A new form of three-body Faddeev equations in the continuum

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Abstract

We propose a novel approach to solve the three-nucleon (3N) Faddeev equation which avoids the complicated singularity pattern going with the moving logarithmic singularities of the standard approach. In this new approach the treatment of the 3N Faddeev equation becomes essentially as simple as the treatment of the two-body Lippmann-Schwinger equation. Very good agreement of the new and old approaches in the application to nucleon-deuteron elastic scattering and the breakup reaction is found.

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I. INTRODUCTION

The three-body Faddeev equations [1] in the continuum put three-body scattering on a firm mathematical basis. The complicated asymptotic boundary conditions in configuration space [2] are fully included. At the time of their formulation the computer power, however, was insufficient to solve them directly given local two-body forces. Therefore one started with forces of finite rank (the simplest being a rank one separable force), which turns the three-body problem in a partial wave basis into a finite set of coupled integral equations in one variable - and thus feasible at that time. Nevertheless the way it was formulated in momentum space the free three-body propagator lead to a complicated singularity structure of the integral kernel, namely to logarithmic singularities, whose position depend on the external variable - so called moving singularities [3]. In order to avoid this obstacle the path of integration in the kernel is moved into the complex plane (contour deformation), which imposes of course conditions on the analytical properties of the two-body separable forces. The need, however, for using the realistic two-body forces, which are dominantly local, enforced their expansion into a series of finite rank forces [4], which was tedious and finally overcome by integrating the logarithmic singularities directly on the real momentum axis [5]. It took until the eighties [6] that fully realistic two-body forces could be handled, now in a set of coupled integral equations in two variables. That approach was based on Spline interpolations [7], which allowed to integrate analytically over the interpolated Faddeev amplitudes under the integral. In this manner in the three-nucleon system the realistic high precision nucleon-nucleon (NN) forces [8, 9, 10], together with three-nucleon forces (3NF) of various types [11, 12] can be handled in a fully reliable manner [13]. These studies are a basic foundation for testing nuclear forces and triggered a tremendously rich set of neutron-deuteron (nd) and proton-deuteron (pd) experiments all over the world. A representative set can be found in [13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30]. Especially the need of 3N forces in conjunction with current two-nucleon (2N) forces was firmly established [31, 32, 33, 34]. In recent years that interplay of 2N and 3N forces has been substantiated by the theoretical insights into nuclear forces gained through effective field theory for π’s and nucleons (∆’s) constrained by chiral symmetry [35].

The feasibility of controlling the 3N continuum in the Faddeev scheme opened the door to evaluate reliably the final state (initial state) interaction in photon induced processes on \(^3\)He (pd capture processes) [36]. This provides important insight into the electromagnetic nucleonic current operator, the interplay with 3N forces and into properties of electromagnetic nucleon form factors.

Despite that achieved technical status of controlling the integral Faddeev kernel in the three-
body continuum it would be desirable to avoid those logarithmic singularities totally. A suggestion in that direction was undertaken in [37] where, however, the presence of the virtual state pole of the 2N t-matrix in the state $^1S_0$ caused problems. It is the purpose of this paper to establish a definite solution of that long lasting technical challenge with the moving logarithmic singularities by formulating the three-body Faddeev kernel in the continuum in such a manner that only trivial poles occur in one variable, which can be handled as easily as in the 2-body Lippmann Schwinger equation.

In section II we give a brief reminder of our standard approach followed by the discussion of all possible alternative choices of variables in the intermediate state integral of the Faddeev integral kernel. One of them sticks out which is free of all technical difficulties and will be displayed in detail. In section III we compare numerical results for 3N scattering in the old and new approach using modern high precision NN forces. The great simplification achieved with the novel approach shows up also in case of finite rank 2-body forces as displayed in section IV. We conclude in section V.

II. DIFFERENT FORMS OF THE 3-BODY FADDEEV EQUATION IN THE CONTINUUM

We use the notation detailed in [13]. The nucleon-deuteron (Nd) breakup amplitude is

$$\langle \vec{p}\vec{q} | U_0 | \Phi \rangle = \langle \vec{p}\vec{q} | (1 + P)T | \Phi \rangle ,$$

(1)

where $\vec{p}$ and $\vec{q}$ are standard Jacobi momenta, $P$ the sum of a cyclical and anticyclical permutation of 3 particles and $|\Phi\rangle$ the initial product state of a deuteron and a momentum eigenstate of the projectile nucleon.

The amplitude $T|\Phi\rangle$ obeys our standard Faddeev type equation

$$T|\Phi\rangle = tP|\Phi\rangle + tPG_0T|\Phi\rangle ,$$

(2)

where $t$ is the 2N off-shell t-operator and $G_0$ the free 3N propagator. Introducing the momentum space 3N partial wave basis $|pq\alpha\rangle$ and projecting [2] on these states we get

$$\langle p\alpha | T | \Phi \rangle = \langle p\alpha | tP | \Phi \rangle + \langle p\alpha | tPG_0T | \Phi \rangle .$$

(3)

The kernel part $\langle p\alpha | tPG_0T | \Phi \rangle$ can be evaluated using the completeness of the states $|pq\alpha\rangle$ as

$$\langle p\alpha | tPG_0T | \Phi \rangle = \sum_{\alpha'} \int p'^2 dp' q'^2 dq' p''^2 dp'' q''^2 dq'' \langle p\alpha | t| p'q'\alpha' \rangle$$
\[
x \times \langle p' q' | T | p'' q'' \rangle = \frac{\langle p'' q'' | T | p' q' \rangle}{E + i\epsilon - \frac{1}{m}(p'^2 + \frac{3}{2}q'^2)}.
\]

Here we use the Balian-Berézin \[38, 39\] approach to calculate the permutation matrix element \( \langle p' q' | P | p'' q'' \rangle \) (see Appendix for details)

\[
\langle p q | P | p' q' \rangle = \int_{-1}^{1} dx \frac{\delta(p - p_1)}{p^2} \frac{\delta(p' - p_2)}{p'^2} G_{\alpha\alpha'}^{BB}(q, q', x).
\]

Moreover the 2-body t-matrix conserves the spectator momentum \( q \) and all discrete quantum numbers except the orbital angular momentum \( l \):

\[
\langle p q | t | p' q' \rangle = \frac{\delta(q - q')}{q^2} t_{l_{\alpha}l_{\alpha}}^{s_{\alpha}j_{\alpha}t_{\alpha}}(p, p'; E(q)) = E - \frac{3}{4m}q^2
\]

\[
\delta_{s_{\alpha}l_{\alpha}}, \delta_{j_{\alpha}l_{\alpha}}, \delta_{t_{\alpha}l_{\alpha}}, \delta_{l_{\alpha}l_{\alpha}}, \delta_{l_{\alpha}l_{\alpha}}, \ldots.
\]

