3-Leibniz bialgebra in $N = 6$ Chern-Simons gauge theories, multiple M2 to D2 branes and vice versa

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Abstract

Constructing M2-brane and its boundary conditions from D2-brane and the related boundary conditions and vice versa has been possible in our recent work by using 3-Lie bialgebra for BLG model with $N = 8$ supersymmetry. This could be generalized for BL model with $N = 6$ by the concept of the 3-Leibniz bialgebra. The 3-Lie bialgebra is an especial case of 3-Leibniz bialgebra, then more comprehensive information will be obtained in this work. Consequently, according to the correspondence of these 3-Leibniz bialgebras with Lie bialgebras, we reduce to D2-brane such that with some restrictions on the gauge field this D2-brane is related to the bosonic sector of an $N = (4,4)$ WZW model equipped with one 2-cocycle in its Lie bialgebra structure. Moreover, the Basu-Harvey equation which is found by considering boundary conditions for BL model containing Leibniz bialgebra structure is reduced to Nahm equation and vice versa using this correspondence.

Keywords: String theory, M-theory, Leibniz bialgebra, 3-Leibniz bialgebra, Manin triple.

1 Introduction

Unification of forces is one of the important issues in high energy physics. M-theory has been suggested for this purpose which can be obtained by developing the string theory [1]. M-theory contains two types of branes, M2-brane and M5-brane [2]. There are the great success [3] in the description of action for single M2-brane. However, according to the existence of a self-dual field in the world volume of M5-brane and in spite of efforts, this theory is still unknown [4]. It seems that some tricks may give us new information as the boundary conditions in the M2-brane ending to an M5-brane do [5]. People attempted to find a Lagrangian in describing multiple M2-branes. Finally, Bagger and Lambert [6–8] and Gustavsson [9] found one successful idea. Although, there was just one example [8] only for two membranes [10], the case has been considered by many authors [11]. Bagger and Lambert inspired by the Basu-Harvey article [12] (see also Ref. [13]) and groping supersymmetry transformations of D2-brane [14,15] expressed $N = 8$ supersymmetry transformations of M2-branes. They obtained a series of equations of motion according to supersymmetric transformations to be closure. They could be derived from a Lagrangian which Bagger and Lambert found it. It was interesting with some limitations as mentioned, so they attempted to find the other one for explaining multiple M2-brane. Using complex fields, Aharony, Bergman, Jafferis, and Maldacena (ABJM) [16] found a Lagrangian for the arbitrary number of membranes and reduced the supersymmetry to $N = 6$. Bagger and Lambert followed this idea and described their own theory using complex scalar fields for arbitrary number of membranes [17]. In this way, they have got to use structure constants of 3-Lie algebra that is not totally antisymmetric. The relationship between those works has been described in Ref. [18].

The world volume of the membrane in M-theory and the equivalent one in string theory (D-brane) are in relation to each other by using the Higgs mechanism in $N = 8$ BLG model [19,20]. The model and relations can be extended to $N = 6$ BL model [21] with some difference, such as attributing vacuum expectation value (VEV) to a real part of one of the complex scalar fields [21] and also, the fields of this model are 3-Leibniz algebra valued.

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instead of 3-Lie algebra one. Note that, the structure constants in 3-Leibniz algebra are antisymmetric only in two indices, 3-Lie algebra will obtain from it by completely antisymmetric structure constants then the results of this work will involve the ones in [39]. On the other hand, 3-Lie algebra can be considered as a special case of 3-Leibniz algebra. $N = 6$ BL mode must have done with gauge fields in U(N) group instead of SU(N). One of the results of studying boundary condition for Lorentzian BL model [22–25] is Basu-Harvey equation [12] (as a boundary condition for the M2-brane ending to M5-brane) which will be turned into Nahm-equation [26] (as a boundary condition for D1-string ending D3-brane). This work is not a simple generalization of $N = 6$ [23]. It seems to be impossible to do the same in the inverse direction, but in this work, correspondence between 3-Leibniz bialgebra [27] and Lie bialgebra [28] we will show the possibility of it.

Blok suggested the notion of Leibniz algebra [29] in 1965; it was rediscovered by J. L. Loday [30] with a non-antisymmetric bilinear bracket. Leibniz bialgebra [31] and 3-Leibniz bialgebra [27] have been introduced recently by using cohomology of Leibniz and 3-Leibniz algebra. We think that the role of 3-Leibniz bialgebra can be important in M-theory if we express $N = 6$ BL model in the 3-Leibniz bialgebra structure as an example. We have shown that there is a relation between these models and WZW model, with Lie bialgebra structure equipped with one 2-cocycle [32], this work with some difference has been done in [35]. In the previous work [35] we have shown there is a relation between Nahm and Basu-Harvey equations and vice versa by considering their boundary conditions. In the other one [39] we have shown that knowledge of the WZW model and its algebraic structure able one in increasing information about the $N = 8$ BLG model and its algebraic structure, i.e., one will be able to construct a relation between string theory and M-theory directly using 3-Leibniz bialgebra [39]. In this work using methods applied in Ref. [39], we will try to extend the results to $N = 6$ BL model by using the 3-Leibniz bialgebra [27]. In this case, our results will be useful since it makes a relation between multiple M2-brane and D2-brane. The relation between Nahm and Basu-Harvey equations has been found by using the relation between 3-Lie algebra and super-Lie algebra [40,41].

The outline of this paper is as follows. We review $N = 6$ BL model in section 2 and consider boundary conditions for it in section 4. We will review and give a special example the 3-Leibniz bialgebra and show the relation between 3-Leibniz bialgebra and Lie bialgebra in section 3. The BL model by 3-Leibniz bialgebra has been constructed as the works in Ref. [38,39]. One of the most results is that BL model has been turned into WZW model with Lie bialgebra structure which has been showed in section 4. Also, boundary conditions of the theory have been mentioned in section 5.

2 $N=6$ BL model

Here for self-containing of the paper, we review the $N = 6$ BL model [17]. Dynamics of M2-brane in M-theory has been studied by maximal supersymmetric 3-dimensional Chern-Simons model [5,6]. Bagger and Lambert extended their model to arbitrary number of membranes with $N = 6$ [17] superymmetry using special 3-Lie algebra [4] and complex scalar fields which have been deduced from the ABJM model [16]. In addition to the fundamental difference between ABJM and BL theories, there is a conventional relation between them [18]. The BL formulation has got four complex scalar fields $Z^a$ (8 real field). Note that in ABJM model complex scalar fields are split into two pairs $A_\alpha$ and $\bar{B}_\alpha$ with $\alpha = 1, 2$, where $(Z^1 = A_1, Z^2 = A_2, Z^3 = B_1, Z^4 = B_2)$ which must be considered complex conjugates of each other $Z_\alpha = (Z_\alpha)^*$. Because of turning SO(8) R-symmetry into $SU(4) \times U(1)$ [17]. One must have for the fermion fields and their complex conjugates $\psi_{a\alpha}, \bar{\psi}_{\alpha a} = (\psi_{a\alpha})^*$ where the raised indice express as the fields in the 4 of SU(4) and a lowered indice convert in the 4. An index is raised or lowered by complex conjugation and supersymmetry generators $\epsilon_{a\alpha}$ that are in the 6 of SU(4) with zero charge of U(1) and must satisfy the relation as $\epsilon\delta = \frac{1}{2}\epsilon\epsilon\gamma\delta\epsilon\gamma\delta$ [17]. The covariant derivative is $D_a Z^a = \partial_a Z^a - \bar{A}_a Z^b c_{ab}^d D_aZ^d + \partial_a Z_{ad} + \bar{A}_a Z_{ad} + \bar{A}_a Z_{ad}$ where $\bar{A}_a^d = f^{cba} a_{ab}^d$ such that $a, b, c, \gamma$ and $\delta$ is 1, 2, 3, 4 the numbers of the scalar fields, $\mu, \nu = 0, 1, 2$ for representation of M2-brane worldvolume, and the supersymmetry transformations are as follows:

\[
\delta Z^a_d = i\epsilon^{a\alpha}\psi_{\beta d} \tag{1}
\]

