Medium-modification of photon-tagged jets in \( AA \) collisions

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We study nuclear modification of the photon-tagged jets in \( AA \) collisions within the jet quenching scheme based on the light-cone path integral approach to the induced gluon emission. The calculations are performed for running coupling. Collisional energy loss is treated as a perturbation to the radiative mechanism. We obtain a reasonable agreement with the recent data from the STAR Collaboration on the mid-rapidity nuclear modification factor \( I_{AA} \) for Au+Au collisions at \( \sqrt{s} = 200 \) GeV for parametrization of running \( \alpha_s \) consistent with that necessary for description of the data on suppression of the high-\( p_T \) spectra.

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I. INTRODUCTION

Results from RHIC and LHC on heavy-ion collisions give strong evidence for production of a deconfined quark gluon plasma (QGP). One of the main signature of the QGP formation in \( AA \) collisions is the discovery at RHIC and LHC of the extremely strong suppression of the high-\( p_T \) hadron spectra. It is commonly believed that this suppression is a consequence of the jet modification (jet quenching) due to the final state interaction with the QGP produced in the initial stage of \( AA \) collisions. The jet quenching is caused by the radiative and collisional energy loss of fast partons in the QGP. The RHIC and LHC data on the nuclear modification factor \( R_{AA} \), characterizing the suppression of the high-\( p_T \) spectra, can be reasonably described by the radiative and collisional parton energy loss in the QGP with dominant contribution from the radiative mechanism due to the induced gluon emission. A consistent analysis of the jet quenching phenomenon requires understanding multiple gluon emission. The available approaches to the induced gluon emission \([1, 2, 4, 5]\) deal with one gluon emission. At the one gluon level in the light-cone path integral (LCPI) \([2]\) approach the spectrum of gluon emission by a quark may be expressed via the retarded Green function of a two dimensional Schrödinger equation, in which the longitudinal coordinates (along the initial quark momentum) plays the role of time, and the imaginary potential is proportional the cross section of interaction of the three-body \( q\bar{q}- \)system with the medium constituent. The diagram technique developed in Refs. \([2]\) allows one to go beyond the one gluon level. However, already for the double gluon emission, even in a crude oscillator approximation \([7, 8]\) (when the potential is approximated by a quadratic form), calculations become extremely complicated \([9]\). And up to now there are no phenomenological schemes for the jet quenching analyses that treat accurately the double gluon emission. Anyway the double gluon level is insufficient for analyses of the jet quenching data from RHIC and LHC. In the presently available analyses of the nuclear modification factor \( R_{AA} \) the effect of multiple gluon emission is usually accounted for in the approximation of independent gluon emission \([10–13]\), similar to the Landau method for multiple soft photon emission in QED. This approximation does not account for the effect of the gluon cascading that may be important for the medium-modified fragmentation functions (FFs) in the soft region \( z \ll 1 \). Nevertheless, this approximations seems reasonable to calculate the nuclear modification factor \( R_{AA} \). Because it depends mostly on the form of the medium-modified FFs for parton–hadron transitions in the region of intermediate and large \( z \), where the main effect of multiple gluon emission is the Sudakov suppression which should not be sensitive to the details of the in-medium parton cascading at \( z \ll 1 \).

Since at small \( z \) the approximation of independent gluon emission becomes questionable, it is of course highly desirable to perform comparison of the theoretical predictions obtained in the approximation of independent gluon emission \([10–13]\) with experimental observables that are sensitive to the form of the medium-modified FFs in a broad range of \( z \). Experimentally the information about the medium jet modification in a broad range of \( z \) can be obtained from measurement of the photon-tagged jet FFs in \( \gamma+jet \) events \([14]\). The medium effects in jet fragmentation in \( \gamma+jet \) events are characterized by the nuclear modification factor \( I_{AA} \), defined as the ratio

\[
I_{AA} = \frac{D_{hAA}^{AA}(z)}{D_{hpp}^{pp}(z)},
\]

where \( D_{hAA}^{AA}(z) \) and \( D_{hpp}^{pp}(z) \) are the \( \gamma \)-triggered jet FFs for \( AA \) and \( pp \) collisions, respectively. The mid-rapidity factor \( I_{AA} \) has been recently measured by the STAR Collaboration \([15]\) at RHIC for central Au+Au collisions at \( \sqrt{s} = 200 \) GeV in a broad range of \( z \) for the photon energies \( 12 < E_{\gamma} < 20 \) GeV.

In the present paper we perform a comparison of the STAR data \([15]\) with the theoretical predictions for \( I_{AA} \) obtained within the model of jet quenching that we de-
veloped in Ref. 13, and previously used in Refs. 16, 18 for successful description of the data on the nuclear modification factor $R_{AA}$. The scheme is based on the LCPI approach 2 to the induced gluon emission. The method allows one to treat accurately the Landau-Pomeranchuk-Migdal (LPM) effect and the finite-size effects. The calculations are performed for running coupling. We perform numerical calculations beyond the oscillator approximation when parton multiple scattering in the medium can be described in terms the well known transport coefficient $\hat{q}$ 1. The model also includes the contribution of the collisional energy loss.

The plan of the paper is as follows. In Sec. 2 we review our theoretical framework. In the first subsection we discuss our model for the in-medium FFs. In the second subsection we discuss the model of the QGP fireball used in our numerical calculations. In the last subsection we discuss calculations of the nuclear modification factor $I_{AA}$. In Sec. 3 we present comparison of our numerical results with the STAR data 15 on the nuclear modification factor. In Sec. 4 summarizes our work.

II. THE THEORETICAL FRAMEWORK

In this section, we review the main aspects of our theoretical framework for calculation of the nuclear modification factor $I_{AA}$ that characterizes the jet medium modification in $AA$ collisions.

A. Medium-modified FFs

As was said in Introduction, currently the first principle analysis of the jet medium modification in $AA$ collisions is impossible. Our treatment of the medium-modified FFs $D_{j/i}^{m}$ for parton → hadron transitions for $AA$ collisions is similar to that developed in our analysis 13 of the nuclear modification factor $R_{AA}$. It is based on the LCPI approach 2 to the induced gluon emission from fast partons in the QGP. For reader’s convenience, and since some of the details have been omitted in our concise paper 13, in this subsection we discuss the important points of our model for calculation of the medium modified FFs.

