Coherent emission from disordered arrays of driven Josephson vortices

Fabio Marchesoni,1,2 Sergey Savel'ev,2,3 Masashi Tachiki,4 and Franco Nori5,2

1Dipartimento di Fisica, Università di Camerino, I-62032 Camerino, Italy
2Advanced Science Institute, The Institute of Physical and Chemical Research (RIKEN), Wako-shi, Saitama, 351-0198, Japan
3Department of Physics, Loughborough University, Loughborough LE11 3TU, United Kingdom
4National Institute for Materials Science, 1-2-1 Sengen, Tsukuba 305-0047, Japan
5Center for Theoretical Physics, Department of Physics, University of Michigan, Ann Arbor, MI 48109-1040, USA

(Dated: May 1, 2014)

We propose a mechanism of coherent emission from driven vortices in stacked intrinsic Josephson junctions. In contrast to super-radiance, which occurs only for highly ordered vortex lattices, we predict resonant radiation emission from weakly correlated vortex arrays. Our analytical results for the THz wave intensity, resonance frequencies, and the dependence of THz emission power on dissipation are in good agreement with the ones obtained by recent simulations.

PACS numbers: 74.50.+r, 74.25.Gz, 03.40.Kf

I. INTRODUCTION

It has been experimentally observed1,2 and confirmed both analytically3,4 and numerically5,6 that moving Josephson vortices (JVs) emit sub-THz electromagnetic radiation. Tera-Hertz radiation has applications in physics, astronomy, chemistry, biology, and medicine. This motivates recent proposals3 for THz filters10,12, detectors13, quantum devices14,15, and emitters3,6 based on highly anisotropic layered superconductors (e.g., Bi2Sr2CaCu2O8+δ), that can be modelled as coupled intrinsic Josephson junctions (IJJs).

The ultimate challenge in this field is to produce coherent THz radiation. It is commonly believed that this goal can be achieved by controlling super-radiance from highly ordered vortex lattices16,17. A vortex lattice is deemed necessary because the constructive interference of Josephson plasma waves from individual JVs is strongly suppressed by small amounts of disorder. Unfortunately, driven periodic lattices are often very unstable (especially in the presence of impurities, defects, and pinning centers), and moving JVs form either a mixture of coexisting different lattices16 or even disordered arrays17. Moreover, a broad radiation spectrum by individual vortices results in a broad spectrum of the emitted radiation (e.g., Ref. 6), in contrast to a desirable resonant IJJ, where coherent radiation is characterized by sharp spectral lines.

In this context, recent interesting simulations by Tachiki and coworkers5,6 show that coherent radiation may be generated by JVs moving as disordered arrays, instead of just ordered ones. This raises the question as under what conditions JVs in layered superconductors emit coherent radiation. Solving this problem is crucial to the effective design of Bi2Sr2CaCu2O8+δ-based THz emitters.

The experimental demonstrations of THz radiation in zero magnetic field and various failed attempts at detecting THz emission in the presence of magnetic fields cast serious doubts on the initial idea that moving JVs can radiate in this frequency domain. Indeed, the now prevailing interpretation is that JVs ought to be considered as perturbing degrees of freedom, which destroy the layer coherence and thus cause the suppression of THz radiation. In this study, however, we reach the conclusion that, under appropriate conditions, applied magnetic fields do help amplify and tune THz emission. This interesting result is also consistent with the recent systematic studies in Ref. 16.

Below, we show that the nonlocal nature of JVs in layered superconductors is responsible for a two-scale dynamics. A longer scale, λEM, characterizes the inter-vortex magnetic interaction; spatial dispersion, or disorder, of vortices up to such a scale has no appreciable impact on the radiation mechanism. In other words, in contrast with super-radiance, which is suppressed by vortex disorder, in our approach radiation coherence is preserved even if the vortex distribution can become appreciably modulated by the radiation itself for wavelengths shorter than λEM. The shorter relevant length scale λG determines the cross-section of the nonlinear vortex core (where the linear approximation sin ϕ ≈ ϕ with the gauge invariant phase difference across the junction having a vortex is not valid). According to this picture, for λG ≪ λEM and for a sufficiently high vortex density, radiation is emitted through a linear mechanism, as from a JV lattice, whereas the vortex-radiation coupling occurs mainly in the inner JV cores. Under these conditions, the magnetic interaction among vortices is much weaker than their interaction with the emitted radiation. Our approach explains the spatial modulation of the JV density numerically found in Ref. 17. Our analytical estimates, based on a one-dimensional sine-Gordon (sG) model, prove to be in good agreement with their simulations and explain their results.

