Two-gluon production of $\phi$ and $\eta'$ mesons in proton-proton collisions at high energies

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Abstract

We discuss gluon-gluon mechanisms for production of mesons with hidden strangeness, such as $\eta'$ and $\phi$ meson, in proton-proton collisions at large energies. The $g^*g^* \rightarrow \eta'$ and $g^*g^* \rightarrow \phi g$ mechanisms are considered only and the corresponding cross sections are calculated in the $k_t$-factorization approach. The $F_{\gamma^*\gamma^* \rightarrow \eta'}$ and $F_{g^*g^* \rightarrow \eta'}$ form factors are calculated from quark-antiquark $\eta'$ light-cone wave function including quark/antiquark transverse momenta. The result for two-photon transition form factor demonstrates that higher twists may survive even to large photon virtualities. The result is compared with the result of a recent leading-twist NLO analysis which uses phenomenological distribution amplitudes fitted to exclusive production of $\eta'$ in $e^+e^- \rightarrow e^+e^- \eta'$ reaction. We calculate transverse momentum distributions of both $\eta'$ and $\phi$ mesons in proton-proton collisions for RHIC and LHC energies. The results are compared to experimental data whenever available. The results of the Lund string model are shown for comparison for $\eta'$ production at LHC energies. It seems that the two-gluon fusion is not the dominant mechanism for both $\phi$ and $\eta'$ production, although the situation for $\eta'$, especially at larger energies, is less clear due to lack of experimental data.

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I. INTRODUCTION

The mechanism of the particle production in proton-proton collisions was studied continuously for five decades. Pions and kaons are believed to be produced in the fragmentation process. The Lund string model is state of art in this context. On the other side quarkonium production at high energies is studied considering two-gluon process. Depending on $C$-parity we have either $g^*g^* \to Q \ (C = +1)$ or $g^*g^* \to Qg \ (C = -1)$ processes when limiting to color singlet mechanisms. Color octet processes are not under full theoretical control. Recently our group showed that the production of $\eta_c$ quarkonium can be understood assuming simple color-singlet $g^*g^* \to \eta_c$ fusion \cite{2}. The situation with resonances, such as $\rho^0$, $f_0(980)$ or $f_2(1270)$ is still different. We have shown that at large isoscalar meson transverse momenta the gluon-gluon fusion may be an important mechanism \cite{3,4}. At low transverse momenta one has to included also coalescence mechanism \cite{4}. For fully heavy tetraquark production the double-parton mechanism may be required \cite{5}.

The situation with hidden strangness mesons was not discussed carefully in the literature. In PYTHIA \cite{6} such mesons are produced within the Lund-string model. Recently there was some works on modification of strange hadron production. Effects of the rope hadronization on strangeness enhancement in $pp$ collisions at the LHC were discussed e.g. in \cite{7}.

Here we wish to explore whether the two-gluon fusion may be also important production mechanism. It was suggested that the hadronic production of $\eta'$ meson could be used to extract two-gluon transition form factor \cite{8,9}. The formalism of $\eta'$ meson production in proton-proton collisions was discussed already some time ago but no explicit calculation was performed so far. In addition, it could not be compared to the data as the latter was not available at that time (2007). In the meantime the PHENIX collaboration measured the $\eta'$ production at $\sqrt{s} = 200$ GeV but no comparison was made with theoretical results according to our knowledge. Here we wish to study the situation more carefully and make comparison to realistic calculation of the gluon-gluon fusion.

Exclusive $pp \to pp\eta'$ production was studied in \cite{12} within KMR perturbative approach and the measured cross section could not be explained at the relatively low $\sqrt{s} = 29.1$ GeV energy of the WA102 collaboration experiment at CERN SPS. On the other hand
soft pomeron/reggeon exchanges can be fitted to describe the experimental data [13].

The presence of the two-gluon component in $\eta'$ may be relevant in the context of its production in the hadronic reaction. The gluon content in a meson may occur e.g. via mixing with glueballs. Mixing of scalar glueball with the scalar-isoscalar “quarkonia” was discussed e.g. in [18]. According to our knowledge there was not such a discussion for $\eta'$. However, according to lattice QCD pseudoscalar glueball has a mass of about 2.6 GeV [10, 11], so the mixing should be rather small. Lower mass pseudoscalar glueball was discussed in [20], which mixes however rather with radial excitations. Review on experimental searches for the light pseudoscalar glueball can be found in [19]. On the other hand axial anomaly may “cause” the presence of gluons in the $\eta'$ wave function [22]. No big gluonic content either in $\eta$ or $\eta'$ was found from the $V \rightarrow P\gamma$ and $P \rightarrow V\gamma$ radiative decays in [21]. The KLOE-2 collaboration found $Z_8^2 \approx 0.11$ probability of the two-gluon component [24].

