Open String Tachyon in Supergravity Solution

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abstract

We study the tachyon condensation of the D\bar{D}-brane system with a constant tachyon vev in the context of classical solutions of the Type II supergravity. We find that the general solution with the symmetry ISO(1, p) × SO(9 − p) (the three-parameter solution) includes the extremal black p-brane solution as an appropriate limit of the solution by fixing one of the three parameters (c_1). Furthermore, we compare the long distance behavior of the solution with the massless modes of the closed strings from the boundary state of the D\bar{D}-brane system with a constant tachyon vev. We find that we must fix c_1 to zero and only two parameters are needed to express the tachyon condensation of the D\bar{D}-brane system. This means that the parameter c_1 does not correspond to the tachyon vev of the D\bar{D}-brane system.

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1 Introduction

In recent years, unstable D-brane systems have been investigated eagerly [1, 2]. Their instability is characterized by the existence of a tachyon field on the world-volume and they decay to the closed string vacuum or the stable BPS states by the tachyon condensation. For example, if we start from the system with a pair of D-brane and D-brane, it flows to the closed string vacuum [3, 4]. Such phenomena are important not only for the string theory but also for the general relativity. In fact, the construction of a dynamical solution which represents the tachyon condensation is significant, since gravitational or observational objects are dynamical in general and such dynamical systems will be essential to describe realistic objects in terms of the string theory [5].

The aim of this paper is to investigate the unstable D-brane systems and their tachyon condensation from the viewpoint of the classical solutions of the Type II supergravity [6]-[15]. In particular, we would like to understand the correspondence between (microscopic) objects in superstring theory and (macroscopic) classical solutions in the supergravity. It is known that there is such a correspondence between the BPS D-branes and the extremal black p-brane solutions [16]. However, for unstable D-brane systems, it is not fully understood what their classical counterparts are, although there are some discussions [17, 18].

Previously, in [19], it is argued that there is no such correspondence for unstable D-brane systems. The main point of their discussion is as follows: the supergravity approximation is valid only when curvature effects are small, and it can be realized by sufficiently large number of coincident D-branes. However, the unstable D-brane systems do not satisfy the no-force condition, unlike BPS cases, then, we cannot consider multiple coincident D-branes. As opposed to this observation, we argue here that it is still possible to study the correspondence. As discussed in [19, 20], once we fix a source of closed strings, we can determine the solution of the equation of motion of the closed string field theory uniquely. Thus, the problem is whether we can construct a source corresponding to an unstable D-brane system where the supergravity approximation is valid. Here, the important fact is that we can express any type of D-branes as boundary states with appropriate boundary interactions [3, 4]. In particular, it is possible to construct many number of coincident unstable D-branes in this formulation, and this argument is
irrespective of whether the D-brane system is stable or not. Since the validity of the supergravity approximation is the same as in the BPS case, the comparison is meaningful as long as sufficiently large number of coincident unstable D-branes. Note that if the source is static, then the solution is also static. From this observation, it is possible to deal with an unstable D-brane system in the context of a classical solution of the supergravity.

As a remark we comment on the no-force condition. The no-force condition is related to dynamical issues, that is, it is the condition that the interaction between several D-branes vanishes, which guarantees that the superposition of several D-branes also becomes a consistent source of bulk closed strings even if we take into account the interaction between D-branes. Actually, an unstable D-brane system does not satisfy the no-force condition and is expected to receive a backreaction effect and decay to some stable one, if we take into account such interactions. The full solution is obtained by solving the equation of motion of gravity coupled with a dynamical source (matter) and the matter equation of motion simultaneously[5]. The classical solution we will consider in this paper is thought to be the zero-th order approximation to the full solution. There are also other issues, such as $\alpha'$ and loop correction, but we will not discuss them here.

In this paper, we consider a system of coincident $N$ D$p$-branes and $\bar{N}$ D$p$-branes (for simplicity we restrict ourselves to $N > \bar{N}$) with an excitation of an open string tachyon on them. This system is a typical example of the unstable D-brane systems. For simplicity, we assume that the tachyon profile on the world-volume is constant. In this setting, the system possesses the global symmetry $ISO(1, p) \times SO(9 - p)$. Although any number of D$\bar{D}$-pairs can be annihilated by the tachyon condensation in general, we concentrate on the case where the final state is BPS saturated, i.e. the system of $(N - \bar{N})$ D-branes. On the other hand, the general solution with the same symmetry $ISO(1, p) \times SO(9 - p)$ in the Type II supergravity is known as the “three-parameter solution”[15]. Therefore, it is natural that the three parameters of the solution have a relationship to physical parameters of the D$\bar{D}$-brane system. In fact, in [17, 18], it is argued that the three parameters correspond to microscopic quantities, that is, the number of D$p$-branes $N$, the number of D$p$-branes $\bar{N}$ and the tachyon vev $\langle T \rangle$, respectively. In this paper, we re-examine these correspondences in two ways. First, we introduce a new parametrization for the three-parameter solution and show that the solution becomes the extremal $p$-brane.

