REVERSE BULLWHIP EFFECT: DUALITY OF A DYNAMIC MODEL OF SUPPLY CHAIN

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ABSTRACT

This study aims to investigate control strategies for the bullwhip effect based on a dynamic model of the linear supply chain, proposed by Helbing and Lammer (2005), which describes the inventory dynamics and production rates of productive units. We simulated the model for instability and stability conditions defined by mathematical analysis. Through these results, we verified both classical and reverse bullwhip effects associated with instability and stability conditions, respectively. The model revealed a duality once the control strategy proposed by Helbing and Lammer (2005) for the classical bullwhip effect ends up causing a reverse effect, which is equally troubling. In the reverse bullwhip effect, we observed amplification of the production rates in the network chains from the supplier to the customer in a way that the upstream chain was not able to meet the needs of the downstream chain. To withhold both effects, we suggest the dynamic control of the parameters that describe the network based on Helbing and Lammer (2005) model.

Keywords: reverse bullwhip effect; bullwhip effect; supply chain; industrial dynamics
1. INTRODUCTION

A supply chain is comprised of a set of productive units involved in the fulfillment of a client’s needs and connected through product flow, information, and financial resources (BASAK et al., 2014). One of the most common phenomena on the supply chain is the bullwhip effect (DAI; PENG; LI, 2017).

The bullwhip effect is the phenomenon in which the variability of the inventory levels and deliveries grows from the end customer to the first stage supplier in the chain network (WANG; DISNEY, 2016). There is evidence on the literature of a phenomenon opposite to the bullwhip effect called reverse bullwhip effect (ÖZELKAN; LIM; ADNAN, 2018; RONG; SNYDER; SHEN, 2017; SVENSSON, 2003).

Characterized by the growing variability in the demands downstream the suppliers, the reverse bullwhip effect is caused, mainly, by interruptions on delivery (RONG; SHEN; SNYDER, 2009).

Even though the reverse bullwhip effect is often experienced in practice (RONG; SNYDER; SHEN, 2017), it is a poorly explored concept in the literature. Most of the studies refer to the reverse bullwhip effect in pricing (ÖZELKAN; ÇAKANYILDIRIM, 2009; ÖZELKAN; LIM; ADNAN, 2018), rather than in products flow throughout the chain.

On a brief search through the main production engineering databases, few results were found. Searching for “Reverse Bullwhip Effect” as titles of papers published from 2014 to 2019 on Web Of Science and Science Direct databases, we had only one result, which referred to the reverse bullwhip effect in pricing.

The reverse bullwhip effect is a phenomenon that causes losses on supply chains but is poorly mentioned in the literature. Therefore, this paper can contribute to the quantitative analysis of the reverse bullwhip effect, for it aims to investigate the reverse bullwhip effect based on a dynamic model of supply chain built by Helbing and Lammer (2005) by computational simulation of inventory dynamics and production rates of the productive units of a linear supply chain.

Modeling the reverse bullwhip effect can help reduce its negative impacts over inventories and the level of service of the supply chain, once it is useful to quantify its intensity (FIORIOLLI; FOGLIATTO, 2009). In this work, we tested control parameters, and we verified the global correlation among them regarding the impacts on the supply chain dynamics.
Results reveal that the same features that cause the classical bullwhip effect can be associated with the reverse bullwhip effect, only in different quantitative correlations. That is significant because, despite the lack of studies about the reverse bullwhip effect, the search for answers to the problem can employ previously studied tools.

2. CONCEPTUAL BASIS

To understand the concepts discussed in Helbing and Lammer’s (2005) model, we will provide a brief literature review about the concepts of the classical and reverse bullwhip effect, and we will also describe the model.

2.1. The Bullwhip Effect

The bullwhip effect is one of the most popular terms in the supply chain area (CHEN; LUO; SHANG, 2017; WANG; DISNEY, 2016). Also known as the Forrester (1961) effect and upstream amplification (DISNEY; TOWILL, 2003; WANG; DISNEY, 2016), it refers to the phenomenon in which there is a distortion of the demand being propagated and amplified upstream the customers (HASSANZADEH; JAFARIAN; AMIRI, 2014).

