Gravitomagnetism and the Earth-Mercury range

Lorenzo Iorio

Ministero dell’Istruzione, dell’Università e della Ricerca (M.I.U.R.)-Istruzione
International Institute for Theoretical Physics and High Mathematics
Einstein-Galilei

Abstract

We numerically work out the impact of the general relativistic Lense-Thirring effect on the Earth-Mercury range $|\vec{\rho}|$ caused by the gravitomagnetic field of the rotating Sun. The peak-to-peak nominal amplitude of the resulting time-varying signal amounts to $1.75 \times 10^{11}$ m over a temporal interval $\Delta t = 2$ yr. Future interplanetary laser ranging facilities should reach a cm-level in ranging to Mercury over comparable timescales; for example, the BepiColombo mission, to be launched in 2014, should reach a $4.5 - 10$ cm level over $1 - 8$ yr. We looked also at other Newtonian (solar quadrupole mass moment, ring of the minor asteroids, Ceres, Pallas, Vesta, Trans-Neptunian Objects) and post-Newtonian (gravitoelectric Schwarzschild solar field) dynamical effects on the Earth-Mercury range. They act as sources of systematic errors for the Lense-Thirring signal which, in turn, if not properly modeled, may bias the recovery of some key parameters of such other dynamical features of motion. Their nominal peak-to-peak amplitudes are as large as $4 \times 10^5$ m (Schwarzschild), $3 \times 10^2$ m (Sun’s quadrupole), $8 \times 10^1$ m (Ceres, Pallas, Vesta), 4 m (ring of minor asteroids), $8 \times 10^{-1}$ m (Trans-Neptunian Objects). Their temporal patterns are different with respect to that of the gravitomagnetic signal.

Key words: Experimental tests of gravitational theories, Ephemerides, almanacs, and calendars, Lunar, planetary, and deep-space probes

---

Email address: lorenzo.iorio@libero.it (Lorenzo Iorio).
URL: http://digilander.libero.it/lorri/homepage_of_lorenzo_iorio.htm (Lorenzo Iorio).

1 Address for correspondence: Viale Unità di Italia 68, 70125, Bari (BA), Italy
1 Introduction

In its slow-motion and weak-field approximation, the Einsteinian General Theory of Relativity (GTR) predicts that a slowly rotating central body of mass $M$ and proper angular momentum $\vec{S}$ induces two kinds of small perturbations on the otherwise Keplerian motion of a test particle orbiting it. The largest one is dubbed “gravitoelectric” (GE) (Mashhoon, 2007), and depends only on the mass $M$ of the body which acts as source of the gravitational field. It is responsible of the well-known anomalous secular precession of the perihelion $\omega$ of Mercury of 42.98 arcsec cty$^{-1}$ in the field of the Sun. There is also a smaller perturbation, known as “gravitomagnetic” (GM) (Mashhoon, 2007), which depends on $\vec{S}$: it causes the Lense-Thirring (LT) precessions of the node $\Omega$ and pericenter $\omega$ of a test particle (Lense & Thirring, 1918). More specifically, the Parameterized Post-Newtonian (PPN) perturbing acceleration $\vec{A}_{PPN}$ to be added to the Newtonian monopole $\vec{A}_{\text{Newton}} = -GM\dot{r}/r^2$ in the equations of motion

$$\frac{d^2\vec{r}}{dt^2} = \vec{A}_{\text{Newton}} + \vec{A}_{\text{PPN}}$$

is (Soffel, 1989, p. 89, p.95), (McCarthy & Petit, 2004, p. 106)

$$\vec{A}_{\text{PPN}} = -\vec{E}_g - 2\left(\frac{\vec{v}}{c}\right) \times \vec{B}_g.$$  \hspace{1cm} (2)

In it (Soffel, 1989, p. 89, p.95), (McCarthy & Petit, 2004, p. 106),

$$\vec{E}_g = -\frac{GM}{c^2 r^3} \left\{ \left[\frac{2(\beta+\gamma)GM}{r} - \gamma v^2 \right] \vec{r} + 2(1 + \gamma) (\vec{r} \cdot \vec{v}) \frac{\vec{v}}{c} \right\},$$ \hspace{1cm} (3)

$$\vec{B}_g = -\left(\frac{1+\gamma}{2}\right) \frac{G}{c} \left[ \vec{S} - 3 \left(\vec{S} \cdot \vec{r}\right) \vec{r} \right],$$

where $\gamma, \beta$ are the usual PPN parameters (Will, 1993) which are both 1 in GTR, $c$ denotes the speed of light in vacuum, $G$ is the Newtonian constant of gravitation, and $\vec{v}$ is the velocity of the test particle moving at distance $r$ from $M$; the unit vector $\dot{r}$ is directed from the central body to the test particle. In eq. (2)-eq. (3) $\vec{E}_g$ is the GE field, while $\vec{B}_g$ is the GM one. The resulting GM
precessions are, in GTR, (Lense & Thirring, 1918)

\[
\dot{\Omega}_{LT} = \frac{2G\Sigma}{c^2a^3(1-e^2)^{3/2}},
\]

\[
\dot{\omega}_{LT} = -\frac{6G\Sigma \cos I}{c^2a^3(1-e^2)^{3/2}},
\]

where \(a, e, I\) are the semi-major axis, the eccentricity and the inclination to the equatorial plane of the central body, respectively, of the orbit of the test particle. The PPN expressions of the GM node and pericenter precessions have a multiplicative \((1 + \gamma)/2\) factor in front of \(\dot{\Omega}_{LT}\) and \(\dot{\omega}_{LT}\) (Peterson, 1997, p. 40).

