Surface Quantum Effects in a Fireball Model of Gamma Ray Bursts

R. Lieu\textsuperscript{1}, Y. Takahashi\textsuperscript{1}, T.W.B. Kibble\textsuperscript{2}, J. van Paradijs\textsuperscript{1,3}, and A.G. Emslie\textsuperscript{1}

\textsuperscript{1} Department of Physics, University of Alabama, Huntsville, AL 35899.
\textsuperscript{2} Blackett Laboratory, Imperial College, London SW7 2BZ.
\textsuperscript{3} Astronomical Institute ‘Anton Pannekoek’, University of Amsterdam, 1098 SJ Amsterdam, The Netherlands.

Recent rapid advances in the observations of Gamma Ray Bursts (GRBs) lend much confidence to the cosmological fireball (FB) model: there is now compelling evidence that the radiation is emitted by a relativistic shock where a high speed upstream flow terminates. The question concerning what generates such an outflow is central to our search for the ultimate trigger mechanism. A key requirement, not well explained by current theories, is that the flow must have high entropy-to-baryon ratio. In this Letter we point out that a quantum discharge induced by the radiation of the initial FB may be the explanation. The effect is likely to be relevant, because the FB energy density inferred from GRB data is large enough that the radiation pressure leads to the formation of a surface electric field which is unstable to pair creation. Under suitable conditions, such as those of a supernova core, the discharge can convert a substantial fraction of the FB energy into surface pairs. This pair plasma is not contaminated by FB baryons because it is formed outside the FB. We demonstrate that the pairs can then develop relativistic bulk expansion, reaching a maximum speed that meets the constraints required to form a GRB.

The discovery of GRB afterglows at X-ray, optical and radio wavelengths (Costa et al 1997, van Paradijs et al 1997, Frail et al 1997) quickly led to a final settlement of the issue concerning distance scale: the high redshift associations of GRB-970508 (Metzger et
al 1997) and GRB-971214 (Kulkarni et al 1998) indicate that GRB must be of cosmological origin, and that its observed fluence corresponds to a total energy release \( \epsilon_o \geq 10^{52-54} \) ergs. In this paper we take \( \epsilon_o \sim 4 \times 10^{52} \) ergs as representative. Since such a value suggests that a process at the end point of stellar evolution (e.g., merging of two neutron stars, collapse of a massive star at its evolutionary end point) is responsible for triggering the GRB, it is reasonable to assume that the radius of the trigger is \( r_o \sim 10 \) km. To the lowest order, therefore, one may consider the uniform and simultaneous release of energy \( \epsilon_o \) over a radius \( r_o \), and deduce an initial energy density \( U_o \sim 10^{34} \) ergs cm\(^{-3}\) (hereafter \( U_{34} \) will be expressed in such a unit, i.e., \( U_{34} \)). Any internal spatial gradients will after all be rapidly smoothed out in the course of the expansion.

The classic difficulty in explaining a GRB lies not with the availability of a violent energy release mechanism, but rather with our ability to account for the basic observed properties of the burst. A FB of energy density \( U_{34} = 1 \) is extremely optically thick to Compton-related quantum processes, so that the thermalization timescale is very short. The plasma explosion (i.e. \( \gamma, e^+e^- \), and baryons) is therefore unobservable electromagnetically, and when the plasma expands outwards to become optically thin, its flow turns relativistic \textit{in situ}. The observed non-thermal spectrum and the long duration (1 - 10 s) of a GRB implies that most of the energy we detect is from the bulk motion, not the radiation, of the outflow (Meszaros and Rees 1993, Paczynski 1993). The conversion back to random energy takes place via the extremely rapid particle acceleration at a relativistic shock (Quenby and Lieu 1989) when the flow is finally terminated by the ambient medium, and the energy is then radiated away more leisurely. This notion of ‘delayed emission’ was predicted (Meszaros & Rees 1997, Paczynski and Rhoads 1993, Vietri 1997) and is confirmed by the discoveries of afterglows at longer wavelengths.

