X-ray Line Diagnostics of Ion Temperature at Cosmic-Ray Accelerating Collisionless Shocks

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Abstract

A novel collisionless shock jump condition is suggested by modeling the entropy production at the shock transition region. We also calculate downstream developments of the atomic ionization balance and the ion temperature relaxation in supernova remnants (SNRs). The injection process and subsequent acceleration of cosmic-rays (CRs) in the SNR shocks are closely related to the formation process of the collisionless shocks. The formation of the shock is caused by wave-particle interactions. Since the wave-particle interactions result in energy exchanges between electromagnetic fields and charged particles, the randomization of particles associated with the shock transition may occur with the rate given by the scalar product of the electric field and current. We find that order-of-magnitude estimates of the randomization with reasonable strength of the electromagnetic fields in the SNR constrain the amount of the CR nuclei and ion temperatures. The constrained amount of the CR nuclei can be sufficient to explain the Galactic CRs. The ion temperature becomes significantly lower than in the case of no CRs. To distinguish the case without CRs, we perform synthetic observations of atomic line emissions from the downstream region of the SNR RCW 86. Future observations by XRISM and Athena can distinguish whether the SNR shock accelerates the CRs or not from the ion temperatures.
1 Introduction

Collisionless shocks of supernova remnants (SNRs) are invoked as the primary sources of Galactic cosmic-rays (CRs); however, the production process of CRs is an unsettled issue despite numerous studies reported. The most generally accepted and widely studied mechanism for CR acceleration is the diffusive shock acceleration (DSA, Bell 1978; Blandford & Ostriker 1978). In the DSA mechanism, we assume energetic particles around the shock, and the particles go into bouncing back and forth between the upstream and downstream regions by scattering particles. The particle scattering results from interactions between plasma waves and the particles. The maximum energy of the accelerated particles depends on the magnetic field strength and turbulence (e.g., Lagage & Cesarsky 1983a; Lagage & Cesarsky 1983b). To explain the energy spectrum of the CR nuclei observed around the Earth, the maximum energy of the accelerated protons should be at least $10^{15.5}$ eV (so-called the knee energy). The knee energy can be achieved in the DSA mechanism by the magnetic field strength of $\gtrsim 100 \mu G$ which is larger than the typical strength of $\sim 1 \mu G$ seen in the interstellar medium (ISM, Myers 1978; Beck 2001). Bell (2004) pointed out that the upstream magnetic field is amplified by the effects of a back reaction from the accelerated protons themselves. This amplification is called the Bell instability, whose growth rate is proportional to the CR energy density. Observations of nonthermal X-ray emissions around the SNR shocks imply the existence of amplified magnetic fields in the downstream region (e.g., Vink & Laming 2003; Bamba et al. 2005; Uchiyama et al. 2007). Hence, in a modern scenario of the CR acceleration, the SNR shock is assumed to inject a considerably large amount of CRs ($\gtrsim 10$% of the shock kinetic energy), and their effects on the background plasma are regarded as one of the most important issues. Since the wave-particle interactions also give the formation of the collisionless shock, the injection of energetic particles, subsequent acceleration by the DSA mechanism, and the amplification of the magnetic field are closely related to the formation process. Although many kinetic simulations studying collisionless shock physics are reported (e.g., Ohira 2013; Ohira 2016b; Ohira 2016a; Matsumoto et al. 2017; Caprioli et al. 2020; Marcowith et al. 2020), self-consistent treatment of the collisionless shock, including these effects, is currently incomplete due to the limitation of too short simulation time compared to actual SNR shocks.

When the SNR shock consumes its kinetic energy to accelerate energetic, nonthermal particles, the downstream thermal energy can be lower than the case of adiabatic shock without the CRs (e.g., Hughes et al. 2000, Helder et al. 2009; Morlino et al. 2013b; Morlino et al. 2013a; Morlino et al. 2014; Hovey et al. 2015; Hovey et al. 2018; Shimoda et al. 2015; Shimoda et al. 2018b). Thus, we observe small downstream ion temperatures if the SNR shocks efficiently accelerate the CR nuclei (protons and heavier ions). In the near future, spatially resolved high energy-resolution spectroscopy of the SNR shock regions will be achieved by the micro-calorimeter array with Resolve (Ishisaki et al. 2018) onboard XRISM (Tashiro et al. 2020) and with the X-IFU onboard Athena (Barret et al. 2018) providing precise line diagnostics of plasmas to represent the effect of the CR acceleration. Note that observations of $\gamma$-ray emissions possibly provide the amount of the CR proton from luminosities. However it may be challenging to determine the amounts of the CR nuclei individually. Thus, in this paper, we study the shock jump conditions of ions, including the effects of CR acceleration. To distinguish the case without the CRs by the future X-ray spectroscopy, we calculate temporal evolutions of the downstream ionization structure and downstream ion temperatures resulting from the Coulomb interactions. From the calculations of the downstream values, we also perform synthetic observations of atomic lines, including effects of downstream turbulence. Since the turbulence affects the line width by the Doppler effect, it is non-trivial whether the observed line width reflects the intrinsic ion temperature.

Typical example of the missing thermal energy measurement was provided at the SNR RCW 86 (Helder et al. 2009). The RCW 86 is considered as the remnant of SN 185 (Vink et al. 2006), so its age is $\sim 2000$ yr. The current radius is $\sim 15$ pc, and the shock velocity is $\sim 3000$ km s$^{-1}$ with an assumed distance of 2.5 kpc (Yamaguchi et al. 2016), where we estimate the angular size as $\sim 40$ arcmin. Since $2000$ yr $\times 3000$ km s$^{-1}$ $\approx 6$ pc $< 15$ pc, the blast wave has already decelerated. The mean expansion speed should be $\sim 10^9$ cm s$^{-1}$ if the radius becomes $\sim 10$ pc within the time of $\sim 1$ kyr. Such high
expansion speed can be maintained during 1 kyr if the progenitor star explodes in a wind-brown cavity created by the progenitor system. Broersen et al. (2014) studied this scenario by comparing the X-ray observations and hydrodynamical simulations. They concluded that the progenitor star of SN 185 exploded as a Type-Ia supernova by the progenitor system. Broersen et al. (2014) studied everywhere (Helder et al. 2013). The Hα emission means that the shock is now propagating into a partially ionized medium (Chevalier et al. 1980). In the case of a stellar wind by a massive star, the ionization front precedes the front of the swept-up matter (e.g., Arthur et al. 2011). Thus, the forward shock of the RCW 86 currently may propagate in the medium not swept up by the wind. In this paper, we suppose such a scenario for the RCW 86 and perform synthetic observations of the X-ray atomic lines.

This paper is organized as follows: In section 2, we review a physical model of the temporal evolutions of the downstream ionization balance and the ion internal energies. The ion temperatures are derived from the equation of state. Section 3 provides shock jump conditions as initial conditions for the downstream temporal evolution. We introduce shock jump conditions usually supposed in the SNRs and a novel condition given by modeling the entropy productions of the ions due to the wave-particle interactions. The latter includes the effects of the CR acceleration, magnetic-field amplification, and ion heating balance. The results of the downstream temporal evolutions are summarized in section 4. In section 5, we perform synthetic observations of atomic lines, including the effects of the downstream turbulence. Finally, we summarize our results and prospects.

2 Physical Model of downstream ionization balance and ion internal energies

Here we review a physical model of the temporal evolutions of the downstream ionization balance and ion internal energy. Let \( V \) be a fluid parcel volume. The parcel contains a mass of \( M \). We assume that species within the parcel always have the Maxwell velocity distribution function with a temperature of \( T_j \), where the subscript \( j \) indicates the species \( j \). Then, the internal energy \( E_j \) and pressure \( P_j \) of the species \( j \) can be written as

\[
E_j = \frac{N_j k T_j}{\gamma - 1}, \quad P_j = n_j k T_j, \quad \text{(1)}
\]

where \( N_j, n_j, \) and \( k \) are the total number of the species \( j \), the number density of the species \( j \) \((n_j = N_j/V)\), and Boltzmann constant, respectively. The adiabatic index is \( \gamma = 5/3 \). From the first law of thermodynamics, we obtain

\[
dE_j \frac{dt}{dt} + P_j \frac{dV}{dt} = dQ_j \frac{dt}{dt}, \quad \text{(3)}
\]

where \( dQ_j/dt \) is the external energy gain or loss per unit time of the species \( j \); we discuss it later. Defining the internal energy per unit volume \( \epsilon_j \equiv E_j/V \) and the external energy gain or loss per unit time per unit volume \( \dot{q}_j \equiv V^{-1} dQ_j/dt \), we rewrite the equation (3) as

\[
\frac{d\epsilon_j}{dt} = \dot{q}_j + \frac{\gamma \epsilon_j}{\rho} \frac{dp}{dt}, \quad \text{(4)}
\]

where \( \rho \equiv M/V \) is the total mass density. In this paper, we suppose the case of young SNRs and approximate their dynamics by the Sedov-Taylor model (Sedov 1959; Vink 2012). Then, we approximate the downstream velocity profile as

\[
v(r, t) = \left(1 - \frac{1}{r_c} \right) \frac{V_{sh}(t)}{R_{sh}(t)} r, \quad \text{(5)}
\]

where \( r \) is the radial distance from the explosion center and \( r_c \) is the compression ratio, respectively. The radius of the SNR and the shock velocity are given by, respectively,

\[
R_{sh}(t) = R_0 \left( \frac{t}{t_0} \right)^{2/5}, \quad \text{(6)}
\]

\[
V_{sh}(t) = \frac{dR_{sh}}{dt} = \frac{2 R_0}{5 t_0} \left( \frac{t}{t_0} \right)^{-3/5}, \quad \text{(7)}
\]

where we have assumed the ambient density structure around the SNR is uniform. The dimensional constants \( R_0 \) and \( t_0 \) are characterized by the combination of the explosion energy of the supernova, the structure of the ejecta, and the ambient density structure. The actual values of \( R_0 \) and \( t_0 \) are not used in our model calculation; we only use \( V_{sh}/R_{sh} = (2/5) t^{-1} \). The temporal evolution of the mass density along the trajectory of the fluid parcel is derived from the continuous equation as

\[
\frac{d\rho}{dt} = -\rho \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 v \right) = -\frac{6}{5} \left( 1 - \frac{1}{r_c} \right) \frac{\rho}{t}. \quad \text{(8)}
\]

To calculate \( \rho \) and \( \epsilon_j \) along the trajectory of the fluid parcel, we introduce the position of the fluid parcel at \( \tilde{r}(t) \) that is derived from the differential equation of

\[
\frac{d\tilde{r}}{dt} = v(\tilde{r}(t), t) = \frac{2}{5} \left( 1 - \frac{1}{r_c} \right) \frac{\tilde{r}}{t}. \quad \text{(9)}
\]

