Using $e^-e^+ \rightarrow b\bar{b}$ to test properties of new interactions at LEP2 and higher energies

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Abstract

We show that in $e^-e^+$ colliders at energies above the $Z$-peak, the process $e^-e^+ \rightarrow b\bar{b}$ becomes very sensitive to the presence of residual New Physics (NP) effects described by the $dim = 6 SU(3) \times SU(2) \times U(1)$ gauge invariant operators $O_{qW}$, $O_{qB}$ and $O_{bB}$. This observation should be combined with the already known great sensitivity of the light fermion production through $e^-e^+$ annihilation above the $Z$-peak, to the bosonic operators $\mathcal{O}_{BW}$ and $\mathcal{O}_{DB}$. It is important to emphasize that the effects of all these operators are largely hidden at the $Z$-peak; while they are enhanced above it through the use the "$Z$-peak subtracted representation". The observability limits for detecting these operators at LEP2 and NLC, through such light fermion production processes, are also established.

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1 Introduction.

It is very reasonable to expect that the part of the New Physics (NP) beyond the Standard Model (SM), which is responsible for the mass generation mechanism, should predominantly affect the scalar sector as well as the sectors most closely related to it; namely the one of the heavy quarks of the third family and the sector of the gauge bosons.

Assuming that the particles responsible for NP are too heavy to be directly produced in the present and forthcoming Colliders, and integrating them out, we end up with a description of the residual NP effects in terms of $SU(3) \times SU(2) \times U(1)$ gauge invariant operators constructed in terms of the fields of the aforementioned sectors. If we also assume that the NP scale is sufficiently larger than the electroweak scale, then one can restrict the list of such operators to those of dimension $= 6$ \cite{1}. In the present discussion we will further assume NP to be CP-invariant.

For purely bosonic CP conserving interactions, the NP effects are then described in terms of 11 $\text{dim} = 6$ operators involving $\gamma$, $W$, $Z$ and Higgs boson fields \cite{2, 3}, and another 3 ones involving also the gluon field \cite{5}. Direct ways to test the presence of such interactions have been proposed by mainly studying boson pair ($W^+W^-$, $HZ$, $H\gamma$) production in the $e^+e^-$ or $\gamma\gamma$ collisions and also at hadron colliders, \cite{4, 6, 7}. Indirect tests through fermion pair production at Z peak and beyond have been done by looking at gauge boson self-energy corrections and also at the vertex corrections induced by these operators \cite{8, 9, 10}.

Related studies concerning the NP induced operators involving the quarks of the third family, which are 34 in number, have also been recently done \cite{11, 12, 13, 5, 14, 15}.

In the study of these operators, one often divides them into two categories, using the concepts of ”blind” and ”non-blind”. ”Non-blind” operators are defined as those inducing tree-level contributions to observables measurable at LEP1/SLC. Such observables may include pair production of any lepton or quark pair, (except the top), as well as studies of the gauge boson propagators properties, that are strongly constrained by the present LEP1/SLC precision measurements. It is commonly believed that the present bounds on ”non-blind” operators are very strong, and that they cannot be substantially improved by higher energy experiments at LEP2 or bigger Colliders.

It turns out that this conclusion is erroneous for the bosonic operators \cite{8, 9, 10}

\[ \overline{O}_{DB} = 2 \left( \partial_\mu B^{\mu \nu} \right) \left( \partial_\nu B_{\nu \rho} \right) , \]
\[ \overline{O}_{DW} = 2 \left( D_\mu \overrightarrow{W}_{\nu \rho}^{\mu \nu} \right) \left( D_\nu \overrightarrow{W}_{\nu \rho} \right) . \]  

Because, these operators give tree level contributions to the fermion production amplitudes that are strongly rising with $q^2$, so that the constraints achievable at LEP2 and higher energies should be substantially stronger than those possible at LEP1/SLC. Concerning the comparison of these operators, to the operators

\[ O_{DW} = \left( D_\mu \overrightarrow{W}_{\nu \rho}^{\mu \nu} \right) \left( D^\rho \overrightarrow{W}_{\nu \rho}^{\nu \rho} \right) , \quad O_{DB} = \left( D_\mu B^{\nu \rho} \right) \left( D^\rho B_{\nu \rho} \right) , \]

defined in \cite{3}, we remark that

\[ O_{DW} = \overline{O}_{DW} + 12g \frac{1}{\text{dimension}} O_W , \quad O_{DB} = \overline{O}_{DB} , \]

where $g$ is the strong coupling constant.
where

\[ O_W = \frac{1}{3!} \left( \vec{W}^\mu_{\nu} \times \vec{W}^\nu_{\lambda} \right) \cdot \vec{W}^\lambda_{\mu} . \]  

(5)

Thus, the operators \( \mathcal{O}_{DW} \), \( \mathcal{O}_{DW} \) give identical tree level contributions to the gauge boson self-energies; and the same is true for the operators \( \mathcal{O}_{DB} \), \( \mathcal{O}_{DB} \).

A contribution to the fermion production amplitudes which is strongly rising with \( q^2 \), is interesting in two aspects. On the one hand it is obviously favouring observation at higher energy colliders. And on the other hand, the use of the so-called "Z-peak subtracted representation", allows a clean disentangling of operators inducing such \( q^2 \) dependent contributions. This procedure, which consists in using as inputs Z-peak measurements and in subtracting NP contributions at \( q^2 = M_Z^2 \), manages to express all observables beyond the Z-peak in terms of only the aforementioned specific operators, while all other possible contributions automatically cancel.

In [9, 10] it was observed that \( \mathcal{O}_{DW} \) and \( \mathcal{O}_{DB} \) are the only purely bosonic such operators, and the process \( e^-e^+ \rightarrow f \bar{f} \), for any light fermion \( f \), was used to study them. In the present work we present a corresponding study for the NP operators involving heavy quarks of the third family. Among them, we find that only

\[ O_{qW} = \frac{1}{2} (q_L \gamma^\mu \tau q_L) \cdot (D_\nu \vec{W}^{\mu\nu}) , \]

(6)

\[ O_{qB} = \bar{q}_L \gamma^\mu q_L (\partial_\nu B_{\mu\nu}) , \]

(7)

\[ O_{bB} = \bar{b}_R \gamma^\mu b_R (\partial_\nu B_{\mu\nu}) , \]

(8)
generate at tree level such a strong \( q^2 \) dependent contribution. In (6-8), \( q_L = (t_L, b_L) \) is the third family left-handed quark doublet and \( D_\nu \) is the usual covariant derivative.

