Dark energy and possible alternatives

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We present a brief review of various approaches to late time acceleration of universe. The cosmological relevance of scaling solutions is emphasized in case of scalar field models of dark energy. The underlying features of a variety of scalar field models is highlighted. Various alternatives to dark energy are discussed including the string curvature corrections to Einstein-Hilbert action, higher dimensional effects, non-locally corrected gravity and \( f(R) \) theories of gravity. The recent developments related to \( f(R) \) models with disappearing cosmological constant are reviewed.

I. INTRODUCTION

The accelerated expansion has played a very important role in the history of our universe. Universe is believed to have passed through inflationary phase at early epochs and there is a growing faith that it is accelerating at present. The late time acceleration of the universe, which is directly supported by supernovae observations, and indirectly, through observations of the microwave background, large scale structure and its dynamics, weak lensing and baryon oscillations, poses one of the most important challenges to modern cosmology.

Einstein equations in their original form, with an energy-momentum tensor for standard matter on the right hand side, cannot account for the observed accelerated expansion of universe. The standard lore aimed at capturing this important effect is related to the introduction of the energy-momentum tensor of an exotic matter with large negative pressure dubbed dark energy in the Einstein equations. The simplest known example of dark energy (for recent reviews, see [1]) is provided by the cosmological constant \( \Lambda \). It does not require adhoc assumption for its introduction, as is automatically present in the Einstein equations, by virtue of the Bianchi identities.

The field theoretic understanding of \( \Lambda \) is far from being satisfactory. Efforts have recently been made to obtain \( \Lambda \) in the framework of string theory, what leads to a complicated landscape of de-Sitter vacua. It is hard to believe that we happen to live in one of the \( 10^{500} \) vacua predicted by the theory. One might take the simplified view that, like \( G \), the cosmological constant \( \Lambda \) is a fundamental constant of the classical general theory of relativity and that it should be determined from large scale observations. It is interesting to remark that the \( \Lambda CDM \) model is consistent with observations at present. Unfortunately, the non-evolving nature of \( \Lambda \) and its small numerical value lead to a non-acceptable fine-tuning problem. We do not know how the present scale of the cosmological constant is related to Planck’s or the supersymmetry breaking scale; perhaps, some deep physics is at play here that escapes our present understanding.

The fine-tuning problem, associated with \( \Lambda \), can be alleviated in scalar field models which do not disturb the thermal history of the universe and can successfully mimic \( \Lambda \) at late times. A variety of scalar fields have been investigated to this end [1]; some of them are motivated by field/string theory and the others are introduced owing to phenomenological considerations. It is quite disappointing that a scalar field description lacks predictive power; given a priori a cosmic evolution, one can always construct a field potential that would give rise to it. These models should, however, not be written off, and should be judged by the generic features which might arise from them. For instance, the tracker models have remarkable features allowing them to alleviate the fine-tuning and coincidence problems. Present data are insufficient in order to conclude whether or not the dark energy has dynamics; thus, the quest for the metamorphosis of dark energy continues [2].

One can question the standard lore on fundamental grounds. We know that gravity is modified at small distance scales; it is quite possible that it is modified at large scales too where it has never been confronted with observations directly. It is therefore perfectly legitimate to investigate the possibility of late time acceleration due to modification of Einstein-Hilbert action. It is tempting to study the string curvature corrections to Einstein gravity amongst which the Gauss-Bonnet correction enjoys special status. A large number of papers are devoted to the cosmological implications of string curvature corrected gravity [3, 4, 5, 6, 7, 8, 9, 10, 11, 12]. These models, however, suffer from several problems. Most of these models do not include tracker like solution and those which do are heavily constrained by the thermal history of universe. For instance, the Gauss-Bonnet gravity with dynamical dilaton might cause transition from matter scaling regime to late time acceleration allowing to alleviate the fine tuning and coincidence problems. But it is difficult to reconcile this model with nucleosynthesis constraint. The large scale modification may also arise in extra dimensional theories like DGP model which contains self accelerating brane. Apart from the theoretical problems, this model is heavily constrained by observation.