In regard to the LT effect, at present, some attempts to measure it in the gravitational fields of the Earth and Mars have been performed with the LAGEOS (Ciufolini et al., 2009) and Mars Global Surveyor (Iorio, 2006) artificial satellites in a series of tests exploiting non-dedicated spacecraft in an “opportunistic” approach. Their status is somewhat uncertain, and the realistic evaluation of the accuracy reached in such tests is matter of controversy (Krogh, 2007; Iorio, 2009; Ciufolini et al., 2009; Iorio, 2010) The data analysis (Everitt et al., 2009) of the Gravity Probe B (GP-B) mission (Everitt et al., 2001), aimed to directly measure another GM effect, i.e. the Schiff (1960) precession of the spin of an orbiting gyroscope, in a dedicated spacecraft-based experiment orbiting the Earth, has recently released its final results (Everitt et al., 2011); it was proposed for the first time in 1961 (Fairbank & Schiff, 1961). Anyway, it will likely not be possible to repeat such an experiment in any foreseeable future because of its extreme sophistication and cost.

Concerning the Sun and its planets, it can be said that, contrary to the planet-spacecraft scenarios, the systematic errors caused by competing classical forces are of relatively less concern with respect to the measurement errors which, at present, represent the major obstacle in measuring the solar LT effect. Indeed, the non-gravitational perturbations are absent, while the gravitational ones are relatively well known and easily to model. Suffices it to say that, in contrast with, e.g., the Earth, only one even zonal harmonic coefficient \(J_2\) of the multipolar expansion of the non-spherical part of its Newtonian gravitational potential has to be taken into account for the Sun. For a long time since its prediction in 1918 within GTR, the LT precessions have been retained too small to be detected with planetary motions. Indeed, by assuming a homogeneous and uniformly rotating Sun, de Sitter (1916) obtained a value of \(-0.01\) arcsec cyt\(^{-1}\) for Mercury in his pioneering work in which he preliminarily worked out the effects of the solar rotation on the planetary perihelia within GTR limiting to ecliptic orbits only; such a figure is also quoted by Soffel (1989, p. 111). With the same assumptions concerning
the rotation of the Sun, Cugusi and Proverbio (1978) obtained $-0.02$ arcsec ct"y$^{-1}$; as a consequence, all such authors concluded that, at their time, it was not possible to measure the solar LT effect; Soffel (1989, p. 23), e.g., reports failed attempts to detect the LT effect by fitting planetary data with modern numerically produced PPN ephemerides. Nowadays, the expected magnitude of the LT planetary precessions is even smaller than before. It is so because recent measurements of the Sun’s proper angular momentum

$$S_\odot = (190.0 \pm 1.5) \times 10^{39} \text{ kg m}^2 \text{ s}^{-1}$$

from helioseismology (Pijpers, 1998, 2003), accurate to 0.8%, yield a value about one order of magnitude smaller than that obtained by assuming a homogeneous and uniformly rotating Sun. Despite such new findings, the perspectives of measuring the GM field of the Sun from planetary motions are more favorable now than in the past. This can be easily recognized from the following simple arguments. The characteristic length with which the accuracy of the determination of the orbits of the particles should be compared is

$$l_g^\odot \doteq \frac{S_\odot}{M_\odot c} = 319 \text{ m};$$

in the case of the GE effects, $l_g$ is usually replaced by the Schwarzschild radius $R_g \doteq 2GM/c^2 = 3 \text{ km}$ for the Sun. The present-day accuracy in knowing, e.g., the inner planets’ mean orbital radius

$$\langle r \rangle = a \left(1 + \frac{e^2}{2}\right),$$

is shown in Table 1. Such values have been obtained by linearly propagating the formal, statistical errors in $a$ and $e$ according to Table 3 of Pitjeva (2008); even by re-scaling them by a a factor of, say, $2 - 5$, the GM effects due to the Sun’s rotation fall, in principle, within the measurability domain. Another possible way to evaluate the present-day uncertainty in the planetary orbital motions consists of looking at different ephemerides of comparable accuracy. In Table 1 we do that for the EPM2006/EPM2008 (Pitjeva, 2008, 2010), and the DE414/DE421 (Standish, 2006; Folkner et al., 2008) ephemerides; although, larger than $\delta \langle r \rangle$, the maximum differences between such ephemerides are smaller than the solar GM length $l_g^\odot$.

Here we will focus on the Earth-planet range $|\vec{r}|$ because it is a direct, unambiguous observable for which great improvements are expected in the near future.

The paper is organized as follows. In Section 2 we deal with the numerical calculation of the impact of several dynamical perturbations on the Earth-
Mercury range after having reviewed the extent of the expected future improvements in measuring it. In particular, Section 2.1 and Section 2.2 are devoted to the general relativistic gravitoelectromagnetic fields of the Sun. In Section 2.3-Section 2.5 the effects of the Newtonian perturbations due to the solar oblateness, the ring of the minor asteroids, Ceres, Pallas and Vesta, and the TNOs are investigated. The conclusions are in Section 3.

2 The interplanetary ranging

Recent years have seen increasing efforts towards the implementation of the Planetary Laser Ranging (PLR) technique accurate to cm-level (Chandler et al., 2005; Smith et al., 2006; Neumann et al., 2006; Degnan, 2006; Turyshev & Williams, 2007; Merkowitz et al., 2007; Degnan, 2008; Zuber & Smith, 2008). It would allow to reach major improvements in three related fields: Solar System dynamics, tests of GTR and alternative theories of gravity, and physical properties of the target planet itself. In principle, any Solar System body endowed with a solid surface and a transparent atmosphere would be a suitable platform for a PLR system, but some targets are more accessible than others. Major efforts have been practically devoted so far to Mercury (Smith et al., 2006) and Mars (Chandler et al., 2005; Turyshev & Williams, 2007), although simulations reaching 93 au or more have been undertaken as well (Degnan, 2006, 2008). In 2005 two interplanetary laser transponder experiments were successfully demonstrated by the Goddard Geophysical Astronomical Observatory (GGAO). The first utilized the non-optimized Mercury Laser Altimeter (MLA) on the Messenger spacecraft (Smith et al., 2006; Neumann et al., 2006), obtaining a formal error in the laser range solution of 0.2 m, or one part in $10^{-11}$. The second utilized the Mars Orbiting Laser Altimeter (MOLA) on the Mars Global Surveyor spacecraft (Abshire et al., 2006; Neumann et al., 2006). A precise measure of the Earth-Mars distance, measured between their centers of mass and taken over an extended period (five years or more), would support, among other things, a better determination of several parameters of the Solar System. Sensitivity analyses point towards measurement uncertainties between 1 mm and 100 mm (Chandler et al., 2005). Concerning Mercury, a recent analysis on the future BepiColombo mission, aimed to accurately determining, among other things, several key parameters of post-Newtonian gravity and the solar quadrupole moment from Earth-Mercury distance data collected with a multi-frequency radio link (Milani et al., 2002, 2010), points toward a maximum uncertainty of 4.5 – 10 cm in determining the Earth-Mercury range over a multi-year time span (1-8 yr) (Milani et al., 2002; Ashby

