THE LARGE-SCALE THREE-POINT CORRELATION FUNCTION OF SLOAN DIGITAL SKY SURVEY LUMINOUS RED GALAXIES

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ABSTRACT

We present new measurements of the redshift-space three-point correlation function (3PCF) of Luminous Red Galaxies (LRGs) from the Sloan Digital Sky Survey (SDSS). Using the largest data set to date, the Data Release 7 LRGs, and an improved binning scheme compared to previous measurements, we measure the LRG 3PCF on large scales up to $\sim 90\, h^{-1}$ Mpc, from the mildly nonlinear to quasi-linear regimes. Comparing the LRG correlations to the dark matter two- and three-point correlation functions, obtained from $N$-body simulations we infer linear and nonlinear bias parameters. As expected, LRGs are highly biased tracers of large-scale structure, with a linear bias $b_1 \sim 2$; the LRGs also have a large positive nonlinear bias parameter, in agreement with predictions of galaxy population models. The use of the 3PCF to estimate biasing helps to also make estimates of the cosmological parameter $\sigma_8$, as well as to infer best-fit parameters of the halo occupation distribution parameters for LRGs. We also use a large suite of public mock catalogs to characterize the error covariance matrix for the 3PCF and compare the variance among simulation results with jackknife error estimates.

Key words: cosmology: observations – galaxies: statistics – large-scale structure of universe – surveys

Online-only material: color figures

1. INTRODUCTION

The large-scale structure traced by galaxies is shaped both by cosmic expansion history, which determines the gravitational evolution of density perturbations, and by the physics of galaxy formation. Comparison of galaxy clustering with the predictions of structure formation models therefore constrains both cosmological and galaxy formation parameters. For a given clustering observable, e.g., the galaxy two-point correlation function (2PCF), $\xi_{gg}(r)$, there are typically significant degeneracies between the inferred cosmological and galaxy formation parameters. Measurement of multiple observables with different relative dependencies on cosmology and galaxy formation can help break such degeneracies (Abazajian et al. 2005; Zheng & Weinberg 2007). The galaxy three-point correlation function (3PCF), the next level up in the hierarchy of $N$-point correlation functions, encodes information that is complementary to that contained in the 2PCF and is therefore a useful second observable for constraining cosmology and galaxy formation (Sefusatti & Scoccimarro 2005; Sefusatti et al. 2006).

The 3PCF has been measured since the advent of the first angular catalogs (Peebles & Groth 1975), and more recently in the last generation of spectroscopic surveys such as 2dFGRS (Jing & Börner 2004; Gaztañaga et al. 2005), and the Sloan Digital Sky Survey (SDSS; Kayo et al. 2004; Nichol et al. 2006; Kulkarni et al. 2007; McBride et al. 2011). The goals of these efforts have been mostly to test predictions from theories of growth of structure and cosmological simulations, as well as to measure the biasing of the galaxies with respect to the dark matter 3PCF. With the large volume of current surveys, we are able to improve the signal-to-noise ratio of the measurements, and obtain more information from the 3PCF to help constrain cosmological and galaxy formation models.

In this paper, we measure the 3PCF for a particular class of galaxies, the Luminous Red Galaxies (LRGs; Eisenstein et al. 2001) observed spectroscopically by the SDSS (York et al. 2000), and use the results to constrain galaxy bias and cosmological parameters. LRGs offer several advantages for this measurement. At the bright end of the galaxy luminosity function, LRGs can be targeted out to relatively large distances, $z \sim 0.5$ (whereas the average redshift is $z \sim 0.3$), in the SDSS spectroscopic survey. Their clustering can therefore be measured over scales large compared to those probed by $L_*$ galaxies in the same survey (Eisenstein et al. 2005; Tegmark et al. 2006; Percival et al. 2007; Gaztañaga et al. 2009b), which are on average at $z \sim 0.1$. In addition, LRGs are more strongly clustered than less-luminous galaxies, making their correlations intrinsically easier to measure on large scales, where shot noise is subdominant.

The physics of galaxy formation is imprinted in the $N$-point galaxy correlation functions via the bias, the relation between the spatial distributions of galaxies and dark matter. Although the halo occupation distribution model (HOD; see, for instance, Jing et al. 1998; Seljak 2000; Scoccimarro et al. 2001; Berlind & Weinberg 2002; Cooray & Sheth 2002; Zheng et al. 2005) has become a popular framework for parameterizing the bias (especially on small scales), here we focus on using the 2PCF and 3PCF LRG measurements on large scales to constrain a simpler, nonlinear, local bias model that appears to adequately capture the features displayed by the clustering data. This approach allows us to break the degeneracy that exists between the linear bias parameter and the rms of density fluctuations at scale of $8\, h^{-1}$ Mpc, $\sigma_8$, when extracting information from the 2PCF, as shown, using different statistics, in Pan & Szapudi (2005) and Ross et al. (2008). Although a complete and detailed HOD analysis using fits to the two-point and three-point functions will be presented elsewhere (Sabiu 2009), we explore the idea of using the LRG bias parameters in constraining HOD models, as suggested by Sefusatti & Scoccimarro (2005).
We note that Gaztaña et al. (2009a) have recently measured the SDSS LRG 3PCF on very large scales, focusing on detection of the baryon acoustic oscillation signature. Reid & Spergel (2009) used a counts-in-cylinders statistic to constrain the LRG HOD. In addition, Ross et al. (2008) measured higher-order angular clustering of a sample of photometrically selected LRGs from the SDSS and used it to constrain nonlinear bias parameters and get constraints on $\sigma_8$.

Our goal here is to present an up-to-date measurement of the 3PCF, show another way to use the 3PCF information on large scales, in this case to calculate the bias, and offer complementary constraints on $\sigma_8$, and show that we can use the 3PCF as a complement to find best-fit HOD parameters.

The outline of the paper is as follows. In Section 2, we review the 3PCF and describe our binning scheme for the 3PCF measurement; we also describe the SDSS LRG sample and the simulations used to create mock catalogs. In Section 3, we present our measurements of the 3PCF in the LRG sample. In Section 4 we discuss the estimation of 3PCF errors, which are critical in constraining parameters. In Section 5, we constrain the nonlinear bias parameters of the LRGs and explore the possibility of constraining the matter power spectrum amplitude $\sigma_8$, as well as relevant HOD parameters. We summarize and conclude in Section 6.

2. METHODS, DATA, AND SIMULATIONS

2.1. The Three-point Correlation Function

The 3PCF describes the probability of finding three objects in a particular triangle configuration, compared to that of a random sample. The joint probability of finding three objects in three infinitesimal volumes $dV_1$, $dV_2$, and $dV_3$ is given by (Peebles 1980)

$$P = [1 + \xi(r_{12}) + \xi(r_{23}) + \xi(r_{31}) + \xi(r_{12}, r_{23}, r_{31})]$$

\[\times \bar{n}^3 dV_1 dV_2 dV_3,\]

where $\bar{n}$ is the mean density of objects, $\xi$ is the 2PCF, and $\zeta$ is the (connected) 3PCF:

$$\xi(r_{12}) = \langle \delta(r_1)\delta(r_2) \rangle,$$

$$\zeta(r_{12}, r_{23}, r_{31}) = \langle \delta(r_1)\delta(r_2)\delta(r_3) \rangle,$$

where $\delta$ is the fractional overdensity of objects or the continuous field studied. The triangle sides $r_{ij}$ are the distances between objects $i$ and $j$ in the triplet. Since the 3PCF depends upon the configuration of the three sides, it is sensitive to the shapes of spatial structures (Scoccimarro & Szalay 2000; Gaztaña & Scoccimarro 2005; Marín et al. 2008). Since the ratio $\zeta/\xi^2$ is both predicted and found to be close to unity over a large range of length scales even though $\xi$ and $\zeta$ each vary by orders of magnitude (Peebles 1980), it is convenient to define the reduced 3PCF:

$$Q(s, u, \theta) \equiv \frac{\zeta(s, u, \theta)}{\xi(r_{12})\xi(r_{23}) + \xi(r_{23})\xi(r_{31}) + \xi(r_{31})\xi(r_{12})}.$$  

Here, $s \equiv r_{12}$ sets the scale size of the triangle, and the shape parameters are given by the ratio of two sides of the triangle, $u \equiv r_{23}/r_{12}$, and the angle between those two sides, $\theta \equiv \cos^{-1}(r_{12} \cdot r_{23})$, where $r_{12}$, $r_{23}$ are unit vectors in the directions of those sides (see Figure 1). By measuring the shape and scale dependence of $Q(s, u, \theta)$ for galaxies and comparing with that predicted for dark matter, we can constrain the galaxy bias.

