Statistical distribution of pentad rainfall over India during monsoon season

D. A. MOOLEY and G. APPA RAO
Institute of Tropical Meteorology, Poona
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ABSTRACT. Statistical distribution of pentad rainfall during southwest and northeast monsoon seasons, at representative stations in India, has been studied. From the histograms it is seen that these distributions are right (positive) skewed. Gamma distribution function has been fitted to rainfall data and the goodness of fit of the distribution to the rainfall data has been tested by Chi-square tests. These tests show that pentad rainfall may be described by Gamma distribution.

1. Introduction

Barger and Thom (1949), Barger, Shaw and Dale (1959), Friedman and Janes (1957) have applied Gamma distribution for obtaining probabilities of weekly precipitation at stations in U.S.

Suzuki (1964) has fitted hypergamma distribution to 5-day rainfall of Tokyo.

In this paper it is proposed to examine the distribution of pentad rainfall during the southwest and northeast monsoon at Indian stations representing different rainfall regimes over the country.

2. Data

Rainfall distribution is proposed to be studied for 11 stations as shown in Fig. 1. Daily rainfall data for these stations for the period 1901-1960 were obtained from the punched cards and 5-day totals were computed from these. For Port Blair, 5 years’ data are not available during this period and the study for this station is based on 55 years data only.

The pentads used are standard pentads, numbered 1 to 73, the first pentad corresponding to 1 to 5 January and seventy-third pentad corresponding to 27 to 31 December; 29 February during leap years being ignored.

3. Pentad rainfall distribution during southwest and northeast monsoon seasons

June to September is normally the period of southwest monsoon over India and pentad numbers 31 to 55 cover this period. The time of onset and of withdrawal of southwest monsoon varies from one year to another and from one part of India to another. By the end of June monsoon is normally established over the whole country and by about mid-September it withdraws from northwest India. In view of this, the question arises what are the pentad numbers for the different stations for which rainfall distribution should be considered, if we want these pentads to be representative of monsoon conditions? Obviously, the pentad numbers representative of monsoon conditions would show some variation from one station to another. It is, therefore, necessary to lay down some objective criterion for deciding upon the pentads whose rainfall distribution should be studied.

3.1. Criteria for selection of pentads

The two criteria which can be considered are as follows —

(i) Normal dates of onset and withdrawal — Pentads may be taken from the set of pentads enclosing the normal date of onset of monsoon and its normal date of withdrawal at the stations under consideration. The normal dates of onset and withdrawal of monsoon may be fixed from the normal charts of onset and withdrawal as given in Climatological Atlas for Airmen (India met. Dep. 1948).

(ii) Rainfall Probability — If the pentads are to be representative of monsoon conditions they should satisfy some limit of probability of non-zero rain. All pentads with probability $P$ (non-zero rain) $>3$ may be considered. More stringent limits of this probability may be set if considered necessary for any specific purpose.

Normal dates of onset of monsoon at different stations were computed by sudden jumps in normal pentad rainfall amounts and not by the sudden jumps in the probability of rainfall and the normal chart of monsoon onset as given in
**Climatological Atlas for Airmen** (loc. cit.) was prepared on the basis of these individual dates for the different stations. In a similar way, the normal chart of withdrawal was prepared, the criterion in this case being marked decrease in pentad rainfall. Normal dates obtained in this way are likely to be biased as a result of very heavy rainfall during a few years in some pentads. Taking these facts into account it was felt advisable to use criterion (ii) given above with $P \geq 3$, in preference to criterion (i).

Ramamurty (see Ref.) in his study of pentad rainfall at 168 stations of India and Pakistan, has investigated into the dates of onset and withdrawal of rainy season at these stations on the basis of significant changes in pentad rainfall. The criteria for significant changes adopted by him were those laid down by Crowe (1933) or by Savur (1936-37 and 1937). These criteria are based on the significant changes of median pentad rainfall. It may be mentioned that in respect of the stations considered in the present study, the pentads of rainy period arrived at by him, and those arrived at during the present study on the basis of criterion (ii) given earlier, are in general agreement.

Utilizing criterion (ii), pentads were selected for each station from amongst pentads 31 to 67 covering the period June to November (southwest and northeast monsoon) and rainfall distribution was considered for each of these.

### 3.2. Testing for normal distribution

It is well known that distribution of daily rain is very far from normal and it does not appear that pentad rainfall may be normally distributed. It was, however, decided to carry out tests of significance of skewness and kurtosis of the frequency distribution of pentad rainfall and also Chi-square tests for normality of pentad rainfall at all the stations under consideration to get an idea of the extent of departure from normality.

The estimates of Fishers' (1932) measures of skewness and of Kurtosis $g_1$, $g_2$ respectively, of pentad rainfall distribution were computed by using the expressions for consistent and unbiased estimates of the second, third and fourth moments of the distribution as given by Cramer's (1946). Significance of $g_1$, $g_2$ was tested by utilising the exact expressions, for mean and variance, first obtained by Fisher (1930) and later by Cramer (loc. cit.).

Significance of skewness, kurtosis and of the frequency distribution of pentad rainfall was tested. The null hypothesis $H_0$ that the distribution of pentad rainfall was not different from normal distribution was tested against the alternative hypothesis $H_1$ that it was different from the normal distribution. The results of these tests show clearly that (i) for 90 per cent of the pentads skewness and Chi-square values are significant at 1 per cent level and (ii) for 70 to 80 per cent of the pentads,
kurtosis values are significant at 1 per cent level.

Thus the departure of the pentad rainfall distribution from normality is highly significant.

