On the Interacting Chiral Gauge Field Theory in $D = 6$ and the Off-Shell Equivalence of Dual Born-Infeld-Like Actions

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**Abstract**

A canonical action describing the interaction of chiral gauge fields in $D = 6$ Minkowski space-time is constructed. In a particular partial gauge fixing it reduces to the action found by Perry and Schwarz. The additional gauge symmetries are used to show the off-shell equivalence of the dimensional reduction to $D = 5$ Minkowski space-time of the chiral gauge field canonical action and the Born-Infeld canonical action describing an interacting $D = 5$ Abelian vector field. Its extension to improve the on-shell equivalence arguments of dual D-brane actions to off-shell ones is discussed.
1 Introduction

A manifestly covariant action for the bosonic $D = 11$ five-brane was constructed in [1] and generalized to a $kappa$-invariant action for the M-theory super five-brane in $D = 11$ supergravity backgrounds [2], the latest theory was independently obtained in [3] from a non-manifestly covariant action. The above results are based on previous work [4].

The authors of reference [4] constructed a $D = 6$ Lorentz invariant interacting theory of a self-dual tensor gauge field, which gives Born-Infeld theory upon reduction to $D = 5$. The fully covariant action for the bosonic 5-branes constructed in [1] may be obtained as an extension of the non covariant action of reference [4] via the PST auxiliary scalar field approach [5] [6]. The five-brane superfield equations were also obtained using the superembedding of the world volume superspace into a target superspace in [7].

The duality between M-Theory and IIA superstring suggests that the double dimensional reduction of the $M5$-brane action should coincide with the duality transformed Dirichlet 4-brane action, this was proven in [8]. The approach followed there is similar to that used in the analysis of duality transformation in [9] [10] [11] [12].

The essential steps involved in world-volume duality transformation of $D$-brane actions may be described for the Born-Infeld theory. The Born-Infeld action in $n$ dimensional Minkowski space-time is given by

$$S_A = - \int d^n x \sqrt{-\det(\eta_{\mu\nu} + F_{\mu\nu})}$$ (1)

where $F$ is the curvature of the connection 1-form $A$. The idea is to formulate the theory in terms of a dual $(n - 3)$-form potential $B$ defined by

$$^*H = - \frac{\delta S}{\delta F}$$ (2)

where $^*H$ is the Hodge dual of the “curvature” $(n - 2)$-form

$$H = dB.$$ (3)

In this formulation the Bianchi identity for the $B$ field is the field equation for the connection $A$, while the Bianchi identity for $A$ is the field equation for the potential $B$. The next step in the approach is to solve for $F$ in terms of $^*H$ and then to find the action which gives the field equations for $B$, a problem which becomes increasingly non-linear and difficult to solve with the
Nevertheless the programme was successfully carried out in [4] and [8] for \( n \leq 5 \) and the action for the \( B \) field turned out to be

\[
S_B = -\int d^nx \sqrt{-\det(\eta_{\mu
u} + i^* H_{\mu\nu})}.
\]

(4)

The equivalence between the action (1) and (4) has been then established over the field equations. The approach just described was extended in [8] to prove on-shell equivalence of duality transformed actions for super D-branes including Wess-Zumino terms.

In this paper we show the off-shell equivalence of the canonical actions associated to (1) and (4), which will be called the Born-Infeld action for a 1-form gauge field and a 2-form gauge field respectively. We consider the particular case \( n = 5 \) which corresponds to the double dimensional reduction of the 5-brane action and the Dirichlet four-brane action. We start from the action proposed in [4] for the interacting theory of self-dual tensor gauge field and look for an action with first class constraints only. The canonical action we propose in a particular partial gauge fixing reduces to the action in [4]. The action is described with the same canonical variables as the one in [4]. It does not require the introduction of auxiliary fields. It has more gauge symmetries, but the manifest \( SO(5,1) \) invariance, which was reduced to a manifest \( SO(4,1) \) in [4] is now reduced to \( SO(4) \). We will then consider the reduction of the action to 5-dimensional Minkowski space-time. In a particular admissible partial gauge fixing we will obtain the canonical formulation of (1) and in a different admissible partial gauge fixing we will obtain the canonical formulation of (4). The equivalence between the canonical formulations will then be established off-shell, without using the field equations. The equivalence between duality transformed quadratic actions is always an off-shell equivalence. The difficulties arises when considering interacting theories, in particular when the actions involved include higher order time derivative terms as in the case we will discuss. In [12] off-shell arguments between dual actions have also been used, by realizing the duality symmetry as canonical transformations. In [13] the standard duality transformation was used to show the off-shell equivalence of the \( D = 11 \) M-2-Supermembrane and the \( D = 10 \) IIA D-brane.

