Reaction cross sections of the deformed halo nucleus $^{31}$Ne

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Using the Glauber theory, we calculate reaction cross sections for the deformed halo nucleus $^{31}$Ne. To this end, we assume that the $^{31}$Ne nucleus takes the $^{30}$Ne + $n$ structure. In order to take into account the rotational excitation of the core nucleus $^{30}$Ne, we employ the particle-rotor model (PRM). We compare the results to those in the adiabatic limit of PRM, that is, the Nilsson model, and show that the Nilsson model works reasonably well for the reaction cross sections of $^{31}$Ne. We also investigate the dependence of the reaction cross sections on the ground state properties of $^{31}$Ne, such as the deformation parameter and the $p$-wave component in the ground state wave function.

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I. INTRODUCTION

Interaction cross sections as well as reaction cross sections are intimately related to the size of nuclei [1]. The root-mean-square radius diverges for $s$ and $p$ waves as the single-particle energy approaches to zero [3], and the halo structure has been ascribed to an occupation of an $l = 0$ or $l = 1$ orbit by the valence neutron [3]. $^{11}$Be, $^{12}$C, $^{13}$C, and $^{19}$F have been regarded as $s$-wave halo nuclei, and $^{6}$He is an example of a $p$-wave halo nucleus.

A large interaction cross section for $^{31}$Ne was recently observed by Takechi et al. [2]. This observation suggests the extended density distribution for $^{31}$Ne, that is, the halo structure, being consistent also with a large Coulomb breakup cross section measured by Nakamura et al. [10]. Takechi et al. have analyzed the data using single particle levels in a deformed potential and argued that $^{31}$Ne is an $s$- or $p$- wave halo nucleus [2].

The ground state properties for $^{31}$Ne have not been known well. For example, the one neutron separation energy $S_{n} = 0.29 \pm 1.64$ MeV [1] has a large uncertainty and the spin and parity have not yet been determined. In the core nucleus $^{30}$Ne, the candidates for the first excited $2^+$ and $4^+$ states have been experimentally observed at excitation energies of 0.801 MeV and 2.24 MeV, respectively [12,13]. The energy ratio $E_{2^+}/E_{4^+} = 2.80$ suggests this nucleus to be a transitional one in comparison with the ratio 3.33 for well-deformed nuclei. Hamamoto has carried out the Nilsson model calculation with a deformed Woods-Saxon potential and argued that the $[330 1/2]$, $[321 3/2]$, and $[200 1/2]$ Nilsson levels occupied by the valence neutron can hold the halo structure [14]. The $[330 1/2]$ and $[321 3/2]$ configurations lead to $I^\pi = 3/2^-$, while the $[200 1/2]$ leads to $I^\pi = 1/2^+$ for the spin and parity of the ground state of $^{31}$Ne in the laboratory frame.

In the previous publication, we used a particle-rotor model (PRM) [15,20] to analyze the experimental data for the Coulomb breakup cross section and discussed the ground state configuration for the $^{31}$Ne nucleus [21]. Notice that the Nilsson model corresponds to the adiabatic limit of PRM. We have shown that the ground state configuration corresponding to the $[321 3/2]$ Nilsson orbit can be excluded if the finite rotational excitation energy of the core nucleus is taken into account [21].

In this paper, we apply the same model to the reaction cross section of $^{31}$Ne. The effect of deformation on the reaction cross section of the $^{31}$Ne nucleus has been discussed recently by Minomo et al. using the microscopic optical potential model [22,23], and has been shown to play an important role. It would thus be of interest to discuss the role of deformation in the reaction cross section of $^{31}$Ne using the PRM as an alternative approach, which has been successful in reproducing the Coulomb breakup cross section. Notice that reaction cross sections with deformed projectiles have been evaluated by Christley and Tostevin with the optical limit Glauber theory [24]. We will extend it to a system of deformed core nucleus plus a valence neutron, based on the formalism given in Ref. [25] for single-nucleon knockout reactions.

