Scanning quantum decoherence microscopy

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Abstract

The use of qubits as sensitive nanoscale magnetometers has been studied theoretically and recently demonstrated experimentally. In this paper we propose a new concept, in which a scanning two-state quantum system is used to probe a sample through the subtle effects of decoherence. Mapping both the Hamiltonian and decoherence properties of a qubit simultaneously provides a unique image of the magnetic (or electric) field properties at the nanoscale. The resulting images are sensitive to the temporal as well as spatial variation in the fields created by the sample. As examples we theoretically study two applications; one from condensed matter physics, the other biophysics. The individual components required to realize the simplest version of this device (characterization and measurement of qubits, nanoscale positioning) have already been demonstrated experimentally.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The ability to image the very small is a powerful tool in the physical and life sciences. For each advance in imaging techniques, there have been equally important steps in understanding the systems they image. We theoretically show how a scanning two-state quantum system provides a unique perspective through monitoring of the subtle effects of decoherence. Previous work has focused on using a coherent two-state probe as a sensitive spatial electrometer or magnetometer. The system proposed here expands this view by simultaneously imaging spatial information and the fluctuations occurring within. To illustrate the technique, we investigate two contrasting applications; one from the realm of condensed matter physics, the other biological. Our results demonstrate that the technique provides a new nano-imaging mode with wide applicability, combining both structure and process information in a way not accessible by other means and, given the recent demonstration of scanning qubit magnetometry, that implementation is well within reach of current technology.

Quantum coherence and entanglement in mesoscopic systems are such delicate effects that we usually treat the environment as an inevitable influence to be mitigated against at all costs. One seeks to isolate the quantum system as far as possible from the environment in order to maintain and observe non-classical effects. The imaging technique we introduce here is based on the inversion of this viewpoint—the inherently quantum properties of a probe system are deliberately exposed to an environment in a controlled fashion. The effects of the environment are monitored and analysed as a function of position. The resulting scan forms a unique image of the position and behaviour of fluctuations in the environment as measured by the direct electromagnetic effects on the probe and its loss of quantum coherence.

The use of qubits as sensitive magnetometers (or electrometers) has previously been discussed\cite{1–5} as the energy levels, and hence coherent evolution, depends very precisely on its electromagnetic environment. Of particular interest to our proposal is the recent demonstration of a room temperature diamond colour-centre qubit scanning magnetometer system\cite{6–8}. The use of a qubit to probe its surrounding environment has also been of general interest recently\cite{9–12} from both a quantum computing and quantum physics point of view.

Here we focus on a novel imaging mode, rather than increased sensitivity. Monitoring the position-dependent probe Hamiltonian and its decoherence allows one to use the scanning quantum system as both a magnetometer/electrometer and...
as a probe of the decoherence environment simultaneously. As we will show, this imaging mode can reveal new features not observable by other means, as it is sensitive to the temporal as well as spatial variation in the fields created by the sample. For a given sample, this provides more than just structural information, it also provides unique information on processes occurring in the sample and where these processes occur.

This paper is organized as follows: we first present the background and concept of scanning quantum decoherence microscopy in a general way and illustrate its operation with a pertinent example. Following this, we present an example using existing technology, where we study a qubit composed of a nitrogen-vacancy defect within a diamond substrate. We simulate the use of this system in imaging a biological macromolecule with non-trivial magnetic properties.

2. Dynamics of a two-state system as a probe of the environment

Throughout this paper, we will be considering the evolution of a two-state system in the presence of environmental effects. Monitoring the dynamical behaviour of this system provides a probe of its environment. In general, the unitary (time-reversible) dynamics of our two-state system (or probe qubit) are described by its Hamiltonian $H_p$, where

$$H_p = H_{\text{int}} + H_{\text{env}},$$

which we decompose into the intrinsic component $H_{\text{int}}$ felt by the probe without the environment and $H_{\text{env}}$, the (unitary) effect of the environment on the probe. As the evolution can be (in general) non-unitary, we must describe the state of the system via the density matrix, $\rho$. The dynamical evolution of the density matrix is then given by the master equation

$$\dot{\rho} = -i[H_p, \rho] + \mathcal{L}[\rho]$$

where $\mathcal{L}[\rho]$ is a superoperator describing the non-unitary component of the evolution, i.e. the decoherence sources felt by the probe. This superoperator can also be decomposed:

$$\mathcal{L}[\rho] = \mathcal{L}_{\text{int}}[\rho] + \mathcal{L}_{\text{env}}[\rho]$$

into intrinsic and environmental components. In this case, what is defined as an intrinsic or environmental source of decoherence depends on the particular set-up, as most systems cannot be completely isolated from the environment and will always suffer from some decoherence.

