Lagrangian formulation of massive fermionic higher spin fields on a constant electromagnetic background

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Abstract

We consider massive half-integer higher spin fields coupled to an external constant electromagnetic field in flat space of an arbitrary dimension and construct a gauge invariant Lagrangian in the linear approximation in the external field. A procedure for finding the gauge-invariant Lagrangians is based on the BRST construction where no off-shell constraints on the fields and on the gauge parameters are imposed from the very beginning. As an example of the general procedure, we derive a gauge invariant Lagrangian for a massive fermionic field with spin 3/2 which contains a set of auxiliary fields and gauge symmetries.

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1 Introduction

Despite the recent progress in higher spin gauge theories (see e.g. [1]–[16] for review of various aspects of the subject) there are still a number of problems to address. Construction of the interacting Lagrangians of massive higher spin fields on various backgrounds and study of the properties of these systems is one of these problems. Apart from being interesting in its own right, it is also important from the string theory perspective [17]. As is well known, string theory contains an infinite tower of massive higher spin modes and therefore it is important to understand on which backgrounds these fields can propagate consistently.

Although many aspects of Lagrangian formulation of free fermionic higher spin fields have been studied well enough (see e.g. [18]–[20] and references therein) the problem of interacting fermionic fields is much less understood than the problem of interacting bosonic fields (see also [16] for a recent review). In particular, that the cubic vertices which include fermionic higher spin fields have been constructed in the light cone framework in [21] and various problems of interaction with gravitational and electromagnetic fields have been addressed in [22]–[34].

When considering interactions of massive fields with spin more than zero with a nontrivial background one faces several difficulties such as superluminal propagation and violation of the number of physical degrees of freedom. The requirement that no superluminal propagation takes place imposes in general certain conditions on the background fields [40]–[41] (see also [42] for a recent discussion). Similarly, when turning on nonzero background fields the invariance of the initial system under its gauge transformations can be partially or completely lost and this means in turn that nonphysical polarizations can appear in the spectrum. The requirement of preserving of physical degrees of freedom generically imposes some extra conditions on the background. The question is therefore to find if a background under consideration is physically acceptable i.e., if it satisfies the constraints imposed by the above mentioned conditions.

In this paper we consider a problem of interaction of massive totally symmetric fermionic higher spin fields with constant electromagnetic (EM) background in Minkowski space of an arbitrary dimension $d$. These higher spin fields are described by tensor–spinors with one spinorial index and an arbitrary number $n = s - 1/2$ of totally symmetric tensorial indices. Our main aim is to derive the gauge invariant Lagrangian using the method of BRST construction in the linear approximation in strength $F_{\mu\nu}$ of the external field. This method in fact yields a gauge invariant Lagrangian description for massive higher spin fields in extended Fock space and therefore the Lagrangian will contain, apart from the basic fields, some extra auxiliary fields such as Stückelberg fields. Some of these fields are eliminated with the help of gauge transformations, some of the others should be eliminated as a result of the equations of motion. Therefore, in order to have a consistent gauge invariant

\footnote{Also one points out the papers [36], [37] where non-Lagrangian equations of motion for higher spin fields in the external fields have been considered.}
description for massive higher spin fields, one should have enough gauge freedom and have the “correct” equations of motion, which ensure the absence of ghost.\footnote{\text{One way to check this is to perform a complete gauge fixing in the equations of motion and obtain the equations in terms of basic fields. As a result one obtains equations defining the spectrum of the theory and check if it is ghost free or not.}} Performing this analysis in a way similar to how it has been done in \cite{43} one can show that the preservation of physical degrees of freedom indeed takes place for the Lagrangian under consideration, provided that the terms containing the strength of the external space are considered as a perturbation. Where the problem of superluminal propagation of higher spin fields is concerned we note that in the linear in $F_{\mu\nu}$ approximation this problem does not arise at all due to antisymmetry of $F_{\mu\nu}$ (see e.g. \cite{40} for a spin $3/2$ field).

The paper is organized as follows. Section 2 contains our main results. After a brief reminder of construction of Lagrangians for free massive fermionic higher spin fields we introduce interaction with background electromagnetic fields by modifying the operators which define the BRST charge. The requirement that the modified operators form a closed algebra determines free parameters which are present in the definition of the operators. Then we present the corresponding BRST charge, construct the Lagrangian and use a part of the BRST gauge transformations to gauge away an infinite number of neutral bosonic ghost variables from the Lagrangian. The remaining components of the basic fields obey the Lagrangian field equations and these equations still posses necessary gauge invariance. Integrating the field equations back into a Lagrangian we complete the construction of gauge invariant Lagrangian and equations of motion in terms of a basic massive fermionic higher spin field and appropriate auxiliary fields interacting with a constant EM background.

Section 3 contains a more generic description in terms of so called “quartet formulation” \cite{38}–\cite{39} (see also \cite{44}). This formulation is obtained from the one given in Section 2 by further use of the BRST gauge transformations to gauge away some auxiliary fields which are originally present in the system. In this way the Lagrangian contains only one physical field and six auxiliary fields three of which are Lagrangian multipliers. Let us note that in both cases the fields and the parameters of gauge transformations do not contain any off-shell conditions, unlike the formulation of \cite{45}.

In Section 4 we give a description of the simplest example of the spin $3/2$ field interacting with a constant EM background.

The final Section contains our conclusions and a discussion of some open problems.

## 2 Construction of gauge invariant Lagrangians

Let us briefly summarize the features of the BRST approach for the construction of the gauge invariant free and interacting Lagrangians (see \cite{6} for a review). First
one introduces a set of operators that define a spectrum of the theory\footnote{In free theory the spectrum is given with the help of the relations defining either reducible or irreducible representations of the Poincare or AdS group.}. Provided these operators form a closed algebra one builds a nilpotent BRST charge $Q$, which in turn yields to a quadratic gauge invariant Lagrangian of the form

$$\mathcal{L} \sim \langle \chi | Q | \chi \rangle$$

where $| \chi \rangle$ is a vector in an extended Fock space. The gauge invariance of the Lagrangian under the linear gauge transformations

$$\delta | \chi \rangle = Q | \Lambda \rangle$$

is guaranteed by the nilpotency of the BRST charge $Q^2 = 0$. This procedure is however slightly modified for the case of fermionic higher spin fields, since the condition of the BRST invariance

$$Q | \chi \rangle = 0$$

cannot be integrated back into a Lagrangian in a straightforward way. Rather one uses a part of the gauge transformations (2.2) to gauge away a part of the auxiliary fields which are contained in $| \chi \rangle$. The resulting field equations turn out to be Lagrangian ones and they still possess enough gauge invariance to remove all nonphysical polarizations (see [19] for the details).

The situation is even more complicated if the closure of the algebra of the initial set of operators requires inclusion of certain additional operators into the system. These extra operators can impose too strong conditions on the field $| \chi \rangle$ so that there will be no nonzero solution to the equation (2.3). A way out of this problem is the following (see [18] [19] [20] for fermionic higher spin fields and [6] for a detailed review of the BRST formulation for higher spin fields). One introduces additional sets of oscillator variables and builds auxiliary representation of the generators of the algebra (i.e, of the operators under consideration) in terms of these new variables. Then one defines a modified set of operators as a sum of new and old ones and therefore considers the problem in an extended Fock space. After that one builds BRST charge for modified generators in the standard way since the generators form a closed algebra. It allows us to construct a Lagrangian of the base of the BRST charge under consideration.

