A new model for settling velocity of non-spherical particles

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ABSTRACT

The settlement of non-spherical particles, such as propagules of plants and natural sediments, are commonly observed in riverine ecosystems. The settling process is influenced by both particle properties (size, density and shape) and fluid properties (density and viscosity). Therefore, the drag law of non-spherical particles is a function of both particle Reynolds number and particle shape. Herein, a total of 828 settling data are collected from the literatures, which cover a wide range of particle Reynolds number (0.008-10000). To characterize the influence of particle shapes, sphericity is adopted as the general shape factor, which varies from 0.421 to 1.0. By comparing the measured drag with the standard drag curve of spheres, we modify the spherical drag law with three shape-dependent functions to develop a new drag law for non-spherical particles. Combined with an iterative procedure, a new model is thus obtained to predict the settling velocity of non-spherical particles of various shapes and materials. Further applications in hydrochorous propagule dispersal and sediment transport are projected based on deeper understanding of the settling process.

Keywords: Drag coefficient; Settling velocity; Sphericity; Non-spherical particles; Particle Reynolds number; shape-dependent functions.
1 Introduction

In riverine ecosystems, hydrochory plays a major role in transporting and depositing freshly-produced plants propagules (predominantly seeds) along the river corridors (Merritt & Wohl, 2002; Yoshikawa et al., 2013). In the way of becoming an essential part of the ecosystem, the non-buoyant seeds have to undergo a long period of settlement, dispersal, germination, and then gradually contribute to the colonization of the riparian zone (Gurnell et al., 2008; Chambert and James, 2009; Koch et al., 2010). Another factor that greatly affects the hydrodynamic process of rivers is the natural sediment. As mentioned by Meier et al. (2013), sediments can promote vegetation growth due to the transportation of nutrients and other organic matters. Moreover, by creating accretional structures in rivers, sediments can also facilitate the development of other species in the new habitats (Gurnell et al., 2012). Thus, as a starting point of the hydrodynamic process, the settlement of both hydrochorous seeds and natural sediments should be paid more attention since the construction of aquatic ecosystem grows more important.

Since all settlement issues are originated from the settlement of spheres, many spherical drag laws, whether explicit or implicit, are proposed by previous studies (Clift & Gauvin, 1971; Haider & Levenspiel, 1989; Brown & Lawler, 2003; Cheng, 2009; Terfous et al., 2013). However, in practical applications, the most commonly encountered particles (e.g., seeds, pebbles and gravels) are non-spherical and even irregular. Thus, the knowledge of drag coefficient and settling velocity of non-spherical particles is essential for solving the settling problem.

Based on numerous experimental and numerical settling data, many models have been proposed to predict the settling velocities of non-spherical particles. In these models, particles are usually separated into several categories, such as regular-shaped particles (Komar, 1980; Haider & Levenspiel, 1989; Ganser, 1993; Gogus et al., 2001; Wang et al., 2011; Lau & Chuah, 2013; Song et al., 2017), natural sediments and crushed rock fragments (Komar & Reimers, 1978; Hallermeier, 1981; Dietrich, 1982; Swamee & Ojha, 1991; Chien, 1994; Cheng, 1997; Alcerreca et al., 2013; Wang et al., 2017), and conglomerates of particles (Tran-Cong et al., 2004). Hence, the performances of most models are restricted in certain types of particles. Moreover, to scale the influence of non-spherical shapes of particles, different shape factors are applied. For instance, the factor, aspect ratio (E) is especially designed for the axisymmetric particles, such as cylinders, spheroids, and ellipsoids (Trans-Cong et al., 2004; Loth, 2008; Wang et al., 2011). The term S, which is defined as the ratio between equivalent sphere area and the projected area of particle settling direction, is also proposed to describe the effect of settling orientation (Song et al., 2017). Therefore, it is difficult to generalize the
effect of shape with one shape descriptor. Another problem for some models is that only part of the flow regimes is covered due to the limitation of experimental conditions. For instance, the model of Wang et al. (2017) is applicable for a certain range of particle Reynolds number, which varies from 0.01 to 3700. The range covers Strokes', intermediate, and early stage of Newton’s flow regime. As for the settlement of particles in higher stage of Newton regime, the prediction ability of the model is unknown.

