Adaptive Sliding Mode Control of Rack Position Tracking System for Steer-by-Wire Vehicles

KWANGIL KIM¹, JAEPOONG LEE¹, MINJUN KIM², AND KYONGSU YI¹, (Member, IEEE)

¹Department of Mechanical and Aerospace Engineering, Seoul National University, Seoul 08826, South Korea
²Hyundai Research and Development Center, Hwasung 18278, South Korea
Corresponding author: Kyongsu Yi (kyi@snu.ac.kr)

This work was supported in part by the Hyundai Motor Group, in part by the Seoul National University Institute of Advanced Machines and Design (SNU-IAMD), BrainKorea21 (BK21) Program, National Research Foundation of Korea (NRF) grant funded by the Ministry of Science, Information and Communication Technology (ICT), and Future Planning [Ministry of Science, ICT, and Future Planning (MSIP)] [NRF-2016R1E1A1A01943543].

ABSTRACT This paper describes adaptive sliding mode control of a steer-by-wire (SBW) system to guarantee rack position tracking performance in various driving situations. The proposed control algorithm was developed using only motor position sensors (MPS) without information on tire/road friction. A stiffness parameter adaptation law was designed to compensate for disturbances in the SBW rack system. It is demonstrated that the proposed adaptation algorithm provides good tracking performance without using an additional gain tuning approach under various road conditions. Moreover, in the proposed algorithm, a dynamic stiffness model has been developed to improve rack position tracking performance under a zero vehicle speed scenario. In a dynamic stiffness model, the stiffness center is not fixed but changes depending on the actual rack position. In the event than an SBW vehicle is parking, it is important to ensure rack position tracking performance at low and zero vehicle speeds. Computer simulations and vehicle tests were performed under various driving situations to test the performance of the proposed control algorithm. The results demonstrate that the proposed control algorithm ensures tracking performance on dry asphalt and wet road conditions, as well as at zero vehicle speed.

INDEX TERMS Steer-by-wire (SBW) system, adaptive sliding mode control, rack position tracking.

I. INTRODUCTION

Many researchers have been engrossed in the development of a next generation steering system, the steer-by-wire (SBW) system. The SBW system eliminates the mechanical linkage between the existing handle and the front steering wheel and directly controls the rack position. With the SBW system, front tires can be controlled through the rack movement. The advantages of the SBW system are that it can create the desired steering feel through electronic actuators and can also create the desired vehicle responses by directly controlling the movement of the front wheels via the rack system. A vehicle stability can be improved for SBW equipped vehicle through active front steering [1]–[4]. In addition to improving vehicle stability, the SBW can also improve vehicle maneuverability such as fast response of the vehicle compared to the conventional mechanical steering system [2], [4]. Therefore, accurate steering tracking control of SBW system in various driving situation is important.

A. RELATED WORK

The SBW system has been studied by many researchers because of its unique advantages. In [5], the difference between a conventional vehicle system and an SBW system was described and the SBW system was modeled using simple second-order dynamics. The accuracy of the proposed model was validated through experiments. In [6], [7] a tracking controller was developed to ensure that the rack system follows the driver’s steering input. However, there are difficulties in online estimation of the lateral forces in real time that can vary with different road conditions. Kazemi and Janbakhsh [8] proposed a nonlinear adaptive sliding mode control algorithm that improves vehicle handling via a SBW system. Although estimation of cornering stiffness has been proposed theoretically using an adaptive sliding mode control algorithm, the difficulty in obtaining tire parameters presents a limitation to the application of the algorithm in real-time.
This is because the detection of tire parameters in real-time is difficult. In [9], Fenglou and Deyu devise a variable structure controller scheme. The sliding mode-based controller is insensitive to parameter variations, and it is robust against nonlinearity and uncertainty of the system. Sun et al. [10] proposed an adaptive dynamic programming (ADP) approach to follow the desired yaw rate in vehicle system. It focuses on eliminating the model complexity and controlling the system through ADP method. In [11], a steering wheel model, front wheel model, and 9-DOF vehicle model are proposed for construction of an SBW system. Two independent feedback motors were used to describe the SBW system. C-J Kim et al. [12] proposed the bond graph method to describe the rack-actuating SBW system. The rack motion is determined by driver’s command angle and the rack movement is described by a second-order dynamics. The tire self-aligning torque was used to model the rack dynamics, which has a limitation in that the tire model must be known accurately when applying the proposed algorithm to the actual vehicle. In [11], [12], and [19], a proportional derivative control algorithm was proposed that allows the front wheel of the rack to track the driver’s command angle. The proposed PD controller is designed to ensure the tracking performance of the rack system in normal driving conditions. However, it is difficult to compensate for the disturbances that change according to various driving situations with PD controller.

The SMC technique is widely used for nonlinear system tracking control, especially in the performance of uncertainty compensation [19]–[22]. In practical applications, it is important to ensure the robustness of the control system since there exist unstructured. The SMC is one of the powerful control schemes and has been used in many practical systems.

Wang et al. [13] proposed a sliding mode control (SMC) algorithm with AC motors for SBW vehicles. Considering the uncertain disturbance from front steering angle, a sliding mode control scheme is desirable. In [13], a cornering stiffness coefficient value that varies depending on road surface conditions was used; however, using the cornering stiffness value makes it difficult to implement the algorithm in real-time. This is because it is difficult to determine the cornering stiffness value in various road conditions. The conventional SMC uses large control gain to compensate for the disturbances, which yields the undesired chattering phenomenon.

Cristi et al. [23] proposed a stability control scheme for a submersible vehicle. The proposed controller combines the adaptability with the robustness of a sliding mode controller. This approach, based on a dominant linear model and bounds on the nonlinear perturbations of the dynamics.

Chen and Huang [24] proposed a sliding controller with a functional approximation scheme for controlling a suspension system of non-autonomous quarter car. A functional approximation technique is developed to describe the unknown disturbance with a combination of various basis functions. However, the number of required basis function is difficult to calculate, and as the number of Fourier series functions increases, the amount of calculation increases severely.

Huang et al. [25] proposed an adaptive sliding mode controller scheme for nonlinear systems with unknown parameters. An adaptive parameter tuning technique is described to deal with unknown but bounded parameter uncertainties. In real-world application, the upper bound of the parameter uncertainty is often not easy to find. The purpose is to compensate the uncertainties through an adaptive gain algorithm without knowing the upper limit of parameter uncertainty.