3.3. Testing for Gamma distribution

Next, Gamma distribution function given by

\[ \int_{0}^{\infty} \frac{x^{\gamma-1} e^{-x/\beta}}{\beta^\gamma \Gamma(\gamma)} dx \]

where \( \gamma > 0, \beta > 0 \) was fitted to non-zero pentad rainfall data. \( \gamma \) is the shape parameter and \( \beta \) is the scale parameter. For \( \gamma < 1 \), the frequency curve is reversed J-shaped and for \( \gamma > 1 \), it is bell-shaped. Maximum likelihood estimates \( \hat{\gamma} \) and \( \hat{\beta} \) which are consistent and efficient were obtained as per method given by Thom (1968). The units of \( x \) and \( \beta \) are the same. Thom's method is given below:

\[ M, \text{ the likelihood function} = \prod_{i=1}^{n} \frac{x^{\gamma-1} e^{-x/\beta}}{\beta^\gamma \Gamma(\gamma)} \]

\[ \frac{\partial}{\partial \gamma} \log_w M = \sum_{i=1}^{n} \frac{x_i}{\beta^\gamma} - n \log \beta - n \psi(\gamma) = 0 \]  

(1)

where, \( \psi(\gamma) = \frac{\partial}{\partial \gamma} \log_w \Gamma(\gamma) \)

\[ \frac{\partial}{\partial \beta} \log_w M = \frac{1}{\beta^2} \sum_{i=1}^{n} x_i - n(\gamma/\beta) = 0 \]

\[ = A, \text{ the arithmetic mean} \]

(2)

Elimination of \( \beta \) from (2) and (1) gives,

\[ \log_w \gamma - \psi(\gamma) + \log_w (G/A) = 0 \]  

(3)

where,

\[ \log_w G = \frac{1}{n} \sum_{i=1}^{n} \log_w x_i \]

\[ = \log_w \left( (x_1, x_2, ..., x_n)^{1/n} \right) \]

Thus \( G \) is the geometric mean of the \( n \) quantities \( x_1, x_2, ..., x_n \). Now \( G/A < 1 \), hence \( \log_w (G/A) < 0 \). The sign of equality holds when \( x_1 = x_2 = ... = x_n = A \), which is a trivial case.

Equation (3) can be solved for \( \gamma \) by using tables of \( \log_w \gamma - \psi(\gamma) \) prepared by Masuyama and Kuroiwa (1961).

Thom (loc. cit.) has approximated \( [\log_w \gamma - \psi(\gamma)] \) to \( [1/25 + 1/12\gamma^2] \)

Substituting this in (3) and solving for \( \gamma \)

\[ \gamma = \frac{-1 - [1 - (4/3) \log_w (G/A)]^{1/2}}{4 \log_w (G/A)} \]

the other root being negative is inadmissible. By substituting this value of \( \gamma \) in (2), we get \( \beta \).

\[ P_1, \text{ the probability of rain} < x, \]

\[ = Q + (1 - Q) \int_{0}^{x} \frac{x^{\gamma-1} e^{-x/\beta}}{\beta^\gamma \Gamma(\gamma)} dx \]

where \( Q \) is empirical probability of no rain, being the ratio of occasions of zero rain to the total number of occasions and \( x \) includes zero rain also.

\[ P_2, \text{ the probability of rain} \geq x = 1 - P_1 \]

\[ = (1 - Q) \left\{ 1 - \int_{x}^{\infty} \frac{x^{\gamma-1} e^{-x/\beta}}{\beta^\gamma \Gamma(\gamma)} dx \right\} \]

Probabilities of rainfall exceeding specified amounts \( x_1, x_2, x_3, ... \) were computed on CDC 3600 TIFR, Bombay. \( x_1, x_2, x_3, ... \) were expressed as percentages of mean pentad rainfall, \( X \), this mean being mean of rainfall for pentads with non-zero rain. The percentages considered are 20 to 200 in steps of 20, 200 to 300 in steps of 50 and 400. From these probabilities, theoretical frequencies for ranges of rainfall like \(< x_1, x_1 \) to \( x_2, x_2 \) to \( x_3, ... \) were obtained. The corresponding observed or empirical frequencies of the pentad rainfall distribution are also obtained; the Chi-square test of goodness of fit to the distribution is then applied with significance level, \( \alpha = 0.05 \). The results of Chi-square tests for the different pentads for the stations under consideration are tabulated under Table 1 as frequencies of \( P(X^2 > x^2) \), i.e., probability of getting \( x^2 \) exceeding or equalling \( x^2 \), the value actually obtained, over different ranges of probability. This table shows that \( P(X^2 > x^2) \) is less than 0.05 in only 2 per cent of the cases. In other words, \( x_1^2 > x_{0.05}^2 \) for appropriate degrees of
freedom in 2 per cent of the cases only. $X_{0.05}^2$ is the value critical at 5 per cent level. Thus the null hypothesis is not contradicted and pentad rainfall distribution is not expected to be different from Gamma distribution. This distribution can be used to describe the pentad rainfall distribution.

Histograms of rainfall of randomly selected pentads, one from each of the stations, are shown in Fig. 2. Fitted smooth curves are superimposed over these histograms.

Table 2 gives $N'$, the number of occasions of non-zero rainfall, $z'$, the mean rainfall (in cm) on occasions of non-zero rain, $s$, the standard deviation (in cm) of pentad rainfall, $\overline{\gamma}$ and $\overline{\beta}$, the M.L. estimates of the parameters of rainfall distribution. $\overline{\beta}$ is in cm and $\overline{\gamma}$ is dimensionless parameter.

