Footholds admissible areas structure of a two-legged walking robot on an inclined cylinder

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Abstract. We consider the walking robot with \textit{n} legs and \textit{m} arms, where each leg and arm contacts the surface in a single foothold. Given the motion of the robot’s legs and arms with respect to its body, we solve the problem of finding the reaction forces, both analytically and numerically. The first step in solving similar problem belongs to N Y Zhukovsky. We describe the robot motion in terms of the general dynamics theorems, with six different equations of the robot’s dynamics from the momentum and angular momentum theorems. In the special case a robot with two legs, the existence of the solution is related to a set of straightforward inequalities. Using numerical simulations we develop the classification of footholds positions for different values of the friction coefficient. This problem equivalent to the problem of curved object grasping by the fingers of the robot-manipulator.

1. Introduction
This paper discussed the problem of robot dynamics on one-side constraint. While the general robot motion on a plane was detail analyzed in [1, 2]. Work [3] discussed model dynamics and control problems. The case of the dynamics on a curved surface is far more complicated. In the special case of a robot with eight legs whose up porting points are restricted to the inner surface of a tube, this problem was considered in [4]. Present work is devoted to the more general case of a robot with two arbitrary supporting points on a rough cylinder and on a curved surface. Equilibrium conditions for a solid on a rough plane was considered in [5]. Walking robot parameters optimization for the motion in tubes was discussed in [6].

2. The problem
Consider the walking robot with \textit{n} legs and \textit{m} arms. Each leg and arm contacts the surface in the one foothold. Suppose that robot legs and arms accomplish the desired motion with respect to the body of the robot. Using general dynamics theorems to describe the robot motion, we obtain six different equations for the robot dynamics from the momentum and angular momentum theorems. Among them there are three equations of the body translation with point \textit{A} and another three describe body rotation about point \textit{A}. For prescribed motion be realized then reaction in \textit{n} + \textit{m} supporting points should satisfy following equations [7]:
\[
\sum_i \vec{R}_i = -\vec{\Phi}, \quad \sum_i \vec{\dot{r}}_i \times \vec{R}_i = -\vec{M}, \quad i = 1, ..., n + m,
\]
where \(\vec{R}_i\) is the reaction component, \(\vec{\dot{r}}_i\) corresponds to the \(i\)-th supporting point vector, \(\vec{\Phi}\) is the sum of the external active forces plus time derivative of desired momentum, and \(\vec{M}\) is the sum of external
active forces momentum and time derivative of desired angular momentum with respect to the point \( O \) (which is defines as the origin fixed in absolute space). In two vector equations (1), the former corresponds to the momentum of the robot (and is equivalent to three scalar equations when projected onto the basis vectors), while the latter defines the desired change of the angular momentum.

Assuming that \( \Phi \) is orthogonal \( \bar{M} \), we obtain: [8] that the system \( \{ \Phi, \bar{M} \} \) can be also used at the point \( C \)

\[
\vec{r}_C \times \Phi = \bar{M}, \quad \vec{r}_C = -\frac{\overrightarrow{M} \times \Phi}{\Phi^2}, \quad \Phi = |\Phi|,
\]

where \( \vec{r}_C \) is the vector \( OC \), and \( C \) corresponds to point at which the resultant of the reactions is acting.

Further problem of reactions distribution \( \overrightarrow{R}_i \) in some fixed point of time is investigated by the proposal that force \( \Phi \) is acting at the point \( C \) and force moment there is zero. Motion equations (1) for finding reactions of walking robot body arms and legs prescribed motion can be transformed [11]:

\[
\sum_i \overrightarrow{R}_i = -\Phi, \quad \sum_i \overrightarrow{r}_i \times \overrightarrow{R}_i = -\overrightarrow{r}_C \times \Phi. \tag{2}
\]

Assuming that the robot footholds are on the surface of a rough cylinder of radius \( \rho \) with a friction coefficient \( k \), we introduce the coordinate system \( Oxyz \) such that the axis \( Ox \) is directed along the cylinder axis (so that the projection of \( \Phi \) on the axis \( Ox \) is negative - see Fig. 1.), the axis \( Oz \) is parallel to the vector \( \Phi \), and the angle between the cylinder axis and the vector \( \Phi \) is \( \alpha \).

