ANALYSIS OF THE $Z_c(4200)$ AS AXIAL-VECTOR MOLECULE-LIKE STATE

Zhī-Gang Wang
Department of Physics, North China Electric Power University, Baoding 071003, P. R. China

Abstract

In this article, we assume the $Z_c(4200)$ as the color octet-octet type axial-vector molecule-like state, and construct the color octet-octet type axial-vector current to study its mass and width with the QCD sum rules. The numerical values $M_{Z_c(4200)} = 4.19 \pm 0.08$ GeV and $\Gamma_{Z_c(4200)} \approx 334$ MeV are consistent with the experimental data $M_{Z_c(4200)} = 4196^{+31+17}_{-29-13}$ MeV and $\Gamma_{Z_c(4200)} = 370^{+70+70}_{-70-132}$ MeV, and support assigning the $Z_c(4200)$ to be the color octet-octet type molecule-like state with $J^{PC} = 1^{−+}$. Furthermore, we discuss the possible assignments of the $Z_c(3900)$, $Z_c(4200)$ and $Z(4430)$ as the diquark-antidiquark type tetraquark states with $J^{PC} = 1^{−+}$.

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1 Introduction

In 2014, the Belle collaboration analyzed the $B^0 \rightarrow K^- \pi^+ J/\psi$ decays with the full $\Upsilon(4S)$ data sample corresponding to $711 fb^{-1}$ data sample collected by the Belle detector at the asymmetric-energy $e^+e^−$ collider, and observed a resonance (named $Z_c(4200)$) in the $J/\psi \pi^+$ mass spectrum with a statistical significance of more than 6.2 $\sigma$, the measured mass and width are $M_{Z_c(4200)} = 4196^{+31+17}_{-29-13}$ MeV and $\Gamma_{Z_c(4200)} = 370^{+70+70}_{-70-132}$ MeV, respectively [4,2]. The preferred assignment of the quantum numbers is $J^P = 1^+$.

In 2007, the Belle collaboration observed a distinct peak in the $\pi^±\psi'$ mass spectrum in the $B^0 \rightarrow K^±\pi^±\psi'$ decays with the statistical significance of 6.5 $\sigma$ [5]. In 2014, the LHCb collaboration analyzed the $B^0 \rightarrow \psi'\pi^- K^+$ decays by performing a four-dimensional fit of the decay amplitude, and provided the first independent confirmation of the $Z(4430)^−$ and established its spin-parity to be $J^P = 1^+$ [4].

In 2013, the BESIII collaboration studied the process $e^+e^- \rightarrow \pi^±\pi^- J/\psi$ and observed a structure $Z_c(3900)$ in the $\pi^± J/\psi$ mass spectrum [6,7]. Then the Belle and CLEO collaborations confirmed the existence of the $Z_c(3900)$ [5,7]. Although the quantum numbers are not measured, the assignment $J^P = 1^+$ is favored if the decay $Z_c(3900)^\pm \rightarrow J/\psi\pi^\pm$ takes place in relative S-wave. In 2014, the BESIII collaboration studied the process $e^+e^- \rightarrow \pi D\bar{D}^*$ and observed a distinct charged structure $Z_c(3885)$ in the $(D\bar{D}^*)^\pm$ mass spectrum, the assignment $J^P = 1^+$ is favored [8]. We tentatively identify the $Z_c(3900)$ and $Z_c(3885)$ as the same particle according to the uncertainties of the masses and widths. For more literatures on the $X$, $Y$, $Z$ mesons, one can consult the recent review [9].

The quark constituents of the $Z_c(3900)$, $Z_c(4200)$ and $Z(4430)$ are $c\bar{c}d\bar{u}$ or $c\bar{c}d\bar{u}$, there are three analogous decays,

\begin{align}
Z_c(3900)^\pm & \rightarrow J/\psi \pi^\pm, \\
Z_c(4200)^\pm & \rightarrow J/\psi \pi^\pm, \\
Z(4430)^\pm & \rightarrow \psi' \pi^\pm,
\end{align}

which take place through fall-apart mechanism. The mass differences are $M_{Z(4430)} - M_{Z_c(3900)} = 576$ MeV and $M_{\psi'} - M_{J/\psi} = 589$ MeV, so it is natural to assign the $Z(4430)$ to be the first radial excitation of the $Z_c(3900)$ if $\Gamma_{Z_c(3900)}$ are large enough. Naively, we expect the tetraquark states have large decay widths, the $Z_c(4200)$ and $Z(4430)$ are good candidates of the tetraquark states according to the

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1E-mail: zgwang@aliyun.com.
widths $\Gamma_{Z_c(4200)} = 370^{+70}_{-70} - 132 \text{ MeV}$ [1] and $\Gamma_{Z_c(4430)} = (172 \pm 13^{+37}_{-34}) \text{ MeV}$ [1]. In Ref. [14], we study the axial-vector hidden charm (and hidden bottom) tetraquark states in details with the QCD sum rules and obtain the value $M_{Z_c(4430)} = (4.44 \pm 0.19) \text{ GeV}$. In Ref. [14], Chen and Zhu study the vector and axial-vector charmonium-like tetraquark states with the QCD sum rules and obtain the value $M_{J^P=1^+} = (4.16 \pm 0.10) \text{ GeV}$. In Ref. [15], Chen et al. assume the $Z_c(4200)$ to be the axial-vector tetraquark state and calculate its decay width with the QCD sum rules. In Ref. [13], the QCD spectral densities are calculated at the special energy scale $\mu = 1 \text{ GeV}$, while in Ref. [14], the energy scale of the QCD spectral densities is not specified. If the $Z_c(3900)$, $Z_c(4200)$ and $Z_c(4430)$ have the same quantum numbers $J^P = 1^+$, they cannot all be the ground state axial-vector tetraquark state with the same quark structure.

The $X, Y$ and $Z$ mesons have been studied extensively by the QCD sum rules [13141516], but the energy scale dependence of the QCD spectral densities are not studied. In previous works [1217181920], we explore the energy scale dependence of the QCD sum rules for the hidden charmed and hidden bottom tetraquark states and molecular states in details for the first time, and suggest a formula,

$$\mu = \sqrt{M_X^2 / \mu^2 - (2M_Q)^2},$$

(2)

with the effective masses $M_Q$ to determine the energy scales of the QCD spectral densities.

