Mach-Zehnder interferometer in the Fractional Quantum Hall regime

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We consider tunneling between two edges of Quantum Hall liquids (QHL) of filling factors \( \nu_0,1 = 1/(2m_{0,1}+1) \), with \( m_0 \geq m_1 \geq 0 \), through two point contacts forming Mach-Zehnder interferometer. Quasiparticle description of the interferometer is derived explicitly through the instanton duality transformation of the initial electron model. For \( m_0 + m_1 + 1 \equiv m > 1 \), tunneling of quasiparticles of charge \( e/m \) leads to non-trivial \( m \)-state dynamics of effective flux through the interferometer, which restores the regular "electron" periodicity of the current in flux. The exact solution available for equal propagation times between the contacts of interferometer shows that the interference pattern depends in this case on voltage and temperature only through a common amplitude.

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An electronic Mach-Zehnder interferometer (MZI) realized recently \(^1\) at Weizmann institute consists of the two point contacts between two single-mode edges of the Integer Quantum Hall liquids. Its transport properties exhibit strongly pronounced electron interference sensitive in experiments to charging effects. MZI in the regime of the Fractional Quantum Hall effect (FQHE) \(^2\) \(^3\) and more complicated structures including it \(^4\) were studied theoretically in search for signatures of the fractional statistics of FQHE quasiparticles. Some of these theories, however, (cf. \(^2\) and \(^3\)) were based on different postulated models of the quasiparticle transport in MZI and obtained conflicting result, e.g., different periods of the tunnel current modulation by external magnetic flux \( \Phi \) through the interferometer. The goal of this work is to develop the theory of symmetric MZI in the FQHE regime that is valid for arbitrary tunneling strength in its point contacts. In this theory, quasiparticles are derived consistently from the standard model of electron tunneling in the weak-tunneling limit. Scaling of electron tunneling amplitudes up with voltage or temperature to the strong-tunneling limit (similar to that in one point contact \(^6\)) generates non-trivial model of quasiparticle tunneling in MZI as a dual to weak electron tunneling.

The main qualitative elements of our approach can be summarized as follows. The phase difference between the two point contacts expressed in terms of the effective flux \( \Phi \) through the MZI contains, in addition to the external flux \( \Phi_{ex} \), a statistical contribution. This contribution emerges, since each electron coherently tunneling at different contacts changes \( \Phi \) by \( \pm m \Phi_0 \), where \( m = m_0 + m_1 + 1 \) and \( \Phi_0 \) is the flux quantum. As a result, the system has \( m \) quantum states which differ by number of flux quanta modulo \( m \) which are not mixed by perturbative electron tunneling. However, in the non-perturbative regime of strong tunneling, the states are mixed as \( \Phi \) is changed by \( \pm \Phi_0 \) by tunneling of individual quasiparticles. This implies that the quasiparticles have to carry the charge \( e/m \) and coincide with the quasiparticles \( e^* = 2\pi\nu_0\nu_1/(\nu_0+\nu_1) = e/m \) in one point contact \(^6\). (Flux dynamics in the MZI is similar to that in the antidot tunneling \(^3\), where, however, \( m = m_0 - m_1 \), and the \( e/m \) quasiparticles are different from those in individual contacts.) The quasiparticle Lagrangian for MZI derived below is a mathematical expression of the flux-induced electron-quasiparticle transmutation. If the times \( t_0 \) and \( t_1 \) of propagation between the contacts along the two edges of the interferometer are equal: \( \Delta t = t_0 - t_1 = 0 \), the quasiparticle Lagrangian can be solved by methods of the exactly solvable models. The resultant expression for the tunneling current shows the crossover from the quasiparticle tunneling at large voltages to the electron tunneling at low voltages. Our results correct Ref. \(^2\) by showing that the quasiparticle model used in that work does not correspond in the weak-tunneling limit to electron tunneling at two separate point contacts, and also restrict the validity of the quasiparticle current found in \(^3\) to the leading term in the large-\( V \) asymptotics.

