Diffractive $\rho^0$ photo- and leptoproduction at high energies *)

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Abstract

We discuss the elastic photo- and leptoproduction of $\rho^0$-mesons from nucleons at $Q^2 \lesssim 1 \text{GeV}^2$ as studied in recent experiments at HERA and FNAL. We find that the mass distribution of the measured $\pi^+\pi^-$ pairs is determined to a large extent by the two-pion contribution to the photon spectral function as given by the pion form factor. With rising $Q^2$ the rate of diffractive events decreases and the $\pi^+\pi^-$ mass distribution approaches a symmetric shape.

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The elastic photo- and leptoproduction of $\rho^0$-mesons from nucleons at high energies and moderate momentum transfers $Q^2 \lesssim 1 \text{GeV}^2$ has been actively discussed for many years. Earlier data are available from fixed target experiments at a photon-nucleon center of mass energy $W \lesssim 20 \text{GeV}$ (for a review see [1]). There is a renewed interest in such processes because of recent collider measurements at HERA taken at $Q^2 \lesssim 2 \cdot 10^{-2} \text{GeV}^2$ and $W \simeq 180 \text{GeV}$ [2, 3]. Cross sections for $\rho^0$ leptoproduction at $Q^2 \lesssim 1 \text{GeV}^2$ are presently deduced from recent data taken by the E665 collaboration at Fermilab [4]. In those experiments $\pi^+\pi^-$ pairs are detected with an invariant mass $M_{\pi\pi} < 1 \text{GeV}$ at small transverse momenta. They result dominantly from the $\rho^0 \rightarrow \pi^+\pi^-$ decay; a small fraction of the measured $\pi^+\pi^-$ pairs is due to the production of uncorrelated pions. Contributions from $\omega$- and $\phi$-mesons (apart from $\rho - \omega$ mixing) are excluded.

At high energies the elastic photoproduction of $\pi^+\pi^-$ pairs is characterized by a weak energy dependence and an exponential decrease of the production cross section with $t$, the squared four-momentum transfer. Furthermore, the vector meson is observed to retain, to a good approximation, the helicity of the incoming photon. Such features are typical of diffractive processes in high energy hadron-hadron collisions. This similarity can be understood by looking at the space-time structure of the high energy photon-nucleon interaction in the laboratory frame, where the target is at rest. Here the dominant contribution to the interaction cross section results from processes in which the photon fluctuates to a hadronic Fock state, in presence of the nucleon target, and subsequently scatters diffractively from this target [1].

Given the new and upcoming data, we find it useful to present in this note an improved and updated study of the mass distribution of diffractively produced $\pi^+\pi^-$ pairs in the $\rho$-resonance region. It was observed already long ago [1] that this mass distribution is skewed compared to a Breit-Wigner shape: there is an enhancement at the low mass side and a suppression of large mass contributions above the resonance. For an explanation several models were proposed (see e.g. [5, 6]), mostly based on the interference of non-resonant and resonant $\pi^+\pi^-$ contributions as a source for the observed asymmetry.

Since then a more detailed understanding of the $\rho$-meson and its coupling to the low mass $\pi^+\pi^-$ continuum has been reached in terms of an effective field theory which approximates QCD in the region of composite hadrons (see e.g. [7] and references therein). We point out that the mass distribution of $\pi^+\pi^-$ pairs in diffractive photoproduction is consistent, up to small corrections, with the one deduced from the pion form factor in the timelike momentum region. The latter is known to high precision from $e^+e^- \rightarrow \pi^+\pi^-$ annihilation. In this process resonant and non-resonant $\pi^+\pi^-$ states are automatically accounted for, and there is no need to separate them, as we shall demonstrate.

To describe the diffractive photoproduction of low mass pion pairs, $M_{\pi\pi} < 1 \text{GeV}$, we apply a generalized vector meson dominance model. Within this framework the corre-

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1 In the following we identify always $\rho$ with the neutral $\rho^0$-meson and drop the index.
sponding cross section reads [3]:

$$\frac{d^2\sigma_{\gamma N \to \pi^+\pi^- N}}{dM_{\pi\pi}^2 dt} = \frac{\alpha}{4} \frac{\Pi(M_{\pi\pi}^2)}{M_{\pi\pi}^2} \frac{|T_{\pi\pi N}(W^2, t)|^2}{W^4},$$  \hspace{1cm} (1)

with $\alpha = 1/137$. Here $\Pi(M_{\pi\pi}^2)$ represents the two-pion contribution to the photon spectral function as measured in $e^+e^- \to \pi^+\pi^-$ annihilation:

$$\Pi(M_{\pi\pi}^2) = \frac{1}{12\pi^2} \frac{\sigma_{e^+e^- \to \pi^+\pi^-}}{\sigma_{e^+e^- \to \mu^+\mu^-}}. \hspace{1cm} (2)$$

