Two-channel spin-chain communication line and simple quantum gates

J. Stolze\textsuperscript{1} and A. I. Zenchuk\textsuperscript{2}

\textsuperscript{1}Technische Universität Dortmund, Fakultät Physik, D-44221 Dortmund, Germany,
\textsuperscript{2}Institute of Problems of Chemical Physics, RAS, Chernogolovka, Moscow reg., 142432, Russia.

Abstract

We consider the remote creation of a mixed state in a one-qubit receiver connected to two two-qubit senders via different channels. Channels are assumed to be chains of spins (qubits) with nearest-neighbor interactions, no external fields are being applied. The problem of sharing the creatable region of the receiver’s state-space between two senders is considered for a communication line with the receiver located asymmetrically with respect to these senders (asymmetric communication line). An example of a quantum register realizing simple functions is constructed on the basis of a symmetric communication line. In that setup, the initial states of the two senders serve as input and control signals, respectively, while the state of the receiver at a proper time instant is considered as the output signal.

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I. INTRODUCTION

The remote creation of mixed states including the control of their state parameters has become an attractive area of quantum information processing. Originating from quantum state transfer (first explored by Bose [1]) this field of research aims at extending the possible applications of quantum communication lines, stressing the quantum nature of the systems considered whose states are (generally) mixed and thus described by density matrices. The power of the density matrix lies in its large dimensionality even for systems of small size (for instance, the density matrix of \( N \) spins 1/2 has dimension \( 2^N \times 2^N \)) and, in addition, in the intricate connection between the parameters of the density matrix (changing one of the parameters, in general, causes changes in all others), which is reflected in such characteristics of quantum systems as quantum entanglement \([2, 3]\) and discord \([4–6]\). Therefore, quantum operations based on the density matrix of a mixed state seem to offer a promising perspective in the development of quantum information devices.

The research on remote state creation started with photon systems \([7–9]\). Other two-level quantum systems were studied less intensively (perhaps because of the obstacles in experimental realization), although there has been some progress in that direction \([10]\). In this regard we mention our recent papers on one-qubit \([11–13]\) and two-qubit \([14]\) state creation.

In order to optimize remote state-creation protocols we use a boundary controlled spin chain for state creation. Originally it was observed that quantum state transfer can be improved if the two sites at opposite ends of a chain are only weakly coupled to the intermediate section \([15, 16]\). Later performance was improved via optimizing one pair \([17–20]\) or even two pairs of boundary coupling constants \([14, 21]\). In the present paper we will only employ the second approach, that is, two-pair boundary-controlled chains.

Since many of the references cited above deal with perfect state transfer (PST) we would like to stress that we are presently considering remote state creation. These two processes differ in various respects. In PST a pure quantum state of (usually) a single qubit is identically transferred to a different qubit. Remote state creation in contrast deals with mixed states of sender and receiver units which may consist of one or more qubits. In order to create a desired state of the receiver, the sender first has to be prepared in an appropriate initial state. Subsequently the desired state of the receiver develops by the natural dynamics
of the system, without external control. Sender and receiver units (which need not be of the same size) are connected by chains of spins 1/2 with nearest-neighbor coupling. In contrast to PST the initial state of the sender and the final state of the receiver usually are neither identical nor even similar to each other.

We pursue the idea of sharing the receiver’s state space between two different senders of a communication line. If the subregions creatable by different senders do not overlap, then the created state can serve, in particular, as an indicator telling us which of the two senders is involved in this process of state creation. In our model, each of the two-qubit senders is connected to the one-qubit receiver by a channel of spin-1/2 particles. Each channel is an optimized boundary controlled chain with two pairs of properly adjusted boundary coupling constants. This optimization reduces significantly the dependence of the creatable region on the length of the channel. To demonstrate this, two communication lines with 20 and 60 node channels are compared. Non-overlapping sharing of the creatable region requires an asymmetric communication line, for instance, with different lengths of the two channels involved. Using, by contrast, a similar communication line with two equivalent channels we construct two quantum gates performing simple manipulations with the eigenvalues and eigenvectors of the receiver’s density matrix.