Sajnani (1964) has shown that correlation between pentad rainfall of Bombay station and pentad rainfall of Colaba district is about 0.9 in June, August and September and 0.7 in July. From his study it may be inferred that during southwest monsoon season pentad rainfall at a station could be easily considered as fully representative of an area covered by 40 km (or 25 mile) radius around the station, which area is about one fourth the area of an average-sized district. As such when the interest lies in an area of this size, the parameters of pentad rainfall at the station could be taken to be applicable to this area. Unless the area considered is several times larger than the area represented by a station, the areal pentad rainfall is expected to be Gamma-distributed. Hence it is plausible to expect pentad district rainfall to be Gamma distributed.
3-4. Application of the additive property of Gamma distribution

If rainfall for different pentads are statistically independent it is possible to utilize the additive property of Gamma distribution for obtaining the parameters of Gamma distribution for the combined rainfall of 2 or more pentads. From the Gamma parameters of the combined rainfall distribution and Pearson's (1934) Tables of Incomplete Gamma Function or the nomograms prepared by Barger and by Thom, both of which are included in a paper by Barger, Shaw and Dale (1959 a), probabilities of rainfall exceeding specified amounts during the combined 5-day periods can be obtained. In combining the rainfall distribution for 2 or more pentads it is necessary that the parameter $\beta$ for the pentads is not different. The observed differences may be due to random sampling fluctuations. We must, therefore, check for significance the difference in the parameters for two pentads. Now Fisher (1941) has shown that M.L. estimates of parameters are normally distributed for large samples and Thom (loc. cit.) has given the following expressions for variance for M.L. estimates from large samples,

$$\text{Var} (\xi) = \frac{1}{N} \left[ \xi \psi' (\xi) - 1 \right]$$
$$\text{Var} (\beta) = \beta^2 \psi' \psi / N \left[ \xi \psi' (\xi) - 1 \right]$$

Digamma, $\psi (\xi)$ and trigamma $\psi' (\xi)$ functions have been tabulated by Davis (1933). By utilising these, Var $\xi$ and Var $\beta$ can be computed. Making an assumption that estimates or M.L. parameters obtained from samples of size 60 are normally distributed we can obtain 90 per cent confidence limits for $\beta$. If the $\beta$ values for the pentads to be combined lie within the 90 per cent confidence limits of $\beta$ for one of the pentads, we can consider that these $\beta$ values are not different and that these pentads can be combined. In this case, the $\beta$ values for the pentads to be combined can be averaged as suggested by Shaw, Barger and Dale (loc. cit.) and Friedman and Janes (loc. cit.). For the combined period, the Gamma parameter is the sum of $\xi$ values for the pentads and the beta parameter is the average of the $\beta$ parameters for the pentads. Masuyamma and Kuroiwa (1951) have tabulated the functions $\xi / [\xi \psi (\xi) - 1]$ and $\psi (\xi) / [\xi \psi (\xi) - 1]$ for different values of $\xi$. Utilising these values of $\psi (\xi) / [\xi \psi (\xi) - 1]$, 90 per cent or any other desired confidence limits on $\beta$ can be obtained.

A separate study has been undertaken to find out if pentad rainfall amounts during monsoon are statistically independent.

3-5. Reduction to exponential distribution

When $\xi = 1$, Gamma distribution, reduces to the special case of exponential distribution. Considering sample of size 60 as large and utilising the table of $\xi / [\xi \psi (\xi) - 1]$ given by Masuyamma and Kuroiwa (1951), the standard error of $\xi$ and 95 per cent confidence limits on $\xi$ have been computed and are given under Table 3 for $\xi = 0.70$ to 1.70. If it is desired to know whether rainfall distribution for any particular pentad is not significantly different from the exponential distribution, $\xi$ value for the pentad is referred to in Table 3. If the corresponding 95 per cent confidence limits on $\xi$ include 1, it may be inferred that the rainfall distribution for the pentad is not significantly different from the exponential distribution. Table 3 suggests that when $0.80 < \xi > 1.45$, the distribution is not significantly different from the exponential distribution. Table 2 shows that some of the values of $\xi$ lie within this range and as such for these pentads rainfall distribution is not likely to be significantly different from exponential distribution.

3-6. Utilisation of Gamma parameters

The values of Gamma parameters for pentad rainfall distribution can be used for computation of probabilities of rainfall exceeding specified amounts as required for planning agricultural activities in the various stages of the crop. The probabilities can be computed by using the Tables of Incomplete Gamma Function (op. cit.) or nomograms of Barger (op. cit.) and of Thom (op. cit.). It is necessary to obtain beforehand the requirements of minimum or maximum rainfall for carrying out the requisite agricultural operations. If any activity covers more than 5 days, requisite number of 5-day periods may be combined, Gamma parameters may be obtained for the combined periods and then requisite rainfall probabilities may be worked out. If agricultural operations are being planned for the district as a whole, Gamma parameters for pentad district rainfall may be worked out and from these, requisite rainfall probabilities may be computed.

4. Conclusion

Gamma distribution can be used to describe pentad rainfall at an Indian station during the southwest and northeast monsoon season.

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### TABLE 1

Frequencies for different ranges of probabilities of getting $\chi^2 > \chi^2_{p}$ for appropriate degrees of freedom

Chi-square test for null hypothesis of five-day rainfall distribution being not different from Gamma distribution

| Station                  | 0.01—0.05 | 0.05—0.09 | 0.10—0.25 | 0.26—0.50 | 0.51—0.75 | >0.75 | Total |
|--------------------------|------------|------------|------------|------------|------------|-------|-------|
| Ahmedabad                | 1          | 0          | 0          | 6          | 8          | 1     | 1     | 17    |
| Allahabad                | 0          | 0          | 0          | 0          | 3          | 3     | 7     | 19    |
| Bombay (Colaba)          | 0          | 1          | 7          | 5          | 6          | 4     | 1     | 21    |
| Calcutta (Allipore)      | 0          | 0          | 2          | 6          | 9          | 6     | 2     | 25    |
| Cochin                   | 1          | 0          | 3          | 7          | 12         | 8     | 4     | 35    |
| Jaipur                   | 0          | 0          | 3          | 4          | 4          | 3     | 0     | 14    |
| Madras                   | 0          | 1          | 3          | 10         | 6          | 9     | 6     | 25    |
| Nagaon                   | 0          | 0          | 3          | 6          | 7          | 2     | 5     | 23    |
| Port Blair               | 0          | 2          | 3          | 7          | 10         | 7     | 8     | 37    |
| Simla                    | 0          | 0          | 4          | 3          | 6          | 8     | 2     | 23    |
| Visakhapatnam            | 0          | 0          | 4          | 2          | 10         | 6     | 5     | 27    |
| **Total**                | **2**      | **4**      | **32**     | **59**     | **81**     | **61** | **40** | **279** |

