Features of mathematical formulation and solution of the problem of optimal division of funds in the construction business

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Abstract: The paper considers a microeconomic model of the construction business economy, assuming that it is a stable object in a crisis. The paper presents the distinctive features of the microeconomic formulation of the problem of finding a method for optimal separation of construction materials into consumed and accumulated parts. This area is constantly paid attention to and described in the media, sometimes with much unexpected outcome for investors. It is quite natural that consumption is optimized, just as in macroeconomics, but not in its pure form. The integral discounted utility of consumption is maximized. The communication equation in this paper differs from the similar macroeconomic equation, as the construction business exists and survives even in crisis conditions not in the same way as it does in social environments.

1. Introduction

The construction business is one of the key factors of sustainable economic development. In the conditions of economic crises, construction business demonstrates stable survival rate, which can be envied by numerous enterprises. The efficiency of the construction industry is determined primarily by the prompt solution of all negative processes that arise. The specifics of the construction sector of the Russian Federation are that there are certain difficulties in obtaining government contracts when conducting electronic auctions. The public procurement website is a priority when carrying out orders in the construction sector, as it allows saving budget funds significantly, ensures the competitiveness of contractors, and minimizes the bureaucratic official component. Auctions are held automatically, which reduces the risks of unfair preliminary collusion between construction companies. However, there is a very significant disadvantage of this procedure. Preference for a construction company is always given only based on the criterion - the price of the contract. This circumstance contributes to numerous abuses. Low-skilled workers, often without experience in the performance of complex construction works, are recruited for the execution. Of course, the customer needs to reduce operating costs to ensure the profitability of the business. Unaffordable requirements are often set for the performer to exclude the majority of tenderers. Numerous one-day fake companies are created. When performing construction work, outdated equipment is applied and manual labor is widely used. Many construction companies are forced to pay salaries using gray schemes to hide their real income.
Therefore, when choosing a contractor, it is necessary to be guided not only by the cost of the contract, but also by competencies and economic quality priorities. It should be borne in mind that the full set of financial statements of a construction company includes: balance sheet; statement of financial results; statement of changes in equity; statement of cash flows; accounting policy, and is included in the main balance sheet equation: Assets = Liabilities + Equity + Income-Expenses + Investments-Withdrawals. In this form, balance sheet equation clearly demonstrates not only the process of increasing capital as a result of the construction company’s own activities, but also the opportunities for changing it, brought from outside. It provides a general view of the funds, operations and results of the construction organization and their reflection in the financial statements.

2. Materials and methods

Similarly to the problem of optimal division of material resources into consumed and accumulated parts [1], it is possible to maximize consumption, but not in its pure form. As in works [2-4], we will maximize the integral discounted utility of consumption. However, in the considered mathematical model of the construction business economy, the ordinary differential balance equation, which plays the role of the connection equation in the optimal control problem, has expressed microeconomic specificity. The economic characteristics encountered in the course of research, both known and unknown, will be considered continuous functions of time. The variation method is used in this work.

3. Results and discussion

1. Connection equation of the problem of optimal division of material resources into consumed and accumulated parts in the construction business. In contrast to [2-4], we will proceed from the basic balance equation [1]

\[ Y = I + C, \]  

where \( Y \) – income, \( I \) – investment and \( C \) – consumption. Let us suppose that now

\[ Y = P + \rho x, I = x'_i, \]  

where \( P \) – proceeds from investors, \( \rho \) – bank rate, \( x \) – accumulated capital of the construction business, and \( t \) – time. Thus, the banking system directs the accumulated capital to the construction business. Substituting (2) in (1) we get

\[ P + \rho x = x'_i + C, \]

or

\[ x'_i = P + \rho x - C. \]  

Equation (3) can also be understood in a different balance-sheet sense than (1), namely: the rate of increment of accumulated capital is equal to the instantaneous difference between income and expenditure [1, 3]. It is (3) that acts as the connection equation in the optimal control problem.
For equation (3), it is natural to set the Cauchy problem for the initial moment of time $t_0$:

$$x(t_0) = x_0 > 0.$$  \hspace{1cm} (4)

2. Maximization of the penalty function. We take the integral discounted utility of consumption as a penalty function

$$J(C) = \int_{t_0}^{t_1} u(C(t)) \exp(-\delta t) dt.$$  \hspace{1cm} (5)

Here $t_1$ – finite time point, $u$ – utility function $\delta$ – discount factor, and (5) – the functional that we need to maximize. Following [4], we consider that the construction business assesses the utility of consumption by function $u(C)$ that describes a constant risk aversion according to Arrow-Pratt:

$$a = \frac{u''(C)C}{u'(C)} \geq 0.$$  \hspace{1cm} (6)

Following [11-13] from (6) we get

$$u'(C) = \frac{\gamma}{C^a} = \gamma C^{-a},$$  \hspace{1cm} (7)

$$u(C) = \begin{cases} \frac{\gamma C^{1-a}}{1-a} + \chi, & a \neq 1; \\ \gamma \ln C + \chi, & a = 1; \end{cases} \gamma = \text{const} > 0, \chi = \text{const}.$$  \hspace{1cm} (8)

Further, the integration constants in (8) will be considered as given ones.

