Extended State Observer Based Robust Position Tracking Control Using Nonlinear Damping Gain for Quadrotors With External Disturbance

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\textbf{ABSTRACT} We propose an extended-state-observer (ESO)-based robust position tracking control method using nonlinear damping gain to improve the control performance under external disturbances and parameter uncertainties for quadrotors. The proposed method consists of an ESO and a nonlinear damping controller (NDC). The ESO is designed to estimate full state and disturbance. The external disturbance, velocity dynamics, and the uncertainty of the input parameter are lumped in the disturbance. The NDC is developed via backstepping procedure to suppress the output tracking error according to the disturbance estimation error. The proposed method is simple and robust against external disturbance and parameter uncertainties. In addition, only the nominal value of the input gain parameters are required. The closed-loop stability is proven by using the input-to-state stability property. The position tracking performance of proposed method was verified by performing hardware-in-the-loop simulations using a quadrotor platform.

\textbf{INDEX TERMS} Extended state observer (ESO), nonlinear damping control (NDC), quadrotor, robust position control, hardware-in-the-loop-simulation (HILS).

\section{I. INTRODUCTION}

Quadrotors are a type of the unmanned aerial vehicle. In recent years, quadrotors have attracted considerable interest owing to their various advantages, such as simplicity in structure, vertical take-off and landing capability, hover capability, rapid maneuverability, and agility. In addition, quadrotors are more effective than conventional helicopters in terms of economics, safety, and size. Applying these advantages, quadrotors have been widely used in fields, such as military services, surveillance, fire fighting, and environmental monitoring [1]–[3]. To accomplish the mission of these industrial applications, the high precision attitude and position control should be necessary. However, it is difficult to control quadrotors for autonomous flight due to high nonlinearity, strong coupled states, and open-loop instability. A quadrotor is a typical underactuated system which the number of actuators is less than the degrees of freedom. And obtaining the exact values of parameters such as moment of inertia, lift and drag coefficients is very hard. In addition, external disturbances, such as fickle winds, degrade the control performance of quadrotors because of their small and light weight [28].

Proportional-integral-derivative controllers and linear quadratic regulators have been widely used in representative linear control methods [4]–[6]. However, the control performance may deteriorate at outside of the region around the equilibrium point owing to strong nonlinear terms. Therefore, various nonlinear control methods have been developed for quadrotors [7]–[25]. Feedback linearization methods were developed to eliminate nonlinear term using feedback [9], [10]. The Lyapunov redesign method was proposed to improve the stability from effect of high order nonlinear term and unmodeled dynamics [11]–[13].
Sliding mode control methods were proposed to improve the robustness in finite time [14]–[18]. Adaptive control methods were developed to compensate for parameter uncertainties such as mass variation [20]–[25]. Even though these methods improve stability and control performance, the external disturbance such as wind gust was not compensated for.

Disturbance observer (DOB)-based control algorithms have been developed via backstepping procedure to compensate for the external disturbances that do not satisfy matching conditions [26]–[28]. Adaptive fuzzy based control methods have been developed to compensate for the disturbances and uncertainties [29]–[31]. However, the full state feedback is required to apply these methods. Thus, an extended state observer (ESO) has been developed to estimate full state and disturbance. Various ESO-based control methods have been implemented for quadrotors [32]–[35]. Only external disturbances or partial unmodeled dynamics, such as the gyroscopic effect are considered as the disturbance in attitude dynamics. A nonlinear robust compensation method was developed using robust filter [36]. A finite-time backstepping controller was proposed to guarantee the finite-time convergence using an augmented sliding mode observer [37]. A nonlinear augmented observer-based control was proposed to reject the stochastic noise based on frequency analysis [38]. In [38], the external disturbance and model dynamics were lumped into disturbance. In practice, it is difficult to estimate the disturbance which include the external disturbance and model dynamics. Therefore, a high observer gain is necessary to obtain precise disturbance estimation performance. However, a high observer gain may result in amplifying high frequency measurement noise and inducing input saturation [39], [40].

In this study, we propose an ESO-based robust position tracking control method using nonlinear damping gain to improve the control performance under external disturbances and parameter uncertainties for quadrotors. The proposed method consists of an ESO and a nonlinear damping controller (NDC). The ESO is designed to estimate full state and disturbances. The disturbance is defined as including external disturbance, model dynamics and the uncertainties of input parameters. The NDC is developed via backstepping procedure to suppress output tracking error according to disturbance estimation error. Although poor disturbance estimation performance may degrade the position tracking performance, the nonlinear damping gain increases to enhance the damping effect. The proposed method is simple and robust against external disturbance and parameter uncertainties. In addition, only nominal values of input gain parameters are required. The closed-loop stability is investigated to prove that the proposed method ensures the uniform ultimate boundedness of position and attitude tracking errors using the input-to-state stability (ISS) property. The position tracking performance of proposed method is verified by performing hardware-in-the-loop-simulation (HILS) using a quadrotor platform.

The main contributions of this study are as follows:

- An NDC is designed to suppress the tracking error according to the disturbance estimation error.
- Only the nominal value of the input gain parameters are required for the proposed method.
- The proposed method is simple and robust against external disturbance and parameter uncertainties.

