BEING CAREFUL WITH GW190412 INTERPRETATIONS

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ABSTRACT

The LIGO/Virgo Scientific Collaboration recently announced the detection of a compact object binary merger, GW190412, as the first asymmetric binary black hole (BBH) merger with mass ratio \( q \approx 0.25 \). Other than the mass ratio, this BBH has shown to have a positive effective spin of around \( \chi_{\text{eff}} \approx 0.28 \). Assuming a field formation channel, associating this effective spin to either the primary or the secondary BH each has its implications: If the spin of the BBH comes form the primary BH, it would imply that the angular momentum transport in the formation of massive BHs operates such that high spin massive BHs are born abundantly. If, on the other hand, the spin is due to the secondary BH through tidal spin up processes, one has to note that such processes have very short delay times and low local star formation rate at sufficiently low metallicities. We show that the predicted merger rate density from this channel is \( \lesssim 0.3 \) Gpc\(^{-3}\)yr\(^{-1}\) and in tension with the rather high local merger rate of such systems which we estimate from this single event to be \( \sim 2.5^{+3.5}_{-0.5} \) Gpc\(^{-3}\)yr\(^{-1}\) (90\% confidence interval). Large natal kicks (\( v \gtrsim 500 \) km/s) would be required to get such BBHs with in-plane spin component to account for the marginal detection of precession in GW190412. However, this would only exacerbate the tension as the estimated local merger rate would be further decreased. Similarly, the formation of such systems through the dynamical assembly is exceedingly rare, leaving this system a dilemma hard to account for with the currently accepted paradigms of BBH formation.

1. INTRODUCTION

The spin distribution of the black holes at birth is largely unknown. The majority of the LIGO-Virgo BBHs in the first and second observing runs have been consistent with having a zero effective spin and only a few events show non-zero effective spin \( \chi_{\text{eff}} \) (Abbott et al. 2018; Belczynski et al. 2017; Roulet & Zaldarriaga 2019). GW151226 and GW170729 have positive values of \( \chi_{\text{eff}} = 0.18^{+0.20}_{-0.12} \) and \( 0.36^{+0.21}_{-0.25} \) respectively. Notably, Zackay et al. (2019) discover a highly spinning BBH, GW151216, from the publicly available LIGO-Virgo first observing run data. Although their estimate of the probability that GW151216 is of astrophysical origin is not very high, 0.71, the discovery of GW151216 favors the field binary scenario Piran & Piran (2020).

The recent discovery of the asymmetric BBH merger, GW190412 (Abbott et al., et al. 2020a), is interesting because of two (perhaps related) facts: i) it is a low FAR event a non-zero effective spin of \( \chi_{\text{eff}} = 0.28^{+0.07}_{-0.05} \) which provides the clue to the formation of spinning BBHs. ii) it has a low mass ratio of \( q = 0.25^{+0.06}_{-0.04} \). This second fact is interesting on its own as dynamical formation channels would have a hard time accounting for such mergers, leaving field formation and its variants as a more promising channel to account for such mergers.

The LIGO/Virgo Scientific Collaboration (LVC) has reported that the dimensionless spin magnitude of the primary is large \( \chi_1 = 0.46^{+0.12}_{-0.15} \), while that of the secondary is unconstrained. Olejak et al. (2020) claim to have produced such mergers through conventional binary population synthesis. They find that in ~10\% of local BBH mergers with \( q < 0.41 \). Recent work by Mandel & Fragos (2020) suggest that the spin of the secondary can instead be large while the primary is non-spinning and find slightly different results, \( q = 0.31^{+0.05}_{-0.04} \) and \( \chi_{\text{eff}} = 0.20^{+0.03}_{-0.04} \). In this scenario the spin of the secondary originates from a tidal spin up process (Kushnir et al. 2016; Zaldarriaga et al. 2017; Hotokezaka & Piran 2017).