The bullwhip effect phenomenon is not new in supply chains (DISNEY; TOWILL, 2003). Since the 90’s it has been acknowledged by several markets and it is commonly observed in most industries (WANG; DISNEY, 2016).

The variability amplification effect was formalized by Forrester (1961) following the industrial dynamics approach when he demonstrated that the dynamics among companies in a supply chain might cause errors and distortions, which amplify upstream the network.

Within the companies, evidence suggests that the expense caused by the bullwhip effect is significant over the years. The phenomenon is associated with machine settings and shut-offs, inactivity, and extra hours in workload (WANG; DISNEY, 2016).

The bullwhip effect has a significant influence on the overall performance of the supply chain, for the players are not aware of the true nature of the demand, which results in undesired consequences, such as imprecise forecasting, inventory overload, unfit use of the production capacity, and lousy customer service (TAI; DUC; BUDDHAKULSOMSIR, 2019).

The bullwhip effect makes it more difficult for companies to realize the market demands, which causes inventory overload and a decrease of operational effectiveness on the whole supply chain (DAI; PENG; LI, 2017). The phenomenon is the primary cause of the logistical inefficiency in the supply chain (FAIZAN; HAQUE, 2015).
Currently, several studies discuss the bullwhip effect in different contexts. One example is Shan’s et al. (2014) work, which showed that more than two-thirds of the Chinese companies listed in Shanghai’s and Shenzhen’s Stock Markets manifest the bullwhip effect.

Lee, Padmanabhan, and Whang (1997a, b) identified the four major sources of the bullwhip effect: the process of updating the demand forecasting of a chain stage based on information of a lower stage, accumulation of the demand due to fixed batch size, price variation, and rationing and market scarcity games.

In a study about supply chain using response surface methodology, Hassanzadeh, Jafarian, and Amiri (2014) showed that the accumulation of the demand due to fixed batch size alone is significant to cause the bullwhip effect. In contrast, the causes related to the rationing factor and demand signal processing solely are not significant.

On the other hand, Vokhmyanina, Zhuravskaya, and Osmólski (2018) point out that the bullwhip effect is caused by the lack of forecasting reliability, which ends up decreasing the effectiveness of inventory planning in supply chains and large logistical systems.

Faizan and Haque (2015) state that if the product offer in the network does not match the demand, the gap between supply and demand enhances in the different stages and might cause either inventory overload or shortages in inventory. In addition to that, the bullwhip effect in a supply chain can cause undesirable service quality to the client due to inefficiencies risen along the process (BUCHMEISTER; FRISCIC; PALCIC, 2014).

To mitigate the bullwhip effect, Vokhmyanina, Zhuravskaya, and Osmólski (2018) suggest employing more advanced demand forecast models to soothe the impact of variability.

The minimum mean square error framework for demand forecast is ideal for reducing the impact of the bullwhip effect when the demand model is autoregressive (MA; ZHANG; ZHU, 2018). When there is no well-specified model for demand or when it changes over time, forecasting models of moving average and exponential smoothing should be used, for they are flexible and adapt best to the variable structure of the demand (MA; ZHANG; ZHU, 2018).

Information sharing can also help reduce the bullwhip effect, mostly when an upper chain stage uses historical data of the lower chain stage to forecast demand (LU et al., 2017). As higher the information sharing rate is, the more significant is the reduction of the bullwhip effect (JEONG; HONG, 2017).
However, sharing information on the end demand is not enough to mitigate the bullwhip effect. Even the wealthiest information conditions lead the decision-makers to cause the bullwhip effect (HAINES; HOUGH; HAINES, 2017).

The stage of the supply chain must decide to adopt the information. The value of adopting the information about the end demand, and about the order is always higher than the information on the end demand (MA et al., 2013). Decision making based in data has become a decisive factor for competitive advantage to companies in a supply chain (VIET; BEHDANI; BLOEMHOF, 2018).

