Multisensor Poisson Multi-Bernoulli Filtering with Uncertain Sensor States

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Abstract—In a typical multitarget tracking (MTT) scenario, the sensor state is either assumed known, or tracking is performed based on the sensor’s (relative) coordinate frame. This assumption becomes violated when the MTT sensor, such as a vehicular radar, is mounted on a vehicle, and the target state should be represented in a global (absolute) coordinate frame. Then it is important to consider the uncertain sensor location for MTT. Furthermore, in a multisensor scenario, where multiple sensors observe a common set of targets, state information from one sensor can be utilized to improve the state of another sensor. In this paper, we present a Poisson multi-Bernoulli MTT filter, which models the uncertain sensor state. The multisensor case is addressed in an asynchronous way, where measurements are incorporated sequentially based on the arrival of new sensor measurements. In doing so, targets observed from a well localized sensor reduce the state uncertainty at another poorly localized sensor, provided that a common non-empty subset of features is observed. The proposed MTT filter has low computational demands due to its parametric implementation. Numerical results demonstrate the performance benefits of modeling the uncertain sensor state in feature tracking as well as the reduction of sensor state uncertainty in a multisensor scenario compared to a per sensor Kalman filter. Scalability results display the linear increase of computation time with number of sensors or features present.

I. INTRODUCTION

Intelligent transportation systems in general, and autonomous driving (AD) in particular, require accurate position information [1]. Measurements provided by various on-board sensors allow to infer the vehicle state, e.g., position and velocity, as well as information about the surrounding environment. For instance, a global navigation satellite system (GNSS) receiver provides absolute position, whereas a radar sensor provides relative position with respect to (w.r.t.) the sensor origin. Furthermore, vehicles have access to a pre-recorded local dynamic map (LDM) containing static features such as, e.g., landmarks [2]. Dynamic features such as pedestrians, cyclists, etc. are not part of the pre-recorded map. For an AD system to be fully aware of the surrounding environment, dynamic features need to be estimated and tracked over time using the vehicles on-board sensors thus allowing to enrich the vehicle’s LDM. In order to incorporate mobile features into the LDM, which contains map features described in a global coordinate frame, location uncertainty of on-board sensors used to track dynamic features needs to be considered, i.e., the vehicle’s state uncertainty such as its location and pose. In an ITS, vehicles communicate through the wireless channel with other vehicles (vehicle-to-vehicle (V2V) communication) or with road infrastructure, such as a roadside unit (RSU), through vehicle-to-infrastructure (V2I) communication, using IEEE 802.11p or 4G/5G cellular communication. Through information exchange, vehicles can make local LDM information available to their neighbors allowing to enrich their LDMs and their situational awareness. Not only can LDM information be shared, it can also be fused to improve every LDM [3], [4]. For the special case, they observe an overlapping set of dynamic features, information from one vehicle can be utilized to increase location accuracy of other vehicles, and vice versa [5], [6]. Note, in this context no V2V measurements are performed, different to a traditional cooperative localization approach [7], [8]. The problem of vehicular localization using locally observed features with unknown observation to feature correspondence, aggregated at an RSU, can be interpreted as an MTT problem. In MTT, a varying number of mobile features (targets) are tracked, using sensors such as for example radars, LIDARS, or cameras [9]. Thereby, it is typically assumed that the state of the observing sensor is known. Although not true in general, this assumption can be motivated by the fact that sensor state uncertainty is negligible in comparison to the sensors’ measurement accuracy. If sensor state uncertainty is significant, it needs to be modeled in the MTT in order not to have negative impact on feature tracking performance.

In this paper, we consider the case of MTT with uncertain sensor state, where there are potentially multiple sensors with varying sensor state uncertainty. To enable accurate feature tracking we model the sensor state uncertainty in the MTT filter. The main contributions are:

- an asynchronous parametric multitarget-multisensor tracking filter with uncertain sensor state information,
- fusion of multitarget-multisensor tracking information with local sensor tracking information,
- and numerical simulation results demonstrating the performance of the filter in a multisensor vehicular scenario.

In the application example, we demonstrate how the proposed filter can be used to transfer location information from a well localized vehicle to a poorly localized vehicle through MTT. Hence, positioning accuracy of the poorly localized vehicle is greatly improved compared to using a local Kalman filter with GNSS measurements alone.

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A. Motivation

In this paper, we consider an urban intelligent transportation system (ITS) scenario, consisting of cooperating vehicles (illustrated in Fig. 1). Each vehicle is equipped with an on-board sensor allowing them to determine their absolute position, e.g., a GNSS receiver, and an on-board sensor to retrieve relative positions of mobile features present in the environment, e.g., a radar. Absolute position measurements are denoted GNSS measurements and measurements taken w.r.t. features are denoted vehicle-to-feature (V2F) measurements. Due to the sensor used to obtain V2F measurements, it is in general not known which feature gave rise to which measurement. A GNSS and a set of V2F measurements from every vehicle are transmitted in a non-time synchronized manner to the RSU, where a centralized filter is run to track the feature as well as the vehicle states. This information can be utilized by the RSU and sent back to the vehicles to increase their situation awareness.

B. Related Work

1) MTT with known sensor state: Many MTT filters have been proposed to track mobile features using radar-like sensors when the sensor state is known [9]. The multi-hypothesis tracking (MHT) filter builds a growing hypothesis tree with feature-to-measurements data association (DA), and needs to be pruned to limit computation complexity [10]. The joint probability data association (JPDA) filter finds the most likely DA, where feature state information is reduced after each update step to a single Gaussian per feature. In the last years, MTT filters based on random finite set (RFS) and finite-set statistics (FISST) (which avoid the inherent DA), originally developed by [11], have gained much attention. The probability hypothesis density (PHD) filter propagates the first moment of the RFS density over time [12]–[14]. The Poisson multi-Bernoulli (PMB) filter approximates the first moment of the RFS density over time [12]–[14]. In [15], a derivation of this PMB filter based on standard single target measurement models, without using probability generating functional (p.g.fl.) and functional derivatives, is presented. Furthermore, a connection to the $\delta$-generalized labelled multi-Bernoulli ($\delta$-GLMB) filter [17], [18] is shown, where the $\delta$-GLMB density is a special case of multi-Bernoulli (MB) density for labeled targets and can therefore be seen as a special case of the PMB filter. In [19], a Gaussian mixture (GM) multitarget-multisensor Bernoulli tracker, with known sensor locations, is developed and compared to a particle filter (PF) implementation for multistatic sonobuoy fields. Target state update from multiple sensors was achieved through sequential sensor updates. In [20], a factor graph (FG) based approach, not using FISST, was proposed for a variant of the JPDA filter. A multi-scan scenario was considered, and the filter was realized by running loopy belief propagation on the FG containing cycles. In [21], a PF based implementation of the PMB filter [15] was presented. This implementation has been used in [21] as a performance comparison of the FG based MTT. There it was found that the PF based PMB filter implementation scales exponentially with an increasing number of features. In [22], vehicles perform local feature tracking and send track information over the wireless channel to a central fusion center. Then, track-to-track fusion [23], [24] is performed, taking care of out-of-sequence measurements which can arise by utilizing a shared communication media. There the DA problem arises on the decision of which local tracks to fuse. The Mahalanobis distance is employed as DA metric.

