Role of deformation in the nonmesonic decay of light hypernuclei

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Abstract

We discuss the nonmesonic decay of deformed p-shell hypernuclei. The Nilsson model with angular momentum projection is employed in order to take into account the deformation effects. The nonmesonic decay rate and the intrinsic Λ asymmetry parameter decrease as a function of the deformation parameter, while the ratio of the neutron- to proton-induced decay rates increases. We find that the deformation effects change these observables by about 10 % for $^9_\Lambda\text{Be}$ from the spherical limit.
I. INTRODUCTION

One of the main issues of nuclear physics is to understand the nucleon-nucleon (NN) interaction. The $|\Delta S| = 1$ NN interaction is particularly important in this respect, since the change of strangeness can be used as a signature to study both the parity-conserving (PC) and the parity-violating (PV) amplitudes. This is in clear contrast to the $\Delta S = 0$ weak NN interaction, where the weak PC signal is masked by the strong interaction.

Due to the lack of stable $\Lambda$-particle beams, the weak decay of $\Lambda$-hypernuclei has been the only source of information on the weak four-baryon $|\Delta S| = 1$ interaction. Single $\Lambda$-hypernuclei are typically produced via either hadronic reactions, as $(K^{*-}\pi^0)$ \(^1\) or $(\pi^+,K^+)$ \(^2\), or electroproduction mechanisms, as $(e,e' K^+)$ \(^3\). These hypernuclei are typically produced in an excited state and reach their ground state by electromagnetic-$\gamma$ and/or particle emission. Once they are stable against strong decay, they decay via weak interaction mechanisms which are nonleptonic in nature and violate isospin, parity and strangeness. Since the mesonic decay mode, $\Lambda \rightarrow \pi N$, is Pauli blocked in the nuclear medium, hypernuclei with $A \gtrsim 5$ predominantly decay through the nonmesonic decay (NMD) mode, $\Lambda N \rightarrow NN$.

In order to learn about the weak $\Lambda N \rightarrow NN$ interaction from the theoretical side, one has to take into account different inputs as accurately as possible. These include the description of nuclear structure, the choice of the strong BB potential model \([4,5]\), $\Delta I = 1/2$ violations \([6]\) and the importance of the 3N emission channel, $\Lambda np \rightarrow nnp$ \([7,8]\). In Refs. \([9,10]\), a one-meson-exchange (OME) model was applied to calculate the nonmesonic decay observables of the $p$-shell \(^{11}\Lambda B\) and \(^{12}\Lambda C\) and the $s$-shell \(^5\Lambda He\) and \(^3\Lambda H\) hypernuclei. We included the virtual exchange of the ground-state pseudoscalar and vector mesons $\rho$, $\eta$, $\omega$, $K$ and $K^*$, in addition to the long-ranged pion. Except for the hypertriton, where the hypernuclear wave function was calculated exactly using the Faddeev formalism, the structure of the initial hypernucleus was described in a shell-model framework which assumed spherical configuration. In these calculations, the strong baryon-baryon (BB) interaction was accounted for using the Nijmegen BB potential model \([4]\). Monopole form factors at each vertex were
included in order to regularize the weak potential, while the weak baryon-baryon-meson coupling constants were derived based on $SU_w(6)$ and soft-meson theorems. The total NMD rate and the asymmetry in the distribution of emitted protons from the decay of polarized hypernuclei were in good agreement with the experimental data. However, the theoretical values for the neutron-to-proton ratio were found to be very small compared to the experimental data. Several attempts have been made to reconcile this discrepancy [6–8,11–13], but none of them has solved this problem yet.

Our aim in this paper is to investigate how much these observables depend on the deformation of hypernuclei. All previous calculations were performed using the spherical configuration, however, it is well known that many $p$-shell nuclei are deformed in the ground state. For instance, the quadrupole deformation parameter extracted from the experimental quadrupole moment [14] is $\beta_2 = 0.65$ for $^{10}\text{B}$ and $-0.71$ for $^{11}\text{C}$. It may be important to take these deformation effects into account in order to describe quantitatively the nonmesonic decay of $p$-shell hypernuclei. Deformed hypernuclei can be described using several models such as the $\alpha$-cluster model [15] or the deformed self-consistent Hartree-Fock method. In fact, one can also use realistic wave functions obtained by a diagonalization of a shell-model Hamiltonian for $p$-shell nuclei, as in Ref. [16]. In the present paper, however, in order to perform a systematic study, we use instead the Nilsson model [17,18] as a simplified Hartree-Fock method.

