Axion Model with Intermediate Scale Fermionic Dark Matter

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Abstract

We investigate a non-supersymmetric $SO(10) \times U(1)_{\text{PQ}}$ axion model in which the spontaneous breaking of $U(1)_{\text{PQ}}$ occurs after inflation, and the axion domain wall problem is resolved by employing the Lazarides-Shafi mechanism. This requires the introduction of two fermion 10-plets, such that the surviving discrete symmetry from the explicit $U(1)_{\text{PQ}}$ breaking by QCD instantons is reduced from $Z_{12}$ to $Z_4$, where $Z_4$ is precisely the center of $SO(10)$. An unbroken $Z_2$ subgroup of $Z_4$ yields intermediate scale topologically stable strings, as well as a stable electroweak doublet non-thermal dark matter candidate from the fermion 10-plets with mass comparable to or somewhat smaller than the axion decay constant $f_a$. We present an explicit realization with inflation taken into account and which also incorporates non-thermal leptogenesis. The fermion dark matter mass lies in the $3 \times 10^8 - 10^{10}$ GeV range and its contribution to the relic dark matter abundance can be comparable to that from the axion.
1 Introduction

An elegant resolution of the strong CP problem is provided by the Peccei-Quinn (PQ) mechanism [1], which also predicts the existence of axion, a compelling dark matter (DM) candidate [2]. In a relatively simple but realistic non-supersymmetric $SO(10) \times U(1)_{PQ}$ axion model constructed sometime ago [3], the well-known axion domain wall problem [4] is resolved without invoking inflation through implementation of the Lazarides-Shafi mechanism [5]. This is achieved by introducing two 10-plets of fermions carrying appropriate charges under the $U(1)_{PQ}$ symmetry [1], such that the residual discrete symmetry from the explicit $U(1)_{PQ}$ breaking by the QCD instantons coincides with the center of $SO(10)$. This construction therefore allows one to implement the spontaneous breaking of $U(1)_{PQ}$ in a post-inflationary phase. Among other things the model evades some thorny issues such as the isocurvature problem [6] that appears if $U(1)_{PQ}$ breaks during inflation. For some recent papers on $U(1)_{PQ}$ breaking during inflation see Ref. [7].

The model in Ref. [3] employs only tensor representations in order to break $SO(10)$ via at least one intermediate stage to the Standard Model (SM) gauge group, and subsequently to $SU(3)_c \times U(1)_{em}$. This means that a discrete $Z_2$ subgroup of $Z_4$, the center of $SO(10)$, remains unbroken [8], which has some important consequences. Firstly, there exist non-superconducting topologically stable cosmic strings which, depending on their mass scale, may survive inflation. Secondly, in the framework of supersymmetry, the leftover $Z_2$ symmetry is precisely matter parity which, among other things, ensures that the lightest supersymmetric particle is stable. Finally, in the non-supersymmetric $SO(10) \times U(1)_{PQ}$ model under discussion, the presence of the unbroken $Z_2$ symmetry [8], offers a new DM particle candidate arising from the 10-plet fermions whose mass is related to the intermediate scale of the PQ symmetry breaking. For an earlier example of intermediate scale DM particles coexisting with axions see Ref. [9]; for superheavy DM (wimpzillas) see Ref. [10]. It should be noted that this unbroken $Z_2$ symmetry [8] has been employed [11], in recent years, to guarantee DM stability in non-supersymmetric $SO(10)$ models. The proposed scenarios, though, are very different from ours with the DM particle masses in the TeV range or so.

The plan of this paper is as follows. In Sec. 2, we present the salient feature of our $SO(10)$ model coupled to the inflationary scheme of Ref. [12], where the inflaton is a gauge singlet scalar field with a Coleman-Weinberg potential. In Sec. 3 we describe the reheating process following the end of inflation and, in Sec. 4, we examine the possibility that the fermion 10-plets provide a novel DM candidate with intermediate scale mass. Sec. 5 is devoted to the discussion of the generation of the baryon asymmetry of the Universe (BAU) via leptogenesis. Our results are summarized in Sec. 6.

2 The model

We consider the non-supersymmetric $SO(10)$ grand unified theory (GUT) model introduced in Ref. [3]. The model contains a global anomalous PQ symmetry $U(1)_{PQ}$ [1]. The fermion content
consists of the following $SO(10)$ multiplets,

$$
\psi^{(i)}_{16} \ (i = 1, 2, 3), \quad \psi^{(\alpha)}_{10} \ (\alpha = 1, 2),
$$

where the subscripts denote the dimension of the representation. The PQ charges of the $\psi^{(i)}_{16}$'s and $\psi^{(\alpha)}_{10}$'s are 1 and -2 respectively. The scalar Higgs fields are

$$
\varphi_{210} (0), \quad \varphi_{126} (2), \quad \varphi_{45} (4), \quad \varphi_{10} (-2)
$$

with the PQ charges $Q_{\text{PQ}}$ indicated in parentheses. Note that the discrete subgroup of $U(1)_{\text{PQ}}$ left unbroken by the instantons is $Z_4$ and coincides with the center of $SO(10)$. Consequently, the axion domain wall problem [4] does not arise [5] even if the PQ symmetry is broken after the end of inflation. The allowed Yukawa couplings are

$$
\psi^{16} \psi^{16} \varphi^{10}, \quad \psi^{16} \psi^{16} \varphi^{\dagger}_{126}, \quad \psi^{10} \psi^{10} \varphi^{45},
$$

while the Higgs couplings include

$$
\varphi_{210} \varphi^{\dagger}_{126} \varphi^{45}, \quad \varphi_{210} \varphi^{\dagger}_{126} \varphi^{10} \varphi^{45}, \quad \varphi_{210} \varphi^{126} \varphi_{10}.
$$

The $SO(10)$ symmetry breaking to the SM in a non-supersymmetric setting usually proceeds via one or more intermediate stages [13]. For definiteness, we assume the following symmetry breaking chain:

$$
SO(10) \times U(1)_{\text{PQ}} \xrightarrow{\varphi_{210}^{10}} SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times U(1)_{\text{PQ}} \xrightarrow{\varphi_{126}^{10}}
$$

$$
SU(3)_c \times SU(2)_L \times U(1)_Y \times Z_2 \times U(1)_{\text{PQ}}^{B-L} \xrightarrow{f_a} SU(3)_c \times U(1)_{\text{em}} \times Z_2,
$$

where the Higgs fields implementing the breaking chain and the corresponding scales are indicated. The first breaking in Eq. (5) is achieved by the vacuum expectation value (VEV) of $\varphi_{210}$ along its $(1,1,1)$ and $(15,1,1)$ components with respect to the subgroup $G_{\text{PS}} = SU(4)_c \times SU(2)_L \times SU(2)_R$ [14]. The PQ symmetry is left unbroken, but superheavy magnetic monopoles are created during this breaking. As we shall see they are inflated away.