\[
\delta\psi_{\beta d} = \gamma^d D_a Z^a_{\alpha \beta} + f_{abc} Z^a_{\alpha \gamma} Z^d_{\gamma \delta} \epsilon_{\alpha \beta \gamma \delta} + f_{abc} Z^a_{\beta \gamma} Z^d_{\gamma \delta} \bar{Z}_{\gamma \delta} \epsilon_{\alpha \beta \gamma \delta} \tag{2}
\]

\[
\delta A_{\mu d} = -i\epsilon^{\gamma\delta}_{(a\alpha}\gamma\delta^{(a\alpha)} Z^a_{\alpha \gamma} f_{\gamma \delta} + i\epsilon^{a\alpha}_{(a\alpha}\gamma\delta^{(a\alpha)} \bar{Z}_{\alpha \gamma} \psi_{\beta d} f_{\gamma \delta} \tag{3}
\]

\[\epsilon_{(a\alpha}(\gamma\delta) Z^a_{\alpha \beta} f_{\gamma \delta} + i\epsilon_{(a\alpha}\gamma\delta^{(a\alpha)} \bar{Z}_{\alpha \gamma} \psi_{\beta d} f_{\gamma \delta} \tag{3}
\]
with gauge transformations as:
\[ \delta_{\lambda} Z_{\alpha}^d = \Lambda_{\alpha \beta} f^{abc} Z_{\alpha}^c \] (4)
where can be showed by \( \delta_{\lambda} Z_{\alpha}^N = N Z_{\alpha}^N - Z_{\alpha}^N N \) such that \( N \) and \( M \) is \( n \times n \) and \( m \times m \) matrix respectively, i.e., gauge group is \( U(n) \times U(m) \) (see details in Ref. [17]). Closure of the supersymmetric transformations terminate the following equations of motion:
\[ 0 = \gamma^\mu D_\mu \psi_{\gamma d} + f^{abc} d_{\psi_{\gamma a}} Z_{\beta}^b \bar{Z}_{\gamma c} - 2 f^{abc} d_{\psi_{\gamma a}} Z_{\gamma c} - c_{\gamma \delta \beta} f^{abc} d_{\psi_{\gamma a}} Z_{\alpha}^c Z_{\beta}^b \] (5)
\[ 0 = \bar{F}_{\mu \nu} \epsilon^d c_{\epsilon \mu \nu} (D^\Lambda Z_{\alpha}^a) \bar{Z}_{\alpha a} - Z_{\alpha}^a (D^\Lambda Z_{\alpha a}) - i \psi_{\gamma a} \gamma^\lambda \psi_{\alpha a} f^{cab} , \] (6)
and equation of motion of the scalar fields achieved by taking the supersymmetric variation of (5). The invariant Lagrangian related to this equations can be expressed as follows [17]:
\[ L = \frac{1}{2} \epsilon_{\mu \nu \lambda} \left( f^{abc} A_{\mu \nu \lambda} A_{\alpha \delta} - \frac{1}{2} \delta_{\beta}^\gamma f^{abc} Z_{\alpha}^a Z_{\beta}^b \bar{Z}_{\gamma c} + \frac{1}{2} \delta_{\beta}^\gamma f^{abc} Z_{\alpha}^a Z_{\beta}^b \bar{Z}_{\gamma c} + \frac{1}{3} \delta_{\beta}^\gamma f^{abc} Z_{\alpha}^a Z_{\beta}^b \bar{Z}_{\gamma c} \right) \] (7)
with
\[ V = 2 \Upsilon_{\beta d} \Upsilon_{\gamma d} \Upsilon_{\gamma d}, \]
where
\[ \Upsilon_{\beta d} = f^{abc} Z_{\alpha}^a Z_{\beta}^b \bar{Z}_{\gamma c} - \frac{1}{2} \delta_{\beta}^\gamma f^{abc} Z_{\alpha}^a Z_{\beta}^b \bar{Z}_{\gamma c} + \frac{1}{2} \delta_{\beta}^\gamma f^{abc} Z_{\alpha}^a Z_{\beta}^b \bar{Z}_{\gamma c} + \frac{1}{3} \delta_{\beta}^\gamma f^{abc} Z_{\alpha}^a Z_{\beta}^b \bar{Z}_{\gamma c} \] (8)
and Chern-Simons term have the following form:
\[ L_{CS} = \frac{1}{2} \epsilon_{\mu \nu \lambda} \left( f^{abc} A_{\mu \nu \lambda} A_{\alpha \delta} + \frac{3}{2} f_{g}^{abcd} f^{ebf} A_{\alpha \beta} A_{\nu \delta} A_{\lambda \mu} \right) \] (9)
where \( f_{g}^{abcd} \) is the structure constants of the 3-Leibniz algebra \( A \) with the following nonantisymmetric 3-bracket:
\[ [T^a, T^b; T^c] = f^{abc} T^d, \quad a, b, c, d = 1, \ldots, \text{dim}\mathcal{A}, \]
(11)
such that we have the following fundamental identity (see for example [22]):
\[ [[T^a, T^b; T^c], T^d; T^e] = [[T^a, T^d; T^e], T^b; T^c] + [T^a, [T^b, T^d; T^e]; T^c] + [T^a, T^b; [T^c, T^d; T^e]], \]
(12)
where it can be redefined by the structure constant of \( A \) in the following form:
\[ f^{abc} f^{fg} f^{def} g - f^{abc} f^{fg} f^{ef} g - f^{abc} f^{def} g = 0. \]
(13)

### 3 3-Leibniz bialgebra

In this section, we review the definition of 3-Leibniz bialgebra that have been given recently in [27].

**Definition:** [28] Lie bialgebra deals with a map \( \delta : \mathcal{G} \rightarrow \mathcal{G} \otimes \mathcal{G} \) such that:

1. \( \delta \) is a one-cocycle, i.e.:
   \[ \delta([T^i, T^j]) = \text{ad}^{(2)}_{T^i} \delta(T^j) - \text{ad}^{(2)}_{T^j} \delta(T^i), \]
   (14)
   where
   \[ \text{ad}^{(2)}_{T^i} = \text{ad}_{T^i} \otimes 1 + 1 \otimes \text{ad}_{T^i}, \]
   (15)

2. \( \mathcal{G} \) is a Lie algebra and \( \{ T^i \} \)s are bases for it,

3. \( \delta : \mathcal{G}^* \otimes \mathcal{G}^* \rightarrow \mathcal{G}^* \) operate as a cocommutator on dual space of \( \mathcal{G} \) i.e. \( \mathcal{G}^* \) with the following relation:
   \[ \delta(T^k) = \delta(T^i \wedge T^j) = \delta(T^k) = (\tilde{T}_i \wedge \tilde{T}_j), \]

\[ \text{Note that the 3- Lie algebra } \mathcal{A} \text{ as vector space is a subspace of } u(n) \] [17].

\[ f^{abc} = -f^{ba} f^{d} \] and \( f^{abc} = f^{dabc}. \)
such that \( \{\tilde{T}_i\} \) are bases of \( G^* \). \( G^* \) is a Lie algebra and there is a pairing between \( G \) and \( G^* \) shown as \( \langle \cdot, \cdot \rangle \).

One can obtain following identities for Lie bialgebra:

\[
\begin{align*}
  f^{ij}k f^{kl}_m - f^{ik}_m f^{jl}_{k} + f^{jk}_m f^{il}_k &= 0, \\
  \tilde{f}^{ij}_k \tilde{f}^{kl}_m - \tilde{f}^{ik}_m \tilde{f}^{jl}_k + \tilde{f}^{jk}_m \tilde{f}^{il}_k &= 0, \\
  -f^{ij} \tilde{f}^{km}_l + f^{ik} \tilde{f}^{jl}_m - f^{jk} \tilde{f}^{il}_m + f^{ik} \tilde{f}^{jl}_m &= 0,
\end{align*}
\]

which are Jacobi identity of \( G \) and \( G^* \) and mix Jacobi identity, respectively.

