On the Calibration of the Temperatures and Colors of Stellar Models for Lower Mass Stars

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Abstract. The temperatures and colors of stellar models are much less secure than predicted luminosities because they depend sensitively on the still very uncertain physics of stellar envelopes and atmospheres. Consequently, it is important to ensure that the models which are used to interpret stellar populations data satisfy existing observational constraints on these properties. As shown in this study, the available $T_{\text{eff}}$ constraints do not pose a problem for evolutionary calculations in which the free parameter ($\alpha_{\text{MLT}}$) in the Mixing-Length Theory of convection (which continues to be widely used), is set to the value required by a Standard Solar Model. That is, it is still not possible to say whether some dependence of $\alpha_{\text{MLT}}$ on mass, metallicity, or evolutionary state should be invoked. On the other hand, the evidence seems compelling that adjustments of the color transformations from model atmospheres are needed to achieve consistency with observations of cool stars. Stellar models that have been normalized to the Sun and transformed to the observed plane using recent empirical color–$T_{\text{eff}}$ relations are able to provide superb fits to the M67 and Hyades color-magnitude diagrams (CMDs). Because the properties of metal-poor stars and star clusters (metallicities, distances, etc.) are more uncertain, it is much harder to constrain the $T_{\text{eff}}$ and color scales at low metallicities than at [Fe/H] $\approx 0.0$. These difficulties are highlighted in the following examination of the constraints provided by Population II subdwarfs and several well-observed globular clusters (M3, M5, M68, M92, and 47 Tuc). It is not clear that significant improvements to current semi-empirical color–$T_{\text{eff}}$ relations for metal-deficient stars will be possible until much tighter observational constraints become available. Brief comparisons of the evolutionary tracks computed by different workers are also included in this investigation.

1. Introduction

Our understanding of stellar populations is only as good as the models that are used to interpret the data and the accuracy to which such basic properties as distance and metallicity are known (assuming that the observations are reliable). Due to the many factors at play, and the many degeneracies between them, it is virtually impossible to find a unique solution to the values of the various quantities of interest simply by incompare synergistic and observed color-magnitude diagrams (CMDs). The lack of a rigorous theory for super-adiabatic convection, in particular, makes it very risky to place too much reliance on predicted effective temperatures, in both an absolute and relative sense (especially at metallicities quite different from solar). This is why there has long been a general consensus that...
ages are best derived from computed turnoff (TO) luminosity versus age relations (because TO luminosities are predicted to be nearly independent of convection theory; e.g., see VandenBerg 1983). Moreover, it has become clear that there must be some process(es) (e.g., turbulence) at work in the envelopes of lower mass, metal-poor stars as, otherwise, it is not possible to explain either the variation of Li with $T_{\text{eff}}$ in field halo dwarfs (see Richard et al. 2002) or the lack of any detectable difference in the chemical compositions of TO and red-giant-branch (RGB) stars in such globular clusters (GCs) as NGC 6397 and 47 Tucanae (see Gratton et al. 2001). The extent to which diffusive processes are inhibited has ramifications for the predicted difference in $T_{\text{eff}}$ between the TO and lower RGB (VandenBerg et al. 2002) that are quite similar to those caused by variations in age, [Fe/H], and [O/Fe] (e.g., VandenBerg & Bell 2001). This makes the interpretation of the length (and slope) of the subgiant branch very complicated indeed.

Further uncertainties, on the theoretical side, can be attributed to the low-temperature opacities, which can alter both the location and the slope of the RGB, and to the treatment of the atmospheric layers. It is conventional to adopt either a gray or scaled-solar $T$–$\tau$ relation to describe the atmospheric structure and to integrate the equation of hydrostatic equilibrium in conjunction with this relation to derive the pressure boundary condition that is used to construct stellar models. Such a procedure is bound to lead to systematic errors in the predicted effective temperatures because a fixed $T$–$\tau$ relation is certainly inappropriate for stars of all masses, metallicities, and evolutionary states. A better approach would be to use proper model atmospheres to provide the outer boundary conditions for stellar models, as well as the color–$T_{\text{eff}}$ relations that are used to transpose the models from the theoretical to the observed plane. However, this opens the door to the many uncertainties inherent in model atmosphere computations and thereby to further sources of systematic error. It is clear from the plots provided by VandenBerg & Clem (2003; hereafter VC03) that the recent color transformations based, in part, on model atmospheres by Lejeune, Cuisinier, & Buser (1998); Castelli (1999); and Houdashelt, Bell, & Sweigart (2000) differ considerably from each other, but it is not obvious whose calculations are the most realistic ones.

Added to this depressing situation is the fact that the photometric data themselves may have both zero-point and systematic errors; and certainly the reddenings, distances, and chemical compositions of even the simplest stellar populations (like GCs) are not as well constrained as they need to be. For instance, as shown by De Santis & Cassisi (1999), current predictions for the luminosities of horizontal-branch (HB) stars, which are one of the favored “standard candles” for distance determinations, do not agree at all well. Moreover, it is not uncommon to find 0.2–0.4 dex variations in the [Fe/H] (and [O/Fe]) values derived by different workers. In the case of NGC 4590 (M 68), for example, recent metallicity estimates range from [Fe/H] = −1.99 (Carretta & Gratton 1997) to −2.43 (Kraft & Ivans 2003). Such differences can have huge implications for the comparisons of isochrones with photometric data.

From time to time, suggestions are made that completely objective techniques should be used to deduce the properties of stellar populations. Perhaps the first such suggestion was made by Flannery & Johnson (1982), who devised
a statistical “goodness of fit” criterion to determine the values of $m - M$, age, [Fe/H], $Y$, and $\alpha_{\text{MLT}}$ (where the latter is the usual mixing-length parameter) from fits of isochrones to the main sequence (MS), TO, and subgiant (SGB) portions of observed CMDs. Not surprisingly, given the many potential problems described above, they concluded that, “in practice, the results are more sensitive to unknown calibration effects, and to uncertainties in the theoretical modelling of convection and stellar atmospheres, than to formal errors in the fits”. One should not expect meaningful results to be obtained if the sizes and shapes of isochrones play the dominant role in determining which isochrone provides the best match to a given CMD, whether or not a least-squares fitting method is used.

Similar ideas have been proposed recently by Wilson & Hurley (2003), but with the important difference that the information which is subjected to a least-squares analysis is the variation in the numbers of stars along the principal sequences of an observed CMD. This variation reflects the changes in the evolutionary timescales in different parts of the H–R diagram, which is widely considered to be a more robust prediction of stellar models (with good reason) than, in particular, surface temperatures. However, we note that theoretical luminosity functions (LFs) appear to have such a weak sensitivity to [Fe/H], [$\alpha$/Fe], and age (see VandenBerg, Larson, & De Propris 1998) that they cannot be used to determine these quantities. (As shown in the same study, LFs may provide a way of determining the helium abundance in a star cluster or of detecting whether member stars have been able to retain significant rotational angular momenta throughout their evolution.)