The evolution of the FB from creation to observability is, however, easily jeopardized
by the presence of trace baryons in the flow, which drastically slow down the bulk motion. Indeed, canonical FB models (Meszaros and Rees 1993) presume the mass \( M_b \) of baryons in the FB obeys \( \xi = M_b/M_\odot < 10^{-4} \approx \xi_c \) (resulting in an outflow that reaches maximum bulk Lorentz factor of a few \( \times 10^2 \)) although hitherto there does not exist a satisfactory explanation of how the requirement may be met. This Letter addresses a new physical mechanism which may provide vital clues.

It has long been recognized (Schwinger 1951) that when a static electric field exceeds a critical value \( E = m_e^2 c^3/e\hbar = E_c \), corresponding to an electron acceleration of \( a_e \sim 2.4 \times 10^{31} \text{ cm s}^{-2} \), it is unstable with respect to pair production. Under the circumstance, a virtual \( e^+e^- \) pair is accelerated in opposite directions by the electric field to the speed of light within a Compton wavelength. This greatly reduces the probability of annihilation: real pair creation may then take place at the energy expense of the field. The process is analogous to a lightning discharge, except that it happens in vacuum. The rate of pair production per unit volume is given by the formula Schwinger derived:

\[
\frac{d^2N}{dVdt} = \frac{\alpha^2 E^2}{\pi^2 \hbar} \sum_{m=1}^{\infty} m^{-2} \exp \left(-\frac{m\pi E_c}{E}\right) \text{ cm}^{-3} \text{ s}^{-1}
\]  

(1)

where \( \alpha \) is the fine structure constant and \( N \) is the number of pairs created. Using this formula, we find that for \( E \sim E_c \) the ‘vacuum breakdown’ causes the field to dissipate its energy in a timescale of \( \leq 10^{-17} \) seconds, resulting in a gamma ray and pair plasma.

Owing to the strength of \( E_c \), this instability has not been realized in the laboratory. However, in the context of a GRB, a short-term but intense surface electric field may develop as a result of the large radiation pressure of the explosion. Consider a FB of energy density \( U \) which expands into its immediately vicinity at speed \( c \), the latter having a proton number density \( n_i \) before the disturbance. Assuming that the fraction of \( U \) due to radiation (i.e. \( \gamma, e^+e^- \)) is \( \sim 100 \% \), the Eddington force on an electron is \( \sim \sigma_c U \) (\( \sigma_c \) being the Compton cross section). Although in an extreme Compton limit the prevalence of forward
scattering tends to undermine this force, the Eddington value remains correct if the mean free path is short enough that repeated Compton events can ‘re-use’ the scattered radiation, which is likely to be the situation just outside a GRB FB. The radiation will accelerate the electron away from its neighbouring protons until a counteracting electric field develops between them. At this point the separation distance $\Delta r$ reaches a value $l$ given by:

$$\sigma_c U = 4\pi n_i e^2 l$$  \hspace{1cm} (2)

Non-relativistically the electrons continue to surge forward until $\Delta r$ is at the maximum value of $2l$, and subsequently $\Delta r$ oscillates between zero and maximum at the plasma frequency $\omega_p = \sqrt{4\pi n_i e^2 / m_e}$ while the two layers of charges accelerate outwards. The ‘equilibrium separation’ $l$ as depicted in (2) is dependent on $n_i$. Now the baryon density of the stationary medium at the surface of FB is unlikely to be orders of magnitude smaller than that inside the FB. Assuming that the density ratio is $\eta \sim 1 - 10 \%$, and that the FB density within is initially $\sim a few \times 10^{36-38} \text{ cm}^{-3}$ (i.e. $\xi \sim 0.01 - 0.5$ for $r_o = 10 \text{ km}$), we shall express $n_i$ in units of $10^{36} \text{ cm}^{-3}$, i.e. $n_{36}$. Since the radiation blast comprises photons of energy $kT \gg m_e c^2$ where $aT^4 = U$, the high energy limit of the Compton cross section may be used:

$$\sigma_c = 8.8 \times 10^{-27} U_{34}^{-\frac{1}{4}} f(U_{34}) \text{ cm}^2$$  \hspace{1cm} (3)

where

$$f(U_{34}) = 1 + 0.04 \ln U_{34}$$  \hspace{1cm} (4)

