Precision determination of the $d\pi \leftrightarrow NN$ transition strength at threshold

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An unusual but effective way to determine at threshold the $d\pi \leftrightarrow NN$ transition strength $\alpha$ is to exploit the hadronic ground-state broadening $\Gamma_{1s}$ in pionic deuterium, accessible by x-ray spectroscopy. The broadening is dominated by the true absorption channel $d\pi \rightarrow nn$, which is related to $s$-wave pion production $pp \rightarrow d\pi^+$ by charge symmetry and detailed balance. Using the exotic atom circumsuits the problem of Coulomb corrections to the cross section as necessary in the production experiments. Our dedicated measurement finds $\Gamma_{1s} = (1171\pm24)\text{meV}$ yielding $\alpha = (252^{+5}_{-11})\mu\text{b}$.

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Meson production and absorption at low energies plays a key role in developing methods within the framework of effective field theories such as chiral perturbation theory (\chi PT). Directly at threshold, experimental access to $NN \leftrightarrow NN\pi$ processes is provided both via the hadronic ground-state broadening $\Gamma_{1s}$ in pionic deuterium ($\pi D$) and pion production in nucleon-nucleon collisions.

Considering only pure hadronic cross sections (denoted by $\hat{\sigma}$), i.e., with the Coulomb interaction switched off to circumvent the divergence problem at threshold but with the particles keeping their physical mass, the production cross section is parametrised by \(\hat{\sigma}_{pp \rightarrow \pi^+d} = \alpha\eta + \beta\eta^2 + \ldots\)

with $\eta = p^*_e c / m_e c^2$ being the reduced pion momentum in the $\pi d$ rest frame. For $\eta \to 0$ higher partial waves ($\beta$, ...) vanish, and only the threshold parameter $\alpha$ contributes owing to pure $s$-wave production. Directly related is the reaction $np \rightarrow \pi^0 d$ because in the limit of charge independence the relation $2 \cdot \sigma_{np \rightarrow \pi^0d} = \sigma_{pp \rightarrow \pi^+d}$ holds.

Values for $\alpha$ derived from pion-production \([2, 4]\) and absorption experiments \([10]\) scatter widely even when comparing recent data (Tab.II and Fig.2). However, sometimes only statistical errors are given. The fluctuations suggest systematic uncertainties of about 10% possibly stemming from normalization. In particular, the Coulomb corrections, mandatory to obtain the pure hadronic cross section, are a significant source of uncertainty \([4, 5, 11]\).

As discussed for example by Lensky et al. \([12, 13]\), phenomenological descriptions \([2, 11, 14, 17]\) may suffer from an incomplete knowledge of the contributing mechanisms, which in principle is avoided within the \chi PT approach if enough terms are considered in the expansion. A recent calculation up to next-to-leading order (NLO) terms yields $\alpha^{NLO} = 220 \mu\text{b}$ \([12]\) and thus $\Delta a_{\pi D} = 5.65 \times 10^{-3} \text{m}^{-2}_a$ \([13]\) for the imaginary part of the $\pi D$ scattering length. The uncertainty of about ±30% is expected to decrease to below ±10% by next-to-next-to-leading order (NNLO) calculations \([18]\).

A measurement of $\Gamma_{1s}$ in pionic deuterium is equivalent to the determination of $\Delta a_{\pi D}$ \([13]\) being predominantly attributed to true pion absorption $\pi^- \rightarrow nn$. Pion absorption at rest on the isospin I=0 nucleon-nucleon pair of the deuteron induces the transition $^3S_1 [^3D_1](I=0) \rightarrow ^3P_1(I=1)$, the inverse of which accounts for $s$-wave pion production in $pp \rightarrow d\pi^+$ \([20]\). Therefore, $\Gamma_{1s}$ is a measure of the $s$-wave pion-production strength.

