Finding Planet Nine: a Monte Carlo approach

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ABSTRACT

Planet Nine is a hypothetical planet located well beyond Pluto that has been proposed in an attempt to explain the observed clustering in physical space of the perihelia of six extreme trans-Neptunian objects or ETNOs. The predicted approximate values of its orbital elements include a semimajor axis of 700 au, an eccentricity of 0.6, an inclination of 30°, and an argument of perihelion of 150°. Searching for this putative planet is already under way. Here, we use a Monte Carlo approach to create a synthetic population of Planet Nine orbits and study its visibility statistically in terms of various parameters and focusing on the aphelion configuration. Our analysis shows that, if Planet Nine exists and is at aphelion, it might be found projected against one out of four specific areas in the sky. Each area is linked to a particular value of the longitude of the ascending node and two of them are compatible with an apsidal anti-alignment scenario. In addition and after studying the current statistics of ETNOs, a cautionary note on the robustness of the perihelia clustering is presented.

Key words: methods: statistical – celestial mechanics – minor planets, asteroids: general – Oort Cloud – planets and satellites: detection – planets and satellites: general.

1 INTRODUCTION

Batygin & Brown (2016) have predicted the existence of a massive planet well beyond Pluto in order to explain the observed clustering in physical space of the perihelia of six extreme trans-Neptunian objects or ETNOs (90377) Sedna, 2004 VN112, 2007 TG422, 2010 GB186, 2012 VP113, and 2013 RF96. Such clustering is fairly obvious in terms of the values of their arguments of perihelion in Table 1 and, even more clear, positionally in Table 2 and Fig. 1 at about 3° in right ascension, α, and 0° in declination, δ. The putative object responsible for inducing this clustering has been provisionally denominated Planet Nine. Batygin & Brown (2016) have provided tentative values for the orbital parameters of the proposed 10 Earth masses planet (semimajor axis, a = 700 au, eccentricity, e = 0.6, inclination, i = 30°, and argument of perihelion, ω = 150°) and discussed its possible location in the sky.1 Planet Nine as characterized by Batygin & Brown (2016) is expected to exhibit a magnitude V in the range 16–21 at perihelion and 20–25 at aphelion. Detailed modelling by Linder & Mordasini (2016) gives a magnitude V from the reflected light of 23.7 at aphelion.

In principle, searching for this putative planet is feasible for currently active moving object surveys and it is already under way. For slow-moving objects, most active surveys can record candidates brighter than V = 22 mag (see e.g. Harris & D’Abramo 2015). The faintest natural moving object observation performed so far corresponds to V = 26.7 mag for asteroid 2008 LG2 (Micheli et al. 2015) and the faintest objects observable with 8-m class telescopes have about 27. V774104 was discovered with magnitude 24 at 103 au (Sheppard, Trujillo & Tholen 2015). Detection of moving objects not only depends on their apparent magnitude but on their rate of motion as well (see e.g. Harris & D’Abramo 2015). However, with an average daily motion of nearly 1 arcsec d−1 at perihelion and almost 0.06 arcsec d−1 at aphelion, this should be a non-issue for Planet Nine; the object’s sky motion is far too slow. Fortunately, its shifting with respect to background stars due to parallax as the Earth moves around the Sun could be as high as 3 arcsec d−1 at aphelion (V774104 was identified via parallax, not daily motion). In any case, if Planet Nine currently moves projected against a rich stellar background (a bright section of the Milky Way galaxy for instance) and/or its apparent magnitude is > 22 in V, its eventual identification could be particularly challenging.

The Planet Nine hypothesis presents a suitable and robust scenario to explain the orbital properties of six ETNOs, but it may not be adequate to account for the apparent clustering of arguments of perihelion around 0° (Trujillo & Sheppard 2014) and inclination around 20° (de la Fuente Marcos & de la Fuente Marcos 2014) observed for ETNOs with values of the semimajor axis in the range 150–250 au. A number of scenarios aimed at explaining the available observational evidence have been proposed since the discovery of 2012 VP113 (Trujillo & Sheppard 2014). They include the possible existence of one (Trujillo & Sheppard 2014; Gomes, Soares & Brassier 2015; Malhotra, Volk & Wang 2016) or more trans-Plutonian planets (de la Fuente Marcos & de la Fuente Marcos 2014; de la Fuente Marcos, de la Fuente Marcos & Aarseth 2015), capture of ETNOs within the Sun’s natal open star cluster (Jílková et al. 2015), stellar encounters (Brasser & Schwamb 2015; Feng

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1 http://www.findplanetnine.com

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in terms of orbital period. The sample is small but this fact may be signalling a different dynamical context for these two objects. It is statistically possible that Sedna and 2012 VP113 are members of a separate dynamical class within the ETNOs and therefore be subjected to a different set of perturbations.

