Shadow Extra Dimensions & Fuzzy Branes
with an Exorcized Spectrum

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Abstract: Particle physics models with extra dimensions of space (EDS’s) and branes shed new light on naturalness of the electroweak and flavor sectors, with a rich TeV-scale phenomenology. They are usually formulated as local effective field theories. This article proposes new model building issues with EDS and branes, arising in the framework of string-inspired infinite-derivative quantum field theories (QFT’s), which are ghost-free and intrinsically weakly nonlocal (WNL) above some UV-scale. It is shown that a 4D field localized on a δ-like 3-brane is still delocalized in the bulk on a small distance from the brane position, with a penetration depth given by the WNL length scale. Fields localized on such distant fuzzy branes are thus allowed to interact directly with suppressed couplings, which is impossible in a local QFT. By using this new possibility, a new realization of split fermions in an EDS is presented, allowing to naturally generate flavor hierarchies. It is also argued that above the WNL scale, it should be much more difficult to probe the Kaluza-Klein (KK) tower in an experiment: a shadow EDS is obtained if the WNL and KK-scales are close to each other, such that the effects of the KK-excitations are screened compared to a local QFT. Moreover, with a warped EDS, the usual warp transmutation of a brane mass term is revisited, where it is shown that the WNL scale is also redshifted and stabilizes the Higgs boson mass at the TeV-scale.

Keywords: Braneworlds; Weak Nonlocality; String Phenomenology; Naturalness; Higgs Physics; Flavor Physics.
1 Introduction

Nowadays, the standard model (SM) of particle physics [1]*, defined as a local renormalizable (commutative) quantum field theory (QFT) on a 4-dimensional (4D) Minkowski spacetime [2, 3], provides a self-consistent theoretical description of the strong, electromagnetic and weak interactions between all known elementary particles, via quantum chromodynamics (QCD) and the electroweak (EW) theory, with a great experimental success [4]*. Nevertheless, the SM should not be the end of the high-energy physics (HEP) story, since we have several good reasons to go beyond the SM (BSM) [5]*. Therefore, the SM should be only an effective field theory (EFT) which is valid up to some unknown UV-cutoff $\Lambda_{SM}$, such as new nonrenormalizable operators become important for energies approaching $\Lambda_{SM}$ from below [6]*.

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1 In the following, references marked with an asterisk “*” are reviews/books/lectures instead of original research articles.
From a bottom-up perspective, HEP models based on local QFT’s with extra dimensions of space (EDS) and p-branes were extensively used to solve or reformulate various BSM issues, with a very rich phenomenology [7]*. From a top-down perspective, these constructions are usually considered as EFT’s that may have ultraviolet (UV) completions in string theory [8, 9]*. Besides, as strings are 1D extended objects, string theory leads naturally to nonlocal effects at the string scale $M_s$ [10]*, such as nonlocal form factors decrease rapidly for distances larger than the string length $\ell_s = 1/M_s$ [11, 12]. This feature is manifest in its string field theory (SFT) formulation [13]*, since it is an infinite-derivative QFT [14, 15].

During these last years, independently of SFT, there has been a revival of interest in the construction of weakly nonlocal (WNL) commutative QFT’s [15–22] and their properties, such as unitarity [15–19, 23–27], causality [16, 17, 23], and their induced positivity bounds on low energy EFT’s [28]. Usually, they can be formulated as an infinite-derivative QFT [15, 17–19, 21]. Weak nonlocality (WNL) in QFT is an old subject, originally motivated by heuristic arguments in quantum mechanics and a better UV-behavior than in local QFT [29, 30]*, which could be very promising to quantize gravity, since a naive quantization of (local) general relativity is not perturbatively renormalizable [31]*. Moreover, several heuristic arguments based on semiclassical gravity suggest that in a fundamental theory including a full quantum theory of gravity, pointlike particles and events are meaningless, and that there should exist a minimal length scale in Nature that one can probe [32]*. Therefore, such fundamental theory should have WNL features [33], as in string theory [10–12]. However, nonlocality in QFT is usually considered as controversial, since it generally involves higher-derivative terms: such theory is expected to have Ostrogradsky ghost(s) [34]* in its spectrum [18, 19], and is thus expected to be pathological [34, 35]. Nevertheless, it is possible to define *exorcized* infinite-derivative QFT’s (EID-QFT’s), with a well-defined Cauchy problem [54–56], which offer a framework to build nonlocal QFT’s of gravity that are free of singularities [57, 58]*. In an EID-QFT, the interactions between fields are not pointlike as in a local QFT but smeared over a length scale $\sim \eta = 1/\Lambda_\eta$, where $\Lambda_\eta$ is the WNL scale, e.g. $\Lambda_\eta = M_s$ in string theory. Therefore, particles are not pointlike anymore but lumplike objects for which the notion of *microcausality* is meaningless and replaced by the more general notion of *macrocausality* [16, 17, 23]. Such fuzzy particles is a feature which is also shared by noncommutative QFT’s [59, 60]*, which constitute another class of EID-QFT’s. In this article, only commutative EID-QFT’s are considered.

Beyond the motivations from quantum gravity, the modern EID-QFT’s implement local gauge symmetries via an infinite tower of covariant derivatives [61], such as one can hope to be able to build WNL UV-completions of the SM. At the moment, there are 2

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*Note that during these last years, some authors proposed several issues to make the presence of ghosts in a higher-derivative QFT harmless [35–51].

*In this context, *exorcized* means free of Ostrogradsky ghost(s).
main families of theories, depending on the WNL form factors:

♠ **String-inspired form factor** (asymptotically exponential) [62]: Such gauge EID-QFT's are claimed to be asymptotically safe above the WNL scale [63–66]. Nevertheless, the procedure to renormalize and keep unitarity that is discussed so far is unusual [67, 68], and should deserve deeper studies. Such features may signal that these theories should be analyzed from a nonperturbative point of view in the WNL regime, e.g. some links between WNL and classicalization [69] have already been suggested [33, 70–72]. Alternatively, these EID-QFT’s may be seen as EFT’s of some SFT-models. Note also that they can be defined as Lee-Wick (LW) extensions of the SM [73]*, in the limit of an infinite numbers of higher-derivatives for each field [74–76].

♠ **Asymptotically Lee-Wick form factors** (asymptotically polynomial) [77]: Since the UV-sensitivity is determined by the asymptotic behavior of the WNL form factor, such gauge EID-QFT’s satisfy the same perturbative renormalization properties as LW theories [78, 79]. They can be made 1-loop superrenormalizable and asymptotically free, or even UV-finite and conformal invariant in any spacetime dimension [77, 103–105].

Nevertheless, a ghost-free Higgs mechanism is a more subtle issue [106, 107]. For that purpose, a recent recipe [21] allows building theories which have the same spectrum and tree level scattering amplitudes than their local limit [108], such as WNL manifests only at loop level, so one can build a ghost-free Higgs mechanism [109].

In the existing literature, effective models of particle physics with EDS’s and branes are usually built in a local QFT framework. In a local higher-dimensional EFT, *thin* (or δ-like) 3-branes are perfectly acceptable objects in the worldvolume of which 4D fields are trapped [110], cf. also Refs [111–113]*. However, locality requires that the brane-localized interaction/kinetic terms appear as pointlike in the transverse EDS’s, which sometimes creates ill-defined situations [114–121], and requires a careful mathematical treatment [122–127]. In an EID-QFT framework, it is clear that point interactions on branes are smeared by the delocalized vertices on δ-like branes. For instance, it has been shown that EID-QFT’s resolve the transverse singular behavior of δ-like sources [128–130] like p-branes [131, 132]. Moreover, higher-dimensional gauge theories are notoriously known to not be perturbatively renormalizable [113]*. If gauge symmetries are really fundamental ingredients in Nature, and not emergent symmetries at low energy, then this situation is odd. The standard point of view is that a UV-completion (like string theory) restores a good UV-behavior, as it does with gravity. Nevertheless, as mentioned previously, asymptotically LW EID-QFT’s can be UV-complete in any spacetime dimension [103, 104]: it opens the

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5 For asymptotically free/safe and local extensions of the SM, cf. Refs [80–102]  
6 A *thin* or δ-like brane is by definition a brane with zero thickness (at least in the regime of validity of the EFT).  
7 For renormalizable and local higher-dimensional gauge theories which are asymptotically safe, cf. Refs. [133–137].
door to find higher-dimensional EID-QFT’s (with gauge symmetries and gravity), which can be candidate to a fundamental theory of Nature, without relying at all on string theory!

The goal of this article is to propose a set of new interesting possibilities relying on EID-QFT’s, in order to solve BSM issues in HEP models with EDS’s and branes. The analysis is restricted to toy models to focus on the new features of WNL at tree level. For that purpose, it is sufficient to limit the study to the example of SFT-like form factors, motivated by a bottom-up approach towards UV-completions in string theory, but the methods developed in this article can be extended to asymptotically LW form factors. Indeed, in the proposed models, WNL may originate either from UV-completions in string theory, such as $\Lambda_\eta \sim M_s$, or from some other UV-complete EID-QFT, or maybe from some other nonlocal UV-theory. It is confirmed that $\delta$-like branes appear as fuzzy, such as their singular behavior is softened, and the nonlocal interactions with brane-localized fields is described for the first time. Moreover, it is discussed that WNL can protect light elementary scalars from radiative corrections above $\Lambda_\eta$, which can be below the nonperturbative UV-cutoff in usual local EFT’s. Furthermore, as the propagators are suppressed above $\Lambda_\eta$ [17] (this feature is known as UV-opaqueness [22]), the cross-sections to produce the Kaluza-Klein (KK) excitations of the higher-dimensional fields is suppressed compared to local models, if the KK-scale is $M_{KK} \gtrsim \Lambda_\eta$. It would then be more challenging to probe directly this kind of shadow EDS’s at colliders.

