A New Free Core Nutation Model with Variable Amplitude and Period

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Abstract

Three most long and dense VLBI nutation series obtained at the Goddard Space Flight Center, Institute of Applied Astronomy, and U.S. Naval Observatory were used for investigation of the Free Core Nutation (FCN) contribution to the celestial pole offset. Some recent studies have showed that the FCN period or/and phase does not remain constant, but varies in a rather wide range of about 410–490 days (for equivalent period). To implement this result in the practice, a new FCN model with variable amplitude and period (phase) is developed. Comparison of this model with observations shows better agreement than existing one. After correction of the differences between observed VLBI nutation series and the IAU2000A model, they decreased to the level about 100 $\mu\text{as}$.

1 Introduction

Free Core Nutation (FCN) of the Earth is predicted more than a century ago as a common rotational mode of a body having an ellipsoidal solid shell and fluid core. Investigation of the FCN is an important scientific task. First, the FCN parameters determined from observations provide valuable, sometimes unique, information about processes in the Earth’s interior. From the practical point of view, accurate modelling the FCN term, including prediction, is necessary to compute celestial pole offset with accuracy compatible with modern VLBI observations.

The IAU2000A model based on the MHB2000 model developed in (Mathews, et al., 2002), and adopted as a new IAU standard can predict a regular part of the nutation with accuracy of about 100 $\mu\text{as}$. However, the FCN contribution is much larger, up to 400 $\mu\text{as}$, which yields degradation of accuracy in modelling celestial pole offset, if FCN not accounted. It is well known also that the FCN contribution gives the prevailing contribution to the power spectrum of the differences between observed nutation series and modern models.

The IAU2000A model, like the previous ones, is constructed as a Poison series, and does not include free mode terms, such as the FCN, which cannot be presented as a Poison series term with predictable parameters. For this reason, FCN parameters are to be determined from the VLBI observations.

Historically, the FCN frequency is considered as a constant fundamental value included in transfer function expression describing the relationship between the amplitudes of nutation terms for real and rigid Earth. Many authors have made an effort to estimate the FCN period and possible reasons for its excitation (see e.g. Mathews and Shapiro, 1995; Brzeziński and Petrov, 1998; Shirai and Fukushima, 2001a; Herring et al., 2002; Malkin and Terentev, 2003a, 2003b). They found the FCN period to be in the range of 425–435 solar days, with average value about 430$^d$.

Recently, it was found from a wavelet analysis of VLBI nutation series that the FCN period likely varies in a range of about 410–490 days (Malkin and Terentev, 2003a, 2003b). This result
is also confirmed by means of another method, Short-time Periodogram with Gabor Function, proposed by T. Shirai (Shirai et al., 2004). Of course, found variability of the FCN period may be fully or partially an transformation of the variations of the FCN phase, which may has more geophysical meaning. However, geophysical considerations of the FCN variability lie beyond of this study.

In this paper we develop an practical model for computation of the FCN contribution to the celestial pole offset taken into account variability both amplitude and period (phase) of the FCN oscillation.

2 Computation of the FCN parameters

Three most long and dense VLBI nutation series obtained at the Goddard Space Flight Center (GSF), Institute of Applied Astronomy (IAA), and U.S. Naval Observatory (USN), each containing more than 3000 estimates of the nutation angles for the period from 1979 up to now were used for investigation of the FCN parameters.

Firstly, estimates of $d\psi$ and $d\epsilon$ w.r.t. the IAU1976/1980 nutation model computed at the GSF and USN were transformed to the $dX_c$ and $dY_c$ w.r.t. IAU2000A model (IAA series already contains this data).