To solve the above-mentioned problems, a reliable model is developed based on a large amount of settling data, including irregular mineral sediments and artificial particles with non-spherical shapes. Note that behaviours of non-buoyant seeds during entrainment and settling are consistent with that of mineral sediments and the settling process of such artificial particles are analogous to that of natural sediments (Zhu et al., 2017). Herein, the shape factor, sphericity, is used as the general descriptor to show the influence of particle shapes. Note that the sphericity of these particles varies from 0.421 to 1.0. In addition, the particle Reynolds number ranges from 0.008 to 10000, which almost covers all regimes that can be encountered in natural processes. Herein, we modify the spherical drag law of Clift and Gauvin (1971) with shape-related functions to produce a new drag law for non-spherical particles. Subsequently, with an iterative procedure, a new model is developed to predict the settling velocity for different types of particles, which can eventually be applied in understanding the deposition, transport and dispersal of seeds and natural sediments.

1.1 In situ settling velocity

For particles settling through a static fluid, the balance between surface (drag) and body forces acting on the particle is expressed as follow:

\[
\frac{1}{2} \rho_f C_d A_p W_i^2 = (\rho_s - \rho_f) g V
\]  

Therefore, the settling velocity of the particle can be obtained through the following formula:

\[
W_i = \sqrt{\frac{4(\rho_s - \rho_f) gd_s}{3\rho_f C_d}}
\]  

where \( \rho_f \) is the fluid density, \( \rho_s \) is the particle density, \( C_d \) is the drag coefficient depending on properties of particle and fluid, \( A_p \) is the projected area of the particle perpendicular to the settling direction, \( W_i \) is the settling velocity of the particle, \( g \) is the gravitational acceleration, and \( V \) is the volume of the particle.

The particle Reynolds number (\( R_{ep} \)) is defined as:
\[ R_{wp} = \frac{\rho_f W_d}{\mu} \]  
(3)

where \( d_n \) is the diameter of the volume equivalent sphere, and \( \mu \) is the dynamic viscosity of the fluid.

### 1.2 Particle shape characterization

For particles that are non-spherical, the diameter can be presented by the above-mentioned nominal diameter \( d_n \) to eliminate the influence of non-spherical shapes (Wadell, 1932).

\[ d_n = \sqrt[3]{6V/\pi} \]  
(4)

To describe the shape of non-spherical particles, multiple shape descriptors are proposed. For instance, the 1D descriptor, corey shape factor (CSF), is proposed by Corey (1963) to describe particles of relatively smooth surfaces. The term, circularity (X), is a 2D descriptor, which is suitable for particles with sharp corners and large obtuse angles (Büttner et al., 2002). However, since sphericity (\( \phi \)) is the most widely used shape descriptor, and can accurately describe the shape of various particles, it is chosen in this study as the best descriptor. Sphericity is defined as the ratio between the surface area of the equivalent sphere \( A_{sph} \) and the particle surface area \( A_s \):

\[ \phi = \frac{A_{sph}}{A_s} \]  
(5)

where \( A_{sph} \) is calculated by \( A_{sph} = 4\pi \left( d_n/2 \right)^2 \).

