Retailer's optimal procurement plan considering random demand, delayed payment and cash discount

Ruiqi Huang¹, Yongbiao Huang²,³,⁴,⁵ and Yueli Lu³

¹School of Business Administration, Northeastern University, Shenyang, 110169, China
²Preparatory Education College, Guangxi University for Nationalities, Nanning, 530006, China
³Xingjian College of Science and Liberal Arts, Guangxi University, Nanning, 530005, China
⁴Email: hybxyf@163.com

Abstract. Many inventory models have been established based on determined demand but ignore the fact that demand can be affected by random factors. A procurement model considering random demand, different trade credit and cash discount is established in this paper. Due to the randomness of demand, possibilities of salvage or stock out at the end of the cycle are considered, and these cases can be dealt as classic news vendor problem. The optimal procurement strategy for the expected profit is derived by some theoretical results in this paper, which are also illustrated by a series of numerical examples and the corresponding sensitivity analyses are also presented.

1. Introduction

Customer demand is typically treated as an exogenous parameter in the prior research. Many inventory models are established based on constant demand rate. In real situation, the customer demand may be interrelated with time [1] or with price [2], or may be influenced by the quantity stocked [3] or marketing effort expended by the buyer [4]. Many extensions to different states of information about demand based on economic order quantity model were proposed, especially in time-varying demand. For instance, the demand rate is increasing of time [5], which demonstrates the demand of product is in the growth stage during the life cycle; the demand rate is decreasing of time [6], which implies that product is in the maturity stage. The demands of above models are all deterministic without considering that demand may be influenced by undetermined factors.

In a rival market, a discounted price generally stimulates volume of sales. Based on the concept, Abad [7], Kim [8], and Burwell [9] proposed the traditional quantity discount models. Abad [10] and Wee [11] proposed the traditional price discount models. Particularly, Arcelus [12] discussed the advantages and disadvantages of a discount in the purchase price under delay in the payment. Teng [13] presented an economical production quantity model for deteriorating items on the assumption that demand is dependent on price and stock. Das [14] extended to the situation that vendor provides the permissible delay and cash discount to the buyer. Li [15] proposed an optimal order quantity model

⁵ Yongbiao Huang is the corresponding author (Email: hybxyf@163.com).
combined effects of selling price, expiration date and credit term affecting demand. All above models are developed on deterministic demand.

Retailer must pay for the products received immediately in traditional EOQ model. In real life, supplier may offer a period for the retailer to settle account within a permitted period, which is called credit period. During credit period, the retailer can earn interests by depositing the accumulate revenues in an interest-bearing account. Goyal [16] first applied the trade credit in the classical EOQ model. Since then, lots of extensions on trade credit have been developed. The representative results in recent years, for instance, Ho [17] developed an integrated inventory model under two-level trade credit, which demand rate is a function of both retail price and credit period provided by retailer. Teng [5] extended the constant demand assumed in EOQ model to a linear non-decreasing time-vary demand under trade credit, which is more suitable for life cycle of product. Min [18] proposed a lot-sizing model for degenerative items under two-level trade credit, in which the demand rate is assumed to be dependent on the retailer’s current-stock level. In practice, suppliers usually offer different credit periods and price discounts to stimulate retailer to order more amounts. For example, Sana [19] considered an EOQ model for varieties of description of deterministic demand under condition of trade credit and discount offered to the retailer. And Tiwari [20] established an ordering model for deteriorating items with permissible partial delayed payment based on the order quantity. Though an amount of research on trade credit has been done, the inventory model with the price-discount rate depending on permissible delay period under stochastic demand has not been developed. More discussion of random demand can see models proposed by Liao [21] and Ebrahimi [22].

In the actual research, the profit-maximizing order strategy for a retailer with the undetermined demand under the credit strategy where supplier offers different discount rates of price at different credit periods has not been proposed till now. So, in this paper, we consider an inventory model for stochastic demand when delay in payment and cash discount is offered by supplier to retailer. Moreover, the supplier offers different discount rates of price to retailer at different credit periods. Due to the stochastic demand, salvage or stock out at the end of the cycle may occur. These cases are dealt as classic news vendor problem, i.e., the salvage will lead to disvalue of productions, and the stock out will bring the punishment cost. The optimal solution for the expected profit is derived by some theoretical results, which are illustrated by a series of numerical examples. The corresponding sensitivity analyses are also presented.