The scenario in which trans-Plutonian planets keep the values of the orbital parameters of the ETNOs in check thanks to a particular case of the Kozai mechanism (Kozai 1962) discussed in Trujillo & Sheppard (2014), de la Fuente Marcos & de la Fuente Marcos (2014) or de la Fuente Marcos et al. (2015) and the resonant coupling mechanism described in Batygin & Brown (2016) create dynamical pathways that in some cases may deliver objects to high inclination or even retrograde orbits. In principle, the mechanism detailed in de la Fuente Marcos et al. (2015) can also produce objects with orbits at steeply inclined angles, it already does it for Jupiter. On the other hand, the scenarios described in de la Fuente Marcos et al. (2015) and Batygin & Brown (2016) are not incompatible, and the Kozai mechanism can operate at high eccentricities (see e.g. Naoz 2016) when the value of the relative longitude of perihelion, ΔΩ, librates about 180° (apsidal anti-alignment). A hypothetical Planet Nine may induce Kozai-like behaviour, i.e. libration of the value of the argument of perihelion of the ETNOs that reproduces the observed clustering. The currently known ETNOs probably represent evolutionary steps within dynamical tracks and some of them are more dynamically evolved than others. The presence of statistical outliers could be a sign of this.

## 3 A MONTE CARLO APPROACH

The study of the visibility of a set of orbits with parameters defined within some boundary values is a statistical problem well suited to apply Monte Carlo techniques (Metropolis & Ulam 1949; Press et al. 2007). A representative sample of the set of orbits under study is systematically explored so the regions in the sky with optimal visibility (highest probability) can be determined; in our case, the equations of the orbits of both the Earth and Planet Nine under the two-body approximation (e.g. Murray & Dermott 1999) are sampled to find the minimum and maximum distance between the Earth and Planet Nine. This technique was used in de la Fuente Marcos & de la Fuente Marcos (2014) to analyse the visibility of the ETNOs, and the details and further references are given there. Using a Monte Carlo approach, we generate a synthetic population of Planet Nines with semimajor axis, a ∈ (650, 750) au, eccentricity, e ∈ (0.55, 0.65), inclination, i ∈ (25, 35)°, longitude of the ascending node, Ω ∈ (0, 360)°, and argument of perihelion, ω ∈ (140, 160)° as no explicit value of Ω is given in Batygin & Brown (2016). We assume that the orbits of the multiple instances of Planet Nine are uniformly distributed in orbital parameter space. Ten million test orbits have been studied focusing on the visibility at aphelion. The analyses in Fienga et al. (2016) and Linder & Mordasini (2016) strongly disfavour a present-day Planet Nine located at perihelion.

The distribution in equatorial coordinates of the set of studied orbits is presented in Fig. 1. In this figure, the value of the parameter in the appropriate units is colour coded following the scale printed on the associated colour box. In panel D (inclination), the locations of the Galactic disc and centre are indicated. The background stellar density is the highest towards these regions in the sky. The distribution of aphelion distances, semimajor axes and eccentricities is rather uniform. The distribution in inclination and argument of perihelion depends on the declination; those orbits with higher values of the inclination reach aphelion at lower declinations, the
Table 1. Various orbital parameters — $q = a(1 - e)$, $Q = a(1 + e)$, $\omega = \Omega + \omega$, $P$ is the orbital period, $\Omega^*$ and $\omega^*$ are $\Omega$ and $\omega$ in the interval $(-\pi, \pi)$ instead of the regular $(0, 2\pi)$ — for the 16 objects discussed in this Letter. The statistical parameters are $Q_1$, first quartile, $Q_3$, third quartile, IQR, interquartile range, OL, lower outlier limit ($Q_1 - 1.5$ IQR), and OU, upper outlier limit ($Q_3 + 1.5$ IQR); see the text for additional details. (Epoch: 2457400.5, 2016-January-13.0 00:00:00.0 UT. 32000.0 ecliptic and equinox. Source: Jet Propulsion Laboratory Small-Body Database. Data retrieved on 27 February 2016.)

