\textbf{\textit{p}-ADIC FAMILIES OF AUTOMORPHIC FORMS OVER SOME UNITARY SHIMURA VARIETIES}

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\textbf{Abstract.} We construct $n$-dimensional eigenvarieties for finite slope overconvergent eigenforms over some unitary Shimura varieties with signature $(1, n-1) \times (0, n) \times \cdots \times (0, n)$ by adapting Andreatta-Iovita-Pilloni’s method. We show that there are some Galois pseudo-characters over these eigenvarieties by studying analytic continuation of finite slope eigenforms over the Shimura varieties.

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1. \textbf{Introduction}

The theory of $p$-adic families of automorphic forms started from the various work of Hida and Coleman. For the related history one can see the introduction of [10]. In loc. cit. Coleman and Mazur constructed the eigencurve for finite slope elliptic modular forms, which parameterizes the system of eigenvalues for the Hecke eigenforms. Since then there have been many authors working on this field to develop a general theory of $p$-adic automorphic forms on higher rank groups. For example, there is the work of Urban [25] based on studying overconvergent cohomology. In particular, Urban constructed eigenvarieties for any reductive groups $G$ over $\mathbb{Q}$ such that $G(\mathbb{R})$ admits discrete series. There is also an approach of Emerton [12] by studying the completed cohomology of arithmetic quotients. For definite unitary groups Chenevier has constructed some eigenvarieties in [8] which have quite useful applications to Galois representations, see for example [9]. Finally, there is a geometric approach of Andreatta, Iovita and Pilloni [1] for finite slope Siegel modular cuspforms. Their method is based on the theory of canonical subgroups, and is quite promising to be generalized to general PEL type Shimura varieties, see for example [2] and [5].
In this paper we work out the construction of \( n \)-dimensional eigenvarieties for finite slope eigenforms over some unitary Shimura varieties with signature \((1, n-1) \times (0, n) \times \cdots \times (0, n)\) by adapting Andreatta-Iovita-Pilloni’s method in [1]. These Shimura varieties are restricted to the class studied by Harris-Taylor in [16] for proving the local Langlands correspondence for \( GL_n \). However see the remark 6.5 for a discussion for more general Shimura varieties.

There were some related works in our set up. In [18] Kassaei studied \( p \)-adic modular forms of integral weights over the Shimura curves, that is the case \( n = 2 \). In [4] Brasca studied non integral weight forms over these Shimura curves and constructed an eigencurve. However, the definitions of modular forms in both [18] and [4] can not be compared with the classical theory of automorphic forms, since their modular forms are sections of some line bundles which are not automorphic vector bundles over the Shimura curves. In [11] Ding studied also non integral weight forms, whose definition is compatible with classical modular forms. He constructed in fact an eigenvariety of dimension two and used it to study the local-global compatibility of Langlands correspondence in this setting. This paper deals with the classical points in our eigenvarieties are dense. As a consequence, there are some Galois pseudo-characters over the eigenvarieties. The main theorem of the paper is the following.  

**Theorem 1.1.** There is a rigid analytic variety \( \mathcal{E} \) over \( L \) (a finite extension of \( \mathbb{Q}_p \)) and a locally finite map to the weight space \( w : \mathcal{E} \to \mathcal{W} \), such that  

1. \( \mathcal{E} \) is equidimensional of dimension \( n \).
2. We have a character \( \Theta : T^{K_p} \otimes T_p \to \mathcal{O}(\mathcal{E}) \). For any \( \kappa \in \mathcal{W} \), \( w^{-1}(\kappa) \) is in bijection with the eigensystems of \( T^{K_p} \otimes T_p \) acting on the space of finite slope locally analytic overconvergent automorphic forms of weight \( \kappa \).
3. For any \( \kappa = (k_1, \ldots, k_{n-1}, k_n) \in \mathbb{Z}_+^{n-1} \times \mathbb{Z} \subset \mathcal{W} \), if \( x \in w^{-1}(\kappa) \) satisfies \( v(\Theta_x(U_i)) < k_{n-i} - k_{n-i+1} + 1 \) for \( 2 \leq i \leq n-1 \) and \( v(\Theta_x(U_1)) < k_n + k_{n-1} - n + 1 \), then the character \( \Theta_x \) comes from a weight \( \kappa \) automorphic eigenform on \( X \).
4. There is a Galois pseudo-character \( T : \text{Gal}(\overline{\mathbb{Q}}/F) \to \mathcal{O}(\mathcal{E}) \) (\( F \) is some CM field, see subsection 2.1), such that for any point \( x \in \mathcal{E} \), there is a continuous semi-simple representation \( \rho_x : \text{Gal}(\overline{\mathbb{Q}}/F) \to GL_n(k(x)) \) (\( k(x) \) is the residue field of \( x \)) and the trace of this Galois representation is \( T_x \). Here \( T_x \) is the composition of \( T \) with the evaluation map \( ev_x : \mathcal{O}(\mathcal{E}) \to k(x) \).

For the definition of the Hecke algebra \( T^{K_p} \otimes T_p \) and the operators \( U_i \) for \( 1 \leq i \leq n-1 \) see section 4. We hope that these eigenvarieties have useful applications to Galois representations as those constructed by Chenevier for definite unitary groups.

This paper is organized as follows. In section 2, we introduce the related Shimura varieties and review automorphic vector bundles on them. In section 3, we first review the
theory of canonical subgroups and the Hodge-Tate maps for them as in [1] section 4, then we construct the overconvergent sheaves \( \omega^w \) by proceeding in the same way as in loc. cit.. In section 4 we define the Hecke operators which act on the spaces of overconvergent automorphic forms. Then in section 5 we study analytic continuation of finite slope overconvergent eigenforms and prove the classicality theorem. In the last section we construct the \( n \)-dimensional eigenvarieties and prove the main theorem.

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2. **Shimura varieties and automorphic vector bundles**

2.1. **Some unitary Shimura varieties.** In this section we introduce the Shimura varieties which will be the main object of study in this paper. These were studied by Harris-Taylor in [16] for proving the local Langlands correspondence for \( GL_n \).

Let \( p \) be a prime number. Fix an imaginary quadratic field \( E \) in which \( p \) splits. The two primes of \( E \) above \( p \) will be denoted by \( u \) and \( v \), and the complex conjugation of \( Gal(E/\mathbb{Q}) \) will be denoted by \( c \). Let \( F^+ \mid \mathbb{Q} \) be a totally real field of degree \( N \). Set \( F = F^+E \), so that \( F \) is a CM-field with maximal totally real subfield \( F^+ \). Let \( \varpi = \varpi_1, \varpi_2, \ldots, \varpi_r \) denote the primes of \( F \) above \( u \), and let \( v = v_1, v_2, \ldots, v_r \) denote their restrictions to \( F^+ \). We will denote the degrees of \( F_{\varpi_i} \approx F_{v_i}^+ \) by \( d_i \) for \( i = 1, 2, \ldots, r \). Let \( B/F \) denote a central division algebra of dimension \( n^2 \) over \( F \) such that

- the opposite algebra \( B^{op} \) is isomorphic to \( B \otimes_{E,c} E \);
- \( B \) is split at \( \varpi \);
- at any place \( x \) of \( F \) which is not split over \( F^+ \), \( B_x \) is split (here and in the following \( B_x = B \otimes F_x \));
- at any place \( x \) of \( F \) which is split at \( F^+ \), either \( B_x \) is split or \( B_x \) is a division algebra;
- if \( n \) is even then \( 1 + Nn/2 \) is congruent modulo 2 to the number of places of \( F^+ \) above which \( B \) is ramified.

As in [16], we can choose an involution of second kind \( * \) on \( B \). Moreover, we can choose some alternating pairing \( \langle \cdot, \cdot \rangle \) on \( V \times V \rightarrow \mathbb{Q} \) for the \( B \otimes_F B^{op} \) module \( V := B \), which corresponds to another involution of second kind \( \sharp \) on \( B \). The associated reductive group \( G/\mathbb{Q} \) is defined by

\[
G(R) = \{(g, \lambda) \in (B^{op} \otimes_{\mathbb{Q}} R)^{\times} \times R^{\times} \mid gg^{\sharp} = \lambda\},
\]

for any \( \mathbb{Q} \)-algebra \( R \). Let \( G_1 \) be the kernel of the map \( G \rightarrow \mathbb{G}_m, (g, \lambda) \mapsto \lambda \), which can be viewed as a group over \( F^+ \). Choose a distinguished embedding \( \tau_0 : F^+ \hookrightarrow \mathbb{R} \). In loc. cit. we can make the choice of the alternating pairing on \( V \times V \rightarrow \mathbb{Q} \) such that

- if \( x \) is a rational prime which is not split in \( E \), then \( G \) is quasisplit at \( x \),
- if \( \sigma : F^+ \hookrightarrow \mathbb{R} \) is an embedding, then \( G_1 \times F^+_{\tau, \mathbb{R}} \mathbb{R} \) is isomorphic to the unitary group \( U(1, n-1) \) if \( \sigma = \tau_0 \) and \( U(n) \) otherwise.

We can say more about the group \( G \). First, we have an isomorphism

\[
Res_{E/\mathbb{Q}} G_E \simeq (B^{op}_Q)^{\times} \times Res_{E/\mathbb{Q}} \mathbb{G}_m,
\]
which is an inner form of the quasi-split group $\text{Res}_{F/Q} \text{GL}_n \times \text{Res}_{E/Q} \text{G}_m$. So the theory of automorphic representations for $G$ can be understood by those for $\text{GL}_n/F$ via the stable base change theorem of Clozel and Labesse, see 1.2.6 in [15]. Second, the local reductive group at $p$ has the form

$$G_{\mathbb{Q}_p} \simeq \prod_{i=1}^r (B_{\mathbb{Q}_p}^\times)^{\times} \times \mathbb{G}_m.$$  

Fix a maximal order $\Lambda_i = O_{B_{\mathbb{Q}_i}}$ in $B_{\mathbb{Q}_i}$ for each $i = 1, 2, \ldots, r$. Our pairing on $V = B$ induces perfect duality between $V_{\mathbb{Q}_i}$ and $V_{\mathbb{Q}_i}^\vee$. Let $\Lambda^\vee \subset V_{\mathbb{Q}_i}^\vee$ be the dual of $\Lambda_i \subset V_{\mathbb{Q}_i}$. Then

$$\Lambda := \bigoplus_{i=1}^r \Lambda_i \oplus \bigoplus_{i=1}^r \Lambda_i^\vee$$

is a $\mathbb{Z}_p$-lattice in $V \otimes_{\mathbb{Q}} \mathbb{Q}_p$ and we have a perfect pairing $\Lambda \times \Lambda \to \mathbb{Q}_p$. Let $O_B \subset B$ be the unique maximal $\mathbb{Z}(p)$-order such that $O_B^\vee = O_B$ and $O_{B;\mathbb{Q}_i} = O_{B;\mathbb{Q}_i}$ for $i = 1, 2, \ldots, r$. Fix an isomorphism $O_{B_{\mathbb{Q}_i}} \simeq M_n(O_{F_0})$. Let $\varepsilon = (\varepsilon_{ij}) \in M_{11}(O_{F_0})$ be the idempotent with $\varepsilon_{11} = 1$ and all the other $\varepsilon_{ij} = 0$. Then $\Lambda_{11} := \varepsilon \Lambda_1 \simeq (O_{F_0}^\times)^\vee$, and

$$\Lambda \simeq ((O_{F_0}^n \otimes \Lambda_{11}) \oplus (O_{F_0}^n \otimes \Lambda_{11})^\vee) \oplus \bigoplus_{i=2}^r (\Lambda_i \oplus \Lambda_i^\vee).$$

Let $K \subset G(\mathbb{A}_f)$ be a sufficiently small open compact subgroup. Then we have a Shimura variety $Sh_K$ over $F$, which is a moduli space of abelian varieties with additional structures. More precisely, for any connected locally noetherian $F$-scheme, $Sh_K(S) = \{(A, \lambda, \iota, \eta)\}/ \simeq$ where

- $A/S$ is an abelian scheme of dimension $Nr^2$;
- $\lambda : A \to A^\vee$ is a polarization;
- $\iota : B \to \text{End}(A) \otimes \mathbb{Q}$ is an action such that $\lambda \circ \iota(b) = \iota(b^*) \circ \lambda$ for all $b \in B$ and $(A, \iota, \varepsilon)$ is compatible (cf. [16] Lemma III.1.2);
- $\bar{\eta}$ is a level structure $\eta : V \otimes \mathbb{A}_f \to V_f(A) (mod K)$.

