The effects of compressible and incompressible states on the FIR-absorption of quantum wires and dots in a magnetic field.

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Abstract

We investigate the effects of compressible and incompressible states on the FIR-absorption of quantum wires and dots in a homogeneous perpendicular magnetic field. The electron-electron interaction is treated in the Hartree approximation at a finite low temperature. The calculated dispersion of the collective excitations reproduces several experimental results.

I. INTRODUCTION

According to Kohn’s theorem \cite{1-4} a spatially homogeneous electric field can only excite the rigid center of mass motion of all the electrons confined parabolically in a quantum dot or wire. In far-infrared (FIR) spectroscopy this condition is usually satisfied due to the small size of the individually isolated dots and wires compared to the wavelength of the radiation. In order to detect some of the internal structure or relative motion of the two-dimensional electron gas (2DEG) through FIR spectroscopy the confining potential has to deviate from the parabolic form. The deviation is found to influence the FIR response of the 2DEG in two different ways, besides a trivial blue or red shift: First, the center of mass mode (the magnetoplasmon) interacts with higher order harmonics of the cyclotron resonance forming complicated anticrossing patterns \cite{5,6}. Second, the formation of compressible and incompressible stripes \cite{7,9} in the electron density modulates slightly the dispersion of the lower lying branch in quantum dots \cite{10}. In addition, effects of the stripe formation in quantum dots have been measured both in tunneling \cite{11} and transport experiments \cite{12}.

In this report we elucidate further the connection between the modulation of the lower magnetoplasmon branch in quantum dots and the formation of compressible and incompressible stripes in the electron density. We explain the apparent weakness of this phenomenon in the upper main magnetoplasmon branch in quantum dots and the main branch in quantum wires. In both systems we find similar modulation in the dispersion of weak higher order collective oscillations.

In order to have consistent results for the absorption we use a corresponding self-consistent method for the ground state properties and the excited states of the 2DEG.
II. MODEL

The ground state properties of a single quantum dot or a wire are calculated using the Hartree approximation for the electron-electron Coulomb interaction \[6,13\]. The spin degree of freedom is neglected. The circular symmetric lateral confinement potential for the 2DEG in the quantum dot is

\[
V_{\text{conf}}(r) = \frac{1}{2} \hbar \omega_0 \left[ \left( \frac{r}{l_0} \right)^2 + a \left( \frac{r}{l_0} \right)^4 \right],
\]

where \( \hbar \omega_0 \) is the characteristic energy of the parabolic part of the potential and the confinement length is \( l_0 = \sqrt{\hbar/(m^* \omega_0)} \). The dielectric constant and the effective mass of an electron are denoted by \( \kappa \) and \( m^* \), respectively. The confinement in the third dimension, the \( z \)-direction, is considered strong enough to justify treating the system as exactly two-dimensional. The combined effects of the perpendicular homogeneous external magnetic field \( \mathbf{B} = B \mathbf{z} \) and \( V_{\text{conf}} \) produce the effective frequency of the dot \( \bar{\omega} = (\omega_c^2 + 4 \omega_0^2)^{\frac{1}{2}} \) and the effective length \( \lambda = [\hbar/(m^* \bar{\omega})]^{\frac{1}{2}} \), replacing the cyclotron frequency \( \omega_c = eB/(m^*c) \) and the magnetic length \( l_c = [\hbar/(m^* \omega_c)]^{\frac{1}{2}} \), respectively. The single electron Hartree energies \( \varepsilon_{n,M} \) and the states \( |n,M\rangle \) are labeled by the Landau-band (LB) index \( n = 0, 1, 2, \cdots \) and the angular quantum number \( M = -n, \cdots, 0, 1, 2, \cdots \).

The confinement potential for the quantum wire extended in the \( x \)-direction is of the form

\[
V_{\text{conf}}(y) = \frac{1}{2} \hbar \omega_0 \left[ \left( \frac{y}{l_0} \right)^2 + a \left( \frac{y}{l_0} \right)^4 \right].
\]

The effective frequency for the transverse motion of the 2DEG is \( \Omega = (\omega_c^2 + \omega_0^2)^{\frac{1}{2}} \) and the effective width is \( L = [\hbar/(m^* \Omega)]^{\frac{1}{2}} \). The single electron Hartree energies \( \varepsilon_{n,y_k} \) and the states \( |n, y_k\rangle \) are labeled by the Landau-band index \( n = 0, 1, 2, \cdots \) and the center coordinate \( y_k = kL^2 = 2\pi L^2 N/L_x \), where \( N \) is an integer and \( L_x \) is the length of the wire in the \( x \)-direction. The coefficient \( a \) determines the deviation from the parabolic confinement.