The paper is organized as follows. Two general schemes of a communication line together with the Hamiltonian governing the spin dynamics are discussed in Sec. II. The general settings for the remote state creation in the two-channel communication line are given in Sec. III. The sharing of the creatable region between two senders is studied in Sec. IV. Simple quantum gates are constructed in Sec. V while Sec. VI contains a general discussion.

II. TWO GENERAL SCHEMES FOR A TWO-CHANNEL COMMUNICATION LINE

We consider two schemes for a two-channel communication line: (i) the asymmetric scheme with the receiver at the last node of the first channel, Fig. 1a, and (ii) the symmetric scheme with the receiver inserted between two equivalent channels, Fig. 1b.

If we want to separate the regions creatable by each of the senders (while the other sender is in its ground state initially) then the asymmetric scheme (Fig. 1a) is preferable. By contrast, the symmetric scheme will be useful for the quantum gates to be discussed in
FIG. 1: The two-channel communication lines with the two-node senders $S_1$ and $S_2$ and the one-node receiver $R$ (a) embedded in the first channel (the communication line consists of $N = 2N_1$ nodes in total) and (b) located between the two channels (the communication line consists of $N = 2N_1 + 1$ nodes in total)

Sec[V]

A. Asymmetric communication line

The communication line in Fig 1(a) consists of two equivalent channels, each with two pairs of properly adjusted end-nodes [14] as shown in Fig 2. The whole system is governed by a nearest-neighbor XY-Hamiltonian with three parts:

$$H = H_1 + H_2 + H_{12},$$

(1)
TABLE I: The coupling constants $\delta_i$, $i = 1, 2, 3$, and the time instant $t_0$ optimizing the state creation in the asymmetric communication line (shown in Fig. 1(a)) with pairs of channels of $N_1 = 20$ and 60 nodes, respectively.

| $N_1$ | $\delta_1$ | $\delta_2$ | $\delta_3$ | $t_0$ |
|-------|-------------|-------------|-------------|--------|
| 20    | 0.550       | 0.817       | 0.28        | 28     |
| 60    | 0.414       | 0.720       | 0.20        | 72.45  |

where $H_i$, $i = 1, 2$, are the Hamiltonians governing the dynamics within each particular channel and $H_{12}$ is the Hamiltonian responsible for the channel interaction:

$$H_1 = \sum_{i=3}^{N_1-3} D(I_{ix}I_{(i+1)x} + I_{iy}I_{(i+1)y}) + \delta_1 \left( I_{1x}I_{2x} + I_{1y}I_{2y} + I_{(N_1-1)x}I_{N_1x} + I_{(N_1-1)y}I_{N_1y} \right)$$

$$H_2 = \sum_{i=2N_1-3}^{2N_1-1} D(I_{ix}I_{(i+1)x} + I_{iy}I_{(i+1)y}) +$$

$$\delta_2 \left( I_{2x}I_{3x} + I_{2y}I_{3y} + I_{(N_1-2)x}I_{(N_1-1)x} + I_{(N_1-2)y}I_{(N_1-1)y} \right),$$

$$H_{12} = \delta_3 \left( I_{N_1x}I_{(N_1+1)x} + I_{N_1y}I_{(N_1+1)y} \right)$$

The system thus consists of two subsystems of $N_1$ sites each. Note that the subsystems are completely identical physically; the asymmetry solely consists in identifying the end node of the left chain (site $N_1$) as the receiver $R$ which receives information from either of the two two-qubit senders $S_1$ and $S_2$, while the other sender is assumed to be in its ground state initially. Note further that each of the two identical subsystems is again physically symmetric in itself, with couplings $\delta_1$ at the first and last nearest-neighbor bonds, and $\delta_2$ at the second and second to last bonds. The remaining nearest-neighbor couplings within each subsystem are all equal to $D$. Henceforth we consider the dimensionless time $Dt$ formally assuming $D = 1$ in eq.(1). The two channels in this scheme are connected via the interaction between the $N_1$th and $(N_1 + 1)$th nodes with the coupling constant $\delta_3$.

