Majorana Neutrinos Production at NLC in an Effective Approach

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Abstract

We investigate the possibility of detecting Majorana neutrinos at the $e^+e^-$ Next Linear Collider (NLC). We study the $l_j^\pm l_k^\mp + \text{jets}$ ($l_j \equiv e, \mu, \tau$) final states which are, due to leptonic number violation, a clear signature for intermediate Majorana neutrino contributions. Such signals (final leptons of the same-sign) are not possible if the heavy neutrinos have Dirac nature. The interactions between Majorana neutrinos and the Standard Model (SM) particles are obtained from an effective Lagrangian approach. As for the background, we considered the SM reaction $e^+e^- \rightarrow W^+W^+W^-W^-$, with two $W$’s decaying into jets and two $W$’s decaying into $l^\pm + \nu(\bar{\nu})$, producing extra light neutrinos which avoid the detection. We present our results for the total cross-section as a function of the neutrino mass and the center of mass energies. We also show the discovery region as a function of the Majorana neutrino mass and the effective coupling.

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I. INTRODUCTION

The standard model of particle physics (SM) only contains left-handed neutrinos, which makes it not possible to generate mass for them. One very important discovery in the field is the neutrino oscillations, which requires the neutrinos to possess a small mass \( m_\nu \gtrsim 0.01 \text{ eV} \). Thus we need to go beyond the SM to solve this issue. One way to do it, is by the seesaw mechanism, which requires one or more right-handed neutrinos, generically \( \nu_R \), with a mass term

\[
\mathcal{L}^\text{mass} = \frac{1}{2} \bar{\nu}_R^c M \nu_R - \bar{L} \tilde{\phi} Y \nu_R + h.c., \tag{1}
\]

where \( L \) denotes the left-handed lepton doublet, \( Y \) the Yukawa coupling matrix, \( \phi \) the doublet higgs boson and \( M \) the Majorana mass matrix. There are many extensions of the SM (Left-right symmetric model, SO(10), E6, ...) with extra right-handed neutrinos which are singlets of the SM gauge group and for which the Majorana mass terms are naturally allowed \[1\].

Upon diagonalization of the mass term we obtain, besides the light neutrinos, heavy Majorana neutrinos (N), which allow for lepton number violation.

By solving the eigenvalue problem, we obtain the masses

\[
m_\nu = m_D M^{-1} m_D^T, \quad \text{with} \quad m_D = Y \frac{v}{\sqrt{2}}, \tag{2}
\]

and the mixing angle \( U_{lN} \sim m_D/M \). In typical seesaw scenarios, the Dirac mass term \( m_D \) are expected to be around the electroweak scale (then \( Y \sim O(1) \) in Eq.(2)), whilst the Majorana mass \( M \) being a singlet under the SM gauge group may be very large, close to the Grand Unification Scale. Thus, the seesaw mechanism can explain the smallness of the observed light neutrino masses \( m_\nu \sim 0.01 \text{ eV} \) and clearly leads to the decoupling of \( N \).

Even a different choice in which \( M \sim 100 \text{ GeV} \) and \( m_D \sim 0.1 \text{ me} \), keeping \( m_\nu \sim 0.01 \text{ eV} \), implies a vanishing mixing \( U_{lN} \sim 10^{-7} \[2\].

This mixing weighs the coupling of \( N \) with the standard model particles and in particular with the charged leptons through the \( V-A \) interaction:

\[
\mathcal{L}_W = -\frac{g}{\sqrt{2}} U_{lN} \bar{\tilde{N}} \gamma^\mu P_L W^+_{\mu} + h.c \tag{3}
\]

This effect is so weak that the observation of Lepton Number Violation (LNV) must indicate new physics beyond the minimal seesaw mechanism, as was indicated in Ref.\[2\]. In view of
the above discussion we consider, in a model independent way, the effective interactions of the Majorana neutrino $N$, of mass lower than 1 TeV and negligible mixing to $\nu_L$.

In the case that heavy neutrinos ($N$) exist, the present and future experiments will be capable of determining their nature. The production of Majorana neutrinos via $e^+e^-$, $e^-\gamma$, $\gamma\gamma$ and hadronic collision have been extensively investigated in the past [2–12].

