Dissipative production of a maximally entangled steady state of two quantum bits

Y. Lin1*, J. P. Gaebler1*, F. Reiter2, T. R. Tan1, R. Bowler1, A. S. Sørensen2, D. Leibfried1 & D. J. Wineland1

Entangled states are a key resource in fundamental quantum physics, quantum cryptography and quantum computation1. Introduction of controlled unitary processes—quantum gates—to a quantum system has so far been the most widely used method to create entanglement deterministically2. These processes require high-fidelity state preparation and minimization of the decoherence that inevitably arises from coupling between the system and the environment, and imperfect control of the system parameters. Here we combine unitary processes with engineered dissipation to deterministically produce and stabilize an approximate Bell state of two trapped-ion quantum bits (qubits), independent of their initial states. Compared with previous studies that involved dissipative entanglement of atomic ensembles3 or the application of sequences of multiple time-dependent gates to trapped ions4, we implement our combined process using trapped-ion qubits in a continuous time-independent fashion (analogous to optical pumping of atomic states). By continuously driving the system towards the steady state, entanglement is stabilized even in the presence of experimental noise and decoherence. Our demonstration of an entangled steady state of two qubits represents a step towards dissipative state engineering, dissipative quantum computation and dissipative phase transitions5–7. Following this approach, engineered coupling to the environment may be applied to a broad range of experimental systems to achieve desired quantum dynamics or steady states. Indeed, concurrently with this work, an entangled steady state of two superconducting qubits was demonstrated using dissipation8.

Trapped ions are one of the leading experimental platforms for quantum information processing, and advanced protocols using unitary quantum gates have been demonstrated (see, for example, refs 9, 10). However, decoherence and dissipation from coupling to the environment remains a challenge. One approach to overcome this relies on active feedback11–17. Such feedback techniques may be extended to quantum error correction, which can stabilize entangled states or realize fault-tolerant quantum computations. This will, however, require high-fidelity quantum gates and large qubit overheads that are beyond the reach of current experiments9. Recently, a complementary approach has been proposed to create entangled states or perform quantum computing by engineering the continuous interaction of the system with its environment5–7,16–20. In our experiment, we take a step towards harnessing dissipation for quantum information processing by producing an entangled state that is inherently stabilized against decoherence by the applied interactions in a setting fully compatible with quantum computation. With this technique, we realize maximally entangled steady states with a fidelity of $F = 0.75(3)$ by simultaneously applying a combination of time-independent fields. We also demonstrate that a stepwise application of these fields can speed up the dynamics of the scheme and achieve a fidelity of $F = 0.89(2)$ after approximately 30 repetitions. In both cases, the errors can be attributed to known experimental imperfections. Although these errors lead to a lower fidelity for entanglement preparation in our system as compared to unitary gates20, the dissipative technique is much less sensitive to certain sources of experimental noise—for example, laser intensity fluctuations common to both qubits—and so may lead to improved performance in other trapped-ion systems where this is the dominant error. In the Supplementary Information we model the errors for the dissipative entanglement preparation and propose how they can be reduced.

