A microscopic approach to Casimir and Casimir-Polder forces between metallic objects

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We consider the Casimir-Polder interaction energy between a metallic nanoparticle and a metallic plate, as well as the Casimir interaction energy between two macroscopic metal plates, in terms of the many-body dispersion interactions between their constituents. Expressions for two- and three-body dispersion interactions between the microscopic parts of a real metal are first obtained, both in the retarded and non-retarded limits. These expressions are then used to evaluate the two- and three-body contributions to the macroscopic Casimir-Polder and Casimir force for two geometries: metal nanoparticle/half-space and half-space/half-space, where all the materials are assumed perfectly conducting. In the case of nanoparticle/half-space, our results fully agree with the corresponding macroscopic results and show the non-applicability of the pairwise approximation for the geometry considered. We also show that the nonapplicability of the pairwise approximation also holds for the case of two metal half spaces, and the issue of the convergence of the many-body expansion is discussed.

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I. INTRODUCTION

Casimir and Casimir-Polder forces are electromagnetic interactions between neutral macroscopic bodies or between atoms and macroscopic bodies, respectively, due to the quantum zero-point fluctuations of the electromagnetic field [1, 2]. Dispersion forces are analogous interactions between neutral microscopic polarizable objects, such as atoms, molecules, condensates or nanoparticles, and described in terms of exchange of virtual photons between them or in terms of vacuum fluctuations [3–6]. Dispersion forces (van der Waals, including the Casimir-Polder regime) are not additive [7–12], and many-body effects may become relevant for dense systems [13, 14].

It is worth to consider Casimir and Casimir-Polder forces involving macroscopic bodies in terms of the dispersion interactions between their constituent parts and investigate the role of non-additive components. This could also give a deeper understanding on the origin of Casimir forces, showing how they derive from microscopic fundamental interactions. A microscopic pairwise summation (PWS) approach, consisting in summing up only the two-body components of the interaction, works correctly only for dilute dielectrics [15–17], but not in general for dense dielectrics and metals. The PWS approximation in fact does not take into account the many-body components of the dispersion interactions, or, equivalently, exchange of more than two virtual photons between the objects involved. The error made by the PWS approximation in some geometric configurations has been recently considered by comparison with exact macroscopic results [18]. A microscopic approach to Casimir interactions could be also relevant in order to understand discordant results in the literature about the attractive or repulsive character of the Casimir force for closed topologies such as a sphere [19, 20].

In this paper we develop a microscopic approach to Casimir and Casimir-Polder forces involving macroscopic metallic bodies, including two- and three-bodies components of the dispersion (van der Waals/Casimir-Polder) force between their constituents (metal nanoparticles). Specifically, we will consider the two following configurations: metal nanoparticle - metal half space and two metal half spaces. This will allow us to compare the role of two- and three-body components for such objects, and also gain some hints about the convergence rapidity of the many-body expansion.

This paper is organized as follows. In Sec. II we outline our microscopic approach and derive the expressions of the two- and three-body interaction between metal nanoparticles. These results will be used in Sec. III to evaluate from a microscopic point of view two- and three-body contributions of dispersion interactions to the nanoparticle-wall Casimir-Polder interaction and to the wall-wall Casimir interaction. Sec. IV will be devoted to our conclusive remarks.

II. THE MICROSCOPIC APPROACH TO CASIMIR-POLDER AND CASIMIR INTERACTIONS

In our microscopic approach we imagine to decompose a macroscopic metallic body into small metal spheres, or nanoparticles, of radius ρ; a typical value of their radius could be of the order of some nanometer. We first evaluate their non-additive van der Waals/Casimir-Polder interaction in terms of their dynamical polarizability. Finally, we sum these dispersion interactions over all the
nanoparticles, maintaining $N$-body components up to a given order.

