Reflections on the Spatiotemporal Aspects of the Quantum World*

Ulrich Mohrhoff
Sri Aurobindo International Centre of Education
Pondicherry-605002 India
ujm@satyam.net.in

The proper resolution of the so-called measurement problem requires a “top-down” conception of the quantum world that is opposed to the usual “bottom-up” conception, which builds on an intrinsically and maximally differentiated manifold. The key to that problem is that the fuzziness of a variable can manifest itself only to the extent that less fuzzy variables exist. Inasmuch as there is nothing less fuzzy than the metric, this argues against a quantum-gravity phenomenology and suggests that a quantum theory of gravity is something of a contradiction in terms—a theory that would make it possible to investigate the physics on scales that do not exist, or to study the physical consequences of a fuzziness that has no physical consequences, other than providing a natural cutoff for the quantum field theories of particle physics.

1 INTRODUCTION

At present the almost single constraint on speculations about the interface of the gravitational and quantum realms is consistency with general relativity (GR) and the standard model (SM) as effective theories for their respective realms. The meaning of quantum mechanics (QM) is not generally considered a constraint. The prevalent attitude seems to be that any interpretation will do as long as it permits asking the pertinent questions, or that the final resolution of interpretational issues must wait for a unified quantum theory of all forces including gravity. I shall argue instead that the proper resolution of the so-called measurement problem requires a “top-down” conception of the quantum world that is opposed to the usual “bottom-up” conception, which builds on an intrinsically and maximally differentiated manifold. The key to that problem is that the fuzziness of

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a variable can manifest itself only to the extent that less fuzzy variables exist. Inasmuch as there is nothing less fuzzy than the metric, this argues against a quantum-gravity phenomenology and suggests that a quantum theory of gravity is something of a contradiction in terms—a theory that would make it possible to investigate the physics on scales that do not exist, or to study the physical consequences of a fuzziness that has no physical consequences, other than providing a natural cutoff for the quantum field theories of particle physics.

Section 2 ties together the probabilistic nature of quantum “states,” the positional fuzziness that “fluffs out” matter, and the extrinsic nature of quantum variables. Section 3 requires nothing more sophisticated than a two-slit interference experiment to show that space cannot be a self-existent and intrinsically partitioned expanse, and that the intrinsically differentiated spatiotemporal background of classical field theory cannot be part of a description of the spatiotemporal aspects of the quantum world. Section 4 disposes of a vicious regress that appears to arise from the extrinsic nature of quantum variables, and establishes the existence of a macroworld—a system of causally connected properties that are effectively detached from the facts by which they are indicated. It further demonstrates that the quantum world is only finitely differentiated both spacewise and timewise, and that as a consequence it ought to be regarded as constructed from the top down, by the spatiotemporal differentiation of an intrinsically undifferentiated reality, rather than from the bottom up, on an intrinsically and maximally differentiated manifold.

Section 5 tries to bring our intuitions in line with the spatiotemporal aspects of the quantum world by stressing that the so-called “point particles” are formless objects, and that space exists between them—it is spanned by their (more or less fuzzy) relations. Section 6 deals with the physical roots of the metric properties of the quantum world and the meaning of “second quantization.” Section 7 discusses how the quantum world relates to its substance. Section 8 argues against the need for a quantum theory of gravity, and Sec. 9 reflects on the possible meaning of “the wave function of the (early) universe.”

2 INDEFINITENESS

Let me begin by denouncing a didactically disastrous approach to QM. This starts with the observation that in classical physics the state of a system is represented by a point \( \mathcal{P} \) in some phase space, and that the system’s possessed properties are represented by the subsets containing \( \mathcal{P} \). Next comes the question, what are the quantum-mechanical counterparts to \( \mathcal{P} \) and the subsets containing \( \mathcal{P} \) qua representations of an actual state of affairs and possessed properties, respectively? Once we accept this as a valid question, we are on a wild-goose chase.

If at all we need to proceed from classical physics, the right way to do so is to point
out that every classical system is associated with a probability measure, that this is represented by a point $P$ in some phase space, that observable properties are represented by subsets, and that the probability of observing a property is 1 if the corresponding subset contains $P$; otherwise it is 0. Next comes the question, what are the quantum-mechanical counterparts to $P$ and the subsets containing $P$ qua representations of a probability measure and observable properties, respectively? Once we have the answer we are ready for the next question: Is it possible to reinterpret the quantum-mechanical counterpart to $P$ as representing an actual state of affairs connoting a set of possessed properties? Because the classical probability measure assigns trivial probabilities (either 0 or 1), it is possible to think of it as an actual state of affairs. Because quantal probability measures generally assign nontrivial probabilities, it is not possible to similarly reinterpret the quantum counterpart to $P$.

