Elastic Enhancement Factor: from Mesoscopic Systems to Macroscopic Analogous Devices.

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Excess of probabilities of the elastic processes over the inelastic ones is a common feature of the resonance phenomena described in the framework of the random matrix theory. This phenomenon is quantitatively characterized by the elastic enhancement factor $F^{(0)}$ that is a typical ratio of elastic and inelastic cross sections. Being measured experimentally, this quantity can supply us with information on the character of dynamics of the intermediate complicated open system. We discuss properties of the enhancement factor in a wide scope from mesoscopic systems to macroscopic analogous devices and demonstrate essential qualitative distinction between the elastic enhancement factor’s peculiarities in these two cases. Complete analytical solution is found for the case of systems without time-reversal symmetry and only a few open equivalent scattering channels.

I. Enhancement of elastic processes as compared to inelastic ones is a remarkable phenomenon inherent to various resonance processes in nuclear and many-electron atomic physics as well as in analogous experiments with 2D open macroscopic devices. Various aspects of this effect have attracted attention of theorists and experimentalist starting from the pioneering papers by Moldauer\textsuperscript{[1]}. The issue has gained a solid foundation in the random matrix approach to the problem of the nuclear composite resonance phenomena described in the framework of the random matrix theory. This phenomenon is quantitatively characterized by the elastic enhancement factor $F^{(0)}$ that is a typical ratio of elastic and inelastic cross sections. Being measured experimentally, this quantity can supply us with information on the character of dynamics of the intermediate complicated open system. We discuss properties of the enhancement factor in a wide scope from mesoscopic systems to macroscopic analogous devices and demonstrate essential qualitative distinction between the elastic enhancement factor’s peculiarities in these two cases. Complete analytical solution is found for the case of systems without time-reversal symmetry and only a few open equivalent scattering channels.

II. Many aspects of the theory of the quantum chaotic scattering can be analyzed and checked experimentally with the aid of macroscopic analogous devices. This method took on wide dissemination after pioneering experiments\textsuperscript{[9]} with open irregularly shaped 2D electromagnetic cavities (see for example\textsuperscript{[10, 11]} and references therein). In particular, the elastic enhancement
factor has thoroughly been measured for both the symmetry classes as well as in the transient regime between them \[ \text{[11]} \]. However, in contrast with the Verbaarschot’s regime of very large number of very weak channels the number of them in the analogous experiments is restricted to only several ones, as a rule even to two. Therefore the ruling parameter \( \eta \) becomes irrelevant. The enhancement factor depends mainly on the transmission coefficients under such conditions. Suggesting equivalence of the channels (see below) the enhancement factor is expressed as

\[
F_M^{(\beta)}(T) = 1 + \delta_{1\beta} + (1 - T) \frac{P_M^{(\beta)}(T)}{F_M^{(\beta)}(T)},
\]

where the functions \( F^{(\beta)} \) and \( P^{(\beta)} \) are given by the well known \( 3- (\beta = 1) \) or \( 2- (\beta = 2) \) fold integrals \[ \text{[2, 4]} \].

A general analytical formula valid for arbitrary number of channels \( M \) does not exist. Nevertheless, in the case of the systems with broken Time-reversal symmetry, \( \beta = 2 \), for any given number of channels the enhancement factor can be expressed explicitly as a ratio of two \( (M-1) \)-order polynomials. Dropping the subscript \( \beta = 2 \), we get for example

\[
F_2(T) = \frac{6T}{T^2}, \quad F_4(T) = 3 \frac{6T + 4T^2 - T^4}{12T^2 + 4T^4 - T^6}
\]

and so on... More examples are shown in Fig.3. It is seen that the larger the number of channels is the faster enhancement factor decays when \( T \) increases. The established in the part I. connection between the enhancement factor and delay time variance that looks now as \[ \text{[12]} \]

\[
K_M(T) = \frac{2}{T} \frac{1-(1-T)^{M+1}}{M-1}
\]

does not exist anymore!

III. Below, we restrict ourselves to the practically most important case of only two open channels. Then complete analytical treatment becomes possible for the systems without Time-reversal symmetry. First of all we will concentrate upon the significance of the equivalent channels assumption. Let us suppose the opposite and define the following two new variables:

\[
T = \frac{1}{2}(T_1 + T_2), \quad \Delta = \frac{1}{2}(T_1 - T_2)
\]

so that \( T_1 = T + \Delta, T_2 = T - \Delta \) and \( 0 < T < 1, -\frac{1}{2} < \Delta < \frac{1}{2} \).

Though explicit analytical solution exists even in this case the appropriate expression is extremely lengthy. Therefore we skip it and rather illustrate the result graphically (see Fig.4.)

As long as \( \Delta \) is noticeably smaller than \( T \), the result is the same as in the case of equivalent channels and this approximation works well. Only when \( \Delta \) is very close to \( T \) enhancement factor can become very large. The reason is quite clear: if one out of the two channels is almost closed everything is going on via the second one.