In this work we study the possibility for the $e^+e^-$ next linear collider (NLC) to produce clear signatures of Majorana neutrinos in the context of interactions coming from an effective lagrangian approach. We study the reaction $e^+e^- \rightarrow l_j^\pm l_k^\mp + \text{jets}$ ($l_j \equiv e, \mu, \tau$), which is divided into two subprocesses depicted in Fig.1 and Fig.2. In the first case we produce two Majorana neutrinos ($N$) which will decay into one charged lepton and jets ($N \rightarrow l + \text{jets}$). In the second case, which is a three body reaction, we consider single neutrino production decaying in the same way as before, and a $W$ decaying into two jets ($W \rightarrow \text{jets}$). We have not considered the pure lepton channels since they involve light neutrinos which escape detection, in which case the Majorana nature of the heavy neutrinos would have no effect on the signal (we should be able to know whether the final state contains neutrinos or antineutrinos). For the decay of the Majorana neutrinos we have calculated the branching ratios of the most important channels. It is possible to parameterize the effects of new physics beyond the standard model by a series of effective operators $\mathcal{O}$ constructed with the standard model and the Majorana neutrino fields and respecting the Standard Model $SU(2)_L \otimes U(1)_Y$ gauge symmetry [13]. These effective operators represent the low-energy limit of an unknown theory. Their effects are suppressed by inverse powers of the new physics scale $\Lambda$ for which we take the value $\Lambda = 1$ TeV. Here we consider the effect of dimension 6

FIG. 1: Diagram contributing to the production of two Majorana neutrinos.
operators which are the dominant.

The total lagrangian is organized as follows:

\[ \mathcal{L} = \mathcal{L}_{SM} + \sum_{J,i} \frac{\alpha_{ij}}{\Lambda^2} \mathcal{O}_i^j \]  

(4)

where \( J \) and \( i \) labels the operators and families respectively. For the considered operators we follow Ref [2] starting with a rather general effective lagrangian density for the interaction of a Majorana neutrino \( N \) with leptons and quarks. All the operators which we list here are of the dimension 6 and could be generated at tree level in the unknown fundamental high energy theory:

\[ \mathcal{O}_{LN\phi}^i = (\phi^\dagger \phi)(\bar{L}_i N \tilde{\phi}), \quad \mathcal{O}_{NN\phi}^i = i(\phi^\dagger D_{\mu} \phi)(\bar{N} \gamma^\mu N), \quad \mathcal{O}_{Ne\phi}^i = i(\phi^T \epsilon D_{\mu} \phi)(\bar{N} \gamma^\mu e_i) \]  

(5)

and for the baryon-number conserving 4-fermion contact terms we have:

\[ \mathcal{O}_{duNe}^i = (\bar{d}_i \gamma^\mu u_i)(\bar{N} \gamma^\mu e_i), \quad \mathcal{O}_{fNN}^i = (\bar{f}_i \gamma^\mu f_i)(\bar{N} \gamma^\mu N), \]  

(6)

\[ \mathcal{O}_{LNLe}^i = (\bar{L}_i N)(\bar{L}_i e_i), \quad \mathcal{O}_{LNDd}^i = (\bar{L}_i N)(\bar{Q}_i d_i), \]  

(7)

\[ \mathcal{O}_{QuNL}^i = (\bar{Q}_i u_i)(\bar{N} L_i), \quad \mathcal{O}_{QNLD}^i = (\bar{Q}_i N)(\bar{L}_i d_i), \]  

(8)

\[ \mathcal{O}_{LN}^i = |\bar{N} L_i|^2 \]  

(9)

where \( e_i, u_i, d_i \) and \( L_i, Q_i \) denote the right handed \( SU(2) \) singlet and the left-handed \( SU(2) \) doublets, respectively.

