| **Title** | Distributed Transmission Probability Control in Fading Channel with State Information |
|-----------|---------------------------------------------------------------------------------|
| **Author(s)** | Xie, S.; Low, Kay Soon; Gunawan, Erry |
| **Citation** | Xie, S., Low, K. S., & Gunawan, E. (2015). Distributed Transmission Probability Control in Fading Channel with State Information. International Journal of Distributed Sensor Networks, 2015, 184629-. |
| **Date** | 2015 |
| **URL** | http://hdl.handle.net/10220/39073 |
| **Rights** | © 2015 S. Xie et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited. |
Distributed Transmission Probability Control in Fading Channel with State Information

S. Xie, K.S. Low, and E. Gunawan

Satellite Research Centre, School of Electrical and Electronic Engineering, Nanyang Technological University, 50 Nanyang Drive, Singapore 639798

Correspondence should be addressed to K.S. Low; k.s.low@ieee.org

Received 4 December 2014; Revised 10 May 2015; Accepted 12 May 2015

Academic Editor: Álvaro Marco

Copyright © 2015 S. Xie et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

A wireless system consisting of a finite number of heterogeneous users transmitting packets over a slotted ALOHA Rayleigh-fading channel is investigated in this paper. In this system, each user tries to minimize its number of transmission attempts while meeting the throughput demand. The user interaction in such an ALOHA network is modeled as a random access game. A novel technique to better exploit the channel state information (CSI) named adaptive CSI transmission method is proposed. Benchmarking with the no-CSI method and threshold-based CSI method, it is revealed that the proposed adaptive CSI method can yield up to 9.9% more throughput and 12.3% power reduction. Two simple yet effective guidelines on selecting among these CSI-related methods are formulated for systems with different SNRs and capture ratios. A distributed algorithm is proposed to find the optimal transmission probability. Both analytical and simulation results show that the algorithm exhibits fast convergence speed and is robustness against changes on the number of active nodes in the network.

1. Introduction

The ALOHA protocol has been attracting a lot of attentions from both academia and industry since its introduction. It has been used for various generations of mobile phone services such as GPRS and SMS [1]. In recent years, various derivatives of ALOHA protocol have been developed and considered promising for a wide spectrum of applications including wireless sensor networks [2], next generation Internet of Things (IoT) [3, 4], cognitive radio [5, 6], and smart grid [7].

The earlier works on ALOHA, for example, in [8, 9], were focused on the system-level performance, for example, the system throughput. With the evolution of telecommunication systems into distributed systems in recent years [10], the focus has been shifted from the system perspective to the user perspective. As the network might be of heterogeneous nature, a system-level approach might be insufficient to differentiate each user’s goal and action. Moreover, many users in the wireless networks are power-constrained and they may prefer power conservation over throughput maximization. As such, new techniques and performance metrics should be developed to better design the network [11, 12].

In [13], MacKenzie and Wicker conducted the analysis of a slotted ALOHA from the perspective of a selfish user. It is shown that a game-theoretic approach [14] was able to provide insights into the equilibrium operating point [15, 16] and the network performance such as throughput from a decentralized perspective. Later works explore more realistic settings by incorporating the CSI into the ALOHA model to enable each node to make channel-aware decisions [10, 12, 17–19]. More sophisticated channel models were also investigated towards a more accurate characterization of the fading channel. A simple collision model had been widely assumed to model the network interaction in a random access network [10, 18, 20, 21]. It was subsequently generalized to power capture model [22] and SINR capture model [17] to better characterize the time-varying fading channels and the multipacket reception capacity [23].

Intuitively, the use of CSI should lead to power-efficient outcomes as a node can choose not to send a packet under bad channel conditions to avoid wasting energy. In [10, 12, 24], CSI is exploited to enhance the throughput performance in a collision channel. However, it was shown in [17, 25] that, for a channel model with capture effect, a threshold-based
CSI method may experience *Braess Paradox* [26]; that is, adding extra information to the network reduces the overall performance. Consequently, the achievable throughput may be even worse than that without CSI.

This paradox does not occur all the time. In ideal cases when the signal-to-noise ratio (SNR) approaches to infinity, it will always occur and it is suggested in [17] not to use CSI in such cases. When the SNR is a finite number, the occurrence of *Braess Paradox* is quite intricate. Thus it is difficult to decide whether to use CSI.

In this work, this *Braess Paradox* has been further investigated and it has been shown that the occurrence of the paradox is purely determined by the radio capture ratio and the SNR. It has been further discovered that there exists an enhanced method on exploiting the CSI, namely, adaptive CSI method, which may result in better system-level performance. Inspired by the threshold-CSI and no-CSI methods in [17], the adaptive CSI method adopts a probabilistic rather than deterministic strategy on how to respond to the instantaneous CSI. With the adaptive CSI method, this paper presents three original contributions.

1. The throughput using the proposed adaptive CSI is derived. The adaptive CSI can achieve up to 9.9% more throughput than the threshold CSI and no-CSI methods for a wide variety of systems. Moreover, it can result in up to 12.3% power reduction than the other two methods.

2. Two simple yet effective guidelines are generalized on selecting the aforementioned three CSI-related methods to achieve maximum throughput or power efficiency for different system parameters and thus to avoid the occurrence of *Braess Paradox*.

3. A distributed algorithm is proposed to tune the transmission probability to the optimal point. It is shown that the algorithm can guarantee fast convergence and is robust against changes on the number of active nodes in the network.

The rest of the paper is organized as follows. Section 2 formulates the problem and states the assumptions on the system. The proposed adaptive CSI is presented in Section 3. Section 4 compares the adaptive CSI with the threshold-based CSI method and no-CSI method. Guideline on the selection scheme for different systems is also derived. Section 5 compares the power efficiency for the three methods. The convergence algorithm is presented in Section 6, together with an update interval analysis. Section 7 presents the simulation results, followed by a conclusion in Section 8.