We calculate the 2PCF using the estimator of Landy & Szalay (1993),

$$\zeta = \frac{DD - 2DR + RR}{2DR + RR}.$$  

Here, DD is the number of data pairs normalized by $N_D \times N_D/2$, DR is the number of pairs using data and random catalogs normalized by $N_D N_R$, and RR is the number of random data pairs normalized by $N_R \times N_R/2$, where $N_D$ and $N_R$ are the number of points in the data and in the random catalog, respectively. The 3PCF is calculated using the Szapudi & Szalay (1998) estimator

$$\zeta = \frac{DDD - 3DDR + 3DRR - RRR}{RRR},$$  

where DDD, the number of data triplets, is normalized by $N_D^3/6$, and RRR, the random data triplets, is normalized by $N_R^3/6$. DDR is normalized by $N_D^2 N_R/2$, and DRR by $N_D N_R^2$. These estimators optimally correct for edge effects by incorporating pair and triplet counts from a random catalog. To compute the correlation functions, we use the NPT software developed in collaboration with the Auton Lab at Carnegie Mellon University. NPT is a fast implementation of $N$-point correlation function estimation using multi-resolution kd-trees to compute the number of pairs and triplets in a data set. For more details and information on the algorithm, see Moore et al. (2001), Gray et al. (2003), and Nichol et al. (2006).

2.2. Measuring the 3PCF: Binning

Since measurements of two- and three-point correlations involve counts of pairs and triplets of objects, the question arises of how to choose the bins of pair and triplet separation (Matsubara & Suto 1994; Chen & Szapudi 2005; Gaztaña & Scoccimarro 2005; Nichol et al. 2006; Kulkarni et al. 2007; Marín et al. 2008). Small bins enable measurements with fine resolution in separation and the possible identification of features in the correlation functions but at the cost of large shot noise due to the small numbers of pairs and triplets per bin, resulting in artificially large diagonal errors and in a poorly determined covariance matrix. Large bins reduce the shot noise but tend to wash out features of the 3PCF. The optimal bin size is therefore a compromise and depends on the relative importance of signal-to-noise ratio versus resolution for the question at hand.

For the 3PCF, there is also the question of which separation parameters to bin in, that is, which triangle configurations to include in each bin. Nichol et al. (2006) and Kulkarni et al. (2007) measured the 3PCF in bins of $s$, $u$, and $\theta$, that is, they counted triplets of galaxies that satisfy $s \in s_0 \pm \Delta s$,
In their measurement of the LRG 3PCF using the DR3 sample, Kulkarni et al. (2007) used very small resolution in $\Delta s = \pm 0.1$ Mpc (1% at $10 h^{-1}$ Mpc) and $\Delta \theta = \pm \pi/100$ rad, and an intermediate value for $\Delta u = \pm 0.1$. With such small resolution the result is that $Q(\theta)$ is measured with low signal-to-noise ratio (since there are few pairs and triplets in each bin) and since $\Delta u$ is not small (for $u = 2$, $s = 10$, the bin size of the second side of the triangle is 12% of the side), there are triplets counted in two or more configurations, or “bins.” With this binning scheme it is not possible to measure the LRG 3PCF on larger scales. But just increasing bin size in this way would lead to very different triangles accepted in the same bin, as shown in the right side of Figure 2.

In this work, we use the parameters $s$, $u$, and $\theta$ to determine the centers of the triplet bins, but we construct the bins themselves directly in each of the pair separations, with $\Delta r_{ij} \propto r_{ij}$, i.e., we use bins of constant $\Delta \log r_{ij}$ (Marín et al. 2008; similar schemes were used also in Matsubara & Suto 1994; Chen & Szapudi 2005; McBride et al. 2011). As Figure 2 shows, this binning scheme groups together triplet configurations that are more similar in shape (both mathematically and colloquially) and size than do bins in $s$, $u$, and $\theta$. As a result, we can achieve either higher resolution in triangle shape and size or higher signal-to-noise ratio for fixed resolution. Also to obtain a better signal in the 3PCF, we increase somewhat the size of the bins, which leads to a bigger correlation between the points, and therefore we do not need to measure the 3PCF for a large number of angles $\theta$ as Kulkarni et al. (2007) did.

We show in Figure 3 the results of using the different binning schemes: using the DR3 LRG sample (see Section 2.4 for a description of the sample), we compare the resultant 3PCF using the binning scheme used in Kulkarni et al. (2007, green solid lines), with the scheme used in this paper, for $\Delta r_{ij} = \pm 0.1 r_{ij}$ (open squares); it is clear that in our binning scheme $Q(\theta)$ fluctuates much less than when using the “old” scheme, it reproduces better the theoretical predictions (note that there is no plateau in the new scheme that happens in the $\theta > 100^\circ$ in the $s = 10 h^{-1}$ Mpc configurations), and has smaller errors (calculated, as in Kulkarni et al. 2007, using jackknife (JK) resampling, which will be discussed in Section 4).

We take advantage of this binning scheme to measure the 3PCF over a large range of triangle sizes and shapes: we study central configurations with $s$ ranging from $7 h^{-1}$ Mpc (which allows us to compare these measurements to previous works) to $30 h^{-1}$ Mpc (on the quasi-linear regime, in order to estimate bias parameters), with $u = 2$ and 15 equally spaced values of $\theta$. For other studies (outside the scope of this paper) such as finding best-fit galaxy population (HOD) parameters from fits to the 3PCF, it is necessary to carry out measurements on smaller scales. The results are limited by shot noise on small scales (due to the small spatial density of the LRG field) and by finite volume on large scales.

In Figure 4, we illustrate the sensitivity of the reduced 3PCF measurement to the choice of binning resolution. We show measurements with $s = 7$ and $15 h^{-1}$ Mpc for the SDSS DR7-Dim LRG sample (see Section 2.4). The blue dotted, solid red, and dashed magenta curves represent results with $\Delta r_{ij}/r_{ij} = \pm 0.05$, 0.1, and 0.2, respectively. We can see significant bin-dependent differences in the reduced 3PCF amplitude, particularly for elongated triangle configurations ($\theta$ close to $0^\circ$ or $180^\circ$). The differences and the covariance increase for larger scales. Proceeding from smaller to larger bins, the configuration dependence of $Q$ is smoothed out, and the measurements for different values of $\theta$ become more strongly correlated. On the other hand, the signal-to-noise ratio for each measurement increases with bin width.

After experimenting with different bin sizes using the LRG data catalogs as well as multiple realizations of artificial data resembling the LRG clustering (i.e., mock catalogs, described in Section 2.5), we found that higher bin sizes ($\Delta r \gtrsim 0.1r$) reduce greatly the configuration dependence of the 3PCF; lower bin sizes (higher resolution, $\Delta r \lesssim 0.05r$) limited our ability to estimate the LRG 3PCF on the largest scales. Therefore, we opted to use a resolution of 14%, i.e., the bin size per triangle side to be $\Delta r = \pm 0.07r$, for all LRG samples we studied (see Section 2.4).
2.3. The HOD Model

The HOD model (e.g., Jing et al. 1998; Seljak 2000; Scoccimarro et al. 2001; Berlind & Weinberg 2002; Cooray & Sheth 2002; Zheng et al. 2005) provides a parameterized prescription for the galaxy spatial distribution. The first component is a model for the distribution of dark halos. One specifies the halo mass function, $n(M)$, the spatial correlations of the halos, and the density profiles for halos of mass $M$, all based on analytical models and fits to $N$-body simulations (e.g., Press & Schechter 1974; White & Rees 1978; Sheth et al. 2001; Tinker et al. 2008). The second component is the HOD itself, a model specifying how galaxies occupy dark halos as a function of halo mass.

The HOD is a parameterized model of the $P(N|M)$, the probability that a halo of mass $M$ contains $N$ galaxies with specified properties. The mean occupation function, $\langle N(M) \rangle$, is well modeled when one separates the contribution from central galaxies, $N_{\text{cen}}(M)$, which is roughly a step function in halo mass (for halos above a certain mass $M_{\text{max}}$), and the contribution of the satellite galaxies, $N_{\text{sat}}(M)$, which appears to be well characterized by a power law in halo mass (Guzik & Seljak 2002; Kravtsov et al. 2004; Zheng et al. 2005), i.e., $N_{\text{sat}} \sim (M/M_1)^{\alpha}$, where $M_1$ characterizes the mass of halos that host satellite galaxies, and the exponent $\alpha$ describes the high-mass slope of the satellite occupation number.

There is no unique model for the galaxy HOD; in fact, there are many functional forms of $\langle N(M) \rangle$, as well as the number of parameters describing a particular HOD. Also, apart from the mean HOD, one needs to specify the higher moments of parameters describing a particular HOD. Also, apart from specifying how galaxies occupy dark halos as a function of halo mass.

2.4. The SDSS LRG Sample

The SDSS LRGs, as selected by the algorithm developed by Eisenstein et al. (2001), are an excellent tracer of dark matter on large scales due to their high luminosity, which allow us to map them up to $z \sim 0.5$ and comoving volumes up to $1.5 \, h^{-1} \, \text{Gpc}^3$. The selection algorithm and the high success rate of the spectroscopy make (average sky completeness of 98%) the LRG sample nearly volume-limited up to $z \sim 0.35$ and flux limited up to $z \sim 0.47$.

