Emergent physics: Fermi-point scenario

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The Fermi-point scenario of emergent gravity has the following consequences: gravity emerges together with fermionic and bosonic matter; emergent fermionic matter consists of massless Weyl fermions; emergent bosonic matter consists of gauge fields; Lorentz symmetry persists well above the Planck energy; space–time is naturally four dimensional; the Universe is naturally flat; the cosmological constant is naturally small or zero; the underlying physics is based on discrete symmetries; ‘quantum gravity’ cannot be obtained by quantization of Einstein equations; and there is no contradiction between quantum mechanics and gravity, etc.

Keywords: Fermi point; emergent physics; cosmological constant

1. Introduction

Astronomical observations suggest the existence of a cosmological constant introduced by Einstein (1917). Its value corresponds to the vacuum energy density (Bronstein 1933; Zeldovich 1967) of the order of $A_{\text{obs}} \sim E_{\text{obs}}^4$ with the characteristic energy scale $E_{\text{obs}} \sim 10^{-3}$ eV. Current theories do not provide any good symmetry explanation for the smallness of this value as compared with a naive theoretical estimation suggesting the Planck scale for the vacuum energy: $A_{\text{theor}} \sim E_{\text{P}}^4$ with $E_{\text{P}} \sim 10^{19}$ GeV. This is the so-called cosmological constant problem. Another huge disagreement between the naive expectations and observations concerns the masses of elementary particles. Naive estimation tells us that these masses should be of the order of the Planck energy scale, $M_{\text{theor}} \sim E_{\text{P}}$, while the masses of observed particles are many orders of magnitude smaller, being below the electroweak energy scale, $M_{\text{obs}} < E_{\text{ew}} \sim 1$ TeV. This is called the hierarchy problem. There should be a general principle, which could resolve both paradoxes. Here, we discuss the principle of emergent physics based on the topology in momentum space.

The momentum-space topology suggests that both in relativistic quantum field theories and in fermionic condensed matter there are several universality classes of ground states, quantum vacua (Horava 2005). One of them contains

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vacua with trivial topology, whose fermionic excitations are massive (gapped) fermions. The natural mass of these fermions is of the order of $E_P$. However, the other classes contain gapless vacua. Their fermionic excitations live either near the Fermi surface (as in metals), or near the Fermi point (as in superfluid $^3$He-A) or near some other topologically stable manifold of zeroes in the energy spectrum. The gaplessness of these fermions is protected by topology, and thus is not sensitive to the details of the microscopic (trans-Planckian) physics. Irrespective of the deformation of the parameters of the microscopic theory, the value of the gap (mass) in the energy spectrum of these fermions remains strictly zero. This solves the main hierarchy problem: for these classes of fermionic vacua, the masses of elementary particles are naturally small.

In the emergent physics, the vacuum energy that is relevant for gravity is naturally small. This can be checked on the example of self-sustained vacua, i.e. vacua that can be in equilibrium in the absence of any surroundings (Klinkhamer & Volovik 2008). The energy density of such vacua is strictly zero if the vacuum is perfect and is isolated from any surroundings. This solves the main cosmological constant problem: $\Lambda$ is naturally small.

2. Fermionic and bosonic contents in vacua with Fermi points

For our Universe, which obeys Lorentz invariance, only those vacua are important that are either Lorentz invariant, or acquire Lorentz invariance as an effective symmetry emerging at low energy. This excludes vacua with a Fermi surface and leaves the class of vacua with a Fermi point of chiral type, in which fermionic excitations behave as left- or right-handed Weyl fermions (Froggatt & Nielsen 1991; Volovik 2003), and the class of vacua with a nodal point obeying $Z_2$ topology, where fermionic excitations behave as massless Majorana neutrinos (Horava 2005; Volovik 2007). The general properties of quantum vacua obeying Lorentz invariance are discussed by Klinkhamer & Volovik (2008).