| Object | $a$ (au) | $e$ | $i$ (°) | $\Omega$ (°) | $\omega$ (°) | $q$ (au) | $Q$ (au) | $P$ (yr) | $\Omega^*$ (°) | $\omega^*$ (°) |
|--------|---------|-----|--------|------------|------------|---------|--------|--------|-------------|-------------|
| (82158) 2001 FP85 | 226.3448 | 0.8486685 | 30.75720 | 179.3004 | 6.9787 | 186.2791 | 34.2531 | 418.4364 | 3405.367 | 179.3004 | 6.9787 |
| (90377) Sedna | 507.5603 | 0.8508124 | 11.92872 | 144.5463 | 31.4614 | 96.0077 | 76.0415 | 939.0792 | 11435.094 | 144.5463 | -48.5386 |
| (148209) 2009 CR10 | 227.9513 | 0.8057223 | 22.71773 | 128.2463 | 31.0219 | 85.4655 | 44.8529 | 411.6646 | 3441.687 | 128.2463 | -42.7807 |
| (445473) 2010 VZ88 | 152.7794 | 0.7753635 | 4.50950 | 117.4524 | 313.8953 | 71.3477 | 34.3198 | 271.2389 | 1888.449 | 117.4524 | -46.1047 |
| 2002 GB32 | 215.7621 | 0.8362043 | 14.17368 | 176.9791 | 36.9855 | 213.9646 | 35.3409 | 396.1834 | 3169.356 | 176.9791 | 36.9855 |
| 2003 HB27 | 164.6181 | 0.7685925 | 15.47644 | 197.8293 | 10.7805 | 208.6098 | 38.0399 | 291.1424 | 2698.666 | 151.1119 | -110.1157 |
| 2003 SS222 | 321.0199 | 0.8525664 | 25.56295 | 66.0107 | 327.1707 | 33.1814 | 47.3291 | 549.7106 | 5751.830 | 66.0107 | -32.8293 |
| 2004 VN112 | 151.1376 | 0.7420410 | 20.46234 | 306.1711 | 32.3890 | 338.5601 | 38.9873 | 236.2879 | 1858.091 | -53.8289 | 32.3890 |
| 2005 RH53 | 492.7277 | 0.9279716 | 18.58697 | 112.9515 | 285.7968 | 38.7438 | 35.5791 | 949.8764 | 10937.517 | 112.9515 | -74.2032 |
| 2007 VJ305 | 188.3373 | 0.8131705 | 12.00306 | 24.3834 | 338.3161 | 2.7445 | 35.181 | 70.3419 | 2584.715 | 24.3834 | -21.6389 |
| 2010 GB104 | 371.1183 | 0.8867090 | 25.03182 | 130.6119 | 347.8124 | 118.4243 | 48.7245 | 693.5121 | 4201.648 | -12.1876 |
| 2012 VP113 | 259.3002 | 0.6896024 | 20.44680 | 90.8179 | 239.7168 | 24.5346 | 80.4862 | 438.1142 | 4715.538 | 90.8179 | -66.2832 |
| 2013 GP16 | 152.4968 | 0.7305475 | 33.48578 | 210.7142 | 42.1284 | 252.8246 | 41.1201 | 263.8736 | 1883.213 | -149.2858 | 42.1284 |
| 2013 RF89 | 390.0738 | 0.8826202 | 26.41402 | 67.5205 | 336.4991 | 24.0166 | 36.3846 | 581.8631 | 4337.774 | 67.5205 | -43.5009 |
| 2015 SO26 | 162.7035 | 0.7967110 | 23.44153 | 33.6221 | 354.9699 | 35.6201 | 33.1637 | 292.2434 | 2112.149 | -162.1707 | 354.9699 |

Table 2. Equatorial coordinates, apparent magnitudes (with filter if known) at discovery time, absolute magnitude, and $\omega$ for the 16 objects discussed in this Letter. (J2000.0 ecliptic and equinox. Source: Minor Planet Center — MPC—Database. —)

| Object | $a$ (AU) | $\delta$ (°, ′, ″) | $m$ (mag) | $H$ (mag) | $\omega$ (°) |
|--------|---------|------------------|--------|---------|------------|
| 2015 SO26 | 162.7035 | 0.7967110 | 23.44153 | 33.6221 | 354.9699 |

same behaviour is observed for the ones with lower values of the argument of perihelion. The distribution in longitude of the ascending node depends on the right ascension; orbits with $\Omega \sim 0^\circ$ reach aphelion at $\alpha \sim 23^\circ$, the ones with $\Omega \sim 90^\circ$ at $\alpha \sim 8^\circ$, and those with $\Omega \sim 270^\circ$ reach aphelion at $\alpha \sim 17^\circ$. In any case, orbits reach perihelion at declination in the range $-20^\circ$ to $50^\circ$ (not shown) and aphelion in the range $-50^\circ$ to $20^\circ$. This is markedly different from the bias found for the ETNOs in de la Fuente Marcos & de la Fuente Marcos (2014); i.e. searching for Planet Nine is not expected to increase the discovery rate of ETNOs and searching for ETNOs is not going to make a direct detection of Planet Nine more likely (indirectly yes, by improving the values of its putative orbital elements).