### TABLE 2

Parameters of pentad rainfall during monsoon

| Pentad No. | $\nu$ | $\bar{X}$ (cm) | $s$ (cm) | $\nu$ (cm) | $\beta$ (cm) |
|------------|-------|----------------|----------|------------|---------------|
| 34         | 40    | 1.88           | 1.88     | 0.718      | 2.769         |
| 35         | 7     | 2.74           | 3.79     | 0.933      | 4.321         |
| 36         | 44    | 3.49           | 3.53     | 0.957      | 3.561         |
| 37         | 46    | 6.25           | 6.63     | 0.764      | 8.319         |
| 38         | 56    | 5.41           | 6.02     | 0.885      | 6.124         |
| 39         | 53    | 5.77           | 7.09     | 0.702      | 8.219         |
| 40         | 54    | 4.70           | 5.31     | 0.617      | 7.244         |
| 41         | 54    | 5.41           | 0.88     | 0.592      | 9.121         |
| 42         | 53    | 8.94           | 16.28    | 0.656      | 13.657        |
| 43         | 56    | 4.81           | 4.57     | 0.767      | 6.022         |
| 35         | 44    | 3.29           | 5.72     | 0.699      | 5.441         |
| 36         | 49    | 3.00           | 3.94     | 0.738      | 4.064         |
| 37         | 56    | 4.24           | 5.51     | 0.759      | 5.575         |
| 38         | 55    | 4.85           | 5.74     | 0.716      | 6.690         |
| 39         | 56    | 4.88           | 4.78     | 0.833      | 5.333         |
| 40         | 54    | 6.15           | 4.95     | 1.419      | 4.329         |
| 41         | 57    | 4.27           | 4.04     | 0.950      | 4.501         |
| 42         | 55    | 8.15           | 5.41     | 1.200      | 5.121         |
| 43         | 79    | 5.66           | 1.05     | 1.056      | 5.118         |
| 72         | 45    | 6.19           | 7.75     | 0.602      | 9.139         |
| 33         | 57    | 8.10           | 8.39     | 0.872      | 9.301         |
| 34         | 58    | 9.86           | 7.09     | 0.786      | 12.634        |
| 75         | 7     | 12.68          | 11.33    | 1.037      | 16.104        |
| 36         | 61    | 5.61           | 16.58    | 0.708      | 16.927        |
| 37         | 60    | 11.46          | 12.29    | 0.957      | 12.482        |
| 39         | 60    | 12.17          | 11.46    | 1.041      | 12.037        |
| 41         | 60    | 10.21          | 10.57    | 0.843      | 7.623         |
| 42         | 60    | 9.47           | 10.36    | 1.193      | 9.568         |
| 43         | 60    | 9.53           | 10.69    | 0.995      | 9.568         |
### TABLE 2 (contd)