On purely phenomenological grounds, one could seek a modification of Einstein gravity by replacing the Ricci scalar by generic function \( f(R) \) [13, 14, 15]. The \( f(R) \) gravity theories giving rise to cosmological constant in low curvature
regime are faced with difficulties which can be circumvented in $f(R)$ gravity models proposed by Hu-Sawicki and Starobinsky \[16,17\] (see Ref.\[18\] on the similar theme). These models can evade solar physics constraints by invoking the chameleon mechanism \[16,17,19\]. An important observation has recently been made in Refs.\[20,21\](see also Ref.\[22\] on the related theme), namely, the minimum of scalaron potential which corresponds to dark energy can be very near to $\phi = 0$ or equivalently $R = \infty$. As pointed out in Ref.\[19\], the minimum should be near the origin for solar constraints to be evaded. Hence, it is most likely that we hit the singularity if the parameters are not properly fine tuned. This may have serious implications for relativistic stars\[23\].

In what follows we shall briefly review the aforesaid developments.

II. LATE TIME ACCELERATION AND COSMOLOGICAL CONSTANT

Einstein equations exhibit simple analytical solutions in a homogeneous and isotropic universe. The dynamics in this case is described by a single function of time $a(t)$ dubbed scale factor,

$$H^2 = \frac{\dot{a}^2}{a^2} = \frac{8\pi G \rho}{3} - \frac{K}{a^2}$$

$$\dot{\rho} + 3H(\rho + p) = 0,$$

where $\rho$ designates the total energy density in the universe. Three different possibilities, $K = 0$, $K > 0$ or $K < 0$ correspond to flat geometry, hyperbolic geometry and geometry of the sphere correspondingly. Evolution can not change the nature of a particular geometry. What geometry we live in, depends upon the energy content of the universe,

$$\frac{K}{a^2} = H^2 (\Omega(t) - 1)$$

$$\Omega = \rho/\rho_c, \quad \rho_c = 3H^2/8\pi G$$

Observations on CMB reveal that we live in a nearly critical universe, $K = 0$ or $\rho = \rho_c$ which is consistent with inflationary paradigm. The equation for acceleration has the following form,

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3p)$$

$$\ddot{a} > 0 \iff p < -\frac{\rho}{3} : \text{DarkEnergy.}$$

Thus an exotic fluid with large negative pressure is needed to fuel the accelerated expansion of universe. Let us note that pressure corrects the energy density and positive pressure adds to deceleration where as the negative pressure contributes towards acceleration. It might look completely opposite to our intuition that highly compressed substance explodes out with tremendous impact whereas in our case pressure acts in the opposite direction. It is important to understand that our day today intuition with pressure is related to pressure force or pressure gradient. In a homogeneous universe pressure gradients can not exist. Pressure is a relativistic effect and can only be understood within the frame work of general theory of relativity. Pressure gradient might appear in Newtonian frame work in an inhomogeneous universe but pressure in FRW background can only be induced by relativistic effects. Strictly speaking, it should not appear in Newtonian gravity; its contribution is negligible in the non-relativistic limit. Indeed, in Newtonian cosmology, acceleration of a particle on the surface of a homogeneous sphere with density $\rho$ and radius $R$ is given by,

$$\ddot{R} = -\frac{4\pi}{3} G \rho R$$

(1)

The simplest possibility of dark energy is provided by cosmological constant which does not require an adhoc assumption for its introduction; it is automatically present in Einstein equations by virtue of Bianchi identities,

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}$$

(2)

The evolution equations in this case become,

$$H^2 = \frac{8\pi G}{3} \rho + \frac{\Lambda}{3}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3p) + \frac{\Lambda}{3}$$

(4)
In case the universe is dominated by $\Lambda$, it follows from the continuity equation that $p_\Lambda = -\rho_\Lambda$ and Eq. (3) tells us that a positive cosmological constant contributes to acceleration.