In details, we start with the electronic model of MZI (Fig. 1) formed by two single-mode edges with filling factors \( \nu_l = 1/(2m_l+1) \), \( l = 0,1 \), which differ from the model studied in \(^7\) only by the direction of propagation of one edge. In the standard bosonization approach \(^8\), Lagrangian of weak electron tunneling in the two contacts \(^8\) can be expressed through two bosonic fields \( \phi_l \) related to the correspondent edge density \( \rho_l(x,\tau) = (\sqrt{\mu_l}/2\pi)\partial_\tau \phi_l(x,\tau) \) as follows:

\[
\mathcal{L}_t = \sum_{j=1,2} \frac{DU_j}{2\pi} e^{i\nu_j} e^{i\lambda \varphi_j} + \text{h.c.} \equiv \sum_{j=1,2} (T_j^+ + T_j^-) \tag{1}
\]

\[
\lambda \varphi_j(t) = \frac{\phi_0(x_j,t)}{\sqrt{\nu_0}} - \frac{\phi_1(x_j,t)}{\sqrt{\nu_1}} , \quad \lambda = \sqrt{2m} , \tag{2}
\]

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where $D$ is a common energy cut-off of the edge modes, $U_j$ and $\kappa_j$ are the absolute values and the phases of the dimensionless tunneling amplitudes. Dynamics of the operators $\varphi_j$ is defined by the Fourier transform of the imaginary-time-ordered correlators $\langle \varphi(x) \varphi_p \rangle_\omega = \delta_{x p} g(x/v_1, \omega)$, where (see, e.g., [4]):

$$g(z, \omega) = \frac{2\pi}{\omega} \text{sgn}(z) \left( -\frac{1}{2} + \theta(\omega z) e^{-\omega z} \right),$$

(3)

and the first term in brackets gives the usual equal-time commutation relation $[\varphi(x), \varphi_p(0)] = i\delta(x) \delta_{p 0}$. The phases $\kappa_j$ include contributions from the external magnetic flux $\Phi_{ex}$ and from the average electron numbers $N_{0,1}$ on the two sides of the interferometer: $\kappa_2 - \kappa_1 = 2\pi[\Phi_{ex}/\Phi_0 + (N_0/\nu_0) - (N_1/\nu_1)] + \text{const} \equiv \kappa$.

If a bias voltage $V$ is applied to the junction, the operator of electron current from the edge 0 into the edge 1 is $I^c = i \sum_{j=1,2} \sum_{\pm} \langle \pm \rangle T_{\pm} e^{i\phi T_{\pm}}$. Its average contains the phase-insensitive part $I^c$ and the phase-sensitive interference term $\Delta I^c(\kappa)$: $I = \langle I^c \rangle = I^c + \Delta I^c(\kappa)$. At finite $V$, the phase difference $\kappa$ acquires additional contribution related to $V$. For instance, if the voltage changes only the electrochemical potential of the edge 0, and for perfect screening by external gates, the phase varies as $\kappa(V) = \kappa + V t_0$. In the lowest non-vanishing order in $U_j$, the phase-sensitive current consists of two contributions from the individual point contacts $I^c = \sum_j I_j^c$, which at temperature $T$ are [3]:

$$I_j^c = (U_j^2 D/2\pi)(2\pi T/D)^{3/2-1} C_{\chi D}(V/2\pi T),$$

(4)

where $C_{\chi D}(v) \equiv \frac{\sinh(\pi v)(\Gamma(g/2 + iv))^2}{\pi^3\Gamma(g)}$, and, for $g$ equal to an even positive integer, reduces to the polynomial $C_{\chi D}(v) = v \prod g^{2-1}(u^2 + v^2)/\Gamma(g)$. The interference current can be written as

$$\Delta I^c = \left( \frac{U_1 U_2 D}{\pi^2} \right) (\pi T/D)^{3/2-1} \text{Im} \left\{ \int_{-\infty}^{\infty} ds \sin(\kappa(V) - V t - \frac{s V}{\pi T}) \prod_{l=0,1} \left[ i \sinh(s - (-1)^l \Delta t + i 0) \right]^{-1/\nu_l} \right\},$$