After this reminder let us turn to a description of massive fermionic higher spin fields. To this end we introduce the Fock space spanned by the oscillators

$$[a_\mu, a_\nu^\dagger] = \eta_{\mu\nu}, \quad \eta_{\mu\nu} = \text{diag}(-1, 1, \ldots, 1)$$

and consider the operators

$$i l'_0 = i \tilde{\gamma}^\mu \partial_\mu, \quad l'_0 = \partial^2 - m^2, \quad l'_1 = ia_\mu^\dagger \partial_\mu, \quad l'_2 = \frac{1}{2} a_\mu^\dagger a_\mu.$$
Here we introduce Grassmann odd “gamma-matrix like objects” $\tilde{\gamma}^\mu$ and $\tilde{\gamma}$ which are connected with the usual Grassmann even gamma-matrices $\gamma^\mu$ by relation \[18\]

$$\gamma^\mu = \tilde{\gamma}^\mu \tilde{\gamma}, \quad \{\tilde{\gamma}^\mu, \tilde{\gamma}^\nu\} = -2\eta^\mu{}^\nu, \quad \{\tilde{\gamma}^\mu, \tilde{\gamma}\} = 0, \quad \{\tilde{\gamma}, \tilde{\gamma}\} = -2.$$  

(2.6)

The first of the operators in (2.5) corresponds to the Dirac operator for the massive fermion, the second operator is the d’Alembertian for a massive field, the third one is a divergence operator, the fourth one is an operator which takes a gamma-trace and the fifth one is an operator which takes a trace. In order to have a hermitian BRST charge we also introduce operators which are hermitian conjugate to the operators $l_1, t_1$ and $l_2$

$$l_1^+ = ia^+{}^\mu \partial_\mu, \quad t_1^+ = \tilde{\gamma}^\mu a^+_{\mu}, \quad l_2^+ = \frac{1}{2} a^+{}^\mu a^+_{\mu};$$  

(2.7)

Finally in order to close the algebra one introduces the extra operators

$$g_0' = a^+{}^\mu a_\mu + \frac{d}{2};$$  

(2.8)

and $g_m' = m^2$. The operator $g_0'$ is a ”particle” number operator and its eigenvalues are always strictly positive. Therefore, we have a situation described earlier in this Section. We introduce three sets of additional oscillator variables: two sets of bosonic oscillator variables with commutation relations

$$[b_1, b_1^+] = 1, \quad [b_2, b_2^+] = 1,$$  

(2.9)

and one set of fermionic oscillator variables

$$\{f, f^+\} = 1.$$  

(2.10)

Using these new variables one can build auxiliary representation for the original operators and define modified operators as \[18\]

$$t_0 = i\tilde{\gamma}^\mu \partial_\mu - \tilde{\gamma} m$$  

(2.11)

$$l_1 = ia^\mu \partial_\mu + mb_1$$  

(2.12)

$$t_1 = \tilde{\gamma}^\mu a_\mu - \tilde{\gamma} b_1 + f^+ b_2 - 2(b_2^+ b_2 + h)f$$  

(2.13)

$$l_2 = \frac{1}{2} a^\mu a_\mu + \frac{1}{2} b_1^2 + (b_2^+ b_2 + f^+ f + h) b_2$$  

(2.14)

$$g_0 = a^+_{\mu} a^\mu + b_1^+ b_1 + 2 b_2^+ b_2 + f^+ f + \frac{d+1}{2} + h$$  

(2.15)

where $h$ is an arbitrary real constant. The algebra of these operators is given by Table 1.

In order to introduce an interaction of the fermionic fields with an external constant EM background field $F_{\mu\nu} = const$ we shall proceed as follows. First we replace all the partial derivatives by the $U(1)$ covariant ones $D_\mu = \partial_\mu - i e A_\mu$ and include into the expressions of the operators\[4\] \[2.11\] - \[2.15\] terms which vanish in the limit $F_{\mu\nu} \to 0$. After that we require that the new operators form a closed algebra.

\[4\]We shall denote these new operators by the corresponding capital letters.
Before writing an ansatz for the operators let us note that since the trace of a field and its traceless part are independent from each other one can shift the trace of a field so that the traceless condition remains unchanged. Thus we suppose that the operators related with the traceless condition \( t_1, t^+_1, l_2, l^+_2 \) as well as the number operator \( g_0 \) remain unchanged.

\[
T_1 = t_1, \quad T^+_1 = t^+_1, \quad L_2 = l_2, \quad L^+_2 = l^+_2 \quad G_0 = g_0. \quad (2.16)
\]

Moreover, since the oscillator variables \( b_2, b^+_2, f, f^+ \) (see also the expressions (2.13)-(2.15) we assume that these variables are not present in the expressions of the operators \( T_0, L_0, L_1, L^+_1 \).

Since we are going to consider only the linear in \( F_{\mu\nu} \) approximation we take the following ansatz for the operators

\[
L_1 = \imath a^\alpha D^{\alpha} + mb_1 + a^\alpha F_{\alpha\sigma} D^\sigma \sum_{k=0}^\infty f_{0k} b_1^{++k} b_1^k + \bar{\gamma} \bar{\gamma}^\tau F_{\tau\sigma} D^\sigma \sum_{k=0}^\infty f_{2k} b_1^{++k} b_1^{k+1} \\
+ a^{+\mu} F_{\mu\sigma} D^\sigma \sum_{k=0}^\infty f_{4k} b_1^{++k} b_1^{k+2} + \bar{\gamma} \bar{\gamma}^\mu F_{\mu\nu} \sum_{k=0}^\infty d_{0k} b_1^{++k} b_1^{k+1} \\
+ \bar{\gamma} \bar{\gamma}^{\sigma} F_{\sigma\alpha} a^\alpha \sum_{k=0}^\infty d_{2k} b_1^{++k} b_1^k \\
+ a^{+\mu} F_{\mu\alpha} a^\alpha \sum_{k=0}^\infty d_{8k} b_1^{++k} b_1^{k+1} + \bar{\gamma} \bar{\gamma}^{\sigma} F_{\sigma\mu} a^{+\mu} \sum_{k=0}^\infty d_{4k} b_1^{++k} b_1^{k+2} \quad (2.17)
\]

\[
T_0 = \imath \bar{\gamma}^{\mu} D_{\mu} - \bar{\gamma} m + \bar{\gamma}^{\tau} F_{\tau\sigma} D^\sigma \sum_{k=0}^\infty c_{0k} b_1^{++k} b_1^k \\
+ \bar{\gamma} a^\alpha F_{\alpha\sigma} D^\sigma \sum_{k=0}^\infty c_{4k} (b_1^{++})^{k+1} b_1^k + \bar{\gamma} a^{+\mu} F_{\mu\sigma} D^\sigma \sum_{k=0}^\infty c_{5k} b_1^{++k} b_1^{k+1} \\
+ \bar{\gamma} \bar{\gamma}^{\mu\nu} F_{\mu\nu} \sum_{k=0}^\infty a_{0k} b_1^{++k} b_1^k + \bar{\gamma} a^{+\mu} F_{\mu\alpha} a^\alpha \sum_{k=0}^\infty a_{4k} b_1^{++k} b_1^k \\
+ \bar{\gamma} a^{+\mu} F_{\mu\sigma} a^\alpha \sum_{k=0}^\infty a_{2k} (b_1^{++})^{k+1} b_1^k + \bar{\gamma} a^{+\mu} F_{\mu\sigma} a^\alpha \sum_{k=0}^\infty a_{3k} b_1^{++k} b_1^{k+1} \quad (2.18)
\]
\begin{align}
L_1^+ &= ia^{+\mu}D_\mu + mb_1^+ + a^{+\mu}F_{\mu\sigma}D^\sigma \sum_{k=0}^{\infty} f_{1k} b_1^{+k}b_1^k + \gamma\tilde{\gamma}^\nu F_{\nu\sigma}D^\sigma \sum_{k=0}^{\infty} f_{5k} (b_1^+)^{k+1}b_1^k \\
&\quad + a^\alpha F_{\alpha\sigma}D^\sigma \sum_{k=0}^{\infty} f_{5k} (b_1^+)^{k+2}b_1^k + \tilde{\gamma}\tilde{\gamma}^\nu F_{\nu\sigma}F_{\sigma\alpha}a^\alpha \sum_{k=0}^{\infty} d_{3k} (b_1^+)^{k+1}b_1^k \\
&\quad + \gamma \tilde{\gamma}^\sigma F_{\sigma\mu}a^{+\mu} \sum_{k=0}^{\infty} d_{3k} b_1^{+k}b_1^k \\
&\quad + a^{+\mu}F_{\mu\alpha}a^\alpha \sum_{k=0}^{\infty} d_{5k} (b_1^+)^{k+2}b_1^k + \gamma \tilde{\gamma}^\sigma F_{\sigma\alpha}a^\alpha \sum_{k=0}^{\infty} d_{5k} (b_1^+)^{k+2}b_1^k
\end{align}

(2.19)

where \(a_{ik}, c_{ik}, d_{ik}, c_{ik}\) are arbitrary complex constants and the rest of the operators (2.13)–(2.15) are unchanged as one can see form the equation (2.16). Let us note that the above relations can be treated as the deformations of the corresponding relations of free theory by the terms linear in \(F_{\mu\nu}\).

Let us point out that the ansatz for the operators \(L_1, T_0, L_1^+\) (2.17)–(2.19) is not the most general one. The ansatz is taken on the basis of the following “minimal” rule. Let us consider the operators (2.17)–(2.19) in free theory, replace the partial derivatives by the covariant ones and calculate the commutators. Obviously the algebra will not be closed. Then one adds to these operators the minimal number of terms linear in \(F_{\mu\nu}\) in such a way that the algebra is closed in the linear approximation. One can see that according to this “minimal” rule the Lorentz indices of the creation and annihilation operators are always contracted with the an index or indices of \(F_{\mu\nu}\). In principle it is possible to consider other deformations of the free theory by the terms linear in \(F_{\mu\nu}\). For example, one can add to \(L_1\) a term of the form \(a^{+\mu}\gamma_\mu\gamma_\nu F_{\nu\sigma}D^\sigma\) but this term does not obey the “minimal” rule.

