Analysis of unsteady flow in compressor cascade based on modal DMD methods

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Abstract: The reduced-order model based on the modal decomposition method can capture the main characteristics of the compressor. In this part, dynamic mode decomposition (DMD) methods are used to analyze the flow in a high-speed compressor cascade. The methods can extract dynamic information of the flow field. Both the design and off-design conditions (−2°, 0°, 4°) are analyzed. The DMD methods can also obtain the global energy, time coefficient and spatial characteristics of the flow field. DMD mode energy of the characteristic frequency is the highest. The mode with the frequency doubling also has high energy at off-design points. The time coefficient obtained by DMD changes uniformly and eigenvalues are located on the unit circle, which indicates that there is no divergent or convergent mode.

1. Introduction

In order to obtain the main characteristics of the flow field, an important method is to construct the flow field reduced-order model. Dynamic mode decomposition is a new reduced-order model.

Dynamic mode decomposition is developed by Schmid in 2008 [1]. DMD computes a set of modes each of which is associated with a fixed oscillation frequency and growth rate. It is more physically meaningful because each mode is associated with a damped sinusoidal behavior in time. It was originally used to extract dominant dynamic mode in simple flows. In recent years, it has been applied to the unsteady analysis of compressors. Hong et al.[2] applied DMD to a centrifugal compressor to obtain coherent structures and the global spectrums. The results show that the coherent structures are homogeneous with the control. Yang et al. [3] used the compressed DMD to analysis the unsteady characteristics of a centrifugal compressor. Zhu et al. [4] used the DMD method to extract the main flow structures in a centrifugal compressor under different conditions. Semlitsch et al. [5] used POD and DMD methods to analyze the flow structures appearing with the surge in a centrifugal compressor. The POD method works well for periodic flow. The DMD method is suitable for the flow field with a specific frequency and can get the mode growth rate and stability.

Many researchers have used POD to study the unsteady flow of the two-dimensional compressor cascade. More attention is paid to the boundary layer separation of the suction surface. However, the cross secondary flow near the end wall of the compressor cascade is also the main source of loss. Many researchers use unsteady flow control methods to inhibit separation. The studies are focused on the actuator parameters and the control effects. The large-scale flow structures that occur in the compressor have not been assessed. In part 1, the POD method has been used to analyze the main flow field structure.

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in the cascade. In this part, the DMD method is also used to analyze the flow field. Both the design and the off-design condition is analyzed. So we can compare the differences between the two methods in the unsteady flow field.

2. DMD method

DMD can extract dynamical features from flow data. First, a space-time velocity field takes the form of a snapshot sequence written as a vector matrix (1). The time step between two snapshots is $\Delta t$. The number of the snapshot is $N$.

$$V_N^1 = \{v_1, v_2, L, v_N\}$$  (1)

Assuming that the velocity field at the latter moment $v_{i+1}$ can be linearly represented at the previous moment $v_i$.

$$v_{i+1} = A v_i$$  (2)

So the system matrix $A$ can translate the flow field along time $\Delta t$. $A$ can describe the dynamical process of $V_N^1$.

$$V_N^2 = \{v_2, v_2, L, v_N\} = \{Av_1, Av_2, L, Av_{N-1}\} = A V_N^{N-1}$$  (3)

In general, the dimension $A$ is higher, so it is difficult to solve it directly. For the matrix $V_N^{N-1}$ with rank $r$. DMD methods require finding the matrix $S \in R^{r\times r}$ to replace $A$. This can be achieved by the singular value decomposition of $V_N^{N-1}$.

$$V_N^{N-1} = U \Lambda W^T$$

$$A = USU^T$$  (4)

$U$ is the right singular vectors of $V_N^{N-1}$. The calculation of $S$ can be seen as a minimum problem of

$$\min \left\| V_N^{N-1} - USU^T \right\|^2$$  (5)

Solve the corresponding eigenvalue $\mu_i$ and eigenvector $y_i$.

$$Sy_i = \mu_i y_i$$  (6)

The dynamic modes $\phi_i$ is defined as:

$$\phi_i = U y_i$$  (7)

The two norms of the DMD mode $\|\phi\|$ can reflect the global energy of the flow field. The real component of the $\lambda_i = \ln(\mu_i) / \Delta t$ can represents the growth rate of the DMD modes. The imaginary part of $\lambda_i / 2\pi$ represents the frequency of the DMD modes.

We are more concerned with the fluctuating velocity fields. Therefore, the time-averaged component is subtracted in this study. The time-averaged flow field is called the zeroth-order mode. The POD method usually normalizes the mode, while the DMD does not. The mode decomposition code used in this study is provided by Higham et al[6]. It has been applied to a two-dimensional shallow flow.

