Semiclassical instability of warp drives

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Abstract. Warp drives, at least theoretically, provide a way to travel at superluminal speeds. However, even if one succeeded in providing the necessary exotic matter to construct them, it would still be necessary to check whether they would survive to the switching on of quantum effects. In this contribution we will report on the behaviour of the Renormalized Stress-Energy Tensor (RSET) in the spacetimes associated with superluminal warp drives. We find that the RSET will exponentially grow in time close to the front wall of the superluminal bubble, hence strongly supporting the conclusion that the warp-drive geometries are unstable against semiclassical back-reaction.

1. Introduction
Since they were introduced by Alcubierre\textsuperscript{[1]}, warp-drive spacetimes have been certainly one of the most studied solutions of the Einstein equations among those requiring exotic matter\textsuperscript{[2]}. They are not only an exciting theoretical test for our comprehension of general relativity and quantum field theory in curved spacetimes, but they might also be, at least theoretically, a way to travel at superluminal speed. The warp drive consists of a bubble containing an almost flat region, moving at arbitrary speed within an asymptotically flat spacetime.

After the proposal of this solution, its most investigated aspect has been the amount of exotic matter (\textit{i.e.} energy-conditions violating matter) required to support such a spacetime\textsuperscript{[3, 4, 5, 6]}. It has been found, using the so called quantum inequalities (QI), that such a matter must be confined in Planck-size regions at the edges of the bubble. This bound on the wall thickness turns into lower limits on the amount of exotic matter required to support the bubble (at least of the order of 1 solar mass).

Less effort has been devoted to other important issue regarding the feasibility of these spacetimes: the study of the warp-drive semiclassical stability. In particular, it was studied in\textsuperscript{[7]} the case of an eternal superluminal warp drive by discussing its stability against quantum effects. It was there noticed that, to an observer within the warp-drive bubble, the backward and forward walls (along the direction of motion) look respectively as the future and past event horizon of a black hole. By imposing over the spacetime a quantum state which is vacuum at the null infinities (\textit{i.e.} what one may call the analogue of the Boulware state for an eternal black hole) it was found that the renormalized stress-energy tensor (RSET) had to diverge on the horizons.

In this contribution we consider the more realistic case of a warp drive which is created with zero velocity at early times and then accelerated up to some superluminal speed in a finite time.
(a more detailed treatment can be found in [8]). We found, as expected, that in the centre of the bubble there is a thermal flux at the Hawking temperature corresponding to the surface gravity of the black horizon. However, this surface gravity is inversely proportional to the wall thickness, leading to a temperature of the order of the Planck temperature, for Planck-size walls. Even worse, we do show that the RSET does increase exponentially with time on the white horizon (while it is regular on the black one). This clearly implies that a warp drive becomes rapidly unstable once superluminal speed are reached.

2. Light-ray propagation

In the actual computation we shall restrict our attention to the 1 + 1 dimensions case (since this is the only one for which one can carry out a complete analytic treatment as explained below)1. Changing coordinates to those associated with an observer at the centre of the bubble, the warp-drive metric in [1] becomes

\[ ds^2 = -c^2 dt^2 + \left[ dr - \bar{v}(r) dt \right]^2, \quad \bar{v} = v - v_c. \] (1)

Here \( v(r) = v_c f(r) \) with \( f \) a suitable smooth function satisfying \( f(0) = 1 \) and \( f(r) \to 0 \) for \( r \to \infty \); \( v_c \) is the velocity of the centre of the bubble and \( r \) the signed distance from the centre of the bubble \( r = 0 \). In our dynamical situation the warp-drive geometry interpolates between an initial Minkowski spacetime \( \hat{v}(t,r) \to 0 \), for \( t \to -\infty \) and a final stationary superluminal \( (v_c > c) \) bubble \( \bar{v}(t,r) \to \bar{v}(r) \), for \( t \to +\infty \). To an observer living inside the bubble this geometry has two horizons, a black horizon \( \mathscr{H}^+ \) located at \( r_1 \) and a white horizon \( \mathscr{H}^- \) located at \( r_2 \). For those interested, in [8] you can find the Penrose diagram of these spacetimes. Here let us just mention that from the point of view of the Cauchy development of \( \mathscr{I}^- \) these spacetimes have Cauchy horizons.

