The difference logical operation for images in optical echo holography

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Abstract. In this paper, we consider the implementation of logical operations on images using the accumulated long-lived echo hologram. It is shown that the accumulated echo hologram could be used for the implementation of the set difference logical operation.

1. Introduction
Echo holography is a process of recording and reconstructing light wave front when reference, object, and reconstructed waves do not match in time [1–6]. Echo holography may be applied when recording, reproducing, and transforming images. As images represent sets of points, logical operations on sets may be applied for their transformation.

The work [7] considers an implementation of logical operations on sets represented as images by means of stimulated echo hologram (SEH). It is shown that stimulated echo hologram could be used for the implementation of the set intersection logical operation. The work [8] examines the implementation of the logical operation of union of sets represented as images using the mode of accumulated long-lived echo hologram (ALEH). In [9] it is shown that, depending on phase difference between exciting pairs of laser pulses, it is possible to implement logical operations of combining sets, symmetric difference, and their superposition.

In case of recording information in ALEH mode, the resonant medium is influenced by a sequence of several pairs of laser pulses. Each such pair together with reading pulse causes the generation of response of long-lived photon echo (LPE) at the same point of time. ALEH signal generated by resonant medium after the impact from N pairs of exciting pulses and one reading pulse represents a superposition of LPE signals.

In this work, we consider an implementation of the logical difference operation using ALEH.

2. Logical difference operation for images in optical echo holography
A sequence of three pairs of exciting pulses used at the implementation of the difference logical operation is presented in figure 1.
Figure 1. Order of exciting laser pulses’ effects at implementation of logical operations on images using ALEH. $P_n^{(i)}$ – pairs of exciting pulses, $P$ – reading pulse, $P_e$ – ALEH signal. $\tau$ – time interval between pulses in pairs, $\tau'$ – time interval between pairs, $\tau''$ – time interval between last recording pair and reading pulse.

Such an ALEH excitation could be achieved in LaF$_3$: Pr$^{3+}$ crystal ($^3H_4(0) - ^3P_0$ transition, $\lambda = 477.7$ nm), where $\tau$ is time interval between pulses in pairs that can reach tens of nanoseconds, $\tau'$ – time interval between pairs, and $\tau''$ – time interval between last recording pair and reading pulse that can reach tens of minutes at helium temperatures in the crystal [10].

Figure 2 presents the set difference logical operation $A \setminus B = \{x | x \in A, x \notin B\}$. To demonstrate the set difference logical operation in ALEH mode, we take transparencies with images inserted in $P_1^{(2)}$, $P_2^{(2)}$ (figure 1).

![Euler-Venn diagrams for set difference logical operation.](image)

Figure 2. Euler-Venn diagrams for set difference logical operation.

Let us write down the electric field strength of $\eta$-th exciting laser pulse transmitted through the corresponding transparency as

$$E_\eta(r, t) = U_\eta(r) e^{i\omega t} + k.c. \quad (0 \leq t \leq \Delta t_\eta),$$

where $\Delta t_\eta$ – duration of $\eta$-th exciting laser pulse, $U_\eta(r)$ describes the spatial structure of $\eta$-th exciting laser pulse.

We consider an image on a transparency as a set of points with $r_n$ radius vectors. Each such point emits a spherical wave. The aggregate of waves in the location of $j$-th optical center in a sample with radius vector $r_{0j}$ provides the value of the optical center’s resonant transition perturbation. Then the electric field strength of an object laser pulse in the point $r_{0j}$ could be written down as an expansion into spherical waves:

$$E_j = \sum_n A_{nj} e^{ik_n(r_{0j} - r_n)} \frac{(r_{0j} - r_n) - i\omega t + i\phi_n}{|r_{0j} - r_n|},$$
where \( \mathbf{k}_n^{(j)} = \frac{\omega}{c} \mathbf{n}_n, \mathbf{n}_n = \frac{\mathbf{r}_{oj} - \mathbf{r}_n}{|\mathbf{r}_{oj} - \mathbf{r}_n|}, \varphi_n \) – initial phase of spherical waves, \( e^{i\varphi_n} \) may be included in \( A_{nj} \) complex amplitudes. If \( |\mathbf{r}_{oj} - \mathbf{r}_n| \) is significantly greater than the sample size, then expansion (2) into spherical waves turns into the expansion into plane waves:

\[
E_j = \sum_n \varepsilon_n e^{i\mathbf{k}_n \mathbf{r}_{oj} - i\omega t},
\]

where \( \varepsilon_n \) – electric field strength amplitudes of plane waves from individual points of an object.

As one exciting laser pulse of each pair is an image carrier, the spatial phase synchronism at the formation of ALEH response will have the form as follows:

\[
k_{en}^{(j)} = -k_{1n}^{(j)} + k_{2n}^{(j)} + k_{3n}^{(j)},
\]

where \( k_{en}^{(j)} \) – propagation vector of plane waves of spatial expansion of wave fronts of object laser pulses for each \( j \)-th pair.

Similarly to the work [4], ALEH response could be received both in inverted and noninverted modes that could be used for a better spatial division of exciting laser pulses and response. It is worth noticing, only those components of the field response expansion will exist for which the amplitudes of the fields expansion of exciting pulses corresponding to the modes that could be used for a better spatial division of exciting laser pulses and response.

