Farfield and nearfield source identification for machine tools using Microphone array imaging systems

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Abstract

Farfield and nearfield microphone arrays are proposed for noise source identification (NSI) and sound field visualization (SFV). Farfield acoustic imaging algorithms including the delay and sum (DAS) algorithm, the minimum variance distortionless response (MVDR) algorithm and the multiple signal classification (MUSIC) algorithm are employed to estimate direction of arrival (DOA). Results demonstrate that the MUSIC algorithm can attain the highest resolution of localizing sound sources positions. In the nearfield array signal processing, one formulation termed the indirect equivalent source model (ESM)-based nearfield acoustical holography (NAH) is derived from discretizing the simple layer potential. As indicated by the experimental results, the proposed technique proved effective in identifying the noise sources from various machine tools such as milling machine, turning lathe and shearing machine.

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1. Introduction

Noise source identification (NSI) and sound field visualization (SFV) using microphone array are the important steps in noise analysis and diagnosis. Microphone array imaging techniques can be realized by using two categories of arrays[1], [2]: farfield beamforming[3]-[5] and nearfield acoustical holography (NAH).[6]-[10] Many farfield beamformers have been proposed in the past. For example, delay-and-sum (DAS) algorithm,[3] the minimum variance distortionless Response (MVDR) array[4] and the multiple signal classification (MUSIC) algorithm[5] are widely used in beamforming applications. The farfield array algorithms are particularly useful for long-distance and large scale sources such as trains and aircrafts. NAH methods appropriate for arbitrarily shaped source were suggested, e.g., the NAH based on inverse boundary element methods (IBEM)[6] the statistically optimal NAH (SONAH)[7] method and the Helmholtz Equation Least Squares (HELS)[8] method. Another method, the indirect equivalent Source model (ESM)[9], [10] also known as wave superposition method, were suggested for sound field calculation with far less complexity. The idea underlying the indirect ESM is to represents sound field with discrete simple sources with no need to perform numerical integration. As opposed to the actual source, these solutions of simple sources deduced from the acoustic wave equation serve as the basis for sound field representation. The simplicity of the indirect ESM lends itself very well to the implementation with digital signal processing and control paradigms. These nearfield techniques are well suited for imaging small-scale sources such as engine compartments and desktop computers by virtue of high resolution focusing algorithms. This study demonstrates that acoustical signal processing algorithms for farfield and nearfield imaging can be implemented using the DAS, MVDR, MUSIC beamformers and the indirect ESM-based NAH, respectively. To validate the proposed methods, experiments were conducted for a number of machine tool examples including milling machine, turning lathe and shearing machine. The experimental results of beam patterns, sound pressure and particle velocity fields using the proposed methods are shown and discussed in the paper.

2. Farfield array signal processing

2.1. Farfield array model

For the farfield array model, we assume that the sources are located far enough from the array that the wave fronts impinging on the array can be modeled as plane waves radiated by a point source. Consider a linear array comprised of \( M \) microphones distributed uniformly with inter-element spacing \( d \) in the \( x \) axis.[3]

With the time-harmonic dependence \( e^{j\omega t} \), the sound pressure field at the \( m \)th microphone can be written as

\[
x_m(x_m, \omega) = s(\omega) e^{j(\omega/\omega) \cdot x_m} + n(x_m, \omega), \quad m = 1, 2, \ldots, M
\]

where \( j = \sqrt{-1}, x_m \) is the position vector of the \( m \)th microphone, \( s(\omega) \) is the Fourier transform of the source signal, wave vector \( \mathbf{k} = -(\omega / c) \mathbf{\kappa} \) is the wave vector with \( \mathbf{\kappa} \) being the unit vector pointing from the array reference to the source, “\( \cdot \)” denotes inner product, \( c \) is the speed of sound, \( \omega \) is the angular frequency, and \( n(x_m, \omega) \) is the uncorrelated sensor noise. Assemble the microphone signals \( x_1(x_1, \omega), \cdots, x_M(x_M, \omega) \) into the data vector

