Optimal Planar Electric Dipole Antennas
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Abstract—Considerable time is often spent optimizing antennas to meet specific design metrics. Rarely, however, are the resulting antenna designs compared to rigorous physical bounds on those metrics. Here we study the performance of optimized planar meander line antennas with respect to such bounds. Results show that these simple structures meet the lower bound on radiation Q-factor (maximizing single resonance fractional bandwidth), but are far from reaching the associated physical bounds on efficiency. The relative performance of other canonical antenna designs is compared in similar ways, and the quantitative results are connected to intuitions from small antenna design, physical bounds, and matching network design.

Index Terms—Antenna theory, optimization methods, numerical methods, Q-factor, efficiency.

I. INTRODUCTION

ANTENNA parameters such as gain, Q-factor, and efficiency are limited by the geometry made available for a given design. Given bounds on these parameters under certain constraints, a designer can rapidly assess the feasibility of design requirements. This feasibility assumes the existence of an “optimal antenna” design which approaches the bounds on certain specified parameters. Synthesis of an optimal antenna is not a trivial task, and it remains to be demonstrated how an antenna designed to be optimal in one parameter (e.g., radiation Q-factor) performs relative to bounds on other parameters (e.g., efficiency). The goal of this paper is to discuss the synthesis and analysis of optimal antennas starting from classical antenna topologies.

Many strategies have been employed to optimize antennas. Heuristic optimization methods such as genetic algorithms [1], [2] and particle swarm optimization have the advantage of generating design geometries outside of the antenna designer’s usual catalog [3]–[5]. Such techniques have been used to design optimal antennas with radiation Q-factors very close to the physical bounds [6], though the resulting designs are computationally expensive to produce and offer only rough insight into guidelines for designing optimal antennas in volumes with arbitrary shapes and electrical size. Conversely, canonical antenna designs were shown [7], [8] to reach the lower bound on radiation Q-factor, but the question remains whether these designs represent optimal solutions over arbitrary electrical sizes and whether they are optimal in other parameters, e.g., radiation efficiency and input impedance. The cost of matching an optimal antenna design to arbitrary impedances is also unclear, regardless if matching is performed on the antenna itself or through external networks.

In this paper we study whether there exists a simple “recipe” for an optimal planar antenna with respect to radiation Q-factor and radiation efficiency. In doing so, we ask whether, when prescribed with some form factor and electrical size, a simple design can be readily employed to achieve an antenna whose properties are sufficiently close to their bounds. The strategy adopted here is to optimize parameterizations of canonical antenna geometries known for good behavior in certain parameters. The examples studied here give quantifiable results to this end, i.e., how to design certain kinds of optimal antennas.

Along the way, we address the crossover of optimality of antennas across different performance parameters, e.g., do minimum radiation Q-factor antennas have inherently high radiation efficiency? Also discussed are the impacts of certain constraints, particularly those related to an antenna’s input impedance, on optimized parameters.

We stress out that this work differs significantly from other works on antenna optimization through parametric, heuristic, or metaheuristic means which typically involves the iterative evaluation and modification of designs until a local optimum or design goal is reached. Here, instead, we focus on designing antenna performance with respect to physical bounds, which provide an absolute measure in judging the quality of the synthesized design.

II. MINIMUM RADIATION Q-FACTOR OF PLANAR TM ANTENNAS

We begin by studying the synthesis of electrically small dipole-like (TM) antennas with minimal radiation Q-factor $Q_{rad}$ (see Box 1 and Box 2). This leads to increased impedance bandwidth, however, the lower bound on radiation Q-factor increases rapidly as an antenna design region becomes smaller (see Box 2). Thus, obtaining low Q-factor $Q_{rad}$ is a key objective and challenge in the design of electrically small antennas.
A. Synthesis of meander line antennas

Drawing from the prevalence of meander line antennas in applications requiring electrically small planar antennas [24], [39], as well as previous work studying their optimality in radiation Q-factor [8], we focus on determining whether meander lines present a consistent, simple solution, to obtaining minimum radiation Q-factor at arbitrary frequencies within rectangular design regions. Here, and throughout Section III, we specify a rectangular design region of fixed aspect ratio (\(L/W = 2\)). The impact of varying aspect ratios is demonstrated and discussed in Section II-B.

