Efficient Density Ratio-Guided Subsampling of Conditional GANs, With Conditioning on a Class or a Continuous Variable

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Abstract

Recently, subsampling or refining images generated from unconditional generative adversarial networks (GANs) has been actively studied to improve the overall image quality. Unfortunately, these methods are often observed less effective or inefficient in handling conditional GANs (cGANs) — conditioning on a class (aka class-conditional GANs) or a continuous variable (aka continuous cGANs orCcGANs). In this work, we introduce an effective and efficient subsampling scheme, named conditional Density Ratio-guided Rejection Sampling (cDR-RS), to sample high-quality images from cGANs. Specifically, we first develop a novel conditional density ratio estimation method, termed cDRE-F-cSP, by proposing the conditional Softplus (cSP) loss and an improved feature extraction mechanism. We then derive the error bound of a density ratio model trained with the cSP loss. Finally, we accept or reject a fake image in terms of its estimated conditional density ratio. A filtering scheme is also developed to increase fake images’ label consistency without losing diversity when sampling from CcGANs. We extensively test the effectiveness and efficiency of cDR-RS in sampling from both class-conditional GANs and CcGANs on five benchmark datasets. When sampling from class-conditional GANs, cDR-RS outperforms modern state-of-the-art methods by a large margin (except DRE-F-SP+RS) in terms of effectiveness (i.e., Intra-FID, FID, and IS scores). Although the effectiveness of cDR-RS is often comparable to that of DRE-F-SP+RS, cDR-RS is substantially more efficient. For example, cDR-RS only requires 1.19% of the storage usage and 77% of the implementation time spent by DRE-F-SP+RS on ImageNet-100. When sampling from CcGANs, the superiority of cDR-RS is even more noticeable in terms of both effectiveness and efficiency. Notably, with the consumption of reasonable computational resources, cDR-RS can substantially reduce Label Score without decreasing the diversity of CcGAN-generated images, while other methods often need to trade much diversity for slightly improved Label Score.

1 Introduction

Generative adversarial networks (GANs) [11] are popular generative models for image synthesis, aiming to estimate the marginal distribution of images. As an extension and an essential family of GANs, conditional GANs (cGANs) [22] intend to estimate the image distribution given some conditions. These conditions are usually categorical variables such as class labels. cGANs with class labels are also known as class-conditional GANs [28, 25, 2, 32]. Recently, [8, 7] propose a new conditional GAN framework, continuous conditional GANs (CcGANs), which take continuous, scalar variables (termed regression labels) as conditions. Recent advances in unconditional GANs [15, 16] and cGANs [2, 8, 7] generally enable these models to generate high-quality images. Nevertheless, low-quality images still appear frequently even with such advanced GAN models during image generation. We would like to clarify that the image quality discussed in this paper are three-fold: (1) visual quality, (2) diversity, and (3) label consistency. Label consistency applies to cGANs only and is defined as the consistency of generated images with respect to the conditioning label. Notably, CcGANs [8, 7] often suffer from low label consistency because it is designed to sacrifice label consistency for better visual quality and higher diversity.

To enhance the image quality of unconditional GANs, increasing attention has been paid to improve the sampling strategy of pre-trained unconditional GANs via subsampling or refining. Refining methods (e.g., Collab [21]) aim to refine visually unrealistic fake images to improve visual quality. For example, Collab [21] refines an intermediate hidden map of a generator to remove artifacts in generated images by using information from a trained discriminator. Subsampling methods (e.g., DRS [1], DRE-F-SP+RS [9], DDLS [4]) remove visually unrealistic images to improve visual quality and adjust the likelihood of generated images to increase diversity. To accomplish subsampling, discriminator rejection sampling (DRS) [1] accepts or rejects a fake image by rejection sampling (RS). DRS requires an accurate density ratio estimation (DRE). However, since the DRE step in DRS relies on the assumption of optimality of the discriminator, DRS may not perform well if the discriminator is far from optimal. DRS is also inapplicable to some GANs such as MMD-GAN [20], [9] improves DRS...
by proposing density ratio estimation in the feature space with Softplus loss (DRE-F-SP), which does not require an optimal discriminator and is applicable to various GANs. Then, with RS as the sampler, [9] introduces DRE-F-SP+RS to subsample GANs. Besides these density ratio-based methods, discriminator driven latent sampling (DDLS) [4] proposes to accept or reject samples via an energy-based model defined in the latent space of the generator in a GAN. The methods discussed above, however, are not designed for cGANs.

An intuitive approach to better sample from cGANs is applying unconditional methods described above for each distinct class or regression label. Unfortunately, this approach may be inefficient or even impractical, especially when many distinct classes or regression labels exist. For example, if we apply DRE-F-SP+RS [9] to sample from BigGAN trained on ImageNet-100 [3] (a subset of ImageNet [5] with 100 classes), we need to fit 100 density ratio models separately. As visualized in Fig. 1(a), this is often time-consuming (84.4 hours) and it requires a large storage space (39.97 GB). Another noticeable example is DDLS [4], which spends 218 hours subsampling BigGAN trained on ImageNet-100. Furthermore, as shown in Fig. 1(b) and Section 4.2, the unconditional approach is usually ineffective in sampling from CcGANs because it is not designed to solve the label inconsistency problem suffered by cGANs. Moreover, although DRE-F-SP+RS may perform well in subsampling class-conditional GANs (e.g., Fig. 1(a)), it is not suitable for subsampling CcGANs for two reasons: it lacks a suitable feature extraction method for regression datasets, and it cannot sample from CcGANs conditional on labels that are unseen in the training phase. The ineffectiveness or inefficiency of applying unconditional sampling methods to cGANs is demonstrated in our empirical study in Section 4.2.2.

Recently, [26] proposes a rejection sampling scheme, called GOLD, to subsample the auxiliary classifier GAN (ACGAN) [28], a special class-conditional GAN. GOLD is based on the gap in log-densities that measures the discrepancy between the actual image distribution and the fake image distribution of given samples. [26] shows that, empirically, GOLD can improve the performance of ACGAN in the class-conditional image synthesis. However, this method does not apply to general class-conditional GANs (e.g., BigGAN [2]), and it is not designed for CcGANs.

To improve the overall image quality of class-conditional GANs and CcGANs effectively and efficiently, we propose the conditional density ratio-guided rejection sampling (cDR-RS). Our contributions can be summarized as follows:

- In Section 3.1, we propose a DRE scheme to estimate an image’s density ratio conditional on a class or regression label. We first introduce a new feature extraction method for images with regression labels, when the feature extraction mechanism in DRE-F-SP [9] is not applicable. Then, we propose a novel conditional Softplus loss (cSP) loss, which enables us to estimate density ratios conditional on different labels by fitting only one density ratio model. This density ratio model takes as input both the high-level features and the class/regression label of an image and outputs the density ratio conditional on the given label.
- To analyze the proposed cSP loss theoretically, we derive in Section 3.2 the error bound of a density ratio model trained with the proposed cSP loss.
- In Section 3.3, we propose a novel rejection sampling scheme to subsample cGANs. A filtering scheme is also proposed in Section 3.4 for CcGANs to increase fake images’ label consistency without losing diversity. By only tuning one hyper-parameter of this filtering scheme, we can easily control the trade-off between label consistency and diversity according to users’ needs.
- In Section 4, extensive experiments on five benchmark

Figure 1: The efficiency and effectiveness comparisons between the proposed cDR-RS and four baseline methods in sampling 100,000 and 60,000 fake images from BigGAN [2] (a) and CcGAN [8, 7] (b), respectively. The dashed red lines denote Intra-FID [25] and Label Score [8, 7] of sampling from cGANs without subsampling or refining in (a) and (b), respectively. Label Score is a metric to evaluate the discrepancy between the actual and conditioning labels of fake images. cDR-RS achieves state-of-the-art performances in sampling from cGANs while requiring only reasonable computational resources.
datasets and different GAN architectures convincingly demonstrate the state-of-the-art performances of the proposed subsampling scheme over baseline methods.

