Comment on ‘A Scaling law beyond Zipf’s law and its relation to Heaps’ law’.

Xiao-Yong Yan1,2 and Petter Minnhagen1

1School of Systems Science, Beijing Normal University, Beijing 100875, China
2Center for Complex System Research, Shijiazhuang Tiedao University, Shijiazhuang 050043, China

It is shown that the scaling law for the text-length dependence of word-frequency distributions proposed by Font-Clos et al in 2013 New J. Phys.15 093033 is fundamentally impossible and leads to incorrect conclusions. Instead the text-length-dependence can be connected to the general properties of a random book with consistent agreement. It is pointed out that this finding has strong implications, when deciding between two conceptually different views on word-frequency distributions, i.e. the ‘Zipf’s-view’ and the ‘Randomness-view’, as is discussed. It is also noticed that the text-length transformation of a random book does have an exact scaling property precisely for the power-law index \( \gamma = 1 \), as opposed to the Zipf’s exponent \( \gamma = 2 \) and the implication of this exact scaling property is discussed. However a real text has \( \gamma > 1 \) and as a consequence \( \gamma \) increases when shortening a real text, contrary to the claim by Font-Clos et al in 2013 New J. Phys.15 093033. The connections to the predictions from the RGF(Random Group Formation) and to the infinite length-limit of a meta-book are also discussed. The difference between ‘curve-fitting’ and ‘predicting’ word-frequency distributions is stressed.

PACS numbers: 89.75.Fb, 89.70-a

I. INTRODUCTION

Font-Clos et al in Ref.[1] proposed and discussed a new scaling relation for the word-frequency distribution of a text. This paper falls into a long tradition of trying to understand what linguistic information is hidden in the shape of the word-frequency distribution. This type of research goes back to the first part of the twentieth century when it was discovered that the word-frequency distribution of a text typically has a broad “fat-tailed” shape, which often can be well approximated with a power law over a large range.

The question is then what special principle or property of a language causes this power law distribution of word-frequencies and this is still an ongoing research. Ref.[1] is related to this scientific tradition, which will here be termed the ‘Zipf’s-view’.

In middle of the twentieth century Simon in Ref.[11] instead suggested that, since quite a few completely different systems also seemed to follow Zipf’s law in their corresponding frequency distributions, the explanation of the law must be more general and stochastic in nature and hence independent of any specific information of the language itself. Instead he proposed a random stochastic growth model for a book written one word at a time from beginning to end. This became a very influential model and has served as a starting point for much later works.

In the ‘Simon-view’ the shape of the word-frequency distribution does not reflect any specific property of a language but is shaped by a random stochastic element. An extreme random model was proposed in the middle of the twentieth century by Miller in Ref.[19]: the resulting text can be described as being produced by a monkey randomly typing away on a type-writer. However the properties of the monkey book are quite unrealistic and different from a real text. This ‘Randomness-view’ was recently developed further in a series of paper in terms of concepts like Random Group Formation, Random Book Transformation and the Meta-book. A crucial difference, compared to the ‘Zipfs-view’, is that the ‘Randomness-view’ is based on the notion that the shape of the word-frequency distribution is a general consequence of randomness which carries no specific information of the language.

Font-Clos et al in Ref.[1] conclude that their proposed scaling law is in contradiction with some of the fundamental results based on the ‘Randomness-view’. In some general sense, this means that a cross-road is reached: if the scaling law proposed by Font-Clos et al is conceptually correct, then the ‘Randomness-view’ is incorrect. And if the ‘Randomness-view’ is conceptually correct, then both the scaling law Font-Clos et al and the ‘Zipf’s-view’ are incorrect.

In this comment we explain the consequences of the ‘Randomness-view’, to what extent this view is borne out by data and why, as a consequence, the scaling law proposed in Ref.[1] is incorrect.

