Recent Developments in Physics Beyond the Standard Model

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In this talk I discuss what I believe are the most interesting recent developments in physics beyond the Standard Model. After some initial comments on neutrino masses, I discuss the status of low-energy supersymmetry and finally turn to describing some recent work in theories with extra spatial dimensions.

1 Neutrinos

The most concrete indication for the existence of physics beyond the Standard Model has recently emerged from the Superkamiokande data [1], which convincingly confirm the presence of an atmospheric neutrino anomaly [2]. The most reasonable explanation for these experimental observations relies on the assumption that neutrinos are massive and that the different flavour eigenstates can oscillate among each other. If this interpretation is correct, it implies evidence for new physics beyond the Standard Model. During this Conference, we have heard much discussion of the experimental status of neutrino masses and oscillations. Let me here make some comments on what I believe are the main lessons for theory we have learnt from these results which, if confirmed, represent one of the most important discoveries in physics in recent years.

- We are finding evidence for a new mass scale much larger than the typical weak scale, but different from the Planck mass $G_N^{-1/2}$. Indeed, including only Standard Model degrees of freedom, neutrino masses are described by dimension-5 operators of the form

$$\frac{1}{\Lambda} \ell_L^T C \ell_L H H,$$

(1)

where $\ell_L$ represents the lepton weak doublet and $C$ is the charge-conjugation matrix. After electroweak symmetry breaking, the Higgs field $H$ gets a non-vanishing vacuum expectation value, and the operator in eq. (1) leads to a Majorana neutrino mass $m_\nu = \langle H \rangle^2 / \Lambda$. According to the most natural interpretation of the atmospheric neutrino anomaly, there exists a neutrino with mass of about $6 \times 10^{-2}$ eV. This

\[\text{footnote reference}\]

\[\text{footnote text}\]

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implies a new-physics mass scale $\Lambda$ at about $10^{15}$ GeV, tantalizingly close to the GUT scale.

- We are finding that one neutrino mixing angle (most likely the one corresponding to $\nu_\mu - \nu_\tau$ oscillations) is large, since the best fit of the Superkamiokande data gives $\sin^2 2\theta = 0.99$ and $\Delta m^2 = 3.1 \times 10^{-3}$ eV$^2$ [3]. This situation is different from the case of the familiar Cabibbo–Kobayashi–Maskawa (CKM) mixing among quarks, and therefore it first appeared as a surprise. To assess if this result contradicts our prejudices on the structure of Yukawa couplings, we have to understand if it is indeed incompatible with hierarchical neutrino masses and with GUTs, which relate the properties of quarks and leptons. Recent investigations [4] have shown that this is not the case; let me explain why.

Let us consider the neutrino mass matrix as it emerges from the see-saw mechanism:

$$m_\nu = h^T \nu M h_\nu \langle H \rangle^2.$$  \hspace{1cm} (2)

Here $h_\nu$ is the $3 \times 3$ Yukawa coupling matrix and $M$ is the right-handed neutrino Majorana mass matrix. If the large mixings reside in the matrix $M$ but not in $h_\nu$, the neutrino oscillation results can be simply made compatible with $SU(5)$ GUTs relations, since the right-handed neutrinos are $SU(5)$ singlets. An interesting alternative [5] is that the large mixings reside instead in the left-handed neutrinos. This is compatible with $SU(5)$ if the Yukawa coupling matrices are highly asymmetric. Indeed, the $SU(5)$ relation between the charged lepton and down quark Yukawa coupling is $h_\ell = h_d^T$. Therefore a large mixing in the left-handed charged lepton sector (which corresponds to large neutrino mixing after an $SU(2)$ rotation) corresponds to a large mixing in the right-handed quark sector (which does not affect the CKM matrix).

It has also been observed [6] that large neutrino mixing angles are not incompatible with hierarchical structures in $h_\nu$ and $M$. For instance, consider the toy model of $2 \times 2$ symmetric matrices

$$h_\nu = \begin{pmatrix} A\epsilon & B\epsilon \\ B\epsilon & 1 \end{pmatrix}, \quad M = \begin{pmatrix} C\epsilon^n & D\epsilon^m \\ D\epsilon^m & 1 \end{pmatrix},$$  \hspace{1cm} (3)

with $A, B, C, D$ parameters of order unity and $\epsilon \ll 1$. From eq. (3), we find that, for $n > 2$ and $m > n/2$, the two eigenvalues of the matrix $m_\nu$ are $\epsilon^{2-n}(A^2 + B^2)/C$ and $\epsilon^n A^2 C/(A^2 + B^2)$, and therefore there is a hierarchy between the two physical neutrino masses. On the other hand, the neutrino mixing parameter $\sin 2\theta = 2AB/(A^2 + B^2)$ is of order unity, as long as $A \simeq B$.

In conclusion, although it could not have been theoretically anticipated, the large mixing angle suggested by atmospheric neutrino data can be easily accommodated both with hierarchical neutrino masses and with GUT relations.
It is well known that neutrino masses have profound consequences in cosmology and astrophysics. I just want to emphasize here that the results of the atmospheric neutrino data strongly suggest a GUT-realized see-saw mechanism and therefore give further justification for leptogenesis [7]. Indeed, I find that at present the best motivated way of explaining the observed baryon asymmetry is to invoke the out-of-equilibrium decay of the right neutrinos. With the natural assumption of the presence of CP-violating phases in the Yukawa couplings, the right-handed neutrino decay modes

\[
N_R \rightarrow \ell_L H, \\
N_R \rightarrow \bar{\ell}_L H^* 
\]

give rise to a cosmic lepton asymmetry. Sphaleron-like interactions, which violate a certain linear combination of lepton and baryon number, are in thermal equilibrium before the electroweak phase transition, and reshuffle the particle populations, creating a baryon asymmetry. It is very encouraging that a large range of reasonable neutrino mass parameters can lead to the correct value of the baryon asymmetry. The leptogenesis can then also be used as a criterion to select or disfavour particular models of fermion mass matrices. However, it is unfortunately hard to translate the conditions for successful leptogenesis into simple constraints on the observed neutrino masses and mixings. The main reason for this is that leptogenesis is driven by the inclusive decay processes (4) and (5), summed over the three generations of \( \ell_L \).

Therefore leptogenesis is mainly sensitive to the mixing angles in the right-handed sector, while experiments observe the properties of mainly left-handed neutrinos.

The ultimate goal of the theoretical activity is to use the experimental information on neutrino masses and mixing in order to unravel the flavour mystery and construct a predictive theory of fermion masses. Although there has been quite an intense research with this aim [4], I believe that we are still far, unfortunately, from understanding the rationale of the flavour structure.

