On the quantum inductance of orbiting electron in the hydrogen atom

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Abstract. With the answer of yes, for the question, is the quantity current can be tagged to the minute loop of electron in the atom? Therefore, regard as the minute current loop. The next related question based on the voltage to current proportionality over operators of different so arise, is the quantity voltage for the current can also be mapped out of minute loop? And if yes, then, what is the corresponding proportionality constant be it holds? If the quantum electrical representative of the atom, for instance the quantum inductance is there, then, in what ways and under what conditions it involve with the de Broglie wavelength of the electron? Keeping these questions and all others with the spherical symmetric potential in case of hydrogen atom, here within this paper the atomic-electricity related issues due to the orbiting electron around the nucleus of hydrogen atom obtained and conclude.

1. Introduction

Electron (charge, $-e = -1.6021766208 \times 10^{-19} C$ [1] and mass, $m_e = 9.1093837015 \times 10^{-31} Kg$ [2]) orbiting around the nucleus of atom with velocity $V_n(V_k \cdot x_k, t)$ determines the minute current loop such that over the proportionality constant $Q_n$ for quantum number $n$, the relation \( \iint (N \cdot V(V_k \cdot x_k, t) \, dV) \, dt^2 \propto \iint (N \cdot d(\partial_t^2 N)) \, dt^2 \), in case of very small time period $\tau_n$ with $|V_n(V_k \cdot x_k, t)| = \text{constant}$ and by $N = 1$ holds to eq. $n$ (1). $x_1, \cdots, x_l, t \equiv (V_k x_k, t)$ is space-time coordinate and $N$ is the charged particle numbers. Over operator $D^y = \partial^y / \partial t^y = \partial^2$ and $l$ belongs to integer, the eq. $n$ (1) from earlier relation is simply by the consideration of kinetic energy $KE$ of electron orbiting around the nucleus with its corresponding current $i_n$ at the primary place represent by $KE(V_k x_k, t) = eQ_n D^2 i_n \equiv Q_n \frac{e^2}{2}$. The velocity $|V_n(V_k \cdot x_k, t)|$ is constant until the eccentricity $\epsilon$ of loop remains unaffected, here let say from the unity. In case of system as a neutral atom the changes introduce in the eccentricity when for instance photon, energy $hv$, collide with the atom such that $hv < \phi_{f \epsilon}$. $v$ is the frequency of incident photon and $\phi_{f \epsilon}$ is the free energy of electron from the site $f$. $h = 6.626070040 \times 10^{-34} Kg m^2/s$ is the Planck’s constant [3]. If the transferred $hv$ completely match up with $\phi_{f \epsilon}$ then the electron available is with the kinetic energy equal to zero. $\phi_{fK}$, where $\phi_{fK} = \phi_{f \epsilon} - \phi_{K \epsilon}$, is the internal transition situation.
with correspondent frequency $\phi_{jk}/\hbar$. The current loops of electrons orbiting around the nucleus of atom as electronic sites are based on the population scheme in such a way that it holds the conicality with vertex towards the nucleus. The conic population scheme irrespective of the cylindrical one shows divergence, and being disappear when the work function $\phi_e$ by amount of same get received.

$$V_n^2 \left( \sum_k x_k, t \right) t_n^2 = Q_n \frac{2e^2}{m_e} \tag{1}$$

The dimension of constant $Q_n$ from eq. (1) is $\text{Henry}$ like of the current carrying wire and is due to the current $e/\tau_n$ in correspondent to the orbital kinetic energy. $Q_n$ here regard as the orbiting inductance of quantum number $n$ (or simply quantum inductance). With respect to the $Q_n$ sometime the function $f(Q_n)$ as an additional during transient may also include such that the total $Q_n + f(Q_n)$ in case of $\epsilon = 1$ limited to $Q_n$ only. Like the quantum inductance of atom, is it possible to have the likewise atomic orbital resistance in consideration with the proportional relation of

$$\int \left( N V(V_k x_k, t) / 2 \right) \partial t V(V_k x_k, t) dt \propto \int \left( N \int d(\partial_t N) \right) dt$$

or by any other? The answer is straightly no, as this corresponds to the dissipative part keeping $\epsilon$ remains same. Therefore, from the notational $\int dt^3 + \int dt^2 + \int dt^1$ for $V(V_k x_k, t)\partial V(V_k x_k, t)$ neither the representative $\int dt^2$ nor the $\int dt^1$ going to be hold as the quantum electrical representative of the atom other than one the quantum inductance of atom. It means not only the real part of the atomic orbital impedance (or quantum impedance) in steady-state condition is zero but the capacitive term of imaginary part is also zero for the current $e/\tau_n$. Keeping this quantum inductance in to continue for the hydrogen atom ($H$-atom), here within this paper the objective is to find and discuss the atomic-electricity related issues of the $H$-atom due to the minute current loop around the nucleus of atom. The structure of this paper is as follows. It begins with introduction in Sect 1, wave particle duality description in Sect 2, then followed by derivation in Sect 3 and ends with conclusion in Sect 4.