2) MTT with uncertain sensor state: In contrast to MTT, simultaneous localization and mapping (SLAM) methods determine the sensor state while mapping features of the environment [2], [26]. Most of the proposed SLAM methods assume static features such as, e.g., walls and street signs. SLAM methods excel through maintaining the correlation between features. A reduction of complexity was achieved in FastSLAM through Rao-Blackwellization (RB), where the feature state, conditioned on the sensor state, is tracked through a Kalman filter and the sensor state through a PF. In [27], a RFS based approach to the SLAM problem was proposed. There the target state is conditioned on the sensor location and then tracked through a PHD filter following a RB. In [28], an MTT with uncertain (single) sensor location is derived for the SLAM problem using FISST and point process theory. Simulation results are shown with a RB PF implementation. In [29], the problem of sensor uncertainty for Bernoulli filtering of at most one target using a single sensor is addressed. Since the scenario is restricted to a single sensor, a suboptimal approach is presented, where the sensor state is updated only by measurements independent of the target state, i.e., target tracking information is not used to update the sensor location. Similar to [27], a FG based approach was considered in [31] for an urban ITS scenario, where the number of features is assumed a priori known, and in [32] a variant of SLAM was considered for indoor environments using time-of-arrival radio signal measurements. There, features are the (static) source locations of line-of-sight and non line-of-sight signal propagation paths of the transmitted radio wave.

C. Notation and Paper Organization

Scalars are described by non-bold letters $r$, vectors by lower-case bold letters $\mathbf{r}$; matrices and sets by upper-case bold...
letters \( X \). The cardinality of set \( X \) is denoted \(|X|\). The set operator \( \cup \) denotes the disjoint union, i.e., \( F^u \cup F^d = F \) means \( F^u \cap F^d = \emptyset \) and \( F^u \cup F^d = F \). The vehicle state is reserved by letter \( x \), the feature state by letter \( f \), and measurements by letter \( z \). The identity matrix of size \( n \times n \) is denoted \( I_n \). The \( L_2 \)-norm of vector \( x \) is \( ||x||_2 \).

The remainder of this paper is organized as follows. Section II gives some background knowledge on RFS, and Section III introduces the problem formulation and system models. Section IV details the proposed MTT filter with uncertain sensor state, numerical results are given in Section V and conclusions are drawn in Section VI.

II. BACKGROUND ON RFS

In this section, we describe some useful properties of an RFS. If not stated otherwise, the source of all these is [15].

A. Random Finite Set Formulation

According to [15], RFS based methods have been developed in [11] to conduct statistical inference in problems in which the variables of interest and/or observations form finite sets. In tracking, they address two major challenges of interest: (i) the number of targets present in the scene is unknown, (ii) measurements are invariant to ordering (measurement-to-target correspondence is unknown). An RFS \( X \) is a finite-set valued random variable, which can be described by a discrete probability distribution \( p(n) \), \( n \geq 0 \) and a family of joint probability densities \( f_n(x_{\pi(1)}, \ldots, x_{\pi(n)}) \) yielding

\[
f(x_1, \ldots, x_n) = p(n) \sum_{\pi} f_n(x_{\pi(1)}, \ldots, x_{\pi(n)}),
\]

where the sum spans over the \( n! \) permutation functions \( \pi(\cdot) \), such that its RFS density \( f(X) \) is permutation invariant. The set integral of a real-valued function \( g(X) \) of a finite-set variable \( X \) is defined as [11, p. 361]

\[
\int g(X) \delta X \triangleq g(\emptyset) + \sum_{n=1}^{\infty} \frac{1}{n!} \int g(x_1, \ldots, x_n)dx_1 \cdots dx_n.
\]

A Bernoulli process \( X \) with probability of existence \( r \) and existence-conditioned probability density function (PDF) \( f(x) \) has RFS density

\[
f(X) = \begin{cases} 1 - r, & X = \emptyset \\ r \cdot f(x), & X = x \\ 0, & \text{otherwise}, \end{cases}
\]

The RFS density of a MB process for the RFS \( X = \{x_1, \ldots, x_n\} \) is [11, p. 368]

\[
f(X) = \prod_{i=1}^{N} (1 - r_i) \sum_{i \leq i_1 \neq \ldots \neq i_n \leq N} \prod_{k=1}^{n} \frac{r_{i_k}}{1 - r_{i_k}} f_{i_k}(x_{i_k}).
\]

A Poisson point process (PPP) with intensity function \( \lambda_c(y) \) has RFS density [15]

\[
f(Y) = \exp(-\lambda_c(1)) \prod_{y \in Y} \lambda_c(y).
\]

with inner product \( \langle \lambda_c, h \rangle \triangleq \int \lambda_c(y)h(y)dy \).

Remark 1: If \( X \) and \( Y \) are independent RFSs such that \( Z = X \cup Y \), then

\[
f_Z(Z) = \sum_{X \cup Y = Z} f_X(X) f_Y(Y).
\]

Note, for RFS \( X \) a MB and RFS \( Y \) a PPP [3] is called a PMB density.

B. State estimation from RFS density

A common way to estimate the set states from a Bernoulli process with RFS density \( f(X) \) is by comparing the probability of existence \( r \) against an existence threshold \( r_{th} \). For \( r > r_{th} \), the target is said to exist and has PDF \( f(x) \) (c.f. [3]). Its state can then be estimated by the mean of \( f(x) \), i.e.,

\[
\bar{x} = \int xf(x)dx.
\]

III. PROBLEM FORMULATION AND SYSTEM MODELS

Here, we first present the problem formulation, and the vehicle and feature dynamics. This is followed by the GNSS and V2F measurement models, and the communication model.

A. Problem Formulation

The goal of the filter, which runs on the RSU, is to track the features and the states of all vehicles in every discrete time step \( t \) through incorporation of all sensor measurements (GNSS and V2F measurements) up until time step \( t \). We are therefore interested in the joint posterior distribution of the feature and vehicle states at every time step \( t \).

B. Vehicle and Feature Dynamics

Vehicle state motion follows independent Markovian processes, where the time-varying vehicle state \( x_{s,t} \in \mathbb{R}^{N_s} \) of each vehicle \( s \in S \) at time step \( t \in \mathbb{N}_0 \) is statistically modeled as \( p(x_{s,t}|x_{s,t-1}) \), with linear state-space model

\[
x_{k,t} = A_{s,t}x_{s,t-1} + w_{s,t},
\]

where \( A_{s,t} \) denotes the state-transition matrix and \( w_{s,t} \sim \mathcal{N}(0, W_{s,t}) \) with error covariance matrix \( W_{s,t} \). A single time-varying feature \( k \in K \) with state \( f_{k,t-1} \in \mathbb{R}_+ \) survives to the next time step \( t \) following an independent identically distributed (IID) Markovian process with survival probability \( p_S(f_{k,t}) \). The feature state motion follows IID Markovian processes and is statistically modeled as \( p(f_{k,t}|f_{k,t-1}) \) with linear state-space model

\[
f_{k,t} = B_{f}f_{k,t-1} + v_{k,t},
\]

where \( B_{f} \) denotes the state-transition model and process noise \( v_{k,t} \sim \mathcal{N}(0, V_{f}) \) with error covariance matrix \( V_{f} \). The state-transition matrices, as well as the error covariance matrices
are assumed equal among the features. Note that vehicle and feature state motion is independent of each other.\footnote{For the sake of brevity, we present linear system and measurement models. In case the true system dynamics and/or measurement model are (mildly) non-linear, linearization steps can be performed similar to the steps taken in an extended Kalman filter (EKF) or the unscented Kalman filter (UKF)\cite{Julier1997}. In doing so, the proposed filter remains valid unaltered.}

In the following, we will drop the subscript indexing on states and measurements w.r.t. vehicle/feature/time whenever the context allows.

C. Measurement models

At time step $t$, vehicle $s \in S$ obtains two different kind of measurements: (i) measurements of the vehicle state $x$ w.r.t. the reference frame, i.e., GNSS-like measurements; and (ii) measurements w.r.t. to features, i.e., from a radar-like (on-board) V2F sensor. Without loss of generality, we assume that the on-board sensors’ state is equal to the vehicle state. Thus an uncertain vehicle location implies an uncertain location for the on-board V2F sensor.