Finally we extract the deuteron pole in the channels \( |\alpha| = |\alpha_d| \) which contain the 2-body \( ^3S_1 - ^3D_1 \) states. Thus we define

\[
t_{l_{\alpha}l_{\alpha}}^{s_{\alpha}j_{\alpha}t_{\alpha}}(p, p'; E(q)) = \frac{t_{l_{\alpha}l_{\alpha}}^{s_{\alpha}j_{\alpha}t_{\alpha}}(p, p'; E(q))}{E + i\epsilon - \frac{3}{4m}q^2 - E_d}
\]

for the deuteron quantum numbers \( s_{\alpha} = j_{\alpha} = 1, t_{\alpha} = 0, l_{\alpha}, l_{\alpha} = 0, 2 \) and keep \( t \) as it is otherwise.

That pole property obviously carries over to the \( T \)-amplitude and we define just for the \( |\alpha| = |\alpha_d| \) channels

\[
\langle p q | T | \Phi \rangle = \frac{\langle p q | \hat{T} | \Phi \rangle}{E + i\epsilon - \frac{3}{4m}q^2 - E_d}.
\]

Using all that the coupled set \[33\] is for \( \alpha \neq \alpha_d \)

\[
\langle p q | \hat{T} | \Phi \rangle = \langle p q | \hat{P} | \Phi \rangle + \sum_{l_{\alpha}} \sum_{\alpha'} \int p r^2 dp' r'^2 dp'' q'^2 dq''
\]

\[
\int_{-1}^{1} dx \, t_{l_{\alpha}l_{\alpha}}^{s_{\alpha}j_{\alpha}t_{\alpha}}(p, p'; E(q)) G_{\alpha\alpha'}^{BB}(q, q', x) \frac{\delta(p' - p_1)}{p'^2} \frac{\delta(p'' - p_2)}{p''^2}
\]

\[
\frac{\langle p'' q'' | \alpha'' | \hat{T} | \Phi \rangle}{E + i\epsilon - \frac{3}{4m}q''^2 - E_d} + \bar{\alpha}_{\alpha'}^{\alpha''} \langle p'' q'' | \alpha'' | \hat{T} | \Phi \rangle)
\]

\[
\frac{1}{E + i\epsilon - \frac{1}{m}(p''^2 + \frac{3}{2}q''^2)}.
\]

where \( \bar{\alpha}_{\alpha'}^{\alpha''} = 1 - \delta_{\alpha''\alpha_d} \), and for \( \alpha = \alpha_d \)

\[
\langle p q | \hat{T} | \Phi \rangle = \langle p q | \hat{P} | \Phi \rangle + \sum_{l_{\alpha}} \sum_{\alpha'} \int p r^2 dp' r'^2 dp'' q'^2 dq''
\]

\[
\int_{-1}^{1} dx \, t_{l_{\alpha}l_{\alpha}}^{s_{\alpha}j_{\alpha}t_{\alpha}}(p, p'; E(q)) G_{\alpha\alpha'}^{BB}(q, q', x) \frac{\delta(p' - p_1)}{p'^2} \frac{\delta(p'' - p_2)}{p''^2}
\]

\[
\frac{\langle p'' q'' | \alpha'' | \hat{T} | \Phi \rangle}{E + i\epsilon - \frac{3}{4m}q''^2 - E_d} + \bar{\alpha}_{\alpha'}^{\alpha''} \langle p'' q'' | \alpha'' | \hat{T} | \Phi \rangle)
\]

\[
\frac{1}{E + i\epsilon - \frac{1}{m}(p''^2 + \frac{3}{2}q''^2)}.
\]
The shifted $p'$ and $p''$ arguments are

$$\pi_1 = \sqrt{q'^2 + \frac{1}{4}q^2 + qq''x} ,$$

$$\pi_2 = \sqrt{q^2 + \frac{1}{4}q'^2 + qq''} .$$

We are left in Eqs. (9) and (10) with four integrations over $p', p'', q''$, and $x$, where any two of them can be performed analytically using the two $\delta$-functions. Thus there are six possibilities, which we regard now in turn and we discuss the advantages or disadvantages to use them. Apparently it is sufficient to discuss only the kernel parts and moreover just one, say in (9), which we shall denote just by "the kernel".

A. Analytical integration over $p'$ and $p''$

The resulting form of "the kernel" is

$$\langle pq\alpha|tPG_0T|\Phi\rangle = \sum_{\tilde{l}a} \sum_{\alpha''} \int q'^2 dq'' \int_{-1}^{1} dx \ t^{\tilde{l}a_\alpha\alpha}'(p, \pi_1; E(q))$$

$$G_{\tilde{\alpha}\alpha''}(q, q'', x)(\tilde{\delta}_{\alpha''\alpha} E + i\epsilon - \frac{1}{4m} q'^2 - E_d)$$

$$+ \tilde{\delta}_{\alpha''\alpha'} \langle \pi_2 q''\alpha''|T|\Phi\rangle - \frac{1}{E + i\epsilon - \frac{1}{4m}(q^2 + q'^2 + qq''x)} .$$

(12)

This is our standard approach [6, 13]. For $q \leq q_{\text{max}} \equiv \sqrt{\frac{4}{3}mE}$ the integration over $x$ leads to logarithmic singularities depending on $q$ and $q''$ - the so called moving singularities. Nevertheless the advantage is, that the complex $q''$-dependence of $\langle \pi_2 q''\alpha''|T|\Phi\rangle$ can be properly mapped out. That nontrivial dependence of the T-amplitude arises from the property of the 2N t-matrix in the state $^1S_0$ and from the 3N breakup threshold behavior [13].

The deuteron pole at $q'' = q_0 = \sqrt{\frac{4m}{3}(E - E_d)}$ can be taken care of in the $q''$-integration using e.g. a subtraction method.