In a more narrow and focused perspective the issue is as follows:

The approach in Font-Clos et al in Ref.[1] is based on a ‘Zipfs-view’: A language is a very special system. Hence, according to the ‘Zipfs-view’, it is likely that the functional form of the distribution of words in a text reflects some special property of the language. Such a special property is the scaling law beyond Zipf’s law proposed in Ref.[1]. From this proposed scaling law the authors
of Ref. [1] conclude that ‘...we have shown that, contrary to claims in previous research [3, 24, 25], Zipf’s law in linguistics is extraordinarily stable under changes in size of the analyzed text. A scaling function ... provides a constant shape for the distribution of frequencies of each text, \( N_M(k) \), no matter its length \( M \), which only enters into the distribution as a scale parameter and determines the size of the vocabulary \( N_M \). The apparent size-dependence exponent found previously seems to be an artifact of the slight convexity...in a log−log plot,...’. A crucial issue here is the constant shape of the word-frequency distribution, \( N_M(k) \), and that the exponent in a power-law approximation of the distribution does not depend on the length of the text. This is crucial because in the ‘Randomness-view’ description the change in shape of the distribution with length and the changing power-law exponent are predictions which follows from randomness [20–24]. This means that if the shape does not change in accordance with the ‘randomness-view’-prediction, then this description is indeed not correct.

We will in the present paper use the following notation: \( N_M(k) (N_M \geq k) \) is the number of distinct words which occur \( k \)-times (\( k \)-times or more) in a text which in total contains \( M \) words. The scaling law proposed by Front-Clos et al can be cast into the form \( N_M(\geq k) = G(k/M) \).

In section 2, we first demonstrate directly from raw data that \( N_M(\geq k) \) does indeed change shape with text-length in a very systematic manner, which means that the proposed scaling-form \( G(k/M) \) is not valid. This means that the idea of a scaling function in Ref. [1] is not a fundamental feature, although it could still ‘fit’ data over some limited parameter-ranges, as shown in Ref. [1]. In section 3, we then compare the systematic length dependence of \( N_M(\geq k) \) with the predictions from the ‘Randomness-view’ and indeed find consistent agreement. We elucidate just how little information you need about the language in order to predict the characteristic features of the data for the word-frequency. This has a crucial and more far reaching consequence: Whenever you need very little information to describe a particular feature, then indeed very little specific information about the system can be extracted from this characteristic feature. In 4, we discuss and show that for a distribution \( N_M(k) \propto 1/k \) the shape is indeed length-invariant under the randomness (more precisely under the RBT=book transformation assumption [20, 21, 23, 24]). In Ref. [21] it was observed that the limit of a very large text by an author seems to approach the limit \( N_M(k) \propto 1/k \). This suggests that an approximate scaling, like the one proposed in Ref. [1] should work better the longer the text is. This is indeed in accordance with the suggestion in Ref. [1] that ‘...the scale invariance of the distribution of frequencies, holds more strongly for longer texts’. Some concluding remarks are added in section 5.

II. SCALING OR NO SCALING?

The first issue is the factual situation. Does or doesn’t the word-frequency distribution, \( N_M(k) \) change shape when shortening the text-lengths \( M' \)? Note that, in the present context, two curves have the same shape provided their log-log-plots can be slid on top of each other, so that one is entirely on top of the other.

Figure 1a gives a first illustration: the number of distinct words in a text, \( N \), increases with the total number of words in the text \( M \). \( N \) as a function of \( 1/M \) is determined for two novels, Moby Dick by H.Melville and Harry Potter 1-7 by J.K. Rowling, by taking averages over fixed text-length \( M \). The data is plotted as \( N \) against \( 1/M \) in a log-log scale. In the limit \( M = 1 \) is \( N = 1 \) (the first word is always a distinct word), so that the curves in Fig.1a overlap at this point (which is the right-most point in Fig.1a). However as \( M \) increases the two curves start to systematically deviate. This demonstrates that these two curves do have different shapes. What conclusions can be drawn from this fact? The first conclusion is that the function \( N(1/M) \) is not a unique function of a language. This leaves you with two possibilities: it could be a unique property of an author writing a text or it could just be a property of a given text. In Ref. [21] it was shown that it is fact to large extent a property of the author. This led to the concept of a Meta book for an author serving as a literary fingerprint. We will come back to this meta book concept in the context of the results of Font-Clos et al. in Ref. [1].