We use galaxies drawn from a sample comprising 105,831 spectroscopically selected LRGs, based on SDSS Data Release 7 (DR7; Abazajian et al. 2009). The LRG galaxy selection algorithm is described in Eisenstein et al. (2001), and the DR7 LRG samples we use are drawn from the “DR7-Full” catalog described in Kazin et al. (2010), which is publicly available. The DR7-Full sample spans the redshift range $0 < z < 0.47$ and includes a range of luminosities in SDSS $g$ band of $-23.2 < M_g < -21.2$, where the magnitudes have been K-corrected and passively evolved to a redshift of $z = 0.3$, near the median redshift of the sample. Although the sample is not volume-limited, the number density is close to uniform for redshifts $z \lesssim 0.36$ and drops off due to the flux limit at higher redshift. The sample covers a sky area of 7908 deg$^2$ (close to 20% of the sky) and includes data from both the northern and southern Galactic hemispheres.

From this main catalog, two important samples are extracted: one that includes all LRGs, but is strictly volume-limited, which we call “DR7-Dim,” which has a maximum redshift of $z = 0.36$ and median spatial density of $\sim 10^4 \, h^{-1} \, \text{Mpc}^{-3}$, and a sample of the bright LRGs, where $M_g < -21.8$, which we call “DR7-Bright,” which reaches redshifts up to $z = 0.44$ and has a small spatial density $\sim 2.5 \times 10^5 \, h^{-1} \, \text{Mpc}^{-3}$. Both of these samples are the same as the ones presented in Kazin et al. (2010), and use data from the northern cap only, in order to have more mock catalogs when estimating the errors and covariance of our measurements (see Section 2.5 for more details).

1 http://cosmo.nyu.edu/~eak306/SDSS-LRG.html
To calculate the correlation functions, we use the same DR7-Full random catalogs as in Kazin et al. (2010), which contain 10 times the number of LRGs as the data. These random catalogs have been built having the same redshift selection function as the data, as well as the same sky completeness. On the smallest scales we ensure that we have enough random triplets in all configurations. For the purposes of calculating the 3PCF in a reasonable time, on the largest scales we lower the number of random points; when measuring the 3PCF on our largest scales, its random catalog has four times the number of data points. This is a conservative choice to ensure we have enough random points from the binning chosen, and, as was shown in Kulkarni et al. (2007), the Poisson errors on large scales are much smaller than the JK errors expected on these distances.

For comparison, we have also analyzed the earlier SDSS DR3 LRG sample used by Kulkarni et al. (2007). That sample covers 3816 deg$^2$ and contains 50,967 LRGs spanning the redshift range $0.15 < z < 0.55$, similar to the DR7-Full sample except that it has a smaller sky area coverage. In this case, we use the data and random catalogs of Eisenstein et al. (2005), kindly provided by the author.

The information on these samples is summarized in Table 1. The LRGs, i.e., giant red ellipticals, are associated with massive halos, and they are highly clustered (see, for instance, Zehavi et al. 2005a; Masjedi et al. 2006). This is illustrated in Figure 5, where we show the redshift-space 2PCF of the DR7-Dim and DR7-Bright LRGs, along with the dark matter redshift 2PCF from $N$-body simulations with a cosmological parameters based on the WMAP5 results (Komatsu et al. 2009, described in Section 2.5). Both LRG samples have a higher 2PCF than the dark matter, and on large scales the offset seems constant, and due to this fact we can say that the LRGs are a biased tracer of dark matter on large scales. The DR7-Bright LRGs have a higher 2PCF than the DR7-Dim LRGs, suggesting that they are associated with more massive halos (Zheng et al. 2009).

### 2.5. Simulations and Mock Catalogs

To help analyze and interpret the 3PCF results, we use mock catalogs drawn from $N$-body simulations. The mock catalogs serve two functions: (1) to measure the 3PCF of dark matter, with which the galaxy 3PCF measurements can be compared to infer the bias and (2) to determine the expected error covariance matrix for the 3PCF from the variance among a large number of mock catalogs. We use different simulations tailored to these different functions.

First, to study the dark matter two- and three-point correlation functions, we use six $N$-body simulations carried out by C. Sabiu et al. (2010, in preparation), kindly provided by the author, using the GADGET code. The cosmological model is a spatially flat $\Lambda$CDM ($A$-cold dark matter) universe, with matter density parameter $\Omega_m = 0.27$, baryon density $\Omega_b = 0.045$, power spectrum amplitude $\sigma_8 = 0.8$, reduced Hubble parameter $h = 0.7$, and initial power spectrum index $n_s = 0.95$. The simulation box volume is $1 (h^{-1} \text{Gpc})^3$ and contains $512^3$ dark matter particles with a particle mass of $5 \times 10^{11} h^{-1} M_\odot$, with a spatial softening of $60 h^{-1} \text{kpc}$, and we use the output at $z = 0.3$, the mean redshift of the LRGs, to compare their correlation functions.

Since using all the dark matter particles for the 3PCF calculations would take a prohibitively long time, we randomly select particles from the simulation, to a degree where we are sure that the dilution will not affect the results. For the 3PCF measurement, this depends on scale, and we use between 5% and 0.5% of the particles (a higher fraction for smaller scales); this fraction is enough to obtain reliable measurements of the 2PCF and 3PCF on the scales studied in this work. To obtain the spatial coordinates in the redshift-space dark matter catalog, we use the long-distance approximation in the $z$-direction (see, e.g., Bernardeau et al. 2002) to account for the effect of peculiar velocities on the measurement of the correlation functions.

We use the dark matter halo catalogs obtained from these simulations to study redshift distortions in the estimation of the

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**Table 1**

| Sample       | Area (deg$^2$) | Redshift | Luminosity (Absolute Mag) | $N_{\text{galaxies}}$ | Average Density ($h^{-1} \text{Mpc}^3$) |
|--------------|----------------|----------|---------------------------|-----------------------|----------------------------------------|
| DR7-Full     | 7,908          | 0.16 < $z$ < 0.47 | $-21.2 > M_r > -23.2$    | 105,831               | 6.70 x $10^{-5}$                      |
| DR7-Dim$^{a,b}$ | 7,189         | 0.16 < $z$ < 0.36 | $-21.2 > M_r > -23.2$    | 61,899                | 9.40 x $10^{-5}$                      |
| DR7-Bright$^{a,b}$ | 7,189       | 0.16 < $z$ < 0.45 | $-21.8 > M_r > -23.2$    | 30,272                | 2.54 x $10^{-5}$                      |
| DR3-Full     | 3,816          | 0.16 < $z$ < 0.55 | $-21.2 > M_r > -23.2$    | 50,987                | 6.50 x $10^{-5}$                      |

Notes.

$^a$ From Kazin et al. (2010).

$^b$ Volume-limited sample.

$^c$ From Eisenstein et al. (2005).
bias parameters (see Section 5.2 below), found using a friends-of-friends algorithm (Davis et al. 1985) with a linking length $b$ times the mean interparticle separation. To populate halos with mock LRGs, we use the best-fit HOD parameters from fits to the projected 2PCF by Zheng et al. (2009) in their five-parameter model. We assign mock galaxy positions, based on halo profiles, and peculiar velocities (based on a Gaussian distribution, which depends on the mass of the host halo) of galaxies inside halos using a code kindly provided by Jeremy Tinker.

Second, to estimate the errors in the 3PCF estimates, we use the covariance among public mock catalogs from the LasDamas simulations (McBride et al. 2011). As will be discussed in Section 4, we prefer this error estimation method to the use of JK resampling. For the LasDamas simulations, the cosmological model is similar but not identical to that above: flat, Λ CDM with $\Omega_m = 0.25$, $\Omega_b = 0.04$, $\sigma_8 = 0.8$, $h = 0.7$, and $n_s = 1$. For this set of parameters, the LasDamas team has carried out $N$-body simulations in boxes of different sizes to model galaxies of different luminosities; for the LRGs, they use 40 boxes with volume of $2400^3 (h^{-1}$ Mpc)$^3$, each one containing $1280^3$ dark matter particles with a particle mass of $4.57 \times 10^{11} M_{\odot}$.

These simulations are used to construct mock galaxy catalogs by placing galaxies in dark matter halos using an HOD model (Berlind & Weinberg 2002), with HOD parameters fit from the observed SDSS LRG two-point clustering. For this main set they create two types of galaxy mock catalogs, one that resembles the LRG DR7-Dim catalog, with $z_{\text{max}} = 0.36$, and one that resembles the Bright LRG sample with $z_{\text{max}} = 0.44$. The mocks reproduce the SDSS DR7 geometry, and they include redshift-space distortions from peculiar velocities. These galaxy mocks have been shown to reproduce well the form of the DR7 LRG 2PCF on large scales (Kazin et al. 2010), and we show below that they are in general agreement with the 3PCF measurements as well. Since our DR7-Dim and DR7-Bright samples use only the northern cap of the SDSS, in each LasDamas box it is possible to extract four mock LRG catalogs; in this way, we have 160 mock catalogs for each sample.