Let $L\mathbb{Q}_p$ be a finite extension which is large enough so that it contains all the embeddings of $F$ into $\mathbb{Q}_p$ and splits all $\mathbb{B}_{\mathbb{Q}_i}$ for $2 \leq i \leq r$. We fix an embedding $F_{\mathbb{Q}_i} \subset L$ and still denote $Sh_K$ the above variety base change to $L$ via the fixed embedding by abuse of notation. Write $K = K^pK_p \subset G(\mathbb{A}_f^p) \times G(\mathbb{Q}_p)$. If $K_p = \prod_{i=1}^r O_{B_{\mathbb{Q}_i}}^\times \times \mathbb{Z}_p^r$, then one can define a proper smooth integral model $S_K$ of $Sh_K$ over $O_L$ by considering a similar integral modular problem. In fact, let $m = (m_1, \ldots, m_r) \in \mathbb{Z}^r_{\geq 0}$, and $K^p(m)$ be the product

$$K^p \times \prod_{i=1}^r \ker(O_{B_{\mathbb{Q}_i}}^\times \to (O_{B_{\mathbb{Q}_i}}^p / \mathbb{Q}_i^m)^\times) \times \mathbb{Z}_p^r,$$

then by introducing the notion of Drinfeld level structures at $p$, one can define a proper flat regular model $S_{K^p(m)}$ of $Sh_{K^p(m)}$ over $O_L$. For $K^p$ and $m$ varies, the group $G(\mathbb{A}_f)$ acts as Hecke correspondences on the tower $(S_{K^p(m)}(K^p(m)))_{K^p,m}$. Over $\mathbb{C}$, $Sh_{K, C}$ is a disjoint union of $ker^1(\mathbb{Q}, G)$ copies of the PEL unitary Shimura variety $Sh_K(G, X)$ associated to the corresponding Shimura data. By abuse of notation, we will call these moduli spaces for $K$ varies Shimura varieties.

2.2. Automorphic vector bundles. Let $(m_2, \ldots, m_r) \in \mathbb{Z}^r_{\geq 0}$ and $K^p$ be fixed. We will be interested in the levels $K_0 = K^p(0, m_2, \ldots, m_r)$ and

$$K = K^p \times Iw \times \prod_{i=2}^r \ker(O_{B_{\mathbb{Q}_i}}^\times \to (O_{B_{\mathbb{Q}_i}}^p / \mathbb{Q}_i^m)^\times) \times \mathbb{Z}_p^r \subset K^p(1, m_2, \ldots, m_r),$$
where \( Iw \subset GL_n(O_{F_w}) \) denotes the Iwahoric subgroup. To simplify notations, let \( X := Sh_{K}, Y := Sh_{K_0} \) and \( \overline{X} \) (resp. \( \overline{Y} \) ) the special and generic fibers of \( S_K \) (resp. \( S_{K_0} \)). We know that \( S_{K_0} \) is smooth, and \( S_K \) has strictly semi-stable reduction (24). We will study automorphic vector bundles on \( X \) and the related theory of automorphic forms. For this we will need some preparations.

Let \( \Sigma = Hom(F^+, L) \), which is bijective to \( Hom(F^+, \mathbb{R}) \) after fixing an embedding \( L \rightarrow \overline{\mathbb{Q}}_p \) and an isomorphism \( \overline{\mathbb{Q}}_p \approx \mathbb{C} \). Let \( \tau \in \Sigma \) be the element which corresponds to \( \tau_0 \) under this bijection for the fixed \( \tau_0 : F^+ \rightarrow \mathbb{R} \) in last subsection. We can and we do take the embedding and isomorphism above such that \( \tau \) comes from an element in \( Hom(F_w, L) \). Consider the Levi subgroup \( M \) of

\[
G_{F_w} \approx \mathbb{G}_m \times \prod_{\sigma:F_w \rightarrow L} GL_n \times \prod_{i=2}^{r} (B_{\sigma_i}^{op})_{F_w}^x
\]
defined by

\[
M = \mathbb{G}_m \times GL_{n-1} \times GL_1 \times \prod_{\sigma \neq \tau:F_w \rightarrow L} GL_n \times \prod_{i=2}^{r} (B_{\sigma_i}^{op})_{F_w}^x.
\]

Then over \( L \), we have the isomorphisms

\[
G_L \approx \mathbb{G}_m \times \prod_{\sigma \in \Sigma} GL_n
\]

\[
M_L \approx \mathbb{G}_m \times GL_{n-1} \times GL_1 \times \prod_{\sigma \neq \tau:F^+ \rightarrow L} GL_n.
\]

Take a maximal torus \( T \subset M_L \subset G_L \), and choose a Borel subgroup \( T \subset B \subset G_L \). Then we have isomorphisms of dominant weights (respective to \( B \) and \( B \cap M_L \))

\[
X^*(T)^{G_L} \approx \mathbb{Z} \times \prod_{\sigma \in \Sigma} \mathbb{Z}_{+}^n
\]

\[
X^*(T)^{M_L} \approx \mathbb{Z} \times \mathbb{Z}_{+}^{n-1} \times \mathbb{Z} \times \prod_{\sigma \neq \tau} \mathbb{Z}_{+}^n.
\]

Therefore each irreducible representation \( \xi \) of \( G_L \) (resp. \( \rho \) of \( M_L \)) corresponds to \( (a_0, (\alpha^x_\sigma)_\sigma) \) (resp. \( (b_0, b_r, b_{\sigma}, (b_{\sigma})_{\sigma \neq \tau}) \) ) with \( \alpha^x_\sigma = (a_{1\sigma}, \ldots, a_{n\sigma}) \in \mathbb{Z}_{+}^n \) (resp. \( b_r = (b_{1r}, \ldots, b_{(n-1)r}) \in \mathbb{Z}_{+}^{n-1}, b_{\sigma} = (b_{1\sigma}, \ldots, b_{n\sigma}) \in \mathbb{Z}_{+}^n \) )

Fix an isomorphism \( \overline{\mathbb{Q}}_p \approx \mathbb{C} \) and an embedding \( L \rightarrow \overline{\mathbb{Q}}_p \). Then we can base change objects over \( L \) to \( \mathbb{C} \) by the fixed choices of isomorphism and embedding. To each irreducible representation \( \xi \) of \( G_C \), we can associate a local system \( \mathcal{L}_\xi \) on \( X_C \) and the corresponding flat vector bundle \( (\mathcal{V}_\xi, \nabla) \). By Faltings’s dual BGG resolution, we have a subcomplex \( \mathcal{E}^* \subset \mathcal{V}_\xi \otimes \Omega^1_{X_C} \) which has the same cohomology and which is a direct summand as a filtered complex, with \( \mathcal{E}^i = \mathcal{E}_{\rho_i} := \mathcal{E}_{w_i}(\xi) \) for \( i = 0, \ldots, n-1 \). Here \( w_i, i = 0, \ldots, n-1 \) are explicitly defined elements in the Weyl group (see [16] III.2, p.103), and \( \mathcal{E}_{\rho} \) is the automorphic vector bundle over \( X_C \) associated to an irreducible representation \( \rho \) of \( M_C \). We have a spectral sequence

\[
E_1^{i,j} = H^i(X_C, \mathcal{E}^j) \Longrightarrow H^{i+j}_{dR}(X_C, \mathcal{V}_\xi),
\]

which degenerates at \( E_1 \). In fact, by construction there is a filtration on the complex \( \mathcal{E}^* \), and if we denote the graded pieces by \( gr^F_{\mathcal{E}^*} \), then there is another spectral sequence

\[
E_1^{i,j} = H^{i+j}(X_C, gr^F_{\mathcal{E}^*}(\mathcal{E}^*)) \Longrightarrow H^{i+j}_{dR}(X_C, \mathcal{V}_\xi).
\]
which also degenerates at $E_1$. By de Rham theorem we have $H^i(X_{\overline{C}}, \mathcal{L}_\xi) \simeq H^i_{dR}(X_{\overline{C}}, V_\xi)$. Here the left hand side is the Betti cohomology (or étale cohomology). We remark that everything above works over $L$, except that we need the $p$-adic comparison theorem instead of de Rham theorem:

$$H^i(X_{\overline{\mathbb{P}_p}}, \mathcal{L}_\xi) \otimes_L B_{dR} \simeq H^i_{dR}(X, V_\xi) \otimes_L B_{dR}.$$ 

Let $A/S_K$ be the universal abelian scheme with unit section $e : S_K \to A$. Recall $S_K$ is the integral Shimura varieties over $O_L$ with generic fiber $X$. Let $\omega := \varepsilon e^* \Omega^1_{A/S_K}$, where $\varepsilon = (\varepsilon_i^+ + \varepsilon_i^-)$, for each $i = 1, \ldots, r$ both $\varepsilon_i^+$ and $\varepsilon_i^-$ are the idempotent matrices with all $(k, j) \neq (1, 1)$-elements $0$ and $1$ for the $(1,1)$ element. We have the decomposition

$$A[p^\infty] = A[\varpi_i^\infty] \oplus \cdots \oplus A[\varpi_r^\infty] \oplus A[\varpi_1^\infty] \oplus \cdots \oplus A[\varpi_r^\infty],$$

with $A[\varpi_i^\infty]$ ind-étale for $2 \leq i \leq r$, and $A[\varpi_1^\infty]^P \simeq A[\varpi_1^\infty]$ for all $1 \leq i \leq r$. This induces a decomposition

$$\omega = \omega_1 \oplus \cdots \oplus \omega_r \oplus \omega_1^c \oplus \cdots \oplus \omega_r^c,$$

with $\omega_i$ comes from $A[\varpi_i^\infty]$ and $\omega_i^c$ comes from $A[\varpi_i^\infty]$. This is also the decomposition of $\omega$ under the action of $O_F \otimes \mathbb{Z}_p$. Thus $\omega_2 = \cdots = \omega_r = 0$, rank $\omega_i^c = d_i n, i \geq 2$, rank $\omega_1^c = d_1 n - 1$, rank $\omega_1 = 1$. For $1 \leq i \leq r$, let $\Sigma_i = Hom(F_{\varpi_i}, L)$. From the next subsection we will assume $F_{\varpi_i} = \mathbb{Q}_p$. Then $\Sigma_1 = \{\tau\}$, and $\Sigma = \Sigma_1 \sqcup \cdots \sqcup \Sigma_r$. If we consider the action of $O_F \otimes O_L$ on $\omega$, we have further decomposition

$$\omega = (\omega_1 \oplus \omega_1^c) \oplus \omega_2 \oplus \cdots \oplus \omega_r \oplus \cdots \oplus \omega_r^c$$

where $\omega_2 = \omega_2^c = \omega_1 \oplus \omega_1^c$ is of rank one, and rank $\omega_1^c = n - 1$. For later use, let $H = \varepsilon_i^+ A[\varpi_i^\infty]$, which is a $p$-divisible group of dimension one and height $n$. We have similar objects over $X, S_{K_0}$ and $\overline{Y}$. By abuse of notation we also denote the restriction of $\omega$ over $X$ by $\omega$, which can be defined directly in the same way.

Consider the subgroup $M' = GL_{n-1} \times GL_1 \times \prod_{\sigma \neq \tau} GL_n \subset M_L$, then every irreducible representation of $M'$ can be viewed as an irreducible representation of $M_L$ via the natural projection $M_L \to M'$. In the following we will concentrate on these representations and the associated automorphic vector bundles. Let

$$\mathcal{T} = Isom(\omega, O_X^{n-1} \oplus O_X \bigoplus_{\sigma \in \Sigma, \sigma \neq \tau} O_X^1),$$

which is a $M'$-torsor over $X$. Denote by $\pi$ the projection $\mathcal{T} \to X$. Let $\kappa = (k_{\tau,1}, \ldots, k_{\tau,n-1}, k_{\tau}, (k_\sigma)_{\sigma \neq \tau}) \in \mathbb{Z}_n^{n-1} \times \mathbb{Z} \times \prod_{\sigma \neq \tau} \mathbb{Z}_n$, $\kappa' = (-k_{\tau,n-1}, \ldots, -k_{\tau,1}, k_{\tau}, (k_\sigma)_{\sigma \neq \tau})$. We define a coherent sheaf

$$\omega^\kappa := \pi_* O_{\mathcal{T}}[\kappa'],$$

which as an automorphic vector bundle corresponds to the irreducible representation $V_{\kappa'}$ of $M'$ of highest weight $\kappa'$. Conversely, any automorphic vector bundle corresponding to an irreducible representation $V_{\kappa'}$ of $M'$ of highest weight $\kappa'$ comes in this way. In the above we have noted $\omega^\kappa = \mathcal{E}_{\kappa'}$.

**Definition 2.1.** We call the elements of $H^0(X, \omega^\kappa)$ as automorphic forms of weight $\kappa$ over $X$. 
By definition, an element \( f \in H^0(X, \omega^\kappa) \) is a functorially defined function \( T(R) \to R \) for any \( L \)-algebra \( R \), such that for all \( b \in B_{M'} := B \cap M' \) we have
\[
f(A, \lambda, \iota, \eta, ((v^+_\tau, v^-_\tau), (v_\sigma)_{\sigma \neq \tau}) \circ b) = \kappa'(b)f(A, \lambda, \iota, \eta, ((v^+_\tau, v^-_\tau), (v_\sigma)_{\sigma \neq \tau})),
\]
where \((A, \lambda, \iota, \eta, ((v^+_\tau, v^-_\tau), (v_\sigma)_{\sigma \neq \tau}))\) is a \( \text{Spec}(R) \)-point of \( T \).

