An Inventory-Dependent Demand Model with Deterioration, All-Units Discount, and Return

N Loedy¹, D Lesmono¹, and T Limansyah¹

¹Department of Mathematics, Universitas Katolik Parahyangan, Jalan Ciumbuleuit 94 Bandung 40141

Abstract. In this paper, we propose an inventory model with inventory-dependent demand, deterioration, all-units discount and return. Conditions considered in the model are reflected in the real life when a retailer deal with products with deterministic demand having constant deterioration rate. Also, occasionally the supplier offers an all-units discount to the retailer and on the other occasion retailer can return some products to the supplier at some costs. We develop the model that will determine the optimal return time and optimal order quantity that minimize the total cost. We also assume that all shortages are backordered and one tenth of the order quantity is returned to the supplier. We provide some numerical examples and find that as the deterioration rate, and demand increase, the optimal return time decreases making the optimal order quantity and the total cost increase. Sensitivity analysis is also conducted to analyse the effect of changes in parameter values to the optimal return time, optimal order quantity and the total cost. It is found that changes in return cost do not significantly change the optimal return time, the optimal order quantity and the total cost, compared with changes in deterioration rate, inventory-dependent demand rate and time between replenishment.

1. Introduction

Inventory for retailers is a number of products that are stored to be used or sold within a certain period of time. In general, American Production Inventory Control Society (APICS) Dictionary defines inventory as items used to support production, supporting activities and customer service [1]. There are some characteristics owned by inventory such as demand, replenishment lead time, inventory level and review times, lifetime and reparability [1]. Managing inventory related with determination of order quantity and ordering time. This task becomes more complicated when we are dealing with other factors such as deterioration, discount, inventory-dependent demand and return. In real world, deterioration almost occur in every product, but especially in food industry, chemical material and dairy products. Inventory-dependent demand relates to the situation where demands are increasing when customers see a lot of product in the shelf such as in department store or supermarket ([2], [3]). This situation will make retailer to provide more products in its shelf but in reality the shelf capacity is limited. Another factor that make retailer tend to provide more product is the amount of discount offered by the supplier and the possibility to return some deteriorated products to the supplier. In this paper, we develop an inventory-dependent demand model with deterioration, all-units discount, and return. The model we develop is an extension and combination of the model in [4], [5] and [6]. We also propose an algorithm to find the optimal solution and provide numerical experiments and sensitivity analysis to study the effect of parameter changes on the optimal solution. The aim of our model is to determine the optimal
return time and order quantity that minimize the total cost.

In the last few decades, there are a lot of research regarding deteriorating goods, making this topic as one of important factors that should be considered by retailers. Managing goods with deteriorating factor or perishable goods needs a good strategy, especially in determining the optimal order quantity. Determining the optimal order quantity is crucial, because it is related with many factors faced by retailers including some costs such as holding, ordering and shortage. Shortage cost is sometimes intangible if it is related to the customer satisfaction. It is also related to the potential profit loss. Several strategies in managing perishable goods include giving discount and the possibility for the retailer to return deteriorated goods to the supplier. Some mathematical models have also been developed and implemented regarding these issues. Nafisah et al. [4] have developed an inventory model for perishable goods by considering return and discount, while Nagare and Dutta [5] developed a continuous review inventory model with inventory-dependent demand and deterioration. Setiawan, et.al [6] proposed an inventory-dependent demand model with deterioration rate and return. Rikardo et al. [7] proposed a continuous review model with deterioration and all-units discount. We use the same model of differential equation to describe the inventory level as in Nagare and Dutta [5], but we add all-unit discount and return in the model to describe more realistic situation. The inventory-dependent demand functions used in Nafisah et.al. are constant in each interval between \(0 \leq t \leq T\), while we use a continuous function that depends on the number of inventory at time \(t\). Comparing with Setiawan, et.al [6], our model has considered all-unit discount and has sensitivity analysis on the optimal solution based on some changes in the parameters’ values. The aim of our model is to determine the optimal return time and optimal order quantity that give the minimum total cost. We extend the model of [4], [5] and [6] by incorporating inventory-dependent demand, all-units discount and constant deterioration rate. The organization of this paper is as follows. In Section 2, we introduce the notations, assumptions, our model and propose an algorithm to find the optimal solution. Section 3 deals with numerical experiments and sensitivity analysis. Conclusions and further research are provided in Section 4.