As stated already, the application of the "Z-peak subtracted representation" to the processes \( e^+e^- \rightarrow f \bar{f} \), uses as inputs the experimental values for the partial widths \( \Gamma(Z \rightarrow f \bar{f}) \) and the Z-peak asymmetry factors \( A_f \). Under these conditions, it can been shown that any further NP contribution to \( e^+e^- \rightarrow f \bar{f} \) must come from (some of) the five operators \( \mathcal{O}_{DW}, \mathcal{O}_{DB}, \mathcal{O}_{qW}, \mathcal{O}_{qB} \) and \( \mathcal{O}_{bB} \) only. The two bosonic ones contribute to all fermion pair production in a universal way, while \( (\mathcal{O}_{qW}, \mathcal{O}_{qB}, \mathcal{O}_{bB}) \) only contribute to \( e^+e^- \rightarrow b \bar{b} \) (and of course to \( e^+e^- \rightarrow t \bar{t} \) to which we are not interested here since there is no definite Z-peak subtraction). Since \( \mathcal{O}_{DB} \) and \( \mathcal{O}_{DW} \) can be discriminated by using the lepton production channels, as well as the channels involving quarks of the first two families, we concentrate in the present work on the process \( e^-e^+ \rightarrow b \bar{b} \), which allows the study of the three operators \( \mathcal{O}_{qW}, \mathcal{O}_{qB}, \mathcal{O}_{bB} \). In the presence of polarized beams, there are four possible observables that can be constructed, namely \( \sigma_b, A^b_E, A^b_L \) and \( A^{pol}(b) \), which allows to test and disentangle the three operators \( \mathcal{O}_{qW}, \mathcal{O}_{qB}, \mathcal{O}_{bB} \). The study therefore of the effects of the operators \( (\mathcal{O}_{DW}, \mathcal{O}_{DB}, \mathcal{O}_{qW}, \mathcal{O}_{qB}, \mathcal{O}_{bB}) \) can go beyond the usual treatment of NP effects, which usually consists in taking one operator at a time, ignoring possible correlations.

In Section 2 we calculate the effects on \( e^-e^+ \rightarrow f \bar{f} \) amplitudes starting by first using the equations of motion, which considerably simplifies the computation. We apply this
technique to all five operators, and check that for $\mathcal{O}_{DB}$ and $\mathcal{O}_{DW}$ it reproduces the results obtained in a different way in ref.\[9, 10\]. We also establish the unitarity constraints for these five operators. They allow to relate their coupling constants to the energy scale at which the two-body scattering amplitudes saturate unitarity. At this energy, new types of effects like the creation of heavy degrees of freedom, should appear in order to restore unitarity. So this energy scale corresponds to the effective NP scale $\Lambda_{NP}$. In Section 3 we compute the contributions to the observables from the $\mathcal{O}_{qW}$, $\mathcal{O}_{qB}$ and $\mathcal{O}_{bB}$ operators, using the $Z$-peak subtracted representation.

In Section 4 we apply the results to the LEP2 and NLC energy ranges and we derive the observability limits for the relevant NP couplings. Finally, Section 5 summarizes the physics issues of such an analysis, for what concerns the search for residual NP effects in $e^-e^+ \rightarrow f\bar{f}$ processes.

2 $e^-e^+ \rightarrow b\bar{b}$ observables for studying $\mathcal{O}_{DB}$, $\mathcal{O}_{DW}$, $\mathcal{O}_{qW}$, $\mathcal{O}_{qB}$, $\mathcal{O}_{bB}$.

As explained in the Introduction, the $Z$-peak subtracted representation allows to disentangle the contributions from the "derivative" operators leading to a strong $q^2$-dependence; namely $\mathcal{O}_{DB}$, $\mathcal{O}_{DW}$, $\mathcal{O}_{qW}$, $\mathcal{O}_{qB}$, $\mathcal{O}_{bB}$. In this framework the effect of the two bosonic operators has already been discussed in \[9, 10\] using the results of \[2\] obtained with the propagator formalism. According to this, all NP contributions are expressed as modifications to the $\gamma$ and $Z$ propagators. In this section we show that the contribution of the above operators to $e^+e^- \rightarrow f\bar{f}$ can be more directly computed in a very simple way using the equations of motion.

The effective Lagrangian describing the NP contribution from the $\mathcal{O}_i$ operators in (1, 2, 6-8), is

$$\mathcal{L}_{eff} = \mathcal{L}_{SM} + \sum_i \frac{f_i}{m_i^2} \mathcal{O}_i .$$

(9)

The SM equations of motion for the $B$ and $W$ fields

$$D_\mu \overline{W}^{\mu\nu} = g J^{\nu} - i \frac{g}{2} [D^\nu \Phi^\dagger \overline{\tau} \Phi - \Phi^\dagger \overline{\tau} D^\nu \Phi] ,$$

(10)

$$\partial_\mu B^{\mu\nu} = g tJ^{\nu}_Z - i \frac{g}{2} [D^\nu \Phi^\dagger \Phi - \Phi^\dagger D^\nu \Phi] ,$$

(11)

with $\overline{J}^{\nu}$ and $J^{\nu}_Z$ being the $SU(2)$ and hypercharge fermionic currents respectively, allow us to express $\mathcal{O}_i$ as

$$\overline{\mathcal{O}}_{DB} = 2g^2 J^\nu \cdot J^\mu W^\nu = \frac{g^2}{c_W} (v + H) J^\nu \cdot J^\mu Z^\nu Z^\mu ,$$

(12)

$$\overline{\mathcal{O}}_{DW} = 2g^2 J^\mu \cdot J^\mu = \frac{g^3}{\sqrt{2}} (v + H)^2 (J^\mu W^\mu + J^\mu W^-) - \frac{g^3}{c_W} (v + H)^2 Z^\mu J^3 \mu .$$
\[ + \frac{g^4}{8} (v + H)^4 \left( 2W^+W^- - \frac{1}{c_W} Z^\mu Z_\mu \right), \]  

