The human ECG – nonlinear deterministic versus stochastic aspects

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We discuss aspects of randomness and of determinism in electrocardiographic signals. In particular, we take a critical look at attempts to apply methods of nonlinear time series analysis derived from the theory of deterministic dynamical systems. We will argue that deterministic chaos is not a likely explanation for the short time variability of the inter-beat interval times, except for certain pathologies. Conversely, densely sampled full ECG recordings possess properties typical of deterministic signals. In the latter case, methods of deterministic nonlinear time series analysis can yield new insights.

1 Introduction

The physiology of the human heart is well understood mechanistically. Clear relations between the heart rate and many regulatory influences are known. The shape of a single cycle ECG is understood in detail and can be related to the different actions of the heart during one cycle. On the other hand, electrocardiographic recordings pose intricate problems of time series analysis due to the different nature of the dynamical processes on different time scales. The long-term evolution has been reported\(^[1]\) to show non-stationarity in the sense of power law correlations. At time scales of a few hundred beats, non-stationarity becomes less pronounced and the dominant feature are rapid, hardly predictable low-amplitude fluctuations in the instantaneous heart rate which is usually defined through inter-beat intervals. At the time scale of a heart beat, high resolution ECG signals show repetitive structures specific of the beating mechanism. A number of attempts have been made to analyse various aspects of cardiac time series in the context of nonlinear deterministic dynamical systems.

Deterministic chaos offers an interesting explanation for the emergence of aperiodicity and unpredictability. Since rather simple systems can exhibit chaos, one is lead to use nonlinear time series methods to verify whether such a source of unpredictability is underlying a given observation. In fact, the concept of deterministic low-dimensional chaos has proven to be fruitful in the understanding of many complex phenomena despite the fact that very few natural systems have actually been found to be low-dimensional deterministic in the sense of the theory. In favourable situations this approach allows one to determine the number of active variables, the stability properties of the system with respect to perturbations, and even the equations of motion, including applications like noise reduction or control.

Deterministic chaos is not the only, and not even the most probable source of aperiodicity. The superposition of a large number of active degrees of freedom can produce extremely complicated signals, which might not be distinguishable from randomness. Stochasticity in the sense that a system is driven by processes whose dynamics are too complex to be inferred from the information stored in the observations is the most frequent source of unpredictability in open systems and field measurements. The fundamentals of deterministic chaos as a theory are by now well established and described in a rich literature (for example\(^[2]\), see\(^[3]\) for an introductory text). Nonlinear time series analysis based on this theoretical paradigm is described in two recent monographs\(^[4,5]\), and a number of conference proceedings volumes\(^[6–8]\).

Several authors have applied nonlinear time series methods to ECG data, with varying success. Many findings are accepted positively by cardiologists, but have been contested by more mathematically oriented researchers. One of the strongest arguments, the ability to apply chaos control techniques\(^[9]\), has recently been put in question by the successful control of a stochastic process\(^[10]\). From the understanding of physiological mechanisms like the regulatory system and the conduction system, one is lead to expect certain deterministic structures. However, the average heart rate of the healthy heart depends deterministically on several different processes, which in turn depend on other influences, so that this dependency altogether might be far too complicated to be observable as a deterministic process through ECG data alone. It is our point of view, and we will give some non-exhaustive support, that ECG data does not prove heart rate variability to be the result of an autonomous deterministically chaotic system. We will however demonstrate that even without determinism there are situations in which nonlinear time series analysis yields new insights.

2 Determinism from time series

Determinism in the mathematical sense means that there exists an autonomous dynamical system, defined typically by a first order ordinary differential equation \(\dot{x} = f(x)\) in a state space \(\Gamma \subset \mathbb{R}^D\), which we assume to be observed through a single measurable quantity \(h(x)\). The system thus possesses \(D\) natural variables, but the measurement is a usually nonlinear projection onto a scalar value (the following discussion can be straightforwardly extended to multi-channel measurements). In
order to recover the deterministic properties of the system, we have to reconstruct an equivalent of the subspace of $\Gamma$ which is explored by the solutions of the system from the observations. The time delay embedding method is a way to do so that can be derived with mathematical rigour [11]. From the sequence of $N$ scalar observations $s_1, s_2, \ldots, s_N$, overlapping vectors $s_n = (s_n, s_{n-\tau}, \ldots, s_{n-(m-1)\tau})$ are formed ($\tau$ is a delay time). For mathematically perfect, noise free observations $s_n$, it is proven that for $m > 2D_f$ there exists a one-to-one relation between $s_n \in \mathbb{R}^m$ and the unobserved vectors $x_n$ in the state space, from which the measurements were taken. Here $D_f$ is the box-counting dimension of the attractor, that is, of the set in state space which is visited asymptotically by a trajectory.

