Cavity-induced coherence phenomena in a Josephson parametric amplifier

Ya-peng Lu, Jia-zheng Pan, Xing-yu Wei, Jun-liang Jiang, Sheng Lu, Zi-shuo Li, Xue-cou Tu, Lin Kang, Chun-hai Cao, Hua-bing Wang, Jian Chen, Wei-wei Xu, Guo-zhu Sun, and Pei-heng Wu
Cavity-induced coherence phenomena in a Josephson parametric amplifier

Cite as: AIP Advances 10, 025135 (2020); doi: 10.1063/1.5128724
Submitted: 21 September 2019 • Accepted: 6 February 2020 • Published Online: 25 February 2020

Ya-peng Lu, Jia-zheng Pan, Xing-yu Wei, Jun-liang Jiang, Sheng Lu, Zi-shuo Li, Xue-cou Tu, Lin Kang, Chun-hai Cao, Hua-bing Wang, Jian Chen, Wei-wei Xu, Guo-zhu Sun, and Pei-heng Wu

AFFILIATIONS
Research Institute of Superconductor Electronics, School of Electronic Science and Engineering, Nanjing University, Nanjing 210093, China and Purple Mountain Laboratories, Nanjing 211111, China

*Author to whom correspondence should be addressed: gzsun@nju.edu.cn

ABSTRACT
By adjusting the frequency of the cavity, we perform a microwave reflection measurement and directly probe the coherence and interference effects in a phase-sensitive Josephson parametric amplifier. We demonstrate the shift in the peak and the dip in the reflection spectra of the amplifier, which operates in the phase-sensitive mode. The behavior of the shift can be precisely controlled by tuning the pump power, the frequency of the cavity, and the relative phase between the incident signal and pump field. Theoretical simulations are in good agreement with the experimental results. These results provide an alternative way of controlling the parametric process by adjusting the frequency of the cavity.

© 2020 Author(s). All article content, except where otherwise noted, is licensed under a Creative Commons Attribution (CC BY) license (http://creativecommons.org/licenses/by/4.0/). https://doi.org/10.1063/1.5128724

Parametric amplification, a result of frequency mixing, is an important process in both classical and quantum domains. Usually, a cavity with a nonlinear component is the key element in such parametric amplifiers. It has been analyzed that coherence and interference effects between the incident and pump fields play an important role in the phase-sensitive amplification process. The effect of the cavity and its coupling to the environment have also been demonstrated. Interesting phenomena like spectrum splitting are clearly demonstrated in various systems such as optical parametric amplification, quantum mechanical resonators, coupled electromechanical oscillators, and coupled optomechanical systems. These results help a lot in the exploration of phase-sensitive switching, information storage, and logic operation. However, the effect of the cavity frequency in a parametric amplifier in the microwave domain has not been investigated in detail yet.

On the other hand, Josephson parametric amplifiers (JPAs) have become crucial components of circuitry in superconducting quantum information technology and well promoted in the quantum information measurements in the microwave domain such as a single-shot readout in dispersive measurement. Also, when operating in a phase sensitive mode and relying on classical or quantum interference, a JPA can amplify one quadrature of the signal while de-amplifying the other one. This means that a squeezed state of the microwave field can be created in a specific phase.

In this paper, we utilize a Josephson phase-sensitive parametric amplifier as a tunable parametric cavity to experimentally and theoretically demonstrate coherent signal squeezing and amplification. In addition to de-amplification and amplification, we also observe a linear phase-sensitive amplitude spectral shift by continuously tuning the cavity frequency. Especially, the phase shift of the squeezing point in the spectra of a phase-sensitive JPA is clearly demonstrated. The symmetric and asymmetric behaviors of the amplitude spectra can be precisely controlled and modulated by tuning the signal phase and the cavity frequency.

Our device consists of a quarter-wavelength coplanar waveguide resonator terminated by a direct current superconducting quantum interference device (dc-SQUID), in which a superconducting loop is interrupted by two Josephson junctions, as shown in Fig. 1(a). The cavity is coupled to the transmission port through an interdigital capacitor, allowing a signal to come in and interact with the intra-cavity field. The frequency of the resonant cavity is tunable with the flux bias, which is applied through an on-chip line inductively coupled to the dc-SQUID. When the dc-SQUID loop is biased...
by an ac pump field with twice the resonant frequency and appropriate amplitude, the phase-sensitive parametric amplification for the incident signal at the cavity resonant frequency occurs.