II. MATHEMATICAL MODEL AND PROBLEM FORMULATION

In this section, the dynamics of quadrotor is explained [15]. A quadrotor can be simplified by considering only the cross-shaped rigid frame with four rotors. The simplified model of the quadrotor is shown in Fig 1. The inertia frame with respect to earth is defined as $P_E = [X_E, Y_E, Z_E]$, and the body-fixed frame with respect to body is defined as $P_B = [X_B, Y_B, Z_B]$. Define $\xi = [\phi, \theta, \psi]^T$ and $\omega = [p, q, r]^T$, where $\phi$, $\theta$, and $\psi$ denote the angle of roll, pitch, and yaw with respect to the inertia frame, respectively, and $p$, q, and $r$ denote the angular velocity of roll, pitch, and yaw with respect to the body-fixed frame, respectively. The coordinate transform matrix, $R_t$, from $P_B$ to $P_E$ can be obtained as follows:

$$
\begin{bmatrix}
C\psi C\theta & C\psi S\theta S\phi - S\psi C\phi & C\psi S\theta C\phi + S\psi S\phi \\
S\psi C\theta & S\psi S\theta S\phi + C\psi C\phi & S\psi S\theta C\phi - C\psi S\phi \\
-S\theta & S\phi C\theta & C\theta C\phi
\end{bmatrix}
$$

where $C* = \cos(*)$ and $S* = \sin(*)$. The relationship between $\dot{\xi}$ and $\omega$ can be represented using the derivatives of $R_t$ with respect to time as follows:

$$
\omega = \begin{bmatrix}1 & 0 & -S\theta \\ 0 & C\phi & C\theta S\phi \\ 0 & -S\phi & C\theta C\phi \end{bmatrix} \dot{\xi}.
$$

The dynamics of the quadrotor can be obtained by using the Newton-Euler equations as follows:

$$
M\ddot{P} = e_3 Mg - F_A + R_t F
$$

$$
\omega = W(\xi)\dot{\xi}
$$

$$
J\ddot{\omega} = -\omega \times J\omega - \tau_{\text{gyro}} - \tau_A + \tau
$$

FIGURE 1. Quadrotors model.
where \( P = [x, y, z]^T \) is the absolute linear position of the quadrotor, \( e_3 = [0, 0, 1]^T \) is the unit vector, \( M \) is the mass diagonal matrix of the quadrotor, \( F_A \) and \( \tau_A \) are the aerodynamic force matrices of translational motion and rotational motion respectively, \( \tau_{gyro} \) is the gyroscopic moment, and \( J \) is the inertia diagonal matrix of the quadrotor. \( F \in \mathbb{R}^4 \) and \( \tau \in \mathbb{R}^3 \) are control inputs which are defined as follows:

\[
\begin{bmatrix}
F \\
\tau
\end{bmatrix} =
\begin{bmatrix}
U1 \\
U2 \\
U3 \\
U4
\end{bmatrix} = \begin{bmatrix}
k(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \\
lk(\Omega_2^2 - \Omega_3^2) \\
lk(\Omega_3^2 - \Omega_4^2) \\
lk(\Omega_4^2 - \Omega_1^2)
\end{bmatrix}
\] (4)

where \( k \) is the drag force coefficient, \( b \) is the lift coefficient, \( l \) is the arm length, and \( \Omega_c \) \( a \in [1, 4] \) denotes the rotor speeds of the front, right, rear, and left rotors. The gyroscopic moment, \( \tau_{gyro} \), is expressed as

\[
\tau_{gyro} = J_f(\omega \times e_3) \hat{\Omega}
\] (5)

where \( J_f \) is the inertia of the rotor, and \( \hat{\Omega} \) is the angular velocity \( \Omega = \Omega_1 + \Omega_3 - \Omega_2 - \Omega_4 \). For representing the state-space equation, state \( X \) is defined as

\[
X = [x_1, \cdots, x_{12}]^T
\]

\[
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6 \\
x_7 \\
x_8 \\
x_9 \\
x_{10} \\
x_{11} \\
x_{12}
\end{bmatrix} = \begin{bmatrix}
a_{1x}x_6 - a_{2x}x_8 - \frac{\tau_{Ax}}{I_x} + \frac{d_1}{I_x} + b_1 U_2 \\
a_{3x}x_6 + a_{4x}x_8 - \frac{\tau_{Ay}}{I_y} + \frac{d_2}{I_y} + b_2 U_3 \\
a_{5x}x_4 - \frac{\tau_{Ax}}{I_z} + \frac{d_3}{I_z} + b_3 U_4 \\
g + \frac{1}{m}[F_{Ax} + d_4] + b_4 U_1 \\
\frac{1}{m}[-F_{Ax} + d_5] + b_5 u_x \\
\frac{1}{m}[-F_{Ay} + d_6] + b_6 u_y
\end{bmatrix}
\]

\[
f(X, U) =
\begin{bmatrix}
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6 \\
x_7 \\
x_8 \\
x_9 \\
x_{10} \\
x_{11} \\
x_{12}
\end{bmatrix} = \begin{bmatrix}
da_1 + b_1^o U_2 \\
da_2 + b_2^o U_3 \\
da_3 + b_3^o U_4 \\
d_4 + b_4^o U_1 \\
d_5 + b_5^o u_x \\
d_6 + b_6^o u_y
\end{bmatrix}
\] (7)

