Comments on Measurements and Calculations of $\hat{q}L$ via transverse momentum broadening in Relativistic Heavy Ion Collisions using di-hadron correlations.

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Presentations at Quark Matter 2018, led me to reread articles on Measurements and Calculations of $\hat{q}L$ from azimuthal broadening of di-hadrons in Au+Au and p+p collisions at $\sqrt{s_{NN}} = 200$ GeV at RHIC. New results will be presented in addition to some corrections to the previous work. Di-jets rather than di-hadrons are proposed as an improved method to measure $\hat{q}L$ and possibly $\bar{q}$.

I. INTRODUCTION

When I was reviewing talks from Quark Matter 2018, a slide in a presentation by Miklos Gyulassy [1] drew my attention because it involved a figure (Fig. 2) from a preprint [2] that I had referenced in my publication on measuring $\hat{q}L$ from di-hadron correlations [3]. I had not paid much attention to that figure previously, but in checking the reference [4] to the two plots labeled PHENIX, I realized that the data in the plots looked nothing like the measurement shown in the quoted reference. When I pressed Bo-Wen, I got the following important additional information: “We used the software called xyscan to get the data points and the error from the figure. Indeed we rescaled the points for both pp and AA data to make the data and fits, as well as previous fit to the STAR data in Ref. [5], to Fig. 2.”

The most notable observation about the fits in Fig. 2 is that for both $p_{T_A}$ ranges, the PHENIX Au+Au fits have smaller $\sigma_{\Delta\phi}$ than the p+p fits, which is more convenient to quote in the variable $\langle p_{out}^2 \rangle = (p_{T_A} \sin \Delta\phi)^2$ as follows: for PHENIX $p_{T_A} = 5 – 10 \text{ GeV/c}$, the values of $\langle p_{out}^2 \rangle$ for $p_{T_A} = 3 – 5 \text{ GeV/c}$ and $p_{T_A} = 5 – 10 \text{ GeV/c}$ are $0.79 \pm 0.04 \text{ (GeV/c)^2}$ for Au+Au 0-20%, $1.54 \pm 0.08 \text{ GeV}^2$ for p+p; and for $p_{T_A} = 5 – 10 \text{ GeV/c}$, $2.12 \pm 1.13 \text{ (GeV/c)^2}$ for Au+Au and 3.92 $\pm 0.33 \text{ GeV}^2$ for p+p. For the STAR Au+Au 00-12%, $p_{T_A} = 12 – 20$, $p_{T_A} = 3 – 5 \text{ GeV/c}$ data, the results are the same as in Ref. [5], namely $\langle p_{out}^2 \rangle = 0.851 \pm 0.203 \text{ (GeV/c)^2}$ for Au+Au and $0.576 \pm 0.167 \text{ (GeV/c)^2}$ for p+p.

From these numbers it is obvious [3] that $\langle \hat{q}L \rangle$ (which corresponds to the $\langle p_{out}^2 \rangle$ on Fig. 2) is negative for the PHENIX data and thus not equal to $\langle \hat{q}L \rangle = 13 \text{ GeV}^2$ quoted on Fig. 2 [6]. For readers who may not understand this as obvious, a review of the method to calculate $\langle \hat{q}L \rangle$ is presented followed by the calculations of $\langle \hat{q}L \rangle$ from the PHENIX and STAR data in Fig. 8 and some other published PHENIX data, leading to an interesting conclusion.

II. A REVIEW AND IMPROVEMENT OF THE METHOD TO MEASURE $\langle \hat{q}L \rangle$ FROM DI-HADRON AZIMUTHAL BROADENING

The BDMPSZ [7] QCD based prediction for detecting the QGP is jet quenching produced by the energy loss, via LPM coherent radiation of gluons, radiated from an outgoing parton with color charge fully exposed in a medium with a large density of similarly exposed color charges (i.e. the QGP). As a parton from hard-scattering in the A+B collision exits through the medium it can radiate

\[ \hat{q}L = \int d\phi \frac{dN}{d\phi} \cos \Delta\phi \]
a gluon; and both continue traversing the medium. It is important to understand that “Only the gluons radiated outside the cone defining the jet contribute to the energy loss.” Also, because of the angular ordering of QCD, the angular cone of any further emission will be restricted to be less than that of the previous emission and will end the energy loss once inside the jet cone. Thus quenching occurs only when the first gluon emitted by a parton is outside the jet cone.