(5)

in the notation $t_{0,1} = t \pm \Delta t$. This expression coincides (up to redefinition of the phase $\kappa_V = \kappa(V) - V t$) with the interference current in the antidot geometry [7, 10]. One can evaluate the integral [5] for integer $1/\nu_l$ by residues, and find the visibility $\text{Vis} \equiv (\max \frac{I}{\min} I)/(\max \frac{I}{\min} I)$. For instance, for $\nu_0 = \nu_1 = \nu$ in the asymptotic regime $V \Delta t \gg 1$ and low temperatures, the visibility decreases with voltage oscillating as

$$\text{Vis} \sim \frac{2(\nu_0 - 1)\nu_0}{(\nu_0 - 1)!} \frac{V}{U_1^2 + U_2^2} \left| \frac{\sin(V \Delta t)}{\pi \Delta t^1/\nu_1} \right|,$$

(6)

In the opposite limit of $V, T < 1/\Delta t$, the integral in Eq. (5) reduces to the same polynomial $C_{\chi D}(V/2\pi T)$ as in Eq. (4), and the full current $\langle I^c \rangle$ is specified by the coherent sum of the two point-contact amplitudes:

$$I = |U_1 + U_2 e^{i\kappa} \sqrt{2} T(2\pi T/D)^{3/2-1} C_{\chi D}(V/2\pi T).$$

(7)

In this regime, the visibility reaches its maximum $2U_1 U_2/(U_1^2 + U_2^2)$. Naively, Eq. (7) seems to suggest that for small $\Delta t$ the two-point-contact model of MZI reduces to one point contact with the new tunnel amplitude. This, however, is true only for weak tunneling. At large voltages or temperatures, the system automatically flows into the regime of strong tunneling, in which the model of two FQHs strongly coupled at two separate points possesses non-trivial topology of quasiparticle tunneling trajectories. Since FQHL is a topological quantum liquid [8], the non-trivial topology of the model implies multiple degeneracy of its ground state which is absent in one point contact.

To derive the dual strong-coupling model for the MZI, we treat the problem in imaginary time and follow a standard instanton technique. The ground states are determined by minimization of the action $S$ that consists of the tunneling part $S_t [1]$ and kinetic term $S_{\text{kin}}$ defined by the correlators [3]. In the limit $U_j \gg 1$, $S_t$ gives the dominant contribution to the action, and it would be natural to fix both tunneling modes $\varphi_j$ at the extrema of their corresponding parts of Eq. (1). These modes, however, do not commute, $[\varphi_2, \varphi_1] = i\pi$, the fact reflected in the interference relation for the transfer terms [1] at the two contacts:

$$T^\pm_{1} T^\mp_{2} = e^{2\pi i \nu_{1} T^\pm_{1} T^\mp_{2}}.$$

(8)

We see that while different $T^\pm_j$ commute, each interchange of electron transfers at the two contacts adds statistical contribution $\pm m \Phi_0$ to the external magnetic flux $\Phi_{ex}$ modifying the interference phase $\kappa$. This flux dynamics affects the perturbative expansion of the partition function in $U_{1,2}$ by changing the phase branch of terms in the expansion according to Eq. (3), when the imaginary times of the transfer operators, $T^\pm_j$ and $T^\mp_j$, change order. In general, we can make different choices of the phase branches multiplying the operators $T^\pm_j$ by some Klein factors $\exp(\pm i\sqrt{S_j} \eta_j)$ with arbitrary integers $\gamma$ and the free zero-energy bosonic modes $\eta_j$ defined by their imaginary-time-ordered correlators: $T_{\tau \eta_j}(\tau) \eta_j(0) = \pm i \Theta(j - i\tau)(1 - \delta_{ij})$. For any $\gamma$,
incorporation of these Klein factors into \( T_j^+ \) does not change the perturbation expansion of the partition function in \( S_l \) in any order. However, it affects the kinetic part of the action and hence the ground-state energy. Indeed, the new tunneling fields \( \Phi_j = \lambda \varphi_j + \sqrt{2\gamma} \eta_j \) are characterized by the kinetic action

\[
S_{\text{kin}} = \frac{1}{2} \int \frac{d\omega}{2\pi} \sum_{i,j} \Phi_i(-\omega) K_{i,j}^{-1}(\omega) \Phi_j(\omega),
\]

where \( \hat{F} \) are the raising and lowering matrices.

