Raising Anti de Sitter Vacua to de Sitter Vacua in Heterotic M-Theory

Evgeny I. Buchbinder

School of Natural Sciences, Institute for Advanced Study
Einstein Drive, Princeton, NJ 08540

Abstract

We explore the possibility of obtaining de Sitter vacua in strongly coupled heterotic models by adding various corrections to the supergravity potential energy. We show that, in a generic compactification scenario, Fayet-Iliopoulos terms can generate a de Sitter vacuum. The cosmological constant in this vacuum can be fine tuned to be consistent with observation. We also study moduli potentials in non-supersymmetric compactifications of $E_8 \times E_8$ theory with anti five-branes and $E_8 \times \tilde{E}_8$ theory. We argue that they can be used to create a de Sitter vacuum only if some of the Kahler structure moduli are stabilized at values much less than the Calabi-Yau scale.
1 Introduction

The moduli stabilization problem is one of the central in search for realistic string theory vacua in four dimensions. On one hand, the existence of massless scalar fields is in conflict with experiment. On the other, the four-dimensional gravitational and gauge coupling constants depend on the values of the moduli. Therefore, in any realistic string compactification, all moduli have to be stabilized in a phenomenologically acceptable range. Another four-dimensional quantity, apparently intrinsically related to the moduli stabilization problem, is the cosmological constant. Recently, substantial progress in this direction was achieved in the work of Kachru, Kallosh, Linde and Trivedy [1]. In the context of Type IIB flux compactifications [2, 3, 4, 5, 6, 7, 8, 9, 10], in [1], it was shown that it is possible to stabilize all moduli in a metastable de Sitter (dS) vacuum. The stabilization procedure in [1] was performed in two steps. At step one, all moduli were stabilized in an anti de Sitter (AdS) minimum. This was done by balancing fluxes against non-perturbative effects [12, 13, 14, 15]. At step two, it was demonstrated that the minimum can be lifted to a metastable dS vacuum, by adding anti D3-branes to the system. A crucial ingredient at this step was the result of [16] that a flux-anti D3-brane system can form a metastable bound state with positive energy. The effect of combining this positive contribution to the potential energy with the negative supergravity contribution can produce a dS vacuum. Furthermore, in [1], it was shown that it is possible to fine tune the cosmological constant and make it consistent with observations. Later, moduli stabilization in this Type IIB scenario was explored in more detail in [17, 18, 19].

It is a natural question whether a similar moduli stabilization procedure can be fulfilled in a more realistic framework of strongly coupled heterotic string theory, or, heterotic M-theory [20, 21, 22]. Such compactifications have a lot of phenomenologically attractive features (see [23] for a recent review on phenomenological aspects of M-theory). Various GUT- and Standard Model-like theories were obtained from heterotic compactifications on a Calabi-Yau manifold [24, 25, 26]. The actual particle spectrum in such theories was recently studied in [27, 28]. Progress towards construction of dS vacua from heterotic M-theory was recently reported in [29], based on the earlier work [30]. A dS vacuum was obtained by balancing two non-perturbative effects, gaugino condensate [32, 33, 34] and open membrane

Footnotes:

1 Flux compactifications were also found promising for moduli stabilization in M-theory on singular G2 manifolds in [11].

2 Questions concerning under what circumstances a dS vacuum can arise in heterotic string theory were also studied in [30].
Despite obvious progress, this method has certain shortcomings. First, not all moduli have been stabilized. In particular, complex structure moduli remained unfixed. In order to stabilize them, apparently, it is necessary to introduce flux-induced superpotentials. However, results from suggest that such superpotentials tend to stabilize moduli in AdS minima. Second, authors in gave vacuum expectation values (vevs) to the charged matter fields. This breaks the low-energy gauge group down to nothing unless the vevs are under control. One more problem is that it seems hard to stabilize all run away moduli this way. This, though, might be avoided by using non-Kahler background. It would be great to overcome these problems because in no fine tuning was required to produce a phenomenologically attractive dS vacuum.

In any case, it seems important to create a heterotic analogue of . The first step associated with stabilizing moduli in an AdS minimum was proposed in in strongly coupled heterotic string theory and in in the weakly coupled case. In , it was shown in a very general set-up that all heterotic moduli, including complex and Kahler structure moduli, the volume modulus, vector bundle moduli and five-brane moduli, could be stabilized in an AdS vacuum in a phenomenologically acceptable range. In this paper, we would like to explore the possibility of performing the second step of and lifting this AdS minimum to a dS minimum. To be more precise, we consider two different ways how such a lift can be achieved. First, we study in detail moduli potentials induced by Fayet-Iliopoulos terms. Such terms arise when the low-energy field theory contains an anomalous $U(1)$ factor. The anomaly cancels by the Green-Schwarz mechanism. The idea that such potentials can produce a dS minimum was suggested by Burgess, Kallosh and Quevedo in . We apply this idea for the case of strongly coupled heterotic string theory and argue that, generically, the order of magnitude of such potentials is small enough, so that it is possible to balance them against fluxes and non-perturbative effects. Then we show that it is indeed possible to obtain a metastable dS vacuum along these lines. At this point, we should note that Fayet-Iliopoulos terms modify the potential for charged matter fields as well and might be important for understanding issues related to the supersymmetry breaking scale. In this paper, we will not address these questions. Matter potentials represent an independent difficult problem which requires a serious study. The goal of this paper is to show that it is possible to stabilize all heterotic string moduli in a dS vacuum.

The second method to raise AdS minima that we consider is more universal as it does not put any restrictions on the structure of the low-energy physics. This method is based on
adding anti five-branes in the bulk. A more general set-up would be to consider the $E_8 \times \bar{E}_8$ theory with (anti) five-branes present in the bulk. The $E_8 \times \bar{E}_8$ theory was introduced by Fabinger and Horava in [50]. It is obtained by a chirality flip at one of the orbifold planes. This is equivalent to having supergravity on $S^1/Z_2$ with the gravitino antiperiodic along the circle. This implies that in the effective field theory, the gravitino will have a mass, whereas in the matter sector, supersymmetry will be preserved, at least at the tree level. In [50], it was argued that the orbifold fixed planes experience the attractive Casimir-type force. Upon compactifying this theory to four-dimensions on a Calabi-Yau manifold, this effect becomes subleading. The leading potential is induced by non-trivial charges, depending on the second Chern classes of the gauge and the tangent bundles, on these planes. Similar potentials arise if one adds (anti) five-branes in either $E_8 \times E_8$ or $E_8 \times \bar{E}_8$ theory. However, we obtain that, for a generic compactification, the order of magnitude of this new potential is too big comparing to the order of magnitude of fluxes and non-perturbative effects. Therefore, it destabilizes the vacuum rather than just modifying it. A way to resolve this problem can be to take a Calabi-Yau which has cycles of various sizes. Then it is possible to decrease this new correction to the potential energy. To do this, it is necessary to stabilize $h^{1,1}$ moduli of the Calabi-Yau manifold in such a way that various cycles are fixed at different scales. This problem seems to be related to understanding non-exponential factors in non-perturbative superpotentials [51, 52] and will not be discussed in this paper.

This paper is organized as follows. In Section 2, we consider a system having an AdS minimum. This system involves the volume modulus, the interval modulus, one five-brane modulus and the complex structure moduli. This is a simplified version of the one studied in [40]. In [40], it was shown that the remaining moduli of the vector bundle can be stabilized as well. It was also argued in [40] that any number of $h^{1,1}$ moduli can be stabilized by similar mechanism. The necessity of a five-brane is dictated by the fact that the non-perturbative superpotential for the interval modulus dies off too fast. Therefore, it is problematic to stabilize it. However, if the five-brane is located close enough to one of the orbifold fixed plane one can stabilize the interval modulus. The complex structure moduli are stabilized by the flux-induced superpotential, two out of three remaining moduli are stabilized by balancing run away moduli against the fluxes. For the last modulus, we obtain a purely algebraic equation. A numeric analysis shows that it is possible to find a solution satisfying all requirements and assumptions. In Section 3, we discuss the contribution to the potential energy induced by Fayet-Iliopoulos terms [48, 49]. The form of the potential is slightly different depending on in what sector there is an anomalous $U(1)$ gauge group. This contribution
depends on the volume as well as on the interval and the five-brane moduli. We estimate its order of magnitude and find that, for a generic compactification, it is comparable with that of fluxes and non-perturbative effects. In Section 4, we consider the system from Section 2 and modify the potential energy by a Fayet-Iliopoulos term. By explicit calculation, we show that it is indeed possible to produce a dS minimum. We also show that it is also conceivable to obtain a small cosmological constant by fine tuning. The reason for this is that the supergravity contribution to the cosmological constant can still be kept negative. However, we note that if the number of moduli is large enough it is very hard to keep the supergravity potential energy negative in the vacuum. As a consequence, in this case, it is still possible to find a dS minimum but the cosmological constant will always be very large. Even though we do not include vector bundle moduli into our analysis, we comment that it is straightforward to add them without facing conceptual difficulties. In Section 5, we move on to the $E_8 \times \bar{E}_8$ theory. By simple anomaly arguments along the lines of [21], we find a necessary condition for anomaly free compactifications. Then, we derive the moduli potential in this theory. We show that a potential with the identical functional structure arises from adding (anti) five-branes. This is, of course, not surprising. In a supersymmetric $E_8 \times E_8$ compactification without anti five-branes, the potential is identically zero as the consequence of the net tension cancellation. It is interesting to note that this potential can be both positive and negative. It depends on the Calabi-Yau volume modulus and on the interval modulus. Even though we do not do explicit calculations with this potential, it is natural to expect that it works as good as Fayet-Iliopoulos terms. We estimate its order of magnitude and find that, for a generic compactification, it is not comparable with that of fluxes and non-perturbative effects. We give a brief discussion on what it takes to decrease the order of magnitude of this potential. The key issue seems to be to learn how to stabilize some of the Kahler structure moduli of a Calabi-Yau manifold at scales sufficiently smaller than the Calabi-Yau scale. This method to introduce a correction to the supergravity Lagrangian is, in a certain sense, more attractive than to use Fayet-Iliopoulos terms. First, it is more universal. It does not impose any constraints on the structure of the low-energy gauge group. Second, this new contribution to the potential energy can be both positive and negative. This might be important for the purposes of fine tuning the cosmological constant.