The absorption of the quantum dot \[13\] and wire \[6,14\] is calculated as a linear response to the self-consistent electrostatic potential \( \phi_{\text{sc}} = \phi_{\text{ext}} + \phi_{\text{ind}} \), where \( \phi_{\text{ind}} \) is the induced potential and \( \phi_{\text{ext}} \) is the external potential. In case of the quantum wire \( \phi_{\text{ext}}(r,t) = y \mathcal{E}_{\text{ext}} \exp(-i \omega t) \) and for the quantum dot \( \phi_{\text{ext}}(r,t) = r \mathcal{E}_{\text{ext}} \exp(-i N_p \varphi - i \omega t) \). The external electrostatic field is linearly polarized transverse to the quantum wire but in case of the quantum dot it is circularly polarized with the choice \( N_p = \pm 1 \). The power absorption of the systems is calculated from the Joule heating of the 2DEG caused by \( \phi_{\text{sc}} \).

Kohn’s theorem states that if the confinement potential is parabolic and the external electric field is spatially homogeneous then only center of mass motion can be excited \[1,4\]. In the wire the dispersion is then

\[
\Omega_p = \Omega = \sqrt{\omega_c^2 + \omega_0^2},
\]

and for the dot the dispersion has two branches

\[
\omega_\pm = \frac{1}{2} \sqrt{\omega_c^2 + 4 \omega_0^2} \pm \frac{\omega_c}{2},
\]

with \( \omega_- \) corresponding to the polarization \( N_p = +1 \) and \( \omega_+ \) to the choice \( N_p = -1 \).

