Hierarchical Small-Worlds in Software Architecture

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Abstract—In this paper, we present a complex network approach to the study of software engineering. We have found universal network patterns in a large collection of object-oriented (OO) software systems written in C++ and Java. All the systems analyzed here display the small-world behavior, that is, the average distance between any pair of classes is very small even when coupling is low and cohesion is high. In addition, the structure of OO software is a very heterogeneous network characterized by a degree distribution following a power-law with similar exponents. We have investigated the origin of these universal patterns. Our study suggest that some features of OO programming languages, like encapsulation, seem to be largely responsible for the small-world behavior. On the other hand, software heterogeneity is largely independent of the purpose and objectives of the particular system under study and appears to be related to a pattern of constrained growth. A number of software engineering topics may benefit from the present approach, including empirical software measurement and program comprehension.

I. INTRODUCTION

It is over fifteen years since Norman Fenton outlined the need for a scientific basis of software measurement [1]. Such a theory is a prerequisite for any useful quantitative approach to software engineering, although little attention has been received from both practitioners and researchers. Measurement is the process that assigns numbers or symbols to attributes of real-world entities. Unfortunately, many empirical studies of software measurements lack a forecast system that combines measurements and parameters in order to make quantitative predictions [2]. How can we overcome these limitations?

Here we present a new approach to software engineering based on recent advances in complex networks [3], [4], [5]. We study graph abstractions of software designs, where nodes represent software entities (i.e., classes and/or methods) and edges represent static relationships between them (i.e., inheritance and association). We measure graph attributes of software designs in order to find universal patterns of software organization. Graph measures are not anew to software [2]. Empirical software studies assume there is a correlation between software design measures (i.e., lines of code, coupling, cohesion, modularity) and external features (like software reliability or development effort). Although good agreement has been observed in some cases, it is difficult to know if empirical mappings hold in general or not without appropriate models [1]. We have found that some graph measurements of software structures are (statistically) predictable. Moreover, they are within a definite range of values. Intriguingly, these patterns are almost independent of functionality and other external features. It seems that strong constraints limit the set of possible patterns that software structures can display. These constraints might be useful to define useful reference models that enable predictive software development processes.

Object-oriented (OO) software systems display small-world (SW) behavior. Many real systems, including the WWW, food webs, and cellular networks [6] are small-worlds, that is, they have high-degree of clustering and a small average distance. Another common property of OO software is that probability distributions of structural attributes tend to follow skewed distributions with long tails [7]. Heterogeneous metrics have been interpreted as an accident or the signal of rare, atypical behavior. In this context, software researchers often avoid heterogeneity by manipulating the original distribution. Unfortunately, this transformation hides important structural information and the true nature of OO software. Here, we show that the probability of a class to participate in $k$ relationships follows a scaling-law, that is, software designs are scale-free (SF) networks. We have validated the SW and SF behavior of OO software in many real systems, and thus suggesting they are universal features of software designs. In this paper, we explore the origin of these patterns. Eventually, we provide some tentative explanations but clearly more work is needed in this direction.

The regularities found here suggest that concepts and theories developed by complex networks studies are useful in other software engineering contexts, like program comprehension.
For instance, OO software and the WWW share many structural features. Recent analyses of web graphs have shown the existence of some key pages called hubs and authorities\[8\]. Hubs are web pages having a large number of links, like web directories or lists of personal pages. Authorities are pages that contain useful information and thus are pointed to by hubs. OO systems display a similar pattern, where a few (hub) classes have a large number of relationships. Hub classes are excellent starting points for the program comprehension process. A node centrality index might enable us to locate key software components very quickly in a very large source code database (i.e., pagerank [9]). In addition, we study a partial software system and suggest that we can obtain useful information by comparing different network representations of the same software system.

This paper is structured as follows. Section II defines class graphs, an abstraction that captures static structural features of object-oriented systems. These class graphs display universal features: they are small-worlds and scale-free networks. Section III investigates the intrinsic origin of small-world behavior in class graph, which seems to be related to the bipartite association between methods and classes. Section IV proposes that class graphs are scale-free because they evolve under constraints and thus claiming for an external cause. Finally, section V concludes the paper and outlines additional implications of the network patterns found here in empirical software engineering and distributed software development.

II. Class Graphs

A class graph (a software network) is a digraph \( D = (V, L) \) that consists of the set \( V \) of classes and the set of relationships \( L = P \cup S \). There are two types of relationships: a membership relationship \( P = \{ (v_i, v_j) \} \), i.e., read “\( v_i \) has part \( v_j \)”; and a reflexive and transitive relationship \( S = \{ (v_i, v_j) \} \), i.e., read “\( v_i \) is a subclass of \( v_j \)”. However, and from now on, we will not make any distinction between these two relationships \( P \) and \( S \) and we only consider the full set of links \( L \). We discard any dynamic class relationship from the graph definition. For instance, method invocation (i.e., uses relationship in fig. [I]) is not represented in the class graph (compare with fig. [II]). Instead, we conceive nodes and links as black boxes hiding internal complexities that do not change the global structure. This bare-bones characterization enables us to detect global patterns in the static software structure. Ultimately, we hope that the analysis of class graphs will provide important insights into high-level processes of software evolution. We also define the undirected class graph (or undirected software network) \( G = (V, E) \) where \( E = \{ (v_i, v_j) \} | (v_i, v_j) \in L \land (v_j, v_i) \in L \) is the set of edges (see fig. [I]).

