The cosmological constant problem: a user’s guide*

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Abstract

We discuss the validity of general relativity at low-energy and relate the threshold below which the theory breaks down with the observed value of the cosmological constant. This suggests the existence of a mass scale of $\mathcal{O}(10^{-3})$ $eV$ and a putative violation of the equivalence principle at about $10^{-14}$ level.

Dedicated to the memory of Maria da Conceição Bento

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1 Introduction

Einstein’s theory of general relativity as established by its field equations is consistent with all experimental evidence to considerable accuracy (see e.g. Refs. [1, 2] for reviews). However, quite fundamental problems lead one to conclude that general relativity cannot be a complete description of gravity. These difficulties are associated with the existence of spacetime singularities, the cosmological constant problem and the incompatibility of the theory with quantum mechanics. Indeed, all attempts to quantize gravity with the techniques of quantum gauge field theories that successfully describe electromagnetic, weak, and strong interactions, were shown to be unfeasible, as beyond one-loop (without matter), the quantum theory emerging from general relativity is not renormalizable. Furthermore, none of the known approaches to quantize gravity do solve these difficulties. Indeed, for instance, in the context of superstring/M-theory, the most studied programme to quantize gravity, singular solutions such as black holes are encountered, and no satisfactory answer to the cosmological constant problem has ever been presented [3].

Furthermore, the curvature-matter connection of Einstein’s theory is not the most general one and, having in mind the Standard Model (SM) of the fundamental interactions and the cosmological standard model, which requires an early period of accelerated expansion, i.e. inflation, it is fairly natural to consider additional fields, most particularly scalar fields. In this respect, scalar-tensor theories of gravity are an interesting possibility given that they capture the main features of many unification models. For instance, the graviton-dilaton system that arises from string/M-theory can be seen as a particular example of a scalar-tensor theory of gravity.

However, likewise general relativity, all proposed alternative gravity theories do not provide a consistent description of our universe given the huge discrepancy between the observed value of the cosmological constant and the one arising from the SM. Several proposals have been advanced to solve this difficulty (see e.g. Refs. [4, 5, 6]). A vanishing cosmological constant suggests a symmetry principle in action. Indeed, it has been suggested that the problem admits a solution likewise the strong CP problem [7], which actually can be somehow implemented in the context of a S-modular invariant $N = 1$ supergravity quantum cosmological model in a closed homogeneous and isotropic spacetime [8]. A connection between the cosmological constant and Lorentz invariance has also been discussed in different contexts [9, 10] as well as an invariance under complex transformations [11, 12]. Finally, an evolving cosmological term has also been considered by some authors [13, 14, 15]. Unfortunately, none of the proposed mechanisms are fully consistent [4, 5, 6].

As already mentioned, within the framework of superstring/M-theory, no fully satisfactory solution for the cosmological constant problem has ever been advanced [3], even though more recently, it has been argued that a solution arises through a suitable choice of the vacuum in the “landscape” of vacua of the theory in its multiverse interpretation (see e.g. [16] and references therein). In this context, each vacuum configuration in the multitude of about $10^{500}$ vacua of the theory [17] is regarded as a distinct universe, a “bubble” in an $a\; pri\; ri\; o$ spacetime. The evolution of each universe follows classical and quantum physics with physical parameters and coupling constants corresponding to each vacuum state. The choice of a suitable vacuum
for our universe requires some selection criteria. Anthropic arguments \cite{18, 19} and quantum cosmological considerations \cite{20} have been proposed as possible vacuum selection procedures.

Another interesting possibility is that these “bubble” universes do interact with each other as it usually occurs in condensate matter systems with multiple vacua. An interaction between universes, based on a Curvature Principle, has been suggested and shown, in the context of a simplified model, that it can drive the cosmological constant of one of the universes toward a vanishingly small value \cite{21}. It has also been conjectured on how this Curvature Principle suggests a solution for the entropy evolution of the universe so to satisfy the generalized Second principle of Thermodynamics \cite{21}.

