Anisotropic pressure in the quark core of a strongly magnetized hybrid star

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Abstract. The impact of a strong magnetic field, varying with the total baryon number density, on thermodynamic properties of strange quark matter (SQM) in the core of a magnetized hybrid star is considered at zero temperature within the framework of the Massachusetts Institute of Technology (MIT) bag model. It is clarified that the central magnetic field strength is bound from above by the value at which the derivative of the longitudinal pressure with respect to the baryon number density vanishes first somewhere in the quark core under varying the central field. Above this upper bound, the instability along the magnetic field is developed in magnetized SQM. The total energy density, longitudinal and transverse pressures are found as functions of the total baryon number density.

1. Introduction

The study of the QCD phase diagram under extreme conditions of temperature or density is continuing to be a hot research issue. The other factor which can significantly influence the structure of the QCD phase diagram is magnetic field. Strong magnetic fields of about $H \sim 10^{18}$ G (RHIC), or even an order of magnitude larger (LHC), are generated in noncentral high-energy heavy-ion collisions [1, 2]. The electric charge separation with respect to the reaction plane of colliding nuclei due to the the chiral magnetic effect [3] could be one of the observable imprints of strong magnetic fields generated in heavy-ion collisions. In the astrophysical context, for a special class of neutron stars, called magnetars, the magnetic field strength at the surface can reach the values of about $10^{14}-10^{15}$ G. Even stronger magnetic fields up to $10^{19}$ G may potentially occur in the inner cores of magnetars [4]. The possible imprint of such ultrastrong magnetic fields could be the large pulsar kick velocities as a result of asymmetric neutrino emission in direct Urca processes in the dense core of a magnetized neutron star [5]. The origin of strong magnetic fields of magnetars is still under discussion, and, among other possibilities, it is not excluded that this can be due to spontaneous ordering of hadron [6, 7, 8, 9], or quark [10] spins in the dense interior of a neutron star.

Thus, the study of thermodynamic properties of nuclear matter in a strong magnetic field is the problem of a considerable interest. In particular, the pressure anisotropy, exhibited in the difference between the longitudinal and transverse (along and perpendicular to the magnetic field) pressures, becomes important for strongly magnetized matter [11, 12, 13, 14, 15]. In this study, we consider strongly magnetized SQM, composed of deconfined up, down and strange quarks, within the framework of the MIT bag model [16, 17]. The relevant astrophysical object...
is a hybrid star whose quark core is surrounded with the hadronic crust. It was clarified earlier that, if the uniform magnetic field exceeds some critical value, the longitudinal pressure becomes
negative resulting in the appearance of the longitudinal instability in SQM [14]. The value of
the corresponding critical field represents, in fact, the upper bound on the magnetic field in the
quark core of a hybrid star when spatial nonuniformity in the field distribution is disregarded. In
this research, we would like to extend the previous consideration [14] to the case of the spatially
nonuniform magnetic field distribution.

2. Numerical results and discussion
For the details of the formalism one can address to Ref. [14]. We use the simplified variant of the
MIT bag model, in which quarks are considered as free fermions moving inside a finite region of
space called a "bag". The effects of the quark confinement are implemented by introducing the
core pressure \( B \). Note that SQM can represent the ground state of matter, or can be metastable.
In order to be absolutely stable, the energy per baryon of magnetized SQM should be less than
that of the most stable \(^{56}\)Fe nucleus under the zero external pressure and temperature. Further
we will be interested in the astrophysical scenario, in which SQM forms the core of a hybrid star
and, hence, is metastable at zero pressure. The gravitational pressure from the outer hadronic
layers stabilizes quark matter in the core.

In the MIT bag model, the total energy density \( E \), the longitudinal \( p_l \) and transverse \( p_t \)
pressures in magnetized quark matter read

\[
E = \Omega + \sum_i \mu_i \varrho_i + \frac{H^2}{8\pi} + B, \\
p_l = -\Omega - \frac{H^2}{8\pi} - B, \\
p_t = -\Omega - HM + \frac{H^2}{8\pi} - B,
\]

where \( M = -\frac{\partial \Omega}{\partial H} \) is the total magnetization, \( \Omega = \sum_i \Omega_i \), \( \Omega_i \) and \( \varrho_i = -\frac{\partial \Omega}{\partial \varrho_i} \) are the
thermodynamic potential and number density, respectively, for fermions of \( i \)th species with
the chemical potential \( \mu_i \), including \( u, d, s \) quarks plus electrons to ensure charge neutrality and
beta equilibrium with respect to the weak processes occurring in the quark core of a hybrid star.

In the previous research [14], the impact of the uniform magnetic field on thermodynamic
properties of SQM was considered. In a more realistic study, it is necessary to take into account
that the magnetic field varies from the core to the surface of a star. Following Ref. [4], we will
model this change by the dependence of the magnetic field on the baryon density of the form

\[
H(\varrho_B) = H_s + H_{cen}\left(1 - e^{-\beta(\varrho_B/\varrho_0)}\gamma\right).
\]

Here \( H_{cen} \) and \( H_s \) are the magnetic field strengths in the center (assuming that the central
baryon density is essentially larger than the nuclear saturation density \( \varrho_0 = 0.16 \text{ fm}^{-3} \)) and at
the surface of a star, respectively; \( \beta \) and \( \gamma \) are the model parameters. We set the surface field
\( H_s = 10^{15} \text{ G} \), and the central density is chosen to be \( \varrho_{cen} = 7\varrho_0 \). In Eq. (4), we use the model
parameter sets \( \beta = 0.02, \gamma = 3 \) from Ref. [18] (slow varying magnetic field), and \( \beta = 0.001, \gamma = 6 \)
from Ref. [15] (fast varying magnetic field). In the subsequent calculations, the same quark
current masses are employed as in the studies [14, 19], and we adopt \( B = 76 \text{ MeV/fm}^3 \), which
is slightly larger than the upper bound \( B_u \approx 75 \text{ MeV/fm}^3 \) from the absolute stability window.

