Optimization design of active suspension of vehicle based on LQR control

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Abstract. In this article, we take the 1/4 vehicle model as an example, according to the requirements of the driving performance of vehicles. We establishes the dynamic model of the vehicle, and use the linear quadratic optimal control theory to design the LQG controller of the active suspension, and use MATLAB/simulink to simulate the vehicle dynamic model. The results show that the active suspension with LQG controller has a good effect on the improvement of vehicle running stability and riding comfort.

1. Introduction
Suspension system is an important component of Vehicles, which has an important impact on the ride, stability and safety of Vehicles. The use of active suspension is the inevitable direction of suspension development. The design of the controller plays an important role in the performance of the active suspension. In this paper, the active suspension of the 1/4-wheel vehicle is taken as the research object, the dynamic model of the Vehicle is established, and the LQG controller algorithm is designed. We appliedMatlab/Simulink performs the control simulation of the Vehicle system.

2. The establishment of vehicle model based on linear two degrees of freedom
2.1. Establishment of passive suspension system
The vehicle suspension system is a multi-input majority system. For the convenience of research and better matching with the driving situation of the vehicle, the 1/4 vehicle model is the research object of this paper. The vehicle system is shown in Figure 1.
Figure 1. Passive suspension vehicle 1/4 model.

According to Figure 1, first establish the differential equation of motion, in order to establish a passive suspension vehicle 1/4 model:

\[
\begin{align*}
    m_b \ddot{x}_b &= -K_s (x_b - x_w) - C_s (\dot{x}_b - \dot{x}_w) \\
    m_w \ddot{x}_w &= -K_t (x_w - x_g) + K_s (x_b - x_w) + C_s (\dot{x}_b - \dot{x}_w)
\end{align*}
\]

Organized:

\[
\begin{align*}
    \ddot{x}_b &= \frac{-C_s}{m_b} x_b - \frac{C_s}{m_b} x_w - \frac{K_s}{m_b} x_b + \frac{K_s}{m_b} x_b \\
    \ddot{x}_w &= \frac{-C_s}{m_w} x_b - \frac{C_s}{m_w} x_w - \frac{K_s}{m_w} x_b + \frac{K_s - K_t}{m_w} x_b + \frac{K_t}{m_w} x_g
\end{align*}
\]

(1)

Where: \( C_s \) is the suspension damping and \( K_s \) is the suspension stiffness.

The selected state variables and input vectors are:

\[
X = [\dot{x}_b \quad \dot{x}_w \quad x_b \quad x_w] U = x_g
\]

Then the system motion equation and road surface excitation can be written in the state space matrix form, as shown in the following:

\[
\dot{X} = AX + BU
\]

Among them, \( A \) is the state matrix, \( B \) is the input matrix, their values are as follows:

\[
A = \begin{bmatrix}
    \frac{-C_s}{m_b} & \frac{-C_s}{m_b} & K_s & K_s \\
    \frac{C_s}{m_b} & \frac{C_s}{m_b} & K_s & -K_s - K_t \\
    m_w & m_w & m_w & m_w \\
    1 & 0 & 0 & 0
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
    0 \\
    K_t/m_w \\
    0 \\
    0
\end{bmatrix}
\]

Taking body acceleration, dynamic tire deformation and suspension dynamic travel as performance indicators, as shown in the following:

\[
Y = [x_b \quad x_w - x_g \quad x_b - x_w]^T
\]
Write the performance indicators as a linear combination of state variables and input signals, as shown in the following:

\[ Y = CX + DU \]

Among them:

\[
C = \begin{bmatrix}
-\frac{Cs}{m_b} & \frac{Cs}{m_b} & -\frac{Ks}{m_b} & \frac{Ks}{m_b}
0 & 0 & 0 & 1
0 & 0 & 1 & -1
\end{bmatrix}
\]

\[
D = \begin{bmatrix}
0
-1
0
\end{bmatrix}
\]

2.2. Establishment of active suspension system

It is shown in Figure 2:

![Figure 2. Active suspension vehicle 1/4 model.](image)

According to Figure 2, first establish the differential equation of motion in order to build an active suspension vehicle 1/4 model:

\[
\begin{cases}
\ddot{m_w} x_w = K_s (x_b - x_w) + K_t (x_b - x_w) - U_g \\
\ddot{m_b} x_b = -K_s (x_b - x_w) + U_g
\end{cases}
\]

(2)

The matrix state matrix is as the following:

\[
A = \begin{bmatrix}
0 & 0 & -\frac{Ks}{m_b} & \frac{Ks}{m_b} \\
0 & 0 & -\frac{Ks}{m_b} & -K_t - Ks \\
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
0 & 1 & 0 & 0
\end{bmatrix}
\]
2.3. Establishment of pavement model

When analyzing the active suspension control process, the road surface input is an important factor that cannot be ignored. In this paper, the white noise signal is used to excite the road surface input.

\[ x_g(t) = -2\pi f_0 x_g(t) + 2\pi \sqrt{G_0 U_0} w(t) \]

Among them, \( f_0 \) is the lower cut-off frequency, Hz; \( G_0 \) is the road roughness coefficient, m3/cycle; \( U_0 \) is the forward speed, m/sec; \( w \) is the random input unit white noise with mean zero. The above formula shows that the road surface displacement can be expressed as a random filtered white noise signal. This representation is derived from the shape of the power spectral density (PSD) curve of the road roughness measured by the test. We can add the road surface input to the model in the form of a state equation:

\[
\begin{align*}
\dot{X}_{\text{road}} &= A_{\text{road}} X + F_{\text{road}} W \\
Y_{\text{road}} &= C_{\text{road}} X 
\end{align*}
\]