An optical micrograph of the device is shown in Fig. 1(b). The coplanar waveguide resonator consists of a 70 nm thick niobium film deposited on a high-resistance silicon wafer. With optical lithography and reactive ion etching, a center conductor with a width of 24 μm is separated from the ground planes by a gap with a width of 12 μm. The center conductor is coupled to the transmission line via an interdigital capacitor with a coupling capacitance of 15 fF. The Josephson junctions in the dc-SQUID are Al–AlOx–Al junctions, which are fabricated with double-angle evaporation. The critical current of each Josephson junction is about 2.0 μA. The tuning of JPA cavity frequency is in the range of 3.95–4.26 GHz. The device is mounted on a dilution refrigerator (Triton2016) with the base temperature lower than 20 mK. A schematic of the measurement setup is shown in Fig. 1(c). The incident signal is heavily attenuated to suppress residual thermal noise, and filters are used to remove the noise from the applied tone. A cryogenic circulator is connected to the device to separate the reflected signal from the incident one. To amplify the reflected signal from the device, we use a cryogenic high electron mobility transistor (HEMT) amplifier mounted on the 4 K plate, providing a gain of 35 dB with a noise temperature of 2 K at 4–8 GHz, as well as amplifiers at room temperature. The amplified signal is measured with a vector network analyzer. The detailed description of the measuring circuits in the cryostat is given in Refs. 23 and 24.

We first characterize the sample by measuring the reflection spectra as a function of the external magnetic flux. The power of the probe signal is low enough (typically −120 dBm) so that the JPA operates in the linear regime. As shown in Fig. 2, the resonant frequency of the cavity depends on the dc flux bias. To study the interference phenomenon with a JPA in a phase-sensitive mode, we set the dc flux bias so that the cavity frequency is 4.244 GHz, as shown in the inset of Fig. 2. A 2π rotation of the phase at the resonant frequency implies that the cavity operates in the over-coupled regime. The quality factor of the JPA cavity measured at 4.244 GHz is about 470. Then, signal amplification is characterized at the frequency of the cavity around 4.244 GHz, where \( f_s = 4.244 \text{ GHz} \), \( f_p = 8.488 \text{ GHz} \), \( P_s = -126 \text{ dBm} \), and \( P_p = -54 \text{ dBm} \).

We investigate the phase dependence of the gain with phase-locked microwave generators by varying the phase of the incident signal while keeping the phase of the pump tone unchanged. Here,
FIG. 3. (a) Phase-sensitive amplification as a function of the relative phase $\phi$ between the incident signal and pump field; for comparison, the trace measured when the pump is turned off is also plotted, (b) the peak measured at the point “a” in (a), where the relative phase is nearly 0° (in phase), and (c) the dip measured at the point “b” in (a), where the relative phase is nearly 90° (out of phase).

we measure the output amplitude in the voltage format by using a heterodyne detection scheme with a waveform digitizer. The output amplitude as a function of the phase of the incident signal is plotted in Fig. 3(a). To avoid the phase drifting between the signal and the sub-harmonic field of the pump tone, we use a vector signal generator with a dual-channel output to keep the relative phase as a constant value. The gain is sensitive to the relative phase between the incident signal and the pump field. For comparison, the trace measured when the pump is turned off is also plotted. Note that the value at the point “b” is nearly but NOT zero. In order to better understand the impact of phase interference, we measure the spectra of the JPA by fixing the cavity frequency and pump frequency and scanning the incident signal frequency around the particular operating point. As shown in Figs. 3(b) and 3(c), which correspond to the relative phase nearly 0° and 90° of the incident signal relative to the pump field, respectively, a peak and a dip appear in the spectra with the Lorentzian profile when the pump field is applied. Here, in order to vary the phase effectively, we use another microwave source to control the output of the dual-channel microwave source, which results in a small deviation in the relative phase, and thus, the maximum and minimum values are a little different from those shown in Fig. 3(a).

We then scan the frequency of the resonant cavity while fixing the frequencies of the signal $f_s$ and the pump field $f_p$ with $f_p = 2f_s$. Figure 4 shows the experimental results of phase-sensitive amplification as a function of the incident signal phase and cavity detuning.