3. RESULTS

We present the redshift-space LRG 3PCF measurements in Figures 6 and 7 for configurations with $s$ between 7 and $30 h^{-1}$ Mpc, with $u = r_2/r_1 = 2$. We chose these scales, which go from the mildly nonlinear to the quasi-linear regime, in order to calculate the large-scale bias of LRGs, and use only triangles with $u = 2$ since that puts all triplet distances (i.e., the sides of the triangle) above the $s$ selected. If we used $u = 1$, then our most collapsed triangles would have been in the strong nonlinear regime, affecting our bias estimates. We could also try triangles with $u = 3$ or more, but then our biggest distances would be comparable to the scale of the survey, and also computing the 3PCF becomes prohibitive. We first discuss the measurements on intermediate and large scales and then explore the 3PCF error estimates.

3.1. LRG 3PCF on Intermediate Scales

Figure 6 shows the reduced 3PCF of the DR7 LRG samples on intermediate scales, for $s = 7, 10, 15 h^{-1}$ Mpc. For clarity, we show diagonal error bars only for the DR7-Dim sample, calculating using the 3PCF variance between the LasDamas mocks. For comparison, we also show the 3PCF for the DR3 LRG sample. The solid black curves show the 3PCF of the dark matter from the $N$-body simulations.

The reduced 3PCF shows the general shape dependence expected from gravitational instability theory (Bernardeau et al. 2002): the amplitude is higher for elongated triangle configurations ($\theta \sim 0^\circ, 180^\circ$), reflecting anisotropic velocity flows along density gradients. The configuration dependence is less
pronounced on smaller scales, where quasi-virialized, more isotropic flows and structures dominate. Since these measurements are in redshift space, the configuration dependence also reflects the effects of redshift distortions due to peculiar velocities.

Since LRGs are biased tracers of the dark matter, the 3PCF of the LRGs differs from that predicted for the dark matter. On small scales, the reduced 3PCF amplitude of the LRGs is smaller by a nearly constant factor from that of the dark matter. On larger scales, the nature of the 3PCF bias changes: there is a shape dependence in the offset between the LRG and dark matter amplitudes, with the galaxy 3PCF showing less configuration dependence than that of the dark matter.

Comparing the DR7-Full results to the DR3 measurements, which cover half the volume of the DR7 sample, we find general consistency in the 3PCF amplitude on these intermediate scales. For $s = 15 h^{-1}$ Mpc, the DR3 amplitude is slightly lower than that of the DR7 sample for all configurations; this is a consequence of the smaller volume sampled to estimate the correlation functions: as observed in other studies (see Marín et al. 2008, and references therein), the 3PCF is more sensitive to the sample volume than the 2PCF.

In general, there are no important differences between the DR7-Full and the volume-limited DR7-Dim sample results. There are differences between these and the DR7-Bright sample: the reduced 3PCF for the bright LRGs fluctuates much more and it deviates from the DR7-Dim sample results for the elongated triangles (large $\theta$) for the $s = 7$ and $15 h^{-1}$ Mpc configurations, and in the collapsed triangles (small $\theta$) for the $s = 10 h^{-1}$ Mpc configurations. Since we have used a binning more tailored to measure the DR7-Dim correlations on the DR7-Bright (a much less dense sample) measurements, we expect more fluctuations due to poorer statistics and effects of large structures (Nichol et al. 2006); we will explore the variance of bright LRGs in Section 3.3.

### 3.2. LRG 3PCF on Large Scales

In Figure 7, we show the reduced 3PCF for LRGs on large scales, for $s = 20$ and $30 h^{-1}$ Mpc, with $u = 2.0$. As expected from second-order perturbation theory, the reduced 3PCF shows a stronger configuration dependence on these scales compared to smaller scales, with a dramatic increase in the difference in amplitude between elongated and rectangular configurations at $s \gtrsim 20 h^{-1}$ Mpc. That behavior is seen for both the dark matter and the LRG samples. For $s = 30 h^{-1}$ Mpc, the DR3 LRG 3PCF dependence in $\theta$ is more asymmetrical between collapsed and elongated configurations, as it happens with the dark matter 3PCF. The errors are greatly increased as well, and there are strong anticorrelations, i.e., negative values of $Q(\theta)$, for the rectangular configurations; this is again a consequence of the filamentary shape of the large-scale structure.

The large-scale 3PCF of the DR7-Bright galaxies is similar to the DR7-Dim 3PCF, with less significant fluctuations on larger scales. On these scales the finite-volume effects can be observed more clearly in the 3PCF: comparing the DR7-Full and DR7-Dim LRG samples, we can see that in general they agree within the error bars, and the small differences are due to a combination of finite-volume effects and biasing effects (the DR7-Full sample includes brighter galaxies at large redshifts, which have a different clustering bias than the DR7-Dim sample). Comparing the DR3-Full results compared to DR7-Full and even DR7-Dim, we can see that the DR3-Full 3PCF deviates from the DR7 samples on the elongated configurations (larger scales).

### 3.3. Comparison with LasDamas Mocks

In Figure 8, we compare the 3PCF measurements for the DR7 LRGs, as shown previously in Sections 3.1 and 3.2 to the ones obtained from the LasDamas mocks, with dashed lines for redshift-space measurements, solid lines for real-space measurements. Here we present a representative set of values, with $s = 7, 15,$ and $30 h^{-1}$ Mpc. In the top panels we show the comparison for DR7-Dim galaxies; in general the agreement is good, and usually within the 1$\sigma$ error bars.

In the lower panels, we compare DR7-Bright galaxies and the mock 3PCFs. It can be seen that the variance of the bright mocks is larger compared to the dim ones, and the agreement between the mocks and the data is not as good, especially on smaller scales for the elongated configurations. Although due to their low density less signal-to-noise ratio and more fluctuations in $Q(\theta)$ are expected, the variance from the mocks should reflect this, and that is not the case for the smaller scales. Since the mocks are built in order to fit the measured two-point statistics, it can happen that the mock misses the higher-order statistics, and for future modeling these statistics have to be considered.

Note also the differences between the real- and redshift-space measurements in the LasDamas mocks. Both have a dependence on shape, but on small scales the real-space 3PCF has a more pronounced shape dependence. On bigger scales, especially around $s = 15 h^{-1}$ Mpc, they are very similar within the errors. Comparing these differences to the ones found by Marín et al. (2008), there are two causes of this: first, the LRGs are galaxies that inhabit massive halos (e.g., Kulkarni et al. 2007; Zheng et al. 2009, among others), which are in the center of the gravitational potential wells of the large matter structures, and therefore they are less affected peculiar velocities. Second, the 3PCF on larger scales is much more affected by the morphology of the structures than from the peculiar velocities, and therefore on the largest scales the distortions of the reduced 3PCF will be smaller than on the small scales. There is also a resolution (or binning) component to this; in very thin bins we might be able to detect better the distortions on large scales. We will use this similarity between the real- and redshift-space 3PCFs below in Section 5.2 to estimate real-space bias parameters from the 3PCF.

### 4. 3PCF ERROR ESTIMATES

Two methods have been commonly employed to estimate errors in clustering measurements: the variance among mock samples and JK sampling. Zehavi et al. (2005b) showed that the JK method can be reliable for obtaining the covariance matrix in the 2PCF, comparing with mock catalogs from independent realizations. In principle, the errors on the 3PCF depend on higher-order correlations up to sixth order (see, e.g., Szapudi 2005) and could be calculated analytically once the latter are measured. In practice, including edge effects and shot noise contributions makes the analytic computation of these errors a complicated and computationally challenging problem. The JK, in which one sequentially removes subvolumes and computes the variance among 3PCF measurements for the remaining volumes, provides a convenient computational short cut, but there is no fundamental basis for assuming it is accurate in this context. Indeed, the JK method assumes that the removed subvolumes are independent, ignoring density perturbation
modes on scales larger than those of the subvolumes. The variance in 3PCF estimates among a large number of simulated realizations of a given model (mock samples) provides a ground-truth measure of the 3PCF error for that model. If the model provides an accurate representation of the actual galaxy clustering, then the mock sample variance should provide the best estimate of the errors. In the following, we compute and compare the mock variance estimates of 3PCF errors, using the 160 LRG-Dim mocks from the LasDamas simulations, to the JK variance estimations from a representative LasDamas LRG-Dim mock. In this case, we divided the mock catalog into many angular regions with equal number of galaxies (at the limit of large number of subsamples, this is equivalent to having regions with equal volume); each JK subsample is built by extracting one of these regions from the LRG mock.

4.1. Diagonal Errors

For the JK and variance methods, the diagonal errors are given by

$$
\sigma^2_Q = \frac{E_{\text{method}}}{N} \sum_{i=0}^{N} (Q_i - \bar{Q})^2.
$$

For the JK method, $N = N_{\text{JK}}$, the number of subvolumes, and $E_{\text{JK}} = N_{\text{JK}} - 1$; for the variance among mocks, $N$ is the number of independent realizations, and $E_{\text{var}} = 1$.