The paper is organized as follows. In Sec. II, we summarize the framework of PRM and the calculation procedure for the reaction cross section. In Sec. III, we present the results of the reaction cross section for $^{30,31}$Ne. We discuss the effect of the finite rotational excitation energy on the reaction cross section. We investigate also the dependence of the reaction cross section on the deformation, the ground state configuration, and the rms radius of $^{31}$Ne. In Sec. IV, we summarize the paper.

II. FORMALISM

A. Particle-rotor Model

In order to compute the reaction cross section of the $^{31}$Ne nucleus, we assume that it consists of the statically deformed core nucleus $^{30}$Ne and one valence neutron as shown in Figure 1. The relevant coordinate systems are also shown in the figure. In this model, the single particle motion of the valence neutron is coupled to the rotation of
the deformed core nucleus. For simplicity, we assume the axially symmetric deformation for the core nucleus with the quadrupole deformation parameter \( \beta_2 \). We consider the same Hamiltonian for this system as in Ref. [21],

\[
H = -\frac{\hbar^2}{2\mu} \nabla^2 + V(\mathbf{r}, \mathbf{r}_c) + H_{\text{rot}},
\]

where \( \mu = m_N A_e/(A_e + 1) \) is the reduced mass of the valence neutron, with \( A_e = 30 \) and \( m_N \) being the mass number of the core nucleus and the nucleon mass, respectively. \( H_{\text{rot}} \) is the rotational Hamiltonian for the core nucleus. \( V(\mathbf{r}, \mathbf{r}_c) \) is the single-particle potential for the valence neutron interacting with the deformed core. \( \mathbf{r} \) and \( \mathbf{r}_c \) are the coordinates of the valence neutron and the direction of the symmetry axis of the core nucleus in the laboratory frame, respectively. \( \mathbf{r}_{cn} \) is the angle between \( \mathbf{r} \) and \( \mathbf{r}_c \).

We use a deformed Woods-Saxon potential for \( V \) and expand it up to the linear order of the deformation parameter \( \beta_2 \) as,

\[
V(\mathbf{r}, \mathbf{r}_c) \sim V_0(\mathbf{r}) + V_{\text{def}}(\mathbf{r}, \mathbf{r}_{cn}),
\]

where \( V_0 \) is a spherical Woods-Saxon potential together with the spin-orbit (\( ls \)) force, and \( V_{\text{def}} \) is the deformed part of the potential given by,

\[
V_{\text{def}}(\mathbf{r}, \mathbf{r}_{cn}) = -R_0\beta_2 \frac{dV_0^{(0)}(\mathbf{r})}{dr} Y_{20}(\mathbf{r}_{cn}),
\]

where \( V_0^{(0)}(\mathbf{r}) \) is the central part of the spherical Woods-Saxon potential, \( V_0(\mathbf{r}) \), and \( \mathbf{r}_{cn} \) is the angle between \( \mathbf{r} \) and \( \mathbf{r}_c \). The deformation of the \( ls \) potential is neglected for simplicity. We have checked the validity of the expansion up to the linear order of \( \beta_2 \) by comparing to the calculation with the higher order terms, and have confirmed that it works well.

Since the calculation of the reaction cross section needs only the ground state wave function of \( ^{31}\text{Ne} \), it is sufficient to expand the wave function on the basis \( \phi_{jllK} \), the eigen-functions of the spherical part \( V_0 \) of the potential, \( \phi_{njl}(\mathbf{r})Y_{l0m}(\mathbf{r}) \), where \( \phi_{njl}(\mathbf{r}) \) is the radial wave function and \( Y_{l0m}(\mathbf{r}) \) is the spin-angular wave function. The continuum spectrum can be discretized within a large box. Together with the rotational wave function \( \phi_{jllK}(\mathbf{r}_c) \), the total wave function for the \( n+^{30}\text{Ne} \) system is expanded as,

\[
\Psi_{JM}(\mathbf{r}, \mathbf{r}_c) = \sum_{njl} \sum_{l_c} \alpha_{njl}^{(I)} R_{njl}(\mathbf{r})|Y_{jl}(\mathbf{r})\phi_{l_c}(\mathbf{r}_c)|^{(1M)},
\]

where \( I \) is the spin of \( ^{31}\text{Ne} \) and \( M \) is its \( z \)-component. The expansion coefficients \( \alpha_{njl}^{(I)} \) as well as the corresponding eigen-energies for the \( ^{31}\text{Ne} \) nucleus are obtained by numerically diagonalizing the Hamiltonian \( H \).

