Linear Stability analysis of hydromagnetic Couette flow with small injection/suction through the modified Orr-Sommerfeld equation

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Abstract

This paper analyses the effects of small injection/suction Reynolds' number, Hartmann number, permeability parameter and wave number on a viscous incompressible electrically conduction fluid flow in a parallel porous channel. The plates of the channel with small constant injection/suction, have constant temperature. The upper plate is allowed to move in flow direction and the lower plate is kept at rest. A magnetic field of uniform strength is also applied normally to the plates what are parallel. The originality of the paper is to study the effect of the above parameter in temporal linear stability analysis of the flow through the modified Orr-Sommerfeld equation.

Keywords : Temporal linear stability, small injection/injection Reynolds number, modified Orr-Sommerfeld equation, hydromagnetic Poiseulle flow.

1 Introduction

The study of Couette flow in a rectangular channel of an electrically conducting viscous fluid under the action of a transversely applied magnetic field has immediate applications in many devices such as magnetohydrodynamic (MHD) power generators, MHD pumps, accelerators, aerodynamics heating, electrostatic precipitation, polymer technology, petroleum industry, purification of crude oil and fluid droplets sprays. Channel flows of a Newtonian fluid with heat transfer were studied with or without Hall currents by many authors [1–11]. The effects of injection/suction through the injection/suction parameter number, Hartmann number, Permeability or Darcy parameter . . . on the stability of the fluids flows were studied by many researchers [3–7] with different approach.

The heat source and the Soret effects on an oscillatory hydromagnetic flow through a porous medium bounded by two vertical parallel porous plates is analyzed by K. Chand et al. [3], where one plate of the channel is kept stationary and the other is moving with uniform velocity. The plates of the channel are subjected to constant injection and suction velocities respectively. A. NAYAK et al. [4] have studied an oscillatory effects on magneto-hydrodynamic flow and heat transfer in rotating horizontal porous channel. N. V. R. V. Prasad et al. [5] have considered the unsteady hydromagnetic incompressible viscous fluid flow through a porous medium in a horizontal channel.

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under prescribed discharge, under the influence of inclined magnetic field. S.S. DAS et al. [6] analyze the effects of constant suction and sinusoidal injection on three dimensional couette flow of a viscous incompressible electrically conducting fluid through a porous medium between two infinite horizontal parallel porous flat plates in presence of a transverse magnetic field. The stationary plate and the plate in uniform motion are, respectively, subjected to a transverse sinusoidal injection and uniform suction of the fluid. Shalini et al. [7] have studied the effects of variable viscosity and heat source on unsteady laminar flow of dusty conducting fluid between parallel porous plates through porous medium with temperature dependent viscosity. It is assumed that the parallel plates are porous and subjected to a uniform suction from above and injection from below.

The objective of the present paper is to study the effects of the injection/suction Reynolds’ number, Hartmann number, Permeability parameter on the linear temporal stability of the flow of a viscous incompressible electrically conducting fluid in a Couette horizontal porous channel in the presence of a uniform transverse magnetic field when it is fixed relative to the fluid through the modified Orr-Sommerfeld equation. The plates of the channel are considered porous and flow within the channel is due to the uniform motion of the upper plate. Such linear temporal stability analysis through a modified Orr-Sommerfeld equation has been made earlier by A. V. Monwanou and J. B. Chabi Orou [2] in a Poiseuille flow without injection/suction. The same problem with a viscous incompressible no-conduction fluid but with small injection/suction in medium through the porous plates has been made by L. hinv et al. [9]. The paper is organized as follows.

In the second section the modified Orr-Sommerfeld equation governing the stability analysis in the Couette horizontal porous plates flow is checked. In the third section an analysis of the effects of small injection/suction Reynolds’ number $R_e\omega$, Hartmann number $H$ and permeability parameter $K_p$ on a viscous incompressible electrically conduction fluid flow in a parallel porous channel linear stability will be investigated with the help of figures and tables. The conclusions is presented in the final section.

