Supernova remnants in clumpy media: particle propagation and gamma-ray emission

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ABSTRACT

Observations from the radio to the gamma-ray wavelengths indicate that supernova remnant (SNR) shocks are sites of effective particle acceleration. It has been proposed that the presence of dense clumps in the environment where supernovae explode might have a strong impact in shaping the hadronic gamma-ray spectrum. Here we present a detailed numerical study about the penetration of relativistic protons into clumps which are engulfed by a SNR shock, taking into account the magneto-hydrodynamical (MHD) properties of the background plasma. We show that the spectrum of protons inside clumps is much harder than that in the diffuse inter-clump medium and discuss the implications for the formation of the spectrum of hadronic gamma rays, which does not reflect anymore the acceleration spectrum of protons, resulting substantially modified inside the clumps due to propagation effects. For the Galactic SNR RX J1713-3946.7, we show that a hadronic scenario including dense clumps inside the remnant shell is able to reproduce the broadband gamma-ray spectrum from GeV to TeV energies. Moreover, we argue that small clumps crossed by the shock could provide a natural explanation to the X-ray variability observed in some hot spots of RX J1713-3946.7. Finally we discuss the detectability of $\gamma$-ray emission from clumps with the upcoming Cherenkov Telescope Array and the possible detection of the clumps themselves through molecular lines.

Key words: ISM: supernova remnants – shock waves – acceleration of particles – instabilities – radiation mechanisms: non thermal – gamma rays: general

1 INTRODUCTION

It is now well established supernova remnants can accelerate electrons and presumably also hadrons, as demonstrated by the non thermal emission detected from the great majority of SNRs. In particular, the pion-bump detected at $\approx 280$ MeV from two middle aged SNRs, IC443 and W44, indicates the presence of accelerated hadrons (Ackermann et al. 2013) (but see also Cardillo, Amato & Blasi (2016) for an alternative explanation). Nevertheless two main questions need to be answered in order to validate the idea that SNRs represent the main source of the galactic cosmic rays (CRs) observed at Earth: which is the total amount of energy channeled into relativistic particles and which is the final energy spectrum of accelerated particles injected into the interstellar medium (ISM). Gamma-ray observations provide a powerful tool to answer these questions, allowing to directly infer the properties of accelerated hadrons.

In some young SNRs it is still unclear whether the detected gamma-ray emission is produced by leptonic processes, via inverse Compton (IC) scattering, or hadronic collisions, through the decay of resulting neutral pions. On a general ground the two scenarios mainly differ in the required magnetic field strength: the IC scenario usually requires a very low magnetic field (of the order of $10$ $\mu$G) in order to simultaneously account for radio, X-ray and $\gamma$-ray emission, while the hadronic scenario requires much larger values, of the order of few hundreds of $\mu$G. Such a large magnetic field cannot result from the simple compression of interstellar magnetic field, but requires some amplification, which is, in turn, a possible signature of efficient CR acceleration itself. The case of RX J1713.7-3946 is of special interest to this respect. This remnant has been considered for long time the best candidate for an efficient acceleration scenario, mainly due to its high $\gamma$-ray flux. The detection of $\gamma$-ray emission in the energy range $[1 – 300]$ GeV by the Fermi-LAT satellite (Abdo et al. 2011) has shown an unusu-
2 SHOCK PROPAGATION THROUGH A CLUMPY MEDIUM

Observations of the interstellar medium have revealed a strong non homogeneity, particularly inside the Galactic Plane. On scales of the order of few parsecs, the dense molecular clouds (MCs) constitute structures, mainly composed of H$_2$ molecules, while their ionized component is composed by C ions. Typical temperatures and masses are respectively of the order of $T_{\text{MC}} \approx 10^2$ K and $M_{\text{MC}} \gtrsim 10^3 M_\odot$. On smaller scales (of the order of a fraction of a parsec), colder and dense molecular clumps are present: they are characterised by typical temperatures of $T \sim 10$ K, masses of the order of $M \approx 0.1 - 1 M_\odot$ and therefore number densities of the order of $n_c \gtrsim 10^3$ cm$^{-3}$. In the following, we will consider the lower bound as a reference value for the density of target gas inside individual clumps. These clumps are mostly composed by neutral particles while the most relevant ions are HCO$^+$. Typical values for the ionized density are at least equal to $n_i \lesssim 10^{-3} n_c$, while the average mass of ions is $m_i = 29 m_n$ ($m_p$ is the proton mass) and that of neutrals is $m_n = 2 m_p$. Therefore the ion to neutral mass density $\epsilon$ in clumps amounts to (Gabici & Montmerle 2016)

$$\epsilon = \frac{m_i n_i}{m_n n_n} \lesssim 1.5 \times 10^{-3} \ll 1$$

(1)

Molecular clouds are strongly influenced by the presence of cosmic rays, since most likely low-energy CRs provide their ionization rate (see Padovani, Galli & Glassgold 2009), which in turns controls both the chemistry of clouds and the coupling of plasma with local magnetic fields, and hence star formation processes.

Shocks propagating through inhomogeneities of the ISM are able to generate MHD instabilities, which modify the thermal properties of the plasma and might be able to disrupt the clumps because of thermal conduction (Orlando et al. 2008). Therefore the dynamical interaction between the shock emitted at the supernova (SN) explosion and the medium surrounding the star is an essential ingredient for the understanding of star formation processes (Hennebelle 2013; Dwarkadas 2007). In particular, the environment where type II SNe explode is most likely populated by molecular clumps: indeed, given their fast evolution, they explode in an environment rich of molecular clouds, the same that generated the star. Moreover, given their massive progenitor, strong winds in the giant phase of the star evolution accelerate the fragmentation of clouds into clumps, while creating a large cavity of hot and rarified gas around them. This scenario could be similar to the one in which the remnant of RXJ1713.7-3946 is evolving (Slane et al. 1999). Previous works in this direction (Kelion, McKee & Colella 1994) have shown that plasma instabilities develop (Fraschetti 2013; Giacalone & Jokipii 2007; Sano et al. 2012) when the upstream medium is not homogeneous. In particular in the presence of dense clumps, the amplification occurs in the contact region among the shock and the clump. These instabilities are able to amplify the background magnetic field in a layer around the clump which has a typical size of half of the clump radius (see Inoue et al. 2012). Moreover, if the density contrast between the clump and the surrounding medium is very large, the clump can survive for long time before to evaporate, even longer than the SNR age, as we will show in the next section.