2. Notations and Assumptions

2.1. Notations

- $Q$: retailer’s order quantity (decision variable)
- $Q^*$: the optimal order quantity
- $M$: variable permissible delay period (decision variable)
- $M_i$: $i$-th permissible delay period in settling the amount
- $T$: ordering cycle
- $D(t)$: cumulative demand
- $c$: purchasing cost per unit which depends on the delay period and supplier’s offer
- $p$: retail price per unit
- $g$: salvage value per unit
- $b$: shortage penalty cost per unit
- $I_e$: interest earned per dollar per unit time
- $I_p$: interest charged per dollar per unit time
- $\delta_i$: discount rate (in percent) of purchasing cost at $i$-th permissible delay period
- $E[\cdot]$: the mathematical expectation operator
- $\epsilon^*$: $\max\{\epsilon, 0\}$
the expected profit per order

2.2. Assumptions

(1) The inventory system contains a single type of products. Lead time is negligible, and replenishment is instantaneous.

(2) The cumulative demand \( D(t) \) can be given by \( D(t) = \alpha(t) \), where \( \alpha(t) \) is an increasing function of time \( t \) with \( \alpha(0) = 0 \), and \( \varepsilon \) is a random variable with a probability density function \( f(\cdot) \) and cumulative distribution function \( F(\cdot) \). Assume that \( \varepsilon \) is independent of time with a mean of one, i.e., \( E[\varepsilon] = 1 \).

(3) Shortages are permitted during the cycle and will lead to a penalty cost of \( b \) per unit. At the end of the cycle, the salvage items are all sold at a reduced price of \( g \) per unit.

(4) Permissible delay in payment to retailer is considered. The supplier offers different discount rates on purchase cost to retailer at different credit periods.

(5) Considering the time during the credit periods, generated sales revenue is deposited in an interest-bearing account. At the end of credit period, the retailer starts paying for the interest charges on the items in stocks.

(6) \( I_p \) is generally greater than \( I_e \) in practice.

The above notations and assumptions are inspired essentially by the study of Sana [19].

3. Model formulation

Inventory model for stochastic demand will be established in this part. And we also consider the supplier offers different discount rates of price to the retailer at different credit periods.

The purchasing costs at different credit periods offered by the supplier as

\[
\begin{align*}
M = M_1, \quad & c_1 (1 - \delta_1), \\
M = M_2, \quad & c_2 (1 - \delta_2), \\
M = M_3, \quad & c_3 (1 - \delta_3), \\
M > M_3, \quad & \infty,
\end{align*}
\]

which are proposed in the model of Sana [19], where \( c_i \) is the maximum retail price per unit, \( M_i (i = 1, 2, 3) \) is decision point where supplier offers \( \delta_i \) discount to the retailer. \( c \to \infty \) when \( M > M_3 \) implies that retailer never order any quantity at an infinite cost. The discount rate \( \delta_i (i = 1, 2, 3) \) is a constant decided by the supplier. \( M_i \) and \( \delta_i \) are best fitted by the supplier, from a priori knowledge of statistics.

From the above discussion, the expected total profit in an order for the retailer can be expressed as \( EP = E[\text{revenue} - \text{purchase cost} + \text{salvage value} - \text{shortage cost} + \text{interest earned} - \text{interest payable}] \), which are listed below:

The purchasing cost is \( cQ \).

The sales revenue is \( p \min(Q, D(T)) \).

The salvage value is \( g(Q - D(T)) \).

The shortage cost is \( b[D(T) - Q] \).

The computation of interest. The interest payable and interest earned are associated with the size of \( M \) and \( T \). Two cases are discussed.

**Case 1.** when \( M < T \)

The interest earned during time-span \([0, M]\) is \( p \int_0^M \min(Q, D(t)) \, dt \).

