On the Observability of Meso- or Macro-scopic Quantum
Coherence of Domain Walls in Magnetic Insulators

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ABSTRACT

Results are presented of a numerical calculation of the tunneling gap for a domain wall moving in the double well potential of a pair of voids in a magnetic insulator. Both symmetric and asymmetric double well potentials are considered. It is found that, even in the absence of dissipation, the prospect for observing quantum coherence on a meso- or macro-scopic scale appears unlikely.

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There has been a great deal of interest recently in the prospect that magnetic systems might provide a new setting in which to observe a macroscopic degree of freedom behaving quantum mechanically [1]. To date, the magnetic systems considered are: (i) magnetic grains [2, 3, 4, 5, 6]; and (ii) solitons in magnetic systems (particularly 180°-Bloch walls) [1, 7, 8, 9, 10, 11]. Attention has focused on macroscopic quantum tunneling since the conditions necessary for its observation appear the most favorable [12]. Arguably, of more fundamental interest is macroscopic quantum coherence (MQC) because of its connections with quantum measurement theory [13]. In MQC the macroscopic object tunnels periodically through the central barrier of a double well potential (DWP). This effect is a direct consequence of the quantum state of the object being in a coherent linear superposition of macroscopically distinguishable states, and it is this aspect of MQC that connects it to the Schrödinger cat paradox [14] and to quantum measurement theory.

In this paper we will examine quantum coherence (QC) for a 180°-Bloch wall (we suppress the “180°” below) moving in the double well potential due to a pair of voids present in a uniaxial magnetic insulator with quality factor $Q = K/2\pi M^2 \gg 1$ ($K =$ magnetic anisotropy constant, $M =$ spontaneous magnetization) at $T = 0$ [15]. Under these conditions: (i) demagnetization effects are small so that the voids do not alter significantly the Bloch wall configuration of the magnetization [16]; and (ii) most dissipative effects are small [8]. Thus our system, under the conditions assumed, is expected to have weak dissipation—a situation we approximate by ignoring dissipation (though see below). We calculate the ground (first excited) state energy $E_0$ ($E_1$) numerically (from which the tunneling gap is $\Delta_0 = E_1 - E_0$) for: (i) identical voids leading to a symmetric double well potential for various wall sizes $N$ (the number of spins in the wall) and void separations
and (ii) non-identical voids leading to an asymmetric double well potential for varying degrees of asymmetry (for a particular choice of $N$ and $L$). We find that observation of quantum coherence on either a macro- or meso-sopic scale appears unlikely. For macro-sopic QC ($N \geq 10^4$), weak stray magnetic fields introduce a bias into the gap which masks $\Delta_0$ except for void separations very close to the value at which the central barrier disappears. At these separations, the tunneling gap varies on a length scale that is less than the coarse graining length scale. One would expect that the experimentally relevant gap would be a coarse grained average which is seen to be less than the bias and so unobservable. For mesoscopic QC ($N \sim 10^2 - 10^3$), although the bias introduced by a stray magnetic field is quite small, the slightest bit of asymmetry in the voids is sufficient to pin the wall to the larger of the two voids. The difficulty here is that an adequately large tunneling gap requires a very small central barrier which is destroyed by the slightest difference in the voids. For spherical voids, we find that pinning of the wall can be avoided only when the difference in the radii is much less than the coarse graining length scale. Averaging the effects of asymmetry over the coarse graining length scale leads one to conclude that a real mesoscopic wall will likely be pinned by asymmetry. Thus even in the dissipationless approximation, one expects that observation of quantum coherence on any scale larger than microscopic appears unlikely due to the severe tolerances imposed on the experimental situation.