The parameters $\delta_i$, $i = 1, 2$, are such that they maximize the probability of the excited state transfer between the end nodes of each individual $N_1$-site channel; the optimal values for channels of lengths $N_1 = 20$ and 60 were found in ref.[14]. Then, having fixed $\delta_1$ and...
FIG. 2: The channel used as a building block in the communication lines in Figs 1(a) and 1(b). There are two pairs of adjusted coupling constants: $D_1 = D_{N_1-1} = \delta_1$ and $D_2 = D_{N_1-2} = \delta_2$. All other coupling constants are equal, $D_i = 1$, $2 < i < N_1 - 2$.

$\delta_2$, we adjust the coupling constant $\delta_3$ and the time instant of the state registration $t_0$ which provide separation of the subregions of the receiver’s state-space creatable by the two different senders while keeping the area of each of the creatable subregions large, as shown in Sec. IV, Figs 3a, 4a. All the parameters $\delta_i$, $i = 1, 2, 3$ and $t_0$ are collected in Table I.

B. Symmetric communication line

The communication line in Fig. 1(b) again employs two chains of $N_1$ sites each as building blocks, but now the one-site receiver $R$ is situated between the two chains, so that the complete system now has $2N_1 + 1$ sites. The Hamiltonian has the same structure (1) as before, with the same $H_1$, physically different $H_{12}$ due to the presence of the additional site $R$, and formally different $H_2$ due to a renumbering of the sites:

$$H_2 = \sum_{i=N_1+4}^{2N_1-2} (I_{ix}I_{(i+1)x} + I_{iy}I_{(i+1)y}) +$$
$$\delta_1 \left( I_{(N_1+2)x}I_{(N_1+3)x} + I_{(N_1+2)y}I_{(N_1+3)y} + I_{(2N_1)x}I_{(2N_1+1)x} + I_{(2N_1)y}I_{(2N_1+1)y} \right) +$$
$$\delta_2 \left( I_{(N_1+3)x}I_{(N_1+4)x} + I_{(N_1+3)y}I_{(N_1+4)y} + I_{(2N_1-1)x}I_{(2N_1)x} + I_{(2N_1-1)y}I_{(2N_1)y} \right),$$

$$H_{12} = \delta_3 \left( I_{N_1x}I_{(N_1+1)x} + I_{N_1y}I_{(N_1+1)y} + I_{(N_1+1)x}I_{(N_1+2)x} + I_{(N_1+1)y}I_{(N_1+2)y} \right).$$

The bulk coupling constant is again $D \equiv 1$ and the parameters $\delta_1$ and $\delta_2$ are the same as before, see Table I while the parameters $\delta_3 = 0.318$ and $t_0 = 29.190$ maximize the probability of the excited state transfer from one of the senders to the receiver (different
from the parameters $\delta_3$ and $t_0$ in the asymmetric communication line, Sec. [II A] which were optimized according to different criteria).

In addition to the two different systems described above we introduce a special symmetric system whose properties will be discussed in Sec. [V] below. In that system the two-node senders $S_1$ and $S_2$ are directly coupled to the receiver $R$ without any intermediate channel. This minimal system then has length $N = 5$. The corresponding nearest-neighbor XY Hamiltonian reads:

$$H = \left( I_1x I_2x + I_1y I_2y + I_4x I_5x + I_4y I_5y \right) + \delta_3 \left( I_2x I_3x + I_2y I_3y + I_3x I_4x + I_3y I_4y \right)$$  \(4\)

For this case, the optimal parameters read: $t_0 = 4.443$, $\delta_3 = 0.707$.

III. GENERAL PROTOCOL OF REMOTE ONE-QUBIT STATE CREATION

We now explain how to use the quantum hardware described in the previous section. Let us consider the asymmetric two-sender communication line with two $N_1$-site channels shown in Fig. IIa. Our protocol is based on the tensor-product initial state

$$\rho_0 = |\Psi_{S_10}\rangle \langle \Psi_{S_10}| \otimes |\Psi_{\text{rest}}\rangle \langle \Psi_{\text{rest}}| \otimes |\Psi_{S_20}\rangle \langle \Psi_{S_20}|,$$

