Specific yield of aquifer evaluation by means of a new experimental algorithm and its applications

Una valutazione della porosità efficace dell’acquifero mediante un nuovo algoritmo sperimentale e sue applicazioni

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Riassunto: Si propone un metodo semplificato per il calcolo della potenzialità specifica (denominata anche porosità efficace) partendo dai dati di conducibilità idraulica determinati con prove di pompaggio. Questo metodo deriva da una rielaborazione di alcuni dati di letteratura e da una calibrazione con diverse prove di pompaggio eseguite in differenti contesti idrogeologici. L’uso dell’algoritmo consente di disporre di valori di potenzialità specifica ($S_r$) significativi per la risoluzione dei problemi di bilancio idrico e di trasporto di contaminanti nelle acque sotterranee. Questo metodo è stato utilizzato a titolo esemplificativo per la determinazione della porosità efficace in un’area vasta (Milano e dintorni). Sono state valutate le conseguenze sulla stima dei volumi di acque immagazzinate nel sottosuolo utilizzando circa settanta anni di misure, le quali rappresentano la storia del territorio dal punto di vista socio-economico.

Abstract: A simplified method to determine specific yield (i.e., effective porosity) from hydraulic conductivity data obtained through pumping tests is proposed. This new method derives from a reprocessing of literature data and a subsequent calibration with results from pumping tests performed in different hydrogeological contexts. The use of the algorithm allows obtaining values of specific yield ($S_r$), which could be useful for the resolution of problems concerning the water balance and the transport of contaminants in groundwater.

The proposed algorithm is applied to a large-scale area (Milan and its suburbs, northwestern Italy) to determine a map of the specific yield of a sandy-gravel aquifer and the effects on the estimation of water volumes stored in the subsoil from a hydrogeological point of view, considering about seventy years of measures. It is demonstrated that the great variation in water volumes reflects the socio-economic history of the territory.

Introduction

In porous media, the evaluation of the specific yield of an aquifer is very important since it quantifies the exploitable groundwater resource and contributes to describe the advection component of both water and solutes, evaluating their migration.

In a porous medium of total volume $V$ [L$^3$], $V_v$ and $V_s$ are the volumes occupied by the voids and the solid matrix, respectively, and the total porosity $n$ [L$^3$/L$^3$] is defined as:

$$n = \frac{V_v}{V} = \frac{V_v}{V_v + V_s}$$  \hspace{1cm} (1)

The total porosity is given by the sum of the immobile water that is adsorbed on the solid particles or lies in the isolated pores and the mobile water in the interstices according to the relation:

$$n = S_r + S_y$$  \hspace{1cm} (2)

where $S_r$ is specific retention and $S_y$ is specific yield, which is also known as effective porosity.

Starting from the soil-water interface, the specific retention is the sum of the “hygroscopic water” (pressure of water < -1000 MPa), wilting point (pressure of water < -3 MPa), and filed capacity (pressure of water < -1.5 MPa), where the wilting point and field capacity describe the “capillary water” in the subsoil. Thus, the specific retention is expressed through the matric potential.

Instead, the specific yield is the free volume of water that is released from or taken into storage and flows according to the
gravity force. Such water is subjected to hydrostatic potential and its variations in space define the hydraulic gradient, which is the engine of groundwater flow.

The Eckis diagram summarizes the theoretical reference values for the parameters illustrated above (Eckis 1934; Fig. 1). It illustrates the distribution of the total porosity, specific yield and specific retention functions of granulometric aggregates. Comparing several porous and fissured aquifer media, it can be noticed that $S_y$ tends to increase with increasing sediment grain size (Table 1; Domenico and Mifflin 1965; Freeze and Cherry 1979; Batu 1998). In 1966, Johnson published a report on specific yields representative of different materials, with the purpose of assisting hydrogeologists in estimating the water storage in aquifers (Johnson 1966). Specific yield was evaluated with different methods at different locations of the world (mainly in the United States). Specific yield ranges between 0 and 35%, according to the grain size (Table 2). Maximum values of $S_y$ range between 30-35% and coincide with coarse sand-to-fine gravel. (Fig. 1, Table 1, Table 2).