**Definition:** \( [27] \) A 3-Leibniz algebra \( A \) equipped with a linear co commutator \( \delta : A \rightarrow A \otimes A \otimes A \) defines a 3-Leibniz bialgebra if:

\( a \) \( \delta \) is a 1-cocycle of \( A \) and get values on \( \otimes^3 A \), i.e:

\[
\delta([T^a, T^b; T^c]) = ad^{(3)}_{T^a \otimes T^b} \delta(T^a),
\]

where \( \delta(T^i) = \tilde{f}^{mn} _i T_i \otimes T_m \otimes T_n \) and

\[
ad^{(3)}_{T^a \otimes T^c} = ad_{T^a \otimes T^c} \otimes 1 \otimes 1 + ad_{T^a \otimes T^c} \otimes 1 \otimes 1 + ad_{T^a \otimes T^c},
\]

with \( \{T^a\} \)s are bases of 3-Leibniz algebra \( A \).

\( b \) 3-Leibniz bracket can be defined as dual map \( ^{t}\delta : \otimes^3 A^* \rightarrow A^* \) and it is a commutator on \( G^* \)

\[
(\tilde{T}_i \otimes \tilde{T}_k \otimes T_m, \delta(T^i)) = (\delta^*(\tilde{T}_j \otimes \tilde{T}_k \otimes \tilde{T}_m), T^j) = \tilde{f}^{km}_i
\]

where \( \{\tilde{T}_i\} \)s are the bases for the space \( A^* \) and there is a natural pairing between \( A \) and \( A^* \). Furthermore from this and \( 20 \) we have the following relation between the structure constants \( [27] \):

\[
f^{abc} g \tilde{f}_{def} g = f^{gbc} f \tilde{f}_{deg} a + f^{gbc} \tilde{f}_{cfg} a - f^{gbc} d \tilde{f}_{fga}
\]

### 3.1 An example

Generally, there is not Manin triple for 3-Leibniz bialgebras, but here we will show that there is Manin triple for a special example of 3-Leibniz bialgebras.

**Proposition:** \( (A_G, A_G^*) \) is a 3-Leibniz bialgebra if and only if \( (G, G^*) \) be a Lie bialgebra\(^5\).

Consider a special example of 3-Leibniz algebra \( A_G \) in relation with Lie algebra \( G \). This 3-Leibniz algebras \( A_G \) (mentioned in \( 20 \) for a first time) have commutation relations as follows:

\[
[T^-, T^a; T^b] = 0, \quad [T^+, T^i; T^j] = f^{ij}k T^k, \quad [T^i, T^j; T^k] = -f^{ijk} T^-,
\]

where \( \{T^i\} \)s and \( f^{ij}k \) are basis and structure constant of the Lie algebra \( G \) respectively with \( [T^i, T^j] = f^{ijk} T^k \) and fundamental identity for \( A_G \) has the following form:

\[
f^{abc} f f^{ede} g - f^{ade} f f^{bce} g - f^{bde} f f^{ace} g - f^{cde} f f^{abf} g = 0,
\]

where \( a, b = i, -, + \).

Now, we consider 3-Leibniz algebra structure on \( A_G^* \):

\[
[\tilde{T}^-, \tilde{T}_a; \tilde{T}_b] = 0, \quad [\tilde{T}^+, \tilde{T}_i; \tilde{T}_j] = \tilde{f}^{ij}k \tilde{T}_k, \quad [\tilde{T}_i, \tilde{T}_j; \tilde{T}_k] = -\tilde{f}^{ijk} T_-,
\]

where \( \tilde{T}_i \) are bases for \( G^* \) with \( [\tilde{T}_i, \tilde{T}_j] = \tilde{f}^{ij}k \tilde{T}_k \), \( i, j, k = 1, 2, ..., dim G^* \), where \( G^* \) is the dual Lie algebra of \( G \), and the fundamental identity for this 3-Leibniz algebra which is obtained as:

\[
\tilde{f}^{abc} f \tilde{f}_{def} g - \tilde{f}^{ade} f \tilde{f}_{bce} g - \tilde{f}^{bde} f \tilde{f}_{ace} g - \tilde{f}^{cde} f \tilde{f}_{abf} g = 0,
\]

---

\(^5\) Note that 3-Leibniz algebra can be right or left and according to left or right 3-Leibniz algebra one can differently characterize the actions of \( A \) on \((\otimes)^3 A \). We use only the right Leibniz algebra in this work, for details see Ref. \( [27] \).

\(^6\) Note that in the forthcoming section, we will consider a Lie algebra \( G \) such that related Lie bialgebra \((G, G^*)\) has a 2-cocycle.

\(^7\) Note that for this 3-Leibniz algebra there is an antisymmetry only between elements \( T^i \) and the commutations such as \( [T^\pm, T^A, T^B] \) are not fully antisymmetric.
is equivalent to the Jacobi identity of the Lie algebra \( G^* \). By considering the following inner product:

\[
(T^i, \tilde{T}_j) = \delta^i_j, \quad (T^i, T^j) = (\tilde{T}_i, \tilde{T}_j) = 0,
\]

one can show that the space \( A_G \oplus A_G^* \), with the commutation relations \((24, 26)\) and continuing that, is a 3-Leibniz algebra if \((A_G, A_G^*)\) is a 3-Leibniz bialgebra\(^8\) and these are equivalent if \((G, G^*)\) construct a Lie bialgebra \((27)\):

\[
\begin{align*}
[T^+, T^i; T^j] &= f^{ij} k T^k, \quad [T^i; T^j; T^k] = -f^{ij} k T^-, \quad [T^i, T^j; T^k] = \tilde{f}_{ij} k T^+, \quad [\tilde{T}_i, T^j; T^k] = \tilde{f}_{ij} k T^-, \\
[T^+, \tilde{T}_i; T^j] &= -\tilde{f}_{ik} j T^k, \quad [\tilde{T}^i, T^j, T^k] = \tilde{f}_{ik} j T^+, \quad [\tilde{T}^i, \tilde{T}^j; T^k] = -f^{ik} j T^-, \quad [\tilde{T}^i, T^j; T^k] = -f^{ik} j T^-, \quad [\tilde{T}_i, \tilde{T}_j; T^k] = \tilde{f}^{ik} j T^+
\end{align*}
\]

if we take \( T^A \) as a basis for the Manin triple of 3-Leibniz algebra with \( A = i, T^i = T^i \) and \( A = i + \dim G + 2, \)

\( T^{A+\dim G+2} = T^i \) and \( A = (-) + \dim G + 2, T^{A-\dim G+2} = T^- \) and \( A = (+) + \dim G + 2, T^{A+\dim G+2} = T^+ \),

these are not exactly commutation relations that showed in \[(39)\]. If we take \( a = \gamma, d = \gamma + 1 \) and other indice from algebra then we will have:

\[
\begin{align*}
\tilde{f}_{ijk} \tilde{f}^{lm} + \tilde{f}^{kli} f_{kj} m + \tilde{f}^{kjm} f_{kj} l &= 0, \\
\text{if } d = \gamma, c = \gamma & \quad \tilde{f}_{kmi} \tilde{f}^{lmj} &= 0, \\
\text{and we take } d = \gamma, c = \gamma & \quad \tilde{f}_{kmi} \tilde{f}^{lmj} &= 0.
\end{align*}
\]

Now, summation of \[(29), (30), (31)\] gives us \[(19)\] which is mix Jacobi for Lie bialgebra.