The operators listed above contribute to the effective lagrangian

\[ e^-, N \rightarrow W, \bar{\nu}_e, e^+ \]  

(1)

\[ e^-, e^+, N \rightarrow W, \bar{\nu}_e, e^+ \]  

(2)

FIG. 2: Diagram contributing to the production of a single Majorana neutrino.
\[ L_{\text{eff}} = \frac{1}{N^2} \left( \frac{v^2}{2} \alpha_\phi \bar{\nu}_{L,i} N_R \Phi - \frac{M^2_w v}{2} \alpha_z Z^\mu N_R \gamma^\mu N_R - \frac{M^2_w v}{\sqrt{2}} \alpha_W W^\dagger \mu \bar{N}_R \gamma_\mu e_{R,i} + \right. \\
\left. \alpha_{v_1} d_{R,i} \gamma^\mu u_{R,i} \bar{N}_R \gamma_\mu e_{R,i} + \alpha_{v_2} \bar{e}_{R,i} \gamma^\mu e_{R,i} \bar{N}_R \gamma_\mu N_R + \alpha_{v_3} \bar{N}_R \gamma_\mu N_R + \alpha_{v_4} \bar{d}_{R,i} \gamma^\mu d_{R,i} \bar{N}_R \gamma_\mu N_R + \alpha_{v_5} Q_{\mu} l_{\gamma} \gamma^\mu Q_{\mu} \bar{N}_R \gamma_\mu N_R + \right. \\
\left. \alpha_{s_0} (\bar{\nu}_{L,i} N_R \bar{e}_{L,i} e_{R,i} - \bar{e}_{L,i} N_R \bar{\nu}_{L,i} e_{R,i}) + \alpha_{s_1} (\bar{u}_{L,i} u_{R,i} \bar{N}_R v_{L,i} + \bar{d}_{L,i} u_{R,i} \bar{N}_{L,i}) + \right. \\
\left. \alpha_{s_2} (\bar{\nu}_{L,i} N_R \bar{d}_{L,i} d_{R,i} - \bar{d}_{L,i} N_R \bar{\nu}_{L,i} d_{R,i}) + \alpha_{s_3} (\bar{u}_{L,i} N_R \bar{e}_{L,i} e_{R,i} - \bar{e}_{L,i} N_R \bar{u}_{L,i} e_{R,i}) + \right. \\
\left. \alpha_{s_4} (N_R \nu_{L,i} \bar{\nu}_{L,i} N_R + \bar{N}_R e_{L,i} e_{L,i} N_R) + h.c. \right) \\
\right) \\
\] (10)

where the sum over \( i \) is understood and the constants \( \alpha_i \) are associated to specific operators

\[
\begin{align*}
\alpha_Z &= \alpha_{NN\Phi}, \quad \alpha_\phi = \alpha_{LN\Phi}, \quad \alpha_W = \alpha_{Ne\Phi}, \quad \alpha_{v_1} = \alpha_{d_{eN}e}, \quad \alpha_{v_2} = \alpha_{e_{NeN}}, \\
\alpha_{v_3} &= \alpha_{u_{NN}}, \quad \alpha_{v_4} = \alpha_{d_{NN}}, \quad \alpha_{v_5} = \alpha_{QNN}, \quad \alpha_{s_0} = \alpha_{LNe}, \\
\alpha_{s_1} &= \alpha_{Qu_{NL}}, \quad \alpha_{s_2} = \alpha_{LNQd}, \quad \alpha_{s_3} = \alpha_{QNld}, \quad \alpha_{s_4} = \alpha_{LN}. 
\end{align*}
\] (11)

We calculate the cross-section for the production of the Majorana neutrino according to the process shown in Fig. 1, valid for the kinematic range \( m_N < \sqrt{5}/2 \)

\[ \sigma^{NN} = \frac{s \beta}{64 \pi \Lambda^2} (C^- + C^+) (1 + \beta^2/3) \] (12)

where

\[ C^- = \left( \frac{M^2_Z \alpha_Z}{s - M^2_Z} C_L + \alpha_{v_2} + \alpha_{s_4}/2 \right)^2, \quad C^+ = \left( \frac{M^2_Z \alpha_Z}{s - M^2_Z} C_R + \alpha_{v_4} \right)^2 \]

\[ \beta = \sqrt{1 - \frac{4m^2_N}{s}} \] (13)