In contrast to production experiments, the extraction of the threshold parameter $\alpha$ or $\Delta a_{\pi D}$ from $\Gamma_{1s}$ avoids the problem of Coulomb corrections to the measured cross sections. However, previous x-ray experiments \([21, 22]\) are of limited statistics, and insufficient knowledge on the experimental resolution and cascade-induced broadening prevents a precise extraction of the pure hadronic width $\Gamma_{1s}$. Hence, a remeasurement of $\Gamma_{1s}$ was performed \([23]\) aiming at an accuracy of at least the one expected from the forthcoming \chi PT calculations.

The complex pion-deuteron scattering length $a_{\pi D}$ is related to the 1s-state shift $\epsilon_{1s}$ and width $\Gamma_{1s}$ in $\pi D$ by

$$\epsilon_{1s} = \frac{2\alpha^3 \mu^2 c^4}{\hbar c} a_{\pi D} \left[ 1 - 2\alpha \mu c^2 / \hbar c (\ln \alpha - 1) \cdot a_{\pi D} \right].$$

The first term corresponds to the classical Deser for-
formula \[19\], yielding the scattering length in leading order \[24\]. The term in brackets corrects for the fact that \(a_{sD}\) is determined from a Coulomb bound state \[25,27\]. In equations \[28\] and \[32\], \(\alpha\) denotes the fine structure constant.

The imaginary part of \(a_{sD}\) is given by

\[
\Im a_{sD} = \frac{h c}{2\alpha^3 m^2 c^4} \cdot \frac{\Gamma_{1s}/2}{1 - 2\alpha m c^2 (\ln \alpha - 1) \cdot 2 \Re a_{sD}}
\]

\[
= 0.010642 m^{-1} eV^{-1} \cdot 1.004 \cdot (\Gamma_{1s}/2).
\]  

(3)

The factor 1.004 stands for the—in this case—small bound-state correction. Hence, it is sufficient to insert the leading order result for the real part of the scattering length in equation (3). The value \(\Re a_{sD} = (26.3 \pm 0.6) \cdot 10^{-3} m^{-1}\) is taken from a previous experiment \[22\].

Detailed balance states that \[20\]

\[
\sigma_{\pi^+ d \rightarrow pp} = \frac{2}{3} \cdot \left(\frac{p^*_p}{p^*_\pi}\right)^2 \cdot \sigma_{pp \rightarrow \pi^+ d},
\]  

(4)

where \(p^*_p\) and \(p^*_\pi\) are the final state momenta of proton and pion in the center-of-mass (CMS). Assuming charge symmetry, for the transition matrix elements \(M_{\pi^- d \rightarrow nn}\) \(= \pm M_{\pi^+ d \rightarrow pp}\) holds. A small difference in the transition rate, \(\sigma_{\pi^- d \rightarrow nn}/\sigma_{\pi^+ d \rightarrow pp} = p^*_n/p^*_\pi = 0.982\), must be taken into account because of the slightly larger phase space for \(\pi^+ d \rightarrow pp\) with \(p^*_n,p\) being the nucleon CMS momenta.

In principle, both electromagnetic and hadronic isospin-breaking effects must be considered in view of the different quark contents in the final states of the processes \(\pi^- d \rightarrow nn\) and \(\pi^+ d \rightarrow pp\). Their magnitude, however, is assumed to be at most a few percent \[18,28\], which is about the precision achieved in this experiment but far below the fluctuations of the pion-production data. The atomic binding energy of the \(\pi^- D\) system is neglected.

Combining optical theorem, charge symmetry, detailed balance and inserting the s-wave part from \[11\], the purely hadronic imaginary part of the scattering length \(a_{\pi^- d \rightarrow nn}\) reads in terms of the threshold parameter \(\alpha\) \[19\]

\[
\Im a_{\pi^- d \rightarrow nn} = \lim_{p^*_\pi \rightarrow 0} \frac{p^*_n}{4\pi} \cdot \sigma_{\pi^- d \rightarrow nn} = \frac{1}{6\pi} \cdot \frac{p^*_p \cdot p^*_n}{m_\pi} \cdot \alpha.
\]  

(5)