Let us now summarize a central idea of our approach. Consider a moving JV lattice emitting radiation. This radiation will bounce back and forth the sample edges, like in a laser cavity. This radiation accumulates and creates a standing wave with a wavelength about λEM.
This standing wave modulates the JV density which is now in resonance with the standing wave. This positive feedback enhances the radiation of vortices. Namely, the JV motion emits radiation, which is weaker at first. This radiation bounced inside the sample (acting as a cavity) locks the collective motion of the JVs. This collective motion produces stronger emission. The JVs then interact more strongly with the electromagnetic standing wave, compared with the now much weaker vortex-vortex interaction. Thus, the triangular vortex lattice, produced by the vortex-vortex interaction, is finally replaced by a more disordered, but still modulated by the radiation, vortex structure.

II. NONLOCAL SINE-GORDON MODEL

Layered superconductors can be considered as stacks of strongly interacting IJJs. As the superconducting layers are only a few nanometers thick, i.e., the inter-layer distance is much smaller than the magnetic field penetration depth $\lambda_{ab}$, the currents flowing through different junctions are coupled. On neglecting, for the time being, external drives and internal dissipation, a system of stacked IJJs is well described by the coupled sine-Gordon equations

$$\left(1 - \frac{\lambda^2 N}{s^2} \Delta_n^2 \right) \left( \frac{\phi^{(n)}}{\omega_p^2} + \sin \phi^{(n)} \right) - \lambda^2 \phi^{(n)}_{xx} = 0, \quad (1)$$

where $\phi^{(n)}$ is the gauge invariant phase difference across the $n$th junction. Here, $\omega_p$ is the Josephson plasma frequency, $\lambda$ the London penetration depth ($\lambda/\lambda_{ab} = \gamma$) along the layers, and the operator $\Delta_n^2 f = f^{(n+1)} - 2 f^{(n)} + f^{(n-1)}$.

A full analysis of this set of equations is a complicated problem which requires numerical simulation. However, if we restrict ourselves to the case of moderate magnetic fields, when JV cores do not overlap, we can reduce Eq. (1) to an effective 1D problem. Indeed, as was shown in\(^\text{11}\) (see Eqs. (21) and (22) and Fig. 2 there), the phase difference $\phi$ decreases very fast away from the junction where a vortex located. Thus, a reasonable strategy could consist in neglecting the nonlinear couplings between junctions at a distance of some $s$ from the vortex center; on solving the linearized equations (1) for such junctions, one would end up with a few coupled nonlinear equations for a few junctions in the vicinity of the vortex center. The case when the nonlinearity was restricted to one junction only has been considered in Ref.\(^\text{5}\). The coupled junction system of Eq. (1) boils down to a 1D Josephson junction described by a nonlocal sine-Gordon equation. We assume below that retaining the nonlinear coupling between more junctions can lead to the same nonlocal 1D sine-Gordon equation with additional noise-like weak perturbations.

For simplicity, let us consider the pair of adjacent junctions $j$ and $(j + 1)$ locating a moving JV. We then reduce the description of the IJJ stack to a 1D problem by assuming the nonlinear coupling to be important only for the paired junctions and linearizing Eq. (1) for all other junctions (i.e., for $n \neq j, j + 1$). It is interesting to note that the importance of the interaction between two neighboring junctions is numerically well established\(^\text{22}\). Moreover, Koshelev\(^\text{22}\) recently reduced the multi-junction system to two coupled junctions, and this model reproduced the simulation data in Ref.\(^\text{21}\) and interpreted the experimental results of Ref.\(^\text{3}\).