Defining the time \( t_\ast \) when the fluid parcel currently at \( \tilde{r}(t) = r \) crosses the shock, i.e., \( \tilde{r}(t_\ast) = R_{sh}(t_\ast) \), we obtain

\[
\ln \frac{\tilde{r}(t_\ast)}{R_{sh}(t_\ast)} = \frac{2}{5} \left( 1 - \frac{1}{r_c} \right) \ln \frac{t}{t_\ast}, \quad \text{(10)}
\]

where we regard \( r_c = \text{const} \). When we observe the atomic line emissions from the fluid parcel at \( r = \tilde{r}(t_{\text{age}}) \), where
the downstream internal energy and the mass density are written as, respectively,

\[
\frac{d\varepsilon_j}{dt} = \dot{q}_j - \frac{6}{5} \left(1 - \frac{1}{n_r} \right) \frac{\varepsilon_j}{t' + t'}.
\]

(12)

\[
\rho(t') = \rho(t_*) \left(1 + \frac{t' - t_*}{t_*} \right)^{-\frac{5}{2}}.
\]

(13)

Integrating the differential equation of \(\varepsilon_j\) from \(t' = 0\) to \(t' = t_{age} - t_*(r)\) with the shock jump conditions given by \(V_{sh}(t_*)\), we obtain the spatial profile of the downstream internal energy at the observed time \(t = t_{age}\). The age is known for a historical SNR (e.g., SNR RCW 86, SN 1006, Tycho’s SNR, Kepler’s SNR). The shock velocity at the current time can be estimated from the proper motion of the shock. To calculate the rate of the Coulomb interactions (see below), the number density needs. The density is evaluated from the surface brightness of the X-ray or H\(\alpha\) emissions, for instance.

Here we consider the energy source or sink term \(\dot{q}_j\). The charged particles exchange their momenta and energies via the Coulomb collision. Although the exchange is negligible during the shock transition, the effect becomes important for the long-time evolution in the downstream region. The energy exchange rate is given by (e.g., Spitzer 1962; Itoh 1984)

\[
\dot{q}_{j,\text{Col}} = \sum_{m} \left( n_j \varepsilon_m - n_m \varepsilon_j \right) \frac{z_m^2 z_j^2}{5.87 A_m A_j} \ln \Lambda \left[ \frac{T_m}{A_m} + \frac{T_j}{A_j} \right]^{-3/2}
\]

(14)

where \(A_j\) and \(\ln \Lambda\) are the particle mass in atomic mass units and the Coulomb logarithm. In this paper, we fix \(\ln \Lambda = 30\) for simplicity. For atoms, the energy transfer due to the ionization or recombination may be given by

\[
\dot{q}_{Z,z} = n_e [R_{Z,z} - 1] \varepsilon_{Z,z} - 1 - (R_{Z,z} + K_{Z,z}) \varepsilon_{Z,z}
\]

(15)

\[+ K_{Z,z+1} \varepsilon_{Z,z+1} + \cdots.
\]

where we introduce the notation \(j = \{Z,z\}\) to represent the species with an atomic number \(Z\) and ionic charge state \(z\), respectively (e.g., \(Z = 2\) and \(z = 1\) indicate He\(^{+1}\) or He\(\text{II}\)). The subscript ‘e’ indicates the electron. The electron-ion collision rate per unit time per particle (s\(^{-1}\) cm\(^3\)) is \(R_{Z,z}(T_e)\), and the recombination rate per unit time per particle is \(K_{Z,z}(T_e)\). In this paper, we omit the charge-exchange reactions and the ion impact ionization for simplicity and consider ten atoms H, He, C, N, O, Ne, Mg, Si, S, and Fe with the solar abundance (Asplund et al. 2009). The atomic data used in this paper are the same as Shimoda & Inutsuka (2021): The ionization cross-sections are given by Janey & Smith (1993) for H, and Lennon et al. (1988) for the others. The fitting functions for those data are given by International Atomic Energy Agency.\(^2\)

Table 1 summarises the literature on the recombination rates. Those data are fitted by the Chebyshev polynomials with twenty terms. For the hydrogen-like atoms, the fitting function is given by Kotelnikov & Milstein (2019). The electron number density is given by the charge neutrality condition as

\[
n_e = \sum_{z=1}^{Z} n_{Z,z}.
\]

(16)

and the total number density \(n\) is given by

\[
n = n_e + \sum_{Z=0}^{Z} \sum_{z=0}^{Z} n_{Z,z}.
\]

(17)

For electrons, by following Shimoda & Inutsuka (2021), the radiative and ionization losses are given by

\[
\dot{q}_e = - \sum_{Z=0}^{Z} \sum_{z=0}^{Z} n_e n_{Z,z} R_{Z,z} I_{Z,z} - n_e \sum_{Z,z} n_{Z,z} W_{Z,z},
\]

(18)

where \(I_{Z,z}\) is the first ionization potential of the species \(j = \{Z,z\}\) (we omit the inner shell ionization). The radiative power \(W_{Z,z}\) includes the bound-bound, free-bound, free-free, and two-photon decay. For the continuum components, the formula given by Gronenschild & Mewe (1978) (free-free and two-photon decays) and Mewe et al. (1986) (free-bound) are used. For the bound-bound component, the radiation power per particle is given by

\[
W_{Z,z} = E_{\text{ul}} C_{lu},
\]

(19)

where the emitted photon energy is the subtraction of the upper level energy \(E_u\) and the lower energy level \(E_l\), \(E_{\text{ul}} = E_u - E_l\). The collisional excitation rate per unit time per particle (s\(^{-1}\) cm\(^3\)) is given by (e.g., Osterbrock & Ferland 2006)

\[
C_{lu} = 8.629 \times 10^{-6} \frac{\Omega_{lu} e^{\frac{E_{\text{ul}}}{T_e}} g_i}{g_{i} \sqrt{T_e}},
\]

(20)

where \(g_i\) is the statistical weight of the lower level. The collision strength is

\[
\Omega_{lu} = \frac{8\pi}{\sqrt{3}} \frac{g_i f_{lu}}{E_{\text{ul,Ryd}}} g(T_e),
\]

(21)

where \(f_{lu}\) is the oscillator strength, and \(E_{\text{ul,Ryd}}\) is the photon energy given in the Rydberg unit. The \(g_i\) is the statistical weight of the lower level. The averaged Gaunt factor is

\(^2\) [https://www.iaea.org/resources/databases/aladdin]
\[ \tilde{g}(T_e) = 0.15 + 0.28 \left[ \log \left( \frac{\chi + 1}{\chi} \right) \right] - \frac{0.4}{(1 + \chi)^2}, \]  

(22)

where \( \chi = E_{\text{au}}/kT_e \), is used for the neutral atoms, while \( \tilde{g} = 1 \) is assumed for the ionized atoms. Note that the cooling function mainly depends on the ionization structure rather than \( \tilde{g} \). For the oscillator strength and energy levels, the data table given by the National Institute of Standards and Technology\(^3\) is used. For the calculation of the radiative cooling rate, it is sufficient to consider only the allowed transitions from the ground state. We obtain the net radiation power and thus the net radiative loss by integrating the photon frequency. Here is a summary of the energy source or sink term: \( \dot{q}_i = \dot{q}_{z,0} \) for the neutral atoms, \( \dot{q}_i = \dot{q}_{j,\text{Col}} + \dot{q}_{z,2} \) for the ions, and \( \dot{q}_i = \dot{q}_{j,\text{Col}} + \dot{q}_e \) for the electrons.

Since the SNR shock may heat the plasma faster than the Coulomb collisions due to the wave-particle interactions in the plasma, the ionization state of atoms can significantly deviate from the ionization equilibrium. Thus, we simultaneously solve the atomic rate equations

\[
\frac{d}{dt} \left( \frac{n_{Z,z}}{\rho} \right) = \nu \left[ R_{Z,z-1} \frac{n_{Z,z-1}}{\rho} - (R_{Z,z} + K_{Z,z}) \frac{n_{Z,z}}{\rho} \right] + K_{Z,z+1} \frac{n_{Z,z+1}}{\rho}.
\]

(23)

Note that in our formulation, the velocity distribution function of the species is always assumed to be the Maxwellian.

### 3 Shock jump conditions

Here we give the initial conditions for the temporal developments of the downstream ionization balance and temperature relaxation by considering shock jump conditions. We introduce the conditions usually supposed in the SNR shocks from analogs of collisional shocks and the novel condition given by modeling the energy exchange between electromagnetic fields and particles.

#### 3.1 collisional shock model (Model 0, 1, and 2)

For the pre-shock gas (denoted by the subscript ‘0’), we set \( T_{j,0} = T_0 = 3 \times 10^4 \text{ K} \) with assuming the collisional ionization equilibrium and temperature equilibrium. In this condition, the fraction of the neutral atoms is \( \sim 1.3 \times 10^{-2} \) in the number. For the downstream values (denoted by the subscript ‘2’), the fraction of the charged particles, assuming a negligibly small magnetic field at the upstream region (or a parallel shock), we consider the total flux conservation laws as

\[
\rho_0 v_0 = \rho_2 v_2, 
\]

(24)

\[
\rho_0 v_0^2 + P_0 = \rho_2 v_2^2 + P_2, 
\]

(25)

\[
\frac{\rho_0 v_0^3}{2} + (\varepsilon_0 + P_0) v_0 = \frac{\rho_2 v_2^3}{2} + (\varepsilon_2 + P_2) v_2, 
\]

(26)

where the total pressure is \( P = \sum_j P_j \), and the total internal energy is \( \varepsilon = \sum_j \varepsilon_j \), respectively. The mass density of the species \( j \) is \( \rho_j = m_j n_j \), where \( m_j \) is the particle mass, and the total mass density is \( \rho = \sum_j \rho_j \). The compression ratio \( r_c \) and total pressure jump \( x_c \) are derived as

\[
r_c \equiv \frac{\rho_2}{\rho_0} = \frac{v_0}{v_2} = \frac{(\gamma + 1) M_s^2}{(\gamma - 1) M_s^2 + 2}, 
\]

(27)

\[
x_c \equiv \frac{P_2}{P_0} = \frac{\varepsilon_2}{\varepsilon_0} = \frac{2\gamma M_s^2 - (\gamma - 1)}{\gamma + 1}, 
\]

(28)

where \( M_s \equiv v_0 / \sqrt{\gamma P_0 / \rho_0} \) is the sonic Mach number defined by the total pressure and mass density. For each species \( j \), we assume the flux conservation laws as

\[
\rho_{j,0} v_0 = \rho_{j,2} v_2, 
\]