To relate \(\Im a_{\pi^- d \rightarrow nn}\) to \(\Im a_{sD}\), a correction for final states other than \(nn\) must be applied. The measured branching ratios \[29,31\] yield for the relative strength of true absorption with respect to all other processes \(S' = nn/(nn\gamma + nne^+e^- + nn\pi^0) = 2.76 \pm 0.04\). Consequently,

\[
\Im a_{sD} = (1 + S') \cdot \Im a_{\pi^- d \rightarrow nn}
\]

\[
= (2.48 \pm 0.01) \cdot 10^{-5} \cdot \alpha \cdot m^{-1} \mu b^{-1}.
\]  

(6)

| TABLE I: Threshold parameter \(\alpha\) derived from the hadronic broadening \(\Gamma_{1s}\) in \(\pi D\) and pion-production and absorption data together with a selection of theoretical approaches. |
|-----------------|-----------------|-----------------|
| pionic deuteron | \(\alpha / \mu b\) |
| \(3p \rightarrow 1s\) | \(220 \pm 45\) \[21\] |
| \(2p \rightarrow 1s\) | \(257 \pm 23\) \[22\] |
| \(3p \rightarrow 1s\) | \(252 \pm 5\)  \[31\] this \exp. |

| pion production/absorption | \(\alpha / \mu b\) |
|----------------------------|-----------------|
| \(pp \rightarrow d\pi^+\) | \(138 \pm 15^*\) \[3\] |
| \(pp \rightarrow d\pi^+\) | \(240 \pm 20^*\) \[4\] |
| \(pp \rightarrow d\pi^+\) | \(180 \pm 20^*\) \[5\] |
| \(pp \rightarrow d\pi^+\) | \(228 \pm 46^*\) \[7\] |
| \(np \rightarrow d\pi^0\) | \(184 \pm 14^*\) \[6\] |
| \(d\pi^+ \rightarrow pp\) | \(174 \pm 3^*\) \[10\] |
| \(p_{\text{pol}}p \rightarrow d\pi^+\) | \(208 \pm 5^*\) \[8\] |
| \(pp \rightarrow d\pi^+\) | \(205 \pm 9^*\) \[9\] |

*experiments reporting statistical uncertainty only*
The πD line shape is determined by the spectrometer response (i), the natural width of the x-ray transition (ii), and Doppler broadening from Coulomb transitions (iii):

(i) The spectrometer response was determined using the narrow M1 x-ray line from helium-like argon at 3.104 keV as outlined in [38, 39]. When scaled to the πD(3p – 1s) energy (3.075 keV), it corresponds to a resolution of 436 ± 3 meV (FWHM) (Fig. 1), which is close to the theoretical value of 403 meV as calculated for an ideal flat crystal (code XOP [40]).

(ii) The natural linewidth is practically given by the hadronic broadening Γ₁ˢ. The 3p-level width is dominated by radiative decay (28 μeV). Nuclear reactions are estimated to be <1 μeV. Likewise, based on calculated transitions rates [41], the induced width due to 3p ↔ 3s Stark mixing turns out to be as small as 1 μeV.

(iii) Coulomb transitions may occur when excited exotic hydrogen atoms penetrate the electron cloud of target atoms, and the energy release of the deexcitation step is converted into kinetic energy shared by the πD system and another D atom [42]. Coulomb deexcitation generates peaks in the kinetic energy distribution, which are at 12, 20, 38, and 81 eV for the πD Δn = 1 transitions (7-6), (6-5), (5-4), and (4-3), respectively. Therefore, subsequent x-ray transitions may be Doppler broadened. Acceleration, however, is counteracted by elastic and inelastic scattering, which may lead to a continuum below the peak energies or even complete deceleration.

Cascade calculations have been extended to follow the velocity change during the deexcitation cascade and, therefore, can provide kinetic energy distributions at the instant of x-ray emission from a specific atomic level (extended standard cascade model ESCM [41]). At present, only calculations for μH and πH are available [41, 43, 44]. Therefore, an approach independent of a cascade model was used to extract the Doppler broadening directly from the data, which was applied successfully in the neutron time-of-flight and μH x-ray analyses. The kinetic energy distribution was modeled by boxes of a few eV width corresponding to the peaks generated by the Coulomb transitions. Their number, width, and position are preset but adjustable parameters of the analysis code. The hadronic width Γ₁ˢ, the total intensity, and the background level are free parameters of the fit, as are the relative intensities, whose sum is normalized to one.

