Lepton Flavor Violating $l \rightarrow l'\gamma$ and $Z \rightarrow l\bar{l}'$ Decays Induced by Scalar Leptoquarks

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Abstract

Motivated by the recent muon $g-2$ data, we study the lepton flavor violating $l \rightarrow l'\gamma$ and $Z \rightarrow l\bar{l}'$ ($l, l' = e, \mu, \tau$) decays with $l \neq l'$ in a scalar leptoquark model. Leptoquarks can produce sizable LFV $l \rightarrow l'\gamma$ decay rates that can be easily reached by present or near future experiments.

Leptoquark masses and couplings are constrained by the muon $g-2$ data and the current $l \rightarrow l'\gamma$ bounds. We predict $Br(Z \rightarrow \tau^\pm e^{\mp})$ reaching the present limit ($10^{-5}$) and $Br(Z \rightarrow \mu^\pm \tau^{\mp})$ reaching $2 \times 10^{-8}$, which will be accessible by future linear colliders, whereas, the current bounds on LFV impose very strong constraints on the $Br(Z \rightarrow \mu^\pm e^{\mp})$ and the ratio is too low to be observed in the near future.

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I. INTRODUCTION

The excess value of the anomalous magnetic moment of muon was reported by the E821 collaboration at BNL \[1\]

\[
\alpha_{\mu}^{\text{exp}} = 116\,592\,080(63) \times 10^{-11}. \quad (1)
\]

The Standard Model prediction for \(\alpha_{\mu}^{\text{SM}}\) with QED, hadronic and electroweak contributions is \[2, 3\]

\[
\alpha_{\mu}^{\text{SM}} = 116\,591\,785(61) \times 10^{-11}. \quad (2)
\]

with the experimental value of \((g - 2)/2\), the comparison gives

\[
\Delta \alpha_{\mu} \equiv \alpha_{\mu}^{\text{exp}} - \alpha_{\mu}^{\text{SM}} = (295 \pm 87.7) \times 10^{-11}, \quad (3.4\sigma) \quad (3)
\]

The 3.4 standard deviation difference between the two, may be a hint of new physics contribution.

It has been shown that contributions from leptoquark (LQ) exchanges are capable to resolve the above deviation \[4, 5, 6\]. Leptoquarks are vector or scalar particles carrying both lepton and baryon numbers. LQs can be quite naturally introduced in the low-energy theory as a relic of a more fundamental theory at some high-energy scale, such as grand unified theories (GUT) \[7, 8\]. In some models, it is possible to have leptoquarks at TeV scale \[9\]. The low-energy LQ phenomenology has received considerable attention. Possible LQ manifestations in various processes have been extensively investigated \[9\]-\[25\]. Various constraints on LQ masses and couplings have been deduced from existing experimental data and prospects for the forthcoming experiments have been estimated. Direct searches of LQs as s-channel resonances in deep inelastic ep-scattering and pair production in hadron colliders placed lower limits on their mass \(M_{\text{LQ}} \geq 73 - 298\,\text{GeV} \[18\] depending on the LQ types and couplings. The interest on leptoquarks has been renewed during the last few years since ongoing collider experiments have good prospects for searching these particles \[27\]. For a recent review of leptoquarks, one is referred to \[28\].

Lepton Flavor Violation (LFV) are powerful tools to search for new physics. The present experimental limits give \[18\]:

\[
\text{Br}(\mu \rightarrow e\gamma) < 1.2 \times 10^{-11}, \quad (4)
\]

\[
\text{Br}(\tau \rightarrow e\gamma) < 1.1 \times 10^{-7}, \quad (5)
\]

\[
\text{Br}(\tau \rightarrow \mu\gamma) < 6.8 \times 10^{-8}. \quad (6)
\]
Since effects of leptoquark interactions can manifest in $a_\mu$, it is very likely that they can also give interesting contributions to these $l \to l'\gamma$ processes \[5, 6\]. There are considerable efforts on experiments that aim at pushing the sensitivity of $\text{Br}(\mu \to e\gamma)$ down by two order of magnitudes \[29\]. B factories and the upgraded super B factory can probe the $\tau \to e\gamma, \mu\gamma$ decays at better sensitivities.