The paper is organized as follows. In Sec. II, we present the relevant formulae to evaluate the NMD observables in a OME model. In Sec. III, we briefly review the deformed shell model based on the Nilsson model. Sec. IV presents the deformation dependence of the nonmesonic observables for the decay of $^9\Lambda\text{Be}$, whose $^8\text{Be}$ core is known to be largely deformed. Although there is no experimental data for this hypernucleus at present, we choose this system as the simplest non-spherical $p$-shell hypernucleus and as a representative example of deformed $p$-shell nuclei. We compare our theoretical predictions with the typical experimental data for other $p$-shell hypernuclei. Sec. V summarizes the paper.
II. NONMESONIC WEAK DECAY IN A ONE-MESON-EXCHANGE MODEL

Assuming that the initial hypernucleus is at rest, the NMD rate is

\[
\Gamma_{nm} = \int \frac{d^3k_1}{(2\pi)^3} \int \frac{d^3k_2}{(2\pi)^3} \sum_{M_I} (2\pi)^3 \delta(M_H - E_R - E_1 - E_2) \frac{1}{(2J+1)} |M_{fi}|^2, \tag{1}
\]

where \(M_{fi}\) is the hypernuclear transition amplitude. The quantities \(M_H, E_R, E_1\) and \(E_2\) are the mass of the hypernucleus, the energy of the residual \((A-2)\)-particle system, and the total asymptotic energies of the emitted nucleons, respectively. The integration variables \(\vec{k}_1\) and \(\vec{k}_2\) are the momenta of the two baryons in the final state. The momentum conserving delta function has been used to integrate over the momentum of the residual nucleus. The sum, together with the factor \(1/(2J+1)\), indicates an average over the initial hypernucleus spin projections, \(M_I\), and a sum over all quantum numbers of the residual \((A-2)\)-particle system, \(\{R\}\), as well as the spin and isospin projections of the exiting particles, \(\{1\}\) and \(\{2\}\).

In general, one can write the total nonmesonic decay rate as \(\Gamma_{nm} = \Gamma_{\Lambda N \to NN} = \Gamma_n + \Gamma_p\), where \(\Gamma_n (\Lambda n \to nn)\) stands for the neutron-induced decay and \(\Gamma_p (\Lambda p \to np)\) for the proton-induced one.

In addition to the total and partial decay rates, we also calculate the intrinsic \(\Lambda\) asymmetry parameter. When working with polarized hypernuclei and in combination with coincidence measurements of the decay particles, one can study the angular distribution of particles coming from the \(\Lambda N \to NN\) weak decay. Due to the interference between the PV and PC amplitudes, the distribution of the emitted protons in the weak decay displays an angular asymmetry with respect to the polarization axis. The asymmetry \(\mathcal{A}\), defined by

\[
\mathcal{A} = P_y \frac{3}{(2J+1)} \frac{Tr(M_{fi} S_y M_{fi}^\dagger)}{Tr(M_{fi} M_{fi}^\dagger)}, \tag{2}
\]

is expressed in terms of the hypernuclear polarization created in the strong production reaction, \(P_y\), the \(J\)-spin operator along the polarization axis, \(S_y\), and the total spin of the initial hypernucleus, \(J\). In Ref. [19] it is shown that the asymmetry follows a simple \(\cos \chi\) dependence, \(i.e., \mathcal{A} = P_y A_p \cos \chi\), where \(\chi\) stands for the angle between the direction of the
proton and the polarization axis. The hypernuclear asymmetry parameter $A_p$ is characteristic of the hypernuclear weak decay process and depends on $J$ and the intensity of protons exiting along the quantization axis for the different spin projections of the hypernucleus.