2.1 MHD simulations of shock-clump interaction

The description of the thermal properties of a classical fluid follows from the solution of the Navier-Stokes equations, coupled to the induction equation for the time evolution of the background magnetic field $B_0$. The system of conservation equations to be solved reads, for non resistive fluids, as

$$\begin{align*}
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) &= 0 \\
\frac{\partial \mathbf{v}}{\partial t} + \rho \mathbf{v} \cdot \nabla \mathbf{v} &= -\frac{1}{\rho} \nabla p + \beta \left( \nabla \times (\nabla \times \mathbf{B}_0) \right) \times \mathbf{B}_0 \\
\frac{\partial \mathbf{B}}{\partial t} &= \nabla \times (\mathbf{v} \times \mathbf{B}_0) \\
\nabla \cdot \mathbf{B}_0 &= 0
\end{align*}$$

(2)

where $\rho$ is the fluid density, $\mathbf{v}$ its velocity, $p$ its pressure and $\gamma = 5/3$ is the adiabatic index of an ideal fluid.

In order to introduce a shock discontinuity, as well as the presence of a clump, a numerical approach has been adopted, though the PLUTO code (see Mignone et al. 2010). The shock-cloud interaction is implemented among one of the possible configurations provided by this code. We are interested in performing a three dimensional simulation, in cartesian coordinates. We set a finite difference scheme for the solution of Eq. (2), based on an unsplit 3rd order Runge-Kutta algorithm with an adaptive time step subject to the Courant condition $C = 0.3$. A strong shock is moving in the...
In order to investigate a situation as much similar as that of high mass star SN explosion, like RX J1713.7-3946, we set in the upstream region a low density medium with temperature \( T = 10^6 \) K and \( n_\text{up} = 10^{-2} \) cm\(^{-3}\), which gets compressed by the shock in the downstream region up to \( n_\text{down} = 4 \times 10^{-2} \) cm\(^{-3}\). As initial condition, we also set the clump in the upstream, with a density as high as \( n_c = 10^3 \) cm\(^{-3}\). Therefore, a density contrast

\[
\chi = \frac{n_c}{n_\text{up}} = 10^3 \tag{3}
\]

is assumed: if the shock speed is \( v = v_s \hat{z} = 4.4 \times 10^6 \) cm s\(^{-1}\), it propagates inside the clump with a velocity \( v_{s,c} = v_s \hat{z} \) equal to (Kelin, McKee & Colella 1994 and Inoue et al. 2012)

\[
v_{s,c} = \frac{v_s}{\sqrt{\chi}} = 1.4 \times 10^6 \text{ cm s}^{-1} \tag{4}
\]

Boundary conditions are set as outflow in all directions, except for the downstream boundary in the \( z \)-direction, where an injection flow is set. A grid of dimensions 2 pc \( \times \) 2 pc \( \times \) 2 pc is used: a spherical clump of radius \( R_c = 0.1 \) pc is located in \( x_0 = y_0 = z_0 = 1 \) pc. Such a characteristic length scale for the clump size follows from MHD simulations and corresponds to the smaller scale where thermal instability is effective (Inoue et al. 2012). All the evolution is followed in the clump reference frame.

In the MHD simulation the clump is assumed to be fully ionized: this is not the real condition, since molecular clumps are mainly composed by neutrals, as discussed in § 2. However, while the shock is passing through the ionized part of the clump, the heated ions are able to ionize the neutral part on a time scale of the order of few years. This condition allows us to consider the neutral clump as if it were completely ionized. In this process ions cool down and the pressure drops accordingly, hence reducing the shock speed to the a value given by \( v_{s,c} \) (Kelin, McKee & Colella 1994). The time needed to the shock to cross the clump is the so called clump crossing time

\[
\tau_{cc} = \frac{2R_c}{v_{s,c}} \approx 1.4 \times 10^4 \text{ yr} \tag{5}
\]

It has been shown through both analytical estimates (Kelin, McKee & Colella 1994; Chevalier 1999) and simulations (Orlando et al. 2005, 2008) that the time required for the clump to evaporate is of the order of few times \( \tau_{cc} \). This time scale is larger than the estimated age for RX J1713.7-3946, which amounts to \( T_{\text{SNR}} \approx 1620 \) yr.

A magnetic field of intensity \( B_0^\mu = 5 \mu G \) is set in the upstream region. We present here a simulation of an oblique shock, representing the most general case where the background magnetic field has two components: along the shock direction and perpendicular to it. We set \( B_0 = (B_0^x, 0, B_0^z) \), \( B_0^x = 5 \mu G \) and \( B_0^z = B_{0z} \). We are interested in the evolution of the background plasma for about 300 years from the first shock-clump interaction, as will be described in § 4. Within this time, results from MHD simulations can be summarized as follows:

I) The clump maintains its density contrast, although the density distribution tends to smoothens, as seen in Fig. 1(a);

II) The shock has not yet crossed the clump, as represented by the plasma velocity field lines in Fig. 3(a);

III) The magnetic field is amplified by about a factor 10 in a region surrounding the clump with a typical size of \( R_c/2 \), as shown in Fig. 1(b) and Fig. 2. We call this region the clump magnetic skin. Furthermore, the magnetic field around the clump is mainly directed along the tangential direction. This is an indication of the fact that it is difficult for accelerated particles to diffuse orthogonally to the clump surface, along the radial direction, as we discuss in § 3.2.

We also performed MHD simulations with purely perpendicular and purely parallel magnetic field configurations. Only in the case when the magnetic field is entirely oriented along the shock direction we see a non significant amplification of the large scale field in the region surrounding the clump.

We explored different density contrasts between the clump and the surrounding medium, performing simulations with \( \chi = 10^2, 10^3 \) and \( 10^4 \). In these cases, for a fixed shock speed, we expect the clump to evaporate on shorter time scales than the time given in Eq. (5). We found that an effective amplification of the magnetic field around the clump is obtained if \( \chi \gtrsim 10^3 \). For less massive clumps, where the evaporation time is comparable to the remnant age, two differences arise: i) once heated, these clumps would contribute to the thermal emission of the remnant and ii) the resulting gamma-ray spectrum would not manifest a pronounced hardening. Hence, detailed spectroscopic and morphological observations are crucial for providing a lower limit on the density contrast of the circumstellar medium (CSM).

