Monopoles and contact 3-manifolds

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Abstract

We propose the study of some kind of monopole equations directly associated with a contact structure. Through a rudimentary analysis about the solutions, we show that a closed contact 3-manifold with positive Tanaka-Webster curvature and vanishing torsion must be either not symplectically semifillable or having torsion Euler class of the contact structure.

1 Statement of results

In this paper we propose some kind of monopole equations directly associated to a contact structure. By studying the solutions of these equations, we can draw a conclusion about the underlying contact structure.

Given an oriented contact structure \( \xi \) on a closed (compact without boundary) 3-manifold \( M \), we can talk about \( \text{spin}^c \)-structures on \( \xi \) or \( \xi^* \). (see §2 for the definition) Furthermore, associated to an oriented pseudohermitian structure, we have the so-called canonical \( \text{spin}^c \)-structure \( c_\xi \). With respect to \( c_\xi \), we consider the equations (3.9) for our “monopole” \( \Phi \) coupled to the “gauge field” \( A \). Here \( A \), the \( \text{spin}^c \)-connection, is required to be compatible with the pseudohermitian connection on \( M \).

The Dirac operator \( D_\xi \) relative to \( A \) is identified with a certain boundary \( \bar{\partial} \)-operator \( \sqrt{2}(\bar{\partial}_a^b + (\bar{\partial}_b^a)^*) \). (cf.(3.10)) In terms of components \((\alpha, \beta)\) of \( \Phi \), (3.9) is equivalent to

\[
(\bar{\partial}_a^b + (\bar{\partial}_b^a)^*)(\alpha + \beta) = 0 \\
(\text{or } \alpha_{a,1}^0 = 0, \beta_{1,1}^0 = 0) \\
da(e_1, e_2) - W = |\alpha|^2 - |\beta_1|^2
\]

\[(3.11)\]

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On the other hand, there are notions of symplectic fillability and symplectic semi-fillability in the study of contact structures due to Eliashberg. (e.g., [ET], [Kro]) A contact 3-manifold \((M, \xi)\) is symplectically fillable if \(\xi\) is positive (i.e. \(\theta \wedge d\theta > 0\) for any contact form \(\theta\)) with respect to the induced orientation on \(M\) as the boundary of the canonically oriented symplectic 4-manifold \((X, \Omega)\) and \(\Omega|_\xi \neq 0\). If \(M\) consists of a union of components of such a boundary, then it is symplectically semifillable.

Let \(e(\xi)\) denote the Euler class of the contact bundle \(\xi\). We say the equations (3.11) have nontrivial solutions if \(\alpha\) and \(\beta\) are not identically zero simultaneously. Our first step to understand the equations (3.11) is the following result.

**Theorem A.** Suppose there is an oriented pseudohermitian structure with vanishing torsion on a closed 3-manifold \(M\) with an oriented contact structure \(\xi\). Also suppose \(\xi\) is symplectically semifillable, and \(e(\xi)\) is not a torsion class. Then the equations (3.11) (for the canonical \(spin^c\)-structure \(c_\xi\)) have nontrivial solutions.

We remark that our \(M\) in Theorem A must be a Seifert fibre space with even first Betti number by an argument of Weinstein. ([CH]) The idea of proving Theorem A goes as follows. The contact structure \(\xi\) being symplectically semifillable implies that its Euler class \(e(\xi)\) is a so-called monopole class in Kronheimer’s terminology. (see Corollary 5.7 in [Kro]) That is to say, \(e(\xi)\) arises as the first Chern class of a usual (i.e. on \(TM\) or \(T^*M\)) \(spin^c\)-structure for which the usual Seiberg-Witten equations admit a solution for every choice of Riemannian metric on \(M\). By choosing a suitable family of Riemannian metrics adapted to our pseudohermitian structure, we prove that the associated solutions admit a subsequence converging to a nontrivial solution of our equations (3.11). (see §4 for details)

On the other hand, associated to an oriented pseudohermitian structure on a contact manifold is the notion of the so-called Tanaka-Webster curvature \(W\). ([Tan], [Web], [CL], see also §5) The Weitzenbock-type formula tells a nonexistence result: (see §3 for details)

**Theorem B.** Let \((M, \xi)\) be a closed 3-manifold with an oriented contact structure \(\xi\). Suppose there is an oriented pseudohermitian structure on \((M, \xi)\) with \(W > 0\). Then the equations (3.11) have no nontrivial solutions with
The solution we find for Theorem A actually satisfies the condition (1.1). Therefore by Theorems A and B, we can conclude

**Corollary C.** Let \((M, \xi)\) be a closed 3-manifold with an oriented contact structure \(\xi\). Suppose there is an oriented pseudohermitian structure on \((M, \xi)\) with vanishing torsion and \(W > 0\). Then either \(\xi\) is not symplectically semifillable or \(e(\xi)\) is a torsion class.

We remark that Rumin ([Rum]) proved that \(M\) must be a rational homology sphere under the conditions in Corollary C by a different method. On the other hand we feel that we haven’t made use of the full power of equations (3.11). Also note that Eliashberg gives a complete list of classes in \(H^2(L(p, 1), \mathbb{Z})\), which can be realized as Euler classes of fillable contact structures on the lens spaces \(L(p, 1)\). ([Eli])

During the preparation of this paper we noticed that Nicolaescu had a similar consideration of the so-called adiabatic limit as in our proof of Theorem A. ([Nic]) But our viewpoint is sufficiently different from his. Also we noticed that Kronheimer and Mrowka ([KM],[Kro]) had studied contact structures on 3-manifolds via 4-dimensional monopole invariants introduced by Seiberg and Witten. ([Wit])

Since our Dirac operator \(D_\xi\) (also \(da(e_1, e_2)\)) is not elliptic (not even subelliptic) from our knowledge about \(\bar{\partial}_b\)-operator, we do not know how to deal with the solution space of (3.11) in general.

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2 \(\textit{Spin}^c\)-structures on contact bundles

Let \((M, \xi)\) be a smooth contact 3-manifold with oriented contact bundle \(\xi\). Choose an oriented pseudohermitian structure \((J, \theta)\) compatible with \(\xi\) (see §5, the Appendix)
so that \( h(u, v) = \frac{1}{2} d\theta(u, Jv) \) for \( u, v \in \xi \) defines a Riemannian structure on \( \xi \). Let \( \xi^* \) denote the dual of \( \xi \). The \( h \) also induces a Riemannian structure on \( \xi^* \), still denoted \( h \). A \( spin^c \)-structure on \( (\xi^*, h) \) (or similarly on \( (\xi, h) \), cf.[Sal]) is a pair \((W, \Gamma)\) where \( W \) is a 2-dimensional complex Hermitian vector bundle and \( \Gamma : \xi^* \to \text{End}(W) \) is a homomorphism which satisfies

\[
(2.1) \quad \Gamma(v)^* + \Gamma(v) = 0, \Gamma(v)^* \Gamma(v) = |v|^2 h I.
\]

Here \( I \) means the identity endomorphism. Let \( C^c(\xi^*) \) denote the bundle of complexified Clifford algebras of \( \xi^* \). Then \( \Gamma \) extends to an algebra (bundle) isomorphism \( : C^c(\xi^*) \to \text{End}(W) \), still denoted \( \Gamma \). A \( spin^c \)-structure on \( (\xi^*, h) \) (or similarly on \( (\xi, h) \)), cf.[Sal] is a pair \((W, \Gamma)\) where \( W \) is a 2-dimensional complex Hermitian vector bundle and \( \Gamma : \xi^* \to \text{End}(W) \) is a homomorphism which satisfies

\[
(2.2) \quad \nabla_v (\Gamma(w) \Phi) = \Gamma(w) \nabla_v \Phi + \Gamma(\nabla_v w) \Phi
\]

for \( \Phi \in C^\infty(M, W) \) and \( w \in C^\infty(M, \xi^*) \), \( v \in C^\infty(M, TM) \). A \( spin^c \)-connection \( \nabla \) on \( W \) is said to be compatible with the pseudohermitian connection on \( \xi^* \) if it satisfies (2.2) with \( \nabla_v w \) denoting the pseudohermitian connection induced on \( \xi^* \). (see §5 and note that we’ll often view \( \xi^* \) as the orthogonal complement of \( \theta \) in \( T^* M \) with respect to the adapted metric \( \theta \otimes \theta + h \))

Let \( e^1, e^2 \) be a positively oriented orthonormal basis of \( \xi^* \). Denote \( \epsilon = e^2 e^1 \). Then \( \epsilon^2 = -1 \) and thus \( \Gamma(\epsilon) \) has eigenvalues \( \pm i \). Let \( W^\pm = \{ \Phi \in W : \Gamma(\epsilon) \Phi = \pm i \Phi \} \). Then \( W = W^+ \oplus W^- \), and \( \dim C W^\pm = 1 \). Note that \( \Gamma(v) \) maps \( W^\pm \) to \( W^\mp \), and every \( spin^c \)-connection \( \nabla \) on \( W \) preserves subbundles \( W^+ \) and \( W^- \) respectively.