The energy loss of the original outgoing parton, \(-dE/dx\), per unit length \((x)\) of a medium with total length \(L\), is proportional to the total 4-momentum transfer-squared, \(q^2(L)\), with the form:

\[
\frac{-dE}{dx} \simeq \alpha_s \langle q^2(L) \rangle = \alpha_s \mu^2 L/\lambda_{\text{mfp}} = \alpha_s \hat{q} L
\]

where \(\mu\) is the mean momentum transfer per collision, and the transport coefficient \(\hat{q} = \mu^2/\lambda_{\text{mfp}}\) is the 4-momentum-transfer-squared to the medium per mean free path, \(\lambda_{\text{mfp}}\).

Also, the accumulated momentum-squared, \(\langle p^2_{T,W} \rangle\) transverse to the parton from its collisions traversing a length \(L\) in the medium is well approximated by

\[
\langle p^2_{T,W} \rangle \approx \langle q^2(L) \rangle = \hat{q} L.
\]

This is strongly correlated to the energy loss Eq. \(\ref{eq:energy_loss}\) [11] and results in the azimuthal broadening of the outgoing parton from its original direction by \(\langle \hat{q} L \rangle / 2\) since only the component \(\langle p^2_{T,W} \rangle / 2\) is in the azimuthal direction, \(\perp\) to the scattering plane. The original parton that scattered had a so-called [12] intrinsic mean transverse momentum \(\langle k_{T} \rangle\) which is perpendicular to the collision axis but can act both perpendicular to the scattering plane and in the scattering plane at random. This means that in a p+p collision, the mid-rapidity di-jets from hard-parton-parton scattering are not back-to-back in azimuth but are acollinear from the random sum of \(\langle k_{T}^2 \rangle\) from both scattered partons or \(\langle k^2_{T,\text{pair}} \rangle = 2 \langle k_{T}^2 \rangle\), of which only half or \(\langle k^2_{T} \rangle\) affects the azimuthal broadening while the other half unbalances the original equal and opposite transverse momenta of the jets [3]. In an A+A collision this di-jet gets further broadened in azimuth by the random sum of the azimuthal component \(\langle p^2_{T,W} \rangle / 2\) from each outgoing jet or \(\langle p^2_{T,W} \rangle = \hat{q} L\), so that the di-jet azimuthal broadening acoplanarity in A+A collisions compared to p+p collisions should be

\[
\langle \hat{q} L \rangle = \langle k^2_{T} \rangle_{AA} - \langle k^2_{T} \rangle_{pp}
\]

since only the component of \(\langle p^2_{T,W} \rangle \perp\) to the scattering plane affects \(k_T\). This is the azimuthal di-jet broadening.

\[\text{Ref. [5] had } \langle \hat{q} L \rangle / 2 = \text{in Eq. [5]} \text{ because I forgot that the di-hadron correlation represents both the trigger and away-side scattered partons.}\]
The signal-to-background ratio for the $k_T^2$ distributions for di-hadrons $(k_T^2)_{pp}$ denotes the intrinsic rms. transverse momentum of the hard-scattered parton in a nucleon in an A+A collision plus any medium effect; and $(k_T^2)_{p+p}$ denotes the reduced value of the $p+p$ comparison di-hadron $(k_T^2)_{pp}$ measurement with $p_{Tt}$ and $p_{Ta}$ correcting for the lost energy of the scattered partons in the QGP [3]. This reduces to the simpler equation when the equation for the $(k_T^2)$ for di-hadrons is substituted [3]:

$$\langle \hat{q}L \rangle = \left[ \frac{x_h}{(z_l)} \right]_{AA}^2 \left[ \frac{(p^2_{out})_{AA} - (p^2_{out})_{pp}}{x_h^2} \right]$$

