Analogue surface gravity near the QCD chiral phase transition

Neven Bilić and Dijana Tolić
Rudjer Bošković Institute,
P.O. Box 180, 10001 Zagreb, Croatia
E-mail: bilic@thphys.irb.hr, dijana.tolic@irb.hr

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Abstract

Using the formalism of relativistic acoustic geometry we study the expanding chiral fluid in the regime of broken chiral symmetry near the QCD chiral phase transition temperature $T_c$. The dynamics of pions below $T_c$ is described by the equation of motion for a massless scalar field propagating in curved spacetime similar to an open FRW universe. The metric tensor depends locally on the soft pion dispersion relation and the four-velocity of the fluid. In the neighbourhood of the critical point an analogue trapped region forms with the analogue trapped horizon as its boundary. We show that the associated surface gravity diverges near the critical point as $\kappa \sim (T_c - T)^{-1}$. Hence, if the horizon forms close to the critical temperature the analogue Hawking temperature may be comparable with or even larger than the background fluid temperature.

Keywords: analogue gravity, chiral symmetry, critical temperature, trapped surface, Hawking effect

1 Introduction

Analog gravity models of general relativity seem promising routes to providing laboratory tests of the foundation of quantum field theory in curved spacetime [1] (for a review and an extensive list of references, see [2]). The models are based on kinematics of waves propagating in inhomogeneously flowing media. Various aspects of these phenomena have been studied in acoustics [3] optics [4] and superfluidity [5]. In this paper we study the analogue gravity model based on massless pions propagating in a hadronic fluid.

Strongly interacting matter is described at the fundamental level by a nonabelian gauge theory called quantum chromodynamics (QCD). At large distances or small momenta, the QCD exhibits the phenomena of quark confinement and chiral symmetry breaking. Chiral symmetry breaking expresses the fact that massless quarks confined in hadrons appear effectively as massive constituents with a dynamically generated mass of several hundred MeV. At low energies, the QCD vacuum is characterized by a nonvanishing expectation value [6, 7]:
\[ \langle \bar{\psi} \psi \rangle \approx (235 \text{ MeV})^3, \] the so-called quark condensate, which describes the density of quark-antiquark pairs found in the QCD vacuum and its nonvanishing value is the manifestation of chiral symmetry breaking. The chiral symmetry is restored at finite temperature above certain critical temperature \( T_c \) of the order of 150 MeV. In the chirally broken phase, below \( T_c \) the pions, although being massless, propagate slower than light \([8, 9, 10]\) with velocity approaching zero at the critical temperature. The pions propagate through the expanding fluid the velocity of which may well be comparable with the speed of light. Hence, it is very likely that the flow velocity will exceed the pion velocity if the temperature of the fluid is below and close to the critical temperature. A similar condition is also realized in cosmological models of expanding universe in which the recession velocity of the expanding spacetime beyond the Hubble horizon exceeds the speed of light.

In order to explore this analogy with cosmology more closely we need to specify the velocity field of the flow in a given spacetime geometry and the fluid parameters such as the density, temperature and the velocity of pions propagating in the fluid. For that purpose we exploit a simple boost invariant Bjorken type spherical expansion \([11]\) of the chiral fluid. A similar model has been studied some time ago in the context of disoriented chiral condensate \([12]\). Furthermore, the velocity of pions in the chiral fluid can be derived using a linear sigma model as an effective low energy model of strong interactions. Then, the propagation of massless pions provides a setting for a geometric analogue of expanding spacetime. Since the velocity of the fluid may exceed the velocity of pions it is conceivable that an apparent horizon may exist with the associated analogue Hawking radiation. In this paper we study the analogue Hawking effect using the Kodama-Hayward definition of surface gravity \([13]\).

In the next two sections we describe our basic concepts and in sec. 4 we derive an expression for surface gravity and study its behaviour near the critical point.