2 Supersymmetry

As we have discussed above, one of the most important consequence of the atmospheric neutrino oscillations is the evidence for a new mass scale \( \Lambda \). In this respect, this result agrees with the other indirect indication for new physics: the unification of gauge coupling constants. They are both hints to the existence of a physical threshold at the GUT scale. Following this line of reasoning, one is almost compelled to believe in low-energy supersymmetry. This is because a simple extrapolation of the Standard Model to energies much larger than the weak scale requires a disturbing fine tuning of the parameters in the Higgs potential, while supersymmetry allows for
such an extrapolation without conflicts with criteria of naturalness. Moreover, the prediction of $\alpha_s$ under unification assumptions fails if the $\beta$ functions contain only the contributions from Standard Model particles, but it correctly reproduces the experimental value when one includes the quantum effects of the supersymmetric partners with masses in the 100 GeV–1 TeV range. Therefore, the theoretical motivations for low-energy supersymmetry are still very strong.

On the other hand, the experimental limits \[8\] are worryingly increasing. The limit on the chargino mass is 100 GeV (except for certain pathological regions of parameter space), the one on the lightest neutralino mass is 37 GeV (assuming GUT-related gaugino masses). In the minimal scheme, the gluino mass limit varies between about 200 GeV (for very large squark mass $\tilde{m}_q$) to 300 GeV (for $\tilde{m}_g \simeq \tilde{m}_q$). A considerable constraint also comes from the Higgs mass limit, which varies between 90 GeV (for large tan $\beta$) to 106 GeV (for small tan $\beta$). It seems appropriate and timely to question whether these limits are compatible with the original motivation for low-energy supersymmetry, i.e. the hierarchy problem.

To obtain a quantitative answer, one has to rely on a naturalness criterion \[9, 10, 11\] which specifies the amount of fine tuning among parameters. Recent analyses quantify the result in different ways and conclude that the present experimental limits rule out “95% of the supersymmetric parameter space” \[12\] or require “fine tunings among parameters at a level of 7% or more” \[13\]. Undoubtedly these statements sound rather grim. However it should be noted that they are based on certain theoretical assumptions and prejudices. For instance, for specific values of the top quark Yukawa coupling (corresponding to not too small values of tan $\beta$) a universal squark and slepton mass contribution at the GUT scale cancels out in the expression of $M_Z$ \[10, 14\]. This means that, in this case, squarks and sleptons can be made heavy without causing serious fine-tuning difficulties. Depending on your favourite point of view, this situation can or cannot be viewed as an indirect fine tuning on the top Yukawa coupling. Another interesting observation \[15\] is that the naturalness bounds on charginos and neutralinos are significantly modified if we abandon GUT relations on gaugino masses. This is because in the expression of $M_Z$ in terms of the supersymmetry-breaking parameters, there is a prominent sensitivity on the gluino mass, but only a mild dependence on the electroweak gaugino masses. Fine tunings of no more than 10% can be achieved for chargino masses as large as 165 GeV, although the gluino has to be lighter than 260 GeV. In conclusion, although the present experimental bounds have severely limited the plausible range of supersymmetric parameters, low-energy supersymmetry is far from being ruled out and we have to wait for the LHC for the final verdict.

Let us now turn to discussing the theoretical developments in supersymmetric model building. Most of the recent activity has focused on understanding the structure of the soft supersymmetry-breaking terms, especially in view of the flavour problem I will illustrate below. The question of the origin of the supersymmetry-breaking
terms is indeed a crucial one, because the soft terms represent the connection between theory (i.e., the mechanism of supersymmetry breaking) and experiment (i.e., the mass spectrum of the new particles).

For many years the paradigm has been that the soft terms are produced by the gravitational couplings between a hidden sector where supersymmetry is originally broken and an observable sector containing the ordinary degrees of freedom [16]. The scale of supersymmetry breaking is determined to lie at an intermediate scale $\sqrt{F} \sim 10^{11}$ GeV by requiring that the observable supersymmetric particle masses $\tilde{m}$ are close to the weak scale:

$$\tilde{m} = \frac{F}{M_{Pl}} \sim \text{TeV}. \quad (6)$$

This mechanism is elegant and theoretically appealing, as gravity is directly participating in electroweak physics. However, in this framework, the soft terms are renormalizable parameters of the effective theory defined at energies below the Planck mass $M_{Pl}$. As such, at the quantum level, they receive corrections that depend on the properties of the underlying theory in the far ultraviolet. Therefore the soft terms cannot be computed, as long as we do not know the ultimate theory including a full description of quantum gravity. This could just be a limitation due to our lack of knowledge but, from a pragmatic point of view, it introduces two main problems. The first one is the lack of theoretical predictivity. This is indeed an acute problem since, even with the minimal field content, the low-energy supersymmetric model contains more than 110 free renormalizable parameters, crippling our ability to give solid guidelines to experimental searches. Secondly, the sensitivity of the soft terms to ultraviolet physics implies that their flavour structure will retain the effects of any (unknown) flavour violation at very high energies [17]. In particular, flavour universality of the soft terms will be spoilt by new interactions, which include, for instance, effects from GUTs [18] or from the dynamics at the (unknown) scale $\Lambda_F$ responsible for the origin of the flavour-violating Yukawa couplings. This is described by the lines indicated with the caption “Gravity mediation” in fig. 1. This figure schematically illustrates the energy dependence of the running squark masses of the three different generations. Even if we hypothetically took mass-degenerate squarks at $M_{Pl}$, high-energy flavour violations would induce large squark splittings, not correlated to the Yukawa couplings, at low energy. This situation is experimentally ruled out because the flavour violations in squarks and sleptons induce, through loop diagrams, unacceptably large contributions to $\Delta m_K$, $\epsilon_K$, $\Delta m_B$, $b \rightarrow s\gamma$, $\mu \rightarrow e\gamma$, and other flavour processes.

To solve the flavour problem in the context of gravity-mediated supersymmetry breaking one needs to have full control of the dynamics even beyond $M_{Pl}$. It is possible that its solution lies in the properties of quantum gravity and its flavour symmetries. However, recently there have been theoretical attempts to pursue alternative solutions, finding mechanisms aimed at eliminating the ultraviolet sensitivity.
Figure 1: A schematic illustration of the energy dependence of the running squark masses belonging to the three different generations, in the context of the various supersymmetric scenarios discussed in the text. In gravity mediation, new dynamics at the scale $\Lambda_F$ and GUT physics tend to induce large flavour-breaking effects in the squark spectrum, even if we start from a universality assumption at $M_{Pl}$. In the case of gauge mediation, the squark masses can be generated at scales sufficiently low to ensure a super-GIM mechanism. In anomaly mediation, the squark spectrum is determined by the low-energy theory and it is insensitive to flavour violations occurring at large scales.
of the soft terms altogether. If such a program succeeds, there are two immediate advantages. First of all, one has control over the flavour violations in the soft terms. Moreover, in this case, the soft terms are necessarily computable (i.e. their quantum corrections are finite in the effective theory below $M_{Pl}$) and therefore one can make definite mass predictions relevant to experimental searches.

