Finite hadronization time and unitarity in quark recombination model

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The effect of finite hadronization time is considered in the recombination model, and it is shown that the hadron multiplicity turns out to be proportional to the initial quark number and unitarity is conserved in the model. The baryon to meson ratio increases rapidly with the initial quark density due to competition among different channels.

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In last few years, the quark recombination (or coalescence) model has aroused intense interest in the communities of both theorists and experimentalists in the field of ultra-relativistic heavy ion collision physics, because of its ability to explain novel phenomena, such as unexpected high proton over pion ratio at $p_T$ about 3 GeV/$c$ and the constituent quark number scaling of the elliptic flow, that are observed in RHIC experiments. Although different physics considerations are input in the investigation of final state particle spectra in different implementations of the quark recombination model by different groups, there are a lot of common points. All implementation of the model, except the latest attempt, considered only quark and antiquark degrees of freedom and gluons are assumed having been converted into quarks and antiquarks before hadronization. Every group assumed implicitly that hadronization occurs instantly. In heavy ion collision processes there are plenty of soft thermalized partons and some hard partons which will evolve into showers of semihard partons. Because of the absence of gluon degree of freedom in the model, quarks (and antiquarks) at the moment just before hadronization must have been dressed up as constituent quarks ready for recombining into final state hadrons. It has been shown that the recombination of thermal quarks dominates in Au+Au collisions at RHIC energies for low $p_T$ pion production and the traditional fragmentation, which can be interpreted as a recombination process from shower partons originated from a hard parton, dominates at very high $p_T$, while the thermal-shower recombination is important for moderately high $p_T$, 3GeV/$c < p_T < 8$GeV/$c$ in central Au+Au collisions. In this way, all hadron production can be considered consistently in the framework of quark recombination model. In every implementation of the quark recombination model, the transverse momentum spectrum of mesons at mid-rapidity can be written, after some algebra, as

$$\frac{dN^M}{p_T dp_T} = \int dp_1 dp_2 F(p_1, p_2) R^M(p_1, p_2, p_T),$$

where a factor $\delta(p_T - p_1 - p_2)$ is included in the recombination function $R^M(p_1, p_2, p_T)$ to ensure momentum conservation and $F(p_1, p_2)$ is associated with the joint quark-antiquark momentum distribution. When we are interested in the total multiplicity of a kind of meson, we can consider the contribution from thermal-thermal recombination only, because most produced hadrons are in the low transverse momentum region where pure thermal recombination dominates. In this range of transverse momentum, the joint distribution for quark-antiquark pair can be written as $F(p_1, p_2) = V \rho^2 f_1(p_1) f_2(p_2)$ with $V$ the spatial volume of the partonic system and $\rho$ the thermal parton density just before hadronization when the (anti)quark transverse momentum distributions $f_{1,2}$ are normalized to 1. A simple conclusion from Eq. (1) is that the total yield of the meson is proportional to the square of quark density just before hadronization. Similarly, one can conclude that the yield of a kind of baryon should be proportional to the cubic of the quark density. On the one hand, if one considers production of pentaquarks along the same line, one has to say that its yield is proportional to $\rho^5$ and higher in heavy ion collisions than in elementary ones. This conclusion seems to contradict to current experimental observations. On the other hand, that the yield in a given coalescence channel scales quadratically/cubically with the constituent number violates unitarity, as argued in 2-12, because the total number of hadrons should be linearly proportional to that of constituent quarks just before hadronization. Apparently, the total number of constituent quarks in final state hadrons should be equal to the quark number just before hadronization. Considering above, one has to conclude that something must be changed in current formulism of the recombination model.

An important observation in this paper is that the numbers of constituent quarks are decreasing at finite rate during the hadronization process if one assumes that the hadronization lasts a nonzero period of time. Then one should study the effect of finite hadronization time on the yields of hadrons in the recombination model. For this purpose, one can interpret the recombination function $R^M$ as the average production rate of meson from a given pair of quark and antiquark and write the joint two-quark distribution as a function of time explicitly. Then the right hand side of Eq. (1) should be $dN^M/p_T dp_T dt$. Here time $t$ can be defined in the laboratory frame. In this paper, we will show that the introduction of the fi-
nite hadronization time in the recombination model can ensure unitarity.

Instead of considering the spectra of different hadrons in the framework of quark recombination models, we only investigate the yields of mesons and baryons, as done in earlier work of the coalescence model [13]. For simplicity, we assume in this paper that quarks and antiquarks have the same density in the almost baryon free central rapidity region and that the shapes of the all transverse momentum distributions involved remain unchanged in the hadronization process. The strange contents are not considered in this paper. If the phase transition from partons to hadrons is of first-order, there is a boundary and hadrons are produced only on the surface while the partons inside have a constant temperature and/or density. If, on the other hand, the phase transition is of second-order, no boundary between parton state and hadron state can be defined, and hadronization takes place in the whole system at the same time. Most event generators for high energy collisions adopt the second scenario in the codes. Such a scenario was also taken in [14]. In ultra-relativistic heavy ion collisions the produced partonic system expands. Then the phase transition may be neither first-order nor second-order. If the expansion retains to the last stage of the evolution and the phase transition is likely first-order, there are two competing trends on the change of the volume. The expansion will make the partonic volume larger, and the hadronization tries to shrink the system. Thus the hadronization dynamics should, generally speaking, be connected with the hydrodynamical calculations. This cannot be included in this paper. Now we simply consider two limiting cases for the evolution of the volume in hadronization process.