The standard approach towards establishing low-dimensional determinism in a scalar time series consists of constructing spaces with increasingly larger $m$ and to search for deterministic structures in each of these spaces. A common tool is the correlation dimension introduced by Grassberger & Procaccia [12], which tests for self-similarity of the set of points and thus for the existence of a finite dimensional attractor. Despite the existence of several pitfalls, this is still the method of choice when a finite dimension is to be established. Since contamination by about 2% of noise usually suffices to destroy all nontrivial self-similarity, dimension estimates per se cannot be recommended in field measurements, even if they stem from a deterministic source plus observational noise. A more promising approach is to establish the existence of nontrivial dynamical correlations by either directly constructing equations of motion (forecast maps) or by proving an enhanced predictability, when the dimensionality of the embedding space is sufficiently high.

3 Inter-beat intervals

In this article we discuss exclusively electrocardiographic (ECG) data [13] measured with surface electrodes. Some arguments can be carried over to invasive recordings as well. The ECG signal reflects the electrochemical activity of the cardiac muscle fibres, integrated over the whole surface with a local weight factor, depending on the relative distance and electrical conductivity between each point and the electrode.

In studies of the heart rate variability, often the long-term ECG recording is reduced to the sequence of inter-beat time intervals (RR-intervals), as defined by the time between consecutive R-waves. (See Goldberger and Goldberger [14] for an introduction to clinical electrocardiography and definitions of the nomenclature.) However, there is a fundamental difference between the full ECG trace and the sequence of inter-beat intervals. In the latter, the “time” index is given by the event number and the observable by the time elapsing in between two events. This time is not obviously a phase space observable in the sense of embedding theory, but recent work [15, 16] implies that in principle RR-intervals may be suited to search for determinism in the ECG. However, attractors reconstructed from inter-beat time intervals of similar duration possess extremely small dimen-

sions. Small errors or noise in the time intervals can easily extinguish all nontrivial sub-structure.

If indications of deterministic structure are weak, one can first try to reject certain null hypotheses. One of the simplest is that the data stem from a stationary linear Gaussian random process (GRP). It is obviously too simple and can be rejected for most interesting data sets, since the marginal distributions are almost never Gaussian. The default explanation for this fact are static nonlinearities, that is, nonlinearities in the measurement process, which have nothing to do with the dynamics, but may hide the original character of the process. Time reflection invariance, which is another property of GRPs, is not affected by static nonlinearities in the measurement. It can, however, be affected by a memory effect of the measurement device, for example by a simple causal low-pass filter on the data.

In order to test against a composite null hypothesis, one can produce new data, so called surrogates, and compare the original data and the surrogates by help of a suitably chosen test statistics. The advantage is that we do not have to specify, for example, which particular process generated the data but we can test if any GRP could be underlying. For this approach, first an ensemble of surrogate data sets has to be produced. If the null hypothesis is that all structure of the data is fully described by their marginal distribution and by their auto-correlation function (equivalently, by their power spectrum), one creates otherwise random data with exactly the same power spectrum and marginal distribution [18, 19]. This simple test will often yield a rejection of the null and thus suggest the existence of additional, nonlinear structure, but can also be caused by non-stationary features which are common in clinical time series and which are not compatible with the null hypothesis.

Figure 1 illustrates the variation of the heart rate (middle trace) of a human at rest in connection with the breath rate (upper trace). Here, the instantaneous heart rate was derived from the RR-interval series by a Fourier interpolation technique, see [14] for details.
The lower trace contains a randomised sequence that has the same autocorrelation structure and the same cross correlation to the fixed breath rate recording. These random surrogate data present much but not all of the structure observed in the measurements. The remaining structure, in particular a slight asymmetry under time-reversal, might be due to the measurement process. In any case the structure is not pronounced enough for any kind of deterministic modeling.

It is hard to find nonlinearities in RR-interval time series beyond those which can be easily accounted for by the measurement process or by the coupling to the slower breath rate. We thus support the conclusion of [21] that there is no clear evidence for determinism in RR-interval data. This does not mean that determinism is positively absent, but that potential determinism in the heart rate variability is too complex to be accessible through RR-interval data alone. The conservative working hypothesis, which is also implied by the widespread use of spectral characteristics, is that the process which governs the initiation of new cardiac cycles is effectively stochastic, superimposed by the regulations of the autonomous nervous system which control the average heart rate. This does not in itself exclude the possibility that methods for the characterisation of deterministic chaos (like dimension estimates, entropies, empirical symbolic encoding) can be exploited for diagnostic purposes. In the next section we will show why despite the supposed lack of determinism certain nonlinear analysis tools can be employed, focusing on the single cycle ECG wave.

