Degradation spectra of electrons in the ionosphere

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Abstract. Theory and numerical simulations of degradation spectra of electrons in gases are presented. Theory is based on the power spectra of degradation charged particles as the spectra with fluxes in energy space. Numerical calculations of the electron energy distribution function have been performed for ionospheric gas mixtures constituted of molecules N₂, O₂ and atom O under influence of high energy electron source with detailed elementary electron collision processes with molecules and atoms being taken into consideration. The energy expenses of electrons into ionization, dissociation and excitation of various levels have been obtained so that to determine the rates of electron collision processes. The dependence of the electron energy expenses into various inelastic electronic processes upon the energy of primary electron source has been revealed. The results are presented for the rates of numerous elementary processes of electron interaction with basic ionospheric components to be suitably determined.

1. Introduction
A central problem of aeronomy and optics of upper aerosphere is the theoretical model for interaction of an ionospheric gas with the high energy electrons erupting out of magnetosphere, which produces ionization, dissociations and excitation of numerous atomic and molecular levels of the gas. The action of any ionization source within gaseous medium can be reduced to that of highly energetic primary electrons with their specified energy distribution. As the electrons interact with gas molecules, there occur successive loss of energy by primary electrons and formation of secondary electrons, that is, the energy of electrons degrades. Such an electron energy distribution function, i.e. the electron degradation spectrum, settles the rates of elementary electron processes. The consistent and accurate determination of rates of electronic processes appears to be a rather complex and laborious problem demanding the numerical research. It is specified by the fact that the electron degradation spectrum represents a strongly non-equilibrium distribution depending both on the cross-sections of numerous elementary electron–molecule collisions and on the gas composition that can vary in the ionosphere.

2. Simulation of ionospheric gas by high energy electrons
2.1. Electron degradation spectrum in ionospheric gas
In present research the numerical calculations of the electron degradation spectrum have been carried out for typical ionospheric gas mixtures of the molecules N₂, O₂ and atom O with detailed consideration of the electron–molecule and electron–atom elementary collisions, all the rates of these processes being obtained. The electron degradation spectrum and rates of electron elementary processes are determined on a basis of the numerical solution of Boltzmann kinetic
equation for an electron distribution function. The spatial distribution of electrons is supposed
to be isotropic, such a supposition is usually reasonable under conditions where the free path
of electrons at threshold energies is less than the standard size of gas spatial inhomogeneity, i.e.
the pressure scale height. Similar conditions are observed at the Earth lower ionosphere up to
altitudes about 250 km approximately. The present analysis of electron distribution function
does not consider the electron–electron, electron–ion collisions and collisions of the second kind,
which is justified enough under the low degree of ionization in the ionosphere, for example, at
the hours of darkness or either at non-intensive fluxes of primary electrons erupting out of the
magnetosphere.

The Boltzmann kinetic equation for the isotropic distribution function of electrons on their
energies \( f(\varepsilon, t) \) within spatially-homogeneous gaseous medium takes into consideration various
electron elementary processes and represents a complicated integro-differential equation with
shifted arguments; its general form is following [1]

\[
\rho \frac{\partial f}{\partial t} = \sum_k S_{tk} f + \sum_i S_{ti} f + S - A.
\]

Here the terms of right side present:

(i) \( S_{tk} f \) are multiple collision integrals of electrons with molecules which include translational
\((m)\), rotational \((r)\), vibrational \((v)\) and electronic \((j)\) processes;
(ii) \( S_{ti} f \) are the rate of secondary electrons created in cascades of primary and other generations
of electrons in different ion states;
(iii) \( S \) is the rate of primary electrons generation by an external ionization source;
(iv) \( A \) is the rate of disappearance of free electrons in all possible reactions of their recombination
with ions and attachment to molecules.

