Abstract. Modern computer systems are awash in a sea of asynchronous events. There is an increasing need for a declarative language that can permit business users to specify complex event-processing rules. Such rules should be able to correlate different event streams, detect absence of events (negative information), permit aggregations over sliding windows, specify dependent sliding windows etc. For instance it should be possible to precisely state a rule such as “Every seventh trading session that DowJones has risen consecutively, and IBM’s stock is off 3% over its average in this period, evaluate IBM position”, “Declare the sensor as faulty if no reading has been received for 500 ms”, etc. Further, the language should be implementable efficiently in an event-driven fashion.

We propose the Timed (Default) Concurrent Constraint, TCC, programming framework as a foundation for such complex event processing. The framework (developed in the mid 90s) interprets computation as deduction in a fragment of linear temporal logic. It permits the programmer to write rules that can react instantaneously to incoming events and determine the “resumption” that will respond to subsequent events. The framework is very powerful in that it permits instantaneous pre-emption, and allows user-definable temporal operators (“multi-form time”).

However, the TCC framework “forgets” information from one instant to the next. We make two extensions. First, we extend the TCC model to carry the store from previous time instants as “past” information in the current time instant. This permits rules to be written with rich queries over the past. Second, we show that many of the powerful properties of the agent language can be folded into the query language by permitting agents and queries to be defined mutually recursively, building on the testing interpretation of intuitionistic logic described in RCC [15]. We show that this permits queries to move “back and forth” in the past, e.g. “Order a review if the last time that IBM stock price dropped by 10% in a day, there was more than 20% increase in trading volume for Oracle the following day.”

We provide a formal semantics for TCC + Histories and establish some basic properties.

Keywords: synchronous programming, concurrent constraint programming, RCC, TCC, HCC, complex event processing

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1 Introduction

1.1 Timed Concurrent Constraint Programming

From about 1985 to about 1995, the programming languages/embedded systems community worked out a very robust programming model for time-based systems, under the framework of “synchronous languages”, such as Esterel, Signal and Lustre \([3,13,2,7]\). In particular, the authors developed the *Timed (Default) Concurrent Constraint Programming Framework*, \([24]\), based on the simple idea of extending “across time” the ideas of concurrent constraint programming, using the Synchrony Hypothesis of Berry \([3]\). One thinks of a reactive system as lying inert, waiting for a stimulus from the outside world. On each stimulation, the system computes an instantaneous response, and prepares itself for further interaction (by computing a resumption). The system is *amnesiac* in that its past state is flushed, only the resumption is kept. Thus the system has an internal notion of time that corresponds to its periodic interaction with the outside world.

This notion of time can be made explicit through certain temporal combinators within the language used to program these agents. TCC is built on just six orthogonal basic combinators:\[4\]

\[
\text{A,B ::= } c \mid \text{if } G \text{ then } A \mid \text{if } c \text{ else } A \mid A \text{ and } B \mid \text{some } V \text{ in } A \mid \text{hence } A \\
\text{Z \mid mu } Z \text{ in } A
\]

\[
\text{G ::= } c \mid G \text{ and } G
\]

Above, \(c\) ranges over constraints; \(X, V\) over first-order variables used in constraints; \(Z\) over Agent variables; \(A, B\) ranges over Agent formulas, and \(G\) over Goal formulas.

The TCC framework is parametric on an underlying notion of *constraint system* \(C\) \([24]\): essentially such a system specifies pieces of partial information, called *tokens* or *constraints*, and an *entailment* relation which specifies which tokens follow from which other sets of tokens. The (tell) \(c\) agent adds the constraint \(c\) to a shared store of constraints. The (positive ask) agent \(\text{if } c \text{ then } A\) reduces to \(A\) if the store is strong enough to entail \(c\). The (negative ask) agent \(\text{if } c \text{ else } A\) reduces to \(A\) only if the final store (at this time instant) will not be strong enough to entail \(c\) (this circularity – the final store is defined in terms of the final store – is characteristic of defaults \([22]\)). The (parallel composition) agent \(A \text{ and } B\) behaves as both \(A\) and \(B\). The agent \(\text{some } X \text{ in } A\) introduces a new local variable \(X\) in \(A\). The agent \(\text{hence } A\) is the only agent with temporal behavior – it reduces to \(A\) at every time instant after the current instant. The agent \(\text{mu } Z \ A\) (taken from the modal mu calculus) behaves like \(A\) with occurrences of \(Z\) replaced by \(\text{mu } Z \ A\).

This language is powerful enough to be the basis for a rich algebra of temporal control constructs. For instance, one can define:

1. *always* \(A\) (run \(A\) at every time step);

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\[4\] In the rest of this paper we will use the acronym TCC to stand for Timed Default Concurrent Constraint Programming.

\[5\] We introduce recursion explicitly through \(\text{mu}\); in fact recursion is definable in TCC.
2. do A watching c (run A until such time instant as the condition c is true, at which point abort the remainder of A);
3. next A (run A only at the next time step);
4. time A on c (run A but on a clock derived from the basic clock by only passing through those ticks at which the condition c is true).

The last combinator in particular is very powerful – it realizes the idea of “multi-form” time, the notion that the basic clock on which an agent is defined may itself be defined by another agent [24].

TCC (and its continuous time extension, HCC, [11]) have been used in modeling complex electro-mechanical systems (photo copiers [12], robots [11]) and biological systems [5]. They have a very well-developed theory – semantic foundations, reasoning framework, implementation techniques, compilation into finite state automata, abstract interpretation, etc. (see Related Works section below).