## 2 Modified Orr-Sommerfeld equation

We considered a Poiseuille viscous incompressible and electrically conduction fluid flow between two porous parallel plates of infinite length, distant $h$ apart in the presence of uniform transverse constant magnetic field $B_0$ applied parallel to $y^*$ axis which is normal to the planes of the plates. We considered the simple case where, $B_0$ is fixed relative to the fluid. We work at constant temperature, the heat transfert aspect of the flow is not studied. We applied a small constant injection $V_\omega$, at the lower plate and a same small constant suction $V_\omega$, at the upper plate. The upper plate is allowed to move with non-zero uniform velocity $U = U_0$ in flow direction and the lower plate is kept at rest. We choose the origin on the plane $(x^*, 0, z^*)$ such as $-h \leq y^* \leq h$ and $x^*$ parallel to the direction of the motion of the upper plate. We assumed the magnetic Reynolds’ number very small for metallic liquids and neglected the induced magnetic field in comparison with the applied one. Initially, $t^* < 0$, both the fluid and plates are assumed to be at rest. When $t^* > 0$, the upper plate starts moving with a constant velocity $U$ in coordinate system with the fluid. The equations of continuity,
motion for the viscous incompressible electically conduction fluid in vector form are:

\[
\nabla \cdot V + (V, \nabla) V = -\frac{1}{\rho} \nabla p + \nu \nabla^2 V + \frac{1}{\rho} J \wedge B - \frac{\mu V}{\rho \kappa},
\]

(1)

\[
\nabla \wedge B = \mu_0 J,
\]

(2)

\[
\nabla \wedge E = \frac{\partial B}{\partial t^*},
\]

(3)

\[
\nabla \cdot B = 0,
\]

(4)

\[
\nabla \cdot J = 0.
\]

(5)

They are continuity, Newton’s second Law, Ampere’s Law, Faraday’s Law, Maxwell’s Law and Gauss Law equations respectively, with

\[
J = \sigma (E + V \wedge B).
\]

(6)

Where \( V (u^*, v^*, w^*) \), \( B \), \( E \), \( J \), \( \mu_0 \) the velocity, the magnetic field, the electric field, the current density vector, the fluid electrical conductivity and the magnetic permeability of the fluid respectively and \( t^* \) denotes time. The physical model of the problem is illustrated in figure 1 below, where \( V (u^*, v^*, w^*) \) is the velocity vector in the \( x^*, y^*, z^* \) directions respectively.

![Physical Model and Coordinate System](image)

Figure 1: physical model and coordinate system

\[
B = (0, B_0, 0),
\]

(7)

\[
E = (E_x, E_y, E_z),
\]

(8)

\[
J = (J_x, 0, J_z);
\]

(9)

where \( B_0 \) is a constant. We assumed that no applied and polarization voltage exists (i.e., \( E = 0 \)). Then

\[
\text{Equation (7) and (10) } \implies J = \sigma B_0 (-w, 0, u)
\]

(10)

and (6) yields

\[
\sigma B_0 \left( \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) = 0
\]

(11)

We introduced the following non-dimensional quantities \( \tilde{x} = \frac{x^*}{h}, \tilde{y} = \frac{y^*}{h}, \tilde{z} = \frac{z^*}{h}, \tilde{t} = \frac{Ut^*}{h}, \tilde{u} = \frac{u^*}{U}, \tilde{v} = \frac{v^*}{U}, \tilde{w} = \frac{w^*}{U}, \tilde{p} = \frac{p^*}{\rho U^2}, \tilde{R}_e = \frac{U h}{\nu} \) (hydrodynamic Reynolds’
number) \( R_{\text{ew}} = \frac{\nu h}{\nu} \) (Reynolds’ injection/suction number), \( M = B_0 h \sqrt{\frac{\nu}{\mu}} \) (Hartmann number), \( K_p = \frac{b}{h} \) (permeability parameter) and the equations (1) and (2) become