Whence the nontrivial probabilities? One of the most obvious features of the world is the stability of matter, by which I mean the existence of spatially extended material objects that neither explode nor implode the moment they are formed. We know that the world owes this feature to the indefiniteness of its relative positions. Together with the exclusion principle it “fluffs out” matter [1]. The proper way of dealing with indefinite values is to make counterfactual probability assignments [2, 3]. If we say that a variable has an “indefinite value,” what we mean is that it does not have a value (inasmuch as no value is indicated) but that it would have a value if one were indicated, and that positive probabilities are associated with at least two possible values. While the reference to counterfactuality cannot be eliminated, it may be shifted from values that are only counterfactually indicated to values that are only counterfactually indefinite: If a certain measurement is performed on an ensemble of identically “prepared” systems and the results exhibit a positive dispersion, the value of the measured variable would be indefinite for each system if the measurement were not performed.

The occurrence of irreducible probabilities in a fundamental physical theory thus is a direct consequence of the positional indefiniteness to which matter owes its stability. So is the extrinsic nature of values. If a variable sometimes does and sometimes does not have a value, a criterion is called for, and this is the existence of a value-indicating fact. Such a variable possesses a value only if, when, and to the extent that, a value is indicated (by an actual event or state of affairs). A fundamental physical theory that is essentially a probability algorithm presupposes actual events (or states of affairs) not once but twice: as events to which probabilities are assigned, and as events on the basis of which probabilities are assigned.

What is a fact? Dictionaries define facts in epistemological terms. The Concise Oxford Dictionary (8th edition, 1990), for instance, defines “fact” as a thing that is known to have occurred, to exist, or to be true; a datum of experience; an item of verified information; a piece of evidence. Are the editors of dictionaries intent on convincing us that the
existence of facts presupposes knowledge or experience? To find out, let’s look up these
terms: “Experience” is the “actual observation of or practical acquaintance with facts or
events”. “Knowledge” is “awareness or familiarity gained by experience (of a person, fact,
or thing)” (italics supplied). These definitions are obviously circular, which might suggest
a mutual dependence: All knowledge is knowledge of facts, and all facts are known facts.
This may well be so, but it is beside the point.

Physics is concerned with laws, and hence with nomologically possible worlds (that is,
with worlds consistent with the laws). It does not tell us which of these possible worlds
corresponds to the actual world. In classical physics the actual world can be identified
by initial or final conditions (or both), which we must gather from observational data. In
quantum physics the identification of the actual world requires the identification of (i) the
actual property-indicating facts (a.k.a. “measurements”) and (ii) the property indicated
by each such fact (a.k.a. the “measurement result”). These too we need to gather from
observations. Does this imply that quantum physics presupposes conscious observers? If
the answer is negative for classical physics, it is equally negative for quantum physics.

The point is that a physical theory cannot identify the actual world, cannot charac-
terize it, cannot distinguish it from the merely possible worlds, and thus cannot define
actuality, reality, existence, or factuality. The existence of an actual world is the ultimate
given. But what matters is that it is given, not for or to whom it is given, nor whether
there is anyone to whom it is given. No theory can explain why there is anything at all,
rather than nothing. This is why QM cannot explain the property-indicating facts, which
it takes for granted.

That QM presupposes the existence of such facts is readily seen: The probability that
a variable $Q$ has the value $v$ is the product of two probabilities—the probability that any
one of the possible values of $Q$ is indicated, and the probability that the indicated value
is $v$, given that a value is indicated. QM is exclusively concerned with probabilities of the
latter type. It does not assign a probability to the occurrence of a value-indicating event,
let alone specify sufficient conditions for such an occurrence. If QM is a fundamental and
universal theoretical framework, this means that the value-indicating events presupposed
by QM are uncaused. Any attempt to “explain why events occur” is therefore a waste
of time.

