A review of exact solutions for conversion of a surface wave into a propagating wave

Svetlana N Tcvetkova\(^1\), Stefano Maci\(^2\), and Sergei A Tretyakov\(^1\)

\(^1\) Department of Electronics and Nanoengineering, Aalto University, P.O. 15500, FI-00076 Aalto, Finland
\(^2\) Department of Information Engineering and Mathematics, University of Siena, Via Roma 56, 53100, Siena, Italy.
E-mail: svetlana.tcvetkova@aalto.fi

Abstract. In this paper, we summarize our recent theoretical and numerical studies of conversion of a surface wave to a propagating wave (and vice versa) with maximized conversion efficiency. We show that the exact solution for the homogeneous plane wave to surface wave conversion in a homogeneous medium by a point-wise lossless metasurface is restricted by Maxwell’s equations. Such performance is possible only with approximations and implies some limitations, including impossibility to fully avoid unwanted scatterings. Nevertheless, an inhomogeneous plane wave can be converted into a surface wave perfectly, and it is described by an exact solution formed by a superposition of only two modes of the Floquet expansion. The results of the studies are expected to be of fundamental importance for leaky-wave antennas efficiency improvement. Besides, they can lead to novel applications in various research areas.

1. Introduction
Conversion between guided and propagating space wave modes at microwave frequencies can be performed by conventional leaky-wave antennas. Their operation is limited due to the presence of higher-order Floquet harmonics, which store reactive fields near the surface limiting the bandwidth. Moreover, the conversion itself is not perfect and causes some radiation into unwanted directions.

Recently, it was recognized that most of reflectarray and transmittarray antennas performing the space-to-space waves transformation are typically accompanied by fundamental imperfections, which lead to unwanted scatterings, similarly to leaky-wave antennas. Studies in the field of non-local metasurfaces have proved the possibility to design devices which can perform transformation of a single plane wave incident along one direction into a single plane wave propagating into another direction with unprecedented efficiency [1]. However, there are no designs available for theoretically perfect guided-to-space wave conversions. Therefore, it is of fundamental and practical importance to find and explore possibilities for perfect and full conversion of a surface-bound mode to propagating plane waves and vice versa.

2. Possibility of exact solution
Let us consider an infinite impenetrable boundary characterized by its surface impedance which is illuminated by a plane wave (for simplicity of discussion, at normal incidence). The boundary
should be engineered in such a way, that all the power carried by the plane wave and impinged on the surface is collected and carried by the surface wave towards the load.

The main obstacle in the realization of a perfect converter is the necessary presence of the active regions on the impenetrable surface. Similarly to anomalous reflectors, the propagating wave and surface wave generally interfere. It leads to the complex value of the required scalar surface impedance, the real part of which corresponds to the locally placed loss and gain [1, 2]. In the average, the surface can be lossless; nevertheless, the realization of such a structure is a difficult task. However, there is a way to overcome this problem: The two waves should be orthogonally polarized. We adopt this approach from the theory of perfect anomalous reflectors to create an ideal space-to-guided wave converter.

2.1. Homogeneous plane wave to surface wave conversion
Let us consider an ideal scenario: One TE-polarized homogeneous plane wave impinges the surface and fully converts to one TM-polarized surface wave. Fields of both waves should satisfy Maxwell’s equations. To satisfy the energy conservation condition, the power carried by the surface wave should grow linearly along the surface. However, the known surface-wave solutions (separable or nonseparable) cannot provide such performance. In other words, the perfect conversion between only one surface wave and one homogeneous plane wave in homogeneous media by any point-wise lossless metasurface is restricted by Maxwell’s equations [2]. Nevertheless, an approximate solution can be found using a separable eigenwave field solution, allowing to reach a nearly perfect conversion. The power carried by an associated single-mode surface wave grows exponentially along the direction of propagation. Slow exponential growth can approximate the required linear increase of collected power. However, it sets limitations on the length of the surface: Only a finite interval of the surface can be considered. Still, the length of the surface sample can reach up to tens of wavelengths.

Due to the approximation, the tensor surface impedance (because of polarization transformation) of the lossless and reciprocal surface can be found in two ways: Using a periodic approximation, which implies restrictions on the attenuation constant, and the use of the least square approximation, which may complicate the practical realization due to aperiodicity. The surface impedance matrices found both ways show good performance in numerical simulations: conversion efficiency (ratio of the power carried by the surface wave to the load and the input power of the incident wave) of 95% and 81% is achieved by the aperiodic and periodic solution, respectively. Figure 1 shows the field distribution of the tangential component of the total electric and magnetic fields obtained by using a numerical simulation tool for the least square solution. Further, the search of the exact solution for the conversion continued.

