Theoretical Analysis of Proppant Embedment and Fracture Conductivity in Fractured Reservoir

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Abstract. Proppant embedment mainly accounts for diminishing fracture aperture and conductivity. The conductivity decrease could greatly restrict fluid flow, thereby affecting the well performance. In this paper, new mathematical models are derived to calculate the proppant embedment and fracture conductivity after hydraulic fracturing. These models take three crucial factors into consideration, including proppant instantaneous elastic deformation, the conductivity change over time, and effective fracture geometry and corresponding proppants amount. Fitting correction is performed for the new theoretical model based on the experimental data. The results show that the theoretical model proposed matches well with the experimental data. The fracture conductivity is in proportion to proppant viscosity, elastic modulus of proppant and in inverse proportion to closure pressure, while elastic modulus of rock and large value of formation rock viscosity have slight impact on fracture conductivity. Moreover, with the increase of proppant viscosity, it takes longer for the conductivity to come to the steady state. These new mathematical models can provide some insights for proppant size selection and sand ratio optimization of fracturing treatment, and it is also a good method to predict the proppant embedment and the fracture conductivity.

Key words: Fracture geometry; Proppant deformation; Creeping deformation model; Embedment; Conductivity

1. Introduction

Hydraulic fracturing refers to a significant stimulation technology, which has been increasingly applied in low permeability and unconventional reservoirs. Fracturing fluids and proppants are the essential materials for fracturing. Proppants, such as sand and ceramic, mixed with fracturing fluids, are injected into the created fractures to resist closure stress, hold fractures open and reach the critical fracture conductivity for fluids to flow. Numerous SMA (surface modification agent) treated proppant and fiber have been applied to enhance the effect of hydraulic fracturing (Warpinski et al 2008; Cipolla CL et al. 2009; Vincent MC 2012; ZHANG JC 2014; Kunnath Aven N et al. 2013; Burukhin AA et al. 2012). However, the overall stimulation performance would be greatly affected due to the proppant embedment. The result of a number of experimental tests in weakly consolidated core by using a new computer-
controlled laboratory technique clearly demonstrates this fact (Lacy et al. 1998). Thus, it is of vital importance to study proppant embedment and conductivity for the simulation of reservoirs and well test.

There are mainly two methods to study the proppant embedment and fracture conductivity: experimental studies and computational modeling. Empirical or semi-empirical equations are considered to be one kind of experimental tests, because multiple empirical or semi-empirical equations were presented to predict and examine the influencing factors of conductivity by the experimental data, and had their correction coefficient verified by experiments.

Mindlin et al. (1953) studied the load-displacement behavior on two adhered elastic spheres for varying stress and analyzed the response of the system, which can be applied to two adhered proppants. Huitt et al. (1958) conducted a number of experimental tests and used the results of experimental data to establish an experiential regression formulate for computing the proppant embedment in the case of known proppant concentration and overburden stress. Volk et al. (1981) formulated empirical equations based on experimental results, and examined several variables concerning embedment, such as proppant concentration, size, distribution, rock type, and embedment surface. Wen QZ et al. (2005) found that the damage of embedment would come out when closure pressure reaches a special value. Wen QZ et al. (2007) conducted experimental research on proppant embedment in formation core, Experimental tests were also made in their work to study the factors influencing proppant embedment, including concentration of proppant-paving, closure pressure, rock property and the degree of embedment. Guo et al. (2008) carried out an experimental research on proppant embedment in core samples. Their study indicates that the fracture width would be significantly diminished due to proppant embedment. Kulkarni et al. (2012) established a model to identify the effectiveness and efficiency of proppants enlisted in hydraulic fracturing treatments, and various mixtures of hard and soft particles are investigated as a function of shape, size and inter-particle friction. Guo et al. (2012) conducted extensive laboratory tests to optimize the proppant size for an effective sand control in weakly consolidated formations, and investigated the effect of fluid viscosity, flow rate, median grain diameter of formation sand, and proppant concentration on proppant embedment and fracture conductivity. There are also many other experimental investigations on proppant embedment and fracture conductivity (Zhang J et al. 2014; Chapman M et al. 2014; Zou Y S et al. 2014). However, although the empirical or semi-empirical models of embedment and conductivity have been derived to calculate the specific reservoir, the accuracy of matching or predicting the proppant embedment and fracture conductivity may be not reasonable in some cases (Li, K. 2014). It is also uneconomic and requires a lot of time to conducted experimental tests for the analysis of the proppant embedment and prediction of fracture conductivity.

