A low temperature derivation of spin-spin exchange in Kondo lattice model

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Using Hubbard-Stratonovich transformation and drone-fermion representations for spin-$\frac{1}{2}$, which is presented for the first time, we make a path-integral formulation of the Kondo lattice model. In the case of weak coupling and low temperature, the functional integral over conduction fermions can be approximated to the quadratic order and this gives the well-known RKKY interaction. In the case of strong coupling, the same quadratic approximation leads to an effective local spin-spin interaction linear in hopping energy $t$.

PACS numbers: 71.27.+a
Keywords: spin-spin exchange, Kondo lattice

I. INTRODUCTION

In systems in which there is little direct overlapping of wave functions of localized magnetic atoms, the spin-spin interaction mediated by itinerant electrons is the dominant magnetic interaction, and has been an interesting topic for several decades. In particular, the study of spin-spin interaction in diluted magnetic semiconductors (DMS) has drawn much attention in the past several years. One of the key issues in explaining the magnetic properties of DMS is how the very diluted itinerant electrons (or holes) mediate the interactions between the localized, randomly distributed holes. When the coupling between itinerant fermions and spins is weak compared to the Fermi energy, perturbation theory applies. Ruderman and Kittel initially considered the indirect exchange coupling of nuclear magnetic moments by conduction electrons. And later, this interaction, conventionally called RKKY exchange, was extended by Bloembergen, Rowland, Kasuya, and Yosida. Usually, RKKY interaction can be obtained using second order perturbation theory. But in the strong coupling regime, the conventional perturbation theory doesn’t work and we need to devise a new way to derive the effective spin-spin exchange. In section II, we present a path-integral formulation of Kondo lattice model, in which local spins are represented by drone-fermions. Here the drone-fermion representation of spin-$\frac{1}{2}$ is presented for the first time. At low temperatures, the spin-spin exchange is obtained. In section III we discuss a two site system and the origin of ferromagnetic interaction linear in $t$. The last section is a brief summary.

II. THE PATH-INTEGRAL AND THE SPIN-SPIN EXCHANGE

We consider Kondo lattice model with Hamiltonian

$$H = \sum_{(i,j,\sigma)} t_{ij} c^\dagger_{i\sigma} c_{j\sigma} + J_K \sum_i S_i \cdot s_i - \mu \sum_i (n_i^\uparrow + n_i^\downarrow)$$

where $c^\dagger_{i\sigma} (c_{i\sigma})$ denotes the creation (annihilation) operator of the itinerant electrons (or holes) and $n_{i\sigma} = c^\dagger_{i\sigma} c_{i\sigma}$ is the number operator. $S_i$ denotes the localized spins, either $S = \frac{1}{2}$ or $S = \frac{3}{2}$. $s = \frac{1}{\sqrt{2}}(\tau c)$ is the electron spin where $\tau = (\tau_1, \tau_2, \tau_3)$ is the vector of the three usual Pauli matrices. Hopping energy $t_{ij} = t > 0$ if $i, j$ are nearest neighbors and zero otherwise, $J_K > 0$ is the Kondo coupling and $\mu$ denotes chemical potential. The symbol $\langle i, j \rangle$ implies that the summation in the first term is taken over nearest neighbors. The other two summations are taken throughout the lattice. Note that there exists a local spin on every site and these local spins do not interact with each other directly. To derive the mediated interaction, we need to separate the conduction fermions and the local spins first. One way to this is to introduce an auxiliary field and use Hubbard-Stratonovich transformation. For this, one needs to express the coupling $S \cdot s$ in terms of squares of Hermitian, bounded operators. There are different ways to do this. For example, one can write

$$S \cdot s = \frac{1}{2}[(S + s)^2 - S^2 - s^2] = -\frac{1}{2}(S - s)^2 - S^2 - s^2]$$