(29)

\[
\rho_{j,0} v_0^2 + P_{j,0} = \rho_{j,2} v_2^2 + P_{j,2}, 
\]

(30)

\[
\frac{\rho_{j,0} v_0^3}{2} + (\varepsilon_{j,0} + P_{j,0}) v_0 = \frac{\rho_{j,2} v_2^3}{2} + \left( \varepsilon_{j,2} + P_{j,2} \right) \frac{\rho_{j,0}}{\rho_0} v_2, 
\]

(31)

where we have assumed that the downstream ion velocities are the same as each other (\( v_{j,2} = v_2 \)), and that the downstream internal energy \( \varepsilon_{j,2} = (\rho_{j,0}/\rho_0) \varepsilon_2 \) and pressure \( P_{j,2} = (\rho_{j,0}/\rho_0) P_2 \) are proportional to the upstream kinetic energy \( \rho_{j,0} v_0^2 / 2 \). The downstream temperature of the species \( j \), \( k T_{j,2} = P_{j,2} / n_{j,2} = (P_2 / \rho_2) m_j \), is derived as

\[
k T_{j,2} = \frac{m_j v_0^2}{r_c} \left( 1 - \frac{1}{r_c} + \frac{1}{\gamma M_s^2} \right) \left( M_s^2 - \frac{\gamma - 1}{\gamma} \right) \left( M_s^2 - 1 \right), 
\]

(32)

where \( v'_0 = v_0 - v_2 \) is the upstream velocity measured in the downstream rest. In the strong shock limit with \( \gamma = 5/3 \), we obtain the relation of \( (3/2) k T_{j,2} = m_j v_0^2 / 2 + \varepsilon_{j,0} \). This corresponds that the widths of the Maxwell velocity distribution function of each species are the same. The neutral particles do not form the shock structure because they do not interact with the electromagnetic fields. Then, for the neutral particles, we approximately adopt \( \varepsilon_{j,2} = \rho_{j,0} v_0^2 / 2 + \varepsilon_{j,0} \). We will refer this collisional shock model to Model 0.
To investigate the effects of the electron heating around the shock transition region, we parameterize the energy exchange between protons and electrons as

$$\varepsilon_{p,2} = \varepsilon_2 \frac{p_{e,0}}{p_0} (1 - f_{eq})$$

$$\varepsilon_{e,2} = \varepsilon_2 \left( \frac{p_{e,0}}{p_0} + \frac{p_{e,0}}{p_0} f_{eq} \right)$$

where the subscript ‘p’ denotes the proton. The degree of equilibrium is represented by the parameter $f_{eq}$ that is related to the temperature ratio as follows

$$\frac{\varepsilon_{e,2}}{\varepsilon_{p,2}} = \frac{n_{e,2}}{n_{p,2}} \frac{f_{e,2}}{f_{p,2}} = \frac{(\varepsilon_{e,0}/\varepsilon_{p,0}) + f_{eq}}{1 - f_{eq}}$$

Introducing $\beta = T_{e,2}/T_{p,2}$, we obtain

$$f_{eq} = \frac{n_{e,0} \beta - (n_{e,0}/n_{p,0}) \beta}{n_{p,0} 1 + (n_{e,0}/n_{p,0}) \beta}$$

We consider the cases of $\beta = 0.01$ (Model 1) and $\beta = 0.1$ (Model 2). Although the electron might exchange its internal energy with other ions, we omit this possibility for simplicity. Complete treatments of the electron heating around the shock may need to solve the nature of electromagnetic fields and wave-particle interactions in detail, and this issue is unsettled yet (e.g., Ohira & Takahara 2007; Ohira & Takahara 2008; Rakowski et al. 2008; Laming et al. 2014).

### 3.2 Collisionless shock model (Model 3, 4, and 5)

Here we consider another way of giving a shock transition with the CR acceleration. We assume that a part of shock kinetic energy is consumed for the generation of the CRs and the amplification of the magnetic field. The generated magnetic field is assumed to be disturbed (not an ordered field). In this model, we consider the randomization of the particles incoming from the far upstream region at the shock transition region. The randomization is quantified by the entropy. We notice that the ‘randomization’ results in a more isotropic particle distribution downstream than the pre-shock one (measured in the shock rest frame). In
the collisional shocks, it may be called as the ‘thermalization’, however, the particle distribution may deviate from the Maxwellian in the collisionless shocks. We use the ‘randomization’ for both collisional and collisionless shocks in the following.

Conservation laws of total mass and momentum flux can be written as

$$\rho_0 v_0 = \rho_2 v_2$$  \hspace{1cm} (37)
$$\rho_0 v_0^2 + P_0 + F_{\text{esc}} = \rho_2 v_2^2 + P_2 + \frac{\delta B^2}{4\pi} + P_{\text{cr}},$$  \hspace{1cm} (38)

where the generated (turbulent) magnetic-field strength is $\delta B$. We regard that the field with $\delta B$ has a coherent length scale (injection scale of turbulence) much larger than the Larmor radius of the thermal particles with a velocity of $\sim v_0$ and that the turbulence cascades to the smaller scale. The disturbances associated with the field are assumed to randomize the thermal particles by the wave-particle interactions. The CR pressure of the species $j$ is defined as $P_{\text{cr},j}$ and the total CR pressure is $P_{\text{cr}} = \sum_j P_{\text{cr},j}$. The net momentum flux of escaping CRs is $F_{\text{esc}} \gtrsim \rho_0 v_0^3/3c$. We neglect the total flux and the flux of each species $j$ in this article ($F_{\text{esc}} = 0$ and $F_{\text{esc},j} = 0$). For each species denoted by the subscript $j$, we give the flux conservation laws as

$$\rho_{j,0} v_0 = \rho_{j,2} v_2,$$
$$\rho_{j,0} v_0^2 + P_{j,0} = \rho_{j,2} v_2^2 + P_{j,2} + \frac{\delta B^2}{4\pi} + P_{\text{cr},j},$$  \hspace{1cm} (40)

where we assume a contribution of the species $j$ for the magnetic field amplification and nonthermal pressure is proportional to the upstream kinetic energy $\rho_{j,0} v_0^2/2$. The compression ratios of the species $j$ are equal ($v_{2,j} = v_2$). From these conservation laws, we can derive the relation between the compression ratio $r_{c,j} = \rho_{j,2}/\rho_{j,0} = \rho_{j,2}/\rho_0 \equiv r_c$ and the jump of internal energy $x_{c,j} \equiv \varepsilon_{j,2}/\varepsilon_{j,0}$ as

$$r_c = \left[1 + \frac{1}{\gamma M_{\text{a},j}^2} - \xi_B - \xi_{\text{cr}} \right]^{-1},$$  \hspace{1cm} (41)

or

$$x_{c,j} = 1 + \frac{1}{\gamma M_{\text{a},j}^2} \left(1 - \frac{1}{r_c} - \xi_B - \xi_{\text{cr}} \right),$$  \hspace{1cm} (42)

where $\xi_B \equiv \delta B^2/(4\pi \rho_0 v_0^2)$, $\xi_{\text{cr}} \equiv P_{\text{cr}}/(\rho_0 v_0^2)$, and $M_{\text{a},j} = v_0/\sqrt{\gamma P_{\text{a},j}/\rho_{j,0}}$ is the sonic Mach number defined by the pressure and density of the species $j$. Thus, once another relation between $r_c$ and $x_{c,j}$ is found, we can derive the shock jump condition with given $\xi_B$ and $\xi_{\text{cr}}$. As usual, the energy flux conservation is considered by modeling the magnetic field amplification and the injection rate of nonthermal particles. Since we focus on the downstream ion temperature, we consider the randomization process of thermal ions rather than modeling the behavior of nonthermal particles. Thus, we consider the entropy production of the thermal particles explicitly.

The entropy of the species $j$ per unit mass is defined as

$$ds_j = \frac{dQ_j}{M_j kT_j},$$  \hspace{1cm} (43)

where $dQ_j$ is the energy transferred from electromagnetic fields to the internal energy of the species $j$ due to the shock transition, and $M_j = N_j m_j$ is the total mass of the species $j$ within the fluid parcel.\footnote{This definition corresponds to a reversible process. The collisionless shock is formed by the wave-particle interaction, which might be a reversible process like a plasma echo, for example. Although it may be an unsettled issue, we apply this definition of entropy in this article. Note that the entropy is defined as a non-dimensional value differing from the usual dimensional definition in thermodynamics, $dS = dQ/T$.} Note that $dQ_j = dE_j + P_j dV$ indicates only the increment of the internal energy rather than the total kinetic energy of the thermal particles (a sum of the bulk motion and the random motion). The upstream total kinetic energy of the thermal particles is divided into $\delta B$ and $P_{\text{cr}}$. Substituting $dQ_j = dE_j + P_j dV$ to the equation (43), and using the relation of $ds_j = d(\rho_1 e_1) = e_j d\rho_j + \rho_j d\varepsilon_j$, where $e_j \equiv E_j/M_j$, we can derive the change of the internal energy per unit volume as

$$\frac{d\varepsilon_j}{\varepsilon_j} = \frac{\gamma}{\rho_j} \frac{d\rho_j}{\rho} + (\gamma - 1) m_j ds_j.$$  \hspace{1cm} (44)

Note that we have presumed that $N_j$ is constant during the shock transition. Thus, we obtain the entropy jump before and after the shock transition, $\Delta s_j = s_{j,2} - s_{j,0}$ as

$$\langle \gamma - 1 \rangle m_j \Delta s_j = \ln \left( \frac{\varepsilon_{j,2}}{\varepsilon_{j,0}} \right) - \gamma \ln \left( \frac{\rho_{j,2}}{\rho_{j,0}} \right) = \ln x_{c,j} - \gamma \ln r_c.$$  \hspace{1cm} (45)