For well-separated contacts, \( t_{0,1}D \gg 1 \), minimization of energy under the strong-tunneling conditions:

\[
\Phi_j = 2\pi n_j - \kappa_j \equiv \Phi_{n_j},
\]

(10)
gives \( \gamma = m \), the choice that also guarantees the commutativity of the tunneling fields \( \Phi_j \).

The standard instanton expansion of the partition function \( Z \) for the degenerate ground states \( (\Phi_{n_1}, \Phi_{n_2}) \) expresses it as \( Z = \sum_{n_1, n_2} Z_{n_1, n_2} \). Substitution of the asymptotic form of the expansion around the state \( (\Phi_{n_1}, \Phi_{n_2}) \): \( \Phi_j(\tau) = \Phi_{n_j} + \sum_{2\pi i\epsilon_j, \theta(\tau - \tau_{ij})} \exp\{ -S(\Phi_1, \Phi_2) \} \), followed by summation over the numbers of instantons/anti-instantons \( 2\pi \epsilon_j = \pm 1 \) and integration over their times \( \tau_{ij} \) gives:

\[
Z_{n_1, n_2} \propto \int \frac{d\tau}{2\pi} \exp\{ -S_{\text{kin}}^D + \sum_j W_j D \int d\tau \cos[\Theta_j(\tau) + \kappa_j - 2\pi n_j \left( \frac{1}{1} \right)/m] \}
\]

(11)

where a constant of proportionality independent of \( n_1, n_2 \), and

\[
S^D_{\text{kin}}(\Theta) = \frac{1}{2} \int \frac{d\omega}{2\pi} \Theta(-\omega) \left( \frac{2\pi}{\omega} K^{1/2} - 1 \right)(\omega)^{-1} \Theta(\omega).
\]

Comparing the \( \Theta \)-correlators defined by action (12) with \( g(z, \omega) \), we separate these fields into two parts: \( \Theta_j = (-1)^j \left[ (2/m)^1/2 \eta_j + (2/\lambda) \vartheta_j \right] \), with statistical terms \( \eta_{1,2} \), and the chiral fields \( \vartheta \):

\[
< \vartheta^2 > = g(0, \omega), \quad < \vartheta \vartheta > = \frac{g(t_0, 0)}{\nu_0 \lambda^2} + \frac{g(t_1, 0)}{\nu_1 \lambda^2}.
\]

(13)

Since the terms \( Z_{n_1, n_2} \) depend on \( n_1, n_2 \) only through their difference modulo \( m \), the partition function \( Z \) becomes a finite sum up to an irrelevant (but divergent) factor. Summation over \( n_1 - n_2 \) combined with integration over the new statistical fields can be reduced to the trace over an \( m \)-dimensional Hilbert space, if a proper \( m \)-dimensional matrix is ascribed to each instanton tunneling exponent in (11). These unitary matrices \( \hat{F}_j \) are characterized by the following relations:

\[
\hat{F}_1 \hat{F}_2 = e^{2\pi i} \hat{F}_2 \hat{F}_1, \quad (\hat{F}_1^{\dagger} \hat{F}_2^{\dagger})^p \hat{F}_2 \hat{F}_1^p = \delta_{kp} \delta_{lq},
\]