In this work one considers the cosmological constant problem from a quite conservative standing point, i.e. from the point of view of general relativity itself and of the SM, regardless of any further unification of the fundamental interactions. It is argued that although the cosmological constant problem is an ultraviolet problem, possibly involving non-local aspects, and its solution cannot be found in the context of general relativity, it is nevertheless, interesting to examine what can be learnt by assessing the validity of the theory at low energies. These considerations lead one to conclude that although the fine tuning related with ultraviolet behaviour of the theory involves a still eluding mechanism, the cosmological constant problem has also a bearing at low energy, and, in particular, it indicates the existence a mass scale of about $O(10^{-3})$ eV and a putative violation of the equivalence principle at $10^{-14}$ level so to account for the residual non-vanishing value of the vacuum energy.

2 Relativistic invariance, four-dimensional cutoff and a fundamental scalar field

In what follows one considers the vacuum energy contribution arising from a field theory calculation of a new fundamental scalar field with mass $m$. But before one addresses this issue it is relevant to point out that the vacuum energy is an extensive quantity and hence it is an additive part of the energy of a physical system. This energy is irrelevant in non-gravitational physics where only energy differences matter; however, the situation is fundamentally different in general relativity given that in its framework any non-vanishing configuration of energy/matter is a source for gravity and curves spacetime. Actually, strictly speaking, the vacuum energy shows up in both sides of Einstein’s field equations. Indeed, it reveals itself as a constant, $\Lambda_E$, at the geometrical side of the field equations and as a source term from the vacuum contribution of the relevant fields:

$$G_{ab} + \Lambda_E g_{ab} = 8\pi G (T_{ab} + \langle T_{ab} \rangle),$$  \hspace{1cm} (1)

where $G_{ab}$ is the Einstein tensor and $\langle T_{ab} \rangle$ is the contribution to vacuum of the field(s) after, for instance, the matter theory undergoes some spontaneous symmetry breaking. The cosmological constant problem consists in the highly unnatural fine tuning between $\Lambda_E$ and $8\pi G < T_{ab} >$ so to fit the cosmological data. Furthermore, given Lorentz invariance, then $\langle T_{ab} \rangle = \rho_V g_{ab}$, where $\rho_V$ is the vacuum energy density. The vacuum pressure corresponds to the space components $\langle T_{ii} \rangle$, from which follows that energy density and pressure must
satisfy the relationship:
\[ \rho_V = -p_V . \]  
(2)
Assuming the observed cosmological value for the vacuum energy, namely, \( \Omega_\Lambda = 0.7 \), or \( \rho_V = 5.65 \times 10^{-47} \text{ GeV}^4 \) for \( h_0 = 0.7 \)\(^1\), comparison with the contribution from the SM after the Higgs field acquires a non-vanishing vacuum expectation value, \( \rho^{SM}_V = \mathcal{O}(250 \text{ GeV})^4 \), yields a discrepancy of \( 10^{56} \) orders of magnitude or an adjustment of 56 decimal places.

In what concerns the field theory computation of the energy density in the context of a real scalar field theory, it has been recently pointed out that only through a fully covariant 4-dimensional procedure one can respect the vacuum equation of state, Eq. (2) \[22\]. Indeed, the zero-point energy of a free real scalar field \( \phi \) of mass \( m \) can be estimated introducing into the energy-momentum tensor \( T_{ab} \) of the field a plane-wave decomposition and taking the vacuum expectation value of the component \( T_{00} \). It then follows the well-known result for the contribution of the field \( \phi \) to the energy density of the vacuum:
\[ \rho_V = \frac{1}{2} \int_{0}^{\Lambda} \frac{d^3k}{(2\pi)^3} \sqrt{k^2 + m^2} , \]  
(3)
for a high-energy cutoff \( \Lambda \). Notice that this is just the sum over all modes of the zero-point energies \( \omega_k / 2 = \sqrt{k^2 + m^2} / 2 \) and that integral in Eq. (3) is clearly divergent as \( \Lambda \to \infty \).

The contribution to the vacuum pressure can be obtained through the same procedure:
\[ p_V = \frac{1}{6} \int_{0}^{\Lambda} \frac{d^3k}{(2\pi)^3} \frac{k^2}{\sqrt{k^2 + m^2}} , \]  
(4)
where the isotropy of the vacuum has been used. The above computations were carried out in Minkowski space, but it can be argued that the spacetime curvature does not affect results Eqs. (3) and (4) \[22\].

