DETECTION OF FLUX EMERGENCE, SPLITTING, MERGING, AND CANCELLATION OF NETWORK FIELD. I. SPLITTING AND MERGING

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ABSTRACT

Frequencies of magnetic patch processes on the supergranule boundary, namely, flux emergence, splitting, merging, and cancellation, are investigated through automatic detection. We use a set of line-of-sight magnetograms taken by the Solar Optical Telescope (SOT) on board the Hinode satellite. We found 1636 positive patches and 1637 negative patches in the data set, whose time duration is 3.5 hr and field of view is 112 × 112′′. The total numbers of magnetic processes are as follows: 493 positive and 482 negative splittings, 536 positive and 535 negative mergings, 86 cancellations, and 3 emergences. The total numbers of emergence and cancellation are significantly smaller than those of splitting and merging. Further, the frequency dependence of the merging and splitting processes on the flux content are investigated. Merging has a weak dependence on the flux content with a power-law index of only 0.28. The timescale for splitting is found to be independent of the parent flux content before splitting, which corresponds to ∼33 minutes. It is also found that patches split into any flux contents with the same probability. This splitting has a power-law distribution of the flux content with an index of −2 as a time-independent solution. These results support that the frequency distribution of the flux content in the analyzed flux range is rapidly maintained by merging and splitting, namely, surface processes. We suggest a model for frequency distributions of cancellation and emergence based on this idea.

Key words: Sun: granulation – Sun: photosphere – Sun: surface magnetism

Online-only material: color figures

1. INTRODUCTION

How a magnetic structure on the solar surface is constructed and maintained is one of the fundamental issues in solar magnetic field observation. It is important for the statistical understanding of solar activities because they are triggered by magnetic activities on the solar surface. It may also give a quantitative restriction to the solar dynamo problem. One approach is to investigate the frequency distribution of the magnetic flux content on the solar surface. Some authors found an exponential distribution of flux content (Schrijver et al. 1997; Hagenaar 1999). On the other hand, other authors found a power-law distribution (Wang et al. 1995; Parnell et al. 2009; Zhang et al. 2010). Parnell et al. (2009) reported a power-law distribution with an index of −1.85 between 2 × 10^{17} Mx and 10^{23} Mx, which means that magnetic patches from large active regions to small patches in quiet networks are described by a single flux distribution. They suggest the idea that either all surface magnetic features are generated by the same mechanism or that they are dominated by surface processes.

The next question is how the frequency distribution is achieved and sustained. Magnetic processes, namely, flux emergence, splitting, merging, and cancellation of magnetic patches on the photosphere, are thought to change and maintain the frequency distribution. The relationship between these magnetic processes and the flux distribution is described by the magnetohemistry equation (Schrijver et al. 1997). Based on this equation and some assumptions, they found a time-independent solution for an exponential frequency distribution of flux content. Furthermore, Parnell (2002) found a particular solution for an arbitrary frequency distribution of flux content. Both solutions assume re-appearances of canceled fluxes and a detailed balance between any two fluxes, which should be verified observationally.

From the view of flux balance on the photosphere, flux emergence and cancellation are especially investigated because they have a direct relation with a flux exchange through the photosphere. Flux emergence, which is observed as a divergence of opposite polarities in the magnetogram, is thought to be a flux ascension from below the photosphere. It produces flux increases in both polarities in line-of-sight magnetograms. The frequency distribution of the emerging flux is investigated by several authors. Hagenaar (2001) found an exponential distribution by using full-disk magnetograms obtained by the Solar and Heliospheric Observatory (SOHO)/Michelson Doppler Imager (MDI). Thornton & Parnell (2011) found a power-law distribution from the active region to the inter-network field by using the Hinode/SOT magnetogram. Flux cancellation is defined as a convergence and a disappearance of magnetic fluxes of positive and negative polarities in the line-of-sight magnetograms (Martin et al. 1985; Livi et al. 1985). It decreases both positive and negative fluxes on the solar surface. Two physical models, U-loop emergence and Ω-loop submergence, are proposed (Zwaan 1987) and many authors have tried to distinguish them (Harvey et al. 1999; Yurchyshyn & Wang 2001; Chae et al. 2002, 2010; Kubo et al. 2010; Iida et al. 2010). Flux replacement timescales by emergence and cancellation are physical quantities representing the importance of these processes on the solar surface. But it varies from several days to several hours (Livi et al. 1985; Schrijver et al. 1998; Hagenaar 2001) and it is not clear why this timescale decreases as the spatial resolution becomes higher.