## 2. Problem Formulation

In the following, a wireless network where there are $n$ nodes transmitting packets at a predetermined power level to a base station over a shared channel is considered. It is assumed that the time is slotted and the amount of information contained in a data packet is fixed and is the same for all the nodes. A data packet, with the possible acknowledgement packet, occupies one time slot. Each node contends for the channel access using the slotted probabilistic ALOHA protocol. In each time slot $k$, each node $i$ transmits with a transmission probability $p_{i,k}$. Each node is assigned with an average throughput demand $\rho_i$. In this network, each node tries to minimize its overall power investment, which is equivalent to minimizing the transmission probability, while meeting the average throughput demand $\rho_i$. It is assumed that all the nodes have packets buffered for transmission at any time and the essential problem for each node is to choose a transmission strategy that fulfills its goal. Moreover, all the nodes are assumed to be well synchronized.

### 2.1. Throughput

For the brevity of the analysis, the average throughput for node $i$ is defined as the ratio of the number of packets that are successfully received by the base station to the total number of time slots. For example, if a node $i$ transmits 100 packets to the base station during a 10000-slot period, of which 67 are successful, then the average throughput for node $i$ is 0.067.

### 2.2. Fading Channel and SINR Capture

Signal-to-interference-and-noise ratio (SINR) capture [17] is considered in this paper to model the fading channel. In the SINR model, node $i$’s packet at time $k$ will be successfully received by the base station if $\text{SINR}_{i,k} > b$ [17], where $b$ is the radio capture ratio and $\text{SINR}_{i,k}$ is the SINR of node $i$ at time slot $k$. $\text{SINR}_{i,k}$ is given by

$$\text{SINR}_{i,k} = \frac{B_i |h_{i,k}|^2 P_T}{\sum_{j \neq i} B_j |h_{j,k}|^2 P_T + N}, \quad (1)$$

where $N$ is the average power of the additive noise at the base station and $P_T$ is the transmission power of each node. $B_{i,k} = 1$ if node $i$ transmits a packet at time slot $k$ and $B_{i,k} = 0$ otherwise. $h_{i,k}$ is the channel gain between node $i$ and the base station at time $k$. The physical intuition of the SINR model can be traced back to the capture effect of a radio receiver. Only when the signal from a particular node $i$ is at least $b$ times stronger than the interference, then it can be amplified and successfully decoded [27].

In the sequel, it is assumed that the channel is an *i.i.d Rayleigh Fading* channel and $x_{i,k} = |h_{i,k}|^2$ are thus assumed to be an *i.i.d* uniformly exponential random variable, fixed within one slot and varying from one slot to another as in [17, 28]. It is further assumed that at the beginning of the time slot $k$, each node $i$ is able to obtain its own instantaneous channel state information, which provides an indication of the quality of the channel. Furthermore, each node can obtain $h_{i,k}$ and thus $x_{i,k}$ from the CSI reading, as in [17].

### 2.3. Adaptive CSI-Based Transmission Method

In this paper, a novel method named adaptive CSI-based method is proposed. Instead of using a fixed threshold-based CSI strategy as in [17], the proposed method considers the fact that $x_{i,k}$ is exponentially distributed. A complementary function $c_i (1 - e^{-c_i x})$ is constructed to dynamically determine the transmission probability $p_{i,k}$ with respect to the instantaneous CSI, where $c_i \in (0, 1)$ is a tuning factor determining the amplitude.
of the transmission probability. The intuition of the adaptive CSI-based transmission is that the transmission probability is made to be proportional to “how good the channel is.” This method can be defined as follows.

**Definition 1.** An adaptive CSI-based transmission strategy for node \(i\) in time slot \(k\) is defined as a mapping from \(x_{i,k}\) to \(p_{i,k} \in [0, 1]\) and has the form of \(p_{i,k} = c_i(1 - e^{-x_{i,k}}).\) Moreover, the node \(i\) transmits a packet in slot \(k\) with a probability \(p_{i,k}^*\).

It can be concluded that \(p_{i,k}^*\) is a monotonically increasing function of the channel state \(x_{i,k}\). It also immediately follows that the average transmission probability of node \(i\) is solely determined by \(c_i\) and is given by

\[
p_i(c_i) = \int_0^\infty c_i \left(1 - e^{-x_i}\right) e^{-x_i} dx_{i,k} = \frac{c_i}{2}.\]  

(2)

**3. Equilibrium Analysis**

**3.1. Power Optimization Problem.** According to the problem formulation, the local optimization problem for each node \(i\) using the adaptive CSI method can be modeled as

\[
\min_{p_i} p_i(c_i),
\]

subject to \(r_i(p_i, p_{-i}) \geq \rho_i,\)

where \(p = (p_1, p_2, \ldots, p_n)\) is the transmission probability vector, \(r_i(p_i, p_{-i})\) is the average throughput of node \(i\) when it transmits with probability \(p_i\) given that others transmit with probability \(p_{-i}\), where \(p_{-i}\) represents the transmission probability vector of all nodes except node \(i\), and \(\rho_i\) is the throughput demand for node \(i\).

The system can be modeled as a constrained noncooperative game. Each node acts as a selfish entity, taking actions, that is, choosing the tuning factor \(c_i\) and thus the transmission probability \(p_i\) [11, 17, 18], to minimize its own power investment subject to the throughput constraint.

**3.2. Nash Equilibria.** It is assumed that each \(p_i\) takes value from [0, 1]. A utility function can be defined as \(U_i(p_i, p_{-i}) = 1 - p_i.\) Higher transmission probability results in higher power investment and thus lower utility. With the utility function, the following definition on Nash equilibrium can be established.

