The role of low air pressure in the variation of negative corona-generated space charge in a rod to plane electrode

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Abstract: The corona-generated space charge may vary in different air pressures, and the quantitative law between the corona-generated space charge density and air pressure has never been investigated. This work utilised computational approaches to explore the role of air pressure in the variation of negative corona-generated space charge. A negative fluid model in which the air pressure could be varied was built up to calculate the charged-particle density and electric field under different applied voltages. The continuity equations were solved by flux corrected transport algorithm and the Poisson equation was computed by the successive over relaxation method. The variations of important parameters due to air pressure were analysed. The particles densities of electrons, positive ions and negative ions at standard and low atmosphere pressures were computed, which increased markedly due to the decrease of atmosphere pressure. At fixed applied voltages, the exponential decay law between the maximal values of electron/positive ion densities at the head of the streamer and air pressure could be obtained. The quantitative law between the maximal values of electron/positive ion densities at the head of the streamer and the applied voltage, relative air density was also achieved.

1 Introduction

Nowadays, with the huge increase in the power energy transmitted over distances greater than 1000 km, numerous high-voltage direct current (HVDC) transmission lines have been constructed all around the world [1]. In some cases, the HVDC transmission lines have to pass through high altitude areas, for example Qinghai-Tibet Plateau in China, therefore, the study of DC corona discharge at various air pressures has been an ongoing subject of research by both physicists and electrical engineers [2–7].

Up until now, much work about self-maintained dc corona discharge at standard air pressure has been done, the research focus has been on the corona inception voltage [8, 9], corona current [10–15], streamer discharge [16–20].

In the recent years, due to the increasingly power grid construction in high altitudes, more and more attention has been paid to the effect of air pressure on the physical mechanism of corona discharge.

Allen et al. [21] investigated the effect of temperature and air pressure change on positive dc corona inception in short air gap. Hu et al. [2, 22] studied the corona onset voltages of bundle conductors under various air pressures by both experiments and computation. Bian et al. [23, 24] developed a method of calculation to determine the corona inception voltage for humid air over a range of pressures and humidities, at the same time, the variation of positive/negative dc corona inception voltages was measured in a perspex chamber. Mikropoulos and Zagkanas [25, 26] investigated the conditions for threshold inception of both positive and negative dc corona in the coaxial cylindrical electrode arrangement under variable atmospheric 20 conditions. Besides the indoor work in the laboratory, Li et al. [27] and Bian et al. [28] also carried out the practical tests to get corona inception voltages of dc power transmission conductors in high altitudes based on a full-scale test line or corona cage, the approximate linear altitude correction formulas were suggested.

Several works have also been done about the effect of pressure on corona current. Bian et al. [23] investigated the variation of positive corona current for humid air over a range of pressures and humidities by calculations and experiments. Grosu et al. [29] calculated the corona current–voltage characteristics at several pressures of helium and nitrogen. Yan et al. [30] measured the corona current of wire–plate electrode configurations while the gas temperature varied from 373 to 1073 K and air pressure varied from 30 to 100 kPa. From these research works, it is found the corona current increased with the decrease of air pressure. Most research work of corona-generated space charge and streamer was carried out in standard atmospheric environment. As early as 1972, Gallimberti [31] utilised an equivalent simplified physical model to compute the streamer propagation through a series of successive equivalent avalanches which develop in front of the streamer tip. In 1980s, Morrow was the first to calculate the streamer propagation and the distributions of charged particles based on Possion’s equation and continuity equations [32, 33]. Afterwards, several scholars worked out many new methods to solve the Possion’s equation and continuity equations, therefore, the streamer discharge of different kinds of electrodes was computed [19, 34–41], at the same time, the characteristics of streamer pulses in corona discharges were also measured [17–18, 42–45]. However, little attention has been paid to the variation of charged particles density and development of streamer during corona discharge. It is therefore worthwhile to consider such physical processes with the decrease of air pressure.