### 4 BL model on Manin of 3-Leibniz algebras ( \( M2 \leftrightarrow D2 \) )

In the previous section, we study the Manin triple \((\mathcal{D}, A_G, A_G^*)\) and correspondence between the 3-Leibniz bialgebra \((A_G, A_G^*)\) and Lie bialgebra \((G, G^*)\) in the special case. In this section, we will use the \( N = 6 \) BL model which was explained in section 2 with one difference on the 3-Leibniz algebras. The 3-Leibniz algebras in our model is \((\mathcal{D})\), i.e., the Manin triple which is \((4 + 2 \dim G)\) dimensional 3-Leibniz algebra\(^9\). We will show its relation with WZW model with the bialgebraic structure equipped with one 2-cocycle. Note that, with this work all provided issues and conditions in the section \[2\] for BL models are generalizable to a Manin triple as a 3-Leibniz algebra, i.e., the shape of the equation of motions, Lagrangian and ... are not modified. So consider the Lagrangian as follows:

\[
\mathcal{L} = -D^\mu \bar{Z}_\alpha^A D_\mu Z_\alpha^A - i\bar{\psi}_\alpha \gamma^\mu D_\mu \psi_\alpha - V + \mathcal{L}_{CS} + iF^{ABC} D^\beta \bar{\psi}_\alpha \psi_\beta Z_\alpha^A Z_\beta^B Z_\gamma^C + 2iF^{ABCD} \bar{\psi}_\alpha \psi_\beta Z_\alpha^A Z_\beta^B Z_\gamma^C + \frac{i}{2} \varepsilon^{\alpha\beta\gamma\delta} F^{ABCD} \bar{\psi}_\alpha \gamma D \bar{\psi}_\beta \gamma Z_\alpha^A Z_\beta^B Z_\gamma^C Z_\delta^\gamma + \frac{i}{2} \varepsilon^{\alpha\beta\gamma\delta} F^{ABCD} \bar{\psi}_\alpha \gamma D \bar{\psi}_\beta \gamma Z_\alpha^A Z_\beta^B Z_\gamma^C Z_\delta^\gamma
\]

such that, structure constants have the following relations obtained from \[(28)\]:

\[
[T^A, T^B; T^C] = F^{ABC} D T^D,
\]

\[
\begin{align*}
F^{-AB}_C &= 0, \\
F^{\alpha AB}_C &= 0, \\
F^{ABC} &= 0, \\
F^{ABC} &= 0,
\end{align*}
\]

\[
\begin{align*}
F^{ij} k &= f^{ij} k, \\
F^{ikj} &= -f^{jk} i, \\
F^{ijk} &= \tilde{f}_{ijk}, \\
F^{ikj} &= -\tilde{f}_{ijk}, \\
F^{ijk} &= -\tilde{f}_{ijk},
\end{align*}
\]

\[
\begin{align*}
F^{\alpha ij} k &= f^{\alpha ij} k, \\
F^{\alpha ikj} &= -f^{\alpha jk} i, \\
F^{\alpha ijk} &= \tilde{f}_{\alpha ijk}, \\
F^{\alpha ikj} &= -\tilde{f}_{\alpha ijk}, \\
F^{\alpha ijk} &= -\tilde{f}_{\alpha ijk},
\end{align*}
\]

\[
\begin{align*}
F^{\alpha ijk} &= f^{\alpha ijk}, \\
F^{\alpha ijk} &= -f^{\alpha jk} i, \\
F^{\alpha ijk} &= -f^{\alpha jk} i, \\
F^{\alpha ijk} &= f^{\alpha ijk}.
\end{align*}
\]

\(^8\) Note that this is not direct sum of 3-Lie algebras.

\(^9\) In general the Manin triple of 3-Leibniz bialgebra \((\mathcal{D}, A_G, A_G^*)\) does not exist and Lie bialgebra \((G, G^*)\) have no correspondence with \((A^*, A^*)\), but for this special example \(\mathcal{D}\) is a 3-Leibniz algebra and there is Manin triple \((\mathcal{D}, A_G, A_G^*)\), then we have this correspondence.

\(^{10}\) \(Z_\gamma^A\) are the fields of model that are 3-Leibniz algebra \((\mathcal{D})\) valued and \((4 + 2 \dim G)\) is the dimension of 3-Leibniz algebra \(\mathcal{D}\).
the gauge transformation for gauge fields will be
\[
\delta \tilde{A}_\mu = \partial_\mu \Lambda + \Lambda \tilde{F}_\mu + \tilde{F}_\mu \Lambda, \quad (35)
\]
where \(\Lambda^C = F^{CB}_D A_{\mu AB}\). By using the eq. (34), we will have following relations:
\[
\delta A_{\mu AB} = \partial_\mu A_2 + [A_{\mu + \delta}, A_2] + [A_{\mu + \delta}, A_2] + [A_{\mu + \delta}, A_1], \quad (36)
\]
\[
\delta A_{\mu + i} = \partial_\mu A_1, \quad (37)
\]
\[
\delta A_{\mu + i} = \partial_\mu A_1, \quad (38)
\]
so that \(A_1\) and \(A_2\) take following relations:
\[
A_1 = 2A_{+ \delta}T^A, \quad A_2 = A_{+ \delta}A^F \quad (39)
\]
Note that we have used \(\tilde{A}\) because we thought it could be confused with \(A\) in 3-Leibniz bialgebra which could get \(i, \tilde{i}, +, \ldots\) values, i.e., \(\{T^\lambda\}\) are the basis of Manin triple of Lie bialgebra with \(\tilde{A} = i, T^i, A = + \ldots\) and \(\Lambda\) is the structure constant of Manin triple of Lie bialgebra \((D_G)\). The other index you will see in the following relation is \(A\). As you know when we separated the structure constants with only \(i, \tilde{i}\), it must remain some of them with a combinational index of them and \(+, \ldots\) which will be shown with \(A\) in the rest of this work. Also from eq. (39), the gauge group for this case is \(D_G \otimes D_G\) and after using the structure constants of 3-Leibniz bialgebra, it reduce to \(D_G\), i.e., Drinfeld double of Lie bialgebra. Now using of the above relations the terms of Lagrangian turn into the following form:
\[
D_\mu Z^D_\nu D^\mu \tilde{Z}^D_\nu = \partial_\mu Z^D_\nu \partial_\mu \tilde{Z}^D_\nu + C^\mu_\nu \tilde{Z}^D_\nu + 4 C^\mu_\nu \tilde{Z}^D_\nu + 4 f_{ijk} f_{ijk} A^j_\mu \tilde{Z}^D_\nu + C^\mu_\nu \tilde{Z}^D_\nu + C^\mu_\nu \tilde{Z}^D_\nu + ... \quad (40)
\]
where \(Z^\alpha = X^\alpha + iX^\alpha^4\). Furthermore, the first term of CS term \(10\) turns into the following forms:
\[
\frac{1}{2} \epsilon^{\mu \nu \lambda} F^{ABCD} A_{\mu \nu \lambda} = 2 \epsilon^{\mu \nu \lambda} F^{BCD} A_{\mu \nu \lambda} = 2 \epsilon^{\mu \nu \lambda} F^{BCD} A_{\mu \nu \lambda} \quad (41)
\]
where
\[
\epsilon^{\mu \nu \lambda} F^{BCD} A_{\mu \nu \lambda} = \frac{1}{3} \epsilon^{\mu \nu \lambda} f_{ijk} A^i_\mu \partial_\mu A^j_\nu \lambda_j + \frac{2}{3} \epsilon^{\mu \nu \lambda} f_{ijk} A^i_\mu \partial_\mu A^j_\nu \lambda_j + \frac{2}{3} \epsilon^{\mu \nu \lambda} f_{ijk} A^i_\mu \partial_\mu A^j_\nu \lambda_j + \frac{2}{3} \epsilon^{\mu \nu \lambda} f_{ijk} A^i_\mu \partial_\mu A^j_\nu \lambda_j \quad (42)
\]
and
\[
\epsilon^{\mu \nu \lambda} F^{BCD} A_{\mu \nu \lambda} = - \frac{1}{2} \epsilon^{\mu \nu \lambda} f_{ijk} A^j_\mu \partial_\mu A^k_\nu \lambda_k - \frac{1}{3} \epsilon^{\mu \nu \lambda} f_{ijk} A^j_\mu \partial_\mu A^k_\nu \lambda_k + \frac{5}{6} \epsilon^{\mu \nu \lambda} f_{ijk} A^j_\mu \partial_\mu A^k_\nu \lambda_k - \frac{5}{6} \epsilon^{\mu \nu \lambda} f_{ijk} A^j_\mu \partial_\mu A^k_\nu \lambda_k + \frac{5}{6} \epsilon^{\mu \nu \lambda} f_{ijk} A^j_\mu \partial_\mu A^k_\nu \lambda_k - \frac{5}{6} \epsilon^{\mu \nu \lambda} f_{ijk} A^j_\mu \partial_\mu A^k_\nu \lambda_k \quad (43)
\]
and the second term of CS term turn into
\[
\frac{1}{3} \epsilon^{\mu \nu \lambda} F^{AEF} F^{BCDG} A_{\mu \nu \lambda} A_{\mu \nu \lambda} A_{\mu \nu \lambda} = - 2 \epsilon^{\mu \nu \lambda} F^{AEF} F^{BCDG} A_{\mu \nu \lambda} A_{\mu \nu \lambda} A_{\mu \nu \lambda} \quad (44)
\]
where