The best known class of theories in which the soft terms are insensitive to the far ultraviolet is given by gauge-mediated models [19]. Here the original supersymmetry breaking is felt at tree level only by some new particles of mass $M$ (the messengers) and then communicated to the observable sector by loop diagrams involving ordinary Standard Model gauge interactions. Quantum corrections to the soft terms vanish for momenta larger than $M$, as schematically illustrated in fig. 1 by the lines indicated with the caption “Gauge mediation”. Any dynamics occurring at energy scales above $M$ do not affect the soft terms. If we assume that $M$ lies below any new flavour dynamics, then the Yukawa couplings provide the only source of flavour violations and we recover a supersymmetric extension of the GIM mechanism. Flavour violations in low-energy hadronic and leptonic processes are fully under control.

In gauge mediation, the soft terms are finite and computable. In the simplest version of the model, the gaugino, squark, and slepton masses are given by

$$\tilde{m}_{\tilde{g}_i} = \frac{\alpha_i F}{4\pi M},$$

$$\tilde{m}^2_{\tilde{f}_i} = 2 \sum_{i=1}^{3} C^i_f \left( \frac{\alpha_i F}{4\pi M} \right)^2.$$  

Here $\alpha_i$ are the Standard Model gauge coupling constants and $C^i_f$ are the corresponding quadratic Casimir coefficients.

Recently a different approach to obtain ultraviolet insensitivity of the soft terms has been pursued. The central observation is that, in the presence of supersymmetry breaking, gravity generates soft terms even if there are no direct couplings between the hidden and observable sectors [20, 21]. This is an effect of the superconformal anomaly and it gives rise to soft terms that are suppressed by loop factors. If tree-level soft terms exist, then the anomaly-induced terms are subdominant. However, in some cases, they can provide the leading contribution. For gauginos, this occurs when the theory does not contain any gauge-singlet superfield that breaks supersymmetry (as for theories with dynamical supersymmetry breaking) [21]. Indeed, in the absence of gauge singlets $X$, one cannot generate the gaugino masses $\tilde{m}_g$ from the usual operator

$$\int d^2\theta \frac{X}{M_{Pl}} \text{Tr} W^\alpha W_\alpha + \text{h.c.},$$

and therefore one has to rely on higher-dimensional operators, which give at most $\tilde{m}_g \sim F^{3/2}/M_{Pl}^2 \sim \text{keV}$. For scalars, the absence of tree-level contributions to their soft
masses can be obtained with specific structures of Kähler potentials. These structures occur when the supersymmetry-breaking and observable sectors reside on different branes embedded into a higher-dimensional space and separated by a sufficiently large distance [20].

Let us assume that the soft terms, for the reasons explained above (or for any other unknown reason), are dominated by the anomaly contribution. In this case, the gaugino masses are given by [20, 21]

\[ \tilde{m}_g = \frac{\beta_g}{g} m_{3/2}, \quad (10) \]

where \( m_{3/2} \) is the gravitino mass (a measure of the supersymmetry-breaking scale) and \( \beta \) is the corresponding gauge-coupling beta function. More explicitly, for the gauginos relative to the three factors of the Standard Model gauge group, eq. (10) gives

\[ M_3 = -\frac{3\alpha_s}{4\pi} m_{3/2} \]
\[ M_2 = \frac{\alpha}{4\pi \sin^2 \theta_W} m_{3/2} \simeq -0.1M_3 \]
\[ M_1 = \frac{11\alpha}{4\pi \cos^2 \theta_W} m_{3/2} \simeq -0.3M_3. \quad (11) \]

This is to be compared with the usual gaugino mass relations under GUT assumptions,

\[ \tilde{m}_g = \left( \frac{g^2}{g_{GUT}^2} \right)\tilde{m}_g(M_{GUT}) \]

which give

\[ M_2 = 0.30 \ M_3 \]
\[ M_1 = 0.17 \ M_3. \quad (12) \]

The anomaly-mediated mass relation in eq. (10) is particularly interesting because it depends only on low-energy coupling constants and it makes no reference on high-energy boundary conditions (GUT, messengers, ...). Indeed the form of eq. (10) is invariant under renormalization group transformations. This entails a large degree of predictivity, since all soft terms can be computed from known low-energy Standard Model parameters and a single mass scale, \( m_{3/2} \). Also, it leads to robust predictions, since the renormalization group invariance guarantees complete insensitivity of the soft terms from ultraviolet physics. As demonstrated with specific examples in ref. [21], heavy states do not affect the soft terms, since their contributions to the \( \beta \) functions and to threshold corrections exactly compensate each other. This means that the gaugino mass predictions in eqs. (11) are valid irrespective of the GUT gauge group in which the Standard Model may or may not be embedded\(^2\). Therefore, although the soft terms are generated at very high-energy scales, their renormalization

\(^2\)However, exceptions to ultraviolet insensitivity appear in the presence of gauge-singlet superfields [22].
group trajectories are determined in such a way that the low-energy values of the soft terms are specified only by low-energy parameters. This is schematically illustrated in fig. 1 by the lines indicated with the caption “Anomaly mediation”. Whatever the dynamics that breaks flavour symmetry at high energies may be, the low-energy soft terms will respect a super-GIM mechanism.

In spite of its great theoretical appeal, a supersymmetric model with anomaly-mediated mass spectrum is not phenomenologically acceptable. The problem lies in the form of the scalar masses

\[ \tilde{m}^2 = -\frac{1}{4} \left( \frac{\partial \gamma}{\partial \beta_y} \beta_y + \frac{\partial \gamma}{\partial \beta_y} \beta_y \right) \frac{m_{3/2}^2}{2}. \]  

(13)

Here \( \beta_y \) and \( \beta_y \) are the beta functions for the gauge and Yukawa coupling \( y \), and \( \gamma \) is the anomalous dimension of the corresponding superfield. In the supersymmetric model \( SU(3) \) is asymptotically free and has a negative \( \beta \) function, but \( SU(2) \) and \( U(1) \) have a positive \( \beta \) function. Therefore, eq. (13) predicts positive squark squared masses, but negative slepton squared masses. This would induce a spontaneous breaking of QED.