Then, the jump conditions are derived by estimating $\Delta s_j$ independently from the equation (45). Since the SNR shock is expected to be formed by the wave-particle interactions, the transferred energy in total $\Delta \tilde{Q}_j$ may be around $\sim J_j \cdot \mathbf{E} \Delta t_j$, where $J_j$ is the electric current of species $j$. The electric field measured in the comoving frame of the ions is $\mathbf{E}$. $\Delta t_j$ is a time taking the shock transition. We estimate each value as $J_j \sim q_j N_j \langle \tilde{v}_j \rangle$, $E \equiv \langle |\tilde{E}| \rangle \sim \langle |\langle \tilde{E} \rangle| \rangle / c \delta B$, and $\Delta t_j \sim m_j c/q_j \delta B$, where $q_j$ is the electric charge of the species $j$, $\langle \tilde{v}_j \rangle = v_0 + \sqrt{2kT_0}/m_j$ is the mean kinetic velocity of the species $j$, and $c$ is the speed of light, respectively. The transition time scale is assumed to be comparable with an inverse of the cyclotron frequency. In a hybrid simulation solving the particle acceleration (e.g., Ohira 2016b), the shock jump seems to occur at a very small length scale despite a significant amplification of turbulent magnetic fields at the ‘upstream’ region (it may correspond to a shock precursor region in our situation). We regard that...
the randomization of particles resulting in the shock transition mainly occurs at such a very small length scale. Thus, we assume the entropy production due to the shock transition as

$$\Delta s_j = \frac{1}{M_j} \frac{J_i E_d}{k T_j}$$

$$= \frac{1}{M_j} \frac{q_j N_j (\bar{v}_j) (\bar{v}_j)}{c} \frac{2 \xi_j}{q_j}$$

$$= \frac{1}{k T_0} \frac{\xi_j}{x_c,j},$$

where we suppose $T_j \sim T_{j,2}$. Substituting the equation (46) to the equation (45), we obtain the relation between $r_c$ and $x_{c,j}$ as

$$f \equiv \frac{x_{c,j}}{r_c} \left[ \ln x_{c,j} - \gamma \ln r_c \right] - \gamma (\gamma - 1) \frac{\bar{v}_j}{v_0}^2 = 0.$$  

(47)

We solve this equation setting $P_{cr}$, $\delta B$ and $M_{s,j} = v_0/\sqrt{\gamma P_{cr}/p_{c,j}}$ with the equation (41) to derive $x_{c,j}$ in the case of the proton by regarding that the most abundant ions form the shock structure. Then, the compression ratio $r_c$ is derived from the equation (41) by using the derived $x_{c,p}$. The downstream pressures of the other species $j$ are derived from the equation (42) by using the derived compression ratio $r_c$. Note that if we supposed small $\delta B$ and $P_{cr}$, the resultant downstream values would be different from the case of the collisional shock (Model 0) reflecting the different randomization process. In this paper, we consider the most efficiently accelerating CR shock feasible. In such a situation, the CR pressure is a practical function of $\delta B$ because of the energy budget of the shock. The upstream kinetic energy is divided into the thermal energy, the magnetic field, and the CRs. The fraction of the thermal energy is given by the entropy production. The fraction of the magnetic field is treated as a free parameter. Thus, the remaining energy is divided into the CRs.

The left panel of figure 1 shows $f = f(x_{c,j})$ (upper part), $r_c = r_c(x_{c,j})$ (middle part), and $\Delta s_j(x_{c,j})/\Delta s_{j,cr}$ (lower part) for the proton with $\gamma = 5/3$. We set parameters as $\xi_{cr} = 0.5$ (purple line), $\xi_{cr} = 0.3$ (black line), and $\xi_{cr} = 0$ (green line) with fixed values of $v_0 = 4000$ km s$^{-1}$, $T_0 = 3 \times 10^4$ K, and $1/\sqrt{\xi_B} = v_0/(\delta B/\sqrt{4\pi p_0}) = 3$. Note that $M_{s,j} = 197$. (right panel)

Solutions of $f = 0$ with fixed $1/\sqrt{\xi_B} = 3$ and $M_{s,j} = 197$ for the proton. The horizontal axis shows the CR fraction $\xi_{cr}$, and the vertical axis shows the pressure jump $x_{c,j}/M_{s,j}^2$. The color indicates the compression ratio $r_c$.

Fig. 1. (left panel) The function of $f = f(x_{c,j})$ defined in the equation (47) (upper part), the compression ratio $r_c = r_c(x_{c,j})$ (middle part), and $\Delta s_j(x_{c,j})/\Delta s_{j,cr}$ (lower part) for the proton with $\gamma = 5/3$. We set parameters as $\xi_{cr} = 0.5$ (purple line), $\xi_{cr} = 0.3$ (black line), and $\xi_{cr} = 0$ (green line) with fixed values of $v_0 = 4000$ km s$^{-1}$, $T_0 = 3 \times 10^4$ K, and $1/\sqrt{\xi_B} = v_0/(\delta B/\sqrt{4\pi p_0}) = 3$. Note that $M_{s,j} = 197$. (right panel)

Solutions of $f = 0$ with fixed $1/\sqrt{\xi_B} = 3$ and $M_{s,j} = 197$ for the proton. The horizontal axis shows the CR fraction $\xi_{cr}$, and the vertical axis shows the pressure jump $x_{c,j}/M_{s,j}^2$. The color indicates the compression ratio $r_c$.

precise explanation about these two solutions that may require a full understanding of the ion heating by the kinetics theory, we may be able to interpret them from resultant downstream values. Let us consider the case of $\xi_{cr} = 0$ in which $\Delta s_j/\Delta s_{j,cr} \approx 1$ around each solution. We will refer to the solution giving $x_{c,j}/M_{s,j}^2 \approx 0.17$ and $r_c \approx 1.27$ as ‘solution A’, while we will refer to the other solution giving $x_{c,j}/M_{s,j}^2 \approx 1.28$ and $r_c \approx 8.31$ as ‘solution B’. The resultant temperature $(T_{j,2}/T_0 = x_{c,j}/r_c \approx 0.1)(v_0^2/\gamma k T_0)$ is almost the same as each other. This means that the speed of particles’ random motion is almost the same as each solution. On the other hand, the difference in the compression ratios indicates that the speed of particles’ bulk motion is significantly different from each other. In a collisional shock in the strong shock limit, the downstream temperature satisfies $(3/2)k T_2 = m v_0^2/2$, where $v_0 = v_0 - v_c$ is the upstream velocity measured in the downstream rest frame and we use $\gamma = 5/3$. This might mean that since our shock consumes its energy for the generation of the nonthermal
components, the random motion speed measured in the downstream rest frame \( \tilde{v}_R \equiv \sqrt{3kT_{j,2}/m_j} \) should be equal or smaller than \( v_0 = v_0 - v_2 \) for the solution representing the shock transition (i.e., \( \tilde{v}_R/v_0 \leq 1 \)). Solution A gives the speed as \( \tilde{v}_R/v_0 \approx 2.3 \), while solution B gives \( \tilde{v}_R/v_0 \approx 0.6 \). Hence, solution B may correspond to the shock transition. Solution A should be rejected because it does not satisfy the energy flux conservation law.

When \( \xi_{\text{cr}} \) becomes large, the two solutions approach with each other, coinciding at \( \xi_{\text{cr}} \approx 0.3 \) (multiple roots), and finally, the solution vanishes. The multiple roots \( \xi_{\text{cr}} \approx 0.3 \) give \( \tilde{v}_R/v_0 \approx 0.7 \) and \( \Delta s_j/\Delta s_{j,\text{ncr}} \approx 0.93 \). Thus, the multiple roots may represent the shock transition giving the maximum \( P_c \) feasible in our shock model. In this article, we set the maximum \( \xi_{\text{cr}} \) to compare the no CR cases with the case of extremely efficient CR acceleration. The maximum \( \xi_{\text{cr}} \) is derived from the multiple roots of \( f = 0 \) with given \( \xi_B \).

For the case of \( v_0 = 4000 \text{ km s}^{-1} \) and \( T_0 = 3 \times 10^4 \text{ K} \) with given \( 1/\sqrt{\xi_B} = 3 \), we obtain the maximum acceptable CR production \( \xi_{\text{cr}} \approx 0.3 \), \( \Delta s_j/\Delta s_{j,\text{ncr}} \approx 0.92-0.95 \) depending on \( m_j, r_c \approx 3.29 \), and \( kT_{p,2} \approx 14.4 \text{ keV} \). Note that in the case of Model 0 (the usual collisional shock case), we obtain \( r_c = 4.00 \) and \( kT_{p,2} = 31.3 \text{ keV} \). The fraction of the CRs \( \xi_{\text{cr}} = 0.3 \) seems to be reasonable for the SNR shocks as sources of Galactic CRs. From the subtraction of the energy fluxes of the thermal particles at the far upstream and downstream, we can regard that roughly 50% of the upstream energy flux is transferred to the nonthermal components. The fraction of magnetic pressure \( 1/\sqrt{\xi_B} = 3 \) corresponds to magnetic-field strength of \( \delta B \approx 611 \mu G (v_0/4000 \text{ km s}^{-1}) (n_{p,0}/1 \text{ cm}^{-3})^{-1/2} \) which is consistent with estimated strength from X-ray observations of young SNRs (e.g., Vink & Laming 2003; Bamba et al. 2005; Uchiyama et al. 2007). Thus, our parameter choice of \( 1/\sqrt{\xi_B} = 3 \) can be reasonable to adopt our model to the young SNR shocks.

Here we consider about the choice of the maximum \( \xi_{\text{cr}} \). In the case of the collisional shock formed by the hard-sphere collisions, for example, the collision result in one of the most efficient randomizations of particles. Thus, the collisional shock can ‘easily’ dissipate its kinetic energy within the mean collision time. In the collisionless plasma, such efficient randomization process is absent. The particles in the plasma tend to behave as ‘nonthermal’ particles resulting in a generation of electromagnetic disturbances by themselves. The collisionless shock is formed by the self-generated disturbances so that almost all particles become thermal particles. Although the number of the nonthermal particles is very smaller than the number of the thermal particles, the efficient randomization caused by the nonthermal particles is required to form the collisionless shock. Our choice of the maximum \( \xi_{\text{cr}} \) corresponds that the effect is minimized per one nonthermal particle.

![Fig. 2](image_url)

**Fig. 2.** The maximum \( \xi_{\text{cr}} \) derived from \( f = 0 \) as a function of \( 1/\sqrt{\xi_B} = v_0/(\delta B/\sqrt{4\pi n_0}) \) for given the shock velocity \( v_0 \) and \( T_0 = 3 \times 10^4 \text{ K} \). The heavy black solid line shows \( M_{e,p} = 197 \) (\( v_0 = 4000 \text{ km s}^{-1} \)). The vertical thin line indicates \( 1/\sqrt{\xi_B} = 3 \). For a comparison, we display \( M_{e,p} = 19.7 \) (purple, \( v_0 = 400 \text{ km s}^{-1} \)), 39.4 (green, \( v_0 = 800 \text{ km s}^{-1} \)), 78.8 (light blue, \( v_0 = 1600 \text{ km s}^{-1} \)), 158 (orange, \( v_0 = 3200 \text{ km s}^{-1} \)), 315 (blue, \( v_0 = 6400 \text{ km s}^{-1} \)), and 492 (red, \( v_0 = 10000 \text{ km s}^{-1} \)).

