Weak measurement from the electron displacement current: new path for applications

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Abstract. The interest on weak measurements is rapidly growing during the last years as a unique tool to better understand and predict new quantum phenomena. Up to now many theoretical and experimental weak-measurement techniques deal with (relativistic) photons or cold atoms, but there is much less investigation on (non-relativistic) electrons in up-to-date electronics technologies. We propose a way to perform weak measurements in nanoelectronic devices through the measurement of the total current (particle plus displacement component) in such devices. We study the interaction between an electron in the active region of a electron device with a metal surface working as a sensing electrode by means of the (Bohmian) conditional wave function. We perform numerical (Monte Carlo) simulations to reconstruct the Bohmian trajectories in the iconic double slit experiment. This work opens new paths for understanding the quantum properties of an electronic system as well as for exploring new quantum engineering applications in solid state physics.

1. Introduction

In the last years an increasing attention has been posed on the utility of weak measurements [1] for exploiting new fascinating quantum phenomena. A weak measurement implies that the measuring apparatus provides a small perturbation on the measured system and that the output result has a large uncertainty. Although quantum mechanics forbids to measure simultaneously the position and the momentum of a quantum particle in a single experiment, weak measurements realized on a large set of identically prepared systems provide well-defined values of momentum and position of the quantum system. Thus, one of the targets of weak measurements techniques has been the Bohmian formulation of quantum mechanics, which explicitly deals with quantum trajectories that have well-defined position and velocity. Therefore this type of measurement has been used, for example, to reconstruct trajectories in recent experiments [2]. In particular, the trajectories reconstructed from weak measurements are exactly the ones predicted by Bohmian mechanics [3, 4]. Up to now, all such experiments are done with (relativistic) photons or cold atoms and, unfortunately, a procedure to perform weak measurements of position and momentum for (non-relativistic) electrons in solid-state devices does not exist. Weak measurements in electronic devices are available in Refs. [5, 6, 7, 8] where a scheme with a quantum point contact weakly coupled to a quantum system was proposed for the
amplification of small signals. But this technique is unable to measure the fundamental position and momentum of a quantum electronic state. For this reason, in this work we investigate the feasibility of doing weak measurements from the (high-frequency) displacement current measured by an ammeter in a solid-state electronic device.

2. Displacement current and momentum measurements

First of all, without specifying how the measurement is explicitly implemented, let us discuss mean (ensemble) values of the displacement current and momentum. The mean value of the total electrical current for an (ideal) quantum system \( \langle I \rangle \) can be computed straightforwardly as the sum of the (mean) particle current plus the (mean) displacement current on a given surface \( S_L \):

\[
\langle I \rangle = \int_{S_L} \langle J_e(\mathbf{r},t) \rangle \cdot ds + \int_{S_L} \epsilon(\mathbf{r}) \frac{d\langle E(\mathbf{r},t) \rangle}{dt} \cdot ds,
\]

where \( \epsilon(\mathbf{r}) \) is the dielectric constant of the material, \( \langle J_e(\mathbf{r},t) \rangle \) the ensemble value of the particle current density and \( \langle E(\mathbf{r},t) \rangle \) the ensemble value of the electric field on the surface. Identically, \( \langle I \rangle \) can be calculated from the Ramo-Schockley-Pellegrini expression [9, 10, 11]. In the particular situation where two metallic surfaces, in which there is no variation of the potential, are placed at distance \( L_x \) the total current from the Ramo-Schockley-Pellegrini becomes:

\[
\langle I(t) \rangle = \frac{q}{mL_x} \langle p(t) \rangle_{\Omega},
\]

