The internal dynamics of the Local Group dwarf elliptical
galaxies NGC147, NGC185, and NGC205 *

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Accepted 1988 December 15. Received 1988 December 14; in original form 1988 October 11

ABSTRACT

We present three-integral dynamical models for the three Local Group dwarf elliptical
galaxies: NGC147, NGC185, and NGC205. These models are fitted to the 2MASS J-
band surface brightness distribution and the major-axis kinematics (mean streaming
velocity and velocity dispersion) and, in the case of NGC205, also to the minor-axis
kinematics. The kinematical information extends out to 2 \( R_e \) in the case of NGC205
and out to about 1 \( R_e \) in the case of NGC147 and NGC185. It is the first time
models are constructed for the Local Group dEs that allow for the presence of dark
matter at large radii and that are constrained by kinematics out to at least one half-
light radius. The B-band mass-to-light ratios of all three galaxies are rather similar,
\((M/L)_B \approx 3 - 4 \ M_\odot/L_\odot\). Within the inner two half-light radii, about 40 – 50% of
the mass is in the form of dark matter, so dEs contain about as much dark matter as
bright ellipticals.

Based on their appreciable apparent flattening, we modeled NGC205 and NGC147
as being viewed edge-on. For NGC185, having a much rounder appearance on the sky,
we produced models for different inclinations. NGC205 and NGC147 have a relatively
isotropic velocity dispersion tensor within the region where the internal dynamics are
strongly constrained by the data. Our estimated inclination for NGC185 is \( i \approx 50^\circ \)
because in that case the model has an intrinsic flattening close to the peak of the
intrinsic shape distribution of dEs and it, like the best fitting models for NGC147 and
NGC205, is nearly isotropic. We also show that the dynamical properties of the bright
nucleus of NGC205 are not unlike those of a massive globular cluster.

Key words: galaxies: individual: NGC147, NGC185, NGC205 – galaxies: dwarf

1 INTRODUCTION

From the days of the great comet-hunter Charles Messier up
to well into the 20th century, NGC205 was the only known
member of the class of objects that we now call dwarf ellipticals (dEs) (Messier 1801). dEs are faint \((M_B \geq -18\) mag) galaxies with diffuse, approximately exponentially declining
surface brightness profiles and smooth elliptical isophotes (Ferguson & Binggeli 1994). NGC205 was resolved into stars and thus confirmed as a Local-Group member by Walter
Baade, using the 100 inch telescope at Mount Wilson Ob-
servatory (Baade 1944a). He correctly identified NGC205, at

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are in good agreement with those presented by Lee et al. (1993) and Lee (1996), and the angular distances between the galaxies, the following picture emerges. Using the distance moduli and the sources of photometric errors given by McConnachie et al. (2005), we find that NGC147 and NGC185 are separated by 63 ± 33 kpc and the couple is situated 160 kpc in front of M31. For all practical purposes, we can therefore treat them as isolated stellar systems. NGC205, on the other hand, is situated 48 ± 30 kpc behind M31.

In this paper, we present dynamical models for NGC147, NGC185, and NGC205. In order to construct a mass model for the luminous matter, we use 2MASS J-band images of these galaxies, downloaded from the NASA/IPAC Infrared Science Archive website. These dwarf galaxies have complex star-formation histories and show evidence for spatial variations of the stellar population, the most striking being the very blue nucleus of NGC205 (Butler & Martínez-Delgado 2005; Han et al. 1997; Peletier 1993). Therefore, the density distribution obtained from near-infrared J-band images should trace the mass distribution of the luminous matter more closely than that obtained from optical images. We measured the surface-brightness profile, position angle, and ellipticity $\epsilon \equiv 1 - b/a$ of all three galaxies as a function of the geometric mean of major and minor axis distance, denoted by $a$ and $b$ respectively. These were obtained using our own software. Basically, the code fits an ellipse through a set of positions where a given surface brightness level is reached. The shape of an isophote, relative to the best fitting ellipse with semi-major axis $a$ and ellipticity $\epsilon$, is quantified by expanding the intensity variation along this ellipse in a fourth order Fourier series with coefficients $S_1$, $S_3$, $C_4$ and $C_5$:

$$I(a, \theta) = I_0(a) [1 + C_3(a) \cos(3\theta) + C_4(a) \cos(4\theta) + S_3(a) \sin(3\theta) + S_4(a) \sin(4\theta)].$$

Here, $I_0(a)$ is the average intensity of the isophote and the angle $\theta$ is measured from the major axis. Using all photometric parameters, evaluated as a function of $a$, a smooth image of each galaxy is reconstructed. We used this smooth image to measure the total J-band magnitude and half-light radii. In each case, the total apparent magnitude we derive is within 0.05 mag of that listed in the 2MASS Large Galaxy Atlas (Jarrett et al. 2003). As to the half-light radii, we find for NGC205 that $R_e(NGC205) = 130''$ (0.52 kpc), for NGC147 that $R_e(NGC147) = 122''$ (0.38 kpc), and NGC185 that $R_e(NGC185) = 90''$ (0.27 kpc).

This smooth image is then deprojected in order to obtain the spatial density distribution of the stars. The spatial density of an axisymmetric stellar system cannot uniquely be reconstructed from the observed projected density if it is not viewed edge-on (Gerhard & Binney 1996). From the infinity of equally plausible positive, axially symmetric spatial densities that project to the observed projected density, we pick the one that can be represented as a weighted sum of basis functions, $\rho(\varpi, z) = \sum_k c_k \rho_k(\varpi, z)$, with $\rho_k(\varpi, z)$ of the form

$$\rho_k(\varpi, z) = \exp \left[ - \left( \frac{\varpi^2 + (z/q)^2}{\sigma_k} \right)^{m_k} \right].$$

Here, $\sigma_k$ is a scale radius, $m_k$ is a shape parameter, making the density profile more (small $m_k$) or less (large $m_k$) centrally concentrated, $q$ is the axis-ratio, controlling the flattening of the basis function. These basis functions produce elliptical isophotes when projected onto the sky. For a given inclination, the unknown coefficients in the expansion of the spatial density are determined by fitting the projected basis functions to the observed photometry, evaluated on an elliptical, logarithmically spaced grid subject to the constraint that the spatial density is positive everywhere using a Quadratic Programming algorithm (Dejonghe 1989). Note that the coefficients $c_k$ can be negative, as long as the weighted sum of basis functions is positive. This algorithm has a library at its disposal that contains hundreds of basis functions to choose from. Typically, the $\chi^2$ does not further improve after co-adding a few tens of components and the procedure can be stopped. This deprojection procedure ensures that the spatial density is smooth, well-behaved, and positive.

Already from the 2MASS image, it is obvious that NGC205 is noticeably influenced by the tidal forces exerted by M31 (see Fig. 1). Beyond a major-axis distance of ≈ 300′′ the isophotes of NGC205 twist from a position angle of 167° to 160° at 400″. At that radius, the surface brightness profile also shows a downward break, as can clearly be seen in Fig. 5 (see also Hodge (1973) and Choi et al. (2002)). In these outer regions, peculiar kinematics, such as the outermost stars moving counter to the rotation of the main body (Geha et al. 2006), provide further evidence for a tidal interaction with M31. We will therefore check if our results for NGC205 are affected by it being so close to M31. The isophotes of NGC147 and NGC185 are much more regular, with negligible isophote twisting, see Figs. 2 and 3.
The internal dynamics of the Local Group dEs

The spectra were obtained with the 1.93-m telescope of the Observatoire de Haute-Provence, equipped with the CARELEC long-slit spectrograph. In combination with a EEV receptor with 2048 × 780 pixels of 15 μm, the instrumental velocity dispersion was \( \simeq 25 \text{ km s}^{-1} \) (R=5050) for a slit width of 1.5″. More details and a log of the observations can be found in Simien & Prugniel (2002). 10-minute exposures on blank fields of sky obtained during the same night as the science exposures were used for sky subtraction. With a Fourier-Fitting technique (Fruscione et al. 1989), the spectrum of a G8III template star (HD5395), convolved with a Gaussian line-of-sight velocity distribution, yielded a good fit to the galaxy spectrum in Fourier space, providing a simultaneous estimate of the mean streaming velocity \( v_p \) and the projected velocity dispersion \( \sigma_p \).