and \( a_1 = (I_y - I_z)/I_x, a_2 = J_f/I_x, a_3 = (I_z - I_y)/I_x, a_4 = J_f/I_x, a_5 = (I_x - I_y)/I_z, b_1 = 1/I_x, b_2 = 1/I_y, b_3 = 1/I_z, b_4 = -\cos(\phi)\sin(\phi)/m, b_5 = -U_1/m \) and \( b_6 = U_1/m, U \) is the input matrix, \( U \) is thrust force, \( U_2 \), \( U_3 \), and \( U_4 \) are the rotational torques with respect to roll, pitch, and yaw, respectively, \( m \) is the mass of the quadrotor, \( I_{x,y,z} \) is the inertial diagonal matrix of the quadrotor, and \( d_{ext} = [d_x, d_y, d_z, d_\phi, d_\theta, d_\psi]^T \) denotes external disturbances. \( u_x \) and \( u_y \) are the orientations of \( U_1 \), and they are defined as follows:

\[
u_x = (\sin \psi \sin \phi + \cos \psi \sin \theta \cos \phi)
\]

\[
u_y = (\cos \psi \sin \phi - \sin \psi \sin \theta \cos \phi).
\] (8)

\( u_x \) and \( u_y \) are used as the virtual control inputs for the translational system to compute \( x_{13} \) and \( x_{14} \). The uncertainty \( \Delta b_i \) and the nominal value \( b_i^o \) of each \( b_i \) \( i \in [1, 6] \) are defined as

\[
b_i = \Delta b_i + b_i^o.
\] (9)

The disturbances, \( d_i \) \( i \in [1, 6] \), which include the external disturbances, dynamics, and the uncertainties of input parameters, are defined as

\[
d_1 = a_{1x}x_6 - a_{2x}x_8 - \frac{\tau_{Ax}}{I_x} + \frac{d_1}{I_x} + \Delta b_1 U_2
\]

\[
d_2 = a_{3x}x_6 + a_{4x}x_8 - \frac{\tau_{Ay}}{I_y} + \frac{d_2}{I_y} + \Delta b_2 U_3
\]

\[
d_3 = a_{5x}x_4 - \frac{\tau_{Ax}}{I_z} + \frac{d_3}{I_z} + \Delta b_3 U_4
\]

\[
d_4 = g + \frac{1}{m}[F_{Ax} + d_4] + \Delta b_4 U_1
\]

\[
d_5 = \frac{1}{m}[-F_{Ax} + d_5] + \Delta b_5 u_x
\]

\[
d_6 = \frac{1}{m}[-F_{Ay} + d_6] + \Delta b_6 u_y.
\] (10)

For simplification, (7) can be rewritten as

\[
f(X, U) =
\begin{bmatrix}
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6 \\
x_7 \\
x_8 \\
x_9 \\
x_{10} \\
x_{11} \\
x_{12}
\end{bmatrix} = \begin{bmatrix}
da_1 + b_1^o U_2 \\
da_2 + b_2^o U_3 \\
da_3 + b_3^o U_4 \\
da_4 + b_4^o U_1 \\
da_5 + b_5^o u_x \\
da_6 + b_6^o u_y
\end{bmatrix}
\] (11)

III. EXTENDED STATE OBSERVER DESIGN

The ESO is designed to estimate full state and disturbances. The extended state variable, \( X_e = [x_{e1}, \cdots, x_{e18}]^T \in \mathbb{R}^{18} \), and the estimation state variable, \( \hat{X}_e = [\hat{x}_{e1}, \cdots, \hat{x}_{e18}]^T \in \mathbb{R}^{18} \), are defined as follows:

\[
X_e = [\phi, \dot{\phi}, d_1, \dot{d}_1, \theta, \dot{\theta}, d_2, \dot{d}_2, \psi, \dot{\psi}, d_3, \dot{d}_3, z, \dot{z}, d_4, x, \dot{x}, d_5, y, \dot{y}, d_6]^T
\]

\[
\hat{X}_e = [\hat{\phi}, \dot{\hat{\phi}}, \hat{d}_1, \dot{\hat{d}}_1, \hat{\theta}, \dot{\hat{\theta}}, \hat{d}_2, \dot{\hat{d}}_2, \hat{\psi}, \dot{\hat{\psi}}, \hat{d}_3, \dot{\hat{d}}_3, \hat{z}, \dot{\hat{z}}, \hat{d}_4, \dot{\hat{d}_4}, \hat{x}, \dot{\hat{x}}, \dot{\hat{d}}_5, \hat{y}, \dot{\hat{y}}, \hat{d}_6]^T.
\] (12)