In general, the master equation can be non-Markovian but for now we will consider a Markovian master equation of the ‘Lindblad’ form [13, 24, 25]. In this case, the superoperator is decomposed further into the sum of various contributions from different decoherence channels, such as relaxation (loss of energy from the probe qubit to its environment) and dephasing (loss of phase coherence). Each decoherence channel is associated with a characteristic decoherence rate $\Gamma$, which defines a timescale $T = 1/\Gamma$ over which the coherence of the system decays, due to that particular decoherence channel.

The imaging set-up we envisage employs a two-state system (a qubit), based on charge or spin degrees of freedom, attached to the end of a cantilever probe, as illustrated in figure 1(a). The quantum probe is governed by the Hamiltonian $H_p = \epsilon \sigma_x + \Delta \sigma_z$, where the $\sigma$’s are the Pauli operators and the coefficients ($\epsilon$ and $\Delta$) are functions of both the operating
conditions and the probe’s electromagnetic environment. The probe is scanned across a sample and, at each point, the Hamiltonian and decoherence felt by the qubit is determined through a series of measurements. The decoherence of the qubit, in particular, is highly sensitive to the fluctuations in the local environment. Thanks (in part) to the ongoing work to construct controllable quantum devices [13], a number of techniques have been developed to measure the state or evolution characteristics of an (open) quantum system. These include techniques to reconstruct an arbitrary quantum state or process [13, 14], quantum channel [13, 15] and the Hamiltonian governing a few-state quantum system [16–21] or even spin echo techniques [22] from magnetic resonance. The simplest realization of decoherence microscopy is to perform a Rabi oscillation or spin echo experiment on a qubit which can be moved relative to the background decoherence source. In this case, the measurement procedure (initialize, evolve, measure) is used to map the (ensemble-averaged) time evolution of the system and the decoherence rate is fitted as an exponential decay. With sophisticated measurement schemes and fitting procedures, the image can be acquired more quickly and efficiently. In this paper, we will focus on the technique of Hamiltonian characterization [16–18] and a general weak continuous measurement of the qubit as an example. It is important to note that the particular choice of continuous measurement and Hamiltonian characterization is not a requirement to perform decoherence microscopy; it simply provides an elegant and efficient realization but other more established experimental techniques could be similarly applied.

We consider a quantum probe, located at position \((x, y)\), suspended a distance \(h_p\) above the sample. The probe is weakly monitored, providing a measurement record \(I_{xy}(t)\). This measurement record is then correlated and the spectral response \(S_{xy}(\omega)\) computed, see figure 1(b). The Hamiltonian and decoherence parameters can then be estimated from this spectral response for each probe position. As the height of the probe above the sample can be controlled, it is possible to calibrate the probe separately from the sample. At this stage, estimates for the intrinsic Hamiltonian and decoherence (\(H_{int}\) and \(L_{cont}[\rho]\)) can be obtained as a reference. The sample-induced effects can then be directly estimated relative to this ‘baseline’.

If the components of the Hamiltonian are plotted as a function of position across the sample (\(H_{xy}\)), the probe acts as a sensitive electrometer or magnetometer (depending on the type of qubit), as given in the \(H_{xy}\) example in figure 1(c). This is due to the environmental effects on the probe Hamiltonian \(H_{env}\), such as a Lamb-, Stark- or Zeeman-like shift. However, complete analysis of the probe evolution allows the decoherence channel(s) to be extracted (\(\Gamma_{xy}\)), giving information about the strength, direction and character of the dynamics of the environment, as well as the induced static field. The resulting decoherence scan, shown in figure 1(c), reveals new information about the fluctuator frequency distribution in the sample which was not apparent in the electrometer/magnetometer image. Combining both sets of data, figure 1(d), provides a direct image of the correlation between the temporal dynamics of the environment and the spatial structure.