In order to implement a relativistically invariant 4-momentum cutoff, one must use manifestly covariant expressions for \( \rho_V \) and \( p_V \). Thus, one must turn the integrals Eqs. (3) and (4) over 3-momentum into 4-momentum integrals. This is not a well defined procedure given that the integrals are divergent and therefore, one must regularize them so not to modify the integrands and not to spoil their covariance properties. A suitable procedure involves differentiating the integrals with respect to \( m^2 \). From Eq. (3) one obtains
\[ \frac{\partial \rho_V}{\partial m^2} = \frac{1}{2} \int_{0}^{\Lambda} \frac{d^3k}{(2\pi)^3} \frac{1}{\omega_k} . \]  
(5)
The r.h.s. integral is Lorentz invariant and can be written in a manifestly invariant form as \[22\]:
\[ \frac{\partial \rho_V}{\partial m^2} = \frac{i}{2} \int_{0}^{\Lambda} \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m^2 + i\varepsilon} . \]  
(6)
\(^1\)One should remark that although cosmological data do not exclude other scenarios, supernovae data, baryon acoustic oscillations, microwave background radiation shift parameter and topological considerations are all consistent with the possibility that the late accelerated expansion of the universe is driven by the cosmological constant (see for instance \[23\] \[24\] and references therein).
Integrating Eq. (6) one gets, up to an $m$-independent constant,

$$\rho_V = \frac{1}{2} \int_0^\Lambda \frac{d^4k_E}{(2\pi)^4} \ln \left(1 + \frac{m^2}{k_E^2}\right),$$

where the integral was Wick-rotated to Euclidean space, so that a 4-momentum cutoff can be introduced. With the now covariant cutoff $\Lambda$ one finds from Eq. (7) 

$$\rho_V = \frac{1}{64\pi^2} \left[ \Lambda^4 \ln \left(\frac{\Lambda^2 + m^2}{\Lambda^2}\right) + \Lambda^2 m^2 - m^4 \ln \left(\frac{\Lambda^2 + m^2}{m^2}\right) \right].$$

This result accounts for the vacuum contribution $< T_{ab} >$ in the r.h.s. of Einstein’s equations. Restricting oneself to the SM, hence the cutoff, $\Lambda = \mathcal{O}(250 \text{ GeV})$.

In what concerns the geometrical side of Einstein’s equations in order to ensure the proper cancelation of the huge vacuum contribution arising from the SM one should consider, up to the $8\pi G$ factor and the still unknown cancelation mechanism, the estimate Eq. (7). However, since it may happen that Einstein’s equations break down at low energies, one considers instead the integration from a low energy cutoff $\Delta$ to $\Lambda$:

$$\Lambda_E = 4\pi G \int_\Delta^\Lambda \frac{d^4k_E}{(2\pi)^4} \ln \left(1 + \frac{m^2}{k_E^2}\right),$$

and hence the resulting cosmological term in Einstein’s equation is the “regularized” one:

$$\Lambda_{\text{Reg.}} = -\frac{G}{8\pi} \left[ \Delta^4 \ln \left(\frac{\Delta^2 + m^2}{\Delta^2}\right) + \Delta^2 m^2 - m^4 \ln \left(\frac{\Delta^2 + m^2}{m^2}\right) \right].$$

Notice that the negative sign reflects the fact that this contribution is in the geometrical side of Einstein’s field equation, but it clearly corresponds to a residual positive vacuum energy density.

In what follows one assumes that the scalar field mass is of the order of the low-energy cutoff and therefore

$$\Lambda_{\text{Reg.}} = -\frac{G}{8\pi} \Delta^4.$$

The scale $\Delta$ can be estimated by comparison with the observed value of the vacuum energy density. It then follows that $\Delta = \mathcal{O}(10^{-3}) \text{ eV}$ and hence

$$\frac{\rho_{\text{Reg.}}}{\rho_{\text{SM}}} \simeq 10^{-56} \equiv \delta^4.$$

Thus, the scale $\delta = 10^{-14}$ corresponds, relative to the SM typical energy scale, the scale below which general relativity breaks down. The most likely broken low energy symmetry to consider is the equivalence principle which states that any energy couples to gravity in the same fashion. This is fairly logical as a violation of the equivalence principle in this instance would mean that the vacuum energy couples to gravity in a special way and curves spacetime somewhat differently. These considerations lead one to conclude that this coupling principle holds down to
\( \mathcal{O}(10^{-3}) \) eV. It is relevant to point out that the current bound on the validity of the equivalence principle is \( 1.4 \times 10^{-13} \) \( \text{[25]} \).