(13)

\[ \mathcal{O}_{qB} = - g' (\bar{q}_L \gamma^\mu q_L) \cdot (J_{Y\mu} + \frac{g}{4c_W} (v + H)^2 Z_\mu), \]  

(14)

\[ \mathcal{O}_{bB} = - g' (\bar{b}_R \gamma^\mu b_R) \cdot (J_{Y\mu} + \frac{g}{4c_W} (v + H)^2 Z_\mu), \]  

(15)

\[ \mathcal{O}_{qW} = - g \left( \frac{\tau}{2} \gamma^\mu q_L \right) \tau_\mu + \]  

\[ \frac{g^2}{4} (v + H)^2 \left\{ \frac{1}{\sqrt{2}} (\bar{q}_L \gamma^\mu t^\tau q_L) W^+_\mu + \frac{1}{\sqrt{2}} (\bar{q}_L \gamma^\mu \tau^- q_L) W^-_\mu + \frac{1}{c_W} (\bar{q}_L \gamma^\mu \tau^3 q_L) Z_\mu \right\}. \]  

(16)

In (12-16) \( \tau^i \) (\( i = 1 - 3 \)) are the usual three Pauli matrices and \( t^\pm \equiv (\tau^1 \pm i\tau^2)/2 \), \( q_L = (l_L, b_L) \) is the doublet of the left-handed quarks of the third family, \( J^{\pm} = J^{1}_\mu \pm iJ^{2}_\mu \), are the charged fermion currents, and \( W^\pm \) are the fields absorbing \( W^\pm \) respectively.

From (12-16), note that these operators provide \( q^2 \)-independent NP contributions to the \( Zf\bar{f} \) couplings. These contributions are irrelevant for our treatment though, since, together with any other NP contributions from all other operators in \( t, j \), they will be absorbed in the \( Z \)-peak observables used as inputs. We also note from (12-13), that \( \tilde{\mathcal{O}}_{DB} \) and \( \tilde{\mathcal{O}}_{DW} \) induce tree level contributions to the \( W \) and \( Z \) masses, but no contribution to the \( \rho \) parameter measuring the neutral to charged current ratio \( \frac{\rho}{\rho_{NP}} \). Finally we also remark that there is no NP contribution to the \( \gamma f\bar{f} \) coupling.

We next turn to the unitarity constraints on the couplings of the \( \mathcal{O}_i \) operators. Note that in (9) we have normalized these couplings to \( m_t^2 \). We apply the same techniques as in (17, 18). We consider the strongest unitarity constraints arising from the two-body scattering amplitudes and we identify the energy at which unitarity is saturated to the scale \( \Lambda_{NP} \). We find:

\( \mathcal{O}_{qW} \): The most stringent constraint for this operators arises from the transitions among the \( j = 1 \) colour-singlet flavour-neutral channels \(|tt - + >, |bb - + >, |uu - + >, |cc - + >, |dd - + >, |ss - + >, |ee - + >, |\mu\mu - + >, |\tau\tau - + >, |\nu_e\nu_e - + >, |\nu_\mu\bar{\nu}_\mu - + >, |\nu_\tau\bar{\nu}_\tau - + >, |ZHL >, |W^+W^-LL > \). The result is

\[ f_{qW} \approx 5.7 \frac{\pi}{g} \left( \frac{m_t}{\Lambda_{NP}} \right)^2 \approx 27.5 \left( \frac{m_t}{\Lambda_{NP}} \right)^2. \]  

(17)

\( \mathcal{O}_{qB} \): The most stringent constraint also comes from the \( j = 1 \) colour-singlet flavour-neutral channels \(|tt - + >, |bb - + >, |uu - + >, |cc - + >, |dd - + >, |ss - + >, |ee - + >, |\mu\mu - + >, |\tau\tau - + >, |\nu_e\nu_e - + >, |\nu_\mu\bar{\nu}_\mu - + >, |\nu_\tau\bar{\nu}_\tau - + >, |ZHL >, |W^+W^-LL > \). The result is

\[ f_{qB} \approx 2.66 \frac{\pi}{g'} \left( \frac{m_t}{\Lambda_{NP}} \right)^2 \approx 23.4 \left( \frac{m_t}{\Lambda_{NP}} \right)^2. \]  

(18)
$O_{bB}$: The most stringent constraint comes from the same $j = 1$ channels as in the $O_{qB}$ case. The result is

$$f_{bB} \simeq 3.45 \frac{\pi}{g^2} \left( \frac{m_t}{\Lambda_{NP}} \right)^2 \simeq 30.4 \left( \frac{m_t}{\Lambda_{NP}} \right)^2 . \quad (19)$$

$O_{dB}$: The most stringent constraint comes from $j = 1$ channels which are singlet under colour, as well as under the horizontal $SU(3)$ group relating the three families. Denoting by $U$ and $D$ the generic up and down quarks and by $E$ and $N$ the charged and neutral leptons, we write these channels as $|U\bar{U}+\rangle$, $|U\bar{E}+\rangle$, $|D\bar{D}+\rangle$, $|D\bar{E}+\rangle$, $|E\bar{E}+\rangle$, $|N\bar{N}+\rangle$, $|ZHL\rangle$ and $|W^+W^-LL\rangle$. After diagonalizing the relevant $9 \times 9$ transition matrix, we obtain the constraint

$$f_{dB} \simeq 0.56 \frac{\pi}{g^2} \left( \frac{m_t}{\Lambda_{NP}} \right)^2 \simeq 13.8 \left( \frac{m_t}{\Lambda_{NP}} \right)^2 . \quad (20)$$

$O_{DW}$: The situation is similar to the previous one, but the channels are somewhat fewer now, namely $|U\bar{U}+\rangle$, $|D\bar{D}+\rangle$, $|E\bar{E}+\rangle$, $|N\bar{N}+\rangle$, $|ZHL\rangle$ and $|W^+W^-LL\rangle$. The diagonalization of the transition matrix gives

$$f_{DW} \simeq \frac{\pi}{g^2} \left( \frac{m_t}{\Lambda_{NP}} \right)^2 \simeq 7.36 \left( \frac{m_t}{\Lambda_{NP}} \right)^2 . \quad (21)$$

