Research into the process of impingement of two plane jets of an ideal fluid with free boundaries

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Abstract. The problem of finding parameters stationary jets, outgoing from a place of impact of two incoming flat ideal jets with free boundaries and possessing the identical speed, but various width, has no decision. Various models are widely used for a conclusion of the missing equation now, but they lead to contradictory results. The new model is offered. Adequacy to the offered model was checked by comparison results with data of the numerical calculations in ANSYS AUTODYN. The dependence approximating results of numerical calculations is developed to increase accuracy in calculations of angular provision of the internal outgoing jet. First of all, the executed researches are interesting to experts, who works on behavior low-value technological errors in shaped charges.

In one of the classic problems of theory of perfect fluid jets (Fig. 1) it is required to calculate the parameters $\delta_3, \delta_4, \beta, \gamma$ of the outgoing jets provided that the values of the parameters $\alpha, \delta_1$ and $\delta_2$ are specified. The solution to this problem is important for designing devices of inject automatics and development of technologies for the manufacture of cones shaped charges.

The problem has no unequivocal solution, for to be able to define 4 unknowns one may write only 3 separate equations based on the mass and momentum conservation laws.

Figure 1. Impingement of two plane jets of a perfect fluid with free boundaries
$\delta_1, \delta_2$ – the width of the impinging jets ($\delta_1 > \delta_2$); $2\alpha$ – the collapse angle of the impinging jets ($2\alpha < \pi$); $\delta_3, \gamma$ – the width and the deflection angle of the outer jet; $\delta_4, \beta$ – the width and the deflection angle of the inner jet ($\beta < \alpha$)

$$\bar{\delta}_3 + \bar{\delta}_4 = 2,$$

$$2\cos\alpha = \bar{\delta}_3 \cos\gamma - \bar{\delta}_4 \cos\beta,$$

$$2\Delta\bar{\delta}\sin\alpha = \bar{\delta}_3 \sin\gamma + \bar{\delta}_4 \sin\beta,$$

where $\bar{\delta}_3 = \delta_3/\delta_0$, $\bar{\delta}_4 = \delta_4/\delta_0$, $\Delta\bar{\delta} = \Delta\delta/\delta_0$, $\delta_0 = (\delta_1 + \delta_2)/2$, $\Delta\delta = (\delta_1 - \delta_2)/2$ – the deflection of the width of the impinging jets from the average value of $\delta_0$.

The problem (1) may be solved without introducing any additional conditions only in two specific cases: if the impinging jets have equal widths $\delta_1 = \delta_2 = \delta_0$ or if the jets of different widths impinge at $\alpha = \pi/2$.

By calculating kinetic energy in a system of interacting jets, A. Palanini has established that it reaches its minimum if the directions of the outgoing jets are opposite:

$$\beta = -\gamma.$$  \hspace{1cm} (2)

From solving (1) and (2) combined, we may derive the following:

$$\beta = -\gamma = -\operatorname{arctg}(\Delta\bar{\delta}\tan\alpha),$$  \hspace{1cm} (3)

$$\bar{\delta}_3 = 1 + \cos\alpha\sqrt{1 + \Delta\bar{\delta}^2 \tan^2\alpha}, \quad \bar{\delta}_4 = 1 - \cos\alpha\sqrt{1 + \Delta\bar{\delta}^2 \tan^2\alpha}.$$  

The condition (2) contradicts the results of later researches [1 – 4]. The model developed on the basis of the concept of elastic/non-elastic interactions of jet flows is regarded as a more advanced. It states that the angles at which the outgoing jets tilt towards the OY axis are equal [1]:

$$\beta = \gamma.$$  \hspace{1cm} (4)

The combined solving of (1) and (4) determines that:

$$\beta = \gamma = \arcsin(\Delta\bar{\delta}\sin\alpha),$$  \hspace{1cm} (5)

$$\bar{\delta}_3 = 1 + \cos\alpha, \quad \bar{\delta}_4 = 1 - \cos\alpha.$$  

While calculating angle deflections and sizes of outgoing jets. J.P. Curtis proceeded from two basic propositions [2]: a slug is formed in the region of the point 0 of the impinging jets (Fig. 1); the transverse sizes of the impinging jets are to each other as $\delta_1$ is to $\delta_2$; it is also assumed that $\beta$ and $\gamma$ are of small value.

$$\beta = \Delta\bar{\delta}\ctg\frac{\alpha}{2}, \quad \gamma = \Delta\bar{\delta}\ctg\frac{\alpha}{2}.$$  \hspace{1cm} (6)

At the same time, the widths of outgoing jets are defined by the relation from the model (5).