Due to the limitations of the measuring instruments, nowadays it is still very complicated to precisely measure the particles densities of electrons, positive/negative ions generated during corona discharge. Therefore, the computational approaches were adopted to explore the role of air pressure in the variation of negative corona-generated space charge in this work. A negative fluid model in which the air pressure could be varied was built up to calculate the charged-particle density and electric field under different applied voltages. The densities of electrons, positive/ negative ions and the electric field in the air gap of the electrode were computed under several air pressure, therefore, the effect of air pressure on these physical quantities were acquired.
2 Fluid model of steamer discharge

2.1 Calculation of normal electric field

A rod-plane electrode was used in the study, and the model is shown in Fig. 1a. The radius of rod is 1 mm. The distance between rod and plane is 30 mm. The tip of the rod and axis of electrode were set to be the original point and z-axis, respectively. In CSM, as presented in Fig. 1c, a single point charge located at the centre of the rod tip and a number of ring charges distributing uniformly in the angular direction inside the hemispherical tip were used to simulate the hemispherical rod tip; the finite line charges extending along the rod axis were used to simulate the cylindrical part of the rod.

The electric potential of rod electrode surface was the voltage applied, and the electric potential of plane electrode surface was zero. The CSM method was used to calculate the electric potentials of boundaries 1 and 2. The boundary condition of inner boundary 3 should satisfy \( \partial \varphi / \partial r = 0 \). The two-dimensional (2D) Poisson equation (without the space charges) as (1) was adopted to calculate the potential and electric field distribution of the model

\[
\frac{\partial^2 \varphi}{\partial z^2} + \frac{\partial^2 \varphi}{\partial r^2} + \frac{1}{r} \frac{\partial \varphi}{\partial r} = 0
\]

where \( \varphi \) is the electrical potential in the space.

2.2 Calculation of particles density

In the process of negative corona steamer development, the distributions of the charged particles and compound electric field are dynamic and may interact with each other. The continuity equations for electrons, positive ions and negative ions are given as (2)–(4), the 2D Poisson equation is given in (5), all in axisymmetric cylindrical coordinates [33, 34]

\[
\frac{\partial N_e}{\partial t} + \frac{\partial}{\partial r} \left( D_e \frac{\partial N_e}{\partial r} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( D_e \frac{\partial N_e}{\partial r} \right) = (\alpha - \eta) N_e |v_e| - \beta_{np} N_e N_p + S_e
\]

\[
\frac{\partial N_p}{\partial t} + \frac{\partial}{\partial r} \left( D_p \frac{\partial N_p}{\partial r} \right) = \alpha N_e |v_e| - \beta_{np} N_e N_p - \beta_{pp} N_p N_p + S_p
\]

\[
\frac{\partial N_n}{\partial t} + \frac{\partial}{\partial r} \left( D_n \frac{\partial N_n}{\partial r} \right) = \eta N_e |v_e| - \beta_{np} N_e N_p + S_n
\]

\[
\frac{\partial^2 \varphi}{\partial z^2} + \frac{\partial^2 \varphi}{\partial r^2} + \frac{1}{r} \frac{\partial \varphi}{\partial r} = -\frac{e(N_p - N_n - N_e)}{\varepsilon_0}
\]

Here \( z \) and \( r \) refer to the axial and radial positions in cylindrical coordinates, respectively; \( t \) is the time; \( D_e \) is the diffusion coefficient; subscripts \( p, n \) and \( e \) refer to positive ions, negative ions and electrons, respectively. \( v \) and \( N \) are the drift velocity of each of the charged particles and the density of the charged particles, respectively. Rate coefficients \( \eta, \alpha, \beta_{np} \) and \( \beta_{pp} \) indicate the attachment, ionisation, positive ion–negative ion recombination and electron–ion recombination, respectively.

