Physics Motivation for a Pilot Dark Matter Search at Jefferson Laboratory

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It has recently been demonstrated that a program of parasitic electron-beam fixed-target experiments would have powerful discovery potential for dark matter and other new weakly-coupled particles in the MeV–GeV mass range. The first stage of this program can be realized at Jefferson Laboratory using an existing plastic-scintillator detector downstream of the Hall D electron beam dump. This paper studies the physics potential of such an experiment and highlights its unique sensitivity to inelastic “exciting” dark matter and leptophilic dark matter scenarios. The first of these is kinematically inaccessible at traditional direct detection experiments and features potential “smoking gun” low-background signatures.

I. INTRODUCTION

Although overwhelming astrophysical and cosmological evidence supports the existence of dark matter (DM) [1], its identity, interactions, and origin remain elusive. There is currently an active program to probe particle DM scattering with direct detection experiments, annihilation with indirect detection telescopes, and production with particle accelerators [2]. However, most of these efforts are designed to find heavy (10–1000 GeV) DM candidates and sharply lose sensitivity to lighter (sub-GeV) states whose signals are either too feeble or lie in high-background regions. Even direct-detection experiments [3–5] and proposals [6–8] that are expanding sensitivity to GeV-scale DM rely on an elastic scattering channel that is absent or highly suppressed in many DM scenarios [9–13].

Recently it was shown that electron-beam fixed target experiments offer powerful sensitivity to a broad class of dark sector scenarios that feature particles in the elusive MeV–GeV mass range [17, 18]. If DM couples to leptonic currents via mediators of comparable mass, it can be produced copiously in relativistic electron-nucleus collisions and scatter in a downstream detector (see Fig. 1). Electron-beam dump experiments are complementary to dedicated efforts at proton beam facilities [19–23], and have comparable DM scattering yield. Electron-beam experiments can run parasitically on a smaller scale and benefit from negligible beam-related backgrounds.

Jefferson Laboratory (JLab) is currently upgrading its 6 GeV electron beam to operate at 12 GeV energies. The new CEBAF (continuous electron beam accelerator facility) is scheduled to begin delivering ~100μA currents in mid-2014 and presents new opportunities to search for new light weakly coupled particles. A possible first step would be a parasitic pilot experiment using an existing plastic-scintillator detector behind the Hall D electron beam dump, which will receive a ~200 nA current [24]. Such an experiment could pave the way for a larger-scale experiment behind a higher-current beam dump [17]. Remarkably, even a small-scale pilot experiment has potential discovery sensitivity to several DM scenarios, which we explore in this paper. A particularly dramatic signal could be seen if DM states are split by \( \gtrsim \) MeV, so that DM scattering produces energetic \( e^+e^- \) pairs (considered in other contexts in [9–11, 14, 16, 25–29]).

The basic production and detection processes we consider here parallel those discussed in [17, 19–20]. Electrons impinging on atomic nuclei in a beam dump can emit light mediator particles that promptly decay to pairs of DM particles or the DM can be radiated via off shell mediator exchange (Figure 1(a)). The pair of DM particles emerge from the beam dump in a highly collimated
beam and pass through the shielding and dirt because their interactions are weak. A fraction of the DM particles scatter off electrons, nucleons, or nuclei via mediator exchange in a downstream detector (Figure 1(b), left). Because the DM particles are relativistic, their scattering can induce multi-MeV recoils of the target which in turn produce scintillation or Čerenkov light.

Our treatment generalizes [17] in three important ways. First, we consider the possibility that the mediator coupling DM to SM matter couples only to leptons, not to nucleons — for example, a vector can couple to the conserved DM states with masses below 68 MeV, this extension faces constraints from BaBar [17], LSND [20], (g − 2)_{e,μ} [55, 61], and rare Kaon decays [57], shown in the bottom panel of Fig. 2.

The interactions in Eq. (2) mediate DM scattering off electrons, coherent scattering off nuclei, quasielastic scattering off nucleons, and inelastic scattering off nuclei.

1 Constraints on a visibly decaying mediator $A' \rightarrow e^+e^-$ are also given in [59], but we do not consider this scenario.
The last process requires substantial momentum transfer and is not included in our simulations. For a detailed discussion of the signals we simulate in our numerical studies, see Sec. [III C] and Appendix A.