For particles that are similar to scalene ellipsoids, the surface area of particle \( A_s \) can be approximately calculated as follow (Taylor et al., 2006):

\[ A_s \approx 4\pi \frac{(ab)^\lambda + (ac)^\lambda + (bc)^\lambda}{3-k(1-27abc(a+b+c)^3)} \]  
(6)

where \( a, b, \) and \( c \) are semi-axes of a triaxial ellipsoid, \( \lambda = 1.5349, k = 0.0942. \)

Except for \( \phi \), another frequently used shape descriptor is \( \psi \), which is defined as the ratio between sphericity and circularity (\( \psi = \phi/X \)). This descriptor is first introduced by Dellino et al. (2005) for highly irregular particles.
2 Materials and methods

2.1 Formulation

To develop a new shape-dependent drag law, we collect 828 sets of settling data from previous studies (Komar & Reimers, 1978; Komar, 1980; Baba & Komar, 1981; Chambert & James, 2009; Koch et al., 2010; Wang et al., 2011; Dioguardi & Mele, 2015; Song et al., 2017; Zhu et al., 2017). The new database mainly contains the settling data of volcanic materials and particles of regular shapes, such as cubes, cylinders, and cuboids. Details of the database are listed in Table 1. Note that within the database the range of $\phi$ is 0.421-1.0, and the value of $R_{ep}$ varies from 0.008 to 10000.

Following the idea of Dioguardi et al. (2018), we construct a new drag law for non-spherical particles based on the spherical drag law of Clift and Gauvin (1971). This spherical drag law is chosen since it can effectively cover the entire $R_{ep}$ range of the standard drag curve.

By comparing the difference between the measured drag coefficient ($C_{d,\text{meas}}$) in our database and the drag of a sphere ($C_{d,\text{sphere}}$) at the same $R_{ep}$, we separate the Eq. (7) into the sum of three terms:

$$C_{d,\text{sphere}} = \frac{24}{R_{ep}} (1 + 0.15R_{ep}^{0.687}) + \frac{0.42}{1 + \frac{42500}{R_{ep}^{1.16}}} \tag{7}$$

By comparing the difference between the measured drag coefficient ($C_{d,\text{meas}}$) in our database and the drag of a sphere ($C_{d,\text{sphere}}$) at the same $R_{ep}$, we separate the Eq. (7) into the sum of three terms:

$$C_{d,\text{sphere}} = \frac{24}{R_{ep}} + \frac{24}{R_{ep}} (0.15R_{ep}^{0.687}) + \frac{0.42}{1 + \frac{42500}{R_{ep}^{1.16}}} \tag{8}$$

In Fig. 1, we plot the measured drag coefficient of particles $C_{d,\text{meas}}$ versus the drag curve for spheres. The drag of non-spherical particles is observed to be larger than $C_{d,\text{sphere}}$ in most cases. Specifically, the measured drag (black dots) is close to $C_{d,\text{sphere}}$ (solid line) for $R_{ep} < 10$. As $R_{ep}$ grows larger than 10, the bias between the measured drag and the standard drag curve also becomes larger. In addition, for black dots that are closely related to the standard drag curve, the corresponding sphericities are close to 1.0. From these observations, we hypothesize that the influence of particle shape can be separately added to the three parts of Eq. (8).

$$C_{d,\text{calc}} = \frac{24}{R_{ep}} f_1(\phi) + \frac{24}{R_{ep}} (1 + 0.15R_{ep}^{0.687}) f_2(\phi) + \frac{0.42}{1 + \frac{42500}{R_{ep}^{1.16}}} f_3(\phi) \tag{9}$$

For the shape-modified functions, the following constraints must be satisfied.

$$f_1(\phi = 1) = f_2(\phi = 1) = f_3(\phi = 1) = 1 \tag{10}$$
Therefore, when the shape of a particle is close to a sphere ($\phi \approx 1$), Eq. (9) can turn back to the spherical drag law (Eq. (8)). Based on the collected database, the three functions are obtained by comparing the difference between $C_{d,\text{meas}}$ and $C_{d,\text{sphere}}$ focusing on the three parts separately. We also assume that the ratio between each terms of $C_{d,\text{sphere}}$ and the total $C_{d,\text{sphere}}$ is equal to the ratio between each part of $C_{d,\text{meas}}$ and the total drag of non-spherical particles (see Eq. (11)).