\( S_e, S_p \) and \( S_n \) are the electrons and positive ions produced by photo-ionisation as time steps during the negative corona, respectively, which can be calculated by (6) [47]

\[
S_e(z) = S_p(z) = \frac{R_e}{4} \frac{P_q}{P_a} \frac{\alpha z^2}{\alpha} \int_z^D d \alpha N_e |v_e| \frac{f(z') - z}{z'} dz'
\]

\[
f(z') = \exp(k_s P_{H_2} z') - \exp(k_s P_{H_2} z') - \exp(k_s P_{H_2} z') - \exp(k_s P_{H_2} z')
\]

\[
f(z') = \exp(k_s P_{H_2} z') - \exp(k_s P_{H_2} z') - \exp(k_s P_{H_2} z') - \exp(k_s P_{H_2} z')
\]

\[
\Delta \tau = \min \left\{ \Delta \tau_{2D}; \Delta \tau_{2D}/2D_e \right\}
\]

In order to calculate the charged particles and compound electric field at different air pressures, the air was considered as a mixture of air molecules and water molecules. It is necessary to have the effective coefficients for the mixture. The mixture coefficients were obtained using the linear expression proposed by Morrow and Lowke [32] and Abdel-Salam [49]

\[
k = \frac{P_w}{P_{H_2}} k_p + \frac{P_d}{P_{H_2}} k_d
\]
pressure taken as the sum of the partial pressures $P_w$ and $P_d$. The dependence on local field divided by number density ($E/N$) of these coefficients in dry air and water vapour were taken from [32, 49], respectively.

The continuity equations were solved by the FCT algorithm proposed by Morrow and Cram [48] for a non-uniform mesh. The Poisson equation was computed by the SOR method proposed by Li [50].

The flowchart of the calculation program is presented in Fig. 2.

The condition of negative corona inception requires that at least one photoelectron is emitted by the cathode to keep the negative corona discharge self-sustaining; it can be obtained by the approach in our previous work in [24].

$U_{\text{app}}$ is the applied voltage was raised gradually in steps of 0.1 kV. When the number of the photoelectron emitted by the photons at the cathode surface reached 1, the corona discharge might occur. The corresponding voltage was the corona inception voltage. For the electrode in Fig. 1, the variation of the calculated corona inception voltages at pressures in the range of 0.05–0.1 MPa (the absolute humidity is 8.7 g/m$^3$, the temperature is 20°C) is shown in Table 2. The relationship between the corona inception voltage and the relative air density could be fitted as (10). The corona inception voltages decrease significantly as the air pressure is lower down

$$U_i = 26.38e^{(0.42)} - 22.68$$ (10)

where $U_i$ is the corona inception voltage in kV, $\delta$ is the relative air density, which can be obtained from (11) ($P$ is the air pressure, $P_0$ is the standard pressure and $t$ is the Celsius temperature). The data has a maximum discrepancy of 1% from (10)

![Flow-process diagram of negative corona streamer model](image)

**Table 2** Corona inception voltage at different air pressures (the absolute humidity is 8.7 g/m$^3$, the temperature is 20 °C)

| air pressure, MPa | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 | 0.1 |
|------------------|------|------|------|------|------|-----|
| corona inception voltage, kV | 7.8  | 8.8  | 9.7  | 10.7 | 11.7 | 12.6 |

3 Results

3.1 Particles densities at standard atmospheric pressure

Fig. 3 presents the schematic development of the charged particles in the electrode. The densities of charged particles may vary with time in less than 1 μs, then become stable.

When the inter-electrode distance ($d$) is 30 mm as shown in Fig. 1a, the calculated electrons, positive ions and negative ions densities along the z-axis are shown in Figs. 4a and 4b for applied voltages of 12.5 and 12.6 kV (corona inception). Fig. 4c presents the calculated electrons, positive ions and negative ions densities along the z-axis, and at the maximum density point of the streamer, when the applied voltage is 14 kV; Fig. 4d is the local enlarged image of Fig. 4c, when z coordinate ranges from 0 to 5 mm. Clearly the density of the charged particles increases quickly with a small rise of applied voltage at the corona inception and the streamer production also be enhanced thereafter.

The particles densities of electrons, positive ions and negative ions along the z-axis at normal atmospheric pressure, under applied voltages of 13, 14, 15, 16, 17 and 18 kV are presented in Fig. 5. It is obvious that the particles densities increase with the raise of the applied voltage and the head of the streamer move towards the plane electrode slowly.