2 Related Works

2.1 Conditional generative adversarial networks

Conditional GANs (cGANs), first proposed in [22], extend GANs [11] to the conditional image synthesis setting, where a condition $y$ is fed into both the generator and discriminator networks. Mathematically, cGANs aim to estimate the density function $p_r(x|y)$ of the actual conditional image distribution. The estimated density function $p_g(x|y)$ is the density of the fake conditional image distribution induced by the generator network. The condition $y$ is often a categorical variable such as a class label and cGANs with class labels as conditions are also known as class-conditional GANs. Class-conditional GANs have been widely studied in [28, 25, 2, 32]. State-of-the-art class conditional GANs (e.g., BigGAN [2]) can generate photo-realistic images for a given class. However, GANs conditional on regression labels have been rarely studied due to two problems. First, very few (even zero) real images exist for some regression labels. Second, since regression labels are continuous and infinitely many, they cannot be embedded by one-hot encoding like class labels. To solve these two problems, [9] propose the CCcGAN framework, which introduces novel empirical cGAN losses and label input mechanisms. The novel empirical cGAN losses, consisting of the hard vicinal discriminator loss (HVDL), the soft vicinal discriminator loss (SVDL), and a new generator loss, are developed to solve the first problem. The second problem is solved by a naive label input (NLI) mechanism and an improved label input (ILI) mechanism. The effectiveness of CCcGAN has been demonstrated on diverse datasets.

2.2 Subsampling GANs by DRE-F-SP+RS

Among existing sampling methods for unconditional GANs, DRE-F-SP+RS proposed by [9] can achieve the state-of-the-art sampling performance. This subsampling framework consists of two components: a density ratio estimation method termed DRE-F-SP and a rejection sampling scheme. DRE-F-SP aims to estimate the density ratio function $r^*(x) := p_r(x)/p_g(x)$ based on $N^r$ real images $x_1^r, x_2^r, \ldots, x_{N^r}^r \sim p_r(x)$ and $N^g$ fake images $x_1^g, x_2^g, \ldots, x_{N^g}^g \sim p_g(x)$, where $p_r(x)$ and $p_g(x)$ are the density functions of the actual and fake conditional image distributions, respectively. Based on the estimated density ratios, to push $p_g$ towards $p_r$, rejection sampling (RS) is used to sample from the trained GAN model. Empirical studies in [9] show that DRE-F-SP+RS substantially outperforms existing sampling methods (e.g., DRS [1]) in subsampling different types of GANs.

As the key component of DRE-F-SP+RS, DRE-F-SP first trains a specially designed ResNet-34 [13] on a set of real images with class labels under the cross-entropy loss. The network architecture of this ResNet-34 is adjusted to ensure that the dimension of one hidden map $h$ equals that of the input image $x$. Thus, this ResNet-34 defines a mapping of an image $x$ to a high-level feature $h$, i.e., $h = \phi(x)$. Then, DRE-F-SP estimates the density ratio of an image in the feature space defined by $\phi$ instead in the pixel space. Specifically, DRE-F-SP models the actual density ratio function in the feature space by a 5-layer multilayer perceptron (MLP-5). [9] also proposes a novel loss function called Softplus (SP) loss to train this MLP-5. Finally, compositing $\phi(x)$ and MLP-5 leads to an estimate of $r^*(x)$.

3 Proposed Method

As discussed in Section 1 due to the ineffectiveness or inefficiency, the unconditional subsampling or refining methods (e.g., DRS [1], Collab [21], DDLS [4] and DRE-F-SP+RS [9]) may be impractical for sampling from cGANs. Moreover, the only existing conditional subsampling method [26] is designed for ACGAN [28] and cannot be applied to other cGANs. Motivated by these limitations, in this section, we propose an effective and efficient dual-functional subsampling method, which is suitable for both class-conditional GANs and CCcGANs regardless of the GAN architectures and the number of distinct class or regression labels.

3.1 Conditional density ratio estimation in feature space with conditional Softplus loss

In this section, we introduce cDRE-F-cSP, a novel conditional density ratio estimation (cDRE) method. Assume we have $N^r$ real image-label pairs $(x_1^r, y_1^r), \ldots, (x_{N^r}^r, y_{N^r}^r)$ and $N^g$ fake image-label pairs $(x_1^g, y_1^g), \ldots, (x_{N^g}^g, y_{N^g}^g)$, where $x_i^r$ and $x_i^g$ can be seen as samples drawn from $p_r(x|y_i^r)$ and $p_g(x|y_i^g)$, respectively. Based on these samples, we aim to estimate $r^*(x|y) := p_r(x|y)/p_g(x|y)$.

Like DRE-F-SP [9], we conduct cDRE in a feature space learned by a pre-trained neural network $\phi$. DRE-F-SP [9] trains a specially designed ResNet-34 on some real images with class labels to extract high-level features for DRE. This mechanism also applies to class-conditional GANs; however, it is inapplicable to CCcGANs since regression datasets may not have class labels. Therefore, when subsampling CCcGANs, we develop a specially designed sparse autoencoder (AE) to extract features whose architecture is visualized in Fig. 2. The encoder with ReLU [10] as the final layer is treated as $\phi$ to extract sparse high-level features from images. The bottleneck dimension of the sparse autoencoder equals the dimension of the flattened input image. The decoding process is trained to reconstruct the input image and predict the regression label of the input image. The training loss of this sparse AE is the summation of three loss components: (1) the mean square error (MSE) between the input image and the reconstructed image; (2) the MSE between the actual regression label and the predicted regression label; (3) the product of a positive constant...
we propose the training data, a natural constraint applied to Eq. (2) is loss [9] to the conditional setting. The empirical approximation p in the feature space by a 5-layer MLP (MLP-5) denoted by features. Based on Eq. (1) and the pre-trained neural network tions of the real and fake condition distributions of high-level density ratio function in the feature space:

\[
\hat{\lambda}(\psi) = \frac{1}{N^g} \sum_{i=1}^{N^g} \psi(h^g_i | y^g_i) = 1.
\]

An empirical approximation to Eq. (5) is

\[
\frac{1}{N^g} \sum_{i=1}^{N^g} \psi(h^g_i | y^g_i) = 1.
\]

Therefore, in practice, we minimize the penalized version of Eq. (3) as follows:

\[
\min_{\psi} \left\{ \mathcal{L}_c(\psi) + \lambda \hat{Q}_c(\psi) \right\},
\]

where

\[
\hat{Q}_c(\psi) = \left( \frac{1}{N^g} \sum_{i=1}^{N^g} \psi(h^g_i | y^g_i) - 1 \right)^2.
\]

An algorithm shown in Alg. 1 is used to implement cDRE-F-cSP in practice. In our experiments, \( \lambda \) is set as \( 10^{-2} \) or \( 10^{-3} \) empirically.

### 3.2 Error bound

In this section, we derive the error bound of a density ratio model \( \psi(h | y) \) trained with the empirical cSP loss \( \mathcal{L}_c(\psi) \). For simplicity, we ignore the penalty term in this analysis.

Firstly, we introduce some notations. Let \( \Psi = \{ \psi : h \rightarrow \psi(h | y) \} \) denote the hypothesis space of the density ratio function \( \psi(h | y) \). We also define \( \bar{\psi} \) and \( \hat{\psi} \) as follows: \( \bar{\psi} = \arg \min_{\psi \in \Psi} \mathcal{L}_c(\psi) \) and \( \hat{\psi} = \arg \min_{\psi \in \Psi} \hat{\mathcal{L}}(\psi) \). Please note that the hypothesis space \( \Psi \) may not cover the actual density ratio function \( \psi^* \). Therefore, \( \mathcal{L}_c(\bar{\psi}) - \mathcal{L}_c(\psi^*) \geq 0 \). Denote by \( \alpha \) all learnable parameters of \( \psi \) and assume \( \alpha \) is in a parameter space \( \mathcal{A} \). Denote \( \sigma(\psi(h | y)) = \psi(h | y) - \eta(\psi(h | y)) \) by \( g(h | y; \alpha) \). Let \( \mathcal{R}_q(\psi(h | y), N^r(\Psi)) \) denote the empirical Rademacher complexity \[27\] of \( \Psi \), which is defined based on independent feature-label pairs \( \{(h^1, y^1), \ldots, (h^r, y^r)\} \) from \( q_r(h, y) \).

