Tests of the Atomki anomaly in lepton pair decays of heavy mesons

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The anomalies recently reported in lepton pair transitions of 8Be* and 4He nuclei may be attributed to the existence of a feeble interacting light vector boson X17. We study the effects of this hypothetical particle in the semileptonic H* → He+e− decays (H a Qq meson) in the framework of the HQET+VDM model. Using current bounds on the X17 boson to quarks, we find that decays of D*+ and D*+ mesons can be enhanced with up to O(15%) corrections relative to the photon-mediated contributions. Dedicated experimental searches at current heavy meson factories may confirm the existence of this light boson or set stronger bounds of their couplings to ordinary matter.

1. Introduction

The existence of a light vector boson weakly coupled to Standard Model (SM) fermions, has been suggested as a solution to the observed discrepancy between the SM prediction and the experimental measurement of the muon g − 2 magnetic moment anomaly (see for example [1, 2]). It may be also a good candidate as a mediator of dark and ordinary matter interactions [1, 2]. Several strategies aiming their detection in different collider and fixed target experiments have not found any signal so far [3, 4], but have excluded different regions in the mass and coupling strenghts of parameter space. Theoretically, different models can accommodate a light vector boson and its required interactions through dimension-four kinetic mixing with SM neutral gauge bosons and their interactions with fermionic currents of SM or dark matter particles [1–3].

The anomalies recently reported in the invariant-mass spectrum and angular distribution of lepton pairs produced in 8Be* transitions to its ground state [5], re-inforces the interest in searches of light vector bosons. The observed anomalies seems to require the existence of a spin-1 boson named X17 [5–7] with mass mX = (16.7 ± 0.35 ± 0.50) MeV and a relative ratio B(8Be* → 8 BeX)/B(8Be* → 8 Beγ) = 5.8 × 10−6 [7]. Couplings to standard model first-generation fermions of O(10−3) (in units of the the electron charge), required to explain this ratio is not discarded by other data. More recently, the same group seems to confirm the X17 particle in studies of the 0− → 0+ transitions of 4He [8].

The almost isosinglet nature and the small mass difference of nucliei involved in 8Be* decay provides an ideal place to observe this light boson, in case it exists. Mixing of nuclear isospin states [7–9] and other nuclear interference effects [10] can only partially explain the observed anomaly. Further studies in analogous systems will be very important in order to establish or discard this light boson. In the present letter, we propose the study of H* → He+e− decays, where H(H*) is a heavy Qq spin-0 (spin-1) meson. Previous related studies include: 1) J/ψ → η, X decays and associated production of H/ψ mesons at BESIII and Belle II experiments, recently reported in [11] and, 2) a search proposal at LHCb of D*0 → D0A′ → D0e+e− with displaced vertex or resonant production of the dark photon A′ was detailed in Ref. [12]. H* → He+e− decays seem to be interesting to further test the Atomki anomaly: on the one hand, the mass-splitting in heavy mesons is large enough (see Table I) to produce the X17 boson on-shell; on the other hand, strong decays of H* are either very suppressed of forbidden by kinematics, leaving electromagnetic decays as dominant. Furthermore, the large amount of data produced at heavy meson factories would allow to test the proposed channels in the near future.

The Lagrangian describing the interaction of quark and lepton flavors f with the photon and the X boson is Lγ,Xff = −eεfεp(εfAp + εpXp)γµνpµpνf, with couplings strenghts εp and εf given in units of the electron charge e. The photon and X bosons couplings to hadrons are described each by a single vector form factor which takes into account their structure in the momentum transfer region 4mε2 ≤ q2 ≤ (mH* − mH)2, with q = pe+ + pe−. The form factors describing the couplings of the off-shell vector particles (V = γ, X) in H*(pH*, εH*) → H(pH) V(q) are defined from the hadronic amplitude

\[ M_μ = iεF_{H*HV}(q^2)e^{μναβ}ε^νε^αε^β. \]  

For on-shell vector particles V, this Lorentz-vector amplitude must be contracted with its vector polarization εV(q). The case of lepton pair production is discussed below.