$$\frac{C_{d,\text{sphere}}(\text{part})}{C_{d,\text{sphere}}(\text{total})} = \frac{C_{d,\text{meas}}(\text{part})}{C_{d,\text{meas}}(\text{total})} \quad (11)$$

With this assumption, the three functions are separately searched by correlating each part of $C_{d,\text{calc}}$ with $\phi$ and $R_{ep}$. The best function is the one with the minimum error when predicting the settling velocity $W_{s,\text{meas}}$. Finally, the three functions satisfying the constraints and assumptions are:

$$f_1(\phi) = (2 - \phi)^{\alpha_1} \quad (12a)$$

$$f_2(\phi) = \phi^{R_{ep}^{\alpha_2}} \quad (12b)$$

$$f_3(\phi) = e^{\alpha_3(1-\phi)} \quad (12c)$$

The values of three exponents $\alpha_1$, $\alpha_2$, and $\alpha_3$ are obtained by iteratively searching for the values that can present the best fit with the measured data. The iterative searching process is conducted with the Matlab script. The best values of $\alpha_1$, $\alpha_2$, and $\alpha_3$ are 1.29, 0.134, and 1.43, respectively.

Thus, Eq. (9) turns to the new drag law:

$$C_{d,\text{calc}} = \frac{24}{R_{ep}} (2 - \phi)^{1.29} + \frac{24}{R_{ep}} (1 + 0.15R_{ep}^{0.687}) \phi^{R_{ep}^{0.134}} + \frac{0.42}{42500} R_{ep}^{1.16} e^{1.4X(1-\phi)} \quad (13)$$

As an implicit drag law depending on $R_{ep}$, an iterative procedure is usually adopted for calculating $C_d$, since both $C_d$ and $R_{ep}$ are relevant to $W_s$. The trial-and-error procedure is presented in the flow chart (Fig. 2). Details of the new drag law can be found in the Online Resource (sheet of “Drag law”).

2.2 Models for comparison

To evaluate the ability of the present model to predict the settling velocity of non-spherical particles, we compare it with previous models (Chien, 1994; Alcerreca et al., 2013; Dioguardi & Mele, 2015; Dioguardi et al., 2018). Details of comparison with other models are given in the Online Resource (sheet of “Model comparison”).
Chien (1994) derived the drag law for irregular particles settling in Newtonian and non-Newtonian fluids. Since this drag law is also implicit, a numerical iteration method is required to predict the settling velocity. The model of Chien (1994) covers a $\phi$ range of 0.2-1.0, and a $R_{ep}$ range of 0.001-10000.

\[
C_d = \frac{30}{R_{ep}} + \frac{67.289}{e^{5.03\phi}}
\] (14)

The drag law of Alcerreca et al. (2013) is derived based on a large amount of settling data of calcareous sand particles. The drag law is explicit since $C_d$ can be expressed as a function of the dimensionless particle diameter $D_s$:

\[
C_d = \frac{4}{3} \frac{D_s^3}{R_{ep}^2}
\] (15a)

\[
R_{ep} = \frac{\rho_f W d_s}{\mu} = \left(\sqrt{22 + 1.13D_s^2} - 4.67\right)^{1.5}
\] (15b)

\[
D_s = d_N \left[\left(g/\nu^2\right)\left(\rho_f/\rho_s - 1\right)^{1/3}\right]
\] (15c)

where $d_N$ is the nominal particle diameter which can be calculated by $d_N = (d_l d_m d_s)^{1/3}$, and $\nu$ is the kinematic viscosity of the fluid. Note that $d_l, d_m, d_s$ means the lengths of the longest, intermediate, and shortest principle axes of the particle, respectively. Consequently, the settling velocity can be calculated directly from basic properties of particles and fluids ($d_l, d_m, d_s, \rho_s, \rho_f, \nu$).