(a) Emergent fermionic matter

The advantage of vacua with Fermi points is that practically all the main physical laws (except for quantum mechanics) can be considered as effective laws, which naturally emerge at low energy. This is the consequence of the so-called Atiyah–Bott–Shapiro construction (Horava 2005), which leads to the following general form of expansion of the inverse fermionic propagator near the Fermi point:

$$G^{-1}(p_\mu) = e^\beta \Gamma^\alpha (p_\beta - p_\beta^{(0)}) + \text{higher order terms.} \quad (2.1)$$

Here, $\Gamma^\mu = (1, \sigma_x, \sigma_y, \sigma_z)$ are Pauli matrices (or Dirac matrices in the more general case); the expansion parameters are the vector $p_\beta^{(0)}$ indicating the position of the Fermi point in momentum space where the Green’s function has a singularity, and the matrix $e^\beta$. This expansion is written for the simplest case of an isolated Fermi point with elementary topological charges, i.e. with either $N = +1$ or $N = -1$. Equation (2.1) can be transformed to the form

$$G^{-1}(p_\mu) = ip_0 + N\sigma \cdot p + \text{higher order terms,} \quad (2.2)$$
where the position of the Fermi point is shifted to \( p_{\beta}^{(0)} = 0 \); the matrix \( e_{\alpha}^{\beta} \) is transformed to the unit matrix; and \( p_{\mu} = (i p_0, \mathbf{p}) \). This form demonstrates that, close to the Fermi point with \( N = +1 \), the low-energy fermions behave as right-handed relativistic particles, while the Fermi point with \( N = -1 \) gives rise to left-handed particles. This scenario agrees with the fermionic content of our Universe, where all the elementary particles, left- and right-handed quarks and leptons, are Weyl fermions.

In principle, the infrared divergences may violate the simple pole structure of the propagator in equation (2.2), and one will have

\[
G(p_{\mu}) \propto \frac{-i p_0 + N \mathbf{\sigma} \cdot \mathbf{p}}{(p^2 + p_0^2)^{\gamma}},
\]

with \( \gamma \neq 1 \). This modification does not change the topology of the propagator; its topological charge is \( N \) for arbitrary parameter \( \gamma \) (Volovik 2007). Fermions without a pole in the Green’s function occur in condensed matter, in particular in Luttinger liquids (Giamarchi 2004), and may also occur in relativistic quantum fields, for example, fermionic unparticles with \( \gamma = 5/2 - d_U \), where \( d_U \) is the scale dimension of the quantum field (Georgi 2007; Luo & Zhu 2008). In the Fermi-point (FP) scenario, the form of the propagator in equation (2.3) is dictated by topology in momentum space.

\((b)\) Emergent gauge fields

The FP scenario gives a particular mechanism for emergent symmetry. The Lorentz symmetry is simply the result of the linear expansion; it becomes better and better when the Fermi point is approached and the non-relativistic higher-order terms in equation (2.2) may be neglected. This expansion demonstrates the emergence of the relativistic spin, which is described by the Pauli matrices. It also demonstrates how gauge fields and gravity emerge together with chiral fermions. The expansion parameters \( p_{\beta}^{(0)} \) and \( e_{\alpha}^{\beta} \) may depend on the space and time coordinates and they actually represent collective dynamic bosonic fields in vacuum with a Fermi point. The vector field \( p_{\beta}^{(0)} \) in the expansion plays the role of the effective \( U(1) \) gauge field \( A_{\beta} \) acting on fermions. For the more complicated Fermi points, the shift \( p_{\beta}^{(0)} \) becomes the matrix field; it gives rise to effective non-Abelian (Yang–Mills) gauge fields emerging in the vicinity of the Fermi point, i.e. at low energy. For example, the Fermi point with \( N = 2 \) may give rise to the SU(2) gauge field \( p_{\beta}^{(0)} = A_{\beta}^{a} \tau_{a} \), where \( \tau_{a} \) are Pauli matrices corresponding to the emergent isotopic spin.

\((c)\) Emergent gravity

The matrix field \( e_{\alpha}^{\beta} \) acts on the quasi-particles as the vierbein field, and thus describes the emergent dynamical gravity field. As a result, close to the Fermi point, matter fields (all ingredients of the standard model: chiral fermions and Abelian and non-Abelian gauge fields) emerge together with geometry, relativistic spin, Dirac matrices and physical laws: Lorentz and gauge invariance, equivalence principle, etc. In this scheme, gravity emerges together with matter (figure 1). This means that the so-called ‘quantum gravity’ should be the unified
theory of the underlying quantum vacuum, where the gravitational degrees of freedom (d.f.) cannot be separated from all other microscopic d.f., which give rise to the matter fields (fermions and gauge fields).