where $|\Psi_{S_10}\rangle$, $|\Psi_{S_20}\rangle$ and $|\Psi_{\text{rest}}\rangle$ are, respectively, the normalized initial states of the first sender $S_1$, of the second sender $S_2$ and of the rest of the communication line (i.e., the two transmission lines $TL_1$, $TL_2$ together with the receiver $R$):

$$|\Psi_{S_10}\rangle = a_{10}|0\rangle + a_{11}|1\rangle + a_{12}|2\rangle,$$

$$|\Psi_{S_20}\rangle = a_{20}|0\rangle + a_{21}|N\rangle + a_{22}|N - 1\rangle,$$

$$|\Psi_{\text{rest}}\rangle = |0\rangle.$$
Here \(|0\rangle\) is the state without excitations and \(|k\rangle\) means the state with the \(k\)th spin excited. The coefficients in the above states can be conveniently parametrized in terms of angles:

\[
\begin{align*}
    a_{10} &= \sin \frac{\alpha_{11}}{2}, \\
    a_{11} &= \cos \frac{\alpha_{11}}{2} \cos \frac{\alpha_{12}}{2} e^{2\pi i \phi_{11}}, \\
    a_{12} &= \cos \frac{\alpha_{11}}{2} \sin \frac{\alpha_{12}}{2} e^{2\pi i \phi_{12}}, \\
    a_{20} &= \sin \frac{\alpha_{21}}{2}, \\
    a_{21} &= \cos \frac{\alpha_{21}}{2} \cos \frac{\alpha_{22}}{2} e^{2\pi i \phi_{21}}, \\
    a_{22} &= \cos \frac{\alpha_{21}}{2} \sin \frac{\alpha_{22}}{2} e^{2\pi i \phi_{22}},
\end{align*}
\]

with

\[
0 \leq \alpha_{ij} \leq 1, \quad 0 \leq \phi_{ij} \leq 1, \quad i, j = 1, 2.
\]

Since the initial state involves at most two excited nodes and the XY Hamiltonian (2) commutes with the \(z\)-projection of the total spin momentum \(I_z\) \(([H, I_z] = 0)\) then the spin dynamics can be described in the subspace spanned by the vectors

\[
|0\rangle, \ |k\rangle, \ k = 1, \ldots, N, \ |ij\rangle, \ j > i, \ i, j = 1, \ldots, N
\]

whose dimensionality is \(N + 1 + \left(\frac{N}{2}\right) = \frac{1}{2}(N^2 + N + 2)\). Again, \(|0\rangle\) is the state without excitations, \(|k\rangle\) means the state with the \(k\)th spin excited, and \(|ij\rangle\) is the state with the \(i\)th and \(j\)th spins excited.

We represent the state of the receiver \(R\) in the following factorized form:

\[
\rho^R = U \Lambda U^+, \quad (12)
\]

where \(\Lambda\) and \(U\) are, respectively, the diagonal matrix of the eigenvalues of \(\rho^R\) and the matrix of its eigenvectors:

\[
\begin{align*}
    \Lambda &= \text{diag}(\lambda, 1 - \lambda), \\
    U &= \begin{pmatrix}
        \cos \frac{\pi \beta_1}{2} & -e^{-2\pi i \beta_2} \sin \frac{\pi \beta_1}{2} \\
        e^{2\pi i \beta_2} \sin \frac{\pi \beta_1}{2} & \cos \frac{\pi \beta_1}{2}
    \end{pmatrix},
\end{align*}
\]

and \(\sigma_i\) are the Pauli matrices:

\[
\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_3 = \text{diag}(1, -1). \quad (15)
\]
Hereafter we study the creatable region in the plane \((\lambda, \beta_1)\) of the receiver’s state space disregarding the value of the parameter \(\beta_2\) for the sake of simplicity.

IV. ASYMMETRIC COMMUNICATION LINE: SHARING THE CREATABLE REGION BETWEEN TWO SENDERs

Every initial state (5) of the system is mapped to a final state (at time \(t_0\)) (12) of the receiver, characterized by parameters \(\lambda\) and \(\beta_1\). As the parameters \(\alpha_{ij}\) and \(\phi_{ij}\) (9) of the initial state vary continuously over some region, the state of the receiver varies over a region in the \((\lambda, \beta_1)\)-plane, which we call the creatable region corresponding to the given region of the \(\alpha_{ij}\) and \(\phi_{ij}\). In this section we study the shape of the creatable region for two types of initial states.