Our scheme uses an ion chain with two qubit ions and at least one ‘cooler’ ion for sympathetic cooling21 of the qubit ions’ motion. We consider a normal motional mode of this ion chain having frequency $v$ and mean motional quanta $n$. We cool the motional mode to $n = 0$ by laser-cooling the coolant ion (or ions) and thus effectively couple the vibration to a zero-temperature bath with the phonon loss rate denoted by $k$. As depicted in Fig. 1, we consider four energy levels of each qubit ion (‘Be’), where $|\downarrow\rangle$ and $|\uparrow\rangle$ are the qubit ‘spin’ states, $|a\rangle$ is an auxiliary state and $|e\rangle$ is a fast-decaying excited electronic state. A sideband excitation, with Hamiltonian $H = \Omega_0(|\downarrow\rangle\langle\downarrow| + |\uparrow\rangle\langle\uparrow|)b^\dagger + \text{h.c.}$ in the atomic and motional rotating frame, couples the two ions’ spins via their motion (here $\Omega_0$ denotes the Rabi frequency, $b^\dagger$ is the motional-mode Fock-state creation operator, the number subscripts denote the qubit ion number, and $\text{h.c.}$ is the Hermitian conjugate). A carrier interaction with Hamiltonian $H_c = \Omega_c(|a\rangle\langle\downarrow| + |a\rangle\langle\uparrow|) + \text{h.c.}$ drives the $|\downarrow\rangle \leftrightarrow |a\rangle$ transition of each ion with Rabi frequency $\Omega_c$, and a repump laser incoherently drives $|a\rangle \rightarrow |\downarrow\rangle$, $|\uparrow\rangle$ by coupling to the intermediate state $|e\rangle$ at a rate $\gamma$ (see Fig. 1a). All the above transitions are homogeneously driven for both qubit ions, such that individual addressing is not needed for this scheme. These couplings ensure that the maximally entangled singlet state $|S\rangle \equiv \frac{1}{\sqrt{2}}(|\downarrow\downarrow\rangle - |\uparrow\uparrow\rangle)$ is the only steady state of the effective dynamics21 in the regime $\gamma, k, \Omega_c \ll \Omega_0$. For an intuitive understanding of the scheme, we first consider only the sideband excitation and the sympathetic cooling (blue lines in Fig. 1a), which, when applied together, have two dark states that are not affected by the interactions $|\downarrow\downarrow\rangle$ and $|\uparrow\uparrow\rangle$. The remaining basis states of the qubits, $|\downarrow\downarrow\rangle$ and $|\uparrow\uparrow\rangle$, are driven by $H_a$ and eventually pumped to $|\downarrow\downarrow\rangle$ by the combination of the sideband drive and the sympathetic cooling (Fig. 1b). The effect of adding the carrier drive $H_c$ is to couple the $|\downarrow\downarrow\rangle$ state to a combination of the $|a\rangle$, $|a\rangle$ and $|aa\rangle$ states and the $S$ state to $|Sa\rangle \equiv \frac{1}{\sqrt{2}}(|a\rangle - |a\rangle)$ state. However, assuming the ions are in the ground state of motion, the dressed states of the sideband Hamiltonian $H_c$ containing $|Sa\rangle$ have eigenenergies $\pm \Omega_c$, while $|S\rangle$, $|\downarrow\downarrow\rangle$, $|\uparrow\uparrow\rangle$ and $|a\rangle$ are dark states of $H_c$ with zero eigenenergy. Thus, the transition from $|S\rangle$ to $|Sa\rangle$ is shifted out of resonance with the carrier drive and therefore suppressed for $\Omega_c \ll \Omega_0$. On the other hand, the transitions from the $|\downarrow\downarrow\rangle$ state to the $|\downarrow\rangle$ and $|\uparrow\rangle$ states are not energy shifted and remain resonant. The repumping laser incoherently transfers the state $|a\rangle$ back to the $|\downarrow\rangle$ and $|\uparrow\rangle$ qubit manifold. Thus, the combination

---

1 National Institute of Standards and Technology, 325 Broadway, Boulder, Colorado 80305, USA. 2QUANTOP, The Niels Bohr Institute, University of Copenhagen, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark.  
*These authors contributed equally to this work.
of $H_e$ and the repumping beam create a process to pump $|\uparrow\uparrow\rangle$ to $|S\rangle$ as well as a depumping process from $|S\rangle$ to $|\uparrow\downarrow\rangle$, $|I\rangle$ and $|\downarrow\downarrow\rangle$, although the latter is significantly slower (Fig. 1b). In the limit where the rate of pumping other states into $|S\rangle$ is much greater than the depumping rate from $|S\rangle$, the steady state will approach $|S\rangle$. The ratio of these rates can be made arbitrarily high by reducing the values of $\gamma$, $\kappa$ and $\Omega_d$ compared to $\Omega_a$ and in steady state the fidelity of the maximally entangled state $|S\rangle$ can approach unity (see Supplementary Information). Fluctuations in the values of these parameters do not reduce the fidelity of the entangled state as long as the values of $\gamma$, $\kappa$ and $\Omega_a$ remain small compared to $\Omega_d$, which is in contrast to the method of entanglement preparation via unitary gates.