III. RESULTS OF MODEL CALCULATION

The calculations for the quantum dot have been performed using GaAs parameters, \( m^* = 0.067m_0 \) and \( \kappa = 12.4 \). The results for a confinement frequency \( \omega_0 = 3.37 \text{ meV} \) are shown in Fig. \[1\]. The number of electrons, \( N_x = 60 \), guarantees that states in more than one Landau band are occupied for \( 1 < B < 6 \text{ T} \). The dispersion of the
\( \omega_- \) mode \((N_p = +1)\) shows slight oscillations that correlate with the average filling factor of the 2DEG or the location of the chemical potential \( \mu \) with respect to the Landau bands. The density profiles of the 2DEG for \( B = 3.7 \) T and 4.9 T corresponding, respectively, to a minimum and a maximum in the oscillations of the \( \omega_- \) mode are presented in Fig. 1a. The radial electron density for the whole range of the magnetic field considered here is seen in Fig. 2. Clearly visible are the “layers” corresponding to electrons in specific Landau bands, that are separated by sharp steps in larger quantum dots in a strong magnetic field, i.e. incompressible regions separated by narrow compressible regions. In the relatively small quantum dot considered here the steps are not quite so exact due to screening effects, finite temperature, and the fact that the effective magnetic length \( \lambda \) is not that very much smaller than the size of the quantum dot \( R \sim 100 \) nm. For integer values of the average filling factor \( \nu \) the chemical potential \( \mu \) lies between the bulk parts of two Landau bands as is seen in Fig. 3a, otherwise \( \mu \) lies in the bulk states of a particular Landau band as Fig. 3b shows. Comparison of Fig. ’s 1,2,3 shows that the oscillations of the \( \omega_- \) mode take a maximum value for an integer \( \nu \) and concurrently the steps in the density are clear. The oscillations of the \( \omega_- \) mode with \( \nu \) have been explained in terms of an oscillating radius of the electronic system [15], an effect due to the screening properties of the system. We prefer to use the picture of compressible and incompressible states to explain the oscillations [10]. For the \( \omega_- \) mode these pictures are probably equivalent for intermediate sized dots, but the latter one is more convenient in case of other modes to be discussed here. Kohn’s theorem is not programmed explicitly into the numerical calculation but in the density response function all relevant single electron-hole transitions are summed over and in the case of a parabolic confinement only the center of mass modes predicted by the theorem are visible. Slight deviations from the parabolic confinement result in other modes that in the case of few electrons [10] can be traced back to certain single electron transitions or groups thereof. The dipole active collective modes of the 2DEG in a quantum dot are such that only transitions of electrons observing \( M \rightarrow M - 1 \) contribute to the \( \omega_+ \) mode, such that just below \( \mu \) a hole state with quantum number \( M \) is formed but above \( \mu \) an electron state with \( M - 1 \) is formed. An inspection of Fig. 3 shows that the single electron transitions involved are almost exclusively interband transitions satisfying \( n \rightarrow n + 1 \). The \( \omega_- \) mode, on the other hand, has strong contributions from intraband \( M \rightarrow M + 1 \), hence it’s much lower energy. This is also supported by the induced density which for the \( \omega_- \) mode is usually concentrated close to the boundary of the quantum dot where the concerning \( M \) states have their heaviest weight. Now the different properties of the \( \omega_- \) mode with respect to an integer filling factor or not should be clear; When \( \nu \) is not close to an integer \( \mu \) is pinned to the bulk states of a particular Landau band, where the band rises above \( \mu \) its slope is much less than at a crossing point with edge states with higher \( M \). The single electron transitions contributing from this area thus lower the total energy of the collective mode. The oscillations of the \( \omega_- \) mode take a maximum for \( \nu \sim \) integer and a minimum for half integers. The states pinned to \( \mu \) are compressible and the induced density for the \( \omega_- \) mode is concentrated around the narrow stripes of compressible states close to the edge of the dot or close to the edge of a broader region of compressible bulk states in a quantum dot with not all bulk states of a particular band filled. The oscillations of the \( \omega_- \) mode are thus a direct consequence of the formation of compressible and incompressible stripes.
The main contribution to the $\omega_+$ mode comes from transitions of occupied bulk states $(n, M)$ to empty $(n+1, M-1)$ states. The energy of these interband transitions oscillates weakly with the filling factor, but the transitions all add up to the collective motion that can be identified as a rigid oscillation of the center of mass of the 2DEG, thus, the dispersion of the main branch of the $\omega_+$ mode shows only much weaker oscillations than are present in the $\omega_-$ mode. Fig. 4 shows the dispersion of the $\omega_+$ mode for a parabolic confinement potential with the same deviation as was used for the calculation of the $\omega_-$ mode in Fig. 1. Around $B = 2$ T the $\omega_+$ mode shows an anticrossing behavior just left of the line $E = 2\hbar\omega_c$, the magnetoplasmon is interacting with the first harmonic of the cyclotron resonance, a Bernstein mode [6]. The emerging lower branch of the $\omega_+$ mode, the branch corresponding to the rigid center of mass motion, becomes independent of $\nu$, but the higher vanishing mode, that takes on the character of a higher order magnetoplasmon, oscillates with $\nu$. The quartic deviation to the parabolic confinement strengthens the contribution of single electron transitions of the form $(n, M) \rightarrow (n+2, M-1)$ to the collective oscillations, thus introducing the higher order magnetoplasmons that are blocked by Kohn’s theorem in a parabolically confined 2DEG. The oscillations of the side branch of the $\omega_+$ mode (just above the main branch) are in antiphase to the oscillations of the $\omega_-$ mode, explanation can be found in Fig. 3. Without an interaction between the electrons the Landau bands would be parallel, and thus equidistant, interaction and screening properties of the 2DEG change this. When $\mu$ lies in the bulk states of a particular Landau band, i.e. a large compressible region is present in the density, then each Landau band develops a flat region, but due to the increasing spatial extent of the wave functions of the higher bands the flat regions shrink with growing $n$. The relative distance between the Landau bands has therefore a maximum when the a band is pinned to $\mu$ and a large compressible region in the electron density is present. The finite size of the wave functions that increases with $n$ also explains why the “layers” in the density of a relatively small quantum dot get thinner and smaller with increasing $n$. A large quantum dot does not exhibit this effect.

The observation of higher order dipole active magnetoplasmons opens the question what effects the deviation from the parabolic confinement has on the quadruple active magnetoplasmon $[13,17,18]$, $N_p = \pm 2$, even though it has not been measured directly in quantum dots. This collective mode has contributions of single electron transitions with $\Delta M = \pm 2$ and is seen in Fig. 5 together with the dipole active modes discussed above. The quadrupole $\omega_+$ mode shows a very clear and simple $2\omega_c$ anticrossing, much simpler than in the dipole case. The quadrupole active $\omega_-$ mode shows stronger filling factor oscillations than the dipole counterpart and the oscillations are in phase since they have the same origin. In the lower split-off branch of the quadrupole active $\omega_+$ mode weak oscillations with $\nu$ are found in phase with the dipole counterpart.

