Maximally Symmetric Minimal Unification Model SO(32) with Three Families in Ten Dimensional Space-time

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Abstract

Based on a maximally symmetric minimal unification hypothesis and a quantum charge-dimension correspondence principle, it is demonstrated that each family of quarks and leptons belongs to the Majorana-Weyl spinor representation of 14-dimensions that relate to quantum spin-isospin-color charges. Families of quarks and leptons attribute to a spinor structure of extra 6-dimensions that relate to quantum family charges. Of particular, it is shown that 10-dimensions relating to quantum spin-family charges form a motional 10-dimensional quantum space-time with a generalized Lorentz symmetry SO(1,9), and 10-dimensions relating to quantum isospin-color charges become a motionless 10-dimensional quantum intrinsic space. Its corresponding 32-component fermions in the spinor representation possess a maximal gauge symmetry SO(32). As a consequence, a maximally symmetric minimal unification model SO(32) containing three families in ten-dimensional quantum space-time is naturally obtained by choosing a suitable Majorana-Weyl spinor structure into which quarks and leptons are directly embedded. Both resulting symmetry and dimensions coincide with the ones of type I string and heterotic string SO(32) in string theory.

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I. INTRODUCTION

The well-known mysterious puzzles in the standard model of elementary particle physics contain: why parity is non-invariant in our world? why the observing matter in our world is consist of three families of quarks and leptons? why the motion of matter in our living universe is limited in an 4-dimensional space-time. Revealing those puzzles may concern more fundamental issues, such as: what is the basic building blocks of nature? what is the basic symmetries of nature? what is the basic dimensions of space-time? what is the origin of CP violation and masses? How about the stability of proton? How about the features of dark matter and dark energy?

The standard model based on quarks and leptons as basic building blocks of matter has well been established\cite{1, 2, 3} and tested by more and more precise experiments at the energy scale of order 100 GeV. The electromagnetic interaction is well described by an U(1) gauge symmetry, and the weak interaction is characterized by a left-handed $SU(2)_L$ gauge symmetry based on the parity violation\cite{4}. The strong interaction of quarks\cite{5, 6} described by the Yang-Mills gauge theory\cite{7} with a gauge symmetry $SU(3)_C$ displays a behavior of asymptotic freedom\cite{8, 9}. Three families of quarks and leptons are necessary for a non-trivial CP-violating phase\cite{10}, which can be originated from spontaneous CP violation\cite{11, 12, 13}. It has also been proved that the standard model is a renormalizable quantum field theory\cite{14}. Of particular, the strength of all forces has been shown to run into the same magnitude at high energy scales\cite{15}, which makes it more attractive for the explorations of grand unification theory (GUT). The simple and widely investigated GUT models include $SU(4) \times SU(2)_L \times SU(2)_R$\cite{16}, $SU(5)$\cite{17} and $SO(10)$\cite{18, 19}. One of the important predictions in the GUT models is proton decay. The current experimental data on proton decays have provided strong constraints on the minimal $SU(5)$ and $SO(10)$ models.

Unification of families has been much investigated by enlarging gauge symmetries, such as $O(14)$\cite{20, 21, 22, 23}, $SO(15)$\cite{24}, $SO(16)$\cite{25}, $SO(18)$\cite{26, 27, 28, 29}, $E_7$\cite{30, 31}, and $E_8$\cite{32, 33, 34, 35, 36, 37, 38}. One of the main difficulties for family unification in four dimensions is the existence of mirror families at the weak scale\cite{39}, which was shown to be avoidable by considering family unification in five and six dimensional space-time\cite{40}. As all these models are the ”bottom-up” models within the framework of quantum field theory, thus the basic building blocks are assumed to be quantum fields of point particles.
An alternative exploration of unification models is via the so-called "top-down" approach. String theory\[41, 42\], which was motivated by extending basic building blocks from point particles to one-dimensional strings, is thought of the most attractive theory for "top-down" model building. Great efforts have been made to construct standard like models with three families from string models\[43\]. While it remains an open question at moment how to uniquely obtain the standard model due to the landscape of string vacua. Nevertheless, string theory has inspired us with many interesting concepts. Two of the most important predictions in string theory are symmetry groups (G) and dimensions (D), i.e., G=SO(32) or $E_8 \times E_8'$, and D=10. Especially, the heterotic string theory\[44, 45\] has mostly been studied and shown its potential for a realistic model building.