Unlike the other systems mentioned above (Esterel, and other reactive languages), TCC, and its parent framework, Concurrent Constraint Programming (CCP) are declarative and rule-based. Computation can be interpreted as deduction corresponding to certain “agent” formulas in linear time temporal logic, defined over a certain notion of defaults [24]. Defaults play a crucial role in permitting agents to detect the absence of information. This is critical for faithfully modeling such computational phenomena as time-outs and strong pre-emption. This logical reading extends the understanding of CCP [28] as computation in intuitionistic logic [26].

1.2 Event processing

Over the last decade a new and interesting application area has emerged, event processing, [17]. The basic computational problem in event processing is to implement a powerful “sense, analyze, respond” system. The system should be capable of receiving multiple (usually discrete) time-varying signals, correlating them in potentially complex ways involving detecting the absence of events, maintaining sliding windows, computing statistics over sliding windows (averages, max values, etc), and comparing these values. If the desired temporal pattern is detected, then appropriate programmer specified action (e.g. issuing an alert) needs to be taken.

For an event processing language to be useful, it should be capable of expressing complex patterns of temporal interactions. For example, it should be possible to support rules of the form:

1. Every tenth time the price drops within an hour emit volatility warning.
2. Every seventh trading session that DowJones rises consecutively, and IBM stock has fallen over this interval, evaluate IBM position.
3. Declare the sensor is faulty if no reading has been received in the past 500s.
4. Declare the room is too cold if the average temperature over the last 100s is below a threshold.
5. Ignore an over global limit notification on an account if an over global limit notification was sent on this account in the past two days.
6. If the merchant has been tenured less than 90 days, and the sum of the transactions in the last 7 days is much higher than the seven day average for the last 90 days, then investigate a 7 day hit and run possibility.

We also desire a language in which programs can be understood declaratively as “rules”. Ultimately the language needs to mesh well with an OPS-like rule language, such as ILOG JRules and ILOG Business Rules. We desire that the programmer should be able to reason rigorously (if informally) about such rules. We require that the rules should be compilable efficiently. For example queries involving sliding window averages should be implemented in an incremental forward-driven fashion (with a rolling average being maintained).

1.3 TCC for event processing

Given the many valuable properties of TCC, it is interesting to consider it as a basis for complex event processing. Incoming events can be represented as atoms to be added to the constraint store. As events arrive, they are buffered while the system is active (executing events it received at the previous tick). Once the system quiesces, and the buffer is not empty, the system is advanced to the next time unit, and all buffered events added.

A fundamental limitation of TCC for complex event processing, however, is that TCC computations do not maintain history. All rules must be written in a “forward looking” fashion, responding to the current events received, and whatever state has been explicitly stored from past interactions. For instance, to express the rule “Trigger an alert whenever it is the case that the stock price of company A falls over 10%, while that of company B has risen over the past 7 days”, the programmer must write code that maintains in the current store the value of the proposition “the stock of company B has risen over the past 7 days”. Now on receipt of a notification that the stock price of A has fallen, a check can be made for the value of the proposition and an alert emitted if necessary.

But this way of writing rules is awkward. In essence, the programmer is being made to work like a compiler – figure out how to write the rule in such a way that it is always event-driven and forward looking. In many cases it is very natural instead to simply write a query over the past that “looks back” and checks if the desired condition is true, on demand.

Our basic move, therefore, is to augment TCC with history. When moving from time step \( t \) to \( t + 1 \), we propose to retain the constraints computed at \( t \), and time-stamp them with \( t \). Thus the store will contain not just the current constraint, but also, separately and equally, past constraints, each tagged with the time at which they were computed.

A simple way to accommodate this view in TCC is simply to work over the constraint system \( H(C) \) built from \( C \) as follows. The tokens in \( H(C) \) are of the form \( \text{time}^i c \) for some \( i \), where \( c \) is a token of \( C \). A multiset \( \Gamma \) of such tokens entails \( \text{time}^k d \) only if \( \Gamma_k \) entails \( d \) (in \( C \)), where \( \Gamma_k \) is the set of all constraints \( c \) such that \( \text{time}^k c \in \Gamma \). Given \( k \), by abuse of notation we will say that \( \Gamma \) entails \( \text{prev} c \) at \( k \) if \( \Gamma_{k-1} \) entails \( c \) (in \( C \)). Using \( H(C) \), the user can write ask agents that query the past. Tell agents must still be prevented from modifying the past by ensuring that they can only assert constraints
about the instantaneous state. The operational semantics is now modified to carry past constraints automatically in the constraint store, and to tag tell constraints with the current time step.

**Example 1 (Querying the past in TCC(H(C))).** The rule:

```plaintext
always if ((prev price(IBM)) > price(IBM)) then signalIBMdrop
```

will trigger if there is a drop in price of IBM stock over successive time instants.

Unfortunately, this simple technique is not powerful enough. What if we wanted to trigger a rule if the current price is less than half the price at any point in the past when MSFT stock was above a certain threshold? In other words, it is natural to require recursive computations in our queries, capable of examining the past at arbitrary depth. [15] in fact develops such a rich framework for CCP, called RCC. RCC is based on the idea that a judgement $A_0, \ldots, A_{n-1} \vdash G$ can be regarded as asking whether the system of concurrently interacting agents $A_i$ ($i < n$) satisfy the query or goal $G$. Appendix ?? provides more details.) Queries are internalized in the agent language through the production $A::= \text{if } G \text{ then } A$.