2. The Model

2.1 Notations and Assumptions

The notations we use in this paper are as follows:

\[TC: \text{total inventory cost} \]
\[PC: \text{purchasing cost} \]
\[OC: \text{ordering cost} \]
\[HC: \text{holding cost} \]
\[SC: \text{shortage cost} \]
\[RC: \text{return cost} \]
\[T: \text{time between order (months)} \]
\[O(t): \text{demand at time } t [O(t) = a + bI(t)] \]
\[b: \text{inventory-dependent demand factor} \]
\[Q: \text{order quantity} \]
\[T_l: \text{return time (months)} \]
\[Q_l: \text{inventory level at returns time} \]
\[I(t): \text{inventory level at time } t \]
\[n: \text{number of inventory during period} \]
\[[0,T_l]: \text{interval} \]
\[S: \text{number of shortage during period } [T_l,T] \]
\[P: \text{purchasing cost per unit} \]
\[\theta: \text{deteriorating rate} \]
\[A: \text{ordering cost per order} \]
\[A_l: \text{return cost per return} \]
\[R: \text{return cost per unit} \]
\[\psi: \text{backorder cost per unit} \]

We have several assumptions in developing our model: (1) The inventory-planning horizon is infinite and the inventory system only deals with one product; (2) The amount of inventory returned \((Q_l)\) is 10% of order quantity \((Q)\); (3) Replenishments are instantaneous with zero lead-time and the entire lot is delivered in one batch (fresh and new); (4) When \(t = T_l\), returns will occur for \(Q_l\) instantaneously; (5) Demand at time \(t (O(t))\) is a linear function of the inventory level at time \(t (I(t))\), (6) Supplier offers discount in terms of all-units discount.

2.2 Model and Algorithm

Based on the fifth assumption, the inventory-dependent demand is given by the following equation.
The inventory level decreases in time due to demand and deteriorating rate in the interval [0, T] until it reaches its minimum at $T_1$. At time $t = T_1$, a return will be made for all deteriorated items at the amount of $Q_1$. In the interval $[T_1, T]$, shortage occurs and will be handled by full back order since all returned product will only be replaced on the next shipment. Based on the condition above, in general, the inventory level during interval $0 \leq t \leq T$ can be described using the following differential equation

$$\frac{dl(t)}{dt} + \theta l(t) = -o(t)$$

Substituting $O(t) = a + bl(t)$, we have

$$\frac{dl(t)}{dt} = -\theta l(t) - (a + b l(t)) , 0 \leq t \leq T_1$$

$$\frac{dl(t)}{dt} = -\theta l(t) - \delta , T_1 \leq t \leq T$$

Using the boundary condition $l(T_1) = 0$, we have

$$\frac{dl(t)}{dt} = -\delta ; l(t) = \delta (T_1 - t)$$

In the interval $0 \leq t \leq T_1$, using the boundary condition $l(T_1) = Q_1$, we have

$$l(t) = -\frac{a}{b + \theta} + e^{-(b+\theta)t} \left( Q_1 + \frac{a}{b + \theta} \right)$$

When $t = 0$, $l(t = 0) = Q - \delta (T_1 - T)$ the following expression for $Q$ is obtained.

$$Q = -\frac{a}{b + \theta} + e^{-(b+\theta)t} \left( Q_1 + \frac{a}{b + \theta} \right) + \delta (T - T_1)$$

By employing the second assumption, after some calculations we have the following

$$Q_1 = \frac{-\delta T_1 e^{-(b+\theta)T_1 \beta} - \delta T_1 e^{-(b+\theta)T_1 \theta} + \delta T e^{-(b+\theta)T_1 \beta} + \delta T e^{-(b+\theta)T_1 \theta} - a e^{-(b+\theta)T_1} + a}{10e^{-(b+\theta)T_1 \beta} + 10e^{-(b+\theta)T_1 \theta} - b - \theta}$$

The number of inventory during interval $[0, T_1]$ is given by

$$n = \int_0^{T_1} l(t) dt ; S = \delta T_1 (T - T_1) - \frac{B(T^2 - T_1^2)}{2}$$

The objective of our model is to find the return time that minimizes the total cost. The total cost ($TC$) consists of purchasing cost ($PC$), ordering cost ($OC$), annual holding cost ($HC$), shortage cost ($SC$) and return cost ($RC$). The formulation of the total cost can be written as:

$$TC = P \frac{Q}{T} + A + \frac{h.n}{T} \left( -\psi \left( BT_1 (T - T_1) - \frac{\delta (T^2 - T_1^2)}{2} \right) + A_1 + Q_1 R \right)$$

The optimal return time is found by using the conditions $\frac{dTC}{dT_1} = 0$ and $\frac{d^2TC}{dT_1^2} > 0$. In order to find the optimal solution, we develop an algorithm in Figure 1 below.