Observations of complimentary nature reveal that,
- $\Omega_{\text{tot}} \simeq 1$ - CMB,
- $\Omega_m \simeq 0.3$ - Large scale structure and its dynamics,
- $\Omega_{DE} \simeq 0.7$ - high redshift Ia Supernovae,

which is independently supported by data on baryon oscillations and weak lensing.

Observations at present do not rule out the *phantom* dark energy with $w < -1$ corresponding to super acceleration. In this case the expanding solution takes the form,

$$a(t) = (t_s - t)^n, \quad (n = 2/3(1 + w))$$

where $t_s$ is an integration constant. It is easy to see that phantom dominated universe will end itself in a singularity known as *big rip* or *cosmic doomsday*,

$$H = \frac{n}{t_s - t}$$

$$R = 6 \left[ \frac{\ddot{a}}{a} + \left( \frac{\dot{a}}{a} \right)^2 \right] = \frac{6(n(n - 1) + n^2)}{(t_s - t)^2}$$

The big rip singularity is characterized by the divergence of $H$ and consequently of the curvature after a finite time in future. It should be noted that when curvature becomes large, we should incorporate the higher curvature corrections to Einstein-Hilbert action which can modify the structure of the singularity [24]. Big rip can also be avoided in specific models of phantom energy [22].

**A. Issues associated with $\Lambda$**

There are important theoretical issues related to cosmological constant. Cosmological constant can be associated with vacuum fluctuations in the quantum field theoretic context. Though the arguments are still at the level of numerology but may have far reaching consequences. Unlike the classical theory the cosmological constant in this scheme is no longer a free parameter of the theory. Broadly the line of thinking takes the following route.

The quantum effects in GR become important when the Einstein-Hilbert action becomes of the order of Planck’s constant; this happens at the Planck’s length $L_p = 10^{-32}$ cm corresponding to Planck energy which is of the order of $M_p^4 \sim 10^{16} GeV^4$. The ground state energy dubbed zero point energy or vacuum energy of a free quantum field is infinite. This contribution is related the ordering ambiguity of fields in the classical Lagrangian and disappears when normal ordering is adopted. Since this procedure of throwing out the vacuum energy is *ad hoc*, one might try to cancel it by introducing the counter terms. The later, however requires fine tuning and may be regarded as unsatisfactory. The divergence is related to the modes of very small wave length. As we are ignorant of physics around the Planck scale, we might be tempted to introduce a cut off at $L_p$ and associate with this a fundamental scale. Thus we arrive at an estimate of vacuum energy $\rho_V \sim M_p^4$ (corresponding mass scale - $M_V \sim \rho_V^{1/4}$) which is away by 120 orders of magnitudes from the observed value of this quantity. The vacuum energy may not be felt in the laboratory but plays important role in GR through its contribution to the energy momentum tensor as $T_{\mu\nu} \equiv -\rho_V g_{\mu\nu}$ ($\rho_V = \Lambda/8\pi G, M_p^2 = 1/8\pi G$) and appears on the right hand side of Einstein equations.

The problem of zero point energy is naturally resolved by invoking supersymmetry which has many other remarkable features. In the supersymmetric description, every bosonic degree of freedom has its Fermi counterpart which contributes zero point energy with opposite sign compared to the bosonic degree of freedom thereby doing away with the vacuum energy. It is in this sense the supersymmetric theories do not admit a non-zero cosmological constant. However, we know that we do not leave in supersymmetric vacuum state and hence it should be broken. For a viable supersymmetric scenario, for instance if it is to be relevant to hierarchy problem, the supersymmetry breaking scale should be around $M_{\text{susy}} \simeq 10^3 GeV$. We are still away from the observed value by many orders of magnitudes. At present we do not know how Planck scale or SUSY breaking scale is related to the observed vacuum scale.