\[
x(\omega) = a(\omega)s(\omega) + n(\omega),
\]
where \( \mathbf{x}(\omega) = \left[ x_1(\mathbf{x}_1, \omega) \ldots x_M(\mathbf{x}_M, \omega) \right]^T \), \( \mathbf{a}(\omega) = \left[ e^{j \omega x_1/c} \ldots e^{j \omega M/c} \right]^T \) is called the array manifold vector, superscript “\( T \)” denotes matrix transpose, and \( \mathbf{n}(\omega) = \left[ n(\mathbf{x}_1, \omega) \ldots n(\mathbf{x}_M, \omega) \right]^T \). For \( D \) sources, we may invoke the principle of superposition to write

\[
\mathbf{x}(\omega) = \sum_{i=1}^{D} \mathbf{a}(\theta_i) s_i(\omega) + \mathbf{n}(\omega) = \mathbf{A} \mathbf{s}(\omega) + \mathbf{n}(\omega). \tag{3}
\]

A beamformer can be regarded as a linear combiner

\[
y(\omega) = \mathbf{w}^H(\omega) \mathbf{x}(\omega), \tag{4}
\]

where \( \mathbf{w}(\omega) = \left[ w_1(\omega) \ldots w_M(\omega) \right]^T \) denotes the array weight vector, superscript “\( T \)” denotes matrix transpose and superscript “\( \mathbb{H} \)” denotes matrix Hermitian transpose.

### 2.2. Delay-And-Sum (DAS) beamformer

We consider a uniform rectangular array (URA) comprising \( I \times J \) microphones in a rectangular lattice with inter-element spacing \( d_x \) and \( d_y \) in the x and y axis, respectively (Fig. 1). Let the element at the upper left corner be the reference. The position vector of each microphone is given by

\[
\mathbf{x}_{ij} = ((i-1)d_x, (j-1)d_y, 0), \tag{5}
\]

where \( i = 1,2,\ldots,I \) and \( j = 1,2,\ldots,J \). The unit vector \( \mathbf{r} \) pointing to a sound source at the look directions \( \theta \) and \( \phi \) can be expressed in spherical coordinates as

\[
\mathbf{r} = (\sin \theta \sin \phi, \sin \theta \cos \phi, \cos \theta). \tag{6}
\]

The corresponding delay of each microphone can be written as

\[
\tau_{ij} = \frac{\mathbf{x}_{ij} \cdot \mathbf{r}}{c}. \tag{7}
\]

Then, the array manifold vector can be expressed as

\[
\mathbf{a}(\omega, \theta, \phi) = \left[ e^{j \omega d_x \sin \theta \sin \phi/c} e^{j \omega d_x \sin \theta \cos \phi/c} \ldots e^{j \omega d_y \sin \phi/c} \ldots e^{j \omega (1-I)d_x \sin \theta \sin \phi+(1-J)d_y \sin \theta \cos \phi/c} \right]. \tag{8}
\]

The output of DAS[3] beamformer can be expressed as
\[ y(t) = \frac{1}{IJ} \sum_{i=1}^{I} \sum_{j=1}^{J} x_{ij}(t - \tau_{ij}), \quad (9) \]

where \( x_{ij}(t) \) is the signal received by \( ij \)th microphone, \( \tau_{ij} \) are the steering delays to focus the array to the look direction.

### 2.3. Minimum variance distortionless response (MVDR)

The rationale underlying the MVDR\[4\] beamformer is to find array weights that satisfy the gain constraint at the look direction, \( \mathbf{w}^H \mathbf{a}(\theta_0) = 1 \), while attempting to minimize the array output power

\[
E\left\{ |y(n)|^2 \right\} = E\left\{ |\mathbf{w}^H \mathbf{x}(n)|^2 \right\} = E\left\{ \mathbf{w}^H \mathbf{x}(n) \mathbf{x}^H(n) \mathbf{w} \right\} = \mathbf{w}^H E\left\{ \mathbf{x}(n) \mathbf{x}^H(n) \right\} \mathbf{w} = \mathbf{w}^H \mathbf{R}_{xx} \mathbf{w},
\]

in order to suppress undesired interference and noise from \( \theta \neq \theta_0 \). This boils down to the constrained optimization problem expressed as follows:

\[
\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R}_{xx} \mathbf{w} \quad \text{st.} \quad \mathbf{w}^H \mathbf{a}(\theta_0) = 1
\]