In this paper, we work in the framework of Calabi-Yau compactifications. It is natural to expect that similar results should hold in the context of non-Kahler compactifications [42, 43, 44, 45, 46, 47]. In such compactifications, the volume of the manifold is stabilized perturbatively. Therefore, the analysis can be very similar from the conceptual viewpoint.
but simpler technically since a fewer number of moduli is involved. However, it is hard
to say exactly what the structure of Kahler potentials and superpotentials is, because the
moduli of non-Kahler compactifications are not known. In particular, since there are no
$h^{1,1}$ moduli, the structure of non-perturbative superpotentials is unclear. These subtleties
require detailed investigations.

2 AdS Vacua

As in [1], we begin with the construction of the AdS minimum. This was done in a very
general setting in [40]. Here we consider a simplified system where we ignore vector bundle
moduli. The details of their stabilization can be found in [40]. We work in the context of the
strongly coupled heterotic string theory [20, 21]. To one of the orbifold fixed planes we will
refer as to the visible brane (or the visible sector), to the other one we will refer as to the
hidden brane (or the hidden sector). The system under study involves the following complex
moduli

\[ S, T, Y, Z_\alpha. \]  

(2.1)

The modulus $S$ is related to the volume of the Calabi-Yau manifold

\[ S = V + i\sigma, \]  

(2.2)

where $\sigma$ is the axion. The real part of the $T$-modulus is the size of the eleventh dimension

\[ T = R + ip, \]  

(2.3)

where $p$ comes from the components of the M-theory three-form $C$ along the interval and the
Calabi-Yau manifold. Here we have assumed that $h^{1,1} = 1$. In [40], it was argued that one
should be able to stabilize any number of $h^{1,1}$ moduli by solving similar but technically more
complicated equations. As in [40], we will do all calculations assuming that there is only one
$h^{1,1}$ modulus which we denote by $T$. $Y$ is the modulus of the five-brane. Throughout the
paper, we assume that there is only one five-brane in the bulk wrapping an isolated genus
zero curve. In this case, there is only one five-brane modulus [53], whose real part is the
position of the five-brane in the bulk

\[ Y = y + i(a + \frac{p}{R}), \]  

(2.4)

where $a$ is the axion arising from dualizing the three-form field strength propagating on
the five-brane world-volume. At last, by $Z_\alpha$ we denote the complex structure moduli. The
actual number of them is not relevant for us. The moduli $V, R$ and $y$ are assumed to be dimensionless normalized with respect to the following reference scales

$$v_{CY}^{-1/6} \approx 10^{16} GeV, \quad (\pi \rho)^{-1} \approx 10^{14} - 10^{15} GeV.$$  \hfill (2.5)

In order to obtain the four-dimensional coupling constants in the correct phenomenological range \[22, 54\], the corresponding moduli should be stabilized at (or be slowly rolling near) the values

$$V \sim 1 \quad R \sim 1.$$  \hfill (2.6)

The Kahler potential for this system is as follows \[55, 56, 57\]

$$\frac{K}{M_{Pl}^2} = K_Z + K_{S,T,Y}, \quad (2.7)$$

where

$$K_Z = - \ln(-i \int \Omega \wedge \bar{\Omega}), \quad (2.8)$$

and

$$K_{S,T,Y} = - \ln(S + \bar{S}) - 3 \ln(T + \bar{T}) + 2\tau_5 \frac{(Y + \bar{Y})^2}{(S + \bar{S})(T + \bar{T})}. \quad (2.9)$$

Here $M_{Pl}$ is the four-dimensional Planck scale and $\tau_5$ is given by

$$\tau_5 = \frac{T_5 v_5 (\pi \rho)^2}{M_{Pl}^2}, \quad (2.10)$$

where $v_5$ is the area of the cycle on which the five-brane is wrapped and $T_5$ is

$$T_5 = (2\pi)^{1/3} \left( \frac{1}{2\kappa_{11}^2} \right)^{2/3}, \quad (2.11)$$

with $\kappa_{11}$ being the eleven-dimensional gravitational coupling constant. It is related to the four-dimensional Planck mass as

$$\kappa_{11}^2 = \frac{\pi \rho v_{CY}}{M_{Pl}^2}. \quad (2.12)$$

Evaluating $\tau_5$ by using (2.12) and (2.5) gives

$$\tau_5 \approx \frac{v_5}{v_{CY}^{1/3}}. \quad (2.13)$$

Generically this coefficient is of order one.

The superpotential for this system consists of three different contributions

$$W = W_f - W_g - W_{np}. \quad (2.14)$$
$W_f$ is the flux-induced superpotential \[ W_f = \frac{M^2_{\text{Pl}}}{v_{\text{CY}}} \int_{\text{CY}} H \wedge \Omega, \] 
where $H$ is the Neveu-Schwarz three form. In M-theory notation it can be written as 
\[ W_f = \frac{M^2_{\text{Pl}}}{v_{\text{CY}} \pi \rho} \int dx^{11} \int_{\text{CY}} G \wedge \Omega, \] 
where $G$ is the M-theory four-form flux. The order of magnitude of $W_f$ was estimated in \[40\] and was found to be, generically, of order $10^{-8} M^3_{\text{Pl}}$. In fact, this is flexible. The superpotential $W_f$ may receive certain higher order corrections from Chern-Simons invariants. In \[41\] it was argued that this Chern-Simons invariants can reduce the order of magnitude of $W_f$. We will assume in this paper that the order of magnitude of $W_f$ is approximately $10^{-10} - 10^{-9}$ in Planck units. It is well known that perturbatively a $(3,0)$ Neveu-Schwarz flux (or, equivalently, a $(3,0,1)$ $G$-flux in M-theory) breaks supersymmetry. Therefore, the idea is to balance the superpotential \[\text{(2.15)}\] against the non-perturbative contributions $W_g$ and $W_{np}$. At this point, it is appropriate to mention that we are not allowing $(2,1,1)$ components of the M-theory $G$-flux which leads to non-Kahler compactifications \[42\] \[43\] \[44\] \[45\] \[46\] \[47\]. The flux introduced above provides only a perturbative deformation of the Calabi-Yau metric.

By $W_g$ we denote the superpotential induced by a gaugino condensate in the hidden sector \[32\] \[33\] \[34\] \[58\]. A non-vanishing gaugino condensate has important phenomenological consequences. Among other things, it is responsible for supersymmetry breaking in the hidden sector. When that symmetry breaking is transported to the observable brane, it leads to soft supersymmetry breaking terms for the gravitino, gaugino and matter fields \[59\] \[60\] \[61\] \[62\]. See \[63\] for a good review on gaugino condensation in string theory. This superpotential has the following structure 
\[ W_g = h M^3_{Pl} \exp(-\epsilon S + \epsilon \alpha^{(2)} T - \epsilon \beta \frac{Y^2}{T}). \] 
The order of magnitude of $h$ is approximately $10^{-6}$ \[34\]. The coefficient $\epsilon$ is related to the coefficient $b_0$ of the one-loop beta-function and is given by \[ \epsilon = \frac{6\pi}{b_0 \alpha_{\text{GUT}}} . \] 
For example, for the $E_8$ gauge group $\epsilon \approx 5$. The coefficient $\alpha^{(2)}$ represents the tension (up to the minus sign) of the hidden brane 
\[ \alpha^{(2)} \sim \frac{\pi \rho}{16 \pi v_{\text{CY}}} \left( \frac{K_{11}}{4\pi} \right)^{2/3} \int_{\text{CY}} \omega \wedge (tr F^{(2)} \wedge F^{(2)} - \frac{1}{2} tr R \wedge R), \] 
(2.19)
where $\omega$ is the Kahler form and $F^{(2)}$ is the curvature of the gauge bundle on the hidden brane. Similarly, the coefficient $\beta$ is the tension of the five-brane. It is given by

$$\beta = \frac{2\pi^2 \rho (k_{11})^{2/3}}{v_{CY}^{2/3} 4\pi} \int_{CY} \omega_1 \wedge W,$$

(2.20)

where $W$ is the four-form Poincare dual to the holomorphic curve on which the five-brane is wrapped. Generically both $\alpha^{(2)}$ and $\beta$ are of order one. In fact, from eqs. (2.10), (2.11) and (2.20) it follows that

$$\beta \approx \tau_5.$$  

(2.21)

The quantity

$$\text{Re}(S - \alpha^{(2)}T + \beta \frac{Y^2}{T})$$

(2.22)

represents the inverse square of the gauge coupling constant in the hidden sector, $\frac{1}{g_{\text{hidden}}^2}$. Therefore, it cannot become negative. This, in particular, says that the superpotential cannot be trusted for large values of the interval length $R$. We believe that higher order corrections to the combination $\frac{1}{g_{\text{hidden}}^2}$ will make the gauge coupling constant well defined for large values of $R$. Partial support for this comes from the work of Curio and Krause [65, 66] who showed that the next order order $T$-correction to $\frac{1}{g_{\text{hidden}}^2}$ is indeed positive.