(4)

where $(p^2_{out}) = p^2_{Ta} \sin^2(\pi - \Delta \phi)$ and the di-hadrons with $p_{Tt}$ and $p_{Ta}$, with ratio $x_h = p_{Ta}/p_{Tt}$, are assumed to be fragments of jets with transverse momenta $p_{Tt}$ and $p_{Ta}$ with ratio $\hat{x}_h = p_{Ta}/p_{Tt}$, where $z_l \approx p_{Tt}/\hat{p}_{Tt}$ is the fragmentation variable, the fraction of momentum of the trigger particle in the trigger jet. For di-jet measurements, Eq. (4) becomes even simpler: i) $x_h \equiv \hat{x}_h$ because the trigger and away ‘particles’ are the jets; ii) $(z_l) \equiv 1$ because the trigger ‘particle’ is the entire jet not a fragment of the jet; iii) $(p^2_{out}) = p^2_{Tt} \sin^2(\pi - \Delta \phi)$. This reduces Eq. (4) for di-jets to:

$$\langle \hat{q}L \rangle = \left[ \frac{(p^2_{out})_{AA} - (p^2_{out})_{pp}}{x_h^2} \right]$$

(5)

I checked Eq. (5) against a prediction [13] for 35 GeV/c jets at RHIC at $\sqrt{s_{NN}} = 200$ GeV with $(\hat{q}L) = 0$ GeV$^2$ (p+p), and for A+A, $(\hat{q}L) = 8$ GeV$^2$ and 20 GeV$^2$. I got 9.7 GeV$^2$ and 21.5 GeV$^2$ respectively for the 8 GeV$^2$ and 20 GeV$^2$ curves subtracting the p+p value of $(p^2_{out})_{pp}$.

### A. How to Find $(z_l)$, $\hat{x}_h$, and the energy loss of $\hat{p}_{Tt}$ for dihadrons

1. $(z_l)$ and $\Delta \hat{p}_{Tt}$

At RHIC, in p+p and Au+Au collisions as a function of centrality the $\pi^0$ $p_T$ spectra with $5 \leq p_T \leq 20$ GeV/c all follow the same power law with $n = 8.10 \pm 0.05$ [14]. The Bjorken parent-child relation and ‘trigger-bias’ [15] then imply that the single particle cross section has the same power law shape, $d^2 \sigma/2 \pi p_T dp_T d\eta \propto p_T^{-n}$, as the parent jet cross section and that large values of $(z_l) = p_{Tt}/\hat{p}_{Tt}$ dominate the single-particle cross section. This means that the shift $(\Delta \hat{p}_{Tt})$ in the A+A Jet $p_T$ spectrum at a given $p_{Tt}$ from the $(T_{AA})$ corrected p+p cross section can be measured from the shift in the trigger hadron spectrum [3] [10]. Similarly, the $(z_l)$ as a function of $p_{Tt}$ can be calculated [17] [18] (Fig. 4) using the measured fragmentation functions from e+p collisions [19] [20] with the value of $n = 8.10 \pm 0.05$ fixed as determined in Ref. [13], where $n$ is the power-law of the inclusive $\pi^0$ spectrum and is observed to be the same in p+p and Au+Au collisions in the $p_T$ range of interest.

