Time variability of AGN and heating of cooling flows

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Accepted 2005 May 3; in original form 2004 October 8

ABSTRACT
There is increasing evidence that AGN feedback is important in the energetics of cooling flows in galaxies and galaxy clusters. It is possible that in most cooling-flow clusters radiative losses from the thermal plasma are balanced, over the cluster lifetime, by mechanical heating from the central radio source. We investigate the implications of the variability of AGN mechanical luminosity $L_m$ on observations of cooling flows and radio galaxies in general.

It is natural to assume that $\ell = \ln(L_m/L_X)$ is a Gaussian process. Then $L_m$ will be log-normally distributed at fixed cooling luminosity $L_X$, and the variance in a measure of $L_m$ will increase with the time-resolution of the measure. We test the consistency of these predictions with existing data for cooling flows and radio galaxies. These tests hinge on the power spectrum $P(\omega)$ of the Gaussian process $\ell(t)$. General considerations suggest that $P$ is a power law $P \sim \omega^{-\beta}$. Long-term monitoring of Seyfert galaxies combined with estimates of the duty cycle of quasars imply that $\beta \approx 1$, which corresponds to flicker noise. The power spectra of microquasars have similar values of $\beta$. We combine a sample of sources in cooling flows that have cavities with the assumption that the average mechanical luminosity of the AGN equals the cooling flow’s X-ray luminosity. Given that the mechanical luminosities are characterized by flicker noise, we find that their spectral amplitudes $\omega P(\omega)$ lie between the estimated amplitudes of quasars and the measured values for the radio luminosities of microquasars.

The model together with the observation that powerful radio galaxies lie within a narrow range in optical luminosity, predicts the luminosity function of radio galaxies. Both the shape and the normalization of the predicted function are in agreement with observations. Forthcoming radio surveys will test the prediction that the luminosity function turns over at about the smallest luminosities so far probed.

Key words: galaxies: clusters: general – cooling flows – galaxies: jets – radio continuum: galaxies – galaxies: luminosity function, mass function

1 INTRODUCTION
In the cores of the majority of rich galaxy clusters the radiative cooling time of the hot, X-ray emitting intracluster medium (ICM) is significantly shorter than the Hubble time. In the absence of heat sources, the gas will cool and flow towards the bottom of the cluster potential well: hence the name ‘cooling-flow’ clusters. In fact, recent X-ray observations – in particular XMM-Newton high-resolution spectroscopy – have ruled out the presence of significant cooling flows (Kaastra et al. 2001; Molendi & Pizzolato 2001; Peterson et al. 2001; Tamura et al. 2001; Kaastra et al. 2004, and references therein): the radiating gas in the cores of these clusters is heated somehow.

The most promising heating mechanism is feedback from the Active Galactic Nucleus (AGN) hosted by the central galaxy of the cluster. The basic idea of this AGN/cooling-flow scenario is that the cooling ICM in the cluster core is heated by outbursts from the central AGN: an unsteady equilibrium may be achieved, in the sense that the energy input from the AGN balances the radiative losses of the ICM over the lifetime of the cluster. Different models have been proposed in which the AGN heat the thermal plasma with either mechanical (Binney & Tabor 1993; Tabor & Binney 1995) or radiative feedback (Ciotti & Ostriker 1997, 2001; Ostriker & Ciotti 2005). Although both processes might be effective to some extent and possibly cooperate, here we focus on mechanical heating, whose relevance to many observed systems is apparent (e.g. Blanton et al. 2004 and Omma & Binney 2004 and references therein).

The cooling luminosity $L_X$ can be determined directly from X-ray observations (e.g. Peres et al. 1998). By contrast, the mechanical power $L_m$ of the radio sources is not only much harder to determine observationally (e.g. Bicknell 1995;
Eilek 2004), but it is expected to be time-variable (e.g. Ulrich, Maraschi & Urry 1997). If \( L_m \) varies with time, how can it be compared with the cooling luminosity in observed systems? This paper addresses this question.

## 2 CONCEPTUAL FRAMEWORK

The mechanical luminosity of an AGN is expected to fluctuate on time-scales that range from months to several gigayears, but from the point of view of the symbiosis between the AGN and its cooling flow, only time-scales in excess of \( \sim 10 \) Myr are of interest. It is clearly impossible to probe such time-scales by monitoring individual AGN. Instead we must draw inferences by studying the demography of AGN and cooling flows. Such studies are still in their infancy, but it is nevertheless worthwhile to develop a methodology that they can employ, and to see what conclusions this methodology and currently available data lead us to.

Cooling flows vary in size from those associated with large clusters of galaxies, such as Abell 1795, to those associated with individual elliptical galaxies, such as NGC 4472. We start by scaling out the variations in the mechanical luminosities \( L_m \) of AGN that simply reflect variations in size rather than temporal variability, by focusing on the dimensionless variable

\[
\tilde{L}_m = \frac{L_m}{L_X}
\]

where \( L_X \) is the X-ray luminosity of the cooling flow, which is expected to vary at most slowly and weakly with time. The logarithm of \( \tilde{L}_m \),

\[
\ell = \ln \frac{L_m}{L_X},
\]

is a continuous random variable defined on the interval \((-\infty, \infty)\). Throughout physics one models such a variable through the Fourier representation

\[
\ell(t) = \int d\omega A_\omega e^{i\omega t}.
\]