For comparison, the hadronization process for $\eta'$ production was not discussed carefully in the literature. Some discussion was presented e.g. in [25] but in the context of $e^+e^-$ collisions.

In this paper we shall discuss possible consequences of gluonic component in $\eta'$ for its production in proton-proton collisions. We shall compare our results both with experimental data as well as with results of the Lund string model.

II. SKETCH OF THE FORMALISM

The main color-singlet mechanism of $\phi$ meson production is illustrated in Fig[1]. In this case $\phi$ is produced in association with an extra “hard” gluon due to C-parity conservation.

We calculate the dominant color-singlet $gg \rightarrow \phi g$ contribution taking into account transverse momenta of initial gluons. In the $k_t$-factorization the NLO differential cross section can be written as:

$$\frac{d\sigma(pp \rightarrow \phi gX)}{dy_{\phi}dy_gd^2p_{\phi,t}d^2p_g,t} = \frac{1}{16\pi^2s^2} \int \frac{d^2q_{1t}}{\pi} \frac{d^2q_{2t}}{\pi} \left| M^{\text{off-shell}}_{g^*s^*\rightarrow \phi g} \right|^2$$

$$\times \delta^2(\vec{q}_{1t} + \vec{q}_{2t} - \vec{p}_{H,t} - \vec{p}_{g,t}) \mathcal{F}_{g}(x_1, q_{1t}^2, \mu^2) \mathcal{F}_{g}(x_2, q_{2t}^2, \mu^2) \quad (2.1)$$

where $\mathcal{F}_{g}$ are unintegrated (or transverse-momentum-dependent) gluon distributions.
FIG. 1: The leading-order diagram for direct $\phi$ meson production in the $k_t$-factorization approach.

The matrix elements were calculated as done e.g. for $J/\psi g$ production in $^{[33]}$. The corresponding matrix element squared for the $gg \rightarrow \phi g$ is

$$|M_{gg \rightarrow \phi g}|^2 \propto \alpha_s^3 |R(0)|^2.$$  \hspace{1cm} (2.2)

Running coupling constants are used in the calculation. Different combination of renormalization scales were tried. Finally we decided to use:

$$\alpha_s^3 \rightarrow \alpha_s(\mu_1^2)\alpha_s(\mu_2^2)\alpha_s(\mu_3^2),$$ \hspace{1cm} (2.3)

where $\mu_1^2 = \max(q_{1r}^2, m_t^2)$, $\mu_2^2 = \max(q_{2r}^2, m_t^2)$ and $\mu_3^2 = m_t^2$, where here $m_t$ is the $\phi$ transverse mass. The factorization scale in the calculation was taken as $\mu_F^2 = (m_t^2 + p_{1g}^2)/2$.

The radial wave function at zero can be estimated from the decay of $\phi \rightarrow l^+l^-$ as is usually done for $J/\psi (c\bar{c})$, see e.g. $^{[34]}$

$$\Gamma(\phi \rightarrow l^+l^-) = 16\pi \frac{\alpha Q_s^2}{M_\phi^2} |\Psi_\phi(0)|^2 \left(1 - \frac{16 \alpha_s}{3 \pi}\right),$$ \hspace{1cm} (2.4)

where $Q_s$ is fractional charge of the $s$ quark. Then

$$|\Psi_\phi(0)|^2 = \frac{\Gamma(\phi \rightarrow l^+l^-)}{16\pi\alpha_{\text{em}} Q_s^2} \frac{M_\phi^2}{1 - 16\alpha_s/(3\pi)}.$$ \hspace{1cm} (2.5)

In the evaluation we use $\alpha_s = 0.3$. Using branching fraction from PDG $^{[15]}$ we get $|\Psi(0)|$. By convention $|R(0)|^2 = 4\pi |\Psi(0)|^2$. Assuming $\alpha_s = 0.3$ we get $|R(0)|^2 = 0.11 \text{ GeV}^3$. 

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We shall use this value to estimate the cross section for production of \( \phi \) meson. For comparison for \( J/\psi \) (real quarkonium) one gets \(|R(0)|^2 \approx 0.8 \text{ GeV}^3\).