Using $S^2 = S(S+1)$ and $s^2 = -\frac{3}{4}(n_\uparrow + n_\downarrow)^2 + \frac{3}{2}(n_\uparrow + n_\downarrow)$

$$S \cdot s = \frac{1}{2}[(S + s)^2 - S(S+1) + \frac{3}{4}(n_\uparrow + n_\downarrow)^2 - \frac{3}{2}(n_\uparrow + n_\downarrow)]$$

or

$$S \cdot s = -\frac{1}{2}[(S - s)^2 - S(S+1) + \frac{3}{4}(n_\uparrow + n_\downarrow)^2 - \frac{3}{2}(n_\uparrow + n_\downarrow)]$$

To formulate the path-integral, we also need a representation for the local spins. For spin-$\frac{1}{2}$, we can use the drone-fermion representation

$$S^z = f^\dagger f - \frac{1}{2}, \quad S^+ = f^\dagger (d + d^\dagger), \quad S^- = (d + d^\dagger) f$$

where $f, d$ are fermionic operators. One of the most important conveniences of this representation is that unlike the Schwinger fermion representation, this representation does not require constraints. For spin-$\frac{3}{2}$, we have a generalization (to our best knowledge, this generalization is new, ref. only represents the spin operators in terms of fermi operators and the representation is obviously not a drone-fermion representation since one term is bosonic and the
other is fermionic: $S^+ = \sqrt{3} f_1^+ (d + d^\dagger) + 2 f_2^+ f_1, S^- = \sqrt{3} (d + d^\dagger) f_2 + 2 f_1^+ f_2 - \frac{3}{2}$. Note that we use 3 fermi fields for spin-$\frac{3}{2}$. In general, this type of fermi representation is valid for $S-$spin when $2^n - 1 < 2S + 1 \leq 2^n$, as counted in [12]. Thus it is seen that for both spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$, we can write the partition function as purely fermionic path-integral. Using expression (3) and the relation (the so-called Hubbard-Stratonovich transformation) $e^{-A^2} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx e^{-(x^2 + 2Ax)}$, which holds for Hermitian and bounded operator $A$, we can write the partition function as

$$Z = \int D\mathbf{A}_1(\tau) D\varphi_1(\tau) e^{-\int_0^\beta d\tau \sum_i (\mathbf{A}_i^2 + \varphi_i^2)} \int D\mathbf{c}_\tau(\tau) D\mathbf{c}^*_\tau(\tau) \int D\mathbf{d}_\tau(\tau) D\mathbf{d}^*_\tau(\tau) \int D\mathbf{\varphi}_\tau(\tau) D\mathbf{\varphi}^*_\tau(\tau) e^{-\int_0^\beta d\tau [\mathbf{c}_\tau \mathbf{c}^*_\tau + \mathbf{d}_\tau \mathbf{d}^*_\tau + \mathbf{\varphi}_\tau \mathbf{\varphi}^*_\tau + \text{int}]}$$

where $D$ denotes differential measure and $\mathbf{A}_1(\tau), \varphi_1(\tau)$ are auxiliary random fields and the extended Hamiltonian

$$\mathcal{H} = t \sum_{(ij),\sigma} c_i^\dagger c_j - \left(\mu - \frac{3}{4} J_K\right) \sum_i (n_i \uparrow + n_i \downarrow) + \sum_i [i \sqrt{2J_K} (s_i + s_i^*) \cdot \mathbf{A}_1 + i \sqrt{\frac{3}{2}} J_K \varphi_1 (n_i \uparrow + n_i \downarrow)]$$