The complex numbers \( A_\omega \) are random variables. The simplest conjecture is that these variables are statistically independent of one another, and have phases that are uniformly distributed in \((0, 2\pi)\). In this case \( \ell(t) \) is a Gaussian random process, and it is completely characterized by the power spectrum \( P(\omega) \equiv \langle |A_\omega|^2 \rangle \). The probability density of the luminosity at any given time \( t \) is given by

\[
P(L_m) d\tilde{L}_m = \frac{1}{\sqrt{2\pi} \sigma} \exp \left[ -\frac{(\ell - \langle \ell \rangle)^2}{2\sigma^2} \right] d\ell,
\]

where \( \sigma^2 = \langle (\ell - \langle \ell \rangle)^2 \rangle \) is the variance of \( \ell \). It is easy to show that with this model the expectation value of the \( n \)th power of \( L_m \) is

\[
\langle \tilde{L}_m^n \rangle = \exp \left( n \langle \ell \rangle + n^2 \frac{\sigma^2}{2} \right).
\]

The appearance of \( \sigma^2 \) in the exponential signals the skewness of the probability distribution. The larger the value of \( n \), the more important this skewness is in the sense that rare excursions to high \( \tilde{L}_m \) more strongly dominate \( \langle \tilde{L}_m^n \rangle \). A measure of the importance of skewness is the probability that an individual measurement of \( \tilde{L}_m \) will return a value smaller than \( \langle \tilde{L}_m \rangle \). One can readily show that

\[
P(\tilde{L}_m < \langle \tilde{L}_m \rangle) = \frac{1}{2} [1 + \text{erf}(\sigma/\sqrt{2})].
\]

\[\text{Figure 1.} \] The probability for a log-normal distribution that a single measurement yields a value smaller than the expectation value.

Figure 1 plots this function of \( \sigma \). When \( \sigma \gtrsim 3 \), there is a high probability of finding the luminosity to be smaller than its mean value because much of the total energy output comes during infrequent outbursts.

We are interested in the properties of the light-curve observed with a finite time-resolution, i.e., averaged over a given time \( \Delta t \). We define this ‘coarse-grained’ light-curve as

\[
\ell_{\Delta t}(t) = \int_0^\infty dt' F_{\Delta t}(|t' - t|) \ell(t'),
\]

where the filter function \( F_{\Delta t}(t) \) satisfies \( \int dt F_{\Delta t} = 1 \) and is non-zero in a region of width \( \Delta t \) around the origin. \( \ell_{\Delta t}(t) \) has average

\[
\langle \ell_{\Delta t} \rangle = \lim_{t \to \infty} \frac{1}{t} \int_0^t dt' \ell_{\Delta t}(t') = \langle \ell \rangle,
\]

and variance

\[
\sigma_{\Delta t}^2 = \lim_{t \to \infty} \frac{1}{t} \int_0^t dt (\ell_{\Delta t}(t') - \langle \ell \rangle)^2 \]

\[= \int_0^\infty d\omega \tilde{F}_{\Delta t}(\omega) P(\omega).
\]

where \( \tilde{F}_{\Delta t}(\omega) \) is the Fourier transform of the filter \( F_{\Delta t}(t) \). Approximating the window function \( F_{\Delta t}(\omega) \) with the Heaviside step function \( \theta(1 - \omega/\omega_{\Delta t}) \), where \( \omega_{\Delta t} = 2\pi/\Delta t \), we get

\[
\sigma_{\Delta t}^2 \approx \frac{\omega_{\Delta t}}{2} \int_0^\infty d\omega P(\omega).
\]

In words, contributions to the variance on time-scales shorter than \( \Delta t \) vanish in the averaging process. Thus, for any power spectrum \( P(\omega) \), \( \sigma_{\Delta t}^2 \) is a non–increasing function of \( \Delta t \), with \( \lim_{\Delta t \to \infty} \sigma_{\Delta t}^2 = 0 \) and \( \lim_{\Delta t \to 0} \sigma_{\Delta t}^2 = \sigma^2 \): the variance is smaller when averaging over longer \( \Delta t \).

### 2.1 The power spectrum

An AGN involves an enormous dynamical range of time-scales, from the few days characteristic of the dynamical time at the event horizon to the hundreds of megayears required for the development of a cooling catastrophe in the core of...
the cooling flow. In view of this great span of time-scales, one might expect the power spectrum of the AGN to be a power law in frequency. So we assume that \( P(t) \) has a power spectrum in the form

\[
P(\omega) = \begin{cases} 
  P_0 \left( \frac{\omega}{\omega_0} \right)^{-\beta} & \text{if } \omega_{\text{min}} \leq \omega \leq \omega_{\text{max}}, \\
  0 & \text{otherwise},
\end{cases}
\]

where \( P_0 \) is the power at some reference frequency \( \omega_0, \omega_{\text{min}} = 2\pi/t_{\text{max}} \) and \( \omega_{\text{max}} = 2\pi/t_{\text{min}} \). \( t_{\text{max}} \) and \( t_{\text{min}} \) are, respectively, the maximum and minimum time-scale of variability.

There is no difficulty in principle in deriving equations for a general power-law index \( \beta \), but we shall argue that \( \beta \) must lie close to unity, and the discussion is greatly simplified if we specialize to \( \beta = 1 \) at this point. The case \( \beta = 1 \) is associated with ‘flicker noise’, which empirically arises in diverse physical phenomena that range from electrical noise, to wind speeds, to earthquake magnitudes (e.g. Press 1978; Milotti 2002). In case of flicker noise the relation between the variance and the power spectrum is

\[
\sigma_{\Delta t}^2 = P(\omega_0) \ln \frac{\omega_{\text{max}}}{\omega_{\text{min}}} = P(\omega_0) \ln \frac{t_{\text{max}}}{\Delta t}.
\]

The amplitude of flicker noise is conveniently characterized by the dimensionless quantity \( \omega P(\omega) \), which is independent of \( \omega \).