It accounts for the probability that the photon fluctuates into a $\pi^+\pi^-$ pair with invariant mass $M_{\pi\pi}$ in presence of the target. The subsequent scattering of the pion pair is described by the amplitude $T_{\pi\pi N}$. This amplitude is normalized such that its imaginary part is related to the effective $\pi\pi$-nucleon cross section as usual by the optical theorem,

$$\sigma_{\pi\pi N} = \frac{\text{Im}T_{\pi\pi N}(W^2, t = 0)}{W^2},$$

at large energies $W$. The picture is that of a high energy $\pi\pi$ pair acting like a “beam” as it scatters from the nucleon. We neglect inelastic contributions (e.g. those involving components with more than two pions in the photon spectral function) in this so-called diagonal approximation.

At the large center of mass energies of interest here one can safely neglect the real part of the $\pi^+\pi^-$-nucleon forward scattering amplitude [3] and finds:

$$\frac{d^2\sigma_{\gamma N \to \pi^+\pi^- N}}{dM_{\pi\pi}^2 dt} \bigg|_{t=0} = \frac{\alpha}{4} \frac{\Pi(M_{\pi\pi}^2)}{M_{\pi\pi}^2} \sigma_{\pi\pi N}^2. \hspace{1cm} (3)$$

The effective $\pi^+\pi^-$-nucleon cross section $\sigma_{\pi\pi N}$ can in principle depend on the invariant $\pi\pi$ mass $M_{\pi\pi}$. We take $\sigma_{\pi\pi N}$ to be a constant, averaged over all (resonant and non-resonant) pion pairs with $M_{\pi\pi} < 1$ GeV. Its magnitude is expected to be of the order of the empirical $\rho$-nucleon total cross section, $\sigma_{\rho N} \sim (20-30)$ mb [1].

From experiment [1, 2, 3] it is known that diffractive amplitudes decrease exponentially, $d\sigma/dt \sim e^{bt}$, with the squared momentum transfer $t < 0$. The diffractive excitation of a $\pi^+\pi^-$ pair with invariant mass $M_{\pi\pi}$ requires a non-zero longitudinal momentum transfer $k_L$. In the laboratory frame with the $z$-axis chosen along the photon momentum, $q = (\nu, 0, \nu)$, one finds $k_L = M_{\pi\pi}^2/2\nu$. At large photon energies $\nu \gtrsim 100$ GeV the squared minimal momentum transfer $t_{\text{min}} \approx -k_L^2 = -M_{\pi\pi}^4/4\nu^2$ is negligible and we obtain for the $t$-integrated differential cross section:

$$\frac{d\sigma_{\gamma N \to \pi^+\pi^- N}}{dM_{\pi\pi}} = \frac{\alpha}{2b} \frac{\Pi(M_{\pi\pi}^2)}{M_{\pi\pi}^2} \sigma_{\pi\pi N}^2. \hspace{1cm} (4)$$

Note that the observed energy dependence of the diffractive photoproduction cross section, $d\sigma/dM_{\pi\pi} \sim W^{4(\alpha(0)-1)} \approx W^{0.32}$ [3], translates into an energy dependence of the effective $\pi\pi N$ cross section, $\sigma_{\pi\pi N} \sim W^{2(\alpha(0)-1)} \approx W^{0.16}$. Such a behavior is similar to the energy dependence of total cross sections in high energy hadron-hadron collisions, usually described by the exchange of a “soft” pomeron with Regge-intercept $\alpha(0) \approx 1.08$ [3].
As an aside we mention that the diffractive cross section in eq.(3) also describes successfully the shadowing contribution from low mass $\pi^+\pi^-$ pairs in high-energy photon-deuteron scattering [10]. To verify this, note that the total photon-deuteron cross section has two contributions: the single scattering contribution which gives twice the photon-nucleon cross section, and the double scattering term in which the photon interacts coherently with both proton and neutron in the deuteron. This latter process reduces the total photon-deuteron cross section as compared to twice the photon-nucleon cross section, i.e. it causes shadowing. Its contribution to the total photon-deuteron cross section is related to the differential cross section
\[ d\sigma^{\gamma N \rightarrow XN}/dM_X^2 \] for the diffractive photoproduction of hadrons with invariant mass $M_X$ from free nucleons (see e.g. [11]):

\[ \sigma^{(2)} = -4\pi \int_{4m^2_\pi}^{W^2} dM_X^2 \frac{d^2\sigma^{\gamma N \rightarrow XN}/dM_X^2}{dM_X^2} \bigg|_{t=0} \mathcal{F}_d(k_L). \] (5)

The diffractive excitation of heavy hadronic states $X$ requires a large momentum transfer $k_L$, which is however suppressed by the longitudinal deuteron form factor $\mathcal{F}_d$. Replacing the diffractive cross section in (5) by (3) yields the contribution of low mass $\pi^+\pi^-$ pairs which dominates the shadowing effect [10].