Li et al. [26] showed an adaptive sliding mode control algorithm with T-S fuzzy approach for nonlinear active suspension vehicle systems. Since the T-S fuzzy algorithm is useful for analyzing the complexity of nonlinear systems, it is powerful in suspension vehicle system. However, the fuzzy sliding mode control algorithm requires a lot of work to determine the fuzzy rules. Fu et al. [27] proposed an adaptive sliding mode controller with direct yaw-moment control for electric vehicles. The proposed algorithm uses various feedback control gains to respond to changes in the tire slip angle in various driving situations. This controller provides robustness against errors in estimating the front/rear tire slip angle.

Bernard and Clover [28] proposed a lateral tire model that simulates a vehicle motion at low speed. This paper derives differential equations that are numerically well-governed at low speeds for longitudinal slip and slip angle. This method can calculate a lateral tire force at low speed and was evaluated via severe braking in a turn maneuver scenario; however, when the vehicle speed is at zero, the slip angle is calculated as zero, so there is a limitation in that the calculated lateral tire force is also zero. Thus, this approach is difficult to apply in a parking scenario.

B. CONTRIBUTIONS

In this paper, an adaptive sliding mode control (ASMC) algorithm is proposed to guarantee the tracking performance of a rack system under different road conditions using only a motor position sensor. The main contribution of the proposed ASMC algorithm is that it does not require an estimation of the cornering stiffness coefficient value. To cope with various road surface conditions, a method of replacing a self-aligning torque model with a stiffness model has been developed. The idea is to change the stiffness coefficient parameter in various road conditions using the parameter adaptation law. In addition, a dynamic stiffness model has been developed to guarantee the steering tracking performance of the rack system in a vehicle at zero speed. The main concept of a dynamic stiffness model is that a stiffness center is not fixed but varying depending on the actual rack position.

II. DYNAMIC MODEL OF A STEER-BY-WIRE RACK SYSTEM

The basic structure of a SBW rack system is shown in Fig. 1 [29]. The SBW system is divided into two parts: the steering wheel system and the rack system. The steering
wheel system consists of the steering wheel, the torque and angle sensor (TAS), and the hand-wheel feedback motor. The rack system includes the rack bar, the pinion angle sensor, the rack and pinion gear, front-wheel steering motor and the front steering wheels. The hand-wheel feedback motor creates the steering reaction feel by the driver in the existing mechanical system. The front-wheel steering motor in rack system is used to control the front steering wheels to follow the driver’s hand-wheel reference command. In this paper, the control target is front-wheel steering motor. The steering system and the rack system are disconnected in a SBW vehicles. Therefore, the movement of the rack bar is made through the steering motor. A motor position sensor (MPS) is used to measure the actual rack position.

The dynamic equation of the rack system is described by the following equation (1):

\[ m_{eq} \ddot{x}_{rack} + b_{eq} \dot{x}_{rack} + f_{eq} \cdot \text{sgn}(\dot{x}_{rack}) + \frac{\tau_e}{(l_p + l_c)} = \frac{\tau_{motor}}{G_{motor}} \]

where, \( m_{eq} \), \( b_{eq} \), and \( f_{eq} \) are the equivalent mass, damping coefficient and the friction applied to the rack system respectively, \( \tau_{motor} \) is the rack motor torque control input, \( x_{rack} \) is the actual rack position, \( \tau_e \) is the self-aligning torque applied to the rack bar from the front steering wheels and \( G_{motor} \) is the gear ratio between the front-wheel steering motor and the rack bar. The main role of the front wheel motor is to control the rack position according to the steering wheel command of the driver. The self-aligning torque which is generated by the lateral tire force is expressed by [15].

\[ \tau_e = (l_c + l_p) \cdot F_{sf} \]

where \( l_c \), \( l_p \) and \( F_{sf} \) is the mechanical trail, the pneumatic trail, and the lateral tire force, respectively. The lateral tire force \( F_{sf} \) is linearly related to the front tire slip angle \( \alpha_f \) and the front tire cornering stiffness coefficient \( C_f \) as follows:

\[ F_{sf} = C_f \cdot \alpha_f \]

Tire self-aligning moment is related to the caster angle determined by steering geometry and kingpin longitudinal offset. The mechanical trail, the distance between the tire center and the point on the ground is a function of caster angle and kingpin longitudinal offset as shown in Fig.2.

\[ l_c = \Delta + r_{eff} \cdot \theta \]

where \( \Delta \) is the kingpin longitudinal offset, \( r_{eff} \) is the effective radius of the tire, and \( \theta \) is the caster angle. Considering that the kingpin longitudinal offset, the effective radius of tire and the caster angle are system design parameters, the mechanical trail can be assumed to be a constant. Also, pneumatic trail is related to a tire slip angle, it can be assumed linear for small angles [5].

As shown in Fig.3, the cornering stiffness value \( C_f \) varies depending on the driving situation, especially the road surface, and the region in which the lateral tire force \( F_{sf} \) is linearly related to the tire slip angle \( \alpha_f \) is at a small slip angle, such as 2 deg or less. Therefore, in order to calculate the self-aligning torque, it is necessary to estimate the \( C_f \) value and also to estimate the \( \alpha_f \) value, which is difficult to obtain in practice.

Assuming that the body slip angle of the vehicle is close to zero, the tire slip angle can be approximated as follows [16]:

\[ \alpha_f \approx -\beta - \frac{\gamma \cdot l_f}{v_x} + \delta_f \]

where \( \alpha_f \), \( \beta \), \( l_f \), \( v_x \), \( \gamma \) and \( \delta_f \) are the tire slip angle, vehicle body slip angle, the distance from front axle to COG, vehicle speed, yaw rate and the front tire angle. The following equation can be derived from (2)-(5).

\[ \tau_e = (l_c + l_p) \cdot F_{sf} = (l_c + l_p) \cdot C_f \cdot \alpha_f \]

\[ = (l_c + l_p) \cdot C_f \cdot \left(-\beta - \frac{\gamma \cdot l_f}{v_x} + \delta_f\right) \]
The steady-state equation can be calculated as follows:

$$\dot{\beta}_{ss} = \frac{1}{1 - \frac{m}{2l_f C_f} \frac{v_x^2}{l_f}} \frac{l_f}{\tau} \delta_f$$

$$\gamma_{ss} = \frac{1}{1 - \frac{m}{2l_f C_f} \frac{v_x^2}{l_f}} \frac{v_x}{\tau} \delta_f$$

By substituting (7) for (6), the steady-state self-aligning torque can be calculated as follows:

$$\frac{\tau_{e,ss}}{(l_p + l_c)} = \frac{m C_f}{2l^2 C_f C_r + m v_x^2 (l_f C_f - l_f C_r)} \frac{l_f}{\tau} \delta_f$$

$$= k_f \cdot \delta_f \quad (k_f > 0)$$

$$= k_{eq} \cdot x_{rack} \quad (\delta_f = g_{rack} \cdot x_{rack}, \ k_{eq} = k_f \cdot g_{rack})$$

where \( \tau_{e,ss}, \delta_f, k_f, x_{rack} \) and \( g_{rack} \) are the steady-state self-aligning torque, front steering wheel angle, proportional value, actual rack position and ratio between the rack position and the hand-wheel steering angle.

