Decoherence induced by an ordered environment

Juliana Restrepo\textsuperscript{1,2}, S. Camalet\textsuperscript{2} and R. Chitra\textsuperscript{2,3}

\textsuperscript{1} Laboratoire de Physique Théorique de la Matière Condensée, UMR 7600, Université Pierre et Marie Curie
4 place Jussieu, 75252 Paris Cedex 05, France, EU
\textsuperscript{2} Grupo de Sistemas Complejos, Universidad Antonio Nariño - Medellín, Colombia
\textsuperscript{3} Theoretische Physik, ETH Zurich - 8093 Zurich, Switzerland

received 12 December 2012; accepted in final form 21 February 2013
published online 19 March 2013

PACS 03.65.Yz – Decoherence; open systems; quantum statistical methods
PACS 73.22.Gk – Broken symmetry phases

Abstract – This letter deals with the time evolution of a qubit weakly coupled to a reservoir which has a symmetry-broken state with long-range order at finite temperatures. In particular, we model the ordered reservoir by a standard BCS superconductor with \( s \)-wave pairing. We study the reduced density matrix of a qubit using both the time-convolutionless and Nakajima-Zwanzig approximations. We study different kinds of couplings between the qubit and the superconducting bath. We find that ordering in the superconducting bath generically leads to an unfavorable non-Markovian faster-than-exponential decay of the qubit coherence. On the other hand, a coupling of the qubit to the non-ordered sector of the bath can result in a Markovian decoherence of the qubit with a drastic reduction of the decoherence rate. Since these behaviors are endemic to the ordered phase, qubits can serve as useful probes of continuous phase transitions in their environment. We also briefly discuss the validity of our main result, faster-than-exponential decay of the qubit coherences, for a qubit coupled to a generic ordered bath with a spontaneously broken continuous symmetry at finite temperatures.

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Introduction. – The past decade has seen tremendous activity devoted to developing experimentally viable qubits for quantum computing. These two-level systems are realized, for example, by directly using the charge or spin degrees of freedom of electrons and quantum dots [1] or more complex entities like flux qubits [2,3] and Cooper boxes [4]. The utility of all these qubits for quantum computation is strongly limited by the influence of their environment which tends to destroy their quantum coherence. Consequently, a lot of recent theoretical studies have focused on ways and means of increasing the coherence time scales [5–10]. Another viewpoint consists of employing such “ancillary qubits” as probes of the environment with which they interact, as typically done in NMR spectroscopy. An example is a spin-glass bath where the reduced dynamics of the qubit was shown to be directly sensitive to the spin-glass order parameter [11]. This is especially useful in the field of quantum optics, where given the difficulty of standard thermodynamic measurements, such small probes can be used to explore phase transitions in Dicke-like models [12] and also to investigate both equilibrium and non-equilibrium properties of cold atoms [13].

The first theoretical studies of decoherence considered environments consisting of independent harmonic oscillators [14] or spins [15,16]. More recently, complex baths have been studied. In particular, intrabath interactions have been taken into account [7,17–19], raising the question of the possible influence of thermodynamic phases and transitions on the decoherence of the qubit. It has been shown that the mitigating impact of intrabath interactions seen in many cases [17,19] breaks down when the bath is in the vicinity of a phase transition [6].

Numerous studies [18,20,21] have addressed the question of what happens when the bath is in the vicinity of a quantum phase transition in the case of one-dimensional spin baths and have found enhanced decoherence near the critical point. Note, however, that the time evolution in these zero-temperature systems is generically non-Markovian and due to the one-dimensional nature of the baths considered, no true long-range order exists. For higher-dimensional baths at finite temperatures, it was shown that the Markovian decoherence rate diverges on the disordered side as one approaches the second-order transition temperature [6] signaling the non-Markovian time evolution in the ordered phase. References [17,22]
argued that once in the ordered phase, symmetry breaking in the reservoir helps reduce decoherence, while ref. [19] found a strong Gaussian decay of quantum coherence. However, these works suffer from different drawbacks ranging from a complete neglect of low-energy modes in ref. [17] to obtaining an order-parameter–independent behavior for the time evolution of the qubit in ref. [19].