To proceed further a good representation of the $\pi^+\pi^-$ contribution to the photon spectral function (4) is needed. The latter is related to the pion form factor $F_\pi$ as follows:

\[ \Pi(M_{\pi\pi}^2) = \frac{1}{48\pi^2} \Theta(M_{\pi\pi}^2 - 4m_\pi^2) \left(1 - \frac{4m_\pi^2}{M_{\pi\pi}^2}\right)^{3/2} \left|F_\pi(M_{\pi\pi}^2)\right|^2. \] (6)

At timelike four-momenta the pion form factor is dominated by the $\rho$-meson resonance. An improved representation of the pion form factor, derived recently [7], gives perfect agreement with the measured form factor. It is based on an effective Lagrangian which combines vector meson dominance and chiral dynamics. The result is [7]:

\[ F_\pi(q^2) = \left(1 - \frac{g_{\rho\pi\pi}}{g_\rho(q^2)} q^2 - m_\rho^2 + im_\rho \Gamma_\rho(q^2)\right) \left(1 + \frac{g_\rho(q^2)}{g_\omega} \frac{\omega}{q^2 - m_\omega^2 + im_\omega \Gamma_\omega}\right). \] (7)

The first term in (7) involves the dominant $\rho$-meson contribution. The width

\[ \Gamma_\rho = \frac{g_{\rho\pi\pi}^2}{48\pi m_\rho \sqrt{q^2}} (q^2 - 4m_\pi^2)^{3/2} \] (8)

reflects the strong coupling of the $\rho$-meson to the $\pi^+\pi^-$ continuum, with $g_{\rho\pi\pi} = 6.05$. The effective $\gamma\rho$ coupling with inclusion of vertex corrections due to the $\pi\pi$ loop is:

\[ \frac{1}{g_\rho(q^2)} = \frac{1}{g_\rho^0} - \frac{m_\rho^2 - \frac{\omega^2}{g_\omega} - im_\rho \Gamma_\rho(q^2)}{g_{\rho\pi\pi} q^2}. \] (9)

The bare and physical $\rho$-meson masses are $m_\rho^0 = 0.81\ GeV$ and $m_\rho = 0.775\ GeV$, respectively. Their difference comes from the real part of the $\rho \rightarrow \pi\pi$ self-energy as explained
in details in ref.\[7\]. The bare $\gamma\rho$ coupling constant is fixed as $g_\rho = 5.44$ to reproduce the $\rho \to e^+e^-$ partial width. The second term in eq.\(7\) yields a fine tuning of $F_\pi$ due to $\rho-\omega$-mixing, with $m_\omega = 0.782\, GeV$, $g_\omega = 17.0$, $\Gamma_\omega = 8.4\, MeV$ and $z_{\rho\omega} = -4.52 \cdot 10^{-3}\, GeV^2$. In Fig.1 the resulting pion form factor from \(7\) is shown together with data from ref.\[12\]. The agreement is evidently very satisfactory.

With eqs.\((4,6,7)\) we have calculated the $\pi^+\pi^-$ mass distribution $d\sigma/dM_{\pi\pi}$. For a comparison with recent data of the ZEUS collaboration \[3\], taken at an average center of mass energy $W = 70\, GeV$, we use the measured value of the slope parameter $b \approx 10\, GeV^{-2}$. With an effective $\pi^+\pi^-$-nucleon cross section $\sigma_{\pi\pi N} = 30\, mb$ the main features of the observed mass distribution are reproduced as shown in Fig.2. We conclude that the mass distribution of diffractively produced $\pi^+\pi^-$ pairs is indeed determined primarily by the two-pion component of the photon spectral function, or equivalently, by the pion form factor in the timelike momentum region. Note again that the latter already includes a substantial contribution from non-resonant pion pairs. Corrections due to a possible mass dependence of $\sigma_{\pi\pi N}$, or from non-diagonal inelastic corrections, are evidently small. To quantify their possible size we fit the experimental data by adding the following correction to the diffractive cross section \(4\):

$$
\sigma_{\pi\pi N}^2 \to \sigma_{\pi\pi N}^2 \left(1 + c \frac{m_\rho^2 - M_{\pi\pi}^2}{M_{\pi\pi}^2} \right),
$$

(10)

with $c = 0.6$. This fit is shown as the dashed curve in Fig.2. The correction in \(10\) is substantially smaller than in previous fits to $\pi^+\pi^-$ photoproduction data (see \[3,13\] and references therein). The reason for this improvement is mainly due to a proper treatment of the energy dependent width of the $\rho$-meson as discussed in detail in \[7\].