The equation uses the normalization

\[
\int_0^{\infty} f(\varepsilon, t) \rho(\varepsilon) d\varepsilon = n(t),
\]

where the density of free electron states \( \rho(\varepsilon) = \varepsilon^{1/2} \) with \( \varepsilon \) being the kinetic energy of electrons,
the time \( t \), the concentration of free electrons \( n \). The equilibrium Maxwellian distribution with
electron temperature \( T_e \) is expressed there as

\[
f_M(\varepsilon) = 2n \exp(-\varepsilon/T_e) / (\pi T_e)^{3/2}.
\]

The kinetic equation considers in consecutive order the following processes of electron–
molecule collisions—elastic \((m)\), inelastic excitation \((k)\) and ionization \((i)\) of molecules. The integrals of appropriate excitations—rotational \((r)\), vibrational \((v)\), and electronic \((j)\) states
of molecules—are summed up. The collision integrals can be introduced through appropriate
electron–molecule collision frequencies \( \nu \) describing the rate of an electron elementary process
and being equal to \( \nu(\varepsilon) = N \nu(\varepsilon) \sigma(\varepsilon) \), where \( N \) is the concentration of particles, \( \nu(\varepsilon) = (2/m)^{1/2} \varepsilon^{1/2} \) is the electron velocity; \( m \) is the electron mass; \( \sigma(\varepsilon) \) is the cross-section of the
appropriate process.

The collisions of electrons with exited molecules (collisions of the second kind) being not
taken into account, the integral of any inelastic collision related to excitation of molecule can
be produced by a similar form for rotational, vibrational and electronic levels

\[
S_{tk} f = f(\varepsilon + \varepsilon_k, t) \rho(\varepsilon + \varepsilon_k) \nu_k(\varepsilon + \varepsilon_k) - f(\varepsilon, t) \rho(\varepsilon) \nu_k(\varepsilon), \nu_k(\varepsilon) = N \nu(\varepsilon) \sigma_k(\varepsilon),
\]
where $\varepsilon_k$, $\sigma_k$ are the threshold and cross-section of $k$-th process of inelastic excitation.

The sum of ionization collision integrals considers the formation of ions in several excited states, each integral has the following form

$$
St_i f = \int_0^\infty f (\varepsilon_p, t) \rho (\varepsilon_p) \nu_{ii} (\varepsilon_p, \min (\varepsilon, \varepsilon_p - \varepsilon_i - \varepsilon)) \, d\varepsilon_p - f (\varepsilon, t) \rho (\varepsilon) \nu_i (\varepsilon),
$$

$$
\nu_{ii} (\varepsilon_p, \varepsilon_s) = Nv (\varepsilon_p) \sigma_{ii} (\varepsilon_p, \varepsilon_s), \nu_i (\varepsilon) = Nv (\varepsilon) \sigma_i (\varepsilon), \sigma_i (\varepsilon_p) = \int_0^{(\varepsilon_p - \varepsilon_i)/2} \sigma_{ii} (\varepsilon_p, \varepsilon_s) \, d\varepsilon_s.
$$

Here $\varepsilon_i$, $\sigma_i$ are the ionization potential and ionization cross-section; $\sigma_{ii}(\varepsilon_p, \varepsilon_s)$ is the differential ionization cross-section; $\varepsilon_p, \varepsilon_s$ are the energies of primary and secondary electrons respectively. The feature of an elementary ionization act is the appearance of two electrons of their different energies, the secondary electron being reputed one of smaller energy; therefore $\varepsilon_p \geq \varepsilon_i + 2\varepsilon_s$, and the energy range of secondary electrons $0, \varepsilon_s - \varepsilon_i/2$.

An important quality of the kinetic equation for distribution function with the integrals written above is its linearity—the electron energy distribution function $f (\varepsilon, t)$ appears to be in direct proportion to the external ionization source of and in inverse proportion to total molecule concentration. Therefore, it must be a natural physical parameter to describe influence of the source of high energy electrons upon gas, and this is the source power per one gas molecule.

