Form Factors and Decay of $\bar{B}_s^0 \to J/\psi \phi$ From QCD Sum Rule

Ying-Quan Peng* and Mao-Zhi Yang†

School of Physics, Nankai University,
Tianjin 300071, People’s Republic of China

Abstract

We calculate $\bar{B}_s^0 \to \phi$ translation form factors $V, A_0, A_1, A_2$ based on QCD sum rule and study the nonleptonic two-body decay of $\bar{B}_s^0 \to J/\psi \phi$ with the form factors obtained. We calculate the time-integrated branching ratio of $\bar{B}_s^0 \to J/\psi \phi$ decay. The results for both the total branching ratio and the cases for the final vectors in longitudinal and transverse polarizations are consistent with experimental data.

PACS numbers: 13.20.He,11.55.Hx,12.15.Lk

Keywords: Decay of $B_s$ meson

* 2120170119@mail.nankai.edu.cn
† yangmz@nankai.edu.cn
I. INTRODUCTION

$CP$ asymmetry arises due to the non-vanishing complex phase of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements \cite{1, 2} in the standard model (SM). The requirement of unitarity of the CKM matrix results in a set of triangles in the complex plane. $\bar{B}_s^0(t) \rightarrow J/\psi \phi$ is a golden decay mode to measure the angle $\beta_s$, one of the angles of the triangle in the $bs$ sector: $V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0$. The angle $\beta_s$ is related to the sides of unitarity triangle by $\beta_s = \arg[-V_{ts}V_{tb}^*/(V_{cs}V_{cb}^*)]$ \cite{3}. $\bar{B}_s(t) \rightarrow J/\psi \phi$ decay stimulate wide interest in both theory and experiment \cite{4–8}. This decay mode is not only interesting for analysis of $CP$ violation, but also useful for studying strong interaction in the decay process. Factorization is a basic method to calculate non-leptonic $B$ meson decays, where it is assumed that the hadronic decay amplitude can be factorized as a product of matrix elements of two local quark-antiquark currents \cite{9, 10}. The QCD non-factorizable corrections to the factorization result can be calculated systematically in perturbation theory, which is called QCD factorization \cite{11, 12}. The hadronic matrix element of the quark-antiquark current in $B$ decays can be decomposed as polynomials of form factors. The form factors are in general non-perturbative quantities in QCD, which can be calculated with non-perturbative method, such as Lattice QCD, QCD sum rule, QCD light-cone sum rule, and quark model, etc. The $\bar{B}_s^0 \rightarrow \phi$ transition form factors involved in $\bar{B}_s^0 \rightarrow J/\psi \phi$ decay have been calculated by QCD light-cone sum rule (LCSR) \cite{13, 14}, Quark model (QM) \cite{15}, and QCD sum rule \cite{16} in literature. Some form factors obtained by QCD sum rule in Ref. \cite{16} are different from other results calculated in Refs. \cite{14, 15} by a sign. Here in this work, we revisit the $\bar{B}_s^0 \rightarrow \phi$ transition form factors with QCD sum rule method. Then the from factors obtained in this work are used to study the nonleptonic decay of $\bar{B}_s^0 \rightarrow J/\psi \phi$. The the transverse, longitudinal and total time-dependent decay widths $\Gamma_L(t)$, $\Gamma_T(t)$ and $\Gamma(t)$ are calculated respectively. The results are consistent with experimental data within experimental and theoretical uncertainties.

The remainder of this paper is organized as follows: In sec. II, we present the method to calculate the form factors in QCD sum rule method. Section III is for the numerical analysis, and Section IV for the application of the form factors in the decay mode $\bar{B}_s^0 \rightarrow J/\psi \phi$. Section V is a brief summary.
II. THEORETICAL FRAMEWORK

1. The method

In the factorization approach, one ingredient of the amplitude of the decay mode $\bar{B}_s^0 \rightarrow J/\psi \phi$ is the hadronic matrix element $\langle \phi | \bar{s} \gamma_\nu (1 - \gamma_5) b | \bar{B}_s^0 (p_1) \rangle$, which can be decomposed as

$$\langle \phi (\varepsilon, p_2) | \bar{s} \gamma_\nu (1 - \gamma_5) b | \bar{B}_s^0 (p_1) \rangle = \varepsilon_\nu \rho_\alpha \rho_\beta \varepsilon^* \rho_1 \rho_2 \frac{2V(q^2)}{m_{B_s} + m_\phi}$$

$$-i(\varepsilon^*_\nu - \frac{\varepsilon^* \cdot q}{q^2} q_\nu)(m_{B_s} + m_\phi) A_1(q^2)$$

$$+i[(p_1 + p_2)_\nu - \frac{m_{B_s}^2 - m_\phi^2}{q^2} q_\nu] \varepsilon^* \cdot q \frac{A_2(q^2)}{m_{B_s} + m_\phi}$$

$$-i\frac{2m_\phi}{q^2} \varepsilon^* \cdot q A_0(q^2),$$

where the parameters $V$, $A_0$, $A_1$, $A_2$ are the transition form factors, and $q = p_1 - p_2$.

The QCD sum rule method was originally developed by Shifman, Vainshtein and Zakharov in the late 1970s \cite{17,18}, which was widely used in hadronic process, see Ref. \cite{19} for a review. In order to calculate the transition form factors of $\bar{B}_s^0$ to $\phi$ in QCD, we consider the three-point correlation function defined by

$$\Pi_{\mu\nu} = i^2 \int d^4 x d^4 y e^{ip_1 \cdot x - ip_2 \cdot y} \langle 0 | T \{ j_5^\phi (x) j_\nu (0) j_\nu (y) \} | 0 \rangle,$$

where the three currents are: (1) $j_5^\phi (y) = \bar{b} (y) \gamma_5 s (y)$, the current of $\bar{B}_s^0$ channel; (2) $j_\nu (0) = \bar{s} \gamma_\nu (1 - \gamma_5) b$, the current of weak transition; (3) $j_\nu^\phi (x) = \bar{s} (x) \gamma_\nu s (x)$, the current of $\phi$ channel.

One can use the double dispersion relation to express the correlation function as

$$\Pi_{\mu\nu} = \int ds_1 ds_2 \frac{\rho(s_1, s_2, q^2)}{(s_1 - p_1^2)(s_2 - p_2^2)},$$

where $\rho(s_1, s_2, q^2)$ is the spectral density. By inserting a full set of intermediate hadronic states into the time-ordered product in the correlation function, one can obtain the spectral density function as

$$\rho(s_1, s_2, q^2) = \sum_X \sum_Y \langle 0 | j_5^\phi | X \rangle \langle X | j_\nu | Y \rangle \langle Y | j_\nu | 0 \rangle \delta(s_1 - m_X^2) \delta(s_2 - m_Y^2) \theta(p_X^0) \theta(p_Y^0),$$

where $X$ and $Y$ denote the full set of hadronic states of $\phi$ and $\bar{B}_s^0$ channels, respectively. Substituting the spectral density $\rho(s_1, s_2, q^2)$ in Eq. (4) into Eq. (3) and integrating over $s_1$
and $s_2$, then we can get

$$\Pi_{\mu\nu} = \sum_X \sum_Y \frac{\langle 0 | j_\mu^0 | X \rangle \langle X | j_\nu | Y \rangle \langle Y | j_5 | 0 \rangle}{(m_Y^2 - p_1^2)(m_X^2 - p_2^2)} + \text{continuum states.} \quad (5)$$

The result of the correlation function in the above equation can be also expressed as a sum of the ground states, excited states and continuum states

$$\Pi_{\mu\nu} = \frac{\langle 0 | j_\mu^0 | \phi \rangle \langle \phi | j_\nu | \bar B_s^0 \rangle \langle \bar B_s^0 | j_5 | 0 \rangle}{(m_{B_s}^2 - p_1^2)(m_\phi^2 - p_2^2)} + \text{excited states + continuum states.} \quad (6)$$

Using the definition of decay constants in the following

$$\langle 0 | \bar{s}_\gamma \mu s | \phi \rangle = m_\phi f_\phi \varepsilon_\mu^{(\lambda)},$$
$$\langle 0 | \bar{s}_\gamma \mu \gamma_5 s | \phi \rangle = 0,$$
$$\langle 0 | si_\gamma_5 b | \bar B_s^0 \rangle = \frac{f_{B_s} m_{B_s}^2}{m_b + m_s},$$

where $f_\phi$ and $f_{B_s}$ are decay constants of the relevant mesons, the correlation function is changed to be

$$\Pi_{\mu\nu} = \frac{m_\phi f_\phi \varepsilon_\mu^{(\lambda)} \langle \phi (\varepsilon_\mu^{(\lambda)}, p_2) | j_\nu | \bar B_s^0(p_1) \rangle f_{B_s} m_{B_s}^2}{(m_{B_s}^2 - p_1^2)(m_\phi^2 - p_2^2)(m_b + m_s)} + \text{excited and continuum states.} \quad (7)$$

Meanwhile the time-ordered current operator in the correlation function in Eq. (2) can be expanded in terms of a series of local operators with increasing dimensions in QCD

$$i^2 \int d^4 x d^4 y e^{ip_2 x - ip_1 y} T \{ j_\mu^0(x) j_\nu(0) j_5(y) \}$$
$$= C_{0\mu\nu} I + C_{3\mu\nu} \bar \Psi \gamma_\mu \Psi + C_{4\mu\nu} G^a_\alpha G^{a\alpha\beta} \Psi + C_{5\mu\nu} \bar \Psi \sigma_\alpha \gamma_\mu T^a G^{a\alpha\beta} \Psi$$
$$+ C_{6\mu\nu} \bar \Psi \Gamma \Psi \bar \Psi \Gamma' \Psi + \cdots, \quad (8)$$

where $C_{i\mu\nu}$ are Wilson coefficients, $I$ the unit operator, $\bar \Psi \Psi$ the local fermion field operator of light quarks, $G^a_\alpha$ gluon strength tensor, $\Gamma$ and $\Gamma'$ the matrices appearing in the procedure of calculating the Wilson coefficients. At deep negative values of $p_1^2$ and $p_2^2$, the Wilson coefficients can be calculated reliably in perturbative QCD, and the operator-product expansion (OPE) in Eq. (9) can converge quickly.

Considering the non-vanishing vacuum-expectation-value of the operators in Eq. (9), we can get the correlation function in terms of Wilson coefficients and condensates of local
operators

\[ \Pi_{\mu\nu} = i^2 \int d^4 x d^4 y e^{ip_2 \cdot x - ip_1 \cdot y} \langle 0 | T \{ j_\mu^\phi(x) j_\nu(0) j_5(y) \} | 0 \rangle \]

\[ = C_{0\mu\nu} + C_{3\mu\nu} \langle 0 | \bar{\Psi} \sigma^a T^a G^{\alpha\beta} \Psi | 0 \rangle + C_{5\mu\nu} \langle 0 | \bar{\Psi} \sigma^a T^a G^{\alpha\beta} \Psi | 0 \rangle + C_{6\mu\nu} \langle 0 | \bar{\Psi} \sigma^a T^a G^{\alpha\beta} \Psi | 0 \rangle + \cdots, \]

(10)

According to the Lorentz structure of the correlation function, Eq. (10) can be re-expressed by six parts

\[ \Pi_{\mu\nu} = f_0 \epsilon_{\mu\nu\alpha\beta} p_1^\alpha p_2^\beta - i \left( f_1 p_1 \mu p_1 \nu + f_2 p_2 \mu p_2 \nu + f_3 p_1 \mu p_2 \nu + f_4 p_1 \nu p_2 \mu + f_5 g_{\mu\nu} \right). \]  

(11)

The coefficients \( f_i \)'s are consisted of perturbative and condensate contributions,

\[ f_i = f_i^{pert} + f_i^{(3)} + f_i^{(4)} + f_i^{(5)} + f_i^{(6)} + \cdots, \]

(12)

where \( f_i^{pert} \) is the perturbative contribution of the unit operator, and \( f_i^{(3)} \), \( f_i^{(4)} \), \( f_i^{(5)} \), \( f_i^{(6)} \), \( \cdots \), are contributions of condensates of operators with increasing dimension in OPE.