Then, we derive the error bound of the conditional density ratio estimate \( \hat{\psi} \) under Eq. (2) as follows:
Theorem 1. If (i) \( N^g \) is large enough, (ii) \( A \) is compact, (iii) \( \forall \theta \leq \psi \) is continuous at \( \theta \), (iv) \( \forall \theta \leq \psi \), \( \exists \) a function \( \psi''(\theta) \) that does not depend on \( \theta \), s.t. \( |\theta(\theta; \alpha)| \leq \psi''(\theta) \), and (v) \( E_{y}[\psi(\theta; \alpha)] < \infty \), then \( \forall \delta \in (0, 1) \) and \( \psi(\theta) \in (0, \delta) \) with probability at least \( 1 - \delta \).

\[
\mathcal{L}_d(\psi) - \mathcal{L}_d(\psi^*) \leq \frac{1}{N^g} + \mathcal{R}_{q_r}(h, \alpha, N^r)(\Psi) + 2 \sqrt{\frac{4}{N^r} \log \left( \frac{2}{\delta} \right) + \mathcal{L}_d(\psi) - \mathcal{L}_d(\psi^*)}.
\]

Proof. The proof is in Supp. S.2.

Remark 1. \( \mathcal{R}_{q_r}(h, \alpha, N^r)(\Psi) \) on the right of Eq. (6) implies that we should not use an overly complicated density ratio model. It supports our proposed cDRE-F-cSP because we just need a small neural network (e.g., a shallow MLP) to model the density ratio function in the feature space.

3.3 cDR-RS: Conditional density ratio-guided rejection sampling for conditional GANs

Based on the cDRE method proposed in Section 3.1, we develop a rejection sampling scheme, termed cDRE-RS, for subsampling conditional GANs. The workflow can be summarized in Fig. 3 and Alg. 1. This rejection sampling scheme is conducted for each distinct label \( y \) of interest. For example, on ImageNet-100 [7], we train only one density ratio model \( \psi(\theta; y) \), based on which we repeat the rejection sampling scheme 100 times for 100 classes respectively.

Figure 3: The workflow of cDRE-RS has two sequential modules: cDRE-F-cSP and rejection sampling. \( M \) in the acceptance probability \( p \) equals to \( \max \{ q_r(h) / q_g(h) \} \), which can be estimated by evaluating \( \psi(h) \) on some burn-in samples before subsampling.

3.4 A filtering scheme to increase label consistency in subsampling CcGANs

To solve the problem of insufficient data, CcGANs [8, 7] use images with labels in a vicinity of a conditioning regression label \( y \) to estimate \( p_g(x|y) \). Consequently, the actual labels of some images sampled from \( p_g(x|y) \) may be far from \( y \) (aka label inconsistency). Unfortunately, the current subsampling scheme in Fig. 3 may still accept these fake images if they have good visual quality or can contribute to the diversity increase (see Table 2). An intuitive solution to this issue is to filter out these fake images with actual labels different from \( y \), but this filtering may substantially decrease the diversity of fake images.

To increase label consistency without losing diversity, we propose a filtering scheme for cDRE-RS, which is inspired by a finding that we may sample images with actual labels equal to \( y \) from both \( p_g(x|y) \) and \( p_g(x|y') \) in the CcGAN sampling, where \( y' \neq y \). Let’s take the subsampling at \( y \) to show the procedure of this filtering scheme. We first replace \( p_g(x|y) \) with \( p_g^{\phi}(x) \) as the density of the proposal distribution, where \( p_g^{\phi}(x) \) stands for the density function of the distribution of fake images with predicted labels in \( Y^\phi_y \) := \{ |y - \zeta, y + \zeta | \} and \( \zeta \) is a hyper-parameter. The predicted labels, assumed to be close to the fake images’ actual labels, are the prediction from the composition of the encoder and predictor in Fig. 2. Afterwards, the density ratio model \( r(x|y) \) is used to model \( p_r(x|y) / p_g^{\phi}(x) \) and it is trained on fake images with predicted labels in \( Y^\phi_y \). In the sampling phase, before conducting the rejection sampling process (Fig. 3 and Alg. 1), we filter out fake images with predicted labels outside of \( Y^\phi_y \). Please note that \( \zeta \) controls the trade-off between label consistency and diversity. As shown in Fig. 3, a smaller \( \zeta \) often leads to higher label consistency but lower diversity, while a larger \( \zeta \) often leads to lower label consistency but higher diversity. We can adjust \( \zeta \) according to our needs, but a good \( \zeta \) should improve label consistency without decreasing diversity. In our experiment, \( \zeta \) is set based on a vicinity parameter \( m_e \) of CcGANs [8, 7]. Please refer to Supp. S.3 for a rule of thumb to select \( \zeta \).

4 Experiments

In this section, we will empirically evaluate the efficiency and the effectiveness of the proposed cDRE-RS scheme in subsam-
pling class-conditional GANs and CcGANs. We compare cDR-RS with four state-of-the-art sampling methods: GOLD [26], Collab [21], DRS [1], and DRE-F-SP+RS [9]. For sampling methods proposed for unconditional GANs, we implement them for each distinct class or regression label. For a fair comparison, extensive experiments are conducted on multiple benchmark datasets and cGAN architectures, and diverse evaluation metrics are utilized to demonstrate the superiority of our cDR-RS method.

4.1 Sampling from class-conditional GANs

Experimental setup: For class-conditional GANs, we conduct experiments on three image datasets: CIFAR-10 [18], CIFAR-100 [18], and ImageNet-100 [3], respectively. ImageNet-100 [3], as a subset of ImageNet [5], has 128,503 RGB images at $128 \times 128$ resolution from 100 classes. In our experiment, we randomly split ImageNet-100 into a training set and a test set, where 10,000 images are for testing (on average 100 images per class) and the rest are for training.

On CIFAR-10, we train three class-conditional GANs: ACGAN [28], SNGAN [24], and BigGAN [2]. On CIFAR-100 and ImageNet-100, we only implement BigGAN because the training of ACGAN and SNGAN is unstable. When sampling from ACGAN, we test four candidate methods, i.e., Baseline, GOLD [26], DRE-F-SP+RS [9], and cDR-RS. When sampling from other GANs, we test six candidates, i.e., Baseline, Collab [21], DRS [1], DDLS [4], DRE-F-SP+RS [9], and cDR-RS. Please note that Collab, DRS, and DDLS are not applicable to ACGAN, and Baseline refers to sampling without subsampling or refining. Please refer to Supp. S.5, S.6, and S.7 for detailed setups.

Evaluation metrics: We evaluate the quality of fake images by Fréchet Inception Distance (FID) [14], Intra-FID [25], and Inception Score (IS) [29]. Intra-FID is an overall image quality metric, which computes FID separately for each class and reports the average FID score. A lower Intra-FID or FID score indicates better image quality or vice versa. Oppositely, a larger Inception Score implies better image quality.

Experimental results: We quantitatively compare the image quality of fake images sampled from class-conditional GANs with different candidate methods. For the CIFAR-10 experiment, we draw 10,000 fake images for each class by each sampling method. For the CIFAR-100 and ImageNet-100 experiments, we sample 1000 fake images for each class. Quantitative results are summarized in Table 1. We can see, in all settings, the proposed cDR-RS and DRE-F-SP+RS perform comparably, and they substantially outperform other candidate methods with a large margin in terms of all three metrics. We also show in Fig. 4 some example fake images for the “indigo bunting” class sampled by Baseline, DRE-F-SP+RS, and cDR-RS with real images as reference. Both DRE-F-SP+RS and cDR-RS can effectively remove some fake images with recognizable birds (marked by red rectangles). Although cDR-RS and DRE-F-SP+RS have similar effectiveness, their efficiency differs significantly. Our efficiency analysis on ImageNet-100, visualized in Fig. 1(a), shows that cDR-RS only requires 1.19% of the storage usage and 77% of the implementation time consumed by DRE-F-SP+RS. Furthermore, since the cDRE-F-cSP is sample-based and does not rely on any properties of cGANs, cDR-RS is applicable to various cGAN architectures, making it more flexible than GOLD, Collab, DRS and DDLS.