1) GNSS measurement: The GNSS measurement $z_G \in \mathbb{R}^{M_G}$ of vehicle $s \in S$ at time $t$ is statistically modeled through the likelihood function $p(z_G|x)$ with linear observation model

$$z_G = H_G x + r,$$

where $H_G$ is the linear observation matrix and $r \sim \mathcal{N}(0, R)$ with error covariance matrix $R$.

2) V2F measurement: Let $Z^F$ be a set of measurements from a tracking sensor that is susceptible to measurement noise, missed detections, and false detections. Examples of such sensors include camera, radar and LIDAR. Consequently,

$$Z^F = Z^{FA} \cup Z^D,$$

where $Z^{FA}$ denotes the set of false alarm measurements due to clutter modeled by a PPP with intensity $\lambda_c(z)$, and $Z^D$ denotes the set of detected features. Let a V2F measurement $z \in Z^D$ with state dimension $\mathbb{R}^{M_{V2F}}$ be obtained through the V2F sensor at vehicle $s \in S$ w.r.t. feature $k$ at time $t$. It is modeled through the likelihood function $\ell(z|x, f)$ with linear observation model

$$z = H_1 x + H_2 f + q,$$

where $H_1$ and $H_2$ denote observation matrices, and $q \sim \mathcal{N}(0, Q)$ with error covariance matrix $Q$. Note that the measurement-to-feature state correspondence denoted DA, is in general not known and needs to be inferred from the measurements. Let the measurement likelihood for a single feature RFS $F$, i.e. $|F| \leq 1$, be

$$\eta(Z^F | x, F) = \begin{cases} 1 & \text{if } F = \emptyset, \ Z^F = \emptyset \\ 0 & \text{if } F = \emptyset, \ Z^F = \{z\} \\ p_D(x, f) \ell(z|x, f) & \text{if } F = \{f\}, \ Z^F = \{z\} \\ 0 & \text{if } |F| > 1, \text{ or } |Z^F| > 1. \end{cases}$$

Note that the proposed filter only predicts over small time-horizons in the order of a few tens of milliseconds. Then the assumption on vehicle and feature states evolving independently is reasonable, because there is very little interaction among them within the prediction horizon.

where $\ell(z|x, f)$ follows the measurement model (11). Note that due to (11), the probability of detection $p_D(x, f)$ in (12) depends on the vehicle state $x$ as well as on the feature state $f$. For instance, a limited sensor field-of-view (FoV) affects the probability of feature detection based on the distance between vehicle and feature.

Remark 2: For the case the radar-like V2F sensor is able to detect features within a radius $r_{\text{max}}$, the probability of detection is defined

$$p_D(x, f) = \begin{cases} \alpha, & \text{if } \|H_1 x + H_2 f\|_2 \leq r_{\text{max}} \\ 0, & \text{otherwise,} \end{cases}$$

where $0 < \alpha \leq 1$ is a constant such that it is a valid PDF. Depending on the specific sensor at hand, the FoV may be different and \cite{13} needs to be adapted.

D. Communication model

We assume that every vehicle is able to communicate all obtained measurements (V2F and GNSS) with the RSU instantaneously and without errors. This implies that at any time $t$ the number of vehicles communicating with the RSU can vary. The incorporation of a realistic V2I channel model and its performance impact is a point for future work.

IV. POISSON MULTI-BERNOULLI FILTERING WITH UNCERTAIN SENSOR STATE

In this section, we formulate the proposed PMB filter with uncertain sensor state. We consider a tracking scenario subject to Section I-A, where there may be multiple features. In Section V-C, we proceed with the asynchronous multisensor case allowing to track multiple features and vehicles with uncertain vehicle state. In Section V-D, a tractable Gaussian density approximation of the proposed filter is given. The vehicle state PDF at time step $t-1$ is indicated by subscript ‘\(-\)’, i.e., $p_-(x)$, the PDF predicted to the current time step $t$ (before updating by a measurement) is indicated by subscript ‘\(+\)’, i.e., $p_+(x)$, and the posterior PDF is stated without subscript. Similar definitions hold w.r.t. the feature RFS density.

The proposed filter is developed within a Bayesian framework with alternating prediction and update steps operating on an RFS $X$ by \cite{11} Ch. 14.

$$f_{+}(X) = \int f(X | X') f_{-}(X') \delta X', \quad (14)$$

and

$$f(X | Z) \propto \ell(Z | X) f_+(X), \quad (15)$$

where $f_{-}(X')$ is the prior RFS density, $f(X | X')$ is the RFS transition density, $f_+(X)$ is the predicted density, and $\ell(Z | X)$ is the RFS measurement likelihood for measurement set $Z$.

From the problem definition stated in Section III-A we are interested in the joint posterior density of the vehicle and features considering all measurements up to the current time step $t$. We now proceed with the development of the proposed filter within this framework.
With a single vehicle, the prior joint vehicle-feature density is of form
\[ f_-(x, F) = p_-(x)f_-(F), \tag{16} \]
where \( p_-(x) \) is the prior PDF on the vehicle state, and \( f_-(F) \) is the prior PMB density. The latter density can be written in terms of an PPP intensity of undetected features \( f^u(F^u) \), i.e., features which are hypothesized to exist but have never been detected [15 Def. 1], and the prior MB RFS density of detected features \( f^d(F^d) \), as [11] p. 484, [15]
\[ f_-(F) = \sum_{F^u \cup F^d = F} f^u(F^u)f^d(F^d). \tag{17} \]
In (17), the PPP density of undetected features is
\[ f^u(F^u) = e^{-\langle D^u,1 \rangle} \prod_{f \in F^u} D^u(f), \tag{18} \]
where \( D^u(f) \) is the intensity of undetected features. We are interested in a low computational complexity method to compute the (posterior) joint vehicle-feature density \( f(x, F) \) in every discrete time step \( t \) through incorporation of all sensor measurements. In doing so, the posterior density should remain in the same form as the prior joint density (16).

A. Prediction step

With the vehicle state \( x \), and existing feature RFS \( F \), the predicted joint vehicle-feature density is
\[ f_+(x, F) = p_+(x)f_+(F). \tag{19} \]
Here, the predicted vehicle state PDF is given by the Chapman-Kolmogorov equation [35]
\[ p_+(x) = \int p(\mathbf{x}|x')p_-(x')dx', \tag{20} \]
where \( p(\mathbf{x}|x') \) is the state transition PDF described by (7), and \( p_-(x') \) is the prior PDF. Similarly, the predicted feature state PMB density is calculated by
\[ f_+(F) = \int f(F|F')f_-(F')dF', \tag{21} \]
where \( f(F|F') \) is the transition RFS density, and \( f_-(F') \) is the prior PMB density. The predicted intensity of undetected features \( D^+_u(f) \) of the predicted PPP density \( f^+_u(F^u) \) is given by
\[ D^+_u(f) = D^u(f) + \int p(f|f')p_S(f')D^u(f')df'. \tag{22} \]
Here, the birth intensity is denoted \( D^b(f) \), feature transition PDF \( p(f|f') \) is described by (8), feature survival probability is denoted \( p_S(f') \), and \( D^+_u(f') \) denotes the prior intensity. The MB RFS density of detected features of (21) is
\[ f^+_d(F^d) = \sum_{\mathcal{F}_d \in F^d} f^+_d(F^d), \tag{23} \]
where \( \mathcal{F}_d \) denotes the set of existing features (before measurement update), and
\[ f^+_d(F^d) = \begin{cases} 1 - r^+_1, & F^d = \emptyset \\ r^+_1p_+(f^d), & F^d = \{f^d\} \\ 0, & \text{otherwise}. \end{cases} \tag{24} \]
Here, the predicted PDF of feature \( i \) is
\[ p_+(f^i) = \int p(f^i|f^d)p_-(f^d)df^d, \tag{25} \]
where \( p_-(f^d) \) is the prior PDF of feature \( i \). The probability of existence of feature \( i \) is [15 Eqn. (40)]
\[ r^+_i = r^-_i \int p_S(f^i)p_-(f^d)df^d, \tag{26} \]
where \( r^+_i \) denotes the prior probability of existence, \( p_-(\cdot) \) the prior PDF, and \( p_S(\cdot) \) the probability of feature survival.