We identify the ground state configuration in the same manner as in the previous work [21]. That is, we first solve the Hamiltonian in the adiabatic limit by setting \( H_{\text{rot}} = 0 \) in Eq. (1), that is, by assuming that all the members of the ground rotational band are degenerate in energy. In this case, the \( K \) quantum number, that is, the projection of the total angular momentum onto the \( z \)-axis in the body-fixed frame, is conserved, and several states with different \( I \), having the same value of \( K \), are degenerate in energy when the maximum value of \( I_c \), included in the calculation is sufficiently large. The wave function in this limit is related to the wave function in the Nilsson model, \( \phi_{jllK} \), in the sense that it is a transformation of the Nilsson wave function from the body-fixed frame to the laboratory frame. The eigen-energies so obtained thus form the single-particle Nilsson levels. The probability for the \((j, l)\) component in the Nilsson wave function is represented in the following relation:

\[
\sum_{j} \sum_{l} |\alpha_{njl}^{(I)}|^2 = \int r^2 dr \phi_{jll}^2(r) = P_{jll}^{(\text{Nil})},
\]

which is independent of \( I \).

In order to construct the ground state, we put two neutrons to each Nilsson orbit from the bottom of the potential well, and seek the Nilsson orbit which is occupied by the last unpaired neutron. We then gradually increase the value of \( E_{2+} \) energy of the core nucleus up to the physical value, \( E_{2+} = 0.801 \text{ MeV} \), and monitor how the Nilsson orbit for the valence neutron evolves. For a finite value of \( E_{2+} \), the \( K \) quantum number is not conserved any more due to the Coriolis coupling, and the degeneracy with respect to \( I \) is resolved. We select the lowest energy state among several \( I \) at \( E_{2+} = 0.801 \text{ MeV} \) as the ground state of \( ^{31}\text{Ne} \). In this way, we take into account the Pauli principle between the valence neutron and the neutrons in the core nucleus.

We consider two configurations with the spin-parity of \( I^\pi = 3/2^- \) in the laboratory frame as candidates for the
ground state of $^{31}$Ne. One with the deformation parameter $\beta_2 = 0.2$ corresponds to the Nilsson level [330 1/2], and the other with $\beta_2 = 0.55$ corresponds to the Nilsson level [321 3/2] in the adiabatic limit where the rotational energy of the core nucleus is neglected. Following Ref. [22], we use the same Woods-Saxon potential parameters as those given in Table I of Ref. [20]. The depth of the Woods-Saxon potential is varied to reproduce the one neutron separation energy. We use a similar value for the energy cut-off for the single particle basis and a similar size of the box to discretize the continuum spectrum as in Ref. [21].

B. Reaction Cross Sections

In this paper, we discuss the reaction cross sections for $^{30,31}$Ne on the carbon target. For simplicity, we neglect the effect of the Coulomb force. To verify this approximation, we have calculated the Coulomb breakup cross sections of $^{31}$Ne for the configuration with $\beta_2=0.2$ at the separation energy $S_n=0.2$MeV on the carbon target using the method given by Ref. [21]. The calculated Coulomb breakup cross section, 0.0033b, is indeed small as compared to the total reaction cross section, suggesting that the nuclear force dominantly contributes to reaction cross sections for $^{31}$Ne on the carbon target. In order to compute the reaction cross sections, we use the Glauber theory where the eikonal approximation and the adiabatic approximation are adopted [27]. We closely follow the formalism in Ref. [22], in which the PRM has been used to evaluate single-nucleon knockout reactions of a deformed odd-A nucleus based on the Glauber theory.