At present there is no satisfactory solution to cosmological constant problem. One might assume that there is some way to cancel the vacuum energy. One can then treat $\Lambda$ as a free parameter of classical gravity similar to Newton constant $G$. However, the small value of cosmological constant leads to several puzzles including the fine tuning and coincidence problems. The energy density in radiation at the Planck scale is of the order of $10^{24} GeV^4$ or $\rho_\Lambda / \rho_r \sim 10^{-120}$. Thus the vacuum energy density needs to be fine tuned at the level of one part in $10^{-120}$ around the Plank epoch, in order to match the current universe. Such an extreme fine tuning is absolutely unacceptable.
log(ρ)

log(a)

FIG. 1: Desired evolution of field energy density ρφ (ρB is the background energy density). The field energy density in case of undershoot and overshoot joins the scaling solution for different initial conditions. At late times, the scalar field exits the scaling regime to become the dominant component.

log(ρ)

log(a)

FIG. 2: Evolution of ρφ and ρB in absence of the scaling solution. The scalar field after its energy density overshoots the background gets into locking regime and waits till its energy density becomes comparable to ρB. It then begin to evolve and over takes the background. similar picture holds in case of the overshoot.

at theoretical grounds. Secondly, the energy density in cosmological constant is of the same order of matter energy density at the present epoch. The question what causes this coincidence has no satisfactory answer.

Efforts have recently been made to understand Λ within the frame work of string theory using flux compactification. String theory predicts a very complicated landscape of about $10^{500}$ de-Sitter vacua. Using Anthropic principal, we are led to believe that we live in one of these vacua! It is easier to believe in God than in these vacua!

III. SCALAR FIELD DYNAMICS RELEVANT TO COSMOLOGY

The fine tuning problem associate with cosmological constant led to the investigation of cosmological dynamics of a variety of scalar field systems such as quintessence, phantoms, tachyons and K-essence. In past years, the underlying dynamics of these systems has been studied in great detail. It is worthwhile to bring out the broad features that makes a particular scalar field system viable to cosmology. The scalar field model aiming to describe dark energy should possess important properties allowing it to alleviate the fine tuning and coincidence problems without interfering with the thermal history of universe. The nucleosynthesis puts an stringent constraint on any relativistic degree of freedom over and above that of the standard model of particle physics. Thus a scalar field has to satisfy several important constraints if it is to be relevant to cosmology. Let us now spell out some of these features in detail. In case the scalar field energy density ρφ dominates the background (radiation/matter) energy ρB, the former should redshift
faster than the later allowing radiation domination to commence which in turn requires a steep potential. In this case, the field energy density overshoots the background and becomes subdominant to it. This leads to the locking regime for the scalar field which unlocks the moment the $\rho_\phi$ is comparable to $\rho_B$. The further course of evolution crucially depends upon the form the potential assumes at late times. For the non-interference with thermal history, we require that the scalar field remains unimportant during radiation and matter dominated eras and emerges out from the hiding at late times to account for late time acceleration. To address the issues related to fine tuning, it is important to investigate the cosmological scenarios in which the energy density of the scalar field mimics the background energy density. The cosmological solution which satisfy this condition are known as scaling solutions,

$$\frac{\rho_\phi}{\rho_B} = \text{const.} \quad (8)$$

The steep exponential potential $V(\phi) \sim \exp(\lambda \phi/M_P)$ with $\lambda^2 > 3(1+w_B)$ in the frame work of standard GR gives rise to scaling solutions. Nucleosynthesis further constraints $\lambda$. The introduction of a new relativity degree of freedom at a given temperature changes the Hubble rate which crucially effects the neutron to proton for temperature of the order of one MeV when weak interactions freeze out. This results into a bound on $\lambda$, namely, $\Omega_\phi \equiv 3(1+w_B)/\lambda^2 < 0.13-0.2$ or $\lambda \gtrsim 4.5$. In this case, for generic initial conditions, the field ultimately enters into the scaling regime, the attractor of the dynamics and this allows to alleviate the fine tuning problem to a considerable extent. The same holds for the case of undershoot, see Fig.1.