Queries are not restricted to primitive constraints $c$. Recursive queries are permitted. **Universal** queries, $G::= \text{all } V \text{ in } G$, are permitted, where $V$ is a first-order variable. Such a query can be thought of as succeeding only when the query $G$ succeeds, where $V$ is a brand-new variable that does not occur in the agents. **Hypothetical** queries are also permitted: $G::= \text{if } A \text{ then } G$ can be thought of as temporarily augmenting the system currently being tested with $A$ and asking if the augmented system satisfies $G$. If so, the guard is satisfied and execution continues, with the temporary augmentation discarded. Hypothetical queries permit “what if” reasoning and allow for a compact representation of very powerful idioms. To implement this, the underlying infrastructure must support the notion of copying the entire concurrent assembly of agents.

[15] shows that the computational interpretation is sound and complete with respect to the obvious logical interpretation of the queries.

**TCC with deep guards.** We now consider how to apply these ideas to TCC. Clearly, we need to augment the power of guards, $G$. To add recursion across time, we introduce $G::= \text{hitherto } G$ (analogously to $A::= \text{hence } A$). The query hitherto $G$ is intended to be true if $G$ is true at every point in the past (excluding the current one). We also introduce recursive queries, $G::= \mu X G$, and require that $X$ be guarded in $G$ (occur inside a hitherto). Similarly, we introduce universal queries $G::= \text{all } V \text{ in } G$.

We could introduce hypothetical queries, $G::= \text{if } A \text{ then } G$. However, we can do something richer. Note that $A$ is not permitted to operate in the “past”, i.e. it is not possible for an agent to spawn an agent to “change the past”. (Concretely, $A::= \text{hitherto } A$ is not allowed. This is fundamental to the basic idea that computation always moves ahead in time.) However, within the scope of hypothetical execution, it does make sense to add agents to the past – these agents are free to participate in “what if” reasoning, exploring what might have been. Therefore we introduce a new category of **nested agents**, $B$, which is the same as $A$ except that it permits $B::= \text{hitherto } B$, and add $G::= \text{if } B \text{ then } G$. 

With these constructs it is possible for a query to "move back in time" arbitrarily deeply, spawn agents in the past, and ask queries of the modified system. Still, the nested agents and queries are asymmetric: nested agents can move back in time (hitherto B) as well as forward (hence B), but queries can only move backwards. Logically, it makes sense, then, to permit queries to also move forward in time; we add \( G ::= \text{hence } G \). This permits us to express a query that checks whether the day after the last time IBM stock fell 10\% it was the case that MSFT stock rose 10\%. The natural formulation of this query would involve moving back in time and then forward.

Table 1 summarizes the language being considered, which we name "TCC, with history". The basic picture of computation supported by this language is as follows: The system interacts with incoming events in a synchronous fashion. The rate at which events arrive is controlled by the environment and not by the system. Each interaction marks the progression of the system down a time-line. At each instant, the state of the system carries the entire state of past interactions. This is accessible to be queried in a very rich way through a query language which permits computations to move backwards and forwards in the past, and also spawn hypothetical queries. However, querying cannot change the actual past, only read it.

This paper takes the first step in studying this language. Section 2 discusses how some interesting idioms can be expressed in this language. For reasons of space we omit standard extensions of the query language with "bag of" operators that permit the collection of some statistic over all answers to a query (these are very important in practice). Section 3 formalizes the informal reasoning presented here. We conclude with an outline of the work that lies ahead.

\[
\begin{align*}
\text{(Agents)} & ~ A ::= c \mid \text{if } G \text{ then } A \mid \text{if } c \text{ else } A \mid A \text{ and } A \mid \text{some } V \text{ in } A \mid \text{hence } A \\
& \quad \mid X \mid \mu X \text{ in } A \\
\text{(Goals)} & ~ G ::= c \mid \text{if } B \text{ then } G \mid G \text{ and } G \mid G \text{ or } G \mid \text{all } V \text{ in } G \mid \text{hence } G \\
& \quad \mid \text{hitherto } G \mid X \mid \mu X \text{ in } G \\
\text{(Nested Agents)} & ~ B ::= c \mid \text{if } G \text{ then } B \mid \text{if } c \text{ else } B \mid B \text{ and } B \mid \text{some } V \text{ in } B \\
& \quad \mid \text{hence } B \mid \text{hitherto } B \mid X \mid \mu X \text{ in } B
\end{align*}
\]

Agents \( A \) are those \( B \)'s which do not have any occurrence of the hitherto combinator.

Table 1. TCC, with history

**Contributions.** The contributions of this paper may be summarized as follows:

- We motivate the use of TCC for complex temporal event processing. TCC is capable of handling the absence of information.
- We extend TCC with a way to capture the past history of the system. This permits a natural declarative style of querying the past.
- We motivate the introduction of recursive queries in TCC. This permits recursive queries that can reach arbitrarily deeply into the past.
Motivated by [15] and the testing interpretation of intuitionistic logic, we further introduce “hypothetical” queries if $B$ then $G$ that ask if the current system augmented with the agent $B$ can answer the query $G$. Unlike TCC, we also permit such nested agents to move backwards in time, allowing speculative augmentation of the past. Together, these two capabilities permit $B$ to move backwards and forwards in time, while confined to the past.

