Road pavement design using the finite element method

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Abstract. The purpose of the work is to calculate the stress-strain state of the III-type road, as well as to determine the load factor for road pavement and roadbed. A model of non-rigid road pavement consisting of 4 layers is considered: dense-graded asphalt concrete, open-graded asphalt concrete, gravel roadbed, ground roadbed (silt sandy loam). A numerical code is developed for calculating the principal physical and mechanical parameters of road pavement. C++ is used as a programming language. A method of evaluating the shear resistance of roadbed is proposed. To calculate the load factor of roadbed, the Drucker-Prager strength criterion is used without the term responsible for the intensity of tangential stress. A T-90 tank fitted with special rubber track pads is considered to be a static load. The computations reveal the maximum vertical deflection, equal to 0.004 m, in the contact area of the tracks. In the same area, the load factor of road pavement is $9 \div 10$, which indicates a high bearing capacity of dense-graded asphalt concrete. The analysis of shear resistance shows the presence of irreversible deformations in gravel roadbed in the contact area of the tracks, while the load factor of silt sandy loam is $3 \div 10$.

1. Introduction

The development of the transport system is inextricably linked with the construction and upgrading of roads, with most of the costs spent on the construction of road pavement (RP). Road pavement has two main layers: covering and roadbed. In addition, each layer can be divided into several intermediate layers, each of which performs a certain role under the load of vehicles [1]. Road pavement is in direct contact with vehicles and carries the main load, distributing it to the lower layers of RP. As a rule, the following requirements are imposed on road pavement: compressive and tensile strength, moisture resistance, and frost resistance. The roadbed requirements are to take and uniformly distribute the ground load. Most often roadbed is made of sand and gravel mixtures. These mixtures can be both weakly bound and reinforced with special substances, which greatly affects the carrying capacity of road.

Scientific literature emphasizes two main directions in the design and engineering of road pavement. The first direction is related to theoretical and experimental strength estimation and the improvement of the methods for RP calculations [2–4]. A lot of works in this area are devoted to the calculation of shear resistance of road pavement and roadbed [2]. Road pavement [2] is subjected to shear deformation, especially in the summer period, when the strength asphalt concrete characteristics decrease by about two times due to elevated temperatures. A decrease in the shear strain resistance
leads to the formation of ruts and local subsidence on road pavement. These defects significantly affect the quality of road and are a serious danger to all road users. After calculating the shear resistance of road pavement, it is necessary to perform the same calculations for roadbed. As a rule, roadbed is made of bulk and weakly bound materials, so there is a high probability of roadbed subsidence under the load of heavy vehicles [2]. Irreversible deformations of the ground are directly related to shear stress, and considering the deformation of the ground in detail, it can be noted that when critical shear stress exceeds the resistance and adhesion forces of the medium, the ground is displaced along one or more sliding surfaces. As a result, in some places the ground may subside or buckle in other places, which leads to the deformation of road pavement. Calculation of shear resistance of the ground is a complicated task, since there is a need in an adequate fracture model. The Mohr – Coulomb and Drucker – Prager strength criteria are often used in calculations, since they best describe the deformation of ground.

The second direction is related to the methods of strengthening road pavement and roadbed. Works in this area are devoted to both theoretical and practical research in strengthening roads [5-7]. Geosynthetic materials provide a possibility of reinforcing the upper layers of road pavement and strengthening slopes and roadbed. At the same time, the main difficulty in the design of such structures is the estimation of their strength. Modification of analytical methods for the calculation of reinforced road pavement does not provide good agreement between numerical results and experimental data; nevertheless, experts in this field continue to improve calculation methods [5].

Reinforcement of roadbed is also applied. This approach increases the elastic deflection resistance and fully ensures redistribution of stress over the entire roadbed. But the most effective method to strengthen roadbed is the use of binding materials such as slags, cement, bitumen, synthetic resins, lime, sodium silicate, organic and polymeric binders [6]. Reinforced roadbed perfectly resists shear strain and has high strength properties, however, there is the problem of selecting a binder for different roads. The fact is that different binders affect the physical and mechanical properties of the ground in different ways, therefore, it is necessary to accurately predict the behavior of reinforced roadbed for certain loads in designing.