One of most important conclusions that may be drawn from the analysis of the models determining the parameters of outgoing jets is as follows: the widths $\delta_4, \delta_3$ either conform to the symmetric case (5) [1, 2] or approximate it. In this connection, one may use as one’s basic proposition allowing one to solve the system of equations (1) the assumption that the widths of outgoing jets is only determined by the mean width $\delta_0$ of the impinging jets and the angle between them and does not depend on the difference $\Delta\delta$ in the width of the impinging jets. Thus, solving the system of equations (1) and assuming that angles $\beta$ and $\gamma$ are low, i. e. accepting that $\sin\beta \approx \beta$, $\sin\gamma \approx \gamma$, $\cos\beta \approx 1 - \beta^2/2$, $\cos\gamma \approx 1 - \gamma^2/2$, yields the following:

$$\beta = 2\Delta\bar{\delta}f_1(\alpha), \quad \gamma = 2\Delta\bar{\delta}f_2(\alpha),$$  \hspace{1cm} (7)

where $f_1(\alpha) = \frac{1}{1 + \theta_2^2\alpha}$, $f_2(\alpha) = \frac{\theta_2^2\alpha}{1 + \theta_2^2\alpha}$.

Dependencies (7) and the already mentioned model (6) ensure that the condition $\beta > \gamma$ is fulfilled.
It is significant that if \( \alpha = 90^0 \) the obtained relations (7) and the models (5), (6) yield identical value of the angles of outgoing jets, which also agrees with the exact solution (3) of the problem (1) provided that \( \Delta \delta \ll 1 \).

In order to compare and analyse the models (3, 5, 6, 7), the parameters of outgoing jets were determined in numerical system ANSYS AUTODYN in dynamic view in the Euler coordinate system taking into consideration the compressibility the medium according to the recommendations provided in the work [4].

The following system of equations was solved numerically:

1) Equations of motion

\[
\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} u + \frac{\partial u}{\partial y} v + \frac{1}{\rho} \frac{\partial p}{\partial x} = 0,
\]

\[
\frac{\partial v}{\partial t} + \frac{\partial v}{\partial x} u + \frac{\partial v}{\partial y} v + \frac{1}{\rho} \frac{\partial p}{\partial y} = 0.
\]

2) Continuity equations

\[
\frac{\partial \rho}{\partial t} + \frac{\partial \rho}{\partial x} u + \frac{\partial \rho}{\partial y} v + \rho \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0,
\]

where \( u, v \) – the components of the velocity vector received as functions of the coordinates of points \( (x, y) \) and time \( t \), \( p \) – pressure, \( \rho \) – density.

The speed of blast wave \( U_s \) in a fluid was defined with this dependency [4]:

\[ U_s = c_0 + s \cdot u_p, \]

where \( c_0 \) – the speed of sound, \( u_p \) – the speed of stream of the fluid, \( s \) – parameter.

During the performance of calculations the value was \( s = 5 \), in accordance with the recommendations in the work [4].

The design field of the problem included three interrelated sub-fields unfilled with medium (white areas in Fig. 2, 3). The lateral rhombus-shaped sub-fields were intended for impinging jets, the central – for outgoing ones. The impinging jets are not restricted in length and flow into design sub-fields uninterruptedly. The speed of stream of the impinging jets was \( U = 3 \) m/s, the density - \( \rho_0 = 1000 \) kg/m\(^3\), the speed of sound - \( c_0 = 1500 \) m/s. The condition \( U \ll c_0 \) is essential for the problem to approximate the model of an uncompressible fluid. The Hydro (Pmin) model was accepted as a failure model of fluid.