Merging and splitting should also play important roles in flux maintenance because magnetic patches change their flux content...
through these processes. The action of convective motion should differ among patches with different flux content. However, there are fewer reports for merging and splitting compared to those of emergence and cancellation. The role of these processes in flux balance still remains unclear.

We investigate the frequencies of magnetic patch processes, namely, flux emergence, splitting, merging, and cancellation based on the observations in this series of papers. This first paper is mainly devoted to investigations of the total flux amount of magnetic processes and frequency distributions on the flux content of merging and splitting. The purpose of this paper is to investigate what magnetic processes play the dominant role in generation and maintenance of frequency distribution of flux content. Our final goal of this series of papers is to understand how the flux distribution is maintained on the solar surface. We concentrate on magnetic processes in the network because most flux is contained there (Martin 1990; Schrijver 1995).

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Figure 1. Example of Na I D 1 magnetograms after the pre-process explained in the text. This magnetogram is taken at 2:21 UT on 2009 November 11, which is the middle of the observational period. Some magnetic networks are contained in the field of view.

Figure 2. Scatter plot of CP obtained by Hinode/SOT and the magnetic flux obtained by SOHO/MDI. Asterisks show the averaged SOT CP signal corresponding to 1 pixel of MDI. The horizontal bars indicate the minimum and maximum value of the SOT CP signal in each MDI pixel. The solid line shows a result of a linear fitting, whose slope is 9067.38 G DN$^{-1}$. The fitting range is from 30 G to 100 G in the absolute magnetic field strength.

We define magnetic processes and explain our method for detecting the magnetic processes. We use the fg_prep.pro procedure in the SolarSoftWare (SSW) package for a correction of the dark current and the flat field of the CCD camera. The data are rotated to the position at November 11 2:03 UT when the region is near the disk center by using the drot_map.pro procedure in the SSW package. We remove a columnwise median offset of the CCD camera (Lamb et al. 2010) by subtracting the median value of each column from all the pixels in the same column. A small spatial fluctuation in the field of view is removed by making a correlation between the consecutive images. Then the magnetograms are averaged over three continuous images and three pixels for smoothing.

The observed circular polarization (CP) signals are converted to the line-of-sight magnetic field strength by the following method. We multiply CP by a conversion coefficient to obtain the actual magnetic field strength in the next step. This conversion coefficient is determined by comparing the SOHO/MDI full disk (FD) magnetogram and CP image on 2009 November 11 1:39 UT. We spatially smear the SOT data to two times the MDI pixel size (R. A. Shine 2010, private communication) and make a linear fitting between CP in the SOT and the magnetic field in the MDI. Because the pixels in SOT data, which are out of linear range are too weak or too strong, we make the fitting in the range from 30 G to 100 G as a magnetic field strength. Figure 2 shows a scatter plot of SOT data numbers and magnetic field strength obtained by the MDI. The solid line shows a result of linear fitting. We obtain 9067.98 G DN$^{-1}$ as a conversion coefficient.

3. IDENTIFICATION OF MAGNETIC PROCESSES

We define magnetic processes and explain our method for the identification of them in this section. Figure 3 represents
schematic pictures of four magnetic processes detected in this study and Figure 4 summarizes our definition of magnetic processes. Note that our definitions of these processes are valid even when more than two patches are involved in the process but invalid when processes that increase and decrease flux content occur at the same time.

3.1. Detection and Tracking of Magnetic Patches

We use a clumping method for the detection of magnetic patches: each patch is picked up as a clump of marked pixels having magnetic strength beyond a given threshold (Parnell 2002). The adopted threshold of $1\sigma$ was obtained by fitting the histogram of the signed magnetic strength in each magnetogram, i.e., $\sigma \sim 5 \text{ G pixel} \sim 6.8 \times 10^{14} \text{ Mx}$. This value is close to that of Parnell et al. (2009). A magnetic patch in a clumping method corresponds to a massif of magnetic patches in a downhill method and a curvature method that are used in some previous studies (Hagenaar et al. 1999; Welsch & Longcope 2003; DeForest et al. 2007). We pick up magnetic patches with sizes beyond 81 pixels for focusing our analysis on the network magnetic field. The validity of this choice is demonstrated in Figure 5, in which the network structure is seen when adopting the 81 pixel threshold.