**Specific yield determination**

$S_y$ can be evaluated both in the laboratory and in the field. Laboratory methods include the “saturation and drainage” or “centrifugation” of the sample and the “mercury porosimeter” technique. Common field methods are slug tests and pumping tests. It is believed that pumping tests are to be preferred, as they cover a more significant volume of the aquifer, obtaining an “average” value of the hydrogeological parameters.

In unconfined aquifers, it is possible to determine $S_y$ values through the interpretation of pumping tests, applying the Neuman method (Neuman 1974). The analysis involves matching type curves to drawdown data collected during a pumping test.

The response described by the Neuman model assumes retarded drainage at the water table and exhibits three distinct drawdown segments. Early-time response is controlled by the transmissivity and storage coefficient and is analogous to the response of a confined aquifer. Late-time response is a function of transmissivity and specific yield. At intermediate time, response is controlled by the aquifer’s vertical hydraulic conductivity. Traditionally, one matches Type A curves to early-time drawdown data (Fig. 2a) and Type B curves to late-time response (Fig. 2b; Neuman 1975).

In particular, the total storage coefficient $S$, dimensionless, is obtained from the relation:

$$S = S_A + S_y \approx S_y$$  \hspace{1cm} (3)

where $S_A$ is the storage coefficient calculated from Type A curve, $S_y$ is the specific yield calculated from Type B curve and being $S_y >> S_A$. In the example of Fig. 2, $S_A = 4 \cdot 10^{-4}$ and $S_y = 1.5 \cdot 10^{-1}$.

However, it is necessary to carry out a long-term test (for example, longer than 24-36 hours) in order to obtain the late-time response of the experimental curve, as in Fig. 2b; without the possibility of interpreting the second part of the curve it is not possible to calculate $S_y$.

Specific yield in solute transport analyses is usually estimated from grain-size distribution and soil-water
characteristic curves (e.g., Eckis 1934). Instead, tracer tests in the field or laboratory are less frequent and relatively expensive and time-consuming. However, Stephens et al. (1998) have demonstrated that estimates of specific yield from textural data or moisture retention were approximately 50-90% greater than values calibrated according to field data. This result is remarkable for the implementation of numerical models with regard to the advection component of solute transport and sorption.

**Specific yield and hydraulic conductivity**

Several hydrogeological and hydrochemical case studies explore the characterization of large-scale areas, where performing numerous traditional pumping tests is not sustainable from both a technical and an economical point of view. Thus, it is necessary a regionalization of the specific yield.

Since the beginning of the XX century, several studies have attempted to determine a mathematical relationship between hydraulic conductivity and the grain size of the aquifer or between permeability and specific yield (the reader is referred to Beretta, 1992, for a complete review). We report here the relationship between hydraulic conductivity and grain size expressed by the empirical Hazen formula (Hazen 1911):

\[
k = B \cdot C \cdot \tau \cdot d_e^2
\]

where:

- \( k \) is the hydraulic conductivity [m/s],
- \( B \) is a dimensionless parameter equal to 1 for \( k \) expressed as cm/s and 0.00116 for \( k \) expressed as m/s,
- \( d_e \) is the effective diameter, usually \( d_{10} \)
- \( \tau \) is a dimensionless parameter of correction according to water temperature \( T \) [°C], \( \tau = 0.70 + 0.03T \), and \( C \) is a dimensionless constant function of the clay content of the soil; \( C \) values range from 800 to 1200 for clean and uniform sands and from 400 to 800 for non-uniform clay sands.

Kozeny (1927) introduced the empirical equation of hydraulic conductivity function of the grain size. Kozeny used the harmonic mean grain size of samples. Carman (1937) later modified Kozeny’s permeability formula. The resultant form of the equation is known as the Kozeny–Carman (Kozeni 1927; Carman 1937):

\[
k = 7.94d_e^2 \cdot \frac{n^8}{(1 - n)^3} \tau
\]

where \( k \) is the hydraulic conductivity [cm/s], \( n \) is the porosity [-] and \( d_e \) is the assumed \( d_{10} \) of the grain-size distribution. \( \tau \) is the temperature coefficient correction [-], which ranges between 0.807 and 1.052, valid for groundwater temperatures ranging between 10 and 20 °C.