The system of interest is a magnetic insulator which is a lattice of spins (with lattice constant $l_0$) coupled to each other via the exchange and dipole-dipole interactions; and to the underlying lattice via the anisotropy interaction which is assumed to be uniaxial with easy axis along $\hat{z}$. For the length scales of interest to us the lattice system can be coarse
grained so that the magnetic state of the system is described by a magnetization $\mathbf{M}(x,t)$ defined on a 3-D spatial continuum. The total static energy in the absence of an external magnetic field is the sum of the exchange, anisotropy and demagnetization energies. A stationary Bloch wall is a soliton configuration of the magnetization $\mathbf{M}(x)$ with vanishing demagnetization energy, subject to the boundary condition $\mathbf{M}(y \rightarrow \pm \infty) = \pm M\hat{z}$. The spatial variation of the wall is localized to a planar region of thickness $\lambda = \sqrt{J/K}$ ($J =$ exchange stiffness constant) which is assumed to be parallel to the xz-plane. The wall coordinate $q$ specifies the distance from the origin to a reference point on the wall (viz. $\mathbf{x}_{wall} = q\hat{y}$) [15].

Voids in the magnetic insulator act as pinning sites for the wall. For materials with $Q \gg 1$, the attraction is due primarily to a reduction in the exchange and anisotropy energies that occurs when the wall sits on the void. For a void of length scale $R$ satisfying $l_0 \ll R \ll \lambda$ ($l_0 =$ lattice constant $\equiv 5\text{Å}$), located at the origin, the pinning potential seen by a flat Bloch wall is $U(q) = -U_0 \text{sech}^2(q/\lambda)$ [17], where $U_0 = 2KV_d$ and $V_d$ is the void volume. In our calculation, $\lambda = 1000\text{Å}$ ($50\text{Å}$) for walls with $N \geq 10^4$ ($300 \leq N \leq 3000$) and $U_0 = 0.1\text{eV}$ ($1.0 \times 10^{-5}\text{eV}$). For LaGaYIG, $Q = 25.2$ and $K \sim 2000\text{ergs/cm}^3$, so that for a spherical void $R \sim 200\text{Å}$ ($10\text{Å}$). We consider two spherical voids located at $\mathbf{x}_\pm = \pm Q_0\hat{y}$ ($L = 2Q_0$) with volumes $V_+ = aV_-$ ($a \geq 1$). They produce the double well pinning potential

$$U(q) = -U_0 \left[ \text{sech}^2\left(\frac{q + Q_0}{\lambda}\right) + a \text{sech}^2\left(\frac{q - Q_0}{\lambda}\right) \right].$$

When $a = 1$ we obtain a symmetric double well potential (SDWP); otherwise, the wall sees an asymmetric double well potential (AsDWP). For an energy $E$ corresponding to QC
there will be 4 turning points $T_1 < T_2 < T_3 < T_4$. We refer to the region $q < T_1$ as the "left barrier"; the region $T_1 < q < T_2$ as the "left well"; the region $T_2 < q < T_3$ as the "central barrier"; the region $T_3 < q < T_4$ as the "right well"; and the region $T_4 < q$ as the "right barrier". Varying the void separation $L$ (viz. $Q_0$) varies the depth of the wells and the height and width of the central barrier. The experimental situation envisioned is either: (1) a thin film or; (2) a very narrow wire of the magnetic insulator in which only one Bloch wall is present [8]. In the thin film case, only a small region (of the wall) of cross-sectional area $A_w$ is involved in tunneling between the pinning sites [8]. In our analysis below, we treat the wall as if it were flat, whereas, for the thin film scenario, it will in fact be curved. Curvature effects will be discussed below (see also [16]). In the case of the very thin wire, curvature effects are not expected to be important because of the large energy required to bend the wall on a length scale of order the cross-sectional dimension of the wire. Flat moving walls have a kinetic energy $M_D \dot{q}^2/2$ [15]. Here $M_D = A_w/2\pi\gamma^2\lambda$ is the Döring mass; and $\gamma$ is the gyromagnetic ratio. The wall Hamiltonian is $H = p^2/2M_D + U(q)$. Introducing the dimensionless length $x = q/\lambda$ and the energy scale $S = \hbar^2/2M_D\lambda^2$, we can write the time independent Schrodinger equation in the dimensionless form

$$\left[\frac{d^2}{dx^2} + U_0 \left\{ \text{sech}^2(x + x_0) + a \text{sech}^2(x - x_0) \right\} + \mathcal{E} \right] \psi = 0.$$  