We assume that the parton → hadron transition consists of the three stages: the DGLAP cascading, the induced gluon emission stage in the QGP, and parton hadronization outside the QGP. For a given jet trajectory in the fireball the FF $D_{j/i}^{m}$ reads

$$D_{j/i}^{m}(z, Q) = \int_{z}^{1} \frac{dz'}{z'} D_{j/j}(z/z', Q_0) D_{j/i}(z', Q),$$

(2)

where $D_{j/j}$ describes parton hadronization outside the QGP, and $D_{j/i}$ corresponds to transition of the initial hard parton $i$ to the parton $j$ escaping from the QGP. The partonic FF $D_{j/i}^{m}$ includes the parton evolution in the DGLAP stage and medium modification in the QGP. We write it as a convolution

$$D_{j/i}^{m}(z, Q) = \int_{z}^{1} \frac{dz'}{z'} D_{j/k}(z/z', E_k) \times D_{k/i}^{DGLAP}(z', Q_0, Q),$$

(3)

where $E_k = z'Q$. The FF $D_{j/k}^{DGLAP}(z, Q_0, Q)$ describes the first DGLAP stage for parton → parton transition in the parton cascading from the initial parton virtuality $Q$ to a small virtuality scale $Q_0$, where the DGLAP cascade is stopped. The FF $D_{k/i}^{DGLAP}(z, Q_0, Q)$ corresponds to the in-medium parton → parton transition in the QGP fireball. It depends on the energy of the parton $k$.

In the absence of the medium the $D_{j/k}^{m}$ in Eq. (3) reduces to the unit operator $\delta_{j/k} \delta(z - 1)$, and $D_{j/i}^{m}(z, Q)$ becomes equal to $D_{j/i}^{DGLAP}(z, Q_0, Q)$. In this case reduces to its vacuum counterpart corresponding to FF for $pp$ collisions

$$D_{j/i}(z, Q) = \int_{z}^{1} \frac{dz'}{z'} D_{j/j}(z/z', Q_0) \times D_{j/i}^{DGLAP}(z', Q_0).$$

(4)

As in Ref. 13 we take $Q_0 = 2$ GeV for the FFs $D_{j/i}(z, Q_0)$ in Eq. (4), describing parton → hadron transition outside the QGP (and in Eq. (1) for $pp$ collisions). For these FFs we use the KKP 19 parametrization. The DGLAP FFs $D_{j/i}^{DGLAP}(z, Q_0, Q)$ have been computed with the help of the PYTHIA event generator 20. It was used to create a grid of values for $D_{j/i}^{DGLAP}(z, Q_0, Q)$ in the $z - Q$ plane. Our method for calculation of the FFs for $pp$ collisions in the form of the convolution of the KKP FF at $Q_0 = 2$ GeV and the DGLAP FFs guarantees that in the limit of the vanishing induced radiation the medium-modified FFs given by Eqs. (2), (3) exactly reduce to the $pp$ FFs 1. We checked that quantitatively our formula (1) reproduces reasonably well the Q-dependence of the KKP FFs 19. Nevertheless, to avoid the effect of a possible difference between the KKP FFs and the FFs given by Eq. (4) on predictions for $I_{AA}$, the use of the form (4) is clearly preferred for numerical calculations of the $I_{AA}$.

The form given by Eqs. (2), (3) assumes that the DGLAP and the induced gluon emission stages are approximately ordered in time. This picture seems to be reasonable for initial parton energies $\lesssim 100$ GeV, because in this region the typical formation time for emission of the first most energetic gluon in the DGLAP cascade turns out to be relatively small. The gluon formation length emitted by a fast quark in vacuum is approximately

$$L_f(x, k_T) \sim 2 E_g x(1 - x)/(k_T^2 + \epsilon^2),$$

(5)

where $x$ is the gluon fractional momentum, and $\epsilon^2 =$
of the interference of the vacuum gluon emission with
for the induced gluon emission. Note that just because
sufficiently small transverse momenta (about the Debye
may be questionable for the DGLAP gluon emission with
ing of the DGLAP and the induced gluon emission stages
parton cascading it is difficult to estimate the theoretical
Q
matches the above formula for
fast parton virtu-
timation of the fast parton virtuality \( \Delta Q \) (which can be obtained from the
uncertainty relation \( \Delta E \Delta t \sim 1 \)). This says that for
L \sim t_0 \sim 0.5 \text{ fm} for the initial partons with \( \varepsilon \sim 10 - 50 \text{ GeV} \) we have \( Q(L) \sim 2 - 4 \text{ GeV} \). For this reason the scale \( Q_0 \sim 2 \text{ GeV} \), that is lower for the lower end of the
DGLAP evolution, at the same time translates to the
longitudinal scale that agrees qualitatively with the
QGP production time where the induced gluon emission
comes into play. Note however, that our numerical cal-
culations show that the results are practically insensitive
to the value of \( Q_0 \). For this reason it does not make
sense to try to find more appropriate value of \( Q_0 \) which
better corresponds to the space-time picture of the QGP
production and transition of the DGLAP stage to the
stage of the induced gluon emission in the QGP. Any-
way, in the DGLAP cascade the time of parton splitting
can be only estimated very roughly. One remark about the
arguments of the FFs in Eqs. (4), (5) is in order. For
the DGLAP and the hadronization stages the FFs in Eqs. (4), (5) are written as functions of the parton
virtualities. But the in-medium FF \( D_{j/k}^{n} \) is a function of the parton energy. It is because we evaluate the
induced gluon spectrum within the old fashioned pertur-
bation theory in the coordinate representation in which
particles are not characterized by virtuality. In this for-
mulation the virtuality may be estimated from the length
scale \( L \) and the parton energy \( q \) with the help the uncer-
tainty relation that gives \( Q \sim \sqrt{E/L} \) which, of course,
matches the above formula for \( Q(L) \).

We are fully aware that the picture with the time order-
ing of the DGLAP and the induced gluon emission stages
may be questionable for the DGLAP gluon emission with
sufficiently small transverse momenta (about the Debye
mass of the QGP) when the vacuum gluon formation length
becomes as large as the typical formation length for the
induced gluon emission. Note that just because of the interference of the vacuum gluon emission with
the induced one the induced gluon spectrum vanishes for
zero medium size. In the form (2), (3) these interfer-
ence effects are assigned in the in-medium FF \( D_{j/k}^{n} \). In
the absence of a consistent approach to the in-medium
parton cascading it is difficult to estimate the theoretical
uncertainties from the use of the representation given by
Eqs. (2), (3). It is worth noting that a formal interchange
in Eq. (4) of the DGLAP and in-medium FFs practically
does not change the results.

We calculate the in-medium FFs \( D_{j/k}^{n}(z, E_{k}) \) in the
approximation of independent induced gluon emission
using for the one gluon emission distribution the induced
gluon spectrum in the form obtained in Ref. [21] within
the LCPI approach [2]. The form of Ref. [21] does not
require calculation of the singular Green’s function as in
the original representation of the spectrum of Refs. [2].
The method of Ref. [21] reduces calculation of the gluon
spectrum to solving a two-dimensional Schrödinger equa-
tion in backward time direction with a smooth boundary
condition. It is convenient for accurate numerical cal-
culations beyond the oscillator approximation. For the
reader’s convenience in Appendix A we give the necessary
formulas.