In this study, analytical models are derived to calculate proppant embedment and fracture conductivity in hydraulic fracturing, and the fracture geometry and proppant amount have been taken into consideration. These models can be closely related to the fracturing treatment in terms of fracture geometry and corresponding proppants amount, and the calculated conductivity may be the real case in field after hydraulic fracturing. Experimental data of fracture conductivity are used to match these models derived in this study, which yields reasonable results. The proppant deformation model and creeping deformation model are adopted to predict the change of proppant embedment and fracture conductivity over time, and some influencing factors, such as closure pressure, elastic-plastic properties, properties of viscoelastic proppant and rock, are investigated to quantify the key parameters affecting proppant embedment and fracture conductivity in coalbed reservoir.

2. Theoretical Analytical model
Given that the proppant is equivalent to elastic sphere, the stress on the upper and lower rocks is equivalent. Proppant and rock can be deformed, but not easily broken. It is also assumed that the proppants are evenly paved on the rocks, and the proppant-packing is regarded as capillary model, as shown in Fig.1.
Those assumptions above were justified by the studies of many researchers, such as Mindlin et al. (1958), Wu et al. (2001), Gao et al. (2012), Gao et al. (2013), Li et al. (2014), Zhang et al. (2015). The classical method presented by Mindlin and Deresiewicz was developed in the assumption that the adhered spheres are elastic materials, and cannot be broken under varying forces. Wu et al. (2001) proposed the similar assumption and demonstrated that the change in distance between two mutually squeezing elastic spheres were influenced by two factors: embedment and deformation. Mindlin (1958) and Wu’s (2001) models laid the foundation for current proppant embedment and fracture conductivity study. Regarding the proppant-packing as capillary model were modified by Gao et al. (2013), Li et al. (2014), and Zhang et al. (2015) in their research of fracture conductivity model.

For the analysis of the impact of main factors on proppant embedment and fracture conductivity, the proppant elastic deformation model and creeping deformation model were applied to drive the mathematical equations in this work. Fracture geometry and corresponding proppants amount are also considered to calculate and predict the conductivity, which are always overlooked in many previous work on proppant embedment and fracture conductivity.

2.1. Number of particles
From the Fig.2, we can see the proppant distribution inside of fracture. Fig.2-a, b, c represents the single layer pattern, double-layer pattern, and three-layer pattern, respectively. Comparatively, the Fig.3 shows the proppant distribution of the laboratory experiment. As shown in Fig. 3-b (the multi-layer pattern) (Zou et al. 2012), the proppants are squeezed successively under the closure pressure. In order to characterize the multi-layer pattern, the proppant arrangement and proppant distribution pattern are proposed to be repetition in every three-layer, and most theoretical analysis on proppant embedment and fracture conductivity are based on this assumption (Gao et al. 2013; Li et al. 2014; Zhang et al. 2015).

![Fig. 1 Model of squeezed sphere and fracture (Gao et al.2012) [23]](image1)

![Fig. 2 Proppant distribution in the front of three layers (a, the single layer pattern; b, double-layer pattern; c, and three-layer pattern)](image2)
Fig. 3 Proppant distribution in single-layer pattern (a) and in multi-layer pattern (b) (Zou et al. 2012)

One can calculate the total number of proppants and the number of embedment proppants with the following equation:

\[
N_{\text{num}} = \left[ n \left( \frac{H_e - D_i}{\sqrt{2}} + 1 \right) \right] - A \times \left( \frac{L}{D_i} \right)_{\text{int}}
\]

(1)

Where \( n \) the number of layers, \( m \) is integer. If \( n=3m-2 \), then \( A=2m-2 \); if \( n=3m-1 \), then \( A=2m-1 \); else \( n=3m, A=2m \).

\[
N_{\text{numo}} = \left[ 2 \left( \frac{H_e - D_i}{\sqrt{2}} + 1 \right) \right] - B \times \left( \frac{L}{D_i} \right)_{\text{int}}
\]

(2)

Where \( n \) the number of layers, \( m \) is integer. If \( n=3m-2 \), then \( B=0 \); if \( n=3m-1 \), then \( B=1 \); else \( n=3m, B=1 \).