Considering the increment of the functional (5): $J(C(t) + h(t)) - J(C(t))$, adding a condition at the right end

$$x(t_1) = x_1 > 0,$$  \hspace{1cm} (9)

and following [3], we obtain the existence of the maximum of the functional (5), and the validity of the Euler equation

$$u'(C(t)) \rho(t) e^{-\delta t} + \frac{d}{dt}\left[u'(C(t)) e^{-\delta t}\right] = 0.$$  \hspace{1cm} (10)

Replacement
\[ w = u'(C(t))e^{-\delta t} \quad (11) \]
simplifies Euler equation (10)

\[ \frac{dw}{dt} + \rho(t)w = 0. \]

The latter is simply integrated:

\[ w = De^{-\int_{t_0}^{t} \rho(s)ds}, \quad D = \text{const}. \quad (12) \]

From (11) and (12), we have

\[ u'(C(t))e^{-\delta t} = De^{-\int_{t_0}^{t} \rho(s)ds}. \]

Expressing the derivative of the utility function from the last equation and comparing it with equation (7), we get

\[ u'(C(t)) = \frac{\delta r - \int_{t_0}^{t} \rho(s)ds}{\gamma [C(t)]^\gamma}. \quad (13) \]

Expressing the consumption from (13) we will have

\[ [C(t)]^\gamma = \frac{\gamma}{D} e^{\delta r - \int_{t_0}^{t} \rho(s)ds}, \quad C(t) = \left( \frac{\gamma}{D} \right) e^{\frac{1}{\gamma} \left( \frac{\delta r - \int_{t_0}^{t} \rho(s)ds}{\gamma} \right)}. \quad (14) \]

From the left formula (14), you can get the right one if the Arrow–Pratt risk aversion (6) is strictly positive.

For the final solution of the problem it remains to find a constant for integration \( D \), which first occurs in equation (12).

3. Use of the condition at the right end. Equation (3) is linear and following [1] we can write

\[ x(t) = \int_{t_0}^{t} e^{\int_{t_0}^{\tau} \rho(s)ds} \left[ P(\tau) - C(\tau) \right] d\tau + x_0 e^{\delta t_0}. \quad (15) \]

Formula (15) gives a solution to the Cauchy problem (3), (4) and is checked directly. The fact that (15) satisfies (4) is obvious, and that (15) satisfies equation (3) follows from the Leibniz formula [3]. Substituting in (15) \( t = t_1 \) and using (9) it can be written

\[ x_1 = \int_{t_0}^{t_1} e^{\int_{t_0}^{\tau} \rho(s)ds} \left[ P(\tau) - C(\tau) \right] d\tau + x_0 e^{\delta t_0}. \quad (16) \]

Now let us transform (16) as follows
Now let us substitute the expression we have obtained for consumption (14) in (17)

\[ \left( \gamma \frac{1}{D} \right) \int_{t_0}^{t_1} e^\tau \left( \int_{t_0}^{\tau} \rho(s) ds \right) e^{-\frac{\tau}{a}} d\tau = \int_{t_0}^{t_1} e^\tau P(\tau) d\tau + x_0 e^{b_0} - x_1. \]

Raising the latter to \( a > 0 \) degree we will have

\[ \gamma \left( \int_{t_0}^{t_1} e^\tau \left( \int_{t_0}^{\tau} \rho(s) ds \right) e^{-\frac{\tau}{a}} d\tau \right)^a = \left( \int_{t_0}^{t_1} e^\tau P(\tau) d\tau + x_0 e^{b_0} - x_1 \right)^a \]

So, from the last equation, we can express the desired constant

\[ D = \gamma \left( \int_{t_0}^{t_1} e^\tau \left( \int_{t_0}^{\tau} \rho(s) ds \right) e^{-\frac{\tau}{a}} d\tau \right)^a \]

of course, if the denominator of the fraction (18) is different from zero.

4. Summary

As in the conditions of economic chaos and instability, the construction business demonstrates a stable survival rate, which can be envied by many enterprises; the study of various aspects of the economy of the construction business is very relevant. In the very important problem of optimal division of material resources into consumed and accumulated parts, both in microeconomics and in macroeconomics, there must be and are similarities and differences. As in macroeconomics [1] it is necessary to maximize consumption, but not in its pure form. As in [5-9], we maximized the integral discounted utility of consumption. In the considered mathematical model of the construction business economy, the ordinary differential balance equation, which plays the role of a communication equation in the optimal control problem, has expressed microeconomic specificity due to the specific nature of the business. However, this work is aimed at identifying and studying general economic trends, both in micro and macro cases. As in work [1], the interesting variation method was used here. Thus, it is proposed to integrate the successes and advantages of various methods of modeling the economic processes of construction business development for monitoring, control and formation of adequate management decisions in advance.

5. References

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