Figure 4: Some example images for the “indigo bunting” class at $128 \times 128$ resolution in the ImageNet-100 experiment. The first row includes six real images. From the second row to the bottom, we show some example fake images generated from Baseline, DRE-F-SP+RS [9], and the proposed cDR-RS, respectively. We often observe some fake images sampled by Baseline without recognizable birds (e.g., two images with red rectangles), while both DRE-F-SP+RS and cDR-RS can effectively remove such images.

4.2 Sampling from CcGANs

Experimental setup: Besides class-conditional GANs, we evaluate the performances of candidate methods in sampling from CcGANs. We experiment on the UTKFace [33] and RC-49 datasets [8, 7], which are two benchmark datasets for CcGANs. UTKFace is an RGB human face image dataset with ages as regression labels. We use the pre-processed UTKFace dataset [8], consisting of 14,760 RGB images at $64 \times 64$ resolution with ages in [1, 60]. The number of images in UTKFace ranges from 50 to 1051 for different ages, and all images are used for training. RC-49 consists of 44,051 RGB images at $64 \times 64$ resolution for 49 types of chairs. Each type of chairs contains 899 images labeled by 899 distinct yaw rotation angles from $0.1^{\circ}$ to $89.9^{\circ}$ with a step size of $0.1^{\circ}$. We first choose yaw angles with odd numbers as the last digit for training. Then, for each chosen angle, we randomly select 25 images to construct a training set. The final training set of RC-49 includes 11250 images and 450 distinct angles. All 44,051 RC-49 images are used in the experiment evaluation.

We follow the official implementation of CcGAN (SVDL+ILI) in [8, 7]. GOLD is not taken as a baseline method because of its incompatibility in subsampling CcGANs. DRE-F-SP+RS is tested on UTKFace but excluded from the RC-49 experiment because it cannot subsample images that are gen-
Table 1: Effectiveness analysis of different methods in sampling various class-conditional GANs in terms of Intra-FID, FID, and IS. For experiments on CIFAR-10 and CIFAR-100, we report the quality of 100,000 fake images (10,000/1000 per class) sampled by each method. For experiment on ImageNet-100, we evaluate the quality of 100,000 fake images (1000 per class) generated from each method. The numbers in the parentheses stand for the standard deviations of FIDs computed within each distinct classes. The quality of real images from each datasets is also provided as references, where the Intra-FID and FID are computed between training and test samples while IS is computed in terms of test samples only. Please note that the test samples of ImageNet-100 is insufficient for computing a reliable Intra-FID. In all settings, cDR-RS is either better than or comparable to DRE-F-SP+RS [9], and these two methods substantially outperform the others.

| Method          | CIFAR-10     | CIFAR-100    | ImageNet-100 |
|-----------------|--------------|--------------|--------------|
|                 | Intra-FID ↓  | FID ↓        | IS ↑         |
| Real Data       | 0.681 (0.284) | 0.134        | 9.981        |
| - ACGAN -       |              |              |              |
| Baseline        | 3.688 (0.674) | 3.676        | 6.563        |
| GOLD [26]       | 3.660 (0.682) | 3.682        | 6.603        |
| DRE-F-SP+RS [9] | 2.733 (0.519) | 2.518        | 8.122        |
| cDR-RS (ours)   | 2.656 (0.461) | 2.209        | 8.294        |
| - SNGAN -       |              |              |              |
| Baseline        | 1.887 (0.265) | 1.426        | 8.947        |
| Collab [21]     | 1.882 (0.274) | 1.433        | 8.941        |
| DRS [1]         | 1.875 (0.281) | 1.417        | 8.958        |
| DDLS [4]        | 1.769 (0.268) | 1.368        | 9.070        |
| DRE-F-SP+RS [9] | 1.164 (0.214) | 0.896        | 9.554        |
| cDR-RS (ours)   | 1.042 (0.233) | 0.715        | 9.577        |
| - BigGAN -      |              |              |              |
| Baseline        | 0.998 (0.403) | 0.478        | 9.352        |
| Collab [21]     | 0.993 (0.403) | 0.472        | 9.355        |
| DRS [1]         | 1.034 (0.389) | 0.497        | 9.341        |
| DDLS [4]        | 0.909 (0.380) | 0.469        | 9.436        |
| DRE-F-SP+RS [9] | 0.700 (0.251) | 0.385        | 9.605        |
| cDR-RS (ours)   | 0.594 (0.227) | 0.282        | 9.648        |
| - BigGAN -      |              |              |              |
| Baseline        | 19.752 (2.926) | 4.319        | 60.846       |
| Collab [21]     | 19.799 (2.882) | 4.387        | 60.719       |
| DRS [1]         | 19.742 (2.857) | 4.354        | 60.468       |
| DDLS [4]        | 19.315 (2.678) | 4.011        | 64.095       |
| DRE-F-SP+RS [9] | 17.554 (1.559) | 1.978        | 82.145       |
| cDR-RS (ours)   | 17.726 (1.618) | 1.993        | 80.393       |
| - SNGAN -       |              |              |              |
| Baseline        | 16.976 (3.271) | 2.729        | 98.855       |
| Collab [21]     | 16.855 (3.192) | 2.674        | 98.123       |
| DRS [1]         | 16.742 (3.147) | 2.624        | 97.789       |
| DDLS [4]        | 16.315 (2.978) | 2.574        | 97.542       |
| DRE-F-SP+RS [9] | 15.554 (1.968) | 2.518        | 96.215       |
| cDR-RS (ours)   | 15.922 (2.074) | 2.462        | 95.871       |

Quantitative results: We quantitatively compare the performances of different candidate methods in sampling from CcGANs. For the UTKFace experiment, we sample 1000 fake images for each age by each method. For the RC-49 experiment, we sample 200 fake images for each of 899 distinct angles by each method (449 angles are unseen in the training set). The comparison results are shown in Table 2. We can see Collab does not take effect in sampling from CcGANs. DRS can improve NIQE in both experiments, but it fails to improve Diversity and Label Score simultaneously. DDLS improves visual quality in the UTKFace experiment, but it sacrifices Diversity for a slightly lower Label Score. The sampling time (11.24 hours) spent by DDLS is also much longer than others, and such a long sampling time makes DDLS infeasible in the RC-49 experiment. DRE-F-SP+RS performs worst among all methods, and it does not apply to RC-49. The proposed cDR-RS with the filtering scheme, denoted by cDR-RS (Filter), outperforms all other candidate methods on both datasets. It can effectively improve Label Score. Meanwhile, it also improves NIQE and Diversity scores. We also show in Fig. 5 some example fake images for age 24 sampled by Baseline, DRE-F-SP+RS, and cDR-RS (Filter) with real images as reference. Fig. 5 shows the effectiveness of cDR-RS (Filter) and the failure of DRE-F-SP+RS. Furthermore, an efficiency analysis is also conducted on UTKFace and visualized in Fig. 1. This efficiency analysis shows that cDR-RS requires only reasonable computational resources but leads to substantial performance improvements. Moreover, in Table 2 we also show the performance of cDR-RS without the filtering scheme, i.e., cDR-RS (No Filter). We can see, without filtering, cDR-RS cannot effectively solve the label inconsistency problem and even makes it worse. Thus, this observation validates the effectiveness of the proposed filtering scheme. In addition, we conduct an ablation study on UTKFace to analyze the effects of ζ of the filtering scheme, and this analysis is visualized in Fig. 6.
Table 2: Effectiveness analysis of different sampling methods with CcGANs (SVDL+ILI) and two regression datasets in terms of Intra-FID, NIQE, Diversity, and Label Score. Values in the parentheses represent the standard deviation of evaluation scores reported at each distinct regression label. The proposed cDR-RS substantially outperforms other candidate methods on both datasets.