The difficulty with understanding $W_g$ for sufficiently large values of $R$ leads to necessity of introducing five-branes and non-perturbative superpotentials. If we ignore the $Y$-modulus and restrict ourselves to the $S, T, Z_a$-system with the superpotential

$$W_f - W_g,$$

(2.23)

it is straightforward to show that the potential energy is strictly positive definite unless

$$\frac{1}{g_{\text{hidden}}^2} < 0.$$ 

(2.24)

This, apparently, implies that we cannot trust $W_g$ in the form (2.17) and have to include higher order corrections. Instead of doing that, we will add the five-brane and show that it is possible to find an interesting solution without running into troubles with the imaginary gauge coupling constant. One more problem with the superpotential (2.23) is that it can stabilize only one linear combination of the imaginary parts of $S$ and $T$ moduli leaving the remaining one flat. In order to be able to balance $W_f$ and $W_g$ with each other, their orders of magnitude should approximately be the same. This is clearly possible, especially with a help of Chern-Simons invariants, if $V$ and $R$ and of order one and $V - \alpha^{(2)} R$ is positive.
The last contribution to the superpotential that we have to discuss is the non-perturbative superpotential \( W_{np} \). In principle, it has three parts

\[
W_{np} = W_{vh} + W_{v5} + W_{5h}.
\]

(2.25)

\( W_{vh} \) is induced by a membrane stretched between the visible and the hidden branes. It behaves as

\[
W_{vh} \sim e^{-\tau T}.
\]

(2.26)

\( W_{v5} \) is induced by a membrane stretched between the visible brane and the five-brane. It behaves as

\[
W_{v5} \sim e^{-\tau Y}.
\]

(2.27)

At last, \( W_{5h} \) is induced by a membrane stretched between the five-brane and the hidden brane. It behaves as

\[
W_{5h} \sim e^{-\tau(T - Y)}.
\]

(2.28)

The coefficient \( \tau \) is given by \[35, 36\]

\[
\tau = \frac{1}{2} (\pi \rho) v_z (\frac{\pi}{2 \kappa_{11}})^{1/3},
\]

(2.29)

where \( v_z \) is the area of the holomorphic curve. Taking \( v_z \approx v_{CY}^{1/3} \) and using (2.5) and (2.12), we obtain

\[
\tau \approx 250.
\]

(2.30)

Since we are interested in the regime \( Re(T) \sim 1 \) it is very difficult to make \( W_{np} \) of order \( W_f \). The only situation when these two contributions to the superpotential can compete is when the five-brane is close to one of the orbifold fixed planes. If the five-brane is close to the visible brane then both \( W_{vh} \) and \( W_{5h} \) will die off very fast and the non-perturbative superpotential will not depend on \( T \). As a consequence, the \( T \)-modulus cannot be stabilized. Therefore, the only way to proceed is to assume that the five-brane is closed to the hidden brane. In this case

\[
W_{np} = W_{5h} = M_{Pl}^3 a e^{-\tau(T - Y)}.
\]

(2.31)

For concreteness we assume that the coefficient \( a \sim 1 \). We will demand that

\[
\tau e^{-\tau(T - Y)} \sim W_f \sim 10^{-10}.
\]

(2.32)

This says that

\[
Re(T - Y) \approx 0.1.
\]

(2.33)
Thus, we will assume that the five-brane is close to the hidden brane and take $W_{np}$ to be given by \((2.31)\). Of course, we have to show that it is possible to stabilize the five-brane at such a distance.

Now we show following \[40\] that the system under consideration indeed has an AdS minimum satisfying

$$D_{all\ fields}W = 0,$$  \hspace{2cm} (2.34)

where $D$ is the Kahler covariant derivative, and all the assumptions stated above. For simplicity, we will look only at the real parts of the moduli. All imaginary parts can be stabilized as well. See \[40\] for details. Furthermore, we will not distinguish between the superpotentials and their absolute values. First, we look at the equation

$$D_{Z_\alpha}W = 0.$$  \hspace{2cm} (2.35)

Assuming that

$$W_f >> W_g, W_{np}$$  \hspace{2cm} (2.36)

in the interesting regime, eq. \((2.35)\) can be written as

$$\partial_{Z_\alpha}W_f + \frac{\partial K_{Z_\alpha}}{\partial Z_\alpha}W_f = 0.$$  \hspace{2cm} (2.37)

In \[40\] it was shown that eq. \((2.36)\) is indeed satisfied. In eq. \((2.37)\), all quantities depend on the complex structure moduli only. We will assume that this equation fixes all the complex structure moduli. Partial evidence that equations of the type \((2.37)\) fix all the complex structure moduli comes, for example, from \[4\]. The next equation to consider is

$$D_SW = 0.$$  \hspace{2cm} (2.38)

By using eqs. \((2.9)\), \((2.17)\) and \((2.36)\) we can rewrite this as

$$2\epsilon Ve^{-\epsilon V} = (1 + \frac{2y^2}{VR})W_f.$$  \hspace{2cm} (2.39)

Eq. \((2.39)\) provides stabilization of the volume $V$. It is conceivable to find a solution

$$V \sim 1.$$  \hspace{2cm} (2.40)

By using eqs. \((2.9)\), \((2.17)\), \((2.31)\), \((2.25)\) and \((2.39)\), eq.

$$D_TW = 0$$  \hspace{2cm} (2.41)
can be rewritten as

$$\tau W_n = \left( \frac{3}{2R} + \tau_5 \frac{y^2}{VR^2} + \frac{1}{2V} (\alpha^{(2)} + \frac{\beta y^2}{R^2})(1 + \frac{2\tau_5 y^2}{VR}) \right) W_f.$$  (2.42)

The left hand side of this equation is

$$\tau e^{-\tau(R-y)}.$$  (2.43)

The right hand side is of order $W_f$. As discussed before, this implies that

$$R - y = 0.1.$$  (2.44)

If it is possible to make the flux-induced superpotential $W_f$ smaller, then the difference $R - y$ can be increased. Eq. (2.44) stabilizes $R$ provided we can stabilize the five-brane. By using eqs. (2.9), (2.17), (2.31), (2.39) and (2.42), eq.

$$D_Y W = 0$$  (2.45)

can be reduced the following purely algebraic equation

$$(1 + \frac{2\tau_5 y^2}{VR}) \left( \frac{\beta y}{R} - \frac{\alpha^{(2)}}{2} - \frac{\beta y^2}{2R^2} \right) - \frac{3}{2} - \frac{\tau_5 y^2}{R^2} + \frac{2\tau_5 y}{R} = 0.$$  (2.46)

A numeric analysis shows that it possible to find a solution for $y$ satisfying (2.44) when $V$ and $R$ are both or order one. For example, if we take

$$\alpha^{(2)} = 1, \quad \beta = 1.5, \quad \tau_5 = 1.5, \quad V = 1.2, \quad R = 0.8,$$  (2.47)

then eq. (2.46) has a unique positive solution for $y$ given by

$$y \approx 0.7.$$  (2.48)

Thus, we have shown that the system (2.34) can have a solution satisfying all our assumptions and requirements. It is easy to see that the combination (2.22) is positive and of order one.

The value of the potential energy in the vacuum is

$$U_{\text{min}} \sim -W_f^2.$$  (2.49)

Our goal will be to raise this vacuum to a metastable dS vacuum. First, we consider to the correction to the potential energy due to Fayet-Iliopoulos terms. Then we will take a look at the $E_8 \times \bar{E}_8$ theory.
Before we conclude this section, let us make some comments on the imaginary parts and of the $S, T$ and $Y$ moduli. Details can be found in [40]. Imaginary parts are stabilized in such a way that the superpotentials $W_g$ and $W_{np}$ are out of phase with respect to $W_f$. We have already indicated this fact in eq. (2.14) by choosing the appropriate minus signs. The imaginary part of the $T$-modulus behaves as

$$Im(T) \sim \frac{1}{\tau} \approx 0$$

since $\tau$ is very large. One can also show that

$$Im(Y) \approx 0.$$  \hspace{1cm} (2.51)

Various slices of the potential energy are schematically shown on Figures [13].

### 3 Moduli Potentials From Fayet-Iliopoulos Terms

In both weakly and strongly coupled heterotic string models there can be anomalous $U(1)$ gauge groups. They can arise in both the visible and the hidden sectors. The anomaly is cancelled by the four-dimensional version of the Green-Schwarz mechanism. To cancel the anomaly the axion $\sigma$ must undergo the gauge transformation of the form

$$\sigma \to \sigma + c\lambda,$$  \hspace{1cm} (3.1)

where $\lambda$ is a parameter and $c$ is a constant whose order of magnitude will be estimated later. The transformation law (3.1) implies [48] that the Kahler potential for the $S$-modulus has to be modified as follows

$$K_S = -M^2_{Pl} \ln(S + \bar{S} + c\mathcal{V}),$$  \hspace{1cm} (3.2)

where $\mathcal{V}$ is the anomalous $U(1)$ vector superfield. From here we find that the kinetic term of the action

$$\int d^4x d^4\theta K_S$$

contains among others the Fayet-Iliopoulos term

$$\int d^4x M^2_{Pl} \frac{c}{S + \bar{S}} D,$$  \hspace{1cm} (3.4)

where $D$ is the auxiliary field of the vector multiplet. This gives rise to the moduli potential energy of the form

$$U_D \sim M^4_{Pl} g^2 \frac{c^2}{V^2}.$$  \hspace{1cm} (3.5)
Figure 1: A schematic slice of the potential near the AdS minimum (multiplied by $10^{12}$) in the $V$ direction.

Figure 2: A schematic slice of the potential near the AdS minimum (multiplied by $10^{12}$) in the $R$ direction.

Figure 3: A schematic slice of the potential near the AdS minimum (multiplied by $10^{12}$) in the $y$ direction.
where $g$ is the gauge coupling which is itself moduli dependent. Note that $U_D$ is strictly positive. Since the gauge coupling constants are different in the visible and in the hidden sectors, the precise form of the potential energy $U_D$ depends on in which sector there appeared an anomalous $U(1)$ gauge group. The gauge coupling constants in the visible and the hidden sectors are given by

\[ g_{\text{visible}}^2 = \frac{g_0^2}{\text{Re}(S + \alpha^{(1)} T + \beta (T - \frac{Y^2}{T}))} \]  
\[ g_{\text{hidden}}^2 = \frac{g_0^2}{\text{Re}(S - \alpha^{(2)} T + \beta \frac{Y^2}{T})} \]

respectively. Here $\alpha^{(1)}$ is the tension of the visible brane

\[ \alpha^{(1)} = \frac{\pi \rho}{16\pi v_{CY}} (\frac{\kappa_{11}}{4\pi})^{2/3} \int_{CY} \omega \wedge (tr F^{(1)} \wedge F^{(1)} - \frac{1}{2} tr \mathcal{R} \wedge \mathcal{R}), \]  

where $F^{(1)}$ is the curvature of the gauge bundle on the visible brane, and $g_0^2$ is a moduli independent parameter of order $\alpha_{GUT}$.