### 2.1.1 Calibration with quasars

The variability of X-ray emission from AGN has been studied on time-scales shorter than \( \sim 10 \) yr. When the X-ray power spectra\(^2\) of Seyfert galaxies at frequencies \( 10^{-8} - 10^{-2} \) Hz are fitted with a power-law, one finds \( \beta \sim 1 \) (for instance, \( \beta \simeq 1.05 \pm 0.20 \) for the Seyfert galaxy NGC-4051), with amplitudes \( P(\omega) \sim 0.01 - 0.04 \) (Uttley, McHardy & Papadakis 2002; McHardy et al. 2004). Hence, the only power-law indices that we can consistently adopt are \( \beta \simeq 1 \).

There is a vast difference between the longest time-scales on which the power spectra have been empirically determined, and the time-scales \( \gtrsim 10^7 \) yr of interest here. Hence to extrapolate blindly from the observed \( \lesssim 10 \) yr fluctuations out to \( 10^7 \) yr ones is a very hazardous proceeding. Fortunately, independent estimates of the duty cycle of quasars provides a degree of reassurance. Ciotti & Ostriker (2001) define the duty cycle of a source with light-curve \( L(t) \) as

\[
\delta \equiv \frac{(L_2)^2}{(L_1)^2} = e^{-\sigma^2},
\]

where the second equality follows from equation (13). Various arguments indicate that quasars, over the lifetime of their host galaxies have duty cycles \( \delta \sim 10^{-3} - 10^{-2} \), with episodic lifetime (i.e., duration of individual major outbursts) \( \gtrsim 10^7 \) yr (Haehnelt, Natarajan & Rees 1998; Yu & Tremaine 2002; Martini & Schneider 2003; Haiman, Ciotti & Ostriker 2004). By equation (13), these duty cycles require variances in the range \( \sim 4.6 - 6.9 \). Inserting these variances and the time-scale range \( 10^4 - 10^{10} \) yr into equation (14), we find that the associated flicker spectra have amplitudes \( P(t_0) \sim 0.3 - 0.5 \). These values are larger by a factor \( \sim 30 \) than the amplitudes measured by Uttley et al. (2002) for Seyfert galaxies at frequencies higher by \( \gtrsim 10^6 \). If \( \beta \) differed significantly from unity, we would expect the amplitudes to be many orders of magnitude different, so we must have \( \beta \simeq 1 \).

### 2.1.2 Calibration from microquasars

Microquasars are radio-jet emitting X-ray binaries made up by a stellar-mass black hole (BH) and a companion star, and seem in many respects small-scale versions of AGN (Mirabel & Rodriguez 1999). It is widely accepted that on small to intermediate scales these objects are identical to AGN scaled down in both length- and time-scales by the ratio of the black holes’ Schwarzschild radii \( r_s \equiv 2GM_{\odot}/c^2 \). Hence, we can study the temporal variability of microquasars on human time-scales to get information about the variability of AGN on time-scales much longer than those that can be directly probed. It seems likely that the scaling holds at out to time-scales \( \sim 10^5 \) yr, which for an AGN black-hole mass \( 10^7 - 10^8 M_{\odot} \) corresponds to a readily observed frequency \( 10^{-5} \) Hz in microquasars. Thus from studies of microquasars we can estimate the amplitude of the AGN power spectrum at frequencies corresponding to \( 10^5 \) yr, and then use our knowledge that \( \beta \) cannot significantly differ from unity to extrapolate it to frequencies lower by a factor \( \sim 100 \).

Most studies of the low-frequency power spectrum of microquasars focus on X-ray emission. Reig, Papadakis & Kyllafis (2002) find that the power spectrum of the microquasar GRS 1915+105 is well fitted by a power law (equation (11)) with index \( \beta \simeq 1 \) in the frequency range \( 10^{-7} \) to \( 10^{-5} \) Hz. A power spectrum with this slope, over the same frequency range, has been also found for the X-ray light curve of Cygnus X-3 by Choudhury et al. (2004).

However, we are interested in the variability of the kinetic power of the jet, which is expected to be traced by radio emission rather than by X-rays. The radio variability of microquasars has not been investigated as much as their X-ray variability. A spectral analysis was carried out by Fiedler et al. (1987) for the radio (2.25 and 8.3 GHz) light curve of the peculiar microquasar SS 433: they found that its power spectrum is consistent with a power law with index \( \beta \simeq 1 \) at frequencies \( 10^{-7} \) to \( 10^{-6} \) Hz (see also Revnivtsev et al. 2005). Nipoti, Blundell & Binney (2005) show that at these low frequencies the slopes of the radio power spectra of the microquasars Cygnus X-3 and GRS 1915+105 are similar to those found in X-rays.

The X-ray light curve of GRS 1915+105 has power spectrum with amplitude \( P(\omega_0) \sim 0.02 \) at frequencies \( 10^{-7} \) to \( 10^{-5} \) Hz (Reig et al. 2002). An amplitude smaller by a factor of \( \sim 10 \) has been measured for the X-ray power spectrum of Cygnus X-3 in the same frequency range (Choudhury et al. 2004). However, there is evidence that the amplitude of the variability of the radio emission of these powerful microquasars is larger than that of their X-ray emission (Nipoti et al. 2005). The radio emission of Cygnus X-3 (at 2.25 and 8.3 GHz) was well fitted with a power law with index \( \beta \simeq 1 \) in the frequency range \( 10^{-7} \) to \( 10^{-6} \) Hz (see also Revnivtsev et al. 2005). Nipoti, Blundell & Binney (2005) show that at these low frequencies the slopes of the radio power spectra of the microquasars Cygnus X-3 and GRS 1915+105 are similar to those found in X-rays.