\[ \epsilon^{\mu \nu \lambda} F^{AB} \tilde{F}_{\mu} e_{\nu} F^F_{\lambda} A_{\mu E} F_{\nu B} A_{\lambda C} = \frac{1}{2} \epsilon^{\mu \nu \lambda} f_{ijk} f_{ilm} A_{ijk} A_{ilm} + \frac{1}{2} \epsilon_{\mu \nu \lambda} f_{ijk} f_{ilm} A_{ijk} A_{ilm} \]

\[ + \frac{1}{2} \epsilon_{\mu \nu \lambda} f_{ijk} f_{ilm} A_{ijk} A_{ilm} \]

\[ + \frac{1}{2} \epsilon^{\mu \nu \lambda} f_{ijkl} f_{ilm} A_{ijkl} A_{ilm} + \frac{1}{2} \epsilon^{\mu \nu \lambda} f_{ijkl} f_{ilm} A_{ijkl} A_{ilm} + \frac{1}{2} \epsilon_{\mu \nu \lambda} f_{ijkl} f_{ilm} A_{ijkl} A_{ilm} \]

\[ + \frac{1}{2} \epsilon^{\mu \nu \lambda} f_{ijkl} f_{ilm} A_{ijkl} A_{ilm} \]

\[ + \frac{1}{2} \epsilon^{\mu \nu \lambda} f_{ijkl} f_{ilm} A_{ijkl} A_{ilm} \]

\[ + \frac{1}{2} \epsilon^{\mu \nu \lambda} f_{ijkl} f_{ilm} A_{ijkl} A_{ilm} + \frac{1}{2} \epsilon^{\mu \nu \lambda} f_{ijkl} f_{ilm} A_{ijkl} A_{ilm} \]

\[ + \frac{1}{2} \epsilon_{\mu \nu \lambda} f_{ijkl} f_{ilm} A_{ijkl} A_{ilm} + \frac{1}{2} \epsilon_{\mu \nu \lambda} f_{ijkl} f_{ilm} A_{ijkl} A_{ilm} \]

\[ + \frac{1}{2} \epsilon_{\mu \nu \lambda} f_{ijkl} f_{ilm} A_{ijkl} A_{ilm} \]

Finally, the gauge group $D_\mu$ is obtained from the special 3-Leibniz algebra $D_\mu$.

The Lagrangian (32) is rewritten as follows:

\[ L_{\text{CS}} = \frac{1}{2} \epsilon_{\mu \nu \lambda} \{ C_{\mu \nu} A_{\lambda} - D_{\nu} A_{\lambda} - A_{\lambda} D_{\nu} - [A_{\lambda}, A_{\mu}]\} + \epsilon_{\mu \nu \lambda} \partial_\mu A^\lambda - \partial_\nu A^\lambda - [A_{\mu}, A_{\nu}]B \]
where $I = 1, ..., 8$ and the ... terms do not affect the Yang-Mills and WZW-like terms. In this way, one can reach to Yang-Mills relation (similar to [19]) with 7 scalar fields by giving vacuum expectation value (VEV) to real part of one of complex scalar fields $Z^\alpha$. This manner imaginary part of it behaves as a Goldstone mode in Higgs mechanism [21]. Now, by defining $B_{\nu\lambda\bar{\alpha}} = \partial_\nu A_{\lambda\bar{\alpha}} - \partial_\lambda A_{\nu\bar{\alpha}} - [A_\nu, A_\lambda]_{\bar{\alpha}}$, $F_{\nu\lambda\bar{\alpha}} = \partial_\nu A_{\lambda\bar{\alpha}} - \partial_\lambda A_{\nu\bar{\alpha}} - [A_\nu, A_\lambda]_{\bar{\alpha}}$, by integrating $C_\mu B$, $C_\mu^B$ in (47) we will obtain the following equation:

$$\mathcal{L} = \frac{1}{4} F_{\nu\lambda\bar{\alpha}} F^{\nu\lambda\bar{\alpha}} + \frac{1}{8} B_{\nu\lambda\bar{\alpha}} B^{\nu\lambda\bar{\alpha}} + \ldots,$$

such that $F_{\nu\lambda\bar{\alpha}}$ and $B_{\nu\lambda\bar{\alpha}}$ are two strength fields on $D_G \oplus D_G^*$, which look like Yang-Mills and B-field of a string, respectively. Note that $B_{\mu\nu\lambda\bar{\alpha}}$ as the antisymmetric Lie algebra $D_G$ valued field obtained from Higgs mechanism while CS term is written to the BL model on Manin triple $D$ not from prior 3-form since the BL Lagrangian do not conclude it. A 3-form can be written as $H = dB + C$ for the M2-M5 system as s mentioned in [19], although there is not a 3-form $C$ in this case, a 2-form $B$ on the boundary of M2-brane ending to M5-brane could be the one appeared in our model. The Lagrangian (47) is a conclusion from low energy limit of DBI action related to the propagating of the string in the background $G_\mu\nu$ and Lie algebra valued fields $A_{\mu\nu\lambda\bar{\alpha}}$ and $B_{\mu\nu\lambda\bar{\alpha}}$. So the antisymmetric Lie algebra valued form $B_{\mu\nu\lambda\bar{\alpha}}$ could be B-field of the string at the boundary of the M2-brane end to M5-brane [49]. This means we have considered boundary conditions of the M2-M5 system which make some restrictions on the gauge fields. Going to the boundary for the supersymmetric problems change the number of supersymmetry so that for the $N = 6$ supersymmetry two anti-chiral fermions on the boundary is disappeared. So we could expect $N = (4, 4)$ supersymmetry to the string [21][22]. In this way, the $N = 6$ BL model related to the bosonic sector of a $N = (4, 4)$ supersymmetric string theory in our model. The $N = (4, 4)$ supersymmetric WZW model is constructed from the group of Manin triple (as a Lie algebra) of Lie bialgebra, i.e., Drinfeld double $D_G$ with one 2-cocycle. Applying 3-Leibniz bialgebra in the BL model make clear relation between two superconformal theory (superconformal BLG model stated in Ref. [42][43]) and one can obtain them from each other, i.e., BL model with 3-Leibniz algebra structure $D$ as a Manin triple is constructed by having 2-dimensional $N = (4, 4)$ supersymmetric WZW models (analyzed [22]) with Lie bialgebra structure equipped one 2-cocycle and vice versa. We meant the vice versa process is only from the algebraic point of view. On the other hand, the algebraic relation between Manin triple of 3- Leibniz algebra and Lie bialgebra (as we describe in the proposition) make possible constructing BL model from the $N = (4, 4)$ supersymmetric WZW model. There exist two gluing matrices $R, \bar{R}$ in an $N = (2, 2)$ WZW which must satisfy $R \in Aut(G)$ and $\bar{R} \in Hom(G_-, G_+)$ then according to these gluing matrices there are two types D-brane, i.e., B-type and A-type respectively. In order to preserve half of the bulk supersymmetry [45] then it is natural to conclude that for the $N = (4, 4)$ WZW. In this case, existence one 2-cocycle which can play an important role, also possible relations between gluing matrices $R$ and $\bar{R}$ according to [47] and [45].

