Comment on ”Interaction-driven Relaxation of Two-level Systems in Glasses” In a series of our previous publications a scaling theory of the interaction stimulated relaxation (ISR) in ensemble of two-level-systems (TLS) coupled as $U_0/R^3$ has been proposed \cite{1} to describe the linear temperature relaxation rate $\tau^{-1}_1 = c \cdot (P_0 U_0)^3 T$ observed in certain ultra-low temperature experiments \cite{2 3 4}. Here $P_0$ characterizes the density of TLS per unit energy interval and the $c$ is a numerical pre-factor which cannot be found within our scaling approach. To obtain a quantitative agreement between the theory and the experiments, the pre-factor should be of the order of $10^3$.

In a recent Letter \cite{5} D. Bodea et al. have attempted to attain the quantitative agreement with the experiment suggesting an alternative scenario for the ISR of TLS. Their approach is based on the resonant triples of TLS. First the authors estimated a coupling amplitude for a triple $J$ as

$$J^2 = \bar{u}_i^2 J_{ik}^2 + \bar{u}_j^2 J_{jk}^2.$$ \hspace{1cm} (1)

Here $\bar{u} = \Delta/E$, with $\Delta$ being the asymmetry energy, $E$ is the energy splitting and $J_{ik} = U_0/R^3$. This estimate has been served as a basis for establishing the energy delocalization mechanism and for calculating the TLS relaxation rate induced by this mechanism. We cannot agree with the results of Ref. \cite{5}, since, in our opinion, the triple transition amplitude \cite{1} is qualitatively overestimated. It seems evident that any many-TLS transition amplitude, in particular the triple amplitude Eq. (1), should decrease when one of the TLS moves off from the others. Yet, it follows from Eq. (1) that, if $R_{ij} \approx R_{jk} \gg R_{ik}$, the triple transition amplitude becomes independent of the $R_{ik}$ instead of vanishing.

Using second order perturbation theory in TLS interaction one can derive the correct triple TLS coupling amplitude $J$ in the form \cite{1}

$$J_{ikj} \approx \frac{\Delta_0 \Delta_0 \Delta_0}{E_i E_j E_k} \times \left[ \frac{U_{ij} U_{ik} \Delta_i}{E_i E_j} - \frac{U_{ij} U_{jk} \Delta_j}{E_j E_k} + \frac{U_{ik} U_{jk} \Delta_k}{E_i E_k} \right],$$ \hspace{1cm} (2)

significantly different from Eq. (1). Using this expression one can readily show that the probability to form a resonant triple is much smaller than unity so the triples, like pairs, can not result in any relaxation. To our mind, these reasonings disprove the conclusions of Ref. \cite{5}.

We understand that the lack of the pre-factor of the order of $10^3$ in our theory can alert the reader. However, obtaining the very underestimated relaxation rate $\tau^{-1}_1$ is a feature of the scaling approach, where the delocalization criterion is about one order stronger than that obtained within the consistent microscopical approach cf. Ref. \cite{5}. Therewith, in our scaling approach \cite{1} we properly consider only resonant interaction when energy level spacing is smaller than the transition amplitude coupling these levels. The contribution of nonresonant interaction cannot qualitatively change the scenario of the many-body ISR relaxation \cite{1}, but might introduce an extra large numerical factor into our result. A similar effect seems to be observed in the system of interacting electrons in disordered wires \cite{10} where the renormalization of the dimensionless disorder parameter $\alpha$ takes place ($\alpha \to \alpha (\ln \alpha)$) when estimating the delocalization criterion. In our case, this could result in the substitution $P_0 U_0 \to P_0 U_0 (\ln (P_0 U_0)) \approx 10P_0 U_0$ and reconcile the theory and the experiment.

A large pre-factor is known to appear when integrate over the phase volumes of many dimensions simultaneously. In particular, the accurate estimate of the swing of spectral diffusion in glasses found in Ref. \cite{8} is about $10^3$ larger than that obtained within the scaling approach \cite{1}. Also, by the same reason, the quantum hopping diffusion coefficient of light impurities in crystals acquire a huge pre-factor $10^7$ \cite{2}.

Thus, the scenario of ISR induced by a long range $R^{-3}$ interaction proposed in Ref. \cite{1} results in the most fast relaxation channel. Within the range of applicability of the scaling approach, the theory correctly predicts the linear temperature behavior for the TLS relaxation rate. One method of experimental verification of our approach is measuring the relaxation rate as a function of the density of TLS $P_0 U_0$.

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