and \( C_L = -1/2 + x_W, \ C_R = x_W, \) with \( x_W = \sin^2 \theta_W \). We also study the single Majorana neutrino production. The corresponding diagrams are shown in Fig. 2 and the result for the square amplitude is

\[ |\mathcal{M}|^2 = \frac{g^2}{8 \Lambda^2 (q^2)^2} \left\{ 8 \alpha_{v_2} \frac{\Pi^2_w}{M^2_w} s \left[ 2(k \cdot p)(2k \cdot q)(l \cdot q) - (k \cdot l)q^2 \right] + M^2_w (2l \cdot q)(p \cdot q) - (l \cdot p)q^2 \right\} \]

\[ + 4 \left( \alpha_{s_0} \right)^2 \frac{(l \cdot p_2)}{M^2_w} \left[ 2(k \cdot p)(2k \cdot q)(p_1 \cdot q) - (k \cdot p_1)q^2 \right] \]

\[ + M^2_w (2p \cdot q)(p_1 \cdot q) - (p \cdot p_1)q^2 \} \] (14)

where \( q = p + k \) and \( p_1, p_2, l, p \) and \( k \) are the 4-momenta of the electron, the positron, the Majorana neutrino \( N \), the charged lepton and the \( W \) boson respectively. The \( W \) propagator is \( \Pi_w = m^2_w / [(p_1 - l)^2 - m^2_w] \).
FIG. 3: In the left panel the Branching ratios for the Majorana neutrino decay with $\alpha_{0\nu\beta\beta} = 0$. In the right panel there is a comparison with the same-coupling case for the $N \rightarrow l^- + jets$ decay.

FIG. 4: The cross-section for the process $e^+e^- \rightarrow l^\pm l^\pm + jets$. The left panel correspond to the zero contribution of the operators related with $0\nu\beta\beta$ ($\alpha_{0\nu\beta\beta} = 0$). In the right panel the curves labeled (a), (b), (c) and (d) correspond to energies of center of mass $\sqrt{s} = 0.5TeV$, $\sqrt{s} = 0.8TeV$ and different values of the constant $\alpha_i^{(j)}$. In both panels the horizontal solid line represent the values of the SM background.

The cross-section is obtained by integrating the phase space in the usual way, using the numerical routine RAMBO [14].

The total cross section is the combination of the two processes mentioned above in the
approximated expression:

\[
\sigma^{(e^+e^->l^++l^+jets)} = 2 \left( \sum_{i,j} \sigma^{(e^+e^->NN)} Br(N \to l_i^+ + jets) Br(N \to l_j^+ + jets) \Theta(\sqrt{s}/2 - m_N) + \sum_i \sigma^{(e^+e^->Ne^+W)} Br(N \to l_i^+ jets) Br(W \to jets) \Theta(\sqrt{s} - m_N) \right) \tag{15}
\]

The factor two in front of Eq. (15) takes into account the possible charges of the final leptons.

As we shall see later, some of the considered operators contribute to the neutrinoless double beta decay \((0\nu\beta\beta\text{-decay})\) and may be strongly constrained. In these conditions we analyze the case where the operators contributing to the \(0\nu\beta\beta\text{-decay}\) vanishes whilst the rest are non-zero and contribute with similar strength. For completion we also analyze the case where all of the operators contribute with similar strength. The branching ratios shown in the left panel of Fig. (3) (the expressions are collected in the Appendix) correspond to the case with non contribution of the \(0\nu\beta\beta\text{-decay}\) related operators \((\alpha_{0\nu\beta\beta} = 0)\). In the right panel of the same figure we show for comparison the branching ratio for \(N \to l^+ + jets\) with \(\alpha_{0\nu\beta\beta} = 0\) and \(\alpha_{0\nu\beta\beta} \neq 0\). As we can see there are not significant differences.