Figure 2 shows the maximum \( \xi_{\text{cr}} \) derived from \( f = 0 \) as a function of \( 1/\sqrt{\xi_B} = M_{e,p} \) for \( M_{e,p} = 197 \). The fraction \( \xi_{\text{cr}} \) drops around \( 1/\sqrt{\xi_B} \leq 3 \) but is flattened for \( 1/\sqrt{\xi_B} \gtrsim 3 \). This depletion of the maximum \( \xi_{\text{cr}} \) is qualitatively obvious in terms of the energy budget of the shock; the upstream kinetic energy is divided into the thermal components, \( P_c \) and \( \delta B \). The fraction of \( \delta B \) is a given parameter. The fraction of the thermal components and the maximum fraction of \( P_c \) are derived from the entropy production. We will refer to this model with \( 1/\sqrt{\xi_B} = 3 \) and the maximum \( \xi_{\text{cr}} \) as Model 3.

![Fig. 3](image_url)

**Fig. 3.** The initial, downstream ion temperatures divided by \( 2Z \) (i.e., the particle mass in atomic mass unit) for Model 0 and Model 3 with \( v_0 = 4000 \text{ km s}^{-1} \) and \( T_0 = 3 \times 10^4 \text{ K} \). The black square shows the results of Model 0, and the red square shows the results of Model 3. The results of Model 1 and Model 2 are the same as the results of Model 0, respectively. The horizontal axis shows the Atomic number \( Z \).
Figure 3 shows the results of downstream ion temperatures divided by $2Z$ (i.e., the particle mass in atomic unit) for Model 0 and Model 3 with $v_0 = 4000 \text{ km s}^{-1}$ and $T_0 = 3 \times 10^4 \text{ K}$. The reduced temperatures $kT_{2,s}/2Z$ of Model 3 do not depend on the particle mass, indicating that the temperature ratios between the ions are equal to their ion mass ratio. Such mass-proportional ion temperatures are observed at SN 1987A (Miceli et al. 2019). The temperature jump $T_{j,2}/T_0$ is given by $x_{c,j}/r_c \sim M_{s,j}$. The relation of $x_{c,j}/r_c \sim M_{s,j}$ is also implied by the condition of $f = 0$. Thus, Model 3 predicts that the ion temperature ratio is given by the mass ratio, similar to the case of Model 0. Note that Model 1 and Model 2 give ion temperatures almost the same as Model 0. On the other hand, $kT_{2,s}/2Z$ of Model 3 is smaller than the case of Model 0 by a factor of 2 due to the generations of the nonthermal components.

The existence of more than two solutions is usually seen in the CR accelerating shock model (e.g., Drury & Voelk 1981; Vink et al. 2010; Vink & Yamazaki 2014, and references therein). The unphysical solution like Solution A of our model, which does not satisfy the energy flux conservation law, also exists in previous studies. The essential difference between our model and previous studies is the randomization process, which determines the downstream thermal energy, may be implicated in the shock transition. In the previous studies, the randomization process, which determines the downstream thermal energy, may be implicitly chosen to satisfy the flux conservation laws with assumed parameters ($P_{cr}$, energy flux of escaping CRs, etc.). Vink & Yamazaki (2014) also derived a critical sonic Mach number $M_{s,cr} = \sqrt{5}$ below which the particle acceleration should not occur. In our model, a similar sonic Mach number may be derived from conditions of $\Delta s_j \leq \Delta s_{j,ncr}$ and $T_{j,2} \leq T_{j,2,ncr}$, where $T_{j,2,ncr}$ is given by Model 0 (the equation 32). The former states that the generated entropy in the collisionless shock should be smaller than in the case of collisional shocks. The latter states that the downstream temperature should be smaller than the case of adiabatic, collisional shocks without the CRs. Note that the entropy and temperature must be determined independently to derive the density or pressure in thermodynamics. In other words, the conditions $\Delta s_j \leq \Delta s_{j,ncr}$ and $T_{j,2} \leq T_{j,2,ncr}$ are independent with each other. From the equations (48) and (46), and using the relation of $m_j v_0^2/kT_0 = (\rho_j/\rho_0)\gamma M_s^2$, we can derive

$$\frac{T_{j,2,ncr}}{T_0} \geq \frac{T_{j,2}}{T_0} \geq \frac{\rho_{j,0}}{\rho_0} \ln \left(\frac{T_{j,2,ncr}}{T_0} - (\gamma - 1)\ln r_{c,ncr}\right),$$

where

$$\frac{T_{j,2,ncr}}{T_0} = \frac{\rho_{j,0}}{\rho_0} \frac{\gamma M_s^2}{r_{c,ncr}} \left(1 - \frac{1}{r_{c,ncr}} + \frac{1}{r_{c,ncr}} \right).$$

and $r_{c,ncr}$ is given by the equation (27). The critical Mach number $M_{s,ncr}$ is given when the equal sign of the inequality holds. Regarding $\langle \tilde{v}_j \rangle \simeq v_0$ and $\rho_j/\rho_0 \simeq 1$ for simplicity, we obtain the numerical value of $M_{s,ncr} \simeq 16.34$ above which we can find sets of $\xi_s$ and $\xi_0$ satisfying the inequality (49). The larger critical Mach number than that derived by Vink & Yamazaki (2014) may be due to the difference in the assumed randomization process. However, the value of $M_{s,ncr}$ may also depend on the Alfvén Mach number, whose effects are not studied in this paper. When the sonic Mach number decreases due to a shock deceleration, the effects of the mean magnetic field at the far upstream region can be important. The shocks with a lower Mach number are seen in the solar wind at coronal mass ejection events, clusters of galaxy, and so on. In predictions of the accelerated particles amount in such cases, we should include the pre-existing ordered magnetic field to the flux conservation laws and evaluation of $J \cdot E$ term, differing from the current approach. We will study a general critical Mach number with more elaborate models in future work.

Finally, we parameterize the electron heating for the case of the extremely efficient CR acceleration as

$$\varepsilon_{p,2} = \varepsilon_{p,2,Model3}(1 - f_{eq}),$$

$$\varepsilon_{e,2} = \varepsilon_{e,2,Model3} + \varepsilon_{p,2,Model3}f_{eq},$$

where $\varepsilon_{p,2,Model3}$ and $\varepsilon_{e,2,Model3}$ are the internal energy calculated by Model 3. Here we have supposed an additional energy transfer: the internal energy of the thermal protons is transferred to the thermal electrons. The fraction of the transferred internal energy is written by the temperature ratio of $\beta = T_{j,2}/T_{p,2}$ as

$$f_{eq} = \frac{\beta(n_{e,0}/n_{p,0}) - (\varepsilon_{e,2,Model3}/\varepsilon_{p,2,Model3})}{\beta(n_{e,0}/n_{p,0}) + 1}.\tag{52}$$

Laming et al. (2014) pointed that the electron temperature can be significantly large ($\beta \sim 0.1$) when the shock accelerates the CRs efficiently. We calculate the cases of $\beta = 0.01$ (Model 4) and $\beta = 0.1$ (Model 5) in this paper. Table 2 shows a summary of our shock models.

### 4 evolution track of the downstream ionization balance and temperatures

Here we show the results of the ionization balance and temperature relaxation in the downstream region, omitting the effects of the expansion ($dV/dt = 0$) as a reference. For convenience, we introduce $d\tau = ndt$, where $n$ is the total number density, so that

$$\frac{d\tilde{v}_j}{d\tau} \approx \frac{\tilde{q}_j}{n},\tag{53}$$

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Table 2. Summary of the shock jump models. We set $v_0 = 4000 \text{ km s}^{-1}$ and $T_0 = 3 \times 10^4 \text{ K}$. From the left-hand side to the right-hand side, the columns indicate the model name, the compression ratio $r_c$, the downstream proton temperature $kT_{p,2}$, the downstream electron temperature $kT_{e,2}$, the fraction of the amplified magnetic field $\xi_B = \delta B^2 / (4\pi \rho_0 v_0^2)$, and the fraction of the CR pressure $\xi_{cr} = P_{cr} / (\rho_0 v_0^2)$.

| Model | $r_c$ | $kT_{p,2}$ | $kT_{e,2}$ | $\xi_B$ | $\xi_{cr}$ |
|-------|-------|------------|------------|---------|------------|
| 0     | 4     | 31.32 keV  | 17.1 eV    | 0       | 0          |
| 1     | 4     | 31.01 keV  | 31.0 eV (0.01) | 0   | 0          |
| 2     | 4     | 28.34 keV  | 2.83 keV (0.1) | 0   | 0          |
| 3     | 3.29  | 14.38 keV  | 8.62 eV (1.1$m_e/m_p$) | 1/9 | 0.30       |
| 4     | 3.29  | 14.23 keV  | 14.2 eV (0.01) | 1/9 | 0.30       |
| 5     | 3.29  | 13.01 keV  | 1.30 keV (0.1) | 1/9 | 0.30       |

Fig. 4. The ionization balance $n_{Z,z}/n_Z$ (left panel) and temperature $kT_{Z,z}$ of the most abundant species among its ionic charge (right panel) for Model 0 with the shock velocity of $v_0 = 4000 \text{ km s}^{-1}$. We display the species He, C, and N. The color represents the ionic charge $z$ of ion. The solid black line and black dots are the temperatures of proton and electron, respectively.

Fig. 5. The ionization balance $n_{Z,z}/n_Z$ (left panel) and temperature $kT_{Z,z}$ of the most abundant species among its ionic charge (right panel) for Model 0 with the shock velocity of $v_0 = 4000 \text{ km s}^{-1}$. We display the species O, Ne, and Mg. The color represents the ionic charge $z$ of ion. The solid black line and black dots are the temperatures of proton and electron, respectively.

$$\frac{dn_{Z,z}}{d\tau} = \frac{n_e}{n} [R_{Z,z-1}n_{Z,z-1} - (R_{Z,z} + K_{Z,z}) n_{Z,z} + + K_{Z,z+1}n_{Z,z+1}],$$

where we use $\rho =$const. Figures 4 (for He, C, and N), 5 (for O, Ne, and Mg), and 6 (for Si, S, and Fe) show $n_{Z,z}/n_Z$ and $kT_{Z,z}$ for Model 0 with the shock velocity of $v_0 = 4000 \text{ km s}^{-1}$, where $n_Z = \sum n_{Z,z}$ is the total number density of the atoms with the atomic number $Z$. Note that $\tau = \int n dt \approx nt$ because of the small neutral fraction. Here we display the ion temperatures for the most abundant species among its ionic charge.