We track the motion of magnetic patches in consecutive images after their detection. Patches are marked as identical when they have a spatial overlapping in continuous images (Hagenaar 1999). It should be noted that the travel distance of magnetic patches in the data interval ($\sim$1 minute) is up to nearly 1 pixel size, namely, $2 \text{ km s}^{-1} \times 1 \text{ minute} = 120 \text{ km} \sim 1$ pixel. In a high-resolution magnetogram, more than one patch in a previous image often have spatial overlappings with one patch in a consecutive image and vice versa. To clear up this problem, we set two conditions when tracking patches. First, we investigate spatial overlappings of patches from those with larger flux contents. This is based on the concept that a smaller patch has a greater tendency to fall below the detection limit of the analysis by splitting and cancellation. Second, we select a patch with the most proximate flux content in case of overlappings of more than one patch. Tracking paths of detected patches become unique with these conditions.

3.2. Merging and Splitting

Merging is a process where more than one patch of the same polarity converge and coalesce into one patch (Figure 3(a)). We define a merging as an event with two conditions. A merging event is defined as a feature that satisfies the following conditions: (1) that there are two or more parent patches in a previous magnetogram overlapping one daughter patch in the consecutive magnetogram, and (2) that more than one of the

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**Figure 3.** Schematic pictures of four magnetic processes between two patches. (a) Merging: two patches converge and coalesce into one patch. (b) Splitting: one patch is divided into two patches. (c) Emergence: two opposite polarities with equal amount of flux content appear. (d) Cancellation: two opposite polarities converge with each other. One of them disappears. The other loses its flux content or disappears.

**Figure 4.** Schematic picture of our patch tracking and magnetic process detection. (1) Detection of patches with a clumping algorithm. (2) Tracking patches by examining overlaps. (3) Detection of merging and splitting by examining overlaps. (4) Detection of cancellation and emergence by making pairs of flux change events. (A color version of this figure is available in the online journal.)
Figure 5. Two-leveled magnetograms taken at 2:21 on 2009 November 11 with different size thresholds. Size thresholds, $S_{th}$, are set as (a) 9 pixels, (b) 36 pixels, (c) 81 pixels, and (d) 144 pixels.

3.3. Emergence and Cancellation

The detailed explanation for the detection technique of emergence and cancellation will be given in the next paper in which many more events are detected by using another data set, making it possible for us to conduct a statistical study of them. Emergence and cancellation are defined as a pair of flux change events of each patch in opposite polarities within a certain distance. We pick up not only complete cancellations but also partial cancellations in this definition.

Panels in the third row of Figure 6 show an example of emergences. In the second panel, positive and negative patches appear but the positive patch is already coalesced with a pre-existing patch. These emerging patches continue to separate against each other and become distant in the last panel. Panels in the last row of Figure 6 show an example of partial cancellations. Opposite polarities are converging in the first and second panel. They start canceling between the second and the third panel. The negative patch totally disappears in the last panel but the positive patch remains after the cancellation in this example.

4. RESULTS

4.1. General Description

Along with the total numbers of patches and their flux density averaged over the field of view and the observational period (in Mx cm$^{-2}$), the frequencies of events in the magnetic flux density (in Mx cm$^{-2}$ s$^{-1}$) are summarized in Table 1. The total numbers of detected negative and positive patches are 1636 and 1637, respectively, enough for a statistical study. As for magnetic processes, merging and splitting are much more frequent than emergence and cancellation. The total numbers of merging for positive and negative patches are 493 and 482. The total numbers of splitting for positive and negative patches are 536 and 535. These are enough for a statistical study. The concerned patches in the previous magnetogram disappear in the time interval. Panels in the first row of Figure 6 show an example of detected mergings. Two patches converge in the first and second images. They coalesce into one patch in the third image.

Splitting is a process where a single patch is divided into more than one patch (Figure 3(b)). A splitting event is defined as a feature that satisfies the following two conditions: (1) that there are one or more daughter patches in a magnetogram overlapping one parent patch in the previous magnetogram, and (2) that more than one of the concerned patches in the latter magnetogram appear in the time interval. Panels in the second row of Figure 6 show an example of the detected splittings. One patch stays in the first and second panel. It splits into two patches between the second and third panel.
number of emergence is 3 and that of cancellation is 86, which are not enough for a statistical study.