In Fig. (4) we show the results for the cross section combining the processes shown in Figs. (1) and (2) with the \(W\)-boson decaying into hadrons and the Majorana neutrino \(N\) decaying according to the Branching Ratios shown in the Appendix. We show the result as a function of the Majorana neutrino mass \(m_N\) and center of mass energies of \(\sqrt{s} = 0.5 \text{ TeV}\) and \(\sqrt{s} = 0.8 \text{ TeV}\). We have considered \(\sqrt{s} < \Lambda\) in order to ensure the validity of the effective lagrangian approach. The cross section is calculated for different values of the constants \(\alpha^{(i)}_J\). In the left panel of the Fig. (4) we have shown the cross section for the case in which the constants related with the operators contributing to \(0\nu\beta\beta\text{-decay}\) are considered zero. These operators are \(O^{1}_{Ne\phi}, O^{1}_{duNe}, O^{1}_{QuNL}, O^{1}_{LNLd}\) and \(O^{1}_{QNLd}\), as will be discussed in section III. In the right panel we plot the cross section where the operators that contribute are the 4-fermion operators (solid line) or the operators involving bosons (dashed line). In both panels, the non-zero coupling constants take the value one. As we can see, the 4-fermion contribution is the most important. In both panels we show with a horizontal solid line the value of the SM background as we will be explained later in the text.

The final leptons can be either of \(e^\pm, \mu^\pm\) or \(\tau^\pm\) since this is allowed by the interaction lagrangian (Eq. (10)). All of these possible final states are clear signals for intermediary Majorana neutrinos, thus we sum the cross section over the flavors of the final leptons. The
partial width of $N$ was determined at tree level considering the dominant decay modes $N \rightarrow l + 2 \text{jets}$, $N \rightarrow l + t\bar{b}$, $N \rightarrow l + \text{leptons}$, $N \rightarrow \nu + H$ and $N \rightarrow \nu + \text{quarks}$ coming from the higgs, the charged $W$-boson and the 4-fermion effective interactions. We present in the Appendix the differential partial-width for its dominant decay channels, where the contributing effective operators are identified by the indicated labels in the couplings.

II. THE STANDARD MODEL BACKGROUND AND THE DISCOVERY REGION

The considered signal is strictly forbidden in the Standard Model. The SM background, which can be confused with the studied reaction, will always involve additional light neutrinos. The dominant SM process arises from the resonant production of four $W^{\pm}$ bosons: $e^{+}e^{-} \rightarrow W^{+}W^{+}W^{-}W^{-}$, the decay of two $W's$ into leptons $W^{\pm} \rightarrow l^{\pm} + \nu(\bar{\nu})$, and the other two into jets, $W \rightarrow \text{jets}$. We calculated the cross section for these processes using the package COMPHEP [19–21] and we multiplied it by the corresponding branching ratios $(\text{BR}[W \rightarrow l\nu])^2 \simeq 0.011$ and the $(\text{Br}[W \rightarrow 2\text{jets}])^2 \simeq 0.46$ and by the factor 18 to take into account the different combinations of the same sign charged final leptons: $l^{\pm}l^{\pm}$ with $l = e, \mu, \tau$. The calculated values are $5.0 \times 10^{-5} \text{ pb}$ and $1.1 \times 10^{-5} \text{ pb}$ for $\sqrt{s} = 0.8 \text{ TeV}$ and 0.5 TeV respectively. In Table I we compare the values of the signal, for different values of the Majorana neutrino mass, with the SM background. In Fig.4 we show along, with the signal cross-section, the corresponding background levels as horizontal lines for $\sqrt{s} = 0.5\text{TeV}$ and $\sqrt{s} = 0.8\text{TeV}$.

In order to investigate the capability of the studied process to discover effects of Majorana neutrinos, we study the region (discovery region) where the signal can be separated from the background with a statistical significance of $5\sigma$. It is done by defining the quantity $S$

$$S = \frac{L[\sigma(\alpha, M_N) - \sigma_B]}{\sqrt{L[\sigma(\alpha, M_N) + \sigma_B]}}$$

(16)

where $L$ is the luminosity and the numerator represents the discrepancy between the signal and the SM background ($\sigma_B$). In Fig.6 we show the discovery region (above the solid curves) where $S \geq 5$ ($5\sigma$ statistical significance) for a luminosity $L = 100 fb^{-1}$.