Moreover, the effect of information sharing is different for each stage of the supply chain. Downstream, the use of reported information about customer demand is associated with better performance, whereas to upstream stages, the performance is not affected (HAINES; HOUGH; HAINES, 2017).

Ma and Ma (2017) suggest stretching the time of the demand forecast to establish an estimate of deliveries for a more extended period so that the bullwhip effect decreases as the demand becomes more accurately forecasted.

Agrawal, Sengupta, and Shanker (2009), and Chen, Luo, and Shang (2017) demonstrated that there will always be the bullwhip effect, and one can only reduce it. Decreasing the lead time is more beneficial compared to information sharing concerning the reduction of the bullwhip effect phenomenon.

2.2. The Reverse Bullwhip Effect

Rong, Shen, and Snyder (2008) pointed out two bullwhip effects. The first one is universally accepted and refers to the amplification of the demand throughout the network (DISNEY; TOWILL, 2003). The second one is the reverse bullwhip effect, opposite to the classical bullwhip effect (RONG; SHEN; SNYDER, 2008).

The term “reverse bullwhip effect” was introduced by Svensson (2003) while studying the bullwhip effect in an intra-organizational context. The reverse bullwhip effect is characterized by the increase of demand variability from the supplier to the customer, and it occurs, generally, due to delivery interruptions (RONG; SHEN; SNYDER, 2009; RONG; SHEN; SNYDER, 2008).

Shukla (2014) stated that the reverse bullwhip effect is caused by the variability in delivery, from supplier to customers through retailers, as opposed by the straight bullwhip
effect, caused by the variability in customer’s demand from the client to the suppliers through retailers.

When studying the bullwhip effect and the reverse bullwhip effect and their relationships with the rationing game, (RONG; SNYDER; SHEN, 2017) concluded that the reverse bullwhip effect is a consequence of a delivery interruption, and, just as the bullwhip effect, it propagates upstream the supply chain until it reaches a stage that does not react to the uncertainty that created it.

Delivery interruptions cause changes in customer’s behavior, and those variations cause the reverse bullwhip effect, once the end customer overreacts in search of the needed product, which inflates demand and leads retailers to order excessively from the supplier (RONG; SHEN; SNYDER, 2008).

In a study about countermeasures for reducing the reverse bullwhip effect in a supply chain of a rural market in China, Liu and Wu (2013) identified that the imprecision in demand forecasting, the long-time period to deliver orders, the discounts in sale prices that inflate the demand, the lack of coordination due to the conflict of interest in the chain, and the low level of information sharing, rather than just the delivery interruptions, are the main causes of the reverse bullwhip effect.

Resende et al. (2009) also pointed out that either the delivery interruption or the production shutdown in an upstream chain echelon causes the reverse bullwhip effect, and that the latter impacts suppliers upstream in reducing their production capability.

The improvement of information sharing among companies in the supply chain, the accuracy of demand forecasting, and the shortening of the orders delivery time are countermeasures that might help the mitigation of the reverse bullwhip effect (LIU; WU, 2013).

The existence of the reverse bullwhip effect shows that, in a company of a supply chain, the managers face uncertainties not only from demand but also from the supply (RONG; SHEN; SNYDER, 2008). The effect can cause losses to supply companies as well as customer’s loss of confidence and sales deterioration (LIU; WU, 2013).

2.3. Description Of The Model

The study of supply chains was carried out based in a dynamic version of Leontief’s input-output model, proposed by Helbing and Lammer (2005), in which the time variation rate
of the number of goods $N_i(t)$ of $i$ available types in the inventory of the production unit is given by:

$$\frac{dN_i}{dt} = Q_i(t) - Q_{i+1}(t) \tag{1}$$

with $i \in \{1, \ldots, u\}$, where $Q_i(t)$ is the rate in which the supplier $i$ receives products ordered from supplier $i - 1$ at time $t$, whereas $Q_{i+1}$ is the rate in which supplier $i$ delivers products to the next supplier $i + 1$. The equation (1) is a continuity equation, and it reflects the conservation of the number of products.