B. Analytical integration over $x$ and $p''$

We rewrite the $\delta$-functions in (9) and (10) as follows

$$\delta(p' - \pi_1) = \frac{2p'}{qq''} \delta(x - x_0) \Theta(1 - |x_0|) ,$$

(13)

with

$$x_0 = \frac{p'^2 - 1/4q^2 - q'^2}{qq''} = \frac{p''^2 - 1/4q'^2 - q^2}{qq''} ,$$

(14)
and

$$\delta(p'' - \pi_2) = \delta(p'' - \sqrt{p''^2 + \frac{3}{4} q^2 - \frac{3}{4} q''^2}) \Theta(p''^2 + \frac{3}{4} q^2 - \frac{3}{4} q''^2).$$  \tag{15}$$

The resulting form of "the kernel" is

$$\frac{\Theta(1 - \frac{|p''^2 - \frac{1}{4} q^2 - \frac{1}{4} q''^2|}{q''}}{\Theta(p''^2 + \frac{3}{4} q^2 - \frac{3}{4} q''^2)} \frac{(q'' q''')^2 \delta_{\alpha'' \alpha''} \langle p'' q'' | | \Phi \rangle}{E + i \epsilon - \frac{1}{m}(p''^2 + \frac{3}{4} q''^2)},$$

with

$$p'' = \sqrt{p''^2 + \frac{3}{4} q^2 - \frac{3}{4} q''^2}.$$

The two \(\Theta\)-functions define the domain \(D\) for the integrations over \(p'\) and \(q''\) which is an open rectangular region in the \(p' - q''\) plane restricted by the straight lines \(q'' = \frac{q}{2} \pm p'\) and \(q'' = p' - \frac{q}{2}\) as displayed in Fig. 1.

The integrations over \(p'\) and \(q''\) in (16) split now in the following way

$$\langle p q | \hat{T} G_0 \hat{T} | \Phi \rangle = \frac{2}{q} \sum_{l_a} \sum_{\alpha''} \int_0^\infty p' dp' q'' dq'' \langle \hat{t}_{\alpha'' l_a} \hat{t}_{l_a} \rangle (p, p'; E(q)) G^{BB}_{\alpha''} (q, q', x_0) \frac{1}{E + i \epsilon - \frac{1}{m}(p''^2 + \frac{3}{4} q''^2)}$$

$$\int_{|q/2 - p'|}^{q/2 + p'} q'' dq'' \langle \hat{t}_{\alpha'' l_a} \hat{t}_{l_a} \rangle (p, p'; E(q)) \frac{1}{E + i \epsilon - \frac{1}{m}(p''^2 + \frac{3}{4} q''^2)}$$

For the channels \(\alpha''\) different from \(\alpha''_q\) only a simple pole in the \(p'\) variable occurs, positioned at \(p_0 = \sqrt{\frac{2}{3}(q_{max}^2 - q^2)}\) and therefore of concern only for \(q \leq q_{max}\). At \(q = q_{max}\) or \(p_0 = 0\) there is no pole since the \(q''\) integral vanishes. For the channels \(\alpha'' = \alpha''_d\) the \(q''\)-integral contains the deuteron pole at \(q'' = q_0 = \sqrt{\frac{2m}{3}(E - E_d)}\). If \(q_0\) does not coincide with the limits of integration \(|q/2 - p'|\) and \(|q/2 + p'|\) that integral generates a smooth function of \(p'\). In case it coincides, however, logarithmic singularities occur. Their positions are determined by

$$q_0 = \frac{q}{2} - p',$$

$$q_0 = \frac{q}{2} + p'.$$

We can avoid them in the following manner. The product of the free propagator and the deuteron pole term can be written as

$$\frac{1}{E + i \epsilon - \frac{1}{m}(p''^2 + \frac{3}{4} q''^2)} \frac{1}{E + i \epsilon - \frac{3}{4} m q''^2 - E_d}.$$

6
Thus (21) yields the contribution to “the kernel” from channels $\alpha_q$. Thus for the
where we used (17). This provides separation of the free propagator and the deuteron pole singularities. Thus for the $\alpha'' = \alpha''_d$ channels alone one has the contribution to “the kernel”

$$\frac{2}{q} \sum_{l_s} \sum_{\alpha''_d} \int_0^\infty p' dp' t^{\alpha''_d \alpha''_{la}}(p, p'; E(q)) \int_{|q/2-p'|}^{q/2+p'} q'' dq'' G^{BB}_{\alpha''_d}(q, q'', x_0)$$

$$\left[ \frac{1}{E + i\epsilon - \frac{1}{m}(p'^2 + \frac{3}{4}q'^2)} \right] \frac{1}{|E_d| + \frac{1}{m}p'^2} \left< p'' q'' \alpha''_d \frac{T}{\Phi} \right>$$

$$- \frac{1}{E + i\epsilon - \frac{3}{4m}q'^2 - E_d} \left< p'' q'' \alpha''_d \frac{T}{\Phi} \right>$$

(21)

The first part has only the free propagator singularity in the $p'$ variable like the part for the channels $\alpha''$ different from $\alpha''_d$ in (18) and in the second part we change the order of integrations. Thus (21) yields the contribution to “the kernel” from $\alpha'' = \alpha''_d$ channels

$$\frac{2}{q} \sum_{l_s} \sum_{\alpha''} \int_0^\infty p' dp' t^{\alpha''_d \alpha''_{la}}(p, p'; E(q)) \int_{|q/2-p'|}^{q/2+p'} q'' dq'' G^{BB}_{\alpha''_d}(q, q'', x_0)$$

$$\left[ \frac{1}{E + i\epsilon - \frac{1}{m}(p'^2 + \frac{3}{4}q'^2)} \right] \frac{1}{|E_d| + \frac{1}{m}p'^2} \left< p'' q'' \alpha''_d \frac{T}{\Phi} \right>$$

$$- \frac{1}{E + i\epsilon - \frac{3}{4m}q'^2 - E_d} \left< p'' q'' \alpha''_d \frac{T}{\Phi} \right>$$

(22)

Now in the second part the $p'$-integration yields a smooth function in $q''$ and the deuteron singularity in the $q''$-integral is a simple pole.