Classical gravity would be a natural macroscopic phenomenon emerging in the low-energy corner of the microscopic quantum vacuum, i.e. it is a typical and actually inevitable consequence of the top (high energy) to the bottom (low energy) coarse-graining procedure. The inverse bottom to top procedure, i.e. from the classical to quantum gravity, is highly restricted. The first steps in the quantization are certainly allowed; it is possible, for example, to quantize gravitational waves to obtain their quanta, gravitons, since in the low-energy corner the results of the microscopic and effective theories coincide. It is also possible to obtain some (but not all) quantum corrections to the Einstein equation; to extend classical gravity to the semi-classical and stochastic (Hu & Roura 2007) levels, etc. But one cannot obtain quantum gravity by full quantization of the Einstein equations, since all other d.f. of the quantum vacuum will be missed.

\[(d)\] Dimension of space and flatness of Universe

In the FP scenario, space–time is naturally four dimensional. This is the property of the FP topology, which, as distinct from string theory, does not require higher dimensional space–times.
In the FP scenario, the Universe is naturally flat. In this scenario, the effective metric emerges as the matrix in the expansion of the Green’s function near the Fermi point. In the homogeneous vacuum, this matrix is constant in space, and thus the homogeneous Universe is flat. In fundamental general relativity, the isotropic and homogeneous Universe is not necessarily flat: it corresponds to the three-dimensional space with a constant curvature. The flatness of the Universe requires either fine tuning or inflation, which irons out the curved space. The observed flatness of our Universe is in favour of emergent gravity.

The effective gravity emerging at low energy may differ essentially from the fundamental gravity even in principle. Since in the effective gravity the general covariance is lost at high energy, the metrics that for low-energy observers look equivalent, since they can be transformed to each other by coordinate transformation, are not equivalent physically. As a result, some metrics, which are natural in general relativity, are simply forbidden in emergent gravity. For example, emergent gravity is not able to incorporate the geodesically complete Einstein universe with spatial section $S^3$ (Klinkhamer & Volovik 2005c). It, therefore, appears that the original static $S^3$ Einstein universe can exist only within the context of fundamental general relativity.

In addition, some coordinate transformations in general relativity are not allowed in emergent gravity; these are either singular transformations of the original coordinates or the transformations that remove some parts of space–time (or add extra parts). The non-equivalence of different metrics is especially important in the presence of the event horizon. For example, in the emergent gravity, the Painlevé–Gullstrand metric is more appropriate for the description of a black hole than the Schwarzschild metric, which is singular at the horizon.

3. Vacuum energy and cosmological constant

There is a huge contribution to the vacuum energy density of order $E_p^4 \approx (10^{28} \text{ eV})^4$, which comes from the ultraviolet d.f., whereas the observed total energy density of approximately $(10^{-3} \text{ eV})^3$ is smaller by many orders of magnitude. In general relativity, the cosmological constant is an arbitrary constant, and thus its smallness requires fine tuning. Thus, observations are in favour of emergent gravity. If graviation were a truly fundamental interaction, it would be hard to understand why the energies stored in the quantum vacuum would not gravitate at all (Nobbenhuis 2006). If, however, graviation were only a low-energy effective interaction, it could be that the corresponding gravitons as quasi-particles do not feel all microscopic d.f. (gravitons would be analogous to small-amplitude waves at the surface of the ocean) and that the gravitating effect of the vacuum energy density would be effectively tuned away and the cosmological constant would be naturally small or zero (Dreyer 2006).

(a) Vacuum as self-sustained medium

A particular mechanism of nullification of the relevant vacuum energy works for such vacua that have the property of a self-sustained medium. A self-sustained vacuum is a medium with a definite macroscopic volume even in the absence of any surroundings. An example is a droplet of quantum liquid at zero

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temperature falling in empty space. The observed near-zero value of the cosmological constant compared with Planck-scale values suggests that the quantum vacuum of our Universe belongs to this class of systems. As any other medium of this kind, the equilibrium vacuum would be homogeneous and extensive. The homogeneity assumption is indeed supported by the observed flatness and smoothness of our Universe (de Bernardis et al. 2000; Hinshaw et al. 2007; Riess et al. 2007). The implication is that the energy of the equilibrium quantum vacuum would be proportional to the volume considered.