Designing the road pavement is a complicated problem, since road pavement layers have to be optimally selected (thickness, type of material) from an economic point of view and the road pavement strength has to ensure reliability. This selection process takes a lot of time; therefore, a more promising direction is numerical simulation based on approaches of deformable solid mechanics, fracture mechanics, and ground mechanics. Modern methods of numerical simulation are widely used in engineering, and in this paper the finite element method (FEM) is considered. The main idea of the method is to discretize the computational domain into finite elements (triangular for this case) connected in nodes, the displacements of which are determined from nodal displacements of the element using linear polynomials [8]. Calculated nodal displacements are used to determine the stress-strain state of the entire structure. One of the advantages of the FEM is the possibility of discretization of any object and the implementation of the method using computing environment.

The use of FEM in the design of road pavement provides high possibilities for analyzing the strain-stress state of road pavement and roadbed, as well as for determining their bearing capacity [9-11]. The road pavement calculation using the finite element method is not limited to the number of layers, their location and type of materials. In addition, several problems can be simultaneously solved: estimating the strength of reinforced road pavement and roadbed, the shear resistance of reinforced and unreinforced roadbed, and many other problems.

In order to show versatility and positive aspects of this approach, the goal was to determine the stress-strain state of all road pavement layers using the finite element method in a static, elastic, plane-strain formulation, estimate the strength of road pavement subjected to elastic deflection and localize irreversible strain sections of roadbed according to the Drucker – Prager strength criterion [12].

2. Formulation of the problem
An example of road pavement was used from the road building requirements “ODN 218.046-01 Design of nonrigid road pavements (Russia)” [13]:
1) road is located in the II road building climatic zone;
2) type of road is III;
3) lifetime of road pavement is $T_{\text{work}} = 15$ years;
4) total load $\Sigma N_p \approx 1.2 \times 10^6$.
5) working roadbed layer is silt sandy loam with a calculated moisture of 0.7 $W_t$, which is qualified as strongly swollen ground;
6) material for roadbed is a gravel mixture;
7) depth of fill is 1.4 m;
8) scheme of moistening the working roadbed layer is III;
9) depth of groundwater is 0.6 m.

The surface of road pavement consists of two 50 mm layers, followed by roadbed that contains a gravel mixture and silt sandy loam. Figure 1 shows the arrangement, height and width of layers. The working third-type road pavement, according to [13], is 7 m in width, and the width of roadside is 1 m. The height of road pavement is 1.4 m, and the slope angle is $45^\circ$. Table 1 presents the main characteristics of road pavement layers. The problem formulation is given in Figure 2.

![Figure 1. Road pavement layout, sizes in mm.](image)

| Depth of layer $h$, mm | Dense-graded asphalt concrete (BND 60/90) | Open-graded asphalt concrete (BND 60/90) | Gravel mixture | Silt sandy loam |
|------------------------|------------------------------------------|------------------------------------------|----------------|----------------|
|                        | 50                                       | 50                                       | 500            | 800            |
| Young's modulus $E$, MPa| 3200                                     | 2800                                     | 240            | 46             |
| Poisson's ratio $\mu$  | 0.1                                      | 0.1                                      | 0.17           | 0.3            |
| Internal friction angle $\varphi$, grad | -                                        | -                                        | 42             | 31             |
| Adhesion coefficient $C$, MPa | -                                        | -                                        | 0.0025         | 0.006          |
| Compression strength $\sigma_s$, MPa at 20°C | 2.2                                      | 2                                        | -              | -              |

A tank $T$-90 is considered to be load acting on road pavement. The load is given by distributed forces (pressure) from the tracks, which simplifies the formulation of the problem.
The reduced load (load per unit length of the track in the longitudinal direction) is calculated by the formula:

\[ Q = \frac{Mg}{2L} \]  

(1)

where \( M \) is the mass of the tank, kg; \( g \) is the acceleration of gravity, m/s\(^2\); \( L \) is the length of the track, m. The total mass of the tank is set equal to 45500 kg, the length of the track is set equal to 4.27 m, therefore, \( Q = 54450 \text{ N/m} \).

Nodal forces \( F \) are calculated as follows. To conduct two-dimensional computations in the plane-strain formulation, the conventional unit length \( l \), equal to 1 m for all elements, is used in the longitudinal direction. Then the force \( F \) acting on each computational node under the assumption that the node masses are equal, is calculated by the formula:

\[ F = \frac{Ql}{n} \]

(2)

where \( l \) is the selected length of computational elements; \( n \) is the number of the nodes subjected to the reduced load \( Q \).