In the eikonal approximation, the final state $\Psi_f$ after the collision is described with the initial state wave function $\Psi_i$, as

$$\Psi_f = S \Psi_i = \exp[i\chi] \Psi_i,$$

where $\chi$ is the phase shift function. The reaction cross section of $^{31}$Ne, defined as the difference between the total cross section and the elastic scattering cross section, is given with the ground state wave function of $^{31}$Ne, $\Psi_{IM}$, as,

$$\sigma_R^{(31)}Ne = \int dB \left( 1 - \frac{1}{2I+1} \sum_M \right)$$

$$\times \left| \langle \Psi_{IM} | S_c S_v | \Psi_{IM} \rangle \right|^2 ,$$

where $B$ is the impact parameter of the center of mass of the projectile nucleus $^{31}$Ne colliding with the target nucleus. The $S$-matrix for the two-body projectile nucleus can be written by $S \sim S_c S_v$ in the Glauber approximation [28]. Here, $S_c$ and $S_v$ are $S$ matrices for the core nucleus and the valence neutron, respectively. Notice that, since the directions of $r$ and $\hat{r}_c$ are integrated in the whole space in Eq. (7) before the integration over $b$ is carried out, the integrand does not depend upon the direction of $\hat{b}$ [25]. The reaction cross section $\sigma_R^{(31)Ne}$ thus reads

$$\sigma_R^{(31)Ne} = 2\pi \int dB \left( 1 - \frac{1}{2I+1} \sum_M \right)$$

$$\times \int \left| \int d\hat{r}_c S_c S_v F(r, \hat{r}_c) \right|^2 ,$$

where

$$F(r, \hat{r}_c) = \sum_{n'j'l' I' J' \ell' L'} \sum_{njl J L} \alpha_{n'j'l' I' J' \ell' L'}^{*} \alpha_{njl J L} R_{n'j'l' I' J' \ell' L'}(r) R_{njl J L}(r)$$

$$\times \langle j' m'_j j' m'_l | IM \rangle \langle j m_j j m_l | IM \rangle$$

$$\times \mathcal{Y}_{n'j'l'}(\hat{r}_c) \mathcal{Y}_{njl}(\hat{r}_c),$$

Using the formula for the product of two spherical harmonics with the same angles, the function $F$ is transformed to [25]

$$F(r, \hat{r}_c) = \sum_{L,m_L L' m_{L'}} U_{LM_L \ell m_L}(\hat{r}_c) U_{LM_L \ell m_L}(r),$$

where

$$U_{LM_L \ell m_L}(r) = \sum_{n'j'l' I' J' \ell' L'} \sum_{njl J L} \alpha_{n'j'l' I' J' \ell' L'}^{*} \alpha_{njl J L} R_{n'j'l' I' J' \ell' L'}(r) R_{njl J L}(r)$$

$$\times \langle j' m'_j j' m'_l | IM \rangle \langle j m_j j m_l | IM \rangle$$

$$\times \langle l' m'_L | L M \rangle \langle l m_L | L M \rangle$$

$$\times W(l' j' l j; \frac{1}{2} L) \left\{ \begin{array}{ccc} l'_L & I' & \ell' \\ I & L & k \end{array} \right\} ,$$

with $j = \sqrt{2j+1}$ and $W$ being the Racah coefficients.