Following the approach in Refs.\(^\text{5,23,24}\), the equation for the averaged phase difference across a junction pair $\phi = (\phi^{(j+1)} + \phi^{(j)})/2$ can be written as

$$\frac{\phi}{\omega_p^2} \sin \phi = \frac{\gamma s}{2 \pi} \int dx' K_0 \left( \frac{|x - x'|}{\lambda} \right) \phi_{xx}(x') + P[\psi] \sin \phi, \tag{2}$$

where $K_0$ is the modified Bessel function and $P[\psi] = 1 - \cos \psi$ with $\psi = (\phi^{(j+1)} - \phi^{(j)})/2$. The length $\lambda_G \equiv \frac{\gamma s}{2} = \frac{\lambda^2 N}{\lambda}$ defines the size of the JV core. Again, the contribution of the next-to-neighbor junctions to the dynamics of the tagged JV weakens fast with their distance from the vortex center, thus, allowing all other nonlinear equations (1) to be replaced by an effective nonlinear medium.

An additional equation for $\psi$ can be derived for a pair of JVs in two adjacent junctions, so that the equations for $\phi$ and $\psi$ form a closed set\(^\text{5,23}\). However, when extending Eq. (2) to describe the collective motion of $N$ traveling JVs (randomly distributed along $N_l$ stacked IJJs of length $l$), the phase $\phi$ can be regarded as a mean-field superposition of the $n = N/N_l$ individual JVs phases, $\phi^{(n)}$, contained in one layer, only.

In our one-IJJ description we assume that $\psi$ is relatively small; this may be the case, for instance, due to the random superposition of the vortex dynamics in different junctions. Anyway, the good agreement between the analytical results reported here and earlier numerical simulations, validates a posteriori our assumption. Thus, the functional $P[\psi]$ can be modelled as a spatial perturbation $\delta + \epsilon P(x)$, where the real function $P(x)$ can be either periodic or random in $x$, depending on the operating conditions. The constant $\epsilon$ is a measure of the strength of the perturbation, while the small offset $\delta$ can be conveniently eliminated by rescaling the dimensional parameters $\omega_p$ and $\lambda_G$, as appropriate. Here, we assimilate such a perturbation as an effective quenched Gaussian disorder along the IJJs; that is, $P(x)$ is modeled as a random, delta-correlated function with

$$\langle P(x) \rangle = 0, \quad \langle P(x) P(x') \rangle = 2 \delta(x - x'), \tag{4}$$

and $\langle \ldots \rangle$ denoting the average over different disorder realizations. The constant $\epsilon$ will be taken as a perturbation parameter and only effects to leading order in $\epsilon$ will
be considered. Moreover, deviations from the Gaussian statistics, implicit in the definition of $P[\psi]$, are assumed to be negligible within this approximation.

### A. Josephson vortex array

We now introduce dimensionless units by expressing $x$ and $t$ in units of the characteristic length $\lambda_{EM}$ and the reciprocal of the plasma frequency $\omega_p$, respectively; that is

$$x \to \tilde{x} = x / \lambda_{EM}, \quad t \to \tilde{t} = \omega_p t.$$  

As a consequence, the system characteristic lengths $\lambda$ and $\lambda_G$ get rescaled as follows:

$$\lambda \to \tilde{\lambda} = \lambda / \lambda_{EM}, \quad \lambda_G \to \tilde{\lambda}_G = \lambda_{EM} / \lambda = 1 / \tilde{\lambda},$$

and, of course, $\lambda_{EM} \to \tilde{\lambda}_{EM} = 1$. Correspondingly, $\varphi(x, t)$ is given in units of the magnetic flux quantum $\phi_0$ and all speeds in units of $\omega_p \lambda$. Hereafter, for the sake of simplicity, we shall only use dimensionless variables and, therefore, omit the “tilde” notation altogether.

The field $\varphi(x, t)$, corresponding to a dense distribution of JVs traveling with speed $V \ll 1$, can be expanded as:

$$\varphi(x, t) = p \left( x - V t \right) - \kappa \sin[p \left( x - V t \right)] + \ldots \quad (5)$$

where $p = 2\pi \rho$, and $\rho = n/L$ denotes the linear JV density with number $n$ of vortices located along the length $L$. The linear term in Eq. (5) corresponds to the phase difference of a uniform vortex spatial distribution, whereas the periodic correction accounts for a residual phase modulation on the lattice scale $1/\rho$, with amplitude $\kappa$ to be determined self-consistently. In the expansion (5) we assume high JV densities, $p \gg 1$, and small amplitudes $\kappa$. On inserting expansion (5) for $\varphi(x, t)$, the nonlinear field equation (2) can be approximated to an effective sine-Gordon equation, where