2.2. Creeping deformation model

Gao et al. (2013) assumed that the proppants are evenly paved in the fracture in conventional fracturing, and derived an analytical model to compute proppant embedment and deformation, change in fracture width, and fracture conductivity with varied closure pressure. Related formulas are as follows:

\[
h = 1.04D_i(K^2P)^{2/3} \left[ \left( \frac{1-v_1^2}{E_1} + \frac{1-v_2^2}{E_2} \right)^{2/3} - \left( \frac{1-v_1^2}{E_1} \right)^{2/3} \right] + D_2 \frac{P}{E_2}
\]

(3)

\[
\beta = 1.04D(K^2P \left( \frac{1-v_1^2}{E_1} \right)^{2/3}
\]

(4)

\[
\alpha = \beta + h
\]

(5)

In this work, however, proppant and formation rock are assumed as viscoelastic material. As the closure pressure almost remains constant after hydro-fracturing, proppant and rock will have creep deformation, in which proppant particles embed into formation rock slowly but constantly with the increase of production time. In the derivation of creep deformation, we use Kelvin-Voigt model to describe the process of viscoelastic creep (Cai et al. 2002):

\[
\xi(t) = \frac{\sigma_0}{E} \left[ 1 - \exp\left( -\frac{t}{\eta / E} \right) \right]
\]

(6)
Where t means production time after hydro-fracturing, in days; The Equation 6 means the pressure changes with time exponentially. When considering creeping deformation of proppant and rock, embedment and deformation are modified into:

\[
h(t) = 1.04D_1 \left[ \left( K^2 P \frac{1 - \frac{v_1^2}{E_1}}{(1 - e^{-\frac{t}{\eta \beta}})} + K^2 P \frac{1 - \frac{v_2^2}{E_2}}{(1 - e^{-\frac{t}{\eta \beta}})} \right)^2 - \left( K^2 P \frac{1 - \frac{v_1^2}{E_1}}{(1 - e^{-\frac{t}{\eta \beta}})} \right)^2 \right]
\]

\[+ D_2 \frac{P}{E_2} (1 - e^{-\frac{t}{\eta \beta}}) \]

\[
\beta(t) = 1.04D \left( K^2 P \frac{1 - \frac{v_1^2}{E_1}}{(1 - e^{-\frac{t}{\eta \beta}})} \right)^2
\]

\[
\alpha(t) = \beta(t) + h(t)
\]

2.3. Fracture conductivity model
The fracture conductivity refers to the product of permeability and fracture width, which is a significant parameter to evaluate the flow capacity of fracture. Fracture permeability is a function of porosity, radius of pore throat and pore tortuosity. In this paper, the Carman-Kozeny equation, a common formula, was applied to calculate the fracture permeability (Yang et al. 2004):

\[
k = \frac{\phi r_k^2}{8 \tau^2}
\]

Based on the assumptions, the porosity formed by the proppants is uniformly distributed and the differences of pore in different layers are ignored. When closure pressure is equal to zero, the expression of fracture porosity is:

\[
\phi_0 = \frac{LH_p D - N_{num} \frac{4\pi R^3}{3}}{LH_p D}
\]

\[
D = (n-1)D_1 \frac{\sqrt{6}}{3} + D_1
\]

Yu et al. (2004) derived an empirical model to study the relationship between porosity and tortuosity, and the expression is:

\[
\tau = \frac{1}{2} \left[ 1 + \frac{1}{2(1-\phi)} \sqrt{1+\phi} \sqrt{\frac{1}{1-\phi} - \frac{1}{4}} \right]
\]