IV. In the analogous experiments discussed here the ohmic losses always play an important role and cannot be neglected. The reasonable way to take them into account has been suggested in \[ \text{[4]} \] and consists in introducing the overall decaying factor \( e^{-\gamma T} \). Analytical solution is still possible in the special case considered and looks as the ratio \( F(T, \gamma) = 1 + (1 - T) \frac{N(T, \gamma)}{\Delta(T, \gamma)} \) where

**Figure 1.** \( F^{(\beta)} \) versus \( \eta \). Top curve \( \beta = 1 \), bottom \( \beta = 2 \).

**Figure 2.** The ratio \( \frac{F^{(\beta)}(T)}{F^{(\beta)}(T)} \).

**Figure 3.** \( F_M \) versus \( T \). From top to bottom \( M = 2, 3, 4, 9, 10 \).
Figure 4. $F(T + \Delta, T - \Delta)$ at different values of $\Delta$: from bottom to top $\Delta = 0, 0.994T, 0.99985T, 0.9999985T$. (Analytical solution.)

$N(T, \gamma) = T(2T^2 - T\gamma + \gamma^2) + \gamma^3 e^{\gamma/T} Ei(-\gamma/T),$

and

$D(T, \gamma) = T\left(T^3 + \gamma^2 - T\gamma(1 + \gamma) + 2T^2 \left(2(\gamma - 1) + \frac{3\gamma}{T}\right)\right) + \gamma \left((1 - T)\gamma^2 - 3T\left(2 - \gamma \coth(\gamma/T)\right)\right) e^{\gamma/T} Ei(-\gamma/T).$

No explicit analytical results can be derived in the case of $\beta = 1$. Therefore we calculated the factor $F_{\beta=1}^{\left(\Delta \rightarrow 0\right)}(T, \gamma)$ numerically to be able to compare the influence of the absorption in these two cases. The results are presented in Fig.4. It is clearly seen that the $T$-invariant systems is somewhat more sensitive to the influence of absorption.

V. Summary
In this letter, we focus on the specific features of the elastic enhancement factor depending on peculiarities of the chaotic open system one is dealing with. On the whole, this factor depends on the number $M$ of scattering channels as well as the channel’s transmission coefficients. However, when the number of channels is very large, what is typical for example of the processes like the resonance nuclear reactions, the enhancement factor is controlled by the only parameter $\eta = MT$ that changes in very wide bounds (Verbaarschot’s regime). Quite opposite situation takes place in the analogous experiments with 2D irregularly shaped billiards that serve for imitation of the quantum chaos. In the experiments of such a kind the number of channels is very restricted. In this case the enhancement factor depends on the number of channels and on transmission coefficients separately. We have juxtaposed in detail the two specified regimes. We have succeeded in finding complete analytical solution valid for any fixed number $M$ of equivalent channels with arbitrary transmission coefficients $0 < T < 1$ in the case of systems without Time-reversal symmetry. More than that, in the practically significant case $M = 2$ the influence of absorption has also received an explicit analytical description.

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[1] P.A. Moldauer, Phys. Rev. 123, 968 (1961) and Phys. Rev. B 135, 642 (1964).
[2] J.F.M. Verbaarschot, H.A. Weidenmüller, and M.R. Zirnbauer, Phys. Rep. 129, 367 (1985).
[3] J.F.M. Verbaarschot, Ann. Phys. 168, 368 (1985).
[4] Y.V. Fyodorov, D.V. Savin, and H.-J. Sommers, J. Phys. A: Math. Gen. 38, 10731 (2005).
[5] Y. A. Kharkov, V.V. Sokolov, Phys. Lett. B 718, 1562 (2013).
[6] N. Lehmann, D.V. Savin, V.V. Sokolov, H.-J. Sommers, Physica D 86, 572 (1995).
[7] M.L. Mehta, Random Matrices, ELSEVIER Ltd.
[8] H.-J. Stöckmann, J. Stein, Phys. Rev. Lett. 65, 2215 (1990).
[9] S. Sridhar, Phys. Rev. Lett. 67, 785 (1991).
[10] Z. Pluhar, H.A. Weidenmüller, J.A. Zak, C.H. Lewenkopf, F.J. Wegner Ann. Phys. 243, 1, (1995); M. Lawniczak, S. Bauch, O. Hul, and L. Sirko, Phys. Rev. E 81, 046204 (2011); M. Lawniczak, A. Nicolaou-Kuklinska, S. Bauch, O. Hul, and L. Sirko, Plenary talk at the International Conference CHAOS2013, Istanbul.
[11] B. Dietz et al., Phys. Rev. E 81, 036205 (2010).
[12] Y.V. Fyodorov, D.V. Savin, and H.-J. Sommers, Phys. Rev. E, 55, R4857 (1997).