If we believe the primary BH’s spin is large, then the formation mechanisms that suggest BHs to be born slowly rotating (Fuller et al. 2019; Fuller & Ma 2019) need to be re-visited (Baibhav et al. 2020; Safarzadeh et al. 2020b). If, on the other hand, we believe the secondary is providing the spin budget of the system, given that such systems have inherently short merging timescales, their local merger rate would be determined by the local star formation rate of the universe, and one has to check whether the predicted merger rate of such systems is consistent with the observations. For example, according to a model investigated by Olejak et al. (2020) in which the overall BH-BH merger rate density in the local universe is: \( 73.5 \) Gpc\(^{-3}\)yr\(^{-1}\), for systems with \( q < 0.21, 0.28, 0.41 \), and 0.59 the rate density is estimated to be: 0.01, 0.12, 6.8, and 22.2 Gpc\(^{-3}\)yr\(^{-1}\), respec-
tively. Therefore, if we believe the median mass ratio $q = 0.25$ for GW190412, binary population predicts a local merger rate of less than $0.1 \ Gpc^{-3} \ yr^{-1}$. One can note that the predicted merger rates are extremely sensitive to the assumed mass ratio and it changes by two orders of magnitude from systems with $q < 0.21$ to systems with $q < 0.41$. However, the situation is even worse with the predicted rates: The quoted rates from Olejak et al. (2020) based on the mass ratio does not impose any other cuts on the primary mass, secondary mass, and the effective spin. Including all these cuts in the mass and spin parameter space reduces the predicted rates for $q < 0.41$ sample with overall merger rate of $6.8 \ Gpc^{-3} \ yr^{-1}$ to $0.11 \ Gpc^{-3} \ yr^{-1}$ Olejak et al. (2020). This is also assuming the primary’s spin lies in the range of $(\chi_1 = 0.17 - 0.59)$ consistent with LIGO’s estimate. Even lower merger rates would be predicted if Olejak et al. (2020) had implemented high efficiency angular momentum transport schemes (Fuller et al. 2019; Fuller & Ma 2019).

But what is the merger rate of GW190412 like systems? There are two simple ways to approximately compute it: the merger rate of BBHs inferred from LIGO’s O1 and O2 runs is in the range of $10^{-100}$ with a tidal spin-up secondary preferably form at low metallicities (i.e., preferably at higher redshifts). In the following, we try to answer the question of whether or not there is a sufficiently large parameter space for local BBH mergers with a tidally spin-up secondary.

Following the description of tidal synchronization by Kushnir et al. (2016); Zaldarriaga et al. (2017); Hotokezaka & Piran (2017), we consider BH-WR binaries as the last stage of binary evolution leading to BH-BH mergers. Here BH-WR binaries are considered as the outcome of a common envelope phase. Therefore, we assume that the initial spin parameter of the WR star and the spin parameter of the primary BH is zero. We compute the evolution of the spin parameter of the WR star, $a_w$, for a given mass loss rate, $\dot{m}_w$, and initial values of $m_2$ and $d$ by using the analytic formula developed by Kushnir et al. (2016). Note that $m_2$ and $d$ evolve with time due to isotropic wind mass loss but the mass and spin parameter of the primary BH, $m_1$ and $a_1$, are assumed to be constant with time. The spin angular momentum is determined by the competition between $t_{\text{syn}}$ and the time scale of angular momentum loss through winds given by $\approx 0.1 m_2 / \dot{m}_2$. For WR stars, Kushnir et al. (2016) show that the tidal synchronization timescale is given by mass ratio, $q = m_2 / m_1$, and coalescence time$^1$, $t_c$ as

$$t_{\text{syn}} \approx 10q^{-1/8} \left( \frac{1 + q}{2q} \right)^{31/24} \left( \frac{1c}{1 \ \text{Gyr}} \right)^{17/8} \ \text{Myr.} \quad (2)$$

The tidal lock occurs on a time scale much shorter than the stellar evolution time scales when $t_c \ll 100 \ \text{Myr}$. The spin parameter of the secondary black hole is given by $\chi_2 = \min(a_w, m_2, 1)$, where $a_w$ is the WR’s spin angular momentum at the end of its life. Since the direction of the spin angular momentum is parallel to the orbit vector the effective spin is simply $\chi_{\text{eff}} = m_2 \chi_2 / (m_1 + m_2)$.

In order to relate the mass loss rate of WR stars to metallicity, we make an assumption that the mass loss rate can be described by the following form

$$\dot{m}_2(m_2, Z) = f_w M_0 \left( \frac{m_2}{M_\odot} \right)^{-a} \left( \frac{Z}{Z_\odot} \right)^{-\beta}, \quad (3)$$

$^1$ We define the coalescence time to be the time of the core collapse of the secondary to the merger.
where $Z$ is the metallicity at ZAMS. Here we fix $M_0 = 10^{-5.73} M_\odot/\text{yr}$ and $\alpha \approx 0.88$ based on the result for the sample of Galactic WN plus WC stars obtained by Nugis \\& Lamers (2000). For our fiducial model, we choose $\beta = 0.8$ and $f_{\text{co}} = 1$. In reality, the index $\beta$ evolve with time because the metallicity at the surface of a WR star increases with time (see, e.g., Eldridge \\& Vink 2006; Yoon 2017 for detailed studies). However, it turns out that our results only weakly depend on $\beta$ for the range of $0.5 \leq \beta \leq 1.1$, and hence, we use $\beta = 0.8$ in the following.