$$\frac{1}{\pi \lambda} \int K_0 \left( \frac{|x - x'|}{\lambda} \right) \varphi_{xx}(x') \, dx' \to c_p^2 \varphi_{xx}, \quad (6)$$

and, consistently,

$$c_p^2 = [1 + (p \lambda)^2]^{-\frac{1}{2}},$$

and

$$\kappa = (\gamma_V / c_p)^2,$$

with

$$\gamma_V = (1 - \beta^2 / c_p^2)^{-\frac{1}{2}}.$$  

In the regime considered in Refs. [24,25], where $\lambda \gg 1$ and $\gamma_V \simeq 1$, the parameter $c_p$ can be further approximated to $(p \lambda)^{-\frac{1}{2}}$.

The validity condition for truncating the expansion (5) to its first order, $\kappa \ll 1$, or equivalently $c_p \rho \gg 1$, implies a direct core-core interaction; that is $1 / \lambda \geqslant 1 / p$. We recall that here $1 / \lambda$ represents the size of a vortex core in dimensionless units. Accordingly, for $\lambda \gg 1$, $c_p$ must be regarded as the maximum velocity of a vortex array in a layered superconductor, to be compared with the maximum dimensionless velocity $1 / \lambda$ of a single vortex (i.e., $\omega_p \lambda G$ in dimensional units).

In the opposite limit, $p / \lambda \ll 1$, vortices only weakly interact on the magnetic length scale $\lambda_{EM}$ (rescaled here to 1); the limiting velocity $c_p$ grows larger than $1 / \lambda$ and the JV array becomes unstable.

### B. Radiation mechanism

The emission of radiation by fast moving JVs also takes place on the magnetic length scale $\lambda_{EM}$. The effective phase difference $\varphi$ associated with an array of JVs moving along an IJJ, thus, obeys the perturbed local sine-Gordon equation

$$\varphi_{tt} - c_p^2 \varphi_{xx} + \sin \varphi = -\beta \varphi_t - f + \epsilon P(x) \sin \varphi. \quad (7)$$

Note that, in leading order, $\varphi \approx p \left( x - V t \right)$, as can be seen from Eq. (5). Here, for completeness, we have restored the viscous term $-\beta \varphi_t$ and the current-induced drive $f$, that allow us to control the net JV speed $V$ (see, e.g., Ref. [24]).

Like in the more conventional single sine-Gordon-soliton perturbation schemes [25,26], we consider the Ansatz

$$\varphi(x, t) = \varphi(x, t) + \chi(x, t), \quad (8)$$

which, inserted in Eq. (7), yields

$$\chi_{tt} - c_p^2 \chi_{xx} + (\cos \varphi) \chi = -\beta \chi_t + \epsilon P(x) \sin \varphi, \quad (9)$$

where $\varphi = 0$ is the ground state and only terms $O(\epsilon)$ have been kept. The wavenumber $q$ and the angular frequency $\omega$ of the unperturbed plasmon modes (i.e., for $\beta = \epsilon = 0$) form a continuum spectrum [22,26], with

$$\omega^2 = 1 + c_p^2 q^2. \quad (10)$$

However, as for the field (5) with $p \gg 1$, the radiation-vortex coupling $(\cos \varphi) \chi$ becomes negligible, and the plasma wave dispersion relation can be approximated to $\omega = c_p |q|$. On introducing the spatial Fourier components of $P(x)$ and $\chi(x, t)$, defined by

$$P(x) = \frac{2}{\pi} \int_{-\infty}^{\infty} P(k) e^{i k x} \, dk, \quad \chi(x, t) = \frac{2}{\pi} \int_{-\infty}^{\infty} \chi_q(t) e^{i q x} \, dq,$$

Eq. (9) can be rewritten as

$$\frac{d}{dt} B(q) - \frac{\beta}{2} B(q) = \frac{\epsilon}{2 i} \left[ e^{i(\omega - V) t} P(q - p) - (p \to -p) \right],$$