![Figure 1. Cylinder.](image)

The problem of finding the reaction forces (2) is similar to the foothold reactions distribution problem in homogeneous gravity field, when the footholds are on the external surface of a rough inclined cylinder where the axis has an angle \( \alpha \) with respect to the vertical vector \( \Phi \). It has been considered in Ref [9], where the problem of searching the reactions components along the cylinder axis when \( \alpha = 0 \) and the work [10] for arbitrary \( \alpha \) was considered.

In the coordinates \( Oxyz \) we define \( \vec{r}_i = (\tilde{x}_i, \tilde{y}_i, \tilde{z}_i) \), \( \overrightarrow{R}_i = (\tilde{R}^x_i, \tilde{R}^y_i, \tilde{R}^z_i) \), \( \vec{r}_C = (\tilde{x}_C, \tilde{y}_C, \tilde{z}_C) \), and \( \Phi = (-\tilde{\phi}\sin\alpha, 0, -\tilde{\phi}\cos\alpha) \). In case of a one-sided surface, we have additional restrictions on normal reactions \( \overrightarrow{N}_i \) [11]:

\[
\overrightarrow{N}_i = \overrightarrow{R} \cdot \overrightarrow{e}^x_i \geq 0, \tag{3}
\]

where \( \overrightarrow{e}^x_i \) is an external normal to \( i \)-th supporting point on the cylinder, while the tangential components are given by \( \overrightarrow{F}_i = \overrightarrow{R}_i - \overrightarrow{N}_i \overrightarrow{e}^y_i \). For the reactions to be in the friction cones (2), we have following inequalities [12]:

\[
|\overrightarrow{F}_i| \leq k\overrightarrow{N}_i, \quad i = 1, n + m, \tag{4}
\]

i.e. the tangential reactions \( \overrightarrow{F}_i \) are restricted by Coulomb limiting friction value. When \( |\overrightarrow{F}_i| \) exceeds this limiting value, the robot legs and arms begin to slide along a surface.

The reaction distribution problem then reduces to the solution of equations (2), and inequalities (3), (4), for reactions limited to the friction cones. The restricted motion can only be realized if the solution of (2)-(4) does exist [13].
The problem of finding the reaction forces (2) is the problem with two-sided constraint [14]. And adding by inequalities (4). For example, let \( n \) is even. And first \( n/2 \) legs are on the cylinder surface, second \( n/2 \) legs are on the inner cylinder surface. The equivalent problem is the same grasp with even supporting points.

If the kinematics of the motion is given, the supporting points reactions are not immediately defined, and we can impose additional conditions or restrictions on the system. For example, we can limit the forces acting on the surface (due e.g. the requirement to avoid its damage), which implies a restoration on \( \vec{N} \).

For vector \( \vec{r}_i = \vec{r}_i/\rho = (x_i, y_i, z_i) \), in the cylinder coordinate: \( \vec{r}_i = (x_i, -\sin\phi_i, \cos\phi_i) \), \( \vec{e}_i^x = (0, -\sin\phi_i, \cos\phi_i) \), \( \vec{N}_i = \vec{N}_i/\Phi = (0, -N_i\sin\phi_i, N_i\cos\phi_i) \), where \( \phi_i \) is the angles between axis \( Oz \) and cylinder normal \( \vec{e}_i^x \). We define \( \vec{e}_x \) as the unitary vector in the \( Ox \) axis, while the vector \( \vec{e}_i^T = (0, \cos\phi_i, \sin\phi_i) \) as the tangential to the cylinder. Then the tangential reaction component: \( \vec{F}_i = \vec{F}_i/\Phi = (F_i^x, F_i^y, F_i^z) \), where \( F_i^x = F_i \cdot \vec{e}_x \), \( F_i^y = F_i \cdot \vec{e}_i^x \), the reaction vector \( \vec{R}_i = \vec{R}_i/\Phi = (R_i^x, R_i^y, R_i^z) \).