For completion we have also considered, although in an approximated way, the bounds on the operators which come from the $0\nu\beta\beta$-decay and from LEP and low energy data. The
FIG. 5: Contribution to $H$ in Eq. (22). In the diagram (1) the solid dot represent the operator $O_{Ne\phi}^1$ and in the diagram (2) the 4-fermion operators $O_{QuNL}^1$, $O_{LQd}^1$ and $O_{QNLd}^1$.

former will be considered in the next section and the latter in the following.

The heavy Majorana neutrino couples to the three flavors families with couplings dependent on the scale $\Lambda$ and the constants $\alpha_{J}^{(i)}$. It is possible to relate this coupling with the mixing between light and heavy neutrinos ($U_{eN}$, $U_{\mu N}$, $U_{\tau N}$) for which the experimental bounds, obtained from LEP and low energy data, have been put in [15–18]. This relation was found in [2] comparing the operator $O_{Ne\phi}^i$ with the strength of the V-A interaction (Eq. (3)). It is $U_{liN} \approx \left(\alpha_{W}^{(i)}/2\right)(v^2/\Lambda^2)$ where $v$ corresponds to the vacuum expectation value: $v = 250 GeV$. In order to keep the analysis as simple as possible we consider that the same bound applies for all the couplings $\alpha_{J}^{(i)}$, generically $\alpha$.

In our case, for one heavy Majorana neutrino, and following [15], we have:

$$\Omega_{ll'} = U_{lN}U_{l'N}$$

(17)

where the allowed values for the parameter are [22]:

$$\Omega_{ee} \leq 0.0054, \quad \Omega_{\mu\mu} \leq 0.0096, \quad \Omega_{\tau\tau} \leq 0.016$$

(18)

For the Lepton-Flavour-Violating process (LFV), e.g. $\mu \rightarrow e\gamma$, $\mu \rightarrow eee$ and $\tau \rightarrow eee$, which are induced by the quantum effect of the heavy neutrinos, we have [23]:

$$|\Omega_{e\mu}| \leq 0.0001, \quad |\Omega_{e\tau}| \leq 0.02, \quad |\Omega_{\mu\tau}| \leq 0.02$$

(19)

These bounds can be translated to the constants $\alpha$ considering, in a simplified way, that all
TABLE I: Comparison between the signal and background.

| $\sqrt{s}$ (GeV) | $\sigma_{SM}$ (pb) | $M_N$ (GeV) | $\sigma$ (pb) |
|-----------------|------------------|------------|-------------|
| 500             | $1.1 \times 10^{-5}$ | 100        | 0.2         |
|                 |                  | 200        | 0.043       |
|                 |                  | 300        | $3.3 \times 10^{-5}$ |
| 800             | $5.0 \times 10^{-5}$ | 200        | 0.5         |
|                 |                  | 300        | 0.33        |
|                 |                  | 500        | $3.2 \times 10^{-4}$ |

the operators satisfy the same constraint

$$\Omega_{e\mu} = U_{eN}U_{\mu N} = \left(\frac{\alpha v^2}{2 \Lambda^2}\right)^2 < 0.0001 \quad (20)$$

For $\Lambda = 1$ TeV we have:

$$\alpha \leq 0.32 \quad (21)$$

This value is shown in both panels of Fig. 6 with a horizontal dot-dashed line.

FIG. 6: Discovery region above the solid curve at 5 $\sigma$ (this work) and below the dot-dashed line for LVF bound (Left panel) or the dotted curve for $0\nu\beta\beta$-decay bound (Right panel).
III. NEUTRINOLESS DOUBLE BETA DECAY BOUNDS

In order to take into account the bounds imposed by the 0νββ-decay experiment on some of the coupling constants $\alpha_{ji}^{(i)}$, we consider, in a general way, the following effective interaction Hamiltonian:

$$\mathcal{H} = G_{\text{eff}} \bar{u} \Gamma d \bar{e} \Gamma N + h.c. \quad (22)$$

where $\Gamma$ represents a general Lorentz-Dirac structure. Following the developments presented in [24, 25] and using the most stringent limits on the lifetime for neutrinoless double beta decay $\tau_{0\nu\beta\beta} \geq 1.9 \times 10^{25}$ yr obtained by the Heidelberg-Moscow Collaboration [26], we have obtained the following bounds for $G_{\text{eff}}$

$$G_{\text{eff}} \leq 8.0 \times 10^{-8} \left( \frac{m_N}{100\text{GeV}} \right)^{1/2} \text{GeV}^{-2} \quad (23)$$