Further we will assume that the quark phase appears at the total baryon density \( \varrho_B \sim 3\varrho_0 \) [20],
and, hence, the baryon density for magnetized SQM changes in the range \( 3\varrho_0 \leq \varrho_B \leq 7\varrho_0 \). The
Figure 1. Transverse $p_t$ (three upper curves) and longitudinal $p_l$ (three lower curves) pressures in magnetized SQM as functions of the total baryon number density, corresponding to Eq. (4) with: (a) $\beta = 0.02, \gamma = 3$ and (b) $\beta = 0.001, \gamma = 6$ for $H_s = 10^{15}$ G and variable central field $H_{cen}$. The full dots correspond to the points where $p_l'(\varrho_B) = 0$.

main strategy in the further calculations of the anisotropic pressure is as follows. We vary the central magnetic field strength $H_{cen}$ in Eq. (4), and like to determine at which baryon density within the above range the onset of the longitudinal instability occurs. Fig. 1 shows the transverse $p_t$ and longitudinal $p_l$ pressures as functions of the total baryon number density for the above parametrizations of the magnetic field strength. The general tendency is that, under increasing the central field $H_{cen}$, the transverse pressure $p_t$ increases while the longitudinal pressure $p_l$ decreases. Also, the transverse pressure always increases with the total baryon density $\varrho_B$, while the dependence of the longitudinal pressure $p_l$ on $\varrho_B$ can be different. Let us consider first the case of the slow varying magnetic field with $\beta = 0.02, \gamma = 3$ in Eq. (4) (Fig. 1a). Under increasing the central field $H_{cen}$, the longitudinal pressure, at first, remains to be increasing function of the baryon density with $p_l'(\varrho_B) > 0$. However, under further increasing the central field, the curve $p_l'(\varrho_B)$ is bending down in its middle part. At $H_{cen} \approx 2.36 \cdot 10^{18}$ G, the derivative $p_l'(\varrho_B)$ vanishes at $\varrho_B \approx 3.74\varrho_0$ (the corresponding point on the curve is marked by the full red dot) while remaining positive for other baryon densities from the interval under consideration. Under further increasing the central magnetic field, there appears the part of the curve characterized by the negative derivative $p_l'(\varrho_B) < 0$, contrary to the stability constraint $p_l'(\varrho_B) > 0$. Hence, the corresponding states of magnetized SQM are unstable and the instability is developed along the magnetic field direction. The onset of instability corresponds to $H_{cen} \approx 2.36 \cdot 10^{18}$ G, at which the derivative $p_l'(\varrho_B)$ vanishes first. This value represents the upper bound on the central magnetic field strength in the quark core of a hybrid star in the case of the slow varying magnetic field with $\beta = 0.02, \gamma = 3$ in Eq. (4). For the fast varying field with $\beta = 0.001, \gamma = 6$, the upper bound on the central magnetic field strength is somewhat smaller, $H_{cen} \approx 1.54 \cdot 10^{18}$ G, for which the derivative $p_l'(\varrho_B)$ vanishes at $\varrho_B \approx 3.27\varrho_0$ (cf. Fig. 1b).

Fig. 2 shows the energy density $E$ of the system and its matter part $E_m \equiv E - E_f$ ($E_f = \frac{H^2}{8\pi}$ being the magnetic field energy density) as functions of the total baryon number density for the above parametrizations of the magnetic field strength. The curves for the matter part are practically indistinguishable for the different values of the central magnetic field, used in calculations with each parameter set, and look almost as one curve. This figure allows to estimate the relative role of the matter $E_m$ and magnetic field $E_f$ contributions to the total energy density.
magnetized SQM, transverse pressure in the core of a hybrid star is bound from above by the value, at which the derivative of the entropy of a spin polarized state [22, 23, 24].

In this study, we have considered the thermodynamic properties of strongly magnetized SQM at zero temperature. It would be of interest also to extend this research to finite temperatures [12, 21], when new interesting effects could occur, such as the unusual behavior of the entropy of a spin polarized state [22, 23, 24].

In summary, we have considered the impact of varying with the total baryon number density magnetic field on thermodynamic properties of SQM at zero temperature under conditions relevant to the cores of strongly magnetized hybrid stars. The total energy density $E$ of magnetized SQM, transverse $p_t$ and longitudinal $p_l$ pressures have been calculated as functions of the total baryon number density. Also, the highly anisotropic EoS has been determined in the form of $p_t(E)$ and $p_l(E)$ dependences. It has been clarified that the central magnetic field in the core of a hybrid star is bound from above by the value, at which the derivative of the longitudinal pressure $p_l'(p_B)$ vanishes first under varying the central field. Above this upper bound, the instability along the magnetic field direction is developed in magnetized SQM.

Because in all cases we choose the central field smaller than the corresponding upper bound, all quantities $p_t$, $p_l$ and $E$ are the increasing functions of the baryon density, and, hence, $p_t(E)$ and $p_l(E)$ are also the increasing functions.

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Figure 3. The transverse $p_t$ (panels (a), (b)) and longitudinal $p_l$ (panels (c), (d)) pressures in magnetized strange quark matter as functions of the total energy density $E$ for the same parametrizations of the magnetic field strength as in Fig. 1 and variable central field $H_{cen}$.

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