From the deep major axis spectrum of NGC205, stellar kinematics out to about 300″ (1.2 kpc or 2.3 half-light radii) could be extracted. The minor axis spectra yields useful kinematic information out to 120″ (0.9 kpc). Thanks to the proximity of this galaxy and the unprecedented depth of the spectra, we can measure kinematics near the center of NGC205 with a spatial resolution of \( \sim 5 - 10 \) parsecs. The major axis spectrum of NGC185 allows extracting kinematics out to \( \sim 100″ \) (0.3 kpc or 1.1 \( R_e \)). NGC147 has the lowest surface brightness of all Local Group dEs and is moreover riddled with foreground stars and was therefore the most difficult object to obtain decent kinematics for. The spectra of NGC147 secured by Simien & Prugniel (2002) turned out to be insufficiently deep. We, therefore, re-observed NGC147 in January 2002 and January 2003, using the same instrumental setup and the same data reduction procedure. With the addition of these new exposures (six one-hour exposures along the southern semi-major axis and four one-hour exposures along the northern semi-major axis), the major-axis kinematics of NGC147 now extend out to \( \sim 150″ \) (0.5 kpc or 1.2 \( R_e \)) along the northern semi-major axis and out to \( \sim 100″ \) (0.3 kpc or 0.8 \( R_e \)) along the southern semi-major axis.

The details of the modeling method are given in Section 2. We present the results for each of the galaxies in Sect. 3. Our conclusions are summarized in Sect. 4.

\section{Two-Integral Dynamical Models}

The internal dynamics of a steady-state oblate stellar system are described by a gravitational potential \( \psi(\varpi, z) \), that determines the stellar orbits, and the distribution function (DF) \( F(\varpi, z) \, d\varpi \, dz \), which gives the number density of stars in phase space ((\varpi, z) are cylindrical coordinates in each meridional plane). Loosely speaking, the DF distributes the stars over all possible orbits in a given potential. In the following, we will work in prolate elliptical coordinates \((\lambda, \nu) \) (Dejonghe & de Zeeuw 1988), defined by

\[ \varpi^2 = \frac{(\lambda + \alpha)(\nu + \alpha)}{\alpha - \gamma}, \quad z^2 = \frac{(\lambda + \gamma)(\nu + \gamma)}{\gamma - \alpha}, \quad (3) \]

with \(-\gamma \leq \nu \leq -\alpha \leq \lambda \). Surfaces of constant \( \lambda \) are confocal prolate spheroids; surfaces defined by constant \( \nu \) are confocal two-sheeted hyperboloids, both with foci at \( z = \pm \Delta = \pm \sqrt{\gamma - \alpha} \). We will approximate the gravitational potential by a Stäckel potential, of the form

\[ \psi(\varpi, z) = \frac{(\lambda + \gamma)G(\lambda) - (\nu + \gamma)G(\nu)}{\lambda - \nu}, \quad (4) \]

with \( G(\lambda) \) the potential in the equatorial plane. Such potentials allow the existence of three integrals of motion: the binding energy \( E \), the z-component of the angular moment
an intrinsic axial ratio which we use in the more convenient form $I_3 = L_z^2/2$, and

\[ I_3 = \frac{1}{2}(L^2 - L_z^2) + (\gamma - \alpha) \left( \frac{1}{2} \nu^2 - \frac{z^2 G(\lambda - G(\nu))}{\lambda - \nu} \right), \quad (5) \]

an integral which is a generalisation of angular momentum conservation in spherical systems. Roughly, for a given $E$ and $I_2, I_3$ determines how high above the equatorial plane an orbit will come. Models that make use of the third integral are called three-integral or 3I models, versus two-integral or 2I models that go without it.

A detailed account of the method we employed to construct the spheroidal coordinate system and Stäckel potential that give the best fit to a given axisymmetric potential can be found in Dejonghe & de Zeeuw (1988), Dejonghe et al. (1996), and in De Bruyne et al. (2001). In brief, we deproject the observed surface brightness distribution, derived from a 2MASS J-band image, assuming the galaxy to be axisymmetric. Since NGC147 and NGC205 have a fairly flattened apparent shape, with an apparent axial ratio $q_{\text{app}} = 0.5$, it is quite unlikely that they are viewed far from edge-on. The intrinsic flattening distribution of dEs peaks around an intrinsic axial ratio $q_{\text{int}} = 0.6$ while galaxies with an intrinsic axial ratio smaller than $q_{\text{int}} = 0.4$ are virtually absent (Binggeli & Popescu 1995). Assuming both galaxies to have an oblate, axisymmetric light distribution that is intrinsically rounder than one with $q_{\text{int}} = 0.4$, the inclination is expected to be larger than $i \approx 70^\circ$. Therefore, we can assume for simplicity that NGC147 and NGC205 are viewed edge-on without significantly affecting any of the results.

NGC185, on the other hand, has a much rounder apparent shape, with an apparent axial ratio $q_{\text{app}} = 0.8$. If we assume that the galaxy has an intrinsic shape that is rounder than $q_{\text{int}} = 0.4$, the inclination is expected to be larger than $i \approx 40^\circ$. In order to assess the influence of the intrinsic shape and the viewing angle of NGC185 on our results, we generated models with $i = 90^\circ$, $i = 60^\circ$, $i = 55^\circ$, $i = 50^\circ$, and $i = 45^\circ$. However, with kinematics along the major axis only and lacking higher-order moments of the line-of-sight velocity distribution, the kinematical data do not allow to constrain the inclination with any degree of precision.

The total mass density, including dark matter, is parameterized as the spatial luminous mass density multiplied by a spatially varying mass-to-light ratio

\[ M/L(\varpi, z) = A \left( 1 + B \sqrt{\varpi^2 + (z/q)^2} \right), \quad (6) \]

with the parameters $A$ and $B$ to be estimated from the data and $q$ the axis ratio of the luminosity density distribution. Attempts to find intermediate-mass black holes in dEs, using either ground-based kinematics (Geha, Guhathakurta, van der Marel 2002) or high spatial resolution kinematical information obtained with HST (Valluri et al. 2005), have so far failed. For this reason, and since we are mostly interested in the total dark-matter content of these galaxies, we did not include a central black hole in our models. The gravitational potential is obtained by decomposing the total mass density in spherical harmonics. Finally, we determine the Stäckel potential, i.e. the focal length $\Delta$ and the function $G$, that best fits this gravitational potential. The Stäckel form of the potential is used only to calculate the third integral $I_3$; elsewhere the potential derived from spherical harmonics is used. The potential is normalised such that $\psi(r) = 1/r$ at large radius $r$, with $r$ expressed in kiloparsecs.