FIG. 4. Phase-sensitive amplitude shift as a function of the incident signal and frequency detuning between the cavity and the signal; the left panel is the experimental result; parameters used for the theoretical simulation (right panel) are $Q = 470$ and $\beta = 0.72\beta_0$. Here, $\beta_0$ depicts the threshold for the amplifier to become a parametric oscillator.
frequency, which is swept by varying the dc flux bias. A linear phase shift in de-amplification and amplification is observed. In order to explain the linear shift induced by frequency detuning between the cavity and the incident signal, we review the systematic Hamiltonian approach for degenerate parametric amplification with a classical pump. \[ H_{DPA} = \hbar \omega_c \hat{a}_i + \frac{i}{2} \beta e^{-i\omega/2} \hat{a}_i^\dagger \hat{a}_i - \hbar \cdot c. \] (1)

Here, \( \hat{a}_i \) and \( \hat{a}_i^\dagger \) are the creation and annihilation operators describing the cavity field, respectively, \( \omega_c = \omega_i + \Delta \) is the resonant frequency of the cavity, \( \Delta \) is the detuning between the cavity frequency and the incident signal frequency, and \( \beta \) is the strength of effective parametric pumping. Here, the pump and signal frequencies will be tuned so that \( \omega_p = 2 \omega_i \). We transform the frame to a rotating one with \( \hat{a} \rightarrow e^{i\Delta/2} \hat{a} \). Solving the Heisenberg equation of motion for the cavity field yields the system’s susceptibility matrix, in which the dip deviates from zero detuning and two peaks become higher. Figures 5(c), 5(d), and 5(f) show the traces when the relative phase deviates from \( \varphi = 90^\circ \). Since the relative phase of the intra-cavity field shifts when the cavity frequency changes, reflection spectra become asymmetric, in which the dip deviates from zero detuning and two peaks have different amplitudes. Hence, one can modulate the symmetry and amplitude of the reflection spectra by controlling the phase of the incident field and the pump power. Note that the amplitude of the reflected signal depends not only on the relative phase \( \varphi \) but also on the detuning frequency \( \Delta \). Hence, it is inconvenient to explain the time-independent operations \( \hat{a} \rightarrow \hat{a} e^{-i\theta/2} \) and \( \hat{a}^\dagger \rightarrow \hat{a}^\dagger e^{i\theta/2} \). The reflected amplitude of the signal is given by

\[
|\varphi(\Delta)| = \frac{\kappa^* \beta^* \left[ (-\Delta + \kappa/2)^2 + \Omega^2 - \beta^2 \right]}{\kappa^* \beta \left[ (-\Delta + \kappa/2)^2 + \Omega^2 + \beta^2 \right]}|\varphi(0)|.
\] (3)

The effective relative phase \( \varphi \) between the incident signal and pump field is equal to the signal phase \( \theta_i \), as the pump tone phase is constant. The numerical simulation result is in good agreement with the experimental result (see Fig. 4).

Because amplitude is distinguishable when the phase of the incident field is exactly out of phase with the pump field, we plot the profile curves when \( \varphi = 90^\circ \) with a different pump power. As shown in Figs. 5(b) and 5(e), the reflection spectra are symmetric like mode-splitting, which are the results of the destructive interference between the incident field and the intra-cavity field. If we increase the pump power slightly under the oscillation threshold, the dip becomes deeper, and two peaks become higher. Figures 5(a), 5(c), 5(d), and 5(f) show the traces when the relative phase deviates from \( \varphi = 90^\circ \). Since the relative phase of the intra-cavity field shifts when the cavity frequency changes, reflection spectra become asymmetric, in which the dip deviates from zero detuning and two peaks have different amplitudes. Hence, one can modulate the symmetry and amplitude of the reflection spectra by controlling the phase of the incident field and the pump power. Note that the amplitude of the reflected signal depends not only on the relative phase \( \varphi \) but also on the detuning frequency \( \Delta \). Hence, it is inconvenient to explain the time-independent operations \( \hat{a} \rightarrow \hat{a} e^{-i\theta/2} \) and \( \hat{a}^\dagger \rightarrow \hat{a}^\dagger e^{i\theta/2} \). The reflected amplitude of the signal is given by

\[
|\varphi(\Delta)| = \frac{\kappa^* \beta^* \left[ (-\Delta + \kappa/2)^2 + \Omega^2 - \beta^2 \right]}{\kappa^* \beta \left[ (-\Delta + \kappa/2)^2 + \Omega^2 + \beta^2 \right]}|\varphi(0)|.
\] (3)

The effective relative phase \( \varphi \) between the incident signal and pump field is equal to the signal phase \( \theta_i \), as the pump tone phase is constant. The numerical simulation result is in good agreement with the experimental result (see Fig. 4).