From the many possible meander line shapes (for example, rectangular, triangular, sinusoidal [39]) we have chosen the simple parametrization from Fig. 1. Thin wire versions of such antennas were previously shown to reach the lower bound on radiation Q-factor \(Q_{\text{rad}}\) for their corresponding rectangular design regions with electrical sizes near \(ka = 0.3\) [8]. Here, we use the parameterization in Fig. 1 to optimize the meander line antenna for resonance by requiring the magnitude of the normalized input reactance \(X_{\text{in}}/R_{\text{in}}\) to be smaller than a specified tolerance, \(|X_{\text{in}}/R_{\text{in}}| < 10^{-3}\). This procedure is repeated at many frequencies (electrical sizes, values of \(ka\)) to obtain a set of antenna designs, each resonant at a specific frequency. The Q-factors \(Q_{\text{rad}}\) of the resulting designs were then calculated in AToM [2] and compared to the bounds discussed in Box 2. The comparison is shown in Fig. 2. Note that the value of Q-factor \(Q_{\text{rad}}\) is just weakly dependent on dissipation factor (see Box 3) provided that dissipation is not exceedingly high.

In order to verify the computed data in Fig. 2, antenna design sample with \(ka = 0.42\) was scaled to 1.4 GHz and fabricated on a 50 \(\mu\)m-thick FR4 substrate to limit dielectric effects. The input impedance of the prototype was measured using a differential technique [40] and has been used to estimate the Q-factor via the \(Q_{\text{Z}}\) formula [12]. Radiation efficiency of the antenna was measured via a multiport near-field method [41] and was used to evaluate radiation Q-factor and its uncertainty, see triangular marker and corresponding error bar in Fig. 2.

Figure 2 illustrates that a simple parametrization, such as the one from Fig. 1, is able to closely approach the radiation Q-factor bound limited to TM radiation \(Q_{\text{rad}}^{\text{TM}}\) (see Box 2) in the entire frequency range of electrically small antennas. From this, it is possible to conclude that a complex design (e.g., the parameterizations found in [42]) is not needed to reach the lower bound.

The absolute lower bound for radiation Q-factor \(Q_{\text{rad}}^{\text{lb}}\) is unreachable by this meander line antenna since its planar geometry and single feed scenario does not allow for an efficient excitation of combined TE and TM radiation. This contrasts to three-dimensional (e.g., spherical) geometries [25], [36], where the dual mode behavior can be realized by a single feed network.

Parameters of the self-resonant designs from Fig. 2 are shown in Fig. 3. Design curves are fit to the optimized parameters data using a polynomial fit with good agreement. While some of the curves from Fig. 3 can be found in [39] for several parametrization, here all the designing curves are

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**Box 1. Q-factor**

The Q-factor of an antenna system tuned to resonance is defined as [9]

\[
Q = \frac{2\omega W_{\text{sto}}}{P_{\text{rad}} + P_{\text{diss}}},
\]

where \(W_{\text{sto}}\) represents the cycle mean stored energy, while \(P_{\text{rad}}\) and \(P_{\text{diss}}\) denote radiated power and power dissipated as heat, respectively. In single-resonance systems, lower Q-factor implies larger fractional impedance bandwidth \(B\) by an inverse relationship [10]–[12]

\[
B \sim Q^{-1}.
\]

Evaluated at a single frequency via (1), Q-factor thus becomes a convenient measure of the frequency selectivity of a system [10]–[22]. Calculation of a system’s Q-factor can be carried out by a variety of approaches, from impedance-based techniques [12] to methods based on the evaluation of stored energy directly [6], [17]. All of these approaches generally agree for electrically small, narrow-band antennas, see [23] for complete discussion and bibliography.