1If we assume $(N - \bar{N}) \gg 1$, the supergravity description is valid.
solution [16] by tuning one of the new parameter (\( \epsilon \)) to zero while fixing another parameter (\( c_1 \)) to an arbitrary value. It means that \( c_1 \) cannot correspond to the tachyon vev \( \langle T \rangle \), as opposed to the proposal which has been made so far [17, 18]. Next, we directly compare the three-parameter solution with a boundary state which expresses the non-BPS D-brane system in considering, in order to obtain more quantitative correspondence between microscopic parameters and macroscopic ones. In this investigation, we use the technique given in [19]. As we mentioned before, we can apply this technique even to non-BPS boundary states although it is originally used to show the correspondence between the extremal black \( p \)-brane solution and the BPS boundary state. The main advantage of this method, compared to the previous approach [17, 18], is the usage of the off-shell boundary state for the \( D\bar{D} \)-brane system with a tachyon vev, which satisfies Sen’s conjecture. As a result, we will find that we need only two parameters to express the \( D\bar{D} \)-brane system. The physical meaning of this parameter is an open question at present, but it will be discussed in [21]. (For recent works, see [22].)

The organization of this paper is as follows. In the next section, we review the ten-dimensional three-parameter solution. In the section 3, which is the main part of this paper, we show that the three-parameter solution becomes the extremal black \( p \)-brane solution by taking an appropriate limit with one of the parameters of the solution fixing to an arbitrary value. We investigate the long distance behavior of the three-parameter solution and compare it with the boundary state corresponding to the system of \( N \) \( D \)-branes and \( \bar{N} \) \( \bar{D} \)-branes with a constant tachyon vev. We determine the values of the three parameters of the solution so that it represents the tachyon condensation of the \( D\bar{D} \)-brane system. Section 4 is devoted to the conclusion and the discussion.

2 Review of the Three-parameter Solution in Ten-dimensions

In this section, we review the three-parameter solution given in [15]. Since we are interested in the string theoretical interpretation of the solution, we only consider the

\(^2\)In [15], a general solution with the symmetry \( ISO(p) \times SO(D - p - 1) \) is constructed as the “four-parameter solution”.

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ten-dimensional Type II supergravity. For the three-parameter solution for an arbitrary
dimensionality, see the appendix A.

We start with the ten-dimensional action,

\[
S = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g} \left[ R - \frac{1}{2} (\partial \phi)^2 - \frac{1}{2(p+2)!} e^{\frac{2\phi}{r}} |F_{p+2}|^2 \right],
\]

(2.1)

where \(F_{p+2}\) denotes the \((p+2)\)-form field strength which relates to the \((p+1)\)-form potential of the RR-field \(A^{(p+1)}\) as \(F_{p+2} = dA_{(p+1)}\). Since we consider the solution with the symmetry \(ISO(1,p) \times SO(9-p)\), we impose the ansatz,

\[
ds^2 = g_{MN} dx^M dx^N = e^{2A(r)} \eta_{\mu\nu} dx^\mu dx^\nu + e^{2B(r)} \delta_{ij} dx^i dx^j,
\]

(2.2)

where \(\mu, \nu = 0, \cdots, p\) are indices of the longitudinal directions of the \(p\)-brane, \(i, j = p+1, \cdots, 9\) express the orthogonal directions, and \(M = (\mu, i)\). Under this ansatz, the equations of motion become

\[
A'' + \left( (p+1)A' + (7-p)B' + \frac{8-p}{r} \right) A' = \frac{7-p}{16} S^2,
\]

\[
B'' + \left( (p+1)A' + (7-p)B' + \frac{15-2p}{r} \right) B' + \frac{p+1}{r} A' = -\frac{p+1}{16} S^2,
\]

\[
(p+1)A'' + (8-p)B'' + (p+1)A'^2 - (p+1)A'B' + \frac{8-p}{r} B' + \frac{1}{2} \phi'^2 = \frac{7-p}{16} S^2,
\]

\[
\phi'' + \left( (p+1)A' + (7-p)B' + \frac{8-p}{r} \right) \phi' = -\frac{3-p}{4} S^2,
\]

\[
\left( \Lambda' e^{\Lambda + \frac{3-p}{4} \phi -(p+1)A + (7-p)B + 8-p} \right)' = 0,
\]

(2.3)

where

\[
S = \Lambda' e^{\Lambda + \frac{3-p}{4} \phi -(p+1)A},
\]

(2.4)

and the prime denotes the derivative with respect to \(r\).

The authors in [15] show that the general asymptotically flat solution of the equations

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(2.3) is given by
\begin{align*}
A(r) &= \frac{(7 - p)(3 - p)c_1}{64}h(r) - \frac{7 - p}{16}\ln[\cosh(kh(r)) - c_2 \sinh(kh(r))], \quad (2.5) \\
B(r) &= \frac{1}{r - p}\ln[f_-(r)f_+(r)] \\
&\quad - \frac{(p + 1)(3 - p)c_1}{64}h(r) + \frac{p + 1}{16}\ln[\cosh(kh(r)) - c_2 \sinh(kh(r))], \quad (2.6) \\
\phi(r) &= \frac{(p + 1)(7 - p)c_1}{16}h(r) + \frac{3 - p}{4}\ln[\cosh(kh(r)) - c_2 \sinh(kh(r))], \quad (2.7) \\
e^\Lambda(r) &= -\eta(c_2^2 - 1)^{1/2}\frac{\sinh(kh(r))}{\cosh(kh(r)) - c_2 \sinh(kh(r))}, \quad (2.8)
\end{align*}