In the approximation of independent gluon emission
the quark fractional energy loss distribution in \( \xi = \Delta E/E \) can be written as [10] (hereafter, for notational
simplicity, we omit argument \( E \))

\[
W(\xi)=W_0 \sum_{n=1}^{\infty} \frac{1}{n!} \left( \prod_{i=1}^{n} \int_{0}^{1} dx_i \frac{dP}{dx_i} \right) \delta \left( \xi - \sum_{i=1}^{n} x_i \right),
\]

(7)

where

\[
W_0 = \exp \left[ - \int_{0}^{1} dx \frac{dP}{dx} \right]
\]

(8)

is the no gluon emission probability, \( dP/dx \) is the prob-
ability distribution for the \( q \rightarrow gq \) transition with \( x = E_g/E_q \). At \( \xi \ll 1 \) the main effect of the multiple gluon
emission is the Sudakov suppression. It is well seen from
the approximate calculation of (7) at the level of two-
gluon emission for the regime when the relative energy
loss \( \Delta E/E = \int_{0}^{1} dx dP/dx \) is much smaller than unity.
In this regime, similarly to the electron energy loss [22],
from (7) one can obtain

\[
W(\xi) \approx \frac{dP}{d\xi} \exp \left[ - \int_{0}^{1} dx \frac{dP}{dx} \right] \left\{ 1 - \frac{1}{2} \int_{0}^{\xi} d\xi_1 \left[ \frac{dP}{dx_1} + \frac{dP}{dx_2} - \frac{dP}{dx_1} \frac{dP}{dx_2} \left( \frac{dP}{d\xi} \right)^{-1} \right] \right\},
\]

(9)

where \( x_1 + x_2 = \xi \). The exponential Sudakov suppression
factor in (9) reflects a simple fact that emission of gluons
with the fractional momentum bigger than \( \xi \) is forbidden.

For accurate numerical calculation of the distribution
\( W(\xi) \) given by Eq. (7) it is convenient to rewrite it as a
series [11]

\[
W(\xi) = \sum_{n=1}^{\infty} W_n(\xi),
\]

(10)

where \( W_n \) are determined by the recurrence relations
\[ W_{n+1}(\xi) = \frac{1}{n+1} \int_0^\xi dx W_n(\xi-x) \frac{dP}{dx} \]  
(11)

with
\[ W_1(\xi) = W_0 \frac{dP}{d\xi}. \]  
(12)

In numerical calculations we set \( dP/dx = 0 \) at \( x < m_q/E_q \) and \( 1 - x < m_q/E_q \).

The expression \( W_1(\xi) \) satisfies the relations
\[ \int_0^\infty d\xi W(\xi) = 1, \]  
(13)

\[ \int_0^\infty d\xi \xi W(\xi) = \int_0^1 dx \frac{dP(x)}{dx}. \]  
(14)

For any value of the ratio \( \Delta E/E \), the formula (11) leads to some leakage of the probability and the fractional momentum to the unphysical region of \( \xi > 1 \). The effect is small at \( \Delta E/E \ll 1 \), but for the conditions of the jet quenching in \( AA \) collisions when \( \Delta E/E \) is not very small, say, \( \sim 0.2 \) for light quarks at \( E \sim 10 - 20 \) GeV for RHIC conditions, the effect may be sizeable. To ensure the flavor conservation
\[ \int_0^1 dz D_{q/q}^{in}(z) = 1, \]  
(15)

we define a renormalized distribution in the physical region \( \xi < 1 \)
\[ W_R(\xi) = K_q q W(\xi) \]  
(16)

with
\[ K_q q = \int_0^\infty d\xi W(\xi) \int_0^1 d\xi W(\xi). \]  
(17)

Then this renormalized distribution is used to define the in-medium \( q \to q \) FF
\[ D_{q/q}^{in}(z) = W_R(1-z). \]  
(18)

We define the FF for \( q \to g \) transition as
\[ D_{q/g}^{in}(z) = K_q g dP/dz. \]  
(19)

We determine the coefficient \( K_{qg} \) from the momentum sum rule
\[ \int_0^1 dz z \left[ D_{q/q}^{in}(z) + D_{q/g}^{in}(z) \right] = 1. \]  
(20)

In the limit \( \Delta E/E \to 0 \), \( K_{qg} \to 1 \). Indeed, at one gluon emission level the \( z \)-distribution for \( q \to g \) transition is connected to that for \( q \to q \) by interchange of the arguments \( z \leftrightarrow 1-z \). Then after setting the upper limit of \( \xi \)-integration in Eqs. (13), (14) to 1, we conclude that the momentum sum rule (20) is satisfied for \( K_{qg} = 1 \).

For the \( g \to g \) transition we use the following procedure. In the first step we define \( D_{g/g}^{in}(z) \) in the region \( z > 0.5 \), where the Sudakov suppression is important, through the independent gluon emission distribution \( W(\xi) \) (with \( \xi = 1-z \)) given by Eq. (7) using for \( dP/dx \) the \( x \)-distribution for \( g \to gg \) induced transition. Note that for \( g \to gg \) transition due to the \( x \leftrightarrow 1-x \) symmetry of the function \( dP/dx \) we can use 0.5 for the upper limit in \( x \)-integrations in Eqs. (7), (8) (it means that we view the softest gluon with \( x < 0.5 \) as a radiated gluon). In the soft region \( z < 0.5 \), where one can expect a strong compensation of the multiple gluon emission and the Sudakov suppression, we use simply the one gluon distribution \( dP/dx \) (with \( x = z \)). This procedure can violate the momentum sum rule
\[ \int_0^1 dz z D_{g/g}^{in}(z) = 1, \]  
(21)

which should be satisfied. To cure this drawback we multiply the FF obtained at the first step by a renormalization coefficient \( K_{gg} \) defined from the momentum conservation (21) Note that for the jet path length in the QGP \( L \sim 5 \) fm and the typical jet energy \( \sim 15 \) GeV, that is relevant to conditions of the STAR experiment \( 15 \), the necessary values of the renormalization coefficients turn out to be not very far from unity (\( K_{qq} \approx 1.1, K_{gg} \approx 0.8 \) and \( K_{qg} \approx 0.7 \)). This says that we are in a regime when the leakage of the probability to the unphysical region \( \xi > 1 \) and the violation of the momentum sum rule for the distribution (17) are relatively small.

Note that in our calculations the process \( g \to q \bar{q} \) is included into the DGLAP FF in Eq. (3), but we neglect the induced gluon conversion into \( q \bar{q} \) pairs in calculations of the in-medium FF \( D_{ij/k}^{in/ij/k} \). Calculations within the LCPI formalism \( 2 \), using the formulas given in Appendix A, show that for light quarks for the QGP produced in \( AA \) collisions for RHIC and LHC conditions the probability of the induced \( g \to q \bar{q} \) transition turns out to be relatively small. For conditions of the STAR data \( 15 \), when the typical gluon energy \( E \sim 15 \) GeV and the typical path length in the QGP \( L \sim 5 \) fm, the probability of the gluon conversion into the \( q \bar{q} \) states \( \lesssim 10\% \). From the point of view of the nuclear modification factors \( I_{AA} \) and \( R_{AA} \) the effect of this process should be negligible since the hadronization of the \( q \bar{q} \) state should be similar to that of a gluon.