Scaling solutions, however, are not accelerating as they mimic the background (radiation/matter). One therefore needs some late time feature in the potential. There are several ways of achieving this: (1) The potential that mimics a steep exponential at early epochs and reduces to power law type $V \sim \phi^{2p}$ at late times gives rise to accelerated expansion for $p < 1/2$ as the average equation of state $< w_\phi > = (p-1)/(p+1) < -1/3$ in this case$^{[26]}$. (ii) The steep inverse power law type of potential which become shallow at large values of the field can support late time acceleration and can mimic the background at early times$^{[27]}$.

The solutions which exhibit the aforesaid features are referred to as tracker solutions. For a viable cosmic evolution we need a tracker like solution.

Recently, a variety of scalar field models such as tachyon and phantom were investigated as candidates of dark energy. In case of tachyon with equation of state parameter ranging from $-1$ to 0, there exists no scaling solution which could mimic the realistic background (radiation/matter). Scaling solution which are possible in this case are associated with negative equation of state and are not of interest. In case of phantom scalar fields (scalar fields with negative kinetic energy), there is no fixed point corresponding to scaling solution. These scenarios suffer from the fine tuning problem; dynamics in this case acquires dependence on initial conditions$^{[28]}$ (see Fig.2).

The second approach to late time acceleration is related to the modification of left hand side of Einstein equations or the geometry of space time. In the past few years, several schemes of large scale modifications have been actively investigated. In what follows, we shall briefly describe the modified theories of gravity and their relevance to cosmology.

### IV. MODIFIED THEORIES OF GRAVITY AND LATE TIME ACCELERATION

In view of the above discussion, it is perfectly legitimate to investigate the possibility of late time acceleration due to modification of Einstein-Hilbert action. Some of these modifications are inspired by fundamental theories of high energy physics where as the others are based upon phenomenological considerations.

#### A. String curvature corrections

It is interesting to investigate the string curvature corrections to Einstein gravity amongst which the Gauss-Bonnet correction enjoys special status. These models, however, suffer from several problems. Most of these models do not include tracker like solution and those which do are heavily constrained by the thermal history of universe. For instance, the Gauss-Bonnet gravity with dynamical dilaton might cause transition from matter scaling regime to late time acceleration allowing to alleviate the fine tuning and coincidence problems. Let us consider the low energy effective action,

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2\kappa^2} R - (1/2) g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) - f(\phi) R^2_{GB} \right] + S_m \quad (9)$$
where \( R_{GB}^2 \) is the Gauss-Bonnet term,
\[
R_{GB}^2 \equiv R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}.
\]
(10)

The dilaton potential \( V(\phi) \) and its coupling to curvature \( f(\phi) \) are given by,
\[
V(\phi) \sim e^{(\alpha\phi)}, \quad f(\phi) \sim e^{-\mu(\phi)}
\]
(11)

The cosmological dynamics of system (9) in FRW background was investigated in Ref.\[5, 6\]. It was shown that scaling solution can be obtained in this case provided that \( \mu = \lambda \). In case \( \mu \neq \lambda \), we have the de-Sitter solution. Hence, the string curvature corrections under consideration can give rise to late time transition from matter scaling regime. Unfortunately, it is difficult to reconcile this model with nucleosynthesis\[5, 6\] constraint.

### B. DGP model

In DGP model, gravity behaves as four dimensional at small distances but manifests its higher dimensional effects at large distances. The modified Friedmann equations on the brane lead to late time acceleration. The model has serious theoretical problems related to ghost modes superluminal fluctuations. The combined observations on background dynamics and large angle anisotropies reveal that the model performs worse than \( \Lambda CDM \[29\].