In order to obtain the whole Casimir force, it is essential to consider non-additive effects too, so all $N$-body dispersion interactions should in principle be taken into account. The Casimir energy in the material is given by a $N$-body expansion or Born series [5]:

$$W = \sum_{N=1}^{\infty} \frac{1}{N!} \frac{1}{\sqrt{V}} \int d^3x_N |x_N| \cdots$$

$$\times \int d^3x_N N(x_N) U^{(N)}(x_1, \ldots, x_N),$$

where $U^{(N)}(x_1, \ldots, x_N)$ is the $N$-body dispersion interaction, and $N$ the number density. We shall now obtain the specific expressions of the two- and three-body components of the dispersion interaction between metal nanoparticles that will be used in the next Section.

We consider here the two- and three-body dispersion interactions between identical metal nanoparticles, both in the non-retarded and retarded limits. Assuming these nanoparticles larger than the electron mean-free path, spatial dispersion can be neglected and their response to an electric field can be represented by a dielectric constant $\varepsilon(\omega)$. We consider the metal as a free-electron gas, where only the electric dipole moments due to free electrons contribute to the macroscopic polarizations. According to the Drude-Sommerfeld model, we have

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\omega\Gamma}$$

where $\omega_p = \sqrt{ne^2/m_e\varepsilon_0}$ is the plasma frequency, $n$ the free electrons density and $\Gamma = v_F/l$ is a damping term, with $v_F$ the Fermi velocity and $l$ the electrons’ mean-free path [21]. The constant $\Gamma$ takes into account dissipative effects of the medium and therefore also its fluctuations. The polarizability for these nanoparticles can be represented in the quasi-static limit (valid for a wavelength of light much greater than the characteristic size of the nanoparticle) as

$$\alpha(\omega) = 4\pi\varepsilon_0\rho^3 \frac{\varepsilon(\omega) - 1}{\varepsilon(\omega) + 2},$$

where $\rho$ is the nanoparticle radius [21, 22]. We consider the nanoparticles as rigid spheres without overlapping of their electron clouds (this condition also allows us to neglect spatial dispersion in the dielectric constant).

The dispersion interaction between two polarizable particles in vacuum in their ground state, at zero temperature, separated by a distance $r$, is [3]

$$U^{(2)}(r) = -\frac{\hbar}{16\pi^3\varepsilon_0} \int_0^{\infty} dx \frac{\alpha^3(x)}{x^6} g_2(x) \frac{(\xi/c)}{r^6},$$

where $g_2(x) = e^{-2x}(3 + 6x + 5x^2 + 3x^3 + x^4)$ and $c$ is the speed of light.

Similarly, the three-body dispersion interaction between three identical polarizable particles in vacuum at zero temperature, is [5, 8, 9]

$$U^{(3)} = \frac{\hbar}{64\pi^3\varepsilon_0} \int_0^{\infty} dx \frac{\alpha^3(x)}{x^6} g_2(x) \frac{1}{a^3b^3c^3}$$

where $a, b, c$ are the distances between the nanoparticles (see Fig. 1), and

$$g_3(x, y, z) = e^{-((x+y+z)^2)} \left[ 3f(x)f(y)f(z) - g(x)f(y)f(z) - f(x)f(y)g(z) + g(x)f(y)g(z) \cos^2\theta_A + g(x)f(y)g(z) \cos^2\theta_B + g(x)f(y)g(z) \cos^2\theta_C + g(x)g(y)g(z) \cos\theta_A \cos\theta_B \cos\theta_C \right],$$

with $f(x) = 1 + x + x^2$ and $g(x) = 3 + 3x + x^2$. $\theta_A, \theta_B$ and $\theta_C$ are the angles opposite to $a, b, c$, respectively, as shown in Fig. 1.

Simplified expressions can be obtained from Eq. (5) in two opposite regimes, where dispersion interactions reduce to simple power laws: the non-retarded regime, when all distances between the particles are much smaller than the plasma wavelength $\lambda_p = 2\pi/\omega_p$ of the nanoparticles; the retarded regime, if at least one distance between the particles is much greater than $\lambda_p$. In the retarded regime, many plasma oscillations occur during the time taken by the virtual photons involved in the interaction to travel between the nanoparticles, and this reduces the strength of the interaction.