To determine what magnetic processes dominate a flux balance in this region, we investigate total flux amounts of patches and those related to the magnetic processes per unit area per unit time. The results are shown in Table 1. The averaged flux densities are 2.53 Mx cm\(^{-2}\) and 3.60 Mx cm\(^{-2}\) for positive and negative patches, respectively. They correspond to the total flux amount of 1.63 \(\times\) 10\(^{20}\) Mx and 2.32 \(\times\) 10\(^{20}\) Mx for our field of view. The rate of averaged flux density involved in merging processes is 1.65 (3.53) \(\times\) 10\(^{-3}\) Mx cm\(^{-2}\) s\(^{-1}\) for positive (negative) patches. We define the flux amount of merging process as that of parent patches. The flux replacement timescale by positive (negative) merging, \(\tau_{\text{mrg}} = \Phi_{\text{tot}}/(\partial\Phi_{\text{tot}}/\partial t)_{\text{mrg}}\), is evaluated as 1.53 (1.02) \(\times\) 10\(^{3}\) s from these values, which is much shorter than the estimated replacement timescale by cancellation and emergence reported in previous studies (Livi et al. 1985; Schrijver et al. 1998; Hagenaar 2001; Thornton & Parnell 2011). The rate of the averaged flux density involved in splitting processes is 1.48 (3.03) \(\times\) 10\(^{-3}\) Mx cm\(^{-2}\) s\(^{-1}\) for positive (negative) patches. We define the flux amount of the splitting process as the sum of those of parent patches. In the same manner as merging, the flux replacement timescale by positive (negative) splitting is evaluated as 1.71 (1.19) \(\times\) 10\(^{3}\) s.

### 4.2. Frequency Distribution of Flux Content

Figure 7 shows a frequency distribution of the flux content. The red and blue dashed histograms denote those of positive and negative polarities, respectively. The solid histogram denotes the sum of them. The black dashed line indicates the fitting result of it. The distribution drops down below \(\Phi_\text{th} = 10^{17.5}\) Mx, which is suggested as a detection limit in this study, below which only a limited number of the patches are detected. The dashed line is a fitted power-law result between 10\(^{17.5}\) Mx

| Positive Polarity | Negative Polarity |
|-------------------|-------------------|
| Patch             | 1636 (2.53 Mx cm\(^{-2}\)) | 1637 (3.60 Mx cm\(^{-2}\)) |
| Merging           | 536 (1.65 \(\times\) 10\(^{-3}\) Mx cm\(^{-2}\) s\(^{-1}\)) | 535 (3.03 \(\times\) 10\(^{-3}\) Mx cm\(^{-2}\) s\(^{-1}\)) |
| Splitting         | 493 (1.48 \(\times\) 10\(^{-3}\) Mx cm\(^{-2}\) s\(^{-1}\)) | 482 (3.53 \(\times\) 10\(^{-3}\) Mx cm\(^{-2}\) s\(^{-1}\)) |
| Emergence         | 3                 |
| Cancellation      | 86                |

**Notes.** The numbers in the parentheses in the first line are the average flux densities of each polarity. Those in the parentheses in the second and third lines are the average rates of the flux density change by each process. These values are averaged over the whole field of view and observational period.

Figure 6. Examples of detected magnetic processes, namely, (a) merging, (b) splitting, (c) emergence, and (d) cancellation. The background shows the magnetic flux density obtained by *Hinode/NFI*. The red (blue) contours indicate positive (negative) patches detected with our threshold. The field of view is 14\(\prime\)4 \(\times\) 14\(\prime\)4 for all the images. (A color version of this figure is available in the online journal.)

![Figure 6](image.png)
and $10^{19}$ Mx. We make a least-squares fitting of the power-law function with a form, $n(\phi) = n_0 (\phi/\phi_0)^{-\gamma}$. We obtain $n_0 = 2.42 \times 10^{-36}$ Mx$^{-1}$ cm$^{-2}$, $\phi_0 = 1.0 \times 10^{18}$ Mx, and $\gamma = 1.78 \pm 0.05$. The error means a 1σ error. The fitted power-law index is sensitive to the fitting range as reported by Parnell et al. (2009). The index varies from $-1.78$ to $-1.91$ with a minimum fitting range of $10^{17.5}$–$10^{18}$ Mx. The total error of our fitting of the power-law index becomes 0.18 when considering this error. This result is consistent with Parnell et al. (2009), which reports $-1.85 \pm 0.14$ as a power-law index of a frequency distribution of the flux content.

### 4.3. Probability Distributions on Flux Content of Merging and Splitting

We investigate frequency distributions on flux content of merging and splitting for one patch, which are defined as frequency distributions of processes divided by a frequency distribution of flux content. The value of these distributions represents the probability of the processes. Further, the detection limit $\phi_{th}$ is taken into account in this study. We call them apparent probability distributions of processes.