The quarks have color $SU(3)$ symmetry, we can construct the tetraquark states according to the routine quark $\rightarrow$ diquark $\rightarrow$ tetraquark,

$$(3 \otimes 3) \otimes (\overline{3} \otimes \overline{3}) = (\overline{3} \otimes 6) \otimes (3 \otimes \overline{3}) = \overline{3} \otimes 3 \otimes \cdots = 1 \otimes 8 \otimes \cdots,$$

(3)

or construct the molecular states and molecule-like states according to the routine quark $\rightarrow$ meson (or meson $\rightarrow$ molecule-like state),

$$(3 \otimes \overline{3}) \otimes (3 \otimes \overline{3}) = (1 \otimes 8) \otimes (1 \otimes 8) \otimes (8 \otimes 8) \otimes \cdots = 1 \otimes 1 \otimes \cdots,$$

(4)

where the 1, 3 ($\overline{3}$), 6 ($\overline{6}$) and 8 denote the color singlet, triplet (antitriplet), sextet (antisextet) and octet, respectively.

In the scenario of tetraquark states, we study the $3 \otimes 3$ type (or the diquark-antidiquark type) scalar, vector, axial-vector, tensor hidden charmed tetraquark states and axial-vector hidden bottom tetraquark states systematically with the QCD sum rules [121718], and assign the $X(3872)$, $Z_c(3900/3885)$, $Z_c(4020/4025)$, $Y(4140)$, $Z(4430)$, $Y(4660/4630)$ and $Z_b(10610/10650)$ to be tetraquark states tentatively;

$$X(3872) = \frac{1}{2} \left( [cu]_A \overline{cs}_A |s + [cd]_A \overline{sd}_A |s + [cu]_S \overline{cs}_A + [cd]_S \overline{sd}_A \right) \text{ (with } 1^{++}),$$

$$Z_c(3900/3885) = \frac{1}{\sqrt{2}} \left( [cu]_A \overline{cs}_A |s - [cu]_S \overline{cs}_A |s \right) \text{ (with } 1^{-+}),$$

$$Z_c(4020/4025) = [cu]_A \overline{cs}_A \text{ (with } 1^{+-} \text{ or } 2^{++}),$$

$$Y(4140) = [cs]_A \overline{cs}_A \text{ (with } 2^{++}),$$

$$Z(4430) = \frac{1}{\sqrt{2}} \left( [cu]_A \overline{cs}_A |s - [cu]_S \overline{cs}_A |s \right) \text{ (with } 1^{-+}),$$

$$Y(4660/4630) = \frac{1}{\sqrt{2}} \left( [cs]_A \overline{cs}_A |s - [cs]_S \overline{cs}_A |s \right) \text{ (with } 1^{-+}),$$

$$Z_b(10610) = \frac{1}{\sqrt{2}} \left( [bu]_A \overline{bd}_A |s - [bu]_S \overline{bd}_A |s \right) \text{ (with } 1^{-+}),$$

$$Z_b(10650) = [bu]_A \overline{bd}_A \text{ (with } 1^{-+}),$$

(5)

where the $[Qq]_S$, $[Qq]_A$ and $[Qq]_P$ denote the scalar, axial-vector and pseudoscalar diquarks in the color antitriplet 3, $q = u, d, s$ and $Q = c, b$. 

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In the scenario of molecular states, we study the meson-meson type (or the $1 \otimes 1$ type) scalar, axial-vector and tensor hadronic molecular states with the QCD sum rules [19][20], and assign the $X(3872)$, $Z_c(3900/3885)$, $Y(4140)$, $Z_c(4020/4025)$ and $Z_b(10610/10650)$ to be the molecular states tentatively,

$$
X(3872) = \frac{1}{\sqrt{2}} \left( D\overline{D}^* - D^*\overline{D} \right) \quad \text{(with $1^{++}$)},
$$

$$
Z_c(3900/3885) = \frac{1}{\sqrt{2}} \left( D\overline{D} + D^*\overline{D}^* \right) \quad \text{(with $1^{+-}$)},
$$

$$
Z_c(4020/4025) = D^*\overline{D}^* \quad \text{(with $1^{+-}$ or $2^{++}$)},
$$

$$
Y(4140) = D^*_s\overline{D}_s^* \quad \text{(with $0^{++}$)},
$$

$$
Z_b(10610) = \frac{1}{\sqrt{2}} \left( B\overline{B} + B^*\overline{B}^* \right) \quad \text{(with $1^{+-}$)},
$$

$$
Z_b(10650) = B^*\overline{B} \quad \text{(with $1^{--}$)}.
$$

(6)

In Ref.[19], we observe that if we determine the energy scales of the QCD spectral densities with the same parameter $M_c$, the $8 \otimes 8$ type molecule-like states have larger masses than the corresponding $1 \otimes 1$ type molecular states. We obtain the masses $M_{Z_c(3900)} = 3.89_{-0.09}^{+0.09}$ MeV and $M_{Z_c(8\otimes 8)} = 4.10_{-0.10}^{+0.09}$ MeV with the QCD sum rules [19]. The upper bound of the predicted mass $M_{Z_c(8\otimes 8)} = 4.10_{-0.10}^{+0.09}$ MeV is consistent with the experimental value $M_{Z_c(4200)} = 4196^{+31}_{-29} - 13$ MeV [1]. Now, we assign the $Z_c(4200)$ to be the $8 \otimes 8$ type molecule-like state tentatively,

$$
Z_c(4200) = \frac{1}{\sqrt{2}} \left( D\overline{D} + D^*\overline{D}^* \right) \quad \text{(with $1^{+-}$)},
$$

(7)

study the mass and decay width with the QCD sum rules in details, and fit the effective mass $M_c$ for the $8 \otimes 8$ type molecule-like states, where the $D$ and $D^*$ have the same quark constituents as the $D$ and $D^*$ respectively, but they are in the color 8 representation.

The article is arranged as follows: we derive the QCD sum rules for the mass of the $8 \otimes 8$ type axial-vector molecule-like state $Z_c(4200)$ in section 2; we derive the QCD sum rules for the width of the $8 \otimes 8$ type axial-vector molecule-like state $Z_c(4200)$ in section 3; we discuss the possible assignments of the $Z_c(3900)$, $Z_c(4200)$ and $Z(4430)$ as the 3 $\otimes 3$ type axial-vector tetraquark states in section 4; section 5 is reserved for our conclusion.