2. Leptophilic $U(1)_{e-\mu}$

The simplest leptonically coupled mediator arises from a $U(1)_{L_i-L_j}$ gauge extension to the SM [62–67], where $\ell_{i,j} = e, \mu, or \tau$ and $i \neq j$ are SM leptons. For concreteness, we consider only $U(1)_{e-\mu}$ as the simplest model that allows mediator couplings to electrons. The lagrangian for this mediator contains

$$\mathcal{L} \supset -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{m^2}{2} V \nu V + V_{\nu} \sum_i g_{\ell_i} \bar{\ell}_i \gamma^\mu \ell_i, \quad (3)$$

where $F_{\mu\nu} = \partial_{\mu} V_{\nu} - \partial_{\nu} V_{\mu}$ is the field strength and $g_{\ell_i}$ is the $U(1)_{e-\mu}$ charge for lepton $\ell_i$.

In this mass range leptophilic invisibly decaying mediators are constrained only by precision QED measurements of $(g-2)_{e,\mu}$ [55, 61] and by neutrino-scattering observations with Borexino [58]. For comparison with the conventional bounds on kinetically mixed gauge bosons, we will present the parameter space in terms of the parameter $\epsilon \equiv g_{\ell}/e$.

B. Dark Species and Spectra

We now consider simplified models of the dark sector that feature either a complex scalar or Dirac fermion coupled to the SM via one of the mediators in Sec. [II A]. For simplicity, in this subsection we use the notation appropriate for the $U(1)_D$ model with an $A'$, but the features discussed below apply equally to a $U(1)_{e-\mu}$ gauge boson or any other spin-1 mediator.

A key feature of these models is that the same spontaneous symmetry breaking that gives the mediator a non-zero mass (for concreteness, we consider a perturbative Higgs mechanism) can also split the bosonic/fermionic matter into two real/Majorana states with different masses. The leading mediator coupling in these cases is generically off-diagonal. Thus the DM production mode shown in Figure [II A] always produces one light and one heavy particle, and its scattering (Figure [II B] left) is always inelastic. The subsequent phenomenology is determined by the excited-state lifetime, which scales as $m_{A'/2}^2/(\Delta^2 \epsilon^2 \alpha \alpha_D)$ and so is very sensitive to the size of the splitting. For large enough splittings, the decay occurs inside the detector and the decay products contribute significantly to (or even dominate) the energy deposition from DM scattering (see [III C]). These inelastic scenarios are especially important to consider because a thermal origin for dark matter in such models is entirely compatible with constraints on light dark matter derived from measurements of the CMB [68].
1. Scalar Spectra

Consider a complex scalar particle Φ coupled to a $U(1)_D$ gauge boson that gets its mass from the symmetry-breaking vev of a second charged scalar, $H_D$. If $\Phi$ and $H_D$ have equal and opposite charges, the most general Lagrangian contains

$$\mathcal{L} \supset \left| (\partial_\mu - ig_D A_\mu) \Phi \right|^2 - (M^2 + \eta |H_D|^2) |\Phi|^2 - \kappa |\Phi|^2 H_D^2 - \lambda |\Phi|^4 + h.c.,$$

where $g_D = \sqrt{4\pi\alpha_D}$ is the $U(1)_D$ coupling constant to dark sector. For $\langle H_D \rangle \neq 0$, the potential contains both diagonal and off-diagonal mass terms for $\Phi$, which split the mass-eigenstates. The mass eigenbasis now features two states $\varphi$ and $\phi$ whose mass splitting $M_\varphi - M_\phi \equiv \Delta$ is generically of order the common mass scale in the dark sector (or smaller for small $\kappa$).

After symmetry breaking, the mass eigenstates couple off-diagonally to the mediator, via the derivative interaction $g_D A_\mu \phi \partial^\mu \varphi + c.c.$ Thus, in the presence of mass splittings, every $A'$ produced in a beam dump yields a ground state $\varphi$ and an excited state $\phi$, which can generate distinct detector signatures. An incident excited state $\phi$ scatters by converting into the ground state and inelastically depositing it energy into the target particle via $A'$ exchange. The ground state $\varphi$ can only interact with the detector by upscattering into the excited state, which for $\Delta < m_{A'}$ decays via $\phi \rightarrow A'^* \rightarrow \varphi e^+ e^-$. In the $\Delta \ll M_\phi$ limit, the width for this process is

$$\Gamma(\phi \rightarrow \varphi e^+ e^-) = \frac{4\epsilon^2 \alpha \alpha_D \Delta^5}{15\pi m_{A'}^4} + \mathcal{O}(\Delta^6),$$

