Simulation of the shape deflection for the spherical shell of the space calibration-adjustment spacecraft

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Abstract. The spherical shell made with high accuracy is one of the possible geometric shapes for passive orbital signal repeaters and calibration-adjustment spacecrafts used to determine the energy potential of the radar channel of the ground-based complex needed to control the space object motion. Under the solar radiation, an uneven temperature distribution over the surface of such a shell causing a deviation of its shape from the spherical one occurs. To determine these deviations, the steady-state temperature distribution for the shell with a fixed orientation relative to the Sun obtained by solving the Fredholm integral equation of the second kind is used. A quantitative analysis of the possible alignment of a quasi-stationary distribution of the shell temperature in case of the shell rotation with a constant angular velocity around the axis perpendicular to the direction to the Sun is carried out.

1. Introduction

One of possible geometric shapes for the calibration-adjustment spacecraft intended for determination and control of the energy potential of the radar channel of the ground-based complex used for control of the space object motion [1, 2] is a spherical shell made with high accuracy. Passive signal repeaters and some types of small reference reflectors used for calibration and adjustment of radar equipment [1, 3–5] have the same shape. Orbits of spacecrafts under consideration can be either circular ones with a height of about 1000 km (some of them may be even close to the polar orbits) or elliptical ones with an apogee up to 2200 km [2, 3].

If a thermal control system for these types of spacecrafts is absent, the main factor determining the temperature state of the spherical shell in the sunlit part of the orbit is the solar radiation. For a fixed location of the shell relatively the direction to the Sun, the resulting uneven temperature distribution over the shell surface leads to the deviation of its shape from the ideal spherical one and that can affect the functionality of the spacecraft. Rotation of the shell relative to an axis perpendicular to the direction to the Sun can reduce the unevenness of the temperature distribution.

Quantitative analysis of the unevenness of the temperature distribution over the surface of the spherical shell being in the near-earth space as well as the influence of this unevenness on the shape deviation from a spherical one may be obtained by means of the methods of mathematical modeling [6–8] and modification of a previously developed mathematical model describing the steady-state temperature state of such a shell in the low near-Earth orbit [9, 10]. In this paper, a shell made of a polymer composite material, the initial spherical shape of which is determined by a relatively low internal pressure, is under consideration. It is assumed that if
some equipment is placed in the shell, the volume occupied by it is small enough that allows not to take into account its influence on the transfer of radiation in the shell cavity. In addition to determining the steady-state temperature distribution over the surface of the shell when it has a fixed orientation in relation to the direction to the Sun, a quasi-stationary temperature state of the shell is obtained for the case of its rotation with a constant angular velocity relative to the axis perpendicular to the mentioned direction. For the calculated temperature distribution over the surface of the non-rotating shell, the deviation of its shape from the spherical one is estimated.

2. Estimations of possible shell parameters

One of the possible design options for the calibration and adjustment spacecraft is a shell made of a polymer composite material, which takes on a spherical shape after being put into near-Earth orbit due to the internal pressure created by the gas filling it. In this case, it is possible to obtain a shell of a sufficiently large diameter, which is characteristic of modern trends of deployment of large-sized transformable structures in orbit [11]. Based on the operating conditions of the spacecraft, such a shell is advisable to perform a multilayer, including several layers, each with different functional purpose. In addition to a relatively thin inner gas-tight layer with a thickness \( h_\ast \approx 0.05 \ldots 0.1 \text{ mm} \), made, for example, from a polyimide membrane PMF-352 with a double-sided coating with polytetrafluoroethylene (Teflon) [12], a layer of material with sufficiently high mechanical characteristics covered with an external layer of material the properties of which are determined directly from the operational requirements for the apparatus is necessary.