The evolution tracks of $n_{Z,z}/n_Z$ and $T_{Z,z}$ for other models are not so different from the case of Model 0. In the case of a higher electron temperature (Model 1 and Model 2), the ions are quickly ionized. Figure 7 shows the electron temperatures for Model 0, 1, 2, and 3 with $v_0 = 4000 \text{ km s}^{-1}$. The relation between Model 4 and Mode 1 (Model 5 and Model 2) is similar to that of Model 3 and Model 0. The ionization balance $n_{Z,z}/n_Z$ becomes the same in each model after the electron temperature coincides. Note that the electron temperature increases within a column density scale of $ntV_{sh} \sim 10^{14} \text{ cm}^{-2}(nt/10^6 \text{ cm}^{-3}s)(V_{sh}/4000 \text{ km s}^{-1})$. This col-
The electron temperatures for Model 0 (black dots), 1 (purple dots), and 2 (green dots) with $v_0 = 4000 \text{ km s}^{-1}$. The solid black line shows the proton temperature for Model 0. The red solid line and red dots are the proton and electron temperatures of Model 3, respectively.

The ionization balance $n_{Z,z}/n_Z$ of the most abundant species among its ionic charge (right panel) for Model 0 with the shock velocity of $v_0 = 4000 \text{ km s}^{-1}$. We display the species Si, S, and Fe. The color represents the ionic charge $z$ of ion. The solid black line and black dots are the temperatures of proton and electron, respectively.

Fig. 7. The electron temperatures for Model 0 with $\rho_0 = 4.08 \times 10^{-3} \text{ g cm}^{-3}$, $t_{age} = 1836 \text{ yr}$, $V_{sh}(t_{age}) = 3000 \text{ km s}^{-1}$, and $r/R_{sh}(t_{age}) = 0.8$. We display the species He, C, N (top left panels), O, Ne, Mg (top right panels), Si, S, and Fe (bottom panels). The color represents the ionic charge $z$ of the ion. The horizontal axis shows the time $t' = t - t_s$, where $t_s$ is the shock transition time of the fluid parcel currently at $r/R_{sh}(t_{age}) = 0.8$.
When the effects of the expansion become important, we cannot characterize the evolution only by \( \tau \approx nt \) and we should introduce parameters to describe the expansion of SNRs and the observed position \( \tau / R_{sh}(t_{age}) \). Here we set \( \rho_0 = 4.08 \times 10^{-2} \text{m}_p \), \( t_{age} = 1836 \text{ yr} \), and \( V_{sh}(t_{age}) = 3000 \text{ km s}^{-1} \) for example. This parameter set will be used in comparisons of our Model to the SNR RCW 86 (discuss later in Sect. 5). Figure 8 shows the downstream ionization structure of He, C, N (top panels), O, Ne, Mg (middle panels), Si, S, and Fe (bottom panels). The fluid parcel currently at \( \tau / R_{sh}(t_{age}) = 0.8 \) crossed the shock at the time of \( t_* \) when the shock velocity was \( V_{sh}(t_*) = 5094 \text{ km s}^{-1} \) for Model 0, 1, and 2 (4565 km s\(^{-1}\) for Model 3, 4, and 5). Since the compression ratio depends on whether the CRs exist, the shock transition time \( t_* \), shock velocity \( V_{sh}(t_*) \), and \( T_e(t_*) \) are different from each model for the fluid parcel currently at \( \tau / R_{sh}(t_{age}) \). The evolution of the ionization balance is similar to the case of the plane-parallel shock until \( t' \sim 10^9 - 10^{10} \text{ s} \sim t_{age} \). The cooling due to the expansion becomes important at \( t' \sim t_{age} \). The ion temperatures decrease before the ions are well ionized due to the expansion (decreasing of the density, ion temperature, and electron temperature). Figure 9 shows the electron temperatures for Model 0 (black dots), 1 (purple dots), 2 (green dots), 3 (red dots), 4 (orange dots), and 5 (blue dots).

5 Synthetic Observations

In this section, we perform synthetic observations of the shocked plasma considering the effects of turbulence for the case of the SNR RCW 86. Since we do not calculate the overall spectrum of the emitted photons which needs enormous calculations about emission lines, we mainly estimate the line shape.

The SNR RCW 86 is one of the best targets for the study of the CR injection via the ion temperatures because the shells of the SNR show different thermal/nonthermal features from position to position (Bamba et al. 2000; Borkowski et al. 2001; Tsubone et al. 2017). The RCW 86 is considered as a historical SNR of SN 185 (Vink et al. 2006). Thus, we set \( t_{age} = 1836 \text{ yr} \). Along the northeastern shell of the RCW 86, the dominant X-ray radiation changes from thermal to synchrotron emission (Vink et al. 2006). The thermal emission-dominated region is referred to ‘E-bright’ region, and the synchrotron one is referred to ‘NE’ region. The ionization age at NE is estimated as \( \tau = (2.25 \pm 0.15) \times 10^9 \text{ cm}^{-3} \text{ s} \) though this estimate potentially contains errors due to the lack of the thermal continuum emissions (Vink et al. 2006). The E-bright region is fitted by two plasma components: (i) \( \tau = (6.7 \pm 0.6) \times 10^9 \text{ cm}^{-3} \text{ s} \), and (ii) \( \tau = (17 \pm 0.5) \times 10^9 \text{ cm}^{-3} \text{ s} \). Both E-bright and NE show clear O VII Heo and Ne IX Heo line emissions. From the width of the synchrotron-emitting region (NE), the magnetic-field strength is estimated as \( \approx 24 \pm 5 \mu \text{G} \) (Vink et al. 2006).

Yamaguchi et al. (2016) measured proper motions around these regions (not exactly the same regions) as \( v_0 = 720 \pm 360 \text{ km s}^{-1} \) (E-bright), \( v_0 = 1780 \pm 240 \text{ km s}^{-1} \) (upper part of NE referred to ‘Ne0’), and \( v_0 = 3000 \pm 340 \text{ km s}^{-1} \) (lower part of NE referred to ‘Ne1’). In the case of Model 3, the fractions of CR pressure \( \xi_{e0} \) become \( \xi_{e0, 720} \approx 0.14 \) for \( v_0 = 720 \text{ km s}^{-1} \), \( \xi_{e0, 1780} \approx 0.24 \) for \( v_0 = 1780 \text{ km s}^{-1} \), and \( \xi_{e0, 3000} \approx 0.28 \) for \( v_0 = 3000 \text{ km s}^{-1} \), respectively. If we simply suppose \( \rho_0 = (\tau / t_{age}) \mu \text{m}_p \) and adopt \( 1/\sqrt{\mu} \), the CR pressure and \( \delta B \) of each region becomes \( P_{e0, 720} \sim 0.2 \text{ keV cm}^{-3} \) and \( \delta B_{720} \sim 51 \mu \text{G} \), \( P_{e0, 1780} \sim 0.3 \text{ keV cm}^{-3} \) and \( \delta B_{1780} \sim 55 \mu \text{G} \), and \( P_{e0, 3000} \sim 1.1 \text{ keV cm}^{-3} \) and \( \delta B_{3000} \sim 93 \mu \text{G} \), where we adopt \( \tau = 12.0 \times 10^9 \text{ cm}^{-3} \text{ s} \) for the E-bright region as an average of the two components and \( \tau = 2.25 \times 10^9 \text{ cm}^{-3} \text{ s} \) for the NE region, respectively. If we adopt \( v_0 = 360 \text{ km s}^{-1} \) for the E-bright region, we obtain \( \xi_{e0, 360} \sim 2.9 \times 10^{-2} \). The thermal-dominated E-bright region results from the higher density than the density at the NE region. The magnetic-field strength \( \delta B \) is the almost same as one another. Note that Vink et al. (2006) estimated the electron density at the E-bright region as \( \sim 0.6-1.5 \text{ cm}^{-3} \) from the emission measure with assuming the volume of the emission region. Our model predicts the downstream density as \( n_e \rho_0 / \mu \text{m}_p \approx 0.55 \text{ cm}^{-3} \) for the E-bright region with \( v_0 = 720 \text{ km s}^{-1} \) that is consistent with the previous estimate. For the NE region, the number density is not well constrained because of the lack of the thermal continuum component. Thus, our choice of model pa-
rameters can be consistent with the observations of the RCW 86. In the following, we apply our model to the NE region setting the parameters as \( t_{\text{age}} = 1836 \text{ yr} \), \( V_{\text{sh}}(t_{\text{age}}) = 3000 \text{ km s}^{-1} \), and \( \rho_0/m_p = \tau/t_{\text{age}} = 4.08 \times 10^{-2} \text{ cm}^{-3} \), where \( \tau = 2.25 \times 10^9 \text{ cm}^{-3} \text{ s} \) is used. We suppose that the downstream region from \( r = R_{\text{obs}} = 0.88R_{\text{sh}}(t_{\text{age}}) \) to \( r = R_{\text{sh}}(t_{\text{age}}) \) is observed. Then, our model supposes that the expansion follows the Sedov-Taylor model during a time of \( \Delta t = t_{\text{obs}} - t_{\text{sh}}(R_{\text{obs}}) = (1 - (R_{\text{obs}}/R_{\text{sh}})^{2/3}) t_{\text{age}} \approx 0.6t_{\text{age}} \), where \( r_c = 4 \) and the equation (11) are used. If the RCW 86 expands with a velocity of \( \sim 10^9 \text{ cm s}^{-1} \) on average before entering the Sedov-Taylor stage, we effectively assume the radius at the transition time of \( t_0 \approx 0.6t_{\text{age}} \) as \( R_0 \approx 10^9 \text{ cm s}^{-1} \times 0.6t_{\text{age}} \approx 11 \text{ pc} \). Then, the radius at the current time \( R_{\text{sh}}(t_{\text{age}}) \approx R_0(1/6.625)^{3/2} \approx 13.5 \text{ pc} \) which can be consistent with the actual radius of \( \sim 15 \text{ pc} \) (the distance is assumed as 2.5 kpc).

Figure 10 shows the radial profile of the electron temperature at \( t_{\text{age}} = 1826 \text{ yr} \) for the NE region of RCW 86 with \( V_{\text{sh}}(t_{\text{age}}) = 3000 \text{ km s}^{-1} \) and \( \rho_0/m_p = 1.29 \times 10^{-2} \text{ cm}^{-3} \). We display the results of Model 0 (black solid line), Model 1 (purple dots), Model 2 (green broken line), Model 3 (red solid line), Model 4 (orange dots), and Model 5 (blue broken line).