3 PARTICLE TRANSPORT

CRs are scattered by MHD waves parallel to the background magnetic field. The equation which describes the temporal and spatial evolution of the CR density function in the phase space \( f(x,p,t) \) is the transport equation. It expresses the conservation of particle number in the phase space and reads as (see Ginzburg & Syrovatsky 1961 and Drury 1983)

\[
\frac{\partial f}{\partial t} + v \cdot \nabla f = \nabla \cdot [D \nabla f] + \frac{1}{3} \frac{\partial f}{\partial p} \nabla \cdot v \tag{6}
\]

where \( D \) is the diffusion coefficient in the background magnetic field and \( p \) is the particle momentum. Advection in the background velocity field is expressed by the left hand side of Eq. (6), together with the time variation of the particle density function, while diffusion and plasma adiabatic compression are expressed on the right hand side. Given the system symmetry, we will solve this equation in cylindrical coordinates, through a finite difference discretization method. A grid of 2 pc \( \times \) 2 pc is set, with a spherical clump located at \( (r_0, z_0) = (0, 0.67) \) pc. A logarithmic step is used in the radial dimension, while a uniform spacing is fixed along the shock direction. The spatial resolution of the grid is set in such a way that, for each simulated momentum, the proton energy spectrum reaches a convergence level better than 5%. We set an operator splitting scheme based on an Alternated Direction Implicit (ADI) method, flux conservative and upwind, second order in both time and space, subject to a Courant condition \( C = 0.8 \). The initial condition of the simulation includes the presence of a shock precursor in the upstream region (everywhere but inside the clump, which starts empty of CRs): it represents the equilibrium solution of the diffusive-advective transport equation, such that

\[
f(r, z, p, t = 0) = f_0(p) \exp \left[ -\frac{(z - z_s)}{D(p)} \right] \tag{7}
\]

where \( f_0(p) \) is the accelerated spectrum at the shock location \( z_s \), which we set following the test-particle predictions from the Diffusive Shock Acceleration (DSA) theory (see Skilling 1975; Bell...
Figure 1. MHD simulation in the oblique shock configuration and $\chi = 10^5$, as described in the text. Panels show the plasma mass density (a) and the magnetic energy density (b), both at a time $t = t_c + 300$ yr ($t_c$ represents the first shock-clump interaction time). Both pictures show a 2D section along $y$, passing through the center of the clump.

Figure 2. MHD simulation in the oblique shock configuration and $\chi = 10^5$. The panel shows magnetic field lines at a time $t = t_c + 300$ yr, along a 2D section along $y$, passing through the center of the clump.

1978; Blanford & Ostriker 1978), as

$$f_0 (p) \propto p^{-4} \exp \left[-p/p_{\text{cut}}\right]$$

Here a cut-off in momentum $p_{\text{cut}} \approx 100$ TeV/c is set in order to mimic a maximum energy attainable from the acceleration mechanism. Boundary conditions are such that a null diffusive flux is set on every boundary, except in the upstream of the $z$-direction, where the precursor shape is set.

When the forward shock hits the clump, we assume that the transmitted shock do not accelerate particle because it is very slow and it is propagating in a highly neutral medium. Given the result shown in § 2.1, we set an analytical velocity field, irrotational and divergence-less through all the space (except at the shock and clump surface). This is obtained solving the Laplace equation in cylindrical coordinates for its velocity potential, with the boundary conditions that in the far field limit the velocity field is directed along the shock direction and equal to $v = v_{\text{down}} \hat{z} = \frac{v_s}{2} v_{\text{down}} \hat{z}$, while at the clump surface the field is fully tangential. The resulting solution reads as

$$v_r (r, z) = -\frac{3}{2} \frac{R_0^3 r z}{(r^2 + z^2)^{3/2}} v_{\text{down}}$$
$$v_z (r, z) = \left[1 + \frac{1}{2} R_0^3 \frac{(r^2 - 2 z^2)}{(r^2 + z^2)^{3/2}}\right] v_{\text{down}}$$

With such a choice of the velocity field, the adiabatic compression term vanishes. Moreover, we set a null velocity field inside the clump, since $v_{s, c} \ll v_s$, and in the upstream region. A schematic view of the velocity vector field adopted is given in Fig. 3(b).
Bohm diffusion through all the space, so that dependent magnetic field

tion of high and very high energy gamma rays. We set a space-

PeV/c

Figure 3. Left: Velocity field resulting from the MHD simulation in the oblique shock configuration and $\chi = 10^5$, at a time $t = t_c + 300$ yr and along a $y$-section passing through the center of the clump. Right: Analytical velocity field adopted in the numerical solution of the transport equation, fully tangential to the clump surface and directed along the $z$-direction in the far field limit. The red dashed line limits the clump position.

Furthermore, we will consider a stationary space-dependent Bohm diffusion through all the space, so that

$$D_{\text{Bohm}}(x, p) = \frac{1}{3} r_L(x, p) v(p) = \frac{1}{3} \frac{p c}{Z e B_0(x)} v(p)$$  \hspace{1cm} (10)$$

where $r_L(x, p)$ is the Larmor radius of a particle with charge $Ze$ in a background magnetic field $B_0(x)$. In the following we will only consider relativistic protons, with momenta $p \in [1\,\text{GeV/c} \cdots 1\,\text{PeV/c}]$, since this is the energy interval relevant for the production of high and very high energy gamma rays. We set a space-dependent magnetic field $B_0(x)$, defining four regions in the space:

I) The unshocked CSM;

II) The shocked CSM, where $B_0 \equiv B_{\text{CSM}} = 10\,\mu\text{G}$;

III) The clump skin, where amplification of magnetic field is realized such that $B_0 \equiv B_s = 100\,\mu\text{G}$, with a size of $R_c = 0.5\,R_s$;

IV) The clump interior, where diffusion is not efficient such that $B_0 \equiv B_c = 1\,\mu\text{G}$, with a size of $R_c = 0.1\,\text{pc}$.

The density profile of accelerated particles diffusing in the region of interaction between the shock and the clump is shown in Fig. 4(a), 4(b) and 4(c) for particles of different energy. The distribution function is flat in the downstream region, while a precursor starts at the shock position, as defined in Eq. (7). Low-energy particles are prevented from penetrating the clump with respect to high-energy particles: this is a time-dependent phenomenon, since the more the clumps get engulfed into the downstream, the more it gets penetrated by CRs. However, on the temporal scales which result interesting for the gamma-ray emission of SNRs (around few hundred years, as explained in § 4), low-energy particles are not able to fill uniformly the clump interior.