At $\chi = 0^\circ$, the asymmetry in the distribution of protons is thus determined by the product $A = P_y A_p$. In the following, we assume a weak coupling scheme where the $\Lambda$ hyperon is coupled only to the ground state of the $(A-1)$-particle core. In this scheme, simple angular momentum algebra relates the hypernuclear polarization $P_y$ to the $\Lambda$ polarization $p_\Lambda$, and the hypernuclear asymmetry parameter $A_p$ to the intrinsic $\Lambda$ asymmetry parameter $a_\Lambda$, such that $A = p_\Lambda a_\Lambda = P_y A_p$.

The nonmesonic decay of hypernuclei proceeds through a two-body mechanism. Therefore in order to evaluate the transition amplitude in Eq. (1), one has to decompose the $(A-1)$-core wave function into a set of states in which a nucleon couples to the residual $(A-2)$-particle state. This can be done using the Coefficients of Fractional Parentage (CFP), which are defined by

$$|JM,TT_z\rangle = \sum_{J_R,T_R,j} \langle JT\{|J_RT_R,jt\} \otimes |jt\rangle |JM,TT_z\rangle,$$

where $J_R$ and $T_R$ are the spin and isospin of the residual nucleus. The weak potential responsible for this transition can be obtained by making a nonrelativistic reduction of the free Feynman amplitude depicted in Fig. 1. In Table I we show the strong and weak vertices for pseudoscalar (PS) and vector (V) mesons. $A,B,\alpha,\beta$ and $\epsilon$ stand for the appropriate baryon-baryon-meson weak coupling constants, while $g$ ($g^V, g^T$) represents the strong (vector, tensor) coupling. Details of the derivation of the transition potential can be found in Ref. [9] and here only the final expression will be presented. For pseudoscalar mesons, the potential is

$$V_{ps}(\vec{q}) = -G_F m_\pi^2 \frac{g}{2M} \left( \hat{A} + \hat{B} \vec{\sigma}_1 \vec{q} \right) \frac{\vec{\sigma}_2 \vec{q}}{\vec{q}^2 + \mu^2},$$

where $G_F m_\pi^2 = 2.21 \times 10^{-7}$ is the Fermi coupling constant, $\vec{q}$ is the momentum carried by the meson directed towards the strong vertex, $\mu$ the meson mass and $M$ ($\overline{M}$) is the average
of the baryon masses at the strong (weak) vertex (the other way around for the exchange of strange mesons). For vector mesons, the potential is

\[
V_v(\vec{q}) = G_F m_n^2 \left( g^V \hat{\alpha} - \frac{(\hat{\alpha} + \hat{\beta})(g^V + g^T)}{4M M} (\hat{\sigma}_1 \times \vec{q}) (\hat{\sigma}_2 \times \vec{q}) \right. \\
\quad \left. - i \frac{\hat{\varepsilon}(g^V + g^T)}{2M} (\hat{\sigma}_1 \times \hat{\sigma}_2) \vec{q} \right) \frac{1}{q^2 + \mu^2}. \tag{5}
\]

The values of the strong and weak couplings are listed in Table III of Ref. [9]. In Eqs. (4) and (5) the operators \( \hat{A}, \hat{B}, \hat{\alpha}, \hat{\beta} \) and \( \hat{\varepsilon} \) contain, apart from the weak coupling constants, the specific isospin dependence of the potential, which is \( \vec{r}_1 \cdot \vec{r}_2 \) for the isovector \( \pi \) and \( \rho \) mesons, \( \hat{I} \) for the isoscalar \( \eta \) and \( \omega \) mesons and a combination of both operators for the isodoublet \( K \) and \( K^* \). In order to derive Eqs. (4) and (5) we assumed the validity of the \( \Delta I = 1/2 \) rule, which is known to experimentally dominate the decay of \( \Lambda \)'s into pions. \( \Delta I = 3/2 \) transitions for vector mesons (\( \rho \) and \( K^* \)) are easily accomodated [6] in the formalism and the results we present here account for such \( \Delta I = 1/2 \) violations.