Similarly we perform calculation for S-wave \( \eta' \) meson production. Here the lowest-order subprocess \( gg \to \eta' \) is allowed by positive C-parity of \( \eta' \) mesons. In the \( k_t\)-factorization approach the leading-order cross section for the \( \eta' \) meson production can be written as:

\[
\sigma_{pp \to \eta'} = \int dy dq^2 p_t d^2 q_t \frac{1}{s x_1 x_2 m_{t,\eta'}} |M^{g^* g^* \to \eta'}|^2 \mathcal{F}_g(x_1, q_1^2, \mu_F^2) \mathcal{F}_g(x_2, q_2^2, \mu_F^2) / 4 ,
\]

that can be also used to calculate rapidity and transverse momentum distribution of the \( \eta' \) mesons. Above \( \mathcal{F}_g \) are unintegrated (or transverse-momentum-dependent) gluon distributions and \( \sigma_{gg \to \eta'} \) is \( gg \to \eta' \) (off-shell) cross section. In the last equation: \( \vec{p}_t = \vec{q}_{1t} + \vec{q}_{2t} \) is transverse momentum of the \( \eta' \) meson and \( \vec{q}_t = \vec{q}_{1t} - \vec{q}_{2t} \) is auxiliary variable which is used in the integration. Furthermore: \( m_{t,\eta'} \) is the so-called \( \eta' \) transverse mass and \( x_1 = \frac{m_{t,\eta'}}{\sqrt{s}} \exp(y), x_2 = \frac{m_{t,\eta'}}{\sqrt{s}} \exp(-y) \). The factor \( \frac{1}{4} \) is the jacobian of transformation from \((\vec{q}_{1t}, \vec{q}_{2t})\) to \((\vec{p}_t, \vec{q}_t)\) variables. The situation is illustrated diagrammatically in Fig. 2.

As for \( \phi \) production the running coupling constants are used. Different combination of scales are tried. The best choice is:

\[
\alpha_s^2 \to \alpha_s(\mu_1^2) \alpha_s(\mu_2^2) ,
\]

where \( \mu_1^2 = \max(q_{1t}^2, m_t^2) \) and \( \mu_2^2 = \max(q_{2t}^2, m_t^2) \). Above \( m_t \) is transverse mass of the \( \eta' \) meson. The factorization scale(s) for the \( \eta' \) meson production are fixed traditionally as \( \mu_F^2 = m_t^2 \).

The \( g^* g^* \to \eta' \) coupling has relatively simple one-term form:

\[
T_{\mu\nu}(q_1, q_2) = F_{g^* g^* \to \eta'}(q_1, q_2) \epsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta ,
\]

where \( F_{g^* g^* \to \eta'}(q_1, q_2) \) object is known as the two-gluon transition form factor. The matrix element to be used in the \( k_t\)-factorization is then:

\[
\mathcal{M}_{ab}^{\mu\nu} = \frac{q_{1t}^\mu q_{2t}^\nu}{|q_1||q_2|} T_{\mu\nu} .
\]

In contrast to the convention for two-photon transition form factor the strong coupling constants are usually absorbed into the two-gluon form factor definition.
The matrix element squared for the $gg \rightarrow \eta'$ subprocess is
\[
|\mathcal{M}_{gg \rightarrow \eta'}|^2 \propto F_{gg \rightarrow \eta'}^2(q_{1l}^2, q_{2l}^2) \propto \alpha_s^2 F_{\gamma^* \gamma^* \rightarrow \eta'}^2(q_{1l}^2, q_{2l}^2),
\]
where $F_{gg \rightarrow \eta'}^2(q_{1l}^2, q_{2l}^2)$ and $F_{\gamma^* \gamma^* \rightarrow \eta'}^2(q_{1l}^2, q_{2l}^2)$ are two-gluon and two-photon transition form factors of the $\eta'$ meson, respectively. It was discussed, e.g. in [17], in leading-twist collinear approximation. Such an approach is valid for $Q_1^2 = q_{1l}^2 \gg 0$ and $Q_2^2 = q_{2l}^2 \gg 0$. Here we need such a transition form factor also for $Q_1^2, Q_2^2 \sim 0$. There is a simple relation between the two-gluon and two-photon form factors for the quark-antiquark systems (see e.g. [2–4]). $\eta'$ meson may have also the two-gluon component in its Fock decomposition [22]. The form factor found there can be approximately parametrized as
\[
\bar{Q}^2 F_{gg \rightarrow \eta'}^2(q_{1l}^2, q_{2l}^2) \approx 0.2 \pm 0.1 \text{ GeV,}
\]
where $\bar{Q}^2 = (Q_1^2 + Q_2^2)/2$. A better approach would be to use their Eqs.(5.13-5.16) with parameters given there. The result from [17] is:
\[
F(Q^2, \omega) = 4\pi\alpha_s \frac{f_p \sqrt{\pi f}}{Q^2 N_c} A(\omega).
\]
In the factorized (in $Q^2$ and $\omega$) formula:
\[
A(\omega) = A_{qq}(\omega) + \frac{N_c}{2n_f} A_{gg}(\omega),
\]
where
\[
A_{qq}(\omega) = \int_0^1 dx \frac{\Phi_1(x, \mu_F^2)}{1 - \omega^2 (1 - 2x)^2},
\]
\[
A_{gg}(\omega) = \int_0^1 dx \frac{\Phi_g(x, \mu_F^2)}{x\bar{x}} \frac{1 - 2x}{1 - \omega^2 (1 - 2x)^2},
\]
and $\Phi_1$ and $\Phi_g$ are singlet and gluon distribution functions, respectively. Above
\[
\omega = \frac{Q_1^2 - Q_2^2}{Q_1^2 + Q_2^2}.
\]
$\Phi_1$ and $\Phi_g$ undergo QCD evolution [17] which is included also in the present paper.