Usually, a self-sustained medium is characterized by an extensive conserved quantity whose total value determines the actual volume of the system (Landau & Lifshitz 1980; Perrot 1998). The Lorentz invariance of the vacuum imposes strong constraints on the possible form this variable can take. One may choose the vacuum variable to be a symmetric tensor $q^{\mu \nu}$ satisfying the following conservation law:

$$\nabla_\mu q^{\mu \nu} = 0.$$  \hfill (3.1)

In a homogeneous vacuum, one has $q^{\mu \nu} = q g^{\mu \nu}$, with $q$ constant in space and time. The quantum vacuum can now be considered as a reservoir of trans-Planckian energies stored in the $q$ field.

Let us consider a large portion of quantum vacuum under external pressure $P$ (figure 2). The volume $V$ of the quantum vacuum is variable, but its total ‘charge’ $Q(t) \equiv \int d^3 r \, q(r, t)$ must be conserved, $dQ/dt = 0$. The energy of this portion of the quantum vacuum at fixed total charge $Q = qV$ is then given by the thermodynamic potential

$$W = E + PV = \int d^3 r \, \epsilon(Q/V) + PV,$$  \hfill (3.2)
where $\epsilon(q)$ is the energy density in terms of $q$. As the volume of the system is a free parameter, the equilibrium state of the system is obtained by variation over the volume $V$, 
\[
\frac{dW}{dV} = 0. \tag{3.3}
\]
This gives an integrated form of the Gibbs–Duhem equation for the vacuum pressure
\[
P_{\text{vac}} = -\epsilon(q) + q \frac{d\epsilon(q)}{dq}, \tag{3.4}
\]
whose solution determines the equilibrium value $q = q_0(P)$ and the corresponding volume $V = V_0(P, Q) = Q / q_0(P)$.

(b) Microscopic versus macroscopic vacuum energy

Since the vacuum energy density is the vacuum pressure with a minus sign, equation (3.4) suggests that the relevant thermodynamic potential or the vacuum energy, which is experienced by the low-energy d.f., is
\[
\tilde{\epsilon}_{\text{vac}} = \epsilon(q) - q \frac{d\epsilon(q)}{dq}. \tag{3.5}
\]
This is confirmed by the example of the dynamic $q$ field, which demonstrates that the energy–momentum tensor of the vacuum is (Klinkhamer & Volovik 2008)
\[
T_{\mu\nu} = g_{\mu\nu} \tilde{\epsilon}_{\text{vac}}. \tag{3.6}
\]
It is thus $\tilde{\epsilon}(q)$ rather than $\epsilon(q)$ that enters the equation of state for the vacuum and thus corresponds to the cosmological constant
\[
\Lambda = \tilde{\epsilon}_{\text{vac}} = -P_{\text{vac}}. \tag{3.7}
\]

(c) Natural value of cosmological constant

While the energy of microscopic quantity $q$ is determined by the Planck scale, $\epsilon(q_0) \sim E_P^4$, the real vacuum energy that sources the effective gravity is determined by a macroscopic quantity, the external pressure. In the absence of any surroundings, i.e. at zero external pressure, one obtains that the pressure of pure and equilibrium vacuum is exactly zero,
\[
\Lambda = -P_{\text{vac}} = -P = 0. \tag{3.8}
\]
Thus, from the thermodynamic arguments, it follows that for any effective theory of gravity the natural value of $\Lambda$ is zero. This result does not depend on the microscopic structure of the vacuum from which gravity emerges, and is actually the final result of the renormalization dictated by macroscopic physics. In the self-sustained vacuum, the huge contribution of zero-point energy of macroscopic fields to the vacuum energy $\tilde{\epsilon}_{\text{vac}}$ is compensated by the microscopic d.f. of the vacuum (Volovik 2008; figure 3). If the cosmological phase transition takes place, the vacuum is readjusted to a new equilibrium and $\Lambda$ approaches zero again (Klinkhamer & Volovik 2008).
Compressibility of the vacuum

Using the standard definition of the inverse of the isothermal compressibility,

\[ \chi_{K}^{-1} \equiv -V \frac{dP}{dV} \] (figure 2), one obtains the compressibility of the vacuum by varying equation (3.4) at fixed \( Q = qV \),

\[ \chi_{\text{vac}}^{-1} \equiv -V \frac{dP_{\text{vac}}}{dV} = \left[ q^2 \frac{d^2 \epsilon(q)}{dq^2} \right]_{q = q_0} > 0. \] (3.9)

A positive value of vacuum compressibility is a necessary condition for stability of the vacuum. In fact, the stability of the vacuum is at the origin of the nullification of the cosmological constant in the absence of any external surroundings.