The above formulas do not include the effect of the the collisional energy loss. Presently there is no a consistent approach for incorporating of the collisional energy loss in jet quenching calculations on an even footing with the radiative mechanism. In the present analysis, as in Refs. \( 13 \), we treat the collisional energy loss (that is relatively small \( 23 \)) as a perturbation to the radiative mechanism, and incorporate it by a small renormalization of the initial QGP temperature for the radiative in-medium FFs \( D_{ij/k}^{in/ij/k} \) according to the change in the \( \Delta E \) due to the collisional energy loss (see Ref. \( 13 \) for details). We evaluate the collisional energy loss using
the Bjorken method \[6\], but with an accurate treatment of kinematics of the binary collisions (the details can be found in Ref. \[22\]).

In calculations of the induced gluon emission distribution \(dP/dx\) we take for parton masses the quasiparticle masses in the QGP. We use the quasiparticle masses \(m_q = 300\) and \(m_g = 400\) MeV supported by the analysis of the lattice data \[24\]. Note that the results are practically insensitive to the quark mass. We use the Debye mass \(\mu_D\) in the QGP obtained in the lattice analysis \[22\], which gives \(\mu_D/T\) slowly decreasing with \(T (\mu_D/T \approx 3\) at \(T \sim 1.5T_c, \mu_D/T \approx 2.4\) at \(T \sim 4T_c\)). However, the sensitivity of the results to the Debye mass is relatively weak. Because the spectrum \(dP/dx\) is mostly controlled by the behavior of the dipole cross section (see Eq. (A5)) in the region \(\rho \lesssim 1/\mu_D\), where it depends on \(\mu_D\) only logarithmically.

Both for radiative and collisional mechanisms we use the one-loop running \(\alpha_s\) frozen at low momenta at some value \(\alpha_s^{Tr}\) (see Appendix A). The use of the same parametrization of running \(\alpha_s\) for the radiative and collisional mechanisms is important for minimizing the theoretical uncertainties in the fraction of the collisional contribution. The results of the analyses of the low-\(x\) structure functions \[26\] and of the heavy quark energy loss in vacuum \[27\] show that for gluon emission in vacuum for this parametrization \(\alpha_s^{Tr} \approx 0.7 - 0.8\). However, the thermal effects can suppress the in-medium QCD coupling, and we treat \(\alpha_s^{Tr}\) as a free parameter.

\[\text{B. Model of the QGP fireball}\]

As in our previous analyses of the nuclear modification factor \(R_{AA}\) \[13, 16, 18\] we use the ideal gas model of the QGP with Bjorken's 1+1D expansion \[28\]. It gives \(T_0^0\tau_0 = T^\alpha_0\tau_0\), where \(\tau_0\) is the thermalization time of the matter. We take \(\tau_0 = 0.5\) fm. As in Refs. \[13, 16, 18\], for simplicity, we neglect variation of \(\tau_0\) with the transverse coordinates. We take the medium density \(\propto \tau\) for \(\tau < \tau_0\). This is just an ad hoc prescription to account for the fact that the QGP production is clearly not an instantaneous process. Note however, that the effect of the region \(\tau < \tau_0\) is small. It is due to a strong finite-size suppression of the induced gluon emission in the regime when the parton path length is smaller than the gluon formation length \[7, 8\].

We fix the initial temperature of the plasma fireball in \(AA\) collisions from the initial entropy density determined via the charged particle multiplicity pseudorapidity density, \(dN_{ch}^{AA}/d\eta\), at mid-rapidity (\(\eta = 0\)) calculated from the two component Glauber wounded nucleus model \[29\]

\[
\frac{dN_{ch}^{AA}}{d\eta} = \left[\frac{1 - \alpha}{2}N_{\text{part}} + \alpha N_{\text{coll}}\right] \frac{dN_{ch}^{pp}}{d\eta}, \tag{22}
\]

where \(dN_{ch}^{pp}/d\eta\) is the multiplicity density for \(pp\) collisions, \(N_{\text{part}}\) and \(N_{\text{coll}}\) for a given impact parameter \(b\) are given by the Glauber formulas

\[
N_{\text{part}}(b) = 2 \int d\rho T_A(\rho) \left\{1 - \exp[-T_A(b - \rho)\sigma_n^{NN}]\right\}, \tag{23}
\]

\[
N_{\text{coll}}(b) = \sigma_n^{NN} \int d\rho T_A(\rho) T_A(b - \rho) \tag{24}
\]

with \(T_A(b) = \int dz \rho_A(b, z)\) the nuclear profile function. The \(N_{\text{part}}\) and \(N_{\text{coll}}\) have been calculated with the Woods-Saxon nuclear distribution

\[
\rho_A(r) = \frac{\rho_0}{1 + \exp((r - R_A)/a)} \tag{25}
\]

with \(R_A = (1.12A^{1/3} - 0.86/A^{1/3})\), and \(a = 0.54\) fm \[30\]. We use \(dN_{ch}^{pp}/d\eta = 2.65\) and \(\sigma_n^{NN} = 35\) mb obtained by the UA1 Collaboration \[31\] for non-single-diffractive events for \(pp\) collisions at \(\sqrt{s} = 200\) GeV. We take for the fraction of the binary collisions \(\alpha = 0.135\) adjusted to reproduce the experimental STAR data \[32\] on centrality dependence of mid-rapidity \(dN_{ch}^{AA}/d\eta\) in \(Au+Au\) collisions at \(\sqrt{s} = 200\) GeV \[33\]. We determine the entropy density with the help of the Bjorken relation \[28\]

\[
s_0 = \frac{C}{\tau_0 \pi S_f} \frac{dN_{ch}^{AA}}{d\eta}. \tag{26}
\]

Here \(C = ds/d\eta /dN_{ch}^{AA}/d\eta \approx 7.67\) \[34\] is the entropy/multiplicity ratio, and \(S_f\) is the transverse area of the QGP fireball. We define it as the overlapping area of the colliding nuclei. In calculating the \(S_f\) we use the nuclear matter disk radius \(r = R_A + ku\) with \(k = 1.5\) used in our analysis \[18\] of the nuclear modification factor \(R_{AA}\). In the physically reasonable range \(k = 1 - 2\), the results for \(I_{AA}\) have a weak dependence on \(k\). This occurs because for the parton energy loss the change in the jet path length with variation of \(k\) is approximately compensated by the corresponding change in the fireball density. For \(Au+Au\) collisions at \(\sqrt{s} = 200\) GeV for \(0-12\%\) centrality bin as in Ref. \[13\] this procedure gives \(T_0 \approx 327\) MeV (we take \(N_f = 2.5\) to account for the mass suppression for the strange quarks in the QGP). For the above value of the initial temperature in \(1+1D\) Bjorken’s expansion the QGP reaches \(T \approx T_c\) (here \(T_c \approx 160\) is the crossover temperature) at \(\tau_{QGP} \approx 4 - 5\) fm. We treat the crossover region as a mixed phase \[28\]. For the relevant values of the proper time \(\tau \gtrsim 8 - 10\) fm (see below) the QGP fraction in the mixed phase is approximately \(\propto 1/\tau\) \[28\]. For this reason we can use in calculating jet quenching the \(1/\tau\) dependence of the number density of the scattering centers in the whole range of \(\tau\) (but with the Debye mass defined for \(T \approx T_c\) at \(\tau > \tau_{QGP}\)).