Class graphs represent an important information space of OO software systems. A prerequisite for software evolution and maintenance is that software engineers recognize and understand the function performed in software. This problem is aggravated in large software systems, where source code navigation can turn easily into a bottleneck. The efficiency of program comprehension depends on general and new knowledge [10]. General knowledge is independent of the particular software application. On the other hand, new knowledge includes all the specific concepts and ideas regarding the particular software application. This includes knowledge encoded in source code, which typically comprises several levels of abstraction. Each level of abstraction defines an information space or subsets of the global information space representing the whole software system[11]. These information spaces display an internal structure that is navigated by software engineers to obtain new knowledge and achieve program comprehension. [12] further decomposes information retrieval in two different strategies: Browsing and Searching. Browsing is an exploration of high-level software entities while searching aims to low-level entities. Efficient browsing requires an adequate software structure. For instance, modular software (i.e., a system that has been subdivided in disjoint chunks or modules with clear boundaries) enhances program comprehension and minimizes the impact of changes. In this context, we think that structural analyses of class graphs might be useful to assess the performance of browsing and program comprehension in general.

A. Data and Methods

We have collected a large sample of 80 different software systems written in Java and C++. This dataset represents a wide variety of different software applications and it is large enough to be statistically significant. We have recovered class graphs according to the definition given in the previous
section. Actually, five systems provide full UML class diagrams: ProRally 2002 (a proprietary C++ videogame), Striker (C++ videogame), JDK-A and JDK-B (two largest connected components of Java 1.5) and Mudsi (a distributed JAVA application). These UML class diagrams are design documents released by their respective software developers. The mapping from UML class diagrams to class graphs is straightforward (see fig.1). The remaining software systems represent a diverse repertoire of Open Source (OS) applications written in C++. These class graphs were reverse-engineered with a simple lexical analysis of the C++/Java source codes (see Appendix I). Fig. 7e is the class graph for the C++ code in fig. 7a. In Table I there is a summary of graph measurements in a subset of software systems. Figure 2 shows the class graph for the C++ videogame ProRally 2002 (see below).

B. Connectance and Linear Growth

The number of links \( L = |L| \) scales with the number of classes \( N = |C| \) in an almost linear way (see fig.3a):

\[
L \sim N^{1.17} \tag{1}
\]

This shows that class graphs are very sparse. In addition, this linear dependence between links and nodes means that every new class attaches (on average) to an approximately constant number of existing classes. This fits very well the assumption of the linear growth in software systems [13]. Define the richness connectance of a graph as the fraction of used links \( L \) compared to the number \( N(N-1) \) of links in the complete graph (self-referencing is avoided). If \( L \) scales linearly with \( N \), then richness connectance will decay approximately as \( 1/N \) (see fig. 3b). Linear growth does not allow for extensive changes to the large-scale class graph structure. Connectance decays very fast and network size quickly saturates to a constant value. This saturation has been associated to a pattern of increasing complexity in software development [14].

C. Class Graphs are Small-Worlds

Watts and Strogatz found that many real networks display short average path length and high clustering (or nonnegligible cliquishness) [6]. A network displaying these properties is called a small world (SW). Given a node \( v_i \) with degree \( k_i \) (i.e., the number of links attached to the node), we define node clustering \( C_i \) as the fraction of actual number of triangles \( t_i \) where the node \( v_i \) participates in:

\[
C_i = \frac{t_i}{k_i(k_i - 1)} \tag{2}
\]

The clustering coefficient \( C \) of a graph measures the proportion of triangles in the graph:

\[
C = \left( \frac{2}{k_i(k_i - 1)} \sum_{j=1}^{N} A_{i,j} \left( \sum_{k=1}^{N} A_{j,k} \right) \right) \tag{3}
\]

where \( A \) is the adjacency matrix for the graph with \( A_{i,j} = 1 \) if node \( v_i \) and \( v_j \) are connected and \( A_{i,j} = 0 \) otherwise. For random graphs, the clustering coefficient is inversely proportional to the graph size:

\[
C_{\text{rand}} \approx \frac{\langle k \rangle}{N} \tag{4}
\]

The clustering coefficient of a SW is significantly larger than the expected clustering coefficient for the random network, \( C > C_{\text{rand}} \). Nodes in the SW are densely connected with its immediate neighborhood. Average path length \( d \) is a measure of the global connectivity, or the mean distance \( d_{ij} \) required to navigate between any pair of nodes \( v_i \) and \( v_j \):

\[
d = \frac{1}{N} \sum_{v_{i,j}} d_{ij} \tag{5}
\]

The average path length in random graphs is proportional to the logarithm of their size:

\[
d_{\text{rand}} \approx \frac{\log N}{\log \langle k \rangle} \tag{6}
\]

The average path length of a SW is as small as in the unrestricted random case \( d \approx d_{\text{rand}} \), due to a few long-range edges (shortcuts) connecting distant regions of the network. Then, small average path length is compatible with a broad range of clustering coefficient values [6]. This is a measure of network spread or compactness that has been observed in different contexts, from the Internet to the social networks. In these systems, it is useful to keep \( d \) as low as possible. For example, shortest paths often enable faster communications. On the other hand, coupled oscillator systems with short average path lengths synchronize much faster than systems displaying longer paths [15], [16].