Dioguardi and Mele (2015) applied the shape factor $\psi$ to describe the irregularity of particles. Their drag law is based on the 340 settling data of volcanic particles, which are in a wide range of $R_{ep}$ (0.03-10000). Developed from the drag law of Dellino et al. (2005), the form of this law is segmented and simple:

\[
C_d = \frac{C_{d,\text{sphere}}}{R_{ep}^2 \exp\left(\frac{R_{ep}}{1.1883}\right)^{0.4826}}
\] (16)

\[
\exp = R_{ep}^{-0.23} \text{ for the Re ranges of 0-50;}
\]

\[
\exp = R_{ep}^{-0.05} \text{ for the Re ranges of 50-10000;}
\]

where $C_{d,\text{sphere}}$ is calculated by the drag law of Clift and Gauvin (1971) (see Eq. (7)). An iterative procedure is needed to construct the final model to predict the terminal settling velocity.

As a development of Dioguardi and Mele (2015), the model of Dioguardi et al. (2018) also used the shape factor $\psi$. Their drag law is obtained by 304 settling velocity measurements,
which are part of the 340 settling data mentioned in Table 1. Note that the structure of this model is similar to the present model (Eq. (13)). Differences between the two models are discussed in Section 3.

\[
C_{d,\text{calc}} = \frac{24}{R_{ep}} \left( \frac{1 - \psi}{R_{ep}} + 1 \right)^{0.25} + \frac{24}{R_{ep}} \left( 0.1806 R_{ep}^{0.6459} \right) \psi^{-(R_{ep}^{0.08})} + \frac{0.4251}{1 + \frac{6880.95}{R_{ep}} \psi^{5.05}} \]  

(17)

2.3 Comparison results

To compare the accuracy of predicting the settling velocity for each model, we first plot the $C_{d,\text{calc}}$ and $C_{d,\text{meas}}$ versus $R_{ep}$ in Fig. 3. After calculating the $C_{d,\text{calc}}$ with the aforementioned laws, we iteratively obtain the terminal settling velocity $W_{s,\text{calc}}$ and compare it with the measured values $W_{s,\text{meas}}$. In Fig. 4, the comparison results are displayed, along with the correlation coefficient of each model. As shown in Fig. 4, most models have similar correlation coefficients, which are approximately 0.94. The performance of Alcerecca et al. (2013) is relatively weak, which may due to the fact that their model is based on settling data of calcareous sand. Thus, for non-spherical particles of other shapes and materials, the ability of this model to predict the settling velocity is limited.

Based on the correlations between $W_{s,\text{calc}}$ and $W_{s,\text{meas}}$, three statistical parameters are used as indicators to assess the ability of different models to predict the settling velocity of non-spherical particles. The first parameter is the R-squared ($R^2$), which is obtained by linear fitting and presented in Fig. 4 for each model. The second parameter is average absolute error ($|\text{err}\%|$), which is calculated as follow:

\[
|\text{err}\%| = \frac{|W_{s,\text{calc}} - W_{s,\text{meas}}|}{W_{s,\text{meas}}} \times 100
\]

(18)

We also evaluate each model with the root-mean-square error (RMSE) by applying the following formula:

\[
RMSE = \sqrt{\frac{\sum_{i=1}^{N} \left( \frac{W_{s,\text{calc},i} - W_{s,\text{meas},i}}{W_{s,\text{meas},i}} \right)^2}{N}} \times 100
\]

(19)

where $N=828$ is the number of settling data in our database.