In order to evaluate the $S$-matrices, we employ the optical limit approximation for simplicity. Using the zero range interaction for the effective nucleon-nucleon interaction, we evaluate the $S$ matrices by folding the densities of the projectile and the target nuclei as,

$$S_c = \exp[-\sigma_{NN}(1 - i\alpha_{NN})\chi_c(b_c, \hat{r}_c)/2],$$

$$S_v = \exp[-\sigma_{NN}(1 - i\alpha_{NN})\chi_v(b_n)/2],$$

where

$$\chi_c(b_c, \hat{r}_c) = \int dz_e \int d\hat{r}' \rho_c(|r' + R_c|) \rho_T(|r' + R_c|),$$

$$\chi_v(b_n) = \int dz_n \rho_T(R_n).$$

Here, $R_c = (b_c, z_e)$ and $R_n = (b_n, z_n)$ are the coordinates of the center of mass of the core nucleus and the
valence neutron from the target nucleus, respectively. \( \rho_c \) and \( \rho_n \) are the densities of the core and the target nucleus, respectively. We construct the density of the core nucleus \(^{30}\)Ne with the Nilsson model. To this end, we use the original values for the potential parameters given in Table I of Ref. [24]. For the density distribution for the target nucleus \(^{12}\)C, we use a one-range Gaussian function whose width parameter is determined so as to reproduce the experimental root mean square radius. \( \sigma_{\text{NN}} \) in Eqs. (12) and (13) is the average value of the total cross sections of the nucleon-nucleon scattering [30]. \( \sigma_{\text{NN}} \) is also the average value of the ratio of the real to the imaginary part of the nucleon-nucleon scattering amplitudes. We use the experimental values for \( \sigma_{\text{pp}}, \sigma_{\text{pn}}, \alpha_{\text{pp}} \) and \( \alpha_{\text{pn}} \) for the incident energy 240 MeV/nucleon listed in Ref. [31].

Notice that the reaction cross section of the core nucleus \(^{30}\)Ne is simply given by

\[
\sigma_R^{(30)\text{Ne}} = \int \frac{db}{4\pi} \int dr_c \left( 1 - |S_c|^2 \right),
\]

with \( S_c \) given in Eqs. (12) and (14) [24].

III. RESULTS

We now numerically evaluate the reaction cross sections for \(^{30,31}\)Ne. The upper and the lower panels of Fig. 2 show the results for the configurations with \( \beta_2 = 0.2 \) and 0.55, respectively. Since the measured one-neutron separation energy \( S_n \) of \(^{31}\)Ne has a large error bar, \( S_n = 0.29 \pm 1.64 \) MeV [11], we show the calculated reaction cross sections as a function of \( S_n \). The upper and the lower shaded regions in each panel indicate the experimental interaction cross sections for \(^{31}\)Ne and \(^{30}\)Ne [9], respectively. The dashed lines are the calculations in the adiabatic limit, while the solid lines are the calculations with the finite rotational energy. The dotted lines denote the reaction cross sections for \(^{30}\)Ne. The upper and the lower shaded regions indicate the experimental interaction cross sections for \(^{31}\)Ne and \(^{30}\)Ne, respectively.

As one can see, the results of PRM are similar to those in the adiabatic limit for both the configurations with \( \beta_2 = 0.2 \) and 0.55. In the adiabatic limit, since each component of \((j, l, I)\) with different values of \( I \) has the same radial wave function, it is the total \( p_{3/2} \) probability, summed with different \( I \) values, that is relevant to the halo structure. On the other hand, due to the non-adiabatic effect, the wave function \( |I_c = 0^+ \otimes p_{3/2}\rangle \) is spatially most extended in the PRM [21]. For the configuration with \( \beta_2 = 0.2 \) and \( S_n = 0.2 \) MeV, the probability for the total \( p_{3/2} \) component is 54.9\% in the adiabatic limit, which is almost equal to the probability for the \( 0^+ \otimes p_{3/2} \) component in the PRM, that is, 54.2\%. The halo structure therefore retains even when the finite excitation energy is taken into account in the PRM. For the configuration with \( \beta_2 = 0.55 \), on the other hand, the total \( p_{3/2} \) probability in the adiabatic limit is 25.7\% and the probability for the \( 0^+ \otimes p_{3/2} \) component in the PRM is 2.1\%. Therefore, the halo structure disappears for this configuration when the finite rotational energy is taken into account [21]. The small difference between the solid and the dashed curves in the lower panel of Fig. 2 reflects this fact. Nevertheless, the halo contribution to the reaction cross section does not seem large in this mass region, and the adiabatic approximation still works for the reaction cross sections.

