Quark spin content of SU(3) light and singly heavy baryons

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We investigate the quark spin content of the SU(3) baryon octet, decuplet, antitriplet, and sextet in a pion mean-field approach or the chiral quark-soliton model, considering the 1/\(N_c\) rotational corrections and SU(3) symmetry breaking effects. We compare the present results with those from various theoretical works and lattice QCD.

I. INTRODUCTION

Understanding the spin structure of the nucleon has been one of the most intriguing issues well over the decades. In quantum chromodynamics (QCD), the spin of the nucleon is known to be made of the intrinsic spin and orbital angular momentum of its partons (quarks and gluons). It attracts considerable attention and becomes one of the most important missions of the upcoming Electron-Ion Collider (EIC) project. A series of experiments on the spin structure has been carried out by measuring the spin asymmetry in the polarized lepton-nucleon deep inelastic scattering (DIS) since the 1980s and has been renewed by consecutive experiments [8–11]. In these experiments, the spin structure function \(g_1\) of the nucleon was extensively measured. The first moment of the structure function \(g_1\) is related to the singlet axial charge \(g_A^{(0)}\). The experimental results imply that only a small fraction of the nucleon spin is carried by quarks. Thus, the Ellis-Jaffe sum rule [12], i.e. \(\Delta s = \Delta \bar{s} = 0\), does not hold anymore. It means that the sea quarks play a crucial role in understanding the spin structure of the nucleon (\(\Delta s \sim -0.10\) [13]). To explore the antiquark flavor asymmetry, the longitudinally polarized semi-inclusive deep inelastic scattering (SIDIS) has been also considered, which enables one to measure the sea quark contributions isolated from the valence quark ones. In addition to the sea quark contributions, one of the ambiguities arises from the QCD axial anomaly \(U(1)_A\) of the nucleon. In contrast to the traditional parton model, the gluon spin contributions are mixed with the quark one, so that they cannot be treated independently. The values of the isotriplet and octet axial charges, \(g_A^{(3)} = 1.2754 \pm 0.0013\) [14] and \(g_A^{(8)} = 0.58 \pm 0.03\) [15], are respectively extracted from the neutron \(\beta\)-decay and the hyperon semileptonic decays (HSDs) with SU(3) symmetry imposed. There are recent studies on the isovector axial charge [16] and flavor octet charge [17]. Note that the value of the \(g_A^{(8)}\) is still under debate. One also has to keep in mind that while they are scale-independent, the isosinglet axial charge depends on the renormalization scale. With knowledge of the isosinglet and octet axial charges, the scale-dependent \(g_A^{(0)}\) was estimated from the structure function \(g_1\). The widely accepted value of the singlet axial charge is given as \(g_A^{(0)} \sim 0.33\) [23]. There have been extensive studies on the axial charge of the baryon octet: for example, chiral perturbation theory [24], the lattice QCD [25–29], a global QCD analysis [30], NNPDF collaboration data [31], and COMPASS data [32]. The axial charges of the \(\Delta\) baryon were also estimated in chiral perturbation theory [33, 34], and those of the baryon octet and decuplet were investigated in the relativistic constituent quark model (RCQM) [35, 36] and the perturbative chiral quark model (PCQM) [37]. Recently, the axial charges of the hyperons and charmed baryons were also obtained from lattice QCD [38, 39].

In the present work, we will investigate the quark spin content of the SU(3) light and singly-heavy baryons within the frame of the chiral quark-soliton model (χQSM) or pion-mean field approach. The model is based on Witten’s seminal idea [39, 40]. He proposed that a light baryon can be viewed as \(N_c\) valence quarks bound by the pion mean field in the large \(N_c\) limit. The χQSM was developed, based on the effective chiral action that was inspired by the QCD instanton vacuum [41, 42]. The axial charges of the nucleon have been studied in both the SU(2) [43–46, 48] and SU(3) χQSMs [45, 48, 55]. The value of the axial charge \(g_A^{(0)}\) derived from the χQSM is comparable to that

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extracted from the experiment. Moreover, the polarization of the strange quark inside the nucleon, $\Delta s$ is in good agreement with the data. The light-cone quantization of the $\chi$QSM having been employed, the same quantities were examined $[21] [56]$. Since there are no gluon degrees of freedom in the $\chi$QSM, the singlet axial charges become the scale dependence. However, it does not mean $g_A^{(0)} = 1$, since the missing contributions to the nucleon spin are attributed to the orbital angular motion of the quarks $[57] [62]$.

While the quark spin content inside the light baryon has been extensively investigated on both the theoretical and experimental sides, only a few works on the spin content of the singly heavy baryons have been carried out $[38]$. Recently, the $\chi$QSM was successfully extended to singly heavy baryons. In the heavy quark mass limit, i.e., $m_Q \to \infty$, a singly heavy baryon is considered as $N_c - 1$ valence quarks bound by the pion mean field. Here, the heavy quark can be taken to be a mere static color source. Accordingly, the allowed SU(3) representations of the lowest-lying colored soliton with spin $J$ are correctly identified as the antitriplet ($3$) with spin $J = 0$ and sextet ($6$) with spin $J = 1$. The color-singlet heavy baryon is then constructed by coupling a heavy quark with spin $J_Q = 1/2$. As a result, the heavy baryon with spin $J'/2$ form one antitriplet ($3$, $J' = 1/2$) and two sextets ($6$, $J' = 1/2$) and ($6$, $J' = 3/2$). The two sextet multiplets are subjected to a hyperfine splitting. Based on the modified pion mean field and the heavy-quark symmetry, various properties of the singly heavy baryons have been described well such as the mass splittings $[63] [65]$, electromagnetic properties $[66] [71]$, strong decays $[72] [73]$, and gravitational form factors $[74]$. Thus, the $\chi$QSM allows one to investigate both the light and singly heavy baryons on an equal footing. In the current work, we want to scrutinize the axial charges of both the light and singly heavy baryons with spin-1/2 and -3/2. We will then discuss the results for the singly heavy baryons compared with those for light baryons.

The present work is organized as follows: In Sec. II, we describe the formalism for the axial charges of both the light and singly heavy baryons within the $\chi$QSM, considering the rotational $1/N_c$ corrections and the effects of flavor SU(3) symmetry breaking. In Sec. III, results for the axial charges and spin contents of the both light and singly heavy baryons are discussed. We summarize the current work and draw conclusions in the last section.

II. AXIAL CHARGES IN THE $\chi$QSM

The axial charges for both the spin-1/2 and -3/2 particles are respectively given by the matrix element of the axial-vector current $A^{(x)}_\mu = \bar{\psi}\gamma_\mu \gamma_5 \lambda^x \psi + \bar{\Psi} \gamma_\mu \gamma_5 \Psi$:

$$
\langle B(p, J'_3) | A^{(x)}_\mu | B(p, J'_3) \rangle = g^{(x)}_A \bar{u}(p, J'_3) \gamma_\mu \gamma_5 u(p, J'_3),
$$

$$
\langle B(p, J'_3) | A^{(5)}_\mu | B(p, J'_3) \rangle = -g^{(5)}_A \bar{u}(p, J'_3) \gamma_\mu \gamma_5 u_3(p, J'_3),
$$

where $p$ and $J'_3$ denote the momentum and the spin polarization of the heavy (light) baryon $B$, respectively. The flavor index runs over $\chi = 0, 3, 8$, and $\lambda^x$ stands for the well-known SU(3) Gell-Mann matrices. We define $\lambda^0$ as $3 \times 3$ unit matrix in the flavor space. The $\psi = (u, d, s)$ and $\Psi$ designate the light-quark and heavy-quark field operators, respectively. Here, one should keep in mind that the heavy quark flavor represents both the charm and bottom quarks. Assuming the heavy quark spin symmetry, we have the degenerate axial charges of the charmed and bottom baryons due to the heavy-quark flavor symmetry. Thus, we consider the singly charmed baryons in the present work. $u(p)$ and $u_3(p)$ are respectively the Dirac spinor and the Rarita-Schwinger spinor that satisfies the subsidiary conditions $\gamma^a u_3(p) = p^a u_3(p) = 0$ $[75]$. These definitions of the axial charges are related to the first moment of the longitudinally polarized quark distributions $q_1$ accessed by the DIS experiment:

$$
g^{(x)}_A = \sum_{q=u,d,s} (\Delta q = \Delta \Sigma), \quad g^{(3)}_A = \Delta u - \Delta d, \quad g^{(8)}_A = \frac{1}{\sqrt{3}} (\Delta u + \Delta d - 2\Delta s).
$$

Here, one has to bear in mind that there are no gluonic degrees of freedom in the $\chi$QSM as mentioned previously, so that we drop the gluon contribution to $g^{(0)}_A$ induced by the U(1) axial anomaly.

In the light-baryon sector, the matrix element of the axial-vector current for the light baryon has been considered in Ref. $[55] [73] [76]$. All dynamical information on the axial-vector properties has been expressed in terms of the collective operators together with the dynamical parameters. They are written by the six dynamical parameters $a_{1...6}$ with the explicit flavor SU(3) symmetry breaking considered. They are reduced to the three dynamical parameters $a_{1,2,3}$ with flavor SU(3) symmetry. The general expressions of the collective operators for the axial-vector charges have been constructed already in previous works $[77] [80]$:}

$$
g^{(x)}_A = a_1 D^{(8)}_{\chi^3} + a_2 D^{(8)}_{\chi^3} J_3 + \frac{a_3}{\sqrt{3}} D^{(8)}_{\chi^3} J_3 + \frac{a_4}{\sqrt{3}} D^{(8)}_{\chi^3} J_3 + a_5 (D^{(8)}_{\chi^3} J_3 + D^{(8)}_{\chi^3} J_3) + a_6 (D^{(8)}_{\chi^3} - D^{(8)}_{\chi^3} J_3),
$$

$$
+ a_6 (D^{(8)}_{\chi^3} J_3 - D^{(8)}_{\chi^3} J_3).
$$

(3)
express our dynamical parameters and $D$. In the exact SU(3) symmetry, baryonic axial-vector amplitudes are given in terms of two reduced matrix elements $\text{Refs. } [55, 76]$:

collective operators between their collective wave functions as follow:

heavy (\( N \)-stable soliton [65]. Thus, in this work, we will concentrate on the axial charges of the light (\( N \)-heavy baryons, the pion mean fields created by the $N_c$ valence quarks are replaced by those with $N_c - 1$ quarks, because the heavy quark inside a singly heavy baryon can be stripped off. The structure of the collective operators is left to be the same. Thus, there is nothing changed from the light-baryon collective operators in Eq. (3) except for the values of the dynamical parameters of which both the light and heavy baryons are listed in the following subsections.