\(^3\) For the accretion disk of a \( \sim 10^8 M_{\odot} \) BH, the characteristic viscous time-scale at \( \sim 10^7 r_s \) is roughly \( 10^5 \) yr. Longer time-scales correspond to larger radii, at which AGN accretion disks might be truncated by self-gravity (e.g. Burderi, King & Szuszkiewicz 1998; Goodman 2003).

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8.3 GHz) has been monitored by the GBI-NASA monitoring program (Waltman et al. 1995, and references therein). We find that the daily-averaged 2.25-GHz radio light-curve of Cygnus X-3 has logarithmic variance \( \sigma^2 \approx 0.96 \) over the period 1982–2001. If we adopt \( t_{\text{max}} = 19 \) yr, \( \Delta t = 1 \) day for the boundaries of the frequency range sampled, equation \( \ref{eq:variance} \) implies that the power spectrum of the radio emission has amplitude \( P_{\omega_0} \approx 0.11 \), about two orders of magnitude larger than that found in X-rays. Similarly, we estimated the amplitude of the radio power spectrum of the microquasar GRS 1915+105, using its 2.25-GHz light-curve obtained by the GBI monitoring program over about 6 yr (see Foster et al. 1996). We found that GRS 1915+105 has amplitude \( P_{\omega_0} \approx 0.12 \) at 2.25 GHz, also much larger than in X-rays. Thus, observations of microquasars suggest \( P_{\omega_0} \approx 0.1 \) as a plausible value of the amplitude of the very low-frequency power spectrum of radio sources. Given the substantial uncertainties involved, this estimate is in remarkably good agreement with the estimate \( P_{\omega_0} \approx 0.4 \) that we obtained by considering the duty cycle of quasars.

These considerations suggest that we can obtain an impression of the long-term variability of AGN by rescaling the radio-frequency light curve of a microquasar. Fig. 2 is a plot of the daily-averaged 2.25 GHz light-curve of Cygnus X-3 rescaled in time by \( 10^8 \). The light curve is characterized by short powerful outbursts, alternating with periods of quiescence.

3 APPLICATION TO COOLING FLOWS

3.1 Systems with cavities

In general, it is not feasible to measure directly the mechanical power of radio sources. An exception is M87 – the central galaxy of the Virgo cluster – for which high-resolution observations of the jet and the very inner hot atmosphere are available (Owen, Eilek & Kassim 2000; Di Matteo et al. 2003). In the case of the central radio sources in galaxy clusters \( L_{\text{in}} \) can be estimated from depressions in the X-ray emission of the surrounding ICM. These ‘cavities’ are interpreted as bubbles produced in the interaction between the jets of the radio source and the thermal plasma. The energy transferred to the ICM by a bubble expanding adiabatically is (e.g. Churazov et al. 2000, 2002)

\[
E_b = \frac{\gamma}{\gamma-1} pV,
\]

where \( V \) is the volume of the bubble, \( p \) is the pressure of the surrounding ICM, and \( \gamma \) is the adiabatic index of the gas in the bubble. Thus, \( E_b = \frac{2}{5} pV \) for a non-relativistic gas (\( \gamma = \frac{5}{3} \)), and \( E_b = 4pV \) for a relativistic fluid (\( \gamma = \frac{4}{3} \)). Given a characteristic time \( \Delta t_b \) associated with the bubble, an estimate of the average kinetic luminosity of the AGN over \( \Delta t_b \) is then

\[
L_{\text{in}} = \frac{E_b}{\Delta t_b}.
\]

The definition of \( \Delta t_b \) is controversial: if we consider \( \Delta t_b \) as the age of the bubble, this is of order the time \( t_{\text{rec}} \) required for the cavity to rise at the speed of sound from the centre to its current location (Gull & Northover 1973; Churazov et al. 2001). Alternatively, the age of the bubble can be defined as the time required for the ICM to refill the displaced volume (McNamara et al. 2000). However, \( \Delta t_b \) is not necessarily the age of the bubble: it could be the recurrence time \( t_{\text{rec}} \) of the radio source, defined as the time elapsing between two subsequent outbursts in which the jet blows bubbles. As we are looking for the characteristic time to be associated with this measure of \( L_{\text{in}} \), i.e., the time over which we average in measuring from the bubbles), it is natural to identify \( \Delta t_b \) with \( t_{\text{rec}} \), so in the rest of the paper we assume \( \Delta t_b \equiv t_{\text{rec}} \).

Birzan et al. (2004, hereafter B04) present radio and X-ray data for a sample of 18 systems with cooling flows that show evidence of cavities in their diffuse X-ray emission. The sample consists of 16 galaxy clusters, the galaxy group HCG 62, and the elliptical galaxy M84. The authors provide for each of these objects the cooling luminosity \( L_X \), the monochromatic radio luminosity at 1.4 GHz \( (L_{1.4}) \), the integrated radio luminosity between 10 MHz and 5 GHz \( (L_{\text{int}}) \), and measures of \( pV \) for each cavity. This sample is suitable for comparison with the AGN/cooling flow model, because for each source different measurements of the mechanical power (averaged on different time-scales) are available.