This optimal weight can be solved by method of Lagrange multiplier

\[
\mathbf{w}_{MVDR} = \lambda \mathbf{R}_{xx}^{-1} \mathbf{a}(\theta_0) = \frac{\mathbf{R}_{xx}^{-1} \mathbf{a}(\theta_0)}{\mathbf{a}(\theta_0)^H \mathbf{R}_{xx}^{-1} \mathbf{a}(\theta_0)} \quad (12)
\]

which is data-dependent on \( \mathbf{R}_{xx} \). In case that \( \mathbf{R}_{xx} \) is rank-deficient, a remedy called “diagonal loading” can be used, i.e., \( \mathbf{R}_{xx} \to (\mathbf{R}_{xx} + \epsilon \mathbf{I}) \). By plotting the array output as the spatial power spectrum \( S_{MVDR}(\theta) \) by continuously varying \( \theta_0 \), we can locate the sources from the peaks of the spectrum.

\[
\theta_s = \arg \max_{\theta_0} S_{MVDR}(\theta_0), \quad (13)
\]

where \( S_{MVDR}(\theta_0) = \frac{1}{\mathbf{a}(\theta_0)^H \mathbf{R}_{xx}^{-1} \mathbf{a}(\theta_0)} \).

### 2.4. Multiple signal classification (MUSIC)

The key step of the MUSIC\[5\] algorithm is to calculate the eigenvalue decomposition (EVD) of the data correlation matrix:

\[
\mathbf{R}_{xx} = \mathbf{A} \mathbf{R}_{\mathbf{e}} \mathbf{A}^H + \sigma_n^2 \mathbf{I} = \mathbf{U} \mathbf{\Lambda} \mathbf{U}^H = \sum_{i=1}^{M} \lambda_i \mathbf{e}_i \mathbf{e}_i^H, \quad (14)
\]

where \( \lambda_1 \geq \cdots \geq \lambda_M \) are eigenvalues of \( \mathbf{R}_{xx} \) and \( \mathbf{U} = [\mathbf{e}_1 \cdots \mathbf{e}_M] \) is a unitary matrix comprising \( M \)
eigenvectors \( \mathbf{e}_i, i = 1, \ldots, M \). Because the signal subspace and the noise subspace are two orthogonal compliments in the \( \mathbb{C}^M \) space, we have

\[
\mathbf{e}_i^H \mathbf{a}(\theta) = 0, \ i = 1, \ldots, D ; j = D + 1, \ldots, M .
\]  

(15)

This property forms the cornerstone for all eigenspace-based methods. An alternative form of MUSIC spectrum was suggested in the literature.[5]

\[
S_{\text{MUSIC}}(\theta) = \frac{1}{\sum_{j=D+1}^{M} \mathbf{e}_j^H \mathbf{a}(\theta)}
\]

(16)

The steering angle that gives the maxima of the MUSIC spectrum corresponds to the source directions.

\[
\theta_s = \arg \max_{\theta} S_{\text{MUSIC}}(\theta)
\]

(17)