### C. Non-local cosmology

An interesting proposal on non-locally corrected gravity involving a function of the inverse d’Almbertian of the Ricci scalar, \( f(\Box^{-1}R) \), was made in Refs.\[30\]. For a generic function \( f(\Box^{-1}R) \sim \exp(\alpha\Box^{-1}R) \), the model can lead to de-Sitter solution at late times. The range of stability of the solution is given by \( 1/3 < \alpha < 2/3 \) corresponding to the effective EoS parameter \( w_{\text{eff}} \) ranging as \( \infty < w_{\text{eff}} < -2/3 \). For \( 1/3 < \alpha < 1/2 \) and \( 1/2 < \alpha < 2/3 \), the underlying system is shown to exhibit phantom and non-phantom behavior respectively; the de Sitter solution corresponds to \( \alpha = 1/2 \). For a wide range of initial conditions, the system mimics dust like behavior before reaching the stable fixed point at late times. The late time phantom phase is achieved without involving negative kinetic energy fields. Unfortunately, the solution becomes unstable in presence of the background radiation/matter\[30\].

### D. \( f(R) \) theories of gravity

On purely phenomenological grounds, one could seek a modification of Einstein gravity by replacing the Ricci scalar by \( f(R) \). The \( f(R) \) gravity theories giving rise to cosmological constant in low curvature regime are plagued with instabilities and on observational grounds they are not distinguished from cosmological constant. The recently introduced models of \( f(R) \) gravity by Hu-Sawicki and Starobinsky (referred as HSS models hereafter) with disappearing cosmological constant\[16, 17\] have given rise to new hopes for a viable cosmological model within the framework of modified gravity. The action of \( f(R) \) gravity is given by\[13\],
\[
S = \int \left[ \frac{f(R)}{16\pi G} + \mathcal{L}_m \right] \sqrt{-g} \; d^4x,
\]
(12)

which leads to the following modified equations,
\[
f' R_{\mu\nu} - \nabla_{\mu} f' + \left( \Box f' - \frac{1}{2} f \right) g_{\mu\nu} = 8\pi GT_{\mu\nu}.
\]
(13)

which are of fourth order for a non-linear function \( f(R) \). Here prime (’) denotes the derivatives with respect to \( R \). The modified Eq.\[13\] contains de-Sitter space time as a vacuum solution provided that \( f(4\Lambda) = 2\Lambda f'(4\Lambda) \). Thus, the \( f(R) \) theories of gravity may provide an alternative to dark energy. The \( f(R) \) gravity theories apart from a spin two object necessarily contain a scalar degree of freedom. Taking trace of Eq.\[13\] gives the evolution equation for the scalar degree of freedom,
\[
\Box f' = \frac{1}{3} (2f' - f'R) + \frac{8\pi G}{3} T.
\]
(14)
It would be convenient to define scalar function $\phi$ as,

$$\phi \equiv f' - 1,$$

(15)

which is expressed through Ricci scalar once $f(R)$ is specified. We can write the trace equation (Eq.(14)) in the terms of $V$ and $T$ as

$$\Box \phi = \frac{dV}{d\phi} + \frac{8\pi G}{3} T.$$

(16)

The potential can be evaluated using the following relation

$$\frac{dV}{dR} = \frac{dV}{d\phi} \frac{d\phi}{dR} = \frac{1}{3} (2f - f'R) f''.$$

(17)

The functional form of $f(R)$ should satisfy certain requirements for the consistency of the modified theory of gravity. The stability of $f(R)$ theory would be ensured provided that,

- $f'(R) > 0$ – graviton is not ghost,
- $f''(R) > 0$ – scalaron is not tachyon.

The $f(R)$ models which satisfy the stability requirements can broadly be classified into categories: (i) Models in which $f(R)$ diverges for $R \to R_0$ where $R_0$ finite or $f(R)$ is non analytical function of the Ricci scalar. These models either can not be distinguishable from $\Lambda CDM$ or are not viable cosmologically. (ii) Models with $f(R) \to 0$ for $R \to 0$ and reduce to cosmological constant in high curvature regime. These models reduce to $\Lambda CDM$ in high redshift regime and give rise to cosmological constant in regions of high density and differ from the latter otherwise; in principal these models can be distinguished from cosmological constant.