The $Z \to ll'$ decays are among the LFV interactions and the theoretical predictions of their branching ratios in the framework of the SM are extremely small \[30, 31, 32\]. These results are far from the experimental limits obtained at LEP1 \[18\]:

$$Br(Z \to e^\pm \mu^\mp) < 1.7 \times 10^{-6}, \quad (7)$$
$$Br(Z \to e^\pm \tau^\mp) < 9.8 \times 10^{-6}, \quad (8)$$
$$Br(Z \to \mu^\pm \tau^\mp) < 1.2 \times 10^{-5}. \quad (9)$$

Better sensitivities are expected from the Giga-Z modes at future colliders, such as International Linear Collider (ILC), to have \[33, 34, 35\]:

$$Br(Z \to e^\pm \mu^\mp) < 2 \times 10^{-9}, \quad (10)$$
$$Br(Z \to e^\pm \tau^\mp) < \kappa \times 6.5 \times 10^{-8}, \quad (11)$$
$$Br(Z \to \mu^\pm \tau^\mp) < \kappa \times 2.2 \times 10^{-8}. \quad (12)$$

with $\kappa \simeq 0.2 - 1.0$. It will be interesting to study the leptoquark contributions to the $Z \to ll'$ processes.

The aim of the present paper is to study the leptoquark effects in various LFV processes including $l \to l'\gamma$ and $Z \to ll'$ decays, while considering leptoquark contribution to $a_\mu$ as a solution to the muon anomalous moment discrepancy. The layout of the present paper is as follows: In Sec. II we introduce the formalism. We then use it in Sec. III to study the leptoquark contributions to $a_\mu$ and LFV processes including $l \to l'\gamma$ and $Z \to ll'$ decays. Sec. IV contains our conclusions. Some formulas and low energy constraints are given in Appendices.
II. FORMALISM

A. Scalar Leptoquark Interactions

In this section we list the relevant parts of the scalar leptoquark Lagrangian. We consider isosinglet scalar leptoquarks. The effective Lagrangian describing the leptoquark interactions in the mass basis is given by \cite{10, 24}:

\[ \mathcal{L}_{\text{LQ}} = \bar{u}_a^c \left( h'_{ai} \Gamma_{k,S_R} P_L + h_{ai} \Gamma_{k,S_L} P_R \right) e_i S_k^* + \bar{e}_j \left( h'_{aj} \Gamma_{k,S_R}^\dagger P_R + h_{aj} \Gamma_{k,S_L}^\dagger P_L \right) u_a^c S_k \]

where \( k = 1, 2 \) are the indices of leptoquark, \( T_3 = -1/2, Q_{u^c} = -2/3 \) are quark’s isospin and electric charge, \( Q_S = -1/3 \) is the electric charge of scalar leptoquarks \( S_k \), \( a \) and \( i, j \) are quarks and leptons flavor indices and we use \( c_W = \cos \theta_W \) and \( s_W = \sin \theta_W \). The \( \Gamma_{k,S_{L,R}} \) are elements of leptoquark mixing matrix that brings \( S_{L,R} \) to the mass basis \( S_k \):

\[ S_L = \Gamma_{k,S_{L,R}}^\dagger S_k, \quad S_R^* = \Gamma_{k,S_{L,R}} S_k^* \]

where the \( S_{L(R)} \) is the field that associates with the \( \bar{e}_j P_{L(R)} u_a^c \) terms in \( \mathcal{L}_{\text{LQ}} \). Note that in the no-mixing case (\( \Gamma = 1 \)), \( S_{1,2} \) reduce to \( S_{L,R} \), which are called chiral leptoquarks, as they only couple to quarks and leptons in certain chirality structures. Finally, the couplings \( h \) and \( h' \) are 3 by 3 matrices, which give rise to various LFV processes and must be subject to experimental constraints.

In this work we do not aim at a comprehensive study of the effects of all possible leptoquark interactions. Instead, we try to demonstrate that a simple scalar leptoquark model can provide rich and interesting LFV phenomenons.