| Method     | Intra-FID | NIQE | Diversity | Label Score |
|------------|-----------|------|-----------|-------------|
| Baseline   | 0.457 (0.171) | 1.722 (0.172) | 1.303 (0.170) | 7.403 (5.956) |
| Collab [21]| 0.457 (0.168) | 1.722 (0.170) | 1.298 (0.180) | 7.420 (6.030) |
| DRS [1]    | 0.455 (0.169) | 1.707 (0.176) | 1.282 (0.185) | 7.487 (6.105) |
| DDLS [4]   | 0.445 (0.166) | 1.712 (0.172) | 1.288 (0.180) | 7.202 (5.888) |
| DRE-F-SP+RS| 0.588 (0.657) | 1.724 (0.173) | 1.293 (0.184) | 7.462 (6.046) |
| Ours: cDR-RS (No filter) | 0.443 (0.183) | 1.703 (0.168) | 1.348 (0.139) | 7.528 (6.125) |
| Ours: cDR-RS (Filter)  | 0.430 (0.176) | 1.708 (0.169) | 1.307 (0.208) | 6.317 (5.026) |

- UTKFace -

- RC-49 -

Figure 5: Some example images for age 24 at 64×64 resolution in the UTKFace experiment. The first row includes ten real images. From the second row to the bottom, we show some example fake images generated from Baseline, DRE-F-SP+RS [9], and the proposed cDR-RS, respectively. By comparing Row 2 and 4, we can see cDR-RS can effectively improve the visual quality. By contrast, DRE-F-SP+RS worsens the visual quality.

Figure 6: The effects of ζ in the filtering scheme of cDR-RS on the relation between Diversity and Label Score of CcGAN-generated samples in the UTKFace experiment. The dotted blue and red lines stand for Diversity and Label Score of Baseline in Table 2, respectively. The vertical grey line specifies the ζ used in Table 2. We can see both Label Score and Diversity increase as ζ increases. If we prefer higher label consistency, we can decrease ζ. Oppositely, if we prefer higher image diversity, we can increase ζ. No matter what the preference is, when choosing ζ, we should ensure that the corresponding Label Score is below the red line while Diversity is above the blue line. In that case, both label consistency and image diversity can be improved. A rule of thumb for the parameter selection is provided in Supp. S.3.

5 Conclusion

In this work, we have presented a novel conditional subsampling scheme to improve the image quality of fake images from cGANs. First, we propose novel conditional extensions of density ratio estimation (cDRE) in the feature space and the Softplus loss function (cSP). Then, we learn the conditional density ratio model through an MLP network. Also, we derive the error bound of a conditional density ratio model trained with the proposed cSP loss. A novel filtering scheme is also proposed in subsampling CcGANs to improve the label consistency. Finally, we validate the effectiveness of the proposed subsampling framework with extensive experiments in sampling multiple conditional GAN models on four benchmark datasets with diverse evaluation metrics.
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Supplementary Material

S.1 GitHub repository

Please find the codes for this paper at Github:

https://github.com/UBCDingXin/cDR-RS

S.2 The Proof of Theorem 1

Theorem 1 provides an error bound of a conditional density ratio model trained with the cSP loss. \( \tilde{R}_{q_i(h,y),N^r}(\Psi) \) on the right of Eq. (S.10) is a constant which implies that, a more complicated density ratio model has a more significant error bound, i.e., a worse generalization performance. Thus, we should not use an overly complicated density ratio model. It supports our proposed cDRE-F-cSP because we just need a small neural network (e.g., a shallow MLP) to model the density ratio function in the feature space.

Our proof of Theorem 1 follows the proof of Theorem 3 in [9].

Proof. Following [9], we decompose \( \mathcal{L}_c(\hat{\psi}) - \mathcal{L}_c(\psi^*) \) as follows:

\[
\mathcal{L}_c(\hat{\psi}) - \mathcal{L}_c(\psi^*) = \mathcal{L}_c(\hat{\psi}) - \mathcal{L}_c(\hat{\psi}) + \mathcal{L}_c(\hat{\psi}) - \mathcal{L}_c(\hat{\psi}) + \mathcal{L}_c(\hat{\psi}) - \mathcal{L}_c(\hat{\psi})
\]

(Since \( \mathcal{L}_c(\hat{\psi}) - \mathcal{L}_c(\hat{\psi}) \leq 0 \))

\[
\leq \mathcal{L}_c(\hat{\psi}) - \mathcal{L}_c(\hat{\psi}) + \mathcal{L}_c(\hat{\psi}) - \mathcal{L}_c(\hat{\psi}) + \mathcal{L}_c(\hat{\psi}) - \mathcal{L}_c(\psi^*)
\]

\[
\leq 2 \sup_{\psi \in \Psi} \left| \mathcal{L}_c(\hat{\psi}) - \mathcal{L}_c(\hat{\psi}) \right| + \left| \mathcal{L}_c(\hat{\psi}) - \mathcal{L}_c(\psi^*) \right|. \tag{S.10}
\]

The second term in Eq. (S.10) is a constant which implies an inevitable error. The first term can be bounded as follows:

\[
\sup_{\psi \in \Psi} \left| \mathcal{L}_c(\hat{\psi}) - \mathcal{L}_c(\hat{\psi}) \right| 
\]

\[
\leq \sup_{\psi \in \Psi} \left| \mathbb{E}_{(h,y) \sim q_y(h,y)} \left[ \sigma(\psi(h|y))\psi(h|y) - \eta(\psi(h|y)) \right] \right|
\]

\[
- \frac{1}{N^r} \sum_{i=1}^{N^r} \left[ \sigma(\psi(h_i^r|y_i^r))\psi(h_i^r|y_i^r) - \eta(\psi(h_i^r|y_i^r)) \right] \right|
\]

\[
+ \left| \mathbb{E}_{(h,y) \sim q_y(h,y)} \left[ \sigma(\psi(h|y)) \right] \right|
\]

\[
- \frac{1}{N^r} \sum_{i=1}^{N^r} \sigma(\psi(h_i^r|y_i^r)) \right| \tag{S.11}
\]

Based on this limit, we can derive an upper bound of the first term of Eq. (S.11) as follows. Since we can generate infinite fake images from a trained cGAN, \( N^9 \) is large enough. Let \( \epsilon = 1/2N^9 \), \( \forall \delta \in (0, 1) \) with probability at least \( 1 - 2^\delta \),

\[
\sup_{\psi \in \Psi} \left| \mathbb{E}_{(h,y) \sim q_y(h,y)} \left[ \sigma(\psi(h|y))\psi(h|y) - \eta(\psi(h|y)) \right] \right|
\]

\[
- \frac{1}{N^9} \sum_{i=1}^{N^9} \left[ \sigma(\psi(h_i^9|y_i^9))\psi(h_i^9|y_i^9) - \eta(\psi(h_i^9|y_i^9)) \right] \right| \leq \frac{1}{2N^9}. \tag{S.12}
\]

The second term of Eq. (S.11) can be bounded based on Lemma 1 and Theorem 2 (the Rademacher bound [19]) in [9] as follows: \( \forall \delta \in (0, 1) \) with probability at least \( 1 - 2^\delta \),

\[
\frac{1}{2N^9} \sum_{i=1}^{N^9} \sigma(\psi(h_i^9|y_i^9)) \right| \leq 2R_{q_i(h,y),N^r}(\sigma \circ \Psi) + \sqrt{\frac{4}{N^9} \log \left( \frac{2}{\delta} \right)} \tag{S.13}
\]

Let \( \delta = \max{\{\delta_1, \delta_2\}} \) and \( \delta' = \delta \), based on Eq. (S.12) and (S.13), we can derive Eq. (9).