Error analyses of the three indicators for different models are summarized in Table 2. From $R^2$ listed in Table 2, the performance of the present model is slightly weaker than the model of Dioguardi et al. (2018). However, when evaluating the performance of models by $|\text{err}\%|$ and RMSE, the present model predicts the settling velocity better than all the other models.
To further explore the differences among these models, we compare the ability of each model to predict settling velocity of particles from different sources. The comparisons of \( |\text{err}\%| \) are given in Table 3 while the results of RMSE are summarized in Table 4. About the two tables, two notations are clarified. Firstly, since the number of settling data of hydrochorous seeds is low, we summarize the settling data of seeds from three sources in one row, which is simply marked as “Hydrochorous seeds” (Chambert & James, 2009; Koch et al., 2010; Zhu et al., 2017). Secondly, since the data of tri-axial lengths are missing for hydrochorous seeds, the shape descriptor \( \psi \) mentioned in the models of Dioguardi and Mele (2015) and Dioguardi et al. (2018) cannot be calculated. Thus, in the last row of both tables, the error analyses for hydrochorous seeds are not given for the two models.

From Table 3 and 4, the present model performs quite well for data collected from Song et al. (2017), Komar and Reimers (1978) and Baba and Komar (1981). Combined with particle shapes and materials listed in Table 1, the present model is found to be especially suitable for regular particles (e.g., cubes and cylinders) and particles with relatively smooth surfaces (e.g., pebbles and irregular glass particles). Nevertheless, the accuracy of the present model to predict the settling velocities for data from Wang et al. (2011), Komar (1980) and Hydrochorous seeds is only acceptable. For data from Wang et al. (2011), Komar (1980), the present model performs better than the other models except for the model of Dioguardi and Mele (2015). The prediction for settling velocity of Hydrochorous seeds, though not accurate enough, is the best among these models. A common reason for the deviations is that all the settling data from the three sources are no more than 50 data points, and thus the errors may be enlarged. At last, the abilities of the models to predict settling velocity of volcanic particles are compared. Since the models of both Dioguardi and Mele (2015) and Dioguardi et al. (2018) are developed from the settling data of volcanic materials, they naturally show smaller errors than the other models. Note that the performance of the present model is just slightly weaker than the two models. To summarize from the above analysis, it can be concluded that the present model shows the best performance when predicting settling velocity for particles of various shapes and materials.

3 Discussion

Among these models for comparison, the differences between the model of Dioguardi et al. (2018) and the present model need to be further discussed, since they are based on similar ideas. Herein, four major differences are clarified. Firstly, as shown in Table 1, the present model contains a larger database, which includes particles of various shapes and materials. On the other hand, the model of Dioguardi et al. (2018) is based on only part of the 340 data points of volcanic particles. Secondly, the shape factor applied by Dioguardi et al. (2018) is
ψ rather than φ. As mentioned in Section 1.2, the calculation of ψ requires the value of circularity (X) in addition to sphericity (φ). Thus, the model of Dioguardi et al. (2018) has more difficulty in application. Thirdly, the first and last shape-modified functions of the present model (f1(φ) and f4(φ) in Eq. (13)) have different forms and positions with that of the model of Dioguardi et al. (2018) (Eq. (17)). Lastly, the model of Dioguardi et al. (2018) is based on the spherical drag law of Haider and Levenspiel (1989), which is suitable for $R_{ep} < 2.6 \times 10^5$.

Developed from the idea of Dioguardi et al. (2018), the present model enhances the applicability in predicting the terminal settling velocity for particles of various shapes and materials, and thus can be considered as an improvement.

4 Conclusion

We propose a new drag law for non-spherical particles based on a total of 828 settling data collected from previous studies (Komar & Reimers, 1978; Komar, 1980; Baba & Komar, 1981; Chambert & James, 2009; Koch et al., 2010; Wang et al., 2011; Dioguardi & Mele, 2015; Song et al., 2017; Zhu et al., 2017). This new database cover a $R_{ep}$ range of 0.008–10000 and a φ range of 0.421–1.0. Thus, the new model is suitable for the settlement of non-spherical particles in both laminar and turbulent flows. In addition, since this new model is developed from settling data from various sources, it can be applied in predicting settling velocity of regular particles (e.g., cubes, cylinders, and cuboids), natural particles (e.g., volcanic particles, pebbles, and glass particles of various shapes), and even non-spherical hydrochorous seeds.