Note that the value of $B_c$ used above is smaller than the strength of the large scale magnetic field expected in molecular clouds. We chose such a smaller value as representative of the effective turbulent magnetic field, which determines the diffusion coefficient in Eq. (10), and is damped by the presence of the ion-neutral friction (see § 3.1).

The diffusion coefficient should be close to the Bohm regime in order to obtain an effective acceleration of protons to multi-TeV energies. From a theoretical point of view, such an efficient diffusion is justified by the self generation of waves from accelerated particles. Nevertheless the correct description of the diffusion in the clump region is not a trivial task. We can distinguish two opposite situations: a) the case where magnetic field lines penetrate inside the clump and b) the case where these lines stay parallel to the clump surface. While in the former case the relevant diffusion coefficient is the one parallel to the magnetic field lines, in the latter we need to account for the perpendicular diffusion as well. In § 3.1 and 3.2 we discuss these two situations separately, showing that in both cases the effective $D$ is reduced.

3.1 Growth and damping of MHD waves

In the framework of non linear theory of DSA, cosmic rays generate MHD waves which are able to scatter them from one side to the other of the shock surface. If this process happens in resonant conditions, a CR particle of momentum $p$ is able to excite only magnetic waves of wavenumber $k_{\text{res}} = 1/r_L(p)$. The wave growth is then due to streaming of CRs. Thus the CR density, obtained as a solution of Eq. (6), affects the amount of turbulence that is generated, which in turn modifies the diffusion properties of the system as (Skilling 1971)

$$D(x, p, t) = \frac{1}{3} r_L(p) c \frac{1}{\mathcal{F}(k, x, t)} |k_{\text{res}}|$$  \hspace{1cm} (11)$$

where $\mathcal{F}(k, x, t)$ is the turbulent magnetic energy density per unit logarithmic bandwidth of waves with wavenumber $k$, normalized to the background magnetic energy density as

$$\left( \frac{\delta B(x, t)}{B_0} \right)^2 = \int \mathcal{F}(k, x, t) d\ln k$$  \hspace{1cm} (12)$$

Given the strong non linearity of the problem, it is computationally prohibitive to solve in a self consistent way the system composed by the transport equation and by the time evolution of the wave power density, which satisfies the following equation

$$\frac{\partial \mathcal{F}}{\partial t} + \mathbf{v} \cdot \nabla \mathcal{F} = (\Gamma_{\text{cr}} - \Gamma_D) \mathcal{F}(k, x, t)$$  \hspace{1cm} (13)$$
where $\Gamma_{\text{CR}}$ is the growth rate of MHD waves and $\Gamma_D$ is the damping rate. We will solve Eq. (6) in a stationary magnetic field and we will evaluate \textit{a posteriori} the contribution of the growth and damping of MHD waves.

The growth rate of the streaming instability strongly depends on the CR density gradient. We therefore expect it to be more pronounced in the clump skin, where magnetic field amplification makes diffusion very efficient, thus increasing the CR confinement time in this region. The rate can be expressed as (Skilling 1971)

$$\Gamma_{\text{CR}} = \frac{16}{3} \pi^2 \frac{v_A}{B_0} \left[ p^4 c \nabla f \right]_{p=p_{\text{res}}}$$

where $p_{\text{res}}$ is the resonant momentum and $v_A$ is the Alfvén speed of MHD waves, equal to

$$v_A = \frac{B_0}{\sqrt{4\pi n_i m_i}}$$

Amplified magnetic field can in turn be damped by non-linear damping (NLD) due to wave-wave interactions or by ion neutral damping (IND) due to momentum exchange between ions and neutrals as a consequence of the charge exchange process. Since we deal with non isolated clumps, we should also account for the typical timescale of the system. Indeed if the age of the clump is shorter than the time for damping to be effective, then waves can grow freely for a timescale equal to the clump age.

The dominant mechanism of wave damping in the clump magnetic skin is the NLD, because we assume the plasma to be completely ionized. Its damping rate can be expressed as (see Ptuskin & Zirakashvili 2003)

$$\Gamma_D = \Gamma_{\text{NLD}} = (2c_k)^{-3/2} k v_A \sqrt{\mathcal{F}}$$

where $c_k = 3.6$. In stationary conditions (when the system age is not a limiting factor), the wave growth rate (due to streaming instability) equals the damping rate, as $\Gamma_{\text{CR}} = \Gamma_D$. Equating Eq. (14) to Eq. (16), the power in the resonant turbulent momentum results in

$$\mathcal{F} = \left[ \frac{16}{3} \pi^2 \frac{v_A^4}{B_0^2} \left( p^4 c \frac{\partial f}{\partial p} \right)_{p=p_{\text{res}}} \right]^{2/3} 2c_k$$

We recall that, in Eq. (17), the CR density gradient is computed within the clump skin, along the radial dimension. We need to verify whether the stationarity assumption is correct or not. Once we insert Eq. (17) into Eq. (16), setting $v_A$ in $B_0 = 10\mu G$ and a typical ion density for a clump of $n_i = 10^{-4}$, $n_e = 10^{-1}$ ions cm$^{-3}$, we obtain that stationary is not valid for CR momenta larger than $p \geq 1$ TeV/c, where the clump age constraints the damping mechanism. Therefore, in this case, the power in turbulence is computed equating the growth rate of the MHD waves, as reported in Eq. (14), to the inverse of the clump age. This gives

$$\mathcal{F} = \left[ \frac{16}{3} \pi^2 \frac{v_A^4}{B_0^2} \left( p^4 c \frac{\partial f}{\partial p} \right)_{p=p_{\text{res}}} \right]^{2/3} \tau_{\text{age}}$$

The result of this computation is shown in Fig. 6(a). The turbulence is generated in the clump magnetic skin, such that CRs with momentum around 100 GeV/c are closer to the Bohm diffusive regime. On the contrary, in the clump interior, where neutral particles are abundant, the most efficient damping mechanism is IND (see Zweibel & Shull 1982 and Nava et al. 2016). Waves dissipate energy because of the viscosity produced in the charge exchange between ions and neutrals, such that previously neutrals start to oscillate with the waves. The frequency of ion-neutral collision is

$$\nu_i = n_i \langle \sigma v \rangle = 8.4 \times 10^{-9} \left( \frac{n_i}{\text{cm}^{-3}} \right) \left( \frac{T_e}{10^4 \text{K}} \right)^{0.4}$$

where an average over thermal velocities is considered. The rate of IND depends on the wave frequency regime, namely whether ions and neutrals are strongly coupled or not. Defining the wave pulsation $\omega_k = kv_A$ in a collision-free medium, then the study of the dispersion relation defines different regimes for ion-neutral coupling. These regimes are as follows:

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**Figure 4.** Density profile of accelerated particle in the shock region along $z$ (at fixed radial position inside the clump) for CR protons of momentum (a) 10 GeV/c, (b) 1 TeV/c and (c) 100 TeV/c. Vertical dashed lines represent the shock position at a given time, as indicated in the legend. The light pink band defines the clump interior, while the dark pink band defines its magnetic skin.
Figure 5. Distribution function of CR protons of momentum (a,c,e) 10 GeV/c and (b,d,f) 10 TeV/c at different times with respect to $t_c$: panel (a) and (b) are at $t = t_c + 50$ yr, (c) and (d) are at $t = t_c + 100$ yr, while (e) and (f) are at $t = t_c + 200$ yr. The precursor presence in front of the shock is well visible at 10 TeV.
I) If $\epsilon < 1/8$, there’s a range of $\omega_k$ for which waves cannot propagate, that is a range of $k$ for which $\omega_k$ is a purely imaginary number. This range is for

$$4\epsilon < \frac{\omega_k^2}{c^2} < \frac{1}{4}$$

(20)

which, within our assumptions ($\epsilon = 1.5 \times 10^{-3}$), equals to CR momenta in 15 GeV/c $< p < 95$ GeV/c;

II) In the intervals $\epsilon \ll 1$ and $(\omega_k/\nu_c)^2 \ll 4\epsilon$, then

$$\Gamma_{\text{IND}} = -\frac{\omega_k^2}{2\nu_c} - \frac{k^2\nu_c^2}{2\nu_c}$$

(21)

III) If $\epsilon \ll 1$ and $(\omega_k/\nu_c)^2 \ll 1/4$, then

$$\Gamma_{\text{IND}} = -\frac{\nu_c}{2}$$

(22)

IV) If $\epsilon \gg 1$, then

$$\Gamma_{\text{IND}} = -\frac{\nu_c}{2} \left[ \left( \frac{\omega_k^2}{\nu_c^2} \right) + \frac{1}{4} \right]$$

(23)

Again, the damping rate $\Gamma_{\text{IND}}$ should be compared with the inverse of the clump age $\tau_{\text{age}}$. For $p = 100$ GeV/c the IND time is shorter than the clump age, therefore setting the equilibrium condition $\Gamma_D = \Gamma_{\text{IND}}$ through Eq. (22), we find

$$F = \frac{16}{3} \pi^2 \nu_A^2 \frac{2}{B_0^2 \nu_c} \left[ 4^p v(p) \frac{\partial f}{\partial v} \right]_{p=p_{\text{max}}}$$

(24)

For $p = 100$ GeV/c, the dominant damping mechanism is still IND. Setting the equilibrium condition $\Gamma_D = \Gamma_{\text{IND}}$ through Eq. (21), we get

$$F = \frac{16}{3} \pi^2 \nu_A^2 \frac{2}{B_0^2 \nu_c} \left[ 4^p v(p) \frac{\partial f}{\partial v} \right]_{p=p_{\text{max}}}$$

(25)

On the other hand, for $p \geq 1$ TeV/c, the clump age is the limiting factor since IND damping requires a longer time. In this case we set $\Gamma_D = \tau_{\text{age}}^{-1}$ and we obtain

$$F = \frac{16}{3} \pi^2 \nu_A^2 \frac{2}{B_0^2 \nu_c} \left[ 4^p v(p) \nabla f \right]_{p=p_{\text{max}}}$$

(26)

As shown in Fig. 6(b), IND is very effective in damping waves resonant with CR particles of momentum lower than 10 GeV/c. On the other hand, Bohm diffusive regime is reached between 100 GeV/c and 1 TeV/c.

### 3.2 Perpendicular diffusion

Since the clump is embedded in a shock environment, turbulence develops because of both MHD instabilities and the CR driven instabilities. The spatial scales involved in these processes are different: in the case of MHD instabilities, the typical scale is the clump size, while in the case of CR driven instabilities the scale is the particle Larmor radius. It is shown in Fig. 6(a) and Fig. 6(b) that diffusion because of CR driven instabilities proceeds almost in the Bohm regime. It implies an isotropic diffusion. However, in the case of MHD instabilities, the magnetic field lines are mostly twisted around the clump, as shown in Fig. 2. Since particles diffuse along magnetic field lines, their heliocoidal motion drives them around clumps, making perpendicular diffusion quite inefficient. According to quasi linear theory, the diffusion perpendicular to the large scale magnetic field, $D_{\perp}$, is related to parallel diffusion, $D_{||}$, through (Casse, Lemoine & Pelletier 2002)

$$D_{\perp} = D_{||} \frac{1}{1 + (\lambda_\parallel/\lambda_L)^2}$$

(27)

where $\lambda_\parallel$ is the particle mean free path along the background field $B_0$. Since $\lambda_\parallel = \tau L (\delta B/B_0)^{-2}$, the perpendicular diffusion coefficient results in

$$D_{\perp} = D_{||} \frac{1}{1 + (\delta B/B_0)^{-2}}$$

(28)

Hence, the radial diffusion into the clump is strongly suppressed with respect to the azimuthal diffusion along the field lines provided that a tiny amplification of the magnetic field is realized. Let’s assume that in the region downstream of the shock $(\delta B/k)/B_0 \approx 1$ at all scale $k$ resonant with accelerated particles. Our results of MHD simulations show that the magnetic field in the clump skin is amplified by a factor of 10 with respect to the downstream field, so that $B_s = 10B_{down}$ (Fig. 1(b)). If the turbulence in the skin is amplified as well, then the Bohm diffusion limit can be reached and the distinction among parallel and transverse diffusion is lost, $D_{\perp} = D_{||}$. However, if the turbulence in the skin is not compressed, then $(\delta B/B_s) \approx 0.1$: this implies that in the skin parallel diffusion holds with $D_{||,s} = 10D_{||,down}$, while for the perpendicular diffusion $D_{\perp, s} = 10^{-3}D_{\perp, down}$. Therefore, in this regime, we can neglect the particle diffusive transverse motion and consider a parallel diffusion coefficient increased by a factor of 10 with respect to its downstream value.