(14)

where the Kronecker symbol \( \delta_{ij} \) is defined modulo \( m \). The first relation in (14) is due to the statistical parts of the fields \( \Theta_j \), while the second one follows from the \( m \)-periodic dependence of (11) on \( n_{1,2} \). Writing \( Z \) as a trace makes possible to interpret it as a partition function of quasiparticles with tunneling Lagrangian \( \mathcal{L}_t \) of the real-time form dual to the Lagrangian (11):

\[
\mathcal{L}_t = \sum_{j=1,2} \left[ \frac{W_j D}{2\pi} \hat{F}_j \exp \left\{ i \left( \kappa_j(V_j + \frac{2\gamma_j}{m} - \frac{V t_j}{m} \right) \right\} + h.c. \right]
\]

(15)

The operators \( \hat{F}_j \) are the quasiparticle Klein factors describing their statistics and acting in the space of the \( m \)-degenerate (in the absence of quasiparticle tunneling) ground state of the MZI. The current associated with the quasiparticle tunneling is expressed as usual in terms of the transfer operators: \( \hat{F}^\dagger = (i/m) \sum_{j=1,2} \sum_{\pm} \sum_{\pm} \hat{T}_{\pm}^j e^{\pm iV t_j/m} \). The model (15) with the Klein factors (17) for quasiparticle tunneling between the edges of the MZI is a direct analogue of the quasiparticle model we derived earlier for the antidot [7]. At \( \nu_0 = \nu_1 \) it coincides with the postulated quasiparticle model of [3] for a special choice of matrices \( \hat{F}_j \).

In the case of symmetric interferometer, with \( \Delta t = 0 \) and equal velocities of the edge modes \( \varphi_j \), both tunneling operators \( \varphi_j \) in Eq. (11) are the operator values of the chiral bosonic field \( \varphi \) at two points \( x_1, x_2 \). The field \( \varphi \) is composed of the incoming edge modes \( \varphi_0, \varphi_1 \) as in Eq. (2). Strong tunneling at the point contacts can be described by imposing the Dirichlet boundary condition on \( \varphi \), the “unfolded” form of which (11) implies a free chiral propagation of the fields \( \text{sgn}(x-x_j)(\varphi_1(x) + \kappa_j/\lambda) \) across the point \( x_j \). Deviation from their free propagation is driven by the dual tunneling terms \( \mathcal{L}_{t,j} = DW_j \cos(2/\lambda(\varphi(x_j + \kappa_j/\lambda))/\pi) \). Successive application of these boundary conditions at the two contacts gives free propagation of the dual chiral field

\[
\vartheta_j(x) = \varphi_j(x) \theta_j(x_1 - x) + (\varphi_j(x) - 2\kappa_j/\lambda) \theta(x - x_2)
\]

(16)

Substitution of this field into \( \mathcal{L}_{t,j} \) and further comparison of the result with Eqs. (12) and (14) prove that \( \vartheta_j = \vartheta_j(x_j) \). Applied voltage changes \( \vartheta_j \) into \( \vartheta_j - V t_j/\lambda \).

The derived quasiparticle model (15) is exactly solvable at \( \lambda = 2 \) (i.e., when \( \nu_0 = 1/3 \) and \( \nu_1 = 1 \)) by fermionization. Indeed, the Klein factors for \( m = 2 \) can be represented by two Pauli matrices and fermionized as \( \hat{F}_j = i\xi_j \hat{S}_0 \) in terms of three Majorana fermions \( \{ \xi_1, \xi_\prime_1 \} = 2 \delta_{n, n'} \). Introducing a chiral fermion field as \( \psi = \xi_0 \sqrt{D}/(2\pi v) \exp(i\vartheta) \) we come to the Hamiltonian

\[
\mathcal{H} = -iv \int dx \psi^+ \partial_x \psi - v \sum_j [w_j \xi_j \psi^+(x_j) + h.c.],
\]

(17)
where the applied voltage is accounted for by the fermion chemical potential equal to \( V/2 \). Note that the Hamiltonian \( \{17\} \), in contrast to the fermionic Hamiltonian of \( \{2\} \), contains two different Majorana fermions at the two tunneling points \( x_{1,2} \). As a result, the Heisenberg equation of motion describes scattering at the two points of the field \( \psi(x,t) \), elsewhere exhibiting a free chiral propagation, with two disentangled matching conditions

\[
i\psi(x)|_{x_j=0}^\pm = w_j \xi_j, \quad \partial_t \xi_j(t) = 2i\nu[w_j \psi^+(x_j, t) - h.c.]. \tag{18}
\]