What needs to be shown instead is that QM is consistent: There are possible events
to which actuality can be attributed consistently. In other words, there are properties
that can be consistently regarded as intrinsic, and hence as capable of indicating extrinsic
properties. The question is not, “how it is that probabilities become facts” but, which
properties of which objects should be considered factual per se. This question is addressed
in Sec. 4.
3 POSITIONS ARE PROPERTIES, NOT SUBSTANCES

Making sense of QM is not so much a question about the ontological status of density operators—they are just sophisticated probability measures—as a question about the ontological status of the space and time coordinates that appear as arguments of density operators in the position representation. Classical field theory has instilled in us the disastrous habit of thinking of anything that depends on positions and times as something that exists at those positions and times, which implies the independent existence of those positions and times. Yet it should be obvious that the probability for something to happen in a given region of space at a given time is not something that exists in that region or at that time, and it stands to reason that the same is true of the algorithm that permits us to calculate this probability. Can we nevertheless postulate the independent existence of the intrinsically differentiated spatiotemporal background of classical field theory?

Showing that the answer is negative takes nothing more elaborate than a two-slit experiment with electrons [6]. No electron is detected in the absence of the electron source in front of the slit plate, and no electron is detected behind the slit plate whenever the two slits are closed. This warrants the inference that each detected electron went through $L \& R$, the regions defined by the slits considered as one region. At the same time the existence of interference fringes demonstrates that each electron went through $L \& R$ without going through a particular slit (and, of course, without having been split into parts that went through different slits). But if space were something that existed by itself, independently of its “material content,” and if it were made up of distinct, separate regions, no object could have a fuzzy position—a position that is counterfactually and probabilistically distributed over disjoint regions. The truth of the proposition “The electron went through $L \& R$”—symbolically, $e \rightarrow L \& R$—would imply the truth of either $e \rightarrow L$ or $e \rightarrow R$.

Interference fringes have been observed using C$_{60}$ molecules and a grating with 50-nm-wide slits and a 100-nm period [7]. Do we need any further proof that $L$ and $R$ cannot be distinct, self-existent “parts of space,” and that, consequently, space cannot be a self-existent and intrinsically partitioned expanse? The proposition $e \rightarrow L \& R$ can be true while both $e \rightarrow L$ and $e \rightarrow R$ lack truth values. In other words, a definite relationship—“inside” or “outside”—can exist between the electron’s position (qua observable) and the region $L \& R$ while no such relationship exists between the electron’s position and either $L$ or $R$. In yet other words, $L \& R$ can be real for the electron while neither $L$ nor $R$ (nor the distinction we make between $L$ and $R$) is real for it.

Although we readily agree that red, or a smile, cannot exist without a red object or a smiling face, we just as readily believe that positions can exist without being properties of material objects. We are prepared to think of material objects as substances, and we are not prepared to think of their properties as substances—except for their posi-
tions. (A substance is anything that can exist without being the property of something else.) There are reasons for these disparate attitudes, but they are psychological and neurobiological. They concern the co-production, by the mind and the brain, of the phenomenal world—the world as we perceive it. They do not apply to the quantum world, but they certainly make it hard to make sense of it. As long as we treat \( L \) and \( R \) as substances that make up \( L \& R \), we cannot comprehend the behavior of electrons in two-slit experiments. This behavior makes sense only if we treat each region of space as a property—the property of being in that region. Then a region exists iff it is possessed by at least one object, and the distinction between a region \( V \) and its complement is real for an object \( O \) iff the proposition “\( O \) is in \( V \)” has a truth value (that is, if a truth value is indicated). If neither \( L \) nor \( R \) exists for the electron, nothing can “compel” the electron to go through either \( L \) or \( R \).

4 THE MACROWORLD

There are objects whose indicated positions are so correlated that each of them is consistent with every prediction that is based on previous indicated positions and a classical law of motion except when it serves to indicate an unpredictable value. Such objects deserve to be labeled “macroscopic.” (Note that this definition does not require that the probability of finding a macroscopic object where classically it could not be, is strictly 0. What it requires is that there be no position-indicating fact that is inconsistent with predictions based on a classical law of motion and earlier position-indicating facts.) Since between those times at which macroscopic objects serve as pointers their indicated positions are predictably correlated, these positions can be considered intrinsic, or factual per se. And since before and after each value-indicating transition the position of a pointer can be considered intrinsic, factuality can also be attributed to the transition itself. These claims will be substantiated in the present section.

The departure of an object \( O \) from a classical trajectory can be indicated only if there are detectors whose position probability distributions are narrower than \( O \)’s. Such detectors do not exist for all objects. Some objects have the sharpest positions in existence. For these objects the probability of a position-indicating event that is inconsistent with a classical trajectory is necessarily very low. It is therefore certain that among these objects there will be macroscopic ones.