Using the Fourier transformation $A_{mk} = N^{-1} \sum_{i} e^{i\omega_n \tau} e^{ik \cdot x} A_i(\tau)$ where $\omega_n^B = \frac{2m}{\beta \pi}, (m = 0, 1, \ldots)$ is the Matsubara frequency for bosonic fields, the electron part of the functional-integral is written as

$$Z_e[\mathbf{A}, \varphi] = \int e^{-\sum n_{m,k,p} c_{m,k,p}^\dagger c_{m,k,p} - \sum \omega_n [i \sqrt{2J_K} (s_i + s_i^*) \cdot \mathbf{A}_1 + i \sqrt{\frac{3}{2}} J_K \varphi_1 (n_i \uparrow + n_i \downarrow)]}$$

where $\omega_n = \frac{2n+1}{\beta \pi}, (n = 0, 1, \ldots)$ is the Matsubara frequency for fermionic fields. Using the path-integral technique for Grassmann variable, we have

$$\ln Z_e[\mathbf{A}, \varphi] = \ln \frac{\text{Det}[(i\omega_n + t_k - \mu - \frac{3}{4} J_K)\delta_{p,k}\delta_{m,n} + i \sqrt{\frac{3}{2N\beta}} J_K \varphi_{m,k} \cdot A_{m,k} - p + i \sqrt{\frac{1}{2N\beta}} J_K \sigma \cdot A_{m,k}]}{\text{Det}(i\omega_n \delta_{m,n})}$$

$$+ \ln \text{Det}[\sqrt{\frac{3}{2N\beta}} J_K \varphi_{k,m} \cdot A_{m,k}]$$

At low temperatures such that $\sqrt{J_K/\beta} \ll$ the largest of $t, J_K$ and $\mu$, the third term can be calculated perturbatively. Using $\ln \text{Det} A = \text{Tr} \ln A$, we have

$$\ln \text{Det}[(i\omega_m + t_k - \mu - \frac{3}{4} J_K)^{-1} i \sqrt{\frac{3}{2N\beta}} J_K \varphi_{m,k} \cdot A_{m,k} - i \sqrt{\frac{J_K}{2N\beta}} \sigma \cdot A_{m,k}]$$

$$= \text{Tr}[(i\omega_m + t_k - \mu - \frac{3}{4} J_K)^{-1} i \sqrt{\frac{3}{2N\beta}} J_K \varphi_{m,k} \cdot A_{m,k} - i \sqrt{\frac{J_K}{2N\beta}} \sigma \cdot A_{m,k}]$$

$$- \frac{1}{2} \text{Tr}[(i\omega_m + t_k - \mu - \frac{3}{4} J_K)^{-1} i \sqrt{\frac{3}{2N\beta}} J_K \varphi_{m,k} \cdot A_{m,k} - i \sqrt{\frac{J_K}{2N\beta}} \sigma \cdot A_{m,k}]^2 + \cdots$$

(9)
To the quadratic order, i.e., neglecting the part \cdots, we have

\[
\ln \text{Det}[\delta_{m,n}d_{k,p} + (i\omega_m + t_k - \mu - \frac{3}{4}J_K)]^{-1} (i\sqrt{\frac{3}{2N^2\beta^2}J_K} \varphi_{m-n,k-p} + i\sqrt{\frac{J_K}{2N\beta^2} \sigma} \cdot A_{m-n,k-p})]
\]

\[
\simeq 2 \sum_n \frac{i}{i\omega_n + t_k - \mu - \frac{3}{4}J_K} - \frac{1}{2} \sum_{p,m,n} \text{Tr} \frac{i}{i\omega_m + t_k - \mu - \frac{3}{4}J_K}
\]

\[
\int \mathcal{D}f_\sigma(\tau) \mathcal{D}f_\sigma^*(\tau) \mathcal{D}d_\sigma(\tau) \mathcal{D}d_\sigma^*(\tau) e^{-\int_0^\beta d\tau \int \left( i\omega_m + t_k + \mu - \frac{3}{4}J_K \right)} \text{Tr} \frac{i}{i\omega_n + t_p - \mu - \frac{3}{4}J_K}
\]