3. Numerical method and validation

3.1. Compressor cascade

3.2. Numerical method and validation

The cascade profile is NACA65-K48 which is design by DLR[7]. Qin et al[8]’s result showed that the fluctuating velocity is small. We get the same conclusions in this study. The chord length ($C$) of the blade is 60mm. The span height ($h$) is 100mm. The inlet angle $\beta_1$ is 132° at the design condition. The outlet angle $\beta_2$ is 90°. The pitch length is 33mm. The Stagger angle is 112.05°. Fig. 1 gives the sketch
A numerical study by means of unsteady Reynolds-averaged Navier–Stokes is performed by a commercial soft ANSYS CFX 14.0. SST k-ω model is used for turbulence treatment. The transition model is the γ-θ model. The computational grids were generated by Gambit. And the number of passage mesh is about 1.5 million. Fig. 2 shows the mesh of compressor cascade. The inlet of the cascade gives the total pressure, total temperature and the flow angle. The inlet Mach number is 0.7. The outlet gives the average static pressure. The wall is set with the adiabatic and non-slip condition. Translational periodic boundary conditions are applied on two sides of the cascade. The time step is $1 \times 10^{-6}$. Maximum inner iterations were set to 15 which produced sufficiently low residual. Fig. 3 shows the monitor points and the sections used in this study. We mainly studied 9 flow planes along the streamwise and 6 planes along spanwise.

![Fig. 1 The sketch of blade profile](image1)

![Fig. 2 Grids of cascade passage](image2)

![Fig. 3 The monitoring points and sections](image3)

The numerical verification work has been described in detail in the first part: POD method. The numerical method used in the paper is reasonable.

### 3.3. DMD analysis

#### 3.3.1. Energy characteristic

![Energy characteristic](image4)

(a) $-2^\circ$
Fig. 4 shows the global energy ratio distribution at different span heights. The energy ratio of the DMD is defined as the \( \| \phi \| / \max \| \phi \| \). The peak of global energy is at the reference frequency of the flow field. The global energy near the endwall is higher. As the frequency increases, the energy coefficient gradually decreases and changes regularly. The peak value appears at the frequency doubling and is lower than that of the reference frequency. For the case of 4 degrees, there is still a similar variation trend at 40\%H. For the cases of 0 and -2 degrees, the peak of frequency doubling is low and the variation trend is not obvious at 40\%H.

Fig. 5 shows the global energy ratio at different streamwise sections. The general trends are basically consistent with the spanwise. The peak still appears at the reference frequency. The DMD mode corresponding to the reference frequency contains most of the energy. For the case of 4 degrees, the energy of the high-frequency mode exceeds 40\% of the reference frequency. For the case of -2 degrees, the energy of the high-frequency mode at the outlet is less than 20\% of the reference frequency. For the case of 0 degrees, the separation flow near the leading edge is weak, so the energy of the low-frequency mode is higher.

Fig. 6 gives the distribution of the eigenvalues at 5\%H. It can be seen that the eigenvalues are basically located on the unit circle. Therefore, the modes are all stable modes. The calculation results also show that the \( \| \lambda \| = 1 \) for the three cases. The mode growth rate is also very small.

![Fig. 4 The energy ratio of mode at different spanwise position](image1)

![Fig. 5 The energy ratio of modes at different streamwise position](image2)

![Fig. 6 DMD mode eigenvalues at 5\%H](image3)
3.3.2. Temporal characteristics

![Graphs showing DMD coefficients at different angles](image)

Fig. 7 The DMD coefficients at 5%H

Fig. 8 The DMD coefficients at 40%H

Fig. 7 and Fig. 8 show the DMD coefficients at 5%H and 40%H. The mode coefficients show a sinusoidal distribution. The amplitude at the midspan is smaller than that of the endwall. The dominant frequency has the highest amplitude, which suggests that the amplitude can reflect global energy. The amplitude of the 4 degrees case is the highest. The time coefficient does not show a tendency to divergence or converge, which shows each mode is stable. The distribution of time coefficients of different axial positions is consistent with that of the spanwise.

3.3.3. Spatial characteristics

![Images showing DMD modes](image)

Fig. 9 DMD mode at 5%H at 4 degrees
The DMD usually considers the real part of the mode. Fig. 9, Fig. 10, and Fig. 11 give the reference frequency mode and high-frequency mode at 5%H. For the case of 4 degrees, the mode distribution is basically the same as the first two order POD mode. The fluctuating velocity near the suction surface separation is still the main structure of the flow field. The high-frequency mode reflects the small-scale structure. Similar conclusions can be extended to other cases and sections.
Fig. 12, Fig. 13, and Fig. 14 show the DMD mode distribution of the 120%C. For the three cases, the DMD method can obtain structures containing the reference frequency and the mode is closer to the first-order mode of the POD. The high-frequency mode reflects the small-scale structure and the magnitude is small.

The DMD method can directly capture the mode containing the unique frequencies and is superior to the POD method in describing the dynamic characteristics of the system. However, the flow field of the cascades studied in the paper has a higher steadiness and a single main frequency, the fluctuating velocity is low. The POD method can capture the dominant frequency structure of the flow field with the first two modes. The DMD method can capture the dominant frequency structure with one mode.

4. Conclusion

This study introduces DMD method to analyze the unsteady flow in a high-speed compressor cascade. Both the design condition and off-design conditions are studied. The main conclusions are as follows.

Fourier transform shows there is a dominant frequency for every incidence angle. The velocity field at the outlet is also well-organized. DMD mode energy of the characteristic frequency is the highest. The mode with the frequency doubling also has high energy at off-design points. The peak of global energy is at the reference frequency of the flow field. The global energy near the endwall is higher.

The time coefficients obtained by the DMD methods exhibit periodic changes. The time coefficient obtained by DMD changes uniformly and eigenvalues are located on the unit circle, which indicates that there is no divergent or convergent mode.

The DMD mode only contains a single frequency. The dominant frequency of the DMD is close to that of the first order POD mode.

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