B. Muon anomalous magnetic moment \((g - 2)_\mu\)

The LQ interaction is capable to generate muon anomalous magnetic moment and resolve the discrepancy between theoretical and experimental results. The one-loop diagrams are shown in Fig. (a)-(b) with \( l = l' = \mu \). The extra contribution to \( a_\mu \) arises from the LQ model due to quark and scalar leptoquark one-loop contribution is given by
\[ a_{LQ}^{\mu} = -\frac{N_c m_\mu^2}{8\pi^2} \sum_{q=1}^{3} \sum_{k=1}^{2} \frac{1}{M_{S_k}^2} \left[ \left( |h_{q\mu} \Gamma_{k,L}^S|^2 + |h'_{q\mu} \Gamma_{k,R}^S|^2 \right) (Q_{(u^c)} F_2(x) - Q_{S} F_1(x)) \right. \\
- \left. \left| \frac{m(u_\mu^c)}{m_\mu} \text{Re}(h'_{q\mu} h'^*_{q\mu} \Gamma_{S,R,k}^+ \Gamma_{k,S,L}) (Q_{(u^c)} F_3(x_{ka}) - Q_{S} F_4(x_{ka})) \right) \right], \]

In the above expression, \( N_c = 3, Q_S = -1/3, Q_{u^c} = -2/3. \) Our expression agrees with that in [6, 15]. The kinematic loop functions \( F_i \) \((i = 1, \ldots, 4)\) depend on the variable \( x = m^2_{(u^c)}/m_{S_k}^2 \) are given in the appendix A.

Using leptoquark contribution to saturate the deviation given in Eq. (3), the leptoquark masses \( M_{S_{1,2}} \), mixing angle \( \theta_{LQ} \) and couplings \( h_{q\mu}^{(l)} \) will be constrained.

**C. \( \ell \rightarrow \ell'\gamma \)**

In this subsection we give the amplitude of \( \ell \rightarrow \ell'\gamma \) from leptoquark exchange. According to the gauge invariance, the amplitude can be written as:

\[ i\mathcal{M}^\gamma = ie\bar{u}(p_2) \left( F_{2RL}^\gamma P_L + F_{2LR}^\gamma P_R \right) (i\sigma_{\mu\nu}q^{\nu}) u(p_1) \bar{\varepsilon}^\mu_{\gamma}, \]

where \( \varepsilon_{\gamma} \) is the polarization vector and \( q = p_1 - p_2 \) is the momentum transfer. For the amplitude of leptoquark exchange at one-loop level as depicted in Figure. 1, we have
\[ F_{2 LR}^\gamma = \frac{N_c}{16\pi^2} \sum_{q=1}^{3} \sum_{k=1}^{2} \frac{1}{M_{S_i}^2} \left[ (m_l h_q h_q^* \Gamma_{S_R,k}^{\gamma} \Gamma_{S_L,k}^{\gamma} + m_\ell h_q h_q^* \Gamma_{S_R,k}^{\gamma} \Gamma_{S_L,k}^{\gamma} ) (Q_{(w)} F_2(x) - Q_S F_1(x)) \right. \\
\left. - m_{(u_R)} (h_q h_q^* \Gamma_{S_R,k}^{\gamma} \Gamma_{S_L,k}^{\gamma} ) (Q_{(w)} F_3(x) - Q_S F_4(x)) \right], \tag{17} \]
\[ F_{2 RL}^\gamma = F_{2 LR}^\gamma (h \leftrightarrow h', R \leftrightarrow L), \tag{18} \]

with \( x = m_{(u_R)}^2 / m_{S_i}^2 \). The branching ratio of \( \ell \to \ell' \gamma \) is:
\[ \text{Br}(\ell \to \ell' \gamma) = \frac{\alpha_{em} (m_\ell^2 - m_{\ell'}^2)^3}{4\Gamma(\ell)} \left( \frac{|F_{2 LR}|^2 + |F_{2 RL}|^2}{|F_{1 LR}|^2 + |F_{1 RL}|^2} \right), \tag{19} \]

In our numerical calculations we analyze the Brs of the decays under consideration by using the total decay widths of the decaying leptons \( \Gamma(\ell) \).