\[2\] It is an ESA mission, including two spacecraft, one of which provided by Japan, to be put into orbit around Mercury. The launch is scheduled for 2014. The construction of the instruments is currently ongoing.
et al., 2007; Milani et al., 2010). A proposed spacecraft-based mission aimed to accurately measure also the GM field of the Sun and its $J_2$ along with other PPN parameters like $\gamma$ and $\beta$ by means of interplanetary ranging is the Astrodynamical Space Test of Relativity using Optical Devices\(^3\) (ASTROD) (Ni, 2008). Another space-based mission proposed to accurately test several aspects of the gravitational interaction via interplanetary laser ranging is the Laser Astrometric Test of Relativity (LATOR) (Turyshev et al., 2009); the GM field of the Sun is one of its goals.

We will concentrate on Mercury both because the magnitude of the GM signal is the largest one and in view of the aforementioned expected improvements in the accuracy in the Earth-Mercury ranging. At present, the 1-way range residuals of Mercury from radar-ranging span 30 yr (1967-1997) and are at a few km-level (Figure B-2 of Folkner et al. (2008)); the same holds for the 1-way Mercury radar closure residuals covering 8 yr (1989-1997, Figure B-3 a) of Folkner et al. (2008)). There are also a pair of Mariner 10 range residuals in the 70s at Mercury at 0.2 km level (Figure B-3 b) of Folkner et al. (2008)).

Here we outline the general approach followed. In order to numerically obtain the effect of a given gravitational perturbation P of the Newtonian Sun’s monopole on the range $|\rho|$ between the Earth-Moon Barycenter (EMB) and a given planet, we used MATHEMATICA to simultaneously integrate with the Runge-Kutta method the equations of motion in Cartesian coordinates of EMB and the planet considered with and without the perturbation P by using the same set of initial conditions. We adopted the ICRF/J2000.0 reference frame, with the ecliptic and mean equinox of the reference epoch, i.e. J2000, centered at the Solar System Barycenter (SSB); the initial conditions at the epoch J2000 were retrieved with the HORIZONSWEB interface by JPL, NASA. The temporal interval of the numerical integration has been taken equal to $\Delta t = 2$ yr because most of the present-day available time series of range residuals approximately cover similar temporal extensions; moreover, also the typical operational time spans forecasted for future PLR technique are similar. The basic model adopted consists of the barycentric equations of motion of the Sun, the eight planets, the Moon, Ceres, Pallas, Vesta, Pluto and Eris, to be simultaneously integrated; the forces acting on them include the mutual Newtonian $N$–body interactions, the perturbation due to the solar quadrupolar mass moment $J_2$, the effect of two rings modeling the actions of the minor asteroids and of the Trans-Neptunian Objects (TNOs), and the post-Newtonian GEM field of the Sun (with its general relativistic value).

In order to preliminarily evaluate the potential measurability of the effects

\(^3\) Its cheaper version ASTROD I makes use of one spacecraft in a Venus-gravity-assisted solar orbit, ranging optically with ground stations (Appourchaux et al., 2009).
considered, the computed differences $\Delta |\vec{\rho}| = |\vec{\rho}_P| - |\vec{\rho}_R|$, where $R$ refers to a reference orbit which does not contain the perturbation $P$, were subsequently compared to the available time series of the range residuals for the inner planets which set the present-day accuracy level in ranging to planets (Folkner et al., 2008; Pitjeva, 2010). About the possibility that a given, unmodeled dynamical effect may show or not its signature in the range residuals, it must be considered that the magnitude of such an effect should roughly be one order of magnitude larger than the range residuals accuracy. This to avoid the risk that it may be absorbed and partially or totally removed from the signature in the process of estimation of the initial conditions and of the other numerous solve-for parameters in the real data reduction.

Depending on the dynamical effect one is interested in, some of the perturbations examined here are to be considered as sources of noise inducing systematic bias on the target signal. For example, if the goal of the analysis is, as in our case, the LT effect, then the range perturbation due to the TNOs is clearly a source of potential systematic error which has to be evaluated. Thus, our plots are useful to assess the level of aliasing of several potential sources of aliasing for some non-Newtonian effects and the correlations that may occur in estimating them. Dynamical effects which are viewed as noise in a given context can also be regarded as main targets in another one; see, e.g., the proposed determination of asteroid masses through the ASTROD mission (Su et al., 1999). In this respect, the LT signature, if not properly modeled, should be regarded as a source of potential bias.

2.1 The gravitomagnetic, Lense-Thirring field of the Sun

Figure 1 depicts the Earth-Mercury range perturbation due to the Sun’s GM field, neither considered so far in the dynamical force models of the planetary ephemerides nor in the BepiColombo analyses. It is important to note that the value of eq. (5) for the Sun’s angular momentum does not come from planetary orbital dynamics, so that there is no risk of a-priori “imprinting” of GTR itself on range tests of the solar LT effect which could, thus, be regarded as genuine and unbiased.