To evaluate the matrix elements of the collective operators, the collective wave functions of both the light and singly heavy baryons should be provided. With $N_Q$ defined as the number of heavy quarks, the wave function of the light-quark state with flavor state (\(Y, T, T'\)) and spin state (\(Y' = -(N_c - N_Q)/3, J, J_3\)) in the flavor SU(3) representation $\nu$ is given by $\psi_{(Y, T, T_3)}(Y', J, J_3)$ in terms of the Wigner rotational $D(R)$ matrices:

$$|B\rangle := \psi_{(Y, T, T_3)}(Y', J, J_3)(R) = \sqrt{\text{dim}(\nu)}(-1)^Q_s[D^{(\nu)}_{(Y, T, T_3)}(-Y', J, -J_3)(R)]^*,$$

(4)

where $\text{dim}(\nu)$ denotes the dimension of the representation $\nu$, and $Q_s$ stands for a charge corresponding to the baryon state $S$, i.e. $Q_s = J_3 + Y'/2$. If one takes $N_Q = 0$, the right hypercharge becomes $Y_R = -Y' = N_c/3$, which corresponds to the well-known light-baryon collective wave function. Furthermore, to construct the heavy-baryon collective wave function, we need to couple this light-quark state to the heavy-quark spinor with the SU(2) Clebsch-Gordan coefficient $C_{J, J_3; J_Q, J_{Q_3}}^{J', J'_3}$. The heavy-baryon collective wave function is then constructed as:

$$|B_{Q}\rangle := \psi_{B_{Q}}^{(\nu)}(R) = \sum_{J_{3}, J_{Q_3}} C_{J, J_3, J_Q, J_{Q_3}}^{J', J'_3} \chi_{J_{Q_3}} \psi_{(Y, T, T_3)}^{(\nu)}(Y', J, J_3)(R),$$

(5)

where $\chi_{J_{Q_3}}$ represents the Pauli spinors. By taking $N_Q = 1$, the right hypercharge becomes $Y_R = -Y' = (N_c - 1)/3$, which corresponds to the singly heavy-baryon collective wave function. Though one can construct the doubly heavy-baryon wave function in the same manner, the corresponding pion mean field is not strong enough to construct the stable soliton [65]. Thus, in this work, we will concentrate on the axial charges of the light ($N_Q = 0$) and singly heavy ($N_Q = 1$) baryons. The axial charges for both the light and heavy baryons can be obtained by sandwiching the collective operators between their collective wave functions as follow:

$$g_A^{(\chi), B_{Q}} = \langle B_{Q}\rangle g_A^{(\chi)} |B\rangle.$$  

(6)

A. Axial charges of the baryon octet and decuplet

All the values of the dynamical parameters for the axial properties in the light-baryon sector are obtained in Refs. [55, 76]:

$$a_1 = -3.74, \quad a_2 = 2.34, \quad a_3 = 0.88.$$  

(7)

In the exact SU(3) symmetry, baryonic axial-vector amplitudes are given in terms of two reduced matrix elements $F$ and $D$, which is drawn from the SU(3) Wigner-Eckart theorem. Since these values $F$ and $D$ are broadly used, we express our dynamical parameters $a_{1,2,3}$ in terms of them:

$$F = -\frac{1}{12} \left( a_1 - \frac{1}{2} a_2 \right) + \frac{1}{24} a_3 = 0.446,$$

$$D = -\frac{3}{20} \left( a_1 - \frac{1}{2} a_2 \right) - \frac{1}{40} a_3 = 0.716,$$

(8)

which is consistent with the results for the $F$ and $D$ presented in Ref. [81].

These results are in agreement with the empirical ones extracted from the experiments [82]: $F = 0.463 \pm 0.008$ and $D = 0.804 \pm 0.008$. There are two different lowest-lying representations of the light baryons: the baryon octet (\(8, J = 1/2\)) and the baryon decuplet (\(10, J = 3/2\)). From Eqs. (6) and (7), we first derive the expressions for the axial charges of the baryon octet in the exact SU(3) symmetry in terms of $F$ and $D$ for the flavor-singlet

$$g_A^{(0), B_8} = 9F - 5D = 0.44,$$

(9)
and for the flavor-triplet
\[ g_A^{(3),N} = 2T_3(F + D), \quad g_A^{(3),A} = 0, \quad g_A^{(3),\Sigma} = 2T_3F, \quad g_A^{(3),\Xi} = 2T_3(F - D), \]
and for the flavor-octet
\[ g_A^{(8),N} = \frac{\sqrt{3}}{3}(3F - D), \quad g_A^{(8),A} = -\frac{2\sqrt{3}}{3}D, \quad g_A^{(8),\Sigma} = \frac{2\sqrt{3}}{3}D, \quad g_A^{(8),\Xi} = -\frac{\sqrt{3}}{3}(3F + D). \]

In the same manner, we derive the axial charges of the baryon decuplet:
\[ g_A^{(0),B_{10}} = 9F - 5D, \quad g_A^{(3),B_{10}} = T_3F, \quad g_A^{(8),B_{10}} = \frac{\sqrt{3}Y}{2}F, \]
where we set all the spin polarization of the baryons to be \( J_3 = 1/2 \) from now on. As discussed in Refs. [48, 50, 72], the dynamical parameters can be expressed in terms of the moments of inertia \( I_1 \) and \( I_2 \) and have their own \( N_c \) factors. While \( a_1 \) consists of both the \( N_c \) and \( N_0 \) order terms, \( a_2 \) and \( a_3 \) are of order \( N_0 \) (see Appendix A). Such a relation can be clearly seen in the non-relativistic (NR) limit, i.e., the small soliton size limit. The dynamical parameters in this limit are found to be
\[ a_1 \xrightarrow{\text{NR}} -(N_c + 2), \quad a_2 \xrightarrow{\text{NR}} 4, \quad a_3 \xrightarrow{\text{NR}} 2. \]

In this limit, we recover the results from the naive quark model for the \( p \) \((J_3 = 1/2)\) and \( \Delta^+ \) \((J_3 = 1/2)\) axial charges
\[ g_A^{(0),p} \xrightarrow{\text{NR}} 1, \quad g_A^{(3),p} \xrightarrow{\text{NR}} \frac{5}{3}, \quad g_A^{(8),p} \xrightarrow{\text{NR}} \frac{1}{\sqrt{3}}, \]
and
\[ g_A^{(0),\Delta^+} \xrightarrow{\text{NR}} 1, \quad g_A^{(3),\Delta^+} \xrightarrow{\text{NR}} \frac{1}{3}, \quad g_A^{(8),\Delta^+} \xrightarrow{\text{NR}} \frac{1}{\sqrt{3}}. \]

### B. Axial charges of singly heavy baryons

In the presence of the \( N_c - 1 \) valence quarks, the pion mean field becomes weaker than that in the light-baryon system, so that all the values of the dynamical parameters related to the axial properties for the singly heavy baryon are also changed. These dynamical parameters are obtained as
\[ a_1 = -3.19, \quad a_2 = 2.98, \quad a_3 = 1.32 \]
in the present model. Keeping the same definition used in Eq. (8), we obtain \( F = 0.446 \) and \( D = 0.671 \) in the heavy-baryon sector. While the value of \( F \) is almost intact, that of \( D \) is slightly changed.

There are three different lowest-lying representations of the singly heavy baryons: the baryon antitriplet \((\bar{3}, J = 0, J_Q = 1/2, J' = 1/2)\), the baryon sextet \((6, J = 1, J_Q = 1/2, J' = 1/2)\) with spin 1\(\overrightarrow{2}\) and the sextet \((6, J = 1, J_Q = 1/2, J' = 3/2)\) with spin 3\(\overrightarrow{2}\). In the case of the antitriplet baryons, their spins are constructed out of light-quark pair with \( J = 0 \) and the heavy quark state with \( J_Q = 1/2 \), which means that there is no light-quark contribution to all the vector and axial-vector quantities, such as the magnetic moments and axial charges of the baryon antitriplet. So, the quark spin contribution to the axial charges of the antitriplet baryons only comes from the heavy quark. Since the results for the baryon antitriplet are trivial, we will not deal with them in this work. The light quarks contribute to the axial charges for the baryon sextet, which dominate over the heavy-quark contributions. The flavor-singlet, \(-\text{triplet}, \) and \(-\text{octet} \) axial charges of the baryon sextet with \( J' = 1/2 \) are given by
\[ g_A^{(0),B_{6,J'=1/2}} = \frac{4}{3}(9F - 5D) - \frac{1}{3}, \quad g_A^{(3),B_{6,J'=1/2}} = \frac{T_3}{15}(27F + 5D), \quad g_A^{(8),B_{6,J'=1/2}} = \frac{\sqrt{3}Y}{30}(27F + 5D). \]

Interestingly, the axial charges of the baryon sextet with \( J' = 3/2 \) can be related to those of the baryon sextet with \( J' = 1/2 \) as follows:
\[ g_A^{(0),B_{6,J'=3/2}} = \frac{1}{2} \left( g_A^{(0),B_{6,J'=1/2}} + 1 \right), \quad g_A^{(x=3,8),B_{6,J'=3/2}} = \frac{1}{2} g_A^{(x=3,8),B_{6,J'=1/2}}. \]
We observe that the light-quark contributions to the axial charges of the baryon sextet with $J' = 1/2$ differ from those with $J' = 3/2$ only by factor 2. Being similar to the light-baryon sector, the $a_1$ consists of both the $(N_c - 1)$ and $N^0_c$ order terms, and $a_2$ and $a_3$ are of order $N^0_c$ (see Appendix A). In the non-relativistic (NR) limit, $a_1$, $a_2$, and $a_3$ are reduced to

$$
a_1 \xrightarrow{\text{NR}} -[(N_c - 1) + 2], \quad a_2 \xrightarrow{\text{NR}} 4, \quad a_3 \xrightarrow{\text{NR}} 2.
$$

Using the values $a_{1,2,3}$ in the NR limit, we show that the $\Sigma^+_{\epsilon}$ and $\Sigma^{++}_{\epsilon}$ axial charges are led to

$$
g^{(0),\Sigma^+_{\epsilon}}_A \xrightarrow{\text{NR}} 1, \quad g^{(3),\Sigma^+_{\epsilon}}_A \xrightarrow{\text{NR}} \frac{4}{3}, \quad g^{(8),\Sigma^+_{\epsilon}}_A \xrightarrow{\text{NR}} \frac{4}{3\sqrt{3}},
$$

and

$$
g^{(0),\Sigma^{++}_{\epsilon}}_A \xrightarrow{\text{NR}} 1, \quad g^{(3),\Sigma^{++}_{\epsilon}}_A \xrightarrow{\text{NR}} \frac{2}{3}, \quad g^{(8),\Sigma^{++}_{\epsilon}}_A \xrightarrow{\text{NR}} \frac{2}{3\sqrt{3}}.
$$

Here the heavy quark makes the flavor-singlet axial charges shifted by $-1/3$ and $1/3$ in the baryon sextets with $J' = 1/2$ and $J' = 3/2$, respectively.