Here $U_0 = U_0/S$; $x_0 = Q_0/\lambda$; and $\mathcal{E} = E/S$. The dimensionless potential strength $U_0$ is related to the wall size $N = A_w\lambda/l_0^3$ by

$$U_0 = \left( \frac{U_0 l_0^3}{\frac{1}{2}g^2\mu_B} \right) N,$$

where $g$ is the electron $g$-factor; and $\mu_B$ is the Bohr magneton.
We utilize a shooting algorithm \[18\] to solve the eigenvalue problem numerically. Details of this calculation will be given in \[16\]. Our results for the tunneling gap \(\Delta_0\) appear in Tables 1 and 2 and correspond to macroscopic and mesoscopic walls of thickness \(\lambda = 1000\,\text{Å}\) and \(50\,\text{Å}\) respectively. So far we have assumed our system of wall and voids to be completely isolated. If a weak stray magnetic field \(H_{\text{ext}}\) were present, it would produce a bias \(\epsilon\) in the gap \(\Delta = \sqrt{\Delta_0^2 + \epsilon^2}\). This bias is a consequence of the Zeeman energy density \(-\mathbf{M} \cdot \mathbf{H}_{\text{ext}}\). For an actual stray field, the direction of \(\mathbf{H}_{\text{ext}}\) is unknown and the experimentally relevant bias \(\bar{\epsilon}\) is obtained by averaging over this direction. It is easily shown that \(\bar{\epsilon} \sim MA_w L H_{\text{ext}}\). We take \(H_{\text{ext}} \sim 10^{-6}G\) as indicative of the magnitude of a stray magnetic field (AC-magnetic fields of \(10^{-5}G\) have been used in measurements of the frequency dependent magnetic susceptibility \[6\]). Clearly, a necessary condition for observable domain wall QC is \(\Delta_0 > \bar{\epsilon}\).

For \(N \geq 10^4\), we find that \(A_w = (1.1 \times 10^{-17}\,\text{cm}^2)N\) (and \(M_D = (5.6 \times 10^{-28}\,\text{gm})N\)), so that for LaGaYIG for which \(M \sim 10G\), \(\bar{\epsilon} = (1.04 \times 10^{-11}K)N\). Thus the bias grows with wall size \(N\), making observation of QC more difficult for the larger walls. To proceed further, note that there exists a limited range of void separations for which \(\Delta_0\) corresponds to QC and still satisfies \(\Delta_0 > \bar{\epsilon}\). For \(L < L_{\text{min}}\), the ground state energy is above the central barrier; while if \(L > L_{\text{max}}\), \(\Delta_0 < \bar{\epsilon}\). Let \(\mathcal{R} = L_{\text{max}} - L_{\text{min}}\) be the size of the allowed range of void separations. Its value is given in Table 1 and is obtained (for a given \(N\)) by comparing \(\Delta_0\) and \(L\) with \(\bar{\epsilon}\). Quantum coherence will be observable only when: (i) the uncertainty in \(L\) satisfies \(\Delta L \ll \mathcal{R}\); and (ii) \(\mathcal{R} \gg \mathcal{C}\), where \(\mathcal{C} \sim (2 - 3)l_0\) is the coarse graining length scale. One expects that \(\Delta L \sim l_0\) and in our calculation \(l_0 = 5\,\text{Å}\). If either (or both) of these conditions is (are) not satisfied one would expect that the experimentally relevant gap would be an average of \(\Delta_0\) over the appropriate length
scale. From Table 1 we see that, for $N \geq 10^4$, such an average is necessary and that any reasonable procedure gives $\bar{\Delta} < \bar{\varepsilon}$. Thus macroscopic QC ($N \geq 10^4$) is not expected to be observable due to the rapid variation of the tunneling gap to small changes in $L$ and the large bias $\bar{\varepsilon}$ introduced by a stray magnetic field. We also see that in the case of the SDWP (in the absence of dissipation), the conditions for observable QC do not rule out walls with $N \sim 10^2 - 10^3$ (see Table 2) which would correspond to mesoscopic quantum coherence. (Here $A_w = (2.67 \times 10^{-16}\text{cm}^2)N$; $M_D = (2.73 \times 10^{-25}\text{gm})N$; and $\bar{\varepsilon}$ is given in Table 2.) We now go on to examine the effects of asymmetry on the case of mesoscopic QC.