From the low-energy point of view, the vacuum compressibility \( \chi_{\text{vac}} \) is as fundamental a physical constant as Newton’s constant \( G \), although \( \chi_{\text{vac}} \) has not yet been observed. While the natural value of the macroscopic quantity \( P_{\text{vac}} \) (and \( \Lambda \)) is zero, the natural values of the parameters \( G \) and \( \chi_{\text{vac}} \) are determined by Planckian physics and are expected to be of order \( 1/E_{\text{P}}^2 \) and \( 1/E_{\text{P}}^4 \), respectively (table 1).

(e) Thermal fluctuations of \( \Lambda \) and the volume of the Universe

The vacuum compressibility \( \chi_{\text{vac}} \), though not measurable at the moment, can be used for estimation of the lower limit for the volume \( V \) of the Universe. This estimation follows from the upper limit for thermal fluctuations of the cosmological constant (Volovik 2004). The mean square of thermal fluctuations of \( \Lambda \) equals that of thermal fluctuations of the vacuum pressure, which in turn is determined by the thermodynamic equation (Landau & Lifshitz 1980)

\[ \langle (\Delta \Lambda)^2 \rangle = \langle (\Delta P)^2 \rangle = \frac{T}{V \chi_{\text{vac}}}. \] (3.10)
Table 1. Natural values of physical quantities. (Natural values of physical quantities following from the FP scenario of emergent physics versus naive estimates and observation. Here $E_{\text{UV}} \gg E_P \gg E_{\text{IR}} \gg R_H^{-1}$ are, respectively, the ultraviolet (Lorentz violating?) energy scale, the Planck energy scale, the infrared cut-off and the inverse Hubble radius.)

| physical quantity                  | naive estimate | natural FP value | observed value |
|-----------------------------------|----------------|------------------|----------------|
| mass of elementary particle       | $E_P$          | $E_P$            | $\ll E_P$      |
| mass of gauge boson               | $E_P$          | $E_P^2/E_{\text{UV}}$ | $\ll E_P$      |
| running coupling constants        | $E_P^{-2}$     | $E_P^{-2}$       | $\ll E_P^2$    |
| Newton’s constant                 | $E_P^{-1}$     | $E_P^{-1}$       | $\ll E_P^2$    |
| temperature of Universe           | $E_P$          | $0$              | $\ll E_P$      |
| cosmological constant             | $E_P^3$ or $R_H^3$ | $\gg R_H^3$ | $\gg R_H^3$ |
| volume of Universe                | $E_P^{-1}$ or $R_H^{-2}$ | $0$              | $\ll R_H^{-2}$ |
| curvature of Universe             | $E_P^{-1}$ or $R_H^{-2}$ | $0$              | $\ll R_H^{-2}$ |
| vacuum compressibility            | $E_P^{-4}$     | $E_P^{-4}$       | —              |

Typical fluctuations of the cosmological constant $\Lambda$ should not exceed the observed value, $<(\Delta \Lambda)^2 < A^2_{\text{obs}}$. Let us assume, for example, that the temperature of the Universe is determined by the temperature $T_{\text{CMB}}$ of the cosmic microwave background radiation. Then, using our estimate for vacuum compressibility $\chi_{\text{vac}}^{-1} \sim E_P^{-1}$, one obtains that the volume $V$ of our Universe greatly exceeds the Hubble volume $V_H = R_H^3$, the volume of visible Universe inside the present cosmological horizon,

$$V > \frac{T_{\text{CMB}}}{\chi_{\text{vac}}A_{\text{obs}}^2} \sim 10^{28} V_H.$$  \hspace{1cm} (3.11)

This demonstrates that the real volume of the Universe is certainly not limited by the present cosmological horizon.

