CLASSICAL/QUANTUM DUALITY

M. J. Duff

Isaac Newton Institute for Mathematical Sciences
University of Cambridge
20 Clarkson Road, Cambridge CB3 0EH, U.K.

ABSTRACT

String theory requires two kinds of loop expansion: classical ($\alpha'$) world-sheet loops with expansion parameter $< T >$ where $T$ is a modulus field, and quantum ($\hbar$) spacetime loops with expansion parameter $< S >$ where $S$ is the dilaton field. Four-dimensional string/string duality (a corollary of ten-dimensional string/fivebrane duality) interchanges the roles of $S$ and $T$ and hence interchanges classical and quantum.

October 1994

1Talk delivered at the International Conference on High Energy Physics, Glasgow, July 1994.
2On leave of absence from the Center for Theoretical Physics, Texas A&M University, College Station, Texas 77843. Research supported in part by NSF Grant PHY-9411543.
1 Classical/Quantum Duality

There is now a consensus that the really important questions of string theory will never be answered within the framework of a weak coupling perturbation expansion. Here I describe some recent work which begins to address this strong coupling problem. It is based on the idea that the same physics may equally well be described by the fundamental four-dimensional superstring or by a dual four-dimensional superstring \[ \text{dual} \] that corresponds to a soliton solution of the fundamental string. In this respect, the idea provides a stringy generalization of the old Montonen-Olive conjecture \[ \text{conjecture} \] of a duality between the electrically charged particles of a fundamental supersymmetric theory and its magnetically charged solitons. Indeed, the latter duality is in fact subsumed by the former in that the solitonic magnetic H-monopoles \[ \text{H-monopoles} \] of the fundamental string are the fundamental electric winding states of the dual string \[ \text{dual string} \].

This four-dimensional string/string duality is a corollary of the ten-dimensional string/fivebrane duality which states that the same physics may equally well be described by the fundamental ten-dimensional superstring (an extended object with one spatial dimension) or by a dual ten-dimensional superfivebrane \[ \text{fivebrane} \] (an extended object with five spatial dimensions) that corresponds to a soliton solution of the fundamental string \[ \text{fundamental string} \]. The pay-off, if such a conjecture proves to be true, is that the strongly coupled string corresponds to the weakly coupled fivebrane. After compactification to four dimensions, the fivebrane will appear as an H-monopole or a dual string according as it wraps around 5 or 4 of the compactified dimensions \[ \text{compactified dimensions} \] which, for concreteness and simplicity, we take to be a six-dimensional torus \[ \text{torus} \]. The inverse tension of the dual string, \( 2\pi \tilde{\alpha}' \), is related to that of the fundamental string, \( 2\pi \alpha' \), by the Dirac quantization rule \[ \text{quantization rule} \]

\[
8G R^2 = n \alpha' \tilde{\alpha}' \quad n = \text{integer}
\]

where \( G \) is Newton’s constant and \( R \) is the compactification scale. One’s first guess might therefore be to assume that the strongly coupled four-

\[ \text{dual string of} \] is not to be confused with the stringy cosmic string of \[ \text{cosmic string} \]. The two solutions are different.

\[ \text{It could in principle also appear as a membrane by wrapping around 3 of the compactified dimensions, but the fundamental four-dimensional string obtained in this way does not admit the membrane soliton} \].
**Fundamental string** | **Dual string**
---|---
**Moduli** | $T = b + ie^{-\sigma}$ | $S = a + ie^{-\eta}$
**Worldsheet coupling** | $< e^{\sigma} > = \alpha'/R^2$ | $< e^{\eta} > = g^2$
**Large/small radius** | $R \rightarrow \alpha'/R$ | $g \rightarrow 1/g$
**T – duality** | $O(6, 22; Z)$ | $SL(2, Z)$
**Axion/dilaton** | $S = a + ie^{-\eta}$ | $T = b + ie^{-\sigma}$
**Spacetime coupling** | $< e^{\eta} > = g^2$ | $< e^{\sigma} > = \alpha'/R^2$
**Strong/weak coupling** | $g \rightarrow 1/g$ | $R \rightarrow \alpha'/R$
**S – duality** | $SL(2, Z)$ | $O(6, 22; Z)$

Table 1: Duality of dualities

The dimensional fundamental string corresponds to the weakly coupled dual string, but in fact something more subtle and interesting happens. The fundamental string exhibits a minimum/maximum length duality, $R \rightarrow \alpha'/R$, called $T$-duality, manifest order by order in perturbation theory. There is also evidence that it exhibits a minimum/maximum coupling constant duality $g \rightarrow 1/g$, called $S$-duality, which is intrinsically non-perturbative.

