A novel online identification algorithm of lithium-ion battery parameters and model order based on a fractional order model

Xiangdong Sun  |  Jingrun Ji  |  Biying Ren  |  Guitao Chen  |  Qi Zhang

Department of Power Electronics and Motors, Xi’an University of Technology, Jinhua South Road, Xi’an, China

Abstract
A fractional order equivalent circuit model with variable order can better reflect the internal reaction mechanism of a lithium-ion battery, however, it is not easy to simultaneously identify the parameters and order of the fractional order model online. In this paper, based on the electrochemical impedance spectroscopy of the lithium-ion battery, the FOM and its discretization are analysed in detail, and a fractional order repeated prediction recursive least square (FORPRLS) method is proposed, the approximate error of fractional derivative is corrected by the repeated identification process with changing the order, so that the parameters and order of the fractional order model can be identified online and accurately at the same time. For the second-order $RC$ equivalent circuit model and fractional order model, different online identification algorithms including the forgetting factor recursive least square and FORPRLS are carried out under the dynamic stress test experiment, and the predicted terminal voltages are compared with the actual measured terminal voltages to judge the identification accuracy of these algorithms. The experimental results verify that the FORPRLS algorithm is superior to the FFRLS algorithm, and the identified order can better match the reaction process of the lithium-ion battery.

1 | INTRODUCTION

In recent years, renewable energy power generation systems such as photovoltaic power generation and wind power generation have been vigorously developed [1]. In order to overcome the contradiction between the intermittence and fluctuation of renewable energy power generation systems and the dispatch of power grid, an energy storage system (ESS) is essential, and the performance of ESS will directly affect the application of the above systems in practice [2]. Due to the advantages of high safety, high energy density and low self-discharge rate, lithium-ion batteries are widely used in the ESS of renewable energy power generation systems. In addition, most of the ESSs can also reuse the retired lithium-ion battery packs of electric vehicles, so as to further improve the resource utilization efficiency and reduce the system cost [3].

During the testing and actual operation of the ESS, it is necessary for accurate estimation of the state of charge (SOC) to exert the battery performance, enhance the system safety and plan the battery energy management [4]. In general, the estimation of SOC mainly includes three aspects: establishing the battery model, identifying the model parameters and estimating the SOC. Among them, an excellent battery model and parameter identification method is prerequisites for accurate SOC estimation, which will determine the effect of SOC estimation. At present, the existing lithium-ion battery models can be mainly divided into electrochemical model [5], equivalent circuit model (ECM) [6], coupling model [7] and black box model [8]. Owing to the flexible structure and moderate complexity of the ECM, many SOC estimation algorithms are based on the ECM in the real-time system, such as extended Kalman filter (EKF) [9], unscented Kalman filter (UKF) [10], particle filter (PF) [11] and state observer method [12]. The ECM simulates the internal reaction (polarization reaction and diffusion effect, etc.) and external electrical response (double-layer capacitance and electrode internal resistance, etc.) through resistance and capacitance elements to describe the dynamic characteristics of the battery. The conventional integer-order ECM...
Different battery models and parameter identification methods

| Electrochemical model | Coupling model | Black box model |
|-----------------------|----------------|-----------------|
| Pseudo-two-dimensional (P2D) model | Electro-thermal aging coupling model | Neural network (NN) Model |
| Single-particle (SP) model | Electrochemical-thermal coupling Model | Recurrent neural network Model |

**Benefits**

- Accurate description of battery dynamic characteristics.
- Flexible structure and high precision.

**Drawbacks**

- Complex modeling and large amount of calculation.
- The complexity increases with precision.

| Integer order model (IOM) | ECM | Fractional order model (FOM) |
|---------------------------|-----|-------------------------------|
| Internal resistance model | Randles model | Single order FOM |
| PNGV model                | $n$ - order RC ECM | Multi-order FOM |
| Thevenin model            |                               | Constant order FOM |
|                           |                               | Variable order FOM |

**Parameter identification method in ECM**

| In frequency domain | Based on EIS | Real-time method | In time domain | Based on chronopotentiometry | Offline method |
|---------------------|--------------|-----------------|----------------|-----------------------------|----------------|
|                      |              | OEIS            | RLS            | Genetic algorithm            | Other optimization algorithms |
|                      |              |                 |                |                              | [19, 23]        |
|                      |              | Levenberg-Marquardt algorithm | [17] |                              | [21, 15, 23]    |
| Characteristic parameter point method | [17] |                              | |               |                              |

**FIGURE 1** Different battery models and parameter identification methods

cannot fully match the electrochemical impedance spectroscopy (EIS) of actual lithium-ion battery, and the SOC estimation error of these algorithms for the integer order ECM is usually less than 3%. With the rise of the fractional calculus theory, the fractional calculus is introduced into ECM, and the fractional order model (FOM) is established to improve the model accuracy. The FOM has been applied to supercapacitors and good results have been obtained [13]. Some researches show that the calculation burden of FOM increases to a certain extent compared with the integer order model (IOM), but FOM can more closely describe the dynamic characteristics of actual lithium-ion battery [14]. Furthermore, the order of FOM can relatively reflect the aging level, internal state and reaction process of the battery, which means that the introduction of fractional order may simultaneously achieve the battery consistency assessment, SOC and state of health (SOH) estimation [15,16]. Figure 1 shows the different battery models and the parameter identification methods of ECM, especially FOM proposed in recent years. In low and middle-high frequency bands, the offline parameter identification is carried out based on chronopotentiometry of time domain and EIS of frequency domain, respectively [17]. On the basis of ensuring the identification accuracy, the measurement speed of EIS in low frequency band is improved by combining the two domains, and a new offline identification strategy of EIS is proposed. The online EIS (OEIS) method can convert the multi-frequency compos-
efficiency of FOM is improved according to the short-term memory principle, the results show that the SOC estimation error of this method can be controlled within 3% [21]. Similarly, a hybrid GA/particle swarm optimization (HGAPSO) method is used offline to identify the parameters of the dual FOM, and a dual fractional-order EKF is proposed to co-estimate SOC and SOH [22], the root-mean-squared error of the predicted voltage response under different conditions is less than 10 mV, and the maximum steady-state error of SOC and SOH is within 1%. All these indicate the superiority of FOM in accuracy. In [23], a frequency fitting method and parameter identification algorithm based on output error are presented, the order of FOM is determined by fitting EIS and other parameters are identified by offline iteration. A fractional order Kalman filter is used in SOC estimation and the simulation results show that FOM can ensure the accuracy of SOC estimation, and the maximum error is 0.5%.