(see Appendix B). For a boost factor of $\gamma$, the decay length is

$$\ell_\phi = \frac{\gamma c}{\Gamma(\phi \rightarrow \varphi e^+ e^-)} \approx 0.01 \text{cm} \left( \frac{2}{\gamma} \right) \left( \frac{10^{-3}}{\epsilon} \right)^2 \left( \frac{0.1}{\alpha_D} \right) \left( \frac{50 \text{ MeV}}{\Delta} \right)^5 \left( \frac{m_{A'}}{50 \text{ MeV}} \right)^4,$$

so for splittings of order the mediator mass, the decay is microscopic on detector length scales and gives rise to a distinctive signal.

2. Fermionic Spectra

If the $A'$ interacts with a dark sector Dirac fermion $\Psi = (\lambda, \xi^\dagger)$ charged under $U(1)_D$, the Lagrangian in Weyl components is

$$\mathcal{L} \supset i\lambda^\dagger \bar{\sigma}^\mu D_\mu \lambda + i\xi^\dagger \bar{\sigma}^\mu D_\mu \xi + m(\xi \lambda + \xi^\dagger \lambda^\dagger) + h.c.$$  

Again, for appropriate charge assignments, there are also Yukawa interactions

$$\mathcal{L} \supset y_\lambda H_D \lambda \lambda + y_\xi H_D \xi \xi + h.c.,$$

which induce Majorana mass terms after spontaneous symmetry breaking. Diagonalizing the fermion masses yields states $\psi$ and $\chi$ with masses $\sim m \pm y \langle H_D \rangle$, respectively and the gauge mediator couples off diagonally to the mass eigenstates via the $g_D A'_\mu \psi^\dagger \bar{\sigma}^\mu \chi + h.c.$ interaction.

Scattering through $A'$ exchange is now necessarily inelastic and the heavier state $\psi$ can now de-excite (see Appendix B) with width

$$\Gamma(\psi \rightarrow \chi e^+ e^-) = \frac{8\epsilon^2 \alpha \alpha_D \Delta^5}{15\pi m_{A'}^4} + \mathcal{O}(\Delta^6),$$

which is parametrically comparable to the corresponding scalar result in Eq. (5).

C. Smoking Gun Signals

In [17] it was shown that electron beam dumps have sensitivity to quasi-elastic DM-nucleon scattering, $\chi n \rightarrow \chi n$, via $A'$ exchange, however, for continuous wave (CW) beams $^2$, exploiting this process typically requires shielding or vetoing environmental backgrounds. However, there are two smoking gun signals with such high energy deposition that the backgrounds can be dramatically reduced or even eliminated (see [11B]); even a $\sim 10$-event signal of these types could suffice for a convincing discovery.

1. High Energy Electron Recoils

For both mediator models in Sec. [1A] and both dark sector scenarios in Sec. [1B] a typical DM particle produced in the beam dump can scatter off detector electrons and produce visible recoil energies. The dominant backgrounds for this channel comes from cosmic muons which either decay in flight or are stopped in the detector and decay at rest. Sec. [11B] will give estimates for these backgrounds and we comment on their reducibility.

2. Inelastic DM Transitions

Models with non-minimal dark scalar (or fermion) spectra offer a unique signature to be exploited at an electron beam-dump experiment. If excited states $\phi$ or

\[\text{For a CW beam, the beam-on live-time coincides with the total duration of the experiment, which is on the order of several-months, so timing is difficult and the detector encounters the maximum flux of environmental backgrounds over that time interval. In contrast, a pulsed beam delivers electrons in small, concentrated bunches, so the beam-on time is typically $\lesssim 10^{-2}$ of the total experimental run, which dramatically reduces the detector’s effective exposure to environmental backgrounds.}\]
psi decay promptly on scales ≲ 10 cm, then a unique handle on DM comes from the ground-states upsampling via the off-diagonal gauge interactions in Sec. II B and transitioning into the short-lived excited states
\[ \chi T \rightarrow \psi T \rightarrow (\psi \rightarrow \chi e^+ e^-)T \] (10)
for fermions \( \chi, \psi \). Similarly for scalars we have
\[ \varphi T \rightarrow \phi T \rightarrow (\phi \rightarrow \varphi e^+ e^-)T, \] (11)
where \( T \) can be a target nucleus, nucleon, or electron. The detector signature of this process is a target recoil accompanied by an energetic \( e^+ e^- \) pair. This final state is difficult to mimic by a beam-originated or cosmic-originated background event.