To arrive at a consistent unification model within a more fundamental theoretical framework, it is very helpful to explore unification models via both "bottom-up" and "top-down" approaches. In this article, we are going to build a general "bottom-up" unification model that coincides for both symmetry and dimensions with some "top-down" models in string theory. The paper is organized as follows: in section 2, we describe in detail the motivations for introducing a maximally symmetric minimal unification (MSMU) hypothesis and a quantum charge-dimension correspondence (QCDC) principle. Then by analyzing the basic quantum charges of quarks and leptons in the standard model, we show that the minimal basic building blocks for a general "bottom-up" model should be Majorana fermions in the spinor representation of (14+6)-dimensions. In section 3, it is further demonstrated that the spin-family-related 10-dimensions form a motional 10-dimensional quantum space-time and possess a generalized Lorentz symmetry SO(1,9). Dual to the spin-family-related 10-dimensions, the isospin-color-related 10-dimensions are found to be motionless and become a 10-dimensional quantum intrinsic space, its corresponding 32-dimensional spinor representation is shown to have a maximal gauge symmetry SO(32). We then arrive at a maximally symmetric minimal unification model (MSMUM) SO(32) containing three families in ten-dimensional quantum space-time. It is further shown in section 4 that the family-related 6-dimensions are actually the minimal extra dimensions and will become a 6-dimensional quantum internal space. Our conclusions and remarks are presented in the last section with emphasizing the coincidence for both symmetry and dimensions between the MSMUM SO(32) and type I or Heterotic String SO(32) in string theory.
II. MAXIMALLY SYMMETRIC MINIMAL UNIFICATION HYPOTHESIS AND QUANTUM CHARGE-DIMENSION CORRESPONDENCE PRINCIPLE

In a general “bottom-up” model building, symmetry should play an important role. This is because symmetry enables one to establish relations among different quantum charges of building blocks. It is well known that basic quantum charges of Dirac fermions are characterized by spin charges and conjugated charges, which are described by the Lorentz symmetry SO(1,3). In the standard model, $SU(2)_L$ symmetry was discovered to describe a symmetry between two isospin charges of leptons, and $SU(3)_C$ symmetry was introduced to characterize a symmetry among three color charges of quarks. $U(1)$ symmetry is known to reflect the electric charge of particles and antiparticles. The orthogonal symmetry SO(10) was found to be a unified symmetry for describing isospin-color charges and their conjugated charges. When treating all the quantum charges of quarks and leptons on the same footing, the symmetry group $SO(1,3) \times SO(10)$ may be regarded as a generalized symmetry which describes the 4-dimensional space-time relating to spin-charges and conjugated charges, and 10-dimensional intrinsic space relating to isospin-color charges and conjugated charges.

From the above analyzes, it is seen that all symmetries are introduced based on the basic quantum charges of building blocks in the standard model. We are then motivated to propose a quantum charge-dimension correspondence (QCDC) as our working principle, namely: dimensions of space and time are directly related to basic quantum charges of building blocks. In this sense, we may mention such resulting space and time as quantum space and time.

In general, the independent degrees of freedom of fermionic building blocks are characterized by the spinor representation of relevant quantum space and time which relate to basic quantum charges of building blocks. It is known that each family of quarks and leptons contains 64 real independent degrees of freedom, which belong to the Majorana-Weyl spinor representation of 14-dimensions that relate to the spin-isospin-color charges. It then raises a question what a symmetry should be introduced to establish possible relations among independent degrees of freedom of building blocks. In the existing GUT models, one has considered symmetries only among basic quantum charges of building blocks rather than among independent degrees of freedom of building blocks. Namely, those introduced symmetries are not large enough to describe possible relations among independent degrees of
From usefulness and economic considerations, we are motivated to make a maximally symmetric minimal unification hypothesis (MSMU-hypothesis) that states that: all independent degrees of freedom of basic building blocks should be treated equally on the same footing and related by a possible maximal symmetry in a minimal unified scheme. A direct deduction from such an MSMU-hypothesis is: fermions as basic building blocks should be Majorana fermions in the (real) spinor representation of high dimensions which is determined by the basic quantum charges of building blocks. The chirality of basic building blocks should also be well defined to understand parity non-invariance. Such a deduction indicates that the possible dimensions should allow a spinor representation that can well define both Majorana and Weyl fermions. It is not difficulty to check that when simultaneously imposing Majorana and Weyl conditions, the allowed dimensions are restricted to be at $D = 2 + 4n$ ($n = 1, 2, \cdots$), i.e., $D = 2, 6, 10, 14, \cdots$.

Inspired from $SO(1, 3) \times SO(10)$ GUT model, the minimal dimension needed for describing each family is $D=14$, i.e., each family of fermions belongs to the Majorana spinor representation of 14-dimensions and has $128 = 2^7$ independent real degrees of freedom, which are twice to the ones in each family of quarks and leptons in the standard model. In fact, each family of quarks and leptons contains 64 real independent degrees of freedom and belongs to the Majorana-Weyl spinor representation of 14-dimensions.

We now come to the issue on the puzzle of three families of quarks and leptons. As all three families have the same quantum charges except their mass hierarchies, it is natural to take three family-charges and conjugated charges to be basic quantum charges. Then by applying for the QCDC-principle, the three family-charges and the conjugated charges must relate to extra 6-dimensions.