The ESO is designed as

\[
\hat{X}_e = A_0 \hat{X}_e + B_0 U + LC(X_e - \hat{X}_e)
\] (13)
where

$$A_o = \begin{bmatrix}
I_o & 0_{3\times3} & 0_{3\times3} & \cdots & 0_{3\times3} \\
0_{3\times3} & I_o & 0_{3\times3} & \cdots & 0_{3\times3} \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
0_{3\times3} & \cdots & \cdots & I_o & 0_{3\times3}
\end{bmatrix} \in \mathbb{R}^{18 \times 18}$$

$$I_o = \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
0 & 0 & 0
\end{bmatrix} \in \mathbb{R}^{3 \times 3}$$

$$B_o = \begin{bmatrix}
B_1 & 0_{3 \times 1} & 0_{3 \times 1} & \cdots & 0_{3 \times 1} \\
0_{3 \times 1} & B_2 & 0_{3 \times 1} & \cdots & 0_{3 \times 1} \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
0_{3 \times 1} & \cdots & \cdots & B_6 & 0_{3 \times 1}
\end{bmatrix} \in \mathbb{R}^{18 \times 6}$$

$$B_1 = \begin{bmatrix}
0 & b_1^o & 0
\end{bmatrix}^T \in \mathbb{R}^{3 \times 1}, \quad i \in [1, 6]$$

$$L = \begin{bmatrix}
L_1 & 0_{3 \times 1} & 0_{3 \times 1} & \cdots & 0_{3 \times 1} \\
0_{3 \times 1} & L_4 & 0_{3 \times 1} & \cdots & 0_{3 \times 1} \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
0_{3 \times 1} & \cdots & \cdots & L_{16} & 0_{3 \times 1}
\end{bmatrix} \in \mathbb{R}^{18 \times 6}$$

$$L_1 = \begin{bmatrix}
L_{i+1} & L_{i+2}
\end{bmatrix}^T \in \mathbb{R}^{3 \times 1}, \quad s \in [1, 4, 7, 10, 13, 16]$$

$$C = \begin{bmatrix}
C_o & 0_{1 \times 3} & \cdots & 0_{1 \times 3} \\
0_{1 \times 3} & C_o & \cdots & 0_{1 \times 3} \\
\vdots & \vdots & \ddots & \vdots \\
0_{1 \times 3} & \cdots & \cdots & C_o
\end{bmatrix} \in \mathbb{R}^{6 \times 18}$$

$$C_o = \begin{bmatrix}
0 & 0 & 0
\end{bmatrix} \in \mathbb{R}^{1 \times 3}$$

and $L$ is the observer gain matrix. The estimation error matrix of the extended state is defined as

$$\hat{X}_e = X_e - \hat{X}_e.$$  \hspace{1cm} (14)

The dynamics of $\hat{X}_e$ becomes

$$\dot{\hat{X}}_e = (A_o - LC) \hat{X}_e + B_d \delta$$  \hspace{1cm} (15)

where

$$\delta = \begin{bmatrix}
\delta_1 & \delta_2 & \delta_3 & \delta_4 & \delta_5 & \delta_6
\end{bmatrix}^T \in \mathbb{R}^6$$

$$B_d = \begin{bmatrix}
I_d & 0_{3 \times 1} & \cdots & 0_{3 \times 1} \\
0_{3 \times 1} & I_d & \cdots & 0_{3 \times 1} \\
\vdots & \vdots & \ddots & \vdots \\
0_{3 \times 1} & \cdots & \cdots & I_d
\end{bmatrix} \in \mathbb{R}^{18 \times 6}$$

$$I_d = \begin{bmatrix}
0 & 0 & 0 & 1
\end{bmatrix}^T \in \mathbb{R}^{3 \times 1}$$

and $\delta_i$, $i \in [1, 6]$ is the derivative of the disturbances. In most practical systems such as quadrotors, vehicles, hydraulic actuators, and motors, all state variables and external disturbances are physically bounded, as the inputs are bounded in practical system [42]. Thus, assumptions 1 and 2 are reasonable.

**Assumption 1:** The disturbance, $d \in \mathbb{R}^6$, and the derivative of the disturbance, $\dot{d} \in \mathbb{R}^6$, are bounded. Thus, the upper bound of $|\delta|$, represented by $\delta_{\text{max}} \in \mathbb{R}^6$, is positive such that

$$\sup_{0 \leq t \leq \infty} |\delta(t)| = \delta_{\text{max}}.$$  \hspace{1cm} (16)

Assumption 2: $\hat{X}_e \in B_{3y} = \|\hat{X}_e\|_2 \leq b_3$, where $b_3$ is an unknown positive constant.

**Theorem 1:** Under Assumptions 1 and 2, if the observer gain matrix, $L$, is selected such that $A_{eo} = A_o - LC$ is a Hurwitz matrix, the perturbation term satisfies the condition as

$$|\delta| \leq \delta_{\text{max}} \leq \sqrt{\lambda_{\text{min}}(P_o) / \lambda_{\text{max}}(P_o)} \eta b_3$$  \hspace{1cm} (17)

for all $t > 0$, $\hat{X}_e \in B_{3y}$ and positive constant $\eta < 1$, where $P_o > 0$ is positive definite such that $A_{eo}^T P_o + P_o A_{eo} = -I$, $\lambda_{\text{max}}(P_o)$ and $\lambda_{\text{min}}(P_o)$ are the maximum and minimum eigenvalues of $P_o$, respectively. Then, for all $\|\hat{X}_e(t_0)\| < \sqrt{\lambda_{\text{min}}(P_o) / \lambda_{\text{max}}(P_o)}) b_3$, $\hat{X}_e(t)$ satisfies

$$\|\hat{X}_e(t)\|_2 \leq k \exp(\rho(t - t_0)) \|\hat{X}_e(t_0)\| \quad \forall t_0 \leq t < t_o + T$$

$$\|\hat{X}_e(t)\|_2 \leq k \frac{2 \lambda_{\text{max}}(P_o) \delta_{\text{max}}}{\eta} \quad \forall t \geq t_o + T$$  \hspace{1cm} (18)

for some finite $T$, where $k = \sqrt{\lambda_{\text{max}}(P_o) / \lambda_{\text{min}}(P_o)})$ and $\rho = 1 - (1 - \eta)$.