III. TEST RUN SETUP

In the test-run set up discussed in this paper, we assume placement of a small detector above ground roughly 10 m behind the electron beam-dump at JLab Hall D. Fig. 3 shows a schematic of possible test-run setups. In a year of normal operations, Hall D will receive currents \( \sim 200 \text{ nA} \) from CEBAF for a few months, which this experiment can use parasitically. We therefore consider a benchmark of \( 10^{13} \) electrons on target (EOT) over a beam-on live time of 90 days. The possibility of an off-axis detector is considered because the beamline into the dump is slightly below ground level. An above-ground experiment would therefore be slightly misaligned with the beam axis.

A. Detector

Inspired by the existing CORMORINO prototype [30], we simulate DM-SM scattering in a \( 40 \text{ cm} \times 30 \text{ cm} \times 30 \text{ cm} \) detector of NE110 polyvinyltoluene (\( C_{27}H_{30} \)) plastic-scintillator. Fig. 4 shows the angular distribution with respect to the beam-line for various mediator masses for both fermion and scalar DM.

B. Backgrounds

The 12 GeV CEBAF at Jefferson Laboratory delivers electrons to experimental Halls A, B, C, and D. The proposed test-run in this article assumes that such an experiment would take place downstream of Hall D and follow the layout in Fig. 3. Given the geography surrounding Hall D, a detector placed 10 meters behind the...
beam dump would be near or at ground level (the latter if run in off-axis mode). The various backgrounds associated with the test-run can be divided into two kinds: those originating from the beam, and those unrelated to the beam (cosmic-originated events). Beam-related backgrounds were estimated to be negligible even with $10^{22}$ EOT [17], so we ignore these for the remainder of this section. In what follows we estimate the beam-unrelated backgrounds for leptophilic (electron channel) and inelastic models.

Models where up-scattering to an excited state is followed by a prompt decay of the excited state leave a unique signature. The signal consists of an $e^+e^-$ pair, collectively depositing $\sim$ GeV energy, and a hard recoil from either an electron, nucleon, or nucleus. If each of these particles could be separately resolved, then this signal would be easily separated from cosmogenic backgrounds. For example, if the excited state lifetime is cm-scale, then the recoil and $e^+e^-$ pair would frequently appear in different cells of the detector. Even for prompt decays and a simple plastic scintillator detector — where the total energy deposited is probably the only observable signal — this energy may be sufficient to stand out over backgrounds. The same is true of the electron-scattering signal.

The most important background process comes from cosmic muons which then decay to an electron. There are two possibilities to consider: stopped and decay-in-flight muons. The former can be removed entirely by vetoing on muon hits in a window as large as 100 $\mu$s and by cutting on $E_R > m_\mu \approx 52.5$ MeV. The timing window can be applied while still having little effect ($\sim$ 1%) on the detector livetime.

The rate of muon decays in flight within the detector can be inferred from measurements of the muon flux at sea-level [69]. For a CORMORINO-sized detector, we estimate a total rate of $\approx 10^{-2}$ Hz. In 90 days of beam-on live time, this gives approximately $10^5$ decay-in-flight muons. While this background component is quite sizeable, it is also reducible with a high efficiency by vetoing events with electronic activity coincident with an incoming charged particle. Furthermore, most of the decaying muons are significantly less energetic than the the multi-GeV signals from electron recoils or de-excitation. For example, requiring $E_\mu > 2$ GeV reduces the decay-in-flight rate to $\approx 6 \times 10^{-3}$ Hz. Assuming a $10^5$ background-rejection efficiency yields $O(10)$ decay-in-flight muon events in 90 days. In contrast, demanding requiring $E_R > 2$ GeV has a weak (negligible) effect on the electron-recoil (inelastic de-excitation) signal efficiencies (See Fig. 5).