**A. $F_{\gamma^* \gamma^* \rightarrow \eta'}$ form factor**

In Ref.[1] we have shown how to calculate the transition form factor from the light-cone $Q\bar{Q}$ wave function of the $\eta_c$ quarkonium. Here we shall follow the same idea but
FIG. 2: The leading-order diagram for $\eta'$ meson production in the $k_t$-factorization approach.

for light quark and light antiquark system. The flavour wave function of $\eta'$ meson can be approximated as [14]

$$|\eta'\rangle \approx \frac{1}{\sqrt{3}} (u\bar{u} + d\bar{d} + s\bar{s}) \ . \quad (2.17)$$

The spatial wave function could be calculated e.g. in potential models. The momentum wave function can be then obtained as a Fourier transform of the spatial one. We shall not follow this path in the present study. Instead we shall take a simple, but reasonable, parametrization of the respective light-cone wave function. In principle, each component in (2.17) may have different spatial as well as momentum wave function. Here for simplicity we shall assume one effective wave function for each flavour component. We shall take the simple parametrization of the momentum wave function

$$u(p) \propto \exp \left( p^2 / (2\beta) \right) \ . \quad (2.18)$$

The light cone wave function is obtained then via the Terentev’s transformation (see e.g. [1]). We shall use the normalization of the light cone wave function as:

$$\int_0^1 \frac{dz}{z(1-z)} \frac{d^2k}{16\pi^3} |\phi(z,k_t)|^2 = 1 \ . \quad (2.19)$$

Above

$$\phi(z,k_t) \propto \sqrt{M_{q\bar{q}}} \exp \left( -p^2 / (2\beta^2) \right) \ . \quad (2.20)$$
and the so-called Terentev’s prescription, relating the rest-frame and light-cone variables, is used:

$$p^2 = \frac{1}{4} \left( M_{q\bar{q}}^2 - 4m_{\text{eff}}^2 \right).$$ (2.21)

Above $M_{q\bar{q}}$ is the invariant mass of the $q\bar{q}$ system.

The parameters in the above equations: $m_{\text{eff}}$ (hidden in $\phi(z,k_t)$) and $\beta$ are in principle free. Here we shall take:

$$m_{\text{eff}} = \left( \frac{2}{3} \right) m_q + \left( \frac{1}{3} \right) m_s, \quad (2.22)$$

where $m_q$ and $m_s$ are constituent masses of light (u,d) and strange quarks, respectively. Therefore $m_{\text{eff}} \sim 0.4 \text{ GeV}$. The weights are from the flavor wave function (2.17). We shall try a few different $\beta$ values in the range (0.4-0.6) GeV. The normalization constant can be then obtained from the light-cone wave function normalization.

Having fixed light-cone wave function one can calculate electromagnetic $\gamma^* \gamma^* \rightarrow \eta'$ transition form factor as:

$$F(Q_1^2, Q_2^2) = -\frac{1}{\sqrt{3}} (e_u^2 + e_d^2 + e_s^2) \sqrt{N_c} 4m_{\text{eff}} \cdot \int \frac{dz d^2k}{z(1-z)16\pi^3} \psi(z,k)$$

$$\left\{ \frac{1}{(k-(1-z)q_2)^2 + z(1-z) q_1^2 + m_{\text{eff}}^2} + \frac{z}{(k+zq_2)^2 + z(1-z) q_1^2 + m_{\text{eff}}^2} \right\}.$$ (2.23)

The $F(0,0)$ is known and can be calculated from the radiative decay width [30].

The present BABAR data [30] are not sufficiently precise to get the parameters of our model ($m_{\text{eff}}$ and $\beta$). They could be adjusted in future to precise experimental data for the $e^+e^- \rightarrow e^+e^- \eta'$ reaction from Belle 2.

The formula (2.23) can be reduced to a single integral

$$F(Q_1^2, Q_2^2) = \frac{1}{\sqrt{3}} (e_u^2 + e_d^2 + e_s^2) f_{\eta'}$$

$$\cdot \int_0^1 dz \left\{ \frac{(1-z)\phi(z)}{(1-z)^2Q_1^2 + z(1-z)Q_2^2 + m_{\text{eff}}^2} + \frac{z\phi(z)}{z^2Q_1^2 + z(1-z)Q_2^2 + m_{\text{eff}}^2} \right\}.$$ (2.24)

when introducing so-called distribution amplitudes $\phi(z)$ and so-called decay constant $f_{\eta'}$ (see e.g. [11]).