We provide a formal operational semantics for the language, based on an interpretation of programs as formulas in linear time temporal logic, and computation as deduction.

We establish that the semantics of this language is conservative over TCC. That is, the behaviors of a program in this language that is also expressible in TCC are exactly the same as in TCC.

1.4 Related Work

Several authors have explored the properties of TCC in the last two decades, extending it in various directions. [30] shows that the synchronous languages Lustre and Argos can be embedded in TCC. Expressiveness is further discussed in [19]: different variants that express recursiveness in different ways are discussed and related. It is shown that equivalence of programs with replication (or parameterless recursive procedures) is decidable. [21][20] propose an extension to TCC (ntcc) that can handle asynchronous communication, and nondeterministic behavior, by providing a guarded-choice operator and an unbounded but finite delay operator. A denotational semantics, and a proof system for temporal properties are presented. Another approach to reasoning about TCC programs is provided in [10,6]. More decidability results for TCC and ntcc are presented in [32]: strongest post-condition equivalence for “locally independent” ntcc programs is shown to be decidable. This language is capable of specifying certain kinds of infinite-state reactive systems. [4] discusses a variant capable of dealing with “soft” constraints and preferences; the intended application area is a collection of agents negotiating over quality of service. Abstract diagnosis for a variant of TCC is considered in [9].

In terms of implementation, [25] describes an initial implementation in Java, for reactive computation. This is currently being extended to an implementation of the language discussed in this paper, on top of X10 [23][18].

2 Programming in TCC with histories

We now consider how several idioms of practical interest can be expressed in this language.

2.1 A concrete TCC language, $V$

To fix intuitions, we work on top of a constraint system which permits (sorted) function and predicate symbols, with equality (“$=$”). Amongst the sorts available are Boolean and Int. Sorts are closed under products and function space, i.e. if $S_1, S_2$ are sorts, then so are $S_1 \times S_2$ and $S_1 \to S_2$. 

8 TCC, with History

(Terms) s,t ::= X | f(t₁,...,tn)
(Constraints) A,B ::= s=t | p(t₁,...,tn) | c,c

The equality predicate is interpreted as a congruence relation (it is symmetric, reflexive and transitive, and equal terms can be substituted for each other in all contexts). A set of constraints c₁,...,cₙ entails p(t₁,...,tk) if and only if it entails s₁=t₁,...,sₖ=tk (for some terms s₁,...,sₖ) and for some i, cᵢ is p(s₁,...,sₖ).

For convenience, we will also permit linear arithmetic constraints, and arithmetic inequality, <, ≤.

We will also find it convenient to permit prev(t) as a term, when t is a term. A constraint store can establish prev(u)=v at time t if it can establish u=v at time t−1, and v is rigid, i.e. does not change value with time. The only rigid terms are the constants – we assume they denote the same value at every time instant.

We shall adopt the convention of specifying named agents through agent clauses of the form a :- A, and named goals through goal clauses of the form g :- G, where a and g are atomic formulas. The predicate names for agents, goals and primitive constraints are understood as being drawn from disjoint spaces.

2.2 Programming in V

Example 2 (past G, next G). We define the query past(X=Y), intended to be true at query time i precisely if X=Y is true at query time i−1.

past(X=Y) :- all U in if (hitherto hitherto U=true) then hitherto (U=true or X=Y)

Here is how we understand it. To establish the goal past(X=Y) in a configuration Γ at query time i, we are permitted to assume U=true (at all times) in [0,i−2], for a brand-new variable U. In turn, we must establish either U=true or X=Y in [0,i−1]. Clearly, the assumption establishes the desired goal in [0,i−2]. Hence we are left with time i−1. No agent in Γ knows about U. Therefore the only way past(X=Y) can be established is if at the previous time instant X=Y can be established.

The past predicate can be defined in a similar way for other constraints of interest, e.g.:

past(X>Y) :- all U in if (hitherto hitherto U=true) then hitherto (U=true or X>Y)

The code next(X=Y) is the dual:

next(X=Y) :- all U in if (hence hence U=true) then hence (U=true or X=Y)

Example 3 (once G). We express the query once G that succeeds only if G can be established at some point in the past. We illustrate for G of the form X=Y.

once(X=Y) :- X=Y or past(once(X=Y)).

This goal can be established in a configuration at i only if G can be established at i, or, recursively, the goal can be established at i−1.

If arithmetic is available, and recursion with parameters, one can program within t do G:
Example 4 (within t do G). We require G to be established within t time units in the past:

within T do X=Y := X=Y or (T > 0 and past(within T-1 do X=Y)).

We show that the query language has enough power to internalize else.

Example 5 (not (X=Y)). We express the query not(X=Y). This query succeeds only if X=Y cannot be established:

not(X=Y) :- all U in if (if X=Y else U=false) then U=false

This goal can be established in a configuration Γ only if Γ, if X=Y else U=false can establish U=false. But this can happen only if Γ can evolve in such a way that X=Y cannot be established (per the semantics of the TCC if/else).

Example 6 (last X then G). We would like to express that G is true at the last time instant at which X was true (assuming there is a time instant at which X is true):

last X=Y then U=V :- prev(last1 X=Y then U=V).
last1 X=Y then U=V :- (X=Y and U=V) or (not(X=Y) and prev(last1 X=Y then U=V).