When using an organoplastics shell based on an epoxy matrix reinforced with a Kevlar-49 type aramid fiber fabric as an intermediate layer material, tensile strength of about 500 MPa and longitudinal elastic modulus (Young’s modulus) \( E \approx 40 \text{ GPa} \) can be calculated [13]. With a thickness \( h_0 = 0.2 \text{ mm} \) of this layer and a radius \( r_0 = 1 \text{ m} \) of its average spherical surface, the internal pressure \( p = 160 \text{ Pa} \) will cause tensile stress \( \sigma = pr_0/(2h_0) = 0.4 \text{ MPa} \) in the layer three times less than the tensile strength. Moreover, the relative elongation of the layer in any direction tangent to the middle surface will be no more than \( 10^{-5} \). The outer layer of the shell can be made, for example, in the form of a polytetrafluoroethylene membrane with a thickness \( h^* \approx 0.05 \ldots 0.1 \text{ mm} \).

The main factor determining the thermal effect on the shell in the illuminated part of the near-Earth orbit is solar radiation. Due to the presence of an intermediate layer of polymer composite material, the shell under consideration is opaque to radiation both in the visible and in the near infrared parts of the spectrum, which have the bulk of the transferred energy. Heat transfer to the inner surface of such a shell occurs through thermal conductivity. The total thermal resistance \( R_T \) of the three-layer shell between its outer and inner surfaces is estimated. For the values of the thermal conductivity coefficients of the polytetrafluoroethylene membrane \( k^* \approx 0.26 \text{ W/m·K} \) [14], organoplastics \( k_0 \approx 0.14 \text{ W/m·K} \) [15] and the polyimide membrane \( k_\ast \approx 0.17 \text{ W/m·K} \) [14] the value

\[
R_T = \frac{h^*}{k^*} + \frac{h_0}{k_0} + \frac{h_\ast}{k_\ast} \approx 0.0022
\]

was obtained.

With an overstated estimate \( q' \approx 10^3 \text{ W/m}^2 \) of the possible value of density of the heat flux passing through the shell, an overstated estimate of the temperature difference \( qR_T \approx 2.2 \text{ K} \) over the thickness of the shell is also obtained. Such an assessment allows to make an assumption about the uniformity of the shell temperature over its thickness.

For a more reliable determination of the value \( q' \) first of all, it is necessary to have reliable information about the so-called optical characteristics of the outer and inner surfaces of the shell. These characteristics include the absorption coefficient of the radiation incident on the
outer surface and the intrinsic emission coefficient (degree of blackness) of this surface, as well as the absorption and intrinsic radiation coefficients of the inner surface. The values of these coefficients in the general case depend on the spectral composition of the incident radiation. Radiation emanating directly from the Sun and solar radiation partially reflected from the Earth are referred to as shortwave, while the Earth’s own radiation, which makes some contribution to the total flux of radiation incident on the outer surface of the shell in low near-Earth orbit, is referred to as long wave. Also, the long-wave length is the intrinsic radiation of the outer and inner surfaces of the shell.

In the spectrum of solar radiation, the ultraviolet region (about 10% of the total radiation energy), the visible and infrared regions (respectively, about 40 and 50% of the total radiated energy) are emitted. The dependence of the spectral intensity of solar radiation outside the Earth’s atmosphere to a first approximation corresponds to the spectrum of an absolutely black body with a temperature $T_S = 5780$ K, which, according to the formula of Wien’s law $\lambda_m T = 2897.8 \mu m K$, has the highest spectral intensity at a wavelength of $\lambda_m \approx 0.5 \mu m$. The Planck’s law determines the distribution of the spectral radiation intensity of a black body, usually measured in $W/(m^2 \cdot \mu m)$, over wavelengths at a fixed temperature $T$

$$I_0(\lambda, T) = 2\pi \frac{hc^2}{\lambda^5} \left( \exp \frac{hc}{\lambda k_B T} - 1 \right)^{-1},$$

where $h = 6.625 \times 10^{-34}$ J·s is Planck’s constant; $c = 2.998 \times 10^8$ s is the speed of light in vacuum; $k_B = 1.380 \times 10^{-23}$ J/K is Boltzmann constant.