In next section we shall know that perturbative contribution and gluon-condensate contribution can be written in the form of dispersion integration

\[ f_i^{pert} = \int d s_1 d s_2 \frac{\rho_i^{pert}(s_1, s_2, q^2)}{(s_1 - p_1^2)(s_2 - p_2^2)}, \]

\[ f_i^{(4)} = \int d s_1 d s_2 \frac{\rho_i^{(4)}(s_1, s_2, q^2)}{(s_1 - p_1^2)(s_2 - p_2^2)}. \]

We can approximate the contribution of excited states and continuum states as integrations over some thresholds \( s_1^0 \) and \( s_2^0 \) in the above two equations. Then equating the two expressions of the correlation function in Eq. (8) and (11), we can get an equation for extracting the form factors. But such an equation may heavily depend on the approximation for the contribution of excited states and the contributions of higher dimensional operators in OPE. To improve such an equation and make the contribution of higher dimensional operator small, one can make Borel transformation over \( p_1^2 \) and \( p_2^2 \) in both sides, which can suppress the contributions of excited states and condensate of higher dimensional operators.

The definition of Borel transformation to any function \( f(x^2) \) is

\[ \hat{B}_{x^2, M^2} f(x^2) = \lim_{k \to \infty, x^2 \to -\infty} \left( -\frac{x^2}{k} \right)^k \frac{\partial^k}{(k-1)!} \frac{\partial (x^2)^k}{(x^2)^k} f(x^2), \]

\[ -x^2/k = M^2 \]

5
Matching these two expressions of the correlation function in Eq. (8) and (11), and performing Borel transformation for both variables $p_1^2$ and $p_2^2$, the sum rules for the form factors can be obtained

$$V(q^2) = - \frac{(m_b + m_s)(m_{B_s} + m_\phi)}{2m_\phi f_B m_{B_s}^2} \frac{e^{m_{B_s}^2/M_1^2} e^{m_\phi^2/M_2^2} M_1^2 M_2^2 (f_1 + f_3)}{B},$$

$$A_1(q^2) = - \frac{(m_b + m_s)}{m_\phi f_B m_{B_s}^2} \frac{e^{m_{B_s}^2/M_1^2} e^{m_\phi^2/M_2^2} M_1^2 M_2^2 (f_1 + f_3)}{2B},$$

$$A_2(q^2) = \frac{(m_b + m_s)(m_{B_s} + m_\phi)}{m_\phi f_B m_{B_s}^2} \frac{e^{m_{B_s}^2/M_1^2} e^{m_\phi^2/M_2^2} M_1^2 M_2^2 (f_1 + f_3)}{2B},$$

$$A_0(q^2) = - \frac{(m_b + m_s)}{2m_\phi f_B m_{B_s}^2} \frac{e^{m_{B_s}^2/M_1^2} e^{m_\phi^2/M_2^2} M_1^2 M_2^2 (f_1 + f_3)}{2B} - \frac{B f_i}{2},$$

where $B f_i$ denotes Borel transformation of $f_i$ for both variables $p_1^2$ and $p_2^2$. $M_1$ and $M_2$ are Borel parameters. After subtracting the contribution of the excited states and continuum states, the dispersion integration for perturbative and gluon condensate contribution should be performed under the threshold

$$f_i^{pert} = \int_{s_1}^{s_0} ds_1 \int_{s_2}^{s_0} ds_2 \frac{\rho_i^{pert}(s_1, s_2, q^2)}{(s_1 - p_1^2)(s_2 - p_2^2)},$$

$$f_i^{(4)} = \int_{s_1}^{s_0} ds_1 \int_{s_2}^{s_0} ds_2 \frac{\rho_i^{(4)}(s_1, s_2, q^2)}{(s_1 - p_1^2)(s_2 - p_2^2)},$$

2. The Calculation of the Wilson Coefficients

In this section, we calculate the Wilson coefficients in the operator-product expansion, then extract the relevant coefficients $f_i$ for the sum rules of the form factors in Eq. (13). The method of the calculation is very similar to that used in our previous work in Ref. [20], where the form factors in $D_s^+ \to \phi \bar{q} \nu$ decay were studied in QCD sum rule method. So we will not give the details of the calculation in the present paper. But for the completeness of this paper we shall give some main points of the calculation in this section.

All of the Feynman diagrams for calculating the Wilson coefficients in OPE in Eq. (9) are shown below. They are: diagram for perturbative contribution in Fig.1, diagrams for contributions of operators $\bar{\Psi}(x)\Psi(y)$ and $\tilde{\Psi}(0)\Psi(x)$ in Fig.2, diagrams for contributions of gluon-gluon operator in Fig.3, diagrams for mixed quark-gluon operators in Fig.4 and diagrams for contributions of four-quark operators in Fig.5.
FIG. 1: Diagram for perturbative contribution.

FIG. 2: Diagrams for the contributions of operators $\bar{\Psi}(x)\Psi(y)$ and $\bar{\Psi}(0)\Psi(x)$.

FIG. 3: Diagrams for contributions of gluon-gluon operator.
2.1 The perturbation contribution

For perturbative contribution shown in Fig.1, only the leading order in $\alpha_s$ expansion is considered here. This contribution is relevant to the Wilson coefficient $C_0$ in OPE of the correlation function in Eq. (10). The amplitude can be written as

$$C_0 = i^2 \int \frac{d^4k}{(2\pi)^4} \left( -1 \right)^F T_T \left[ i\gamma_5 \frac{i(k + m_s)}{k^2 - m_s^2 + i\varepsilon} \gamma_\mu \frac{i(k + p_2 + m_s)}{(k + p_2)^2 - m_s^2 + i\varepsilon} \gamma_\mu (1 - \gamma_5) \right].$$

(14)

We can re-write the integration of Eq. (14) in the form of dispersion integration

$$C_0 = \int ds_1 ds_2 \frac{\rho(s_1, s_2, q^2)}{(s_1 - p_1^2)(s_2 - p_2^2)}.$$  

(15)
The spectral density \( \rho(s_1, s_2, q^2) \) can be calculated according to Cutkosky’s rule [21], i.e., replacing the denominators of the quark propagators with \( \delta \) functions and putting all the quark lines on-mass-shell, \( 1/(k^2 - m^2 + i\varepsilon) \to -2\pi i\delta(k^2 - m^2) \). Then the spectral density can be calculated from

\[
\rho(s_1, s_2, q^2) = \frac{(-2\pi i)^3}{4\pi^2} \int \frac{d^4 k}{(2\pi)^4} \text{Tr}[\gamma_5(k + m_s)\gamma_\mu(k + p_2 + m_s)\gamma_\nu(1 - \gamma_5) \\
\times (k + p_1 + m_b)\delta(k^2 - m_0^2)\delta[(k + p_1)^2 - m_1^2] \\
\times \delta[(k + p_2)^2 - m_2^2] \bigg| p_1^2 \to s_1, p_2^2 \to s_2 \bigg].
\]

(16)

Some basic formulas are needed to perform the integration in Eq. (16), which have been obtained in Refs. [20, 22]. They are given in Appendix A. With results of \( I, I_\mu \) and \( I_\mu\nu \) given in Eqs. (A1) ∼ (A3), the integration in Eq. (16) can be performed without difficulty.

### 2.2 Contributions of the quark-quark operator

The diagrams for the contribution of “quark-quark” condensation are shown in Fig. 2. The contribution of Fig. 2 (b) is zero after the double Borel transformation for both variables \( p_1^2 \) and \( p_2^2 \), because only one variable left in the denominator \( 1/(p_2^2 - m_s^2) \). So Fig. 2 (b) can be ignored.

The contribution of Fig. 2(a) is

\[
\Pi^{3a}_{\mu\nu} = i^2 \int d^4 x d^4 y e^{ip_2 x - ip_1 y} \langle 0|\bar{\Psi}(x)\gamma_\mu S_s^a(x)\gamma_\nu(1 - \gamma_5)S_b^b(-y)i\gamma_5 \Psi(y)|0\rangle,
\]

(17)

where

\[
S_s^a(x) = \int \frac{d^4 k_2}{(2\pi)^4} \frac{i}{k_2 - m_s} e^{-ik_2 x}, \quad S_b^b(-y) = \int \frac{d^4 k_1}{(2\pi)^4} \frac{i}{k_1 - m_b} e^{ik_1 y}
\]

are the propagators of \( s \) and \( b \) quarks, respectively. Moving the quark field operators \( \bar{\Psi}(x) \) and \( \Psi(y) \) together then we can obtain

\[
\Pi^{3a}_{\mu\nu} = i^2 \int d^4 x d^4 y e^{ip_2 x - ip_1 y} \langle 0|\bar{\Psi}_\alpha(x)\Psi_\beta(y)|0\rangle [\gamma_\mu S_s^a(x)\gamma_\nu(1 - \gamma_5)S_b^b(-y)i\gamma_5]\alpha\beta,
\]

(18)

where \( \alpha \) and \( \beta \) are Dirac spinor indices. The matrix element \( \langle 0|\bar{\Psi}_\beta(x)\Psi_\alpha(y)|0\rangle \) can be treated in the fixed-point gauge [23–25], the result of which up to the order of \( x^3 \) and \( y^3 \) has been
given in Ref. [20]

\[
\langle 0 | \bar{\Psi}_a^\alpha(x) \Psi_b^\beta(y) | 0 \rangle = \delta_{ab} \left[ \langle \bar{\Psi} \Psi \rangle \left( \frac{1}{12} \delta_{\beta \alpha} + \frac{i}{48} (\not{x} - \not{y})_{\beta \alpha} - \frac{m^2}{96} (x - y)^2 \delta_{\beta \alpha} \right) 
- \frac{i}{3!} \frac{m^3}{96} (x - y)^2 (\not{x} - \not{y})_{\beta \alpha} \right] 
+ g \langle \bar{\Psi} \sigma_T G \Psi \rangle \left( \frac{1}{192} (x - y)^2 \delta_{\beta \alpha} \right) 
+ \frac{i}{3!} \frac{m}{192} (x - y)^2 (\not{x} - \not{y})_{\beta \alpha} 
+ \cdots \right], 
\]

(19)

where \(a\) and \(b\) in the above equation are the color indices, \(m\) is the quark mass. From Eq. (19) one can see that Fig. 2 (a) contributes not only to the coefficients of quark condensate \(\langle \bar{\Psi} \Psi \rangle\), but also to mixed quark-gluon condensate \(g \langle \bar{\Psi} \sigma_T G \Psi \rangle\) and the four-quark condensate \(\langle \bar{\Psi} \Psi \rangle^2\).

### 2.3 Contributions of the gluon-gluon operator

The diagrams for the contribution of gluon-gluon operator are shown in Fig. 3. It is convenient to calculate these diagrams in the fixed-point gauge, in which the gauge fixing condition is taken as \(z^\mu A^a_\mu(z) = 0\) [23–25]. Then the external color field can be expressed in terms of its strength tensor [24],

\[
A^a_\mu(z) = \int_0^1 d\beta \beta z^\rho G^a_\rho(z). 
\]

(20)

Expanding the above expression to the first order of \(z\), one can get

\[
A^a_\mu(z) = \frac{1}{2} z^\rho G^a_\rho(0) + \cdots. 
\]

(21)

Another equation useful for the calculation of the contributions of the diagrams depicted in Fig. 3 is

\[
\langle 0 | G^{a\alpha}_{\alpha\sigma} G^{b\beta}_{\beta\rho} | 0 \rangle = \frac{1}{96} \langle GG \rangle \delta_{ab} (g_{\alpha\beta} g_{\sigma\rho} - g_{\alpha\rho} g_{\sigma\beta}), 
\]

(22)

where \(\langle GG \rangle\) is the abbreviation of \(\langle 0 | G^{a\mu}_{\mu\nu} G^{a\mu\nu} | 0 \rangle\). Using this equation we can decompose the matrix element \(\langle 0 | G^{a\alpha}_{\alpha\sigma} G^{b\beta}_{\beta\rho} | 0 \rangle\) to obtain the gluon-gluon condensate.