B. Measurement update step

Updating the joint vehicle-feature density (19) by any of the two types of different measurements, GNSS and V2F measurements, involves the application of Bayes’ theorem. In the following, we describe the update calculations using the different type of measurements.

1) Update with vehicle state measurement: Let \( z_G \) be a measurement related to the vehicle state \( x \), and unrelated to the set of features \( F \).
\[ p(z_G|x, F) = p(z_G|x). \tag{27} \]
For example, \( z_G \) could be a GNSS and/or an inertial measurement unit (IMU) measurement. Given a predicted vehicle-feature density (19), and by Bayes’ theorem the updated density is
\[ f(x, F|z_G) = p(x|z_G)f_+(F). \tag{28} \]
In other words, the vehicle state density is updated with the measurement \( z_G \), the feature set density is unaffected by the update, and the independent form is retained (c.f. (15)). Note, this update step can be omitted in the absence of GNSS-like measurements, e.g., in a pure SLAM application [26].

2) Update with clutered set of feature measurements: Let the set of measurements \( Z^F \) subject to the measurement model of Section [III-C2] be indexed by \( M \), and let \( A \) be the space of all DAs \( A \) for the predicted MB. A DA \( A \in A \) is an assignment of each measurement in \( Z^F \) to a source, either to the background (clutter or new feature) or to one of the existing features indexed by \( I \). It is therefore a partition of \( M \cup I \) into non-empty disjoint subsets \( C \in A \), called index cells.

Remark: Due to the standard MTT assumption that the features generate measurements independent of each other

For example, let \( M = \{m_1, m_2, m_3\} \) and \( I = \{i_1, i_2\} \), i.e., three measurements and two features. One valid partition of \( M \cup I \), i.e., one of the possible associations, is \( \{m_1\}, \{m_2, i_1\}, \{m_3\}, \{i_2\} \). The meaning of this is that measurement \( m_2 \) is associated to feature \( i_1 \), feature \( i_2 \) is not detected, and measurements \( m_1 \) and \( m_3 \) are not associated to any previously detected feature, i.e., measurements \( m_1 \) and \( m_3 \) are either clutter or from new features.
an index cell contains at most one feature index, i.e., $|C \cap I| \leq 1$ for all $C \in A$. Any association in which there is at least one cell, with at least two feature indices, will have zero likelihood because this violates the independence assumption. Further, due to the point feature assumption, any feature generates at most one measurement in each time step, i.e., $|C \cap M| \leq 1$ for all $C \in A$. Any association in which there is at least one cell, with at least two measurement indices, will have zero likelihood because this violates the point feature assumption. If the index cell $C$ contains a feature index, then let $i_C$ denote the corresponding feature index. Further, if the index cell $C$ contains a measurement index, then let $m_C$ denote the corresponding measurement index. Measurements in $C$ not assigned to any feature are associated to the background.

With the help of Bayes’ rule, the updated joint vehicle-feature density is

$$ f(x, F|Z^F) = \sum_{F^u \cup F^d = F} f^u(F^u|x) \times \sum_{A \in A} w_A p^A(x|Z^F) f^{d,A}(F^d|x, Z^F), \quad (29) $$

where $w_A$ denotes the weight of DA $A \in A$ with $\sum_{A \in A} w_A = 1$, and $p^A(x|Z^F)$ denotes the vehicle state posterior stated in Appendix A. For now, let us assume the DA weights are given. The undetected feature density $f^u(F^u|x)$ and the detected feature density $f^{d,A}(F^d|x, Z^F)$ are stated in Appendix B. Equation (29) does not factorize as $f(x, F|Z^F) = p(x|Z^F)f(F|Z^F)$ where $f(F|Z^F)$ is a multi-Bernoulli density such that it remains in the same form as the prior (c.f. [19]); on the contrary, there are many dependencies between the feature state RFS and the vehicle state. This means that existing independence-assuming tracking frameworks cannot be applied directly [19], or introduce a significant increase in computational complexity [28]. To overcome this, we approximate

$$ f^u(F^u|x) \approx \hat{f}^u(F^u), \quad (30) $$

$$ f^{d,A}(F^d|x, Z^F) \approx \hat{f}^{d,A}(F^d|Z^F), \quad (31) $$

where the functions on the right hand side need to be found. Towards this end, we make the following approximations for the vehicle and feature dependent probability of detection

$$ p_{D}(x, f) \approx \left\{ \begin{array}{ll} \hat{p}_{D}^u, & f \in F^u \\ \hat{p}_{D}^d, & f \in F^d, \end{array} \right. \quad (32) $$

with $F^u \cup F^d = F$, and where

$$ \hat{p}_{D}^u = \int p_D(x, f) p^u(x) dxdf, \quad (33) $$

$$ \hat{p}_{D}^d = \int p_D(x, f) p^d(x) p^u(x) dxdf. \quad (34) $$

Here, (33) is the expected probability of detection for an undetected feature under the predictive distributions for $x$ and $f$, and (34) is the expected probability of detection for a detected feature. An alternative (and stronger) approximation for (33) would be $\hat{p}_{D}^u = pD(\hat{x}, f)$ with $\hat{x} = \int x p^u(x) dx$ and $\hat{f}$ the estimated feature state (c.f. Section II-B), and similarly for $\hat{p}_{D}^d$.

Under approximation (32), the updated density (29) becomes

$$ \hat{f}(x, F|Z^F) \propto \sum_{F^u \cup F^d = F} \hat{f}^u(F^u) \times \sum_{A \in A} w_A p^A(x|Z^F) \hat{f}^{d,A}(F^d|x, Z^F) \quad (35) $$

with $\hat{f}^u(F^u)$ and $\hat{f}^{d,A}(F^d|x, Z^F)$ given in Appendix C. We observe in (35) that the undetected feature density $\hat{f}^u(F^u)$ depends only on the undetected feature RFS $F^u$, and is independent of the other stochastic variables. What remains are dependencies between the detected features, and the vehicle. To remove the dependency on the vehicle state in the detected feature density $\hat{f}^{d,A}(F^d|x, Z^F)$ in (35), we map the vehicle state uncertainty onto the V2F measurement uncertainty. This is done by averaging the V2F measurement likelihood by the vehicle state uncertainty. In doing so, the detected feature density under association $A$ becomes independent of the vehicle state $x$ leading to the approximation (31), where the approximated updated feature set density $\hat{f}^{d,A}(F^d|Z^F)$ is given in Appendix D. The updated joint vehicle-feature density is approximated as

$$ f(x, F|Z^F) \propto \sum_{F^u \cup F^d = F} \hat{f}^u(F^u) \times \sum_{A \in A} w_A p^A(x|Z^F) \hat{f}^{d,A}(F^d|Z^F). \quad (36) $$

In this form, the vehicle state PDF is now independent on the feature RFS, and so is the feature density on the vehicle state. This allows to state the weights $w_A$, which are given in Appendix E. Note, (36) is a Poisson multi-Bernoulli mixture (PMBM) density, where each DA $A \in A$ denotes a hypothesis on the posterior vehicle state $x$ and the detected feature state $F^d$, weighted by $w_A$. It can be reduced to a PMB density using, e.g., the variational approximation presented in [36], or based on the marginal DA probabilities [15]. We apply the latter approach, where for the reduction of (36) to the form of (16), the track-oriented marginal MemBM/Poisson (TOMB/P) algorithm (c.f. [15]) is used. This results in a single hypothesis per detected feature described by a Bernoulli process (c.f. [3]), and per vehicle described by its PDF; as well as the intensity of undetected feature described by a PPP (c.f. [3]). This means, the summation over the DA space $A$ has vanished in (36), retaining the form of (16).