This article is organized as follows. In Section 2, the notations and conventions used in all sections are specified. In Section 3, a WNL toy model with bulk and brane-localized scalar fields is studied, where the concepts of fuzzy branes and shadow EDS’s are introduced. The goal is to provide for the first time tools for building WNL braneworlds. In Section 4, after reminding the main features of the split fermion scenario with a domain wall [138, 139] (whose goal is to obtain suppressed couplings among fermions strongly localized at different points of an EDS), a new formulation based on multiple fuzzy branes is given. In Section 5, the main ideas to solve the gauge hierarchy problem with WNL and braneworlds are reviewed, and a WNL version of a warped EDS is presented, where it is shown that the WNL scale is redshifted on the brane, in the same way as the EW scale in the original Randall–Sundrum (RS) model [140]. In Section 6, the results are summarized and some perspectives for future works are given.

2 Notations & Conventions

♠ The 4D reduced Planck scale is noted $\Lambda_P = 1/\ell_P \sim 10^{18}$ GeV, the EW scale is $\Lambda_{EW} \sim 100$ GeV, the string scale is $M_s = 1/\ell_s$, the KK-scale is $M_{KK}$, and the WNL scale is $\Lambda_\eta = 1/\eta$.

♠ The EID-QFT’s considered in this article must be defined on spacetime with Euclidean signature $(+ + + + +)$, such as only the momenta of the external states are Wick rotated to Minkowskian signature $(− + + + +)$, in order to preserve unitarity [15, 17, 23–27].
The Gaussian function $\delta^{(d)}(\xi)$ on $\mathbb{R}^d$ with $d \in \mathbb{N}^*$ (noted simply $\delta_d(\xi)$ when $d = 1$) can be expressed as the kernel of an infinite-derivative operator:

\[ \forall \xi \in \mathbb{R}^d, \quad \delta^{(d)}(\xi) = e^{\eta^2 \Delta_d} \delta^{(d)}(\xi), \]

\[ = \left( \frac{1}{4\pi\eta^2} \right)^{d/2} \exp \left( -\frac{\|\xi\|^2}{2\eta^2} \right), \quad (2.1) \]

where $\Delta_d$ is the Laplacian on $\mathbb{R}^d$, $\| \cdot \|$ is the Euclidean norm, and the Dirac generalized function $\delta^d(\xi)$ on $\mathbb{R}^d$ (of mass dimension $d$, and noted simply $\delta_d(\xi)$ when $d = 1$) is normalized as

\[ \int d^d\xi \delta^{(d)}(\xi) = 1. \quad (2.2) \]

In this study, the analysis is limited to Euclidean spacetime with 1 flat/warped EDS, whose metric can be put in the form:

\[ ds^2 = g_{\mu\nu}(y) dx^\mu dx^\nu + dy^2, \]

\[ g_{\mu\nu}(y) = e^{-2A(y)} \delta_{\mu\nu}, \quad (2.3) \]

where $\mu \in [0, 3]$, and $e^{-A(y)}$ is the warp factor. One has $A(y) = 0$ in the flat case, and $A(y)$ is an increasing function in the warped case. In this choice of coordinate system, both the determinant of the 5D metric, and the one of the induced metric on a 3-brane at the coordinate $y$, are given by $g = e^{-8A(y)}$. The 5D Laplacian can thus be split as

\[ \Delta = \Delta_\parallel + \Delta_\perp, \quad (2.4) \]

such as when it acts on a scalar field $\Phi(x, y)$, one has

\[ \Delta_\parallel \Phi = g^{\mu\nu}(y) \partial_\mu \partial_\nu \Phi, \]

\[ = e^{2A(y)} \partial_\mu^2 \Phi, \quad (2.5) \]

and

\[ \Delta_\perp \Phi = \sqrt{\frac{1}{g}} \partial_y \left( \sqrt{g} \partial_y \Phi \right), \]

\[ = e^{4A(y)} \partial_y \left( e^{-4A(y)} \partial_y \Phi \right), \quad (2.6) \]

where $\partial_\mu^2 = \partial_\mu \partial_\mu$ is the Laplacian on $\mathbb{R}^4$.

The orbifold $S^1/\mathbb{Z}_2$ is obtained by modding out the circle $S^1$ of radius $\rho \in \mathbb{R}^*_+$ by the group $\mathbb{Z}_2$, cf. Ref. [112]*. A point on $S^1$ is labeled by the coordinate $y \in (-\pi\rho, \pi\rho]$. There are 2 fixed points localized at $y = 0, \pi\rho$, which have the features of $\delta$-like

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*Usually called the heat kernel.

*For the theory of generalized functions (also usually called distributions), cf. Ref. [141]*.
3-branes. This orbifold is topologically equivalent to the interval $[0, \pi \rho]$. The Dirac
generalized function $\delta(y)$ (mass dimension 1) on the circle $S^1$ is normalized as

$$\oint dy \, \delta(y) = 1.$$ \hfill (2.7)

The 4D Euclidean Dirac matrices are taken in the Weyl representation:

$$\gamma_\mu = \begin{pmatrix} 0 & -i\sigma_\mu^- \\ i\sigma_\mu^+ & 0 \end{pmatrix},$$ \hfill (2.8)

with

$$\sigma_\mu^\pm = (\mp i, \sigma_\mu), \quad \{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu},$$ \hfill (2.9)

where $(\sigma_i)_{i \in [1,3]}$ are the 3 Pauli matrices:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$ \hfill (2.10)

A 4D Euclidean Dirac spinor $\psi$ can be decomposed into its chiral components:

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}.$$ \hfill (2.11)

The operator $\slashed{D}$ is defined as

$$\slashed{D}\psi(x) = \gamma_\mu \partial_\mu \psi(x),$$
$$\slashed{D}\psi_L(x) = \sigma_\mu^+ \partial_\mu \psi_L(x),$$
$$\slashed{D}\psi_R(x) = \sigma_\mu^- \partial_\mu \psi_R(x).$$ \hfill (2.12)

### 3 Ultraviolet Fuzziness in Braneworlds

#### 3.1 Weak Nonlocality with an Extra Dimension & Branes

#### 3.1.1 Toy Model

**Fields & Symmetries** – Consider a 5D Euclidean spacetime $\mathbb{R}^5 = \mathbb{R}^4 \times S^1/\mathbb{Z}_2$. The field and symmetry content of the WNL toy model is given in the following list:

- 1 real 5D scalar field $\Phi(x, y)$ (mass dimension $3/2$) propagates into the whole bulk, and is even under the $\mathbb{Z}_2$ orbifold symmetry.

- 2 real 4D scalar fields $\theta(x)$ and $\omega(x)$ (mass dimension 1) are localized respectively on the branes at $y = 0, \pi \rho$. From these 4D fields, one defines the localized “5D fields” $\Theta(x, y) = \theta(x) \delta(y)$ and $\Omega(x, y) = \omega(x) \delta(y - \pi \rho)$, which describe 4D degrees of freedom sharply localized on the 3-branes.
The model has an exchange symmetry between the brane fields \( \theta(x) \leftrightarrow \omega(x) \).

As the EDS is flat, the KK-scale is defined as \( M_{KK} = 1/\rho \).

There is a UV-scale \( \Lambda_{UV} \), which is identified with the usual UV-cutoff of a braneworld EFT in the local limit \( \eta \to 0 \).

Weak Nonlocality – In this class of EID-QFT’s inspired by SFT [15], it is always possible to transfer WNL effects from the kinetic to the interaction terms by field redefinition [17] (the converse in generally not true). In the simple models of Refs. [16, 17], WNL is introduced in the interaction terms via local products of delocalized/smeared fields, which are defined from the original local fields appearing in the kinetic and mass terms. By using the same procedure in the toy model of this section, the following 5D smeared fields \( \tilde{S} = \tilde{\Phi}, \tilde{\Theta}, \tilde{\Omega} \) are defined by acting a SFT-like smearing operator on the local 5D fields \( S = \Phi, \Theta, \Omega \), such as

\[
\tilde{S}(x,y) = e^{\eta^2 \Delta} S(x,y).
\]

In the case of the quasilocalized 5D fields, one can also split them as \( \tilde{\Theta}(x,y) = \tilde{\theta}(x) \delta_\eta(y) \) and \( \tilde{\Omega}(x,y) = \tilde{\omega}(x) \delta_\eta(y - \pi \rho) \), with

\[
\tilde{s}(x) = e^{\eta^2 \Delta_\parallel} s(x),
\]

\[
\delta_\eta(y) = e^{\eta^2 \Delta_\perp} \delta(y),
\]

where \( s = \theta/\omega \). These quasilocalized “5D fields” describe 4D degrees of freedom localized on the 3-branes, with a penetration depth \( \eta \) in the bulk, such as they have no KK-excitations. Note that, in general, each field species (labeled by \( i \)) has its own WNL length scale \( \eta_i \), such as \( \forall i \neq j, \eta_i \neq \eta_j \). Moreover, since the 5D spacetime symmetries are locally broken to the 4D ones at the fixed point positions, a given smeared field involved in an interaction term on a brane should have a different WNL scale in the directions parallel \( \eta_\parallel \) or transverse \( \eta_\perp \) to the brane, such as \( \eta_\parallel \neq \eta_\perp \). For simplicity, only one universal WNL length scale \( \eta \) is introduced in this model.