### 3. Measurement models in the strong and weak regimes

In order to estimate signal-to-noise ratios and image acquisition times, we need to consider the measurement model for the qubit. For the case of strong (von Neumann) measurements, this has been previously studied for the case of system identification [16–18] and we can use the existing estimates for parameter uncertainties as a function of the number of measurements. For qubits which have a ‘strong’ measurement channel such as a cycling transition (NV centre) or a destructive measurement of the qubit such as spin-dependent tunnelling out of a quantum dot, this is the more natural implementation. In such systems, a conventional Rabi oscillation experiment, consisting of repeated initialize–evolve–measure cycles, is sufficient to construct the spectral response of the qubit. We refer to such a measurement as a projective or ‘DC’ measurement.

Alternatively, in many systems, a weak continuous model is more appropriate. Such systems include continuous measurement of the state of a confined electron using a single-electron transistor or quantum-point contact [26–30]. As we are still ultimately interested in the qubit’s coherent and incoherent properties and the rate at which this information can be extracted, we will detail an equivalent weak measurement model. The results of this model can be directly matched to those from the previous projective model as the end result is still the qubit’s spectral response.

To make our proposal quantitative, we introduce a model for weak continuous measurement [23] which captures all of the essential physics. Here, we assume the measurement of the qubit can be modelled as a inefficient (or weak) positive operator valued measure (POVM) in the \(\sigma_z\) basis. The density matrix after measurement, \(\rho’\), is given by

\[
\rho’ = \frac{A_\pm \rho A_\pm^\dagger}{\text{Tr}[A_\pm^\dagger A_\pm \rho]},
\]

for a measurement operator

\[
A_\pm = \frac{1}{\sqrt{2}}(\sqrt{1 \pm \kappa}|0\rangle\langle 0| + \sqrt{1 \mp \kappa}|1\rangle\langle 1|),
\]

with some measurement strength \(\kappa\). The measurement process consists of repeated weak POVM measurements separated by a time interval \(\Delta t\), during which time the system undergoes normal evolution. The measurement repetition interval is then a measure of the bandwidth of the detector, \(\text{BW} = 1/\Delta t\).

The measurement record \(I(t)\) is the result, +1 or −1, of a measurement at time \(t\). The steady state autocorrelation of this measurement signal \(I(t)\) is then given by

\[
\langle I(t)I(t + \tau)\rangle_0 = \text{Tr}[\sigma_z e^{i\tau} \sigma_z \rho_\text{ss}]
\]

via the quantum regression theorem [24, 25]. Here \(\rho_\text{ss} = \rho(\infty)\) is the steady state density matrix and \(e^{i\tau}\) is the
solution, at time $\tau$, to the density matrix evolution governed by equation (2).

At this point, we address a key difference between the weak and strong measurement schemes. Using weak continuous measurement, it is not necessary to reinitialize the system; rather we take the autocorrelation function of the measurement record. This allows the measurement process to be performed continuously and does not require the measurement apparatus to be switched on and off. In contrast, strong measurement of the system requires a repetitive initialize–evolve–measure cycle which involves fine control of the measurement switch-on time. This added complexity comes with a benefit, namely that the system is no longer affected by measurement-induced decoherence as the measurement process is turned off during the evolution part of the cycle.

For a single two-state system, the spectrum of the signal is then

$$S(\omega) = \mathcal{F} \left( \frac{\langle I(t)I(t+\tau) \rangle_{\text{ss}} - \langle I^2 \rangle_{\text{ss}}}{\langle I(t)I(t) \rangle_{\text{ss}} - \langle I^2 \rangle_{\text{ss}}} \right) = \mathcal{F}[\langle \sigma_z(t) \rangle],$$

(7)

where $\mathcal{F}[\langle \sigma_z(t) \rangle]$ is the Fourier transform of the (ensemble-averaged) expectation value of the $\sigma_z$ operator (starting from the state $\langle \sigma_z(0) \rangle = 1$) [16, 17]. Here we see the utility of developing both strong and weak measurement models which are compatible, as the autocorrelation of the weak measurement signal can be equated directly to the ensemble-averaged result for strong measurement via the quantum regression theorem.

From the response spectrum, we extract the Hamiltonian and decoherence parameters directly [17, 18] for each spatial location across the sample. When performing this parameter estimation process, it is not necessary to make any assumptions about the source or mechanisms producing the decoherence. Simply mapping the induced decoherence as a function of position provides a dynamical image of the sample fluctuations.