Considering the effect of proppant embedment, porosity and pore tortuosity can be redefined as (Li et al., 2014):

\[
\phi = \frac{D \phi_0 - 2 \beta}{D - 2 \beta}
\]

\[
\tau = \sqrt{1 + \left( \frac{D - 2 \beta}{D} \right)^2 (\tau_0^2 - 1)}
\]

The radius of pore throat is a function of proppant embedment. Regarding the proppant-packing as capillary model, we divide the pore throat into two parts, one part is the pore throat in the fracture surface and the other is within the flow channel (between the two red lines connecting center of circles, as shown in Fig.4.) of fracture.
Fig. 4 schematic diagram of proppant embedment

By comparison of many previous studies, this model can be derived to calculate the situation in which the length of proppant embedment is longer than the radius of proppant in weakly formation, and the expressions are:

1) When \(2nR < h < (2n+1)R\) \((n=1, 2, 3, \ldots)\), as shown in Fig. 4a:

\[
r_1 = 1000 \times \frac{LH(D_1 - 2h_1) - N_{num0} \times \frac{2\pi R^3}{3} + V_E}{N_1 \pi L} + \frac{r_0}{h_1}\]
\[
r_2 = 1000 \times \frac{LH[\sqrt{\frac{6}{3}} (n-1)D_1] - N_{num} \times \frac{4\pi R^3}{3} + N_{num0} \times \frac{2\pi R^3}{3}}{N_1 \pi L}
\]

Where \(h_1, N_1\) and \(N_2\) are defined as

\[
h_1 = h - 2nR
\]
\[
N_1 = 2[(H - 2R) / (\frac{\sqrt{3}}{2} \times D_1)]_{int} - E
\]
\[
N_2 = (n-1)[(H - 2R) / (\frac{\sqrt{3}}{2} \times D_1)]_{int} - F
\]

Where \(n\) the number of layers, \(m\) is integer. if \(n=3m-2\), then \(E=0\); if \(n=3m-1\), then \(E=1\); else \(n=3m, E=1\).

Where \(n\) the number of layers, \(m\) is integer. if \(n=3m-2\), then \(F=2(m-1)\); if \(n=3m-1\), then \(F=2(m-1)\); else \(n=3m, F=2m-1\).

The volume of a single proppant embedment:

\[
V_s = \pi(R - \frac{h}{3})h^2
\]

The volume of total proppants embedment:

\[
V_E = N_{num0}V_S
\]

So the radius of pore throat can be derived as:

\[
r_0 = \frac{2}{n+1}r_1 + \frac{n-1}{n+1}r_2
\]

2) When \((2n-1)R < h < 2nR\) \((n=1, 2, 3, 4, \ldots)\), as shown in Fig. 4b, considering all of the pore throat is inside the two red line, so one can derive the expression of pore throat:
r_0 = r_2 = 1000 \times \sqrt[6]{\frac{LH}{3(n-1)D} - \frac{4\pi R^3}{3} + \frac{2\pi R^3}{3} N_a \pi L} \quad (24)

Considering the effect of proppant deformation, the expression of pore throat should be modified as:

\[ r = \left( \frac{D-2\beta D}{D} \right) r_0 \quad (25) \]

Combining the Eqs. 7, 8, 9, 10, 14, 15, 23 and 24, we can obtain the following equation:

\[ F_{RCD} = c_0 kw D \frac{[D(LH, D - N_{num} \frac{4\pi R^3}{3}) - 2\beta(t) LH, D D](D-2\beta(t)r_0^2 c_0)}{8D^2(1 + (\frac{D-2\beta(t)}{D})^2 (r_0^2 - 1))LH, D} (D - 2a(t)) \quad (26) \]

3. Validation of theoretical model

It is important to validate the reliability of theoretical model with experimental data. The model validated can be used to carry out the critical parameters as well as to observe its effect on fracture conductivity and proppant embedment for a long-time period. In this work, two experiments data are used to validate the model: conductivity variation with closure pressure on the condition of different types of proppants and in three different lithologies, respectively.