Then combined series was computed. Since preliminary analysis showed that three input series are of very similar quality, no weighting was applied, however formal errors reported in the input series were scaled for uniformity. After that, input series were averaged, saving only epochs present in all the series. Band-pass Gaussian filtering was applied to the combined series. Transfer function of the filter at the FCN frequency is 0.988. Figure 1 shows obtained smoothed differences between observed nutation series and the IAU2000A model, and spectrum of the differences is presented in Figure 2.

$$A(t) = \sqrt{(dX_c(t))^2 + (dY_c(t))^2},$$  \hspace{1cm} (1)

Indeed, using such an approach we suppose that all differences can be attributed to the FCN, but this seems to be a good approximation to reality. However, a resonance impact on the nuta-
tion terms at the frequencies close to the FCN one, evidently should be accounted for in future developments.

Finally, the FCN period variation was computed using a wavelet analysis as described in (Malkin and Terentev, 2003a, 2003b).

3 Computation of the FCN contribution

Let us consider how a model with variable amplitude and period (phase) can be used in practice. We can describe the FCN term as

\[
\begin{align*}
    dX_c &= A(t) \sin(\Phi(t)) , \\
    dY_c &= A(t) \cos(\Phi(t)) .
\end{align*}
\]  

Mathematically (not geophysically, indeed!), we can suppose three equivalent models for the FCN phase \( \Phi(t) \)

\[
\Phi(t) = \begin{cases} 
    \frac{2\pi}{P(t)} t + \Phi_0 , &  \\
    \frac{2\pi}{P_0} t + \Phi(t) , &  \\
    \frac{2\pi}{P(t)} t + \Phi(t) , &
\end{cases}
\]  

where \( P \) is the FCN period, and zero subscripts mean constant values. In other words, we can consider three models: with variable period and constant phase, variable phase and constant period, or variable both period and phase. Of course, this is a subject of geophysical consideration, but doesn’t matter for an empiric FCN model using time variations of the FCN parameters found from analysis of the observed data.

In practice, one can compute \( \Phi(t) \) as

\[
\Phi(t) = \int_{t_0}^{t} \frac{2\pi}{P(t)} \, dt + \varphi_0 ,
\]  

where \( \varphi_0 \) is the parameter to be adjusted.
Variations of the FCN amplitude $P(t)$ and phase $\Phi(t)$ are shown in Figure 3 along with the corresponding FCN parameters included in the MHB2000 model which is, in fact, also a model with variable phase and amplitude, though this is not stated explicitly (we used the text of the FCN_NUT routine included in the MHB2000 code to extract the FCN(MHB) amplitude and phase variations). One can see that both models show similar behavior of the FCN parameters, however new approach allow us to get continues, non-inflecting and predictable functions $A(t)$ and $\Phi(t)$. Comparing these two models one should keep in mind that MHB2000 model is developed only for the period till 2001.4, and after this epoch the difference between the models grows rapidly.

Figure 3: The FCN amplitude and phase variations found in this study (solid line), and a comparison with the MHB2000 model (dashed line).

Figure 4 shows spectra of the differences between observed nutation series and the IAU2000A model computed for raw differences and after removing the FCN contribution. One can see that the FCN signal is completely eliminated.

Figure 4: Spectrum of the differences between observed nutation series and the IAU2000A model, period in days, amplitude in $\mu$as.

However, the differences between observed nutation series and model have a noise of various origins with the rms compatible with the FCN contribution. To estimate the actual contribution of the FCN model to this noise we have computed rms of differences between the observations and the IAU2000A model after applying three different FCN models: no FCN (raw differences), extracting the MHB2000 FCN term, and extracting the FCN term according to the model described here. The results are shown in Table 1.

One can see that accounting for the FCN contribution leads to decreasing of the differences. Especially interesting is the last part of the table corresponding to the period of observations 2002–2003. Using the MHB2000 FCN model for this period leads to degradation of differences between observations and the IAU2000A model, which is natural for this model is developed only for epochs until 2001.4.
Table 1: WRMS of differences with two FCN models, $\mu$as (No – no FCN model, MHB – MHB2000 FCN model, New – model proposed in this paper applied; NEOS – NEOS-A VLBI sessions observed in 1993–2001, R1R4 – IVS R1 and R4 VLBI sessions only observed since 2002).