When calculating process $e^-e^+ \rightarrow f\bar{f}$ ($f \neq t$), at energies higher than the $Z$ peak, all fermion masses can be neglected. In such a case, the only NP contributions to which $e^-e^+ \rightarrow f\bar{f}$ is sensitive, are those which can interfere with the SM ones and are therefore characterized by the fact that the antifermions $e^+$ and $\bar{f}$ have helicities opposite to those of $e^-$ and $f$ respectively. Thus in the helicity basis, the transition matrix is fully characterized by just the helicity of the outgoing $f$ and the incoming $e^-$. Restricting to $f \neq e$, the differential cross section for $e^-e^+ \rightarrow f\bar{f}$ is then written as

$$\frac{d\sigma}{dcos\theta} = \left( \frac{\pi N_f}{2q^2} \right) \left\{ (1 - P_e P'_e)(1 + cos^2\theta)U_{11} + 2cos\theta U_{12} \right\} \left( P'_e - P_e \right)[(1 + cos^2\theta)U_{21} + 2cos\theta U_{22}], \quad (22)$$

where $P_e$ ($P'_e$) denote twice the average helicity of the incoming $e^-$ ($e^+$) beams, $N_f$ the QCD factor $N_f \simeq 3(1 + \frac{m_t}{\Lambda_{NP}})$ for quarks and $N_f = 1$ for leptons, and

$$U_{11} = \frac{1}{4} \left[ |F_{LL}|^2 + |F_{RR}|^2 + |F_{RL}|^2 + |F_{LR}|^2 \right] , \quad (23)$$

$$U_{12} = \frac{1}{4} \left[ |F_{LL}|^2 + |F_{RR}|^2 - |F_{RL}|^2 - |F_{LR}|^2 \right] , \quad (24)$$

$$U_{21} = \frac{1}{4} \left[ |F_{LL}|^2 - |F_{RR}|^2 + |F_{RL}|^2 - |F_{LR}|^2 \right] , \quad (25)$$

$$U_{22} = \frac{1}{4} \left[ |F_{LL}|^2 - |F_{RR}|^2 - |F_{RL}|^2 + |F_{LR}|^2 \right] . \quad (26)$$
The $F_{ij}$ in (23-26) denote the "reduced" helicity amplitudes, where the angular dependence is removed. The first index $i$ describes the helicity of the outgoing fermion $f$, while the second index $j$ represents the helicity of the incoming $e^-$. As is seen from (22) and remarked in [19], the angular dependence in the differential cross section for fully polarized beams allows the complete separation of all four independent $|F_{ij}|$ quantities.

Applying these for $f = b$, we have that the usual measurable quantities at any $q^2$; namely the integrated cross section, the forward-backward asymmetry, the longitudinal polarization asymmetry and the polarized forward-backward asymmetry, are obtained as

$$
\sigma_b = \frac{4\pi}{q^2} \left( 1 + \frac{\alpha_s(q^2)}{\pi} \right) U_{11}, \quad A_{FB}^b = \frac{3U_{12}}{4U_{11}}, \quad A_{LR}^b = \frac{U_{21}}{U_{11}}, \quad A_{polFB}^b = \frac{3U_{22}}{4U_{11}}
$$

(27)

As emphasized in [10], in order to be able to take into account any possible additional NP contribution not described by the operators in (12-15), we have to express the $Z$-peak contribution to $F_{ij}$ directly in terms of the measured observables at LEP1/SLC. Only then we are able to isolate the $q^2$-enhanced high energy contribution induced by the aforementioned 5 operators. Apparently there are two equivalent ways to do this. In [10] this was done by exactly absorbing all NP contributions, either on or off the $Z$-peak, to modifications of the $\gamma$ and $Z$ propagators. The value of the $Z$-propagator on the $Z$-peak is then fixed by experiment, and the NP induced by the five operators above, is completely described in terms of the off-shell behaviour of the $\gamma - Z$ propagator. This technique was then applied to the study of operators $\mathcal{O}_{DB}$ and $\mathcal{O}_{DW}$. An alternative technique consists in using the SM equations of motion, which naturally leads to a representation where all NP is expressed either in terms $Z$-peak contributions, or in contact-term effects increasing with $q^2$. In the next Section both techniques will be applied to study the operators $\mathcal{O}_{qW}$, $\mathcal{O}_{qB}$ and $\mathcal{O}_{bB}$.

3 Z-peak subtracted forms for the $e^- e^+ \rightarrow b \bar{b}$ Observables.

The "reduced" helicity amplitudes $F_{ij}$ appearing in (23-26) and describing $e^- e^+ \rightarrow f \bar{f}$ with ($e^-$-helicity = $j$) and ($f$-helicity = $i$), are written in the effective Born approximation as

$$
F_{LL} = -\alpha(q^2)Q_f - \frac{3\sqrt{T_f T_e}}{M_Z \sqrt{N_f}} \chi \frac{(1 + \tilde{v}_f)(1 + \tilde{v}_e)\epsilon(Q_f)}{\sqrt{(1 + \tilde{v}_f^2)(1 + \tilde{v}_e^2)}}
$$

\begin{align*}
&+ \frac{q^2}{m_t^2} \left[ -\alpha(q^2) \left( \frac{f_{DW} r_f^3 + 2f_{DB} Y_f}{s_W^2 + c_W^2} \right) + \delta_{bf} \sqrt{\frac{\alpha(q^2)}{4\pi}} \left( -\frac{f_{qW}}{4s_W^2} - \frac{f_{qB}}{2c_W^2} \right) \right], \\
F_{RL} = -\alpha(q^2)Q_f + \frac{3\sqrt{T_f T_e}}{M_Z \sqrt{N_f}} \chi \frac{(1 - \tilde{v}_f)(1 + \tilde{v}_e)\epsilon(Q_f)}{\sqrt{(1 + \tilde{v}_f^2)(1 + \tilde{v}_e^2)}}
\end{align*}

