Toroidal dipole moment of the LSP in the cMSSM

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Abstract. We study the toroidal dipole moment of the lightest neutralino in the constrained Minimal Supersymmetric Standard Model. The toroidal dipole moment is the only electromagnetic property of the neutralino. Since the neutralino is the LSP in many versions of the MSSM and therefore a candidate for dark matter, its characterization through its electromagnetic properties is important both for particle physics and for cosmology. We perform a scan in the parameter space of the cMSSM and find that the toroidal dipole moment is different from zero, albeit very small, in all the parameter space, and reaches a value around $10^{-3}$ GeV$^{-2}$ in a particular region of the parameter space, well below experimental bounds.

1. Introduction

One of the best motivated extensions of the Standard Model (SM) is the Minimal Supersymmetric Standard Model (MSSM), since, besides giving a solution to the hierarchy problem, provides us with a good candidate for cold dark matter (CDM), namely, the lightest neutralino.

There are currently several experiments under way, and more planned for the future for direct and indirect detection of dark matter (DM). If detected, it will be necessary to discriminate between different candidates. To this end, it will be important to characterize as much as possible the different candidates. The neutralino is at present one of the best candidates for DM, and its electroweak properties can give us some insight into its nature. Because it is neutral, these properties appear only radiatively.

One of the least studied electromagnetic properties of a particle is the Toroidal Dipole Moment (TDM), which is directly related to the anapole moment. The anapole moment corresponds to a $T$ invariant interaction, which is \textit{C} and \textit{P} non-invariant [1]. The electromagnetic vertex of a particle can be expressed in a multipole parametrization, including the toroidal moments, which provides a one to one correspondence between the form factors and the multipole moments.

Pospelov and ter Veldhuis have obtained an upper limit for the anapole moment of WIMPS [2], using results from the DAMA and CDMS experiments [3, 4]. In case the neutralino is the main component of dark matter, its anapole moment should comply with this limit.

In this work we calculate the TDM of the neutralino within the constrained Minimal Supersymmetric Standard Model. We do a scan in the five parameter space of the cMSSM, and compare the results with the above mentioned experimental limit.
2. The MSSM and the neutralino as candidate for dark matter

The minimal supersymmetric extension of the Standard Model (MSSM) provides us with one of the best WIMP candidates for dark matter, namely the lightest neutralino (for reviews on SUSY see for instance [5, 6]). The MSSM requires two complex Higgs electroweak doublets to give mass to the up and down type quarks and to avoid chiral anomalies. After electroweak symmetry breaking five physical Higgs states remain: two neutral CP invariant that give mass to the up and down type quarks and to avoid chiral anomalies. After electroweak symmetry breaking five physical Higgs states remain: two neutral CP invariant (\(h^0, H^0\)), two charged CP invariant (\(H^+, H^-\)), and one neutral CP-odd (\(A^0\)).

The MSSM has a new discrete symmetry, R parity, defined as \(R = (-1)^{3B+2S+L}\), where \(B\) and \(L\) are the baryonic and leptonic numbers respectively. This symmetry assigns a charge +1 to the SM particles and -1 to the supersymmetric partners, thus making the lightest supersymmetric particle (LSP) stable.

Supersymmetry has to be broken, or it would have already been observed. To break supersymmetry explicitly, without the reappearance of quadratic divergencies, a set of super-renormalizable terms are added to the Lagrangian, the so-called soft breaking terms. The Lagrangian for the soft breaking terms is given by

\[
\mathcal{L}_{\text{soft}} = -\frac{1}{2} M_\alpha \lambda^\alpha - \frac{1}{6} A^{ijk} \Phi_i \Phi_j \Phi_k - \frac{1}{2} B^{ij} \phi_i \phi_j + c.c. - (m^2)^{ij} \phi^i \phi^j ,
\]

where \(M_\alpha\) are the gaugino masses, \(A^{ijk}\) and \(B^{ij}\) are trilinear and bilinear couplings, respectively, and \((m^2)^{ij}\) are scalar squared-mass terms. It is assumed that supersymmetry breaking happens in a hidden sector, which communicates to the observable one only through gravitational interactions, and that the gauge interactions unify. This means that at the GUT scale the soft breaking terms are “universal”, i.e., the gauginos \(M_\alpha\) have a common mass, as well as the scalars \((m^2)^{ij}\) and the trilinear couplings, \(A^{ijk}\). Requiring electroweak symmetry breaking fixes the value of \(B^{ij}\) and the absolute value of the Higgsino mixing parameter \(|\mu|\). This is known as the constrained MSSM, or cMSSM, which is described by five parameters: the unified gaugino mass \(m_{1/2}\), the universal scalar mass \(m_0\), the value of the universal trilinear coupling \(A_0\), the sign of Higgsino mass parameter \(\mu\), and the ratio of the vacuum expectation values of the two Higgses, \(\tan \beta\).