The interest payable during time-span \([M, T]\) is \( c \int_M^T [Q - D(t)] \, dt \).
Therefore, the total profits \( (T_{P_i}, M \in \{M_1, M_2, M_3\}) \) in this case are
\[
T_{P_i} = p \min \{Q, D(T)\} - cQ + g[Q - D(T)]^+ - b[D(T) - Q]^+
+ pI(\int_0^{\infty} \min \{Q, D(t)\} \, dt - cL + \int_0^{T} [Q - D(t)] \, dt).
\] (2)

**Case 2.** when \( M \geq T \)

In this case, the interest payable is zero. The interest earned during time-span \([0, M]\) is
\[
pI(\int_0^{\infty} \min \{Q, D(t)\} \, dt + (M - T) \min \{Q, D(T)\}).
\]

Therefore, the total profits \( (T_{P_i}, M \in \{M_1, M_2, M_3\}) \) in this case are
\[
T_{P_i} = p \min \{Q, D(T)\} - cQ + g[Q - D(T)]^+ - b[D(T) - Q]^+
+ pI(\int_0^{\infty} \min \{Q, D(t)\} \, dt + (M - T) \min \{Q, D(T)\}).
\] (3)

Since \( D(t) = \alpha(t) \), it is easy to show that
\[
E[\min \{Q, D(t)\}] = Q - \alpha(t) \int_{-\infty}^{Q(\alpha(t))} F(\epsilon) \, d\epsilon
\] (4)
\[
E[Q - D(T)]^+ = \alpha(T) \int_{-\infty}^{Q(\alpha(T))} (1 - F(\epsilon)) \, d\epsilon
\] (5)
\[
E[D(T) - Q]^+ = \alpha(T) \int_{-\infty}^{Q(\alpha(T))} (1 - F(\epsilon)) \, d\epsilon
\] (6)

Therefore, the expected total profits for the case of \( M < T \) are
\[
EP_i(Q) = p[Q - \alpha(T)] \int_{0}^{Q(\alpha(T))} F(\epsilon) \, d\epsilon - cQ + g[Q - \alpha(T)]^+ - b[\alpha(T)]^+
+ pI(\int_0^{\infty} \alpha(t) \int_{0}^{Q(\alpha(t))} F(\epsilon) \, d\epsilon \, dt - cL + \int_0^{T} \alpha(t) \int_{0}^{Q(\alpha(t))} F(\epsilon) \, d\epsilon \, dt) \text{ for } M \in \{M_1, M_2, M_3\}.
\] (7)

The expected total profits for the case of \( M \geq T \) are
\[
EP_i(Q) = p[Q - \alpha(T)] \int_{0}^{Q(\alpha(T))} F(\epsilon) \, d\epsilon - cQ + g[Q - \alpha(T)]^+ - b[\alpha(T)]^+
+ pI(\int_0^{\infty} \alpha(t) \int_{0}^{Q(\alpha(t))} F(\epsilon) \, d\epsilon \, dt - (M - T) \alpha(T) \int_{0}^{Q(\alpha(T))} F(\epsilon) \, d\epsilon) \text{ for } M \in \{M_1, M_2, M_3\}.
\] (8)

Now, our objective is to solve the following integer non-linear program
\[
(p) \max EP(Q, M) \text{ subject to } Q \geq 0
\]

where
\[
EP(Q, M) = \text{Sup} \{EP_i(Q), EP_i(Q), M \in \{M_1, M_2, M_3\})
\] (9)

**4. Theoretical results**

Then we will give the calculation method of the optimal order quantity and payment time.

For convenience, let \( \theta_1 = p - c + b + pL, \theta_2 = p - g + b + pL, \theta_3 = p - g + b + pL, \theta_4 = \alpha(T - M) \).

**4.1. Case 1. when \( M < T \)**

Taking the first-order derivative of \( EP_i(Q) \) with respect to \( Q \), we obtain
\[
EP'_i(Q) = p - c + b + pL, M - (p - g + b)F(Q / \alpha(T))
\]
\[
- pI(\int_0^{\infty} F(Q / \alpha(t)) \, dt - cL + \int_0^{T} F(Q / \alpha(t)) \, dt).
\] (10)

Since \( g < p, f(x) > 0 \), we have
\[
EP^*_i(Q) = - \frac{p - g + b}{\alpha(T)} f\left(\frac{Q}{\alpha(T)}\right) < 0.
\] (11)
then \( EP_n(Q) \) is strictly concave in \( Q \), which implies that we can find the optimal order quantity through the first order optimality condition. Theorem 1 below indicates that there exists a unique \( \tilde{Q} \) satisfied \( EP_n'(\tilde{Q}) = 0 \).