In linear supply chains (figure 1), each productive unit has an $N_i$ inventory level. The productive unit of raw materials has an $N_0$ inventory and delivers products in a $Q_1$ rate. The customer is the last echelon and consumes products at a $Q_{u+1}$ rate.

![Figure 1: Linear supply chain](http://creativecommons.org/licenses/by/3.0/us/)

Source: Helbing and Lammer (2005, p. 5)

It is quite reasonable to assume that the temporal variation of deliveries rate is proportional to the deviation of the observed rate compared to the expected rate $W_i$, the ordering rate, as its adaptation occurs somewhere in the $T_i$ time interval (HELBING; LAMMER, 2005) so that

$$\frac{dQ_i}{dt} = \frac{1}{T_i} \left[ W_i(t) - Q_i(t) \right] \tag{2}$$

Function $W_i(t)$ reflects the managing strategy, i.e., how to make sure the expected delivery rate matches the observed and anticipated inventory levels (HELBING; LAMMER, 2005). Equation 2 is a simple, special case of $W_i$, in which the production rate is controlled in order to reach an optimal inventory $N_i^0$ and optimal production $Q_i^0$ (HELBING; LAMMER, 2005). Therefore, equation 2 can be written as follows:

$$\frac{dQ_i}{dt} = \frac{1}{T_i} \left[ \frac{N_i^0 - N_i(t)}{\tau_i} - \beta_i \frac{dN_i}{dt} - \epsilon_i \left( Q_i^0 - Q_i(t) \right) \right] \tag{3}$$
where $\tau_i$, $\beta_i$, and $\varepsilon_i$ are system's parameters related to times of inventory adjustment, inventory anticipation, and the adjustment of balance production rate, respectively (HELBING; LAMMER, 2005).

Using $n_i(t) = N_i(t) - N^0_i$ to represent the deviation of the observed and stationary inventory, and $q_i(t) = Q_i(t) - Q^0_i$ to represent the deviation of delivery rate, we obtain

$$\frac{d^2 q_i}{dt^2} + \frac{(\beta_i + \varepsilon_i)}{\tau_i} \frac{dq_i}{dt} + \frac{1}{\tau_i \tau_i} q_i(t) = \frac{1}{\tau_i} \left[ \frac{q_{i+1}(t)}{\tau_i} + \beta_i \frac{dq_{i+1}}{dt} \right]$$

Equation 4 describes the behavior of delivery or production rates of the linear supply chain echelons, and it is similar to the equation of a forced damped harmonic oscillator.

Forced damped oscillations are those in which an oscillator dissipates energy and is subjected to a periodical external force so that there is compensation of the dissipation through constant energy supply (NUSSENZVEIG, 2014). The equation that describes the behavior of a forced damped oscillator is

$$\frac{d^2 x}{dt^2} + \gamma \frac{dx}{dt} + \omega_0^2 x = \frac{F(t)}{m} = \frac{F_0}{m} \cos(\omega t)$$

where $x$ is the oscillator position, $\gamma$ is the damping coefficient, $\omega_0$ is the undamped angular frequency of the oscillator, and $F(t)$ is the external acting force, defined as $F_0 \cos(\omega t)$ (NUSSENZVEIG, 2014).

Similarly, equation 4 of the linear supply chain can be written as:

$$\frac{d^2 q_i}{dt^2} + 2\gamma_i \frac{dq_i}{dt} + \omega_i^2 q_i(t) = f_i(t)$$

where $\gamma_i$ is the damping coefficient, $\omega_i$ is the undamped angular frequency, and

$$f_i(t) = \frac{1}{\tau_i} \left[ \frac{q_{i+1}(t)}{\tau_i} + \beta_i \frac{dq_{i+1}}{dt} \right]$$

is the external force.