For a given potential, we wish to find the DF that best reproduces the kinematical information. As a first step, the DF is written as a weighted sum of basis functions, called “components”. Here, we will use components of the form

\[ F^{\ast,i}(E, I_2, I_3) = (E - E_{0i})^\tau_i (I_2 - I_{0i})^\tau_i I_3^\nu, \]

if $E > E_{0i}$, $\epsilon_i I_2 > I_{0i}$, $\geq 0$

\[ = 0, \]

if $E \leq E_{0i}$ or $\epsilon_i I_2 \leq I_{0i}, \quad (7) \]

with $\sigma_i$, $\tau_i$, and $\rho_i$ integer numbers. $E_{0i}$ is a lower bound on the binding energy, which defines the outer boundary within which the component is non-zero. The parameter $\epsilon_i$ indicates whether stars are placed only on orbits with positive angular momentum ($\epsilon_i = +1$) or only on orbits with negative angular momentum ($\epsilon_i = -1$) or on both ($\epsilon_i = 0$). We refer the reader to Appendix A for a discussion of the properties of these components. The DF then takes the form

\[ F(E, I_2, I_3) = \sum c_i F^{\ast,i}(E, I_2, I_3). \]

The coefficients $c_i$ are determined by minimizing the quantity

\[ \chi^2 = \sum_i \left( \frac{\text{obs}_i - \sum c_i \text{obs}^i}{\sigma_i} \right)^2, \quad (8) \]

with obs an observed data point, $\sigma_i$ the 1-σ errorbar on that data point, and obs$^i$ the corresponding value calculated from the basis function $F^{\ast,i}$, subject to the constraint that the DF be positive everywhere in phase space. Note that the coefficients $c_i$ can be negative, as long as the weighted sum of basis functions is positive.

The projected velocity moments, $\mu_n(\Omega)$ can be obtained by integrating suitably weighted combinations of the spatial velocity moments over a line of sight in a direction $\Omega$:

\[ \mu_n(\Omega) = \int_{\Omega} v_p^n F(E, I_2, I_3) d\Omega d\xi \]

\[ \int_{\Omega} v_p F(E, I_2, I_3) d\Omega d\xi \]

\[ = \sum c_i \int_{\Omega} v_p^{n+\tau_i} F^{\ast,i}(E, I_2, I_3) d\Omega d\xi, \quad (9) \]

with $v_p$ the line-of-sight velocity and $\xi$ a coordinate along the line of sight. From the observed quantities, the projected luminosity density $\rho_L = 10^{(25.26 - m_{NIJ})/2.5} L_{\odot}/pc^2$ with $\mu_J$ the observed surface brightness expressed in magnitude arcsec$^{-2}$, the mean projected velocity $v_p$, and the projected velocity dispersion $\sigma_p$, one can easily construct observed values for the lowest-order velocity moments $\mu_0 = \mu_p$, $\mu_1 = \rho_p v_p$, and $\mu_2 = \rho_p (v_p^2 + \sigma^2_p)$ along each observed line of sight and insert these values into eqn. (8), which is minimized using a Quadratic Programming algorithm (Dejonghe 1980). The minimisation algorithm has a library at its disposal that contains hundreds of components to choose from. Since the algorithm itself determines which and how many components are used, this is a virtually non-parametric method of retrieving the distribution function. Typically, the $\chi^2$ does not further improve after co-adding a few tens of components and the procedure can be stopped. This finally yields the coefficients $c_i$ and consequently the distribution function that best reproduces the data for a given gravitational potential. If the basis functions $F^{\ast,i}$ are smooth and well-behaved functions of the integrals of motion, any combination of a finite number of basis functions will by construction also be smooth and well-behaved in phase space.
It is well known that in a spherical geometry even complete kinematical knowledge does not suffice to uniquely constrain both the gravitational potential and the distribution function. On the contrary: for each potential, there is a unique distribution function that reproduces the data (Dejonghe & Merritt 1992). That distribution function, however, need not be positive everywhere in phase-space. The positivity constraint we are imposing on the distribution function evidently requires a spatially varying mass-to-light ratio to reproduce the data. Models with a constant mass-to-light ratio, i.e. with $B = 0$ kpc$^{-1}$ are barely acceptable at the 95% confidence level. The best fit model, however, need not be positive everywhere in phase-space. The positivity constraint we are imposing on the distribution function therefore helps to constrain the gravitational potential. The degeneracy in the general axisymmetric case, however, can be expected to be a lot smaller than in the spherical case. For 2I models, the even part of the distribution function can be derived directly from the augmented stellar density $\tilde{\rho}(x, \psi)$ which, in the absence of dark matter, can in principle be constructed directly by deprojecting the observed surface brightness distribution. This is clearly not possible in the spherical case. Therefore, even if the kinematical data are restricted to the three lowest order moments of the distribution function along the major (and sometimes minor) axis, we can expect to be able to put meaningful constraints on the dark matter content and mass, a point already made in Dejonghe et al. (1996). 3I models have a larger freedom in distributing stars over all possible orbits than 2I models, widening somewhat the range of possible solutions that is consistent with the data.

We fitted models to the data of each galaxy with various values for the parameters $A$ and $B$ in eqn. (6). The model with the lowest $\chi^2$-value, which most closely reproduces the data, is retained as the best model. The uncertainties on the dynamical quantities presented in this paper are derived from the range of allowed models (all models within the 95% confidence level are deemed acceptable).

## 3 RESULTS

### 3.1 NGC205

3.1.1 New and previous kinematical work

The major and minor axis J-band photometry and kinematics of NGC205 as measured by Simien & Prugniel (2002) are presented in Fig. 5. These data were derived from spectra with a 25 km s$^{-1}$ velocity resolution, using a Fourier-Fitting technique. Detailed simulations showed that template mismatch has little effect on the kinematical parameters (Kolcova et al. 2006). There is a slight degeneracy between the velocity dispersion and the metallicity: a template of too high a metallicity will give too large a dispersion. The template star used has a metallicity [Fe/H]= −0.5 (Prugniel & Soubiran 2001). Our simulations show that for a galaxy with [Fe/H]= −1 and a template with [Fe/H]= −0.5, the dispersion can be overestimated by ~ 12% at most. However, the fitting procedure includes an additive continuum which decreases the mismatch by an order of magnitude. Age mismatch, as is likely to be the case in the peculiar nucleus of NGC205, is not expected to bias the kinematics.

NGC205 has a very steep central density cusp or nucleus (see panel a of Fig. 5). Within the inner 3 arcseconds, the J-band surface brightness becomes almost 2 mag arcsec$^{-2}$ brighter. This sudden density jump is reflected in a corresponding drop of the velocity dispersion by about 10 km s$^{-1}$ towards the center; from $\sigma_p = 30 \pm 2$ km s$^{-1}$ at a radius of 10$''$ down to a central velocity dispersion of $\sigma_p = 20 \pm 1$ km s$^{-1}$ (see panels b and c of Fig. 5). The nucleus of NGC205 therefore appears to be a round and very dense but dynamically cold substructure. Outside the nucleus, the velocity dispersion was found to rise outwardly from 30±2 km s$^{-1}$ near the center up to about 45±7 km s$^{-1}$ at 300$''$. The rotation velocity rises to a maximum of 20±5 km s$^{-1}$ at 200$''$ and declines again beyond that radius. This was the first clear detection of rotation in this galaxy. There is evidence that the mean velocity reverses sign at a radius of about 300$''$. This feature agrees with other observations (Hodge 1973; Geha et al. 2006) that suggest that the tidal forces of M31 only appreciably affect the outer regions of NGC205, beyond 5$'$. Moreover, the absence of minor-axis rotation (Simien & Prugniel 2002) and the presence of a very dense nucleus, which is likely to scatter stars off box-orbits, which are the backbone of any slowly rotating triaxial mass-distribution, into loop-orbits (Evans & Collett 1994; Merritt & Quinnan 1998) both argue against NGC205 having a strongly triaxial mass distribution, justifying our assumption of an oblate geometry for the construction of dynamical models.