Compared with the electrons and positive ions, the densities of negative ions are much lower, therefore, they could be neglected in the subsequent analysis in this paper.

3.2 Variations of important parameters due to air pressure

As shown in Fig. 4a, $L_z$, the length of ionisation zone, is determined as the distance from the boundary of critical volume ($z_c$, where $a$–$y = 0$) to the position of maximum net charge density in the head of the streamer. At corona inception, the variation of $L_z$ with air pressure is presented in Fig. 6, as might be expected, the ionisation zone extends out a distance of the order of the radius of the pointed electrode. The values of $a$–$y$ increase significantly as the air pressure becomes lower at constant humidity [24], so the critical avalanche size turns to be larger, and hence corona inception is reached at a slightly lower voltage (Table 2).

It can be found in Figs. 7a–e that the drift velocities of electrons, positive/negative ions increase significantly as the air pressure is lower down, and the velocity of electrons is much higher than that of the positive/negative ions; hence the corona-generated electrons will move quickly to the head of the streamer, accordingly, the ionisation processes will be enhanced, the density of space charge may be raised and the streamer discharge will be accelerated forward.

It can be also noticed in Fig. 7d that the diffusion motion of electrons is more obvious at lower air pressure, hence, more and more electrons will spread to the head of the streamer which may also contribute to the development of the streamer.

As present in Fig. 7e, $S_p$ and $S_n$ (equal to each other), the electrons and positive ions produced as time steps during the negative corona are significantly affected by air pressure, they will become much higher with the decrease of air pressure, therefore, the density of space charge may increase and the formation of streamer will be enhanced.

3.3 Particles densities at various air pressures

The particles densities of electrons, positive ions and negative ions along the z-axis under the same applied voltage of 13 kV, at atmosphere pressures of 0.1, 0.09, 0.08, 0.07, 0.06 and 0.05 MPa

$$\delta = \frac{P}{P_0} \frac{293}{273 + t} \quad (11)$$
are presented in Fig. 8a. When the electrode is in corona status, similar rule could also be found under different applied voltages, therefore, the results of particle densities under 13 kV is provided as a general example. It can be clearly found that the particle densities increase markedly due to the decrease of atmosphere pressure.

The maximal values of positive ion and electron densities at the head of the streamer under 13 kV applied voltages, for different air pressures are displayed in Fig. 8b. The exponential decay rule between the electron densities, positive ion and air pressure could be obtained as

\[ N_e = C_e(e^{-\delta_e/\delta_{e0}} + C_{e1}) \]  \hspace{1cm} (12)

\[ N_p = C_p(e^{-\delta_p/\delta_{p0}} + C_{p1}) \]  \hspace{1cm} (13)

where \( N_e \) and \( N_p \) are the positive ions and electron densities in m\(^{-3}\); \( \delta \) is the relative air density, the coefficients in (12) and (13) are listed as follows:

\( \delta_{e0} = 0.37275, \quad C_{e1} = 1.2233 \times 10^{-2}, \quad C_{e2} = 3.42206 \times 10^{20} \),

\( \delta_{p0} = 0.3401, \quad C_{p1} = -1.6204 \times 10^{-3}, \quad C_{p2} = 3.67361 \times 10^{20} \).

These coefficients are related to the structure and material of electrodes.

Fig. 3 Charged particles in the rod to plane electrode (figure not drawn to scale)
(a) Charged particles and ionisation zone, (b) Schematic development of charged particles

Fig. 4 Particle density along the axis z at standard atmospheric pressure
(a) At 12.5 kV applied voltage, (b) At corona inception voltage 12.6 kV, (c) At 14.0 kV applied voltage, (d) At 14.0 kV applied voltage (0–5 mm)
When the air pressures are 0.05, 0.075 and 0.1 MPa, the positive ion and electron densities at the head of the streamer for several voltages above corona inception are shown in Fig. 9a. It could be found in Fig. 9a that the positive ion and electron densities increase with the raise of applied voltage. The approximate linear rule between the positive ion, electron densities and applied voltage is discovered, which can be described in (14) and (15) and Table 3.

\[
N_e = D_{e1} + D_{e2}U_{app} \quad (14)
\]

\[
N_p = D_{p1} + D_{p2}U_{app} \quad (15)
\]

where the coefficients \(D_{e1}, D_{e2}, D_{p1}\) and \(D_{p2}\) are shown in Table 3.