The first term vanishes in the thermodynamic limit \( N \to \infty \). Therefore the leading term is the second term

\[
\sum_{p,k,m,n} \frac{3}{2N^2} J_K \varphi_{m-n,k-p} \varphi_{n+m-p-k} + i\overline{A}_{m-n,k-p} A_{m-n,p-k}
\]

\[
= - \frac{J_K}{2t} \sum_{m,k} (3\varphi_{m,k} \varphi_{m,-k} + A_{m,k} \cdot A_{m,-k}) Q_{mk}
\]

where

\[
Q_{mk} = -\frac{1}{\beta N} \sum_{n,p} \frac{t}{i\omega_m + t_k + \mu - \frac{3}{4}J_K}(i\omega_n + t_p - \mu - \frac{3}{4}J_K)
\]

so the partition function

\[
Z = \int \mathcal{D}f_\sigma(\tau) \mathcal{D}f_\sigma^*(\tau) \mathcal{D}d_\sigma(\tau) \mathcal{D}d_\sigma^*(\tau) e^{-\int_0^\beta d\tau \int \left( i\omega_m + t_k + \mu - \frac{3}{4}J_K \right)} \text{Tr} \frac{i}{i\omega_n + t_p - \mu - \frac{3}{4}J_K}
\]

\[
\times \exp\left( -\frac{J_K}{2t} \sum_{m,k} (3\varphi_{m,k} \varphi_{m,-k} + A_{m,k} \cdot A_{m,-k}) Q_{mk} \right) e^{-\frac{1}{J_K} \sum_m S_{mk}^* A_{m,k}}
\]

where \( df_{\sigma} d\varphi_{\sigma} \) are just ordinary integration measures as \( A_{mk}, \varphi_{mk} \) are now Fourier components. We have used replacement \( A_{mk} \rightarrow iA_{mk}, A_{mk}^* \rightarrow -iA_{mk} \); i.e., redefine the Fourier component such that

\[
A_{mk} = \frac{1}{N} \sum_i \frac{1}{\sqrt{\beta}} \int_0^\beta d\tau e^{i\omega_m \tau} e^{-i}\tau A_i(\tau)
\]

\[
A_{m,-k} = \frac{1}{N} \sum_i \frac{1}{\sqrt{\beta}} \int_0^\beta d\tau e^{-i\omega_m \tau} e^{-i}\tau (-i) A_i(\tau)
\]

Hence the induced spin-spin interaction comes from the integral over \( A \), which gives

\[
\int \mathcal{D}A_{mk} e^{-\sum_{m,k} A_{mk}^* A_{m,k} - \frac{J_K}{2} \sum_{m,k} (S_{mk}^* A_{m,k}^* + A_{m,k} S_{mk})} = e^{-\frac{1}{J_K} \sum_{m,k} S_{mk}^* \frac{1}{1 + \frac{1}{2}Q_{mk}} S_{mk}}
\]

Now since

\[
Q_{mk} = -\frac{1}{\beta N} \sum_p \frac{t}{i\omega_m + t_k + \mu - \frac{3}{4}J_K} \sum_n \frac{1}{i\omega_m + t_k + \mu - \frac{3}{4}J_K} - \frac{1}{i\omega_n + t_p - \mu - \frac{3}{4}J_K}
\]

, using the Matsubara frequency sum \( \sum_{n=-\infty}^{\infty} \frac{1}{e^{\beta \omega_n + x}} = f(x) = \frac{1}{e^{x+1}} + 1 \) is the Fermi function. In the low temperature limit, we can take \( \omega_n^B \simeq 0 \) so \( Q_{mk} \simeq Q_k \), 

\[
Q_{mk} = \frac{t}{N} \sum_p f(t_k + p - \mu - \frac{3}{4}J_K) - f(t_p - \mu - \frac{3}{4}J_K)
\]

(16)
where

\[ Q_k \simeq \frac{t}{N} \sum_p f(t_{k+p} - \mu - \frac{3}{4}J_k) - f(t_p - \mu - \frac{3}{4}J_k) \]

\[ \frac{1}{t_p - t_{k+p}} \]  \hspace{1cm} (17)

(In the case \( t_k = k^2/(2m) \), this is nothing but the function \( f_3(q) \) [13] and we reproduce the RKKY interaction). If \( t \gg J_k \), the induced interaction is

\[- \int_0^\beta d\tau \sum_{i,j} F(i-j) S_i \cdot S_j \simeq - \frac{J^2}{4t} \sum_{m,k} S^\ast_{mk} Q_k \cdot S_{mk} \]  \hspace{1cm} (18)

which is RKKY-like since it is proportional to \( J_k^2 \) and inversely to \( t \) which corresponds to Fermi energy.