From measures of \( pV \) we derive \( L_{\text{in}} \) and \( L_{\text{in}} = L_{\text{in}}/L_X \) using equation \( \ref{eq:recurrence} \), given the time-scale \( \Delta t_b = t_{\text{rec}} \). In general, we do not know the recurrence time for individual sources. In the Perseus cluster, in which two generations of bubbles are observed (Fabian et al. 2000), the recurrence time-scale is of the order of 100 Myr. Estimating \( t_{\text{rec}} \) for other sources, in which just one generation of cavities is observed, is problematic, but statistical arguments suggest \( t_{\text{rec}} \) ~ few × 100 Myr as a typical value (B04). So we assume that \( \Delta t_b \) is the same for all sources, and we parameterize \( L_{\text{in}} \) as

\[
L_{\text{in}} = f_b pV/10^8 \text{ yr},
\]

where

\[
f_b \equiv \frac{\gamma}{\gamma-1} \left( \frac{\Delta t_b}{10^8 \text{ yr}} \right)^{-1}.
\]
We then use the Kolmogorov–Smirnov (KS) test to estimate how well the measured values of $L_m$ fit the predicted log-normal distribution with $\langle L_m \rangle = 1$. Fig. 3 plots the 90 and 95 per cent confidence levels, derived from the KS test, in the space of the parameters $f_b$ and $\sigma^2$. For instance, if we fix $\gamma/(\gamma - 1) = 4$, and allow $\Delta t_b$ in the range $1 \leq \Delta t_b/10^8$ yr $\leq 4$, the factor $f_b$ lies in the interval $1 \leq f_b \leq 4$. Such a range in $f_b$ corresponds to values of the variance $0.2 \leq \sigma^2 \leq 3.9$ that give distributions consistent with the data at the 95 per cent confidence level.

### 3.2 Calibrating the radio luminosity

The number of systems in which cavities have been accurately observed is small, while the radio luminosities of thousands of radio galaxies have been measured. It is clear that the radio power represents a very small fraction of the kinetic power emitted by the AGN (e.g. Eilek 2004), but the radio luminosity may be a useful indicator of the mechanical power. Hence, much larger samples could be assembled if it were possible to establish a relation between the mechanical and radio luminosities of sources.

Cavities in the X-ray emitting ICM are classified as radio-filled if non-thermal radio emission at a given frequency coincides with the X-ray-depressed area; otherwise they are called ‘ghost’ cavities. If reacceleration of the synchrotron electrons is negligible, the age of the radio-filled bubbles will be of the order of the synchrotron lifetime, or smaller, while the age of the ghosts will be longer. Thus, from radio-filled cavities we can independently measure $L_m \simeq E_b/t_c$, and $L_{1.4}$ and thus determine the relation that holds between them.

The top panel of Fig. 4 plots data for the nine objects with radio-filled cavities presented by B04. The values of $pV/t_c$ are from Chandra, and $t_c$ is from B04. $L_{1.4} = P_{1.4} \times 1.4$ GHz, where $P_{1.4}$ is the monochromatic 1.4 GHz power measured by the VLA. The straight line is the relation

$$L_m = \frac{\gamma pV}{\gamma - 1} t_c \approx \kappa_{1.4} L_{1.4}, \quad (18)$$

with the best–fitting dimensionless conversion factor $\kappa_{1.4} \approx 89\gamma/(\gamma - 1)$. The lower panel of Fig. 4 shows the result of repeating this exercise using the bolometric radio luminosity $L_{\text{rad}}$ instead of the monochromatic luminosity. $L_{\text{rad}}$ is estimated by integrated between 10 MHz and 5 GHz, assuming constant spectral index for each source (B04). The straight line is the relation

$$L_m = \kappa_{\text{rad}} L_{\text{rad}}, \quad (19)$$

with best–fitting $\kappa_{\text{rad}} \approx 11.1\gamma/(\gamma - 1)$. B04 found that a non-linear relation – the mechanical luminosity scaling as $L_{\text{rad}}^{0.62\pm0.1} –$ provides a better fit to their data (including Abell 478) radio-filled by B04, but are more likely to be already fading to ghosts, as suggested by Sun et al. (2003). Also B04 point out the anomalous high ratio of mechanical to radio power of this source.

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4 This classification depends on the frequency of the radio emission. Cavities classified as ghosts on the basis of $\sim$ GHz radio data might appear as radio-filled at lower frequencies, depending on the break frequency of the radio spectrum. This has been observed in Perseus by Fabian et al. (2002).

5 We do not include Abell 478, whose cavities are considered

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**Figure 3.** 90 and 95 per cent confidence levels in the space of the parameters $f_b = [\gamma/(\gamma - 1)]/[(\Delta t_b/10^8$ yr)] and variance $\sigma^2$ for the distribution of $L_m$ (measured from bubbles) for B04’s sample of 18 objects with cavities. The null hypothesis is that the distribution is log-normal with $\langle L_m \rangle = 1$.

**Figure 4.** The ratio $pV/t_c$ as a function of the monochromatic 1.4 GHz luminosity (top) and the bolometric radio luminosity (bottom) for radio-filled cavities (data from B04). The solid lines are the best–fitting linear relations: equations (18) and (19) with $\kappa_{1.4} = 89\gamma/(\gamma - 1)$, and $\kappa_{\text{rad}} = 11.1\gamma/(\gamma - 1)$, respectively, assuming $L_m = E_b/t_c$.\(^{4}\)\(^{5}\)
than the simple proportionality $L_m \propto L_{\text{rad}}$ considered here. However, the sample is very small, and one can show that, excluding Abell 478 and the peculiar radio source Cygnus A – the only FR II (Fanaroff & Riley 1974) source in the sample – the slope of the best-fitting power-law is close to 1. We think that a larger sample is needed to verify the existence of a non-linear relation between kinetic and radio power.