In Sec. III C we discuss the details of the signal simulation. In Sec. IV we give sensitivity estimates for the two classes of signals studied so far. These assume sensitivity at the 10-event level based on this estimates given above. Though we do not explicitly model energy thresholds or beam degradation, these are expected to be at most $O(1)$ corrections to the signal yield.

![Graph](image.png)

**FIG. 5:** Top: Differential recoil spectra in arbitrary units for fermion DM scattering inelastically off electrons with different mass splittings $\Delta$ and a thick blue curve (color online) denoting the nucleon recoil distribution, whose shape does not change visually for the parameters we consider in this paper. Each differential cross section is convolved with a monte carlo distribution of incoming $\chi$ energies that pass through the detector. Bottom: Lab frame distribution of the combined electron and positron energies after $\psi \rightarrow \chi e^+e^-$ de-excitation for various $\psi$ energies in the $m_\psi \gg m_\chi$ limit. Note that beam degradation (not simulated) would broaden the distribution and pull it towards lower energies. Moreover, for $\Delta \lesssim m_\chi$ the peak energy scales as $E_{peak} \approx \frac{\Delta}{m_\psi}$.

### C. Simulation

The calculation of the signal yield is factorized into two reactions which are analogous to QED processes: production and re-scattering. On the production side, we use a modified version of Madgraph 4 to simulate the process depicted in the top panel of Fig. 4

$$eZ \rightarrow (A')^* \rightarrow \chi \bar{\chi}eZ,$$

where $Z$ stands for an individual nucleus in the beam dump target, made mostly of aluminum. A nuclear form factor from [70] was used in the modified Madgraph version. The production simulation is used to extract the
The energy profile for DM particles that pass through the detector. We do not model the effects of beam degeneration as it passes through the dump on $dN/dE$, but instead model only the production in the first radiation length of the dump. The resulting signal yield is given by

$$Y = n_T \ell D \int_{E_c}^{\infty} dE_R \int_{E_m(E_R)}^{\infty} dE \frac{dN}{dE} \frac{d\sigma}{dE dE_R},$$

(13)

for each scattering channel. Here $n_T$ is the target particle density, $\ell D$ is the longitudinal detector length, $E_c$ is the experimental cut on target recoils, $E$ is the incoming DM energy, $E_m(E_R)$ is the minimum energy for an incident particle to induce a target recoil energy $E_R$, and $d\sigma/dE dE_R$ is the differential cross section for a given channel – see Appendix A.

The detector reactions considered in this article are depicted schematically in Fig. 1 (bottom) for fermionic DM; analogous processes apply in the scalar scenario. Following the notation of sections II B 2 and II B 1, for the fermionic and scalar DM scenarios, the signatures of interest are

$$\chi T \rightarrow (\psi \rightarrow \chi e^+ e^-) T,$$

(14)

$$\varphi T \rightarrow (\phi \rightarrow \varphi e^+ e^-) T,$$

(15)

where $T$ is a target nucleus (coherent scattering), nucleon (quasi-elastic scattering), or electron. In Fig. 3 (top) we show the electron and nucleon recoil distributions for different values of $\Delta$ using a monte carlo distribution of incoming DM energies that pass through the detector and (bottom) the lab frame $e^+ e^-$ energy distribution for different energies of the excited state.

Unless otherwise specified, the recoil energy thresholds used in the analysis are 100 MeV for incoherent and electron scattering, and 100 keV for coherent nuclear scattering. For nuclear coherent scattering, a lower threshold is used to enhance the signal and get the $Z^2$ enhancement. In addition to a neutral current coherent scatter, one or more of the electrons from the decay of the excited state are required to scatter in the detector. This signature - an electron signal and a coherent scatter - renders a search for these classes of signals background-free.

**IV. RESULTS**

The test-run set-up discussed in this paper can have discovery potential for new dark matter scenarios. In what follows, we discuss the sensitivity levels to the two smoking gun signals discussed in Sec. II C.

**A. Leptophilic scenario potential**

One kind of new physics that a test-run at JLab can be sensitive to is that of a leptophilic mediator between DM and the SM. Fig. 6 shows the 10-event signal yields for electron scattering in the context of a leptophilic $A'$. The coupling between the DM and the $A'$ is given by $g_D$ and is assumed to be 1 for this scenario. Existing constraints, particularly those coming from solar neutrinos experiments already set strong bounds on the parameter space of this scenario. However, a full-scale experiment as discussed in [17] can cover significant new ground.