Next we investigate the number of distinct words, \( N_M(\geq k) \), which occur more or equal to \( k \) times in a text of length \( M \). First one may note that since any distinct word must occur at least one time it follows that \( N_M(\geq 1) = N(M) \). This means that if we instead change the number of occurrences \( k \) to the relative number of occurrences \( k/M \), then \( N_M(\geq k) \) becomes \( N_M(\geq k/M) \) and \( N_M(\geq 1/M) = N(1/M) \). This latter form should, according Font-Clos et al, be scale-invariant, i.e. \( N_M(\geq 1/M) = G(k/M) \). To check this we again start with Moby Dick. First one notes that \( N_M(\geq k_{max}/M) = 1 \), because the most common word ‘the’ is a single word. This means that if one plots \( N_M(\geq k/M) \), as a function of \( k \), in the same plot as \( N \), as a function of \( 1/M \), then these two curves will coalesce at the left end points by definition. Next one takes the homogeneity of a text into account, which means that the ratio between the number of ‘the’ and the total number of words \( M' \) is to good approximation constant (\( \approx 0.066 \) for Moby Dick, see inset in Fig 2a). It follows that a text-part of length \( M_1 = M/k_{max} \) on the average contains one ‘the’, so at \( N(1/M_1) \approx 15 \) for Moby Dick. Or, in other words, \( N_M(\geq (k = k_{max})/M) = 1 \) is not equal to \( N_M(\geq (k = 1)/(M/k_{max})) \approx 15 \), clearly showing that \( N_M(\geq k/M) \) is not a scaling function in the variable \( k/M \). This inequality is illustrated in Fig 1b for Moby Dick and in Fig 1c for Potter 1-7.

In Fig.2 we investigate this discrepancy in more detail:
the novels are divided into text-lengths of given sizes $M$ and the $N_M(\geq k/M) = G(k/M)$ is obtained as the average over such fixed text-lengths. A first observation is that $N_M(\geq 1/M) = N(1/M)$, which means that the left-most point for each text-length falls on the respective $N(1/M)$ curve shown in Fig.1. Fig.2a shows the result for Moby Dick. The complete Moby Dick contains $M = 212473$ words and corresponds to the lowest curve in Fig.2a. The texts parts correspond to $M/10$, $M/100$, $M/500$, $M/1000$, and $M/3000$, respectively. One notes that all these text parts to good approximation starts from the same right-most point $k_{\text{max}}/M$. The reason for this is the following: the most frequent word in an English text is the word the. To good approximation the density of the’s is independent of where in the novel you are. In Moby Dick the density of the is $k_{\text{max}}/M = 0.066$ and is to good approximation constant and independent of the text lengths. The inset in Fig.2a shows this constancy. You can again ask if this density is a property of the language, author or just the text. As shown in Ref.[21] it is to good approximation a property of the author and not of the language itself.

If the scaling proposed in Font-Clos al in Ref.[1] was entirely correct, then all the data in Fig.2a should fall on the full $N(1/M)$ (highest curve in Fig.1a). This is clearly not the case. Next you can ask if the data for the parts of Moby Dick can be slid so that the overlap with the data for the full Moby Dick. However this is not possible because the curves describing the data do in fact have different shapes. This confirms that the scaling proposed by Font-Clos et al is fundamentally incorrect. Figure 2b contains the same analysis as Fig.2a but for the Harry Potter novels 1-7. Combined into one text these novels contain $M = 1012790$ words and is hence about five times larger than Moby Dick. However, the conclusions are just the same as for Moby Dick: the proposed scaling is fundamentally incorrect. Yet, one notes that the full Harry Potter and a twentieth-part of Harry Potter, for larger values of $k/M$, overlap to good approximation in Fig.2b. We will come back to the issue of what might be implied by this particular overlap.