The next breaking at the intermediate scale $M_I$ is achieved by the VEV of $\varphi_{126}$ along its $(10,1,3)$ component. This leaves the $Z_2$ subgroup of $Z_4$ unbroken, leading to the formation of intermediate scale topologically stable $Z_2$ cosmic strings [8], which are not superconducting [15]. The PQ symmetry, however, is not broken at this stage. It is merely rotated to $U(1)'_{\text{PQ}}$, with $Q'_{\text{PQ}} = (5Q_{\text{PQ}} + \chi)/4$ and $\chi = -3(B - L) + 4T^3_R$ is the generator of the $U(1)_\chi$ subgroup of $SO(10)$ which is not contained in $SU(5)$. In order to see this, note that the $\chi$ charges of the usual quarks and leptons in $\psi_{10}$ are $\nu^c (-5)$, $u^c, q, e^c (-1)$, $d^c, l (3)$, and the $\chi$ charges of the color anti-triplets $D^c$ (triplets $\bar{D}^c$) and $SU(2)_L$ doublets $L$ (anti-doublets $\bar{L}$) in $\psi_{10}$ are -2 (2). The $\nu^c \nu^c$-type component of $\varphi_{126}$ has charges $Q_{\text{PQ}} = 2, \chi = -10$, as one can see from the $\chi$ charge of $\nu^c$. So the unbroken PQ symmetry is indeed $U(1)'_{\text{PQ}}$ with the $Q'_{\text{PQ}}$ charges of the...
achieved by the VEVs of QCD instanton effects. Since they couple with D triplets and anti-triplets with r ≳ U of Z altogether.

The VEVs and the masses of the scalar fields are represented by the same symbol as the field. The VEVs and the masses of the scalar fields are where the real canonically normalized component of a scalar field which acquires a VEV is ν various fields given by νc(0), uc, q, ec(1), d, l(2), Dc, L(−3), D, T(−2). The discrete subgroup ZN of U(1)PQ left unbroken by the QCD instantons is Z5, since from q, uc we have 3 × 3 color triplets and anti-triplets with Q′PQ = 1, from d three color anti-triplets with Q′PQ = 2, from Dc two anti-triplets with Q′PQ = −3, and from D two triplets with Q′PQ = −2. Therefore, altogether N = 3 × 3 + 3 × 2 − 2 × 3 − 2 × 2 = 5. This Z5 coincides with the Z5 subgroup of U(1)Y generated by exp[(i2π/5)Y] and, therefore, is not a genuine discrete symmetry. It is instructive to further rotate the PQ symmetry to U(1)PQ′′ with Q′′PQ = (−6Y + Q′PQ)/5. The Q′′PQ charges of the various fields are given by νc, q, d, L, D, L(0), uc, l(1), ec, Dc, T(−1). In this case, N = 3 − 2 = 1 and therefore only the identity element of U(1)PQ′′ is left unbroken by the QCD instanton effects.

The PQ breaking at a scale fα (the axion decay constant), which can be close to M1, is achieved by the VEVs of ϕ45 along its (15,1,1) and (1,1,3) components. These VEVs have Q′PQ = 1 since they couple with DcD and LTL respectively. Consequently, they break spontaneously U(1)PQ to its identity element. The axion strings from the U(1)PQ breaking acquire, at the QCD transition, just one axion domain wall (for walls bounded by strings see Ref. [17]) and, thus, the string-wall network decays [5]. Therefore, the troublesome axion domain wall cosmological problem [4] is avoided. Finally, the electroweak symmetry breaking is achieved by the VEV of a linear combination of the (1,2,2) component of ϕ10 and the (15,2,2) component of ϕ126. Note that the Z2 subgroup of U(1)B − L is neither in U(1)PQ nor the SM gauge group. It is a genuine discrete symmetry which is left unbroken by all the VEVs and, thus, the corresponding intermediate scale cosmic strings [8] can survive until the present time. These strings are not superconducting [15].

Next, we merge our SO(10) GUT model with the inflationary model of Refs. [12, 18], where inflation is driven by a SO(10) × U(1)PQ singlet real scalar field φ with a Coleman-Weinberg potential and with minimal coupling to gravity:

\[ V(\phi) = A\phi^4 \left( \ln \left( \frac{\phi}{M} \right) - \frac{1}{4} \right) + V_0. \]  

(6)

Here M is the VEV of φ and V0 = AM4/4. This model predicts that the tensor-to-scalar ratio r ≥ 0.01 [19]. The important couplings which induce the VEVs of the various scalar fields φθ (θ = 210, 126, 45, 10) as the inflaton acquires its final VEV, starting below M, are

\[ -\frac{c_\theta}{4}\phi^2\phi_\theta + \frac{\alpha_\theta}{4}\phi^4, \]  

(7)

where the real canonically normalized component of a scalar field which acquires a VEV is represented by the same symbol as the field. The VEVs and the masses of the scalar fields are

\[ \langle \phi_\theta \rangle^2 = \frac{c_\theta M^2}{2\alpha_\theta}, \quad m^2_\theta = c_\theta M^2. \]  

(8)

To be more specific, we will consider a particular viable realization of this inflationary scenario which appears in the fourth line of Table 4 in Ref. [20]. In this case, the inflationary scale \( V_0^{1/4} \simeq 1.75 \times 10^{16} \) GeV, \( A \simeq 1.43 \times 10^{-14} \), and the VEV of the inflaton M \( \simeq 7.17 \times 10^{19} \) GeV.
Following Ref. [12], we evaluate the coefficient $A$ of the Coleman-Weinberg potential in Eq. (6) from the radiative corrections arising from the term $(-1/4) c_{210} φ^2 ϕ_{210}^2$, which are the dominant ones. We find

$$A = \frac{210}{32\pi^2} c_{210}^2,$$

which gives $c_{210} \approx 1.47 \times 10^{-7}$ and, from Eq. (8) with $α_{210} = 1/2$, $M_G \equiv \langle ϕ_{210} \rangle \approx 2.75 \times 10^{16}$ GeV. From Eq. (6), we find the inflaton mass $m_φ \approx 1.7 \times 10^{13}$ GeV.

It is important to note that the dimensionless coupling constants of the two scalar quartic interactions in Eq. (11) should be suppressed by at least $m_{45}/M_G$ for the model to remain perturbative. One can see this by considering, for example, the first coupling in this equation with dimensionless coefficient $λ$ and replace $ϕ_{210}$ by its VEV along its $(1,1,1)$ component. We then obtain a trilinear scalar coupling with coefficient of order $λ M_G$. The $(1,1,3)$ component of $ϕ_{45}$ can decay via this trilinear coupling to a pair of electroweak Higgs fields contained in the $(15,2,2)$ component of $ϕ_{126}$ (see Fig. 1). The vertex diagram in this figure can be radiatively corrected by inserting the same trilinear coupling on the two external $ϕ_{126}$ lines with an exchange between them of the $(1,1,3)$ component of $ϕ_{45}$. The radiative correction acquires a factor of order $λ^2 M_G^2/m_{45}^2$ relative to the tree diagram. The requirement of perturbativity then implies that $λ \lesssim m_{45}/M_G$.