Figure 8 shows the apparent probability distribution of merging. We make a least-squares fitting in a range of $10^{17.5}$–$10^{19}$ Mx, where the number of the merging event is enough for the fitting. The fitting form is

$$\frac{\partial P_{\text{APP}}}{\partial t} = p_{0,\text{merg}} (\frac{\phi}{\phi_0})^{\beta_{\text{merg}}},$$

where $p_{0,\text{merg}}$ is the reference probability, $\phi_0$ is the reference flux content, and $\beta_{\text{merg}}$ is the power-law index of the probability distribution of merging. We obtained $p_{0,\text{merg}} = (2.52 \pm 0.08) \times 10^{-4}$ s$^{-1}$ and $\beta_{\text{merg}} = 0.28 \pm 0.05$ for the positive patches, $p_{0,\text{merg}} = (2.52 \pm 0.08) \times 10^{-4}$ s$^{-1}$ and $\beta_{\text{merg}} = 0.26 \pm 0.05$ for the negative patches, and $p_{0,\text{merg}} = (5.12 \pm 0.11) \times 10^{-4}$ s$^{-1}$ and $\beta_{\text{merg}} = 0.28 \pm 0.04$ for both patches with $\phi_0 = 10^{18}$ Mx. The errors mean a 1σ error of least-squares fitting.

Figure 9 shows the apparent probability distribution of splitting. The strong increase in the range larger than $10^{19}$ Mx is caused by the lack of a patch number in the analysis. On the other hand, there is a drop in the flux range near $\phi_{th}$, where the number of patches is enough for a statistical study. We interpret this dropping as an effect of splitting into the area below $\phi_{th}$. This effect is evaluated in the discussion in Section 5.1. We see that the probability of splitting is almost constant as $1.0 \times 10^{-3}$ s$^{-1}$, which means a timescale of 33 minutes, in the range enough.
above \( \phi_0 = 3.0 \times 10^{18} - 1.0 \times 10^{19} \) Mx. It means that the frequency of splitting is independent of the parents’ flux content.

5. DISCUSSION

5.1. Probability Density Distributions of Merging and Splitting

We evaluate the probability density distributions of merging and splitting terms in a magneto-chemistry equation from the observational results. Because the probability distributions, which we obtained in this study, are obtained by integrating them in the flux content once, we have to put at least one assumption to evaluate them.

The merging function \( l(x, y) \) is given as follows. We obtain the form as a probability distribution from the definition of \( l(x, y) \) as

\[
\frac{\partial P_{\text{mg}}}{\partial t} = \int_{\phi_0}^{\infty} n(x) l(\phi, x) d\phi.
\]

(2)

By comparing with the observational result, we obtain

\[
\int_{\phi_0}^{\infty} n(x) l(\phi, x) d\phi = p_{0,\text{mg}} \left( \frac{\phi}{\phi_0} \right)^{\beta_{\text{mg}}} (\phi \geq \phi_0).
\]

(3)

In the left-hand side of this equation, the variable \( \phi \) appears only in the \( l(\phi, x) \). We assume a simple form satisfying this relation, namely,

\[
l(x, y) \propto x^{\beta_{\text{mg}}}.
\]

(4)

From the symmetry of \( l(x, y) = l(y, x) \), this relation deduces

\[
l(x, y) = l_0 \left( \frac{x}{\phi_0} \right)^{\beta_{\text{mg}}} \left( \frac{y}{\phi_0} \right)^{\beta_{\text{mg}}}.
\]

(5)

Substituting it and the observational result of \( n(\phi) \) into Equation (3), we obtain

\[
l_0 = \frac{p_{0,\text{mg}}}{n_0 \phi_0^{\gamma-\beta_{\text{mg}}} \int_{\phi_0}^{\infty} x^{-\gamma+\beta_{\text{mg}}} d\phi}.
\]

(6)

The upper value of the integration is limited on the actual Sun and we put it as \( \phi_{\text{max}} \). We substitute the value obtained in our study, namely, \( n_0 = 1.21 \times 10^{-36} \) Mx\(^{-1} \) cm\(^{-2} \), \( \gamma = 1.78 \), \( \rho_{0,\text{mg}} = 2.56 \times 10^{-4} \) s\(^{-1} \), \( \beta_{\text{mg}} = 0.28 \), \( \phi_{\text{max}} = 10^{19} \) Mx, \( \phi_0 = 10^{17.5} \) Mx, and \( \phi_0 = 10^{18} \) Mx, and obtain \( l_0 = 7.24 \times 10^{13} \) cm\(^2 \) s\(^{-1} \).