where

$$B(q) = (\hat{\chi}_q - i |q| \chi_q) e^{i \omega t}$$
is directly related to the spectral density of the array emission power,
\[ W(q) = \frac{4}{\pi} \frac{d}{dt}|B(q)|^2, \]
that is
\[ W(q) = \frac{2\epsilon^2}{\pi} \left[ |P(q-p)|^2 \frac{\beta/2}{(\beta/2)^2 + (\omega - pV)^2} + (p \rightarrow -p) \right]. \] (11)
Here we use that \( P(x) \) is a real function, so that \( P^*(k) = P(-k) \). The notation “\( p \rightarrow -p \)” denotes the symmetric term obtained by replacing \( p \rightarrow -p \) in the first term inside the square brackets. This means that two waves propagate in opposite directions with the same frequency \( \omega \); for \( |P(q-p)| = |P(q+p)| \), they generate standing plasma oscillations, like those reported in Ref. 6,7. To simplify our notation, hereafter we restrict ourselves to JVs driven in one assigned direction, say \( x > 0 \).

The spectral emission power (s.e.p.) \( W(q) \) is key to our analysis of a resonant IJJ. The spectrum \( W(q) \) can be easily specialized for any choice of \( P(x) \). In the case of quenched Gaussian disorder, see Eq. (11),
\[ \langle |P(k)|^2 \rangle = \frac{1}{8}, \] (12)
so that on disorder-averaging Eq. (11) we obtain the IJJ spectral emission power per unit of length
\[ w(\omega) = \frac{\epsilon^2}{4\pi} \left[ \frac{\beta/2}{(\beta/2)^2 + (\omega - pV)^2} + (\omega \rightarrow -\omega) \right]. \] (13)
This spectrum holds for \( \beta \ll \omega \), or, equivalently, for \( V \gg \beta/p \), and has a sharp resonance maximum
\[ w^\text{max} = \frac{\epsilon^2}{2\pi\beta} \] (14)
for
\[ \omega_r = pV. \] (15)

**C. Vortex dynamics**

Subject to a drive \( f \) produced by an externally-applied electrical current, the vortices in an IJJ flow with an average speed \( V \) and, simultaneously, their cores interact with the electromagnetic waves they radiate. For the relatively weak vortex core repulsion, \( 1/\lambda \gtrsim 1/p \), simulated in Ref. 26, we expect that the vortex array can be modulated, both in space and time, by the resonant plasma modes.

To express the average speed \( V \) of a JV array with \( p/\lambda \gg 1 \) as a function of the drive \( f \), from Eq. (17) we derive the energy balance equation per unit of length of radiating IJJ,
\[ w(V) + \beta(pV)^2 = pVf. \] (16)
Equation (16) tells us that the rate at which the drive pumps energy into the system (right-hand-side), must be equilibrated by the radiative, \( w(V) \), and the viscous loss, \( \beta(pV)^2 \), of the soliton array \( \varphi(x,t) \) (left-hand-side).

For \( V \gg \beta/p \), the total emission power of the radiating sine-Gordon solitons,
\[ w(V) = \frac{\epsilon^2}{2}, \] (17)
is computed by integrating the spectral emission power (11); solving the ensuing Eq. (16) with respect to \( V \), we obtain
\[ V(f) = \frac{f \pm (f^2 - 2\beta^2)^{\frac{3}{2}}}{2\beta p}, \] (18)
where only the rising branch with the + sign is stable29. Therefore, the observable velocity-drive characteristic \( V(f) \) is expected to show a step at
\[ f_{\text{th}} = (2\beta)^4 \epsilon \] (19)
and to grow linearly with \( f \) for \( f \gg f_{\text{th}} \), when the radiation loss becomes negligible, namely
\[ V = \frac{f}{\beta p}. \] (20)

Note that, at variance with an emitting JV lattice29, no multiple hysteretic steps in the \( V(f) \) are predicted. Indeed, the condition \( f > f_{\text{th}} \) simply implies that the effective phase \( \varphi \) is not pinned by disorder29; for \( f \lesssim f_{\text{th}} \), instead, the JV array can move only by creeping, namely, through the nucleation and the subsequent migration of array defects31,33. Creeping is likely responsible for the smooth low-current \( J-V \) characteristics shown in Fig. 4 of Ref. 6. Moreover, in the linear regime (20) the wavelengths \( \lambda_r \) of the emitted radiation are expected to be much shorter than the length \( L \) of the IJJ (see below), so that corrections due to the appropriate standing-wave periodic boundary conditions are of the order of \( \lambda_r/L \).