From (8), the self-aligning torque \( \tau_r \) is proportional to the rack position \( x_{rack} \) and can be expressed as a stiffness model. Therefore, (1) can be rewritten by introducing a stiffness model as follows:

$$m_{eq} \ddot{x}_{rack} + b_{eq} \dot{x}_{rack} + k_{eq} x_{rack} + f_{eq} \cdot \text{sgn}(\dot{x}_{rack}) = \frac{\tau_{motor}}{G_{motor}}$$

(9)

In actual systems, the \( m_{eq}, b_{eq} \) and \( f_{eq} \) values can be obtained from design parameters, but the coefficient \( k_{eq} \) is difficult to obtain since it is affected by the cornering stiffness value \( C_f, C_r \) and vehicle speed. Therefore, the dynamics of the rack system can be expressed as follows:

$$m_{eq} \ddot{x}_{rack} + b_{eq} \dot{x}_{rack} + (k_{eq} + \Delta k_{eq}) x_{rack} + f_{eq} \cdot \text{sgn}(\dot{x}_{rack}) = \frac{\tau_{motor}}{G_{motor}}$$

(10)

The \( \gamma \) and \( \Delta \) in (10) represent the nominal and uncertainty values, respectively. The equivalent mass \( m_{eq} \) can be calculated as follows:

$$m_{eq} = m_{rack} + \left( \frac{G_{motor}}{r} \right)^2 m_{motor}$$

(11)

where \( m_{rack}, m_{motor}, G_{motor} \) and \( r \) are the mass of the rack bar, the mass of the motor, the gear ratio between the front-wheel steering motor and the rack bar, and the ratio that converts the linear motion of the rack bar into a rotary motion, respectively. The parameters of the SBW system are listed in Table 1.

### III. TARGET RACK POSITION TRACKING ALGORITHM

In this section, a rack position tracking controller for the SBW rack system is described. As shown in (8), the stiffness...
coefficient $k_{eq}$ is affected by cornering stiffness coefficient and vehicle speed. Therefore, a parameter adaptation law has been developed to improve the tracking performance of the rack system in various road surface conditions. In (6), however, the longitudinal vehicle speed term is in the denominator. Therefore, a dynamic stiffness model has been developed to guarantee the tracking performance of the rack system at zero vehicle speed. The proposed dynamic stiffness model utilizes the stiffness center, which varies depending on the actual rack position.

A. STIFFNESS PARAMETER ADAPTATION LAW

An adaptive sliding mode controller (ASMC) was used to control the rack system with uncertain dynamics. The tracking error and the sliding surface can be formulated as:

\[ e(t) = x_{rack}(t) - x_d(t) \]

\[ x_{rack}(t) = \theta_{rack}(t) \cdot g_{rack} \]

\[ x_d(t) = \theta_{driver}(t) \cdot g_{rack} \]

(12)

where $e(t)$ is the tracking error, $\lambda$ is the time constant of the error dynamics, $x_{rack}(t)$ is the actual rack position, $x_d(t)$ is the target rack position, $\theta_{rack}(t)$ is the angle from front-wheel motor position sensor (MPS), $\theta_{driver}(t)$ is the steering wheel angle from the driver and the $g_{rack}$ is the ratio between the rack position and the hand-wheel steering angle. The $g_{rack}$ can be determined by hardware specification. Since the maximum steering wheel angle of this SBW vehicle is 540[°deg] and the maximum rack position of the rack bar is 80[mm], the $g_{rack}$ can be calculated as 6.75(=540/80 [deg/mm]). The steering wheel angle from the driver is measured in hand-wheel feedback motor position sensor. The actual rack position value is determined by the front-wheel steering motor position angle $\theta_{rack}$ and the $g_{rack}$.

The time constant value $\lambda = 0.03$s in (13) is experimentally determined. The Lyapunov function $V_1$ for equivalent control input is as follows:

\[ V_1(t) = \frac{1}{2}s^2(t) \]

(14)

Differentiating $V_1$ with respect to time is as follows:

\[ \dot{V}_1(t) = s(t) \cdot \dot{s}(t) \]

\[ = s(t) [\dot{e}(t) + \lambda \frac{1}{m_{eq}} (\tau_{eq}(t) - b_{eq} \cdot \dot{x}_{rack}(t) \]

\[ - \cdots f_{eq} \cdot \text{sgn}(\dot{x}_{rack}(t) - \dot{k}_{eq} \cdot x_{rack}(t) - \dot{x}_d(t))] = 0 \]

(15)

The control objective is to keep the sliding surface $s(t)$ as zero. This can be accomplished by choosing the equivalent control input as follows:

\[ u_{eq}(t) = \tau_{eq}(t) \]

\[ = b_{eq} \dot{x}_{rack} + \dot{k}_{eq} x_{rack}(t) + f_{eq} \cdot \text{sgn}(\dot{x}_{rack}) \]

\[ + m_{eq} \ddot{x}_d(t) - \frac{x_{rack}}{\lambda} \dot{e}(t) \]

(16)

To achieve robustness of the controller, a switching control input can be included as follows:

\[ u_N(t) = -K \cdot \text{sat}[s(t)/\phi] \]

(17)

where $K$ is the positive sliding mode control gain, $\phi$ is the positive tunable constant that controls the boundary layer of the sliding surface, and sat is the saturation function, defined as follows:

\[ \text{sat}[s(t)/\phi] = \begin{cases} 1 & \text{if } s(t) > \phi \\ -1 & \text{if } s(t) < -\phi \\ s(t)/\phi & \text{else} \end{cases} \]