$l$ in (2) is then given explicitly by:

$$l \sim 3 \times 10^{-11} n_{36}^{-1} U_{34}^{\frac{3}{4}} f(U_{34}) \text{ cm}$$  \hspace{1cm} (5)

Note that because in the case of an initial GRB FB, $U$ is large enough to induce ‘super-Schwinger’ acceleration $a \gg a_c$ (i.e. $U_{34} \geq 1$), the electron will reach speed $c$
having only traversed a small fraction of \( l \), and relativistic corrections apply to most of the motion. Nonetheless there remains the oscillatory nature of \( \Delta r \), now defined as the charge separation measured with respect to our laboratory system \( \Sigma \), so pertinent results may still be estimated. For instance, at maximum \( \Delta r \) the electrons are at rest with respect to \( \Sigma \), and the radiation and electric forces are once again given by \( \sigma_c U \) and \( 4\pi n_i e^2 \Delta r \) respectively, with the latter having exceeded the former. Thus \( l \) as given by (5) provides a lower limit to the growth of the electric field. The field will oscillate between \( E = 0 \) and \( E \geq \sigma_c U/e \) as the separation oscillates between \( \Delta r = 0 \) and \( \Delta r \geq l \).

If \( \sigma_c U/e > E_c \) the field can, in principle, discharge into pairs once the separation \( \Delta r \) is large enough that \( E \sim E_c \). In reality, however, timescale comparisons indicate that the field can reach peak strength, with discharge happening efficiently at this point only if conditions are favorable (see below). Microscopically the pair production process returns an electron to the proton side of the double-layer and creates a positron on the electron side (see Figure 1). The pairs are free to escape, as they are created outside the FB, and as as we ignore the effects of a magnetic field in this first attempt on the problem. Moreover, the discharge is sustained at the FB surface so long as a super-Schwinger surface field continues to be regenerated by the radiation pressure of a homogeneously expanding FB.

To further investigate the Schwinger mechanism, equation (1) may be expressed in energy units since the temperature \( T \) of the post-discharge \( e^+e^- \) pairs is obtainable by equating the energy density at peak field

\[
\frac{E^2}{8\pi} \sim \frac{1}{8\pi} \left( \frac{\sigma_c U}{e} \right)^2
\]

with \( aT^4 \). The mean energy of each pair, \( 3kT (\gg m_e c^2) \), is then multiplied by (1) to give a peak volume energy loss rate due to discharge:

\[
\frac{d^2\varepsilon}{dVdt} = 4.7 \times 10^{52} U_{34}^{4/3} [f(U_{34})]^{2/3} g(U_{34}) \text{ ergs cm}^{-3} \text{ s}^{-1}
\]
where
\[ g(U_{34}) = \sum_{m=1}^{\infty} m^{-2} \exp\left\{ -7.51 \times 10^{-4} U_{34}^{-\frac{7}{3}} [f(U_{34})]^{-1} m \right\} \]  
(8)
is \sim 1 \text{ for } U_{34} \geq 10^{-4} \text{ and } \ll 1 \text{ otherwise (the latter means } E < E_c, \text{ i.e. the low radiation pressure only induces a sub-Schwinger surface field, so no discharge takes place). A simple division of (6) by (7) yields the timescale of peak discharge, while the time for the field to reach this maximum is } \sim l/c. \text{ The ratio of these two times is given by:}
\[ \frac{\tau_{\text{dissipation}}}{\tau_{\text{formation}}} = 29.0 n_{36} U_{34}^{-\frac{2}{3}} f^{-\frac{3}{2}} g^{-1} \]  
(9)
and is \geq 1 \text{ for the parameter regime of concern. The power of pair production at the FB surface, } \frac{d\xi}{dt}, \text{ is the product of } \frac{d^2 \rho}{dV dt} \text{ and } 4\pi r^2 l \text{ where } r \text{ is the FB radius and } l \text{ is given by (5). This yields:}
\[ \left[ \frac{d\xi}{dt} \right]_{\text{pairs}} = 1.8 \times 10^{55} n_{36}^{-1} r_{10}^{-2} U_{34}^{\frac{24}{5}} [f(U_{34})]^{\frac{7}{2}} g(U_{34}) \text{ ergs s}^{-1} \]  
(10)
where \( r_{10} \) is \( r \) in units of 10 km.