It has been shown that the sum of the contributions of the diagrams in Fig. 3 cancel in \(D_S \rightarrow \phi\) transitions in Ref. [20]. Similar case occurs in the calculation for \(\bar{B}_s^0 \rightarrow \phi\) transitions. Therefore there are still no contributions of gluon-gluon condensate in \(\bar{B}_s^0 \rightarrow \phi\) transition.
2.4 Contributions of the quark-gluon mixing and four-quark operators

The diagrams for non-local quark-gluon mixing and four-quark contributions are shown in Figs. 4 and 5, respectively. The methods to calculate the contributions of these diagrams are similar to that for other diagrams. Two different vacuum-expectation values of the non-local quark-gluon mixing operators should be used. They are:

(1) The vacuum expectation value of quark-gluon mixing operator \( \bar{\Psi}(x)\Psi(y)G_{\mu\nu}^a \), which is expanded to be \[ \langle 0 | \bar{\Psi}_{\alpha}^i(x)\Psi_{\beta}^j(y)G_{\mu\nu}^a | 0 \rangle \]

\[ = \frac{1}{192} \langle \bar{\Psi}\sigma TG\Psi \rangle \langle \sigma_{\mu\nu} \rangle_{\beta\alpha} T_{ji}^a + \left[ -\frac{g}{96} \times 9 \langle \bar{\Psi}\Psi \rangle^2 (g_{\rho\mu} \gamma_\nu - g_{\rho\nu} \gamma_\mu) (x + y)^\rho \right. \]

\[ + i (y - x)^\rho \left( \frac{g}{96} \times 9 \langle \bar{\Psi}\Psi \rangle^2 + \frac{m}{96} \times 4 \langle \bar{\Psi}\sigma TG\Psi \rangle \right) \epsilon_{\rho\mu\nu\sigma} \gamma_5 \gamma^\sigma \] \( \beta\alpha \)

\( T_{ji}^a \), \hspace{1cm} (23)

where \( \langle \bar{\Psi}\sigma TG\Psi \rangle \) and \( \langle \bar{\Psi}\Psi \rangle^2 \) are the abbreviations of \( \langle 0 | \bar{\Psi}\sigma_{\mu\nu} T^a G_{\mu\nu}^a \Psi | 0 \rangle \) and \( \langle 0 | \bar{\Psi}\Psi \rangle^2 \) respectively, and \( g \) is the strong coupling constant.

(2) The other matrix element needed is \[ \langle 0 | \bar{\Psi}_{\alpha}^i \Psi_{\beta}^j \hat{D}_{\xi} G_{\alpha\rho}^a | 0 \rangle = -\frac{g}{3^3 \times 2^4} \langle \bar{\Psi}\Psi \rangle^2 (g_{\xi\rho} \gamma_\sigma - g_{\xi\sigma} \gamma_\rho)_{\beta\alpha} T_{ji}^a \), \hspace{1cm} (24)

and the external color field in fix-point gauge expanded up to the second order is used

\[ A_{\mu}^a (z) = \int_0^1 d\beta \beta z^\rho G_{\rho\mu}^a (\beta z) \]

\[ = \frac{1}{2} z^\rho G_{\rho\mu}^a (0) + \frac{1}{3} z^\rho z^\rho \hat{D}_{\alpha} G_{\rho\mu}^a (0) + \cdots \], \hspace{1cm} (25)

here \( \hat{D}_{\alpha} \) is the covariant derivative in the adjoint representation, \( (\hat{D}_{\alpha})^{mn} = \partial_{\alpha} \delta^{mn} - gf^{amn} A_{\alpha}^a \).

After calculating all of the diagrams in Figs.4 and 5, we find that the contributions of Fig.4 (c), (d) and Fig.5 (c), (d) are vanishes after double Borel transformation for both variables \( p_1^2 \) and \( p_2^2 \), because only one variable appearing in the denominator. For example, \( 1/q^2 (p_1^2 - m_1^2) \). The Borel transformation for \( p_2^2 \) will eliminate such terms.

Using the above method, we get the coefficients \( \hat{B} f_0 \), \( \hat{B} (f_1 + f_3) \), \( \hat{B} (f_1 - f_3) \) and \( \hat{B} f_5 \) needed in Eq. (13), which are given in the Appendix B.
III. NUMERICAL CALCULATION OF THE FORM FACTORS

For the numerical calculation, the standard values of the condensates at the renormalization point $\mu = 1$GeV are used [17–19],

$$\langle \bar{q}q \rangle = -(0.24 \pm 0.01 \text{GeV})^3, \quad \langle \bar{s}s \rangle = (0.8 \pm 0.2)\langle \bar{q}q \rangle,$$

$$g\langle \bar{\Psi}\sigma T G \Psi \rangle = m_0^2\langle \bar{\Psi}\Psi \rangle, \quad \alpha_s\langle \bar{\Psi}\Psi \rangle^2 = 6.0 \times 10^{-5} \text{GeV}^6,$$

(26)

$$m_0^2 = 0.8 \pm 0.2 \text{GeV}^2.$$  

The quark masses are $m_s = 95 \text{ MeV}$, $m_b = 4.18 \text{ GeV}$ [3], the meson masses are $m_\phi = 1.02 \text{ GeV}$, $m_{J/\psi} = 3.097 \text{ GeV}$, $m_{B_s} = 5.367 \text{ GeV}$ [3]. The decay constants for $\phi$ and $J/\psi$ mesons are extracted from the experimental measurement of the branching ratios of $\phi \to \ell^+\ell$ and $J/\psi \to \ell^+\ell$ [3], which are $f_\phi = 0.228 \text{ GeV}$ and $f_{J/\psi} = 0.416 \text{ GeV}$. For the decay constant of $B_s$ meson we take $f_{B_s} = 0.266 \pm 0.019 \text{GeV}$ [26]. The threshold parameters $s_1^0$ and $s_2^0$ for $B_s$ and $\phi$ mesons are taken to be $s_1^0 = 34.9 - 35.9 \text{GeV}^2$, $s_2^0 = 1.9 - 2.1 \text{GeV}^2$, respectively.

The physical result should not depend on the Borel parameters $M_1$ and $M_2$ if the OPE were calculated up to infinite order. However, in practice OPE can only be calculated up to finite orders. So Borel parameters have to be selected in some “windows” to get the best stability of the physical results. The criterion to choose the region for $M_1$ and $M_2$ is: (1) The contributions of the excited and continuum states should be effectively suppressed to make sure that the sum rule does not depend on the approximation for the excited and continuum states sensitively. This requires that the Borel parameters should not be too large; (2) The contribution of the condensates of higher dimensional operators should be small to make sure the truncated OPE is effective. The series in OPE generally depends on Borel parameters in the denominator $1/M_{1,2}^n$, where $n$ is positive integer. The higher the dimension of the operator, the larger the integer $n$. This requires that the Borel parameters should not be too small. After numerical analysis, we find the optimal stability in accord with the requirements which are shown in Table I. The regions are shown in Fig.6 as two-dimensional diagram of $M_1^2$ and $M_2^2$. Within these regions we find good stabilities for the form factors.
TABLE I: Requirements to select Borel Parameters $M_1^2$ and $M_2^2$ for each form factors $V(0)$, $A_0(0)$, $A_1(0)$ and $A_2(0)$

| Form Factors | contribution of condensate | continuum of $B_s$ channel | continuum of $\phi$ channel |
|--------------|---------------------------|---------------------------|---------------------------|
| $V(0)$       | $\leq 56.7\%$            | $\leq 11\%$              | $\leq 56\%$              |
| $A_0(0)$     | $\leq 14\%$              | $\leq 10\%$              | $\leq 50\%$              |
| $A_1(0)$     | $\leq 56\%$              | $\leq 17.5\%$            | $\leq 50\%$              |
| $A_2(0)$     | $\leq 5.2\%$             | $\leq 17.2\%$            | $\leq 54\%$              |

FIG. 6: Selected regions of $M_1^2$ and $M_2^2$.

The final results for the form factors at $q^2 = 0$ are

\[
V(0) = 0.45 \pm 0.10, \quad A_0(0) = 0.30 \pm 0.25, \quad A_1(0) = 0.32 \pm 0.07, \quad A_2(0) = 0.30 \pm 0.07.
\] (27)
We compare our results with other nonperturbative approaches such as LCSR [14] and CQM [15] in Table II. The form factors for semileptonic decays of $B_s^0$ to $\phi$ meson have also been calculated by QCD sum rule in Ref. [16]. We do not list the value of $A_2^*(0) = -0.44$ of Ref. [16] in Table II, because the form factor $A_2^*(0)$ defined in [16] does not directly correspond to the definition in our work. The relations of the form factors defined in Ref.[16] and ours are

\[ V^*(q^2) = (-i)V(q^2), \quad A_0^*(q^2) = (-i)A_1(q^2), \quad A_1^*(q^2) = (-i)A_2(q^2), \]
\[ A_2^*(q^2) = \frac{(-i)(m_{B_s}+m_\phi)}{q^2} [(m_{B_s} - m_\phi)A_2(q^2) - (m_{B_s} + m_\phi)A_1(q^2) + 2m_\phi A_1(q^2)] , \] (28)

where $V^*(q^2)$, $A_0^*(q^2)$, $A_1^*(q^2)$ and $A_2^*(q^2)$ denote the form factors defined in [16].

**TABLE II:** Comparison of our results of form factors with other work

|       | $A_0(0)$ | $A_1(0)$ | $A_2(0)$ | $V(0)$ |
|-------|----------|----------|----------|--------|
| LCSR  | 0.474    | 0.311    | 0.234    | 0.434  |
| CQM   | 0.42     | 0.34     | 0.31     | 0.44   |
| SR    | $A_2^*(0)$ | -0.34    | 0.35     | -0.47  |
| This work | 0.30 $\pm$ 0.25 | 0.32 $\pm$ 0.07 | 0.30 $\pm$ 0.07 | 0.45 $\pm$ 0.10 |

Table II shows that our results for $A_1$, $A_2$ and $V$ are more consistent with the results of LCSR in Ref. [14] and CQM in Ref. [15]. Only $A_0$ is slightly smaller than theirs. The difference between the results of the form factors in Ref. [16] and ours is large. The reason is checked, that is: for the contribution of the condensate of the operator of dimension 3, the leading contribution is at the order of $(m_s/M_i)^0$ in our calculation, which comes from the first term $\frac{1}{12}\delta_{\beta\alpha}$ of Eq. (19). But there are no such terms in the result of Ref. [16], only terms like $(m_\phi m_s/M_i^2)^n$ or $(m_s^2/M_i^2)^n$ with $n \geq 1$ exist. The contributions of the operators of dimension 5 are also different.

In the next section we can see that the branching ratios of $B_s \to J\psi\phi$ calculated with the form factors obtained in this work are consistent with experimental data.

For the $q^2$-dependence of the form factors, we varied the value of $q^2$ by keeping it slightly larger than 0. We find that the $q^2$-dependence of $V(q^2)$, $A_0(q^2)$ and $A_2(q^2)$ are well compat-
ible with the pole-model [27], which can be expressed as

\[ V(q^2) = \frac{V(0)}{1 - q^2/m_{\text{pole}}^V}, \]

\[ A_0(q^2) = \frac{A_0(0)}{1 - q^2/m_{\text{pole}}^{A_0}}, \]

\[ A_2(q^2) = \frac{A_2(0)}{1 - q^2/m_{\text{pole}}^{A_2}}, \]

while the \( q^2 \) dependence of and \( A_1(q^2) \) is very weak.