To have a qualitative estimate of the differences in the diagonal errors between the two methods, we calculate, for each sample, the average of the reduced 3PCF for all angles in a particular configuration, i.e., we calculate $Q(s = 15, u = 2, \theta)$, the mean of $Q(\theta)$ for triangles with $s = 15.0$ $h^{-1}$ Mpc, $u = 2.0$, as a function of the number $N$ of samples. We do this for three values of $\theta$, one corresponding to an almost collapsed triangle ($\theta = 18^\circ$), another for a rectangular configuration ($\theta = 90^\circ$), and one for an almost completely extended configuration ($\theta = 160^\circ$). We chose this scale since the 3PCF here is well behaved and does not have big diagonal errors, but also is large enough to make visible the effects of the JK resampling in the 3PCF (i.e., to test the “independent regions” hypothesis).

The results are shown in Figure 9. In the left panel, we compare $Q(s = 15, u = 2, \theta)$ for mocks and for JK subsamples. The average values converge very fast with $N$, and the fact that the means of the JK samples and the mocks are close to each other (differences at the level of 5% on these scales) is a good signal that we can use the mocks for the covariance measurements. In the right panel, we show the variance (standard deviation) of $Q(s, u, \theta)$ as a function of $N$, for the different methods. We
notice that while the mock’s variance seems to converge at large \( N \), the JK errors tend to increase when we increase the number of subsamples, with differences reaching close to 50% on the elongated configurations.

### 4.2. Covariance Matrices

For \( N \)-point correlation functions, measurements for different configurations are strongly correlated, therefore the off-diagonal errors must be included in model fitting and parameter estimation. Having as many mock realizations or subsamples as possible will help us define better the covariance matrix. Since we cannot have an infinite number of subsamples, if our number of data points is larger than the number of mocks used, when calculating the inverse of the covariance matrix for constraining bias parameters, we need to take into account the singular and numerically ill-defined matrix modes. We describe our approach later in Section 5.

Figure 10 shows the correlation matrices of the 3PCF for configurations with \( s = 15 \, h^{-1} \) Mpc, \( u = 2 \), and varying \( \theta \), comparing the JK (top row) and mock-covariance (middle row) methods, for different numbers of JK subvolumes and mock realizations. For both methods, the off-diagonal correlation coefficients increase with increasing \( N \), which goes from 10 to 160 subsamples/realization.

For these configurations, the off-diagonal terms tend to converge when \( N \geq 40 \). As can be seen in the bottom row of Figure 10, where we show the ratio between the off-diagonal terms of the covariance matrices, \( |C_{ij}^{mocks}(N)/C_{ij}^{JK}(N)| \), both methods yield similar results except for the terms relating very collapsed and very elongated configurations, where the JK configurations have higher correlations. This enhanced covariance for the JK methods is due to the fact that this method uses only one “realization”; while on small scales the regions can be effectively independent, on large scales (i.e., when examining large triangles) these regions are correlated, and then the JK assumption breaks down.

Even though for large \( N \) the covariance matrix converges for the JK method, and we would be able to extract more information from the covariance matrix, we would be introducing artificial correlations since the bigger \( N \) is, the smaller and “less independent” the volumes of the subsamples are. From this analysis we can conclude that the JK method leads to significant biases in the estimation of the covariance matrix for the 3PCF, and therefore using adequate mock catalogs is preferable. In our analysis of large-scale clustering (for bias and cosmological parameters) we will use the 3PCF error covariance matrices from the variation among the 160 mock catalogs.

### 5. LRG BIASC

In the following, we quantify the differences between dark matter and LRG clustering, known as galaxy biasing. As we have seen above, there are important differences between them, and, in a similar fashion to what can be done from two-point correlations, we would like to measure the bias of the LRG 3PCF to obtain cosmological information. One way to relate dark matter and galaxy clustering in real space is to adopt a deterministic and local bias model (e.g., Fry & Gaztanaga 1993; Frieman & Gaztanaga 1994),

\[
\delta_{\text{gal}} = f(\delta_{\text{dm}}) = b_1\delta_{\text{dm}} + \frac{b_2}{2}\delta_{\text{dm}}^2 + \cdots, \tag{8}
\]

where \( \delta_{\text{gal}} \) and \( \delta_{\text{dm}} \) are the local galaxy and dark matter overdensities smoothed over some scale \( R \). To leading order, this bias prescription leads to a relation between the galaxy and dark matter reduced 2PCF and connected 3PCF amplitudes,

\[
\xi_{\text{gal}}(r) \approx b_1^2\xi_{\text{dm}}(r) \tag{9}
\]

\[
\xi_{\text{gal}}(r_{12}, r_{23}, r_{31}) \approx b_1^3\xi_{\text{dm}}(r_{12}, r_{23}, r_{31}) + b_2b_1^2[\xi_{\text{dm}}(r_{12})\xi_{\text{dm}}(r_{23}) + \text{perm.}] \tag{10}
\]

In addition to the bias, the 3PCF can help breaking the degeneracy between the bias and the rms variance in density of spheres with radius \( R = 8 \, h^{-1} \) Mpc, \( \sigma_8 \). This parameter is related to the overall amplitude of the dark matter 2PCF. If we use the 2PCF in Equation (9), adopting a fiducial \( \sigma_8^{\text{fid}} \), then
there is a degeneracy between $\sigma_8$ and $b_1$ in linear theory (Pan & Szapudi 2005):

$$
\xi_{\text{gal}} = b_1^2 \left( \frac{\sigma_8^{\text{fid}}}{\sigma_8^8} \right)^2 \xi_{\text{dm}, \sigma_8^8}. \tag{11}
$$

For the 3PCF, there is a factor $(\sigma_8/\sigma_8^8)^4$ that needs to be multiplied the right-hand side of Equation (11). To find the best fit for the bias parameters and $\sigma_8$, we use dark matter correlations in a particular fiducial cosmology, and we use Equation (12) for the 2PCF fit; for the reduced 3PCF, in this prescription reads

$$
Q_{\text{gal}} = \frac{1}{c_1} (Q_{\text{dm}} + c_2), \tag{12}
$$

where $c_1 = b_1$ and $c_2 = b_2/b_1$, where the dependence on $\sigma_8$ cancels, thus allowing us to break the degeneracy between $b_1$ and $\sigma_8$ from the 2PCF. We have to remember, however, that these approximations and expansions are applicable only on large scales and in real space, whereas our measurements are made in redshift space, where correlations are distorted due to peculiar velocities. Therefore, we have to test the validity of these formulas, and try to quantify their effects on the scales we are using for these comparisons. We will test how well this bias prescription captures the clustering statistics by fitting these relations to the dark matter and galaxy two- and three-point correlation functions.

5.1. Constraints on Bias Parameters and $\sigma_8$ in Redshift Space

We compare the dark matter (from the $N$-body simulations described in Section 2.5) and LRG 2PCF and 3PCF in order to constrain bias parameters using relations (11) and (12); in this case assuming these relations are valid in redshift space, i.e., $\xi_{\text{r-space}} \rightarrow \xi_{\text{z-space}}$ and $Q_{\text{r-space}} \rightarrow Q_{\text{z-space}}$. We use all configurations with $s \geq 10 h^{-1}$ Mpc, and $s \geq 15 h^{-1}$ Mpc with $u = 2$ to measure the joint likelihood of the bias parameters and $\sigma_8$.

As described by Gaztañaga & Scoccimarro (2005) and used in Gaztañaga et al. (2005) and Marín et al. (2008), we minimize

$$
\chi^2 = \sum_{i=1}^{N_b} \sum_{j=1}^{N_b} \Delta_i C_{ij,SVD}^{-1} \Delta_j, \tag{13}
$$

where $N_b$ is the number of configurations used ($N_b = 60$ for triangles with $s \geq 10 h^{-1}$ Mpc and $N_b = 45$ for triangles with $s \geq 15 h^{-1}$ Mpc). Since we use both two- and three-point correlations we have

$$
\Delta_i = (\xi(r_3)^{\text{obs}} - \xi(r_3)^{\text{model}})/\sigma_{\xi(r_3)^{\text{obs}}}, \quad \text{for } i \leq N_b, \tag{14}
$$

$$
\Delta_i = (Q(i)^{\text{obs}} - Q(i)^{\text{model}})/\sigma_{Q(i)}, \quad \text{for } i > N_b, \tag{15}
$$

where $\xi(r_3)^{\text{model}}$ and $Q^{\text{model}}$ are given by Equations (9) and (12) for the galaxies. The matrix $C_{ij,SVD}$ is the normalized covariance matrix. This covariance matrix is calculated from the LasDamas LRG mock catalogs; $\sigma_{\xi(r_3)^{\text{obs}}}$ and $\sigma_{Q(i)}$ are the uncertainties in $\xi(r_3)$ and $Q(i)$, respectively, and we assign them the error bars from the mocks. As mentioned in Section 4.2, we need to use a number of modes smaller than the number of mocks to avoid adding or subtracting noise from places where the inverse of covariance matrix is singular. Using $N_{\text{mocks}} = 160$, in principle we do not have that problem, but still we are subject to numerical rounding errors when inverting the covariance matrix. Therefore, we follow the suggestion of Gaztañaga & Scoccimarro (2005): when using a limited number of mocks, we recalculate the inverse of the covariance matrix with the highest modes obtained from a singular value decomposition (SVD), and use the modes where its SVD eigenvalues are $\lambda < \sqrt{2/N_{\text{mocks}}}$. This in practice lowers the number of degrees of freedom when estimating $\chi^2$; they will be only equal to the number of eigenmodes used minus the parameters constrained.