C. Multi-scan scenario using multiple vehicles with uncertain state

Up to this point, we discussed PMB filtering with a single vehicle and uncertain state, where GNSS and V2F measurements are used. To achieve feature tracking as described in Section I-A where sensors are mounted on several vehicles, we have to consider the multisensor case. Furthermore, depending on the infrastructure, sensors are time synchronized, i.e., take measurements at the same time step $t$ or are not synchronized, i.e., measurements from a sensor arrives timestamped, but the time the sensor acquires the measurement is independent of other sensors. Let there be a single RFS $F$
modeling the features state, \( S = |S| \) vehicles with uncertain vehicle state \( x_s \), with \( s = 1, 2, \ldots, S \). The set of vehicles taking a measurement at time step \( t \) is given by \( C \subseteq S \), where each vehicle \( c \in C \) provides a vector with a GNSS measurement \( \tilde{z}_c \) and V2F measurements \( \tilde{Z}_c^F \). Furthermore, let \( \bar{x} = [\bar{x}_1^T, \bar{x}_2^T, \ldots, \bar{x}_|S|_c^T]^T, \tilde{z}_G = [z_{G1}^T, z_{G2}^T, \ldots, z_{GC}^T]^T, \) and \( \tilde{Z}_G^F = \bigcup_{c \in C} \tilde{Z}_c^F \).

In the multisensor case, the PMB filter with uncertain sensor state follows the unisensor case proposed in Section IV: the joint vehicle-feature density is predicted, and updated by the GNSS and the V2F measurements. Thereby, the predicted vehicle-feature density (19) becomes

\[
f_+(\tilde{x}, F) = f_+(F)p_+(\tilde{x}). \tag{37}
\]

Updating the joint density (19) by the GNSS measurement results in (28). In the multisensor case, it becomes

\[
f(\tilde{x}, F|\tilde{z}_G) = f_+(F)p(\tilde{x}|\tilde{z}_G). \tag{38}
\]

After incorporating the GNSS measurements we proceed with the V2F measurements. In the unisensor case, this resulted in the approximated joint sensor-feature density (38). In the multisensor case, this density becomes

\[
f(\tilde{x}, F|\tilde{Z}_G^F) \propto \sum_{F^{u}|F^{d}=F} \hat{f}^u(F^{u}) \times \sum_{A \in A} w^A p^A(\tilde{x}|\tilde{Z}_G^F) \hat{f}^d,A(F^{d}|\tilde{Z}_G^F), \tag{39}
\]

where

\[
p^A(\tilde{x}|\tilde{Z}_G^F) \propto p_+(\tilde{x})\hat{p}^A(\tilde{Z}_G^F|\tilde{x}), \tag{40}
\]

and \( \hat{f}^d,A(F^{d}|\tilde{Z}_G^F) \) involves the marginalization over the vehicle prior PDF \( p_+(\tilde{x}) \), i.e., containing only vehicles which provide a V2F measurement, according to (70). Note that here (38) is used as prior in the joint sensor-feature density (39). Furthermore, the space \( A \) of all DA increases with an increase in the number of communicating sensors |\( C \) for the predicted MB. In terms of complexity, this increase can be significant, because of the increase of possible feature state-to-measurement associations.

**Remark 4:** Several different approaches exist to tackle this DA problem in a tractable manner. For instance, by employing sequential sensor-by-sensor measurement updates on (38), or by performing variational inference (37), or by solving the DA in parallel on a sensor-by-sensor basis (21). Here, we employ the sequential measurement update strategy to limit the size of the DA space \( A \). In doing so, subsequent sensors will benefit from updated vehicle and feature information of preceding sensors. In our application example (c.f. Section I-A), this means that an update of the joint vehicle-feature density, with measurements from a well-localized vehicle (certain vehicle state), results in an improvement of feature tracking performance when prior information on the features is low. An update of the joint vehicle-feature density, with measurements from a poorly localized vehicle (uncertain vehicle state), allows to reduce the uncertainty of its own vehicle state when prior information on the features is high.

### D. Gaussian Density Approximation

In order to obtain a low complexity implementation, we describe the vehicle state PDF by a Gaussian with PDF \( p(x) \) \( \sim N(\mu, \Sigma) \) with mean parameter \( \mu \) and covariance matrix \( \Sigma \). Similarly, the RFS density of a MB process of feature \( i \) is described by a Bernoulli random variable \( r_i \) and a Gaussian PDF \( p(f_i) \) \( \sim N(\mu_f, \Sigma_f) \) with parameters \( \mu_f \) and \( \Sigma_f \). Under this description and the system models of Section III, we can express the prediction and update steps of the proposed filter in closed form with low computational complexity, whose steps are described next.

1) **Prediction Step:** The predicted vehicle state PDF of (19) is

\[
p_+(x) = N(\mu_{x+}, \Sigma_{x+}), \tag{41}
\]

where with the use of (7)

\[
\mu_{x+} = A\mu_{x-}, \tag{42}
\]

\[
\Sigma_{x+} = A\Sigma_{x-}A^T + W. \tag{43}
\]

The notation above means that if at time step \( t-1 \) the vehicle state PDF \( p_{-}(x) = N(\mu_{x-}, \Sigma_{x-}) \), then at time step \( t \), the predicted state PDF (before updating by a measurement) is \( p_{+}(x) = N(\mu_{x+}, \Sigma_{x+}) \).

The intensity of undetected features (22) is modeled by GM consisting of newborn features with weight \( \lambda^{n}(f_i) = \lambda^b \) and PDF \( p(f_i) = N(\mu^b, \Sigma^b) \), where \( \mu^b \) and \( \Sigma^b \) are the birth mean and covariance matrix; and undetected features survived to the current time step with prior parameters \( \{\lambda^n, \mu_{f-}, \Sigma_{f-}\} \) and predicted parameters

\[
\lambda^n = \langle p_S, \lambda^b \rangle, \tag{44}
\]

\[
\mu_{f+} = B\mu_{f-}, \tag{45}
\]

\[
\Sigma_{f+} = B\Sigma_{f-}B^T + V. \tag{46}
\]

The predicted MB density of detected features, stated in (21), has single feature Bernoulli parameters \( r_i \) predicted using (26), and single feature PDF \( p_{+}(f_i) \) calculated similarly to (45) and (46).

2) **Update step:** The joint vehicle-feature state density (28) is computed by updating the predicted vehicle-feature density (19) with the GNSS measurement \( z_G \) through the Kalman update step (33), (35), where the vehicle state PDF is given by

\[
p(x|z_G) = N(\mu_x, \Sigma_x). \tag{47}
\]

Here,

\[
\mu_x = \mu_{x+} + K(z_G - H_G\mu_{x+}), \tag{48}
\]

\[
\Sigma_x = \Sigma_{x+} - KH_G\Sigma_{x+}, \tag{49}
\]

\[
K = \Sigma_{x+}H_G^T S, \tag{50}
\]

\[
S = H_G\Sigma_{x+}H_G^T + R. \tag{51}
\]

The matrices \( H_G \) and \( R \) are defined in (9).

