Development of a Three Dimensional Mathematical Model of the Electromagnetic Casting of Steel

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(Received on May 17, 2001; accepted in final form on October 16, 2001)

The development of a mathematical model for the electromagnetic casting of steel is described. The model is three dimensional and computes the evolution of the electromagnetic field, the turbulent liquid metal flow and the free surface of the metal pool with time. This is achieved by simultaneous solution of the MHD form of Maxwell’s equations, Ohm’s law, the Navier–Stokes and continuity equations (by large eddy simulation) and an equation for the free surface. Solution is by a combination of finite element (field and flow equations) and finite difference (surface equation) methods on an Eulerian-Lagrangian grid. The model was first tested against measurements, by others, of magnetic fields and induced currents in an apparatus akin to an electromagnetic caster, and then against computations, performed using finite difference methods by other investigators, of laminar flow within a 2D rectangular cavity. The model was also tested against classical equations for the oscillations of a free surface in a rectangular trough and then against measurements, at Nippon Steel Corporation, of the surface oscillations of a mercury pool surrounded by an inductor carrying alternating current. Use of the model to predict the behavior of a steel caster is to be described in a subsequent paper.

KEY WORDS: electromagnetic casting; continuous casting; steel casting; electromagnetic fields; computational fluid dynamics; mathematical modeling of free surfaces.

1. Introduction

Electromagnetic casting has reached commercial success in the aluminum industry, although not to the level of displacing the older “direct chill” (DC) method of casting aluminum ingots. Recently the electromagnetic casting of steel has been extensively investigated. In this technology, sometimes known as “soft” casting of steel, the molten metal at the head of the caster is not fully supported by electromagnetic forces (as in electromagnetic casting of aluminum) but is partly supported electromagnetically and partly by a mold. The concept is illustrated in Fig. 1 where a comparison is made between the occurrence of oscillation marks in conventional steel casting and soft casting. In conventional casting the slag rim impacts the thin solid shell at the periphery of the liquid pool as the mold moves downward.

Fig. 1. Comparison of solidification in conventional continuous casting with that in electromagnetic casting of steel.
during “negative strip”. Deformation of the shell leads to mold oscillation marks on the finished ingot together with associated cracks and segregation. In the case of soft casting electromagnetic forces arise from the interaction between a magnetic field, developed by alternating currents flowing in an inductor surrounding the mold, and currents induced in the metal as the magnetic field alternates. These forces serve to support the molten steel against hydrostatic pressure, keeping it away from a height of a few tens of millimeters and avoiding, or minimizing, impact of the rim on the shell during negative strip. Takeuchi and co-workers demonstrated the improvement of the surface quality of stainless steel ingots resulting from the application of the electromagnetic force and soft casting is becoming a reality in the steel industry in Japan.

The objective of the present investigation was to determine the dynamics of the liquid metal and the liquid surface so as to aid in development and design of electromagnetic casters for steel. The approach has been numerical modeling with verification of model predictions where possible. The paper describes the equations to be solved, the numerical techniques for their solution and comparison of model predictions with experimental results and computations of others.

2. Previous Investigations

The literature on electromagnetic casting of aluminum is vast and space is insufficient to fully cover it here. The reader is referred to recent papers for a longer discussion of prior work on electromagnetic casting of aluminum. Much of that work, and other work on the interaction of alternating fields with molten metals, has been on the support of the metal free surface (e.g. the deformation of the metal meniscus by electromagnetic forces). An issue is whether the liquid metal surface is stable (suffering only decaying oscillations once disturbed) or unstable (growing oscillations). It is clear that this question is important in the operation of electromagnetic casters. An analytical approach to an answer was described by Garnier and Moreau who concluded that an alternating field would not destabilize a liquid surface. A more extensive analysis by Gupta and Evans reached a different conclusion. Faurelle and Sned extended these two investigations to a low frequency case and also included damping due to Ohmic losses. These analytical models were necessarily idealized and Kageyama and Evans carried out a coupled numerical and experimental study of the oscillation of the surface of a cylindrical mercury pool within an alternating magnetic field. It was found that the pool surface oscillated with frequencies close to the natural (gravitational) frequencies of the pool, as predicted by classical theory, and that the magnitudes of the oscillations were greater with the field present.