We obtain a regularized potential by including a monopole form factor at each vertex,

\[
F(q^2) = \frac{\Lambda^2 - \mu^2}{\Lambda^2 + q^2},
\]

where the value of the cut-off, \( \Lambda \), different for each meson, is taken from the Jülich hyperon-nucleon interaction [3]. To incorporate the effects of the strong NN interaction, we solve a T-matrix scattering equation in momentum space for the outgoing nucleons using the Nijmegen [4] potential models. For the initial bound two-baryon system we use a spin independent parametrization of the type

\[
f(r) = \left( 1 - e^{-r^2/a^2} \right)^n + b r^2 e^{-r^2/c^2}, \tag{6}
\]

with \( a = 0.5 \) fm, \( b = 0.25 \) fm\(^{-2} \), \( c = 1.28 \) fm, \( n = 2 \). The results obtained with this parametrization [20] lay in between of the ones obtained with a microscopic finite-nucleus G-matrix calculation [21] using the soft-core and hard-core Nijmegen [22] models.

### III. DEFORMED SHELL MODEL

As we mentioned in the previous section, we use a weak coupling scheme for the \( \Lambda \) hyperon in the initial hypernucleus. To this end, we must describe the ground state of the core nucleus
which may be deformed. The Nilsson model provides a simple and convenient framework to describe deformed nuclei, and has been widely used in the literature \[17,18,23–26\]. Its Hamiltonian consists of an anisotropic harmonic oscillator with the spin-orbit interaction as well as an angular momentum dependent term, which mocks up the deviation of the mean field potential from the harmonic oscillator

\[
H = H_0 - \frac{4}{3} \sqrt{\frac{\pi}{5}} \delta m \omega_0^2 Y_{20}(\theta),
\]

\[
= -\frac{\hbar^2}{2m} \nabla^2 + \frac{1}{2} m \omega_0^2 r^2 + C \vec{l} \cdot \vec{s} + D (\vec{l}^2 - \langle \vec{l}^2 \rangle_N) - \frac{4}{3} \sqrt{\frac{\pi}{5}} \delta m \omega_0^2 Y_{20}(\theta).
\]

(7)

(8)

Here \(\delta\) is a deformation parameter, \(\vec{l}\) and \(\vec{s}\) are the single-particle orbital and the spin angular momenta, and \(C\) and \(D\) are adjustable parameters. \(\langle \vec{l}^2 \rangle_N = N(N+3)/2\) is the expectation value of \(\vec{l}^2\) averaged over one major shell with quantum number \(N\). The relation between \(\delta\) and \(\beta_2\) is given by \(27\)

\[
\beta_2 = \frac{4}{3} \sqrt{\frac{\pi}{5}} \frac{\delta}{1 - 2\delta/3}.
\]

(9)

Since the Nilsson Hamiltonian \(\mathbf{8}\) violates rotational invariance, the total angular momentum \(\vec{j} = \vec{l} + \vec{s}\) is not a good quantum number. However, the projection of \(\vec{j}\) on to the \(z\)-direction, \(k\), is conserved and the single-particle levels are characterized by \(k\) and other quantum numbers. We expand a Nilsson single-particle level, \(\psi_{k(q)}\), in terms of the eigenfunctions of the spherical harmonic oscillator Hamiltonian \(H_0\), \(\phi_{nljk}\), as

\[
\psi_{k(q)} = \sum_{nlj} x_{nljk}^{(q)} \phi_{nljk},
\]

(10)

where \(q\) are quantum numbers other than \(k\). We choose \(x_{nlj-k}^{(q)} = (-)^{j-k} x_{nljk}^{(q)}\) so that the eigenvalues of the Nilsson Hamiltonian do not depend on the sign of the projection of total angular momentum \(26\). We denote the creation operator of \(\psi_k\) as \(a_k^\dagger\) and that of \(\phi_{jk}\) as \(b_{jk}^\dagger\).