S.3 A rule of thumb to select \( \zeta \) in the filtering scheme

Let’s first review the rule of thumb for selecting the vicinity hyper-parameter in CcGANs [8, 7]. In [8, 7], real images with labels in a vicinity of \( y \) are used to estimate \( p_r(x|y) \). \( \kappa \) and \( \nu \) are the vicinity hyper-parameters to control the width of the hard and soft vicinities of CcGANs, respectively. We quote or rephrase the suggestions of [8, 7] here: “To select \( \kappa \) and \( \nu \), regression labels are first normalized to \([0, 1]\). Then, let \( \kappa_{\text{base}} = \max{\{y_{[1]} - y_{[2]}, \ldots, y_{[N_{\text{ny}}-1]} - y_{[N_{\text{ny}}-2]}\}}, \) where \( y_{[i]} \) is the \( i \)-th smallest normalized distinct real label and \( N_{\text{ny}} \) is the number of normalized distinct labels in the training set. [8, 7] set \( \kappa \) as a multiple of \( \kappa_{\text{base}} \) (i.e., \( \kappa = m_{\kappa}\kappa_{\text{base}} \)) where the multiplier \( m_{\kappa} \) stands for 50% of the minimum number of neighboring labels used for estimating \( p_r(x|y) \) given a label \( y \). For example, \( m_{\kappa} = 1 \) implies using 2 neighboring labels (one on the left while the other one on the right). In [8, 7], \( m_{\kappa} \) is often set as 1 or 2. For the soft vicinity, \( \nu = 1/\kappa^2 \) often works well.”

Based on above rule of thumb, we propose a scheme to select the hyper-parameter \( \zeta \) of the filtering scheme in cDR-RS. To be specific, we let \( \zeta = 3 \times m_{\kappa}\kappa_{\text{base}} \). In other words, we let the vicinity \( \mathcal{V}_{\zeta} = [y - \zeta, y + \zeta] \) to be three times wider than the hard vicinity in CcGANs. Our experiments in Section 5.2 show that this rule of thumb can let cDR-RS effectively increase label consistency without losing diversity.

As we discussed in Section 3.3, the filtering scheme is inspired by a finding that we may sample images with actual labels equal to \( y \) from both \( p_g(x|y) \) and \( p_g(x|y') \), where \( y' \neq y \).
Unfortunately, in practice, it is difficult to know the actual label of a given fake image. Therefore, we treat predicted labels of fake images as their actual labels. However, the predicted labels may also deviate from the actual labels, but such deviation is assumed not significant because the supervised learning (i.e., training a regression CNN to predict label) is usually considered less difficult than the generative modeling (i.e., the CcGAN training). Although the prediction error may be small, it still exists and won’t be zero. Therefore, when designing the proposal distribution in the rejection sampling, instead of using fake images with predicted labels exactly equal to \( y \) (i.e., \( \zeta = 0 \)), we consider fake images with predicted labels in a vicinity of \( y \), i.e., \( \mathcal{Y}_y^\zeta \). We expect that the actual labels of fake images in \( \mathcal{Y}_y^\zeta \) should be equal or close to \( y \). Furthermore, if we let \( \zeta = 0 \), the training and sampling time may be very long. On UTKFace, we conduct an ablation study to show the effect of \( \zeta \) on the training and sampling. Relevant results are summarized in Fig. S.3.7. We can see a smaller \( \zeta \) substantially increases the training and sampling time, which may make cDR-RS less efficient. Therefore, in practice, we usually use a relatively large \( \zeta \), e.g., \( 3 \times m_{\kappa \text{base}} \).

![Figure S.3.7: The effect of \( \zeta \) in the filtering scheme on the implementation time of cDR-RS. Larger \( \zeta \) implies shorter implementation. The implementation time here consists of the training time of MLP-5 and the sampling time. The training time of the sparse AE for feature extraction is not influenced by \( \zeta \) and excluded from this analysis.](image)

**S.4 Resources for Implementing cGANs and Sampling Methods**

To implement ACGAN, we refer to [https://github.com/christiancosgrove/pytorch-spectral-normalization-gan](https://github.com/christiancosgrove/pytorch-spectral-normalization-gan) and [https://github.com/pfnet-research/sngan_projection](https://github.com/pfnet-research/sngan_projection).

To implement CcGANs, we refer to [https://github.com/UBCDingXin/improved_CcGAN](https://github.com/UBCDingXin/improved_CcGAN).

To implement GOLD, we refer to [https://github.com/sangwoomo/GOLD](https://github.com/sangwoomo/GOLD).

To implement Collab, we refer to [https://github.com/JHpark1677/CGAN-DDLS](https://github.com/JHpark1677/CGAN-DDLS).

To implement DDLS, we refer to [https://github.com/UBCDingXin/DDRE_GANS](https://github.com/UBCDingXin/DDRE_GANS) and [https://github.com/Daniil-Selikhanovych/ebm-wgan/blob/master/notebook/EBM_GAN.ipynb](https://github.com/Daniil-Selikhanovych/ebm-wgan/blob/master/notebook/EBM_GAN.ipynb).

To implement Collab, following [21], we conduct discriminator shaping for 5000 and 2500 iterations for SNGAN and BigGAN, respectively. We use the architecture described in Fig. 15 of [2] for BigGAN.

To implement Collab, following [21], we conduct discriminator shaping for 5000 and 2500 iterations for SNGAN and BigGAN, respectively. We use the architecture described in Fig. 15 of [2] for BigGAN.

**S.5 More Details of Experiments on CIFAR-10**

In the CIFAR-10 experiment, we first train ACGAN, SNGAN, and BigGAN with setups shown as follows. We train ACGAN for 100,000 iterations with batch size 512. We train SNGAN for 50,000 iterations with batch size 512. BigGAN is trained for 39,000 with batch size 512. We use the architecture described in Fig. 15 of [2] for BigGAN.

To implement Collab, following [21], we conduct discriminator shaping for 5000 and 2500 iterations for SNGAN and BigGAN, respectively. We do the refinement 30 times in a middle layer of SNGAN and 20 times in a middle layer of BigGAN. The step size of refinement is set as 0.1 for both SNGAN and BigGAN.

To implement DDLS, we run the Langevin dynamics procedure for SNGAN and BigGAN within each class with step size \( 10^{-4} \) up to 1000 iterations.

To implement DRE-F-SP+RS, we first train the specially designed ResNet-34 on the training set for 350 epochs with the SGD optimizer, initial learning rate 0.1 (decayed at epoch 150 and 250 with factor 0.1), weight decay \( 10^{-4} \), and batch size 256. Ten MLP-5 models for modeling the density ratio function within each class are trained on the training set with the Adam optimizer [17], initial learning rate \( 10^{-4} \) (decayed at epoch 100 and 250), batch size 256, 400 epochs, and \( \lambda = 10^{-2} \). The network architecture of MLP-5 is shown in Table S.5.3.

To implement cDR-RS, we use the specially designed ResNet-34 in the implementation of DRE-F-SP+RS to extract features from images. The MLP-5 to model the conditional density ratio function within each class are trained on the training set with the Adam optimizer [17], initial learning rate \( 10^{-4} \) (decayed at epoch 80 and 150), batch size 256, 200 epochs, and \( \lambda = 10^{-2} \).

For a more accurate evaluation, we do not use Inception-V3 [31] that was pre-trained on ImageNet [5] to compute Intra-FID, FID, and IS. Instead, following [9], we train Inception-V3 from scratch on CIFAR-10 to evaluate fake images.
Table S.5.4: The 5-layer MLP for cDRE in feature space for CIFAR-10 and CIFAR-100.

| Layer | Output Size | Activation |
|-------|-------------|------------|
| fc → 2048, GN (8 groups), ReLU, Dropout(p = 0.5) | | |
| fc → 1024, GN (8 groups), ReLU, Dropout(p = 0.5) | | |
| fc → 512, GN (8 groups), ReLU, Dropout(p = 0.5) | | |
| fc → 256, GN (8 groups), ReLU, Dropout(p = 0.5) | | |
| fc → 128, GN (8 groups), ReLU, Dropout(p = 0.5) | | |
| fc → 1, ReLU | | |