3. Nearefield array signal processing

The nearfield array model is based on the indirect ESM in which the sound field is represented with an array of virtual monopole sources, as depicted in Fig. 2. In this figure, \( \mathbf{x}_m \) is the \( m \)th microphone position on the hologram surface \( S_h \); \( \mathbf{z}_i \) is the \( i \)th source point on the actual source surface \( S_s \); \( \mathbf{y}_l \) is the \( l \)th virtual source point on the virtual surface \( S_v \). By assuming the time-harmonic dependence \( e^{j\omega t} \), we may obtain a nearfield array ESM model in a matrix form.[9]
\[ p_h(\omega) = G_{hv}(\omega)a_v(\omega), \]  

where \( p_h \) represents the hologram pressure vector, \( a_v \) represents the virtual source amplitude vector, and \( G_{hv} \) is the propagation matrix relating the source amplitude and the hologram pressure[10]

\[ \{G_{hv}\}_{ml} = \frac{e^{-jk_{ml}}}{r_{ml}}, \]  

where \( r_{ml} = |x_m - y_l| \). The unknown virtual source strengths can be calculated by inverting Eq. (18)

\[ \hat{a}_v = G_{hv}^+p_h, \]  

where \( \hat{a}_v \) is the estimated source amplitude vector and \( G_{hv}^+ \) is the pseudo-inverse matrix of \( G_{hv} \). Truncated singular value decomposition (TSVD) or Tikhonov regularization (TIKR)[11] can be used to deal with the ill-conditioned inversion process. The inversion distance (sum of the reconstruction and retreat distances) is chosen to keep the condition number of \( G_{hv} \) below 1000.[10]

Once the source amplitudes \( a_v \) are obtained, the sound pressure on \( S_s \) can be reconstructed as

\[ p_s = G_{sv}\hat{a}_v, \]  

where \( G_{sv} \) denotes the propagation matrix relating the virtual source strength and the actual source surface pressure. The normal velocity at the position \( z_i \) on the source surface is given by

\[ u_s(z_i, \omega) = \frac{1}{j\rho_0\omega} \sum_{l=1}^{L} (n \cdot e_e)(jk + 1/r_{ll}) p_s(z_i, \omega), \]  

where \( \rho_0 \) is air density, \( n \) is unit normal vector, and \( e_e = (z_i - y_l)/r_{ll} \). Due to singularity of virtual sources, we need a non-zero retreat distance \( (R_d) \) between \( S_v \) and \( S_s \) to assure reconstruction quality.

4. Experimental investigations of machine tools

The indirect ESM algorithm is utilized to reconstruct the sound field in the nearfield array, while the DAS, MVDR and MUSIC algorithms are used to visualize the beam pattern in the farfield array. The machine tools are employed to validate the nearfield and farfield array algorithms by using a \( 5 \times 6 \) URA and a thirty-channel random array, as shown in Fig. 3. For nearfield application, the microphone spacing \( d \) is selected to be 0.1m (\( \lambda/2 \) corresponding to \( f_{max} = 1.7 \) kHz). The actual source and microphone planes are located at \( z = 0 \)m and \( z = 0.1 \)m. The retreat distance is set to be \( d/2 \). Virtual microphone technique[12] was applied to enhance image quality by interpolate and extrapolate the pressure field on the microphone surface and increase the number of microphones and focal points from \( 5 \times 6 \) to \( 13 \times 15 \). A bandpass filter (20 Hz ~ 1.7 kHz) is used to prevent aliasing and errors occurring in the out-of-band frequencies. For farfield application, we adopted a
The focal source points are positioned in a rectangular lattice (1m × 1m) in the plane at z = 1m with uniform spacing \( d_v = 0.1 \) m. The microphone plane is located at z = 0m. The observed frequencies in the MVDR and MUSIC algorithms are chosen to be 1 kHz. The sampling frequencies of nearfield and farfield arrays were assumed to be 5 and 20 kHz.

4.1. Milling machine

In the first experiment, both the nearfield and farfield methods were applied to reconstruct the sound field of the small-sized milling machine running at the idle speed test. The rms sound pressure and particle velocity reconstructed by using the indirect ESM is shown in Fig. 4 (a) and (b). With indirect ESM, the bright area in the reconstructed particle velocity field reveals that the spindle motor at the top-left was the target noise. In addition to the spindle motor, the reconstructed sound fields indicate that there were secondary sources at the bottom-left on the working platform. From Fig. 4 (c)-(e), DAS has a poor result that is unable to distinguish the two sources. In high resolution algorithms, MVDR and MUSIC, can localize the two sources precisely. The performance of MUSIC is better than MVDR, as shown in Fig. 4 (d) and (e).