### C. Effects of flavor SU(3) symmetry breaking

In this subsection, we will consider the effects of flavor SU(3) symmetry breaking on the axial charges of both the light and singly heavy baryons. As discussed already, the terms proportional to $a_4, a_5, a_6$ arise from the perturbative treatment of the strange current quark mass, so that the collective wave functions for the soliton are no longer pure state but are admixed with higher representations. Thus, to derive the collective wave functions, we need to diagonalize the collective Hamiltonian with flavor SU(3) symmetry breaking. The collective Hamiltonian is given by

$$
H_{\text{coll}} = H_{\text{sym}} + H_{\text{sh}},
$$

where

$$
H_{\text{sym}} = M_{cl} + \frac{1}{2I_1} \sum_{i=1}^{3} j_i^2 + \frac{1}{2I_2} \sum_{p=4}^{7} j_p^2, \quad H_{\text{sh}} = \alpha D^{(8)}_{s8} + \beta \hat{Y} + \gamma \sqrt{\frac{3}{3}} \sum_{i=1}^{3} D^{(8)}_{si} \hat{j}_i.
$$

Here, $I_1$ and $I_2$ stand for the moments of inertia for the soliton, and $D^{(8)}_{si}$ denotes SU(3) Wigner $D$ functions. The dynamical parameters $\alpha$, $\beta$, and $\gamma$, which arise from flavor SU(3) symmetry breaking, are expressed in terms of the moments of inertia $I_1$ and $I_2$, and the anomalous moments of inertia $K_1$ and $K_2$

$$
\alpha = \left( -\frac{\Sigma_{\pi N}}{3m} + \frac{K_2}{I_2} \right) m_s, \quad \beta = -\frac{K_2}{I_2} m_s, \quad \gamma = 2 \left( \frac{K_1}{I_1} - \frac{K_2}{I_2} \right) m_s,
$$

where $\Sigma_{\pi N}$ stands for the pion-nucleon $\Sigma$ term. By diagonalizing the collective Hamiltonian with flavor SU(3) symmetry breaking and by setting $N_Q = 0$ in the dynamical parameters listed in Appendix A we obtain the following wave functions for baryon octet and decuplet

$$
|B_8\rangle = |8, B\rangle + c^{B}_{10} \langle 10, B| + c^{B}_{27} \langle 27, B|,
$$

$$
|B_{103/2}\rangle = |10_{3/2}, B\rangle + a^{B}_{27} \langle 27_{3/2}, B| + a^{B}_{35} \langle 35_{3/2}, B|,
$$

with the mixing coefficients

$$
c^{B}_{10} = c^{B}_{10} \begin{bmatrix} \sqrt{5} \\ 0 \end{bmatrix}, \quad c^{B}_{27} = c^{B}_{27} \begin{bmatrix} \sqrt{3} \\ 2 \sqrt{6} \end{bmatrix}, \quad a^{B}_{27} = a_{27} \begin{bmatrix} \sqrt{15/2} \\ 2 \sqrt{3/2} \end{bmatrix}, \quad a^{B}_{35} = a_{35} \begin{bmatrix} 5/\sqrt{14} \\ 2 \sqrt{5/7} \end{bmatrix},
$$

respectively, in the basis $[N, \Lambda, \Sigma, \Xi]$ for the octet and $[\Delta, \Sigma^*, \Xi^*, \Omega]$ for the decuplet. The coefficients $c^{B}_{10}$, $c^{B}_{27}$, $a_{27}$, and $c^{B}_{35}$ are written as

$$
c^{B}_{10} = -\frac{I_2}{15} \left( \alpha + \frac{1}{2} \gamma \right), \quad c^{B}_{27} = -\frac{I_2}{25} \left( \alpha - \frac{1}{6} \gamma \right), \quad a_{27} = -\frac{I_2}{8} \left( \alpha + \frac{5}{6} \gamma \right), \quad a_{35} = -\frac{I_2}{24} \left( \alpha - \frac{1}{2} \gamma \right).
$$
By setting $N_Q = 1$ in the dynamical parameters listed in Appendix A, the collective wave functions for the baryon sextets are obtained as

$$|B_6⟩ = |6_{11}, B⟩ + q^6_{15}⟨15_{11}, B⟩ + q^6_{10}⟨10_{11}, B⟩,$$

with the mixing coefficients

$$q^6_{15} = \begin{bmatrix} \sqrt{5}/5 \\ \sqrt{30}/20 \\ 0 \end{bmatrix}, \quad q^6_{10} = \begin{bmatrix} -\sqrt{15}/10 \\ -\sqrt{15}/10 \\ -\sqrt{15}/10 \end{bmatrix},$$

respectively, in the basis of $|Σ_c(Σ_c^†), Ξ_c(Ξ_c^†), Ω_c^0(Ω_c^0)⟩$ for the baryon sextets. The coefficients $q^6_{15}$ and $q^6_{10}$ are given in terms of the dynamical parameters $α$ and $γ$

$$q^6_{15} = -\frac{1}{\sqrt{2}} (α + \frac{2}{3}γ) I_2, \quad q^6_{10} = \frac{4}{5\sqrt{10}} (α - \frac{1}{3}γ) I_2.$$

The numerical results for the dynamical parameters are found to be

$$a_1 = -3.75, \quad a_2 = 2.34, \quad a_3 = 0.88, \quad a_4 = -0.29, \quad a_5 = -0.01, \quad a_6 = 0.02$$

for the light baryons, and

$$a_1 = -3.20, \quad a_2 = 2.98, \quad a_3 = 1.32, \quad a_4 = -0.23, \quad a_5 = -0.02, \quad a_6 = 0.03$$

for the singly heavy baryons. Note that values of the $a_i$ are slightly different from those with flavor SU(3) symmetry in Eqs. (33) and (34). As discussed in the previous subsections, the leading contributions to the axial charges are expressed in terms of $F$ and $D$ or $a_{1,2,3}$. We now introduce the contributions from the flavor SU(3) symmetry breaking. They consist of two different terms, i.e., that from the effective chiral action and that from the collective wave functions. Here, we will refer to them respectively as “operator corrections” and “wave function corrections” and denote them by the superscripts (op) and (wf). For convenience, we introduce the following combinations of the dynamical parameters from the effective chiral action ($x', y', z'$), and from the collective wave functions (octet $p', q'$; decuplet $s', n'$; sextets $l', m'$):

$$x' = a_4, \quad y' = a_5, \quad z' = a_6,$$

$$p' = c_{15}(a_1 + a_2 + \frac{1}{2}a_3), \quad q' = c_{10}(a_1 + 2a_2 - \frac{3}{2}a_3),$$

$$s' = a_{27}(2a_1 + a_2 + 3a_3), \quad n' = a_{35}(2a_1 + 5a_2 - 5a_3),$$

$$l' = q_{15}(2a_1 + a_2 + 2a_3), \quad m' = q_{10}(a_1 + 2a_2 - 2a_3).$$

For the singlet axial charges, the expressions of the operator corrections to them for the baryons octet and decuplet are found to be:

$$\langle \hat{g}^{(0),N}_A⟩^{(op)} = \frac{1}{5}(y' - z'),$$

$$\langle \hat{g}^{(0),Λ}_A⟩^{(op)} = \frac{3}{5}(y' - z'),$$

$$\langle \hat{g}^{(0),Σ}_A⟩^{(op)} = -\frac{3}{5}(y' - z'),$$

$$\langle \hat{g}^{(0),Ξ}_A⟩^{(op)} = \frac{4}{5}(y' - z'),$$

$$\langle \hat{g}^{(0),B_{10}}_A⟩^{(op)} = -\frac{Y}{4}(y' - z').$$

For the wave function corrections to the singlet axial charges, they vanish, because the inner product between the states with the different representations is not allowed.

$$\langle \hat{g}^{(0),B_a}_A⟩^{(wf)} = 0, \quad \langle \hat{g}^{(0),B_{10}}_A⟩^{(wf)} = 0.$$
For the triplet axial charges, the expressions for the baryons octet and decuplet from all the operator corrections are proportional to $T_3$:

\[
\hat{g}^{(3),N}_{A}^{(op)}(op) = T_3 \left( -\frac{11}{135}x' - \frac{2}{9}y' - \frac{2}{15}z' \right),
\]

\[
\hat{g}^{(3),\Lambda}_{A}^{(op)}(op) = 0,
\]

\[
\hat{g}^{(3),\Sigma}_{A}^{(op)}(op) = T_3 \left( -\frac{1}{30}x' - \frac{1}{15}y' + \frac{1}{15}z' \right),
\]

\[
\hat{g}^{(3),\Xi}_{A}^{(op)}(op) = T_3 \left( \frac{2}{135}x' + \frac{4}{45}y' - \frac{2}{15}z' \right),
\]

\[
\hat{g}^{(3),\Delta}_{A}^{(op)}(op) = T_3 \left( -\frac{11}{756}x' - \frac{5}{126}y' \right),
\]

\[
\hat{g}^{(3),\Sigma^*}_{A}^{(op)}(op) = T_3 \left( -\frac{5}{252}x' - \frac{1}{42}y' \right),
\]

\[
\hat{g}^{(3),\Xi^*}_{A}^{(op)}(op) = T_3 \left( -\frac{19}{756}x' - \frac{1}{126}y' \right),
\]

\[
\hat{g}^{(3),\Omega}_{A}^{(op)}(op) = 0.
\]

(36)

and those of the wave function corrections are also found to be

\[
\hat{g}^{(3),N}_{A}^{(wf)}(wf) = T_3 \left( -\frac{2}{3}y' - \frac{4}{45}q' \right),
\]

\[
\hat{g}^{(3),\Lambda}_{A}^{(wf)}(wf) = 0,
\]

\[
\hat{g}^{(3),\Sigma}_{A}^{(wf)}(wf) = -\frac{T_3}{3}p',
\]

\[
\hat{g}^{(3),\Xi}_{A}^{(wf)}(wf) = \frac{4T_3}{45}q',
\]

\[
\hat{g}^{(3),\Delta}_{A}^{(wf)}(wf) = T_3 \left( -\frac{5}{72}m' - \frac{1}{168}n' \right),
\]

\[
\hat{g}^{(3),\Sigma^*}_{A}^{(wf)}(wf) = T_3 \left( -\frac{1}{12}m' - \frac{1}{84}n' \right),
\]

\[
\hat{g}^{(3),\Xi^*}_{A}^{(wf)}(wf) = T_3 \left( \frac{7}{72}m' - \frac{1}{56}n' \right),
\]

\[
\hat{g}^{(3),\Omega}_{A}^{(wf)}(wf) = 0.
\]

(37)