We will start our analysis using the non-manifestly covariant action in [4], we think the approach will follow also from the covariant action in [1]. In fact, the canonical analysis of the covariant action for a free chiral tensor gauge field in [1] shows that in this formulation there are first class constraints only.
However, the constraints as presented in [6] have a peculiar behaviour over the submanifold $\partial_a a = 0$, as explained in that paper. Since in any case we will break the manifest Lorentz invariance in our argument, we prefer to start directly from the action in [4]. Recently, a canonical analysis of the five-brane action was performed in [14] and a canonical Hamiltonian obtained. The second class constraints are, however, still present in that formulation. The mechanism we will implement to eliminate the second class constraints of the action in [4], works exactly in the same way for the elimination of the second class constraints of the five-brane actions in [3]. It seems important to have a canonical formulation of the five-brane action with first class constraints only, in order to have control over all the gauge symmetries and to determine the physical Hamiltonian of the theory. It is well known that the physical Hamiltonian of the $D = 11$ M-2-supermembrane has very peculiar properties which yield a continuous spectrum from 0 to $\infty$ when the M-2-brane maps the world-volume onto $D = 11$ Minkowski target space [15]. The situation may change however when the membrane maps over compactified target space. Particularly interesting is the case when the target space is $M_9 \times S^1 \times S^1$. In that case, the infinite dimensional “valleys”, responsible of the continuity of the spectrum, disappear at least from the minima of the Hamiltonian [16]. In spite of several improvements in the analysis, the spectrum of the M-2-brane in the compactified case is still unknown [17]. It is then of interest the analysis of the five-brane physical Hamiltonian, together with a canonical formulation free of second class constraints. In this paper we construct that action for the case of the interacting theory of chiral gauge bosons on 6-dimensional Minkowski space-time and use its additional gauge symmetries to show the off-shell equivalence between its reduction to 5-dimensional Minkowski space-time and the Born-Infeld theory over that base space.

2 Analysis of the Free Theory

We consider a 6-dimensional Minkowski space-time, with coordinates $x^M$, $M = 0, ..., 5$, where $x^5$ denotes the temporal coordinate while $x^\mu$, $\mu = 0, 1, 2, 3, 4$, denote spatial coordinates. The “curvature” 3-form $H$ in terms of the “connection” 2-form $B$ is given by:

$$H = dB,$$

and in component notation:
\[ H = H_{LMN}dx^L \wedge dx^M \wedge dx^N = \frac{1}{3}(\partial_L B_{MN} + \partial_N B_{LM} + \partial_M B_{NL})dx^L \wedge dx^M \wedge dx^N, \]  
(6)

The Hodge dual 3-form \( *H \) has the expression:

\[ *H = \frac{\sqrt{-G}}{6} \epsilon_{RSTMNP} H^{MNP} dx^R \wedge dx^S \wedge dx^T \]  
(7)

Where \( G \) is in general the determinant of the six dimensional metric. In our case \(-1\).

The self-duality field equations:

\[ *H = H, \]  
(8)

were obtained in [4] from an action, which is not manifestly covariant, expressed in terms of the 5-dimensional Hodge dual to \( H \), with components:

\[ *H_{\mu\nu} = \frac{1}{2} \sqrt{g} \epsilon_{\mu\nu\rho\lambda\sigma} H^{\rho\lambda\sigma}. \]  
(9)

In (9) \( H^{\rho\lambda\sigma} \) is obtained from the primitive spatial components \( H_{\mu\nu\lambda} \) by raising the indices with the 5-dimensional spatial components of the metric, in our case \( \delta_{\mu\nu} \). In distinction, the indices in (6) are raised using the six dimensional metric \( \eta_{MN} \).

The action proposed in [4] is:

\[ S_{PS} = \int_{M_6} (*H^{\mu\nu} \partial_5 B_{\mu\nu} - *H^{\mu\nu} *H_{\mu\nu}) d^6x, \]  
(10)

where again the 5-dimensional metric has been used to raise the indices.