There was some hope that the color distributions of subgiant-branch stars could be used to determine distance-independent absolute ages for globular clusters (see Bergbusch & VandenBerg 1997), but a follow-up study by P. A. Bergbusch (in preparation) has found that, here too, the sensitivity of so-called “color functions” (CFs) to quantities of interest is likely too low to make this approach competitive with other methods. For instance, preliminary indications are that GC ages derived from CFs have $1\sigma$ uncertainties of at least $\sim \pm 3$ Gyr. As a result of such work, it seems doubtful that any major breakthroughs will be forthcoming from efforts (like that by Wilson & Hurley 2003) which are based on predictions of the numbers of stars along isochrones.

The simplest stellar populations (open clusters, GCs) provide the best tests of stellar models (along with well observed binary stars, and field stars with accurately determined distances and chemical compositions), and they will continue to do so as our understanding of the physics of stars and the basic properties of such stellar systems improves over time. Indeed, it is largely as a result of work on star clusters that the deficiencies of current stellar models are revealed (e.g., the need for convective core overshooting in models of intermediate-mass stars; see the review by Chiosi 1999). They also provide the means to “calibrate” stellar models so that the latter predict, for instance, the right MS and RGB slopes. It makes no sense to try to interpret the observations for complex stellar populations, such as the Magellanic Clouds and dwarf spheroidal galaxies, using stellar models that are incapable of reproducing the CMDs of simple stellar populations to within some reasonable tolerance.
It is the purpose of the present paper to review recent work that has been carried out at the University of Victoria to try to produce a set of models (VandenBerg, Bergbusch, & Dowler 2005) having well constrained temperatures and colors (see VC03), in the hope that they will offer new and/or improved insights into the stellar content of complex stellar populations. Hopefully this work will provide an instructive example of the degree to which predicted temperatures can be trusted and of how observations may be used to calibrate the colors of stellar models for lower mass stars (in particular) over a wide range in metal abundance.

2. The Calibration of Stellar Models for [Fe/H] ≈ 0.0

The Hyades has long been a pillar of stellar astronomy and of efforts to establish the extragalactic distance scale because its distance can be determined by direct geometric means. As a result of Hipparcos secular and trigonometric parallaxes, its color–$M_V$ diagrams have been established to unprecedented precision over $\sim 6$ magnitudes of its main sequence (see de Bruijne, Hoogerwerf, & de Zeeuw 2001; VC03). Moreover, both its metallicity ([Fe/H] ≈ +0.13) and helium abundance (just slightly subsolar) are known to high accuracy; the former from high-resolution spectroscopy (e.g., Boesgaard & Friel 1990) and the latter from analyses of its binary stars (Lebreton, Fernandes, & Lejeune 2001; VC03). A further advantage of the Hyades over many other systems is that it suffers from negligible foreground reddening. Because so many of its properties involve little uncertainty, the Hyades obviously provides a fundamental test of stellar models. Its main deficiency in this regard is that it is too young to have an extended giant branch, and consequently, does not constrain models for low-gravity stars.

For this reason, and because Hyades stars are super-metal-rich, VC03 opted to rely on M67 to derive the color–$T_{\text{eff}}$ relations for stars having [Fe/H] ≈ 0.0, and to use the Hyades as a secondary check (see below) of the color transformations so obtained. M67 has a well-defined CMD (Montgomery, Marschall, & Janes 1993), its metallicity is close to [Fe/H] = −0.04 according to the results of high-resolution spectroscopy (Hobbs & Thorburn 1991; Tautvaisiene et al 2000), and there is general agreement that its reddening is within 0.01 mag of $E(B-V)$ = 0.04 (Nissen, Twarog, & Crawford 1987; Schlegel, Finkbeiner, & Davis 1998; Sarajedini et al 1999). The cluster helium content is not known, but presumably it is close to that of the Sun given the similarity in metal abundance. The calculation of a Standard Solar Model provides the default value of $Y$ for solar abundance stars (as well as the value of the mixing-length parameter, $\alpha_{\text{MLT}}$, that is assumed for models of stars in all other parts of the H-R diagram). This estimate of $Y_{\odot}$, together with an adopted helium enrichment law (e.g., $\Delta Y/\Delta Z = 2.2$; see VandenBerg et al. 2005) provide the means to determine the helium abundance that is to be assumed at other metallicities.

As the blanketing in the $B$ bandpass is much more severe than in $V$ and $I$, one expects that the $B-V$ colors predicted by model atmospheres will be less trustworthy than $V-I$. Indeed, it is comforting to find that the latter, when derived from MARCS (VandenBerg & Bell 1987; Bell & Gustafsson 1989) and Kurucz (Castelli 1999) model atmospheres, are in very good agreement for dwarf stars having [Fe/H] = 0.0 and temperatures above $\approx 5000$ K. This is illustrated
in Figure 1. Furthermore, when the color transformations (from either source) are applied to models that allow for convective core overshooting, it is found that a 4.0 Gyr isochrone for [Fe/H] = −0.04 (and Y = 0.2735) provides a very good match to the upper MS and TO of M 67 (see Figure 2, which is reproduced from VC03). In order for that isochrone to provide a good fit of the entire CMD, the model-atmosphere-based colors for lower MS and RGB stars were arbitrarily corrected, as necessary. (However, as discussed below, there is considerable justification for such adjustments.)

VC03 found that the fit of the same isochrone to BV observations of M 67 (by Montgomery et al. 1993) was much more problematic: the synthetic colors based on MARCS model atmospheres seemed to give results that were consistent with the fit to VI data only very near the turnoff, while those derived from Kurucz model atmospheres were too red. In order for the 4 Gyr isochrone to reproduce the observed [(B − V)⊙, M⊙]-diagram, VC03 chose to apply whatever corrections to the theoretical color transformations seemed to be necessary. However, it turned out that the resultant (B − V)−T eff relation for the MS stars in M 67 agrees very well with what is perhaps the best empirical relation which has been derived to date for dwarfs having close to the solar metallicity (that by Sekiguchi & Fukugita 2000). As shown in Figure 3, there are no significant differences between the two relationships over a large range in temperature (or color). Note that the values of (B − V)⊙ implied by the VC03 and Sekiguchi & Fukugita color transformations are, respectively, 0.637 (cf. VandenBerg & Poll 1989) and 0.626 ± 0.018.
Figure 2. Fit of a 4.0 Gyr isochrone for [Fe/H] = −0.04 to the $VI$ photometry of M 67 by Montgomery et al. (1993), on the assumption of the indicated reddening and distance modulus. The solid curve assumes the color transformations given by VC03, which are consistent with the predictions from MARCS model atmospheres (VandenBerg & Bell 1985; Bell & Gustafsson 1989) only at temperatures $\geq 5000$ K (see Fig. 1), whereas the dashed curve is obtained if Castelli (1999) color–$T_{\text{eff}}$ relations are used.