The sheaf \( \omega^\kappa \) has a decomposition as tensor product
\[
\omega^\kappa = \omega^\kappa_{\tau} \otimes \bigotimes_{\sigma \neq \tau} \omega^\kappa_{\sigma},
\]
which corresponds to the factors of \( \kappa = (\kappa_\tau, (\kappa_\sigma)_{\sigma \neq \tau}) \). Let \( T' = T_\tau \times \prod_{\sigma \in \Sigma, \sigma \neq \tau} T_{\sigma} \) be the maximal torus of \( M' \). Let \((k_\sigma)_{\sigma \neq \tau} = (\kappa_\sigma)_{\sigma \neq \tau}) \in \prod_{\sigma \neq \tau} \mathbb{Z}_{\mathbb{p}}^n\) be fixed. Under the assumption \( F_{\mathbb{p}} = \mathbb{Q}_p \), \( T_{\tau} \) is defined over \( \mathbb{Q}_p \). Let \( W \) be the rigid analytic space over \( L \) associated to the Iwasawa algebra \( O_L[[T_{\tau}(\mathbb{Z}_{\mathbb{p}})]] \), then it is defined over \( \mathbb{Q}_p \). It can also be defined as \( W := \text{Hom}(T_{\tau}(\mathbb{Z}_{\mathbb{p}}), \mathbb{C}^{\tau\text{rig}}_{\mathbb{p}}L) \), such that for any affinoid algebra \( A \) over \( L \), \( W(A) \) is the set of continuous homomorphisms \( \text{Hom}(T_{\tau}(\mathbb{Z}_{\mathbb{p}}), A^\times) \). We have an embedding:
\[
W(\mathbb{C}_p) = \text{Hom}(T_{\tau}(\mathbb{Z}_{\mathbb{p}}), \mathbb{C}_p^\times) \hookrightarrow \text{Hom}(\bigotimes_{\sigma \in \Sigma} T_{\sigma}, \mathbb{C}_p^\times) \quad \kappa_{\tau} \mapsto (\kappa_\tau, (k_\sigma)_{\sigma \neq \tau}).
\]
We will identify \( W(\mathbb{C}_p) \) with its image under the above embedding, and call it the weight space. In particular, it contains the integral weights \( \mathbb{Z}^{n-1}_{\mathbb{p}} \times \mathbb{Z} \times (k_\sigma)_{\sigma \neq \tau} \). The goal of this paper is to put the automorphic bundles associated to weights in the above set into a family. More precisely, we will define a sheaf \( \omega^{\kappa_{\tau}} \) for any \( \kappa_{\tau} \in W \) over the rigid analytic Shimura variety \( X^{\tau\text{rig}} \) associated to \( X \) over \( L \), which after tensor product with \( \bigotimes_{\sigma \neq \tau} \omega^\kappa_{\sigma} \) interpolates the classical automorphic bundles in some suitable sense. In fact, we can only define these sheaves over some admissible opens of \( X^{\tau\text{rig}} \). So we need better understanding of the geometry of \( X^{\tau\text{rig}} \), which is closely related to the geometry of the special fiber \( \overline{Y} \).

Recall by GAGA, \( H^0(X, \omega^\kappa) = H^0(X^{\tau\text{rig}}, \omega^\kappa) \) for the integral weight \( \kappa \).

### 2.3. The geometry of special fibers and rigid generic fibers

Before going further to the theory of automorphic forms, we shall review some basic geometric facts on the special fibers and rigid generic fibers of \( S_{K_0} \) and \( S_K \).

The Newton stratification of \( \overline{Y} \) was studied in details in [16]. Recall this stratification
\[
\overline{Y} = \bigoplus_{h=0}^{n-1} \overline{Y}^h
\]
with \( \dim \overline{Y}^h = h \), a point \( x \in \overline{Y}^h \) if and only if the étale part of the corresponding one dimensional \( p \)-divisible group \( H_x \) has height \( h \). In particular \( \overline{Y}^{n-1} \) is the \( \mu \)-ordinary locus, which is open and dense in \( \overline{Y} \). The usual ordinary locus of \( \overline{Y} \) is non empty if and only if \( F_{\mathbb{p}} = \mathbb{Q}_p \) ([24]), in which case it is also open and dense. In the following of this paper, we will assume \( F_{\mathbb{p}} = \mathbb{Q}_p \) to use the theory of canonical subgroups. However, see the remark in the final section for general case. Consider the rigid analytic space \( Y^{\tau\text{rig}} \) associated to \( Y \). We have the stratification of rigid analytic spaces over \( L \)
\[
Y^{\tau\text{rig}} = \bigoplus_{h=0}^{n-1} Y^{\tau\text{rig}}^h,
\]
here \( Y^{\tau\text{rig}} \subset Y^{\tau\text{rig}} \) is the tube over \( \overline{Y}^h \). For a point \( x \in Y^{\tau\text{rig}}^h \), we have the associated one dimensional \( p \)-divisible group \( H_x/O_{k(x)} \) with \( htH_x = n \). Here and in the following \( k(x) \) is
the residue field of $x$ and $O_{k(x)}$ is the integer ring. We have the local-étale exact sequence of $p$-divisible groups over $O_{k(x)}$:

$$0 \to H^0_x \to H_x \to H^{et}_x \to 0,$$

with $htH^0_x = n - h$, $htH^{et}_x = h$.

Recall $S_K$ parameterizes the set of total flags of $\mathcal{H}[p]$, where $\mathcal{H}/S_{k_0}$ is the universal one dimensional $p$-divisible group over $S_{k_0}$, see [24]. As in last subsection, let $X^{rig}$ be the rigid analytic space associated to $X$ over $L$. Now for the varieties $\overline{X}, X^{rig}$, we have also the Newton stratification

$$\overline{X} = \coprod_{h=0}^{n-1} \overline{X}^h, X^{rig} = \coprod_{h=0}^{n-1} |\overline{X}^h|.$$

In our special unitary case of signature $(1, n - 1) \times (0, n) \times \cdots \times (0, n)$, the Newton stratification and the $p$-rank stratification coincide. As in [24], we know the irreducible components of $\overline{X}$ are

$$X_i = \{ x \in \overline{X} | \text{Fil}_{ix} \text{ is connected} \}$$

for the filtration $0 \subseteq \text{Fil}_{1x} \subseteq \cdots \subseteq \text{Fil}_{(n-1)x} \subseteq H_x[p]$. For any non empty subset $S \subset \{1, \ldots, n\}$, let $X_S = \bigcap_{i \in S} X_i$, $X^0_S = X_S - \bigcup_{i \in S, j \in \{1, \ldots, n\} - S} (X_i \cap X_j)$, then by loc. cit.

$$\overline{X}^h = \coprod_{S : \text{satisfies } n - h} X^0_S.$$

In particular, $|\overline{X}^{n-1}| = \coprod_{i=1}^{n} |X^0_i|$. The locus

$$|X^0_1| \subset |\overline{X}^{n-1}|$$

is sometimes called “multiplicative-ordinary” locus in our case $F_\omega = \mathbb{Q}_p$.

3. OVERCONVERGENT AUTOMORPHIC FORMS

3.1. Canonical subgroups and applications. Let the notation and assumption be as before. Let $\mathcal{H}/\overline{Y}$ be the universal $p$-divisible group of dimension one over the special fiber $\overline{Y}$. Then we have the Frobenius and Verschiebung morphisms $F : \mathcal{H} \to \mathcal{H}^{(p)}, V : \mathcal{H}^{(p)} \to \mathcal{H}$. Let

$$V^* : \omega_\mathcal{H} \to \omega^{\otimes p}_\mathcal{H}$$

be the induced morphism on cotangent bundles. Then the determinant of $V^*$ defines a section $Ha \in H^0(\overline{Y}, \omega^{\otimes (p-1)}_\mathcal{H})$. We know that the non-vanishing locus of $Ha$ is the ordinary locus $\overline{Y}^{n-1}$.

By the construction, there is natural lift of $Ha$ to characteristic 0. Let $K$ be a complete valued extension of $\mathbb{Q}_p$ for a valuation $v : K \to \mathbb{R} \cup \{\infty\}$ such that $v(p) = 1$ (Here and in the following, since the level is fixed, by abuse of notation $K$ will denote a complete extension of $\mathbb{Q}_p$. In any case, the precise meaning should be clear from the context.). We denote by $O_K$ the ring of integers of $K$ and let $v : O_K / pO_K \to [0, 1]$ be the truncated valuation. For any $w \in v(O_K)$ we set $\mathfrak{m}(w) = \{x \in K | v(x) \geq w\}$ and $O_{K,w} = O_K / \mathfrak{m}(w)$. Let $H/O_K$ be a $p$-divisible of dimension one, and $G = H[p]$ be the truncated $p$-divisible group of level one. Consider the group $G_{O_{K,w}}$. Then we have an element in the $O_{K,w}$-module $Ha(H) \in \omega^{\otimes (p-1)}_G$. Let $Hdg(H) := v(Ha(H)) \in [0, 1]$. Recall we have the following theorem.
Theorem 3.1 ([14], Théorème 6). Let $m \geq 1$ be an integer. Assume that $Hdg(H) < \frac{1}{2p^m}$ (resp. $\frac{1}{3p^m}$ if $p = 3$). Then the first step of the Harder-Narasimhan filtration of $H[p^m]$, denoted $C_m$ is called the canonical subgroup of level $m$ of $H$. It has the following properties.

1. $C_m(\mathcal{K}) \simeq \mathbb{Z}/p^m\mathbb{Z}$.
2. $\deg C_m = 1 - \frac{v^m}{p-1}Hdg(H)$.
3. For any $1 \leq k < m$, $C_m[p^k]$ is the canonical subgroup of level $k$ of $H$.
4. In $H_{O_{K,1-Hdg(H)}}$ we have that $C_{1O_{K,1-Hdg(H)}}$ is the kernel of Frobenius.
5. For any $1 \leq k < m$, $Hdg(H/C_k) = p^kHdg(H)$ and $C_m/C_k$ is the canonical subgroup of level $m - k$ of $H/C_k$.
6. Let $H^D$ be the Cartier-Serre dual of $H$. Denote by $C_m^\perp$ the annihilator of $C_m$ under the natural pairing $H[p^m] \times H^D[p^m] \to \mu_{p^m}$. Then $Hdg(H^D) = Hdg(H)$ and $C_m^\perp$ is the canonical subgroup of level $m$ of $H^D$.

Let $\mathcal{Y}/\text{Spf}O_L$ be the formal completion of $S_{K_0}$ along the special fiber $\overline{Y}$. Since the variety $Y$ is proper, we have $\mathcal{Y}^{rig} = Y^{rig}$. Then by [14] 2.2.2 there is a continuous function $Hdg : Y^{rig} \to [0, 1]$ such that for any point $x \in Y^{rig}$, $Hdg(x) = Hdg(x)$. Here $H_x$ is the $p$-divisible group over $O_{k(x)}$ associated to $x$. By our construction, we have

$$|\mathcal{Y}^{rig}| = Hdg^{-1}(0),$$

and for any $\varepsilon \in [0, 1] \cap v(L^X)$,

$$Y(\varepsilon) := Hdg^{-1}([0, \varepsilon])$$

is an admissible open subset of $Y^{rig}$, which is a strict neighborhood of $|\mathcal{Y}^{rig}|$.

As in [1], we let $\text{Adm}$ be the category of admissible $O_L$-algebras, i.e. flat $O_L$-algebras which are quotients of rings of restricted power series $O_L(X_1, \ldots, X_r)$ for some $r > 0$. Let $\mathbf{NAdm}$ be the category of normal admissible $O_L$-algebras. For any object $R$ of $\text{Adm}$, we let $R-\text{Adm}$ be the category of $R$-algebras which are admissible as $O_L$-algebras. We define similarly $R-\mathbf{NAdm}$.

Fix an object $R$ of $\mathbf{NAdm}$. Let $S = \text{Spec}R$ and $S^{rig}$ be the rigid analytic space associated to the formal scheme $\hat{S} := \text{Spf}R$. Let $H/S$ be a $p$-divisible group of dimension one and constant height $n$. Assume that there is a $v < \frac{1}{2p^m}$ (resp. $v < \frac{1}{3p^m}$ if $p = 3$) such that for any $x \in S^{rig}$, $Hdg(x) < v$. Then for any point $x \in S^{rig}$, $H_x$ has a canonical subgroup of level $m$. By the properties of the Harder-Narasimhan filtration there is a finite flat subgroup $C_{m,L} \subset H^{S^{rig}}$ interpolating the canonical subgroups of level $m$ for all the points $x \in S^{rig}$. If $w \in v(O_L)$ we set $R_w = R \otimes O_{L,w}$ and for any $R$-module $M$, set $M_w = M \otimes R_w$.

Proposition 3.2 ([1], Prop.4.1.3, 4.2.1, 4.2.2).

1. The subgroup $C_{m,L}$ extends to a finite flat subgroup scheme $C_m \subset H[p^m]$ over $S$.
2. Let $w \in v(O_L)$ with $w \leq m - \frac{v^m}{p-1}$. The morphism of coherent sheaves $\omega_{H[p^m]} \to \omega_{C_m}$ induces an isomorphism $\omega_{H[p^m],w} \to \omega_{C_m,w}$.
3. Assume $C_m^D(R) \simeq \mathbb{Z}/p^m\mathbb{Z}$. Then the cokernel of the linearized Hodge-Tate map

$$HT_{C_m^D} \otimes 1 : C_m^D(R) \otimes R \to \omega_{C_m}$$

is killed by $p^{\frac{n}{m-1}}$.