Intuitively, at the last time instant, a check is made for X=Y. If it is true, then U=V must be true, else the goal will fail. If it is not known to be true, then the goal succeeds provided that the same goal can be established at the previous time instant.

We turn now to using these general constructions to show how a complex event query can be formulated.

Example 7. An example of the use of this goal is the query that returns the previous price of a stock. We shall imagine that if in a time instant an event arrives that specifies the price P of a stock S, then the constraint price(P)=S is added to the store. Note that many stock price events may arrive at the same time instant – we assume that all are for different stocks. It is not necessary that each time instant contains a constraint about the price of a given stock S. In this case, we may wish to determine the previous prices of the stock S, which is the price of the stock at the first instant before the current one at which a price event was received.

prevPriceOfStock(S)=P :-
    (prev(price(S))>0 and P=prev(price(S))) or
    (not(prev(price(S))>0) and prev(prevPriceOfStock(S)=P)).

Now one can use this query to determine whether the price has dropped. The query checks that there is a price event at the current time instant, and the price it specifies for the stock S is less than the previously known price for S.

priceDropped(S) :- prevPriceOfStock(S) > price(S).

Such a query can now be used to time an agent. The agent

time next`10 emitVolatilityWarning on priceDropped(S)

will emit a volatility warning at the tenth time instant at which the price has dropped. Using standard TCC idioms, this agent can be packaged up thus:
to precisely capture the rule “Every tenth time the price drops within an hour, emit a volatility warning”.

The above provides a flavor of the richness of this system.

### 3 Semantic model

#### 3.1 Transition relations

The central problem we address is the temporal evolution of (mutually dependent) agents and guards.

We add a new formula $B ::= \text{time}^i B$, to keep track of formulas that are intended to hold at a point in time in the past. We abuse notation slightly by permitting $\text{time}^0 B$ and treating it indistinguishably from $B$. Below, $\Gamma, \Delta, \Pi$ range over (possibly empty) multisets of $B$ formulas. For a multiset of formulas $\Delta = B_0, \ldots, B_{n-1}$ we let $\text{time}^i \Delta$ stand for $\text{time}^i B_0, \ldots, \text{time}^i B_{n-1}$. Similarly for $\text{hence} \Delta$ and $\text{hitherto} \Delta$.

We define three transition relations. All of them are indexed with the current time instant $j$ and the query time instant $i$ (with $i \leq j$). The main relation of interest is $\Gamma \models_{b}^{i, j} G$ (read: “$\Gamma$ proves $G$ at (past) query time $i$ (with quiescent store $b$) when the current time is $j$ ($i \leq j$)”). We need to carry $j$ in the relation because in order to prove a goal of the form $\text{hence} G$ we only need to consider time steps upto $j$. Note that $\Gamma$ will, in general, contain formulas active at different time instants $k \leq j$ (i.e. $\Gamma$ will contain formulas of the form $\text{time}^k B$). $G$ however, is never explicitly timed, since at query time $i$ we care only about queries holding at time $i$.

To define this relation, we need two auxiliary relations that define evolution within time instants in the past ($\Gamma \rightarrow_{b}^{i, j} \Gamma'$), and across time instants in the past ($\Gamma \rightsquigarrow_{b}^{i, j} \Gamma'$). Note that these auxiliary relations may work with hypothetical pasts, since they may reflect the presence of assumptions $B$ made by the goal $G$ being solved at $j$.

We let $\sigma^i(\Gamma)$ stand for the set of all formulas $c$ s.t. $\text{time}^i c \in \Gamma$, i.e. the subset of constraints known to be in effect at time $i$.

**The provability relation for goals** The logical rules are straightforward, and correspond to RHS rules for the appropriate logical connective, in a sequent-style presentation. Rule $\text{Rule 1}$ uses $\sigma^i$ to pick out the constraints in effect at query time $i$ from the current configuration. Rule $\text{Rule 2}$ ensures that in order to prove a goal of the form if $B$ then $G$ at
query time $i$, the assumption $B$ is added at time $i$ to the current configuration.

\[
\frac{\sigma^i(\Gamma) \vdash c}{\Gamma \vdash c} \quad \frac{\Gamma \vdash \mu X G[X]}{\Gamma \vdash \mu X G}
\]

\[
\Gamma \vdash \mu X G \quad G_0 \quad \Gamma \vdash \mu X G \quad G_1
\]

\[
\frac{\Gamma \vdash \text{time}^i B \vdash G_0 \quad G_0 \text{ and } G_1}{\Gamma \vdash \text{if } B \text{ then } G}
\]

\[
\frac{\Gamma \vdash \text{time}^i B \vdash G_0 \quad G_0 \text{ or } G_1}{\Gamma \vdash \text{time}^i B \vdash G_1}
\]

\[
\frac{\Gamma \vdash \mu X G}{\Gamma \vdash \mu X G \quad \text{(V not free in } \Gamma)}
\]

Note that $b$ is not used in these rules; we will see later that it is used when specifying how the LHS evolves within a time instant.