Now, it remains to show that $B_{\mu\nu\lambda\bar{\alpha}}$ is a B-field. As we said a self-dual string appears on the boundary of M5-brane which will construct a 3-form on the worldvolume. Appeared B-field in our model is a result of 3-form that play background field role. Born-Infeld action for propagating string in a background with $G_{\mu\nu}, A_\mu$ and $B_{\mu\nu}$ fields resulting action is DBI action [22][48]:

$$S = \int d^{p+1}x \sqrt{-det(G_{\mu\nu}h_{AB} + 2\pi\alpha' F_{\mu\nu AB} + B_{\mu\nu AB})},$$

$$= \frac{1}{2} F_{\nu\lambda \bar{\alpha}} F^{\nu\lambda \bar{\alpha}} + \frac{1}{4} B_{\nu\lambda \bar{\alpha}} B^{\nu\lambda \bar{\alpha}} + \ldots.$$ (48)

We must show that this B-field can be related to the B-field of WZW action (as in Ref. [39]), for this reason, we use Lie algebra valued fields:

$$S_{\text{WZW-like}} = \int d^3x \epsilon^{\alpha\beta\gamma} L_\mu L_\nu L_\lambda L_\rho \partial_\alpha X^{\mu} \partial_\beta X^{\nu} \partial_\gamma X^{\lambda} \Gamma([T_1 T_2 T_3 T_4], T_K T_N),$$ (49)

by some calculations we obtain

$$S_{\text{WZW-like}} = \int d^2x \left\{ \frac{1}{6} \epsilon^{\alpha\beta\gamma} B_{\nu\lambda\bar{\alpha}} Q_{\beta} X^{\nu} \partial_\gamma X^{\lambda} \Gamma([T_1 T_2 T_3] + \frac{1}{6} \epsilon^{\alpha\beta\gamma} B_{\nu\lambda\bar{\alpha}} Q_{\beta} X^{\nu} \partial_\gamma X^{\lambda} \Gamma([T_1 T_2 T_3] \right\} + \ldots,$$ (50)

12 Note that, this method for obtaining of WZW is different of the method mentioned in [45][47].
13 Note that $G_+$ and $G_-$ are Lie algebra and dual of it, respectively, that make the Lie bialgebra $G$, i.e. $G = G_+ \oplus G_-$. 

8
where \( B_{\nu\mu}^{Q} = L_{\nu}^{L} L_{\lambda}^{N} f_{N L}^{Q} T_{I}^{P} x^{I} f_{P J}^{Q} L_{\mu}^{L} X^{I\mu} T_{I}^{L} |_{\text{boundary}} = x^{L} T_{L} |_{\text{boundary}} \).

5 Boundary conditions of BL model on Manin triple of 3-Leibniz algebra

In this section, we obtain the boundary conditions of \( N = 6 \) BL model with 3-Leibniz bialgebra structure, as we have shown for the \( N = 8 \) BLG model. The difference between them is in the number of supersymmetry and structure constants which are not totally antisymmetric. For this purpose, we must be careful in decreasing the number of supersymmetries. The normal component of supercurrent to boundary direction must be discarded in order to preserve maximum unbroken supersymmetry, i.e., we obtain the supercurrent and reset to zero normal component of it. For this purpose, we will need the other presentation of BL model which have been applied by Bagger and Lambert in Ref. [17]. They have shown that \( F^{A B C D} \) sets up a Lie algebra \( G \) with the following form:

\[
G = \otimes_{\lambda} G_{\lambda} \tag{51}
\]

such that \( G_{\lambda} \)s are commuting Lie algebras and this allows one to rewrite the Lagrangian \( 32 \) as follows:

\[
L = -Tr(D^{\mu} \bar{Z}_{\alpha}, D_{\mu} Z^{\alpha}) - iTr(\bar{\psi}^{\alpha}, \gamma^{\mu} D_{\mu} \psi_{\alpha}) - V + L_{CS} \\
- iTr(\bar{\psi}^{\alpha}, [\psi_{\alpha}, Z^{\beta}; \bar{Z}_{\beta}]) + 2iTr(\bar{\psi}^{\alpha}, [\psi_{\beta}, Z^{\beta}; \bar{Z}_{\alpha}]) \\
+ \frac{i}{2} \varepsilon^{\alpha\beta\gamma\delta} Tr(\bar{\psi}^{\alpha}, [Z^{\gamma}, Z^{\delta}; \psi_{\beta}]) - \frac{i}{2} \varepsilon^{\alpha\beta\gamma\delta} Tr(\bar{Z}_{\delta}, [\bar{\psi}_{\alpha}, \psi_{\beta}; \bar{Z}_{\gamma}]), \tag{52}
\]

with

\[
V = \frac{2}{3} Tr(\Upsilon_{\gamma\delta}, \bar{\Upsilon}_{\gamma\delta}), \tag{53}
\]

and

\[
\Upsilon_{\gamma\delta} = [Z^{\gamma}, Z^{\delta}; \bar{Z}_{\beta}] - \frac{1}{2} \delta_{\gamma}^{\delta} [Z^{\alpha}, Z^{\beta}; \bar{Z}_{\alpha}] + \frac{1}{2} \delta_{\beta}^{\delta} [Z^{\alpha}, Z^{\gamma}; \bar{Z}_{\alpha}]. \tag{54}
\]

Supersymmetry transformations are redressed as in Ref. [23, 24] and the supercurrent is given as follows:

\[
J_{\mu} = \varepsilon^{I} J_{\mu}^{I} = Tr(\delta \bar{\psi}_{\alpha} \gamma_{\mu}, \psi^{\alpha}) + Tr(\delta \bar{\psi}^{\alpha} \gamma_{\mu}, \psi_{\alpha}). \tag{55}
\]

Boundary conditions of \( N = 6 \) BLG theory [38] was investigated in the same method as for boundary conditions of \( N = 8 \) BLG theory [38] was done. The attainment of vanishing the normal component of supercurrent in the boundary conditions is following relation:

\[
0 = \Gamma_{\alpha\beta}^{I} \varepsilon^{I} \gamma^{\mu} D_{\mu} Z^{\alpha} \gamma_{2} \psi^{\alpha} - \Gamma_{\alpha\beta}^{I} [Z^{\gamma}, Z^{\delta}; \bar{Z}_{\gamma}] \varepsilon^{I} \gamma_{2} \psi^{\alpha} - \Gamma_{\gamma\delta}^{I} [Z^{\gamma}, Z^{\delta}; \bar{Z}_{\alpha}] \varepsilon^{I} \gamma_{2} \psi^{\alpha} \\
- \Gamma_{\alpha\beta}^{I} \varepsilon^{I} \gamma^{\mu} D_{\mu} \bar{Z}_{\beta} \gamma_{2} \psi_{\alpha} - \Gamma_{\alpha\beta}^{I} [\bar{Z}_{\gamma}, \bar{Z}_{\beta}; Z^{\gamma}] \varepsilon^{I} \gamma_{2} \psi_{\alpha} - \Gamma_{\gamma\delta}^{I} [\bar{Z}_{\gamma}, \bar{Z}_{\delta}; Z^{\alpha}] \varepsilon^{I} \gamma_{2} \psi_{\alpha}. \tag{56}
\]

As we told the method is the same method in [38] with some difference. One of them is decomposition of \( SU(4) \) as Lorentzian symmetry into two groups. in this case, one way is decompose 4 complex scalar field into
where \( \tilde{\Gamma} \) is the Basu-Harvey example. The result of applying that example and using commutation relations mentioned in (28) is the Basu-Harvey example. The result of applying that example and using commutation relations mentioned in (28) is the Basu-Harvey example.

\[
X^\alpha = Z^1, Z^2 \quad \text{and} \quad Y^{\alpha'} = Z^2, Z^3 \quad \text{as} \quad SO(1, 1) \times SU(2) \times SU(2) \subset SO(1, 2) \times SU(4) \quad [22]:
\]

\[
0 = \Gamma_I^{\alpha\beta} \epsilon^{I\gamma\delta} D_\mu X^\beta \gamma^2 \psi^\alpha - \Gamma^I_{\alpha\beta} \epsilon^{I\gamma\delta} D_\mu \tilde{X}^\gamma \gamma^2 \psi^\alpha, \quad \text{(57)}
\]

where \( \tilde{\Gamma}_2 \psi^\alpha = \tilde{\psi}_\alpha \). Different Lorentzian symmetry for each has been considered, individually, then one can take zero each one with different Lorentzian symmetry and solve. In order to this purpose, the different type of boundary conditions can be provided which we have investigated only the Dirichlet boundary conditions in half of the scalar fields, i.e., \( D_\mu Y = 0 \) and simplest solution for it i.e. \( Y = 0 \), maintain equations of (57) are as follows:

\[
0 = \Gamma_I^{\alpha\beta} \epsilon^{I\gamma\delta} D_\mu X^\beta \gamma^2 \psi^\alpha \quad (58)
\]

\[
0 = \tilde{\Gamma}^{I\alpha\beta} \epsilon^{I\gamma\delta} D_\mu \tilde{X}^\gamma \gamma^2 \psi^\alpha, \quad (59)
\]

\[
0 = \Gamma_I^{I\alpha\beta} \epsilon^{I\gamma\delta} D_\mu \tilde{X}^\gamma \gamma^2 \psi^\alpha, \quad (60)
\]

\[
0 = \tilde{\Gamma}^{I\alpha\beta} \epsilon^{I\gamma\delta} D_\mu \tilde{X}^\gamma \gamma^2 \psi^\alpha, \quad (61)
\]

\[
0 = \Gamma_I^{\alpha\beta} \epsilon^{I\gamma\delta} D_\mu X^\beta \gamma^2 \psi^\alpha, \quad (62)
\]

\[
0 = \Gamma_I^{\alpha\beta} \epsilon^{I\gamma\delta} D_\mu X^\beta \gamma^2 \psi^\alpha, \quad (63)
\]