Away from the tunneling points, the field \( \psi(x,t) \) propagates freely and can be represented as \( \psi(x,t) = \int dk \psi_k \exp\{ik(x-vt)\}/(2\pi) \). Solution of each of the condition \( \{18\} \) defines then a \((2 \times 2)\) scattering matrix \( S_{j,k}^\pm \) and particle and hole \( \psi_k, \psi_k^- \) at contact \( j \):

\[
S_{j,k}^\pm = \frac{k}{k + i2|w_j|^2}, \quad S_{j,k}^{-+} = \frac{2iw_j^2}{k + i2|w_j|^2}. \tag{19}
\]

Successive particle-hole scattering at the two points is governed by the scattering matrix \( S_k = S_{2,k} S_{1,k} \) which determines the average tunneling current:

\[
I = \int_0^{2\pi} \frac{vdk}{2\pi} \sum_j \left| \frac{2ik}{\sum_j |w_j|^2} \right|^2. \tag{20}
\]

Introducing \( \Gamma_j = DW_j^2/\pi \), one expresses the current as

\[
I = \frac{\Gamma_1 e^{i\nu V} + \Gamma_2}{\Gamma_1 - \Gamma_2} \left[ I_{1/2}(V, \Gamma_2) - I_{1/2}(V, \Gamma_1) \right], \tag{21}
\]

where \( I_{1/2}(V, \Gamma) \) is the tunneling current in a single point contact \( \{3\} \): \( I_{1/2}(V, \Gamma) = V^{\pm}/(4\pi) - \Gamma/(2\pi) \arctan(V/2\Gamma) \). The low voltage asymptotics of \( I \) is proportional to \( V^3 \) and coincides with the electron tunneling current in Eq. \( \{7\} \) under the condition \( U_j = \pi W_j^2/2 \) expected from the single-point-contact duality. Equation \( \{21\} \) holds at finite temperature \( T \).

This calculation of the current can be generalized to other values of \( \lambda^2 = 2m \), for which a thermodynamic Bethe ansatz solution is known \( \{13\} \) for a single point contact. The solution exploits a set of quasiparticle states describing \( \vartheta_- (x) \) excitations and introduced through the massless limit of a sine-Gordon model. These quasiparticles are kinks, antikinks, and breathers of height defined by the sine-Gordon interaction and equal to \( \pi \lambda \). They remain interacting in the massless limit as described by a bulk \( S \)-matrix, but undergo separate one by one scattering on the point contact specified with a one particle boundary \( S \)-matrix. Their scattering on two point contacts occurs successively and separately at different points as follows from chiral dynamics of the local \( \vartheta_- (x) \) fluctuations at the point contacts derived above through application of "unfolded" Dirichlet boundary conditions. Therefore it is described with product of two boundary \( S \)-matrices dependent on \( \kappa_{1,2} \), respectively. To obtain these matrices from the one found in \( \{13\} \), in the case of \( \kappa = 0 \), we notice that each phase \( \kappa_j \) in Eq. \( \{13\} \) results from the shift of the \( \vartheta_- \) by the constant \( \kappa_j/\lambda \). Hence the operators \( \exp(\pm i\lambda \vartheta_- /2) \) of the \( \vartheta_- \) - kinks/antikinks acquire just constant phase factors \( e^{\pm i\nu/\lambda} \). The boundary \( S \)-matrix of \( \{13\} \) transforms into

\[
S_{j,k}^\pm = \frac{(k/T_{jB})^{m-1} e^{i\nu k}}{1 + i(k/T_{jB})^{m-1}}, \quad S_{j,k}^{-+} = \frac{e^{i(\kappa_k - \kappa_j \nu)}}{1 + i(k/T_{jB})^{m-1}},
\]

and the tunneling current produced by the kink-antikink transitions breaking charge conservation takes for the two point contact the following form

\[
I = \int_0^\infty dk |(S_j S_k)^{-+}|^2 n[f_+ - f_-]. \tag{22}
\]