\section{Z \to \ell \ell'}

The Feynman diagrams of LFV Z decay process are shown in Fig. 1. The total contribution of all diagrams (c) and (d) can be written as
\[ iM_\mu^Z = iem_Z^2 \bar{u}(p_2) \left[ \left( F_{1L}^Z P_R + F_{1R}^Z P_L \right) \left( - g_{\mu\nu} + \frac{q_\mu q_\nu}{m_Z^2} \right) \gamma^\nu \right. \tag{20} \]
\[ + \frac{1}{m_Z^2} \left( F_{2 RL}^Z P_L + F_{2 LR}^Z P_R \right) \left( i\sigma_{\mu\nu} q^\nu \right) \right] u(p_1) \epsilon_\mu^Z(q) \]

where \( q_\mu \) is the Z four-momentum. The decay rates involve both \( F_{1 LR}^Z \) and \( F_{2 LR}^Z \):
\[ \text{Br}(Z \to \ell \ell') = \frac{\alpha_{em} m_Z}{6\Gamma_Z} \left[ \left( |F_{1 L}^Z|^2 + |F_{1 R}^Z|^2 \right) \right. \tag{21} \]
\[ + \left. \frac{1}{2m_Z^2} \left( |F_{2 LR}^Z(Z)|^2 + |F_{2 RL}^Z(Z)|^2 \right) \right], \]

where the form factors \( F_{1 LR}^Z \) and \( F_{2 LR}^Z \) are given by
\[ F_{1 L}^Z = \frac{N_c}{16\pi^2} \sum_{q=1}^{3} \sum_{k=1}^{2} \frac{1}{M_{S_i}^2} \left[ h_q h_q^* \Gamma_{S_R,k}^{\gamma} \Gamma_{S_L,k}^{\gamma} (g_S G_1(x) + g_R G_2(x)) \right. \tag{22} \]
\[ - m_{(u_R)} (h_q h_q^* \Gamma_{S_R,k}^{\gamma} \Gamma_{S_L,k}^{\gamma} ) (g_L - g_R) \left( h_q h_q^* \Gamma_{S_R,k}^{\gamma} \Gamma_{S_L,k}^{\gamma} m_\ell - h_q h_q^* \Gamma_{S_R,k}^{\gamma} \Gamma_{S_L,k}^{\gamma} m_\ell \right) G_3(x) \right], \]
\[ F_{1 R}^Z = F_{1 L}^Z (h \leftrightarrow h', L \leftrightarrow R), \tag{23} \]
and
\[
F_{2\text{LR}}^Z = \frac{N_c}{16\pi^2} \frac{1}{M_{S_1}^2} \left[ h_{qL} h_{qL}^* \Gamma_{S_{R}, R}^{\dagger} \Gamma_{S_{L}, L} m_{\alpha} (g_R + g_L) G_3(x) \\
+ (g_R h'_{qL} h'_{qL}^* \Gamma_{S_{R}, R}^{\dagger} \Gamma_{S_{L}, L} m_{\ell} + g_L h_{qL} h_{qL}^* \Gamma_{S_{L}, L} m_{\ell}) G_4(x) \\
- g_S \left( (h'_{qL} h_{qL}^* \Gamma_{S_{R}, R}^{\dagger} \Gamma_{S_{L}, L} m_{\ell} + h_{qL} h_{qL}^* \Gamma_{S_{L}, L} m_{\ell}) G_5(x) + m_{\alpha} h_{qL} h_{qL}^* \Gamma_{S_{R}, R}^{\dagger} \Gamma_{S_{L}, L} G_6(x) \right) \right],
\]
\[F_{2\text{RL}}^Z = F_{2\text{LR}}^Z (h \leftrightarrow h', L \leftrightarrow R),\]

where we have \( x = m_{\alpha}^2 / m_{S_1}^2 \) and the couplings \( g_{R,L} \) and \( g_S \) are given by
\[
g_R = - \frac{2}{\sin \theta_W \cos \theta_W} (T_{3(u)} - Q(u) \sin^2 \theta_W),
\]
\[
g_L = Q(u) \tan \theta_W, \quad g_S = Q_S \tan \theta_W.
\]

In the above expressions of \( F_{1\text{L(R)}}^Z \), we keep only the leading term in \( m_{S_2}^2 / m_{S_1}^2 \). The explicit expressions of one loop functions \( G_n \) (\( n = 1, \ldots, 6 \)) can be found in the appendix A.