Introducing projection operator for solving these equations is the main work now [22]. Relation (63) is the only condition independent of \( \Gamma \) on scalar fields, which is Basu-Harvey equation with \( F^{ABC} D \) as the structure constant of the Manin triple of 3-Leibniz algebra \( D \):

\[
0 = D_2 X^I_A + \frac{1}{6} J^{IJK} X^I_B X^K_C X^L_D F^{ABC} D, \quad (64)
\]

In the previous sections, we introduced a special example of 3-Leibniz bialgebra in relation to Lie bialgebra. The result of applying that example and using commutation relations mentioned in [23] is the Basu-Harvey...
equation with the following form:

\[
0 = D_2 X^I - \frac{1}{6} \epsilon^{IJKL} X^K X^L F_{BCD}, \tag{65}
\]

\[
0 = D_2 X^I - \frac{1}{6} \epsilon^{IJKL} X^K X^L F_{BCD}, \tag{66}
\]

\[
0 = D_2 X^I + \frac{1}{6} \epsilon^{IJKL} X^K X^L D F_{BCD}
= D_2 X^I + \frac{1}{2} g_{YM} \epsilon^{IJK} X^K X^L F_{BCK}, \tag{67}
\]

\[
0 = D_2 X^I + \frac{1}{6} \epsilon^{IJKL} X^K X^L D F_{BCD}
= D_2 X^I + \frac{1}{2} g_{YM} \epsilon^{IKL} X^K X^L D F_{BCK}. \tag{68}
\]

Vanishing equations (65, 66) is a result of existence of two totally antisymmetric coefficients with \((I, J, K, L)\) and \((A, B, C = -, +, i, \bar{i}, j, \bar{j})\) index \((A, B, C = -, +, i, \bar{i}, j, \bar{j})\), i.e., \(\epsilon^{IJKL}\) and \(F_{BCD}\). Equations (65, 66) can be combined and written in following form:

\[
\partial_{\sigma} X^I = \frac{1}{2} \epsilon^{IJKL} X^K X^L F_{BCD} A, \tag{69}
\]

where \(F_{BCA}\) is the structure constant for \(D_{\sigma}\) with \(\bar{A}, \bar{B}, \bar{C} = i, \bar{i}\) which is the Manin triple of Lie algebra \(G\). It has been considered as Nahm equation a result of considering boundary conditions in D1-branes \[19\] with the Lie bialgebra representation. So, one of the boundary conditions of the BL model for multiple membranes is Basu-Harvey equation is in relation to Nahm equation \[19\] and vice versa as BLG one in Ref. \[35\] for 3-Lie bialgebra. According to the obtained relation between Nahm and Basu-Harvey equations, there is the relation between M2 and D2 actions and vice versa. As you know, Nahm equation is a result of considering boundary condition for DBI action \[19\] and according to the relation obtained between Nahm and Basu-Harvey equations, it could manifest relation between M2 and D2 actions and vice versa. This show relation between M-theory and string theory and vice versa because BL model is a model for membranes which have been derived from supergravity action as low energy limit of M-theory and DBI is the similar one in the string theory.

Conclusions

We have constructed \(N = 6\) Chern-Simons gauge theory (BL model) on a special Manin triple \((D, A, A^*)\) as a 3-Leibniz bialgebra. Then using the correspondence between 3-Leibniz bialgebra \((A, A^*)\) and Lie bialgebra \((G, G^*)\), we have shown a relation between \(N = 6\) BL model with 3-Leibniz algebra structure and \(N = (4, 4)\) WZW model over Lie group with Manin triple (as Lie algebra) of Lie bialgebra structure. Also, boundary conditions for BL model on Manin triple were considered and the Basu-Harvey equation reduced to Nahm equation, in this way.

Conversely; if \(N = (4, 4)\) supersymmetric WZW action with a Lie bialgebra equipped one 2-cocycle is constructed by helping DBI-action for D2-brane, then constructing M2-model on the special 3-Leibniz algebra is possible according to the relation between Lie bialgebra and 3-Leibniz bialgebra. The other result of this correspondence is obtaining Basu-Harvey equation by considering Nahm equation as the boundary conditions of the D1-string ending to D3-brane in the following relation with DBI action.

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\[14\] Relation between Nahm and Basu-Harvey has mentioned in Ref. \[41\] by using of Lie superalgebra \[41\].


References

[1] J. Polchinski, String theory. Vol. 2: Superstring theory and beyond.
[2] E. Cremmer, B. Julia, J. Scherk, "Supergravity Theory in Eleven-Dimensions", Phys.Lett. B76 (1978) 409.
[3] E. Bergshoeff, E. Sezgin, P. Townsend, "Supermembranes and Eleven-Dimensional Supergravity", Phys.Lett. B189 (1987) 75; M. Duff, T. Inami, C. Pope, E. Sezgin, K. Stelle, "Semiclassical Quantization of the Supermembrane", Nucl.Phys. B297 (1988) 515; P. K. Townsend, "D-branes from M-branes", Phys. Lett. B373 (1996) 68. [hep-th/9512062]; C. Schmidhuber, "D-brane actions", Nucl.Phys. B467 (1996) 146. [hep-th/9601003].
[4] P. S. Howe, N. D. Lambert, P. C. West, "The self-dual string soliton", Nucl. Phys. B515 (1998) 203216. [hep-th/9709014].
[5] C. S. Chu and E. Sezgin, "M-fivebrane from the open supermembrane", JHEP 9712, 001 (1997) [hep-th/9710223]; P. Horava and E. Witten, "Eleven-Dimensional Supergravity on a Manifold with Boundary" Nucl. Phys. B 475 (1996) 94 [hep-th/9603142]; M. Cederwall, "Boundaries of 11-dimensional membranes", Mod. Phys. Lett. A 12 (1997) 2641 [hep-th/9704161].
[6] J. Bagger, N. Lambert, "Modeling multiple M2s", Phys. Rev. D75 (2007) 045020, [arXiv:hep-th/0611108].
[7] J. Bagger and N. Lambert, "Gauge symmetry and supersymmetry of multiple M2-branes", Phys. Rev. D77 (2008) 065008, [arXiv:0711.0955 [hep-th]].
[8] J. Bagger, N. Lambert, "Comments On Multiple M2-branes", JHEP 02 (2008) 105, [arXiv:0712.3738 [hep-th]].
[9] A. Gustavsson, "Algebraic structures on parallel M2-branes", Nucl. Phys. B811 (2009) 66, [arXiv:0709.1260 [hep-th]].
[10] M. Van Raamsdonk, "Comments on the Bagger-Lambert theory and multiple M2-branes", JHEP 05 (2008) 105, [arXiv:0803.3803 [hep-th]].
[11] E. A. Bergshoeff, M. de Roo, O. Hohm, D. Roest, "Multiple Membranes from Gauged Supergravity", JHEP 0808 (2008) 091, [arXiv:0806.2584]; J. P. Gauntlett, J. B. Gutowski, "Constraining Maximal Supersymmetric Membrane Actions", JHEP 06 (2008) 053, [arXiv:0804.3078 [hep-th]].
[12] A. Basu, J. A. Harvey, "The M2-M5 brane system and a generalized Nahm's equation", Nucl. Phys. B713 (2005) 136, [hep-th/0412310].
[13] M. M. Sheikh-Jabbari, Tiny Graviton Matrix Theory: DLCQ of IIB Plane-Wave String Theory, A Conjecture, JHEP 0409:017,2004, [arXiv:hep-th/0406214].
[14] E. Witten, "Bound States Of Strings And p-Branes" Nucl.Phys. B460:335 (1996), [hep-th/9510135].
[15] J. Bagger, N. Lambert, S. Mukhid, C. Papageorgakis, "Membranes in M-theory", [arXiv:1203.3546v1 [hep-th]].
[16] O. Aharony, O. Bergman, D. L. Jafferis, J. Maldacena, "N=6 superconformal Chern-Simons-matter theories, M2-branes and their gravity duals", JHEP 10 (2008) 091, [arXiv:0806.1218 [hep-th]].
[17] J. Bagger, N. Lambert, "Three-Algebras and N=6 Chern-Simons Gauge Theories" Phys. Rev. D79:025002 (2009), [arXiv:0807.0163 [hep-th]].
[18] N. Lambert and C. Papageorgakis, "Relating U(N) x U(N) to SU(N) x SU(N) Chern-Simons Membrane theories",JHEP 1004:104,2010, [arXiv:1001.4779v3 [hep-th]].
[19] P-M Ho, Y. Imamura, Y. Matsuo, "M2 to D2 revisited", JHEP 07 (2008) 003, [arXiv:0805.1202 [hep-th]].
[20] J. Gomis, G. Milanesi, J. G. Russo, "Bagger-Lambert Theory for General Lie Algebras," JHEP 06 (2008) 075, arXiv:0805.1012[hep-th]; Y. Pang, T. Wang, "From N M2s to N D2s," Phys. Rev. D78 (2008) 125007, arXiv:0807.1441[hep-th]; T. Li, Y. Liu, D. Xie, "Multiple D2-Brane Action from M2-Branes", Int.J.Mod.Phys. A24 (2009) 3039, arXiv:0807.1183[hep-th];