The lowest order contribution to 0νββ-decay from the considered effective operators comes from those that involve the $W$ field and the 4-fermion operators with quarks $u, d$, the lepton $e$ and the Majorana neutrino $N$:

$$\mathcal{O}_{1}^{N\phi}, \mathcal{O}_{1}^{d\bar{u}Ne}, \mathcal{O}_{1}^{QuNL}, \mathcal{O}_{1}^{LNQd}, \mathcal{O}_{1}^{QNLd} \quad (24)$$

The contribution of these operators to the effective Hamiltonian in eq. (22) is shown in Fig. 5.

For the coupling constant associated with each operator we use the generic name $\alpha_{0\nu\beta\beta}$, that is to say

$$\alpha_{0\nu\beta\beta} = \alpha_{N\phi}^{(1)} = \alpha_{d\bar{u}Ne}^{(1)} = \alpha_{QuNL}^{(1)} = \alpha_{LNQd}^{(1)} = \alpha_{QNLd}^{(1)} \quad (25)$$

In order to estimate the bounds on the different $\alpha_{ji}^{(i)}$ we consider two different situations. First, we suppose that the contribution of all the operators involved in the 0νββ-decay adds constructively. In this case we expect strong limits on the couplings, then we may assume them to be negligible. Thus, we consider that all of the constants associated with the operators in Eq. (22) vanish ($\alpha_{0\nu\beta\beta} = 0$) and that the other constants, which are not bounded by neutrinoless double beta decay, are non-zero and have similar magnitude. This situation is shown in the left panel of the Fig. 6.

Second, we consider the individual contributions of each operator as acting alone. In this case it is obvious to relate the coupling with the $G_{\text{eff}}$ in Eq. (22)

$$G_{\text{eff}} = \frac{\alpha_{0\nu\beta\beta}}{\Lambda^2} \quad (26)$$
Thus we can translate the limit which came from $G_{\text{eff}}$ to $\alpha_{0\nu\beta\beta}$. For, $\Lambda = 1 TeV$, it is

$$\alpha_{0\nu\beta\beta} \leq 8.0 \times 10^{-2} \left( \frac{m_N}{100 \text{GeV}} \right)^{1/2}$$

(27)

Taking a conservative point of view, in the right panel of Fig.6, we present this bound considering that it is the same for all the constants $\alpha_{J}^{(i)}$ (generically $\alpha$) and show it with the dotted curve. The solid curve, which is the contribution of this work, represent the lower limit for the discovery region. In the right panel, it was calculated considering that all the constants $\alpha_{J}^{(i)}$ have similar magnitude. On the other hand, in the left panel it was calculated considering $\alpha_{0\nu\beta\beta} = 0$. We also show the bound from Lepton Flavors Violating process with the dot-dashed line in the same figure.

Summarizing, we calculated the cross-section for the process $e^{+}e^{-}\rightarrow l_{j}^{+}l_{k}^{-} + jets$ where $l_1$, $l_2$ and $l_3$ are light leptons ($e, \mu, \tau$) respectively. We show the total unpolarized cross-section using the calculated Branching ratios for different values of $m_N$ and the coupling $\alpha_{J}^{(i)}$. We showed the discovery regions at $5\sigma$ statistical significance combining with the $0\nu\beta\beta$ and the LFV bounds. We found that it will be possible to discover Majorana neutrinos with masses lower than 250 $GeV$ and 400 $GeV$ at $e^{+}e^{-}$ colliders with center of mass energy of 0.5 $TeV$ and 0.8 $TeV$ respectively.