The updated joint vehicle-feature density (36) is computed by updating the predicted vehicle-feature density (19) with the V2F measurement \( \tilde{Z}_G^F \). Note that depending on the time difference between the GNSS and V2F measurements, (28) may be used instead of (19) as prior on the joint vehicle-feature.
density. In order to calculate (36), the vehicle measurement-state likelihood \( \ell(z|x) \) used in (64) and (65), and the feature measurement-state likelihood \( \ell(z|f) \) used in (74) are needed. The measurement-feature state likelihood (74) for \( z \) given, can be written in terms of \( f \) in closed form by

\[
\ell(z|f) \propto N(\mu_z|f, \Sigma_z|f) \quad (52)
\]

with

\[
\Sigma_z|f = H_2^T \left( Q + H_1 \Sigma_{x+} H_1^T \right)^{-1} H_2, \quad (53)
\]

and

\[
\mu_z|f = H_2^T \left( z - H_1 \mu_z \right). \quad (54)
\]

Here, \( p_+(x) = N(\mu_{x+}, \Sigma_{x+}) \), and \((\cdot)^+\) denotes the Moore-Penrose pseudo inverse.

Proof: See Appendix [F]

The vehicle measurement-state likelihood (64) for a given \( z \) and marginalized over a detected feature, and in (65) marginalized over an undetected feature, can be written in terms of \( x \) in closed form by

\[
\ell(z|x) = N(\mu_z|x, \Sigma_z|x) \quad (55)
\]

with

\[
\Sigma_z|x = H_2^T \left( Q + H_2 \Sigma_{x+} H_2^T \right)^{-1} H_1, \quad (56)
\]

and

\[
\mu_z|x = H_2^T \left( z - H_1 \mu_z \right). \quad (57)
\]

Here, (55) equals (64) for feature PDF \( p(f) \triangleq p_c^V(f) = N(\mu_{f+}, \Sigma_{f+}) \), and in (65) for feature PDF \( p(f) \triangleq D_2^V(f) = N(\mu_{f-}, \Sigma_{f-}) \).

Proof: The proof is analogous to the proof of the feature measurement-state likelihood (52) with the only difference that the unknown is \( x \) instead of \( f \).

E. Computational complexity, memory footprint, and communication demand

Computational complexity is dominated by the matrix inversion needed to update the feature and vehicle densities, and the measurement-to-feature state DA. Updating the joint vehicle-feature density \( f(x,F|z_G) \) using GNSS measurement \( z_G \) requires a matrix inversion which scales as \( O(N_G^2) \). The update of an MB component in \( f^{d,A}(F|x, Z^F) \) by V2F measurement \( z^{mc} \in Z^F \) scales as \( O(N_G^3) \), and consequently as \( O(|Z^F|N_G^2) \) for the whole measurement set. Computational complexity of DA is \( O(|F||Z^F|) \) \cite{38}. Hence, the update of the joint vehicle-feature density (36) by V2F measurement set \( Z^F \) scales as \( O(|F||Z^F|^2 + N_{ij}^F|Z^F|^2 + N_{ij}^F|Z^F|) \), where the last term comes from vehicle state update of \( Z^F \).

In each time step \( t \), the size of undetected features RFS \( F^u \) increases by \( |D^b(f)| \) new born targets. The number of existing feature-tracks increases by \( |Z^F| \), a new feature-track per measurement \cite{15}, using a Bernoulli component per track. For each existing feature-track \( |Z^F| + 1 \) hypotheses (plus one for a missed detection) are computed. The TOMB/P algorithm reduces each feature-track to a single hypothesis track. Pruning of feature-tracks with low probability of existence \( r \) allows to keep the number of feature-tracks tractable. Each hypothesis, described by a Bernoulli component (c.f. (3)) with Gaussian PDF, has a memory footprint of \( a \) Bytes needed to store \( \{r, f, \text{cov}(f)\} \), where \( f \in \mathbb{R}^{N_f} \). Storing the vehicle state requires \( b \) Bytes, where the state of each vehicle is described by a Gaussian PDF with parameters \( \{x, \text{cov}(x)\} \) with \( x \in \mathbb{R}^{N_x} \).

Without pruning of low-probability Bernoulli components in \( F \), the memory footprint of the proposed filter is \( a|F| + b \) Bytes.

In the multisensor approach of Section IV-C, the RSU receives GNSS measurement \( z_G \) and V2F measurement \( Z^F \) from a vehicle and then performs the filter update computation. The RSU broadcasts the vehicle state estimates either whenever a new measurement has been processed, or based on a fixed schedule. Should information of detected (and tracked) features be required at the vehicles, then the single-feature pdfs need to be transmitted as well (c.f. Section II-B).

V. NUMERICAL RESULTS

We consider a scenario similar to the one outlined in Fig. 1 where we apply the proposed multifile-feature-multisensor state tracking filter presented in Section IV.

A. Setup

The state of a vehicle at time step \( t \) is \( x = [p^T, v^T]^T \) with position \( p \in \mathbb{R}^2 \) and velocity \( v \in \mathbb{R}^2 \). Vehicle dynamics follow a linear constant velocity (CV) model described by (7) with

\[
A = \begin{bmatrix} 1 & T_s \\ 0 & 1 \end{bmatrix} \otimes I_2, \quad (58)
\]

where \( T_s = 0.5 \) s, and

\[
W = r \begin{bmatrix} T_s^3/3 & T_s^2/2 \\ T_s^2/2 & T_s \end{bmatrix} \otimes I_2 \quad (59)
\]

with \( r = 0.05 \) m²/s, and \( \otimes \) denoting the Kronecker product. The state of a feature at time \( t \), denoted \( f \in \mathbb{R}^4 \), is comprised of Cartesian position and velocity, similar to the vehicle state \( x \).

There are maximal five features present, if not noted otherwise. Furthermore, feature dynamics follow the CV model with the same parameters used for the vehicles. To generate a challenging scenario for DA, we initialize the feature states \( f \sim N(0, 0.25 I_4) \) at \( t = 175 \) for all features and run the CV model forward and backward in time similar to [15] Sec. VI. The first feature enters the scene after \( t = 0 \), the second after \( t = 20 \) and so on. Once present, features stay alive for the remaining simulation time. Vehicle and feature trajectories are shown in Fig. 2. The observation matrix of the GNSS measurement model (9) is

\[
H_G = \begin{bmatrix} 1 & 0 \end{bmatrix} \otimes I_2, \quad (60)
\]

where \( R = \sigma_G^2 I_2 \). For vehicle 1, we assume it has low location uncertainty with \( \sigma_G^2 = 0.1 \) m² and for vehicle 2 high location uncertainty with \( \sigma_G^2 = 10 \) m², corresponding to a vehicle with high quality GNSS receiver and one with low quality GNSS receiver. In the single sensor case, only vehicle 1 is present, and in the multisensor case both vehicles are present, if not noted otherwise. The V2F measurement model follows (11), where \( H_1 \triangleq H_G, \ H_2 \triangleq -H_G \) and
Furthermore, the probability of survival is $\text{probability of detection is } \lambda = 20 \text{ m}$ sub-pattern assignment (OSPA) metric with cut-off parameter $c$. After that, we consider the multitarget-on feature tracking performance using a single vehicle and multiple-features. After that, we consider the multitarget-multisensor tracking performance: The features and vehicle trajectories are outlined in Fig. 2 with V2F measurements in Fig. 3. In Fig. 4, the feature state OSPA is plot for each time step $t$. We observe that there are peaks with a high OSPA value when a new feature enters the scene. These peaks are due to a cardinality mismatch between the feature RFS estimate and the true feature set. Furthermore, there is a high OSPA value around time step $t = 175$. At this point in time, features are closely spaced together w.r.t. the V2F measurement variance $\sigma_{V2F}^2$ resulting in a challenging scenario for measurement-to-state DA. In Fig. 5, the cardinality of the feature RFS is plotted over time. Around time step $t = 175$, the filter overestimates the feature RFS cardinality, which may be caused by clutter measurements. Note, different Monte-Carlo (MC) realizations produce a slightly different outcome and feature OSPA value, but with the same tendency. This behavior w.r.t. feature appearance and the effect when they are spatially close agrees with the findings in [15] for a known sensor (vehicle) state. Furthermore, we observe in Fig. 4 that the OSPA is low for time steps where features are spatially separated and already present in the scene. Then the MTT filter is able to produce feature estimates with low error.