Several possible solutions have been suggested in order to cure this problem [20, 22, 23]. All of these solutions of course require new positive contributions to the slepton masses. These new terms necessarily spoil the most attractive feature of anomaly mediation, i.e. the renormalization group invariance of the soft terms and the consequent ultraviolet insensitivity. This is the most disappointing aspect of this scenario. At present, it is too early to assess if some of the appealing features of anomaly mediation have any relevance in the description of the elementary particle world.

2.1 Experimental Consequences

The realization that there are several possible schemes of supersymmetry-breaking communication has profound experimental implications, not only because of the different patterns of the superpartner mass spectrum, but also because each scheme has very distinctive signatures at high-energy collisions. Therefore, the search for supersymmetry requires different experimental analyses aimed at identifying quite different signals.

The stereotype missing-energy signature of supersymmetry is specific to gravity-mediated scenarios, in which the produced supersymmetric particles cascade decay into the invisible lightest neutralino.

In gauge-mediated scenarios, the gravitino is the lightest supersymmetric particle, because its mass is determined by gravitational interactions instead of gauge interactions as in the case of the other superpartners. The experimental signals are then determined by the nature of the next-to-lightest supersymmetric particle (either a
neutralino or a stau, depending on model-dependent parameters) and the scale of supersymmetry breaking $F$ (which determines the lifetime of the next-to-lightest supersymmetric particle). For $\sqrt{F} \lesssim 10^6$ GeV, the next-to-lightest supersymmetric particles promptly decay into their Standard Model partners and gravitinos, leaving topologies containing tau leptons and missing energy (in the case of the stau) or photons and missing energy (in the case of the neutralino). On the other hand, for $\sqrt{F} \gtrsim 10^6$ GeV, the next-to-lightest supersymmetric particle is quasi-stable, since its decay length is typically longer than the detector size. The experimental signature is given by missing energy in the case of the neutralino, while in the case of the stau there is a more unconventional signal coming from a heavy charged particle crossing the apparatus, leaving anomalous ionization tracks.

The gaugino mass relations in eqs. (11), characteristic of anomaly mediation, lead to quite peculiar experimental signals. Indeed, eqs. (11) predict $M_2 < M_1$ (in contrast to the usual case of eqs. (12), in which $M_1 < M_2$). This and the electroweak-breaking conditions imply that, in realistic models, the $SU(2)$ W-ino triplet is almost degenerate in mass. The mass splitting inside the triplet is dominated by loop effects and the charged particle is heavier than the neutral one, with $m_{\chi^\pm} - m_{\chi^0}$ in the range between the pion mass and about 1 GeV [24, 25]. The (mainly W-ino) neutralino is the lightest supersymmetric particle, and the first chargino decays into a neutralino and a relatively soft pion $\chi^\pm \to m_{\chi^0} \pi^\pm$. The experimental difficulty lies in triggering such events, although kinks in the vertex detector could be revealed at the analysis stage. Different strategies consist in tagging high-energy jets or photons [24] or focus on the production and decay of other supersymmetric particles [25].

From this brief discussion, it should be clear that very different experimental strategies and analyses are necessary to look for the diversified ways in which supersymmetry could reveal itself in high-energy collisions.

As we have previously discussed, the various schemes of supersymmetry-breaking communication differ in the way they address the flavour problem. Therefore it is not surprising that experiments searching for rare flavour-violating or CP-violating processes are of great value for discriminating between the different supersymmetry scenarios. We can distinguish between two classes of supersymmetry-breaking models: i) those (like gauge mediation or anomaly mediation with a universal extra contribution to scalar masses) which satisfy a super-GIM mechanism, and flavour or CP violation is originating only from CKM angles and phases; ii) those (like gravity mediation) which rely on some flavour symmetry valid at some very large scale in the proximity of $M_{Pl}$, and necessarily contain some new sources of flavour and CP violation in the supersymmetry-breaking parameters.

In models belonging to class i), we can expect only rather moderate deviations from the Standard Model predictions in flavour processes. The only exceptions could come from processes that are accidentally suppressed in the Standard Model and are more sensitive to new physics corrections (as in the case of the rare decay $B \to X_s \gamma$).
On the other hand, in the models of class \(ii\), it appears almost unavoidable that new flavour-violating and CP-violating effects should lurk just behind the present experimental limits [18]. In this respect, the rôle of B factories will be crucial in helping theorists to sort out the way in which supersymmetry breaking is realized. Similarly, improvements in the sensitivity on lepton-family violating processes (like \(\mu \to e\gamma\) and \(\mu - e\) conversion in nuclei) and CP-violation (like electron and neutron electric dipole moments) will bring very valuable information.

Recently, the KTeV [26] and NA48 [27] collaborations have announced new results for \(\epsilon'/\epsilon\), leading to a world average [28] of \(\text{Re}\ \epsilon'/\epsilon = (21.4 \pm 4.0) \times 10^{-4}\). This value is higher than the predictions made within the Standard Model [29], and stirred some interest on the possibility that supersymmetric effects had been observed [30, 31, 32, 33].

However, it is not impossible for the Standard Model to accommodate the measured value of \(\epsilon'/\epsilon\). For instance, this can be done by taking the hadronic parameter \(B_6\) to be about 1.5. This moderate enhancement of \(B_6\) with respect to the traditional expectations is not unreasonable. Large contributions to \(B_6\) are found from \(\mathcal{O}(p^2/N_c)\) corrections in the \(1/N_c\) expansion [34] and in the chiral quark model [35]. This could be the result of a \(\Delta I = 1/2\) rule for the operator \(Q_6\), analogous to the one that applies to the operators \(Q_1\) and \(Q_2\). It has also been found that isospin-violating effects arising from the mass difference between up and down quark [36] and final-state interactions [37] both contribute to increasing the estimate of \(\epsilon'/\epsilon\).

Therefore, it appears likely that the discrepancy between theory and experiment is just caused by our present poor knowledge of the hadronic matrix elements. Nevertheless, it is interesting to wonder whether supersymmetry can be responsible for a significant enhancement of the prediction for \(\epsilon'/\epsilon\).