2. H*H-Vector vertices. The form factors F_{H*HV}(q) are evaluated in the framework of the heavy quark effective theory suplemented with vector meson dominance model (HQET+VDM) [13, 14], which has shown to give a good description of H* → Hγ decays. Since we will normalize results for our observables to this
radiative decay, we use the ratio of decay rates because they are rather insensitive to the specific $q^2$-dependency of the form factor. This is due to the smallness of the $H^* - H$ mass splitting (see Table I) compared to typical hadronic scales (∼1 GeV$^2$).

The vector $H^*$ and pseudoscalar $H$ heavy mesons are composed of a $Q\bar{q}$ pair, with $Q = b, c$ and $q = u, d, s$. The hadronic matrix element of the electromagnetic current is given by [13]:

$$ \langle H^*(P_{H^*}, \epsilon_{H^*}) | J^{[\mu]}_\mu | H^*(P_H, \epsilon_H) \rangle = e \langle H(P_H) | e_Q \bar{Q} \gamma_\mu Q + e_q \bar{q} \gamma_\mu q | H^*(P_{H^*}, \epsilon_{H^*}) \rangle$$

(2)

with $e_Q(e_q)$ the electric charge of the heavy quark (light antiquark), and similarly, $$ \langle H(P_H) | J^{[\mu]}_\mu | H^*(P_{H^*}, \epsilon_{H^*}) \rangle = e \langle e_Q J^Q_{\mu} + e_q J^q_{\mu} \rangle$$ for the $X$ boson current.

A straightforward evaluation of the form factors in the HQET+VMD model [13] leads to

$$ F_{H^*H\gamma}(q^2) = \sqrt{\frac{m_{H^*}}{m_H}} \left[ \frac{e_Q}{m_{H^*}} + \frac{e_q}{m_q(q^2)} \right] $$

(3)

$$ F_{H^*HX}(q^2) = \sqrt{\frac{m_{H^*}}{m_H}} \left[ \frac{\xi}{m_{H^*}} + \frac{\varepsilon}{m_q(q^2)} \right] $$

(4)

with the effective light “quark mass” parameter

$$ m_q(q^2)^{-1} = -\sum_V \left( 2\sqrt{2} g_V \lambda \frac{f_V}{m_V} \right) \left( 1 - \frac{q^2}{m_V^2} \right)^{-1} $$

(5)

The sum extends over light vector-meson resonances ($V = \rho^0, \omega, \phi$) according to the light-quark content of heavy mesons [13]. For $u$ and $d$ quarks the sum extends over the $\rho$ and $\omega$ mesons, and for the quark $s$ only contributes the $\phi$ meson (we assume ideal mixing of vector mesons). Numerical inputs for couplings constants can be found in Ref [13] and are reproduced here for reference: $g_V = 5.8, \lambda = -0.289 \pm 0.016$ GeV$^{-1}$ (updated from new experimental inputs [15]) and $f_V (m_V)$ the decay constant (mass) of vector meson $V$. Using current experimental data for lepton-pair decays of vector mesons $V \to e^-e^+$ [15], one gets $(f_{\mu}, f_\omega, f_\phi) = (0.171, 0.155, 0.232)$ GeV$^2$, with very small uncertainties.

In Table I we list values for the electromagnetic form factor predicted in the HQET+VMD model at $q^2 = 0$. The quoted uncertainty is dominated by the input on $\lambda$ coupling, the $H^*HV$ strong coupling in this model. A comparison with the magnitude of the measured form factor (within square brackets), obtained from the measurement of the radiative decay $D^{*+} \to D^{\pm} \gamma$ branching fraction [15], give confidence on this model.

Let us define the following ratio of two-body decay rates:

$$ R_{X/\gamma}(H^*) = \frac{\Gamma(H^* \to HX)}{\Gamma(H^* \to H\gamma)} = \frac{F_{H^*HX}(m_X^2)}{F_{H^*H\gamma}(0)} \frac{[\vec{p}_X]^3}{[\vec{p}_\gamma]^3} $$

(6)

where $\vec{p}_X$ is the momentum of the final state boson in the rest frame of $H^*$. This ratio exhibits two important differences with respect to the similar ratio defined in $^8\text{Be}^* \to ^8\text{Be}$ nuclear transitions [6]. First, since $m_{H^*} \gg m_q(q^2)$ one would expect a suppression of the heavy quark contribution relative to the light quarks; this makes $R_{X/\gamma}(H^*)$ more sensitive to the $Xq\bar{q}$ couplings, which are relatively well bounded from other processes [6]. On the other hand, given the larger phase-space in heavy meson decays, this ratio is not suppressed by kinematics, as it happens for decay of $^8\text{Be}$ nucleus.