Now let us estimate the order of magnitude of the parameter $c$. The axion transformation law is inherited from the variation of the $B$-field in ten dimensions (or the $C$-field in eleven dimensions). In the case of strongly coupled heterotic string it is of the form

\[ \delta B \sim \frac{1}{4\sqrt{2} \pi^2 \rho} (\frac{\kappa_{11}}{4\pi})^{2/3} (\Omega_{YM} - \frac{1}{2} \Omega_L), \]  

where $\Omega_{YM}$ and $\Omega_L$ are the usual Chern-Simons forms. By using eq. 2.12, we get

\[ c \sim \frac{1}{(4\pi)^{5/3}} (\frac{1}{M_{Pl}^2 \pi \rho})^{4/3}. \]  

Now we can write the expressions for $U_D$. If the anomalous $U(1)$ is in the visible sector, we get

\[ U_D = M_{Pl}^4 \frac{b}{V^2 \text{Re}(S + \alpha^{(1)} T + \beta (T - \frac{Y^2}{T}))}. \]  

whereas if the anomalous $U(1)$ appeared in the hidden sector we get

\[ U_D = M_{Pl}^4 \frac{b}{V^2 \text{Re}(S - \alpha^{(2)} T + \beta \frac{Y^2}{T})}. \]  

The coefficient $b$ is given by

\[ b \sim \frac{g_0^2}{(4\pi)^{10/3}} (\frac{1}{M_{Pl}^2 \pi \rho})^{8/3} \sim 10^{-18}. \]
We see that generically $b \sim \frac{W^2}{M_{Pl}^2}$. This means that $U_D$ is of the same order of magnitude as the supergravity potential energy. This, in turn, means that it is potentially possible to obtain a vacuum with very small cosmological constant by fine tuning.

To conclude this section, let us make some remarks on the charged matter fields. In principle, $U_D$ contains terms involving the charged matter fields. Giving them generic vacuum expectation values breaks the low-energy gauge group down to nothing. Therefore we will set them to zero and concentrate on the moduli potential. We will not discuss potentials for the charged matter fields in this paper.

4 dS Vacua

In this section, we will show that the AdS vacuum constructed in Section 2 can indeed be raised to a dS vacuum. We will see that two of the equations of motion can be solved by balancing the run away potentials against fluxes. The remaining equation will be purely algebraic and will be solved numerically. Despite the fact that the potential energy is a rather complicated function of three variables, almost all calculations can be performed analytically.

The potential energy of the system under study is given by

$$\frac{U}{M_{Pl}^4} = U_0 + U_D,$$

where $U_0$ is the supergravity contribution

$$U_0 = e^K (G^{-1}|DW|^2 - 3W\bar{W})$$

and $U_D$ is the contribution coming from Fayet-Iliopoulos terms. For concreteness, we assume that the anomalous $U(1)$ is in the visible sector. In this case we have

$$U_D = \frac{b}{V^2Re(S + \alpha^{(1)}T + \beta(T - \frac{Y^2}{T}))}.$$

To simplify our notation, we have redefined

$$\frac{K}{M_{Pl}^2} \rightarrow K, \quad \frac{W}{M_{Pl}^2} \rightarrow W.$$

We will argue that this potential energy can admit a minimum with a positive cosmological constant. We will show that the order of magnitude of the Kahler covariant derivatives in the minimum is sufficiently less than one. Therefore, the supergravity contribution to the
vacuum energy is still negative. This leads a possibility of obtaining a dS vacuum with a small cosmological constant.

As in Section 2, for simplicity, we will ignore all the imaginary parts of the fields $S, T$ and $Y$. To stabilize them one can invoke similar arguments as in the case of the AdS stabilization [40]. In particular, it is possible to show that we still have

$$\text{Im}(T) \sim \frac{1}{\tau} \approx 0, \quad \text{Im}(Y) \approx 0.$$  \tag{4.5}$$

Without loss of generality we can assume that both $W_g$ and $W_{np}$ are out of phase with respect to $W_f$, so that the superpotential for this system is still given by (2.14). Furthermore, since $U_D$ does not depend on the complex structure moduli $Z_\alpha$, it is natural to treat $W_f$ as a constant with all the moduli $Z_\alpha$ frozen at values solving eqs. (2.37). Thus, we can think of $U$ as of a function of three variables $V, R$ and $y$. We cannot assume that any of these moduli are frozen near the old AdS minimum since $U_D$ depends on all three of them. To simplify our calculation, we will also assume that the five-brane Kahler potential (the last term in eq. (2.9)) is sufficiently less than the $S$- and $T$-Kahler potentials (the first two terms in eq. (2.9)), so that we can ignore the off-diagonal components of the inverse Kahler metric. The necessary conditions for this is

$$2\tau_5 \frac{y^2}{VR} << 1.$$  \tag{4.6}$$

To satisfy it, we will search for a solution with $V \approx 2 - 3$ and $\tau_5 \approx 0.5$. To be able to do this, we will assume that $W_f$ is of order $10^{-10}$ or less (in $M_{Pl}^3$ units). We found that the easiest way to analyze the equations of motion is to observe how the Kahler covariant derivatives get disturbed by the presence of the Fayet-Iliopoulos contribution. Therefore, we define

$$A \equiv D_S W = \epsilon W_g - \frac{1}{2V} W_f,$$

$$B \equiv D_T W = \epsilon W_{np} - \epsilon (\alpha^{(2)} + \frac{\beta y^2}{R^2}) W_g - \frac{3}{2R} W_f,$$

$$C \equiv D_Y W = -\tau W_{np} + \frac{2\epsilon \beta y}{R} W_g + \frac{2\tau_5 y}{VR} W_f,$$  \tag{4.7}$$

where eqs. (2.9), (2.14), (2.17), (2.31) and (2.36) have been used. As in Section 2 (see eqs. (2.39) and (2.42)), we will be working in the regime

$$2\epsilon W_g \sim \tau W_{np} \sim W_f.$$  \tag{4.8}$$

In order to be able to fine the cosmological constant we have to have

$$W_f \sim \sqrt{b}.$$  \tag{4.9}$$
It was argued in the previous section that this is generically the case.

Before we write the equations of motion, let us take a brief look at the structure of the derivatives of the supergravity contribution to the potential energy $U_0$. When we differentiate $U_0$ we will obtain two kinds of terms. Some terms will involve derivatives of $A$, $B$ and $C$. We will call such terms “leading”. The other terms we will call “subleading”. It is easy to see that the “leading” terms contain extra factors of $2\epsilon$ and $\tau$ comparing to the “subleading” ones. This means that the “leading” terms have a higher order of magnitude. Clearly, in the equations of motion, it is enough to keep only the “leading” terms (unless they cancel out). First, consider the equation

$$\frac{\partial U}{\partial T} = 0.$$ 

(4.10)

The “leading” terms look as follows

$$\frac{\partial U_0}{\partial T} = e^K[(2\epsilon V)^2(\alpha^{(2)} + \frac{\beta y^2}{R^2})W_g A - \frac{4}{3}R^2(\tau^2 W_{np} + \epsilon^2(\alpha^{(2)} + \frac{\beta y^2}{R^2})W_g)B + \frac{VR}{\tau_5}(\tau^2 W_{np} + \frac{2\epsilon^2 \beta y}{R^2}(\alpha^{(2)} + \frac{\beta y^2}{R^2})W_g)C].$$

(4.11)

Recall that by $A$, $B$ and $C$ we have denoted the Kahler covariant derivatives. In eq. (4.8), the two terms involving $\tau^2 W_{np}$ are greater than the other terms by a factor of $2\epsilon \sim 20$. This means that

$$\frac{\partial U_0}{\partial T} \approx e^K(\frac{VR}{\tau_5}C - \frac{4}{3}R^2 B)\tau^2 W_{np}.$$ 

(4.12)

Furthermore,

$$\frac{\partial U_D}{\partial T} = \frac{-b(\alpha^{(1)} + \beta + \frac{by^2}{R^2})}{2V^2(V + \alpha^{(1)} R + \beta R - \frac{\beta y^2}{R})} \equiv -b_T.$$ 

(4.13)

From eq. (4.8) and (4.11) it follows that in the interesting range of the fields

$$\frac{\partial U_0}{\partial T} \sim \frac{\partial U_D}{\partial T} >> \frac{\partial U_D}{\partial T}.$$ 

(4.14)

Therefore, in the minimum the Kahler covariant derivatives $B$ and $C$ are related to each other as follows

$$\frac{V}{\tau_5} C \approx \frac{4R}{3} B.$$ 

(4.15)

Now let us consider the equation

$$\frac{\partial U}{\partial S} = 0.$$ 

(4.16)

By using eqs. (4.7), (4.11) and (4.15) and keeping the “leading” terms in $\frac{\partial U_0}{\partial S}$, we get

$$e^K[-(2\epsilon V)^2 W_g A + \frac{\epsilon^2 VR}{\tau_5}(\alpha^{(2)} + \frac{\beta y^2}{R^2}) - \frac{2\epsilon^2 V y}{\tau_5}]W_g C - b_S = 0.$$ 

(4.17)
where
\[
b_S = -\frac{\partial U_D}{\partial S} = \frac{b(\alpha^{(1)} + \beta(1 + \frac{y^2}{R^2}))}{2V^2(V + \alpha^{(1)}R + \beta(R - \frac{y^2}{R}))}. \tag{4.18}
\]

By using eqs. (4.17) and (4.15), eq. (4.17) can be written as follows
\[
-\mu W_g^2 + 2\nu W_f W_g - b_S e^{-K} = 0, \tag{4.19}
\]
where
\[
\mu = V\epsilon^3 \left[4V + \frac{((\alpha^{(2)} + \frac{\beta y^2}{R^2}) - 2\beta y)((\alpha^{(2)} + \frac{\beta y^2}{R^2}) - 2\beta y + 2\epsilon V(1 - \frac{7}{5} \tau_5))}{R\tau_5 + \frac{3V}{4}}\right] \tag{4.20}
\]
and
\[
\nu = V\epsilon^2 \left[1 + \frac{(\alpha^{(2)} + \frac{\beta y^2}{R^2}) - 2\beta y)(\frac{8\beta y}{V} - 2)}{\frac{4\epsilon V}{3\tau_5} + V}\right]. \tag{4.21}
\]