Since no object has an exact position, it might be argued that even for a macroscopic object \( M \) there always exists a small enough region \( V \) such that the proposition \( M \rightarrow V \) lacks a truth value. But this is an error. Among the objects that have the sharpest positions in existence there are macroscopic objects, and they are macroscopic (in the sense defined above: their nonindicating positions are predictably correlated) because there isn’t any object that has a (significantly) sharper position. Hence there isn’t any
object for which \( V \) is real. But a region exists only if it is real for at least one object. It follows that there exists no region \( V \) such that the proposition \( M \rightarrow V \) lacks a truth value. Such a region may exist in our imagination, but it does not exist in the real world.

Now recall why positions are extrinsic: The proposition \( O \rightarrow V \) may or may not have a truth value. One therefore needs a criterion for the existence of a truth value—a truth value must be indicated. But one doesn’t need a criterion for the existence of a truth value if for every existing region \( V \) the proposition \( M \rightarrow V \) has a truth value. Since macroscopic objects satisfy this condition, their positions can be consistently considered intrinsic. Since every existing region has a trivial probability of containing \( M \), we can make the transition from a probability measure to an actual state of affairs, exactly as in classical physics (Sec. 4). We can think of the positions of macroscopic objects (macroscopic positions, for short) as forming a system of causally connected properties that are effectively detached from the facts by which they are indicated. We can think of this system as a self-contained causal nexus interspersed with transitions (of value-indicating positions) that are causally linked to the future but not to the past. And it is to this system—and this system alone—that an independent reality can be attributed. Everything else exists because it is indicated by the goings-on within this system, for without it no indicatable properties exist. (The function of a detector is not only that of indicating a position but also that of making real an indicatable position—the detector’s sensitive region—by possessing it as an intrinsic property—a predictably evolving shape.)

Macroscopic positions are so abundantly and so sharply indicated that they are only counterfactually fuzzy. Their fuzziness never evinces itself, through uncaused transitions or in any other manner. It exists solely in relation to an imaginary spatial background that is more differentiated than the real world. The space over which the position of a macroscopic object is “smeared out” is never probed. This space is undifferentiated; it contains no smaller regions. We may imagine smaller regions, but they have no counterparts in the real world. The distinctions we make between them are distinctions that nature does not make.

It follows that the quantum world is only finitely differentiated spacewise, and that we ought to regard it as constructed from the top down, by a finite process of differentiation, rather than from the bottom up, on an intrinsically and maximally differentiated manifold. And much the same applies to the world’s temporal aspect. Time, as everyone knows, is not an independent observable. Time has to be read off of deterministically evolving positions—the positions of macroscopic clocks. If these bear a residual fuzziness, so do all indicated times. The upshot: The quantum world is maximally differentiated neither spacewise nor timewise.
5 SPACE AND THE QUANTUM WORLD

Quarks and leptons are often described as “pointlike.” It is important to be clear about what this means. It expresses the fact that such an object lacks internal structure. Nothing in the formalism of QM refers to the shape of an object that lacks internal structure, and the empirical data cannot possibly do so. All that experiments can reveal in this regard is the absence of evidence of internal structure. The idea that a so-called “point particle” is an object that not only lacks internal relations but also has the shape of a point thus is unwarranted both theoretically and experimentally. It is, besides, seriously misleading, inasmuch as the image of a pointlike object suggests the existence of an infinitesimal neighborhood in an intrinsically and maximally differentiated manifold. To bring our intuitions in line with the spatiotemporal aspects of the quantum world, we ought to conceive of all so-called “point particles” as formless objects. What lacks internal relations also lacks a shape.

It follows that the shapes of material objects resolve themselves into sets of (more or less fuzzy) spatial relations between formless objects, and that space itself is the totality of such relations—relative positions and relative orientations. It further follows that the corresponding relata do not exist in space. Space contains, in the proper, set-theoretic sense of “containment,” the forms of all things that have forms—for forms are sets of spatial relations—but it does not contain objects over and above their forms; a fortiori it does not contain the formless constituents of matter. Instead, space exists between them; it is spanned by their relations.

Apart from being of philosophical interest, these conclusions may be important for a better understanding of the quantum/gravity interface, inasmuch as they provide us with a way of thinking about the spatial and temporal aspects of the quantum world that is consistent with their finite differentiation. The quantum world with its fuzzy spatial relations does not “fit” into the self-existent and maximally differentiated expanse of classical space; the possibility of thinking of the relata as points and embedding them in a single manifold exists only for definite (“sharp”) spatial relations. A clear distinction should therefore be made between the existing (more or less fuzzy) spatial relations that constitute space, and the purely imaginary space that comes with each localizable object $O$ and contains the unpossessed exact positions relative to $O$. These imaginary spaces are delocalized relative to each other: The unpossessed exact positions relative to $O$ are fuzzy relative to any object other than $O$.