Predictions for the $H^* \to HX$ decay fractions require an estimate of the $e_{Q,q}$ couplings. Bounds for the couplings of the $X17$ boson to the quarks of the first generation needed to explain the $^8\text{Be}$ anomaly were obtained in Refs. [6, 7]: $\varepsilon_u \simeq 3.7 \times 10^{-3}$, $\varepsilon_d \simeq 7.4 \times 10^{-3}$ (‘proto-phobic’ assumption, see however [16]), and $0.2 \times 10^{-3} \lesssim |\varepsilon_c| \lesssim 1.4 \times 10^{-3}$. Our study requires the knowledge of second- and third-generation couplings, namely strange $\varepsilon_s$, charm $\varepsilon_c$, and bottom $\varepsilon_b$. A priori these parameters are independent [7], and need not be related to the first-generation couplings. Our simplest starting assumption is universality of down- and up-type quark $\varepsilon_f$ couplings, thus, we will take $\varepsilon_c = \varepsilon_u$ and $\varepsilon_b = \varepsilon_s = \varepsilon_d$; henceforth, our results will be obtained under this assumption [6, 7]. Values of the $H^*HX$ couplings and $R_{X/\gamma}(H^*)$ ratios for these transitions are given in Table II. The ratios are larger than the ones in the nuclear case mainly due to the unsuppressed phase space for $X17$ production.

| Transition                  | $m_{H^*} - m_{H}\text{(MeV)}$ | $F_{H^*H\gamma}(0)\text{ (GeV}^{-1})$ |
|-----------------------------|--------------------------------|--------------------------------------|
| $D^{*+} \to D^+ \gamma$    | 140.603(15)                   | -0.54 ± 0.05 [−0.47 ± 0.06 [15]]      |
| $D^{*0} \to D^0 \gamma$    | 142.014(30)                   | 2.11 ± 0.10 [< 10.8 [15]]             |
| $D^*+ \to D^+ \gamma$      | 143.8(4)                      | -0.04 ± 0.01 [−16.4 [15]]            |
| $B^{*+} \to B^+ \gamma$    | 45.37(21)                     | 1.64 ± 0.09                           |
| $B^{*0} \to B^0 \gamma$    | 45.37(21)                     | -0.92 ± 0.12                           |
| $B^*\to B^0 \gamma$        | 48.6([1, 2], 3)               | -0.91 ± 0.05                           |

TABLE I. Mass splittings of heavy mesons and electromagnetic couplings of $H^* \to H\gamma$ transitions in the HQET+VMD model. Within square brackets we show experimental values when available.
3. Lepton pair production. The decay amplitude for lepton pair production $H^+ (P_{H^+}) \rightarrow H (P_H)e^+ (p_+)^- (p_-)$ is the coherent sum of the photon and $X$-boson mediated amplitudes $\mathcal{M} (H^+ \rightarrow He^+ e^-) = \mathcal{M}_\gamma + \mathcal{M}_X$, where $(V = \gamma, X)$:

$$\mathcal{M}_V = -e^2 G_{H^+HV} (q^2) \epsilon_{\mu\nu\alpha\delta} e^\nu e^\mu V^\alpha P_{H^+}^\nu P_{H^+}^\mu, \quad (7)$$

where $\epsilon_{\mu} = \bar{u} (p_-) \gamma_{\mu} v (p_+)$ is the leptonic current and $G_{H^+HV} (q^2) = -F_{H^+HV} (q^2) / q^2$, $G_{H^+XV} (q^2) = \varepsilon_{\mu} F_{H^+HX} (q^2) / (q^2 - m_X^2 + i m_X \Gamma_X)$. As in Ref. [6, 7], we assume negligible decays of the $X17$ boson into neutrino channels, such that its full width is given by $\Gamma_X = \Gamma (X \rightarrow e^+ e^-) = (\alpha_{em} e^2 / m_X^3) (1 + 2 r_e) \sqrt{\theta e} = 8.0 \times 10^{-8}$ MeV ($r_e = m_e^2 / m_X^2$), corresponding to the maximum value for $\varepsilon_e$. Given this very narrow width of $X17$, the lepton pair invariant mass distribution, normalized to the radiative decay width, becomes the sum of the non-interfering photon and $X$-boson mediated distributions, namely $(\alpha_{em})$ is the fine structure constant, and $\lambda (x, y, z) = x^2 + y^2 + z^2 - 2 xy - 2 xz - 2 yz$:

$$\frac{d\Gamma (H^+ \rightarrow He^+ e^-)}{dq^2} = \frac{\alpha_{em}^2}{72 \pi \Gamma (H^+ \rightarrow H\gamma)} \left[ |G_{H^+H\gamma} (q^2)|^2 + |G_{H^+HX} (q^2)|^2 \right] q^2 \left[ \frac{\lambda (m_{H^+}^2, m_X^2, q^2)}{m_{H^+}} \right]^{1/2} \left[ \frac{\lambda (m_{H^+}^2, m_X^2, q^2)}{m_{H^+}} \right]^{1/2}. \quad (8)$$

The lepton-pair invariant mass distributions due to photon (solid-red) and $X17$-boson (dashed-blue) exchange are shown separately in Figure 1 for the six different decay channels under consideration. The shaded bands around each curve represents the theoretical error. The peak due to the production of the $X17$ boson in each channel is not located very close to the end of the lepton-pair spectrum as it happens in the nuclear case, avoiding in this way possible end-point kinematical effects. In contrast to the on-shell $X17$ production, the effect of this boson is the largest for the $D^{*+} (D^{*+}) \rightarrow D^+ (D^{*+}) e^+ e^-$ decay. The corresponding peaks of this boson contribution is suppressed by one or two orders of magnitude in all other cases, relative to the photon contribution. Note that we are assuming universality bounds on heavier quark $\varepsilon_{c, b, b}$ couplings; since this is a conservative assumption, the experimental study of heavy mesons transitions involving lepton pairs may serve to set bounds on these unknown couplings of the hypothetical $X17$ boson.

Table III displays the decay rates of the lepton-pair production in $H^+ \rightarrow H$ transitions for different heavy mesons. These rates are normalized to the radiative width $\Gamma (H^+ \rightarrow H\gamma) = (\alpha_{em} / 3) |F_{H^+H\gamma} (0)|^2 |p_e|^3$ in order to cancel remaining model-dependent terms in the form factors (all other lepton-pair and angular distributions in the following are normalized to this radiative width). The largest contribution of the $X17$ boson is observed for the $D^{*+}$ and $D^{*+}$, making these channels the most sensitive for the observation of this light boson effects. Our prediction for the $D_s$ channel normalized to the radiative decay is $\mathcal{B} (D_s \rightarrow D_s e^+ e^-) = (7.8 \pm 0.6) \times 10^{-3}$, in good agreement with the corresponding ratio measured by the CLEO collaboration $(7.2^{+1.0}_{-0.5}) \times 10^{-3}$ [18]. A preliminary estimate of this ratio $\mathcal{B} (D_s \rightarrow D_s e^+ e^-) = 6.5 \times 10^{-3}$ was estimated in Ref. [18] based on the model proposed in [19] which includes only the electromagnetic contribution.

Figure 2 displays the constraints (light-red shaded band) on the $(\varepsilon_e, \varepsilon_s)$ plane obtained from the experimental branching fraction of Ref. [18] and the result of integrating Eq. (8) for $D_s \rightarrow D_s e^+ e^-$. For comparison, we also show the two black lines corresponding to the allowed values obtained from $^8$Be* results [6, 7] and the region allowed from the so-called ‘protophobic condition’ from NA48/2 experiment [7, 20] (blue-band). The different sensitivities observed from these measurements to the up- and down-type quark couplings makes worth an improved measurement of the heavy mesons decays discussed in this letter.

Finally, let us comment that the angular distribution of the $e^+ e^-$ pair, in the rest frame of the decaying particle, will be peaked closer to the collinear configuration compared to the nuclear case of $^8$Be* transitions, where $\theta (e^+ e^-) \sim 140^0$. This happens because the $X17$ boson is produced with a larger velocity, while in nuclear transitions this boson is produced almost at rest.