In models of class \(i\), where the flavour and CP violations originate purely from CKM effects, supersymmetric contributions to \(\epsilon'/\epsilon\) are very small and, moreover, in general they tend to reduce the Standard Model prediction [30]. In models of class \(ii\), the new sources of flavour violations are usually parametrized by (complex) flavour non-diagonal entries in the squark mass matrices. Constraints from \(\Delta m_K\) and \(\epsilon\) imply that flavour-violating mass insertion in the left or in the right squark sectors cannot give significant enhancements to \(\epsilon'/\epsilon\). On the other hand, a mass insertion mixing left and right squarks is less constrained and it can give a contribution to \(\epsilon'/\epsilon\) of the size of the measured value. It is interesting that one does not need to rely on unexpectedly large left–right squark mixings to obtain this result. Indeed, the experimental result can be explained with a “theoretically reasonable” guess for the flavour-violating left–right mixing [31]

\[
\tilde{m}^2_{L \leftrightarrow R} \sim m_{3/2} m_s \sin \theta_c e^{i\delta}.
\]

Here \(m_{3/2}\) is the typical supersymmetry-breaking mass, \(m_s\) is the strange quark mass, \(\theta_c\) is the Cabibbo angle, and \(\delta\) is a phase of order 1. The use of similar “reasonable”
estimates for the squark and slepton mass matrices leads to distinctive predictions, which will allow us to test these assumptions. Indeed, the neutron electric dipole moment and the branching ratio for $\mu \to e\gamma$ should lie just beyond the present experimental limits.

Recently, Kagan and Neubert [33] have made the very interesting observation that, in the presence of mass splittings between the squarks $\tilde{u}_R$ and $\tilde{d}_R$, gluino box diagrams can generate $\Delta I = 3/2$ amplitudes that are enhanced by the $\Delta I = 1/2$ selection rule. This gives a potentially very large effect on $\epsilon'/\epsilon$, which can explain the experimental result even for squark masses in the TeV region.

### 3 Extra Dimensions

One of the greatest scientific successes of the last twenty years has been the precise verification of the Standard Model as the correct theory describing elementary particle interactions up to the weak scale. Following the idea of grand unified theories, we are used to extrapolating our knowledge to much smaller length scale, of the order of $M_{GUT}^{-1} \sim 10^{-32}$ m. Moreover, string theory suggests a way to unify gauge and gravity forces at an even smaller distance scale, $M_S^{-1} \sim 2/(\sqrt{k}\alpha_{GUT}M_{Pl})$. Figure 2 illustrates the presumed behaviour of the gauge and gravitational couplings emerging from these conjectures.

These are certainly courageous theoretical extrapolations, but nevertheless are not at present experimentally confirmed. In particular, gravity has been experimentally tested only up to scales of the order of $\lambda \sim \text{mm} \sim (2 \times 10^{-4} \text{ eV})^{-1}$, i.e. 30 orders of magnitude larger than $M_S^{-1}$. It is therefore legitimate to question the scenario illustrated in fig. 2, and wonder whether the gravitational coupling $\alpha_G$ could evolve, at energies above $\lambda^{-1}$, quite differently from our traditional expectations. In particular, one could imagine that the gravitational coupling becomes of the order of the gauge couplings already at the weak scale, eliminating the need for the large mass parameter $M_{Pl}$ or, in other words, eliminating the notorious hierarchy problem.

Arkani-Hamed, Dimopoulos, and Dvali [38] have suggested a physical setting in which this radical point of view can actually be realized. Their construction assumes that our 4-dimensional world, in which ordinary particle processes occur, is actually embedded into a higher-dimensional space, in which only gravitons are free to roam. Let us define the total number of dimensions as $D = 4 + \delta$ and assume that the $\delta$ extra dimensions are compactified in a space with volume $V_\delta$. It is a simple geometrical exercise to prove that the effective Newton constant in the 4-dimensional theory is related to the fundamental energy scale $M_D$ of the full $D$-dimensional gravitational theory by the equation

$$G_N^{-1} \equiv M_{Pl}^2 = M_D^{2+\delta}V_\delta.$$

(15)
Figure 2: The behaviour of the three gauge coupling constants and the gravitational coupling $\alpha_G \sim E^2 / M_{Pl}^2$, as a function of the energy $E$ in the traditional scenario with grand unification at the scale $M_{GUT}$ and superstrings at the scale $M_S$. 

$\alpha_1, \alpha_2, \alpha_3, \alpha_G$ 

$M_Z, M_{GUT}, M_S$ 

$10^{-3}, 10^{-6}, 10^{-9}$ 

$E$
From this, we infer the typical radius of the compactified space

$$R \sim V_\delta^{1/\delta} \sim \frac{1}{M_D} \left(\frac{M_{Pl}}{M_D}\right)^{2/\delta}.$$  \hfill (16)

If we want to realize the scenario in which the fundamental gravitational mass parameter is roughly equal to the weak mass scale, we have to insist that $M_D \sim \text{TeV}$, and therefore the typical size of the compactification radius is

$$R = (5 \times 10^{-4} \text{ eV})^{-1} \sim 0.4 \text{ mm} \quad \text{for } \delta = 2,$$

$$R = (20 \text{ keV})^{-1} \sim 10^{-5} \mu\text{m} \quad \text{for } \delta = 4,$$

$$R = (7 \text{ MeV})^{-1} \sim 30 \text{ fm} \quad \text{for } \delta = 6. \quad \hfill (17)$$

For $\delta = 1$ the size of $R$ is of astronomical length and therefore excluded by standard observations. The case $\delta = 2$ is marginally allowed and therefore interesting for experiments aiming at improving gravitational tests at small distances. As $\delta$ grows, $R$ approaches the inverse of the fundamental mass scale $M_D$.

Before proceeding, we have to discuss whether the construction of ref. [38] can be realized in a physical system. Localizing fields on subspaces with lower dimensions can be achieved in a field theoretical context, but requires the introduction of certain scalar fields with particular potential; it is therefore possible but not straightforward. The great interest in the proposal of ref. [38] has been stirred by the observation that this situation is rather generic in the context of string theory. Indeed, Dirichlet branes (the space defined by the end-points of open strings [39]) are defects intrinsic to string theory on which the gauge theory is confined. The picture of ordinary particles (open strings) localized on the brane with gravity (closed strings) propagating in the bulk can be realized in string models [40]. This observation could actually help in bringing closer two lines of research in theoretical physics (one more phenomenologically oriented and one more formally oriented), which seemed to follow different paths in the last years. Indeed many theoretical speculations intended for Planck energy scales could now be relevant at the TeV scale, and therefore experimentally tested.

As evident from eq. (16), in the higher-dimensional context, the weakness of gravity or, in other words, the smallness of the ratio $G_N/G_F$ is related to the largeness of the number $R M_D$, which measures the compactified radius in its natural units. The hierarchy problem is not completely solved unless we understand why $R^{-1} \ll M_D \sim \text{TeV}$. There have been several attempts to find dynamical explanations for the radius stabilization [41]. This problem may be connected with the cosmological constant puzzle.