Models belonging to the second category were proposed by Hu-Sawicki and Starobinsky [16, 17]. The functional form of $f(R)$ in Starobinsky parametrization is given by,

$$f(R) = R + \lambda R_0 \left[ \left( 1 + \frac{R^2}{R_0^2} \right)^{\frac{n-1}{2}} - 1 \right].$$

(18)

Here $n$ and $\lambda$ are positive. And $R_0$ is of the order of presently observed cosmological constant, $\Lambda = 8\pi G \rho_{vac}$. The properties of this model [17] can be summarized as follows:

1. Stability conditions are satisfied as $f', f'' > 0$
2. Flat space time is an unstable solution of the model.
3. For $|R| \gg R_0$, $f(R) = R - 2\Lambda(\infty)$. The high-curvature value of the effective cosmological constant is $\Lambda(\infty) = \lambda R_0/2$.

In the Starobinsky model, the scalar field $\phi$ is given by

$$\phi(R) = -\frac{2n\lambda R}{R_0(1 + \frac{R^2}{R_0^2})^{n+1}}.$$

(19)

Notice that $R \to \infty$ for $\phi \to 0$. One can easily compute $V(R)$ for a given value of $n$. For instance, in case of $n = 1$, we have

$$\frac{V}{R_0} = \frac{1}{24(1 + y^2)} \left\{ (-8 - 40y^2 - 56y^4 - 24y^6) \lambda + (3y + 11y^3 + 21y^5 - 3y^7) \lambda^2 \right\} - \frac{\lambda^2}{8} \tan^{-1} y,$$

(20)

where $y = R/R_0$. In the FRW background, the trace equation (Eq. [14]) can be rewritten in the convenient form

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = \frac{8\pi G}{3} \rho.$$

(21)
The time-time component of the equation of motion \(H^2 + \frac{d}{dt} \ln f' \frac{d}{dt} H + \frac{1}{6} \frac{f - f'R}{f'} = \frac{8\pi G}{3f'} \rho. \) (22)

It should be noted that the stability conditions ensure that the effective gravitational constant \(G_{\text{eff}} = G/f'\) appearing in Eq. (22) is positive. The simple picture of dynamics which appears here is the following: above infrared modification scale \(R_0\), the expansion rate is set by the matter density and once the local curvature falls below \(R_0\) the expansion rate gets effect of gravity modification. For pressure less dust, the effective potential has an extremum at,

\[2f - Rf' = 8\pi G \rho. \] (23)

For a viable late time cosmology, the field should be evolving near the minimum of the effective potential. The finite time singularity inherent in the class of models under consideration severely constraints dynamics of the field.

The curvature singularity and fine tuning of parameters

The effective potential has minimum which depends upon \(n\) and \(\lambda\). For generic values of the parameters, the minimum of the potential is close to \(\phi = 0\) corresponding to infinitely large curvature. Thus while the field is evolving towards minimum, it can easily oscillate to a singular point. However, depending upon the values of parameters, we can choose a finite range of initial conditions for which scalar field \(\phi\) can evolve to the minimum of the potential without hitting the singularity.

We find that the range of initial conditions allowed for the evolution of \(\phi\) to the minimum without hitting singularity shrinks as the numerical values of parameters \(n\) and \(\lambda\) increase. This is related to the fact that for larger values of
\(n\) and \(\lambda\), the minimum fast moves towards \(\phi = 0\), see figure[4]. The numerical values should be accurately chosen to avoid hitting the singularity.

### Avoiding singularity with higher curvature corrections

We know that in case of large curvature, the quantum effects become important leading to higher curvature corrections. Keeping this in mind, let us consider the modification of Starobinsky’s model,

\[
f(R) = R + \frac{\alpha}{R_0} R^2 + R_0 \lambda \left[-1 + \frac{1}{(1 + \frac{R_0}{R_0})^n}\right],
\]

then \(\phi\) becomes

\[
\phi(R) = \frac{R}{R_0} \left[2\alpha - \frac{2 n \lambda}{(1 + \frac{R_0}{R_0})^{n+1}}\right].
\]