For our experimental implementation, we confine a $^9$Be$^{+}$-$^{24}$Mg$^{+}$-$^{24}$Mg$^{+}$-$^{9}$Be$^{-}$ four-ion chain in a linear radio-frequency Paul trap$^{10}$. The two $^9$Be$^{+}$ ions serve as qubit ions while the two $^{24}$Mg$^{+}$ ions are used for sympathetic cooling. The ion chain lies along the axis of the trap, the axis of weakest confinement, and has an extent of approximately 11 $\mu$m. We label the four-ion axial modes $1, 2, 3, 4$, and they have mode frequencies $\nu_{1-4}$ $\approx$ [2.0, 4.1, 5.5, 5.8 MHz], respectively. An internal-state quantization magnetic field $B$ $\approx$ 11.964 mT is applied along a direction that is at 45° to the trap axis, which breaks the degeneracy of the magnetic sub-levels of $^9$Be$^{+}$ and $^{24}$Mg$^{+}$. As depicted in Fig. 1a, we utilize the $^9$Be$^{+}$ internal states $|F = 1, m_F = 1\rangle = |\uparrow\rangle$, $|2, 2\rangle = |\downarrow\rangle$, $|2, 1\rangle = |a\rangle$. To create the sideband coupling term $H_a$ we apply two 313 nm laser beams in a Raman configuration tuned approximately 270 GHz below the $2S_{1/2}^2$ to $2P_{1/2}^2$ transition with a frequency difference equal to $f_0 + \nu_3$ where $f_0 \approx$ 1.018 GHz is the resonant transition frequency between the $|\uparrow\rangle$ and $|\downarrow\rangle$ states. The two beams are derived from the same laser and frequency-shifted using acousto-optic modulators. The difference wave vector of the two beams is parallel to the trap axis. Microwaves are used to drive resonant transitions between the $|\uparrow\rangle$ state and the $|a\rangle$ state ($f = 1.121$ GHz) to create $H_e$. We also apply a repump laser beam to drive the $|a\rangle$ state to the $2P_{1/2}^2$ state, which subsequently spontaneously emits a photon and decays to $|\uparrow\rangle$, $|\downarrow\rangle$ $|a\rangle$ with a branching ratio of approximately $5:4:3$. Phonon excitations due to the photon recoil are removed by the sympathetic cooling. To cool the $^{24}$Mg$^{+}$ ions, a Doppler cooling beam, two Raman-sideband beams, and a repump beam co-propagate with the $^9$Be$^{+}$ Raman beams. These beams ($\lambda \approx$ 280 nm) interact negligibly with the internal states of the $^9$Be$^{+}$ ions. We initialize each experiment by first applying Doppler cooling to $^9$Be$^{+}$ and $^{24}$Mg$^{+}$, followed by $^{24}$Mg$^{+}$ sideband cooling of all the axial modes to near the ground state of motion. An optical pumping pulse initializes the $^9$Be$^{+}$ ions to the $|\downarrow\rangle$ state. We then apply the dissipative entanglement preparation operations, as detailed below. Finally, we perform spin-state analysis to measure the populations of the $|S\rangle$, $|I\rangle$, $|\uparrow\downarrow\rangle$ and $|\downarrow\downarrow\rangle$ spin states (see Methods).

We implement the entanglement scheme using mode 3, where the $^9$Be$^{+}$ ions oscillate in phase with each other but out of phase with the $^{24}$Mg$^{+}$ ions (which oscillate in phase). In one implementation of the experiment, we apply the laser-induced sideband excitation, microwave-induced carrier excitation, repumping and sympathetic cooling simultaneously (see Methods for parameter values) for a duration $t$ and obtain a steady-state singlet state fidelity of 0.75(3), as shown in Fig. 2.