As a small technical note, we mention that we rotated the reference frame to the mean ecliptic at the epoch to the Sun’s equator by the Carrington angle $i = 7.15$ deg (Beck & Giles, 2005) because eq. (3) holds in a frame with its $z$ axis aligned with $\vec{S}$.

The peak-to-peak amplitude of the LT signal is up to 17.5 m over 2 yr, which, if on the one hand is unmeasurable from currently available radar-ranging to Mercury, on the other hand corresponds to a potential relative
Fig. 1. Difference $\Delta |\vec{\rho}| = |\vec{\rho}_P| - |\vec{\rho}_R|$ in the numerically integrated EMB-Mercury ranges with and without the perturbation due to the Sun’s GM field over $\Delta t = 2$ yr. The same initial conditions (J2000) have been used for both the integrations. The state vectors at the reference epoch have been retrieved from the NASA JPL Horizons system. The integrations have been performed in the ICRF/J2000.0 reference frame, with the reference $\{xy\}$ plane rotated from the mean ecliptic of the epoch to the Sun’s equator, centered at the Solar System Barycenter (SSB).

accuracy in measuring it with BepiColombo of $2 - 5 \times 10^{-5}$; this clearly shows that the solar GM field should be taken into account in future analyses and data processing. Otherwise, it may alias the recovery of other effects, as it will become clearer later.

2.2 The gravitoelectric, Schwarzschild field of the Sun

In Figure 2 we plot the effect of the GE field of the Sun on the Earth-Mercury range.

Figure 2 can be compared with Figure 1 of Milani et al. (2010), obtained for unspecified initial conditions$^4$: they are quite similar.

The maximum variation of the signal is of the order of $4 \times 10^5$ m, corresponding to a measurement accuracy of about $2.5 \times 10^{-7}$. The expected realistic accuracy in determining $\beta$ and $\gamma$ is $2 \times 10^{-6}$ in BepiColombo (Milani et al., 2002). To this aim, let us note that, since the LT effect depends on $\gamma$, neglecting it may alias the determination of $\gamma$ through the larger GE signal at $4 \times 10^{-5}$ level.

$^4$ It also includes the Shapiro delay contribution.
Fig. 2. Difference \( \Delta |\vec{\rho}| = |\vec{\rho}_P| - |\vec{\rho}_R| \) in the numerically integrated EMB-Mercury ranges with and without the perturbation due to the Sun’s Schwarzschild field over \( \Delta t = 2 \) yr. The same initial conditions (J2000) have been used for both the integrations. The state vectors at the reference epoch have been retrieved from the NASA JPL Horizons system. The integrations have been performed in the ICRF/J2000.0 reference frame, with the ecliptic and mean equinox of the reference epoch, centered at the Solar System Barycenter (SSB).

2.3 The Newtonian effect of the oblateness of the Sun

Figure 3 shows the nominal effect of the Sun’s quadrupolar mass moment on the Earth-Mercury range for \( J_2 = 2 \times 10^{-7} \). Its action has been modeled as (Vrbik, 2005)

\[
A_{J_2} = -\frac{3J_2 R^2 GM}{2r^4} \left\{ \left[ 1 - 5 \left( \hat{r} \cdot \hat{k} \right)^2 \right] \hat{r} + 2 \left( \hat{r} \cdot \hat{k} \right) \hat{k} \right\},
\]

(8)

where \( R \) is the Sun’s mean equatorial radius and \( \hat{k} \) is the unit vector of the \( z \) axis directed along the body’s rotation axis.

Since eq. (8) holds in a frame with its \( \{xy\} \) plane coinciding with the body’s equator, also in this case we rotated the the mean ecliptic at the epoch to the Sun’s equator.

The signal of Figure 3 has a maximum span of 300 m, corresponding to an accuracy measurement of \( 3 \times 10^{-4} \). A determination of the solar \( J_2 \) accurate to \( 10^{-2} \) is one of the goals of BepiColombo (Milani et al., 2002); knowing precisely \( J_2 \) would yield important insights on the internal rotation of the Sun. At present, the uncertainty in it is about 10% (Fienga et al., 2010). The solar quadrupole mass moment may play the role of source of systematic bias with respect to, e.g., some non-Newtonian dynamical effects. Concerning the GE signal previously analyzed, the mismodeled \( J_2 \) signature would impact it
Fig. 3. Difference $\Delta |\vec{\rho}| = |\vec{\rho}_P| - |\vec{\rho}_R|$ in the numerically integrated EMB-Mercury ranges with and without the nominal perturbation due to the Sun’s quadrupole mass moment $J_2 = 2.0 \times 10^{-7}$ over $\Delta t = 2$ yr. The same initial conditions (J2000) have been used for both the integrations. The state vectors at the reference epoch have been retrieved from the NASA JPL Horizons system. The integrations have been performed in the ICRF/J2000.0 reference frame, with the reference $\{xy\}$ plane rotated from the mean ecliptic of the epoch to the Sun’s equator, centered at the Solar System Barycenter (SSB).

a $7.5 \times 10^{-5}$ level. It is important to note that the patterns of the two signals are rather different. Conversely, the determination of $J_2$ at the desired level of accuracy may be affected by other unmodeled/mismeasured dynamical effects acting as systematic sources of aliasing on it. For example, the GM signature may affect the determination of $J_2$ at 12% level. On the other hand, in order to allow for a determination of the LT effect itself, the Sun’s quadrupole mass moment should be known with an accuracy better than the present-day one by at least one order of magnitude; this is just one of the goals of BepiColombo. Anyway, also the GM and the $J_2$ patterns are different.