(18)

The Lyapunov function $V_2$ for parameter adaptation law is as follows:

\[ V_2(t) = \frac{1}{2} (s^2(t) + \rho \cdot \Delta k_{eq}^2) \]

(19)

Differentiating $V_2$ with respect to time is as follows:

\[ \dot{V}_2(t) = s(t) \cdot \dot{s}(t) + \rho \cdot \Delta k_{eq}(t)(-\dot{k}_{eq}(t)) \]

\[ = s(t)[\dot{e}(t) + \lambda \frac{1}{m_{eq}} (\tau_{eq}(t) - b_{eq} \cdot \dot{x}_{rack}(t) \]

\[ - \cdots f_{eq} \cdot \text{sgn}(\dot{x}_{rack}(t) - \dot{k}_{eq} \cdot x_{rack}(t) - \dot{x}_d(t)] + \rho \cdot \Delta k_{eq}(t)(-\dot{k}_{eq}(t)) \]

(20)

The stiffness parameter adaptation law is defined as follows:

\[ \dot{k}_{eq}(t) = -\frac{s(t) \cdot x_{rack}(t) \cdot \lambda}{m_{eq} \cdot \rho} \]

(21)

where $\rho$ is the positive design value that determines the convergence speed of the parameter adaptation. Finally, the control input $\tau_{motor,1}$ can be expressed as follows:

\[ \tau_{motor,1}(t) = u_{eq}(t) + u_N(t) \]

\[ = b_{eq} \dot{x}_{rack} + \dot{k}_{eq} x_{rack}(t) + f_{eq} \cdot \text{sgn}(\dot{x}_{rack}(t)) \]

\[ \text{feedforward adaptation} \]

\[ + \cdots m_{eq} \ddot{x}_d(t) - \frac{x_{rack}}{\lambda} \dot{e}(t) - K \cdot \text{sat}[s(t)/\phi] \]

(22)

Substituting the parameter adaptation law in (21) and the control input in (22) into (20) yields:

\[ \dot{V}_2(t) = -s(t) \cdot K \cdot \text{sat}\left(\frac{s(t)}{\phi}\right) < 0 \]

(23)

Since the derivative of the Lyapunov function in (23) is strictly negative, the proposed adaptive sliding mode controller is asymptotically stable [32].

B. RACK POSITION TRACKING AT ZERO VEHICLE SPEED

In this section, a new stiffness model is presented. As shown in (6), the longitudinal vehicle speed is in the denominator of the side slip angle term, so numerical problems can result when calculating the side slip angle at zero vehicle speed. Previously, this was not an issue as vehicle dynamics analysis was mainly done only in high-speed situations.
However, low speed tire analysis has become more important in situations where the vehicle has stopped, such as when parking. In a parking situation, the vehicle speed could be zero in the process of adjusting the position of the vehicle. The rack system dynamics derived experimentally through the vehicle steering motion tests at zero vehicle speed are as follows:

\[
m_{eq} \ddot{x}_{rack} + b_{eq} \dot{x}_{rack} + f_{eq} \cdot \text{sgn}(\dot{x}_{rack}) + f_{\text{stiff}}(t) = \frac{\tau_{\text{motor}}}{G_{\text{motor}}}
\]  

(24)

The stiffness term \(f_{\text{stiff}}\) is defined as follows:

\[
f_{\text{stiff}}(t) = k_0 \cdot (\theta_{\text{rack}}(t) - \theta_0(t))
\]

\[
\begin{align*}
\dot{\theta}_0(t) & = 0 & \quad & |\theta_{\text{rack}}(t) - \theta_0(t)| < \theta_{\text{limit}} \\
\dot{\theta}_0(t) & = k_0 (\theta_{\text{limit}} - \theta_0(t)) & \quad & |\theta_{\text{rack}}(t) - \theta_0(t)| \geq \theta_{\text{limit}}
\end{align*}
\]

(25)

where \(k_0\) is the stiffness coefficient at zero velocity, \(\theta_{\text{rack}}\) is the actual angle from front-wheel MPS, \(\theta_0\) is the elastic equilibrium point, and \(\theta_{\text{limit}}\) is the elastic limit which is the distance between the actual angle from front-wheel MPS and elastic equilibrium point.

Equation (24)-(25) are the results of reverse modeling through the responsiveness of the rack system that occurs when the steering wheel command is applied to the vehicle system at zero velocity. When the speed of the vehicle is not zero, the elastic equilibrium point \(x_0\) becomes zero, as shown in (8). This is explained by the phenomenon that if the steering wheel is released while the vehicle is driving, it will return to the center immediately. However, when the vehicle is at zero speed, return to the center of the steering wheel is not guaranteed. Therefore, an elastic equilibrium point \(x_0\) that is affected by the rack position is introduced.

Fig.4 and Fig.5 shows the return-to-center performance of the steering wheel at zero vehicle speed. In a stop situation, when the steering wheel is released, the rack bar does not return to the center but stops before that point. Therefore, the elastic limit value \(x_{\text{limit}}\) can be determined by how much the rack has moved towards the center. Fig.5 shows the actual vehicle test data of the return-to-center steering performance.

In the case of \(|\theta_{\text{rack}} - \theta_0| < \theta_{\text{limit}}\), the elastic equilibrium point \(x_0\) remains constant as shown in Fig.4(b), Fig.4(a) and Fig.5(b). When the steering wheel angle is gradually increases and satisfies the condition \(|\theta_{\text{rack}} - \theta_0| \geq \theta_{\text{limit}}\), the elastic equilibrium point \(\theta_0\) changes as shown in Fig.4(c) and Fig.5(d).