Figure 1 shows a plot of $I_0(\lambda, T) = F(\lambda T)$ against $\lambda T$. The abscissas of the vertical dashed lines are equal to the upper boundary $\lambda^* T$ of the interval of values $\lambda T \in (0, \lambda^* T)$, within which a black body with temperature $T$ emits a percentage of the total energy. At temperature $T_S$ 25% of the radiated energy falls on the interval $(0, \lambda_m T_S)$, i.e. $\lambda^*_S = \lambda_m \approx 0.5 \mu m$, $\lambda^*_S \approx 0.711 \mu m$ corresponds to a fraction of 50%, which is close to the conditional boundary of the visible part of the spectrum, $\lambda^*_S \approx 1.064 \mu m$ corresponds to a fraction of 75% and $\lambda^*_S \approx 4.017 \mu m$ to 99%. Measurements outside the Earth’s atmosphere show that 95% of the total energy of the Sun emits in the wavelength range from $\lambda = 0.3 \mu m$ to $\lambda = 3 \mu m$.

The maximum radiation intensity of a completely black body at a temperature $T_0 = 293 K$ corresponds, according to Wien’s law, to a wavelength of about $10 \mu m$, i.e. the value $\lambda^* \approx 10 \mu m$ corresponds to the fraction of the total radiated energy of 25%. From the plot in figure 1 it follows that with a fraction of 1% of the total energy emitted $\lambda^* \approx 4.942 \mu m$, 50% — $\lambda^* \approx 14.02 \mu m$, 75% — $\lambda^* \approx 20.99 \mu m$ and 99% — $\lambda^* \approx 79.25 \mu m$. Thus, when the emission spectrum of an absolutely black body is superimposed at temperatures close to the value of $T_0 = 293 K$, the
solar radiation spectrum can be neglected from overlapping these spectra from an energy point of view. A partial overlap of these spectra becomes significant only at temperatures \( T > 1000 \text{K} \), which are not characteristic of the surface of the shell under consideration.

The ratio \( I_\lambda(\lambda, T)/I_{0\lambda}(\lambda, T) = \varepsilon_\lambda(\lambda, T) \leq 1 \), where \( I_\lambda(\lambda, T) \) is the spectral radiation intensity of the surface with temperature \( T \), is called the spectral emissivity (spectral degree of blackness) of this surface. If the dependence of \( \varepsilon_\lambda \) on \( \lambda \) and \( T \) is known for a specific material and the state of its surface, then the integral emissivity \( \varepsilon \) (integral degree of blackness) of the surface and the integral absorption coefficient \( A_S \) by this surface of solar radiation outside the Earth’s atmosphere can be calculated using the formulas [16]

\[
\varepsilon(T) = \frac{1}{\sigma_0 T^4} \int_\lambda^\infty \varepsilon_\lambda(\lambda, T) I(\lambda, T) \, d\lambda, \quad A_S(T) = \frac{1}{\sigma_0 T^4} \int_\lambda^\infty \varepsilon_\lambda(\lambda, T) I_{0\lambda}(\lambda, T_S) \, d\lambda,
\]

where \( \sigma_0 \approx 5.6693 \times 10^{-8} \text{W/m}^2\text{K} \) is Stefan–Boltzmann constant. It follows from these formulas that, due to the absence of an overlapping of the radiation spectra of the Sun and black body with the temperature characteristic of the surface of the shell under consideration, from the energy point of view, the relationship between the values of \( A_S \) and \( \varepsilon \) is decisively influenced by the change in \( \varepsilon_\lambda \) when \( \lambda \) passes through the gap separating the regions of these spectra with a significant fraction of the total radiation energy. In the particular case of the independence of the spectral emissivity from the wavelength, these formulas lead to the equality \( A_S(T) = \varepsilon(T) \), because in accordance with the Stefan–Boltzmann law

\[
\int_0^\infty I(\lambda, T) \, d\lambda = \sigma_0 T^4, \quad \int_0^\infty I_{0\lambda}(\lambda, T_S) \, d\lambda = \sigma_0 T_S^4.
\]

The assumption that \( \varepsilon_\lambda \) is not dependent on \( \lambda \) underlies the model of the so-called gray body. It should be noted that for a membrane of polytetrafluoroethylene and a polyimide membrane with a double-sided Teflon coating, the values of \( A_S \) and \( \varepsilon \) vary over a rather wide range due to the presence of various additives to polytetrafluoroethylene depending on the field of its application. With some combinations of these additives, the properties of the inner and outer surfaces of the shell may correspond to the gray body model.