Let us now consider light-ray propagation in the above described geometry. Only the behaviour of right-going rays determines the universal features of the RSET, just like outgoing modes do in the case of a black hole collapse [8, 9, 10]. Therefore, we need essentially the relation between the past and future null coordinates \( U \) and \( u \), labelling right-going light rays. Following [9], this relation can be found by integrating the right-going-ray equation

\[ \frac{dr}{dt} = c + \hat{v}(r,t). \] (2)

There are two special right-going rays defining, respectively, the asymptotic location of the black and white horizons. In terms of the right-going past null coordinate \( U \) let us denote these two rays by \( U_{BH} \) and \( U_{WH} \), respectively. The finite interval \( U \in (U_{WH}, U_{BH}) \) is mapped to the infinite interval \( u \in (-\infty, +\infty) \) covering all the rays travelling inside the bubble. For rays which are close to the black horizon, in [8] the present authors proved that the relation between \( U \) and \( u \) can be approximated as a series of the form

\[ U(u \to +\infty) \simeq U_{BH} + A_1 e^{-\kappa_1 u} + \frac{A_2}{2} e^{-2\kappa_1 u} + \ldots. \] (3)

Here \( A_n \) are constants (with \( A_1 < 0 \) and \( \kappa_1 > 0 \) represents the surface gravity of the black horizon. This relation is the standard result for the formation of a black hole through gravitational collapse. As a consequence, the quantum state which is vacuum on \( \mathscr{I}^- \) will show, for an observer inside the warp-drive bubble, Hawking radiation with temperature \( T_H = \kappa_1/2\pi \).

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Equivalently, we find that the corresponding expansion in the proximity of the white horizon is

\[ U(u \to -\infty) \simeq U_{WH} + D_1 e^{\kappa_2 u} + \frac{D_2}{2} e^{2\kappa_2 u} + \ldots, \]

where \( D_1 > 0 \) and \( \kappa_2 \) is the white hole surface gravity and is also defined to be positive (\( \kappa_2 \) could be different from \( \kappa_1 \) in general, although it is expected, in principle, to be comparable with \( \kappa_1 \)). The interpretation of this relation in terms of particle production is not as clear as in the black horizon case. For this reason, we shall consider now the RSET.

3. Renormalized stress-energy tensor

For the calculation of the RSET inside the warp-drive bubble we use the method proposed in [10]. In past null coordinates \( U \) and \( W \) the metric can be written as \( ds^2 = -C(U,W)dUdW \). In the stationary region at late times, we use the previous future null coordinate \( u \) and \( \tilde{w} \), defined as

\[ \tilde{w}(t,r) = t + \int_0^r \frac{dr}{c - \tilde{v}(r)}, \]

to write the metric as

\[ ds^2 = -\tilde{C}(u, \tilde{w})dud\tilde{w}, \quad C(U,W) = \tilde{C}(u, \tilde{w}) \]

where \( U = p(u) \) and \( W = q(\tilde{w}) \). In this way, \( \tilde{C} \) depends only on \( r \) through \( u, \tilde{w} \).

For concreteness, we refer to the RSET associated with a quantum massless scalar field living on the spacetime. When particularized to the stationary region, the RSET components [11] acquire the following form:

\[ T_{UU} = -\frac{1}{12\pi} C^{1/2} \partial_u^2 C^{-1/2} = -\frac{1}{48\pi} \frac{1}{\tilde{p}^2} \left[ \tilde{\varphi}^2 + (1 - \tilde{\varphi}^2) \tilde{v}\tilde{\varphi}'' \right] - f(u), \]

\[ T_{WW} = -\frac{1}{12\pi} C^{1/2} \partial_W^2 C^{-1/2} = -\frac{1}{48\pi} \frac{1}{q^2} \left[ \tilde{\varphi}^2 + (1 - \tilde{\varphi}^2) \tilde{v}\tilde{\varphi}'' \right] - g(\tilde{w}), \]

\[ T_{UW} = T_{WU} = \frac{1}{96\pi} C \frac{R}{\tilde{p}^2} = -\frac{1}{48\pi} \frac{1}{\tilde{p}q} \left[ \tilde{\varphi}^2 + (1 - \tilde{\varphi}^2) \tilde{v}\tilde{\varphi}'' \right]. \]