2.2. Inhomogeneous plane wave to surface wave conversion
Another way to find an exact solution and ensure the power balance is to change the structure of the propagating wave. An inhomogeneous plane wave possesses an exponential power dependence similarly to the separable field solution of the surface wave. Let us consider an ideal scenario, however, reciprocal to the previous one (for practicality): One TM-polarized surface wave launched along an impenetrable surface is converted to one TE-polarized inhomogeneous plane wave radiating from the surface. Fields of both waves satisfy Maxwell’s equations. The metasurface is reactive and characterized by a periodic tensor surface impedance. In the absence of sources, a complete eigenmode basis set is given by Floquet-wave modes. Therefore, the surface wave is set as the 0-indexed mode of the Floquet expansion, while the −1-indexed mode corresponds to the inhomogeneous plane wave radiating from the surface. Both waves have the same attenuation along the surface, ensuring perfect power balance at every point. The surface impedance matrix can be directly found without any approximations and limitations. Figure 3 depicts the field distribution of the tangential component of the total electric and magnetic...
Figure 1. Numerically computed distributions of the tangential components of the total (a) magnetic field and (b) electric field with the following wave parameters: the attenuation constant along the surface is $\alpha_x = 0.0117k_0$, the attenuation constant of the surface wave along the normal is $\alpha_y^{sw} = 0.3791k_0$, the propagation constant of the surface wave along the surface is $\beta_x^{sw} = 1.069k_0$, the propagation constant of the surface wave along the normal is $\beta_y^{sw} = -0.033k_0$, where $k_0$ is the free-space wavenumber.

fields obtained by using a numerical simulation tool. The conversion efficiency (ratio of the power carried by the propagating wave and the power of the launched surface wave) of this device is more than 99%. It is important to note that the exact solution of the boundary value problem for an impedance surface does not include any higher-order Floquet modes of the field expansion. It means that there is no energy stored by the higher-order modes: all the power carried by the surface wave is used for the leaky wave creation [4].

Figure 2. Numerically computed distributions of the tangential components of the total (a) magnetic field and (b) electric field with the following wave parameters: $\alpha_x = 0.0425k_0$ (equal for surface wave and propagating wave), $\alpha_y^{sw} = 0.4671k_0$, $\beta_x^{sw} = 1.1k_0$, $\beta_y^{sw} = -0.1k_0$, $\beta_y^{lw} \approx k_0$.

2.3. What can we expect in case of conserved polarization?
The exact solution for surface-to-propagating-wave conversion was found under the following conditions: The waves are orthogonally polarized and the propagating wave is a single inhomogeneous plane wave which has an exponential power dependence similar to a single surface wave. Let us explore what happens if the polarization is conserved in the mode conversion.

The surface impedance, in this case, is scalar and allows an exact solution, which consists of real and imaginary parts. It means that the surface is not point-wise lossless and has active/lossy regions; however, on the average it can be lossless. The distribution of the tangential component of the total magnetic field obtained by using the active/lossy surface impedance solution is depicted in Figure 2(a). The integral over the surface of the normal component of the total Poynting vector at the boundary of such metasurface is equal to zero, which means that all the regions of absorption are balanced and compensated by the generation areas. The efficiency
of the conversion, in this case, is over 99%. Unfortunately, it may be a complicated and disadvantageous task to realize a metasurface with active/lossy regions. What happens if we force the real part of the surface impedance to be equal to zero and use only the imaginary part of such solution? Obviously, conversion cannot be perfect in this case, but the question is how much degradation will take place. The distribution of the tangential component of the total magnetic field obtained by using a lossless metasurface whose reactance is given by the imaginary part of the active/lossy surface impedance solution is depicted in Figure 2(b). The conversion efficiency is as well over 99%, and the surface is lossless. However, when the real part of the impedance is suppressed, the surface wave propagating along the metasurface is not a single harmonic anymore, but a complex superposition of higher-order modes. The metasurface designed this way is advantageous for realization, but the solution is not exact.

Another possibility to find an exact solution while maintaining the same polarization of the two waves is to use a penetrable surface, backed by a perfectly conducting boundary. Such problem formulation gives more freedom and a chance to utilize energy transfer inside the volume of the structure, which may also allow a practical and passive realization, equivalent to the theoretically perfect active/lossy scenario.

3. Conclusions
The use of metasurfaces as converters of a surface wave into a propagating plane wave (and vice versa) has been examined theoretically and numerically. All the considered solutions may open a new way to enhance leaky-wave antennas, including the antenna bandwidth and conversion efficiency. Since the design methods are based on the use of equivalent surface impedance, the theoretical results are applicable across the whole electromagnetic spectrum.

3.1. Acknowledgments
The work has been supported in part by the Academy of Finland (project 287894) and Nokia Foundation (project 20190452).

References
[1] Asadchy V S, Albooyeh M, Tcvetkova S N, Díaz-Rubio A, Ra’di Y, and Tretyakov S A 2016 Phys. Rev. B 94 075142
[2] Tcvetkova S N, Kwon D-H, Díaz-Rubio A, and Tretyakov S A 2018 Phys. Rev. B 97 115447
[3] Tcvetkova S N, Maci S, and Tretyakov S A 2018 2018 12th International Congress on Artificial Materials for Novel Wave Phenomena (Metamaterials) 489
[4] Tcvetkova S N, Maci S, and Tretyakov S A 2019 IEEE Trans. Antennas Propag., 67 no 5 pp. 3200 - 3207