We further define the supporting points the coordinate differences along the cylinder, and angles difference: \( \Delta x = x_2 - x_1 \), \( \Delta \phi = \phi_2 - \phi_1 \). We then project system (2) onto the axes \( Oxyz \). For arbitrary surface we find that the second equation of (2) (corresponding to the angular moment) has the skew-symmetric matrix with respect to the component \( R_i^x \) [10]. These are two independent equations, while the third equation corresponds to the restriction of the point \( C \) to the plane containing the two supporting points. As a result, the system (2) yields five independent equation and a restriction:

\[
\begin{align*}
R_1^x &= -R_2^x + \sin\alpha, \quad R_1^y = -R_2^y, \quad R_1^z = -R_2^z + \cos\alpha, \\
(x_1 - x_2)R_1^x + (x_2 - x_1)(R_1^z - \cos\alpha) &= (z_2 - z_1)\sin\alpha, \\
(y_2 - y_1)R_1^z - (x_2 - x_1)R_1^y &= (y_2 - y_1)\sin\alpha, \\
(x_2 - x_1)(\sin\phi_2 + y_C) + (x_C - x_2)(\sin\phi_2 - \sin\phi_1) &= 0.
\end{align*}
\]

(5)

If \( p = R_2^x - R_1^x \), then from (5):

\[
\begin{align*}
R_1^x &= \frac{p(y_2 - y_1)}{2(x_2 - x_1)}, \quad R_2^x = \frac{(p - y_C - y_1)}{2(x_2 - x_1)}, \\
R_1^y &= \frac{2(y_2 - y_1)}{2(x_2 - x_1)}, \quad R_2^y = \frac{2(y_2 - y_1)}{2(x_2 - x_1)}, \\
R_1^z &= \frac{(x_2 - x_1)\cos\alpha + p(z_2 - z_1)}{2(x_2 - x_1)}, \quad R_2^z = \frac{2(z_2 - z_1) + z_2 - z_1}{2(x_2 - x_1)}.
\end{align*}
\]

(6)

\[\tan\alpha = \frac{z_C(\sin\phi_2 - \sin\phi_1) + \sin(\phi_1 - \phi_2) + y_C(\cos\phi_2 - \cos\phi_1)}{x.C(\sin\phi_2 - \sin\phi_1) + \sin(\phi_1 - \phi_2) + y_C(\cos\phi_2 - \cos\phi_1)}\]

2.1. Two-legged Robot

During the robot motion one-supporting and two-supporting points phases are changed. First, we consider the one-supporting phase. Let \( n = 1 \), then the motion existing condition is reaction is equal to force \( \vec{F} \) and supporting point the point \( C \) are on line along force \( \vec{F} \), while the angle between \( \vec{F} \) and the normal do not exceed the friction angle.

Let \( n = 2 \), and \( x_1 \neq x_2 \). Then \( p = F_2^x - F_1^x \), and from (6):

\[
F_1^x = \frac{(p + \sin\alpha)}{2}, \quad F_2^x = \frac{(\sin\alpha - p)}{2},
\]
\[
N_1 = \frac{(x_2 - x_C) \cos \varphi_1 - \sin^2 \varphi_2 \varphi_1 - \varphi_1}{\text{a}} \cos a + N_1^a,
\]
\[
N_2 = \frac{(x_C - x_1) \cos \varphi_2 - \sin^2 \varphi_2 \varphi_1 - \varphi_1}{\text{a}} \cos a + N_2^a,
\]
\[
F_{1yz} = \frac{2(x_2 - x_C) \sin \varphi_1 - \sin(\varphi_2 - \varphi_1)}{2(x_2 - x_1)} \cos a + F_{1(yz)}^a,
\]
\[
F_{2yz} = \frac{2(x_C - x_1) \sin \varphi_2 - \sin(\varphi_2 - \varphi_1)}{2(x_2 - x_1)} \cos a + F_{2(yz)}^a,
\]
\[
\tan a = \frac{(x_2 - x_1)(\sin \varphi_2 + y_C) + (x_C - x_2)(\sin \varphi_2 - \sin \varphi_1)}{z_C(\sin \varphi_2 - \sin \varphi_1) + \sin(\varphi_1 - \varphi_2) + y_C(\cos \varphi_2 - \cos \varphi_1)}
\]