A. One-sided initial state

First, we consider the two initial states, \(IS_{10}\) and \(IS_{01}\), where one of the senders is in the ground state:

\[
IS_{10} : |\Psi_{S0}\rangle = |0\rangle \text{ and } |\Psi_{S10}\rangle \text{ is state (6) with } \phi_{11} = 0 \tag{16}
\]

\[
IS_{01} : |\Psi_{S0}\rangle = |0\rangle \text{ and } |\Psi_{S20}\rangle \text{ is state (7) with } \phi_{2i} = 0, i = 1, 2
\]

Thus, we keep \(\phi_{12}\) in \(IS_{10}\) free and use this parameter to control the shape of the creatable region, while the creatable region is fixed for \(IS_{01}\). The creatable region is then mapped out by varying the parameters \(\alpha_{11}\) and \(\alpha_{12}\) for state \(IS_{10}\) and \(\alpha_{21}\) and \(\alpha_{22}\) for state \(IS_{01}\), respectively. The creatable regions corresponding to both of these initial states are shown in Fig.3 \((N_1 = 20)\) and Fig.4 \((N_1 = 60)\). The structure of these regions can be read off from the lines of constant \(\alpha_{11}\) and \(\alpha_{12}\) (solid lines) for state \(IS_{10}\) and the lines of constant \(\alpha_{21}\) and \(\alpha_{22}\) (dashed lines) for state \(IS_{01}\). In particular, the monotonic solid and dashed lines emanating from the lower right corners of the figures correspond, respectively, to values \(\alpha_{11}\) and \(\alpha_{21}\) varying (from left to right) from 0 (these two lines coincide with the absciss axis) to 1 with the interval \(\Delta\alpha = \frac{1}{8}\). The remaining solid and dashed lines correspond to values, respectively, \(\alpha_{12}\) and \(\alpha_{22}\) varying (from top to bottom) from 0 to 1 with the same interval \(\Delta\alpha\). Figs.3(a) and 4(a) show data for \(\phi_{12} = 0\). The two creatable subregions for \(IS_{10}\) and
FIG. 3: The creatable regions of the receiver’s state space in the plane $(\lambda, \beta_1)$ for the asymmetric communication line of $N = 2N_1 = 40$ nodes. The larger (solid lines) and smaller (dashed lines) creatable regions correspond, respectively, to $IS_{10}$ and $IS_{01}$ given in [16].

(a) $\phi_{12} = 0$, the two creatable regions do not overlap. (b) $\phi_{12} = 0.6$, there is some overlap of the two creatable regions. (c) $\phi_{12} = 0.75$, the smaller creatable region is completely embedded in the larger one.

$IS_{01}$ do not overlap so that the registered state informs us which sender was used for its creation. For $\phi_{12} = 0.6$, Fig. 3(b), 4(b), there is some overlap of the two creatable subregions so that the states in this overlap can be controlled by both senders. Finally, for $\phi_{12} = 0.75$, Fig. 3(c), 4(c), the smaller creatable region is completely embedded in the larger one, so that all states in the smaller subregion can be created by both senders.
FIG. 4: The same as in Fig.3 for the communication line of $N = 2N_1 = 120$ nodes. (a) $\phi_{12} = 0$, the two creatable regions do not overlap. (b) $\phi_{12} = 0.6$, there is some overlap between the two creatable regions. (c) $\phi_{12} = 0.75$, the smaller creatable region is completely embedded in the larger one.