2. Protrudal Superposition Principle

State, either of rest or of motion of a particle, the position $r(V_k x_k, t)$ or the velocity vector $V(V_k x_k, t)$ at a time so attain is not by the consideration of simply protrudal as inactive but by the protrudal as packet of lines $l_u$ for $u = 1, 2, \ldots$. active thus that any let say $l_{u}$ if define by the function $\phi_u = \phi_u(r - \delta r_u t, e_u)$ over variable $e_u$ in unit of peculiar which determine the strength, it satisfy the equation like oscillation, for instance wave equation of the string etc. $\delta r_u = \delta r_u(V_k \delta x_k, t)$ is the referential point of $l_u$ action with respect to $r(V_k x_k, t)$. The particle as protrudal with no packet of lines means there are no any fluctuation within, whereas the particle as protrudal with packet of lines means there are intrinsic fluctuations within at the state of all and may get alter also when disturbed. It is all the lines $l_u$ describe by $\phi_u$ which collectively determines the packet and stamped by the resultant, let say $\psi$, as $\psi \equiv \psi(r, \delta r_1, \delta r_2, t; e_1, e_2, \ldots)$ given by eq. (2).

With respect to the position vector $r(V_k x_k, t)$ at any time where all the lines of packet localise, the notation $\Lambda$ of eq. (2) is for all $l_u$ collectively within the volume, let say $B$, as $B$ also denote by $B(V_k x_k, t)$ about.

$$\psi \left( r, \sqrt{\delta r_u, t}; \sqrt{e_u} \right) = \lim_{m \to \infty} \left[ \begin{array}{c} c_1 \phi_1(r - \delta r_1 t; e_1) \\ \wedge \\ c_2 \phi_2(r - \delta r_2 t; e_2) \\ \wedge \\ \vdots \\ \wedge \\ c_m \phi_m(r - \delta r_m t; e_m) \end{array} \right]$$

(2)
\[ V_u \delta r_u \text{ and } V_u e_u \text{ of eq}_n(2) \text{ are } \delta r_1, \delta r_2, \ldots \text{ and } e_1, e_2, \ldots \text{ respectively. } c_1, c_2, \ldots \text{ of eq}_n(2) \text{ are constants and } \phi_1(\mathbf{r} - \delta r_1 t; e_1), \phi_2(\mathbf{r} - \delta r_2 t; e_2), \ldots \text{ are the \textit{lines-set mutually orthogonal with the}} \\
\text{limiting condition } B(V_k x_k, t) \text{ tends to the infinitesimal } dB(V_k x_k, t) \text{ such that the relative} \\
\mathbf{r}(V_k x_k, t) - \delta r_u(V_k x_k, t) \text{ for all } u \text{ of } \psi(\mathbf{r}, V_u \delta r_u, t; V_u e_u) \text{ as } \lim_{(r-\delta r_u) \to r} \psi(\mathbf{r}, V_u \delta r_u, t; V_u e_u) \text{ equal to } \lim_{(r-\delta r_u) \to r} \lim_{m \to \infty} \sum_{u=1}^m c_u |\phi_u|_B dt \text{ tends to } \mathbf{r} \text{ and gives eq}_n(3). \]

\[ \psi(\mathbf{r}t) = \lim_{m \to \infty} \sum_{u=1}^m c_u \phi_u(\mathbf{r}t) \]  

\[ (3) \]

From eq_n (3), the orthogonality over the spatial elements describe as \[ \int_1^\infty \int_1^\infty \phi_\alpha(\mathbf{r}t) \phi_\beta(\mathbf{r}t) \] 
\[ dx_1 \cdots dx_2 = \delta(a-b) \] is due to the reason that the string-based obeying fluctuations to all of the lines of packet are independent from each other, but the resultant \( \psi(\mathbf{r}t) \) is such that over the period \( T \) 

in no disturbance condition holds \( \psi(\mathbf{r}t) - \psi(\mathbf{r}t+nT) = 0 \). \( n \) is the integer and \( \delta(a-b) \) is the Dirac delta function equals to 1 when \( a = b \) otherwise 0. The length in correspondent to \( \Gamma \) of this wave type oscillation due to the velocity is \( \lambda \).