**Definition 2.** An action profile \(p\) is a constrained Nash Equilibrium point if, for all \(i = 1, 2, \ldots, n\), there exists \(r_i(p_i, p_{-i}) \geq \rho_i,\)

\[
U_i(p_i, p_{-i}) \geq U_i(\bar{p}_i, p_{-i}),
\]

(4)

all \((\bar{p}_i) \in \{ \bar{p}_i : r_i(\bar{p}_i, p_{-i}) \geq \rho_i \},\)

where \(\bar{p}_i\) is the transmission probability of node \(i\) such that its throughput demand can be fulfilled.

Equivalently, \(p\) is a Nash equilibrium if

\[
p_i \in \arg \min_{0 \leq \bar{p}_i \leq 1} \{ \bar{p}_i : r_i(\bar{p}_i, p_{-i}) \geq \rho_i \}.
\]

(5)

Since \(r_i(p_i, p_{-i})\) and \(U_i(p_i, p_{-i})\) are nondecreasing functions of \(p_i\) under the SINR model, it immediately follows as in [17] that \(p\) is a Nash equilibrium if and only if it is a solution to the set of equations:

\[
r_i(p_i, p_{-i}) = \rho_i, \quad i = 1, 2, \ldots, n.
\]

(6)

**3.3. Existence of Nash Equilibrium.** When there are \(s\) (\(s \leq n\)) nodes using the adaptive CSI strategy to transmit a packet to the base station at the same time slot, the probability that the packet received successfully from a particular node \(i\) is given as

\[
\int_0^\infty \cdots \int_0^\infty c_i \left(1 - e^{-x_i}\right) \int_0^\infty \prod_{j=1}^s c_j \left(1 - e^{-x_j}\right) \prod_{j \neq i}^{\xi} (1 + 2b)(2b + 1) e^{-x_i} dx_i dx_j.
\]

(7)

Solving (7) yields

\[
e^{-b(N/P_i)} \left( \prod_{j=1}^s \frac{c_j}{1 + b} \right) - \frac{1}{2} \left( \frac{c_i}{2} \right)^2.
\]

(8)

Using (2) and denoting \(x_1 = (1 + b)(2 + b)\) and \(x_2 = (1 + 2b)(2b + 2)\) for brevity, the following theorem on the average throughput of node \(i\) using the proposed adaptive CSI method can be formulated.

**Theorem 3.** Under the SINR capture model with capture ratio \(b\) and i.i.d., Rayleigh fading channels between all nodes and the base station, the average throughput of node \(i\) when the transmission probability vector is \(p = (p_1, p_2, \ldots, p_n)\) and when each node uses the adaptive CSI transmission strategy is as follows:

\[
r_i(p_i, p_{-i}) = p_i \left( e^{-b(N/P_i)} - e^{-2b(N/P_i)} \right) \prod_{j \in Q_i} (1 - p_j)
\]

\[+ \frac{p_i e^{-b(N/P_i)}}{x_1} 2 \prod_{j \in Q_i} \left( \frac{2p_j}{x_1} + 1 - p_j \right) \]

\[- \prod_{j \in Q_i} \left( \frac{2p_j}{x_2} + 1 - p_j \right).
\]

(9)
where $Q_i$ is the set of all the nodes in the network except node $i$. The Nash equilibrium is achieved when $r_i(p_i, p_{-i}) = \rho_i$ for every node $i$ in the network.

Proof. The theorem can be proved by Binomial theorem and is similar to Lemma 1 in [17]. Thus the details are omitted here for brevity.

From (9), the following 2 corollaries can be given.

**Corollary 4.** The average packet success rate of node $i$ is given as

$$P_{PSR_i} = (e^{-b(N/P_i)} - e^{-2b(N/P_i)}) \prod_{j \in Q_i} (1 - p_j)$$

$$+ e^{-b(N/P_i)} \left( 2 \prod_{j \in Q_i} \left( \frac{2p_j}{x_1} + 1 - p_j \right) \right) \quad (10)$$

**Corollary 5.** When all the nodes have the same throughput demand $\rho$, the average throughput of each node can be simplified as

$$r = p \left( e^{-b(N/P_i)} - e^{-2b(N/P_i)} \right) (1 - p)^{n-1}$$

$$+ pe^{-b(N/P_i)} \left( 2 \left( \frac{2p}{x_1} + 1 - p \right) \right) \tag{11}$$

To determine the existence of Nash equilibrium, the derivative of $r$ is derived as

$$\frac{dr_{a-CSi}}{dp} = (1 - np) \left( e^{-b(N/P_i)} - e^{-2b(N/P_i)} \right) (1 - p)^{n-2}$$

$$+ e^{-b(N/P_i)} \left( 2 \left( \frac{2p}{x_1} + 1 - p \right) \right)^{n-2} \cdot \left( 1 - \frac{2}{x_1} np \right) \cdot \left( \frac{2p}{x_2} + 1 - p \right)^{n-2} \tag{12}$$

It can be seen that $\frac{dr_{a-CSi}}{dp} > 0$ when $0 < p < 1/n$ and there exists a $p^* > 1/n$ such that when $1/n < p < p^*$ and $\frac{dr_{a-CSi}}{dp} > 0$ and when $p > p^*$, $\frac{dr_{a-CSi}}{dp} < 0$. Thus, when the throughput demand is achievable (i.e., $r_{max} \geq \rho$), there exists one (i.e., when $r_{max} = \rho$) Nash equilibrium points.

When there are 2 equilibrium points, the one associated with smaller transmission probability is superior to the other in terms of minimizing power investment. This immediately follows from the fact that power investment is an increasing function of transmission probability.