The rate at which electrostatic field energy develops to its peak value at the FB surface is given by the product \( \frac{E^2}{8\pi} \times 4\pi r^2 l \times \xi \), and may be expressed as:
\[ \left[ \frac{d\xi}{dt} \right]_{\text{field}} = 5.1 \times 10^{56} r_{10}^{-2} U_{34}^{\frac{24}{5}} [f(U_{34})]^2 \text{ ergs s}^{-1} \]  
(11)
where use has been made of (3) and (4). The excess of (11) over (10) is by the same ratio as that of the field discharge to formation times given in (9). However, for an efficient conversion of the FB energy into surface pairs, one not only must equalize the two rates (10) and (11) at peak field, but this rate must also be \geq \text{ the outward energy flux of the explosion itself. For the parameters of the initial FB used here, the total energy flux across a given surface due to the expanding FB is higher than the rates in (10) and (11), and is given by:}
\[ \left[ \frac{d\xi}{dt} \right]_{\text{FB}} = 4\pi r^2 cU = 3.8 \times 10^{57} r_{10}^2 U_{34} \text{ ergs s}^{-1} \]  
(12)
In fact, when parity between (11) and (12) is reached, we have the radius independent relation:

\[
\frac{1}{8\pi} \left( \frac{\sigma_c U_{e}}{e} \right)^2 = U
\]

which requires an energy density of the surface electric field (6) equal to that of the FB. For \( U_{34} \sim 1 \) the former falls short of the latter by a factor \( \sim 10 \). We shall, however, discuss two possible scenarios (not exhaustive) under which the rates (10) and (11) may be comparable, and be on par with (12) at some early phase of the explosion. The Schwinger mechanism may then be important.

The first scenario maintains our default values of \( r_o \) and \( U_o \), but takes advantage of the fact that in the foregoing development \( U \) refers to the energy density of the FB apparent to a laboratory observer at rest w.r.t. the ambient medium, the only exception being (3) and (4) where \( U \) is the black body energy density at the mean laboratory frequency of the radiation blast. As the FB expands a little, and its bulk flow develops a Lorentz factor \( \gamma \), volume expansion reduces the co-moving number densities of the FB by \( 1/\gamma^3 \), while adiabatic cooling scales (Meszaros, Laguna and Rees 1993) the co-moving energy density as \( 1/\gamma^4 \) and the energy of a typical photon as \( 1/\gamma \). Upon transformation to the laboratory system, the length contraction effect, relativistic beaming, and the ‘blueshift’ of particle energies by a factor \( \sim \gamma \) imply that the photon frequency, number density, and energy density are increased from their corresponding co-moving values by a factor of \( \gamma \), \( \gamma^5 \) and \( \gamma^6 \) respectively. Thus, at finite \( \gamma \) the effect of the FB on the ambient medium should be calculated using number and energy densities which are higher than their initial values by \( \gamma^2 \) times (e.g. \( U \sim \gamma^2 U_o \)), except \( \sigma_c \), which remains unaffected because the photon blueshift is counteracted by adiabatic losses.