We have measured \( d \) and \( C \) in all the class graphs described above. Comparison with random predictions shows that class graphs are instances of small-worlds (see table I). For every class graph, we have observed that \( C > C_{\text{rand}} \) and \( d \approx \)
TABLE I

| Dataset | N | L | d | d_{rand} | C | C_{rand} |
|---------|---|---|---|----------|---|----------|
| Mqsdl   | 168| 231| 2.88| 3.95 | 0.244| 0.017|
| JDK-B   | 1364| 1947| 5.97| 6.80 | 0.225| 0.002|
| JDK-A   | 1376| 2162| 5.40| 6.28 | 0.159| 0.002|
| Prorally| 1993| 4987| 4.85| 4.71 | 0.211| 0.003|
| Striker | 2356| 6748| 5.90| 4.46 | 0.282| 0.002|
| gchempaint | 27 | 41 | 2.85| 3.26 | 0.204| 0.102|
| 4yp     | 54  | 90 | 3.28| 3.44 | 0.069| 0.059|
| Prospectus | 99 | 168 | 3.80| 3.77 | 0.14 | 0.034|
| eMule   | 129 | 218 | 3.87| 4.16 | 0.237| 0.025|
| Aime    | 143 | 319 | 2.66| 3.34 | 0.413| 0.031|
| Openvrd | 159 | 335 | 3.53| 3.53 | 0.08 | 0.026|
| gpfd    | 162 | 300 | 4.02| 3.93 | 0.303| 0.022|
| Dim     | 162 | 254 | 4.32| 4.45 | 0.304| 0.19 |
| Bochs   | 164 | 339 | 3.15| 3.60 | 0.335| 0.025|
| Quanta  | 166 | 239 | 4.31| 5.03 | 0.198| 0.017|
| Fresco  | 189 | 277 | 4.73| 4.89 | 0.228| 0.015|
| Freetype| 224 | 363 | 4.29| 4.71 | 0.193| 0.014|
| Yahoopops | 373 | 711 | 5.57| 4.47 | 0.336| 0.01 |
| Blender | 495 | 834 | 6.54| 5.14 | 0.155| 0.007|
| GTK     | 748 | 1147| 5.87| 5.91 | 0.081| 0.004|
| OIV     | 1214| 3903| 3.99| 3.82 | 0.122| 0.005|
| wxWindows | 1309| 3144| 4.03| 4.62 | 0.235| 0.004|
| CS      | 1488| 3526| 3.92| 4.74 | 0.135| 0.003|

**Fig. 4.** Log-log plots validating the small-world model of class graphs. Every point corresponds to a single class graph. (A) Average path length vs class graph size. Normalized path length grows with the logarithm of number of classes, as expected in small world networks (see text). (B) Normalized clustering for the systems analysed here strongly departs from the predicted scaling relation followed by random graphs (dashed line). Class graphs are much more clustered (by orders of magnitude) than their random counterparts.

\[ d_{rand} \]

In fig.4 we can clearly appreciate this result: the clustering coefficient in class graphs is well above the random expectation while the average path length is rather small. Actually, the values of \( C \) seem rather independent from system size \( N \). This is a common feature in hierarchical networks [17].

**D. Small-World and Breakdown of Modularity**

Software systems are constantly evolving. A goal of software engineers is to minimize the cost of software evolution by limiting the consequences of changes. Some designs are better than others in this regard because they allow software engineers to make small changes without propagating many secondary changes to other software components. In this context, it would be desirable to have reliable estimates of future change costs. Unfortunately, we still do not understand very well the properties of software development and maintenance. Here, we propose that software maintenance is a global process and thus, it is very difficult to predict the spreading of software changes.

For example, compare the modular graph in fig.5B and the random graph in fig.5A. The graph in fig.5B displays three, highly clustered, modules (i.e, a module is a subset of nodes that exchange many more links among them than with the rest of the network). Here, modules are interconnected to other modules by a single link and thus suggesting that internal changes in a module cannot affect other modules. On the other hand, fig.5A is an example of highly coupled, loosely modular architecture. This is a random graph and all nodes belong to the same module. Changes in the random graph (fig.5A) are more likely to affect many more nodes than changes in the modular graph (fig.5B). Notice that local measures cannot separate random and modular structures (i.e., the random graph and the modular graph have the same average degree \( \langle k \rangle \)). This suggests why empirical studies do not report significant correlations between local software measures and change impact [18]. The state of a class relies on the state of all the other classes it references, including these classes referenced through a chain of intermediate classes.