3. Quantum Inductance

After the wave related description of particle given by eq_ns (2) and (3), taking eq_n (1) in to account with the de-Broglie wavelength \[ \lambda_n \equiv \lambda_n(V_k x_k, t) \] with \( \lambda_n = h/m_e \cdot \mathbf{p}_n \) it gives eq_n (4) which involves \( \tau_n \) along with it such that they are in direct proportion to each other. In case of no perturbation the \( \lambda_n \) for state \( n \) is independent of the space-time coordinate \( (V_k x_k, t) \), however for the sake of generality here it carries on irrespective of the condition \( \varepsilon \) equal to unity. The value of eq_n \( (4) \), \( \lambda_n^{-1} \tau_n^1 \), is \( \sqrt{\mathbf{p}_n} \times 0.326372729 \).

\[ \lambda_n^{-1}(V_k x_k, t) \tau_n^1 = \sqrt{Q_n} \cdot \frac{2m_e e^2}{\hbar^2} \]  

\[ (4) \]

Like this eq_n (4) where \( \lambda_n(V_k x_k, t) \) and \( \tau_n = \tau_n^1 \) proportionality by \( \lambda_n^{-1}(V_k x_k, t) \tau_n^1 \) holds to constant in \( H^{-}\text{atom} \), when consider by the condition that the momentum \( P_n(V_k x_k, t) \) over the elemental length \( dl \) as \( P_n(V_k x_k, t) dl \) for stationary orbit in a complete period around the nucleus holds to \( nh \) then the \( \lambda_n(V_k x_k, t) \) and \( \tau_n \) proportionality in this case become by \( \lambda_n^{-2}(V_k x_k, t) \tau_n^1 \) and given by eq_n (5). This \[ \frac{Q_n}{P_n}(V_k x_k, t) dl(V_k x_k) \] integral relation of \( H^{-}\text{atom} \) looks similar to the phase integral condition of Wilson-Sommerfeld quantization rule, and here is due to the reason that for the acquired region about the nucleus determine by \( n = 1, 2, 3, \ldots \) the wavelength of the associated wave being in a way that it satisfy the condition of integral times of its periodicity define spatially in one complete time period of \( \tau_n \). The period, let say \( \Gamma_n \) of \( \lambda_n(V_k x_k, t) \) with \( \tau_n \) hold as \( \tau_n = n\tau_n^1 \) being in a way that it corresponds the electromagnetic theory of enclosed radiation in a hollow enclosure where the nodal planes as reflecting walls by the superposition of incident and reflected waves support stationary vibration of mode \( n \). In other words, like the exchange of energy between radiation and matter cannot takes place continuously but are limited to the discrete set of values, for instance \( hv, 2hv, \ldots, nhv \), in similar way about the nucleus the radius acquire cannot takes place continuously but limited to the discrete time condition at which \( \tau_n = n\tau_n^1 \) holds. The value of eq_n (5), \( \lambda_n^{-2} \tau_n^1 \), is \( n \times 1.37477927 \times 10^3 \).

\[ \lambda_n^{-2}(V_k x_k, t) \tau_n^1 = n \frac{m_e}{\hbar} \]  

\[ (5) \]
Again, like $\lambda_n^{-1}(\mathbf{V}_k x_k, t)\tau_n^1$ and $\lambda_n^{-2}(\mathbf{V}_k x_k, t)\tau_n^1$ proportionality relations given by eqns (4) and (5) respectively by the consideration of wave related description under the condition of different is there any other over different order possible by the condition of different, for instance $\lambda_n^{-3}(\mathbf{V}_k x_k, t)\tau_n^1$ or any, with the help of which $Q_n$ can be determine and all other? The answer is yes and it is the $\lambda_n^{-3}(\mathbf{V}_k x_k, t)\tau_n^1$ proportionality simply under the spherical symmetric potential of $H$ atom. The proportionality $\lambda_n^{-3}(\mathbf{V}_k x_k, t)\tau_n^1$ in case of multi-electron atom does not hold even if the proportionality $\lambda_n^{-2}(\mathbf{V}_k x_k, t)\tau_n^1$ be somehow there, as the force experiences under spherical symmetric potential be not there likewise by the wave function of single orbiting electron in $H$ atom. Now with the consideration of close surface integral $\oint_S \mathbf{E} \cdot d\mathbf{S}$ is equal to $\Phi$ over surface $S$, the value of $\Phi$ remains same even if the field distribution of $\mathbf{E}$ on $S$ is varied, as any of the value of like $\int \mathbf{E}_j \cdot d\mathbf{S}_j$ due to changing of $\mathbf{E}_j$ from $\mathbf{E}_f$ is compensated by all other open integrals and vice-versa. $\mathbf{E} \equiv \mathbf{E}(\mathbf{V}_k x_k, t)$ is the electric field vector at a point $\mathbf{V}_k x_k$ due to enclosed charge with $\Sigma_j \oint \mathbf{E}_j \cdot d\mathbf{S}_j$ is equal to the $\oint \mathbf{E}(\mathbf{V}_k x_k, t) \cdot d\mathbf{S}$. Using Taylor series over time variable $t$, the $\Phi$ become as $\Phi \equiv \Phi(\mathbf{V}_k x_k, t)$ is for the every point $\mathbf{V}_k x_k$ in continuum and radial. This force surface integral in equality to the term, $\frac{1}{\epsilon_o} \int_{in} dq$, which is the Gauss law of electrostatic form is given by eqn (6).