2.4 The Newtonian effect of the ring of the minor asteroids and Ceres, Pallas and Vesta

In Figure 4 we depict one potential source of systematic bias, i.e. the action of the ring of minor asteroids (Fienga et al., 2010). We modeled it following Fienga et al. (2008). In general, for those planets for which $r < R_{\text{ring}}$, by posing $\alpha = r/R_{\text{ring}}$, we obtained

$$\vec{A}_{\text{outer ring}} \simeq \frac{Gm_{\text{ring}}}{2rR_{\text{ring}}^2} \left( \alpha + \frac{9}{8} \alpha^3 + \frac{75}{64} \alpha^5 \right) \vec{r},$$  \hspace{1cm} (9)
Fig. 4. Difference $\Delta |\vec{\rho}| = |\vec{\rho}_P| - |\vec{\rho}_R|$ in the numerically integrated EMB-Mercury ranges with and without the nominal perturbation due to the ring of minor asteroids with $m_{\text{ring}} = 1 \times 10^{-10} M_\odot$ (Fienga et al., 2010) and $R_{\text{ring}} = 3.14$ au (Fienga et al., 2010) over $\Delta t = 2$ yr. The same initial conditions (J2000) have been used for both the integrations. The state vectors at the reference epoch have been retrieved from the NASA JPL Horizons system. The integrations have been performed in the ICRF/J2000.0 reference frame, with the ecliptic and mean equinox of the reference epoch, centered at the Solar System Barycenter (SSB).

Recall that the Laplace coefficients are defined as

$$b_s^{(j)}(\alpha) \doteq \frac{1}{\pi} \int_0^{2\pi} \cos j\psi d\psi \left[ \frac{1}{(1 - 2\alpha \cos \psi + \alpha^2)^s} \right], \quad (11)$$

where $s$ is a half-integer; a useful approximate expression in terms of a series can be found in Murray & Dermott (1999, p. 237)

$$b_s^{(j)} \simeq \frac{s(s+1)\ldots(s+j-1)}{1\cdot3\ldots(j-1)} \alpha^j \left[ 1 + \frac{s(s+j)}{(1+j)} \alpha^2 + \frac{s(s+1)(s+j)(s+j+1)}{1\cdot2(j+1)(j+2)} \alpha^4 \right]. \quad (12)$$

By assuming for the ring of the minor asteroids a nominal mass of $m_{\text{ring}} = 1 \times 10^{-10} M_\odot$ (Fienga et al., 2010) and a radius $R_{\text{ring}} = 3.14$ au (Fienga et al., 2010), it would impact the Mercury range at 4 m level (peak-to-peak amplitude), which will be, in fact, measurable. Its nominal bias on the
Schwarzschild, $J_2$ and LT signals would be $1 \times 10^{-5}, 1.3 \times 10^{-2}, 2.3 \times 10^{-1}$, respectively. Anyway, the present-day level of uncertainty in the mass of the ring is $\delta m_{\text{ring}} = 0.3 \times 10^{-10} M_\odot$ (Fienga et al., 2010). Thus, the impact of such a mismodeling would be, $3 \times 10^{-6}, 4 \times 10^{-3}, 7 \times 10^{-2}$, respectively; it cannot be considered negligible.

The effect of Ceres, Pallas and Vesta on the determination of some Newtonian and non-Newtonian parameters with BepiColombo has been preliminarily investigated in Ashby et al. (2007). Here in Figure 5 we show the nominal perturbation on the Earth-Mercury range due to the combined actions of Ceres, Pallas and Vesta; the values for their masses have been retrieved from Pitjeva & Standish (2009). Its peak-to-peak amplitude amounts to 80 m; thus, their signature would be measurable at a $0.6 - 1 \times 10^{-3}$ level. Anyway, the modeled solar quadrupole mass moment would bias their signal at $4 \times 10^{-1}$ level. The LT effect, if unmodeled, would have an impact at $2.2 \times 10^{-1}$ level. The present-day relative uncertainties in their masses are $6 \times 10^{-3}, 3 \times 10^{-2}, 2 \times 10^{-2}$ respectively (Pitjeva & Standish, 2009). This implies a mismodeled signal with a peak-to-peak amplitude of 50 cm. It would impact the Schwarzschild, $J_2$ and LT range perturbations at $1 \times 10^{-6}, 2 \times 10^{-3}, 3 \times 10^{-2}$ level, respectively.
2.5 The Newtonian effect of the Trans-Neptunian Objects

The situation is different for another potential source of systematic uncertainty, i.e. the Trans-Neptunian Objects (TNOs). Figure 6, obtained by modeling them as a ring with $m_{\text{ring}} = 5.26 \times 10^{-8} M_\odot$ (Pitjeva, 2010) and $R_{\text{ring}} = 43$ au (Pitjeva, 2010), shows that their maximum effect would amount to 80 cm. We used the same formulas as for the asteroid ring. Such an effect, not taken into account so far, would be better measurable than that by the minor asteroids. This implies a bias of $2 \times 10^{-6}$ on the Schwarzschild signal, $3 \times 10^{-3}$ for $J_2$ and $4.5 \times 10^{-2}$ for the Lense-Thirring effect. A major concern is that the mass of the TNOs is far from being accurately known, so that an uncertainty as large as 100% should be applied.

3 Summary and Conclusions

In view of a possible future implementation of some interplanetary laser ranging facilities accurate to cm-level ($4.5 - 10$ cm for BepiColombo in $1 - 8$ yr), we have numerically investigated how the ranges between the Earth and Mercury are affected by certain Newtonian and non-Newtonian dynamical effects by simultaneously integrating the equations of motion of all the major bodies of the Solar System plus some minor bodies of it (Ceres, Pallas, Vesta)
in the SSB reference frame over a time span two years long.

It turns out that the general relativistic gravitomagnetic Lense-Thirring effect of the Sun, not modeled so far either in the planetary ephemerides or in the analyses of some spacecraft-based future missions like, e.g., BepiColombo, does actually fall within the measurability domain of future cm-level ranging facilities. The more favorable situation occurs for Mercury because the relative measurement accuracy is of the order of $2 - 5 \times 10^{-3}$ by assuming a $4.5 - 10^{-5}$ cm uncertainty in the Earth-Mercury ranging, as expected for BepiColombo over some years of operations.