### III. NUMERICAL RESULTS AND DISCUSSION

We are now ready to give some numerical results. The quark mass are evaluated at the scale of the \( \mu = 300 \) GeV [26], which is the typical leptoquark mass used in this work,
\[
m_t = 161.4 \text{ GeV}, \quad m_c = 0.55 \text{ GeV}, \quad m_u = 11.4 \times 10^{-3} \text{ GeV},
\]
and for the following quantities we use [18]
\[
\alpha_{em} = 1/137.0359, \quad M_W = 80.45 \text{ GeV}, \quad M_Z = 91.1875 \text{ GeV}.
\]

For simplicity, we assume that the couplings \( h \) and \( h' \) are real and equal to each other, i.e.
\[
h = h' = h^*.
\]

We use leptoquark mass splitting \( \Delta = 500 \) GeV in our analysis, where \( \Delta \) is defined as \( \sqrt{M_{S_2}^2 - M_{S_1}^2} \). Consequently, the remaining parameters in the leptoquark model are the mass of the light scalar leptoquark \( M_{S_1} \), the mixing angle \( \theta_{LQ} \), and the couplings \( h_{qL} \).

#### A. Muon Anomalous Magnetic Moment \( a_\mu \)

In this section we discuss a few phenomenological aspects of the leptoquark contributions to \( a_\mu \). In the left panel of Fig. 2, we present an scatter plot in the \((M_{S_1} - |h_{qL}|^2)\) plane
for top quark contribution (red) and charm quark contribution (green), which are allowed by $a^{LQ}_\mu = \Delta a_\mu = (295 \pm 87.7) \times 10^{-11}$ [see Eq. (3)] within the 1σ range of data. We note that it is not possible to use the up quark loop contribution alone for the $a^{LQ}_\mu = \Delta a_\mu$, since the mixing angle and couplings $h_{u\mu}$ are strongly constrained by the π leptonic decays (see Appendix B).

In order to see the impact of the mixing angle, we present in the right panel of Fig. 2 the allowed regions $a^{LQ}_\mu = \Delta a_\mu$ in the $(M_{S_1} - \sin 2\theta_{LQ})$ plane. We use $\alpha_{em} \leq h_{q\mu}^2 \leq 1$. The contribution dominates around $\sin 2\theta_{LQ} \sim 0.7$ both for top and charm quark contributions. We see that the constraint from $a_\mu$ confines the allowed range of $M_{S_1}$ to $M_{S_1} \lesssim 950$ GeV for top quark contribution and to $M_{S_1} \lesssim 350$ GeV for charm quark contribution at the 1σ level. These parameter space will be used for later study of LFV processes. The light leptoquark mass should be below 1 TeV, if leptoquarks with couplings of electromagnetic strength are responsible to the deviation $\Delta a_\mu$. It is interesting that LHC may have good chance to observe these particles [27].

**B. Lepton Flavor Violating $l \rightarrow l'\gamma$ and $Z \rightarrow l\bar{l}$ Decays**

In this section, we investigate the LFV decay processes generated by the same leptoquark scalar interactions. We consider only parameter space that corresponds to $a^{LQ}_\mu = \Delta a_\mu$ when
it is appropriate. We discuss $\mu \to e\gamma$ and $\tau \to e\gamma$, $\mu\gamma$ decays first.

In Fig. 3 and Fig. 4 we show scatter plots of the allowed parameters in $(M_{S_1}, h_{q\ell}h_{q\ell'})$ planes from bounds of $\tau \to \mu\gamma$, $\tau \to e\gamma$ and $\mu \to e\gamma$ rates. Note that in the plots we use
\[1.5 \times 10^{-13} \leq \text{Br}(\mu \to e\gamma) < 1.2 \times 10^{-11},\]
\[1 \times 10^{-9} \leq \text{Br}(\tau \to e\gamma) < 1.1 \times 10^{-7},\]
\[1 \times 10^{-9} \leq \text{Br}(\tau \to \mu\gamma) < 6.8 \times 10^{-8}.\]  

(31)

where the upper bounds are from the current limits: Eqs. (4)-(6), while the lower bound for $\text{Br}(\mu \to e\gamma)$ is from [29] and the lower bounds for $\tau \to l\gamma$ are for illustration. For the $\tau \to \mu\gamma$ and $\mu \to e\gamma$ cases the $(g-2)_\mu$ constraint is taken into account.