In Fig. 6, the feature state OSPA averaged over all 351 time steps is plot for different values of GNSS measurement variance $\sigma_G^2$. The increase of GNSS measurement variance leads to an increased vehicle state uncertainty with the effect of an increase of the average feature OSPA. This OSPA increases results in features staying spatially close together w.r.t. the V2F measurement variance $\sigma_{V2F}^2$. Second, this results in features staying spatially close together w.r.t. the feature state measurement uncertainty ($\sigma_c^2$ and $\sigma_{V2F}^2$ together) for a longer period of time around time step $t = 175$. Hence, DA is more challenging with the effect of an increased feature OSPA in this regime. In the same figure, the average feature OSPA without modeling the present vehicle state uncertainty is plotted using the conventional PMB filter [15]. We observe that not modeling the present vehicle state uncertainty has a negative effect on feature tracking performance.

To assess feature tracking performance, we use the optimal sub-pattern assignment (OSPA) metric with cut-off parameter $c = 20$ m and order $p = 2$ [39]. The vehicle tracking performance is assessed in terms of the root mean square error (RMSE).

**B. Discussion**

First, we discuss the impact of an uncertain vehicle state on feature tracking performance using a single vehicle and multiple-features. After that, we consider the multitarget-multisensor case from Section IV-C and show scaling results in terms of numbers of features and vehicles tracked.

1) **Impact of uncertain vehicle state on feature tracking performance**: The features and vehicle trajectories are outlined in Fig. 2 with V2F measurements in Fig. 3. In Fig. 4, the feature state OSPA is plot for each time step $t$. We observe that there are peaks with a high OSPA value when a new feature enters the scene. These peaks are due to a cardinality mismatch between the feature RFS estimate and the true feature set. Furthermore, there is a high OSPA value around time step $t = 175$. At this point in time, features are closely spaced together w.r.t. the V2F measurement variance $\sigma_{V2F}^2$ resulting in a challenging scenario for measurement-to-state DA. In Fig. 5, the cardinality of the feature RFS is plotted over time. Around time step $t = 175$, the filter overestimates the feature RFS cardinality, which may be caused by clutter measurements. Note, different Monte-Carlo (MC) realizations produce a slightly different outcome and feature OSPA value, but with the same tendency. This behavior w.r.t. feature appearance and the effect when they are spatially close agrees with the findings in [15] for a known sensor (vehicle) state. Furthermore, we observe in Fig. 4 that the OSPA is low for time steps where features are spatially separated and already present in the scene. Then the MTT filter is able to produce feature estimates with low error.

In Fig. 6, the feature state OSPA averaged over all 351 time steps is plot for different values of GNSS measurement variance $\sigma_G^2$. The increase of GNSS measurement variance leads to an increased vehicle state uncertainty with the effect of an increase of the average feature OSPA. This OSPA increase results in features staying spatially close together w.r.t. the V2F measurement variance $\sigma_{V2F}^2$. Second, this results in features staying spatially close together w.r.t. the feature state measurement uncertainty ($\sigma_c^2$ and $\sigma_{V2F}^2$ together) for a longer period of time around time step $t = 175$. Hence, DA is more challenging with the effect of an increased feature OSPA in this regime. In the same figure, the average feature OSPA without modeling the present vehicle state uncertainty is plotted using the conventional PMB filter [15]. We observe that not modeling the present vehicle state uncertainty has a negative effect on feature tracking performance.

In Fig. 7, the average feature state OSPA is plotted for different values of V2F measurement noise variance $\sigma_{V2F}^2$. We observe, that a higher V2F noise variance leads to an increased OSPA value. This is because the single feature state estimation error increases and DA becomes more challenging. Note, the results of Fig. 6 and Fig. 7 are averages over 10 MC realizations.

2) **Multitarget-multisensor tracking performance**: The RMSE of the vehicle state is plotted for each time step $t$ for vehicle 1 with low location uncertainty ($\sigma_G^2$ small) in Fig. 8 and in Fig. 9 for vehicle 2 with high location uncertainty ($\sigma_G^2$ high). As a benchmark, results from a centralized Kalman filter (KF) are plot as well, where measurement-to-feature DA is known and where the augmented state vector contains all vehicle and all feature states. Furthermore, the tracking performance using a local KF is plot. The local KF performs filtering only on the individual vehicle state separately using...
Fig. 4. Feature OSPA with $\sigma_G^2 = 10^{-3} \text{ m}^2$ and $\sigma_{V2F}^2 = 0.25 \text{ m}^2$.

Fig. 5. Feature cardinality with $\sigma_G^2 = 10^{-3} \text{ m}^2$ and $\sigma_{V2F}^2 = 0.25 \text{ m}^2$.

Fig. 6. Average feature OSPA for different values of GNSS measurement variance $\sigma_G^2$. The V2F noise variance is set to $\sigma_{V2F}^2 = 0.25 \text{ m}^2$.

Fig. 7. Average feature OSPA for different values of V2F measurement variance $\sigma_{V2F}^2$. The GNSS noise variance is set to $\sigma_G^2 = 10^{-3} \text{ m}^2$.

Fig. 9. Vehicle state RMSE of vehicle 2.

Fig. 10. CDF plot of vehicle state RMSE.

only GNSS measurements and does not estimate feature states. Note, the performance of the local KF can be considered as the worst-case performance on vehicle state estimation, since V2F measurements are not considered at all. We observe from Fig. 8 that for vehicle 1, which has low GNSS measurement noise, all three filter methods deliver a similar performance. The reason for this is that, due to the high accuracy of GNSS measurements, not a lot of information (to improve the vehicle state) is provided from feature tracking, i.e., feature tracking error is high w.r.t. the vehicle state tracking error of vehicle 1 (after updating by the GNSS measurement). In Fig. 10 the cumulative distribution function (CDF) of the RMSE is plotted. Here, the low RMSE of vehicle 1 using the three different filters can be observed as well. Moving the focus to vehicle 2, we observe that the RMSE of the local KF is much higher compared to the central KF, which is caused by the high noise in the GNSS measurements. Due to the low RMSE of vehicle 1’s state there is relevant position information in the system, which can be transferred from vehicle 1 to vehicle 2 via the features utilizing the V2F measurements. In 80% of all cases, the RMSE of vehicle 2 is below 0.5 m with the proposed filter, compared to 1.4 m with the local KF. Despite this great improvement of the proposed filter over the local KF, it does not achieve the performance of the central KF, where the RMSE is below 0.3 m. The reason for the difference is that the central KF has knowledge of the correct DA, knows the true number of features present, and ignores clutter V2F measurements. Furthermore, it tracks any present correlations between features and vehicles not modeled
by the proposed filter. The proposed filter needs to infer the measurement-to-feature DA, estimate the number of features currently present, and needs to appropriately handle clutter in the V2F measurement set \( Z^F \).