Here we have put \( c = 1 \) and we have defined

\[ f(u) \equiv \frac{3\tilde{p}^2(u) - 2p(u)\tilde{p}'(u)}{\tilde{p}^2(u)}, \]

\[ g(\tilde{w}) \equiv \frac{3q^2(\tilde{w}) - 2q(\tilde{w})\tilde{q}'(\tilde{w})}{\tilde{q}^2(\tilde{w})}. \]

One can show [8] that \( \tilde{q} \) contains solely information associated with the dynamical details of the transition region. Moreover, for simple dynamical interpolations between Minkowski and the final warp drive, \( \tilde{q}(\tilde{w}) \) goes to a constant at late times, such that \( g(\tilde{w}) \to 0 \). From now on, we will neglect this term.

We want to look at the energy density inside the bubble, in particular at the energy \( \rho \) as measured by a set of free-falling observers, whose four velocity is \( u^\mu = (1,\tilde{v}) \) in \( (t,r) \) components. For these observers we obtain \( \rho = T_{\mu\nu}u^\mu u^\nu = \rho_{st} + \rho_{dyn} \) where we define a static term \( \rho_{st} \), which depends only on the \( r \) coordinate through \( \tilde{v}(r) \), and a dynamic term \( \rho_{dyn} \) as

\[ \rho_{st} \equiv -\frac{1}{24\pi} \left[ \frac{(\tilde{\varphi}^4 - \tilde{\varphi}^2 + 2)}{(1 - \tilde{\varphi}^2)^2} \tilde{\varphi}^2 + \frac{2\tilde{\varphi}}{1 - \tilde{\varphi}^2} \tilde{\varphi}'' \right], \quad \rho_{dyn} \equiv \frac{1}{48\pi} \frac{f(u)}{(1 + \tilde{\varphi})^2}. \]

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This latter term, depending also on $u$, corresponds to energy travelling on right-going rays, eventually red/blue-shifted by a term depending on $r$.

By studying now the behaviour of $\rho$ at late times in the centre of the bubble and at the horizons $H^+$ and $H^-$, we reach the following conclusions.

4. Summary of results
(1) We found that the central region of the warp drive behaves like the asymptotic region of a black hole: In both of these regions the static term $\rho_{st}$ vanishes and the whole energy density is due to the Hawking radiation generated at the black horizon. If one trusts the QI [3, 4], the wall thickness for a warp drive with $v_c \approx c$ would be $\Delta \lesssim 10^2 L_P$, and its surface gravity $\kappa_1 \gtrsim 10^{-2} t_P^{-1}$, where $t_P \approx 10^{-43}$ s is the Planck time. Hence, the Hawking temperature of this radiation would be unacceptably large: $T_H \sim \kappa_1 \gtrsim 10^{-2} T_P$.

(2) The formation of a white horizon produces a transient radiation which accumulates on the white horizon itself. This causes the energy density $\rho$, as seen by a free-falling observer, to grow unboundedly with time on this horizon. The semiclassical backreaction of the RSET will make the superluminal warp drive to become rapidly unstable, in a time scale of the order of $1/\kappa_2$ (i.e. of the inverse of the surface gravity of the white horizon). In fact, in order to get even a time scale $\tau \sim 1$ s for the growing rate of the RSET, one would need a wall as large as $3 \times 10^8$ m. Thus, most probably, one would be able to maintain a superluminal speed for just a very short interval of time.

(3) The formation of a Cauchy horizon gives rise to an instability, similar to the inner horizon instability in black holes. This is due to the blue-shift suffered by the Hawking radiation produced by the black horizon when it gets accumulated in the white horizon.

On the base of these evidences, we think that this work is convincingly ruling out the semiclassical stability of superluminal warp drives.

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