Potentially dangerous trilinear scalar couplings can also arise from the first term in Eq. (7). For instance, setting $ϕ = M + δϕ$, we obtain the trilinear scalar coupling $(-1/2)c_{45} M ϕ_45^2$. Considering the decay of the inflaton via this coupling into a pair of $ϕ_{45}$’s and repeating the argument of the previous paragraph, we find that the radiative correction acquires a factor of order $c_{45}^2 M^2/m_ϕ^2 = (m_{45}^2/M m_ϕ)^2$ relative to the tree diagram. However, this factor is much smaller than unity as we will see below. This conclusion remains true even if we replace $ϕ_{45}$ by $ϕ_{126}$ or $ϕ_{10}$. In the case of $ϕ_{210}$, this danger is not encountered since the inflaton mass is much smaller than $M_G$ (see below), and the inflaton decay into two $ϕ_{210}$’s is kinematically blocked. One could also insert $ϕ_θ = \langle ϕ_θ \rangle + δϕ_θ$ in the first term in Eq. (7) to obtain the trilinear coupling $(-1/2)c_θ φ^2(ϕ_θ)\deltaϕ_θ$. The only field which is kinematically allowed to decay to two inflatons via
Figure 2: Diagram for the inflaton decay to a pair of right-handed neutrinos $\nu^c$. Solid lines represent fermions, while dashed lines represent scalar bosons.

This coupling is $\delta \varphi_{210}$. Perturbativity then requires that $c_{210} = M_\nu^2 / M^2 \ll 1$, which is well satisfied. Finally, we should mention that, substituting $\phi = M + \delta \phi$ and $\varphi_\theta = \langle \varphi_\theta \rangle + \delta \varphi_\theta$ in the first term in Eq. (7), we obtain a bilinear mixing term between $\delta \phi$ and $\delta \varphi_\theta$:

$$-M_c \langle \varphi_\theta \rangle \delta \phi \delta \varphi_\theta.$$

(10)

As we will see later, these scalar couplings will be important in the inflaton decay.

3 Reheating

With the quartic scalar couplings in Eq. (4) adequately suppressed as discussed above, the main decay mode of the inflaton $\delta \phi$ will be to a pair of right-handed neutrinos $\nu^c$ via the diagram depicted in Fig. 2. This decay has to be out of equilibrium and to the second heaviest right-handed neutrino $\nu_{c2}$ in order to provide the possibility of generating the BAU via the scenario of non-thermal [21, 22] leptogenesis [23] (for a review see Ref. [24]). The cross sign in Fig. 2 represents the coefficient $-M_{c126} \langle \varphi_{126} \rangle$ of the effective bilinear coupling between $\delta \phi$ and $\delta \varphi_{126}$ (see Eq. (11)), $z$ is the Yukawa coupling constant of $\varphi_{126}$ with $\nu_{c2}$ and $\langle \varphi_{126} \rangle$ as well as the propagating $\delta \varphi_{126}$ lie along the $\nu^c \nu^c$-type component of $\varphi_{126}$. For definiteness, we take $\alpha_{126} = 1/2$. Eq. (8) then implies that

$$\langle \varphi_{126} \rangle = m_{126}, \quad c_{126} = \left( \frac{m_{126}}{M} \right)^2.$$

(11)

An important requirement for the mass $M_2$ of $\nu_{c2}$ is that

$$M_2 \equiv M_{c2} = \langle \varphi_{126} \rangle z \leq \frac{1}{2} m_\phi.$$

(12)

This guarantees that the decay $\delta \phi \to \nu_{c2}^c \nu_{c2}^c$ is kinematically possible.

For $m_{126} \leq m_\phi$, the propagator of $\delta \varphi_{126}$ is dominated by the mass of the inflaton and the decay width is given by

$$\Gamma_{\phi \to \nu^c} \simeq \frac{1}{16\pi} \left( \frac{M_{c126} \langle \varphi_{126} \rangle z}{m_\phi^2} \right)^2 m_\phi = \frac{1}{16\pi} \left( \frac{m_{126}^2 M_2}{M m_\phi^2} \right)^2 m_\phi.$$

(13)
Figure 3: Diagram for the inflaton decay to a pair of electroweak Higgs fields and a SM singlet scalar field. The conventions are as in Fig. 1.

Saturating the inequality in Eq. (12) and taking \( m_{126} = m_\phi \), we find the maximal allowed \( \Gamma_{\phi \rightarrow \nu^c} \) in this case:

\[
\Gamma_{\phi \rightarrow \nu^c} \simeq \frac{1}{16\pi} \left( \frac{m_\phi}{2M} \right)^2 m_\phi \simeq 4.84 \times 10^{-3} \text{ GeV.}
\] (14)

For \( m_{126} \geq m_\phi \), on the other hand, the propagator should be replaced by \( 1/m_{126}^2 \), which gives

\[
\Gamma_{\phi \rightarrow \nu^c} \simeq \frac{1}{16\pi} \left( \frac{M c_{210}}{m_{126}^2} \langle \phi_{126} \rangle \frac{\lambda}{2} \right)^2 m_\phi = \frac{1}{16\pi} \left( \frac{M_2}{M} \right)^2 m_\phi.
\] (15)

This is again maximized at the same value of \( M_2 \) as in the previous case. The corresponding (maximal) reheat temperature turns out to be

\[
T_r \simeq \left( \frac{45}{2\pi^2 g_\ast} \right)^{\frac{1}{4}} \left( \Gamma_{\phi \rightarrow \nu^c} m_P \right)^{\frac{1}{2}} \simeq 4.15 \times 10^7 \text{ GeV},
\] (16)

for an effective number of degrees of freedom \( g_\ast = 106.75 \) corresponding to the SM spectrum (\( m_P \) is the reduced Planck mass). Note that the values of \( \Gamma_{\phi \rightarrow \nu^c} \) in Eq. (14) and \( T_r \) in Eq. (16) are independent of \( \langle \phi_{126} \rangle = m_{126} \) provided that \( \langle \phi_{126} \rangle \geq m_\phi \).