Additional methods describing grain-size distribution and hydraulic conductivity computation are illustrated in Vukovic and Soro (1992).

It should be remembered that a correct application of the Neuman method (Neuman 1975) allows evaluating the degree of aquifer anisotropy \( k_{d} \), in terms of the relationship between radial and vertical hydraulic conductivity using the parameter \( \beta \) (value of Neuman curve) given by:

\[
\beta = \frac{k_d}{k_r} \cdot \frac{r^2}{b^2}
\]

where \( k_d \) is the vertical hydraulic conductivity [L/T], \( k_r \) is the radial (horizontal) hydraulic conductivity [L/T], \( b \) is the aquifer thickness [L] and \( r \) is the distance [L]. The degree of aquifer anisotropy is fundamental for the calibration of flow and transport models, where a value of 1/10 for this ratio is assumed by default in absence of experimental data.

In the last years, several methodologies have been proposed for the estimation of the specific yield (Ramsahoye and Lang 1961; Morris and Johnson 1967; U.S. Department of the Interior 1978; Harter 2005; Kresic 2007). Others have attempted to determine a correlation between regional hydrogeological parameters with the inverse process, obtaining hydraulic conductivity values from specific yield data (e.g., Ahuja et al. 1989).

The method proposed in this paper is derived from the adaptation of the previous methodologies, followed by an integration and calibration with data collected through pumping tests carried out during several hydrogeological studies. This methodology aims to identify a statistical
correlation between hydraulic conductivity and specific yield, as a greater ease of water transmission and, consequently, circulating flow is in direct relation with the percentage of free connected voids.

The new algorithm, derived from a regression interpolation of field data, is the following (Fig. 3):

\[ S_y = -0.0014 (\ln k)^2 - 0.0003 \ln k + 0.2973 \]  

(8)

where \( S_y \) is the specific yield [-] and \( k \) is the hydraulic conductivity [m/s], derived from \( k_r \) (radial hydraulic conductivity).

The hydrogeological parameters represented in Fig. 3 derive from several pumping tests executed in unconfined aquifers and the graph shows a good reliability of the interpolation results in the range \( 10^{-3} - 10^{-2} \) m/s of the hydraulic conductivity.

A first validation of the algorithm was carried out with values of specific yield and hydraulic conductivity from the most standard hydrogeological textbooks and the scientific literature: starting from Neuman (1972), who proposed the method of interpretation of the pumping test in an unconfined aquifer, to Fetter (1980), Beretta (1992) and Kresic (2007).

The comparison between experimental and calculated values showed positive results. The proposed method can be improved with the acquisition of field data derived from pumping tests executed in different aquifer media.

**An example of practical application of the algorithm: the Milan case study**

One of the possible applications of the proposed method consists in evaluating the water volumes in the aquifers. The method is illustrated for the shallow, unconfined aquifer in the area of Milan and its surroundings (Fig. 4a, b).

The plain subsoil of the study area is characterized by Plio-Pleistocene sediments, whose upper units form the unconfined aquifer. Four main aquifer units can be identified and are denominated Aquifer Groups A, B, C and D, from the top down-ward (Fig. 4c; Regione Lombardia and ENI 2002). The upper Groups A and B are constituted by gravels and sands with few interbedded layers of silt and clay, whose thickness increases from north to south. The lower Groups C and D are constituted by sandy-clayey silt and silty-clay. Group A is separated from the underlying Group B by a clay-
silt layer (aquitard) of variable thickness and lateral continuity over the study area. The unconfined aquifer, corresponding to the Group A, is the one considered in this study. It is characterized by high transmissivity (higher than $10^{-2}$ m$^2$/s), hydraulic conductivity ranging from $10^{-4}$ to $10^{-3}$ m/s and an average thickness of about 50-60 m.

During the XX and XXI centuries, this area has been subjected to a great variation of the exploitation of groundwater resources, especially in the northern part of the city of Milan (Fig. 5). Until the end of the 1950s, groundwater levels were in a low exploitation condition, decreasing from north-west to south-east towards the base-level defined by the Po River (Fig. 6a). Subsequently to groundwater exploitation for industrial and drinking water supply in the 1960s, groundwater levels decreased dramatically forming a cone of depression in correspondence of the city of Milan (Fig. 6b). After 1975, moment of the minimum acme of groundwater levels, the industrial activities reduced the exploitation of groundwater and groundwater levels began to raise again thanks to natural recharge (Fig. 6c). Currently, groundwater levels in the city of Milan are maintained stable at a lower height with respect to the 1950s levels to preserve the underground infrastructures, such as buildings, tunnels and subway stations (Fig. 6d).