Next we’ll define a canonical \( spin^c \)-structure and connection associated to an oriented pseudohermitian structure \( (J, \theta) \) on our contact manifold \( (M, \xi) \). Let \( \Lambda^{0,1} \xi^* \) be the bundle of complex 1-forms of type \((0, 1) \). (a typical element is \( \theta^1 = e^1 - ie^2 \)) Let \( C(= \Lambda^{0,0}) \) denote the trivial complex line bundle. Consider

\[
(2.3) \quad W_{can} = C \oplus \Lambda^{0,1} \xi^*
\]

with the natural Hermitian structure induced by \( h \). Define \( \Gamma_{can} : \xi^* \to \text{End}(W_{can}) \) by

\[
(2.4) \quad \Gamma_{can}(e^1) \tau = \frac{1}{\sqrt{2}} \theta^1 \wedge \tau - \sqrt{2} \epsilon(e_1) \tau
\]
\[ (2.5) \quad \Gamma_{\text{can}}(e^2)e^\tau = \frac{1}{\sqrt{2}} i\theta^1 \land e^\tau - \sqrt{2} i(e_2)e^\tau \]

where \( \{e_1, e_2\} \) in \( \xi \) is a dual basis of \( \{e^1, e^2\} \), and \( \iota \) denotes the interior product. The above definition is independent of the choice of bases. It is a direct verification that \((W_{\text{can}}, \Gamma_{\text{can}})\) is a \( spin^c \)-structure on \((\xi^*, h)\). Also \( W_{\text{can}}^+ = \mathcal{C}, W_{\text{can}}^- = \Lambda^{0,1}\xi^* \). We call \((W_{\text{can}}, \Gamma_{\text{can}})\) the canonical \( spin^c \)-structure on \((\xi^*, h)\), denoted \( c_\xi \).

We know that the pseudohermitian connection preserves the subspaces \( \Lambda^{0,k}\xi^* \), hence \( W_{\text{can}} \). When it is restricted to \( W_{\text{can}} \), we denote it by \( \nabla_{\text{can}} \).

**Proposition 2.1** \( \nabla_{\text{can}} \) is a \( spin^c \)-connection on \( W_{\text{can}} \), which is compatible with the pseudohermitian connection on \( \xi^* \).

Proof: It is enough to verify (2.2) for \( w = e_1, e^2 \). Let \( f \) be a smooth section of \( \mathcal{C} \), i.e. a smooth complex-valued function on \( M \). Let \( v \) be a tangent vector of \( M \). For simplicity, we use \( \Gamma, \nabla \) instead of \( \Gamma_{\text{can}}, \nabla_{\text{can}} \), respectively. We compute by (2.5), (2.4)

\[
\Gamma(e^1)\nabla_v f + \Gamma(\nabla_v e^1) f = \frac{1}{\sqrt{2}} df(v)\theta^1 + \frac{1}{\sqrt{2}} i f \omega(v)\theta^1 \\
\text{(write } \nabla_v e^1 = \omega(v)e^2 \text{ and } \nabla_v e^2 = -\omega(v)e^1 \text{ where } \omega \text{ is the connection } 1 \text{- form}) \\
= \nabla_v(\frac{1}{\sqrt{2}} f \theta^1) = \nabla_v(\Gamma(e^1)f) \text{ (note that } \nabla_v \theta^1 = i\omega(v)\theta^1) \]

For \( \tau \) being a smooth section of \( \Lambda^{0,1}\xi^* \), we compute

\[
\nabla_v(\Gamma(e^1)\tau) = -\sqrt{2} \nabla_v(\iota(e_1)\tau) \text{ (by (2.4))} \\
= -\sqrt{2} \nabla_v(\tau(e_1)) \\
= -\sqrt{2}(\nabla_v \tau)(e_1) + \tau(\nabla_v e_1) \\
= \Gamma(e^1)\nabla_v \tau + \Gamma(\nabla_v e^1)\tau 
\]

Similarly we can verify (2.2) for \( w = e^2 \).

Q.E.D.
Let $E$ be a Hermitian line bundle over $M$. Let $W = W_{can} \otimes E$, $\Gamma = \Gamma_{can} \otimes \text{id}$. Then $(W, \Gamma)$ defines a $spin^c$-structure on $\xi^*$. (we often suppress the metric $h$) Conversely, we have

**Proposition 2.2** Any $spin^c$-structure $(W, \Gamma)$ on $\xi^*$ is isomorphic to

$$(W_{can} \otimes E, \Gamma_{can} \otimes \text{id})$$

for some Hermitian line bundle $E$.

Proof: Define $\# : \xi \to \xi^*$ by $\#(v) = h(v, \cdot)$. Let $\tilde{\Gamma} = \Gamma \circ \#$. Since $\tilde{\Gamma}(Jv) = \tilde{\Gamma}(v)\Gamma(\varepsilon)$ for $v \in \xi$, we have $\tilde{\Gamma}(Jv)\Phi = -i\tilde{\Gamma}(v)\Phi$ for $\Phi \in W^-$. So $\tilde{\Gamma}(\cdot)\Phi$ is a section of the bundle $\Lambda^{0,1}\xi^\otimes W^+$. Furthermore, the map given by

$$\Phi \mapsto -\frac{1}{\sqrt{2}}\tilde{\Gamma}(\cdot)\Phi$$

is a unitary isomorphism from $W^-$ onto $\Lambda^{0,1}\xi^\otimes W^+$.

Now choose $E = W^+$. (note that $W^+$ is a Hermitian line bundle) It follows that $W^+ \simeq \mathcal{C} \otimes W^+ = \mathcal{C} \otimes E$ and $W^- \simeq \Lambda^{0,1}\xi^\otimes W^+ = \Lambda^{0,1}\xi^\otimes E$. Also it is easy to verify that $\Gamma \simeq \Gamma_{can} \otimes \text{id}$.

Q.E.D.

We remark that if $M$ is a homology sphere, then there exists one and only one $spin^c$-structure on $\xi^*$ (or $\xi$), which is the canonical one.

Let $C_2(\xi^*)$ denote the subspace of $C(\xi^*)$ (the real Clifford algebra of $\xi^*$), consisting of elements of degree 2.

**Proposition 2.3** Given a $spin^c$-structure $(W, \Gamma)$ on $\xi^*$. Let $\nabla^1$, $\nabla^2$ be two $spin^c$-connections on $W$. Then there exists a 1-form $\alpha$ with value in $C_2(\xi^*) \oplus i\mathbb{R}$ so that

$$\nabla^1 - \nabla^2 = \Gamma(\alpha)$$

Conversely, if $\nabla$ is a $spin^c$-connection, so is $\nabla + \Gamma(\alpha)$ for any $C_2(\xi^*) \oplus i\mathbb{R}$-valued 1-form $\alpha$.

The proof of Proposition 2.3 is similar to the usual case for $spin^c$-structures on the tangent bundle. We include a proof for the reference.
Proof: Write $\nabla^1 - \nabla^2 = A$ for some $\text{End}(W)$-valued 1-form $A$ and express the difference of corresponding connections on $\xi^*$ by $a$, a $\text{End}(\xi^*)$-valued 1-form. Taking the difference of (2.2) for $\nabla^1$, $\nabla^2$ gives

$$A(v)\Gamma(w) - \Gamma(w)A(v) = \Gamma(a(v)w)$$

for $v \in TM$, $w \in \xi^*$. Put $A(v) = \Gamma(\alpha_v)$ for some $\alpha_v \in C^c(\xi^*)$. Then the above formula says

(2.6) \[\alpha_v w - w\alpha_v = a(v)w\]

On the other hand, $A(v)$ is skew-Hermitian since $\nabla^1$ and $\nabla^2$ are Hermitian. It follows that

(2.7) \[\alpha_v + \tilde{\alpha}_v = 0\]

where $\tilde{\alpha}_v$ denotes the involution of $\alpha_v$. Now (2.6),(2.7) implies $\alpha_v \in C_2(\xi^*) \oplus i\mathbb{R}$. Let $\alpha(v) = \alpha_v$. Then $\alpha$ is the required 1-form.

For the second part of the Proposition, we define an $\text{End}(\xi^*)$-valued 1-form $a$ by the formula (2.6). Then $\nabla + \Gamma(\alpha)$ is a $\text{spin}^c$-connection on $W$, compatible with $\nabla + a$ on $\xi^*$.

Q.E.D.

Corollary 2.4 Suppose $\nabla^1$, $\nabla^2$ are compatible with the pseudohermitian connection. Then they differ by an imaginary valued 1-form.

Note that in this case, the $a$ in the above proof vanishes.

3 The Weitzenbock formula and the equations

Given a $\text{spin}^c$-structure $(W, \Gamma)$ on the dual contact bundle $\xi^*$ and a $\text{spin}^c$-connection $\nabla$ on $W$, compatible with the pseudohermitian connection on $\xi^*$. We define the associated Dirac operator $D_\xi$ by

$$D_\xi \Phi = \sum_{j=1}^2 \Gamma(e^j) \nabla_{e^j} \Phi$$
for $\Phi$ being a section of $W$ and $\{e^j, j = 1, 2\}$ being the dual of an orthonormal basis $\{e_j, j = 1, 2\}$ in $\xi$.