Moreover, simulation results show that, for heterogeneous network, there exists at least one equilibrium point given that the throughput demand is feasible.

### 4. Maximum Throughput Analysis

In Section 3, the Nash equilibrium of (3) is analyzed. The relationship between throughput demand and transmission probability is derived for the adaptive CSI scheme. In this section, the maximum achievable throughput of the adaptive CSI scheme is compared with the 2 other schemes in [17], namely, threshold-based CSI method and no-CSI method.

For the no-CSI method, the throughput is given by [17] as follows:

$$r_{no-CSi} = e^{-b(N/P_i)} p \left( 1 - \frac{b p}{1 + b} \right)^{n-1} \quad (13)$$

It can be seen that the maximal throughput is obtained when $p = (b + 1) / bn$ and the throughput when $p = (b + 1) / bn$ is as follows:

$$r_{max} = e^{-b(N/P_i)} \frac{b + 1}{bn} \left( 1 - \frac{1}{n} \right)^{n-1} \quad (14)$$

For the threshold-based CSI method, the throughput is given by [17]:

$$r_{th-CSi} = e^{-b(N/P_i)} \left( \frac{p^{b+1}}{1 + b} + (1 - p) \right) \tag{15}$$

$$+ (1 - p)^{n-1} \min \{ p - e^{-b(N/P_i)}, 0 \}.$$
It can be seen that there exists a unique solution \( p = p^* \) for \( \frac{dr_{t,\text{CSI}}}{dp} = 0 \). When \( p < p^* \), \( \frac{dr_{t,\text{CSI}}}{dp} > 0 \) and when \( p > p^* \), \( \frac{dr_{t,\text{CSI}}}{dp} < 0 \). When \( p = p^* \), the maximum throughput is achieved. However, \( p^* \) is difficult to obtain analytically due to the complexity of (16) and (17). Notice that when \( b \) is large (e.g., \( b = 3 \)), \( p^* \approx 0 \) as the transmission probability \( p \) is typically very small. Thus (16) can be approximated as

\[
\frac{dr_{t,\text{CSI}}}{dp} \approx (e^{-b(N/P_T)}(1 - n) + (1 - p) - (n - 1)(p - e^{-b(N/P_T)})(1 - p)^{n-2} \tag{18}
\]

and it can be seen that \( p^* = 1/n \).

Moreover, (17) can be approximated as

\[
\frac{dr_{t,\text{CSI}}}{dp} = -e^{-b(N/P_T)}(1 - p)^{n-2}(n - 1), \tag{19}
\]

where \( dr_{t,\text{CSI}}/dp < 0 \) given that \( 0 < p < 1 \).

Combining (18) and (19), it can be shown that \( p^* = \min\{1/n, e^{-b(N/P_T)}\} \). In practical systems, it is almost certain that \( 1/n < e^{-b(N/P_T)} \). Thus, it is sufficient to discuss the case \( p^* = 1/n \) only.

When \( 1/n < e^{-b(N/P_T)} \), the maximum throughput of the threshold-based CSI is given by

\[
r_{t,\text{CSI}}^{\text{max}} = e^{-b(N/P_T)} \left( \frac{n^{-b+1}}{1 + b} + \left( \frac{1}{1 + b} - \frac{1}{n} \right) \right)^{n-1} \tag{20}
\]

For the proposed adaptive CSI method, similar to the threshold-based CSI method, when \( b \) is large (e.g., \( b = 3 \)), \( 1/x_1 \approx 1/x_2 \approx 0 \), (12) can be approximated as

\[
\frac{dr_{a,\text{CSI}}}{dp} \approx (1 - np) \left( e^{-b(N/P_T)} - e^{-2b(N/P_T)} \right)(1 - p)^{n-2} + e^{-b(N/P_T)}(1 - np)(1 - p)^{n-2} \tag{21}
\]

The unique solution such that \( \frac{dr_{a,\text{CSI}}}{dp} = 0 \) is \( p^* = 1/n \) and the maximum throughput is given by

\[
r_{a,\text{CSI}}^{\text{max}} = \frac{1}{n} \left( e^{-b(N/P_T)} - e^{-2b(N/P_T)} \right) \left( 1 - \frac{1}{n} \right)^{n-1} + \frac{1}{n} \cdot e^{-b(N/P_T)} \left( 2 \left( 1 - \frac{1}{x_1} - \frac{1}{x_2} \right) \right)^{n-1} \tag{22}
\]

Comparing (22) with (14) and (20), the following theorem on the maximum throughput can be established.

**Theorem 6.** Given a capture ratio \( b \) and the network size \( n \), when

\[
m_l(b, n) < SNR < m_h(b, n). \tag{23}
\]

the adaptive CSI method results in higher maximum throughput than the no-CSI method and the threshold-based CSI method, where

\[
m_l(b, n) = \frac{P_T}{N},
\]

\[
m_h(b, n) = \frac{-b}{\ln((a_1 - a_2)/a_2)},
\]

\[
a_1 = \left( \frac{n^{-b+1}}{1 + b} + \left( \frac{1}{1 + b} - \frac{1}{n} \right) \right)^{n-1},
\]

\[
a_2 = \left( \frac{1}{1 + b} - \frac{1}{n} \right)^{n-1},
\]

\[
a_3 = 2 \left( 1 - \left( \frac{1}{x_1} - \frac{1}{x_2} \right) \right)^{n-1} - \left( 1 - \left( \frac{1}{x_1} - \frac{1}{x_2} \right) \right)^{n-1}.
\]

**Proof.** The proof can be completed by enforcing \( r_{a,\text{CSI}}^{\text{max}} > r_{t,\text{CSI}}^{\text{max}} \) and \( r_{a,\text{CSI}}^{\text{max}} > r_{n,\text{CSI}}^{\text{max}} \) with proper algebraic manipulations. The details are omitted for brevity.