The canonical analysis of (10) may be performed in a straightforward way. The canonical action is constrained by the mixture of first and second class constraints:

\[ \varphi^{\mu\nu} \equiv P^{\mu\nu} - 2 *H^{\mu\nu} = 0. \]  
(11)

The canonical Hamiltonian is:

\[ \mathcal{H} = \mathcal{H}_0 + \lambda_{\mu\nu} \varphi^{\mu\nu} \]  
(12)

where \( \mathcal{H}_0 = *H^{\mu\nu} H_{\mu\nu} \) is positive definite.
Although the action (10) proposed in [4] yields the required field equations (8), it has second class constraints, therefore it should exist an action with first class constraints only, from which (10) could be obtained by partial gauge fixing. When we consider dynamical systems with a finite number of degrees of freedom this is always possible, however, when the dynamics involves infinite degrees of freedom the new action may involve non-localities or an infinite number of first class constraints [18]. In the case under consideration, there is however a closed solution to the new action. It is given, in canonical form, by:

\[ S = \int_{M_6} \frac{1}{2} P^{\mu\nu} \partial_5 B_{\mu\nu} - \frac{1}{4} (\ast H^{ij} + \frac{1}{2} P^{ij})(\ast H_{ij} + \frac{1}{2} P_{ij}) - 2^* H^{0i}* H_{0i}]d^6x, \quad (13) \]

subject to the first class constraints:

\[ \phi^i \equiv P^{0i} - 2^* H^{0i} = 0, \quad (14) \]

\[ \Lambda^\nu \equiv \partial_\mu P^{\mu\nu} = 0, \quad (15) \]

where \( i = 1, 2, 3, 4 \) and \( \mu = 0, 1, 2, 3, 4 \).

The first class constraints commute between themselves and with:

\[ H_0 = \frac{1}{4} (\ast H^{ij} + \frac{1}{2} P^{ij})(\ast H_{ij} + \frac{1}{2} P_{ij}) - 2^* H^{0i}* H_{0i}, \quad (16) \]

We emphasize that \( x^0 \) is an spatial coordinate in this formulation.

The constraints (14) and (15) are not independent. In fact

\[ \partial_i \phi^i - \Lambda_0 = 0, \quad (17) \]

\[ \partial_\mu \Lambda^\mu = 0, \quad (18) \]

There are seven independent first class constraints. The original set of constraints (11) consists of ten independent constraints. Six of them are second class constraints which come from the partial gauge fixing of (14). The whole set (14) arises then from partial gauge fixing of (14), (15). Moreover in this particular gauge (13) reduces to (10). We have obtained then an action whose dynamics is restricted by first class constraints only which in a particular gauge fixing yields (10). The new action has more gauge symmetries, and we will exploit this fact shortly, however we have lost the manifest Lorentz invariance of (10) on the 5-dimensional space-time. The full Lorentz invariance will, of course, be recovered at the S-matrix level.
3 Equivalence Between the Chiral Gauge Field Theory and Born-Infeld Theory.

3.1 The Free Case.

We will discuss in this section the quantum equivalence between the free theory of a 2-form gauge field and the Maxwell theory over a 5-dimensional Minkowski space-time. The equivalence arises directly by using the standard duality procedure which we briefly review. However, this approach fails as an off-shell argument when we consider the equivalence of the dimensional reduced interacting chiral gauge field theory and the Born-Infeld action for a curvature 2-form $F(A)$. In fact, the equivalence between both theories has only been established on-shell [4] and [8]. We will follow a different approach, which may be extended to show the off-shell equivalence of the interacting theories under consideration. In this section we apply the approach to the free theories, to show how it works in a straightforward case. The standard duality approach, when applied to the action:

$$S_c = - \int_{M_5} \ast H^{ab} \ast H_{ab} d^5 x, \quad a, b = 1, 2, 3, 4, 5, \quad (19)$$

yields the master action:

$$S_d = \int_{M_5} (-L^{ab} L_{ab} + iF_{ab}(A)L^{ab}) d^5 x, \quad (20)$$

where $F_{ab}(A)$ is the curvature of the connection 1-form $A$. Functional integration on $A$ in (20) yields

$$\partial_a L^{ab} = 0, \quad (21)$$

which implies

$$L^{ab} = \ast H^{ab}. \quad (22)$$

After substitution in (20), we obtain (19). If instead we functionally integrate on $L$ in (20), we obtain the quantum equivalent action:

$$S_d = - \int d^5 x. \frac{1}{4} F_{ab}(A) F^{ab}(A) \quad (23)$$

(19) and (23) are then off-shell equivalent actions. The duality approach as described is very useful when analyzing global properties of the theories,
but has the drawback that becomes, in most of the cases, useless as an off-shell argument, when the theories describe non-linear interactions. We consider now a different argument. We start from the dimensional reduction of (13) to 5-dimensional Minkowski space-time. We compactified $X^0$ and take the zero mode of the Fourier expansion. We will consider two different partial gauge fixing conditions. (13), (14), (15) under one of the gauge fixing will reduce to (19) while under the other one becomes (23).