**B. Inelastic transitions potential**

A small test run has particularly dramatic sensitivity to non-minimal dark sectors, where a DM excited state can decay in the detector, depositing over a GeV of energy. Both the ground ($\chi$ or $\varphi$ for fermion and scalar DM, respectively) and the excited states ($\psi$ and $\phi$) are produced in the beam-dump through an $A'$ radiated by an
electron. For prompt excited states de-excitations, only the ground state makes it to the detector downstream of the beam dump, where it can up-scatter to the excited state. The latter then de-excites within the detector for certain regions of the parameter space. The top and middle plots in Fig. 7 show the 10-event level sensitivity at a test-run for the scalar DM scenario, for fixed choices of $\Delta = M_\phi - M_\varphi$. The $\Delta$ is chosen so as to have a prompt de-excitation within the detector. Thus, at least one of the $e^+e^-$ pair is visible, regardless of whether the ground state $\varphi$ up-scatters off of a nucleus, nucleon, or electron in the detector, and regardless of the recoil energy. Note that B-factory and rare Kaon decay searches are insensitive to this scenario, because these analyses veto on extra event activity. Constraints from LSND for $m_A < m_\pi^0$ do apply to this scenario, but are difficult to model; we simply indicate the kinematic limit to LSND sensitivity in Fig. 7 (middle). Fig. 7 (bottom) shows similar projections varying $\Delta$ in the leptophilic scenario.

V. CONCLUSION

In this paper we have shown that a test run for a parasitic fixed-target experiment to search for DM at Jefferson Laboratory could have sensitivity to several well motivated scenarios in only a few months of livetime. Motivated by efforts to launch such a test run experiment \cite{24}, we considered signal yields for a small (sub meter-scale) plastic scintillator detector positioned above ground 10 meters downstream of a fixed target – a geometry similar to that at the existing Hall D beam dump. With $10^{19}$ electrons on target, signal yields are sufficiently high to give a test run experiment unprecedented sensitivity to DM that couples to the visible sector through leptophilic mediators. The same experiment can also probe scenarios where the DM upscatters into an excited state. In this case, the excited state’s decay into $e^+e^-$ deposits GeV-scale energy in the detector, irrespective of the target electron, nucleus, or nucleon’s recoil energy. These signals can deposit considerably higher energies than the dominant cosmogenic backgrounds. These findings suggest that a small test-run demonstrating the viability of electron beam dump searches for light dark matter will provide new sensitivity to unexplored dark matter scenarios.

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Appendix A: Detector Scattering

Scalar Amplitude

The amplitude for scattering $\varphi_1(p_1) + T(p_2) \rightarrow \varphi_2(k_1) + T(k_2)$ through a kinetically mixed photon is

$$A = \frac{eeG}{(t - m_A^2)} \left( \bar{u}(k_2)\gamma^\mu u(p_2) \right) \left( \bar{u}(k_1)\gamma^\mu u(p_1) \right)$$  \hspace{1cm} (A.1)$$

where $t \equiv (p_2 - k_2)^2$ is the usual Mandelstam variable and $\varphi_i$ carries mass $m_i$. Squaring and averaging (summing) initial (final) state spins

$$|\mathcal{A}|^2 = \frac{2(eeG)^2}{(t - m_A^2)^2} \left\{ (k_2 \cdot p_2)(p_2 \cdot p_1) + (k_2 \cdot p_1)(p_2 \cdot p_1) - (k_2 \cdot p_2)(p_1 \cdot p_1) + (k_2 \cdot p_1)(p_2 \cdot k_1) - (k_2 \cdot p_2)(p_1 \cdot k_1) + (k_2 \cdot k_1)(p_2 \cdot p_1) - (k_2 \cdot p_1)(k_1 \cdot p_1) + m_T^2 [m_1^2 + m_2^2 + 2(p_1 \cdot k_1)] \right\}$$ \hspace{1cm} (A.2)$$