A closer physical simulation of the soft caster was achieved in the work of Asai’s group. Using a fiber scope camera these investigators studied the meniscus of a 30 mm diameter gallium pool deformed by the vertical oscillation of a graphite mold. Silicone oil was used to simulate the mold flux. Experiments were carried out with and without a magnetic field (maximum flux density of 0.04 T). It was observed that the meniscus remained convex throughout the cycle of the model oscillation when the field was on. With the field off, the meniscus was convex during only part of the cycle and surface ripples were seen.

Schwerdtfeger’s group has used a physical model to study the effect of mold oscillation. Mercury was used to simulate steel and silicone oil to simulate the mold flux. These were contained in a 60.5 mm diameter cylindrical hole in a block of Plexiglass® which represented the mold. The interface between the oil and mercury was captured with a high speed camera as the block was oscillated. On the down-stroke the interface was observed to be convex but on the up-stroke it was pulled up at the edges and developed waves. No magnetic fields were applied.

In their investigation of soft casting of stainless steel, Takeuchi and colleagues used a low frequency (60 Hz) field in order to minimize inductive heating of the mold, with its associated loss of field at the metal where it is needed. They pointed out that this low frequency resulted in significant electromagnetically driven flow in the liquid metal and that this flow perturbed the meniscus. There have been two approaches to avoiding this excessive flow: 1) the application of the field intermittently, use of much higher frequencies (e.g. above 20 kHz), 2) use of much higher frequencies (e.g. above 20 kHz).

Li and colleagues cast a 30 mm diameter tin billet under an intermittent 20 kHz magnetic field and reported that oscillation marks were not found when the field was applied. They also measured the oscillations of the meniscus and reported that oscillation due to the magnetic field dominated over oscillation due to the mold movement at higher field strength.

Tani et al. examined the effect on oscillation marks of synchronizing the imposed magnetic field with the mold oscillation. The magnetic field was imposed: 1) continuously or 2) only during the positive strip or 3) only during the negative strip in the casting of 100 mm diameter Sn–Pb alloy ingots at 200 Hz field frequency.

They reported that the depth of oscillation marks was less for all three fields than when no magnetic field was applied. Furthermore, they found that the oscillation mark depth was least for 1, larger for 2 and largest for 3.

Nakata and colleagues determined the surface roughness of 0.15% carbon steel ingots cast under a 20 kHz field. They used a 150 mm square segmented mold (to minimize induction of currents in the mold at this high frequency). The magnetic field was observed to improve the surface roughness. However, it was found that there was an optimum electric power, supplied to the inductor, beyond which the surface roughness was worse.

These previous studies indicate that the proper design of the mold and inductor, together with the application of high frequency or intermittent magnetic fields, are important in the development of soft casting. One objective of the present investigation has been to assist in such development by providing a tested mathematical model of the EMC of steel.

3. Development of the Mathematical Model

There is space only for an outline of the model develop-
3.1. The Equations to Be Solved

The model entails the simultaneous solution of the electromagnetic field equations together with equations for the melt flow and surface oscillation. The equations describing the electromagnetic field are the MHD forms of Maxwell's equations (which are well known and therefore not reproduced here) and the differential form of Ohm's law

$$ J = \sigma (E + V \times B) $$

Here $J$ is the induced current density, $\sigma$ is the electrical conductivity, $E$ is the electric field, $V$ is the velocity and $B$ is the magnetic flux density.

In the model the convective term $V \times B$ is omitted on the grounds that the magnetic Reynolds number is small.