Although the superb fit of the 4 Gyr isochrone to the M 67 CMD obtained by VC03 (see their Fig. 4) was contrived, a virtually identical fit to the lower main sequence would be obtained using the Sekiguchi & Fukugita (2000) transformations. Thus, the adjustments that VC03 applied to the synthetic colors in order for the models to match the MS of M 67 are not at all ad hoc: they are needed, in fact, to satisfy empirical constraints. It then follows that, in order to reproduce the observed $B - V$ versus $V - I$ relation (i.e., as defined by the cluster dwarf stars), the corrections to the synthetic $V - I$ colors adopted by VC03 (essentially the difference between the solid and dashed curves in Fig. 2) are also necessary. Other considerations went into the determination of the color transformations for the faintest MS stars (see VC03), which resulted in
Figure 3. Comparison of the $B - V - T_{\text{eff}}$ relation derived by Sekiguchi & Fukugita (2000; solid curve) for $[\text{Fe/H}] = 0.0$ with that adopted by VC03. The filled circles represent computed ZAMS models for 0.6–1.2$M_\odot$ stars having the solar metallicity.

the isochrone that was fitted to the M67 CMD being too red at $M_V \geq 8$ (see Fig. 2). The correct explanation for this discrepancy is not clear.

As far as the transformations given by VC03 for giants are concerned, it is evident in Fig. 2 that the difference between the adopted $(V - I) - T_{\text{eff}}$ relations and those derived from Kurucz model atmospheres are not large (except near the RGB tip). Moreover, as shown by VC03 (see their Fig. 27), the predicted temperatures along the isochrone used to fit the M67 CMD agree very well with those derived from the empirical relations between $V - K$ and $T_{\text{eff}}$ by Bessell, Castelli, & Plez (1998) and van Belle et al. (1999). Since there are no obvious problems with the predicted temperatures, the adjustments to the synthetic $B - V$ and $V - I$ color indices adopted by VC03 in order for the models to match the colors of the cluster giants are readily justified. Strong support for this calibration of the colors for solar metallicity stars came from a follow-up study by Clem et al. (2004), who found that they were able to obtain a completely consistent fit of the same isochrone to $uvby$ data for M67 using their semi-empirical transformations to the Strömgren photometric system. The reason why this is such an encouraging finding is that stellar models played no role in the development of their color–$T_{\text{eff}}$ relations.

Because its properties are so well determined (as noted at the beginning of this section), the Hyades provides the best available constraint on the color transformations appropriate to super-metal-rich dwarf stars. VC03 set up color tables for $[\text{Fe/H}] = +0.3$ using the dependence of the various color indices on $[\text{Fe/H}]$ predicted by model atmospheres as a guide, and then refined them so that interpolations between these tables and those for $[\text{Fe/H}] = 0.0$ enabled a computed
isochrone for the Hyades metallicity to match the cluster CMD. The result of this exercise is shown in Figure 4. In this case, as well, there is independent support for the color–$T_{\text{eff}}$ relations so obtained. Lejeune, Cuisinier, & Buser (1998) had previously developed their own semi-empirical transformations, completely independently of theoretical stellar models. As shown by VC03, their results also agree very well with the Sekiguchi & Fukugita (2000) relation between $B - V$ and $T_{\text{eff}}$ for solar abundance stars — and if their color table for $[\text{Fe/H}] = +0.3$ is substituted for the one derived by VC03, and the isochrone appropriate to the Hyades is transposed to the observed plane using the revised color transformations, the result is the dashed curve in Fig. 4. The differences between it and the solid curve are clearly not very significant. The color–$T_{\text{eff}}$ relations for $[\text{Fe/H}] \geq 0.0$ by both VC03 and Lejeune et al. appear to be quite robust.

![Figure 4](image.png)

**Figure 4.** Fit of an isochrone for the indicated parameters onto the de Bruijne et al. (2001) CMD for the “high-fidelity” sample of single Hyades members. The VC03 color transformations were used for the solid curve. Interpolations between their table for $[\text{Fe/H}] = 0.0$ and that by Lejeune et al. (1998) for $[\text{Fe/H}] = +0.3$ were used to calculate the colors along the dashed curve (see the text).

### 2.1. Comparison of Solar Evolutionary Tracks

The calibration of the color transformations for stars having $[\text{Fe/H}] = 0.0$ has the distinct advantage that the temperature scale of stellar models can be constrained using the Sun. Ensuring that the models reproduce one point on the H-R diagram does not, however, imply that they will agree elsewhere. Differ-
ences in the assumed physics, boundary conditions, and the metal abundance mixture, as well as the treatment (or not) of diffusive processes, among other things, will all affect the values of $Y$ and $\alpha_{\text{MLT}}$ that are derived from a Standard Solar Model. This (and any differences in the assumed physics) will have implications for, e.g., the location of the giant branch of a computed track relative to its MS location. But just how large would one expect these differences to be?

The answer to this question is given in Figure 5, which compares the evolutionary tracks computed for the Sun by several researchers (as noted) using their own codes. (The goal here was not to compare the output of the different evolutionary codes when as close to the same input physics is assumed, but rather to illustrate the variations in the solar tracks that result from the net effect of differences in the codes, adopted metallicities, and input assumptions.) Some relevant information about these tracks is provided in Table 1, including, in particular, the age at the tip of the RGB. Needless to say, it is very comforting to find that the predicted ages agree so well.

| Code           | $Z$       | Diffusion? | $Y$   | $\alpha_{\text{MLT}}$ | Age  |
|---------------|-----------|------------|-------|------------------------|------|
| Cassisi       | 0.01981   | yes        | 0.2734| 1.89                   | 12.10|
| Girardi       | 0.01900   | no         | 0.2730| 1.68                   | 12.25|
| VandenBerg    | 0.01880   | no         | 0.2768| 1.90                   | 12.18|
| Weiss         | 0.01998   | yes        | 0.2754| 1.74                   | 12.07|
| Yi            | 0.01810   | yes        | 0.2670| 1.74                   | 12.03|

It is apparent in Fig. 5 that the largest differences occur near the tip of the RGB, where the tracks span a range of $\approx 0.02$ in $\log T_{\text{eff}}$ at a fixed $M_{\text{bol}}$. Much of this can be attributed to the diversity in the adopted values of $\alpha_{\text{MLT}}$. For instance, the tracks computed by Cassisi and by VandenBerg assume the highest value of the mixing-length parameter and, consistent with expectations, their RGBs are hotter than those computed by the other workers. (It is, however, a little surprising that these two tracks agree so well given that one allows for diffusive processes, but the other does not — see Table 1. There must be compensating effects due to other factors; e.g., the assumed $Z$.) The track by Girardi assumes the lowest value of $\alpha_{\text{MLT}}$, and it is the coolest track at the base of the RGB (as it should be, all else being equal), though it ends up close the middle of the five tracks near the RGB tip (which also indicates that other factors must be having an impact). Still, the level of agreement is quite satisfactory between the different computations.

Indeed, all of the tracks probably comply with current constraints on the temperatures of solar abundance giants to within their uncertainties. They should also be able to reproduce the CMDs of M 67 and the Hyades quite satisfactorily, simply by employing slightly different transformations than those reported by VC03. Indeed, it would be interesting to see how much of a discrepancy between theory and observation would be found if the same color–$T_{\text{eff}}$ relations were used by all the different grids of isochrones currently in use. Of course, in the case of M 67, in particular, there is enough leeway in the chemical composition and reddening that comparably good fits to the cluster photome-
try could well be obtained using most of the available sets of models if, e.g., somewhat different metallicities were assumed.