As pointed out in Font-Clos et al in Ref.[1], the implication from the scaling function is that, if the full text can be approximated by a power law, then all the text-parts should be well approximated with the same power-law. In other words the implication is that the power-law index $\gamma$ does not change with text-size, in direct contradiction to the prediction from the ‘Randomness-view’ [21]. Figure 2c shows that the full Moby Dick can indeed be approximated by a straight-line. The slope of this line is $\gamma - 1 = 0.97$. However, also the text-parts can to good approximation be approximated by such lines, but these lines become steeper the smaller the text part. Thus the power law index $\gamma$ does systematically increase with smaller text size. Figure 2d shows this systematic increase in the power-law index with decreasing text length. Note that as the text-length limit $M_1 = M/k_{\text{max}}$ is approached $\gamma$ diverges (see Figs.1b and c). The prediction, based on invoking the scaling assumption, is that $\gamma$ is constant (the horizontal line in Fig.2d). This means that inferences based on invoking the scaling law proposed by Font-Clos et al leads to incorrect conclusions.

In this section we have demonstrated that the scaling law proposed by Font-Closet in Ref.[1] is fundamentally incorrect and taken seriously leads to incor-
Note that the right-most point because \( n \) denotes the average over an \( n^{th} \)-part of the novel. All these \( n^{th} \)-part curves to good approximation starts from the same right-most point because \( k_{\text{max}}/M \) is to good approximation independent of the text length, as shown by the inset. However all curves by definition ends on the corresponding \( N(1/M) \)-curve (same curve as in Fig.1b). Clearly all these \( n^{th} \)-part curves have different shapes and are not connected by a scaling relation; b) same as Fig.2a for Potter 1-7. c) power law approximation for the \( n^{th} \)-parts for Moby Dick. Note that for the full novel this approximation is close to the Zipf's law value \( \gamma \approx 2 \). However \( \gamma \) systematically increases with shortening the text-length.; d) shows this systematic increase of \( \gamma \) for Moby Dick and Potter. Note that \( \gamma \) diverges, as the limit text-length corresponding to one \( \text{the} \) on the average is approached (compare Fig.1 a and b). The length-independent \( \gamma \) predicted by invoking a scaling relation corresponds to the horizontal lines in Fig.2d.

FIG. 2. Test of the scaling form \( G(x) = N_M(x = k/M) \) by comparing different text-lengths: a) data for parts of Moby Dick, where \( n \) denotes the average over an \( n^{th} \)-part of the novel. All these \( n^{th} \)-part curves to good approximation starts from the same right-most point because \( k_{\text{max}}/M \) is to good approximation independent of the text length, as shown by the inset. However all curves by definition ends on the corresponding \( N(1/M) \)-curve (same curve as in Fig.1b). Clearly all these \( n^{th} \)-part curves have different shapes and are not connected by a scaling relation; b) same as Fig.2a for Potter 1-7. c) power law approximation for the \( n^{th} \)-parts for Moby Dick. Note that for the full novel this approximation is close to the Zipf's law value \( \gamma \approx 2 \). However \( \gamma \) systematically increases with shortening the text-length.; d) shows this systematic increase of \( \gamma \) for Moby Dick and Potter. Note that \( \gamma \) diverges, as the limit text-length corresponding to one \( \text{the} \) on the average is approached (compare Fig.1 a and b). The length-independent \( \gamma \) predicted by invoking a scaling relation corresponds to the horizontal lines in Fig.2d.

rect conclusions. Before discussing what might be implied by the findings by Font-Clos et al, we will in the next section widen the perspective and describe what the 'Randomness-view' predicts and implies.

III. RANDOMNESS-PREDICTIONS

The 'Randomness view' of word-frequency distributions is based on two assumptions. The first is that a text written by an author is homogeneous and the second randomness assumption enters through the maximum entropy principle. The first means that if you enumerate the word positions, \( M_i \) in the text by \( i = 1, \ldots M \) then the probability to find a word which occurs \( k \) times in the text is to good approximation independent of the position \( i \) within the text. For example you do not find more rare words at the end of the text than in the beginning and the most common word 'the' is to good approximation evenly spread through the text. This assumption can be expressed by a mathematical transformation, the Random Book Random Transformation(RBT), which will
be described and discussed below in the present section.\cite{22,23} However, the basic consequence of the homogeneous assumption is that if you take an \( n^{th} \)-part of the text the word-frequency distribution is to good approximation the same as if you just randomly deleted words from the text until only an \( n^{th} \)-part remain. This means that the homogeneous assumption immediately leads to a prediction for the word-frequency of the \( n^{th} \)-part of the text. This prediction for Moby Dick is given in Fig.3a. The point is that the homogeneous assumption to good approximation predicts how the word-frequency changes when you take a part of the text. It also predicts that the power-law index \( \gamma \) increases with decreasing text size. The conclusion inferred from Fig.3a is that the randomness implied by the homogeneous assumption gives both a qualitative and a quantitative agreement with the data.