Lastly, the expressions for the operator corrections to the octet axial charges are given by

\[
\hat{g}^{(8),N}_{A}^{(op)}(op) = \frac{1}{30\sqrt{3}}x',
\]

\[
\hat{g}^{(8),\Lambda}_{A}^{(op)}(op) = -\frac{1}{10\sqrt{3}}x',
\]

\[
\hat{g}^{(8),\Sigma}_{A}^{(op)}(op) = \frac{1}{18\sqrt{3}}x' + \frac{2}{15\sqrt{3}}y',
\]

\[
\hat{g}^{(8),\Xi}_{A}^{(op)}(op) = -\frac{1}{15\sqrt{3}}x' - \frac{1}{5\sqrt{3}}y',
\]

\[
\hat{g}^{(8),\Delta}_{A}^{(op)}(op) = \frac{5}{168\sqrt{3}}x' + \frac{1}{28\sqrt{3}}y',
\]

\[
\hat{g}^{(8),\Sigma^*}_{A}^{(op)}(op) = -\frac{1}{126\sqrt{3}}x' + \frac{1}{42\sqrt{3}}y',
\]

\[
\hat{g}^{(8),\Xi^*}_{A}^{(op)}(op) = -\frac{5}{168\sqrt{3}}x' - \frac{1}{28\sqrt{3}}y',
\]

\[
\hat{g}^{(8),\Omega}_{A}^{(op)}(op) = -\frac{1}{28\sqrt{3}}x' - \frac{1}{7\sqrt{3}}y',
\]

(38)
and those of the wave function corrections are written as

\[
\begin{align*}
(\hat{g}^{(8)}_A)_{(\text{wf})} = & \, \frac{1}{\sqrt{3}} p', -\frac{2}{5\sqrt{3}} q', \\
(\hat{g}^{(8)}_A)_{(\Sigma)} = & \, -\frac{\sqrt{3}}{5} q', \\
(\hat{g}^{(8)}_A)_{(\Xi)} = & \, \frac{1}{\sqrt{3}} p', -\frac{4}{15\sqrt{3}} q', \\
(\hat{g}^{(8)}_A)_{(\Delta)} = & \, -\frac{2}{5\sqrt{3}} q', \\
(\hat{g}^{(8)}_A)_{(\Sigma^*)} = & \, \frac{5}{16\sqrt{3}} s' - \frac{5}{112\sqrt{3}} n', \\
(\hat{g}^{(8)}_A)_{(\Xi^*)} = & \, \frac{1}{6\sqrt{3}} s' - \frac{1}{14\sqrt{3}} n', \\
(\hat{g}^{(8)}_A)_{(\Omega)} = & \, \frac{1}{14\sqrt{3}} n'.
\end{align*}
\]

When it comes to the singly heavy baryons, we also obtain the expressions for both the operator and wave function corrections to the singlet axial charges

\[
\begin{align*}
(\hat{g}^{(0)}_A)_{(\text{op})} = & \, -\frac{3Y}{5} (y' - z'), \\
(\hat{g}^{(0)}_A)_{(\text{wf})} = & \, 0,
\end{align*}
\]

and the triplet axial charges

\[
\begin{align*}
(\hat{g}^{(3)}_A)_{(\Sigma)} = & \, T_3 \left( -\frac{1}{27} x' - \frac{4}{45} y' \right), \\
(\hat{g}^{(3)}_A)_{(\Xi)} = & \, T_3 \left( -\frac{14}{27} x' - \frac{2}{45} y' \right), \\
(\hat{g}^{(3)}_A)_{(\Omega)} = & \, 0, \\
(\hat{g}^{(3)}_A)_{(\Sigma^*)} = & \, T_3 \left( -\frac{2}{9\sqrt{10}} t' - \frac{1}{45} m' \right), \\
(\hat{g}^{(3)}_A)_{(\Xi^*)} = & \, T_3 \left( -\frac{5}{9\sqrt{15}} t' - \frac{4}{45\sqrt{6}} m' \right), \\
(\hat{g}^{(3)}_A)_{(\Omega^*)} = & \, 0,
\end{align*}
\]

and the octet axial charges

\[
\begin{align*}
(\hat{g}^{(8)}_A)_{(\Sigma)} = & \, -\frac{1}{27\sqrt{3}} x' - \frac{4}{45\sqrt{3}} y', \\
(\hat{g}^{(8)}_A)_{(\Xi)} = & \, \frac{1}{90\sqrt{3}} x' + \frac{1}{15\sqrt{3}} y', \\
(\hat{g}^{(8)}_A)_{(\Omega)} = & \, \frac{4}{45\sqrt{3}} x' + \frac{2}{15\sqrt{3}} y', \\
(\hat{g}^{(8)}_A)_{(\Sigma^*)} = & \, \frac{2}{3\sqrt{30}} t' - \frac{2}{15\sqrt{3}} m', \\
(\hat{g}^{(8)}_A)_{(\Xi^*)} = & \, -\frac{1}{2\sqrt{30}} t' - \frac{2}{15\sqrt{2}} m', \\
(\hat{g}^{(8)}_A)_{(\Omega^*)} = & \, -\frac{2}{15\sqrt{2}} m'.
\end{align*}
\]

Interestingly, since the baryon sextets with the spins $J' = 1/2$ and $J' = 3/2$ arise from the same $N_c - 1$ soliton with spin $J = 1$, one can easily expect that they share the same dynamics. Indeed, the operator and wave function
corrections to both the triplet and octet axial charges for \( J' = 1/2 \) and \( J' = 3/2 \) are related to each other by the overall factor \( 1/2 \):

\[
(g_A^{(3,8), B_6, J'=3/2})^{(\text{wt}, \text{op})} = \frac{1}{2} g_A^{(3,8), B_6, J'=1/2})^{(\text{wt}, \text{op})},
\]

III. NUMERICAL RESULTS

We now present the results for the axial charges of both the light and singly heavy baryons and discuss their physical implications. We will first examine how the \( \chi \)QSM interpolates between the naive NR quark and Skyrme models [10, 17]. We then provide a distinctive feature of the present framework in contrast to the Skyrme model. To see those features explicitly, we draw the axial charges of the light baryons as a function of the soliton size in the left panel of Fig. 1. Note that the flavor-singlet axial charges of the baryon octet and baryon decuplet degenerate in the exact flavor SU(3) symmetry. When the size of the soliton becomes zero, the value of the axial charge in the \( \chi \)QSM reproduces exactly that in the naive NR quark model in the exact flavor SU(3) symmetry. On the other hand, the axial charge becomes zero when the size of the soliton increases, which coincides with the result from the Skyrme model [83]. Note that the flavor-singlet axial charge has a contribution (\( B \) or \( a_3 \)) from the imaginary part of the effective chiral action, which does not appear in a typical Skyrme model. The non-vanishing feature of this term in the \( \chi \)QSM causes the non-zero singlet axial charge. This term is suppressed in the limit of a large soliton size, so that the result of the Skyrme model is reproduced. In the right panel of Fig. 1 we depict the axial charges of the singly heavy baryons as functions of the soliton size. Similar to the light baryon, we can restore the results from the naive NR quark model by taking the limit of the small size soliton. Interestingly, as the size of the soliton increases, the axial charge of the baryon sextet with \( J' = 1/2 \) becomes negative and finally arrives at the value of the axial charge that solely comes from the heavy-quark part, i.e., \( \Delta u, d, s = 0, \Delta c = -1/3 \). On the other hand, the axial charge of the baryon sextet with \( J' = 3/2 \) is always kept positive and approaches \( g_A^{(0)} = 1/3 \) that solely arises from the heavy-quark part \( \Delta u, d, s = 0, \Delta c = 1/3 \) again. These features are distinguished from the Skyrme model. Here we want to mention that the predictions of the \( \chi \)QSM lie in between those from the naive NR quark model and Skyrme model.

It is of great interest to compare the quark spin content of both the light and singly heavy baryons. In Table I we listed the numerical results for the axial charges and their flavor decompositions of the baryon octet, decuplet, and sextet with \( J' = 1/2 \) and \( J' = 3/2 \), considering the exact flavor SU(3) symmetry and its breaking. It should be noted that we choose the spin-polarization to be \( J'' = 1/2 \) for the light baryons and \( J'' = 1/2 \) for the heavy baryons, so that we can easily compare the axial charges of all the baryons.

The values of \( g_A^{(3),p} \) and \( g_A^{(8),p} \) are in agreement with those extracted from the experimental data on the neutron \( \beta \) decay and in the HSDs, respectively. Note that the value of the \( g_A^{(8),p} \) is still under debate. However, since \( g_A^{(0),p} \) cannot be obtained directly from the HSDs, one should relate it to the first moment of the polarized quark distribution to extract the phenomenological value. It was obtained to be \( g_A^{(0),p} \sim 0.33 \) [23], which is smaller than the prediction of the
TABLE I: Axial charges and their flavor decompositions of the baryon octet, the baryon decuplet, and the baryon sextet with \( J' = 1/2, J' = 3/2 \)

| \( J_3 = 1/2 \) | \( g_A^{(0)} \) (sym) | \( g_A^{(0)} \) (sym) | \( g_A^{(0)} \) (sym) | \( g_A^{(0)} \) (sym) | \( g_A^{(0)} \) (sym) | \( g_A^{(0)} \) (sym) | \( g_A^{(0)} \) (sym) | \( \Delta u \) (sym) | \( \Delta u \) (sym) | \( \Delta d \) (sym) | \( \Delta s \) (sym) | \( \Delta s \) (sym) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| \( p \) | 0.437 | 0.444 | 1.163 | 1.188 | 0.360 | 0.332 | 0.831 | 0.838 | -0.332 | -0.350 | -0.062 | -0.044 |
| \( n \) | 0.437 | 0.444 | -1.163 | -1.188 | 0.360 | 0.332 | -0.332 | -0.350 | 0.831 | 0.838 | -0.062 | -0.044 |
| \( \Lambda \) | 0.437 | 0.418 | 0.000 | 0.000 | -0.827 | -0.807 | -0.093 | -0.094 | -0.094 | 0.623 | 0.606 |
| \( \Sigma^+ \) | 0.437 | 0.457 | 0.893 | 0.919 | 0.827 | 0.794 | 0.831 | 0.841 | -0.062 | -0.078 | -0.332 | -0.306 |
| \( \Sigma^0 \) | 0.437 | 0.457 | 0.000 | 0.000 | 0.827 | 0.794 | 0.384 | 0.381 | 0.384 | 0.381 | -0.332 | -0.306 |
| \( \Sigma^- \) | 0.437 | 0.457 | -0.893 | -0.919 | 0.827 | 0.794 | -0.062 | -0.078 | 0.831 | 0.841 | -0.332 | -0.306 |
| \( \Xi^0 \) | 0.437 | 0.412 | -0.270 | -0.274 | -1.187 | -1.173 | -0.332 | -0.338 | -0.062 | -0.064 | 0.831 | 0.814 |
| \( \Xi^- \) | 0.437 | 0.412 | 0.270 | 0.274 | 1.187 | 1.173 | -0.332 | -0.338 | -0.062 | -0.064 | 0.831 | 0.814 |
| \( \Delta^{++} \) | 0.437 | 0.461 | 0.670 | 0.700 | 0.387 | 0.340 | 0.592 | 0.602 | -0.077 | -0.098 | -0.077 | -0.043 |
| \( \Delta^+ \) | 0.437 | 0.461 | 0.223 | 0.233 | 0.387 | 0.340 | 0.369 | 0.368 | 0.146 | 0.135 | -0.077 | -0.043 |
| \( \Delta^0 \) | 0.437 | 0.461 | -0.223 | -0.233 | 0.387 | 0.340 | 0.146 | 0.135 | 0.369 | 0.368 | -0.077 | -0.043 |
| \( \Delta^- \) | 0.437 | 0.461 | -0.670 | -0.700 | 0.387 | 0.340 | -0.077 | -0.098 | 0.592 | 0.602 | -0.077 | -0.043 |
| \( \Sigma^{++} \) | 0.437 | 0.437 | 0.446 | 0.472 | 0.000 | -0.021 | 0.369 | 0.376 | -0.077 | -0.096 | 0.146 | 0.158 |
| \( \Sigma^+ \) | 0.437 | 0.437 | 0.000 | 0.000 | 0.000 | 0.000 | 0.146 | 0.104 | 0.146 | 0.146 | 0.146 | 0.158 |
| \( \Sigma^0 \) | 0.437 | 0.437 | -0.446 | -0.472 | 0.000 | 0.000 | 0.146 | 0.144 | 0.146 | 0.146 | 0.146 | 0.158 |
| \( \Xi^{++} \) | 0.437 | 0.413 | 0.223 | 0.238 | -0.387 | -0.390 | 0.146 | 0.144 | -0.077 | -0.094 | 0.369 | 0.363 |
| \( \Xi^+ \) | 0.437 | 0.413 | 0.223 | 0.238 | 0.387 | 0.390 | -0.077 | -0.094 | 0.146 | 0.144 | 0.369 | 0.363 |
| \( \Xi^0 \) | 0.437 | 0.389 | 0.000 | 0.000 | -0.073 | -0.076 | -0.077 | -0.091 | 0.014 | 0.014 | 0.592 | 0.572 |