\[
\frac{\partial \tilde{u}}{\partial t} + \tilde{u} \frac{\partial \tilde{u}}{\partial x} + \frac{R_{\text{ew}} \tilde{u}}{R_e} \frac{\partial \tilde{v}}{\partial y} + \frac{R_{\text{ew}} \tilde{v}}{R_e} \frac{\partial \tilde{w}}{\partial z} = - \frac{\partial \tilde{p}}{\partial x} + \frac{\nu^2}{R_e} \tilde{u} - \frac{M^2}{R_e} \tilde{u} - \tilde{u}, \quad (13)
\]

\[
\frac{\partial \tilde{v}}{\partial t} + \tilde{u} \frac{\partial \tilde{v}}{\partial x} + \frac{R_{\text{ew}} \tilde{v}}{R_e} \frac{\partial \tilde{v}}{\partial y} + \frac{R_{\text{ew}} \tilde{w}}{R_e} \frac{\partial \tilde{w}}{\partial z} = - \frac{R_e}{R_{\text{ew}}} \frac{\partial \tilde{p}}{\partial y} + \frac{\nu^2}{R_e} \tilde{v} - \frac{M^2}{R_e} \tilde{v} - \tilde{v}, \quad (14)
\]

\[
\frac{\partial \tilde{w}}{\partial t} + \tilde{u} \frac{\partial \tilde{w}}{\partial x} + \frac{R_{\text{ew}} \tilde{w}}{R_e} \frac{\partial \tilde{w}}{\partial y} + \frac{R_{\text{ew}} \tilde{w}}{R_e} \frac{\partial \tilde{w}}{\partial z} = - \frac{R_e}{R_{\text{ew}}} \frac{\partial \tilde{p}}{\partial z} + \frac{\nu^2}{R_e} \tilde{w} - \frac{M^2}{R_e} \tilde{w} - \tilde{w}, \quad (15)
\]

For the stability analysis, the flow is decomposed into the mean flow and the disturbance according to

\[
\tilde{u}_0(r, t) = U_0(r) + u_0(r, t), \quad (17)
\]

\[
\tilde{p}(r, t) = P(r) + p(r, t). \quad (18)
\]

studied and analyzed with the help of figures and tables. We take the dimensional base flow for small suction and injection \([1, 10]\)

\[
U^*(y) = \frac{U}{2} \left( \frac{y^*}{h} + 1 \right) \quad (19)
\]

\[
V^* = \frac{V}{2} \quad (20)
\]

\[
W^* = 0. \quad (21)
\]

By scaling these velocities as above, we obtain with \( h = \pm 1 \ (-1 \leq y^* \leq 1) \) the no-dimensional base flow

\[
U(y) = \frac{y + 1}{2} \quad (22)
\]

\[
V = 1 \quad (23)
\]

\[
W = 0. \quad (24)
\]

To obtain the stability equations for the spatial evolution of three-dimensional, we take the dependent on time disturbances

\[(u(x, y, z, t); v(x, y, z, t); w(x, y, z, t); p(x, y, z, t)); \quad (25)\]

which are scaled in the same way as above.

We replace the equations (17) - (25) in the equations (14) - (16), after linearization and neglect of quadratic terms we find

\[
\frac{\partial u}{\partial t} + U \frac{\partial u}{\partial x} + \frac{R_{\text{ew}} u}{R_e} \frac{\partial v}{\partial y} + \frac{R_{\text{ew}} u}{R_e} \frac{\partial w}{\partial z} = - \frac{\partial p}{\partial x} + \frac{\nu^2}{R_e} u - \frac{M^2}{R_e} u - \frac{u}{K_p R_e}, \quad (26)
\]

\[
\frac{\partial v}{\partial t} + U \frac{\partial v}{\partial x} + \frac{R_{\text{ew}} v}{R_e} \frac{\partial v}{\partial y} + \frac{R_{\text{ew}} v}{R_e} \frac{\partial w}{\partial z} = - \frac{R_e}{R_{\text{ew}}} \frac{\partial p}{\partial y} + \frac{\nu^2}{R_e} v - \frac{M^2}{R_e} v - \frac{v}{K_p R_e}, \quad (27)
\]