3) Scaling results: In Fig. 11, the average computation time per time step \( t \) is plotted for a simulation with two vehicles and different number of present features. We observe that computation time increases linearly as the number of present features increases. This scaling result is different to the PF based implementation of the PMB filter in [21] with a known vehicle state. There, the authors reported an exponential increase of computation time. In Fig. 12, the average computation time per time step \( t \) is plotted for a simulation with five features and different number of vehicles. Here, computation time increases linearly as the number of vehicles increases. Furthermore, we investigated the average computation time per time step \( t \) for different values of the GNSS measurement variance \( \sigma_G^2 \) and of the V2F measurement variance \( \sigma_{V2F}^2 \). In the simulation scenario with five features and two vehicles, the average computation time remained constant around \( 2.4 \times 10^{-2} \text{s} \) for \( 10^{-3} \text{m}^2 \leq \sigma_G^2 \leq 10 \text{m}^2 \). For a V2F measurement variance \( 10^{-3} \text{m}^2 \leq \sigma_{V2F}^2 \leq 10 \text{m}^2 \), the average computation time linearly increased from \( 2 \times 10^{-2} \text{s} \) to \( 3 \times 10^{-2} \text{s} \) as \( \sigma_{V2F}^2 \) increased.

VI. CONCLUSIONS

This paper presented a Poisson multi-Bernoulli filter for multisensor-multitarget tracking with uncertain sensor states. Two different kind of measurements, observations of the sensor state and observations of the features, were used to obtain accurate feature and sensor state tracking. The proposed parametric filter implementation scales linearly with the number of features or sensors. Information from multiple sensors can be incorporated either in a time-synchronized manner where sensor measurements are aggregated in a super-sensor state, or in a non time-synchronized manner through asynchronous update steps, executed whenever sensor measurements arrive at the central node. Simulation results showed for a unisensor-multitarget tracking scenario with known sensor state that tracking performance assessed by the OSPA distance metric is equivalent to William’s PMB filter.

In a scenario with present vehicle state uncertainty, the proposed filter showed superior feature tracking performance over the conventional PMB filter due to the modeling of this type of additional uncertainty. In a multisensor-multitarget tracking scenario, feature information from a well localized vehicle (sensor) allows to significantly reduce the vehicle state uncertainty of (previously) poor localized vehicles. This improvement is possible through joint observation of a subset of the present features and is supported by simulation results.

APPENDIX

A. Vehicle State Posterior

The vehicle state posterior of (29) and (36) is proportional to the vehicle’s prior PDF \( p_+(x) \) times the measurement likelihood \( \ell^A(Z^F|x) \) as

\[
p^A(x|Z^F) \propto p_+(x) \ell^A(Z^F|x), \tag{61}
\]

where

\[
\ell^A(Z^F|x) \propto \prod_{C \in A: C \cap \emptyset \neq \emptyset} \int \ell(z^{MC}|x,f) \rho_C^A(f) df \\
\times \prod_{C \in A: C \cap \emptyset = \emptyset} \int \ell(z^{MC}|x,f) D^A_+(f) df \tag{62}
\]

\[
\propto \prod_{C \in A: C \cap \emptyset \neq \emptyset} \ell^C(z^{MC}|x) \prod_{C \in A: C \cap \emptyset = \emptyset} \ell^U(z^{MC}|x) \tag{63}
\]

with

\[
\ell^C(z|x) = \int \ell(z|x,f) p^A_C(f) df, \tag{64}
\]

\[
\ell^U(z|x) = \int \ell(z|x,f) D^A_+(f) df. \tag{65}
\]

Here, (64) and (65) map the feature uncertainty on the V2F measurement likelihood.

B. Updated Undetected and Detected Feature Density

The updated joint vehicle-feature density (29) has undetected feature density

\[
f^u(F^u|x) \propto \prod_{f \in F^u} (1 - p_0(x,f)) D^A_+(f) \tag{66}
\]
the first product considers the cases where no measurement is associated to any of the existing features, the second product considers the cases where a measurement is associated to an existing feature, and the last line considers the case where a measurement is associated to the background (clutter or undetected feature).

**C. Updated Undetected and Detected Feature Density Approximation**

The joint vehicle-feature density approximation (35) has undetected feature density

\[
\tilde{f}^u(F^u) \propto \prod_{f \in F^u} (1 - \tilde{p}_D^u) D_+(f) \tag{68}
\]

and detected feature density

\[
\tilde{f}^d,\lambda(F^d | x, Z^F) \propto \sum_{\mathcal{C} \subseteq A} \prod_{\mathcal{C} \in A} (1 - \tilde{p}_D^c \Delta|F_C|) f_{+}^{i_c} (F_C) \times \prod_{\mathcal{C} \in A} \eta(\{z^{mC}\} | x, F_C) f_{+}^{u} (F_C) \times \prod_{\mathcal{C} \in A} \eta(\{z^{mc}\} | x, F_C) f_{+}^{u} (F_C), \tag{69}
\]

where \(\Delta|F_C| = 1\) for \(F_C \neq \emptyset\), and zero otherwise. Here, the three products consider similar cases to (67).

**D. Approximated Updated Feature Set Density**

The approximated updated feature set density of (31) and (36) is a MB density,

\[
\tilde{f}^{d,\lambda}(F^d | Z^F) = \sum_{\mathcal{C} \subseteq A} \prod_{\mathcal{C} \in A} \tilde{f}^{i_c}(F_C | Z^F) \tag{70}
\]

with

\[
\tilde{f}^{i_c}(F_C | Z^F) = \int \left[ \int \tilde{f}^{d,\lambda}(F^d | x, Z^F) p_+ (x) \prod_{C' \in A : C' \neq C} \delta F_{C'} \right] dx, \tag{71}
\]

where we average over the predicted vehicle state and marginalize over all subsets \(F_C\) not equal to \(F^C\). Eqn. (70) has Bernoulli parameters [15] Eqns. (45) to (57)

\[
\tilde{p}^{i_c} [Z^F] = \begin{cases} (1-\tilde{p}_D^{i_c})^i_C + \tilde{p}_D^{i_c} \lambda_c (z^{mC}, D_+) & \text{if } C \cap I \neq \emptyset, C \cap M = \emptyset \\ \tilde{p}_D^{i_c} (z^{mC}, D_+) & \text{if } C \cap I \neq 0, C \cap M \neq 0, \\ \lambda_c (z^{mC}, D_+) & \text{if } C \cap I = 0, C \cap M \neq 0, \\ \end{cases} \tag{72}
\]

\[
\tilde{p}^{i_c} (f | Z^F) = \begin{cases} \tilde{p}_D^{i_c} (f) & \text{if } C \cap I \neq \emptyset, C \cap M = \emptyset \\ \tilde{p}_D^{i_c} (f) & \text{if } C \cap I \neq 0, C \cap M \neq 0, \\ \lambda_c (z^{mC}, D_+) & \text{if } C \cap I = 0, C \cap M \neq 0, \\ \end{cases} \tag{73}
\]

where

\[
\ell_z (f) = \ell (z | f) = \int \ell (z | x, f) p_+ (x) dx. \tag{74}
\]

**E. Approximated Updated Feature Density Weights**

The weight of a global association hypothesis \(A \in \mathcal{A}\) stated in (36) is [15] Eqn. (67)

\[
w_A \propto \prod_{\mathcal{C} \subseteq A} (1 - \tilde{p}_D^{i_c}) \prod_{\mathcal{C} \in A} \tilde{p}^{i_c} (f | Z^F) \times \prod_{\mathcal{C} \subseteq A} \lambda_c (z^{mC}, D_+) \tag{75}
\]

**F. Proof of Measurement-Feature State Likelihood**

Here, we prove (52). With (11) and \(p_+ (x) = \mathcal{N}(\mu_{x+}, \Sigma_{x+})\) known, define

\[
y \triangleq z - H_1 x - q, \tag{76}
\]

and consequently

\[
p(y) = \mathcal{N}(z - H_1 \mu_{x+}, H_1 \Sigma_{x+} H_1^T). \tag{77}
\]

Now, we have \(y - H_1 f = 0\) and solve for \(f\) with the help of [40] Rule 4 in Table 3, which results in eqs. (52) to (54).

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