The results are shown in Figures 11 and 12, where we use the covariance matrix from the 160 LasDamas mocks for each sample to constrain $c_1$, $c_2$, and $\sigma_8$. In Figure 11 we show

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**Figure 10.** Normalized correlation matrices for the reduced, redshift-space 3PCF $Q(s = 15 h^{-1}$ Mpc, $u = 2, \theta$) measured in LasDamas mock catalogs, using jackknife resampling for one LasDamas LRG-Dim mock (top), and the covariance of mocks from independent realizations (middle), for $N = 10, 20, 40, 80, 160$. The bottom row shows the ratio of each matrix element $|C_{ij}^{\text{mocks}}(N)/C_{ij}^{\text{fid}}(N)|$. Each matrix element $i, j$ corresponds to the correlation coefficient of $Q(\theta = \pi/30 + \pi(i - 1)/15)$ and $Q(\theta = \pi/30 + \pi(j - 1)/15)$. The results are shown in Figures 11 and 12, where we use the covariance matrix from the 160 LasDamas mocks for each sample to constrain $c_1$, $c_2$, and $\sigma_8$. In Figure 11 we show...
Figure 11. Best-fit marginalized likelihood of bias parameters and $\sigma_8$ from the 2PCF and 3PCF using the LRG DR7-Dim sample. In the top plots, contours represent $\Delta \chi^2 = 1.0, 2.3,$ and $6.2$. In the bottom plots, LRG DR7-Dim 3PCF in symbols and lines represent biased dark matter 3PCF values, see best-fit parameters in Table 2.

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the results for the DR7-Dim sample, and in Figure 12, the results for the DR7-Bright sample. In the top panels of each figure we show constraints on the bias parameters and $\sigma_8$, for two sets of configurations: solid lines use triangles with $s \geq 10\, h^{-1}\text{Mpc}$ and dotted lines use only the largest triangles $s \geq 15\, h^{-1}\text{Mpc}$. The contours correspond to $\Delta \chi^2 = 1.0, 2.3,$ and $6.2$ (corresponding to $1\sigma$ limits for one-parameter fit, $1\sigma$ limit for two-parameter fit, and $2\sigma$ limit for two parameters, respectively).

We can see that in both samples a zero nonlinear bias is excluded at $2\sigma$ significance. We can see that, as expected, DR7-Dim galaxies have a lower linear bias $c_1$ than the DR7-Bright, and there is a bigger difference in the nonlinear bias parameter $c_2$. Using different sets of triangles changes not only the area of the confidence contours, but also the best-fit results; going to smaller scales put us in the direction of the nonlinear regime, where the relations in Equations (11) and (12) are less valid. Observe that the sizes of the confidence intervals are not small: this is an effect of using a fraction of the total number of eigenmodes due to the limited number of mock catalogs, and also that our errors increase on large scales.

With respect to the constraints on $\sigma_8$, in general they agree with the WMAP5 results (Komatsu et al. 2009), but they are more dependent on which configurations are used.

The best-fit values and $1\sigma$ errors are also presented in Table 2. We can see that the values of $\chi^2/\text{dof}$ are robust for the DR7-Dim sample and show that the model is a good fit for the data. For the highly biased DR7-Bright galaxies the $\chi^2/\text{dof}$ values are somewhat worse; we believe this is primarily due to the higher uncertainties and systematics when measuring the 3PCF for the brighter galaxies rather than an inadequacy of the bias model used.

In the lower two panels we show the LRG 3PCF along with the dark-matter-biased 3PCF, using the best-fit bias values mentioned above. For the DR7-Dim sample the fits are quite good, and there are no significant differences between the fits from using different triangles (solid line corresponds to fit using $s \geq 10\, h^{-1}\text{Mpc}$ triangles whereas dotted lines use only $s \geq 15\, h^{-1}\text{Mpc}$ triangles). For the brighter sample, the fits are less good for configurations with $\theta > 100^\circ$.

We explore how the constraints on the bias parameters change if $\sigma_8$ is fixed (instead of being marginalized), and compare results to the analytical bias prediction from the HOD models. In Figure 13 we show these results, using only $s \geq 15\, h^{-1}\text{Mpc}$ configurations. On the left side, the vertical ellipses correspond to constraints on the bias parameters using both the 2PCF and 3PCF, using a model where $\sigma_8$ is fixed. As expected, a lower $\sigma_8$ means a higher $c_1$. The constraints on $c_1$ are tight, due to the lower errors in $\xi$, leaving mostly the uncertainty on $c_2 = b_2/b_1$.

We compare these bias parameters with that expected from the analytical predictions of the best-fit HOD from Zheng et al. (2009, using projected 2PCF only) for both linear and nonlinear biases in the last row of Table 2, and we have good agreement.

Our error bars, in the case of $\sigma_8 = 0.8$, confirm that a zero
Figure 12. Best-fit marginalized likelihood of bias parameters and $\sigma_8$ from the 2PCF and 3PCF using the LRG DR7-Bright sample. In the top plots, contours represent $\Delta \chi^2 = 1, 2, 3, \text{and } 6.2$. In the bottom plots, LRG DR7-Bright 3PCF in symbols and lines represent biased dark matter 3PCF values, see best-fit parameters in Table 2. (A color version of this figure is available in the online journal.)

Table 2
Best-fit Bias Parameters in Redshift Space

| Sample       | $s_{\text{min}}^a$ | Fit                        | $c_1$       | $c_2$       | $\sigma_8$ | $\chi^2_{\text{dof}} / \text{dof}^b$ |
|--------------|--------------------|----------------------------|-------------|-------------|------------|--------------------------------------|
| DR7-Dim      | 10                 | 2PCF and 3PCF, three-parameter | $1.92^{+0.3}_{-0.1}$ | $0.38^{+0.02}_{-0.08}$ | $0.82^{+0.05}_{-0.13}$ | 1.31/(70 − 3)                      |
| DR7-Dim      | 15                 | 2PCF and 3PCF, three-parameter | $2.05^{+0.1}_{-0.1}$ | $0.58^{+0.12}_{-0.08}$ | $0.76^{+0.05}_{-0.07}$ | 0.95/(57 − 3)                      |
| DR7-Bright   | 10                 | 2PCF and 3PCF, three-parameter | $1.92^{+0.2}_{-0.1}$ | $0.55^{+0.12}_{-0.13}$ | $0.84^{+0.04}_{-0.05}$ | 1.76/(68 − 3)                      |
| DR7-Bright   | 15                 | 2PCF and 3PCF, three-parameter | $1.95^{+0.2}_{-0.1}$ | $0.27^{+0.15}_{-0.08}$ | $0.87^{+0.06}_{-0.07}$ | 1.44/(55 − 3)                      |
| DR7-Dim      | 15                 | 2PCF and 3PCF, $\sigma_8 = 0.9$ | $1.77^{+0.2}_{-0.02}$ | $0.31^{+0.05}_{-0.05}$ | $\cdots$ | 0.79/(46 − 2)                      |
| DR7-Dim      | 15                 | 2PCF and 3PCF, $\sigma_8 = 0.8$ | $1.98^{+0.02}_{-0.02}$ | $0.44^{+0.05}_{-0.05}$ | $\cdots$ | 0.73/(47 − 2)                      |
| DR7-Dim      | 15                 | 3PCF, two-parameter           | $1.83^{+0.12}_{-0.05}$ | $0.3^{+0.12}_{-0.05}$ | $\cdots$ | 0.85/(129 − 2)                     |
| DR7-Bright   | 15                 | 2PCF and 3PCF, $\sigma_8 = 0.9$ | $1.98^{+0.02}_{-0.02}$ | $0.44^{+0.05}_{-0.05}$ | $\cdots$ | 0.73/(47 − 2)                      |
| DR7-Bright   | 15                 | 2PCF and 3PCF, $\sigma_8 = 0.8$ | $2.18^{+0.02}_{-0.02}$ | $0.5^{+0.1}_{-0.0}$ | $\cdots$ | 1.28/(52 − 2)                      |
| DR7-Bright   | 15                 | 3PCF, two-parameter           | $2.05^{+0.1}_{-0.1}$ | $0.46^{+0.1}_{-0.1}$ | $\cdots$ | 1.01/(31 − 2)                      |
| DR7-Dim      | ⋯                  | Z08 HOD fit, $\sigma_8 = 0.8$ | $2.22$ | $0.55$ | $\cdots$ | ⋯                                   |
| DR7-Bright   | ⋯                  | Z08 HOD FoF fit, $\sigma_8 = 0.8$ | $2.36$ | $0.79$ | $\cdots$ | ⋯                                   |

Notes.

a All triangles used with $u = 2.0$ where the first side $s \geq s_{\text{min}}$ in $h^{-1}$ Mpc.

b Degrees of freedom (dof) are equal to the number of modes used minus parameters constrained.