4. Conclusions. The hypothetical light vector boson $X17$, proposed as a solution for the anomaly observed in lepton-pair production of $^8$Be* and $^4$He transitions, can be studied in the clean environment provided by vector to pseudoscalar heavy mesons transitions in Belle, Belle II and BESIII factories. These $H^+ (Q\bar{q}) \rightarrow H (Q\bar{q}) e^+ e^-$ decays are free from theoretical uncertainties associated to nuclear effects. We have used the HQET+VMD framework to model the hadronic form factors of $1^- \rightarrow 0^-$ meson transitions, however our results are little-dependent on the model’s uncertainties because the rates are normalized to the dominant $H^+ \rightarrow H\gamma$ electromagnetic decays. Although all branching fractions of the considered heavy meson channels exhibit some sensitivity to the effects of the $X17$ boson, decays of $D^{*+}$
and $D_s^{∗+}$ mesons turn out to be the most sensitive ones. Improved measurements of these leptonic decay channels can set additional powerful constraints on the $X_{17}$ boson couplings to ordinary fermions or, eventually, confirm the existence of this light boson.

**FIG. 1.** Lepton pair invariant mass distributions of $H^* \rightarrow H e^+ e^-$ transitions normalized to the radiative $H^* \rightarrow H \gamma$ decay width: (a) $D^{∗+} \rightarrow D^+ e^+ e^−$, (b) $D^{∗0} \rightarrow D^0 e^+ e^−$, (c) $D_s^{∗+} \rightarrow D_s^+ e^+ e^−$, (d) $B^{∗+} \rightarrow B^+ e^+ e^−$, (e) $B^{∗0} \rightarrow B^0 e^+ e^−$ and (f) $B_s^{∗0} \rightarrow B_s^0 e^+ e^−$. The red-solid plot denotes the virtual photon contribution, while the $X_{17}$ boson contribution is represented by the blue-dashed curve. The shaded bands account for the theoretical uncertainties in form factors.

| Transition       | Photon     | $X_{17}$ boson | Total               |
|------------------|------------|----------------|---------------------|
| $D^{∗+} \rightarrow D^+ e^+ e^−$ | $(6.7 \pm 0.3) \times 10^{-3}$ | $(1.1 \pm 0.1) \times 10^{-3}$ | $(7.7 \pm 0.3) \times 10^{-3}$ |
| $D^{∗0} \rightarrow D^0 e^+ e^−$ | $(6.7 \pm 0.3) \times 10^{-3}$ | $(3.0 \pm 0.1) \times 10^{-5}$ | $(6.7 \pm 0.3) \times 10^{-3}$ |
| $D_s^{∗+} \rightarrow D_s^+ e^+ e^−$ | $(6.8 \pm 0.6) \times 10^{-3}$ | $(1.0 \pm 0.1) \times 10^{-3}$ | $(7.8 \pm 0.6) \times 10^{-3}$ |
| $B^{∗+} \rightarrow B^+ e^+ e^−$ | $(4.9 \pm 0.3) \times 10^{-3}$ | $(1.9 \pm 0.1) \times 10^{-5}$ | $(4.9 \pm 0.3) \times 10^{-3}$ |
| $B^{∗0} \rightarrow B^0 e^+ e^−$ | $(4.9 \pm 0.2) \times 10^{-3}$ | $(4.0 \pm 0.2) \times 10^{-4}$ | $(5.3 \pm 0.2) \times 10^{-3}$ |
| $B_s^{∗0} \rightarrow B_s^0 e^+ e^−$ | $(5.0 \pm 0.3) \times 10^{-3}$ | $(4.1 \pm 0.2) \times 10^{-4}$ | $(5.4 \pm 0.3) \times 10^{-3}$ |

**TABLE III.** Photon and $X_{17}$ boson exchange contributions to the branching ratio of $H^* \rightarrow H e^+ e^-$, normalized to the $H^* \rightarrow H \gamma$ decay width.
FIG. 2. Allowed regions in the parameter space of up-type ($\varepsilon_U$) and down-type ($\varepsilon_D$) X-quark couplings from $D_s^+ \rightarrow D_s^0 e^+e^-$ (red band), $^8\text{Be}^*$ [5–7] (black lines), and NA48/2 [7, 20] (blue band) experimental results.

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