\[
\frac{\partial w}{\partial t} + U \frac{\partial w}{\partial x} + \frac{R_{\text{ew}} w}{R_e} \frac{\partial v}{\partial y} + \frac{R_{\text{ew}} w}{R_e} \frac{\partial w}{\partial z} = - \frac{R_e}{R_{\text{ew}}} \frac{\partial p}{\partial z} + \frac{\nu^2}{R_e} w - \frac{M^2}{R_e} w - \frac{w}{K_p R_e}. \quad (28)
\]
The pressure terms can be eliminated from Navier-Stokes equations. For such a mean profile (base flow), the divergence of Navier-Stokes equations and continuity, gives

\[ \nabla^2 p = -2 \frac{R_{ew} \frac{\partial U}{\partial y}}{\frac{\partial v}{\partial x}} + M^2 \frac{R_{ew}}{\frac{\partial^2 v}{\partial y^2}}. \]  

The equations \( \nabla^2 [27] \) and \( 29 \) after linearization give

\[ \left[ \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} + \frac{1}{K_p R_e} \frac{\nabla^2 v}{\frac{\partial^2 v}{\partial y^2}} \right] \nabla^2 v = \frac{d^2 U}{dy^2} \frac{\partial v}{\partial x} + \frac{M^2}{R_e} \frac{\partial^2 v}{\partial y^2}. \]  

The disturbances are taken to be periodic in the streamwise, spanwise directions and time, which allow us to assume solutions of the form

\[ f(x, y, z, t) = \hat{f}(y) e^{i(\alpha x + \beta z - \omega t)}; \]  

where \( f \) represents either one of the disturbances \( u, v, w \) or \( p \) and \( \hat{f} \) the amplitude function, \( k, \alpha = k_x = k \cos \theta \) and \( \beta = k_y = k \sin \theta \) are the wave numbers, \( \omega = \omega c \) the pulsation of the wave. With \( i^2 = -1, \theta = (\vec{k}_x, \vec{k}_y), c = c_r + i c_i \) wave velocity which is taken to be complex, \( \alpha \) and \( \beta \) are real because of temporal stability analysis consideration. Then with the equation \( 31 \), the equation \( 30 \) becomes

\[ i\alpha \left( U - c - i \frac{R_{ew} D}{\alpha R_e} \right) \frac{i}{\alpha K_p R_e} + i \left( \frac{D^2 - k^2}{\alpha R_e} \right) (\frac{D^2 - k^2}{R_e} - i \alpha U'') \hat{v} = \]  

where \( D = \frac{d}{dy} \); with boundary conditions for all \((x, \pm 1, z, t > 0)\)

\[ \hat{u}(\pm 1) = 1 \text{or} 0 \]  
\[ \hat{v}(\pm 1) = 1 \]  
\[ \hat{w}(\pm 1) = 0 \]  
\[ \hat{u}'(\pm 1) = 0 \]  
\[ \hat{v}'(\pm 1) = 0. \]  

Taking

\[ v_p(x, y, z, t) = \hat{v}(y) e^{i(\alpha x + \beta z - \omega t)} - 1; \]  

the equation \( 32 \), the boundary conditions \( 34 \) and \( 37 \), take the forms

\[ \left( U - i \frac{R_{ew}}{\alpha R_e} \right) \frac{i}{\alpha K_p R_e} + i \left( \frac{D^2 - k^2}{\alpha R_e} \right) (\frac{D^2 - k^2}{R_e} - i \alpha U'') \hat{v}_p = \]  

\[ - \left( U'' + \frac{i M^2 D^2}{\alpha R_e} \right) \hat{v}_p; \]  

5
\[ \dot{v}_p(\pm 1) = 0 \]  
\[ \dot{v}_p(\pm 1) = 0. \]  

The equation (40) is a flow equation modified by the small injection/suction Reynolds number \( R_{\omega} \), the Hartmann number \( (M = B_0 h \sqrt{\frac{\sigma}{\mu}}) \), and permeability parameter \( (K_p = \frac{\kappa h}{h}) \) which we call modified Orr-Sommerfeld equation, rewritten as an eigenvalue problem, where \( c \) is the eigenvalue and \( \dot{v}_p \) the eigenfunction.

\[
\left[ \left( U - i \frac{R_{\omega}D}{\alpha R_e} - i \frac{D^2 - k^2}{\alpha R_e} \right) \left( D^2 - k^2 \right) - U'' - i \frac{M^2 D^2}{\alpha R_e} \right]
\]

and \((D^2 - k^2)\) are the operators.