Parameters of pentad rainfall during monsoon

| Pentad No. | $N'$ | $X'$ (cm) | $\sigma$ (cm) | $\xi$ | $\beta$ (cm) |
|------------|------|-----------|---------------|------|-------------|
| **Calcutta (Alipore)**
| 31 | 45 | 3.48 | 5.89 | -665 | 5.229 |
| 32 | 51 | 4.70 | 6.43 | -844 | 5.629 |
| 33 | 53 | 5.13 | 5.05 | -834 | 6.122 |
| 34 | 57 | 6.32 | 7.59 | -1049 | 6.651 |
| 35 | 60 | 5.10 | 5.16 | -1044 | 4.903 |
| 36 | 59 | 5.54 | 6.16 | -1293 | 4.385 |
| 37 | 60 | 4.37 | 4.29 | -1024 | 4.290 |
| 38 | 60 | 5.97 | 5.77 | -1374 | 4.346 |
| 39 | 60 | 4.70 | 3.23 | -1725 | 2.437 |
| 40 | 60 | 5.03 | 5.05 | -1382 | 4.445 |
| 41 | 60 | 5.82 | 5.11 | -1737 | 3.297 |
| 42 | 60 | 6.17 | 6.10 | -1289 | 4.867 |
| 43 | 60 | 5.03 | 4.47 | -1695 | 3.089 |
| **Ahmedabad**
| 44 | 53 | 4.12 | 4.55 | -611 | 6.734 |
| 45 | 50 | 5.08 | 5.46 | -647 | 7.622 |
| 46 | 53 | 3.40 | 4.95 | -650 | 5.183 |
| 47 | 54 | 2.57 | 2.95 | -699 | 3.683 |
| 48 | 55 | 4.52 | 4.78 | -725 | 6.249 |
| 49 | 49 | 4.19 | 5.56 | -517 | 8.190 |
| 50 | 45 | 4.32 | 5.99 | -533 | 7.780 |
| **Allahabad**
| 44 | 59 | 6.69 | 5.79 | -1097 | 6.375 |
| 45 | 58 | 5.41 | 4.65 | -1072 | 6.030 |
| 46 | 57 | 4.39 | 4.92 | -817 | 5.864 |
| 47 | 57 | 5.64 | 8.99 | -901 | 6.556 |
| 48 | 54 | 4.65 | 5.65 | -924 | 5.023 |
| 49 | 55 | 4.72 | 4.44 | -1080 | 4.329 |
| 50 | 52 | 5.74 | 6.76 | -932 | 6.163 |
| 51 | 53 | 3.99 | 5.00 | -71.3 | 5.657 |
| 52 | 44 | 4.37 | 4.55 | -947 | 4.690 |
| 53 | 40 | 3.05 | 2.79 | -7.6 | 4.059 |
| **Bombay (Colaba)**
| 44 | 60 | 6.43 | 8.35 | -891 | 7.324 |
| 45 | 59 | 5.03 | 5.00 | -931 | 5.409 |
| 46 | 60 | 5.08 | 7.09 | -892 | 5.034 |
| 47 | 59 | 5.00 | 5.49 | -1083 | 4.676 |
| 48 | 60 | 6.43 | 9.30 | -795 | 8.063 |
| 49 | 60 | 5.99 | 7.32 | -764 | 7.333 |
| 50 | 59 | 5.18 | 5.20 | -840 | 6.166 |
| 51 | 57 | 5.23 | 11.33 | -985 | 7.971 |
| 52 | 57 | 5.76 | 7.62 | -933 | 7.568 |
| 53 | 55 | 5.09 | 8.03 | -924 | 8.940 |
| 54 | 53 | 4.04 | 9.27 | -498 | 8.135 |
| 55 | 49 | 4.19 | 8.97 | -600 | 7.026 |
| **Calcutta (Allipore)**
| 44 | 59 | 6.05 | 6.10 | 1.592 | 3.387 |
| 45 | 60 | 5.64 | 4.52 | 1.430 | 3.949 |
| 46 | 59 | 4.70 | 4.67 | 1.489 | 3.690 |
| 47 | 59 | 5.00 | 3.61 | 1.740 | 2.827 |
| 48 | 60 | 4.73 | 4.22 | 1.297 | 3.660 |
| 49 | 59 | 5.33 | 4.24 | 1.680 | 3.183 |
| 50 | 60 | 4.17 | 5.23 | 1.155 | 3.022 |
| 51 | 60 | 5.94 | 5.16 | 1.377 | 4.097 |
| 52 | 59 | 4.42 | 4.72 | -789 | 5.018 |
| 53 | 56 | 3.83 | 4.34 | -836 | 4.690 |
| 54 | 59 | 4.32 | 5.46 | -991 | 4.399 |
| 55 | 59 | 3.73 | 5.16 | -765 | 4.990 |
TABLE 2 (continued)

Parameters of pentad rainfall during monsoon

| Pentad No. | $N'$ | $\bar{X}'$ (cm) | $s$ (cm) | $\bar{y}$ | $\bar{p}$ (cm) |
|------------|------|-----------------|----------|----------|----------------|
| Jaipur     |      |                 |          |          |                |
| 37         | 43   | 2.52            | 2.72     | -909     | 2.756          |
| 38         | 52   | 2.02            | 2.57     | -821     | 3.147          |
| 39         | 49   | 3.66            | 3.43     | -978     | 3.749          |
| 40         | 50   | 3.94            | 4.27     | -902     | 4.371          |
| 41         | 51   | 4.04            | 3.71     | -920     | 4.707          |
| 42         | 51   | 4.02            | 4.57     | -746     | 6.183          |
| 43         | 55   | 3.48            | 3.80     | -803     | 4.351          |
| Nagpur     |      |                 |          |          |                |
| 32         | 44   | 1.98            | 3.07     | -640     | 3.112          |
| 33         | 51   | 3.38            | 6.05     | -729     | 4.580          |
| 34         | 52   | 3.63            | 6.17     | -814     | 4.468          |
| 35         | 55   | 5.57            | 5.56     | -945     | 5.883          |
| 36         | 56   | 7.11            | 7.30     | -934     | 7.447          |
| 37         | 60   | 6.81            | 6.02     | 1.377    | 4.935          |
| 38         | 60   | 5.82            | 4.90     | 1.168    | 4.971          |
| 39         | 57   | 7.47            | 7.14     | 1.030    | 7.115          |
| 40         | 59   | 5.59            | 5.52     | 1.021    | 5.983          |
| 41         | 57   | 4.90            | 4.02     | -973     | 5.034          |
| 42         | 56   | 6.71            | 5.16     | 1.521    | 4.402          |
| 43         | 59   | 5.26            | 4.47     | -980     | 5.352          |
| Simla      |      |                 |          |          |                |
| 31         | 44   | 1.74            | 1.80     | -927     | 1.837          |
| 32         | 47   | 1.85            | 2.04     | -830     | 2.233          |
| 33         | 46   | 2.29            | 3.06     | -870     | 2.619          |
| 34         | 51   | 3.00            | 3.06     | 1.076    | 2.779          |
| 35         | 57   | 3.33            | 3.07     | 1.032    | 3.162          |
| 36         | 55   | 4.47            | 5.28     | -735     | 3.683          |
| 37         | 55   | 5.13            | 4.42     | 1.304    | 3.924          |
| 38         | 58   | 6.13            | 4.57     | 1.707    | 3.767          |
| 39         | 60   | 6.75            | 5.00     | 1.734    | 3.894          |
| 40         | 59   | 8.26            | 5.39     | 1.843    | 5.029          |
| 41         | 60   | 8.83            | 4.07     | 2.078    | 3.284          |
| 42         | 60   | 7.26            | 5.23     | 1.964    | 3.093          |
| 43         | 60   | 7.49            | 4.95     | 2.034    | 2.935          |
| Jaipur     |      |                 |          |          |                |
| 44         | 51   | 3.93            | 4.82     | -743     | 5.022          |
| 45         | 43   | 5.13            | 5.46     | -860     | 5.904          |
| 46         | 45   | 3.94            | 5.31     | -914     | 6.408          |
| 47         | 52   | 3.61            | 4.19     | -893     | 4.034          |
| 48         | 46   | 4.78            | 4.62     | -780     | 6.828          |
| 49         | 50   | 3.40            | 4.59     | -589     | 5.890          |
| 50         | 41   | 2.17            | 2.09     | -925     | 2.337          |
| Nagpur     |      |                 |          |          |                |
| 44         | 59   | 4.98            | 5.46     | -863     | 5.781          |
| 45         | 57   | 3.94            | 4.59     | -732     | 6.592          |
| 46         | 54   | 4.34            | 4.32     | 1.012    | 4.290          |
| 47         | 50   | 5.06            | 4.99     | -780     | 6.499          |
| 48         | 60   | 4.83            | 4.52     | -910     | 6.203          |
| 49         | 50   | 4.95            | 4.98     | -817     | 6.990          |
| 50         | 60   | 4.22            | 3.63     | 1.047    | 4.011          |
| 51         | 55   | 4.94            | 4.11     | -971     | 4.166          |
| 52         | 55   | 3.31            | 4.05     | -800     | 4.382          |
| 53         | 52   | 2.74            | 2.24     | 1.148    | 2.380          |
| 54         | 46   | 3.71            | 3.51     | -756     | 4.895          |
# TABLE 2 (contd)