The proppants tend to spread and are nearly next to each other due to the effect of the squeezing force, so K is set to be unity in the following analytical model. Cutler et al. (1985) carried out an experimental research on the fracture conductivity of three types of proppants at different closure pressures, which are made of porcelain, bauxite and alumina-mullite. In this case, it is assumed that the known conditions are: \( v_1 = v_2 = 0.2 \); \( E_1 = 3460 \text{MPa} \) (alumina-mullite); \( E_1 = 9570 \text{MPa} \) (porcelain); \( E_1 = 71220 \text{MPa} \) (bauxite); \( E_2 = 100000 \text{MPa} \); \( D_1 = 0.635 \text{mm} \), \( D_2 = 25.4 \text{mm} \), \( D = 6.35 \text{mm} \); \( p = 10 \) to \( 90 \text{MPa} \). Fig.5 presents the relationships between closure pressure and fracture conductivity. Comparisons were conducted between the experimental study and the model calculation in our work. The analytical model illustrates good agreement with the experimental data for both three types of proppants, and the Co (the matching parameter) in bauxite, porcelain and alumina-mullite model are 0.63, 0.7, and 0.78, respectively.

![Fig. 5](compare.png)

**Fig. 5** Comparison of fracture conductivity in theoretical model and experimental data with three types of proppants (experimental data from Cutler et al. 1985)

Zhang et al. (2008) conducted experimental tests to calculate conductivity under different closure pressures on the fractures of steel plate, coalbed and sandstone. In this case, it is assumed that the known
conditions are: \( v_1 = v_2 = 0.2; \ E_1 = 1836 \text{MPa}; \ E_2 = 680 \text{MPa} \) (coalbed); \( E_2 = 1120 \text{MPa} \) (sandstone); \( E_2 = 100000 \text{MPa} \) (steel plate); \( D_1 = 0.635\text{mm}; \ D_2 = 15\text{mm}; \ D = 6.35\text{mm}; \ p = \text{from 10 to 60MPa} \). Fig. 6. exhibits the relationship between conductivity and closure pressure. It can be found that the theoretical model fits well with experimental data in three different lithologies. The Co in coalbed, sandstone and steel plate are 0.55, 0.92, and 0.97, respectively. The results illustrates consistency between the above two experimental tests and the new analytical models, which verifies great accuracy of the model proposed in the paper for calculating fracture conductivity.

4. Analysis of the factors of influence

Parameters sensitivity analysis is then performed to quantify the key factors affecting proppant embedment and fracture conductivity, such as closure pressure, elastic-plastic properties, properties of viscoelastic proppant and rock. Taking coalbed as the research target, base model are proposed under the known conditions below: \( v_1 = v_2 = 0.2; \ E_1 = 1836; \ E_2 = 680; \ D_1 = 0.625; \ D_2 = 15; \ D = 6.35; \ L = 200000; \ H = 40000; \ \eta_1 = 15000\text{MPa}; \ \eta_2 = 10000\text{MPa} \) (Gao et al. 2013, and Zhang et al. 2014).

(1) Closure pressure

Fig. 7. exhibits the relationship between conductivity and closure pressure and can also show the law of fracture conductivity failure. It can be found that with an increase of closure pressure, the conductivity decreases. The decline rate increases along with the increase of closure pressure, and the biggest decrease happens between 1 to 10 days. The initial fracture conductivity in hydraulic fracturing keeps at a same value as closure pressure varies. Nearly 100 days will be needed for conductivity to achieve steadiness in hydraulic fracturing, in which law of fracture conductivity failure shows a good agreement to previous study (Zhang et al. 2008 and Wen et al. 2007). In the subsequent modeling investigations, we perform sensitivity study on the condition of 20 MPa closure pressure.

Fig. 7 Effect of closure pressure
(2) Different elastic modulus of proppant

The conductivity as a function of closure pressure for different elastic modulus of proppant (E1) are studied (see Fig.8.). As depicted, the higher fracture conductivity can be obtained at greater elastic modulus of proppant. This could result from the fact that when the E1 increases, the proppant deformation and embedment decrease, so do the changes in fracture aperture, porosity, and the radius of pore throat. It can be found that the initial conductivity keeps the same value if the closure stress is equal to zero. In light of the general assumption in reservoir simulation that the conductivity is constant, the larger the elastic modulus of proppant, the more accurate the result could be.