For different quark contribution the couplings are bounded in the following ranges: $10^{-4} \lesssim h_{q\tau}h_{q\mu} \lesssim 10^{-2}$, $10^{-3} \lesssim h_{c\tau}h_{ce} \lesssim 1$, $10^{-4} \lesssim h_{t\tau}h_{te} \lesssim 1$ and $10^{-7} \lesssim h_{q\mu}h_{qe} \lesssim 10^{-6}$. For the $\tau \to \mu\gamma$ and $\mu \to e\gamma$ cases, the allowed leptoquark masses are $m_{S_1} \lesssim 250 - 300$ GeV and 1 TeV for $c$-quark and $t$- quark loop contribution, respectively. These region are determined from the bounds and the muon $g-2$ constraint (see also Fig. 2) at the same time. On the other hand the couplings governing $\tau \to e\gamma$ decay and those generating muon $g-2$ contribution are decoupled, the parameters corresponding to the former bounds are free from the latter constraint. The resulting allowed regions are larger in this cases. The parameters in these allowed regions will be used to predict $Z \to l^+l^-$ decays. To have an idea of the size the allowed couplings, we give that upper bound on $h_{q\ell}h_{q\ell'}$ obtained form the present $l \to l'\gamma$ limits in Table I. We see that the $\mu \to e\gamma$ constraint is more effective in restricting the sizes of $h_{q\ell}h_{q\ell'}$.

| Decay mode | $h_{c\ell}h_{c\ell'}$ | $h_{t\ell}h_{t\ell'}$ |
|------------|-----------------|-----------------|
| $\tau \to \mu\gamma$ | $< 5.29 \times 10^{-3}$ | $< 9.11 \times 10^{-3}$ |
| $\tau \to e\gamma$ | $< 0.81$ | $< 0.82$ |
| $\mu \to e\gamma$ | $< 1.45 \times 10^{-6}$ | $< 1.92 \times 10^{-6}$ |

TABLE I: Constraints on the parameters $h_{q\ell}h_{q\ell'}$ ($q = t, c$) coming from radiative FCNC processes induced by the scalar leptoquark using the present experimental bounds.
FIG. 3: Scatter plots of leptoquark parameters in $(M_{S1}, h_{q}h_{q'})$ planes from $(\ell \to \ell'\gamma)$ bounds given in Eq. (31). The left (right) figure is for the $\tau \to \mu\gamma$ ($\tau \to e\gamma$) case with top and charm quark contributions.

FIG. 4: Same as Fig. 3 except for the $\mu \to e\gamma$ case.

In Fig. 5 and 6, we give the predicted $Z \to l\bar{l}'$ rates in correlation with $\text{Br}(l \to l'\gamma)$. We see that $\text{Br}(Z \to \tau^{\pm}e^{\pm})$ can reach $1.95 \times 10^{-5}$, which is comparable with the present bound, and $\text{Br}(Z \to \mu^{\pm}\tau^{\pm})$ can reach $2.34 \times 10^{-8}$, which will be accessible by future linear
colliders. On the contrary, the current bound on the $\mu \to e\gamma$ decay imposes very strong constraints on the related couplings as shown in Table I. Hence the predicted $\text{Br}(Z \to \mu \bar{\nu} e^{\pm})$ is rather small and is too low to be observed in the near future. In Fig. 5, we see that the $Z \to l\bar{l}'$ rates are roughly positively correlating with the $l \to l'\gamma$ rates and the top quark loop contributions are larger than the charm quark’s ones. To have observable $Z \to \tau^{\pm}\mu^{\pm}$ and $Z \to \tau^{\pm}e^{\pm}$, the $\tau \to \mu\gamma$, $e\gamma$ rates are predicted to be close to the present bounds.