Finally we extend our considerations to diffractive leptoproduction processes at moderate $Q^2 = -q^2 \lesssim 1\, GeV^2$. It is an empirical fact that in this kinematic region the exclusive leptoproduction of $\rho$-mesons from nucleons is well described within the vector meson dominance picture \[1\], which gives:

$$
\frac{d\sigma_{\gamma N\to\pi^+\pi^- N}}{dM_{\pi\pi}}(Q^2) = \frac{d\sigma_{\gamma N\to\pi^+\pi^- N}}{dM_{\pi\pi}}(Q^2 = 0) \left(\frac{M_{\pi\pi}^2}{M_{\pi\pi}^2 + Q^2}\right)^2 \left(1 + \epsilon \xi^2 \frac{Q^2}{M_{\pi\pi}^2}\right),
$$

(11)

Here $\xi$ is the ratio of the longitudinal to transverse $\pi^+\pi^-$-nucleon forward amplitudes and $\epsilon$ measures the longitudinal polarization of the virtual photon. We investigate the diffractive cross section \(11\) in the kinematic range of recent measurements performed by the E665 collaboration at Fermilab \[1\]. Here the average center of mass energy is $W = 15\, GeV$. Consequently we have to re-scale the previously determined value for the effective $\pi^+\pi^-$-nucleon cross section: $(\sigma_{\pi\pi N})_{E665} \approx (\sigma_{\pi\pi N})_{ZEUS} \cdot (W_{E665}/W_{ZEUS})^{0.16} \approx 24\, mb$. Furthermore we use $\xi^2 = 0.5$ \[1\] and $\epsilon = 0.9$. \[4\]. In Fig.3 we show the results for the diffractive cross section \(11\) for different values of $Q^2$, including the correction term from \(10\). As expected, the diffractive cross section decreases rapidly with rising $Q^2$. Furthermore the mass distribution approaches a more symmetric shape as $Q^2$ increases.
Finally we study the ratio of the diffractive to the inelastic scattering cross section at moderate $Q^2$:
\[
\frac{\sigma_{\text{diff}}}{\sigma_{\text{inel}}} = \frac{1}{\sigma_{\gamma^* N}} \int_{4m^2_\pi}^{1 GeV^2} dM_{\pi \pi} \frac{d\sigma_{\gamma^* N \rightarrow \pi^+ \pi^- N}}{dM_{\pi \pi}}.
\] (12)

At $W = 15 GeV$ and $Q^2 = 0$ we obtain $\sigma_{\text{diff}}/\sigma_{\text{inel}} \approx 0.1$. With increasing $Q^2$ we observe a decrease of diffractive events compared to inelastic ones. For example at $Q^2 = 1 GeV^2$ we find $\sigma_{\text{diff}}/\sigma_{\text{inel}} \approx 0.06$.

In summary, we find that the mass distribution of $\pi^+ \pi^-$ pairs in diffractive photoproduction is determined to a large extent by the two-pion contribution to the photon spectral function as given by the pion form factor. Corrections due to a possible mass dependence of the effective $\pi^+ \pi^-$–nucleon cross section, or from non-diagonal inelastic processes, are small. When applied to diffractive leptoproduction at moderate $Q^2$ we observe a rapid decrease of the production cross section, while the mass distribution of $\pi^+ \pi^-$ pairs approaches a symmetric shape with rising $Q^2$. As $Q^2$ increases we find a decrease of diffractive events as compared to inelastic ones.

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Figure Captions

Figure 1: The pion form factor $F_\pi(q^2)$ in the region of timelike $q^2$. The data are from ref. [12]. The solid line shows the result using eq. (7).

Figure 2: The mass distribution $d\sigma/dM_{\pi\pi}$. The data are from ref. [3]. The solid line is the result using eq. (4). The dashed line includes the correction in eq. (10).

Figure 3: The mass distribution $d\sigma/dM_{\pi\pi}$ for different values of $Q^2$ for $W = 15 GeV$ and $\epsilon = 0.9$. 

