Statistics of Composite Fermions in Quantum Hall Effect

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The high Landau level filling fractions 5/2, 7/3 and 8/3 are interpreted by using the angular momentum model. It is found that for the odd number of flux quanta, the quasiparticles called the “composite fermions” are fermions but for even number-, the quasiparticles are a mixture of bosons and fermions. Therefore, the theory of “composite fermions” is internally inconsistent.

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1 Introduction

In 1980 von Klitzing et al [1] have shown that the Hall current is given by an integer multiple of $e^2/h$ so that an accurate value of $e^2/h$ can be measured. Later on, it was found by Tsui et al [2] that not only the integer but also the fractional multiples of $e^2/h$ can be identified at higher magnetic fields than in von Klitzing’s case. The integers as well as the fractions which multiply $e^2/h$ may be treated as spectra and hence as quasiparticles. It was thought that these quasiparticles obey fractional statistics [3]. The electric and the magnetic field vectors of the Maxwell equations are decoupled so that magnetic flux quanta are attached to the electrons which then obey fractional statistics [4]. At this time, we found [5] the series which gives the fractions at which quasiparticles occur. All of the predicted fractions are the same as those observed in various experiments. In one of the experimental measurements, it was found that masses of two quasiparticles are equal for different values of the fractions. Such an equality of masses can be interpreted [6] by means of particle-hole symmetry based on the earlier theory [5]. Several years later, flux quantization was applied [7] to the present problem which also, gave fractions. One of the series of fractions obtained from this work is the same as in our paper [5] and there is one more series which is slightly shifted from our’s. Laughlin [8] wrote the wave function for the quasiparticle of a fractional charge. In this theory, the vacuum state is correctly defined from which the quasiparticles of fractional charge are created and when sufficient number of particles are created they may be annihilated subject to the condition that the flux area is held rigid so that the quasiparticles are incompressible and hence the hardest objects in the world which GaAs is not. Our work predicts the masses in terms of particle-hole symmetry by using the Kramers theorem [9,10].

| $l$ | $l/(2l+1)$ | $(l+1)/(2l+1)$ | $(nl/(2l+1))$ | $n(l+1)/(2l+1)$ |
|-----|------------|----------------|--------------|----------------|
| $\infty$ | 1/2        | 1/2            | 5/2          | 5/2            |
| 7    | 7/15       | 8/15           | 7/3          | 8/3            |

Table 1: Interpretation of 5/2, 7/3 and 8/3 in terms of theory of ref. 5.
In this communication, we interpret the fractions $7/3$, $5/2$ and $8/3$ seen in the measurements. We report that $7/3$ and $8/3$ are particle-hole conjugates and $5/2$ is the $n = 5$ state of the level at $1/2$. Since we used angular momentum theory to interpret the quantum Hall effect we are able to attach spin to the effective fractional charges found in the data which leads to identification of statistics. The odd fluxes are always fermions whereas even number of fluxes are mixtures of fermions and bosons. Therefore the “composite fermion” theory of Jain is incorrect.

2 Fractions $7/3$, $5/2$ and $8/3$

Our model [5] has several features which are in agreement with experimental measurements. The fractions predicted by us are the same as those experimentally found by Eisenstein and Störmer [11], Willett et al [12] Du et al [13] and others. The fractions occur in two groups. One group of fractions lies on high field side of $1/2$ and the other on the low field side of $1/2$. The value of $1/2$ occurs in both the series and hence represents a fluid of two components. The grouping of fractions in our theory is the same as experimentally found [11]. Our model also gives $\nu = 1/2$ for very large values of $l$ and hence there is a limiting value $n/2$ where $n$ is the Landau level quantum number. In our model, one of the series is,

$$\nu = \frac{l + (1/2) - s}{2l + 1} = \frac{l}{2l + 1} \quad (1)$$

which predicts one group of fractions $0, 1/3, 2/5, 4/9, 5/11$, etc. which are also observed by Willett et al [12]. Another group of fractions is predicted [5] by the expression,