The line shape was constructed by convoluting the crystal response, imaging properties of the bent crystal, and Doppler contributions with the natural linewidth by means of Monte-Carlo ray-tracing. Following the experience of the μH(3p – 1s) analysis [34], one tries to identify consecutively individual Doppler contributions starting with one single box only and moving it through the range of possible kinetic energies [34]. A χ² analysis using the MINUIT package [45] shows the necessity of a low-energy contribution. It was found that the upper bound of this box must not exceed 8 eV, a result obtained independently for the spectra taken at 10 bar and the 17.5 bar equivalent density. The result for Γ₁ˢ turned out to be insensitive to the upper boundary of the kinetic-energy box for values ≤ 8 eV. The low-energy component was set to the range 0 – 2 eV in further analysis.

Searches for any contributions of Coulomb transitions leading to higher energies failed—also for the sum of the spectra measured at 10 and 17.5 bar equivalent density. Tentatively, we used the kinetic energy distribution from the ESCM calculation for the πH(3p – 1s) case, after scaling to πD energies. This distribution, where a fraction of 25% has energies above 15 eV, is unable to describe the line shape, which is in strong disagreement with the findings for muonic [33] and pionic hydrogen [32, 46], where sizeable contributions from higher energy Coulomb transitions are mandatory. There is no explanation yet for this different behavior in pionic deuterium.

Detailed Monte-Carlo studies have been performed to quantify which amount of high-energy components may be missed for the statistics achieved. It was found that a contribution of 25% can be excluded at the level of 99% for the component around 80 eV corresponding to the (4 – 3) Coulomb transition. The chance to identify a 10% contribution is about 2/3 corresponding to 1 μeV. The value for Γ₁ˢ itself hardly varies with the width of any assumed high-energy box.

Omitting any high-energy contribution yields the upper limit for Γ₁ˢ. Defining the limit of sensitivity to 10% – according to the above-mentioned 1 μeV criterion – results in a lower bound 43 meV lower than the upper limit and, hence, in an asymmetric systematic error for Γ₁ˢ.

Systematic discrepancies (bias) as arising in maximum likelihood fits [47] have been studied according to the statistics of the measured spectra. The bias has been determined to be -32±2 meV (3.3 bar data) and -2±2 meV.
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\[ 4.2\% \text{ achieved reaches the expected (5 of forthcoming NNLO χPT calculations. It is noteworthy } \]

\[ \Re \alpha \] in good agreement with previous measurements which found \( \Gamma_{1s} = 1020 \pm 210 \) \[21\] and \( 1194 \pm 105 \) meV \[22\].

This result leads to an imaginary part \[3\]

\[ \Im \alpha_D = \left( 6.26^{+0.12}_{-0.26} \right) \cdot 10^{-3} m_{\pi}^{-1}. \] (8)

The corresponding value for \( \alpha \) is given in Table \[4\].

In summary, the \( \pi D(3p - 1s) \) x-ray transition in pionic deuterium has been studied to determine the strong-interaction broadening of the 1s state and from that the threshold pion-production strength \( \alpha \). The accuracy of 4.2% achieved reaches the expected (5–10)% uncertainty of forthcoming NNLO χPT calculations. It is noteworthy that at the 10% level no components from high-energetic Coulomb transitions could be identified.

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\[\text{FIG. 2: (color online). Threshold parameter } \alpha. \text{ Points are shown in the same order as listed in Table \[4\].}\] (sum of 10 and 17.5 bar). For each set of conditions 400 Monte-Carlo generated spectra were analyzed.

The weighted average for the hadronic broadening,

\[ \Gamma_{1s} = \left( 1171^{+23}_{-49} \right) \text{ meV}, \] (7)

is in good agreement with previous measurements which found \( \Gamma_{1s} = 1020 \pm 210 \) \[21\] and \( 1194 \pm 105 \) meV \[22\].