The Bjorken \(1+1D\) model \[28\], used in our analysis, neglects the transverse expansion of the matter that becomes very important at \(\tau \sim R_A \approx 6\) fm. However, from the point of view of the parton energy loss its effect should not be large due to compensation between the enhancement of the energy loss caused by increase of the
medium size and its suppression caused by reduction of the medium density. The fact that the transverse expansion of the fireball does not affect strongly jet quenching in AA collisions was demonstrated, for the first time, in Ref. [35] within the BDMPs approach [1].

C. Calculation of $I_{AA}$

In leading order (LO) pQCD the energy of the hard parton produced in the direction opposite to the tagged direct photon, $E_T$, coincides with the photon energy $E'_T$. For $E_T = E'_T$ the medium modification factor for a given photon energy reads

$$I_{AA}(z, E'_T) = \frac{D_{h}^{AA}(z, E'_T)}{D_{h}^{pp}(z, E'_T)},$$

where the nominator and the denominator are the FFs for the photon-tagged jets in $AA$ and $pp$ collisions, respectively. For $pp$ collisions

$$D_{h}^{pp}(z, E'_T) = \sum_{i} r_{i}^{pp}(E'_T) D_{h,i}^{pp}(z, E'_T),$$

where $D_{h,i}^{pp}$ is the FF for $i \to h$ process defined by Eq. (11) with $Q = E'_T$, and $r_{i}^{pp}$ is the fraction of the $\gamma + i$ parton state in the $\gamma$+jet events in $pp$ collisions. The FF for $AA$ collisions can be written as

$$D_{h}^{AA}(z, E'_T) = \langle \sum_{i} r_{i}^{AA}(E'_T) D_{h,i}^{m}(z, E'_T) \rangle,$$

where $D_{h,i}^{m}$ is the medium modified FF for $i \to h$ process defined by Eqs. (2), (3) with $Q = E'_T$, and $r_{i}^{AA}$ is the fraction of the $\gamma + i$ parton state in the $\gamma$+jet events for $AA$ collisions. $\langle \rangle$ means averaging over the impact parameter of $AA$ collision, the jet direction and the position of jet production. The distribution of the jet production points in the transverse plane for a given impact vector $b$ is $\propto I_{A}(\rho)T_{A}(b - \rho)$. This averaging procedure leads to fluctuations of the fast parton path length $L$ in the QGP. In Fig. 1 we show the distribution in $L$ for $Au+Au$ collision for the $0 – 12\%$ centrality bin corresponding to the conditions of the STAR experiment [15] calculated for our model of the QGP fireball.

For the STAR experiment [15] $E'_T \lesssim 20$ GeV. In this case the sum over all relevant types of partons on the right hand side of Eqs. (28), (29) is dominated by gluon and light quarks. Since the medium effects for all light quarks are very similar, we consider all light quarks as one effective light quark state $q$ with $r_{q} = 1 - r_{g}$. We calculate the $r_{g,q}$ using the LO pQCD with the CTEQ6 [30] parton distribution functions. As in the PYTHIA event generator [20], to simulate the higher order effects in calculation of the partonic cross sections we take for the virtuality scale in $\alpha_s$ the value $cQ$ with $c = 0.265$. For $AA$ collisions we account for the nuclear modification of the parton densities (which leads to a small deviation of $I_{AA}$ from unity even without parton energy loss) with the help of the EKS98 correction [37]. This calculation gives for $E'_T \sim 12 – 20$ GeV $r_{q}/r_{q}^{pp} \sim 0.27 – 0.33$, and somewhat smaller values for $AA$ collisions $r_{q}^{AA}/r_{q}^{AA} \sim 0.24 – 0.3$.

The LO pQCD formulas do not account for the effect of the intrinsic parton transverse momentum on the photon-tagged jet FFs. The intrinsic parton transverse momenta lead to fluctuation of the jet energy $E_T$ around the photon energy $E'_T$, both for $pp$ and $AA$ collisions [14]. In the pQCD the intrinsic parton transverse momenta in PDFs emerge in NLO calculations due to the initial state radiation in jet production. The NLO calculations performed in Ref. [38] show that for $AA$ collisions the smearing correction, $\Delta_{sm}$, to the medium modification factor $I_{AA}(z)$ at $E'_T \sim 7 – 8$ blows up at $z \lesssim 0.8 – 0.9$. In Appendix B we demonstrate that $\Delta_{sm} \approx F(z, E'_T) dI_{AA}/dz/E'_T^2$, where $F(z, E'_T)$ is a smooth function of $E'_T$. Using this formula with the help of the results of Ref. [38] we can obtain the smearing correction to $I_{AA}$ for the conditions of the STAR experiment [15]. The STAR data are obtained for $z \sim 0.15 – 0.8$. The results of Ref. [38] show that at $E'_T \sim 7 – 9$ GeV, in the potentially dangerous region $z \sim 0.8$ $\Delta_{sm}$ increases $I_{AA}$ by $\sim 30\%$. For the photon energy interval $12 < E'_T < 20$ GeV studied in Ref. [15] from the formula for $\Delta_{sm}$ we conclude that the smearing correction to $I_{AA}$ should be $\lesssim 8\%$ at $z \sim 0.8$ (in fact even at $z = 0.9$ it is relatively small), and for $z \lesssim 0.6$ it should be very small. For this reason for the photon energy region studied in Ref. [15] accuracy of the LO pQCD predictions for $I_{AA}$ should be quite good.
III. NUMERICAL RESULTS AND DISCUSSION

The data of Ref. [13] are obtained for the photon energy region $12 < E_T^h < 20$ GeV. We define the nuclear modification factor $I_{AA}$ for the photon energy window $E_1 < E < E_2$ (hereafter, for notational simplicity, we omit the $T$ and $\gamma$ indices of the photon energy) as

$$I_{AA}(z, E_1, E_2) = \frac{D^{AA}_h(z, E_1, E_2)}{D^{pp}_h(z, E_1, E_2)}, \quad (30)$$

where the FFs in the denominator and numerator are averaged over the photon energy with the help of the LO pQCD cross section for photon-jet events $\sigma/dy$ defined as

$$D^i_h(z, E_1, E_1) = \frac{\int_{E_1}^{E_2} dE \int \frac{d\sigma}{dy} \left( \frac{d\sigma}{dy} \right)_{\text{jet}}} \int_{E_2}^{E_2} dE \int \frac{d\sigma}{dy} \left( \frac{d\sigma}{dy} \right)_{\text{jet}} \quad (31)$$

with superscript $i = AA, pp$. The cross section $\sigma^{AA}/dy$ is calculated with the EKS98 [37] correction to the nuclear PDFs.