To obtain the distribution of the spin parameters of BBH mergers at a given redshift, we integrate the BBH merger rates arising from different metallicities and redshifts. We assume the formation rate of merging BBH is proportional to the cosmic star formation rate (Madau \\& Dickinson 2014):

$$\dot{\rho}(z) = \frac{\rho_0 (1 + z)^{2.7}}{1 + ((1 + z)/2.9)^{3.5}},$$

where $z$ is redshift and $\rho_0$ is the present-day star formation rate. We also assume that the fraction of star formation at and below metallicity mass fraction of Z at given redshift is described by (Langer \\& Norman 2006):

$$\Psi(z, Z) = \frac{\hat{\Gamma}(0.84, (Z/Z_\odot)^2 10^{0.3z})}{\Gamma(0.84)},$$

where $\hat{\Gamma}$ and $\Gamma$ are incomplete and complete gamma-functions, respectively. At higher metallicities, the mass of the primary at the ZAMS is larger, and therefore the formation efficiency of more massive stars is reduced according to the initial mass function. Here we use a relation between the initial mass of a zero-age WR star to the mass at the ZAMS (Limongi \\& Chieffi 2018). Then we assume that the distribution of the primary masses at the ZAMS follows the Salpeter initial mass function.

Finally, we use the delay time distribution of BBH mergers

$$\text{DTD}(t_c) = \begin{cases} C_0 t_c^{-\gamma} & \text{for } t_c > t_{c, \text{min}}, \\ 0 & \text{otherwise} \end{cases}$$

where $C_0$ is a normalization constant and we use $\gamma = 1$ for the fiducial model, which is motivated by the semi-major axis distribution of massive binary (e.g., Sana et al. 2012). Later we also study a case with $\gamma = 1.5$. Note that, however, such a steep delay time distribution requires an extremely steep distribution in the initial separation of the binaries.

3. RESULTS

Figure 1 shows the spin parameter of the secondary black hole, $a_2/m_2$, as a function of coalescence time $t_c$ and the metallicity $Z$. The distribution of $a_2/m_2$ values depends very weakly on the mass ratio. The coalescence time required for BBHs composed of a tidally spun-up secondary is $\lesssim 100 \text{ Myr}$, which is much shorter than the typical lookback time of BBH mergers. Also depicted by a curve in figure 1 is the minimum coalescence time, $t_{c, \text{min}}$, derived based on an assumption that the initial semi-major axis must be larger than the one at which the stellar radius equals the Roche radius. This may be a good estimate of $t_{c, \text{min}}$ unless the secondary ejects a fraction of its mass or receives a significant kick at the core collapse. With this assumption, the $t_{c, \text{min}}$ value becomes larger at higher metallicities because the mass loss effect on the semi-major axis is more significant at higher metallicities but in general, there is a reasonably large parameter space for a tidally spin up secondary. Note that the $t_{c, \text{min}}$ values are smaller for the $32-8 M_\odot$ case than the equal mass case. The reasons are that the semi-major axis at the Roche radius decreases with mass and that the mass loss effect on the orbit is less significant for asymmetric binaries. Consequently, the parameter space where a BBH consists of a tidally-locked secondary is slightly larger for more asymmetric BBHs.

The fraction of BBH mergers consisted of a tidally-locked secondary is obtained by comparing the area of the high spin region of figure 1 to the total area with a weight function that takes into account for the cos-
mic BBH formation history (equation 4), cosmic metallicity evolution (equation 5), and delay time distribution of BBH mergers (equation 6). Figure 2 shows the normalized BBH merger rate at $z = 0.15$ as a function of the metallicity at their formation in the case of $32-8M_\odot$. We find that the merger rates sharply drop around the solar metallicity, which is consistent with the results of binary population synthesis (Eldridge & Stanway 2016). The fraction of BBH mergers with a highly spinning secondary, $\chi_{\text{eff}}(m_1 + m_2)/m_2 > 0.8$, strongly depends on the exponent of the delay time distribution, $\gamma$.