Action – The action of the 5D toy model is

\[
S_{5D} = \int d^4x \int dy (L_B + L_b + L_{Bb} + L_{bb}),
\]

where:

\( L_B \) is the bulk Lagrangian of the 5D field \( \Phi(x,y) \), with a cubic self-interaction term:

\[
L_B = \frac{1}{2} \left[ -\Phi \Delta_\parallel \Phi + (\partial_y \Phi)^2 \right] + \frac{\lambda_B}{3!} \Phi^3,
\]

where \( \lambda_B \) is a real coupling (mass dimension 1/2) that scales as \( \sqrt{\Lambda_{UV}} \);
\( \mathcal{L}_b \) is the free brane Lagrangian of the 4D fields \( \theta/\omega(x) \):
\[
\mathcal{L}_b = \delta(y) \left( -\frac{1}{2} \theta \Delta_\parallel \theta \right) + \delta(y - \pi \rho) \left( -\frac{1}{2} \omega \Delta_\parallel \omega \right); \tag{3.5}
\]

\( \mathcal{L}_{Bb} \) is the bulk-brane Lagrangian of the interaction terms between the bulk field \( \Phi(x, y) \) and the brane fields \( \Theta/\Omega(x, y) \):
\[
\mathcal{L}_{Bb} = \frac{\lambda_{Bb}}{2} \Phi \left( \tilde{\Theta}^2 + \tilde{\Omega}^2 \right), \tag{3.6}
\]
where \( \lambda_{Bb} \) is a real coupling (mass dimension \(-1/2\)) that scales as \( 10 \eta \sqrt{\Lambda_{UV}} \);

\( \mathcal{L}_{bb} \) is the brane-brane Lagrangian of the interaction terms between the brane fields \( \Theta(x, y) \) and \( \Omega(x, y) \):
\[
\mathcal{L}_{bb} = \frac{\lambda_b}{3!} \left( \tilde{\Theta}^3 + \tilde{\Omega}^3 \right) + \frac{\lambda_{bb}}{2} \left( \tilde{\Theta} \tilde{\Omega}^2 + \tilde{\Omega} \tilde{\Theta}^2 \right), \tag{3.7}
\]
where \( \lambda_b \) and \( \lambda_{bb} \) are real couplings (mass dimension \(-1\)) that scales as \( \eta^2 \Lambda_{UV} \).

**String-Inspired Heat Kernel** – The above Lagrangians appear naively local in terms of the local and smeared fields. To understand why the infinite-derivative feature of the smearing operators introduce WNL, remember that the Gaussian function \( \delta^{(d)}(\eta) \) is the kernel of the SFT-like smearing operator on \( \mathbb{R}^d \), cf. Eq. (2.1). Consider the case where \( \eta \ll \rho \), such as one can neglect the compactification effects on the shape of the kernels, and one can use the approximation \( E^5 \simeq \mathbb{R}^5 \) as far as only WNL features are concerned. As a consequence, the smeared fields can be expressed as the convolution product:
\[
\tilde{S}(x, y) = \left( \delta^{(5)}(\eta) * S \right)(x, y), \tag{3.8}
\]
involved in the interaction terms, where WNL is now manifest. In the local limit \( \eta \to 0 \), one gets the local fields \( \tilde{S}(x, y) = S(x, y) \) as required, since in the theory of generalized functions [141]*, one has the weak limit:
\[
\lim_{\eta \to 0} \delta^{(5)}(x, y) = \delta^{(5)}(x, y). \tag{3.9}
\]

### 3.1.2 Kaluza-Klein Dimensional Reduction

**Local Fields** – If the couplings are weak, one can perform a perturbative analysis. From the bulk Lagrangian of Eq (3.4), the free Euler-Lagrange equation in the bulk is
\[
\Delta \Phi(x, y) = 0, \tag{3.10}
\]

\(^{10}\)The scaling of the couplings of the brane-localized operators in the WNL model are chosen to match the ones of the corresponding local model when \( \eta \to 0 \), cf. Section 3.2.1.
which is *local* in spacetime. As usual, one can perform a KK-decomposition of the 5D field $\Phi(x, y)$:

$$\forall (x, y) \in \mathbb{R}^5, \Phi(x, y) = \sum_n \phi_n(x) f_n(y).$$

(3.11)

The 4D fields $\phi_n(x)$ describe an infinite tower of KK-modes obeying to 4D Klein-Gordon equations:

$$\left( \partial^2_{\mu} - m^2_n \right) \phi_n(x) = 0,$$

(3.12)

such as each of them has a KK-wave function $f_n(y)$ describing the localization of the KK-mode along the EDS. The $f_n(y)$’s are solutions of the wave equations:

$$\left( \partial_y^2 + m^2_n \right) f_n(y) = 0,$$

(3.13)

with the Neumann boundary conditions:

$$\partial_y f_n(0, \pi \rho) = 0,$$

(3.14)

and the orthonormalization conditions:

$$\forall (n, m) \in \mathbb{N}^2, \int dy \ f_n(y) f_m(y) = \delta_{nm}.$$  

(3.15)

There is a flat 0-mode solution:

$$f_0(y) = \sqrt{\frac{1}{2\pi \rho}},$$

(3.16)

and an infinite tower of excited modes:

$$\forall n \in \mathbb{N}^*, f_n(y) = \sqrt{\frac{1}{\pi \rho}} \cos \left( \frac{ny}{\rho} \right),$$

(3.17)

with the mass spectrum:

$$\forall n \in \mathbb{N}, \ m_n = \frac{n}{\rho},$$

(3.18)

from which one can see why the KK-scale has been defined as $M_{KK} = 1/\rho$.

**Smeared Fields** – One can also define a KK-decomposition of the smeared 5D field $\tilde{\Phi}(x, y)$:

$$\forall (x, y) \in \mathbb{R}^5, \tilde{\Phi}(x, y) = \sum_{n=0}^{\infty} \tilde{\phi}_n(x) \tilde{f}_n(y),$$

(3.19)

with the smeared KK-fields and wave functions:

$$\tilde{\phi}_n(x) = e^{\eta^2} \partial^2 \phi_n(x),$$

$$\tilde{f}_n(y) = (\delta_n \ast f_n)(y),$$

(3.20)
such as
\[ \tilde{f}_0(y) = f_0(y), \]
\[ \forall n \in \mathbb{N}^*, \tilde{f}_n(y) = \exp \left[ -\left( \frac{n\eta}{\rho} \right)^2 \right] f_n(y), \] (3.21)
by using Eqs. (2.1), (3.16) and (3.17). The mass spectrum of the KK-modes \( \tilde{\phi}_n(x) \) is ghost-free and still given by Eq. (3.18). Therefore, with \( \rho \gg \eta \), only the KK-modes with \( n \gg 1 \) (with \( m_n \gg \Lambda_\eta \)) have their smeared wave functions with a coefficient which is significantly suppressed with respect to the local case.

### 3.2 Aspects of Weakly Nonlocal Braneworlds

#### 3.2.1 Fuzzy versus \( \delta \)-like Brane

**Fuzzy Brane** — Perhaps, the most interesting aspects of WNL in braneworlds, compared to the 4D EID-QFT’s discussed in the literature, are the new features of fields localized on \( \delta \)-like branes. Indeed, as the brane-localized fields have their interactions a bit delocalized in the bulk with a penetration depth \( \eta \), a \( \delta \)-like brane does not appear anymore as a singular object in the transverse dimensions, but have a small width \( \eta \): the terminology of *fuzzy* brane is introduced. Therefore, WNL regularizes the transverse behavior of the branes, and avoids the typical delicate problems of EFT’s with \( \delta \)-like branes. It is instructive to consider the local limit of the interaction terms of the toy model of Section 3.1.