The second randomness assumption enters through the maximum entropy principle. This is the same principle which in physics gives rise to the ideal gas law by assuming that the collisions between the gas-molecules in a container are random or the Gauss-distribution by assuming that the deviations around some average are random. In the present context it can be formulated as the Random Group Formation (RGF)\cite{23}. RGF predicts the word-frequency distribution from the sole knowledge of the total number of words \( M \), the number of distinct words \( N \) and the frequency of the most common word \( k_{max} \).\cite{23} The point is that the RGF-prediction is a general prediction where the randomness is incorporated into the maximum entropy principle: it predicts the probability \( P(k) \) that an object belongs to a group containing \( k \) objects provided that you know that the total number of objects is \( M \), the total number of groups is \( N \) and that the number of objects in the largest group is \( k_{max} \).\cite{23} Thus the RGF-prediction involves no linguistic information other than the identification between objects and words and between groups and distinct words. What is reflected in the word-frequency distribution is some general property, which texts share with many other completely unrelated phenomena.\cite{22,23} Thus RGF predicts the probability \( P_M(k) = N(k)/N \) for the full text without any explicit linguistic information and the homogeneity assumption transforms this expectation into the text-parts of length \( M/n \) using the Random Book Transformation (RBT) inherit in the text homogeneity assumption.\cite{21}

\[
P_{M/n}(k) = B \sum_{k'=k}^{M} A_{k'k} P_M(k'), \tag{1}
\]

where \( P_{M/n} \) and \( P_M \) are column matrices corresponding to \( P_{M/n} \) and \( P_M \). The transformation matrix \( A_{k'k} \) is given by

\[
A_{k'k} = (n-1)^{k-k'} n^{-k'} C_k^{k'}, \tag{2}
\]

where \( C_k^{k'} \) is binomial coefficient. \( B \) is given by the normalization condition

\[
B^{-1} = \sum_{k}^{M} \sum_{k'=k}^{M} A_{k'k} P_M(k'). \tag{3}
\]

Thus, by combining the maximum entropy randomness with the homogeneity randomness, the word-frequency for parts of Moby Dick can be entirely predicted from the sole knowledge of \( M, N \) and \( k_{max} \) for the full text. These predictions are given by the dashed curves in Fig.3b. The agreement with the data (full drawn curves in Fig.3b) is excellent. The fact that you to good accuracy can predict the features of the word-frequency from two very general assumptions of randomness and without any specific linguistic information suggests that you can extract basically no linguistic information from the shape of the word-frequency distribution. This is very far from the general ‘Zipf’s view’ that ‘A language is a very special system. Hence, it is likely that the functional form of the distribution of words in a text reflects some special property of the language.’\cite{21}

### IV. SHAPE INVARANCY UNDER THE RANDOM BOOK TRANSFORMATION

The RBT-transformation given in Eqs. \( \text{[1,23]} \) predicts how the probability distribution \( P_M(k) = N(k)/N \) for the full text changes into \( P_{M'}(k) \) for the \( n^{th} \)-part when assuming text-homogeneity. At the same time the RGF-function \( P_M(k) \propto \exp(-kb)/k^{\gamma} \) with \( \gamma \) typically in the range [1.5, 2] gives both a qualitatively and quantitative account of word-frequency distributions in real texts.\cite{23} This means that one can use the functional form \( P_M(k) \propto \exp(-kb)/k^{\gamma} \) in order to understand how the general shape of the frequency distribution influences what happens when one takes an \( n^{th} \)-part of the text. This leads to two exact mathematical results which helps clarifying the situation. The first result is that \( P_M(k) \propto \exp(-kb)/k \) under the RBT transforms as

\[
P_{M/n}(k) \propto \frac{\exp(-k \ln(n(k^{\gamma} - 1) + 1))}{k} \tag{4}
\]