Similarly to [4], the spatial structure of ALEH response will be defined by:

\[
I \sim EE^*,
\]

where

\[
E \approx \sum_{j=1}^{n} E_j(r)e^{i\Delta \varphi_{1j}},
\]

\[
E_j \approx \frac{1}{V} \sum_{n',n'',n'''} \int_V dV \int_{-\infty}^{\infty} g(\Delta) d\Delta \sin \theta_1^{(j)} \sin \theta_2^{(j)} \sin \theta_3 \times
\]

\[
x \varepsilon_{1n'}^{(j)} \varepsilon_{2n''}^{(j)} \varepsilon_{3n'''}^{(j)} \left| \sum_{n'} \varepsilon_{1n'}^{(j)} e^{-ik_{1n}^{(j)} r} \right| \left| \sum_{n''} \varepsilon_{2n''}^{(j)} e^{-ik_{2n''}^{(j)} r} \right| \left| \sum_{n'''} \varepsilon_{3n'''}^{(j)} e^{-ik_{3n'''}^{(j)} r} \right| e^{-i\Delta \varphi_{12} - i\Delta \varphi_{13}}
\]

\[
\theta_1^{(j)}, \theta_2^{(j)} \text{ – area of the first and second pulses in } j-\text{th pair}, V \text{ – volume of sample part excited, } g(\Delta) \text{ – frequency distribution of optical centers, } \Delta = \omega - \Omega_0, \omega \text{ – laser emission frequency, } \Omega_0 \text{ – resonance transition frequency, } \varepsilon_{in}^{(j)} \text{ – amplitudes of electric field strength of plane waves of spatial expansion of wave fronts of object laser pulses in each } j-\text{th pair, } \Delta \varphi_{12} \text{ – phase of the second pair of pulses in relation to the first one, } \Delta \varphi_{13} \text{ – phase of the third pair of pulses in relation to the first one, } \Delta \varphi_{11} = 0.
\]

In work [11], it is shown that creation of phase difference between pairs of exciting pulses may lead to decrease or disappearance of frequency modulation of population at the response formation, which leads to its disappearance. Thus, if phases between pairs of exciting pulses differ by \( \pi \), ALEH signal intensity significantly reduces for image elements identical on transparencies in the first and second, in the first and third pairs of exciting pulses.

Figure 3 presents an order of laser pulses at the ALEH excitation in the case when a corresponding image is included in each pair of pulses \( (P_1^{(j)}, P_2^{(j)}) \) (figure 1) and phase difference of the exciting pairs of laser pulses changes. Regions in the form of ellipses with different orientation are chosen as images.
| A | B | C | D | E | F | G | F1 |
|---|---|---|---|---|---|---|----|
| $p_1^{(1)}$ | $p_1^{(2)}$ | $p_2^{(1)}$ | $p_2^{(2)}$ | $p_3^{(1)}$ | $p_3^{(2)}$ | $P$ | $P_e$ |

$\Delta \phi_1 = 10^\circ$, $\Delta \phi_2 = 180^\circ$

$\Delta \phi_1 = 20^\circ$, $\Delta \phi_2 = 180^\circ$

$\Delta \phi_1 = 30^\circ$, $\Delta \phi_2 = 180^\circ$

$\Delta \phi_1 = 40^\circ$, $\Delta \phi_2 = 180^\circ$

$\Delta \phi_1 = 50^\circ$, $\Delta \phi_2 = 180^\circ$

$\Delta \phi_1 = 60^\circ$, $\Delta \phi_2 = 180^\circ$

$\Delta \phi_1 = 70^\circ$, $\Delta \phi_2 = 180^\circ$

$\Delta \phi_1 = 80^\circ$, $\Delta \phi_2 = 180^\circ$
$\Delta \varphi_1 = 90^\circ, \Delta \varphi_2 = 180^\circ$

$\Delta \varphi_1 = 100^\circ, \Delta \varphi_2 = 180^\circ$

Figure 3. Recordable and reproducible images in ALEH mode, Pn(i) – exciting, P – reading pulse, Pe – echo signal. A, B, C, D, E, F, G – sets, F1 is the result of logical operation.

The creation of echo holographic processor implies the development of methods for the implementation of logical operations by the processor itself. Numerical calculation of ALEH response using the expressions (5) – (6) will contain an image that excludes all the coinciding transparent parts of the B, D, and F sets at the corresponding choice of the phase difference of laser pulses’ exciting pairs, which corresponds to the implementation of the set difference logical operation. The responses produced in ALEH mode contain images that are the result of the set difference logical operation at the change of phase difference $\Delta \varphi_1$. At the values of phase difference $\Delta \varphi_1 = 60^\circ, \Delta \varphi_2 = 180^\circ$ one observes the best reproduction of the set difference logical operation. At the values of phase difference $\Delta \varphi_1 = 100^\circ, \Delta \varphi_2 = 180^\circ$ the symmetric difference logical operation is implemented. At other values of phase difference $\Delta \varphi_{12}, \Delta \varphi_{13}$ one observes the superposition of those logical operations.

3. Conclusions
In this work, we have considered the implementation of the set difference logical operation using ALEH. It is shown that by changing the phase difference at the exciting pairs of laser pulses one may implement the set difference logical operation. The implementation of logical operations on images may be used at the creation of echo holographic processor, image processing, changing their contrast range, and overlaying images.

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