In the strong coupling case as in diluted magnetic semiconductors, \( t \ll J_k \). Keeping only the quadratic term of the functional determinant in (10), the resulting spin-spin effective action at low temperatures is of the form

\[ \sum_{m,k} S^\ast_{mk}(\frac{t}{2J_k} + Q_k)^{-1} S_{mk} = - \int_0^\beta d\tau (-t) \sum_{i,j} J_{ij} S_i \cdot S_j \]  \hspace{1cm} (19)

if \( Q_k \gg \frac{2t}{J_k} \) for all \( k \), where \( J_{ij} \) is the Fourier transform of the \( (\frac{t}{2J_k} + Q_k)^{-1} \). This interaction is linear in \( t \). The linearity in \( t \) of the induced spin-spin interaction is intuitive since when \( J_k \) is very large, then \( J_k \) is irrelevant and the only energy scale left is \( t \). A more formal discussion of induced spin-spin interactions for a two site system is given in the next section.

III. PERTURBATIVE DERIVATION OF MOMENT-MOMENT EXCHANGE

(1) Exchange between classical moments at half-filling

We now consider a half-filled lattice with a classical moment \( S \) on each site, where \( n \) is a unit vector. The relation of this system to the Kondo lattice model described by Hamiltonian (1) is as follows. The quantum-mechanical states of the local spin \( S \) can be expressed in terms of the eigenstate \( |n \rangle \) of \( S \cdot S |n \rangle = S_n |n \rangle \).

We can then obtain the energy levels by considering different orientations of \( n \). In the strong-coupling limit, the hopping term is taken as a perturbation while the on-site interaction is the unperturbed Hamiltonian. Using the canonical transformation

\[ c_{i \uparrow} = \frac{1}{\sqrt{2 - 2n_{iz}}} (n_{i\uparrow} + (n_{iz} - 1)n_{i\downarrow}) \]  \hspace{1cm} (20)

\[ c_{i \downarrow} = \frac{1}{\sqrt{2 - 2n_{iz}}} ((1 - n_{iz})n_{i\uparrow} + n_{i\downarrow}) \]  \hspace{1cm} (21)

where \( n_{iz} = n_{iz} \pm n_{iy} \), and \( \alpha_i, \beta_i \) are new fermi fields, the on-site term becomes

\[ J_K S \sum_i \langle \tau | n_i(\tau) = \frac{1}{2J_K S} \sum_i \langle \alpha_i^\dagger(\tau)\alpha_i(\tau) - \beta_i^\dagger(\tau)\beta_i(\tau) \]  \hspace{1cm} (22)

Hence at half-filling (the number of particles equals the number of lattice sites), the unperturbed ground-state is (for \( J_K > 0 \))

\[ |\text{Gnd}\rangle = \prod_i |\beta_i^\dagger|0\rangle \]  \hspace{1cm} (23)

with energy \( E_{\text{Gnd}} = -\frac{1}{2}J_K SN \). \( N \) is the number of lattice sites. Since the hopping part reads \( H' = \sum_{i,j} H_{ij} \) where

\[ H_{ij} = t_{ij} \frac{1}{\sqrt{(1 - n_{iz})(1 - n_{iz})}} \frac{1}{\sqrt{(1 - n_{iz})(1 - n_{iz})}} \]

\[ \{ (n_{i\uparrow}n_{j\downarrow} + (1 - n_{iz})(1 - n_{iz})[\alpha_i^\dagger\alpha_j^\dagger + [n_{i\downarrow}(n_{iz} - 1) + (1 - n_{iz})n_{j\downarrow}]\alpha_i^\dagger\beta_j