3. The Calibration of Stellar Models for Metal-Deficient Stars

Not having an extremely metal-poor counterpart to the Sun that can be similarly used as a high-precision calibration point, or a low-metallicity star cluster whose properties are known to exceedingly high accuracy, makes it very difficult to assess the reliability of the temperatures and colors predicted by stellar models at low $Z$. As discussed below, the temperatures determined for field Population II dwarfs having good distance estimates from *Hipparcos* can sometimes vary by up to 100–150 K, depending on how they are derived. This is quite comparable to the uncertainties in $T_{\text{eff}}$ that are found for field and GC giants from infrared

![Solar Tracks](image)
photometry. Coupled with the $\sim 0.3$ dex uncertainties in [Fe/H] that exist for many objects, it is relatively easy for stellar models to satisfy many, if not most, observational constraints. All that may be needed to accomplish this is to choose a particular metallicity scale — e.g., that by Zinn & West (1984) in preference to the one by Carretta & Gratton (1997), or vice versa, in the case of Galactic GCs. It is fair to say that significant improvements to the prediction of the temperatures and colors of metal-poor stars may not be possible until we can confidently measure their chemical compositions to within at least $\pm 0.1$ dex. We are still a long ways from achieving that.

This section will bring to the fore some of the difficulties, apparent inconsistencies, etc., that hinder further progress at the present time. On the one hand, it is very encouraging that model-atmosphere-based color–$T_{\text{eff}}$ relations appear to fine for solar metallicity stars warmer than $\sim 5000$–5500 K. One would naively expect that they should do at least as well at lower metal abundances given the concomitant decrease in the blanketing of stellar atmospheres. On the other hand, predicted temperatures undoubtedly suffer from systematic errors that are not easily quantified. Consequently, it is not clear that stellar models which are transposed to the observational plane using synthetic color–$T_{\text{eff}}$ relations will accurately reproduce the properties of real metal-poor stars. (Even when there appears to be good agreement between theory and observations, the possibility exists that errors in the various factors that play a role have conspired to produce it.)

![Figure 6](image-url)

Figure 6. Similar to Fig. 1, except that the color transformations are for stars having [Fe/H] = −2.0.

The obvious first step in testing the available color transformations that apply to stars of low metallicity is to simply try them out. As illustrated in Figure 6, the $(V-I)-T_{\text{eff}}$ relations derived from MARCS and Kurucz model atmospheres for [Fe/H] = −2.0 agree quite well in a systematic sense, though Kurucz colors are redder by $\sim 0.02$–$0.03$ mag over most of the ranges in tem-
perature and gravity that have been considered therein. (The differences in the predicted $B - V$ color indices from the two sources tend to be somewhat larger, and to be stronger functions of $T_{\text{eff}}$ and $\log g$; see Fig. 3 by VC03.) It is difficult to say which of the two sets of transformations is the most realistic one. Comparisons of isochrones with observed CMDs may indicate a clear preference, but that could well be more of a reflection on the models than on the color transformations. In effect, choosing to use a particular set of color–$T_{\text{eff}}$ relations from among several possible alternatives is equivalent to calibrating the stellar models so that they yield a particular interpretation of, e.g., the data for a few key stars or star clusters. (Only when the same model atmospheres are used to evaluate the surface boundary conditions of the stellar interior models and to accomplish their transformation to the various observational planes can it be claimed that they represent a purely theoretical prediction.}

For this reason, it is important to choose the “calibrating” objects very carefully. Among the most metal-deficient GCs, M68 provides perhaps the best benchmark because it has well-defined CMDs (Walker 1994) based on observations taken in three different bandpasses ($BVI$) during the same run and subjected to the same reduction and standardization procedures. Its reddening is fairly low — $E(B-V) = 0.06$ according to the Schlegel et al. (1998) dust maps — and, at least until recently, its metallicity has not been the subject of much controversy. On the Zinn & West (1984) scale, it has $[\text{Fe/H}] = -2.09$, while Carretta & Gratton (1997) obtained $[\text{Fe/H}] = -1.99$ from high-resolution spectroscopy of cluster giants. Unfortunately, a metallicity near $-2.0$ has not been found by Kraft & Ivans (2003) from their analysis of the equivalent widths of Fe II lines: they obtained $[\text{Fe/H}] = -2.43$. The consequent uncertainty in the metal abundance of M68 obviously makes any comparison of its CMD with stellar models rather insecure.

Still, it is worthwhile to proceed and to explore the implications of the isochrones computed by Bergbusch & VandenBerg (2001), in particular, as an instructive example. Throughout this study, the adopted distance modulus for any GC that is considered will be based on a fit of a ZAHB locus calculated by VandenBerg et al. (2000) to the lower bound of the distribution of cluster HB stars. According to the recent work by De Santis & Cassisi (1999) and by Cacciari, Corwin, & Carney (2005), the luminosities of the RR Lyrae stars in these systems, as derived from analyses of their pulsational properties, are close to those predicted by the VandenBerg et al. models. Indeed, as will become apparent below, ZAHB-based distance estimates agree quite well with those determined using other methods.

3.1. M68

If the evolutionary calculations for $[\alpha/\text{Fe}] = 0.3$ and $[\text{Fe/H}] = -2.01$ are converted to the observational planes using the transformations from MARCS and Kurucz model atmospheres, and overlayed onto the M68 CMDs assuming $E(B-V) = 0.06$ and $(m-M)_V = 15.18$, the results shown in Figure 7 are obtained. The solid curves, which employ the color–$T_{\text{eff}}$ relations from MARCS atmospheric models, clearly provide the best match to the cluster CMD. If Kurucz colors are used, the isochrones are generally too red (at least for the adopted cluster parameters); and while most of the discrepancies on the $[(V-I)_0, M_V]$-
plane appear to be consistent with a small zero-point shift, which would not be a serious concern, the fit of the isochrones to the cluster fiducial on the \([(B-V)_0, M_V]\)-diagram is much more problematic. Granted, the computed RGBs are still a bit too red when MARCS transformations are adopted, but the differences are small enough that it might easily be a mistake to attribute them to errors in the predicted temperatures and/or the color–\(T_{\text{eff}}\) relations. The remaining discrepancies may instead be a consequence of, e.g., the assumed cluster properties (reddening, distance, chemical composition) not being quite right.

![Figure 7](image.png)

**Figure 7.** Overlay of isochrones (Bergbusch & VandenBerg 2001) and a fully consistent ZAHB locus (VandenBerg et al. 2000) for the indicated parameters onto the CMD of M68 by Walker (1994), as represented by the fiducial sequences (filled circles) determined by DAV from the published photometry. The solid and dashed curves are obtained when the color transformations from MARCS and Kurucz model atmospheres are used, respectively. Note that \(E(V-I) = 1.25 E(B-V)\) (Dean et al. 1978) has been assumed.