## Parameters of pentad rainfall during monsoon

| Pentad No. | \( N' \) | \( \bar{X} \) (cm) | \( s \) (cm) | \( \bar{S} \) | \( \bar{\beta} \) (cm) |
|------------|---------|-----------------|----------|-----------|-----------------|
| **Simla**  |         |                 |          |           |                 |
| 44         | 60      | 6.83            | 5.06     | 1.772     | 3.861           |
| 45         | 60      | 7.03            | 5.46     | 1.776     | 4.293           |
| 46         | 58      | 7.72            | 5.79     | 1.684     | 4.539           |
| 47         | 58      | 7.10            | 6.00     | 1.680     | 4.567           |
| 48         | 58      | 5.56            | 4.90     | 1.595     | 3.995           |
| 49         | 58      | 5.18            | 4.27     | 1.485     | 4.390           |
| 50         | 56      | 3.76            | 3.35     | 1.081     | 3.455           |
| 51         | 55      | 4.21            | 4.01     | 0.886     | 4.745           |
| 52         | 47      | 4.67            | 4.32     | 0.925     | 4.999           |
| 53         | 44      | 3.10            | 3.38     | 0.761     | 4.994           |
| **Cochin** |         |                 |          |           |                 |
| 31         | 60      | 10.12           | 8.19     | 1.135     | 8.938           |
| 32         | 60      | 13.56           | 8.28     | 1.895     | 7.160           |
| 33         | 60      | 14.20           | 9.07     | 2.355     | 6.033           |
| 34         | 60      | 11.12           | 7.24     | 2.365     | 4.709           |
| 35         | 60      | 11.48           | 6.96     | 2.022     | 5.677           |
| 36         | 60      | 12.45           | 8.36     | 1.954     | 6.305           |
| 37         | 60      | 10.72           | 6.30     | 2.501     | 4.290           |
| 38         | 60      | 10.21           | 6.78     | 1.902     | 6.368           |
| 39         | 60      | 9.73            | 7.26     | 1.794     | 5.537           |
| 40         | 60      | 9.63            | 7.34     | 1.382     | 6.465           |
| 41         | 59      | 10.34           | 8.66     | 1.426     | 7.254           |
| 42         | 60      | 8.31            | 5.64     | 2.024     | 4.097           |
| 43         | 60      | 7.49            | 5.87     | 1.239     | 5.812           |
| **Madras** |         |                 |          |           |                 |
| 33         | 40      | 1.94            | 1.07     | 0.777     | 1.194           |
| 34         | 42      | 1.04            | 1.17     | 0.782     | 1.331           |
| 35         | 50      | 1.30            | 1.40     | 0.933     | 1.302           |
| 36         | 51      | 1.22            | 1.32     | 0.735     | 1.674           |
| 37         | 58      | 1.09            | 1.42     | 0.910     | 1.196           |
| 38         | 53      | 1.47            | 1.63     | 0.816     | 1.791           |
| 39         | 53      | 1.27            | 1.65     | 0.828     | 1.834           |
| 40         | 52      | 1.75            | 2.21     | 0.862     | 2.032           |
| 41         | 56      | 1.73            | 1.88     | 0.940     | 1.836           |
| 42         | 54      | 2.24            | 2.97     | 0.834     | 2.672           |
| 43         | 54      | 1.55            | 1.72     | 0.873     | 1.769           |
| **Port Blair** |      |                 |          |           |                 |
| 31         | 53      | 10.30           | 8.74     | 1.208     | 8.580           |
| 32         | 54      | 8.76            | 7.72     | 1.197     | 7.361           |
| 33         | 54      | 8.97            | 6.15     | 1.708     | 5.255           |
| 34         | 54      | 8.09            | 6.88     | 1.523     | 5.584           |
| 35         | 54      | 9.30            | 7.37     | 1.291     | 7.739           |
| 36         | 54      | 8.71            | 8.23     | 1.258     | 6.924           |
| 37         | 54      | 6.15            | 5.49     | 1.205     | 4.714           |
| 38         | 54      | 6.20            | 6.12     | 0.927     | 6.703           |
| 39         | 53      | 6.95            | 5.84     | 1.169     | 5.926           |
| 40         | 54      | 6.22            | 5.21     | 1.070     | 5.850           |
| 41         | 54      | 6.93            | 7.54     | 0.888     | 7.803           |
| 42         | 54      | 6.15            | 5.21     | 1.450     | 4.155           |
| 43         | 54      | 7.34            | 7.04     | 1.457     | 4.938           |