3. Numerical Experiments and Sensitivity Analysis

In this section, we will give numerical experiments to illustrate and give a better understanding of our model. Next, sensitivity analysis in order to study the effect of parameters changes to the optimal solution is also performed. First, we consider the following parameters in order to find the optimal return time, the optimal order quantity and the minimum total cost.

$T = 6$ months, $a = 250, A = $10, $h = $5, $\psi = $2, $R = $2, $A_1 = $10, $b = 0.1, \theta = 0.25$ and $\delta = 5000$

The purchasing cost with all-units discount offer is as follows
Based on our algorithm, using Maple, we found the following optimal return time and order quantity.

| $i$ | Price/unit | $T^*_i$ | $Q$ | Status |
|-----|------------|---------|-----|--------|
| 0   | $5         | 3.51    | 21,519 | Valid  |
| 1   | $3.5       | 3.37    | 21,867 | Not valid |

Figure 1. Algorithm in finding the optimal solution

Figure 1 showed that Algorithm in finding the optimal solution. The optimal order quantity $Q$ is 21,519 units at the purchasing cost of $5. In the all-units discount condition, this number is valid because it is in the interval of discount scheme that is less than 25,000 units. Therefore, this $Q$ is valid. However, for the purchasing cost of $3.5, the obtained $Q$ of 21,867 units is not valid since it must be higher than 25,000 units must. This $Q$ is not valid; therefore, we choose $Q = 25,001$ units. The optimal return time, order quantity and minimum total cost are given below. The minimum cost of $38,235.90 occurs for $Q = 21,519$ units and $T^*_i = 3.51$ months.

We also perform sensitivity analysis to study the effect of parameters changes in the optimal solution. We consider the effect of parameters changes of $R, A_t, b, T$ and $\theta$ on the optimal solution. In this sensitivity analysis, we increase parameters’ values by 10% and 15% and decrease them with the same percentage. The results are given in Table 1. Based on Table 1, we can see that increasing the deterioration rate $\theta$, will shorten the optimal return time $T^*_i$, increase the optimal order quantity $Q$ and total cost $TC$. When $\theta$ increases, the number of deteriorated goods increases, therefore $Q$ increases to fulfill demand and $T^*_i$ decreases making then total cost $TC$ increases. Increasing the inventory-dependent demand factor $b$, will have the same effect as increasing the deterioration rate $\theta$. This is because as $b$ increases, demands are increasing and hence the total cost are also increasing. We can also see from
Table 1 that $A_l$ does not significantly affect the total cost. This is because $A_l$ is relatively small (between $0.85$ and $1.15$) compared with other costs. Therefore its changes does not substantially affect $Q$, $T_l^*$ and $TC$. For return cost per unit $R$, as we can see in Table 1, decreasing $R$ will make optimal return time $T_l^*$ shorter, increasing the optimal order quantity $Q$ and decreasing the total cost $TC$ although the increase is not too significant. Also, when time between replenishment $T$ becomes longer, the optimal return time $T_l^*$ becomes shorter, making $Q$ and $TC$ increase. We can also infer from Table 1 that all-units discount does not necessarily make the total cost decreases.

4. Conclusions
We have developed an inventory-dependent demand model by considering deterioration rate, all-units discount and return and introduced the algorithm to find the optimal solution. Based on our numerical experiments and sensitivity analysis, we found that the minimum total cost is obtained when the deterioration rate $\theta$, inventory-dependent demand factor $b$, time between replenishment $T$ and return cost per unit $R$ are lower. Furthermore, we also found that the total cost is not sensitive to the changes of return cost per each return ($A_l$). All-units discount does not guarantee minimum total cost since there are other factors affecting the total cost such as deterioration rate, return time, time between replenishment, the optimal order quantity and inventory-dependent demand factor. In our model, we assume that the model is deterministic where demands, the rate of inventory-dependent demand and deterioration rate are known. Incorporating probabilistic demands and deterioration rate in the model are possible further research directions in this area. Another avenue for further research direction is by considering multi-item inventory problem, where the replenishment decision can be in terms of individual replenishment policy or joint replenishment policy. Using another discount scheme such as incremental discount is also another interesting topic to pursue. The Sensitivity Analysis shown in Table 1 below.