We consider now the temporal rules. These rules have a (finite) set of assumptions, indicated by the for all quantifier. A goal \textit{hitherto} $G$ can be proved at query time $i$ if it can be proven at every time in $[0, i)$. A goal \textit{hence} $G$ can be proved at query time $i$ if it can be proven at every time in $(i, j]$.

\[
\frac{\Gamma \vdash \mu X G \quad \text{V in } G}{\Gamma \vdash \mu X G \quad \text{all V in } G}
\]

Now, since we permit $B$’s to occur on the LHS, and these could evolve, we must also have the following rules. In Rule 5, the configuration is partitioned into three groups of formulas – $\Gamma_i$ which are all the formulas \textit{time}^k B with $k < i$, $\Gamma_i$ which are all the formulas $\text{time}^k B$ with $k \geq i$, and \textit{hitherto} $\Delta$. The rule captures the notion that to prove a formula $G$ at time $i$ one must “go back” to time 0 in order to account for the effects of any \textit{hitherto} $B$ formulas in $\Gamma$. In this traversal into the past, \textit{hitherto} $B$ agents are carried “backwards” (exactly as \textit{hence} $B$ agents are carried forward in TCC, see Rule 15), together with “past” state. The recursion is stopped by Rule 8.

\[
\begin{align*}
\Gamma_i \text{, time}^{i-1} \Delta, \text{hitherto } \Delta & \rightarrow i^{-1} j \Gamma' \quad \Gamma_i, \Delta & \rightarrow i^{-1} j \Gamma'' \quad \Gamma' \vdash i^{-1} j \Gamma'' \\
& \Gamma' \vdash i^{-1} j \Gamma'' \quad \Gamma' \vdash i^{-1} j \Gamma'' \\
\end{align*}
\]
The evolution relation The rules for \( \rightarrow \) (evolution within a time instant) are as in [24], changed in an appropriate way to consider the more general notion of execution at possibly past time points. (The special case of TCC execution is obtained by considering the relation \( \Gamma \rightarrow^{i,j} \Gamma' \) and restricting ask agents to check primitive constraints.) At query time \( i \), an agent \( \text{time}' \) if \( G \) then \( B \) can be reduced to \( \text{time}' B \) provided that the goal \( G \) can be proved from the current configuration. To reduce an \( \text{time}' \) if \( a \) else \( B \) agent, we use the quiescent information \( b \) (associated with query time \( i \)), as usual for Default CC.

\[
\frac{\Gamma \vdash l_{b}^{i,j} G}{\Gamma, \text{time}' \text{if } G \text{ then } B \rightarrow^{l_{b}^{i,j}} \Gamma, \text{time}' B}
\]

\[
\frac{b \nmid a}{\Gamma, \text{time}' \text{if } a \text{ else } B \rightarrow^{l_{b}^{i,j}} \Gamma, \text{time}' B}
\]

\[
\Gamma, \text{time}' B_{0} \text{ and } B_{1} \rightarrow^{l_{b}^{i,j}} \Gamma, \text{time}' B_{0}, \text{time}' B_{1}
\]

(\( Y \) not free in \( B, \Gamma, \Pi \))

\[
\frac{\Gamma, \text{time}' \text{some } V \text{ in } B \rightarrow^{l_{b}^{i,j}} \Gamma, \text{time}' B[V / V]}{\Gamma, \text{time}' \mu X B \rightarrow^{l_{b}^{i,j}} \Gamma, \text{time}' B[\mu X B / X]}
\]

Note that there is no rule for \( \text{hitherto } B \) – it does not contribute to instantaneous evolution, or to the step relation. It is of use in the proves relation when moving backwards in time.

The rules for evolution across time instances are as follows. They differ from the rules for TCC only in that the final constraint at the previous time step is explicitly carried forward into the configuration at the next time step (\( \text{time}' b \)) with the appropriate time index (\( i \)) to distinguish it from the constraints that will be generated at other time steps. The first rule is used to advance time in a query computation, the second to advance time for the overall (top-level) computation. Below, let \( \Pi \) consist of formulas of the form \( \text{time}' c \) for some \( i \), and \( \Gamma' \) does not contain such formulas.

\[
\frac{\Gamma \rightarrow^{i,j} \Gamma', \Pi, \text{hence } \Delta \quad \Gamma', \Pi, \text{hence } \Delta \nrightarrow^{i,j} \Gamma}{\Gamma \rightarrow^{i,j+1} \Gamma', \Pi, \text{time}' \Delta, \text{hence } \Delta}
\]

(9)

\[
\frac{\Gamma \rightarrow^{i,j} \Gamma', \Pi, \text{hence } \Delta \quad \Gamma', \Pi, \text{hence } \Delta \nrightarrow^{i,j} \Gamma}{\Gamma \rightarrow^{i,j+1} \Gamma', \Pi, \text{time}' \Delta, \text{hence } \Delta}
\]

(10)

\[
\frac{\Gamma \rightarrow^{i,j} \Gamma', \Pi, \text{hence } \Delta \quad \Gamma', \Pi, \text{hence } \Delta \nrightarrow^{i,j} \Gamma}{\Gamma \rightarrow^{i,j+1} \Gamma', \Pi, \text{time}' \Delta, \text{hence } \Delta}
\]

(11)

(12)

(13)

Proposition 1 (TCC+history does not change the past.). Let \( \Gamma_{0}, \Gamma_{1}, \ldots, \Gamma_{n}, \ldots \) be an execution. Then for every \( j > 0 \) and \( m, n > j \) it is the case that \( \Gamma_{m} \mid j \) and \( \Gamma_{n} \mid j \) are multisets of constraints that are equivalent.
The following proposition relies on the fact that a multiset of nested TCC+history agents cannot have sub-formulas of the form \( \text{if } G \text{ then } B \) (unless \( G \) is a constraint), \( \text{if } B \text{ then } G \), \( \text{hitherto } G \), \( \text{hitherto } B \). Therefore in any proof of the judgement \( \Gamma \vdash^{i,j} c \) only judgements of the form \( \Gamma' \vdash^{i,j} c' \) are generated. No “travel” in time is possible.