\( \chi QSM \) \( g_A^{(0),p} \) \( \sim 0.44 \). This discrepancy may arise from the U(1) axial anomaly, since there is no gluonic contribution from the \( \chi QSM \). We also present all the axial charges of the baryon octet. Interestingly, the axial charges of the \( \Lambda^0 \) and \( \Xi^0 \) baryons have the opposite signs to each other. Although these baryons have the same quark content, each quark contribution shows a different value because of the different isospin. In Table I we find that the present results are consistent with those from lattice QCD \[29,38\], chiral perturbation theory \[24\], the relativistic constituent quark model (RCQM) \[35,36\], the perturbative chiral quark model (PCQM) \[37\].

In Table II we also list the flavor-decomposed quark spin content. The separate spin contributions of the \( u, d, \) and \( s \) quarks to the nucleon are estimated empirically in Ref. \[23\]: \( \Delta u \sim 0.84, \Delta d \sim -0.43 \), and \( \Delta s \sim -0.08 \) where the normalization point is taken to be \( Q^2 \rightarrow \infty \). Interestingly, we predict the value of \( s \)-quark contribution \( \Delta s \sim -0.05 \)
for the proton, which is comparable to the estimation of Ref. [23], $\Delta s \sim -0.08$. This non-zero value of the s-quark contribution breaks the Ellis-Jaffe sum rule, so that it plays an essential role in understanding the quark spin content of the proton. This feature can be observed for other hyperons as well. For example, the “pure” sea-quark contributions to the axial charges of the $n$, $\Sigma$, and $\Xi$ are also estimated to be less than $-0.08$. Note that “pure” sea-quark stand for the Dirac continuum in the $\chi$QSM. In Table[1] we find that the present results are consistent with those from lattice QCD [25, 27, 28], a global QCD analysis [30], NNPDF collaboration data [31], and COMPASS data [32].

In contrast to the baryon octet, there are no experimental data on the axial charges of the baryon decuplet. However, since we see that the present work has successfully described the proton axial charges, we can proceed to compute the axial charges of the baryon decuplet. While the flavor-singlet axial charge of the baryon decuplet is identical with that of the baryon octet $g_A^{(0),B_{10}} = g_A^{(0),B_8}$, flavor-triplet and -octet axial charges are different. This leads to the different combinations of the quark-spin contributions to the baryons. Compared to the quark spin content of the proton, the $\Delta^+$ isobar has a smaller value $\Delta u \sim 0.37$ with the same sign, whereas it has the opposite sign and smaller value $\Delta d \sim 0.14$. The value $\Delta s$ of $\Delta^+$ is almost the same as that of the proton. In the exact flavor SU(3) symmetry, the axial charges of the baryon decuplet can be related to each other [12]. In Table [III] we compare the results for the axial charges of the baryon decuplet with those from the other models. Here we take the spin polarization to be $J_z = 3/2$ instead of $J_z = 1/2$ to easily compare the present results with those from other models. We find that the current results are consistent with those from chiral perturbation theory [33, 34], the relativistic constituent quark model (RCQM) [35, 36], the perturbative chiral quark model (PCQM) [37], and lattice QCD [38].

In the limit of $m_Q \to \infty$, all the dynamics are governed by the light quarks, whereas the heavy quark merely carries its charge and spin. Employing the dynamical parameters obtained in the model calculation, we can predict the axial charges of the singly heavy baryons in the exact flavor SU(3) symmetry and with the effects of its breaking. It should be noted that the spin of the antitriplet (3) heavy baryon is solely carried by the charm quark, i.e., $\Delta c = 1$, as mentioned previously. The flavor-triplet and -octet axial charges equally vanish. We want to mention that the light quark contributions to the axial charges of the antitriplet (3) baryons are estimated to be null in lattice QCD [38], i.e., $g_A^{(0),B_{3,J'=1/2}} \sim 0.9, g_A^{(3),B_{3,J'=1/2}} \sim 0, g_A^{(8),B_{3,J'=1/2}} \sim 0$, which strongly supports the present results.

In Table [I] we also list the axial charges of the baryon sextet (6) with $J' = 1/2$ and $J' = 3/2$, respectively. The flavor-singlet axial charges of the singly heavy baryons with $J' = 1/2$ and $J' = 3/2$ are respectively enhanced by $\sim 20\%$ and $\sim 40\%$ in comparison with those of the light baryons. In the naive NR quark model, the charm quark yields $-1/3$ ($1/3$) for the spin of the baryon sextet with $J' = 1/2$ ($J' = 3/2$), so the light quarks should be strongly (weakly) polarized to keep the sum of the quark spins unity, i.e., $g_A^{(0)} = 1$. We expect that such a behavior may remain in the $\chi$QSM, though the relativistic effects further suppress the value of $g_A^{(0)}$. Thus, we may conclude that the light quarks inside the baryon sextet (6) with $J' = 1/2$ ($J' = 3/2$) are more strongly (weakly) polarized than those in the light baryons. We have encountered this feature already in the calculation of the heavy baryon spin out of the energy-momentum tensor [74]. We also found an interesting physics that the “pure” sea quark contributions to the axial charges of the singly heavy baryon are smaller than those of the light baryons. It implies that the “pure” sea quark contributions inside the singly heavy baryons are relatively more suppressed compared to the light baryon. In addition, one can clearly see that the light-quark contributions to the flavor-singlet axial charge of the baryon sextet with $J' = 1/2$, i.e., $\Delta u + \Delta d + \Delta s = 0.876$, is twice larger than the baryon sextet with $J' = 3/2$, i.e., $\Delta u + \Delta d + \Delta s = 0.439$ in flavor SU(3) symmetry. Therefore, each quark contribution in both representations shows the same ratio except for the heavy-quark contributions in the exact flavor SU(3) symmetry. In Tables IV and V we compare the values of the axial charges of the singly heavy baryons with those from lattice QCD. We find that the present results are consistent with them [38].

In Fig. 2, we plot the spin content of both the lowest-lying light and singly heavy baryons so that one can easily see how much fraction of the spin is carried by $u$, $d$, and $s$ quarks. The composite quark (say, in the naive NR quark model) spin content of all the baryons has the same signs. However, those of the baryon octet have opposite signs. It means that the polarizations of the composite quark spins inside the octet baryon are antiparallel to each other, whereas those inside the other baryons belonging to the other representations are parallel to each other. Meanwhile, all the “pure” sea quark contributions are found to be negative. Here, one should keep in mind that the sea quark contributions are included in $u$, $d$, and $s$ contributions. So, for the proton, $u$ and $d$ quark contributions to its spin already include the sea quark contributions, i.e., $\Delta \bar{u}$ and $\Delta \bar{d}$. They can not be identified as the “pure” valence quark contributions. This gives a hint that the valence quarks are more dominant in the singly heavy baryons than those in the light baryons.
TABLE II: Axial charges and their flavor decompositions of the baryon octet

| $J_3 = 1/2$ | $g_A^{(0)}$ | $g_A^{(3)}$ | $g_A^{(6)}$ | $\Delta a$ | $\Delta d$ | $\Delta s$ |
|------------|-------------|-------------|-------------|-------------|-------------|-------------|
| $p$        | 0.444       | 1.188       | 0.332       | 0.838       | −0.350      | −0.044      |
| $N^{21}$   | −           | 1.18        | −           | −           | −           | −           |
| $N^{25}$   | −           | −           | −           | 0.415 ± 0.013 ± 0.002° −0.193 ± 0.008 ± 0.003° −0.021 ± 0.005 ± 0.001° |
| $N^{20}$   | −           | 1.278 ± 0.021 ± 0.026 | − | − | − | − |
| $N^{27}$   | 0.405 ± 0.025 ± 0.037 | 1.254 ± 0.016 ± 0.030 | 0.510 ± 0.027 ± 0.039 | 0.847 ± 0.018 ± 0.032 | −0.407 ± 0.016 ± 0.018 | −0.035 ± 0.006 ± 0.007 |
| $N^{22}$   | 0.286 ± 0.062 | 1.218 ± 0.025 ± 0.030 | − | 0.777 ± 0.025 ± 0.030 | −0.438 ± 0.018 ± 0.030 | −0.053 ± 0.008 |
| $N^{30}$   | 0.360_{−0.013}^{+0.018} | − | − | 0.795_{−0.012}^{+0.011} | −0.416_{−0.009}^{+0.011} | −0.012_{−0.024}^{+0.020} |
| $p$(EGBE)  | −           | 1.15        | −           | −           | −           | −           |
| $p$(psGGE) | −           | 1.15        | −           | −           | −           | −           |
| $p$(OGE)   | −           | 1.11        | −           | −           | −           | −           |
| $N^{31}$   | 0.25 ± 0.10  | −           | −           | 0.76 ± 0.04 | −0.41 ± 0.04 | −0.10 ± 0.08 |
| $N^{32}$   | [0.26, 0.36] | 1.22 ± 0.05 ± 0.10 | − | [0.82, 0.85] | [−0.45, −0.42] | [−0.11, −0.08] |
| $N^{37}$   | −           | 1.263       | −           | −           | −           | −           |
| $N^{29}$   | −           | 1.18 ± 0.04 ± 0.06 | − | − | − | − |
| $\Lambda$ | 0.418       | 0.000       | −0.007      | −0.094      | −0.094      | 0.606       |
| $\Lambda^{[4]}$ | 0.6361 ± 0.0180 | 0.0851 ± 0.0145 | −1.5169 ± 0.238° | 0.0035 ± 0.0105 | −0.0861 ± 0.0106 | 0.7185 ± 0.0092 |
| $\Sigma^+$ | 0.457       | 0.919       | 0.794       | 0.841       | −0.078      | −0.306      |
| $\Sigma^{[2]}$ | −           | 0.73        | −           | −           | −           | −           |
| $\Sigma^+$(EGBE) | −           | 0.65^{b}   | −           | −           | −           | −           |
| $\Sigma^+$(psGGE) | −           | 0.65^{b}   | −           | −           | −           | −           |
| $\Sigma^+$(OGE) | −           | 0.65^{b}   | −           | −           | −           | −           |
| $\Sigma^{[7]}$ | 0.0896     | −           | −           | −           | −           | −           |
| $\Sigma^{[5]}$ | 0.4984 ± 0.0244 | 0.7629 ± 0.0218 | 1.2885 ± 0.0288° | 0.7629 ± 0.0218 | − | −0.2634 ± 0.0101 |
| $\Xi^0$    | 0.412       | −0.274      | −1.173      | −0.338      | −0.064      | 0.814       |
| $\Xi^{[4]}$ | −           | 0.23^{d}   | −           | −           | −           | −           |
| $\Xi^0$(EGBE) | −           | −0.21      | −           | −           | −           | −           |
| $\Xi^0$(psGGE) | −           | −0.22      | −           | −           | −           | −           |
| $\Xi^0$(OGE) | −           | −0.22      | −           | −           | −           | −           |
| $\Xi^{[7]}$ | −           | −0.275      | −           | −           | −           | −           |
| $\Xi^{[5]}$ | 0.6735 ± 0.0162 | −0.2479 ± 0.0087 | −2.1092 ± 0.236° | −0.2479 ± 0.0087 | − | 0.9266 ± 0.0121 |
| $\Xi^{[2]}$ | −           | −0.277 ± 0.015 ± 0.019 | − | − | − | − |