4 PLANET NINE AT APHELION

Detection of Planet Nine would be much easier if it is close to perihelion at present, but the analysis of Cassini radio ranging data carried out in Fienga et al. (2016) strongly suggests that Planet Nine as characterized by Batygin & Brown (2016) cannot be at perihelion. This conclusion is consistent with the analysis of its expected photometric properties in e.g. Linder & Mordasini (2016). A location in or near aphelion is however compatible with both the analysis of planetary ephemerides and the outcome of the many surveys completed in recent years. If Planet Nine is currently near aphelion, its declination will be in the range $-50^\circ$ to $20^\circ$ (see Fig. 1), but not all the values of $\delta$ are equally probable. Figure 2 shows that the distribution is somewhat uniform in right ascension, but the probability of finding an orbit reaching aphelion at declinations in the ranges $(40^\circ, 30^\circ)$ and $(0, 10^\circ)$ is nearly 1.7 times higher than that of doing it in the range $(30^\circ, 0^\circ)$. The effect of the bias in declination is analysed in more detail in Fig. 3. Locations near the Galactic Centre are possible if $\Omega \sim 30^\circ$; the Galactic Centre is approximately $\Omega \sim 270^\circ$.
at $\alpha \sim 17^\circ$ and $\delta \sim -29^\circ$. A more exhaustive analysis shows that if $\delta \in (5,7)^\circ$ then $\Omega \in (145,155)^\circ$ and $\alpha \in (8,9)^\circ$, which is located in Hydra and away from the main bulk of the Milky Way. Unfortunately, another optimal solution is possible—albeit slightly less probable—if $\delta \in (-35,-33)^\circ$ then $\Omega \in (265,275)^\circ$ and $\alpha \in (15.5,16.5)^\circ$, which is located towards the Galactic bulge between the constellations of Scorpion and Lupus, but relatively far from the Galactic Centre. These two areas are associated with values of $\Omega$ that give $\Delta \sigma$ equal to $180^\circ$ ($\Omega = 150^\circ$) and $60^\circ$ ($\Omega = 270^\circ$), respectively. This second value is inconsistent with the pseudo resonant scenario described in Batygin & Brown (2016). Figures 2 and 3 show that two other solutions are possible but less probable. One with $\delta \in (5,7)^\circ$, $\Omega \sim 100^\circ$ and $\alpha \in (3.5,4.5)^\circ$ —in Taurus—and the second one with $\delta \in (-35,-33)^\circ$, $\Omega \sim 310^\circ$ and $\alpha \in (20,21)^\circ$ —between Microscopium and Sagittarius. The solution with $\Omega \sim 100^\circ$ is favoured by Batygin and Brown (2016; see footnote 1) and subsequently by Fienga et al. (2016). Such solution gives a value of $\Delta \sigma \sim 200^\circ$ which is also consistent with apsidal anti-alignment.

5 CASSINI HINTS: FIENGA ET AL. (2016)

Fienga et al. (2016) have carried out an analysis of Cassini radio ranging data. Their extrapolation of the Cassini data indicates that Planet Nine as characterised by Batygin & Brown (2016) cannot exist in the interval of true anomaly ($-132, 106.5)^\circ$. This automatically excludes the perihelion (see their fig. 6). The aphelion is included in the uncertainty zone where the Cassini data do not provide any constraints, i.e. the residuals are compatible with zero. Fienga et al. (2016) indicate that from the point of view of the Cassini residuals, the most probable position of Planet Nine —assuming a value of $\Omega = 113^\circ$— is at true anomaly equal to $117.8^\circ+11^\circ$. An analysis similar to that in Section 3 gives Fig. 4; this prediction places Planet Nine at $\alpha \sim 2^\circ$ and $\delta \sim -20^\circ$, in Cetus.