We impose the partial gauge fixing condition:

$$B_{0i} = 0,$$

associated to the first class constraints (14). We may then use (24) and (14) to perform a canonical reduction of (13). we obtain:

$$S_1 = \int_{M_5} \left[ \frac{1}{2} P_{ij} \partial_5 B_{ij} - \frac{1}{16} P_{ij} P_{ij} - 2*H^{0i} H_{0i} + \rho_j \partial_i P_{ij} \right] d^5x,$$

where we have used

$$*H^{ij} = \frac{1}{2} \epsilon^{ijkl} \partial_0 B_{kl} + \epsilon^{ijk\ell} \partial_k B_{0\ell} = 0$$

and we have introduced the remaining first class constraints into the action by means of a Lagrange multiplier which may be reinterpreted as:

$$\rho_i = B_{5i}.$$

Functional integration on $P_{ij}$ gives, after some calculations,

$$S_1 = -\int_{M_5} *H^{ab} H_{ab} d^5x.$$  

We impose now a different admissible gauge fixing condition. Instead of (24) we consider:

$$P_T^{ij} = 0$$

$$B_{Li} = 0,$$

Where the transverse $T$ and longitudinal $L$ parts are defined in terms of the derivative operator. That is, for an anti-symmetric tensor, we have
\[ V^{ij} = V_{ij}^T + \partial^i V^j_L - \partial^j V_i^L \]  
\[ \partial_i V^{ij}_T = 0, \]

(30) 

(31) 

(29) together with (14) and the subset of (15)

\[ \partial_j P_{ji} + \partial_0 P_{0i} = 0 \]

(32) 

allow the elimination of the canonical conjugate pair \((P^{ij}, B_{ij})\). The canonical reduced action is then:

\[ S_2 = \int_{\mathcal{M}_5} \left[ P_{0i} \partial_5 B_{0i} - \frac{1}{4} \ast H^{ij} \ast H_{ij} - \frac{1}{2} P_{0i} P_{0i} + \lambda \partial_i P_{0i} \right] d^5 x, \]

(33) 

Functional integration of \(P^{0i}\) yields, after some calculations,

\[ S_2 = - \int \frac{1}{4} F_{ab}(A) F^{ab}(A) d^5 x. \]

(34) 

Where \(F_{ab} = \partial_a A_b - \partial_b A_a\) and \(A_a = B_{0a}\).

We have thus obtained the off-shell equivalence of (28) and (34) since both arise from different admissible gauge fixing of the action (13), more precisely from its dimensional reduction to \(\mathcal{M}_5\). The factors in front of the actions (28) and (34) are exactly the same as the ones obtained from the duality transformation.

### 3.2 The Interacting Theories

Following [4] we consider the interacting chiral gauge field theory governed by the action:

\[ S = \int_{\mathcal{M}_6} \left[ \ast H^{\mu\nu} \partial_5 B_{\mu\nu} - 4 \sqrt{\text{det}(\delta_{\mu\nu} + \ast H_{\mu\nu})} \right] d^6 x. \]

(35) 

Since in our formulation \(X^5\) is the time coordinate, the metric to be used inside the square root is \(\delta_{\mu\nu}\). Consequently there is no \(i\) factor in front of \(\ast H_{\mu\nu}\), otherwise an indefinite expression will arise from the terms 1+ quadratic terms. The \(-\) sign in front of the square root is required to have a positive definite Hamiltonian of the resulting theory. It is straightforward to see that the expansion of (35) up to quadratic terms on the fields agrees exactly with (10). The changes in (33) with respect to the action proposed in [4] are then
the necessary ones when $X^5$ is considered as a temporal coordinate instead
of a spatial coordinate as taken in [4]. The canonical analysis of (35) yields
the mixture of first and second class constraints (11) as in the free case.
The subset of first class constraints is (15), representing four independent
constraints. The remaining six constraints in (11) are second class ones. The
canonical Hamiltonian is given by (12) where $H_0$ is in this case

$$H_0 = 4 \sqrt{\det(\delta_{\mu\nu} + *H_{\mu\nu})},$$

(36)

$\Lambda^\mu$ commutes with $*H^{\sigma\lambda}$ and hence with $H_0$. We may now look for an action
with first class constraints only, from which (35) may be obtained by partial
gauge fixing. It is given by

$$S = \int_{M_6} \left[ \frac{1}{2} P^{\mu\nu} \partial_5 B_{\mu\nu} - 4 \sqrt{\det(\delta_{\mu\nu} + M_{\mu\nu})} + \lambda_i \phi^i + \rho_\nu \Lambda^\nu \right] d^6 x$$

(37)

Where

$$M_{0i} = *H_{0i}$$
$$M_{ij} = \frac{1}{2}(H_{ij} + \frac{1}{2} P_{ij}).$$

(38)

$\phi^i$ and $\Lambda^\nu$ are given by (14) and (15) respectively. $\phi^i$ and $\Lambda^\mu$ commute
between themselves and with the matrix elements $M_{\mu\nu}$. They hence commute
with the canonical Hamiltonian.