Considering that $f_i(t) = f_i^0 \cos(\alpha t)$, we have

$$\frac{d^2 q_i}{dt^2} + 2\gamma_i \frac{dq_i}{dt} + \omega_i^2 q_i(t) = f_i^0 \cos(\alpha t)$$

The general solution (HELBING; LAMMER, 2005, p. 8) of equation 8 is

$$q_i(t) = f_i^0 F_i \cos(\alpha t + \phi_i) + D_i^0 e^{-\eta_i t} \cos(\Omega_i t + \theta_i)$$
For long times in which $t \gg 1/y_i$, the first term of equation 9 becomes dominant and the behavior or the network is described by $q_i(t) \approx f_i^0F_i^0\cos(at + \phi_i)$.

Forced damped oscillators might exhibit a phenomenon known as resonance. Resonance is characterized by the increase of the oscillation amplitude as undamped angular frequency approaches external force frequency (NUSSENZVEIG, 2014).

In supply chain, the upstream amplification of the demand, which characterizes the bullwhip effect (WANG; DISNEY, 2016), can be interpreted as a resonance phenomenon (HELBING; LAMMER, 2005).

Considering that the external force $f_i(t)$ in equation 6 for productive unit $i$ is $f_i^0\cos(at + \phi_i)$, and for unit $i - 1$ is $f_{i-1}^0\cos(at + \phi_i + \delta_i)$, in which $\delta_i$ is a coefficient of lagging, the oscillation amplitude of a productive unit is smaller than the amplitude of a previous unit when the amplification factor is defined as

$$\frac{f_{i-1}^0}{f_i^0} = \frac{\sqrt{(1/\tau_i)^2 + (\alpha \beta_i)^2}}{\sqrt{(1/\tau_{i-1})^2 + (\alpha \beta_i)^2}} > 1$$

Therefore, in order to that condition to be valid

$$0 < \alpha^2 < \frac{2}{\tau_i \tau_{i-1}} - \frac{\epsilon_i (\epsilon_i + 2\beta_i)}{\tau_{i-1}^2}$$

In other words, in linear supply chains, there will be a bullwhip effect when adaptation time $\tau_i$ of the observed delivery rate compared to the ideal rate, as presented in the management function (2), is very long, or when $\epsilon_i$ and $\tau_i$ are equal to zero, i.e., when adaptation to a balanced production rate is not considered, and the reactions of the observed inventory level to the ideal are very abrupt, respectively (HELBING; LAMMER, 2005).

3. RESEARCH METHODOLOGY

To investigate the existence of the bullwhip effect based on Helbing and Lammer’s (2005) model, we designed a quantitative research methodology based on a simulation model. Simulation models are compelling, broadly employed into complex systems analyses, and may be either continuous or discrete (MIGUEL, 2012). Continuous models mimic systems with temporal-continuous behavior, whereas the discrete models represent systems in which changes occur in specific moments in time (MIGUEL, 2012).
For the present study, we adopted a continuous simulation model that mimics the behavior of the delivery rates of linear supply chain echelons.

For simulation, we used a Python code (v. 3.5.2) with the numpy, sdeint, and matplotlib libraries to create the arrays, integrate the stochastic differential equation, and plot the graphs, respectively.

Based on the numerical integration of equation 4, we obtained the delivery rates and the delivery rates amplitudes of each productive unit in the supply chain.

To simulate a random behavior in market consumption, the external force associated to customer $f_{u+1}(t)$ was employed as white noise (NAGATANI; HELBING, 2004), denoted by $\xi(t)$, i.e., $f_{u+1}(t) = \xi(t)$, which has the following features:

I. Average value $\langle \xi(t) \rangle = 0$;

II. Correlation of time given by $\langle \xi(t)\xi'(t) \rangle = \delta_{r,r'}/4$.

To analyze if the model proposed by Helbing and Lammer (2005) controls the bullwhip effect when the definition of parameters does not match the instability condition of equation 12, we simulated the code for both instability and stability conditions.

The supply chain we analyzed has five productive units, i.e., $u = 5$, and the simulation time was established for every simulation with a 0.2 seconds gap, recognizing the limits concerning the applicability of solution for long times.