Let $e_0$ or $T$ denote the vector field characterized by $\theta(T) = 1$ and $\mathcal{L}_T\theta = 0$. Define the divergence $\text{div}(v)$ of a vector field $v$ with respect to the pseudohermitian connection $\nabla^{\psi,h}$ by

$$\text{div}(v) = \Sigma_{i=0}^2 < \nabla^{\psi,h}_{e_i} v, e^i >$$

(note that $e^0 = \theta$, $<,>$ is the pairing, and the definition is independent of the choice of general bases) It follows that $\mathcal{L}_v(\theta \land d\theta) = \text{div}(v)\theta \land d\theta$. So we have

$$\int \text{div}(v)\theta \land d\theta = 0$$

for $M$ being closed (i.e. compact without boundary). (hereafter, we’ll make this assumption)

Since the $\text{spin}^c$-connection $\nabla$ is Hermitian, it is easy to show by (3.1) that its adjoint $\nabla^*$ satisfies the following formula

$$\nabla^* \Phi = -\nabla \Phi - \text{div}(v)\Phi$$

for a section $\Phi$ of $W$. Let $D^*_\xi$ denote the adjoint of $D_\xi$. Writing $D^*_\xi = \Sigma_{i=1}^2 \nabla^*_{e_i} (\Gamma(e^i))^*$ and using (2.1) and (3.2), we obtain $D^*_\xi = D_\xi$, i.e. $D_\xi$ is self-adjoint. (we may assume $\nabla^{\psi,h}_{e_i} e^j = 0$, hence $\text{div}(e_i)=0$, at a point in the computation [Le1])

Now we compute

$$D^*_\xi D_\xi \Phi = D^2_\xi \Phi \text{ (} D_\xi \text{ being self - adjoint)}$$

$$= \Gamma(e^i)\nabla_{e_i}(\Gamma(e^j)\nabla_{e_j} \Phi) \text{ (summation convention)}$$

$$= \Gamma(e^i)\Gamma(e^j)\nabla_{e_i} \nabla_{e_j} \Phi \text{ (} \nabla^{\psi,h}_{e_i} e^j = 0 \text{ at a point } p)$$

$$= \nabla^*_{e_i} \nabla_{e_i} \Phi + \Sigma_{i<j} \Gamma(e^i)\Gamma(e^j)(\nabla_{e_i} \nabla_{e_j} - \nabla_{e_j} \nabla_{e_i}) \Phi$$

(by (3.2) evaluated at $p$)

It is easy to show from the structural equations of pseudohermitian geometry that $[e_1, e_2] = -2T$ at $p$. (cf. (5.8) in §5) Using this, we can rewrite (3.3) as follows:
\[ D_\xi^k D_\xi \Phi = \sum_{i=1}^{2} \nabla_\xi^* \nabla_\xi \Phi + \Gamma(e^1)\Gamma(e^2) F_\nabla(e_1, e_2) \Phi + \Gamma(e^1)\Gamma(e^2) \nabla_{-2T} \Phi \]

where \( F_\nabla(e_1, e_2) = [\nabla_{e_1}, \nabla_{e_2}] - \nabla_{[e_1, e_2]} \) is the curvature operator in the directions \( e_1, e_2 \).

For \( (W, \Gamma) = (W_{\text{can}}, \Gamma_{\text{can}}) \), we can have more precise description with respect to \( \{ 1, \frac{1}{\sqrt{2}} \theta^1 \} \), a basis of \( W_{\text{can}} \). Write \( \Phi \) as a column vector with respect to this basis:

\[ \Phi = \begin{pmatrix} \alpha \\ \beta_1 \end{pmatrix} \text{ for } \Phi = \alpha + \beta_1 \frac{1}{\sqrt{2}} \theta^1. \]

By (2.4), (2.5), we can write \( \Gamma = \Gamma_{\text{can}} \) as matrices:

\[ \Gamma(e^1) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad \Gamma(e^2) = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}. \]

The canonical \( \text{spin}^c \)-connection \( \nabla_{\text{can}} \) has the connection form:

\[ \begin{pmatrix} 0 & 0 \\ 0 & i\omega \end{pmatrix} \]

where \( \omega \) is the pseudohermitian connection form: \( \nabla^{\psi, h} e^1 = \omega e^2 \) as in the proof of Proposition 2.1. So by Corollary 2.4, our \( \text{spin}^c \)-connection \( \nabla \) (compatible with \( \nabla^{\psi, h} \)) equals \( d + A \) with

\[ A = \begin{pmatrix} ia & 0 \\ 0 & i(\omega + a) \end{pmatrix} \]

where \( a \) is a real-valued 1-form. Let \( Z_1 = \frac{1}{2}(e_1 - ie_2) \). A direct computation shows

\[ D_\xi \Phi = \begin{pmatrix} -2\beta^a_{1,1} \\ 2\alpha^a_{1,1} \end{pmatrix} \]

in which \( \beta^a_{1,1} = \beta_{1,1} + ia(Z_1)\beta_1, \quad \alpha^a_{1,1} = \alpha_{1,1} + ia(Z_1)\alpha. \) (covariant derivative without upper index “\( a \)” is with respect to the pseudohermitian connection)

Observe that \( d\omega(e_1, e_2) = -2W \) where \( W \) denotes the Tanaka-Webster curvature. ([CL], [Tan], [Web], or (5.4) in §5) We compute

\[ F_\nabla(e_1, e_2) = dA(e_1, e_2) \]
= \begin{pmatrix} ida(e_1, e_2) & 0 \\ 0 & -2iW + ida(e_1, e_2) \end{pmatrix}

Taking the Hermitian inner product with $\Phi$ in (3.4) and using (3.6), we obtain

\begin{equation}
\|D_\xi \Phi\|^2 = \sum_{j=1}^2 \|\nabla_{e_j} \Phi\|^2 + 2\int_M W|\beta_1|^2 dv_\theta +
\int_M da(e_1, e_2)(|\alpha|^2 - |\beta_1|^2)dv_\theta + 2i\int_M (\alpha_0^a \bar{\alpha} - \beta_1^a \bar{\beta}_1)dv_\theta
\end{equation}

in which $dv_\theta = \theta \wedge d\theta$. (here $''$, $0''$ means the covariant derivative in the $T$-direction)

Define $\pi_\xi$ from 2-forms to functions by $\pi_\xi(\eta) = \eta(e_1, e_2)$, i.e. projecting $\eta$ onto its $e_1 \wedge e_2$-component. It is easy to see ($tr$ means trace)

\begin{equation}
\frac{1}{2} \pi_\xi \circ tr(F^\nabla - \nabla_{can}) = ida(e_1, e_2)
\end{equation}

Let $\Phi^\sigma = (\alpha, -\beta)$ for $\Phi = (\alpha, \beta) \in C \oplus \Lambda^{0,1} \xi^*$. Now we can define our “monopole” equations for $(A, \Phi)$ as follows:

\begin{equation}
\begin{cases}
D_\xi \Phi = 0 \\
\frac{1}{2} \pi_\xi \circ tr(F^\nabla) = i < \Phi^\sigma, \Phi >_h
\end{cases}
\end{equation}

in which $<,>_h$ denotes the Hermitian inner product induced by $h$ on $W_{can}$. Recall that on a $CR$ or pseudohermitian manifold, we have $\bar{\partial}_b$-operator mapping $\Lambda^{p,q}$ to $\Lambda^{p,q+1}$. Also with respect to the connection $\nabla = \nabla_{can} + ia$, we have the associated covariant differentiation $\bar{\partial}^a_b$. For our case, $\bar{\partial}^a_b \alpha = \alpha_1^a \theta^1$ for $\alpha$ being a function while $(\bar{\partial}^a_b)^* \beta = -\sqrt{2} \beta_{1,1}$ for $\beta = \beta_1 \frac{1}{\sqrt{2}} \theta^1$. (note that $\frac{1}{\sqrt{2}} \theta^1$ has length 1 with respect to $<,>_h$)

\begin{equation}
D_\xi = \sqrt{2}(\bar{\partial}^a_b + (\bar{\partial}^a_b)^*)
\end{equation}

Therefore in terms of $(a, \alpha, \beta = \beta_1 \frac{1}{\sqrt{2}} \theta^1)$, (3.9) is equivalent to

\begin{equation}
\begin{cases}
(\bar{\partial}^a_b + (\bar{\partial}^a_b)^*)(\alpha + \beta) = 0 \\
(\text{or } \alpha_1^a = 0, \beta_{1,1}^a = 0) \\
da(e_1, e_2) - W = |\alpha|^2 - |\beta_1|^2
\end{cases}
\end{equation}
by (3.10), (3.5), and (3.8).

**Proof of Theorem B:** Substituting (3.11) and (1.1) in (3.7) gives

\[
0 = \sum_{j=1}^{2} \| \nabla_{e_j} \Phi \|^2 + \int_{M} W(|\alpha|^2 + |\beta_1|^2) dv_{\theta} \\
+ \int_{M} (|\alpha|^2 - |\beta_1|^2)^2 dv_{\theta}
\]

Now the theorem follows from (3.12).