Similarly, we have a finite flat subgroup scheme $C_m^D \subset H^D[p^m]$ over $S$ with the same properties as in the above (2) and (3), whose generic fiber interpolates the canonical subgroups of level $m$ in $H^D_x$ for all points $x \in S^{rig}$. 
We fix a rational number $v$ such that $v < \frac{1}{2p^{m-1}}$ (resp. $v < \frac{1}{3p^{m-1}}$ if $p = 3$) with the property that for any point $x \in S^{\nu^g}$, $Hdg(x) < v$. Let $C_m$ denote the canonical subgroup of $H$ of level $m$ over $S$. Consider the dual, we have the canonical subgroup $C^*_m$ of $H_D$ of level $m$ over $S$. In the following, we will assume $C^*_m(R) \simeq \mathbb{Z}/p^m\mathbb{Z}$, $(C^*_m)^D(R) \simeq (\mathbb{Z}/p^m\mathbb{Z})^{n-1}$. Then Prop. 4.3.1 of loc. cit. gives us the following sheaves.

- There is a free sub-sheaf of $R$-modules $\mathcal{F}^+$ of $\omega_H$ of rank 1 containing $p^{\frac{v}{p-1}} \omega_H$ which is equipped, for all $w \in [0, m - v \frac{p^m}{p-1}]$, with a map

$$HT^+_w : C^*_m(R[1/p]) \to \mathcal{F}^+ \otimes_R R_w$$

deduced from $HT_{C^*_m}$ which induces an isomorphism

$$HT^+_w \otimes 1 : C^*_m(R[1/p]) \otimes R_w \to \mathcal{F}^+ \otimes_R R_w.$$  

- There is a free sub-sheaf of $R$-modules $\mathcal{F}^-$ of $\omega_{H_D}$ of rank $n - 1$ containing $p^{\frac{v}{p-1}} \omega_{H_D}$ which is equipped, for all $w \in [0, m - v \frac{p^m}{p-1}]$, with a map

$$HT^-_w : (C^*_m)^D(R[1/p]) \to \mathcal{F}^- \otimes_R R_w$$

deduced from $HT_{(C^*_m)^D}$ which induces an isomorphism

$$HT^-_w \otimes 1 : (C^*_m)^D(R[1/p]) \otimes R_w \to \mathcal{F}^- \otimes_R R_w.$$  

Then we have the sum maps

$$HT_w = HT^+_w \oplus HT^-_w : C^*_m(R[1/p]) \oplus (C^*_m)^D(R[1/p]) \to \mathcal{F}^+ \otimes_R R_w \oplus \mathcal{F}^- \otimes_R R_w,$$

$$HT_w \otimes 1 : C^*_m(R[1/p]) \otimes R_w \oplus (C^*_m)^D(R[1/p]) \otimes R_w \to \mathcal{F}^+ \otimes_R R_w \oplus \mathcal{F}^- \otimes_R R_w.$$

Let the notations and assumptions be as in the above proposition. Let $\mathcal{GR} \to S$ be the flag variety parameterizing all flags $\text{Fil}_0 \mathcal{F}^- = 0 \subset \text{Fil}_1 \mathcal{F}^- \subset \cdots \subset \text{Fil}_{n-1} \mathcal{F}^- = \mathcal{F}^-$ of the free module $\mathcal{F}^-$. We can view it parameterizes flags of $\mathcal{F} := \mathcal{F}^- \oplus \mathcal{F}^+$ of the form $\text{Fil}_i(\mathcal{F}) = \text{Fil}_i \mathcal{F}^-$ for $i = 0, 1, \ldots, n - 1$ and $\text{Fil}_n \mathcal{F} = \mathcal{F}$. Note that when $n = 2$, we have $\mathcal{GR} = S$. Let $\mathcal{GR}^+$ be the $T^*_\tau = (\mathbb{G}_m)^n$-torsor over $\mathcal{GR}$ which parameterizes flags $\text{Fil}_\bullet \mathcal{F}$ together with basis $v_i$ of the graded pieces $G_{\tau_i} \mathcal{F}^-$ for $1 \leq i \leq n - 1$ and basis $v_n$ of $\mathcal{F}^+$.

We fix isomorphisms $\psi^+ : \mathbb{Z}/p^m\mathbb{Z} \cong C^*_m(R[1/p])$, $\psi^- : (\mathbb{Z}/p^m\mathbb{Z})^{n-1} \cong (C^*_m)^D(R[1/p])$. Let $x_1, \ldots, x_{n-1}, x_n$ be the $\mathbb{Z}/p^m\mathbb{Z}$-basis of $(C^*_m)^D(R[1/p]) \oplus (C^*_m)^D(R[1/p])$ corresponding to the canonical basis of $(\mathbb{Z}/p^m\mathbb{Z})^{n-1} \oplus \mathbb{Z}/p^m\mathbb{Z}$. By $\psi = (\psi^+, \psi^-)$ we obtain a flag $\text{Fil}_\psi = \{0 \subset (x_1) \subset (x_1, x_2) \subset \cdots \subset (x_1, \ldots, x_n) = (C^*_m)^D(R[1/p]) \oplus C^*_m(R[1/p])\}$. Let $\overline{v_i}$ be the basis of the graded pieces. Let $R'$ be an object in $R-\text{Adm}$. We say that an element $\text{Fil}_\bullet \mathcal{F} \otimes R' \in \mathcal{GR}(R')$ is $w$-compatible if $\text{Fil}_\bullet \mathcal{F} \otimes R'_w = HT_w(\text{Fil}_\psi) \otimes R'_w$. We say that an element $(\text{Fil}_\bullet \mathcal{F} \otimes R', \{v_i\}) \in \mathcal{GR}^+(R')$ is $w$-compatible if $\text{Fil}_\bullet \mathcal{F} \otimes R'_w = HT_w(\text{Fil}_\psi) \otimes R'_w$ and $v_i$ mod $p^w \mathcal{F} \otimes R' + \text{Fil}_{i-1} \mathcal{F} \otimes R' = HT_w(\overline{v_i})$.

We define functors

$$\mathcal{M}_w : R - \text{Adm} \to \text{SET}$$

$$R' \to \{w-\text{compatible } \text{Fil}_\bullet \mathcal{F} \otimes R' \in \mathcal{GR}(R')\},$$

$$\mathcal{M}^+_w : R - \text{Adm} \to \text{SET}$$

$$R' \to \{w-\text{compatible } (\text{Fil}_\bullet \mathcal{F} \otimes R', \{v_i\}) \in \mathcal{GR}^+(R')\}.$$  

These two functors are representable by affine formal schemes, for more detail description see [1] 4.5. We only remark that $\mathcal{M}^+_w$ is a torsor over $\mathcal{M}_w$ under $\mathcal{M}_w$. Where $\mathfrak{T}_w$ is the formal torus defined by

$$\mathfrak{T}_w(R') = \text{Ker}(T\tau(R') \to T\tau(R'/\omega^w R'))$$
for any object \( R' \) in \( \text{Adm} \). All these constructions are functorial in \( R \). They do not depend on \( m \) but only on \( w \).

3.2. The overconvergent sheaves \( \omega_{w}^{\text{nc}} \). Recall we have the Shimura variety \( Y \). On the associated rigid space \( Y^{\text{rig}} \), we have a continuous function \( Hdg : Y^{\text{rig}} \to [0, 1] \). For \( v \in [0, 1] \), we have the open subset \( Y(v) = Hdg^{-1}([0, v]) \). There is a formal model \( \mathcal{Y}(v) \) of \( Y(v) \) by suitable blow-up and normalization, see [1] 5.2.

Let \( m \geq 1 \) be an integer and \( v \in v(\mathcal{O}_{K}) \) such that \( v < \frac{1}{2p^{m}} \) (resp. \( v < \frac{1}{dp^{m}} \) if \( p = 3 \)). We have canonical subgroups \( C_{m}, C_{m}^{\perp} \) of level \( m \) over \( Y(v) \). Let

\[
X_{1}(p^{m})(v) = \text{Isom}_{Y(v)}(\mathbb{Z}/p^{m}\mathbb{Z}, C_{m}^{D}) \times \text{Isom}_{Y(v)}((\mathbb{Z}/p^{m}\mathbb{Z})^{n-1}, (C_{m}^{\perp})^{D})
\]

be the finite étale cover of \( Y(v) \). Let \( X_{1}(p^{m})(v) \) be the normalization of \( \mathcal{Y}(v) \) in \( X_{1}(p^{m})(v) \).

Let \( B_{\tau} \subset GL_{n-1} \times GL_{1} \) be the Borel subgroup which contains \( T_{\tau} \) with unipotent radical \( U_{\tau} \) (when \( n = 2 \), \( B_{\tau} = T_{\tau}, U_{\tau} \) is trivial). Set \( \mathfrak{X}(p^{m})(v) = X_{1}(p^{m})(v)/B_{\tau}(\mathbb{Z}/p^{m}\mathbb{Z}) \). We have the following modular interpretations for the formal schemes \( \mathfrak{X}(p^{m})(v) \) and \( \mathfrak{X}(p^{m})(v) \).

**Proposition 3.3.** For any object \( R \) in \( N\text{Adm} \),

1. \( \mathfrak{X}(p^{m})(v)(R) \) is the set of isomorphic classes of \( (A, \iota, \lambda, \eta, \psi^{+}, \psi^{-}) \), where \( (A, \iota, \lambda, \eta) \in \mathcal{Y}(R) \), and for any rigid point \( \tau \) in \( R \), \( Hdg(H_{\tau}) \leq v \); \( \psi^{+} : \mathbb{Z}/p^{m}\mathbb{Z} \simeq C_{m}^{D}, \psi^{-} : (\mathbb{Z}/p^{m}\mathbb{Z})^{n-1} \simeq (C_{m}^{\perp})^{D} \) are trivialization of canonical subgroups of level \( m \) over \( R_{[p]}^{1} \).

2. \( \mathfrak{X}(p^{m})(v)(R) \) is the set of isomorphic classes \( (A, \iota, \lambda, \eta, \text{Fil}^{+}, \text{Fil}^{-}) \), where \( (A, \iota, \lambda, \eta) \in \mathcal{Y}(R) \), and for any rigid point \( \tau \) in \( R \), \( Hdg(H_{\tau}) \leq v \); \( \text{Fil}^{+} \) (resp. \( \text{Fil}^{-} \)) is a full flag \( \text{Fil}^{+} \) of \( H[p^{m}] \) over \( R_{[p]}^{1} \) such that \( \text{Fil}^{1} = C_{m} \).

**Proof.** (1) is clear from the construction. For (2), by definition \( \mathfrak{X}(p^{m})(v)(R) \) is the set of isomorphic classes \( (A, \iota, \lambda, \eta, \text{Fil}^{+}, \text{Fil}^{-}) \), where \( (A, \iota, \lambda, \eta) \in \mathcal{Y}(R) \), and for any rigid point \( \tau \) in \( R \), \( Hdg(H_{\tau}) \leq v \); \( \text{Fil}^{+} \) (resp. \( \text{Fil}^{-} \)) is a full flag of \( C_{m} \) (resp. \( C_{m}^{\perp} \)). One can easily translate \( \text{Fil}^{+} \) and \( \text{Fil}^{-} \) as a full flag of \( H[p^{m}] \) such that \( \text{Fil}^{1} = C_{m} \). \( \square \)

We will identify the formal scheme \( \mathfrak{X}(p)(v) \) as a sub formal scheme of \( \mathfrak{X}(v) \), and simply write it as \( \mathfrak{X}(v) \).