From the requirement the \(T_0\) and \(L_0\) to be hermitian, from the condition \((L_1)^+ = L_1^+\) and from the requirement that the total system of operators forms a closed algebra in the linear approximation one finds the expressions for constants which are present in (2.17)–(2.19). These expressions are summarized in the Appendix.

Note that a similar problem was considered in [22], but we found two more arbitrary constants because, unlike [22], we do not require from the very beginning that the coefficients in (2.17)–(2.19) must satisfy reality conditions. As one can see from the Appendix, the complex coefficients are also acceptable.

The new operators form the algebra which is the same as in the free case and is given in Table I.

After we have achieved the closure of the algebra for the operators, the next step is to construct the corresponding BRST charge. This procedure follows closely the one developed for the fermionic fields in [13, 19] to which we refer for more details. First
we construct the standard BRST operator on the basis of the operators \( (2.16) \text{--} (2.19) \)

\[
Q = q_0 T_0 + q_1^+ T_1 + q_1 T_1^+ + \eta_0 L_0 + \eta_1 L_1 + \eta_1 L_1^+ + \eta_2^+ L_2 + \eta_2 L_2^+ + \eta_G G_0
+ 2q_0(q_1^+ \mathcal{P}_1 + q_1 \mathcal{P}_1^+) + (q_1^+ \eta_1 - \eta_1^+ q_1) \partial_0 + (\eta_2^+ q_1 - q_0^2) \mathcal{P}_0 + 2q_1^2 \mathcal{P}_2
+ 2q_1^2 \mathcal{P}_2^+ + q_1^+ \eta_2 \partial_1 + (q_1^+ q_1 - \eta_2^+ \eta_2) \mathcal{P}_G
+ \eta_G(q_1^+ \partial_1 - q_1 \partial_1^+ + \eta_1 \mathcal{P}_1 - \eta_1 \mathcal{P}_1^+ + 2\eta_2^+ \mathcal{P}_2 - 2\eta_2 \mathcal{P}_2^+) \quad (2.20)
\]

Here, \( q_0, q_1, q_1^+ \) and \( \eta_0, \eta_1^+, \eta_1, \eta_2, \eta_G \) are, respectively, the bosonic and fermionic ghost “coordinates” corresponding to their canonically conjugate ghost “momenta” \( p_0, p_1, \mathcal{P}_0, \mathcal{P}_1, \mathcal{P}_1^+, \mathcal{P}_2, \mathcal{P}_2^+, \mathcal{P}_G \). They obey the (anti)commutation relations

\[
\{ \eta_1, \mathcal{P}_1^+ \} = \{ \mathcal{P}_1, \eta_1^+ \} = \{ \eta_2, \mathcal{P}_2^+ \} = \{ \mathcal{P}_2, \eta_2^+ \} = \{ \eta_0, \mathcal{P}_0 \} = \{ \eta_G, \mathcal{P}_G \} = 1,
\]

\[
[q_0, p_0] = [q_1, p_1^+] = [q_1^+, p_1] = i \quad (2.21)
\]

and possess the standard ghost number distribution, \( gh(q, \eta) = -gh(p, \mathcal{P}) = 1 \), which gives \( gh(Q) = 1 \).

For the subsequent computations it is convenient to present the BRST operator

\[
\begin{array}{cccccccccc}
| \downarrow, \to \rangle & T_0 & T_1 & T_1^+ & L_0 & L_1 & L_1^+ & L_2 & L_2^+ & G_0 \\
T_0 & 2L_0 & -2L_1 & -2L_1^+ & 0 & 0 & 0 & 0 & 0 & 0 \\
T_1 & -2L_1 & -4L_2 & -2G_0 & 0 & 0 & T_0 & 0 & T_1^+ & T_1 \\
T_1^+ & -2L_1^+ & -2G_0 & -4L_2^+ & 0 & -T_0 & 0 & -T_1 & 0 & -T_1^+ \\
L_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
L_1 & 0 & 0 & T_0 & 0 & 0 & -L_0 & 0 & L_1^+ & L_1 \\
L_1^+ & 0 & -T_0 & 0 & 0 & L_0 & 0 & -L_1 & 0 & -L_1^+ \\
L_2 & 0 & 0 & T_1 & 0 & 0 & L_1 & 0 & G_0 & 2L_2 \\
L_2^+ & 0 & -T_1^+ & 0 & 0 & -L_1^+ & 0 & -G_0 & 0 & -2L_2^+ \\
G_0 & 0 & -T_1 & T_1^+ & 0 & -L_1 & L_1^+ & -2L_2 & 2L_2^+ & 0 \\
\end{array}
\]

Table 1: The algebra of the operators.
\[ Q = \bar{Q} + \eta_G (N + \frac{d-3}{2} + h) + (2q_1^+ q_1 - \eta_2^+ \eta_2) \mathcal{P}_G \]
\[ N = \delta_{\mu}^+ a_{\mu}^+ + b_{1}^+ b_1 + 2b_{2}^+ b_2 + f^+ f \]
\[ + q_1^+ i p_1 - i p_1^+ q_1 + \eta_1^+ \mathcal{P}_1 + \mathcal{P}_1^+ \eta_1 + 2\eta_2^+ \mathcal{P}_2 + 2\mathcal{P}_2^+ \eta_2 \]
\[ \bar{Q} = q_0 \bar{T}_0 + \eta_0 L_0 + \Delta Q + (q_1^+ \eta_1 - \eta_1^+ q_1) i p_0 + (\eta_1^+ \eta_1 - q_0^2) \mathcal{P}_0 \]
\[ \Delta Q = q_1^+ T_1 + q_1 T_1^+ + \eta_1^+ L_1 + \eta_1 L_1^+ + \eta_2^+ L_2 + \eta_2 L_2^+ + 2q_1^2 \mathcal{P}_2 + 2q_1^2 \mathcal{P}_2^+ \]
\[ + q_1^+ \eta_2 i p_1 - \eta_2 q_1 i p_1 - \eta_2^+ \eta_1 \mathcal{P}_1 - \eta_1^+ \eta_2 \mathcal{P}_1^+ \]
\[ \bar{T}_0 = T_0 + 2q_1^+ \mathcal{P}_1 + 2q_1 \mathcal{P}_1^+ \]

Next we choose the following representation for the vacuum in the Hilbert space
\[ (p_0, q_1, p_1, \mathcal{P}_0, \mathcal{P}_G, \eta_1, \mathcal{P}_1, \eta_2, \mathcal{P}_2) |0\rangle = 0, \quad (2.22) \]

and suppose that the vectors and gauge parameters do not depend on \( \eta_G \).

\[ |\chi\rangle = \sum_{k_1, k_2, k_3, k_4, k_5, k_6, k_7, k_8, k_9, k_{10}, k_{11}} (q_0)^{k_1} (q_1^+)^{k_2} (p_1^+)^{k_3} (\eta_0)^{k_4} (f^+)^{k_5} (\eta_1^+)^{k_6} (\mathcal{P}_1^+)^{k_7} (\eta_2)^{k_8} (\mathcal{P}_2^+)^{k_9} (b_1^+)^{k_{10}} (b_2^+)^{k_{11}} \times \]
\[ \times a^{+\mu_1} \cdots a^{+\mu_{10}} \chi^{k_1 \cdots k_{11}} (x) |0\rangle. \quad (2.23) \]

The sum in (2.23) is taken over \( k_1, k_2, k_3, k_4, k_5, k_6, k_7, k_8, k_9, k_{10}, k_{11} \), running from 0 to infinity, and over \( k_4, k_5, k_6, k_7, k_8, k_9 \), running from 0 to 1. Then, we derive from the equations (2.23) as well as from the reducible gauge transformations, (2.22) a sequence of relations

\[ \bar{Q} |\chi\rangle = 0, \quad (N + \frac{d-3}{2} + h)|\chi\rangle = 0, \quad (\epsilon, gh) (|\chi\rangle) = (1, 0), \quad (2.24) \]
\[ \delta |\chi\rangle = \bar{Q} |\Lambda\rangle, \quad (N + \frac{d-3}{2} + h)|\Lambda\rangle = 0, \quad (\epsilon, gh) (|\Lambda\rangle) = (0, -1), \quad (2.25) \]
\[ \delta |\Lambda\rangle = \bar{Q} |\Lambda^{(1)}\rangle, \quad (N + \frac{d-3}{2} + h)|\Lambda^{(1)}\rangle = 0, \quad (\epsilon, gh) (|\Lambda^{(1)}\rangle) = (1, -2), \quad (2.26) \]
\[ \delta |\Lambda^{(i-1)}\rangle = \bar{Q} |\Lambda^{(i)}\rangle, \quad (N + \frac{d-3}{2} + h)|\Lambda^{(i)}\rangle = 0, \quad (\epsilon, gh) (|\Lambda^{(i)}\rangle) = (i, -i - 1). \quad (2.27) \]

Here \( \epsilon \) defines a Grassmann parity of corresponding fields and parameters of gauge transformations as \((-1)^\epsilon\).