The fluid flow equations are the continuity equation for a fluid of constant density

$$ \nabla \cdot V = 0 $$

and the Navier–Stokes equations

$$ \frac{DV}{Dt} = -\frac{\mu}{\rho} \nabla^2 V - \frac{\nabla P}{\rho} + g + \frac{F}{\rho} $$

where $\rho$ is density, $t$ is time, $\mu$ is viscosity, $P$ is pressure, $g$ is acceleration due to gravity and $F$ is the electromagnetic body force given by:

$$ F = J \times B $$

The equation for the position of the free surface is

$$ \frac{Dh}{Dt} = \frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} + v \frac{\partial h}{\partial y} = w $$

where $h$ is the height of the free surface at a point on it, $u$ is the velocity at that point in the horizontal direction $x$, $v$ is the velocity at that point in the horizontal direction $y$, $w$ is the velocity at that point in the vertical direction $z$.

The electromagnetic field equations are solved by treating the dependent variables as phasors. The time averaged value of the electromagnetic body force is then given by

$$ F = \frac{1}{2} \text{Re}(J \times B^*) $$

where Re indicates the real part of its argument and * indicates the complex conjugate.

This time averaged force is used in the fluid flow equations. The solution of the equations is inherently unsteady. As the surface of the melt moves it changes the electromagnetic field and thereby the electromagnetic force distribution; this affects the flow in the melt and the surface of the melt, and so on. The equations are solved with the boundary conditions of zero velocity at liquid-wall interfaces and zero shear at free surfaces.

3.2. The Numerical Solution

The electromagnetic field equations and fluid flow equations were solved by finite element methods while a finite difference solution was used for the position of the free surface. First the electromagnetic field equations were transformed by introducing a scalar potential $A$ and a vector potential $\phi$, such that

$$ B = \nabla \times A $$

and

$$ E = -\frac{\partial A}{\partial r} - \nabla \phi $$

The electromagnetic field equations then become

$$ \nabla \times \frac{1}{\mu} (\nabla \times A) = -\sigma j \omega - \sigma \nabla \phi + J_s $$

and

$$ \nabla \cdot (-\sigma j \omega A - \sigma \nabla \phi + J_s) = 0 $$

Here $\mu$ is the permeability of vacuum and $J_s$ is the current density resulting from externally applied potential differences (e.g. the power supply driving the inductor) and is zero everywhere except in the inductor where it is input to the calculations. These last two equations were solved simultaneously using “brick” elements with the variables $A$ and $\phi$ calculated at the eight nodes (corners) of each element. A co-ordinate transformation was used so that the deformation of the elements in the vertical direction, as the liquid surface moves, could be accommodated. The resulting set of linear equations was solved numerically by the incomplete Cholesky gradient method.

The fluid flow equations were solved on a Lagrangian–Eulerian grid, that is one where elements had fixed horizontal co-ordinates but where the vertical co-ordinates varied (e.g. the power supply driving the inductor) and is zero everywhere except in the inductor where it is input to the calculations. These last two equations were solved simultaneously using “brick” elements with the variables $A$ and $\phi$ calculated at the eight nodes (corners) of each element. A co-ordinate transformation was used so that the deformation of the elements in the vertical direction, as the liquid surface moves, could be accommodated. The resulting set of linear equations was solved numerically by the incomplete Cholesky gradient method.

The fluid flow equations were solved on a Lagrangian–Eulerian grid, that is one where elements had fixed horizontal co-ordinates but where the vertical co-ordinates varied as the surface of the melt moved. The flow is turbulent and large eddy simulation was used to represent the turbulence.