[21] B. Ezhuthachan, S. Mukhi and C. Papageorgakis "The Power of the Higgs Mechanism Higher-Derivative BLG Theories", JHEP 101(2009) 0904, arXiv:0903.0003[hep-th].

[22] D. S. Berman, M. J. Perry, E. Sezgin, D. C. Thompson, "Boundary Conditions for Interacting Membranes" JHEP 1004:20, arXiv:0912.3504[hep-th].

[23] F. Passerini, "M2-Brane Superalgebra from Bagger-Lambert Theory", arXiv:0806.0363v1[hep-th].

[24] A. M. Low, "N = 6 Membrane Worldvolume Superalgebra", JHEP 0904:105,2009, arXiv:0903.0988[hep-th].

[25] K. Hosomichi and S. Lee, "Self-dual Strings and 2D SYM", JHEP 1501 (2015) 076, arXiv:1406.1802[hep-th].

[26] W. Nahm, A Simple Formalism for the BPS Monopole, Phys. Lett. B90 (1980) 413.

[27] A. Rezaei-Aghdam, L. Sedghi-Ghadim, "3-Leibniz bialgebras (3-Lie bialgebras)", arXiv:1604.04475v1[math.RA].

[28] Y. K. Schwarzbach, "Lie bialgebras, Poisson-Lie groups and dressing transformations, Integrability of nonlinear systems, second edition", Lecture notes in physics 038, Springer-Verlag (2004)107.

[29] A.M. Blokh, "On a generalization of the concept of a Lie algebra", Dokl. Akad. Nauk SSSR 165(1965)471.

[30] J. L. Loday, "Une version non commutative des algebres de Lie: les algebres de Leibniz", Enseign Math. 39(1993)269.

[31] A. Rezaei-Aghdam, GH. Haghighatdoost, L. Sedghi-Ghadim, "Leibniz bialgebras", arXiv:1401.6845v4[math-ph].

[32] R.G. Leigh, "Dirac-Born-Infeld action from the Dirichlet sigma model", Mod. Phys. Lett. A 4 (1989) 2767.

[33] M. Kato, T. Okada, "D-branes on group manifolds", Nucl.Phys. B499 (1997) 583-595, arXiv:hep-th/9612148.

[34] M.Aali-Javanangrouh, A. Rezaei-Aghdam, "Algebraic Structures of N=(4,4) and N=(8,8) SUSY Sigma Models on Lie groups and SUSY WZW Models", arXiv:1402.5600v2[hep-th].

[35] C.S. Chu and D.J. Smith, "Multiple self-Dual strings on M5- Branes" JHEP 01 (2010) 001, arXiv:0909.2333[hep-th].

[36] T. Okazaki and D.J. Smith, "Topological M- string and Supergroup WZW Models", Phys.Rev. D94 (2016) 65016, arXiv:1512.06646[hep-th].

[37] S.Elitzur, G.W. Moore, A. Schwimmer and N. Seiberg, "Remark on the Canonical Quantization of Chern-Simons- Witten Theory", Nucl. Phys. B326 (1989) 108.

[38] M.Aali-Javanangrouh, A. Rezaei-Aghdam, "From Basu-Harvey to Nahm equation via 3-Lie bialgebra", arXiv:1604.05181[hep-th].

[39] M.Aali-Javanangrouh, A. Rezaei-Aghdam, "M2 to D2 and vice versa by 3-Lie and Lie bialgebra", Eur. Phys. J. C 76 (2016) 632, arXiv:1604.05183[hep-th].

[40] R. Bielawski, "Nahm’s, Basu-Harvey-Terashima’s equations and Lie superalgebras.", arXiv:1503.03779[math-ph].
[41] P. de Medeiros, J. Figueroa-O’Farrill, E. Mendez-Escobar, and P. Ritter, ”On the Lie-algebraic origin of metric 3-algebras,” Commun.Math.Phys. 290 (2009) 871, arXiv:0809.1086[hep-th].

[42] J. F. O’Farrill, ”Three lectures on 3-algebras”, EMPG-08-23, arXiv:0812.2865 [hep-th]; J. F. O’Farrill, ”Metric 3-Lie algebras for unitary Bagger-Lambert theories”, JHEP 0904:037,2009, arXiv:0902.4674 [hep-th].

[43] M. A. Bandres, A. E. Lipstein, J. H. Schwarz, ”N = 8 Superconformal Chern-Simons Theories”, JHEP 0805:025,2008, arXiv:0803.3242 [hep-th]; de Azcarraga, J. A. and Izquierdo, J. M., cohomology of Fillippov algebras and an analogue of withhead s lemma, J. Phys. Conf. Scr. 175 (2009) 012001, arXiv:0905.3033[math-ph]; de Azcarraga, J. A. and Izquierdo, J. M. n-ary algebra: a review with applications, J. Phys, A: Math. Theor. 43 (2010) 293001, arXiv:1005.1028[math-ph].

[44] Y. Michishita, ” The M2-brane soliton on the M5-brane with constant 3-form”, JHEP 0009 (2000) 036, Arxiv:hep-th/0008247.

[45] D.Youm, ”BPS Solitons in M5-brane Worldvolume Theory with Constant Three-Form Field”, Phys.Rev. D63 (2001) 045004, arXiv:hep-th/0009082.

[46] U. Lindström and M. Zabzine, ”D-branes in N=2 WZW models”, Phys. Lett. B560 (2003) 108, hep-th/0212042.

[47] S. Parkhomenko, ”Extended superconformal current algebras and finite-dimensional Manin triples”, Sov. Phys. JETP. 7 (1992), 1; S. Parkhomenko, ”Quasi Frobenius Lie algebras construction of \( N = 4 \) superconformal field theory”, Mod. Phys. lett. A11 (1996) 445.

[48] D. Tong, ”Lectures on String Theory ”,February 2012; J. Polchinski,” TASI lectures on D-branes”, arXiv:hep-th/9908144.

[49] C.S Chu, D.J. Smith, ”Towards the Quantum Geometry of the M5-brane in a Constant C-Field from Multiple Membranes”, JHEP 0904:097,2009, arXiv:0901.1847 [hep-th]; David S. Berman, ”M-theory branes and their interactions ”, Phys.Rept.456(2008) 89, arXiv:0710.1707 [hep-th].