Q.E.D.

### 4 Proof of Theorem A

We define an almost complex structure \( \tilde{J} \) on \( M \times R \), the “symplectification” of the contact manifold \( (M, \xi) \) as follows: \( \tilde{J} = J \) on \( \xi \), \( \tilde{J}(e_3) = e_0, \tilde{J}(e_0) = -e_3 \). Here \( e_3 = \partial/\partial t \), \( t \) being the coordinate of \( R \), and recall that \( e_0 \) is just the vector field \( T \). (see §3 or §5) Let \( g = (dt)^2 + h \) where \( h \) is the adapted metric. (§5) Let \( \{e^j, j = 0, 1, 2, 3\} \) be the dual basis of the orthonormal basis \( \{e_j, j = 0, 1, 2, 3\} \) with respect to the metric \( g \). (recall that \( e_j \) in \( \xi \) and \( e^j \) in \( \xi^* \) for \( j = 1, 2 \) are defined in §2. Of course we have viewed \( \xi^* \) as a subset of \( T^*(M \times R) \) \( \tilde{J} \) also acts on cotangent vectors by \( (\tilde{J}v)(w) = v(\tilde{J}w) \) as usual. Associated to \( \tilde{J} \), we have a canonical spin\(^c \)-structure on \( (M \times R, g) \). The differential forms of type \( (0, \ast) \) constitute the spinors. The Clifford multiplication is defined by

\[
\Gamma(w)^\tau = \frac{1}{\sqrt{2}} w'' \wedge \tau - \sqrt{2} i(w_\#)^\tau.
\]

(cf. [Sal], for instance) Here \( w_\# \) denotes the corresponding tangent vector of the cotangent vector \( w \) with respect to \( g \), and \( w'' = w + i \tilde{J}w \). Let \( \theta^2 = e^3 - ie^0 \). It is easy to compute that \( (e^3)'' = \theta^2, (e^0)'' = i\theta^2 \). (similarly, \( (e^1)'' = \theta^1, (e^2)'' = i\theta^1 \). Recall that \( \theta^1 = e^1 - ie^2 \)) Define a map

\[
\varpi : \mathcal{C} \oplus \Lambda^{0,2}T^*(M \times R) \to W_{can} = \mathcal{C} \oplus \Lambda^{0,1} \xi^*
\]
by deleting the $\sqrt{2}\theta^2$ factor and restricting its domain of definition. (the first $\mathcal{C}$ denotes the trivial complex line bundle over $M \times R$ while the second $\mathcal{C}$ means the trivial complex line bundle over $M$) In practice, we write $\varpi(\alpha + \beta_1 \theta^1 \wedge \theta^2) = \alpha + \sqrt{2} \beta_1 \theta^1$.

Conversely by extending the domain of definition and wedging $\sqrt{2}\theta^2$ in the second component, we get a map $\Xi : W_{can} \to \mathcal{C} \oplus \Lambda^{0,2}T^\ast(M \times R)$ with $\varpi \circ \Xi$ being the identity. We often write $\tilde{\Phi}$ instead of $\Xi(\Phi)$. Now we can define $\rho : T^\ast M \to \text{End}(W_{can})$ by

$$\rho(w)(\Phi) = \varpi \Gamma(e^3) \Gamma(w)(\tilde{\Phi}).$$

Let $\Phi_0$ be the canonical section $(1,0)$ in $W_{can}$. Let $\Phi_1 = \sqrt{2} \theta^1$. A direct computation shows that $\rho(e^0)(\Phi_0) = -i \Phi_0$, $\rho(e^0)(\Phi_1) = i \Phi_1$, $\rho(e^1)(\Phi_0) = -\Phi_1$, $\rho(e^1)(\Phi_1) = \Phi_0$, $\rho(e^2)(\Phi_0) = -i \Phi_1$, $\rho(e^2)(\Phi_1) = -i \Phi_0$. In matrix form with respect to the orthonormal basis $\{\Phi_0, \Phi_1\}$, we have

$$\rho(e^0) = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}, \quad \rho(e^1) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \rho(e^2) = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix}.$$  

Now it is clear that $\rho$ defines a Clifford multiplication. And from the above construction

- $(W_{can}, \rho)$ is isomorphic to the restriction to $M$ of the canonical $\text{spin}^c$-structure induced by $\tilde{J}$. (see the definition of “restriction” in the proof of Proposition 4.3 in [Kro])

There is a canonical $\text{spin}^c$-connection $\nabla_{can}$ on $\mathcal{C} \oplus \Lambda^{0,2}T^\ast(M \times R)$ which is compatible with the Levi-Civita connection $\nabla^g$ of $g$. ([Sal]) We define a connection $\nabla_{can}$ on $W_{can}$ by

$$\nabla_{can}^v \Phi = \varpi(\tilde{\nabla}_{can}^v \tilde{\Phi})$$

for $v$ in $TM \subset T(M \times R)$. Let $\nabla^h$ denote the Levi-Civita connection of the metric $h$ on $M$. Noting that $\nabla^g_v e^3 = 0$, $\nabla^g_v w = \nabla^h_v w$ for $v$ in $TM$, $w$ in $T^*M$ (viewed as a subset of $T^\ast(M \times R)$), we can easily verify that $\nabla_{can}$ is a $\text{spin}^c$-connection on $(W_{can}, \rho)$, compatible with the Levi-Civita connection $\nabla^h$. Note that $\nabla_{can}$ is different from $\nabla_{can}$ in §2 which is compatible with the pseudohermitian connection on $\xi^\ast$. To use the “monopole class” condition, we will choose a special family of Riemannian metrics on $M$. Let
and let \( g_\epsilon = (dt)^2 + h_\epsilon \) be the corresponding metric on \( M \times R \). (recall that \( e^3 = dt \))

So \( e_\epsilon^0 = e\epsilon^0, e_\epsilon^1, e_\epsilon^2 \), and \( e_\epsilon^3 \), resp.) form an orthonormal coframe for \( h_\epsilon \) (\( g_\epsilon \), resp.)

Now with \( e_\epsilon^0, g_\epsilon, h_\epsilon \) replacing \( e_\epsilon^0, g, h \) resp., we can go through the above procedure again to get \( J_\epsilon, \Gamma_\epsilon, \Theta_\epsilon \), \( \omega_\epsilon \), \( \Xi_\epsilon, \Phi_\epsilon, \rho_\epsilon, \nabla_\epsilon^{can} \), and \( \nabla_\epsilon^{can} \). Note that the hermitian metric on \( W^{can}_\epsilon \) does not change. It is easy to verify that \( \rho_\epsilon(e_\epsilon^j) = \rho(e^j) \) for \( j = 0, 1, 2 \).

Here \( e_\epsilon^j = e_\epsilon^0 \) if \( j = 0 \); \( = e^j \) otherwise. Also \( \nabla_\epsilon^{can} \) is a \( \text{spin}^c \)-connection on \( (W^{can}_\epsilon, \rho_\epsilon) \), compatible with the Levi-Civita connection \( \nabla^{hc}_\epsilon \). Recall (see §5, the Appendix) that \( A_{ij}^1 = A_{i1}^1 (h_{11} = 1) \) denotes the pseudohermitian torsion with respect to \((J, \theta)\).

**Proposition 4.1** (1) \( \nabla_\epsilon^{can} \Phi_0 = \frac{i}{\sqrt{2}} \epsilon^{-1} A_{i1} \theta^1 \otimes \Phi_1. \)

(2) \( \nabla_\epsilon^{can} \Phi_1 = \frac{i}{4} \epsilon^{-1} A_{i1} \theta^1 \otimes \Phi_0 + i(\omega + \epsilon \theta) \otimes \Phi_1. \)

**Proof:** Let us review how to obtain \( \nabla_\epsilon^{can} \) from the Levi-Civita connection \( \nabla^{gc} \) on \( M \times R \). ([Sal]) Let \( \Psi \) be an endomorphism of the tangent bundle. Define \( \iota(\Psi) \) acting on a \( k \)-form \( \tau \) by

\[
\iota(\Psi)\tau(v_1, \ldots, v_k) = \sum_{j=1}^k \tau(v_1, \ldots, v_{j-1}, \Psi v_j, v_{j+1}, \ldots, v_k)
\]

for tangent vectors \( v_1, \ldots, v_k \). \( (\iota(\Psi)\tau = 0 \) if \( \tau \) is a function) Let \( N_\epsilon \) denote the Nijenhuis tensor of \( J_\epsilon \). Our canonical \( \text{spin}^c \)-connection \( \nabla_\epsilon^{can} \) is defined by

\[
(4.1) \nabla_\epsilon^{can}_v \tau = \nabla_v^{gc}\tau + \frac{1}{2} \iota(J_\epsilon \nabla_v^{gc} J_\epsilon) \tau + \frac{1}{8} \Theta_\epsilon \wedge \tau + \frac{1}{2} \iota(\Theta_\epsilon) \tau
\]

in which \( \Theta_\epsilon \) is a \((0,2)\)-form defined by \( \Theta_\epsilon(x, y) = g_\epsilon(v, N_\epsilon(x, y)) \), and \( \iota \) in the last term is just the usual interior product (of forms).