We present the probability distribution of $\chi_{\text{eff}}(m_1 + m_2)/m_2$ values for BBHs merging at $z = 0.15$ in figure 3. With the fiducial parameters, we find 15% and 10% of mergers have a tidally spun-up secondary for 30-30\,$M_\odot$ and $30-30\,$M$\odot$ cases, respectively. The dependence of the fraction of tidally spun-up BBH mergers on the mass ratio is rather weak. We also find that total mass dependence is also quite weak. Increasing the wind mass-loss rate $f_w$ results in the smaller fraction roughly as $\propto f_w^{-1}$. Thus, we conclude that 10–15% of BBH mergers with $q \approx 0.25$ at $z \approx 0.15$ have a tidally spun-up secondary within reasonable ranges of the model parameters. Finally, we find that the high spin fraction is significantly high if the delay time distribution is very steep, e.g., 50% when $\text{DTD}(t_c) \propto t_c^{-1.5}$.

Based on the tidal spin up model presented in this work, we can calculate what is the expected rate of GW190412 type systems:

$$R_{\text{GW190412}} = R_{\text{BBH}} f_{\text{tidal}} f_{m_1,m_2},$$

where $R_{\text{BBH}}$ is the local BBH merger rate estimated to be in the range of $10-100$ Gpc$^{-3}$yr$^{-1}$. $f_{\text{tidal}}$ is the tidal fraction of systems with mass ratio and masses similar to GW190412, which we compute to be about 10%. $f_{m_1,m_2}$ is the fraction of the BBH mergers with primary mass $m_1 = 30\,$M$\odot$, and secondary mass $m_2 = 8\,$M$\odot$. To compute this we assume a primary BH mass function following $\rho(m_1) \propto m_1^{3/2}$ bounded between $5\,$M$\odot$ and $50\,$M$\odot$ (to account for both the lower mass gap limit of the BHs (Farr et al. 2011), and upper limit due to pair-instability supernovae (Woosley 2017)). We set $\gamma = 2$ and compute the fraction of BBHs with masses above $30\,$M$\odot$ over the total population resulting in about 15% (assuming $\gamma = 1$ would slightly increase the fraction to 20%). Furthermore, assuming flat distribution for the mass function of the secondary, the fraction of systems with $q < 0.25$ equals 0.25. The combination of these two makes $f_{m_1,m_2} \approx 0.04$. Therefore the expected rate based on first-order calculation suggests $R_{\text{GW190412}} \approx 0.03 - 0.3$ Gpc$^{-3}$yr$^{-1}$ which is between one or two orders of magnitude smaller than the observed rate of this system.

With a steeper delay time distribution $t_c^{-1.5}$, the expected rate of GW190412-like events can be consistent with the observed rate. Interestingly, the distribution of galactic binary pulsars indicates such a steep delay time distribution for binary neutron star mergers Beniamini & Piran (2019). However, BBH mergers with $q \approx 1$ cannot have a steep delay time distribution because it predicts too many highly spinning equal-mass BBH mergers. Thus, a high rate of GW190412 may suggest that the steepness of the delay time distribution depends on $q$. Note that such models would be detected through stochastic gravitational background as they would predict a very high background level (Safarzadeh et al. 2020a).

4. SUMMARY AND DISCUSSION

GW190412 is unusual due to its two potentially related observed facts: i) low mass ratio ($q \approx 0.25$), ii) relatively high effective spin of $\chi_{\text{eff}} = 0.28 \pm 0.07$. We study a field binary origin scenario in which the effective spin is dominated by the tidally spun-up secondary. This scenario works effectively at low metallicities, and predicts...
that: \( \omega \sim 0 \), the Roche radius, we find \( \chi_{\text{eff}} \sim 0 \) to be detected in the future observing runs. This fraction for BBH mergers with \( q \approx 0.25 \) is by a factor of \( \sim 1.5 \) larger than the equal mass case.

- A steep delay time distribution such as \( DTD \propto t_c^{-1.5} \) results in the mergers composed of a tidally spun-up secondary that would dominate over non-spin-up mergers.

Figure 3. Probability distribution of the secondary spin parameter \( a_2/m_2 \) for BBH merger at \( z = 0.15 \). The bi-modal distribution in all panels is indicative of a subset of the BBHs that undergo a tidal spin up process, and therefore emerge as a high effective spin BBH merger, and those that due to their large initial binary separation do not experience a tidal spin up phase and therefore merge with near zero effective spin parameter. Top left: shows the case for a \( m_1, m_2 = 32, 8 \, M_\odot \) system with fiducial values for wind mass loss and delay time distribution. A nearly 10-15\% of the BBHs experience a tidal spin up process in our fiducial model. Top right: The same is shown for an equal mass ratio binary. Nearly similar result is obtained indicating that the mass ratio plays a minor role in our results to first order. Bottom left: Increasing the mass loss rate by a factor of 2. This decreases the tidally spun-up binaries in that more angular momentum is carried away through the winds and the final spin is subsequently dropped. However, the change in the ratio of the tidally spun-up systems over the total population remains similar to the fiducial case. Bottom right: Changing the underlying delay time distribution to a steeper functional form. This will increase the fraction of the tidally spun-up systems to about 50\% and remains a parameter that our model is most sensitive to.