4. RESULTS AND DISCUSSION

Based on a Python code, we simulated situations in which the answer given by equation 11 was true or false, i.e., for when the bullwhip effect might or might not exist, in theory. To simplify the tests, we assigned random values to the $T_i, \varepsilon_i, \tau_i, \text{ and } \beta_i$ parameters, and assumed those same values to all productive units $T_i = T, \varepsilon_i = \varepsilon, \tau_i = \tau, \beta_i = \beta$.

For cases in which the answer given by equation 11 was true, we simulated the supply chain with the following set of parameters: $T = 150.00, \varepsilon = 0.00, \tau = 0.05, \beta = 1.00, \alpha = 0.05$. The end time of simulation was 200 seconds so that we could be able to realize the general behavior of the network.

Based on the simulation, we obtained the graphs that associate the echelons’ delivery rate with time. Figure 1 illustrates the delivery rates of each productive unit according to time, and figure 2 shows the phase portrait relative to the delivery rates.
Figure 1: Echelons’ delivery rates in an unstable linear supply chain.
Source: The authors.

Figure 1 shows that the echelons’ delivery rate grows with time, mostly that of the raw material supplier, which grows in a higher proportion compared to the other echelons, as shown in the phase portrait of figure 2.

Figure 2: Phase portrait for echelons’ delivery rates in an unstable linear supply chain
Source: The authors.

As for the delivery rate amplitude, figure 3 illustrates the evolution of amplitude for each echelon in specific instants in time. The time instants were strategically selected to represent moments, in the beginning, and at the end of the simulation. Each graph in figure 3 is entitled “amplitude,” followed by the instant in time to which it refers.
Figure 3: Evolution of the delivery rates amplitudes of echelons in an unstable linear supply chain.
Source: The authors.

The graphs in figure 3 illustrate the range of delivery rates of five productive units (four suppliers and one customer). The graphs show that, through time, the delivery rate amplitude of the raw material supplier becomes much higher than the amplitudes of the other chain echelons, as confirmed in figure 1.

These results show that, with temporal evolution, the prime supplier of the supply chain has its delivery rates affected in a much higher proportion with chain instability. It consequently becomes the most affected echelon and produces much more than it should to provide the demand.

There is also an upstream amplification of these values, where the supplier 2, neighbor to the raw material supplier, has an amplitude of delivery rate higher than the supplier 3, and so on. In theory, this situation defines the classical bullwhip effect.

Therefore, it is possible to state that the instability condition determined by Helbing and Lammer’s (2005) model suitably leads to the classical bullwhip effect, once the supply chain exhibits distortion of delivery rates in echelons of productive units and there is amplitude amplification of the delivery rates, from the customer to the raw material supplier.

Based on equation 1, if the supplier’s delivery rate downstream is higher than that of the upstream supplier, the variation rate of the inventory is positive. In other words, if the supplier receives more products than it delivers, its inventory level will rise. In an instability
situation, upper levels suppliers would have excessive inventories, which could cause obsolete materials and make unnecessary expenses.

In cases in which the answer given by equation 11 is non-true, i.e., the bullwhip effect might not occur according to the model, we simulated the supply chain was with the following set of parameters: $T = 5.00$, $\varepsilon = 2.00$, $\tau = 0.10$, $\beta = 10.0$, $\alpha = 2.0$. The end time of the simulation was 80 seconds, so that it could be possible to realize the behavior of the network.

Based on the simulation, we obtained the graphs that relate the echelons’ delivery rate according to time. Figure 4 shows the evolution of delivery rates according to time for five productive units.

![Figure 4: Delivery rates of echelons in an unstable linear supply chain.](Source: The authors.)

As time progresses, deviations in delivery rates $q(t)$ of the supply productive units adapt to customer behavior. Thus, it is possible to state that the customer imposes a production rhythm throughout the chain, for the supplier 4 produces enough to supply the customer, and the supplier 3 produces enough to supply the supplier 4 and so on.