Using the derived equation (25) of rack system dynamics for a zero vehicle speed, motor control input is determine as follows:

\[
\tau_{\text{motor}}(t) = m_{eq} \ddot{x}_d + b_{eq} \dot{x}_{\text{rack}} + f_{eq} \cdot \text{sgn}(\dot{x}_{\text{rack}}) + \cdots f_{\text{stiff}}(t) - \frac{x_{\text{rack}}}{\lambda} \dot{\phi} - K \cdot \text{sat}\left[\frac{\phi}{\phi}\right]
\]

(26)
Finally, the total control input can be obtained by the linear interpolation of (22) and (26).

\[
\tau_{motor}(t) = (1 - \alpha) \cdot \tau_{motor,1}(t) + \alpha \cdot \tau_{motor,2}(t)
\]

where

\[\alpha = \begin{cases} 
0 \cdot v_x(t) > 3 \text{ kph} \\
1 \cdot v_x(t) = 0 \text{ kph} \\
-(1/3) \cdot v_x(t) + 1 \cdot \text{else}
\end{cases}
\]

\[
(27)
\]

\[
\text{IV. COMPUTER SIMULATIONS}
\]

Computer simulations using Matlab/Simulink were performed to evaluate the proposed algorithm. As shown in Fig.6, the overall control scheme consists of an adaptive sliding mode controller, a rack plant, and a vehicle model using Carsim software. The plant model was implemented using Matlab/Simulink Simscape components [28], which reflect physical properties such as mass, friction and damping in the rack system.

In simulations, four open-loop scenarios have been conducted to validate the proposed algorithm.

The first scenario is the periodic sinusoidal signal with an amplitude of 45 deg and a frequency of 0.5 Hz. The sinusoidal test is one of the most frequently used for evaluating the tracking performance of the SBW system [5], [7], [11] and [13]. This experiment is to confirm that the convergence of the parameter adaptation algorithm is guaranteed under a constant vehicle speed.

The second scenario is the same sinusoidal test as the first experiment. The aim of this experiment is to confirm that the convergence of the parameter adaptation algorithm is ensured when the vehicle speed changes.

The third and fourth test were conducted to show the good tracking performance of the proposed algorithm under various road surface condition. For comparison, PD control with feedforward compensation method [5], [29] and adaptive control gain sliding mode control [27] was adopted.

\[
\tau_{PD}(t) = k_p \cdot e(t) + k_d \cdot \dot{e}(t) + m_{eq} \ddot{x}(t) + b_{eq} \dot{x}_{rack}(t)
\]

\[
(28)
\]

where \(k_p\) and \(k_d\) are the proportional and derivative gains, respectively.

\[
T_{SMC(control \ gain)} = m_{eq} \ddot{x}_d + b_{eq} \dot{x}_{rack} - b_{eq} \dot{f} \sgn(s)
\]

\[
(29)
\]

The feedback adaptive law is applied as follows:

\[
\dot{\alpha} = \frac{1}{\alpha} |s|, \quad \alpha > 0
\]

\[
(30)
\]

where \(\alpha\) is the adaptation rate of convergence.

When applying the adaptation approach in practice, the initial setting of the adaptation parameter is needed. The initial parameter of the ‘control gain adaptive’ method should be smaller than the upper bound of the system uncertainties. This approach is effectively used when analysis of disturbance is difficult and only the upper bound is known. If the initial value is large, the control performance increases, but it may cause chattering in motor input signal, and if the initial value is small, the control performance decreases. Therefore, it is necessary to set an appropriate initial value in order to increase the control performance. The initial parameter of the proposed ‘ASMC’ method is determined by the equation (8). In general driving conditions, nominal \(C_f\) and \(C_r\) values and vehicle speed \(v_x\) are used to set the initial value of \(k_f\).

All the control parameters used in the simulation are listed in Table 2.

From (27) – (29), 1st and 2nd derivative of the actual rack position and target rack position are required in practice. However, it is difficult to measure velocity and acceleration of the rack movement. Therefore, first order and second order infinite impulse response (IIR) filtering method [30] was used to estimate the velocity and acceleration signals. Since the input steering angle signal is measured by using MPS, it is quantized and the white noise could be neglected. The advantage of the IIR filtering methodology can attenuate the quantization noise optimally and provide an easy differentiating tool.

Fig.7 shows the results of open-loop sinusoidal signal (Amplitude: 45 deg, Frequency: 0.5 Hz) on dry road (\(\mu = 1.0\)). The speed of the vehicle is set 70 kph. Fig.7(a) shows the steering tracking performance. Fig.7(b) presents...
the stiffness parameter adaptation. Fig.8(c) shows the rack force which is the disturbance in rack system from self-aligning torque. Fig.7(d) shows the control motor torque. As shown in (8), external disturbance \( r_e \) can be represented by the stiffness model \( k_{eq} \cdot x_{rack} \). Therefore, the actual disturbance from the vehicle model and the calculated stiffness value were compared to show the effectiveness of the stiffness parameter adaptation law.

As shown in Fig.7(b), the stiffness coefficient can converge at 3.5 s. After the coefficient parameter converges, it can be seen from Fig.7(a) that the actual steering wheel angle can successfully tracks the desired steering wheel angle. The effectiveness of the parameter adaptation law can be seen from Fig.7(c). After the convergence of the stiffness parameter in 3.5 s, the calculated stiffness \( k_{eq} \cdot x_{rack} \) can tracks the actual external disturbance well.

It is seen from (8) that the stiffness coefficient \( k_{eq} \) is affected by vehicle speed \( v_s \) and road surface condition. Therefore, open-loop sinusoidal signal (Amplitude: 45 deg, Frequency: 0.5 Hz) test when the vehicle speed changes was conducted to validate whether the proposed adaptive algorithm can cope with varying vehicle speed situation. This test was implemented on dry road (\( \mu = 1.0 \)) and the initial speed was 40kph.

Fig.8 shows the control performance with the proposed adaptive sliding mode algorithm. Fig.8(a) shows the tracking performance. Fig.8(b) presents the stiffness coefficient adaptation. The rack force from the vehicle model and calculated stiffness force were illustrated in Fig.8(c). The proposed algorithm was conducted in a situation of increasing vehicle speed as seen in Fig.8(d). The motor control force is depicted in Fig.7(e).

As illustrated in Fig.8(a), the steering tracking performance of the rack system has been guaranteed when the vehicle speed has been increased. It can be seen from Fig.8(b) that the stiffness coefficient \( k_{eq} \) also increases when the vehicle speed increases. From the aforementioned equation (8), the stiffness coefficient relation for the vehicle speed is as follows:

\[
k_{eq} = \frac{(mC_f) \left( l_rC_r - l_fC_f + l_fC_r \right) \cdot v_s^2}{2l^2 \left( l_rC_r + m v_s^2 \left( l_rC_r - l_fC_f \right) \right)} \quad (31)
\]

It is noted from (31) that the stiffness coefficient increases as the vehicle speed increases under the condition of constant road condition. Differentiating stiffness coefficient \( k_{eq} \) with respect to vehicle velocity \( v_s \) is as follows:

\[
\frac{\partial k_{eq}}{\partial v_s} = \frac{4ml^2C_f^2C_r \left( l_rC_r - l_fC_f + l_fC_r \right)}{2l^2 \left( l_rC_r + m v_s^2 \left( l_rC_r - l_fC_f \right) \right)^2} \cdot v_s > 0 \quad (32)
\]

When the vehicle is driving, the (32) is strictly positive. Therefore, as the vehicle speed increases, the stiffness coefficient also increases.