where

\[
N_1^a = \frac{z_C \cos \varphi_1 - y_C \sin \varphi_1 - \cos^2 \varphi_2 \varphi_1 - \varphi_1}{\text{a}} \sin a,
\]
\[
N_2^a = \frac{y_C \sin \varphi_2 - z_C \cos \varphi_2 + \cos^2 \varphi_2 \varphi_1 - \varphi_1}{\text{a}} \sin a,
\]
\[
F_{1(yz)}^a = \frac{2(z_C \sin \varphi_1 + y_C \cos \varphi_1) + \sin(\varphi_2 - \varphi_1)}{2(x_2 - x_1)} \sin a,
\]
\[
F_{2(yz)}^a = \frac{\sin(\varphi_2 - \varphi_1) - 2(z_C \sin \varphi_2 + y_C \cos \varphi_2)}{2(x_2 - x_1)} \sin a.
\]

From the conditions (3)

\[
p \leq \frac{(x_2 - x_C) \cos \varphi_1 \cos a + (x_2 - x_1)N_1^a}{\sin^2 \varphi_2 \varphi_1 - \varphi_1^2},
\]
\[
p \leq \frac{(x_C - x_1) \cos \varphi_2 \cos a + (x_2 - x_1)N_2^a}{\sin^2 \varphi_2 \varphi_1 - \varphi_1^2}.
\]

**Figure 2.** The parameter diagram.

We further define \( s \varphi_{21} = \sin \varphi_2 - \sin \varphi_1, c \varphi_{21} = \cos \varphi_2 - \cos \varphi_1 \). The conditions (4) can be displayed in the form:

\[
E p^2 + B_1 p + C_1 \leq 0, \quad E p^2 + B_2 p + C_2 \leq 0,
\]

where
\[ E = \Delta x^2 + \sin^2 \Delta \varphi - 4k^2 \sin^4 \frac{\Delta \varphi}{2}, \]

\[ B_1 = 4 \sin \frac{\Delta \varphi}{2} \left( x_2 \cos \varphi_1 \left[ \sin \varphi_1 \cos \frac{\Delta \varphi}{2} - k^2 \cos \varphi_1 \sin \frac{\Delta \varphi}{2} \right] + B_1^\alpha \sin \Delta \varphi \right), \]

\[ B_2 = 4 \sin \frac{\Delta \varphi}{2} \left( x_1 \cos \varphi_2 \left[ \sin \varphi_2 \cos \frac{\Delta \varphi}{2} + k^2 \cos \varphi_2 \sin \frac{\Delta \varphi}{2} \right] + B_2^\alpha \sin \Delta \varphi \right), \]

\[ C_1 = x_2^2 \left[ \sin^2 \varphi_1 - k^2 \cos^2 \varphi_1 \right] \cos^2 \alpha + (x_2 - x_c)C_{11}^\alpha \sin^2 \alpha + C_{12}^\alpha \sin^2 \alpha, \]

\[ C_2 = x_1^2 \left[ \sin^2 \varphi_2 - k^2 \cos^2 \varphi_2 \right] \cos^2 \alpha + (x_c - x_1)C_{21}^\alpha \sin 2\alpha + C_{22}^\alpha \sin^2 \alpha, \]

\[ B_1^\alpha = \cos \frac{\Delta \varphi}{2} \left( y_c \cos \varphi_1 + z_c \sin \varphi_1 \right) - k^2 \sin \frac{\Delta \varphi}{2} \left( z_c \cos \varphi_1 - y_c \sin \varphi_1 - 1 \right) - 2 \Delta x^2, \]