During the course of calculation, the following parameters varied: \( k = \delta_2/\delta_1 = (1 - \Delta \delta)/(1 + \Delta \delta) \), which assumed the following values/magnitudes \( k = 0,5; k = 0,667; k = 0,75; k = 0,85 \); the angle between the incoming jets \( 2\alpha \), which varied within the range of \( 90^0 \) to \( 180^0 \) with a \( 10^0 \) step. Numerical calculations with different angles of the impingement of jets differed in the tilt of the lateral sub-fields (Fig. 2):
In the course of performing the calculations, it was observed that from the moment $t \approx 25$ ms the non-stationary stage of the jet interaction transforms into a stationary one and the jet flow configuration doesn’t change henceforth (Fig. 3).

The comparison of the results of numerical calculations of the outgoing jets’ width with the model of invariability in size of the outer $\delta_3$ and the inner $\delta_4$ of the jets during the transition from a symmetric to an asymmetric impingement that has been proposed in the article has shown that the assumption made in this article that it is possible to use the dependencies (2) to estimate the width of outgoing jets is acceptable not only if $\Delta \delta \ll 1$, but also if there are substantial distinctions in the overall dimensions of the impinging jets.

A comparison has been drawn between the results of the numerical calculations of the angle deflections of the inner $\beta = \beta(\alpha)$ of the outgoing jet (curve 2 in Fig. 4) and calculations according to the developed model (7) and the already known models (5), (6) (curves 1, 3, 4 in Fig. 4 respectively). The calculations for the outer jet $\gamma = \gamma(\alpha)$ agree among themselves and are therefore not adduced in the article.
Figure 4. Angle deflection $\beta$ of the inner jet depending on the angle $2\alpha$ between the impinging jets:

a) $k = 0.5$; b) $k = 0.667$; c) $k = 0.75$; d) $k = 0.85$

The analysis of Fig. 4 reveals that if $k = 0.85$ then the results of the calculations according to different models are quite similar. However, if the difference in the widths of the impinging jets is substantial (for example, in case $k = 0.5$), one may observe not only considerable discrepancy in calculations, but also contradictory tendencies in the character of the change of curves $\beta = \beta(\alpha)$ in different models (for example, graphs 3, 4 in Fig. 4a have different signs of curvature).

A special attention should be paid to errors in numerical calculations during the estimation of the angle $\beta$. For instance, if $2\alpha = 180^\circ$ then numerical calculations should agree with the dependency (3), as well as the models (5), (6), (7) (Fig. 4). The observable discrepancies of the calculation results allow one to assume random errors connected with inaccuracies in the measurement of the angle $\beta$ in design graphs of stationary flow of plane jets (Fig. 2, 3b). To compensate the errors, the following approach may be adopted. Given that the developed (7) and the already known (5), (6) analytical dependencies indicate proportionality between $\beta$ and $\Delta\delta$, it may be accepted that proportionality must also be maintained in numerical calculations, i.e.

$$\beta = \Delta\delta f(\alpha). \quad (10)$$

From processing the results of numerical calculations, the following function $f(\alpha)$ is derived.

$$f(\alpha) = 6(2\alpha)^{10} e^{-4.43(2\alpha)} \quad (11)$$

In the equations (10), (11), the angles $\alpha$ and $\beta$ are measured in radians.
The comparison of the results of numerical calculations and calculations through dependencies (10), (11) illustrates their agreement if the condition $k \geq 0.667$ ($\Delta \delta \leq 0.2$) is fulfilled.

Such discrepancy $\Delta \delta$ in the width of impinging jets is quite acceptable for solving practical problems, for example, appearing in the process of estimation of how low-value/magnitude errors of shaped charges affect the angle deflection of the forming inner jet. Besides, the results of the conducted research may be used for developing fluidics devices, as well as for setting conditions explosion welding of metals.

References
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