5.2. LRG Bias in Real Space

We explore here the validity of using redshift-space correlations to calculate galaxy bias parameters. When mapping the galaxies in the sky, we use the redshift of the galaxy as an
indicator of distance. But since galaxies (and dark matter) are subject to gravitational forces from the surrounding structures, peculiar velocities are added to the cosmological redshift, distorting our mapping of the large-scale structure and, as a consequence, the correlation functions are affected as well (see Hamilton 1998, for a review).

In the case of the 3PCF, there are differences between the real- and redshift-space 3PCF of galaxies and dark matter (e.g., Gaztañaga & Scoccimarro 2005; Marín et al. 2008), and they will affect bias parameters as well. Marín et al. (2008), using mock galaxy catalogs, found good agreement, but not perfect, on the bias parameters in the $L_*$ range, between using real- or redshift-space correlations, using $s \sim 10 h^{-1}$ Mpc configurations, and poorer agreement for brighter galaxies. That was caused by a combination of studying the bias using smaller scales (in the mildly nonlinear regime) and by the small size of the box (only 200 $h^{-1}$ Mpc on the side) for studying these galaxies which have a low spatial density.

As shown in Figure 8, on large scales (with $s \gtrsim 15 h^{-1}$ Mpc), for the mocks, the difference between the redshift- and real-space 3PCFs is small and falls within the errors. In the case of the dark matter, there are differences, but because of the large bias, the mocks indicate that the LRG 3PCF will not show significant differences between the redshift- and real-space measurements on large scales.

In Figure 14, we show the differences in the correlation functions of dark matter and mock LRG galaxies from the N-body simulations by C. Sabiu et al. (2010, in preparation), and how the bias parameters and $\sigma_8$ constraints change from using real- and redshift-space correlations. Note that on small scales there are significant differences between redshift- and real-space 3PCFs, but on larger scales, they are very similar. We have to keep in mind that these similarities occur when using this particular binning scheme, and might not be necessarily true with smaller binning 3PCF resolutions. For the 2PCF, the differences are small, but significant. For the bias constraints, note that real-space measurements have a higher linear and nonlinear bias, and usually the best-fit value of $\sigma_8$ is lower in the real-space fits.

There have been analytical attempts to relate the real- and redshift-space three-point function in Fourier space (i.e., bispectrum, Scoccimarro et al. 1999; Smith et al. 2008), but there are practically no attempts of this in the configuration space, since nonlinearities and mixing of large- and small-scale modes make the calculations very complicated unless one takes very simplified cases or does not take full advantage of the shape
dependence of the three-point statistics (e.g., Pan & Szapudi 2005; Sefusatti et al. 2006). In our case, we take an empirical approach. Using the similarity between the mock LRG 3PCF in real and redshift space for the large scales, we estimate constraints on the bias and $\sigma_8$ parameters using the real-space dark matter correlations, and identifying the real-space 3PCF with the redshift-space 3PCF. In the case of the 2PCF, since the differences between real- and redshift-space correlations are significant, we will use the first-order linear approximation in the case of redshift distortion (Kaiser 1987). Following Pan & Szapudi (2005),

$$\xi_{\text{z-space}} = \left(1 + \frac{2}{3} f + \frac{1}{5} f^2 \right) c_1 \left(\frac{\sigma_8}{0.8}\right)^2 \xi_{\text{r-space}}^2,$$

where $f = \Omega_m^{0.6}/c_1$; we tested this prescription for our HOD mocks and it works well on the largest scales $r > 10/h^{-1}$ Mpc. The results are shown in Figure 15, for both the LRG-Dim and LRG-Bright samples, using triangles with $s \gtrsim 15/h^{-1}$ Mpc. The best-fit values are summarized in Table 3. In general, we see the same trend that was seen in Figure 14: both $c_1$ and $c_2$ are higher in real space, and we get reasonable constraints on $\sigma_8$. We notice that here the constraints are in good agreement with the values obtained from the HOD fit of by Zheng et al. (2009; shown in Table 2).

5.3. HOD Constraints from Large-scale Bias

It is possible to use the bias parameter constraints to estimate best-fit values on the LRG HOD parameters. In the Appendix, we show how it is possible to extract the large-scale bias parameters from the HOD, where only the average of the HOD function $\langle N(M) \rangle$ is needed to obtain the bias parameters $b_1$ and $b_2$. Given the uncertainties in $c_1$, $c_2$ from the 3PCF, and for a fixed $\sigma_8$, it is possible in principle to obtain constraints on HOD parameters (Sefusatti & Scoccimarro 2005) from large-scale bias parameters. We exemplify this using the LRG DR7-Dim sample.

There are many models for the mean $\langle N(M) \rangle$, with different number of parameters. It is worth mentioning that even though the meaning of common HOD parameters such as $M_{\text{min}}$, $M_1$, and $\alpha$ is similar for different functional forms of $\langle N(M) \rangle$, the best-fit parameters can change significantly depending on the function used (Zheng et al. 2005). For the sake of simplicity, and to avoid marginalizing over many parameters, we use a simple model that uses three parameters, with a soft function for $N_{\text{cen}}(M)$ (Sefusatti et al. 2006; Kulkarni et al. 2007):

$$\langle N(M) \rangle = N_{\text{cen}}(M) + N_{\text{sat}}(M),$$

$$N_{\text{cen}} = \exp(-M_{\text{min}}/M_1),$$

$$N_{\text{sat}} = \exp(-M_{\text{min}}/M_1) \left(\frac{M}{M_1}\right)^\alpha.$$
Figure 15. Marginalized bias parameters and $\sigma_8$ confidence intervals from joint 2PCF–3PCF fit for LRGs in real space for the DR7-Dim (top) and DR7-Bright (bottom) LRG samples using the prescription described in Section 5.2. Only $s \geq 15 h^{-1}$ Mpc configurations are used. The contours are $\Delta \chi^2 = 1$, 2.3, and 6.2.

(A color version of this figure is available in the online journal.)

Table 3

| Sample        | $s_{\text{min}}$ | Fit                      | $c_1$       | $c_2$       | $\sigma_8$ | $\chi^2_{\text{dof}} \over \text{dof}^b$ |
|---------------|------------------|--------------------------|-------------|-------------|-------------|------------------------------------------|
| DR7-Dim       | 10               | 2PCF and 3PCF, three-parameter | $2.51^{+0.15}_{-0.1}$ | $0.8^{+0.12}_{-0.04}$ | $0.75^{+0.03}_{-0.02}$ | 1.17/(70 − 3) |
| DR7-Dim       | 10               | 2PCF and 3PCF, three-parameter | $2.38^{+0.2}_{-0.1}$ | $0.9^{+0.1}_{-0.1}$ | $0.77^{+0.02}_{-0.05}$ | 0.79/(57 − 3) |
| DR7-Bright    | 15               | 2PCF and 3PCF, three-parameter | $2.22^{+0.2}_{-0.1}$ | $0.75^{+0.1}_{-0.1}$ | $0.91^{+0.1}_{-0.1}$ | 1.65/(68 − 3) |
| DR7-Bright    | 15               | 2PCF and 3PCF, three-parameter | $2.45^{+0.45}_{-0.25}$ | $0.79^{+0.4}_{-0.1}$ | $0.85^{+0.1}_{-0.1}$ | 1.22/(55 − 3) |

Notes.

a All triangles used with $u = 2.0$ where the first side $s \geq s_{\text{min}}$, in $h^{-1}$ Mpc.

b Degrees of freedom (dof) are equal to the number of modes used minus parameters constrained.

6. SUMMARY AND CONCLUSIONS

The purpose of this work was to show that the LRG 3PCF provides a complementary way to estimate biasing and place constraints on cosmological models. With the advent of new and larger surveys, it is possible to obtain constraints comparable to other methods, in cosmology and galaxy population models. Our main findings are as follows.

1. With a new binning scheme, it is possible to have a better measurement of $\bar{Q}(\theta)$ and go to larger scales, making it possible to use the LRG 3PCF to learn about galaxy bias and $\sigma_8$.

2. The measurement of error in the 3PCF is non-trivial. From measuring the 3PCF both in mocks and JK subsamples, we find that the covariance matrices have noticeable differences, and JKs tend to overestimate the errors and covariance compared to using mocks.