6 THE PHYSICAL ROOTS OF THE METRIC

The formal expression of indefiniteness (fuzziness) through nontrivial probability assignments determines the kinematical aspects of QM up to the number field $\mathbb{F}$. Together
with the conservation of probability for freely propagating particles, the kinematics determines the dynamical framework of QM including the number field. Here is (in outline) how the latter may be shown:

Consider a series of measurements. We assign probabilities to the possible results of the final measurement in conformity with the probability algorithm that constitutes the kinematics. If the intermediate measurements are not performed but the histories that lead to a possible result of the final measurement are defined in terms of the possible results of the intermediate measurements, the probability of any possible final result will be given by the (absolute) square of a sum over all histories that lead to this result. Each history contributes an amplitude, which may be real or complex.

Next consider the limit of a series of unperformed position measurements on an isolated scalar particle, in which the histories become continuous trajectories. Suppose $s$ parametrizes such a trajectory, and $ds_1$ and $ds_2$ label adjoining infinitesimal segments. The probability algorithm requires us to multiply the amplitudes associated with successive segments of a history, so that

$$A(ds_1 + ds_2) = A(ds_1)A(ds_2).$$

Hence the amplitude for propagation along an infinitesimal path segment can be written as $A(ds) = \exp(z\,ds)$, and the amplitude for propagation along a path $C$ can be written as $A(s[C]) = \exp(zs[C])$. But if $z$ had a real part, the probability of finding the particle anywhere would not be conserved; it would either increase or decrease exponentially with time. Thus $A(s[C])$ must be a phase factor $\exp(ibs[C])$.

It should perhaps be stated that no contradiction exists between (i) concluding that the intrinsically differentiated spatiotemporal background of classical field theory does not exist in the quantum world and (ii) introducing parametrized continuous trajectories. For one, each trajectory is defined counterfactually as a sequence of results of unperformed (and even unperformable) position measurements. For another, the particle trajectories “exist” in the purely imaginary space of unpossessed exact positions mentioned at the end of the previous section. (In this case the origin of that “space” is any object relative to which the particle got precisely localized by the first measurement.) By summing over all continuous paths leading from $(x_1, t_1)$ to $(x_2, t_2)$, we can calculate the propagator $K(x_2, t_2; x_1, t_1)$, which determines the probability of detecting at $(x_2, t_2)$ a particle having last been “seen” at $(x_1, t_1)$ \[11, 12\]. The probability for something to happen at $(x_2, t_2)$, recall, is not something that exists at $(x_2, t_2)$, nor is the spacetime location $(x_2, t_2)$ something that exists by itself. It exists (for the particle) if something indicates that at the exact time $t_2$ the particle has the exact position $x_2$. The very fact that in calculating $K(x_2, t_2; x_1, t_1)$ we sum over all particle trajectories connecting the two locations, implies that the distinctions we make between the trajectories are distinctions that nature does not make. If the particle is detected at $(x_1, t_1)$ and $(x_2, t_2)$, and if there isn’t any matter of
fact about its intermediate whereabouts, the particle travels along “all of them” without
traveling along any of them, in the same sense in which the electron goes through $L\&R$
without going through either $L$ or $R$ [13].

We are now ready to address the physical roots of the metric properties of the quantum
world. Consider a scalar particle and a particular path. As the particle travels along this
path (counterfactually), $s$ increases, and $\exp(ibs)$ rotates in the complex plane. Let us say
that every time this phase factor completes a cycle, the particle “ticks.” If the particle is
free, it singles out a class of uniform time parameters—those for which the number of ticks
per second is constant. Different particles may tick at different rates, which are related
to the standard rate of one tick per second by the species-specific constant factor $b$.

So much for the physical roots of mass and proper time. Our next task is to determine
the physical origin of the spatial part of the metric. If space is a set of spatial relations
then there are no absolute positions, nor is there anything like absolute rest. Hence all
inertial coordinate systems are created equal. It can be shown [14] that, as a consequence,
the proper-time interval $ds$ and inertial coordinates are related via

$$ds^2 = dt^2 + K(dx^2 + dy^2 + dz^2),$$

where $K$ is a universal constant, which may be positive, zero, or negative. Here are some
of the reasons why $K > 0$ can be excluded: (i) Ubiquitous causal loops; reversing an
object’s motion in time is as easy as changing its direction of motion in space. (ii) The
nonexistence of an invariant speed—a speed that is independent of the inertial frame in
which it is measured—rules out massless particles, long-range forces, and the possibility
of resting the spatial part of the metric on the cyclic behavior of particles (the rates at
which they tick).