Altogether “the kernel” (18) is

$$< pq\alpha | tPG_0T | \Phi > = \frac{2}{q} \sum_{l_s} \sum_{\alpha''} \int_0^\infty p' dp' t^{\alpha''_d \alpha''_{la}}(p, p'; E(q)) \left[ \frac{1}{E + i\epsilon - \frac{1}{m}(p'^2 + \frac{3}{4}q'^2)} \right]$$

$$\int_{|q/2-p'|}^{q/2+p'} q'' dq'' G^{BB}_{\alpha''_d}(q, q'', x_0) \left< p'' q'' \alpha''_d \frac{T}{\Phi} \right> + \delta_{\alpha''_d \alpha''} < p'' q'' \alpha''_d \frac{T}{\Phi} >$$

$$- \frac{1}{E + i\epsilon - \frac{3}{4m}q'^2 - E_d} \left< p'' q'' \alpha''_d \frac{T}{\Phi} \right>$$

(23)

It is the best form out of the six possible ones for the numerical performance. Also the complex $q$-dependence of the $T$-amplitude near $q'' = q_{\text{max}}$ in the channels $\alpha$ containing the $^1S_0$ two-body component can be properly mapped out like in case (1A)
C. Analytical integration over $x$ and $p'$

We choose one $\delta$-function in (23) as in (13) and the other one as

$$\delta(p'' - \pi_2) = \frac{p''}{p'} \delta(p' - \sqrt{p'^2 + 3/4q'^2 - 3/4q^2}) \Theta(p'^2 + 3/4q'^2 - 3/4q^2). \quad (24)$$

This leads to "the kernel"

$$\langle pq \alpha | tP G_0 T | \Phi \rangle = \frac{2}{q} \sum_{l_\alpha} \sum_{\alpha''} \int p'' dp'' \int q'' dq'' t^{\bar{a} \alpha' \alpha}_{l_\alpha l_\alpha} (p, p'; E(q)) G^{BB}_{\bar{a} \alpha''} (q, q'', x_0) \Theta(qq'' - p'^2 - 4/3q'^2 - q^2) \Theta(p'^2 + 3/4q'^2 - 3/4q^2) \Theta(qq'' - p'^2 - 4/3q'^2 - q^2)$$

$$\left( \delta_{\alpha'' \alpha''} \frac{\langle p'' q'' \alpha'' | T | \Phi \rangle}{E + i\epsilon - \frac{4}{3m} q'^2 - E_d} + \bar{\delta}_{\alpha'' \alpha''} \langle p'' q'' \alpha'' | T | \Phi \rangle \right) \frac{1}{E + i\epsilon - \frac{1}{m} (p'^2 + 3/4q'^2)} \quad (25)$$

with

$$p' = \sqrt{p'^2 + 3/4q'^2 - 3/4q^2}. \quad (26)$$

The two $\Theta$-functions define the domain $D$ for the integrations over $p''$ and $q''$ which is an open rectangular region in the $p''$-$q''$ plane restricted by straight lines $q'' = 2(q \pm p'')$ and $q'' = 2p'' - 2q$ and which leads to the integrals

$$\langle pq \alpha | tP G_0 T | \Phi \rangle = \frac{2}{q} \sum_{l_\alpha} \sum_{\alpha''} \int_0^\infty dp'' p'' \int_{2q - 2p''}^{2q + 2p''} dq'' t^{\bar{a} \alpha' \alpha}_{l_\alpha l_\alpha} (p, p'; E(q)) G^{BB}_{\bar{a} \alpha''} (q, q'', x_0) \Theta(qq'' - p'^2 - 4/3q'^2 - q^2) \Theta(p'^2 + 3/4q'^2 - 3/4q^2) \Theta(qq'' - p'^2 - 4/3q'^2 - q^2)$$

$$\left( \delta_{\alpha'' \alpha''} \frac{\langle p'' q'' \alpha'' | T | \Phi \rangle}{E + i\epsilon - \frac{4}{3m} q'^2 - E_d} + \bar{\delta}_{\alpha'' \alpha''} \langle p'' q'' \alpha'' | T | \Phi \rangle \right) \frac{1}{E + i\epsilon - \frac{1}{m} (p'^2 + 3/4q'^2)} \quad (27)$$

One can proceed analogously like in (13) and would arrive also at a form free of logarithmic singularities. The form (14) might appear, however, more favorable since each propagator appears only with one integration variable.

D. Analytical integration in $q''$ and $p'$

We keep the first $\delta$-function in (23) as it is and rewrite the second one as

$$\delta(p'' - \pi_2) = \frac{2p''}{\sqrt{p'^2 - q^2(1 - x^2)}} \delta(q'' - (-2qx + 2\sqrt{p'^2 - q^2(1 - x^2)}) \Theta(p'^2 - q^2). \quad (28)$$

The resulting form of "the kernel" is

$$\langle pq \alpha | tP G_0 T | \Phi \rangle = 2 \sum_{l_\alpha} \sum_{\alpha''} \int p'' dp'' \Theta(p'^2 - q^2) \int_{-1}^1 dx \frac{t^{\bar{a} \alpha' \alpha}_{l_\alpha l_\alpha} (p, \pi_1; E(q))}{\sqrt{p'^2 - q^2(1 - x^2)}}$$
\[
G_{\bar{\alpha}\alpha}^{BB}(q, q'', x)(\delta_{\alpha''\alpha'}|E + i\epsilon - \frac{3}{4m}q'^2 - E_d) + \bar{\delta}_{\alpha''\alpha'}^{\alpha''}(p'' q''|T|\Phi) \]
\[
\frac{\langle p'' q'' \alpha''|\hat{T}|\Phi \rangle}{E + i\epsilon - p'^2 - 3(\sqrt{p'^2 - q'^2(1 - x^2)} - qx)^2}
\]

with
\[
q'' = 2(\sqrt{p'^2 - q^2(1 - x^2)} - qx) .
\]

This form appears quite complicated in the pole structure of the free propagator as well as for the deuteron pole and the \(^1S_0\) virtual state pole singularity close to \(q'' = q_{\text{max}}\) and we do not consider it further.

E. Analytical integration over \(q''\) and \(p''\)

We rewrite the first \(\delta\)-function in (9) as
\[
\delta(p' - \pi_1) = \frac{p'\delta(q'' - (\sqrt{p'^2 - \frac{1}{4}q^2(1 - x^2)} - \frac{qx}{2}))}{\sqrt{p'^2 - \frac{1}{4}q^2(1 - x^2)}}\Theta(p'^2 - 1/4q^2) ,
\]
and keep the second as
\[
\delta(p'' - \sqrt{(q - 1/2q'')^2 + qq''(1 + x)}) .
\]

Thus \(p''\) is always greater equal zero. It is zero for \(x = -1\) and \(p' = 3/2q\).