In the stationary state, we verify that each productive unit has its $q(t)$ values oscillating to supply the random consumption rate. The oscillations are around a $q(t)$ value equals to zero. That means the delivery rates of all echelons are close to the optimal value, given that the oscillation amplitude is low.

Also, supplier 1, which delivers raw material, has the lowest oscillation amplitude, followed by supplier 2, and so on, so that the customer has the highest oscillation amplitude in the network. That can also be seen in the phase portrait in figure 5.
The phase portrait of $\frac{dq}{dt}$ by $q(t)$ shows that after the system adjusts, the customer has the highest oscillation amplitude, its closest neighbor has the second-highest, and so on to the supplier 1, which delivers raw material, has the lowest amplitude. In order to better observe this effect, figure 6 illustrates the evolution of the delivery rate amplitudes in specific moments in time, which were determined so that it could be possible to verify the behavior of the chain from the beginning to the end of the simulation.

The evolution of the delivery rates amplitudes of echelons in a supply chain shows that when reaching a time of stationary state, there is an amplification of those rates from the raw
material supplier to the customer, in which the customer has a consumption rate higher than its supplier’s delivery rate.

This result indicates the existence of a phenomenon opposite to the analyzed in the unstable supply chain. In other words, the model indicates that in stability, there is the reverse bullwhip effect, in which the suppliers are not able to supply the demand, rather than the control of the bullwhip effect, as proposed.

We also simulated the model for when the set of parameters leads to amplitudes ratio equal to 1, in order to verify if the amplitudes of delivery rates would not rise throughout the network in a stationary state. The assigned set of parameters were: $T = 3.00, \varepsilon = 1.00, \tau = 0.50, \beta = 1.00, \alpha = 1.00$, which provide a relation in which $
abla_2 = 1 = \frac{2}{\tau \varepsilon \tau_i} \frac{\varepsilon_i (\varepsilon_i^2 + 2\beta_i)}{\tau_i^2}$. The end time of the simulation was 200 seconds.

Figure 7 shows the evolution of deviations of delivery rates under these conditions according to time.

![Figure 7: Delivery rates of echelons in a linear supply chain.](source: The authors.)

Based on figure 7, we observe that the behavior of the supply chain for the parameter assigned is similar to the behavior of a stable supply chain, as can be observed in figure 8, which illustrates the evolution of amplitudes of delivery rates for some instants in time.
We can then observe that, for a time of stationary state, the behavior of the model for when the delivery rates amplitudes of echelons in a network should be equal to one, is similar to the behavior in which the ratio of these amplitudes is lower than one, i.e., there is an indication of existence of the reverse bullwhip effect.

Therefore, based on the analyzed conditions, we verify that the Helbing and Lammer’s (2005) model describes linear supply chains that can show only two behaviors: stability with the existence of the reverse bullwhip effect, and instability with the existence of the classical bullwhip effect.

The reverse bullwhip effect phenomenon occurs when the ratio of delivery rates amplitudes of a productive unit and its posterior is lower or equal to one \((\frac{f_{i+1}}{f_i} \leq 1)\), and the classical bullwhip effect occurs when that ratio is lower than one \((\frac{f_{i+1}}{f_i} > 1)\).

These results indicate that the Helbing and Lammer’s (2005) model fails to propose control of the supply chain once the stability condition obtained by mathematical analysis leads the supply chain to exhibit the reverse bullwhip effect, as stated by (AGRAWAL; SENGUPTA; SHANKER, 2009; CHEN; LUO; SHANG, 2017), that a supply chain will always face problems related to demand variability.

Based on equation 1, if the delivery rate of the downstream supplier is lower than that of the upstream supplier, the inventory variation rate is negative. In other words, if the supplier delivers more products than it receives, its inventory levels will decrease.
In a stability situation, the suppliers from upper levels would have decreasing inventories with time. The inventories of a company are not infinite. Thus, if a stability condition is maintained, the inventory levels of a supply chain echelon could be zero, and, at some point, the company could suffer the lack of products in inventory.