2 The mass of the $8 \otimes 8$ type axial-vector molecule-like state

In the following, we write down the two-point correlation function $\Pi_{\mu\nu}(p)$ in the QCD sum rules,

$$
\Pi_{\mu\nu}(p) = i \int d^4xe^{ipx} \langle 0| T \left\{ J_\mu(x)J_\nu^\dagger(0) \right\} |0\rangle,
$$

(8)

$$
J_\mu(x) = \frac{\bar{u}(x)i\gamma_5\lambda^a c(x)\bar{c}(x)\gamma_\mu\lambda^a d(x) + \bar{u}(x)\gamma_\mu\lambda^a c(x)\bar{c}(x)i\gamma_5\lambda^a d(x)}{\sqrt{2}},
$$

(9)

where the $\lambda^a$ is the Gell-Mann matrix in the color space. We construct the $8 \otimes 8$ type current $J_\mu(x)$ (see Ref.[21]) to study the molecule-like state $Z_c(4200)$. Under charge conjugation transform $\hat{C}$, the current $J_\mu(x)$ has the property,

$$
\hat{C} J_\mu(x) \hat{C}^{-1} = -J_\mu(x) |_{u+d}.
$$

(10)

We insert a complete set of intermediate hadronic states with the same quantum numbers as the current operator $J_\mu(x)$ into the correlation function $\Pi_{\mu\nu}(p)$ to obtain the hadronic representation
\[ M^2_{Z_c(4200)} \exp \left( -\frac{M^2_{Z_c(4200)}}{T^2} \right) = \int_{4m^2_s}^{s_0} ds \rho(s) \exp \left( -\frac{s}{T^2} \right). \] (13)

One can consult Ref.\[19\] for the explicit expression of the QCD spectral density \( \rho(s) \). We differentiate Eq.(13) with respect to \( \tau = \frac{1}{T^2} \), then eliminate the pole residue \( \lambda_{Z_c(4200)} \) to obtain the QCD sum rule for the mass,

\[ M^2_{Z_c(4200)} = \frac{\int_{4m^2_s}^{s_0} ds \left( \frac{d}{ds} \right) \rho(s)e^{-\tau s}}{\int_{4m^2_s}^{s_0} ds \rho(s)e^{-\tau s}}. \] (14)

We take the standard values of the vacuum condensates \( \langle \bar{q}q \rangle = -(0.24 \pm 0.01) \text{ GeV}^3 \), \( \langle \bar{g}_s G G \rangle = m_0^2 \langle \bar{q}q \rangle \), \( m_0^2 = (0.8 \pm 0.1) \text{ GeV}^2 \), \( \langle \frac{\alpha_s G G}{\pi} \rangle = (0.33 \text{ GeV})^4 \) at the energy scale \( \mu = 1 \text{ GeV} \) \[22, 23, 26\], and choose the \( M^2 \) mass \( m_c(m_c) = (1.275 \pm 0.025) \text{ GeV} \) from the Particle Data Group \[24\]. Furthermore, we take into account the energy-scale dependence of the input parameters.

\[ \langle \bar{q}q \rangle(\mu) = \langle \bar{q}q \rangle(Q) \left[ \frac{\alpha_s(Q)}{\alpha_s(\mu)} \right]^{\frac{b_0}{b_1}}, \]

\[ \langle \bar{g}_s G G \rangle(\mu) = \langle \bar{g}_s G G \rangle(Q) \left[ \frac{\alpha_s(Q)}{\alpha_s(\mu)} \right]^{\frac{b_2}{b_1}}, \]

\[ m_c(\mu) = m_c(m_c) \left[ \frac{\alpha_s(\mu)}{\alpha_s(m_c)} \right]^{\frac{12}{25}}, \]

\[ \alpha_s(\mu) = \frac{1}{b_0t} \left[ 1 - \frac{b_1}{b_0^2} \log t + \frac{b_1^2(\log^2 t - \log t - 1) + b_0 b_2}{b_0^2 t^2} \right]. \] (15)

where \( t = \log \frac{s}{\Lambda^2} \), \( b_0 = \frac{33 - 2n_f}{12\pi} \), \( b_1 = \frac{153 - 19n_f}{24\pi} \), \( b_2 = \frac{2857 - 5033n_f + 325n_f^2}{144\pi} \), \( \Lambda = 213 \text{ MeV} \), \( 296 \text{ MeV} \) and \( 339 \text{ MeV} \) for the flavors \( n_f = 5, 4 \) and 3, respectively \[23\]. We tentatively take the continuum threshold parameter as \( \sqrt{s_0} = (4.7 \pm 0.1) \text{ GeV}^2 \), i.e. \( \sqrt{s_0} = M_{Z_c(4200)} + (0.4 - 0.6) \text{ GeV} \), and search for the optimal Borel parameter to satisfy the two criteria (pole dominance and convergence of the operator product expansion) of the QCD sum rules.

In Fig.1, we plot the mass of the \( Z_c(4200) \) with variations of the Borel parameters \( T^2 \) and energy scales \( \mu \) for the threshold parameter \( \sqrt{s_0} = 4.7 \text{ GeV} \). From the figure, we can see that the masses decrease monotonously with increase of the energy scales. We can reproduce the experimental value \( M_{Z_c(4200)} = 4196^{+31+17}_{-29-13} \text{ MeV} \) \[1\] approximately at the energy scale \( \mu = (1.3 - 1.5) \text{ GeV} \). In this article, we take the energy scale \( \mu = 1.4 \text{ GeV} \). In the Borel window \( T^2 = (3.0 - 3.4) \text{ GeV}^2 \), the pole contribution is about \((43 - 64)\%\), it is reliable to extract the ground state mass.
Figure 1: The mass of the $Z_c(4200)$ with variations of the Borel parameters $T^2$ and energy scales $\mu$, where the horizontal line denotes the experimental value.