**Renormalized Brane Couplings** — It is useful to remind that
\[ \forall N \in \mathbb{N}^*, \delta^N_\eta(y) \propto \eta^{1-N} \delta_\eta \left( \sqrt{N}y \right). \] (3.22)
If a brane-localized operator has a coupling \( \alpha(\eta) \propto \eta^{N-1} \), one can define a renormalized coupling \( \alpha_R \in \mathbb{R} \) such as
\[ \lim_{\eta \to 0} \alpha(\eta) \delta^N_\eta(y) = \alpha_R \delta(y). \] (3.23)
In the following, one can thus formally perform the following replacement:
\[ \forall N \in \mathbb{N}^*, \delta^N_\eta(y) \overset{\eta \to 0}{\to} \delta(y), \] (3.24)
with a renormalization of the coupling of the corresponding brane-localized operator. One can check that the couplings of the toy models considered in this article scale with powers of \( \eta \) which are consistent with this discussion on renormalized couplings in the local limit \( \eta \to 0 \). In the special case of operators involving fields localized on different fuzzy branes (which have a trivial local limit), the scaling can be checked by considering an analogous operator involving these fields localized on the same fuzzy brane, and then by taking the local limit.
Bulk-Brane Interactions – From Eq. (3.6), one can write the interaction terms between bulk and brane fields as

\[ L_{Bb} = \frac{\lambda_{Bb}}{2} \Phi \left[ \delta^2_\eta(y) \tilde{\theta}^2 + \delta^2_\eta(y - \pi \rho) \tilde{\omega}^2 \right]. \]  

(3.25)

By integrating over the EDS, one gets the effective 4D Lagrangian of the KK-modes:

\[ \oint dy \, L_{Bb} = \sum_{n=0}^{\infty} \frac{\lambda_{Bb}^{(n)}}{2} \tilde{\phi}_n \left( \tilde{\theta}^2 + \tilde{\omega}^2 \right), \]  

(3.26)

with the effective 4D couplings (mass dimension 1):

\[ \forall n \in \mathbb{N}, \lambda_{Bb}^{(n)} = \lambda_{Bb} \oint dy \, \tilde{f}_n(y) \delta^2_\eta(y). \]  

(3.27)

From Eqs. (2.1) and (3.21), one gets (with a natural coupling \( \Lambda_{Bb} \sim \eta \sqrt{\Lambda_{UV}} \)):

\[ \lambda_{Bb}^{(0)} \sim \frac{1}{4\pi} \sqrt{\frac{\Lambda_{UV}}{\rho}}, \]

\[ \forall n \in \mathbb{N}^*, \lambda_{Bb}^{(n)} \sim \frac{1}{2\pi} \sqrt{\frac{\Lambda_{UV}}{2\rho}} \exp \left[ -\frac{3}{2} \left( \frac{n \eta}{\rho} \right)^2 \right]. \]  

(3.28)

Only the couplings \( \lambda_{Bb}^{(n)} \) to KK-modes with \( m_n \gg \Lambda_\eta \) are significantly suppressed via WNL. If one takes the local limit \( \eta \to 0 \) in Eq. (3.25) and uses the formal replacement in Eq. (3.24), one gets

\[ L_{Bb} \propto \Phi \left[ \delta(y) \theta^2 + \delta(y - \pi \rho) \omega^2 \right], \]  

(3.29)

which is exactly the form of an interaction term in a local braneworld EFT with a \( \delta \)-like brane.

Brane-Brane Interactions – From Eq. (3.7), one can write the interaction terms between brane fields as

\[ L_{bb} = \frac{\lambda_b}{3!} \left[ \delta^3_\eta(y) \tilde{\theta}^3 + \delta^3_\eta(y - \pi \rho) \tilde{\omega}^3 \right] + \frac{\lambda_{bb}}{2} \left[ \delta_\eta(y) \left( \delta^2_\eta(y - \pi \rho) \tilde{\omega}^2 + \delta_\eta(y - \pi \rho) \delta^2_\eta(y) \tilde{\theta} \tilde{\omega} \right) \right]. \]  

(3.30)

Same Brane – One can take \( \lambda_b \neq 0 \) and \( \lambda_{bb} = 0 \) to discuss the first term, which can be rewritten as

\[ \oint dy \, L_{bb} = \oint dy \, L'_{bb} \]  

(3.31)

where

\[ L'_{bb} = \frac{\lambda'_b}{3!} \left[ \delta(y) \tilde{\theta}^3 + \delta(y - \pi \rho) \tilde{\omega}^3 \right], \]  

(3.32)
with the effective 4D coupling (mass dimension 1):
\[
\lambda'_b = \lambda_b \int dy \, \delta(y) \, \delta(y - \pi r) ;
\]
\[
\sim \sqrt{\frac{T}{3}} \frac{\Lambda_{UV}}{4\pi} , \tag{3.33}
\]
by using Eq. (2.1), and with a natural coupling \( \lambda_b \sim \eta^2 \Lambda_{UV} \). Therefore, for an interaction term between 4D degrees of freedom localized on the same fuzzy brane, the effect of WNL along the EDS is just to rescale\(^{11}\) the coupling constant of the corresponding brane operator with respect to the same interaction in a local model with a \( \delta \)-like brane. Indeed, if one takes the local limit \( \eta \to 0 \) in Eq. (3.30) with the formal replacement in Eq. (3.24), one obtains
\[
\mathcal{L}_{bb} \propto \eta \to 0 \left[ \delta(y) \, \delta(y - \pi \rho) \right] , \tag{3.34}
\]
whose localized fields have the same singular transverse features as in the Lagrangian of Eq. (3.32).

**Different Branes** – One considers \( \lambda_b = 0 \) and \( \lambda_{bb} \neq 0 \) to discuss the second term in Eq. (3.30), such as
\[
\lambda'_{bb} = \lambda_{bb} \int dy \, \delta(y) \, \delta(y - \pi \rho) ,
\]
with the effective 4D coupling (mass dimension 1):
\[
\sim \sqrt{\frac{T}{3}} \frac{\Lambda_{UV}}{4\pi} \exp \left[ -\frac{1}{6} \left( \frac{\pi \rho}{\eta} \right)^2 \right] \ll 1 , \tag{3.36}
\]
by using Eq. (2.1), and with a natural coupling \( \lambda_{bb} \sim \eta^2 \Lambda_{UV} \). Therefore, when \( \rho \gg \eta \), it is possible to get naturally suppressed couplings between the fields localized on the different fuzzy branes: the transverse smearing kernels \( \delta_{\eta}(y) \) and \( \delta_{\eta}(y - \pi \rho) \) have a tiny overlap. Note that in the local limit \( \eta \to 0 \), one has \( \lambda'_{bb} \to 0 \), and one recovers that fields localized on different \( \delta \)-like branes do not couple directly in a local EFT: one needs a bulk mediator field for that purpose. Another way to obtain this result is by taking the local limit \( \eta \to 0 \) directly in Eq. (3.30) with the formal replacement in Eq. (3.24), such as
\[
\mathcal{L}_{bb} \propto \eta \to 0 \delta(y) \, \delta(y - \pi \rho) \left( \theta \omega^2 + \omega \theta^2 \right) = 0 . \tag{3.37}
\]
since
\[
\delta(y) \, \delta(y - \pi \rho) = 0 , \tag{3.38}
\]
because these 2 Dirac generalized functions have disjointed pointlike supports \( \rho > 0 \).

\(^{11}\)However, WNL cannot be used to generate a suppressed coupling with respect to its natural value for interacting fields localized on the same fuzzy brane, since the rescaling factor in Eq. (3.33) is not very small.
3.2.2 Shadow Extra Dimensions

In order to have a taste of the phenomenology of the KK-exitations with WNL, it is useful to consider the smeared 4D fields \( \tilde{\phi}_n(x) \) for the KK-modes, which appear in the WNL interaction terms of the EFT after dimensional reduction to 4D. In this class of EID-QFT, it is possible to rewrite the Lagrangians and Euler-Lagrange equations only in terms of the smeared fields [17], such as WNL then appears only in the kinetic terms of the smeared KK-fields:

\[
-\frac{1}{2} \tilde{\phi}_n e^{-2\eta^2(\partial_\mu^2 - m_n^2)} (\partial_\mu - m_n^2) \tilde{\phi}_n ,
\]  

(3.39)

such as one can easily extract their propagators:

\[
\Pi_n(p^2) = -\frac{i e^{-2\eta^2(p^2 + m_n^2)}}{p^2 + m_n^2} ,
\]  

(3.40)

which are exponentially suppressed in the UV, i.e. when \( p^2 > \Lambda_\eta^2 \): this is what one calls the UV-opaqueness of the EID-QFT’s. Moreover, in Section 3.2.1, it was shown that these KK-modes have also suppressed couplings with brane fields. Therefore, any contribution to a process from a KK-particle whose mass is \( m_n > \Lambda_\eta \) will be suppressed compared to a local model\(^{12}\).

Nevertheless, once the S-matrix elements are computed, there is the issue of the Wick-inverse rotation of the external momenta to real energies. Then, the SFT-like smearing operator of the toy model considered here is known to have a strong coupling problem for energies above \( \Lambda_\eta \): S-matrix elements blow up at high center of mass energy in the WNL regime [25, 27]. Nevertheless, it was proposed in Ref. [68] that in a gravity-inspired EID-QFT, the amplitudes can still be softened for high real energies by an unusual dressing of the propagators and vertices. It was also mentioned in Refs. [15, 25] that in a realistic SFT-derived model (well beyond this toy model), there is compensation between different vertices such as this problem does not appear. Moreover, the qualitative results of this article should not be affected if one takes another WNL form factor of the literature (beyond SFT), which gives UV-suppression in both real and imaginary time directions [25, 27].

As a consequence, in a realistic model, the effects of such KK-excitations will be much more difficult to probe in an experiment. In full analogy with the discussion in Ref. [62], the experimenters will see that the cross-sections predicted in the SM are suppressed for energies \( E > \Lambda_\eta \), concluding to a WNL UV-completion of the SM. However, they will have more difficulties to observe the KK-tower if \( \eta \sim \rho \), and then to conclude to the existence of the EDS. Note that in this particular case, WNL smears the brane all along the EDS, such as the difference between bulk and brane fields is meaningless: all poles of the KK-excitations are in the UV-opaque regime, and one should better understand this.