We shall use these known expressions for the dispersion interactions in order to describe the interaction between our metallic nanoparticles. In the non-retarded limit (van der Waals), the $\xi$ integral in (4) and (5) restricts to a region where $g_2(\xi/c) \approx g_2(0)$, $g_3(\xi/c) \approx g_3(0, 0, 0)$, so the two- and three-body dispersion in-

![FIG. 1: Geometrical arrangement of the three metal nanoparticles.](image-url)
actions take the simpler forms

\[ U_{\text{non-ret}}^{(2)} = -\frac{\sqrt{3}}{4} \hbar \left( \omega_p - \frac{2 \sqrt{3} \Gamma}{\pi} + \ldots \right) \rho^9 \frac{1}{r^7}, \]  

and

\[ U_{\text{non-ret}}^{(3)} = \frac{3 \sqrt{3}}{16} \hbar \left( \omega_p - \frac{8 \sqrt{3} \Gamma}{3 \pi} + \ldots \right) \rho^9 \times \frac{1 + 3 \cos \theta_A \cos \theta_B \cos \theta_C}{\alpha^3 b^3 c^3}. \]

These expressions are indeed Taylor expansions around \( \Gamma / \omega_p = 0 \), since for most metals \( \Gamma \ll \omega_p \) (for example, for gold \( \omega_p = 1.38 \cdot 10^{16} \text{s}^{-1} \) and \( \Gamma = 1.075 \cdot 10^{14} \text{s}^{-1} \)).

In the retarded limit the exponential factor in (4) restricts the \( \xi \)-integral to a region where the atomic polarizability can be approximated by its static value \( \alpha (i \xi) \approx \alpha (0) \). In this case, the two- and three-body dispersion interactions become

\[ U_{\text{rel}}^{(2)} = -\frac{23 \hbar c}{4 \pi} \rho^6 \frac{1}{r^7}, \]  

\[ U_{\text{rel}}^{(3)} = \frac{4 \hbar c \rho^9}{\pi} f(a, b, c), \]

where \( f(a, b, c) \) is a function depending only from the geometrical arrangement of the three nanoparticles

\[ f(a, b, c) = \frac{1}{a^3 b^3 c^3 (a + b + c)} \left[ f_1 + f_2 (a, b, c) \cos^2 \theta_A + f_2 (b, c, a) \cos^2 \theta_B + f_2 (c, a, b) \cos^2 \theta_C + f_3 \cos \theta_A \cos \theta_B \cos \theta_C \right], \]

with

\[ f_1 = 9 - 20 \frac{\sigma_2}{\sigma_1} + 22 \frac{\sigma_3}{\sigma_1} + 54 \frac{\sigma_2^2}{\sigma_1} - 65 \frac{\sigma_2 \sigma_3}{\sigma_1}, \]  

\[ f_2 (a, b, c) = 3 \left[ \frac{a^2}{\sigma_1} + \frac{3 a^2 (b + c)}{\sigma_1} + \frac{4 b c (3 a^2 - b c)}{\sigma_1} \right], \]  

\[ f_3 = 1 + 39 \frac{\sigma_2}{\sigma_1} - 17 \frac{\sigma_3}{\sigma_1} - 72 \frac{\sigma_2^2}{\sigma_1} + 75 \frac{\sigma_2 \sigma_3}{\sigma_1} - 20 \frac{\sigma_2^2}{\sigma_1}, \]

and \( \sigma_i = a^i + b^i + c^i \).

The dependence on the geometrical parameters is the same as for dielectrics, well-known in the literature [5, 8, 9]. The three-body dispersion interaction can be attractive or repulsive. For example, for an equilateral triangular or a right triangle configuration the non retarded force is repulsive, while the force is attractive when the three spheres are collinear. In general, the three-body contribution can be attractive or repulsive, depending on the geometry considered [5, 9, 12].

For perfect conductors \( (\lambda_p \to 0) \), dispersion interactions between nanoparticles are always in the retarded regime regardless their distance. This is the case we shall consider in the next Sections.

## III. TWO- AND THREE-BODY CONTRIBUTIONS OF DISPERSION INTERACTIONS TO CASIMIR-POLDER AND CASIMIR FORCES

We will now consider explicitly the microscopic approach to Casimir-Polder and Casimir forces for two specific cases: a metallic nanoparticle near a perfectly conducting metal half-space, and two metallic half spaces made by a perfect conductor.