Fermonic Amplitude

The generic matrix element for fermion scattering $\chi_1(p_1) + T(p_2) \rightarrow \psi(k_1) + T(k_2)$ is

$$A = \frac{eeG}{(t - m_A^2)} \left( \bar{u}(k_2)\gamma^\mu u(p_2) \right) \left( \bar{u}(k_1)\gamma^\mu u(p_1) \right)$$ \hspace{1cm} (A.3)$$

where $\chi_i$ carries mass $m_i$. Squaring and averaging initial state spins

$$|\mathcal{A}|^2 = \frac{8(eeG)^2}{(t - m_A^2)^2} \left\{ (k_1 \cdot k_2)(p_1 \cdot p_2) + (k_2 \cdot p_1)(p_2 \cdot k_1) - m_1 m_2 (k_2 \cdot p_2) - m_2^2 (p_1 \cdot k_1) + 2m_1 m_2 m_T^2 \right\}$$ \hspace{1cm} (A.4)$$

Cross Section

The differential cross section in the CM frame is

$$\frac{d\sigma}{d\Omega^*} = \frac{|\mathcal{A}|^2}{64\pi^2 s} |\vec{p}^*|^2$$ \hspace{1cm} (A.5)$$

In terms of the lab frame recoil energy, the angular measure is $d\cos \theta^* = (m_n/|\vec{p}^*||\vec{k}^*|)dE_R$, where the quantities

$$|\vec{k}^*|^2 = \frac{(s - m_2^2 - m_1^2)^2 - 4m_T^2 m_2^2}{4s}$$ \hspace{1cm} (A.6)$$

$$|\vec{p}^*|^2 = \frac{(s - m_T^2 - m_1^2)^2 - 4m_T^2 m_1^2}{4s}$$ \hspace{1cm} (A.7)$$

are the CM frame momenta for each particle in the initial and final state, respectively.

If the target is a detector nucleus, there is additional form factor suppression, so we modify the differential cross section with the replacement

$$\frac{d\sigma}{dE_R} \rightarrow F(E_R) \frac{d\sigma}{dE_R},$$ \hspace{1cm} (A.8)$$

where, for momentum transfer $q \equiv \sqrt{2m_N E_R}$, the Helm form factor is $F(E_R) = \left( \frac{3j_1(qr)}{qr} \right)^2 e^{-2r^2}$ \hspace{1cm} (A.9)$$

$$r = \left( c^2 + \frac{7}{3}\pi^2 a^2 - 5s^2 \right)^{1/2},$$ \hspace{1cm} (A.10)$$

where $c = (1.23 A^{2/3} - 0.6) \text{ fm}$, $s = 0.9 \text{ fm}$ and $a = 0.52 \text{ fm}$.

Total Event Rate

For each target species $T$ in the detector (e.g. electrons, nucleons, or nuclei), the total event rate is formally

$$Y = n_T \ell_D \int_{E_{R,c}}^\infty dE_R \int_{E_{m}(E_R)}^\infty dE dN \frac{d\sigma}{dE} \frac{dE}{dE_R},$$ \hspace{1cm} (A.11)$$

where $\ell_D$ is a characteristic detector length scale, $E_{R,c}$ is the experimental cut on recoil energies; inelastic kinematics require there to be a minimum recoil energy for a given splitting regardless of the cut, but this is typically far below any feasible experimental cut. The minimum incoming energy required for an incident particle of mass $m_1$ to scatter into a state of mass $m_2$ for a fixed recoil energy $E_R$

$$E_m = \frac{(E_R - m_T) \left[ m_2^2 - m_1^2 + 2m_T(E_R - m_T) \right]}{4m_T(E_R - m_T)} + \sqrt{G},$$ \hspace{1cm} (A.12)$$

where we define

$$G \equiv (E_R - m_T)(E_R + m_T) \left\{ (m_1 - m_2)^2 + 2m_T(E_R - m_T) \right\} \times \left( (m_1 + m_2)^2 + 2m_T(E_R - m_T) \right),$$ \hspace{1cm} (A.13)$$

and the energy profile $dN/dE$ is normalized to the number of DM particles passing through the detector.