**Proof:** We define the Lyapunov candidate function $V_o$ as

$$V_o = \hat{X}_e^T P_o \hat{X}_e.$$  \hspace{1cm} (19)

The derivative of $V_o$ with respect to time is

$$\dot{V}_o = \hat{X}_e^T [A_{eo}^T P_o + P_o A_{eo}] \hat{X}_e + 2 \hat{X}_e^T P_o B_d \delta \leq -\|\hat{X}_e(t)\|_2^2 + 2 \delta_{\text{max}} \|P_o\|_2 \|\hat{X}_e(t)\|_2$$

$$= -(1 - \eta) \|\hat{X}_e(t)\|_2^2 - \eta \|\hat{X}_e(t)\|_2^2$$

$$+ 2 \delta_{\text{max}} \lambda_{\text{max}}(P_o) \|\hat{X}_e(t)\|_2$$

$$\leq -(1 - \eta) \|\hat{X}_e(t)\|_2^2 \quad \forall \|\hat{X}_e(t)\|_2 \geq \frac{2 \lambda_{\text{max}}(P_o) \delta_{\text{max}}}{\eta}.$$  \hspace{1cm} (20)

The proof is completed by applying Theorem 4.18 in [43].

**IV. NONLINEAR DAMPING CONTROLLER DESIGN**

This section describes the design of the NDC via backstepping to suppress the position tracking error when estimation error increases owing to the disturbances. As the dynamics of the quadrotor consists of six second-order single-input-single-output systems (11), the general form is represented as second-order system. The tracking error, denoted by $e = [e_1 \ e_2 \ \cdots \ e_11 \ e_{12}]^T \in \mathbb{R}^{12}$, is defined as

$$e_i = x_i - x_{id}, \quad i \in [1, 12]$$  \hspace{1cm} (21)

where $x_{id}$ is yet to be defined. The estimated tracking error $\hat{e}_i$ is defined as

$$\hat{e}_i = \hat{x}_i - x_{id}, \quad i \in [1, 12].$$  \hspace{1cm} (22)

The tracking error dynamics can be defined as

$$\dot{\hat{e}}_i = e_{i+1} + x_{i+1d} - \hat{x}_{id}, \quad i \in [1, 3, 5, 7, 9, 11]$$

$$\dot{\hat{e}}_{i+1} = b_i^o u_{id} + d_i - \hat{x}_{i+1d}, \quad j = [1, 6].$$  \hspace{1cm} (23)
To ensure the boundedness of the tracking error \( e_i \), the desired state and NDC input are designed as
\[
x_{i+1;l} = -k_i e_i + \hat{x}_i, \quad i \in \{1, 3, 5, 7, 9, 11\}, \quad j = [1, 6]
\]
\[
u_j = \frac{1}{b_j^2} (-k_{i+1} e_i + \hat{x}_i - \hat{d}_i)
\]
where control gains \( k_i, k_{i+1}, k_d, k_{d+1}, \gamma_i, \) and \( \gamma_{i+1} \) are positive constants. The control input \( u_j \) in (24) consists of two parts, i.e., the stabilization part, \( u_a \), and the nonlinear damping part, \( u_b \). The nonlinear damping part is considered to suppress the tracking error \( e_i \) according to disturbance estimation error. The disturbances include the external disturbances, unmodeled dynamics, and parameter uncertainties. Thus, it is difficult to accurately estimate \( d_i \) when the disturbance increases. Generally, if \( u_a \) is used exclusively, the tracking error, \( e_i \), increases as much as the estimation error of disturbance, \( \hat{d}_i \). The nonlinear damping gain, \( u_b \), can improve the damping effect of \( d_i \) to \( e_i \), when \( \hat{e}_i \) and \( \hat{d}_i \) increase, because \( \hat{d}_i \) becomes larger with the increase of \( d_i \). We define nonlinear damping gain \( k_d(\hat{e}_i, \hat{d}_i) \) as
\[
k_d(\hat{e}_i, \hat{d}_i) = \left( k_d \sqrt{\hat{e}_i^2 + \gamma_i} + k_{d+1} \sqrt{\hat{d}_i^2 + \gamma_{i+1}} \right). \quad (25)
\]

**Theorem 2:** Consider the tracking error dynamics (23). If the proposed control input (24) is applied to (23), the tracking error dynamics (23) is the serial interconnected system of ISS systems with the following property:
\[
|e_i(t)| \leq \exp \left( -\frac{k_i}{2} \right) |e_i(0)| + \frac{2}{k_i} \sup_{0 \leq \tau \leq t} |e_i(\tau)|
\]
\[
|e_{i+1}(t)| \leq \exp \left( -\frac{k_{i+1}}{2} \right) |e_{i+1}(0)| + \sup_{0 \leq \tau \leq t} \sigma(\tau) \quad (26)
\]
where
\[
\sigma = \frac{\|\hat{d}_i\|}{0.5k_{i+1} + k_d(\hat{e}_i, \hat{d}_i)}. \quad (27)
\]

