Heat Transfer of $\text{Ag-Al}_2\text{O}_3/\text{Water}$ Hybrid Nanofluid on a Stagnation Point Flow over a Stretching Sheet with Newtonian Heating

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Abstract. The present study investigated the flow and heat transfer at a stagnation point past a stretching sheet. The premium silver nanoparticles Ag and economic alumina nanoparticles Al$_2$O$_3$ suspended in water to form $\text{Ag-Al}_2\text{O}_3/\text{Water}$ hybrid nanofluid are numerically examined. The analysis started with transforming the mathematical model which is in non-linear partial differential equations to a more convenient form by similarity transformation approach before being solved numerically using the Runge-Kutta-Fehlberg (RKF45) method. The characteristics and effects of the stretching parameter, conjugate parameter and the nanoparticle volume fraction for Al$_2$O$_3$ and Ag on the variation of wall temperature, heat transfer coefficient and reduced skin friction coefficient are analyzed and discussed.

1. Introduction

Nanofluid is a fluid containing the engineered colloidal suspensions of ultrafine particles (nanoparticles) in which the diameter is smaller than 50 nanometer size in a base fluid [1]. Nanofluid is experimentally proven in enhancing the thermal conductivity, viscosity, thermal diffusivity and convective heat transfer compared to those base fluids like water and oil [2].

The metal nanoparticles and carbon nanomaterials performed better in heat transfer compared to oxide nanoparticles. Unfortunately, this type of nanomaterial is expensive and not practical in mass production. Therefore, the used of oxide nanoparticles is more realistic to the economy. The evolution study on nanofluid found that the incorporation of a small amount of metal nanoparticles and oxide nanoparticles suspended in a based-fluid can significantly improve the thermal properties. This type of fluid is called as hybrid nanofluid [3]. The hybrid nanofluid specifically provided higher effective thermal conductivity and heat transfer abilities compared to oxide nanofluid but low in production cost than metal nanofluid.

In considering the manufacturing process which involved the stretching sheet activity, the quality of the final product depends to a large extent on the stretching rate and the rate of heat transfer on
stretching sheet. The convection flow past a stretching sheet was first studied by Crane [4]. The investigations on the flow of a stretching sheet were then extended to the stagnation region with considering other types of fluids such as viscoelastic fluid, nanofluid, micropolar fluid, Jeffrey fluid, Casson fluid and ferrofluid [5, 6, 7, 8, 9, 10, 11, 12]. This topic becoming more attractive year by years with extending it with other external effects like the thermal radiation effect, the chemical reaction, the slip flow, the viscous dissipation, the suction/injection, the magnetohydrodynamic (MHD) field as well as the Newtonian heating boundary conditions[13, 14, 15, 16, 17, 18].

The experimental study regarding this topic is expensive and difficult to be realized hence provided limited findings and knowledge. Thus, the approach from a mathematical model is the alternative and relevant way to be considered. This approached is cheap, fast and provided the theoretical knowledge for the hybrid nanofluid therefore proposed an early idea about the fluid flow and heat transfer characteristics. Motivated from the above literature, the aim of this study is to investigate the fluid flow and heat characteristics of the premium silver Ag nanoparticles blend with economic Alumina Al2O3 in Ag-Al2O3/Water hybrid nanofluid on a stagnation point over a stretching sheet. A study on hybrid nanofluid on a stretching sheet combined with the Newtonian heating has never been done before, so the reported results in this study are new.

2. Mathematical Formulation

Figure 1 illustrates a steady two-dimensional stagnation point flow over a stretching sheet immersed in hybrid nanofluid with ambient temperature, \( T_\infty \). Assuming that \( u \) and \( v \) are the velocity components along the \( x \) and \( y \) axes, respectively. Next, the stretching velocity \( u_s(x) = ax \) and the free stream velocity \( U_s = bx \) are assumed in linear forms where \( a \) and \( b \) are positive constants [19]. Further, the stretching sheet is subjected to a Newtonian heating boundary condition as proposed by Merkin [20]. The Navier-Stoke equations can be governed as follows:

\[
\begin{align*}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0, \\
\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= U \frac{dU_s}{dx} + \nu \frac{\partial^2 u}{\partial y^2}, \\
\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= \frac{\partial^2 T}{\partial y^2},
\end{align*}
\]

with boundary conditions

\[
\begin{align*}
u &= u_s, \quad v = 0, \quad \frac{\partial T}{\partial y} = -hT \quad \text{at} \quad y = 0, \\
u &\to U_s, \quad T \to T_\infty, \quad \text{as} \quad y \to \infty.
\end{align*}
\]

The hybrid nanofluid kinematic viscosity, dynamic viscosity and its density is denoted as \( \nu_{\text{hf}} \), \( \mu_{\text{hf}} \), and \( \rho_{\text{hf}} \), respectively. \( T \) is the temperature inside the boundary layer, \( (\rho C_p)_\text{hf} \) is the heat capacity of hybrid nanofluid and \( k_{\text{hf}} \) is the thermal conductivity of hybrid nanofluid. Other properties related to base fluid and the nanoparticles are denoted with subscript , and \( s \) respectively as follows[3]:

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nanofluid to form the hybrid nanofluid namely Ag/Al₂O₃. In this study, initially 0.06 vol. solid nanoparticle of Ag (ϕ₂ = 0.06) is added into water based-fluid to form Ag/Water nanofluid. Next, the 0.1 solid nanoparticle of Al₂O₃ (ϕ₁ = 0.1) is added into Ag/Water nanofluid to form the hybrid nanofluid namely Ag-Al₂O₃/Water. Noticed that the equations (1)-(3) are non-linear partial differential equations which consist many dependent and independent variables. It is also in dimensional forms which is difficult to solve directly. Therefore, the similarity transformation approach is applied:

\[
\eta = \left[ \frac{y}{v_f} \right], \quad \psi = \left( b v \right)^{1/3} f \left( \eta \right), \quad \theta \left( \eta \right) = \frac{T - T_w}{T_{\infty}}.
\]

Equation (6) shows the similarity variables where \( \eta, \psi \) and \( \theta \) is a non-dimensional variable, stream function and temperature, respectively. The similarity variables (6) satisfy the continuity equation (1) by definition

\[
u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = - \frac{\partial \psi}{\partial x}.
\]

Next, substitute the similarity variables equations (6) and (7) into governing equations (2) and (3) gives the following transformed ordinary differential equations:

\[
\frac{1}{1-\phi} \left[ \left( 1 - \phi \right) \left( 1 - \phi + \phi \left( \rho C_p \right)_{1s} \right) + \phi \left( \rho C_p \right)_{s2} \right] \frac{\partial f}{\partial \eta}^{2} + \frac{f}{2} - \frac{\partial f}{\partial \eta}^{2} + 1 = 0
\]

\[
\frac{\partial \theta}{\partial \eta} + Pr \frac{\partial f}{\partial \eta} \theta' = 0.
\]
The boundary conditions becomes

\[ f(0) = 0, \quad f'(0) = \varepsilon, \quad \theta'(0) = -\gamma (1 + \theta(0)), \]
\[ f'(\eta) \to 1, \quad \theta(\eta) \to 0, \text{ as } \eta \to \infty. \]  

(10)

By definition, \( \Pr = \frac{k_f}{\nu_f} \) is a Prandtl number which will be set as 6.2 in calculation with respect to water-based fluid, \( \varepsilon = \frac{a}{b} \) (\( \varepsilon > 0 \)) is a stretching parameter and \( \gamma = h \left( \frac{b}{\nu_f} \right)^{1/2} \) is a conjugate parameter. The physical quantities interested are the wall temperature \( \theta(0) \), the heat transfer rate \( -\theta'(0) \) and the skin friction coefficient \( C_f \) which given by

\[ C_f = \frac{\tau_w}{\nu_f}, \]

with the surface shear stress \( \tau = \mu \left( \frac{\partial \theta}{\partial y} \right) \). The skin friction coefficient \( C_f \) can be reduced

\[ C_f \Re_f^{1/2} = \frac{f''(0)}{(1 - \phi)^{2.5} (1 - \phi)^{1.5}}, \]

(12)

where \( \Re_x = \frac{U_x x}{\nu_f} \) is the Reynolds number.

### 3. Results and Discussion

The system of ordinary differential equations (8) and (9) with boundary conditions (10) were solved numerically using the Runge-Kutta-Fehlberg (RKF45) technique. The numerical results obtained for the wall temperature \( \theta(0) \), the heat transfer rate \( -\theta'(0) \) and the reduced skin friction coefficient \( C_f \Re_f^{1/2} \) for a various values of stretching parameter \( \varepsilon \), conjugate parameter \( \gamma \) and the nanoparticle volume fraction for alumina \( Al_2O_3 \) \( (\phi_1) \) and silver \( Ag \) \( (\phi_2) \). For computing purposes, the boundary layer thickness from 4 to 12 is considered to provide the accurate numerical results for \( Ag/\text{Water} \) nanofluid and \( Ag-Al_2O_3/\text{Water} \) hybrid nanofluid. The values of thermophysical properties of water and nanoparticles consider are tabulated in Table 1.