Since $U_0 = 2KV_d$, the larger the void, the more strongly it attracts the wall. Thus, if asymmetry is sufficiently pronounced, QC is lost because the larger void pins the wall. This effect can be seen by following the ground state energy as we increase $a$ from 1 (see Table 3). Imagine $a = 1$ (corresponding to a SDWP) and that $\{N, U_0, L\}$ are such that, in the absence of dissipation, we have QC. Imagine further that we increase $a$ so that the void at $q = Q_0$ attracts the wall more strongly than the other void. This stronger attraction causes $E_0$ to decrease (i.e. become more negative) as the probability distribution in the ground state begins to shift towards $q = Q_0$. As we continue to increase $a$, we reach a critical value $a_*$ at which $E_0$ is equal to the value of the AsDWP at the metastable minimum of the left well $U_{meta}$. For $a > a_*$, $E_0$ drops below the metastable minimum which corresponds to the pinning of the wall at $q = Q_0$ and the destruction of QC. Intuitively, we expect that when $\Delta U_0 = U_0^+ - U_0^- = (a - 1)U_0^-$ is approximately equal to the barrier height $U_{bh}$ of the SDWP, the larger defect will pin the wall. For mesoscopic walls, $U_{bh}$ is given in Table 2. For $N = 300$, $L = 75\text{Å}; U_{bh} = 2.8 \times 10^{-7}\text{eV}$ (note the difference in units relative to Table
Thus $\Delta U_0 = U_{bh}$ corresponds to $\bar{a} \approx 1.008$. A numerical calculation of $E_0$ for this case gives $a_\ast = 1.038 > \bar{a} \ast$ (see Table 3). For the spherical voids we have been considering, if $R_\ast = 10\,\text{Å}$, then $R_\ast^{1/2}R_\ast = 10.1\,\text{Å}$. Thus if asymmetry is not to destroy QC, the radii of the two voids must satisfy $\Delta R = R_\ast - R_\ast < 0.1\,\text{Å}$. Such a tolerance is clearly unattainable and since $\Delta R \ll C$ we must average the effects of asymmetry over the coarse-graining length scale $C \sim 10 - 15\,\text{Å}$. As the majority of $\Delta R$ values entering into the average correspond to pinning of the wall, we conclude that asymmetry in the voids acts to destroy QC in this case. We might hope to overcome this difficulty by increasing $L$ and so increasing $U_{bh}$. For $L = 103\,\text{Å}$, $N = 300$, the SDWP tunneling gap is $\Delta_0 = 7.7 \times 10^{-9}\,K$ (see Table 2). At this separation, QC is marginally observable in the absence of dissipation. In this case $U_{bh} = 2.8 \times 10^{-6}\,\text{eV}$ (see Table 2). Then $\Delta U_0 = U_{bh}$ gives $\bar{a} \ast = 1.085$. We did not determine $a_\ast$ numerically for this case. In the previous example we saw that $(a_\ast - 1) \sim 5(\bar{a} \ast - 1)$ so that we will estimate $(a_\ast - 1) \sim 10(\bar{a} \ast - 1)$ for this case. This gives $a_\ast \sim 1.85$. For $R_\ast = 10\,\text{Å}$, $\Delta R \sim 2.3\,\text{Å}$. Again $\Delta R < C$ so that an average of the effects of asymmetry over $C \sim 10 - 15\,\text{Å}$ is necessary. As in the previous case, the majority of the $\Delta R$ values correspond to pinning of the wall so that we again conclude that asymmetry in the voids will act to destroy QC in this already marginal case. Larger values of $L$ lead to $\Delta_0 < \bar{\epsilon}$. We see that asymmetry of the voids will be sufficient to destroy any remaining vestige of domain wall QC—even in the absence of dissipation. The basic difficulty is that maximizing the tunneling gap requires a very small central barrier; so small in fact, that the most minute asymmetry in the two voids produces a bias in the double well which is of order of the height of the central barrier and so capable of pinning the wall (viz. destroying QC). We suspect that this is generally true of macroscopic QC: large objects require small
barriers which are easily removed by small imperfections in the experimental set-up.