B. Two-sided initial state

Now we consider initial states of the communication line in which neither of the two senders $S_1$ and $S_2$ is in the ground state. In that situation one of the senders can, by fixing its control parameters $\alpha_{ij}, \phi_{ij}$, be used to control the position and form of the creatable region that the other sender can create as its initial state parameters vary over some range.
As examples we consider the following three initial states:

\[ IS_1 : \ |\Psi_{S_10}\rangle \text{ is state (6) with } \phi_{1i} = 0 \text{ (input state);} \]
\[ IS_2 : \ |\Psi_{S_20}\rangle \text{ is state (7) with } \alpha_{12} = \phi_{2i} = 0, \alpha_{11} = \frac{1}{2} \text{ (controlling state),} \]
\[ IS_3 : \ |\Psi_{S_10}\rangle \text{ and } |\Psi_{S_20}\rangle \text{ are states, respectively, (6) and (7) with } \alpha_{1i} = \alpha_{2i}, \]
\[ (\phi_{1i} = \phi_{2i} = 0, \text{ equivalent initial states of } S_1 \text{ and } S_2). \]

In \( IS_1 \) the sender \( S_1 \) is in a normalized superposition of \( \ket{0}, \ket{1}, \text{ and } \ket{2} \) with arbitrary positive coefficients (a convex combination), while the controlling state of \( S_2 \) is fixed at \( \ket{N} \).

In \( IS_2 \), in contrast, \( S_2 \) varies over all convex combinations of \( \ket{0}, \ket{N - 1}, \text{ and } \ket{N} \) while \( S_1 \) is in the fixed state \( \frac{1}{\sqrt{2}} (\ket{0} + \ket{1}) \). Finally, in \( IS_3 \) \( S_1 \) and \( S_2 \) are assumed to be in the same convex combination of \( \ket{0}, \ket{1}, \text{ and } \ket{2} \) and \( \ket{0}, \ket{N}, \text{ and } \ket{N - 1} \), respectively.

The creatable regions corresponding to \( IS_1, IS_2 \) and \( IS_3 \) are shown, respectively, in Fig.5a, Fig.5b and Fig.5c. This figure demonstrates the strong effect of the controlling state on the position and area of the creatable region. On this figure, the solid and dashed lines are the lines of constant, respectively, \( \alpha_{12} \) and \( \alpha_{11} \) which vary from 0 to 1 passing, respectively, (i) from right to left-downward and from left to right (Fig.5b); therewith the line \( \alpha_{11} = 0 \) covers the lines \( \lambda = 0 \) and (partially) \( \beta_1 = 0, \beta_1 = 1 \), and (ii) from top to bottom and from left to right (Fig.5b,c)

V. SYMMETRIC COMMUNICATION LINE AND SIMPLE QUANTUM GATES

The remote creation of states may be considered a task in quantum communication, as we did in the previous section. We now change perspectives slightly and look at the symmetric communication lines of Sec. II B from the point of view of quantum computing, or quantum control. In that perspective, we assume the senders \( S_1 \) and \( S_2 \) to initially contain some kind of input, while the state of \( R \) at a certain time \( t_0 \) is the output of a (generalized) quantum gate. In what follows, \( S_1 \) is assumed to contain the input data while \( S_2 \) is a generalized control. (Note that in standard quantum algorithms the control qubit is usually assumed to be in either of the two basis states \( \ket{0} \) or \( \ket{1} \), while here \( S_2 \) has a vastly wider range of possibilities.)
FIG. 5: The creatable regions of the receiver’s state space in the plane \((\lambda, \beta_1)\) for the chain of \(N = 2N_1 = 40\) nodes: (a) the initial state \(IS_1\); (b) the initial state \(IS_2\); (c) the initial state \(IS_3\).

The systems we consider are the communication line in Fig.1b with \(N_1 = 20\) and the minimal system of Eq. (11) which we will henceforth denote by \(N_1 = 2\) for brevity, and three types of initial states:

\[
IS_{10} : \quad |\Psi_{S_10}\rangle \text{ is state (6), (10) and } |\Psi_{S_20}\rangle = |0\rangle
\]

\[
IS_{01} : \quad |\Psi_{S_20}\rangle \text{ is state (7), (10) and } |\Psi_{S_10}\rangle = |0\rangle
\]

\[
IS_{11} : \quad |\Psi_{S_10}\rangle \text{ is state (6), (10) and } |\Psi_{S_20}\rangle \text{ is state (7), (10)}.
\]

In \(IS_{10}\) and \(IS_{01}\) we explore the effect of exchanging input and control, with one being a superposition state containing at most one excited qubit and the other one being the ground state. \(IS_{11}\) describes a situation where input and control states are identical superpositions.
with one or two excitations. To characterize the parameters $\beta_1$ and $\lambda$ of the output state (12) as they depend on the nature of the input state we use the following notation:

$$\beta_{ij} = \beta_1|_{IS_{ij}}, \quad \lambda_{ij} = \lambda|_{IS_{ij}}.$$  (19)

Throughout this section we restrict ourselves to positive real values of the coefficients (9), that is, $\phi_{ij} = 0$ for all $i, j$.