The splitting function \( k(x, y) \) is given as follows. From our observations, the probability distribution of the splitting events \( \partial P_{\text{split}}(\phi)/\partial t \) is suggested to be independent of the parent patch flux:

\[
\frac{\partial P_{\text{split}}}{\partial t} (\phi) = k_0 = \text{constant (for all } \phi). \tag{7}
\]

This claim is observationally supported at least in the range \( \phi > \phi_0 \) (Figure 9). The drop off below \( \phi_0 \) is discussed immediately below. If the splitting ratio between the daughter patches is randomly determined, i.e.,

\[
\frac{\partial}{\partial x} [k(x, \phi - x)] = 0 \text{ (for } 0 < x < \phi),
\]

then we obtain,

\[
k(x, y) = \frac{k_0}{x + y}. \tag{9}
\]

When the flux content of the daughter patch is below \( \phi_0 \), such events are not recognized as a splitting event in our procedure. The probability distribution will be given as

\[
\frac{\partial P_{\text{split}}}{\partial t}(\phi) = \int_{\phi_0}^{\phi-\phi_0} k(x, \phi - x) dx = k_0 \left( 1 - \frac{2\phi_0}{\phi} \right). \tag{10}
\]

The black and blue dashed curves in Figure 9 indicate analytical curves with \( k_0 = 1.0 \times 10^{-3} \) s\(^{-1} \) and \( k_0 = 5.0 \times 10^{-4} \) s\(^{-1} \), respectively. This curve fits the drop of the observational line well, which supports the above assumptions. We obtain \( k(x, y) = k_0/(x + y) \) as a splitting function, where \( k_0 = 5.0 \times 10^{-4} \) s\(^{-1} \).

5.2. The Time-independent Solution of the Splitting Process

Since our observations show that merging and splitting are much more frequent than emergence and cancellation, it suggests that the former two have much influence on the maintenance of the power-law distribution. We show the time-independent solution by splitting, although we have not found the time-independent solution by merging and splitting. The magneto-chemistry equation is consistent with the source (emergence) term, merging terms, splitting terms, and cancellation terms (see the right-hand side of Equation (3) in Schrijver et al. 1997). The frequency of emergence, merging, splitting, and cancellation are represented by \( S(\phi), l(x, y), k(x, y), \) and \( m(x, y) \), respectively. We use the magneto-chemistry equation only, including the splitting terms, by setting \( S(\phi) = 0, l(x, y) = 0, m(x, y) = 0 \), namely,

\[
\frac{\partial n(\phi)}{\partial t} = 2 \int_0^\infty n(x) k(\phi, x - \phi) dx - \int_0^\phi n(\phi) k(x, \phi - x) dx, \tag{11}
\]

where \( n(\phi) \) is the frequency distribution of the flux content.3

After substituting \( k(x, y) = k_0/(x + y) \) into Equation (11) and differentiating with \( \phi \), we obtain

\[
\frac{\partial^2 n(\phi)}{\partial \phi^2} = -k_0 \frac{\partial}{\partial \phi} [\phi^2 n(\phi)]. \tag{12}
\]

This equation has a time-independent solution \( n(\phi) \propto \phi^{-2} \). This power-law index of the flux content is in good agreement with the observational result. This scale-free distribution comes from constancies of splitting, namely, that splitting has a constant timescale independent of the flux content and a constant probability of splitting the flux content. These constancies may come from convection dominating patch stability or flux tube instability with a constant timescale. We need further theoretical and observational studies to determine which of the hypotheses is the actual scenario on the solar surface.

5.3. Relationship Among the Frequency Distribution of the Flux Content, Cancellation, and Emergence

We suggest a relationship model among the frequency distribution of the flux content, cancellation, and emergence. The important hypothesis in this model is that the frequency distribution of the flux content is rapidly maintained regardless of

\[\text{Terms on the right-hand side of this equation are different from those of Equation (3) in Schrijver et al. (1997) at the point of the splitting from } \phi, \text{ namely, } m(x)(\phi, x - \phi) \text{ in this paper and } n(x)(\phi, x) \text{ in Schrijver et al. (1997). The term in Schrijver et al. (1997) should be a typo because we have to multiply the pre-splitting number density here.}\]
cancellation and emergence, which is supported by this study. Figure 10 shows a schematic view of this model. We put a power-law distribution of the flux content in units of patches Mx$^{-1}$ cm$^{-2}$ as

$$n(\phi) = n_0 \left( \frac{\phi}{\phi_0} \right)^{-\gamma}, \quad (13)$$

where $\phi_0$ is a reference value of the flux content and $n_0$ is a reference frequency of the flux content. The power-law index, $\gamma$, is derived as 1.5 < $\gamma$ < 2 by our observation and previous studies (Parnell et al. 2009; Zhang et al. 2010). The maximum flux content in the system ($\phi_{\text{max}}$) is assumed to be much larger than the minimum ($\phi_{\text{min}}$). We calculate $N(\phi)$, a total patch number with a flux content larger than $\phi$, by integrating a flux distribution from $\phi$ to $\phi_{\text{max}}$.