We fit \( V(q^2) \), \( A_0(q^2) \) and \( A_2(q^2) \) with the pole model to our numerical results. Then the relevant fitted pole masses are

\[ m_{\text{pole}}^V = 5.59 \pm 0.27 \text{ GeV}, \]

\[ m_{\text{pole}}^{A_0} = 5.62 \pm 2.38 \text{ GeV}, \]

\[ m_{\text{pole}}^{A_2} = 9.20 \pm 0.40 \text{ GeV}. \] (29)

IV. THE APPLICATION OF THE FORM FACTORS TO THE BRANCHING RATIOS

We use the form factors obtained in this work to calculate the time-dependent decay width and branching ratio of \( B_s^0 \to J/\psi \phi \) mode. For simplicity in checking whether the form factors obtained in this work can give predictions consistent with experiment, we only calculate the branching ratio in naive factorization approach here. The Feynman diagrams for \( B_s^0 \to J/\psi \phi \) decay are shown in Fig.7.

The effective amplitude of \( B_s^0 \to J/\psi \phi \) is

\[ \mathcal{A}_{\text{eff}} = \langle J/\psi \phi | \mathcal{H}_{\text{eff}} | B_s^0 \rangle, \] (30)

where the effective Hamiltonian is

\[ \mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[ V_{ub}V_{cs}^*(C_1O_1 + C_2O_2) - V_{ub}V_{ts}^* \left( \sum_{j=3}^{10} C_j O_j \right) \right], \]

with

\[ O_1 = (\bar{c}_ib_j)_{V-A}(\bar{s}_j c_i)_{V-A}, \quad O_2 = (\bar{c}_ib_i)_{V-A}(\bar{s}_j c_j)_{V-A}, \]

\[ O_3 = (\bar{s}_ib_i)_{V-A}(\bar{c}_j c_j)_{V-A}, \quad O_4 = (\bar{s}_ib_j)_{V-A}(\bar{c}_j c_i)_{V-A}, \]
FIG. 7: The Feynman diagrams for the decay of $\bar{B}_s^0 \to J/\psi \phi$, "•" denotes the effective vertex for the operator insertion.

\[
O_5 = (\bar{s}_i b_i)_{V-A}(\bar{c}_j c_j)_{V+A}, \quad O_6 = (\bar{s}_i b_j)_{V-A}(\bar{c}_j c_i)_{V+A},
\]

\[
O_7 = \frac{3}{2}(\bar{s}_i b_i)_{V-A} \frac{2}{3}(\bar{c}_j c_j)_{V+A}, \quad O_8 = \frac{3}{2}(\bar{s}_i b_j)_{V-A} \frac{2}{3}(\bar{c}_j c_i)_{V+A},
\]

\[
O_9 = \frac{3}{2}(\bar{s}_i b_i)_{V-A} \frac{2}{3}(\bar{c}_j c_j)_{V-A}, \quad O_{10} = \frac{3}{2}(\bar{s}_i b_j)_{V-A} \frac{2}{3}(\bar{c}_j c_i)_{V-A}.
\]

For Wilson coefficient $C_i(\mu)$, we take the value calculated by naive dimensional regularization (NDR) scheme up to the next-to-leading-order at renormalization scale $\mu = m_b$ as [28]

\[
C_1 = -0.176; \quad C_2 = 1.078; \quad C_3 = 0.014;
\]

\[
C_4 = -0.034; \quad C_5 = 0.008; \quad C_6 = -0.039;
\]

\[
C_7 = -0.011\alpha; \quad C_8 = 0.055\alpha; \quad C_9 = -1.341\alpha; \quad C_{10} = 0.264\alpha
\]

where $\alpha$ is the electromagnetic coupling constant, which takes $\alpha = 7.297 \times 10^{-3}$.

We can divide the total effective amplitude into three parts

\[
\tilde{A}_{eff} = \tilde{A}_1 + \tilde{A}_2 + \tilde{A}_3,
\]

where $\tilde{A}_1$ denotes the contribution of the two tree diagrams of Fig.7 (a) and (b), $\tilde{A}_2$ the contribution of Fig.7 (c), and $\tilde{A}_3$ the contribution of Fig.7 (d).
The amplitude of the two tree diagrams is
\[
\bar{A}_1 = \frac{G_F}{\sqrt{2}} \left[ V_{ub}V_{us}^* (C_1 + \frac{C_2}{N_c}) - V_{tb}V_{ts}^* (C_3 + \frac{C_4}{N_c} + C_5 + \frac{C_6}{N_c} + C_7 + \frac{C_8}{N_c} + C_9 + \frac{C_{10}}{N_c}) \right] 
\]

\[
\langle J/\psi | \bar{c} \gamma^\nu (1 - \gamma_5) c | 0 \rangle \langle \phi | \bar{s} \gamma_\nu (1 - \gamma_5) b | \bar{B}_s^0 \rangle, 
\]
while the amplitudes of the two penguin diagrams are
\[
\bar{A}_2 = \frac{G_F}{\sqrt{2}} C_1 \left[ V_{ub}V_{us}^* \frac{Q^2_u \alpha}{\pi} \int_0^1 2\bar{x}x(1 + \ln \frac{a^2}{\mu^2})dx + V_{cb}V_{cs}^* \frac{Q^2_c \alpha}{\pi} \int_0^1 2\bar{x}x(1 + \ln \frac{b^2}{\mu^2})dx \right] 
\]

\[
\langle J/\psi | \bar{c} \gamma^\nu (1 - \gamma_5) c | 0 \rangle \langle \phi | \bar{s} \gamma_\nu (1 - \gamma_5) b | \bar{B}_s^0 \rangle, 
\]
and
\[
\bar{A}_3 = \frac{G_F}{\sqrt{2}} C_1 \left[ V_{ub}V_{us}^* \frac{Q^2_u \alpha}{\pi} \int_0^1 2\bar{x}x \ln \frac{a^2}{\mu^2} dx + V_{cb}V_{cs}^* \frac{Q^2_c \alpha}{\pi} \int_0^1 2\bar{x}x \ln \frac{b^2}{\mu^2} dx \right] 
\]

\[
\langle J/\psi | \bar{c} \gamma^\nu (1 - \gamma_5) c | 0 \rangle \langle \phi | \bar{s} \gamma_\nu (1 - \gamma_5) b | \bar{B}_s^0 \rangle. 
\]

Because of \( \bar{B}_s^0 - B_s^0 \) mixing, we should consider the decay amplitude of \( B_s^0 \rightarrow J/\psi \phi \) in the analysis of time-dependent decays. Similarly we denote \( A_{eff} = \langle J/\psi | \phi | A_{eff} | B_s^0 \rangle \) and

\[
A_{eff} = A_1 + A_2 + A_3, 
\]
with
\[
A_1 = \frac{G_F}{\sqrt{2}} \left[ V_{ub}V_{us}^* (C_1 + \frac{C_2}{N_c}) - V_{tb}V_{ts}^* (C_3 + \frac{C_4}{N_c} + C_5 + \frac{C_6}{N_c} + C_7 + \frac{C_8}{N_c} + C_9 + \frac{C_{10}}{N_c}) \right] 
\]

\[
\langle J/\psi | \bar{c} \gamma^\nu (1 - \gamma_5) c | 0 \rangle \langle \phi | \bar{s} \gamma_\nu (1 - \gamma_5) s | B_s^0 \rangle, 
\]

\[
A_2 = \frac{G_F}{\sqrt{2}} C_1 \left[ V_{ub}V_{us}^* \frac{Q^2_u \alpha}{\pi} \int_0^1 2\bar{x}x(1 + \ln \frac{a^2}{\mu^2})dx + V_{cb}V_{cs}^* \frac{Q^2_c \alpha}{\pi} \int_0^1 2\bar{x}x(1 + \ln \frac{b^2}{\mu^2})dx \right] 
\]

\[
\langle J/\psi | \bar{c} \gamma^\nu (1 - \gamma_5) c | 0 \rangle \langle \phi | \bar{s} \gamma_\nu (1 - \gamma_5) s | B_s^0 \rangle, 
\]

\[
A_3 = \frac{G_F}{\sqrt{2}} C_1 \left[ V_{ub}V_{us}^* \frac{Q^2_u \alpha}{\pi} \int_0^1 2\bar{x}x \ln \frac{a^2}{\mu^2} dx + V_{cb}V_{cs}^* \frac{Q^2_c \alpha}{\pi} \int_0^1 2\bar{x}x \ln \frac{b^2}{\mu^2} dx \right] 
\]

\[
\langle J/\psi | \bar{c} \gamma^\nu (1 - \gamma_5) c | 0 \rangle \langle \phi | \bar{s} \gamma_\nu (1 - \gamma_5) s | B_s^0 \rangle, 
\]

where \( \bar{x} = 1 - x \), \( a^2 = m_u^2 - x(1-x)q^2 \), \( b^2 = m_c^2 - x(1-x)q^2 \), and \( q \) is the transition momentum. \( G_F \) is the Fermi constant, \( N_c = 3 \) the color quantum number of quarks, \( Q_q (q = u, c) \) the charge of relevant quarks, \( C_i (i = 1, 2, ..., 10.) \) the Wilson coefficients, and \( V_{qb}, V_{qs} (q = u, c, t) \) the relevant CKM matrix elements, respectively.
There are three polarization states for $\phi$ meson: one longitudinal state and two transverse polarization states (right-handed and left-handed). We define

$$h_\lambda \equiv \langle J/\psi | \bar{c} \gamma^\nu (1 - \gamma_5) c | 0 \rangle \langle \phi | \bar{s} \gamma_\nu (1 - \gamma_5) b | \bar{B}_s^0 \rangle, \quad (39)$$

then using Eq. (1) and the following matrix elements for $J/\psi$ meson

$$\langle 0 | \bar{c} \gamma_\mu c | J/\psi \rangle = m_{J/\psi} f_{J/\psi} \epsilon_\mu (\lambda),$$

$$\langle 0 | \bar{c} \gamma_\mu \gamma_5 c | J/\psi \rangle = 0, \quad (40)$$

we can obtain, for the longitudinal polarization states of the vector mesons

$$h_0 = \frac{if_{J/\psi}}{2m_\phi} \left[ (m_{B_s}^2 - m_\phi^2 - m_{J/\psi}^2)(m_{B_s} + m_\phi)A_1(q^2) - \frac{4m_{B_s}^2 p_\phi^2}{m_{B_s} + m_\phi} A_2(q^2) \right], \quad (41)$$

and for the transverse polarization states

$$h_+ = if_{J/\psi} m_{J/\psi} \left[ (m_{B_s} + m_\phi)A_1(q^2) \mp \frac{V(q^2)}{m_{B_s} + m_\phi} \right]. \quad (42)$$

where $p_\phi = \frac{1}{2m_{B_s}} \sqrt{\left[ m_{B_s}^2 - (m_\phi + m_{J/\psi}^2) \right] \left[ m_{B_s}^2 - (m_\phi - m_{J/\psi}^2) \right]}$ is the momentum of $\phi$ meson in the rest frame of $B_s$.