A vortex is sensitive to the radiation field only when the wavelengths \( \lambda_r \) excited in the IJJ are larger than its size. For the parameter choice of Refs. 6,7, this can only occur on the JV core scale \( \lambda_G \), because \( \lambda_G \lesssim \lambda_r \).

The interaction between the radiation standing wave, say
\[ \chi(x,t) = \chi_0 \cos(qx) \cos(\omega t + \phi), \] (21)
and a single JV solution of the nonlocal sine-Gordon equation (24),
\[ \varphi(x) = \pi + 2 \arctan(\lambda x) \] (22)
(both in dimensionless units) is well described by the nonrelativistic quasi-particle approach of Ref. 30. The JV center of mass with coordinate \( X(t) \) is subject to an oscillating sinusoidal trap
\[ \ddot{X} = -\beta \dot{X} + \frac{q^2}{\lambda} e^{-|q|/\lambda} \cos(qX) \cos(\omega t + \phi), \] (23)
with an amplitude which is exponentially suppressed at short wavelengths, i.e., for $q/\lambda \gg 1$. However, for sufficiently large trap amplitudes, the vortices in each layer get spatially distributed with wavevector $q$.

III. COMPARISON WITH NUMERICAL RESULTS

The results in Refs. 6, 7 can be easily analyzed within the above theoretical framework. To make contact with their numerical data, one must express: all lengths in units of $\lambda$, with $\lambda = 200 \mu m$; the velocities in units of the light speed in the dielectric $c = c_0/\sqrt{\epsilon}$, where $c_0$ is the speed of light in vacuo, and $\epsilon_c = 10$ is the simulated dielectric constant; the forces in units of $J/J_c$, where $J_c$ is the critical JJ current and $J$ is the superconducting current across the IJJs; and the angular frequencies in units of the plasma gap frequency $\nu_p = \omega_p/2\pi = c/\lambda = 0.47 \times 10^{12}$ Hz. Moreover, the actual layer JV density is $\rho = n/L \simeq 0.6 \mu m^{-1}$ with $L = 100 \mu m$, the layer thickness is $s = 15 \AA$, and the penetration length ratio $\gamma = 500$. For this choice of numerical parameters, the length scales we introduced in the previous section read, respectively, $\lambda_G = 0.38 \mu m$, $\lambda_{EM} = 8.7 \mu m$ and $1/p = 0.27 \mu m$.

First, we note that the simulations of Refs. 6, 7 correspond to the physical condition where $\lambda_G \lesssim \lambda_r$. As for the resonant modes $\chi_0 \propto \epsilon$, see Eq. (14), the amplitude of the driving force in Eq. (23) turns out to scale like $\epsilon(\lambda/\lambda_G)^{1/2}$, which is strong enough to drag a JV against the disorder field [4] and the array of restoring forces. This explains the disordered spatial distribution of the emitting JVs, which, far from forming any ordered lattice, seem rather to get trapped by the plasma standing waves. In spite of the coherent nature of the plasma radiation, the vortex distributions in each IJJ can differ from one another because of the intrinsic disorder brought about by the layer-layer coupling.

In Fig. 1 we compare the current-voltage characteristics from simulation, reported in Fig. 4 of Ref. 6, with the force-velocity ($f$-$V$) curve of Eq. (24). In the units of Ref. 6

$$\frac{J}{J_c} = 2\pi \beta \left( \frac{\rho \lambda}{c} \right) = 2\pi \beta \left( \frac{\mathcal{V}}{V_p} \right),$$

(24)

where $\mathcal{V} = \rho \lambda (V/c)$ is the flux-flow voltage across a IJJ layer and $V_p \equiv \nu_p \Phi_0$, with $\Phi_0 = h/2e$ denoting the flux quantum. The agreement is quite good in the linear regime, whereas the depinning threshold (19) is clearly visible for $J/J_c \simeq 0.2$, which, in our units, corresponds to setting $\epsilon = 1$.

The resonant plasma radiation is investigated in Refs. 6, 7 on the linear branch of the $J$-$V$ characteristics. Three plots of the plasma standing waves, two in Ref. 6 and one in Ref. 5, are shown for different $J/J_c$; from there we read out the corresponding resonance wavelengths $\lambda_r$.