In this paper we have carried out a numerical analysis of domain wall quantum coherence in a uniaxial magnetic insulator with quality factor $Q \gg 1$ at $T = 0$. We find that QC on any scale larger than microscopic appears unlikely due to the combined effects of stray magnetic fields and asymmetry in the voids which are responsible for producing the double well potential seen by the domain wall. Our calculation assumed a flat wall although curved walls are expected in the thin film scenario. For this scenario, and for voids of given size, tunneling will only occur if $L < L_{\text{crit}}$ when curvature effects are included. This is because the curvature energy acts to raise the minima of the DWP relative to the top of the central barrier, thus reducing $U_{\text{bh}}$. When $L = L_{\text{crit}}$, the central barrier has disappeared and we are no longer in the QC regime. If $L_{\text{crit}} > L_{\text{max}}$, the tunneling gap becomes unobservable before curvature effects become significant. Otherwise, $L_{\text{crit}} < L_{\text{max}}$ and curvature effects act to reduce the range of void separation $R \rightarrow R_{\text{new}} = L_{\text{crit}} - L_{\text{min}}$ corresponding to QC. For macro-walls, $R$ was already small enough to rule out macro-QC so that decreasing $R \rightarrow R_{\text{new}}$ acts to strengthen this conclusion. For meso-walls, if $R_{\text{new}} < C$, then curvature effects have reduced the range of QC sufficiently that stray magnetic fields are expected to make meso-QC unobservable. Finally, if $R_{\text{new}} > C$, asymmetry in the void sizes can more easily destroy meso-QC since curvature effects act to reduce $U_{\text{bh}}$. Thus curvature effects are not expected to modify our conclusion that observation of meso- or macro-QC of domain walls appears unlikely. Our calculation ignores dissipation; a proper inclusion of its effects will also act to strengthen our conclusion. Although we have considered a particular type of defect, we expect our conclusion to also apply when: (1) the defects are such that the pinning potential is primarily due to a reduction in the
exchange and anisotropy energies of the wall (when pinned to the defect); and (2) the wall configuration $\mathbf{M}(y)$ depends only on the wall normal coordinate $y$. Under these conditions we continue to expect $U(q) \sim (J/\lambda^2 + K)V_d\text{sech}^2(q/\lambda)$ and our results to be indicative of the scale of QC to be expected. It should be noted that the phase coherences necessary for establishing quantum coherence are much more delicate than those necessary for establishing quantum tunneling so that our work does not preclude a priori the possibility of observing macroscopic quantum tunneling of domain walls in magnetic insulators. I would like to thank Philip Stamp for useful discussions, T. Howell III for support, and NSERC of Canada for financial support.

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1. Tunneling gap $\Delta_0$; width $R$ of range of observable void separations; central barrier height $U_{bh}$; and bias $\bar{\epsilon}$ for a macroscopic Bloch wall moving in a SDWP.

2. Tunneling gap $\Delta_0$; width $R$ of range of observable void separations; central barrier height $U_{bh}$; and bias $\bar{\epsilon}$ for a mesoscopic Bloch wall moving in a SDWP.