One could alternatively consider the decay of the inflaton to a pair of electroweak Higgs fields and a SM singlet scalar field from \( \varphi_{45} \). The relevant diagram is depicted in Fig. 3 and uses the bilinear coupling between \( \delta \phi \) and \( \delta \varphi_{210} \). The electroweak Higgs fields are contained in the \( \varphi_{126} \) or \( \varphi_{10} \) external lines. The decay width is estimated to be

\[
\Gamma_{\phi \rightarrow h} \simeq \frac{1}{192\pi^3} \left( \frac{M c_{210} \langle \varphi_{210} \rangle \lambda}{m_{210}^2} \right)^2 m_\phi \simeq 422 \lambda^2 \text{ GeV.}
\] (17)

Here \( \lambda \) is the typical coefficient of the quartic scalar couplings in Eq. (11), which, as we have shown, should not exceed \( m_{45}/M_\ast \). As we will see later \( m_{45} \sim 10^{10} - 10^{11} \text{ GeV} \), which implies that \( \Gamma_{\phi \rightarrow h} \lesssim 5.6 \times 10^{-11} \text{ GeV} \ll \Gamma_{\phi \rightarrow \nu^c} \). So this decay mode is much less important than the decay into \( \nu^c \)'s. Note that \( M_2 = m_\phi/2 \simeq 8.5 \times 10^{12} \text{ GeV} \gg T_r \), and thus the out-of-equilibrium
condition for its subsequent decay is well satisfied. It is finally important to keep in mind that we can consider smaller values of $M_2$ with any value of $m_{126} \geq m_\phi$. However, this will reduce $T_r$ by the same proportion.

4 Intermediate Scale Dark Matter

We will now examine the possibility that the two neutral Dirac fermions contained in $\psi_{10}^{(\alpha)}$ ($\alpha = 1, 2$) can provide a new DM particle whose mass is of intermediate scale. The fermion $\psi_{10}^{(\alpha)}$’s are the only fields in the model that are odd under an unbroken $Z_2$ symmetry, arising as a combination of the unbroken $Z_2$ subgroup of the $Z_4$ center of $SO(10)$ and the $Z_2$ fermion number symmetry, under which all fermions are odd. Thus, the lightest components of the $\psi_{10}^{(\alpha)}$’s, which are assumed to be their neutral components, cannot decay, but can only annihilate in pairs, which makes them potentially viable DM candidates. Note that each of the color triplets and anti-triplets in the two fermion 10-plets can decay into the $SU(2)_L$ doublet in the same 10-plet plus SM particles through the exchange of a superheavy gauge boson. We estimate their lifetime to be $\sim 10^{-5}$ sec, and so these decays take place well before Big Bang Nucleosynthesis – compare with the last paper in Ref. [11]. Also, each of the charged members of the $SU(2)_L$ doublets in the fermion 10-plets can decay into the neutral member of the same doublet plus SM particles through the exchange of a $W^+_L$ gauge boson. Even a moderate mass splitting between the members of these doublets induced by loops of SM gauge bosons is enough to insure that these decays are very rapid – see second paper in Ref. [25].

The $(1,2,2)$ components of the two $\psi_{10}$’s can be written as

$$H_{(\alpha)} = \begin{pmatrix} \bar{N}_{(\alpha)} & E_{(\alpha)}^- \\ E_{(\alpha)}^+ & N_{(\alpha)} \end{pmatrix}, \quad \alpha = 1, 2,$$

where $\bar{N}_{(\alpha)}$ and $N_{(\alpha)}$ are electrically neutral. The field $\varphi_{45}$ is antisymmetric and thus couples only with $\psi_{10}^{(1)} \psi_{10}^{(2)}$. Consequently, the coupling of its $(1,1,3)$ component with the $H_{(\alpha)}$’s yields the mass term

$$y \langle \varphi_{45} \rangle \text{Tr} \left( H_{(1)} \epsilon \sigma_3 \epsilon \tilde{H}_{(2)} \right) + \text{h.c.},$$

where $y$ is the Yukawa coupling constant, $\langle \varphi_{45} \rangle$ will, from now on, represent the VEV of the $(1,1,3)$ component of $\varphi_{45}$, $\epsilon$ is the antisymmetric $2 \times 2$ matrix with $\epsilon_{12} = 1$, and tilde denotes transposition. This, in turn, yields the mass term

$$y \langle \varphi_{45} \rangle \left( \bar{N}_{(1)} N_{(2)} - N_{(1)} \bar{N}_{(2)} \right) + \text{h.c.}$$

for the neutral components. At tree level we obtain two neutral Dirac fermions of equal mass $m_{\text{DM}} = y \langle \varphi_{45} \rangle$. This degeneracy is broken through loop corrections (see third paper in Ref. [11]), but the splitting in our case is tiny. Higher dimensional operators may induce a larger splitting which we will not pursue here.
These neutral fermions interact with the $Z$ boson since they belong to electroweak doublets. Therefore, they can scatter off nucleons by exchanging a $Z$ boson in the $t$-channel. The spin-independent cross section is [25]

$$\sigma_{\text{SI}} = \frac{\mu^2}{\pi} \left( \frac{Zf_p + (A-Z)f_n}{A} \right)^2,$$

(21)

where $\mu$ is the reduced mass of the DM particle-nucleon system, which can be approximated by the proton mass $m_p$ for $m_{\text{DM}} \gg m_p$. Also, $Z$ and $A$ are the atomic and mass numbers of the nucleus, and

$$f_p = \frac{g_{\text{DM}}}{m_Z^2} (2g_u + g_d), \quad f_n = \frac{g_{\text{DM}}}{m_Z^2} (g_u + 2g_d).$$

(22)

Here $g_{\text{DM}} = g_Z/2$, $g_u = (1/2 - 4\sin^2\theta_W/3)g_Z$, and $g_d = (-1/2 + 2\sin^2\theta_W/3)g_Z$ with $g_Z = m_Z/\sqrt{2}v$, $v = 174$ GeV. For $Z = 54$ and $A = 131$ corresponding to $^{131}\text{Xe}$ used in the XENON 1T experiment [26], one finds

$$\sigma_{\text{SI}} \simeq 2.85 \times 10^{-12} \text{ GeV}^{-2}.$$  

(23)

The DM in our model can, in general, consist of intermediate scale fermions and axions, with the axion fraction given by $R_a = \Omega_a h^2/\Omega_{\text{DM}} h^2$. Here $\Omega_a h^2$ is the relic axion abundance and $\Omega_{\text{DM}} h^2 \simeq 0.12$ [27] is the total relic DM abundance. In the presence of axions, the current experimental bound on $\sigma_{\text{SI}}$ from the XENON 1T experiment [26] can be written as

$$\sigma_{\text{SI}} \lesssim 2.21 \times 10^{-18} \left( \frac{m_{\text{DM}}}{1 \text{ TeV}} \right) \text{ GeV}^{-2},$$

(24)

which implies that

$$m_{\text{DM}} \gtrsim 1.29 \times 10^9 (1 - R_a) \text{ GeV}. $$

(25)

We see that the XENON 1T constraint on the SI cross section of DM scattering off nuclei requires that the mass of the DM particles is at least of intermediate scale exceeding $T_r$ in Eq. (16). Consequently, thermal DM is excluded and we are led to consider non-thermal production of DM particles with at least intermediate scale masses via the inflaton decay. (Non-thermal superheavy DM particles, called wimpzillas, were previously discussed in Ref. [10].)