Fig. 8 Relationships between facture conductivity and elastic modulus of proppant

(3) Different elastic modulus of rock

To our basic knowledge, in conventional fracturing, the elastic modulus of rock plays a vital role in conductivity variation, especially in the weakly consolidated sand reservoir, shale (oil and gas) formation, and coal formation. As demonstrated in Fig.9, a distinctive increase in conductivity could be observed when the E2 increases from 250 MPa to 1000 MPa. The formation of this phenomenon may be due to the large value of proppant embedment when the value of E2 is lower, which fits well with the real case in hydraulic fracturing treatment. It also can be found that the initial conductivity is the same at different elastic modulus of rock, and it shares the identical characteristics with elastic modulus of proppant. This may be because all of the properties affecting fracture conductivity are the same when the closure stress is equal to zero. However, the conductivity increment is smaller as the elastic modulus of formation rock (E2) increases. Note that in the Fig.9, a slight increment of the E2 exerts little influence for conductivity when E2 reaches a critical value (about 10000MPa). The fracture conductivity would be failure with small E2 and high closure pressure, this can explain why in weakly formation the hydraulic fracturing does not work well to enhance oil and gas production.

Fig. 9 Relationships between facture conductivity and elastic modulus of rock
(4) Proppant viscosity

Effect of proppant viscosity on fracture conductivity is investigated in this part. Proppant and formation rock are considered to be elastic material in this model. Fig.10. depicts the fracture conductivity as a function of time for the given proppant viscosity ranging from 5,000 Mpa to 100,000 Mpa. The basic finding is that proppant conductivity decreases over time, with the biggest difference presented of 80µm²cm. In light of this, the former assumption of constant conductivity in reservoir simulation is far from reasonable; variation in fracture conductivity should be taken into consideration in this regard. Moreover, it takes longer for the conductivity to come to the steady state with the increase of proppant viscosity; and that's why proper and economic material selection is essential to well performance.

![Fig. 10 Effect of proppant viscosity](image)

(5) Formation rock viscosity

Fig.11. presents the effect of formation rock viscosity on the fracture conductivity. Formation rock viscosity was varied from 5,000 MPa to 100,000 MPa. It can be observed that the conductivity increases slightly with the increase of formation rock viscosity. This could be explained by the fact that for a given closure pressure creep deformation changes slightly due to the large number of elastic modulus of rock despite that rock viscosity is relative to embedment. Unlike the effect of proppant viscosity on fracture conductivity, the difference of conductivity between four kinds of formation rock viscosity happens after 1 day production. The biggest gap in conductivity at different rock viscosity is 9 percent of the value of initial conductivity, thus, formation rock viscosity can be ignored for proppant conductivity in hydraulic fracturing design.

![Fig. 11 Effect of rock viscosity](image)
5. Conclusions

Corresponding conclusion may be drawn based on the study above:

In this paper, mathematical models were derived to calculate the proppant embedment and fracture conductivity in hydraulic fracturing, in which fracture geometry, the proppant deformation model and the creeping deformation model have been integrated. Experimental data were applied to validate the reliability of theoretical model at different closure pressures for the condition of different types of proppants and in three different lithologies. By adjusting related coefficients for different test conditions, the model proposed in this study shows good agreement with the experimental data, though the decline rate of experimental data is a little bit faster than that of the model.

Factors affecting fracture conductivity and proppant embedment are analyzed in this paper, including closure pressure, elastic-plastic properties, properties of viscoelastic proppant and rock. The fracture conductivity is directly proportional to proppant viscosity, elastic modulus of proppant and inversely proportional to closure pressure, while elastic modulus of rock and large value of formation rock viscosity have slight impact on fracture conductivity. The larger the elastic modulus of proppant, the less the conductivity varies, leading to a more accurate reservoir simulation model. Although closure pressure varies, the initial fracture conductivity in all models keeps at a same value. The results also show that with the increase of proppant viscosity and rock viscosity, it takes longer for the conductivity to come to the steady state. These factors should be considered and managed during fracturing treatment design and the proppant selection to improve the well performance.

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