Note that the expressions for the axial charges in Ref. [25] are different from the present one by 1/2.

Note that the expressions for the axial charges in Ref. [35] are different from the present one by 1/√3.

Note that the expressions for the axial charges in Ref. [29] are different from the present one by 1/2.

Note that the expressions for the axial charges in Ref. [24] are different from the present one by −1.

The values in Ref. [35] are obtained by averaging over various isospin partners.

IV. SUMMARY AND CONCLUSIONS

In this work, we aimed at investigating the axial charges and quark spin content of both the lowest-lying light and singly heavy baryons, based on the chiral quark-soliton model in the exact SU(3) symmetry and with the effects of its breaking. We first provided the relevant formalism for both the light and singly heavy baryons. In the case of the light baryons, their internal dynamics are described by the pion mean field in the presence of the $N_c$ valence quarks. The dynamical parameters $a_{1,6}$ related to the axial properties were determined. On the other hand, In the limit of the infinitely heavy quark mass ($m_Q \to \infty$), the heavy quark can be regarded as a static color source, so that it merely carries its spin inside a singly heavy baryon. Thus, the $N_c − 1$ light quarks govern the internal dynamics of the singly heavy baryon. The pion mean field was lessened in the presence of the $N_c − 1$ valence quarks, so that the dynamical parameters $a_{1,6}$ were revised. By taking the small soliton-size limit, we reproduced the axial charges of both the light and singly heavy baryons from the naive NR quark model in flavor SU(3) symmetry. When the soliton
size increased, we recovered the results from the Skyrme model, i.e., \( g_A^{(0)} = 0 \). However, those of the heavy baryons become \( g_A^{(3)} = \Delta c \), which is a distinguishable feature of the chiral quark-soliton model. In fact, the prediction of the singlet axial charges of this model lies in between those of the naive quark and the Skyrme models. We presented all the axial charges of both the light and singly heavy baryons and compared them to each other. We found that the singlet axial charges of the singly heavy baryons are enhanced in comparison with that of the light baryon, since

### TABLE III: Axial charges and their flavor decompositions of the baryon decuplet

| \( J_A = 3/2 \) | \( g_A^{(0)} \) | \( g_A^{(3)} \) | \( g_A^{(8)} \) | \( \Delta u \) | \( \Delta d \) | \( \Delta s \) |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| \( \Delta^{++} \) | 1.336 | 2.101 | 1.020 | 1.790 | −0.311 | −0.144 |
| \( \Delta^{++} \) (EGBE) | − | −4.48 | − | − | − | − |
| \( \Delta^{++} \) (psGBE) | − | −4.47 | − | − | − | − |
| \( \Delta^{++} \) (OGE) | − | −4.30 | − | − | − | − |
| \( \Delta \) | 1.37 | 1.863 | − | − | − | − |
| \( \Delta \) (37) | − | −4.50 | − | − | − | − |
| \( \Sigma^+ \) | 1.312 | 1.415 | −0.062 | 1.127 | −0.288 | 0.473 |
| \( \Sigma^+ \) (EGBE) | − | −1.06 | − | − | − | − |
| \( \Sigma^+ \) (psGBE) | − | −1.06 | − | − | − | − |
| \( \Sigma^+ \) (OGE) | − | −1.00 | − | − | − | − |
| \( \Sigma^+ \) (37) | − | 1.242 | − | − | − | − |
| \( \Sigma^+ \) (38) | 1.8616 ± 0.0498 | 1.1740 ± 0.0380 | −0.1925 ± 0.0336 | 1.1740 ± 0.0380 | − | 0.6852 ± 0.0171 |
| \( \Xi^0 \) | 1.288 | 0.714 | −1.169 | 0.449 | −0.265 | 1.104 |
| \( \Xi^0 \) (EGBE) | − | −0.75 | − | − | − | − |
| \( \Xi^0 \) (psGBE) | − | −0.75 | − | − | − | − |
| \( \Xi^0 \) (OGE) | − | −0.70 | − | − | − | − |
| \( \Xi^0 \) (37) | − | −0.62 | − | − | − | − |
| \( \Xi^0 \) (38) | 1.9571 ± 0.0379 | 0.5891 ± 0.0198 | −2.1321 ± 0.0415 | 0.5891 ± 0.0198 | − | 1.3637 ± 0.0245 |
| \( \Omega^- \) | 1.264 | 0.000 | −2.299 | −0.242 | −0.242 | 1.749 |
| \( \Omega^- \) (38) | 2.0338 ± 0.0310 | − | −4.0677 ± 0.0620 | − | − | 2.0338 ± 0.0310 |

- The expressions for the axial charges in Refs. [34, 35] are different from the present one by \( \Delta c \).
- The expressions for the axial charges in Ref. [34] are different from the present one by \( -1/\sqrt{2} \).
- Note that the expressions for the axial charges in Ref. [35] are different from the present one by \( -1 \).
- Note that the expressions for the axial charges in Ref. [34] are different from the present one by \( \sqrt{3} \).
- The values in Ref. [38] are obtained by averaging over various isospin partners.

### TABLE IV: Axial charges and their flavor decompositions of the baryon sextet with \( J' = 1/2 \)

| \( J_A = 1/2 \) | \( g_A^{(0)} \) | \( g_A^{(3)} \) | \( g_A^{(8)} \) | \( \Delta u \) | \( \Delta d \) | \( \Delta s \) |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| \( \Sigma^0 \) | 0.566 | 1.055 | 0.568 | 0.991 | −0.064 | −0.028 | −0.333 |
| \( \Sigma^0 \) (38) | 0.4094 ± 0.0199 | 0.7055 ± 0.0191 | 0.7055 ± 0.0191 | 0.7055 ± 0.0191 | − | − | −0.2970 ± 0.0113 |
| \( \Xi^e_0 \) | 0.531 | 0.503 | −0.274 | 0.505 | −0.087 | 0.447 | −0.333 |
| \( \Xi^e_0 \) (38) | 0.4872 ± 0.0127 | 0.3433 ± 0.0085 | −0.5596 ± 0.0099 | 0.3433 ± 0.0085 | − | 0.4530 ± 0.0055 | −0.3133 ± 0.0069 |
| \( \Omega^0_c \) | 0.497 | 0.000 | −1.198 | −0.069 | −0.069 | 0.968 | −0.333 |
| \( \Omega^0_c \) (38) | 0.5428 ± 0.0118 | − | −1.7108 ± 0.0233 | − | − | 0.8554 ± 0.0117 | −0.3125 ± 0.0054 |

- The values in Ref. [38] are obtained by averaging over various isospin partners.
TABLE V: Axial charges and their flavor decompositions of the baryon sextet with $J' = 3/2$

| $J' = 3/2$ | $g_A^{(0)}$ | $g_A^{(3)}$ | $g_A^{(8)}$ | $\Delta u$ | $\Delta d$ | $\Delta s$ | $\Delta c$ |
|------------|-------------|-------------|-------------|------------|------------|------------|------------|
| $\Sigma^{++}$ | 2.349       | 1.583       | 0.852       | 1.487      | −0.096     | −0.042     | 1.000      |
| $\Sigma^+_c$ [38] | 2.0004 ± 0.0346 | 1.0899 ± 0.0308 | 1.0899 ± 0.0308$^*$ | 1.0899 ± 0.0308$^*$ | − | − | 0.9043 ± 0.0090 |
| $\Xi^+_c$ [38] | 2.297       | 0.889       | −0.411      | 0.758      | −0.131     | 0.670      | 1.000      |
| $\Omega^{'0}$ | 2.1192 ± 0.0254 | 0.5466 ± 0.0150 | −0.7581 ± 0.183$^*$ | 0.5466 ± 0.0150 | − | 0.6587 ± 0.0104 | 0.9103 ± 0.0075 |
| $\Omega^{'0}_c$ [38] | 2.1961 ± 0.0261 | − | −2.5817 ± 0.0408$^*$ | − | − | 1.2904 ± 0.0204 | 0.9026 ± 0.0090 |

$^*$ Note that the expressions for the axial charges in Ref. [38] are different from the present one by $\sqrt{3}$.
$^+$ The values in Ref. [38] are obtained by averaging over various isospin partners.

FIG. 2: Quark spin contents of both the lowest-lying light and singly heavy baryons.
the heavy quark may provide a significant contribution to the heavy baryon spin. As a result, the light quarks inside
the heavy baryons with \( J' = 1/2 \) (\( J' = 3/2 \)) are more strongly (weakly) polarized compared to those inside the light
baryons. Finally, we observed that the “pure” sea quark contributions to the spin of the singly heavy baryons are
always kept to be negative and smaller than those of the light baryons. It implies that the sea quark contributions
inside a heavy baryon are relatively more suppressed than inside a light baryon. Thus, the valence quarks come into
more dominant play, compared to the light-baryon sector. We found that the present results are in good agreement
with those from the experimental and lattice QCD.

