Hurricane Ball Movement Analysis

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Abstract. The problem addressed in this paper originates from IYPT 2019 Question 6: By starting with hand rotation and using a tube to blow up towards it, two steel balls connected together can be rotated at a very high frequency and explain to explore this phenomenon. It is found in the pre-experiment that even without the use of the tube blowing, the two-steel-ball system can maintain a fairly long rotation, the kinetic energy decreases slowly, which leads to the steel ball system and the table between the pure roll without friction. The equation of motion of the steel ball system is then derived from the pure roll condition analytically, and the equation illustrates that the angle $\theta$ of buckling of the steel ball system is related to the angular velocity of rotation $\omega$ and the radius of the ball $r$. The experimental results are verified with good agreement with the expected results.

Keywords: hurricane ball; IYPT; rotation of Rigid Body.

1. Introduction
The "hurricane ball" question originated from the sixth question of the 2019 IYPT and was described as follows: Two steel balls that are joined together can be spun at incredibly high frequency by first spinning them by hand and then blowing on them through a tube, e.g. a drinking straw. Explain and investigate this phenomenon.

Andersen used rigid-body dynamics such as the Euler equation of dynamics to describe the hurricane ball phenomenon [1], which is mathematically complex and cumbersome. This paper describes how to find the equations of motion for a two-ball system under pure rolling conditions in an easy-to-understand method. The correctness of the equations is tested, and the experimental results are in good agreement with the expected results.

Figure 1. Hurricane Ball
2. Pre-Experiment and Pure Rolling condition

2.1. Pure rolling condition
A hurricane ball system was made by sticking two small steel balls of radius \((r = 0.50\,\text{cm}, m = 17\,\text{g})\) together. Without adding airflow, this two-ball system is rotated by hand on the surface of a glass-topped, and it is observed that one end of the hurricane ball is cocked, and the center of mass of the two balls remains unchanged for a short period of time, and the system can be maintained for a long time of rotation, and the kinetic energy decreases slowly.

The center-of-mass motion theorem shows that

\[
F_c = 2ma_c
\]  

(1)

The pre-experiment observed that the position of the center of mass of the hurricane ball system is almost constant, indicating that the combined external force is approximately 0 at this time, regardless of air resistance, friction is the only possible force on the hurricane ball system at this time, so there is

\[
f = 0
\]  

(2)

The ball of the hurricane ball system in contact with the table is called the lower ball, and the warped end is called the upper ball. From (1.2), it can be seen that the lower ball and the glass-topped table is approximately frictionless, which is the pure rolling condition.

2.2. Geometric significance of the pure roll condition

Figure 2. Hurricane Ball Pure Roll Model

Let a hurricane ball system do pure rolling motion on the glass plate, clockwise rotation when looking down, as in Figure 1.2, \(m\) refers to the mass of a single ball, \(r\) refers to the radius of the two balls. The center of the lower ball \(C\) and \(\theta\) refers to the included angle between the vertical direction and the line connecting \(C\) (the center of the lower ball) and the center of the upper ball, \(A\) is the intersection point between the lower ball and the glass top, and \(N\) stands for the supporting force given by the glass top to the mechanical system. With the rotation of the hurricane ball, the trajectory of \(A\)’s movement is a circle with \(r = \sin \theta\) (B being the centre of the circle). From the geometric relationship, the perimeter of the circle \(B\) is

\[
C_B = 2\pi r \sin \theta
\]  

(3)
The hurricane ball is rotating at the same time, in order to maintain its own pure roll, the hurricane ball is also rotating. With the rotation of the hurricane ball, the trajectory of the distribution of contact points on the lower ball is a circle of radius $PA$ with point $P$ as the center. From the geometric relationship
\[ PA = r \sin \theta \] (4)
Therefore the circumference of the circle $P$ is
\[ C_P = 2\pi r \sin \theta \] (5)
From (1.3)(1.5), we know that $C_B = C_P$, the hurricane ball rotates for one week, and the circle $P$ corresponds to the points on the circle $B$ one by one, which means that the angular velocity of the hurricane ball's revolution and the angular velocity of its rotation are equal, and let $\Omega$ and $\omega$ be the angular velocity of the hurricane ball's revolution and the angular velocity of its rotation, respectively, that is
\[ \Omega = \omega \] (6)
This is the hurricane ball angular velocity relationship constrained by the pure roll condition.