We first estimate the relative number density of these DM particles $Y_{\text{DM}} = n_{\text{DM}}/s$ required to reproduce a fraction $(1 - R_a)$ of the present DM abundance $\Omega_{\text{DM}} h^2 \simeq 0.12$ [27] ($n_{\text{DM}}$ is the number density of DM particles and $s$ is the entropy density) using the relation

$$(1 - R_a) \Omega_{\text{DM}} h^2 = \frac{m_{\text{DM}} Y_{\text{DM}} s_0}{\rho_c},$$

(26)

where $s_0 \simeq 2890 \text{ cm}^{-3}$ is the present entropy density and $\rho_c \simeq 1.05 \times 10^{-5}$ GeV cm$^{-3}$ is the present critical density. We obtain

$$m_{\text{DM}} Y_{\text{DM}} \simeq 4.36 \times 10^{-10} (1 - R_a) \text{ GeV}. $$

(27)

From the energy density of the DM fermions at reheating, $\rho_{\text{DM}} = m_{\text{DM}} Y_{\text{DM}} s(T_r)$, we then find that at $T_r$,

$$\frac{\rho_{\text{DM}}}{\rho_c} \simeq \frac{5.81 \times 10^{-10}}{T_r} (1 - R_a),$$

(28)
Figure 4: Diagram for the inflaton decay into DM (doublets in the two 10-plets). The conventions are as in Fig. 2.

where $\rho_r$ is the radiation energy density. Assuming that the total energy of the inflaton at reheating is transferred to $\rho_{DM}$ and $\rho_r$, the inflaton decay width $\Gamma_{\phi \rightarrow DM}$ to a pair of DM fermions should satisfy the requirement $\Gamma_{\phi \rightarrow DM}/\Gamma_{\phi \rightarrow \nu_c} \simeq \rho_{DM}/\rho_r \simeq 1.4 \times 10^{-17} (1 - R_a)$. This yields

$$\Gamma_{\phi \rightarrow DM} \simeq 6.78 \times 10^{-20} (1 - R_a) \text{ GeV}. \quad (29)$$

We should now check whether our model can reproduce this $\Gamma_{\phi \rightarrow DM}$.

The diagram for the inflaton decay to a pair of DM fermions is given in Fig. 4. The cross sign represents the coefficient $-Mc_{45}\langle \varphi_{45} \rangle$ of the bilinear coupling between $\delta\phi$ and $\delta\varphi_{45}$ (see Eq. (10)), and $y$ is the Yukawa coupling constant of $\phi_{45}$ with the two $\psi_{10}$'s. The DM fermions are the neutral components of the $SU(2)_L$ doublets in the two $\psi_{10}$'s. The propagating $\delta\varphi_{45}$ as well as $\langle \varphi_{45} \rangle$ are along the $(1,1,3)$ component of $\varphi_{45}$. The inflaton also decays into a pair of charged fermions contained in the $SU(2)_L$ doublets via the diagram of Fig. 4 and into a color triplet and an anti-triplet contained in the $\psi_{10}$'s via a similar diagram where the propagating $\delta\varphi_{45}$ and the $\langle \varphi_{45} \rangle$ are taken along the $(15,1,1)$ component of $\varphi_{45}$. The VEV of the $(15,1,1)$ component has to be somewhat larger than the VEV of the $(1,1,3)$ component in order for the color (anti-)triplets to be heavier than the doublets and be able to decay into them. However, the required mass difference is much smaller than the (anti-)triplet mass since the decay byproducts are SM particles. For simplicity, we will assume that the VEVs along the $(1,1,3)$ and $(15,1,1)$ components are about equal. As mentioned, the color (anti-)triplet and the charged fermions eventually decay into neutral DM fermions. Therefore, the inflaton decays either into two neutral, or two charged, or two color triplet Dirac fermions with about the same width, and all these particles yield neutral fermions contributing to DM. The decay width of the inflaton into a pair of Dirac fermions should then be multiplied by $\approx 10$ to obtain the total decay width.

For $m_\phi \geq m_{45}$ (see below), the $\delta\varphi_{45}$ propagator is dominated by the inflaton mass $m_\phi$ and the total decay width of the inflaton to a pair of DM fermions is given by

$$\Gamma_{\phi \rightarrow DM} \simeq \frac{10}{16\pi} \left( \frac{Mc_{45}\langle \varphi_{45} \rangle y}{m_\phi^2} \right)^2 m_\phi = \frac{10}{16\pi} \left( \frac{m_{45}^2 m_{DM}}{M m_\phi^2} \right)^2 m_\phi, \quad (30)$$
where \( m_{\text{DM}} = y \langle \varphi_{45} \rangle \). Eq. (29) then yields
\[
m_{45}^2 m_{\text{DM}} \simeq 2.96 \times 10^{30} (1 - R_a)^{\frac{1}{2}} \text{ GeV}^3,
\]
where \( m_{45} \) is the Higgs mass in the \((1,1,3)\) direction of \( \varphi_{45} \), which is greater than \( \langle \varphi_{45} \rangle \), its VEV along its \((1,1,3)\) component (see below). The misalignment angle \( \theta \) lies in the interval \([-\pi, +\pi]\) since, in our case, \( N \), the sum of the \( Q^a_{\text{PQ}} \) charges of all fermion color triplets and anti-triplets, is equal to unity. All \( \theta \)'s in this interval are equally probable. The function \( f(\theta) \) accounts for the anharmonicity of the axion potential, and the average \( \langle \theta^2 f(\theta) \rangle \) is evaluated in the above interval and turns out to be about 8.77. For definiteness, we take \( \alpha_{45} = 1/2 \), which implies \( m_{45} = \langle \varphi_{45} \rangle \). We then substitute \( R_a \) in Eq. (31) by using Eq. (32) and solve the resulting equation to find \( m_{45} = \langle \varphi_{45} \rangle \) for given values of \( m_{\text{DM}} \) and \( f_a \). Recall that the VEV of \( \varphi_{45} \) along the \((15,1,1)\) component should be somewhat greater than its VEV along the \((1,1,3)\) component, so that the color (anti-)triplets in \( \psi_{10} \) can decay into the \( SU(2)_L \) doublets. Consequently, the axion decay constant \( f_a \geq \sqrt{2} \langle \varphi_{45} \rangle \). For definiteness, we have chosen these VEVs to be about equal, which fixes \( f_a \) close to \( \sqrt{2} \langle \varphi_{45} \rangle \). Then, for \( y = 1 \), we obtain
\[
m_{45} = \langle \varphi_{45} \rangle = m_{\text{DM}} \simeq 1.4 \times 10^{10} \text{ GeV}, \quad f_a \simeq 2 \times 10^{10} \text{ GeV}. \tag{33}
\]
DM is composed of 17.6% axions and 82.4% intermediate scale fermions. For \( m_{\text{DM}} = 3 \times 10^9 \text{ GeV} \), we find
\[
m_{45} = \langle \varphi_{45} \rangle \simeq 2.76 \times 10^{10} \text{ GeV}, \quad f_a \simeq 3.9 \times 10^{10} \text{ GeV}, \quad y \simeq 0.11. \tag{34}
\]
DM is made up of 39.2% axions and 60.8% intermediate scale fermions. For \( m_{\text{DM}} = 10^9 \text{ GeV} \), we find
\[
m_{45} = \langle \varphi_{45} \rangle \simeq 4.2 \times 10^{10} \text{ GeV}, \quad f_a \simeq 6 \times 10^{10} \text{ GeV}, \quad y \simeq 2.38 \times 10^{-2}. \tag{35}
\]
DM consists of 63.9% axions and 36.1% intermediate scale fermions. Finally, for \( m_{\text{DM}} = 3 \times 10^8 \text{ GeV} \), we find
\[
m_{45} = \langle \varphi_{45} \rangle \simeq 5.61 \times 10^{10} \text{ GeV}, \quad f_a \simeq 7.93 \times 10^{10} \text{ GeV}, \quad y \simeq 5.35 \times 10^{-3}. \tag{36}
\]
89.6% of DM consists of axions and 10.4% of intermediate scale fermions. These values of \( m_{45} \) clearly satisfy the requirement that the inflaton decay into a pair of electroweak Higgs fields and a SM singlet scalar (see Fig. 3) is subdominant. Also, the values of \( m_{\text{DM}} \) satisfy the requirement from direct detection of DM in Eq. (25) and the kinematic constraint \( m_\phi \geq 2m_{\text{DM}} \) which makes the decay possible. Moreover, \( m_{\text{DM}} \) exceeds the reheat temperature and \( m_{45} \) is smaller than \( m_\phi \).
consistent with our assumption in deriving Eq. (33). We can also differentiate $m_{45}$ and $\langle \varphi_{45} \rangle$ by taking $\alpha_{45} \neq 1/2$, so as to increase $\langle \varphi_{45} \rangle$ and, thus, the axion decay constant $f_a$.