This means that the limit case \( b = 0 \) and \( P_M(k) \propto 1/k \) is invariant under the transformation. However, \( 1/k \) is not normalizable, so \( b \) has to be larger than zero. If \( b \) is small enough then Eq.\( \text{[4]} \) reduces to

\[
P_{M'=M/n}(k) = A(n) \exp(-kb)/k. \tag{4'}
\]

where \( M' = M/n \). The average \( <k> \) for \( M' = M/n \) then becomes \( <k> = A(n) \exp(-bn)/bn \), so that \( M'/N' = <k> = A(n)/bn \) for small \( bn \). It follows that \( M' N' P_{M'}(k) = M'' \exp(-bmk)/(kn) \). Consequently, in this special case and provided \( n \) is small enough, \( M' N' P_{M'}(k/M') \) obeys the scaling proposed by Font-Clos et al in Ref.\( \text{[1]} \). Fig.4a shows the result for Eq.\( \text{[4]} \) for the same \( M \) and \( b \) as for Moby Dick. As seen from the figure, in this special case of \( \gamma = 1 \), the approximate scaling is according to the “randomness-view”
predicted to hold to good approximation down to a tenth of the original text-length. However, for larger $n$ the situation shown in Fig.1b and c is recovered, as it must from general considerations.

The second analytical solution to Eqs (1,2,3) is $P_M(k) \propto \exp(-kb)/(k-1)$ which transforms into

$$P_M(k) \propto \frac{\exp(-(k-1) \ln(ne^b - n + 1))}{k(k-1)}$$

One notes that this form diverges at $k = 1$. The point is that if you start with $P_M(k) \propto \exp(-kb)/k^\gamma$ and $\gamma > 1$ then you approach this divergent form with increasing $n$. This tendency is illustrated in Fig.4b for the case $P_M(k) \propto \exp(-kb)/k^{1.5}$ with the same $M$ and $b$ as for Moby Dick. In this case the deviation from the approximate scaling form is significant already for $n = 3$. The point is that since a real text corresponds to $1.5 < \gamma < 2$, it follows that it will always change shape whenever $n$ becomes large enough. It also follows that the discrepancy between a scaling curve and the data for a given $n$ will depend on the starting shape. The larger $\gamma$ the starting shape has, the larger discrepancy for a given $n$. This explains why the discrepancy in case of Potter 1-7 for $n = 20$ is smaller than for Moby Dick (compare Figs 2a and b). Another aspect, which is to some extent reflected in the difference of the 20th-parts for Moby Dick and Potter 1-7, is related to the Meta-book concept discussed in Ref.[21]. The meta-book of an author is all novels written by an author added together to a single text. The larger part of this text you analyze, the smaller the power law index $\gamma$ [21]. It was suggested that, in the limit of an infinite text, $\gamma$ approaches 1 and the distribution $P_M(k)$ approaches the limit form $1/k$ [21]. This means that the longer the text, the smaller the $\gamma$ and consequently also the smaller the difference in functional form, when taking an $n^{th}$-part. Thus the fact that Potter 1-7 is about five times longer than Moby Dick suggests that the discrepancy between the 20th-part and the full text should be larger for Moby Dick than for Potter 1-7, in accordance with Figs 2a and b.

In conclusion we find that an approximate scaling form in a special case indeed emerges from the randomness assumption that words with a given frequency are equally likely to appear anywhere in a text.

V. CONCLUDING REMARKS

This comment discusses both general and specific issues in connection with the proposed scaling law by Font-Closet al. in Ref.[1]. The first general issue concerns the two seemingly not compatible views on word-frequencies i.e. the ‘Zipf’s view’ and the ‘Randomness view’. We have here shown that the ‘Randomness view’ predicts word-frequency distributions both qualitatively and quantitatively. The conclusion drawn from this is that the shapes of word-frequency-distributions are general features, which words distribution share with a mul-
Clos et al falls neatly into the ‘Randomness view’. Furthermore the approximate scaling discussed by Font-Clos et al could well have been such a feature. However, as shown here, there is no such fundamental scaling law. The scaling law beyond Zipf’s law proposed by Font-Clos et al could well have been such a feature. However, as shown here, there is no such fundamental scaling law. The second general issue is the conceptual difference between predicting and fitting: The ‘Randomness view’ predicts word-frequency distributions from general assumptions, whereas the proposed form of the scaling function by Font-Clos et al is based on fitting a limited set of data. The conclusion is that word-frequency distributions for real texts change shapes with the length of the text analyzed, which is in agreement with and a consequence of the randomness-assumption that a text written by an author to good approximation is homogeneous.