Although most of the methods mentioned above consider the time-varying characteristics of resistance-capacitance parameters, the fractional order change under dynamic conditions is hardly discussed. The battery is a non-linear system, its internal reaction state will change with time and operating conditions. Whether in frequency domain or time domain, the fractional order should be identified as a variable parameter to improve the prediction accuracy of the model. In order to describe the strong nonlinearity of the lithium-ion battery voltage response, the classical second-order RC equivalent circuit model is improved by introducing a fractional order capacitor, and a fractional variable-order equivalent circuit model (FV-ECM) is proposed [24]. The model parameters and order under different SOC are identified offline by interpolation/lookup table method and LS method. The experimental result shows that the mean absolute error (MAE) and root mean square error (RMSE) of FV-ECM are less than those of the IOM and the fixed order FOM, and the nonlinear characteristic of the battery is accurately reflected. Using LS method with moving window and gradient-based method to identify the parameters and order of FOM respectively, an online identification method of variable order FOM is proposed in [25]. The voltage prediction accuracy is improved by order adaption in this method, but the moving window length slows down the convergence speed of variable order.

In a word, both IOM and the fixed order FOM have limitations in describing the internal reaction mechanism of the lithium-ion battery, while FOM with variable order can better reflect the dynamic characteristics. In addition, the fractional order is comprehensively affected by the factors, such as frequency, temperature, SOC and polarization. Thus, the variable order obtained by offline identification is unreliable under the actual random condition, but few literatures have studied the online identification method of variable order. For this reason, a FOM with a variable order is established and a fractional order repeated prediction recursive least square method (FORPRLS) is proposed to realize the online identification of fractional order in this paper. Compared with the online parameter identification algorithms of the second-order RC equivalent circuit model and FOM with a fixed order, the validity and superiority of the proposed algorithm are verified.

The specific arrangement of this paper is as follows: the establishment of the FOM and the discretization of the state space equation are introduced in Section II. The forgetting factor recursive least square (FFRLS) algorithm and the proposed FORPRLS algorithm are discussed in Section III. In Section IV, the proposed algorithm is verified by the experiments, and the terminal voltage prediction accuracies of different models and algorithms are compared and analysed. The conclusion is drawn in Section V.

2 MODELLING OF A FRACTIONAL 
ORDER EQUIVALENT CIRCUIT MODEL 
OF LITHIUM-ION BATTERIES

EIS is a common method to study the dynamic characteristics of the battery, which contains the precise impedance of the electrochemical battery at different frequencies. By extracting the EIS curve and characteristic impedance, the reaction process of the battery can be analysed and the battery model can be established. Although the EIS of the lithium-ion battery is slightly different under different battery systems, SOCs and aging levels, the curve trend is basically the same. Blue solid line in Figure 2 shows the EIS of general lithium-ion battery [26-28]. The curve in the low frequency area is a straight line, which indicates the semi-infinite diffusion (Warburg) impedance generated by the diffusion of lithium ions in the solid electrode of the battery. In an ideal situation, the inclination of the straight line is 45°, while there will be deviations in actual non-ideal situations. It should be noted that the low frequency area with the only Warburg impedance is incomplete. If lower frequency is considered, the EIS of diffusion process in low frequency area is actually a semicircle [17]. Because of the long period of EIS measurement in the ultra-low frequency band, only the
Warburg impedance at the linear part of low frequency band is generally measured. The curve in the mid-frequency area is a semicircle, indicating the impedance caused by the charge transfer process and double layer capacitor effect inside the battery. The curve in the high frequency area is approximately a vertical line, which represents the parasitic inductance caused by the current collector of the battery and the test cable. Owing to high frequency, the parasitic inductance is generally ignored in the battery model. Moreover, due to the differences in the film-forming process of solid electrolyte interface (SEI) in different lithium-ion battery systems, the resistance of lithium ion across the SEI is also different. Therefore, the EIS of some lithium-ion batteries will have a slightly small semicircle between the midfrequency area and the high-frequency area, which represents the impedance formed by the SEI. And for the EIS in the middle and high frequency areas, two semicircles will appear simultaneously. In general, the lower the temperature is, the more difficult it is for lithium ions to cross the SEI. Hence, the semicircle corresponding to the SEI is more obvious. However, at normal temperature, the SEI impedance of the lithium-ion battery becomes weak, which is not as obvious as the semicircle generated by both the charge transfer process and the double layer capacitor effect. Therefore, only one semicircular in the EIS of the lithium-ion battery is taken into consideration in this paper.

