On the dynamics of maximum extractable entanglement for open systems

E Isasi, D Mundarain and J Stephany

Departamento de Física, Sección de Fenómenos Ópticos, Universidad Simón Bolívar, Apartado Postal 89000, Caracas 1080A, Venezuela

Received 5 June 2008, in final form 3 October 2008
Published 20 November 2008
Online at stacks.iop.org/JPhysB/41/235504

Abstract
In this work we study the dynamics of the maximum extractable entanglement for a system composed of two qubits interacting either with two independent thermal baths, a common thermal bath or a common squeezed bath. The states with maximum entanglement are found by applying filtering operations which transform each state to a state in Bell diagonal form. We observe a revival of the maximum extractable entanglement for common baths. It is also shown that for some particular states in two independent baths at zero temperature, one can partially recover the initial entanglement at any time.

1. Introduction

Entanglement in systems of few particles is one of the most characteristic aspects of quantum dynamics [1, 2]. With its counterpart, decoherence, it is a key element in quantum computing, quantum cryptography and quantum teleportation [3]. Quantification of the degree of entanglement [2, 4] corresponding to a given quantum state and understanding of how it changes due to interactions with other quantum systems or with the environment are of great importance, both from a fundamental point of view or to envisage possible applications. For systems with many degrees of freedom recent developments have stressed approximate methods for the separation of the entangled component of a given configuration [2, 4, 5], but a complete characterization has not been achieved. In contrast, for two qubits the works of Peres, Horodecki, Hill and Wootters [6–9] established the base for a complete discussion of entanglement in terms of algebraic properties of the density matrix. In particular, the Peres–Horodecki criterion [6, 7] and concurrence as defined by Wootters [9] allow us to quantify entanglement for arbitrary states.

When interacting with the environment, entanglement between subsystems tends to fade away. Interaction may be represented using a bath with chosen specific properties. In some cases a complete suppression of entanglement may be observed at finite time, an effect which is referred to as entanglement sudden death (ESD) [10]. Since in general, interaction with some kind of bath is inevitable and one is interested in having maximum entanglement at our disposal it is important to device strategies to preserve or recuperate entanglement during the evolution of an open system. Among the strategies to minimize the influence of the environment are the use of the quantum Zeno effect [11], the identification and use of free decoherence spaces [12] and the application of quantum error correcting codes in quantum computation [13].

In this work we are interested in the use of local operations which allows us to recover at least partially the entanglement which has been lost in the interaction with the bath. As is known, there are situations where it is possible to improve the degree of entanglement of a pair of systems by means of filtering operations consisting of local non-unitary operations and classical communication (LSOCC) [14]. Moreover, as we discuss below for each state there exists an optimal filtering operation for which the image state is that with maximum entanglement among the accessible states [14]. The concurrence of this state is called the maximum extractable entanglement of the original state. In this paper we propose to study the evolution of the maximum extractable entanglement for two qubits in contact with a bath in order to identify conditions for which local observers using LSOCC at adequate times may recover maximum final entanglement. In the following section we review some of the fundamental concepts discussed in this paper. Then in section 3 we discuss the explicit form of the optimal filtering operation. In section 4 the dynamics of the maximum extractable entanglement for a system composed of two qubits in the presence of two independent thermal baths, a common thermal bath or a common squeezed bath is studied. In section 5 the specific case of interaction with the vacuum of two independent baths is discussed in some detail.
2. Concurrence, filtering operations and entropy

For a composed system with subsystems $A$ and $B$ in a pure state, a good measure of entanglement is the entropy of either of the two subsystems. The entanglement of formation of a mixed state $\rho$ defined as the average entanglement of the pure states appearing in its decomposition minimized over all possible decompositions [15] is also a good measure of the entanglement of such a system. It is a monotonically increasing function of concurrence introduced in [8, 9] which consequently may be taken as an entanglement measure too.

For two qubits, with density matrix $\rho$, concurrence [9] is calculated in terms of the eigenvalues $R_1, R_2, R_3, R_4$ of the related matrix $R$ defined by

$$R = \rho \sigma_y \otimes \sigma_y \rho^* \sigma_y \otimes \sigma_y.$$ (1)

It is given by

$$C = \max\{0, 2\sqrt{R_m} - (\sqrt{R_1} + \sqrt{R_2} + \sqrt{R_3} + \sqrt{R_4})\},$$ (2)

where

$$R_m = \max\{R_1, R_2, R_3, R_4\}.$$ (3)

Entanglement quantified, for example, by concurrence diminishes in general by the effect of decoherence but may be preserved in some particular situations when decoherence free subspaces are allowed. It may be also partially, but in general not totally, recovered by applying on the system some specific operations. Among them, filtering operations of the form

$$\tilde{\rho} = \frac{(A \otimes B)\rho(A \otimes B)^\dagger}{Tr[(A \otimes B)\rho(A \otimes B)^\dagger]}$$ (4)

are important because they represent the only way to increase the entanglement of the bipartite system using local operations and classical communication. Under the action of these operations the concurrence transform as [16]

$$\tilde{C} = C \frac{|\det(A)||\det(B)|}{Tr(A^4 \otimes B^4 \rho)},$$ (5)

which in particular means that separable and entangled states are mapped onto their own kind.

As we mentioned earlier it is of practical interest to find the state with maximum entanglement which can be obtained from the initial state via filtering operations. For two qubits, the density matrix can be represented in terms of Pauli matrices $(\sigma_\mu \mapsto \sigma_0 = 1_{2 \times 2} \sigma_i)$ as

$$\rho = \frac{1}{4} \left( \sum_{\mu, \nu = 0}^3 e^{i \mu \nu} \sigma_\mu^A \otimes \sigma_\nu^B \right).$$ (6)

It was shown [14] that the optimal filtering operation maps the initial state on a Bell diagonal state. These are states which can be written in the standard form

$$\tilde{\rho} = \frac{1}{4} \left( 1 + \sum_{i=1}^3 C_i \sigma_i^A \otimes \sigma_i^B \right),$$ (7)

and define a vector subspace of three dimensions with coordinates $C_i$. In this subspace, physical states form a tetrahedron whose vertices $(C_1, C_2, C_3) = \{(1, 1, 1), (0, 1, 0), (1, 0, 0), (0, 0, 1)\}$ represent pure Bell states. Separable states of this subspace form an octahedron [17]. For matrices written in the standard form it is easy to show that

$$R = \rho^2.$$ (8)

The eigenvalues of $R$ are $\{\rho_1^2, \rho_2^2, \rho_3^2, \rho_4^2\}$, and concurrence in terms of the eigenvalues $\{\rho_1, \rho_2, \rho_3, \rho_4\}$ of $\rho$ is

$$C = \max\{0, 2\rho_m - (\rho_1 + \rho_2 + \rho_3 + \rho_4)\}$$ (9)

where

$$\rho_m = \max(\rho_1, \rho_2, \rho_3, \rho_4).$$ (10)

In the parametrization (7) of the Bell diagonal states the eigenvalues of $\rho$ can be written as

$$\rho_1 = \frac{1}{2}(1 + C_1 - C_2 + C_3)$$ (11)

$$\rho_2 = \frac{1}{2}(1 - C_1 + C_2 + C_3)$$ (12)

$$\rho_3 = \frac{1}{2}(1 + C_1 + C_2 - C_3)$$ (13)

$$\rho_4 = \frac{1}{2}(1 - C_1 - C_2 - C_3).$$ (14)

To quantify the degree of mixing of the states during their evolution we use von Neuman entropy defined in terms of the eigenvalues $\rho_i$ of the density matrix as

$$S = -\sum_i \rho_i \ln \rho_i.$$ (15)

For pure states $S = 0$.

3. Optimal filtering operation

The practical problem we have to address is, given an initial state, to find explicitly at any time the maximum entangled state and the optimal filtering operation leading to it. The concurrence of this optimal state is called the maximum extractable entanglement of the state at this time. There are different approaches to compute it which depend on the rank of the initial density matrix [18–20]. Following Leinaas et al [17] we discuss an explicit procedure to perform this mapping, for a four-parameter family of states and show how the maximum extractable entanglement evolves in the presence of either two independent thermal baths, a common thermal bath or a common squeezed bath. Verstraete et al [18] showed that filtering operations (4) on two qubits correspond to Lorentz transformations

$$\tilde{\epsilon}^{\mu \nu} = L_A^\mu \tilde{L}_B^\nu e^{i \theta}$$ (16)

on the real parametrization (6) of the density matrix. The Lorentz transformations $L_A$ and $L_B$ are related with $A$ and $B$ in (4) by

$$L_A = T(A \otimes A^*)T^\dagger/|\det(A)|,$$ (17)

$$L_B = T(B \otimes B^*)T^\dagger/|\det(B)|,$$

with $T$ being the fixed matrix

$$T = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & i & -i & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix}.$$ (18)
To identify the optimal filtering operations one should first find [17] the four vectors \( \tilde{m} \) and \( \tilde{n} \) that minimize the function

\[
F(m, n) = e^{i\theta} m_n n_\nu,
\]

assuming the normalization condition \( m_\mu m_\mu = n_\nu n_\nu = 1 \) and \( m_0 \geq 0, n_0 \geq 0 \). Then the optimal Lorentz transformation is given so that

\[
L_A^0 = \tilde{m}_\mu, \quad L_B^0 = \tilde{n}_\mu.
\]

In general this transformation does not map directly the initial state to a state written in the standard form (7). An additional local unitary transformation which does not modify the entanglement of the system is necessary to this end. Nevertheless for the class of states defined by \( C_{4\times 4} \) matrices of the form

\[
C_{4\times 4} = \begin{pmatrix}
1 & 0 & 0 & d \\
0 & a & 0 & 0 \\
0 & 0 & b & 0 \\
d & 0 & 0 & c
\end{pmatrix}
\]  

which we consider in this work, the optimal filtering operation leads directly to a state in the standard form. For this class of states using the Lagrange’s multipliers method the extremal leads directly to a state in the standard form. For this class which we consider in this work, the optimal filtering operation

\[
\rho_{AB} = \frac{1}{2} \begin{pmatrix}
\alpha & 0 & 0 & 0 \\
0 & \beta & 0 & 0 \\
0 & 0 & \beta & 0 \\
0 & 0 & 0 & \alpha
\end{pmatrix}
\]

which diagonalize the real parametrization of the density matrix. This is given by

\[
L_A = L_B \begin{pmatrix}
\beta & 0 & 0 & \alpha \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
\alpha & 0 & 0 & \beta
\end{pmatrix}
\]

where \( \alpha \) satisfies both

\[
\alpha = \frac{-d(1 + 2\alpha^2)}{(1 + c)\sqrt{1 + \alpha^2}}
\]

and

\[
\alpha^2 = -\frac{1}{2} + \frac{1}{2} \sqrt{1 - \frac{4d^2}{4d^2 - (1 + c)^2}}
\]

with

\[
\beta = \sqrt{1 + \alpha^2}.
\]

Since the Lorentz transformations are not unitary the transformed density matrix must be normalized after each step. One then obtains the following nonzero entries for the final Bell diagonal state:

\[
C_1 = \frac{a}{\beta^2 + 2\alpha\beta d + \alpha^2c}, \quad C_2 = \frac{b}{\beta^2 + 2\alpha\beta d + \alpha^2c}, \quad C_3 = \frac{\alpha^2 + 2\alpha\beta d + \beta^2c}{\beta^2 + 2\alpha\beta d + \alpha^2c}, \quad C_4 = \frac{\beta^2 + 2\alpha\beta d + \alpha^2c}{\beta^2 + 2\alpha\beta d + \alpha^2c}.
\]

Using these expressions and equations (9)–(14) one can obtain the concurrence of the optimum state.

4. Maximum extractable entanglement evolution

In this section we discuss the evolution of the maximum extractable entanglement of a two qubits system in three different situations. We consider the two particles interacting with two independent thermal baths, a common thermal bath or a common squeezed bath.

We note that although final states in the presence of thermal baths are independent of initial conditions, the details of evolution may differ depending on the starting point. Since we are interested in configurations with a high entanglement degree we choose in each case a pure Bell state as the initial configuration. In what follows we show the results for the initial states \( |\Psi(0rangle = \frac{1}{\sqrt{2}}((++|--) + |--++) \) and \( |\Psi(0rangle = \frac{1}{\sqrt{2}}((++|--) + |--++) \).

If we are in the presence of independent thermal baths, the evolution of the system starting from each of the remaining Bell states is equal to one of the discussed cases. If we are in the presence of a common thermal bath, the evolution for initial states \( (1, -1, 1) \) and \( (-1, 1, 1) \) are equal.

On the other hand, the singlet belongs to the decoherence free subspace, and its evolution is trivial.

The master equation for a pair of two-level particles in the presence of two independent thermal baths is

\[
\dot{\rho} = \frac{1}{\gamma} \{(a_{\sigma_\alpha} \rho_{\sigma_\beta} - a_{\sigma_\alpha} \rho_{\sigma_\beta}) + n(2a_{\sigma_\alpha} \rho_{\sigma_\beta} - a_{\sigma_\alpha} \rho_{\sigma_\beta})
\]

\[
+ n(2a_{\sigma_\alpha} \rho_{\sigma_\beta} - a_{\sigma_\alpha} \rho_{\sigma_\beta}) + (n + 1)(2a_{\sigma_\alpha} \rho_{\sigma_\beta} - a_{\sigma_\alpha} \rho_{\sigma_\beta})
\]

\[
+ n(2a_{\sigma_\alpha} \rho_{\sigma_\beta} - a_{\sigma_\alpha} \rho_{\sigma_\beta})\}
\]

where \( \sigma_\alpha = \sigma_\alpha \otimes 1, \sigma_\beta = 1 \otimes \sigma_\beta \) and \( \gamma \) is the vacuum decay constant. We assume that both baths have the same temperature i.e. they have the same average number \( n \) of thermal photons.

It is easy to show that evolution with this master equation preserves the form of states defined by (21). When Bell states are taken as initial conditions one obtains for the nonzero elements \((a, b, c, d)\) of (21) a set of equations whose solutions can be found either analytically or numerically. In the particular case \( n = 0 \) the analytical solutions are discussed in section 5. Meanwhile, using the results of the previous section, we obtain numerically the evolution of the entanglement and maximum extractable entanglement. In figures 1 and 2 we plot the evolution of concurrence and maximum extractable entanglement for two different initial states. In the first case we take \( |\Psi(0rangle = \frac{1}{\sqrt{2}}((++|--) + |--++) \) as the initial state and in the second the initial state is \( |\Psi(0rangle = \frac{1}{\sqrt{2}}((++|--) + |--++) \).

These figures clearly show that although entanglement evolution is quite similar for both initial conditions, the behaviour of maximal extractable entanglement is radically different. In particular one can observe a better preservation of maximum extractable entanglement for the first initial condition. This may be understood as the result of a balance between two characteristics of the state: the degree of mixing and the entanglement. For a pure state with non-vanishing concurrence the maximum extractable entanglement is always one. Correspondingly the filtering operations are more efficient on states which are almost pure. On the other hand
This is illustrated with the use of entropy defined in (15). In operation is not able to improve the degree of entanglement. For a mixed state with a very small concurrence the filtering evolution for the initial Bell states $(1, 1, -1), |\Psi(0)\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |\rangle)$ in two independent thermal baths $(n = 0.001)$. for a mixed state with a very small concurrence the filtering operation is not able to improve the degree of entanglement. This is illustrated with the use of entropy defined in (15). In figure 1 the entropy decays sufficiently fast and the maximum extractable entanglement remains larger until entanglement sudden death when of course it also vanishes. For the second case (figure 2) the entropy for the second initial condition decays too slowly and when the state finally is almost pure (entropy almost zero) there is no longer sufficient entanglement to be enhanced. Note also that in this case, as in the case of a common bath discussed below, entanglement sudden death appears sharply, and the ESD time may be computed (see the appendix).

For a pair of two level particles in the presence of a common thermal bath the master equation becomes

\[
\dot{\rho} = \frac{\gamma}{2} [(n + 1)(2\sigma\rho\sigma^\dagger - \sigma^\dagger\sigma\rho - \rho\sigma^\dagger\sigma) \\
+ n(2\sigma^\dagger\rho\sigma - \sigma\rho\sigma^\dagger - \rho\sigma\sigma^\dagger)] \tag{30}
\]

where $\sigma = \sigma_a + \sigma_b$. The form of the matrix (21) is also preserved in this evolution and as in the previous case one can obtain the differential equations for the nonzero elements of the real parametrisation of the density matrix. In figure 3 the evolution of concurrence and maximum extractable entanglement are displayed for the initial state $|\Psi(0)\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |\rangle)$ and in figure 4 for the case in which the initial state is $|\Psi(0)\rangle = \frac{1}{\sqrt{2}}(|++\rangle + |\rangle)$. In figure 4 we observe that although in this case concurrence always decreases there is a sector in which we obtain a revival of the extractable entanglement. In other words a decreasing of concurrence does not always imply decreasing of maximum extractable entanglement. This is a more dramatic consequence of the competition between entanglement and mixing, discussed in the case above.