In this paper, we revisit the problem of ordered baths at finite temperatures to have a clearer understanding of their effect on qubits. We consider a superconducting bath at finite temperature in three dimensions described by the Bardeen-Cooper-Schrieffer (BCS) theory. Though the BCS Hamiltonian does not capture the fluctuations in the disordered phase, it provides a very good description of the ordered phase. In line with ref. [6], where it was shown that the impact of the ordering on the qubit depends crucially on the relation between the qubit-bath interaction and the order parameter, we study different kinds of interactions between the qubit and the bath. We shall show below that the ordered phase, characterized by a spontaneously broken continuous symmetry, is synonymous with a non-Markovian time evolution of the qubit density matrix with interesting anomalous features. Moreover, the ordered superconducting phase has a rich variety of behaviors not seen in the disordered phase, including faster-than-exponential decay of the coherence. The latter makes it unfavorable from the point of view of quantum computing. But, there are interesting exceptional qubit states which decohere slower when the bath orders. This sensitivity of the qubit to the order in the bath makes it a good probe of the transition in the bath.

**Model.** — The combined system of the qubit and the superconducting bath is described by the Hamiltonian

$$H = \sigma_q \cdot V + H_B,$$

where $\sigma_q$ is the vector Pauli operator for the qubit, whose components are the usual $2 \times 2$ Pauli matrices, $V$ is some bath vector operator that will be specified later, and $H_B$ is the conventional BCS Hamiltonian $H_B = \sum_{\mathbf{k} \epsilon} E_{\mathbf{k}} \alpha^\dagger_{\mathbf{k} \epsilon} \alpha_{\mathbf{k} \epsilon}$ [23]. The Bogoliubov operators $\alpha_{\mathbf{k} \epsilon}$ are related to the electron annihilation and creation operators by

$$\alpha^\dagger_{\mathbf{k} \epsilon} = u_k c^\dagger_{\mathbf{k} \epsilon} + v_k c_{\mathbf{k} \epsilon - \epsilon},$$

$$\alpha_{\mathbf{k} \epsilon} = u_k c_{\mathbf{k} \epsilon - \epsilon} - v_k c^\dagger_{\mathbf{k} \epsilon},$$

where $c^\dagger_{\mathbf{k} \epsilon}$ creates an electron with momentum $k$ and spin $\epsilon = \uparrow, \downarrow$. The BCS dispersion relation is $E_k = sgn(e_k) \sqrt{\frac{\epsilon^2}{\epsilon^2 + \Delta^2}}$, where $e_k$ is the underlying electronic dispersion and $\Delta$ is the superconducting gap. The coefficients in (2) obey $(u, v)^2_k = (1 \pm e_k/E_k)/2$. We set $h = k_B = 1$ in the rest of the paper. The superconducting order parameter $\Delta$ at temperature $T$ is self-consistently determined by

$$gN \int_0^\infty \frac{d\omega}{\sqrt{\omega^2 + \Delta^2/T}} \frac{\tanh(\sqrt{\omega^2 + \Delta^2}/2T)}{\sqrt{\omega^2 + \Delta^2}} = 1,$$

where $g$ is the strength of the phonon-mediated electron-electron interaction, $\omega$ is the Debye frequency and $N$ is the electronic density of states at the Fermi surface. As is well known, this equation determines a critical temperature $T_c$ which separates a high-temperature metallic phase where $\Delta = 0$ and a low-temperature phase where $\Delta$ increases monotonically as $T$ decreases.