Let \( \{Z_1, Z_2, Z_3, Z_4\} \) be a basis dual to \( \{\theta^1, \theta^1, \theta^2, \theta^3\} \). A direct computation using the formula \([Z_1, T] = A_{i1} Z_1 = -\omega_1^1(T)Z_1 \) (cf. (5.9) in §5) shows that \( N_\epsilon(Z_1, Z_2) = 2i\epsilon^{-1} A_{i1} Z_1 \). Hence

\[
(4.2) \Theta_\epsilon = g_\epsilon(v, 2i\epsilon^{-1} A_{i1} Z_1) \theta^1 \wedge \theta^2
\]

(1) follows from (4.1) easily. To compute \( \nabla_\epsilon^{can}_v \Phi_1 \), we need to know \( \nabla_\epsilon^{can}_v (\theta^1 \wedge \theta^2_\epsilon) \).

Let \( \omega^j_{i(\epsilon)} \) be the Riemannian connection forms for \( h_\epsilon \) so that \( \nabla^{hc}_\epsilon e^j = -\omega^j_{i(\epsilon)} \otimes e^i_\epsilon \). Then in the tangent direction of \( M \), we compute

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Using (4.5)
\[ \nabla^g \theta^1 = \nabla^{h_\epsilon} \theta^1 = -\omega^1_{J(\epsilon)} \otimes e^1_\epsilon - i(-\omega^2_{J(\epsilon)} \otimes e^2_\epsilon) \]
\[ = i(\omega^2_{\theta(\epsilon)}(e_1 - i\epsilon^2) - (\omega^1_{\theta(\epsilon)} - i\omega^2_{\theta(\epsilon)}) \otimes e^0_\epsilon \]
\[ = i(\omega + \epsilon\theta) \otimes \theta^1 + (i\epsilon\theta^1 - A^1_1 \theta^1) \otimes \theta \]
by (5.6) and (5.7) for the metric $h_\epsilon$. Note that $e^0_\epsilon = \epsilon \theta$ and the torsion $A^1_1$ for $(J, e^0_\epsilon)$ equals $\epsilon^{-1} A^1_1$.

Let \( \{e^j_\epsilon, j = 0, 1, 2, 3\} \) denote the basis dual to \( \{e^j_\epsilon, j = 0, 1, 2, 3\} \). Then it is easy to see that $e^0_\epsilon = \epsilon^{-1} T$, $e^j_\epsilon = e_j$ for $j = 1, 2, 3$, and $Z^1_1 = Z_1 = \frac{1}{2}(e_1 - i e_2)$, $Z^2_1 = \frac{1}{2}(e_3 - i e_0)$. Using $\nabla^g v^3 = 0$ for $v$ in $TM$ and (5.6), (5.7), we can show that
\[
\nabla^g Z_1 = -i(\omega + \epsilon\theta) Z_1 - \frac{1}{2}(i\theta^1 + \epsilon^{-1} A^1_1 \theta^1) \epsilon^{-1} T.
\]
It follows that
\[
\begin{align*}
(J_\epsilon \nabla^g J_\epsilon) Z_1 &= (-i) J_\epsilon \nabla^g Z_1 + \nabla^g Z_1 \\
&= (\theta^1 - i\epsilon^{-1} A^1_1 \theta^1) \otimes Z^1_1
\end{align*}
\]
Similarly using $\omega^1_0 + i\omega^2_0 = i\theta^1 + A^1_1 \theta^1$ (the complex version of (5.7)) for $h_\epsilon$, we can easily obtain
\[
(J_\epsilon \nabla^g J_\epsilon) Z^1_2 = (\theta^1 + i\epsilon^{-1} A^1_1 \theta^1) \otimes Z^1_1
\]
Since $\iota(J_\epsilon \nabla^g J_\epsilon \theta^1)(w) = \theta^1((J_\epsilon \nabla^g J_\epsilon)(w))$, it follows from the above two formulas that
\[
\iota(J_\epsilon \nabla^g J_\epsilon \theta^1) = (-\theta^1 - i\epsilon^{-1} A^1_1 \theta^1)(v) \otimes \theta^2_\epsilon
\]
for $v$ in $TM$. Replacing $\theta^1$ by $\theta^2_\epsilon$ in the previous computation, we obtain
\[
\nabla^g \theta^2_\epsilon = -\frac{1}{2}(i\epsilon^{-1} A^1_1 \theta^1 + \theta^1)(v) \theta^1 + \frac{1}{2}(\theta^1 - i\epsilon^{-1} A^1_1 \theta^1)(v) \theta^1
\]
\[
\iota(J_\epsilon \nabla^g J_\epsilon \theta^2_\epsilon) = (\theta^1 + i\epsilon^{-1} A^1_1 \theta^1)(v) \theta^1
\]
for \( v \) in \( TM \). On the other hand, it is easy to see that

\[
(4.7) \quad \iota(\tilde{\Theta}_e)^\theta_1 \wedge \theta_2^2 = i\epsilon^{-1}A_1^1\theta^1(v)
\]

by (4.2). Let \( \tilde{\nabla}^{g_e} \) denote the sum of \( \nabla^{g_e} \) and \( \frac{1}{2} \iota(\tilde{J}_e\nabla^{g_e},\tilde{J}_e) \). Now we can compute

\[
\begin{align*}
\epsilon \nabla^{can}\Phi_1 &= \frac{1}{2} \epsilon \tilde{\nabla}^{can}(\theta_1^1 \wedge \theta_2^2) \\
&= \frac{1}{2} \epsilon [\tilde{\nabla}^{g_e}\theta_1^1 \wedge \theta_2^2 + \theta_1^1 \wedge (\tilde{\nabla}^{g_e}\theta_2^2)] + \frac{1}{4} \iota(\tilde{\Theta}_e)^\theta_1^1 \wedge \theta_2^2 \\
&= i(\omega + \epsilon\theta) \otimes \Phi_1 + \frac{1}{4} i\epsilon^{-1}A_1^1\theta^1 \otimes \Phi_0
\end{align*}
\]

by (4.3),(4.4),(4.5),(4.6), and (4.7).

Q.E.D.

Next we’ll deal with the Dirac operator \( D_{A_\epsilon} \) associated to the canonical \( spin^c \)-connection \( \epsilon \nabla^{can} \). Here \( A_\epsilon \) denotes the connection form with respect to the basis \( \{\Phi_0, \Phi_1\} \):

\[
\left( \begin{array}{cc}
0 & \frac{i}{4} \epsilon^{-1}A_1^1 \theta^1 \\
\frac{i}{\sqrt{2}} \epsilon^{-1}A_1^1 \theta^1 & i(\omega + \epsilon\theta)
\end{array} \right).
\]

The Clifford multiplication \( \rho_\epsilon \) of \( \eta = de^0 = 2e^1 \wedge e^2 \) can be easily computed:

\[
(4.8) \quad \rho_\epsilon(\eta)\Phi_0 = 2\rho_\epsilon(e^1)\rho_\epsilon(e^2)\Phi_0 = -2i\Phi_0 \\
\rho_\epsilon(\eta)\Phi_1 = 2\rho_\epsilon(e^1)\rho_\epsilon(e^2)\Phi_1 = 2i\Phi_1
\]

Let \( \ast_\epsilon \) denote the Hodge star-operator with respect to the metric \( h_\epsilon \). Since \( \rho_\epsilon(e^j)\rho_\epsilon(\Omega) = \rho_\epsilon(e^j \wedge \Omega - \iota(e^j)\Omega) \) for an arbitrary function or form \( \Omega \), we can compute that for a scalar function or forms \( \gamma \)

\[
(4.9) \quad \rho_\epsilon(e^j)\rho_\epsilon(\nabla^{h_\epsilon}_{e^j}\gamma) = \rho_\epsilon(e^j \wedge \nabla^{h_\epsilon}_{e^j}\gamma - \iota(e^j)\nabla^{h_\epsilon}_{e^j}\gamma) \\
= \rho_\epsilon((d + d^\ast)\gamma).
\]
Note that \( d^\ast = \ast d \ast \varepsilon \) on 2-forms (changes sign on 1-forms). So for \( \eta = d e^0 = 2 e^1 \wedge e^2 \), we have

\[
(4.10) \quad d^\ast \eta = 2 \ast d \varepsilon = 4 \varepsilon \ast (e^1 \wedge e^2) = 4 \varepsilon^2 e^0.
\]

Now we can compute \( D_A \varepsilon \Phi_0 \) as follows:

\[
-2iD_A \varepsilon \Phi_0 = D_A (\rho_\varepsilon (\eta) \Phi_0) \quad \text{(by (4.8))}
\]
\[
= \sum_{j=0}^2 \rho_\varepsilon (e^j_\varepsilon) [\rho_\varepsilon (\nabla e^h_j \varepsilon) \Phi_0 + \rho_\varepsilon (\eta)^r \nabla^\text{can} e^r_j \Phi_0]
\]
\[
= \rho_\varepsilon ((d + d^\ast) \eta) \Phi_0 + 2iD_A \varepsilon \Phi_0 \quad \text{(by (4.9), Prop.4.1(1), and (4.8))}
\]
\[
= -4i \varepsilon \Phi_0 + 2iD_A \varepsilon \Phi_0 \quad \text{(by (4.10) and } d \eta = 0).
\]