In order to complete the DM discussion, we have to show that the pair annihilation of DM fermions is out of equilibrium at all temperatures smaller than the reheat temperature so that their abundance remains constant. A dominant diagram for this annihilation is shown in Fig. 5. The propagating $\varphi_{45}$ lies along its (1,1,3) direction and the decay products are a pair of electroweak Higgs fields contained in $\varphi_{126}$. The cross section is estimated to be

$$\sigma_{\text{DM}} \simeq \frac{1}{16\pi^2} \left( \frac{y\lambda M_G}{4m_{\text{DM}}^2} \right)^2.$$  \hspace{1cm} (37)

The out-of-equilibrium condition reads as follows:

$$n_{\text{DM}} \sigma_{\text{DM}} \lesssim H,$$  \hspace{1cm} (38)

for all $T \lesssim T_r$ ($H$ is the Hubble parameter). From the Friedmann equation, we find

$$H = \frac{\rho_{\text{c}}^{1/2}}{\sqrt{3} m_P} = \frac{\pi g_s^{1/2} T^2}{3\sqrt{10} m_P},$$  \hspace{1cm} (39)

and Eq. (27) gives

$$n_{\text{DM}} = s Y_{\text{DM}} = \frac{2\pi^2 g^* T^3}{45} \left( 1 - R_a \right) \left( \frac{m_{\text{DM}}}{4.36 \times 10^{-10} \text{ GeV}} \right)^{-1}.$$  \hspace{1cm} (40)

For the case in Eq. (33) and using Eqs. (37), (39), and (40) with the maximal allowed value of $\lambda$ which is of order $m_{45}/M_G$, we see that the condition in Eq. (38) is very well satisfied for all $T \lesssim T_r$. We conclude that the pair annihilation of DM fermions in Fig. 5 is utterly suppressed at all relevant temperatures. We should note that one could instead consider the DM pair annihilation into SM particles via the exchange of a $Z$ boson in the $s$-channel. The
corresponding cross section is of the same order of magnitude as the cross section in Eq. (37) and our conclusion therefore would be the same.

The spontaneous breaking of the PQ symmetry at a scale of order $10^{10} - 10^{11}$ GeV takes place after the end of inflation. Indeed, in the numerical example under consideration, inflation terminates when the inflaton field reaches the value $\phi_e \approx 6.83 \times 10^{19}$ GeV [20]. From Eq. (6), we find that $V(\phi_e)^{1/4} \approx 6.31 \times 10^{15}$ GeV, yielding the Hubble parameter $H_e \approx 9.42 \times 10^{12}$ GeV. The field $\varphi_\theta$ develops a VEV and the corresponding phase transition takes place when $c_g \varphi_\theta^2 / 2 \sim \sigma T_H^2$ [12], where $\sigma \sim 1$ and $T_H = H / 2\pi$ is the Hawking temperature. Consequently, for a phase transition which occurs before the end of inflation we must have $c_g \geq H_e^2 / 2\pi^2 \varphi_e^2 \approx 9.63 \times 10^{-16}$. This implies that the corresponding scale $\langle \varphi_\theta \rangle \gtrsim 2.23 \times 10^{12}$ GeV, which excludes the PQ transition. Therefore, the presence of the two fermion 10-plets which lead to a solution of the axion domain wall problem via the Lazarides-Shafi mechanism [5] is vital. At reheating, the masses of the scalar fields $\varphi_\theta$ acquire temperature corrections which, however, are subdominant compared to the first term in Eq. (7). Indeed, the decaying inflaton oscillates about $M$ and, thus, $c_{45} = (m_{45} / M)^2 \approx 7.78 \times 10^{-20}$ for $m_{45}$ in Eq. (33). Consequently, $c_{45} \varphi^2 / 2 \gg T_r^2$, for $T_r$ in Eq. (16). The PQ symmetry is already broken at reheating and the DM fermions have acquired their masses.