### A. Two- and three-body contribution to the Casimir-Polder energy between a metallic nanoparticle and a perfectly conducting half-space

Let us first consider the Casimir-Polder interaction between a metallic nanoparticle \( C \) at a distance \( d \) from a perfectly conducting half-space, as shown in Fig. 2.

We first consider the two-body contribution to the Casimir-Polder force for this geometry, summing all two-body interactions between the metallic nanoparticle \( C \) and a generic nanoparticle of the metallic half-space. We need to consider only the retarded interaction (9) because both the nanoparticle and the half-space are perfect conductors. The coordinates are chosen such that the origin is kept fixed in \( C \), and the \( z \)-axis is orthogonal to the half-space surface. The two-body contribution to the nanoparticle-metallic half-space Casimir-Polder force is

\[ W_{CP}^{(2)} = \int U_{\text{rel}}^{(2)} N dV = -\frac{69 \hbar c \rho^3}{160 \pi d^4}, \]

where \( N = (4 \pi \rho^3 / 3)^{-1} \) is the number of nanoparticles for unit volume and the integration over the volume has the following ranges: \( 0 \leq \theta < \pi/2, 0 \leq \varphi \leq 2\pi \) and \( r \geq d / \cos \theta \).

We now evaluate the three-body contribution to the Casimir-Polder interaction, by summing all three-body dispersion interactions between the nanoparticle \( C \) and two generic nanoparticles in the metallic half-space (see Fig. 2). Three-body interactions among three nanoparticles of the half space do not contribute to the interaction energy between the nanoparticle \( C \) and the metal half-space, and thus they will be not considered. We use a spherical coordinate system in which the position of two
generic nanoparticles of the half-space are identified by their radial distance from C and the polar and azimuthal angles in

\[
\vec{a} = (a, \theta_A, \varphi_A)
\]

\[
\vec{b} = (b, \theta_B, \varphi_B).
\]

(14)

Here \(0 \leq \theta_A < \pi/2\), \(a \geq d/\cos \theta_A\), \(0 \leq \varphi_A \leq 2\pi\), and analogous conditions hold for the nanoparticle B. In this coordinate system, the distance \(c\) between the nanoparticles A and B is given by

\[
c = \sqrt{a^2 + b^2 - 2ab \cos \theta_A \cos \theta_B + \sin \theta_A \sin \theta_B \cos \varphi}.
\]

(15)

where \(\varphi = \varphi_A - \varphi_B\). The cosine of the angles \(\theta_A, \theta_B, \vartheta_C\) can be obtained in terms of \(a, b, c\), using straightforward trigonometrical relations. The system is invariant under a generic rotation around the \(x\)-axis and thus all geometrical parameters depend on the angles \(\varphi_A\) and \(\varphi_B\) only via their difference \(\varphi = \varphi_A - \varphi_B\).

Summing over the retarded three-body dispersion interactions (10), we obtain the three-body contribution to the Casimir energy in the following form

\[
W_{CP}^{(3)} = \frac{1}{2!} \int \frac{U_{ret}^{(3)} N^2 dV_A dV_B}{\Omega} = \frac{2\hbar c}{\pi} \rho^3 N^2 K(d, \lambda)
\]

(16)

where \(\rho\) is the nanoparticle radius, \(N = (4\pi\rho^3/3)^{-1}\) is the number of nanoparticles per unit volume and \(K(d, \lambda)\) is the following function

\[
K(d, \lambda) = \int_0^{\pi/2} d\theta_A \int_0^{2\pi} da \int_0^{\pi/2} d\varphi_A \int_0^{\pi/2} d\theta_B \int_0^{2\pi} db \int_0^{2\pi} d\varphi_B
\]

\[
\times \Theta(c) \Theta(a-d/\sin \theta_A) \Theta(b-d/\sin \theta_B)
\]

\[
\times f(a, b, c) a^2 b^2 \sin \theta_A \sin \theta_B.
\]

(17)

In Eq. (17), \(\Theta(x)\) is the Heaviside function, \(\lambda\) the interparticle distance in the metal half-space and the function \(f(a, b, c)\) has been defined in (11). The first Heaviside function in the second line of Eq. (17) gives the condition \(\varphi \geq \lambda\), because the distance between two nanoparticles in the half-space must be larger than the interparticle distance; the two other Heaviside functions give the conditions \(z_A \geq d\) and \(z_B \geq d\) for the \(z\) component of the particles A and B position, respectively.