We shall use also a simple parametrization of the transition form factor called non-factorized monopole for brevity

$$F^{nf, \text{monopole}}(Q_1^2, Q_2^2) = F(0,0) \frac{\Lambda^2}{\Lambda^2 + Q_1^2 + Q_2^2}.$$ (2.25)
This two-parameter formula can be correctly normalized at $Q_1^2 = 0$ and $Q_2^2 = 0$ [30]. It has also correct asymptotic dependence on $Q^2 = (Q_1^2 + Q_2^2)/2$. This is very similar to the approach done long ago by Brodsky and Lepage [31] in the case of neutral pion.

The so-called vector meson dominance model (factorized monopole)

$$F^{VDM}(Q_1^2, Q_2^2) = F(0, 0) \frac{m_V^2}{m_V^2 + Q_1^2} \frac{m_V^2}{m_V^2 + Q_2^2}$$

has incorrectly strong $Q^2$ dependence [30]. We shall compare results obtained with the form factor calculated with the light-cone wave function (2.23) with the parametrization (2.25) for $\Lambda = 1$ GeV. Results of such a calculation will be treated as a reference ones for other approaches.

The effect of internal transverse momenta of quarks and antiquarks in a meson was discussed long ago [38] postulating some wave function of $\pi^0$ in the impact parameter space and including suppression due to so-called Sudakov form factor.

In Ref. [32] the authors tried to adjust the coefficient of the lowest-order Gegenbauer polynomials to describe the BABAR data [30] for two virtual photons within the leading-twist collinear approximation. However, the corresponding error bars on expansion coefficients are very large.

The two-gluon transition form factor is closely related to two-photon transition form factor provided the meson is of the quark-antiquark type i.e. its wave function as in Eq. (2.17). Then

$$|F_{g^*g^*\rightarrow\eta'}(Q_1^2, Q_2^2)|^2 = |F_{\gamma^*\gamma^*\rightarrow\eta'}(Q_1^2, Q_2^2)|^2 \frac{g_{em}^2}{g_{em}^2} 4N_c (N_c^2 - 1) \frac{1}{e_q^2} \frac{1}{\langle e_q^2 \rangle^2}.$$  

(2.27)

Above $g_{em}^2$ must be taken provided it is included in the definition of $F_{\gamma^*\gamma^*\rightarrow\eta'}$ transition form factor. Usually it is not.

In Fig. 3 we show $q\bar{q}$ and $gg$ distribution amplitudes from [17] for different evolution scales. Such distribution amplitudes can be used to calculate $F_{g^*g^*\rightarrow\eta'}$ (see Eq. (2.13)) needed in calculating $\eta'$ production in proton-proton collisions.
III. RESULTS

In this section we present our results for $\phi$ and $\eta'$ meson production.

A. $\phi$ production

In this subsection we show the cross section for $\phi$ meson production for $\sqrt{s} = 200$ GeV, $\sqrt{s} = 2.76$ GeV and $\sqrt{s} = 8$ TeV (see Fig. 4). Our results are shown together with the PHENIX [35] and ALICE [36, 37] experimental data, respectively. For each considered case the result of calculation is below the experimental data. This suggests that the gluon-gluon fusion is not the dominant production mechanism of $\phi$ meson production. The fragmentation mechanism was considered in [42, 43] and it may be the dominant mechanism of $\phi$ meson production.
FIG. 4: Invariant cross section for $\phi$ production at $\sqrt{s} = 200$ GeV, 2.76 GeV and 8 TeV. We show the experimental data of the PHENIX collaboration [35], ALICE collaboration [36] and results from [37]. Here $\Psi(0)$ was calculated from Eq. (2.5).

B. $F_{\gamma^*\gamma^*\rightarrow\eta'}$ form factor

Before presenting our results for the $\eta'$ production we wish to show our results for the $F_{\gamma^*\gamma^*\rightarrow\eta'}$ form factor.

We will start with our results obtained from the LCWF for $F_{\gamma\gamma\rightarrow\eta'}(0,0)$. In Fig5 we present $F_{\gamma\gamma\rightarrow\eta'}(0,0)$ as a function of $\beta$ and $m_{\text{eff}}$. There is a small dependence on the
parameters. The experimental value is

$$F_{\gamma^*\gamma^*\rightarrow\eta'}(0,0) = \left[\frac{4\Gamma_{\eta'\rightarrow2\gamma}}{\pi\alpha_{em}^2 m_{\eta'}^2}\right] = 0.342 \pm 0.006 \text{ GeV}^{-1}.$$  \hspace{1cm} (3.1)$$

A broad range of $\beta$ and $m_{\text{eff}}$ is allowed taken into account the simplicity of our approach.