Around the time of this Conference, many new ideas in theories with extra dimensions are being proposed. Some of them are very interesting alternatives to the scenario of ref. [38] as a solution to the hierarchy problem.
Randall and Sundrum [42] have proposed a higher-dimensional scenario in which the hierarchy problem is solved without the need for large \((R \gg M_{Pl}^4)\) extra dimensions. They consider a 5-dimensional non-factorizable geometry (i.e. the 4-dimensional metric is not independent of the extra coordinates) in which the line element is given by

\[
ds^2 = e^{-2kr_c \Phi} \eta_{\mu\nu} dx^\mu dx^\nu + r_c^2 d\Phi^2.
\] (18)

Here \(k\) is an energy scale of the order of the 5-dimensional Planck mass \(M_5\) and \(\Phi\) \((0 \leq \Phi \leq \pi)\) is the coordinate of the compactified extra dimensions of size \(r_c\). This metric is the solution of the Einstein equation in a model with two 3-branes (at \(\Phi = 0\) and \(\Phi = \pi\)) with opposite tensions tuned to preserve 4-dimensional Poincaré invariance. In this situation, the 4-dimensional Planck mass is given by

\[
M_{Pl}^2 = \frac{M_5^3}{k} \left(1 - e^{-2\pi kr_c}\right).
\] (19)

We will be interested in the limit \(kr_c \gg 1\), in which the exponential factor in eq. (19) is irrelevant, and we take \(M_5 \sim k \sim \mathcal{O}(M_{Pl})\). The exponential factor is however important for the mass parameters of the fields confined on the 3-brane at \(\Phi = \pi\) representing our world. As apparent from eq. (18), the exponential \(e^{-2kr_c \Phi}\) acts as a conformal factor in the 4-dimensional theory and therefore it is not surprising that the physical mass parameters on the brane are given by \(m_0 e^{-\pi kr_c}\), if \(m_0 \sim \mathcal{O}(M_{Pl})\) is the mass parameter in the 5-dimensional theory. For the moderate number \(kr_c \simeq 50\), the large hierarchy between the weak and the gravitational mass can be reproduced.

The emerging physical picture is the following. Because of the non-factorizable form of the geometry, the gravitational field configuration is highly non-trivial. Gravitons are localized on one brane, while the Standard Model particles live on the other brane. The small overlap of the graviton wave-function with our brane explains the weakness of gravity. No hierarchically small numbers are required because of the exponential suppression. The mass gaps and the mass scale in the effective interactions of the Kaluza–Klein gravitons are both of the order of the weak scale, since the weak scale is the only relevant mass in this physical picture.

This proposal has been further elaborated and an alternative scenario for a solution to the hierarchy problem has been suggested in ref. [43]. The crucial observation [44] is that, in the presence of non-factorizable metrics we can envision non-compact extra dimensions without conflicting with observations. The graviton is again localized, but its Kaluza–Klein spectrum has no mass gap. Nevertheless this is not problematic, because all excited Kaluza–Klein modes give corrections to the gravitational couplings of the order of \(E^2/M_{Pl}^2\), where \(E\) is the typical process energy. It is now possible to consider a setup in which the Standard Model resides on one brane while gravity is localized on a different brane, and both branes have positive tensions. The separation between the two branes reproduces the hierarchy \(M_W/M_{Pl}\) and the fifth dimension is infinitely large.
Another very interesting proposal was recently suggested by Cohen and Kaplan [45]. They consider a 6-dimensional setup consisting of gravity and one scalar field $\Phi$, with a scalar potential that allows a 3-dimensional global “cosmic string” solution. The string core is identified with our 4-dimensional space-time. After solving the Einstein equations for this system, one finds that the effective Planck mass in 4 dimensions is given by

$$M^2_{Pl} = \pi \Gamma(3/8) \left( \frac{M_6}{f} \right)^{9/2} e^{(M_6/f)^4} M_6^2.$$  \hspace{1cm} (20)

Here $M_6$ is the fundamental mass of the underlying 6-dimensional theory and $f$ is the asymptotic vacuum expectation value of the scalar field $\Phi$. A ratio $M_6/f \sim 2.7$ is sufficient to generate the large hierarchy between the weak and gravitational scale, because of the steep functional dependence ($\sim e^{x^4}$) in eq. (20). The resulting effective theory looks very similar to the one proposed in ref. [38], but the hierarchy $M_W/M_{Pl}$ is now dynamically explained.

### 3.1 Opening New Problems

The idea of having a unique fundamental mass scale, of the order of the TeV, for both weak and gravity interactions clearly requires a complete rethinking of much of the accepted understanding of the high-energy behaviour and of early cosmology.

First of all, one has to abandon a very successful feature of the traditional constructs: certain symmetry-breaking interactions are small because they arise from physics at very large scales. Usually one describes these symmetry-breaking effects with effective operators suppressed by an unspecified mass scale $\Lambda$, such as

$$\begin{align*}
\text{neutrino masses} & \rightarrow \frac{1}{\Lambda}\ell\ell HH \\
\text{proton decay} & \rightarrow \frac{1}{\Lambda^2}qqq\ell \\
\text{flavour violation} & \rightarrow \frac{1}{\Lambda^2}\pi\delta\delta d \\
\text{lepton family violation} & \rightarrow \frac{1}{\Lambda} \bar{\nu}\sigma_{\mu\nu}eF^{\mu\nu}.
\end{align*}$$  \hspace{1cm} (21)

The smallness of the observed violation of the corresponding exact or approximate symmetries implies that the scale $\Lambda$ is much larger than the weak scale. In theories with quantum gravity at the TeV scale, we cannot rely on such an explanation. These theories therefore require new mechanisms to understand small parameters. One possibility is that small parameters are not the consequence of approximate symmetries, as in the examples above, but instead in what I will call “locality and geometry”. As suggested in ref. [46], suppose that all unwanted symmetry-breaking
effects can only occur locally on branes that are physically separated by a distance $d$ from the 3-dimensional brane of our world. In this case, the effective couplings of the symmetry-breaking interactions will be suppressed by a factor $e^{-m/d}$, where $m$ is the typical mass of the bulk particle that mediates the interaction from one brane to the other. Large suppression factors can be obtained with moderate ratios of $m/d$.

The same mechanism can be used to obtain the flavour structure of the Yukawa couplings [47]. One can also extend this picture and place the three quark and lepton families on different locations in the directions orthogonal to the ordinary 3-dimensional space [48]. Depending on the profile of the fermion wave-functions along the extra dimensions, large hierarchies in the Yukawa couplings could be obtained from numbers of order 1, using the above-mentioned exponential factor. If this conjecture were true, we could even hope to unravel unsuspected properties of the flavour symmetries. The pattern of Yukawa couplings could look much simpler when viewed in terms of exponential factors or some other functional dependence.