As shown in Fig.8(c), the calculated rack force \( k_{eq} \cdot x_{rack} \) can successfully match to the actual rack force. This aspect also shows that the disturbance can be compensated through the stiffness coefficient adaptation algorithm without tire-road friction information.

The normalized errors \( e_z(i) \) are utilized to show the performance of the proposed control algorithm [36].

\[
e_z(i) = \frac{100 \cdot |x_eq(i) - x_{rack}(i)|}{\max |x_{rack}(i)|} \quad (33)
\]

where \( i \) represents each time sample. The sampling interval is 0.001 s (=1 ms) and the measured quantization level of the steering angle sensor is 0.07 deg. The mean and maximum value of the normalized errors were used as a tracking performance evaluation index.

\[
\text{MEAN} (e_z) = \frac{\sum_{i=1}^{N} e_z(i)}{N}, \quad \text{MAX} (e_z) = \max (e_z(i)) \quad (34)
\]

where \( N \) is the sample number. To prove that the proposed adaptive sliding mode control algorithm can cope with various road conditions, simulation tests of dry road (\( \mu = 1.0 \))
and wet road (µ = 0.5) were conducted. The variable µ represents the tire-road friction coefficient and it represents the different road surface condition [37]. For comparison, a PD control with feedforward compensation method and an adaptive control gain SMC control were used as aforementioned.

Fig.9 shows the tracking performance of the rack system using a proposed algorithm, a PD controller and an adaptive control gain SMC on dry road at constant vehicle speed.

In the dry asphalt situation, it can be seen that the proposed algorithm, PD controller and adaptive control gain SMC can guarantee the rack tracking performance.

This can be easily seen from Fig.9(a) and (d). Fig.9(b) shows that the stiffness coefficient values remain constant during the simulation. This is because the parameter is
FIGURE 9. Open-loop sinusoidal steering (Amplitude: 90 deg, Frequency: 0.5 Hz) simulation on dry road ($\mu = 1.0$) at constant speed ($v = 70$ kph).

FIGURE 10. Open-loop sinusoidal steering (Amplitude: 90 deg, Frequency: 0.5 Hz) simulation on wet road ($\mu = 0.5$) at constant speed ($v = 70$ kph).

affected by vehicle speed and road conditions, the values do not change under constant vehicle speed and constant dry road conditions. The rack force in Fig.9(c) shows that the calculated stiffness $k_{eq} \cdot x_{rack}$ can match the disturbance in this scenario.
Fig.10 illustrates the steering tracking performance using a proposed algorithm, a PD control and control gain adaptive SMC on wet road at constant vehicle speed. The target steering wheel angle is the same as the steering input used in the dry road situation. This test was conducted to show the effectiveness of the adaptive sliding mode control algorithm under different road surface: wet road.

As shown in Fig.10(a), the tracking performance with the PD controller was not guaranteed, especially at 1.2s - 1.7 s and at 2.2 s - 2.7s. This result was obtained since the front tire reaches saturation under wet road surface. When the front tires are saturated, the rack force is also saturated. The control gains of the PD controller are tuned to deal with the dry road surface condition. Therefore, the PD controller that uses a fixed gain value cannot cope with the situations where the road surface changes. In order to improve the tracking performance, the control gains of the PD controller need to be adjusted according to the road surface condition. However, it is difficult to obtain the tire/road surface condition accurately.

In the case of a rack force is saturated on wet road surface, the stiffness parameter adaptation process is started as shown in Fig.10(b). Fig.10(c) shows that the adapted stiffness model $k_{eq} \cdot x_{rack}$ can successfully compensate the external disturbance. As illustrated in Fig.19(d), the steering tracking performance using proposed adaptive sliding mode control algorithm has been improved compared with the one using PD control with feedforward and control adaptive SMC. The proposed algorithm adjusts the feedforward term through adaptation, but the control gain adaptation SMC method has a difference in adjusting the feedback term through adaptation. Also, with the proposed feedforward adaptation method, the initial parameter can be determined based on the disturbance analysis. In the case of vehicle system, the self-aligning torque disturbance could be modeled using the bicycle model and tire model, therefore, it is possible to control the rack system more accurately with the proposed ‘ASMC’ algorithm.

V. EXPERIMENTAL TEST
The proposed adaptive sliding mode control algorithm has been validated via the steer-by-wire vehicle tests. A motor position sensor (MPS) is used to measure the target steering wheel angle and actual position of the rack. The sampling time and resolution of the MPS (by Hyundai Motors) is 1 ms and 0.07 deg respectively. The PM ac motor (by Hyundai
Motors) with output torque of 9.5 N · m and gear reduction ratio of 15 is installed on the rack bar for steering the front wheels. The experimental steer-by-wire vehicle is illustrated in Figure 11. The micro-autobox(1401/1511/1513) was used to validate the control algorithms in real-time.

Four scenarios have been conducted. The first scenario is a manual driving test on dry road \( (\mu = 1.0) \) using proposed ASMC algorithm. It was performed to validate the performance of the parameter adaptation in a situation where the vehicle speed is changing. In this scenario, steering tracking performance has been secured not only at low speed, but also at high speed.

The second scenario is a manual sinewave steering driving test on dry road. The aim of this scenario is to attest the performance of the proposed algorithm, PD control and control gain adaptive SMC in normal driving situation. Fig.13 – 15 shows the tracking performance of PD control, control gain adaptive SMC and proposed ASMC respectively.

The third scenario is a manual sinewave steering driving test on wet road \( (\mu = 0.5) \) to attest the performance of the ASMC algorithm regardless of road surface condition. Fig.16 – 18 shows the tracking performance of PD control, control gain adaptive SMC and proposed ASMC respectively.

The fourth test was conducted to validate the proposed algorithm under zero vehicle speed. All the control parameters used in the experiment are listed in Table 2.