In Fig.2, we plot the contributions of different terms in the operator product expansion with variations of the Borel parameters $T^2$ for the threshold parameter $\sqrt{s_0} = 4.7$ GeV and energy scale $\mu = 1.4$ GeV. In the Borel window $T^2 = (3.0 - 3.4)$ GeV$^2$, the $D_3 \gg D_0 \approx |D_5| \gg D_0 \gg |D_8|$, and the $D_4$, $D_7$ and $D_{10}$ play a less important role, where the $D_i$ with $i = 0, 3, 4, 5, 6, 7, 8, 10$ denote the contributions of the vacuum condensates of dimensions $D = i$, and the total contributions are normalized to be 1. The operator product expansion is well convergent.

We take into account all uncertainties of the input parameters, and obtain the values of the mass and pole residue of the $Z_c(4200)$, which are shown explicitly in Figs.3-4,

\[ M_{Z_c(4200)} = 4.19 \pm 0.08 \text{ GeV}, \]
\[ \lambda_{Z_c(4200)} = (5.25 \pm 0.71) \times 10^{-2} \text{ GeV}^5. \]  

(16)

The predicted mass $M_{Z_c(4200)} = 4.19 \pm 0.08$ GeV is consistent with the experimental value $M_{Z_c(4200)} = 4196^{+31}_{-29} \pm 17$ MeV within uncertainties. The QCD sum rules favor assigning the $Z_c(4200)$ to be the $8 \otimes 8$ type $D \bar{D}^* + D^* \bar{T}$ molecule-like state. Now we can obtain the parameter $M_c = 1.98$ GeV for the $8 \otimes 8$ type molecule-like states according to the energy scale formula $\mu = \sqrt{M_{X/Y/Z}^2 - (2M_c)^2}$.

3 The width of the $8 \otimes 8$ type axial-vector molecule-like state

We can study the strong decays $Z_c^\pm(4200) \rightarrow J/\psi \pi^\pm, \eta_c \rho^\pm$ and $(D \bar{D}^*)^\pm$ (or $(D^* \bar{D})^\pm$) with the following three-point correlation functions $\Pi^1_{\alpha\mu}(p, q)$, $\Pi^2_{\alpha\mu}(p, q)$ and $\Pi^3_{\alpha\mu}(p, q)$, respectively,

\[ \Pi^1_{\alpha\mu}(p, q) = i^2 \int d^4x d^4y e^{ipx} e^{iqy} \langle 0 | T \left\{ J_\alpha^{J/\psi}(x) J_5^\pi(y) J_\mu(0) \right\} | 0 \rangle, \]

(17)

\[ \Pi^2_{\alpha\mu}(p, q) = i^2 \int d^4x d^4y e^{ipx} e^{iqy} \langle 0 | T \left\{ J_5^{\rho}(x) J_\alpha^\rho(y) J_\mu(0) \right\} | 0 \rangle, \]

(18)

\[ \Pi^3_{\alpha\mu}(p, q) = i^2 \int d^4x d^4y e^{ipx} e^{iqy} \langle 0 | T \left\{ J_\alpha^{D^*}(x) J_5^{D^*}(y) J_\mu(0) \right\} | 0 \rangle, \]

(19)
Figure 2: The contributions of different terms in the operator product expansion with variations of the Borel parameter $T^2$, where the 0, 3, 4, 5, 6, 7, 8 and 10 denotes the dimensions of the vacuum condensates.

Figure 3: The mass of the $Z_c(4200)$ with variations of the Borel parameter $T^2$, where the horizontal line denotes the experimental value.
Figure 4: The pole residue of the $Z_c(4200)$ with variations of the Borel parameter $T^2$.

where the currents

\begin{align}
J_{\psi}^J(x) & = \bar{c}(x)\gamma_\alpha c(x), \\
J_5^\alpha(y) & = \bar{u}(y)i\gamma_5 d(y), \\
J_5^{\rho}(x) & = \bar{c}(x)i\gamma_5 c(x), \\
J_5^\eta(y) & = \bar{u}(y)\gamma_\alpha d(y), \\
J_5^{D^*}(x) & = \bar{c}(x)\gamma_\alpha d(x), \\
J_5^D(y) & = \bar{u}(y)i\gamma_5 c(y),
\end{align}

(20)

interpolate the mesons $J/\psi$, $\pi$, $\eta_c$, $\rho$, $D^*$ and $D$, respectively. At the leading order $\mathcal{O}(\alpha_s)$, $\Pi_{3\mu}(p,q) = 0$ at the QCD side.

We insert a complete set of intermediate hadronic states with the same quantum numbers as the current operators into the three-point correlation functions $\Pi_{1\mu}(p,q)$, $\Pi_{2\mu}(p,q)$ and isolate the
ground state contributions to obtain the following results,

\[
\Pi_\alpha^1(p, q) = \frac{f_\pi M_\pi^2 f_{J/\psi} f_{J/\psi}^* \lambda Z \zeta G_{Z, J/\psi}}{m_u + m_d} \frac{1}{(M_{Z_c}^2 - p^2)(M_{J/\psi}^2 - p^2)(M_{\pi}^2 - q^2)} \left( -g_{\alpha\beta} + \frac{p_\alpha p_\beta}{p^2} \right) \\
\left( -g_\beta^\beta + \frac{p_\beta p_\beta}{p^2} \right) + \cdots
\]

\[
= \left\{ \frac{f_\pi M_\pi^2 f_{J/\psi} f_{J/\psi}^* \lambda Z \zeta G_{Z, J/\psi}}{m_u + m_d} \frac{1}{(M_{Z_c}^2 - p^2)(M_{J/\psi}^2 - p^2)(M_{\pi}^2 - q^2)} \\
+ \frac{1}{(M_{Z_c}^2 - p^2)(M_{J/\psi}^2 - p^2)} \int_{0}^{\infty} dt \rho_{Z, \pi}(p^2, t, p^2) \\
+ \frac{1}{(M_{Z_c}^2 - p^2)(M_{J/\psi}^2 - p^2)} \int_{0}^{\infty} dt \rho_{Z, J/\psi}(t, q^2, p^2) + \cdots \right\} (g_{\alpha\mu} + \cdots) + \cdots
\]

\[
(23)
\]