3. Hurricane Ball Equations of Motion

3.1. Calculation of angular momentum

The position of the center of mass $O$ of the two balls is found to be constant in the pre-experiment, and the system is considered as a fixed-point rotation with the fixed point being the point $O$. The angular momentum $\vec{L}_1$ of the lower ball is written as
\[ \vec{L}_1 = \vec{L}_C + \vec{L}_{CM} \] (7)
Where $\vec{L}_C$ is the angular momentum of the lower sphere's center of circle $C$ with respect to $O$, and $\vec{L}_{CM}$ is the angular momentum of the lower sphere with respect to the center of circle $C$. These two angular momenta are perpendicular to each other, as in Figure 2.1.
\[ \vec{L}_{CM} = \sum_i \Delta m_i \vec{r}_i \times (\vec{\omega} \times \vec{r}_i) \] (8)
\[ \vec{L}_C = m(\vec{r} \times \vec{v}) \] (9)
The angular momentum $\vec{L}_1$ of the upper ball is calculated in the same way as in equation (2.2) (2.3), so the total angular momentum of the hurricane ball system is
\[ \vec{L} = \vec{L}_1 + \vec{L}_1 = 2\vec{L}_C + 2\vec{L}_{CM} \] (10)

![Figure 3. Two Components of the Angular Momentum of The Lower Ball](image)

3.2. Moment of force calculation

The position of the center of mass $O$ of the two spheres remains unchanged, the system is balanced in the vertical direction, and the support force is equal to the gravitational force, i.e.
\[ N = 2mg \] (11)

The rigid body rotates around the fixed point \( O \), the moment of gravity is 0, and the hurricane ball is only subject to the action of the support force \( N \)

\[ \bar{M} = \hat{O}\hat{A} \times \hat{N} = 2mg r \sin \theta \hat{n} \] (12)

where \( \hat{n} \) is the unit vector whose direction points from \( O \) to \( A \) and \( n^* \) is pointing vertically into the paper.

3.3. Hurricane ball equation of motion

\[ \frac{dL_1}{dt} = \frac{d\bar{L}_1}{dt} + \frac{d\bar{L}_2}{dt} = \bar{M} \] (13)

The rate of change of the angular momentum is calculated below, as shown in the figure by the geometric relation that the change of angular momentum satisfies the following relation, that

\[ \frac{dL_1}{dt} = L_1 \sin \theta \frac{d\phi}{dt} \] (14)

\[ \frac{dL_2}{dt} = -L_2 \cos \theta \frac{d\phi}{dt} \] (15)

Where \( d\phi \) is the instantaneous rotational angle increment during the rotation of the hurricane ball, and the rate of change of angular momentum is

\[ \frac{dL}{dt} = (L_1 \sin \theta - L_2 \cos \theta) \frac{d\phi}{dt} \] (16)

From the angular momentum theorem, we know that

\[ 2mgr \sin \theta = (L_1 \sin \theta - L_2 \cos \theta) \cdot \Omega \] (17)

In the two-ball system of the hurricane ball, \( L_1, L_2 \) satisfy the following relationship

\[ L_1 = \frac{4}{5} mr^2 \omega \] (18)

\[ L_2 = 2mr^2 \sin \theta \cos \theta \cdot \Omega \] (19)

Bringing (1.6)(2.12)(2.13) into (2.11), it follows that

\[ \frac{g}{\omega^2 r} = \frac{2}{5} - \cos \theta \] (20)
This formula specifies the lower ball center $C$ and the two ball center of mass $O$ of the line and the vertical direction of the angle of $\theta$ must meet $\theta > 66.42^\circ$; In addition, the upper ball cannot contact with the table, otherwise it cannot meet the pure roll condition, friction will quickly consume the kinetic energy of the two balls, so the value of $\theta$ range is $66.42^\circ < \theta < 90^\circ$ (21)

It is deduced that when $\omega^2 > \frac{5g}{2r}$, the hurricane ball can enter the movement. Eqs. (2.14) and (2.15) are the equations satisfied by the hurricane ball into motion.

4. Experimental verification

4.1. Experimental operation

In the experiment, a small steel ball with diameter $d = 1.500cm, m = 0.0124kg$ was selected as the upper and lower balls of the hurricane ball, and the two balls were glued together with wax glue melted by a glue gun. The sphere was dipped in the coating solution to facilitate the observation of the hurricane ball rotation.

The experiment selected the model FASTCAM SA-X2 type 200K-C3 high-speed camera to the rotating hurricane ball, and the camera frame rate is 3000fps, the final recorded video compared with the actual scene will be slowed down 2000 times to play.