As a consequence, the minimal basic building blocks in our general ”bottom-up” unification model should be Majorana fermions in the spinor representation of (14+6)-dimensions.

III. EXPLICIT CONSTRUCTION OF MSMUM SO(32) IN TEN DIMENSIONS

We shall explicitly demonstrate in this section that when choosing a suitable spinor structure into which quarks and leptons as building blocks of standard model are directly embedded, the Majorana condition in the spinor representation of (14 + 6)-dimensions will
naturally lead to an interesting "bottom-up" MSMUM with a generalized Lorentz symmetry SO(1,9) in a motional 10-dimensional quantum space-time and a gauge symmetry SO(32) for each family in the spinor representation of a motionless 10-dimensional quantum intrinsic space. Of particular, it will be shown that the motional 10-dimensional quantum space-time is related to the quantum spin-family charges, and the motionless 10-dimensional quantum intrinsic space is related to the quantum isospin-color charges.

Let us now construct in detail a general "bottom-up" model based on the MSMU-hypothesis and QCDC-principle. Denoting \( \hat{\Psi} \) to be the fermionic building block in the spinor representation of (14+6)-dimensions, the Majorana condition implies that

\[
\hat{\Psi} = \hat{\Psi}^c = \hat{C} \bar{\Psi}^T
\]

where \( \hat{\Psi}^c \) defines the charge conjugation in (14+6)-dimensions. \( \hat{C} \) is the charge conjugate matrix and satisfies \( \hat{C}^\dagger = \hat{C}^{-1} = -\hat{C}^T = -\hat{C} \) and \( \hat{C} \hat{C}^\dagger = 1 \).

As the first step of a general "bottom-up" model building, it is crucial to find out an appropriate spinor structure of Majorana fermions in the spinor representation of high dimensions. This is because different spinor structures of building blocks reflect different geometries of space-time, which then result in different physics phenomena. As a natural and realistic consideration, a spinor structure of building blocks should be chosen in such a way that quarks and leptons as building blocks of standard model must directly be embedded into such a spinor structure. For this purpose, it turns out that the 8 \times 128-dimensional spinor representation of Majorana fermion \( \hat{\Psi} \) in (14+6)-dimensions takes the following spinor structure

\[
\hat{\Psi} = \begin{pmatrix} \Psi + i\tilde{\Psi} \\ \Psi - i\tilde{\Psi} \end{pmatrix}
\]

where \( \Psi^T = (\Psi_1, \Psi_2, \Psi_3, \Psi_0)^T \) and \( \tilde{\Psi}^T = (\tilde{\Psi}_1, \tilde{\Psi}_2, \tilde{\Psi}_3, \tilde{\Psi}_0)^T \) with \( \Psi_i \) and \( \tilde{\Psi}_i \) (i=1,2,3,0) being the fermionic building block in the spinor representation of 14-dimensions. They satisfy the Majorana condition in 14-dimensions

\[
\Psi_i = \Psi_i^c = C\Psi_i^T, \quad \tilde{\Psi}_i = \tilde{\Psi}_i^c = C\tilde{\Psi}_i^T
\]

Where \( \Psi_i^C \) ( \( \tilde{\Psi}_i^C \) ) define the charge conjugation in 14-dimensions, and the charge conjugate matrix \( C \) also satisfies \( C^\dagger = C^{-1} = -C^T = -C \) and \( CC^\dagger = 1 \). \( \Psi_i \) and \( \tilde{\Psi}_i \) (i=1,2,3,0) are Majorana fermions in the 128-dimensional spinor representation of 14-dimensions. The
spinor structures $\Psi + i\tilde{\Psi}$ and $\Psi - i\tilde{\Psi}$ in the spinor representation of extra 6-dimensions are corresponding to the 4- and 4-representations of SU(4) which characterizes a family symmetry for the basic building blocks. Here we have considered the facts that $SO(6)$ is isomorphic to $SU(4)$ and all families have the same spin-isospin-color charges. The explicit spinor structure of Majorana fermions $\Psi_i$ and $\tilde{\Psi}_i$ ($i=1,2,3,0$) in 14-dimensions is given by the following form

$$\Psi_i = \begin{pmatrix} F_{iL} + F'_{iR} \\ F_{iR} + F'_{iL} \end{pmatrix}$$  \hspace{1cm} (4)$$

with $F_{iL,R}$ being defined as

$$F^T_{iL,R} = \begin{bmatrix} U_r, U_b, U_g, N, D_r, D_b, D_g, E, & D^c_r, D^c_b, D^c_g, E^c_c, -U^c_r, -U^c_b, -U^c_g, -N^c \end{bmatrix}^T_{i L,R}$$