Therefore we obtain

\[
(4.11) \quad D_A \varepsilon \Phi_0 = \varepsilon \Phi_0.
\]

Before computing \( D_A \Phi \) for a general section \( \Phi \) we need two more preparatory formulas. Let \( \alpha \) be a scalar function. It follows easily from (4.9) that

\[
(4.12) \quad \sum_{j=0}^2 \rho_\varepsilon (e^j_\varepsilon)^r \nabla^\text{can} (\rho_\varepsilon (\alpha) \Phi_0) = \rho_\varepsilon (d \alpha) \Phi_0 + \alpha D_A \varepsilon \Phi_0.
\]

Also a direct computation shows

\[
(4.13) \quad \rho_\varepsilon (\theta^1) \Phi_0 = 0, \quad \rho_\varepsilon (\theta^2) \Phi_0 = -2 \Phi_1
\]
\[
\rho_\varepsilon (\theta \wedge \theta^1) \Phi_0 = 0
\]
\[
\rho_\varepsilon (\theta \wedge \theta^1) \Phi_0 = -2i \varepsilon^{-1} \Phi_1
\]

Let \( \Phi = \alpha \Phi_0 + \beta_1 \Phi_1 \) be a section of \( W_{\text{can}} \). (recall \( \Phi_1 = \frac{\theta^1}{\sqrt{2}} \)) Under the condition \( A_{11} = 0 \), \( \varepsilon \nabla^\text{can} \Phi_0 = 0 \) by Proposition 4.1(1). We compute, under this condition,

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\[ D_{A_\epsilon} \Phi = D_{A_\epsilon}[(\rho_\epsilon(\alpha) - \frac{1}{2}\beta_1 \rho_\epsilon(\theta^1)) \Phi_0] (\text{by (4.13)}) \]
\[ = \rho_\epsilon(da) \Phi_0 + \alpha D_{A_\epsilon} \Phi_0 - \frac{1}{2}\rho_\epsilon((d + d^*)((\beta_1 \theta^1)) \Phi_0 (\text{by (4.12), (4.9)}) \]
\[ = -i\epsilon^{-1}\alpha_0 \Phi_0 - 2\alpha_1 \Phi_1 + \alpha \epsilon \Phi_0 + \beta_{1,1} \Phi_0 + i\epsilon^{-1}\beta_{1,0} \Phi_1 + \beta_{1,1} \Phi_0 \]
(4.11), (4.13) and \( \epsilon \theta^1 = i\theta^1 \wedge e^0 \)
\[ = (2\beta_{1,1} - i\epsilon^{-1}\alpha_0 + \epsilon \alpha) \Phi_0 + (i\epsilon^{-1}\beta_{1,0} - 2\alpha_1) \Phi_1 \]

It is known that any two spin\(^c\)-connections compatible with the Levi-Civita connection differ by an imaginary valued 1-form. (e.g. [Sal]) So we can assume a general spin\(^c\)-connection (on \( W_{con} \)) compatible with \( \nabla^{h^c} \) has the connection form \( A_\epsilon + iaI \) (with respect to the basis \{\Phi_0, \Phi_1\}) with \( a \) being a real valued 1-form and \( I \) being a 2\( \times \)2 identity matrix. Now we compute

\[ D_{A_\epsilon + iaI} \Phi = D_{A_\epsilon} \Phi + \Sigma_{j=0}^2 \rho_\epsilon(e_j^F)(ia(e_j^F) \Phi) \]
\[ = (2\beta_{1,1} - i\epsilon^{-1}\alpha_0 + \epsilon \alpha) \Phi_0 + (i\epsilon^{-1}\beta_{1,0} - 2\alpha_1) \Phi_1 \]

in which \( \alpha_0 = \alpha_0 + ia(T) \alpha, \alpha_1 = \alpha_1 + ia(Z_1) \alpha, \beta_{1,0} = \beta_{1,0} + ia(T) \beta_1, \beta_{1,1} = \beta_{1,1} + ia(Z_1) \beta_1 \). So the Dirac equation \( D_{A_\epsilon + iaI} \Phi = 0 \) is equivalent to

\[ \begin{align*}
2\beta_{1,1} - i\epsilon^{-1}\alpha_0 + \epsilon \alpha = 0 \\
i\epsilon^{-1}\beta_{1,0} - 2\alpha_1 = 0
\end{align*} \]

(4.14)

Next we’ll express the second one of Seiberg-Witten monopole equations in a workable form. Let \( b = \frac{1}{2} \text{tr}(A_\epsilon + iaI) \). It follows from Proposition 4.1 that

\[ b = \frac{1}{2}i(\omega + \epsilon \theta) + ia. \]

(4.15)

Let \( F_A \) denote the curvature 2-form of \( A \). Write \( F_b = iF_b^{12} e^1 \wedge e^2 + iF_b^{01} e^0 \wedge e^1 + iF_b^{02} e^0 \wedge e^2 \). It is easy to see

\[ \rho_\epsilon(F_b) = \begin{pmatrix}
F_b^{12} \\
\epsilon^{-1}(F_b^{01} + iF_b^{02}) \\
\epsilon^{-1}(F_b^{01} - iF_b^{02}) \\
-F_b^{12}
\end{pmatrix} \]

(4.16)
with respect to the orthonormal basis \( \{ \Phi_0, \Phi_1 \} \). On the other hand, the trace free part of the endomorphism \( \Phi \otimes \Phi^* = h(\Phi, \cdot) \Phi \), denoted \( \{ \Phi \otimes \Phi^* \} \), reads

\[
\{ \Phi \otimes \Phi^* \} = \left( \begin{array}{cc}
\frac{1}{2}(|\alpha|^2 - |\beta_1|^2) & \frac{\alpha \beta_1}{\bar{\alpha} \beta_1} \\
\frac{1}{2}(|\beta_1|^2 - |\alpha|^2) & \frac{\bar{\alpha} \beta_1}{\alpha \beta_1}
\end{array} \right)
\]

with respect to the orthonormal basis \( \{ \Phi_0, \Phi_1 \} \). From (4.16), (4.17) the equation \( \rho_\nu(F_b) = \rho_\nu(\frac{1}{2}tr F A_{\nu i} + i) = \{ \Phi \otimes \Phi^* \} \) is equivalent to the following system:

\[
\begin{aligned}
F_{12}^b = & \frac{1}{2}(|\alpha|^2 - |\beta_1|^2) \\
\bar{\epsilon} - \epsilon^{-1} (F_{01}^b + i F_{02}^b) = & \bar{\alpha} \beta_1
\end{aligned}
\]

Before analyzing the behavior of solutions for the Seiberg-Witten monopole equations (4.14), (4.18) as \( \epsilon \to 0 \), we need one more result which relates the scalar curvature \( R_{h_\epsilon} \) of the metric \( h_\epsilon \) to the Tanaka-Webster curvature \( W \) of the background pseudo-hermitian structure \((J, \theta)\).

**Lemma 4.2:** \( R_{h_\epsilon} = 4W - \epsilon^2 - \epsilon^{-2}|A_{11}|^2 \).

**Proof:** We use the notation in [CH]. Consider a new coframe \( \tilde{\omega}_3 = \epsilon^2 \omega_3, \tilde{\omega}_1 = \epsilon \omega_1, \tilde{\omega}_2 = \epsilon \omega_2 \). The corresponding connection forms in the structural equations for the adapted metric \( \epsilon^2 h_\epsilon = (\tilde{\omega}_3)^2 + (\tilde{\omega}_1)^2 + (\tilde{\omega}_2)^2 \) read \( \tilde{\psi}_3 = \psi_3, \tilde{\psi}_1 = \epsilon^{-1} \psi_1, \tilde{\psi}_2 = \epsilon^{-1} \psi_2 \).

