and $f_X$ are arbitrary fermions and $X$ is a spin-zero particle with arbitrary couplings to $Z$ bosons and fermions. This process with $f = e$ and $X = h$ ($h$ denotes the Higgs boson) is planned to be measured at the ILC. Using the derived fully differential cross section, we can obtain an analytical expression for an observable connected with the Higgs-strahlung and convenient to be measured at the ILC. Thus, as soon as this observable is measured, we will be able to put some constraints on the Higgs boson couplings to a pair of $Z$ bosons.
I. INTRODUCTION

Since the discovery \cite{1, 2} of the Higgs boson (denoted by $h$ in this paper) in 2012, it has been important to measure its couplings and CP properties. These measurements can prove or disprove the Higgs mechanism \cite{3} and can let us describe processes involving the Higgs boson more precisely.

The couplings of the Higgs boson are measured at the LHC by the CMS and ATLAS collaborations (see, for example, \cite{4, 5}). These couplings are also planned \cite{6} to be measured at the ILC. The latter measurements are expected to have lower backgrounds and to yield stricter constraints on a number of the Higgs boson parameters.

At the ILC, the Higgs couplings to a pair of $Z$ bosons ($hZZ$ couplings) will be measured in the Higgs-strahlung process $e^-e^+ \rightarrow Z \rightarrow Z(\rightarrow fZ\bar{f}_Z)h(\rightarrow f_h\bar{f}_h)$, where $f_Z = e, \mu, u, d, s, c, b$ and $f_h = \tau, b$. Many papers (see, for example, Refs. \cite{7–10}) concern the process $e^-e^+ \rightarrow Z \rightarrow Z(\rightarrow fZ\bar{f}_Z)h$, not considering a decay of the Higgs boson. Consideration of such a decay allows for the fact that the Higgs boson is off-shell, thereby allowing us to study the Higgs-strahlung more precisely. Moreover, the Higgs-strahlung with a decay of the Higgs boson is studied in Ref. \cite{11} (see Eq. (A2) there). We generalize Eq. (A2) to the case of an arbitrary polarization of the positron beam.

In Section \ref{II} we derive the fully differential cross section of this process for a spin-zero Higgs boson, accounting for the beyond the Standard Model (SM) $hZZ$ and $hff$ couplings and for arbitrary electron and positron polarizations. The $hZZ$ couplings can be extracted from the derived formula once some observables are measured. Such observables can be defined using the MELA approach (see, for instance, Refs. \cite{12–15}), which is used \cite{16, 17} by the CMS and ATLAS collaborations. As an example, in Section \ref{III} we define an asymmetry whose non-zero value would imply a non-zero value of the $hZZ$ CP-odd coupling.

II. FULLY DIFFERENTIAL CROSS SECTION

We consider the process

$$ff \rightarrow Z \rightarrow ZX \rightarrow f_Z\bar{f}_Zf_X\bar{f}_X$$

(see Fig. \ref{fig:process}), where $f$, $f_Z$, and $f_X$ are some fermions, $f_Z \neq f_X$, and $X$ is a particle with zero spin, arbitrary couplings to a pair of $Z$ bosons and arbitrary couplings to a fermion-
antifermion pair. This scattering is going to be measured at the ILC in the case \( f = e, \) \( X = h, \) \( f_Z = e, \mu, u, d, s, c, b, \) and \( f_h = \tau, b. \)

Due to the energy-momentum conservation in process (1),

\[
a_1 = s, \quad a_2 \in (4m_{f_Z}^2, (\sqrt{a_1} - \sqrt{a_X})^2), \quad a_X \in (4m_{f_X}^2, (\sqrt{a_1} - 2m_{f_Z})^2),
\]

where \( a_1, a_2, \) and \( a_X \) are the squared invariant masses of the \( Z \) boson (called \( Z_1 \)) produced by the fermion-antifermion pair \( f \bar{f}, \) of the \( Z \) boson (called \( Z_2 \)) produced together with the boson \( X, \) and of the boson \( X \) itself, respectively, \( s \) is the squared invariant energy of \( f \bar{f}, \)

\( m_{f_Z} \) and \( m_{f_X} \) are the masses of the fermions \( f_Z \) and \( f_X \) respectively.

The amplitude \( A(\lambda_1, \lambda_2) \) of the transition \( Z \to ZX \) is analogous to the \( HVV \) amplitude in Ref. [4] (see Eq. (1) there) and to the \( XZZ \) amplitude in Ref. [18] (Eq. (7)) and in [19] (Eq. (3)):

\[
A(\lambda_1, \lambda_2) = \sqrt{2G_Fm_Z^2} \left( g_1(a_1, a_2) - \frac{a_1 + a_2 - a_X}{m_Z^2} (e_1 \cdot e_2^*) + 2g_2(a_1, a_2) \frac{m_Z^2}{m_Z^2} - (e_1 \cdot q_2)(e_2^* \cdot q_1) \right.
\]

\[
+ \left. 2g_3(a_1, a_2) \frac{m_Z^2}{m_Z^2} \varepsilon_{\mu \nu \rho \sigma} q_1^\mu q_2^\nu e_1^\rho (e_2^*)^\sigma \right),
\]

where \( \lambda_j, e_j, \) and \( q_j \) are the helicity, polarization 4-vector, and 4-momentum of the boson \( Z_j \) respectively \( (j = 1, 2), \) \( G_F \) is the Fermi constant, \( m_Z \) is the pole mass of the \( Z \) boson, \( g_1(a_1, a_2), g_2(a_1, a_2), \) and \( g_3(a_1, a_2) \) are some complex-valued functions on \( a_1 \) and \( a_2 \) — we call these functions \( XZZ \) couplings, \( \varepsilon_{\mu \nu \rho \sigma} \) is the Levi-Civita symbol \( (\varepsilon_{0123} = 1). \) In Ref. [4]
the XZZ couplings are denoted as \( a_1, a_2, \) and \( a_3 \) — we denote them as \( g_1, g_2, \) and \( g_3 \) respectively to avoid confusion.

At the tree level, the XZZ couplings are connected with the CP parity of the boson \( X \), as shown in Table I. In the SM \( g_1 = 1 \) and \( g_2 = g_3 = 0 \).

| \( g_1 \) | \( g_2 \) | \( g_3 \) | \( CP_X \) |
|--------|--------|--------|--------|
| any    | any    | 0      | 1      |
| 0      | 0      | \( \neq 0 \) | \( -1 \) |
| \( \neq 0 \) | any    | \( \neq 0 \) | indefinite |
| any    | \( \neq 0 \) | \( \neq 0 \) | |

The amplitude of the decay \( X \rightarrow f_X \bar{f}_X \) is [12]:

\[
A_{X\rightarrow f_X f_X} = -\sqrt{2} G_F m_{f_X} \bar{u}_{f_X} (\rho_1 + \rho_2 \gamma^5) v_{f_X},
\]

where \( u \) and \( v \) are the Dirac spinors, \( \rho_1 \) and \( \rho_2 \) are some complex numbers which we call the \( Xff \) couplings.

Using the helicity formalism and neglecting \( m_f, m_{f_Z}, \) and \( m_{f_X} \) everywhere save the \( m_{f_X} \) factor in Eq. (4), we derive the fully differential cross section of process (1):

\[
\frac{d^7\sigma}{da_2 da_X d\theta_Z d\theta_{f_Z} d\phi_{f_X} d\phi_{f_X}} = G_4^4 m_Z^6 m_{f_X}^2 \frac{a_2 a_X \lambda^{1/2}(s, a_2, a_X)}{s D(s) D(a_2) D_X(a_X)} (a_{f_Z}^2 + v_{f_Z}^2) (a_{f_Z}^2 + v_{f_Z}^2)
\times (|\rho_1|^2 + |\rho_2|^2) \sin \theta_Z \sin \theta_{f_Z} \sin \theta_{f_X} \left[ (|A_{||}|^2 + |A_{\perp}|^2) \left( P_1 (1 + \cos^2 \theta_{f_Z}) (1 + \cos^2 \theta_{f_Z}) + 4 P_2 A_{f_Z} \cos \theta_{f_Z} \cos \theta_{f_Z} \right) + \text{Re}(A_{||}^* A_{\perp}) \cdot 4 \left( P_1 A_{f_Z} \cos \theta_{f_Z} (1 + \cos^2 \theta_{f_Z}) + P_2 \cos \theta_{f_Z} (1 + \cos^2 \theta_{f_Z}) \right) + |A_0|^2 \cdot 4 P_1 \sin^2 \theta_Z \sin^2 \theta_{f_Z} - 4 \sqrt{2} \sin \theta_Z \sin \theta_{f_Z} \sin \phi \right]
\times \left( (\text{Re}(A_{||}^* A_{||}) \cos \phi + \text{Im}(A_{||}^* A_{\perp}) \sin \phi)(P_2 A_{f_Z} + P_1 \cos \theta_Z \cos \theta_{f_Z}) + (\text{Im}(A_{||}^* A_{||}) \sin \phi + \text{Re}(A_{||}^* A_{\perp}) \cos \phi)(P_1 A_{f_Z} \cos \theta_Z + P_2 \cos \theta_{f_Z}) \right) + P_1 \sin^2 \theta_Z \sin^2 \theta_{f_Z}
\times \left( (|A_{||}|^2 - |A_{\perp}|^2) \cos 2\phi + \text{Im}(A_{||}^* A_{\perp}) \cdot 2 \sin 2\phi \right),
\]

where (see Fig. [2])
• $\theta_Z$ is the angle between the momentum $p_f$ of the fermion $f$ in the $f \bar{f}$ rest frame and the momentum $q_2$ of the Z boson $Z_2$ in the same frame;

• $\theta_{fz}$ is the angle between $q_2$ and the $f_Z$ momentum $p_{fz}$ in the $Z_2$ rest frame;

• $\phi$ is the azimuthal angle between the plane spanned by the vectors $p_f$ and $q_2$ and the plane spanned by $q_2$ and $p_{fz}$;

• $\theta_{fx}$ is the angle between the $X$ momentum in the $f \bar{f}$ rest frame and the $f_X$ momentum in the $X$ rest frame;

• $\phi_{fx}$ is the azimuthal angle of the fermion $f_X$;

• $\lambda(x, y, z) \equiv x^2 + y^2 + z^2 - 2(xy + xz + yz)$;

• $D(x) \equiv (x - m_Z)^2 + (m_Z \Gamma_Z)^2$, $D_X(x) = (x - m_X)^2 + (m_X \Gamma_X)^2$;

• $\Gamma_Z (\Gamma_X)$ is the total width of the Z (X) boson;

• $a_f$ is the projection of the weak isospin of a fermion $f$, $v_f \equiv a_f - 2 q_f e \sin^2 \theta_W$, $q_f$ is the electric charge of the fermion $f$, $e$ is the electric charge of the positron, $\theta_W$ is the weak mixing angle, $A_f \equiv \frac{2 a_f v_f}{a_f^2 + v_f^2}$;

• $A_0 \equiv g_1 \frac{s + a_2 - a_X}{2 \sqrt{s a_2}} g_2$, $A_{\parallel} \equiv \sqrt{2} \left( g_1 - \frac{s + a_2 - a_X}{m_Z^2} g_2 \right)$, $A_{\perp} \equiv i \sqrt{2} \frac{\lambda^{1/2}(s, a_2, a_X)}{m_Z^2} g_3$;

• $P_1 \equiv 1 - P_{e^-} P_{e^+} - (P_{e^-} - P_{e^+}) A_f$, $P_2 \equiv (1 - P_{e^-} P_{e^+}) A_f - P_{e^-} + P_{e^+}$;

• $P_{e^-} (P_{e^+})$ is the electron (positron) beam longitudinal polarization.

According to Eq. (5), all the possible directions of the $f_X$ momentum in the $X$ rest frame are equiprobable. Having measured an observable describing Higgs-strahlung (1), one will be able to use Eq. (5) and to put some constraints on the functions $A_0$, $A_{\parallel}$, and $A_{\perp}$, thus getting possible intervals for the $XZZ$ couplings $g_1$, $g_2$, and $g_3$.

Integration of Eq. (5) with the narrow-width approximation for both $Z$ and $X$ boson yields the total cross section of the Higgs-strahlung:

$$
\sigma = \frac{G_F^4 m_Z^2 m_X m_X^2}{288 \pi^4 \Gamma_Z \Gamma_X} \frac{\lambda^{1/2}(s, m_Z^2, m_X^2)}{s D(s)} (a_f^2 + v_f^2) (a_{fz}^2 + v_{fz}^2) (|\rho_1|^2 + |\rho_2|^2) P_1 \times \sum_{p=0,\|,\perp} |A_{p}|^2 |_{a_2=m_Z^2, a_X=m_X^2}.
$$

(6)

Since $\sigma \sim P_1$, $\sigma$ has its largest value at $P_{e^-} = -1$ and $P_{e^+} = 1$ if $A_f > 0$. 