There is ample evidence that many software projects have a natural tendency to become disordered structures [19]. This code degradation is often associated with a breakdown of modularity that happens when changes are widely dispersed and affect many unrelated classes in apparently distant modules[20]. We suggest that such breakdown of modularity might be related to the emergence of the small-world behavior. Recall that a highly-clustered class graph (i.e., a modular graph) becomes a small-world by the addition of a few shortcut links between dissimilar nodes (i.e., a relationship between unrelated classes in different software modules). Once the system displays small-world behavior, its average path length gets near the minimal value \( d_{rand} \) and the software project might be closer to a breakdown of modularity. In this context, we propose that software engineers evaluate the risk of code degradation by measuring any significant deviation of average path length (\( d \)). This global measure could be a better indicator of code degradation because it takes into account indirect effects.

**E. Class Graphs are Scale-Free Networks**

Class graphs are highly heterogeneous networks, where a very few classes participate in many relationships and the majority of classes have one or two relationships [21]. Highly connected classes are key software components that keep the whole software system as a coherent entity. In this context, software designs are remarkably similar to many other complex networks, like the WWW, the Internet and many biological networks [3]. They are all examples of scale-free (SF) networks, that is, they have a degree distribution that follows a scaling law, \( P(k) \sim k^{-\gamma} \). As shown in figure 6 and in table II, class graphs are nice instances of scale-
free (SF) networks. The fact that all the graphs analysed here display SF structure, in spite of the obvious differences in size, functionality and other features, is an indication that strong constraints are at work in software evolution. However, and contrary to the small-world feature of class graphs, we suggest this scale-free behavior has an exogenous origin (see below).

The cumulative degree distribution $P_0(k)$ reduces noise levels during the estimation of the scaling exponent $\gamma$,

$$P_0(k) = \sum_{k' > k} P(k')$$  \hspace{1cm} (7)

If $P(k) \approx k^{-\gamma}$ then we have $P_0(k) \approx \int P(k') dk' \approx k^{-\gamma + 1}$. The exponent $\gamma$ is estimated by linear regression in the log-log plot (see figure 3b and figure 9). For class graphs analyzed here, we obtain $\gamma \approx 2.5$. On the other hand, in-degree and out-degree distributions of directed class graphs also follow power-laws, $P_{\text{in}}(k) \sim k^{-\gamma_{\text{in}}}$ and $P_{\text{out}}(k) \sim k^{-\gamma_{\text{out}}}$. Directed degree distributions display different exponents from the undirected version. Typically, we observe $\gamma_{\text{in}} < \gamma$ and $\gamma_{\text{out}} > \gamma$. In other words, if we look at the number of outgoing and incoming links, the resulting degree distributions are different. The in-degree distribution has a clear power-law tail while the out-degree distribution decays much faster. A similar pattern has been observed in the web graph [22]. An extensive study of the entire WWW in October 1999 used the webcrawl from AltaVista to obtain empirical in-degree and out-degree distributions for a subset of the full web graph[23]. They have shown that in- and out-degree distributions of the web graph are fitted by scaling laws with exponents $\gamma_{\text{in}} = 2.1$ and $\gamma_{\text{out}} = 2.7$. These exponents are very close to the average in-degree exponent $\langle \gamma_{\text{in}} \rangle = 2.2$ and the average out-degree exponent $\langle \gamma_{\text{out}} \rangle = 2.8$ taken over all class graphs in table II.

**F. Related Work**

In order to measure software cohesion and coupling, [24] proposed to represent software designs with graphs, where nodes represent software entities and edges are relationships between entities. In this framework, a software module is a subset of nodes (or subgraph) more densely connected than with the rest of the network. [24] explores what are the desirable properties of any cohesion and coupling measurement. At the coarsest level of description of a software system (or software architecture), a measure of coupling is the number of edges exchanged between modules. The cohesion of a module comprising $k$ elements scales with the ratio $2t/k(k-1)$ where $t$ is the number of edges within the module. These coupling and cohesion definitions correlate with the number of edges $L$ and local clustering $C_i$ respectively. In this context, a statistically valid model of class graph will provide useful estimators of coupling and cohesion in real systems.

Measurements are needed to assess the best design solution among different alternatives. From all the candidates in the solution space, we want to pick the one with the highest metric value [25]. It has been proposed that change impact defines one of the axes of this solution space. Some graph measurements from the logical structure of OO systems can be used as (static) estimators of change impact. A related definition is alteration visibility or the size of the set of classes affected by the change to a single class [26]. A more detailed approach to impact analysis uses approximate algorithms to compute ripple effects from low-level source code features (see [27][28]). A distance measure very similar to path length was used in [26] in order to select the best choice when restructuring large software designs. However, [26] proposed...
to measure path length between classes belonging to a single module (i.e., intra-distance) while here we propose to measure path length between all the classes in the class graph (i.e., inter-distance).