$$F'^T_{iL,R} = \begin{bmatrix} U'_r, U'_b, U'_g, N', D'_r, D'_b, D'_g, E', & D'^c_r, D'^c_b, D'^c_g, E'^c_c, -U'^c_r, -U'^c_b, -U'^c_g, -N'^c \end{bmatrix}^T_{i L,R}$$  \hspace{1cm} (5)$$

where the subindexes ‘r’, ‘b’, ‘g’ denote three colors. Similar forms hold for $\tilde{\Psi}_i$ with just replacing $F_{iL,R}$ by $\tilde{F}_{iL,R}$. All the fermions $\psi = U, D, E, N, \tilde{U}, \tilde{D}, \tilde{E}, \tilde{N}, \ldots$ are four complex component Dirac fermions defined in the 4-dimensions. The indices “L” and “R” denote the left-handed and right-handed fermions in 4-dimensions, i.e., $\psi_{L,R} = \frac{1}{2}(1 \mp \gamma_5)\psi$. The index “c” represents the charge conjugation in 4-dimensions, $\psi^c = c\tilde{\psi}^T$ with $c = i\gamma_0\gamma_2 = i\sigma_3 \otimes \sigma_2$.

With the above explicitly given spinor structure of Majorana fermions in $(14+6)$-dimensions, it is not difficult to find out the charge conjugate matrix $\hat{C}$ in $(14+6)$-dimensions

$$\hat{C} = i\sigma_1 \otimes 1 \otimes \sigma_1 \otimes \sigma_2 \otimes \sigma_2 \otimes 1 \otimes 1 \otimes \sigma_3 \otimes \sigma_2,$$  \hspace{1cm} (6)$$

and the corresponding charge conjugate matrix $C$ in 14-dimensions

$$C = i\sigma_1 \otimes \sigma_2 \otimes \sigma_2 \otimes 1 \otimes 1 \otimes \sigma_3 \otimes \sigma_2$$  \hspace{1cm} (7)$$

From the above spinor structure of Majorana fermions in $(14+6)$-dimensions and the charge conjugate matrices, we can explicitly write down twenty gamma matrices $\hat{\Gamma}_A = (\hat{\gamma}_A, \Gamma_1) = (\gamma_a, \tilde{\gamma}_m, \Gamma_1)$ corresponding to $(4+6+10)$-dimensions. Here $\hat{\gamma}_A = (\gamma_a, \tilde{\gamma}_m)$ ($a =$.
0, 1, 2, 3; \( A = 0, 1, \cdots, 9 \) are the gamma matrices in 10-dimensions relating to the spin-family charges and conjugated charges. Their explicit forms are given by

\[
\begin{align*}
\gamma_0 &= 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 , \\
\gamma_1 &= i \ 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes \sigma_2 \otimes \sigma_1 , \\
\gamma_2 &= i \ 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes \sigma_2 \otimes \sigma_2 , \\
\gamma_3 &= i \ 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes \sigma_2 \otimes \sigma_3 , \\
\hat{\gamma}_4 &= i \ \sigma_1 \otimes 1 \otimes \sigma_2 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes \sigma_3 \otimes 1 , \\
\hat{\gamma}_5 &= i \ \sigma_2 \otimes \sigma_3 \otimes \sigma_2 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes \sigma_3 \otimes 1 , \\
\hat{\gamma}_6 &= i \ \sigma_1 \otimes \sigma_2 \otimes \sigma_3 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes \sigma_3 \otimes 1 , \\
\hat{\gamma}_7 &= i \ \sigma_2 \otimes \sigma_2 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes \sigma_3 \otimes 1 , \\
\hat{\gamma}_8 &= i \ \sigma_1 \otimes \sigma_2 \otimes \sigma_1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes \sigma_3 \otimes 1 , \\
\hat{\gamma}_9 &= i \ \sigma_2 \otimes \sigma_1 \otimes \sigma_2 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes \sigma_3 \otimes 1 \quad (8)
\end{align*}
\]

and \( \Gamma_I \) (\( I = 1, 2, \cdots, 10 \)) are the gamma matrices in 10-dimensions relating to the isospin-color charges and conjugated charges. Their explicit forms read