4. Energy scales and physical laws

(a) Hierarchy of energy scales

In the emergent physics, the energy scale in our Universe is not limited by the characteristic Planck scale (figure 4). To obtain the observed high precision of physical laws, the Lorentz symmetry must persist well above the Planck energy. This requirement represents the most crucial test for the emergent scenario. In the case when the Lorentz-violating scale $E_{\text{Lorentz}} < E_P$, the metric field does not obey the Einstein equations; instead, it is governed by the hydrodynamic type equations (figure 5). The Einstein equations emerge in the limit $E_{\text{Lorentz}} \gg E_P$. This can be seen in the example of the Frolov–Fursaev version of Sakharov-induced gravity (Frolov & Fursaev 1998), where the ultraviolet cut-off is much larger than the Planck energy, and the Einstein equations are reproduced. The accuracy of the Einstein equations is determined by the small parameter $E_P^2/E_{\text{Lorentz}}^2 \ll 1$. The same parameter would enter the mass of the emergent gauge bosons, $M \sim E_P^2/E_{\text{Lorentz}}$ (Klinkhamer & Volovik 2005a; table 1). Experimental bounds on the violation of Lorentz symmetry can be obtained from ultra-high-energy cosmic
rays. For example, according to conservative estimates, the relative value of the Lorentz-violating terms in the Maxwell equation is smaller than $10^{-18}$ (Klinkhamer & Risse 2008). This suggests that $E_{\text{Lorentz}} \ll 10^{16} K \sim 10^{19} E_P$. All this implies that physics continues far beyond the Planck scale, and this opens new possibilities for the construction of microscopic theories. Since in the FP scenario bosons are composite objects, the ultraviolet cut-off may be different for fermions and bosons (Klinkhamer & Volovik 2005a). The smaller (composite) scale can be associated with $E_P$, while the ‘atomic’ structure of the quantum vacuum will be revealed only at the much higher Lorentz-violating scale $E_{\text{Lorentz}}$.

A first step towards the elusive theory of quantum gravity would be to identify the microscopic constituents (‘atoms’) of space. At the moment, we are not able to do this, but we can estimate the number of the underlying atoms of the ether, whatever they are. This is the volume of our Universe within the cosmological

**Figure 4.** The characteristic high-energy scale in the vacuum of the ‘natural Universe’ is the Planck energy $E_P$. Compared with that energy, high-energy physics and cosmology operate at extremely ultra-low temperatures.

**Figure 5.** Equations for the metric field $g_{\mu\nu}$ emerging near the Fermi point depend on the hierarchy of ultraviolet cut-offs: Planck energy scale $E_P$ versus Lorentz-violating scale $E_{\text{Lorentz}}$. The classical Einstein equations for $g_{\mu\nu}$ emerge only in the limit when the Lorentz invariance is fundamental at the Planck scale, i.e., when $E_{\text{Lorentz}} \ll E_P$. For example, according to conservative estimates, the relative value of the Lorentz-violating terms in the Maxwell equation is smaller than $10^{-18}$ (Klinkhamer & Risse 2008). This suggests that $E_{\text{Lorentz}} > 10^9 E_P$.

All this implies that physics continues far beyond the Planck scale, and this opens new possibilities for the construction of microscopic theories. Since in the FP scenario bosons are composite objects, the ultraviolet cut-off may be different for fermions and bosons (Klinkhamer & Volovik 2005a). The smaller (composite) scale can be associated with $E_P$, while the ‘atomic’ structure of the quantum vacuum will be revealed only at the much higher Lorentz-violating scale $E_{\text{Lorentz}}$.

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horizon divided by the elementary Planck volume: $N \sim R_H^3/l_{\text{Planck}}^3 \sim 10^{180}$. At least an extra 30 orders of magnitude must be added if the real volume of the Universe in equation (3.11) is considered: $N \sim V/l_{\text{Planck}}^3 \sim 10^{210}$. Finally, if we relate the minimum length to the Lorentz-violating scale $l_{\text{Lorentz}}$ and take $E_{\text{Lorentz}} \sim 10^{20} E_{\text{Planck}}$, we obtain $N \sim V/l_{\text{Lorentz}}^3 \sim 10^{270}$ constituents.

The large number $N$ of the constituents of a system means that the system is macroscopic and must obey the laws of emergent macroscopic physics. The most general physical laws that do not depend on the details of the underlying microscopic system are the laws of thermodynamics. The huge number $N$ of constituents could lead to the hierarchical structure of physics, with different physical laws emerging at different levels of the hierarchical structure with highly separated length scales,

$$l_{\text{Lorentz}} \ll l_P \ll l_{\text{ew}} \ll l_{\text{QCD}} \ll \cdots \ll R_H \ll V^{1/3}. \quad (4.1)$$

The accuracy of the physical laws at a given scale $l_n$ is determined by the parameter $l_{n-1}/l_n \ll 1$. If this parameter is not very small, the physical laws at scale $l_n$ are rather crude and contain a lot of phenomenological parameters coming from the smaller length scales.