Figure 5. High-Speed Camera Recording Hurricane Ball Movement

In order to keep the hurricane ball center of mass does not move, only to increase the angular momentum of the two balls, the experiment uses two tubes on both sides of the simultaneous blowing method. Horizontal and vertical lines are pasted on the background wall in the screen to facilitate post-processing, as in Figure 3.1.

Keep the camera lens and the two centers of mass roughly in a horizontal plane, to facilitate the post-processing measurement of the two ball center of the line and vertical direction angle $\theta$.

4.2. Experimental phenomena

In the recorded video, it was found that the center of mass was not significantly displaced during the rotation of the hurricane ball, and the center of mass remained at the same point in the picture after 20 revolutions of the hurricane ball, counting from a certain point in the video.
Figure 6. The Hurricane Ball Rotates One Week and Rotates One Week at The Same Time

In the video, it can be seen that for each week of the hurricane ball's rotation, the marker rotates by one week, as in Figure 3.2. The equation (1.6) is verified.

4.3. Experimental data processing

4.3.1. Measurement of angle $\theta$. The hurricane ball inclination angle was measured using the software Geogebra. The measurement method is to fit the boundary of the two balls in the video with a circle, you can find the center of the two balls; when the distance between the centers of the two balls in the screen is the largest, that is, when the long side of the two balls is facing the camera lens, at this time the software will calculate the line of the center of the two balls and the vertical direction angle $\theta$, as in Figure 3.3. The angle of inclination of the hurricane ball in the video is measured once every 10 seconds of rotation.

Figure 7. Using the Software Geogebra to Measure the Hurricane Ball Inclination Angle

4.3.2. Measurement of angular velocity $\omega$. In order to measure the angular velocity at a certain moment, take 10 revolutions before and after the rotation of the hurricane ball at this moment for calculation. For example, a measured video of the hurricane ball rotation 20 laps in 1 minute 29 seconds, that is, the reality of a turn in 0.045 seconds, the angular velocity is
\[ \omega = \frac{2\pi}{T} = 139.62 \text{ rad/s} \]

The angular velocities of the hurricane ball at different moments were measured by this method, respectively, as shown in Table 1.

It can be seen that \( \theta \) satisfies \( \theta > 66.42^\circ \) when the hurricane ball rotates the fastest, which can verify that the formula (2.15) holds.

| Rotation Time (S) | Angle of Precession \( \theta \) | Angular Velocity (\( \omega \)) |
|------------------|------------------|------------------|
| 5.0              | 69.78            | 154.05           |
| 15.0             | 70.44            | 139.62           |
| 25.0             | 72.56            | 116.15           |
| 35.0             | 76.87            | 86.01            |
| 45.0             | 82.11            | 71.25            |

Drawing the image of the function of equation (2.14) with Mathematica software and bringing in the experimental data points for comparison, it can be found that the theory matches the experiment very well, as shown in Figure (3.4).

**Figure 8.** The Experimental Value and the Theoretical Value Are in General Agreement

5. Conclusion

By starting with hand rotation and using a tube blowing up towards it, two steel balls connected together can rotate at a very high frequency. The essence of this phenomenon is that the system does a pure rolling motion on the table with little kinetic energy consumption. And the role of the hurricane is to give the two-ball system to replenish the kinetic energy consumed due to friction, air resistance.

Hurricane ball on the table to do into the motion, the angular velocity of self-rotation and the angular velocity of rotation equal, into the angle \( \theta \) and the relationship between the angular velocity of self-(male) rotation \( \omega \) for the (2.14) type. Equation (2.14) also puts a constraint on the incoming angle \( \theta \) of the hurricane ball, i.e., \( 66.42^\circ < \theta < 90^\circ \), and the mass of the hurricane ball is independent of the angular speed of rotation of the system. It can be seen in the subsequent experiments that equation (2.14) agrees quite well with the experimental values.

References

[1] Su, X., Zhang, Y., Cheng, L., Cause and Dynamic Analysis of The Hurricane Balls Rising[J]. Physics and Engineering, 2020, 30(05):127-131.
[2] ANDERSEN W L, WERNER S. The dynamics of Hurricane-balls [J]. European Journal of Physics, 2015, 36(5): 055013.
[3] Zhou,Y., Tutorial of theoretical mechanics [M]. 3 edition. Beijing: Higher Education Press, 2009.
[4] JACKSON D P, DAVID M, PEARSON B J. Hurricane balls: a rigid-body-motion project for undergraduates [J]. 2015, 83(11):959-968.