The middle equation in (2.24) is a constraint on possible values of \( h \)
\[ h = 2 - s - \frac{d}{2}, \quad (2.28) \]

By fixing the value of spin, we also fix the parameter \( h \), according to (2.28). Having fixed a value of \( h \), we then substitute it into each of the expressions (2.24)–(2.27).

Analogously to the free case [16] the equation of motion (2.24) cannot be obtained from a Lagrangian. In order to extract from (2.24) a Lagrangian set of equations of
motion we decompose the state vector and gauge parameters in terms of powers of neutral Grassmann even $q_0$, and Grassmann odd $\eta_0$ ghosts

$$ |\chi\rangle = \sum_{k=0}^{\infty} q^k_0 (|\chi^0_0\rangle + \eta_0 |\chi^1_0\rangle), \quad |\Lambda\rangle = \sum_{k=0}^{\infty} q^k_0 (|\Lambda^0_0\rangle + \eta_0 |\Lambda^1_0\rangle).$$

Then we remove all fields except $|\chi^0_0\rangle$ and $|\chi^1_0\rangle$ using a part of the initial gauge symmetries or using their own equations of motion. As a result of this procedure the equation (2.24) is reduced to

$$ \Delta Q |\chi^0_0\rangle + \frac{1}{2} \{ \bar{T}_0, \eta^+_1 \eta_1 \} |\chi^1_0\rangle = 0, \quad \bar{T}_0 |\chi^0_0\rangle + \Delta Q |\chi^0_0\rangle = 0 \quad (2.29)$$

These equations are invariant under the gauge transformations

$$ \delta |\chi^0_0\rangle = \Delta Q |\Lambda^0_0\rangle + \frac{1}{2} \{ \bar{T}_0, \eta^+_1 \eta_1 \} |\Lambda^1_0\rangle, \quad \delta |\chi^1_0\rangle = \bar{T}_0 |\Lambda^0_0\rangle + \Delta Q |\Lambda^1_0\rangle \quad (2.30)$$

The parameters of gauge transformations are in turn invariant under the chain of transformations with a finite number of reducibility stages $i_{\text{max}} = s - 3/2$

$$ \delta |\Lambda^{(i)0}_0\rangle = \Delta Q |\Lambda^{(i+1)0}_0\rangle + \frac{1}{2} \{ \bar{T}_0, \eta^+_1 \eta_1 \} |\Lambda^{(i+1)1}_0\rangle, \quad |\Lambda^{(0)0}_{0/n}\rangle = |\Lambda^0_0\rangle, \quad (2.31)$$

$$ \delta |\Lambda^{(i)1}_0\rangle = \bar{T}_0 |\Lambda^{(i+1)0}_0\rangle + \Delta Q |\Lambda^{(i+1)1}_0\rangle, \quad |\Lambda^{(0)1}_{0/n}\rangle = |\Lambda^1_0\rangle, \quad (2.32)$$

$$ i_{\text{max}} = s - 3/2 \quad (2.33)$$

where $\{ \bar{T}_0, \eta^+_1 \eta_1 \} = \bar{T}_0 \eta^+_1 \eta_1 + \eta^+_1 \eta_1 \bar{T}_0$.

It is straightforward to check that the equations (2.29) can be obtained from the following Lagrangian

$$ \mathcal{L} = \langle \bar{\chi}^0_0 | K_h \left( \bar{T}_0 |\chi^0_0\rangle + \Delta Q |\chi^1_0\rangle \right) + \langle \bar{\chi}^1_0 | K_h \left( \Delta Q |\chi^0_0\rangle + \frac{1}{2} \{ \bar{T}_0, \eta^+_1 \eta_1 \} |\chi^1_0\rangle \right) \rangle \quad (2.34)$$

In (2.34) operator $K_h$

$$ K_h = \sum_{n=0}^{\infty} \frac{1}{n!} \left( |n\rangle \langle n| C(n, h) - 2 f^+ |n\rangle \langle n| f C(n + 1, h) \right), \quad (2.35) $$

$$ C(n, h) = h(h+1) \cdots (h+n-1), \quad C(0, h) = 1, \quad |n\rangle = (b^+_2)^n |0\rangle$$

is needed to maintain hermiticity of the Lagrangian since as one can see from the auxiliary representations for operators (2.13) – (2.13) one has $(l^+_2) \neq l^+_2$ and $(t^+_1) \neq t^+_1$. The fields $|\bar{\chi}^0_0\rangle$, $|\bar{\chi}^1_0\rangle$ are defined as follows

$$ \langle \bar{\chi}^0_0 \rangle = (|\chi^0_0\rangle)^+ \bar{\gamma}^0, \quad \langle \bar{\chi}^1_0 \rangle = (|\chi^1_0\rangle)^+ \bar{\gamma}^0. \quad (2.36)$$

The Lagrangian (2.34) describes the interaction of massive fermionic fields with constant electromagnetic field and it is our main result. It contains, apart from the physical field $\psi_{\mu_1 \ldots \mu_n}(x)$ in $|\chi^0_0\rangle$

$$ |\chi^0_0\rangle = \psi_{\mu_1 \ldots \mu_n}(x)a^{+\mu_1} \ldots a^{+\mu_n}|0\rangle + \ldots \quad (2.37)$$
a number of auxiliary fields\footnote{In decomposition (2.23) they are coefficients in summands which contain at least one creation operator different from $a^{+\mu}$.}, whose number increases with spin value. One can partially or completely fix gauge invariance and obtain different Lagrangian formulations with a smaller number of auxiliary fields, as we shall do it in the next Section.

## 3 Lagrangian formulations with a smaller number of auxiliary fields

In this Section we are going to obtain from (2.34) different Lagrangian formulations partially fixing the gauge invariance.

First we derive a quartet Lagrangian formulation\footnote{Another similar formulation (so-called triplet formulation) of fermionic fields on Minkowski and $AdS_d$ backgrounds contains one physical and two auxiliary fields\cite{46–50} (see also \cite{51} for a recent discussion) and corresponds to a description of reducible representations of the Poincare or $SO(d-2;2)$ groups.}. Initially this formulation was developed for the massless higher spin fields in flat and AdS background in\cite{38}. Its fermionic version contains seven unconstrained fields (one physical field and six auxiliary fields three of which are Lagrangian multipliers) and one unconstrained gauge parameter\footnote{Using dimensional reduction one can obtain the quartet formulation for massive higher spin fields in Minkowski space\cite{39}.}.

To obtain this formulation from the Lagrangian (2.34) we partially fix gauge invariance just as it was done in\cite{20}, except we will not fix gauge invariance corresponding to gauge parameter $|\varepsilon\rangle$

\begin{equation}
|\Lambda_{00}^{(0)}\rangle = |\varepsilon\rangle + \ldots
\end{equation}

\begin{equation}
|\varepsilon_{n-k-1}\rangle = \frac{1}{(n-k)!} a^{+\mu_1} \ldots a^{+\mu_{n-k-1}} \varepsilon_{\mu_1 \ldots \mu_{n-k-1}}(x)|0\rangle.
\end{equation}

Next one can show that after the gauge fixing some of the remaining fields can be removed with the help of the equations of motion and the nonvanishing fields in the quartet formulation are

\begin{equation}
|\chi_0\rangle = |\Psi^{(n)}\rangle + \eta_1^+ \mathcal{P}_1^+ |D^{(n-2)}\rangle + q_1^+ \mathcal{P}_1^+ |E^{(n-2)}\rangle + i\eta_1^+ p_1^+ |\Sigma^{(n-2)}\rangle
\end{equation}

\begin{equation}
|\chi_1\rangle = \mathcal{P}_1^+ |C^{(n-1)}\rangle - ip_1^+ |A^{(n-1)}\rangle + ip_1^+ \eta_1^+ \mathcal{P}_1^+ |\Omega^{(n-3)}\rangle
\end{equation}