It also follows that when \( SNR \leq m_l(b, n) \), the threshold-based CSI method yields higher maximum throughput; when \( SNR \geq m_h(b, n) \), it suffices to use the no-CSI method.
The intuition is that when the channel is in general good enough, there is no need to confine the transmission to the “good” slots; when the channel is bad, the transmission should not be conducted at those bad slots at all; when the channel state is in between, there exists a wide gap where the CSI should be exploited in the proposed way.

Using (14), (20), and (22), the maximum throughput of each method under different capture ratios $b$ and network sizes $n$ can be obtained. Figure 1 compares the maximum throughput of the three methods. It is shown that there exists a red region in terms of SNR and capture ratio where the proposed adaptive CSI result is highest among the three methods. This improvement in throughput can be up to 9.9% as compared with the highest throughput that can be achieved by the no-CSI and threshold-based CSI methods.

5. Power Efficiency Analysis

Given the same transmission probability $p$ (i.e., the same power investment) and using (11)–(13), the adaptive CSI method outperforms the no-CSI method when $r_{a-CSI} > r_{no-CSI}$. Solving the inequality for SNR yields

$$\text{SNR} < t_h(b, n, p),$$

where

$$t_h(b, n, p) = -b \ln \left( \frac{2 \left( (2p/x_1) + 1 - p \right)^n - (2p/x_2) + 1 - p \right)^n + (1 - p)^n - (1 - (bp/(1 + b)))^n \right) / (1 - p)^n.$$  \tag{26}

Similar comparisons can be made between (11) and (15). Solving the inequality $r_{a-CSI} > r_{t-CSI}$ for SNR yields

$$\text{SNR} > t_l(b, n, p),$$

where

$$t_l(b, n, p) = -b \ln \left( -\left( a'_1 - a'_2 - a'_3p - a'_4p \right) - \sqrt{(a'_1 - a'_2 - a'_3p)^2 - 4(a'_2p)^2} \right) / (2(a'_2p)).$$  \tag{28}

Combining (25) and (27), the following theorem can be formulated.

**Theorem 7.** Given a capture ratio $b$, the network size $n$, and a fixed power investment in terms of transmission probability $p$, the power investment of each node can be minimized using the adaptive CSI method when the SNR satisfies both (25) and (27). When $\text{SNR} \leq t_h(b, n, p)$, the threshold-based CSI method is more power-efficient than the others. When $\text{SNR} \geq t_h(b, n, p)$, a node should use no-CSI method for power efficiency.

It should be noted that the intersection of (25) and (27) may be an empty SNR set for a given $(b, n, p)$. It means that, under such circumstances, either no-CSI or threshold-based CSI is superior to the others.

As an illustrative example, Figure 2 shows the throughputs of the three methods for a given power investment (i.e., transmission probability) $p = 0.05$ and $n = 10$. Figure 2 shows that, using the adaptive CSI, the power investment can be minimized for a wide variety of systems with different SNRs and capture ratios. For this particular example, the power reduction can go up to 9.3%. Figure 3 traverses through all the possible power investment schemes, it can be found that the power reduction can go up to 12.3%.

6. Convergence Analysis

6.1. ACK-Based Game Strategy. In Section 3, the Nash equilibrium is derived for the adaptive CSI method. In this section, an ACK-based algorithm is proposed for each node to find the power-efficient Nash equilibrium in a distributed manner.

Each node $i$ in the network updates the transmission probability $p_i$ synchronously every $I$ time slot and each node transmits with this probability for the current update interval.
For node $i$, the new transmission probability for the $t + 1$th update is determined by

$$p_{i}^{(t+1)} = \frac{\rho_i}{P_{PSR_i}^{(t)}},$$  

(29)

where $P_{PSR_i}^{(t)}$ is the packet success rate for node $i$ from the $t$th update to the $t + 1$th update (the $t$th update interval). $P_{PSR_i}^{(t)}$ can be obtained from ACK information.

In particular, the initial transmission probability $p_{i}^{(1)}$ can be any value smaller than the Nash equilibrium point. In practice, each node can start with $p_{i}^{(1)} = \rho_i$ for simplicity and fast convergence.

The following theorem shows each node will converge to the Nash equilibrium using the strategy in (29). It is assumed that the throughput demand profile $\rho$ is feasible.

**Theorem 8.** Using the ACK-based strategy in (29), the transmission probability of each node in the network asymptotically converges to the power-efficient Nash equilibrium point of the game (3).

**Proof.**

**Case 1.** First consider the case where the number of nodes in the network is fixed. For this scenario, the following should be satisfied.

(i) The sequence of the transmission probability is increasing.
(ii) Any two arbitrary entries $p_i$ and $p_k$ in the transmission probability vector $p$ asymptotically follow $p_i / p_k = (\rho_i / P_{PSR_i}) / (\rho_k / P_{PSR_k})$ regardless of their initial values.
(iii) The actual throughput of node $i$ at any update interval $t$ will not exceed its corresponding throughput demand.

Factor (i) must be satisfied because only when the sequence of the transmission probability increases, then it is possible that it converges to the equilibrium point. Factor (ii) must be met because, at the equilibrium point, according to (9) and Corollary 4, $p_i / p_k = P_i P_{PSR_i} / P_k P_{PSR_k}$ and thus factor (ii) must follow. Factor (iii) guarantees that the transmission probability will not go to the region where the actual throughput is larger than what is required and thus it can avoid converging to the unfavorable energy-inefficient equilibrium point.