\[ B_2^\alpha = \cos \frac{\Delta \varphi}{2} \left( \sin \varphi_2 - y_c \cos \varphi_2 - z_c \sin \varphi_2 \right) + k^2 \sin \frac{\Delta \varphi}{2} \left( y_c \sin \varphi_2 - z_c \cos \varphi_2 + \cos \Delta \varphi \right), \]

\[ C_{11}^\alpha = \left( y_c \cos \varphi_1 + z_c \sin \varphi_1 \right) \sin \varphi_1 - k^2 \left( z_c \cos \varphi_1 - y_c \sin \varphi_1 - 1 \right) \cos \varphi_1, \]

\[ C_{12}^\alpha = \left( y_c \cos \varphi_1 + z_c \sin \varphi_1 \right)^2 - k^2 \left( z_c \cos \varphi_1 - y_c \sin \varphi_1 - 1 \right)^2, \]

\[ C_{21}^\alpha = \left( \sin \varphi_2 - z_c \sin \varphi_2 \right) \sin \varphi_2 - k^2 \left( y_c \sin \varphi_2 - z_c \cos \varphi_2 + \cos \Delta \varphi \right) \cos \varphi_2, \]

\[ C_{22}^\alpha = \left( \sin \varphi_2 - z_c \sin \varphi_2 \right)^2 - k^2 \left( y_c \sin \varphi_2 - z_c \cos \varphi_2 + \cos \Delta \varphi \right)^2. \]

**Figure 3.** The case of \( \alpha = 0; x_2 = -x_1 = \rho = k = 1. \)

The boundaries between different regimes can be determined analytically. For example, in the case of significant friction when \( E < 0, \) the solution exists, and can be obtained analytically [10], as shown in Fig 2. Note that in this case it is limited to the range \( x_c - x_1 \leq 2k \rho. \) In contrast to this behavior, for \( E \geq 0 \) there is no such restriction and an additional step is required to address the question of the existence of the solution. At the point \( (0, 0) \) we find: \( E = 0, \) which means that two supporting points are orthogonal to the cylinder axis. Here, two possible solutions are either identical, or limited to a single diameter. In the latter case, point C and the reaction have to be in one plane, parallel to force \( \Phi, \) and the problem has a solution.

**Figure 4.** The case of \( \alpha = 0; x_2 = -x_1 = \varphi_2 = -\varphi_1. \)

For the desired legs and arms configurations and given point C, the problem can be solved numerically. In Fig 3, we present the numerical solution for the example when \( \alpha = 0, x_2 = -x_1 = \rho = = k = 1. \) Note that in this case \( E > 0. \)

In the numerical simulations, we use a \( 300 \times 300 \) array for the points \( (\varphi_1, \varphi_2), \) in the interval \([-\pi, \pi]\), and for each point verify the conditions (8), (9). Specifically, the condition (9) was analyzed.
in two cases, when $E = 0$ and $E > 0$, and when the solution of the problem does exist, the solutions were shown in the plot.

When $E = 0$, the reaction distribution problem reduces to the linear inequalities (8), (9) for the parameter $p$. To avoid zero division, signs of the coefficients in cycle were verified. For example if $B_1 = 0$, then necessary condition of the reaction distribution problem is $C_1 \leq 0$.

\[ \sin^2 \frac{\Delta \varphi}{2} \]

**Figure 5.** The case of $\alpha = \pi/4$; $x_2 = -x_1 = \varphi_2 = -\varphi_1$.

For $E > 0$, we need to consider two conditions. First is the restriction on the determinants $D_1 \geq 0$, while the second is the requirement of a non-empty intersection of the set of points of the intervals between the roots of quadratic equations. If the point $(\varphi_1, \varphi_2)$ satisfies the conditions, then it shown on plot that shows the existing field of the reaction distribution problem. From this plot we see that, if two points are on one diameter, then the solution of the reaction distribution problem exists. The two lines in the plot, correspond to $\varphi_1 = \varphi_2 + \pi$ or $\varphi_1 = \varphi_2 - \pi$. The rhombus form represents the requirement on the determinants $D_1 \geq 0$, while additional conditions further restrict the range. This is further illustrated in Fig 3, on the right, where the 3D-plot shows parameter range for the existence of the reactions distribution problem.