We can write $|\tilde{A}_{eff}|^2$ in terms of the sum of one longitudinal and two transverse polarization amplitudes squared

$$|\tilde{A}_{eff}|^2 = |(\tilde{A}_{eff})_L|^2 + |(\tilde{A}_{eff})_+|^2 + |(\tilde{A}_{eff})_-|^2, \quad (43)$$

where

$$|(\tilde{A}_{eff})_L|^2 = |(\tilde{A}_1)_L|^2 + |(\tilde{A}_2)_L|^2 + |(\tilde{A}_3)_L|^2,$$

$$|(\tilde{A}_{eff})_+|^2 = |(\tilde{A}_1)_+|^2 + |(\tilde{A}_2)_+|^2 + |(\tilde{A}_3)_+|^2.$$ 

(44)

In the same way, $|A_{eff}|^2$ can be written as

$$|A_{eff}|^2 = |(A_{eff})_L|^2 + |(A_{eff})_+|^2 + |(A_{eff})_-|^2,$$

where

$$|(A_{eff})_L|^2 = |(A_1)_L|^2 + |(A_2)_L|^2 + |(A_3)_L|^2,$$

$$|(A_{eff})_+|^2 = |(A_1)_+|^2 + |(A_2)_+|^2 + |(A_3)_+|^2.$$ 

(45)
Similarly, we can also obtain the total time-dependent decay width

\[
\Gamma(B_s^0(t) \to J/\psi\phi)_{\pm} = \frac{p_\phi}{8\pi m_{B_s}^2} \frac{1}{2} e^{-\Gamma_{B_s} t} \left[ (|(A_{eff})_\pm|^2 + |(A_{eff})_\pm|^2) \cosh \frac{\Delta \Gamma}{2} t \right. \\
\left. - (|(A_{eff})_\pm|^2 - |(A_{eff})_\pm|^2) \cos \Delta mt + 2 \text{Re}(\frac{p}{q} (A_{eff})_\pm (A_{eff})_\pm^* \sinh \frac{\Delta \Gamma}{2} t - 2 \text{Im}(\frac{p}{q} (A_{eff})_\pm (A_{eff})_\pm^* \sin \Delta mt) \right],
\]

and the longitudinal time-dependent decay width

\[
\Gamma(B_s^0(t) \to J/\psi\phi)_L = \frac{p_\phi}{8\pi m_{B_s}^2} \frac{1}{2} e^{-\Gamma_{B_s} t} \left[ (|(A_{eff})_L|^2 + |(A_{eff})_L|^2) \cosh \frac{\Delta \Gamma}{2} t \right. \\
\left. - (|(A_{eff})_L|^2 - |(A_{eff})_L|^2) \cos \Delta mt + 2 \text{Re}(\frac{p}{q} (A_{eff})_L (A_{eff})_L^* \sinh \frac{\Delta \Gamma}{2} t - 2 \text{Im}(\frac{p}{q} (A_{eff})_L (A_{eff})_L^* \sin \Delta mt) \right].
\]

We take \( \frac{p}{q} = \frac{\bar{V}_{tb} V_{ts}^*}{\bar{V}_{tb} V_{ts}} = e^{-i2\beta_s}, \beta_s = 0.0185, \frac{\Delta \Gamma}{\Gamma_{B_s}} = 0.122, \frac{\Delta m}{\Gamma_{B_s}} = 26.79 \) and the total decay width of \( B_s \) meson is \( \Gamma_{B_s} = 4.362 \times 10^{-13} \text{ GeV} \). Finally, the combined transverse and total time-dependent decay widths are

\[
\Gamma(B_s^0(t) \to J/\psi\phi)_T = \Gamma(B_s^0(t) \to J/\psi\phi)_T^+ + \Gamma(B_s^0(t) \to J/\psi\phi)_T^-, \\
\Gamma(B_s^0(t) \to J/\psi\phi)_L = \Gamma(B_s^0(t) \to J/\psi\phi)_L + \Gamma(B_s^0(t) \to J/\psi\phi)_L^-.
\]

And the total time-dependent decay width is

\[
\Gamma(B_s^0(t) \to J/\psi\phi) = \frac{p_\phi}{8\pi m_{B_s}^2} \frac{1}{2} e^{-\Gamma_{B_s} t} \left[ (|A_{eff}|^2 + |A_{eff}|^2) \cosh \frac{\Delta \Gamma}{2} t \right. \\
\left. - (|A_{eff}|^2 - |A_{eff}|^2) \cos \Delta mt + 2 \text{Re}(\frac{p}{q} A_{eff} A_{eff}^* \sinh \frac{\Delta \Gamma}{2} t - 2 \text{Im}(\frac{p}{q} A_{eff} A_{eff}^* \sin \Delta mt) \right].
\]

Similarly, we can also obtain the total time-dependent decay width of \( B_s^0(t) \to J/\psi\phi \), which is

\[
\Gamma(B_s^0(t) \to J/\psi\phi) = \frac{p_\phi}{8\pi m_{B_s}^2} \frac{1}{2} e^{-\Gamma_{B_s} t} \left[ (|A_{eff}|^2 + |A_{eff}|^2) \cosh \frac{\Delta \Gamma}{2} t \right. \\
\left. + (|A_{eff}|^2 - |A_{eff}|^2) \cos \Delta mt + 2 \text{Re}(\frac{q}{p} A_{eff}^* A_{eff} \sinh \frac{\Delta \Gamma}{2} t - 2 \text{Im}(\frac{q}{p} A_{eff}^* A_{eff} \sin \Delta mt) \right].
\]
Integrating the above time-dependent decay widths over \( t \) from zero to infinity, we can get the relevant branching ratios [29, 30]

\[
\begin{align*}
Br(B_s \to J/\psi \phi)_T &= \frac{1}{2} \int_0^{\infty} \left[ \Gamma(\bar{B}_s^0(t) \to J/\psi \phi)_T + \Gamma(B_s^0(t) \to J/\psi \phi) \right] dt, \\
Br(B_s \to J/\psi \phi)_L &= \frac{1}{2} \int_0^{\infty} \left[ \Gamma(\bar{B}_s^0(t) \to J/\psi \phi)_L + \Gamma(B_s^0(t) \to J/\psi \phi) \right] dt, \quad (52) \\
Br(B_s \to J/\psi \phi)_{\text{total}} &= \frac{1}{2} \int_0^{\infty} \left[ \Gamma(\bar{B}_s^0(t) \to J/\psi \phi) + \Gamma(B_s^0(t) \to J/\psi \phi) \right] dt.
\end{align*}
\]

Substituting the values for the relevant parameters and quantities into the above equation we can get

\[
\begin{align*}
Br(B_s \to J/\psi \phi)_L &= (0.42 \pm 0.17) \times 10^{-3}, \\
Br(B_s \to J/\psi \phi)_T &= (0.50 \pm 0.09) \times 10^{-3}, \quad (53)
\end{align*}
\]

and the total total decay branching ratio is

\[
Br(B_s \to J/\psi \phi) = (0.92 \pm 0.26) \times 10^{-3}, \quad (54)
\]

which are in good agreement with experimental data within uncertainties [3]:

\[
\begin{align*}
Br(B_s \to J/\psi \phi)^{\text{exp}}_L &= (0.56 \pm 0.04) \times 10^{-3}, \\
Br(B_s \to J/\psi \phi)^{\text{exp}}_T &= (0.52 \pm 0.04) \times 10^{-3}, \\
Br(B_s \to J/\psi \phi)^{\text{exp}} &= (1.08 \pm 0.08) \times 10^{-3}.
\end{align*}
\]

V. SUMMARY

We calculate the \( \bar{B}_s^0 \to \phi \) transition form factors by QCD sum rule method. The form factors are expressed in terms of two Borel parameters \( M^2_1, M^2_2 \) and relevant Borel transformation coefficients. We take the two Borel parameters \( M^2_1, M^2_2 \) as independent parameters and find the “stable windows” in the two-dimensional area of \( M^2_1 \) and \( M^2_2 \) for the transition form factors \( V, A_0, A_1 \) and \( A_2 \). Our results are compatible with that obtained by LCSR and CQM methods in the literature. Finally, we apply the results of the transition form factors \( V, A_0, A_1 \) and \( A_2 \) to the nonleptonic decay process of \( \bar{B}_s^0 \to J/\psi \phi \). We calculate the branching ratios for all the possible polarization states of the vector mesons. The branching ratios we obtained are well consistent with experimental data.
ACKNOWLEDGMENTS

This work is supported in part by the National Natural Science Foundation of China under Contracts No. 11875168 and No. 11375088.
Appendix A

Some basic formulas are needed to perform the integration in Eq. (16) are given here.

\[ I = \int d^4k \delta(k^2 - m_0^2)\delta((k + p_1)^2 - m_1^2)\delta((k + p_2)^2 - m_2^2) = \frac{\pi}{2\sqrt{\lambda}}, \]  
(A1)

\[ I_{\mu} = \int d^4kk_{\mu} \delta(k^2 - m_0^2)\delta((k + p_1)^2 - m_1^2)\delta((k + p_2)^2 - m_2^2) \equiv a_1 p_{1\mu} + b_1 p_{2\mu}, \]

\[
\begin{aligned}
a_1 &= -\frac{\pi}{2\lambda^{3/2}}[s_2(-s_1 + s_2 - q^2) + (s_1 + s_2 - q^2)(m_0^2 - m_1^2) \\
&- 2s_2(m_0^2 - m_1^2)], \\
b_1 &= -\frac{\pi}{2\lambda^{3/2}}[s_1(-s_2 + s_1 - q^2) + (s_1 + s_2 - q^2)(m_0^2 - m_1^2) \\
&- 2s_1(m_0^2 - m_1^2)],
\end{aligned}
(A2)

\[ I_{\mu\nu} = \int d^4kk_{\mu}k_{\nu}\delta(k^2 - m_0^2)\delta((k + p_1)^2 - m_1^2)\delta((k + p_2)^2 - m_2^2) \equiv a_2 p_{1\mu}p_{1\nu} + b_2 p_{2\mu}p_{2\nu} + c_2(p_{1\mu}p_{2\nu} + p_{1\nu}p_{2\mu}) + d_2 g_{\mu\nu}, \]

\[
\begin{aligned}
D_1 &\equiv s_1 - m_1^2 + m_0^2, \quad D_2 \equiv s_2 - m_2^2 + m_0^2, \\
a_2 &= \frac{\pi}{\lambda^{3/2}}m_0^2 s_2 + \frac{1}{\lambda}[3s_2 D_1 a_1 - (s_1 + s_2 - q^2)D_2 b_1 + s_2 D_2 b_1], \\
b_2 &= \frac{\pi}{\lambda^{3/2}}m_0^2 s_1 + \frac{1}{\lambda}[s_1 D_1 a_1 - (s_1 + s_2 - q^2)D_1 b_1 + 3s_1 D_2 b_1], \\
c_2 &= -\frac{\pi}{\lambda^{3/2}}m_0^2 \frac{1}{2}(s_1 + s_2 - q^2) \\
&- \frac{1}{\lambda}[\frac{1}{2}(s_1 + s_2 - q^2)D_1 a_1 - 2s_2 D_1 b_1 + \frac{3}{2}(s_1 + s_2 - q^2)D_2 b_1], \\
d_2 &= \frac{\pi}{4\sqrt{\lambda}} + \frac{1}{4}[D_1 a_1 + D_2 b_1],
\end{aligned}
(A3)

where \( \lambda(s_1, s_2, q^2) = (s_1 + s_2 - q^2)^2 - 4s_1 s_2. \)

Appendix B

The results of relevant Borel transformed Coefficients for the transition form factors in Eq. (13) are given here.