$$N(\phi) = \int_{\phi}^{\phi_{\text{max}}} n(\phi') \, d\phi' \approx \frac{n_0 \phi_0}{\gamma - 1} \left( \frac{\phi}{\phi_0} \right)^{-\gamma+1}. \quad (14)$$

Schrijver et al. (1997) evaluated a collision rate of opposite patches from a total patch number density with assumptions of a constant velocity and a randomness of patch motion along a network. They obtained the collision frequency, $v$, as

$$v = \frac{v_0}{4\sqrt{\rho}} N_0^2, \quad (15)$$

where $v_0$, $\rho$, and $N_0$ mean a typical velocity of patches, a number density of network cells, and a number density of patches, respectively. We multiply 1/2 taking the double counting into account. We obtained that the frequencies of merging and splitting are larger than that of cancellation in this study. It can be deduced that the frequency distribution is maintained rapidly by merging and splitting compared to the timescale of cancellation. This enables us to treat that the number density of patches is time independent in the evaluation of cancellation and apply the same analogy to the number density expanded in the dimension of the flux content. The collision frequency, $\partial N(\phi)/\partial t|_{\text{col}}$, is evaluated as

$$\left. \frac{\partial N(\phi)}{\partial t} \right|_{\text{col}} = -\frac{v_0 n_0^2 \phi_0^2}{4(\gamma - 1) \sqrt{\rho}} \left( \frac{\phi}{\phi_0} \right)^{-2\gamma+2}. \quad (16)$$

Note that this total collision number becomes time independent with an assumption of maintenance of a power-law flux distribution. We assume that the total number of cancellation, $\partial N(\phi)/\partial t|_{\text{cnc}}$, equals the total number of collision events of the opposite polarities, $\partial N(\phi)/\partial t|_{\text{col}}$, namely,

$$\left. \frac{\partial N(\phi)}{\partial t} \right|_{\text{cnc}} = \left. \frac{\partial N(\phi)}{\partial t} \right|_{\text{col}}. \quad (17)$$

The frequency distribution of cancellation, $\partial n(\phi)/\partial t|_{\text{cnc}}$, is given by a differentiating Equation (16) with $\phi$, namely,

$$\left. \frac{\partial n(\phi)}{\partial t} \right|_{\text{cnc}} = \frac{\partial^2 N(\phi)}{\partial \phi \partial t} \bigg|_{\text{cnc}} = -\frac{v_0 n_0^2 \phi_0^2}{2(\gamma - 1) \sqrt{\rho}} \left( \frac{\phi}{\phi_0} \right)^{-2\gamma+1}. \quad (18)$$

This assumption means that there are no patches passing through the patches of opposite polarity once they collide, which is difficult to check and we justify this assumption from the comparison of the obtained frequency distribution of emergence in this model and that in the observation. We also evaluate a frequency distribution of emergence with assumptions of a small amount of flux supply from outside of the system and re-emergences of canceled fluxes. These assumptions lead to the relationship that a frequency distribution of emergence nearly equates that of cancellation,

$$\left. \frac{\partial n(\phi)}{\partial t} \right|_{\text{emrg}} \approx -\left. \frac{\partial n(\phi)}{\partial t} \right|_{\text{cnc}} = \frac{v_0 n_0^2 \phi_0^2}{2(\gamma - 1) \sqrt{\rho}} \left( \frac{\phi}{\phi_0} \right)^{-2\gamma+1}. \quad (19)$$

We compare the power-law index and the absolute value of the frequency distribution of emergence in this model with the observational results. Based on the above discussion, our observation suggests a frequency distribution of emergence as $\partial n/\partial t|_{\text{emrg}} \sim 4.3 \times 10^{-35} \times (\phi/1.0 \times 10^{16} \text{ Mx})^{-2.6} \text{ Mx}^{-1} \text{ cm}^{-2} \text{ s}^{-1}$, where we adopted $\rho = 1.0 \times 10^{-19} \text{ cm}^{-2}$ (Hagenaar et al. 1997) and $v_0 = 2.0 \text{ km s}^{-1}$. Thornton & Parnell (2011) reports the power-law frequency of emergence as $\partial n/\partial t|_{\text{emrg}} \sim 2.5 \times 10^{-35} \times (\phi/1.0 \times 10^{16} \text{ Mx})^{-2.7} \text{ Mx}^{-1} \text{ cm}^{-2} \text{ s}^{-1}$. The steepness of the power-law distribution and the absolute value in our model are in good agreement with the observational results.