$$\oint_S \sum_l \left( \frac{t_l}{l!} \right) \frac{\mathbf{E}(\mathbf{V}_k x_k, t)}{\int_{ex} dq} \cdot d\mathbf{S} \left( \mathbf{V}_k x_k \right) = \frac{1}{\epsilon_o} \int_{in} dq \quad (6)$$

$\epsilon_o$ is the permittivity of the free space and is equal to $8.8541878128 \times 10^{-12} F/m [2]$. Eqn (6) with the force $\mathbf{F}(\mathbf{V}_k x_k, t)$ as centripetal on radius vector $\mathbf{r}(\mathbf{V}_k x_k)$ gives eqn (7) as the integral of wavelength to charge throughout the entire referential surface $S(\mathbf{V}_k x_k)$ due to the de-Broglie wavelength function $\lambda(\mathbf{V}_k x_k, t)$ of external charge $\int_{ex} dq$ in consideration with the corresponding velocity function $\mathbf{V}(\mathbf{V}_k x_k, t)$ at every point $\mathbf{P}(\mathbf{V}_k x_k)$.

$$\oint_S \sum_l \left( \frac{t_l}{l!} \right) \frac{h^2/m_e \int_{ex} dq}{\lambda^2(\mathbf{V}_k x_k, t) r^2(\mathbf{V}_k x_k)} \mathbf{r}(\mathbf{V}_k x_k) \cdot d\mathbf{S} \left( \mathbf{V}_k x_k \right) = \frac{1}{\epsilon_o} \int_{in} dq \quad (7)$$

In time period $\tau$ the radius $|\mathbf{r}(\mathbf{V}_k x_k)|$ of circle $C$ (which determines the minute current loop) on surface $S$ gives eqn (7) in simplified form as eqn (8) with neglection of higher order terms. $\int_{in} dq = e$ is the proton charge and $\int_{ex} dq = | -e |$ is the electron charge. This eqn (8) in case of $H$ atom is the required $\lambda_n^{-3}(\mathbf{V}_k x_k, t)\tau_n^1$ third proportionality under the condition of spherical symmetric potential and is independent of the quantum number $n$. The value of eqn (8), $\lambda_n^{-3}\tau_n^1$, is $n \times 0.0413477591 \times 10^{14}$.

$$\lambda_n^{-3} \left( \mathbf{V}_k x_k, t \right) \tau_n^1 = \frac{m_e^2 e^2}{2\epsilon_o h^3} \quad (8)$$
From eqs (5) and (8), the $\lambda_n$ of moving electron and $\tau_n$ of the referential orbit related to the same around nucleus in the $H-$atom over $n$ are given by $n \times 33.2491845 \times 10^{-11}$ meter and $n^2 \times 151.982985 \times 10^{-18}$ sec respectively. $33.2491845 \times 10^{-11}$ and $151.982985 \times 10^{-18}$ are the minimum wavelength and minimum circulating time respectively. Unlike the $n^3$ order of referential orbiting period the $T_n$ holds $n^2$ only whatever the $n$ be, same as that by the circumference also. Using $\lambda_n$ and $\tau_n$ in eqs (5) the quantum inductance of hydrogen atom due to the orbiting electron around the nucleus of atom is given by eq. (9).

$$Q_n = n^4 \times 1961.55378 \times 10^{-15} \text{Henry}$$

The circulating current $i_n$ is $n^{-3} \times 1.05418157 \times 10^{-3}$ Ampere. With respect to the orders of $\lambda_n$, $\tau_n$ and $i_n$ the order $n^4$ of $Q_n$ is the greater one even if it compares with the other possible like voltage $j_n$ and power $O_n$. The voltage, better to say quantum voltage, which is not the case of extraneous but the intrinsic embedded one is given by $n^{-2} \times 13.605693$ volt. Employing $i_n$ and $j_n$ the power correspondingly is $n^{-5} \times 14.3428708 \times 10^{-3}$ watt.

4. Conclusions

Based upon the three required suppositions describe by eqs (4), (5) and (8) the objective of this paper is to talk about the quantum electrical representative of the $H-$atom due to the orbiting electron referentially about the nucleus under spherical symmetric potential case. Including all the current in discreteness, correspondent representative voltage, power (as quantum voltage, quantum power) etc eq. (9) is the obtained quantum inductance of $H-$atom which increases with the increase in quantize level as well describe by $n$. Eq. ns (2) and (3) are the related wave function description which in case of circulating electron around the nucleus has referential circumference.

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