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**Appendix A: Dynamical parameters within the \( \chi \)QSM**

To keep using the definition of the Ref. 55, the dynamical parameters are expressed in terms of \( \mathcal{A}, \mathcal{B}, \mathcal{C}, \text{and } \mathcal{D} \):

\[
\begin{align*}
    a_1 &= -\frac{1}{\sqrt{3}} A - \frac{1}{3\sqrt{2}} D - \frac{2}{3\sqrt{3}} m_s \mathcal{H}, \\
    a_2 &= -\frac{1}{\sqrt{3}} C, \\
    a_3 &= \frac{1}{3} B, \\
    a_4 &= -2 \frac{K_2}{\sqrt{3}} C + 2 \frac{m_s J}{\sqrt{3}}, \\
    a_5 &= \frac{1}{3\sqrt{3}} m_s \mathcal{H} + \frac{1}{9} K_1 m_s \mathcal{B} - \frac{1}{9} m_s \mathcal{I}, \\
    a_6 &= \frac{1}{3\sqrt{3}} m_s \mathcal{H} - \frac{1}{9} K_1 m_s \mathcal{B} + \frac{1}{9} m_s \mathcal{I},
\end{align*}
\]
with

\[ A = \left( N_c - N_Q \right) \sum_{n \neq val} \frac{\phi_{\text{val}}(r)\sigma \cdot \tau \phi_{\text{val}}(r) + N_c \sum_{n} \phi_{\text{val}}(r)\sigma \cdot \tau \phi_{\text{val}}(r) R_1(E_n)}{E_{\text{val}} - E_n} \right], \]

\[ B = \left( N_c - N_Q \right) \sum_{n \neq val} \frac{1}{E_{\text{val}} - E_n} \phi_{\text{val}}(r)\sigma \cdot \tau \phi_{\text{val}}(r) \cdot \langle n | \tau | \text{val} \rangle \]

\[ - \frac{1}{2} N_c \sum_{n,m} \phi_{\text{val}}(r)\sigma \cdot \tau \phi_{\text{val}}(r) \cdot \langle m | \tau | n \rangle R_5(E_n, E_m) \right], \]

\[ C = \left( N_c - N_Q \right) \sum_{n_0 \neq val} \frac{1}{E_{\text{val}} - E_{n_0}} \phi_{\text{val}}(r)\sigma \cdot \tau \phi_{\text{val}}(r) \cdot \langle n_0 | \text{val} \rangle \]

\[ - N_c \sum_{n,m,n_0} \phi_{\text{val}}(r)\sigma \cdot \tau \phi_{\text{val}}(r) \cdot \langle m | \tau | n \rangle R_5(E_n, E_{n_0}) \right], \]

\[ D = \left( N_c - N_Q \right) \sum_{n \neq val} \frac{\text{sgn}(E_n)}{E_{\text{val}} - E_n} \phi_{\text{val}}(r)(\sigma \times \tau)\phi_{\text{val}}(r) \cdot \langle n | \tau | \text{val} \rangle \]

\[ + \frac{1}{2} N_c \sum_{n,m} \phi_{\text{val}}(r)\sigma \cdot \tau \phi_{\text{val}}(r) \cdot \langle m | \tau | n \rangle R_4(E_n, E_m) \right], \]

\[ \mathcal{H} = \left( N_c - N_Q \right) \sum_{n \neq val} \frac{1}{E_{\text{val}} - E_n} \phi_{\text{val}}(r)\sigma \cdot \tau \langle r | n \rangle \langle n | \gamma^0 | \text{val} \rangle \]

\[ + \frac{1}{2} N_c \sum_{n,m} \phi_{\text{val}}(r)\sigma \cdot \tau \phi_{\text{val}}(r) \cdot \langle m | \gamma^0 | n \rangle R_2(E_n, E_m) \right], \]

\[ I = \left( N_c - N_Q \right) \sum_{n \neq val} \frac{1}{E_{\text{val}} - E_n} \phi_{\text{val}}(r)\sigma \cdot \tau \langle r | n \rangle \langle n | \gamma^0 | \text{val} \rangle \]

\[ + \frac{1}{2} N_c \sum_{n,m} \phi_{\text{val}}(r)\sigma \cdot \tau \phi_{\text{val}}(r) \cdot \langle m | \gamma^0 | n \rangle R_2(E_n, E_m) \right], \]

\[ J = \left( N_c - N_Q \right) \sum_{n_0 \neq val} \frac{1}{E_{\text{val}} - E_{n_0}} \phi_{\text{val}}(r)\sigma \cdot \tau \phi_{\text{val}}(r) \cdot \langle n_0 | \gamma^0 | \text{val} \rangle \]

\[ + N_c \sum_{n,m,n_0} \phi_{\text{val}}(r)\sigma \cdot \tau \phi_{\text{val}}(r) \cdot \langle m | \gamma^0 | n \rangle R_2(E_n, E_{n_0}) \right). \] (A2)

The moments of inertia \((I_1, I_2, K_1, K_2)\) are expressed respectively as

\[ I_1 = \frac{(N_c - N_Q)}{6} \sum_{n \neq val} \frac{1}{E_{n} - E_{\text{val}}} \langle \text{val} | \tau | n \rangle \cdot \langle n | \tau | \text{val} \rangle + \frac{N_c}{12} \sum_{m,n} \langle m | \tau | n \rangle \cdot \langle n | \tau | m \rangle R_5(E_n, E_m), \]

\[ I_2 = \frac{(N_c - N_Q)}{4} \sum_{n} \frac{1}{E_{n} - E_{\text{val}}} \langle \text{val} | n^0 \rangle \langle n^0 | \tau | \text{val} \rangle + \frac{N_c}{4} \sum_{n^0,m} \langle m | \tau | n^0 \rangle \langle n^0 | m \rangle R_5(E_{n^0}, E_m), \]

\[ K_1 = \frac{(N_c - N_Q)}{6} \sum_{n \neq val} \frac{1}{E_{n} - E_{\text{val}}} \langle \text{val} | \gamma^0 | \tau | n \rangle \cdot \langle n | \tau | \text{val} \rangle + \frac{N_c}{12} \sum_{n,m \neq n} \langle m | \tau | n \rangle \cdot \langle n | \gamma^0 | \tau | m \rangle R_5(E_n, E_m), \]

\[ K_2 = \frac{(N_c - N_Q)}{4} \sum_{n^0} \frac{1}{E_{n^0} - E_{\text{val}}} \langle \text{val} | \gamma^0 | n^0 \rangle \langle n^0 | \tau | \text{val} \rangle + \frac{N_c}{4} \sum_{n^0,m} \langle m | \tau | n^0 \rangle \langle n^0 | \gamma^0 | m \rangle R_5(E_{n^0}, E_m), \] (A3)
where the regularization functions are defined by

\[ R_1(E_n) = -\frac{1}{2\sqrt{\pi}} E_n \int_{1/\Lambda^2}^{\infty} \frac{du}{\sqrt{u}} e^{-uE_n^2}, \]

\[ R_2(E_n, E_m) = \frac{1}{2\sqrt{\pi}} \int_{1/\Lambda^2}^{\infty} \frac{du}{\sqrt{u}} \frac{E_m e^{-uE_m^2} - E_n e^{-uE_n^2}}{E_n - E_m}, \]

\[ R_3(E_n, E_m) = \frac{1}{2\sqrt{\pi}} \int_{1/\Lambda^2}^{\infty} \frac{du}{\sqrt{u}} \left[ e^{-uE_m^2} - e^{-uE_n^2} \right] \frac{E_m e^{-uE_m^2} + E_n e^{-uE_n^2}}{E_n + E_m}, \]

\[ R_4(E_n, E_m) = \frac{1}{2\pi} \int_{1/\Lambda^2}^{\infty} d\alpha \frac{e^{-uE_m^2} + u(2E_n - E_m)}{4\pi \sqrt{\alpha(1 - \alpha)}}. \]

\[ R_5(E_n, E_m) = \frac{\text{sign}(E_n) - \text{sign}(E_m)}{2(E_n - E_m)}, \]

with the proper-time regulator \[ \Lambda. \] Since the quark-loop brings about the diverging low-energy constant in the effective theory, we need to regularize this theory by introducing a cutoff mass \( \Lambda \). It is fixed by reproducing the experimental value of the pion decay constant \( f_\pi = 93 \) MeV. While \( |\text{val}\rangle \) and \( |\text{sea}\rangle \) denote the state of the valence and sea quarks with the corresponding eigenenergies \( E_{\text{val}} \) and \( E_{\text{sea}} \) of the single-quark Hamiltonian \( h(U_c) \), respectively, \( |n_0\rangle \) stands for the state of sea quarks with the corresponding eigenenergy \( E_{n_0} \) of the free Dirac Hamiltonian \( h_0(1) \). The Dirac Hamiltonian is given by

\[ h(U_c) = -i\gamma^0 \gamma^k \partial_k + \gamma^0 M U^\gamma + \gamma^0 m, \]

where \( M \) is the dynamical quark mass taken to be 420 MeV in this work, and the isospin symmetry breaking effects are neglected, i.e., \( m_u = m_d = m \). Note that the current quark mass \( m \) is determined by reproducing the physical pion mass \( m_\pi = 140 \) MeV. Here, the chiral field is defined as \( U^\gamma = \exp i\gamma_5 \tau^a \pi^a \) with \( U = \exp i\pi^a \). The free Dirac Hamiltonian \( h_0(1) \) can be simply written by replacing \( U^\gamma \) to unity in Eq. (A6).

### Appendix B: Supplementary

**TABLE VI: Axial vector constant and flavor decomposition of the proton**

| \( J_3 = 1/2 \) | \( g_\lambda^{(0)} \) | \( g_\lambda^{(2)} \) | \( g_\lambda^{(3)} \) | \( \Delta u \) | \( \Delta d \) | \( \Delta s \) |
|---|---|---|---|---|---|---|
| \( p \) | 0.444 | 1.188 | 0.332 | 0.838 | -0.350 | -0.044 |
| \( N \) | 2.21 | 1.18 | - | - | - | - |
| \( N \) | 0.415 ± 0.013 ± 0.002 | -0.193 ± 0.008 ± 0.003 | -0.021 ± 0.005 ± 0.001 |
| \( N \) | 1.278 ± 0.021 ± 0.026 | - | - | - | - |
| \( N \) | 0.405 ± 0.025 ± 0.031 ± 0.046 | 0.016 ± 0.030 ± 0.051 | 0.027 ± 0.039 ± 0.047 | 0.018 ± 0.032 | -0.407 ± 0.016 ± 0.018 | -0.035 ± 0.006 ± 0.007 |
| \( N \) | 0.286 ± 0.062 | 1.218 ± 0.025 ± 0.030 | - | 0.777 ± 0.025 ± 0.030 | -0.438 ± 0.018 ± 0.030 | -0.053 ± 0.008 |
| \( N \) | 0.366 ± 0.015 | - | - | 0.793 ± 0.012 | -0.416 ± 0.011 | -0.012 ± 0.020 |
| \( p(\text{EGBE}) \) | 1.15 | - | - | - | - | - |
| \( p(\text{psGBE}) \) | 1.15 | - | - | - | - | - |
| \( p(\text{OGE}) \) | 1.11 | - | - | - | - | - |
| \( N \) | 0.25 ± 0.10 | - | - | 0.76 ± 0.04 | -0.41 ± 0.04 | -0.10 ± 0.08 |
| \( N \) | 0.26 ± 0.06 | 1.22 ± 0.05 ± 0.10 | - | 0.82 ± 0.85 | [-0.45, -0.42] | [-0.11, -0.08] |
| \( N \) | 1.263 | - | - | - | - | - |
| \( N \) | 1.18 ± 0.04 | - | - | - | - | - |

*a Note that the expressions for the axial charges in Ref. [25] are different from the present one by 1/2.*

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TABLE VII: Axial vector constant and flavor decomposition of $\Lambda$

| $J_3 = 1/2$ | $g_A^{(0)}$ | $g_A^{(3)}$ | $g_A^{(8)}$ | $\Delta u$ | $\Delta d$ | $\Delta s$ |
|------------|------------|------------|------------|------------|------------|------------|
| $\Lambda$  | 0.418      | 0.000      | −0.807     | −0.094     | −0.094     | 0.606      |
| $\Lambda$  | 0.6361 ± 0.0180 | 0.0851 ± 0.0145 | −1.5169 ± 0.238° | 0.0035 ± 0.0105 | −0.0861 ± 0.0106 | 0.7185 ± 0.0092 |

° Note that the expressions for the axial charges in Ref. [38] are different from the present one by $\sqrt{3}$.
† The values in Ref. [38] are obtained by averaging over various isospin partners.