**Proposition 2.** Suppose \( \Gamma \) is a multiset of nested TCC+history agents such that \( \Gamma \upharpoonright j \) is a multiset of TCC agents. Then \( \Gamma \vdash^{i,j} c \iff \Gamma \upharpoonright j \vdash_b c \), where \( \vdash_b \) represents the TCC entailment relation with \( b \) the final resting point.

**Theorem 1.** TCC + History is conservative over TCC.

### 4 Conclusion

This paper represents the first step in the study of TCC, augmented with history and a rich notion of queries. A number of areas of work open up.

**Expressiveness.** Does this language realize the intuition that queries can be multi-form in time, just as agents can be multi-form in time? Is it semantically meaningful to consider deep negative guards?

**Denotational semantics.** The basic semantic intuition is that this language permits rich querying of the past, with a deep interplay between agents and guards. Since the past is not modified it should be possible to adapt the denotational semantics of [24] (based on prefix-closed sets of traces) to this setting.

**Finitary implementations.** For many uses of the language, it would be valuable to bound the amount of past information that needs to be carried in the state. Does this language admit of finite state compilability (a la TCC)? If not, what restrictions need to be placed to achieve finite state compilability?

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A Background

The basic idea of TCC may be summarized as follows:

\[ \text{TCC} = \text{CCP} + \text{Synchrony hypothesis} \]

CCP, concurrent constraint programming, is a simple view of parallel computation that arises from multiple interacting agents sharing a common store of constraints. Constraints are expressions (such as \( X >= Y + Z \)) over a finite set of free variables. Each constraint is associated with a solution set, a set of mappings from variables to values (called valuations) that makes the constraint “true”. e.g. the set of valuations that makes \( X >= Y + Z \) true is the set of valuations \( T \) s.t. \( T(X) \geq T(Y) + T(Z) \).

Two fundamental operations on constraints are used in CCP – `tell c` (add \( c \) to the current store), and `ask c` (check if \( c \) is entailed by the current store). Note that addition is conjunctive – the solution set of \( c, d \) is the intersection of the solution sets of \( c \) and \( d \). Say that \( c \) entails \( d \) if the solution set of \( c \) is contained in that of \( d \) (that is, if \( v \) is a solution for \( c \), then it is a solution for \( d \)) and disentails \( d \) if the solution sets of \( c \) and \( d \) are disjoint. The operation `ask c` succeeds if the store entails \( c \), fails if the store disentails \( c \), and suspends otherwise.

In CCP, the programmer specifies a set of agents over shared variables that interact with each other by telling and asking constraints on the shared variables. The fundamental property of CCP is that computations are determinate – the result is the same, regardless of the order in which agents are executed. Furthermore, programs have a declarative interpretation, they can be read as formulas in logic and have the property that if a program \( \exists \) logically entails a constraint \( c \), then execution of \( \exists \) will result in a store that entails \( c \).
CCP is a rich and powerful framework for (asynchronous) concurrent computation. TCC arises from CCP by “extending” CCP across time. We add the new control construct \textit{next}: if \( A \) is an agent, then so is \textit{next} \( A \). The intuitive idea is that computation progresses in a series of steps. In each step, some input is received from the environment (an “event”), and added to the store. The program is then run to quiescence. This will yield a store of constraints, this provides the “instantaneous response”. In addition it will yield a set of \textit{next} \( A_1 \), \ldots, \textit{next} \( A_n \) agents. (Note some of these agents can be simple constraints.) These are precisely the agents that are used to respond to the next event, at the next time instant.

Notice that this view is concerned with a logical notion of time – time is just a sequence of ticks arriving from the environment (with additional input). There is no intrinsic association of this sequence of ticks with “real” time, e.g. msecs. This is the powerful insight that underlies the notion of multiform time. This notion says that the temporal constructs in the language can all be used for any user-defined notion of time, not just the “built-in” notion of time. In TCC, this is captured by the \textit{time} \( A \) on \( B \) combinator. For the agent \( A \), the agent \( B \) defines the notion of time – only those time ticks that “pass” the test \( B \) are passed on to \( A \). Thus \( A \) is executed with a “programmer supplied” clock. Of course, these constructs can be nested, thus \textit{time} \textit{time} \( A \) on \( B_1 \) on \( B_2 \) will supply to \( A \) only those time ticks that pass \( B_1 \) and \( B_2 \).

This flexibility of the basic formalism permits a large number of combinators to be definable by the user. Combinators such as the following are definable in \( A \) and \( B \):

- \textbf{do} \( A \) \textbf{watching} \( c \): Execute \( A \), across time instants but abort it as soon as there is a time instant which satisfies the constraint \( c \).
- \textbf{suspend} \( c \) \textbf{activate} \( d \) \textbf{A}: Execute \( A \), across time instants, suspending it as soon as a time instant is reached in which \( c \) is true. Then activate it as soon as a time instant is reached in which \( d \) is true.