By using eq. (2.21), it is easy to see that $\mu > 0$. Without loss of generality we can assume that $\nu$ is greater than zero. The sign of $\nu$ depends on the relative phase of $W_g$ and $W_f$ which in turn depends on the imaginary parts of the moduli. Without loss of generality we can assume that $\nu > 0$ and the imaginary parts of the moduli are stabilized in such a way that $W_g$ and $W_f$ are out of phase. One can show that without the simplifying assumption (4.6), the structure of eq. (4.19) would be exactly the same but the coefficients $\mu$ and $\nu$ would be much more complicated. If $b_S e^{-K}$ is not very large, this equation has two solutions for $W_g$ and, hence, for $V$. It is easy to realize that the smaller solution is the minimum of $U$ and the bigger one is the maximum of $U$. In the minimum $W_g$ is given by
\[
W_{g_{\text{min}}} = \frac{1}{\mu} \left(\nu W_f - \sqrt{(\nu W_f)^2 - \mu b_S e^{-K}}\right). \tag{4.22}
\]

From eqs. (4.9), (4.20) and (4.21) it follows that
\[
(\nu W_f) \sim \epsilon^4 W_f^2, \tag{4.23}
\]
whereas
\[
\mu b_S \sim \epsilon^3 W_f^2. \tag{4.24}
\]

Therefore, for interesting relative values of $W_f$ and $b$, the discriminant of the quadratic (with respect to $W_g$) equation (4.19) is positive that guarantees the existence of the minimum. It is clear that if $b$ is relatively large comparing to $W_f$, the minimum disappears. Eq. (4.22) is the analogue of eq. (2.39) in Section 2. It stabilizes $V$ at the value of order one (provided $R$
and $y$ are stabilized). Now we go back to eq. (4.15). By using eqs. (4.7), it can be written as

\[
\left(\frac{4R^2}{3} + \frac{VR}{\tau_5}\right)e^{-\tau(R-y)} = e\left(\frac{2\beta y V}{\tau_5} + \frac{4R^2}{3}(\alpha^{(2)} + \frac{\beta y^2}{R^2})\right)W_g + 2(R + y)W_f,
\]

(4.25)

where $W_g$ is given by eq. (4.22). This equation is the analogue of eq. (2.42) in Section 2. As it was discussed before, we obtain

\[
R - y \approx 0.1.
\]

(4.26)

Now we consider the last equation

\[
\frac{\partial U}{\partial Y} = 0.
\]

(4.27)

If, as before, we take only the “leading” terms in $\frac{\partial U}{\partial Y}$, they turn out to be a linear combination of the “leading” terms of the equations $\frac{\partial U}{\partial S}$ and $\frac{\partial U}{\partial T}$. Therefore, we have to include all the “subleading” terms. After tedious calculations one can derive the following equation

\[
(-2AW_f - \frac{y}{V}W_f - 2V A^2 - \frac{2R^2}{3V}B^2)(\alpha^{(2)} + \frac{\beta y^2}{R^2} - \frac{2\beta y}{R}) -
2BW_f - \frac{y}{R}W_f C - \frac{2R}{3}B^2 - \frac{V}{\beta}C^2 - 2CW_f + \frac{8\tau_5 y}{RV}A^2 +
\frac{8\tau_5 y}{3RV}B^2 + 2yC^2 = [b_S(\alpha^{(2)} + \frac{\beta y^2}{R^2} - \frac{2\beta y}{R}) + b_T - b_Y]e^{-K}.
\]

(4.28)

In this equation, $A, B$ and $C$ are Kahler covariant derivatives (4.7), $W_g$ is given by eq. (4.22), (4.21) and (4.20), $C$ and $B$ are related by eq. (4.15), $b_S$ and $b_T$ are given by eqs. (4.18) and (4.13) respectively and $b_Y$ is given by

\[
b_Y = \frac{b\beta y}{V^2 R(V + \alpha^{(1)}R + \beta(R - \frac{y^2}{R}))^2}.
\]

(4.29)

Eq. (4.28) is the analogue of eq. (2.46) in Section 2. A numeric analysis shows that it is possible to find a solution to eq. (4.28) satisfying all the right conditions. In order to justify our neglecting the off-diagonal components of the inverse Kahler metric, we have to take $V \approx 2 - 3$. We also would like to prove that it is conceivable to fine tune the cosmological constant to zero. For this to be true, it is necessary to remain the supergravity contribution to the cosmological constant $U_{0min}$ negative. If we take

\[
\beta = 0.5, \quad \tau_5 = 0.5, \quad \alpha^{(1)} = 0.5, \quad \alpha^{(2)} = 1, \quad R \approx 0.8, \quad V \approx 2.3, \quad \frac{b e^{-K}}{W_f} \approx 4.2,
\]

(4.30)

we find that there is a unique positive solution for $y$

\[
y \approx 0.7.
\]

(4.31)
The Kahler covariant derivatives (multiplied by the corresponding components of the inverse Kahler metric) are given by

\[
G^{-1}_{SS} A^2 \approx 0.7 W_f^2, \quad G^{-1}_{TT} B^2 \approx 0.4 W_f^2, \quad G^{-1}_{YY} C^2 \approx 0.2 W_f^2.
\] (4.32)

This means that the supergravity contribution to the cosmological constant is negative. Various slices of the potential energy near the minimum are schematically shown on Figures 4-6. Clearly, since the supergravity contribution to the cosmological constant is negative, by fine tuning the ratio \( b W_f \), it is possible to obtain the cosmological constant of order of the experimentally observed value

\[
\Lambda \sim 10^{-120} M_{Pl}^4.
\] (4.33)

However, note that if the number of moduli is sufficiently big, it is very likely that the supergravity contribution to the potential energy will become positive. This means that even though it is still possible to stabilize the moduli in a dS minimum, the value of the cosmological constant cannot be made small even by fine tuning.

This concludes our analysis. We have shown that by using Fayet-Iliopoulos terms, it is conceivable to obtain a vacuum with a positive cosmological constant which can be fine tuned to the experimentally observed value. In this section, we have used the expression for the Fayet-Iliopoulos contribution to the potential energy given by eq. (3.11). That is, we assumed that the anomalous \( U(1) \) was in the visible sector. Clearly, with the same success we could assume that the anomalous \( U(1) \) was in the hidden sector and use the Fayet-Iliopoulos contribution to the potential energy given by eq. (3.12). Let us make a brief remark on the vector bundle moduli. It seems straightforward to add them to our analysis. Since the Fayet-Iliopoulos contributions do not depend on them, one can argue that they will be frozen roughly at the same values as in the old AdS vacuum. No conceptually new difficulties are expected.

5 Moduli Potentials in the \( E_8 \times \bar{E}_8 \) Theory

In this section, we consider what kind of moduli potentials in the low-energy field theory we get if we break supersymmetry in the bulk. This can be achieved either by adding antibranes or by a chirality change at one of the orbifold fixed planes. The theory obtained in the latter case was called the \( E_8 \times \bar{E}_8 \) theory in [50]. In both cases, the functional form of the moduli potential is the same. We will concentrate mostly on the \( E_8 \times \bar{E}_8 \) theory. We will assume that the fermions on the visible brane have positive chirality, whereas the fermions on the
Figure 4: A schematic slice of the potential near the dS minimum (multiplied by $10^{12}$) in the $V$ direction.

Figure 5: A schematic slice of the potential near the dS minimum (multiplied by $10^{12}$) in the $R$ direction.

Figure 6: A schematic slice of the potential near the dS minimum (multiplied by $10^{12}$) in the $y$ direction.
hidden brane have negative chirality. We will refer to the sector on the visible brane as to the $E_8$ sector and to the sector on the hidden brane as to the $\overline{E}_8$ sector. The $E_8 \times \overline{E}_8$ theory is, clearly, non-supersymmetric. In the effective field theory, supersymmetry is broken explicitly by the gravitino mass. Indeed, the $E_8 \times \overline{E}_8$ theory can be viewed as the eleven-dimensional supergravity on $S^1/Z_2$ with the gravitino antiperiodic along the circle $[50]$. This is equivalent to saying that, in the effective field theory, the gravitino has a mass. On the other hand, in the gauge theory sector, supersymmetry is not broken, at least at the tree level. This implies that when we compactify the theory on a Calabi-Yau threefold to obtain the effective theory in four-dimensions, supersymmetry will be broken in the gravity sector and preserved in the gauge and the matter sectors. Despite the fact that in the effective field theory the gravitino is massive, the theory still might suffer from local anomalies. They are cancelled by arguments similar to ones discussed in [21]. Let us briefly go through them. As before, we will denote the gauge field strength on the visible brane by $F^{(1)}$ and on the hidden brane by $F^{(2)}$. In the $E_8$ sector, the anomaly is described [21] by the twelve-form

$$I_{12} = I_4 \wedge I_8,$$  

(5.1)

where

$$I_4 = \frac{1}{2} tr \mathcal{R} \wedge \mathcal{R} - tr F^{(1)} \wedge F^{(1)}$$  

(5.2)

and

$$I_8 = -\frac{1}{4} tr I_4^2 + I_8',$$  

(5.3)

where

$$I_8' = -\frac{1}{4} I_4^2 + \left[-\frac{1}{8} tr \mathcal{R}^4 + \frac{1}{32} (tr \mathcal{R} \wedge \mathcal{R})^2 \right].$$  

(5.4)

Locally $I_{12}$ can be written as

$$I_{12} = d(I_3) \wedge I_8 = d(I_3 \wedge I_8),$$  

(5.5)

with $I_3$ being the difference of the Chern-Simons forms

$$I_3 = \frac{1}{2} \Omega_3(\omega_L) - \Omega_3(A^{(1)}),$$  

(5.6)

where $\omega_L$ is the spin connection. Under gauge and locally Lorentz transformations the polynomial $I_3 \wedge I_8$ transforms as follows

$$\delta(I_3 \wedge I_8) = d(I_2 \wedge I_8),$$  

(5.7)
where
\[ I_2 = \frac{1}{2} tr(\theta R) - tr(\epsilon F^{(1)}). \] (5.8)

This allows us to conclude that the variation of the effective action on the visible brane is given by
\[ (\delta \Gamma)|_{x^{11}=0} \sim \int (I_2 \wedge I_8) = \int d^{10}x \left( \frac{1}{2} tr(\theta R) - tr(\epsilon F^{(1)}) \right) \wedge I_8. \] (5.9)

To cancel this anomaly, it necessary to modify the Bianchi identity for the \( \mathcal{G} \)-flux [21]
\[ (dG)|_{x^{11}=0} \sim \frac{1}{2} tr \mathcal{R} \wedge \mathcal{R} - tr F^{(1)} \wedge F^{(1)}. \] (5.10)