\(U_{app}/U_i\) might be defined as ‘corona discharge level’ in our previous work [51]. It could be discovered in Fig. 9b that the electrons and positive ion densities increase approximately linearly with the raise of \(U_{app}/U_i\) at atmosphere pressures of 0.05, 0.075 and 0.1 MPa. And also, the effect of air pressure on charged particles densities is not as obvious as that in Fig. 9a. The ‘coincidence phenomenon’ could be found in Fig. 9a for electrons and positive ions, respectively, at different air pressures. Hence, based on the data in Fig. 9c, the electron and positive ion densities could be found to be in accordance with (16) and (17), both of which are functions of applied voltage and relative air density/air pressure, furthermore, (18) and (19) can be obtained if the corona inception voltages expressed in (10) is integrated. Equations (18) and (19), which express the relationship between the maximal values of electron/positive ion densities at the head of the streamer and the applied voltage, relative air density, are applicable to negative generated corona at air pressure range from 0.05 to 0.1 MPa. And the corona impulse current amplitude could be calculated from charged particles density, diffusion coefficient and drift velocity, which might give guiding significance for the study of high altitude effect on corona current in the future.

For the rod-plane electrode in this research, the quantitative law of the relationship between the electron/positive ion densities and air pressure, applied voltage has been discovered; it is of great significance for the investigators to penetrate into the effect of air pressure on negative corona discharge.

\[
N_e = C_{e1} \frac{U_{app}}{U_i} + C_{e2} = N_e(U_{app}, \delta) \quad (16)
\]

Fig. 5 Particles densities of electrons, positive ions and negative ions along the z-axis at normal atmosphere pressure, under applied voltages of 13, 14, 15, 16, 17 and 18 kV.

Fig. 6 Variation of \(L_1\) with air pressure at corona inception (8.7 g/m³ humidity, 20 °C).

Fig. 7 Parameters as a function of electric field at 8.7 g/m³ humidity, at 20 °C, at atmosphere pressures of 0.05, 0.075 and 0.1 MPa
(a) Drift velocities of electrons, (b) Drift velocities of positive ions, (c) Drift velocities of negative ions, (d) Diffusion coefficients for electrons, (e) \(S\) as a function of electric field under 13 kV applied voltage.
4 Conclusions

A negative fluid model in which the air pressure could be varied was built up to calculate the charged-particle density under different applied voltages. The continuity equations were solved by FCT algorithm and the Poisson equation was computed by the SOR method.

The particles densities of electrons, positive ions and negative ions along the z-axis at normal atmosphere pressure, under several applied voltages were calculated. The variations of important parameters due to air pressure were analysed, such as the length of ionisation zone, drift velocities of electrons, positive/negative ions, diffusion motion of electrons and electrons/positive ion produced as time steps during the negative corona.

The particles densities of electrons, positive ions and negative ions along the z-axis at low atmosphere pressures were computed, which increased markedly due to the decrease of atmosphere pressure. At fixed applied voltages, the exponential decay rule between the maximal values of electron/positive ion densities at the head of the streamer and air pressure could be obtained. The relationship between the maximal values of electron/positive ion densities at the head of the streamer and the applied voltage, relative air density was also acquired. The law discovered in this paper is of great significance for the investigators to penetrate into the effects of air pressure on both negative corona discharge and electromagnetic environment of high-voltage power equipment in high altitude.

The measurement of the particles densities of electrons, positive/negative ions generated during gas discharge would be of great value to further verify the accuracy of the computation. However, this was outside the scope of the present investigation although it might provide an area for future investigations.

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