Appendix B: Three Body Decays

In this appendix we generalize the results from [23] and compute the de-excitation decays in the inelastic scenario where a DM particle up scatters into a heavier dark-sector state and promptly de-excites to a three-body final state inside the detector.
**Fermion Decay**

For fermions, the amplitude for de-excitation via \(\psi(\ell_i) \to \chi(\ell_f) e^+(p_+) e^-(p_-)\) is

\[
\mathcal{M}_\psi = e e g_D \frac{\langle \bar{u}(p_2) \gamma^\mu u(p_1) \rangle \langle [\bar{u}(p_+)^\gamma_\mu v(p_-)] \rangle}{2(p_+ \cdot p_-) - m_{A'}^2}, \tag{B.1}
\]

Squaring and summing spins, we have

\[
|\mathcal{M}_\psi|^2 = \frac{16 (e e g_D)^2}{m_{A'}^2 - 2(p_+ \cdot p_-)^2} \left[ (p_+ \cdot p_2)(p_- \cdot \ell_i) 
+ (p_+ \cdot \ell_i)(p_- \cdot \ell_f) - m_2 m_1 (p_+ \cdot p_-) \right]. \tag{B.2}
\]

**Scalar Decay**

For scalar decays, the three-body amplitude for \(\phi(\ell_i) \to \varphi(\ell_f) e^+(p_+) e^-(p_-)\) is

\[
\mathcal{M}_\phi = \frac{e e g_D \bar{u}(p_+)(\ell_f^\dagger + \ell_f) v(p_-)}{2(p_+ \cdot p_-) - m_{A'}^2}, \tag{B.3}
\]

Squaring and summing leptons spins yields

\[
|\mathcal{M}_\phi|^2 = \frac{16 (e e g_D)^2 [2(p_+ \cdot \ell_i)(p_- \cdot \ell_f) - m_2^2 (p_+ \cdot p_-)]}{(m_{A'}^2 - 2p_+ \cdot p_-)^2}. \tag{B.4}
\]

**Total Width**

The width for both cases can be expressed as

\[
\Gamma(\phi/\psi) = \frac{1}{2\pi^3 (8m_{\phi/\psi})} \int_0^\Delta dE_+ \int_{E_+ - \varepsilon}^\varepsilon dE_- |\mathcal{M}_{\phi/\psi}|^2, \tag{B.5}
\]

where the parameter

\[
\varepsilon \equiv \frac{\Delta - E_+}{1 - 2E_+ / m_{\phi/\psi}}, \tag{B.6}
\]

is the maximum energy of the final state \(e^-\) for a fixed \(E_+\). Using the kinematic identities in the limit \(\Delta \ll m_{\phi/\psi}\)

\[
p_+ \cdot p_- = m_{\phi/\psi} (E_+ + E_- - \Delta) \tag{B.7}
\]

\[
\ell_f \cdot p_{\pm} = m_{\phi/\psi} (\Delta - E_\mp) \tag{B.8}
\]

\[
\ell_i \cdot p_{\pm} = m_{\phi/\psi} E_{\pm} \tag{B.9}
\]

we obtain

\[
\Gamma(\phi \to \varphi e^+ e^-) = \frac{4e^2(a_0 A_D)^5}{15\pi m_{A'}^5} + O(\Delta^6), \tag{B.10}
\]

\[
\Gamma(\psi \to \chi e^+ e^-) = \frac{8e^2(a_0 A_D)^5}{15\pi m_{A'}^5} + O(\Delta^6), \tag{B.11}
\]

which confirm Eqs. \[3\] and \[9\].

**Decay-Signal Yield**

In addition to the target recoil yield, If the coupling to the \(A'\) is off-diagonal between different mass eigenstates, there is a signal from the decay of the excited state inside the detector. Following the conventions in Eq. \[A.11\], the yield of de-excitation events is

\[
Y = n_T \ell_D \int_{E_{R,c}}^\infty dE_R \int_{E_{m}(E_R)}^\infty dE \xi(E, E_R) \frac{dN}{dE} \frac{d\sigma}{dE} dE_R, \tag{B.12}
\]

where \(\xi \equiv \mathcal{P} \mathcal{F}\) is an efficiency factor for which

\[
\mathcal{P}(E, E_R) = 1 - e^{-\ell_D / \ell_{\phi,\psi}}, \tag{B.13}
\]

is the decay probability inside the detector, \(\ell_{\phi,\psi} \equiv c\gamma / \Gamma_{\phi,\psi}\) is the decay length, and \(\gamma\) is the decaying particle’s boost factor in the lab frame. The function

\[
\mathcal{F}(E, E_R) \equiv \frac{1}{\Gamma_{\phi,\psi}} \int_{E_{\min}}^{\gamma \Delta} dE_+ \frac{d\Gamma_{\phi,\psi}}{dE_+} \tag{B.14}
\]

ensures that only the visible fraction of decay byproduct is counted.

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