The numerical calculations of electron distribution function have been carried out for standard gas composition of the lower ionosphere at various altitudes 100–250 km. Numerous (about sixty) elementary processes of electron collisions with molecules $N_2$ and $O_2$, and atom $O$ have been taken into account separately, they are as follows:

- inelastic collisions with molecule $N_2$—total rotational excitation, excitation of 8 vibrational and 10 electronic levels, ionization to 5 separate ion $N_2^+$ states and dissociative ionization;
- inelastic collisions with molecule $O_2$—total rotational excitation, excitation of 3 vibrational and 6 electronic levels, ionization to 4 separate ion $O_2^+$ states and dissociative ionization;
- inelastic collisions with atom $O$—excitation of the ground state fine structure, excitation of 7 electronic levels, ionization to 3 separate ion $O^+$ states;
- elastic collisions of electrons with molecules $N_2$, $O_2$ and atom $O$.

The excitation of high Rydberg states has been considered as an integrated process with the appropriate averaged potential. Selection of cross sections for the required elementary processes of electron–molecule/atom collisions refers to up-to-date database of the International Atomic Energy Agency [2].

It should be noted that at the altitude of 120 km approximately, firstly, there is a maximum of photopower electron flux due to photoionization of the ionospheric gas by solar radiation; secondly, the maximum intensity of auroras is frequently observed around this altitude as well. Thus, to illustrate the results of calculations a typical mixture of gaseous species $[N_2] : [O_2] : [O] = 0.75 : 0.15 : 0.10$ has been accepted as the ionospheric gas, it fairly corresponds to the Earth ionosphere altitude of about 120 km.

The whole energy range where the electron degradation spectrum has been numerically computed, from high energy electrons 1000 eV down to subthreshold energies 0.1 eV, contains four orders of magnitude, with both logarithmic and linear scales of energy being used in the calculation procedure. The reliability of calculations is controlled by the conservation of electron energy to be fulfilled within 1%.

The primary electron source energy has varied considerably over the range $E = 50$–1000 eV. A known circumstance should be emphasized that the shape of electron degradation spectrum at its threshold energy range is practically independent of the primary source energy $E$ in case
it exceeds 300 eV approximately; it is the threshold energy range $\sim (1–30)$ eV that is most significant for the rates of electron inelastic collisions. But at minor primary source energies $E < 300$ eV the electron degradation spectrum, generally speaking, becomes formed by this particular source and then loses its universal feature.

The results of typical calculations for the electron distribution function $f(\varepsilon)$ are shown in figures 1 and 2. There are three samples of permanent ionization source producing the primary electrons of different energies $E = 50$, 100, 1000 eV, with the source power per a gas molecule being equal to unity,

$$q = \int S(\varepsilon) \, d\varepsilon / N = 1 \text{ eV s}^{-1};$$

the expression $f(\varepsilon)^{1/2} \, d\varepsilon$ to present the number of free electrons in the volume unit within the kinetic energy interval $(\varepsilon, \varepsilon + d\varepsilon)$.

2.2. Electron power distribution percentage

The percentage of power contributed by the electron degradation spectrum into ionization, dissociation and excitation of various molecule/atom levels has been calculated so that to determine the rates of these electron processes in the Ionospheric gas mixture.

The relative electron power $p_k$ contributed into $k$-th inelastic process of electron–molecule collisions is expressed immediately through the electron degradation spectrum by the following
\[
p_k = \frac{(2/m)^{1/2}}{\varepsilon_k} \int_{\varepsilon_k}^{\infty} f(\varepsilon) \varepsilon \sigma_k(\varepsilon) \, d\varepsilon,
\]
where \( f(\varepsilon) \) is the electron degradation spectrum calculated under the source power per a gas molecule \( q = 1 \, \text{eV s}^{-1} \); \( \varepsilon_k, \sigma_k \) are the threshold and cross-section of \( k \)-th inelastic process; the coefficient value \((2/m)^{1/2} = 5.931 \times 10^7 \, \text{cm s}^{-1} \text{eV}^{-1/2}\).