*The study for this station is based on 55 years’ data*
## Table 2 (con't)

Parameters of pentad rainfall during monsoon

| Pentad No. | \(N'\) | \(\bar{X}'\) (cm) | \(s\) (cm) | \(\hat{L}\) (cm) | \(\hat{P}\) (cm) |
|------------|-------|----------------|--------|------------|------------|
| 32         | 46    | 1.80           | 2.30   | 0.89       | 3.056      |
| 33         | 41    | 1.65           | 1.83   | 0.619      | 2.713      |
| 34         | 48    | 3.23           | 4.24   | 0.651      | 4.928      |
| 35         | 47    | 2.03           | 3.43   | 0.749      | 2.095      |
| 36         | 52    | 2.36           | 2.24   | 0.944      | 2.499      |
| 37         | 54    | 2.71           | 3.20   | 0.756      | 3.599      |
| 38         | 50    | 2.11           | 2.13   | 0.999      | 2.118      |
| 39         | 53    | 2.74           | 2.74   | 0.761      | 2.845      |
| 40         | 50    | 1.75           | 2.40   | 0.914      | 1.918      |
| 41         | 52    | 2.39           | 4.32   | 0.775      | 3.086      |
| 42         | 55    | 1.93           | 1.96   | 1.023      | 1.890      |
| 43         | 54    | 2.77           | 3.48   | 0.718      | 3.863      |
| **Visakhapatnam** | | | | | |
| 44         | 59    | 8.23           | 6.10   | 1.328      | 6.200      |
| 45         | 60    | 6.10           | 5.87   | 1.464      | 4.158      |
| 46         | 65    | 6.55           | 6.93   | 1.168      | 5.913      |
| 47         | 59    | 6.29           | 3.94   | 1.734      | 3.226      |
| 48         | 60    | 4.65           | 5.00   | 0.828      | 5.494      |
| 49         | 59    | 3.20           | 3.23   | 0.987      | 3.244      |
| 50         | 59    | 3.96           | 4.65   | 0.839      | 4.785      |
| 51         | 59    | 4.19           | 4.32   | 0.894      | 6.025      |
| 52         | 55    | 3.25           | 3.27   | 0.861      | 3.787      |
| 53         | 56    | 4.06           | 3.43   | 1.190      | 3.414      |
| 54         | 56    | 5.51           | 5.51   | 1.057      | 5.217      |
| 55         | 57    | 6.25           | 6.10   | 0.785      | 8.260      |
| **Cochin** | | | | | |
| 44         | 52    | 2.54           | 2.34   | 1.118      | 2.271      |
| 45         | 52    | 1.88           | 1.96   | 1.051      | 1.778      |
| 46         | 50    | 2.13           | 2.74   | 0.730      | 2.032      |
| 47         | 52    | 2.97           | 3.12   | 0.803      | 3.703      |
| 48         | 54    | 1.68           | 1.70   | 1.128      | 1.493      |
| 49         | 56    | 2.46           | 2.02   | 0.633      | 3.894      |
| 50         | 50    | 2.08           | 1.91   | 1.027      | 2.040      |
| 51         | 51    | 2.56           | 2.67   | 0.926      | 2.764      |
| 52         | 48    | 2.51           | 3.22   | 0.646      | 3.884      |
| 53         | 51    | 2.51           | 2.52   | 0.765      | 3.294      |
| 54         | 48    | 1.73           | 2.02   | 0.668      | 2.637      |
| 55         | 47    | 2.09           | 2.41   | 1.156      | 2.329      |
| **Madras** | | | | | |
| 44         | 54    | 6.91           | 7.06   | 0.924      | 10.018     |
| 45         | 53    | 6.88           | 6.53   | 1.351      | 5.100      |
| 46         | 54    | 5.49           | 4.90   | 1.249      | 4.387      |
| 47         | 54    | 6.58           | 5.00   | 1.305      | 4.714      |
| 48         | 54    | 7.44           | 5.02   | 1.469      | 5.033      |
| 49         | 53    | 7.90           | 6.35   | 1.185      | 6.668      |
| 50         | 54    | 8.20           | 6.81   | 1.228      | 6.668      |
| 51         | 52    | 7.95           | 6.22   | 1.323      | 6.012      |
| 52         | 54    | 7.29           | 5.44   | 0.533      | 4.639      |
| 53         | 53    | 8.07           | 7.62   | 1.058      | 8.213      |
| 54         | 53    | 7.70           | 5.87   | 1.929      | 5.049      |
| 55         | 54    | 6.33           | 5.28   | 1.063      | 5.959      |
| **Fort Blair** | | | | | |