**Theorem 1.** When \( M < T \), there exists a unique \( \tilde{Q} \in [Q_{a_i}, Q_{c_i}] \) satisfied \( EP_n'(\tilde{Q}) = 0 \) where

\[
\hat{Q}_i = F^{-1}(\theta_i / \theta_2), \quad Q_{a_i} = \hat{Q}_i \quad \text{and} \quad Q_{c_i} = \hat{Q}_i \alpha(T).
\]

**Proof.** Since \( \alpha(t) \) is increasing in \( t \) and \( t \leq T \), we have \( \alpha(t) \leq \alpha(T) \). Note that \( F(\cdot) \) is a cumulative distribution function, it follows that

\[
F(Q_{a_i} / \alpha(t)) \leq F(\hat{Q}_i / \alpha(t)) \leq F(Q_{c_i} / \alpha(t)), \quad \int_0^T F(\hat{Q}_i / \alpha(t)) dt = MF(\hat{Q}_i) = \int_0^T F(Q_{c_i} / \alpha(t)) dt.
\]

Let \( \phi(Q) = EP_n'(Q) \), then \( \phi(Q) \) is a continuous and monotonic function by the continuity and the monotonicity of \( F(\cdot) \). Since \( p > c > g \), \( M < T \), then \( \theta_1 \) and \( \theta_2 \) are all positive and \( \theta_1 < \theta_2 \). Moreover, we have

\[
\phi(Q_{a_i}) = \theta_1 - (p - g + b)F(\hat{Q}_i / \alpha(T)) - pl \int_0^T F(\hat{Q}_i / \alpha(t)) dt - cl \int_0^T F(\hat{Q}_i / \alpha(t)) dt \\
\leq \theta_1 - (p - g + b)F(\hat{Q}_i) - plMF(\hat{Q}_i) - clF(T - M)F(\hat{Q}_i) \\
= \hat{\theta}_1 - \hat{\theta}_2 F(\hat{Q}_i) = 0.
\]

\[
\phi(Q_{c_i}) = \theta_1 - (p - g + b)F(\hat{Q}_i) - pl \int_0^T F(\hat{Q}_i / \alpha(T)) dt - cl \int_0^T F(\hat{Q}_i / \alpha(t)) dt \\
\leq \theta_1 - (p - g + b)F(\hat{Q}_i) - plMF(\hat{Q}_i) - clF(T - M)F(\hat{Q}_i) \\
= \hat{\theta}_1 - \hat{\theta}_2 F(\hat{Q}_i) = 0,
\]

which implies that there is a unique \( \tilde{Q} \in [Q_{a_i}, Q_{c_i}] \) satisfied \( \phi(\tilde{Q}) = 0 \), i.e., \( EP_n'(\tilde{Q}) = 0 \).

4.2. **Case 2. when \( M \geq T \)**

Taking the first-order derivative of \( EP_n(Q) \) with respect to \( Q \), we obtain

\[
EP_n'(Q) = p - c + b + plM - (p - g + b)F(Q / \alpha(T)) \\
- pl \int_0^T F(Q / \alpha(t)) dt + (M - T)F(Q / \alpha(T)).
\]

Since \( g < p \), \( f(x) > 0 \), we have

\[
EP_n'(Q) = -\left(\frac{p - g + b}{\alpha(T)} - \frac{Q}{\alpha(T)}\right) - pl \int_0^T F(Q / \alpha(t)) dt + \frac{M - T}{\alpha(T)} - \frac{Q}{\alpha(T)} < 0 \tag{15}
\]

then \( EP_n(Q) \) is strictly concave in \( Q \), which implies that we can find the optimal order quantity through the first order optimality condition. Theorem 2 below indicates that there exists a unique \( \tilde{Q} \) satisfied \( EP_n'(\tilde{Q}) = 0 \).