Following the idea of Dioguardi et al. (2018), the new model adopts three shaped-modified functions in the spherical drag law of Clift and Gauvin (1971). With these functions based on $R_{ep}$ and φ, the new model exhibits better performance than the other models. With the wide range of applicability in both flow regimes and particle shapes, the present model may be practical in solving problems concerning the riverine ecosystem. On the one hand, by relating the settling process of seeds with the dispersal process, more detailed information can be obtained about the germination of seeds and establishment of aquatic vegetation, which is crucial for flow characteristics in river systems (Meier et al., 2013). On the other hand, the advanced settling model can estimate the rate of sediment transport for various particles and flow regimes by being applied in predicting the distribution of sediment concentration (Fu et al., 2005; Graf and Cellino, 2002). Such application may be verified when provided with further sediment concentration profile measurements of irregular particles. In addition to its
application in the riverine ecosystems, the new settling model may be further improved by extending from the drag of single irregular particle to conglomerates of particles.

Declarations

Ethical approval and consent to participate
Not applicable

Consent for publication
Not applicable

Availability of data and materials
All data generated or analysed during this study are included in the supplementary information files, which can be found in the online version of this article.

Competing interests
The authors declare that they have no competing interests

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Authors' contributions
All authors contributed to the model conception and derivation. F. Yang: data curation, methodology, formal analysis, writing - original draft; Y.H. Zeng: conceptualization, funding acquisition, supervision, writing - review & editing; W.X. Huai: conceptualization, writing - review & editing. All authors have read and approved the final manuscript.

Notation
\(a, b, c\) = Semi-axes of a tri-axial ellipsoid (m)
\(A_p\) = Projected area perpendicular to the flow direction (m²)
\(A_s\) = Total surface area of the particle (m²)
\(A_{sph}\) = Surface area of equivalent spheres (m²)
\( C_d \) = Drag coefficient (-)
\( C_{d,\text{calc}} \) = Calculated drag coefficient (-)
\( C_{d,\text{meas}} \) = Measured drag coefficient (-)
\( C_{d,\text{sphere}} \) = Drag coefficient of sphere (-)
\( d_n \) = Diameter of the volume equivalent sphere (m)
\( d_N \) = Nominal particle diameter (m)
\( d_{l}, d_{m}, d_{s} \) = Lengths of the longest, intermediate, and shortest principle axes of the particle (m)
\( D_* \) = Dimensionless particle diameter (-)
\( E \) = Aspect ratio (-)
\( g \) = Gravitational acceleration (m s\(^{-2}\))
\( R_{\text{ep}} \) = Particle Reynolds number (-)
\( S \) = The ratio between equivalent sphere area and the projected area of particle settling direction (-)
\( V \) = Volume of the particle (m\(^3\))
\( W \) = Settling velocity of particles (m s\(^{-1}\))
\( W_{s,\text{calc}} \) = Calculated settling velocity of particles (m s\(^{-1}\))
\( W_{s,\text{meas}} \) = Measured settling velocity of particles (m s\(^{-1}\))
\( X \) = Particle circularity (-)
\( \rho_\text{p} \) = Particle density (kg m\(^{-3}\))
\( \rho_\text{f} \) = Fluid density (kg m\(^{-3}\))
\( \mu \) = Dynamic viscosity (kg m\(^{-1}\) s\(^{-1}\))
\( \nu \) = Kinematic viscosity (m\(^2\) s\(^{-1}\))
\( \phi \) = Particle sphericity (-)
\( \psi \) = Shape descriptor (-)
\( \lambda, k \) = Parameters in Eq. (6) (-)
\( \alpha_1, \alpha_2, \alpha_3 \) = Exponents in Eq. (12) (a, b, c) (-)