As we previously mentioned, the gauge symmetry breaking at the intermediate scale $M_I$ generates topologically stable $Z_2$ cosmic strings. The dimensionless string tension $G\mu_s$, where $G$ is Newton’s gravitational constant and $\mu_s$ the string tension, i.e. the energy per unit length of the string, is given by

$$G\mu_s \approx \frac{1}{8} \left( \frac{M_I}{m_P} \right)^2.$$  \hspace{1cm} (41)

Here we assumed that these strings are close to the Bogomol’nyi limit of the Abelian Higgs model [30]. A recent pulsar timing array 95% confidence level limit on the dimensionless string tension is $31$

$$G\mu_s \lesssim 1.5 \times 10^{-11},$$  \hspace{1cm} (42)

which holds for strings surviving until the present time. Eq. (42) implies the following upper bound on the intermediate scale

$$M_I \lesssim 2.67 \times 10^{13} \text{ GeV}.$$  \hspace{1cm} (43)

Note that strings corresponding to such intermediate scales are not inflated away as shown in Ref. [15] and, thus, the limit in Eq. (42) applies. These strings are possibly measurable by LISA in the future. Applying the analysis of Ref. [15], we find that, for the strings to be inflated away, the number of e-foldings following their generation should exceed about 68 and, thus, $c_{126} \gtrsim 1.94 \times 10^{-12}$ and

$$M_I \gtrsim 10^{14} \text{ GeV}.$$  \hspace{1cm} (44)

In this case, the cosmic strings are not restricted by Eq. (42). The value of the inflaton field $\phi_I$ at which the intermediate transition takes place is found from the relation $c_{126} \phi_I^2 / 2 \sim T_H^2$ and, thus, $\phi_I \lesssim 1.2 \times 10^{19}$ GeV $\approx 4.9 m_P$. The GUT magnetic monopoles are certainly inflated away since $M_G \gg 10^{14}$ GeV [15].
5 Non-thermal Leptogenesis

The observed BAU $Y_B = n_B/s \approx 8.69 \times 10^{-11}$ [27] can be reproduced in our model via non-thermal leptogenesis [21, 22], i.e. the generation of a primordial lepton asymmetry $Y_L = n_L/s$ [23] at reheating which, at the electroweak transition, is partially converted into the observed BAU via sphaleron effects. For non-supersymmetric SM, $Y_B \approx -0.35 Y_L$. As we have already discussed, the inflaton predominantly decays into a pair of $\nu_2^c$’s, where $\nu_2^c$ is the second heaviest right-handed neutrino with mass $M_2 \approx 8.5 \times 10^{12}$ GeV. The primordial lepton asymmetry will be produced non-thermally [21, 22] via the subsequent out-of-equilibrium decay of this right-handed neutrino into an electroweak Higgs doublet and a lepton or anti-lepton via the exchange of the heaviest $\nu^c$ with mass $M_3 = z\langle \varphi_{126} \rangle$. The relevant one-loop diagrams are both of the vertex and self-energy type [32]. Recall that $\langle \varphi_{126} \rangle$ can have any value greater than or equal to $m_\phi$, and thus $M_2/M_3$ can be adjusted at any value smaller than unity. However, it should not be too small since it will suppress the BAU (see below), but also not too close to unity since the validity of our calculation requires [33] that $M_2 \ll M_3$ and $\Gamma_{\nu_2^c \rightarrow \nu^c} \ll (M_3^2 - M_2^2)/M_2$.

Under these assumptions and considering only the two heavier generations, $Y_B$ can be approximated as [22]

$$Y_B \approx 0.35 \left( \frac{9}{16\pi} \frac{T_i}{m_\phi} \frac{M_2 c^2 s^2 \sin 2\delta (m_3^{D^2} - m_2^{D^2})^2}{v^2 (m_3^{D^2} s^2 + m_2^{D^2} c^2)} \right).$$

(45)

Here $v \approx 174$ GeV, $c = \cos \theta$, $s = \sin \theta$, with $\theta$ and $\delta$ being the rotation angle and phase which diagonalize the Majorana mass matrix of $\nu^c$’s in the basis where the Dirac neutrino mass matrix is diagonal with eigenvalues $m_2^D$ and $m_3^D$. The determinant and trace invariants of the light neutrino mass matrix imply [22] that the neutrino parameters should satisfy the following constraints:

$$m_2 m_3 = \frac{(m_2^D m_3^D)^2}{M_2 M_3},$$

(46)

$$m_2^2 + m_3^2 = \frac{\left( m_2^{D^2} c^2 + m_3^{D^2} s^2 \right)^2}{M_2^2} + \frac{\left( m_3^{D^2} c^2 + m_2^{D^2} s^2 \right)^2}{M_3^2} + \frac{2 (m_3^{D^2} - m_2^{D^2}) c^2 s^2 \cos 2\delta}{M_2 M_3}.$$  

(47)

Here we assume a normal hierarchy of light neutrino masses $m_i$ ($i = 1, 2, 3$) [34], with $m_3 \approx 5.05 \times 10^{-2}$ eV, $m_2 \approx 8.73 \times 10^{-3}$ eV, and $m_1 \approx 0$.

For a rough estimate of a possible solution of the system of Eqs. (45), (46), and (47), we take $c^2 \approx s^2 \approx 1/2$, $\sin 2\delta \approx 1$. Note that the latter choice maximizes $Y_B$. Substituting $Y_B$ with its observed value in Eq. (45), we are left with just three unknown variables $\beta \equiv M_2/M_3$, $m_2^D$, $m_3^D$, and we can solve the system of these three equations to determine them. To this end, we find from Eq. (47) that

$$m_2^{D^2} + m_3^{D^2} \approx 876.36 \left( 1 + \beta^2 \right)^{-1/2} \text{GeV}^2,$$

(48)
while Eq. (46) gives
\[
(m_2^D - m_3^D)^2 \simeq 10^5 (7.68(1 + \beta^2)^{-1} - 0.645\beta^{-1}) \text{ GeV}^4.
\] (49)
Substituting these two equations in Eq. (45), we obtain
\[
0.303 \simeq 7.68\beta(1 + \beta^2)^{-1/2} - 0.645(1 + \beta^2)^{1/2},
\] (50)
which is solved numerically and yields $\beta \simeq 0.125$. This implies that
\[
M_3 \simeq 6.84 \times 10^{13} \text{ GeV}.
\] (51)
From Eqs. (48) and (49), we estimate the Dirac neutrino masses:
\[
m_2^D \simeq 14 \text{ GeV}, \quad m_3^D \simeq 26.2 \text{ GeV}.
\] (52)
Clearly, this is just an example to show that the observed BAU can be generated in our model in accord with the neutrino experimental data. A more complete and accurate calculation including all three generations of neutrinos should be carried out. In any case, since the neutrino Dirac mass matrix has a certain degree of freedom, we believe that more realistic solutions can be found. Note that the requirements $M_2 \ll M_3$ and $\Gamma_{\phi \to \nu^c} \ll (M_2^2 - M_3^2)/M_2$ are well satisfied. Also $M_2 \gg T_\tau$ and so the second heaviest $\nu^c$ decays out of equilibrium to generate the primordial lepton asymmetry.