"The kernel" is
\[
\langle pq\alpha|tPG_0 T|\Phi \rangle = \sum_{l\alpha} \sum_{\alpha''} \int dp' \Theta(p'^2 - 1/4q^2)
\]
\[
\int_{-1}^{1} dx \ t_{\bar{\alpha}\alpha l\alpha'} t_{\alpha''}(p, p'; q) \frac{p'q'^2}{\sqrt{p'^2 - \frac{1}{4}q^2(1 - x^2)}} G_{\bar{\alpha}\alpha}^{BB}(q, q'', x)
\]
\[
(\delta_{\alpha''\alpha'}|p'' q''\alpha''|\hat{T}|\Phi \rangle + \bar{\delta}_{\alpha''\alpha'}^{\alpha''}(p'' q''\alpha''|T|\Phi) \frac{1}{E + i\epsilon - \frac{3}{4m}(p'^2 + \frac{3}{4}q'^2)} ,
\]

with
\[
p'' = \sqrt{(q - \frac{1}{2}q'')^2 + qq''(1 + x)} ,
\]
ad
\[
q'' = \sqrt{p'^2 - \frac{1}{4}q^2(1 - x^2)} - \frac{qx}{2} .
\]

Though the free propagator singularity is again a simple pole in \(p'\) a two-fold integration in the \(T\)-amplitude is required. This appears to be a disadvantage (though surmountable) against the case II.B. However, in the deuteron pole both integration variables \(p'\) and \(x\) occur, which is quite unfavorable.
F. Analytical integration over $x$ and $q''$

The first $\delta$-function in (9) is taken as in (13) and the second one as

$$\delta(p'' - \sqrt{q^2 + 1/4q'^2 + qq''x}) = \frac{4p''}{3q''} \delta(p'' - \sqrt{4/3(p'^2 - p''^2) + q^2}) \Theta(4/3(p'^2 - p''^2) + q^2).$$  
(36)

It results in ”the kernel”

$$\langle pq|tPG_0T|\Phi\rangle = \frac{8}{3q} \sum_{l_\alpha} \sum_{\alpha'} \int dp' p' \int dp'' p'' \Theta(4/3(p'^2 - p''^2) + q^2) \Theta(1 - |x_0|)$$

$$t^{\alpha j,\alpha}_{\alpha l}(p, p'; E(q)) G^{BB}_{\alpha\alpha'}(q, q'', x_0)$$

$$\langle p''q''|\alpha''|T|\Phi\rangle$$

$$\delta_{\alpha''\alpha'} \frac{1}{E - E_d + i\epsilon - \frac{1}{m}(p'^2 - p''^2) - \frac{3}{4m}q^2} + \delta_{\alpha''\alpha'} \langle p''q''|\alpha''|T|\Phi\rangle + i\epsilon - \frac{1}{m}(p'^2 + \frac{3}{4}q^2),$$  
(37)

with

$$q'' = \sqrt{\frac{4}{3}(p'^2 - p''^2) + q^2}. \quad (38)$$

Though the free propagator singularity is just a simple pole in one variable in the deuteron pole both integration variables $p'$ and $p''$ occur, which is less favorable.

We conclude that case $\text{BB}$ is clearly the most favorable choice and we compare in the next section results for different 3N observables choosing our standard approach, case $\text{AA}$ and that new one, case $\text{BB}$.

III. COMPARISON OF THE NEW (BB) AND STANDARD (AA) APPROACHES

Since the complicated singularity pattern in the old approach exists only for $q'' \leq q_{max}$ we applied the new approach only there and kept the old one for $q'' > q_{max}$, where only a simple deuteron pole is present. We used the CD Bonn [9] potential restricted to act in the two-nucleon partial wave states with total angular momentum $j \leq 1$. In Fig. 2 we show the resulting nd elastic scattering angular distributions for the cross section and various analyzing powers at an incoming neutron lab. energy $E_{\text{lab}}^n = 13$ MeV. The agreement obtained with the two approaches is very good. The cross sections at the same energy for two geometries of the Nd breakup are shown in Fig. 3. Again the agreement is very good.
IV. FINITE RANK FORCES

Using the choice of II B simplifies the treatment of the 3N continuum in case of finite rank 2-body forces also very significantly since there are no longer logarithmic singularities. For the sake of a simple notation we keep only s-waves and restrict the 2-body force to act only in the states \(^1S_0\) and \(^3S_1\). This leads to two coupled equations for the two amplitudes \(T_k(pq)\) where \(k = 1\) (\(k = 2\)) goes with \(^1S_0\) (\(^3S_1\), respectively. Choosing the kernel of the type \([23]\) one obtains explicitly

\[
T_1(pq) = T_1^0(pq) + \frac{2}{q} \int_0^{\infty} dp' p' t_1(p, p'; E(q)) \frac{1}{E + i\epsilon - \frac{1}{m}(p'^2 + \frac{3}{4}q'^2)} \\
\int_{|q/2-p'|}^{\infty} dq'' q''(G_{11}T_1(p''q'') + G_{12} \frac{\hat{T}_2(p''q'')}{E_d + \frac{1}{m}p'^2}) \\
- \frac{2}{q} \int_0^{\infty} dq'' q'' \frac{1}{E + i\epsilon - \frac{3}{4m}q'^2 - E_d} \int_{|q/2-q''|}^{\infty} dp' p' t_1(p, p'; E(q)) \frac{\hat{T}_2(p''q'')}{E_d + \frac{1}{m}p'^2} \tag{39}
\]

\[
\hat{T}_2(pq) = \hat{T}_2^0(pq) + \frac{2}{q} \int_0^{\infty} dp' p' \hat{t}_2(p, p'; E(q)) \frac{1}{E + i\epsilon - \frac{1}{m}(p'^2 + \frac{3}{4}q'^2)} \\
\int_{|q/2-p'|}^{\infty} dq'' q''(G_{21}T_1(p''q'') + G_{22} \frac{\hat{T}_2(p''q'')}{E_d + \frac{1}{m}p'^2}) \\
- \frac{2}{q} \int_0^{\infty} dq'' q'' \frac{1}{E + i\epsilon - \frac{3}{4m}q'^2 - E_d} \int_{|q/2-q''|}^{\infty} dp' p' \hat{t}_2(p, p'; E(q)) \frac{\hat{T}_2(p''q'')}{E_d + \frac{1}{m}p'^2} \tag{40}
\]

with

\[
p'' = \sqrt{p'^2 + \frac{3}{4}q'^2 - \frac{3}{4}q''^2}. \tag{41}
\]