Chidamber and Kemerer (C&K) presented a suite of object-oriented metrics [29] that seems to be related to our suite of graph measurements in class graphs. Several histograms of C&K metrics display highly skewed distributions, i.e., fig. 2 for the WMC (Weighted Methods per Class) metric, fig. 14 for the CBO (Coupling between Object Classes) metric and fig. 16 for RFC (Response for a Class) metric in [29]. These histograms resemble power-laws like the degree distribution of class graphs (see previous subsection). Unfortunately, while C&K metrics appear to be related to some of our graph measurements, no further comparison is possible because no regression analysis of histograms is available from their study. Still, C&K made a qualitative interpretation of extreme metric values in terms of “outliers” [29], which provides some evidence of heterogeneous software metrics.

In this context, there is a close relationship between depth of inheritance tree (or DIT, see [29]) and degree of difficulty in understanding and comprehending the organization of object-oriented systems. Dvorak claimed that “the deeper the level of the hierarchy, the greater the probability that a subclass will not consistently extend and/or specialize the concept of its superclass” [30], that is, excessively deep class hierarchies are complex to develop. Evidence of positive correlation between DIT and the likelihood of faults in OO systems is given in [31]. We should expected some correlation between DIT and average path length $d$ of class graphs because inheritance tree is a subset of the class graph, suggesting how DIT might be closely related to the small-world. Very low values of DIT found in the study of Li and Henry [32] provide some empirical support to this hypothesis.

### III. CLASS-METHOD ASSOCIATION GRAPHS

We have shown that class graphs are small-worlds. Here, we investigate the possibility that small-world can spontaneously emerge in an evolving OO software system. We provide empirical and theoretical support to this conjecture by modeling the hierarchical structure of OO software with a bipartite association between classes and methods. In Java and C++, we conceive a software system as a set of interrelated classes. These classes are further decomposed into data members (or variables) and code methods (a method is the OO equivalent of subroutines in Fortran, C or Pascal). This hierarchical organization of OO software can be represented with a bipartite association graph $B = (V, U, E)$ where $V = \{v_i\}$ is the set of classes and $U = \{m_i\}$ is the set of methods and $E = \{(v_i, m_i)\}$ is the set of dependencies between classes and methods. Also, $N = |V|$ is the number of classes and

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**Fig. 7.** (a) An example of C++ code and its: (b) bipartite association graph $B$, (c) its class projection $B_c$, (d) its method projection $B_m$ and (e) its class graph $G$.

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**Table II**

| Dataset | Degree | In-degree | Out-degree |
|---------|--------|-----------|------------|
| Mudsii  | 1.74 ± 0.04 | 1.20 ± 0.08 | 2.00 ± 0.05 |
| JDK-B   | 1.55 ± 0.08 | 1.39 ± 0.05 | 2.30 ± 0.14 |
| JDK-A   | 1.41 ± 0.02 | 1.18 ± 0.02 | 2.39 ± 0.14 |
| Prorally| 1.72 ± 0.03 | 1.44 ± 0.02 | 1.88 ± 0.10 |
| Striker | 1.70 ± 0.04 | 1.54 ± 0.03 | 1.73 ± 0.06 |
| gchempaint | 1.63 ± 0.31 | 1.11 ± 0.35 | 1.41 ± 0.12 |
| 4yp     | 1.54 ± 0.09 | 1.30 ± 0.05 | 1.59 ± 0.18 |
| Prospectus | 1.67 ± 0.09 | 1.13 ± 0.09 | 1.92 ± 0.27 |
| eMule   | 1.58 ± 0.03 | 1.51 ± 0.07 | 1.42 ± 0.08 |
| Aim     | 1.43 ± 0.05 | 1.30 ± 0.04 | 1.48 ± 0.07 |
| Openvrm | 1.34 ± 0.06 | 0.94 ± 0.05 | 1.59 ± 0.23 |
| gpdf    | 1.64 ± 0.11 | 1.23 ± 0.10 | 1.76 ± 0.17 |
| Bochs   | 1.37 ± 0.08 | 1.17 ± 0.09 | 1.64 ± 0.20 |
| Quanta  | 1.69 ± 0.10 | 1.55 ± 0.13 | 1.87 ± 0.13 |
| Fresco  | 1.66 ± 0.09 | 1.14 ± 0.10 | 1.76 ± 0.19 |
| Freetype| 1.65 ± 0.07 | 1.42 ± 0.04 | 1.82 ± 0.16 |
| Yahooops| 1.67 ± 0.05 | 1.46 ± 0.06 | 1.69 ± 0.05 |
| Blender | 1.64 ± 0.04 | 1.36 ± 0.05 | 2.04 ± 0.09 |
| GTK     | 1.51 ± 0.04 | 1.22 ± 0.02 | 2.38 ± 0.20 |
| OIV     | 1.43 ± 0.02 | 1.14 ± 0.03 | 2.10 ± 0.12 |
| wxWindows | 1.41 ± 0.03 | 1.11 ± 0.02 | 2.18 ± 0.12 |
| CS      | 1.58 ± 0.02 | 1.22 ± 0.03 | 1.96 ± 0.09 |
$M = |U|$ is the number of methods. We have an edge \( \{v_i, m_j\} \in E \) when class \( v_i \) appears in the parameter list of method \( m_j \). In addition, a class is always a parameter of its own collection of methods (i.e., self or this keyword). We can recover this bipartite graph with a simple algorithm (see Appendix II). Figure 7 illustrates a small C++ code (see fig. 7b) and its corresponding bipartite association graph (see fig. 7a).