\[
\Pi_\alpha^2(p, q) = \frac{f_\eta M_\eta^2 f_\rho f_\pi \lambda Z \zeta G_{Z, J/\psi}}{2m_\eta} \frac{1}{(M_{Z_c}^2 - p^2)(M_{\eta_c}^2 - p^2)(M_{\pi}^2 - q^2)} \left( -g_{\alpha\beta} + \frac{q_{\alpha} q_{\beta}}{q^2} \right) \\
\left( -g_\beta^\beta + \frac{p_\beta p_\beta}{p^2} \right) + \cdots
\]

\[
= \left\{ \frac{f_{\eta} M_{\eta}^2 f_{\rho} f_{\pi} \lambda Z \zeta G_{Z, J/\psi}}{2m_\eta} \frac{1}{(M_{Z_c}^2 - p^2)(M_{\eta_c}^2 - p^2)(M_{\pi}^2 - q^2)} \\
+ \frac{1}{(M_{Z_c}^2 - p^2)(M_{\eta_c}^2 - p^2)} \int_{0}^{\infty} dt \rho_{Z, \eta_c}(p^2, t, p^2) \\
+ \frac{1}{(M_{Z_c}^2 - p^2)(M_{\eta_c}^2 - p^2)} \int_{0}^{\infty} dt \rho_{Z, \eta_c}(t, q^2, p^2) + \cdots \right\} (g_{\alpha\mu} + \cdots) + \cdots
\]

\[
(24)
\]

where \( p' = p + q \), the \( f_{J/\psi}, f_{\eta_c}, f_\rho \) and \( f_\pi \) are the decay constants of the mesons \( J/\psi, \eta_c, \rho \) and \( \pi \), respectively, the \( G_{Z, J/\psi} \) and \( G_{Z, \eta_c, \rho} \) are the hadronic coupling constants. In this article, we choose the tensor \( g_{\alpha\mu} \) to study the coupling constants \( G_{Z, J/\psi} \) and \( G_{Z, \eta_c, \rho} \).
In the following, we write down the definitions,

\[
\begin{align*}
(0|J_{\alpha}^{J/\psi}(0)|J/\psi(p)) &= f_{J/\psi}M_{J/\psi}\xi_\alpha, \\
(0|J_\rho^{2}(0)|\rho(q)) &= f_{\rho}M_{\rho}\varepsilon_\alpha, \\
(0|J_{\pi}^{2}(0)|\pi_c(p)) &= \frac{f_{\pi}M_{\pi}^{2}}{2m_c}, \\
(0|J_{\pi}^{2}(0)|\pi(q)) &= \frac{f_{\pi}M_{\pi}^{2}}{m_u + m_d}, \\
(J/\psi(p)\pi(q)|Z_c(p')) &= i\xi^*(p)\cdot(\zeta(p')G_{Z_c, J/\psi\pi}), \\
(\eta_c(p)\rho(q)|Z_c(p')) &= i\varepsilon^*(q)\cdot(\zeta(p')G_{Z_c, \eta_c, \rho}),
\end{align*}
\]

(25)

the \( \xi, \zeta \) and \( \varepsilon \) are polarization vectors of the \( J/\psi, Z_c(4200) \) and \( \rho \), respectively. The four unknown functions \( \rho_{Z_{c_{\pi}}}(p^2, t, p'^2), \rho_{Z_{c_{J/\psi}}}(t, q^2, p'^2), \rho_{Z_{c_{\rho}}}(p^2, t, p'^2) \) and \( \rho_{Z_{c_{\eta_c}}}(t, q^2, p'^2) \) have complex dependence on the transitions between the ground states and the high resonances or the continuum states. We introduce the notations \( C_{Z_{c_{\pi}}}, C_{Z_{c_{J/\psi}}}, C_{Z_{c_{\rho}}} \) and \( C_{Z_{c_{\eta_c}}} \) to parameterize the net effects,

\[
\begin{align*}
C_{Z_{c_{\pi}}} &= \int_{s_0}^{\infty} dt \rho_{Z_{c_{\pi}}}(p^2, t, p'^2) \frac{t - q^2}{t - q^2}, \\
C_{Z_{c_{J/\psi}}} &= \int_{s_0}^{\infty} dt \rho_{Z_{c_{J/\psi}}}(t, q^2, p'^2) \frac{t - p^2}{t - p^2}, \\
C_{Z_{c_{\rho}}} &= \int_{s_0}^{\infty} dt \rho_{Z_{c_{\rho}}}(p^2, t, p'^2) \frac{t - q^2}{t - q^2}, \\
C_{Z_{c_{\eta_c}}} &= \int_{s_0}^{\infty} dt \rho_{Z_{c_{\eta_c}}}(t, q^2, p'^2) \frac{t - p^2}{t - p^2}.
\end{align*}
\]

(27)

Some terms parameterizing the transitions between the ground states and the high resonances are not suppressed compared to the ground state contributions in the limit \( M_{\pi}^2 = 0, M_{\rho}^2 = 0 \) and \( q^2 \to 0 \), as

\[
\frac{C_{Z_{c_{\pi}}}}{(M_{Z_{c_{\pi}}}^2 - p^2)(M_{J/\psi}^2 - p'^2)} + \frac{C_{Z_{c_{J/\psi}}}}{(M_{Z_{c_{J/\psi}}}^2 - p^2)(M_{Z_{c_{\rho}}}^2 - p'^2)} \to \frac{C_{Z_{c_{J/\psi}}}}{(M_{Z_{c_{J/\psi}}}^2 - p'^2)(-q^2)},
\]

\[
\frac{C_{Z_{c_{\rho}}}}{(M_{Z_{c_{\rho}}}^2 - p^2)(M_{Z_{c_{\eta_c}}}^2 - p'^2)} \to \frac{C_{Z_{c_{\eta_c}}}}{(M_{Z_{c_{\eta_c}}}^2 - p'^2)(-q^2)},
\]

(28)

we should take them into account. In numerical calculations, we smear the dependencies of the \( C_{Z_{c_{\pi}}}, C_{Z_{c_{J/\psi}}}, C_{Z_{c_{\rho}}}, \) and \( C_{Z_{c_{\eta_c}}} \) on the variables \( p^2, p'^2, q^2 \), take the \( C_{Z_{c_{\pi}}}, C_{Z_{c_{J/\psi}}}, C_{Z_{c_{\rho}}}, \) and \( C_{Z_{c_{\eta_c}}} \) as free parameters, and choose the suitable values to eliminate the contaminations to obtain the stable sum rules with the variations of the Borel parameters [25].