2.1 Fractional order equivalent circuit model

ECMs are widely used in the real-time battery management system (BMS) due to low calculation burden, flexible modeling and high accuracy. At present, the integer order ECM mainly includes: $R_{\text{int}}$ model [29], PNGV model [30], Thevenin model [10] and second-order $RC$ equivalent circuit model [31]. Compared with other models, the second-order $RC$ equivalent circuit model has been widely concerned due to its high accuracy and moderate complexity, its ECM and EIS diagram are shown in Figure 2. It can be seen from Figure 2 that the second-order $RC$ equivalent circuit model basically conforms to the EIS of the actual lithium-ion battery in the mid-frequency area, while the EIS deviation in the low frequency area is obvious. Moreover, in the actual measurement of the EIS curve of the lithium-ion battery, it is found that the impedance curve in the mid-frequency area always deviates from the ideal semicircular trajectory more or less and appears as an arc shape, which is called capacitance arc. The capacitance arc is caused by the dispersion effect, and it is generally considered to be related to the solution conductivity, the adsorption layer and inhomogeneity of the electrode surface, reflecting the property of the double layer capacitor deviating from the ideal capacitance. As the property of capacitance arc is between the resistance and the capacitance, it is difficult for the traditional IOM to describe the capacitance arc. Nevertheless, the fractional calculus can describe the non-ideal physical system very well. Therefore, some researchers introduce the constant phase element (CPE) into the ECM to describe the Warburg impedance and the dispersion effect under non-ideal conditions by fractional calculus. The CPE is defined as:

$$Z_{\text{CPE}} = \frac{1}{C \cdot (\omega i)^\alpha} \quad (1)$$

Where, $C$ and $\alpha$ are two parameters of CPE. $\alpha$ is the order of CPE, and its value range is $[-1, 1]$. $\omega$ is the angular frequency and $i$ is the unit of the imaginary part. The electrical characteristic expression of CPE is the same as that of the inductance and capacitance, except that the order of the derivative is fractional. The EIS curve of CPE is a straight line with an angle of $\alpha \times 90^\circ$. When $\alpha$ is equal to 1, -1 and 0, the CPE represents the capacitance, inductance and resistance, respectively. When $\alpha = 0.5$, the inclination of the straight line is 45°, which represents an ideal Warburg impedance existing in the lithium-ion battery system. Considering the ultra-low frequency, the whole EIS should be two semicircles, corresponding to the charge transfer process in the middle frequency area and the diffusion process in the low frequency area. Since the parallel structure $R_i//\text{CPE}$ can fit the semicircle well, the charge transfer process and diffusion process can be combined into one time-varying $R_i//\text{CPE}$ structure. Hence, a variable order FOM is established, and the model structure of FOM is shown in Figure 3(a), where $Z_1 = 1/\text{C}_1 \cdot (\omega i)^-\alpha$. $R_0$ and $R_1$ represent ohmic resistance and polarization resistance, respectively. It should be pointed out that the model parameters $\alpha$, $R_0$, $R_1$ and $C_1$ all change with

![Figure 3](image-url)
time. When the input current frequency is in the low frequency range, $\alpha$ will change to around 0.5, and the parallel structure of $R_1 || Z_1$ will describe the Warburg impedance during the diffusion process. When the input current frequency changes to the middle frequency, $\alpha$ will increase to about 1, and the $R_1 || Z_1$ at this time is used to describe the dispersion effect in the process of charge transfer. It indicates that the dynamic model established has strong applicability. Even in the actual situation with complex operating conditions, the time-varying model parameters can ensure the fitting degree of the battery EIS in a wide range. In Figure 3(b), it can be seen that the EIS of the FOM is highly consistent with the EIS of the actual lithium-ion battery measured in fixed working condition, and the error in the entire frequency range is small. The shadow in Figure 3(b) reflects the EIS range that can be fitted by the variable order FOM under dynamic conditions.

Assuming that the discharge current is in the positive direction, the electrical characteristic equation and the state space equation of FOM are expressed by Equations (2) and (3).

$$\begin{align*}
U_1 &= U_{oc} \langle SOC(t) \rangle - U_1 - (t) \cdot R_0 \\
C_1 \cdot \frac{d^\alpha U_1}{dt^\alpha} &= I(t) - \frac{U_1}{R_1} \\
x(t) &= A_1 \cdot x(t) + B_1 \cdot u(t) \\
y(t) &= C_1 \cdot x(t) + D_1 \cdot u(t)
\end{align*}$$

Where, $x(t) = [\frac{d^{\alpha} SOC(t)}{dt^{\alpha}} U_1(t)]^T$, $y(t) = SOC(t) U_1(t)^T$, $u(t) = I(t)$, $A_1 = diag[0, -1/(R_1 \cdot C_1)]$, $B_1 = [-1/Q_1, 1/C_1]^T$, $C_1 = [U_{oc}(SOC(t))/SOC(t) \cdot -1]$, and $D_1 = -R_0 \cdot Q_1$ is the battery capacity. $U_{oc}$ is the open circuit voltage (OCV) of the battery. $U_1$ is the terminal voltage of the battery. $U_1$ is the voltage of $R_1$. $diag[a,b]$ is a diagonal matrix with diagonal elements $a$ and $b$ respectively.

Both parameter identification and SOC estimation involve the discrete state space equation. Thanks to the existence of fractional calculus, FOM cannot be discretized by the state transition matrix directly. Thus, the fractional calculus must be discretized first, and the discrete state space equation of the FOM can be obtained.