Again, as in the free case, by gauge fixing of the symmetries generated
by $\phi^i$ and use of the first class constraints, we obtain

$$P^{\mu\nu} = 2*H^{\mu\nu}$$

(39)

which after its substitution in (37) yields exactly (35).

We will study now the quantum equivalence between the dimensional
reduction of the interacting theory of chiral gauge fields (37) and the Born-
Infeld action in $D = 5$ Minkowski space-time.

We proceed as in the free case, we will show that there are two admissible
partial gauge fixing of (37) which give rise in one case to the canonical
formulation of the Born-Infeld action for interacting 1-form gauge fields, and
in the other case to the canonical formulation of the Born-Infeld type action
for interacting 2-form gauge fields. The canonical actions are equivalent at
the quantum level since they will arise from different admissible gauge fix-
ing of the master action (37) (provided there is no gauge anomaly in the
quantum formulation of the theory). The off-shell equivalence is established
between the canonical formulations of the actions not between the covariant
Lagrangian formulations. The on-shell equivalence between the covariant
formulations was proven in [8] and on this aspect we do not have anything
to add.

However, from the point of view of the quantum theory formulated in
terms of functional integrals, one must start with the canonical formulation
for the actions and our equivalence argument implies then the first step
towards the quantum equivalence of the theories.

We consider first the partial gauge fixing (24), that is

\[ B_{0i} = 0 \]  

Its Poisson bracket with \( \phi^j \) is

\[ \{ B_{0i}(x), \phi^j(x') \} = \delta^6(x - x')\delta_i^j \]  

We then eliminate \( P_{0i} \) from (14),

\[ P_{0i} = 2^{*} H^0_i. \]  

The canonical reduction of (37) is given by

\[ S = \int_{M_6} \left[ \frac{1}{2} P^{ij} \partial_5 B_{ij} - 4 \sqrt{det(\delta_{\mu\nu} + M_{\mu\nu}) + \rho_j \Lambda^j} \right] d^6 x \]  

where \( ^*H_{ij} = \frac{1}{2} \epsilon_{ijkl} \partial^0 B^{kl} \).

If we consider the dimensional reduction to 5-dimensional Minkowski
space-time, we then have

\[ ^*H_{ij} = 0, \]  

and

\[ \Lambda^j = \partial_0(2^{*} H^{0j}) + \partial_i P^{ij} = \partial_i P^{ij}. \]  

We notice that, because of (17), we are left with only the four first class
constraints
\[ \Lambda^j = 0 \]  \hspace{1cm} (46)

If we take the quadratic approximation in (13), after the reduction to \( M_5 \) we exactly obtain (25). That is, (13) describes the dynamics of the same number of degrees of freedom of the interacting 2-form gauge field theory over \( M_5 \). The Lagrange multiplier in (13) may be interpreted as \( B_5^i \), see (27). We will now show that it is in fact, exactly the same dynamics. Moreover, we will now prove that the reduction to \( M_5 \) of (13) is the canonical formulation of the covariant action (4). From (13) we obtain

\[
\frac{1}{2}(\partial_5 B_{ij} + \partial_j B_{5i} - \partial_i B_{5j}) \sqrt{\det(\delta_{\mu\nu} + M_{\mu\nu})} = \frac{\delta \det(\delta_{\mu\nu} + M_{\mu\nu})}{\delta P_{ij}} \]  \hspace{1cm} (47)

We would like to obtain \( P_{ij} \) in terms of \( B_{kl} \) from this equation. The problem is not straightforward since it is a higher order equation on the \( P_{ij} \). However a general solution may be given. In fact, consider the action

\[
-4 \int_{M_5} \sqrt{-\det(\eta_{ab} + i^* H_{ab})}, \quad a, b = 1, 2, 3, 4, 5. \]  \hspace{1cm} (48)

Where, as in 3.1, the indices \( a, b \) are related to 5-dimensional Minkowski space-time. It exactly reduces to (28) when the quadratic approximation of the square root of the determinant is taken. Consider the conjugate momenta to \( B_{ij} \)

\[
\mathcal{P}^{ij} = 6 \sqrt{-\det(\eta_{ab} + * H_{ab})} \frac{\delta \det(\eta_{ab} + * H_{ab})}{\delta \partial_5 B_{ij}}. \]  \hspace{1cm} (49)

The right hand side of (49) contains higher order time derivatives of the \( B_{ij} \) field.