where
\begin{align*}
f_\pm(r) &\equiv 1 \pm \frac{r_{7-p}^0}{r_{7-p}^0}, \quad (2.9) \\
h(r) &\equiv \ln\left(\frac{f_-}{f_+}\right), \quad (2.10) \\
k &\equiv \pm \sqrt{\frac{2(8 - p)}{7 - p} - \frac{(p + 1)(7 - p)}{16}c_1^2} \\
&\equiv \pm \sqrt{\frac{(p + 1)(7 - p)}{4}} \sqrt{c_m^2 - c_1^2}, \quad \left( c_m = \sqrt{\frac{32(8 - p)}{(p + 1)(7 - p)^2}} \right) \quad (2.11) \\
\eta &= \pm 1. \quad (2.12)
\end{align*}

The three parameters,\(^3\) \(r_0, c_1, c_2,\) are the integration constants that parametrize the solution. As discussed in [17], the region of the parameters in the solution (2.5)–(2.8) is
\begin{align*}
c_1 &\in (0, c_m), \quad (2.13) \\
c_2 &\in (-\infty, -1) \cup (1, \infty), \quad (2.14) \\
r_{7-p}^0 &\in \mathbb{R}, \quad (2.15)
\end{align*}

where we have already fixed the \(\mathbb{Z}_2\) symmetries of the solution,
\begin{align*}
(r_{7-p}^0, c_1, c_2, \text{sgn}(k), \eta) &\rightarrow (r_{7-p}^0, c_1, -c_2, -\text{sgn}(k), -\eta), \\
(r_{7-p}^0, c_1, c_2, \text{sgn}(k), \eta) &\rightarrow (-r_{7-p}^0, -c_1, c_2, -\text{sgn}(k), \eta), \quad (2.16)
\end{align*}

by choosing \(c_1 \geq 0\) and \(k \geq 0.\) Furthermore, we have a degree of freedom to choose the signs of \(r_{7-p}^0\) and \(c_2.\) In the next section, we will discuss the extremal black \(p\)-brane limit

\(^3\)We have labeled \(c_3\) of [15] as \(c_2\) and \(k\) as \(-k\) according to [17].
of the solution. To take this limit consistently, we must choose the branch,

\[(r_0^{7-p} \geq 0, c_2 \geq 0), \quad \text{or} \quad (r_0^{7-p} \leq 0, c_2 \leq 0).\]  
(2.17)

For simplicity, we choose \((r_0^{7-p} \geq 0, c_2 \geq 0)\) in this paper.\(^4\)

From the viewpoint of the gravity theory, the three-parameter solution describes a charged dilatonic black object. Thus, the RR-charge \(Q\) and the ADM mass \(M\) are natural quantities to characterize the solution.\(^5\) For convenience, we consider wrapping the spatial world-volume directions on a torus \(T^p\) of volume \(V_p\). The RR-charge is given by an appropriate surface integral over the sphere-at-infinity in the transverse directions\(^[17, 23]\):

\[Q = 2\eta N_p (c_2^2 - 1)^{1/2} kr_0^{7-p},\]  
(2.18)

where

\[N_p = \frac{(8-p)(7-p)\omega_{8-p}V_p}{16\kappa^2},\]  
(2.19)

and \(\omega_d = \frac{2\pi^{(d+1)/2}}{\Gamma((d+1)/2)}\) is the volume of the unit sphere \(S^d\). The ADM mass is defined as\(^[24, 25]\)

\[g_{00} = -1 + \frac{2\tilde{\kappa}^2 M}{(8-p)\omega_{8-p} r_0^{7-p}} + \mathcal{O}\left(\frac{1}{r_0^{2(7-p)}}\right),\]  
(2.20)

where the metric is written in Einstein frame and \(\tilde{\kappa}^2 \equiv \kappa^2/V_p\). Using this definition, we see that the ADM mass of the three-parameter solution is\(^[17]\)

\[M = N_p \left(\frac{3-p}{2} c_1 + 2c_2 k\right) r_0^{7-p}.\]  
(2.21)

### 3 D\(\bar{D}\)-brane System in the Three-parameter Solution

In the previous section, we reviewed the ten-dimensional three-parameter solution and defined the physical quantities \(Q\) and \(M\). In this section, we first show that we can obtain

\(^4\)In\(^[17]\), the authors choose \((r_0^{7-p} \leq 0, c_2 \leq 0)\) for \(p = 0, \cdots, 3\) and \((r_0^{7-p} \geq 0, c_2 \geq 0)\) for \(p = 3, \cdots, 6\) by imposing that \(M\) should decrease monotonically as \(c_1\) increases, since they identify \(c_1\) as the tachyon vev. In this paper, however, we conclude that \(c_1\) is not the vev of the open string tachyon, and thus, there is still an ambiguity in the signs of \(r_0^{7-p}\) and \(c_2\).