In order to see the relation between the halo structure and the reaction cross section more clearly, Fig. 3 shows the rms radii for those configurations as a function of \( S_n \). The behaviors of the rms radii are qualitatively the same as the reaction cross sections shown in Fig. 2. The rms radius is almost constant for \( \beta_2 = 0.55 \) when the finite rotational energy is taken into account, that is consistent with the disappearance of the halo structure. The rms radii increase as the one-neutron separation energy, \( S_n \), decreases for the other cases, indicating the halo structure. Notice that in contrast to the rms radii and the Coulomb breakup cross section, the reaction cross section is less sensitive to the extended density distribution.
since the inner part of the density distribution also contributes to the cross section.

The calculated density distribution of $^{31}$Ne for the configuration with $\beta_2=0.2$ and $S_n=0.2$ MeV is shown in Figure 4. The dashed and the solid lines are the results for $^{30}$Ne and $^{31}$Ne, respectively. Since the effects of the finite excitation of the core nucleus on the reaction cross sections and rms radii are small for this configuration, we calculate the density for $^{31}$Ne in the adiabatic limit, that is, Nilsson model, which is defined in the body-fixed frame. The upper and the lower panels show the density distributions in the direction of the symmetry axis of the core nucleus $^{30}$Ne and in the direction perpendicular to the symmetry axis, respectively. The density distribution has an exponentially extended tail, indicating the halo structure for this nucleus. Notice that the density distribution is proportional to $Y_{00}(\theta_{cn})+Y_{20}(\theta_{cn})/\sqrt{5}$ for the pure $p_{3/2}$ state with $K=1/2$, and it is extended more in the direction of the symmetry axis compared to the direction perpendicular to it.

In Figure 2, the calculated reaction cross sections appear to reproduce the experimental data for both the configurations with $\beta_2=0.2$ and 0.55. However, the increase of the calculated reaction cross section from $^{30}$Ne to $^{31}$Ne is much smaller for the configuration with $\beta_2=0.55$ than with $\beta_2=0.2$. This is because the probability of the $p_{3/2}$ component is much larger at $\beta_2=0.2$ than at $\beta_2=0.55$.

Notice that these different $p_{3/2}$ probabilities stem from the Nilsson levels $[330 1/2]$ at $\beta_2=0.2$ and $[321 3/2]$ at $\beta_2=0.55$ in the adiabatic limit, respectively. The difference between the reaction cross sections for $^{31}$Ne and $^{30}$Ne may be approximately identified as the one-neutron removal cross section for $^{31}$Ne \cite{27},

$$\sigma_{-1n}^{(31)Ne} \sim \sigma_R^{(31)Ne} - \sigma_R^{(30)Ne}. \quad (17)$$

Figure 5 shows the one-neutron removal cross sections for $^{31}$Ne on the carbon target so obtained as a function of the one-neutron separation energy for $^{31}$Ne. The shaded region indicates the experimental data with the incident energy of 230 MeV/nucleon \cite{10}. The thick and the thin lines are the one-neutron removal cross sections for the configuration with $\beta_2=0.2$ and 0.55, respectively. The dashed and the solid lines are the results in the adiabatic limit and with the finite rotational excitation, respectively. One can clearly see that the results with the configuration with $\beta_2=0.2$ reproduces the experimental data, while the configuration with $\beta_2=0.55$ is inconsistent with the experimental one-neutron removal cross section. We conclude that the configuration with $\beta_2=0.2$ is a very promising candidate for the ground state of the deformed halo nucleus $^{31}$Ne, which is consistent with the analysis of the Coulomb dissociation cross section of $^{31}$Ne with the PRM \cite{21}.