Figure 2. Formulation of the problem, sizes in mm

The boundary conditions of the problem are set as follows: computational mesh nodes located along the x axis at \( y = 0 \) are rigidly fixed in the vertical direction \( \delta_y = 0 \) (Figure 2). The remaining nodes have free boundary conditions.

3. Results and Discussion

All computations were performed using a numerical code that is written in the C++ programming language and includes dynamic arrays and vectors, which significantly reduces the requirements for computer resources. The number of elements of the grid is \( N = 5152 \).

Figure 3 shows the displacement of nodes \( \delta_y \). The maximum displacements correspond to the contact area of the tracks with road pavement, however, the heterogeneity of the movement fields is observed. This is explained by the fact that the left load area is located at roadside, that is, where dense-graded asphalt concrete pavement and gravel-sand mixture roadside are in contact. This effect is caused by a sharp decrease in the elastic modulus from 3200 MPa to 240 MPa, in contrast to the right area, where the elastic modulus gradually decreases from 3200 MPa to 2800 MPa.
Figure 3. Displacement fields $\delta_y$ (m)

Figure 4 shows the graph of $\delta_y(x)$ at $y = 1.4$ m, which corresponds to the upper boundary of road pavement. This figure demonstrates how road pavement is deformed under asymmetric loading relative to the center of road. Buckling of the surface in the positive direction is observed on the right side of road, as a result, ruts may be formed.

Figure 4. Graph of $\delta_y(x)$ (m) at $y = 1.4$ m.

1. solution obtained using ANSYS Mechanical,
2. solution obtained using the authors' numerical code

In addition to vertical movements, road pavement is subjected to horizontal movements. Figure 5 shows the displacement fields $\delta_x$, in this case it should be noted that the value of maximum ($1.3 \times 10^{-4}$ m) and minimum ($-1.8 \times 10^{-4}$ m) displacements $\delta_x$ differs in module by 40%, since load is located asymmetrically relative to the center of road.

Figure 5. Displacement fields $\delta_x$ (m)
For the analysis of the bearing capacity of road pavement and roadbed, the load factor \( k = \sigma_i / \sigma \) is used, where \( \sigma_i \) is the admissible limit value, and \( \sigma \) is stress calculated by the selected criterion. For the estimation of the strength of road pavement, it is necessary to take into account that the material is subjected to compressive stress, therefore, \( \sigma_i \) is critical compressive stress. The von Mises stress intensity is considered as \( \sigma \) (3):

\[
I = \frac{1}{\sqrt{2}} \sqrt{\left(\sigma_1 - \sigma_2\right)^2 + \left(\sigma_2 - \sigma_3\right)^2 + \left(\sigma_3 - \sigma_1\right)^2}
\]

where \( \sigma_1, \sigma_2, \sigma_3 \) are principal stresses. Figure 6 shows the graphs of the load factor plots \( k(x) \) for two-layer road pavement. In the graph it is seen that in the contact area of the tank tracks and road pavement \( k \) reaches 9 ÷ 10.

Figure 6. Graph of \( k(x) \).

(1) open-graded asphalt concrete, (2) dense-graded asphalt concrete

For \( x = 3.25 \div 5.25 \) m, that is, between the tank tracks, the load factor increases to 13 ÷ 15 on average. On the right side of roads, the load factor increases to 200 and higher, since the load is distributed on the left side of roadbed.

Determination of the shear resistance of roadbed is also an integral part of strength calculations for non-rigid road pavement. For this purpose, the Drucker-Prager strength criterion (4) is used in computations [12]:

\[
D = \frac{6C \cos \alpha}{\sqrt{3(3 + \sin \alpha)}} + \frac{2C \sin \alpha}{\sqrt{3(3 + \sin \alpha)}} (\sigma_1 + \sigma_2 + \sigma_3) - \frac{1}{6} \left[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right]
\]

where \( C \) is the coefficient of adhesion, and \( \alpha \) is the angle of internal friction. According to this criterion, irreversible deformations occur when \( D \) is less than or equal to zero, that is, when stress acts on or outside the yield surface. This criterion accurately describes the behavior of bulk materials, ground, gravel and clay roadbeds.