We carry out the operator product expansion up to the vacuum condensates of dimension 5 and neglect the tiny contribution of the gluon condensate, one can see Fig.2 as an example. The double Borel transformed QCD sum rules converge much faster than the single Borel transformed QCD sum rules at the operator product expansion side. We obtain the QCD spectral densities through dispersion relation, take the quark-hadron duality below the continuum thresholds, then set \( p'^2 = p^2 \) and take the double Borel transforms with respect to the variable \( P^2 = -p^2 \) and
\( Q^2 = -q^2 \) respectively to obtain the following QCD sum rules,

\[
\frac{f_f M_f^2 f_{J/\psi} M_{J/\psi} \lambda_{Z_c} G_{Z_c J/\psi \pi}}{m_u + m_d} \left[ \frac{1}{M_{Z_c}^2 - M_{J/\psi}^2} \exp\left( -\frac{M_{J/\psi}^2}{T_1^2} \right) - \exp\left( -\frac{M_{Z_c}^2}{T_1^2} \right) \right] \exp\left( -\frac{M_{Z_c}^2}{T_2^2} \right) + C_{Z_c J/\psi} \exp\left( -\frac{M_{Z_c}^2}{T_1^2} - \frac{M_{Z_c}^2}{T_2^2} \right) = \frac{1}{12 \sqrt{2} \pi^4} \int_{4m_c^2}^{s_\eta} ds \int_{0}^{s_\eta} du (s + 2m_c^2) \sqrt{1 - \frac{4m_c^2}{s}} \exp\left( -\frac{s}{T_1^2} \right),
\]

(29)

\[
\frac{f_{\eta, M_{\eta}} M_{\eta} f_{\rho} M_{\rho} \lambda_{Z_c} G_{Z_c \eta, \rho}}{2m_c} \left[ \frac{1}{M_{Z_c}^2 - M_{\eta}^2} \exp\left( -\frac{M_{\eta}^2}{T_1^2} \right) - \exp\left( -\frac{M_{Z_c}^2}{T_1^2} \right) \right] \exp\left( -\frac{M_{Z_c}^2}{T_2^2} \right) + C_{Z_c \eta} \exp\left( -\frac{M_{Z_c}^2}{T_1^2} - \frac{M_{Z_c}^2}{T_2^2} \right) = \frac{1}{12 \sqrt{2} \pi^4} \int_{4m_c^2}^{s_\rho} ds \int_{0}^{s_\rho} du (s + 2m_c^2) \sqrt{1 - \frac{4m_c^2}{s}} \exp\left( -\frac{s}{T_1^2} - \frac{u}{T_2^2} \right) + m_c \langle \bar{q}q, \sigma G_q \rangle \int_{4m_c^2}^{s_\eta} ds \sqrt{1 - \frac{4m_c^2}{s}} \exp\left( -\frac{s}{T_1^2} \right),
\]

(30)

where the \( s_\eta^2, s_\rho^2 \) and \( s_\rho^0 \) are the continuum threshold parameters for the \( Z_c(4200) \), \( \pi \) and \( \rho \), respectively.

The hadronic parameters are taken as \( M_z = 0.13957 \) GeV, \( M_{\rho} = 0.77526 \) GeV, \( M_{J/\psi} = 3.0969 \) GeV, \( M_{\eta, \rho} = 2.9836 \) GeV [24], \( f_\pi = 0.130 \) GeV, \( f_\rho = 0.215 \) GeV [26], \( f_{J/\psi} = 0.418 \) GeV, \( f_{\eta, \rho} = 0.387 \) GeV [27], \( \sqrt{s_\eta^2} = 0.85 \) GeV, \( \sqrt{s_\rho^2} = 1.3 \) GeV [26], \( \sqrt{s_\eta^0} = 4.7 \) GeV, \( \lambda_{Z_c} = 5.25 \times 10^{-2} \) GeV \( ^5 / \) GeV, \( T_1^2 = (3.0 - 3.4) \) GeV \( ^2 \) (present work), \( T_2^2 = (0.8 - 1.2) \) GeV \( ^2 \) [26], \( f_\pi M_{Z_c}^2/(m_u + m_d) = -2 \langle \bar{q}q \rangle / f_\pi \) from the Gell-Mann-Oakes-Renner relation. The unknown parameters are chosen as \( C_{Z_c J/\psi} = 0.01 \) GeV \( ^5 / \) and \( C_{Z_c \eta, \rho} = 0.09 \) GeV \( ^5 / \) to obtain platforms in the Borel windows \( T_1^2 = (3.0 - 3.4) \) GeV \( ^2 \). The parameters at the QCD side are chosen as the same in the two-point QCD sum rules for the \( Z_c(4200) \). Then it is easy to obtain the values of the hadronic coupling constants,

\[
G_{Z_c J/\psi \pi} = 3.34 \pm 0.07 \pm 0.25 \) GeV,
\]

\[
G_{Z_c \eta, \rho} = 11.31 \pm 0.07 \pm 1.06 \) GeV,
\]

(31)

where the uncertainties originate from the Borel parameters \( T_1^2 \) and \( T_2^2 \), respectively, see Figs.5-6. As the largest uncertainties originate from the Borel parameter \( T_2^2 \), we neglect the uncertainties of the parameters other than the Borel parameters. The uncertainties of the \( G_{Z_c J/\psi \pi} \) and \( G_{Z_c \eta, \rho} \), lead to the uncertainties \( \delta \Gamma(Z_c^+(4200) \rightarrow J/\psi \pi^+) / \Gamma(Z_c^+(4200) \rightarrow J/\psi \pi^+) = 2\delta G_{Z_c J/\psi \pi} / G_{Z_c J/\psi \pi} \approx 8\% \) and \( \delta \Gamma(Z_c^+(4200) \rightarrow \eta, \rho^+) / \Gamma(Z_c^+(4200) \rightarrow \eta, \rho^+) = 2\delta G_{Z_c \eta, \rho} / G_{Z_c \eta, \rho} \approx 9\% \).