We define the (discrete) generating functions \( \mu(n) \) and \( \nu(n) \) for the bipartite graph:

\[
\mu(n) = \sum_k k^n P_u(k)
\]

and

\[
\nu(n) = \sum_k k^n P_v(k)
\]

where \( n = 1, 2, \ldots \) and \( P_u(k) \) is the fraction of \( U \) nodes having \( k \) edges and \( P_v(k) \) is the fraction of \( V \) nodes having \( k \) edges. First moments \( \mu = \mu(1) \) and \( \nu = \nu(1) \) indicate the average method degree and the average class degree, respectively. It is easy to check that \( M \nu = N \mu \).

The one-mode projection (or unipartite) network expresses connections between nodes of the same kind (see fig. 7c,d). We have two one-mode projections \( B_v = (V, E_v) \) (i.e., so-called class projection) and \( B_u = (U, E_u) \) (i.e., so-called method projection) from the bipartite association method-class graph. Formally, we define \( A \) as the adjacency matrix of the bipartite network \( B \), where \( A_{i,j} = 1 \) if \( \{v_i, u_i\} \in E \) and \( A_{i,j} = 0 \) otherwise. The adjacency matrix \( A^V \) for the one-mode projection \( B_v \) is related to the adjacency matrix \( A \) by:

\[
A^V_{i,j} = \sum_k A_{ik} A_{jk}
\]

A similar relation holds between the adjacency matrix \( A^U \) of projection \( B_u \) and the adjacency matrix \( A \) of the bipartite network \( B \):

\[
A^U_{i,j} = \sum_k A_{ki} A_{kj}
\]

Netwman et al. have shown that one-mode projections must be small-worlds even when the bipartite association is random[33]. Social networks display high-clustering coefficients because agents follow a natural tendency to group together in communities. Moreover, the addition of a few shortcuts between distant agents in clustered communities yields to small average path lengths. Assuming that bipartite association \( B \) is random, we have that the average path length between two classes in \( B_v \) will be very small,

\[
d(B_v) = \frac{\log N}{\log z}
\]

and correspondingly for the method projection \( B_u \) we have,

\[
d(B_u) = \frac{\log M}{\log z}
\]

where \( z = \mu \nu \) is the expected average degree for the one-mode projection. The clustering coefficient for the one-mode projection \( B_v \) will be very high:

\[
C(B_v) = \frac{1}{\mu + 1}
\]

and for \( B_u \),

\[
C(B_u) = \frac{1}{\nu + 1}
\]

Then, the above suggests that partitioning a OO software system into classes and methods is very likely to result into a highly-clustered software structure with small average path lengths. Moreover, this seems to be largely independent of the specific association between methods and classes. The clustering coefficient and mean path length only depend on the average connectivity of classes and methods. The small-world behavior of class graphs does not appear to be an additional requirement selected by software engineers but an unavoidable feature associated to the hierarchical nature of OO software systems.

A. Comparison between Class Graphs and Class Projections

Beyond specific topological patterns displayed in any OS software system we can investigate how well (in a statistical sense) the class projection explains structural patterns displayed by the class graph. Here, we are more interested in the structural properties of the average OS software system. In this context, we have found very good agreement between local and global measures of class graphs and class projections. In order to enable a meaningful comparison between the class projection \( B_v \) and the class graph \( G \), we must ensure they have the same number of nodes and links. First, we obtain the filtered class graph \( \bar{G} = (\bar{V}, \bar{E}) \) by removing \( G \) nodes without edges in \( B_v = (V, E_v) \). On the hand, the class projection \( B_v \) often displays more edges than the filtered class graph \( \bar{G} \), that is, \( |E_v| \geq |\bar{E}| \). Then, we remove a fraction \( p = 1 - |\bar{E}|/|E_v| \) from the original class projection to obtain
the filtered class projection $\tilde{B}_v = (\tilde{V}, \tilde{E}_v)$ having the same number of nodes and edges in the filtered class graph $\tilde{G}$. For the systems analyzed here, the average edge removal probability is $(p) \approx 0.54$.

Clustering coefficient of (filtered) class graphs scales almost linearly with clustering coefficient measured in the (filtered) class projections (see fig. 8A):

$$C(G) = 0.92C(\tilde{B}_v) ± O(1) \quad (16)$$

Edge removal can leave disconnected nodes in the class projection. Average path length $d$ cannot be computed in disconnected networks because $d_{i,j} = \infty$. Fortunately, we can use the global efficiency $E_{glob}$ measure that is formally equivalent to average path length. Global efficiency of an undirected graph $G$ is defined as follows [4]:

$$E_{glob}(G) = \frac{1}{N(N-1)} \sum_{i \neq j \in G} \frac{1}{d_{i,j}} \quad (17)$$

where $0 \leq E_{glob}(G) \leq 1$. Note that the maximum value of global efficiency is attained when $G$ is the complete graph having $N(N-1)/2$ possible edges and the minimum value indicates that $G$ has no edges, i.e., the graph is completely disconnected. We have found that global efficiency in class graphs scales with global efficiency of class projections:

$$E_{glob}(\tilde{G}) = 1.15E_{glob}(\tilde{B}_v) ± O(1) \quad (18)$$

Good agreement of local and global measurements in the class graph $G$ and the class projection $\tilde{B}_v$ provides support that the SW behavior of class graphs is an invariant feature of any OO software system. OO programming requires that related code and data cluster together in the same class, and thus resulting in high clustering coefficients. However, methods cross class boundaries when they use data (and methods) from other external classes. These eventual interactions among unrelated software entities yield small average path lengths.