\(^{12}\)Note that the status of such trans-nonlocal states is not clear, and it is argued in Ref. [17] that no external states can be associated to them.
UV-regime of EID-QFT's to be able to really discuss the phenomenology of such shadow EDS. Moreover, in the case of a UV-completion in string theory, where $M_s = \Lambda_\eta$, one expects that Regge excitations also appear in the WNL regime, and one should use the full stringy UV-completion to study the phenomenology.

4 Flavor Hierarchy from Fuzzy Split Fermions

4.1 Flavor Puzzle & Split Fermions

Flavor Puzzle – In the SM of particle physics, the EW interactions are described by the Glashow-Salam-Weinberg (GSW) theory [142–144], where the EW gauge symmetry $SU(2)_W \times U(1)_Y$ is spontaneously broken to the electromagnetic one $U(1)_{EM}$ by the so-called Higgs mechanism [145–149], i.e. by the vacuum expectation value (VEV) of a scalar field (Higgs field) that is a $SU(2)_W$ doublet, and implies the existence of the $H^0$ boson (SM Higgs boson). This mechanism is also responsible for the masses of the fermions in the SM via Yukawa couplings to the Higgs field. Nevertheless, the observed fermion mass spectrum [4]* needs to introduce a very hierarchical pattern for the Yukawa couplings, implying a naturalness issue [150]*. This is not a fine-tuning problem, since a small Dirac mass is protected by chiral symmetry, so it is technically natural. However, from a Dirac naturalness point of view, one can wonder why the Yukawa couplings are so hierarchical, and why the CKM matrix appears so close to the identity, since no particular texture is preferred in the SM. A related issue is the smallness of the neutrino masses (of Dirac/Majorana type), which involves another hierarchy with the charged leptons. Why the textures of the CKM and PMNS matrices are so different is also another issue, since it suggests that the quark and lepton sectors are treated differently in a UV-theory of flavor. All these issues are known as the flavor puzzle.

Split Fermions – Among all the creative proposals to solve the flavor puzzle in the literature, one of them is the split fermion scenario, originally proposed by Arkani-Hamed and Schmaltz (AS) in Ref. [138]. This idea was then studied by many authors, to build realistic models of flavor with interesting phenomenological consequences (cf. Ref. [139, 151–160] for instance). The central idea behind the AS proposal (cf. Ref. [138] for details) is that the different species of SM fermions are “stuck” at different points along at least 1 (flat) EDS, with the SM gauge and Higgs fields identified with the flat zero mode of bulk fields. In this way, the 4D effective Yukawa couplings are suppressed by the tiny overlap between the wave functions along the EDS of the 2 chiral fermions and the Higgs field.

Arkani-Hamed–Schmaltz Model – In the original AS model [138], one has:

- a flat EDS compactified on an interval $\pi \rho$, or a fat brane of width $\pi \rho$ outside which only gravity propagates;

- a background 5D scalar field (mass dimension 3/2) with a kink profile $\Phi(y)$ along the EDS, i.e. a domain wall, which can be well approximated by a linear function;

13A fat brane is by definition a brane with nonzero thickness (the brane width is larger than the inverse of the EFT UV-cutoff), such as a fuzzy brane can be considered as a new type of fat brane.
5D fermions propagating on the interval $[0, \pi \rho]$, with Yukawa couplings to $\Phi(y)$ and bulk masses;

a 5D Higgs field propagating on the interval $[0, \pi \rho]$, with Yukawa couplings to the 5D fermions;

5D gauge fields propagating on the interval $[0, \pi \rho]$, whose 4D zero modes are identified with the SM gauge bosons which have flat wave functions along the EDS.

Because of the Yukawa couplings to $\Phi(y)$, the 4D zero modes of the 5D fermions (identified with the SM fermions) are trapped inside the domain wall, with Gaussian wave functions peaked at different positions along the EDS that are controlled by the fermion bulk masses. The width of these Gaussian functions are given by the scale of the domain wall width and vary with the Yukawa couplings to $\Phi(y)$. The 5D Higgs field has a flat VEV profile along the EDS, and its 4D zero mode is identified with the SM Higgs boson. Since the overlap of Gaussian wave functions is tiny, one gets naturally hierarchies in the effective 4D Yukawa couplings, which are identified with the SM ones.

4.2 Split Fermions via Multiple Fuzzy Branes

General Picture – The aim of this Section 4.2 is to realize the AS idea, but without the need of a domain wall in the 5D EFT to trap the chiral fermions. Each 4D fermion is now localized on a different fuzzy 3-brane, such as they are delocalized in the bulk by a Gaussian smearing form factor originating from WNL. In the proposed class of WNL models inspired by an AS extension of the SM, one can consider:

a 5D Euclidean spacetime $\mathbb{E}^5 = \mathbb{R}^4 \times [0, \pi \rho]$.

Gauge and Higgs fields propagating in the bulk, with the same features as the original AS model briefly described in Section 4.1.

4D chiral fermions (identified with the SM fermions) localized on different 3-branes which are “stuck” at different points in the EDS.

Toy Model – In order to illustrate the above idea, it is enough to consider a toy model with a real 5D Higgs-like scalar field $H(x, y)$ with a potential $V(H)$, and Yukawa couplings to 2 brane-localized Weyl fermions of opposite chirality, i.e. a 4D left/right-handed fermion $\psi_{L/R}(x)$ localized on a 3-brane at $y = y_{L/R}$. All these fields have a mass dimension $3/2$. As we will see, the crucial feature of the model is that the Weyl fermions are smeared in the bulk. Therefore, it is enough to assume that the WNL scales associated to the fermion fields are lower than the one associated to $H(x, y)$ in the bulk. Then, one can consider a 5D EFT where the WNL features of the Higgs-like field decouple, and only the 4D fermion fields are smeared by WNL, with a universal WNL length scale $\eta$ for simplicity.
**Action** – The smeared fields $\tilde{\Psi}_{L/R}(x,y)$ are defined in terms of the 5D localized fields $\Psi_{L/R}(x,y) = \psi_{L/R}(x) \delta(y - y_{L/R})$, such as

$$\tilde{\Psi}_{L/R}(x,y) = e^{\eta^2 \Delta_\parallel} \Psi_{L/R}(x,y),$$

$$= \tilde{\psi}_{L/R}(x) \delta_\eta(y - y_{L/R}), \quad (4.1)$$

with

$$\tilde{\psi}_{L/R}(x) = e^{\eta^2 \Delta_\parallel} \psi_{L/R}(x),$$

$$\delta_\eta(y - y_{L/R}) = e^{\eta^2 \Delta_\perp} \delta(y - y_{L/R}), \quad (4.2)$$

where $\delta_\eta(y)$ is well approximated by a 1D Gaussian function in Eq. (2.1) if $\eta \ll \rho$, and the brane positions are sufficiently far from the EDS boundaries. The width of these Gaussian profiles along the EDS is given by the WNL length scale $\eta$, which plays the same role as the domain wall width in the original AS model [138]. The 5D action is

$$S_{5D} = \int d^4x \int_0^{\pi \rho} dy \, (\mathcal{L}_H + \mathcal{L}_L + \mathcal{L}_R + \mathcal{L}_Y), \quad (4.3)$$

with the Lagrangians

$$\mathcal{L}_H = \frac{1}{2} \left[ -H \Delta_\parallel H + (\partial_y H)^2 \right] - V(H),$$

$$\mathcal{L}_L/R = \delta(y - y_{L/R}) \psi_{L/R}^\dagger (-i\partial_y) \psi_{L/R},$$

$$\mathcal{L}_Y = -Y i \tilde{\Psi}_L H \tilde{\Psi}_R + \text{H.c.}, \quad (4.4)$$

where the real 5D Yukawa coupling $Y$ has a mass dimension $-3/2$ and scales as $\eta \sqrt{\ell_{UV}}$, where $\ell_{UV}$ is some UV-length scale defined as the EFT UV-cutoff in the local limit $\eta \to 0$.

**Effective 4D Hierarchy** – By performing a KK-dimensional reduction, the 5D Higgs-like field can be decomposed around its flat VEV in 4D KK-modes $h_n(x)$ (mass dimension 1), such as

$$H(x,y) = \frac{v}{\sqrt{\pi \rho}} + \sum_n h_n(x) f_n(y), \quad (4.5)$$

where $v$ is a mass scale, the $f_n(y)$'s are the KK-wave functions (mass dimension 1/2) associated to the KK-modes $h_n(x)$, and orthonormalized with the conditions

$$\forall(n,m) \in \mathbb{N}^2; \int_0^{\pi \rho} dy f_n(y) f_m(y) = \delta_{nm}. \quad (4.6)$$

The EDS is flat, so the KK-scale is defined as $M_{KK} = 1/\rho$. As in the original AS model, $V(H)$ is chosen such that the 0-mode (identified with a SM-like Higgs boson) has a flat wave function:

$$f_0(y) = \sqrt{\frac{1}{\pi \rho}}, \quad (4.7)$$
In the 4D EFT obtained by integrating over the EDS, the action includes the terms:

\[
S_{4D} \supset -i \int d^4x \; \psi^\dagger \left( \partial + m_\psi + y_0 h_0 \right) \psi,
\]

where \( m_\psi = y_0 v \) is the mass of the Dirac fermion

\[
\psi(x) = \begin{pmatrix} \psi_L(x) \\ \psi_R(x) \end{pmatrix},
\]

and the effective 4D Yukawa coupling \( y_0 \) (mass dimension 0) between \( \psi(x) \) and \( h_0(x) \) is given by the overlap:

\[
y_0 = Y \int_0^{\pi \rho} dy \; f_0(y) \delta_\eta(y - y_L) \delta_\eta(y - y_R).
\]

By assuming a natural 5D Yukawa coupling \( Y \sim \eta \sqrt{\ell_{UV}} \), and an interbrane distance \( r = |z_L - z_R| \gg \eta \), one gets

\[
y_0 \sim \frac{1}{2\pi} \sqrt{\frac{\ell_{UV}}{2\rho}} \exp \left[ -\frac{1}{8} \left( \frac{r}{\eta} \right)^2 \right] \ll 1,
\]

by using Eqs. (2.1) and (4.7). Therefore, \( y_0 \) is naturally exponentially suppressed for sufficiently separated fuzzy branes.