**Figure 2** | Steady-state entanglement. Measured populations of the singlet, triplet, $|\uparrow\downarrow\rangle$ and $|\downarrow\downarrow\rangle$ states (respectively squares, crosses, circles and triangles) are shown as a function of duration; the duration is the length of time during which all elements of the dissipative entanglement scheme are applied simultaneously. The system reaches a steady state with a 0.75(3) population in the target singlet state after a few milliseconds. The solid lines are the result of a simulation based on the experimental parameters (see Methods). The slow decrease in the singlet state fidelity at long times visible in the simulation is due to a leak of the qubits to spin states outside the $|\downarrow\downarrow\rangle$, $|\downarrow\rangle$, $|a\rangle$ manifold caused by spontaneous emission from the lasers that generate the sideband coupling (see Methods and Supplementary Information). Strictly speaking this depumping means that the state is only a quasi-steady state. For our parameters there is, however, a clear separation of the preparation and depumping timescales, justifying the description as a steady state. Data are shown as mean $\pm$ s.d.
We model the experiment (solid lines in Fig. 2) taking into account: (1) the additional spontaneous emission due to the off-resonant $^9\text{Be}^+$ sideband laser beams, (2) the position fluctuations of those beams at the ions’ location, which leads to unequal sideband Rabi rates for the two $^9\text{Be}^+$ ions, (3) off-resonant coupling of the sideband excitation to other motional modes, and (4) heating processes (see Methods). The model is in close agreement with the data and suggests that the resulting state is an incoherent mixture of $|S\rangle$ and other states (mainly $|\uparrow\uparrow\rangle$) and the dominant errors come from the spontaneous emission induced by the sideband laser beams and unequal sideband Rabi rates.

We also implement the scheme in a stepwise manner. In this case, we can take advantage of coherences to speed up the entanglement creation process and thereby reduce the effect of the spontaneous emission induced by the $^9\text{Be}^+$ sideband laser beams. Specifically, we apply a sequence of steps, with each step consisting of a coherent pulse with $H_{coh} = H_r + H_s$, followed by the dissipative processes of repumping and sympathetic cooling, applied sequentially (the order does not matter). In the steady-state entanglement procedure outlined above, we required $\Omega_c, \gamma_j < \Omega_{coh}$ to suppress transitions from $|S\rangle$ to $|S_o\rangle$. However, when $H_{coh}$ is applied without any dissipation, ions initially in the $|S\rangle$ state will oscillate between $|S\rangle$ and a superposition of $|S\rangle$ and $|S_o\rangle$, which is dressed by $H_s$ with a period of $2\pi/\sqrt{\Omega^2_c + \Omega^2_s}$, assuming the ions are in the motional ground state. Thus, by applying $H_{coh}$ for a full oscillation period the interaction will be an identity operation for the $|S\rangle$ state while all other states will be partially transferred to the auxiliary level $|\alpha\rangle$, which can then be repumped to create $|S\rangle$. However, if $n \neq 0$ some population will be transferred out of the $|S\rangle$ state because the oscillation period is dependent on $n$. By taking advantage of the coherent evolution, we relax the requirement $\Omega_c, \gamma_j < \Omega_{coh}$ and the entanglement preparation timescale can be shortened, which reduces the error due to spontaneous emission induced by the sideband laser beams. During the coherent process, the entangled state $|S\rangle$ is no longer strictly a steady state; however, if the ratio $\Omega_c/\Omega_s$ is small, the evolution of the state away from $|S\rangle$ will be correspondingly small and $|S\rangle$ remains an approximate steady state.

The results of the stepwise experiment are shown in Fig. 3. We obtain the singlet state with fidelity 0.89(2). We use the same model as for the continuous case to predict the outcome of the stepwise scheme, and find good agreement with the data (solid lines in Fig. 3), with the largest sources of error coming from heating processes, unequal sideband Rabi rates, spontaneous emission caused by the $^9\text{Be}^+$ sideband lasers and off-resonant coupling of the sideband to mode 4.