**Proof:** The tracking error dynamics (23) with the control law (24) becomes
\[
\dot{e}_i = -k_i e_i + e_{i+1}, \quad i \in \{1, 3, 5, 7, 9, 11\}
\]
\[
\dot{e}_{i+1} = -k_{i+1} e_{i+1} - k_d(\hat{e}_i, \hat{d}_i) e_{i+1} + \hat{d}_i. \quad (28)
\]
From (28), the derivative of \( \frac{e_{i+1}^2}{2} \) becomes
\[
\frac{d}{dt} \left( \frac{e_{i+1}^2}{2} \right) = -k_{i+1} e_{i+1}^2 - k_d(\hat{e}_i, \hat{d}_i) e_{i+1}^2 + \hat{d}_i e_{i+1} \leq \frac{k_{i+1}}{2} e_{i+1}^2 - \left( \frac{k_{i+1}}{2} + k_d(\hat{e}_i, \hat{d}_i) \right) |e_{i+1}|(|e_{i+1}| - \sigma) \leq \frac{-k_{i+1}}{2} e_{i+1}^2 \forall |e_{i+1}| \geq \sigma. \quad (29)
\]

We can derive the following result using [44], Th. C.2:
\[
|e_{i+1}(t)| \leq \exp \left( -\frac{k_{i+1}}{2} t \right) |e_{i+1}(0)| + \sup_{0 \leq \tau \leq t} \sigma(\tau). \quad (30)
\]
From (30), we can show that the relationship between \( e_{i+1} \) and \( \sigma \) satisfies the ISS property. Similarly, the derivative of \( e_i^2 \) becomes
\[
\frac{d}{dt} \left( \frac{e_i^2}{2} \right) = -k_i e_i^2 + e_i e_{i+1} \leq \frac{k_i}{2} e_i^2 - \left( \frac{k_i}{2} \right) |e_i|(|e_i| - \frac{2}{k_i} |e_{i+1}|) \leq \frac{k_i}{2} e_i^2 \forall |e_i| \geq \frac{2}{k_i} |e_{i+1}|. \quad (31)
\]
Then,
\[
|e_i(t)| \leq \exp \left( -\frac{k_i}{2} t \right) |e_i(0)| + \frac{2}{k_i} \sup_{0 \leq \tau \leq t} e_{i+1}(\tau). \quad (32)
\]
Equation (32) shows that the relationship between \( e_{i+1} \) and \( e_i \) satisfies the ISS property. From (30) and (32), the ISS property of the overall tracking error system is represented by (26). Thus, the tracking error dynamics (28) is the serial interconnected system of ISS systems.

**Remark 1:** The expression for \( \sigma \) (27) includes \( k_d(\hat{e}_i, \hat{d}_i) \) in the denominator. Therefore, the \( \sigma \) decreases when \( \hat{e}_i \) and \( \hat{d}_i \) increase because of external disturbance. In other words, the nonlinear damping gain, \( u_b \), suppresses the effects of \( |\hat{d}_i| \) to \( e_{i+1} \). According to the ISS property (26), as \( t \to \infty \)
\[
|e_i(\infty)| \leq \frac{2}{k_i} \sup_{0 \leq \tau \leq \infty} |e_{i+1}(\tau)| \leq \frac{2}{k_i} \sup_{0 \leq \tau \leq \infty} \sigma(\tau). \quad (33)
\]

The tracking error, \( e_i \), can be effectively suppressed even though \( |\hat{d}_i| \) increase.

**Remark 2:** The schematic of proposed method is shown in Fig. 2. As mentioned previously, a quadrotor is a typical underactuated system in which the number of actuators is less than the degrees of freedom. Thus, \( u_a \) and \( u_b \) are used to generate the desired attitude as the virtual control input for
the position control. Based on (8) and simple calculation, \(x_{1d}\) and \(x_{3d}\) can be obtained as
\[
x_{1d} = \arcsin(u_x \sin x_{5d} + u_y \cos x_{5d})
\]
\[
x_{3d} = \arcsin\left(\frac{u_x \cos x_{5d} - u_y \sin x_{5d}}{\cos x_{1d}}\right).
\]

**V. ANALYSIS OF CLOSED-LOOP STABILITY**

This section describes closed-loop stability analysis. Note that only the output feedback is available. In (24), the actual states must be substituted with in estimated state. The estimated control input, \(\hat{u}_j\), \(j = [1, 6]\), is defined as
\[
\hat{u}_j = \frac{1}{b_j} \left( -k_{i+1} \hat{e}_{i+1} + \hat{k}_{i+1} \hat{d}_i \right) + \frac{1}{b_j} \left( -k_d \sqrt{\hat{e}_i^2 + \gamma_i} + k_{d,i+1} \sqrt{\hat{d}_i^2 + \gamma_{i+1}} \right) \hat{e}_{i+1}.
\]