We now take

\[
P^{ij} = \mathcal{P}^{ij} \]  \hspace{1cm} (50)

and then, one may check that (47) is identically satisfied. This means that (50) is a solution of (47). It is not the unique solution to (17), but it seems to be the only one giving rise to covariant field equations. We now eliminate \( P^{ij} \) in terms of \( B_{ij} \) and \( \partial_5 B_{ij} \) in the reduction to \( M_5 \) of (13). It turns out that
\[ \det(\eta_{\mu\nu} + M_{\mu\nu}) = \frac{[1 + (H_{i5})^2 - \frac{1}{3!} \epsilon^{ijkl} \epsilon^{emnpq} H_{mi} H_{nj} H_{pk} H_{ql}]^2}{-\det(\eta_{ab} + i^* H_{ab})}. \] (51)

The kinetic terms in (43) have the same denominator as the square root of the determinant, that is

\[ [-\det(\eta_{ab} + i^* H_{ab})]^{\frac{1}{2}}. \] (52)

We choose the square root of (51) as

\[ [\det(\eta_{\mu\nu} + M_{\mu\nu})]^{\frac{1}{2}} = \frac{[1 + (H_{i5})^2 - \frac{1}{3!} \epsilon^{ijkl} \epsilon^{emnpq} H_{mi} H_{nj} H_{pk} H_{ql}]^2}{[-\det(\eta_{ab} + i^* H_{ab})]^{\frac{1}{2}}}. \] (53)

It combines with the kinetic terms of (43) to give in the numerator

\[ -4[-\det(\eta_{ab} + i^* H_{ab})] \] (54)

We end up exactly with (48). The field equations of the action

\[ S_5 = \int_{M_5} \frac{1}{2} P^{ij} \partial_5 B_{ij} - 4 \sqrt{\det(\delta_{\mu\nu} + N_{\mu\nu})} + \rho_j \Lambda^j] d^5 x \]

\[ N_{0i} = * H_{0i} \]

\[ N_{ij} = \frac{1}{4} P_{ij}, \] (55)

and the field equations of (48) are then equivalent. (55) is then a canonical formulation of (48). The latest contains higher order time derivatives. Generically the canonical formulation is in that cases not necessarily unique (13).

We now consider the partial gauge fixing (29), that is

\[ P^i_T = 0 \]

\[ B_{Lij} = 0 \] (56)

(29) is an admissible partial gauge associated to the constraints (14),

\[ \phi^i = P^{0i} - 2* H^{0i} = P^{0i} - \epsilon^{0ijkl} \partial_j B_{kl} = 0 \] (57)
In fact, only the transverse part of $B_{kl}$ appears in that constraint. (29) and (14) allow to eliminate the conjugate pair ($B_{Tij}, P_{T}^{ij}$) from the canonical formulation. Consider now the constraints

$$\Lambda^i = \partial_0 P^{0i} + \partial_j P^{ji} = 0. \quad (58)$$

They only involve the longitudinal part of $P^{ij}$. An admissible gauge fixing condition associated to it is then (29). (29) and (58) allow then to eliminate the canonical conjugate pair ($B_{Lij}, P_{L}^{ij}$). We notice that, after the reduction to 5-dimensional Minkowski space-time, (29) and (58) imply over a contractible manifold. (it is the case for $M_5$), with appropriate boundary conditions,

$$P^{ij} = 0 \quad (59)$$

The canonical reduction of (37), over $M_5$, becomes then

$$S = \int_{M_5} \left[ P^{0i} \partial_5 B_{0i} - 4 \sqrt{\text{det}(\delta_{\mu\nu} + L_{\mu\nu}) + \lambda \phi^0} \right] d^5x \quad (60)$$

Where $L_{0i} = \frac{1}{2} P_{0i}$ and $L_{ij} = \frac{1}{2} H_{ij}$. The only constraint left is the first class Gaussian one

$$\partial_j P^{0j} = 0 \quad (61)$$

The quadratic approximation of (60) yields exactly (33). (60) describes then the same degrees of freedom of the Born-Infeld action for a gauge field of rank one. We will now show that not only the degrees of freedom are the same but the dynamics is exactly equivalent. We consider, as before, the field equations.