We explicitly express only the \(j\) and \(k\) quantum numbers to simplify the notation. Intrinsic wave functions, i.e., eigenfunctions of the Nilsson Hamiltonian \(\mathbf{8}\) are given by

\[
|\chi_k\rangle = a_{k_1}^\dagger a_{k_2}^\dagger \cdots a_{k_n}^\dagger |0\rangle = \prod_{i=1}^{n} \left( \sum_{j} x_{jk_i} b_{jk_i}^\dagger \right) |0\rangle.
\]

(11)
where the $K$ quantum number is the sum over all $k_i$. The intrinsic wave function (11) is not an eigenstate of the total angular momentum $\vec{J}$, and thus has to be projected out to a good angular momentum state. This can be achieved by using the projector given by [18,23,24]

$$\hat{P}^{JM}_{MK} = \frac{2J + 1}{8\pi^2} \int d\Omega \ D^{JM}_{MK}(\Omega) \hat{R}(\Omega),$$

where $\Omega$ are Euler angles, and $D^{JM}_{MK}(\Omega)$ and $\hat{R}(\Omega)$ are the Wigner $D$-function and the rotation operator, respectively.

For systems with a single-Nilsson level, such as $^8\text{Be}$ which we discuss in the next section, the CFP can be analytically obtained [23,24]. Note that a single Nilsson level can accommodate up to four nucleons, i.e., two protons and two neutrons. For three particle systems (2 neutrons and 1 proton, for example), the wave function is given by

$$\Psi_{JM} = [N(3)J]^{-1} \hat{P}^{JM}_{M,K=0} a_{k\nu}^+ a^+_{-k\nu} a^+_{k\pi} |0\rangle,$$

$$= [N(3)J]^{-1} \sum_{j_1,j_2,j_3,j_12} x_{j_1k} x_{j_2k} x_{j_3k} \langle j_1k{j}_2 - k|J_{12}0 \rangle \langle J_{12}0j_3k|Jk \rangle \times \left( \left[ a^+_{j_1\nu} a^+_{j_2\nu} \right]_{J_{12}} a^+_{j_3\pi} \right)_{JM} |0\rangle,$$

where $\pi$ stands for proton and $\nu$ for neutron. The normalization factor $N(3)J$ is given by [25]

$$[N(3)J]^2 = \sum_{j,j_{12}} \delta_{J_{12,even}} (x_{jk})^2 U(J_{12},k) \langle J_{12}0j_3k|Jk \rangle^2,$$

where

$$U(J,k) = 2 \sum_{j_1,j_2} (x_{j_1k})^2 (x_{j_2k})^2 \langle j_1k{j}_2 - k|J0 \rangle.$$

The isospin of this system is $1/2$. For four nucleon systems, the wave function reads

$$\Psi_{JM} = [N(4)J]^{-1} \hat{P}^{JM}_{M,K=0} a_{k\nu}^+ a^+_{-k\nu} a^+_{k\pi} a^+_{-k\pi} |0\rangle,$$

$$= [N(4)J]^{-1} \sum_{j_1,j_2,j_3,j_4,J_{12},J_R} x_{j_1k} x_{j_2k} x_{j_3k} x_{j_4k} \langle j_1k{j}_2 - k|J_{12}0 \rangle \langle J_{12}0j_3k|J_Rk \rangle \langle J_Rk{j}_4 - k|J0 \rangle \times \left( \left[ a^+_{j_1\nu} a^+_{j_2\nu} \right]_{J_{12}} a^+_{j_3\pi} a^+_{j_4\pi} \right)_{JM} |0\rangle,$$
with the normalization given by $[25]$

$$[N(4),J]^2 = \sum_{J_{12},J_{34}} \delta_{J_{12},\text{even}} \delta_{J_{34},\text{even}} U(J_{12},k) U(J_{34},k) \langle J_{12}0,J_{34}0|J0 \rangle^2. \quad (19)$$

The isospin of this wave function is 0. Comparing Eqs. (14) and (18), the CFP for the four particle system reads

$$\mathcal{C}(j) = \langle JT|J_R T_R, j t \rangle = -\sqrt{2} x_{j-k} \langle J_R k j - k|J0 \rangle N(3)_{J_R} N(4)_{J}. \quad (20)$$

IV. NONMESONIC DECAY OF $^9\Lambda\text{Be}$

Let us now apply the deformed shell model of Sec. III to the nonmesonic decay of $^9\Lambda\text{Be}$. The quadrupole moment of the neighbour nucleus $^9\text{Be}$ was measured to be 5.86 $\text{e fm}^2$ $[14]$, from which we extract the quadrupole deformation parameter $\beta_2=1.00$ using the radius parameter $r_0=1.2$ fm. Several theoretical calculations suggest that the core nucleus $^8\text{Be}$ and the $^9\Lambda\text{Be}$ hypernucleus also have similar deformation parameters with the same sign $[15]$. Our interest is to discuss such deformation effects on nonmesonic decay observables.