After the electroweak symmetry breaking the neutral and charged states in the MSSM can mix. In the case of the neutral ones they give rise to a set of four mass eigenstates, the neutralinos. It is the lightest one of these that is the LSP and a good candidate to dark matter in many SUSY models. The lightest neutralino, in the gauge eigenstate basis, is thus a function of the neutral Higgsinos and the neutral gauginos (Wino and Bino)

\[
\psi_0 = (\tilde{B}, \tilde{W}^0, \tilde{H}_u^0, \tilde{H}_d^0) .
\]

The properties of the neutralinos will depend on the mixing, which in turn depends on the soft breaking parameters. Thus, the lightest neutralino can range from almost pure Bino to almost pure Higgsino.

3. Toroidal Dipole Moment

For 1/2-spin particles the most general expression for the electromagnetic vertex function, which characterizes the interaction between the particle and the electromagnetic field, is:

\[
\Gamma_\mu(q) = f_Q(q^2)\gamma_\mu + f_\mu(q^2)i\sigma_{\mu\nu}q^\nu\gamma_5 - f_E(q^2)\sigma_{\mu\nu}q^\nu + f_A(q^2)(q^2\gamma_\mu - \not{q}\gamma_5),
\]

where \(f_Q(q^2), f_\mu(q^2), f_E(q^2)\) and \(f_A(q^2)\) are the so called charge, magnetic dipole, electric dipole and anapole form factors, respectively; where \(q_\mu = p'_\mu - p_\mu\) is the transferred 4-momentum; and
\[ \sigma_{\mu\nu} = (i/2) [\gamma_\mu, \gamma_\nu] \quad [7, 8]. \] These form factors are physical observables when \( q^2 \to 0 \), and their combinations define the well known magnetic dipole (\( \mu \)), electric dipole (\( d \)) and anapole (\( a \)) moments.

The electromagnetic properties of Majorana fermions (like the neutralino) are described by a unique form factor, the anapole, \( f_A(q^2) \). This is a consequence of CPT-invariance and the C, P, T properties of \( \Gamma_\mu(q^2) \) and the interaction Hamiltonian. Thus, the electromagnetic vertex function of a neutralino can be written as

\[ \Gamma_\mu(q^2) = f_A(q^2)(q^2\gamma_\mu - q\gamma_\mu)\gamma_5. \] (4)

The anapole moment was introduced by Zel’dovich to describe a T-invariant interaction that does not conserve P and C parity [1]. The anapole moment does not have a simple classical analogue, since \( f_A(q^2) \) does not correspond to a multipolar distribution. A more convenient quantity to describe this interaction was proposed by V. M. Dubovik and A. A. Cheshkov [9]: the toroidal dipole moment (TDM), \( \tau(q^2) \).

The TDM and the anapole moment coincide in the case of \( m_i = m_f \), i.e. the incoming and outgoing particle are the same. This type of static multipole moments does not produce any external fields in vacuum but generate a free-field (gauge invariant) potential [7], which is responsible for topological effects like the Aharonov-Bohm one.

The simplest TDM model (anapole) was given by Zel’dovich as a conventional solenoid rolled up in a torus and with only one poloidal current. For such stationary solenoid, without azimuthal components for the current or the electric field, there is only one magnetic azimuthal field different from zero inside the torus.

4. One-loop calculation
The TDM of the neutralino may be defined in the one-loop approximation in the cMSSM by the Feynman diagrams shown in figs. 1 and 2, where \( f \) represents the charged fermions of the SM. Taking each fermionic family separately we obtain 94 Feynman diagrams in total: 66 corresponding to self-energy and 28 to vertex corrections.

We use FeynCalc to calculate the amplitude of these diagrams. Since we are only interested in the terms that contribute to the anapole form factor, we isolate the ones that have the Lorentz structure \( \gamma_\mu\gamma_5 \). It is important to notice here that we work in the t’Hooft-Feynman gauge (\( \xi = 1 \)). One of the first results we obtain is that the self-energies \( \gamma H^0, \gamma h^0, \gamma A^0 \) and \( \gamma G^0 \) do not contribute to the calculation. If we call \( \Xi_i \) the coefficient that multiplies \( \gamma_\mu\gamma_5 \) for the \( i \)th diagram, then we have that

\[ \sum_i \Xi_i = f_A(q^2)q^2. \] (5)

To obtain the toroidal dipole moment \( \tau = f_A(0) \) we use the l’Hopital rule and get

\[ \tau = f_A(0) = \lim_{q^2 \to 0} \frac{\sum_i \Xi_i}{q^2} = \frac{\partial}{\partial q^2} \Big|_{q^2 \to 0}. \] (6)

The contributions to the self-energies have two point Passarino-Veltman scalar functions of the type \( B_0(q^2, x^2, x^2) \) and \( B_0(0, x^2, x^2) \). Likewise, the contributions to the vertex corrections have two and three point scalar functions of the type \( B_0(q^2, x^2, x^2) \), \( B_0(M_{\tilde{\chi}_1}^2, y^2, x^2) \) and \( C_0(q^2, M_{\tilde{\chi}_1}^2, M_{\tilde{\chi}_2}^2, x^2, y^2) \). In both cases \( x \) and \( y \) represent the masses of the particles in the loop.
When evaluating (6), derivatives of the Passarino-Veltman functions appear. To evaluate the $B_0$’s, as well as their derivatives, we use LoopTools[10]. To evaluate the $C_0$’s and their derivatives we expand them in a power series around $q^2 = 0$.

