Gravitational Lensing of Relativistic Fireball

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60th October Anniversary Prospect 7a
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Abstract

The gravitational lensing of a relativistic fireball can produce the time delayed multiple images with quite different spectra and temporal patterns in contrast with a nonrelativistic source and hence can imitate the source recurrence. In particular, the enigmatic four multiple gamma-ray bursts detected during two days at October 1996 in the same sky region may be due to a single fireball event multiply imaged by the foreground galactic nucleus or cluster of galaxies.

PACS numbers: 95.30.Sf; 95.85.P; 98.62.Sb
Keywords: astrophysics, general relativity, gravitational lenses, gamma-ray bursts

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The identical spectra and time histories of multiple images are generally considered as necessary consequences of the gravitational lens (GL) phenomenon (for review see e.g. [1]). However these signatures of gravitational lensing are suitable only in the case of both nonrelativistic source and GL when multiple images are generated by the same region on the surface of the source albeit shifted in time. Meanwhile we demonstrate in the following that boosting of light in the relativistic source such as a gamma-ray burst (GRB) fireball after an appropriate lensing may provide the multiple images without any similarities of spectra and light-curves.

The observed cosmological GRBs are the promising targets for the gravitational lensing of relativistic sources. The similarities of spectra and temporal patterns of multiple images were used until now as observation signatures in searches of the possible gravitational lensing of GRBs with a hope to determine or confine e.g. the contents and density of compact objects in dark matter and average redshift of GRBs [2, 3, 4]. The GRB, within the framework of a standard cosmological scheme of their origin, is generated by the nearly spherical or beamed low mass leptonic relativistic fireball resulting from the coalescence of tight neutron star binary or collapse of some massive star in distant galaxy (for reviews see e.g. [6, 7]). A typical expected recurrence rate of cosmological GRBs is one per million years per galaxy. Hence if possible observation of GRB repetition were not due to the time delayed multiple lensed images, it would create hard problems for the standard cosmological scheme of GRB origin. The hunting for GRB recurrence is the difficult task because of the low angular resolution of nowadays gamma-telescopes. Recent statistical analysis of GRB samples provide only the upper limits on the possible repetition of GRBs [8, 9, 10]. Meanwhile among more than 2500 GRBs detected nowadays there are surprising multiple GRBs coming at October 1996. At that time the orbital telescopes BATSE, KONUS, TGRS and Ulysses detected independently four GRB events from the same sky region during two days [13, 14]. The probability of accidental projection of these four GRBs is very small, $3.1 \cdot 10^{-5}$ ÷ $3.3 \cdot 10^{-4}$, whereas the clustering is not so significant if the four events combine into the three bursts [15]. However in the latter case one of the bursts would be more than 20 minutes in duration. Nevertheless the recurrence of cosmological GRBs is a rather natural if some part of GRBs are generated not by the coalescence of neutron star binaries but by the accidental collisions of neutron stars in the dense central stellar clusters of distant galactic nuclei [17]. In this case the observation of multiple GRBs is a consequence of a serendipitious event of massive black hole formation accompanied by the four events of neutron star collisions during dynamical collapse of extremely dense central stellar cluster in some distant galactic nuclei [16].

Here we propose the other resolution of the ‘fast GRB recurrence’ problem. We demonstrate in the following that a cluster of four GRB events observed during two days at October 1996 can be all due to a single relativistic fireball event at cosmological distance expanding with a large bulk Lorentz factor $\Gamma \gg 1$ and multiply imaged by the GL. The basic idea of specific gravitational lensing with resulting different spectra and temporal patterns of separate time delayed
Figure 1: Schematic view of the gravitational lensing of relativistically expanding fireball with a center of deflector (d) shifted to distance x from the line connecting the source (s) and observer (o) in the absence of deflector. Rays 1 and 2 leave the fireball with a separation angle $\theta$ and have correspondingly the impact parameters $r_1$ and $r_2$ which define deflection angle $\alpha$. Relativistic boosting confines each visible image by the narrow region of the expanding fireball within an open angle $\theta_c = \Gamma^{-1} \ll 1$ relative to the fireball center.

The emission of the fireball beyond the confinement angle $\theta_c$ comes to the detector without the boosting and generally sinks down to the noise. In this case two ‘rays’ separated by the angle $\theta > \theta_c$ originate from the different parts of the fireball connected by the space-like world lines. These regions are causally disconnected and their emission may be generated under different physical conditions. An additional supposed requirement for the realization of this model is a small-scale turbulent structure of emitting shocks in the fireball with a typical length much less than the instant fireball radius. In this case the outgoing rays would not retain information on the temporal structure of the central source of energy. This highly turbulent structure seems quite reasonable for the GRB fireball because of developing of instabilities in the relativistic shocks (see e.g. [6]).
Gravitational lensing of *nonrelativistic* source provides multiple images only with time-shifted but similar spectral and temporal structures. On the contrary outgoing rays from the *relativistic* fireball separated by the angle $\theta > \theta_c$ in general can be generated under the different physical conditions. After an appropriate gravitational lensing they would appear to the distant low-resolution telescope as recurrent separate events from the same prospective source on the sky but with the quite different spectral and temporal structures. This specific feature of the gravitational lensing of relativistic fireballs may influence the results of statistical searches of GRB repetition of [8, 9, 10]. In the following we determine the necessary physical parameters of the possible GL to imitate the recurrent multiple GRBs with different spectra and light curves from the same GRB event.