Figure 5 shows a fit of Eq. (6) to the PHENIX $x_E$ Au+Au 0-20% and p+p distributions in a region with $(p_{Tt}) \approx 7.8$ GeV/c, close to the $5 \leq p_{Tt} < 10$ GeV/c region in Fig. 5 with $(p_{Tt}) \approx 6.5$ GeV/c. The results are $\hat{x}_h = 0.86 \pm 0.03$ in p+p and $\hat{x}_h = 0.47 \pm 0.07$ Au+Au (dashes). What is more interesting is a fit to Eq. (6) for $N$ and $\hat{x}_h$ plus another term of Eq. (6) with $\hat{x}_h = 0.86$ fixed at the p+p value, with the normalization $N_p = 0.22 \pm 0.08$ fitted, compared to the $N = 1.5_{-0.4}^{+1.4}$ for the partons that have lost energy. The result is the solid Au+Au curve with a much better $\chi^2$ which is notably parallel to the p+p curve for $x_E \geq 0.4$ ($p_{Ta} \approx p_{Tt} \times x_E = 3.1$ GeV/c).
All show the same effect as in Fig. 6, they fall in the range $\langle qL \rangle = 0.86 \pm 0.04$, $\langle z_t \rangle = 0.80 \pm 0.05$ are given in Table I. The value of $\langle qL \rangle$ is $0.86 \pm 0.87$ GeV$^2$ for the fit to the $3 \leq p_{Ta} \leq 5$ GeV/c STAR data shown in Fig. 3 is consistent with zero and clearly in significant disagreement with the proposed $\langle qL \rangle = \langle p_T^2 \rangle / (2N)$ = 13 GeV$^2$ quoted on Fig. 2. The value of $\langle qL \rangle = 4.21 \pm 3.24$ GeV$^2$ in the lower $p_{Ta}$ bin is closer to the prediction, within 2.7 standard deviations, but also consistent with zero.

The calculations of $\langle qL \rangle$ from the fits to the PHENIX data in Fig. 6 with $\tilde{z}_a = 0.51 \pm 0.06$ and $\langle z_t \rangle = 0.64 \pm 0.64$ are given in Table I. The values of $\langle qL \rangle = -2.24 \pm 2.01$ and $-1.68 \pm 1.20$ GeV$^2$ are negative, as noted above, and both consistent with zero but inconsistent with the predicted 13 GeV$^2$. 

The sharp-eyed reader will notice that the $\langle qL \rangle$ values in Ref. 5 were $8.41 \pm 2.66$ and $1.71 \pm 0.67$ GeV$^2$ for two reasons: first is the $\langle qL \rangle / 2$ in Eq. 5 [4] there, second was a miscalculation of the error which should have been obvious from the errors in $\langle p_{Tout}^2 \rangle$ which are unchanged.

3. This effect is well known under a different name

One possible explanation is that this region for $p_{Ta} \geq 3$ GeV/c, which is at a fraction $\approx 1\%$ of the $dP/\pi E_{pp}$ distribution, is from a scattered parton where the first gluon emission was inside the jet cone, so that the jet lost no energy in the QGP and had a $dP/\pi E_{pp}$ distribution the same as in a pp collision which begins to dominate the $x_E$ distribution for $p_{Ta} \geq 3$ GeV/c. An unlikely possibility is from tangential parton-parton collisions at the periphery of the A+A overlap region which has probability much smaller than the $N_p/N$ ratio.

Either possibility is consistent with measurements of the ratio of the Au+Au to pp $x_E$ (or $p_{Ta}$) distributions for a given $p_{Tt}$ which are called $I_{AA}$ distributions (Fig. 6) [22]. All $I_{AA}$ distributions ever measured show the same effect as in Fig. 6 they fall in the range $0 < p_{Ta} < 3$ GeV/c and then remain constant. The same effect can be seen in an $I_{AA}$ measurement in $\sqrt{s_{NN}} = 200$ TeV pp and Pb+Pb 0-10% by ALICE at the LHC [24]. The fact that $I_{AA}$ remains constant above $p_{Ta} \geq 3$ GeV/c means that the ratio of the away-jet to the trigger jet transverse momenta in this region remains equal in A+A and pp, i.e. no apparent suppression via energy loss in this region. This effect also causes problems in the following calculations of $\langle \hat{q}L \rangle$ from the di-hadron correlations.
TABLE II. Tabulations for $\langle qL \rangle$—PHENIX Fig. [3]