[1] Font-Close F and Boleda G and Corral A A scaling law beyond Zipf’s law and its relation to Heaps’ law New J. Phys. 15 093033
[2] Estoup J B 1916 Les Gammes Sténographiques 4th edn (Paris: Institut Sténographique de France)
[3] Zipf G K 1932 Selective Studies of the Principle of Relative Frequency in Language (Cambridge, MA: Harvard University Press)
[4] Zipf G K 1935 The Psycho-Biology of Language: an Introduction to Dynamic Philology (Boston, MA: Mifflin)
[5] Zipf G K 1949 Human Behavior and the Principle of Least Effort (Reading, MA: Addison-Wesley)
[6] Mandelbrot B 1953 An Informational Theory of the Statistical Structure of Languages (Woburn, MA: Butterworth)
[7] Li W 1992 Random texts exhibit Zipf’s-law-like word frequency distribution IEEE Transactions on Information Theory 38 1842
[8] Baayen R H 2001 Word Frequency Distributions (Dordrecht, Netherlands: Kluwer Academic)
[9] i Cancho R F and Solé R V 2003 Least effort and the origins of scaling in human language Proc. Natl. Acad. Sci. U.S.A. 100 788
[10] Montemurro M A 2001 Beyond the Zipf-Mandelbrot law in quantitative linguistics Physica A 300 567
[11] Simon H 1955 On a class of skew distribution functions Biometrika 42 425
[12] Kanter I and Kessler D A 1995 Markov processes: linguistics and Zipf’s Law Phys. Rev. Lett. 74 4559
[13] Dorogovtsev S N and Mendes J F F 2001 Language as an evolving word web Proc. R. Soc. Lond. B 268 2603
[14] Zanette D H and Montemurro M A 2005 Dynamics of text generation with realistic Zipf’s distribution J. Quant. Linguistics
[15] Wang D H, Li M H and Di Z R 2005 Ture reason for Zipf’s law in language Physica A 358 545 12 29
[16] Masucci A and Rodgers G 2006 Networks properties of written human language Phys. Rev. E 74 26102
[17] Cattuto C, Loreto V and Servedio V D P 2006 A Yule-Simon process with memory *Europhys. Lett.* **76** 208
[18] Lü L, Zhang Z-K and Zhou T 2013 Deviation of zipf’s and heaps’ laws in human languages with limited dictionary sizes *Scientific reports* **3** 1082
[19] Miller G A 1957 Some effects of intermittence silence *American Journal of Psychology* **70** 311
[20] Bernhardsson S, da Rocha L E C and Minnhagen P 2010 Size dependent word frequencies and the translational invariance of books *Physica A* **389** 330
[21] Bernhardsson S, da Rocha L E C and Minnhagen P 2009 The meta book and size-dependent properties of written language *New J. Phys.* **11** 123015
[22] Bernhardsson S, Baek S K and Minnhagen P 2011 A paradoxical property of the monkey book *J. Stat. Mech.* **7** P07013
[23] Baek S K, Bernhardsson S and Minnhagen P 2011 Zipf’s law unzipped *New J. Phys.* **13** 043004
[24] Yan X Y and Minnhagen P 2014 Maximum Entropy, Word-Frequency, Chinese Characters, and Multiple Meanings ArXive preprint 1402.1939
[25] Powers D M W 1998 NeMaP3/CoLL’98: Proc.of Joint-Conf. on Methods in Language Processing and Computational Natural Language Learning (Stroudsburg, PA: Association for Computational Linguistics) pp 151-60
[26] Bokma F, Baek S K and Minnhagen P 2014 50 years of inordinate fondness *Syst. biol.* **63** 251