The inventory available in a company is highly related to the performance of the supply chain, which can be measured by the attendance rate, the level of pending orders, lost sales, and the probability of delay (HOPP; SPEARMAN, 2013).

The attendance rate is the ratio of attended demand by the available inventory. In a situation of chain stability in which the inventory is zero, the attending rate will be low, and the client service might be affected.

The lost sales are the potential number of client orders lost by the lack of products in inventory. Hence, in a situation of stability, lost sales might be high if the client is not attended right away, which could lead to orders in a rival company.

The probability of delay is the possibility of an activity to delay due to the lack of products in inventory. In a stable network from Helbing and Lammer’s (2005) model, the products might come to lack in inventory and raise that measure.

Therefore, we verify that Helbing and Lammer’s (2005) model, when proposing a condition of stabilizing the supply chain, also causes performance problems and affects the efficiency of the whole network as it causes the reverse bullwhip effect.

Given the little knowledge about the reverse bullwhip effect, demonstrated through the lack of scientific articles in the bibliography review, this study contributes towards indicating that the possible causes of the reverse bullwhip effect are not from a different origin compared to the causes of the classical bullwhip effect.

Helbing and Lammer’s (2005) model demonstrates that the same features, represented by parameters which are taken into consideration in the management function, in different quantitative relations, might cause either one or the other effect.

This dynamic control can only be implemented with the improvement of information sharing throughout the network, which enables that each echelon of the chain could adjust its time of inventory reposition, and/or its delivery rate, throughout the time, for example.

5. CONCLUSION
This study aimed to investigate the phenomena of the reverse bullwhip effect based on a dynamic model of supply chain built by Helbing and Lammer (2005) by computationally simulating the inventory dynamics in productive units of a linear supply chain.

For the simulation, we wrote a Python code, by which we obtained the delivery rates and the amplitude of delivery rates of each productive unit of the supply chain.

We verified that Helbing and Lammer’s (2005) model describes supply chains that might exhibit only two behaviors: stable and unstable, which characterize the existence of both reverse and classical bullwhip effect, respectively.

In the supply chain presenting the reverse bullwhip effect, we observed amplification of the delivery rate from the raw material supplier to the customer, in which the echelons upstream were not able to supply the demand of the echelons downstream.

In the supply chain presenting the classical bullwhip effect, we observed amplification of the delivery rates from the customer to the raw material supplier, so that the echelons produced more than they should to supply the demand.

In the simulation of the model, the reverse bullwhip effect occurred when the ratio of the amplitudes of production rates in a productive unit and its posterior was lower or equal to one, and the classical bullwhip effect occurred when this ratio was lower that one. The phenomenon can significantly affect the performance of a supply chain once it could cause a lack of products in inventory.

We also verified that, even though Helbing and Lammer’s (2005) model proposes the control of the classical bullwhip effect, there is the reverse bullwhip effect under the same parameters, only differing in values.

The presented duality suggests that the model does not resolve the problematic dynamics of echelons in a supply chain once when controlling one phenomenon, it causes another one equally harmful.

Therefore, to withhold both effects, we suggest the dynamic control of the parameters that describe the model, so that they become controlled as time progresses, enabling that there is no amplification of the delivery rates neither upstream nor downstream the supply chain.

The dynamic control of the parameters can optimize the relationship of the echelons of the supply chain as it proposes a mode to supply the demand when there is the reverse bullwhip effect.
effect, and to decrease the unnecessary production rates when there is the classical bullwhip effect.

This work showed that Helbing and Lammer’s (2005) model does not resolve the problem of the bullwhip effect. It also contributed to the understanding of the reverse bullwhip effect, a very little discussed theme in the literature, but on that can significantly affect the performance of a supply chain.

We hope the results we obtained can contribute to the literature about problems faced in a supply chain, and that it becomes the ground for the development of models which incorporate technics to increase the global performance of the network, towards mitigating both the classic and the reverse.

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