In this work the analysis has been performed for the scalar leptoquark case. It is possible that vector leptoquarks may also contribute to $(g - 2)\mu$ and LFV processes. As shown in Ref. [4, 20], quite often $(g - 2)\mu$ and LFV processes provide more stringent constraints on vector leptoquark couplings and masses than on scalar leptoquark ones. For example, using the measured $m_t$ and the formula given in [4], the present $\Delta a_{\mu}$ leads to a very large mass scale $\Lambda \simeq 500$ TeV in the vector leptoquark case, where $\Lambda$ was defined from the relation: $4\pi/\Lambda^2 \equiv g_{LQ}^2/m_{LQ}^2$. The mass scale is much larger than the corresponding mass scale exhibited in Fig. 2, which is found to be $\Lambda \simeq \text{few} - \mathcal{O}(10)$ TeV. Similarly, in $l \to l'\gamma$ processes, the constraints on vector leptoquark parameters are usually more severe [20].
FIG. 6: The correlation between $\text{Br}(\mu \rightarrow e\gamma)$ and $\text{Br}(Z \rightarrow \mu e)$.

IV. CONCLUSION

Motivated by the reported discrepancy of the muon $g - 2$ results, we studied the lepton flavor violating $\ell \rightarrow \ell'\gamma$ and $Z \rightarrow \ell\ell'$ decays in the LQ model. We showed that the $g - 2$ anomaly favors LQ masses in rather low-energy regime, e.g. $< 1$ TeV, which is within the reach of the forthcoming Large Hadron Collider.

We found that leptoquarks can generate sizable LFV $l \rightarrow l'\gamma$ decays. The present experimental limits are used to confined the leptoquark parameter space. On the other hand, it is interesting to search for these LFV effects in experiments, such as MEG, B factories and the super B factory.

We predict $\text{Br}(Z \rightarrow \tau^+\tau^-)$ reaching $10^{-5}$ and $\text{Br}(Z \rightarrow \mu^+\mu^-)$ reaching $2 \times 10^{-8}$, which can be accessible by present experiments and future linear colliders, such as ILC. On the contrary, the current bounds on LFV impose very strong constraints on the $\text{Br}(Z \rightarrow \mu^+\mu^-)$ and the ratio is too low to be observed in the near future. In this case, it is much useful to search for the LFV effects in $\mu \rightarrow e\gamma$ decay.
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APPENDIX A: ONE LOOP FUNCTIONS

The loop functions $F_i$ and $G_i$ used in Sec. II are given by

\[
F_1(x) = \frac{\left[2 + 3x - 6x^2 + x^3 + 6x \log(x)\right]}{12(1 - x)^4},
\]

\[
F_2(x) = \frac{\left[1 - 6x + 3x^2 + 2x^3 - 6x^2 \log(x)\right]}{12(1 - x)^4},
\]

\[
F_3(x) = \frac{-1}{2(1 - x)^3} \left[3 - 4x + x^2 + 2 \log(x)\right],
\]

\[
F_4(x) = \frac{1}{2(1 - x)^3} \left[1 - x^2 + 2x \log(x)\right],
\]

and

\[
G_1(x) = \frac{\left[-2 + 9x^2 - 18x^4 + 11x^6 - 12x^6 \log(x)\right]}{36(x^2 - 1)^4},
\]

\[
G_2(x) = \frac{1}{36(x^2 - 1)^4}
\]

\[
\times \left[16 - 45x^2 + 36x^4 - 7x^6 + 12(-2 + 3x^2) \log(x)\right]
\]

\[
G_3(x) = \frac{3 - 4x^2 + x^4 + 4 \log(x)}{4(x^2 - 1)^3},
\]

\[
G_4(x) = \frac{2 + 9x^2 - 6x^4 + x^6 + 12x^2 \log(x)}{12(x^2 - 1)^4},
\]

\[
G_5(x) = \frac{1 - 6x^2 + 3x^4 + 2x^6 - 12x^4 \log(x)}{12(x^2 - 1)^4},
\]

\[
G_6(x) = \frac{1}{2(x^2 - 1)^3} \left[-1 + x^4 - 4x^2 \log(x)\right].
\]
APPENDIX B: CONSTRAINT FORM $\pi \rightarrow e\nu_e$ AND $\pi \rightarrow \mu\nu_\mu$ DECAYS