Let \( w \in v(\mathcal{O}_{L}) \cap \mathbb{Z}[m-1 + \frac{w}{p^{m}} - m - \frac{w}{p^{m}}] \). Let \( H/\mathfrak{X}_{1}(p^{m})(v) \) be the universal \( p \)-divisible group. Applying the construction in last subsection, we have locally free sub-sheaves \( \mathcal{F}^{\perp} \subset \omega_{H, \mathfrak{X}_{1}(p^{m})(v)}, \mathcal{F}^{-} \subset \omega_{H, \mathfrak{X}_{1}(p^{m})(v)} \). They are equipped with isomorphisms:

\[
(HT_{w} \circ \psi^{+}) \otimes 1 : \mathbb{Z}/p^{m}\mathbb{Z} \otimes \mathcal{O}_{\mathfrak{X}_{1}(p^{m})(v)}/p^{m\mathcal{O}_{\mathfrak{X}_{1}(p^{m})(v)}} \simeq \mathcal{F}^{\perp} \otimes \mathcal{O}_{L, w},
\]

\[
(HT_{w} \circ \psi^{-}) \otimes 1 : (\mathbb{Z}/p^{m}\mathbb{Z})^{n-1} \otimes \mathcal{O}_{\mathfrak{X}_{1}(p^{m})(v)}/p^{w\mathcal{O}_{\mathfrak{X}_{1}(p^{m})(v)}} \simeq \mathcal{F}^{-} \otimes \mathcal{O}_{L, w}.
\]

We have a chain of formal schemes:

\[
\mathfrak{M}_{w}^{+} \xrightarrow{\mathfrak{F}_{-}} \mathfrak{M}_{w}^{\perp} \xrightarrow{\mathfrak{F}_{-}} \mathfrak{X}_{1}(p^{m})(v) \xrightarrow{\mathfrak{F}_{-}} \mathfrak{X}(p^{m})(v) \xrightarrow{\mathfrak{F}_{-}} \mathfrak{X}(v).
\]

Recall that \( \mathfrak{M}_{w}^{+} \) is a torsor over \( \mathfrak{M}_{w}^{\perp} \) under the formal torus \( \mathfrak{T}_{w} \). Let \( \mathfrak{B}_{w} \) be the formal group defined by

\[
\mathfrak{B}_{w}(R) = \text{Ker}(B_{\tau}(R) \to B_{\tau}(R/p^{w}R))
\]

for all \( R \in \text{Adm} \). Then there is a surjective map \( \mathfrak{B}_{w} \to \mathfrak{T}_{w} \) with kernel \( \mathfrak{T}_{w} \). Then we have an action of \( B_{\tau}(\mathbb{Z}_{p})\mathfrak{B}_{w} \) on \( \mathfrak{M}_{w}^{+} \) over \( \mathfrak{X}(p^{m})(v) \) (with \( \mathfrak{T}_{w} \) acting trivially).

Recall our weight space \( \mathcal{W} \) with \( \mathcal{W}(\mathbb{C}_{p}) = \text{Hom}(T_{\tau}(\mathbb{Z}_{p}), \mathbb{C}_{p}^{\times}) \). As in the definition 2.2.1 of [1], for \( w \in \mathbb{Q}_{>0} \), a character \( \kappa \in \mathcal{W}(\mathbb{C}_{p}) \) is called \( w \)-analytic if it extends to an analytic map \( \kappa : T_{\tau}(\mathbb{Z}_{p})(1 + p^{m}\mathbb{O}_{\mathbb{C}_{p}^{\times}})^{n} \to \mathbb{C}_{p}^{\times} \). Moreover, \( \mathcal{W} \) has an increasing cover by affinoids \( \mathcal{W} = \bigcup_{w>0} \mathcal{W}(w) \), such that the restriction of the universal character \( \kappa^{\text{un}} \) of \( \mathcal{W} \) to \( \mathcal{W}(w) \) is \( w \)-analytic. Let \( \kappa = \kappa_{w} \in \mathcal{W}(K) \) be a \( w \)-analytic character, where \( K|L \) is a finite extension.
Then we have the total character \( \tilde{\kappa} = (\kappa, (\kappa_\sigma)_{\sigma \neq \tau}) \), with the characters \((\kappa_\sigma)_{\sigma \neq \tau}\) fixed as in the subsection 2.2. The involution

\[
\kappa = (k_1, \ldots, k_{r-1}, k_r) \mapsto \kappa' = (-k_{r-1}, \ldots, -k_1, k_r)
\]

of \( X^*(T_\tau) \) extends to an involution of \( W \) mapping \( w \)-analytic characters to \( w \)-analytic characters. Let \( \mathfrak{W} \) be the formal model of \( W \) defined by \( \text{Hom}(T_\tau(\mathbb{Z}_p), \mathcal{O}_m) \). Then \( \mathcal{W}(K) = \mathfrak{W}(O_K) \). The character \( \kappa' : T_\tau(\mathbb{Z}_p) \to O_K^\times \) extends to a character \( \kappa' : B_\tau(\mathbb{Z}_p)\mathfrak{B}_w \to O_K^\times \) with \( U_\tau(\mathbb{Z}_p)\mathfrak{U}_w \) acting trivially. Set \( \pi = \pi_1 \circ \pi_2 \circ \pi_3 \circ \pi_4 \). Let \( \hat{\omega}^\tau \) be the formal completion of the integral bundle \( \otimes_{\sigma \neq \tau} \omega_{w,\sigma}^T \) over \( S_K \). This is a sheaf over \( X \). Since \((\kappa_\sigma)_{\sigma \neq \tau}\) is fixed, the weights of our automorphic forms depend only on \( \kappa = \kappa_\tau \).

**Definition 3.4.**

1. The formal Banach sheaf of \( w \)-analytic, \( v \)-overconvergent automorphic forms of weight \( \kappa \) is

\[
\mathfrak{m}^{1\kappa}_w = \pi_\ast O_{\mathfrak{W}_w}[\kappa'] \otimes \hat{\omega}^\tau|_{X(v)}.
\]

2. The space of integral \( w \)-analytic, \( v \)-overconvergent automorphic forms of weight \( \kappa \) over \( X \) is

\[
M^{1\kappa}(X(v)) = H^0(X(v), \mathfrak{m}^{1\kappa}_w).
\]

Let \( \kappa, m, v, w \) satisfy all the compatible conditions for the existence of \( \mathfrak{m}^{1\kappa}_w \). If \( v' < v \) then \( \kappa, m, v', w \) satisfy also the conditions and the sheaf \( \mathfrak{m}^{1\kappa}_{v'} \) on \( X(v') \) is the restriction of the sheaf on \( X(v) \). For any \( w' > w \), one can find \( m' \) such that \( \kappa, m', v, w' \) satisfy the conditions, and one has a map \( \mathfrak{m}^{1\kappa}_w \to \mathfrak{m}^{1\kappa}_{w'} \) and thus a map \( M^{1\kappa}_{w}(X(v)) \to M^{1\kappa}_{w'}(X(v)) \).

**Definition 3.5.** Let \( \kappa \in \mathcal{W} \). The space of integral locally analytic overconvergent automorphic forms of weight \( \kappa \) over \( X \) is

\[
M^{+\kappa}(X) = \lim_{v \to 0, w \to \infty} M^{1\kappa}_w(X(v)).
\]

Let \( \mathcal{D}_w^+ \) and \( \mathcal{D}_w \) be the rigid spaces associated to \( \mathfrak{W}_w^+ \) and \( \mathfrak{W}_w \) respectively. They are admissible opens of the rigid spaces associated to \( GR^+ \) and \( GR \) respectively. Let \( \mathfrak{T}_w \) be the rigid space associated to the formal torus \( \mathfrak{T}_\tau \). Then \( \mathcal{D}_w^+ \) is a \( T_w \)-torsor over \( \mathcal{D}_w \). The rigid spaces associated to \( X(p^m)(v) \) (resp. \( X(v) \)) will be denoted by \( X(p^m)(v) \) (resp. \( X(v) \)). Note that \( X(v) \) is a strict neighborhood of the tube \( |X^0| \) over the multiplicative-ordinary locus, see subsection 2.3. Moreover, \( X(0) = |X^0| \). We have a chain of rigid spaces:

\[
\mathcal{D}_w^+ \to \mathcal{D}_w \to X_1(p^m(v)) \to X(p^m)(v) \to X(v).
\]

As in \([1]\), let \( X^+(p^m)(v) = X_1(p^m)(v)/U_\tau(\mathbb{Z}/p^m\mathbb{Z}) \), which is the rigid space associated to \( X^+(p^m)(v) = X_1(p^m)(v)/U_\tau(\mathbb{Z}/p^m\mathbb{Z}) \), then \( \mathcal{D}_w^+ \) (resp. \( \mathcal{D}_w \)) descends to a rigid space \( \mathcal{D}_w^0 \) (resp. \( \mathcal{D}_w^0 \)) over \( X(p^m)(v) \) (resp. \( X(p^m)(v) \)). Moreover, \( \mathcal{D}_w^0 \) is a \( T_w \)-torus over \( \mathcal{D}_w^0 \). Recall the \( GL_{n-1} \times GL_1 \times \prod_{\sigma \neq \tau} GL_n \)-torsor \( T \) over \( X \). We have the decomposition \( T = T_\tau \times T^\tau \), where \( T_\tau \) (resp. \( T^\tau \)) is the \( GL_{n-1} \times GL_1 \)-torsor (resp. \( \prod_{\sigma \neq \tau} GL_n \)-torsor) over \( X \). Let \( T^{rig}, T^{rig}_\tau, T^{rig,rig}_\tau \) be the rigid analytic spaces associated to \( T, T_\tau \) and \( T^\tau \) respectively. For \( w > m - 1 + \frac{v}{p-1} \), we have open immersions \((1)\text{ Prop. 5.3.1})

\[
\mathcal{D}_w^{0+} \hookrightarrow (T^{rig}_\tau / U_\tau)|_{X(v)}, \mathcal{D}_w^{0} \hookrightarrow (T^{rig}_\tau / B_\tau)|_{X(v)}.
\]

Let \( \omega_{w,\tau}^+ \) be the generic fiber of the formal Banach sheaf \( \mathfrak{m}^{1\kappa}_w \). It can be defined by using the morphism \( \mathcal{D}_w^{0+} \to X(v) \) in the same way as in the definition of \( \mathfrak{m}^{1\kappa}_w \).

**Definition 3.6.** Let \( \kappa \in \mathcal{W} \). The space of \( w \)-analytic, \( v \)-overconvergent automorphic forms of weight \( \kappa \) is

\[
M^{1\kappa}_w(X(v)) = H^0(X(v), \omega_{w,\tau}^+).
\]
The space of locally analytic overconvergent automorphic forms of weight $\kappa$ is

$$M^{\kappa}(X) = \lim_{v \to 0, w \to \infty} M^{\kappa}_w(X(v)).$$

Concretely, we can describe a $w$-analytic, $v$-overconvergent automorphic form $f$ of weight $\kappa$ as an element in $H^0(\mathcal{V}^0_w + \mathcal{T}^{\tau, \rig}, \mathcal{O}^{\mathcal{V}^0_w + \mathcal{T}^{\tau, \rig}})$ in the following way (here and in the following we write simply $\mathcal{V}^0_w + \mathcal{T}^{\tau, \rig}$ for the space $\mathcal{V}^0_w + \mathcal{T}^{\tau, \rig}|_{X(v)}$ over $X(v)$). For any finite extension $K'/L$, a $K$-value point of $\mathcal{V}^0_w + \mathcal{T}^{\tau, \rig}$ has the form

$$(A, \lambda, \eta, \overline{\eta}, \Fil_h H[p], \Fil_s F, (v_\tau, (v_\sigma)_{\sigma \neq \tau})), $$

where $(A, \lambda, \eta, \overline{\eta}) \in Y(v)(K)$. $\Fil_h H[p]$ is a full flag of the $p$-torsion subgroup of the associated one dimensional $p$-divisible group $H$ over $O_K$ such that $\Fil_1 H[p] = C_1$, $(\Fil_s F, v_\tau)$ is a full flag in $(\mathcal{T}^{\tau, \rig}/B_\tau)(K)$ with a basis $v_\tau = (v_{1\tau}, \ldots, v_{(n-1)\tau}, v_{n\tau})$ for the graded pieces, such that there is a trivialization $\psi = (\psi^+, \psi^-) : C_0^D(\mathcal{K}) \oplus (C^1_m)^D(\mathcal{K}) \simeq \mathbb{Z}/p^m \mathbb{Z} \oplus (\mathbb{Z}/p^m \mathbb{Z})^{n-1}$ (for some integer $m \geq 1$) which is compatible with $\Fil_h H[p]$, and $(\Fil_s F, v_\tau)$ is $w$-compatible with $\psi$; finally $(v_\sigma)_{\sigma \neq \tau}$ is an element in $\mathcal{T}^{\tau, \rig}(K)$ over $X^{\rig}(K)$. Then

$$f(A, \lambda, \eta, \overline{\eta}, \Fil_h H[p], \Fil_s F, (v_\tau, (v_\sigma)_{\sigma \neq \tau})) \in K$$

such that for all $b \in B_{M'}$ we have

$$f(A, \lambda, \eta, \overline{\eta}, \Fil_h H[p], \Fil_s F, (v_\tau, (v_\sigma)_{\sigma \neq \tau}))(b) = \kappa'(b)f(A, \lambda, \eta, \overline{\eta}, \Fil_h H[p], \Fil_s F, (v_\tau, (v_\sigma)_{\sigma \neq \tau})).$$

The space $M^{\kappa}_w(X(v))$ is a Banach space, with the unit ball $M^{\kappa}_w(\mathcal{X}(v))$. The sheave $\omega^\kappa_w$ has fibres isomorphic to the space $V^w_{\kappa^w-an} \otimes_{\otimes_{\sigma \neq \tau}} V_{\kappa^w}$, where $V^w_{\kappa^w-an}$ is the locally $w$-analytic representation of the Iwahori subgroup $I$ of $GL_{n-1}(\mathbb{Z}_p) \times GL_1(\mathbb{Z}_p)$, which is defined as

$$V^w_{\kappa^w-an} = \{ f : I \to L | f(ib) = \kappa'(b)f(i) \forall (i, b) \in I \times B_\tau(\mathbb{Z}_p), f|_{N^0} \in F^{w-an}(N^0, L) \}.$$ 

Here are some explanations. Recall $I$ has the Iwahori decomposition $I = B_\tau(\mathbb{Z}_p) \times N^0$, where $N^0$ is defined in the following way: let $B_0^0 \subset GL_{n-1} \times GL_1$ be the opposite Borel subgroup of $B_\tau$ with unipotent radical $U_\tau$, then $N^0$ is the subgroup of $U_\tau(\mathbb{Z}_p)$ of matrices which reduce to the identity modulo $p$. $F^{w-an}(N^0, L)$ is the set of $w$-analytic functions, i.e. the functions from $N^0$ to $L$ which are the restrictions to $N^0$ of the unique analytic functions on $N^0 := \bigcup_{x \in N^0} B(x, p^{-w})$, where $B(x, p^{-w})$ is the closed ball with center $x$ radius $p^{-w}$ in the rigid analytic affine space $A^{(n-1)(n-2)/2, \rig}$ over $\mathbb{Q}_p$. Here we identify $N^0$ with $(p\mathbb{Z}_p)^{(n-1)(n-2)/2} \subset A^{(n-1)(n-2)/2, \rig}$. In the case $n = 2$, we have $N^0 = \{ 1 \}$, $I = B_\tau(\mathbb{Z}_p) = T_\tau(\mathbb{Z}_p) = (\mathbb{Z}_p^*)^2$, $V^w_{\kappa^w-an}$ is just the space of $w$-analytic characters.