While Factor (ii) immediately follows from the game strategy and Corollary 4, and it can be met starting from the second update, factor (i) can be proved by induction. It has
been assumed that $p_i^{(1)}$ is not greater than $\rho_i$. Thus, $p_i^{(2)} = p_i^{(1)}/p_{PSR}^{(1)} > p_i^{(1)}$. Assuming that $p_i^{(2)} < p_i^{(3)} < \cdots < p_i^{(t)}$ for all $i$, one needs to prove that $p_i^{(t)} < p_i^{(t+1)}$ holds.

Recall that $p_{PSR}$ is a decreasing function of $p_i$, $\forall p_i \in \mathbb{R}$. As $p_j^{(t-1)} < p_j^{(t)}$, $p_{PSR}^{(t)} > p_{PSR}^{(t+1)}$, it thus follows that $p_j^{(t)} < p_j^{(t+1)}$.

As for the proof of (iii), since $p_{PSR}^{(t+1)} < p_{PSR}^{(t)}$, it follows that $r_j^{(t+1)} = p_j^{(t+1)}/p_{PSR}^{(t+1)} < p_j^{(t)}/p_{PSR}^{(t)}$. Thus $r_j^{(t+1)} < \rho_k$, according to (29).

Combining (i) and (iii), the transmission probability $p_i$ of node $i$ will increase monotonically and is always smaller (yet getting closer) to the equilibrium point. Meanwhile, factor (ii) ensures that for each update $p_i$ and the transmission probability of any other node $k$ will “move” up in an appropriate step and thus they asymptotically converge to the equilibrium point at the same time.

Case 2. Next consider the case when there are newly joined nodes during the network operation. In this case, a new equilibrium point must be sought and the new equilibrium probability is expected to be greater than the previous one as more packet transmissions are expected. This situation can be viewed as a special case of case 1. The convergence can be achieved as long as the new throughput demand is feasible.

Case 3. Finally the case where there are nodes dissociating from the network or stopping transmitting data during the network operation is examined. Contrary to case 2, the new equilibrium probability of each node should be smaller than the previous one as there is less traffic in the network. For this case, which is similar to case 1, the following guidelines must be met.

(i) The sequence of the transmission probability of any node $i$ decreases over time after the nodes leave the network.

(ii) Any two arbitrary entries $p_i$ and $p_k$ in the transmission probability vector $\mathbf{p}$ asymptotically follow $p_i/p_k = (p_i/p_{PSR})/(p_k/p_{PSR})$, regardless of their values at the time the nodes leave the network.

The rationales and proofs are similar to case 1 and omitted for brevity. In this case, $p_i$ will decrease. Thus it will not converge to the energy-inefficient equilibrium point, which is always greater than the value of $\rho_i$ at the time the nodes leave the network.

This completes the proof of Theorem 8. □

Remark 9. This proposed ACK-based updating strategy can also apply to threshold-based CSI and noCSI methods. The proofs are very similar and are omitted.

Remark 10. Through simulations, it is observed that the condition that the initial transmission probability $p_i^{(1)}$ should be any value smaller than the Nash equilibrium point can be relaxed. Even for initial transmission probabilities greater than the equilibrium point, each node can still adjust the probability to the optimum using the proposed algorithm. The simulations also show that the transmission probability vector can always converge to the equilibrium point with negligible errors within 3 updates.

6.2. Update Interval Analysis. For the proposed algorithm, the update interval $I$ is a key parameter that can affect its performance. There exists a tradeoff: frequent updates allow each node to respond promptly to the traffic condition. However, a short update interval may result in a high randomness in calculating the $p_{PSR}$ and thus the transmission probability of each node may deviate from the optimal point. A statistical characterization of the packet transmission is given here in order to provide a guideline on choosing the update interval that allows timely update while reducing the randomness to a desired level.

Each packet transmission is considered i.i.d with success probability $p_{PSR}$, where $p_{PSR}$ can be obtained from (10). In other words, each packet transmission can be seen as a Bernoulli trial. If we take $m$ consecutive transmissions, the number of successes out of $m$ transmissions follows binomial distribution $B(m, p_{PSR})$. Furthermore, when $mp_{PSR} > 5$ and $m(1 - p_{PSR}) > 5$, the binomial distribution can be approximated using the corresponding normal distribution $N(mp_{PSR}, mp(1 - p_{PSR}))$ [29].

According to the $p_{PSR}$ for $m$ consecutive transmissions is also a random variable following normal distribution $N(p_{PSR}, p_{PSR}(1 - p_{PSR})/m)$. Denote each set of $m$ consecutive transmissions as a minimal unit. For $w$ consecutive minimal units, the sample mean of PSR is also approximately normally distributed with mean $\overline{x} = p_{PSR}$ and standard deviation $\sigma = \sqrt{p_{PSR}(1 - p_{PSR})/mw}$.

The standard deviation of the sample mean, $\sigma$, can be used to characterize the significance of the randomness and show how "steady" is the $p_{PSR}$ obtained. It can be shown that a larger $w$, which translates to a larger update interval, will bring about smaller standard deviation of the $p_{PSR}$ and vice versa.