Most of the studies on transition form factors has been concentrated on the case of only one virtual photon. In Fig. 6 we show $Q^2 F_{\gamma^*\gamma^*\rightarrow\eta'}(Q^2)$ for different values of model parameters $\beta = 0.4, 0.5, 0.6 \text{ GeV}$. In leading twist approach, without QCD evolution of distribution amplitudes, one should get a constant at large photon virtualities. In our approach this happens at extremely large virtualities. Below $Q^2 < 50 \text{ GeV}^2$ our model clearly contains higher twists. In collinear leading twist approach the rise of $Q^2 F_{\gamma^*\gamma^*\rightarrow\eta'}(Q^2)$ is caused by the evolution (see e.g. [26]).

In Fig. 7 we show the two-photon $\eta'$ form factor as a function of both photon virtualities. The form factor drops quickly from $F_{\gamma^*\gamma^*\rightarrow\eta'}(0,0)$ in the region $Q_1^2 < 10 \text{ GeV}^2$, $Q_2^2 < 10 \text{ GeV}^2$. The change beyond this region is rather mild. We show our result obtained using the light-cone wave function (left panel) and for comparison the result from Ref. [32] (right panel). The leading-twist result is realiable only for larger virtualities. Both the results are similar for larger virtualities.

In order to better visualize our result we show in Fig. 8 also the ratio:

$$R(Q_1^2, Q_2^2) = \frac{F_{\eta',\gamma^*\rightarrow\gamma^*\eta'}^{\text{LC}}(Q_1^2, Q_2^2)}{F_{\eta',\text{mono pole}}^{\text{nf}}(Q_1^2, Q_2^2)}.$$  \hspace{1cm} (3.2)$$
FIG. 6: $Q^2 F(Q^2)$ as a function of one photon virtuality. We show results for $m_{\text{eff}} = 0.4$ GeV and for different values of $\beta = 0.4, 0.5, 0.6$ GeV (from bottom to top). For comparison we show experimental data from [27-29].

FIG. 7: $F_{\gamma^* \gamma^* \to \eta'}(Q_1^2, Q_2^2)$ obtained with the light-cone wave function for $m_{\text{eff}} = 0.4$ GeV and $\beta = 0.5$ GeV (left panel) and the leading-twist result from Ref.[32] (right panel).

and similar obtained when using formula (2.24).

We observe that the form factor calculated from (2.23) deviates only slightly from the simple parametrization (2.25). The two parametrizations almost coincide in the broad range of $(Q_1^2, Q_2^2)$. A similar result is obtained when using Eq.(2.24) with asymptotic distribution amplitude.
FIG. 8: $R(Q_1^2, Q_2^2)$ (see Eq. (3.2)) with the $\gamma^*\gamma^* \to \eta'$ form factor calculated with $\eta'$ light-cone wave function (left panel). In this calculation $m_{\text{eff}} = 0.4$ GeV and $\beta = 0.5$ GeV is used for example. In the right panel we show similar ratio obtained from Eq. (2.24) with asymptotic distribution amplitude.

In Fig. 9 we show the two-photon transition form factor as a function of asymmetry parameter $\omega$ (see Eq. (2.16)) for different values of $Q^2$ specified in the figure caption. In contrast to the non-factorizable monopole form factor (2.25) we get some dependence on asymmetry parameter $\omega$. This dependence is somewhat similar to the result of Ref. [32]. In contrast to [32] our dependence on $\omega$ is not universal, i.e. different for different values of $Q^2$.

The two-photon form factor will be transformed to two-gluon form factor and the latter will be used in the calculation of $\eta'$ production. For this purpose first a grid for $F_{\gamma^*\gamma^* \to \eta'}$ in the $(Q^2, \omega)$ plane is prepared. The grid is then used in the interpolation when calculating differential distributions of $\eta'$ meson in proton-proton collisions.

C. $\eta'$ production

In this subsection we discuss the $\eta'$ production considering the simple gluon-gluon fusion mechanism illustrated in Fig. 2.

What are typical gluon transverse momenta for hadronic $g^*g^* \to \eta'$ process. In Fig. 10...
FIG. 9: The $\gamma^* \gamma^* \rightarrow \eta'$ form factor as a function of $\omega$ for a fixed values of $Q^2$ (from top to bottom: 2, 5, 10, 20, 50, 100 GeV$^2$) calculated with $\eta'$ light-cone wave function. In this calculation $m_{\text{eff}} = 0.4$ GeV and $\beta = 0.5$ GeV.

we show the distribution of the cross section integrated over $\eta'$ transverse momenta in the $(q_{1t}, q_{2t})$. Both small and large gluon virtualities enter into the $p_t$-integrated cross section.

In Fig[11] we show similar distributions as above but for two different regions of meson transverse momentum $p_t$. The larger $p_t$ the larger gluon transverse momenta enter into the game.