We next investigate the constraints that can be placed on $\kappa_{1.4}$ and $\kappa_{\text{rad}}$ by assuming that $\langle L_m \rangle = L_X$. We use the whole sample presented by B04, without any distinction between radio-filled and ghost cavities, and assume that the conversion coefficients $\kappa_{1.4}$ and $\kappa_{\text{rad}}$ are the same for all sources. We use the KS test to evaluate the consistency between the values of $L_m$ obtained using trial values of these coefficients and the log-normal probability distribution (top) with $\langle L_m \rangle = 1$. Fig. 6 shows contours of 90 and 95 percent confidence levels in the planes of $\sigma^2$ versus $\log(\kappa_{1.4})$ (top) or $\log(\kappa_{\text{rad}})$ (bottom). The dotted line in each panel shows the value of $\kappa$ determined above from Fig. 4 under the assumption that $\gamma/(\gamma - 1) = 4$. The data are consistent for a wide range of parameters. Larger variances $\sigma^2$ are required for smaller coefficients $\kappa$ because the current values of $L_m$ are proportional to $\kappa$, and if these are small the assumption $\langle L_m \rangle = 1$ requires that significant energy is released during excursions to high luminosity. For the values of $\kappa$ that we derived from radio-filled cavities, $\sigma^2_{\text{rad}} \simeq 3$ and $\sigma^2_{1.4} \simeq 4$ are favoured variances. These values are intermediate between the values $\approx 1$ that we inferred from microquasars and $\approx 5$ that we estimated from quasars.

### 3.3 Dependence of $\sigma^2$ on time-scale

As discussed in Section 2, the variance $\sigma^2$ associated with any measurement of $L_m$ should decrease with increasing timespan over which that measurement effectively averages $L_m$. Using radio data we estimate the kinetic power averaged over the lifetime of synchrotron-emitting relativistic electrons, which depends on the radio frequency. A reference time-scale for synchrotron radiation at frequency $\nu$ is the synchrotron cooling time

$$t_{\text{sync}} \approx 50 \left( \frac{B}{10^{-5} \text{G}} \right)^{-3/2} \left( \frac{\nu}{\text{GHz}} \right)^{-1/2} \text{Myr}, \quad (20)$$

where $B$ is the modulus of the magnetic field (e.g. Carilli et al. 1991). $t_{\text{sync}}$ depends strongly on $B$, which is usually poorly constrained in observed radio sources, and for fixed strength of the magnetic field decreases by about one order of magnitude from $\nu = 70$ MHz to $\nu = 5$ GHz. Actually, $t_{\text{sync}}$ represents an upper limit on the lifetime of the radio-emitting electrons because adiabatic expansion losses are expected to be dominant over radiative losses (e.g. Blundell & Rawlings 2000). As the effects of adiabatic expansion losses are difficult to quantify in general, in our treatment we simply assume that the averaging time-scale is $\leq t_{\text{sync}}(\nu)$, when $L_m$ is measured from radio luminosity at frequency $\nu$.

Fig. 6 plots the three ranges of acceptable values of $\sigma^2$ that we have derived above against the characteristic averaging time $\Delta t$ associated with each measurement. For the mechanical luminosity measured from the bubbles, we have $0.2 \leq \sigma^2 \leq 3.9$, and we adopt $100 \leq \Delta t_\text{mech} \leq 400$ Myr as the range for the recurrence time (see Section 3.1). For the mechanical luminosity derived from 1.4 GHz radio data, $1.8 \leq \sigma^2 \leq 6.9$ and $3 \leq \Delta t_{1.4} \leq 36$ Myr, where the range in $\Delta t_{1.4} \equiv t_{\text{sync}}(1.4 \text{GHz})$ has been computed from equation (20) for $B$ in the range $(1 - 5) \times 10^{-5}$ G – consistent with minimum-energy magnetic fields measured in the radio lobes of 3C 218 (Hydra A; Taylor et. al. 1990), 3C 84 (Perseus; Fabian et al. 2002), and 3C 405 (Cygnus A; Carilli et al. 1991). Considering the bolometric radio luminosity, we get more stringent limits on the variance, $1.9 \leq \sigma^2 \leq 4.6$, while the corresponding $\Delta t_{\text{rad}}$ spans the same range as $\Delta t_{1.4}$, because in this case the bolometric luminosity is measured from 1.4 GHz radiation. We assume that the relevant limits on the variance are those found for $\sigma^2_{\text{rad}}$, because the spread in the slopes of the radio spectra is likely to contribute to $\sigma^2_{1.4}$.

The dashed curves in Fig. 6 show the dependence of $\sigma^2$ upon $\Delta t$ that is expected for flicker noise spectra (equation (10)) with two different amplitudes, $P_{\omega \omega} = 0.24$ and 0.84. The model and the data are consistent for amplitudes that lie within this range, which overlaps the interval between the amplitudes of the radio spectra of microquasars.
and those inferred from the duty cycle of quasars. Note that for $\Delta t_{\text{rad}}$ we used the synchrotron cooling time, which is an upper limit. A shorter $\Delta t_{\text{rad}}$ would result in lower value of $P_0\omega_0$; for instance, reducing $\Delta t_{\text{rad}}$ by a factor of 10 we get $0.19 < P_0\omega_0 < 0.59$.

4 APPLICATION TO THE RADIO-GALAXY LUMINOSITY FUNCTION

In the previous section we have shown that, under plausible assumptions, observed data of central radio galaxies in cooling-flow clusters are consistent with the hypothesis that AGN heating balances radiative cooling. Here we discuss the consequences of extending the considered picture to cooling-flow systems in general. In particular, applying our model to luminous early-type galaxies leads to predictions on the local radio luminosity function.