When \(|R|\) is large, the first term which comes from \(\alpha R^2\) dominates. In this case, the curvature singularity \(R = \pm \infty\) corresponding to \(\phi = \pm \infty\), see Fig[5]. Hence, in this modification, the minimum of the effective potential is separated from the curvature singularity by the infinite distance in the \(\phi, V(\phi)\) plane. In case of \(n = 2\), the expressions for \(\phi\) and \(V(\phi)\) are given by,

\[
\phi(y) = 2\alpha y - \frac{4\lambda y}{(1 + y^2)^3},
\]

\[
\frac{V}{R_0} = -\frac{1}{480(1 + y^2)^6} \left\{ \lambda^2 y \left(-105 - 595 y^2 - 2154 y^4 + 106 y^6 + 595 y^8 + 105 y^{10}\right) \right\} - \frac{1}{3(1 + y^2)^3} \left\{ 1 + 5 y^2 + \alpha (3 + 8 y^2 + 9 y^4 + 4 y^6) \right\} + \frac{1}{3} \alpha y^2 + \frac{1}{32} (32\alpha - 7\lambda) \tan^{-1}(y).
\]

For \(n = 2, \lambda = 2\) and \(\alpha = 0.5\), we have a large range of the initial condition for which the scalar field evolves to the minimum of the potential. Though the introduction of \(R^2\) term formally allows to avoid the singularity but can not alleviate the fine tuning problem as the minimum should be brought near to the origin to respect the solar constraints. Last but not the least one could go beyond the approximation (see Eq.(23)) by iterating the trace equation and computing the corrections to \(R\) given by equation (23). As pointed by Starobinsky[17], such a correction might become large in the past. This may spoil the thermal history and thus needs to be fine tuned. The aforesaid discussion makes it clear that HSS models are indeed fine tuned and hence very delicate.

In case of any large scale modification of gravity, one should worry about the local gravity constraints. The \(f(R)\) theories belong to the class of scalar tensor theories corresponding the Brans-Dicke parameter \(\omega = 0\) or the PPN parameter \(\gamma = (1 + \omega)/(2 + \omega) = 1/2\) unlike GR where \(\gamma = 1\) consistent with observation (\(|\gamma - 1| \lesssim 10^{-4}\)). This problem can be circumvented by invoking the so called chameleon mechanism. In case, the scalar degree of freedom is coupled to matter, the effective mass of the field depends upon the matter density which can allow to avoid the conflict with solar physics constraints.

### V. SUMMARY

We have briefly summarized here various approaches to understand the late time acceleration of universe. In case of scalar field models of dark energy, we emphasized the relevance of scaling solutions in alleviating the fine tuning and the coincidence problems. We hope that the future data would reveal the metamorphosis of dark energy.

Amongst the various alternatives to dark energy, the \(f(R)\) gravity models have received considerable attention in past years. There are broadly two classes of \(f(R)\) models, namely, those in which \(f(R)\) diverges as \(R \to R_0\) (\(R_0\) is finite) or \(f(R)\) is non-analytic in \(R\). And those with \(f(R) \to 0\) as \(R \to 0\), they reduce to \(\Lambda CDM\) in the limit of high redshift and give rise to cosmological constant in high density regime. These models can evade local gravity constraints with the help of the so called chameleon mechanism and have potential capability of being distinguished from \(\Lambda CDM[31]\).

Unfortunately, the \(f(R)\) models with chameleon mechanism are plagued with curvature singularity problem which may have important implications for relativistic stars[23]. The model could be remedied with the inclusion of higher...
FIG. 5: Plot of the effective potential for \( n = 2, \lambda = 2 \) and \( \alpha = 1/2 \) in presence of \( R^2 \) correction. the minimum of the effective potential in this case is located at \( \phi_{\text{min}} = 3.952 \) (\( R_{\text{min}} = 3.958 \)).

curvature corrections\([32]\). At the onset, it seems that one needs to invoke fine tunings to address the problem\([33]\). The presence of curvature singularity certainly throws a new challenge to \( f(R) \) gravity models. In our opinion, the problem requires further investigations. It would also be interesting to look for a realistic scenario of quintessential inflation in the frame work of \( f(R) \) gravity.

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