Now we assume the finite rank forms

\[
t_1(pp'; E(q)) = g_1(p)\tau_1(E(q))g_1(p'), \\
t_2(pp'; E(q)) = g_2(p)\frac{\tau_2(E(q))}{E + i\epsilon - \frac{3}{4m}q'^2 - E_d}g_2(p'), \tag{42}
\]

and obtain

\[
T_1(pq) = T_1^0(pq) + g_1(p)\tau_1(E(q)) \frac{2}{q} \int_0^{\infty} dp' p' g_1(p') \frac{1}{E + i\epsilon - \frac{1}{m}(p'^2 + \frac{3}{4}q'^2)} \\
\int_{|q/2-p'|}^{\infty} dq'' q''(G_{11}T_1(p''q'') + G_{12} \frac{\hat{T}_2(p''q'')}{E_d + \frac{1}{m}p'^2}) \\
- \frac{2}{q} \int_0^{\infty} dq'' q'' \frac{1}{E + i\epsilon - \frac{3}{4m}q'^2 - E_d} \int_{|q/2-q''|}^{\infty} dp' p' \frac{g_1(p')}{E + i\epsilon - \frac{3}{4m}q'^2 - E_d} \frac{\hat{T}_2(p''q'')}{E_d + \frac{1}{m}p'^2} \tag{43}
\]

\[
\hat{T}_2(pq) = \hat{T}_2^0(pq) + g_2(p)\tau_2(E(q)) \frac{2}{q} \int_0^{\infty} dp' p' g_2(p') \frac{1}{E + i\epsilon - \frac{1}{m}(p'^2 + \frac{3}{4}q'^2)} \\
\int_{|q/2-p'|}^{\infty} dq'' q''(G_{21}T_1(p''q'') + G_{22} \frac{\hat{T}_2(p''q'')}{E_d + \frac{1}{m}p'^2}) \\
- \frac{2}{q} \int_0^{\infty} dq'' q'' \frac{1}{E + i\epsilon - \frac{3}{4m}q'^2 - E_d} \int_{|q/2-q''|}^{\infty} dp' p' \frac{g_2(p')}{E + i\epsilon - \frac{3}{4m}q'^2 - E_d} \frac{\hat{T}_2(p''q'')}{E_d + \frac{1}{m}p'^2} \tag{44}
\]

\[\text{11}\]
where going through the same steps it results

\[
T_1^0(pq) = N g_1(p) \tau_1(E(q)) F_1^0(q), \\
T_2^0(pq) = N g_2(p) \tilde{\tau}_2(E(q)) \hat{F}_2^0(q).
\] (45)

The normalisation factor \(N\) provides the dependence on spin and isospin quantum numbers \((m_d, m_0)\) are spin projections of the initial deuteron and nucleon, respectively, and \(\nu_0\) the nucleon’s isospin projection), on the initial momentum \(q_0\) and the deuteron normalisation factor \(N_d\) defined as \(\phi_d(p) = N_d g_2(p)\),

\[
N = \frac{1}{\sqrt{4\pi}} \delta_{M_T,\nu_0} (1 \frac{1}{2}, m_d, m_0, M) \frac{N_d}{q_0}.
\] (46)

Further \(F_1^0(q)\) and \(\hat{F}_2^0(q)\) are given as

\[
F_1^0(q) = G_{12} \frac{2}{q} \int_{|q_0 - q/2|}^{q_0 + q/2} dp' p' g_1(p') \frac{g_2(\sqrt{p'^2 + \frac{3}{q_0^2}} - \frac{3}{q_0^2})}{E_d - \frac{p'^2 + \frac{3}{q_0^2} - \frac{3}{q_0^2} - \frac{3}{q_0^2}}}{}, \]

\[
\hat{F}_2^0(q) = G_{22} \frac{2}{q} \int_{|q_0 - q/2|}^{q_0 + q/2} dp' p' g_2(p') \frac{g_2(\sqrt{p'^2 + \frac{3}{q_0^2}} - \frac{3}{q_0^2})}{E_d - \frac{p'^2 + \frac{3}{q_0^2} - \frac{3}{q_0^2} - \frac{3}{q_0^2}}}{}. \] (47)

It follows the structures

\[
T_1(pq) = g_1(p) \tau_1(E(q)) F_1(q), \\
\hat{T}_2(pq) = g_2(p) \tilde{\tau}_2(E(q)) \hat{F}_2(q),
\] (48)

and therefore one obtains the two coupled one-dimensional equations inserting explicitly the integration limits

\[
F_1(q) = F_1^0(q) + \frac{2}{q} \int_{0}^{\infty} dp' p' g_1(p') \frac{1}{E + i\epsilon - \frac{1}{m}(p'^2 + \frac{3}{q_0^2})} \int_{|q_2 - p'|}^{q_2 + p'} dq'' q'' (G_{11} g_1(p'') \tau_1(E(q'')) F_1(q'')) + G_{12} g_2(p'') \tilde{\tau}_2(E(q'')) \hat{F}_2(q'')) \frac{G_{12}}{E_d + \frac{1}{m}p'^2} \int_{|q_2 - q'|}^{q_2 + q''} dp' p' g_1(p') \frac{g_2(p'') \tilde{\tau}_2(E(q'')) \hat{F}_2(q''))}{E_d + \frac{1}{m}p'^2}, \]

\[
\hat{F}_2(pq) = \hat{F}_2^0(q) + \frac{2}{q} \int_{0}^{\infty} dp' p' g_2(p') \frac{1}{E + i\epsilon - \frac{1}{m}(p'^2 + \frac{3}{q_0^2})} \int_{|q_2 - p'|}^{q_2 + p'} dq'' q'' (G_{21} g_1(p'') \tau_1(E(q'')) F_1(q'')) + G_{22} g_2(p'') \tilde{\tau}_2(E(q'')) \hat{F}_2(q'')) \frac{G_{22}}{E_d + \frac{1}{m}p'^2} \int_{|q_2 - q'|}^{q_2 + q''} dp' p' g_2(p') \frac{g_2(p'') \tilde{\tau}_2(E(q'')) \hat{F}_2(q''))}{E_d + \frac{1}{m}p'^2}. \] (49)
It remains to provide the factors

\[ G_{11} = \frac{\sqrt{2}}{8} = G_{22}, \]
\[ G_{12} = -\frac{3\sqrt{2}}{8} = G_{21}. \] (51)

Note for \( E = \frac{3}{4m}q^2 \) there is no singularity at \( p' = 0 \) since the \( q'' \)-integral vanishes at \( p' = 0 \). Also there occur only simple poles, which can be treated by subtraction. Using for instance Spline interpolation for \( F_1(q'') \) and \( \hat{F}_2(q'') \) based on a set of grid points the two integrals can be trivially performed and one obtains a low dimensional inhomogeneous algebraic set of equations.

V. SUMMARY AND CONCLUSIONS

Starting from a partial wave decomposed form of the 3N Faddeev equation for a breakup amplitude we discussed the six possible choices of integrating over internal angular and momentum variables in the integral kernel. While our standard approach integrates over the moving logarithmic singularities along the real momentum axis we found a new one, which totally avoids that technical obstacle. It is very simple. The free 3N propagator singularity appears as a pole in a single variable, which can be taken care of trivially (like in the 2-body Lippmann-Schwinger equation). The deuteron pole singularity is again a simple pole and moreover the two poles are cleanly separated in two different integration variables.