\(^5\)In addition to \(Q\) and \(M\), it is often useful to define the “dilaton charge” \(D\). In the case of the three-parameter solution\(^[11]\),

\[D = N_p \left[16c_2 k - 4(p+1)c_1\right] r_0^{7-p}.\]

Instead of \((c_1, c_2, r_0)\), we can use \((Q, M, D)\) to parametrize the solution.
the extremal black $p$-brane solution as an appropriate limit of the three-parameter solution for arbitrary $c_1$ by using a reparametrization. Then we introduce another convenient parametrization for dealing with the tachyon condensation, in the sense that we can change the solution without changing the value of the RR-charge. By comparing the behavior of the solution with the boundary state corresponding to the system of $N$ D-branes and $\bar{N}$ $\bar{D}$-branes with a constant tachyon vev, we show that we must choose $c_1 = 0$ to express the tachyon condensation of the D$\bar{D}$-brane system by the three-parameter solution.

### 3.1 Black $p$-brane solution in the three-parameter solution and new parametrizations

Let us first show that the three-parameter solution coincides with the extremal black $p$-brane solution under a certain limit for arbitrary $c_1$.$^6$ The ten-dimensional extremal black $p$-brane solution ($p < 7$) is given by \[16\]

\[
ds^2 = f_p(r)^{-\frac{7-p}{8}} \eta_{\mu\nu} dx^\mu dx^\nu + f_p(r)^{\frac{p+1}{8}} \delta_{ij} dx^i dx^j,
\]

\[
e^\phi(r) = f_p(r)^{\frac{3-p}{4}},
\]

\[
e^\Lambda(r) = -\eta(f_p(r)^{-1} - 1),
\]

where $f_p(r)$ is defined as

\[
f_p(r) = 1 + \frac{\mu_0}{r^{7-p}},
\]

and $\mu_0$ is the only parameter of this solution that is proportional to the RR-charge of the D$p$-branes. To realize the solution \(3.1\)–\(3.3\) in the three-parameter solution, we introduce a parametrization by $r_0^{7-p}$ and $\epsilon$ as

\[
r_0^{7-p} = \epsilon r_0^{7-p}, \quad c_2 = \frac{1}{\epsilon}, \quad (0 \leq \epsilon \leq 1)
\]

\[6\text{After almost completing this work, we found that the same idea has been developed independently by the authors of \[22\].}
When $\epsilon \to 0$, the three-parameter solution coincides with the extremal black $p$-brane solution (3.1)-(3.3) with the identification,

$$
\mu_0 = 2k r_0^{7-p}.
$$

In the parametrization (3.5), the RR-charge (2.18) and the ADM mass (2.21) are written as

$$
Q = 2\eta N_p (1 - \epsilon^2)^{1/2} k r_0^{7-p},
$$

$$
M = 2N_p(1 + \frac{3 - p}{4k} c_1 \epsilon) k r_0^{7-p}.
$$

If we set $c_1 = 0$, $M$ is invariant under the change of $\epsilon$, and $Q$ and $M$ satisfy the relation,

$$
M^2 - Q^2 = (2N_p k r_0^{7-p} \epsilon)^2,
$$

thus the parameter $\epsilon$ can be regarded as a “non-extremality parameter”. Note that, in the $\epsilon \to 0$ limit, the RR-charge and the ADM mass become

$$
|Q| = M = 2N_p k r_0^{7-p},
$$

for arbitrary $c_1$. Thus we conclude that $c_1$ has nothing to do with the tachyon vev, since the solution becomes BPS for arbitrary $c_1$.

The above parametrization, however, is not suitable for our purpose to analyze the tachyon condensation of the D$\bar{D}$-brane system, since the RR-charge depends on $\epsilon$. As is explained in detail in the next subsection, not the RR-charge but the mass of the D$\bar{D}$-brane system will change as the tachyon condensates. Thus, we define another set of parameters $(v, \mu_0)$ to fix $Q$ as

$$
\frac{r_0^{7-p}}{2k} = \frac{v \mu_0}{k}, \quad c_2^2 - 1 = \frac{1}{v^2}, \quad (0 \leq v \leq \infty)
$$

It is easy to show that the $v \to 0$ limit of the solution is again the extremal black $p$-brane solution (3.1)-(3.3). In this parametrization, the RR-charge and the ADM mass

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5In [17], the black $p$-brane solution is given as the $\epsilon \to 0$ limit of

$$
r_0^{7-p} = \epsilon^{1/2} r_0^{7-p}, \quad k = \epsilon^{1/2} k, \quad c_2 = \frac{\tau_2}{\epsilon}.
$$

Since $k \to 0$ means $c_1 \to c_m$, this is a particular example of (3.5).

6This is the case of the D$\bar{D}$-brane system we consider in this article. See the next subsection.
are expressed as
\begin{align}
Q &= \eta N_p \mu_0, \quad (3.12) \\
M &= N_p \left( \sqrt{1 + v^2} + \frac{3 - p}{4k} c_1 v \right) \mu_0, \quad (3.13)
\end{align}
in which the RR-charge does not depend on \( v \) and \( c_1 \) as announced. Note that \( v \) works as the “non-extremality parameter” as \( \epsilon \) does, but \( v \) denotes how much the ADM mass exceeds the RR-charge.