In the limit of small $\kappa$, the model of equation (4) is equivalent [23] to more complicated master equation models [23, 27–30]. Expanding the evolution to first order in both $\kappa$ and $\Delta t$, we can derive an equivalent Lindbladian master equation with an effective $\sigma_z$ decoherence channel of strength

$$\Gamma_{\text{meas}} = \frac{\kappa^2}{4\Delta t},$$

(8)

which corresponds to the measurement-induced decoherence. To maximize the rate of information extraction, the measurement strength should be chosen such that this is smaller than both the sample-induced decoherence and other intrinsic decoherence sources.\textsuperscript{3} The intrinsic noise sources include those introduced by the tip supporting the qubit, the measurement (and/or reset) process and any other noise introduced due to imperfections in the dynamical control of the system [31].

\textsuperscript{3} This equates to operating well above the Korotkov–Averin bound [26, 27] so that the sample decoherence effects dominate the signal.

We can also calculate the information extracted at each measurement step, $\Delta t$, by looking at the reduction in entropy, $\Delta S_E$, of the system. Expanding for small $\kappa$, this gives

$$\Delta S_E = S_E(\rho) - S_E(\rho') = \frac{\kappa^2}{4\log(4)} + O(\kappa^3)$$

(9)

as the information obtained (in bits) from a single measurement of an initially mixed state $\rho = (|0\rangle\langle 0| + |1\rangle\langle 1|)/2$. As we increase $\kappa$ or the bandwidth, the amount of information obtained in a given time interval is increased, equation (9), at the expense of greater measurement-induced decoherence, equation (8).

The spatial resolution of the probe system is ultimately governed by the effective strength of the environmental decoherence as a function of distance. Most decoherence channels (for solid-state qubits) depend, in some way, on the inverse of the separation between qubit and environment. The coupling between a single decoherence (point) source located at $x_s$ and the probe qubit is, in general, proportional to $1/r^n$, hence the response of the qubit as a function of $x$ position across the sample is given by a exponentiated Lorentzian:

$$R(x) = \left\lceil \frac{1}{h_p^2 + (x - x_s)^2} \right\rceil^{\eta/2}. \quad (10)$$

The full width at half-maximum (FWHM) of this function gives the spatial resolution $\Delta x = 2h_p\sqrt{2^{\eta/n} - 1}$, where $h_p$ is the height of the probe above the sample. By inspection, we see an electric-dipole-induced potential (1/r$^3$) has an FWHM $\Delta x = 2h_p$ whereas a magnetic dipolar interaction (1/r$^3$) has slightly better resolution, $\Delta x \approx 1.53h_p$. This link between probe/sample separation and spatial resolution must be balanced against the need to maintain a well-defined probe. Ideally, the effects of the environment should always be a perturbation on the probe system ($H_{\text{env}} \ll H_{\text{tot}}$). This means we must maintain a probe distance which minimizes effects such as quantum tunnelling or a direct exchange interactions between probe and sample.

We now consider two examples in which we simulate the effects of a decohering environment on an example probe. These examples provide both a straightforward illustration of the power of imaging the sample-induced decoherence and a test of the feasibility using current and near-future technology.

\textbf{4. Example I: imaging the distribution of background charge fluctuators}

Our first example system comprises an electrostatic qubit interacting with a sample containing background charge fluctuators. This example is of particular interest as background charge fluctuations have been the subject of intense scrutiny due to their relevance to solid-state quantum devices [33, 34]. In this case, we can probe the spatial and frequency distribution of these fluctuators in a way which is not possible using current microscopy techniques.

A suitable probe qubit would be any of the myriad of charge-based qubit designs such as quantum dots [35], donors [36] or Cooper-pair boxes [37–39]. A probe based
on a Cooper-pair-box (CPB) system provides a particularly good example as CPB qubits are now regularly produced experimentally [40–42] and the bias point of the system can be varied, resulting in a change in the sensitivity to different components of the environmental decoherence [33]. For generality, we model the qubit purely as a two-state system which interacts via a field-dependent component in its Hamiltonian and thereby all the dimensions in this example are given in terms of a normalized length scale (L).

We wish to use a simple model of the decoherence felt by a probe (charge) qubit interacting with a bath of 1/f fluctuators [43, 34, 33]. The numerical parameters used for such a model vary greatly depending on the system and even from sample to sample. Here we are interested in the functional dependence, rather than the explicit values. We draw heavily from [43] and [34] as an example.

Consider a qubit interacting with a bath of fluctuators via an interaction Hamiltonian \( H_{\text{int}} \) which has the form

\[
H_{\text{int}} = \sigma_z \sum_j v_j b_j^\dagger b_j
\]  

(11)

where \( \sigma_z \) acts on the qubit and \( b_j, b_j^\dagger \) destroys (creates) an electron in a localized state in the bath.