Furthermore, the resonance frequencies $\nu_r = 2\pi \omega_r$ are either given explicitly in the text or shown in the figure (see, e.g., Fig. 5 of Ref. 6). The product $\lambda_r \nu_r$ appears to define a Swihart velocity, denoted here by $c_S$, independent of the simulation parameters $J/J_c$ and $\beta$, that is

$$\frac{c_S}{c} \simeq 0.04.$$  \hspace{1cm}

(25)

In view of our radiation mechanism [4], the ratio $c_S/c$
can be identified with $c_p$ in Eq. (6); accordingly, for $\lambda \gg 1$ and $\gamma_V \simeq 1$, 
\begin{equation} 
\frac{c_S}{c} \simeq \frac{1}{\sqrt{\lambda p}} 
\end{equation} 
is predicted to be of the order of 0.036, which is reasonably close to the result in Refs. 6,7 given the accuracy of the data available.

The average JV speed in a resonant IJJ structure is proportional to the resonance frequency, that is, from Eq. (15), 
\begin{equation} 
V = \frac{\nu_r}{\rho} 
\end{equation}
This equation holds for all different choices of the simulation parameters presented in Refs. 6,7. Note that the measured JV speeds are relatively small, $V \ll c$, as assumed in our nonrelativistic treatment of Eq. (9), where $\gamma_V \simeq 1$. Moreover, when combined with Eq. (24), this equation yields the dependence of $\nu_r$ on the simulation control parameters $J/J_c$ and $\beta$. The ensuing law 
\begin{equation} 
\frac{\nu_r}{\nu_p} = \frac{1}{2\pi \beta} \frac{J}{J_c} 
\end{equation}
closely matches all spectral resonance peaks reported in Ref. 6, as shown in Fig. 2. Note that combining Eqs. (24) and (28) yields the simple $\beta$-independent relation 
\begin{equation} 
\frac{\nu_r}{\nu_p} = \frac{V}{V_p}. 
\end{equation}
Finally, we notice from Eqs. (14) and (20) that $w_{\text{max}}$ is proportional to $\epsilon^2/\beta$ and $V$ is proportional to $f/\beta$; as a consequence, one would expect that on decreasing $\beta$ the IJJ spectral emission band shifts to lower $J/J_c$ while growing in intensity, both inversely proportional to $\beta$. This is exactly the dependence displayed in Fig. 6 of Ref. 6.

IV. CONCLUSIONS

We propose a new mechanism of coherent radiation from the moving Josephson vortices in layered superconductors. We show, that due to the two-scale structure of Josephson vortices, they radiate THz radiation on a characteristic scale $\lambda_{EM}$, which is much longer than the Josephson vortex core size $\lambda_G \sim \gamma s$. Among all emitted waves, only standing modes in the sample (working as a cavity) survive. These standing modes produce modulation of the density of JVs. This, in turn, make vortices mainly radiate with wavelengths corresponding to standing waves. Such positive feedback can result in relatively strong radiation with well pronounced maxima in the spectra. All our analytical estimates are in a good agreement with numerical data.

The experimental demonstration of THz radiation in zero magnetic field and various failed attempts at detecting THz emission in the presence of magnetic fields cast serious doubts on the initial idea that moving JVs can radiate in this frequency domain. Indeed, the now prevailing interpretation is that JVs ought to be considered as perturbing degrees of freedom, which destroy the layer coherence and thus cause the suppression of THz radiation. In this study, however, we reach the conclusion that, under appropriate conditions, applied magnetic fields do help amplify and tune THz emission. This interesting result is also consistent with the recent systematic studies in Ref. 10.

Acknowledgments

FN acknowledges partial support from the National Security Agency (NSA), Laboratory for Physical Sciences (LPS), Army Research Office (ARO), National Science Foundation (NSF) grant No. EIA-0130383. FN and SS acknowledge partial support from JSPS-RFBR 06-02-91200, and Core-to-Core (CTC) program supported by the Japan Society for Promotion of Science (JSPS). S.S. acknowledges partial support from the UK EPSRC via Nos. EP/D072581/1 and EP/F005482/1.