### 2.2 Discretization of the fractional order model

There are multiple definitions of the discrete fractional calculus, and the most commonly used is the discrete recursive equation of fractional calculus defined by Grunwald–Letnikov. The definition is shown in Equation (4).

$$D^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \sum_{j=0}^{k} (-1)^j \binom{\alpha}{j} f((k-j) T)$$

Where, $D^\alpha$ is a fractional calculus operator, which represents the integration or differentiation of any real order, and its Laplace transform is $D^\alpha f(t) = \hat{f}(s)$, $T$ is the sampling time, $k$ is the current moment and $j$ is an integer with a value range of $[0, k]$. $\binom{\alpha}{j}$ represent the coefficient of Newton binomial, and its expansion is expressed by Equation (5).

$$\binom{\alpha}{j} = \frac{\Gamma(\alpha+1)}{\Gamma(j+1)\Gamma(\alpha-j+1)} = \left\{ \begin{array}{ll}
1 & j = 0 \\
\frac{\alpha(\alpha-1)\ldots(\alpha-j+1)}{\Gamma(\alpha-j+1)} & j > 0
\end{array} \right.$$

where, $\Gamma(n)$ is the Gamma function, which has factorial properties: $\Gamma(n+1) = n\Gamma(n)$. When $n$ is a positive integer, $\Gamma(n) = n!$. As can be seen from Equation (4), the fractional calculus at the current moment contains the historical information of all previous moments, which is unfavourable for the real-time operation in the digital processor. In order to reduce the calculation burden of FOM, an appropriate data memory length is selected based on the short-term memory principle in this paper, and the historical data beyond the memory length is ignored. When the memory length $L$ is longer, the FOM is more accurate and the calculation burden is increased. Therefore, the calculation burden and accuracy need to be comprehensively considered to select an appropriate memory length $L$ in practical applications.

It is known from Equation (4) that $D^\alpha f(k)$ cannot obtain the information of the time $k+1$. Thus, in order to predict the terminal voltage at the next moment, $D^\alpha U_1(k+1)$ is assumed to be approximately equal to $D^\alpha U_1(k)$ within one sampling interval, and $D^\alpha U_1(k)$ is the fractional derivative of $U_1$ at time $k$. The discrete state space equation of the FOM can be obtained from Equation (6).

$$\begin{align*}
x(k+1) &= A \cdot x(k) + B \cdot u(k) + S_{k+1} \\
y(k) &= C \cdot x(k) + D \cdot u(k)
\end{align*}$$

Where $x(k) = [SOC(k) \ U_1(k)]^T$, $y(k) = U_1(k)$, $u(k) = I(k)$, $A = diag[1, -Q_1/\tau_1]$, $B = [-T/Q_1, T^\alpha/C_1]^T$, $C = [U_{oc}(SOC(k))/SOC(k) \cdot -1]$, $D = -R_0$, $\tau_1 = R_1 \cdot C_1$, $S_{k+1} = \sum_{j=1}^{\infty} (-1)^j \binom{\alpha}{j} U_1(k-j+1)$ is the remainder term of $D^\alpha U_1(k+1)$.

### 3 ONLINE PARAMETER IDENTIFICATION OF THE FRACTIONAL ORDER EQUIVALENT CIRCUIT MODEL

Online parameter identification is to identify the parameters recursively on the basis of the data collected at the current moment, and the identification result can be corrected in real
time as the system input and output change. Under different conditions, the internal physical and chemical reaction degree of the battery is different, and the external impedance characteristics are also different. Therefore, the parameters of the battery model should continuously change with various factors in the actual operation process.

### 3.1 Forgetting factor recursive least square algorithm

FFRLS is one of the commonly used online parameter identification algorithms for lithium-ion battery models. By introducing the forgetting factor λ into the RLS algorithm, FFRLS redistributes the weight of the old and new data in the recursion process, and reduces the proportion of the old data in the recursion result, so that the algorithm can quickly converge nearby the actual value when the input changes. The advantage of FFRLS is that it requires less computation and storage space, and can realize dynamic identification of model parameters and rapid tracking of system output. FFRLS is suitable for the time-varying nonlinear battery system.

In order to use FFRLS algorithm for the FOM, Equation (7) is obtained by Laplace transformation of Equation (2).

\[
G(s) = \frac{U_{oc}(s) - U_L(s)}{I(s)} = R_0 + \frac{R_1}{1 + R_1 C_1 s^2}
\]

(7)

Let \(E = U_{oc} - U_L\) and transform Equation (7) into the time domain, and the discretization expression of the system to be identified is expressed by Equation (8).

\[
E(k) = \theta_1 D^2 E(k) + \theta_2 I(k) + \theta_3 D^2 I(k)
\]

(8)

Where, \(\theta_1 = -R_1 C_1, \theta_2 = R_0 + R_1\) and \(\theta_3 = R_0 R_1 C_1\), \(E(k)\) is the difference between the open circuit voltage and the terminal voltage of the battery at time \(k\), \(D^2 E(k)\) and \(D^2 I(k)\) are the fractional derivatives of \(E(k)\) and input current \(I(k)\) at time \(k\), respectively. Define \(y(k) = [E(k)], \phi(k) = [D^2 E(k) I(k) \ D^2 I(k)]^T\), \(\theta(k) = [\theta_1 \ \theta_2 \ \theta_3]^T\), and bring them into Equation (8). The matrix of parameter identification of FOM is expressed by Equation (9). When the coefficient vector \(\theta(k)\) is updated at each moment, the resistance–capacitance parameters \(R_0, R_1,\) and \(C_1\) can be inversely solved by \(R_0 = -\theta_3/\theta_1 \theta_2,\)

\[
R_1 = \theta_2 + \theta_3/\theta_1\) and \(C_1 = -\theta_1^2/(\theta_1 \theta_2 + \theta_3),\)

(9)

Similarly, the matrix of parameter identification of the second-order RC equivalent circuit model can also be obtained by Laplace transformation and bilinear transformation, and its resistance–capacitance parameters can be inversely solved by the coefficient vector \(\theta(k)\) [31].

The specific implementation steps of the FFRLS algorithm are shown in Table 1.

### 3.2 Fractional order repeated prediction recursive least square method

It is seen from Equation (6) that there is a complex nonlinear relationship between the order \(\alpha\) and the output vector. In order to simultaneously identify the fractional order and resistance–capacitance parameters online, the FORPRLS algorithm is proposed in this paper, which can improve the accuracy of the predicted terminal voltage.