When high energy electrons are slowing down in gaseous medium, the rate of \( k \)-th inelastic process of electron–molecule collisions is suitable to be characterized by such quantities as the relative power \( p_k \) contributed into \( k \)-th inelastic process and also the value of mean energy \( U_k \) expended by the slowing electrons for a singular act of this \( k \)-th inelastic process. In the radiation chemistry it is conventional to characterize the rate of an elementary process by the non-dimensional quantity \( G_k \) that determines the number of acts of the given elementary process due to each 100 eVof the energy supplied by an ionizer. For any inelastic electron process of excitation or ionization of a molecule/atom with the threshold energy \( \varepsilon_k \) there are simple relationships between the quantities \( U_k, G_k \), and the appropriate relative power \( p_k \) (units are specified in brackets):

\[
U_k = \varepsilon_k/p_k, \quad G_k = 100/U_k[\text{eV}], \quad p_k[\%] = G_k \varepsilon_k[\text{eV}].
\]

Thus the total energy value of electron–ion pair \( U_{\text{ion}} \) (the price of electron–ion pair) in a
Table 1. Power contribution (%) of electron degradation spectrum into the ionospheric gas at different source energies $E$.

| $E$, eV | Ionization | Electron excitation | Vibration excitation |
|---------|------------|---------------------|---------------------|
|         | $N_2$ | $O_2$ | O | $N_2$ | $O_2$ | O | $N_2$ | $O_2$ |
| 50      | 28.3  | 4.9  | 2.9 | 40    | 9.8  | 4.6 | 6.6  | 1.4   |
| 100     | 35.4  | 5.9  | 2.7 | 35.1  | 9.0  | 3.8 | 5.6  | 1.2   |
| 200     | 39.3  | 6.8  | 2.7 | 32.3  | 8.2  | 3.4 | 5.1  | 1.1   |
| 1000    | 42.7  | 7.4  | 2.7 | 29.7  | 7.3  | 3.1 | 4.8  | 1.0   |

Table 2. Energy value of electron–ion pair $U_{\text{ion}}$ in the ionospheric gas at different source energies $E$.

| $E$, eV | 50  | 100 | 200 | 1000 |
|---------|-----|-----|-----|------|
| $U_{\text{ion}}$, eV | 47.9 | 40.5 | 37.3 | 34.5 |

gaseous mixture is determined by the relationship

$$U_{\text{ion}}^{-1} = \sum_i U_i^{-1} = \sum_i \frac{p_i}{\varepsilon_i},$$

with all the ionization processes to be summed up.

The energy balance of the electron degradation spectrum in the lower ionosphere is specified almost completely by the inelastic processes of ionization, electronic and vibrational excitation of molecule/atom levels by electron impacts, the minor processes—rotational excitation of molecules, attachment of electrons to molecules, and elastic electron–molecule collisions—in aggregate constitute but 1% of total balance. The calculated power contribution of the primary electron source at its different energies $E$ into various inelastic electron–molecule interactions is illustrated by table 1.

The results obtained can be used for making of artificial plasma clouds in the ionosphere by a source of high energy electrons.

3. Conclusions
The electron energy distribution function (electron degradation spectrum) within the threshold range of energies, principally forming the rates of electron inelastic processes, is practically independent of the primary electron source energy if that energy exceeds roughly 300 eV.

At minor primary source energies the electron degradation spectrum fails its universality and becomes formed by an actual ionization source. At its minor energies the primary electron source decreases its contribution to ionization of gas molecules, so such an ionizer being less efficient.

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References

[1] Aleksandrov N L, Konovalov V P and Son E E 2000 *Encyclopedia of Low-Temperature Plasma* vol 1 (Moscow: Nauka) pp 548–564

[2] International Atomic Energy Agency Databases on Atomic and Molecular Data, Vienna, Austria URL [www-amdis.iaea.org](http://www-amdis.iaea.org)