+ [(n_{iz} - 1)n_{j\uparrow} + n_{i\downarrow}(1 - n_{iz})]\beta_i^\dagger\alpha_j + [(n_{iz} - 1)(n_{iz} - 1) + n_{i\downarrow}n_{j\downarrow}]\beta_i^\dagger\beta_j \} \]  \hspace{1cm} (24)

the only non-vanishing term in \( H'|\text{Gnd}\rangle \) is,

\[ \sum_{ij} t_{ij} \frac{1}{\sqrt{(1 - n_{iz})(1 - n_{iz})}} \frac{1}{\sqrt{(1 - n_{iz})(1 - n_{iz})}} [n_{i\downarrow}(n_{j\downarrow} - 1) + (1 - n_{iz})n_{j\downarrow}]\alpha_i^\dagger\beta_j \]  \hspace{1cm} (25)

Defining states

\[ |ij\rangle = |\beta_1^\dagger, ..., \beta_{i\uparrow}^\dagger, ..., (j), ..., \beta_{N\downarrow}^\dagger \rangle \]  \hspace{1cm} (26)

in which site \( j \) is empty, site \( i \) is doubly occupied and all other sites are singly occupied by \( \beta \) quasi-particles. Therefore the second-order perturbation is

\[ \sum_{kl} \frac{\langle \text{Gnd} | \sum_{ij} H_{ij} | kl \rangle \langle kl | \sum_{ij} H_{ij} | \text{Gnd} \rangle}{E_{\text{Gnd}} - E_m} \]  \hspace{1cm} (27)
Note that

\[ (ij|H'|Gnd) = (-1)^{i+j} t_{ij} \frac{1}{2} \frac{1}{\sqrt{(1-n_{ix})(1-n_{iz})}} [n_{i-}(n_{j_2}-1) + (1-n_{iz})n_{j-}] \]  

(28)

\[ (Gnd|H'|n_{ij}) = (-1)^{i+j} t_{ij} \frac{1}{2} \frac{1}{\sqrt{(1-n_{ix})(1-n_{iz})}} [n_{i+}(n_{j_2}-1) + (1-n_{iz})n_{j+}] \]  

(29)

we get the second-order correction to the ground-state energy

\[ \Delta E_{Gnd}^{(2)} = - \sum_{ij} \frac{t_{ij}^2}{4J_KS} (2 - 2n_{ix}n_{jz} - n_{ix}n_{j+} - n_{ix}n_{j-}) = \sum_{ij} \frac{t_{ij}^2}{2J_KS} (\frac{S_i \cdot S_j}{S^2} - 1) \]  

(30)

We see that this is an antiferromagnetic interaction, which agrees with references [14] and [15]. For less than half-filling, the ground-state is highly degenerate and we must use degenerate perturbation theory, which is in fact not practical since the degeneracy is often too high to be handled. Eq.(3) indicates that the effective spin-spin exchange which is linear in \( t \) obtained in section (II) applies only to systems away from half-filling.

(2) Two sites energy-levels

In the case of two sites, the Hamiltonian (1) is

\[ H = \begin{pmatrix} c_{1\uparrow} & c_{1\downarrow} & c_{2\uparrow} & c_{2\downarrow} \end{pmatrix} \begin{pmatrix} \frac{J_K S}{2} & \frac{J_K S}{2} & t & 0 \\ \frac{J_K S}{2} & -\frac{J_K S}{2} & 0 & t \\ t & 0 & \frac{J_K S}{2} & -\frac{J_K S}{2} \\ 0 & t & -\frac{J_K S}{2} & \frac{J_K S}{2} \end{pmatrix} \begin{pmatrix} c_{1\uparrow} \\ c_{1\downarrow} \\ c_{2\uparrow} \\ c_{2\downarrow} \end{pmatrix} \]  

(31)

with eigenvalues

\[ \pm \frac{1}{2} \sqrt{4t^2 + J_K^2 S^2 \pm 2J_KS \sqrt{2 + 2n_1 \cdot n_2}} \]  