For the later discussion, we write down the long distance behavior of this solution; up to the order \( 1/r^{7-p} \). The result is
\begin{align}
e^{2A(r)} &= 1 - \frac{7 - p}{8} \left( \sqrt{1 + v^2} + \frac{3 - p}{4k} c_1 v \right) \frac{\mu_0}{r^{7-p}} + O \left( 1/r^{2(7-p)} \right), \quad (3.14) \\
e^{2B(r)} &= 1 + \frac{p + 1}{8} \left( \sqrt{1 + v^2} + \frac{3 - p}{4k} c_1 v \right) \frac{\mu_0}{r^{7-p}} + O \left( 1/r^{2(7-p)} \right), \quad (3.15) \\
\phi(r) &= \left( \frac{3 - p}{4} \sqrt{1 + v^2} - \frac{(p + 1)(7 - p)}{16} c_1 v \right) \frac{\mu_0}{r^{7-p}} + O \left( 1/r^{2(7-p)} \right), \quad (3.16) \\
e^\Lambda = \eta \frac{\mu_0}{r^{7-p}} + O \left( 1/r^{2(7-p)} \right). \quad (3.17)
\end{align}

### 3.2 Comparison with the boundary state

In this subsection, we show that the D\( \bar{D} \)-brane system with a constant tachyon vev corresponds to the three-parameter solution of \( c_1 = 0 \). This means that we need only two parameters to express the D\( \bar{D} \)-brane system in the low-energy gravity theory. To show it, we use the technique to reveal the correspondence between the extremal black \( p \)-brane solution and the boundary state established in [19].

Let us first briefly review the system of \( N \) D\( p \)-branes and \( \bar{N} \) \( \bar{D}p \)-branes with a constant tachyon vev. The gauge symmetry of the low energy effective theory on the world-volume is \( U(N) \times U(\bar{N}) \). There is a complex tachyon field \( T(x) \) on it that is in the bi-fundamental representation of the gauge group. In this paper, we consider the case where the \( \bar{N} \) \( \bar{D} \)-pairs vanish and \((N - \bar{N}) \) D\( p \)-branes remain. Namely, we decompose the \( \bar{N} \times N \) matrix by \( \bar{N} \times (N - \bar{N}) \) and \( \bar{N} \times \bar{N} \) components and set the tachyon profile as
\begin{equation}
T(x) = \begin{pmatrix}
0 \\
T
\end{pmatrix}, \quad (3.18)
\end{equation}
where $T$ is a constant $\bar{N} \times \bar{N}$ matrix. Note that, since the other open string excitations, gauge fields, scalar fields and a non-constant tachyon break the global symmetry $ISO(1, p) \times SO(9 - p)$, we do not consider them here. It is known that this system can be expressed by the boundary state on which the boundary interaction for the tachyon field is turned \[3, 4\]. For our case, the NSNS part of the boundary state is then given by

$$|B_p; T\rangle_{NSNS} = \frac{1}{\bar{b}_{2/2}^N} \sum_{n=1}^{\infty} \frac{1}{n} \alpha_{-n}^M S_{MN} \bar{\alpha}_{-n}^N e^{-|T|^2} \delta^{(9 - p)}(x^i)$$

where $\bar{N} = (\eta_{\mu\nu}, -\delta_{ij})$, $|0\rangle$ is the Fock vacuum of a closed string in the NSNS sector, and $\alpha_{-n}^M$ and $b_{-r}^M$ are the creation operators of the modes of the world-sheet bosons and fermions, respectively. The trace is taken over $\bar{N} \times \bar{N}$ matrices. Note that only the difference from the BPS boundary state is the effect of the constant tachyon vev, which contributes only to the overall numerical factor $T_p[(N - \bar{N}) + 2\text{tr} e^{-|T|^2}]$ in \[3.19\]. The first term of this factor corresponds to the tension of the $(N - \bar{N})$ D-branes, and the second term is the tachyon potential $V(T)$ for the $\bar{N}$ D$\bar{D}$-pairs, which satisfies Sen’s conjecture $V(T = 0) = V(T_{\text{min}}) = 2\bar{N}T_p$. Thus, when $T = 0$ (i.e. $\text{tr} e^{-|T|^2} = \bar{N}$), the factor is the tension of the sum of $N$ D-branes and $\bar{N}$ $\bar{D}$-branes, while, when $|T| \sim \infty$, the tension becomes that of $(N - \bar{N})$ D-branes. On the other hand, the boundary state of the RR-sector is exactly the same as that of the BPS D$p$-brane under the excitation of the constant tachyon $T$, since the RR $(p + 1)$-form charge conserves and other kinds of charges do not appear.

According to the discussion in \[19\], the long distance behavior of the classical solution of the Type II supergravity corresponding to the D$\bar{D}$-brane system can be read off from the boundary state \[3.19\]. Following \[19\], we first define the quantity,

$$J^{MN}(k) = \langle 0; k | b_{1/2}^M \bar{b}_{1/2}^N D | B_p; T\rangle_{NSNS}$$

$$= -\frac{T_p}{2} \left[ (N - \bar{N}) + 2\text{tr} e^{-|T|^2} \right] \frac{V_{p+1}}{k_i^2} S^{MN}.$$  \(3.20\)