Because amplitude is distinguishable when the phase of the incident field is exactly out of phase with the pump field, we plot the profile curves when \( \varphi = 90^\circ \) with a different pump power. As shown in Figs. 5(b) and 5(e), the reflection spectra are symmetric like mode-splitting, which are the results of the destructive interference between the incident field and the intra-cavity field. If we increase the pump power slightly under the oscillation threshold, the dip becomes deeper, and two peaks become higher. Figures 5(a), 5(c), 5(d), and 5(f) show the traces when the relative phase deviates from \( \varphi = 90^\circ \). Since the relative phase of the intra-cavity field shifts when the cavity frequency changes, reflection spectra become asymmetric, in which the dip deviates from zero detuning and two peaks have different amplitudes. Hence, one can modulate the symmetry and amplitude of the reflection spectra by controlling the phase of the incident field and the pump power. Note that the amplitude of the reflected signal depends not only on the relative phase \( \varphi \) but also on the detuning frequency \( \Delta \). Hence, it is inconvenient to explain the time-independent operations \( \hat{a} \rightarrow \hat{a} e^{-i\theta/2} \) and \( \hat{a}^\dagger \rightarrow \hat{a}^\dagger e^{i\theta/2} \). The reflected amplitude of the signal is given by

\[
|\varphi(\Delta)| = \frac{\kappa^* \beta^* \left[ (-\Delta + \kappa/2)^2 + \Omega^2 - \beta^2 \right]}{\kappa^* \beta \left[ (-\Delta + \kappa/2)^2 + \Omega^2 + \beta^2 \right]}|\varphi(0)|.
\] (3)
adjust the relative phase, one can manipulate the parametric process by controlling the frequency of the cavity. The controllable phase-sensitive behavior can be applied to some phase-sensitive microwave experiments in analogy to those in quantum optical and mechanical parametric systems.

In conclusion, we have experimentally demonstrated coherence phenomena with a phase-sensitive Josephson parametric device. Besides the usual tunable parameters such as the pump power and the relative phase between the incident signal field and pump field, we have observed the coherent squeeze, amplification, and phase-sensitive reflection amplitude spectra of a JPA by tuning the cavity frequency. The experimental results can be understood quantitatively with an analytical model and may be applied to quantum information processing.

This work was partially supported by the NKRDP of China (Grant No. 2016YFA0301801), NSFC (Grant Nos. 11474154, 61521001, and 61571219), PAPD, Dengfeng Project B of Nanjing University, and the Natural Science Foundation of the Jiangsu Higher Education Institutions of China (Grant No. 17KJB140009).

REFERENCES

1. K. Sundqvist, S. Kintan, M. Simoen, P. Krantz, M. Sandberg, C. Wilson, and P. Delsing, Appl. Phys. Lett. 103, 102603 (2013).
2. H. Ma, C. Ye, D. Wei, and J. Zhang, Phys. Rev. Lett. 95, 233601 (2005).
3. M. Zalalutdinov, A. Olkhovets, A. Zehnder, B. Ilc, D. Czaplewski, H. G. Craighead, and J. M. Parpia, Appl. Phys. Lett. 78, 3142 (2001).
4. S. Wu, J. Sheng, X. Zhang, Y. Wu, and H. Wu, AIP Adv. 8, 015209 (2018).
5. D. W. Carr, S. Evoy, L. Sekaric, H. G. Craighead, and J. M. Parpia, Appl. Phys. Lett. 77, 1545 (2000).
6. M. Rossi, N. Kralj, S. Zippilli, A. Borrielli, G. Pandraud, E. Serra, G. Di Giuseppe, and D. Vitali, Phys. Rev. Lett. 120, 073601 (2018).
7. Sheng, M. Xiao, and U. Khadka, Phys. Rev. Lett. 109, 223906 (2012).
8. U. Khadka, J. Sheng, and M. Xiao, Phys. Rev. Lett. 111, 223601 (2013).
9. V. Fiore, Y. Yang, M. C. Kuzyk, R. Barbour, L. Tian, and H. Wang, Phys. Rev. Lett. 107, 133601 (2011).