In Figs 4, 5 and 6, we present the results for $E > 0$ and $E \leq 0$, when $x_2 = -x_1$, $\varphi_2 = -\varphi_1$. Fig 4 shows the case of $\alpha = 0$, Fig 5 corresponds to $\alpha = \pi/4$, and Fig 6 describes the case when $\alpha$ increased to $\pi/2$. Note that when $\alpha = \pi/2$, the solution exists only for diametrical footholds.

\[ \sin^2 \frac{\Delta \varphi}{2} \]

**Figure 6.** For $\alpha$ increased to $\pi/2$; $x_2 = -x_1 = \varphi_2 = -\varphi_1$.

The boundaries between different regimes can be determined analytically. For two-legged robot when $E$ is negative, the solution exists, and obtained analytically. Using numerical simulations we explain the reaction distribution problem existing and build this problem solution existing fields for given supporting points and point $C$ positions. For example, for two supporting points phase, we consider symmetric, about point $C$, along and orthogonal cylinder axis, robot configurations. For first of these configurations examined three cases with nonnegative $E$ coefficient, for distance $x$, between point $C$ and footholds: 0,9; and 1,1 at $p$ and $k$ equal 1, $\alpha$ from 0 to $\pi$ (in all 13 different values cylinder inclination angles). Reactions distribution problem solution existing fields constructed on the two angles plane, corresponds to supporting points projections on the cylinder base and three dimensional fields which supplement this plane by point $C$ $z$-coordinate altitude. When $\alpha$ equals to 0, $x$ equals to 1, the field consist of three separate situated subregions. On the angle plane each of pair parallel lines
corresponds to support on the cylinder diameter plane section contained point C. There is connected field between these lines. It contains the line segment corresponding to the equality, robot supported above on the line which is parallel to cylinder axis and satisfy force direction deviation restriction. The indicated segment on the plot disappear when \( x \) equals to 0, 9 for \( \alpha \) equals \( \pi/4 \), and at increasing \( x \), later, for \( 4\pi/9 \). It corresponds to the robot beginning sliding down the cylinder. When \( x \) equals to 1, 1 for \( \alpha \) equals \( \pi/3 \) in three-dimensional fields observed bundles of separate points, Fig 7. That means that the point C altitude position more harsh change while changing the angles.

Figure 7. Admissible areas for \( \alpha = \pi/3; \Delta x = 1,1 \).

2.1.1. An objects grasping problem. Let consider the problem of curved object grasping by the fingers of the robot-manipulator. An object grasping problem is equivalent to the problem of the walking robot with \( n \) legs. Consider a grasp with \( n \) fingers. Each finger contacts an object in one point. Let \( C \) is the object center of mass. An object grasp kinetostatic described by general dynamics theorems. The momentum and angular momentum theorems give six differential kinetostatic equations. Among them there are three equations of the body translation with point \( A \) and another three describe body rotation about point \( A \). Let the point \( C \) is an origin fixed in absolute space. For a grasp of an object be realized then reaction in \( n \) contact points should satisfy following equations investigated by [1, 7, 8, 12 − 14]:

\[
\sum_{i} \vec{R}_i = -\vec{P}, \quad \sum_{i} \vec{r}_i \times \vec{R}_i = -\vec{r}_c \times \vec{P}, \quad i = 1, ..., n, \tag{10}
\]

where \( \vec{R}_i \) is reaction component, \( \vec{r}_i \) - \( i \)-th finger supporting point vector, \( \vec{P} \) is gravity force, \( \vec{r}_c \times \vec{P} \) is gravity force momentum about point \( O \), \( \vec{r}_c - OC \) is the center of mass vector. First vector equations express object momentum. It equivalents to three scalar equations in basic vectors projections. The second equation of (10) defines angular momentum.

We introduce right axes \( O\xi\eta\zeta \) so that an axis \( O\zeta \) be parallel \( \vec{P} \).