This choice was made because of the reported success of LES in modeling recirculating flows. The key equations in the LES model are

$$ \frac{\partial \tilde{u}_i}{\partial t} + (\tilde{u}_j - u_j) \frac{\partial \tilde{u}_i}{\partial x_j} = \frac{1}{\rho} \frac{\partial \tilde{p}}{\partial x_i} + \frac{\tilde{F}_i}{\rho} $$

and

$$ \frac{\partial \tilde{u}_i}{\partial x_i} = 0 $$

with

$$ v_t = (C_s \Delta)^2 \left[ \frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} \right]^{1/2} $$
\[ \Delta = f (\Delta x \cdot \Delta y \cdot \Delta z)^{1/3} \] .................(14)

the \( f \) denoting a wall damping function given by the usual law of the wall

\[ f = 1 - \exp \left( -\frac{y^+}{25} \right) \] .................(15)

In these equations the Einstein convention has been used and the overbar over a variable indicates a value averaged over one grid spacing (\( \Delta X, \Delta Y, \Delta Z \)). \( u_i \) is the grid velocity in the direction \( i \), \( C_s \) is the Smagorinsky constant and \( y^+ \) is the reduced distance from the wall.

The mixed element formulation of the flow equations was used, that is the velocities were computed at the nodes (corners) of the brick elements while the pressures were calculated at the element centers. Pressure was determined by employing a Poisson equation. A predictor–corrector method was used to march forward in time. The discretization of the differential equations for flow was carried out using the streamline upwind/Petrov–Galerkin method described by Brooks and Hughes\(^{23}\) to represent the convective terms.

The Eq. (5) describing the surface movement was solved using a finite difference technique with the spatial derivatives approximated by third order upwind differencing. Integration in time was implicit. Filtering was applied to eliminate a grid size dependent surface fluctuation.\(^{24}\) The boundary condition on the equation was that the radial gradient of the surface height was zero at the walls.

**Figure 2** is a flowchart illustrating the numerical procedure. The software incorporating the model was written in FORTRAN77. The usual checks were carried out to ensure independence from grid size and time step size.

### 4. Testing of the Model

The model was first tested against measurements of magnetic field and induced currents that were reported by Konuka \textit{et al.}\(^{25}\) Their apparatus is illustrated in **Fig. 3**. It consisted of a coil with a ferrite core. The coil and core were rectangular (seen from above) with aluminum plates positioned above and below the coil. Two kinds of aluminum plate were used, ones without holes and ones with

![Flowchart](image)

**Fig. 2.** The flow chart of the simulation.

![Apparatus](image)

**Fig. 3.** The apparatus used to provide experimental data for testing of mathematical models of EMC.
holes. From symmetry it was only necessary to measure electromagnetic variables in one quadrant of the experiment. Current density measurements were made along a horizontal line on the underside of the upper aluminum plate, parallel to the wide faces of the ferrite core and at the center of the plate. A second set of measurements was made along a second centerline parallel to the narrow faces of the core. Magnetic field measurements were made along horizontal lines that were half-way between the upper surface of the core and the aluminum plate, parallel to the vertical center plane (in turn parallel to the wide faces of the core) and at distances (y) of 25, 55 and 85 mm from it. The coil was operated at 1000 Amp·turns (RMS) at a frequency of 50 Hz.

Figure 4 is representative of the fit between the computed and measured magnetic flux density components. These results are for y=25 mm, x is the second horizontal co-ordinate and z the vertical co-ordinate. It is seen that the model is able to reproduce the magnetic field quite well, including the sharp peak in the vertical field component just above the edge of the ferrite core. Figure 4 is for plates without holes ("model 1") while Fig. 5 gives some computed and measured results for the case where the plates have holes (model 2). The latter, for y=85 mm, shows the worst fit between the model and measured results with a significant discrepancy in the vertical component. However, the measured values for the other (larger) components are still fairly well reproduced by the model.

Measured and computed induced currents appear in Figs. 6 and 7 for models 1 and 2 respectively. The model appears to satisfactorily match the experimental results. In the case of Fig. 6 the large current density in the aluminum plate just above the coil is seen in both the experimental results and the calculations. In Fig. 7 the coil lies under the plate between y=75 and 100 mm and the induced currents are seen to be high here for both measurements and computed results.