(32)

For \( J_K S \gg 2t \), they are

\[ E_1 = \frac{J_K S}{2} - \frac{|t|}{2} \sqrt{2 + 2n_1 \cdot n_2} - \frac{t^2}{J_K S} \left[ 1 - \frac{1}{4} (2 + 2n_1 \cdot n_2) \right] \]  

(33)

\[ E_2 = \frac{J_K S}{2} + \frac{|t|}{2} \sqrt{2 + 2n_1 \cdot n_2} - \frac{t^2}{J_K S} \left[ 1 - \frac{1}{4} (2 + 2n_1 \cdot n_2) \right] \]  

(34)

\[ E_3 = \frac{J_K S}{2} - \frac{|t|}{2} \sqrt{2 + 2n_1 \cdot n_2} + \frac{t^2}{J_K S} \left[ 1 - \frac{1}{4} (2 + 2n_1 \cdot n_2) \right] \]  

(35)

\[ E_4 = \frac{J_K S}{2} + \frac{|t|}{2} \sqrt{2 + 2n_1 \cdot n_2} + \frac{t^2}{J_K S} \left[ 1 - \frac{1}{4} (2 + 2n_1 \cdot n_2) \right] \]  

(36)

If there is only one electron, only \( E_1 \) is filled in the ground-state with energy

\[ E_0 = - \frac{J_K S}{2} - \frac{|t|}{2} \sqrt{2 + 2n_1 \cdot n_2} - \frac{t^2}{J_K S} \left[ 1 - \frac{1}{4} (2 + 2n_1 \cdot n_2) \right] \]  

(37)

and the correction due to hopping is

\[ \Delta E_0 \approx - \frac{|t|}{2} \sqrt{2 + 2n_1 \cdot n_2} \]  

(38)

which is always ferromagnetic. When there are two electrons, i.e., the system is half-filled, \( E_1, E_2 \) are filled in the ground-state and the energy is

\[ E = E_1 + E_2 = -J_K S - \frac{t^2}{J_K S} (1 - n_1 \cdot n_2) \]  

(39)

So the correction due to hopping is

\[ \Delta E_0 = - \frac{t^2}{J_K S} (1 - n_1 \cdot n_2) = \sum_{ij} \frac{t_{ij}^2}{2J_K S} (n_i \cdot n_j - 1) \]  

(40)

This agrees with the conclusion (30) for a general half-filled lattice. It is seen from (38) and (40) that whether...
the induced moment-moment interaction is linear or quadratic in $t$ depends on the number of particles. This justifies our previous argument.

IV. CONCLUSIONS

In this paper, we present a path-integral formulation for Kondo lattice model with localized spins ($S = \frac{1}{2}$ or $S = \frac{3}{2}$). In both cases, the system can be described in terms of fermions conveniently since spin operators are difficult to deal with in either path-integral or Feynman diagram calculations. In the low-temperature limit, the conduction fermions can be integrated perturbatively, and the resulting effective spin-spin exchange is RKKY-like interaction in the weak-coupling regime. In the strong-coupling regime, the effective spin-spin exchange is linear in $t$ if $Q_k \gg 2t$ for all $k$ where $Q_k$ is the function $F_3(k)$ in the lattice case. Since $Q(k)$ depends on $\mu$ which is the chemical potential, we see qualitatively that whether or not the leading order of the induced spin-spin interaction is linear in $t$ depends on the particle density in the system. At half-filling, the leading term of interaction is antiferromagnetic and quadratic in $t$ according to quantum-mechanical perturbation theory. It should be noted that though the path-integral formalism agrees with quantum-mechanical perturbative analysis qualitatively, the former does not show that half-filling is of criticality. Therefore further study on the path-integral formalism is still called for.

This work is supported in part by the Army High Performance Computing Research Center (AHPCRC) under the auspices of the Department of the Army, Army Research Laboratory (ARL) under Cooperative Agreement number DAAD19-01-2-0014.

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