Neutrino masses cannot be any longer explained by the see-saw mechanism and require some new higher-dimensional mechanism. One possibility is that right-handed neutrinos, in contrast with the other Standard Model particles, live in the full $D$-dimensional space [49]. The Yukawa interaction between left- and right-handed neutrinos is localized on the brane. Since the wave-function of $\nu_R$ is spread in the bulk space, the effective Yukawa coupling is suppressed by the square root of the compactified volume $V_δ$. The neutrino mass is then given by

$$m_\nu = \frac{\lambda \langle H \rangle}{\sqrt{V_\delta M_D^6}} \sim \frac{\lambda \langle H \rangle M_D}{M_{Pl}} \sim 10^{-4} \text{ eV} \left( \frac{M_D}{\text{TeV}} \right),$$

where we have assumed that the Yukawa coupling $\lambda$ in the $D$-dimensional theory is of order 1. Notice that the resulting neutrino mass is of the Dirac type and it is in the correct ballpark to explain the atmospheric neutrino data.

Although it first appears that gauge-coupling unification is irremediably lost, it is nevertheless possible to conceive new higher-dimensional schemes in which the success of the supersymmetric prediction is recovered. One possibility [50] is to assume that Standard Model particles have Kaluza–Klein excitations (with masses larger than a few TeV). Their effects in the $\beta$ functions change the logarithmic dependence on the energy into a power dependence and speed up the unification, which can now occur at energies not much larger than the weak scale. From the field-theoretical point of view, one loses control of the theory, but nevertheless it is possible that an actual gauge-coupling unification is achieved in a string theory with TeV scale. Another possibility [51] is to use field variations in the large extra dimensions to achieve a logarithmic unification.

The early cosmology of theories with quantum gravity at the TeV scale will also look drastically different from what has been traditionally assumed. In the scenario of ref. [38], a problem arises. During the early phase of the Universe, energy can
be emitted from the brane into the bulk in the form of gravitons. The gravitons propagate in the extra dimensions and can decay into ordinary particles only by interacting with the brane, and therefore with a rate suppressed by \(1/M_{Pl}^2\). Their contribution to the present energy density exceeds the critical value unless \(T^\star < M_D \delta^{-2} \delta^{+2} \text{MeV} \). \(T^\star \) is the maximum temperature to which we can simply extrapolate the thermal history of the Universe, assuming it is in a stage with completely stabilized \(R\) and with vanishing energy density in the compactified space. As a possible example of its origin, \(T^\star \) could correspond to the reheating temperature after an inflationary epoch. The bound in eq. (23) is very constraining. In particular, for \(\delta = 2\), only values of \(M_D\) larger than about 6 TeV can lead to \(T^\star > 1 \text{ MeV}\) and allow for standard nucleosynthesis. Moreover, even for larger values of \(\delta\), eq. (23) is very problematic for any mechanism of baryogenesis [52].

The graviton emission is also dangerous in an astrophysical context. extradimensional gravitons would speed up supernova cooling in contradiction with the neutrino observation from SN1987A, unless \(M_D > 50 \text{ TeV} \) for \(\delta = 2\), \(M_D > 4 \text{ TeV} \) for \(\delta = 3\). \(M_D > 110 \text{ TeV} \) for \(\delta = 2\), \(M_D > 5 \text{ TeV} \) for \(\delta = 3\). These limits are determined by the infrared behaviour of the gravitational theory. Therefore they do not apply to theories that have large Kaluza–Klein graviton gaps. They can also be evaded in the scenario of ref. [38], in the case of very particular compactified spaces which enhance the masses of the first Kaluza–Klein excitations.

\[
T^\star < \frac{M_D}{\text{TeV}} 10^{\frac{\delta-15}{2+\delta}} \text{MeV}.
\] (23)

\[
M_D > 50 \text{ TeV} \quad \text{for} \quad \delta = 2,
M_D > 4 \text{ TeV} \quad \text{for} \quad \delta = 3.
\] (24)

\[
M_D > 110 \text{ TeV} \quad \text{for} \quad \delta = 2,
M_D > 5 \text{ TeV} \quad \text{for} \quad \delta = 3.
\] (25)

This bound is very constraining in the case of two extra dimensions, and it rapidly decreases with \(\delta\), because of the power-law suppression of graviton interactions. Notice that these limits are determined by the infrared behaviour of the gravitational theory. Therefore they do not apply to theories that have large Kaluza–Klein graviton gaps. They can also be evaded in the scenario of ref. [38], in the case of very particular compactified spaces which enhance the masses of the first Kaluza–Klein excitations.

3.2 Experimental Tests

The idea that quantum gravity resides at the weak scale can be put under experimental scrutiny. We started our discussion on the motivations of extra dimensions by pointing out that gravity has been tested only to scales just below the millimetre. It is therefore clear that improvements in the experimental sensitivity will be of great importance. Indeed there are ongoing experiments [55] that aim at testing gravity up to distances of several tens of microns.
Unfortunately, the astrophysical bounds presented in eq. (25) can be translated into a limit on the Compton wavelength of the first graviton Kaluza–Klein mode of $5 \times 10^{-2} \, \mu m$. The possibility of experimentally observing a deviation of gravity caused by higher-dimensional gravitons is then ruled out, at least in near-future experiments. Any modification of the compactified space capable of avoiding the astrophysical bound will also exclude a visible signal at short-distance gravitational experiments. Nevertheless, in many models realizing the idea of low-scale quantum gravity, there exist other light bulk particles, which could lead to observable signals [38]. A possible effect could also come from other light particles in scenarios with low-energy supersymmetry breaking [56].

High-energy collider experiments can directly probe the new dynamics of quantum gravity at the weak scale. At first, one may believe that the experimental signal should depend on the specific quantum gravitational theory, and therefore no solid prediction could be made. However, in a certain kinematical regime, it is possible to make rather model-independent estimates of graviton production in high-energy collisions. The strategy is to use an effective theory [57, 58], valid below the fundamental mass scale $M_D$, where one can perform an expansion in $E/M_D$ (here $E$ is the typical process energy) and use our knowledge of the infrared properties of gravity.

In the scenario of ref. [38], gravitons are massless particles propagating in $D$ dimensions. Therefore, the relation between their energy $E$ and their momentum is $E^2 = \vec{p}^2 + p_{\text{extra}}^2$, where $\vec{p}$ describes the usual 3-dimensional components and $p_{\text{extra}}$ is the momentum along the extra dimensions. This relation gives an intuitive explanation of how a $D$-dimensional particle can be described by a collection of 4-dimensional modes (called the Kaluza–Klein excitations) with mass $m = |p_{\text{extra}}|$.