Notice that both, the density of states \( n(k, V) \) and the distribution functions \( f_\pm \) for kinks and antikinks, are defined by the "bulk" of the system and do not depend on the scattering on impurities. Then the tunneling current in \( \{22\} \) takes the form that generalizes Eq. \( \{21\} \)

\[
I = \left| T_{1B}^{m-1} e^{i\nu V} + T_{2B}^{m-1} \right|^2 \left( I_{1/m}(V, T_{2B}) - I_{1/m}(V, T_{1B}) \right), \tag{23}
\]

where \( I_{1/m}(V, T_{jB}) \) is the tunneling current through a single point contact between two effective edges of the filling factor \( 1/m \). The energy scales \( T_{jB} \) are related to both correspondent electron and quasiparticle tunneling amplitudes \( U_j, W_j \) in the same way as in the case of the individual point contact \( \{13\} \). This matches the low voltage dependence of the current in \( \{23\} \) with the electron tunneling current asymptotics in Eq. \( \{7\} \). Meanwhile, its high voltage dependence asymptotically coincides with the quasiparticle calculation in \( \{3\} \). Finally, the expression \( \{23\} \) for the current shows that the visibility of Eq. \( \{7\} \) does not vary with temperature and voltage, while \( V \Delta, T \Delta t \ll 1 \). The interference dependence of the current on the external magnetic flux has the same form of a simple on mode modulation, which is not affected by the change of regimes from electron to quasiparticle tunneling.

In conclusion, we have derived the quasiparticle model \( \{Eqs. \{17\}, \{14\}, \text{and } \{15\}\} \) of the Mach-Zender interferometer in the FQHE regime for arbitrary filling factors of interferometer edges of MZI from its electron tunneling model. In the limit \( \Delta t = 0 \), this model allows an exact solution, which describes the crossover from electron to quasiparticle tunneling and shows that the interference pattern remains the same in both regimes and is independent of voltage and temperature.

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[1] Y. Ji, Y. C. Chung, D. Sprinzak, M. Heiblum, D. Mahalu, and H. Shtrikman, Nature 422, 415 (2003); I. Neder, M. Heiblum, Y. Levinson, D. Mahalu, and V. Umansky, Phys. Rev. Lett. 96, 016804 (2006).
[2] T. Jonckheere, P. Devillard, A. Crepieux, and T. Martin, Phys. Rev. B 72, 201305(R) (2005).
[3] K.T. Law, D.E. Feldman, and Y. Gefen, Phys. Rev. B 74, 045319 (2006).
[4] C.L. Kane, Phys. Rev. Lett. 90, 226802 (2003).
[5] C.L. Kane and M.P.A. Fisher, Phys. Rev. B 46, 15233 (1992).
[6] C.C. Chamon and E. Fradkin, Phys. Rev. B 56, 2012 (1997); N.P. Sandler, C.C. Chamon, and E. Fradkin, Phys. Rev. B 57, 12324 (1998); Phys. Rev. B 59, 12521 (1999).
[7] V.V. Ponomarenko and D.V. Averin, Phys. Rev. B 71, 241308(R) (2005).
[8] X.G. Wen, Adv. Phys. 44, 405 (1995).
[9] V.V. Ponomarenko and D.V. Averin, Phys. Rev. B 70, 195316 (2004).
[10] M.R. Geller and D. Loss, Phys. Rev. B 56, 9692 (1997).
[11] Ch. Nayak et al, Phys. Rev. B 59, 15694 (1999).
[12] V.V. Ponomarenko and D.V. Averin, Phys. Rev. B 67, 35314 (2003); JETP Lett. 74, 87 (2001).
[13] P. Fendley, A.W.W. Ludwig, and H. Saleur, Phys. Rev. Lett. 74, 3005 (1995); Phys. Rev. B 52, 8934 (1995).