\[
\begin{align*}
\Gamma_1 &= 1 \otimes 1 \otimes 1 \otimes \sigma_1 \otimes \sigma_1 \otimes 1 \otimes 1 \otimes \sigma_2 \otimes \sigma_3 \otimes 1 , \\
\Gamma_2 &= 1 \otimes 1 \otimes 1 \otimes \sigma_1 \otimes \sigma_2 \otimes 1 \otimes \sigma_3 \otimes \sigma_2 \otimes \sigma_3 \otimes 1 , \\
\Gamma_3 &= 1 \otimes 1 \otimes 1 \otimes \sigma_1 \otimes \sigma_1 \otimes 1 \otimes \sigma_2 \otimes \sigma_3 \otimes \sigma_3 \otimes 1 , \\
\Gamma_4 &= 1 \otimes 1 \otimes 1 \otimes \sigma_1 \otimes \sigma_2 \otimes \sigma_2 \otimes 1 \otimes \sigma_3 \otimes \sigma_3 \otimes 1 , \\
\Gamma_5 &= 1 \otimes 1 \otimes 1 \otimes \sigma_1 \otimes \sigma_3 \otimes 1 \otimes \sigma_2 \otimes \sigma_1 \otimes \sigma_3 \otimes 1 , \\
\Gamma_6 &= 1 \otimes 1 \otimes 1 \otimes \sigma_1 \otimes \sigma_2 \otimes \sigma_1 \otimes \sigma_2 \otimes \sigma_3 \otimes 1 , \\
\Gamma_7 &= 1 \otimes 1 \otimes 1 \otimes \sigma_1 \otimes \sigma_3 \otimes \sigma_1 \otimes 1 \otimes 1 \otimes \sigma_3 \otimes 1 , \\
\Gamma_8 &= 1 \otimes 1 \otimes 1 \otimes \sigma_1 \otimes \sigma_3 \otimes \sigma_2 \otimes 1 \otimes 1 \otimes \sigma_3 \otimes 1 , \\
\Gamma_9 &= 1 \otimes 1 \otimes 1 \otimes \sigma_1 \otimes \sigma_3 \otimes \sigma_3 \otimes 1 \otimes 1 \otimes \sigma_3 \otimes 1 , \\
\Gamma_{10} &= 1 \otimes 1 \otimes 1 \otimes \sigma_2 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes \sigma_3 \otimes 1 \quad (9)
\end{align*}
\]

where \( \sigma_i \) (\( i = 1, 2, 3 \)) are Pauli matrices, and “1” is understood as a \( 2 \times 2 \) unit matrix. From
the above gamma matrices, one may define three type of chiral gamma matrices

\[
\hat{\gamma}_{11} = \gamma_0 \cdots \gamma_9 = -\sigma_3 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \\
\Gamma_{15} = -i\gamma_0 \cdots \gamma_3 \Gamma_1 \cdots \Gamma_{10} = 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \\
\Gamma_{11} = \Gamma_1 \cdots \Gamma_{10} = 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1
\]

with \( \hat{\gamma}_{11} \hat{\gamma}_A = -\hat{\gamma}_A \hat{\gamma}_{11} \), \( \Gamma_{15} \Gamma_I = -\Gamma_I \Gamma_{15} \), \( \Gamma_{15} \hat{\gamma}_A = -\hat{\gamma}_A \Gamma_{15} \) and \( \Gamma_{11} \Gamma_I = -\Gamma_I \Gamma_{11} \).

The Majorana-Weyl representation of \( \Psi_i \) in 14-dimensions is then defined through the projection operators \( P_{W,E} = (1 \mp \Gamma_{15})/2 \) with \( P_{W,E}^2 = P_{W,E} \)

\[
\Psi_{Wi} = P_W \Psi_i = \frac{1}{2} (1 - \Gamma_{15}) \Psi_i \equiv \begin{pmatrix} F_{Li} \\ F_{Ri} \end{pmatrix}, \\
\Psi_{Ei} = P_E \Psi_i = \frac{1}{2} (1 + \Gamma_{15}) \Psi_i \equiv \begin{pmatrix} F'_{Ri} \\ F'_{Li} \end{pmatrix}
\]

In order to distinguish with the left- and right-handed fermions defined via the projection operators \( P_{L,R} = (1 \mp \gamma_5)/2 \) in 4-dimensional space-time, the Majorana-Weyl fermions \( \Psi_{Wi} \) and \( \Psi_{Ei} \) are mentioned as ‘westward’ and ‘eastward’ fermions. Similar considerations are applicable to \( \tilde{\Psi}_i \). It is of interest to notice the following fact: each westward Majorana-Weyl fermion \( \Psi_{Wi} \) contains 64 independent degrees of freedom, which exactly represent 64 independent degrees of freedom of quarks and leptons in each family when including the right-handed neutrino. The eastward Majorana-Weyl fermions \( \Psi_{Ei} \) are regarded as mirror particles, i.e., mirroquarks and mirroleptons.