## 2 Dynamics of the chiral fluid

In the following we assume that the chiral fluid undergoes a boost invariant Bjorken-type spherical expansion. A Bjorken-type expansion is a simple and very useful hydrodynamic model that reflects the boost invariance of the deep inelastic scattering in high energy collisions. The original model \([11]\) was introduced to describe the longitudinal expansion only. A more realistic model of heavy ion collisions involves a transverse expansion superimposed on the longitudinal boost invariant expansion \([14]\). In order to draw the analogy with cosmology, here we consider a spherically symmetric Bjorken expansion\(^1\) which is invariant under radial boosts. In this model the radial three-velocity in radial coordinates \( x^\mu = (t, r, \vartheta, \varphi) \) is a simple function \( v = r/t \). Then the four-velocity is given by

\[ u^\mu = (t/\tau, r/\tau, 0, 0), \]

where \( \tau = \sqrt{t^2 - r^2} \) is the proper time. With the substitution

\[ t = \tau \cosh y, \]
\[ r = \tau \sinh y, \]

\(^1\)From the high energy physics perspective, a spherical expansion is more appropriate for \( e^+e^- \) collisions \([15]\) because in this case the jets are produced with no directional preference.
the radial velocity is expressed as
\[ v = \tanh y, \quad (3) \]
and the four-velocity as
\[ u^\mu = (\cosh y, \sinh y, 0, 0). \quad (4) \]
The substitution (2) may be regarded as a coordinate transformation from ordinary radial coordinates to new coordinates \((\tau, y, \vartheta, \varphi)\) in which the flat background metric takes the form
\[ g_{\mu\nu} = \text{diag} (1, -\tau^2, -\tau^2 \sinh^2 y, -\tau^2 \sinh^2 y \sin^2 \theta), \quad (5) \]
and the velocity components become \(u^\mu = (1, 0, 0, 0)\). Hence, the new coordinate frame is comoving. The metric corresponds to an FRW expanding cosmological model with cosmological scale \(a = \tau\) and negative spatial curvature. The transformation (2) maps the spatially flat Minkowski spacetime into an expanding FRW spacetime with cosmological scale \(a = \tau\) and negative spatial curvature. The resulting flat spacetime with metric (5) is known in cosmology as the Milne universe [16].

The temperature of the expanding chiral fluid, to a good approximation, is proportional to \(\tau^{-1}\). This follows from the fact that the chiral matter is dominated by massless pions, and hence, the density of the fluid may be approximated by the density \(\rho = (g\pi^2/30)T^4\) of an ideal massless boson gas [17]. Using this and the energy-momentum conservation one finds
\[ T = \frac{c_0}{\tau}, \quad (6) \]
where the constant \(c_0\) may, in principle, be fixed from the phenomenology of high energy collisions.

Next we derive the pion velocity in the chiral fluid. The dynamics of mesons in a medium is described by a chirally symmetric Lagrangian of the form [9, 10, 18]
\[ \mathcal{L} = \frac{1}{2} (a g^{\mu\nu} + b u^\mu u^\nu) \partial_\mu \varphi \partial_\nu \varphi - \frac{m_0^2}{2} \varphi^2 - \frac{\lambda}{4} (\varphi^2)^2, \quad (7) \]
where \(u_\mu\) is the velocity of the fluid, and \(g_{\mu\nu}\) is the background metric. The mesons \(\varphi \equiv (\sigma, \pi)\) constitute the \((1/2, 1/2)\) representation of the chiral SU(2) \(\times\) SU(2). The parameters \(a\) and \(b\) depend on the local temperature \(T\) and on the parameters of the model \(m_0\) and \(\lambda\) and may be calculated in perturbation theory. At zero temperature the medium is absent in which case \(a = 1\) and \(b = 0\).

If \(m_0^2 < 0\) the chiral symmetry will be spontaneously broken. At the classical level, the \(\sigma\) field develops a nonvanishing expectation value such that \(\langle \sigma \rangle = f_\pi\). At nonzero temperature the expectation value \(\langle \sigma \rangle\), usually referred to as the chiral condensate, is temperature dependent and vanishes at the chiral transition point. Hence, the quantity \(\langle \sigma \rangle\) serves as an order parameter. For temperatures below the chiral transition point the meson masses are given by
\[ m_\pi^2 = 0; \quad m_\sigma^2 = 2\lambda \langle \sigma \rangle^2, \quad (8) \]
in agreement with the Goldstone theorem. The temperature dependence of the chiral condensate \(\langle \sigma \rangle\) is obtained by minimizing the thermodynamical potential \(\Omega = -(T/V) \ln Z\).
with respect to $\langle \sigma \rangle$ at fixed temperature $T$. At one loop order the solution to the extremum condition as a function of temperature exhibits a weak first-order phase transition [19, 20]. However, Pisarski and Wilczek have shown on general grounds that the phase transition in SU(2)×SU(2) chiral models should be of second order [21]. Hence, it is generally believed that a first-order phase transition in this case is an artifact of the one loop approximation. Two loop calculations [22] make an improvement and confirm the general analysis of [21].