We have presented deterministic steady-state pumping into a maximally entangled state with fidelities that are limited by known experimental imperfections. This result can be extended to other systems where two-qubit quantum logic gates may not be feasible owing to strong dissipation and represents a step towards harnessing dissipation for quantum information processing.

METHODS SUMMARY

The Methods section includes detailed descriptions of (1) the state detection and analysis procedure, (2) the experimental parameters for continuous and stepwise implementation of the scheme, and (3) the theoretical model used to produce the solid lines in Figs 2 and 3.

Online Content

Any additional Methods, Extended Data display items and Source Data are available in the online version of the paper; references unique to these sections appear only in the online paper.

Received 18 July; accepted 16 October 2013. Published online 24 November 2013.

1. Nielsen, M. A. & Chuang, I. L. Quantum Computation and Quantum Information (Cambridge Univ. Press, 2000).
2. Ladd, T. D. et al. Quantum computers. Nature 464, 45–53 (2010).
3. Krauter, H. et al. Entanglement generated by dissipation and steady state entanglement of two macroscopic trapped ions. Phys. Rev. Lett. 107, 080503 (2011).
4. Barreiro, J. T. et al. An open-system quantum simulator with trapped ions. Nature 470, 486–491 (2011).
5. Kraus, B. et al. Preparation of entangled states by quantum Markov processes. Phys. Rev. A 78, 042307 (2008).
6. Diehl, S. et al. Quantum states and phases in driven open quantum systems with cold atoms. Nature Phys. 4, 578–583 (2008).
7. Verstraete, F., Wolf, M. M. & Cirac, J. I. Quantum computation and quantum-state engineering driven by dissipation. Nature Phys. 5, 633–636 (2009).
8. Shankar, S. et al. Autonomously stabilized entanglement between two superconducting quantum bits. Nature http://dx.doi.org/10.1038/nature12802 (this issue).
9. Lanyon, B. P. et al. Universal digital quantum simulation with trapped ions. Science 334, 57–61 (2011).
10. Hanneke, D. et al. Realization of a programmable two-qubit quantum processor. Nature Phys. 6, 13–16 (2010).
11. Sayrin, C. et al. Real-time quantum feedback prepares and stabilizes photon number states. Nature 477, 73–77 (2011).
12. Vijay, R. et al. Stabilizing Rabi oscillations in a superconducting qubit using quantum feedback. Nature 490, 77–80 (2012).
13. Ristè, D., Bultink, C. C., Lehner, K. W. & DiCarlo, L. Feedback control of a solid-state qubit using high-fidelity projective measurement. Phys. Rev. Lett. 108, 240502 (2012).
14. Brakhane, S. et al. Bayesian feedback control of a two-atom spin-state in an atom-cavity system. Phys. Rev. Lett. 109, 173601 (2012).
15. Schindler, P. et al. Quantum simulation of dynamical maps with trapped ions. Nature Phys. 9, 361–367 (2013).
16. Campagne-Ibarcq, P. et al. Persistent control of a superconducting qubit by stroboscopic measurement feedback. Phys. Rev. X 3, 021008 (2013).
17. Ristè, D. et al. Deterministic entanglement of superconducting qubits by parity measurement and feedback. Nature 502, 350–354 (2013).
18. Poyatos, J. F., Cirac, J. I. & Zoller, P. Quantum reservoir engineering with laser cooled trapped ions. Phys. Rev. Lett. 77, 4728–4731 (1996).
19. Plenio, M. B., Huelga, S., Beige, A. & Knight, P. L. Cavity-loss-induced generation of entangled number states. Phys. Rev. A 59, 2468–2475 (1999).
20. Endrik, S., Peng, A., Gu, M. & Parkins, S. Unconditional preparation of entanglement between atoms in cascaded optical cavities. Phys. Rev. Lett. 91, 177901 (2003).
21. Parkins, A. S., Solano, E. & Cirac, J. I. Unconditional two-mode squeezing of separated atomic ensembles. Phys. Rev. Lett. 96, 053602 (2006).
22. Kastoryano, M. J., Reiter, F. & Sørensen, A. S. Dissipative preparation of entanglement in optical cavities. Phys. Rev. Lett. 106, 090502 (2011).
23. Cho, J., Bose, S. & Kim, M. S. Optical pumping into many-body entanglement. Phys. Rev. Lett. 106, 020504 (2011).
24. Bermudez, A., Schäetz, T. & Plenio, M. B. Dissipation-assisted quantum information processing with trapped ions. Phys. Rev. Lett. 110, 110502 (2013).
25. Leghtas, Z. et al. Stabilizing a Bell state of two superconducting qubits by dissipation engineering. Phys. Rev. A 88, 032317 (2013).
26. Corman, C., Bermudez, A., Huelga, S. F. & Plenio, M. B. Dissipative ground-state preparation of a spin chain by a structured environment. New J. Phys. 15, 073027 (2013).
27. Ticozzi, F. & Viola, L. Steady-state entanglement by engineered quasi-local Markovian dissipation: Hamiltonian-assisted and conditional stabilization.