Here $k_i$ is the momentum of the transverse direction to the brane. We can evaluate the long distance behavior of the graviton and the dilaton by multiplying the corresponding
polarization tensors,
\[ \epsilon_{MN}^{(h)} = \epsilon_{MN}^{(h)}, \quad \epsilon_{MN}^{(h)} \eta^{MN} = \epsilon_{MN}^{(h)} k^M = 0, \]  
(3.21)
\[ \epsilon_{MN}^{(\phi)} = \frac{1}{2\sqrt{2}} [\eta_{MN} - k_M l_N - k_N l_M], \quad k \cdot l = 1, \quad k^2 = l^2 = 0, \]  
(3.22)
to \( J_{MN}(k) \);
\[ \bar{h}_{MN}^{(1)}(k) = 2\kappa \times \left( J_{MN}(k) - \frac{J^{MN}(k) \epsilon_{MN}^{(\phi)}}{\epsilon_{MN}^{(\phi)}} \eta_{MN} \right) \]
\[ = \left[ (N - \bar{N}) + 2\text{tr} e^{-|T|^2} \right] \frac{2\kappa T_p V_{p+1}}{k_i^2} \left( -\frac{7 - p}{8} \eta_{\mu\nu}, \quad \frac{p + 1}{8} \delta_{ij} \right), \]  
(3.23)
\[ \bar{\phi}^{(1)}(k) = \sqrt{2}\kappa \times J^{MN}(k) \epsilon_{MN}^{(\phi)} \]
\[ = \left[ (N - \bar{N}) + 2\text{tr} e^{-|T|^2} \right] \frac{2\kappa T_p V_{p+1}}{k_i^2} \frac{3 - p}{4}, \]  
(3.24)
where the additional factors, \( 2\kappa \) and \( \sqrt{2}\kappa \), are necessary to compare with the classical solution. (For detail, see [19].) For the RR \((p + 1)\)-form field, the similar calculation leads to the result,
\[ e^{\Lambda^{(1)}}(k) = \eta(N - \bar{N}) \frac{2\kappa T_p V_{p+1}}{k_i^2}. \]  
(3.25)
To compare the result from the boundary state (3.23)–(3.25) with the three-parameter solution, we use the Fourier transformation,
\[ \int d^{10} x e^{ik \cdot x} \frac{1}{(7 - p)\omega_{8-p} r^{7-p}} \frac{1}{k_i^2} = \frac{V_{p+1}}{k_i^2}, \]  
(3.26)
then we rewrite (3.23)–(3.25) as functions of \( r \);
\[ h_{MN}^{(1)}(r) = \left[ (N - \bar{N}) + 2\text{tr} e^{-|T|^2} \right] \frac{2\kappa T_p}{(7 - p)\omega_{8-p} r^{7-p}} \frac{1}{k_i^2} \left( -\frac{7 - p}{8} \eta_{\mu\nu}, \quad \frac{p + 1}{8} \delta_{ij} \right), \]  
(3.27)
\[ \phi^{(1)}(r) = \left[ (N - \bar{N}) + 2\text{tr} e^{-|T|^2} \right] \frac{2\kappa T_p}{(7 - p)\omega_{8-p} r^{7-p}} \frac{1}{k_i^2} \frac{3 - p}{4}, \]  
(3.28)
\[ e^{\Lambda^{(1)}}(r) = \eta(N - \bar{N}) \frac{2\kappa T_p}{(7 - p)\omega_{8-p} r^{7-p}}. \]  
(3.29)
Let us compare the long distance behavior of the classical solution (3.14)–(3.17) with the result from the boundary state (3.27)–(3.29). First, from the results of the RR field (3.17) and (3.29), we can identify \( \mu_0 \) as
\[ \mu_0 = \frac{2\kappa T_p(N - \bar{N})}{(7 - p)\omega_{8-p}}. \]  
(3.30)
Next, from the graviton and the dilaton, we see that the parameters must satisfy the following relations simultaneously:

\[ 1 + \frac{2}{N - \bar{N}} \text{tr} e^{-|T|^2} = \sqrt{1 + v^2 + \frac{3 - pc_1 v}{4 k}} = \sqrt{1 + v^2 - \frac{(p + 1)(7 - p) c_1 v}{4(3 - p) k}}, \]  

(3.31)

which are consistent only when \( c_1 = 0 \). From this consideration, we conclude that the three-parameter solution with

\[
c_1 = 0, \\
\mu_0 = \frac{2\kappa T_p (N - \bar{N})}{(7 - p)\omega_{8-p}}, \\
\sqrt{1 + v^2} = 1 + \frac{2}{N - \bar{N}} \text{tr} e^{-|T|^2},
\]

corresponds to the system of \( N \) D-branes and \( \bar{N} \) Ð-branes with a constant tachyon vev. We emphasize that the vev of the open string tachyon between the D-branes and the Ð-branes does not correspond to \( c_1 \), but corresponds to \( v \), as opposed to the previous expectation.

Once this correspondence is obtained, we can easily relate the ADM mass \( M \) and the RR-charge \( Q \) to the quantities of the DD-brane system as

\[
Q = \eta T_p (N - \bar{N}) \frac{(8 - p)V_p}{8\kappa}, \\
M = T_p \left[(N - \bar{N}) + 2\text{tr} e^{-|T|^2}\right] \frac{(8 - p)V_p}{8\kappa}.
\]

(3.33)

(3.34)

This is quite natural result that the overall coefficient of the NSNS and the RR boundary state is directly related to the mass and the RR-charge, respectively, as in the BPS cases.