Previous work on the kinematics of NGC205 has yielded contradictory results. Held et al. (1990) present major axis velocity and velocity dispersion profiles out to about 1$'$, measured from spectra with a 83 km s$^{-1}$ per pixel spectral resolution and using a G5 star spectrum as template. The mean

![Figure 4. Contours of constant $\chi^2$ for the models fitted to the NGC205 data as a function of the total mass within a 2 $R_e$ radius, $M(2 R_e)$, and the scale parameter $B$ of equation (6). The white dots mark the parameter values of the models we constructed. The contours indicate the 64%, 90%, 95%, and 99% confidence levels. At the 95% confidence level, the mass within a 2 $R_e$ radius is $M(2 R_e) = 10.2 \pm 3.4 \times 10^8 M_\odot$. For the scale parameter, we find $B = 1.1^{+1.5}_{-1.1}$ kpc$^{-1}$, at the same confidence level. The best fit model evidently requires a spatially varying mass-to-light ratio to reproduce the data. Models with a constant mass-to-light ratio, i.e. with $B = 0$ kpc$^{-1}$ are barely acceptable at the 95% confidence level. The vertical lines indicate the locus of models for which the mass-to-light ratio at 2 $R_e$ equals 1, 1.5, 2, and 2.5 times the estimated stellar mass-to-light ratio ($\langle M/L \rangle = 1.32$, see section 3.1.2). All acceptable models, even the ones with a zero $B$, have mass-to-light ratios well above that of a purely stellar system.

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Figure 5. Fit to the kinematics of NGC205. Panel a: the J-band surface brightness, $\mu_J$, along major and minor axis; panel b: the major-axis velocity dispersion, $\sigma_p$; panel c: the minor-axis velocity dispersion; panel d: the mean streaming velocity along the major axis, $v_p$. The black dots are the data points, the grey curves correspond to the best fitting dynamical model. The top axis of each panel is labeled with radius expressed in kiloparsecs. The model was fitted to the surface brightness on a logarithmically spaced grid covering the whole face of the galaxy but for clarity we only plot the major and minor axis profiles in this figure. The model accurately reproduces the central surface brightness peak (see insets in panel a), and the corresponding central drop in the velocity dispersion (see insets in panels b and c). The major-axis velocity dispersion appears to keep rising very slowly out to the last data point, at 5′.

Figure 6. Contours for the radial velocity dispersion, $\sigma_\varpi$, the tangential velocity dispersion, $\sigma_\phi$, the difference between the radial and the vertical velocity dispersion $\sigma_\varpi - \sigma_z$, and the mean tangential velocity, $v_\phi$, in the meridional plane of the best fit dynamical model for NGC205. All contours are labeled in km s$^{-1}$. 

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velocity dispersion is about 60-70 km s\(^{-1}\), with only a very slight decline towards the center, reaching about 50 km s\(^{-1}\) in the very center. No significant rotation was detected. These authors deduce a mass-to-light ratio of \((M/L)_B \sim 7 \, M_\odot/L_\odot, B\). Bender et al. (1991) on the other hand found the velocity dispersion to rise almost linearly with radius, from \sim 30 km s\(^{-1}\) near the center to \sim 70 km s\(^{-1}\) at one half-light radius. At the very center, the velocity dispersion was found to drop to 20 km s\(^{-1}\). Again, rotation turned out to be negligible. The kinematics were derived at a 30 km s\(^{-1}\) spectral resolution using a single G8II stellar template spectrum. These authors estimate the B-band mass-to-light ratio at \((M/L)_B = 8 \pm 2 \, M_\odot/L_\odot, B\). In Held et al. (1992), the mean value of the major-axis velocity dispersion of NGC205, \(\sigma = 42 \, km/s\), is used to arrive at \((M/L)_B \sim 3.5 \, M_\odot/L_\odot, B\). Carter & Sadler (1990) on the other hand retrieved an almost flat velocity dispersion out to 1\(\prime\), constant at about 45 km s\(^{-1}\). Very near the center, the velocity dispersion was observed to drop to \sim 15 km s\(^{-1}\). These kinematics were measured using a composite stellar template template from a galaxy spectrum with a very high spectral resolution (5 km s\(^{-1}\)). These authors estimate the mass-to-light ratio at \((M/L)_B \sim 9.4 \, M_\odot/L_\odot, B\) with an uncertainty of a factor of 2.

\[3.1.2 \quad \text{The dynamical mass and mass-to-light ratio}\]

From the range of models that is compatible with the data at the 95% confidence level, we estimate the mass within a 2\(R_e\) radius sphere at \(M(2 \, R_e) = 10.2^{+3.3}_{-2.2} \times 10^8 \, M_\odot\), corresponding to a J-band mass-to-light ratio within the inner 2\(R_e\) of \((M/L)_J = 2.2^{+0.7}_{-0.5} \, M_\odot/L_\odot, J\) or a B-band mass-to-light ratio of \((M/L)_B = 4.5^{+1.5}_{-1.0} \, M_\odot/L_\odot, B\), using a B\(-J\) color, corrected for Galactic reddening (de Vaucouleurs et al. 1991; Jarrett et al. 2003). The mean metallicity of NGC205 is quite well determined at [Fe/H] \approx -0.9, using various techniques (McConnachie et al. 2005; Butler & Martínez-Delgado 2005; Richer et al. 1998). In the central regions of our dynamical models, mass follows light. If the central mass-to-light ratio of the best fit model, \((M/L)_J = 1.32 \, M_\odot/L_\odot, J\), is indicative of that of the stellar population and if we adopt a mean metallicity [Fe/H] \approx -0.9, we estimate the mean age of the stellar population at \sim 9 Gyr (Worthey 1994), which is not unreasonable. The model with the lowest mass that is still consistent with the data at the 95% confidence level has a constant mass-to-light ratio \((M/L)_J = 1.8 \, M_\odot/L_\odot, J\), which is significantly higher than that estimated for the stellar population above. Hence, we can state that NGC205 is not a pure stellar system, completely devoid of dark matter. Within a 2\(R_e\) sphere, NGC205 consists of about 60% luminous matter, by mass, and of 40% dark matter.

\[3.1.3 \quad \text{The internal dynamics}\]

The best fitting 3I model for NGC205 very accurately reproduces the photometric and kinematical data, as can be judged from Fig. 5, where the kinematics of the best fitting model are overplotted onto the data. Since the dynamics of this galaxy are constrained by kinematical information along both the major and minor axes, it is instructive to more closely examine some internal dynamical quantities. In Fig. 6, the spatial kinematics are presented in the meridional plane: the radial velocity dispersion \(\sigma_\rho\), the tangential velocity dispersion \(\sigma_\phi\), the difference between the radial and vertical velocity dispersions \(\sigma_\rho - \sigma_\phi\) (in the case of a 21 model: \(\sigma_\rho - \sigma_z = 0\)), and the mean tangential velocity \(v_\phi\).