Propagation of pions is governed by the equation of motion

$$\frac{1}{\sqrt{-g}} \partial_{\mu} \left( \sqrt{-g} \left( a g^{\mu\nu} + b u^{\mu} u^{\nu} \right) \partial_{\nu} \pi \right) + V(\sigma, \pi) \pi = 0,$$

which follows from the effective Lagrangian obtained from (7) by spontaneous chiral symmetry breaking. The quantity $V$ in (9) is the interaction potential the form of which is irrelevant for our consideration. From (9) we obtain the pion velocity squared as [18]

$$c_{\pi}^2 = \frac{a}{a + b}.$$

The parameters $a$ and $b$ at nonzero temperature may be derived from the finite temperature self energy $\Sigma(q, T)$ of the pion in the limit when the external momentum $q$ approaches 0. For a flat background geometry $g_{\mu\nu} = \eta_{\mu\nu}$, the inverse pion propagator $\Delta^{-1}$ is given by

$$\Delta^{-1} = a q^2 - b (q^2) - m_{\pi}^2; \quad (11)$$

or in comoving frame, i.e., in a reference frame in which $u^{\mu} = (1, 0, 0, 0)$,

$$\Delta^{-1} = (a + b) q_0^2 - a q^2; \quad (12)$$

Hence, the parameters $a$ and $b$, can be expressed in terms of second derivatives of $\Sigma(q, T)$ with respect to $q_0$ and $q_i$ evaluated at $q^{\mu} = 0$.

The pion velocity at nonzero temperature was calculated at one loop level by Pisarski and Tytgat in the low temperature approximation [8]. At one loop level the only diagram that gives a nontrivial $q$-dependence of $\Sigma(q, T)$ is the bubble diagram and the calculation may be performed for the hole range of temperatures below the chiral critical point [10, 18]. As we are particularly interested in the behaviour near the critical point of the chiral phase transition we make use of the exact results near the critical temperature based on scaling and universality analysis of Son and Stephanov [9]. In their notation our parameters $a$ and $b$ are expressed as

$$a = \frac{f^2_s}{\langle \sigma \rangle^2}; \quad a + b = \frac{f^2_t}{\langle \sigma \rangle^2}, \quad (13)$$

where $f_s$ is equal to the temperature dependent pion decay constant and $f^2_t$ coincides with the isospin susceptibility $\chi_{I5}$. The chiral condensate $\langle \sigma \rangle$ up to a factor of the order $f^2_s$ equals the quark condensate $\langle \bar{\psi} \psi \rangle$ at zero quark masses. In the limit $T \rightarrow T_c$ the quantity $f^2_t$ goes to a nonzero constant whereas, contrary to naive expectations, the scaling of $f_s$ turns out to be different than that of the order parameter $\langle \sigma \rangle$:

$$f^2_s \sim (T_c - T)^{(d-2)\nu}; \quad \langle \sigma \rangle \sim (T_c - T)^{\beta}, \quad (14)$$
where $d$ is the number of space dimensions and $\nu$ and $\beta$ are positive critical exponents. Hence, we obtain

$$a \sim (T_c - T)^{(d-2)\nu - 2\beta}, \quad a + b \sim (T_c - T)^{-2\beta}, \quad (15)$$

so the pion velocity scales as

$$c_\pi^2 \sim (T_c - T)^{(d-2)\nu}, \quad (16)$$

as one approaches the critical temperature. In $d = 3$ dimensions, the critical exponents for the O(4) universality class are $\nu = 0.73$ and $\beta = 0.38$ [23].