Fig.12(a) shows the tracking performance of the proposed ASMC at various vehicle speeds. Fig.12(b) demonstrates the parameter adaptation in the proposed algorithm in various driving speeds. The steer-by-wire system using the proposed ASMC algorithm shows a maximum error amplitude of 6 deg, which means a 3% error of the target steering angle. As shown in Fig.12(b), the \( k_{eq} \) value increases when the vehicle speed increases and decreases when the vehicle speed decreases. This can be confirmed in the simulation results as shown in Fig.8(b), (d) and the Eq.(32).

Fig.13 – 15(a), (b) demonstrate the tracking performance of the three controllers in dry road at 40kph. Not surprisingly, in dry road surface condition, the tracking performance is not significantly different depending on the type of the controller.
As shown in Fig.15(b), the mean normalization error performance of the ASMC is 2.4 deg, which shows better performance than PD and control gain adaptive SMC. Both the PD control and control gain adaptive SMC shows the performance with the mean normalized error of 4.7 deg and 3.2 deg.

Fig.16 - 18 shows the steering tracking performance of the all three controllers on wet road ($\mu = 0.5$) at 40kph. Unlike dry road situation, the difference in tracking performance according to the controller is evident on wet road. Fig.16(a)-(d) shows the tracking performance with PD control, tracking error control force and tracking in 11.5th second respectively. Fig.16(b) shows that in the case of a PD controller, the tracking performance is not ensured as much as using the control gain adaptive SMC and the proposed ASMC. It is because of the PD controller is gain-tuned for dry road surface condition, it could not cope with road surface changes. This point could be seen from Fig.16(d) that...
the tracking performance at 11.5 s is seriously deteriorated. The PD controller shows the maximum error of 40 deg, mean normalized error of 12.5 deg and maximum phase delay of 70 ms.

Fig.17(a)-(d) shows the tracking performance with control gain adaptive SMC, tracking error control force and tracking in 11.6th second respectively. It is clearly seen from Fig.16(b) and Fig.17(b) that the control gain adaptive method shows a satisfactory tracking performance with the maximum error of 10 deg, mean normalized error of 7.2 deg and maximum phase delay of 40 ms. It demonstrates that the control gain
adaptive SMC method shows robust characteristics to road surface changes. However, adaptively reducing the errors through feedback method has a performance limitation. In order to obtain high-precision tracking performance, a high feedback gain value is required, but it could cause chattering problem in motor control input.

Fig.18(a)-(d) shows the tracking performance with proposed ASMC algorithm, tracking error, parameter adaptation,
control force and tracking in 10.1th second respectively. It can be seen from Fig.16(b), Fig.17(b) and Fig.18(b) that the proposed ASMC shows the remarkable tracking performance with the maximum amplitude error of 7 deg, mean normalized error of 4.1 deg and maximum phase delay of 25 ms, which is better than the ones of the PD control and control gain adaptive SMC algorithm. It can be observed from Fig.18(c) that the parameter $k_{eq}$ decreases on low-friction road. This is because, as can be seen from Fig.3, the lateral tire force decreases on low-friction road.

Fig.19, Fig.20 and Fig.21 are the steering tracking performance at zero vehicle speed using PD controller, control gain adaptive SMC and the proposed ASMC algorithm, respectively.

Fig.19-21(a), (b), (c) represent the steering wheel tracking performance, error and control motor force of the three controllers at zero vehicle speed on dry road.

It is noted from Fig.19(a) that the performance of the PD controller is poor at zero vehicle speed even in a dry road condition with the maximum error of 10 deg and mean normalized error of 5.7 deg. This is because in a stationary situation, the reaction rack force coming through the front tires is greater than the one in a driving situation. Therefore, an appropriate feedforward compensation is needed.

Fig.21(a) shows that with the proposed dynamic stiffness model in (25), the tracking performance at zero vehicle speed has been improved. As illustrated in Fig.21(b) and (c), the presented dynamic stiffness model in (25) not only improves the tracking performance, but also eliminates the chattering of motor control force.

Fig.21(b) shows that the tracking performance of the proposed ASMC algorithm with the maximum error of 2.8 deg and mean normalized error of 1.2 deg at zero vehicle speed. The control gain adaptive SMC method get worse performance with the maximum error of 7.5 deg and mean normalized error of 4.7 deg at zero vehicle speed.

A comparison of the proposed ASMC with PD control and control gain adaptive SMC is shown in Table 3. From the data, it can be seen that the simulation result performance is better than the actual vehicle test performance. This is because, hardware characteristics such as backlash, motor efficiency and gear efficiency can affect the tracking accuracy.

The above experimental vehicle test results show that the tracking performance of the proposed ASMC algorithm has been improved compared with the PD control and control gain adaptive SMC approach. In addition, with the proposed adaptation control algorithm, the good tracking performance can be obtained not only in a wet road condition but also in a stopped situation. Therefore, the proposed ASMC algorithm can be used as a steer-by-wire rack steering controller in various driving situations.

VI. CONCLUSION

In this paper, a dynamic modelling of a steer-by-wire (SBW) rack system has been studied and an adaptive sliding mode control (ASMC) algorithm has been proposed to cope with various road surface conditions. It has been demonstrated that the adaptation sliding mode control algorithm can effectively compensate for the rack force acting as a disturbance. The proposed ASMC algorithm does not require knowledge of road surface conditions. It replaces the lateral tire force with a stiffness and changes the value of the stiffness coefficient according to the road surface conditions through the adaptation law. In addition, a dynamic stiffness model was introduced in the proposed algorithm to improve rack position tracking performance in a zero vehicle speed scenario. In a dynamic stiffness model, the stiffness center is not fixed, but changes depending on the actual rack position.

Both a simulation and a vehicle test were conducted to verify the tracking performance of the proposed algorithm. Both the simulation and vehicle test results showed that the proposed algorithm ensures the tracking performance in wet road conditions without using any road surface information. In addition, it has been shown that the tracking performance at zero vehicle speed is improved by use of the proposed stiffness model. This paper only deals with the tracking control
algorithm using one actuator. However, the SBW system is
dependent of the motor safety, the future study on designing a
fault tolerant control system using dual steering actuators are
under the authors’ investigation. The fault-tolerant method of
SBW system can be found in another paper, [38]–[40].