Geometrically, it is clear that the minor-axis velocity dispersion puts strong constraints on the behaviour of the radial velocity dispersion along the symmetry axis while the major-axis kinematics constrain both the radial and the tan-

\[\text{Figure 7. Panel (a): the distribution function of the best fit dynamical model for NGC205, plotted in turning-point space for loop orbits in the equatorial plane, characterized by } L_3 = 0. \text{ The pericenter distance is denoted by } \rho = 0 \text{ axis while circular orbits lie along the diagonal. } \rho_+ \text{ has the same sign as the angular momentum } L_\phi. \text{ The contour labels are expressed in units of } \log(L_{\odot, J}) (50 \text{ km s}^{-1})^{-3}, \text{ from } -4.5 \text{ to } 1.0 \text{ with a } 0.5 \text{ interval. The orbital distribution within the equatorial plane is clearly very nearly isotropic, with only a very slight excess of stars with positive angular momentum } L_\phi, \text{ causing the rotation of NGC205. Panel b: to have a more detailed look at this slight excess of stars on positive angular momentum orbits, we plotted } F(E, L_\phi > 0, 0) - F(E, L_\phi < 0, 0). \text{ The contour labels are expressed in units of } \log(L_{\odot, J}) (50 \text{ km s}^{-1})^{-3}. \text{ At small radii, there is almost exact counter-rotation, with stars on positive and negative angular momentum orbits almost cancelling any rotation. At larger radii, the number of stars on positive angular momentum orbits starts to dominate and at a radius of } \sim 1 \text{ kpc, there are about twice as many stars on right-handed near-circular orbits as there are on left-handed ones.}\]
The anisotropy parameter $\beta''$ radial dispersion within the inner 0.8 kpc (200"") is only marginally smaller than the vertical velocity dispersion is only very weakly constrained by the kinematical data. Beyond a radius of 0.8 kpc, the tangential anisotropy increases to keep up with the observed rise of the projected velocity dispersion along the major axis.

The mean tangential velocity $v_\phi$ rises only very slowly as a function of radius, as does the projected mean velocity $v_p$ along the major axis. The velocity reaches a maximum at 0.8 kpc (200"") and declines beyond that radius (see Fig. 8). Extrapolating somewhat beyond the extent of our data, the velocity is expected to reverse sign at about 300"", in agreement with Geha (2006). From Fig. 7, it is clear that there is only a slight excess of stars with positive angular momentum $L_z$ causing the observed rotation. Only beyond ~ 0.7 kpc does the number of stars with $L_z > 0$ clearly dominate that of $L_z < 0$ stars. At a radius of ~ 1 kpc, there are about twice as many stars on right-handed near-circular orbits as there are on left-handed ones.

NGC205 has a very distinct nucleus, which shows up in the dynamical model as a spherically symmetric, dynamically very cold substructure (see Figs. 6 and 8). By singling out those basis functions for the distribution function that make up the nucleus, i.e. the basis functions with a very small outer boundary, we can study the nucleus in more detail. The nucleus can be modeled as an isotropic star cluster with a one-dimensional velocity dispersion $\sigma \approx 7$ km s$^{-1}$ and a truncation radius of about 50 pc. By integrating the spherically symmetric mass density of the nucleus, we estimate its total mass at $1.4 \times 10^6 M_\odot$. This is in excellent agreement with a previous estimate based on a King-model fit to the surface brightness profile of the nucleus and its projected velocity dispersion (Carter & Saillier 1990; Jones et al. 1996). It should be noted, however, that the mass-to-light ratio of the nucleus is probably different from that of the main body of NGC205 (Valluri et al. 2005). The functional form for the mass-to-light ratio adopted by us, eq. (6), cannot reproduce a central $M/L$-variation. However, this is not our intention: we are interested in quantifying the amount of dark matter at large radii. Since the nucleus contributes only of the order of 0.1% of the total mass of the galaxy, our total mass estimates are robust.

Thus, the nucleus of NGC205 is structurally and dynamically not unlike a massive globular cluster. The orbital decay time scale for a globular cluster in NGC205 by dynamical friction is of the order $10^8 - 10^9$ years, significantly less than a Hubble time (Lotz et al. 2001; Jones et al. 1996; Tremaine, Ostriker, Spitzer 1975), which makes it plausible that one or more star clusters have spiraled inwards and settled at the bottom of the gravitational well. Though detailed N-body simulations of this process are still lacking, the observed properties of the NGC205 nucleus are not inconsistent with this scenario.

3.1.4 The tidal influence of M31

The TRGB distance estimates from McConnachie (2005) place NGC205 48 ± 33 kpc behind M31. The 1σ error on the TRGB estimates for the distances of NGC205 and M31 are large enough that it is quite plausible that NGC205 and

\[ \beta_{\phi, z} = 1 - \frac{\sigma_z}{\sigma_\phi} \]
are unbound. If NGC205 has passed by M31 with a pericentric distance as small as 5 kpc, stars are unbound beyond a radius of $\sim 1.5$ kpc. These values can be compared with the analytical estimate

$$r_{\text{tidal}} \simeq r_{\text{peri}} \left( \frac{M_{\text{sat}}}{x M_{\text{gal}}} \right)^{1/3},$$

with $r_{\text{peri}}$ the orbital radius of a satellite galaxy with mass $M_{\text{sat}}$ around a galaxy with mass $M_{\text{gal}}$, and $x = 9$ or 1, for stars inside the satellite galaxy on prograde or retrograde circular orbits, respectively (Read et al. 2006). For a pericentric distance of 10 kpc, this yields $r_{\text{tidal}} = 1.0$ to 2.0 kpc (depending on $x$); for a pericentric distance of 5 kpc, this yields $r_{\text{tidal}} = 0.6$ to 1.3 kpc. We substituted the mass of M31 enclosed within a radius $r_{\text{peri}}$ for $M_{\text{gal}}$. These are also the radii where in Fig. 9 the “effective” potential starts to deviate significantly from that of NGC205 alone. This makes us confident that we are providing an adequate description of the tidal influence of M31 on NGC205.

Small pericentric distances, i.e. smaller than $\sim 5 \sim 10$ kpc, which may have occurred in the past, lead to a large effect on the force field within the inner 1.2 kpc but also give rise to a tidal radius well within the region constrained by the photometric and kinematical data, in which case the construction of equilibrium models itself becomes meaningless. In short, our results are robust if NGC205 has kept a distance of at least $\sim 10$ kpc from M31.

### 3.2 NGC147

#### 3.2.1 New and previous kinematical work

The major and minor axis J-band photometry and the major-axis kinematics of NGC147 as measured by Simien & Prugniel (2002), including the additional exposures obtained in January 2002 and 2003, are presented in Fig. 10. The kinematics of NGC147 extend out to $\sim 150''$ (0.5 kpc or 0.8 $R_e$). The projected velocity dispersion is roughly constant, at about $\sigma_v \approx 25$ km s$^{-1}$. The rotation curve is consistent with NGC147 having zero rotation. The velocity dispersion profile is in good agreement with the one presented by Bender et al. (1991). However, these authors found significant rotation in NGC147, with an amplitude of $\sim 10$ km s$^{-1}$ at a radius of 100$''$. They estimated the B-band mass-to-light ratio of NGC147 at $(M/L)_B = 7 \pm 3 M_\odot/L_\odot$. B.