As the output states are available, the same desired states (24) designed by the backstepping control law are used in (35). The estimated control input (35) is applied to (23), and the closed-loop tracking error dynamics can be obtained as
\[
\dot{\tilde{e}}_i = A_{ei} \tilde{e}_i + B_{ei} \tilde{d}_i \quad i \in \{1, 3, 5, 7, 9, 11\}
\]
where
\[
A_{ei} = \begin{bmatrix} e_i & e_{i+1} \end{bmatrix}^T
\]
\[
B_{ei} = \begin{bmatrix} -k_i & 1 \\ 0 & -k_{i+1} \end{bmatrix}
\]
\[
\tilde{d}_i = -k_d(\hat{e}_i, \hat{d}_i) e_{i+1} + d_i - \hat{d}_i + b'_{i} \hat{u} - b'_{i} u.
\]

The closed-loop system is represented as
\[
\dot{\tilde{X}}_e = A_{e\delta} \tilde{X}_e + B_{d\delta} \quad i \in \{1, 3, 5, 7, 9, 11\}.
\]

As the actual output states and the same desired states are used in \(u\) (24) and \(\hat{u}\) (35), the positive value, \(\gamma\), can be defined as
\[
|d_i - \hat{d}_i + b'_{i} \hat{u} - b'_{i} u| \leq \gamma \|\tilde{X}_e\|.
\]

**Theorem 3:** Consider the closed-loop tracking error dynamics (36). Based on the estimated control input, \(\hat{u}\) (35), the closed-loop tracking error dynamics (36) is the serial interconnected system of ISS systems with the following property:
\[
|e_i(t)| \leq \exp\left(\frac{-k_{i+1}}{2} t\right) |e_i(0)| + \frac{2}{k_i} \sup_{0 \leq \tau \leq t} |e_{i+1}(\tau)|
\]
\[
|e_{i+1}(t)| \leq \exp\left(\frac{-k_{i+1}}{2} t\right) |e_{i+1}(0)| + \sup_{0 \leq \tau \leq t} \sigma_c(\tau)
\]
where
\[
\sigma_c \leq \frac{\gamma \|\tilde{X}_e\|}{0.5k_{i+1} + k_d(\hat{e}_i, \hat{d}_i)}.
\]

**Proof:** In (36), the dynamics of the \(e_{i+1}\) with estimated control input becomes
\[
\dot{e}_{i+1} = -k_{i+1} e_{i+1} - k_d(\hat{e}_i, \hat{d}_i) e_{i+1} + d_i - \hat{d}_i + b'_{i} \hat{u} - b'_{i} u.
\]

The derivative of \(e_{i+1}^2\) is obtained from (43), in a similar manner as the aforementioned proof of theorem 2, as follows:
\[
\frac{d}{dt} \left(\frac{e_{i+1}^2}{2}\right) \leq \frac{k_{i+1}}{2} e_{i+1}^2 - \left(k_{i+1} \frac{e_{i+1}}{2} + k_d(\hat{e}_i, \hat{d}_i) \right) |e_{i+1}| - \sigma_c
\]
\[
\sigma_c = \frac{|d_i - \hat{d}_i + b'_{i} \hat{u} - b'_{i} u|}{0.5k_{i+1} + k_d(\hat{e}_i, \hat{d}_i)}
\]

Thus, equation (30) becomes
\[
|e_{i+1}(t)| \leq \exp\left(\frac{-k_{i+1}}{2} t\right) |e_{i+1}(0)| + \sup_{0 \leq \tau \leq t} \sigma_c(\tau).
\]

From equation (46), we can show that the relationship between \(e_{i+1}\) and \(\sigma_c\) satisfies the ISS property. From equation (46) and (32), the ISS property of the overall closed-loop tracking error system is shown in equation (41). Thus, the closed-loop tracking error dynamics (36) is the serial interconnected system of ISS systems.

**Remark 3:** \(\sigma_c\) (42) depends on the magnitude of estimation error. From equation (17) and (20), the upper bound of estimation error is determined by \(b_k\) and eigenvalue of \(P_q\). Generally, as \(b_k\) is unknown and constant, a high observer gain is required to decrease the upper bound of estimation error. However, a small \(b_k\) and a high observer gain are not necessary to obtain a small \(\sigma_c\) in the proposed method. This is because, when \(\hat{e}_i\) and \(\hat{d}_i\) increase with \(\|\tilde{X}_e\|\), and hence, \(k_d(\hat{e}_i, \hat{d}_i)\) increases. In other words, even though estimation performance is insufficient, a small \(\sigma_c\) can be obtained from...
The following three cases were examined:

Case 1: Conventional backstepping control.

The control gains were selected as $k_1 = 400$, $k_2 = 1.2$, $k_3 = 400$, $k_4 = 1.2$, $k_5 = 100$, $k_6 = 1$, $k_7 = 2.5$, $k_8 = 1.5$, $k_9 = 2.5$, $k_{10} = 2.5$, $k_{11} = 2.5$, and $k_{12} = 2.5$. The desired states were same as those in (35). The control input is designed as follows:

$$U2 = \frac{1}{b^2} (-k_2 \dot{e}_2 + \dot{x}_3 - a_1 x_4 x_6 + a_2 x_4 \dot{\omega})$$

$$U3 = \frac{1}{b^2} (-k_4 \dot{e}_4 + \dot{x}_4 - a_3 x_2 x_6 - a_4 x_2 \dot{\omega})$$

Case 2: ESO-based backstepping control.