4 Conclusion and Discussion

In this paper, we discussed the tachyon condensation of the system of \( N \) D-branes and \( \bar{N} \) Ð-branes with a constant tachyon vev \( T \) using the general solution of the Type II supergravity with the symmetry \( ISO(1, p) \times SO(9-p) \). Since the solution is parametrized by the three parameters \( c_1, c_2 \) and \( r_0 \), it is called as the “three-parameter solution”. Introducing the new parameters \( \bar{r}_0 \) and \( \epsilon \) instead of \( c_2 \) and \( r_0 \), we found that the three-parameter solution becomes the extremal black \( p \)-brane solution by taking \( \epsilon \to 0 \) for arbitrary \( c_1 \). This means that \( c_1 \) is not related to tachyon vev.
In order to relate the three-parameter solution with the D\(\bar{D}\)-brane system more directly, we compared the long distance behavior of the three-parameter solution with the massless modes of the closed string from the D\(\bar{D}\)-brane boundary state. We can apply the relation between the classical solution and the boundary state to our case, although the D\(\bar{D}\)-brane system is a non-BPS state except at the end of the tachyon condensation. In this connection, we can similarly apply this method to any other sources that are non-BPS or unstable.

When we compared the solution with the boundary state, we introduced the parametrization \(\{\mu_0, v\}\) so that we can fix the RR-charge. In fact, the RR-charge is conserved during the tachyon condensation, and neither the parametrization \(\{c_2, r_0\}\) nor the parametrization \(\{\epsilon, \bar{r}_0\}\) are suitable to such a situation. In contrast, the parametrization \(\{\mu_0, v\}\) is convenient for our purpose because it makes the RR-charge depend only on \(\mu_0\), and we can change the ADM mass without changing \(\mu_0\).

By comparing the long distance behavior of the three-parameter solution with the massless modes of the closed string from the boundary state of the D\(\bar{D}\)-brane system, we found that we must fix \(c_1\) to zero and only two parameters \(\mu_0\) and \(v\) (or \(c_2\) and \(r_0\)) are needed to express the D\(\bar{D}\)-brane system. This result means that we can see the tachyon vev not as an independent parameter in the supergravity solution but as a part of the ADM mass. This fact is consistent with the form of the boundary state, i.e. the tachyon vev is included in the coefficient of the NSNS sector of the boundary state as shown in (3.19), and the coefficient is nothing but the ADM mass of the solution. So we concluded that the D\(\bar{D}\)-brane system corresponds to the three-parameter solution of \(c_1 = 0\). If \(c_1\) turned on, the long distance behavior of the three-parameter solution can not be reproduced by the boundary state of the D\(\bar{D}\)-brane system. This means that \(c_1\) relates to another physical quantity in the string theory, which will be discussed in the forthcoming paper [21].

At the end of this paper, we would like to note that the parameter \(c_1\) is not so unfamiliar at least in the general relativity. In fact, \(c_1\) has something to do with the quantity which has been known as a “scalar charge” in the general relativity. To see this explicitly, let us see the Janis-Newman-Winicour solution or the Wyman solution [26]-[28]. It is the four-dimensional and SO(3) symmetric solution of the Einstein equation with a free scalar
field $\phi$. The concrete form is

$$ds^2 = - \left[ \frac{f_-(r)}{f_+(r)} \right]^{2\gamma} dt^2 + \left[ f_-(r) \right]^{2-2\gamma} \left[ f_+(r) \right]^{2+2\gamma} \left( dr^2 + r^2 d\Omega_2^2 \right), \quad (4.1)$$

$$\phi(r) = \sqrt{\frac{4q^2}{m^2 + q^2}} \ln \left[ \frac{f_-(r)}{f_+(r)} \right], \quad (4.2)$$

where $f_{\pm}(r)$ is defined as

$$f_{\pm}(r) = 1 \pm \sqrt{\frac{m^2 + q^2}{r}}, \quad \gamma = \sqrt{\frac{m^2}{m^2 + q^2}}. \quad (4.3)$$

Here $q$ is the so-called scalar charge, which relates to the scalar field $\phi$ as shown in (4.2). If we set $q = 0$, the scalar field $\phi$ vanishes and it reduces to the four-dimensional Schwarzschild metric.

Compared this solution with the four-dimensional three-parameter solution of $c_2 = 1$ (see appendix A, where we give the three-parameter solution for an arbitrary dimensionality), we find that they are the same solution under the following identification

$$r_0^2 = m^2 + q^2, \quad c_1^2 = \frac{4q^2}{m^2 + q^2}. \quad (4.4)$$

It is true that the three-parameter solution we have considered in this paper is a ten-dimensional one due to the consistency with the superstring theory, but we can understand that the parameter $c_1$ relates to the scalar charge $q$ and this fact would be something helpful when we consider the physical meaning of $c_1$. Of course, this relation is not enough to clear the physical meaning of $c_1$, because this relation does not explain $c_1$ from the viewpoint of the source term of the three-parameter solution. That is, we need to know how $c_1$ is expressed in the Born-Infeld-type action, which makes it possible to relate $c_1$ to the boundary state. This issue will be also discussed in our next paper [21].