We next investigate the deformation dependence of the reaction cross sections of $^{30,31}$Ne for the configuration...
may be the ground state even at around uncertainties of the potential parameters, this configuration the one neutron separation energy is assumed to be $S_n$. As in Fig. 2, the meaning of each line is the same as in Fig. 2.

which reproduces the experimental data of the reaction and the one-neutron removal cross section at $\beta_2 = 0.2$. With the method explained in section II A, this configuration remains the ground state in the range of the deformation parameter, $0.17 \lesssim \beta_2 \lesssim 0.33$. Given the uncertainties of the potential parameters, this configuration may be the ground state even at around $\beta_2 = 0.4$, as suggested by the Anti-symmetrized Molecular Dynamics (AMD) calculation for $^{29-31}$Ne [33,34], see Table II in Ref. [22]. Figure 6 shows the reaction cross sections for $^{30,31}$Ne for the separation energy of $S_n = 0.2$ MeV, as a function of the deformation parameter in the region of $0.2 \lesssim \beta_2 \lesssim 0.4$. The dashed line is the result in the adiabatic limit, while the solid line is the result with the finite rotational energy. The result for $^{30}$Ne is shown with the dotted line. The reaction cross sections for $^{30,31}$Ne smoothly increase only by about 0.01 b from $\beta_2 = 0.2$ to $\beta_2 = 0.4$ due to the deformation of the core density. This may be understood in terms of the deformation dependence of the rms radius, see e.g., Eq. (1) in Ref. [24]. The total $p_{1/2}$ probability in the adiabatic limit varies from 54.9% to 57.0% as the deformation parameter changes from $\beta_2 = 0.2$ to $\beta_2 = 0.4$. Consequently, the deformation dependence of the reaction cross section of $^{31}$Ne is small, as far as the same configuration is concerned, i.e., the Nilsson level $[330 \, 1/2]$ in the adiabatic limit. The experimental reaction cross section can thus be reproduced within the region of $0.2 \lesssim \beta_2 \lesssim 0.4$.

IV. SUMMARY

We have discussed the reaction cross sections for $^{30,31}$Ne with the particle-rotor model. Assuming the system with the deformed core nucleus and one valence neutron for $^{31}$Ne, the finite rotational excitation energy of the core nucleus $^{30}$Ne is taken into account. In order to calculate the reaction cross section on the carbon target, we have used the optical limit approximation of the Glauber theory. We have considered two configurations with the spin-parity of $I^\pi = 3/2^-$ at $\beta_2 = 0.2$ and 0.55 as candidates for the ground state of $^{31}$Ne, corresponding to the Nilsson levels $[330 \, 1/2]$ and $[321 \, 3/2]$ in the adiabatic limit, respectively. The effect of the finite rotational energy changes the probability of each component in the wave function, especially the proportion of the $[0^+ \otimes p_{3/2}]$ component as well as the $[2^+ \otimes p_{3/2}]$ component. We have found that the non-adiabatic effects on the reaction cross sections for these two configurations are small, and it is concluded that the Nilsson model works reasonably well for the reaction cross section for $^{31}$Ne. We have also found that the difference of the reaction cross sections between $^{31}$Ne and $^{30}$Ne is much larger for the configuration with $\beta_2 = 0.2$ than for the configuration with $\beta_2 = 0.55$, leading to a consistent description for one-neutron removal cross section for $\beta_2 = 0.2$.

Interaction cross sections of Ne isotopes have been measured from $^{26}$Ne to $^{32}$Ne by Takechi et al. [9]. The data show a large odd-even staggering for $^{30,31,32}$Ne, which has been understood in terms of the pairing anti-halo effect [35]. As we have found, the adiabatic approximation works well for the reaction cross sections for neutron-rich Ne isotopes. It would thus be interesting to describe the deformed nucleus $^{32}$Ne with e.g., the Hartree-Fock-Bogoliubov (HFB) method taking into ac-
count the pairing interaction and then evaluate the interaction cross section in the adiabatic approximation of the Glauber theory. A work toward this direction is now in progress.

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