\( X_{\text{road}} = x_g, A_{\text{road}} = -2\pi f_0, B_{\text{road}} = 2\pi \sqrt{G_0 U_0}, C_{\text{road}} = 1 \); \( D = 0 \); Considering that the road surface is a common road surface, the road roughness coefficient \( G_0 = 5 \times 10^{-6} \)m3/cycle; vehicle speed \( U_0 = 20 \)m/s; in modeling, random white noise on the road surface can be generated with random numbers (Random Number) or band-limited white noise (Band-Limited White Noise). The noise researched in this article has been given by the TIN4 file. The simulation model established by MATLAB/simulink is as follows:

![Figure 3. Pavement model.](image)

3. Determination of performance index of armored vehicle suspension

The target performance index \( J \) in the LQG control design is the integral value of the weighted square sum of body acceleration, suspension travel and displacement, which is expressed as follows:

\[
J = \lim_{T \to \infty} \frac{1}{T} \int_0^T \left[ q_1 (x - x_g)^2 + q_2 (x - x_u)^2 + q_3 x_h \right] dt
\]

In order to solve the state feedback gain based on this, the above equation must be expressed in terms of state variables and input variables:
\[ J = \lim_{T \to \infty} \frac{1}{T} \int_0^T X^T Q X + U^T R U + 2X^T N U \, dt \]

Where Q: the weight matrix corresponding to the state variable; R: the weight matrix that constrains the size of the input signal; N: the coupling term.

For the performance functions expressed by \( q_1 \), \( q_2 \), and \( q_3 \), they can be organized as:

\[ J = \lim_{T \to \infty} \frac{1}{T} \int_0^T [q_1(x_w - x_g)^2 + q_2(x_b - x_w)^2 + q_3 x_b] \, dt \]

\[ = \lim_{T \to \infty} \frac{1}{T} \int_0^T \left[ \begin{array}{c} x_b \\ x_w - x_g \\ x_b - x_w \end{array} \right] Q_0 \left[ \begin{array}{c} x_b \\ x_w - x_g \\ x_b - x_w \end{array} \right] \, dt \]

Among them

\[
Q_0 = \begin{bmatrix} q_3 & 0 & 0 \\ 0 & q_1 & 0 \\ 0 & 0 & q_2 \end{bmatrix}
\]

Because the following

\[ Y = CX + DU \]

and so

\[ Y^T Q_0 Y = (CX + DU)^T Q_0 (CX + DU) \]

\[ = X^T C^T Q_0 C X + U^T D^T Q_0 D U + X^T C^T Q_0 D U + U^T D^T Q_0 C X \]

Where \( q_1 \), \( q_2 \), and \( q_3 \) are the weighting coefficients of tire displacement, suspension travel, and vertical acceleration of the vehicle body, respectively. Therefore, Q, R, N can be found:

\[
Q = C^T Q_0 C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & q_2 + \frac{K_w^2}{m_b} & -q_2 - \frac{K_w^2}{m_b} & 0 \\ 0 & 0 & -q_2 - \frac{K_b^2}{m_b} & q_1 + q_3 + \frac{K_b^2}{m_b} & -q_1 \\ 0 & 0 & 0 & -q_1 & q_1 \end{bmatrix} ;
\]

\[
R = D^T Q_0 D = \frac{1}{m_b} ; \quad N = C^T Q_0 D = -K_i ;
\]

Use the MATLAB function LQR to calculate the state feedback K:
\[ [K \quad S \quad E] = lqr(A, B, Q, R, N) \]

Among them, K is the optimal state feedback matrix; S is the solution of Riccati equation; E is the system eigenvalue. Therefore, the active control force \( U = -KX \) can be obtained.

\[
U = -(k_1 x_b + k_2 x_w + k_3 x_b + k_4 x_w + k_5 x_{\dot{r}})
\]

The selection of the weighting coefficient determines the performance of the suspension. If the weighting coefficient of the vertical acceleration of the vehicle body is larger, the riding comfort can be improved; if the weighting coefficient of the dynamic displacement of the tire is larger, the handling stability of the vehicle is better.

4. **Simulation analysis of armored vehicle handling**

The parameters of the armored vehicle will be given in the appendix program, and the simulink model is established according to the requirements for comparative analysis based on the following figures:

![Figure 4. Body acceleration comparison.](image-url)
Figure 5. Suspension workspace comparison.

Figure 6. Tire displacement comparison.
Figure 7. Amplitude-frequency comparison of active suspension and passive suspension.

5. Conclusions
In this article, the two-degree-of-freedom 1/4 car body model is first established. The LQG controller of vehicle active suspension is designed by using linear quadratic optimization theory. Simulation analysis is carried out by virtue of MATLAB/Simulink software. The analysis results show that the designed optimal active suspension significantly reduces the vertical vibration acceleration of the car body. Compared with the passive suspension, the suspension dynamic travel and wheel dynamic displacement using the LQG controller have also been improved. Therefore, the design of the active suspension LQG controller based on the linear quadratic optimal control theory is effective.

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