If not properly modeled and solved-for, the Lense-Thirring effect may also impact the determination of other Newtonian and post-Newtonian parameters at a non-negligible level, given the high accuracy with which their measurement is pursued. For example, in the case of BepiColombo the expected accuracy in determining $\gamma$ and $\beta$ from the range perturbation due to the Schwarzschild field of the Sun is of the order of $10^{-6}$; the Lense-Thirring range perturbation would impact the Schwarzschild one at $4 \times 10^{-5}$ level. Another goal of the BepiColombo mission is a measurement of the Sun’s quadrupole mass moment $J_2$ accurate to $10^{-2}$; the unmodeled Lense-Thirring effect would bias it at $10^{-1}$ level.

From the point of view of a measurement of the Sun’s gravitomagnetic field itself, it results that a major concern would be the solar oblateness; it should be known at a $10^{-2}$ level of accuracy-which is just the goal of BepiColombo-to allow for a reduction of its aliasing impact on the Lense-Thirring signal down to just 17%. The ring of the minor asteroids should be taken into account as well because its mismodeling would impact the gravitomagnetic signal at about $7 \times 10^{-2}$. The lingering uncertainty in the masses of Ceres, Pallas, Vesta translates into a potential bias of about $4.5 \times 10^{-2} - 10^{-3}$. The TNOs, not modeled so far apart from the EPM ephemerides, would nominally affect it at a $4.5 \times 10^{-2}$ level; it must be considered that there is currently a high uncertainty in their mass. However, it must be noted that the previous figures have been obtained by comparing the peak-to-peak amplitudes of the various time-dependent range signals. Actually, the time signatures of such sources of systematic bias are different with respect to the gravitomagnetic one; this would greatly help in disentangling it from the noisy effects.

Table 2 summarizes our findings.

Finally, we point out that the analysis presented here should be regarded just as a preliminary investigation. Actually, extensive simulations should be performed in which a gravitomagnetic-dedicated parameter should be estimated along with the wealth of other ones which are routinely solved-for in the data reduction procedures in order to check how the relativistic signal of
interest may be affected by the estimation of, say, the initial conditions of the planets/spacecraft whose data are processed. Indeed, it must be recalled that an unmodelled dynamical effect may be, partially or totally, absorbed in some of the estimated parameters, especially if its magnitude is not large enough with respect to the characteristic accuracy level of the observations. Anyway, the implementation of such a non-trivial task is beyond the scopes of the present paper, and may well constitute the subject for further researches.

References

Abshire, J.B., Sun, X., Neumann, G.A., McGarry, J., Zagwodzki, T.W., Jester, P., Riris, H., Zuber, M.T., & Smith, D.E., Laser Pulses from Earth Detected at Mars, Conference on Lasers and Electro-Optics (CLEO-06), Long Beach, California, May 2006.

Appourchaux, T., Burston, R., Chen, Y., Cruise, M., Dittus, H., Foulon, B., Gill, P., Gizon, L., Klein, H., Kliener, S., Kopeikin, S., Krüger, H., Lämmerzahl, C., Lobo, A., Luo, X., Margolis, H., Ni, W.-T., Patón, A.P., Peng, Q., Peters, A., Rasel, E., Rüdiger, A., Samain, É., Selig, H., Shaul, D., Sumner, T., Theil, S., Touboul, P., Turyshev, S.G., Wang, H., Wang, L., Wen, L., Wicht, A., Wu, J., Zhang, X., & Zhao, C., Astrodynamical Space Test of Relativity Using Optical Devices I (ASTROD I)A class-M fundamental physics mission proposal for Cosmic Vision 2015-2025, Experimental Astronomy, 23, 491-527, 2009.

Ashby, N., Bender, P., & Wahr, J.M., Future gravitational physics tests from ranging to the BepiColombo Mercury planetary orbiter, Phys. Rev. D, 75, 022001, 2007.

Beck, J.G., & Giles, P., Helioseismic determination of the solar rotation axis, Astrophys. J. Lett., 621, L153-L156, 2005.

Chandler, J.F., Pearlman, M.R., Reasenberg, R.D., & Degnan, J.J., Solar-System Dynamics and Tests of General Relativity with Planetary Laser Ranging, in: Noomen R., Davila, J.M., Garate, J., Noll, C., & Pearlman, M. (eds.) Proc. 14-th International Workshop on Laser Ranging, San Fernando, Spain, 2005, http://cddis.nasa.gov/lw14/docs/papers/sci7b_jcm.pdf.

Ciufolini, I., Paolozzi, A., Pavlis, E.C., Ries, J.C., Koenig, R., Matzner, R.A., Sindoni, G., & Neumayer, H., Towards a One Percent Measurement of Frame Dragging by Spin with Satellite Laser Ranging to LAGEOS, LAGEOS 2 and LARES and GRACE Gravity Models, Space Sci. Rev., 148, 71-104, 2009.

Cugusi, L., & Proverbio, E., Relativistic Effects on the Motion of Earth’s Artificial Satellites, Astron. Astrophys., 69, 321–325, 1978.

de Sitter, W., Einstein’s theory of gravitation and its astronomical consequences, Mon. Not. R. Astron. Soc., 76, 699728, 1916.

Degnan, J.J., Simulating Interplanetary Transponder and Laser Communications Experiments Via Dual Station Ranging To SLR Satellites in:
Degnan, J.J., Laser Transponders for High-Accuracy Interplanetary Laser Ranging and Time Transfer, in: Dittus, H., Lämmerzahl, C., & Turyshhev, S.G. (eds.) Lasers, Clocks and Drag-Free Control Exploration of Relativistic Gravity in Space, Springer Verlag (Berlin, 2008), pp. 231-242.