1 G. Hechtfischer, R. Kleiner, A. V. Ustinov, and P. Müller, Phys. Rev. Lett. 79, 1365 (1997).
2 J. Zitzmann, A.V. Ustinov, M. Levitchev, S. Sakai, Phys. Rev. B 66, 064527 (2002).
3 L. Ozyuzer, A.E. Koshelev, C. Kurter, N. Gopalsami, Q. Li, M. Tachiki, K. Kadowaki, T. Yamamoto, H. Minami, H. Yamaguchi, T. Tachiki, K.E. Gray, W.-K. Kwok, and U. Welp, Science 318, 1291 (2007).
4 E. Goldobin, A. Wallraff, N. Thysen, A.V. Ustinov, Phys. Rev. B 57, 130 (1998).
5 S. Savel’ev, V. Yampol’skii, A. Rakhmanov, F. Nori, Phys. Rev. B 72, 144515 (2005); Physica C 437–438, 281 (2006); Physica C 445, 175 (2006).
6 M. Tachiki, M. Iizuka, K. Minami, S. Tejima, and H. Nakamura, Phys. Rev. B 71, 134515 (2005).
7 M. Tachiki, M. Iizuka, K. Minami, S. Tejima, and H. Nakamura, Physica C 426–431, 8 (2005); Physica C 437–438, 299 (2006).
For a recent review on Terahertz Josephson plasma waves in layered superconductors, see: S.E. Savel’ev, V.A. Yampol’skii, A.L. Rakhmanov, and F. Nori, Reports on Progress in Physics 73 026501 (2010).

See, e.g., the special issue of Philosophical Transactions: Mathematical, Physical & Engineering Science 362, No 1815 (2004).

S. Savel’ev, A.L. Rakhmanov, F. Nori, Phys. Rev. Lett. 94, 157004 (2005); Physica C 445, 180 (2006); S. Savel’ev, A.L. Rakhmanov, V.A. Yampol’skii, F. Nori, Nature Physics 2, 521 (2006); V.A. Yampol’skii, S. Savel’ev, O.V. Usatenko, S.S. Mel’nik, F.V. Kusmartsev, A.A. Krokhin, and F. Nori, Phys. Rev. B 75, 014527 (2007).

S. Savel’ev, A.L. Rakhmanov, F. Nori, Phys. Rev. B 74, 184512 (2006); S. Savel’ev, A.O. Sboychakov, A.L. Rakhmanov, and F. Nori, Phys. Rev. B 77, 014509 (2008); A. Barone and G. Paternò, Physics and Applications of the Josephson Effect (Wiley, New York, 1982).

M. Machida, T. Koyama, A. Tanaka, and M. Tachiki, Physica C 330, 85 (2000).

M.-H. Bae, H.-J. Lee, and J.-H. Choi, Phys. Rev. Lett. 98, 269901 (2007); S. Savel’ev, A.L. Rakhmanov, X. Hu, A. Kasumov, F. Nori, Phys. Rev. B 75, 165417 (2007); S. Savel’ev, A.O. Sboychakov, A.L. Rakhmanov, F. Nori, Phys. Rev. B 77, 014509 (2008); A.O. Sboychakov, S. Savel’ev, A.L. Rakhmanov, and F. Nori, EPL 80 17009 (2007).

M. Machida, T. Koyama, A. Tanaka, and M. Tachiki, Physica C 330, 85 (2000).

A.E. Koshelev, I.S. Aranson Phys. Rev. Lett. 85, 3938 (2000).

A.L. Rakhmanov, S.E. Savel’ev, and F. Nori, Phys. Rev. B 79, 184504 (2009).

For instance, L.N. Bulaevskii, M. Zamora, D. Baeriswyl, H. Beck, and J.R. Clem, Phys. Rev. B 50, 12831 (1994); S.N. Artemenko and S.V. Remizov, Physica C 362, 200 (2001).

S. Lin and X. Hu, Phys. Rev. Lett. 100, 247006 (2008).

A.E. Koshelev, Phys. Rev. B 78, 174509 (2008).

A. Gurevich, Phys. Rev. B 46, 3187 (1992).

A. Barone and G. Paternò, Physics and Applications of the Josephson Effect (Wiley, New York, 1982).

V.E. Zakarov, S.V. Manakov, S.P. Novikov, and L.P. Pitaevskii, Soliton Theory (Nauka, Moscow, 1980).

O.M. Braun and Y.S. Kivshar, Phys. Rep. 306, 1 (1998).

J.F. Currie, J.A. Krumhansl, A.R. Bishop, and S.E. Trullinger, Phys. Rev. B 22, 477 (1980).

F. Marchesoni and C.R. Willis, Phys. Rev. A 36, 4559 (1987).