The spectral response from the \( j \)th fluctuator (in the fast fluctuator limit) is given by

\[
s_j(\omega) \propto \frac{v_j^2 \gamma'_j}{\gamma'_j^2 + \omega^2}
\]  

(12)

where \( v_j \) is the strength of the fluctuator which fluctuates with a rate \( \gamma'_j \). The total spectral response is then a sum over each of the fluctuators:

\[
S(\omega) = \sum_j s_j(\omega)
\]  

(13)

and a simplified golden rule model gives the relaxation rate (\( \Gamma^-_{\text{GR}} \)) and dephasing rate (\( \Gamma^+_{\text{GR}} \)) as

\[
\Gamma^+_{\text{GR}} = \frac{1}{2} \Gamma^-_{\text{GR}} = \frac{1}{4} S(E_f)
\]  

(14)

where \( E_f \) is the tunnelling energy of the qubit. Given a functional form for how \( v_j(r) \) depends on the fluctuator/qubit separation \( r \), one can then compute the effective decoherence rate felt by the qubit.

Let us consider a 2D surface which contains background charge fluctuators that we wish to image. We take the potential felt by the qubit as a simple electrostatic potential due to a charge defect dipole and simulate the effective decoherence felt by the qubit due to the fluctuator bath (defects in the sample). In figure 2(a), we have a fictitious sample comprised of regions (outlined) containing a uniform fluctuator distribution with an area density of 1000 defects per square (all distances are in normalized units). The frequencies of these fluctuators are then assumed to be distributed with a 1/f distribution. We have also included a ‘calibration’ region on the left of the sample, where the frequency of fluctuators is varied in a controlled fashion. This will illustrate the frequency selective nature of the measurement process.

The variation of the probe Hamiltonian, \( H_{\text{xy}} \), due to the presence of background charges is shown in figure 2(b). The background charges are taken to be charge dipoles [43, 34] coupling to \( \Delta \), the \( \sigma_z \) component of the qubit, as \( 1/r^2 \) and the probe is positioned \( h_p = 0.05L \) above the sample. Initially assuming the fluctuators to be static, the spatial variation in the Hamiltonian coupling term is then a measure of the electric potential induced by the dipoles (a simple electrometer). The state of the fluctuators will not be static (in general) and the total induced field will result from an average over the fluctuator states. In figure 2(c), we calculate the decoherence felt by the probe due to the combined effects of all the fluctuators.
fluctuators in our fictitious sample. The resolution for this example increases by a factor of 1.55, compared to measuring the Hamiltonian, as $\Gamma \propto S(\omega) \propto 1/r^4$. Comparing these images, we see that the decoherence measurement is more sensitive to the fluctuators whose frequencies are closest to the probe energy. It is known that, for a $1/f$ bath, a relatively small fraction of the total fluctuators contributed a large amount of the total decoherence [33] but here we see it strikingly depicted in the images.

To further demonstrate the utility of mapping both Hamiltonian and decoherence components simultaneously, figure 2(d) is a combined plot of both figure 2(b) and figure 2(c). In this case, the effective field (static) is a surface plot with coloration corresponding to the decoherence rate (dynamics). The vertical scale of this ‘mountain range’ plot corresponds to the varying electric potential across the sample. The red ‘mountains’ are fluctuators with frequency close to the qubit transition frequency while blue corresponds to far-off resonant fluctuators. In principle, the frequency sensitivity of the qubit can be tuned to probe different components of the decoherence, to obtain more information about the spectral characteristics ($1/f$, ohmic, etc) as a function of position.

As illustrated via equation (10), the probe is most sensitive to decoherence sources in its immediate environment. In general this adds two important complications. One is that the usual arguments of ensemble averaging, which allow for a Markovian description of the dynamics of the probe, may not apply. As the probe volume of the system is now finite, one cannot assume an infinite number of fluctuators are interacting with the probe at once. The second complication is that the assumption that the probe does not disturb the bath (weak coupling) may also not apply. In this case the measured environment will be that of the combined sample plus probe, not just the sample.