(28)
\[
F_{LR} = -\alpha(q^2)Q_f + \frac{3\sqrt{\Gamma_f \Gamma_e}}{M_Z \sqrt{N_f}} \left[ 1 + \tilde{v}_f(1 - \tilde{v}_e)\epsilon(Q_f) \right] \left( 1 + \tilde{v}_f^2 \right) \left( 1 + \tilde{v}_e^2 \right)
\]
\[
F_{RR} = -\alpha(q^2)Q_f - \frac{3\sqrt{\Gamma_f \Gamma_e}}{M_Z \sqrt{N_f}} \left[ 1 - \tilde{v}_f(1 - \tilde{v}_e)\epsilon(Q_f) \right] \left( 1 + \tilde{v}_f^2 \right) \left( 1 + \tilde{v}_e^2 \right)
\]

In (28-31),
\[
\chi = \frac{q^2}{q^2 - M_Z^2 + iM_Z \Gamma_Z},
\]
\(\Gamma_f\) is the \(Z \rightarrow f \bar{f}\) partial width, and
\[
\tilde{v}_f = \frac{g
u_f}{g_{Af}} = 1 - 4|Q_f|s_f^2,
\]
where \(s_f^2\) is the effective Weinberg angle for the \(f\)-fermion. Thus \(s_e^2\) (and \(s_l^2\) assuming lepton universality) is defined by the longitudinal polarization asymmetry through
\[
A_{LR} = \frac{2\tilde{v}_e}{1 + \tilde{v}_e^2},
\]
while \(s_f^2\) is defined by the so called polarization forward-backward asymmetry for the final \(f\)-fermion through
\[
A_f = \frac{2\tilde{v}_f}{1 + \tilde{v}_f^2}.
\]

In the following we will apply this for \(f = b\). In practice, as we are only interested in first order manifestations of the NP effects we can safely identify \(s_W\) with \(s_f\) or \(s_e\) (or \(s_l\)) inside the coefficients multiplying these NP terms.

In the expressions for \(F_{ij}\) given in (28-31), the first term comes from the \(\gamma\) exchange, the second from \(Z\)-exchange and the term proportional to \(q^2\) arises from the contact \(4\)-fermion interactions induced by the \(\mathcal{O}_i\) operators we are studying. One way to apply the \(Z\)-subtraction technique of [16] in these expressions is to neglect the photon and contact contributions for \(q^2 = M_Z^2\), and this way fix the coefficient of the term proportional to \(\chi\) using the LEP1/SLC measurements. This is what was done in (28-31) and its validity is based on the dominance of the \(Z\)-peak. Substituting then these to (23-26), keeping only terms linear in the NP couplings, we get through (27) the predictions for the four possible observables.
An alternative, and in principle more general way to ensure the correct Z-peak subtraction is the one suggested in [16, 9, 10] using the results of [2]. To ensure the correct Z-peak subtraction, we first decompose the NP contact terms in [25, 26] to a superposition of terms having the photon and Z Lorentz structures; i.e. $Q_f \gamma^\mu$ and $\gamma^\mu (g_{V_f} - g_{AF} \gamma_5)$. This way, we identify the $\gamma \gamma, ZZ, \gamma Z, Z \gamma$ contributions to the neutral gauge boson propagator, with the Z-peak subtracted quantities $\Delta^{lf} \alpha(q^2)$, $R^{lf}(q^2)$, $V^{lf}_{\gamma Z}(q^2)$ and $V^{lf}_{Z \gamma}(q^2)$ defined in [10]. This is done in a straightforward way, leading for $e^+ e^- \to b \bar{b}$ (i.e. $f \equiv b$) to the results

\[
\Delta^{ib} \alpha(q^2) = \frac{q^2}{m_t^2} \{4(c^2_{lW} f_{DB} + s^2_{lW} f_{DW}) - \frac{s^2_{lW}}{g} f_{qW} + \frac{2 s_{lW} c_{lW}}{g} f_{qB} + \frac{3}{g'} (1 - 2 s^2_{lW} / 3) f_{bB}\} \tag{36}
\]

\[
R^{ib}(q^2) = \frac{(q^2 - M_Z^2)}{m_t^2} \{ -4 (s^2_{lW} f_{DB} + c^2_{lW} f_{DW}) + \frac{c^2_{lW}}{g} f_{qW} + \frac{2 s_{lW} c_{lW}}{g'} (f_{qB} - f_{bB})\} \tag{37}
\]

\[
V^{ib}_{\gamma Z}(q^2) = \frac{(q^2 - M_Z^2)}{m_t^2} \{4 s_{lW} c_{lW} (f_{DB} - f_{DW}) + \frac{1}{4 g} f_{qW} - \frac{2 s_{lW} c_{lW}}{g} (f_{qB} - f_{bB})\} \tag{38}
\]

\[
V^{ib}_{Z \gamma}(q^2) = \frac{(q^2 - M_Z^2)}{m_t^2} \{4 s_{lW} c_{lW} (f_{DB} - f_{DW}) + \frac{1}{4 g} f_{qW} - \frac{2 s_{lW} c_{lW}}{g} (f_{qB} - f_{bB})\} \tag{39}
\]

Using then (22, 23), we have that the general expression of the polarized $e^+ e^- \to b \bar{b}$ angular distribution is determined by

\[
U_{11} = \frac{\alpha^2(0)}{9} \{1 + 2 \delta \Delta^{ib} \alpha(q^2)\} + \frac{2 \alpha(0)}{3} \frac{q^2 - M_Z^2}{q^2 ((q^2 - M_Z^2)^2 + M_Z^2 \Gamma_Z^2)} \frac{3 \Gamma_{l1}}{M_{l1}^{1/2}} \frac{3 \Gamma_{l3}}{M_{l3}^{1/2}} \frac{1}{N_b M_Z^2} \frac{\tilde{v}_l \tilde{v}_b}{(1 + \tilde{v}_l^2)^{1/2} (1 + \tilde{v}_b^2)^{1/2}} \times [1 + \Delta^{ib} \alpha(q^2) - R^{ib}(q^2) - 4 s_{lC} \{ \frac{V^{ib}_{\gamma Z}(q^2)}{\tilde{v}_l} + \frac{1}{3 \tilde{v}_b} V^{ib}_{Z \gamma}(q^2)\}] \tag{40}
\]