The radiation Q-factor \(Q_{\text{rad}}\), in which only radiated power is considered, can be expressed in terms of Q in (1) and radiation efficiency \(\eta\) (see Box 3, (5)) as

\[
Q_{\text{rad}} = Q/\eta \sim (B\eta)^{-1}.
\]

**Box 2. Lower bounds on radiation Q-factor**

The approximate inverse proportionality between the Q-factor and the fractional bandwidth (see Box 1) induced considerable effort in lowering the Q-factor for spherical [10], [13], [24]–[27] and arbitrarily shaped antennas [28]–[34]. The sole focus on the radiation Q-factor \(Q_{\text{rad}}\) in these works avoids the undesired possibility of reducing Q-factor \(Q\) by degrading radiation efficiency.

For electrically small antennas, the lower bound on the \(Q_{\text{rad}}\), here denoted as \(Q_{\text{rad}}^{\text{lb}}\), is a combination of electric and magnetic dipoles [10], [26], [32], [35], [36]. In general, it is challenging to excite such a current with a single feeding position. A constrained minimization which is more representative for single port antennas is to restrict the radiation to TM (electric dipole) modes, yielding the lower bound \(Q_{\text{rad}}^{\text{lb}}\), see, e.g., [37].

The importance of Q-factor bounds arises from two key properties. First, the Q-factor bound represents the physical lower bound among all possible currents contained within the considered region. It thus presents an absolute measure against which to compare the performance of different antenna designs. Practical feasibility of designing antennas which reach various bounds remains an open question. Second, both \(Q_{\text{rad}}\) and \(Q_{\text{rad}}^{\text{lb}}\) scale approximately as \((ka)^{-3}\) for electrically small antennas (\(ka < 1\), cf. [38]). Here \(k\) is the free-space wavenumber and \(a\) is the radius of the smallest circumscribing sphere. This \((ka)^{-3}\) scaling is the root cause of the limited bandwidth in electrically small antennas. The associated geometry coefficients for certain shapes are shown in Table I, where \(\gamma_0\) denotes free-space impedance and \(R_s\) denotes surface resistance.
Table I

| Lower bounds on radiation Q-factors and efficiency in the limit of electrical size $ka \to 0$ for a sphere, a cylindrical tube, a rectangle, a thin strip dipole and a square loop. |
|-----------------|-----------------|-----------------|
| $(ka)^3 Q^{lb}_{rad}$ | $(ka)^3 Q^{lb,\text{TM}}_{rad}$ | $(ka)^4 i_0/R_s \delta$ |
| 1 | 3/2 | 3 |
| $\ell/2\pi$ | 4.5 | 4.6 | 59 |
| $\ell$ | 4.3 | 5.2 | 42 |
| $\ell/50$ | 16 | 16 | 3400 |
| $0.9\ell$ | 5.3 | 7.4 | 130 |
| $\ell$, $W$ |

related back to Fig. 2 in which the Q-factor $Q_{rad}$ is minimized. The presented data series can therefore be used for designing meandered dipoles approaching lower bounds on radiation Q-factor for TM antennas. It should, however, be noted that design curves from Fig. 3 depends on the used parametrization and are valid only for $L/W = 2$ and $w/s \approx 1$.

B. Varying aspect ratios

Meander line antennas, introduced in the previous section, are now studied for various $L/W$ and $w/s$ aspect ratios and compared against the fundamental bounds calculated for given form factor.

In all cases, the value of radiation Q-factor $Q_{rad}$ is normalized with respect to the minimal TM radiation Q-factor. Generally, Fig. 4 shows that the minimal values can closely be approached for various $L/W$ aspect ratios. Slightly better performance is observed for higher $L/W$ ratios, however, at the cost of higher absolute bound on radiation Q-factor see top panel of Fig. 4.