While such interactions between a qubit and a finite bath are an interesting topic in their own right, this does not fundamentally change the operation of our device. In such a situation, the process of system characterization can proceed as long as one can describe the dynamical evolution with a master equation, even though this is most probably non-Markovian. The red ‘mountains’ are fluctuators with frequency close to the qubit transition frequency while blue corresponds to far-off resonant fluctuators. In principle, the frequency sensitivity of the qubit can be tuned to probe different components of the decoherence, to obtain more information about the spectral characteristics ($1/f$, ohmic, etc) as a function of position.

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5. Example II: imaging the position and spin occupation of ferritin molecules

Now, we choose to use both a probe qubit and sample which have been well studied experimentally. In contrast to the previous example, where all parameters were dimensionless, here we investigate a specific implementation to evaluate the technical feasibility of the proposal. We consider the imaging of (bio)molecules with large uncompensated spin, such as horse-spleen ferritin [44, 45] or Fe$_3$ [46]. Here, the point is neither to image an individual spin [47] or image the location of the molecules [48, 32, 49], as both can be done with existing technology. We show that qubit probe imaging can both map the location of the spins and probe their magnetic dynamics.

![Figure 3. Variation in strength of the maximum signal ($\phi = 0$) as a function of probe height for different-sized spins. The signal is the fractional variation in the probe spin Hamiltonian due to the influence of the sample spin. The upper and lower threshold limits come from the measurement bandwidth and decoherence, respectively. The inset illustrates the simplified magnetic dipolar coupling model, which depends on the separation, orientation and magnetic moment of both the probe and sample spins.](image)

If we assume that the sample spins are flipping ‘quickly’ relative to the Rabi frequency, then we can perform a similar analysis to the previous example. Instead, we consider a more physically realistic example, where the flipping processes are ‘slow’ and therefore the characterization of the Hamiltonian (rather than the decoherence) provides detailed information about the system. In this sense, the final results are similar to those provided by spectroscopy of the system (read out via the qubit state). However, this is a continuous scale—by using exactly the same experimental set-up and basic analysis, one can smoothly move from a regime where decoherence provides useful information, to one where simply measuring Hamiltonian components provides an important insight into the system.

The decoherence introduced at the probe qubit will be a function of both the interaction strength and the flipping rate of the sample spin(s). As we are considering large sample spins in a static magnetic field at low temperature, we will assume that the flipping rate is slow on the scale of the probe Hamiltonian. The spectral response of the qubit is split, with the separation between the peaks giving the effective difference in the Hamiltonian between the two sample spin states.

To aid in visualizing this peak ‘splitting’, the insets to figure 4(c) show example qubit spectra, highlighting the contribution from several sample spins. Although the measured response of the qubit will be the simple summation of the contributions from each spin, the spatial dependence of the peak positions allows each spin to be uniquely identified. We compute the coupling strength between the spins, given the simplified magnetic dipolar model, as illustrated in the inset of figure 3. The ability to resolve the induced coupling is ultimately limited by the total effective decoherence rate of the qubit. The ratio of the peaks also gives a measure of the
relative spin populations, which in turn relates to the effective
temperature and/or flipping mechanism.

We use a nitrogen-vacancy (NV) centre in diamond as our
probe spin, as this has been shown to be a controllable, well-
isolated spin system which displays stable quantum coherent
properties up to room temperature [50–57] and has even
been used as a nanoscale scanning magnetometer [6, 8], in
a similar cantilever configuration to that which we envisage.
We then couple this probe to a bulk spin of order \( M_0 = 50–
200 \, \mu_B \) and include the effects of intrinsic decoherence and
finite measurement bandwidth. Using experimentally realistic
parameters for both the probe qubit and the sample spin, we
plot the response of the system as it passes over the spin. An
alternative probe qubit system would be to use a micro-SQUID
or flux qubit [2, 3, 37].

For this analysis, we will use a simplified (but quite
general model) consisting of a probe spin interacting via
the magnetic dipolar interaction with a much larger sample
spin (see the inset of figure 3). The sample spin can be
considered as a ferritin, Fe_8 or other mesoscopic molecule
with a net magnetic moment \( M_0 \gg 1 \, \mu_B \). As the spin
is large (and to simplify the analysis of decoherence) we
will assume that it is in thermal equilibrium with the sample
environment and therefore the average magnetization and spin
flip rates are given by the standard thermodynamic quantities.

In making this assumption, we ignore quantum mechanical
effects between the probe and sample spins and treat the effect of the sample spin on the probe as a fluctuating classical field.