Everitt, C.W.F., Buchman, S., Debra, D.B., Keiser, G.M., Lockhart, J.M., Muhlfelder, B., Parkinson, B.W., Turneaure, J.P., & other members of the Gravity Probe B team, Gravity Probe B: Countdown to Launch, in: Lämmerzahl, C., Everitt, C.W.F. & Hehl, F.W. (eds.) Gyros, Clocks, Interferometers...: Testing Relativistic Gravity in Space, Springer Verlag (Berlin, 2001), pp. 52-82.

Everitt, C.W.F., Adams, M., Bencze, W., Buchman, S., Clarke, B., Conklin, J.W., Debra, D.B., Dolphin, M., Heifetz, M., Hipkins, D., Holmes, T., Keiser, G.M., Kolodziejczak, J., Li, J., Lipa, J., Lockhart, J.M., Mester, J.C., Muhlfelder, B., Ohshima, Y., Parkinson, B.W., Salomon, M., Silbergleit, A., Solomonik, V., Stahl, K., Taber, M., Turneaure, J.P., Wang, S., & Worden, P.W., Gravity Probe B Data Analysis, Space Sci. Rev., 148, 53-69, 2009.

Everitt, C. W. F., DeBra, D. B., Parkinson, B. W., Turneaure, J. P., Conklin, J. W., Heifetz, M. I., Keiser, G. M., Silbergleit, A. S., Holmes, T., Kolodziejczak, J., Al-Meshari, M., Mester, J. C., Muhlfelder, B., Solomonik, V., Stahl, K., Worden, P., Bencze, W., Buchman, S., Clarke, B., Al-Jadaan, A., Al-Jibreen, H., Li, J., Lipa, J. A., Lockhart, J. M., Al-Suwaidan, B., Taber, M., & Wang, S., Phys. Rev. Lett., Gravity Probe B: Final Results of a Space Experiment to Test General Relativity, 106, id 221101, 2011.

Fairbank, W.M., & Schiff, L.I., Proposed Experimental Test of General Relativity, Proposal to NASA, Stanford University (Stanford, 1961).

Fienga, A., Manche, H., Laskar, J., & Gastineau, M., INPOP06: a new numerical planetary ephemeris, Astron. Astrophys., 477, 315-327, 2008.

Fienga, A., Laskar, J., Kuchynka, P., Le Poncin-Lafitte, C., Manche, H., & Gastineau, M., Gravity tests with INPOP planetary ephemerides, in: Klioner, S.A., Seidelmann, P.K., & Soffel, M.H. (eds.) Relativity in Fundamental Astronomy: Dynamics, Reference Frames, and Data Analysis, Proceedings IAU Symposium No. 261, Cambridge University Press (Cambridge, 2010), pp. 159-169.

Folkner, W.M., Williams, J.G., & Boggs, D.H., The Planetary and Lunar Ephemeris DE 421, Memorandum IOM 343R-08-003, (California Institute of Technology, Jet Propulsion Laboratory, 2008).

Iorio, L., COMMENTS, REPLIES AND NOTES: A note on the evidence of the gravitomagnetic field of Mars, Class. Quantum Grav. 23, 5451-5454, 2006.

Iorio, L., An Assessment of the Systematic Uncertainty in Present and Future
Tests of the Lense-Thirring Effect with Satellite Laser Ranging, Space Sci. Rev., 148, 363-381, 2009.

Iorio, L., On the Lense-Thirring test with the Mars Global Surveyor in the gravitational field of Mars, Centr. Eur. J. Phys., 8, 509-513, 2010.

Krogh, K., COMMENTS, REPLIES AND NOTES: Comment on ‘Evidence of the gravitomagnetic field of Mars’, Class. Quantum Grav., 24, 5709-5715, 2007.

Lense, J., & Thirring, H., Über den Einfluss der Eigenrotation der Zentralkörper auf die Bewegung der Planeten und Monde nach der Einsteinschen Gravitationstheorie, Phys. Z., 19, 156-163, 1918.

Mashhoon, B., Gravitoelectromagnetism: A Brief Review, in: Iorio, L. (ed.) The Measurement of Gravitomagnetism: A Challenging Enterprise, Nova Publishers (Hauppauge, New York, 2007), pp. 29-39.

McCarthy, D.D., & Petit, G., IERS Conventions, Verlag des Bundesamtes für Kartographie und Geodäsie (Frankfurt am Main, 2004).

Merkowitz, S.M., Dabney, P.W., Livas, J.C., McGarry, J.F., Neumann, G.A., & Zagwodzki, T.W., Laser Ranging for Gravitational, Lunar and Planetary Science, Int. J. Mod. Phys. D, 16, 2151-2164, 2007.

Milani, A., Vokrouhlický, D., Bonanno, C., & Rossi, A., Testing general relativity with the bepicolombo radio science experiment, Phys. Rev. D, 66, 082001, 2002.

Milani, A., Tommei, G., Vokrouhlický, D., Latorre, E., & Cicalò, S., Relativistic models for the BepiColombo radioscience experiment, in: Klioner, S.A., Seidelmann, P.K, & Soffel M.H. (eds.) Relativity in Fundamental Astronomy: Dynamics, Reference Frames, and Data Analysis, Proceedings of the International Astronomical Union, IAU Symposium, Volume 261, Cambridge University Press (Cambridge, 2010), pp. 356-365.

Murray, C.D., & Dermott, S.F., Solar System Dynamics, Cambridge University Press (Cambridge, 1999).

Neumann, G., Cavanaugh, J., Coyle, B., McGarry, J., Smith, D., Sun, X., Zagwodzki, T., & Zuber, M.T., Laser Ranging at Interplanetary Distances, in: Proc. 15-th International Workshop on Laser Ranging, Canberra, Australia, 2006, http://cddis.gsfc.nasa.gov/1w15/docs/papers/Laser_Ranging_at_Interplanetary_Distances.pdf

Ni, W.-T., ASTROD and ASTROD I-Overview and Progress, Int. J. Mod. Phys. D, 17, 921-940, 2008.