It is noteworthy that very similar interpretations of the data are found in both panels of Fig. 7 and that, if the distance is set using the HB, the lower MS of M68 is well matched by the models. [As pointed out by VC03, comparable consistency is not obtained if the Lejeune et al. (1998) transformations are used. Either there are some problems with the latter or with the M68 observations.] If, for instance, M68 is significantly more metal poor than \([\text{Fe/H}] \approx -2.0\), then the HB stars should be somewhat brighter if they conform to theoretical expectations, implying an increased distance modulus, and the lower MS stars would lie above the ZAMS for the lower metallicity. In this case, redder color
transformations would be needed. To be sure, it is also possible that M 68 actually has both a lower metal abundance and a shorter distance than assumed, in which case, an even better fit to the cluster CMD than that shown in Fig. 7 might be found using the same MARCS color transformations. (A lower metallicity isochrone for the same or slightly higher age will have a bluer RGB as well as a reduced difference in color between the turnoff and the RGB.) These few comments serve to illustrate some of the difficulties that are encountered when trying to establish the color–$T_{\text{eff}}$ relations for very metal poor stars using observational constraints that have appreciable uncertainties.

It goes without saying that any fine-tuning of the color transformations to obtain a “perfect” match to the CMD of M 68 (or any other globular cluster) cannot be justified. What is encouraging is that, without applying any corrections at all to the transformations from MARCS model atmospheres, it is possible to get quite a respectable fit of isochrones to the entire CMD of a very metal-deficient GC like M 68, and to have a consistent interpretation of both $BV$ and $VI$ data. Indeed, VC03 found that similar good agreement could be obtained for more metal-rich systems, provided that some adjustments were made to the predicted colors for upper RGB and lower MS stars: the higher the metallicity, the warmer the temperature at which these corrections seemed to be necessary. However, such a trend is not unexpected given that substantial corrections to the color–$T_{\text{eff}}$ relations are needed at rather warm temperatures if models for $[\text{Fe/H}] \approx 0.0$ are to satisfy empirical constraints (see §2). Importantly, no adjustments were applied to the theoretical color transformations at temperatures appropriate to turnoff stars at any metallicity. Indeed, the main effect of correcting the synthetic colors for cooler stars is to ensure that the predicted RGB and MS slopes are in good correspondence with those observed. Models that fail to satisfy at least these constraints will not be very useful for the interpretation of the photometric data for complex stellar populations.

### 3.2. M 92 and 47 Tucanae

Let us now consider some comparisons of isochrones with the well-defined CMDs for M 92 and 47 Tuc that were obtained by Stetson & Harris (1988) by Hesser et al. (1987), respectively. In the lower panel of Figure 8, the fiducial sequences for these two GCs have been superposed onto a number of Bergbusch & VandenBerg (2001) isochrones for $[\text{Fe/H}]$ values ranging from $-2.31$ to $-0.40$ (assuming $[\alpha/\text{Fe}] = 0.3$ in each case) and ages that approximately reflect the GC age–metallicity relation derived by VandenBerg (2000). (To take into account the effects of diffusive processes, which were not treated in the models that have been plotted, the ages should be reduced by $\sim 10\%$; see VandenBerg et al. 2002.) The adopted reddenings are from the Schlegel et al. (1988) dust maps, and the assumed $(m-M)_V$ values follow from the fits of computed ZAHBs (VandenBerg et al. 2000) to the cluster HB stars that are shown in the upper panel. All of the models have been transposed to the $[(B-V)_0, M_V]$–plane using the calibrated color–$T_{\text{eff}}$ relations given by VC03.

Clearly, the predicted and observed RGB and MS slopes are in good agreement and, moreover, there is excellent consistency between the ZAHBs and the lower MS locations insofar as the assumed/implied $[\text{Fe/H}]$ values are concerned. (If, for instance, lower metallicity ZAHBs were fitted to the cluster HB stars, the
Figure 8. The lower panel superimposes on a plot of isochrones, for the indicated [Fe/H] values and ages, the fiducial sequences for M 92 (Stetson & Harris 1988; filled circles) and 47 Tucanae (Hesser et al. 1987; open circles) assuming the reddenings from the Schlegel et al. (1998) dust maps and distance moduli based on fits of computed ZAHBs (VandenBerg et al. 2000) to the cluster counterparts, as shown in the upper panel. The location of the field Population II subgiant, HD 140283, is given by the open square.

Thus, if the actual metallicities of M 92 and 47 Tuc are as assumed in Fig. 8, the fine agreement between theory and observations which is evident therein would provide a strong argument in support of the colors of the models that have been used in both an absolute and relative sense (provided that, e.g., the photometric zero-points and reddenings are accurate).

Unfortunately, current estimates of the metallicity of M 92 vary from [Fe/H] = −2.16 (Carretta & Gratton 1997) to −2.38 (Kraft & Ivans 2003). (An intermediate value, −2.24, was obtained by Zinn & West 1984). Even though there are good reasons to favor the Kraft & Ivans determination (e.g., being based on
Fe II lines, their results are largely unaffected by departures from LTE), it is not impossible that M92 has a metallicity as high as [Fe/H] = −2.14, or even higher. Recent work on the local subgiant, HD140283, which is one of the best observed field Population II stars, has called into question metal abundance determinations based on 1-D model atmospheres. (The location of this star has been plotted in Fig. 8 to support the ZAHB-based distance to M92. This star should be slightly brighter than the cluster subgiants at the same color, by approximately the amount shown, if it is the same age as, but slightly more metal poor than, the globular cluster.) Whereas nearly all spectroscopic studies of HD140283 over the years have found [Fe/H] ≈ −2.4 (see, e.g., the [Fe/H] catalogue by Cayrel de Strobel et al. [1997], Shchukina, Trujillo Bueno, & Asplund [2005] have suggested from model atmospheres which take 3-D effects into account that a reduction of ≈ 100 K in the $T_{\text{eff}}$ usually adopted for this star together with an assumed [Fe/H] ≈ −2.0 result in much improved agreement between the predicted and observed spectra. It remains to be seen what the implications of the more sophisticated model atmospheres will be for the metallicity of M92, but an upward revision is conceivable.

The possible ramifications of 3-D model atmospheres notwithstanding, there is a general consensus that 47 Tuc has an iron content very close to [Fe/H] = −0.70 (Zinn & West [1984]; Carretta & Gratton [1997]; Kraft & Ivans [2003]). This poses a bit of a problem for the models plotted in Fig. 8, which indicate that a consistent interpretation of the HB and MS observations is obtained only if the cluster metallicity is [Fe/H] ≈ −0.8. There are at least a couple of possible ways of explaining this (admittedly small) discrepancy. First, based on a careful analysis of published photometry for 47 Tuc, using the secondary cluster standards established by Stetson [2000], Percival et al. [2002] have suggested that, in the mean, the Hesser et al. [1987] observations are too blue by ≈ 0.02 mag. Indeed, an analogous plot to Fig. 8, but with the 47 Tuc fiducial shifted to the red by 0.02 mag, shows that the observed lower MS coincides very well with a that of 12 Gyr, [Fe/H] = −0.71 isochrone if, as required by the fit of the ZAHB for this metallicity to the cluster HB stars, $(m - M)_V = 13.30$ is assumed.¹ This plot has not been included here because it is already quite obvious from Fig. 8 that such a redward adjustment of the 47 Tuc CMD would result in a close match to the isochrone for [Fe/H] = −0.71.)