3. Deviation of shell shape from spherical

If the temperature is constant along the shell thickness and the shell is only loading by the internal pressure \( p \), it is possible to assume that the spherical shell is a membrane one. The shell temperature states discussed above are axis-symmetric ones relative to the coordinate axis \( OX_1 \), which coincides with the direction to the Sun. Then the meridian section of the shell can be formed by any plane containing this axis, and the meridian stress \( \sigma_1 \) will be directed tangentially to the contour of the shell in this section. The direction of the circumferential stress \( \sigma_2 \) will coincide with the tangent to the contour of the shell in the cross-section perpendicular to the axis \( OX_1 \). For a spherical shell with thickness \( h \) and radius of the middle surface \( r_0 \) that is loaded with internal pressure \( p \)

\[
\sigma_1 = \sigma_2 = \frac{pr_0}{2h}, \quad (1)
\]

Let us estimate the deviation of the shell shape from the spherical one assuming in the first approximation that the stresses \( \sigma_1 \) and \( \sigma_2 \) are independent of these deviations and determined by the formula (1). Let us suppose as well, that the isotropic shell material is linearly elastic one. Then, according to the generalized Hooke’s law and taking into account formula (1), we can write for equal deformations \( \varepsilon_1 \) and \( \varepsilon_2 \) respectively in the meridian and circumferential directions

\[
\varepsilon_1 = \varepsilon_2 = pr_0 \frac{1 - \nu}{2Eh} + \alpha \Delta T, \quad (2)
\]
where \( \nu, E, \) and \( \alpha \) are respectively the Poisson’s ratio, the longitudinal modulus of elasticity (Young’s modulus) and the temperature coefficient of linear expansion of the shell material, \( \Delta T = T - T_0 \) is the increment of the shell temperature \( T \) in comparison with the homogeneous over the shell surface temperature \( T_0 \), at which the radius of its average surface is equal to \( r_0 \).

If denote displacements of points of the shell middle surface in the directions of the normal and tangent to its contour in the meridian section by \( u \) and \( w \) respectively, the next equations will be valid in the linear approximation

\[
\varepsilon_1 = \frac{1}{r_0} \left( w - \frac{du}{d\chi} \right), \quad \varepsilon_2 = \frac{w + u \tan \chi}{r_0} = \frac{u_r}{r},
\]

(3)

where \( u_r \) is the displacement in the radial direction in the cross-section of the shell, and \( r = r_0 \cos \chi \) is the radius of the middle surface contour in this section. From the equality (2) and the second equation in (3) follows

\[
u_r = \left( pr_0 \frac{1 - \nu}{2Eh} + \alpha \Delta T \right) r_0 \cos \chi.
\]

(4)

Equations (2) and (3) allow us to write the equality \( du/d\chi = -u \tan \chi \), to which the function \( u = C \cos \chi \), where \( C = \text{const} \) satisfies. Then from the equality (2) and the first equation in (3) we can get

\[
w = pr_0 \frac{1 - \nu}{2Eh} + \alpha r_0 \Delta T - C \sin \chi.
\]

The displacement along the axis \( Ox_1 \) is equal to \( u_1 = u \cos \chi - w \sin \chi \), and taking into account the formulas for displacements \( u \) and \( w \), we can write

\[
u_1 = C - \left( pr_0 \frac{1 - \nu}{Eh} + \alpha \Delta T \right) r_0 \sin \chi.
\]

If take the point \( M_1 \) on the axis \( Ox_1 \) with coordinate \( x_1 = r_0 \) that corresponds to the value \( \chi = \pi/2 \) as the zero count for the displacement \( u_1 \), we will get \( C = pr_0^2 (1 - \nu)/(2Eh) + \alpha r_0 \Delta T \) and

\[
u_1 = \left( pr_0 \frac{1 - \nu}{Eh} + \alpha \Delta T \right) r_0 (1 - \sin \chi).
\]

(5)