### 3.3 Proton spectrum

Once the proton distribution function is known from the solution of Eq. (6), it is possible to obtain the proton energy spectrum inside the clump $J_p(p, t)$ at different times with respect to the first shock-clump contact, that we will indicate as $t = t_c$. Therefore the spectrum reads as

$$J_p(p, t) = \frac{1}{V_c} \frac{d^3N_c(t)}{dp^3}$$

(29)

where $V_c = \frac{4}{3} \pi R_c^3$ is the clump volume and $d^3N_c(t)/dp^3$ is the number of protons inside the clump at a time $t$ per unit volume in momentum space. The spectrum can be computed summing upon all the discretized bins which define the clump volume. In this way, we obtain

$$J_p(p, t) = \frac{2\pi}{V_c} \sum_{i \in \text{clump}} f_i(r_i, z_i, p, t) r_i \Delta r_i \Delta z_i$$

(30)

Results are shown in Fig. 7, where a proton cut-off momentum of $p_{\text{cut}} = 70$ TeV/c was set in order to reproduce the high-energy gamma-ray data. The spectrum of particles from younger clumps is much harder than the DSA spectrum, introduced at the shock as defined in Eq. (8). This is explained by the prevention of penetration of low-energy CRs into the clump due to the amplified magnetic field at the skin and because of the linear dependency of the diffusion coefficient with the particle momentum. In this way, the entrance of CRs into the clump is delayed. The spectral index of protons below 100 GeV/c is as hard as $\alpha = -3.5$ when the clump age is 50 yr, moving to $\alpha = -3.54$ when the clump age is 150 yr, and finally $\alpha = -3.57$ when the clump is 300 yr old. On the other hand, CRs with $p \geq 100$ TeV/c are quite unaffected by the presence of the clump.
of CRs with the ambient matter. In such a case the gamma-ray spectrum would reflect the spectrum of particles inside the clumps rather than the one outside as produced by the shock acceleration. In this section we calculate the total gamma-ray spectrum due hadronic interactions, assuming that clumps are uniformly distributed over the CSM where the shock expands.

The emissivity rate of gamma rays from a single clump, given the differential flux of protons inside the clump $\phi_c(T_p, t)$ and the density of target material, is

$$\epsilon_c(E_{\gamma}, t) = 4\pi n_0 \int dT_p \frac{d\sigma_{pp}}{dE_\gamma}(T_p, E_\gamma) \phi_c(T_p, t)$$

where $d\sigma_{pp}/dE_\gamma$ is the differential cross section of the interaction while $\phi_c(T_p, t)$ is obtained from the spectrum in Eq. (30). We use the analytical parametrization for the p-p cross section provided by the LibPRegm library (see Kafexhiu et al. 2016) and, specifically, we chose the parametrization resulting from the fit to Sibyll 2.1.

In order to evaluate the cumulative distribution resulting from a fixed distribution of clumps we need to include clumps that satisfy the following two conditions: I) they should survive (not evaporated); II) they should be located between the position of the contact discontinuity (CD) $R_{cd}$ and the shock position $R_s$. Indeed we will assume that, once a clump passes through the CD, either it is destroyed by MHD instabilities or it soon gets emptied of CRs. Therefore we should consider the minimum time between the evaporation time $\tau_{ev}$, and the time elapsed between the moment the clump crosses the forward shock and the moment it crosses the contact discontinuity $\tau_{cd}$. As estimated in § 2, the evaporation time is of the order of few times the cloud crossing time (see Eq. (5)). For the parameters we chose, this time is always larger than the SNR age. In the following, we will consider the conservative value of $\tau_{cd} = \tau_c$. The CD radial position, instead, can be estimated imposing that all the compressed matter is contained in a shell of size $\Delta R = R_{SNR} - R_{cd}$, so that

$$\frac{4}{3}\pi R_{SNR}^3 n_{cl} = 4\pi R_s^2 \Delta R n_{down}$$

which for strong shock amounts to $\Delta R = \frac{1}{12} R_{SNR}$. Therefore, the time that a clump takes to be completely engulfed in the CD is

$$\tau_{cd} = \frac{2R_c + \Delta R}{\frac{4}{3}v_s}$$

The oldest clumps in the remnant shell will therefore have an age

$$T_{c, max} = \min(\tau_{ev}, \tau_{cd})$$

In the following, we will consider a uniform spatial distribution of clumps, with number density $n_0 = 0.2$ clumps pc$^{-3}$, inside the remnant of age $T_{SNR}$. Therefore, the total number of clumps at a distance among $r$ and $r + dr$ from the source is equal to

$$\frac{dn(r)}{dr} = 4\pi n_0 r^2 \frac{dv_c(t)}{dt} = 4\pi n_0 (v(t)) v_s(t)$$

Furthermore, we will assume constant shock speed. We are interested in the number of clumps with a given age $t_{age}(r) = T_{SNR} - t_c(r)$. The number of clumps with an age between $t_{age} - \Delta t$ and $t_{age}$, namely $N(t_{age})$, is equal to the number of clumps that the shock has encountered between $T_{SNR} - t_{age}$ and $T_{SNR} - t_{age} + \Delta t$. It is equal to

$$N(t_{age}) = 4\pi n_0 \int_{T_{SNR} - t_{age}}^{T_{SNR} - t_{age} + \Delta t} r(t')^2 v_s(t') dt'$$

The total number of clumps with $t_{age} \leq T_{c, max}$ is equal to $N_c \simeq$
440, which corresponds to the total mass in clumps inside the remnant shell equal to \( M_e \simeq 45M_\odot \). Consequently, the total gamma-ray emissivity due to these clumps is

\[
\epsilon_\gamma(E_\gamma, T_{\text{SNR}}) = \sum_{\text{age}=0}^{T_{\text{max}}} N(\text{age}) \epsilon_\gamma(E_\gamma, t_{\text{age}})
\]  
(37)

We also account for the emissivity from the downstream region of the remnant \( \epsilon_{\text{down}}(E_\gamma) \), which is constant with time. The gas target in the downstream is considered with an average density \( n_{\text{down}} \), that satisfies mass conservation in the whole remnant

\[
\frac{4}{3} \pi R_{\text{SNR}}^3 n_{\text{up}} = \frac{4}{3} \pi (R_{\text{SNR}}^3 - R_{\text{cl}}^3) n_{\text{down}}
\]  
(38)

Therefore we compute the gamma-ray flux from the source located at a distance \( d \) as

\[
\phi_\gamma(E_\gamma, T_{\text{SNR}}) = \frac{1}{d^2} \left[ V_c \epsilon_\gamma(E_\gamma, T_{\text{SNR}}) + V_{\text{down}} \epsilon_{\text{down}}(E_\gamma) \right]
\]  
(39)

where \( V_{\text{down}} = V_{\text{shell}} - V_c \) and \( V_{\text{shell}} = \frac{4}{3} \pi (R_{\text{SNR}}^3 - R_{\text{cl}}^3) \).  