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IV. APPENDIX

We present here the partial decay widths of a heavy Majorana neutrino $N$ for its dominant decay channels. They were calculated using the effective interactions shown above in the text.
\[ \frac{d\Gamma^{(N\rightarrow l^+\bar{d}d)}}{dx} = \frac{m_N}{256\pi^3} \left( \frac{m_N}{\Lambda} \right)^4 x^2 \left\{ \frac{3}{2} \sum_{i=1,2} \left( \alpha^2_{s_1,i} + \alpha^2_{s_2,i} - \alpha_{s_2,i} \alpha_{s_3,i} \right) (1-x) \right. \\
+ \left. \frac{1}{4} \sum_{i=1,2} \left( \alpha^2_{s_3,i} + 4\alpha^2_{t_0,i} \right) (3-2x) \right\} \text{ with } 0 < x < 1. \]

\[ \frac{d\Gamma^{(N\rightarrow l^+tb)}}{dx} = \frac{m_N}{256\pi^3} \left( \frac{m_N}{\Lambda} \right)^4 \frac{(1-x-y)^2 x^2}{(1-x)^3} \left\{ \left[ \frac{3}{2} (\alpha^2_{s_1,3} + \alpha^2_{s_2,3} - \alpha_{s_2,3} \alpha_{s_3,3}) (1-x)^2 \right. \right. \\
+ \left. \left. \frac{1}{4} (\alpha^2_{s_3,3} + 4\alpha^2_{t_0,3}) ((3-2x)(1-x) + y(3-x)) \right] \right. \\
+ \left. \left. \left( \sum_{i=1,3} \alpha^2_{W,i} \right) \frac{(3-2x)(1-x) + y(3-x)}{w + (1-(1-x)z)^2} \right \} \text{ with } 0 < x < 1 - y. \]

\[ \frac{d\Gamma^{(N\rightarrow \nu dd)}}{dx} = \frac{m_N}{256\pi^3} \left( \frac{m_N}{\Lambda} \right)^4 x^2 \left[ \sum_{i=1,3} \left( \alpha^2_{s_2,i} - \alpha_{s_2,i} \alpha_{s_3,i} \right) \right. \\
+ \left. \left( \sum_{i=1,3} \alpha^2_{s_3,i} \right) (3-2x) \right] \text{ with } 0 < x < 1. \]

\[ \frac{d\Gamma^{(N\rightarrow \nu uu)}}{dx} = \frac{m_N}{256\pi^3} \left( \frac{m_N}{\Lambda} \right)^4 \left( \sum_{i=1,3} \alpha^2_{s_1,i} \right) \frac{3}{2} x^2 (1-x) \text{ with } 0 < x < 1. \]
\[ \frac{d\Gamma^{(N\to\nu tt)}}{dx} = \frac{m_N}{256\pi^2} \left( \frac{m_N}{\Lambda} \right)^4 \left( \frac{\alpha_{s_{1,2}}^2}{2} \right) \frac{3}{2} x^2 \sqrt{1 - \frac{4y}{1 - x}} (1 - x - 2y) \Theta(1 - x - 4y) \]

with \( 0 < x < 1 - 4y \)

\[ \frac{d\Gamma^{(N\to t^+\text{leptons})}}{dx} = \frac{m_N}{256\pi^2} \left( \frac{m_N}{\Lambda} \right)^4 \frac{x^2}{12} (3 - 2x) \left[ \left( \sum_{i=1,3} \alpha_{s_{i}}^2 \right) \right] \]

\[ + \left( \sum_{i=1,3} \alpha_{W,i}^2 \right) \frac{12}{w + (1 - (1 - x)z)^2} \]

with \( 0 < x < 1 \)

\[ \frac{d\Gamma^{(N\to\nu H)}}{dx} = \frac{m_N}{256\pi^2} \left( \frac{m_N}{\Lambda} \right)^4 \left( \sum_{i=1,3} \alpha_{s_{i}}^2 \right) \left( \frac{v}{m_N} \right)^4 2\pi^2 (1 - z_{\phi}) \]

with \( 0 < x < 1 \)

where \( x = 2p_{\text{lepton}}^0/m_N \), and \( y = (m_t/m_N)^2 \), \( z_{\phi} = (m_{\phi}/m_N)^2 \), \( z = (m_N/m_W)^2 \), \( w = (\Gamma_w/m_W)^2 \).

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