**Phenomenology** – Since the transverse Gaussian kernels of the fermions have the same features as the bulk wave functions of the trapped fermions in the original AS model [138], one can localize the fermions at the same EDS points and reproduce the same mass matrix and mixing angles as in Ref. [139] to explain the SM Yukawa hierarchy. For energies below the WNL scale \( \Lambda_\eta \), the phenomenology of this WNL model is completely identical to a AS one [138], so one has the same constraints as in Ref. [139]. One needs to probe the scale associated to the transverse Gaussian fermion profiles to resolve the microscopic details of the model, i.e. fermions trapped inside a domain wall or localized on fuzzy branes. Note that for another WNL form factors, the transverse kernels are not Gaussian anymore but are still sharply localized, so this qualitative discussion still holds.

5 Scale Hierarchy from a Fuzzy Warped Throat

5.1 Gauge Hierarchy Problem & String-Inspired Solutions

**Gauge Hierarchy Problem** – In an EFT with a UV-cutoff \( \Lambda_{UV} \), the existence of an elementary scalar field of mass \( m \ll \Lambda_{UV} \) is expected to require fine-tuning in the UV-completion, if the limit \( m \to 0 \) does not enhance the number of symmetries of the action, which is known as technical naturalness [150]. A Higgs-like \( H^0 \) boson [4] was discovered at the Large Hadron Collider (LHC) with a mass \( M_H \approx 125 \text{ GeV} \), which is not protected by any symmetry in the SM action. Then, if \( \Lambda_{SM} \gg M_H \), there is a technical naturalness
issue [161–163]. By extrapolating the validity of the SM up to $\Lambda_P \sim 10^{18}$ GeV (where one expects new physics to regulate the UV-behavior of quantum gravity), standard EFT arguments [6, 150]* suggest that there should be an incredible fine-tuning between the parameters of the fundamental theory at the Planck scale, in order to get such a light $H^0$ boson at the EW scale $\Lambda_{EW} \sim 100$ GeV $\ll \Lambda_P$. This issue is known as the gauge hierarchy problem. The standard solution is to assume that $\Lambda_{SM}$ is at the TeV scale (terascale), such as in the UV-completion, a light $H^0$ boson appears as natural.

**Weak Nonlocality** – The paradigm of EID-QFT’s gives a potential solution to the fine-tuning problem related to a light $H^0$ boson:

♠ As originally noticed by Krasnikov and Moffat in the WNL theories with a string-inspired form factor [61, 171], which was reexamined more recently by Biswas and Okada [62], the radiative corrections to the mass of the $H^0$ boson scales as $\delta M^2_H \propto \Lambda^2_\eta$, instead of $\delta M^2_H \propto \Lambda^2_{SM}$ in the local SM. A light EW Higgs sector is then natural if $\Lambda_\eta$ is at the terascale, even if a larger new scale like $\Lambda_P \gg \Lambda_\eta$ exists.

♠ Since the UV-sensitivity of $M_H$ to higher-scales is controlled by the UV-properties of the WNL form factor, it is expected that asymptotically polynomial ones [24] stabilize the weak scale as in Lee-Wick theories [172–174]: $\delta M^2_H \sim M^2_{LW}$, where $M_{LW}$ is the mass of the first LW-partner. In a WNL theory with an asymptotically LW form factor, the WNL scale has the role of the LW-scale, so one expects the replacement $M_{LW} \mapsto \Lambda_\eta$.

It would thus be very interesting to look for WNL at the LHC along the lines of Refs. [62, 175]. Nevertheless, this way of having a natural light $H^0$ boson does not explain the gauge hierarchy $\Lambda_P \gg \Lambda_\eta$, which is also the case in the popular paradigm of weak scale supersymmetry without extra ingredients, whose purpose is also to have a technically natural light EW Higgs sector [176]*.

**Large Extra Dimensions** – From another perspective, the paradigm of large EDS’s, proposed by Arkani-Hamed, Dimopoulos and Dvali (ADD) in Refs. [177, 178], offers a way to reformulate the gauge hierarchy problem. If $q$ EDS’s are compactified on a space with a large volume $V$, and the SM fields are 4D degrees of freedom trapped into the worldvolume of a 3-brane, gravity is diluted in the bulk and appears feebly coupled on our brane. The fundamental gravity scale $\Lambda_G$ is the higher-dimensional one, and is related to the 4D effective reduced Planck scale $\Lambda_P$ as $\Lambda^2_P = V \Lambda^q_G$ [177–179]. With a sufficiently large $V$, $\Lambda_G$ may then be at the terascale, such as a light $H^0$ boson is natural [177, 178]. If right-handed neutrinos propagate into the bulk, the observed neutrinos can have naturally light brane-localized Dirac mass terms [180, 181], and the same mechanism can be used to have interesting models of QCD axions [182, 183]. Some realizations of the ADD-models can be

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*Note that in absence of any concrete UV-completion of the SM, these technical naturalness arguments are only heuristic, i.e. based on the comparison with other well known success of technical naturalness such as the pion mass in QCD. Nevertheless, there is no guarantee that the naturalness issues of the fundamental theory of Nature is solved in the same way as what we are used to, i.e. with new physics near the EW scale, cf. Refs. [164–170] for discussions.
UV-completed in superstring theory\textsuperscript{15} [188] with $q = 6$, but the stabilization of the large bulk in toroidal/orbifold compactifications is difficult in practice [194, 195], such as the gauge hierarchy problem is reformulated into a geometrical hierarchy problem, between the fundamental length scale $1/\Lambda_G$ and the compactification radii. More involved geometries [196–198] were proposed, that may provide a path to solve this problem. Nevertheless, a low $\Lambda_G$ is complicated to reconcile with flavor physics [188], since one expects new higher-dimensional operators at the terascale inducing flavor changing neutral currents (FCNC’s).

Gauging all approximate flavor symmetries in the SM, and breaking them on distant branes with messenger fields in the bulk (shining flavor), is a solution [199, 200], but still there is no evidence at the LHC [4]* of the spectacular signatures of strongly coupled gravitational phenomena with $\Lambda_G$ at the terascale, like mini-black holes [201–203]*. It seems to imply a little hierarchy problem: $\Lambda_{EW} \ll 10 \text{ TeV} \ll \Lambda_G$. Therefore, a WNL scale $\Lambda_\eta \sim \mathcal{O}(1) \text{ TeV}$ could reduce the amount of fine-tuning [62, 175], at the price of a mild hierarchy between $\Lambda_G$ and $\Lambda_\eta$ (which is technically natural since $\Lambda_\eta$ is radiatively stable). Note that it is similar to what happens in UV-completions of ADD-models in perturbative superstring theory [188], where $\Lambda_\eta = M_s \ll \Lambda_G$.

**Warped Extra Dimension** – The paradigm of warped EDS’s, originally proposed by Randall and Sundrum in Ref. [140], offers a powerful alternative to the ADD-models, in order to considerably reduce the fine-tuning issue of a light scalar, like the $H^0$ boson. In the original RS-model (RS1) [140], the Higgs field of the GSW theory is a 4D field localized at the IR boundary (IR-brane) of a slice of a 5D anti-de Sitter (AdS$^5$) spacetime (RS throat). The scale $\Lambda_{IR}$, at which gravity becomes strongly coupled on the IR-brane, and the VEV of the EW Higgs field are exponentially redshifted from the 5D gravity scale $\Lambda_{UV} \sim \Lambda_P$ on the UV-brane, by the AdS$^5$ warp factor: one could talk about warp transmutation of scales. One needs $\Lambda_{IR}$ at the terascale to avoid unnatural fine-tuning. The proper length of the AdS$^5$ slice is usually stabilized at a small value (i.e. not large with respect to the fundamental length $\sim \ell_P$) by a classical scalar field via the Goldberger-Wise mechanism [204–206]. In the literature, the SM fermions and gauge fields can be 4D fields localized on the IR-brane, as in the RS1-model [140]. The right-handed neutrinos and/or the SM fermions and gauge fields can also be promoted to 5D fields in extended RS-models [207–213], providing robust geometrical models to solve the flavor puzzle with suppressed FCNC’s (cf. Refs. [214–222] for instance). Recent more realistic constructions with a warped EDS (beyond the RS-throat and motivated by top-down approaches) can be found in Refs. [223–226].