The Lagrangian and the gauge transformation for the massive fermionic higher spin field interacting with constant electromagnetic field in the quartet formulation
are\footnote{In order to obtain triplet formulation \cite{46,50} one should to discard field $|E^{(n-2)}\rangle$ and Lagrangian multipliers $|\Lambda^{(n-1)}\rangle$, $|\Sigma^{(n-2)}\rangle$, $|\Omega^{(n-3)}\rangle$ in \eqref{3.5}.}
\begin{align}
\mathcal{L} & = \langle \bar{\Psi}^{(n)} | \{ T_0 | \Psi^{(n)} \rangle + L_1^+ | C^{(n-1)} \rangle + T_1^+ | \Lambda^{(n-1)} \rangle \} \\
& \quad - \langle \bar{C}^{(n-1)} | \{ T_0 | C^{(n-1)} \rangle - L_1 | \Psi^{(n)} \rangle + L_1^+ | D^{(n-2)} \rangle - | \Lambda^{(n-1)} \rangle - T_1^{\prime+} | \Sigma^{(n-2)} \rangle \} \\
& \quad - \langle \bar{D}^{(n-2)} | \{ T_0 | D^{(n-2)} \rangle + L_1 | C^{(n-1)} \rangle + 2 | \Sigma^{(n-2)} \rangle - T_1^{\prime+} | \Omega^{(n-3)} \rangle \} \\
& \quad + \langle \bar{\Lambda}^{(n-1)} | \{ T_1^\prime | \Psi^{(n)} \rangle + | C^{(n-1)} \rangle + L_1^+ | E^{(n-2)} \rangle \} \\
& \quad + \langle \bar{\Sigma}^{(n-2)} | \{ T_1^\prime | C^{(n-1)} \rangle - 2 | D^{(n-2)} \rangle + T_0 | E^{(n-2)} \rangle \} \\
& \quad + \langle \bar{\Omega}^{(n-3)} | \{ T_1^\prime | D^{(n-2)} \rangle + L_1 | E^{(n-2)} \rangle \} \\
& \quad + \langle \bar{E}^{(n-2)} | \{ L_1 | \Lambda^{(n-1)} \rangle + T_0 | \Sigma^{(n-2)} \rangle + L_1^+ | \Omega^{(n-3)} \rangle \} \tag{3.5}
\end{align}

\begin{align}
\delta | \Psi^{(n)} \rangle & = L_1^+ | \Psi^{(n-1)} \rangle, & \delta | C^{(n-1)} \rangle & = -T_0 | \Psi^{(n-1)} \rangle, \tag{3.6} \\
\delta | D^{(n-2)} \rangle & = L_1 | \Psi^{(n-1)} \rangle, & \delta | E^{(n-2)} \rangle & = -T_1^{\prime} | \Psi^{(n-1)} \rangle \tag{3.7}
\end{align}

The fields and the gauge parameter $| \Psi^{(n-1)} \rangle \equiv | \psi^{(n-1)} \rangle$ depend only on the oscillators $(a_\mu, b_1)$. In particular in \eqref{3.5}--\eqref{3.7} the fields and the gauge parameter have uniform decomposition
\begin{align}
| \Phi^{(m)} \rangle & = \sum_{k=0}^{m} \frac{1}{k!} (b_1^\dagger)^k | \phi_{m-k} \rangle \tag{3.8} \\
| \phi_{m-k} \rangle & = \frac{1}{(n-k)!} a^{+\mu_1} \ldots a^{+\mu_{m-k}} \phi_{\mu_1 \ldots \mu_{m-k}} (x) | 0 \rangle \tag{3.9}
\end{align}

and the operators $T_1^\prime$ and $T_1^{\prime+}$ are the $(a_\mu, b_1)$ parts of the operators $t_1$ and $t_1^+$ \eqref{2.13}
\begin{align}
T_1^\prime = \tilde{\gamma}^\mu a_\mu - \tilde{\gamma} b_1 \\
T_1^{\prime+} = a^{+\mu} \tilde{\gamma}^\mu - \tilde{\gamma} b_1^+. \tag{3.10}
\end{align}

Next we will show that the Lagrangian formulation, which obtained in \cite{22}, is a particular case of our general result \eqref{3.5}. To get such Lagrangian formulations we first partly fix the gauge, removing the field $|E^{(n-2)}\rangle$ with the help of gauge transformations \eqref{3.7} and then integrate out all the fields except the field $|\Psi^{(n)}\rangle$. The result is
\begin{align}
\mathcal{L} & = \langle \bar{\Psi}^{(n)} | \{ T_0 - L_1^+ T_1^\prime - T_1^{\prime+} L_1 - T_1^{\prime+} T_0 T_1^\prime \\
& \quad - \frac{1}{4} T_1^{\prime+} T_1^\prime T_1^\prime T_1^\prime - \frac{1}{2} T_1^{\prime+} T_1^\prime L_1 T_1^\prime - \frac{1}{4} T_1^{\prime+} T_1^\prime T_0 T_1^\prime \} | \Psi^{(n)} \rangle \tag{3.11} \\
\delta | \Psi^{(n)} \rangle & = L_1^+ | \Psi^{(n-1)} \rangle \tag{3.12}
\end{align}
where the state $|\Psi^{(n)}\rangle$ and the parameter of gauge transformations $|\Upsilon^{(n-1)}\rangle$ obey the constraints

$$
(T'_1)^3|\Psi^{(n)}\rangle = 0, \quad T'_1|\Upsilon^{(n-1)}\rangle = 0. \quad (3.13)
$$

Such a partial form of the Lagrangian was obtained in [22], but with another (less generalootnote{It should be noted that in [22] was considered deformation of the operators corresponding to the gamma-traceless conditions as well. But this deformation is proportional to an arbitrary constant and as we said at the beginning of our paper can be removed by a field redefinition.}) expressions for the operators $(2.16)$–$(2.19)$.

We can proceed to obtain more Lagrangian formulations. For example, we can resolve constraints on the field and the gauge parameter $(3.13)$. Using decomposition $(3.9)$ for $|\Psi^{(n)}\rangle$ and $|\Upsilon^{(n-1)}\rangle$

$$
|\Psi^{(n)}\rangle = \sum_{k=0}^{n} \frac{1}{k!}(b_1^+)^k|\psi_{n-k}\rangle \quad |\Upsilon^{(n-1)}\rangle = \sum_{k=0}^{n-1} \frac{1}{k!}(b_1^+)^k|\epsilon_{n-1-k}\rangle \quad (3.14)
$$

we find that gauge parameter $|\epsilon_{n-1}\rangle$ is not restricted and the other parameters $|\epsilon_k\rangle$ are expressed in terms of its gamma-traces $|\epsilon_k\rangle = (\gamma^\mu a^\mu)^{n-1-k}|\epsilon_{n-1}\rangle$, so we may make gauge transformation using the unrestricted gauge parameter $|\epsilon_{n-1}\rangle$. One can do the same for the field $|\Psi^{(n)}\rangle$. Due to restriction $(3.13)$ there are only three independent fields $|\psi_n\rangle, |\psi_{n-1}\rangle, |\psi_{n-2}\rangle$ and all the other fields are expressed through these three fields

$$
|\psi_{n-2k-1}\rangle = -k(\gamma^\mu a^\mu)^{2k+1}|\psi_n\rangle + (\gamma^\mu a^\mu)^{2k}|\psi_{n-1}\rangle + (k+1)(\gamma^\mu a^\mu)^{2k-1}|\psi_{n-2}\rangle \quad (3.15)
$$

$$
k \geq 1,
$$

$$
|\psi_{n-2k-2}\rangle = -k(\gamma^\mu a^\mu)^{2k+2}|\psi_n\rangle + (k+1)(\gamma^\mu a^\mu)^{2k}|\psi_{n-2}\rangle. \quad (3.16)
$$

Thus one can obtainootnote{Since the Lagrangian formulation is very large, we do not present it here.} a gauge invariant Lagrangian formulation for a massive fermionic field interacting with constant electromagnetic field with the help of three fields $|\psi_n\rangle, |\psi_{n-1}\rangle, |\psi_{n-2}\rangle$ and one gauge parameter $|\epsilon_{n-1}\rangle$.

Finally, using the remaining unrestricted gauge parameter $|\epsilon_{n-1}\rangle$ one can remove field $|\psi_{n-1}\rangle$ and obtain a Lagrangian formulation in terms of two traceful unrestricted fields: one physical $|\psi_n\rangle$ field and one auxiliary $|\psi_{n-2}\rangle$ field. This Lagrangian has no gauge invariance since we have already used entire gauge freedom. It should be noted that if we decompose the tracefull fields $|\psi_n\rangle$ and $|\psi_{n-2}\rangle$ in a series of traceless fields we obtain set of the fields which coincide with the set of fields of Singh and Hagen [52].