To predict the terminal voltage at the next moment, \(D^\alpha U_1(k+1)\) is assumed to be equal to \(D^\alpha U_1(k)\) in Equation (6). Considering \(D^\alpha U_1(k+1)\) is probably not equal to \(D^\alpha U_1(k)\) in fact, the terminal voltage predicted by Equation (6) will produce a slight approximation error. If the assumption in Equation (6) is cancelled, the error caused by approximation will disappear, and the discrete electrical characteristic equation of the FOM is expressed by Equation (10) according to Equations (2) and (4). However, it can be seen from Equation (10) that the predicted terminal voltage at the next moment is not available, and only the current information is available. Therefore, Equation (10) can be used to correct the fractional order and make it converge to the optimal value in the online identification process, so as to reduce the approximation error and obtain the dynamic fractional order. The traditional FFRLS algorithm only performs one recursive process at each moment and cannot correct the order. After the improvement, the FORPRLS algorithm repeats the recursive process of FFRLS at each moment, and different fractional orders are used to correct the approximation error of fractional derivative. In the repetitive process at one moment, the current terminal voltage of each order is predicted by Equation (10). Meanwhile, the error between the predicted and measured terminal voltages is calculated. Taking the minimum absolute value of all errors as the evaluation criterion, the order and resistance–capacitance parameters obtained are the optimal identification results at current moment, and they are used to predict the terminal voltage of next moment through Equation (6). Then the process will be repeated at the next moment. At each moment, the repeated recursive process with different
orders will be reset to the optimal identification value of the previous moment. By continuously correcting the approximation error of $D^\alpha U_1(k+1)$, the order $\alpha$ will gradually approach the optimal value.

\[
\begin{align*}
    D^\alpha U_1(k) &= \frac{1}{\alpha} \sum_{j=0}^{\alpha-1} (-1)^j \binom{\alpha}{j} U_1(k-j) \\
    U_2(k) &= U_{oc}(SOC(k)) - U_1(k) - I(k) \cdot R_0 \quad (10)
\end{align*}
\]

Simplify and convert Equation (10) into the discrete state space equation, and Equation (11) is obtained.

\[
\begin{align*}
    x(k) &= A_2 \cdot x(k-1) + B_2 \cdot u(k) + S_k \\
    y(k) &= C \cdot x(k) + D \cdot u(k)
\end{align*}
\]

Where $x(k) = [SOC(k), U_1(k)]^T$, $y(k) = U_{1a}(k)$, $u(k) = I(k)$, $A_2 = \text{diag}(1, \alpha/2)$, $B_2 = \begin{bmatrix} -T/L_1 \cdot \alpha/2 \cdot R_0/(T_0 + T_1) \end{bmatrix}$, $C = [U_{oc}(SOC(k))/SOC(k)]$, $D = [-R_0]$, $S_k = [0 \ q_{k-2} \ q_{k-2}]^T$, $q_{k} = \sum_{j=2}^k (-1)^{j/2} U_1(k-j)$ is the remainder term of $D^\alpha U_1(k)$. The evaluation function $F(k)$ in the repeated identification process is expressed by Equation (12), and $U_{1a}(k)$ is the predicted terminal voltage at time $k$. The measured terminal voltage $U_1(k)$ at time $k$ is used to evaluate $U_{1a}(k)$. When $F(k)$ is minimum in the repeated identification process, the order and resistance–capacitance parameters are the optimal identification result at this moment.

\[
F(k) = \min\{U_1(k) - U_{1a}(k)\} \quad (12)
\]

Suppose the order at time $k$ is $\alpha_k$, the number of times to try different orders is $N$, and the change step of the order in each attempt is $\Delta\alpha$. Defining $\alpha_k$ as the centre of order change in the repeated identification process, and the change range of the order $\alpha_k$ at each moment is represented by Equation (13).

\[
\begin{align*}
    \alpha_k - (N - 1)\Delta\alpha/2 &\leq \alpha \leq \alpha_k + (N - 1)\Delta\alpha/2, \quad N \text{ is odd} \\
    \alpha_k - (N - 2)\Delta\alpha/2 &\leq \alpha \leq \alpha_k + N\Delta\alpha/2, \quad N \text{ is even}
\end{align*}
\]

In the FORPRLS algorithm, a set of the order and resistance–capacitance parameters with the minimum prediction error in the order range are taken as the identification result at each moment. When the number of attempts $N$ is larger and the order change step $\Delta\alpha$ is smaller, the identification result is more accurate, and the calculation burden will increase at each moment. Therefore, to reduce the calculation amount, $\Delta\alpha$ is generally taken as 0.01 and $N$ is within 10. The order identification of FORPRLS is a non-linear identification process. By attempting different orders continuously, the online identification of order $\alpha$ is realized. Figures 4 and 5 are the flow chart and overall block diagram of the FORPRLS algorithm, respectively.

### 4 EXPERIMENTAL VERIFICATION AND ANALYSIS

Figure 6 shows the experimental platform of the battery. The special power supply is the BTS-60V100A battery testing equipment produced by Neware Electronics Co., Ltd. The battery is the 3.2 V/36 Ah lithium iron phosphate battery produced by Shandong Wina Green Power Co., Ltd., and the parameters of the battery are shown in Table 2. During the experiments, the change of battery aging level and ambient temperature is minor.
hence the influence of them on the performance of the experimental battery is not considered.

The detailed control process of the experimental platform is:
(1) Set the experimental operation condition by the host computer. (2) Start the experiment and sample the voltage, current and temperature information in real time through the special power supply. (3) The real-time information is transmitted to the host computer through the serial port for online identification algorithm.