For a general mass distribution in the GL (deflector) with the Newtonian potential $\phi(\vec{r})$ the deflection angle of a separate ray can be expressed (see e. g. [11]) as a two dimensional vector

$$\vec{\alpha} = \frac{2}{c^2} \int ds \, \vec{n} \times (\vec{n} \times \nabla \phi),$$

where $\vec{n}$ is a unit vector along the ray and the integral is performed along the ray too. Fig. 1 represents the schematic view of the possible GRB (i. e. relativistic fireball) lensing geometry with a center of GL (deflector) in general shifted from the line connected the observer and source which is the same as the path of the undeflected ray (i. e. in the absence of deflector). The estimation for potential is $\phi \sim \sigma^2$ for the GL composed of constituent masses (e. g. stars or galaxies in the cluster) moving with a virial velocity dispersion $\sigma$. It follows then from Eq. (1) that a value of the deflection angle $\alpha = |\vec{\alpha}|$ is always small, $\alpha \sim \phi/c^2 \sim \sigma^2/c^2 \ll 1$. In the limit of a small deflection angle there are general relations for a geometric time delay $\Delta t_g$ which is conditioned by the path difference between the deflected and undeflected rays

$$\Delta t_g = (1 + z_d) \frac{\alpha \xi}{2c},$$

and for a separation $\xi$ between deflected and deflected rays

$$\xi = \frac{D_s D_d}{D_s} \alpha,$$

where $D_s$, $D_d$, and $D_{ds}$ are the angular diameter distances from the observer to the source of GRB, to the deflector (lens) and from the deflector to the source correspondingly and $z_d$ is a deflector redshift. Besides the geometric time delay $\Delta t_g$ there is an additional general relativistic time delay $\Delta t_{gr}$ due to traversing through the region with a gravitational field (“Shapiro delay”)

$$\Delta t_{gr} = -\frac{2}{c^2} \int ds \, \phi.$$

The both geometrical $\Delta t_g$ and gravitational $\Delta t_{gr}$ time delays are of the same order of magnitude for the general ray position as can be verified from Eqs. (2)
and (4) and using estimation $\alpha \sim \phi/c^2$ from Eq. (1). So the corresponding total time delay $\Delta t = \Delta t_g + \Delta t_{gr} \sim GM(\xi)/c^3$ is defined by the effective mass $M \sim M(\xi)$ of the GL inside the radius $r = \xi$. It must be taken in mind that in some degenerate cases of GL symmetry this estimation of time delay between images may provide only the upper limit on $\Delta t$ because the time delays of different rays tend to compensate each other.

Now we can formulate two necessary requirements for production of lensing images of causally disconnected regions of relativistic fireball. For the general case of both the lens (deflector) and relativistic fireball (source) at comparable (cosmological) distances $D_d \sim D_s$ the angle between two outgoing rays is $\theta = \alpha(D_d/D_s) \simeq \alpha$. Using deflection angle estimation $\alpha \sim \sigma^2/c^2$ and causal disconnection condition $\theta > \theta_c \sim \Gamma^{-1}$ we find the first requirement on the velocity dispersion $\sigma$ in the GL: $\sigma \geq c/\Gamma^{1/2}$ with expected $\Gamma \sim 10^3$ for GRB case.

The second requirement on the effective total mass $M$ of the GL follows from the estimation of the time delay $\Delta t(1, 2) = \Delta t(1) - \Delta t(2)$ between two images: $M \sim c^3 \Delta t(1, 2)/G$ with $\Delta t(1, 2) \sim 1$ day. One of this requirements may be replaced by the equivalent one for the GL radius $R < \Gamma c \Delta t(1, 2)$ with the use of virial relation $R \sim GM/\sigma^2$.

To specify more explicitly these requirements we consider a simple model of the gravitational lensing of relativistic fireball by the singular isothermal sphere (SIS) \[18, 19\] with a center of SIS shifted to distance $x$ from the line connecting the source and observer (which is the path of undeflected ray) as shown on Fig. 1. It is convenient to suppose that SIS is confined within some finite radius $R$. The spherically symmetric and constant temperature mass distribution $M(r)$ in this model of GL is connected with a current radius $r$ by the relation

$$r = \frac{GM(r)}{\sigma^2},$$

where $\sigma = \text{const} \ll c$ is a line-of-sight velocity dispersion in the SIS. The deflection angle of this GL is independent on the impact parameter of the ray (while it is much less than $R$) and equals

$$\alpha = 4\pi \left(\frac{\sigma}{c}\right)^2.$$  

This Eq. specifies the first requirement for causal disconnection of images, $\alpha \geq \Gamma^{-1}$, which provides the restriction on the velocity dispersion in the GL:

$$\sigma \geq \frac{c}{(4\pi\Gamma)^{1/2}} \simeq 3 \cdot 10^3 \Gamma_3^{-1/2} \text{ km s}^{-1},$$

where $\Gamma_3 = \Gamma/10^3$.