4 Embedding of single-cycle ECG waves

The fact that the different parts of the single cycle ECG wave mirror well understood physiological processes which are repeated with considerable precision introduces determinism-like properties in the ECG signal during the time between the beginning of the P-wave and the end of the T-wave. A simplified model of the ECG signal could consist of a concatenation of identical PQRST complexes, with a time interval of short but random duration in between. This is motivated by Fig. 2, where 4 consecutive QRS-complexes from a long ECG recording are superimposed. Although in general there is much more variability in the wave-form, Fig. 3 shows impressively the reproducibility of the wave-form with random TP intervals in between. A more detailed study which is in progress shows that the whole variability of a single cycle ECG wave can be parametrised by a few, typically about 5, parameters.

Single-cycle ECG-waves which can be well approximated by lower dimensional surfaces. This fact allows to make use of methods of nonlinear time series analysis which exploit the concept of phase space. In Fig. 3 we show an ECG signal in a two-dimensional time delay embedding space. In this representation, the data seem to fall onto a two-dimensional ribbon, but including more delay coordinates one would find a higher dimensional hyper-tube. We do not say that the data lie on an “attractor”, since the object we see is not invariant in the strict sense: when accumulating more and more data, due to non-stationarity, they will fill more and more of the plane. Nevertheless, the heart rate variability and the unpredictability of the onset of the next heart cycle in this plot is hidden in the cluster around the origin, so that we see essentially the few-parameter set of single cycle segments. This will allow to approximate this set locally by smooth manifolds.

Predictability. The existence of (approximate) constraints like a containing manifold can be used to make short time predictions inside an ECG-cycle. Strict determinism would mean that a point in state space possesses a unique future, and that nearby points have a similar future. Thus, collecting neighbours of the point whose future should be determined, we can average over the future of the neighbours to learn how the present signal will continue. This simplest implementation of phase space predictability exploits much more than just linear temporal correlations. The present state is characterized by a delay vector of sufficient length, or, otherwise said, by a segment of measurements. To obtain the predictions discussed below we select such a 50 ms data segment, look for similar segments in the data base, and take the average over the continuations of all these similar segments as a prediction of the future of the this data segment. Let \( s_n = (s_{n}, \ldots, s_{n-49}) \) be the segment whose future should be determined, and \( (s_{n'}, \ldots, s_{n'-49}) \) a similar segment, i.e. \( |s_{n-i} - s_{n'-i}| < \delta \) for all \( i = 0, \ldots, 49 \), then the prediction \( \hat{s}_{n+k} \) ms ahead is \( s_{n'+k} \) averaged over all similar segments \( n' \). In Fig. 4 we show a few typical results of such predictions (continuous lines) up to \( k_{\text{max}} = 600 \) ms ahead (to be compared
Small fluctuations of this part of the signal are mis-
soon, leading to large errors even a few steps ahead. In panel A, predictions start 160 ms before the R-
mination, amplitude and width of the T-wave. Of course, all predic-
tions do not become worse with increasing prediction
ment, in size). All forecasts/data segments were synchronised by
error (units of Fig. 4)
Figure 4: Phase space predictions (continuous lines), com-
pared to measurements (dotted lines): Using the last 50 me-
asurements represented by the broken lines to the left of the
arrow to characterise the present state, the future was pre-
dicted. In panel A, predictions start 160 ms before the R-
wave, in B 100 ms. In each panel, four different cycles are
shortened.
Figure 5: Average forecast errors for predictions from 1 to
600 ms ahead. Predictions start from segments \( s_n \) consisting
of the 50 data items immediately before the dotted lines
relating the position to the point in the ECG-cycle (reduced
in size). All forecasts/data segments were synchronised by
the zero crossing of the descending part of the R-wave.
interpreted by the algorithm to indicate the beginning
of the next P-wave. On this time scale, ECG data do
not contain much information about the onset of the
next heart beat. (2) As soon as the true beginning of
the P-wave is covered by \( s_n \) (the three other curves in
Fig. 5), predictions are very robust and accurate: Even
the end of the T-wave (250 ms ahead) is predicted quite
well, and the forecast errors are almost zero up to 400 ms
ahead, the beginning of the next P-wave. (3) The pre-
dictions do not become worse with increasing prediction
time (up to the above mentioned 400 ms), as it would be
for chaotic data. Altogether, these findings show that
each cycle can be characterised by a single state vector.
The initial part of the P-wave already determines posi-
tion, amplitude and width of the T-wave. Of course, all
this is in agreement with physiological knowledge.