The first case we consider is for fixed volume for partonic system. In other words, the expansion of the system is assumed to be compensated by the shrinking from hadronization. Because the hadronization time is very short, this case can be regarded as the limit of extremely rapid expansion.

First of all, we assume that only one species of meson can be produced in the process. Then one can integrate over the momentum of produced hadrons and get the production rate of the meson as

$$\frac{dN^M}{dt} = AV \rho^2(t) ,$$

and the information of both the shape of quark distribution and the recombination function is encoded in $A$. No reverse term for the process from hadron to quarks is considered here, because (i) most of the produced hadrons will move quickly to the detectors, thus the hadron density in the reaction zone is much smaller than that for quarks, and (ii) at the energy scale corresponding to the phase transition (about 170 MeV), it is very unlikely for hadrons to dissolve into quarks and antiquarks in the interactions. As a consequence, the interactions among the produced hadrons will be elastic and no lose term appear in Eq. (2). With the production of the meson, the constituent quark number decreases accordingly, and we have

$$\frac{d(V \rho)}{dt} = -\frac{dN^M}{dt} .$$

This equation comes from the model assumption that just before hadronization all partons have been converted into constituent quarks ready for recombination into final state hadrons and no more constituent quarks can be created. Finally, all those $N_q + N_{\bar{q}}$ constituent quarks have to be inside some hadrons, because of color confinement. Thus as time goes to infinity, $\rho$ must go to zero from initial density $\rho_0 = N_q/V$ just before hadronization. Equations corresponding to Eqs. (2) and (3) can be found for any chemical reaction $A + B \rightarrow C$ which is obviously a combination process similar to the process $q + \bar{q} \rightarrow \pi$ studied in the quark recombination model.

For chemical reactions the amount of final product always depends linearly on the initial density. From above two equations, one can easily deduce the yield as

$$N^M = V \rho_0 = N_q .$$

Therefore, the total multiplicity of the meson turns out to be equal to the initial number of constituent quark, and unitarity is maintained in the recombination process. A careful reader may have noticed that the parameter $A$ introduced in Eq. (2) is not relevant to the total multiplicity of the meson, as shown in Eq. [11]. This is expected from quark number counting.

If only one species of hadrons consisting of $n$ constituent quarks is considered, the total multiplicity $N^n$ can be shown, along the same line as for mesons, to be

$$N^n = \frac{2V\rho_0}{n} = \frac{2N_q}{n} .$$

Again the total multiplicity is proportional to the initial number of constituent quark and there is no violation of unitarity.

In real high energy collisions, many different species of particles are produced. In the recombination model, the production rates for different hadrons depend on quark distribution differently. For mesons the production rate depends on quark density quadratically, while for baryons the dependence is of cubic. So there exist competitions among different channels and the effect of competition depends on the initial number of quarks. To get a parameter free expression, we assume in this investigation that only non-strange mesons and (anti)baryons are produced. Then we have

$$\frac{dN^M}{dt} = A_1 V \rho^2(t) ,$$
$$\frac{dN^B}{dt} = A_2 V \rho^3(t) ,$$
$$\frac{d(V \rho)}{dt} = - \left( \frac{dN^M}{dt} + 3 \frac{dN^B}{dt} \right) .$$
with initial conditions $N^{M,B} = 0$ at $t = 0$, $\rho(0) = \rho_0$. There are exactly the same number $N^B$ anti-baryons. The third expression in last equation ensures the unitarity in hadronization process. Because unitarity condition was not taken into account in former applications of the recombination model, the result in this paper will not be reduced to the old ones even in the limit of zero hadronization time. From the reaction-rate theory \cite{15}, $A_1$ and $A_2$ are proportional to the corresponding cross-sections and the degeneracy factors. Above equations can be solved numerically. First we can define dimensionless quantities $u = 3A_2\rho_0/A_1$, $r = \rho(t)/\rho_0$, $\tau = A_1\rho_0 t$. Here variable $u$ has encoded initial quark density and the competition between meson and baryon channels. With these scaled variables above equations can be normalized to

$$
\frac{dN^M}{ud\tau} = \frac{VA_1}{3A_2}r^2(u, \tau), \quad (7)
$$

$$
\frac{dN^B}{ud\tau} = \frac{VA_1}{9A_2}ur^3(u, \tau), \quad (8)
$$

where the scaled quark density $r(u, \tau)$ at scaled time $\tau$ is the solution of the equation

$$
\tau = u \ln r + \frac{1}{r} - 1 - u \ln \frac{1 + ur}{1 + u}. \quad (9)
$$

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig1}
\caption{Behavior of scaled density as a function of scaled time for fixed $u = 1$ and 5.}
\end{figure}