#### 3.2.2 The dynamical mass and mass-to-light ratio

The total mass within a 2 $R_e$ radius sphere is estimated at $M(2 R_e) = 3.0^{+2.4}_{-1.8} \times 10^9 M_\odot$, corresponding to a J-band mass-to-light ratio within the inner 2 $R_e$ of $(M/L)_J = 3.4^{+1.7}_{-1.0} M_\odot/L_\odot$, or a B-band mass-to-light ratio of $(M/L)_B = 4.0^{+1.2}_{-1.4} M_\odot/L_\odot$. The mean metallicity of the stellar population is $[\text{Fe/H}] \approx -1.0$ (Han et al. 1997; McConnachie et al. 2005). There is no evidence that NGC147 contains a population of intermediate-age stars (Renzini 1998); Mould, Kristian, and Da Costa (1983) estimate that 90% of the stellar population is older than 12 Gyr. Using 12 Gyr as a rough estimate for the mean age of the stellar population, the stellar mass-to-light ratio is $(M/L)_J \sim 1.6 M_\odot/L_\odot$ or $(M/L)_B \sim 2.6 M_\odot/L_\odot$.
The major-axis velocity dispersion remains approximately constant at about 30 km s\(^{-1}\), in good agreement with Bender et al. (1991). There is a hint of rotation, at variance with Bender et al. (1991), but, as in the case of NGC147, sky subtraction at these low surface-brightness levels is very difficult and this might affect the measurements. Bender et al. (1991) estimate the B-band mass-to-light ratio of NGC185 at \((M/L)_{B} = 5 \pm 2 \, M_{\odot}/L_{\odot,B}\). Held et al. (1992) found an essentially flat velocity dispersion profile, with \(\sigma \approx 30\) km/s. Interestingly, their major-axis velocity dispersion profile shows a central peak, with the dispersion rising to a central value of 50 km/s. These authors used a spherically symmetric, isotropic model with an exponentially declining density profile to estimate the central mass-to-light ratio of NGC185 at \((M/L)_{B} \sim 3\, M_{\odot}/L_{\odot,B}\).

### 3.3 NGC185

#### 3.3.1 New and previous kinematical work

The major and minor axis J-band photometry and major-axis kinematics of NGC185 as measured by Simien & Prugniel (2002) are presented in Fig. 12. The kinematics of NGC185 extend out to \(\sim 100'' (0.3\) kpc or \(1.1\) \(R_{e}\)). The major-axis velocity dispersion is roughly constant at about 30 km s\(^{-1}\), in good agreement with Bender et al. (1991). There is a hint of rotation, at variance with Bender et al. (1991), but, as in the case of NGC147, sky subtraction at these low surface-brightness levels is very difficult and this might affect the measurements. Bender et al. (1991) estimate the B-band mass-to-light ratio of NGC185 at \((M/L)_{B} = 5 \pm 2 \, M_{\odot}/L_{\odot,B}\). Held et al. (1992) found an essentially flat velocity dispersion profile, with \(\sigma \approx 30\) km/s. Interestingly, their major-axis velocity dispersion profile shows a central peak, with the dispersion rising to a central value of 50 km/s. These authors used a spherically symmetric, isotropic model with an exponentially declining density profile to estimate the central mass-to-light ratio of NGC185 at \((M/L)_{B} \sim 3\, M_{\odot}/L_{\odot,B}\).

#### 3.3.2 The dynamical mass and mass-to-light ratio

The best fitting model, with \(i = 50^\circ\) and an intrinsic axial ratio \(q_{\text{int}} = 0.62\), yields our best estimate for the mass of NGC185. The error-bar on this quantity is estimated taking into account all inclination angles. We estimate the mass within a 2 \(R_{e}\) radius sphere at \(M(2\, R_{e}) = 2.6^{+0.8}_{-0.6} \times 10^{10} \, M_{\odot}\), corresponding to a J-band mass-to-light ratio within the inner 2 \(R_{e}\) of \((M/L)_{J} = 2.2^{+0.5}_{-0.5} \, M_{\odot}/L_{\odot,J}\) or a B-band mass-to-light ratio of \((M/L)_{B} = 3.0^{+1.0}_{-0.7} \, M_{\odot}/L_{\odot,B}\). This result is in good agreement with the one obtained by Held et al. (1992). The mean metallicity of the stellar population is estimated at \([\text{Fe/H}] = -1.43 \pm 0.15\) by Martínez-Delgado & D. & Aparicio (1998). Lee et al. (1993) find NGC185 to

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(Worthey 1994). Comparing this with the dynamical mass-to-light ratio of the best fit dynamical model, we find that NGC147 consists of about 50% luminous matter, by mass, and of 50% dark matter.

### 3.2.3 The internal dynamics

The stellar density and the radial, tangential, and vertical components of the velocity dispersion tensor of the best fit model for NGC147 are presented in Fig. 11. The velocity dispersion tensor is almost isotropic, with \(\sigma \approx 20 - 25\) km s\(^{-1}\), up to a radius of 0.3 kpc, with the vertical velocity dispersion being somewhat lower to help flatten the galaxy. Outside 0.3 kpc, the tangential velocity dispersion rises up to...
3.3.3 The internal dynamics

The stellar density and the radial, tangential, and vertical components of the velocity dispersion tensor of the best fit models for NGC185 are presented in Fig. 13. The velocity dispersion tensor of the edge-on model (i = 90°, see left column of Fig. 13) has a remarkably large vertical velocity dispersion, peaking up to $\sigma_z \sim 40$ km s$^{-1}$, in the inner parts of the galaxy. This feature is not just an oddity of this one dynamical model but is common to all edge-on models we fitted to NGC185. Since within the context of axisymmetric models, viewed edge-on, the vertical velocity dispersion is determined almost solely by the photometry, we have to conclude that if NGC185 is viewed edge-on, it can only maintain its round shape by having the vertical velocity dispersion much larger than the radial velocity dispersion, which is set by the observed major-axis velocity dispersion. The model with $i = 50°$ and an intrinsic axial ratio $q_{tot} = 0.62$, is the most nearly isotropic of all models we constructed for NGC185, see the middle column of Fig. 13. The rotation velocity of the stars flattens at about 22 km s$^{-1}$ in this model, versus a rotation velocity of 8 km s$^{-1}$ in the edge-on model. Since (i) the intrinsic axial ratio of this model is still fairly close to the peak of the intrinsic shape distribution of dEs, and (ii), like the best fitting models for NGC147 and NGC205, it is very nearly isotropic, this is our preferred best model for NGC185. The intrinsically very flattened model, with $q_{tot} = 0.5$ and $i = 45°$, see the right column of Fig. 12, has a much larger rotation velocity, its peak projected rotation velocity is somewhat higher than that of the $i = 90°$ model but still safely within the error-bars. The vertical velocity dispersion $\sigma_z$ is now much smaller than in the less inclined models. Although this model agrees with the observations to within the (quite substantial) error-bars, it fails to reproduce the outward rise of the major-axis velocity dispersion. Hence, with better data it should be possible to better constrain the inclination of NGC185 on purely dynamical grounds.

4 CONCLUSIONS

In this paper, we present dynamical models for NGC147, NGC185, and NGC205. NGC205 and NGC147 have already a flattened apparent shape so we modeled them assuming them to be viewed edge-on (i = 90°). NGC185, on the other hand, has a much rounder projected shape than the two other Local Group dEs, so we produced models for it assuming inclination angles $i = 90°, 60°, 55°, 50°$, and 45°.