![Figure 3](image-url) Entanglement with stepwise scheme. The measured populations of the singlet, triplet, $|\uparrow\uparrow\rangle$ and $|\downarrow\downarrow\rangle$ states (respectively squares, crosses, circles and triangles) are shown as a function of the number of applied steps. Each step has a duration of approximately 220 μs. The solid lines are the results of a model (see Methods). Data are shown as mean ± s.d.
Supplementary Information is available in the online version of the paper.

Acknowledgements This research was funded in part by the Office of the Director of National Intelligence (ODNI), Intelligence Advanced Research Projects Activity (IARPA). All statements of fact, opinion or conclusions contained herein are those of the authors and should not be construed as representing the official views or policies of IARPA or the ODNI. This work was also supported by ONR, by the NIST Quantum Information Program, and by the European Union’s Seventh Framework Program through SIQS (grant no. 600645) and through the ERC grant QIOS (grant no. 306576). We thank D. Allcock and B. Sawyer for comments on the manuscript and E. Knill for conversations. F.R. acknowledges conversations with B. Lanyon, R. Blatt and J. Home and support from the Studienstiftung des deutschen Volkes. This Letter is a contribution of NIST and is not subject to US copyright.

Author Contributions Y.L. and J.P.G. performed the experiments, analysed the data and developed the numerical model. F.R. proposed the entanglement scheme and developed the analytic rate model described in Supplementary Information under the guidance of A.S.S. T.R.T. contributed to the numerical model and the experimental apparatus. R.B. contributed to the experimental apparatus. D.L. and D.J.W. directed the experiments. All authors provided important suggestions for the experiments, discussed the results and contributed to the manuscript.

Author Information Reprints and permissions information is available at www.nature.com/reprints. The authors declare no competing financial interests. Readers are welcome to comment on the online version of the paper. Correspondence and requests for materials should be addressed to Y.L. (yiheng.lin@colorado.edu).
and 3 we measured the probability to find at least one ion outside the qubit manifold to be of the order of the square of the probability to find one ion outside the three-state manifold is at most 5% for the data in Fig. 2 and 3% for data in Fig. 3. In both cases, the ion spacing was set by adjusting the strength of the harmonic confinement, such that $\lambda_1z = 2\pi m$ where $\lambda_1 \sim 2\sqrt{2} 313 \times 10^{-6}$ $m^{-1}$ is the wavevector difference of the $^9$Be Raman sideband lasers, $z$ is the distance between the $^9$Be ions, and $m$ is an integer, such that the phase of the sideband excitation was equal on both ions. For our confinement strength, $z = 11 \mu m$ such that the value of $m$ was near 300. For modes where the qubit ions move in phase, the integer value of $m$ ensures $H_m$ is as defined in the main text. However, in the general case $H_m = \Omega_m\left(\left|1\right\rangle\left\langle1\right| + e^{i\phi}\left|2\right\rangle\left\langle2\right|\right)b^+ + h.c.$, where $\phi$ is the phase difference between the two $^9$Be ions of the sideband coupling, and the steady state of the system (including the cooling, repumping and microwave carrier) is $\left|D_m\right\rangle \equiv \left|1\right\rangle - e^{i\phi}\left|1\right\rangle\sqrt{2}/\sqrt{2}$.