In going from the string to the dual string, these two dualities trade places leading to a *duality of dualities* as illustrated in Table 1.

String theory requires two kinds of loop expansion: classical ($\alpha'$) worldsheet loops with expansion parameter $< e^{\sigma} >$ where $\sigma$ is a modulus field, and quantum ($\hbar$) spacetime loops with expansion parameter $< e^{\eta} >$ where $\eta$ is the dilaton field. Introducing the axion field $a$ and another pseudoscalar modulus field $b$, four-dimensional string/string duality interchanges the roles of $S = a + ie^{-\eta}$ and $T = b + ie^{-\sigma}$, and hence interchanges classical and quantum. Thus this duality of dualities exhibited by four-dimensional strings is entirely consistent with the earlier result that ten-dimensional string/fivebrane duality interchanges the spacetime and worldsheet loop expansions, and is entirely consistent with the Dirac quantization rule that follows from an earlier string/fivebrane rule. Thus, for $n = 1$, we have

\[
< e^{\eta} > = g^2 = 8G/\alpha' = \tilde{\alpha}'/R^2
\]
\[
< e^{\sigma} > = \tilde{g}^2 = 8G/\tilde{\alpha}' = \alpha'/R^2
\]

where $\tilde{g}$ is the dual string spacetime loop expansion parameter.
Group theoretically, these dualities are given by $O(6, 22; Z)$ in the case of $T$-duality and $SL(2, Z)$ in the case of $S$-duality. It has been suggested \cite{17, 4} that these two kinds of duality should be united into a bigger group $O(8, 24; Z)$ which contains both as subgroups. This would have the bizarre effect of eliminating the distinction between classical and quantum.

\section{Acknowledgments}

I am grateful to the Director and Staff of the Isaac Newton Institute, and to the organizers of the \emph{Topological Defects} programme, for their hospitality.
References

[1] M. J. Duff and R. R. Khuri, Nucl. Phys. B411 (1994) 473.

[2] B. R. Greene, A. Shapere, C. Vafa, and S. T. Yau, Nucl. Phys. B340 (1990) 33.

[3] C. Montonen and D. Olive, Phys. Lett. B72 (1977) 117.

[4] R. R. Khuri, Phys. Lett. B259 (1991) 261; Nucl. Phys. B387 (1992) 315.

[5] J. Gauntlett, J. H. Harvey and J. Liu, Nucl. Phys B409 (1993) 363.

[6] M. J. Duff, R. R. Khuri, R. Minasian and J. Rahmfeld, Nucl. Phys. B418 (1994) 195.

[7] M. J. Duff and J. Rahmfeld, CTP-TAMU-25/94, hep-th/9406103.

[8] M. J. Duff, Class. Quantum Grav. 5 (1988) 189.

[9] A. Strominger, Nucl. Phys. B343 (1990) 167.

[10] M. J. Duff and J.X. Lu, Nucl. Phys. B354 (1991) 141.

[11] A. Font, L. Ibanez, D. Lust and F. Quevedo, Phys. Lett. B249 (1990) 35.

[12] C. Vafa and E. Witten, hep-th/9408074, HUTP-94/A017, IASSNS-HEP-94-54.

[13] J. H. Schwarz and A. Sen, Phys. Lett. B312 (1993) 105.

[14] P. Binetruy, Phys. Lett. B315 (1993) 80.

[15] A. Sen, TIFR/TH/94-03, hep-th/9402002.

[16] M. J. Duff and J.X. Lu, Nucl. Phys. B357 (1991) 534.

[17] M. J. Duff and J.X. Lu, Nucl. Phys. B347 (1990) 394.