$$\nu = \frac{l + (1/2) + s}{2l + 1} = \frac{l + 1}{2l + 1} \quad (2)$$

which are $1, 2/3, 3/5, 4/7$, etc in complete agreement with the experimental data [11-13]. When $l = \infty$ both the above series approach $1/2$ except that one series approaches from the right hand side and the other from the left hand side exactly as observed [11]. The left and the right side approaches arise from the Kramers conjugate states and the
predicted approach is exactly as observed. Due to the limit, there is a Fermi surface at 1/2. However, we can shift from the Fermi surface to higher values when higher Landau levels are occupied. The fraction 1/2 becomes \( n/2 \) with \( n \) as the Landau level quantum number. Thus 1/2, 3/2, 5/2, 7/2, 9/2, etc., become allowed. This predicted feature with an odd numerator with 2 in the denominator is also exactly as observed. Thus for \( n = 1 \), we have two series, one merging from left while the other merging from right at 1/2 and the same picture is repeated for different values of \( n \). The entire pattern of pairwise series is observed exactly as predicted. Many fractions with 2 in the denominator have been observed by Lilly et al [14] and by Yeh et al [15]. The series (1) and (2) above can be used to explain the higher Landau levels easily. Eisenstein et al [16] have found that at higher values of the Landau level quantum number, \( n \), the number of fractions observed is much less than at the lowest Landau level. At the magnetic field of 4 to 5 Tesla only a small number of fractions are observed, the strongest ones are at 8/3, 5/2 and 7/3. Since there is a charge versus Landau level quantum number product, it is not possible to distinguish large charge from a large Landau level quantum number. In the angular momentum series, \( l/(2l + 1) \) is the particle-hole conjugate of \( (l + 1)/(2l + 1) \) by virtue of Kramers theorem. For \( l = 7 \) two values, 7/15 and 8/15 are predicted and \( l = \infty \) values are \( \pm 1/2 \). When the same particle occurs in different levels, its charge remains unchanged. We can multiply the values by \( n = 5 \) so that the predicted values of 1/2, 7/15 and 8/15 become 5/2, 7/3 and 8/3. These predicted values are exactly the same as those observed experimentally by Eisenstein et al[16]. Thus 7/3 is the particle-hole conjugate of 8/3 as seen in Table 1 for \( n = 5 \). Here the particle-hole symmetry is obtained by reversing the spin, -1/2 for one and +1/2 for the other as in Kramers conjugate pairs due to strong spin-orbit interaction whereas in ordinary semiconductors the particle has the same spin as the hole and they are separated in energy by a gap in the range of optical frequencies. Thus the present problem is different from the usual optical absorption in semiconductors. The present spin-orbit interaction has \( 1/c \) in the coupling constant and
both spin orientations are permitted to determine the gyromagnetic ratios whereas the ordinary spin-orbit interaction has $1/c^2$ and the gyromagnetic ratio is determined by the Lande’s formula. Thus the angular momenta series (1) and (2) given by ref. 5 explain the quantum Hall effect correctly.

3 Composite Fermions

We introduce the flux quantization such that the field inside the material is smaller than outside. The reduction in the field is linearly proportional to the electron density, the number of electrons per unit area, $\rho$. We use the even number of fluxons, $2\phi_o$ or $2p\phi_o$ where $p$ is an integer so that the effective field becomes,

$$B^* = B - 2p\rho\phi_o .$$  \hspace{1cm} (3)

The quasiparticles which experience the field $B^*$ are called composite fermions, CFs, and the field $B$ is experienced by the usual electrons. As far as the flux quantization is concerned the fractions are completely symmetric,

$$\nu = \frac{\rho\phi_o}{B}$$  \hspace{1cm} (4)

for electrons and

$$\nu^* = \frac{\rho\phi_o}{B^*}$$  \hspace{1cm} (5)

for the CFs. We substitute (3) in (5) and then (4) in the resulting equation so that

$$\nu_+ = \frac{\nu^*}{2p\nu^* + 1}.$$  \hspace{1cm} (6)