**String-Inspired Warped Throats** – Attempts to UV-complete the RS-like models in string theory exist, e.g. with warped throat geometries à la Klebanov-Strassler [227], but it remains very challenging to find a fully realistic construction [228–230]*. Note that in such string theory models, the geometry near the IR-tip of the throat in not AdS$^5$, but this does not spoil the RS-mechanism of warp transmutation of scales. After integrating

\textsuperscript{15}Note that the idea of EFT’s with large EDS’s is originally motivated by the possibility of a low string scale and TeV scale EDS’s, as suggested in some superstring constructions of the 90s [184–193].

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out the Regge excitations, stringy effects can be encapsulated into the 5D EFT through higher-dimensional operators, and the extended features of the strings remains through WNL form factors [11, 13]. In the following, the choice is to stay agnostic about the UV-origin of these WNL form factors, and the aim of the Section 5.2 is to show that a warped EDS realizes the warp transmutation of both the brane scalar mass and the WNL scale\textsuperscript{16}. In order to illustrate the mechanism, it is enough to focus on a toy model with a real Higgs-like scalar field in a SFT-inspired EID-QFT, and it is straightforward to generalize the discussion to other EID-QFT Higgs-like models\textsuperscript{17}.

### 5.2 Warp Transmutation of Scales

#### 5.2.1 Weak Nonlocality in a Warped Extra Dimension

**Geometrical Background** – Let a model with a warped EDS compactified on the orbifold $S^1/Z_2$ of proper length $\pi \rho$, i.e. a 5D Euclidean spacetime with a metric as in Eq (2.3). The boundaries at $y = 0$ and $y = \pi \rho$ are called UV- and IR-branes, respectively. It is assumed that this kind of background can arise from some WNL extension of Einstein-Hilbert gravity\textsuperscript{18}. The 5D reduced Planck scale, which is also the UV-cutoff on the UV-brane, is noted $\Lambda_{\text{UV}}$.

**IR-Brane Field** – A 4D real scalar field $H(x)$ (mass dimension 1) is localized on the IR-brane, with a quartic self-interaction. Since it is a brane-brane interaction, it is enough to consider a smeared field $\tilde{H}(x)$, which is delocalized only in the directions parallel to the IR-brane:

$$\tilde{H}(x) = e^{\eta^2 \Delta_{\parallel}} H(x). \quad (5.1)$$

Indeed, as discussed in Section 3.2.1, a delocalization transverse to the brane is equivalent to a rescaling of the self-coupling $\lambda \in \mathbb{R}_+$ (mass dimension 0). The brane action is

$$S_H = \int d^4x \int dy \, \delta(y - \pi \rho) \sqrt{\tilde{g}} L_H, \quad (5.2)$$

where the Lagrangian $L_H$ is invariant under the $Z_2$ transformation $H(x) \mapsto -H(x)$:

$$L_H = -\frac{1}{2} H \left( \Delta_{\parallel} - \mu_H^2 \right) H + \frac{\lambda}{4!} \tilde{H}^4, \quad (5.3)$$

with the mass parameter $\mu_H^2 \in \mathbb{R}$. When $\mu_H^2 \geq 0$, $H(x)$ is a Klein-Gordon field (vanishing VEV), whereas for $\mu_H^2 < 0$, it has a nonvanishing VEV which spontaneously breaks the $Z_2$ symmetry, like a Higgs field for a continuous gauge symmetry. The choice of a SFT-like form factor is not important for the qualitative statements of this section, e.g. one can instead consider an asymptotically LW form factor [24].

\textsuperscript{16}For another mechanism of WNL scale transmutation without EDS’s, cf. Ref. [71].

\textsuperscript{17}Note that an infinite tower of ghost-like poles appear at tree level in SFT-inspired Higgs mechanism above $\Lambda_{\text{UV}}$, when one expands the Higgs field around the physical vacuum [107]. Nevertheless, a more involved theory can cure this potential sources of instabilities [107, 109]. Moreover, as argued in Ref. [17], these spurious poles are in the UV-opaque regime, where microcausality is meaningless, such that one should use an Euclidean prescription for spacetime where ghost-like poles are harmless.

\textsuperscript{18}Remember that warped throat solutions are expected to exist in string theory [228–230].
**Bulk Field** — A 5D scalar field $\Phi(x,y)$ (mass dimension $3/2$) propagates all along the EDS, and interacts with the brane field $H(x)$, such as the 5D action is

$$ S_{5D} = \int d^4x \oint dy \sqrt{g} [\mathcal{L}_B + \delta(y - \pi \rho) \mathcal{L}_b] , \quad (5.4) $$

with the bulk Lagrangian

$$ \mathcal{L}_B = \frac{1}{2} \left[ -\Phi \Delta_{\parallel} \Phi + (\partial_y \Phi)^2 \right] , \quad (5.5) $$

and the brane Lagrangian

$$ \mathcal{L}_b = \lambda_{\Phi H} \tilde{O}_\Phi \tilde{O}_H . \quad (5.6) $$

The brane-localized generic operators $\tilde{O}_\Phi$ and $\tilde{O}_H$ involve the smeared fields

$$ \tilde{\Phi}(x,y) = e^{\eta^2 \Delta_{\parallel}} \Phi(x,y) \quad (5.7) $$

and $\tilde{H}(x)$, respectively, and the coupling $\lambda_{\Phi H} \in \mathbb{R}$ is weak. Note that for simplicity, the fields are smeared only in the directions parallel to the IR-brane, and this choice is allowed by the spacetime symmetries at the brane position. Considering instead a sufficiently narrow fuzzy IR-brane (i.e. with a proper width $\ll \rho$) would change nothing to the qualitative discussion on the warp transmutation of scales which is discussed in the following. The goal of this spinless bulk field is just to model some generic 5D field coupled to a brane-localized scalar (the spin degrees of freedom of this bulk field are irrelevant for the following discussion).

**Warp Transmutation of Scales** — By using the same method as in Section 3.1.2, the study of the KK-dimensional reduction to 4D of the free local field $\Phi(x,y)$ can be found in Ref. [231] in the case of a RS-throat. The KK-decomposition of the smeared field $\tilde{\Phi}(x,y)$ is formally the same as in Eq. (3.19), where the smeared KK-mode interaction is

$$ \tilde{\phi}_n(x) = e^{\eta^2 \partial^2 \mu} \phi_n(x) , \quad (5.8) $$

where $\eta' = 1/\Lambda'_{\eta}$, and

$$ \Lambda'_{\eta} = e^{-A(\pi \rho)} \Lambda_{\eta} . \quad (5.9) $$

One needs also to rescale the brane field $H(x)$, in order to get a canonically normalized kinetic term [140]. The following field redefinition is thus performed:

$$ h(x) = e^{-A(\pi \rho)} H(x) , \quad (5.10) $$

such as the 4D action involving only the field $h(x)$ is

$$ S_h = \int d^4x \mathcal{L}_h , \quad (5.11) $$

with the Lagrangian

$$ \mathcal{L}_h = -\frac{1}{2} h \left( \partial^2 \mu_h^2 \right) h + \frac{\lambda}{4!} \tilde{h}^4 . \quad (5.12) $$

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where
\[ \mu_h = e^{-A(\pi \rho)} \mu_H, \]  
and the following smeared field appears in the self-interaction term:
\[ \tilde{h}(x) = e^{y^2 \eta^2} h(x), \]  
where
\[ \mu_h = e^{-A(\pi \rho)} \mu_H. \]  
For a warp factor satisfying \( A(\pi \rho) \gg 1 \), both \( \mu^2_H \) and \( \Lambda_\eta \) appear redshifted from their natural 5D values: this result generalizes the warp transmutation of the RS1-model to the case of the WNL scale\(^{19}\). It is also useful to introduce the redshifted nonperturbative scale \( \Lambda_{IR} \) on the IR-brane:
\[ \Lambda_{IR} = e^{-A(\pi \rho)} \Lambda_{UV}. \]  
In the following, the scale hierarchy of the model is studied in the particular case of a RS-throat.

### 5.2.2 Scale Hierarchy in a Randall–Sundrum Throat

**Randall–Sundrum Throat** — It is instructive to discuss the particular case of an Euclidean RS-throat\(^ {20}\) (i.e. a slice of EAdS\(_5\)), where \( A(y) = ky \) with the curvature scale \( k < \Lambda_{UV} \). The KK-scale is thus
\[ M_{KK} = e^{-A(\pi \rho)} k. \]  
The class of WNL theories of gravity considered here are the ones where one gets the usual 5D Einstein-Hilbert gravity of the RS-model for energies \( E \ll \Lambda_{UV} \). By performing the KK-dimensional reduction to 4D, one will be able to relate the 4D and 5D reduced Planck scales. In the local RS1-model, one gets \([140]\):
\[ \Lambda_P^2 \sim \frac{\Lambda_{UV}^3}{k}. \]  
Here, after a Taylor expansion of the WNL form factors in the gravity action, corrections in powers of \( k/\Lambda_\eta \) to this relation are expected \([232, 233]\). In the following application, only the case \( k/\Lambda_\eta \leq 1 \) will be considered, such that \( \Lambda_P/\Lambda_{UV} \) should still be of the same order of magnitude as in the RS1-model. As discussed in Ref. \([234]\) in the local RS1-model, the couplings between the 4D fields localized on the IR-brane and the KK-modes become nonperturbative at the scale \( \Lambda_{IR} \), which is interpreted as the UV-cutoff of the EFT on the IR-brane. This one is also redshifted from the natural 5D UV-cutoff \( \Lambda_{UV} \), cf. Eq. (5.16).