Taking variations with respect to $P^{0i}$, we get

$$\frac{1}{2} \left[ \partial_5 B_{0i} - \partial_i B_{05} \right] \sqrt{\text{det}(\delta_{\mu\nu} + L_{\mu\nu})} = \frac{\delta \text{det}(\delta_{\mu\nu} + L_{\mu\nu})}{\delta P^{0i}} \quad (62)$$

We now present a solution to this highly non-linear equation on $P^{0i}$. We start from the covariant action

$$-4 \int_{M_5} \sqrt{-\text{det}(\eta_{ab} + \frac{1}{2} F_{ab})}, \quad a, b = 1, 2, 3, 4, 5 \quad (63)$$
which exactly reduces to (34) when the quadratic approximation for the square root is used. We consider now the canonical conjugate momenta to $A_a = B_{0a}$,

$$\mathcal{P}^i = 2 \sqrt{-\det(\eta_{ab} + \frac{1}{2} F_{ab})} \frac{\delta \det(\eta_{ab} + \frac{1}{2} F_{ab})}{\delta \partial_5 B_{0i}}$$  \hspace{1cm} (64)$$

The numerator of the right hand side member of (64) is linear in the time derivatives, but the denominator is a non-linear expression in terms of them. We now take

$$P_{0i}^i = \mathcal{P}^i$$  \hspace{1cm} (65)$$

and replace it into (62). It is identically satisfied. Having obtained a solution to (62), we substitute it into the action.

It turns out that

$$[\det(\delta_{\mu\nu} + L_{\mu\nu})] = \left[ 1 + \frac{1}{8} f_{ij}^2 + \frac{1}{16} \epsilon_{ijkl} \epsilon^{mnpq} f_{mi} f_{nj} f_{pk} f_{ql} \right]^2 \frac{-\det(\eta_{ab} + \frac{1}{2} F_{ab})}{-\det(\eta_{ab} + \frac{1}{2} F_{ab})}$$  \hspace{1cm} (66)$$

The kinetic terms in (60) have the same denominator as the square root of the determinant, that is

$$(-\det(\eta_{ab} + \frac{1}{2} F_{ab}))^{-\frac{1}{2}}$$  \hspace{1cm} (67)$$

we choose the square root of (67) as

$$[\det(\delta_{\mu\nu} + L_{\mu\nu})]^\frac{1}{2} = \left[ 1 + \frac{1}{8} f_{ij}^2 + \frac{1}{16} \epsilon_{ijkl} \epsilon^{mnpq} f_{mi} f_{nj} f_{pk} f_{ql} \right]^2 \frac{-\det(\eta_{ab} + \frac{1}{2} F_{ab})}{-\det(\eta_{ab} + \frac{1}{2} F_{ab})}$$  \hspace{1cm} (68)$$

It combines with the kinetic terms of (60) to give in the numerator

$$-4[-\det(\eta_{ab} + \frac{1}{2} F_{ab})]$$  \hspace{1cm} (69)$$

We notice that there is a different sign in the fourth order term of (68) compared with the one in (53). This is so because in (43) there also contributions of fourth order of the same form coming from the kinetic terms. The overall fourth order term has the right coefficient. In (60) instead, the fourth order terms coming form the square root, have only space like derivatives and sum in a direct way to the kinetic terms.
We obtain exactly (63). (60) is then a canonical formulation of (63).

We have thus obtained canonical formulations of (48) and (63) given by (55) and (60) respectively, and show they are equivalent off-shell. We have thus realized the duality transformation in terms of two different gauge fixing conditions of a master action. Several interesting aspects of the duality symmetry in the Born-Infeld and chiral bosons actions have been discussed in the literature [20].

4 Conclusions

We constructed an action describing the interaction of chiral gauge fields in 6-dimensional Minkowski space-time, which in a particular partial gauge fixing reduces to the Born-Infeld type action found in [4]. The dynamics of the new action is restricted by first class constrains only, in distinction to the Perry-Schwarz formulation of the theory. The new action is used to prove the off-shell equivalence of the canonical formulation of the dimensional reduced interacting chiral theory and Born-Infeld theory for 1-form gauge fields, on \( D = 5 \) space-time. In particular it provides a canonical formulation for the interacting 2-form gauge field theory, whose Lagrangian depends on higher order time derivatives. The action we presented, over \( M_5 \), has the peculiar property that there are two admissible partial gauge fixing which give rise to two covariant theories. The usual situation is that for a given canonical action there is only one gauge fixing giving rise to a covariant action. It is clear that the peculiarity is due to the duality transformation between the two covariant actions. It suggests then that the approach we presented could be generalized and used to improve the on-shell equivalence of non-linear dual theories to off-shell ones.