Given a standard deviation requirement, say $T$, the minimal sample size $w_{opt}$ can be readily computed as $w_{opt} = p_{PSR}(1 - p_{PSR})/mT^2$. Subsequently, as the update interval $I = mw/p_i$, the optimal update interval can be computed as $I_{opt} = [p_{PSR}(1 - p_{PSR})/T^2 p_i] = [p_{PSR}^2(1 - p_{PSR})/T^2 p_i]$. It can be concluded that if the imposed standard deviation requirement is the same for every node, the node with the lowest throughput, say node $i$, will require the largest update interval. Assuming that all nodes update their transmission probabilities synchronously, the update interval will depend on node $i$, that is,

$$I_{net} = [p_{PSR}^2(1 - p_{PSR})/T^2 p_i].$$ (30)

Determining the update interval online in an adaptive manner will theoretically lead to optimal results. By doing this, however, node $i$ will have to broadcast the new update interval to the rest, which introduces additional traffic overheads and more power investment. Alternatively, the update interval can be fixed offline assuming the worst-case scenario that $p_{PSR}^2(1 - p_{PSR})$ is maximum. It can be shown that when
\( p_{\text{PSR}} = \frac{2}{3}, p_{\text{PSR}}^2(1 - p_{\text{PSR}}) \) is at its peak \( \frac{4}{27} \). As such, (30) can then be approximated as

\[
I_{\text{net}} = \left\lceil \frac{4}{27T^2\rho_i} \right\rceil.
\] (31)

7. Simulation Results

In this section, various Monte Carlo simulation results are presented to validate the proposed distributed convergence algorithm. A scenario where there are \( n \) nodes transmitting fixed-length packets to a base station in a probabilistic manner is considered. Each transmission, with the possible acknowledgement, takes up one time slot. The update interval is determined using (31) with a standard deviation requirement of 0.04.

The applicability of the convergence algorithm for different CSI-based methods is first evaluated. In this case the capture ratio is \( b = 3 \) and the SNR is 10. The network size is \( n = 10 \). The convergence algorithm is used for all the three methods. Different throughput demands ranging from 0.005 per node to 0.04 per node have been evaluated. Each set of simulation is run for 10 times and the transmission probability of each node at equilibrium is recorded.

Figure 4 shows the actual throughput and transmission probability of both the simulation and the theoretical analysis. The simulation results match quite well with the theoretical ones for all the three methods, indicating that the proposed ACK-based convergence algorithm can achieve accurate results.

Figure 5 examines the accuracy of the convergence algorithm for different heterogeneous networks in detail. Such a network consists of 5 classes of nodes, with a throughput demand of 0.002, 0.004, 0.006, 0.008, and 0.01, respectively. The capture ratio is \( b = 5 \) and the SNR is 50. Simulations have been conducted for the networks with different number of nodes per class, ranging from 1 to 4. Each set of simulation is repeated 10 times. It can be shown that nodes are able to meet the throughput demand regardless of the number of nodes in the network, with only an average error of 1.1% and a standard deviation of no more than 0.1% of its throughput demand. The transmission probability increases as the number of nodes increases. This is because more frequent transmissions are needed to compensate for the higher packet loss rate due to the increment of the number of nodes in the network.

As an illustrative example, Figure 6 presents the convergence speed for a heterogeneous network with 5 nodes and a capture ratio of 5 and a SNR of 50. The throughput demand is set to be 0.01, 0.02, 0.03, 0.04, and 0.05 for each node, respectively. It is observed that each node can reach the equilibrium point from the 3rd update instant and is stable subsequently with negligible deviations after 2 updates.

Lastly, the robustness of the algorithm against changes on the number of nodes in the network is examined. The capture ratio and the SNR are the same in the previous simulations. In the following study 10 nodes are evaluated, each with a throughput demand of 0.001\( r \), where \( r \) is the node ID labeled from 1 to 10. At Phase I (from 1st to 7th update intervals), only the first 5 nodes transmit packets. At Phase II (from the 7th to the 10th update intervals), two more nodes, namely, Node 6 and Node 7, are active in the network. At Phase III (from the 10th to the 13th update intervals), all nodes actively perform transmissions. At Phase IV, Nodes 6–10 leave the network while Nodes 1–5 remain active in the network.
Figure 6: Performance of the convergence algorithm.

Figure 7 plots the transmission probability versus the update interval. It can be observed that both groups of node adapt well with the changes and the new equilibrium points can be reestablished in a distributed manner. Table 1 shows the actual throughput at the equilibrium. It is assumed that the equilibrium can be established one update interval later after each change on the number of nodes. The results show that each node matches well with its corresponding throughput demand, with an average relative error of 1.26%. Compared with the case that there is no change on the number of nodes (e.g., Figure 5, with average error of 1.1%), the error is still insignificant, indicating that the proposed algorithm works well with such changes.

8. Conclusion

A new method to exploit the channel state information (CSI), namely, adaptive CSI, has been presented. The throughput performance of the proposed adaptive CSI method has been analyzed and compared with the threshold-based CSI and the no-CI method. It is shown that the proposed method is able to achieve up to 9.9% throughput improvement and 12.3% power reduction for a wide variety of systems as compared to the other two approaches. Two guidelines have been derived on selecting these three methods under different system scenarios. The proposed method can be readily implemented in a distributed manner with the use of the acknowledgement packets issued by the base station. Both the analytical and simulation results show that this distributed implementation is able to guarantee fast convergence and is robustness against changes on the number of nodes in the network.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

References

[1] N. Abramson, C. Sacchi, B. Bellalta, and A. Vinel, “Multiple access communications in future generation wireless networks,” Eura- sip Journal on Wireless Communications and Networking, vol. 2012, article 45, 2012.
[2] S. Sen, D. J. Dorsey, R. Guerin, and M. Chiang, “Analysis of slotted ALOHA with multipacket messages in clustered surveillance networks,” in Proceedings of the IEEE Military Communications Conference (MILCOM '12), pp. 1–6, Orlando, Fla, USA, November 2012.
[3] S.-Y. Lien, T.-H. Liu, C.-Y. Kao, and K.-C. Chen, “Cooperative access class barring for machine-to-machine communications,” *IEEE Transactions on Wireless Communications*, vol. 11, no. 1, pp. 27–32, 2012.

[4] H. Wu, C. Zhu, R. J. La, X. Liu, and Y. Zhang, “FASA: accelerated S-ALOHA using access history for event-driven M2M communications,” *IEEE/ACM Transactions on Networking*, vol. 21, no. 6, pp. 1904–1917, 2013.