The central values of the decay widths are

\[
\Gamma(Z_c^+(4200) \rightarrow J/\psi \pi^+) = \frac{p(M_{Z_c}, M_{J/\psi}, M_\pi)}{24 \pi M_{Z_c}^2} \left[ 3 + \frac{p(M_{Z_c}, M_{J/\psi}, M_\pi)^2}{M_{J/\psi}^2} \right]\]

\[
= 24.6 \) MeV,
\]

\[
\Gamma(Z_c^+(4200) \rightarrow \eta, \rho^+) = \frac{p(M_{Z_c}, M_{\eta, \rho}, M_\rho)}{24 \pi M_{Z_c}^2} \left[ 3 + \frac{p(M_{Z_c}, M_{\eta, \rho}, M_\rho)^2}{M_\rho^2} \right]\]

\[
= 309.1 \) MeV,
\]

(32)

where \( p(a, b, c) = \frac{\sqrt{(a^2 - (b+c)^2)[a^2 - (b-c)^2]}}{2a} \). If we saturate the width of the \( Z_c(4200) \) with the strong decays to \( J/\psi \) and \( \eta, \rho \), then \( \Gamma(Z_c(4200)) \approx 334 \) MeV, which is consistent with the experimental
Figure 5: The coupling constants $G_{Z_c J/\psi\pi}$ (A) and $G_{Z_c \eta_c \rho}$ (B) with variations of the Borel parameters $T_1^2$.

Figure 6: The coupling constants $G_{Z_c J/\psi\pi}$ (A) and $G_{Z_c \eta_c \rho}$ (B) with variations of the Borel parameters $T_2^2$. 
value $\Gamma_{Z_c(4200)} = 370^{+70+70}_{-70-122}$ MeV from the Belle collaboration [1], the present calculations support assigning the $Z_c(4200)$ to be the $8 \otimes 8$ type axial-vector molecule-like state. Due to the special structure of the $Z_c(4200)$, the decays to the final states $D\bar{D}$ and $D^*\bar{D}$ can only take place through re-scattering mechanism $Z_c(4200) \rightarrow J/\psi\pi$, $\eta_c\rho \rightarrow D\bar{D}$, $D^*\bar{D}$, the decay widths $\Gamma(Z_c(4200) \rightarrow DD^*, D^*D)$ are expected to be small.

4 The masses of the $3 \otimes 3$ type axial-vector tetraquark states

In the following, we write down the two-point correlation function $\Pi_{\mu\nu}(p)$ in the QCD sum rules,

$$\Pi_{\mu\nu}(p) = i \int d^4x e^{ipx} \langle 0| T \left\{ \eta_\mu(x) \eta_\nu^*(0) \right\} |0\rangle,$$  \hspace{1cm} (33)

$$\eta_\mu(x) = \frac{\epsilon^{ijk}\epsilon^{imn}}{\sqrt{2}} \left\{ w^j(x) C\gamma_5 c^k(x) d^m(x) \gamma_\mu C d^n(x) - w^j(x) C\gamma_\mu c^k(x) d^m(x) \gamma_5 C d^n(x) \right\} , \hspace{1cm} (34)$$

the $i, j, k, m, n$ are color indexes, the $C$ is the charge conjugation matrix. We choose the current $\eta_\mu(x)$ to interpolate the $3 \otimes 3$ type axial-vector tetraquark states. Under charge conjugation transform $\hat{C}$, the current $\eta_\mu(x)$ has the property,

$$\hat{C}\eta_\mu(x)\hat{C}^{-1} = -\eta_\mu(x) |_{\alpha\neq\alpha} . \hspace{1cm} (35)$$

We carry out the operator product expansion up to the vacuum condensates of dimension 10 and obtain the correlation function at the QCD side,

$$\Pi_{\mu\nu}(p) = \int_{4m_0^2}^{\infty} ds \frac{\rho(s)}{s - p^2} \left( -g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2} \right) + \Pi_0(p^2) \frac{p_\mu p_\nu}{p^2} , \hspace{1cm} (36)$$

the expression of the QCD spectral density $\rho(s)$ is shown explicitly in Ref.[17], the component $\Pi_0(p^2)$ is irrelevant in the present analysis.

In case 1, the $Z_c(3900)$ and $Z(4430)$ are the ground state and the first radial excited state of the $3 \otimes 3$ type axial-vector tetraquark states, respectively, the $Z_c(4200)$ is not the $3 \otimes 3$ type axial-vector tetraquark state, then the current couples potentially to the $Z_c(3900)$ and $Z(4430)$, $\langle 0|\eta_\mu(0)|Z_c(3900)/Z(4430)\rangle = \lambda_{Z_c(3900)/Z(4430)}\epsilon_\mu$, where the $\epsilon_\mu$ are the polarization vectors of the $Z_c(3900)$ and $Z(4430)$. Now we retain the ground state and the first radial excited state and write down the QCD sum rule [12],

$$\lambda_{Z_c(3900)}^2 \exp \left( -\frac{M_{Z_c(3900)}^2}{T^2} \right) + \lambda_{Z(4430)}^2 \exp \left( -\frac{M_{Z(4430)}^2}{T^2} \right) = \int_{4m_0^2}^{s_0} ds \rho(s) \exp \left( -\frac{s}{T^2} \right) . \hspace{1cm} (37)$$

We differentiate Eq.(37) with respect to $\tau = \frac{s}{T^2}$ and obtain three additional QCD sum rules,

$$\lambda_{Z_c(3900)}^2 M_{Z_c(3900)}^2 \exp \left( -\frac{M_{Z_c(3900)}^2}{T^2} \right) + \lambda_{Z(4430)}^2 M_{Z(4430)}^2 \exp \left( -\frac{M_{Z(4430)}^2}{T^2} \right)$$

$$= \int_{4m_0^2}^{s_0} ds \rho(s) \exp \left( -\frac{s}{T^2} \right) , \hspace{1cm} (38)$$

$$\lambda_{Z_c(3900)}^2 M_{Z_c(3900)}^4 \exp \left( -\frac{M_{Z_c(3900)}^2}{T^2} \right) + \lambda_{Z(4430)}^2 M_{Z(4430)}^4 \exp \left( -\frac{M_{Z(4430)}^2}{T^2} \right)$$