5 APPLICATION TO RX J1713.7-3946

The Galactic supernova remnant RX J1713.7-3946 (also called G347.3-0.5) represents one of the brightest TeV emitters on the sky. The origin of its gamma-ray flux in the GeV-TeV domain (see Abdal et al. 2011 and Abdal et al. 2016) has been object of a long debate, since both hadronic and leptonic scenarios are able to reproduce, under certain circumstances, the observed spectral hardening. The presence of accelerated leptons is guaranteed by the detected X-ray shell (see Slane et al. 1999 and Tanaka et al. 2008), which shows a remarkable correlation with the TeV gamma-ray emission. The presence of accelerated leptons is guaranteed by the detected X-ray shell (see Slane et al. 1999 and Tanaka et al. 2008), which shows a remarkable correlation with the TeV gamma-ray emission.  

The multi-wavelength observations point towards the clear presence of a non homogeneous environment, where the young SNR is expanding.

The estimated distance of the remnant is about \( d \simeq 1 \) kpc (Fukui et al. 2003), while the radial size of the detected gamma-ray shell today extends up to \( R_\gamma \simeq 0.6 \) deg (Abdal et al. 2016). The remnant is supposed to be associated to the Chinese detected type II SN explosion of 203 AD (Chen, Qu & Wang 1997) and would also be to the remnant an age of \( T_{\text{SNR}} \simeq 1620 \) yr. The distance, age and detected size yield an average shock speed of about \( v_s \simeq 6.3 \times 10^8 \) cm s\(^{-1}\). Measurements of proper motion of X-ray structures indicate that the shock speed today should be \( v_s \simeq 4.5 \times 10^8 \) cm/s (Uchiyama et al. 2007), meaning that the shock has slightly slown down during its expansion. This is expected in SNR evolution (Truelove & McKee 1999) during both the ejecta-dominated (ED) and the Sedov-Taylor (ST) phases. RX J1713.7-3946 is nowadays moving from the ED to the ST phase, therefore we can safely assume a constant shock speed through the time evolution up to now, with a value of \( v_s = 4.4 \times 10^8 \) cm/s (Gabici & Aharonian 2014). At this speed, the time that the CD takes to completely engulf a clump is, following Eq. (33), \( t_{\text{cd}} \simeq 300 \) yr. On the other hand, the evaporation time would be much longer, indicating that the relevant clumps contributing to the gamma-ray emission are younger than \( T_{\text{max}} = 300 \) yr.

With the parameters representing RX J1713.7-3946, as defined above, we can compute the gamma-ray flux of the remnant shell, through Eq. (39). We fix the normalization \( k \) of the resulting gamma-ray flux by minimizing the \( \chi^2 \) of the Fermi LAT and H.E.S.S. data with respect to our model. We investigate two different configurations. The first model explores a configuration with the magnetic field inside the clump reduced by a factor of 10 with respect to the CMS value, in order to account for the effect of IND, therefore it is set to \( B_1 = 1 \) G. The second model, instead, explores the situation where no IND is acting, therefore the magnetic field inside the clump is set to \( B_1 = 10 \) G, as in the CMS. Results are shown in Fig. 8. The GeV data from two years of data-taking of the Fermi LAT satellite (Abdo et al. 2011) are reported, together with the H.E.S.S. TeV data (Abdal et al. 2016) and with the H.E.S.S. Collaboration analysis of five years of Fermi data, as reported in Abdal et al. (2016). The two models predict a different trend in the GeV emission of the remnant. A more pronounced hardening in the case of \( B_1 = 10 \) G better reproduces the GeV data, while a flatter trend is visible in case diffusion would act less efficiently inside the clump. In this respect, electrons are more suitable to derive constraints on the magnetic field properties of the remnant. A more quantitative study on secondary electrons from pp interactions will be discussed elsewhere.

The normalization constant \( k \), obtained fitting the gamma-ray data, defines the amount of ram pressure \( P_{\text{ram}} = \rho_{\text{up}} v_{\text{up}}^2 \) that is instantaneously converted into CR pressure. The latter is defined, for relativistic particles, as

\[
P_{\text{CR}} = \frac{k}{3} \int_{m_p e}^{\infty} 4\pi p^2 dp f_0(p) \rho_{\text{CR}}
\]  
(40)

The efficiency of the pressure conversion mechanism from bulk motion to accelerated particles equals to \( \eta = P_{\text{CR}}/P_{\text{ram}} \simeq 2 \% \). Such a value is somewhat smaller than the efficiency estimated by other works in the context of hadronic scenarios, where usually \( \eta \simeq 10 - 20 \% \) (see, e.g Morlino, Amato & Blasi 2009A; Gabici & Aharonian 2014). Compared to Morlino, Amato & Blasi (2009A) the main differences are due to the highest total target density we use here (\( \sim 45M_\odot \) vs. \( \sim 15M_\odot \)) which is close to the total mass in
SNRs in clumpy media

5.1 Observing clumps through molecular lines

An interesting possibility to detect clumps is provided by radio observations. Secondary electrons emit synchrotron radiation in the radio domain. On top of this continuum, the molecular gas emits lines. For instance, rotational CO lines are often observed in these systems (Fukui et al. 2003). In the case of RX J1713.7-3946, a few arcsecond angular resolution is needed to probe the spatial scales of clumps, which is well within the performance of radio instruments, such as the Atacama Large Millimeter Array (ALMA). A precise pointing is however required, since the instrument field of view of $\leq 35^\prime \prime$ would not entirely cover a region as extended as the remnant RX J1713.7-3946.