### 3 Linear Stability analysis

We consider our three-dimensional disturbances. We use a temporal stability analysis as mentioned above. With \( c \) complex as we have defined above, when \( c_i < 0 \) we have stability, \( c_i = 0 \) we have neutral stability and elsewhere we have instability. We employ Matlab 7.8.0.(R2009a) in all our numerical computations to find the eigenvalues. The Couette horizontal porous plates flow with the basic velocity profile

\[ U = \left( \frac{y + 1}{2}, 1, 0 \right) \]  

for \( R_{\omega} \) small (i.e. small suction) is considered. The eigenvalue problem (40) is solved numerically with the suitable boundary conditions. The solutions are found in a layer bounded at \( y = \pm 1 \) with \( U(\pm 1) = (0, 1, 0) \). The results of calculations are presented in the figures below. We present the figures related to the eigenvalue problem (40).

For all these figures the black, red, green and blue colors are respectively, the curves I,II,III,IV and the yellow color is for \( c_i = 0 \). For all frame a,b,c,d we have fixed free parameters and we gave the curves \( C_i vs. R_e \) for sequential values of other parameters.

Figure 2 presents the effect of Reynods’ injection/suction number \( R_{\omega} \) on linear temporal stability of viscous incompressible non electrically conduction fluid \((M = 0)\) flow for different values of wave number. It is observed that for \( k = 1 \) and \( k = 1.02 \) (see figure 2 frames a and b), the stability is not affected by \( R_{\omega} \) and the flow is instable but for \( k = 2 \) and \( k = 3 \) (see figure 2 frames c and d), \( R_{\omega} \) affects it and the flow stays stable, also increase of \( R_{\omega} \) doesn’t contribute to the satability.

Figure 3 exhibits the effect of Permeability parameter \( K_p \) on linear temporal stability of viscous incompressible non electrically conduction fluid flow for different values of wave number. It is observed that \( K_p \) affects the stability. For \( k = 1 \) and \( k = 1.02 \), the frames a and b show that for \( K_p = 0.045 \) the flow is instable (see curves I) and stable for \( K_p = 1.000 \) (see curves IV) but for \( K_p = 0.048 \) and \( K_p = 0.130 \) (see curves II,III), we have the transition of the flow (See tabular below for the criticales Reynolds’number values). For \( k = 2 \) and \( k = 3 \) (see frames c and d) the flow is completely stable. On careful observation, we remark that for \( R_e < 12500 \), \( K_p \) ’s increase contributes to the stability in frame d case and the opposite is noticed in the frame c
case, but when $R_c > 12500$, $K_p$'s increase contributes in the two cases. Thus, it may be concluded that except the frame $c$ case where, the $K_p$'s increase doesn’t contribute to stability for $R_c < 12500$, the Permeability parameter increasing contributes to the flow stability.

Figure 8 shows the effect of Hartmann number $M$ on linear temporal stability of viscous incompressible electrically conduction fluid flow for different values of wave number. It is observed that $M$ affectes the stability. For $k = 1$ and $k = 1.02$, the frames $a$ and $b$ show that for $M = 10$ the flow is instable (see curves I) and stable for $M = 100$ (see curves IV) but for $M = 50$ and $M = 80$ (see curves II, III), we have the transition of the flow (Seen tabular below for the criticals Reynolds’ number values). For $k = 2$ and $k = 3$ (see frames $c$ and $d$) the flow is completely stable. Thus, we may concluded that the Hartmann number increasing contribute more to the flow stability.

Figure 5 depicts the effet of phase angle $θ$ for different values of the wave number on the flow stability. For $k = 1$ and $k = 1.02$, the frames $a$ and $b$ show that the flow is instable and the instability increases when the angle $θ$ increases. But, for $k = 2$ and $k = 3$ the flow is completely stable except the curve IV frame $c$), which presents a transition initialy and stays instable after.