**Theorem 2.** When \( M \geq T \), exists for a unique \( \tilde{Q}_2 \in [Q_{a_2}, Q_{c_2}] \) satisfied \( EP_n'(\tilde{Q}_2) = 0 \) where

\[
\hat{Q}_2 = F^{-1}(\theta_2 / \theta_1), \quad Q_{a_2} = \hat{Q}_2 \quad \text{and} \quad Q_{c_2} = \hat{Q}_2 \alpha(T).
\]

**Proof.** Since \( \alpha(t) \) is increasing in \( t \) and \( t \leq T \), we have \( \alpha(t) \leq \alpha(T) \). Note that \( F(\cdot) \) is a cumulative distribution function, it follows that

\[
F(Q_{a_2} / \alpha(t)) \leq F(\hat{Q}_2) \quad \text{and} \quad \int_0^T F(Q_{a_2} / \alpha(t)) dt \leq TF(\hat{Q}_2) \leq \int_0^T F(Q_{a_2} / \alpha(t)) dt.
\]
Let $\varphi_2(Q) = ETP_2'(Q)$, then $\varphi_2(Q)$ is a continuous and monotonic function by the continuity and the monotonicity of $F(\cdot)$. Since $p > c > g$, then $\theta_1$ and $\theta_2$ are all positive and $\theta_1 < \theta_2$. Moreover, we have

$$
\varphi_2(Q_{12}) = \theta_1 - (p - g + b)F\left(\hat{Q}_1 / \alpha(T)\right) - pI_t\left[\int_0^T F\left(\hat{Q}_1 / \alpha(T)\right)dt + (M - T)F\left(\hat{Q}_1 / \alpha(T)\right)\right] \\
\geq \theta_1 - \theta_2 F(\hat{Q}_2) \\
= 0,
$$

(16)

$$
\varphi_2(Q_{22}) = \theta_1 - (p - g + b)F(\hat{Q}_2) - pI_t\left[\int_0^T F(\hat{Q}_2 / \alpha(T))dt + (M - T)F(\hat{Q}_2)\right] \\
\leq \theta_1 - \theta_2 F(\hat{Q}_2) \\
= 0,
$$

(17)

which implies that there is a unique $\hat{Q}_2 \in [Q_{12}, Q_{22}]$ satisfied $\varphi_2(\hat{Q}_2) = 0$, i.e., $ETP_2'(\hat{Q}_2) = 0$.

From above discussion, we have

$$
\{Q', M'\} = \arg\max\{EP_0(\hat{Q}_1), EP_2(\hat{Q}_2), i \in \{1, 2, 3\}\},
$$

(18)

4.3. The solution procedure

Although we are unable to provide a closed-form solution of the problem (P) due to its complexity, we can present its numerical solution through the following optimization method.

Step 1. Enter the values of the parameters.

Step 2. When $M < T$, determine the value of $Q_{11}$ and $Q_{21}$ according to the Theorem 1, and compute the value of $\hat{Q}_1$ by using a non-linear program search method such as newton iteration method, then figure out the corresponding expected profits $EP_1(\hat{Q}_1)$. Similarly, when $M \geq T$, determine the value of $Q_{12}$ and $Q_{22}$ according to the Theorem 2, and compute the value of $\hat{Q}_2$, then figure out the expected profits $EP_2(\hat{Q}_2)$.

Step 3. Determine the optimal value of $Q'$ and $M'$ by comparing the maximal expected profits, $EP_0(\hat{Q}_1)$, $EP_2(\hat{Q}_2)$, $i \in \{1, 2, 3\}$.

5. Numerical examples

In this section, the present study provides the following numerical examples to illustrate all the theoretical results as reported in Section 4. For convenience, the values of the parameters are selected randomly. We consider the values of the parameters as follows: $c_i = $15/unit , $p = $50/unit , $g = $12/unit , $b = $10/unit , $I_s = 0.12/S/year , $I_p = 0.18/S/year , $T = 150/360 year , $M_1 = 60/360 year , $\delta_1 = 10\%$ , $M_2 = 120/360 year , $\delta_2 = 5\%$ , $M_3 = 180/360 year , $\delta_3 = 0\%$ and the cumulative demand function is $D(t) = 10e^{0.5t}$, where $e$ is normally distributed with a mean of 1 and a standard deviation of 2.