$$= \int_{4m_0^2}^{s_0} ds \rho(s) \exp \left( -\frac{s}{T^2} \right) , \hspace{1cm} (39)$$
We differentiate Eq.(42) with respect to $\tau$

$$Z$$

In Fig.7, we plot the mass of the $\mu$ scales

We differentiate Eq.(45) with respect to $\tau$

and $Z$

We solve the coupled equations consistently and obtain the values of the masses of the $Z_c(3900)$ and $Z(4430)$,

$$M_{Z_c(3900)} = 3.91^{+0.21}_{-0.17} \text{ GeV}, \text{ Experimental value } 3899.0 \pm 3.6 \pm 4.9 \text{ MeV} [5],
M_{Z(4430)} = 4.51^{+0.17}_{-0.09} \text{ GeV}, \text{ Experimental value } 4475 \pm 7_{-25}^{+15} \text{ MeV} [2],$$

which favors assigning $Z_c(3900)$ and $Z(4430)$ to be the ground state and the first radial excited state of the $3 \otimes 3$ type axial-vector tetraquark states, respectively. For the technical details, one can consult Ref.[12]. We can assign the $Z_c(4200)$ to be the $8 \otimes 8$ type axial-vector molecule-like state, or it is possible to assign the $Z_c(4200)$ to be the $8 \otimes 8$ type axial-vector molecule-like state.

In case II, the $Z_c(4200)$ is the ground state of the $3 \otimes 3$ type axial-vector tetraquark state, the $Z_c(3900)$ and $Z(4430)$ are not the $3 \otimes 3$ type axial-vector tetraquark states, then the current couples potentially to the $Z_c(4200)$, $\langle 0|\eta(0)|Z_c(4200)\rangle = \lambda_{Z_c(4200)}\epsilon_\mu$, where the $\epsilon_\mu$ is the polarization vector of the $Z_c(4200)$. Now we write down the QCD sum rule,

$$\lambda^2_{Z_c(4200)} \exp \left(-\frac{M^2_{Z_c(4200)}}{T^2}\right) = \int_{4m^2}^{s_0} ds \rho(s) \exp \left(-\frac{s}{T^2}\right).$$

We differentiate Eq.(42) with respect to $\tau = \frac{1}{T^2}$, then eliminate the pole residue $\lambda_{Z_c(4200)}$ to obtain the QCD sum rules for the mass,

$$M^2_{Z_c(4200)} = \frac{\int_{4m^2}^{s_0} ds \left(-\frac{d}{ds}\right) \rho(s)e^{-\tau s}}{\int_{4m^2}^{s_0} d\rho(s)e^{-\tau s}}.$$

In Fig.7, we plot the mass of the $Z_c(4200)$ with variations of the Borel parameters $T^2$ and energy scales $\mu$ for the threshold parameter $\sqrt{s_0} = 4.7 \text{ GeV}$. From the figure, we can see that the mass decreases monotonously with increase of the energy scales, the experimental value $M_{Z_c(4200)} = 4196^{+31}_{-20}^{+17}_{-13} \text{ MeV} [1]$ can be reproduced approximately at the energy scales $\mu = (1.1 - 1.4) \text{ GeV}$. If the $Z_c(4200)$ is the ground state of the $3 \oplus 3$ type axial-vector tetraquark state, we have to assign the $Z_c(3900)$ to be the $1 \otimes 1$ type molecular state [19],

$$Z_c(3900/3885) = \frac{1}{\sqrt{2}} \left(D\overline{D} + D^*\overline{D}\right),$$

however, it is odd to assign the $Z(4430)$ to be the excited $1 \otimes 1$ type molecular state.

In case III, the $Z(4430)$ is the ground state of the $3 \otimes 3$ type axial-vector tetraquark state, the $Z_c(3900)$ and $Z(4430)$ are not the $3 \otimes 3$ type axial-vector tetraquark states, then the current couples potentially to the $Z(4430)$, $\langle 0|\eta(0)|Z(4430)\rangle = \lambda_{Z(4430)}\epsilon_\mu$, where the $\epsilon_\mu$ is the polarization vector of the $Z(4430)$. Now we write down the QCD sum rule,

$$\lambda^2_{Z(4430)} \exp \left(-\frac{M^2_{Z(4430)}}{T^2}\right) = \int_{4m^2}^{s_0} ds \rho(s) \exp \left(-\frac{s}{T^2}\right).$$

We differentiate Eq.(45) with respect to $\tau = \frac{1}{T^2}$, eliminate the pole residue $\lambda_{Z(4430)}$ to obtain the QCD sum rules for the mass,

$$M^2_{Z(4430)} = \frac{\int_{4m^2}^{s_0} ds \left(-\frac{d}{ds}\right) \rho(s)e^{-\tau s}}{\int_{4m^2}^{s_0} d\rho(s)e^{-\tau s}}.$$
In Fig. 8, we plot the mass of the $Z(4430)$ with variations of the Borel parameters $T^2$ and energy scales $\mu$ for the threshold parameter $\sqrt{s_0} = 5.0$ GeV. From the figure, we can see that the experimental value $M_{Z(4430)} = 4475 \pm 7_{-25}^{+15}$ MeV [4] can be reproduced approximately at the energy scale $\mu = 1$ GeV [13]. At the energy scales $\mu > 1$ GeV, the predicted mass $M_{Z(4430)}$ is much smaller than 4475 MeV, assigning the $Z(4430)$ to be the ground state of the $3 \otimes 3$ type axial-vector tetraquark state is not favored.

5 Conclusion

In this article, we assume the $Z_c(4200)$ as the $8 \otimes 8$ type axial-vector molecule-like state, and construct the $8 \otimes 8$ type axial-vector current to study its mass and width with the QCD sum rules. The numerical result supports assigning the $Z_c(4200)$ to be the $8 \otimes 8$ type molecule-like state with $J^{PC} = 1^{+-}$. Furthermore, we discuss the possible assignments of the $Z_c(3900)$, $Z_c(4200)$ and $Z(4430)$ to be the $3 \otimes 3$ type tetraquark states with $J^{PC} = 1^{+-}$.

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Figure 8: The mass of the $Z(4430)$ with variations of the Borel parameters $T^2$ and energy scales $\mu$, where the horizontal line denotes the experimental value.

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