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D-dimensional three-parameter solution

In [15], the D-dimensional three-parameter solution is given\(^9\). It is derived from the action,

\[
S = \frac{1}{2\kappa^2} \int d^D x \sqrt{-g} \left[ R - \frac{1}{2} (\partial\phi)^2 - \frac{1}{2(p+2)!} e^{a\phi} |F_{p+2}|^2 \right], \tag{A.1}
\]

where \(F_{p+2}\) denotes the \((p+2)\)-form field strength and it relates to the \((p+1)\)-form potential of RR-field \(A_{(p+1)}\) as \(F_{p+2} = dA_{(p+1)}\). The parameter \(a\) is a dilaton coupling to the RR field strength.\(^10\)

The D\(\bar{D}\) system has the \(ISO(1,p) \times SO(D-p-1)\) symmetry and the three-parameter solution also keeps it. The ansatz which follows this symmetry is

\[
ds^2 = e^{2A(r)} \eta_{\mu\nu} dx^\mu dx^\nu + e^{2B(r)} \delta_{ij} dx^i dx^j
\]
\[
= e^{2A(r)} \eta_{\mu\nu} dx^\mu dx^\nu + e^{2B(r)} (dr^2 + r^2 d\Omega^2_{(D-p-2)}), \tag{A.2}
\]

\[
\phi = \phi(r), \tag{A.3}
\]

\[
A^{(p+1)} = e^{\Lambda(r)} dx^0 \wedge dx^1 \wedge \cdots \wedge dx^p. \tag{A.4}
\]

We represent the D-dimensional coordinates by \(x^M (M = 0, 1, \cdots, D - 1)\), the \(p\)-brane world-volume coordinates \(x^\mu (\mu = 0, 1, \cdots, p)\) and the transverse coordinates by \(x^i (i = p + 1, \cdots, D - 1)\) or, equivalently, by the polar coordinates \(r, \theta_1, \cdots, \theta_{D-p-2}\) as we did in

\(^9\)The D-dimensional four-parameter solution which possesses \(ISO(p) \times SO(D-p-1)\) symmetry is also given in [15].

\(^{10}\)If we assume the gravity theory has stringy origin, the parameter \(a\) is determined as \(a = (D - 2p - 4)/(\sqrt{2}(D - 2))\).
Sec. 2. \( r \) is defined as \( r \equiv \sqrt{x^i x_i} \). The equations of motions of this system become

\[
A'' + \left( (p + 1)A' + (D - p - 3)B' + \frac{D - p - 2}{r} \right) A' = \frac{D - p - 3}{2(D - 2)} S^2,
\]

(A.5)

\[
B'' + \left( (p + 1)A' + (D - p - 3)B' + \frac{2(D - p - 3) + 1}{r} \right) B' + \frac{p + 1}{r} A' = -\frac{p + 1}{2(D - 2)} S^2,
\]

(A.6)

\[
(p + 1)A'' + (D - p - 2)B'' + (p + 1)A^2 + \frac{D - p - 2}{r} B'
\]

\[
- (p + 1) A'B' + \frac{1}{2} \phi'^2 = \frac{D - p - 3}{2(D - 2)} S^2,
\]

(A.7)

\[
\phi'' + \left( (p + 1)A' + (D - p - 3)B' + \frac{D - p - 2}{r} \right) \phi' = -\frac{a}{2} S^2,
\]

(A.8)

\[
\left( \Lambda' e^{\Lambda + a\phi - (p + 1)A + (D - p - 3)B_r (D - p - 2)} \right)' = 0,
\]

(A.9)

where

\[
S^2 = \Lambda'^2 e^{2\Lambda + a\phi - 2(p + 1)A}.
\]

(A.10)

The \( D \)-dimensional solution is given by

\[
A(r) = \frac{ac_1 (D - p - 3)}{\Delta(D - 2)} h(r) - \frac{2(D - p - 3)}{\Delta(D - 2)} \ln[\cosh(kh(r)) - c_2 \sinh(kh(r))],
\]

(A.11)

\[
B(r) = \frac{1}{D - p - 3} \ln[f_-(r)f_+(r)]
\]

\[
- \frac{ac_1 (p + 1)}{\Delta(D - 2)} h(r) + \frac{2(p + 1)}{\Delta(D - 2)} \ln[\cosh(kh(r)) - c_2 \sinh(kh(r))],
\]

(A.12)

\[
\phi(r) = \frac{2c_1 (D - p - 3)(p + 1)}{\Delta(D - 2)} h(r) + \frac{2a}{\Delta} \ln[\cosh(kh(r)) - c_2 \sinh(kh(r))],
\]

(A.13)

\[
e^{\Lambda(r)} = -2\eta \left( \frac{c_2^2 - 1}{\Delta} \right)^{1/2} \frac{\sinh(kh(r))}{\cosh(kh(r)) - c_2 \sinh(kh(r))}.
\]

(A.14)
where
\[ h(r) \equiv \ln \left( \frac{f_-}{f_+} \right), \quad (A.15) \]
\[ f_\pm(r) \equiv 1 \pm \frac{r_0^{D-p-3}}{r^{D-p-3}}, \quad (A.16) \]
\[ \Delta \equiv \frac{2(D-p-3)(p+1)}{D-2} + a^2, \quad (A.17) \]
\[ k \equiv \pm \sqrt{\frac{\Delta}{4} \left[ \frac{2(D-p-2)}{D-p-3} + \left( \frac{a^2}{\Delta} - 1 \right) c_1^2 \right]}, \quad (A.18) \]
\[ \eta = \pm 1. \quad (A.19) \]

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