It is observed that under the transformation of parity and charge conjugation, the gamma matrices \( \hat{\gamma}_A \) and \( \Gamma_I \) transform with an opposite sign, i.e.,

\[
\hat{C} \hat{\gamma}_A \hat{C}^\dagger = -\hat{\gamma}_A^T, \quad \gamma_0 \hat{\gamma}_A \gamma_0 = \hat{\gamma}_A^\dagger \\
\hat{C} \Gamma_I \hat{C}^\dagger = \Gamma_I^T, \quad \gamma_0 \Gamma_I \gamma_0 = -\Gamma_I^\dagger
\]

which indicates that the 10-dimensions relating to the quantum spin-family charges must be different from the 10-dimensions relating to the quantum isospin-color charges. Such a difference should directly reflect their geometries. The former turns out to be motional with a nontrivial kinetic term and forms a high dimensional quantum space-time, the latter is found to be motionless and becomes a quantum intrinsic space. This can be seen more
explicitly from the following identities

\[ \tilde{\Psi} \gamma^A i \partial_A \Psi = \frac{1}{2} [\tilde{\Psi} \gamma^A i \partial_A \Psi - i \partial_A (\tilde{\Psi}) \gamma^A \tilde{\Psi}] \]

\[ \tilde{\Psi} \Gamma^I i \partial_I \Psi = \frac{1}{2} \partial_I \left( \tilde{\Psi} i \Gamma^I \Psi \right) \]

with \( A = 0, 1, \ldots, 9 \), and \( I = 1, \cdots, 10 \). In obtaining the above identities, we have used the Majorana condition \( \hat{\Psi} = \hat{\Psi}^c \). As the second identity is given by a total derivative, which means that no kinetic term can exist in 10-dimensions relating to the isospin-color charges. Namely, the corresponding space is motionless and will be mentioned as an 10-dimensional quantum intrinsic space.

As the spin-family-related 10-dimensional quantum space-time is motional, it possesses a generalized maximal Lorentz symmetry \( \text{SO}(1,9) \). In contrast, since the isospin-color-related 10-dimensional quantum intrinsic space is motionless, its 32-component fermions in the spinor representation turn out to get a maximal symmetry \( \text{SO}(32) \) when they are moving in 10-dimensional quantum space-time. Note that in a motional 4-dimensional space-time without considering three families and introducing extra 6-dimensions relating to three-family charges, the maximal gauge symmetry was found to be a unitary symmetry \( \text{SU}(32) \)\cite{47}.

The generators of the symmetry group \( \text{SO}(32) \) are given by the following tensors constructed from \( \Gamma \)-matrices

\[ T^U \equiv (\Gamma_{11}, \Sigma^{IJ}, \Gamma_{11} \Sigma^{JKL}), \quad T^V_5 \equiv (\Sigma^{JK}, i \Gamma_{11} \Sigma^{JK}) \]

\[ T^\hat{U} \equiv (T^U, i \Gamma_{15} T^V_5), \quad (\hat{U} = 1, \cdots, 496) \]

where \( \Sigma^{IJ} = \frac{i}{4} [\Gamma^I, \Gamma^J] \) is a two-rank antisymmetric tensor which forms the generators of subgroup \( \text{SO}(10) \), and others are the high-rank antisymmetric tensors with \( T^U \ (U = 1, \cdots, 256) \) being the generators of subgroup \( \text{U}(16) \). The generators of the generalized Lorentz symmetry \( \text{SO}(1,9) \) in 10-dimensional quantum space-time are given by a two-rank antisymmetric tensor of gamma matrices

\[ \Sigma^{AB} = \frac{1}{4i} [\gamma^A, \gamma^B] \]

Under the charge conjugation and parity transformation, the tensors of gamma matrices transform as follows

\[ \hat{C} \Sigma^{AB} \hat{C}^\dagger = C \Sigma^{AB} C^\dagger = - (\Sigma^{AB})^T, \quad \gamma^0 \Sigma^{AB} \gamma^0 = (\Sigma^{AB})^\dagger, \]

\[ \hat{C} T^\hat{U} \hat{C}^\dagger = C T^U C^\dagger = - (T^U)^T, \quad \gamma^0 T^U \gamma^0 = (T^U)^\dagger \]
We are now in the position to explicitly construct an Lagrangian for the general "bottom-up" MSMUM with a gauge symmetry SO(32) and a generalized Lorentz symmetry SO(1,9) in the motional 10-dimensional space-time. For our purpose in this article, the 10-dimensional space-time is treated as a flat space-time without considering gravity. With the Majorana-Weyl fermions in the 10-dimensional space-time, the Lagrangian is found to be

$$\mathcal{L}^{10D} = \frac{1}{4} \bar{\Psi} \gamma^A i D_A \Psi - \frac{1}{4g_{U}^2} \mathcal{F}_{AB} \mathcal{F}^{AB}$$

(17)

which is self-Hermitian due to Majorana condition. $D_A$ is the covariant derivative corresponding to the gauge field $A^\Upsilon_A$ ($A = 0, \cdots, 9$) in 10-dimensions

$$D_A = \partial_A - ig_{U} A^\Upsilon_A T^\Upsilon$$

(18)

and its commutation relation defines the field strength $\mathcal{F}^\Upsilon_{AB}$

$$i[D_A, D_B] = \mathcal{F}^\Upsilon_{AB} T^\Upsilon$$

(19)

with $T^\Upsilon$ ($\Upsilon = 1, \cdots, 496$) being the generators of SO(32) and $g_{U}$ the coupling constant.