We follow [11, 20] to constrain leptoquark parameters using pion decay data. Form the interactions given in Eq. (13), we obtain the effective four-Fermi interaction

$$\mathcal{L}_{\text{eff}} = -\frac{h'_{ai}h'_{bj}^*}{M^2_{S_k}}(e_i^cP_Lu_a)(\bar{d}_bP_R\nu_i^c)$$ (B1)

$$- \frac{h_{ai}h_{bj}^*}{M^2_{S_k}}(e_i^cP_Ru_a)(\bar{d}_bP_R\nu_j^c)$$

By using the Fierz transformation, we can rewriting the Eq. (B1) as

$$\mathcal{L}_{\text{eff}} = -\frac{1}{2M^2_{S_k}}h'_{ai}h'_{bj}^*\Gamma^\dagger_{R,k}\Gamma_{k,R}(\bar{d}_L\gamma^\mu b u_L,a)(\bar{\nu}_{L,j}\gamma^\mu e_{L,i})$$

$$+ \frac{1}{2M^2_{S_k}}h_{ai}h_{bj}^*\Gamma^\dagger_{R,k}\Gamma_{k,L}(\bar{d}_L\gamma^\mu b u_{R,a})(\bar{\nu}_{L,j}e_{R,i})$$ (B2)

On the other hand, the conventional interaction for the $\pi \rightarrow l\nu_l$ decay in the SM is given by

$$\mathcal{L}_{\text{eff}} = -\frac{G_FV_{ud}}{\sqrt{2}}[\bar{\nu}\gamma_\mu(1-\gamma_5)l][\bar{d}\gamma^\mu(1-\gamma_5)u] + \text{h.c.}$$

where $|V_{ud}|$ is the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements between the constituent of the pion meson, $G_F$ is the Fermi couplings constant. The ratio $R_{th}$ of the electronic and muonic decay modes is

$$R_{th} = \frac{\Gamma_{SM}(\pi^+ \rightarrow e\nu_e)}{\Gamma_{SM}(\pi^+ \rightarrow \mu\nu_\mu)}$$

$$= \left(\frac{m_e^2}{m_\mu^2}\right)^2\left(\frac{m^2_\pi - m^2_e}{m^2_\pi - m^2_\mu}\right)^2\left(1 + \delta\right)$$

$$= (1.2352 \pm 0.0001) \times 10^{-4}$$ (B3)

where $\delta$ is the radiative corrections. Thus the ratio $R_{th}$ is very sensitive to non standard model effects (such as multi-Higgs, non-chiral leptoquarks). The experimental ratio is

$$R_{exp} = (1.2302 \pm 0.004) \times 10^{-4}$$ (B4)

The interference between the standard model and LQ model can be expressed by

$$R_{SM-LQ} = R_{th} + R_{th} \frac{m^2_\pi}{m_u + m_d} \left(\frac{1}{\sqrt{2}G_FV_{ud}M^2_{S_k}}\frac{\text{Re}(h_{ue}h'^*_{ue})}{m_e} - \frac{\text{Re}(h_{u\mu}h'^*_{u\mu})}{m_\mu}\right)\Gamma^\dagger_{R,k}\Gamma_{k,L}\] (B5)

At $2\sigma$ level, we get

$$R_{min} < \sum_{k=1}^2 \left(\frac{m_e \text{Re}(h_{ue}h'^*_{ue})}{M^2_{S_k}} - \frac{m_\mu \text{Re}(h_{u\mu}h'^*_{u\mu})}{M^2_{S_k}}\right)\Gamma^\dagger_{R,k}\Gamma_{k,L} < R_{max}$$ (B6)
where,

\[ R_{\text{min}} = -1.06 \times 10^{-8} \text{GeV}^{-2}, \]  
\[ R_{\text{max}} = 2.45 \times 10^{-9} \text{GeV}^{-2}. \]  

(B7)  
(B8)

The total contribution to \( R_{\text{SM-LQ}} \) must be smaller than the differences between SM and experiment within the error limits allow.
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