From the range of models that is compatible with the data at the 95% confidence level, we estimate the mass of NGC205 within a 2 $R_e$ radius sphere at $M(2R_e) = 10.2^{+3.3}_{-2.0} \times 10^8 M_\odot$, corresponding to a J-band mass-to-light ratio within the inner 2 $R_e$ of $(M/L)_J = 2.2^{+0.7}_{-0.5} M_\odot/L_\odot,J$ or a B-band mass-to-light ratio of $(M/L)_B = 4.5^{+1.5}_{-1.0} M_\odot/L_\odot,B$. The best fitting 3I model for NGC205 very accurately reproduces the photometric and kinematical data, such as the very bright nucleus and the central drop in the velocity dispersion. Overall, the velocity dispersion tensor is quite isotropic. Only beyond a radius of 0.8 kpc does the tangential anisotropy increase to keep up with the observed rise of the projected velocity dispersion along the major axis. The very bright nucleus shows up in the model as spherically symmetric, dynamically very cold substructure, which can be modeled as an isotropic star cluster with a total mass of $1.36 \times 10^8 M_\odot$, a one-dimensional velocity dispersion $\sigma \approx 7$ km s$^{-1}$ and a truncation radius of about 50 pc. Thus, the nucleus of NGC205 is structurally and dynamically not unlike a massive globular cluster.
Figure 13. The internal dynamics of the best fit models for NGC 185. In the left column, all quantities are plotted for the edge-on model (inclination $i = 90^\circ$); in the middle column, the same dynamical quantities are plotted for the best model with inclination $i = 60^\circ$; in the right column, the best model with inclination $i = 45^\circ$ is presented. Top panels: the logarithm of the luminosity density of the stars, $\rho$; middle panels: the rotation velocity, $v_\phi$; bottom panels: the radial, tangential, and vertical components of the velocity dispersion tensor. All quantities are plotted as a function of radius within the equatorial plane.
We estimate the total mass of NGC147 within a 2 $R_e$ radius sphere at $M(2 \, R_e) \sim 3 \times 10^8 M_\odot$, corresponding to a J-band mass-to-light ratio within the inner 2 $R_e$ of $(M/L)_J = 2 \times 10^8 M_\odot / L_\odot$. As in the case of NGC205, the velocity dispersion tensor of the best fit model for NGC147 is remarkably isotropic.

We estimate the total mass of NGC185 within a 2 $R_e$ radius sphere at $M(2 \, R_e) \sim 2 \times 10^8 M_\odot$, corresponding to a J-band mass-to-light ratio within the inner 2 $R_e$ of $(M/L)_J = 4 \times 10^8 M_\odot / L_\odot$. The model with $i = 50^\circ$ has an intrinsic flattening that is still close to the peak of the intrinsic shape distribution of dEs and it, like the best fitting models for NGC147 and NGC205, is nearly isotropic. Therefore, this is our preferred model for NGC185.

Summarizing, our dynamical models for the three Local Group dEs show them to have nearly isotropic velocity dispersion tensors. They have very similar mass-to-light ratios, of the order of $(M/L)_J \sim 4 \times 10^8 M_\odot / L_\odot$. While this is still larger than the expected mass-to-light ratio of the stellar populations of these galaxies, it is about a factor of two smaller than the mass-to-light ratios derived by many previous studies. This rather large difference is due in part to differences in the data analysis. Depending on the spectral resolution, the signal-to-noise ratio of the spectra, and the differences in the data analysis. Depending on the spectral resolution, the signal-to-noise ratio of the spectra, and the method of extracting kinematical data from the spectra, the velocity dispersion estimates for the same galaxy can differ by a factor of 1.5 -- 2 between different authors, with lower resolution observations tending to yield higher values for the velocity dispersion. This, exacerbated by the difficulty of measuring core radii and surface brightness of such diffuse objects, required for use in the standard King formula (Richstone & Tremaine 1986), seems to account for at least part of the large spread on the $M/L$ estimates. Therefore, these dE galaxies contain much less dark matter than was believed to be the case up to now. Still, within the inner two half-light radii, about 40 -- 50% of the mass is in the form of dark matter, so dEs contain as much dark matter as bright ellipticals. Indeed, Gerhard et al. (2001) produced dynamical models for a sample of 21 luminous ellipticals and found that the dark matter contributes 10 -- 40% of the mass within 1 $R_e$ and equal interior mass of dark and luminous matter occurs at 2 -- 4 $R_e$.

**ACKNOWLEDGMENTS**

This research has made use of the NASA/IPAC Infrared Science Archive, which is operated by the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration. We thank the referee for his constructive remarks.

**REFERENCES**

Baade, W., 1944, ApJ, 100, 137
Baade, W., 1944, ApJ, 100, 147
Bender, R., Paquet, A., Nieto, J.-L., 1991, A&A, 246, 349
Binggeli, B. & Popescu, C., 1995, A&A, 298, 63
Butler, D. J. & Martínez-Delgado, D., 2005, AJ, 129, 2217
Carter, D. & Sadler, E. M., 1990, MNRAS, 245P, 12
Choi, P. I., Guhathakurta, P., Johnston, K., 2002, AJ, 124, 310
De Bruyne, V., Dejonghe, H., Pizzella, A., Bernardi, M., Zeilinger, W. W., 2001, ApJ, 546, 903
Dejonghe, H., & de Zeeuw, T., 1988, ApJ, 329, 720
Dejonghe, H., 1989, ApJ, 343, 113
Dejonghe, H. & Merritt, D., 1992, ApJ, 391, 531
Dejonghe, H., De Bruyne, V., Vauterin, P., Zeilinger, W. W., 1996, A&A, 306, 363
de Vaucouleurs, G., de Vaucouleurs, A., Corwin JR., H. G., Buta, R. J. Paturel, G., Fouque, P., 1991, Third reference catalogue of bright galaxies, version 3.9
Evans, N. W., & Collett, J. L., 1994, ApJ, 420, 647
Ferguson, H. C., & Binggeli, B. 1994, A&ARv, 6, 67
Franx, M., Illingworth, G., Heckman, T., 1989, ApJ, 344, 613
Geha, M., Guhathakurta, P., van der Marel, R. P., 2002, AJ, 124, 3073
Geha, M., Guhathakurta, P., Rich, R. M., Cooper, M. C., 2006, AJ, 131, 332
Gerhard, O. & Binney, J., 1996, MNRAS, 279, 993
Gerhard, O., Kronawitter, A., Saglia, R. P., Bender, R., 2001, AJ, 121, 1936
Han, M., Hoessel, J. G., Gallagher, J. S., J., Hotsman, J., Stetson, P. B. et al., 1997, AJ, 113, 1001
Held, E. V., Mould, J. R., de Zeeuw, P. T., 1990, AJ, 100, 415
Held, E. V., de Zeeuw, T., Mould, J., Picard, A., 1992, AJ, 103, 851
Hodge, P. W., 1973, ApJ, 182, 67
Jarrett, T. H., Chester, T., Cutri, R., Schneider, S. E., Huchra, J. P., 2003, AJ, 125, 525
Jones, D. H., Mould, J. R., Watson, A. M., Grillmair, C., Gallagher J., J., Ballester, G. E., Burrows C. J., Casertano, S., et al., 1996, ApJ, 466, 742
Kim, S. C. & Lee, M. G., 1998, JKAS, 31, 51
Klypin, A., Zhao, H., Somerville, R. S., 2002, ApJ, 573, 597
Koleva, M., Bivouzet, N., Chilingarian, I., Prugniel, P., 2006, in "Scientific prospectives for 3D spectroscopy", Kissler-Patig, M. and Walsh, J. (Eds), in press.
Lee, M. G., Freedman, W. L., Madore, B. F., 1993, AJ, 106, 964
Lee, M. G., 1996, AJ, 112, 1438
Lutz, J. M., Telford, R., Ferguson, H. C., Miller, B. W., Stiavelli, M., Mack, J., 1995, ApJ, 552, 572
Martínez-Delgado, D., Aparicio, A., Gallart, C., 1999, ApJ, 118, 2229
Martínez-Delgado, D. & Aparicio, A., 115, 1462
McConnachie, A. W., Irwin, M. J., Ferguson, A. M. N., Ibata, R. A., Lewis, G. F., Tanvir, N., 2005, MNRAS, 356, 979
Merritt, D., & Quinlan, G. D., 1998, ApJ, 498, 625
Messier, C., 1770, Observations Astronomiques, 1770-1774, Connaissance des Temps pour l’an IX, 461
Mould, J. R., Kristian, J., Da Costa, G. S., 1983, ApJ, 270, 471
Peletier, R. F., 1993, A&A, 271, 51
Prugniel, P. & Soubiran, C., 2001, A&A, 369, 1048
Read, J. I., Wilkinson, M. I., Evans, N. W., Gilmore, G., Kleyena, J. T., 2006, MNRAS, 366, 429
Renzini, A., 1993, AJ, 115, 2459
Richer, M. C., McCall, M. L., Stasińska, G., 1998, A&A, 340, 67
Richstone, D. O. & Tremaine, S., 1986, AJ, 92, 72
Simien, F. & Prugniel, Ph., 2002, A&A, 384, 371
Tremaine, S. D., Ostriker, J. P., Spitzer, L., 1975, ApJ, 196, 407
Valluri, M., Ferrarese, L., Merritt, D., Joseph, C. L., 2005, ApJ, 628, 137
van den Bergh, S., 1998, AJ, 116, 1688
Worthey, G., 1994, ApJS, 95, 107
APPENDIX A: THE THREE-INTEGRAL COMPONENTS