If $K$ is not positive, causal loops are ruled out by the existence of an invariant speed.
For $K = 0$ this is infinite, and for negative $K$ it is $c = \sqrt{1/|K|}$. The problem with the
nonrelativistic case $K = 0$ is that the rates at which free particles tick cannot fix the
spatial part of the metric. They just define a universal inertial time scale via $ds = dt$.
Nor are light signals available for converting time units into space units. Nor do we get
interference from free particles since $ds = dt$ implies that $\exp(ibs[C])$ is the same for all
paths with identical endpoints—and without interference QM is inconsistent (with the
existence of a macroworld, which it presupposes). Yet all there is to fix the spatial part of
the metric—for every inertial frame—is the rates at which free or freely falling particles
tick. Hence these ought to be invariant, and this requires negative $K$ or a finite invariant
speed.

The rates at which particles tick and the correlations between events in null separation
not only underlie the metric properties of the world but also make it possible to influence
the behavior of particles by influencing their propagators. The only way of influencing the
probability of finding at one space-time location a scalar particle last “seen” at another
location, is to modify the rate at which it ticks as it travels along each path connecting the two locations. The number of ticks associated with a path defines a species-specific Finsler geometry \(dS(dt, dr, t, r)\) \[15, 16\]. By invoking again the stability of matter (Sec. 2), it can be shown that there are just two ways of influencing the Finsler geometry that goes with a scalar particle.

The stability of matter rests on the exclusion principle \[1\]. For this to hold, the ultimate constituents of matter must be indistinguishable members of one or several species of fermions. The necessary indistinguishability requires that all free particles belonging to the same species of fermions tick at the same rate. This guarantees the possibility of a global system of spacetime units \[17\]: While there may be no global inertial frame, there will be local ones, and they will mesh with each other as described by a pseudo-Riemannian spacetime geometry. Accordingly there is a species-specific way of influencing the Finsler geometry associated with a scalar particle that bends geodesics relative to local inertial frames, and there is a universal way that bends the geodesics of the pseudo-Riemannian spacetime geometry. In natural units:

\[
dS = m\sqrt{g_{\mu\nu}dx^\mu dx^\nu} + qA_\nu dx^\nu.
\]

Here the one-form \(A\) and the tensor \(g\) [of type \((0,2)\)] represent the possible effects on the motion of scalar particles—any effects, whatever the causes may be \[15, 19\]. If the sources of these fields have no definite positions, the fields themselves cannot have definite values. We take this into account by summing over histories of \(A\) and \(g\), and for consistency with the existence of a macroworld we make sure that a unique history is obtained in the classical limit. Obvious and well-known constraints then uniquely determine the terms that we need to add to \(dS\) in order to obtain a definite field history in this limit (except for a possible cosmological term).

A note on the quantization of particle fields (“second quantization”): We know from Huygens’ principle that a sum over space-time trajectories can be replaced by a wave equation. A relativistic wave equation has “negative energy” solutions corresponding to particles for which proper time decreases as inertial time increases \[20\], and it conserves charge rather than probability. Particle numbers are therefore variables, and variables that are not sharply and continuously monitored by the macroworld (a.k.a. the environment) are fuzzy. To accommodate fuzzy particle numbers we sum over the histories of a field the Lagrangian of which yields the wave equation in the classical limit. This turns the field modes into harmonic oscillators the quanta of which represent individual particles with definite energies and momenta. Expanding the interaction part of \(\exp(iS)\) yields a sum over histories in which free particles are created and/or annihilated, and by using the appropriate wave equation for spin-1/2 particles we arrive at the Feynman rules for QED.

I mention this to emphasize that quantum field theory (QFT) is a method of calcu-
lating (multi-)particle propagators. There is no discernible reason for endowing quantum fields with a special ontological status. Since this is nevertheless frequently done, I shall devote the following section to questions of ontology.

7 ONTOLOGY

Wilczek creates the impression that what Misner, Thorne, and Wheeler call “the miraculous identity of particles of the same type” [21] is explained by the fact that quantum fields, rather than individual particles, are the primary reality: “We understand [the identity of two electrons] as a consequence of the fact that both are excitations of the same underlying ur-stuff, the electron field. The electron field is thus the primary reality” [22]. Given that the self-existent and intrinsically differentiated spatiotemporal background of classical field theory is not a feature that can be consistently attributed to the quantum world, the assignment of ontological primacy to fields is a nonstarter.