Suppose a grasp supporting points be on a rough cylinder as for walking robot. The force may be \( \vec{P} \). Or the up theory for arbitrary force \( \vec{F} \) and up built point \( C \) as for walking robot.

In case of the one-side surface, we have adding restrictions on normal reactions \( N_i \) as [7, 10, 11]. For reactions be in the friction cones (2) we have inequalities (3), (4). Inequalities (3) in case of the grasp inside the object, for example, the cylinder. And opposite sign in inequalities (3) if the grasp is out the object. That is the tangential reactions \( \vec{F}_t \) are restricting by Coulomb limiting friction value. When \( |\vec{F}_t| \) became more then limiting value, sliding of the cylinder begun along the grasp.

The reaction distribution problem formulated so. We want to find the reaction corresponding to equations (10), inequalities (3) or opposite sign, and (4), which be in the friction cones.

For two-sided constraint does not matter in which direction. For one-sided constraint the reaction must be on the side permitted for living.
The equations (10) be added by square inequalities, for reactions be in the friction cones. And the properties of the coefficient $E$ are the same as for walking robot in case of the grasp inside the cylinder or an another surface object.

If the solution of (10), and pointed inequalities exists than restricted motion can be realized. If not exists, than can not be realized. This problem equivalent to the problem of a walking robot on the rough cylinder.

For the cylinder the equations (10) are:

$$
\sum_{i}(N_i + F_i^{x} + F_i^{yz}) = -P, \quad \sum_{i} r_i \times (N_i + F_i^{x} + F_i^{yz}) = -r_c \times P, \quad i = 1, ..., n
$$

(11)

where $F_i^{x} = F_i \cdot e_x$, $F_i^{yz} = F_i \cdot e_z$.

Then projection on Oxyz the first and the second vector equations (11):

$$
\sum_{i} F_i^{x} = \sin\alpha,
\sum_{i} (N_i \sin\varphi_i - F_i^{yz} \cos\varphi_i) = 0,
\sum_{i} (N_i \cos\varphi_i + F_i^{yz} \sin\varphi_i) = \cos\alpha,
$$

$$
\sum_{i} F_i^{yz} = -y_c \cos\alpha,
\sum_{i} (x_i N_i \cos\varphi_i + x_i F_i^{yz} \sin\varphi_i - F_i^{x} \cos\varphi_i) = x_c \cos\alpha - z_c \sin\alpha,
\sum_{i} (x_i N_i \sin\varphi_i - x_i F_i^{yz} \cos\varphi_i - F_i^{x} \sin\varphi_i) = y_c \sin\alpha.
$$

Robot can hold the surface object by one finger. If the finger inside the cylinder, the center mass of an object is under the finger. If the finger out the cylinder, the center mass of an object is up the finger. And the angle between $\Theta$ and the normal not exceed friction angle.

3. Conclusion

During the robot motion, one-foothold and two-foothold phases are changed. Reaction distribution problem have a solution in following cases.

1. One foothold phases. So the motion existing condition is reaction is equal to force $\Phi$ and foothold and the point $C$ are on the line along $\Phi$. And the angle between $\Phi$ and the normal not exceed friction angle.

2. Two-foothold phases. The point $C$ and the reactions have to be in the plane parallel to force $\Phi$.

2.1. If footholds are on one diameter.

2.2. When the coefficient

$$
E = \Delta x^2 + \sin^2 \Delta \varphi - \frac{4k^2 \sin^4 \Delta \varphi}{2} < 0.
$$

And in some fields with connected set of points, when $E \geq 0$. So robot can move along the cylinder changing one and two-footholds phases.

3. And for example monkey-robot with 20 fingers or two legged human-robot with 10 arms fingers can hold the object by one and grasp by two-fingers.

Robot can hold the surface object by one finger. If the finger inside the cylinder, the center mass of an object is under the finger. If the finger out the cylinder, the center mass of an object is up the finger. And the angle between $P$ and the normal not exceed friction angle.
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