REFERENCES

[1] B. Zheng and S. Anwar, “Yaw stability control of a steer-by-wire equipped
vehicle via active front wheel steering,” Mechatronics, vol. 19, no. 6, pp. 799–804, Sep. 2009.
[2] K. Nam, S. Oh, H. Fujimoto, and Y. Hori, “Robust yaw stability control
for electric vehicles based on active front steering control through a steer-
by-wire system,” Int. J. Automot. Technol., vol. 13, no. 7, pp. 1169–1176, Dec. 2012.
[3] W. Zhao, X. Qin, and C. Wang, “Yaw and lateral stability control for four-
wheel Steer-by-Wire system,” IEEE/ASME Trans. Mechatronics, vol. 23, no. 6, pp. 2628–2637, Dec. 2018.
[4] S. Oh, The Development of an Advanced Control Method for the Steer-
by-Wire System to Improve the Vehicle Maneuverability and Stability,
Warrendale, PA, USA: SAE, 2003.
[5] P. Yih and J. C. Gerdes, “Modification of vehicle handling characteristics via steer-by-wire,” IEEE Trans. Control Syst. Technol., vol. 13, no. 6, pp. 965–976, Nov. 2005.
[6] P. Seffur, D. Dawson, J. Chen, and J. Wagner, “A nonlinear tracking
controller for a haptic interface steer-by-wire systems,” in Proc. 41st IEEE Conf. Decision Control, 2002, p. 45.
[7] Y. Yamaguchi and T. Murakami, “Adaptive control for virtual steering
characteristics on electric vehicle using Steer-by-Wire system,” IEEE Trans. Ind. Electron., vol. 56, no. 5, pp. 1585–1594, May 2009.
[8] R. Kazemi and A. A. Janbakhsh, “Nonlinear adaptive sliding mode control
for vehicle handling improvement via steer-by-wire,” Int. J. Automot. Technol., vol. 11, no. 3, pp. 345–354, Jun. 2010.
[9] F. Zou, D. Song, Q. Li, and B. Yuan, “A new intelligent technology of
Steering-by-Wire system by variable structure control with sliding mode,”
in Proc. Int. Joint Conf. Artif. Intell. Apr. 2009, pp. 1–8.
[10] W. Sun, X. Wang, and C. Zhang, “A model-free control strategy for vehicle
lateral stability with adaptive dynamic programming,” IEEE Trans. Ind. Electron., vol. 67, no. 12, pp. 10693–10701, Dec. 2020.
[11] S.-W. Oh, H.-C. Chae, S.-C. Yun, and C.-S. Han, “The design of a
controller for the steer-by-wire system,” JSME Int. J. Ser. C, Mech. Syst. Mach. Elements Manuf., vol. 47, no. 3, pp. 896–907, Sep. 2004.
[12] C.-T. Kim, I.-H. Jung, S. Oh, and K. J. Hedrick, “Development of a control algorithm for a rack-actuating steer-by-
wire system using road information feedback,” Proc. Inst. Mech. Eng. D, J. Automobil. Eng., vol. 222, no. 9, pp. 1559–1571, Sep. 2008.
[13] H. Wang, H. Kong, Z. Man, D. M. Tuan, Z. Cao, and W. Shen, “Sliding
mode control for Steer-By-Wire systems with AC motors in road vehicles,”
IEEE Trans. Ind. Electron., vol. 61, no. 3, pp. 1596–1611, Mar. 2014.
[14] H. Wang, Z. Man, H. Kong, and W. Shen, “Terminal sliding mode control
for steer-by-wire system in electric vehicles,” in Proc. 7th IEEE Conf. Ind. Electron. Appl. (ICIEA), Jul. 2012.
[15] M. T. Do, Z. Man, C. Zhang, H. Wang, and F. S. Tay, “Robust sliding mode-
based learning control for Steer-By-Wire systems in modern vehicles,”
IEEE Trans. Veh. Technol., vol. 63, no. 2, pp. 580–590, Feb. 2014.
[16] R. Rajamani, Vehicle Dynamics and Control, New York, NY, USA:
Springer-Verlag, 2006.
[17] E. Bakker, B. Hans, and L. Lidner, A new tire model with an application in
drive vehicles studies, Warrendale, PA, USA: SAE, 1989.
[18] Y. Yasui, A new tire model with an application in vehicle dynamics studies, Warrendale, PA, USA: SAE, 2004, pp. 632–637.
[19] A. Levant, “Sliding order and sliding accuracy in sliding mode control,”
Int. J. Control, vol. 58, no. 6, pp. 1247–1263, Dec. 1993.
[20] K. D. Young, V. I. Utkin, and U. Ozguner, “A control engineer’s guide to
sliding mode control,” in Proc. IEEE Int. Workshop Variable Struct. Systems, Dec. 1996, pp. 1–14.
[21] V. I. Utkin, “Sliding mode control design principles and applications to
electric drives,” IEEE Trans. Ind. Electron., vol. 40, no. 1, pp. 23–36, Feb. 1993.
[22] Y. Shitessel, Sliding Mode Control and Observation, New York, NY, USA:
Springer, 2014.

K. Kim

Kwangil Kim received the B.S. degree in electrical engineering from Seoul National University, South Korea, in 2015, where he is currently pursuing the Ph.D. degree in mechanical engineering. His research interests include vehicle stability control, vehicle dynamics, adaptive sliding mode control with its applications, and steer-by-wire vehicles.

VOLUME 8, 2020 163499
JAEPOONG LEE received the B.S. and M.S. degrees in mechanical engineering from Seoul National University, South Korea, in 2015 and 2018, respectively, where he is currently pursuing the Ph.D. degree in mechanical engineering. His research interests include steer-by-wire vehicles, haptic control, steering feel generation, and fail safe.

MINJUN KIM received the B.S. degree in mechanical engineering from Busan University, South Korea, in 2006. He is currently with the Hyundai Research and Development Center. His research interests include steer-by-wire rack system dynamics.

KYONGSU YI (Member, IEEE) received the B.S. and M.S. degrees in mechanical engineering from Seoul National University, South Korea, in 1985 and 1987, respectively, and the Ph.D. degree in mechanical engineering from the University of California at Berkeley, in 1992. He is currently a Professor with the School of Mechanical and Aerospace Engineering, Seoul National University. His research interests include control systems, driver-assistance systems, active safety systems, and automated driving of ground vehicles. He also serves as a member for the Editorial Board of Mechatronics and the Chair of the KSME IT Fusion Technology Division.

* * *