The other four choices for internal integration variables turned out to be less favorable.

We numerically compared the two approaches evaluating some nd elastic and breakup observables and found very good agreement. This should open the door to handle the 3N continuum as simply as solving the 2-body Lippmann-Schwinger equation - though of course some more variables occur. It will also simplify the application to electromagnetic processes in the 3N system, where initial and final state interactions have to be treated properly.

We also draw attention to the Balian-Berézin method, which was proposed long time ago, and which deserves much more attention than it received up to now.

The inclusion of 3N forces do not change the singularity structure of the kernel and can be equally well treated in the new approach but is left to a forthcoming study.

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The numerical calculations were performed on the IBM Regatta p690+ of the NIC in Jülich, Germany.

**APPENDIX A: PERMUTATION OPERATOR**

In view of using the Balian-Berézin method \[38, 39\] for evaluation of the permutation matrix element \(|p'q'\alpha'|p''q''\alpha''\rangle\) it is adequate to change from \(jJ\) coupling to \(LS\) coupling

\[
|pq\alpha\rangle = \sum_\beta |pq\beta\rangle <\beta|\alpha\rangle , \tag{A1}
\]

where

\[
<\beta|\alpha\rangle = <(l\lambda)L(s12)S(LS)J(t12)T|(ls)j(12)I|(jI)J(t12)T > = \sqrt{jILS} \begin{pmatrix} l & s & j \\ \lambda & \frac{1}{2} & I \end{pmatrix} \begin{pmatrix} L \\ S \\ J \end{pmatrix} . \tag{A2}
\]

The permutation matrix element is \[2\]

\[
\langle p'q'\beta'|P|p''q''\beta''\rangle = 2\delta_{SS'}\delta_{T'T'}\delta_{L'L'}\delta_{\mu'\mu''} \langle -s''\sqrt{s' s''} \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & s' \\ \frac{1}{2} & \frac{1}{2} & t' \end{pmatrix} (-t''\sqrt{t' t''}) \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & s'' \\ \frac{1}{2} & \frac{1}{2} & t'' \end{pmatrix} \langle p'q'(l'\lambda')L'\mu'P_{12}P_{23}|p''q''(l''\lambda'')L''\mu' > . \tag{A3}
\]

Since the momentum space matrix element in \[A3\] is independent of \(\mu\) one can put

\[
\langle p'q'(l'\lambda')L'\mu'P_{12}P_{23}|p''q''(l''\lambda'')L''\mu' > = \frac{1}{2L' + 1} \sum_\mu \langle p'q'(l'\lambda')L'\mu|P_{12}P_{23}|p''q''(l''\lambda'')L''\mu > . \tag{A4}
\]

Further we use

\[
<\vec{p}'\vec{q}'|pq(l\lambda)L\mu > = \frac{\delta(p' - p)\delta(q' - q)}{p^2 q^2} \gamma^{L\mu}_{l\lambda}(\vec{p}'\vec{q}') , \tag{A5}
\]

and the linear relations among the Jacobi momenta of different types and obtain

\[
\langle p'q'(l'\lambda')L'\mu|P_{12}P_{23}|p''q''(l''\lambda'')L''\mu > = \frac{1}{2L' + 1} \sum_\mu \int d\vec{p}'d\vec{q}'d\vec{p}''d\vec{q}'' \gamma^{L'\mu}_{l'\lambda'}(\vec{p}'\vec{q}')\gamma^{L''\mu}_{l''\lambda''}(\vec{p}''\vec{q}'') \delta(\vec{p}' - 1/2\vec{q}' - \vec{q}'')\delta(\vec{p}'' + \vec{q}' + 1/2\vec{q}'') . \tag{A6}
\]

Here enters the idea of Balian-Berézin \[38\]. The sum over the products of spherical harmonics is a scalar and depends only on the scalar products among the four unit vectors. Due to the two
\[ X(p' q' p'' q'') \equiv \frac{1}{2L^* + 1} \sum_{\mu} Y^L_{\mu, \lambda}(\hat{p}' \hat{q}') Y^L_{\mu, \lambda}(\hat{p}'' \hat{q}''). \]

Consequently \( X \) can be taken out of the integral and one obtains

\[ \langle p' q'(l' \lambda') L' | P_{12} P_{23} | p'' q''(l'' \lambda'') L' \rangle = \frac{8 \pi^2}{x} \int_{-1}^{1} dx \ X(p' q' p'' q'') \delta(p' - |1/2\hat{q}'' + \hat{q}'|) \delta(p'' - |\hat{q}'' + 1/2\hat{q}'|), \]

where \( x = \hat{q}' \cdot \hat{q} \).

Finally we combine geometrical factors and write the permutation matrix element in the form

\[ \langle pq\alpha | P | p' q' \alpha' \rangle = \int_{-1}^{1} dx \ \frac{\delta(p - \pi_1)}{p^2} \frac{\delta(p' - \pi_2)}{p'^2} G_{\alpha \alpha'}^{BB}(q, q', x), \]

with

\[ \pi_1 = \sqrt{q^2 + \frac{1}{4}q^2 + qq'x} \]
\[ \pi_2 = \sqrt{q^2 + \frac{1}{4}q^2 + qq'x}, \]

and

\[ G_{\alpha \alpha'}^{BB}(q, q') = (4\pi)^{3/2} \delta_{TT'}\delta_{M_T M_{T'}} \sqrt{\hat{j}_I \hat{s}_I} \sqrt{\hat{j}' I' \hat{s}' I'} (-1)^{s' + t} \left\{ \begin{array}{c} 1/2 \ 1/2 \ t \\ 1/2 \ T \ t' \end{array} \right\} \sum_{LS} \hat{S}_{\lambda \lambda'} \left\{ \begin{array}{c} l \ s \ j \ \\ \frac{1}{2} \ I \ \\ L \ S \ J \end{array} \right\} \left\{ \begin{array}{c} l' \ s' \ j' \ \\ \frac{1}{2} \ I' \ \\ L' \ S' \ J' \end{array} \right\} \frac{1/2}{1/2} \frac{1/2}{s} \sum_{m_l m_{\lambda'} m_{\lambda'}} (l \lambda L, m_l 0 m_{\lambda})(l' \lambda' L, m_{\lambda'}, m_{\lambda'}) \left( \hat{\delta}_{m_l m_{\lambda} m_{\lambda'}} \right). \]