4 Example: spin 3/2

In this section we apply a general procedure described in the previous Sections for the simplest example of spin-3/2 field.
In the case of spin-3/2 field we have \( h = \frac{4d}{d-2} \) (see eq. (2.28)) and since according to (2.33) we have \( i_{\text{max}} = 0 \). Therefore the corresponding Lagrangian formulation is an irreducible gauge theory. Due to \( gh(|\Lambda_{0}^{1}| = -2) \), the nonvanishing fields \( |\chi_{0}^{1}| \) and the gauge parameter \( |\Lambda_{0}^{1}| \), (we have \( |\Lambda_{0}^{1}| \equiv 0 \), possess the following Grassmann grading and ghost number distributions:

\[
(\varepsilon, gh) (|\chi_{0}^{1}|) = (1, 0), \quad (\varepsilon, gh) (|\chi_{0}^{1}|) = (1, -1), \quad (\varepsilon, gh) (|\Lambda_{0}^{1}|) = (0, -1). \quad (4.1)
\]

These conditions determine the dependence of the fields and of the gauge parameters on the oscillator variables in a unique form

\[
|\chi_{0}^{1}| = [ia^{+\mu}(\psi_{\mu}(x) + f^{+}(\bar{\gamma}(x) + b_{1}^{+}\varphi(x))]|0>, \\
|\chi_{0}^{1}| = \langle 0 | [-\psi_{\mu}^{+}(x)ia^{+\mu} + (\psi^{+}(x))\bar{\gamma}f + \varphi^{+}(x)b_{1}]\bar{\gamma}^{0}, \\
|\chi_{0}^{1}| = [\mathcal{P}^{+}_{\lambda}(x) - ip^{+}_{\lambda}(x)]|0>, \\
|\chi_{0}^{1}| = \langle 0 | \{\chi^{+}_{\mu}(x)ip_{\mu} + \chi^{+}(x)\bar{\gamma}P_{1}\}\bar{\gamma}^{0}, \\
|\Lambda_{0}^{1}| = [\mathcal{P}^{+}_{\lambda}(x) - ip^{+}_{\lambda}\gamma\lambda_{1}(x)]|0>. \quad h = -\frac{d-1}{2}.
\]

Substituting these expressions for the fields and the gauge parameters in (2.33) and (2.31) one finds the Lagrangian and gauge transformations for the physical spin-3/2 field \( \psi^{\mu} \) and for the auxiliary fields

\[
\mathcal{L}_{3/2} = \psi^{\mu}\left([i\gamma^{\nu}D_{\nu} - m\right)_{\psi} + \frac{ie}{2m^{2}}(2\zeta_{1} + \xi_{1})\gamma^{\nu}F_{\nu\sigma}D_{\sigma}^{\psi}_{\psi} - \frac{ie}{8m}(1 + 4\zeta_{0})\gamma^{\sigma\nu}F_{\nu\sigma}\psi_{\mu} + \frac{e}{m^{2}}(\zeta_{1} - 2i\zeta_{0})F^{\mu\sigma}_{\sigma\varphi} - \frac{e}{2m}(1 - 2i\zeta_{1})\gamma^{\nu}F_{\nu\mu}\psi_{\nu} + D_{\mu\chi} + \frac{e}{m^{2}}(\zeta_{1} + i\zeta_{0})F_{\mu\sigma}D_{\sigma}\chi - \frac{e}{4m}(1 + 2i\zeta_{1})\gamma^{\nu}F_{\nu\mu}\chi - \chi_{1} - \frac{e}{2m^{2}}(2\zeta_{0} + i\zeta_{0})\gamma^{\nu}F_{\nu\sigma}D^{\sigma}\chi \right. - \frac{e}{8m}(1 + 4\zeta_{0})\gamma^{\mu\nu}F_{\mu\nu}\varphi + \frac{e}{2m^{2}}(2\zeta_{1} + \xi_{1})\gamma^{\nu}F_{\nu\sigma}D^{\sigma}\psi + \frac{e}{8m}(1 + 4\zeta_{0})\gamma^{\mu\nu}F_{\mu\nu}\chi - \chi_{1} - \frac{ie}{2m^{2}}(2\zeta_{1} + \xi_{1})\gamma^{\nu}F_{\nu\sigma}D^{\sigma}\psi + \frac{e}{8m}(1 + 4\zeta_{0})\gamma^{\mu\nu}F_{\mu\nu}\psi_{\mu} + \frac{ie}{2m^{2}}(2\zeta_{0} + i\zeta_{0})\gamma^{\nu}F_{\nu\sigma}D^{\sigma}\varphi + \frac{ie}{8m}(1 + 4\zeta_{0})\gamma^{\mu\nu}F_{\mu\nu}\varphi \right.
\]

\[
- \zeta_{1}\left\{[i\gamma^{\nu}D_{\nu} + m + \frac{ie}{2m^{2}}(2\zeta_{1} + \xi_{1})\gamma^{\nu}F_{\nu\sigma}D^{\sigma} + \frac{ie}{8m}(1 + 4\zeta_{0})\gamma^{\mu\nu}F_{\mu\nu}\right\}\chi - \chi_{1} + \{D_{\mu} + \frac{e}{m^{2}}(\zeta_{1} - i\zeta_{0})F_{\mu\sigma}D^{\sigma} + \frac{e}{4m}(1 - 2i\zeta_{1})\gamma^{\nu}F_{\nu\mu}\psi_{\nu} - m\varphi + \frac{e}{2m^{2}}(2\zeta_{0} - i\zeta_{0})\gamma^{\nu}F_{\nu\sigma}D^{\sigma}\varphi + \frac{ie}{8m}(1 - 4\zeta_{0} - 4i\zeta_{1})\gamma^{\mu\nu}F_{\mu\nu}\varphi \right. \right.
\]

\[
+ \zeta_{1}\left[\chi + ii\gamma^{\mu}\psi_{\mu} - \varphi + (d - 1)\psi\right]
\]
Here we have used that $K_h f^+|0\rangle = -2hf^+|0\rangle$ with substitution $-2h \to (d - 1)$.

Thus we have derived from the general Lagrangian the one which contains component fields and the corresponding gauge transformations. This Lagrangian describes a massive field with spin 3/2, coupled to a constant electromagnetic background in the linear approximation and contains a number of free parameters. The relations $(4.2) - (4.3)$ are our final results. One can further eliminate the auxiliary fields using the gauge freedom and some of the equations of motion and thus obtain the field equations for only physical field $\psi^\mu$.

5 Conclusions

In the present paper we have developed the BRST approach to construct and analyze a Lagrangian description of massive higher spin fermionic fields interacting with constant electromagnetic field in the linear approximation. To this end, we modified the operators underlying the BRST charge which corresponds to the noninteracting fermionic massive higher spin fields by terms depending on the electromagnetic field. The obtained Lagrangian contains the auxiliary Stückelberg fields which provide the gauge invariant description for massive theory, and the number of these fields grows with the value of the spin.

We also showed that one can partially or completely fix the gauge invariance and obtain a family of different Lagrangian formulations with a smaller number of auxiliary fields. As an example we derived a Lagrangian formulation for the massive fermionic higher spin fields interacting with a constant electromagnetic background in the quartet formulation [38, 39] and obtained the results of paper [22] as a particular case. Also we gave a detailed description of the component Lagrangian and gauge

10The problem of Lagrangian formulation for spin-3/2 field coupled to EM field in a linear approximation has been studied in [35] where the Lagrangian also contains a number of free parameters. However, unlike our paper, it is has been assumed in [35] that the electromagnetic field is dynamical and moreover, the model under consideration possesses a certain amount of supersymmetries. These requirements impose the some strong restrictions on the structure of the Lagrangian. As a result, the Lagrangian (4.2) contains more free parameters in comparison with the Lagrangian given in [35].
transformations for a simplest example of the spin $\frac{3}{2}$ field interacting with a constant electromagnetic background.

Since in the present paper we have considered fermionic higher spin fields it would be naturally interesting to generalize the present results for the case of supersymmetric systems as well as to consider higher order interactions. Inclusion of a nontrivial gravitational background is yet another interesting problem to consider (see for example [54]–[56] for recent progress in these directions). It would be interesting also to establish more connection with the recent studies in conformal higher spin fields (see for example [57]–[61]). We hope to address these questions in future publications.