We assume that, at time $t = 0$, the qubit and the bath are uncorrelated and that the bath is in thermal equilibrium at temperature $T$. The initial state of the combined system is thus $\Omega = \rho(0) \otimes \rho_B$, where $\rho_B \propto \exp(\mathcal{H}_B/T)$ and $\rho(0)$ is any qubit density matrix. The time evolution of the reduced density matrix of the qubit is given by

$$\rho(t) = \text{Tr}_B [e^{-i\mathcal{H}_B t} \rho_0 e^{i\mathcal{H}_B t}],$$

where $\text{Tr}_B$ denotes the partial trace over the bath degrees of freedom. Since the superconducting bath has long-range order, this implies that the typical correlation times of the bath are very long. This automatically precludes the use of Markovian master equations, which relies on short bath correlation times, to study the time evolution of the reduced density matrix $\rho$. Since we anticipate non-Markovian behavior in the present problem, in the limit of weak coupling between the qubit and the bath that we are interested in, one can use two main methods to calculate $\rho(t)$: i) the time-convolutionless (TCL) projection operator technique and ii) the Nakajima-Zwanzig (NZ) approximation [24,25]. Though both methods can deal with non-Markovian time evolution, TCL gives local-in-time equations of motion for $\rho(t)$ whereas the NZ approximation gives an integro-differential dynamical equation for $\rho(t)$. The accuracy of these methods depends on the problem studied and it is difficult to assert a priori which one is more appropriate [26]. In this letter, we focus on the asymptotic evolution predicted by the second-order TCL approximation, and briefly discuss the results obtained using the NZ technique at the end. To write the master equation given by the TCL approximation to second order, it is convenient to first rewrite the Hamiltonian (1) as

$$H = H_B + \sigma_q \cdot (V(t) + H_I(t)),$$

where $\langle \ldots \rangle = \text{Tr}(\rho_B \ldots)$ and $H_I = \sigma_q \cdot [V(t) - \langle V(t) \rangle]$. With these notations, we obtain using the Born approximation

$$\partial_t \rho = -i[\sigma_q \cdot (V(t)), \rho(t)] - \int_0^t \text{d}t' \text{Tr}_B [H_I(t'), [H_I(-\tau), \rho(t) \otimes \rho_B]]$$

where the time-dependent $H_I$ is given in the interaction picture by

$$H_I(t) \equiv e^{i\mathcal{H}_B t} \mathcal{H}_I e^{-i\mathcal{H}_B t} = \sigma_q \cdot (V(t) - \langle V(t) \rangle).$$

The TCL equation, eq. (6), is our starting point for the calculations which follow. We remark that, replacing the upper limit of the integral $t$ in (6) by $+\infty$ leads
to the well-known Markovian master equation. However, as we will show below, the time evolution of $\rho(t)$ can be non-Markovian in the ordered phase precluding the use of such Markovian master equations. We note that Markovian evolution means a simple exponential decay of the elements of the density matrix. To obtain the equivalent equation for the reduced density matrix within the NZ scheme, it suffices to replace $\rho(t)$ in the integral on the right-hand side of (6) by $\rho(t - \tau)$. This results in a time non-local equation for the reduced density matrix.

**Kondo coupling.** – We first consider a Kondo-like coupling where the qubit couples to the electronic spin density at the origin $S(0)$, i.e.,

$$V_\alpha = \lambda S_\alpha(0) \equiv \lambda \sum_{k,k',\sigma,\sigma'} c^\dagger_{ke}\sigma_{\alpha k'}c_{k'\sigma'}, \quad (8)$$

where $\alpha \in \{x, y, z\}$, $\lambda$ is the coupling strength, and $\sigma^\sigma_{\alpha k'}$, are the matrix elements of the Pauli matrix $\sigma^\sigma$. The Hamiltonian (1) is effectively the same as that of a magnetic impurity embedded in a superconductor. Similar Kondo couplings were studied experimentally in multiwalled carbon nanotubes [27] and in spin half quantum dots coupled to two superconducting reservoirs in [28].

Contrary to the situation of a metallic bath where one really does not have a true weak-coupling regime because of the dynamical Kondo effect, here we have a weak-coupling regime [29]. The isotropy of the total Hamiltonian (1) and the absence of any net moment in the bath lead to the following simplifications: since $V = 0$, the first term in (6) vanishes and all components of the effective spin-1/2 corresponding to the qubit, i.e., $s_\alpha(0) = \text{Tr}(\rho(t)\sigma^\alpha_n)$, satisfy the same equation of motion. Consequently, the qubit evolution is characterized by a unique time function $M$ defined by

$$s_{\alpha}(t) = M(t)s_{\alpha}(0). \quad (9)$$

We see that with a Kondo-like coupling, both decoherence and relaxation exhibit the same time evolution. By writing the electron operators $c_{ke}$ in terms of the Bogoliubov quasi-particle operators (2), we find

$$\ln M(t) \simeq -4\lambda^2 \int_{-\infty}^{\infty} d\omega \frac{\sin(\omega t/2)^2}{\omega^2} \Gamma^+(\omega), \quad (10)$$

with

$$\Gamma^\pm(\omega) = [S^\pm(\omega) + S^\pm(-\omega)] \quad (11)$$

and the functions $S^\pm$ are given by

$$S^\pm(\omega) = \int_{-\infty}^{\infty} de f^\pm(e, \omega) \rho(e) \rho(e - \omega) n(e) n(\omega - e), \quad (12)$$

where $f^\pm(e, \omega) \equiv 1 \pm \Delta^2/|e(\omega - e)|$, $\rho(e) = |e|(\omega^2 - \Delta^2)^{-1/2}$ is the superconducting density of states, and $n(e) = 1/(\exp(e/T) + 1)$ is the Fermi function. We note that $S^+(\omega)$ and $S^-(\omega)$ are the dynamical spin and charge structure factors of the superconducting bath.

For $T > T_c$, the bath is a simple metal ($f^\pm(e) = 1$) and we obtain the usual asymptotic Markovian decay

$$\ln M(t) = -\gamma t$$

with a decay rate $\gamma = 4\pi\lambda^2 S^+(0)$ which increases and saturates to a density-of-states–dependent value at very high temperatures [7,8]. As one approaches the transition temperature $T \to T^+_c$, we expect the growing fluctuations to result in a divergent rate $\gamma$ at $T_c$ [6] though this is not captured by the mean-field BCS theory used here. We now analyze the asymptotic qubit evolution in the ordered phase $0 \leq T < T_c$. At $T = 0$, we find that $S^+(\omega) = 0$ for all $\omega > 2\Delta$, where $2\Delta$ is the gap to two-particle excitations. This gap leads to an incomplete decoherence of the qubit where the function $M(t)$ approaches a constant as a power law in the asymptotic limit $t \to \infty$ just as seen in the case of an insulating bath [8]. The resulting asymptotic density matrix $\rho$ does not lose the memory of the initial conditions since $M(t) \neq 0$ implying that the qubit is not in a simple statistical mixture with equal probabilities of spin-up and spin-down states. At temperature $0 < T < T_c$, due to the divergence present in the superconducting density of states, $S^+$ is infrared divergent: $S^+(\omega) \sim -r(T)\ln|\omega/T|$ as $\omega \to 0$. This divergence stems from the existence of Goldstone modes in the ordered phase. The pre-factor $r(T) = (\Delta^2/2\cosh^2(\Delta/2T))$, shown in the inset of fig. 1, is a non-monotonic function of temperature which vanishes at $T = 0$ and $T = T_c$. This infrared divergence results in

$$\ln M(t) \simeq -2\pi\lambda^2 r(T)t \ln t \quad (13)$$

for times $t \gg t_0 \equiv \text{Max}(1/T, 1/\Delta)$. We see that, contrary to naive expectations, the ordered bath leads to a novel faster-than-exponential loss of coherence of the qubit. This can be attributed to the fact that the qubit couples to the spin fluctuations and hence order parameter fluctuations via the singlet Cooper pairs. Moreover, we see an interesting reentrance in the asymptotic regime because the
coefficient $r(T)$ which dictates the asymptotic decoherence is the same for two different temperatures (cf. fig. 1).

**Order coupling.** – To further understand the physical origin of the ultra-fast decoherence seen above for a Kondo-coupled qubit, we now consider a direct coupling of the qubit to the order parameter of the bath of the form

$$\lambda [\sigma_q^- c_{0\uparrow}^\dagger + \sigma_q^+ c_{0\downarrow} c_{0\uparrow}]$$

where $\sigma_q^\pm$ are the qubit spin raising and lowering operators and $c_{0\uparrow}^\dagger c_{0\uparrow}$ is the superconducting order parameter at the origin. This coupling can be rewritten in momentum space and in terms of the operators $V$ such that $V_x = 0$ and

$$V_x = \lambda \sum_{k,k'} (c_{k\uparrow}^\dagger c_{k'\uparrow} + c_{k'\downarrow} c_{k\uparrow})$$

$$V_y = i\lambda \sum_{k,k'} (c_{k\uparrow}^\dagger c_{k'\downarrow} - c_{k'\downarrow} c_{k\uparrow})$$

These operators are directly related to the superconducting order parameter operator at the origin, $O = \sum_{k,k'} c_{k\uparrow}^\dagger c_{k'\downarrow}$. Such couplings can be realized in qubits made from Cooper pair boxes which are capacitively tunnel coupled to a superconducting reservoir [4,30]. In this case, the interaction Hamiltonian describes the tunneling of a Cooper pair box into the superconductor with a temperature-dependent gap [8].

For $T > T_c$, all the components $s_\alpha$ are uncoupled and decay asymptotically as $\ln s_\alpha = -\gamma_\alpha t$ with the rates $\gamma_x = \gamma_y = 8\pi \lambda^2 S^(-) (0)$ and $\gamma_z = 2\gamma_y$. The existence of two different rates is a direct consequence of the spin anisotropy of the order coupling, whereas in the spin isotropic Kondo case all rates are the same. In the ordered phase, at $T = 0$, solving eq. (16), we find that the presence of a gap, both in $\Sigma_-$ and $\Sigma_+$, leads to an incomplete decay of the central spin coherences. For $T \neq 0$, since $S^-$ is always regular and finite at low frequencies this results in a Markovian decay in $s_x \simeq -\gamma_x t$ for times $t \gg t_\alpha$. The full temperature dependence of the rate $\gamma_x$ is shown in the inset of fig. 2. Its behavior for $T \to 0$ is $\gamma_x \propto T^{-\Delta/T}$. This is an important result of our work because it shows that in the ordered phase, the Markovian rate for the component $s_x$ is strongly suppressed compared to a simple metallic bath. This is very similar to the relaxation induced by a coupling to the charge fluctuations, studied in the context of NMR by Fulde and Black [32], which is described by $\sigma_q \cdot V \propto \sigma_q \tilde{n}$ where $\tilde{n}$ is the number of electrons at the origin. For $T > T_c$, the rate $\gamma_x$ coincides with the rate of the equivalent metallic bath with $\Delta = 0$ and saturates to a finite DOS-dependent value proportional to $\int dE \rho(E)^2$ as expected. If the qubit’s entire evolution were determined by this component, then the bath would be effectively a semiconductor with a temperature-dependent gap [8]. On the other hand, we find that the components $s_y$ and $s_z$ exhibit

$$\begin{pmatrix} s_x \\ s_y \\ s_z \end{pmatrix} \begin{pmatrix} \Sigma_- (t) & 0 & 0 \\ 0 & \Sigma_+ (t) & h \\ 0 & -h & \Sigma_- (t) + \Sigma_+ (t) \end{pmatrix} \begin{pmatrix} s_x \\ s_y \\ s_z \end{pmatrix} = \partial_t \begin{pmatrix} s_x \\ s_y \\ s_z \end{pmatrix}$$