[5] X. Li, H. Liu, S. Roy, J. Zhang, P. Zhang, and C. Ghosh, “Throughput analysis for a multi-user, multi-channel ALOHA cognitive radio system,” *IEEE Transactions on Wireless Communications*, vol. 11, no. 11, pp. 3900–3909, 2012.

[6] K. Cohen, A. Leshem, and E. Zehavi, “Game theoretic aspects of the multi-channel ALOHA protocol in cognitive radio networks,” *IEEE Journal on Selected Areas in Communications*, vol. 31, no. 11, pp. 2276–2288, 2013.

[7] E. Yaacoub and A. Abu-Dayya, “Automatic meter reading in the smart grid using contention based random access over the free cellular spectrum,” *Computer Networks*, vol. 59, pp. 171–183, 2014.

[8] I. M. I. Habbab, M. Kaveh, and C.-E. W. Sundberg, “ALOHA with capture over slow and fast fading radio channels with coding and diversity,” *IEEE Journal on Selected Areas in Communications*, vol. 7, no. 1, pp. 79–88, 1989.

[9] M. Zorzi, “Mobile radio slotted ALOHA with capture and diversity,” *Wireless Networks*, vol. 1, no. 2, pp. 227–239, 1995.

[10] Y. Jin, G. Kesidis, and J. W. Jang, “A channel aware MAC protocol in an ALOHA network with selfish users,” *IEEE Journal on Selected Areas in Communications*, vol. 30, no. 1, pp. 128–137, 2012.

[11] I. Menache and N. Shimkin, “Noncooperative power control and transmission scheduling in wireless collision channels,” *ACM SIGMETRICS Performance Evaluation Review*, vol. 36, no. 1, pp. 349–358, 2008.

[12] I. Menache and N. Shimkin, “Rate-based equilibria in collision channels with fading,” *IEEE Journal on Selected Areas in Communications*, vol. 26, no. 7, pp. 1070–1077, 2008.

[13] A. B. MacKenzie and S. B. Wicker, “Selfish users in Aloha: a game-theoretic approach,” in *Proceedings of the IEEE 54th Vehicular Technology Conference (VTC ’01)*, pp. 1354–1357, October 2001.

[14] R. B. Myerson, *Game Theory*, Harvard University Press, Cambridge, Mass, USA, 2013.

[15] S. Sengupta, M. Chatterjee, and K. A. Kwiat, “A game theoretic framework for power control in wireless sensor networks,” *IEEE Transactions on Computers*, vol. 59, no. 2, pp. 231–242, 2010.

[16] Y. Wang, G. Miao, and X. Wang, “An energy-efficient non-cooperative game approach for channel-aware distributed medium access control,” in *Proceedings of the IEEE 80th Vehicular Technology Conference (VTC Fall ’14)*, pp. 1–6, Vancouver, Canada, September 2014.

[17] F.-T. Hsu and H.-J. Su, “Random access game in fading channels with capture: equilibria and braess-like paradoxes,” *IEEE Transactions on Signal Processing*, vol. 59, no. 3, pp. 1158–1169, 2011.

[18] I. Menache and N. Shimkin, “Efficient rate-constrained nash equilibrium in collision channels with state information,” in *Proceedings of the 27th IEEE Conference on Computer Communications (INFOCOM ’08)*, IEEE, Phoenix, Ariz, USA, April 2008.

[19] Y.-N. Yeh, K.-C. Chen, and Y.-C. Chen, “Throughput in a cooperative network and channel state information,” *Wireless Personal Communications*, vol. 81, no. 4, pp. 1481–1510, 2015.

[20] P. Park, S. C. Ergen, C. Fischione, and A. Sangiovanni-Vincentelli, “Duty-cycle optimization for IEEE 802.15.4 wireless sensor networks,” *ACM Transactions on Sensor Networks*, vol. 10, no. 1, article 12, 2013.

[21] S. Xie, K. S. Low, and E. Gunawan, “A distributed transmission rate adjustment algorithm in heterogeneous CSMA/CA networks,” *Sensors*, vol. 15, no. 4, pp. 7434–7453, 2015.

[22] F.-T. Hsu and H.-J. Su, “Throughput improvement in ALOHA networks with power capture and a cheat-proof access control,” in *Proceedings of the IEEE 21st International Symposium on Personal Indoor and Mobile Radio Communications*, pp. 1447–1451, September 2010.

[23] Y. H. Bae, B. D. Choi, and A. S. Alfa, “Achieving maximum throughput in random access protocols with multipacket reception,” *IEEE Transactions on Mobile Computing*, vol. 13, no. 3, pp. 497–511, 2014.

[24] S.-H. Wang and Y.-W. P. Hong, “Transmission control with imperfect CSI in channel-aware slotted ALOHA networks,” *IEEE Transactions on Wireless Communications*, vol. 8, no. 10, pp. 5214–5224, 2009.

[25] S. Jun and E. Modiano, “Opportunistic power allocation for fading channels with non-cooperative users and random access,” in *Proceedings of the 2nd International Conference on Broadband Networks (BROADNETS ’05)*, pp. 366–374, IEEE, October 2005.

[26] D. Braess, A. Nagurney, and T. Wakolbinger, “On a paradox of traffic planning,” *Transportation Science*, vol. 39, no. 4, pp. 446–450, 2005.

[27] P. Cardieri, “Modeling interference in wireless ad hoc networks,” *IEEE Communications Surveys & Tutorials*, vol. 12, no. 4, pp. 551–572, 2010.

[28] J. G. Proakis, *Digital Communications*, McGraw–Hill, New York, NY, USA, 4th edition, 2001.

[29] E. Kreyszig, *Advanced Engineering Mathematics*, John Wiley & Sons, 2007.