Before presenting our results for the medium modification factor $I_{AA}$, it is instructive to first compare our results for the $pp$ photon-tagged FF and to illustrate the $z$ and $L$ dependence of the medium effects in our model. In Fig. 2 we compare our results for the $pp$ photon-tagged FF for charged hadrons for the STAR energy window between $E_1 = 12$ and $E_2 = 20$ GeV with that measured in Ref. [13]. From Fig. 2 one sees that the theoretical photon-tagged FF agrees reasonably with that from STAR [13]. In Fig. 2 in addition to the prediction for the photon-tagged jet FF calculated using FFs given by Eq. (4) we also plotted the curve obtained using the pure KKP FFs $D^{h/i}_h(z, Q)$. One can see that the results for both the methods agree reasonably well. However, in principle, from the point of view of the theoretical predictions for $I_{AA}$ the quality of description of the $pp$ photon-tagged FF is not very important.

To illustrate the relative contribution from gluon and quarks to the photon-tagged FFs in Fig. 3 we plot fraction of the gluon contribution to $D^{AA}_h$. The results for $D^{AA}_h$ are calculated for $0 - 12\%$ central Au+Au collisions (as we noted in Sec. 2 it corresponds in our model of the QGP fireball to the initial QGP temperature $T_0 \approx 327$ MeV at $\tau_0 = 0.5$ fm). We used $\alpha_s^{1-f} = 0.5$ that gives a reasonable description of $I_{AA}$ (see below). To illustrate better the medium effects we present predictions for $D^{AA}_h$ obtained with and without the EKS98 corrections to the nuclear PDFs. One can see that both for $pp$ and $AA$ collisions the relative gluon contribution decreases strongly with increasing $z$. The effect of the nuclear modification of the PDFs is relatively small. From the curves for the Au+Au collision, one can see that the suppression of the gluon fraction at large $z$ is considerably stronger than for $pp$ collisions. This is a consequence of a bigger energy loss in the QGP for gluons.

In order to better demonstrate the difference between the strength of jet quenching for gluon and quark jets in

![Fig. 2: The photon-tagged jet FF for $12 < E_T^h < 20$ GeV for $pp$ collisions at $\sqrt{s} = 200$ GeV calculated using FFs given by Eq. (4) (solid) and obtained for pure KKP FFs $D^{h/i}_h(z, Q)$ (dotted). Data points are from Ref. [13].](image-url)
as for the factor $I_{AA}$ given by Eqs. (30), (31). From Fig. 5 one sees that the $L$-dependence of the medium modification becomes rather weak at $L \sim 8 - 10$ fm. One can see that, in agreement with the decrease of $\langle L_{q,g}(z) \rangle$ with increase of $z$ seen from Fig. 4, the photon-tagged FF at large $z$ should be biased to the events with smaller jet path lengths through the QGP.

Finally, after presenting illustrative results, in Fig. 6 we confront our results for the medium modification factor $I_{AA}$ obtained for $\alpha_{fr}^r = 0.4$, 0.5 and 0.6 with the data from STAR [15]. Our previous analyses [18, 39] of the RHIC data on the nuclear modification factor $R_{AA}$ in Au+Au collisions at $\sqrt{s} = 200$ GeV at $p_T \sim 10 - 20$ GeV support $\alpha_{fr}^r \sim 0.5$. From Fig. 6 one sees that $\alpha_{fr}^r \sim 0.5$ gives also a reasonable agreement with the STAR data [15] on the nuclear modification factor $I_{AA}$ in the whole range of $z$ studied in Ref. [15] from $z \sim 0.8$ down to $z \sim 0.15 - 0.25$. Note that at $z \lesssim 0.4$ our results depend weakly on the value of $\alpha_{fr}^r$.

In Fig. 6 we included the region of very small $z$ down to $z \sim 0.03$ that corresponds to hadrons with momentum $\sim 0.4 - 0.5$ GeV. It is done just to illustrate the flow of the jet momentum into the soft region in our model, which satisfies the momentum sum rule. Of course, the production of such low energy hadrons cannot be treated in the jet fragmentation picture, because it should involve fragmentation of partons with energy comparable to the energy of the thermal partons in the produced QGP.

As we noted in Introduction, the approximation of independent gluon emission, based on Landau’s method [10], has no a rigorous theoretical justification at $z \ll 1$, where cascading of the primary gluons radiated from fast partons may come into play. The typical energy for partons fragmenting to hadrons in the region $z \sim 0.15 - 0.25$ is $\sim 3 - 6$ GeV (we take the jet energy $E \sim 15$ GeV that is approximately the mean energy for the STAR window $E \sim 12 - 20$ GeV [13]). It corresponds to the parton fractional momentum $x \sim 0.2 - 0.4$. The formation length for the induced radiation, $L_{fr}^r$, of a primary gluon with the fractional momentum $x$ from a fast quark can be esti-
mated as \[ L_f^{pp} \sim \frac{2E_x(1-x)}{\epsilon^2} S_{LPM} \quad (34) \]

where as in Eq. (5) \( \epsilon^2 = m_{\gamma}^2 x^2 + m_{\gamma}^2 (1-x) \), \( S_{LPM} \) is the LPM suppression factor. From the formula \[ \] one can obtain \( L_f^{pp} \sim 3 - 4 \) fm. We used \( S_{LPM} \sim 0.3 \) that corresponds to \( \tau \sim 3 \) fm that is interesting to us. The formation length for radiation of the secondary gluons with energy about half of that for the primary gluons is about \( \sim 2 - 3 \) fm. It means that typically splitting of the primary gluons can occur at \( \tau \sim 5 - 6 \) fm. In this region the density of the QGP is at least by a factor of \( \sim 10 \) smaller than that at the initial time \( \tau_0 \). For this reason the induced gluon splitting of the primary radiated gluons, that are neglected in our analysis, may be a relatively weak effect.