The magnetic dipolar interaction is given by [22]

\[ H_{\text{dip}} = \left( \frac{\mu_0}{4\pi} \right) \hbar^2 \gamma_p \gamma_s \left[ \frac{\vec{P} \cdot \vec{S}}{r^3} - \frac{3(\vec{P} \cdot \vec{r})(\vec{S} \cdot \vec{r})}{r^5} \right] \]

(15)

where \( \gamma_p \) and \( \gamma_s \) are the probe and sample spin gyromagnetic ratios, \( \vec{r} \) is the vector separation between the spins, and \( \vec{P} \) and \( \vec{S} \) are the probe and sample spin operators.

We assume the system is bathed in a global magnetic field \( B \) which orients both the sample and probe spins and sets their energy scales. The dipolar interaction is then given by the separation between the spins \( r \) and the angle subtended between the spin orientation and the vector separating the spins \( \phi \), as illustrated in the inset to figure 3. The simplified coupling Hamiltonian is therefore

\[ H_{\text{dip}} = \left( \frac{\mu_0}{4\pi} \right) \hbar^2 \gamma_p \gamma_s \left( 3 \cos^2 \phi - 1 \right) P_z \cdot S_z, \]

(16)

which is a purely Ising-type interaction whose strength depends on both the separation and angle between the spins. The perturbing effect of this dipolar Hamiltonian can then be measured in the spectral response of the qubit, giving a direct link to both the spin state and magnetization of the sample spin.

For our example, we use known system parameters for an NV centre driven by a microwave loop and readout via a laser probe measurement [58]. We take the Rabi frequency of the qubit to be 10 MHz and the measurement bandwidth \( BW = 100 \text{ MHz} \) but not necessarily ‘strong’. The intrinsic decoherence rate is approximately 100 times slower than the Rabi frequency. The measurement strength \( \kappa \) is chosen such that the measurement-induced decoherence is weaker than the intrinsic decoherence, for a given detector bandwidth. In choosing specific values for the cantilever-induced intrinsic decoherence, the measurement-induced decoherence and a finite measurement time, we are able to include the effects of finite signal-to-noise ratio of the imaging process.

Figure 3 shows the maximum fractional Hamiltonian component as a function of probe height for three different spin samples. The measurement window is defined between the decoherence rate and the measurement bandwidth. The probe height can vary over almost 100 nm and still provide a detectable signal. In figure 4, we create an image from a fictitious sample consisting of four mesoscopic spins with varying net magnetization. We assume the spins are in thermal equilibrium with the sample substrate (which we have set at \( T = 4 \text{ K} \)) and that they are flipping due to thermal processes. The magnetizations are \( M_0 = 50, 70, 100 \) and 200 \( \mu_B \) and the average population of the excited state is given by a Boltzmann distribution for a background magnetic field of \( B = 0.1 \text{ T} \) and temperature \( T = 4 \text{ K} \). The spatial resolution of the probe

| Pixel variance \( N_{\text{m}} \) per pixel | Strong measurement \( N_{\text{m}} \) total | Weak continuous measurement \( t_{\text{dwell}} \) | \( t_d \) |
|---|---|---|---|
| \( 10^{-1} \) | \( 2 \times 10^2 \) | \( 5 \times 10^5 \) | \( 2 \mu \text{s} \) | 5 ms |
| \( 10^{-3} \) | \( 2 \times 10^4 \) | \( 5 \times 10^7 \) | \( 200 \mu \text{s} \) | 0.5 s |
| \( 10^{-5} \) | \( 2 \times 10^6 \) | \( 5 \times 10^9 \) | \( 20 \mu \text{s} \) | 50 s |

Table 1. Achievable pixel variances for a given number of strong measurements or equivalently a given continuous measurement dwell time.

position is a 50 \( \times \) 50 grid, giving 2500 points over 10,000 nm\(^2\) and the probe height was set to \( h_0 = 20 \text{ nm} \). Figure 4(a) shows the measured magnetic field over the sample. Note that the probe in this mode (purely acting as a magnetometer) does not successfully resolve two of the spins.

Figure 4(b) shows the measured peak height ratios over the same sample. As each spin has a different magnetization, the environmental effects (in this case splitting of the spectral response peak) resulting from each spin are different. The ratio of the two split peaks provides the population of the spin states, which is directly related to the magnetization and effective temperature of the sample spin. In this plot, the ratio of the split peaks has been used to colour code the data, with blue indicating a large spin magnetization (or low effective temperature) and red a small magnetization (high temperature). The intensity of the colour is given purely by the amount of signal available from each spin (compared to the probe spin’s intrinsic decoherence), whereas in figure 4(a), the intensity was proportional to the total induced field. Finally, we can combine this data to produce a plot showing the field intensity with each detectable component of the environment (in this case mesoscopic spin) tagged based on its effective temperature. This is shown in figure 4(c), where the existence of all four spins can be detected based on the colour tagging, in contrast to the magnetometer scan alone.