Although the integral in (17) is not defined for \(\lambda = 0\) (continuum case), we will consider its limit for \(\lambda \to 0\). We will show that the integral in \(K(d) = \lim_{\lambda \to 0} K(d, \lambda)\) is finite in this limit apart from an additive constant that, being independent from the distance \(d\) between the particle and the metal half space, does not contribute to the Casimir-Polder energy we are interested to. This also indicates that the divergence in the continuum limit is related to a Casimir self-energy related to the metallic half-space only, as it is expected from previous results for perfect conductors or field sources [23, 24].

In order to simplify the evaluation of the integral \(K(d)\), we first take its derivative with respect to \(d\), using the relation \(\Theta'(x) = \delta(x)\). We obtain

\[
\frac{\partial K(d)}{\partial d} = -2 \int_0^{\pi/2} d\theta_A \int_0^{\infty} da \int_0^{2\pi} d\varphi_A \int_0^{\pi/2} d\theta_B \int_0^{2\pi} d\varphi_B
\]

\[
\times \Theta(c) f(a, b, c) a^2 b^2 \sin \theta_A \tan \theta_B |_{b=d/\cos \theta_B}.
\]

(18)

The factor 2 in (18) has been introduced because both cases \(z_A = d\) and \(z_B = d\) equally contribute. With this procedure, the dimensionality of the integral has been reduced, fixing the position of the nanoparticle B on the plane \(z_B = b \cos \theta_B = d\). The integrand depend on \(\varphi_A\) and \(\varphi_B\) only through their difference \(\varphi = \varphi_A - \varphi_B\), due to the symmetry of our system. With the substitutions \(\varphi = \varphi_A - \varphi_B\) and \(\varphi' = \varphi_A + \varphi_B\), we can use the relation

\[
\int_0^{2\pi} d\varphi_A \int_0^{2\pi} d\varphi_B g(\varphi_A - \varphi_B) = 2 \int_0^{2\pi} g(\varphi) d\varphi
\]

(19)

where \(g(\varphi)\) is a periodic function with period \(2\pi\). Using this expression in Eq. (18) and renaming \(\varphi\) as \(\varphi_A\), we obtain

\[
\frac{\partial K(d)}{\partial d} = -4 \int_0^{\pi/2} d\theta_A \int_0^{\infty} da \int_0^{2\pi} d\varphi_A \int_0^{\pi/2} d\theta_B \Theta(c)
\]

\[
\times f(a, b, c) a^2 b^2 \varphi_A \sin \theta_A \tan \theta_B |_{b=d/\cos \theta_B, \varphi_B=0}
\]

(20)

In the continuum limit \(\lambda \to 0\), the function \(\partial K(d)/\partial d\) can depend only from the distance scale \(d\), because there are no other distance scales involved, and it has the dimension of a length to the power \(-5\); thus it must have the following form

\[
\frac{\partial K(d)}{\partial d} = \frac{\alpha}{d^5}.
\]

(21)
It follows that:

\[ \alpha = -8.5 \pm 0.3. \]  

(22)

It follows that:

\[ K(d) = (2.1 \pm 0.1) \frac{1}{d^4}. \]  

(23)

The (diverging) integration constant has not been considered because it yields a distance-independent self-energy, that does not contribute to the Casimir-Polder force. The overall three-body contribution to the Casimir-Polder energy is then

\[ W_{CP}^{(3)} \simeq (7.6 \pm 0.4) \cdot 10^{-2} \hbar c \rho^3 \frac{1}{d^4}. \]  

(24)

Although the two-body contribution is always attractive (see Eq. (13)), the sign of (24) shows that the overall three-body contribution turns out to be repulsive and about one half of the two-body contribution. The distance dependence as \( d^{-4} \) is the same.