Additional comparisons between the measurements and model, physical properties used in the calculations and details of the numerical simulation can be found in Kageyama's dissertation.16)

The fluid flow aspects of the model were tested next by using the model to predict the (two-dimensional) flow, within a cavity, that is induced by movement of the upper wall of the cavity. This flow is illustrated in Fig. 8 where the calculations are for a Reynolds number of 1000. The upper wall is moving from left to right as is the fluid adjacent to it. The flow is laminar (the turbulence model is not invoked) and the computed streamlines are shown. There is a major recirculation loop with flow in a clockwise direction. Additionally the model predicts two smaller counter-

![Figure 4](image1.png)

**Fig. 4.** Comparison of the measured and simulated magnetic flux densities at \( y = 25 \) mm in the first physical model (aluminum plate without holes).

![Figure 5](image2.png)

**Fig. 5.** Comparison of the measured and simulated magnetic flux densities at \( y = 85 \) mm in the second physical model (aluminum plate with holes).

![Figure 6](image3.png)

**Fig. 6.** Comparison of the measured and simulated induced current density at \( y = 0 \) in the first physical model.

![Figure 7](image4.png)

**Fig. 7.** Comparison of the measured and simulated induced current density at \( x = 0 \) in the second physical model.
clockwise loops. These results are in agreement with the computed results of Ghia et al.\textsuperscript{26} who used a finite difference approach to solving the flow equations in stream function/vorticity form. For example, these investigators computed a flow towards the upper left corner as seen in the upper left of Fig. 8. They also calculated a larger minor recirculation loop at the lower right than at the lower left, again as seen in Fig. 8. A more quantitative comparison appears in Fig. 9 where the horizontal velocity is plotted versus vertical position along a line passing up through the center of the cavity. The “finite difference method” is that of Ghia et al. The present model is seen to match the finite difference results, particularly when the elements have a non-uniform size designed to put more of them in locations where velocity gradients are steep.

Next, the computations of free surface oscillations were tested by calculating the oscillation of a liquid surface in an infinitely long “trough” of rectangular cross section. The liquid was treated as inviscid in these 2D computations, so that the oscillations should not decay. Figure 10 shows the computed height of the surface at the wall of the trough following an initial disturbance where the trough is tilted by 1% of the liquid depth. Pleasingly the oscillations do not decay. The dominant frequency evident in the results is 2.67 Hz. The analytical solution for the eigenfrequency can be obtained from potential flow theory\textsuperscript{27} and gives

$$\nu = \left( \frac{g}{2\pi^2} \tanh \left( \frac{2\pi h}{\lambda} \right) \right)^{1/2}$$

where $\nu$ is the eigenfrequency, $g$ is the acceleration due to gravity, $h$ is the liquid depth, and $\lambda$ is the wavelength.

For the lowest eigenfrequency, this equation gives a value of 2.67 Hz, exactly matching the computed results.

Finally the model was tested against the experimental data obtained by Wajima et al.\textsuperscript{28} on a mercury pool within a square inductor. Their apparatus is shown in Fig. 11. Surface oscillations were measured at the center of the pool surface and 28 mm from the center. The magnetic flux density was measured without the mercury present and at the level of the coil mid-height. Measurements were on the pool centerline, at the edge (on the centerline of the face) and at the corner. The inductor had 120 turns and was driven with a current of 180 A at 200 Hz. In the finite element calculations of the electromagnetic field the mercury was represented by 30 by 30 by 18 elements (the last being the number in the vertical direction). The elements were of uneven size so that eight elements were contained within the skin depth (35.6 mm) of the mercury.

Table 1 shows the comparison between measured and computed magnetic flux density. The agreement is to within 8% suggesting that the discretization of the field equations is satisfactory.

In simulating the flow and surface oscillations of the Wajima experiment, a time step of 2 ms was used. The Smagorinsky constant was set to 0.1 and a predictor-corrector method was employed to integrate in time.