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A Expressions for free parameters

Below we give the expressions for free parameters which are present in the equations (2.17)–(2.19)

\[ a_0(0) = -\frac{ie}{8m} - \frac{ie}{2m} \zeta_0 \]
\[ a_2(0) = \frac{ie}{2m} - \frac{e}{m} \xi_1 \]
\[ a_3(0) = -\frac{ie}{2m} - \frac{e}{m} \xi_1 \]
\[ a_4(0) = -\frac{ie}{2m} + \frac{2ie}{m} \zeta_0 \]
\[ a_0(k) = 0 \quad k \geq 1 \]
\[ a_2(k) = -\frac{(-2)^k e}{k! m} \xi_1 \quad k \geq 1 \]
\[ a_3(k) = -\frac{(-2)^k e}{k! m} \xi_1 \quad k \geq 1 \]
\[ a_4(k) = 0 \quad k \geq 1 \]

\[ c_0(0) = \frac{ie}{2m^2} (2 \zeta_1 + \xi_1) \]
\[ c_4(0) = \frac{e}{m^2} (-2 \zeta_0 + i \xi_1) \]
\[ c_5(0) = \frac{e}{m^2} (2 \zeta_0 + i \xi_1) \]
\[ c_0(k) = \frac{(-2)^k ie}{k! m^2} \xi_1 \quad k \geq 1 \]
\[ c_4(k) = \frac{(-2)^k ie}{k! m^2} \xi_1 \quad k \geq 1 \]
\[ c_5(k) = \frac{(-2)^k ie}{k! m^2} \xi_1 \quad k \geq 1 \]

\[ ^1 \text{Lagrangian formulation of free supersymmetric massive higher spin theory was done in [53].} \]
\[ d_{0(0)} = -\frac{ie}{8m} + \frac{ie}{2m}(\zeta_0 + i\xi_1) \]
\[ d_{1(0)} = -\frac{ie}{8m} + \frac{ie}{2m}(\zeta_0 - i\xi_1) \]
\[ d_{2(0)} = -\frac{ie}{4m} + \frac{e}{2m}\xi_1 \]
\[ d_{3(0)} = -\frac{ie}{4m} + \frac{e}{2m}\xi_1 \]
\[ d_{0(k)} = \frac{(-2)^{k-1}e}{k!} \frac{m}{m}\xi_1 \quad k \geq 1 \]
\[ d_{1(k)} = -\frac{(-2)^{k-1}e}{k!} \frac{m}{m}\xi_1 \quad k \geq 1 \]
\[ d_{2(k)} = \frac{(-2)^{k-1}e}{k!} (k + 1) \frac{m}{m}\xi_1 \quad k \geq 1 \]
\[ d_{3(k)} = -\frac{(-2)^{k-1}e}{k!} (k + 1) \frac{m}{m}\xi_1 \quad k \geq 1 \]
\[ d_{4(k)} = \frac{(-2)^k e}{k!} \xi_1 \quad k \geq 0 \]
\[ d_{5(k)} = -\frac{(-2)^k e}{k!} \xi_1 \quad k \geq 0 \]
\[ d_{8(0)} = \frac{ie}{m} - \frac{ie}{m}(2\zeta_0 + i\xi_1) \]
\[ d_{9(0)} = \frac{ie}{m} - \frac{ie}{m}(2\zeta_0 - i\xi_1) \]
\[ f_{0(0)} = \frac{e}{m^2}(\zeta_0 + i\xi_1) \]
\[ f_{0(1)} = \frac{e}{m^2}(2\zeta_0 - i\xi_1) \]
\[ f_{1(0)} = \frac{e}{m^2}(-\zeta_0 + i\xi_1) \]
\[ f_{1(1)} = -\frac{e}{m^2}(2\zeta_0 + i\xi_1) \]
\[ f_{0(k)} = -\frac{(-2)^{k-1}ie}{k!} \frac{m^2}{m}\xi_1 \quad k \geq 2 \]
\[ f_{1(k)} = -\frac{(-2)^{k-1}ie}{k!} \frac{m^2}{m}\xi_1 \quad k \geq 2 \]
\[ f_{2(0)} = \frac{e}{m^2}(\zeta_0 - \frac{i}{2}\xi_1) \]
\[ f_{3(0)} = \frac{e}{m^2}(\zeta_0 + \frac{i}{2}\xi_1) \]
\[ f_{4(0)} = -\frac{2e}{m^2}\zeta_0 \]
\[ f_{5(0)} = \frac{2e}{m^2}\zeta_0 \]
\[ f_{4(k)} = 0 \quad k \geq 1 \]
\[ f_{5(k)} = 0 \quad k \geq 1 \]

Here \( \zeta_0, \zeta_1, \xi_1 \) are arbitrary real dimensionless constants.

References

[1] M. A. Vasiliev, “Higher spin gauge theories: Star product and AdS space,” In *Shifman, M.A. (ed.): The many faces of the superworld* 533-610 [hep-th/9910096].
[2] M. A. Vasiliev, “Higher spin gauge theories in various dimensions,” Fortsch. Phys. 52, 702 (2004) [PoS jhw 2003, 003 (2003)] [hep-th/0401177].

[3] D. Sorokin, “Introduction to the classical theory of higher spins,” AIP Conf. Proc. 767, 172 (2005) [hep-th/0405069].

[4] N. Bouatta, G. Compere and A. Sagnotti, “An Introduction to free higher-spin fields,” [hep-th/0409068].

[5] X. Bekaert, S. Cnockaert, C. Iazeolla and M. A. Vasiliev, “Nonlinear higher spin theories in various dimensions,” [hep-th/0503128].

[6] A. Fotopoulos and M. Tsulaia, “Gauge Invariant Lagrangians for Free and Interacting Higher Spin Fields. A Review of the BRST formulation,” Int. J. Mod. Phys. A 24 (2009) 1 [arXiv:0805.1346 [hep-th]]; M. Tsulaia, “On Tensorial Spaces and BCFW Recursion Relations for Higher Spin Fields,” Int. J. Mod. Phys. A 27, 1230011 (2012) [arXiv:1202.6309 [hep-th]].

[7] A. Campoleoni, “Metric-like Lagrangian Formulations for Higher-Spin Fields of Mixed Symmetry,” Riv. Nuovo Cim. 33, 123 (2010) [arXiv:0910.3155 [hep-th]].

[8] X. Bekaert, N. Boulanger and P. Sundell, “How higher-spin gravity surpasses the spin two barrier: no-go theorems versus yes-go examples,” Rev. Mod. Phys. 84, 987 (2012) [arXiv:1007.0435 [hep-th]].

[9] D. Francia, “On the Relation between Local and Geometric Lagrangians for Higher spins,” J. Phys. Conf. Ser. 222, 012002 (2010) [arXiv:1001.3854 [hep-th]].

[10] A. Sagnotti, “Notes on Strings and Higher Spins,” J. Phys. A 46, 214006 (2013) [arXiv:1112.4285 [hep-th]].

[11] M. R. Gaberdiel and R. Gopakumar, “Minimal Model Holography,” J. Phys. A 46, 214002 (2013) [arXiv:1207.6697 [hep-th]].

[12] S. Giombi and X. Yin, “The Higher Spin/Vector Model Duality,” J. Phys. A 46, 214003 (2013) [arXiv:1208.4036 [hep-th]].

[13] M. Taronna, “Higher Spins and String Interactions,” [arXiv:1005.3061 [hep-th]]. M. Taronna, “Higher-Spin Interactions: three-point functions and beyond,” [arXiv:1209.5755 [hep-th]].

[14] V. E. Didenko and E. D. Skvortsov, “Elements of Vasiliev theory,” [arXiv:1401.2975 [hep-th]].

[15] M. A. Vasiliev, “Higher-Spin Theory and Space-Time Metamorphoses,” Lect. Notes Phys. 892, 227 (2015) [arXiv:1404.1948 [hep-th]].
[16] G. Lucena Gómez, “Aspects of Higher-Spin Theory with Fermions,” arXiv:1406.5319 [hep-th].

[17] M. Porrati, R. Rahman and A. Sagnotti, “String Theory and The Velozwanziger Problem,” Nucl. Phys. B 846, 250 (2011) arXiv:1011.6411 [hep-th].

[18] I. L. Buchbinder, V. A. Krykhtin, L. L. Ryskina and H. Takata, “Gauge invariant Lagrangian construction for massive higher spin fermionic fields,” Phys. Lett. B 641 (2006) 386 [hep-th/0603212].

[19] I. L. Buchbinder, V. A. Krykhtin and A. Sagnotti, “BRST approach to Lagrangian construction for fermionic massless higher spin fields,” Nucl. Phys. B 771 (2005) 367 [hep-th/0410215].

[20] I. L. Buchbinder, V. A. Krykhtin and A. A. Reshetnyak, “BRST approach to Lagrangian construction for fermionic higher spin fields in (A)dS space,” Nucl. Phys. B 787 (2007) 211 [hep-th/0703049].