For centuries philosophers have argued over the existence of intrinsically distinct substances. QM has settled the question for good: There are no intrinsically distinct substances. The concept of substance betokens existence; it never betokens individuality. Individuality is strictly a matter of properties.

If you think that QM is about regularities in sensory experience or about experimental “interventions into the course of Nature” [23], you don’t need the concept of “substance.” You need it if you want to think of the quantum world as a free-standing reality, inasmuch as it is the concept of “substance” that betokens independent existence. And you need to know how the quantum world relates to its substance. Since it would be absurd to substantialize a probability algorithm, substantiality can’t be attached to a state vector or a wave function. Nor can it be attached to the points of a space-time manifold. Nor can the substance of the quantum world be decomposed into a multiplicity of intrinsically distinct substances, as we just saw.

If the property of being here and the property of being there are simultaneously possessed, how many substances does that make? The correct answer is one, for the substance that betokens the reality of the property of being here also betokens the reality of the property of being there. QM does not permit us to interpose a multiplicity of distinct substances between the substance that betokens existence and the multiplicity of possessed positions. If particles are distinct, they are so by virtue of distinguishing properties. If there are no distinguishing characteristics, there is multiplicity without distinctness. The constituents of a Bose-Einstein condensate are many without being distinct. Their multiplicity is not a multiplicity of substances but the property of a single substance. When particles of the same type possess distinct positions, there exists a multiplicity of positions, not a multiplicity of substances. When the particles belong to different species, the distinct positions are correlated with different species-specific
properties, but there never exists a multiplicity of substances. Treating particles as a multiplicity of substances, and hence as distinct by virtue of being a multiplicity of substances, rather than by virtue of possessing distinguishing properties, inevitably leads to the wrong statistics in situations in which distinguishing properties do not exist, as is well known.

QM thus lends unstinting support to the constitutive idea of all monistic ontologies: Ultimately there is only one substance (that is, only one thing that exists by itself, rather by virtue of something else). As physicists we are not concerned with the intrinsic nature of this substance. (It arguably plays an important role in the emergence of consciousness). What is of interest to us is how it acquires the aspect of a spatiotemporal expanse teeming with quarks and leptons. In broad outline the answer is simple enough: By entering into spatial relations with itself, this substance acquires at one stroke the aspect of a multiplicity of spatial relations, which constitute space, and the aspect of a multiplicity of formless relata, which constitute matter. And if you allow the spatial relations to change, you’ve got time as well, for change and time imply each other. (In a timeless world nothing can change, and a world in which nothing changes is a world without temporal relations; such a world is temporally undifferentiated and therefore timeless, just as a world without spatial relations is spatially undifferentiated and therefore spaceless.)

The title of this letter may give the impression that the quantum world owes its nonclassical aspects to the nonclassical nature of space or spacetime, rather than to the nonclassical nature of matter. This impression rests on a false opposition. Since space—the totality of spatial relations that exist between particles—does not exist in the absence of particles, the nonclassical “nature of space” is one (and only one) aspect of the nonclassical nature of matter. Another such aspect is the radical unity of substance pointed out in this section. The overarching principle is logical. It consists in the impossibility of objectifying all the distinctions we make, whether they be spatial, temporal, or substantial. It finds its formal expression in the necessity of summing amplitudes. Whenever QM requires us to do this, it is because the distinctions we make between the corresponding histories have no counterparts in the physical world. When we sum over histories with swapped particle identities, it is because the particles lack identities: The distinction between this particle and that particle (over and above the distinction between this property and that property) is a distinction that nature does not make.

8 A QUANTUM THEORY OF GRAVITY?

The uncertainty principle implies that at finite energies particles cannot be brought arbitrarily close to each other, even if it made sense to consider arbitrarily small distances. The fuzziness of the metric implies that arbitrarily small distances do not exist. It thus provides a natural high-energy cutoff. Renormalization probably only makes sense be-
cause of this natural cutoff \[24\]. It allows us to follow the scale-dependent parameters of a renormalizable theory down to where the “uncertainty” in a distance is of the same order of magnitude as the distance itself, and the concept of “scale” loses its meaning. ("Uncertainty" mistranslates Heisenberg’s term “Unschärfe,” which means “fuzziness.”) This suggests to me that even if a renormalizable quantum theory of gravity existed, it wouldn’t make sense, owing to the nonexistence of another fuzzy variable providing a natural cutoff for such a theory.