It follows from the formulas (4) and (5) that the internal pressure does not distort the shape of the spherical shell, but it leads only to the equal increment \( \Delta r_0 = pr_0^2 (1 - \nu)/(2Eh) \) of its radius \( r_0 \) at all points of the middle surface. The deviation of the shell shape from the spherical one is directly related to the uneven temperature distribution over the shell surface. In the case of a non-rotating shell this irregularity occurs only on the sunlit part of its surface. For such a shell, in figure 2 deviations of the meridian section contour of the shell middle surface caused only by this unevenness are shown in the enlarged scale by the continuous curve. These deviations are shown relative to the circle arc (dashed line with light circles) corresponding to the contour of the shell middle surface in its meridian section at the selected fixed temperature \( T_0 \approx 290.4 \).

The placements of the dark circles on the solid curve relative to the corresponding light circles allow us to compare the local distortions of the middle surface contour. In the shaded part of the shell the temperature is \( T_{\text{min}} \approx 236.5 = \text{const} \) and therefore the middle surface keeps the hemispherical shape, but of a smaller radius because of \( T_{\text{min}} < T_0 \).

Conclusions

The quantitative analysis of the mathematical model constructed to describe the temperature state of the inflatable spherical shell for the calibration and adjustment spacecrafts allowed us...
Figure 2. The shape deviations of the middle surface contour for the non-rotating shell to receive the degree of unevenness of the temperature distribution over the surface of the shell fixed relative to the direction to the Sun and the temperature state of the shell rotating with a constant angular velocity relative to the axis perpendicular to this direction. On the base of calculated results for the temperature state of the non-rotating shell, the deviation of its shape from the ideal spherical one was estimated.

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References
[1] Fateev V F (Editor) 2010 Small Spacecrafts for Information Support (Moscow: Radiotekhnika) [in Russian]
[2] Mashchenko A N, Pappo-Korystin V N, Pashchenko V A, Vasil’ev V G 2000 Rockets and Spacecraft from the Engineering Department “Yuzhnoe” (Dnepropetrovsk: GKB “Yuzhnoe” im. M.K. Yangelya) [in Russian]
[3] Tarasenko M V 1992 Military aspects of the soviet astronautics (Moscow: TOO “Nikol”) [in Russian]
[4] Kuzenov V V and Ryzhkov S V 2018 Approximate calculation of convective heat transfer near hypersonic aircraft surface J. Enhanced Heat Transf. 25 (2) 181–93
[5] Zarubin V S, Kuvyrkin G N, and Savelyeva I Yu 2015 Radiative–conductive heat transfer in a spherical cavity High Temp. 53 (2) 234–9
[6] Zarubin V S, Kuvyrkin G N, and Savel’eva I Yu 2016 Critical and optimal thicknesses of thermal insulation in radiative–convective heat transfer High Temp. 54 (6) 831–6
[7] Savelyeva I Yu 2018 Dynamic temperature stresses in elastic body with curved boundary Herald of the Bauman Moscow State Tech. Univ., Nat. Sci. 1 (76) 38–46
[8] Kuvyrkin G N, Savelyeva I Y, and Kuvshevnikova D A 2018 Mathematical model of the heat transfer process taking into account the consequences of nonlocality in structurally sensitive materials J. Phys.: Conf. Ser. 991 012050
[9] Zarubin V S 1963 Temperature state of a thin spherical shell Zh. Prikl. Mekh. Tekh. Fiz. No. 6 169–71 (In Russian)
[10] Zarubin V S, Kuvyrkin G N, and Savelyeva I Yu 2018 Two-sided thermal resistance estimates for heat transfer through an anisotropic solid of complex shape Int. J. Heat Mass Transf. 116 833–9
[11] Zimin V N 2006 Oboronn. Tekhnol. 1 123
[12] Bühler K-U 1984 Heat and Thermal Resistant Polymers (Moscow: Khimiya Publ.) [in Russian]
[13] Lubin G (Editor) 1982 Handbook of Composites (Springer US)
[14] Grigor’yev I S and Meylikhov Ye Z 1991 Physical Parameters (Moscow: Energoatomizdat) [in Russian]
[15] Vasiliev V V and Tarnopolsky Yu M 1990 Composite Materials (Moscow: Mashinostroyenie) [in Russian]