The relative density $r$ are shown in Fig. 1 as a function of scaled time $\tau$ for $u = 1$ and 5, respectively. For larger $u$, $r$ decreases more quickly with $\tau$. This is not surprising because of the quadratical and cubical dependence on $r$ of the rates for meson and baryon production. From the fact that the constituent quark density decreases with $\tau$ very rapidly, we can see that the average production time for baryons is shorter than that for mesons. Or in other words, baryons are produced earlier than mesons in the hadronization process. It has been shown that the pion size measured by interferometry increases with the pion multiplicity \cite{16}, steadily with bombarding energies in similar systems \cite{17}. For protons, the measurements of NA49 \cite{18} together with measurement at lower energies suggested a very weak dependence of the two-proton correlation function on bombarding energies. If protons and pions share the same collective expansion, smaller source size means earlier production. Our results agree qualitatively with those experimental conclusions.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig2}
\caption{Dependence of the scaled multiplicities of mesons and baryons on the scaled initial density $u$.}
\end{figure}

Once we know the relative quark density as a function of scaled time $\tau$, one can integrate Eqs. (7) and (8) to get the multiplicities of mesons and baryons produced in the hadronization process. There exists an unknown constant factor $VA_1/A_2$. In the following, we define scaled multiplicities of mesons and baryons as

$$
N_s^{M,B} = \frac{3N^{M,B}A_2}{uA_1V}. \quad (10)
$$

One can easily see that $N_s$ is proportional to hadron yield divided by the quark density just before hadronization. $N_s^{M,B}$ are shown in Fig. 2 as functions of $u$. As the initial quark density increases, the scaled yield for mesons decreases, indicating that the meson yield increases slower than the constituent quark density does. Meanwhile the scaled yield for baryons increase quickly at small $u$, therefore the baryon yield increases faster than the quark density. The difference between the trends of $N^M$ and $N^B$ as $u$ increases can be shown more clearly by the baryon to meson ratio $N_B^B/N_M^M$, as in Fig. 3. The ratio increases with $u$ almost linearly in the range shown. In the near future at LHC, it can be predicted from Fig. 3 that the baryon to meson ratio will become even larger, since the initial constituent quark density (or $u$) will be higher. If the scaled constituent quark density can become high enough (about $u = 8$), the yield of baryons will be about the same as that for mesons, and the baryon to meson ratio can be more than 2 at $u = 20$, if it can be achieved.
For the second case we assume that there is no expansion at all. Thus the system shrinks to 0 from initial volume $V_0$ while the parton density remains the same in hadronization. Since $\rho$ is now a constant, we get from Eq. (10)

$$V_0 = V_0 \rho e^{-(A_1 \rho + 3 A_2 \rho^2) t}.$$  \hspace{1cm} (11)

In this case the hadronization is much faster than in the first case, because the volume of parton phase decreases exponentially. The multiplicities of produced mesons and baryons are

$$N^M = \frac{A_1 N_q}{A_1 + 3 A_2 \rho}, \quad N^B = \frac{A_2 N_g \rho}{A_1 + 3 A_2 \rho}. \hspace{1cm} (12)$$

Then we have the baryon to meson ratio $N^B/N^M = u/3$. Now the ratio turns out to be proportional to the initial parton density and is larger than the corresponding values for the first case, as shown in Fig. 3.

In real ultra-relativistic heavy ion collisions the parton system is always expanding in hadronization. The expansion will slow down the shrinking of the parton volume in hadronization. Thus the two cases considered in this paper are the two limiting evolutions of the system. The real dependence of hadron yield will be in between the results for the two limiting cases.

One may have noticed that we only considered in this paper the contribution to particle production from the recombination of plenty soft quarks, which is the most important at low $p_T$. This discussion may be applied to particle production at SPS, RHIC and LHC energies with different initial parton density or volume as long as a dense parton phase is created. Thus the rapid increase of baryon to meson ratio can be a signal of the formation of the dense medium of soft partons, and the density of the medium can be deduced approximately from the measured $p/\pi$ ratio. The initial parton density just before hadronization may depend only on the colliding energy. So the baryon to meson ratio for collisions at fixed center of mass energy will be proportional to the ratio of the reaction rates, or in other words, proportional to the ratio of the degeneracy factors, as claimed by the statistical hadronization model.

It should be pointed out that the shape of constituent quark distributions can, in reality, be changing in the hadronization process. As a result, above results may be modified. However, one can easily imagine that the shape changes more slowly than the normalization of the distribution. Then the modification will not be too big. In Eqs. (7) and (8) the shape dependence can be canceled partially in $A_1/A_2$. So, qualitatively, the results from this investigation will not be changed when the evolution of shapes of quark distributions is taken into account.

In conclusion, we showed in this paper the importance of the evolution of constituent quark density in the hadronization process. When the evolution is taken into account, unitarity can be maintained, and the baryon to meson ratio is shown to increase monotonically with the initial constituent quark density.

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