In light of the results above for quantum dots it would not be unexpected to find some filling factor dependent effects in quantum wires, even though the magnetoplasmon in a parabolically confined system has only one branch corresponding to the $\omega_+$ mode of a quantum dot. The electronic density of a quantum wire develops clearly separate compressible and incompressible regions with increasing wire width, and the steps are also clear in the Hartree energy spectra. In the case of a dipole excitation the single electron transitions fulfill the conservation of the center coordinate, $\Delta y_k = 0$. The calculation for the wire has been performed with the follow-
ing parameters, $\hbar \omega_0 = 3.94$ meV, $a = 0.03$, and $\kappa = 12.53$, other parameters are the same as for the quantum dot. The results for the dispersion are seen in Fig. 6. Again we have the $2\omega_c$ anticrossing, now just right of the $E = 2\hbar \omega_c$ line. For $B > 3$ T the lower branch gains oscillator strength and corresponds to the rigid center of mass motion of the whole 2DEG. The upper branch is a higher order magnetoplasmon as the induced density confirms. The dispersion has a very similar overall appearance as the dispersion for the $\omega_+$ mode of the quantum dot seen in Fig. 4, and the Hartree energy spectra show similar properties as the spectra for quantum dots, thus leading to the same explanation of the origin of the oscillations with $\nu$.

IV. SUMMARY

We have been able to explain filling factor dependent oscillations occurring in the magnetoplasmon dispersion for quantum dots and wires in terms of the formation of compressible and incompressible regions in the electronic density of the systems. The oscillations have been found in the $\omega_-$ mode of quantum dots [10] and some indications of them have been found in wires and in the $\omega_+$ mode for dots. Their appearance in wires and in the $\omega_+$ mode for quantum dots seems to depend on the concerning system to be of intermediate size, i.e. the effective magnetic length needs to be not much less than one order of magnitude smaller than the system. The far-infrared measurements are therefore yet another method to observe the internal structure of not parabolically confined quantum dots and wires, the stripes formed by the compressible and incompressible states.

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FIGURES

FIG. 1. (a) Results of the Hartree approximation for the density profile vs. the spatial coordinate $x$ in a quantum dot in units of $10^{11} \text{cm}^{-2}$. The flat regions for $B = 4.9 T$ indicate the incompressible regimes. (b) Resonance frequency of the $\omega_-$ mode in RPA. (c) Ratio of the RPA and the classical result ($\omega_{\text{RPA}}/\omega_-$, vs $B^{-1}$. $T = 1 \text{ K}$, $m^* = 0.067m_0$, $\kappa = 12.4$, $\omega_0 = 3.37 \text{ meV}$, and $a = 0.0674$.

FIG. 2. The radial electron density in units of $10^{11} \text{cm}^{-2}$ for a quantum dot in the Hartree approximation as a function of the magnetic field $B$ and the radius $r$. Same parameters as in Fig. [1].

FIG. 3. The Hartree single-electron energy spectra vs the angular momentum quantum number $M$ for (a) $B = 4.9 T$, and (b) $B = 5.7 T$. Same parameters as in Fig. [1].

FIG. 4. The power absorption $P(\hbar \omega)$ for the $\omega_+$ mode ($N_p = -1$) of quantum dot. The calculation was performed for $B \sim 1 - 6 T$. A constant Lorentzian linewidth $\hbar \eta = 0.01 \omega_0 \text{ meV}$ is used. Other parameters are as in Fig. [1].

FIG. 5. The dispersion of the quadrupole modes in a quantum dot labelled with $N_p = -2$ and $N_p = +2$, and the dipole modes labelled with $N_p = -1 \ (\omega_+)$ and $N_p = +1 \ (\omega_-)$. The straight lines represent the energies $E = \hbar \omega_c$, and $E = 2\hbar \omega_c$. The calculation was performed for $B \sim 1 - 6 T$. Other parameters are as in Fig. [1].

FIG. 6. The dispersion of dipole modes in a quantum wire with $a = 0.03$. The one dimensional electron density $n_{1D} = 1.25 \times 10^6 \text{ cm}^{-1}$, $T = 1 \text{ K}$, $m^* = 0.067m_0$, $\kappa = 12.53$, and $\omega_0 = 3.94 \text{ meV}$.
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