At the ambient temperature of 25 °C, the 0.33C constant current intermittent charge/discharge experiment and dynamic stress test (DST) experiment are conducted respectively, and the sampling time is 10 s. The DST is a simplified condition for the battery test of electric vehicles under the federal urban driving schedule in the United States. The cyclic charge current rates are 0.22C, 0.33C and 0.5C, and the cyclic discharge current rates are 0.22C and 0.33C, respectively. The corresponding input current and terminal voltage curves are shown in Figure 7(a–c).

4.1 OCV-SOC curve

The input current of the constant current intermittent charge/discharge experiment is 12 A (0.33C) recommended by the battery specification, as shown in Figure 7(a,b), which are used to obtain the non-linear relationship between OCV and SOC. Since the time of the rest stage is long enough, the terminal voltage reaches a stable state, which is approximately equal to the OCV at the end of the static placement. The OCV-SOC curve of the lithium iron phosphate battery can be obtained by extracting the SOC value and terminal voltage data at the end of each rest stage and using Matlab software for polynomial fitting. The OCV-SOC curve provides the open circuit voltage $U_{oc}$ for the FFRLS and FORPRLS algorithms. In view of the difference between the actual terminal voltage and the ideal OCV, the hysteresis characteristic of the measured OCV is obvious. In order to reduce the negative effect of hysteresis caused by the error, the average value of OCV in the charge/discharge experiment is selected for fitting. The obtained OCV-SOC fitting result is expressed by Equation (14), and the fitting curve is shown in Figure 8.

$$U_{oc} = -284.4 \times SOC^8 + 1266.3 \times SOC^7 - 2331.8 \times SOC^6 + 2298.1 \times SOC^5 - 1312.3 \times SOC^4 + 440.4 \times SOC^3 - 84.64 \times SOC^2 + 8.81 \times SOC + 2.861$$

4.2 Comparative analysis of the prediction effect of terminal voltage of the lithium-ion battery model

Figures 9–12 are experimental results under the DST condition shown in Figure 7(c). Figure 9 shows the results of parameter identification using the FFRLS and FORPRLS algorithms based on FOM. Where, the FOM-FFRLS means the FFRLS algorithm with the fractional order of 0.5, and the FORPRLS refers to the algorithm proposed in this paper with variable fractional order. When the initial values of $R_0$, $R_1$, $C_1$ are set as zero, the parameters can rapidly converge to a stable state through the recursive process. Since the OCV curve is located in the platform zone and the polarization voltage changes little before 9000 s, the parameters identified are basically stable. As the
SOC further decreases after 9000 s, the electrochemical reaction of lithium-ion battery becomes more complex. The OCV curve enters the exponential zone and the polarization voltage changes dramatically, at this time, the results of the parameter identification fluctuate violently, which reflects the impedance variation caused by the degree of electrochemical reaction under different SOCs.

Figure 10 shows the comparison curve of the predicted terminal voltage using the FFRLS algorithm based on FOM (parameters identified by the FOM-FFRLS algorithm show in Figure 9 and IOM (second-order RC equivalent circuit model) under the DST experiment. Where, the measured terminal voltage refers to the actual battery voltage. It can be seen from Figure 10(b) that most of the relative errors of two models are within ±2%, indicating that the online identification algorithm has good prediction accuracy under the real-time dynamic condition and most of the relative errors of FOM are less than that of the second-order RC equivalent circuit model, which shows that FOM has higher model prediction accuracy than the second-order RC model with integer order.

Figure 11 shows the comparison curve of the predicted terminal voltage using FFRLS and FORPRLS algorithms corresponding to the parameters identified in Figure 9. The relative error of two algorithms in Figure 11(b) is mostly within ±1%, while the terminal voltage prediction accuracy using the FORPRLS algorithm is more precise. Except for some of the terminal voltage error spikes caused by the instantaneous switch of input current, the relative error fluctuates around zero.

Figure 12 is the curve of input current and the fractional order obtained by FORPRLS. As shown in Figure 12, the order $\alpha$ increases with the raise of current change rate. When the input current suddenly changes, the order $\alpha$ increases rapidly, which corresponds to the process of model impedance in the EIS moving from the low frequency range to the mid-frequency range. The charge transfer impedance gradually becomes apparent, indicating that the main reaction control step inside the battery is changed from the ion diffusion process to the charge transfer process. On the other hand, when the current retains constant, the order $\alpha$ reduces to around 0.5, which reflects the increase of the Warburg impedance and indicates that the internal reaction of the battery is gradually controlled by the diffusion of lithium ions. Due to the Warburg impedance of the battery is slightly different under different SOC conditions, the order $\alpha$ fluctuates around 0.5. When the SOC is close to 0 or 100%, the concentration difference of lithium ions on the positive and negative electrodes is large, and their diffusion is promoted. Therefore, the charge transfer process is more obvious, and the order $\alpha$ is closer to 1. The above inference is identical with the theoretical analysis in Section 2.1. Moreover, other research results also show that the fractional order vary within a certain range under different current frequency bands [32, 33], and the variation range is consistent with Figure 12. The curve of order $\alpha$ and input current further verifies the validity of the FORPRLS algorithm in identifying the fractional order.

Table 3 compares the predicted terminal voltage errors of IOM and FOM obtained by various identification algorithms under the DST experiment. It is seen from Table 3 that the mean
relative error (MRE) and RMSE of FOM are both less than the second-order RC equivalent circuit model, indicating that the FOM is more consistent with the actual battery characteristics and the availability of introducing the CPE into FOM is proved. In addition, the MRE and RMSE of FORPRLS algorithm are minimal, which verifies the validity and superiority of the proposed algorithm. The average single calculation time (ASCT) refers to the average time spent for parameter identification at each moment. Although the ASCT of FORPRLS is longer than that of the other two algorithms, it is acceptable since the actual sampling time is in the range of milliseconds to seconds.