As is discussed in Sec. II, the use of CFP allows us to write the hypernuclear transition amplitude $\mathcal{M}_{fi}$ in terms of elementary two-body amplitudes. Therefore, our first task is to compute these coefficients for the core nucleus $^8\text{Be}$. We assume the inert spherical $^4\text{He}$ core and explicitly work with only the four valence nucleons. Diagonalizing the Nilsson Hamiltonian $[8]$, one finds that the lowest Nilsson level for the valence nucleons has $k=1/2$ for prolate deformation $[18]$. We diagonalize the Nilsson Hamiltonian in the $\Delta N=0$ states. Contributions from the $\Delta N = 2$ can be neglected unless the deformation is large. The $k=1/2$ state is thus

$$|\psi_{k=1/2} \rangle = x|\phi_{p_{3/2,1/2}} \rangle + y|\phi_{p_{1/2,1/2}} \rangle, \quad (21)$$

where $x$ and $y$ are determined by diagonalizing the Nilsson Hamiltonian within this configuration space and depend upon the deformation of $^8\text{Be}$. Using Eq. (20), the CFP’s are found to be
\[ [C(p_{3/2})]^2 = \frac{3x^8 + 3x^6y^2 + 9x^4y^4}{3x^8 + 4x^6y^2 + 18x^4y^4 + 10y^8} \] (22)

for the \( p_{3/2} \) state, and

\[ [C(p_{1/2})]^2 = \frac{x^6y^2 + 9x^4y^4 + 10y^8}{3x^8 + 4x^6y^2 + 18x^4y^4 + 10y^8} \] (23)

for the \( p_{1/2} \) state. Note that \( [C(p_{3/2})]^2 + [C(p_{1/2})]^2 = 1 \). In the spherical limit, \( x=1 \) and \( y=0 \), so the CFP become \( [C(p_{3/2})]^2 = 1 \) and \( [C(p_{1/2})]^2 = 0 \). The CFP for the deeply bound \( 1s_{1/2} \) state is just equal to 1 since \(^4\text{He}\) is a spin-isospin saturated nucleus.

Our results for the nonmesonic decay rate, \( \Gamma_{\text{nm}} \), in units of the free \( \Lambda \) decay rate, \( \Gamma_{\Lambda} = 3.8 \times 10^9 \text{s}^{-1} \), the neutron-to-proton ratio, \( \Gamma_n/\Gamma_p \), and the \( \Lambda \) asymmetry parameter, \( a_\Lambda \), are shown in Fig. 2 as a function of the deformation parameter \( \beta_2 \). We use an oscillator length \( b_N \) of 1.65 fm for nucleons, so that the experimental root mean square radius of \(^9\text{Be}\) is reproduced. Following Refs. \[17,18\], the parameters \( C \) and \( D \) in the Nilsson Hamiltonian \( (8) \) are taken to be \(-0.16\hbar\omega_0 \) and 0, respectively. As for the oscillator length \( b_\Lambda \) for the \( 1s_{1/2} \) wave function of the \( \Lambda \) hyperon, we estimate it to be 1.5 fm in order to reproduce its binding energy in \(^9\Lambda\text{Be} \) (= 6.71 ± 0.04 MeV \[15\]). From the figure, we see that \( \Gamma_{\text{nm}}/\Gamma_{\Lambda} \) is a decreasing function of \( \beta_2 \), while \( \Gamma_n/\Gamma_p \) and \( a_\Lambda \) are increasing functions. As we have already mentioned, the deformation parameter of \(^8\text{Be}\) is expected to be close to 1. We notice that the nonmesonic decay observables are altered by about 10% from the spherical limit at \( \beta_2 = 1 \).