Radio galaxies, though spanning several orders of magnitude in radio power, represent a very homogeneous family of objects when the properties of their host stellar systems are considered, being early-type galaxies with optical luminosity larger than $L^*$. For instance, Magliocchetti et al. (2002) derived the local radio luminosity function for AGN and early-type galaxies of the FIRST-2dF Galaxy Redshift Survey (2dFGRS), finding that the optical counterparts have B-band luminosity characterized by a very narrow distribution, with average $M_B \sim 5 \log h = -19.95$ and scatter 0.41 mag [$h \equiv H_0/(100 \text{ km s}^{-1} \text{ Mpc}^{-1})$, where $H_0$ is the Hubble constant].

Let us assume that all bright elliptical galaxies have a cooling core, a central supermassive BH, and that, over the galaxy lifetime, the radiative losses from their hot plasma are balanced by the mechanical energy emitted in radio outbursts from the central AGN ($L_m = L_X$). The radio luminosity of the AGN will be strongly variable, and poorly correlated with the cooling luminosity of the host cooling core. However, under the assumption that the monochromatic 1.4 GHz radio luminosity traces the kinetic luminosity ($L_m = \kappa_4 L_{1.4}$), for a sample of galaxies with the same cooling luminosity $L_X$, we would expect $(L_{1.4}) = L_X/\kappa_4$. If $\Psi(L_X)$ is the distribution of the cooling luminosities of bright early-type galaxies, the radio luminosity function of early-type galaxies will be given by

$$\Phi(L_{1.4}) = \int L_{1.4} \Psi(L_X)P(L_{1.4};L_X),$$ (21)

where $P(L_{1.4};L_X)$ is a log-normal distribution with $(L_{1.4}) = L_X/\kappa_4$ and variance $\sigma_{L_{1.4}}^2$. The luminosity function $\Psi(L_X)$ has yet to be determined observationally, because of the lack of complete X-ray samples of early-type galaxies. As well known, the diffuse X-ray luminosity of early-type galaxies with similar optical luminosity is characterized by significant scatter (Fabbiano, Gioia & Trinchieri 1989; O’Sullivan, Forbes & Ponman 2001). Given that the optical luminosity is roughly the same for all sources, it is natural to model the distribution of $L_X$ as

$$\Psi(L_X)dL_X = \frac{N_0}{\sqrt{2\pi}\sigma_X} \exp \left[ -\frac{(\ell_X - \langle \ell_X \rangle)^2}{2\sigma_X^2} \right] d\ell_X,$$ (22)

where $\ell_X = \ln(L_X)$, $\sigma_X^2$ is the logarithmic variance and $N_0$ is the number density. Substituting $\Psi(L_X)$ and $P(L_{1.4};L_X)$ in equation (21) we get

$$\Phi(L_{1.4})dL_{1.4} = \frac{N_0}{\sqrt{2\pi}\sigma_{L_{1.4}}} \exp \left[ -\frac{(\ell_{1.4} - \ln L_{1.4})^2}{2\sigma_{L_{1.4}}^2} \right] d\ell_{1.4},$$ (23)
where \( t_{1.4} = \ln(L_{1.4}), \sigma_z^2 = \sigma_{L_{1.4}}^2 + \sigma_\chi^2, \) and \( \ln L_{1.4} = \langle \chi \rangle - \ln \kappa_1 \approx \sigma_{L_{1.4}}^2/2. \) Thus, our model predicts that the radio luminosity function of early-type galaxies is log-normal with variance \( \sigma_{L_{1.4}}^2 + \sigma_\chi^2, \) where \( \sigma_{L_{1.4}}^2 \) is the variance due to radio-variability of each source and \( \sigma_\chi^2 \) is the variance in the distribution of X-ray emission of bright ellipticals.

We test this prediction by fitting the local radio luminosity function of early-type galaxies of the FIRST-2dFGRS (Magliocchetti et al. 2002) with equation \( 23, \) leaving \( \sigma_z^2, L_{1.4}^\star \) and \( N_0^\star \) as free parameters. We found best-fitting parameters (with 1 \( \sigma \) uncertainties) \( \sigma_z^2 = 7.3 \pm 1.0, \)

\[
L_{1.4}^\star = 10^{36.5\pm0.2} \text{ erg s}^{-1}, \quad \text{and} \quad N_0^\star = (1.2 \pm 0.3) \times 10^{-10} h^2 \text{Mpc}^{-3}. \]

Fig. 7 plots the data and error bars as reported by Magliocchetti et al. (2002) together with our best-fitting log-normal function. We note that the best-fit gives \( \sigma_{L_{1.4}}^2 + \sigma_\chi^2 \approx 7.3, \) consistent with values of \( \sigma_{L_{1.4}}^2 \) found analysing B04’s sample of central radio galaxies in cooling cores (Section 3.2) plus a contribution from intrinsic scatter in X-ray luminosity.

We based our analysis of the radio luminosity function on the assumption that most bright elliptical and S0 galaxies have AGN-powered radio emission. We can check this assumption a posteriori by comparing the best-fitting number density \( N_0^\star \) with the number density of luminous early-type galaxies. We assume that the B-band optical luminosity of early-type galaxies \( \phi(L_B) \) is a Schechter function with slope \( \alpha, \) characteristic luminosity \( L_B^\star, \) and normalization \( \phi^\star. \) Then the number density of objects brighter than \( L_B \) is

\[
N_B^\star(L_B) = \int_{L_B^\star}^{\infty} dL_B' \phi(L_B') = \phi^\star \Gamma(\alpha+1, L_B/L_B^\star). \]

If we consider the best-fitting B-band luminosity function determined by Madgwick et al. (2002) for the early-type galaxies of the 2dFGRS \( \alpha = -0.54\pm0.02, \phi^\star = (9.9\pm0.5)\times10^{-3} h^3 \text{Mpc}^{-3}, \) and we assume that all early-type galaxies with \( L_B > L_B^\star \) are radio galaxies, we find a number density \( N_B^\star(L_B) = (2.7 \pm 0.1) \times 10^{-4} h^3 \text{Mpc}^{-3}. \) Thus, \( N_B^\star(L_B) \) is a factor of \( \sim 2 \) larger than \( N_0^\star \) found from our best-fitting log-normal distribution. The discrepancy between the two number densities is easily explained by our arbitrary choice of \( L_B^\star \) as the minimum B-band luminosity of a radio galaxy, and by uncertainties on the faint-end of radio luminosity function.