In order to validate the numerical results obtained, the comparison has been made. Table 2 shows the comparison values of \( C_f \Re_f^{1/2} \) with previous results by Bachok et al. [5] and Yacob et al. [6]. It is found that the numerical results are in good agreement.

| Physical Properties | Water \((f)\) | \(Al_2O_3(\phi_1)\) | \(Ag(\phi_2)\) |
|---------------------|--------------|------------------|---------------|
| \(\rho(\text{kg/m}^3)\) | 997          | 3970             | 10500         |
| \(C_p(\text{J/kg·K})\) | 4179         | 765              | 235           |
| \(k(\text{W/m·K})\)    | 0.613        | 40               | 429           |
Table 2. Comparison values of $C_{Re}^{1/2}$ for some values of $\varepsilon$ and $\phi$ for $Al_2O_3/ Water$ nanofluid.

| $\varepsilon$ | $\phi_1$ | Bachok et al. [5] | Yacob et al. [6] | Present      |
|-----------|-------|-----------------|-----------------|-------------|
| 0         | 0.1   | 1.6019          | 1.6019          | 1.602081    |
| 0.2       | 2.0584| 2.0584          | 2.058376        |             |
| 0.5       | 0.1   | 0.9271          | -               | 0.927121    |
| 0.2       | 1.1912| -               | 1.191179        |             |

Figure 2. Variation of $\theta(0)$ with various values of $\varepsilon$ when $Pr = 6.2$ and $\gamma = 1$.

Figure 3. Variation of $C_{Re}^{1/2}$ with various values of $\varepsilon$ when $Pr = 6.2$ and $\gamma = 1$.

Figure 4. Variation of $\theta(0)$ with various values of $\gamma$ when $Pr = 6.2$ and $\varepsilon = 1$.

Figure 5. Variation of $-\theta'(0)$ with various values of $\gamma$ when $Pr = 6.2$ and $\varepsilon = 1$.

Figure 2 shows the variation of the wall temperature $\theta(0)$ for various values of the stretching parameter $\varepsilon$. It is found that the wall temperature is decreasing with the increase of $\varepsilon$ same as reported by Mohamed et al. [19]. In considering the effect of the nanoparticles on fluid, from Figure 2, the temperature raised when 0.06 vol. of silver $Ag$ nanoparticles is added up into water-based fluid. The temperature then drastically increases with the adding of $Al_2O_3$ nanoparticles into $Ag/ Water$ to form the $Ag – Al_2O_3 / Water$ hybrid nanofluid. The effects of hybrid nanoparticles are more
significant at a small ratio of stretching velocity over free stream velocity. As stretching velocity domination ($\varepsilon > 1$), the differences goes negligible. The variation of the reduced skin friction coefficient $C_f \sqrt{Re}$ for various values of $\varepsilon$ are illustrates in Figure 3. Generally, the skin friction is positive from $0 \leq \varepsilon < 1$ then turn negatives as $\varepsilon > 1$ due to the changes in velocity direction as stated in boundary conditions (4). As $\varepsilon = 1$, the values of $C_f \sqrt{Re}$ approaches $0$. This is realistic as the fluid flow at the ambient is equal to a fluid flow at a stretching sheet thus produce zero velocity gradient which reflects to $C_f \sqrt{Re}$. Next, it is suggested that the presence of nanoparticles has increase the fluid friction with surface. This is confirmed as $Ag-Al_2O_3/Water$ hybrid nanofluid with 0.06 vol. $Ag$ and 0.1 vol. $Al_2O_3$ nanoparticles has the highest $C_f \sqrt{Re}$ than 0.06 vol. $Ag/Water$ nanofluid and water-based fluid.

Lastly, Figures 4 and 5 show the variation of the wall temperature $\theta(0)$ and the heat transfer $-\theta'(0)$ coefficient for various values of the conjugate parameter $\gamma$, respectively. From Figures 4 and 5, it is observed that the values of $\theta(0)$ and $-\theta'(0)$ are increase as the values of $\gamma$ increases. Logically, the pattern of variation between $\theta(0)$ and $-\theta'(0)$ are same due to the heat transfer rate from the bounding surface with a finite heat capacity is proportional to the local surface temperature [20]. Further, from both Figures 4 and 5, the $Ag/Water$ nanofluid ($\phi_1 = 0, 0.01 < \phi_2 < 0.06$) produced higher $\theta(0)$ and $-\theta(0)$ than the water-based fluid. This physical quantities then raised drastically with $Ag-Al_2O_3/Water$ hybrid nanofluid ($\phi_1 = 0.1, 0.01 < \phi_2 < 0.06$). The effects of nanoparticles are more significant as $\gamma$ increases. From numerical computation, the increase in $\gamma$ gave no effect on the $C_f \sqrt{Re}$.

4. Conclusion
In this paper, the stagnation point flow of a $Ag-Al_2O_3/Water$ hybrid nanofluid on a stretching sheet with Newtonian heating was numerically studied. It was shown how the stretching parameter $\varepsilon$, conjugate parameter $\gamma$ and the nanoparticle volume fraction for $Al_2O_3$ and $Ag$ ($\phi_1, \phi_2$) affect the wall temperature, the heat transfer coefficient and the skin friction coefficient. It is found that the wall temperature and the heat transfer coefficient decrease as a stretching parameter increases while the conjugate parameter does the contrary. Next, it is observed that both quantities increase drastically when $Al_2O_3$ is added into $Ag/Water$ nanofluid. From the numerical calculation, it is suggested that the effects of hybrid nanoparticles are more significant at a lower stretching parameter and a higher conjugate parameter values. Lastly, the increase of nanoparticle volume fraction has increased the fluid friction with the surface.

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