\[
U_{12} = \frac{2 \alpha(0)}{3} \frac{q^2 - M_Z^2}{q^2 ((q^2 - M_Z^2)^2 + M_Z^2 \Gamma_Z^2)} \frac{3 \Gamma_{l1}}{M_{l1}^{1/2}} \frac{3 \Gamma_{l3}}{M_{l3}^{1/2}} \frac{1}{N_b M_Z^2} \frac{\tilde{v}_l \tilde{v}_b}{(1 + \tilde{v}_l^2)^{1/2} (1 + \tilde{v}_b^2)^{1/2}} \times [1 + \Delta^{ib} \alpha(q^2) - R^{ib}(q^2)] + \frac{4 \tilde{v}_l \tilde{v}_b}{(q^2 - M_Z^2)^2 + M_Z^2 \Gamma_Z^2 (1 + \tilde{v}_l^2)(1 + \tilde{v}_b^2)} \times [1 - 2 R^{ib}(q^2) - 4 s_{lC} \{ \frac{1}{\tilde{v}_l} V^{ib}_{\gamma Z}(q^2) + \frac{1}{3 \tilde{v}_b} V^{ib}_{Z \gamma}(q^2)\}] \tag{41}
\]

\[
U_{21} = \frac{2 \alpha(0)}{3} \frac{q^2 - M_Z^2}{q^2 ((q^2 - M_Z^2)^2 + M_Z^2 \Gamma_Z^2)} \frac{3 \Gamma_{l1}}{M_{l1}^{1/2}} \frac{3 \Gamma_{l3}}{M_{l3}^{1/2}} \frac{1}{N_b M_Z^2} \frac{\tilde{v}_b}{(1 + \tilde{v}_l^2)^{1/2} (1 + \tilde{v}_b^2)^{1/2}} \times [1 + \Delta^{ib} \alpha(q^2) - R^{ib}(q^2)] + \frac{4 \tilde{v}_l \tilde{v}_b}{(q^2 - M_Z^2)^2 + M_Z^2 \Gamma_Z^2 (1 + \tilde{v}_l^2)(1 + \tilde{v}_b^2)} \times [1 - 2 R^{ib}(q^2) - 4 s_{lC} \{ \frac{1}{\tilde{v}_l} V^{ib}_{\gamma Z}(q^2) + \frac{1}{3 \tilde{v}_b} V^{ib}_{Z \gamma}(q^2)\}] \tag{42}
\]

\[
U_{22} = \frac{2 \alpha(0)}{3} \frac{q^2 - M_Z^2}{q^2 ((q^2 - M_Z^2)^2 + M_Z^2 \Gamma_Z^2)} \frac{3 \Gamma_{l1}}{M_{l1}^{1/2}} \frac{3 \Gamma_{l3}}{M_{l3}^{1/2}} \frac{1}{N_b M_Z^2} \frac{\tilde{v}_l}{(1 + \tilde{v}_l^2)^{1/2} (1 + \tilde{v}_b^2)^{1/2}} \times [1 + \Delta^{ib} \alpha(q^2) - R^{ib}(q^2)] + \frac{4 \tilde{v}_l \tilde{v}_b}{(q^2 - M_Z^2)^2 + M_Z^2 \Gamma_Z^2 (1 + \tilde{v}_l^2)(1 + \tilde{v}_b^2)} \times [1 - 2 R^{ib}(q^2) - 4 s_{lC} \{ \frac{1}{\tilde{v}_l} V^{ib}_{\gamma Z}(q^2) + \frac{1}{3 \tilde{v}_b} V^{ib}_{Z \gamma}(q^2)\}] \tag{43}
\]
the implied lower bounds on the associated unitarity scales are (comp are (20, 21))

has been discussed in [9, 10]. The sensitivity limits on the couplings found in [9, 10] and

precision measurements obtained through lepton pair and light hadron production. This

luminosity of 500 fb for the LEP2, NLC500 (unpolarized) and NLC500 (polarized) cases respectively, with a

calculating

In practice, at the accuracy at which the NP effects can be observed, both ways of

operations give identical results.

\[ \begin{align*}
U_{22} &= \frac{2\alpha(0)}{3} \left[ q^2 - M_Z^2 \right]^{1/2} + \left[ \frac{3\Gamma_t}{M_Z^2} \right]^{1/2} \frac{\tilde{v}_t}{1 + \tilde{v}_b^2} \left[ 1 + \tilde{\Delta}_Z \right] \\
&\times \left[ 1 - 2R^{(bb)}(q^2) - 4slct \left\{ \frac{1}{v_t} V^{(ib)}_{\gamma Z}(q^2) + \frac{2\tilde{v}_b}{3(1 + \tilde{v}_b^2)} V^{(bb)}_{\gamma Z}(q^2) \right\} \right], \quad (42) \\
&\times \left[ 1 - 2R^{(bb)}(q^2) - 4slct \left\{ \frac{2\tilde{v}_t}{1 + \tilde{v}_t^2} V^{(ib)}_{\gamma Z}(q^2) + \frac{1}{3\tilde{v}_b} V^{(bb)}_{\gamma Z}(q^2) \right\} \right]. \quad (43)
\end{align*} \]

In practice, at the accuracy at which the NP effects can be observed, both ways of calculating \( U_{ij} \) give identical results.

4 Observability limits at LEP2 and NLC

The operators \( \mathcal{O}_{DB} \) and \( \mathcal{O}_{DW} \) contribute in a universal way to \( e^+e^- \rightarrow f\bar{f} \) for any (light) fermion \( f \). The best constraints on their couplings obviously come from the more accurate precision measurements obtained through lepton pair and light hadron production. This has been discussed in [3, 10]. The sensitivity limits on the couplings found in [3, 10] and the implied lower bounds on the associated unitarity scales are (compare (20, 21))

\[ \begin{align*}
&\frac{|f_{DB}|}{m_t^2} (TeV^{-2}) \lesssim 0.05, \quad 0.011 \quad 0.0056 \\
&\Lambda_{NP}(DB) (TeV) \gtrsim 17 \quad 35 \quad 50
\end{align*} \]

\[ \begin{align*}
&\frac{|f_{DW}|}{m_t^2} (TeV^{-2}) \lesssim 0.026, \quad 0.005 \quad 0.0028 \\
&\Lambda_{NP}(DW) (TeV) \gtrsim 17 \quad 38 \quad 51
\end{align*} \]

for the LEP2, NLC500 (unpolarized) and NLC500 (polarized) cases respectively, with a luminosity of 500 fb\(^{-1}\) for LEP2 and 20 fb\(^{-1}\) for NLC.