The pair efficiency requirement (13) may then be met at \( \gamma \geq 3[f(U_{34})]^{-1} U_{34} \). For a FB with \( U_o \) of \( U_{34} \sim 1 \) the expansion can easily reach \( \gamma \geq 3 \) (at which point \( \geq 50 \% \) of \( U \) is
due to radiation). The criterion on $\xi$ for this to happen, $\xi \leq 8.8 \times 10^{-3} = \xi_o$, is consistent with an outflow from neutron star merging (Lattimer and Schramm 1976), and is much less stringent than that of canonical FB models, since $\xi/\xi_c \sim 100$. On the requirement of rapid discharge the ratio in (9), now modified by the fact that $\sigma_c$ does not scale with $\gamma$, is $\sim 1$ for $n_{36} \geq 1$. Under these conditions most of the FB energy $\epsilon_o$ will be converted to pairs, and will eventually emerge as a GRB. Since the initial FB baryon density is $\geq 2.5 \times 10^{36}(\xi/\xi_o)r_{10}^{-3} cm^{-3}$, and the laboratory density of the FB at the time of discharge is $\gamma^2$ times higher, the density ratio across the active front is a reasonable $\eta \sim$ a few %.

In our second scenario an efficient discharge occurs at the initial explosion $\gamma = 1$. A larger value for $U_o$ at the trigger, $U_{34} \geq 20$, or $\epsilon_o \sim 9 \times 10^{53}$ ergs for $r_o \sim 10$ km, is assumed. This will be a limit where (11) is slightly less than (12), so that even though (10) can be as large as (11) for $n_{36} \geq 2$, only $\sim 80\%$ of $\epsilon_o$ is available to drive a GRB. Under optimal conditions, therefore, the burst will have total energy $\sim 7 \times 10^{53}$ ergs, which may be relevant to the more energetic events such as the one detected recently (Kulkarni et al 1998) There is no restriction at all on $\xi$; but for $\xi \leq 1$ the FB is radiation dominated and has energy $< M_\odot c^2$. The FB has number density of baryons $\leq 3 \times 10^{38}\xi$ cm$^{-3}$ and $kT \geq 200$ MeV, which renders it fortuitously resembling the inner core of a supernova. The density ratio $\eta \geq 0.7/\xi \%$.

Since the region outside the created pairs is essentially the ambient medium which is likely to be optically thin to the pairs, a conservative estimate of the baryon contamination is obtained by assuming that all the baryons of the original electric double-layer are dragged out after discharge. Then the Entropy-per-baryon of the flow is $> U/(n_i m_p c^2)$, and is $> 1/\eta$ times higher than that of the FB itself. The formula provided by Meszaros, Laguna and Rees (1993) shows that the flow will reach a maximum speed given by $\gamma > 2.4 \times 10^{-2}(\epsilon_o/4 \times 10^{52} ergs)\xi^{-1}\eta^{-1}$. In the first scenario where $\epsilon_o \sim 1$ and both $\eta, \xi$ are
\[ \sim 1 \%, \ \gamma > \text{a few} \times 10^2, \text{well within the range of values capable of delivering a GRB at the terminal relativistic shock. Similarly in the second scenario where } \epsilon_o \sim 20 \text{ and } \eta \sim 1 \%, \text{ we have } \gamma > 100 \text{ if } \xi \sim 0.5. \]

In conclusion, this Letter demonstrates that a fundamental quantum process which takes place at the surface of a powerful celestial explosion has hitherto been ignored. The resulting pair outflow can have the right properties to form a GRB at large radii. If the proposed mechanism is relevant, further research on the details of its operational environment may shed light on the origin of GRBs.

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Figure Caption

Figure 1. Left: radial outflux of radiation induces a surface electric field (sometimes called a Pannekoek-Rosseland field) by accelerating ambient electrons away from protons. Right: if this electric field exceeds $E_c$ its discharge will result in the creation of $e^+e^-$ pairs outside the FB. The pairs can thus expand without encountering more FB matter.