From the point of view of the quantum formulation of the fields theories in terms of the functional integral approach, one has to start with their canonical actions. The off-shell equivalence between the canonical formulations yields then a first step towards the proof of the quantum equivalence between the dual theories. We notice that, since the dependence on the momenta of the canonical action is not quadratic, we cannot functionally integrate the momenta to obtain the functional integral in terms of the covariant Lagrangian. From this point of view, the off-shell equivalence between the canonical actions is the one that has to be established.

The elimination of the second class constraints of the action presented in
may be performed on the same lines we have presented. We expect the
same programme will then follow to show the off-shell equivalence of dual
D-brane actions.

References

[1] P. Pasti, D. Sorokin and M. Tonin, Phys. Lett. B 398 (1997) 41.
[2] I. Bandos, K. Lechner, A. Nurmagambetov, P. Pasti, D. Sorokin and M.
    Torin, Phys. Rev. Lett. 78 (1997) 4332., I. Bandos, N. Berkovits and D.
    Sorokin, Nucl. Phys. B 522 (1988) 214.
[3] M. Aganagic, J. Park, C. Popescu, J. H. Schwarz, Nucl. Phys. B 496
    (1997) 191.
[4] M. J. Perry and J. H. Schwarz, Nucl. Phys. B 498 (1997) 47.
[5] P. Pasti, D. Sorokin, M. Tonin, Phys. Lett. B 352 (1995) 59.
[6] P. Pasti, D. Sorokin and M. Tonin, Phys. Rev. D 55 (1997) 6292.
[7] P. S. Howe and E. Sezgin, Phys. Lett. B 394 (1997) 62., P. S. Howe, E.
    Sezgin and P. West, Phys. Lett. B 399 (1997) 49.
[8] M. Aganagic, J. Park, C. Popescu ans J. H. Schwarz, Nucl. Phys. B 496
    (1997) 215
[9] C. Schmidhuber, Nucl. Phys. B 467 (1996) 146.
[10] A. A. Tseytlin, Nucl. Phys. B 469 (1996) 51.
[11] S. P. de Alwis and K. Sato, Phys. Rev. D 53 (1996) 7187.
[12] Y. Lozano, Phys. Lett. B 399 (1997) 233.
[13] P. K. Townsend, Phys. Lett. B 373 (1996) 68.
[14] E. Bergshoeff, D. Sorokin and P. K. Townsend, hep-th/9805065.
[15] B. de Wit, M. Lüscher an H. Nicolai, Nucl. Phys. B 320 (1989) 135.
[16] I. Martin, A. Restuccia, R. Torrealba, Nucl. Phys. B 521 (1998) 117.
[17] B. de Wit, K. Peeters, J. Plefka, Phys. Lett. B 409 (1997) 117.

[18] I. A. Batalin and E. S. Fradkin, Nucl. Phys. B 279 (1987) 514, B. McClain, Y. S. Wu and F. Yu, Nucl. Phys. B 343 (1990) 689, I. Martin and A. Restuccia, Phys. Lett. B 323 (1994) 311, I. Bengtsson, Int. J. Mod. Phys. A 12 (1997) 4869, F. P. Devecchi and M. Henneaux, Phys. Rev. D 45 (1996) 1600, N. Berkovits, Phys. Lett. B 388 (1996) 743; Phys. Lett. B 395 (1997) 28; Phys. Lett. B 398 (1997) 79, R. Gianvittorio, A. Restuccia and J. Stephany, Mod. Phys. Lett. A 6 (1991) 2121, A. Restuccia and J. Stephany, Phys. Rev. D 47 (1993) 3437, A. Restuccia and J. Stephany, Phys. Lett. B 343 (1995) 147.

[19] M. Lledó and A. Restuccia, Annals of Physics 224 (1993) 1; Nucl. Phys. B 434 (1995) 231.

[20] H. O. Girotti, M. Gomes, V. O. Rivelles, A. J. da Silva, Phys. Rev. D 56 (1997) 6615., D. Berman, Phys. Lett. B 409 (1997) 153, M. Gaillard and B. Zumino, hep-th/9705226, A. Khoudeir and Y. Parra, Phys. Rev. D 58 (1998) 025010. A. Maznytsia, C. Preitschopf and D. Sorokin hep-th/9805110; hep-th/9808049.