Fig[12] shows a similar distributions but in the $(q_{1t}^2, q_{2t}^2)$ space used usually for presentation of transition form factors. One can observe that at large transverse momentum $p_t$ one is sensitive to the region of $Q_1^2$ very small and $Q_2^2$ very large or $Q_1^2$ very large and $Q_2^2$ very small. These are regions relevant for the leading-twist collinear approach to two-gluon transition form factor. This is also the region of the phase space where the meson light-cone approach gives a small relative enhancement compared to the naive monopole parametrizations (see Fig[8]). We observe that at $p_t \sim 10$ GeV the gluon virtualities $Q_1^2 >$
FIG. 10: Two-dimensional map in \((q_{1t}, q_{2t})\) for the full range of \(\eta'\) transverse momentum. Here \(\sqrt{s} = 200\) GeV and the KMR UGDF was used.

20 GeV^2 or \(Q^2 > 20\) GeV^2 are clearly in the domain of leading-twist approach [17].

This is shown better in Fig.13 where we display distribution in \(Q^2\). Only large \(Q^2\) occur for \(p_t > 10\) GeV. This situation is generic, independent of the form factor used.
FIG. 12: Two-dimensional map in \((q_1^2, q_2^2)\) for \(4.5 \text{ GeV} < p_t < 5.5 \text{ GeV}\) (left panel) and \(9.5 \text{ GeV} < p_t < 10.5 \text{ GeV}\) (right panel). Here \(\sqrt{s} = 200 \text{ GeV}\). In this calculation the KMR UGDF was used and the light-cone wave function with \(\beta = 0.5 \text{ GeV}\).

FIG. 13: Distribution in \(Q_{\text{ave}}^2 = \bar{Q}^2\) for the two distinct cases from the previous figure: \(p_t = 5 \pm 0.5 \text{ GeV}\) (solid line) and \(p_t = 10 \pm 0.5 \text{ GeV}\) (dashed line). Here \(\sqrt{s} = 200 \text{ GeV}\). In this calculation the KMR UGDF was used and the light-cone wave function with \(\beta = 0.5 \text{ GeV}\).

Finally in Fig. 14 we show invariant cross section for the \(\eta'\) meson production for the RHIC energy \(\sqrt{s} = 200 \text{ GeV}\) relevant for the PHENIX experiment \s{35}. In the left panel we show different results:
(a) with non factorized monopole form factor (solid line),
(b) with the form factor calculated from the LCWF with \(\beta = 0.5 \text{ GeV}\) (dashed line),
(c) leading twist parametrization (2.11) of the $F_{g^*g^*\to\eta'}$ result from [17] (dash-dotted line),
(d) without $F_{g^*g^*\to\eta'}$, except of normalization constant (dotted line).

The result obtained with the leading-twist parametrization (2.11) can be taken serious only for $p_t > 5$ GeV, when $Q_1^2$ or $Q_2^2$ are bigger than 5 GeV$^2$. We also used the formalism
of collinear distribution amplitudes from [17], including their QCD evolution, to calculate
$F_{g^*g^*\to\eta'}(q_1^2, q_2^2)$ from evolved $q\bar{q}$ and $gg$ distribution amplitudes. The evolution scale of
distribution amplitudes is taken as $\mu^2 = \bar{Q}^2 + \mu_0^2$, where $\mu_0^2 = 1$ GeV$^2$ is used in our
calculation. The corresponding result is shown by the red thick solid line. The line is
below other lines in the region where the experimental data exist. For comparison we
show somewhat arbitrarily also result with $q\bar{q}$ alone (red thick dashed line) and $gg$ alone
(red thick dotted line) in Eq.(2.13). Both the results are much bigger than the result when
both components are included coherently. Clearly a strong destructive interference effect
of both contributions is observed. The opposite sign of the $gg$ distribution amplitude
would cause constructive interference in Eq.(2.13).

In all cases we get less cross section than measured by the PHENIX collaboration at
RHIC. This suggests that the gluon-gluon fusion is probably not the dominant mecha-
nism of $\eta'$ production, at least in the measured region of transverse momenta. A natural
candidate is fragmentation process, which was not discussed in the literature so far in the
context of $\eta'$ production.

Neglecting the form factor at all leads to overestimation of the cross section at large
transverse momenta of $\eta'$ (see the dotted line in the left panel of Fig.14). Such a result
one could expect in the TMD (transverse momentum dependent gluon distributions) ap-
proach [41] where the incoming gluons should be taken on mass shell. The present result
shows therefore shortcomings of the TMD approach in the context of meson production.

To better illustrate the role of the initial $gg$ component in the approach with distri-
bution amplitudes in Fig.15 we show the final (including QCD evolution) result with
different initial $gg$ component: as in [17] (plus), with opposite sign (minus) and with ini-
tial $gg$ component put to zero. The final results are quite different. We conclude that the $\eta'$
transverse momentum distribution is very sensitive to the unknown nonperturbative
$gg$ distribution amplitude.