The particle-wall geometry has been already considered in the literature using a microscopic approach [4]. We wish to stress the agreement between our microscopic approach based on the field zero-point energy [1] and the macroscopic models in this particular geometry. In fact, the retarded total Casimir energy between a metallic nanoparticle and a conductor half-space, described by a static dielectric function \( \varepsilon \) is [4]

\[
\begin{align*}
W & = -\frac{3}{16\pi} \int_{\varepsilon}^{\infty} dv \left[ \left( \frac{2}{v^2} - 1 \right) \frac{v + \sqrt{\varepsilon - 1 + v^2}}{v - \sqrt{\varepsilon - 1 + v^2}} + 
\frac{1}{v^4} \frac{v - \sqrt{\varepsilon - 1 + v^2}}{v + \sqrt{\varepsilon - 1 + v^2}} \right] \hbar c \rho^3 \frac{1}{d^4}. 
\end{align*}
\]  

(25)

Many-body contributions can be extracted from Eq. (25) by expanding the function around the point \( x = 4\pi \alpha N \) and expressing the relative dielectric function in terms of the quantity \( \varepsilon = (3 + 2x)/(3 - x) \), obtained from the Clausius-Mossotti formula. For example, for our system we obtain the following exact two- and three-body contributions

\[
\begin{align*}
W_{CP}^{(2)} & = -\frac{69}{160\pi} \hbar c \rho^3 \frac{1}{d^4} \simeq -0.137 \hbar c \rho^3 \frac{1}{d^4}, \\
W_{CP}^{(3)} & = \frac{111}{448\pi} \hbar c \rho^3 \frac{1}{d^4} \simeq 0.0789 \hbar c \rho^3 \frac{1}{d^4}. 
\end{align*}
\]  

(26) 

(27)

On the other hand, the total energy obtained from (25) is

\[
W_{CP} = -\frac{3}{8\pi} \hbar c \rho^3 \frac{1}{d^4} \simeq -0.119 \hbar c \rho^3 \frac{1}{d^4}. 
\]  

(28)

Comparison of our results (13) and (24), obtained through our microscopic approach of summing respectively the two- and three-body components of dispersion forces, with the macroscopic-approach results (26) and (27) clearly shows that the two approaches are fully compatible, at least for the two- and three-body contributions to the Casimir-Polder energy. However, the sum of two and three-body contributions is not sufficient to obtain the total Casimir-Polder force between the nanoparticle and the half-space given by (28), indicating a slow convergence of the many-body expansion (1). Thus, we can conclude that higher-order non-additive components play a significant role in determining the total interaction energy, and that the Born series indeed converges quite slowly for metals, contrarily to the case of dilute dielectrics [15].

### B. Two- and three-body contribution to the Casimir force between two perfectly conducting metal half-spaces

We now consider the common and relevant geometry of two half-spaces of a perfect conductor, whose distance is \( d \) along the \( z \) direction. For this geometry, the Casimir energy for unit area \( A \) was evaluated using a macroscopic approach based on the field zero-point energy [1]

\[ W_{Cas} \frac{A}{\hbar c} \simeq -\pi^2 \left( \frac{207}{1920} \right) \frac{\hbar c}{d^3} \simeq -0.0137 \frac{\hbar c}{d^3}. \]  

(29)

It follows from (29) that the interaction of the two metal half-spaces with the zero-point electromagnetic field fluctuations leads to an attractive force. This is also consistent with a recent general theorem, according to which the Casimir force between two bodies related each other by a reflection is always attractive [20].