TABLE VIII: Axial vector constant and flavor decomposition of $\Sigma^+$

| $J_3 = 1/2$ | $g_A^{(0)}$ | $g_A^{(3)}$ | $g_A^{(8)}$ | $\Delta u$ | $\Delta d$ | $\Delta s$ |
|------------|------------|------------|------------|------------|------------|------------|
| $\Sigma^+$ | 0.457      | 0.919      | 0.794      | 0.841      | −0.078     | −0.306     |
| $\Sigma$   | 0.65b      | 0.65b      | 0.896      |            |            |            |
| $\Sigma^+(\text{EGBE})$ | 0.4984 ± 0.0244 | 0.7629 ± 0.0218 | 1.2885 ± 0.0288° | 0.7629 ± 0.0218 | −0.2634 ± 0.0101 | −0.2634 ± 0.0101 |
| $\Sigma^+(\text{OGE})$ | 0.450 ± 0.021 ± 0.027° |            |            |            |            |            |

° Note that the expressions for the axial charges in Ref. [38] are different from the present one by $1/\sqrt{2}$.
† The values in Ref. [38] are obtained by averaging over various isospin partners.

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TABLE IX: Axial vector constant and flavor decomposition of $\Xi^0$

| J3 = 1/2 | $g_A^{(0)}$ | $g_A^{(3)}$ | $g_A^{(8)}$ | $\Delta u$ | $\Delta d$ | $\Delta s$ |
|----------|-------------|-------------|-------------|------------|----------|----------|
| $\Xi^0$  | 0.412       | -0.274      | -1.173      | -0.338     | -0.064   | 0.814    |
| $\Xi^0$ (EGBE) | 0.23°      | -0.21       | -0.22       | -0.22      | -0.22    | -0.22    |
| $\Xi^0$ (psGBE) | -0.22       | -0.22       | -0.22       | -0.22      | -0.22    | -0.22    |
| $\Xi^0$ (OGE) | -0.22       | -0.22       | -0.22       | -0.22      | -0.22    | -0.22    |
| $\Xi^0$ | 0.79 ± 0.016 | -0.2479 ± 0.0087 | -2.1092 ± 0.236° | -0.2479 ± 0.0087 | -0.9266 ± 0.0121 |
| $\Xi^0$† | -0.277 ± 0.015 ± 0.019 | -0.277 ± 0.015 ± 0.019 | -0.277 ± 0.015 ± 0.019 | -0.277 ± 0.015 ± 0.019 | -0.277 ± 0.015 ± 0.019 |

* Note that the expressions for the axial charges in Ref. [24] are different from the present one by $-1$.
* Note that the expressions for the axial charges in Ref. [38] are different from the present one by $\sqrt{3}$.
† The values in Ref. [38] are obtained by averaging over various isospin partners.

TABLE X: Axial vector constant and flavor decomposition of $\Delta^{++}$

| J3 = 3/2 | $g_A^{(0)}$ | $g_A^{(3)}$ | $g_A^{(8)}$ | $\Delta u$ | $\Delta d$ | $\Delta s$ |
|----------|-------------|-------------|-------------|------------|----------|----------|
| $\Delta^{++}$ | 1.336       | 2.101       | 1.020       | 1.790      | -0.311   | -0.144   |
| $\Delta^{++}$ (EGBE) | -4.48°      | -4.47°      | -4.30°      | -4.50°     | -4.50°   | -4.50°   |
| $\Delta^{++}$ (psGBE) | -4.48°      | -4.47°      | -4.30°      | -4.50°     | -4.50°   | -4.50°   |
| $\Delta^{++}$ (OGE) | -4.48°      | -4.47°      | -4.30°      | -4.50°     | -4.50°   | -4.50°   |
| $\Delta$ | 1.863       | -          | -           | -          | -        | -        |
| $\Delta$† | -4.50°      | -          | -           | -          | -        | -        |

* The expressions for the axial charges in Refs. [34, 35] are different from the present one by $-2$.

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Note that the expressions for the axial charges in Ref. [38] are different from the present one by $-1/\sqrt{2}$.

$\xi^+$ (EGBE) [35]

$\xi^+$ (psGBE) [35]

$\xi^+$ (OGE) [35]

$\xi^*$ [37]

$\xi^*$ [38]$^\dagger$

The values in Ref. [38] are obtained by averaging over various isospin partners.

$\xi^0$ (EGBE) [35]

$\xi^0$ (psGBE) [35]

$\xi^0$ (OGE) [35]

$\xi^*$ [37]

$\xi^*$ [38]$^\dagger$

The values in Ref. [38] are obtained by averaging over various isospin partners.

$\Omega^-$

$\Omega^-$ [38]

The values in Ref. [38] are obtained by averaging over various isospin partners.

$\Sigma^{++}$

$\Sigma^- [38]$

The values in Ref. [38] are obtained by averaging over various isospin partners.
TABLE XV: Axial vector constant and flavor decomposition of $\Xi_c^+$

| $J'_3 = 1/2$ | $g_A^{(0)}$ | $g_A^{(3)}$ | $g_A^{(8)}$ | $\Delta u$ | $\Delta d$ | $\Delta s$ | $\Delta c$ |
|--------------|-------------|-------------|-------------|------------|------------|------------|------------|
| $\Xi_c^+$    | 0.531       | 0.593       | $-0.274$    | 0.505      | $-0.087$   | 0.447      | $-0.333$   |
| $\Xi_c^+$ [38] | 0.4872 ± 0.01270.3433 ± 0.0085 − $0.5596$ ± 0.0099o $0.3433$ ± 0.0085 − 0.4539 ± 0.0055 $-0.3133$ ± 0.0069 |

$^o$ Note that the expressions for the axial charges in Ref. [38] are different from the present one by $\sqrt{3}$.
$^\dagger$ The values in Ref. [38] are obtained by averaging over various isospin partners.

TABLE XVI: Axial vector constant and flavor decomposition of $\Omega_c^0$

| $J'_3 = 1/2$ | $g_A^{(0)}$ | $g_A^{(3)}$ | $g_A^{(8)}$ | $\Delta u$ | $\Delta d$ | $\Delta s$ | $\Delta c$ |
|--------------|-------------|-------------|-------------|------------|------------|------------|------------|
| $\Omega_c^0$ | 0.497       | 0.00        | $-1.198$    | $-0.069$   | $-0.069$   | 0.968      | $-0.333$   |
| $\Omega_c^0$ [38] | 0.5428 ± 0.0118 − $-1.7108$ ± 0.0233o − − 0.8554 ± 0.0117 $-0.3125$ ± 0.0054 |

$^o$ Note that the expressions for the axial charges in Ref. [38] are different from the present one by $\sqrt{3}$.
$^\dagger$ The values in Ref. [38] are obtained by averaging over various isospin partners.

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### TABLE XVII: Axial vector constant and flavor decomposition of $\Sigma_c^{++}$

| $J'_3 = 3/2$ | $g_A^{(0)}$ | $g_A^{(3)}$ | $g_A^{(8)}$ | $\Delta u$ | $\Delta d$ | $\Delta s$ | $\Delta c$ |
|--------------|-------------|-------------|-------------|------------|------------|------------|------------|
| $\Sigma_c^{++}$ | 2.349 | 1.583 | 0.852 | 1.487 | -0.096 | -0.042 | 1.000 |
| $\Sigma_c^{*}$ [38] | 2.0004 ± 0.0346 | 1.0899 ± 0.0308 | 1.0899 ± 0.0308 | 1.0899 ± 0.0308 | 0.9043 ± 0.0090 |

*Note that the expressions for the axial charges in Ref. [38] are different from the present one by $\sqrt{3}$.† The values in Ref. [38] are obtained by averaging over various isospin partners.

### TABLE XVIII: Axial vector constant and flavor decomposition of $\Xi_c^{*+}$

| $J'_3 = 3/2$ | $g_A^{(0)}$ | $g_A^{(3)}$ | $g_A^{(8)}$ | $\Delta u$ | $\Delta d$ | $\Delta s$ | $\Delta c$ |
|--------------|-------------|-------------|-------------|------------|------------|------------|------------|
| $\Xi_c^{*+}$ | 2.297 | 0.889 | -0.411 | 0.758 | -0.131 | 0.670 | 1.000 |
| $\Xi_c^{*}$ [38]† | 2.1192 ± 0.0254 | 0.5466 ± 0.0150 | -0.7581 ± 0.183° | 0.5466 ± 0.0150 | 0.6587 ± 0.0104 | 0.9103 ± 0.0075 |

*Note that the expressions for the axial charges in Ref. [38] are different from the present one by $\sqrt{3}$.† The values in Ref. [38] are obtained by averaging over various isospin partners.

### TABLE XIX: Axial vector constant and flavor decomposition of $\Omega_c^{*0}$

| $J'_3 = 3/2$ | $g_A^{(0)}$ | $g_A^{(3)}$ | $g_A^{(8)}$ | $\Delta u$ | $\Delta d$ | $\Delta s$ | $\Delta c$ |
|--------------|-------------|-------------|-------------|------------|------------|------------|------------|
| $\Omega_c^{*0}$ | 2.245 | 0.000 | -1.797 | -0.104 | -0.104 | 1.452 | 1.000 |
| $\Omega_c^{*0}$ [38]† | 2.1961 ± 0.0261 | - | -2.5817 ± 0.0408° | - | - | 1.2904 ± 0.0204 | 0.9026 ± 0.0090 |

*Note that the expressions for the axial charges in Ref. [38] are different from the present one by $\sqrt{3}$.† The values in Ref. [38] are obtained by averaging over various isospin partners.