The present analysis assumes that the medium effects are present only in \( AA \) collisions. It is possible that a small-size QGP is produced in \( pp \) collisions as well. The idea that the QGP may be produced in hadron collisions is very old [40]. Application of the Bjorken relation [26] to \( pp \) collisions shows that at RHIC energy \( \sqrt{s} = 200 \) GeV the initial temperature of the mini-QGP fireball may be as large as \( \sim 200 - 230 \) MeV [39] which is well above the deconfinement temperature. In the scenario with the mini-QGP production the theoretical nuclear modification factor \( R_{AA} \) should be divided by the medium modification factor for \( pp \) collisions \( R_{pp} \) [39] that accounts for jet quenching in the mini-QGP produced in \( pp \) collisions. For RHIC energy \( \sqrt{s} = 200 \) GeV numerical calculations within the LCPI approach give \( R_{pp} \sim 0.7 - 0.8 \) at \( p_T \sim 10 - 20 \) GeV [39]. The results of Ref. [39] show that in the scenario with mini-QGP production some increase of the nuclear modification factor \( R_{AA} \) due to the additional factor \( 1/R_{pp} \) can be imitated by reduction of the \( \alpha^f \). Since a direct measurements of \( R_{pp} \) is impossible, it is practically impossible to distinguish the scenarios with and without the mini-QGP production in \( pp \) collisions using the data on the \( R_{AA} \). For the nuclear modification factor \( I_{AA} \) the situation is similar. In the scenario with mini-QGP production in \( pp \) collisions the theoretical \( I_{AA} \) should be divided by the medium modification factor \( I_{pp} \) for \( pp \) collisions, which, similarly to \( R_{pp} \), is unobservable quantity. And its effect on the theoretical \( I_{AA} \) can be imitated by some change in \( \alpha_{s} \). However, contrary to the medium effects in \( pp \) collisions on the high-\( p_T \) spectra, the photon-tagged jet FF, at least in principle, enables us [11] to study the effect of mini-QGP by measuring its variation with the multiplicity of soft off-jet particles (the so-called underlying events, see Ref. [42] for a review). But this requires measurements of the photon-tagged jet FF for very high underlying event multiplicities \( dN_{ch}^{UE}/d\eta \sim 40 \) [11]. Such multiplicities are too high to be measured at RHIC energies in the \( \gamma \)-jet events.

\section{IV. SUMMARY}

We have calculated the nuclear modification factor \( I_{AA} \) for the photon-tagged jets within the jet quenching scheme based on the LCPI approach [2] to the induced radiation with the collisional energy loss treated as a perturbation to the radiative mechanism. The calculations are performed for running \( \alpha_{s} \) frozen at low momenta. Our scheme for calculation of the medium-modified photon-tagged jet FF function in \( AA \) collisions preserves the flavor and the momentum conservation. We have compared the theoretical predictions with the recent data from STAR [13] for \( Au+Au \) collisions at \( \sqrt{s} = 200 \) GeV for \( 12 < E_T < 20 \) GeV. We obtained a reasonable agreement with the STAR data in the whole range of \( z \) from \( z \sim 0.8 \) down to \( z \sim 0.15 \) for running coupling constant frozen at the value \( \alpha^f_s \sim 0.5 \). This value agrees well with that necessary for description of the RHIC data on the nuclear modification factor \( R_{AA} \).

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Appendix A. Formulas for the spectrum of the induced $a \rightarrow bc$ transition

In this appendix for the reader convenience and completeness we give formulas for calculations of the probabilities of $q \rightarrow gq$, $q \rightarrow gq$, and $q \rightarrow q\bar{q}$ transitions in the LCPI approach [2]. We use the representation of the induced spectrum obtained in Ref. [21] which is convenient for numerical calculations. In general in the LCPI approach the $x$-distribution for the induced $a \rightarrow bc$ partonic transition (hereafter $x = E_a/E_\rho$) for parton $a$ with momentum along the $z$-axis produced in a hard process at $z = 0$ in the matter of thickness $L$ can be written as

$$\frac{dP}{dx} = \int_0^L n(z) \frac{d\sigma_{BH}^{\text{eff}}(x,z)}{dx},$$  \hspace{1cm} (A1)

where $n(z)$ is the medium number density, $d\sigma_{BH}^{\text{eff}}/dx$ is an effective Bethe-Heitler cross section. It accounts for both the LPM and finite-size effects and can be written as

$$\frac{d\sigma_{BH}^{\text{eff}}(x,z)}{dx} = -\frac{P^b_a(x)}{\pi M} \text{Im} \int_0^\infty d\xi \alpha_s(Q^2(\xi))$$

$$\times \exp \left(-i\frac{\xi}{L_f} \frac{\partial}{\partial \rho} \left(\frac{F(\xi, \rho)}{\sqrt{\rho}}\right)\right) \bigg|_{\rho=0}. \hspace{1cm} (A2)$$

Here $P^b_a(x)$ is the usual pQCD splitting function for $a \rightarrow bc$ transition, $M = E_a/(1-x)$, $L_f = 2M/c^2$ with $\epsilon^2 = m_\rho^2(1-x) + m_{\bar{q}}^2 x - m_a^2 (1-x)$, $Q^2(\xi) = aM/\xi$ with $a \approx 1.85$ (this value of the parameter $a$ was fixed in Ref. [23] by comparison of the induced gluon spectrum calculated in the coordinate representation with that obtained in the momentum representation for the dominant $N = 1$ rescattering contribution). $F$ is the solution to the radial Schrödinger equation for the azimuthal quantum number $m = 1$

$$i \frac{\partial F(\xi, \rho)}{\partial \xi} = \left[ -\frac{1}{2M} \left(\frac{\partial}{\partial \rho}\right)^2 + v(\rho, x, z - \xi)\right. $$

$$\left. + \frac{4m^2 - 1}{8M \rho^2}\right] F(\xi, \rho), \hspace{1cm} (A3)$$

with the potential

$$v(\rho, x, z) = -i \frac{n(z)\sigma_3(\rho, x, z)}{2}.$$  \hspace{1cm} (A4)

Note, that in terms of the original longitudinal variable $z$ along the fast parton momentum we solve the Schrödinger equation backward in time/$z$. The point $\xi = 0$ corresponds to the last rescattering of the $bc\bar{a}$ system on a medium constituent located at $z$. This technical trick allows us to have a smooth boundary condition for $F$ at $\xi = 0$: $F(\xi = 0, \rho) = \sqrt{\rho} \sigma_3(\rho, x, z) e K_1(\epsilon \rho)$ ($K_1$ is the Bessel function). The function $\sigma_3(\rho, x, z)$ is the cross section of interaction of the $bc\bar{a}$ system with a medium constituent located at $z$ (the argument $\rho$ is the transverse distance between $b$ and $c$). In the transverse plane the parton $\bar{a}$ in the $bc\bar{a}$ system is located at the center of mass of the $bc$ pair. In the Schrödinger equation (A3) $M$ plays the role of the reduced "Schrödinger mass" for the $bc$ pair (since the “masses” for the $b$ and $c$ partners are $E_{\rho}x$ and $E_{\rho}(1-x)$, respectively).