3 Chiral geometry

We now introduce the analogue gravity metric which describes the effective geometry of the expanding chiral fluid using the the formalism of relativistic acoustic geometry [24, 25, 26]. The equation of motion (9) may be written in the form

$$\frac{1}{\sqrt{-G}} \partial_\mu (\sqrt{-G} G^{\mu\nu}) \partial_\nu \pi + \frac{c_\pi^2}{a} V(\sigma, \pi) \pi = 0, \quad (17)$$

with the analogue metric tensor, its inverse, and its determinant given by

$$G_{\mu\nu} = \frac{a}{c_\pi} [g_{\mu\nu} - (1 - c_\pi^2) u_\mu u_\nu], \quad (18)$$

$$G^{\mu\nu} = \frac{c_\pi}{a} \left[ g^{\mu\nu} - (1 - \frac{1}{c_\pi^2}) u^\mu u^\nu \right], \quad (19)$$

$$G = \frac{a^4}{c_\pi^2} g. \quad (20)$$

Hence, the pion field propagates in a (3+1)-dimensional effective geometry described by the metric $G_{\mu\nu}$. It is convenient to work in comoving coordinates $(\tau, y, \vartheta, \varphi)$ with background metric $g_{\mu\nu}$ defined by (5). In these coordinates the analogue metric tensor (18) is diagonal with components

$$G_{\mu\nu} = \frac{a}{c_\pi} \text{diag} \left( c_\pi^2, -\tau^2, -\tau^2 \sinh^2 y, -\tau^2 \sinh^2 y \sin^2 \theta \right), \quad (21)$$

where the parameters $a$ and $c_\pi$ are functions of the temperature $T$ which in turn is a function of $\tau$ by (6). In the following we assume that these functions are positive.

In contrast to [25], where it was assumed that both the background geometry and the flow were stationary, in an expanding fluid the flow is essentially time dependent. Hence, the acoustic geometry formalism must be adapted to a non-stationary spacetime. To this end we introduce the concept of analogue marginally trapped surface or analogue apparent horizon.

To define the analogue apparent horizon we need to examine the behaviour of radial null geodesics of the analogue metric (21) in which $a$ and $c_\pi$ are functions of $\tau$. In the following we assume spherical symmetry and denote by $l^\mu_+$ and $l^\mu_-$ the vectors tangent to outgoing and
ingoing affinely parameterized radial null geodesics normal to a spherical two-dimensional surface. The tangent vectors are null with respect to the metric (18), i.e.,

\[ G_{\mu\nu} l_+^\mu l_+^\nu = G_{\mu\nu} l_-^\mu l_-^\nu = 0. \]  

(22)

Using the geodesic equation

\[ l^\mu \nabla_\mu l^\nu = 0, \]  

(23)

where the symbol \( \nabla_\mu \) denotes a covariant derivative associated with the metric (18), one easily finds the tangent null vectors corresponding to future directed radial null geodesics,

\[ l_\mu = \frac{1}{a\tau} \left( 1, \pm \frac{c_\pi}{\tau}, 0, 0 \right), \]  

(24)

The null vectors \( l_+^\mu \) and \( l_-^\mu \) point towards increasing and decreasing \( y \), respectively. Hence, we adopt the usual convention and refer to \( l_+^\mu \) and \( l_-^\mu \) (and the corresponding null geodesics) as outgoing and ingoing.

The key element in the study of trapped surfaces is the expansion parameter \( \varepsilon_{\pm} \) of null geodesics. A two-dimensional surface \( S \) with spherical topology is called a trapped surface if the families of ingoing and outgoing null geodesics normal to the surface are both converging or both diverging. More precisely, the expansion parameters

\[ \varepsilon_{\pm} = \nabla_\mu l_{\pm}^\mu \]  

(25)

on a trapped surface \( S \) should satisfy \( \varepsilon_+ \varepsilon_- > 0 \). A two-dimensional surface \( H \) is said to be future inner marginally trapped if the future directed null expansions on \( H \) satisfy the conditions: \( \varepsilon_+|_H = 0 \), \( l_+^\mu \partial_\mu \varepsilon_+|_H > 0 \) and \( \varepsilon_-|_H < 0 \). We shall refer to this surface as the apparent horizon since it is equivalent to the apparent horizon in cosmological context.

From (25) with (24) we find

\[ \varepsilon_{\pm} = \frac{2}{a\tau^2} \left( \partial_\tau \left( \tau \sqrt{a/c_\pi} \right) \pm \frac{c_\pi}{v} \right). \]  

(26)

From (26) one finds the condition for the apparent horizon

\[ \frac{c_\pi}{v} \pm \frac{\partial_\tau \left( \tau \sqrt{a/c_\pi} \right)}{\sqrt{a/c_\pi}} = 0. \]  

(27)

This equation defines a hypersurface which we refer to as the analogue trapping horizon. The condition (27) provides a functional relation between \( y \) and \( \tau \):

\[ \tanh y = f(\tau) \equiv \pm \sqrt{ac_\pi} \partial_\tau \left( \tau \sqrt{a/c_\pi} \right), \]  

(28)

which by the coordinate transformation (2) yields an implicit functional dependence of \( r \) on \( t \):

\[ r/t = f(\sqrt{t^2 - r^2}). \]  

(29)
Any solution to this equation, e.g., in terms of \( r \) for an arbitrary fixed \( t = t^* \), gives the location of the analogue apparent horizon \( r_H \).

The critical behaviour of the null expansions \( \varepsilon_+ \) and \( \varepsilon_- \) and of the derivative \( l_\mu \partial_\mu \varepsilon_+ \) near the trapping horizon is of particular interest. Using (14)-(16) in the neighbourhood of the horizon we find

\[
\varepsilon_+ \simeq \frac{2}{a r_c^2} \left( \frac{c \pi}{v} - \frac{\eta r_c}{\tau - r_c} \right),
\]

\[
\varepsilon_- \simeq -\frac{4\eta}{a r_c (\tau - r_c)},
\]

\[
l_\mu \partial_\mu \varepsilon_+ \simeq \frac{(d - 2) \nu \eta + 2\eta(1 + \eta)}{a^2 r_c^2 (\tau - r_c)^2},
\]

where

\[
\eta = \beta - (d - 2)\nu/4.
\]

and \( \tau_c = c_0/T_c \) is the critical proper time corresponding to the critical temperature \( T_c \). Note that for an arbitrary fixed \( t = t^* \), the outgoing null expansion \( \varepsilon_+ \) vanishes at the radius \( r = r_H \) at which \( v = c \pi (\tau - r_c)/(\eta r_c) \). The ingoing null expansion is obviously negative whereas the derivative of \( \varepsilon_+ \) given by (32) is positive at \( r_H \) and, according to the standard convention [27], the region \( \{ r > r_H, t = t^* \} \) is future trapped with the apparent horizon located at \( r_H \) as its inner boundary.

At this point, it is worthwhile examining the effective Hubble parameter in the neighbourhood of \( \tau_c \). The effective Hubble parameter for the spacetime defined by the metric (21) is given by

\[
H = \frac{\partial_\tau \left( \tau \sqrt{a/c \pi} \right)}{a \tau}.
\]

Using (15) and (16) with (6) one finds the scaling behavior of \( H \) in the neighbourhood of \( \tau_c \):

\[
H \propto -(\tau - \tau_c)^{-1 + \beta - 3(d - 2)\nu/4}.
\]

With the critical exponents \( \nu = 0.73 \) and \( \beta = 0.38 \) in \( d = 3 \) dimensions we find \( H \propto -(\tau - \tau_c)^{-1.17} \), so the effective Hubble parameter is negative and diverges as \( \tau \) approaches \( \tau_c \). Hence, our analogue spacetime describes a shrinking universe with a singularity at the critical point.