1) Borel transformed \( f_0 \):

\[ \hat{B} f_0 = \hat{B} f_0^{pert} + \hat{B} f_0^{(3)} + \hat{B} f_0^{(5)} + \hat{B} f_0^{(6)}. \]
where,

$$\hat{B} f_0^{pert} = \int_{s_1}^{s_2} ds_2 \int_{s_1}^{s_2} ds_1 \frac{3e^{-s_1/M_1^2-s_2/M_2^2}}{4M_1^4 M_2^4 \pi^2 \lambda s_1/2} [-s_2 m_b (2m_s^2 + q^2)$$

$$+ s_1 - s_2) - 2s_2 m_b^2 m_s + 2s_2 m_b^3 + m_s (\lambda + 2s_2 m_s^2$$

$$+ q^2 s_2 + s_1 s_2 - s_2^2)],$$

where $\lambda = (s_1 + s_2 - q^2)^2 - 4s_1 s_2$. The lower limit of the integration $s_1$ is determined by requiring that all internal quarks are on their mass shell [27]

$$s_1 = \frac{m_b^2}{m_b^2 - q^2} s_2 + m_b^2,$$

and

$$\hat{B} f_0^{(3)} = -\frac{e^{-m_b^2/M_1^2-m_s^2/M_2^2}}{6 M_1^2 M_2^2} [M_1^2 M_2^2 m_b^2 (m_s^2 (M_1^2 + M_2^2)) (3M_2^2 - m_s^2)$$

$$+ M_1^2 M_2^2 m_b m_s (M_2^2 m_s^2 (M_1^2 + M_2^2) + q^2) - m_s^4 (M_1^2 + M_2^2)$$

$$- 3M_1^2 M_2^4) - M_1^2 m_s^3 m_s^3 (M_1^2 + M_2^2) + M_1^4 (-3M_2^4 m_s^2 (M_1^2$$

$$+ q^2) + M_2^2 m_s^4 (4M_1^2 + 4M_2^2 + q^2) - m_s^6 (M_1^2 + M_2^2)$$

$$+ 6M_1^2 M_2^6)] \times \langle \bar{s}s \rangle,$$

$$\hat{B} f_0^{(5)} = -\frac{e^{-m_b^2/M_1^2-m_s^2/M_2^2}}{12 M_1^2 M_2^2} [M_1^2 M_2^2 m_b m_s (m_s^2 (M_1^2 + M_2^2)$$

$$+ M_1^2 M_2^2 m_b m_s (M_2^2 m_s^2 (M_1^2 + M_2^2)) - M_1^2 M_2^2 m_b^2 (M_1^2$$

$$+ M_1^2 M_2^2 m_b m_s (M_2^2 m_s^2 (M_1^2 + M_2^2)) + M_2^2 (3M_2^2 - m_s^2)$$

$$+ M_2^4 (-3M_2^4 m_s^2 (M_1^2 + M_2^2) + m_s^6 (M_1^2 + M_2^2)$$

$$+ M_2^4 (3(M_2^2 + q^2) - M_1^2))] \times g \langle \bar{s}s T Gs \rangle,$$

$$\hat{B} f_0^{(6)} = \frac{e^{-m_b^2/M_1^2-m_s^2/M_2^2}}{81 M_1^8 M_2^8 (m_b^2 - q^2) m_s^2} [M_1^2 M_2^2 m_b^2 m_s^4 (M_1^2 + M_2^2)$$

$$+ M_2^4 m_b^2 m_s^4 (M_1^2 + M_2^2) + M_2^4 m_s^3 m_s^3 (M_1^2 m_s^2 (M_1^2 + M_2^2)$$

$$- M_2^2 (-2M_1^4 + M_1^2 (13M_2^2 + 2q^2) + M_2^2 q^2))$$

$$+ M_1^2 M_2^2 m_b m_s (36M_1^4 M_2^4 (e^{-m_b^2/M_1^2} - 1) + M_2^2 q^2 m_s^2 (-2M_1^2$$

$$+ 13M_2^2 + q^2) - q^2 m_s^4 (M_1^2 + M_2^2)) + m_b^2 (54M_1^6 M_2^4 m_s^4$$

$$- 54M_1^6 M_2^6 (e^{-m_b^2/M_1^2} - 1) - M_1^2 M_2^2 m_s^4 (M_1^4 + 2M_1^2 (5M_2^2 + q^2)$$

$$+ M_2^2 q^2) M_1^4 m_s^6 (M_1^2 + M_2^2)) + M_1^4 (M_2^2 q^2 m_s^4 (M_1^2 + 10M_2^2$$

$$+ q^2) - q^2 m_s^6 (M_1^2 + M_2^2) + 54M_1^2 M_2^6 q^2 (e^{-m_b^2/M_1^2} - 1)$$

$$- 18M_1^2 M_2^4 m_s^2 (M_2^2 (e^{-m_b^2/M_1^2} - 1) + 3q^2)] \times g^2 \langle \bar{s}s \rangle^2.$$. 

23
2) Borel transformed result for $f_1 + f_3$:

$$\hat{B}(f_1 + f_3) = \hat{B}f_{1+}^{\text{pert}} + \hat{B}f_{1+}^{(3)} + \hat{B}f_{1+}^{(5)} + \hat{B}f_{1+}^{(6)},$$

where

$$\hat{B}f_{1+}^{\text{pert}} = \int_{4m_2^2}^{s_2} ds_2 \int_{s_1}^{s_2} ds_1 \frac{3e^{-s_1/M_1^2-s_2/M_2^2}}{4M_1^2M_2^2 \pi^2 \lambda^{5/2}} \left\{ 2s_2m_s^3(2q^2(3m_s^2 + s_1 + s_2) + 2q^4 \\
+ \lambda - 6m_s^2(s_1 - s_2) - 4s_1^2 + 8s_1s_2 - 4s_2^2) + 2s_2m_s^2m_s(-2q^2(3m_s^2 + s_1 + s_2) \\
- 2q^4 - 3\lambda + 6m_s^2(s_1 - s_2) + 4s_1^2 - 8s_1s_2 + 4s_2^2) + m_s(q^2(-2m_s^2(\lambda + 2s_1s_2 + 2s_2^2) \\
- 6s_2m_s^4 + s_2(-\lambda + 2s_1^2 - 6s_1s_2 + 4s_2^2)) - 2s_2q^4(2m_s^2 + 2s_1 + s_2) + 2m_s^2(4s_1^2s_2 \\
+ \lambda s_1 - 8s_1s_2^2 + 4s_2^3 - 2\lambda s_2) + 6s_2m_s^4(s_1 - s_2) + s_2(s_1 - s_2)(-\lambda + 2s_1^2 \\
- 4s_1s_2 + 2s_2^2)) + 6s_2m_s^4m_s(q^2 - s_1 + s_2) - 6s_2m_s^5(q^2 - s_1 + s_2) \\
+ m_s(q^2(2m_s^2(\lambda + 2s_1s_2 + 2s_2^2) + 6s_2m_s^4 + s_2(3\lambda - 2s_1^2 + 6s_1s_2 - 4s_2^2)) \\
+ 2s_2q^4(2m_s^2 + 2s_1 + s_2) + \lambda^2 - 2m_s^2(4s_1^2s_2 + \lambda s_1 - 8s_1s_2^2 + 4s_2^3 - 4\lambda s_2) \\
+ 6s_2m_s^4(s_2 - s_1) - 2s_1^3s_2 + 6s_1^2s_2^2 - 6s_1s_2^3 + 3\lambda s_1s_2 + 2s_2^4 - 3\lambda s_2^2) \right\},$$

$(B5)$
\[
\hat{B}f_+^{(3)} = -\frac{e^{-m_s^2/M_1^2 - m_b^2/M_2^2}}{6 M_1^8 M_2^2} \{ -M_1^2 M_2^2 m_b m_s (M_2^2 m_s^2 (q^2 + M_1^2 + M_2^2) \\
+ m_s^4 (M_1^2 + M_2^2) + 3 M_1^2 M_2^4) + M_1^4 (M_2^2 m_s^4 (q^2 + 4 (M_1^2 + M_2^2)) \\
- 3 M_2^4 m_s^2 (q^2 + M_1^2 - 2 M_2^2) - m_s^6 (M_1^2 + M_2^2) + 6 M_1^2 M_2^6) \\
+ M_1^2 M_2^2 m_b^2 m_s^2 (M_1^2 + M_2^2) (3 M_2^2 - m_s^2) - M_2^4 m_b^3 m_s^3 (M_1^2 + M_2^2) \\
+ M_2^2) \} \times \langle \bar{s}s \rangle ,
\]

\[
\hat{B}f_+^{(5)} = \frac{e^{-m_s^2/M_1^2 - m_b^2/M_2^2}}{12 M_1^8 M_2^2} \{ q^2 (M_1^2 M_2^2 m_b m_s + M_1^4 M_2^2 (m_s^2 - 3 M_2^2)) \\
- M_1^2 M_2^2 m_b m_s (m_s^2 (M_1^2 + M_2^2) + 2 M_2^2 (M_1^2 + 2 M_2^2)) \\
+ M_1^2 M_2^2 m_b^5 (M_1^2 + M_2^2) (3 M_2^2 - m_s^2) - M_2^4 m_b^3 m_s (M_1^2 + M_2^2) \\
+ M_1^4 (-m_s^4 (M_1^2 + M_2^2) + m_b^2 (5 M_1^2 M_2^2 + 7 M_2^4) \\
+ M_2^4 (M_1^2 + 3 M_2^2)) \} \times \langle \bar{s}s T G s \rangle ,
\]

\[
\hat{B}f_+^{(6)} = \frac{e^{-m_s^2/M_1^2 - m_b^2/M_2^2}}{81 M_1^8 M_2^2 (m_b^2 - q^2) m_s^8} \{ M_1^2 M_2^2 q^4 m_s^4 (M_2^2 m_b + M_1^2 m_s) \\
+ q^2 (-M_1^2 M_2^2 m_b m_s^5 (M_1^2 + M_2^2) + M_1^2 M_2^2 m_s^4 (-m_b^2 (2 M_1^2 + M_2^2) \\
+ M_1^4 + 10 M_1^2 M_2^2) - M_2^4 m_b m_s^3 (m_b^2 (2 M_1^2 + M_2^2) + 4 M_1^4 - 11 M_1^2 M_2^2) \\
- 54 M_1^6 M_2^4 m_b^6 + 54 M_1^6 M_2^6 (e^{m_b^2} - 1) - M_1^4 m_s^6 (M_1^2 + M_2^2)) \\
+ M_1^2 M_2^2 m_b^4 m_s^2 (M_1^2 + M_2^2) + M_2^4 m_b^3 m_s^3 (M_1^2 + M_2^2) \\
+ M_1^2 M_2^2 m_b^2 m_s^4 (M_1^2 + M_2^2) + 4 M_1^2 M_2^2 - 11 M_2^4) \\
+ M_1^4 m_b^2 (54 M_1^2 M_2^2 m_b^4 - M_2^2 m_s^4 (M_1^2 + 10 M_2^2) + m_s^6 (M_1^2 + M_2^2) \\
- 54 M_1^2 M_2^6 (e^{m_b^2} - 1)) - 18 M_1^6 M_2^4 m_b^6 (e^{m_b^2} - 1) \} \times g^2 \langle \bar{s}s \rangle^2 .
\]
3) Borel transformed result for $f_1 - f_3$:

$$\hat{B}(f_1 - f_3) = \hat{B} f_{\text{pert}} + \hat{B} f_{(3)} + \hat{B} f_{(5)} + \hat{B} f_{(6)},$$

where

$$\hat{B} f_{\text{pert}} = \int_{4m_s^2}^{q_0^2} ds_2 \int_{s_1^0}^{s_1^0} ds_1 \frac{-3e^{-s_1/M^2_s-s_2/M^2_s}}{4M^2_s M^2_s \pi^2 \lambda^{5/2}} \{2s_2m^2_b(2q^2(3m^2_s + s_1 - 5s_2) + 2q^4 + \lambda - 6m^2_s(s_1 + 3s_2) - 4s_1^2 - 4s_1s_2 + 8s_2^2) + 2s_2m^2_m_m_s(-2q^2(3m^2_s + s_1 - 5s_2) - 2q^4 + \lambda + 6m^2_s(s_1 + 3s_2) + 4s_1^2 + 4s_1s_2 - 8s_2^2) + m_b(q^2(-2m^2_s(\lambda + 2s_1s_2 - 10s_2^2) - 6s^2_m_m_s - s_2(\lambda - 2s_1^2 - 10s_1s_2 + 4s_2^2)) + 2s_2q^4(-2m^2_s - 2s_1 + s_2) + 2m^2_m_m_s(s_1 + 2s_2)(\lambda + 4s_1s_2 - 4s_2^2) + 6s_2m^2_m_m_s(s_1 + 3s_2) + s_2(s_1 - s_2)(-\lambda + 2s_1^2 - 2s_2^2) - 6s_2m^2_m_m_s(-q^2 + s_1 + 3s_2) + 6s_2m^2_b(-q^2 + s_1 + 3s_2) - m_s(q^2(-2m^2_s(\lambda + 2s_1s_2 - 10s_2^2) - 6s_2m^2_m_m_s + s_2(\lambda + 2s_1^2 + 10s_1s_2 - 4s_2^2)) + 2s_2q^4(-2m^2_s - 2s_1 + s_2) - \lambda^2 + 2m^2_b(4s_1s_2 + \lambda s_1 + 4s_1s_2^2 - 8s_2^3 + 4\lambda s_2) + 6s_2m^2_m_m_s(s_1 + 3s_2) + 2s_1^3s_2 - 2s_1^2s_2 + 2s_2^3 + \lambda s_1s_2 + 2s_2^4 - \lambda s_2^2)\}, \quad (B9)$$