\(^{19}\)For the warped throats in string theory, where \( \ell_s = \eta \), the mass threshold of the Regge excitations is also redshifted \([230]\), which means that the string scale \( M_s \) is indeed warped down, and it confirms the consistency of the results of this article from a top-down approach.

\(^{20}\)This example is just to illustrate the proposed mechanism with a well-known warp factor, but it is not crucial for the claims of this article to have a RS-throat.
**Scale Hierarchy** – In order to further discuss the scale hierarchy in this WNL RS1-model, one can introduce the following quantities:

\[
\begin{align*}
\kappa_k &= \frac{k}{\Lambda_{UV}} = \frac{M_{KK}}{\Lambda_{IR}}, \\
\kappa_\mu &= \frac{|\mu_H|}{\Lambda_{UV}} = \frac{|\mu_h|}{\Lambda_{IR}}, \\
\kappa_\eta &= \frac{\Lambda_\eta}{\Lambda_{UV}} = \frac{\Lambda'_\eta}{\Lambda_{IR}},
\end{align*}
\]

(5.19)
such as:

♠ In order to be in the classical regime for the background metric, one needs \(\kappa_k \ll 1\).
In practice, a mild hierarchy \(\kappa_k \sim 10^{-1}\) or \(10^{-2}\) is sufficient.

♠ The natural value of \(\kappa_\mu\) depends on the sensitivity of \(|\mu_H|\) on the higher scales \(\eta\) and/or \(\Lambda_{UV}\).

♠ 3 benchmark points will be considered for \(\kappa_\eta\), i.e. \(\kappa_\eta \to \infty\), \(\kappa_\eta = 1\), and \(\kappa_\eta = \kappa_k\).

Note that both \(\Lambda_\eta\) and \(k\) are stable under radiative corrections (as well as their redshifted quantities). A mild hierarchy with \(\Lambda_{UV}\) is then natural from a technical naturalness point of view (no fine-tuning), but not from a Dirac naturalness one (i.e. one may want to explain this mild hierarchy by a UV-mechanism).

**Benchmark Limit** \(\kappa_\eta \to \infty\) – By keeping \(\Lambda_{UV}\) fixed, it means that the WNL scale \(\Lambda_\eta\) decouples, and one gets a local RS-like model, so the radiative corrections \(\delta \mu_h^2\) to \(\mu_h^2\) scale as [140, 234]

\[
\delta \mu_h^2 \sim \Lambda_{IR}^2 \quad \Rightarrow \quad \kappa_\mu \sim 1 \quad (\text{naturalness}).
\]

(5.20)
For a realistic model of EW symmetry breaking, \(H(x)\) is promoted to the Higgs field in the GSW theory. When the RS1-model was published in 1999 [140], \(\Lambda_{IR}\) could be as low as few TeV with \(k\pi\rho \approx 34\). Nowadays, since \(M_{KK}\) is constrained by the LHC to be at the TeV scale [4], there is a little hierarchy problem, which is usually worse than in other BSM scenarios as weak scale supersymmetry because \(k \ll 1 \Rightarrow \Lambda_{IR} \gg 1 \text{ TeV} \text{ and } |\mu_h| \sim \Lambda_{EW}\).

**Benchmark Point** \(\kappa_\eta = 1\) – The WNL scale \(\Lambda_\eta\) is redshifted to \(\Lambda'_\eta\) on the IR-brane. Any field localized on the IR-brane will then give contributions to \(\delta \mu_h^2\) such as [62]

\[
\delta \mu_h^2 \sim \Lambda'_\eta^2 \quad \Rightarrow \quad \delta \mu_h^2 \sim \Lambda_{IR}^2 \\
\Rightarrow \quad \kappa_\mu \sim 1 \quad (\text{naturalness}).
\]

(5.21)
The situation is thus the same as for the local RS-like models (\(\kappa_\eta \to \infty\)).

**Benchmark Point** \(\kappa_\eta = \kappa_k\) – In this case, \(\eta \sim k \ll \Lambda_{UV}\) (it may be possible that a UV-mechanism set this mild hierarchy). The radiative corrections \(\delta \mu_h^2\) to \(\mu_h^2\), from the 4D
fields localized on the IR-brane, will give [62]

\[ \delta \mu_h^2 \sim \Lambda_h^2 \Rightarrow \delta \mu_h^2 \sim M_{KK}^2 \]
\[ \Rightarrow \kappa_{\mu} \ll 1 \text{ (naturalness).} \]  

(5.22)

In a realistic model with the Higgs field of the GSW theory, even if \( \Lambda_{IR} \gg \Lambda_{EW} \), the Higgs boson mass is protected above the redshifted WNL scale \( \Lambda'_{\eta} \). With \( \kappa_{\eta} = \kappa_{k} \), one gets a shadow warped EDS, which reduces the little hierarchy problem in RS-like models if \( \Lambda'_{\eta} \) is at the TeV-scale, which seems to be roughly the current constraints, at least in the toy 4D models of the literature [62, 175]. The situation should thus be similar to the RS-like models where the Higgs sector is embedded in a gauge-Higgs unification framework [235–237] (instead to being brane-localized), such as \( \delta \mu_h^2 \sim M_{KK}^2 \). Nevertheless, a complete realistic model is needed for a quantitative statement of the remaining fine-tuning, which should be sensitive to the form of the WNL form factor.

6 Conclusion & Perspectives

In this article, new possibilities for model building with EDS’s and branes have been presented, based on EID-QFT’s which introduce the amazing features of WNL. This class of WNL braneworld models is also the natural framework when one wants to integrate in an EFT the first Reggeons of a stringy UV-completion (in the spirit of Refs. [230, 238–241] in a local EFT), since the states have their masses \( M \gtrsim M_s \), and are thus in the regime where WNL cannot be neglected (\( \Lambda_{\eta} = M_s \) in SFT).

In Section 3, a WNL braneworld action has been studied for the first time. It has been shown, in this 5D toy model, that WNL smears the interactions of 4D fields localized on a \( \delta \)-like 3-brane, such as one can talk about a fuzzy brane. The main results are:

- It is then possible that 4D fields localized on 2 different fuzzy branes can interact, which is not possible in a local 5D EFT, unless they are identified with the quasilocalized zero modes of some 5D fields [121]. Therefore, in a WNL framework, it is possible to have quasilocalized 4D modes which do not come with KK-excitations [126], which is a rather unique feature of WNL braneworlds which distinguish them from their local cousins.

- KK-excitations of bulk fields whose masses are above the WNL scale have sizable suppressed couplings with the brane-localized fields. If the WNL scale coincide with the KK-scale, one thus expect to be able to suppress the effect of the KK-excitations on the observables, compared to the local braneworld models (shadow EDS). Nevertheless, a better understanding of the phenomenology of this UV-nonlocal regime is needed to make solid conclusions.

In Section 4, an application of fuzzy branes to flavor physics is proposed. Inspired by the AS models of split fermions [138], suppressed effective 4D Yukawa couplings are
realized by localizing Weyl fermions on different fuzzy brane with a bulk Higgs, such one should be able to reproduce the same phenomenology as in Ref. [139]. One can imagine other applications in BSM models where feeble interactions are important, such as in dark matter and hidden sectors.

In Section 5, a model with a warped EDS has been considered. A 4D scalar field is localized on the IR-brane, and it has been shown that both the mass of the scalar field and its WNL scale undergo a warp transmutation à la RS. This feature provides a built-in perturbative UV-cutoff to stabilize physical scales provided by a Higgs mechanism, such as in the standard EW theory. Therefore, WNL offers a new alternative to gauge-Higgs unification [235–237] to alleviate the UV-sensitivity of the Higgs sector in warped EDS models.

For future research tracks, one can mention the following points (for instance):

- ♠ It is important to understand if the theories presented in this article are meaningful above the WNL scale without relying on some UV-completion, as in shadow EDS’s. For that purpose, one needs to better understand the WNL regime in EID-QFT in order to determine which subclass of these theories are UV-complete. Theories with asymptotically LW form factors seem to be the most promising candidates at the moment, since gauge and gravity theories are superrenormalizable/finite and asymptotically free/safe [103, 104].

- ♠ Since the only ghost-free Higgs mechanism of the literature [109] is built from the recipe of Ref. [21], it is important to revisit the conclusions of the present article in this framework where WNL is apparent only at loop level in scattering amplitudes [108].

- ♠ It would be interesting to build a realistic WNL extension of the traditional RS-like scenarios to solve the gauge and flavor hierarchy problems [213]*, in order to study how much one can weaken the various constraints on the KK-scale in these models, and if it is possible to reduce significantly the little hierarchy problem as in gauge-Higgs unification [235–237].

- ♠ Another open issue would be to see what happens when other WNL features such as noncommutative geometry are added to these new WNL braneworlds. For instance, in the framework of QFT on $\kappa$-Minkowski spacetime [242]*, recent investigations [243–245] have shown that the consistency of gauge theories requires that gauge fields propagate in 1 EDS!

Acknowledgments

My research work is supported by CEA Paris-Saclay, IPhT. Thanks to my family and friends for their support during those difficult times.
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