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**List of tables**

Table 1 Sources of data

| Source                      | Number of points | Particle types                  |
|-----------------------------|------------------|---------------------------------|
| Song et al. (2017)          | 276              | Cube, cylinder                  |
| Dioguardi and Mele (2015)   | 340              | Volcanic materials              |
| Wang et al. (2011)          | 48               | Cuboids                         |
| Komar and Reimers (1978)    | 51               | Ellipsoidal pebbles             |
| Baba and Komar (1981)       | 72               | Irregular glass particles       |
| Komar (1980)                | 27               | Cylindrical-shaped grains       |
| Chambert and James (2009)   | 8                | Non-spherical seeds             |
| Koch et al. (2010)          | 3                | Non-spherical seeds             |
| Zhu et al. (2017)           | 3                | Non-spherical seeds             |

Table 2 Performance comparison of models

| Model                        | $R^2$ | $|\text{err}|$% | RMSE  |
|------------------------------|-------|----------------|-------|
| Chien (1994)                 | 0.930 | 13.347         | 18.664|
| Alcerreca et al. (2013)      | 0.904 | 18.270         | 28.098|
| Dioguardi and Mele (2015)    | 0.941*| 16.186*        | 20.142*|
| Dioguardi et al. (2018)      | 0.951*| 11.652*        | 15.800*|
| Present model                | 0.943 | 9.624          | 15.061|

Note: Data marked with asterisk are obtained without considering the settlement of hydrochorous seeds. The data are still convincible since the settling data of seeds (14 data points) only account for 1.69% of the total database (828 data points).
### Table 3 Comparisons of models by \(|\text{err}\%|\)

| Data source                  | Chien (1994) | Alcerecca et al. (2013) | Dioguardi and Mele (2015) | Dioguardi et al. (2018) | Present model |
|------------------------------|--------------|-------------------------|---------------------------|-------------------------|---------------|
| Song et al. (2017)           | 22.790       | 13.507                  | 17.314                    | 12.23                   | 7.234         |
| Dioguardi and Mele (2015)    | 15.233       | 14.701                  | 11.123                    | 10.69                   | 11.587        |
| Wang et al. (2011)           | 16.747       | 15.646                  | 20.800                    | 16.25                   | 17.627        |
| Komar and Reimers (1978)     | 8.407        | 4.847                   | 24.692                    | 7.35                    | 2.590         |
| Baba and Komar (1981)        | 9.733        | 3.418                   | 25.356                    | 10.30                   | 3.790         |
| Komar (1980)                 | 54.307       | 27.107                  | 19.701                    | 21.394                  | 20.435        |
| Hydrochorous seeds           | 18.455       | 24.914                  | /                         | /                       | 16.424        |

### Table 4 Comparisons of models by RMSE

| Data source                  | Chien (1994) | Alcerecca et al. (2013) | Dioguardi and Mele (2015) | Dioguardi et al. (2018) | Present model |
|------------------------------|--------------|-------------------------|---------------------------|-------------------------|---------------|
| Song et al. (2017)           | 36.102       | 18.560                  | 21.313                    | 17.143                  | 13.058        |
| Dioguardi and Mele (2015)    | 19.240       | 19.040                  | 13.777                    | 14.362                  | 16.465        |
| Wang et al. (2011)           | 21.374       | 22.618                  | 27.009                    | 19.958                  | 21.339        |
| Komar and Reimers (1978)     | 10.406       | 6.100                   | 25.512                    | 8.980                   | 3.165         |
| Baba and Komar (1981)        | 11.173       | 4.268                   | 27.401                    | 11.693                  | 4.863         |
| Komar (1980)                 | 68.173       | 32.323                  | 25.876                    | 26.451                  | 26.529        |
| Hydrochorous seeds           | 21.001       | 31.771                  | /                         | /                       | 19.830        |