Errors in the adopted reddening or the model colors could also explain why the deduced [Fe/H] value from Fig. 8 is at variance with spectroscopic metallicity estimates. For instance, Gratton et al. [2003] have recently obtained $E(B-V) = 0.024 \pm 0.004$ for 47 Tuc, which is ≈ 0.01 mag smaller than the value assumed here. This supplies approximately one-half of the shift needed to reconcile the cluster CMD with models for [Fe/H] ≈ −0.7. While it is quite possible that the remaining difference can be traced to the color–$T_{\text{eff}}$ relations that have been used, it is just as likely (if not more so) that the isochrones are too red because the effective temperatures predicted by the models for metal-poor

¹Note that an apparent distance modulus of 13.30–13.35, as inferred from HB models, is in excellent agreement with recent determinations based on the cluster white dwarfs (13.27 ± 0.14; Zoccali et al. [2001]) and on MS fits to field subdwarfs (13.33 ± 0.04 ± 0.1; Grundahl, Stetson, & Andersen [2002]).
stars are too low. As discussed by VandenBerg et al. (2000), stellar models that are computed using MARCS model atmospheres as boundary conditions will be ∼ 50–100 K warmer (depending on the metal abundance and evolutionary state) than those which assume a scaled-solar \( T-\tau \) relation to describe the atmospheric structure.

Figure 9. Tracks for 0.8\( M_\odot \) having \([\alpha/Fe] = 0.0\) and the \([Fe/H]\) values specified adjacent to the various loci, in which the boundary conditions are derived either from model atmospheres (dashed curves) or by integrating the hydrostatic equation in conjunction with the Krishna Swamy (1966) scaled-solar \( T-\tau \) relation (solid curves). Note that slightly different values of the mixing-length parameter, as indicated, are implied by a Standard Solar Model, and that the dashed curves are less complete than the solid curves because model atmospheres were available only for \( T_{\text{eff}} \geq 5500 \) K.

Their results are presented on the \([ (B-V)_0, M_V ] \)–plane in Figure 9. This shows that evolutionary tracks for 0.8\( M_\odot \) stars having \([Fe/H] = -2.31, -1.31, -0.83, \) and \(-0.30\) are 0.005–0.02 mag bluer if the surface boundary conditions of the individual stellar models are derived from model atmospheres rather than from integrations of the hydrostatic equation in conjunction with the Krishna Swamy (1966) \( T-\tau \) relation. (This prediction should be regarded as "preliminary" as it is based on a small number of model atmospheres that were made available ∼ 6 years ago by B. Gustafsson and B. Edvardsson. A collaborative project with the Uppsala group is presently in progress to more fully analyze the impact of using their latest model atmospheres to describe the outermost layers of stellar models.) The degree to which isochrones, as opposed to evolutionary tracks, are affected remains to be seen, but it seems proba-
ble that predicted turnoff color versus age relations will be bluer by one- or two-hundredths of a magnitude (at a fixed age) if the effects of proper model atmospheres are taken into account.

While such offsets seem desirable for our understanding of 47 Tuc (at the present time, anyway), they would appear to pose a problem for the most metal-deficient systems. Fig. 9 suggests that all of the isochrones in Fig. 8 more metal poor than \( \sim -0.7 \) should be shifted to the blue by at least 0.01 mag. However, any blueward correction of the isochrones appropriate to M92 (and M68, for that matter) would have the consequence that a consistent explanation of its CMD (i.e., of both its HB and MS stars) would be possible only if a somewhat higher metallicity were assumed (closer to \([\text{Fe/H}] = -2.0\) than to \(-2.14\)), which would conflict with current spectroscopic estimates. To be sure, with so many sources of uncertainty, it is easily possible that errors in one or more quantities are compensating for errors in others. Being able to predict the colors of stars of any metallicity to within \( \sim 0.02 \) mag could well be the best that we can do (and will do for the foreseeable future), and once that point is reached, further debate may not be very meaningful. Realistically, one cannot say with any certainty that there is, or is not, a problem with the comparison between theory and observations presented in Fig. 8.

### 3.3. M3 and M5

Stetson et al. (1999) fiducials for M3 and M5 are compared with a set of 12 Gyr isochrones spanning the range in metallicity, \(-2.14 \leq [\text{Fe/H}] \leq -1.14\), in Figure 10, which is very similar to the lower panel in Fig. 8 except that the \([(V-I)_{0}, M_{V}]\)-plane is considered. The fiducial sequence for M68 from Fig. 7 has been re-plotted here to reiterate that it can be reproduced quite well by models for \([\text{Fe/H}] \approx -2.0\) — i.e., for the metal abundance derived by Carretta & Gratton (1997). The assumed reddenings are from the Schlegel et al. (1998) dust maps and, although not shown, the adopted distance modulus in each case is such that a theoretical ZAHB, for that metallicity which is implied by the overlay of the predicted and observed lower main sequences, provides a good fit to the lower bound of the distribution of cluster HB stars. Thus, the models provide consistent fits to the MS and HB stars in M3 and M5 only if their metallicities are \([\text{Fe/H}] \approx -1.6\) and \(-1.4\), respectively.

This is an intriguing result because these estimates are not in good agreement with the [Fe/H] values derived by Carretta & Gratton (1997) using high-resolution spectroscopy (even though there is no such conflict in the case of the most metal-deficient clusters, like M68). According to Carretta & Gratton, the iron abundance of M3 is \([\text{Fe/H}] = 1.34\), while that of M5 is \(-1.11\). However, it is well known that, although the Zinn & West (1984) and Carretta & Gratton (1997) [Fe/H] scales are very similar below \([\text{Fe/H}] \approx -2.0\) and above \( \approx -1.0\), they tend to differ by \( \sim 0.3\) dex at intermediate metallicities. In fact, the [Fe/H] values of M3 and M5 as inferred from the models (see Fig. 10) agree very well with the Zinn & West determinations; namely, \(-1.66\) and \(-1.40\), respectively. Interestingly, metal abundances derived from Fe II lines are close to the mean of the Carretta–Gratton and Zinn–West results: Kraft & Ivans (2003) have obtained \([\text{Fe/H}] = -1.50\) for M3 and \(-1.26\) for M5. (Consistency with the findings
Figure 10. Similar to the lower panel in Fig. 8, except that 12 Gyr isochrones for the indicated [Fe/H] values are compared with the fiducial sequences of M3 (filled triangles) and M5 (open squares) from Stetson et al. (1999). The same fiducial of M68 that appeared in Fig. 7 has been replotted here as filled circles.