Note that the other models (e.g., Model 0) also result in a large O\,\text{VII} abundance. Model 2 predicts the smaller abundance of O\,\text{VII} than the case of Model 5 because the higher electron temperature results in a faster ionization. The temperature of O\,\text{VII} is approximately \( kT_{Z,Z}(r) \approx 250 \text{ keV} \times |r/R_{\text{sh}}(t_{\text{age}})| \) for Model 2 and \( kT_{Z,Z}(r) \approx 140 \text{ keV} \times |r/R_{\text{sh}}(t_{\text{age}})| \) for Model 5.

We estimate the line emission as follows: the observed specific intensity per frequency \( I_r \) at the sky position \( \mathcal{X} \) from the center of the SNR is calculated as

\[
I_r(\mathcal{X}) = \int_{-L}^{L} \int_{-\infty}^{\infty} J_{\nu}(w_1, v_2) \, dw_1 \, dZ, \tag{55}
\]

where \( L = \sqrt{R_{\text{sh}} - \mathcal{X}^2} \). The position along the line of sight is \( Z \) so that \( r = \sqrt{\mathcal{X}^2 + Z^2} \). \( v_2(r) = (Z/r)v(r) \) is the line of sight velocity. The probability distribution function of the turbulence \( G \) is supposed to be a Gaussian as

\[
G(w_1, v_2) = \frac{1}{\sqrt{\pi \nu_{\text{turb}}}} \exp\left(-\frac{mZ(w_1 - v_2)^2}{2KZ}\right), \tag{56}
\]

where \( \nu_{\text{turb}} \) is a typical turbulent velocity and \( K_Z \equiv (1/2)m_Z\nu_{\text{turb}}^2 \). Note that \( w_1 \) is the variable for the integration. In this paper, we assume that the intensity of the turbulence is proportional to the proton sound speed as \( \nu_{\text{turb}}(r) = \delta \sqrt{\gamma kT_p(r)/m_p} \). Supposing the incompressi-
ible turbulence is driven in the downstream region (see Shimoda et al. 2018a), we calculate the case of $\delta = 0.5$ and the case without the turbulent Doppler broadening $\delta = 0$ for a comparison. The emissivity of the line is given by

$$ J_\nu = \frac{W_{\nu,l}}{4\pi} n_e n_{Z, z} \phi_\nu, \quad (57) $$

where we have neglected the cascade from the higher excitation levels. The line profile function is defined as

$$ \phi_\nu \equiv \frac{1}{\sqrt{\pi} \Delta \nu} \exp \left[ - \left( \frac{\nu - \nu_0}{\Delta \nu} \right)^2 \right], \quad (58) $$

$$ \Delta \nu = \nu_0' \sqrt{\frac{2kT_{Z,z}}{m_Z c^2}}, \quad (59) $$

$$ \nu_0 = \nu_0' \left( 1 + \frac{v_Z}{c} \right), \quad (60) $$

where $\nu_0'$ is the frequency of the line measured in the rest frame of the atom. Then, we obtain

$$ I_\nu(X) = \int_{-L}^{L} \frac{n_e n_{Z, z} W_{\nu,l}}{4\pi \Delta \nu} \sqrt{\frac{1 + M_{Z,z}^2}{\pi}} \exp \left[ - \left( \frac{\nu - \nu_0'(1 + v_Z/c)}{\Delta \nu \sqrt{1 + M_{Z,z}^2}} \right)^2 \right] dZ, \quad (61) $$

where $M_{Z,z}^2 \equiv K_{Z}/kT_{Z,z} = (\gamma \delta^2/2)(m_Z/m_\rho)(T_\rho/T_{Z,z})$. The line shape is broadened by the bulk Doppler effect $(1 + v_Z/c)$ and the turbulent Doppler effect $\sqrt{1 + M_{Z,z}^2}$. Figure 12 shows $I_\nu(X)/I_\nu(0.8R_{sh})$ at the line center for Model 5 (solid lines) and Model 2 (dots). We also display profiles of the column density (green) of O VII. The difference between the column density profile and the intensity profile results from the excitation. The spatial variation of the electron temperature is relatively less important in this case because the excitation rate depends on $\exp(-E_{ul}/kT_e)$, which is not so sensitive to $T_e$ unless $kT_e \ll E_{ul}$.

Figure 13 shows the calculated O VII Heα line for Model 5 (blue solid line) and Model 2 (green solid line) derived from $\int I_\nu dX$ with $\delta = 0.5$. We assume the distance of the RCW 86 is 2.5 kpc and the observed area is $0.2R_{sh} \times 0.2R_{sh}$, where $R_{sh} = 15.27$ pc. We also display the results of Model 0 (black dots), Model 1 (purple dots), Model 3 (red dots), and Model 4 (orange dots). The results show a good agreement with the observed photon counts $\sim 0.15$ counts s$^{-1}$ keV$^{-1}$ (Vink et al. 2006). Table 3 shows a summary of the calculated O VII Heα line. The derived temperatures reflect the effects of the efficient CR acceleration. From the comparison of $\delta = 0.5$ to $\delta = 0$, the turbulent Doppler broadening results in the higher observed temperatures by a factor of $\sim 1.05$. The degree of the broadening can be estimated as $\sqrt{1 + M_{Z,z}^2} \approx 1.1$ for $\delta = 0.5$ with approximating $T_\rho/T_{Z,z} \approx m_\rho/m_Z$. Since the observed line consists of multiple temperature populations, and since a higher temperature population less contributes around the line center, a lower temperature population is accentuated around the line center. The contribution of the higher temperature population appears far from the line center like a 'wing'. If we measure the temperature using the full width at the e-folding scale, the difference in the derived temperatures becomes large. Hence the observed FWHM is smaller than that expected from $\sqrt{1 + M_{Z,z}^2}$.

The RCW 86 also shows bright Ne IX Heα however, our model predicts a faint Ne IX Heα emission (the intensity is smaller than a tenth of O VII Heα intensity). The line intensity also depends on the ion abundance. In this paper, we use the solar abundance that reflects the condition of our galaxy $\approx 4.6$ Gyr ago. Moreover, De Cia et al. (2021) found large variations of the chemical abundance of the neutral ISM in the vicinity of the Sun over a factor of 10 (Si, Ti, Cr, Fe, Ni, and Zn they analyzed). Their findings
magnetic-field amplification driven by the CRs is assumed. The amount of the downstream thermal energy is given. The feature which is given by modeling each ion species’ entropy contribution function are almost the same as each other for $\delta = 0$. We suggest the novel collisionless shock jump condition, $\frac{kT_{\text{Z,n}}}{kT_{\text{Z,n}}/2Z}$ for $\delta = 0$.5, and the O VII temperature derived from the FWHM of the line for the case of $\delta = 0.5$, and the O VII temperature derived from the FWHM of the line for the case of $\delta = 0$.

| Model | $kT_{\text{Z,n}}$ (kT$_{\text{Z,n}}$/2Z) for $\delta = 0.5$ | for $\delta = 0$ |
|-------|-------------------------------------------------|----------------|
| 0     | 325.9 keV (20.4 keV)                            | 312.3 keV (19.5 keV) |
| 1     | 325.6 keV (20.3 keV)                            | 312.4 keV (19.5 keV) |
| 2     | 306.4 keV (19.2 keV)                            | 296.5 keV (18.5 keV) |
| 3     | 160.6 keV (10.0 keV)                            | 153.8 keV (9.62 keV) |
| 4     | 160.6 keV (10.0 keV)                            | 154.2 keV (9.63 keV) |
| 5     | 157.7 keV (9.85 keV)                            | 152.5 keV (9.53 keV) |

Figure 14 represents the line shape with 5 eV resolution for $\delta = 0.5$. We additionally show O VII Ly$\alpha$, Ne IX He$\alpha$, and Ne X Ly$\alpha$ for Model 5 (black solid line) and Model 2 (green solid line).

implies that the gaseous matter is not well mixed. The predicted faint Ne IX-He$\alpha$ might reflect a different abundance pattern from the solar abundance pattern.

### 6 Summary and Discussion

We suggest the novel collisionless shock jump condition, which is given by modeling each ion species’ entropy production at the shock transition region. As a result, the amount of the downstream thermal energy is given. The magnetic-field amplification driven by the CRs is assumed. For the given strength of the amplified field, the amount of the CRs is constrained by the energy conservation law. The constrained amount of the CRs can be sufficiently large to explain the Galactic CRs. The ion temperature is lower than the case without the CRs because the upstream kinetic energy is divided into the CRs and the amplified field. The strength of the field around the shock transition region is assumed to be $1/\sqrt{\xi B} = \delta v_0/(\delta B/\sqrt{4\pi \rho_0}) \simeq 3$. Downstream developments of the ionization balance and temperature relaxation are also calculated. Using the calculated downstream values, we perform synthetic observations of atomic lines for the SNR RCW 86, including the Doppler broadening by the turbulence. Our model predictions can be consistent with the previous observations of the SNR RCW 86, and the predicted line widths are sufficiently broad to be resolved by the XRISM’s micro-calorimeter. Future observations of the X-ray lines can distinguish whether the SNR shock accelerates the CRs or not from the ion temperatures.

Our shock model constrains the maximum fraction of the CRs depending on the shock velocity, the upstream density, and the sonic Mach number (see figure 2). Since the SNR shock decelerates gradually, we can predict a history of the CR injection and related nonthermal emissions, especially the hadronic $\gamma$-ray emissions. Although the injection history of the CRs is essential to estimate the intrinsic injection of the CRs into our galaxy per one supernova explosion, this issue currently remains to be resolved (e.g., Ohira et al. 2010; Ohira & Ioka 2011). The injected CRs will contribute to the dynamics of the ISM as a pressure source, leading to a feedback effect on the star formation rate, for example (e.g., Hopkins et al. 2018; Girichidis et al. 2018; Shimoda & Inutsuka 2021). The origin of $\gamma$-ray emissions in the SNRs is also unsettled, whether the hadronic origin or leptonic origin (Abdo et al. 2011, but see Fukui et al. 2021). We will study them in a forthcoming paper.

For distinguishing the case of extremely efficient CR acceleration (Model 3) from the case of no CRs (Model 0), a comparison of the FWHM to other values is required in general (e.g., the difference between the ionization states, the shock velocity, and so on). The FWHM of Model 3 becomes smaller than Model 0 at a given shock velocity and $nt$, and abundant ions of Model 3 tend to be less ionized than in the case of Model 0 because of the lower electron temperature. The lower electron temperature and lower ionization states of Model 3 may result in a different X-ray spectrum from the case of other Models, especially the equivalent widths, recombination lines, Auger transitions due to the inner shell ionization, and so on. We will attempt further investigations by calculating the overall photon spectrum in future work.