This implies that the three-form potential \( C \) is not gauge invariant and transforms as follows
\[ (\delta C)|_{x^{11}=0} \sim -\frac{1}{2} tr(\theta R) + tr(\epsilon F^{(1)}). \] (5.11)

The anomaly (5.9) is cancelled by the Chern-Simons coupling of the eleven-dimensional supergravity
\[ \Gamma_{CS} \sim \int C \wedge \mathcal{G} \wedge \mathcal{G}, \] (5.12)

as well as proposed Green-Schwarz interaction
\[ \Gamma_{GS} \sim C \wedge I_8''. \] (5.13)

Here \( I_8'' \) is given by
\[ I_8'' = -\frac{1}{4} \left( \frac{1}{2} tr \mathcal{R} \wedge \mathcal{R} - tr F^{(1)} \wedge F^{(1)} - tr F^{(2)} \wedge F^{(2)} \right)^2 + \left[ -\frac{1}{8} tr \mathcal{R}^4 + \frac{1}{32} (tr \mathcal{R} \wedge \mathcal{R})^2 \right]. \] (5.14)

On the visible brane we have
\[ I_8'' = I_8'. \] (5.15)

The explicit proportionality coefficients in eqs. (5.9)-(5.13) are not important here and can be found in [21]. It is straightforward to see that
\[ (\delta \Gamma)|_{x^{11}=0} + \delta(\Gamma_{CS} + \Gamma_{GS})|_{x^{11}=0} = 0. \] (5.16)

In the \( \tilde{E}_8 \) sector, the anomaly consideration is analogous. Due to a chirality change, the anomaly polynomial \( \bar{I}_{12} \) is given by
\[ \bar{I}_{12} = \bar{I}_4 \wedge \bar{I}_8, \] (5.17)

where
\[ \bar{I}_4 = -(\frac{1}{2} tr \mathcal{R} \wedge \mathcal{R} - tr F^{(2)} \wedge F^{(2)}). \] (5.18)
and
\[ \bar{I}_8 = -\frac{1}{4} \bar{I}_4^2 + \bar{I}_8', \]  
(5.19)

where
\[ \bar{I}_8' = -\frac{1}{4} \bar{I}_4^2 + \left[ -\frac{1}{8} tr R^4 + \frac{1}{32} (tr R \wedge R)^2 \right]. \]  
(5.20)

The variation of the effective action on the hidden brane is given by the expression similar to eq. (5.9)
\[ (\delta \Gamma)|_{x^{11}=\pi \rho} \sim \int d^{10}x \left( -\frac{1}{2} tr (\theta R) + tr (\epsilon F^{(2)}) \right) \wedge I_8. \]  
(5.21)

The modified Bianchi identities and the transformation law of the three-form field \( C \) become
\[ (dG)|_{x^{11}=\pi \rho} \sim -\frac{1}{2} tr R \wedge R + tr F^{(2)} \wedge F^{(2)} \]  
(5.22)

and
\[ (\delta C)|_{x^{11}=\pi \rho} \sim \frac{1}{2} tr (\theta R) - tr (\epsilon F^{(2)}). \]  
(5.23)

By using eqs. (5.12)-(5.14), (5.21) and (5.23), it is straightforward to see that
\[ (\delta \Gamma)|_{x^{11}=\pi \rho} + \delta (\Gamma_{GS})|_{x^{11}=\pi \rho} = 0. \]  
(5.24)

Therefore, the local anomaly cancels in the \( \bar{E}_8 \) sector as well. From the modified Bianchi identities (5.10) and (5.22), it follows that if we consider a Calabi-Yau compactification to four dimensions, the anomaly cancellation condition reads
\[ c_2(V_1) = c_2(V_2), \]  
(5.25)

where by \( V_1 \) and \( V_2 \) we denoted the gauge bundles on the visible and on the hidden branes respectively. This condition gets modified by the presence of (anti) five-branes (wrapped on holomorphic cycles) in the bulk. Recall [38], that every five-brane contributes to the anomaly as one instanton in the \( E_8 \) sector and every anti five-brane contributes as one instanton in the \( \bar{E}_8 \) sector. Hence, the modified anomaly cancellation condition reads
\[ c_2(V_1) - c_2(V_2) + [W] - [\bar{W}] = 0, \]  
(5.26)

where the last two terms represent the five-brane and the anti five-brane classes respectively. For completeness and further reference it is useful to recall that, in the standard \( E_8 \times E_8 \) theory with (anti) five-branes, the anomaly cancellation condition is given by [22]
\[ c_2(V_1) + c_2(V_2) - c_2(T X) + [W] - [\bar{W}] = 0. \]  
(5.27)
The moduli potential in the four-dimensional effective field theory will come from the boundary terms of the eleven-dimensional supergravity

\[ S_{\text{boundary}} = - \frac{1}{8 \pi \kappa^2_{11}} \left( \frac{\kappa_{11}}{4 \pi} \right)^{2/3} \int d^{10} x \sqrt{-G_{10}} \left( tr(F^{(1)})^2 - \frac{1}{2} tr\mathcal{R}^2 \right) \]

\[ - \frac{1}{8 \pi \kappa^2_{11}} \left( \frac{\kappa_{11}}{4 \pi} \right)^{2/3} \int d^{10} x \sqrt{-G_{10}} \left( tr(F^{(2)})^2 - \frac{1}{2} tr\mathcal{R}^2 \right). \]

(5.28)

Upon the compactification to four-dimensions the metric is written as follows [56]

\[ ds^2_{11} = R^{-1} V^{-2/3} g_{4\mu\nu} dx^\mu dx^\nu + V^{1/3} g_{CY,AB} dx^A dx^B + V^{-2/3} R^2 (dx^{11})^2. \]

(5.29)

This form of the metric guarantees that the Einstein in the action is properly normalized [56]. Substituting this metric into eq. (5.28) and using the identities

\[ \int_{\text{CY}} \sqrt{g_{CY}} tr F_{AB}^2 = -2 \int_{\text{CY}} \omega \wedge F \wedge F = 32 \pi^2 \int_{\text{CY}} \omega \wedge c_2(V) \]

(5.30)

and

\[ \int_{\text{CY}} \sqrt{g_{CY}} tr \mathcal{R}_{AB}^2 = -2 \int_{\text{CY}} \omega \wedge \mathcal{R} \wedge \mathcal{R} = 32 \pi^2 \int_{\text{CY}} \omega \wedge c_2(TX), \]

(5.31)

for \( F \) and \( R \) satisfying the Hermitian Yang-Mills equations, we obtain the following moduli dependent potential

\[ \Delta U = \frac{8 \pi}{\kappa^2_{11}} \left( \frac{\kappa_{11}}{4 \pi} \right)^{2/3} \int_{\text{CY}} \omega \wedge (c_2(V_1) + c_2(V_2) - c_2(TX)) \frac{1}{VR^2}. \]

(5.32)

In the absence of five-branes in the bulk, in the \( E_8 \times E_8 \) theory this potential vanishes identically since the combination

\[ J = c_2(V_1) + c_2(V_2) - c_2(TX) \]

(5.33)

vanishes by the anomaly cancellation condition [56]. In the presence of five-branes, this potential cancels against similar terms coming from the five-brane world volume theory (see below). On the other hand, in the \( E_8 \times \bar{E}_8 \) theory, the anomaly cancellation condition looks different. In the absence of (anti) five-branes it is given by eq. (5.25) and the combination [56] does not vanish anymore. Terms with the identical functional structure come from the theory on the (anti) five-brane world-volume. The relevant part of the action of the (anti) five-brane is given by

\[ S_5 = -T_5 \int d^6 \zeta \sqrt{h} + \ldots, \]

(5.34)
where $T_5$ is the (anti) five-brane tension and $h_{rs}$ is the pullback of the eleven-dimensional metric \(5.29\)

\[
h_{rs} = \frac{\partial x^M}{\partial \zeta^r} \frac{\partial x^N}{\partial \zeta^s} G_{MN}.
\]

Now we impose the gauge

\[
x^\mu = \zeta^\mu, \quad \mu = 0, 1, 2, 3, \tag{5.36}
\]

and identify $x^{11}$ with the real part of five-brane modulus $y$. Then, by using eq. \(5.29\), the components of the induced metric $h_{rs}$ can be written as follows

\[
h_{\mu\nu} = V^{-2/3} R^{-1} g_{4\mu\nu} + V^{-2/3} (\pi R)^2 \partial_\mu y \partial_\nu y \tag{5.37}
\]

and

\[
h_{\sigma\tau} = \frac{\partial x^A}{\partial \zeta^\sigma} \frac{\partial x^B}{\partial \zeta^\tau} V^{1/3} g_{CY,AB}, \quad \sigma, \tau = 1, 2. \tag{5.38}
\]

The metric $h_{\sigma\tau}$ is just the induced metric on the holomorphic curve on which the (anti) five-brane is wrapped. Performing the integration over the area of the holomorphic curve in \(5.34\), we obtain a potential of the functional form \(5.32\). As we mentioned before, in the $E_8 \times E_8$ compactification, the potential \(5.32\) vanishes, whereas in the $E_8 \times \bar{E}_8$ theory it does not. Let us also point out that if one takes care of the terms involving two derivatives of $y$ one obtains the Kahler potential for this modulus given by the last term in eq. \(2.9\). The moduli potential \(5.32\) has a similar functional structure as the Fayet-Iliopoulos potentials \(3.11\) and \(3.12\). Therefore, potentially, it can be used to raise the AdS vacuum discussed in Section 2. However, we will see that it is not very easy to use this potential to stabilize the moduli. The problem comes from the overall scale of the potential. The potential \(5.32\) has the following structure

\[
\Delta U = \frac{a}{V R^2}, \tag{5.39}
\]

where the coefficient $a$ is generically of order

\[
a \sim \frac{8\pi v_{CY}^{1/3}}{\kappa_{11}^2} \left(\frac{\kappa_{11}}{4\pi}\right)^2. \tag{5.40}
\]

By using eqs. \(2.5\) and \(2.12\), we can estimate $a$ and obtain

\[
a \sim M_{Pl}^4 10^{-10}. \tag{5.41}
\]