[21] R. R. Metsaev, “Cubic interaction vertices for fermionic and bosonic arbitrary spin fields,” Nucl. Phys. B 859 (2012) 13 arXiv:0712.3526 [hep-th].

[22] S. M. Klyshhevich, “Massive fields of arbitrary half integer spin in constant electromagnetic field,” Int. J. Mod. Phys. A 15 (2000) 609 [hep-th/9811030].

[23] S. Deser, V. Pascalutsa and A. Waldron, “Massive spin 3/2 electrodynamics,” Phys. Rev. D 62 (2000) 105031 [hep-th/0003011].

[24] S. Deser and A. Waldron, “Inconsistencies of massive charged gravitating higher spins,” Nucl. Phys. B 631 (2002) 369 [hep-th/0112182].

[25] R. R. Metsaev, “Gravitational and higher-derivative interactions of massive spin 5/2 field in (A)dS space,” Phys. Rev. D 77 (2008) 025032 [hep-th/0612279].

[26] Y. M. Zinoviev, “Frame-like gauge invariant formulation for mixed symmetry fermionic fields,” Nucl. Phys. B 821, 21 (2009) arXiv:0904.0549 [hep-th].

[27] E. D. Skvortsov and Y. M. Zinoviev, “Frame-like Actions for Massless Mixed-Symmetry Fields in Minkowski space. Fermions,” Nucl. Phys. B 843, 559 (2011) arXiv:1007.4944 [hep-th].

[28] M. Henneaux, G. Lucena Gómez and R. Rahman, “Higher-Spin Fermionic Gauge Fields and Their Electromagnetic Coupling,” JHEP 1208 (2012) 093 arXiv:1206.1048 [hep-th].

[29] M. Henneaux and R. Rahman, “Note on Gauge Invariance and Causal Propagation,” Phys. Rev. D 88, 064013 (2013) arXiv:1306.5750 [hep-th].
[30] M. Henneaux, G. Lucena Gómez and R. Rahman, “Gravitational Interactions of Higher-Spin Fermions,” JHEP 1401 (2014) 087 [arXiv:1310.5152 [hep-th]].

[31] R. Rahman, “Helicity-1/2 mode as a probe of interactions of a massive Rarita-Schwinger field,” Phys. Rev. D 87 (2013) 6, 065030 [arXiv:1111.3366 [hep-th]].

[32] M. Porrati and R. Rahman, “Causal Propagation of a Charged Spin 3/2 Field in an External Electromagnetic Background,” Phys. Rev. D 80 (2009) 025009 [arXiv:0906.1432 [hep-th]].

[33] M. Porrati and R. Rahman, “A Model Independent Ultraviolet Cutoff for Theories with Charged Massive Higher Spin Fields,” Nucl. Phys. B 814 (2009) 370-404 [arXiv:0809.2807 [hep-th]].

[34] I. L. Buchbinder, T. V. Snegirev and Y. M. Zinoviev, “Frame-like gauge invariant Lagrangian formulation of massive fermionic higher spin fields in $AdS_3$ space,” Phys. Lett. B 738 (2014) 258 [arXiv:1407.3918 [hep-th]].

[35] I. L. Buchbinder, T. V. Snegirev and Y. M. Zinoviev, “Formalism of gauge-invariant curvatures and constructing the cubic vertices for massive spin-3/2 field in $AdS_4$ space,” Eur. Phys. J. C 74 11, 3153 [arXiv:1405.7781].

[36] I. Cortese, R. Rahman and M. Sivakumar, “Consistent Non-Minimal Couplings of Massive Higher-Spin Particles,” Nucl. Phys. B 879, 143 (2014) [arXiv:1307.7710 [hep-th]].

[37] M. Kulaxizi and R. Rahman, “Higher-Spin Modes in a Domain-Wall Universe,” JHEP 1410, 193 (2014) [arXiv:1409.1942 [hep-th]].

[38] I. L. Buchbinder, A. V. Galajinsky and V. A. Krykhtin, “Quartet unconstrained formulation for massless higher spin fields,” Nucl. Phys. B 779 (2007) 155 [hep-th/0702161].

[39] I. L. Buchbinder and A. V. Galajinsky, “Quartet unconstrained formulation for massive higher spin fields,” JHEP 0811 (2008) 081 [arXiv:0810.2852 [hep-th]].

[40] G. Velo and D. Zwanziger, “Propagation and quantization of Rarita-Schwinger waves in an external electromagnetic potential,” Phys. Rev. 186 (1969) 1337.

[41] G. Velo and D. Zwanziger, “Noncausality and other defects of interaction lagrangians for particles with spin one and higher,” Phys. Rev. 188, 2218 (1969).

[42] I. L. Buchbinder, V. A. Krykhtin, P. M. Lavrov, “On manifolds admitting the consistent Lagrangian formulation for higher spin fields,” Mod Phys. Lett. A 26 (2011) 1183-1196 [arXiv:1101.4860 [hep-th]].
[43] I. L. Buchbinder, P. Dempster and M. Tsulaia, “Massive Higher Spin Fields Coupled to a Scalar: Aspects of Interaction and Causality,” Nucl. Phys. B 877, 260 (2013) [arXiv:1308.5539 [hep-th]].

[44] A. I. Pashnev, “Composite Systems and Field Theory for a Free Regge Trajecto-ry,” Theor. Math. Phys. 78, 272 (1989) [Teor. Mat. Fiz. 78, 384 (1989)].

[45] J. Fang and C. Fronsdal, “Massless Fields with Half Integral Spin,” Phys. Rev. D 18, 3630 (1978).

[46] D. Francia and A. Sagnotti, “On the geometry of higher spin gauge fields,” Class. Quant. Grav. 20, S473 (2003) [Comment. Phys. Math. Soc. Sci. Fenn. 166, 165 (2004)] [PoS JHW 2003, 005 (2003)] [hep-th/0212185].

[47] A. Sagnotti and M. Tsulaia, “On higher spins and the tensionless limit of string theory,” Nucl. Phys. B 682, 83 (2004) [hep-th/0311257].

[48] D. Francia, “String theory triplets and higher-spin curvatures,” Phys. Lett. B 690, 90 (2010) [arXiv:1001.5003 [hep-th]].

[49] A. Campoleoni and D. Francia, “Maxwell-like Lagrangians for higher spins,” JHEP 1303, 168 (2013) [arXiv:1206.5877 [hep-th]].

[50] D. P. Sorokin and M. A. Vasiliev, “Reducible higher-spin multiplets in flat and AdS spaces and their geometric frame-like formulation,” Nucl. Phys. B 809, 110 (2009) [arXiv:0807.0206 [hep-th]].

[51] X. Bekaert, N. Boulanger and D. Francia, “Mixed-symmetry multiplets and higher-spin curvatures,” [arXiv:1501.02462 [hep-th]].

[52] L. P. S. Singh, C. R. Hagen, “Lagrangian formulation for arbitrary spin. 2. The fermionic case,” Phys. Rev. D9 (1974) 910-920.

[53] Yu. M. Zinoviev, “Massive N = 1 supermultiplets with arbitrary superspins,” Nucl. Phys. B 785 (2007) 98-114 [arXiv:0704.1535 [hep-th]].

[54] I. Florakis, D. Sorokin and M. Tsulaia, “Higher Spins in Hyperspace,” JHEP 1407, 105 (2014) [arXiv:1401.1645 [hep-th]].

[55] I. Florakis, D. Sorokin and M. Tsulaia, “Higher Spins in Hyper-Superspace,” Nucl. Phys. B 890, 279 (2014) [arXiv:1408.6675 [hep-th]].

[56] X. Bekaert, J. Erdmenger, D. Ponomarev and C. Sleight, “Towards holographic higher-spin interactions: Four-point functions and higher-spin exchange,” [arXiv:1412.0016 [hep-th]].
[57] R. R. Metsaev, “Gauge invariant two-point vertices of shadow fields, AdS/CFT, and conformal fields,” Phys. Rev. D 81, 106002 (2010) [arXiv:0907.4678 [hep-th]].

[58] R. R. Metsaev, “CFT adapted approach to massless fermionic fields, AdS/CFT, and fermionic conformal fields,” arXiv:1311.7350 [hep-th].

[59] T. Nutma and M. Taronna, “On conformal higher spin wave operators,” JHEP 1406, 066 (2014) [arXiv:1404.7452 [hep-th]].

[60] R. R. Metsaev, “BRST invariant effective action of shadow fields, conformal fields, and AdS/CFT,” arXiv:1407.2601 [hep-th].

[61] R. R. Metsaev, “Mixed-symmetry fields in AdS(5), conformal fields, and AdS/CFT,” arXiv:1410.7314 [hep-th].