Table S.5.4: The 5-layer MLP for cDRE in feature space for CIFAR-10 and CIFAR-100. The embedded class label is appended to the extracted feature $h$.

```
Input: extracted feature $h \in \mathbb{R}^{3072}$ and embedded class label $y \in \mathbb{R}^{10}$, where $C = 10$ for CIFAR-10 and $C = 100$ for CIFAR-100

Concatenate $[h, y] \in \mathbb{R}^{3082}$

| Layer | Output Size | Activation |
|-------|-------------|------------|
| fc → 2048, GN (8 groups), ReLU, Dropout(p = 0.5) | | |
| fc → 1024, GN (8 groups), ReLU, Dropout(p = 0.5) | | |
| fc → 512, GN (8 groups), ReLU, Dropout(p = 0.5) | | |
| fc → 256, GN (8 groups), ReLU, Dropout(p = 0.5) | | |
| fc → 128, GN (8 groups), ReLU, Dropout(p = 0.5) | | |
| fc → 1, ReLU | | |
```

S.6 More Details of Experiments on CIFAR-100

In the CIFAR-100 experiment, we only test candidate sampling methods on BigGAN because both ACGAN and SNGAN are unstable on CIFAR-100 and somehow suffer from the model collapse problem [12]. BigGAN is trained for 38,000 iterations with batch size 512 on the training set of CIFAR-100. We use the architecture described in Figure 15 of [2] for BigGAN. We also adopt DiffAugment [34] (a data augmentation method for the GAN training with limited data) to improve the performance of BigGAN. The strongest data augmentation policy, “color, translation, cutout”, is used for DiffAugment.

To implement Collab, following [21], we conduct discriminator shaping for 2500 iterations for BigGAN. We do the refinement 16 times in a middle layer of the generator network of BigGAN. The step size of refinement is set as 0.5.

To implement DRS, we fine-tune the discriminator of BigGAN for 5 epochs with batch size 128.

To implement DDLS, we run the Langevin dynamics procedure for BigGAN within each class with step size $10^{-4}$ up to 1000 iterations.

To implement DRE-F-SP+RS, we first train the specially designed ResNet-34 on the training set for 350 epochs with the SGD optimizer, initial learning rate 0.1 (decayed at epoch 150 and 250 with factor 0.1), weight decay $10^{-4}$, and batch size 256. Ten MLP-5 models for modeling the density ratio function within each class are trained on the training set with the Adam optimizer [17], initial learning rate $10^{-4}$ (decayed at epoch 100 and 250), batch size 256, 400 epochs, and $\lambda = 10^{-2}$. The network architecture of MLP-5 is shown in Table S.5.3.

To implement cDR-RS, we use the specially designed ResNet-34 in the implementation of DRE-F-SP+RS to extract features from images. The MLP-5 to model the conditional density ratio function is similar to Table S.5.4. It is trained with the Adam optimizer [17], initial learning rate $10^{-4}$ (decayed at epoch 80 and 150), batch size 256, 200 epochs, and $\lambda = 10^{-2}$.

For a more accurate evaluation, we do not use Inception-V3 [31] that was pre-trained on ImageNet [5] to compute Intra-FID, FID, and IS. Instead, following [9], we train Inception-V3 from scratch on CIFAR-100 to evaluate fake images.

Please refer to our codes for more detailed setups such as network architectures and hyperparameter settings.

S.7 More Details of Experiments on ImageNet-100

S.7.1 Setups of training, sampling, and evaluation

In the ImageNet-100 dataset, we implement BigGAN with the BigGAN-deep architecture described in Figure 16 of [2]. BigGAN is trained for 96,000 iterations with batch size 1024. We also adopt DiffAugment [34] (a data augmentation method for the GAN training with limited data) to improve the performance of BigGAN. The strongest data augmentation policy, “color, translation, cutout”, is used for DiffAugment.

To implement Collab, following [21], we conduct discriminator shaping for 3000 iterations for BigGAN. We do the refinement 16 times in a middle layer of the generator network of BigGAN. The step size of refinement is set as 0.5.

To implement DRS, we fine-tune the discriminator of BigGAN for 5 epochs with batch size 128.

To implement DDLS, we run the Langevin dynamics procedure for BigGAN within each class with step size $10^{-4}$ up to 1000 iterations.

To implement DRE-F-SP+RS, we first train the specially designed ResNet-34 on the training set for 350 epochs with the SGD optimizer, initial learning rate 0.1 (decayed at epoch 150 and 250 with factor 0.1), weight decay $10^{-4}$, and batch size 256. Ten MLP-5 models for modeling the density ratio function within each class are trained on the training set with the Adam optimizer [17], initial learning rate $10^{-4}$ (decayed at epoch 100 and 250), batch size 256, 400 epochs, and $\lambda = 10^{-2}$. The network architecture of MLP-5 is similar to Table S.5.3. Please note that, when implementing DRE-F-SP+RS, we use more epochs than cDR-RS does (400 epochs vs 200 epochs) to ensure that all density ratio models are well-trained.

To implement cDR-RS, we use the specially designed ResNet-34 in the implementation of DRE-F-SP+RS to extract features from images. The MLP-5 to model the conditional density ratio function is similar to Table S.5.4. It is trained with the Adam optimizer [17], initial learning rate $10^{-4}$ (decayed at epoch 80 and 150), batch size 128, 200 epochs, and $\lambda = 10^{-2}$. The network architecture of MLP-5 is shown in Table S.5.3.
For a more accurate evaluation, we fine-tune on the training set of ImageNet-100 [3] an Inception-V3 network that was pre-trained on ImageNet [5]. We fine-tune the Inception-V3 with the SGD optimizer, 50 epochs, batch size 128, and initial learning rate $10^{-4}$. The fine-tuned Inception-V3 is then used to compute Intra-FID, FID, and IS.

S.7.2 More details of the efficiency analysis

In order to compare the efficiency of candidate sampling methods, we summarize their storage usage and training and sampling time in Table S.8.9 based on which we plot Fig. [1] Some pie charts are also plotted to show more detailed storage usage and training time for DRE-F-SP+RS and cDR-RS in Fig. [S.7.8]

For DRE-F-SP+RS, the 100 MLP-5 models take a lot of disk space (almost 40 GB), making it less efficient in subsampling class-conditional GANs with many classes.

S.8 More Details of Experiments on UTKFace

S.8.1 Setups of training, sampling, and evaluation

We follow [8, 7] to implement CcGANs (VDL-ILI) with the SNGAN architecture. The detailed setups can be found in [8, 7] or our codes.

To implement Collab, we conduct discriminator shaping for 3000 iterations for CcGAN. We do the refinement 16 times in a middle layer of the generator network of CcGAN. The step size of refinement is set as 0.5.

To implement DRS, we fine-tune the discriminator of CcGAN for 2000 iterations with batch size 128.

To implement DDLs, we run the Langevin dynamics procedure for CcGAN for each age with step size $10^{-4}$ up to 400 iterations. More iterations will make the sampling process too time-consuming.

To implement DRE-F-SP+RS, we first train the specially designed sparse AE on the training set for 200 epochs with the SGD optimizer, initial learning rate 0.01 (decayed every 50 epochs with factor 0.1), weight decay $10^{-4}$, batch size 256, and sparsity parameter $\lambda = 10^{-3}$. The network architecture of this sparse AE is shown in Table S.8.6 and Table S.8.7.

Sixty MLP-5 models for modeling the density ratio function are trained on the training set with the Adam optimizer [17], initial learning rate $10^{-4}$ (decayed at epoch 80 and 150), batch size 256, 200 epochs, and $\lambda = 10^{-2}$. The $\zeta$ in the filtering scheme of cDR-RS is set to be 0.1. The models for the computation of Intra-FID, NIQE, Diversity, and Label Score are consistent with those used by [8, 7]. We quote and rephrase the definitions of these metrics in [8, 7] as follows.

- **Intra-FID** [25]: “We take Intra-FID as the overall score to evaluate the quality of fake images and we prefer the small Intra-FID score. At each evaluation angle, we compute the FID [14] between real images and 1000 fake images in terms of the bottleneck feature of the pre-trained AE.”

- **NIQE** [23]: “NIQE is used to evaluate the visual quality of fake images with the real images as the reference and we prefer the small NIQE score.”