Peterson, G.E., Estimation of the Lense-Thirring Precession Using Laser-Ranged Satellites, CSR-97-1, Center for Space Research (Austin, Texas, 1997).

Pijpers, F.P., Helioseismic determination of the solar gravitational quadrupole moment, Mon. Not. R. Astron. Soc., 297, L76-L80, 1998.

Pijpers, F.P., Asteroseismic determination of stellar angular momentum, Astron. Astrophys., 402, 683-692, 2003.

Pitjeva, E.V., Use of optical and radio astrometric observations of planets, satellites and spacecraft for ephemeris astronomy, in: Jin, W.J., Platais, I,
& Perryman, M.A.C. (eds.) *A Giant Step: from Milli- to Micro-arcsecond Astrometry*, Cambridge University Press (Cambridge, 2008), pp. 20-22.

Pitjeva, E.V., EPM ephemerides and relativity, in: Klioner, S.A., Seidelmann, P.K., & Soffel, M.H. (eds.) *Relativity in Fundamental Astronomy: Dynamics, Reference Frames, and Data Analysis, Proceedings IAU Symposium No. 261*, Cambridge University Press (Cambridge, 2010), pp. 170-178.

Pitjeva, E.V., & Standish, E.M., Proposals for the masses of the three largest asteroids, the Moon-Earth mass ratio and the Astronomical Unit, Celest. Mech. and Dyn. Astron., 103, 365-372, 2009.

Schiff, L.I., Possible New Experimental Test of General Relativity Theory, Phys. Rev. Lett., 4, 215-217, 1960.

Smith, D.E., Zuber, M.T., Sun, X., Neumann, G.A., Cavanaugh, J.F., McGarry, J.F., & Zagwodzki, T.W., Two-Way Laser Link over Interplanetary Distance, Science, 311, 53, 2006.

Soffel, M.H., *Relativity in Astrometry, Celestial Mechanics and Geodesy*, Springer Verlag (Berlin, 1989).

Standish, E.M., JPL Planetary and Lunar Ephemerides, DE414, *Interoffice Memo IOM 343R-06-002*, (California Institute of Technology, Jet Propulsion Laboratory, 2006).

Su, Z.-Y., Wu, A.-M., Lee, D., Ni, W.-T., & Lin, S.-C., Asteroid perturbations and the possible determination of asteroid masses through the ASTROD space mission, Planet. Space Sci., 47, 339-343, 1999.

Turyshev, S.G., & Williams, J.G., Space-Based Tests of Gravity with Laser Ranging, Int. J. Mod. Phys. D, 16, 2165-2179, 2007.

Turyshev, S.G., Shao, M., Nordtvedt, K., Dittus, H., Lämmerzahl, C., Theil, S., Salomon, C., Reynaud, S., Damour, T., Johann, U., Bouyer, P., Touboul, P., Foulon, B., Bertolami, O., & Páramos, J., Advancing fundamental physics with the Laser Astrometric Test of Relativity. The LATOR mission, Experimental Astronomy, 27, 27-60, 2009.

Vrbik, J., Zonal-Harmonics Perturbations, Celest. Mech. and Dyn. Astron., 91, 217-237, 2005.

Will, C.M., *Theory and experiment in gravitational physics. Revised edition*, Cambridge University Press (Cambridge, 1993).

Zuber, M.T., & Smith, D.E., One-Way Ranging to the Planets, in: *Proc. 16-th International Workshop on Laser Ranging, Pozna, Poland, 2008*, http://cddis.gsfc.nasa.gov/llr_9_Zuber.pdf
Table 1
First line: uncertainties (in m) in the average heliocentric distances of the inner planets obtained by propagating the formal errors in $a$ and $e$ according to Table 3 of Pitjeva (2008); the EPM2006 ephemerides were used by Pitjeva (2008). Second line: maximum differences (in m) between the EPM2006 and the DE 414 (Standish, 2006) ephemerides for the inner planets in the time interval 1960-2020 according to Table 5 of Pitjeva (2010). Third line: maximum differences (in m) between the EPM2008 (Pitjeva, 2010) and the DE 421 (Folkner et al., 2008) ephemerides for the inner planets in the time interval 1950-2050 according to Table 5 of Pitjeva (2010). They have to be compared with the characteristic gravitomagnetic length of the Sun $l_g^2 = 319$ m.

| Type of orbit uncertainty | Mercury | Venus | Earth | Mars |
|---------------------------|---------|-------|-------|------|
| $\delta \langle r \rangle$ (EPM2006) | 38      | 3     | 1     | 2    |
| EPM2006 – DE414           | 256     | 131   | 17.2  | 78.7 |
| EPM2008 – DE421           | 185     | 4.6   | 11.9  | 233  |

Table 2
Maximum peak-to-peak nominal amplitudes, in m, of the Earth-Mercury range signals over $\Delta t = 2$ yr due to the dynamical effects listed. We adopted the standard value $J_2^\odot = 2.0 \times 10^{-7}$ (Fienga et al., 2010) for the quadrupole mass moment of the Sun. It is presently known at a 10% level of accuracy. For its proper angular momentum we used $S_\odot = 190.0 \times 10^{39}$ kg m$^2$ s$^{-1}$ (Pijpers, 1998, 2003) from helioseismology.

| Dynamical effect | Peak-to-peak amplitude (m) |
|------------------|----------------------------|
| Solar Schwarzschild | $4 \times 10^5$ |
| Solar $J_2^\odot$    | 300                        |
| Ceres, Pallas, Vesta | 80                         |
| Solar Lense-Thirring | 17.5                       |
| Ring of minor Asteroids | 4                         |
| TNOs               | 0.8                        |