To satisfy (36) in [CH], the \( L_{ij} \)'s transform as below: \( \tilde{L}_{ij} = \epsilon^{-4} L_{ij} \) for \( i, j \) in \( \{1, 2\} \); \( \tilde{L}_{ij} = \epsilon^{-3} L_{ij} \) if one of indices is 3. To determine \( \tilde{L}_{33} \) we group the coefficients of \( \omega_1 \wedge \omega_2 \) in the right-hand side of the third equation in (36) of [CH] to get

\[
\frac{1}{2} \epsilon^{-2}(L_{11} + L_{22}) + \epsilon^2 \tilde{L}_{33} = 4W.
\]

(here we have used \( d\psi_3 = 4W \omega_1 \wedge \omega_2 \) and note that \( \psi_3 \) is just \(-\omega \) in our notation)

Now we can compute the scalar curvature of the metric \( \epsilon^2 h_\epsilon \):

\[
R_{\epsilon^2 h_\epsilon} = \tilde{L}_{11} + \tilde{L}_{22} + \tilde{L}_{33} - 1 ([CH])
\]

\[
= \epsilon^{-2}(4W - \epsilon^2 - \epsilon^{-2}|A_{11}|^2).
\]

(by (4.19) and \( \frac{L_{11} + L_{22}}{2} = -|A_{11}|^2 \) due to (38), (40) in [CH])

Our result follows from the above formula and the dilation relation: \( R_{h_\epsilon} = \epsilon^2 R_{\epsilon^2 h_\epsilon} \).
PROOF OF THEOREM A:

According to Corollary 5.7 and the proof of Proposition 4.3 in [Kro], the contact structure \( \xi \) being symplectically semifillable implies that the Euler class \( e(\xi) \) is a monopole class for the restriction to \( M \) of the canonical \( spin^c \)-structure of “bounded” symplectic 4-manifold. The \((W_{can}, \rho_\epsilon)\) provides such a \( spin^c \)-structure. (note that they are isomorphic to each other for different \( \epsilon \)'s and the first Chern class of \( W_{can} \) is just \( e(\xi) \)) So for the given metric \( h_\epsilon \), we have a solution \((\Phi = \Phi_\epsilon, a = a_\epsilon)\) of (4.14) and (4.18). Recall that \( \Phi = \alpha \Phi_0 + \beta \bar{\Phi}_1 \), and we sometimes write \( \alpha_\epsilon, \beta_1^\epsilon \) instead of \( \alpha, \beta_1 \) to indicate the \( \epsilon \)-dependence.

Now an application of the Weitzenbock formula for the Seiberg-Witten equations ([Kro] or [Sal]) gives the following estimate: \( \Phi \equiv 0 \), or, under the assumption \( A_{11} = 0 \),

\[
(4.20) \quad \sup | \Phi |^2_{h_\epsilon} \leq \sup(-R_{h_\epsilon}) = \sup(-4W) + \epsilon^2
\]

by Lemma 4.2. The situation \( \Phi \equiv 0 \) is ruled out by the assumption that \( e(\xi) \) is not a torsion class: \( \Phi \equiv 0 \) implies \( F_b = 0 \) which represents the first Chern class \( c_1(W_{can}) \) of \( W_{can} \) up to a constant. But \( c_1(W_{can}) \) is just \( e(\xi) \).

The (4.20) tells that \( \alpha \) and \( \beta_1 \) are uniformly bounded (i.e. there is an upper bound independent of \( \epsilon \)). We’ll use \( O(1) \) to mean an uniformly bounded function or form. Also we use \( O(\epsilon^k) \) to mean a function or form bounded by a constant times \( \epsilon^k \). By (4.18) we have

\[
(4.21) \quad F_b^{12} = O(1), F_b^{01} = O(\epsilon), F_b^{02} = O(\epsilon).
\]

From (4.21) and a theorem of Uhlenbeck (e.g. [Sal]), \( b \) is uniformly bounded in the \( L^p \)-norm for any \( p > 1 \) in Coulomb gauges. (all our norms and the star-operator are with respect to the fixed metric \( h \)) It follows from (4.19) that

**Lemma 4.3:** For a sequence \( \epsilon_j \to 0 \), \( a_{\epsilon_j} \) converges weakly in \( L^1_{\epsilon_j} \subset C^\alpha \) to \( \hat{a} \).

On the other hand we write (4.14) in a matrix form as follows:

\[
(4.22) \quad (\epsilon^{-1} \nabla_T^\epsilon + \nabla_\Xi^\epsilon) \Phi = -\Lambda_\epsilon \Phi
\]
in which

\[
\nabla^c_T = \begin{pmatrix}
-i\nabla^a_T & 0 \\
0 & i\nabla^a_T
\end{pmatrix}, \quad \nabla^c = \begin{pmatrix}
0 & 2\nabla^a_{Z_1} \\
-2\nabla^a_{Z_1} & 0
\end{pmatrix}, \quad \Lambda_\epsilon = \begin{pmatrix}
\epsilon & 0 \\
0 & 0
\end{pmatrix}.
\]

Taking the square $L^2$-norm of both sides of (4.22) and noting that $\nabla^c_T, \nabla^c, \Lambda_\epsilon$ are all self-adjoint, we obtain

\[
\tag{4.23}
\epsilon^{-2}\|\nabla^c_T \Phi\|^2_{L^2} + \|\nabla^c \Phi\|^2_{L^2} + \epsilon^{-1} < \{\nabla^c_T, \nabla^c\}\Phi, \Phi > = < \Lambda^2_\epsilon \Phi, \Phi > = \epsilon^2\|\alpha\|^2_{L^2}.
\]

where $< \cdot, \cdot >$ denotes the $L^2$-inner product induced by the metric $h$ and $\{\nabla^c_T, \nabla^c\} = \nabla^c_T \nabla^c + \nabla^c \nabla^c_T$.

**Lemma 4.4:** Let $F^0_b = F^0_{b1} + iF^0_{b2}$. Then

\[
\{\nabla^c_T, \nabla^c\} = \begin{pmatrix}
0 & 2iA_{11}\nabla_{Z_1} \\
2iA_{11}\nabla_{Z_1} & 0
\end{pmatrix} + \begin{pmatrix}
0 & iA_{11,1} - 2A_{11}a(Z_1) \\
F^0_b + iA_{11,1} - 2A_{11}a(Z_1) & 0
\end{pmatrix}.
\]

**Proof:** A direct computation shows

\[
\tag{4.24}
\{\nabla^c_T, \nabla^c\} \begin{pmatrix}
\alpha \\
\beta_i
\end{pmatrix} = i \begin{pmatrix}
2(\beta_{i,01} - \beta_{i,10}) \\
2(\alpha_{i,01} - \alpha_{i,10})
\end{pmatrix}.
\]

Using the commutation relations: $\alpha_{01} - \alpha_{i,0} = A_{11}\alpha, 1$ and $\beta_{i,01} - \beta_{i,10} = \beta_{i,1}A_{11} + \beta_1A_{11,1}$ ([Le2]), we can compute

\[
\tag{4.25}
\beta_{i,01} - \beta_{i,10} = \beta_{i,1}A_{11} + \beta_1A_{11,1} + i(a_{0,1} - a_{1,0})\beta_i \\
\alpha_{i,01} - \alpha_{i,10} = A_{11}\alpha, 1 + i(a_{0,1} - a_{1,0})\alpha
\]

Here $a_0 = a(T), a_1 = a(Z_1), a_1 = a(Z_1)$. By (4.15) and (5.3) we can easily obtain

\[
\tag{4.26}
a_{1,0} - a_{0,1} = \frac{1}{2}(F^0_b + iA_{11,1}) - a_1A_{11}.
\]

Now Lemma 4.4 follows from (4.24),(4.25),(4.26).
Applying our assumption $A_{11} = 0$ and (4.18) to Lemma 4.4 and substituting the result in (4.23), we obtain

\[(4.27) \quad \epsilon^2 \|\alpha\|^2_{L^2} = \epsilon^{-2} \|\nabla_T^\epsilon \Phi\|^2_{L^2} + \|\nabla_{\Xi}^\epsilon \Phi\|^2_{L^2} + 2 \|\alpha \beta_1\|^2_{L^2}\]

where $\beta_1$ is the complex conjugate of $\beta_1$. It follows that

\[\|\alpha \beta_1\|^2_{L^2} = O(\epsilon^2).\]

Substituting this in (4.27), we obtain

\[(4.28) \quad \|\nabla_T^\epsilon \Phi\|^2_{L^2} = O(\epsilon^4), \quad \|\nabla_{\Xi}^\epsilon \Phi\|^2_{L^2} = O(\epsilon^2).\]

Let $\hat{\nabla}_T$, $\hat{\nabla}_{\Xi}$ denote the following operators:

\[\hat{\nabla}_T = \begin{pmatrix} -i\nabla_T^\epsilon & 0 \\ 0 & i\nabla_T^\epsilon \end{pmatrix}, \quad \hat{\nabla}_{\Xi} = \begin{pmatrix} 0 & 2\nabla_{Z_1}^\epsilon \\ -2\nabla_{Z_1}^\epsilon & 0 \end{pmatrix}.\]