To determine the required scales of time delay $\Delta t(1, 2) \sim 1$ day and required deflection angle $\alpha$ we consider two images produced by the lensing of two rays, 'ray 1' and 'ray 2', coming through opposite sides of the SIS with corresponding impact parameters $r_1$ and $r_2$ such that $\max(r_1, r_2) \ll R$. The gravitational potential inside the SIS (at $r < R$) is

$$\phi(r < R) = -\sigma^2 \left(1 + \ln \frac{R}{r}\right).$$
Because of $\alpha = \text{const}$ any two rays coming through opposite sides of the SIS have the same separation $\xi_1 = \xi_2 = \xi = (r_1 + r_2)/2$. As a consequence the geometric time delay between ray 1 and ray 2 equals zero according to Eq. (3). So the only time delay between considered two rays results from the difference of gravitational time delays $\Delta t(1,2) = \Delta t_{gr}(1) - \Delta t_{gr}(2)$. After the simple but lengthy calculations we find

$$\Delta t(1,2) = 4\pi(1 + z_d) \left( \frac{\sigma}{c} \right)^2 \frac{x}{c} = 4\pi(1 + z_d) \frac{GM(x)}{c^3},$$

(9)

where the last equality follows from Eq. (5). According to limitation of Eq. (7) the shift of SIS center $x = (r_2 - r_1)/2$ is

$$x \leq \frac{c\Delta t(1,2)}{1 + z_d} \Gamma \simeq \frac{0.84}{1 + z_d} \frac{\Delta t(1,2)}{\text{1 day}} \Gamma_3 \text{ pc.}$$

(10)

Using $\Delta t(1,2)$ from Eq. (9) as an observed time delay between different GRB lensing images of the same relativistic fireball we find the required mass of the GL:

$$M \geq M(x) \simeq \frac{1.4 \cdot 10^9}{1 + z_d} \frac{\Delta t(1,2)}{\text{1 day}} \text{M}_\odot;$$

(11)

This estimation with an order of magnitude accuracy is valid for the more complex mass distributions inside the GL and in fact provides only the low limit of the possible GL mass because of the possible time delay compensation between different images.

It follows from Eqs. (6) and (11) that the necessary restrictions on the parameters of GL which produce (i) the one day time delayed and (ii) causally independent images (with different spectra and temporal histories) of the same single GRB fireball are the following: the required velocity dispersion of the constituent masses in the GL for the generation of multiple GRB events at October 1996 is $\sigma \gtrsim 3 \cdot 10^3 \text{ km s}^{-1}$ and the total mass of the GL $M \geq 10^9 \text{M}_\odot$ for the effective redshift of GRBs $z \simeq 1 \div 2$. These required values of $M$ and $\sigma$ may be realized in the real case of lenses such as a dense enough cluster of galaxies or massive central stellar cluster in galactic nucleus. In addition Eqs. (3) and (7) provide another restriction: for a rather general case of $D_s \sim D_d$ the deflector must be situated at the distance from the source $D_{ds} \sim \xi/\alpha \leq 10^2 \Gamma_3 R$. For the angular diameter distance $D_{ds}$ of cosmological scale the suitable $R \sim 1 \text{ Mpc}$ is a characteristic scale for the cluster of galaxies and respectively for $D_{ds} \sim 1 \text{ kpc}$ (GRB originated in distant galactic disk) the suitable $R \sim 1 \text{ pc}$ is a scale of the central stellar clusters in the host galactic nucleus. These both examples of promising GL candidates are again in agreement with other previous requirements. At the same time these both deflectors are the most common examples of the nowadays observed cosmological lenses. Meanwhile the considered model works for fireballs emerging inside (or closely) to deflectors: whether (i) in one of the galaxies in the host cluster or (ii) in the galactic disk near the host galactic nucleus. This proximity of the source to the deflector increases the probability of the lensing with respect, for example, to the lensing of quasars (by distant foreground galaxy or cluster of galaxies).
The gravitational potential of real GL must be nonspherical to generate 4 or more images. The simplest model is the SIS potential perturbed by a quadratic share [2], e.g. caused by rotation, which in general generates one faint and four bright images. Massive rotating black hole can also produces the suitable nonspherical gravitational field.

In general we demonstrate that identities of spectra and time histories cannot serve as specific signatures of gravitational lensing in the case of relativistic sources. In particular the gravitational lensing of relativistic fireball from a single GRB event may imitate the GRB recurrence.

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