It is clearly very hard to assess the accuracy of the temperatures and colors of stellar models from comparisons of isochrones with observed CMDs until, at the very least, metallicity determinations are placed on a much firmer footing. Even then, uncertainties in the basic cluster properties (distance, reddening, and photometric zero-points) limit what can be done. Perhaps the main value of Fig. 10 is to show how precisely everything must be known in order to obtain the correct interpretation (whatever that may be) of any given CMD: note that the separation in the lower MS segments of the isochrones at $M_V \geq 5.5$ is only $\sim 0.075$ mag in $V - I$ for a 1 dex change in [Fe/H].

3.4. Globular Cluster Standard Field Photometry (Stetson 2000)

When VandenBerg (2000) determined the ages of a sample of 26 GCs, ZAHB-based distance moduli and reddenings from the Schlegel et al. (1998) dust maps were adopted (as in this study). On the basis of these assumptions, the observed CMDs were converted to $[(B - V)_0, M_V]$- or $[(V - I)_0, M_V]$-diagrams. The age of each cluster was then obtained by first adjusting the isochrones for...
the assumed metallicity in color until the predicted MS overlaid that observed, and then noting which isochrone best reproduced the location of the turnoff and the subgiant branch. The advantage of this approach is that errors in the model temperatures or colors, or problems with the photometric zero-points or reddening, do not affect the derived age. As noted in the plots provided by VandenBerg, the model colors were sometimes adjusted to the blue and sometimes to the red, by typically \( \sim 0.02 \) mag. Moreover, the corrections needed to fit \( BV \) and \( VI \) photometry for the same cluster were not always in the same direction. Only for a few clusters, like M 68 (Walker 1994), was it possible to obtain consistent interpretations of the data on two different color planes. (As already noted, Walker’s \( BVI \) observations were taken in the same observing run, and subjected to the same reduction and calibration procedures.)

Rosenberg et al. (1999), among others, have emphasized the importance of having a homogeneous photometric database (in their case, to obtain self-consistent relative GC ages). Indeed, the lack of homogeneity could well be the main reason why there was no obvious pattern to the corrections that VandenBerg (2000) applied to the isochrones in his investigation. Needless to say, the possibility of zero-point and/or systematic errors in cluster photometry is a real concern for any attempts that are made (e.g., that by VC03) to use such data to calibrate synthetic colors. Fortunately, P. B. Stetson (see Stetson 2000) is engaged in a monumental, on-going effort to homogenize a large fraction of the \( BVRI \) CCD photometry that has been acquired by many researchers over the years. In particular, he is in the process of establishing many “standard fields” in GCs that can be used for calibration purposes. Given their availability, an obvious question to ask is: “How well are Bergbusch & VandenBerg (2001) isochrones, using VC03 color transformations, able to reproduce the CMDs defined by his secondary standards in, say, 47 Tuc, M 5, and M 92?”.

The answer to this question, as revealed in Figure 11, is “not as well as one would have hoped”. As far as 47 Tuc is concerned, the predicted MS (on both color planes) is about 0.01 mag too blue, suggesting that models for a slightly higher \([\text{Fe/H}]\) value than \(-0.83\) would be more appropriate. (Isochrones for \([\text{Fe/H}] = -0.71\), which is the next highest metallicity in our grid, provided a less satisfactory fit to the observations than that shown.) Curiously, the RGB segments of the isochrones are 0.01–0.02 mag too blue on the \([ (B-V)_0, M_V] \) plane, but too red by about the same amount on the \([ (V-I)_0, M_V] \) diagram. It is quite possible that these discrepancies are due to minor problems with the color transformations, particularly those for the \( B-V \) color index.

The offsets between the computed and observed RGBs are much bigger (\( \approx 0.04 \) mag) in the case of M 5. They are larger than any found in the extensive study of GC CMDs by VandenBerg (2000), though it must be acknowledged that most of his fits of isochrones to photometric data were discrepant in the same sense. It seems unlikely that the color–\( T_{eff} \) relations are at the root of this difficulty because the predictions from MARCS model atmospheres, which were adopted without appreciable modifications by VC03 for \([\text{Fe/H}] \) values \( \leq -1.0 \), tend to be bluer than other transformations currently in use (e.g., see Fig. 6).

\(^2\)The data for these clusters were downloaded from the Canadian Astronomy Data Center web site [http://cadcwww.hia.nrc.ca/standards] on April 25, 2005.
Either the model temperatures along the giant branch are too cool, which is possible (though they appear to be consistent with empirical constraints, as shown in the next section), or our understanding of M5 is somehow lacking (assuming, of course, that the CMDs of M5 presented in Fig. 11 are trustworthy). Isochrones for a lower metallicity, a higher oxygen abundance, or an increased age should, for instance, provide a better match to the observed (small) difference in color between the TO and the RGB.

It is, perhaps, especially disheartening that isochrones which had previously provided a superb match to the CMD of M92 by Stetson & Harris (1988; see Fig. 8) do not fare nearly as well when confronted by the standard field data. As shown in the lower left-hand panel of Fig. 11, the observed MS lies slightly to the
red of the isochrones while the cluster giants are, in the mean, \( \sim 0.02 \) mag bluer than the models. Of particular concern is the fact that, in the \([\(V - I\)]_0, M_V\)-diagram (see the lower right-hand panel), the deviation between the predicted and observed MS of M\(92\) is in the opposite sense, and the discrepancies along the giant branch are appreciably larger (though in the same direction as in the left-hand panel). Thus, the models do not provide consistent interpretations of the \(BV\) and \(VI\) observations. Does this mean that Walker’s (1994) photometry for M\(68\) suffers from systematic errors (given that, as shown in Fig. 7, no such inconsistency is found for this cluster, which has a very similar metal abundance as M\(92\))? There are clearly some mysteries here that need to be understood. Perhaps the main conclusion to be drawn from this brief examination of the standard field data is that, when using the CMDs for GCs to test the predicted colors of stellar models, one must be wary of the very real possibility that the observations suffer from significant zero-point and/or systematic errors.

### 3.5. Giant-branch Temperatures

A valuable test of the predicted temperatures of giants is provided by empirical \((V - K) - T_{\text{eff}}\) relations. Due to the extensive work by [Frogel, Persson, & Cohen](1983, and references therein), fiducial sequences on the \((\log T_{\text{eff}}, M_{\text{bol}})\)-plane have been derived for many star clusters from \(V - K\) photometry. Those reported by [Frogel, Persson, & Cohen](1981) for M\(92\), M\(3\), 47 Tuc, and M\(67\) are plotted in Figure 12, together with the RGB segments of the isochrones that appeared in Fig. 8. (The distance moduli adopted by Frogel et al have been corrected by \(\approx 0.2\) mag, or less, to be consistent with the ZAHB-based distances that have been adopted throughout this study. Because the giant branches rise so steeply, such adjustments have no more than minor consequences for the comparisons with theoretical models.) It is immediately apparent that the predicted and “observed” RGB slopes are nearly the same, which suggests that the value of the mixing-length parameter, \(\alpha_{\text{MLT}}\), does not vary by much, if at all, during the first ascent of the giant branch. Furthermore, the empirically derived temperatures for M\(92\), M\(3\), and 47 Tuc are well reproduced by the models for \([\text{Fe/H}] \approx -2.3, -1.55, \text{and} -0.7\), respectively, though the size of the \(1\sigma\) error bar is large enough to accommodate larger or smaller \([\text{Fe/H}]\) values by \(\sim 0.15-0.3\) dex. There is clearly no conflict with current spectroscopic determinations, and hence no indication that \(\alpha_{\text{MLT}}\) varies with metallicity. Indeed, much tighter constraints on the temperatures of giants are needed if they are to provide more exacting tests of stellar models.

