The effect of $r$-mode instability on the evolution of isolated strange stars

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ABSTRACT
We studied the evolution of isolated strange stars (SSs) synthetically, considering the influence of $r$-mode instability. Our results show that the cooling of SSs with non-ultrastrong magnetic fields is delayed by heating due to $r$-mode damping for millions of years, while the spin-down of the stars is dominated by gravitational radiation (GR). Especially for the SSs in a possible existing colour–flavour locked (CFL) phase, the effect of $r$-mode instability on the evolution of stars becomes extremely important because the viscosity, neutrino emissivity and specific heat involving pairing quarks are blocked. It leads to the cooling of these colour superconducting stars being very slow and the stars can remain at high temperature for millions of years, which differs completely from previous understanding. In this case, an SS in CFL phase can be located at the bottom of its $r$-mode instability window for a long time, but does not spin-down to a very low frequency for hours.

Key words: stars: evolution – stars: neutron – stars: rotation.

1 INTRODUCTION
The $r$-modes in a perfect fluid star succumb to gravitational radiation (GR) driven Chandrasekhar–Friedman–Schutz instability for all rates of stellar rotation, and they arise due to the action of the Coriolis force with positive feedback increasing GR (Andersson 1998; Friedman & Morsink 1998). However, actually, the viscosity of stellar matter can hold back the growth of the modes effectively. Based on the competition between the destabilizing effect of GR and the damping effect of viscosity, Owen et al. (1998) modelled the evolution of the $r$-modes and described the spin-down of neutron stars as losing angular momentum to GR. Afterwards, Ho & Lai (2000) improved the model considering the conservation of wave action and containing the magnetic braking due to magnetic dipole radiation (MDR).

The viscosity of stellar matter is decisive for the $r$-mode evolution and hence for the spin-down and cooling of the star. The bulk viscosity of strange quark matter in the normal phase is several orders of magnitude stronger than that of normal nuclear matter. As a consequence, the instability in a strange star (SS) can only be driven at relatively lower temperature ($<10^8$ K), which means not very early ages, differing from the case of a neutron star in which the instability acts immediately at the birth of the star (Owen et al. 1998; Watts & Andersson 2002). Thus Madsen (2000) argued the evolution of SSs can be very different from that of neutron stars. Andersson, Jones & Kokkotas (2002) calculated the evolution of SSs thoroughly using the model of Ho & Lai (2000) (but without a magnetic field) in the presence of accretion to point out that SSs may be persistent sources of GR. However, most works always focused on the stars with $r$-mode instability as sources of GR, although the authors also claimed that the $r$-modes could change the spin evolution of the stars due to GR and induce heating due to viscous dissipation of the modes.

We also note the work of Gusakov, Yakovlev & Gnedin (2005) who studied the thermal evolution of a pulsating neutron star, taking into account the pulsation damping due to viscosity and accompanying heating of the star. This idea leads us to pay attention to the effect of the $r$-mode instability on the evolution of compact stars. As shown in fig. 5 in Watts & Andersson (2002), by heating due to $r$-mode damping, the cooling of a nascent neutron star is delayed in the first few decades. In addition, the calculation in Andersson et al. (2002) implies that the history of the $r$-modes in isolated SSs could be longer than neutron stars. According to the above works, we consider that a careful investigation of the effect of the $r$-mode instability on the evolution of SSs is an interesting topic, which we will focus on in this paper. As expected, our results show that, although the $r$-modes may be too small to induce a detectable GR at the age of a million years, the accompanying heating effect still can not be ignored in SSs.

In particular, our attention is attracted by the same consideration to colour superconducting SSs. For strange quark matter at a sufficiently high density, phenomenological and microscopic studies predict that it could undergo a phase transition into a colour superconducting state, such as the typical cases of two-flavour colour superconductivity (2SC) and the colour–flavour locked (CFL) phase (Shovkovy 2005; Alford 2004). Theoretical approaches also concur that the pairing gap ($\Delta$) in the quark spectrum is approximately $\sim 100$ MeV for baryon densities existing in the interior of compact stars. In this case, the reaction rates involving two quarks are reduced by a factor exp ($-2\Delta/k_B T$), assuming equal behaviour for...
all quark flavours. Because the quarks in the 2SC phase pair in part, the corresponding SSs will have similar behaviours to SSs in normal phase (NSS hereafter; Blaschke et al. 2000; Madsen 2000; Yu & Zheng 2006). Therefore, here we only account for SSs in CFL phase (CSS hereafter) with \( \Delta = 100 \text{ MeV} \), in which all flavour quarks pair. As a consequence, all of the viscosity, the neutrino emissivity and the quark specific heat of CSSs are reduced. Thus the effect of the r-mode instability on the evolution of CSSs may become more important. As shown in Section 5, the dramatic variation in the stellar evolution changes our previous understanding of CSSs that the stars are very cold and slow revolving objects (Blaschke et al. 2000; Madsen 2000).

This paper is organized as follows: In Sections 2 and 3 we recall the r-mode instability and the model of spin evolution due to gravitational and magnetic braking respectively. The thermal evolution equation of SSs considering the heating due to the viscous dissipation of CSSs that the stars are very cold and slow revolving objects (Blaschke et al. 2000; Madsen 2000).

### 2 THE R-MODE INSTABILITY

The r-modes of rotating Newtonian stars are generally defined to be solutions of the perturbed fluid equations having (Eulerian) velocity perturbations of the form (Lindblom, Owen & Morsink 1998)

\[
\delta \mathbf{v} = \alpha R \Omega \left( \frac{r}{R} \right)^l Y_m^0 e^{il\phi},
\]

where \( R \) and \( \Omega \) are the radius and angular velocity of the unperturbed star, \( \alpha \) is an arbitrary constant considered as the amplitude of the r-modes (Owen et al. 1998), and \( Y_m^0 \) is the magnetic-type vector spherical harmonic defined by

\[
Y_m^0 = \left[ \frac{l(l+1)}{2(l+1)} \right]^{1/2} r V \times \nabla Y_m(r).
\]

Papaloizou & Pringle (1978) first showed that the Euler equation for r-modes determines the frequencies as

\[
\omega = -\frac{(l-1)(l+2)}{l+1} \Omega.
\]

Further use of the Euler equation (as first noted by Provost, Berthomieu & Rocca 1981) in the barotropic case determines that only \( l = m \) r-modes exist and that \( \delta \mathbf{v} \) must have the radial dependence given by equation (1). These modes represent large-scale oscillating currents that move (approximately) along the equipotential surfaces of the rotating star. The density perturbation associated with the r-modes can be deduced by evaluating the inner product of \( \mathbf{v} \) (the unperturbed fluid velocity) with the perturbed Euler equation, and the equation for the perturbed gravitational potential (Lindblom et al. 1998):

\[
\delta \rho = \alpha R^2 \Omega^2 \rho \frac{d\rho}{dp} \left[ \frac{2l+1}{2l+1} \right]^{1/2} \left[ \frac{l(l+1)}{l+1} \left( \frac{r}{R} \right)^{l+1} + \delta \Psi(r) \right] Y_{l+11} e^{il\phi}.
\]

The quantity \( \delta \Psi \) is proportional to the perturbed gravitational potential \( \delta \Phi \) and is the solution to the ordinary differential equation

\[
\frac{d^2 \delta \Psi(r)}{dr^2} + \frac{2}{r} \frac{d \delta \Psi(r)}{dr} + \left[ \frac{4\pi G \rho}{r^2} \right] \left( \frac{l(l+1)(l+2)}{r^2} \right) \frac{d \rho}{dp} \frac{d \rho}{dp} - \frac{l(l+1)}{r^2} \right] \delta \Phi(r) = -\frac{8\pi G T l M R^2}{2l+1} \left[ \frac{l}{l+1} \frac{d \rho}{dp} \left( \frac{r}{R} \right)^{l+1} \right].
\]

The r-modes evolve with time dependence \( e^{i\omega t/\tau_v} \) as a consequence of ordinary hydrodynamics and the influence of the various dissipative processes. The real part of the frequency of these modes, \( \omega \), is given in equation (3), while the imaginary part \( 1/\tau_v \) is determined by the effects of GR, viscosity, etc. The simplest way to evaluate \( 1/\tau_v \) is to compute the time derivative of the energy \( \dot{E} \) of the mode (as measured in the rotating frame). \( \dot{E} \) can be expressed as a real quadratic functional of the fluid perturbations:

\[
\dot{E} = \frac{1}{2} \int \left[ \rho \delta \mathbf{v} \cdot \delta \mathbf{v} + \left( \frac{\delta p}{\rho} - \delta \Phi \right) \delta \rho \right] d^3 \mathbf{x}.
\]

Thus, the time derivative of \( \dot{E} \) is related to the imaginary part of the frequency \( 1/\tau_v \), which can be conveniently decomposed into two parts associated with GR and viscosity, respectively. It reads

\[
\frac{d \dot{E}}{d \mathbf{r}} = -\frac{2 \dot{E}}{\tau_v} = -\frac{2 \dot{E}}{\tau_{g}} - \frac{2 \dot{E}}{\tau_{v}}.
\]

For \( n = 1 \) polytrope, the time-scale for GR is calculated by Lindblom, Mendell & Owen (1999)

\[
\tau_{g} = 3.26(\Omega/\sqrt{\pi G \rho})^{-6},
\]

where \( \rho \) is the mean density of the star. The viscous time-scale is contributed by shear and bulk viscosity as

\[
\tau_{v} = \left( \tau_{v}^{-1} + \tau_{v1}^{-1} \right)^{-1}.
\]

For strange quark matter, we can give (Lindblom et al. 1999; Madsen 2000)

\[
\tau_{v1} = 5.41 \times 10^{9} \alpha_{c,0.1} T_{9}^{3/2},
\]

\[
\tau_{v2} = 0.886(\Omega/\sqrt{\pi G \rho})^{-2} T_{9}^{-2} m_{100}^{-4},
\]

where \( \alpha_{c,0.1} \) and \( T_{9} \) are the strong coupling in units of 0.1, the mass of strange quark in units of 100 MeV and the interior temperature of the stars in units of 10^9 K, respectively. When these viscosities due to quark reactions are blocked in the CFL phase, the dissipation of the r-modes will be mainly dominated by a smaller shear viscosity due to electron–electron scattering. The corresponding time-scale is (Madsen 2000)

\[
\tau_{v} = 2.95 \times 10^{9} (\mu_{c}/\mu_{u})^{-1/3} T_{9}^{2}.
\]

From equation (7), it can be seen that the r-modes are unstable if \( \tau_{g}^{-1} + \tau_{v}^{-1} < 0 \). In this case, a small perturbation will lead to an unbounded growth of the modes. The competition between the destabilizing effect of GR that is dependent on spin frequency and the damping effect of the temperature-dependent viscosity gives an instability window in the \( \nu–\Omega \) plane, as shown by the shadow in the figures below. We can see that, compared with the window of a NSS, the window of a CSS expands significantly because its viscosity involving quarks is blocked.

### 3 SPIN EVOLUTION

We employ a simple phenomenological spin evolution model proposed by Ho & Lai (2000), which is analogous to that devised by Owen et al. (1998). In this case, the total angular momentum of a star can be obtained as the sum of the bulk angular momentum and the canonical angular momentum of the r-modes \( J_c \)

\[
J = \tilde{I} \Omega + J_c.
\]

where \( \tilde{I} = \tilde{I} M R^2 (\tilde{I} = 0.261 \) for \( n = 1 \) polytrope) represents the moment of inertia of the star. The canonical angular momentu
an $r$-mode can be expressed in terms of the velocity perturbation $\delta v$ by (Friedman & Schutz 1978)

$$J_c = -\frac{l}{2(\omega + i\Omega)} \int \rho \delta v \cdot \delta u^* d^3x. \quad (14)$$

For the (dominant) $l = m = 2$ current multipole of the $r$-modes, the above expression reduces (at the lowest order in $\Omega$) to

$$J_c = -\frac{3}{2} \alpha^2 \tilde{J} M R^2 \Omega, \quad (15)$$

where $\tilde{J} = 1.635 \times 10^2$ for $n = 1$ polytrope. The canonical angular momentum of the mode can increase through GR back reaction and decrease by transferring angular momentum to the star through viscosity:

$$\frac{dJ_c}{dt} = -\frac{2J_c}{\tau_g} - \frac{2J_c}{\tau_v}. \quad (16)$$

On the other hand, adding the reverse angular momentum transferred from the $r$-mode, the bulk angular momentum of the star decreases. At the same time, the star also undergoes braking by MDR. Thus the rotation of the star is determined by

$$\frac{dJ}{\tau_\Omega} = \frac{2J_c}{\tau_v} - \frac{I\Omega}{\tau_m}. \quad (17)$$

where $\tau_m = 1.69 \times 10^6 B_{12}^{-3}(\Omega/\sqrt{\pi G \rho})^{-2}$ s is the magnetic braking time-scale, and $B_{12}$ is the magnetic field intensity in units of $10^{12}$ G.

Submitting equation (15) into equations (16) and (17), we can obtain the coupled evolution equations of the amplitude $\alpha$ of the $r$-mode and the angular velocity $\Omega$:

$$\frac{d\alpha}{dt} = -\alpha \left( \frac{1}{\tau_g} + \frac{1 - \alpha^2 Q}{\tau_v} - \frac{1}{2\tau_m} \right), \quad (18)$$

$$\frac{d\Omega}{dt} = -\Omega \left( \frac{2\alpha^2 Q + 1}{\tau_v} + \frac{1}{\tau_m} \right), \quad (19)$$

where $Q = 3\tilde{J}/2\tilde{I} = 0.094$. In some cases, the small initial perturbation could increase to a large value so that non-linear effects can no longer be ignored. As postulated by Owen et al. (1998), there may exist a saturation amplitude $\alpha^2 = \kappa$. When $\alpha^2 > \kappa$, the growth of the amplitude of the mode stops, then

$$\frac{d\alpha}{dt} = 0. \quad (20)$$

In this stage, the spin evolution will be described as

$$\frac{d\Omega}{dt} = \frac{2\Omega}{\tau_g} \frac{\kappa Q}{1 - \kappa Q} - \frac{\Omega}{\tau_m} \frac{1}{1 - \kappa Q}. \quad (21)$$

Of fundamental importance in judging the astrophysical relevance of the instability is the determination of the saturation amplitude of the mode. Lindblom, Tohline & Vallisneri (2001, 2002) suggested that the non-linear saturation amplitude may be set by dissipation of energy in the production of shock waves. However, Gressman et al. (2002) argued that the decay of the amplitude of the order unity is due to leaking of energy into other fluid modes, leading to a differential rotation configuration. Afterwards, the coupling between the $r$-modes and other modes was analysed (Arras et al. 2003). On the other hand, the role of differential rotation in the evolution of the $r$-mode was also studied thoroughly (Sá & Tomé 2005). All these works obtained a credible conclusion that the saturation amplitude of the $r$-mode may be not larger than the small value of $10^{-3}$. Thus, we take $\kappa = 10^{-6}$ in our calculations. The saturation amplitude is important for the evolution of CSSs because the $r$-mode in CSSs will retain the saturated state for a long time. However, to a certain extent, the uncertainty of the saturation value can only influence our numerical results quantitatively, but cannot change the conclusions essentially. Note that, for NSSs, this artificial cut of the amplitude is needless, as is also claimed by Andersson et al. (2002).
4 THERMAL EVOLUTION

An SS can be divided roughly into an inner, approximately isothermal, quark core and a thin, solid, nuclear crust (Alcock, Farhi & Olive 1986). For the relationship between the interior temperature $T$ and the surface temperature $T_s$, we apply the result given by Gudmundsson, Pethick & Epstein (1983), which is valid for a crust with the density at the base that is just larger than $10^8$ g cm$^{-3}$. Thus, the emissivity due to the surface photon emission can be written as

$$E_{\text{phot}} = 1.24 \times 10^{22} T_s^{2.2} \text{ erg cm}^{-2} \text{ s}^{-1}.$$  \hspace{1cm} (22)

Besides the contribution from this thermal radiation, SSs also cool via internal neutrino emission. In strange quark matter, there are three main neutrino processes: (i) direct Urca processes $d \rightarrow u e \bar{v}$ and $ue \rightarrow d \nu$; (ii) modified Urca processes $dq \rightarrow uqe\bar{v}$ and $uqe \rightarrow dq\nu$; (iii) and the quark bremsstrahlung processes $q_i q_j \rightarrow q_i q_j \nu e\bar{v}$. The corresponding neutrino emissivities read respectively (Iwamoto 1982)

$$E_{\text{neutrino}}^{(D)} = 8.8 \times 10^{26} a_c \left( \frac{\rho_e}{\rho_0} \right) Y_e^{1/3} T_6^8 \text{ erg cm}^{-3} \text{ s}^{-1},$$  \hspace{1cm} (23)

$$E_{\text{neutrino}}^{(M)} = 2.83 \times 10^{26} a_c^{8/3} \left( \frac{\rho_e}{\rho_0} \right) T_6^n \text{ erg cm}^{-3} \text{ s}^{-1},$$  \hspace{1cm} (24)

$$E_{\text{neutrino}}^{(B)} = 2.98 \times 10^{26} \left( \frac{\rho_e}{\rho_0} \right) T_6^3 \text{ erg cm}^{-3} \text{ s}^{-1},$$  \hspace{1cm} (25)

where $\rho_0$ is the baryon number density and $\rho_0 = 0.17$ fm$^{-3}$ is the nuclear saturation density. $Y_e = \rho_e/\rho_0$ is the electron fraction. On the other hand, SSs also can be heated. During GR, in addition to the losing of angular momentum, the $r$-modes also can lose energy via GR and neutrino emission (from bulk viscosity) and can deposit energy into the thermal state of the star via shear viscous dissipation (Owen et al. 1998). As defined in equation (6), for the $l = m = 2$ current multipole, the energy of the $r$-mode is $E = \frac{1}{2} \alpha J M R^2 \Omega^2$ and the heating rate is

$$H_{\nu} = \frac{2E}{\tau_{\nu}} = \frac{\alpha J M R^2 \Omega^2}{	au_{\nu}}.$$  \hspace{1cm} (26)

Thus the thermal evolution equation is written as

$$C \frac{dT}{dt} = -L_{\text{neutrino}} - L_{\text{photon}} + H_{\nu},$$  \hspace{1cm} (27)

where $L_{\text{neutrino}}$ and $L_{\text{photon}}$ are the neutrino luminosity and the surface photon luminosity, respectively, and $C$ is the total specific heat, which is mainly contributed by quarks and electrons (Blaschke et al. 2000):

$$c_q \simeq 2.5 \times 10^{20} \left( \frac{\rho_e}{\rho_0} \right)^{2/3} T_9 \text{ erg cm}^{-3} \text{ K}^{-1},$$  \hspace{1cm} (28)

$$c_e \simeq 6 \times 10^{20} \left( \frac{Y_e \rho_e}{\rho_0} \right)^{2/3} T_9 \text{ erg cm}^{-3} \text{ K}^{-1}.$$  \hspace{1cm} (29)

In the calculation for CSSs, the reductive factor due to the pairing gap should be added in equations (23)–(25) and (28). In other words, the electron specific heat, the surface photon emission and the heating term become more important for the thermal evolution of CSSs.

5 EVOLUTION CURVES

In our calculations, we take the initial temperature $T_0 = 10^{10}$ K, the initial angular velocity $\Omega_0 = \frac{1}{3} \sqrt{\pi G \rho}$, and a representative

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**Figure 2.** The evolution of the amplitude of the $r$-mode in NSSs for different magnetic fields.
at the age of over a million years. Finally, the evolution tracks will depart from the instability windows at the moment that the magnetic braking effect exceeds that of GR (see the upper panel of Fig. 3). Thus, the stronger the field, the earlier the departure. Synchronously, this departure also leads to the end (a catastrophic decay) of the \( r \)-mode as shown in Fig. 2. Before this abrupt drop, however, the variation of the mode is small apart from the first several decades, during which the amplitude of the \( r \)-mode oscillates greatly until it converges to a value of \( \sim 10^{-3} \) as discussed by Andersson et al. (2002). At the extreme, a star with an ultrastrong \((B > 10^{13})\) field decelerates so rapidly due to MDR that the track will not come into contact with the instability window as shown in Fig. 1(a). Thus, no \( r \)-mode arises in the star throughout its life as shown in Fig. 2(a).

To be clear, Fig. 3 shows the evolution curves of spin frequency (upper panel) and surface temperature (lower panel) corresponding to the cases shown in Fig. 1. Not surprisingly, being governed by equation (19), the spin history of the stars with non-ultrastrong \((B < 10^{13})\) fields can be divided into two stages: (i) the stage due to angular momentum transfer by viscosity (losing angular momentum via GR ultimately) at early ages in the presence of the \( r \)-mode; and (ii) the stage due to MDR in the absence of the \( r \)-mode. On the other hand, as shown in the lower panel, the cooling of the stars with weak fields is delayed under the impact of heating due to the viscous dissipation of the \( r \)-mode, whereas the curve of the strong field superposes the curve without the consideration of the \( r \)-mode instability.

In the second situation, colour superconductivity suppresses the cooling effect extremely, whereas the heating effect is enhanced considerably. The evolution tracks of CSSs in the \( v-T \) plane for different magnetic fields and the corresponding amplitude evolution curves of the \( r \)-mode are shown in Figs 4 and 5, respectively. The same as Fig. 3 but for CSSs is also shown in Fig. 6. Differing from NSSs, the \( r \)-mode in CSSs can appear at the birth of the stars and quickly \((\sim 1000\) s, as shown clearly in Fig. 5) rise to the saturated state because the bulk viscosity is blocked. However, at the same time, the temperature nearly does not decrease because the neutrino cooling effect is suppressed. However, because the specific heat is only contributed from electrons, the temperature decreases finally in the following several hundred years until the photon luminosity is nearly compensated by the heat gain from the shear viscous dissipation of the \( r \)-mode. From then on, the stars should experience a long-term slow cooling stage. As discussed by Blaschke et al. (2000) and Yu & Zheng (2006) previously, CSSs will become very cold at early ages \((< 1000\) yr) if the heating effect is ignored. However, the presence of the long-term slow cooling stage can change the situation completely as shown in Fig. 6 (lower panel). We can see the surface temperature can remain at a high value of \( \sim 10^{6} \) K lasting several million years. The spin-down of CSSs that is shown in the upper panel of Fig. 6 is similar to the one of NSSs, but the stage dominated by GR is much longer because the life of the \( r \)-mode in CSSs is longer as shown in Fig. 5.

### 6 Conclusion and Discussions

We have studied the evolution of SSs synthetically, considering the influence of \( r \)-mode instability. The spin history is divided into two stages and the heating effect due to viscous dissipation of the \( r \)-mode can delay the cooling of SSs with non-ultrastrong fields remarkably. Our prime objective is to investigate the behaviour of SSs in the colour superconducting phase (to be specific, the CFL phase). This problem is estimated simply by Madsen (2000), who argued that CSSs should spin-down to a very low spin frequency for hours due

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1 We provide a brief review of the relevant properties of strange quark matter in the previous sections in order to discuss the evolution characteristic of SSs in this section. However, a more comprehensive investigation of these properties is needed to compare the theoretical curve with the observational data. Comments on an early version of this paper helped us to note that the contribution to viscosity in the CFL phase from superfluid phonons may be more important than that of electrons (Manuel, Dobada & Llanes-Estrada 2005), although the physical inputs left unmentioned in this paper may not affect the general astronomical picture.
Figure 4. The evolution of CSSs (with canonical parameters $M = 1.4 \, M_\odot$ and $R = 10\, \text{km}$) for different magnetic fields. The labels for the open cycles represent the value of $\log (\text{yr}^{-1})$.

Figure 5. The evolution of the amplitude of the $r$-mode in CSSs for different magnetic fields.

to the $r$-mode instability. Conversely, our results indicate that the stars can be located at the bottom of the window for millions of years due to their characteristic cooling behaviour. A CSS could be a rapid revolving pulsar. Differing from NSSs, the cooling of CSSs is changed completely by heating due to $r$-mode damping. Except for the first several hundred years, CSSs cool very slowly. In other words, the stars can remain at a high temperature over several million years but do not become very cold at early ages as calculated in the
show that the heating due to the spin-down can impact the cooling behaviour indirectly. Looking at more detailed calculations, previous modelling of the thermal evolution of SSs based on the assumption that the spin-down is only determined by MDR needs to be reconsidered because it is GR but not MDR that dominates the first spin-down of the star in the presence of the $r$-modes.  

Using NSSs, several authors (Madsen 2000; Andersson et al. 2002) try to explain the clustering of low mass X-ray binaries in the $v$–$T$ plane. Our results (Fig. 1) show that, for weak-field ($B \leq 10^{10}$ G) cases, MDR has no influence on the spin-down of stars that are younger than 10 million years old. This implies the existence of a weak magnetic field in millisecond pulsars if we follow the same philosophy that has been used by Madsen (2000). However, in our opinion, the final clarification of this problem should be dependent on careful consideration of the accretion of stars as Watts & Andersson (2002) did for neutron stars.

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