Numerical model. We modelled our experiment using a master equation with a coherent component describing the $^9$Be sideband and microwave carrier drives and Lindblad operators describing the sympathetic cooling, repumping and spontaneous emission due to the $^9$Be sideband lasers. The coherent Hamiltonian is

$$H_{coh} \equiv \Omega_m\left(\left|1\right\rangle\left\langle1\right| + \left|2\right\rangle\left\langle2\right|\right)b^+ + \Omega_r\left(\left\langle a\right| + \left\langle a\right|\right)b + h.c.$$  

where $r$ describes the Rabi-rate imbalance of the sideband on the two ions. The Lindblad operator describing sympathetic cooling is given by $L_{s}\equiv\sqrt{\kappa}\delta b$, and the repumping is given by $L_{\phi}\equiv\sqrt{\kappa}b\phi$, where $\phi$ is either the $^1S_0$ state and $\kappa$ is determined experimentally by measuring $n$ for mode 3 after sympathetic cooling (no other interactions are turned on). The heating rate is given by $r = \kappa/\hbar$. For the continuous cooling used for the data in Fig. 2 we found $n = 0.11(1)$ and for the stepwise case of Fig. 3 we found $n = 0.08(1)$. We take into account spontaneous emission that incoherently changes population from the state $i$ to the state $j$ (i $\neq$ j) caused by the $^9$Be sideband laser beams with Lindblad operators of the form $L_{ij} = \sqrt{\gamma_j}\delta b_j$, where $\gamma_j$ can be calculated using the Kramers-Heisenberg formula. The error caused by Rayleigh scattering (i $\neq$ j) is negligible. Off-resonant coupling of mode 4 is taken into account with an additional Hamiltonian term $H_4 = \Omega_4\left(\left|1\right\rangle\left\langle1\right| + \left|2\right\rangle\left\langle2\right|\right)c + e^{-i\omega t} + h.c.,$ where $c$ is the raising operator for the fourth mode, $\delta = 2\pi \times 250 kHz$ is the splitting between modes 3 and 4, and $\eta_4 = 0.180$ and $\eta_4 = 0.155$ are the Lamb-Dicke parameters of modes 3 and 4, respectively. The continuous implementation of the scheme is modelled by numerically solving a master equation that includes all terms for a variable duration and a given value of $r$. We then obtain the theoretical prediction shown in Fig. 2 by averaging simulations with different values of $r$ using a Gaussian distribution with a r.m.s. value of 0.014. This r.m.s. value was determined from fits to qubit Rabi flopping curves for a single $^9$Be ion and for the $^9$Be$^{-}\text{Be}^{+}$ chain. Percent-level fluctuations of $\Omega_m$ cause negligible changes to the predicted fidelity. The result of the calculation at the end of each step is plotted in Fig. 3. In both cases, the initial state of the $^9$Be ions was taken to be $\left|1\right\rangle/\sqrt{2}$. The particular initial state chosen affects the dynamics only at short times and does not affect the steady state. All numerical models were implemented by use of the quantum optics toolbox.

31. Monroe, C. et al. Resolved-sideband Raman cooling of a bound atom to the 3D zero-point energy. Phys. Rev. Lett. 75, 4011–4014 (1995).
32. Jost, J. D. et al. Entangled mechanical oscillators. Nature **459**, 683–685 (2009).
33. Özeri, R. et al. Errors in trapped-ion quantum gates due to spontaneous photon scattering. Phys. Rev. A **75**, 042329 (2007).
34. Uys, H. et al. Decoherence due to elastic Rayleigh scattering. Phys. Rev. Lett. **105**, 200401 (2010).
35. Tan, S. M. A computation toolbox for quantum and atomic optics. J. Opt. B **1**, 424–432 (1999).