We conclude that available radio data are consistent with the proposed scenario, in which each bright elliptical oscillates, over its lifetime, along the radio luminosity function, spanning several orders of magnitude in radio power. Interestingly, results of high-redshift radio surveys also suggest that most massive ellipticals have experienced one or more episodes of powerful radio activity during their lifetime (Rawlings 2003). Finally, we stress that the proposed picture naturally explains the very large scatter in radio luminosity for radio galaxies with the same BH mass (Lacy et al. 2001; McLure et al. 2004, and references therein) as a consequence of the strong time-variability of their radio emission (see also Nipoti et al. 2005).

5 SUMMARY AND CONCLUSIONS

Since the cosmic density of matter is very inhomogeneous on small scales, it is not possible to derive its global properties from individual measurements of the density in small regions: the characteristics of large-scale structure must be determined from a model of the fluctuating density field, and the values that one derives from it of statistical quantities, such as the mass variance. Similarly, the temporal variability of AGN is too large to ignore. We cannot hope to understand the radio luminosity function, or to test the AGN/cooling-flow model, without a model of AGN variability.

The normalized mechanical luminosities of a sample of AGN, \( L_m = L_m/L_X, \) should show scatter only because of temporal variability, and be free of differences in the scales of systems. Since the variability of AGN is known to be large and \( L_m \) is an inherently positive variable, the natural variable to model is \( t = \ln(L_m), \) and the default assumption to make is that \( t(t) \) is a Gaussian random process. This assumption implies that individual measures of \( L_m \) should have a log-normal distribution, so the time-sequence of \( L_m \) is completely characterized by the power spectrum \( P(\omega) \) of \( t(t). \)

The enormous range of physical scales involved in AGN suggests that the power spectrum will be a power law \( P(\omega) \sim \omega^{-\beta}. \) Several considerations suggest that \( \beta \approx 1, \) the index characteristic of flicker noise. First, on the short time-scales that are directly accessible, monitoring of AGN yields \( \beta \approx 1. \) Second, a comparison of the amplitudes measured in these studies with that inferred from the short duty cycle of quasars requires \( \beta \approx 1. \) Third, microquasars, whose power spectra can be probed over a comparatively wide range of frequencies, are found to have \( \beta \approx 1. \) Finally, diverse physical phenomena are found to be characterized by flicker-noise power spectra, so it would not be surprising if AGN conformed to this pattern.

We estimate the spectral amplitude of the mechanical luminosity of AGN at the centres of cooling flows by assuming that energy radiated by the thermal plasma is on the average replaced by the mechanical input of the central AGN, and that X-ray cavities probe the mechanical luminosity averaged over time-scales \( \sim 10^8 \text{yr}. \) The amplitudes required are slightly larger than those that characterize the radio output of two microquasars, and of the order of those estimated from the duty cycle of quasars.

We estimate the coefficients \( \kappa \) in an assumed proportionality between the mechanical luminosity of an AGN and its radio power, either at 1.4 GHz or bolometric. We do this in two ways. In the first we directly fit a straight line to the correlation between radio and mechanical luminosities of sources that have radio-loud cavities. Alternatively, we fit our model of AGN variability to the radio data of all sources with cavities, whether radio-loud or silent, under the assumption that \( L_m = L_X. \) The two methods yield compatible ranges of values of \( \kappa. \)

The available observations fail to constrain the model’s parameters strongly, both because the currently available sample of sources is small and because our understanding of the physics of these sources is poor. Despite the significant advance that was made possible by the discovery of numbers of X-ray cavities, measurements of the ratio of radio to mechanical luminosity are still extremely uncertain. Also, our analysis clearly shows the importance of determining the duty cycle of the radio sources in order to quantify their contribution in the energetics of cooling flows: in particular, it is crucial to estimate the amplitude of their power spectrum on time-scales \( \gtrsim 10^8 \text{Myr}. \) We conclude that larger samples of AGN/cooling-flow pairs and better theoretical models of radio sources are needed to challenge the AGN/cooling-flow model.

In a scenario in which AGN radio activity is strongly variable with time, the local luminosity function of radio galaxies should be interpreted as the distribution of the time

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spent by a galaxy at a given radio luminosity rather than a distribution of sources with different intrinsic properties. This picture is supported by the observational finding that early-type galaxies hosting supermassive BHs with the same mass span several orders of magnitude in radio power. We showed that the FIRST-2dFGRS luminosity function of radio galaxies can be fitted by a log-normal distribution, consistent with the hypothesis that in all early-type galaxies, the cooling flow is balanced by a strongly variable radio source. Our model predicts a turnover at luminosities similar to the lowest currently probed. Any turnover in the luminosity function of AGN will be to a large extent masked by radio emission from nuclear starbursts. Nonetheless, the prediction of a turnover will be tested by forthcoming radio surveys with instruments such as the Square Kilometer Array.

ACKNOWLEDGMENTS

We are grateful to Katherine M. Blundell and Steve Rawlings for helpful discussions. We used data made publicly available by the GBI-NASA monitoring program. The Green Bank Interferometer (GBI) was a facility of the National Science Foundation operated by the National Radio Astronomy Observatory in support of NASA High Energy Astrophysics programs.

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