B. Case Study: Stellarium

We illustrate a detailed comparison between class graph and class projection (see above) with the OS software Stellarium (http://stellarium.free.fr). This comparison suggests how useful is to analyze and compare several network representations during reverse engineering and program comprehension. Stellarium is written in C++ and computes the position of stars and other space bodies in real-time. Figure 9 shows the largest connected components of class graph $G$ and class projection $\tilde{B}_v$ recovered from the C++ source code our reconstruction algorithm. Looking at the class graph (see fig. 9A) we notice two well-defined communities or modules in Stellarium. Comparing between the class graphs and the class projection indicates that modularity is preserved across multiple levels of the software hierarchy (see fig. 9B). Indeed, every software system analyzed here follows this pattern: global features are preserved across levels while individual nodes might play different roles depending on the network representation. Table III and table IV summarize individual node measurements (classes are highlighted with solid circles in figure 9).

For instance, Class Projector belongs to the same module in $G$ and in $\tilde{B}_v$. However, Projector is a hub in $\tilde{B}_v$ but has few connections in $G$ (see fig. 9). An opposite example is the class stel_core, which is a hub in $G$ but has only two connections in $\tilde{B}_v$ (in fact, this node belongs to a disconnected subgraph not shown in the above figure). Class stel_core relies in many other classes ($k_{out} = 22$) and is the main application dispatcher in Stellarium. stel_core is the starting point of many code reviews and thus, is frequently visited by Stellarium engineers. Consequently, this class dis-
...the graphical application state (i.e., projection matrix, observer coordinates, etc.).

class of any user interface control in Stellarium. On the other hand, displayed by class

The second largest centrality value is referenced for the class graphs comprising two years of development. From this dataset, we have analyzed time series of the number of nodes in very good agreement with the exponent obtained in section IV.C) and the final class graph has \( N = 1993 \) and \( L = 4987 \) links (see fig. 2). Table I and II reports some network measurements for the final ProRally 2002 class graph.

The time evolution of average path length \( d \) in ProRally 2002 quickly saturates with \( d \approx 5 \) after a brief transient (see fig. 10). This constant growth pattern in the time evolution of ProRally 2002 yields an heterogeneous \( P(k) \). As shown by Puniyani and Lukose, growing random networks under the constraint of constant diameter must display scale-free architecture and with a scaling exponent \( \gamma \in [2,3] \) [37]. Specifically, they found that:

\[
P(k) \approx k^{3-\frac{\alpha}{\beta}}
\]

where \( \alpha \leq 1 \) is an exponent relating network size \( N \) with degree fluctuations:

\[
N^\alpha = \frac{1}{\langle k \rangle} \int k^2 P(k)dk
\]

and \( \beta \) is an exponent linking the degree cutoff \( k_c \) (see subsection II.D) with network size \( N \), i.e.,

\[
k_c \approx N^\beta
\]

Using our dataset, we estimate \( \beta = 0.62 \pm 0.09 \) and \( \alpha = 0.42 \pm 0.08 \). This predicts the scaling exponent \( \gamma \approx 2.59 \) to be compared with the average exponent over all systems \( \langle \gamma \rangle = 2.57 \pm 0.07 \) (computed from table II). The scaling law in the cutoff \( k_c(N) \) allows us to provide an analytic calculation of the scaling between \( L \) and \( N \). The following integral gives the general relationship between \( L \) and \( N \):

\[
L = N \int_0^{\infty} k P(k)dk
\]

Here, we have,

\[
L \approx \frac{N}{k_c^{1-\gamma}}
\]

approx \( N(N^\beta)^{\gamma-2} = N^{1+\beta(\gamma-2)} \approx N^{1.22} \)

in very good agreement with the exponent obtained in section II.E for class graphs. Keeping the average path length constant during class graph evolution yields an heterogeneous degree distribution with the observed exponent. However, we were unable to find any intrinsic explanation to this constraint. A possible explanation is an exogenous pressure related to communication constraints in distributed software teams [38] (see below).
An influence in the organization and structuring of source code measurements.