To maintain four-dimensional Poincare invariance and fit the so-called no-go theorem [48], we shall take the 10-dimensional space-time to be of the form $M^4 \times K$, where $M^4$ is 4-dimensional Minkowski space and $K$ is a compact 6-dimensional internal space. The symmetry in the product space $M^4 \times K$ is corresponding to $SO(1,3) \times SO(6)$ which is a subgroup of SO(1,9).

In the 10-dimensional quantum space-time, one can define the Majorana-Weyl fermions via the projection operators $P_\pm = (1 \pm \gamma_{11})/2$ with $P_\pm^2 = P_\pm$

$$\Psi_\pm = P_\pm \bar{\Psi} = \frac{1}{2}(1 \pm \gamma_{11}) \bar{\Psi} = \begin{pmatrix} \Psi_{L,R} + i \tilde{\Psi}_{L,R} \\ \Psi_{R,L} - i \tilde{\Psi}_{R,L} \end{pmatrix}$$

(20)

where

$$\Psi_L = \begin{pmatrix} F_L \\ F'_L \end{pmatrix}, \quad \Psi_R = \begin{pmatrix} F'_R \\ F_R \end{pmatrix}$$

(21)

Similar forms hold for $\tilde{\Psi}_{L,R}$ with replacing $F_{L,R}$ and $F'_{L,R}$ by $\tilde{F}_{L,R}$ and $\tilde{F}'_{L,R}$ respectively. Where $F^T_{L,R} = (F_1, F_2, F_3, F_0)_{L,R}$ and $\tilde{F}^T_{L,R} = (\tilde{F}_1, \tilde{F}_2, \tilde{F}_3, \tilde{F}_0)_{L,R}$ are two kinds of fermionic building blocks, the corresponding $F'_{L,R}$ and $\tilde{F}'_{L,R}$ are their mirror fermionic building blocks.
Here $F_{iL,R}$ ($F'_{iL,R}$) and $\tilde{F}_{iL,R}$ ($\tilde{F}'_{iL,R}$) ($i=1,2,3,0$) belong to the spinor representation in (4+8)-
dimensions.

It is noticed that the 10-dimensional Weyl fermions $\hat{\Psi}^-$ and $\hat{\Psi}^+$ can be related by a kind of F-parity operation $\hat{F}$ defined as follows

$$\hat{\Psi}_- = \hat{\Psi}_+^F = \hat{\Gamma}_0 \hat{\Psi}_+(\hat{\Psi} \to -\hat{\Psi}) \tag{22}$$

where $\hat{\Gamma}_0 = \sigma_1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1$

which implies that one may take one of the 10-dimensional Majorana-Weyl fermions $\hat{\Psi}_+$ as the basic building blocks for constructing the Lagrangian, namely

$$\mathcal{L}^{10D} = \frac{1}{2} \bar{\hat{\Psi}}^+ \dot{A}^A \hat{D}_A \hat{\Psi}^+ - \frac{1}{4g_U^2} \tilde{F}^{\hat{U}}_{AB} \tilde{F}^{\hat{U}AB} \tag{23}$$

**IV. MINIMAL EXTRA 6-DIMENSIONS AND QUANTUM FAMILY-CHARGES**

In the above considerations, we have introduced extra 6-dimensions by relating to three family charges and conjugated charges based on the QCDC-principle. Here we shall directly demonstrate that the extra 6-dimensions are actually the minimal ones in addition to the 14-dimensions relating to the spin-isospin-color charges. The reason can be found from the fact that since the fermionic basic building blocks in the 128-dimensional spinor representations of 14-dimensions are Majorana fermions, for extra dimensions to be nontrivial with kinetic term, it requires that the corresponding gamma matrices in the spinor representation of extra dimensions must be antisymmetric. The question then becomes that starting from which dimension the possible antisymmetric gamma matrices that satisfy the anticommutation relations can be equal to the corresponding dimensions. It is not difficult to check that in two dimensions there is only one antisymmetric gamma matrix (i.e., $\sigma_2$), and in four dimensions there are at most three antisymmetric gamma matrices that satisfy anticommutation relations (i.e., $\sigma_2 \times \sigma_1$, $\sigma_2 \times \sigma_3$, $1 \times \sigma_2$). Namely, in two and four dimensions the number of antisymmetric gamma matrices is less than the number of dimensions. Only up to six dimensions, there exist six antisymmetric gamma matrices which satisfy the anticommutation relations. The explicit forms of six antisymmetric gamma matrices corresponding
to six dimensions can be expressed as follows

$$\hat{\gamma}_1 = i \sigma_1 \otimes 1 \otimes \sigma_2 ,$$
$$\hat{\gamma}_2 = i \sigma_3 \otimes \sigma_3 \otimes \sigma_2 ,$$
$$\hat{\gamma}_3 = i \sigma_1 \otimes \sigma_2 \otimes \sigma_3 ,$$
$$\hat{\gamma}_4 = i \sigma_3 \otimes \sigma_2 \otimes 1 ,$$
$$\hat{\gamma}_5 = i \sigma_1 \otimes \sigma_2 \otimes \sigma_1 ,$$
$$\hat{\gamma}_6 = i \sigma_3 \otimes \sigma_1 \otimes \sigma_2 ,$$