Recall we have the total character $\tilde{\kappa} = (\kappa, (\kappa_\sigma)_{\sigma \neq \tau})$. When $\kappa \in \mathbb{Z}_+^{n-1} \times \mathbb{Z}$ we have the automorphic vector bundle $\omega^\kappa$ over $X$, which we will denote simply as $\omega^\kappa$ since $(\kappa_\sigma)_{\sigma \neq \tau}$ is fixed. We denote also the corresponding vector bundle over the rigid analytic space $X^{\rig}$ by $\omega^\kappa$. By construction we have the following proposition and corollary.

**Proposition 3.7.** If $\kappa \in \mathbb{Z}_+^{n-1} \times \mathbb{Z}$, then there is a canonical restriction map

$$\omega^\kappa|_{X(v)} \hookrightarrow \omega^\kappa_w$$

induced by the open immersion $\mathcal{V}^0_w \hookrightarrow (\mathcal{T}^{\tau, \rig}/U_\tau)|_{X(v)}$. Locally for the étale topology, this map is isomorphic to the inclusion

$$V^w_{\kappa^w} \otimes V^{\tau} \hookrightarrow V^w_{\kappa^w-an} \otimes V^{\tau}$$

of the algebraic induction into the analytic induction, where $V^{\tau} = \otimes_{\sigma \neq \tau} V_{\kappa_{\sigma}}$. 


Corollary 3.8. For any \( \kappa \in \mathbb{Z}_+^{n-1} \times \mathbb{Z} \), we have an inclusion

\[
H^0(X, \omega_\kappa) \hookrightarrow M^w_\kappa(X(v))
\]

from the space of classical forms of weight \( \kappa \) into the space of \( w \)-analytic, \( v \)-overconvergent automorphic forms of weight \( \kappa \).

4. Hecke operators

4.1. Hecke operators outside \( p \). Consider the set of prime to \( p \) Hecke correspondences \( K^p \setminus G(A^p) / K^p \). For any \( K^p \) \( K^p \), there is an algebraic correspondence \( p_1, p_2 : C_g \to X \) over \( X \). We consider the rigid analytification of this correspondence, then we restrict it over \( X(v) \), so we have the diagram

\[
\begin{array}{ccc}
C_g(v) & \xrightarrow{p_1} & X(v) \\
\vspace{0.5cm}
\end{array}
\]

Over \( C_g \) there is a universal prime to \( p \) isogeny \( \pi : A \to A' \), which induces a map \( \pi^* : \omega_{A'} \to \omega_A \), hence a map \( p^*_2(T^{rig}_\tau / U_\tau \times T^{rig}_\tau) \to p^*_1(T^{rig}_\tau / U_\tau \times T^{rig}_\tau) \) which is an isomorphism. For \( w \in ]m-1 + \frac{n}{p-1}, m - \frac{v}{p-1} \] , the map \( \pi^* \) induces an isomorphism (see [1] Lemma 6.1)

\[
\pi^* : p^*_2(\mathcal{IW}^{0+}_w \times T^{rig}_\tau) \simeq p^*_1(\mathcal{IW}^{0+}_w \times T^{rig}_\tau).
\]

We define the Hecke operator \( T_g \) as the composition:

\[
T_g : H^0(X(v), \omega_\kappa^{w}) \xrightarrow{p_2^*} H^0(C_g(v), p^*_2 \omega_\kappa^{w}) \xrightarrow{\pi^{w+1}} H^0(C_g(v), p^*_1 \omega_\kappa^{w}) \xrightarrow{T_{p_1}} H^0(X(v), \omega_\kappa^{w}).
\]

These operators generate the prime to \( p \) Hecke algebra \( \mathbb{T}^p = \mathcal{H}(G(A^p) / K^p) \), which is the restricted tensor product of all local Hecke algebras \( \mathbb{T}_l = \mathcal{H}(G(A^p) / K_l) \) for primes \( l \neq p \). We know there are only finite primes \( l \) such that \( K_l \) is not hyperspecial maximal. Let \( \mathbb{T}^{K^p} \) be the sub algebra of \( \mathbb{T}^p \) which is the restricted tensor product of all \( \mathbb{T}_l \) such that \( K^p \) is hyperspecial maximal at \( l \). Then \( \mathbb{T}^{K^p} \) is commutative.

4.2. Hecke operators at \( p \). We will define \( n-1 \) Hecke operators at \( p \) as follows. Recall in section 2 the primes of the CM field \( F \) over \( p \) are \( \varpi = \varpi_1, \varpi_2, \ldots, \varpi_r, \varpi_r^2, \ldots, \varpi_r^e \), and we have assumed \( F_{\varpi} = \mathbb{Q}_p \). The \( p \)-divisible group associated to the universal abelian scheme \( A \) over \( X \) has the decomposition

\[
A[p^\infty] = A[\varpi_1^\infty] \oplus \cdots \oplus A[\varpi_r^\infty] \oplus A[\varpi_r^{e+1}] \oplus \cdots \oplus A[\varpi_r^{e+1}],
\]

with \( A[\varpi_i^\infty]D \simeq A[\varpi_i^{e+1}] \) for all \( 1 \leq i \leq r \). The \( p \)-divisible group \( H = \varepsilon^+_1 A[\varpi] \) has dimension one, where \( \varepsilon^+_1 \) is the idempotent introduced in subsection 2.2. For a finite locally free subgroup \( L \subset H[p] = A[\varpi] \), we denote by \( \overline{L} \subset A[p] \) the subgroup such that under the induced decomposition \( \overline{L} = \overline{L}_1 \oplus \cdots \oplus \overline{L}_r \oplus \overline{L}_1^\infty \oplus \cdots \oplus \overline{L}_r^\infty \), we have

\[
\overline{L}_i = \mathbb{Q}_p^n \otimes \overline{L}_i, \quad 2 \leq i \leq r, \quad \overline{L}_1^\infty = \mathbb{Q}_p^n \otimes (A[\varpi]/L)^D, \quad \overline{L}_i^\infty = 0, \quad 2 \leq i \leq r.
\]

Note \( \varepsilon^+_1(A/L)[\varpi^\infty] = H/L \).

Now for \( i = 1, \ldots, n-1 \), let \( C_i \) be the moduli scheme over \( X \) parameterizing finite locally free subgroups \( L \subset H[p] \) such that \( L \oplus \text{Fil}_i H[p] = H[p] \). There are two projections \( p_1, p_2 : C_i \to X \). The first projection is defined by forgetting \( L \). The second projection is defined by mapping \( (A, \lambda, \iota, \eta, \text{Fil}_i H[p], L) \) to \( ((A/L), \lambda', \iota', \eta', \text{Fil}_i(H/L)[p]) \), where the filtration on \( (A/L)[\varpi] = (H/L)[p] \) is defined as follows:

- For \( j = 1, \ldots, i \), \( \text{Fil}_j(H/L)[p] \) is simply the image of \( \text{Fil}_j H[p] \) in \( H/L \),
• For \(j = i + 1, \ldots, n-1\), \(\text{Fil}_j(H/L)[p]\) is the image of \(\text{Fil}_jH[p] + p^{-1}(\text{Fil}_jH[p] \cap L)\) in \(H/L\).

We consider the analytifications \(p_1, p_2 : C^{rig}_i \to X^{rig}\). For an admissible open subset \(V \subset X^{rig}\), we denote the image of \(V\) under the correspondence \(C_i\) by \(U_i(V) := p_2(p_1^{-1}(V))\), which is also an admissible open subset of \(X^{rig}\).

**Remark 4.1.** In the definition of the correspondences \(C_i\) for \(i = 1, \ldots, n-1\), we have taken them by parameterizing \(L\) with \(L \oplus \text{Fil}H[p] = H[p], \tilde{L}_j = A[\omega_j]\) and \(\tilde{L}_j^c = 0\) for \(2 \leq j \leq r\). In fact, as in [22], it is more natural to define the correspondences \(C_i, \omega_j\) for \(i = 1, \ldots, n-1\) by parameterizing \(L\) with \(L \oplus \text{Fil}H[p] = H[p], \tilde{L}_j = \tilde{L}_j^c = 0\) for \(2 \leq j \leq r\). At the places \(\omega_j\) for \(2 \leq j \leq r\), we define correspondences \(C_{\omega_j}\) by parameterizing \(L\) with \(L_j = A[\omega_j], \tilde{L}_k = 0\) for all \(1 \leq k \leq r, k \neq j\). Then \(C_{\omega_j} = X\) for \(2 \leq j \leq r\) and the morphism \(p_2\) is by taking the quotient by \(L = A[\omega_j]\). For \(2 \leq j \leq r\), by construction these quotients do not change the part \(\otimes_{\sigma \neq \omega} \omega^k\). Therefore when passing to operators on the space of automorphic forms (see below), there is no difference between \(U_i = U_{i, \omega_j} \circ \prod_{j=2}^r U_{\omega_j}\) and \(U_{i, \omega_j}\) for \(i = 1, \ldots, n-1\).

We first consider the correspondence \(C_1\). If \(v < \frac{p-2}{2p-2}\), then by the theory of canonical subgroups we know ([14] Prop. 17) \(U_1(X(v)) \subset X(v)_p\).

Let \(C_1(v) = C^{rig}_1 \times_{p_1, X^{rig}} X(v)\). We have the diagram

\[
\begin{array}{ccc}
C_1(v) & \xrightarrow{p_1} & X(v) \\
\downarrow & & \downarrow \\
X(v) & \xrightarrow{p_2} & X(v)_p.
\end{array}
\]

Let \(\pi : A \to A'\) be the universal isogeny over \(C_1(v)\). Then as before it induces an isomorphism

\[
\pi^* : p_2^*(\mathcal{I}W_w^{0+} \times \mathcal{T}^{r,rig})|_{X(v)_p} \simeq p_1^*(\mathcal{I}W_w^{0+} \times \mathcal{T}^{r,rig}).
\]

We define the Hecke operator \(U_1\) as the composition:

\[
H^0(X(v)_p, \omega^k) \xrightarrow{p_2^*} H^0(C_1(v), p_2^*\omega^k_{w'}) \xrightarrow{\pi^*} H^0(C_1(v), p_1^*\omega^k_{w'}) \xrightarrow{p_1^{-1}\mathcal{T}^{r,p_1}} H^0(X(v), \omega^k_{w'}).
\]

By abuse of notation we also denote by \(U_1\) the endomorphism of \(H^0(X(v), \omega^k_{w'})\) defined as the composition of the above operator we just defined with the restriction map \(H^0(X(v), \omega^k_{w'}) \to H^0(X(v)_p, \omega^k_{w'})\).

Next we consider the correspondences \(C_i\) for \(2 \leq i \leq n-1\). Let \(C_i(v) = C^{rig}_i \times_{p_i, X^{rig}} X(v)\). If \(v < \frac{p-2}{2p-2}\), we have a diagram

\[
\begin{array}{ccc}
C_i(v) & \xrightarrow{p_1} & X(v) \\
\downarrow & & \downarrow \\
X(v) & \xrightarrow{p_2} & X(v).
\end{array}
\]

As in [15] 5.6, for \(v < \frac{1}{2^{m-r}}\) (resp. \(v < \frac{1}{3^{m}}\) if \(p = 3\)) and \(w = (w_{k,j})_{1 \leq j \leq k \leq n} \in \Sigma_{p-1,m}^v\), \(w = w_{k+1,j} \geq w_{k,j}, w_{k,j-1} \geq w_{k,j}\), we can introduce a space \(\mathcal{I}W^0_{w'}\) over \(X(v)\) such that for \(w\) with all \(w_{k,j} = w\), \(\mathcal{I}W^0_{w'} = \mathcal{I}W^0_{w'}\). Let \(\pi : A \to A'\) be the universal
isogeny over $C_i(v)$. We have a map $\pi^*: \omega_A \to \omega_A$. It induces a map $\bar{\pi}^*: \mathcal{T}_r \mapsto \mathcal{T}_r$ which sends a basis $v_1, \ldots, v_n$ of $\omega_A \otimes \mathbb{R}$ to $p^{-1}\pi^*v_1, \ldots, p^{-1}\pi^*v_n$. This map is an isomorphism. It induces an isomorphism $\bar{\pi}^{*\pi^*}: p_1^*T_{r,\pi} \mapsto p_1^*T_{r,\pi}$.