For $\nu^* = 0, 1, 2, 3, \cdots$ etc. we can get $\nu = 0, 1/3, 2/5, 3/7, \cdots$ etc. When we reverse the sign of the field shift by reversing the sign of $2p\rho\phi_o$, the series generated is,

$$\nu_- = \frac{-\nu^*}{2p\nu^* - 1}.$$  \hspace{1cm} (7)

which gives, $\nu_- = 0, 1, 2/3, 3/5, 4/7, \cdots$, etc. Both of these series are the same as those of Shrivastava [5] except that the $\nu_-$ series is shifted in the value of $\nu^*$. Because of this shift,
there is a serious loss because (1) and (2) add up to one but (6) and (7) do not. If both series in (6) and (7) are correct, when we put \( p = 1 \), the effective field of (3) becomes \( B^* = B - 2\rho\phi_o \) so that two (even number) of fluxes, \( 2\phi_o \), is multiplied to the areal density of electrons [7]. For \( p = 2, 4\phi_o \) and for \( p = 3, 6\phi_o \) are needed. Here \( \nu = 1/3 \) means that effective charge is \((1/3)e\), etc. However, the quantity which enters the definition of \( \nu \) is the area multiplied by charge. Hence we can not know whether the area or the charge has to become 1/3. The fractions \( \nu \) for electrons and \( \nu^* \) for CFs are interchangeable in (6) and (7),

\[
\nu^* = \frac{\nu}{2p\nu + 1}
\]

and

\[
\nu^* = \frac{-\nu}{2p\nu - 1}.
\]

By comparing (6) with (8) and (7) with (9), we find that \( \nu \) and \( \nu^* \) are completely interchangeable. One of the CF series (6) is identical to (1) and the other series (7) can be made equal to (2) if

\[
\frac{l + 1}{2l + 1} = \frac{\nu^*}{2p\nu^* - 1}
\]

The solution of which is

\[
\nu^* = l + 1.
\]

This is an important result because it brings CF series in agreement with those obtained by use of the angular momentum [5]. The series in (1) is,

\[
\frac{l + \frac{1}{2} - s}{2l + 1} = \frac{l}{2l + 1}
\]

for \( s = \frac{1}{2} \) and that in (2) is \( [l + \frac{1}{2} + s]/(2l + 1) \) which for \( s = \frac{1}{2} \) becomes \( (l + 1)/(2l + 1) \). Therefore (11) is equivalent to

\[
l + \frac{1}{2} + s = \nu^*.
\]

Substituting this relation in (5) and writing \( \rho = n_o/A \), the number of electrons \( n_o \) in the area \( A \), we obtain,

\[
l + \frac{1}{2} + s = \frac{n_o\phi_o}{AB^*}.
\]
For \( l = 0, s = \frac{1}{2} \), the above gives

\[ AB^* = n_o \phi_o \]  

(15)

where \( n_o \) can be even or odd. Thus both the even as well as old values of \( n_o \) correspond to fermion statistics due to spin, \( s = \frac{1}{2} \). When \( l = 0, s = 0 \), the result (14) gives

\[ AB^* = 2n_o \phi_o \]  

(16)

which means that bosons (\( s = 0 \)) are having even number of flux quanta. Thus CFs are bosons for even number of \( \phi_o \) and fermions for both even and odd number of flux quanta. Therefore, for odd number of flux quanta, the quasi particles are fermions and for even number, they are mixtures of bosons and fermions. The field \( B^* \) is zero at,

\[ B^* = B - 2p \rho \phi_o = \frac{\rho \phi_o}{\nu} - 2p \rho \phi_o = 0 \]  

(17)

so that

\[ \frac{1}{\nu} = 2p \]  

(18)

which for \( p = 1 \), gives \( \nu = \frac{1}{2} \). Therefore, for half filled Landau level there is a possibility of zero resistivity but the experimental zeros of \( \rho_{xx} \) are not located at \( \nu = 1/2 \). If the flux is reversed, \( \nu = -1/2 \) occurs which does not have a physical interpretation in terms of band filling. Therefore, Jain’s formula of \( B^* \) given by (3) is not in agreement with the experimental data of \( \rho_{xx} \). Therefore, attachment of \( 2 \phi_o \), even number of fluxes is not consistent with the data. We require that \( \rho_{xx} \) should touch zero at \( \nu = 1/3 \) but that is not born out from the value of \( B^* \) which attaches even number of flux quanta, when \( \nu = 1/3, p = 3/2 \) but \( p \) is required to be an integer. Therefore, the electric field (resistivity) is not understood if even fluxes are attached to electrons, when \( \nu = 1/3, p = 3/2 \), the effective field becomes,

\[ B^* = B - 3 \rho \phi_o \]  