(24)

which shows that the minimal extra dimensions are truly six dimensions. As the Lorentz symmetry group SO(6) of extra 6-dimensions is isomorphic to SU(4), the corresponding gamma matrices can be chosen in a complex spinor representation of 6-dimensions. Thus the gamma matrices corresponding to spin-family-related 10-dimensions can also be given in a complex spinor representation, which has actually been constructed explicitly in the previous section.

With the above explicit demonstrations, we are led to the conclusion that three family-charges are in fact the minimal basic quantum charges in addition to the spin-isospin-color charges. Consequently, the spin-family-related 10-dimensional quantum space-time becomes a minimal quantum space-time in a general "bottom-up" MSMUM.

V. CONCLUSIONS AND REMARKS

Based on the maximally symmetric minimal unification hypothesis and the quantum charge-dimension correspondence principle, we have built, by analyzing the basic quantum charges of quarks and leptons in the standard model, a maximally symmetric minimal unification model SO(32) containing three families in ten dimensional quantum space-time. It is of interest to notice that both resulting symmetry and dimensions coincide with the ones of type I and Heterotic string SO(32) in string theory. In addition, the present motional 10-dimensional quantum space-time gets a physical meaning as it directly relates to the basic quantum spin-family charges of quarks and leptons. Also the symmetry group SO(32) characterizes a maximal gauge symmetry among the independent degrees of freedom for each family of fermionic building blocks which belong to the Majorana spinor representation of 10-dimensional quantum intrinsic space. The 10-dimensional quantum intrinsic space has
turned out to be motionless and directly relate to the basic quantum isospin-color charges of quarks and leptons. The quantum spin-family charges and quantum isospin-color charges appear to be a kind of dual quantum charge. In the real world, the spin-color charges are conserved by symmetries, while the isospin-family charges are no longer conserved as the corresponding symmetries have to be broken down.

It has been seen that the most crucial point in building a general "bottom-up" MSMUM is to find out a suitable spinor structure into which quarks and leptons as building blocks of standard model are directly embedded. It is also interesting to observe that all interactions in the MSMUM SO(32) are self-Hermitian due to Majorana condition, thus CP symmetry is preserved in the model. In general, parity is going to be broken down when quarks and leptons get different masses with mirroquarks and mirroleptons, and CP symmetry has to be broken down spontaneously.

Similar to GUT models, the key issue becomes how to obtain the standard model with three families of quarks and leptons from MSMUM SO(32) in ten dimensions, which will depend on the compactification of extra dimensions and the symmetry breaking pattern of SO(32). As the families are manifestly related to the extra 6-dimensions in the 10-dimensional MSMUM SO(32) with a maximal family symmetry SO(6) which is isomorphic to SU(4), thus the observing three families of quarks and leptons must be resulted from SU(4) symmetry broken down to SU(3) family symmetry, which can be very similar to the Parti-Salam SU(4) symmetry broken down to SU(3) color symmetry of strong interaction. More specifically, three families of quarks and leptons characterized by the spinor fermions \( F_{iL} \) (i=1,2,3) within the spinor structure defined in eqs.(2-5) can simply be realized as zero modes of extra 6-dimensions. In general, it is more interest to realize compactifications on 6-dimensional Calabi-Yau manifolds of SU(3) holonomy.

Before ending, we would like to address that when applying the MSMU-hypothesis to space-time symmetry, it should naturally lead to a supersymmetric MSMUM since the supersymmetry is the maximal symmetry for space-time. It is then intriguing to extend the MSMUM SO(32) in 10-dimensions to a supersymmetric 10-dimensional MSMUM SO(32) and study its possible relation with the type I string or heterotic string SO(32) in string theory.

We have laid in this article only the foundation for obtaining a natural "bottom-up" MSMUM SO(32) in ten dimensions. Many puzzles such as stability of proton, dark matter
of universe, origin of masses and mixing, flavor physics at low energies, all require us to figure out a suitable compactification of extra 6-dimensions and symmetry breaking pattern of the gauge group SO(32). We shall investigate those puzzles and more interesting features of MSMUM SO(32) elsewhere.

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