Proposition 4.2. Let $v < \inf\{\frac{1}{p^{n+1}}, \frac{p-2}{2p-1}\}$ and $w = (w_{k,j})_{1 \leq j \leq k \leq n}$ with $w_{k,j} \in ]\frac{v}{p^{n+1}}, m - 2 - v\frac{2n}{p-1}]$. Then the isogeny $\pi$ induces $\bar{\pi}^{*\pi^*}: p_1^*\mathcal{T}_r \mapsto p_2^*\mathcal{T}_r$ where $w'_{k,j} = w_{k,j}$ for $j \leq k$ and $w'_{k,j} = 1 + w_{k,j}$ for $j > k$. Let $\omega^{\pm\pi^*\pi}_w$ be the composition of the above operator we just defined with the restriction map $\bar{\pi}^*\pi$.

By abuse of notation we also denote by $U_i$ the endomorphism of $H^0(X(v), \omega^{\pm\pi^*\pi}_w)$ as the composition of the above operator we just defined with the restriction map $H^0(X(v), \omega^{\pi^*\pi}_w) \mapsto H^0(X(v), \omega^{\pi^*\pi}_w)$.

Let $m \geq 1$ be some large integer. For $v < \inf\{\frac{1}{p^{n+1}}, \frac{p-2}{2p-1}\}, w \in ]\frac{v}{p^{n+1}}, m, v\frac{2n}{p-1}]$, and $\kappa$ a $w$-analytic character, the composite operator $\prod_{i=1}^{n-1} U_i$ induces a map $H^0(X(v/w), \omega^{\pi^*\pi}_w) \mapsto H^0(X(v/w), \omega^{\pi^*\pi}_w)$ for some suitable $w'$ with $w'_{k,j} \geq w$. Let $U$ be the composition of $\prod_{i=1}^{n-1} U_i$ with the restriction map $H^0(X(v), \omega^{\pi^*\pi}_w) \mapsto H^0(X(v), \omega^{\pi^*\pi}_w)$. Then we have the following proposition.

Proposition 4.3. $U$ is a compact endomorphism of $M^{\pi^*\pi}_w(X(v)) = H^0(X(v), \omega^{\pi^*\pi}_w)$.

Proof. The operator $U_1$ improves the radius of overconvergence, and for $2 \leq i \leq r$ the operators $U_i$ improve analyticity, therefore all of them are compact. Moreover, the natural restriction map is compact. Hence their composition $U$ is compact.

Let $\mathcal{T}_p$ be the algebra generated by $U_1, \ldots, U_{n-1}$, and $\mathcal{T}^K_p$ be the commutative Hecke algebra defined in the end of last subsection. For the above $v, w, \kappa$, we have the morphism of algebras $\mathcal{T}^K \otimes \mathcal{T}_p \mapsto \operatorname{End}(M^{\pi^*\pi}_w(X(v)))$.

5. Analytic continuation and classicality

Recall we have fixed the weights $(k_\sigma)_{\sigma \neq r} = (k_1, \ldots, k_n)$ and $k_{\sigma \neq r} \in \mathbb{Z}_{\mathbb{P}}$. Let $\kappa = \kappa_r = (k_1, \ldots, k_n, k_n) \in \mathbb{Z}_{\mathbb{P}}^{n-1} \times \mathbb{Z}$ be $w$-analytic for some rational number $0 < w \leq 1$. Then we have the inclusions $H^0(X, \omega^{\kappa}) \mapsto H^0(X(v), \omega^{\kappa}) \mapsto H^0(X(v), \omega^{\pi^*\pi}_w)$. We will establish a criterion for an element in $H^0(X(v), \omega^{\pi^*\pi}_w)$ to be classical, i.e. in the image of $H^0(X, \omega^{\pi^*\pi})$. For $\omega = (a_1, \ldots, a_{n-1}) \in [0, +\infty]^{n-1}$, we set $M^{\pi^*\pi}_w(X(v))^{\omega}$ for the union of the generalized eigenspaces where $U_i$ has finite slope $< a_i$ for $1 \leq i \leq n - 1$. 
Theorem 5.1. Let \( \kappa = (k_1, \ldots, k_{n-1}, k_n) \in \mathbb{Z}_+^{n-1} \times \mathbb{Z} \) be \( w \)-analytic for some rational number \( 0 < w \leq 1 \). Let \( \alpha = (a_1, \ldots, a_{n-1}) \in \mathbb{R}_{\geq 0}^{n-1} \) with \( a_i = k_{n-i} - k_{n-i+1} + 1 \) when \( 2 \leq i \leq n-1 \) and \( a_1 = k_n + k_{n-1} - n + 1 \). Then we have the inclusion
\[
M^w_\kappa(X(v))^< \alpha \subset H^0(X, \omega^\kappa).
\]

The proof of this theorem consists of the following two propositions.

Proposition 5.2. Let \( \alpha' = (+\infty, a_2, \ldots, a_{n-1}) \in [0, +\infty]^{n-1} \) with \( a_i \) the same as in the above theorem for \( 2 \leq i \leq n-1 \). Then we have the inclusion
\[
M^w_\kappa(X(v))^< \alpha' \subset H^0(X(v), \omega^\kappa).
\]

Proof. We argue as [\ref{1}] 7.2 and 7.3. First, there is an exact sequence of sheaves over \( X(v) \)
\[
0 \rightarrow \omega^\kappa \xrightarrow{d_0} \omega^\kappa_w \xrightarrow{d_1} \bigoplus_{\alpha \in \Delta} \omega^\kappa_{w^\alpha \cdot \kappa}.
\]

Here \( \Delta \) is the set of simple roots of \( GL_{n-1} \times GL_1 \), \( s_\alpha \) is the associated element in the Weyl group to \( \alpha \in \Delta \), \( s_\alpha \cdot \kappa \) is the usual dot action, \( d_1 = \bigoplus_{\alpha \in \Delta} \Theta_\alpha : \omega^\kappa_w \rightarrow \bigoplus_{\alpha \in \Delta} \omega^\kappa_{w^\alpha \cdot \kappa} \) is the sum of the maps \( \Theta_\alpha \) which are defined as in subsection 7.2 of loc. cit.. Next, for \( 2 \leq i \leq n-1 \) we have the operators \( \delta_i \) for \( V^{w-an} \) as defined in loc. cit. 2.5 for \( d_i = \left( \begin{array}{c} p^{-1}n_i \\ 1 \\ 1 \end{array} \right) \in GL_{n-1}(\mathbb{Q}_p) \times GL_1(\mathbb{Q}_p) \). \( \delta_i \) extends naturally to an operator on \( V^{w-an}_{\kappa'} \otimes V^\tau \) (by acting trivially on \( V^\tau \), where \( V^\tau \) is as in Prop. 3.7). Then one checks that locally on the fibers, the operators \( U_1 \delta_i \) and \( \delta_i \) are compatible for \( 2 \leq i \leq n-1 \). Using this we get
\[
\Theta_\alpha U_1(f) = \alpha(d_i^{-1} \kappa, \alpha < \kappa, \kappa > \kappa + 1) U_1 \Theta_\alpha(f)
\]
from the corresponding equalities for the representation \( V^{w-an} \) (subsection 2.5 of loc. cit.). One then uses the slope condition as the proof of loc. cit. Prop. 7.3.1 to conclude. \( \Box \)

Proposition 5.3. Let \( H^0(X(v), \omega^\kappa)^{<k_n+k_{n-1}-n+1} \) be the eigenspace where \( U_1 \) has slope \( < k_n + k_{n-1} - n + 1 \). Then it is included in the space of classical forms
\[
H^0(X(v), \omega^\kappa)^{<k_n+k_{n-1}-n+1} \subset H^0(X, \omega^\kappa).
\]

We begin the preparation for the proof of the above proposition. It will be finished in Prop. 5.7. The method which we use here is by studying the analytic continuation of finite slope overconvergent eigenforms as in [\ref{22}]. We remark that there is a related result of Johansson in [\ref{17}] by studying the rigid cohomology for a more restricted sub class of our Shimura varieties.

We define a function of \( \deg \) over \( X^{rig} \) as follows
\[
\deg : X^{rig} \rightarrow [0, 1], \quad x \mapsto \deg(\Fil_1 H_x[p]).
\]

Then it is continuous, and note that \( X(v) = \deg^{-1}([1 - v, 1]) \), \( X(v)^1 = \deg^{-1}(1) \). For any point \( y \in U_1(x) \), we have \( \deg(y) \geq \deg(x) \). Moreover, the equality holds if and only if \( \Fil_1 H_x[p] \) is a truncated \( p \)-divisible group of level one, see [\ref{22}] propositions 3.1.1 and 3.1.2. Therefore any finite slope eigenform for \( U_1 \) in \( H^0(X(v), \omega^\kappa) \) extends to the analytic domain \( \deg^{-1}([0, 1]) \). We would like to extend them further to the whole space \( X^{rig} \). Since in our special case
\[
H^0(X^{rig}, \omega^\kappa) = H^0([X^{n-1}], \omega^\kappa),
\]
it suffices to extend these forms to the tube \(|X^{n-1}\prod X^{n-2}|\). We know that we can do this for the domain
\[ \text{deg}^{-1}([0, 1]) \cap (|X^{n-1}\prod X^{n-2}|). \]
To extend to the remaining domain
\[ \text{deg}^{-1}(0) \cap (|X^{n-1}\prod X^{n-2}|), \]
we will use the condition that the slope for \(U_1\) satisfies \(v(\alpha) < k_n + k_{n-1} - n + 1\) to define some Kassaei series and then glue.

Consider a strict neighborhood \(|X^{n-1}|_\varepsilon\) of \(|X^{n-1}|\) such that the canonical subgroup \(C \subset H[p]\) inside the universal \(p\)-divisible group \(H\) exists. Then the correspondence \(C_1^{\text{rig}}\) has the decomposition
\[ C_1^{\text{rig}} \cap p_1^{-1}(|X^{n-1}|_\varepsilon) = C_1^\leq \sqcup C_1^> \]
when restricts to \(|X^{n-1}|_\varepsilon\), where \(C_1^\leq\) parameterizes the subgroups \(L \subset H[p]\) which has trivial intersection with \(C\), and \(C_1^>\) parameterizes the subgroups \(L \subset H[p]\) which contains \(C\). Then the degree of \(L\) is \(v(\alpha) < k_n + k_{n-1} - n + 1\) to define some Kassaei series and then glue.

Remark 5.4. Here we use our special signature condition \((1, n-1) \times (0, n) \times \cdots \times (0, n)\) to have the decompositions of the correspondence \(C_1^{\text{rig}}\) over \(|X^{n-1}|_\varepsilon\) and \(|X^{n-2}|\). In general, one should work on the tube over each Kottwitz-Rapoport strata to get the decompositions of the related Hecke correspondence, see [22].

Let \(f \in H^0(X(v), \omega^\infty)\) be an overconvergent eigenform for \(U_1\) with eigenvalue \(\alpha \neq 0\). We know that \(f\) extends to a form over \(\text{deg}^{-1}(0) \cap (|X^{n-1}\prod X^{n-2}|),\) which we still denote by \(f\). Assume that \(v(\alpha) < k_n + k_{n-1} - n + 1\). We claim that there exists an overconvergent form \(\tilde{f}\) over \(|X^{n-1}\prod X^{n-2}|\) which extends \(f\).

Let \(\varepsilon\) be a real number such that \(k_n + k_{n-1} - n + 1 - v(\alpha) > \varepsilon > 0\). We choose an \(\varepsilon_0 > 0\) such that over any admissible open \(V \subset |X^{n-1}|_{\varepsilon_0}\), the norm of \(U_1^>\) has the estimate (cf. [22] Lemme. 5.4.6)
\[ \|U_1^>\|_V \leq p^{-k_n - k_{n-1} - n + 1 + \varepsilon}, \]
where for an operator $T : H^0(T(V), \mathcal{F}) \to H^0(V, \mathcal{F})$ ($\mathcal{F}$ is a locally free sheaf equipped with a norm $| \cdot |$), its norm over $V$ is defined as

$$\|T\|_V = \inf \{ \beta \in \mathbb{R}^1 | \|T(f)\|_V \leq \beta |f|_T(V), \forall f \in H^0(T(V), \mathcal{F}) \}.$$ 

Let $\varepsilon_k = (\frac{1}{k})^k \varepsilon_0$ for all $k \geq 1$. Here $N \geq 1$ is such that $U_1^N(\overline{X}^{n-1}[\varepsilon_0]) \subset \overline{X}^{n-1}[N\varepsilon_0]$ (loc. cit. Prop. 5.4.5). For $k \geq 1$, $f$ as above, we define

$$f_k = \sum_{j=0}^{k-1} \alpha^{-j-1}(U_1^N)^j f \circ U_1^\varepsilon(f),$$

which is a well defined element in $H^0(\overline{X}^{n-1}[\varepsilon_k, \omega^\alpha])$. We have the following proposition.

**Proposition 5.5.** For all $k \geq 1$, we have the estimate

$$|f_{k+1} - f_k|_{\overline{X}^{n-1}[\varepsilon_{k+1}] \setminus \overline{X}^{n-1}[\varepsilon_k]} = O(p^{-k(n + k_{n-1} - n) - v(\alpha - \varepsilon)}).$$

**Proof.** By definition over $|\overline{X}^{n-1}[\varepsilon_{k+1}] \setminus \overline{X}^{n-1}[\varepsilon_k]|$

$$f_{k+1} - f_k = \alpha^{-k-1}(U_1^N)^k f \circ U_1^\varepsilon(f).$$

Since the norm of $U_1^\varepsilon$ is always bounded, we can conclude by the estimate for the norm of $U_1^N$. \qed

Over the tube $|\overline{X}^{n-2}|$ we have also the following definition. For $k \geq 1$, $f$ as above, we define

$$f'_k = \sum_{j=0}^{k-1} \alpha^{-j-1}(U_1^N)^j f \circ U_1^\varepsilon(f),$$

which is a well defined element in $H^0(|\overline{X}^{n-2}|, \omega^\alpha)$. For any admissible open $V \subset |\overline{X}^{n-2}|$ we have the following norm estimate

$$\|U_1^N\|_V \leq p^{-k_{n-1} + n - 1}.$$

Similar to the above, we have the proposition.

**Proposition 5.6.** For all $k \geq 1$, we have the estimate

$$|f'_{k+1} - f'_k|_{|\overline{X}^{n-2}|} = O(p^{-k(n + k_{n-1} - n) - v(\alpha)}).$$

Now we can prove the proposition, which will finish the analytic continuation of $f$ to the whole space $X^{rig}$ under the slope condition. Thus it is classical.

**Proposition 5.7.** There exists a unique section of $\omega^\alpha$ over

$$|\overline{X}^{n-1}[\bigcup \overline{X}^{n-2}|$$

which extends $f$.

**Proof.** We consider the larger domain

$$|\overline{X}^{n-1}[\varepsilon_k] \cup |\overline{X}^{n-2}|$$

for all $k \geq 1$. By the above we can define $f_k$ and $f'_k$ over $|\overline{X}^{n-1}[\varepsilon_k]$ and $|\overline{X}^{n-2}|$ respectively. Let $V = |\overline{X}^{n-1}[\varepsilon_0] \cap |\overline{X}^{n-2}|$. We study the norm of $f_k - f'_k$ over $V$. First we consider $V$ as a sub domain of $|\overline{X}^{n-1}[\varepsilon_k]$. Then the operator $U_1$ has a decomposition

$$U_1 = U_1^N \cup U_1^\varepsilon$$
according to the decomposition of the correspondence $C_1^\text{rig}$ when restricting to $p_1^{-1}(V)$. Now we consider $V$ as a sub domain of $X^{n-1}$. The operator $U_1^\geq$ has a further decomposition

$$U_1^\geq = U_1' \cup U_1''$$

where the part $U_1'$ corresponds to the sum over the subgroups $L$ of $H[p]$ such that $L \cap \text{Fil}_1 H[p] = 0$ and $L^{ct} = H[p]^{ct}$ (which have height $n-2$), and the part $U_1''$ corresponds to the sum over the rest subgroups $L$ of $H[p]$ such that $L \cap \text{Fil}_1 H[p] = 0$ and $L^{ct} \subsetneq H[p]^{ct}$ (which have degree 1). Therefore, the image of $U_1'$ is included in $\text{deg}^{-1}([0,1])$. We have

$$f_k - f_k' = -\alpha^k \sum_{j=1}^{k} (U'''_1)^{k-j} \circ (U'_1)^{j}(f).$$

By using similar estimate for the norms of the operators $U_1'$ and $U_1''$, we have

$$|f_k - f_k'|_V = O(p^{-k(n+k+1)-1-v(o)-\varepsilon}).$$

This will suffice to prove the proposition by using Kassaei’s gluing lemma in [19].

6. Families of overconvergent automorphic forms

Let $\mathcal{U} = Sp(A) \subset \mathcal{W}$ be any affinoid open subset. Then there exists $w_{\mathcal{U}} > 0$ such that the universal character $\kappa_{\text{un}} : T_\tau(\mathbb{Z}_p) \times \mathcal{W} \to \mathbb{C}_p^\times$ restricted to $\mathcal{U}$ extends to an analytic character $\kappa_{\text{un}} : T_\tau(\mathbb{Z}_p)(1 + p^m \mathcal{O}_{\mathbb{C}_p}) \times \mathcal{U} \to \mathbb{C}_p^\times$, cf. [1] Prop. 2.2.2. Let $m \in \mathbb{N}, v \leq \frac{1}{2p^{m+1}}$ (resp. $\frac{1}{3p^{m-1}}$ for $p = 3$) and $w \leq m-1 + \frac{v}{p-1}, m-v\frac{p}{p-1}$ satisfying $w \geq w_{\mathcal{U}}$. We have the following proposition which says the construction in section 3 works in families, see also loc. cit. Prop. 8.1.1.1.

**Proposition 6.1.** There exists a sheaf $\omega^{\kappa_{\text{un}}} w$ on $X(v) \times \mathcal{U}$ such that for any weight $\kappa \in \mathcal{U}$, the fiber of $\omega^{\kappa_{\text{un}}} w$ over $X(v) \times \kappa$ is $\omega^{\kappa} w$.

**Proof.** Consider the projection $\pi : T\mathcal{W}^0 v \times \mathcal{U} \to X(v) \times \mathcal{U}$. Recall we have the bundle $\omega^\tau = \otimes_{\sigma \neq \tau} \omega^{\sigma_{\tau}} w$ over $X(v)$ for the fixed weight $(\kappa_{\sigma})_{\sigma \neq \tau}$ apart from $\tau$. We take $\omega^{\kappa_{\text{un}}} w = (\pi \times 1)_{*} \mathcal{O}_{T\mathcal{W}^0 v} \times \mathcal{U}[(\kappa_{\text{un}})] \otimes \omega^\tau$.

Let $M_{v,w}$ be the Banach $A$-module $H^0(X(v) \times \mathcal{U}, \omega^{\kappa_{\text{un}}} w)$. Then similar to section 4, there is an action of the Hecke algebra $\mathbb{T}^K \otimes \mathbb{T}_p$ on $M_{v,w}$ for $v$ small enough. As $\mathcal{W}(w)$ is affinoid, we can consider the admissible open subspace $\mathcal{U} \subset \mathcal{W}$ in the form $U = \mathcal{W}(w) = Sp(A)$.

**Proposition 6.2.** (1) The Banach $A$-module $M_{v,w}$ is projective.

(2) For any $\kappa \in \mathcal{U}$, the specialization map

$$M_{v,w} \to H^0(X(v), \omega^{\kappa} w)$$

is surjective.

**Proof.** The proof is similar to the proof of Prop. 8.2.3.3 in [1], except that our Shimura varieties are proper, so the things here are simpler. First, note that $m_{v,w}^{\kappa_{\text{un}}} w$ over $X(v) \times \mathcal{W}(w)$ is a formal Banach sheaf, where $\mathcal{W}(w)$ is a formal model of $\mathcal{W}(w)$, and $m_{v,w}^{\kappa_{\text{un}}} w$ is constructed in the same way as $m_{v,w}^{\kappa} w$, which is a formal model of the sheaf $\omega^{\kappa_{\text{un}}} w$ in the above proposition. Next, since $X(v)$ is affinoid, we can take a finite affine covering $U = (\mathfrak{U}_j)_{1 \leq j \leq k}$ of $X(v)$. Let $i = (i_1, \ldots, i_j)$ be an index with $1 \leq i_1 < \cdots < i_j \leq k$, and $\mathfrak{U}_i$ be the intersection of $\mathfrak{U}_j$ for $1 \leq l \leq j$ which is again an affine formal scheme. Let $M_i = H^0(\mathfrak{U}_i \times \mathcal{W}(w), m_{v,w}^{\kappa_{\text{un}}} w)$, then it is isomorphic to the $p$-adic completion of a free $B$-module, where $\mathcal{W}(w) = Spf(B)$ (so $A = B[\frac{1}{p}]$). Let $M = M_{v,w} = H^0(X(v) \times \mathcal{W}(w), m_{v,w}^{\kappa_{\text{un}}} w)[\frac{1}{p}]$. Since the rigid analytic fiber of
the covering $\mathcal{U}$ forms a covering of $X(v)$ and $X(v) \times W(w)$ is affinoid, we get a resolution of $M$ by the projective modules $M[\frac{1}{\xi_0}]$. As a result $M$ is projective. Finally, the specialization map is surjective by considering the Koszul resolution of $A/m$ and the double complex obtained by taking the Cech complex of the Koszul complex, see the proof of corollary 8.2.3.2 of loc. cit..

By the above proposition, one can apply Coleman’s spectral theory as in [1] 8.1.2 or in [6] to construct an equidimensional eigenvariety over $W$. More precisely we have the following theorem.

**Theorem 6.3.** There is a rigid analytic variety $E$ over $L$ and a locally finite map to the weight space $w : E \to W$, such that

1. $E$ is equidimensional of dimension $n$.
2. We have a character $\Theta : T^{K^p} \otimes T_p \to O(E)$. For any $\kappa \in W$, $w^{-1}(\kappa)$ is in bijection with the eigensystems of $T^{K^p} \otimes T_p$ acting on the space of finite slope locally analytic overconvergent automorphic forms of weight $\kappa$.
3. For any $\kappa = (k_1, \ldots, k_{n-1}, k_n) \in \mathbb{Z}^{n-1} \times \mathbb{Z} \subset W$, if $x \in w^{-1}(\kappa)$ satisfies $v(\Theta_x(U_i)) < k_{n-i} - k_{n-i+1} + 1$ for $2 \leq i \leq n - 1$ and $v(\Theta_x(U_i)) < k_n + k_{n-1} - n + 1$, then the character $\Theta_x$ comes from a weight $\kappa$ automorphic eigenform on $X$.
4. There is a Galois pseudo-character $T : Gal(\overline{Q}/F) \to O(E)$, such that for any point $x \in E$, there is a continuous semi-simple representation $\rho_x : Gal(\overline{Q}/F) \to GL_n(k(x))$ ($k(x)$ is the residue field of $x$) and the trace of this Galois representation is $T_x$. Here $T_x$ is the composition of $T$ with the evaluation map $ev_x : O(E) \to k(x)$.

**Proof.** (1) and (2) come from the construction. (3) was proved in section 5. For (4) we use the density of classical points as proved in (3) and the results of Harris-Taylor in [16] to get the desired Galois pseudo-character.

**Remark 6.4.** It will be interesting to study the properties of the Galois representations $\rho_x$ for $x \in E$. For example, when it is crystalline or potentially semi-stable? By the main results of [16] and [23], it is true that the crystalline points (that are the points $x \in E$ such that $\rho_x$ are crystalline) are dense in $E$, see [3]. Also it will be interesting to find applications of our eigenvarieties to the construction of Galois representations as in [9].

**Remark 6.5.** In this paper we worked out the construction of some equidimensional eigenvarieties for the Shimura varieties studied by Harris-Taylor with local reflex field $\mathbb{Q}_p$. An immediate extension to the Shimura varieties studied in [23] works. One may try to work in the general case that the local reflex field is not necessary $\mathbb{Q}_p$. However, for this one will need a theory of canonical subgroups for $\varpi$-divisible $O$-modules in the sense of section II.1 of [16] ($O$ is the integer ring of some finite extension of $\mathbb{Q}_p$, $\varpi$ is a fixed uniformizer of $O$). Here one should consider the Faltings dual instead of the usual Cartier-Serre dual, and the expected canonical subgroups should be strict in the sense of [13]. This theory should be as good as the theory of canonical subgroups for usual $p$-divisible groups as in [14] (the case for dimension one may be easier, see [18] for the case of height two). Once such a theory is available, combined with the theory of arithmetic compactification of PEL type Shimura varieties (with good or bad reductions, see [20]), one can deal with any PEL type Shimura varieties (compact or not) with the usual ordinary loci empty or not to construct geometrically the associated eigenvarieties. For example, one can look at the unitary Shimura varieties with signature $(d, n - d) \times (0, n) \times \cdots \times (0, n)$ and construct $n[E : \mathbb{Q}_p]$-dimensional eigenvarieties, where $E$ is the local reflex field.
In the future work we will compare the eigenvariety $E$ with those introduced by Emerton in [12] and Chenevier in [8]. In particular we will study the completed cohomology of these Shimura varieties, and the possible $p$-adic Jacquet-Langlands correspondence for $E$ and the corresponding eigenvariety constructed in [8] as in the case of curves studied in [7]. See [21] for some results in this direction in the case of Shimura curves.

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