We will be interested in the production of the Kaluza–Klein graviton modes in high-energy collisions. The single production of a graviton with non-vanishing $|p_{\text{extra}}|$ violates momentum conservation along the extra dimensions. This is not surprising, since the presence of the 3-brane breaks translational invariance in the directions orthogonal to the brane. It is like playing tennis against a wall: the momentum along the direction orthogonal to the wall is not conserved. Gravitons cannot be directly detected. Therefore the signal in collider experiments is missing energy and imbalance in final-state momenta, caused by the graviton escaping in the extra-dimension compactified space. Just for illustration, we can visualize elementary-particle interactions as the collisions of balls on a pool table. The balls can only move on a 2-dimensional surface (the brane), but as they knock each other they can emit a sound wave (the graviton), which travels in the air (the bulk). Because of this energy loss, an observer living on the surface of the table can infer the existence of the extra dimension by measuring the kinematics of the balls before and after the collision.

Each graviton Kaluza–Klein mode $G_n$ has a production probability proportional
to $E^2/M_{Pl}^2$, which gives rise to a cross section at hadron colliders of

$$\sigma(pp \to G_n \text{jet}) \simeq \frac{\alpha_s}{\pi} G_N = 10^{-28} \text{ fb}. \quad (26)$$

This is hopelessly small and it cannot be observed. However, experiments are sensitive to inclusive processes, in which we sum over all kinematically accessible Kaluza–Klein modes. Because of the large volume in the extra dimensions, the number of graviton Kaluza–Klein modes with mass less than a typical energy $E$ is very large \( \sim E^\delta M_{Pl}^2/M_{Pl}^{2+\delta} \). As a result, the dependence of the inclusive cross section on $M_{Pl}$ cancels out, and we find

$$\sum_n \sigma(pp \to G_n \text{jet}) \simeq \frac{\alpha_s}{\pi} \frac{E^\delta}{M_{D}^{2+\delta}}. \quad (27)$$

By studying final states with photons and missing energy, LEP has already set bounds on the fundamental quantum gravity scale $M_{D}$ of about 1 TeV (for a number of extra dimensions \( \delta = 2 \)) \cite{8}. Future studies at the Tevatron, LHC, linear colliders or muon colliders can significantly extend the sensitivity region of $M_{D}$ by analysing final states with jets and missing energy or photons and missing energy \cite{57, 58}.

It should be stressed that in a complete quantum gravity theory there will certainly exist other experimental signals, quite different from the graviton signal considered above. However, these new signatures are model-dependent and cannot be predicted without a complete knowledge of the final theory. Therefore, the effective-theory signal discussed here, although it does not necessarily represent the discovery mode, is best suited for setting reliable bounds on $M_{D}$.

In general, one can parametrize new physics effects at the scale $M_{D}$ with all possible effective interactions with couplings of order 1 in units of $M_{D}$. However, there is one particular operator that could play a special role,

$$\mathcal{T} \equiv T_{\mu\nu} T^{\mu\nu} - \frac{1}{\delta + 2} T_{\mu}^{\mu} T_{\nu}^{\nu}. \quad (28)$$

Here $T_{\mu\nu}$ is the energy–momentum tensor. The operator in eq. (28) is induced by tree-level virtual graviton exchange and it will appear in the effective Lagrangian with a coefficient of order $1/M_{Pl}^4$. Unfortunately the precise form of this coefficient cannot be computed by using only the effective theory, because it depends on ultraviolet properties. Nevertheless, experimental searches on the existence of this operator are interesting because they represent a test on the spin-2 nature of the particle that mediates the effective interactions. The operator $\mathcal{T}$ gives rise to a variety of experimental signals, which include, in $e^+e^-$ colliders, $d$-wave contributions to fermion pair production, $\gamma\gamma$ and multijet final states and, in hadron colliders, dilepton or $\gamma\gamma$ production \cite{57, 59}. All these signals are in principle related, because they originate from the same interaction.
The graviton-production signal is characteristic of theories with large extra dimensions $R \gg M_D^{-1}$. In models in which the graviton Kaluza–Klein gaps are of the order of $M_D$ (as for instance in the scenario of ref. [42]), the interesting experimental signal is given by the production of the new gravitational excitations with weak-scale masses. Actually, it is possible that all Standard Model particles have Kaluza–Klein modes at the TeV scale [60]. This is the case, for instance, in the proposal of ref. [50] to achieve gauge-coupling unification at low-energy scales. This situation is not inconsistent with the large extra dimension scenario. The Standard Model could live in a $D'$-dimensional space with $4 < D' < D$ and with compactification radius $R' \sim \text{TeV}$. Gravity propagates also in the extra $D - D'$ dimensions characterized by a radius $R \gg R'$. Precision electroweak measurements constrain at present $R'^{-1}$ to be above about 3–4 TeV [61, 62]. Nevertheless, LHC still has the chance of observing the first Kaluza–Klein excitations of Standard Model particles or, at least, of setting bounds on $R'^{-1}$ of more than 6 TeV [62, 63].

If indeed quantum gravity sets in at the electroweak scale, future collider experiments will directly test the structure of its unknown dynamics. For instance, if string theory becomes relevant at $M_D$ [64], experiments could observe Regge recurrences with higher masses and spins. It is certain that, whatever the underlying weak-scale quantum gravity theory may be, collider experiments in the TeV range will be quite exciting.

4 Conclusions

We are now entering a phase in which searches for new physics are becoming the main experimental goal. The community in theoretical physics beyond the Standard Model is therefore facing a special responsibility. I believe that we are responding to this challenge, since in the last few years numerous new theoretical ideas have arisen to question some of the traditional beyond-the-Standard-Model assumptions. It is too early to make definite assessments, but it is very plausible to believe that some of these ideas may lead to a profound revision of our views on the underlying high-energy theory.

In this talk, I first made a few theoretical comments on neutrino oscillation data, the first direct indication of physics beyond the Standard Model. Then, I turned to discussing supersymmetry and showed how recent research has focused on the problem of the ultraviolet sensitivity of the soft terms. Solutions to this problem yield control over flavour violations and calculability of the supersymmetric mass spectrum. Finally, I discussed some recent developments in theories with extra dimensions, aiming at bringing the gravitational scale down to the TeV region. These proposals require a complete rethinking of the high-energy behaviour in theories beyond the Standard Model. Therefore they have deep physical and cosmological implications,
beside the more sociological implication of bringing closer together formal research and phenomenology. If these theories are true, collider experiments will observe a great deal of surprises above the TeV.

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