The line diagnostics of the thermal plasma of young...
SNRs on the effect of CR acceleration will be a good science objective for the XRISM mission (Tashiro et al. 2020), which will provide high-resolution X-ray spectroscopy. Since the micro-calorimeter array is not a distributed-type spectroscope like grating optics on Chandra and/or XMM-Newton, theResolve onboard XRISM (Ishisaki et al. 2018) can accurately measure the atomic-line profiles in the X-ray spectra from diffuse objects like SNRs. The XRISM will have the energy resolution of 7 eV (as the design goal), and the calibration goals on the energy scale and resolution are 2 eV and 1 eV, respectively (Miller et al. 2020). Therefore, the line broadening values from multiple elements with/without CRs in figure 3 can be distinguished by XRISM. Another importance of XRISM is the wider energy coverage, with which atomic lines not only from light elements (C, N, O, etc.) but also Fe will be measured. So, the intensity of the turbulence demonstrated in section 5 will be constrained with XRISM. The preparation for the instruments (Nakajima et al. 2020; Porter et al. 2020) and in-orbit operations (Terada et al. 2021; Loewenstein et al. 2020) are proceeding smoothly for the launch in 2022/2023, and several young SNRs, including RCW86 are listed as the target during the performance verification phase of XRISM. ⁵ We expect to verify our predictions observationally soon.

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References

Abdo, A. A., Ackermann, M., Ajello, M., et al. 2011, ApJ, 736, 131
Altun, Z., Yumak, A., Yavuz, I., et al. 2007, A&A, 474, 1051
Arnaud, M., & Rothenflug, R. 1985, A&AS, 60, 425
Arthur, S. J., Henney, W. J., Mellemma, G., de Colle, F., & Vázquez-Semadeni, E. 2011, MNRS, 414, 1747
Asplund, M., Grevesse, N., Sauval, A. J., & Scott, P. 2009, ARA&A, 47, 481
Bamba, A., Koyama, K., & Tomida, H. 2000, PASJ, 52, 1157
Bamba, A., Yamazaki, R., Yoshida, T., Terasawa, T., & Koyama, K. 2005, ApJ, 621, 793
Barret, D., Lam Trong, T., den Herder, J.-W., et al. 2018, in Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series, Vol. 10699, Space Telescopes and Instrumentation 2018: Ultraviolet to Gamma Ray, ed. J.-W. A. den Herder, S. Nikzad, & K. Nakazawa, 106991G
Beck, R. 2001, Science, 293, 94
Bell, A. R. 1978, MNRAS, 182, 147
—. 2004, MNRAS, 353, 550
Blandford, R. D., & Ostriker, J. P. 1978, ApJL, 221, L29
Borkowski, K. J., Rho, J., Reynolds, S. P., & Dyer, K. K. 2001, ApJ, 550, 334
Broersen, S., Chiotellis, A., Vink, J., & Bamba, A. 2014, MNRS, 441, 3040
Caprioli, D., Haggerty, C. C., & Blasi, P. 2020, ApJ, 905, 2
Chevalier, R. A., Kirshner, R. P., & Raymond, J. C. 1980, ApJ, 235, 186
De Cia, A., Jenkins, E. B., Fox, A. J., et al. 2021, Nature, 597, 206
Drury, L. O., & Voelk, J. H. 1981, ApJ, 248, 344
Fukui, Y., Sano, H., Yamane, Y., et al. 2021, ApJ, 915, 84
Girichidis, P., Naab, T., Hanasz, M., & Walch, S. 2018, MNRS, 479, 3042
Gronenschild, E. H. B. M., & Mewe, R. 1978, A&AS, 32, 283
Hahn, M., Badnell, N. R., Grieser, M., et al. 2014, ApJ, 788, 46
Helder, E. A., Vink, J., Bamba, A., et al. 2013, MRAS, 435, 910
Helder, E. A., Vink, J., Bassa, C. G., et al. 2009, Science, 325, 719
Hopkins, P. F., Wetzel, A., Kereˇ s, D., et al. 2018, MNRS, 480, 800
Hovey, L., Hughes, J. P., & Eriksen, K. 2015, ApJ, 809, 119
Hovey, L., Hughes, J. P., McCully, C., Pandya, V., & Eriksen, K. 2018, ApJ, 862, 148
Hughes, J. P., Rakowski, C. E., & Decourchelle, A. 2000, ApJL, 543, L61
Ishisaki, Y., Ezoe, Y., Yamada, S., et al. 2018, Journal of Low Temperature Physics, 193, 991
Itoh, H. 1984, ApJ, 285, 601
Janev, R. K., & Smith, J. J. 1993, Cross Sections for Collision Processes of Hydrogen Atoms with Electrons, Protons and Multiply Charged Ions, 192
Kotelnikov, I. A., & Milstein, A. I. 2019, Phys. Scr., 94, 055403
Lagage, P. O., & Cesarsky, C. J. 1983a, A&AS, 543, L61
Lee, E., & Gendreau, K. M. 2018, ApJ, 862, 119
Lening, M. A., Bell, K. L., Gilbody, H. B., et al. 2004, MNRAS, 353, 550
Lennon, M. A., Bell, K. L., Gilbody, H. B., et al. 1988, Journal of Physical and Chemical Reference Data, 17, 1285
Lestinsky, M., Badnell, N. R., Bernhardt, D., et al. 2009, ApJ, 698, 648
Loewenstein, M., Hill, R. S., Holland, M. P., et al. 2020, in Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series, Vol. 11444, Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series, 114445D
Marcowith, A., Ferrand, G., Grech, M., et al. 2020, Living Reviews in Computational Astrophysics, 6, 1

⁵ (https://xrism.isas.jaxa.jp/research/proposer/approved/pv/index.html)
Matsumoto, Y., Amano, T., Kato, T. N., & Hoshino, M. 2017, Phys. Rev. Lett., 119, 105101
Mewe, R. 1972, A&A, 20, 215
Mewe, R., Lemen, J. R., & van den Oord, G. H. J. 1986, A&AS, 65, 511
Mewe, R., Schrijver, J., & Sylwester, J. 1980a, A&AS, 40, 323
—. 1980b, A&A, 87, 55
Miceli, M., Orlando, S., Burrows, D. N., et al. 2019, Nature Astronomy, 3, 236
Miller, E. D., Sawada, M., Guainazzi, M., et al. 2020, in Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series, Vol. 11444, Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series, 1144426
Mitnik, D. M., & Badnell, N. R. 2004, A&A, 425, 1153
Morlino, G., Blasi, P., Bandiera, R., & Amato, E. 2013a, A&A, 557, A142
—. 2014, A&A, 562, A141
Morlino, G., Blasi, P., Bandiera, R., Amato, E., & Caprioli, D. 2013b, ApJ, 768, 148
Murakami, I., Kato, T., Kato, D., et al. 2006, Journal of Physics B Atomic Molecular Physics, 39, 2917
Myers, P. C. 1978, ApJ, 225, 380
Nahar, S. N. 1995, ApJS, 101, 423
—. 1998, Phys. Rev. A, 58, 3766
—. 2000, ApJS, 126, 537
—. 2006, ApJS, 164, 280
Nahar, S. N., & Pradhan, A. K. 1997, ApJS, 111, 339
—. 1999, A&AS, 135, 347
Nahar, S. N., Pradhan, A. K., & Zhang, H. L. 2001, ApJS, 133, 255
Nakajima, H., Hayashida, K., Tomida, H., et al. 2020, in Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series, Vol. 11444, Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series, 1144423
Novotný, O., Badnell, N. R., Bernhardt, D., et al. 2012, ApJ, 735, 57
Ohira, Y. 2013, Phys. Rev. Lett., 111, 245002
—. 2016a, ApJ, 827, 36
—. 2016b, ApJ, 817, 137
Ohira, Y., & Ioka, K. 2011, ApJL, 729, L13
Ohira, Y., Murase, K., & Yamazaki, R. 2010, A&A, 513, A17
Ohira, Y., & Takahara, F. 2007, ApJL, 661, L171
—. 2008, ApJ, 688, 320
Osterbrock, D. E., & Ferland, G. J. 2006, Astrophysics of gaseous nebulae and active galactic nuclei
Porter, F. S., Eckart, M. E., Leutenegger, M., et al. 2020, in Space Telescopes and Instrumentation 2020: Ultraviolet to Gamma Ray, ed. J.-W. A. den Herder, S. Nikzad, & K. Nakazawa, Vol. 11444, International Society for Optics and Photonics (SPIE)
Rakowski, C. E., Laming, J. M., & Ghavamian, P. 2008, ApJ, 684, 348
Savin, D. W., Behar, E., Kahn, S. M., et al. 2002, ApJS, 138, 337
Sedov, L. I. 1959, Similarity and Dimensional Methods in Mechanics
Shimoda, J., Akahori, T., Lazarian, A., Inoue, T., & Fujita, Y. 2018a, MNRAS, 480, 2200
Shimoda, J., Inoue, T., Ohira, Y., et al. 2015, ApJ, 803, 98
Shimoda, J., & Inutsuka, S.-i. 2021, arXiv e-prints, arXiv:2112.04762
Shimoda, J., & Laming, J. M. 2019, MNRAS, 485, 5453
Shimoda, J., Ohira, Y., Yamazaki, R., Laming, J. M., & Katsuda, S. 2018b, MNRAS, 473, 1394
Spitzer, L. 1962, Physics of Fully Ionized Gases
Tashiro, M., Maejima, H., Toda, K., et al. 2020, in Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series, Vol. 11444, Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series, 1144422
Terada, Y., Holland, M., Loewenstein, M., et al. 2021, Journal of Astronomical Telescopes, Instruments, and Systems, 7, 037001
Tsubone, Y., Sawada, M., Bamba, A., Katsuda, S., & Vink, J. 2017, ApJ, 835, 34
Uchiyama, Y., Aharonian, F. A., Tanaka, T., Takahashi, T., & Maeda, Y. 2007, Nature, 449, 576
Vink, J. 2012, A&AR, 20, 49
Vink, J., Bleeker, J., van der Heyden, K., et al. 2006, Ap.JL, 648, L33
Vink, J., & Laming, J. M. 2003, ApJ, 584, 758
Vink, J., & Yamazaki, R. 2014, ApJ, 780, 125
Vink, J., Yamazaki, R., Helder, E. A., & Schure, K. M. 2010, ApJ, 722, 1727
Yamaguchi, H., Katsuda, S., Castro, D., et al. 2016, ApJL, 820, L3
Zatsarinny, O., Gorczyca, T. W., Fu, J., et al. 2006, A&A, 447, 379
Zatsarinny, O., Gorczyca, T. W., Korista, K. T., Badnell, N. R., & Savin, D. W. 2003, A&A, 412, 587
—. 2004, A&A, 417, 1173