This order of magnitude is too big. For a generic compactification, it does not seem to be possible to balance $\Delta U$ against the non-perturbative and the flux-induced superpotentials to provide an interesting moduli stabilization, at least if $h_{1,1} = 1$. Let us now assume that $h_{1,1}$
is greater than one and try to determine whether it is ever possible to decrease the coefficient $a$. Let us look more carefully at the combination (5.33). Expanding the Kahler form $\omega$ in the basis of harmonic forms $\{\omega_I\}$

$$\omega = \sum_{I=1}^{h^{1,1}} b_I \omega_I,$$

where $b_I$ are related to the real parts of the $h^{1,1}$ moduli (see [69] for details), we can write the integral of $J$ more explicitly as

$$\int_{CY} \omega \wedge J = -\frac{1}{16\pi^2} \int_{CY} \sum_{I=1}^{h^{1,1}} b_I \omega_I \wedge (\text{tr} F^{(1)} \wedge F^{(1)} + \text{tr} F^{(2)} \wedge F^{(2)} - tr \mathcal{R} \wedge \mathcal{R})$$

$$= -\frac{1}{16\pi^2} \sum_{I=1}^{h^{1,1}} b_I v_I \int_{C_I} (\text{tr} F^{(1)} \wedge F^{(1)} + \text{tr} F^{(2)} \wedge F^{(2)} - tr \mathcal{R} \wedge \mathcal{R})$$

$$= \sum_{I=1}^{h^{1,1}} b_I v_I \int_{C_I} (c_2(V_1) + c_2(V_2) - c_2(TX)),$$

where $\{C_I\}$ are the four-cycles Poincare dual to the basis $\{\omega_I\}$ and $v_I$ is the volume of the two-cycle, Poincare dual to the four-form

$$c_2(V_1) + c_2(V_2) - c_2(TX),$$

measured with respect to the Kahler forms $\omega_I$. Apparently, the only way to decrease the order of magnitude of $\Delta U$ is to consider a Calabi-Yau threefold in which there is at least one very small cycle and to take $c_2(V_1) + c_2(V_2) - c_2(TX)$ localized precisely on such a cycle. Of course, the four-form that has to be localized on a small cycle does not necessarily have to be $c_2(V_1) + c_2(V_2) - c_2(TX)$. It does only if there are no (anti) five-branes in the bulk. In general, in the $E_8 \times \bar{E}_8$ theory this four-form is given by

$$J' = c_2(V_1) + c_2(V_2) - c_2(TX) + [\mathcal{W}] + [\bar{\mathcal{W}}] = 2c_2(V_1) - c_2(TX) + 2[\mathcal{W}],$$

where the anomaly cancellation condition (5.26) has been used. Note that $J'$ (more precisely $\int_{CY} \omega \wedge J'$) is not necessarily positive. Therefore, the potential (5.39) combined with the supergravity one can generate a phenomenologically attractive dS minimum even if the number of moduli is large. A potential of the form (5.39) also exist in the $E_8 \times E_8$ theory with anti five-branes in the bulk. It has the form

$$\Delta \tilde{U} = \frac{8\pi}{\kappa_{11}^2} \left( \frac{\kappa_{11}}{4\pi} \right)^{2/3} \int_{CY} \omega \wedge J'' \frac{1}{VR^2},$$

(5.46)
where
\[ J'' = c_2(V_1) + c_2(V_2) - c_2(TX) + [\mathcal{W}] + [\tilde{\mathcal{W}}] = 2[\tilde{\mathcal{W}}], \] (5.47)
where the $E_8 \times E_8$ anomaly cancellation condition eq. (5.27) has been used. This potential is strictly positive. In any case, this type of potentials can be used only if we can find a Calabi-Yau manifold with relatively small 2-cycles. Of course, what we mean by this is that one has to be able to stabilize at least one of the $h^{1,1}$ moduli at a very small scale comparing to the Calabi-Yau scale. We will not attempt to solve this problem in this paper. Instead, let us speculate on what could be the key to solving this problem. The non-perturbative superpotentials discussed in Section 2 have factors depending on complex structure and vector bundle moduli [15, 35, 36]. We ignored them as they were not important for our purposes. For certain geometries those factors were explicitly calculated in [51, 52] and found to be high degree polynomials. If one manages to show that the values of such factors associated to different isolated curves in the Calabi-Yau threefold can vary a lot, it could stabilize various cycles at different scales. The high degree polynomials might provide a considerable help. Unfortunately, to study systematically the vector bundle moduli contribution to non-perturbative superpotentials is a very challenging problem.

6 Conclusion

In this paper, we considered a problem of moduli stabilization in a metastable dS vacuum in the context of strongly coupled heterotic string theory. We showed that, as in type IIB theory [1], dS vacua can be constructed by adding various correction to the supergravity potential energy. We studied two types of such corrections. The first type corrections are generated by Fayet-Iliopoulos terms [48, 49]. They appear if the low-energy gauge group in the visible or in the hidden sector contains an anomalous $U(1)$ factor. The form of the moduli potential energy is slightly different depending on in which sector there is such a factor. This potential is always positive. We showed that the total potential energy indeed can have a dS minimum. Moreover, since the supergravity part of the potential energy can be kept negative, it is possible, by fine tuning, parameters to get a cosmological constant consistent with observations. However, if the number of moduli is sufficiently large, one can expect that both supergravity and Fayet-Iliopoulos contributions to the potential energy will become positive and it will not be possible to obtain a small cosmological constant even by fine tuning. Corrections of the second type can be generated if we add anti five-branes in the bulk or, more generally, consider $E_8 \times \tilde{E}_8$ compactifications with both five-branes and
anti five-branes involved. A moduli potential arises since the net tension of various branes
does not cancel anymore (though the net charge does). This potential can, in principle, be
both positive and negative. It is also more universal because its existence does not depend
on details of the compactification. Roughly, it has a similar functional form as the Fayet-
Iliopoulos terms. Therefore, it can generate a dS minimum in a similar way. However,
we noticed that, in a generic compactification, its order of magnitude is not small enough
and not comparable with that of fluxes and non-perturbative superpotentials. To make it
comparable, it is necessary to learn how to stabilize some of the Kahler structure moduli at
a scale sufficiently smaller than the Calabi-Yau scale. One can speculate that this problem
is related to understanding of non-exponential factors in non-perturbative superpotentials.
The results of this paper can be considered as the heterotic version of the type IIB moduli
stabilization mechanism [1]. However, there are some new non-trivial elements. Generic
heterotic models always have more types of moduli and, hence, more types of superpotentials
have to be included. Some of them can be trusted only in a certain range of the moduli
space. One more important feature is that it seems hard to avoid five-branes.

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References

[1] S. Kachru, R. Kallosh, A. Linde and S. P. Trivedi, _de Sitter Vacua in String Theory_,
Phys.Rev. D68 (2003) 046005 [hep-th/0301240].

[2] K. Dasgupta, G. Rajesh and S. Sethi, _M Theory, Orientifolds and G-Flux_, JHEP 9908
(1999) 023 [hep-th/9908088].

[3] S. B. Giddings, S. Kachru and J. Polchinski, _Hierarchies from Fluxes in String Com-
pactifications_, Phys.Rev. D66 (2002) 106006 [hep-th/0105097].

[4] S. Kachru, M. B. Schulz and S. Trivedi, _Moduli Stabilization from Fluxes in a Simple
IIB Orientifold_ [hep-th/0201028].
[5] S. Kachru, M. B. Schulz, P. K. Tripathy and S. P. Trivedi, *New Supersymmetric String Compactifications*, JHEP 0303 (2003) 061 [hep-th/0211182].

[6] A. R. Frey and J. Polchinski, *N=3 Warped Compactifications*, Phys.Rev. D65 (2002) 126009 [hep-th/0201029].

[7] S. Gukov, C. Vafa and E. Witten, *CFT’s From Calabi-Yau Four-folds*, Nucl.Phys. B584 (2000) 69-108; Erratum-ibid. B608 (2001) 477-478 [hep-th/9906070].

[8] T.R. Taylor and C. Vafa, *RR Flux on Calabi-Yau and Partial Supersymmetry Breaking*, Phys.Lett. B474 (2000) 130-137 [hep-th/9912152].

[9] G. Curio, A. Klemm, D. Luest and S. Theisen, *On the Vacuum Structure of Type II String Compactifications on Calabi-Yau Spaces with H-Fluxes*, Nucl.Phys. B609 (2001) 3-45 [hep-th/0012213].

[10] A. Giryavets, S. Kachru, P. K. Tripathy and S. P. Trivedi, *Flux Compactifications on Calabi-Yau Threefolds*, JHEP 0404 (2004) 003 [hep-th/0312104].

[11] B. S. Acharya, *A Moduli Fixing Mechanism in M theory* [hep-th/0212294].

[12] M. Dine, N. Seiberg, X.G. Wen and E. Witten, *Nonperturbative Effects On The String World Sheet. 2*, Nucl.Phys. B278 (1986) 769.

[13] M. Dine, N. Seiberg, X.G. Wen and E. Witten, *Nonperturbative Effects On The String World Sheet. 2*, Nucl.Phys. B289 (1987) 319.

[14] K. Becker, M. Becker and A. Strominger, *Fivebranes, Membranes and Non-Perturbative String Theory*, Nucl.Phys. B456 (1995) 130-152 [hep-th/9507158].

[15] E. Witten, *Non-Perturbative Superpotentials In String Theory*, Nucl.Phys. B474 (1996) 343-360 [hep-th/9604030].

[16] S. Kachru, J. Pearson and H. Verlinde, *Brane/Flux Annihilation and the String Dual of a Non-Supersymmetric Field Theory*, JHEP 0206 (2002) 021 [hep-th/0112197].

[17] A. Saltman and E. Silverstein, *The Scaling of the No Scale Potential and de Sitter Model Building* [hep-th/0402135].

[18] D. Robbins and S. Sethi, *A Barren Landscape* [hep-th/0405011].
[19] F. Denef, M. R. Douglas and B. Florea, \textit{Building a Better Racetrack} [hep-th/0404257].

[20] P. Horava and E. Witten, \textit{Heterotic and Type I String Dynamics from Eleven Dimensions}, Nucl.Phys. B460 (1996) 506-524 [hep-th/9510209].