We now want to consider the Casimir interaction for this geometry from a different point of view, evaluating the two- and three-body contributions of dispersion interactions to the Casimir energy, similarly to the case discussed in the previous Section. Our microscopic approach will also clarify the importance of non-additive effects for this geometry. The two-body contribution to the Casimir energy per unit area is straightforward

\[
\begin{align*}
\frac{W_{Cas}^{(2)}}{A} & = \int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{\infty} dy_1 \int_{-\infty}^{\infty} dz_1 \int_{d}^{\infty} dz_2 N^2 U_{rel}^{(2)}(|r_1 - r_2|) \\
& = -\frac{207}{1920} \frac{\hbar c}{d^3} \simeq -0.0109 \frac{\hbar c}{d^3}.
\end{align*}
\]  

(30)

where subscripts 1 and 2 refer to coordinates relative to the two half spaces. In the derivation of (30), we have integrated only the retarded two-body dispersion interactions, given by Eq. (9), because the two half-spaces are made of a perfect conductor. This result is known in the literature and can be found in Ref. [2].
Comparison of (30) with (29) shows that pairwise summation accounts only for about 80% of the total macroscopic Casimir force. Thus pairwise summation does not hold for this geometry, and non-additive effects play an important role to achieve a compatibility between macroscopic and microscopic models.

Next-order (non-additive) contribution to the Casimir energy is the three-body contribution. When we sum up retarded three-body dispersion interactions between three generic nanoparticles in the two conducting half-spaces, we must consider the three different cases shown in Fig. 3.

![Fig. 3: Different possible cases of a three-body interaction in the two metal half-spaces.](image)

Case 1 (and the similar one relative to the other half-space) gives a distance-independent contribution to the Casimir energy, and thus it does not contribute to the Casimir force between the half spaces: hence, it will be neglected in our following evaluation. Cases 2 and 3 give equal contributions, and we can exploit the result (27) obtained in the previous subsection for nanoparticle/half-space case. Using (27), after summation over all half-space nanoparticles, we obtain the three-body contribution to the Casimir energy

$$W_{\text{Cas}}^{(3)} = \frac{222}{1792\pi^2} \frac{\hbar c}{d^3} \approx 0.0126 \frac{\hbar c}{d^3}. \quad (31)$$

The overall three-body contribution to the Casimir force is thus repulsive, contrarily to the overall two-body contribution which is attractive. Also, two- and three-body contributions are of the same order of magnitude showing, also in this case, a slow convergence of the many-body expansion of the macroscopic Casimir energy. Therefore higher-order many-body contributions are expected to play an essential role too. Our result also indicates that both attractive and repulsive contributions play an essential role in this geometric configuration and that, for dense systems such as metal bodies, three-body and higher-order contributions have a role comparable to the usual two-body components.

IV. CONCLUSIONS

In this work we have developed a microscopic model for Casimir and Casimir-Polder forces for metal bodies, summing up the dispersion interactions between their constituents. In particular, two- and three-body dispersion interactions between metal nanoparticles have been considered, both in the nonretarded and retarded regime. Summing up these interactions, we have evaluated the two- and three-body contributions to the macroscopic Casimir-Polder and Casimir energy for two different geometries of metallic bodies: nanoparticle/half-space and two half-spaces. Our expression of the interaction for the nanoparticle/half-space geometry is fully consistent with known macroscopic model results, and shows a quite slow convergence of the many-body expansion.

In the literature, at the best of our knowledge, the microscopic evaluation of the force has so far been developed only for dilute dielectrics, where only two-body interactions play an important role. Our model considers metals, in which the introduction of three-body (and higher many-body) interactions is necessary in order to obtain a deeper understanding of the physical problem and agreement with results obtained from macroscopic approaches. We wish to point out that our model can be also extended to non-dilute dielectrics, and clearly shows the importance of many-body dispersion interactions in the cases considered.

Our evaluation shows that a pairwise approximation for the geometries we have considered is not valid, and that non-additive effects must be taken into account. We have also shown that, while the two-body dispersion interactions always lead to attractive contributions, the overall three-body dispersion interaction may lead to a repulsive contribution. Furthermore, overall three-body contributions are of the same magnitude as overall two-body contributions (in the cases here considered). Finally, we expect that the introduction of non-additive effects in the microscopic model could also clarify discordances in the literature concerning with the attractive or repulsive character of the Casimir force for some connected geometries, such as the perfectly conducting sphere [19, 20]. We shall discuss this point in a future publication.

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