We calculate the flux replacement time by cancellation and emergence. The total flux amount in the system is calculated

![Figure 10. Relationship model among the flux distributions of the flux content, cancellation, and emergence with 1.5 < $\gamma$ < 2. The solid arrows indicate mathematical relations, while the dashed arrows indicate relationships with some assumptions.](image-url)
from Equation (13) as
\[
\Phi_{\text{tot}} = \int_{\phi_{\text{min}}}^{\phi_{\text{max}}} \phi' n(\phi') d\phi' \approx n_0 \rho_0^2 \frac{\phi_{\text{max}}}{2 - \gamma} \left( \frac{\phi_{\text{max}}}{\phi_0} \right)^{-\gamma+2}.
\]

(20)
Since \( \gamma < 2 \), this result shows that the total flux is dominated by patches with a larger flux content. On the other hand, we obtain a total flux loss amount by cancellation, \( \partial \Phi_{\text{tot}}(\phi)/\partial t|_{\text{cnc}} \), from Equation (18) as
\[
\frac{\partial \Phi_{\text{tot}}}{\partial t}|_{\text{cnc}} = \int_{\phi_{\text{min}}}^{\phi_{\text{max}}} \phi' \frac{\partial n(\phi')}{\partial t}|_{\text{cnc}} d\phi' \approx -\frac{\nu_0 \rho_0^2}{2(\gamma - 1)(2\gamma - 3)} \sqrt{\rho} \left( \frac{\phi_{\text{min}}}{\phi_0} \right)^{-2\gamma+3}.
\]

(21)
The total flux supply by recycled emergence is also evaluated in the same manner from Equation (19) as
\[
\frac{\partial \Phi_{\text{tot}}}{\partial t}|_{\text{emrg}} = \int_{\phi_{\text{min}}}^{\phi_{\text{max}}} \phi' \frac{\partial n(\phi')}{\partial t}|_{\text{emrg}} d\phi' \approx \frac{\nu_0 \rho_0^2}{2(\gamma - 1)(2\gamma - 3)} \sqrt{\rho} \left( \frac{\phi_{\text{min}}}{\phi_0} \right)^{-2\gamma+3}.
\]

(22)
Then flux replacement time is evaluated as
\[
\tau_{\text{replace}} = \frac{\Phi_{\text{tot}}}{\partial \Phi_{\text{tot}}/\partial t|_{\text{cnc}}} = \frac{\Phi_{\text{tot}}}{\partial \Phi_{\text{tot}}/\partial t|_{\text{emrg}}} = \frac{2(\gamma - 1)(2\gamma - 3)}{(2 - \gamma)\nu_0 \rho_0^2} \left( \frac{\phi_{\text{max}}}{\phi_0} \right)^{-\gamma+2} \left( \frac{\phi_{\text{min}}}{\phi_0} \right)^{2\gamma-3}.
\]

(23)
This result is qualitatively consistent with the previous result that the flux replacement time becomes shorter with higher resolution (Martin et al. 1985; Schrijver et al. 1998; Hagenaar 2001).

We summarize our interpretation from the discussion. In our interpretation, a power-law frequency distribution of the flux content is rapidly maintained by merging and splitting, which is supported by comparing the change rate of the flux amount in each process. In addition, we have found that a power-law frequency distribution with an index of \(-2\) is a time-independent solution of splitting. Cancellation is caused by convective dominant motion and frequency distribution of cancellation should naturally become a steep power-law distribution. Emergence is interpreted as re-emergence of submerged flux by cancellation, which is consistent with a steep frequency distribution of emergence (Thornton & Parnell 2011). This should be one of the possibilities but the important point of this model is that the apparent flux transport through the photospheric layer becomes much more drastic when investigating emergence and cancellation, although the injected flux amount from the deeper layer is small or even zero. A recent paper, Meyer et al. (2011), reports their numerical simulations of the magnetic carpet on the photosphere. They show that a power-law distribution of the flux content is maintained with an input of a steep power-law frequency of emergence. The induced frequency of cancellation becomes a steep power-law one, which is consistent with our model.

What should be investigated for a further understanding of flux transport is a statistical investigation of cancellation on the photosphere. Another important question is whether there is a stable solution of our model with four magnetic processes or not. We will investigate these questions in this series of papers.

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