3. Asymmetry parameter $a$; ground state energy $E_0$; metastable minimum $U_{meta}$ of AsDWP; and barrier height $U_{bh}$ for a mesoscopic Bloch wall with $N = 300$ and void separation $L = 75\,\AA$. 
| \(N(\text{spins})\) | \(L(\text{Å})\) | \(\Delta_0(K)\) | \(\mathcal{R}(\text{Å})\) | \(U_{bh}(K)\) | \(\bar{\epsilon}(K)\) |
|---|---|---|---|---|---|
| \(10^4\) | 1320 | \(\sim 6\) | — | — | \(1.04 \times 10^{-7}\) |
| 1322 | \(4.0 \times 10^{-4}\) | 1.8 \(\times 10^{-2}\) |
| 1324 | \(2.0 \times 10^{-5}\) | 3.6 \(\times 10^{-2}\) |
| 1326 | \(5.0 \times 10^{-7}\) | 5.9 \(\times 10^{-2}\) |
| \(10^5\) | 1318 | \(\sim 2\) | — | — | \(1.04 \times 10^{-6}\) |
| 1319 | \(1.8 \times 10^{-4}\) | 3.0 \(\times 10^{-3}\) |
| 1320 | \(9.6 \times 10^{-6}\) | 6.7 \(\times 10^{-3}\) |
| 1321 | \(2.0 \times 10^{-7}\) | 1.2 \(\times 10^{-2}\) |
| \(10^6\) | 1317 | \(\sim 1\) | — | — | \(1.04 \times 10^{-5}\) |
| 1318 | \(2.4 \times 10^{-5}\) | 7.9 \(\times 10^{-4}\) |
| 1319 | \(9.1 \times 10^{-9}\) | 3.0 \(\times 10^{-3}\) |
| 1320 | \(< 7.2 \times 10^{-11}\) | 6.7 \(\times 10^{-3}\) |
| \(10^7\) | 1317.2 | \(< 0.1\) | — | — | \(1.04 \times 10^{-4}\) |
| 1317.3 | \(2.0 \times 10^{-5}\) | 8.5 \(\times 10^{-5}\) |
| 1317.4 | \(8.2 \times 10^{-6}\) | 1.4 \(\times 10^{-4}\) |
| 1317.6 | \(4.0 \times 10^{-7}\) | 3.0 \(\times 10^{-4}\) |
| 1318.0 | \(2.2 \times 10^{-10}\) | 7.9 \(\times 10^{-4}\) |

*\(\text{AB} = \text{ground state Above Barrier}\)

Table 1:
| N(spins) | L (Å) | $\Delta_0$ (K) | $R$ (Å) | $U_{bb}$ (K) | $\bar{c}$ (K) |
|---------|-------|----------------|--------|-------------|--------------|
| 300     | 74    | AB*            | $\sim 30$ | —           | $5.2 \times 10^{-9}$ |
| 75      | 1.1 $\times 10^{-3}$ | 3.3 $\times 10^{-3}$ |
| 80      | 2.5 $\times 10^{-4}$ | 7.2 $\times 10^{-3}$ |
| 103     | 7.7 $\times 10^{-9}$ | 3.2 $\times 10^{-2}$ |
| 110     | 2.4 $\times 10^{-10}$ | 4.0 $\times 10^{-2}$ |
| 120     | 1.8 $\times 10^{-12}$ | 5.2 $\times 10^{-2}$ |
| 3000    | 69    | AB             | $\sim 10$ | —           | $4.3 \times 10^{-8}$ |
| 75      | 3.1 $\times 10^{-6}$ | 3.3 $\times 10^{-3}$ |
| 80      | 6.1 $\times 10^{-9}$ | 7.2 $\times 10^{-3}$ |

*AB = ground state Above Barrier

Table 2:
| $a$   | $E_0 (K)$      | $U_{meta} (K)$ | $U_{bh} (K)$ |
|------|---------------|---------------|-------------|
| 1.000| $-1.4638 \times 10^{-1}$ | $-1.4859 \times 10^{-1}$ | $3.33 \times 10^{-3}$ |
| 1.010| $-1.4719 \times 10^{-1}$ | $-1.4894 \times 10^{-1}$ | $2.96 \times 10^{-3}$ |
| 1.020| $-1.4814 \times 10^{-1}$ | $-1.4930 \times 10^{-1}$ | $2.59 \times 10^{-3}$ |
| 1.030| $-1.4914 \times 10^{-1}$ | $-1.4966 \times 10^{-1}$ | $2.23 \times 10^{-3}$ |
| 1.038| $-1.4997 \times 10^{-1}$ | $-1.4995 \times 10^{-1}$ | $1.94 \times 10^{-3}$ |

Table 3: