Finite element reduction modeling and vibration analysis of mistuned coating thickness blisk

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Abstract: The application of hard coating damping technology to the blades of blisk can effectively reduce the vibration stress and improve its service life. In this paper, a three-dimensional eight-node laminated incompatible element is proposed, which only needs the node coordinates of the structure without the coating. In the coating damping design process, it could avoid reconstructing the finite element mesh when changes the coating thickness. Based on the cyclic symmetry algorithm and the SNM method, a finite element reduction model to analyze the vibration characteristics of the coated blisk with coating thickness mistuning was established and verified by ANSYS. The results indicate that the proposed finite element model can be used to analyze the vibration characteristics with the changed coating thickness and it shows great consistence with the result of ANSYS. On the other side, the SNM still has high accuracy when the coating thickness varies from 0 mm to 0.3 mm.

1. Introduction

The blisk is widely used because it can simplify the structure, reduce the weight and can improve the thrust weight ratio of aeroengines significantly [1,2]. However, the blisk detuning due to the manufacturing errors, material defects and abrasion, which leads to obvious vibration localization and makes the blade bear more than the allowable vibration stress and resulting in fatigue damage [3-4]. Therefore, it is necessary to reduce the excessive vibration of blisks as far as possible.

In recent years, many scholars have studied the technology of increasing blade damping, such as friction damping ring, piezoelectric ceramic materials [5,6], passive surface treatment [7] and active damping control [8], these methods are of great significance to reduce the vibration level of blade and improve the life cycle of blisk. At present, a new kind of hard coating damping technology has been reported which can improve the surface wear resistance of the structure [9,11] and reduce the vibration stress of the structure in high temperature and corrosive environment [12,13]. Therefore, the research on modeling method and vibration characteristic analysis of hard coated blisk has certain engineering significance for its damping design.

At present, the dynamic modeling methods for mistuned blisk mainly include lumped parameter model, continuous parameter model, and high fidelity finite element model [14]. Among them, the lumped...
The parameter model and continuous parameter model are too simplified to simulate the actual blade and disk, so they are only used in the study of mechanism [15-18]. The high fidelity finite element model can accurately simulate the real blisk, but it is inefficient because it contains more degrees of freedom. Therefore, some scholars obtain high fidelity finite element models based on commercial finite element software such as ANSYS, and then use model reduction technology to model and analyze the mistuning characteristics of blisk [19-22]. Relatively speaking, the model reduction technology for blisk is relatively mature, such as SNM [23], FMM [24], CMM [25], AMM [26], N-PRIME [27], MMDA [28], etc. These model reduction techniques can significantly improve the computational efficiency, and have a certain reference value for the design of hard coating damping of actual bladed disks.

Hard coated damping materials have been widely used to improve the damping properties of beams, plates and shells [29-32]. In recent years, some references [32-34] have tentatively studied the high fidelity finite element modeling method and the vibration characteristics analysis of hard coated blades. However, in the above studies, the stiffness and mass matrices are usually obtained from commercial finite element software such as ANSYS. In general, the coating thickness must be changed to improve the damping performance. However, when the coating thickness is changed in commercial finite element software, the finite element mesh of the coated bladed disk must be reconstructed to obtain the new stiffness and mass matrix. In this way, it is very inconvenient in the process of hard coating damping design. Therefore, the modeling method of hard coated blisk with variable coating thickness needs to be further studied.

In this paper, based on the research and development of 3D eight-node laminated nonconforming element, a finite element model of hard coated blisk with variable coating thickness is proposed. Furthermore, the rotation period algorithm is used to solve the inherent characteristics of the blisk. Based on the SNM reduction method, the solution of the vibration characteristics of the mistuned blisk with coating thickness is given. Finally, a blisk coated with 0.3 mm NiCoCrAlY + YSZ material on both sides is taken as an example to verify the above modeling and reduction method.

2. Finite element modeling of coated blisk

2.1 Stiffness and mass matrix

For the hard coated blisk (see Figure 1), a 3D eight-node (each node has three freedom) laminated incompatible element is proposed, in which nine additional freedom are introduced to avoid shear locking, and the additional freedom can be eliminated by static condensation in solving the stiffness matrix. The corresponding displacement function is:

\[ u = \sum_{i=1}^{k} N_i \mu_i + N_i \alpha_i + N_i \beta_i, \]
\[ v = \sum_{i=1}^{k} N_i v_i + N_i \alpha_i + N_i \beta_i, \]
\[ w = \sum_{i=1}^{k} N_i w_i + N_i \alpha_i + N_i \beta_i. \]

Among them, nine internal freedom are introduced as \( \alpha_j = 1,\cdots, 9 \). The above shape functions are expressed as follows:

\[ N_i = \frac{1}{8} (1 + \xi_i \xi_j)(1 + \eta_i \eta_j)(1 + \zeta_i \zeta_j), \quad (i = 1, 2, \cdots, 8) \]
\[ N_{10} = 1 - \xi^2, \quad N_{11} = 1 - \eta^2, \quad N_{11} = 1 - \zeta^2. \]
Fig.1 Coated Blisk

Fig.2 Element sector of coated blade

For the composite structure, the static equilibrium equation can be expressed as:

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} \delta' \\ \alpha \end{bmatrix} = \begin{bmatrix} F' \\ 0 \end{bmatrix}$$

(3)

Where, $\delta'$ and $\alpha$ are the displacement vectors corresponding to the principal and additional freedom respectively, and $F'$ is the force vector. The specific solution formula of other components are as follows:

$$K_{11} = \sum_{m=1}^{n} \int_{S_{m}} B' D_{m} B \ dx dS$$

$$K_{12} = \sum_{m=1}^{n} \int_{S_{m}} B' D_{m} B \ dx dS$$

(4)

$$K_{22} = K_{12}^T$$

Here, $K'$ and strain matrix $B'$ are consistent with the conforming hexahedral element. $B'$ is the enhanced strain matrix generated by introducing additional degrees of freedom. The following table $m$ refers to layer $m$ materials.

Static condensation is used to eliminate the internal additional degrees of freedom, and the stiffness matrix of the element is obtained:

$$K'_c = K'_1 - K'_2 \left( K'_{22} \right)^{-1} K'_1$$

(5)

It should be noted that the mass matrix with strain enhancement element is obtained by the shape function of conforming element, Specific for:

$$M = \sum_{m=1}^{n} \int_{S_{m}} \rho_{m} N^T N \ dx dS$$

(6)

Here, $N$ is a Shape function matrix, $\rho_{m}$ is the density of the material which belong to the layer $m$. 
In order to use gauss integral for each layer, the coordinate transformation of equations (4) and (6) is carried out again, and the coordinate transformation of the layer \( m \) is expressed as:

\[
\xi = \frac{\xi_m + \xi_{m+1}}{2}, \quad \eta = \frac{\eta_m + \eta_{m+1}}{2}
\] (8)

Here, \( \xi_m \) and \( \eta_m \) stand for the coordinates in the thickness direction of the layer \( m \) which place in the local coordinate system \( \xi, \eta, \zeta \). And then we can get:

\[
K_{11} = \sum_{n=1}^{N} \int \int B^{T} D_{m} B \left| J \right| \frac{\xi_{m+1} - \xi_{m}}{2} d\xi d\eta d\zeta
\] (9)

\[
K_{12} = \sum_{n=1}^{N} \int \int B^{T} D_{m} B \left| J \right| \frac{\eta_{m+1} - \eta_{m}}{2} d\xi d\eta d\zeta
\] (10)

\[
K_{22} = \sum_{n=1}^{N} \int \int B^{T} D_{m} B \left| J \right| \frac{\zeta_{m+1} - \zeta_{m}}{2} d\xi d\eta d\zeta
\] (11)

\[
M = \sum_{n=1}^{N} \int \int \rho_{n} N^{T} N \left| J \right| \frac{\zeta_{m+1} - \zeta_{m}}{2} d\xi d\eta d\zeta
\] (12)

Here, the jacobian matrix \( J \) is:

\[
J = \begin{bmatrix}
\frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} & \frac{\partial z}{\partial \xi} \\
\frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} & \frac{\partial z}{\partial \eta} \\
\frac{\partial x}{\partial \zeta} & \frac{\partial y}{\partial \zeta} & \frac{\partial z}{\partial \zeta}
\end{bmatrix}
\] (13)

It should be noted that in the calculation of \( K_{11} \), jacobian matrix \( J \) is \( J|_{\zeta=0, \eta=0, \zeta=0} \).

For the 3D eight-node laminated incompatible element which is shown as Fig. 2, only the node coordinates before the coating of the blisk are needed, that is, when the coating thickness changes, just need to change the parameters in equation (7) which is set up in the local coordinate system. The unit can also be used for uncoated parts of the roulette, and the relevant material parameters of the first and third layers are set to 0.

3. Vibration characteristics analysis of mistuned blisk with coating

For the blisk with detuning coating thickness, the dynamic equation is

\[
\left[ (K + \Delta K_b) + i(\eta_b \Delta K_b + \eta_b K_b + \eta_b K_b) \right] + \omega^2 (M + \Delta M_b) X = F
\] (14)

Here, \( K \) and \( M \) stand for harmonic integral disc stiffness and mass matrix, respectively. \( \eta_b \) and \( \eta_b \) stand for loss factors of coating and disk substrate, respectively. \( K_b \) and \( K_b \) refer to the contribution matrix of coating and disk substrate to stiffness matrix. \( \Delta K_b \), \( \eta_b \Delta K_b \), \( \Delta M_b \) stand for the stiffness, damping and mass detuning induced by coating thickness detuning. \( X \) and \( F \) stand for the displacement vector and harmonic exciting force.

3.1 Solution of stiffness matrix and mass matrix of coated blade

For the harmonized blisk, its overall stiffness matrix and mass matrix can be expressed as:
\[
K = G^2 \left( k_s \otimes I_N \right) G = G^2 \tilde{K} G ^{(13a)} \\
M = G^2 \left( m_s \otimes I_N \right) G = G^2 \tilde{M} G ^{(13b)}
\]

Here, \(G\) stands for the assembly matrix of the structure. \(\tilde{K}\) and \(\tilde{M}\) stand for the uncoupled stiffness and mass matrices respectively. \(I_N\) stands for the equal identity matrix of dimension and sector number of the blisk. \(\otimes\) stands for the Kronecker product. \(k_s\) and \(m_s\) refer to the stiffness matrix and mass matrix of a single sector, which are sorted according to interface degree of freedom \(t\), internal degree of freedom \(g\) and interfacedegree of freedom as shown in Fig. 3. Specific for:

\[
k_s = \begin{bmatrix}
k_{11} & k_{1g} & k_{1p}
k_{g1} & k_{gg} & k_{gp}
k_{p1} & k_{pg} & k_{pp}
\end{bmatrix},
m_s = \begin{bmatrix}
m_{11} & m_{1g} & m_{1p}
m_{g1} & m_{gg} & m_{gp}
m_{p1} & m_{pg} & m_{pp}
\end{bmatrix}
\] (15)

Fig. 3 Blisk and sector in Cartesian coordinate system

The assembly matrix \(G\) is further solved as follows. For the uncoupled systems, stiffness matrix \(\tilde{K}\) and mass matrix \(\tilde{M}\), the displacement vector is

\[
x_n = \begin{bmatrix}
x_{n1}^T & x_{ng}^T & x_{np}^T & x_{n2}^T & x_{ng}^T & \cdots & x_{nM}^T & x_{ng}^T & x_{np}^T
\end{bmatrix}^T
\] (16)
Here, \( \mathbf{x}_t^j \), \( \mathbf{x}_g^j \), and \( \mathbf{x}_p^j \) refer to the displacement vector of sector \( j \) corresponding to the freedom of interface \( t \), freedom of internal \( g \) and freedom of interface \( p \). In fact, the freedom of interface \( t \) which belongs to the previous sector and the freedom of interface \( p \) which belongs to the next sector are exactly the same, that is, there are repeated displacement vectors in equation (16). By eliminating the repeated displacement vector, the actual displacement vector of the bladed disk can be obtained as:

\[
\mathbf{x}_{\text{basic}} = \begin{bmatrix}
\mathbf{x}_{1t}^T \\
\mathbf{x}_{1g}^T \\
\mathbf{x}_{2t}^T \\
\mathbf{x}_{2g}^T \\
\vdots \\
\mathbf{x}_{M_t}^T \\
\mathbf{x}_{M_g}^T
\end{bmatrix}^T
\]  

(17)

So we can get:

\[
x_{\text{ex basic}} = \mathbf{G} \mathbf{x}_{\text{basic}}
\]  

(18)

In the above formula:

\[
\mathbf{G} = \begin{bmatrix}
\mathbf{H} & \mathbf{E}_1 & \mathbf{E}_2 & \cdots & \mathbf{E}_N \\
0 & \mathbf{E}_1 & \mathbf{E}_2 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\mathbf{H} & 0 & 0 & \cdots & \mathbf{E}_N
\end{bmatrix}
\]  

(18)

Here:

\[
\mathbf{H} = \begin{bmatrix}
\mathbf{I}_t & \mathbf{I}_g \\
\mathbf{R}
\end{bmatrix}
\]

\[
\mathbf{E}_i = \begin{bmatrix}
\mathbf{I}_t & 0 \\
0 & \mathbf{I}_g \\
0 & 0
\end{bmatrix}
\]

(19)

Here, \( \mathbf{I}_t \) and \( \mathbf{I}_g \) stand for the identity matrix with dimensions equal to the interface degree of freedom \( t \) and the interface degree of freedom \( g \). \( \mathbf{R} \) stand for the rotation transformation matrix of stiffness and mass matrix in rectangular coordinate system. Specific for:

\[
\mathbf{R} = \mathbf{I}_t \otimes \begin{bmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta \\
0 & 1
\end{bmatrix}, \quad \theta = \frac{2\pi}{N}
\]  

(20)

In the above formula, \( \theta \) refers to the sector angle. According to the above process, the stiffness matrix and mass matrix of the system can be obtained. When the thickness of the coating is detuning, the sum of the detuning values of the stiffness matrix and the mass matrix \( \Delta K_b \) and \( \Delta M_b \) can be expressed as:

\[
\Delta K_b = \mathbf{G}^T \mathbf{Bdiag} \left[ \mathbf{k}_j \right] \mathbf{G} \quad (21a)
\]

\[
\Delta M_b = \mathbf{G}^T \mathbf{Bdiag} \left[ \mathbf{m}_j \right] \mathbf{G} \quad (21b)
\]

In the above formula:

\( \mathbf{Bdiag}[\cdot] \) stands for a pseudo block diagonal matrix. \( \mathbf{k}_j \) and \( \mathbf{m}_j \) stand for detuning of stiffness and mass matrix of the sector \( j \). Specific for:

\[
\mathbf{k}_j = \begin{bmatrix}
\Delta k_{bb,b}^j \\
\Delta k_{bg,b}^j \\
\Delta k_{gb,b}^j \\
\Delta k_{gg,b}^j \\
0
\end{bmatrix}, \quad \Delta k_{bb,b}^j = \begin{bmatrix}
0 & 0 \\
0 & \Delta k_{bb,b}^j
\end{bmatrix}
\]  

(22a)

\[
\mathbf{m}_j = \begin{bmatrix}
\Delta m_{bb,b}^j \\
\Delta m_{bg,b}^j \\
\Delta m_{gb,b}^j \\
\Delta m_{gg,b}^j \\
0
\end{bmatrix}, \quad \Delta m_{bb,b}^j = \begin{bmatrix}
0 & 0 \\
0 & \Delta m_{bb,b}^j
\end{bmatrix}
\]  

(22b)

Here, \( \Delta k_{bb,b}^j \) and \( \Delta m_{bb,b}^j \) stand for variation of stiffness and mass matrix induced by coating thickness detuning.
3.2 response solution

It is very convenient to obtain the mode matrix $\Phi$ and eigenvalue matrix $\Lambda$ of the system from the stiffness matrix and mass matrix of a single sector by using the cyclic symmetry characteristics of the tuned bladed disk, which will not be described in detail here.

Furthermore, based on the thought of SNM, the equation (14) can be rewritten as:

$$
\left( A + \Delta K_b \right) p - \omega^2 \left( I + \Delta M_b \right) p + i \left( \eta_i \Delta K_b + \eta_h \Delta K_b + \eta_r \Delta K_b \right) p = f
$$

Where, the specific expression of each component are:

$$
\begin{align*}
\Delta K_b &= \Phi^{(b)} \Delta K_b \Phi \\
\hat{K}_b &= \Phi^{(b)} K_b \Phi \\
\hat{K}_p &= \Phi^{(b)} K_p \Phi \\
\Delta M_b &= \Phi^{(b)} \Delta M_b \Phi \\
\hat{f} &= \Phi F
\end{align*}
$$

Equation (23) is a full array because of the existence of detuning, if it is solved directly, it will be inefficient. Therefore, solve the characteristic equation for equation (23). Then we can obtain the mode matrix $\Phi^{(b)}$ and eigenvalue matrix $\Lambda^{(b)}$ of the subspace.

$$
\left( A + \Delta K_b \right) = \left( I + \Delta M_b \right) \Phi^{(b)} A^{(b)}
$$

So the mode matrix of mistuned bladed disk can be obtained as follows:

$$
\phi^{(b)}_{new} = \phi^{(b)}_{new} \Phi^{(b)}
$$

Furthermore, decoupling the formula of (14) by the use of $\phi^{(b)}_{new}$, it can be concluded that:

$$
\left[ \hat{A}_{new} - \omega^2 I_{new} \right] \alpha = f_{new}
$$

In the above equation, each component is:

$$
\begin{align*}
\hat{A}_{new} &= \Phi^{(b)} \hat{A}_{new} \Phi^{(b)} \\
I_{new} &= \Phi^{(b)} I_{new} \Phi^{(b)} \\
f_{new} &= \Phi^{(b)} f \Phi^{(b)}
\end{align*}
$$

Finally, the forced response of the mistuned bladed disk can be obtained by the use of the classical mode superposition method.

4. The Model validation

Take a titanium alloy blisk with hard coating on both sides as an example to verify the developed finite element model. The young’s modulus, density, loss factor and poisson’s ratio of the titanium alloy are set to 103.213 GPa, 4200 kg / m$^3$, 0.0007 and 0.33 respectively. The material parameters of NiCoCrAlY + YSZ coating are set to 54.45 GPa, 5600 kg / m$^3$, 0.0212 and 0.3 GPa respectively. It should be noted that if the thickness of the coating is too large, the corresponding additional mass will be introduced. Therefore, the thickness of the coating is taken as 0.3mm (10% of the blade thickness).

4.1 Finite element model validation

In order to verify the rationality of the finite element model which is proposed in this paper, the vibration characteristics of the unreduced coated blisk are analyzed and compared with the results of ANSYS, in which the unit solid185 is used and the coated blade is simulated by a solid element offset on both sides of the blade. It should be noted that the number of elements and degrees of freedom of the developed finite element model are 21870 and 91692, while the number of elements and degrees of freedom of ANSYS model are 25758 and 105732 respectively.
Figure 4 shows the natural frequencies of the pitch diameter arrangement of the coated blisk based on the developed finite element model and ANSYS, and the first three natural frequencies of pitch diameter 3 and pitch diameter 6 are listed in Table 1. At the same time, the forced response under the third and sixth order excitation is shown in Fig. 5. The corresponding resonance vector peaks are listed in Table 2. Here, the excitation point and pick-up point of order excitation are the tip of each blade.

Table 1 The first three natural frequencies (Hz) of pitch diameter 3 and pitch diameter 6

| order | ANSYS  | R & D model | Deviation |
|-------|--------|-------------|-----------|
| 3     | 688.3  | 690.98      | 0.39%     |
|       | 1960.3 | 1956.39     | 0.20%     |
|       | 2464.3 | 2470.36     | 0.25%     |
| 6     | 711.56 | 714.81      | 0.46%     |
|       | 2431.6 | 2437.96     | 0.26%     |
|       | 3864.9 | 3856.48     | 0.22%     |

Through comparative analysis, the maximum error between the natural frequency obtained by the developed finite element model and ANSYS is 0.46%. Through comparative analysis, while the
maximum error of response is 1.23%. Therefore, the developed model can be used to analyze the vibration characteristics of blisk with variable coating thickness.

![Graph showing response and frequency](image)

**Fig.6 The response of 6 EO**

| order | ANSYS | R & D Model | Deviation |
|-------|-------|-------------|-----------|
| 3     | 2.65 × 10^{-4} | 4.78 × 10^{-5} | 3.25 × 10^{-5} |
| ANSYS | 2.55 × 10^{-4} | 4.64 × 10^{-5} | 3.16 × 10^{-5} |
| R & D Model | 3.77% | 2.93% | 2.76% |

**Table 2 Resonance response of 3EO and 6EO**

4.2 Analysis on the applicability of SNM

This paper analyzes the applicability of SNM in the case of “large detuning”. It is considered that the blisk with 0.3mm coating is a harmonious blisk, while the uncoated blisk is a “mistuned blisk”. The natural frequency and forced response of mistuned bladed disk are obtained respectively based on the SNM. The deviation between the natural frequencies obtained by SNM method and those analyzed by the complete finite element model is shown in Figure 7. Figure 8 and Figure 9 show the forced responses of the reduced model and the complete model under 3EO and 6EO excitation respectively.

![Graph showing deviation](image)

**Fig.7 Natural frequency deviation between reduced model and complete model**
Reduced Model

Complete Model

Response(m)

Frequency(Hz)

500 3300 1200 1900 2600

-610

-810

-410

-210

Fig.8 Response of reduced model and complete model of 3EO

Complete Model

Reduced Model

Response(m)

Frequency(Hz)

500 1200 1900 2600 3300

-510

-710

-410

-610

-810

-910

-310

-210

Fig.9 Response of reduced model and complete model of 6EO

Through comparative analysis, it can be found that the natural frequency deviation between the reduced model and the complete model is only -0.5% ~0%. While the forced response of resonance and nonresonance is basically the same. Therefore, it can be considered that SNM can satisfy the analysis accuracy of damping design when the coating thickness is within 0 ~ 0.3mm.

5. Conclusion

(1) Based on the developed 3D eight-node laminated incompatible element, a finite element model of hard coated blisk with variable coating thickness is proposed, which only needs the node coordinates before coating, and only needs to modify the thickness of each layer during local coordinate transformation when the coating thickness changes. Therefore, the reconstruction of finite element mesh due to the change of coating thickness can be avoided.

(2) When the thickness of the coating changes from 0 to 0.3 mm, SNM can still meet the calculation accuracy. Combining the developed finite element model with SNM, the vibration characteristics of the blisk with hard coating can be obtained quickly, which is very beneficial to the design of damping and vibration reduction with hard coating.

References

[1] Chan Y J, Ewins D J. Prediction of vibration response levels of mistuned integral bladed disks (blisks): Robustness studies[J]. Journal of Turbomachinery, 2012, 134(4): 044501.

[2] Klinger H, Lazik W, Wunderlich T. The engine 3E core engine[C]. ASME Turbo Expo 2008: Power for Land, Sea, and Air, Berlin, Germany, 2008.
[3] Laxalde D, Thouerez F, Sinou J J, et al. Qualitative analysis of forced response of blisks with friction ring dampers[J]. European Journal of Mechanics-A/Solids, 2007, 26(4): 676-687.

[4] Beirow B, Kühhorn A, Nipkau J. An equivalent blisk model considering the influence of the airflow on blade vibrations of a mistuned compressor blisk[M]. Vibration Problems ICOVP 2011. Springer Netherlands, 2011: 549-555.

[5] Li L, Deng P, Liu J, et al. Theoretical Study of the Vibration Suppression on a Mistuned Bladed Disk Using a Bi-periodic Piezoelectric Network[J]. International Journal of Turbo & Jet-Engines, 2016.

[6] Castanier M P, Pierre C. Modeling and analysis of mistuned bladed disk vibration: Status and emerging directions[J]. Journal of Propulsion and Power, 2006, 22(2): 384-396.

[7] Kenyon J A, Griffin J H, Kim N E. Sensitivity of tuned bladed disk response to frequency veering[J]. Journal of Engineering for Gas Turbines and Power, 2005, 127(4): 835-842.

[8] Hou J. Cracking-induced mistuning in bladed disks[J]. AIAA Journal, 2006, 44(11): 2542-2546.

[9] Yu C B, Wang J J, Li Q H. Investigation of the combined effects of intentional mistuning, damping and coupling on the forced response of bladed disks[J]. Journal of Vibration and Control, 2011, 17(8): 1149-1157.

[10] Turcotte J S, Hollkamp J J, Gordon R W. Vibration of a mistuned bladed-disk assembly using structurally damped beams[J]. AIAA Journal, 1998, 36(12): 2225-2228.

[11] Chen Y G, Zhai J Y, Han Q K. Vibration and damping analysis of the bladed disk with damping hard coating on blades[J]. Aerospace Science and Technology, 2016, 58: 248-257.

[12] Hassan M. Vibratory analysis of turbomachinery blades[J]. RPI Master's Project, 2008.

[13] Zhou K, Hegde A, Cao P, et al. Design Optimization Toward Alleviating Forced Response Variation in Cyclically Periodic Structure Using Gaussian Process[J]. Journal of Vibration and Acoustics, 2017, 139(1): 011017.

[14] Beirow B, Giersch T, Kühhorn A, et al. Optimization-aided forced response analysis of a mistuned compressor blisk[J]. Journal of Engineering for Gas Turbines and Power, 2015, 137(1): 012504.

[15] Bladh R, Castanier M P, Pierre C. Component-mode-based reduced order modeling techniques for mistuned bladed disks—part i: Theoretical models[J]. Journal of Engineering for Gas Turbines and Power, 2001, 123(1): 89-99.

[16] Yang M T, Griffin J H. A reduced order model of mistuning using a subset of nominal system modes[C]//ASME 1999 International Gas Turbine and Aeroengine Congress and Exhibition. American Society of Mechanical Engineers, 1999: V004T03A032-V004T03A032.

[17] Feiner D M, Griffin J H. A fundamental model of mistuning for a single family of modes[C]//ASME Turbo Expo 2002: Power for Land, Sea, and Air. American Society of Mechanical Engineers, 2002: 953-964.

[18] S. H. Lim, M. Castanier, C. Pierre, Vibration modeling of bladed disks subject to geometric mistuning and design changes, in: Proceedings of the Structures, Structural Dynamics, and Materials and Co-located Conferences, American Institute of Aeronautics and Astronautics, 2004

[19] A Sinha. Reduced-order model of a bladed rotor with geometric mistuning. Journal of Turbomachinery, 2009, 131(3): 031007.

[20] Martel C, Corral R. Asymptotic description of maximum mistuning amplification of blisked disk forced response[J]. Journal of engineering for gas turbines and power, 2009, 131(2): 022506.

[21] A Madden, BI Epureanu, S Filippi. Reduced-order modeling approach for blisks with large mass, stiffness, and geometric mistuning. AIAA journal, 2012, 50(2): 366-374.

[22] W Tang, BI Epureanu, S Filippi. Models for blisks with large blends and small mistuning. Mechanical Systems and Signal Processing, 2017, 87: 161-179.

[23] Sinha A. Vibration absorbers for a mistuned bladed disk[J]. Journal of Vibration & Acoustics, 2018.
[24] Zhou K, Hegde A, Cao P, et al. Design Optimization Toward Alleviating Forced Response Variation in Cyclically Periodic Structure Using Gaussian Process[J]. Journal of Vibration and Acoustics, 2017, 139(1): 011017.

[25] Beirow B, Giersch T, Kühhorn A, et al. Optimization-aided forced response analysis of a mistuned compressor blisk[J]. Journal of Engineering for Gas Turbines and Power, 2015, 137(1): 012504.

[26] Beck J A, Brown J M, Slater J C, et al. Probabilistic mistuning assessment using nominal and geometry based mistuning methods[J]. Journal of Turbomachinery, 2013, 135(5): 051004.

[27] Khemiri O, Martel C, Corral R. Asymptotic description of damping mistuning effects on the forced response of turbomachinery bladed disks[J]. Journal of Sound and Vibration, 2013, 332(20): 4998-5013.

[28] Blackwell C, Palazotto A, et al. The evaluation of the damping characteristics of a hard coating on titanium[J]. Shock and Vibration, 2007, 14(1): 37-51.

[29] Ivancic F, Palazotto A. Experimental considerations for determining the damping coefficients of hard coatings[J]. Journal of Aerospace Engineering, 2005, 18(1): 8-17.

[30] Chen Y, Zhai J, Han Q. Vibration and damping analysis of the bladed disk with damping hard coating on blades[J]. Aerospace Science and Technology, 2016, 58: 248-257.

[31] Gao F, Sun W. Vibration characteristics and damping analysis of the blisk-deposited hard coating using the Rayleigh-Ritz method[J]. Coatings, 2017, 7(8): 108.

[32] Gao F, Sun W, Gao J. Forced vibration analysis of the hard-coating blisk considering the strain-dependent manner of the hard-coating damper[J]. Aerospace Science and Technology, 2018.