$$\hat{B} f_{(3)} = \frac{e^{-m^2_b/M^2_s-m^2_s/M^2_s}}{6M^8_s M^2_s} \{ -M^2_1 M^2_2 m_6 m_s(M_2^2 m^2_s(-q^2 + M^2_1 - 3M^2_2) + m^4_1(M_1^2 + M_2^2) + 3M^4_1 M_2^4) + M_1^4(M_2^2 m^4_s(q^2 + 4(M_1^2 + M_2^2)) - 3M^4_1 m^2_s(q^2 + M_1^2 + 2M_2^2) - m^6_s(M_1^2 + M_2^2) + 6M^2_1 M_2^6) + M_1^2 M_2^2 m^2_b m^2_s M_2^2 + M^2_2)(3M^2_2 - M^2_s) - M^4_2 m^2_b m^3_s M_1^2 + M^2_2 \} \times \langle \bar{ss} \rangle, \quad (B10)$$

$$\hat{B} f_{(5)} = -\frac{e^{-m^2_b/M^2_s-m^2_s/M^2_s}}{12M^8_1 M^8_s M^2_s} \{ q^2(M_1^2 M_2^2 m_6 m_s + M_1^4 M_2^2 (m^2_s - 3M^2_2)) + M_1^2 M_2^2 m^2_b(M_1^2 + M_2^2)(3M^2_2 - M^2_s) - M^2_2 m^3_b m_6 m_s(M_1^2 + M_2^2) - m_b(2M_1^4 M_2^2 m_6 m_s + M_1^2 M_2^2 m^3_b M_2^2) + M_1^4(-m^4_s M_1^2 + M^2_2) + M^2_2) + m^2_s(5M_1^2 M_2^2 - M^4_2) + M^2_1^4 (M_1^2 - 9M^2_2) \} \times g \langle \bar{ss} \sigma T G_s \rangle, \quad (B11)$$
\[ \hat{B}f^{(6)} = \frac{e^{-m_b^2/M_1^2 - m_s^2/M_2^2}}{81M_1^8M_2^8(2M_0^2 - q^2)m_2^2} \{ M_1^2M_2^2q^4m_s^3(M_2^2m_b + M_1^2m_s) \\
+ q^2(2M_1^2M_2^2m_b m_s^5(2M_1^2 + M_2^2) + M_1^2M_2^2m_s^4(-m_b^2(2M_1^2 + M_2^2) + M_1^4 + 10M_1^2M_2^2) - M_2^4m_b m_s^3(M_b^2(2M_1^2 + M_2^2) + 4M_1^4 \\
- 15M_1^2M_2^2 - 54M_1^6M_2^4m_s^2 + 54M_1^6M_2^6(e^{-\frac{m_b^2}{M_2^2}} - 1) - M_1^4m_b^2(M_1^2 \\
+ M_2^2) + M_1^2M_2^2m_b^3m_s^4(M_1^2 + M_2^2) + M_2^4m_b^5m_s^3(M_1^2 + M_2^2) \\
+ M_1^2M_2^2m_b^3m_s^3(m_s^2(M_1^2 + M_2^2) + 4M_1^2M_2^2 - 15M_1^4 \\
+ M_1^4m_b^2(54M_1^2M_2^4m_s^2 - M_2^2m_b^4(M_1^2 + 10M_2^2) + m_b^6(M_1^2 \\
+ M_2^2) - 54M_1^2M_2^6(e^{-\frac{m_b^2}{M_2^2}} - 1) + 54M_1^6M_2^6m_s^2(e^{-\frac{m_b^2}{M_2^2}} - 1) \} \\
\times g^2(\bar{s}s)^2. \]

4) Borel transformed result for \( f_5 \):

\[ \hat{B}(f_5) = \hat{B}f_5^{pert} + \hat{B}f_5^{(3)} + \hat{B}f_5^{(5)} + \hat{B}f_5^{(6)}, \]

where,

\[ \hat{B}f_5^{pert} = \int_{4m_b^2}^{s_0} ds_2 \int_{s_1}^{s_0} ds_1 \frac{-3e^{-s_1/M_1^2 - s_2/M_2^2}}{8M_1^4M_2^4\pi^2\lambda^3/2} \{ m_b(2s_2q^2(m_s^2 + s_1) + 2m_s^2(\lambda \\
+ s_1s_2 - s_2^2) + 2s_2m_s^4 + \lambda s_2) - q^2(2m_s^2 + s_1 - s_2) + 2s_2m_b^2m_s(q^2 + 2m_s^2 + s_1 - s_2) \\
+ 2s_2m_b^2m_s(q^2 + 2m_s^2 + s_1 - s_2) - m_s(q^2(\lambda + 2s_2m_s^2 + 2s_1s_2) \\
+ 2m_s^2(\lambda + s_1s_2 - s_2^2) + 2s_2m_s^4 - \lambda s_1) - 2s_2m_b^2m_s + 2s_2m_b^5 \}, \]
\[ \hat{B}_f^{(3)} = -\frac{e^{-m_b^2/M_1^2} - m_b^2/M_2^2}{12 M_1^2 M_2^8} \{ M_1^2 M_2^2 q^4 m_s^2 (M_2^2 m_b m_s + M_1^2 (m_s^2 - 3 M_2^2)) + q^2 (M_1^2 M_2^2 m_b^2 m_s^2 (3 M_2^2 (2 M_1^2 + M_2^2) - m_s^2 (2 M_1^2 + 3 M_2^2))) + M_1^2 M_2^2 m_b m_s (m_s^2 (3 M_1^2 + 2 M_2^2) + m_s^2 (9 M_1^2 M_2^2 + 2 M_2^4)) - 3 M_1^2 M_2^4) - M_2^4 m_b^2 m_s^3 (2 M_1^2 + M_2^2) + M_1^4 (-m_s^6 (M_1^2 + 2 M_2^2) - 3 m_s^4 (2 M_1^2 M_2^2 + M_2^6) + m_s^4 (5 M_1^2 M_2^2 + 7 M_2^4) + 6 M_1^2 M_2^6) \}

\[ \hat{B}_f^{(5)} = \frac{e^{-m_b^2/M_1^2} - m_b^2/M_2^2}{24 M_1^8 M_2^8} \{ q^4 (M_1^2 M_2^2 m_b m_s + M_1^4 M_2^2 (m_s^2 - 3 M_2^2)) - q^2 (M_1^2 M_2^2 m_b^2 (m_s^2 (2 M_1^2 + 3 M_2^2) - 3 (2 M_1^2 M_2^2 + M_2^4)))) + M_1^2 M_2^2 m_b m_s (m_s^2 (3 M_1^2 + 2 M_2^2) - 10 M_1^2 M_2^2 + M_2^4)) + M_2^4 m_b^2 m_s (2 M_1^2 + M_2^2) + M_1^4 (m_s^2 (M_1^2 + 2 M_2^2) - 2 m_s^2 (3 M_1^2 M_2^2 + 2 M_2^4)) + 2 M_2^4) + M_2^4 (M_1^2 + 3 M_2^2)) \} - m_s^2 (M_1^2 + 2 M_2^2) + M_2^4 m_s^2 m_s^2 (M_1^2 + 2 M_2^2) + M_2^2 m_b^2 m_s^2 (3 M_1^2 + 4 M_1^2 M_2^2 + 2 M_2^4)) - 2 M_1^2 M_2^2 (5 M_1^2 + 3 M_2^2)) + M_1^2 m_b m_s (m_s^2 (2 M_1^2 + 3 M_2^2)) + 3 M_1^2 M_2^2 + M_2^4) + m_s^2 (-13 M_1^4 M_2^2 - 8 M_1^2 M_2^4 + M_2^6) + 4 M_1^2 M_2^4 (M_1^2 + 3 M_2^2)) + m_s^2 (-2 m_s^2 (3 M_1^2 M_2^2 + 5 M_1^4 M_2^4) + M_1^4 (m_s^2 (M_1^2 + 2 M_2^2) + M_2^4 m_b^2 (M_1^2 + 2 M_2^2) - m_s^4 (5 M_1^2 M_2^2 + M_2^4)) + 4 M_1^2 M_2^6) \} \times \langle s \bar{s} \rangle ,

28
\[ \hat{B} f_s^{(6)} = -\frac{g^2/M_T^2 - m_s^2/M_T^2}{162 M_T^2 m_s^2 (m_b^2 - q^2) m_b^2} \left\{ M_2^4 (M_1^2 + M_2^2) m_s^3 m_b^2 + M_2^2 (M_1^4 + 2 M_2^4) m_s^3 m_b^2 + 2 (M_1^4 + 3 M_2^4) m_s^5 m_b^2 - 15 M_2^6 (m_s^3 m_b^2 + M_1^2 (-54 (-1 + e^{m_s^2/M_T^2}) M_1^4 M_2^6 + 54 M_1^4 m_s^2 M_2^4 + (4 M_2^6 + 3 M_1^4 M_2^4) m_s^6 - (33 M_2^6 + 14 M_1^2 M_2^4) + 2 M_2^4 m_s^2) + 2 (M_1^2 + M_2^2) m_s^6 + 2 M_1^2 M_2^2 q^6 m_s^3 (m_s M_1^2 + M_2^2 m_b^2 + q^4 (-54 (-1 + e^{m_s^2/M_T^2}) M_2^6 M_1^4 + 54 M_2^4 m_s^2 M_1^6 + M_2^2 (3 M_2^4 + 3 M_1^2 M_2^4) + 2 M_1^4 + M_2^4) m_s^5 m_b^2 - 2 (M_1^4 + 5 M_2^4 M_1^2) M_2^4 m_s^4 M_1^2 + (M_1^4 + 2 M_2^2 M_1^4) m_s^6 + (4 M_2^6 + 3 M_1^4 M_2^4) m_b^2 - 14 M_1^2 M_2^6 m_b^2 + m_s^3) + q^2 (-M_2^4 (3 M_1^2 + 2 M_2^2) m_s^5 m_b^2 - M_2^4 (3 M_1^4 + 6 M_2^4 M_1^2 + 2 M_2^4) m_s^4 m_b^2 - (2 M_2^4 + 6 M_1^4 M_2^4 + 6 M_1^2 M_2^4) m_s^5 m_b^2 - (108 (-1 + e^{m_s^2/M_T^2}) M_1^4 M_2^6 - 108 M_1^4 m_s^2 M_2^4 + (2 M_1^4 + 6 M_2^4 M_1^2 + 3 M_2^4) m_s^6 + (33 M_2^6 + 24 M_1^2 M_2^4 + 4 M_1^4 M_2^2) m_s^5 m_b^2 - M_1^4 m_s^2 (-36 (-1 + e^{m_s^2/M_T^2}) M_2^6 + 2 M_1^4 + 3 M_2^4 M_1^2 + M_2^4) m_s^6 - (14 M_2^6 + 40 M_1^2 M_2^4 + 7 M_1^4 M_2^4) m_s^4 m_b^2 + 2 (10 M_1^2 M_2^6 + 51 M_1^4 M_2^4) m_s^5 m_b^2 - M_1^4 m_s^2 (18 (-1 + e^{m_s^2/M_T^2}) M_2^6 - (M_1^2 + M_2^2) m_s^6 + 2 (31 M_1^2 M_2^4 - 8 M_2^6) m_s^2)) \right\} \times g^2 \langle s \bar{s} \rangle^2. \] 