| Series | All sessions | NEOS | R1R4 |
|--------|--------------|------|------|
|        | FCN model    | FCN model | FCN model |
|        | No MHB New No | MHB New No | MHB New No |
| GSF    | 166 146 138 138 122 120 134 150 102 |
| IAA    | 170 152 144 140 123 123 138 154 111 |
| USN    | 161 144 136 138 122 122 136 156 107 |
| Mean   | 156 136 126 131 113 112 129 146 97 |

A FCN model with variable period and phase allow us to try a new approach to FCN prediction. One can consider two possibilities. The first one is a prediction of actual FCN contribution, which is developed e.g. in (Brzeziński and Kosek, 2004). Another possibility is to predict functions $A(t)$ and $\Phi(t)$ separately, and then use predictions to construct the FCN contribution using the formulas given above. Figure 5 presents a variant of such a prediction obtained using ARMA method. It is interesting to compare both approaches to a FCN prediction in details. Please note that in our final computation we replaced observed series for the period 1979 with predicted one which seems to be more accurate taking into account relatively large errors in the VLBI observations made before 1984.

Figure 5: Examples of predictions of the FCN amplitude and phase.

4 Conclusions

We have developed a new FCN model with variable amplitude and period (phase) which provides a computation of a continuous FCN contribution to the celestial pole offset with good accuracy for whole interval of the VLBI observations, and convenient prediction of the FCN contribution. Using this model allow us to reduce the differences between VLBI observations and model to the level 100$\mu$as.

It is clear that the proposed model is a pure empiric one. Considerable efforts should be made to understand the physical origin of the variability of the FCN period and/or phase, and its consequences on a theory of nutation.

The proposed model is routinely used in the VLBI processing at the Institute of Applied Astronomy since September 2003.
References

[1] Brzeziński, A., and W. Kosek, (2004), Free core nutation: stochastic modelling versus predictability, in Proc. Journées Systèmes de Référence Spatio-temporels 2003, St. Petersburg, Russia, Sep 22-25, 2003, edited by N. Capitaine, and A. Finkelstein, in press.

[2] Brzeziński, A., and S. Petrov (1999), Observational evidence of the free core nutation and its geophysical excitation, in: Proc. Journées Systèmes de Référence Spatio-temporels 1998, Paris, France, Sep 21–23, 1998, edited by N. Capitaine, 169–174.

[3] Herring, T. A., P. M. Mathews, and B. A. Buffet, (2002), Modelling of nutation-precession: Very long baseline interferometry results, J. Geophys. Res., 107, doi:10.1029/2001JB000165.

[4] Malkin, Z., and D. Terentev, (2003a), Preliminary analysis of the Free Core Nutation from VLBI data, in Proc. 16th Working Meeting on European VLBI for Geodesy and Astrometry, Leipzig, Germany, 9–10 May 2003, 227–235.

[5] Malkin, Z., and D. Terentev, (2003b), Investigation of the Parameters of the Free Core Nutation from VLBI data, Comm. IAA RAS, 149.

[6] Mathews, P.M., and I. I. Shapiro (1996) Recent advances in nutation studies, in: Proc. Journées Systèmes de Référence Spatio-temporels 1995, Warsaw, Poland, Sep 18–20, 1995, edited by N. Capitaine, 61–66.

[7] Mathews, P. M., T. A. Herring, and B. Buffett, (2002), Modeling of nutation and precession: New nutation series for nonrigid Earth and insights into the Earth’s interior, J. Geophys. Res., 107, doi:10.1029/2001JB000390.

[8] Shirai, T., and T. Fukushima, (2001a), Construction of a New Forced Nutation Theory of the Nonrigid Earth, Astron. J., 121, 3270–3283.

[9] Shirai, T., and T. Fukushima, (2001b), Did Huge Earthquake Excite Free Core Nutation?, J. Geodetic Soc. Japan, 47, 198–203.

[10] Shirai, T., T. Fukushima, Z. Malkin (2004), Short-time Periodogram with Gabor Function and its Application to Free Core Nutation Period Analysis (in press).