The first-order term $h \equiv -2V_y = 2\lambda \Delta / gN$ and the functions $\Sigma_{\pm}$ are given by

$$\Sigma_{\pm} (t) = -4\lambda^2 \int_0^\infty d\omega \frac{\sin (\omega t)}{\omega} \Gamma^{\pm} (\omega)$$

with the $\Gamma^{\pm}$ defined earlier in (12). Unlike the Kondo case previously studied, here the equations for the components $y$ and $z$ are coupled in the ordered phase where $\Delta \neq 0$. A closer look shows that $\Sigma_-$ and hence $s_x$ are related to the dynamic charge correlation function which is non-singular in the superconducting phase and $\Sigma_+$ is related to the spin correlation function which shows singular behavior in the superconducting phase. As will be discussed below, this leads to a complex time evolution for the different components of the qubit density matrix.

![Fig. 2](image-url)
non-Markovian behavior. In the limit of weak coupling between the qubit and the bath, eq. (16) can be solved for $s_y$ and $s_z$ [33]. The full solution is given by $(s_y(t), s_z(t)) = (s_y(0), s_z(0)) \exp(A)$ where the $2 \times 2$ matrix $A$ is typically an infinite series in the coefficient $\lambda$. However, since we are interested in the weak-coupling limit, it suffices to retain terms only up to second order in the coupling $\lambda$ in the matrix $A$. This is equivalent to the TCL result for the Kondo coupling seen in the previous section (cf. eq. (10)). Doing the algebra, we find,

$$s_y \simeq e^{\omega_0 t} \left[ a_0 s_0(\Theta) + s_z(0) h t \sin \Theta \right],$$

$$s_z \simeq e^{\omega_0 t} \left[ - s_y(0) h t \sin \Theta + s_z(0) \cos \Theta \right],$$

(18)

where the time-dependent functions $a_0$ and $a_1$ are given by

$$a_\mu(t) = \int_0^t dt' [\mu \Sigma_+(t') + (\mu - 1/2) \Sigma_-(t')],$$

$$\Theta(t) = ([ht] - a_0(0))^2)_{1/2} \simeq h t$$

and $\sin \Theta = \Theta^{-1} \sin \Theta$. Note that both $s_y$ and $s_z$ show oscillatory behavior in the ordered phase with a frequency proportional to the order parameter. At $T = 0$, $s_y$ and $s_z$ oscillate at a maximal frequency and at finite temperatures $0 < T < T_c$, these oscillations are damped with the envelope $a_\mu(t) \propto -1/T \ln t$ for $t \gg t_a$ and we recover the faster-than-exponential decay (13) with reentrant temperatures encountered for the Kondo coupling (cf. fig. 2). The appearance of oscillations of the qubit can be used as a tool to demarcate the phase diagram of the superconductor.

Because of the very different asymptotic behaviors of the components $s_y$ found above, the decay time scale of the qubit depends crucially on the initial conditions. Assume the qubit is initially prepared in an eigenstate of $\sigma_z$. Then the components $s_y(t) = s_z(t) = 0$ are constant, and the asymptotic time evolution of the qubit, determined by $s_y(t)$, is non-oscillatory and Markovian with a highly reduced rate in the ordered phase. These pure states are thus relatively stable in the environment considered here. Moreover, a combination of good initial preparation and pulse sequences which repeatedly orient the qubit in the x-direction could efficiently reduce decoherence times [34,35]. For an initial qubit state such that $s_x(0) = 0$, this component remains zero and the asymptotic behavior of the qubit consists of an oscillatory and faster-than-exponential decay. In this case, due to the coupling between y and z components, we have situations where even if one of the components $s_y$ or $s_z$ is initially zero, this component can grow with time and show oscillatory behavior. For generic initial states, since $s_y$ and $s_z$ decay much faster than $s_x$, the asymptotic time evolution of the qubit is Markovian. The qubit state first decoheres to a statistical mixture of the eigenstates of $\sigma_z$ and then relaxes exponentially into the maximally mixed state.