Notice that this conjectured violation of the equivalence principle is, at least at first glance, unrelated with the recently reported one encountered at cluster and cosmological scales, which are due to dark matter-dark matter interaction \( \text{[26]} \) or to the interaction of dark matter to dark energy \( \text{[27, 28]} \). However, given that dark energy might be the vacuum energy density, one can thus speculate that these pieces of evidence about the violation of the equivalence principle might have a common origin. If so, then this violation should arise at the typical “length scale” of dark energy or vacuum energy \( \text{[29]} \):

\[
L_{DE} = \left[ \frac{hc}{\rho_V} \right]^{1/4} = 8.5 \times 10^{-5} \text{ m} = 85 \mu\text{m} ,
\]

where one expects Yukawa-type deviations from Newton’s gravitation potential. No deviations have been found that are at least as important as gravity for distances of about 56 \( \mu\text{m} \) \( \text{[30]} \). For smaller distances, no conclusions can be drawn as contributions significantly greater than gravity are still consistent with the experimental results, but they cannot be disentangled from microscopical forces of electrostatic nature and the Casimir effect \( \text{[31]} \). Of course, our conclusions suggest however, that deviations should be found corresponding to violations of the equivalence principle of \( 10^{-14} \) and at separation distances no very different than \( (13) \).

It is interesting to point out that similar conclusions have been drawn by arguing that the vacuum energy has its origin in a scalar field \( \text{[32]} \) or in tensor fields through the spontaneous violation of Lorentz invariance \( \text{[9]} \).

### 3 Discussion and Conclusions

The cosmological constant problem challenges the current understanding about the vacuum of the SM and its relationship with gravity. In the context of general relativity, the cosmological constant that appears in the geometrical side of the Einstein’s field equations must be employed to cancel out the contribution arising from the contribution to the vacuum energy of the matter fields. In the SM, the Higgs field contribution yields a discrepancy of 56 orders of magnitude with the observed value of the vacuum energy on cosmological scales. This requires an adjustment between the two quantities in Einstein’s equation of 56 decimal places. Of course, further unifying schemes designed to encompass the strong interaction, and eventually gravity, do turn this tuning much tighter. It is believed that only in the context of a theory of quantum gravity this adjusting problem can be properly addressed. From this point of view, the cosmological constant problem is a touching stone as any theory that does not satisfactorily solve this riddle cannot be regarded as fundamental.

In this work it has been considered the possibility that the besides its well known ultraviolet inadequacy, general relativity may turn out to be unsuitable at low energies and that this may account for the residual value of the vacuum energy. After remarking that only through the implementation of Lorentz invariant 4-dimensional cutoff one can satisfy the invariance of the
vacuum under Lorentz transformations, one has shown, in the context of a field theory of a real scalar field with mass $m$, that if the low-energy cutoff $\Delta$ is equal to $m$, than one should expect a violation of the equivalence principle at the level of the observed vacuum energy density, that is at about $O(10^{-12}) \ eV^4$. The typical length scale associated with this energy is about $85 \ \mu m$. This violation might appear as Yukawa-type deviations from the Newtonian gravitational potential.

Searching for this violation is therefore of great relevance. However, confirmation of the proposed scenario requires the detection of the fundamental scalar field$^2$. In this respect, it is particularly interesting to conjecture that this scalar field corresponds to the quantum excitation of the cosmological quintessence field responsible for the late time accelerated expansion of the universe. This is a quite exciting possibility as it has been argued elsewhere$^{35}$ that these excitations could, under conditions, be detected at LHC or at the next generation of colliders$^{35}$.

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$^2$Notice that a long lived fundamental scalar field has also been proposed as a candidate for dark matter$^{33, 34}$. 
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