Finally we consider evolution of a pair of two level particles in the presence of a common squeezed bath. The master equation is

\[
\dot{\rho} = \frac{\gamma}{2} [(n + 1)(2\sigma\rho\sigma^\dagger - \sigma^\dagger\sigma\rho - \rho\sigma^\dagger\sigma) \\
+ n(2\sigma^\dagger\rho\sigma - \sigma\rho\sigma^\dagger - \rho\sigma\sigma^\dagger)] \\
- \frac{\gamma m}{2} [e^{i\psi}(2\sigma^\dagger\rho\sigma - \sigma\rho\sigma^\dagger - \rho\sigma\sigma^\dagger) \\
- e^{-i\psi}(2\sigma\rho\sigma - \sigma^\dagger\sigma\rho - \sigma^\dagger\sigma^\dagger)] \tag{31}
\]

where $m = \sqrt{n(n + 1)}$ and $\psi$ are the squeezing parameters.
Moreover in the stationary regime almost all the entanglement and of the maximum extractable entanglement is recoverable. One can also partially recover the entanglement by the master equation (31)\[21–23\]. Moreover for states with nonzero components of the density matrix satisfy the following equations:

\[
\begin{align*}
\dot{d} &= -\gamma(1 + d + 2nd) \\
\dot{a} &= -\gamma(a + 2na) \\
\dot{b} &= -\gamma(b + 2nb) \\
\dot{c} &= -2\gamma(d + c + 2nc).
\end{align*}
\]

For \(n \neq 0\) the solutions of these differential equations are

\[
\begin{align*}
d(t) &= \frac{e^{-\gamma(1+2n)t} - 1}{1 + 2n} \\
\dot{a}(t) &= e^{-\gamma(1+2n)t} \\
\dot{b}(t) &= e^{-\gamma(1+2n)t} \\
\dot{c}(t) &= -e^{-2\gamma(1+2n)t} - \frac{2e^{-\gamma(1+2n)t} - e^{-2\gamma(1+2n)t} - 1}{(1 + 2n)^2}.
\end{align*}
\]

For any finite \(t\) when \(n \to 0\)

\[
4d^2(t) - (1 + c(t))^2 \to 0.
\]

Then from (23–24) and (26–28) one obtains

\[
\alpha \to \infty \quad C_1 \to 1 \quad C_2 \to 1 \quad C_3 \to -1.
\]

The optimal state at any time is equal to the initial Bell state \((1, 1, -1)\). In the limit case the complete initial entanglement is recoverable. One can also partially recover the entanglement to any desired degree at any finite time using a finite boost. By
numerical analysis one verifies that with the boost $\gamma t \approx \sqrt{a}$ one recovers almost all the entanglement at any finite time $t$. In figure 7 we show the extracted entanglement with $\alpha = 9$. In this case one recovers almost all the entanglement if $\gamma t \approx 3$.

6. Conclusion

In this work we studied the evolution of the maximum extractable entanglement for an open system of two qubits considering three different baths. For two independent thermal baths at zero temperature we show that it is possible to recover almost all the initial entanglement using finite operations of local filtering. In the case of a common thermal bath we observed during the evolution an increasing of the maximum extractable entanglement when entanglement was in fact diminishing. Related to this it is important to note that the maximum extractable entanglement is a property of the state and not of the evolution. In this case what is happening is that evolution drove the system to states with less entanglement but more extractable entanglement. This suggest as a strategy to manipulate efficiently the entanglement to set the conditions of interaction of the system with the environment in such a way as not to preserve maximum entanglement but maximum extractable entanglement.

Acknowledgments

This work was supported by Did-Usb Grant Gid-30 and by Fonacit Grant No. G-2001000712.