4 Surface gravity and analogue Hawking effect

On suggestion by Hajicek [28] it was recently argued [29, 30, 31, 32, 33, 34] that the Hawking effect might be associated with the apparent horizon rather than the event horizon. This observation is irrelevant for a stationary spacetime where the apparent and event horizons coincide. In this case the apparent horizon is Killing and the surface gravity is uniquely defined as a parameter that measures the inaffinity of the properly normalized Killing vector \( \xi^\mu \). The surface gravity \( \kappa \) of the Killing horizon can be defined by

\[
\xi^\nu \nabla_\nu \xi_\mu = \kappa \xi_\mu,
\]
evaluated on the horizon. If the geometry were stationary, i.e., if the components of $G_{\mu\nu}$ in (18) were time independent, the apparent horizon would coincide with the chiral event horizon at the surface defined by $v = c_\pi$. In that case the surface gravity would read [25]

$$\kappa = \frac{1}{1 - c_\pi^2} \frac{\partial}{\partial r} (v - c_\pi),$$

(37)

where the derivative is to be taken at the horizon.

In the case of nonstationary spacetime, the apparent horizon is neither Killing nor null. The definition of surface gravity in this case is not unique [31] and several ideas have been put forward how to generalize the definition of surface gravity for the apparent horizon [13, 27, 29, 35]. In this paper we adopt the prescription of [13] which, we believe, is most suitable for spherical symmetry. This prescription involves the so-called Kodama vector $K^\mu$ [36] which generalizes the concept of the time translation Killing vector to non-stationary spacetimes. The Kodama vector we define as [27, 37]

$$K^\alpha = k \epsilon^{\alpha\beta} n_\beta \text{ for } \alpha = 0, 1; \quad K^i = 0 \text{ for } i = 2, 3,$$

(38)

where $\epsilon^{\alpha\beta}$ is the covariant two-dimensional Levi-Civita tensor in the space normal to the surface of spherical symmetry and $n_\alpha$ is a vector normal to that surface. For the metric (21) $n_\alpha$ is given by

$$n_\mu = \partial_\mu \left( \tau \sqrt{\frac{a}{c_\pi}} \sinh y \right).$$

(39)

Our definition differs from the original one [27, 37] by a normalization factor $k$ which we have introduced in order to meet the requirement that $K^\mu$ should coincide with the time translation Killing vector $\xi^\mu$ for a stationary geometry. Using the definition (38) with metric (21) we find

$$K^\tau = k a^{-1} \cosh y; \quad K^y = -k \frac{\partial_\tau \left( \tau \sqrt{\frac{a}{c_\pi}} \right)}{\tau a} \sinh y,$$

(40)

with norm squared

$$|K|^2 \equiv h_{\alpha\beta} K^\alpha K^\beta = k^2 \sinh^2 y \left[ \coth^2 y - \left( \frac{\partial_\tau \left( \tau \sqrt{\frac{a}{c_\pi}} \right)}{\sqrt{ac_\pi}} \right)^2 \right].$$

(41)

The vector $K^\alpha$ is spacelike in the trapped region and vanishes on the trapping horizon.

In order to fix $k$, consider a general spherically symmetric analogue geometry with metric (18). In $(t, r)$ coordinates

$$n_\mu = \partial_\mu \left( r \sqrt{a/c_\pi} \right).$$

(42)

Using (38) with metric (18) we find

$$K^t = ka^{-1} \partial_r \left( r \sqrt{a/c_\pi} \right); \quad K^r = -ka^{-1} r \partial_t \sqrt{a/c_\pi}.$$

(43)
In the case of stationary geometry $K^r = 0$ because the quantities $a$ and $c_\pi$ depend only on $r$. Then, if we set

$$k = \sqrt{ac_\pi} \left( 1 + \frac{c_\pi}{ar} \partial_r \sqrt{a/c_\pi} \right)^{-1}, \quad \text{(44)}$$

we have $K^\mu = \xi^\mu$ in the stationary case. In $(\tau, y)$ coordinates

$$k = \sqrt{ac_\pi} \left( \cosh^2 y - \frac{\partial_\tau \left( \tau \sqrt{a/c_\pi} \right)}{\sqrt{a/c_\pi}} \sinh^2 y \right). \quad \text{(45)}$$