In the second experiment, the farfield array methods was used to reconstruct the sound field of the large-sized milling machine in the idle speed and in-process tests, as depicted in Fig. 5 and Fig. 6. The map trend in-process test has better focusing effect than the idle speed test in the all farfield array methods. The noise maps obtained using DAS have very large main lobes but cannot correctly point the source positions. Fig. 6 (d) and (e) show the noise maps obtained using MVDR and MUSIC algorithms with random array configuration. As predicted, the results validated that the MVDR and MUSIC methods can achieve the higher resolutions, especially MUSIC. They can correctly localize the noise source with narrow main lobes. The side-lobes of DAS are higher than the MVDR and MUSIC.

4.2. Turning lathe

In this experiment, a small-sized turning lathe in the idle speed served as a practical source to examine the capability of nearfield and farfield microphone arrays, as shown in Fig. 7. In Fig. 7 (a) and (b), the turning lathe is mounted on a table, where the major noise source appeared to be at the spindle motor position (bottom-
As can be seen in the particle velocity reconstructed by nearfield array, indirect ESM is able to identify the major source at the motor and the vibration on the surface. The result of DAS method was quite poor, while noise source was successfully identified using the MVDR and MUSIC methods, as shown in Fig. 7 (c)-(e).

Fig. 4 The proposed algorithms were applied to reconstruct the sound field of the small-sized milling machine running at the idle speed. (a) The reconstructed rms sound pressure image of indirect ESM, (b) the reconstructed rms particle velocity image of indirect ESM, (c) the beam pattern of DAS method, (d) the beam pattern of MVDR method, (e) the beam pattern of MUSIC method.

Fig. 5 The proposed algorithms were applied to reconstruct the beam pattern of the large-sized milling machine in the idle speed test. (a) DAS method, (b) MVDR method, (c) MUSIC method.
Fig. 6 The proposed algorithms were applied to reconstruct the beam pattern of the large-sized milling machine in-process test. (a) DAS method, (b) MVDR method, (c) MUSIC method.

Fig. 7 The proposed algorithms were applied to reconstruct the sound field of the small-sized turning lathe running at the idle speed. (a) The reconstructed rms sound pressure image of indirect ESM, (b) the reconstructed rms particle velocity image of indirect ESM, (c) the beam pattern of DAS method, (d) the beam pattern of MVDR method, (e) the beam pattern of MUSIC method.

4.3. Shearing machine

In this experiment, the shearing machine in the idle speed and in-process tests are employed to validate the nearfield and farfield array algorithms. The bright areas on the velocity plot revealed that the material inlet and outlet are the major sources. The noise distribution in-process test may move from material outlet to material inlet. The map trend in-process test has better focusing effect than the idle speed test in the all farfield array methods. Results show that the MUSIC algorithms can attain the highest resolution of localizing sound sources
positions than the DAS and MVDR methods. The nearfield images apparently yielded more reliable information about noise sources than the farfield images.

5. Conclusions

Farfield and nearfield sound imaging techniques including DAS, MVDR, MUSIC and inverse filter-based method indirect ESM have been developed to estimate DOA in this paper. In the experiment of machine tools, farfield and nearfield array algorithms were compared in terms of image resolution. The nearfield array algorithm, indirect ESM, is more flexible in that it is capable of reconstructing the sound field radiated by sources of arbitrary geometries. As expected, high resolution methods such as MVDR and MUSIC attained better quality images than DAS method in localizing noise sources. While MVDR and MUSIC deliver better resolution, the inverse filter-based method is more computationally efficient due to the Fast Fourier Transform. Indirect ESM enable reconstructing acoustic variables such as sound pressure, particle velocity and active intensity, whereas the MUSIC spectrum gives no direct physical interpretation.

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