In the work the load factor of roadbed was calculated as follows. The strain-stress state computations were carried out to determine the yield radius of each component using the Drucker-Prager yield criterion, and in expression (4) the last term corresponding to the intensity of tangential stresses \( \sigma \) is removed. Tangential octahedral diagonal stresses (5) are used as \( \sigma \):

\[
\tau_{oct} = \frac{1}{3} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}
\]

Then the load factor of roadbed is given by (6):
\[
k_b = \frac{6C \cos \alpha}{\sqrt{3(3 + \sin \alpha)}} + \frac{2C \sin \alpha}{\sqrt{3(3 + \sin \alpha)}} \left( \sigma_1 + \sigma_2 + \sigma_3 \right)
\]

Figure 7 shows the load factor fields for gravel and silt sandy loam roadbed. Analysis of the computations shows large irreversible deformations of roadbed. The main reason is likely to be the weakly bound material and the large angle of internal friction. The top right part of roadbed is not subjected to tangential strain and performs its bearing function, since the main load acts on the left side of road pavement. For silt sandy loam, \( k_b \) is within the range of \( 3 \div 10 \), that is, irreversible deformations are not observed in this zone.

![Figure 7. Load factor for roadbed](image)

To verify the obtained solution, the same problem was solved using ANSYS Mechanical. The governing comparison parameter is the displacement component \( \delta \). Analysis of the graphs shows the qualitative and quantitative agreement of the results. For example, the magnitude of vertical displacement of the left point on road pavement in both cases is \( 0.5 \times 10^{-4} \) m, and in the right part, with \( x \) greater than 7 m, buckling of road pavement takes place. Maximum deflection in Figure 4, curve 1, is \( 0.35 \times 10^{-3} \) m, and \( 0.425 \times 10^{-3} \) m for the solution obtained by the authors.

4. Conclusion

A numerical code has been developed for designing non-rigid road pavement using the finite element method. The stress-strain state and the load factor were calculated for road pavement and roadbed under static loading, and the vertical displacements of road pavement were determined. Subjected to such load, road pavement is capable of resisting compressive stresses and has a load factor in the range of \( 10 \div 15 \).

An approach for calculating the load factor of road pavement using the Drucker – Prager strength criterion is presented. This approach revealed a low bearing capacity of gravel-sand roadbed over the entire width of pavement road, except for the free side part, where tangential stresses are minimal. Bottom roadbed (silt sandy loam) of road pavement has \( k_b \) in the range of \( 3 \div 10 \), which indicates sufficient shear strain resistance in this area.

The obtained solution was compared to the solution obtained using ANSYS Mechanical. The governing comparison parameter was the vertical displacement component \( \delta_y \). Verification revealed qualitative and quantitative agreement of the results.

References

[1] Babkov V F and Andreev O V 1987 (Moscow: Transport)
[2] Aleksandrov A S, Dolgikh G V and Kalinin A 2016 Magazine of Civil Engineering 2(62) 51
[3] Aman, Awaluddin A, Ahmad A and Olivia M 2018 IOP Conference Series: Materials Science and Engineering 345 012018
[4] Katman H Y, Ibrahim M R, Matori M Y, Norhisham S and Ismail N 2013 IOP Conference Series: Earth and Environmental Science 16 012091
[5] Matveev S A, Martynov E A and Litvinov N 2015 *The Russian Automobile and Highway Industry Journal* 5(45) 72-6

[6] Al-Jumaili M A and Issmael O D 2018 *IOP Conference Series: Materials Science and Engineering* 433 012013

[7] Vyrozhemskyi V, Krayushkina K and Bidenko N 2017 *IOP Conference Series: Materials Science and Engineering* 236 012031

[8] Zienkiewicz O C 1971 *The finite element method in engineering science* (McGraw-Hill London)

[9] Calvarano L S, Palamara R, Leonardi G and Moraci N 2017 *IOP Conference Series: Earth and Environmental Science* 95 022024

[10] Li S and Hu C 2018 *IOP Conference Series: Earth and Environmental Science* 113 012188

[11] Leonardi G 2015 Bulletin of the Polish Academy of Sciences: Tech. Sciences 63(2) 397-403

[12] Drucker D C, Prager W 1952 Soil mechanics and plastic analysis for limit design *Quarterly of Applied Mathematics* 10(2), 157-165

[13] ODN 218.046-01 2001 (Moscow: Informvtodor) p. 146