For a given potential, we wish to find the DF that best reproduces the kinematical information. As a first step, the DF is written as a weighted sum of basis functions, called "components". Here, we use components of the form

\[ F^{i+}(E, I_2, I_3) = (E - E_{0,i})^{\sigma_i} (I_2 - I_{0,i})^i I_3^i, \]

if \( E > E_{0,i}, \sigma_i > 0, \) \( I_2 > I_{0,i}, \sigma_i > 0, \)

\[ = 0, \]

if \( E \leq E_{0,i} \) or \( \sigma_i I_2 \leq I_{0,i}, (A1) \)

with \( \sigma_i, \tau_i, \) and \( \rho_i \) integer numbers. \( E_{0,i} \) is a lower bound on the binding energy, which defines the outer boundary within which the component is non-zero. The condition \( \sigma_i I_2 > I_{0,i} \geq 0, \) with \( \sigma_i \in [-1, +1], \) defines the "handedness" of the component. If \( \sigma_i = +1, \) only orbits with positive angular momentum \( I_2 > I_0 \) are populated and we call the component \( F^{ii+} \) "right-handed". If \( \sigma_i = -1, \) only orbits with negative angular momentum \( I_2 < I_0 \) are populated and the component \( F^{i-} \) is "left-handed". Components that populate prograde and retrograde orbits equally, labeled with \( \sigma_i \), are constructed as \( F^{i+0}(E, I_2, I_3) = F^{i+}(E, I_2, I_3) + F^{i-}(E, I_2, I_3). \) Components with \( I_0 \neq 0 \) and \( E_0 \neq 0 \) populate orbits within a torus-shaped region of space.

The spatial velocity moments, \( \mu^{i+}_{m,n}(\varpi, z) \), of the component \( F^{i+}(E, I_2, I_3) \) are given by:

\[
I_{2,m,n}^{i+}(\varpi, z) = \int F^{i+}(E, I_2, I_3) v_\varpi^2 v_z^m v_{\varphi}^n \, dE v_\varpi v_z \tag{A2}
\]

\[
= \frac{\rho_{n+m+1}}{\varpi^{m+1} l_{m}^{n+1} (\lambda - \nu)^{l_n + n}} \int_{E_{0,i}}^{\infty} (E - E_{0,i})^{\sigma_i} \, dE 
\times \int_{I_{0,i}}^{I_2} (I_2 - I_{0,i})^{\tau_i} \, dI_2 
\times \int_{I_3}^{I_3^{+}} (I_3^{+} - I_3)^{(l+1)/2} \, dI_3, \tag{A3}
\]

with

\[
I_3^+ = (\lambda + \gamma) \left( G(\lambda) - E - \frac{I_2}{\lambda + \alpha} \right), \tag{A4}
\]

\[
I_3^- = (\nu + \gamma) \left( G(\nu) - E - \frac{I_2}{\nu + \alpha} \right). \tag{A5}
\]

This integration can be performed analytically by repeatedly using the lemma

\[
\int_a^b x^i (b - x)^j (x - a)^k \, dx = B(k + 1, j + 1) 
\times (b - a)^{j+k+1} b \, x F_1 (-i, j + 1; j + k + 2; 1 - \frac{b}{a}) \tag{A6}
\]

with \( B \) the Euler beta function and \( F_1 \) the hypergeometric function. Thus, one obtains the following expression for the spatial velocity moments:

\[
\mu^{i+}_{m,n}(\varpi, z) = \frac{\rho_{n+m+1}}{\varpi^{m+1} l_{m}^{n+1} (\lambda - \nu)^{l_n + n+1+2}} \left( \frac{(\lambda + \gamma)(G(\lambda) - E_{0,i})}{\alpha} \right)^{l+1} 
\times (\psi - E_{0,i})^{(m-1)/2} [\lambda + \gamma]^{l+n+1+2} \Gamma(l+1) \Gamma(n+1) \Gamma(l+\frac{2}{3}) \Gamma(n+\frac{2}{3}) \tag{A7}
\]

with the forefactor \( F_{l,m,n} \) given by

\[
F_{l,m,n} = \sqrt{\frac{(\lambda + \gamma)(\lambda - \nu)}{(\lambda - \nu)(\lambda - \nu)}} \frac{\Gamma(l+1) \Gamma(n+1) \Gamma(l+n+1)}{\Gamma(l+1+n+1+2)} \tag{A8}
\]

The velocity moments of the left-handed component \( F^{i-}(E, I_2, I_3), \mu^{i-}_{m,n} \), are linked to those of its right-handed analog \( F^{i+}(E, I_2, I_3) \) via the relation \( \mu_{m,n}^{i+} = (-1)^m \mu_{m,n}^{i-} \). The spatial velocity moments with respect to the cylindrical velocity components \( v_{\varpi}, v_\phi, \) and \( v_z \) can be constructed from the moments \( \mu^{i,\rho}_{m,n} \) using the transformation

\[
\begin{pmatrix} v_{\varpi} \\ \varpi \\ \cos \Theta \\ \sin \Theta \end{pmatrix} = \begin{pmatrix} \cos \Theta & -\sin \Theta \\ \sin \Theta & \cos \Theta \end{pmatrix} \begin{pmatrix} v_\varpi \\ v_\phi \end{pmatrix} \tag{A9}
\]

with

\[
\cos \Theta = \sqrt{\frac{(\nu + \alpha)(\lambda - \nu)}{(\lambda + \gamma)(\lambda - \nu)}}, \quad \sin \Theta = \sqrt{\frac{(\lambda + \alpha)(\nu + \gamma)}{(\lambda - \alpha)(\lambda - \nu)}} \tag{A10}
\]

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