- **Diversity**: “Diversity is used to evaluate the intra-label diversity and the larger the better.” In UTKFace, there are 6 races. At each age, we ask a pre-trained classification-oriented ResNet-34 to predict the races of the 1000 fake images and an entropy is computed based on these predicted races. The Diversity score is the average of the entropies computed on all ages.

- **Label Score**: “Label Score is used to evaluate the label consistency and the smaller the better.” We ask the pre-trained regression-oriented ResNet-34 to predict the ages of all fake images and the predicted ages are then compared with the conditioning ages. The Label Score is defined as the average absolute distance between the predicted ages and conditioning ages over all fake images, which is equivalent to the Mean Absolute Error (MAE).

Please refer to the official implementation of CcGANs at https://github.com/UBCDingX1n/improved_CcGAN for more details.

S.8.2 Why the Diversity score is improved?

It may be counter-intuitive that the Diversity score is increased when cDR-RS may reject some generated images. We show Fig. S.8.9 to illustrate. In this figure, we visualize the distributions of 1000 fake images sampled from Baseline and cDR-RS for age 36 over 5 races, respectively. To make the illustration more clearer, we increase $\zeta$ to 0.183, so that the Label Scores of Baseline and cDR-RS are comparable and the improvement caused by cDR-RS focuses on Diversity. Please note that, in the evaluation of the UTKFace experiment, Diversity is defined as entropy of the ages of fake images that are predicted by a pre-trained classification CNN (races are taken as class labels). Therefore, more balanced race distribution implies higher Diversity. From Fig. S.8.9, we can see, after applying cDR-RS, the frequency of Race 1 decreases while the...
Figure S.7.8: Pie charts for the efficiency analysis of DRE-F-SP+RS and cDR-RS on ImageNet-100.
Table S.7.5: Efficiency analysis of different sampling methods on ImageNet-100 based on Two NVIDIA V100. For DRE-F-SP+RS and cDR-RS, the training time includes the time spent on the ResNet-34 training and the MLP-5 network training.

| Methods        | Total storage usage (GB) | Total training time (hours) | Total sampling time (hours) | Total implementation time (hours) |
|----------------|--------------------------|----------------------------|----------------------------|-----------------------------------|
| Collab         | 0.13                     | 5.58                       | 5.23                       | 10.81                             |
| DRS            | 0.13                     | 1.47                       | 0.75                       | 2.22                              |
| DDLS           | 0.13                     | 0                         | 218.05                     | 218.05                            |
| DRE-F-SP+RS    | 38.94                    | 84.41                      | 2.43                       | 86.84                             |
| cRS-RS         | 0.75                     | 65.69                      | 1.18                       | 66.87                             |

Table S.8.6: The architecture of the encoder in the sparse autoencoder for reconstructing 64 × 64 RGB images. In convolutional (Conv) operations, ch denotes the number of channels, k/s/p denote kernel size, stride and number of padding, respectively.

Table S.8.7: The architecture of the decoder in the sparse autoencoder for reconstructing 64 × 64 RGB images. In transposed-convolutional (ConvT) operations, ch denotes the number of channels, k/s/p denote kernel size, stride and number of padding, respectively.

Table S.8.8: The architecture of the label prediction branch in the sparse autoencoder for 64 × 64 images.

```
Input: extracted sparse features h ∈ \mathbb{R}^{12288}
```

```
fc → 1024, BN, ReLU
fc → 512, BN, ReLU
fc → 256, BN, ReLU
fc → 1, ReLU
```

Output: the predicted label \( \hat{y} \)

Figure S.8.9: The distributions of fake images sampled from Baseline and cDR-RS, respectively, at age 36 over 5 races in the UTKFace experiment. After subsampling by cDR-RS, the distribution of fake images for 5 races are more balanced, resulting in higher diversity.

**S.8.3 More details of the efficiency analysis**

Similar to the ImageNet-100 experiment, we analyze the efficiency of all candidate methods, and summarize the results in Table S.8.9 and Fig. S.8.10. The 60 MLP-5 models take a lot of disk space for DRE-F-SP+RS.
Figure S.8.10: Pie charts for the efficiency analysis of DRE-F-SP+RS and cDR-RS on UTKFace.
Table S.8.9: Efficiency analysis of different sampling methods on UTKFace based on One NVIDIA V100. For DRE-F-SP+RS and cDR-RS, the training time includes the time spent on the sparse AE training and the MLP-5 network training.

| Methods      | Total storage usage (MB) | Total training time (hours) | Total sampling time (hours) | Total implementation time (hours) |
|--------------|--------------------------|-----------------------------|-----------------------------|----------------------------------|
| Collab       | 82.8                     | 1.05                        | 0.27                        | 1.32                             |
| DRS          | 82.8                     | 0.16                        | 0.13                        | 0.29                             |
| DDLS         | 82.8                     | 0                           | 11.24                       | 11.24                            |
| DRE-F-SP+RS  | 6,671                    | 1.92                        | 5.69                        | 7.61                             |
| cRS-RS       | 303                      | 1.99                        | 0.49                        | 2.48                             |

S.9 More Details of Experiments on RC-49

Similar to the UTKFace experiment, we follow [8, 7] to implement CcGANs (SVDL+ILI) with the SNGAN architecture. The detailed setups can be found in [8, 7] or our codes.

To implement Collab, we conduct discriminator shaping for 3000 iterations for CcGAN. We do the refinement 16 times in a middle layer of the generator network of CcGAN. The step size of refinement is set as 0.5.

To implement DRS, we fine-tune the discriminator of CcGAN for 1000 iterations with batch size 128.

DDLS is not implemented on RC-49 due to a too long sampling time.

DRE-F-SP+RS is not applicable to this scenario, where we need to generate images conditional on labels that are unseen in the training phase.

To implement cDR-RS, first train the specially designed sparse AE on the training set for 200 epochs with the SGD optimizer, initial learning rate 0.01 (decayed every 50 epochs with factor 0.1), weight decay $10^{-4}$, batch size 256, and sparsity parameter $\lambda = 10^{-3}$. The MLP-5 to model the conditional density ratio function is similar to Table S.5.4. It is trained with the Adam optimizer [17], initial learning rate $10^{-4}$ (decayed at epoch 80 and 150), batch size 256, 200 epochs, and $\lambda = 10^{-3}$. The $\zeta$ in the filtering scheme of cDR-RS is set to be 0.13.

In the testing phase, we generate 179,800 fake images (200 per angle) from each candidate method over 899 distinct labels. This experiment adopts four evaluation metrics—(i) Intra-FID [25] is an overall image quality metric; (ii) Naturalness Image Quality Evaluator (NIQE) [23] evaluates the visual quality of fake images. Please note again that the visual quality is only one aspect of image quality; (iii) Diversity measures the diversity of fake images; and (iv) Label Score (LS) evaluates label consistency.

Specifically, the four metrics are computed as follows. (i) For the Intra-FID index, at each of the 899 angles ($0.1^\circ - 89.9^\circ$), we compute the FID [14] value between 49 real images and 200 fake images in terms of the bottleneck feature of the pre-trained autoencoder. The Intra-FID score is the average FID over all 899 evaluation angles. (ii) For the NIQE index, firstly we fit an NIQE model with the 49 real rendered chair images at each of the 899 angles which gives 899 NIQE models. We then compute an average NIQE score for each evaluated angle using the NIQE model at that angle. Finally, we report the average of the 899 average NIQE scores over the 899 yaw angles. The block size and the sharpness threshold are set to 8 and 0.1 respectively in this experiments. We employ the built-in NIQE library in MATLAB. (iii) For the Diversity index, at each evaluation angle, firstly we use a pretrained classification-oriented ResNet-34 to predict the chair types (49 types in total) of these 200 fake images. Then, an entropy value can be computed based on the chair type predictions at this angle. Finally, the Diversity index is defined as the average of the entropies at all 899 angles. (iv) For the Label Score index, at each evaluation angle, firstly we ask a pretrained regression-oriented ResNet-34 to predict the yaw angles of all fake image samples and the predicted angles are then compared with the assigned angles. The Label Score value is defined as the average absolute distance between the predicted angles and assigned angles over all fake images.