(B16)

Appendix C

The amplitudes in Eq. (44) and Eq. (46) are given here.

\[ (\bar{A}_1)_L = \frac{g_v}{\sqrt{2}} \left[ V_{cb} V_{c*} (C_1 + \frac{C_9}{N_e}) - V_{tb} V_{t*} (C_3 + \frac{C_6}{N_e} + C_5 + \frac{C_9}{N_e} + C_7 + \frac{C_6}{N_e} + C_9 + \frac{C_8}{N_e}) \right] h_0. \]

(C1)
\[
A_1 \pm = \frac{G_F}{\sqrt{2}} \left[ V_{cb} V_{cs} (C_1 + \frac{C_4}{N_c}) - V_{\bar{u}b} V_{\bar{c}s} (C_3 + \frac{C_4}{N_c} + C_5 + \frac{C_6}{N_c} + C_7 + \frac{C_8}{N_c} + C_9 + \frac{C_{10}}{N_c}) \right] h_\pm.
\]  \tag{C2}

\[
A_2 \pm = \frac{G_F}{\sqrt{2}} C_1 \left[ V_{ub} V_{us} \frac{Q^2_{\alpha}}{\pi} \int_0^1 2 \bar{x}x (1 + \ln \frac{a^2}{\mu^2}) dx + V_{cb} V_{cs} \frac{Q^2_{\alpha}}{\pi} \int_0^1 2 \bar{x}x (1 + \ln \frac{b^2}{\mu^2}) dx \right] h_0.
\]  \tag{C3}

\[
A_2 \pm = \frac{G_F}{\sqrt{2}} C_1 \left[ V_{ub} V_{us} \frac{Q^2_{\alpha}}{\pi} \int_0^1 2 \bar{x}x (1 + \ln \frac{a^2}{\mu^2}) dx + V_{cb} V_{cs} \frac{Q^2_{\alpha}}{\pi} \int_0^1 2 \bar{x}x (1 + \ln \frac{b^2}{\mu^2}) dx \right] h_\pm.
\]  \tag{C4}

\[
A_3 \pm = \frac{G_F}{\sqrt{2}} C_1 \left[ V_{ub} V_{us} \frac{Q^2_{\alpha}}{\pi} \int_0^1 2 \bar{x}x \ln \frac{a^2}{\mu^2} dx + V_{cb} V_{cs} \frac{Q^2_{\alpha}}{\pi} \int_0^1 2 \bar{x}x \ln \frac{b^2}{\mu^2} dx \right] h_0.
\]  \tag{C5}

\[
A_3 \pm = \frac{G_F}{\sqrt{2}} C_1 \left[ V_{ub} V_{us} \frac{Q^2_{\alpha}}{\pi} \int_0^1 2 \bar{x}x \ln \frac{a^2}{\mu^2} dx + V_{cb} V_{cs} \frac{Q^2_{\alpha}}{\pi} \int_0^1 2 \bar{x}x \ln \frac{b^2}{\mu^2} dx \right] h_\pm.
\]  \tag{C6}

\[
A_1 \pm = \frac{G_F}{\sqrt{2}} \left[ V_{cb} V_{cs} (C_1 + \frac{C_4}{N_c}) - V_{\bar{u}b} V_{\bar{c}s} (C_3 + \frac{C_4}{N_c} + C_5 + \frac{C_6}{N_c} + C_7 + \frac{C_8}{N_c} + C_9 + \frac{C_{10}}{N_c}) \right] h_0.
\]  \tag{C7}

\[
A_1 \pm = \frac{G_F}{\sqrt{2}} \left[ V_{cb} V_{cs} (C_1 + \frac{C_4}{N_c}) - V_{\bar{u}b} V_{\bar{c}s} (C_3 + \frac{C_4}{N_c} + C_5 + \frac{C_6}{N_c} + C_7 + \frac{C_8}{N_c} + C_9 + \frac{C_{10}}{N_c}) \right] h_\pm.
\]  \tag{C8}

\[
A_2 \pm = \frac{G_F}{\sqrt{2}} C_1 \left[ V_{ub} V_{us} \frac{Q^2_{\alpha}}{\pi} \int_0^1 2 \bar{x}x (1 + \ln \frac{a^2}{\mu^2}) dx + V_{cb} V_{cs} \frac{Q^2_{\alpha}}{\pi} \int_0^1 2 \bar{x}x (1 + \ln \frac{b^2}{\mu^2}) dx \right] h_0.
\]  \tag{C9}

\[
A_2 \pm = \frac{G_F}{\sqrt{2}} C_1 \left[ V_{ub} V_{us} \frac{Q^2_{\alpha}}{\pi} \int_0^1 2 \bar{x}x (1 + \ln \frac{a^2}{\mu^2}) dx + V_{cb} V_{cs} \frac{Q^2_{\alpha}}{\pi} \int_0^1 2 \bar{x}x (1 + \ln \frac{b^2}{\mu^2}) dx \right] h_\pm.
\]  \tag{C10}

\[
A_3 \pm = \frac{G_F}{\sqrt{2}} C_1 \left[ V_{ub} V_{us} \frac{Q^2_{\alpha}}{\pi} \int_0^1 2 \bar{x}x \ln \frac{a^2}{\mu^2} dx + V_{cb} V_{cs} \frac{Q^2_{\alpha}}{\pi} \int_0^1 2 \bar{x}x \ln \frac{b^2}{\mu^2} dx \right] h_0.
\]  \tag{C11}

30
\[ (A_3)_{\pm} = \frac{G_F}{\sqrt{2}} C_1 \left[ V_{ub}^* V_{us} \frac{Q_2^a}{a} \int_0^1 2 \bar{x} x \ln \frac{a^2}{\mu^2} dx + V_{cb}^* V_{cs} \frac{Q_2^c}{c} \int_0^1 2 \bar{x} x \ln \frac{b^2}{\mu^2} dx \right] h_{\pm}, \quad (C12) \]

[1] M. Koayashi and T. Maskawa, “CP Violation in the Renormalizable Theory of Weak Interaction”, Prog. Theor. Phys. 49 (1973) 652.

[2] N. Cabibbo, “Unitary Symmetry and Leptonic Decays”, Phys. Rev. Lett. 10 (1963) 531.

[3] M. Tanabashi et al. (Particle Data Group), “Review of particle physics”, Phys. Rev. D 98 (2018) 030001.

[4] M. Artuso, G. Borissov, A. Lenz, ”CP violation in the $B_0^s$ system”, Rev. Mod. Phys. 88 (2016) 045002.

[5] CMS Collaboration, “CP-Violation studies at the HL-LHC with CMS using $B_0^s$ decays to $J/\psi\phi(1020)$”, CMS Physics Analysis Summary CMS-PAS-FTR-18-041 , 2018. http://cdsweb.cern.ch/record/2650772.

[6] O. Leitner, J.-P. Dedonder, and B. Loiseau, B. El-Bennich, “Scalar resonance effects on the $B_s - \bar{B}_s$ mixing angle”, Phys. Rev. D82 (2010) 076006.

[7] P. Colangelo, F. De Fazio, and W. Wang, “Nonleptonic $B_s$ to charmonium decays: Analysis in pursuit of determining the weak phase $\beta_s$”, Phys. Rev. D83 (2011) 094027.

[8] X. Liu, W. Wang, Y. Xie, “Penguin pollution in $B \to J/\psi V$ decays and impact on the extraction of the $B_s - \bar{B}_s$ mixing phase”, Phys. Rev. D89 (2014) 094010.

[9] D. Fakirov, B. Stech, “F and D Decays”, Nucl. Phys. B133 (1978) 315.

[10] N. Cabibbo, L. Maiani, “Two-Body Decays of Charmed Mesons”, Phys. Lett. B73 (1978) 418, Erratum: Phys.Lett. B76 (1978) 663.

[11] M. Beneke, G. Buchalla, M. Neubert, and C.T. Sachrajda, “QCD Factorization for $B \to \pi \pi$ Decays: Strong Phases and CP Violation in the Heavy Quark Limit”, Phys. Rev. Lett.,83 (1999) 1914.

[12] M. Beneke, G. Buchalla, M. Neubert, and C.T. Sachrajda, “QCD Factorization for exclusive non-leptonic $B$-meson decays: general arguments and the case of heavy-light final states”, Nucl. Phys., B591 (2000) 313.
[13] P. Ball, V.M. Braun, “Exclusive semileptonic and rare $B$ meson decays in QCD”, Phys. Rev. D58 (1998) 094016.

[14] Patricia Ball, Roman Zwicky, “$B_{d,s} \to \rho \omega, K^*$, $\phi$ decay form factors from light-cone sum rules reexamined”, Phys. Rev. D71,014029(2005).

[15] D. Melikhov and B. Stech, “Weak form-factors for heavy meson decays: An Update”, Phys. Rev. D62, 014006(2000).

[16] R. Khosravi, F. Falahati, “Semileptonic decays of $B_s$ to $\phi$ meson in QCD”, Phys.Rev. D88 (2013) no.5, 056002.

[17] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, “QCD and Resonance Physics. Theoretical Foundations”, Nucl. Phys. B147 (1979) 385.

[18] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, “QCD and Resonance Physics: Applications”, Nucl. Phys. B147 (1979) 448.

[19] P. Colangelo, A. Khodjamirian, “QCD Sum Rules, A Modern Perspective”, eprint hep-ph/0010175.

[20] D.S. Du, J.W. Li, M.Z. Yang, “Form factors and semileptonic decays of $D_s^+ \to \phi \ell \nu$ from QCD sum rule”, Eur.Phys.J.C 37 (2004) 173.

[21] R.E. Cutkosky, “Singularities and discontinuities of Feynman amplitudes”, J. Math. Phys. 1 (1960) 429.

[22] B.L. Ioffe and A.V. Smilga, “Meson Widths and Form-Factors at Intermediate Momentum Transfer in Nonperturbative QCD”, Nucl. Phys. B216 (1983) 373.

[23] J. Schwinger, “Particles, Sources, and Fields”, Addison-Wesley (1973).

[24] M.A. Shifman, Wilson Loop in Vacuum FieldsNucl. Phys. B173 (1980) 13.

[25] M.S. Dubovikov and A.V. Smilga, Analytical Properties of the Quark Polarization Operator in an External Selfdual Field”, Nucl. Phys. B185 (1981) 109.

[26] Hao-Kai Sun, Mao-Zhi Yang, “Decay Constants and Distribution Amplitudes of B Meson in the Relativistic Potential Model”, Phys. Rev. D95 (2017) no.11, 113001.

[27] L.D. Landau, “On analytic properties of vertex parts in quantum field theory”, Nucl. Phys. 13 (1959) 181.

[28] G. Buchalla, A.J. Buras, M.E. Lautenbacher, “Weak decays beyond leading logarithms”, Rev. Mod. Phys.68 (1996) 1125-1144.

[29] I. Dunietz, R. Fleischer, U. Nierste, “In pursuit of new physics with $B_s$ decays”, Phys. Rev.
D63 (2001) 114015.

[30] K. De Bruyn, R. Fleischer, R. Knegjens, P. Koppenburg, M. Merk, and N. Tuning, “Branching ratio measurements of $B_s$ decays”, Phys. Rev. D86 (2012) 014027.

[31] CKM fitter Group Collaboration, ”CP violation and the CKM matrix: Assessing the impact of the asymmetric $B$ factories”, Eur.Phys.J. C41 (2005) no.1, 1-131.