An important question is whether this effect is significant when comparing to the experimental data. We note that the typical experimental uncertainties for nonmesonic decay of p-shell hypernuclei are: 7% – 17.5% for the total decay rate \[28,30\], 46.2% – 84.2% for the neutron-to-proton ratio \[29,30\], and 50% – 1000% for the asymmetry \[31\]. These experimental uncertainties are much larger than the theoretical one originating from the deformation effects. Thus we conclude that the spherical approximation gives a good estimate of the nonmesonic decay of p-shell nuclei, at least within the present experimental precision.

Before we close this section, we would like to stress that our conclusion is not altered qualitatively even if more realistic wave functions are used instead of the present schematic
ones. For instance, using the shell-model C.F.P. of Cohen and Kurath \cite{10} for the decay of $^{12}_\Lambda C$, we obtain $\Gamma^\text{nm}/\Gamma_\Lambda = 0.76$ and $\Gamma_n/\Gamma_p = 0.22$. Those numbers have to be compared to the spherical limit values of 0.74 and 0.25, respectively. As we see, the amount of deviation of those observables with respect to the spherical limit is of the same order of the one given here, although their behaviour is opposite due partly to the fact that the residual $^{11}C$ is oblate while $^8\text{Be}$ is prolate.

**V. SUMMARY**

We have discussed the role of nuclear structure in the nonmesonic decay of p-shell hypernuclei, especially focusing on the effects of deformation. To this end, we have used the Nilsson model with explicit angular momentum projection. We have studied the nonmesonic decay of $^9_\Lambda \text{Be}$ as a typical example of deformed p-shell hypernuclei. We have shown that the deformation effects change the total NMD rate, the neutron-to-proton ratio and the $\Lambda$ asymmetry parameter by about 10 % from the spherical limit. Although this value is not negligible, it still is smaller than the present typical experimental uncertainty and smaller than other theoretical uncertainties, e.g., the effects of SU(3) symmetry breaking \cite{11,12} or $\Delta I = 1/2$ violations \cite{13}. This indicates that the existing discrepancies between the experimental and theoretical values of hypernuclear weak decay observables cannot be attributed solely to deviations from the spherical configuration, and still remain an open question. New experiments are urged in order to reduce the large experimental error bars, which prevent any definite conclusion about the reliability of the theoretical models.

Our conclusions may not be the same for heavier hypernuclei such as $^{238}_{\Lambda} \text{U}$ \cite{32,33}. There are a lot of intruder states in such heavy deformed systems, unlike p-shell nuclei where there is only a few, or maybe zero, intruder states. Therefore, an interesting future work would be to discuss the nonmesonic decay of heavy hypernuclei including the deformation effects. For that purpose, the projected shell model developed in Refs. \cite{23,24}, which also uses the Nilsson model with angular momentum projection, would provide a powerful tool to describe
the structure of deformed hypernuclei.

ACKNOWLEDGEMENTS

We thank David Brown and Amour Margarian for useful and illuminating discussions. This work was supported by the U.S. Dept. of Energy under Grant No. DE-FG03-00-ER41132.
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FIGURES

FIG. 1. Free Feynman diagrams for the $\Lambda N \to NN$ transition mediated by the exchange of the nonstrange $\pi, \eta, \rho, \omega$ (left) and strange $K, K^*$ (right) mesons. The shaded circle (filled) stands for the weak (strong) vertex.
FIG. 2. The nonmesonic decay observables for $^9\Lambda$Be as a function of the deformation parameter $\beta_2$. 
TABLE I. Vertices entering the expression of the Feynman amplitude of Fig. [1] for pseudoscalar (PS) and vector (V) mesons. The weak vertices are in units of $G_F m_\pi^2$.

|       | PS                  | V                                      |
|-------|---------------------|----------------------------------------|
| Strong| $ig\gamma_5$       | $g^V\gamma^\mu + ig\frac{T}{2M}\sigma^\mu\nu q_\nu$ |
| Weak  | $i(A + B\gamma_5)$ | $\alpha\gamma^\mu - \beta\frac{\sigma^\mu\nu q_\nu}{2M} + \varepsilon\gamma^\mu\gamma_5$ |