Figure 12 gives a comparison between the oscillation of the free surface, at 28 mm from the center, as measured by Wajima et al. for an inductor current of 120 A, and as computed by the present model. The agreement is fair; the period of the oscillations, over the 1 s after the current is turned on, is accurately computed but the amplitude of the oscillations is underestimated in the simulation. Somewhat better agreement is obtained for the results at the center of the surface.\textsuperscript{16}

The computed shape of the free surface at 0.5 s after
turning on the current is plotted in Fig. 13. At this time the height at the center is close to a minimum. The computed instantaneous velocities at this time are given in Fig. 14 for

The apparatus used by Wajima et al. in physically modeling EMC.

Table 1. Comparison between measured\(^{(20)}\) and computed magnetic flux density at the level of the mercury pool half height. Current=180 A, pool absent.

| Location with respect to pool | Measured flux density (G) | Computed flux density (G) |
|-------------------------------|---------------------------|---------------------------|
| Pool center                   | 1383                      | 1320                      |
| Corner                        | 2027                      | 1877                      |
| Edge center                   | 1733                      | 1605                      |

Fig. 11.

The apparatus used by Wajima et al. in physically modeling EMC.

Fig. 12.

Comparison of the surface oscillations measured by Wajima et al. with the results of the present simulation.

Fig. 13.

Computed surface shape 0.5 s after current starts to flow in the inductor. Scales are in cm.

Fig. 14.

Computed velocity vectors in the vertical plane y=0 at 0.5 s. Velocity scale in cm/s. Deformation of free surface not depicted.

The vertical plane passing through the center of the melt and parallel to one face. [Limitations of the vector plotting software prevented simultaneous plotting of vectors and interface deformation so the latter is not depicted in Figs. 14 and 15.] At first the velocities may appear symmetric about a vertical axis passing through Fig. 14, which would be unusual for a turbulent flow computed by LES. However close examination reveals that the flow is not quite symmetric. These results are for one second after the (initially symmetric) electromagnetic forces are applied and there is further development of asymmetry later in the simulation (see Kayeyama’s dissertation\(^{(16)}\) for examples). These velocities

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| Edge center                   | 1733                      | 1605                      |
range up to about 20 mm/s. They show the toroidal recirculation loops (towards the top of the pool) that are commonly encountered in inductively stirred melts (e.g. Tarapore et al. 29,30). However there is also an unusual strong downward and inward flow at approximately 150 mm above the bottom of the pool. This flow can be seen returning upwards close to the corner in the instantaneous velocities of the vertical diagonal plane shown in Fig. 15.

5. Concluding Remarks

It appears that electromagnetic “soft” casting of steel will follow the electromagnetic casting of aluminum into commercial reality. This paper has described the development of a mathematical model for electromagnetic casting of steel. The relevant equations were summarized as were the numerical techniques entailed in the dynamic 3D computation of electromagnetic fields, flow and surface oscillation.

The resulting mathematical model was tested in four ways with satisfactory results. It was tested against the measurements of magnetic fields and induced currents carried out by Konuka and coworkers on an apparatus intended to mimic electromagnetic (but steady state) aspects of soft casting. It was tested against the results of Ghia et al., who used a numerical technique different from that of the present study to compute 2D flows in a cavity with a moving upper surface. The free surface movements predicted by the model were compared with classical results from potential flow theory which describe the oscillation of a liquid surface in a long rectangular trough, resulting in an excellent match. Finally the model was compared with the measurements of Toh and colleagues on a mercury pool intended to simulate the liquid steel in electromagnetic casting.

It is suggested that the model is now a credible one and the use of the model in predicting the behavior of steel in the field of an electromagnetic caster, including behavior when an intermittent field is applied will be the subject of a subsequent paper.

Acknowledgements

The authors are grateful for the support of Nippon Steel Corporation and for permission to publish this paper.

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