A.1 RCC– Combining agent execution and testing

The key intuition was the recognition that CCP corresponds to “computation on the left”, or forward chaining, and (definite clause) logic programming corresponds to backward chaining. This is illustrated by the following characterization of CCP agents as formulas in intuitionistic logic:

\begin{align*}
\text{(Agents)} & \quad A ::= c \mid G \Rightarrow A \mid E \mid \text{some} \ V \ \text{in} \ A \\
\text{(Goals)} & \quad G ::= c \mid \text{all} \ G \ \text{and} \ G \\
\text{(Clauses)} & \quad P ::= E \Rightarrow D \mid \text{all} \ P \ \text{and} \ P
\end{align*}

Computation is initiated on the presentation of an initial agent, \( A \), and progresses in the “forward” direction. One thinks of a sequent \( A_1, A_n \to \) as a multiset of interacting agents operating on a store of constraints (the subset of the \( A_i \) that are constraints). If the store is powerful enough to entail the condition \( G \) of an agent \( G \Rightarrow A \), then \( G \Rightarrow A \) can be replaced by \( A \). This corresponds to the application of the left hand rule for implication. Recursive calls \( E \) are replaced by the body \( A \) of their defining clauses \( E \Rightarrow D \). Computation terminates when no more implication can be discharged.

This is logically sound. Clearly if we start computing with an agent \( A \) and terminate in a state with the subset of constraints \( \sigma \) then we have \( A \vdash \sigma \), where \( \vdash \) represents...
provability in Intuitionistic Logic (IL), augmented with axioms from the underlying constraint system, \( C \). Is this logically complete? Indeed – [26] shows that if there is a constraint \( d \) that is entailed by \( \lambda \), then in fact it is entailed by \( \sigma \) the constraint store of the final configuration obtained by executing \( \lambda \) as a CCP agent. Hence CCP operational semantics is sound and complete with respect to entailment of constraints.

Note that this language corresponds to “flat” guards. In the early development of concurrent logic programming languages [29,27,31] a lot of attention was paid to “deep” guards. How can deep guards be integrated into CCP?

One idea is to look at definite clause logic programming. The logical picture here is well known.

\[
\begin{align*}
\text{(Goals)} & \quad G ::= c \mid \text{all } G \text{ and } G \mid \text{all } G \text{ or } G \mid H \mid \text{some } V \text{ in } G \\
\text{(Clauses)} & \quad P ::= H \Rightarrow G \mid \text{all } P \text{ and } P
\end{align*}
\]

Computation corresponds to posing a query, or a goal, against a database of clause of the form \( H \Rightarrow G \) [14]. A configuration consists of a collection of \( G \) formulas. In each step, a conjunction is replaced by its components, an existential \( \text{some } V \text{ in } G \) by \( G \), with \( V \) a “new” variable, and an atom \( H \) by the body \( G \) of a clause \( H \Rightarrow G \) from the program. A disjunct is non-deterministically replaced by one of its disjuncts. Computation terminates when the configuration contains only constraints. Of particular interest are terminal configurations in which the constraints are jointly satisfiable, these correspond to answers for the original query.

Is there a reasonable way to combine the two? In fact, it is possible to do this, and a lot more. It is possible to give an intuitive operational semantics for the following system of agents and goals.

\[
\begin{align*}
\text{(Agents)} & \quad D ::= c \mid G \Rightarrow D \mid E \mid E \Rightarrow D \mid \text{all } D \text{ and } D \mid \text{some } V \text{ in } D \mid \text{all } V \text{ in } D \\
\text{(Goals)} & \quad G ::= c \mid A \Rightarrow G \mid H \mid G \Rightarrow H \mid \text{all } G \text{ and } G \mid \text{all } G \text{ or } G \mid \text{some } V \text{ in } G \\
\text{(Clauses)} & \quad P ::= H \Rightarrow G \mid E \Rightarrow D \mid \text{all } P \text{ and } P
\end{align*}
\]

Note that richness of interplay between agents and goals – agents can be defined in terms of goals, and goals can be defined in terms of agents.

What is the underlying programming intuition? We think of \( D \) as representing a concurrent, interacting system of agents (interacting through a shared constraint store). We think of \( G \) as a test of such a system. We think of a sequent \( D \vdash G \) as establishing that the system \( D \) passes the test \( G \). With this interpretation, we can think of an agent \( G \Rightarrow D \) as saying: if the current system of agents can pass the test \( G \), then reduce to \( D \). Conversely, one thinks of the goal \( D \Rightarrow G \) as a “what if” test: Suppose the existing system is augmented with the agent \( D \). Does it now pass the test \( G \)? Similarly, \( \text{all } V \text{ in } G \) is a generic goal: it asks the question “Does the system pass the test \( G \)” for some completely unknown variable \( V \) (hence for all possible values of \( V \)).

We showed further that this semantics is sound and complete with respect to the interpretation of agents and goals as formulas in Intuitionistic Logic (IL). The key insight is to “segregate” the atomic formulas that occur in agents (\( E \)) and in goals (\( H \)) – these must come from disjoint vocabularies. Therefore the “left hand side” (LHS) and the “right hand side” (RHS) of a sequent can no longer communicate through the application of identity rules (\( \Gamma \vdash A \Rightarrow A \)). Rather the replacement constraint inference rule must be used. Computation can be performed in potentially arbitrary combinations of LHS
steps and RHS steps, corresponding to evolution of the concurrent agent and simplification of the test, respectively.

Subsequent work by Liang and Miller [16] established a connection between [15] and the notion of focussing proofs developed by Andreoli [18]. Indeed, the notion of combining forward and backward chaining in the very flexible way described above has seen significant recent work.