Simple and accurate physical laws may emerge only for a large number of constituents and/or for a large ratio between adjacent scales (Bjorken 2001).

(b) Natural values of physical quantities

Both in particle physics and in condensed matter, the natural value of a quantity depends on whether this quantity is determined by macroscopic or microscopic physics (table 1).

The first column in table 1 contains naive estimates of physical quantities. They follow from the dimensional analysis assuming that the role of the fundamental scale is played either by the Planck energy $E_P$ or by the size of the Universe $R_H$. The second column shows the natural values of these quantities, which follow from the FP scenario. In most cases, the naive estimate contradicts both the values dictated by the FP scenario and the observations in the third column.

The naive estimates are consistent with natural values for those quantities that are determined by microscopic physics and are expressed in terms of the corresponding microscopic scale, which is the Planck scale $E_P$ in our Universe or the atomic scale in condensed matter systems. An example is Newton’s constant $G = a_G E_P^{-2}$. For emergent gravity, the dimensionless prefactor $a_G$ depends on the vacuum content and is of the order of unity in units $\hbar = c = 1$. In principle, the parameter $a_G$ can be zero, but this requires fine tuning between different scalar, vector and spinor fields in vacuum. That is why the natural value of $G$ is $E_P^{-2}$. The compressibility of the vacuum is also determined by microphysics. The running coupling constants $\alpha_n$ also fall into this category, since they depend on the ultraviolet cut-off together with the infrared cut-off $E_{\text{IR}}$: $\alpha_n^{-1} \sim \ln(E_P/E_{\text{IR}})$.

Temperature, pressure and the volume of the Universe belong to the category determined by macroscopic physics, thermodynamics. These thermodynamic quantities do not depend on the microphysics or the momentum-space topology; they only depend on the surroundings. In the absence of forces from the surroundings, the pressure and the temperature of any system relax to zero. The same should hold for the temperature of the Universe and for the vacuum

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pressure. The vacuum pressure is the cosmological constant with a minus sign, $\Lambda = \epsilon_{\text{vac}} = -p_{\text{vac}}$. Whatever the vacuum content, and independently of the history of the phase transitions in the quantum vacuum, the cosmological constant must relax to zero or to a small value that compensates the other partial contributions to the total pressure of the system, since it is the total pressure of the system that must be zero in equilibrium in the absence of any surroundings.

\( (c) \) Mass is naturally zero

The masses of elementary fermions are quantities that most crucially depend on the momentum-space topology. The naive estimation tells us that these masses should be of the order of the Planck energy scale: $M_{\text{theor}} \sim E_P \sim 10^{19}$ GeV. This highly contradicts observations: the observed masses of known particles are many orders of magnitude smaller, being below the electroweak energy scale $M_{\text{obs}} < E_{\text{ew}} \sim 1$ TeV. This represents the main hierarchy problem. In the ‘natural’ Universe, where all masses are of the order of $E_P$, all fermionic d.f. are completely frozen out owing to the Boltzmann factor $e^{-M/T}$, which is about $e^{-E_P/E_{\text{ew}}} \approx e^{-10^{16}}$ already at a temperature corresponding to the highest energy reached in accelerators (figure 4). There is no fermionic matter in such a Universe.

That we survive in our Universe is not the result of the anthropic principle (the latter chooses universes that are fine-tuned for life but have an extremely low probability). On the contrary, this simply indicates that our Universe is also natural, and its vacuum is generic, though it belongs to a different universality class of vacua, the vacua with Fermi points. In such vacua, the masslessness of fermions is protected by topology (combined with symmetry; §5).

As for masses of gauge bosons, they may appear either due to symmetry breaking occurring at low energy or due to the higher-order corrections to the effective action emergent close to the Fermi point. In the latter case, the mass is determined by the hierarchy of scales $E_P$ and $E_{\text{UV}}$, say by $E_P^2 / E_{\text{UV}}$, where the higher energy scale may correspond to the Lorentz-violating scale $E_{\text{Lorentz}}$ (Klinkhamer & Volovik 2005a).