**Table 1. Sensitivity Analysis**

| Parameter | % Difference of TC | $P$ ($) | $T_l^*$ (months) | $Q$ (unit) | $TC$ ($/year$) |
|-----------|--------------------|--------|-----------------|------------|----------------|
| $\theta$  |                    |        |                 |            |                |
| +15       | 0.0913             | 3.5    | 4.58            | 25,001     | 35,399.12      |
| +10       | 0.0750             | 3.5    | 5.01            | 25,001     | 35,162.28      |
| -10       | Not valid          | 3.5    | 7.3             | 25,001     | Not valid      |
| +15       | 0.0345             | 5.517  | 22,727          | 39,556     |
| +10       | 0.0235             | 5.346  | 22,348          | 39,135     |
| -10       | 0.0258             | 3.710  | 20,574          | 37,251     |
| $b$       |                    |        |                 |            |                |
| +15       | 0.4812             | 3.5    | 5.346           | 25,001     | 56,633.20      |
| +10       | 0.5410             | 3.5    | 5.517           | 25,001     | 58,920.91      |
| -10       | 0.8192             | 3.5    | 6.225           | 25,001     | 69,557.87      |
| -15       | 0.9001             | 3.5    | 6.410           | 25,001     | 72,650.36      |
| +15       | 0.0000             | 5.515  | 21,519          | 38,235.74  |
| +10       | 0.0000             | 5.515  | 21,519          | 38,235.74  |
| $A_t$  | -10 | 5  | 3.515 | 21,519 | 38,235.74 | 0.0000 |
|--------|-----|----|-------|--------|-----------|--------|
|        | -15 | 5  | 3.515 | 21,519 | 38,235.74 | 0.0000 |
|        | +15 | 3.5| 5.866 | 25,001 | 63,918.76 | 0.6717 |
|        | +10 | 3.5| 5.866 | 25,001 | 63,918.76 | 0.6717 |
|        | -10 | 3.5| 5.866 | 25,001 | 63,918.76 | 0.6717 |
|        | +15 | 5  | 3.518 | 21,512 | 38,343.48 | 0.0028 |
|        | -10 | 5  | 3.514 | 21,523 | 38,164.17 | 0.0019 |
|        | -15 | 5  | 3.513 | 21,525 | 38,128.30 | 0.0028 |
| $R$    | +15 | 3.5| 5.866 | 25,001 | 64,043.58 | 0.6750 |
|        | +10 | 3.5| 5.866 | 25,001 | 64,001.84 | 0.6739 |
|        | -10 | 3.5| 5.866 | 25,001 | 63,835.17 | 0.6695 |
|        | -15 | 3.5| 5.866 | 25,001 | 63,793.50 | 0.6684 |
|        | +15 | 5  | 3.17  | 28,852 | 45,337.56 | 0.1857 |
|        | +10 | 5  | 3.26  | 26,521 | 43,107.20 | 0.1274 |
|        | -10 | 5  | 4.03  | 15,341 | 32,302.27 | 0.1552 |
|        | -15 | 5  | 7.55  | -      | -         | -      |
| $T$    | +15 | 3.5| 3.09  | 28,859 | 39,064.00 | 0.0217 |
|        | +10 | 3.5| 3.17  | 27,707 | 38,561.37 | 0.085  |
|        | -10 | 3.5| 7.8   | 25,001 | Not valid | -      |
|        | -15 | 3.5| 8.01  | 25,001 | Not valid | -      |

5. References

[1] Shenoy D and Rosas R 2018 Problems & Solutions in Inventory Management Switzerland: Springer

[2] Levin R I, McLaughlin C P, Lamone R P, and Kottas J F 1972 Productions/Operations Management: Contemporary Policy for Managing Operating Systems New York: McGraw-Hill

[3] Silver E A and Peterson R 1985 Decision Systems for Inventory Management and Production Planning, 2nd ed (New York: Wiley)

[4] Nafisah L, Sally W and Purviani 2016 Model persediaan pada produk yang mendekati masa kadaluwarsa: mempertimbangkan diskon penjualan dan retur Jurnal Teknik Industri 18 63-72

[5] Nagare M and Dutta P 2012 Continuous review model for perishable products with inventory dependent demand. Proc. of the International MultiConference of Engineers and Computer Scientists March 24-16 2012 pp. 1513-17

[6] Setiawan, S W, Lesmono D and Limansyah T 2018 A perishable inventory model with return. IOP Conf. Ser: Mater. Sci. Eng 335 012049

[7] Rikardo C, Lesmono D and Limansyah T 2017 Pengembangan model continuous review dengan all-unit discount dan faktor kadaluwarsa Jurnal Teknik Industri 19 29-38

Acknowledgement

Funding of this research from The Institute of Research and Community Service Universitas Katolik Parahyangan are gratefully acknowledged.