A. First gate: switching unitary transformations of receiver

We first assume that the nontrivial initial sender state is the same in all three initial states (18). That means, we have

$$\alpha_{2i} = \alpha_{1i}$$  (20)

between $|\Psi_{S_{10}}\rangle$ and $|\Psi_{S_{20}}\rangle$ within $IS_{11}$ and between $|\Psi_{S_{10}}\rangle$ in $IS_{10}$ and $|\Psi_{S_{20}}\rangle$ in $IS_{01}$. The output of the gate is the mixed state $\rho^R$ (12) of the receiver, characterized by parameters $\lambda$, $\beta_1$, and $\beta_2$. We concentrate on the parameters $\lambda$ and $\beta_1$, disregarding the phase angle $\beta_2$. The action of the gate that we can achieve is described by the mapping

$$IS_{10} \Rightarrow \rho^R(\lambda, \beta_{10}(\lambda))$$
$$IS_{01} \Rightarrow \rho^R(\lambda, \beta_{10}(\lambda))$$
$$IS_{11} \Rightarrow \rho^R(\lambda, \beta_{11}(\lambda)).$$  (21)

Thus, the eigenvalues of $\rho^R$ are the same in all three cases ($\lambda_{10} = \lambda_{01} = \lambda_{11} = \lambda$), while the parameter $\beta_1$ takes either of two values $\beta_{10}$ or $\beta_{11}$, depending on the initial state. These values are uniquely related to $\lambda$, as shown in Fig 6. Thus, the considered gate switches between two possible sets of eigenvectors of $\rho^R$ while keeping the same eigenvalues $\lambda$ and $1 - \lambda$.

In order for the gate (21) to work properly, the input parameters $\alpha_{11}$ and $\alpha_{12}$ must have specific values related to $\lambda$. Fig. 6 shows these values, along with the output parameters $\beta_{10}$, $\beta_{11}$ as functions of $\lambda$ for the chains of $N = 5$ and $N = 41$ nodes.

These figures show that there is a restricted interval of the parameter $\lambda$ where the gate (21) is realizable. In addition, this interval is uniquely related to the appropriate intervals
FIG. 6: Parameters $\beta_{10}$ (solid lower line), $\beta_{11}$ (solid upper line), $\alpha_{11}$ (dashed increasing line) and $\alpha_{12}$ (dashed decreasing line), for (a) the communication line of $N = 2N_1 + 1 = 5$ nodes, and (b) the communication line of $N = 2N_1 + 1 = 41$ nodes.

of all other parameters:

$$N = 5 \quad N = 41$$

$$0.666667 < \lambda < 0.933064 \quad 0.66667 < \lambda < 0.918869$$

$$0 < \beta_{10} < 0.304033 \quad 0 < \beta_{10} < 0.305756$$

$$1 > \beta_{11} > 0.695726 \quad 1 > \beta_{11} > 0.708911$$

$$0 < \alpha_{11} < 0.500114 \quad 0 < \alpha_{11} < 0.470208$$

$$0.391827 > \alpha_{12} > 0 \quad 0.368291 > \alpha_{12} > 0.$$ (22)

Comparison of the intervals for the communication lines of $N = 5$ and $N = 41$ nodes shows that there is a minor shrinking of all intervals with an increase in $N$.

B. Second gate: switching of eigenvector-parameter $\beta_1$ and eigenvalue $\lambda$

The second gate we study involves only two of the initial states (18) and its action on the output state is somewhat unusual:

$$IS_{10} \Rightarrow \rho^R(\lambda_{10}, \beta_{10})$$

$$IS_{11} \Rightarrow \rho^R(1 - \beta_{10}, \lambda_{10}).$$ (23)
FIG. 7: The domain of the function (24) in the plane \((\lambda_{10}, \beta_{10})\), for (a) the communication line of \(N = 2N_1 + 1 = 5\) nodes, and (b) the communication line of \(N = 2N_1 + 1 = 41\) nodes.