Maps of piezometric levels have been derived from the interpolation of measurements, through the kriging methodology (Fig. 6). Piezometric levels data were derived from multiple sources of information and were different in type, accuracy, and survey scale. Seventy years of measurements have been collected for the interpretation of groundwater levels variations in the study area.

Hydraulic conductivity of the saturated zone was determined from the available pumping tests executed on several wells homogeneously distributed in the study area. Field data were interpolated through kriging methodology to obtain the map of the distribution of hydraulic conductivity in the study area. The proposed algorithm (Equation 8) was applied for the evaluation of the specific yield and the generation of a $S_y$ map (Fig. 7). In the study area, specific yield ranges from 0.14 to 0.30. Higher values are located in the central sector, in correspondence of the southeastern area of Milan, and in the northwestern sector, while, lower values are mainly found in the northeastern sector.
Fig. 6 - Piezometric levels (m a.s.l.), every 5 m, in 1952 (a), 1975 (b), 1994 (c), 2014 (d). Coordinates refer to Monte Mario - Gauss Boaga 1940 projection.

Fig. 6 - Livelli piezometrici (m s.l.m.), ogni 5 m, nel 1952 (a), 1975 (b), 1994 (c), 2014 (d). Sistema di coordinate: Monte Mario – Gauss Boaga 1940.
The evaluation of the water volume variations in the period 1952-2014 has been computed by subtracting the groundwater levels for each time step (1952, 1975 and 1994) from the levels of the following time step (1975, 1994 and 2014). To obtain the water stored in the aquifer, considering the grain size distribution, the water volume variations have been multiplied for the specific yield obtained from the proposed algorithm (Fig. 8). It is possible to estimate a loss of 2215 Mm³ during the great exploitation for industrial purposes and population increase in the cities (1960-1975). Considering that the great exploitation occurred mainly in 15 years, from 1960 to 1975, as shown in Fig. 5, an average deficit of 138 Mm³/year in the water balance was recorded in this period.

Subsequently, a recover of about 1050 Mm³ occurred. However, it is clear that only a partial recovery of groundwater levels occurred. As, nowadays, a complete recovery is no more possible due to the managed system of water abstraction for the preservation of underground infrastructures interfering with groundwater (Colombo et al. 2017).

Conclusions

The determination of the specific yield is particularly important for both qualitative and quantitative studies of groundwater resources, such as the transport of contaminants and water balance evaluations. Specific yield cannot be reliably estimated from particle size and specific yield or from measurements of soil-water retention. Whereas laboratory or field tracer tests and pumping tests provide the most direct methods for obtaining specific yield, but often they are relatively expensive and time-consuming.

The proposed new algorithm allows determining the specific yield from hydraulic conductivity, with advantages in large-scale studies. In particular, this study demonstrates the importance of a valid estimation of the specific yield (i.e., effective porosity) for the evaluation of groundwater volumes in unconfined aquifers. In fact, these unconfined aquifers are characterized by short recharging times and a high vulnerability to contamination.

In addition, variations in groundwater levels and volumes reflect the socio-economic history of a territory. The application to the city of Milan and its surroundings emphasizes the necessity of a correct management of groundwater resources, identifying a reference piezometric level, or a reference storage volume, above which the resource needs to be removed and below which it needs to be restored. For the Milan area, such reference level needs to be maintained such that groundwater exploitation could be allowed, underground infrastructures are preserved and a land-use development at the ground and underground level is permitted.

An improvement of the proposed algorithm can be carried out adding hydraulic conductivity data from new pumping tests or calibrating it using Sₜ values derived from tracer tests, reducing the uncertainties of the interpolation. At present, the proposed algorithm provides a quick evaluation of the specific yield in practical applications, when technical and economic resources are limited in order to characterize the aquifer in terms of groundwater flow and solute transport.

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