These bounds imply that the effect of the \( \mathcal{O}_{DB} \) (\( \mathcal{O}_{DW} \)) operators on the \( b \) quark observables defined in [27], are at LEP2 at most

\[ \begin{align*}
&\frac{\delta \sigma_{tb}}{\sigma_{tb}} \approx 0.0016 \quad (0.0054) \quad (46) \\
&|\delta A_{FB}^b| \approx 0.0013 \quad (0.0008) \quad (47) \\
&|\delta A_{LR}^b| \approx 0.0040 \quad (0.0022) \quad (48) \\
&|\delta A_{pol,b}^f| \approx 0.0026 \quad (0.0012) \quad (49)
\end{align*} \]
while for NLC500(unpol,pol) they are at most

\[
\frac{\delta \sigma_b}{\sigma_b} \approx 0.0031, 0.0016 \quad (0.0084, 0.0047) , \quad (50)
\]

\[
|\delta A_{FB}^b| \approx 0.0021, 0.0011 \quad (0.0010, 0.0006) , \quad (51)
\]

\[
|\delta A_{LR}^b| \approx 0.0071, 0.0036 \quad (0.0032, 0.0018) , \quad (52)
\]

\[
|\delta A_{pol,b}^{FB}| \approx 0.0052, 0.0027 \quad (0.0023, 0.0013) . \quad (53)
\]

Thus, the effects of the $\mathcal{O}_{DB}$ and $\mathcal{O}_{DW}$ operators on the $b\bar{b}$ observables, expected on the basis of the sensitivity limits derived from the light fermion processes, turn out to be much smaller than the expected experimental uncertainties listed in Table 1 for a $b\bar{b}$ tagging efficiency of 25%. As shown in this Table, these uncertainties appear to be of the order of a few percent. So in the following analysis of the $e^+e^- \rightarrow b\bar{b}$ process, working with the $Z$-peak subtracted representation, we can ignore the uncertainties brought by the $\mathcal{O}_{DB}$ and $\mathcal{O}_{DW}$ operators. Therefore, we restrict to a 3-free parameter case involving $f_{qW}$, $f_{qB}$ and $f_{bB}$ only and proceed to the derivation of the observability limits. We write for each of the above four observables, $A^i$, ($i = 1, ..4$), the inequality

\[
| \sum_{j=1}^{3} K^i_j f_j | \geq \frac{\delta A^i_{exp}}{A^i_{SM}} , \quad (54)
\]

where

\[
K^i_j = \frac{d((A^i - A^i_{SM})/A^i_{SM})}{df_j} . \quad (55)
\]

In (54), $\delta A^i_{exp}$ is assumed to be given by the expected statistical uncertainty for the nominal collider luminosity, assuming that the mean value of $A^i$ is given by SM. We then combine quadratically all such information coming from the $l$ available observables ($l = 2$ at LEP2 and $l = 2$ or 4 at NLC). At one standard deviation this gives the observability domain which is outside the ellipsoid surface

\[
\sum_{i=1}^{l} \left[ \sum_{j=1}^{3} [K^i_j f_j] \left[ \frac{\delta A^i_{exp}}{A^i_{SM}} \right] \right]^{2} = 1 . \quad (56)
\]

The projections of this ellipsoid on the 3 planes spanned by pairs of couplings $(f_j, f_k)$ are shown in Fig.1. For the unpolarized case, only two observables are available, namely $\sigma_b$ and $A_{FB}^b$. Thus, experiments provide only two (linear) constraints on the system of the three NP couplings $f_j$. The system is therefore not fully constrained, and the ellipsoid degenerates into bands, in (some) planes at least. In Fig.1a,b,c, these bands are indicated with dotted lines for the unpolarized LEP2 case at 190GeV, and with solid lines for the unpolarized NLC case at 500GeV. We should remark that in passing from the LEP2 case to the unpolarized NLC one, an important reduction of the widths of the bands occurs, from $O(1)$ to a few $10^{-2}$ TeV$^{-2}$. This is of course due to the strong increase with $q^2$, of the contributions of the above operators.
Finally if $e^\pm$ beam polarization is available, then two more physical observables, namely $A_{LR}^b$ and $A_{LR}^{pol,b}$ become possible, which transforms the band into the ellipses shown in Fig.1a,b,c, and magnified in Fig.2a,b,c.

These results can be compared with a treatment of the $O_{qW}$, $O_{qB}$ and $O_{bB}$ operators one by one, deriving the corresponding sensitivities on the NP couplings and the related lower bounds on the unitarity scales. Thus, for the LEP2, NLC(unpol), and NLC(pol) cases, we get the respective results

$$\frac{f_{qW}}{m_f^2} (\text{TeV}^{-2}) \lesssim 0.60 \quad 0.036 \quad 0.032 \quad (57)$$

$$\Lambda_{NP} (\text{TeV}) \gtrsim 6.8 \quad 27.6 \quad 29.3$$

$$\frac{f_{qB}}{m_f^2} (\text{TeV}^{-2}) \lesssim 0.41 \quad 0.030 \quad 0.018 \quad (58)$$

$$\Lambda_{NP} (\text{TeV}) \gtrsim 7.6 \quad 27.9 \quad 36.0$$

$$\frac{f_{bB}}{m_f^2} (\text{TeV}^{-2}) \lesssim 0.58 \quad 0.030 \quad 0.013 \quad (59)$$

$$\Lambda_{NP} (\text{TeV}) \gtrsim 7.3 \quad 31.8 \quad 48.4$$

At NLC, if polarization is available, one can observe that the bounds obtained are less than a factor 2 stronger than those obtained in the 3-parameter case. This illustrates the quality of the disentangling provided by the four observables.

5 Conclusions

In this paper we have studied a special set of $\dim = 6\ SU(3) \times SU(2) \times U(1)$ gauge invariant operators dubbed $O_{DB}$, $O_{DW}$, $O_{qW}$, $O_{qB}$ and $O_{bB}$. These operators are essentially characterized by the two properties of being "non-blind" (i.e. affecting Z-peak observables at tree level), and of involving many derivatives which lead to strong energy dependencies.