The disturbances, ESO, and control input were defined as in (10), (13), and (35), respectively. The control gains as those for Case 1 were used. The nonlinear damping gains were selected as $k_{dl} = 0$, and $\gamma_i = 0$, $i \in \{1, 12\}$. The ESO gains were selected as $l_i = 53$, $l_{s+1} = 895$ and $l_{s+2} = 4875$, $s \in \{1, 4, 7, 10, 13, 16\}$. The initial values used in the simulations are as follows: $x_i(0) = 0$, $i \in \{1, 12\}$ and $\dot{x}_{i}(0) = 0$, $j \in \{1, 18\}$.

Case 3: Proposed method.

The disturbances, ESO, and control input are defined as in (10), (13), and (35), respectively. The control gains were selected as $k_1 = 400$, $k_2 = 1$, $k_3 = 400$, $k_4 = 1$, $k_5 = 100$, $k_6 = 1$, $k_7 = 2.5$, $k_8 = 1$, $k_9 = 2.5$, $k_{10} = 2$, $k_{11} = 2.5$, and $k_{12} = 2$. The nonlinear damping gains were selected as $k_{dl} = 1$, and $\gamma_i = 0.1$, $i \in \{1, 12\}$. The same ESO gains and initial values are the same as those used in Case 2.

To evaluate the control performance of the proposed method, the control gains of in Case 1 and 2 were set to be larger than the control gains in Case 3. Wind was injected along the $X$-$Y$ axis as the external disturbance, as follows:

$$d_x = \begin{cases} 
0, & 0 \leq t \leq 30 \\
-0.5(1 - e^{-0.5t}), & 30 \leq t.
\end{cases}$$
\[ d_y = \begin{cases} 0, & 0 \leq t \leq 30 \\ 0.5(1 - e^{-0.5t}), & 30 \leq t \end{cases} \quad (49) \]

The desired positions were defined such as
\[ x_d = \begin{cases} 0, & 0 \leq t \leq 2.5 \\ (1 - e^{-0.5t}) \sin(0.1\pi t), & 2.5 \leq t \end{cases} \]
\[ y_d = \begin{cases} 0, & 0 \leq t \leq 7.5 \\ (1 - e^{-0.5t}) \cos(0.1\pi t), & 7.5 \leq t \end{cases} \]
\[ z_d = \begin{cases} (1 - e^{-0.5t}), & 0 \leq t \leq 1.5 \\ (1 - e^{-0.5t}) + 0.02t, & 1.5 \leq t \end{cases} \quad (50) \]

The estimation performances of proposed methods are shown in Figs. 4 and 5. The position, velocity, attitude, and angular velocity were accurately estimated using only nominal input gain parameters. The estimated disturbances are shown in Fig. 5. After \( t = 30 \) s, the offset is observed in \( d_5 \) and \( d_6 \) because of the injected wind. In addition, a small wave is observed in \( d_5 \) and \( d_6 \) because of air drag force. The desired and actual trajectories of the quadrotor in Cases 1-3 are shown in Fig. 6. The quadrotor takes off from the origin of the inertial reference frame, \( P_E \), and follows the desired a cylindrical spiral trajectory. The position tracking performance in Case 3 is better than those in Cases 1 and 2. The position tracking errors in Cases 1-3 are shown in Fig. 7. In Case 1, a large steady-state error was observed at \( t = 30 \) s, because the wind disturbance is not compensated for. In contrast, steady-state
The X tracking error in Case 3 was smaller than that in Case 2 at \( t = 46s, 58s, \) and \( 68s \), because the nonlinear damping gain (25) was simultaneously increased as much as increment of estimated disturbance. Similar to the X tracking error, the Y tracking error in Case 3 was smaller than that in Case 2, because the increase in external disturbance leads to the increase in nonlinear damping gain (25) in order to indirectly suppress the effect of the disturbance. Z tracking error in Case 3 was reduced by the nonlinear damping gain at hovering. In addition, the settling time for Case 3 was smaller than that for Cases 1 and 2. Videos of the HILS experiments are available at the following web-site.

- Case 1: https://youtu.be/FOW_w5BJmLU
- Case 2: https://youtu.be/BMs28YT90RY
- Case 3: https://youtu.be/_RLg3J2gbRw

Error comparison of Cases 1-3: https://youtu.be/4ZcLGSsDIPk

VII. CONCLUSION

We proposed an ESO-based robust position tracking control method using nonlinear damping gain to improve control performance under external disturbance and parameter uncertainties. The ESO was designed to estimate full state and disturbances, which included the external disturbance, model dynamics and the uncertainties of the input parameters. An NDC was developed via backstepping to suppress position and attitude tracking error according to disturbance estimation error. In experimental results, showed that the ESO accurately estimated the actual states and disturbances. Therefore, the tracking errors of ESO-based control methods did not have steady-state error by compensating for estimated disturbance. In addition, when the external disturbance increased, the nonlinear damping gain simultaneously increased to suppress the effect of the disturbance. Therefore, the position tracking error of proposed method was lesser than that of the conventional backstepping control and ESO-based backstepping control with larger control gains. In future work, we will focus on setting up a real quadrotor experimental environment with a position sensor which has centimeter resolution, i.e., real-time kinematic global navigation satellite system, and evaluating the proposed control strategy via outdoor flight experiments.

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(Sesun You and Kwanyeon Kim are co-first authors.)

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