Noise reduction. Due to the sharp QRS-complexes,
the power spectrum of an ECG signal is broad banded.
Therefore, filters in Fourier space cannot be employed
with great success. They might smooth the signal on
average, but also destroy the structure of the QRS-
complex. However, nonlinear techniques originally de-
designed for the treatment of noise in low-dimensional
chaotic data can be applied, as long as the approxima-
tion by a manifold introduces less errors than there are
measurement errors on the data.

Let us emphasise three observations: (1) When mak-
ing predictions for segments which contain only data be-
fore the beginning of the P-wave (the two curves starting
leftmost in Fig. 5) the predictions fluctuate quite
soon, leading to large errors even a few steps ahead. Small
fluctuations of this part of the signal are mis-
conceived. In Fig. 5 we show an average of the prediction errors,
the modulus of the difference between the predicted and
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Figure 6: Illustration of the local projection scheme. For each point to be corrected, a neighbourhood is formed (grey shaded area) which is then approximated locally by an ellipsoid. An approximately two-dimensional manifold embedded in a three-dimensional space could for example be cleaned by projecting onto the first two principal directions.

Figure 7: Nonlinear noise reduction applied to electrocardiogram data. Upper trace: original recording. Middle: the same contaminated with typical baseline noise. Lower: the same after nonlinear noise reduction. The enlargements on the right show that indeed clinically important features like the small downward deflection of the P-wave preceding the large QRS-complex are recovered by the procedure. Note that the noise and the signal have very similar spectral contents and could thus not be separated by Fourier methods.

Figure 8: Signal separation by locally linear projections in phase space. The original recording (upper trace) contains the fetal ECG hidden under noise and the large maternal signal. Projection onto the manifold formed by the maternal ECG (middle) yields fetus plus noise, another projection yields a fairly clean fetal ECG (lower trace). The data was kindly provided by J. F. Hofmeister [22].

5 Non-stationarity

The last issue we want to discuss in this paper is the inevitable problem of non-stationarity. As mentioned before, the ECG seems to show structures on all time scales, formalised by the notion of $1/f$-noise. Most obviously, the heart rate changes depending on the physical activity of an individual. On the other hand, from a diagnostic point of view, important information is contained in the variability which is summarised under the term non-stationarity. It is therefore important not to regard non-stationarity only as a complication that arises for the statistical description of ECG data but as an important source of information. So far, there is no general methodology of extracting the relevant aspects from the vast amount of data collected in long-term recordings. A representation of the ECG in a (reconstructed) phase space may offer a powerful basis to trace changes in the waveform. Consider an increase in the instantaneous heart rate. The ECG cycle will appear more condensed which is however not a simple rescaling described by one parameter. The waveform will occupy a different part of the time delay embedding space. Searching for neighbours in this space, one will be able to identify similar situations in a long recording and thus be able to subdivide it into different phases [26, 27]. Due to the lack of chaos in the ECG case, the cross-prediction errors used in [27] can be replaced by phase space distances between data segments covering...
ing the part of a cycle from the beginning of the P-wave until the end of the T-wave. In Fig. 3 we show a typical outcome of such a similarity study. The representation is similar to the well-known recurrence plot [28] where only recurrences of full P-to-T segments are considered.

6 Conclusions

One or two decades ago, chaos theory was received with great enthusiasm: It seemed to supply concepts for the analysis and understanding of aperiodic temporal evolution in arbitrary environments. During the last years disillusion prevailed, since it became clear that chaos in a stronger sense is hardly to be expected outside well controlled laboratory experiments. Much effort has been spent, or wasted, on the discussion “is it chaos or is it noise”. In almost all interesting dynamical problems in nature, stochastic fluctuations, intrinsic instability, and a changing environment act together to produce the intriguing patterns we observe. Given that the human heart is also likely to be such a mixed system, we want to promote a different approach towards the issue by asking what is a useful framework for the analysis of cardiac data. Due to the presence of approximately deterministic structures in the cardiac cycle, state space concepts can be applied with the necessary care when the typical time scales of a cycle are involved in the analysis. On longer time scales, the reduction of the available information to a single number, the RR-interval is too severe to still permit a useful study with phase space methods. When this reduction is avoided, comparisons in phase space may well be suitable to study the variability of the cardiac dynamics over longer times.

We have argued that the heart dynamics contains a component which – at least on the basis of the information supplied by ECG recordings – has to be considered as stochastic. On the other hand, we have shown that the single cycle ECG wave can be represented by only a few parameters, or, equivalently, by state vectors in a low-dimensional embedding space. We presented two applications, noise reduction and fetal ECG extraction. They demonstrate that the concepts of nonlinear time series analysis are powerful also for non-deterministic systems, if one is able to isolate a specific problem.

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