5. Symmetry versus topology

\( (a) \) Discrete symmetries in the underlying physics

The standard model above the electroweak transition contains 16 chiral fermions in each generation; eight right-handed fermions with topological charge $N = +1$ each and eight left-handed fermions with $N = -1$ each. The vacuum of the standard model above the electroweak transition is marginal; there is a multiply degenerate Fermi point at $p = 0$ with the total topological charge $N = +8 - 8 = 0$. The absence of topological stability means that even a small mixing between fermions may lead to annihilation of the marginal Fermi point. In the standard model, mixing leading to the fully gapped vacuum is prohibited by discrete symmetries; electroweak $Z_2$ symmetry (subgroup of $2\pi$ rotations of SU(2) group; see Volovik 2003) and CPT symmetry.

This means that the underlying physics must contain discrete symmetries (figure 6). Their role is extremely important. The main role is to prohibit the cancellation of the Fermi points with opposite topological charges (figure 7). As a
side effect, in the low-energy corner, discrete symmetries are transformed into gauge symmetries and give rise to an effective non-Abelian gauge field. In particular, the $Z_2$ symmetry produces the $SU(2)$ gauge field (Volovik 2003). Discrete symmetries also reduce the number of massless gauge bosons and the number of metric fields. To justify the FP scenario, one should find such discrete symmetry that leads in the low-energy corner to one of the grand unification (GUT) or Pati–Salam models.

(b) Discrete symmetries and splitting of Fermi points

Explicit violation or spontaneous breaking of one of the two discrete symmetries transforms the marginal vacuum of the standard model into one of two topologically stable vacua. If, for example, the electroweak $Z_2$ symmetry is broken, the marginal Fermi point disappears and the fermions become massive (figure 7, bottom left). This is assumed to happen below the symmetry-breaking electroweak transition caused by the Higgs mechanism, where quarks and charged leptons acquire the Dirac masses. If, on the other hand, the CPT symmetry is violated, the marginal Fermi point splits into topologically stable Fermi points that protect massless chiral fermions (figure 7, bottom right). One can speculate that in the standard model the latter happens with the electrically neutral leptons, the neutrinos (Klinkhamer & Volovik 2005b). Most interestingly, FP splitting of neutrinos may provide a new source of T and CP violation.
in the leptonic sector, which may be relevant for the creation of the observed cosmic matter–antimatter asymmetry (Klinkhamer 2006). Examples of discrete symmetry and splitting of Fermi and Majorana points in condensed matter are discussed in the review paper by Volovik (2007).

6. Conclusion

There are two complementary schemes for the classification of quantum vacua, both based on quantum mechanics, which is assumed to be a fundamental theory (figure 8). The traditional classification, the GUT scheme, assumes that fermionic and bosonic fields and gravity are also fundamental phenomena. They obey the fundamental symmetry, which becomes spontaneously broken at low energy, and is restored when the Planck energy scale is approached from below.

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The FP scenario provides a complementary anti-GUT scheme in which the 'fundamental' symmetry and fundamental fields of GUT gradually emerge together with fundamental physical laws when the Planck energy scale is approached from above. The emergence of the fundamental laws of physics is provided by the general property of topology—robustness to details of the microscopic trans-Planckian physics. In these schemes, fermions are primary objects. Approaching the Planck energy scale from above, they are transformed to the standard model chiral fermions and give rise to the secondary objects, gauge fields and gravity. Below the Planck scale, the GUT scenario intervenes, giving rise to symmetry breaking at low energy. This is accompanied by the formation of composite objects, Higgs bosons and tiny Dirac masses of quarks and leptons.

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In the GUT scheme, general relativity is assumed to be as fundamental as quantum mechanics, while in the second scheme general relativity is a secondary phenomenon. In the anti-GUT scheme, general relativity is the effective theory describing the dynamics of the effective metric experienced by the effective low-energy fields. It is a side product of quantum field theory or of quantum

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mechanics in vacuum with a Fermi point. As a result, in the FP scenario, there are no principle contradictions between quantum mechanics and gravity. That is why the emergent gravity cannot be responsible for the issues related to the foundations of quantum mechanics, and in particular for the collapse of the wave function. Also, the hierarchy of scales implies that, if quantum mechanics is not fundamental, the scale at which it emerges should be far above the Planck scale.

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