Table 4 shows the comparison between the proposed algorithm and the latest FOM parameter identification method under DST experimental conditions. For the convenience of comparison, the RMSE of FORPRLS algorithm is added when the memory length is $L = 100$ and $L = 200$. It can be seen from Table 4 that: (1) the accuracy of FOM parameter identification algorithm is obviously affected by the memory length $L$. The larger the value of $L$ is, the more accurate the predicted terminal voltage is. (2) The HGAPSO algorithm in [22] is an...
TABLE 4 Comparison of the latest FOM parameter identification method

| Algorithm       | Type      | Order $\alpha$ | Memory length ($L$) | RMSE (DST) |
|-----------------|-----------|----------------|---------------------|------------|
| HGAPSO [22]     | Offline   | Fixed          | 10                  | 10.4 mV    |
| FFRELS [34]     | Online    | Fixed          | 200                 | 4.31 mV    |
| GMWLS [25]      | Online    | Variable       | 100                 | 5.21 mV    |
| FORPRLS         | Online    | Variable       | 50                  | 9.97 mV    |
|                 |           |                | 100                 | 5.37 mV    |
|                 |           |                | 200                 | 4.18 mV    |

FIGURE 11 Comparison of the terminal voltages predicted by FFRLS and FORPRLS algorithms based on FOM under DST experiment. (a) Terminal voltage. (b) Relative error

FIGURE 12 Change curve of fractional order $\alpha$ for FORPRLS algorithm under DST experiment

offline identification algorithm. Compared with other online algorithms, the prediction accuracy of this algorithm is slightly lower, and it is not suitable for dynamic random conditions. (3) When $L = 200$, the prediction accuracy of FORPRLS algorithm is slightly better than that of the forgetting factor recursive expanded least squares (FFRELS) algorithm in [34], and when $L = 100$, the prediction accuracy of FORPRLS algorithm is slightly lower than that of gradient based moving window least square (GMWLS) algorithm in [25]. However, in addition to the memory length, the GMWLS algorithm introduces a moving window length of 300. Therefore, in fact, the data storage length of GMWLS algorithm is higher than 100, and the longer window length also hinders the convergence speed of order. The above comparison results show that the proposed FORPRLS algorithm has advantages over the existing methods, which can realize online order identification with less computational resources, and achieve high prediction accuracy.

For the robustness analysis of FORPRLS algorithm, the abrupt change of parameters and noise interference are mainly considered in this paper. On the one hand, the DST experiment show that the FORPRLS algorithm can accurately predict the terminal voltage and rapidly track the optimal parameters under the complex condition, which verifies the excellent robustness of the proposed algorithm in terms of parameter dynamic change. On the other hand, the Gaussian noise with a standard deviation of 10 mV is added to the real-time sampled data by using MATLAB software, and the FORPRLS algorithm is executed with the constant initial setting. The curve of predicted terminal voltage is shown in Figure 13. In the case of noise interference, the MRE between the predicted value and the noise free value is 0.301%, and the RMSE is 0.0139. The experimental
result shows that the error of FORPRLS with Gaussian noise is almost the same as that of IOM-FFRLS method without Gaussian noise, and this result verifies the strong robustness of proposed algorithm in anti-interference.

5 CONCLUSIONS

The variable order FOM is established on the basis of the EIS of the lithium-ion battery in this paper, and the FORPRLS algorithm is proposed to realize online identification of the FOM parameters. Under the DST experiment, the predicted terminal voltage results of the second-order \( RC \) equivalent circuit model, FOM with the fixed order and FOM with the variable order are compared to show that FOM has better model accuracy than the second-order \( RC \) equivalent circuit model, and the FORPRLS algorithm has more superior prediction accuracy than the fixed-order FFRLS algorithm. The MRE of the predicted terminal voltage is about 0.158\%. Therefore, the FORPRLS algorithm can better reflect the dynamic process of battery reaction through the change of order. Compared with other algorithms, the FORPRLS algorithm takes less computing resource and has higher prediction accuracy. Moreover, the proposed algorithm has strong robustness regardless of parameter changes or noise interference.

In the near future, the SOC estimation algorithm suitable for FOM will be taken as a research focus to maximize the accuracy advantages of FOM. Besides, the relationship between the fractional order and the battery reaction state is also worthy of further research, which has profound significance for the consistency classification, charge/discharge management and aging level prediction of the battery.

ACKNOWLEDGEMENTS

This research was funded by the Natural Science Foundation of Shaanxi Province (2020JM-449).