So far we used only one unintegrated gluon distribution. In Fig.16 we compare results
obtained using different UGDFs. The result obtained with the Jung-Hautmann UGDF is
FIG. 14: Invariant cross section as a function of meson transverse momentum. Here $\sqrt{s} = 200$ GeV and the KMR UGDF was used in the calculation. In the left panel results for the nonfactorized monopole, LCWF with $\beta = 0.5$ GeV, and simple LT parametrization. In the right panel we show results obtained using distribution amplitudes from [17]. We show the full result as well as result when only $q\bar{q}$ or only $gg$ components in (2.13) are included.

FIG. 15: Invariant cross section as a function of meson transverse momentum in the approach with distribution amplitudes and different initial $\Phi_{gg}$. Here $\sqrt{s} = 200$ GeV and the KMR UGDF was used in the calculation.
FIG. 16: Invariant cross section as a function of meson transverse momentum. Here $\sqrt{s} = 200$ GeV. We show results with the KMR (solid line), Jung-Hautmann (dashed line) and GBW (dash-dotted line) UGDFs. In this calculation the form factor based on the LCWF with $\beta = 0.5$ GeV is used for illustration.

similar to that obtained with the KMR UGDF. The GBW UGDF gives sizeable cross section only at low $\eta'$ transverse momenta as it does not include higher order perturbative effects.

What about larger energies? The number of $\eta'$ per event as a function of $\eta'$ transverse momentum is shown in Fig.17 for $\sqrt{s} = 8$ TeV. We show the result for the non-factorized monopole (2.25) two-photon transition form factor (solid line), light-cone wave function with $\beta = 0.5$ GeV (dashed line), the result with the simple leading twist parametrization (2.11) of the two-gluon transition form factor and the results of collinear approach with evolution of distribution amplitudes (see Eq.(2.12)). For comparison we show also result from the Lund string model. The two-gluon mechanism gives much smaller cross section than that from the Lund-string model. So even at the LHC we do not find any region of the phase space $(y, p_t)$ where the two-gluon fusion is the dominant mechanism of $\eta'$ production.
FIG. 17: Number of $\eta'$ mesons per event as a function of meson transverse momentum. Here $\sqrt{s} = 8$ TeV and the KMR UGDF was used in the calculation. The result of the Lund string model simulations is shown as “data points” for comparison.

IV. CONCLUSIONS

In this study we have considered production of two ($\phi$ and $\eta'$) isoscalar mesons with hidden strangeness via gluon-gluon fusion in proton-proton collisions for different collision energies relevant for RHIC and the LHCb. The calculations have been performed within $k_t$-factorization approach with the KMR UGDF which is known to include effectively higher-order corrections [39, 40].

For the $\phi$ production we extend the calculation performed earlier for the $J/\psi g$ production by using effective spatial wave function at the origin $R_{ss}(0)$ which can be estimated from the decay $\phi \rightarrow e^+e^-$ by adjusting it to experimental branching ratio. Having found the parameter we have compared results of our calculation with the PHENIX and ALICE experimental data. In both cases the calculated cross section stays below the experimental data by two (PHENIX) and by one (ALICE) order of magnitude. This shows that another mechanism is more important. The fragmentation of $s/\bar{s} \rightarrow \phi$ is a natural candidate.

Inspired by the successful description of $\eta_c$ production in proton-proton collisions [2], here we have considered the $g^* g^* \rightarrow \eta'$ fusion with off-shell initial gluons. The coupling can be described by the two-gluon nonperturbative transition form factor. For the quark-
antiquark states the latter object is closely related to the two-photon transition form factor, studied theoretically and measured by the CLEO, L3 and BABAR collaborations.

The two-photon form factor has been calculated using a light-cone wave function for different values of model parameters. The so-obtained form factor has been compared with a simple non-factorized monopole parametrization as well as the results obtained recently by Kroll and Passek-Kumericki in the leading-twist collinear NLO approach.

The two-photon form factors have been translated to the two-gluon ones assuming the dominance of the quark-antiquark components in the Fock $\eta'$ wave function expansion. Then it was used in the $k_t$-factorization approach to calculate the cross section for $\eta'$ production in $pp$ collisions. The results have been compared with the PHENIX experimental data. In spite of the expectation of the community the calculated cross section is definitely smaller than the measured one obtained by the PHENIX collaboration. The situation may improve at larger energies but the relevant cross section at the LHC was not measured so far. We have presented our predictions for the LHC and has compared our two-gluon fusion result with the result form the Pythia generator. For $\sqrt{s} = 8$ TeV the cross section from the Lund string model is much above that for the two-gluon fusion mechanism. Respective data from the ALICE collaboration would be very important to clarify the situation.

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