In other words,

\[
\begin{align*}
\beta_{11} &= \lambda_{10}, \\
\lambda_{11} &= 1 - \beta_{10}.
\end{align*}
\]

Thus, the eigenvalue of the receiver’s state corresponding to the initial state \(IS_{11}\) is linearly expressed in terms of the eigenvector parameter \(\beta_{10}\) of the receiver’s state corresponding to the initial state \(IS_{10}\), while the eigenvector-parameter \(\beta_{11}\) is proportional to the eigenvalue \(\lambda_{10}\). Thus, the considered gate switches parameters between eigenvalues and eigenvectors. The receiver’s state corresponding to the initial state \(IS_{01}\) is not of interest here, unlike with the first gate in Sec.IVA.

It is to be expected that the transformation (24) cannot be achieved for arbitrary values of the input parameters. The domain of the function (24) in the plane \((\lambda_{10}, \beta_{10})\) is shown in Fig.7 for the communication lines of \(N = 5\) and \(N = 41\) nodes. These domains have been constructed using random solutions of system (24). For this purpose, we consider the mapping (24) as a system of equations for the control parameters \(\alpha_{ij}\) \((i, j = 1, 2; \text{note that we no longer assume the relation (20) to hold})\) and find 1567 (for \(N = 5\)) and 1653 (for \(N = 41\)) random solutions. Each of the domains displayed in Fig.7 is creatable by the parameters \(\alpha_{ij}\) from corresponding domains in the planes \((\alpha_{11}, \alpha_{12})\) and \((\alpha_{21}, \alpha_{22})\) and we
observe that there are almost one-to-one (except for the near-boundary subregion) mappings

\[
\{\alpha_{21}, \alpha_{22}\} \Rightarrow \{\lambda_{10}, \beta_{10}\} \tag{25}
\]

\[
\{\alpha_{21}, \alpha_{22}\} \Rightarrow \{\alpha_{11}, \alpha_{12}\}.
\]

These mappings are represented in Fig.8 for the case \(N = 41\), where the parameters \(\lambda_{10}\), \(\beta_{10}\), \(\alpha_{11}\) and \(\alpha_{12}\) are shown as surfaces over the plane of the parameters \(\alpha_{21}\) and \(\alpha_{22}\). The discontinuity of the domain in Fig.7a at \(\lambda_{10} \sim 0.92\) is related to the form of the creatable subregion of the receiver’s state space.

VI. CONCLUSION

We have considered the problem of remote state creation using two senders connected to a single receiver by boundary-optimized channels. Such a communication line can be used, for instance, to share the creatable region of the receiver’s state space between two senders so that the subregions creatable by each of the sender (provided that the other one is in the ground state) do not overlap. Therefore, if the state is registered at the receiver, we know from which sender the creation process originated. Varying the phase of the initial state we can change the position of the creatable subregions.

Another application of the two-sender communication line is a quantum gate operating on the mixed state of the receiver. In that case, the initial state of one of the senders is regarded as an input signal, the initial state of the other sender is a controlling signal. The state of the receiver at the proper time instant is the output signal. There is a large family of functions which can be realized by such a gate of which we here consider only two functions.

In the first function the controlling state changes the parameter of the eigenvector matrix of the receiver state keeping the eigenvalues unchanged. In the second function the controlling state switches parameters between the eigenvectors and eigenvalues of the output state. Each of these functions can be realized if the control parameters \(\alpha_{ij}\) of the initial state and the creatable parameters of the receiver’s state belong to the proper domain of their values; we characterize all these domains. Further study of such gates would be of interest.

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FIG. 8: The one-to-one (except for the near-boundary region) mapping \[\lambda_{10}\] for the communication line of \(N = 41\) nodes. (a) The parameter \(\lambda_{10}\), (b) the parameter \(\beta_{10}\), (c) the parameter \(\alpha_{11}\), and (d) the parameter \(\alpha_{12}\) over the plane \((\alpha_{21}, \alpha_{22})\).

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