6 Conclusions

We have explored some predictions of a non-supersymmetric $SO(10) \times U(1)_{\text{PQ}}$ model in which the spontaneous breaking of $U(1)_{\text{PQ}}$ takes place after inflation. A pair of 10-plet fermions with intermediate scale masses comparable to or somewhat smaller than the axion decay constant $f_a$ are introduced in order to evade the axion domain wall problem. The electroweak doublets from these 10-plets provide a novel non-thermal dark matter candidate whose stability is guaranteed by an unbroken $Z_2$ symmetry. We discuss an explicit realization of this scenario by incorporating inflation driven by an $SO(10) \times U(1)_{\text{PQ}}$ singlet scalar field with a Coleman-Weinberg potential. The dark matter fermions have mass on the order of $3 \times 10^8 \sim 10^{10}$ GeV and, in addition, non-thermal leptogenesis is realized. The model also yields topologically stable intermediate mass scale cosmic strings which survive inflation and emit possibly observable gravity waves. Last, but not least, the tensor-to-scalar ratio $r$, a canonical measure of gravity waves generated during inflation, cannot be smaller than 0.01 and therefore should be accessible in the next generation experiments.

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References

[1] R.D. Peccei and H.R. Quinn, Phys. Rev. Lett. 38, 1440 (1977); S. Weinberg, Phys. Rev. Lett. 40, 223 (1978); F. Wilczek, Phys. Rev. Lett. 40, 279 (1978).

[2] J. Preskill, M.B. Wise, and F. Wilczek, Phys. Lett. 120B, 127 (1983); L.F. Abbott and P. Sikivie, Phys. Lett. 120B, 133 (1983); M. Dine and W. Fischler, Phys. Lett. 120B, 137 (1983).

[3] R. Holman, G. Lazarides, and Q. Shafi, Phys. Rev. D 27, 995 (1983).

[4] P. Sikivie, Phys. Rev. Lett. 48, 1156 (1982).

[5] G. Lazarides and Q. Shafi, Phys. Lett. 115B, 21 (1982).

[6] M. Kawasaki, E. Sonomoto, and T.T. Yanagida, Phys. Lett. B 782, 181 (2018) and references therein.

[7] G. Ballesteros, J. Redondo, A. Ringwald, and C. Tamarit, JCAP 08, 001 (2017); G. Lazarides, C. Panagiotakopoulos, and Q. Shafi, Phys. Rev. D 95, 055017 (2017); S.M. Boucenna and Q. Shafi, Phys. Rev. D 97, 075012 (2018); N. Okada, D. Raut, and Q. Shafi, arXiv:1910.14586 [hep-ph].

[8] T.W.B. Kibble, G. Lazarides, and Q. Shafi, Phys. Lett. 113B, 237 (1982).

[9] G. Lazarides and Q. Shafi, Phys. Lett. B 489, 194 (2000).

[10] E.W. Kolb, D.J.H. Chung, and A. Riotto, AIP Conf. Proc. 484, 91 (1999) [hep-ph/9810361].

[11] M. Kadastik, K. Kannike, and M. Raidal, Phys. Rev. D 81, 015002 (2010); Y. Mambrini, N. Nagata, K.A. Olive, J. Quevillon, and J. Zheng, Phys. Rev. D 91, 095010 (2015); S.M. Boucenna, M.B. Krauss, and E. Nardi, Phys. Lett. B 755, 168 (2016); S. Ferrari, T. Hambye, J. Heeck, and M.H.G. Tytgat, Phys. Rev. D 99, 055032 (2019); N. Okada, D. Raut, and Q. Shafi, Eur. Phys. J. C 79, 1036 (2019).

[12] Q. Shafi and A. Vilenkin, Phys. Rev. Lett. 52, 691 (1984).

[13] J. Chakrabortty, R. Maji, and S.F. King Phys.Rev. D 99, no.9, 095008 (2019) and references therein.

[14] J.C. Pati and A. Salam, Phys. Rev. D 8, 1240 (1973), Erratum: Phys. Rev. D 11, 703 (1975).
[15] G. Lazarides and Q. Shafi, J. High Energy Phys. 10, 193 (2019).

[16] E. Witten, Nucl. Phys. B249, 557 (1985).

[17] T.W.B. Kibble, G. Lazarides, and Q. Shafi, Phys. Rev. D 26, 435 (1982); A. Vilenkin and A.E. Everett, Phys. Rev. Lett. 48, 1867 (1982).

[18] G. Lazarides and Q. Shafi, Phys. Lett. 148B, 35 (1984).

[19] Q. Shafi and V.N. Şenoğuz, Phys. Rev. D 73, 127301 (2006); M.U. Rehman, Q. Shafi, and J.R. Wickman, Phys. Rev. D 78, 123516 (2008).

[20] N. Okada, V.N. Şenoğuz, and Q. Shafi, Turk. J. Phys. 40, 150 (2016).

[21] G. Lazarides and Q. Shafi, Phys. Lett. B 258, 305 (1991); G. Lazarides, R.K. Schaefer, and Q. Shafi, Phys. Rev. D 56, 1324 (1997).

[22] G. Lazarides, Q. Shafi, and N.D. Vlachos, Phys. Lett. B 427, 53 (1998); G. Lazarides and N.D. Vlachos, Phys. Lett. B 459, 482 (1999); G. Lazarides, NATO Sci. Ser. II 34, 399 (2001) [hep-ph/0011130].

[23] M. Fukugita and T. Yanagida, Phys. Lett. B 174, 45 (1986).

[24] W. Buchmüller, P. Di Bari, and M. Plüümacher, Annals Phys. 315, 305 (2005).

[25] M.W. Goodman and E. Witten, Phys. Rev. D 31, 3059 (1985); M. Cirelli, N. Fornengo, and A. Strumia, Nucl. Phys. B753, 178 (2006); D.G. Cerdeño and A.M. Green, in Bertone, G. (ed.): Particle dark matter, pp. 347-369 [arXiv:1002.1912]; T.M. Undagoitia and L. Rauch, J. Phys. G 43 013001 (2016); J. Ellis, N. Nagata, and K.A. Olive, Eur. Phys. J. C 78, 569 (2018); M. Schumann, J. Phys. G 46, 103003 (2019).

[26] XENON Collaboration (E. Aprile et al.), Phys. Rev. Lett. 121, 111302 (2018).

[27] Planck Collaboration (N. Aghanim et al.), arXiv:1807.06209 [astro-ph.CO].

[28] L. Visinelli and P. Gondolo, Phys. Rev. D 80, 035024 (2009).

[29] K. Dimopoulos, G. Lazarides, D. Lyth, and R. Ruiz de Austri, J. High Energy Phys. 05, 057 (2003).

[30] N. Bevis, M. Hindmarsh, M. Kunz, and J. Urrestilla, Phys. Rev. D 75, 065015 (2007); Phys. Rev. D 76, 043005 (2007); Phys. Rev. Lett. 100, 021301 (2008).

[31] J.J. Blanco-Pillado, K.D. Olum, and X. Siemens, Phys. Lett. B 778, 392 (2018).

[32] L. Covi, E. Roulet, and F. Vissani, Phys. Lett. B 384, 169 (1996).

[33] A. Pilaftsis, Phys. Rev. D 56, 5431 (1997).

[34] P.F. de Salas, D.V. Forero, C.A. Ternes, M. Tórtola, and J.W.F. Valle, Phys. Lett. B 782, 633 (2018).