6. Image acquisition rate and noise

While we have demonstrated that new information can be obtained by measuring the probe Hamiltonian and decoherence, this is only useful if the information can be obtained within an experimentally accessible time. Using the measurement model discussed earlier, we can estimate the parameter uncertainties in the Hamiltonian characterization process [17, 18]. Retaining the parameters from Example II, we calculate the noise expected for a finite number of strong measurements \( (N_{\text{m}}) \) or equivalently for continuous weak measurement, a dwell time \( (t_{\text{dwell}}) \) on each pixel and the total image acquisition time \( (t_d) \). To give a feel for these acquisition times, table 1 gives the number of measurements required for a given pixel variance and to acquire an image consisting of 2500 pixels. These acquisition times are given both in terms of the number of measurements (for the case of strong measurement) and as dwell times for high-bandwidth continuous weak measurement with \( BW = 100 \text{ MHz} \).
Figure 5 replicates figure 4(a) with the noise resulting from a finite bandwidth and dwell time or equivalent number of strong measurements. The image acquisition times (for 2500 pixels) varies between 5 ms and 50 s for pixel error variances which range from $10^{-1}$ to $10^{-5}$. These correspond to a total of between $10^5$ and $10^9$ strong measurements, as given in table 1.

To allow fast image acquisition times (using a continuous measurement mode), it is important to have both large detector bandwidth and a large ratio of bandwidth to intrinsic decoherence rate $\text{BW}/\Gamma_0$. The qubit transition frequency is less important, provided it is at least an order of magnitude greater than the intrinsic decoherence. However, a tunable qubit frequency is advantageous as many types of decoherence depend strongly on the frequency at which they are probed.

To estimate detector noise as a measure (rather than acquisition time), we take recent noise limit estimates for magnetometry based on NV centres [5, 4] as a guide. In this case, the measurement is a ‘DC’ measurement ($\text{BW} = 10 \text{kHz}$) with a sensitivity of $20 \text{nT}/\sqrt{\text{Hz}}$ and a 10 MHz qubit Rabi frequency [5]. This results in an additional contribution to the pixel variance of $\approx 10^{-5}$ but still results in a clear image (see figure 5).

Finally, some general comments on the limitations of a decoherence probe microscope are given. The performance of any such device depends strongly on the particular components but will generally share common characteristics. The detectable environmental signal will be bounded below by the intrinsic behaviour of the probe while measured (but without the sample present) and above by the requirement that the probe is only perturbed by the environment. The spatial resolution is ultimately controlled by the physical extent of the probe qubit and the probe/sample separation distance. The requirements of measurement and/or state preparation will also place restrictions on the fabrication of the probe tip, especially in the case of electrical (as opposed to optical) detection. A slightly more abstract issue is that the results must be able to be interpreted accurately, which may prove difficult in the case of complicated (possibly non-Markovian) evolution. In almost all types of system/environment interactions, a signal can be obtained (given sufficient hardware) but what this signal indicates at the microscopic level may not be straightforward.

7. Conclusions

We have proposed and theoretically investigated a new and complementary imaging mode which takes advantage of current qubit technology and modern techniques for characterizing few-state quantum systems. Mapping the induced decoherence across a sample indirectly images the dynamics of the environment, providing a new window into the microscopic world with wide applications to spin and charge fluctuations and transport in both solid-state and biological systems. In contrast to a scanning qubit magnetometer, the decoherence signature is sensitive to fluctuations even when the time-averaged field is close to zero. Such processes abound in nature, in which the time-dependent behaviour of electromagnetic fields stems directly from dynamical processes of interest, such as ion flow, spin fluctuations or conformational change. The device may be realized using a range of architectures, each of which will be sensitive to a particular electromagnetic effect within a sample. While we have discussed several specific examples using near-future technology, the basic requirements for a decoherence microscopy already exist in the form of nanoscale positioning and individual two-state quantum systems which can be initialized, controlled and measured. Hence, this technique has the potential to open a new window for imaging nanoscale processes in the physical and life sciences.

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