Then the optimal solutions are: $\{EP_{11} = $670.81 , $\hat{Q} = 24.09$ units $\}, \{EP_{12} = $668.92 , $\hat{Q} = 24.00$ units $\}, \{EP_{21} = $659.09 , $\hat{Q} = 23.92$ units $\}$. Among those optimal solutions, the better optimal solution is $\{EP_{11} = $670.81 , $Q' = 24.09$ units, $M' = 60/360 year \}$. It is clear that there is tiny difference in the optimal order quantity of three strategies offered by supplier, but when the retailer takes a shorter credit period, he can get more expected profit.

When supplier offers different discount rates on purchase cost to retailer and the other arguments are unchanged, the optimal solutions are shown in Table 1. This result shows that the optimal expected profit is influenced distinctly by different discount rates and credit periods. When the price discount offered by supplier is adequately high, the retailer should choose a short credit period.
Table 1. The optimal solutions of different discount strategies.

| $i$ | $\delta_i$ | $\bar{Q}$ | $EP(\bar{Q})$ | $\bar{Q}$ | $EP(\bar{Q})$ |
|-----|------------|-----------|---------------|-----------|---------------|
| 1   | 20%        | 25.49     | 741.67        | 10%       | 25.06         | 702.83        |
| 2   | 10%        | 25.17     | 722.94        | 5%        | 25.17         | 722.94        |
| 3   | 0%         | 25.33     | 730.98        | 0%        | 25.33         | 730.98        |

Next, we present the effects of random variable contained in the cumulative demand function to the optimal solutions. Suppose $\delta_1 = 20\%$, $\delta_2 = 10\%$, $\delta_3 = 0\%$, the other arguments are the same in example 1. When $\varepsilon$ is normally distributed with different standard deviation (denote as $\delta$), the optimal solutions are shown in Table 2 and Figure 1. It is worth to be mentioned that the optimal credit period $M^* = M_2 = 120/360$ year are all the same under above assumptions.

From Table 2, we know that when the random variable distributes on a larger interval, the optimal order quantity increases gradually, but the expected profit decreases. This implies retailer need order more quantities to satisfy the demand when demand is affected deeply by random factors, which may lead to high possibilities of salvage or shortage. Since the salvage will lead to disvalue of productions, and the stockout will bring the punishment cost, the expected profit reduces naturally, which illustrates that the influence of the random demand to inventory system cannot be ignored.

Table 2. Sensitivity analysis on $\delta$.

| $\delta$ | $\bar{Q}$ | $EP(\bar{Q})$ |
|----------|-----------|---------------|
| 2        | 25.49     | 741.67        |
| 3        | 38.14     | 983.24        |
| 4        | 50.78     | 1224.6        |
| 5        | 63.43     | 1465.8        |
| 6        | 76.08     | 1707.1        |

Figure 1. Sensitivity analysis on $\delta$.

In conclusion, in order to get more expected profits, the retailer should order more quantities to satisfy the demand affected deeply by random factors.
6. Conclusions
Inventory can maintain the stability of product sales, balance the company’s capital flow, and help companies achieve economies of scale. Enough inventory can ensure the quality of the company’s services and reduce out-of-stock costs, but inventory hoarding may increase management costs. So efficient inventory management is an important part of business operations. In the proposed model, we analyze an inventory model of undetermined demand for a retailer where supplier offers different trade credit and price-discount on the ordering products. Due to the randomness of demand, two possibilities of salvage or stock out at the end of the cycle are considered. The optimal order quantity is obtained by maximizing the expected profit. A series of numerical examples are presented to demonstrate that it is beneficial for retailer to take a shorter credit period and order more quantities to satisfy the demand. To the best of authors’ knowledge, the proposed model has not considered in the literature before. This has been done in the present paper. Future extensions of the proposed model are possible in several ways. First, our work could be improved by considering the effect of inflation on the economic order quantity. And we may consider a modified inventory model by integrating deterioration and imperfect quality for items based on the same assumption in the future. Also, we can further consider this inventory model with limited inventory capacity.

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