In both the Kondo and order coupling cases, we find interesting intermediate-time behaviors but these will be discussed elsewhere. We now briefly discuss the results obtained using the NZ approximation [8]. To second order, the reduced density matrix satisfies (6) with $\rho_z(t)$ in the integral in the second term replaced by $\rho_z(t - \tau)$. Using the results for $S_2$ given in eq. (12), we find that in the case of the Kondo coupling, the asymptotic faster-than-exponential behavior seen earlier is replaced by a much slower non-Markovian behavior

$$M_{\text{NZ}}(t) \sim \int dt \cos(\omega t) / \ln |\omega| \sim -1/t \ln t$$

for times $t \gg t_n$. We find $t_n \gg t_a$ and an intermediate regime $t_a \ll t < t_n$ characterized by a quantitatively faster decay than that predicted by the TCL approach, where the qubit becomes practically incoherent. In the case of the order coupling, we find that the conclusions for the asymptotic Markovian behavior seen for the component $s_x$ remain unchanged whereas, the components $s_y$ and $s_z$ show the same functional form of decay given by $M_{\text{TCL}}$. We plot the full time evolution predicted by both approximation methods in fig. 3. We can safely conclude that anomalously fast decoherence seems to be a feature of both NZ and TCL methods in the ordered phase.

**Conclusion.** – To summarize, we have studied the influence of a true long-range–ordered bath on the state of a qubit. For two different qubit-bath couplings and generic initial conditions, we found a faster-than-exponential decoherence of the qubit in the ordered phase leading to the conclusion that ordered baths are often disastrous for qubits. This non-Markovian time evolution essentially stems from the contribution of the Goldstone modes to the bath correlation functions in the symmetry-broken phase. However, for a direct coupling of the qubit to the ordering operator, some particular pure states of the qubit with initial values $\langle s_y \rangle = \langle s_z \rangle = 0$ lead to pure decoherence with a highly reduced Markovian decoherence rate when the bath orders, making such states potentially useful in experiments using echo sequences or other related
techniques. This result constitutes another example of the important role played by the nature of the coupling to a bath that can order [6]. For generic initial states of the qubit, we expect, if fluctuations in the disordered phase are properly taken into account, a divergence of the Markovian rate at the transition from the disordered side, followed by a faster-than-exponential loss of coherence in the ordered phase. Exceptions are possible, as our study shows.

We believe that this picture of non-Markovian time evolution in the ordered phase should be valid for all ordered baths where a continuous symmetry is spontaneously broken, provided the qubit couples in some way to the order parameter. In the problem studied here, the faster-than-exponential non-Markovian behavior in the ordered phase is intricately linked to the existence of the soft Goldstone modes in the ordered phase due to the spontaneous breaking of the continuous \( U(1) \) symmetry. Typically, the non-Markovianity could be a faster-than-exponential decay or a power law decay. Power law decay (see, for example, in the spin boson problem at \( T = 0 \)) however, would imply slow decoherence. To obtain such power law decays for the case or ordered baths at finite temperatures would require a \( \Gamma^{\pm}(\omega) \propto |\omega| \). Given that \( \Gamma^{\pm} \) is related to the dynamical structure factors of the bath, \( S^{\pm}(\omega) \), the fluctuation dissipation theorem tells us that \( S^{\pm}(\omega = 0) \neq 0 \) at finite temperatures. This effect rules out any power law asymptotic behavior and any resulting non-Markovian behavior has to be faster than exponential. The full quantitative time evolution will of course depend on the particular physical problem studied. An obvious question is, whether one would obtain such enhanced non-Markovian loss of coherence in other ordered systems where the order arises from a spontaneously broken discrete symmetry like \( Z_N \) as in the latter one would not expect the formation of Goldstone modes. It would be interesting to generalize our model to a non-degenerate qubit and to study the effect of such anomalous dissipation on the tunneling of the qubit [14]. These questions are beyond the scope of the present paper and are left for future work.

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