(19)

so that odd number of flux quanta, \( 3 \phi_o \) are attached. This creates the problem of statistics. If we put \( \nu^* = 1/3 \) in (13), quasiparticles of different statistics must be mixed.
It is true that Laughlin’s theory [8] of fractional charge has one-body interaction summed over all the particles from 1 to \( N \) and a factor which has two-body interaction and the product is made from 1 to \( N \). This forms the many body wave function using which the matrix elements of the hamiltonian containing electron-electron repulsive interaction, electron-nuclear attraction and respective kinetic energies, are computed to find the energy. The charge of \( 1/3 \) is assumed while writing the two-body correlation. This value of the charge is taken from the experimental measurements of the plateau in the Hall resistivity. On the other hand, our theory of ref.[5] is surely a single-particle theory and gives the correct charge. At \( l \to \infty \), we get \( \nu = 1/2 \) for both (1) and (2). At this point our energy is \( -(1/2)l \) for \( j = l + 1/2 \) and \( +(1/2)(l + 1) \) for \( j = l - 1/2 \) so that the energy diverges at \( l \to \infty, \nu = 1/2 \). The interaction of the form \( \hat{n} \times \vec{v} \cdot \vec{s} \) where \( \hat{n} \) is a unit vector in the direction of \( \vec{r} \), \( \vec{v} \) the velocity and \( \vec{s} \) the spin, has a distance dependent coupling constant, \( f(\vert \vec{r} \vert) \) so that \( -f(\vert \vec{r} \vert)\vec{l} \cdot \vec{s} \) depends on the distance. We can introduce the summation over \( N \) particles to make it into many-body hamiltonian but it is not required to get the fractional charge. Similarly, for one electron per unit area, the flux is quantized without the need for many-body interactions.

**Inconsistencies.** It is agreed that even number of flux quanta when attached to an electron give “composite fermions” which must be fermions. Similarly, odd number of flux quanta attached to the electron are called “composite bosons” and they are bosons. In eq.(15) integer number of flux quanta give fermions and in eq.(16) even number of flux quanta give bosons. Therefore, there are inconsistencies in the Jain’s theory of composite fermions. In the composite fermions spin is completely arbitrary. Therefore, at \( \nu = 1/2 \) four states, one singlet and three components of the triplet should occur. On the other hand \( \nu = 1/3 \) is found to be polarized and \( \nu = 1/2 \) gives only two states. This experimental observation is consistent with the theory of Shrivastava [5] and not with that of the composite fermion theory of Jain [7]. The claim of Melinte et al [17] that the composite fermion theory agrees with the data of polarization of electrons in the NMR...
experiment is therefore incorrect.

It was found[10] that a lot of data on the quantum Hall effect is in fact in accord with the theory of Shrivastava [5] and the theoretical Table 1 of Shrivastava [5] is the same as the experimental Fig.18 of Störmer’s Nobel lecture[18]. The experimentally found high Landau levels are in agreement with our theory [19]. We find that there is a change in the magnetic moment of the electron [20] due to the new way of looking at the spin-orbit interaction. There is an important effect of sweep rate on the quantum Hall effect resistivity minima[21]. The fact that our predicted results are in complete agreement with the data is clearly brought out in a recent article [22].

In conclusion, we are able to understand the high Landau levels by using the theory of angular momentum given in ref. 5. We find that the composite fermions are not able to explain the zeroes in the transverse resistivity. By means of flux quantization, the odd number of fluxes give Fermi statistics but even number of fluxes are obtained for both the Fermi as well as Bose statistics. Therefore, the CF theory of Jain [7] is internally inconsistent.
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