Another important observational feature of GW190412 is the marginal detection of precession, suggestive of a non-zero in-plane spin (Abbott et al. et al. 2020a). To obtain a non-zero in-plane spin that is sufficiently large to induce observable precession, for any field binary scenario, requires a black hole natal kick of which magnitude is comparable to the orbital velocity at the collapse. For the tidal spin up model, this corresponds to an extreme natal kick, \( \gtrsim 500 \, \text{km/s} \) in the case of \( \chi_{\text{eff}}(m_1 + m_2)/m_2 \sim 1 \). Such large natal kicks are not expected from observations of Galactic X-ray binaries. For instance, the natal kicks for Cyg X-1 and GRO J1655-40 are constrained to \( \lesssim 80 \, \text{km/s} \) and \( \lesssim 120 \, \text{km/s} \), respectively (Wong et al. 2012; Willems et al. 2005). Therefore, if non-zero \( \chi_p \) of GW190412 is really significant, it disfavors the tidal spin up model. However, such a strong natal kick perpendicular to the orbital plane may result from the launch of a one-side jet from a BH-accretion disk system (Barkov & Komissarov 2010). Note that the formation of the accretion disk and mass ejection naturally occur at the collapse of tidally spun-up secondary because the spin parameter of the collapsing star may exceed unity (Batta & Ramirez-Ruiz 2019).

While LVC’s analysis results in the primary BH to be spinning with no constraint on the secondary’s mass, if we assume GW190412 is a tidal spin up system, the spin of the binary will be largely attributed to the secondary BH. The tidal spin up scenario faces two challenges: 1) The predicted merger rate of such system would be too low in this formation channel: Olejak et al. (2020) predicts less than 0.1 \( \text{Gpc}^{-3}\text{yr}^{-1} \) for binaries with \( q < 0.28 \), 0.11 \( \text{Gpc}^{-3}\text{yr}^{-1} \) for binaries with \( q < 0.41 \) and similar mass and spin to GW190412. This is similar to our results presented in this work. However, the observed merger rate of such systems is about 10 times more common than the predictions suggesting of a large tension between the tidal spin up model for this event. 2) This formation channel can not explain the observed in-plane spin of GW190412 if we assume the spin direction of the BHs at birth are aligned with their orbital angular momentum vector. Imposing random natal kicks on the BHs at birth to account for the observed precession of GW190412 would only exacerbate the tension in the es-
estimated local merger rate and the observed high merger rate for such systems.

Any successful model attempting to explain GW190412 should take into account the rather high \(2.5^{+3.5}_{-0.5}\) Gpc\(^{-3}\)yr\(^{-1}\) local merger rate of this system. Separate from the tidal spin-up channel that we challenge in this work, dynamical formation channels of the BBHs (Samsing et al. 2018; Rodriguez et al. 2019) would have a difficulty predicting such high merger rate for such low mass ratio systems (Gerosa et al. 2020). For example, based on a scenario in which GW190412 is formed through repeated mergers in the Globular Clusters, Rodriguez et al. (2020) arrive at a merger of about 0.1 Gpc\(^{-3}\)yr\(^{-1}\) which is an order of magnitude below the observed merger rate of GW190412 type systems. Gerosa et al. (2020) argue that second-generation mergers in such environments is exceedingly rare and one might need super dense environments such as super star clusters or AGN disks (Bartos et al. 2017) to account for such events, although we expect the rates of such channels to be low as well. Other scenarios based on, for example, the quadruple configuration for the birth of such systems (Hamers & Safarzadeh 2020), although successful in reproducing all the observables of GW190412, would imply that birth rate of the quadruples are high in the universe which would encourage a more scrutinized look into the birth rate of such systems. Moreover, the estimates provided in Olejak et al. (2020) can not re-produce the precession (in plan spin) for the binaries, which would require impose larger BH natal kicks. However, imposing natal kicks to get merging systems with precession will further decrease the local estimated rates in such a model and therefore increasing the tension with the observations.

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