| PHENIX PRC77 | $\sqrt{s_{NN}}$ = 200 | $\langle p_{Tt}^1 \rangle$ | $\langle p_{Tt}^2 \rangle$ | $\langle qL \rangle$ |
|--------------|-------------------|------------------|----------------|----------------|
| Reaction     | GeV/c             | GeV/c            | (GeV/c)$^2$    | GeV/$^2$       |
| p+p          | 8.08              | 3.75             | 1.54 ± 0.08   |               |
| p+p          | 8.08              | 6.68             | 3.92 ± 0.33   |               |
| Au+Au 0-20% | 8.08              | 3.75             | 0.79 ± 0.64   | -2.24 ± 2.01  |
| Au+Au 0-20% | 8.08              | 6.68             | 2.12 ± 1.13   | -1.68 ± 1.21  |

TABLE III. Tabulations of $\langle qL \rangle$—PHENIX 9-12 GeV/c Fig. [7]

| PHENIX PRL104 | $\sqrt{s_{NN}}$ = 200 | $\langle p_{Tt}^1 \rangle$ | $\langle p_{Tt}^2 \rangle$ | $\langle qL \rangle$ |
|--------------|-------------------|------------------|----------------|----------------|
| Reaction     | GeV/c             | GeV/c            | (GeV/c)$^2$    | GeV/$^2$       |
| p+p          | 10.22             | 1.30             | 0.319 ± 0.023 |               |
| p+p          | 10.22             | 2.31             | 0.491 ± 0.052 |               |
| p+p          | 10.22             | 3.55             | 1.256 ± 0.166 |               |
| p+p          | 10.22             | 5.73             | 2.884 ± 1.376 |               |
| Au+Au 0-20% | 10.22             | 1.30             | 0.86 ± 0.339  | 13.3 ± 10.4   |
| Au+Au 0-20% | 10.22             | 2.31             | 0.299 ± 0.190 | -1.5 ± 1.7    |
| Au+Au 0-20% | 10.22             | 3.55             | 0.394 ± 0.189 | -2.9 ± 1.6    |
| Au+Au 0-20% | 10.22             | 5.73             | 4.08 ± 2.83   | 1.5 ± 4.0     |

Although not discussed in Ref. [6], the PHENIX measurement of $I_{AA}$ shown in Fig. 6 also provided values of $\sigma_{away}$ for Au+Au and p+p plotted clearly (Fig. 7) so that values of $\tilde{q}L$ can be read off practically by inspection. While $\sigma_{away}$ is apparently larger in Au+Au than in p+p for $p_{Tt} < 2$ GeV/c it is smaller or equal to the p+p value for $p_{Tt} > 2$ GeV/c, i.e. $\tilde{q}L$ consistent with zero. Details for $p_{Tt} = 9 - 12$ GeV/c are given in Table III.

IV. CONCLUSION

When calculated with fits to the measured distributions in Fig. 3 the values of $\tilde{q}L$ are inconsistent with the calculation of $\tilde{q}L = 13$ GeV$^2$ claimed in Fig. 2 [3], for $p_{Tt} \geq 3$ GeV/c. For values of $p_{Tt} < 3$ GeV/c, separating the flow background causes the errors in the measurement of $\tilde{q}L$ to be too large to obtain a reasonable value.

The measurement of $\tilde{q}L$ and possibly $\tilde{q}$ can be greatly improved by measuring di-jet angular distributions rather than di-hadron distributions. The energy loss of the trigger jets can be determined by the shift in the $p_{Tt}$ spectrum from p+p to A+A the same way as for $\pi^0$ [10]. Then a plot of the $\tilde{p}_{Tt}$ of the away jets for a given trigger jet with $\tilde{p}_{Tt}$ analogous to Fig. 5 and an evaluation of $\tilde{q}L$ as a function of $\tilde{p}_{Tt}$ by Eq. 3 might allow the $L$ dependence to be factored out or determined which would lead to a experimental measurement of $\tilde{q}$.

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