The expression obtained for the toroidal dipole moment depends on various parameters of the MSSM, including the supersymmetric particles masses as well as the mass mixing matrix elements, the value of $\tan \beta$, and the values of the soft breaking terms. We evaluate the TDM within the cMSSM using Suspect[11], by fixing the value of $A_0$, $\tan \beta$ and $\text{sign } \mu$, and scanning over the other two parameters $m_0$ and $m_{1/2}$, from 0 to 1500 GeV and 250 to 1500 GeV, respectively.

Figure 3 shows the neutralino toroidal dipole moment for $\tan \beta = 10$, $\mu > 0$, and three different values of $A_0$, $-1000$, 0 and 1000 GeV (top to bottom). Comparing the three different plots, no dependence on $A_0$ is shown. The TDM is very low for almost every region of the parameter space scanned, with values between $10^{-5}$ and $10^{-8}$ GeV$^{-2}$. However the TDM increases for increasing $m_0$ and decreasing $m_{1/2}$, reaching values over $10^{-3}$ GeV$^{-2}$ for high $m_0$ ($\geq 800$ GeV) and low $m_{1/2}$ ($\leq 400$ GeV). A similar behavior can be seen in the results for $\tan \beta = 50$, $\mu > 0$, and the same three different values of $A_0$, $-1000$, 0 and 1000 GeV (4). The TDM reaches values around $10^{-3}$ GeV$^{-2}$.

Figure 5 shows a comparison of two plots for different $\tan \beta$ but same $\text{sign } \mu$ and $A_0$. This figure shows the dependence of the TDM on $\tan \beta$. Figure 6 shows a comparison of two plots for different values of $\text{sign } \mu$ but same $\tan \beta$ and $A_0$. $\text{Sign } \mu > 0$ may solve the problem of the discrepancy between the measured value of $g - 2$ of the muon and the one predicted by the SM. However, this does not mean negative $\text{sign } \mu$ is ruled out since others mechanisms could solve this problem, and therefore $\text{sign } \mu < 0$ should be taken into consideration. This figure shows no
dependence of the TDM on sign\(\mu\).

Notice that in all the plots the region for which \(M_{\tilde{\chi}_1^0} = M_{\tilde{\tau}}\) is suppressed since we are not considering this possibility. This condition \(M_{\tilde{\chi}_1^0} = M_{\tilde{\tau}}\) separates the region where the
neutralino $\chi_0^1$ is the LSP and the one where the stau $\tilde{\tau}$ is the LSP.

5. Conclusions
We calculated the only electromagnetic property of the lightest neutralino: its toroidal dipole moment. Its characterization is extremely valuable for discriminating different models which have the neutralino as dark matter candidate. We performed the calculation in the framework of the cMSSM, however a similar analysis can be performed for other models (work in progress). We found that the TDM of the neutralino is highly sensitive to $m_0$, $m_{1/2}$ and $\tan \beta$, but very weakly or practically non-dependent on $A_0$ and $\text{sign} \mu$.

All points in the parameter space we scanned give a TDM consistent with the upper limit ($\sim 10^{-2} \text{ GeV}^{-2}$) obtained by Pospelov and ter Veldhuis [2] for WIMPs interacting with heavy nuclei using data from the CDMS and DAMA experiments. However, this data can and will be improved in the next few years helping to refine the upper limit, likely ruling out some regions of the parameter space.

The TDM analysis can be used as another criteria to constrain the parameter space of a given model which has a neutralino as candidate for dark matter. Thus, according to our results, if a non-zero (around $10^{-4}$-$10^{-3}$ GeV$^{-2}$) TDM could be measured for the neutralino, that would indicate that the favored region of the parameter space of the cMSSM would be high $m_0$ ($\geq 800$ GeV) and low $m_{1/2}$ ($\leq 400$ GeV). Otherwise, other regions are compatible with a TDM lower than $10^{-5}$ GeV.

If combined with other criteria (such as dark matter relic density, or frequentist analysis of data from the LHC) the parameter space can be reduced even further. In fact, the combination with these other criteria favours a region with low $m_0$, which is compatible with a $10^{-5}$-$10^{-8}$ GeV$^{-2}$ TDM for the neutralino. This means, among other things, that if a TDM higher than $10^{-4}$ GeV$^{-2}$ is measured for a WIMP it would exclude some regions of parameter space of the cMSSM (and other more specific models), at least if the neutralino is the only component of dark matter.

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