### TABLE 2 (contd)

#### Parameters of pentad rainfall during monsoon

| Pentad No. | $N'$ | $\bar{X}$ (cm) | $s$ (cm) | $\bar{\bar{X}}$ | $\bar{\bar{s}}$ (cm) |
|------------|------|----------------|----------|-----------------|---------------------|
| **Visakhapatnam** | | | | | |
| 44 | 56 | 1.85 | 2.95 | 1.070 | 2.835 |
| 45 | 52 | 1.62 | 1.83 | 0.901 | 2.062 |
| 46 | 55 | 1.85 | 2.83 | 0.776 | 3.216 |
| 47 | 47 | 3.18 | 3.63 | 0.773 | 2.184 |
| 48 | 57 | 2.49 | 2.39 | 1.142 | 2.454 |
| 49 | 52 | 2.49 | 2.39 | 0.940 | 5.273 |
| 50 | 53 | 2.74 | 2.36 | 0.748 | 5.716 |
| 51 | 56 | 1.91 | 2.49 | 0.943 | 5.372 |
| 52 | 54 | 3.40 | 5.26 | 0.719 | 4.736 |
| 53 | 56 | 3.40 | 3.26 | 0.753 | 5.273 |
| 54 | 53 | 4.84 | 4.85 | 0.727 | 5.273 |
| 55 | 52 | 3.84 | 4.95 | 5.913 | 5.987 |
| **Cochin** | | | | | |
| 56 | 54 | 5.01 | 6.60 | 0.950 | 5.913 |
| 57 | 57 | 5.69 | 5.89 | 0.897 | 5.334 |
| 58 | 53 | 6.46 | 5.94 | 1.209 | 5.334 |
| 59 | 58 | 5.08 | 4.39 | 0.860 | 6.076 |
| 60 | 67 | 5.18 | 4.83 | 0.863 | 6.076 |
| 61 | 56 | 5.49 | 5.03 | 1.127 | 4.882 |
| 62 | 57 | 3.63 | 2.79 | 1.019 | 4.768 |
| 63 | 50 | 4.57 | 4.62 | 0.960 | 6.109 |
| 64 | 47 | 5.21 | 4.62 | 0.763 | 4.768 |
| 65 | 46 | 3.94 | 4.68 | 5.964 | 5.964 |
| **Madras** | | | | | |
| 56 | 48 | 4.53 | 7.04 | 0.963 | 8.999 |
| 57 | 46 | 4.32 | 5.07 | 0.912 | 7.084 |
| 58 | 45 | 4.42 | 5.66 | 0.912 | 7.118 |
| 59 | 50 | 5.97 | 7.44 | 0.664 | 8.837 |
| 60 | 48 | 7.23 | 5.81 | 0.902 | 8.837 |
| 61 | 50 | 9.35 | 5.28 | 1.033 | 9.559 |
| 62 | 44 | 10.93 | 9.27 | 0.921 | 11.890 |
| 63 | 46 | 9.12 | 8.09 | 0.925 | 9.855 |
| 64 | 45 | 7.31 | 6.86 | 1.013 | 9.855 |
| 65 | 42 | 7.21 | 6.86 | 0.768 | 9.814 |
| 66 | 43 | 4.47 | 5.00 | 0.846 | 8.543 |
| 67 | 41 | 5.59 | 4.70 | 0.836 | 5.359 |
| **Port Blair** | | | | | |
| 56 | 54 | 6.88 | 5.08 | 1.168 | 5.946 |
| 57 | 54 | 5.26 | 5.18 | 1.046 | 5.034 |
| 58 | 52 | 4.19 | 3.61 | 1.360 | 3.076 |
| 59 | 53 | 3.84 | 3.43 | 1.343 | 2.969 |
| 60 | 54 | 4.12 | 4.87 | 0.866 | 4.759 |
| 61 | 50 | 4.22 | 4.34 | 0.774 | 6.453 |
| 62 | 53 | 4.27 | 4.93 | 0.910 | 4.679 |
| 63 | 51 | 4.72 | 6.88 | 0.764 | 6.988 |
| 64 | 50 | 4.50 | 4.72 | 0.764 | 6.377 |
| 65 | 48 | 4.80 | 4.82 | 0.888 | 5.596 |
| 66 | 49 | 4.91 | 4.82 | 0.689 | 5.682 |
| 67 | 39 | 4.80 | 4.91 | 0.934 | 5.138 |
| **Visakhapatnam** | | | | | |
| 56 | 44 | 4.32 | 5.97 | 0.500 | 7.839 |
| 57 | 47 | 4.17 | 4.78 | 0.743 | 5.906 |
| 58 | 44 | 5.64 | 5.77 | 0.861 | 6.029 |
TABLE 3

Standard error of $\hat{\gamma}$ ($\sigma_\hat{\gamma}$) and 95 per cent confidence limits of $\hat{\gamma}$

| $\hat{\gamma}$ | $\sigma_\hat{\gamma}$ | 95% confidence limits for $\hat{\gamma}$ | $\hat{\gamma}$ | $\sigma_\hat{\gamma}$ | 95% confidence limits for $\hat{\gamma}$ |
|----------------|-----------------------|----------------------------------------|----------------|-----------------------|----------------------------------------|
| 0.70           | 0.109                 | 0.486–0.914                            | 1.25           | 0.205                 | 0.849–1.651                            |
| 0.75           | 0.117                 | 0.520–0.989                            | 1.20           | 0.214                 | 0.880–1.720                            |
| 0.80           | 0.126                 | 0.555–1.047                            | 1.35           | 0.223                 | 0.914–1.756                            |
| 0.85           | 0.134                 | 0.587–1.113                            | 1.40           | 0.231                 | 0.940–1.884                            |
| 0.90           | 0.143                 | 0.620–1.180                            | 1.45           | 0.240                 | 0.970–1.921                            |
| 0.95           | 0.152                 | 0.652–1.218                            | 1.50           | 0.249                 | 1.011–1.980                            |
| 1.00           | 0.161                 | 0.686–1.255                            | 1.55           | 0.258                 | 1.044–2.056                            |
| 1.05           | 0.169                 | 0.718–1.382                            | 1.60           | 0.267                 | 1.077–2.123                            |
| 1.10           | 0.178                 | 0.751–1.449                            | 1.65           | 0.276                 | 1.109–2.191                            |
| 1.15           | 0.187                 | 0.783–1.517                            | 1.70           | 0.285                 | 1.140–2.259                            |
| 1.20           | 0.196                 | 0.815–1.584                            |               |                       |                                        |

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