In analogy with (36) the surface gravity $\kappa$ is defined by [13, 38]

$$K^\alpha \nabla_{[\alpha} K_{\beta]} = \kappa K_\beta, \quad \text{(46)}$$

or equivalently by

$$\kappa = \frac{1}{2} \frac{1}{\sqrt{-h}} \partial_\alpha (\sqrt{-h} h^{\alpha\beta} k_{\beta}), \quad \text{(47)}$$

where the quantities on the right-hand side should be evaluated on the trapping horizon. Using (42) and (44) it is easy to check that this expression reduces to (37) for a stationary spherical flow, i.e., a steady flow where the velocity $v$ and the constants $a$, $b$, and $c_\pi$ are time independent. By making use of the horizon condition (27) for our time dependent Bjorken spherical flow we find from (47)

$$\kappa = \frac{\sqrt{1 - v^2}}{(1 + c_\pi v)^2} \left[ \frac{c_\pi}{2\tau v} + \frac{c_\pi^2 - c^2_\pi v - c_\pi v^2}{2\tau} - \frac{c^2_\pi v^2}{2\tau} + \frac{\partial_\tau^2 \left( \tau \sqrt{a/c_\pi} \right)}{\sqrt{ac_\pi}} \right], \quad \text{(48)}$$

where $v = \tanh y$ and it is understood that the right-hand side should be evaluated on the trapping horizon. In other words, the function $\kappa = \kappa(\tau, y)$ in (48) is a function of two dependent variables $y$ and $\tau$ subject to the constraint (27). Hence, $\kappa$ is effectively a function of only one variable, e.g., $\kappa = \kappa(\tau, y(\tau))$ through the explicit dependence on $\tau$ and implicit dependence via $y(\tau)$.

The critical behaviour of $\kappa$ is determined by the behaviour of the quantity $\sqrt{a/c_\pi}$ in the neighbourhood of the critical point. From (14)-(16) we have

$$\sqrt{\frac{a}{c_\pi}} \propto (\tau - \tau_c)^{-\eta} \quad \text{(49)}$$

in the neighbourhood of the critical point, where $\eta$ is defined in (33). Then, on the trapping horizon in the limit $\tau \to \tau_c$ the two dominant terms in square brackets in (48) scale as

$$\frac{c_\pi}{2\tau v} = -\frac{\partial_\tau \left( \tau \sqrt{a/c_\pi} \right)}{2\tau \sqrt{a/c_\pi}} \simeq \frac{1}{2} \eta (\tau - \tau_c)^{-1}, \quad \text{(50)}$$

$$\frac{\partial_\tau^2 \left( \tau \sqrt{a/c_\pi} \right)}{\sqrt{ac_\pi}} \simeq (\eta + 1)(\tau - \tau_c)^{-1}. \quad \text{(51)}$$
Plugging these expressions in (48) we find that the surface gravity in the neighbourhood of $\tau_c$ behaves as

$$\kappa \simeq (\eta + 1/2)(\tau - \tau_c)^{-1}.$$  \hspace{1cm} (52)

From (33), with the critical exponents $\nu = 0.73$ and $\beta = 0.38$ for the O(4) universality class in $d = 3$ dimensions [23], we find $\eta = 0.1975$.

The temperature

$$T_H = \frac{\kappa}{2\pi}$$  \hspace{1cm} (53)

associated with the analogue apparent horizon is the analogue Hawking temperature of thermal pions emitted at the apparent horizon as measured by an observer near infinity. Since the background geometry is flat, this temperature equals the locally measured Hawking temperature at the horizon. From (52) it follows that the analogue Hawking temperature may be arbitrary large in the limit when the analogue horizon approaches the critical point. However, Eq. (52) holds for an ideal spherical expansion of the chiral fluid and in a realistic scenario one could only expect that the Hawking temperature may be comparable with or larger than the background fluid temperature.

5 Discussion

It is tempting to speculate about possible signals for the analogue Hawking effect in a hadronic fluid. In principle, one could measure the temperature by fitting the pion spectrum to the thermal Planck distribution. However, one must be cautious before making any prediction for a realistic physical system. First, one must invent a reliable signal to distinguish between the thermal pions produced above the critical temperature from those emitted as an analogue Hawking radiation from the apparent horizon below the critical temperature. Second, a spherically symmetric expansion model considered here is not realistic for high energy heavy ion collisions. A more realistic hydrodynamic model would involve a transverse expansion superimposed on a longitudinal boost invariant expansion. In this case the calculations become rather involved as the formalism for general nonspherical spacetimes is not yet fully developed. This work is in progress.

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