[21] P. Horava and E. Witten, \textit{Eleven-Dimensional Supergravity on a Manifold with Boundary}, Nucl.Phys. B475 (1996) 94-114 [hep-th/9603142].

[22] E. Witten, \textit{Strong Coupling Expansion Of Calabi-Yau Compactification}, Nucl.Phys. B471 (1996) 135-158 [hep-th/9602070].

[23] A. E. Faraggi, \textit{Phenomenological survey of M-theory}, to appear in the proceedings of SUGRA 20 conference, Boston, 17-21 March 2003 [hep-th/0307037].

[24] R. Donagi, A. Lukas, B. A. Ovrut and D. Waldram, \textit{Holomorphic Vector Bundles and Non-Perturbative Vacua in M-Theory}, JHEP 9906 (1999) 034 [hep-th/9901009].

[25] R. Donagi, B. A. Ovrut, T. Pantev and D. Waldram, \textit{Standard-Model Bundles on Non-Simply Connected Calabi–Yau Threefolds}, JHEP 0108 (2001) 053 [hep-th/0008008].

[26] R. Donagi, B. A. Ovrut, T. Pantev and R. Reinbacher, \textit{SU(4) Instantons on Calabi-Yau Threefolds with }\mathbb{Z}_2 \times \mathbb{Z}_2\textit{ Fundamental Group} [hep-th/0307273].

[27] R. Donagi, Y.-H. He, B. A. Ovrut and R. Reinbacher, \textit{Moduli Dependent Spectra of Heterotic Compactifications} [hep-th/0403291].

[28] R. Donagi, Y.-H. He, B. A. Ovrut and R. Reinbacher, \textit{The Particle Spectrum of Heterotic Compactifications} [hep-th/0405014].

[29] M. Becker, G. Curio and A. Krause, \textit{De Sitter Vacua from Heterotic M-Theory} [hep-th/0403027].

[30] R. Brustein and S. P. de Alwis, \textit{Moduli potentials in string compactifications with fluxes: mapping the Discretuum} [hep-th/0402088].

[31] G. Curio and A. Krause, \textit{G-Fluxes and Non-Perturbative Stabilisation of Heterotic M-Theory}, Nucl.Phys. B643 (2002) 131-156 [hep-th/0108220].

[32] M. Dine, R. Rohm, N. Seiberg and E. Witten, \textit{Gluino Condensation In Superstring Models}, Phys.Lett. B156 (1985) 55.
[33] P. Horava, *Gluino Condensation in Strongly Coupled Heterotic String Theory*, Phys.Rev. D54 (1996) 7561-7569 [hep-th/9608019]

[34] A. Lukas, B. A. Ovrut and D. Waldram, *Gaugino Condensation in M-theory on $S^1/Z_2$*, Phys.Rev. D57 (1998) 7529-7538 [hep-th/9711197]

[35] E. Lima, B. A. Ovrut, J. Park and R. Reinbacher, *Non-Perturbative Superpotentials from Membrane Instantons in Heterotic M-Theory*, Nucl.Phys. B614 (2001) 117-170 [hep-th/0101049]

[36] E. Lima, B. A. Ovrut and J. Park, *Five-Brane Superpotentials in Heterotic M-Theory*, Nucl.Phys. B626 (2002) 113-164 [hep-th/0102046]

[37] G. Moore, G. Peradze and N. Saulina, *Instabilities in heterotic M-theory induced by open membrane instantons*, Nucl.Phys. B607 (2001) 117-154 [hep-th/0012104]

[38] K. Behrndt and S. Gukov, *Domain Walls and Superpotentials from M Theory on Calabi-Yau Three-Folds*, Nucl.Phys. B580 (2000) 225-242 [hep-th/0001082]

[39] M. Becker and D. Constantin, *A Note on Flux Induced Superpotentials in String Theory*, JHEP 0308 (2003) 015 [hep-th/0210131]

[40] E. I. Buchbinder and B. A. Ovrut, *Vacuum Stability in Heterotic M-Theory*, Phys.Rev. D69 (2004) 086010 [hep-th/0310112]

[41] S. Gukov, S. Kachru, X. Liu and L. McAllister, *Heterotic Moduli Stabilization with Fractional Chern-Simons Invariants*, Phys.Rev. D69 (2004) 086008 [hep-th/0310159]

[42] K. Becker, M. Becker, K. Dasgupta and P. S. Green, *Compactifications of Heterotic Theory on Non-Kahler Complex Manifolds: I*, JHEP 0304 (2003) 007 [hep-th/0301161]

[43] K. Becker, M. Becker, K. Dasgupta and S. Prokushkin, *Properties Of Heterotic Vacua From Superpotentials*, Nucl.Phys. B666 (2003) 144-174 [hep-th/0304001]

[44] K. Becker, M. Becker, K. Dasgupta, P. S. Green and E. Sharpe, *Compactifications of Heterotic Strings on Non-Kahler Complex Manifolds: II*, Nucl.Phys. B678 (2004) 19-100 [hep-th/0310058]

[45] G. L. Cardoso, G. Curio, G. Dall’Agata, D. Lust, P. Manousselis and G. Zoupanos, *Non-Kaehler String Backgrounds and their Five Torsion Classes*, Nucl.Phys. B652 (2003) 5 [hep-th/0211118]
[46] G. L. Cardoso, G. Curio, G. Dall’Agata and D. Lust, *BPS Action and Superpotential for Heterotic String Compactifications with Fluxes*, JHEP 0310 (2003) 004 [hep-th/0306088].

[47] G. L. Cardoso, G. Curio, G. Dall’Agata and D. Lust, *Heterotic String Theory on non-Kaehler Manifolds with H-Flux and Gaugino Condensate* Fortsch.Phys. 52 (2004) 483-488 [hep-th/0310021].

[48] M. Dine, N. Seiberg and E. Witten, *Fayet-Iliopoulos Terms in String Theory*, Nucl. Phys. B 289 (1987) 589.

[49] C.P. Burgess, R. Kallosh and F. Quevedo, *de Sitter String Vacua from Supersymmetric D-terms*, JHEP 0310 (2003) 056 [hep-th/0309187].

[50] M. Fabinger and P. Horava, *Casimir Effect Between World-Branes in Heterotic M-Theory*, Nucl.Phys. B580 (2000) 243-263 [hep-th/0002073].

[51] E. I. Buchbinder, R. Donagi and B. A. Ovrut, *Superpotentials for Vector Bundle Moduli*, Nucl.Phys. B653 (2003) 400-420 [hep-th/0205190]

[52] E. I. Buchbinder, R. Donagi and B. A. Ovrut, *Vector Bundle Moduli Superpotentials in Heterotic Superstrings and M-Theory*, JHEP 0207 (2002) 066 [hep-th/0206203].

[53] R. Donagi, B. A. Ovrut and D. Waldram, *Moduli Spaces of Fivebranes on Elliptic Calabi-Yau Threefolds*, JHEP 9911 (1999) 030 [hep-th/9904054].

[54] T. Banks and M. Dine, *Couplings and Scales in Strongly Coupled Heterotic String Theory*, Nucl.Phys. B479 (1996) 173-196 [hep-th/9605136].

[55] P. Candelas and X. de la Ossa, *Moduli Space of Calabi-Yau Manifolds*, Nucl.Phys. B355, 455 (1991).

[56] A. Lukas, B. A. Ovrut and D. Waldram, *On the Four-Dimensional Effective Action of Strongly Coupled Heterotic String Theory*, Nucl.Phys. B532 (1998) 43-82 [hep-th/9710208].

[57] J.-P. Derendinger and R. Sauser, *A Five-brane Modulus in the Effective N=1 Supergravity of M-Theory*, Nucl.Phys. B598 (2001) 87-114 [hep-th/0009054].

[58] A. Lukas, B. A. Ovrut and D. Waldram, *Non-standard embedding and five-branes in heterotic M-Theory*, Phys.Rev. D59 (1999) 106005 [hep-th/9808101].
[59] V. Kaplunovsky and J. Louis, *Model-Independent Analysis of Soft Terms in Effective Supergravity and in String Theory*, Phys.Lett. B306 (1993) 269-275 [hep-th/9303040].

[60] A. Brignole, L.E. Ibanez and C. Munoz, *Towards a Theory of Soft Terms for the Supersymmetric Standard Model*, Nucl.Phys. B422 (1994) 125-171; Erratum-ibid. B436 (1995) 747-748 [hep-ph/9308271].

[61] H.P. Nilles, M. Olechowski and M. Yamaguchi, *Supersymmetry Breaking and Soft Terms in M-Theory*, Phys.Lett. B415 (1997) 24-30 [hep-th/9707143].

[62] Z. Lalak and S. Thomas, *Gaugino Condensation, Moduli Potentials and Supersymmetry Breaking in M-Theory Models*, Nucl.Phys. B515 (1998) 55-72 [hep-th/9707223].

[63] H. P. Nilles, *Gaugino Condensation and SUSY Breakdown*, Lectures at Cargese School of Physics and Cosmology, Cargese, France, August 2003, [hep-th/0402022].

[64] R. Donagi, J. Khoury, B. A. Ovrut, P. J. Steinhardt and N. Turok, *Visible Branes with Negative Tension in Heterotic M-Theory*, JHEP 0111 (2001) 041 [hep-th/0105199].

[65] G. Curio and A. Krause, *Four-Flux and Warped Heterotic M-Theory Compactifications*, Nucl.Phys. B602 (2001) 172-200 [hep-th/0012152].

[66] G. Curio and A. Krause, *Enlarging the Parameter Space of Heterotic M-Theory Flux Compactifications to Phenomenological Viability* [hep-th/0308202].

[67] E. Witten, *World-Sheet Corrections Via D-Instantons*, JHEP 0002 (2000) 030 [hep-th/9907041].

[68] M. J. Duff, R. Minasian and E. Witten, *Evidence for Heterotic/Heterotic Duality*, Nucl.Phys. B465 (1996) 413-438 [hep-th/9601036].

[69] A. Lukas, B. A. Ovrut, K. S. Stelle and D. Waldram, *Heterotic M-theory in Five Dimensions*, Nucl.Phys. B552 (1999) 246-290 [hep-th/9806051].