6 DISCUSSION

The Planet Nine hypothesis represents an exciting opportunity to survey the outskirts of the Solar system with a purpose and improve our current knowledge of that distant region as well as of testing the correctness of models and long-standing assumptions. Our visibility analyses provide clues on the most probable location of the putative planet given a set of assumed orbital parameters based on the published data. The aphelion configuration gives two preferred present locations with nearly the same degree of probability and two others with lower probability. If any of the assumed parameters is grossly in error, the preferred locations computed via Monte Carlo would be different but perhaps not too far from the values discussed here. Less probable locations are mostly close to the regions where ETNOs have already been found and the lack of detections there suggests that their lower values of the probability are confirmed by the available observational data. Figure 1 shows that for finding ETNOs, the region enclosed between galactic latitude $-30^\circ$ and $30^\circ$ has been so far carefully avoided. In general, this cannot be the case for any serious search for Planet Nine.

7 CONCLUSIONS

In this Letter, we have explored the visibility of Planet Nine. This study has been performed using Monte Carlo techniques. In addi-
On the visibility of Planet Nine

Figure 3. Frequency distribution in $\Omega$ (top panels), $\omega$ (middle panels), and right ascension (bottom panels) of virtual orbits with $\delta$ in the ranges ($-40$, $-30$)° (left-hand panels), ($-30$, $0$)° (central panels), and ($0$, $10$)° (right-hand panels). The black circles correspond to objects in Tables 1 and 2.

Figure 4. As Fig. 1 but for the orbital solution in Fienga et al. (2016). Here, panel A shows the value of the geocentric distance.

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REFERENCES

Batygin K., Brown M. E., 2016, AJ, 151, 22
Brasser R., Schwamb M. E., 2015, MNRAS, 446, 3788
Bromley B. C., Kenyon S. J., 2014, ApJ, 796, 141
Brown R. B., Firth J. A., 2016, MNRAS, 456, 1587
Cowan N. B., Holder G., Kaib N. A., 2016, ApJ, submitted (arXiv:1602.05963)
de la Fuente Marcos C., de la Fuente Marcos R., 2014, MNRAS, 443, L59
de la Fuente Marcos C., de la Fuente Marcos R., Aarseth S. J., 2015, MNRAS, 446, 1867
Feng F., Bailer-Jones C. A. L., 2015, MNRAS, 454, 3267
Fienga A., Laskar J., Manche H., Gastineau M., 2016, A&A, 587, L8
Ginzburg S., Sari R., Loeb A., 2016, ApJ, submitted (arXiv:1603.02876)
Gomes R. S., Soares J. S., Brasser R., 2015, Icarus, 258, 37
Harris A. W., D’Abramo G., 2015, Icarus, 257, 302
Iorio L., 2014, MNRAS, 444, L78
Jilkoviř L., Portegies Zwart S., Pijloo T., Hammer M., 2015, MNRAS, 453, 3157
Kenyon S. J., Bromley B. C., 2015, ApJ, 806, 42
Kozai Y., 1962, AJ, 67, 591
Li G., Adams F. C., 2016, ApJ, submitted (arXiv:1602.08496)
Linder E. F., Mordasini C., 2016, A&A, submitted (arXiv:1602.07465)
Luhman K. L., 2014, ApJ, 781, 4
Madigan A.-M., McCourt M., 2016, MNRAS, 457, L89
Malhotra R., Volk K., Wang X., 2016, ApJ, submitted (arXiv:1603.02196)
Metropolis N., Ulam S., 1949, J. Am. Stat. Assoc., 44, 335
Micheli M., Borgia B., Drolshagen G., Koschny D., Perozzi E., 2015, IAU Gen. Assem. 29, p. 2249572
Murray C. D., Dermott S. F., 1999, Solar system Dynamics. Cambridge Univ. Press, Cambridge, p. 97
Naoz S., 2016, ARA&A, XXX, XXX (arXiv:1601.07175)
Paužo R., Klčař J., 2016, A&A, XXX, XXX (arXiv:1602.03151)
Press W. H., Teukolsky S. A., Vetterling W. T., Flannery B. P., 2007, Numerical Recipes: The Art of Scientific Computing, 3rd edn. Cambridge Univ. Press, Cambridge
Sheppard S. S., Trujillo C., Tholen D., 2015, Am. Astron. Soc. – DPS meeting, 47, 203.07
Trujillo C. A., Sheppard S. S., 2014, Nature, 507, 471
Whitmire D. P., 2016, MNRAS, 455, L114

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