We use our standard notation \( I \equiv 2l + 1 \). It is assumed that the z-axis is along \( \hat{q} \) and the momentum \( \hat{q}' \) lies in the x-z plane. That leads to the following components of the \( \hat{q}, \hat{q}', \hat{p}, \) and \( \hat{p}' \) vectors

\[ \hat{q} = [0, 0, q], \]
\[ \hat{q}' = [q'\sqrt{1-x^2}, 0, q'x], \]
\[ \hat{p} = [q'\sqrt{1-x^2}, 0, q'x + \frac{1}{2}q], \]
\[ \hat{p}' = [-\frac{1}{2}q'\sqrt{1-x^2}, 0, -q - \frac{1}{2}q'x]. \]

\[ \text{(A12)} \]
[1] L.D. Faddeev, Sov. Phys. JETP 12, 1014 (1961); L.D. Faddeev, Mathematical aspects of the three body problem in quantum scattering theory (Davey, New York, 1965); L.D. Faddeev, S.P. Merkuriev, Quantum scattering theory for several particle systems (Kluwer Academic Publishers, Dordrecht, 1993).

[2] W. Glöckle, The Quantum Mechanical Few-Body Problem (Springer-Verlag 1983).

[3] E.W. Schmidt, H. Ziegelmann, The Quantum Mechanical Three-Body Problem (Pergamon Press, Oxford, 1974).

[4] J. Haidenbauer, Y. Koike, W. Plessas, Phys. Rev. C 33, 439 (1986).

[5] W.M. Kloet, J.A. Tjon, Nucl. Phys. A 210, 380 (1973).

[6] H. Witala, W. Glöckle, Th. Cornelius, Few-Body Systems, Suppl. 2, 555 (1987).

[7] W. Glöckle, G. Hasberg, A.R. Neghabian, Z. Phys. A 305, 217 (1982).

[8] R.B. Wiringa, V.G.J. Stoks, R. Schiavilla, Phys. Rev. C 51, 38 (1995).

[9] R. Machleidt, F. Sammarruca, and Y. Song, Phys. Rev. C 53, R1483 (1996).

[10] V.G.J. Stoks, R.A.M. Klomp, C.F. Terheggen, J.J. de Swart, Phys. Rev. C 49, 2950 (1994).

[11] S.A. Coon et al., Nucl. Phys. A 317, 242 (1979); S.A. Coon and W. Glöckle, Phys. Rev. C 23, 1790 (1981).

[12] B.S. Pudliner et al., Phys. Rev. C 56, 1720 (1997).

[13] W. Glöckle, H. Witala, D. Hüber, H. Kamada, J. Golak, Phys. Rep. 274, 107 (1996).

[14] C.R. Howell et al., Phys. Rev. Lett. 61, 1565 (1988).

[15] M. Stephan et al., Phys. Rev. C 39, 2133 (1989).

[16] G. Rauprich et al., Nucl. Phys. A 535, 313 (1991).

[17] J.E. McAninch et al., Phys. Lett. B 307, 13 (1993).

[18] L. Sydow et al., Nucl. Phys. A 567, 55 (1994).

[19] M. All et al., Phys. Lett. B 376, 255 (1996).

[20] H.R. Setze et al., Phys. Lett. B 388, 229 (1996).

[21] H. Rohdjess et al., Phys. Rev. C 57, 2111 (1998).

[22] W.P. Abfalterer et al., Phys. Rev. Lett., 81, 57 (1998).

[23] H. Sakai et al. Phys. Rev. Lett. 84, 5288 (2000).

[24] R. Bieber et al., Phys. Rev. Lett. 84, 606 (2000).

[25] R.V. Cadman et al., Phys. Rev. Lett. 86, 967 (2001).

[26] K. Ernisch et al., Phys. Rev. Lett. 86, 5862 (2001).

[27] K. Hatanaka et al., Phys. Rev. C 66, 044002 (2002).

[28] K. Sekiguchi et al., Phys. Rev. C 70, 014001 (2004).

[29] K. Ernisch et al., Phys. Rev. C 71, 064004 (2005).

[30] H.R. Amir-Ahmadi et al., Phys. Rev. C 75, 041001(R) (2007).

[31] H. Witala, W. Glöckle, D. Hüber, J. Golak, and H. Kamada, Phys. Rev. Lett. 81, 1183 (1998).
[32] H. Witala et al., Phys. Rev. C 63, 024007 (2001).
[33] H. Witala, J. Golak, W. Glöckle, H. Kamada, Phys. Rev. C 71, 054001 (2005).
[34] K. Sekiguchi et al., Phys. Rev. Lett. 95, 162301 (2005).
[35] E. Epelbaum, Prog. Part. Nucl. Phys. 57, 654 (2006).
[36] J. Golak, R. Skibiński, H. Witala, W. Glöckle, A. Nogga, and H. Kamada, Phys. Rep. 415, 89 (2005).
[37] D. Hüber, H. Kamada, H. Witala, W. Glöckle, Few-Body Systems 16, 165 (1994).
[38] R. Balian, E. Berézin, Nuovo Cimento B II2, 403 (1969).
[39] B.D. Keister and W.N. Polyzou, Phys. Rep. C 73, 014005 (2003).
FIG. 1: The domain for the integrations over $p'$ and $q''$ (rectangular region contained between the three lines) in the case II B.
FIG. 2: The $^7$Li elastic scattering angular distribution and analyzing powers at an incoming neutron lab. energy $E_{\text{lab}}^n = 13$ MeV. The solid line is the CD Bonn potential prediction using our standard approach of handling the logarithmic singularities. The dotted line is obtained with the new approach without logarithmic singularities. The 2-nucleon states are kept up to $j_{\text{max}} = 1$.
FIG. 3: Cross sections for the exclusive d(n,nn)p breakup at an incoming neutron lab. energy $E_{\text{lab}}^n = 13$ MeV. Curves as in Fig 2. The upper configuration is the final-state-interaction (FSI) geometry with the polar angles of the detected neutrons $\theta_1 = 39^\circ$, $\theta_2 = 62.5^\circ$ and the azimuthal angle $\phi_{12} = 180^\circ$. The lower configuration is the quasi-free-scattering (QFS) geometry with the polar angles of detected neutrons $\theta_1 = \theta_2 = 39^\circ$ and the azimuthal angle $\phi_{12} = 180^\circ$. 

$\frac{d\sigma}{d\Omega_1 d\Omega_2 dS} \text{[mb sr}^{-2} \text{MeV}^{-1}]$