It is easy to see that $\hat{\nabla} = \hat{\nabla}_T + \hat{\nabla}_{\Xi}$ is an elliptic operator. (independent of $\epsilon$) So we can compute

\[(4.29) \quad \|\Phi\|_{L^2_1} \leq C_1 (\|\hat{\nabla}\Phi\|_{L^2} + \|\Phi\|_{L^2}) \quad (elliptic \ estimate)\]

\[
\leq C_1 (\|\nabla_T^\epsilon \Phi\|_{L^2} + \|\nabla_{\Xi}^\epsilon \Phi\|_{L^2} + \|(\hat{\nabla}_T - \nabla_T^\epsilon)\Phi\|_{L^2} \\
+ \|(\hat{\nabla}_{\Xi} - \nabla_{\Xi}^\epsilon)\Phi\|_{L^2} + \|\Phi\|_{L^2})
\]

\[
\leq C_2 \quad (by \ (4.28), \ (4.20))
\]

in which $C_1, C_2$ are constants independent of $\epsilon$, and we can use the covariant derivative $\nabla^h$ to define the Sobolev norm $L^2_1$. By (4.29) $\Phi = \Phi_\epsilon$ (indicating the $\epsilon$-dependence) converges strongly in $L^2$ for some sequence $\epsilon_j$ tending to 0. Moreover, applying the first inequality of (4.29) to $\Phi_{\epsilon_j} - \Phi_{\epsilon_k}$ and using (4.28) for $\epsilon_j, \epsilon_k$ to show $\|\hat{\nabla}\Phi_{\epsilon_j}\|_{L^2}$ and $\|\hat{\nabla}\Phi_{\epsilon_k}\|_{L^2}$ are small as $\epsilon_j, \epsilon_k$ are small enough, we conclude that $\Phi_{\epsilon_j}$ is Cauchy in $L^2_1$. Therefore $\Phi_{\epsilon_j}$ converges strongly in $L^2_1$ to $\Phi$ ($\alpha_\epsilon, \beta_1^\epsilon$ converge to $\hat{\alpha}, \hat{\beta}_1$, resp.) as $\epsilon_j$ goes to 0. It follows from (4.28) that
\[ \hat{\nabla}_T \hat{\Phi} = 0, \hat{\nabla}_\xi \hat{\Phi} = 0 \]

i.e. \( \hat{\alpha}_{1} = \hat{\beta}_{1} = 0, \hat{\alpha}_{0} = \hat{\beta}_{0} = 0 \).

We’ll show the \( C^\infty \)-smoothness of \( \hat{a} \) and \( \hat{\Phi} \) by the usual bootstrap argument. First \( \hat{a} \in L^p_1 \) (\( p > 1 \)) and \( \hat{\Phi} \in L^2_1 \) imply \( \hat{a} \hat{\Phi} \in L^2_1 \) since \( L^2_1 \times L^2_1 \subset L^2_1 \) in dimension 3. It follows that \( \hat{\Phi} \in L^2_3 \) by the elliptic regularity. \( \nabla \hat{\Phi} = 0 \) by (4.30). Since \( L^k_2 \) is an algebra for \( 2k > \text{dimension} = 3 \), \( F_\hat{a} \) is in \( L^2_3 \) by (4.18), (4.15). So \( \hat{a} \) is in \( L^2_3 \). (note that \( d^* \hat{a} = -\frac{1}{2}d^* \omega \) is smooth by (4.15) and \( b_\xi \) having been taken in Coulomb gauges) Now repeating the above argument, we obtain \( \hat{a} \hat{\Phi} \in L^2_3 \), then \( \hat{\Phi} \in L^2_3, F_\hat{a} \in L^2_3, \) and \( \hat{a} \in L^2_4, \) etc. So \( \hat{a}, \hat{\Phi} \) are \( C^\infty \) smooth.

On the other hand, taking the limit of the first equation of (4.18) gives

\[ d\hat{a}(e_1, e_2) - W = \frac{1}{2}( |\hat{a}|^2 - |\hat{\beta}|^2 ). \]

From (4.30),(4.31), \( (\hat{\alpha} / \sqrt{2}, \hat{\beta} / \sqrt{2}, \hat{a}) \) is a \( (C^\infty\) smooth) solution of (3.11). Suppose both \( \hat{\alpha} \) and \( \hat{\beta}_1 \) are identically zero. Then by (4.18), \( c_1(W_{\text{can}}) = e(\xi) \) vanishes in \( H^2(M, C) \), contradicting our assumption.

5 Appendix: a brief introduction to pseudohermitian geometry

Let \( M \) be a smooth (paracompact) 3-manifold with an oriented contact structure \( \xi \).

We say a contact form \( \theta \) is oriented if \( d\theta(u, v) > 0 \) for \( (u, v) \) being an oriented basis of \( \xi \). There always exists a global oriented contact form \( \theta \), obtained by patching together local ones with a partition of unity. The characteristic vector field of \( \theta \) is the unique vector field \( T \) such that \( \theta(T) = 1 \) and \( \mathcal{L}_T \theta = 0 \) or \( d\theta(T, \cdot) = 0 \). A \( CR \)-structure compatible with \( \xi \) is a smooth endomorphism \( J : \xi \to \xi \) such that \( J^2 = -\text{identity} \).

We say \( J \) is oriented if \( (X, JX) \) is an oriented basis of \( \xi \) for any nonzero \( X \in \xi \). A pseudohermitian structure compatible with \( \xi \) is a \( CR \)-structure \( J \) compatible with \( \xi \) together with a global contact form \( \theta \).
Given a pseudohermitian structure \((J, \theta)\), we can choose a complex vector field \(Z_1\), an eigenvector of \(J\) with eigenvalue \(i\), and a complex 1-form \(\theta^1\) such that \(\{\theta, \theta^1, \theta^\dagger\}\) is dual to \(\{T, Z_1, Z_1\}\). \((\theta^\dagger = (\theta^1), Z_1 = (Z_1))\) It follows that \(d\theta = i h_{11} \theta^1 \wedge \theta^\dagger\) for some nonzero real function \(h_{11}\). If both \(J\) and \(\theta\) are oriented, then \(h_{11}\) is positive. In this case we call such a pseudohermitian structure \((J, \theta)\) oriented, and we can choose a \(Z_1\) (hence \(\theta^1\)) such that \(h_{11} = 1\). That is to say

\[
d\theta = i \theta^1 \wedge \theta^\dagger. \tag{5.1}\]

The pseudohermitian connection of \((J, \theta)\) is the connection \(\nabla^{\psi,h}\) on \(TM \otimes \mathbb{C}\) (and extended to tensors) given by

\[
\nabla^{\psi,h} Z_1 = \omega_1^1 \otimes Z_1, \quad \nabla^{\psi,h} Z_\dagger_1 = \omega_\dagger_1 \otimes Z_\dagger_1, \quad \nabla^{\psi,h} T = 0
\]

in which the 1-form \(\omega_1^1\) is uniquely determined by the following equation with a normalization condition:

\[
d\theta^1 = \theta^1 \wedge \omega_1^1 + A_{1\dagger} \theta^\dagger \wedge \theta^\dagger \\
\omega_1^1 + \omega_\dagger_1 = 0. \tag{5.2}\]

The coefficient \(A_{1\dagger}\) in (5.2) is called the (pseudohermitian) torsion. Since \(h_{11} = 1\), \(A_{1\dagger} = h_{11} A_{1\dagger} = A_{1\dagger}\). And \(A_{1\dagger}\) is just the complex conjugate of \(A_{1\dagger}\). Differentiating \(\omega_1^1\) gives

\[
d\omega_1^1 = \mathcal{W} \theta^1 \wedge \theta^1 + 2i \text{Im}(A_{1\dagger} \theta^\dagger \wedge \theta) \tag{5.3}\]

where \(\mathcal{W}\) is the Tanaka-Webster curvature. Write \(\omega_1^1 = i \omega\) for some real 1-form \(\omega\) by the second condition of (5.2). This \(\omega\) is just the one used in previous sections. Write \(Z_1 = \frac{1}{2}(e_1 - ie_2)\) for real vectors \(e_1, e_2\). Now the real version of (5.3) reads:

\[
d\omega(e_1, e_2) = -2\mathcal{W}. \tag{5.4}\]

Let \(e^1 = \text{Re}(\theta^1), e^2 = \text{Im}(\theta^1)\). Then \(\{e^0 = \theta, e^1, e^2\}\) is dual to \(\{e_0 = T, e_1, e_2\}\). The oriented pseudohermitian structure \((J, \theta)\) induces a Riemannian structure \(h\) on \(\xi\):
$h(u, v) = \frac{1}{2} d\theta(u, Jv)$. The adapted metric of $(J, \theta)$ is the Riemannian metric on $TM$ defined by $\theta^2 + h = (e^0)^2 + (e^1)^2 + (e^2)^2$, still denoted $h$. The Riemannian connection forms $\omega^i_j$ are uniquely determined by the following equations:

\begin{equation}
\begin{aligned}
d e^i &= e^j \wedge \omega^i_j \\
\omega^i_j + \omega^j_i &= 0.
\end{aligned}
\end{equation}

Comparing (5.5) with (5.1), (5.2), we can relate $\omega^i_j$ to the pseudohermitian connection $\omega^1_1 = i\omega$ and torsion $A^1_1 = \lambda + i\mu$ ($\lambda, \mu$ being real) as follows:

\begin{equation}
\begin{aligned}
\omega^1_1 &= \omega + \theta, \\
\omega^1_0 &= \lambda e^1 + (\mu - 1)e^2 \\
\omega^2_0 &= (\mu + 1)e^1 - \lambda e^2.
\end{aligned}
\end{equation}

Observe that (5.1) and (5.2) imply

\begin{equation}
\begin{aligned}
\left[ \frac{-i}{2} [e_1, e_2] = [Z_1, Z_1] = iT + \omega^1_1(Z_1)Z_1 - \omega^1_1(T)Z_1, \\
[Z_1, T] = A^1_1Z_1 - \omega^1_1(T)Z_1.
\end{aligned}
\end{equation}

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