where \( \langle p(t) \rangle_{\Omega} \) is the mean value of the momentum in the volume \( \Omega \) of the device. In summary, the measurement of the total current can be interpreted as a measurement of the momentum of the quantum system (only for the particular geometry of the electron device mentioned above).
3. How to treat the system-apparatus interaction?
In this section, we briefly explain the interaction between the quantum system and the rest of the parts of the measuring setup (cables, ammeter, etc) to justify why the measurement of the displacement current can be considered as a weak measurement. In particular, we want to simulate the interaction between an electron in the active region of a device and the electrons composing the metal surface. In figure 1 we depict the studied system: It is considered the interaction between an electron in the device with \( N = 1 \) in the device and \( M \) electrons in the metal surface (of area \( S_m \) and width \( d_m \)), and the interaction between the \( M \) electrons in the metals through Coulomb interaction. The many particle Schrödinger equation can not be numerically solved exactly for \( N + M \) degrees of freedom due to computer memory limitations; thus, we have to choose a suitable numerical method to have an appropriate approximate solution of the equation. We use the method reported in Ref. [12, 13], which uses the notion of Bohmian conditional wave function [3, 14] because it is able to keep track of the dynamical and variable interaction given by the Coulomb potential. Then we solve numerically the following single-particle (conditional) Schrödinger-type equation for the electron in the device (\( x_1 \) in figure 1):

\[
i\hbar \frac{\partial \psi(x_1, t)}{\partial t} = [H_0 + V] \psi(x_1, t), \tag{3}
\]

where \( V = V(x_1, X_2(t), ..., X_{M+1}(t)) \) is the conditional Coulomb potential felt by the system and \( H_0 \) is its free Hamiltonian. With capital letter, \( X(t) \), we denote the actual positions of the (Bohmian) particles. The \( M \) electrons in the metal surface are simulated semi-classically taking into account the interactions with a bath of phonons at room temperature. We assume for simplicity that the electron 1 can move only in the transport direction \( x \).

The output total current in the surface \( S_L \) is then computed through the Bohmian trajectories by means of the following equation

\[
I_T(t) = \int_{S_L} \epsilon(r) \frac{dE}{dt} ds = \sum_{i=1}^{N} \nabla F(X_i(t)) v_i(t), \tag{4}
\]

where the flux \( F \) depends on each electron position and \( v_i \) is the Bohmian velocity [15, 16, 17]. We note that in (4) we have inserted only the displacement component of the total current because the electrons are not crossing the surface \( S_L \). By means of numerical simulations performed along the ideas exposed above we show that the measurement of the total current in a large surface \( S_L \) turns out to be weak (long-range Coulomb interaction) with almost negligible perturbation of the electron wave function [16, 17].

4. Measurement of the local (Bohmian) velocities
Identically to the last section, it can be demonstrated that the total current measured in a sensing electrode with a small surface provides output current only when the electron is very close to the electrode (short-range Coulomb interaction). This latter type of measurement provides a strong measurement of the position [17]. The two measurements just exposed provide a way to reconstruct the Bohmian trajectories in a solid state device [4, 18].

By means of numerical (quantum Monte Carlo) experiments of the system plus measuring apparatus, we test the feasibility of the previous weak measurements. As an example, in figure 2, the electron trajectories in the iconic double-slit scenario are reconstructed from displacement current measurements in a three terminal solid-state device. It can be seen that the trajectories are more dense near the maxima of the interference pattern, while no trajectories are found around the minima: this feature is in accordance with the mentioned experiment for photons [2] and with the general Bohmian theory which states that the trajectories follow the wave function probability density and are distributed according to the modulus squared of the wave function.
Figure 2. Reconstructed trajectories for an electron from an ensemble of weak measurements of the displacement current. The initial electron wave function is composed of two space separately wave packets that later interfere. It can be seen that the trajectories are more dense around the maxima of the wave function while no trajectories are found around the minima.

5. Conclusions
In this conference, we show how the weak measurements of the displacement current (in multi terminal devices with different geometries) allow the weak measurement of the momentum of a quantum system or its strong measurement of position. From these results, one can expect a new and unexplored path for solid-state quantum engineering applications (quantum tomography, quantum computing, etc.). As an example, the viability of measuring local (Bohmian) velocities from these techniques is shown through a numerical experiment.

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