3. Using the redshift-space 2PCF and 3PCF, we estimate constraints on the linear and nonlinear bias parameters.

The results shown above demonstrate the potential of 3PCF measurements to complement and enhance our understanding of galaxy biases and their relationship with cosmological parameters.
We show that the nonlinear bias is non-zero at the 2σ level for LRGs.

4. Using \(N\)-body simulations, for this binning scheme, we find that the predicted LRG 3PCF is very similar (within the errors) for real- and redshift-space measurements on large scales. Using this fact, we estimate bias parameters in real space, as well as \(\sigma_8\), which gives similar results to WMAP5 estimates.

5. From the large-scale bias parameter constraints, we estimate HOD parameters. Our constraints are not very strong, but the best-fit values are similar to what is found by other methods.

Using a new binning scheme that preserves the shape dependence of \(Q(\theta)\), it is possible to measure the 3PCF on a larger range of scales with a better signal-to-noise ratio than previous schemes. We treated here the statistical errors only, but future investigations (outside the scope of this particular work) need also to address the issue of how different resolutions, or binning schemes, can lead to systematic errors in the 3PCF measurements and on bias constraints.

The measurement of errors shows that, provided that the mocks we use describe adequately the correlations of the sample which is analyzed, it is preferable to use these instead of JK resampling as a method to calculate the covariance of 3PCF measurements. As opposed to the case of the 2PCF (e.g., Zehavi et al. 2005b), where the JK provides a good description, in the 3PCF we would need much larger volumes and even there, measure only on a limited range of scales, to be able to trust the JK error measurements. This is more of a problem when we are talking about the covariance of the 3PCF measurements. For diagonal errors, it is possible to obtain them using a small number of subsamples \(N \sim 10\), but to have a converged covariance matrix, \(N \gtrsim 100\) are needed.

The aspects mentioned above will affect the model constraints from the 3PCF, and we will work on these aspects as well in future investigations. We showed that LRGs, or equivalently, high-mass galaxies have a high (non-zero) positive nonlinear bias, and a confirmation from what is predicted by the halo and HOD models: that high mass galaxies (and halos) have large values of the linear and nonlinear bias parameters.

For larger scales, we take advantage of the fact that the LRG real-space 3PCF is similar to the redshift space counterpart. With improved, high-resolution measurements this might not be possible to do, especially if the purpose is to obtain more precise constraints on \(\sigma_8\), but the constraints obtained are consistent with what is expected from simulations and, moreover, they allow us to put complementary constraints on galaxy occupation models.

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This work is presented as a dissertation to the Department of Astronomy and Astrophysics, The University of Chicago, in partial fulfillment of the requirements for a PhD degree.
APPENDIX

LRG HOD AND LARGE-SCALE BIAS

A.1. Halo Mass Function

The distribution of dark matter halos is biased with respect to the dark matter (Press & Schechter 1974), therefore the galaxies, formed inside these halos, are biased tracers as well. As mentioned above, in the HOD model we proceed in two steps: first, we described how dark matter halos are distributed in the universe, and second, how galaxies populate these halos.

To describe the mass function of dark matter halos (i.e., the spatial density of halos as a function of mass) we use the Sheth & Tormen mass function (Sheth et al. 2001; Sheth & Tormen 2002):

$$n_h(M) = \frac{\bar{\rho}_m}{M^2} \frac{d \ln \sigma}{d \ln M} f(v),$$  \hspace{1cm} (A1)

where $\bar{\rho}_m \equiv \Omega_m \rho_c$ is the average matter density in the universe at the epoch of observation, $v = \delta_c / \sigma(M, z)$, with $\delta_c = 1.69$ the threshold for growth of linear fluctuations, and $\sigma$, the rms variance of a sphere of radius $R(M)$ at redshift $z$:

$$\sigma^2(R, z) = \int \frac{d^3k}{(2\pi)^3} |W(k, R)|^2 P(k, z).$$  \hspace{1cm} (A2)

Here, $P(k, z)$ is the linear matter power spectrum and $W(k, R)$ is the Fourier transform of a top-hat window of radius $R$. The function $f(v)$ is motivated by the ellipsoidal collapse model (Sheth & Tormen 2002) and has the form:

$$f(v) = A\left[\frac{2av^2}{\pi}\left[1 + (av^2)^{-p}\right] e^{-av^2/2}\right]^{1/2},$$  \hspace{1cm} (A3)

where for normalization $A = 0.322$, $p = 0.3$, and $a = 0.707$.

A.2. HOD Bias

Knowing the probability function $P(N|M)$ of how galaxies populate halos as a function of mass, we can infer the clustering at any scale. The HOD model allows us to infer the mean occupation number $\langle N(M) \rangle$, in the large-scale limit to estimate the bias parameters:

$$b_i = \frac{1}{n_g} \int_{M_{min}}^{\infty} dM n_h(M) b_i^h(M)\langle N_g(M) \rangle,$$  \hspace{1cm} (A4)

where $i = 1, 2$ represent the linear and first nonlinear bias term, $n_h(M)$ the dark matter halo mass function, $b_i^h(M)$ the halo bias and $\langle N_g(M) \rangle$ is the mean number of galaxies for a halo of mass $M$, and $n_g$ is the galaxy number density, calculated as:

$$n_g = \int_{M_{min}}^{\infty} dM n_h(M) \langle N(M) \rangle.$$  \hspace{1cm} (A5)

The dark matter halo bias parameters are given by (Scoccimarro et al. 2001)

$$b_1^h(M) = 1 + \epsilon_1 + E_1,$$  \hspace{1cm} (A6)

$$b_2^h(M) = \frac{8}{21} (\epsilon_1 + E_1) + \epsilon_2 + E_2,$$  \hspace{1cm} (A7)

where the coefficients are given by:

$$\epsilon_1 = \frac{av^2 - 1}{\delta_c},$$  \hspace{1cm} (A8)

$$\epsilon_2 = \frac{a^2v^2 - 3}{\delta_c^2},$$  \hspace{1cm} (A9)

$$E_1 = \frac{\delta_c^2}{2\bar{\rho}_m},$$  \hspace{1cm} (A10)

$$E_2 = 1 + \frac{2\delta_c}{\bar{\rho}_m} + 2\epsilon_1.$$  \hspace{1cm} (A11)

It is worth emphasizing that the bias relations mentioned in Equation (A4) are only valid on large scales, in the quasi-linear regime, and they are scale independent. On smaller scales the bias becomes very scale dependent and higher moments of the $P(N|M)$ function need to be known.

REFERENCES

A.25. HOD Bias

Knowing the probability function $P(N|M)$ of how galaxies populate halos as a function of mass, we can infer the clustering at any scale. The HOD model allows us to infer the mean occupation number $\langle N(M) \rangle$, in the large-scale limit to estimate the bias parameters:

$$b_i = \frac{1}{n_g} \int_{M_{min}}^{\infty} dM n_h(M) b_i^h(M)\langle N_g(M) \rangle,$$  \hspace{1cm} (A4)

where $i = 1, 2$ represent the linear and first nonlinear bias term, $n_h(M)$ the dark matter halo mass function, $b_i^h(M)$ the halo bias and $\langle N_g(M) \rangle$ is the mean number of galaxies for a halo of mass $M$, and $n_g$ is the galaxy number density, calculated as:

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REFERENCES
Sefusatti, E., & Scoccimarro, R. 2005, Phys. Rev. D, 71, 063001
Seljak, U. 2000, MNRAS, 318, 203
Sheth, R. K., Mo, H. J., & Tormen, G. 2001, MNRAS, 323, 1
Sheth, R. K., & Tormen, G. 2002, MNRAS, 329, 61
Smith, R. E., Sheth, R. K., & Scoccimarro, R. 2008, Phys. Rev. D, 78, 023523
Szapudi, I. 2005, arXiv:astro-ph/0505391v1
Szapudi, I., & Szalay, A. S. 1998, ApJ, 494, L41
Tegmark, M., Eisenstein, D. J., Strauss, M. A., et al. 2006, Phys. Rev. D, 74, 023507
Tinker, J., Kravtsov, A. V., Klypin, A., et al. 2008, ApJ, 688, 709
White, S. D. M., & Rees, M. J. 1978, MNRAS, 183, 341
York, D. G., Adelman, J., Anderson, J. E., Jr., et al. 2000, AJ, 120, 1579
Zehavi, I., Eisenstein, D. J., Nichol, R. C., et al. 2005a, ApJ, 621, 22
Zehavi, I., Weinberg, D. H., Zheng, Z., et al. 2004, ApJ, 608, 16
Zehavi, I., Zheng, Z., Weinberg, D. H., et al. 2005b, ApJ, 630, 1
Zheng, Z., Berlind, A. A., Weinberg, D. H., et al. 2005, ApJ, 633, 791
Zheng, Z., & Weinberg, D. H. 2007, ApJ, 659, 1
Zheng, Z., Zehavi, I., Eisenstein, D. J., Weinberg, D. H., & Jing, Y. P. 2009, ApJ, 707, 554