Worse than “uncertainty” (in lieu of “fuzziness”) is the widespread use of “fluctuations,” a term that refers to the statistical consequences of the fuzziness of a variable. Recall that the fuzziness of a variable finds expression as a statistical distribution over the possible results of unperformed measurements. If the measurements are performed, the fuzziness evinces itself counterfactually, as something that would have been had the measurements not been made (Sec. 2). As a rule, the indefiniteness of a variable cannot evince itself, through statistically distributed and hence unpredictable results, unless the results are distributed over something more definite. The fuzziness of the position of a material object, for instance, can evince itself through statistical fluctuations only to the extent that detectors with sharper positions exist. This is why the fuzziness of the positions of macroscopic objects cannot evince itself: Detectors with sharper positions do not exist.

The fuzziness of positions thus has factual consequences up to a point, and so has the fuzziness of the electromagnetic field. The indefiniteness of \(A\) induces a fuzziness in the species-specific lengths of the “possible” trajectories of electrically charged particles. On all observationally accessible scales this fuzziness exceeds the fuzziness that is induced by the indefiniteness of the metric \(g\). This is why the fuzziness of \(A\) can have factual consequences such as the Lamb shift. This effect exists not simply because a 2S electron is closer to the proton on average than is a 2P electron but essentially because on atomic scales “closer” is still extremely well defined. Because the 2S electron probes the electromagnetic field on a smaller scale than the 2P electron, it “sees” a fuzzier field, and thus more contributions from the perturbation expansion of the interaction part of \(\exp(iS)\).

On the other hand, there is nothing less fuzzy than the metric. Hence the fuzziness of the metric has no statistical consequences in the realm of facts, for essentially the same reason that the fuzziness of macroscopic positions has none. Nor can there be anything comparable to the Lamb shift, for it is essential for this kind of effect that distinct scales exist—such as the distinct scales on which electrons in the aforementioned states probe the field. This suggests to me that the only physical consequence of the fuzziness of the metric is that it provides a natural cutoff for the quantum field theories of particle physics. (There is another: the very existence of GR. In the quantum world, everything that is not completely indefinite, and therefore nonexistent, is based on the interference of “histories” that are not objectively distinct.)
GR has been said to be an effective theory \cite{25}, which suggests the existence of a more correct theory for scales on which the fuzziness of the metric becomes significant. But on such scales the very concept of “scale” loses its meaning. That is why a “quantum theory of gravity” may be a contradiction in terms. Such a theory would make it possible to investigate the physics on scales that do not exist. It would allow us to study the physical consequences of a fuzziness lacks physical consequences, other than the aforesaid ones.

9 “INITIAL CONDITIONS”

As we approach the cosmological time $t = 0$, we reach a point at which the concept of “distance” loses its meaning and the possibility of spatiotemporal structure ceases to exist. Well before we reach this point, we enter an era in which there is as yet no macroworld. Yet nothing happens or is the case unless it is indicated by what goes on in the macroworld. The properties that make up the quantum domain, including the properties of the universe at pre-macroscopic times, exist only to the extent that they can be inferred from the goings-on in the macroworld. Hence whatever happened before the onset of the macroworld did so only because it is indicated by something that happened later.

For the rest, QM allows us to make counterfactual probability assignments. As long as the indefiniteness of the metric does not void the concepts of “distance” and “duration,” we can counterfactually consider regions of space that are not probed by any detector, and times that are not indicated by any clock. Such regions and times would exist if they were probed or indicated, which cannot be the case before the onset of the macroworld.

Finally, probabilities that are assigned to the possible results of measurements at pre-macroscopic times cannot be based on a “preparation.” QM allows us to assign probabilities on the basis of any relevant set of data, not only data pertaining to earlier times but also data pertaining to later times and data involving events in spacelike separation \cite{3}. However, where the early universe is concerned, there are no earlier relevant data, so probability assignments can only be based on data pertaining to later times. Hence the only density operator that we can meaningfully associate with the early universe is an advanced or “retropared” one—a density operator that “evolves” toward the past in the same (spurious) sense in which a retarded or “prepared” density operator “evolves” futurewards \cite{2, 26, 27}. The notion that the density operator of the early universe causally determines the later universe is therefore as absurd as the idea that a “prepared” density operator causally determines its “preparation.” If we had an ensemble of early universes, we would be dealing with a post-selected rather than a pre-selected ensemble. We would have final conditions—macroscopic data relevant to probability assignments to possible events at pre-macroscopic times—but no initial conditions.
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