Appendix. Entanglement sudden death time

In the case of two independent baths the analytical solutions for the nonzero entries of the real parametrization of the density matrix has a simple structure. If the initial state is the Bell state $(1, -1, 1)$, the time evolution equations reads

\[ d(t) = e^{-\gamma(1+2n)t} - 1 \quad \text{(A.1)} \]

\[ a(t) = e^{-\gamma(1+2n)t} \quad \text{(A.2)} \]

\[ b(t) = -e^{-\gamma(1+2n)t} \quad \text{(A.3)} \]

\[ c(t) = e^{-2\gamma(1+2n)t} - 2 e^{-\gamma(1+2n)t} - e^{-2\gamma(1+2n)t} - 1/(1+2n)^2 \quad \text{(A.4)} \]

The associated Bell diagonal state is given by equation (7), and the corresponding diagonal entries $C_i$ are given by equations (26)–(28). Since filtering operations map entangled states to entangled states and separable states to separable states, it is possible to compute the ESD time using the associated Bell diagonal state, instead of the non-filtered states. Following equations (9) to (14) it is easy to show that the concurrence of a Bell diagonal state is equal to $C = 2\rho_m - 1$, which, for the initial state $(1, -1, 1)$ gives as the resulting expression $C = 1/2(C_1 - C_2 + C_3 - 1)$. This relation allows us to compute the ESD time straightforwardly as

\[ \gamma t \approx \sqrt{a} \quad \text{for small } n \quad \text{and a slow decreasing for high } n. \]

The dependence of $T_{\text{ESD}}$ with $n$ for the two initial states is displayed in figure 8. We can appreciate a fast decreasing (from infinity) for small $n$ and a slow decreasing for high $n$. In both cases the limits are equal to zero.
In the case of a system of two particles interacting with a common thermal bath, the evolution of the real parametrization of the density matrix can also be determined analytically. These equations allow us, in principle, to compute the ESD time in the same way as was done for the independent thermal baths. However, the resulting relations are very involved and to express the ESD time in terms of a transcendental equation should be solved. Instead of the analytical expression we show in figure 9 the evolution of $T_{\text{ESD}}$ computed numerically, for both initial states. We note that the qualitative dependence in this case is similar to that obtained in the previous case.

References

[1] Einstein A, Podolsky B and Rosen N 1935 *Phys. Rev.* **47** 777
[2] Horodecki R, Horodecki P, Horodecki M and Horodecki K 2007 arXiv:quant-ph/0702225
[3] Bennett C H, Brassard G, Crépeau C, Jozsa R, Peres A and Wootters W K 1993 *Phys Rev Lett.* **70** 1895
[4] Plenio M B and Virmani S 2007 *Quant. Inf. Comput.* **7** 1
[5] Lewenstein M and Sanpera A 1998 *Phys. Rev. Lett.* **80** 2261
[6] Peres A 1996 *Phys. Rev. Lett.* **77** 1413
[7] Horodecki M, Horodecki P and Horodecki R 1996 *Phys. Lett. A* **223** 1
[8] Hill S and Wootters W K 1997 *Phys. Rev. Lett.* **78** 5022
[9] Wootters W K 1998 *Phys. Rev. Lett.* **80** 2245
[10] Yu T and Eberly J H 2004 *Phys. Rev. Lett.* **93** 140404
[11] Maniscalco S, Francica F, Zaffino R L, Gullo N Lo and Plastina F 2008 *Phys. Rev. Lett.* **100** 090503
[12] Lidar D A, Chuang I L and Whaley K B 1998 *Phys. Rev. Lett.* **81** 2594
[13] Nielsen M A and Chuang I L 2000 *Quantum Computation and Quantum Information* (Cambridge: Cambridge University Press)
[14] Kent A, Linden N and Massar S 1999 *Phys. Rev. Lett.* **83** 2656
[15] Bennett C H, DiVincenzo D P, Smolin J A and Wootters W K 1996 *Phys. Rev. A* **54** 3824
[16] Verstraete F, Audenaert K and De Moor B 2001 *Phys. Rev. A* **64** 012316
[17] Leinaas J M, Myrheim J and Ovrum E 2006 *Phys. Rev. A* **74** 012313
[18] Verstraete F, Dehaene J and De Moor B 2001 *Phys. Rev. A* **64** 010101
[19] Bennett C H, Berstein H J, Popescu S and Schumacher B 1996 *Phys. Rev. A* **53** 2046
[20] Cen L-X, Wu N-J, Yang F-H and An J-H 2002 *Phys. Rev. A* **65** 052318
[21] Palma G M and Knight P L 1989 *Phys. Rev. A* **39** 1962
[22] Ekert A K, Palma G M, Barnett S M and Knight P L 1989 *Phys. Rev. A* **39** 6026
[23] Mundarain D and Orzag M 2007 *Phys. Rev. A* **75** 040303