We have emphasized that this second property allows to disentangle the effects of these operators from all other ones ("blind" or "non-blind"); provided the Z-peak subtraction technique is employed. This technique consists in fixing the values of the inputs using the LEP1/SLC data, which then implies that the high energy behaviour of $e^- e^+ \to f \bar{f}$ is only sensitive to the five operators just mentioned. We stress that this does not include any assumption on the other 43 $\dim = 6\ SU(3) \times SU(2) \times U(1)$ gauge invariant and CP symmetric operators [2, 5], since their effect is fully removed [10, 11].

We have then studied the observability of the effects of these five operators at LEP2 and NLC. To appreciate the physical importance of these, we have also established the unitarity constraints for the two-body scattering amplitudes induced by the same operators. This allows to relate the coupling constant of each of them, to an effective operator-dependent NP scale; (defined as the energy at which new degrees of freedom should be created in order to restore unitarity). The observability limits can then be expressed as lower bounds for these NP scales.

To disentangle the 5 operators above, we may proceed as follows. In a first step, the processes $e^- e^+ \to f \bar{f}$, (where $f$ is a charged lepton or a light quark $u, d, s, c$),

...
may be used to study the possible appearance of the $\mathcal{O}_{DW}$ and $\mathcal{O}_{DB}$ operators. Once
the situation concerning them is clarified, the process $e^-e^+ \rightarrow b\bar{b}$ may be used to study
the remaining three operators $\mathcal{O}_{qW}, \mathcal{O}_{qB}$ and $\mathcal{O}_{bB}$. We have shown that the uncertainties
which may affect the $\mathcal{O}_{DW}$ and $\mathcal{O}_{DB}$ effects on the process $e^-e^+ \rightarrow b\bar{b}$ are weaker than the
experimental errors affecting the corresponding observables and thus the determination
of the $\mathcal{O}_{qW}, \mathcal{O}_{qB}$ and $\mathcal{O}_{bB}$ effects.

In order to achieve the goal to fully study and discriminate these 3 operators, it is
mandatory to have polarized beams. Because only then, we will have at least three
independent $b\bar{b}$ observables. This should be possible at the NLC500 Collider.

The resulting observability limits presented in Fig.2abc, show a possible determination
of the $f_q$, $f_{qB}$, $f_{bB}$ couplings at the percent level, which means NP scales in the 20–30 TeV range. These bounds are just slightly weaker than those obtained, under the same
conditions, for the operators $\mathcal{O}_{DW}$ and $\mathcal{O}_{DB}$, which were in the 50 TeV range.

At LEP2, where no polarization is available, we only have two constraints affecting
the contributions from the three operators $\mathcal{O}_{qW}, \mathcal{O}_{qB}$ and $\mathcal{O}_{bB}$, which makes their full
determination impossible. Non trivial constraints in the form of bands for pairs of couplings
are nevertheless obtained and shown in Fig.1abc. The widths of these bands are of order $O(1 \text{ TeV}^{-2})$.

For comparison we have also treated the above operators one by one. In the NLC case
the bounds obtained are not much stronger than in the 3-parameter case. This illustrates
the quality of the disentangling provided by the four observables. In the LEP2 case, the
independent bounds for each coupling separately are at the 0.4–0.6 $\text{TeV}^{-2}$ level, which
means NP scales in the 6–8 TeV range. This is comparable with what is expected for
other operators and better than what is obtained from a similar one by one treatment at
LEP1/SLC. This point was also emphasized in a recent paper [14].

In conclusion, we would like to reiterate on the fact that, due to the lack of knowledge
of the underlying dynamics, NP manifestations can take many different forms (just note
the large number of possible $\text{dim} = 6$ operators) and that it is therefore essential to
look for ways of disentangling the various classes of effects. An analysis of experimental
data at present and future colliders along the lines presented in this paper, should bring
significant information in this direction, as it singles out a special class of operators.

Table 1: Expected accuracy on $e^-e^+ \rightarrow b\bar{b}$ observables
(for 25% $b\bar{b}$ tagging efficiency.)

| Observable | $\delta\sigma_b/\sigma_b$ | $\delta A^b_{FB}$ | $\delta A^b_{LR}$ | $\delta A^b_{\text{pol,}0}$ |
|------------|------------------------|-----------------|-----------------|-----------------|
| LEP2(190GeV, $\mathcal{L} = 500 fb^{-1}$) | 0.05 | 0.04 | – – | – – |
| NLC(500GeV, $\mathcal{L} = 20 fb^{-1}$) | 0.012 | 0.01 | 0.01 | 0.01 |

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Figure captions

**Fig.1** Constraints from $e^+e^- \rightarrow b\bar{b}$ observables in the 3-free parameter case, at LEP2 (without polarization) (*dotted*), at NLC (without polarization) (*solid*), at NLC (with polarization) (*ellipse*).
(a) projection on the $(f_{qB}, f_{bB})$ plane.
(b) projection on the $(f_{qW}, f_{bB})$ plane.
(c) projection on the $(f_{qW}, f_{qB})$ plane.

**Fig.2** Constraints from $e^+e^- \rightarrow b\bar{b}$ observables in the 3-free parameter case, at NLC with polarization.
(a) projection on the $(f_{qB}, f_{bB})$ plane.
(b) projection on the $(f_{qW}, f_{bB})$ plane.
(c) projection on the $(f_{qW}, f_{qB})$ plane.
$e^+e^- \rightarrow b\bar{b}$ constraints on $f_{qB}, f_{bB}$

Fig.1a
$e^+ e^- \rightarrow b\bar{b}$ constraints on $f_{qW}$, $f_{bB}$

Fig. 1b
\[ e^+ e^- \rightarrow b\bar{b} \] constraints on \( f_{qW}, f_{qB} \)
$e^+ e^- \rightarrow b \bar{b}$ constraints on $f_{qB}, f_{bB}$

Fig. 2a
\[ e^+ e^- \rightarrow b \bar{b} \text{ constraints on } f_{qW}, f_{bB} \]
$e^+e^- \rightarrow b\bar{b}$ constraints on $f_{qW}, f_{qB}$