Finally, the Figures 5, 7 and 8 ($M \neq 0$, electrically conduction fluid) show that for $k = 1$ and $1.02$ the small injection/suction has no effect on the linear temporal stability of the flow. But for $k = 2$ and 3, we remark a little influence of the small injection/suction on the stability only in a little interval of $R_c$.

| $R_{cw}$ | $k$ | $K_p$ | $M$ | $θ(π)$ | $R_{sc}$ |
|----------|----|------|-----|--------|--------|
| 0.00     | 1.00 | 0.130 | 0.00 | 0.00   | 05347  |
| 0.00     | 1.00 | 0.048 | 0.00 | 0.00   | 19370  |
| 0.00     | 1.02 | 0.130 | 0.00 | 0.00   | 05139  |
| 0.00     | 1.02 | 0.048 | 0.00 | 0.00   | 18640  |
| 0.00     | 1.00 | 0.045 | 50  | 0.00   | 18650  |
| 0.00     | 1.00 | 0.045 | 80  | 0.00   | 07178  |
| 0.00     | 1.02 | 0.045 | 50  | 0.00   | 18010  |
| 0.00     | 1.02 | 0.045 | 80  | 0.00   | 06845  |
| 0.00     | 1.00 | 0.045 | 50  | 0.10   | 19610  |
| 0.50     | 1.00 | 0.045 | 50  | 0.10   | 19610  |
| 0.75     | 1.00 | 0.045 | 50  | 0.10   | 19610  |
| 1.00     | 1.00 | 0.045 | 50  | 0.10   | 19610  |
| 0.00     | 1.02 | 0.045 | 50  | 0.10   | 18940  |
| 0.50     | 1.02 | 0.045 | 50  | 0.10   | 18940  |
| 0.75     | 1.02 | 0.045 | 50  | 0.10   | 18940  |
| 1.00     | 1.02 | 0.045 | 50  | 0.10   | 18940  |
| 0.00     | 1.00 | 0.048 | 50  | 0.10   | 17750  |
| 0.50     | 1.00 | 0.048 | 50  | 0.10   | 17750  |
| 0.75     | 1.00 | 0.048 | 50  | 0.10   | 17750  |
| 1.00     | 1.00 | 0.048 | 50  | 0.10   | 17750  |
| 0.00     | 1.02 | 0.048 | 50  | 0.10   | 17140  |
| 0.50     | 1.02 | 0.048 | 50  | 0.10   | 17140  |
| 0.75     | 1.02 | 0.048 | 50  | 0.10   | 17140  |
| 1.00     | 1.02 | 0.048 | 50  | 0.10   | 17140  |
Figure 2: Ci vs. Re for $M, K_p, \theta$ fixed and $R_e\omega, k$ variable
Figure 3: Ci vs. Re for $R_{ncr}$, $M$, $\theta$ fixed and $K_p$, $k$ variable

Figure 4: Ci vs. Re for $R_{ncr}$, $K_p$, $\theta$ fixed and $M$, $k$ variable
Figure 5: $\text{Ci vs. Re for } R_{\omega\kappa}, K_p, M$ fixed and $\theta, k$ variable

Figure 6: $\text{Ci vs. Re for } \theta, M$ fixed and $R_{\omega\kappa}, K_p, k$ variable
Figure 7: $C_i$ vs. $Re$ for $\theta$, $K_p$, $M$ fixed and $R_{ew}$, $k$ variable

Figure 8: $C_i$ vs. $Re$ for $\theta$, $K_p$, $M$ fixed and $R_{ew}$, $k$ variable
Conclusion

The conclusions of the study are:

- The small injection/suction Reynolds’ number has a little effect on the linear temporal stability of the couette flow only for the high waves numbers (k ≥ 2).
- The permeability parameter i.e. Darcy number, the Lorentz force parameter i.e. the Hartmann number and the wave number contribute to the linear temporal stability of the couette flow.
- For the small wave number (k ≃ 1), θ doesn’t contribute to the stability of the couette flow but for k = 2, 3 his effect is opposite.

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