The three-body cross section $\sigma_3$ can be written in terms of the well known dipole cross section for the color singlet $q\bar{q}$ pair [43] that is given by

$$\sigma_{q\bar{q}}(\rho, z) = C_T C_F \int dq q^2 q^2 [1 - \exp(iq\rho)] \left| q^2 + \mu_D(z)^2 \right|^2. \hspace{1cm} (A5)$$

Here $C_F, T$ are the color Casimir for the quark and thermal parton (quark or gluon), and $\mu_D(z)$ is the local Debye mass. In terms of the dipole cross section (A5) the three-body cross sections for $q \rightarrow gq$, $g \rightarrow gq$, and $g \rightarrow q\bar{q}$ transitions read

$$\sigma_3(\rho, x, z)|_{q\rightarrow gq} = \frac{9}{8} [\sigma_{q\bar{q}}(\rho, z) + \sigma_{q\bar{q}}((1-x)\rho, z)]$$

$$- \frac{1}{8} \sigma_{q\bar{q}}(x\rho, z), \hspace{1cm} (A6)$$

$$\sigma_3(\rho, x, z)|_{g\rightarrow gq} = \frac{9}{8} [\sigma_{q\bar{q}}(\rho, z) + \sigma_{q\bar{q}}((1-x)\rho, z)]$$

$$+ \sigma_{q\bar{q}}(x\rho, z), \hspace{1cm} (A7)$$

$$\sigma_3(\rho, x, z)|_{g\rightarrow q\bar{q}} = \frac{9}{8} [\sigma_{q\bar{q}}(x\rho, z) + \sigma_{q\bar{q}}((1-x)\rho, z)]$$

$$- \frac{1}{8} \sigma_{q\bar{q}}(\rho, z). \hspace{1cm} (A8)$$

The LPM suppression and finite-size suppression for the in-medium $a \rightarrow bc$ process is characterized by the factor

$$S(M, x) = \frac{dP/dx}{dP_{BH}/dx}, \hspace{1cm} (A9)$$

where the denominator is the Bethe-Heitler spectrum calculated using in Eq. (A1) the real Bethe-Heitler cross section for $a \rightarrow bc$ transition given by [44]

$$\frac{d\sigma_{BH}^{\text{eff}}(x,z)}{dx} = \int d\rho |\Psi_a^{\text{BH}}(\rho, x)|^2 \sigma_3(\rho, x, z), \hspace{1cm} (A10)$$

where $\Psi_a^{\text{BH}}(\rho, x)$ is the light-cone wave function for $a \rightarrow bc$ transition. Since $|\Psi_a^{\text{BH}}(\rho, x)|^2$ contains the splitting function $P^b_a(x)$, the factor $S(M, x)$ for a given value of

\footnote{It is worth noting that the formula for the Bethe-Heitler cross section (A10) exactly corresponds to that for the effective Bethe-Heitler cross section given by Eq. (A2) with the infinite upper limit of the $\xi$-integration and the function $F$ calculated for $v = 0$.}
M has a smooth dependence on x. It occurs because the x-dependence of the integrand of $\langle A_2 \rangle$, that comes only from the x-dependence of $\hat{e}^2$ and of the three-body cross section $\sigma_3$, is relatively weak. This fact allows one to reduce considerably CPU time in numerical calculations by creating a grid of values of $S(M,x)$ in the $M-x$ plane with relatively small number of points in x. Then this grid can be used for calculation of the spectrum $dP/dx$ by performing interpolation in x and using the Bethe-Heitler spectrum that does not require much computing power.

In the above formulas for the effective Bethe-Heitler cross section $\langle A_2 \rangle$ and for the dipole cross section $\langle A_5 \rangle$ we use the following parametrization for $\alpha_s(Q^2)$

$$
\alpha_s(Q^2) = \begin{cases}
\alpha_s^{fr} & \text{if } Q \leq Q_{fr}, \\
\frac{4\pi}{9 \log(Q^2/\Lambda^2_{QCD})} & \text{if } Q > Q_{fr},
\end{cases}
$$

(A11)

with $Q_{fr} = \Lambda_{QCD} \exp(2\pi/9\alpha_s^{fr})$, $\Lambda_{QCD} = 300$ MeV.

\section*{Appendix B. The smearing correction to $I_{AA}$}

In this appendix we discuss the energy dependence of the smearing correction to the LO predictions for the nuclear modification factor $I_{AA}$. This correction potentially may be important at $z$ close to unity. The question is where the regime of large smearing correction begins. To understand this we use as a plausible estimate of the smearing effect the results of the NLO model \cite{38}. This analysis shows that for AA collisions the smearing correction to the medium modification factor $I_{AA}(z)$ blows up at $z \gtrsim 0.8 - 0.9$ for $E \sim 8$ GeV. The results of Ref. \cite{38} can be easily rescaled to our conditions. Indeed, let us write the LO $I_{AA}$ as

$$
I_{AA}(z) = D_h(z + \Delta z)/D_h(z),
$$

(B1)

where $D_h$ is the FF for $pp$ collisions, $\Delta z = z\Delta E/E$ is shift of $z$ due to the radiative parton energy loss $\Delta E$ in the QGP (here, for simplicity, we omit averaging over $\Delta z$ which should be made, also for clarity we omit the argument $E$ and superscript $pp$ on the FF). The $I_{AA}$ in the presence of the smearing (we denote it $\tilde{I}_{AA}$) can be written as

$$
\tilde{I}_{AA}(z) = [D_h(z + \Delta z) + D_h^\prime(z + \Delta z)\langle \delta z^2 \rangle/2] \times [D_h(z) + D_h^\prime(z)\langle \delta z^2 \rangle/2]^{-1},
$$

(B2)

where $\delta z = zq/E$ and $q$ is the shift of the jet energy ($E_{jet} = E + q$) (we assume that the smearing correction is not very large, and keep only the second order terms in $\delta z$). From (B2) we obtain an equation which does not contain $\Delta z$

$$
\tilde{I}_{AA}(z) \approx I_{AA}(z) + \Delta_{sm},
$$

(B3)

$$
\Delta_{sm} \approx [2I_{AA}(z)D_h^\prime(z) + I_{AA}'(z)D_h(z)] \times z^2(q^2/2E^2).
$$

(B4)

The second term in the square brackets in Eq. (B3) can be neglected (since $I_{AA}$ is a smooth function as compared to $D_h$). Then we obtain

$$
\Delta_{sm} \approx F(z,E)I_{AA}'(z)/E^2,
$$

(B5)

where $F(z,E)$ is a smooth function of energy, and does not depend on the strength of the medium suppression at all. The fact that $\Delta_{sm} \propto dI_{AA}/dz$ is quite natural. For a flat $I_{AA}$ the smearing effect should vanish since the numerator and denominator are affected in the same way. Note that the $\Delta_{sm} \propto dI_{AA}/dz$ scaling agrees with the results for $\Delta_{sm}$ from Ref. \cite{38} obtained for different magnitudes of the energy loss (shown in Fig. 2 of Ref. \cite{38}). So now with the help of (B5) we can rescale the smearing correction of Ref. \cite{38} to the higher energy region corresponding to the STAR experiment \cite{15}.

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