With respect to the varying $w/s$ ratio, slightly better performance is observed for higher values, i.e., wider metallic strips. The differences become negligible for small values of $ka$, see Fig. 5. Notice, however, that this behavior is substantially changed while the ohmic losses are introduced, mainly since the spatial proximity of out-of-phased currents degrades the radiation and enhance the ohmic losses [43].

C. The impact of impedance matching on Q-factor $Q_{rad}$

The designs obtained above are all self resonant ($X_{\text{in}} \approx 0$), but no constraint was placed on the value of the input resistance $R_{\text{in}}$. In most practical cases, the objective antenna input resistance is not driven by any antenna consideration but is set by the radio frequency electronic equipment to be interfaced with a particular antenna. Transmission lines and active receivers based on Low Noise Amplifiers (LNA) often require matching to 50 Ω. However, where devices with complex impedances are used, antenna resonance may not be ideal for conjugate matching and maximum power transfer. For example, a typical Power Amplifier (PA) output
impedance is complex [44], with an input resistance lower than 50 Ω and an inductive (positive) reactive component. Similarly, passive RFID receivers based on Schottky diode rectifiers typically exhibit input resistances lower than 50 Ω and strong capacitive (negative) reactance [45]. Examples of nominal impedances $Z_0$ for these systems are listed in Table II.

Antennas may be designed to have input impedances which conjugate match a desired load. However, any of the designs shown in Fig. 2 can be conjugate matched to an arbitrary complex impedance $Z_0$ through an L-network consisting of two reactive components [46]. In many instances, the stored energies within these reactances will raise the radiation Q-factor of the system. To assess the cost of this form of simple matching, we select the design in Fig. 2 corresponding to self-resonant frequency and lower the overall radiation Q-factor.

A typical frequency dependence of this cost is depicted in Fig. 6 while the dependence on matching impedance is depicted in Fig. 7. We observe that it is generally possible to transform the resonant antenna impedance to an arbitrary real value with minimal increase in radiation Q-factor. As expected, adding a reactive component to the real-valued (resonant) antenna impedance necessarily increases radiation Q-factor, though this increase is on the order of 30% for the most extreme case examined here. Additionally, Fig. 6 shows that it is often possible to move slightly away from the self-resonant frequency and lower the overall radiation Q-factor by a small amount. Nonetheless, the minimum radiation Q-factor of the matched antenna is, for practical values of the matching impedance, within the vicinity of the self-resonance of the antenna.
The importance of Q-factor is its relation to fractional bandwidth which is predicated on simple, single resonance behavior [12]. We demonstrate that the low variance in Q-factor corresponds to consistent realized bandwidth when L-networks are used to conjugate match an antenna to an arbitrary impedance. Figure 8 shows the power delivered $P_{del}$ to the meander line antenna studied above using a matched source ($Z_0 = R_{in}$) as well as with L-networks designed to match the antenna to the three complex impedances of practical interest in Table II. In each case, a network tunes the antenna to the desired (possibly complex) impedance at its natural resonant frequency. The frequency profile of the mismatch factor [46], [47]

$$\tau = \frac{P_{del}}{P_{cm}} = \frac{4R_{in}R_0}{|Z_{in}^m + Z_0|^2}$$

is nearly identical in all four cases, in agreement with the predictions based on the relatively invariant Q-factor across these cases. Here, $Z_{in}^m$ is the antenna impedance including the tuning network, $P_{del}$ is the power delivered to the antenna, and $P_{cm}$ is the power delivered under a conjugate match condition. It is necessary to point out that we have assumed non-dispersive matching impedances, i.e., $\partial Z_0/\partial \omega = 0$. In practice, the matching impedance may be dispersive within the band of interest, in which case the relation between Q-factor and bandwidth described in [12] ceases to be valid. However, inclusion of a dispersive load impedance may not necessarily cause major changes to the realized bandwidth due to the already heavily frequency-dependent nature of the impedance of high Q-factor antennas.

The results in Figs. 7 and 8 numerically suggest that there is little cost in bandwidth to match a self-resonant antenna to arbitrary impedances. However, further considerations reveal why it is of practical importance to design an antenna with a given impedance, rather than relying on this form of matching. First, the use of lumped components increases complexity and cost of an antenna system and the required component values for the L-networks described in this section may not be realizable. Second, lumped components made of any practical, lossy material (e.g., metallic inductors) increase the net loss in antenna system while not adding any potential radiation mechanism. This guarantees a decrease in overall efficiency, particularly in high Q-factor antennas [48]. Additionally, tunability or the use of broadband multiple resonance matching may benefit from the design of an antenna with specific impedance characteristics, e.g., to increase the radiation resistance [8], [49].

Fig. 6. Matched radiation Q-factor of a selected meander line antenna for several impedance matching scenarios listed in Table II. Matching to each impedance is accomplished via a two-element reactive L-network. In cases where several L-networks exist for a given matching impedance, the network with the lowest stored energy has been used. Abrupt jumps of the matched radiation Q-factor curves result from non-existence of matching by two inductances in certain frequency ranges. This double inductance matching condition. It is necessary to point out that we have assumed non-dispersive matching impedances, i.e., $\partial Z_0/\partial \omega = 0$. In practice, the matching impedance may be dispersive within the band of interest, in which case the relation between Q-factor and bandwidth described in [12] ceases to be valid. However, inclusion of a dispersive load impedance may not necessarily cause major changes to the realized bandwidth due to the already heavily frequency-dependent nature of the impedance of high Q-factor antennas.

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Fig. 7. Matched radiation Q-factor of a selected meander line antenna for varying complex matching impedance normalized to radiation Q-factor of bare antenna. For each complex impedance, the meander line is conjugate matched at its self-resonance (circle mark in Fig. 6, $\kappa a \approx 0.479$) using the lossless L-network matching circuit with lowest Q-factor. The markers denote the three impedances from Fig. 6 and Table II.

Fig. 8. Frequency dependence of normalized power delivered $P_{del}/P_{cm}$ to the meander line antenna studied in Figs. 6 and 7. The separate curves correspond to devices exhibiting the three practically relevant impedances from Table II and to a device with impedance corresponding to that of the meander line at its self-resonance (“Matched”). In each case, the antenna is conjugate matched using an L-network at its self resonant frequency, $\kappa a \approx 0.479$. The $-3$ dB bandwidths 50% power delivered bandwidths $B_{-3 \, dB}$ for each scenario are also listed.
III. RADIATION EFFICIENCY OF Q-OPTIMAL ANTENNAS

The previous section demonstrated that meander line antennas are nearly optimal with respect to radiation Q-factor, including the cases when matching to realistic complex impedances is desired. This section studies how these antennas perform with respect to another critical antenna metric: radiation efficiency (see Box 3). Specifically, we examine their performance with respect to radiation efficiency bounds (see Box 4).

Before presenting the radiation efficiency of matched meander line antennas it is necessary to deal with losses in the matching circuit since, similarly to the case of Q-factor, any matching circuit with finite losses will worsen the overall efficiency of the antenna system. Throughout this section we will assume that all matching networks are composed of lossless capacitors and lossy inductors\(^1\). The inductors are further assumed to be planar, made of the same material (metallic sheet, surface resistivity \(R_s\)) as the antenna itself. Under such restrictions it is possible to estimate the loss added by a matching network quite precisely using data from Fig. 9, which shows the normalized reactance, \((10^3/k_\alpha L) (R_s/\eta_0) (X_L/R_L)\), of several spiral inductors as a function of their electrical size. Here \(\eta_0\) denotes the free space impedance. The normalized reactance in Fig. 9 is independent on surface resistance \(R_s\) and, at small electrical size, just weakly dependent on number of turns and frequency, consistent with classical relations for helical air-core inductors [50]. A conservative value \((10^3/k_\alpha L) (R_s/\eta_0) (X_L/R_L) = 66\) will be used in this section to determine losses of all inductors within the L-matching network, assuming further that inductors are always ten times smaller in electrical size than the antenna, i.e., \(a_L = a/10\). This last assumption enforces the use of an electrically small, approximately lumped element, matching network.

\(^1\)Q-factors of lossy capacitors are typically much higher than those of lossy inductors.

Lossy elements with the above mentioned specifications are used to match the meander studied in Figs. 6–8 to impedance \(Z_0 = 50\,\Omega\) over a band of interest near the meander line’s self-resonant frequency. The resulting radiation Q-factor and efficiency (here presented in the form of dissipation factor, \(\delta\)) are depicted in Fig. 10 as functions of frequency (scaled as electrical size \(k_\alpha\)). The figure reiterates the previously-observed near-optimal performance of meander line antennas with respect to radiation Q-factor, but, surprisingly, shows a rather poor performance with respect to radiation efficiency. This metric is, at the self-resonance frequency of the antenna, almost one order of magnitude worse than the value of the physical bound (see Box 4). Similarly to radiation Q-factor, dissipation factor reaches its minimum in the vicinity of the resonance frequency, at least in the case of realistic values of matching impedances used here.

Within the used normalization of dissipation factor and radiation Q-factor, it is reasonable to represent the data from Fig. 10 as a two dimensional curve (radiation Q-factor vs. dissipation factor) parametrized by frequency, see Fig. 11. The figure also shows the Pareto bound (represented by the black line) evaluated by the method from [51], which demonstrates the optimal trade-off between radiation Q-factor and dissipation factor for the given design geometry and frequency. The Pareto bound has been evaluated at \(k_\alpha = 0.5\), but, due to the used normalization, it is almost independent on electrical size. The Pareto bound was evaluated for a combination of TM and TE modes which, as normalized to the TM bound \(Q_{rad, TM}\), gives values lower than one. The reason for this particular normalization is that TM bounds represent meaningful limit of one-port planar antennas.

The two-dimensional plot in Fig. 11 represents a complete comparison of various antenna designs with respect to matched efficiency and matched radiation Q-factor. An example of such comparison is shown in Fig. 12, where the normalized and frequency-parameterized \(Q–\delta\) curves are drawn for several small antenna designs within the same

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**Fig. 9.** Normalized reactances of selected rectangular spiral inductors. The quantities \(X_L, R_L,\) and \(R_s\) denote input reactance, input resistance, and surface resistance, respectively. The radius \(a_L\) defines the smallest sphere circumscribing the inductor.

**Fig. 10.** Normalized radiation Q-factor and normalized dissipation factor of a selected meander line antenna matched to 50\,\Omega over a range of frequencies (scaled here as electrical size \(k_\alpha\)). The self-resonance of the antenna \((k_\alpha = 0.47)\) is denoted on each trace with a circular marker while the minimum of each trace is also marked. The same data are also plotted as a curve parameterized by frequency in Fig. 11.
design specifications. Figure 12 clearly presents the superior performance in efficiency and Q-factor of simple meander line antennas shown in Fig. 1 with respect to other designs. It also shows that although there exist other meander lines which perform slightly better in radiation efficiency (Palmier pastry type, [52]) this improvement costs much in the radiation Q-factor. In conclusion, simple meander line antennas present the best trade-off between radiation Q-factor and dissipation factor from the depicted antennas when matching to real impedances is demanded. As in the previous section, we note that the use of more advanced matching topology (e.g., folding or impedance transformer) may benefit from alternative antenna designs.

Figures 11 and 12 show that the considered antenna structures, which are close to optimal in radiation Q-factor, are far away from the efficiency bounds. This is puzzling since resonant modes optimal in radiation Q-factor and efficiency are similar in nature. However, there are important differences. Radiation Q-factor restricted to TM modes is minimized by separation of charges and inducing dipole like currents [37]. These modes can be tuned to resonance by inducing edge loops along the structure. TM efficiency, on the other hand, is minimized by inducing homogeneous currents [53]. These are similar in nature to the dipole like currents minimizing Q-factor, but the loop currents which minimize TE Q-factor and maximize TE efficiency are fundamentally different. Where low Q-factor loops tend to be confined towards the edges of the structure, high efficiency loops are spread across the whole area [51, Fig. 4]. Such loop currents are naturally restricted as an original simply connected object fully filling a prescribed bounding box is perforated, forcing the current distribution into more inhomogeneous forms. Thus, low Q-factor loops are tolerant of alterations to a structure whereas high efficiency loops are harshly disrupted.

In Fig. 13, the optimal resonant Q-factor and dissipation factor are plotted normalized to the corresponding bounds of a rectangular plate. Data for different shapes made by removing portions of the plate are shown. The currents on the structures in Fig. 13 have been calculated with current optimization without physical feeding. It is clear that removing metal does not greatly affect the achievable radiation Q-factor, at worst reducing it to the TM-only bound. However, when metal is removed from the plate the loss factor is significantly increased, especially for small electrical sizes. Thus, while optimal radiation Q-factor and radiation efficiency modes are fairly similar, removing design space has a much greater effect on the loss factor than the Q-factor in relation to the physical bounds. This can be seen in Fig. 13 where the loss factor of the optimal resonant currents is very high for the structures with slots in them. Consider the meander line antenna which has significantly higher loss factor at electrical sizes $ka < 0.4$, here the loop modes are extremely disrupted, however, the Q-factor is hardly affected. The sharp change in the meander line’s loss at around $ka = 0.6$ is due to its resonance, where it is possible to induce a resonant dipole mode on the structure. This example illustrates a fundamental challenge in designing efficient small resonant antennas: many of the strategies normally utilized to induce resonance, such as meandering, harshly limit the achievable efficiency.

IV. ANTENNAS OPTIMAL IN OTHER PARAMETERS

Determining the best possible Q-factor can be formulated as a minimization problem. Therefore it is possible to add different or additional constraints to such an optimization. So far, in this paper, we have considered the constraints of
efficiency and impedance matching. Another type of constraints are different kinds of field-shaping requirements of near and/or far-fields [60], [61]. For small antennas it is well known that the radiated far-field tends to resemble a dipole pattern. However, with these types of Q-factor optimization procedures it is possible to determine the Q-factor cost, to have the antenna radiating with a certain front-to-back ratio or (super-) directivity in a given direction. These classes of bounds indicate that for a limited bandwidth cost it is possible to extend, e.g., the directivity beyond the traditional dipole pattern, see [60]–[67].

To illustrate bounds on superdirectivity, Q-factor optimization for a given directivity described in [60], [61], was solved for a small antenna with length to width ratio of 2:1, infinitesimal thickness, and electrical sizes $ka \in \{0.2, 0.5, 0.8\}$. The bounds for low directivities are identical to the lower bound on the Q-factor, where the radiation pattern changes from that of an elliptically polarized dipole with $D \approx 1.5$ to that of a Huygens source with directivity just below $D = 3$ and the main beam pointing in the direction of the longest side [21]. Higher directivities require quadrupole and higher order modes which increases the Q-factor rapidly [10]. The direction of the main beam changes from the longest side of the antenna to an endfire pattern along the shortest side at $Q_{rad}/Q_{rad}^{lb} \approx 3$ as indicated by circles in Fig. 14.

Much like bounds on other parameters (e.g., efficiency), it is an open problem if the directivity-constrained limits are achievable for all sizes and desired directivity, even under idealized

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**Box 3. Radiation efficiency**

The radiation efficiency of an antenna is defined as

$$\eta = \frac{P_{\text{rad}}}{P_{\text{rad}} + P_{\text{diss}}} = \frac{1}{1 + \delta},$$

where, as in (1), $P_{\text{rad}}$ and $P_{\text{diss}}$ are radiated and ohmic dissipated power, respectively, and $\delta = P_{\text{diss}}/P_{\text{rad}}$ is the dissipation factor [54]. Along with bandwidth radiation efficiency is a key antenna performance parameter, particularly in electrically small systems where it is known to decrease rapidly with antenna size.

For objects with homogeneous loss properties, e.g., uniform surface resistance or conductivity, the dissipation factor $\delta$ is a linear function of those properties. As such, values of dissipation factor can be normalized by surface resistivity for ease of comparison.

**Box 4. Lower bounds to dissipation factor**

Two different paradigms for minimization of dissipation factor exist. The first assumes that tuning or general impedance matching of the antenna can be performed in a lossless manner. Under this assumption, the optimal current density minimizing dissipation factor is the result of a generalized eigenvalue problem [53]–[56]. Such lower bounds were shown to scale with electrical size as $(ka)^{-2}$ and are straightforward to calculate. Their major drawback, however, is that, by neglecting matching network losses, the resulting dissipation factors are overly optimistic and unachievable by realistic designs where some form of matching is required [48].

One solution to the aforementioned drawback is a paradigm in which the optimal currents are calculated while taking into account the dissipation cost of achieving resonance or general matching [48], [57]. Dissipation factors coming from this second paradigm are generally closer to realistic designs and scale with electrical size as $(ka)^{-1}$ [51], [57]–[59]. Lower bounds to tuned dissipation factor for several selected shapes are shown in Table I.
lossless conditions. As a demonstration of one possible high directivity, low Q-factor design, a three port array composed of a meander line and a loop structure with optimized feeding is presented in relation to the bounds, see Fig. 14. However, high directivity for single port antennas remains, as of yet, far from the bound and new designs ideas that allow a high directivity with larger bandwidth are desired.

V. CONCLUSION

The possibilities how to approach the fundamental bounds on selected antenna metrics were investigated. A planar region of rectangular form factor was considered. It was observed that the lower bound on Q-factor with the radiation restricted to TM modes only is closely approached by a meander line antenna for a broad range of electrical sizes. The optimal design parameters were depicted and various aspect ratios of the bounding rectangle were studied together with selected ratios of the strip and slot widths. The simulated results were verified by a measurement of a fabricated prototype. The impedance matching and its impact on the Q-factor of the antenna was studied, concluding that the effect of the impedance matching on radiation Q-factor is minor and, in some cases, that matching the antenna slightly away from its self-resonance can even decrease its Q-factor. Radiation efficiency of the meander line antennas optimal in Q-factor was evaluated, taking into account ohmic losses dissipated in the matching circuit. It was observed that the radiation efficiency of the studied meander line antennas is far from its upper bound. Several other planar antennas were similarly evaluated against fundamental bounds yielding consistent conclusions: synthesizing antenna designs which approach the upper bound on radiation efficiency is more difficult than designing those which reach the lower bound on Q-factor. The reason was identified in the high sensitivity of radiation efficiency to the perturbation of ideal constant current density. Namely, when an initial structure fully filling the prescribed bounding box is perforated (as is done in a practical synthesis procedure), the performance of maximum efficiency current distributions drops much faster than that of a minimum Q-factor distribution. Finally, a Pareto-type bound between Q-factor and directivity has been calculated and compared to meander line antennas. An attempt has been made to find an antenna with reasonably low Q-factor and directivity higher than that of an electric dipole type antenna. Nevertheless, no planar antenna with one feed fulfilling these contradictory constraints was found. This task and its feasibility remains as a subject for ongoing research.

The fundamental bounds, i.e., the lower bounds on Q-factor, the upper bounds on radiation efficiency, the Pareto-optimality between Q-factor and efficiency, or Q-factor and directivity, were demonstrated to be powerful tools for judging the performance of the radiating devices. If the realistic designs are compared to the fundamental bounds, designer can assess how far from the optima the design is, therefore, if further improvement is needed. Furthermore, incremental progress in design improvement can be put into context by considering the remaining distance between an antenna’s realized performance and the fundamental bounds. It is the normalized ratio of the actual device’s performance to the fundamental bounds what reveals the real quality of the design.

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