As mentioned in §2 (also see VC03), the predicted temperatures along the giant-branch segment of the 4 Gyr isochrone used to fit the M\(67\) CMD (see Fig. 2) agree well with those determined from \(V - K\) photometry and the latest available \((V - K) - T_{\text{eff}}\) relations (Bessell et al. 1998; van Belle et al. 1999). The upper part of this isochrone has also been plotted in Fig. 12 (the dashed curve) in order to determine how much it differs from the temperatures derived for M\(67\) giants by Frogel et al. (1981; the open circles) nearly 25 years ago. The agreement between the two is obviously quite good, which indicates that the transformation between \(V - K\) and \(T_{\text{eff}}\) has withstood the test of time rather well.
3.6. Population II subdwarfs

Although our comparisons of isochrones with the CMDs of M3 and M5 (see Figs. 10 and 11) indicate a preference for the Zinn & West [1984] [Fe/H] scale over that by Carretta & Gratton [1997] in the intermediate metal-poor regime, this is not a consequence of the color–Teff relations that have been used. Indeed, the VC03 transformations are completely consistent with those used by R. G. Gratton and colleagues to determine the temperatures of the Population II subdwarf standards with [Fe/H] values between −1.0 and −2.0 that have often been used to derive GC distances via the MS fitting technique. As shown in Figure 13, the $B - V$ colors that are determined from interpolations in the VC03 color tables for the values of $T_{\text{eff}}$ and [Fe/H] derived by Gratton, Carretta, & Castelli [1996] and Clementini et al. [1999] for some of the best observed subdwarfs (also see Bergbusch & VandenBerg [2001]), using
log g values from stellar models, are in exceedingly good agreement with the observed colors. (In the case of dwarf stars, colors are not very sensitive to the assumed gravity; consequently, very similar results would obtained for any log g value between 4.3 and 4.7.)

![Graph](image)

**Figure 13.** Comparison of the predicted and observed colors of several Population II subdwarfs that are identified by their HD numbers. The former are derived by interpolations in the VC03 color tables using the temperatures and metallicities (which range between [Fe/H] = −0.97 and −1.91, as indicated) determined for the subdwarfs by Gratton et al. (1996) and Clementini et al. (1999), and using values of log g from stellar models.

The agreement is remarkable considering that (i) the assumption of higher or lower temperatures by as little as 50 K would affect the predicted colors by ∼ 0.02 mag for the coolest stars, and (ii) the temperatures derived for the same star from color–T eff relations (e.g., Gratton et al. 1996), the infrared-flux method (e.g., Alonso, Arribas, & Martínez-Roger 1996), the fitting of Balmer line profiles (e.g., Axer, Fuhrmann, & Gehren 1994), and fits to the uv–flux distributions (e.g., Allende Prieto & Lambert 2000) sometimes differ by up to ∼ 150 K. (However, for many of the stars considered in Fig. 13, the temperatures obtained via the different techniques agree quite well — see Table 5 by Clem et al. 2004.) To the extent that the assumed properties of the subdwarfs have been accurately estimated, Fig. 13 provides additional support for the color transformations championed by VC03.

### 3.7. Comparison of 0.8M⊙ Tracks for [Fe/H] = −2.3

Figure 14 presents a comparison of the computed tracks from completely independent stellar evolution codes for a very low value of [Fe/H]. As for the calculations shown in Fig. 5 for solar abundances, there are a number of differences
in the assumed chemistry and the adopted physics, some of which are noted in Table 2. (The assumed values of $\alpha_{\text{MLT}}$ are the same as in Table 1.) The main intent of such a comparison is to determine how much of an effect the different assumptions and physics have on, in particular, the predicted temperatures.

![Graph](image)

Figure 14. Similar to Fig. 5, except that tracks for $0.8M_\odot$ stars having $[\text{Fe/H}] = -2.3$ are compared.

In fact, the differences are larger than one might have anticipated. Curiously, the Girardi and VandenBerg tracks coincide almost exactly between the ZAMS and the lower RGB, but they diverge thereafter: Girardi’s track has the warmest giant branch of all those plotted in Fig. 14, while VandenBerg’s is the coolest. [Without a detailed examination of the respective codes that were used, it is not possible to explain the cause(s) of these differences.] On the one hand, models that predict warmer RGBs should do a better job of matching GC CMDs if the VC03 color–$T_{\text{eff}}$ relations are used to transform the models to the observed plane. On the other hand, the empirical $T_{\text{eff}}$ scale for giants is supportive of the VandenBerg predictions (as shown in §3.5) — though the uncertainties are probably large enough to encompass all of the theoretical models.
that have been plotted. It is not surprising that the track computed by Yi has the coolest TO given that his calculations were the only ones to take diffusion into account. On the whole, there seems to be reasonable consistency between the different computations; e.g., the age is approximately inversely correlated with the assumed He abundance and the subgiant luminosity, as expected.

4. Conclusions

For simple stellar populations like those found in open and globular star clusters, it really does not matter very much if there are problems with the colors of the isochrones that are compared with the observations. Distance determinations are considered to be far more reliable, for instance, if they are based on so-called “standard candles” (lower MS stars having accurate distances and metallicities, RR Lyraes, white dwarfs) than on fits to stellar models. Indeed, it may be possible to find an isochrone that appears to provide a very good match to all of the principal sequences in an observed CMD, but that isochrone would probably not be the correct explanation of the data. There are too many factors that affect the shapes of isochrones, including the overall metallicity, the detailed heavy element mixture (notably of the CNO and α elements), the treatment of convection, the low-temperature opacities, the color–Teff relations, physics usually not taken into account (e.g., turbulence at the base of the envelope convection zone, rotation), and more. Each of these variables has its own uncertainty and, collectively, they will certainly permit substantial variations in e.g., the predicted slope of the subgiant branch, or the difference in color between the TO and the lower RGB. It should not be too surprising, or too disturbing, to find that stellar models do not provide as satisfactory a fit to color-magnitude data as one would like. Fortunately, TO luminosities are subject to far fewer (and better understood) uncertainties, and they can be used to obtain well-constrained ages for those systems having good distance estimates and metal abundances that have been determined spectroscopically.

The situation is quite different in the case of complex stellar populations; i.e., systems containing stars that span wide ranges in age and chemical composition. In order to unravel their star formation and chemical evolution histories, both the luminosity and color distributions in the observed CMDs must be explained.
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