In the following, we will evaluate the radio flux for the $J = 1 \rightarrow 0$ rotational line of the CO molecule. This transition is located at $\nu = 115$ GHz (band 3 of ALMA receivers) and radiates photons with a rate equal to $A_{10} = 6.78 \times 10^{-8}$ Hz. Assuming clumps estimated by HD simulations (Inoue et al. 2012). A weaker effect is also due to the fact that we are neglecting adiabatic losses, which leads to a smaller acceleration efficiency by less than a factor of two. Also, comparing our result with Gabici & Aharonian (2014) we should note a few more differences. We consider a con-
5.3 Resolving the gamma-ray emission

The deep morphological and spectroscopic studies of SNRs are among the highest priority scientific goals of the forthcoming Cherenkov telescope Array (CTA) (Actis et al. 2011; Acero et al. 2017). Despite its great potential, CTA will be not able to resolve the gamma-ray emission from individual clumps. For a SNR at a distance of 1 kpc, the angular extension of a clump with a typical size of $\sim 0.1$ pc does not exceed 20 arcsec, which is one order of magnitude smaller than the angular resolution of CTA. Nevertheless, the component related to the superposition of gamma-ray emission of several clumps on the line of sight in principle could be resolved. Hence, we estimate the number of clumps $dN_\rho/d\rho$ which overlap along the line of sight $l$, when observing the annular region extending from a distance $\rho = \sqrt{r^2 - l^2}$ up to $\rho + d\rho$, with respect to the center of the SNR. Assuming a uniform distribution of clumps, as described in § 4, and integrating it along the line of sight (Morlino & Caprioli 2012), we obtain

$$dN_\rho(\rho) = 2 n_0 \left( \sqrt{R^2 - \rho^2} - \max[0, \sqrt{R^2 - \rho^2}] \right)$$

where the emission is assumed to come from the shocked ISM located between the contact discontinuity and the forward shock. Considering a uniform map of the whole remnant by CTA, we compute the gamma-ray flux from several circular regions centered at a given $\rho$ and with radius equal to $\sigma_{CTA} = 0.037$ deg $\text{ }^1$ (at $E_\gamma = 10$ TeV) (Acharya et al. 2017). We considered $\rho$ spanning from 0 to $R_\ast$. Moreover, we should take into account the different ages of clumps, since they produce gamma rays with different spectral shapes, as shown in § 3.3. These fluxes are represented in Fig. 9, where also shown is the sensitivity curve of the CTA Southern array, for a 50 hr observation of a point-like source centered in the instrument field of view (FoV). The predicted flux clearly shows that CTA will be able to resolve the gamma-ray emission from clumps contained in a circle of radius equal to its high-energy PSF over about one decade in energy. However, the gamma-ray fluxes expected at different pointing regions strongly reflect the number of overlapping clumps, which represent the main contributors to the emission. Such a number is maximum in correspondence of $\rho = R_{cd}$, where $N_{cd} = 2.6$. Given the limited number of overlapping clumps in each pointing region, large fluctuations are expected, according to the Poissonian statistics. Therefore, the detection of such fluctuations constitutes a characteristic signature of the presence of clumps. The amount of the fluctuations depends on the clump density $n_0$ in the CSM. In fact, once the mass of the target gas is fixed, more massive clumps with $n_c = 10^4$ cm$^{-3}$ would require a lower clump density and therefore would produce much stronger fluctuations on the scale of $\sigma_{CTA}$. Such kind of morphological studies are hence crucial to derive constraints on the number density of clumps in the remnant region. Large fluctuations on the scale of $\sigma_{CTA}$ are not expected if the SNR is expanding into a uniform medium or into a medium where the density contrast is such that the clump evaporates soon after the shock crossing, namely if $\tau_{ev} \ll T_{SNR}$. The latter condition can be rearranged using Eqs. (4) and (5) to give an upper limit for the density contrast which reads $\chi \ll (T_{SNR} \nu_c^2 / 2R_c^2)^{1/2} \sim 10^3 (R_c / 0.1 \text{ pc})^{-2}$. Such a small density contrast also implies a smaller amplified magnetic field and, as a consequence, a flattening of the gamma-ray spectrum.

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Footnote 1: http://www.cta-observatory.org/science/cta-performance/

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Figure 9. CTA Southern array sensitivity curve for point-like sources located in the FoV center (zenith $\theta = 20$ deg, pointing average) for an observation time equal to 50 hr (black solid line). Also shown are the gamma-ray fluxes due to the overlapping clumps (indicated in the legend) in different circular sky regions of radius $\sigma_{CTA}$, located at a distance $\rho$ from the SNR center.

6 CONCLUSIONS

The presence of inhomogeneities in the CSM, in the form of dense molecular clumps where a shock propagates, strongly affects the plasma properties, in such a way that MHD instabilities develop and amplify the large scale magnetic field around the clumps. As a consequence, the propagation of particles accelerated at the shock proceeds in such a way that low-energy particles needs more time to penetrate the clump compared to high-energy ones. The resulting energy spectrum of particles inside the clump is significantly harder than the spectrum accelerated at the shock through DSA. Such a scenario is very common in the case of core-collapse SNe, where the remnant typically expands in a region populated by dense molecular clouds. It is then necessary to account for the inhomogeneous CSM to correctly predict the $\gamma$-ray spectrum from the SNRs. In this work we numerically solved the propagation of accelerated particles in the shock region at the presence of clumps, taking into account the amplification of magnetic filed produced by MHD instabilities and its effect on the particle diffusion. The most important parameter in this scenario is the density contrast between the diffuse CSM and the clumps. Using MHD simulation we showed that the magnetic amplification is effective only when the density contrast is larger than $10^3$.

Given that clumps contain most of the target gas, the gamma-ray spectrum produced in hadronic collisions of accelerated particles appears to be much harder than the parent spectrum. We demonstrate this effect for the brightest SNR in gamma rays, RX J1713.7-3946. The cumulative contribution of clumps embedded between the contact discontinuity and the current shock position is able to reproduce the observed GeV hardening, though some degeneracy in the parameter space of the model is present. Remarkably, for the gas density inside the clump assumed here ($n_c =$...
$10^3 \text{ cm}^{-3}$) the evaporation time is much longer than the SNR age. As a consequence, the clumps crossed by the forward shock do not produce significant thermal X-ray emission, in agreement with observations. We argue that such a scenario can naturally account for the fast variability in non-thermal X-ray reported in some hot-spots inside RX J1713.7-3946, thanks to the magnetic field amplified around the clumps. The electrons entering these regions rapidly lose their energy because of synchrotron emission. An independent signature on the 'clump' origin of the gamma-ray emission could be revealed from a morphological study performed with the next generation of ground-based gamma-ray observatory CTA. The superb sensitivity and the high angular resolution of CTA could allow to resolve regions small enough that contains only few clumps. This implies that we do expect a large spatial fluctuation of the gamma-ray flux, unlike a scenario where the SNR is expanding into a uniform medium.

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