REFERENCES

1. Mohammad, F., et al.: Review of energy storage system technologies in microgrid applications: Issues and challenges. IEEE Access. 6, 35143–35164 (2018)
2. Strasser, T., et al.: A review of architectures and concepts for intelligence in future electric energy systems. IEEE Trans. Ind. Electron. 62(4), 2424–2438 (2015)
3. Jinlei, S., et al.: Economic operation optimization for 2nd use batteries in battery energy storage systems. IEEE Access. 7, 41852–41859 (2019)
4. Hidalgo-Reyes, J.I., et al.: Battery state-of-charge estimation using fractional extended Kalman filter with Mittag-Leffler memory. Alex. Eng. J. 59(4), 1919–1929 (2019)
5. Bartlett, A., et al.: Electrochemical model-based state of charge and capacity estimation for a composite electrode lithium-ion battery. IEEE Trans. Control Syst. Technol. 24(2), 384–399 (2016)
6. Xiong, R., et al.: A systematic model-based degradation behavior recognition and health monitoring method for lithium-ion batteries. Appl. Energy. 207, 372–383 (2017)
7. Mohajer, S., et al.: A fractional-order electro-thermal aging model for lifetime enhancement of lithium-ion batteries. IFAC-PapersOnLine. 51(2), 220–225 (2017)
8. Hannan, M.A., et al.: Neural network approach for estimating state of charge of lithium-ion battery using backtracking search algorithm. IEEE Access. 6(2018), 10069–10079 (2018)
9. Haus, B., Mercorrell, P.: Polynomial augmented extended Kalman filter to estimate the state of charge of lithium-ion batteries. IEEE Trans. Veh. Technol. 69(2), 1452–1463 (2020)
10. Wu, X., Li, X., Du, J.: State of charge estimation of lithium-ion batteries over wide temperature range using unscented Kalman filter. IEEE Access. 6, 41993–42003 (2018)
11. Zhang, K., et al.: State of charge estimation for lithium battery based on adaptively weighting cubature particle filter. IEEE Access. 7, 166657–166666 (2019)
12. Li, W., et al.: State of charge estimation of lithium-ion batteries using a discrete-time nonlinear observer. IEEE Trans. Ind. Electron. 64(11), 8557–8565 (2017)
13. Hidalgo-Reyes, J.I., et al.: Determination of supercapacitor parameters based on fractional differential equations. Int. J. Circ. Theor. Appl. 47(8), 1225–1253 (2019)
14. Hidalgo-Reyes, J.I., et al.: Classical and fractional-order modeling of equivalent electrical circuits for supercapacitors and batteries, energy management strategies for hybrid systems and methods for the state of charge estimation: A state of the art review. Microelectron. J. 85, 109–128 (2019)
15. Li, X., et al.: A physics-based fractional order model and state of energy estimation for lithium ion batteries. Part II: Parameter identification and state of energy estimation for LiFePO4 battery. J. Power Sources. 367, 202–213 (2017)
16. Lu, X., et al.: An indicator for the electrode aging of lithium-ion batteries using a fractional variable order model. Electrochim. Acta. 299, 378–387 (2019)
17. Nasser-Eddine, A., et al.: A two steps method for electrochemical impedance modeling using fractional order system in time and frequency domains. Control Eng. Pract. 86(5), 96–104 (2019)
18. Kuipers, M., et al.: An algorithm for an online electrochemical impedance spectroscopy and battery parameter estimation: Development, verification and validation. J. Energy Storage. 30, 101517 (2020)
19. Mawonou, K.S.R., et al.: Improved state of charge estimation for Li-ion batteries using fractional order extended Kalman filter. J. Power Sources. 435, 226710 (2019)
20. Xiong, R., et al.: A novel fractional order model for state of charge estimation in lithium ion batteries. IEEE Trans. Veh. Technol. 68(5), 4130–4139 (2019)
21. Mu, H., et al.: A novel fractional order model based state-of-charge estimation method for lithium-ion battery. Appl. Energy. 207, 384–393 (2017)
22. Hu, X., et al.: Co-estimation of state of charge and state of health for lithium-ion batteries based on fractional-order calculus. IEEE Trans. Veh. Technol. 67(11), 10319–10329 (2018)
23. Ma, Y., et al.: Fractional modeling and SOC estimation of lithium-ion battery. IEEE/CAA J. Autom. Sin. 3(3), 281–287 (2016)
24. Zhang, Q., et al.: A novel fractional variable-order equivalent circuit model and parameter identification of electric vehicle Li-ion batteries. ISA Trans. 97, 448–457 (2020)
25. Tian, J., et al.: Online simultaneous identification of parameters and order of a fractional order battery model. J. Clean Prod. 247, 119147 (2020)
26. Wang, Q., et al.: State of charge-dependent polynomial equivalent circuit modeling for electrochemical impedance spectroscopy of lithium-ion batteries. IEEE Trans. Power Electron. 33(10), 8449–8460 (2018)
27. Bizeray, A.M., et al.: Identifiability and parameter estimation of the single particle lithium-ion battery model. IEEE Trans. Control Syst. Technol. 27(5), 1862–1877 (2019)
28. Stroe, D., et al.: Lithium-ion battery power degradation modelling by electrochemical impedance spectroscopy. IET Renew. Power Gener. 11(9), 1136–1141 (2017)
29. Yang, J., et al.: Lithium-ion battery internal resistance model based on the porous electrode theory. IEEE Vehicle Power and Propulsion Conference (VPPC), Coimbra, 1–6 (2014)
30. Zhang, S., Sun, H., Lyu, C.: A method of SOC estimation for power Li-ion batteries based on equivalent circuit model and extended Kalman filter. 13th IEEE Conference on Industrial Electronics and Applications (ICIEA), Wuhan, pp. 2683–2687 (2018)
31. Liu, Y., et al.: State-of-charge co-estimation of Li-ion battery based on on-line adaptive extended Kalman filter carrier tracking algorithm. 44th Annual Conference of the IEEE Industrial Electronics Society, Washington, DC, pp. 1940–1945 (2018)
32. Wang, Y., Li, M., Chen, Z.: Experimental study of fractional-order models for lithium-ion battery and ultra-capacitor: Modeling, system identification, and validation. Appl. Energy. 278, 115736 (2020)
33. Sun, Y., et al.: Variable fractional order - A comprehensive evaluation indicator of lithium-ion batteries. J. Power Sources 448, 227411 (2020)
34. Tian, J., Xiong, R., Yu, Q.: Fractional-order model-based incremental capacity analysis for degradation state recognition of lithium-ion batteries. IEEE T. Ind. Electron. 66(2), 1576–1584 (2019)

How to cite this article: Sun, X., et al.: A novel online identification algorithm of Lithium-Ion battery parameters and model order based on a fractional order model. IET Renew. Power Gener. 15, 2396–2408 (2021). https://doi.org/10.1049/rpg2.12172