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FIG. 2. Kinematics of process (1). The momenta of $f$, $\bar{f}$, $Z_1$, $X$, and $Z_2$ are shown in the $ff\bar{f}$ rest frame, the momenta of $f_{Z_1}$ and $\bar{f}_{Z_2}$ are displayed in the $Z_2$ rest frame, the momenta of $f_X$ and $\bar{f}_X$ are described in the $X$ rest frame. The $z$ axis is co-directional with the $X$ momentum while the $x$ and $y$ axes are arbitrary axes forming a right-handed system with the $z$ axis.

III. FORWARD-BACKWARD ASYMMETRY

The forward-backward asymmetry with respect to the polar angle $\theta_Z$ is defined as

$$A_{FB} \equiv \frac{1}{\sigma} \left( \int_{\pi/2}^{\pi} d\theta_Z \frac{d\sigma}{d\theta_Z} - \int_{\pi/2}^{\pi} d\theta_Z \frac{d\sigma}{d\theta_Z} \right).$$  \hspace{1cm} (7)

In the narrow-width approximation for the $Z$ and $X$ bosons

$$A_{FB} = \frac{3P_2}{4P_1} \frac{\text{Re}(A_{\parallel}^* A_{\perp})}{\sum_{\rho=0, \parallel, \perp} |A_{\rho}|^2} \bigg|_{a_2=m_{Z_1}^2,a_X=m_X^2}. \hspace{1cm} (8)$$

In Fig. 3 we show a plot of Eq. (8) for $f = e$ and $X = h$. To demonstrate the sensitivity of $A_{FB}$ to the anomalous $hZZ$ couplings, we choose $g_1 = 1$, $g_2 = 0$, $g_3 = 0.5i$ because this set of values of the $hZZ$ couplings is not excluded by the present experimental data (see Ref. [4]). In Table III we list some values used for plotting Eq. (8). The presented electron and positron beam polarizations are planned to take place at the ILC for $\sqrt{s} = 250, 350,$ or $500$ GeV. The values of $A_e$, $m_h$, and $m_Z$ are taken from Ref. [20].
FIG. 3. $A_{FB}$ as a function of $\sqrt{s}$ in the case $f = e$, $X = h$, $g_1 = 1$, $g_2 = 0$, $g_3 = 0.5i$.

TABLE II. The experimental values we used while plotting Eq. (8).

|                  |   |
|------------------|---|
| $P_{e^-}$        | $-0.8$ |
| $P_{e^+}$        | $0.3$  |
| $A_e$            | $0.1515$   |
| $m_h$            | $125.09 \text{ GeV}$ |
| $m_Z$            | $91.1876 \text{ GeV}$ |

In the SM $A_{FB} = 0$, according to Eq. (8). If $g_1 = 1$, $g_2 = 0$, and $g_3 = 0.5i$, $A_{FB}$ significantly differs from zero (see Fig. 3). That is why measurements of the forward-backward asymmetry can put constraints on the $hZZ$ couplings.

We can define a different observable connected with process (1) and derive an analytical expression for this observable, using Eq. (5). Thus, once the observable is measured, we can obtain constraints on $g_1$, $g_2$, and $g_3$.

IV. CONCLUSIONS

We have derived the fully differential cross section of the Higgs-strahlung process $f\bar{f} \rightarrow Z \rightarrow ZX \rightarrow f_Z\bar{f}_Zf_X\bar{f}_X$, where $f$, $f_Z$, and $f_X$ are arbitrary fermions, $X$ is a particle with zero spin and arbitrary $ZZ$ and $ff$ couplings. This process with $f = e$, $X = h$, $f_Z = e, \mu, u, d, s, c, b$, and $f_h = \tau, b$ is going to be measured at the ILC. Once an observable related to the Higgs-strahlung process and suitable for measurement at the ILC is defined, we can obtain an analytical expression for this observable, using the derived fully differential cross section. Such an observable can be constructed with the MELA approach (see Refs. [12–15]).
When this observable is measured, one can derive constraints on the $hZZ$ couplings.

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