Software systems. For example, degree distributions predict this view by large-scale statistical characterizations preserving inevitability yield inaccurate predictions. In conclusion, we must that using non power-law expressions of degree distribution simple (instructions) to the large and complex (the modular architecture). The signature of complex software organization partly explains why it is difficult to find a clear, nice decomposition of software systems. Moreover, the role of broad distributions in software measurements have been largely dismissed. Researchers treat these distributions like normally distributed [7]. Such transformation losses a significant amount of information and hides the true nature of software. Instead, we must address heterogeneous distributions with appropriate tools. This knowledge could be crucial to develop future software systems. For example, degree distributions predict how many classes have more than, say, a hundred, data members in a future class diagram of doubled size. Notice that using non power-law expressions of degree distribution inevitably yield inaccurate predictions. In conclusion, we must abandon reductionistic descriptions of OO systems and replace this view by large-scale statistical characterizations preserving the structural variability.

A more general question concerns the usefulness of single-valued metrics. For example, the distribution of class sizes (measured as number of lines of code, NLOC) encodes more information than the integral value or system size. Given a NLOC value (say, 10KLOC), there are many distributions satisfying this integral value. That is, NLOC is an ambiguous measure that provides less information than the original size distribution. We can compute average path length and the average clustering coefficient from probability distributions of basic graph metrics, i.e. connectivity. There is an important source of information in the probability distributions of software measurements.

On the other hand, large-scale software development is a social task. Interaction between software engineers might have an influence in the organization and structuring of source code bases. For example, open-source developments are geographically distributed. These software teams face pressing communication and coordination problems that require specific software structures according to the social organization: "organizations which design systems are constrained to produce designs which are copies of the communications structure of these organizations" (Conway’s Law)[38]. Under this social perspective, software is viewed simultaneously as the product and the vehicle that enables efficient communication between software engineers, i.e., the communication medium. Separated software modules minimize communication overheads, which is the bottleneck in distributed software developments [39]. In this context, it should be interesting to study how the patterns described here relate to distributed software development.

Finally, while our study focused on static analyses of source code, recent studies on object graphs have revealed similar patterns in the distribution of run-time connections between objects in several programs [40]. This similarity suggests a link between structural and dynamical features of OO software that should be investigated in future studies.

V. CONCLUSION

The structure and operation of software systems can be studied at different levels of organization, from the small and simple (instructions) to the large and complex (the modular architecture). The signature of complex software organization is an heterogeneous and hierarchical network. This pattern partly explains why it is difficult to find a clear, nice decomposition of software systems. Moreover, the role of broad distributions in software measurements have been largely dismissed. Researchers treat these distributions like normally distributed [7]. Such transformation losses a significant amount of information and hides the true nature of software. Instead, we must address heterogeneous distributions with appropriate tools. This knowledge could be crucial to develop future software systems. For example, degree distributions predict how many classes have more than, say, a hundred, data members in a future class diagram of doubled size. Notice that using non power-law expressions of degree distribution inevitably yield inaccurate predictions. In conclusion, we must abandon reductionistic descriptions of OO systems and replace this view by large-scale statistical characterizations preserving the structural variability.

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APPENDIX I

CLASS GRAPH RECONSTRUCTION ALGORITHM

The following algorithm reconstrucsthe class graph from a collection of Java/C++ header files (comments are highlighted in italics). The class Digraph implements a directed graph. The method Digraph::AddLink(c1, c2) tests if class names c1 and c2 have been already inserted in the graph. If not, they are inserted correspondingly. We discard repeated links (c1, c2). There is distinction between public, private or protected attributes. Finally, the algorithm outputs the directed (and undirected) network versions to a file.

Digraph D; // class graph D = (C,L)
String c1, c2; // class names
FOR every header file DO
WHILE (not end of file) DO
    // Find class declaration
    c1 = get_class_name(); //c1 ∈ C
    // Test if inheritance relationship ("is a")
    IF (next sequence is ': public') THEN
        c2 = get_parent_class(); //c2 ∈ C
        D.AddLink(c1, c2); //c1, c2) ∈ L
    ENDIF
    // get attributes ("has a")
    WHILE (not end of class) DO
        Look for attribute declaration;
        c2 = get_attribute_class(); //c2 ∈ C
        D.AddLink(c1, c2); //c1, c2) ∈ L
    END
END
D.Output();

APPENDIX II

ASSOCIATION GRAPH RECONSTRUCTION ALGORITHM

The following algorithm recovers the bipartite association class-method graph from a collection of C++ header files. The class Bipartite implements a bipartite graph. There
is a method Bipartite::AddLink\((u, v)\) that checks if method \(u\) and class \(v\) have been already inserted in the graph. We assume that methods have unique identifiers. Methods having the same name can still be differentiated because they belong to different classes (and classes cannot have the same name).

Bipartite \(B\) \(\triangleq\) association graph \(B = (V, U, E)\)

String \(v_1, v_2\) \(\triangleq\) // classes

String \(u\) \(\triangleq\) // method name

FOR every header file DO

WHILE (not end of file) DO

// Find class declaration

Look for 'class' keyword;

\(v_1 = \text{get\_class\_name}(); \quad // v_1 \in V\)

WHILE (not end of class) DO

Look for method declaration;

\(u = \text{get\_method\_name}(); \quad // u \in U\)

D.AddLink\((u, v_1); \quad // \{u, v_1\} \in E\)

END

END

B. Output();

END

END

END

END

END

END

END

END

END

END

END

END

END

END

END

END

END

END

END

END

END

END

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