Scale Hierarchies, supersymmetry and cosmology

I. Antoniadis

Albert Einstein Center, University of Bern

and

LPTHE, Sorbonne Université, CNRS Paris

INPP, Athens, 12 July 2018
Problem of scales

- describe high energy (SUSY?) extension of the Standard Model
- unification of all fundamental interactions
- incorporate Dark Energy
  - simplest case: infinitesimal (tuneable) +ve cosmological constant
- describe possible accelerated expanding phase of our universe
  - models of inflation (approximate de Sitter)

\[ \Rightarrow 3 \text{ very different scales besides } M_{Planck} : \]

| DarkEnergy | ElectroWeak | Inflation | QuantumGravity |
|------------|-------------|-----------|----------------|
| meV        | TeV         | $M_I$     | $M_{Planck}$   |
Relativistic dark energy  70-75% of the observable universe

negative pressure: \[ p = -\rho \] \Rightarrow \text{cosmological constant}

\[ R_{ab} - \frac{1}{2} R g_{ab} + \Lambda g_{ab} = \frac{8\pi G}{c^4} T_{ab} \Rightarrow \rho_\Lambda = \frac{c^4 \Lambda}{8\pi G} = -p_\Lambda \]

Two length scales:

- \([\Lambda] = L^{-2} \leftarrow \text{size of the observable Universe}
  \[ \Lambda_{\text{obs}} \simeq 0.74 \times 3H_0^2/c^2 \simeq 1.4 \times (10^{26} \text{ m})^{-2} \]
  \text{Hubble parameter} \simeq 73 \text{ km s}^{-1} \text{ Mpc}^{-1}

- \(\frac{\Lambda G \times c^3}{\hbar} = L^{-4} \leftarrow \text{dark energy length} \simeq 85 \mu\text{m} \)
Problem of scales

1. they are independent
2. possible connections
   - $M_I$ could be near the EW scale, such as in Higgs inflation
     
     but large non minimal coupling to explain
   - $M_{Planck}$ could be emergent from the EW scale
     
     in models of low-scale gravity and TeV strings
   → connect inflation and SUSY breaking scales
     
     while accommodating observed vacuum energy
Inflation in supergravity: main problems

- slow-roll conditions: the eta problem $\Rightarrow$ fine-tuning of the potential
  \[ \eta = \frac{V''}{V}, \quad V_F = e^K(|DW|^2 - 3|W|^2), \quad DW = W' + K'W \]

  $K$: Kähler potential, $W$: superpotential

- canonically normalised field: $K = X\bar{X} \Rightarrow \eta = 1 + \ldots$  

- trans-Planckian initial conditions $\Rightarrow$ break validity of EFT

  no-scale type models that avoid the $\eta$-problem \[ K = -3\ln(T + \bar{T}) \]

- stabilisation of the (pseudo) scalar companion of the inflaton

  chiral multiplets $\Rightarrow$ complex scalars

- moduli stabilisation, de Sitter vacuum, \ldots
Starobinsky model of inflation

\[ \mathcal{L} = \frac{1}{2} R + \alpha R^2 \]

Lagrange multiplier \( \phi \Rightarrow \mathcal{L} = \frac{1}{2} (1 + 2\phi) R - \frac{1}{4\alpha} \phi^2 \]

Weyl rescaling \( \Rightarrow \) equivalent to a scalar field with exponential potential:

\[ \mathcal{L} = \frac{1}{2} R - \frac{1}{2} (\partial \phi)^2 - \frac{M^2}{12} \left( 1 - e^{-\sqrt{\frac{2}{3}} \phi} \right)^2 \quad M^2 = \frac{3}{4\alpha} \]

Note that the two metrics are not the same

supersymmetric extension:

add D-term \( R \bar{R} \) because F-term \( \bar{R}^2 \) does not contain \( R^2 \)

\( \Rightarrow \) brings two chiral multiplets
SUSY extension of Starobinsky model

\[ K = -3 \ln(T + \overline{T} - C\overline{C}) \quad ; \quad W = MC(T - \frac{1}{2}) \]

- \( T \) contains the inflaton: \( \text{Re} \ T = e^{\sqrt{\frac{2}{3}} \phi} \)

- \( C \sim R \) is unstable during inflation

  \Rightarrow \text{add higher order terms to stabilize it}

  e.g. \( C\overline{C} \rightarrow h(C, \overline{C}) = C\overline{C} - \zeta(C\overline{C})^2 \quad \text{Kallosh-Linde '13} \)

- SUSY is broken during inflation with \( C \) the goldstino superfield

  \rightarrow \text{model independent treatment in the decoupling sgoldstino limit}

  replace \( C \) by a constrained superfield \( X \) satisfying \( X^2 = 0 \)

  \Rightarrow \text{sgoldstino} = (\text{goldstino})^2/F

  \Rightarrow \text{minimal SUSY extension that evades stability problem} \ [12]
Non-linear supersymmetry \(\Rightarrow\) goldstino mode \(\chi\)

Volkov-Akulov '73

Effective field theory of SUSY breaking at low energies

Analog of non-linear \(\sigma\)-model \(\Rightarrow\) constraint superfields

Rocek-Tseytlin '78, Lindstrom-Rocek '79, Komargodski-Seiberg '09

Goldstino: chiral superfield \(X_{NL}\) satisfying \(X_{NL}^2 = 0 \Rightarrow\)

\[
X_{NL}(y) = \frac{\chi^2}{2F} + \sqrt{2}\theta\chi + \theta^2 F \\
y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}
\]

\[
= F\Theta^2 \\
\Theta = \theta + \frac{\chi}{\sqrt{2}F}
\]

\[
\mathcal{L}_{NL} = \int d^4\theta X_{NL}\bar{X}_{NL} - \frac{1}{\sqrt{2}\kappa} \left\{ \int d^2\theta X_{NL} + h.c. \right\} = \mathcal{L}_{Volkov–Akulov}
\]

R-symmetry with \([\theta]_R = [\chi]_R = 1\) and \([X]_R = 2\)

\[
F = \frac{1}{\sqrt{2}\kappa} + \ldots \quad [11]
\]
Non-linear SUSY transformations:

\[ \delta \chi^\alpha = \frac{\xi^\alpha}{\kappa} + \kappa \Lambda^\mu_\xi \partial_\mu \chi^\alpha \quad \Lambda^\mu_\xi = -i \left( \chi \sigma^\mu \bar{\xi} - \xi \sigma^\mu \bar{\chi} \right) \]

\( \kappa \): goldstino decay constant (SUSY breaking scale) \( \kappa = (\sqrt{2}m_{\text{susy}})^{-2} \)

Volkov-Akulov action:

Define the ‘vierbein’:

\[ E^a_\mu = \delta^a_\mu + \kappa^2 t^a_\mu \quad t^a_\mu = i \chi \stackrel{\leftrightarrow}{\partial}_\mu \sigma^a \bar{\chi} \]

\[ \delta(\det E) = \kappa \partial_\mu \left( \Lambda^\mu_\xi \det E \right) \Rightarrow \text{invariant action:} \]

\[ S_{VA} = - \frac{1}{2\kappa^2} \int d^4x \det E = - \frac{1}{2\kappa^2} - \frac{i}{2} \chi \sigma^\mu \partial_\mu \bar{\chi} + \ldots \]
Non-linear SUSY in supergravity

\[ K = -3 \log(1 - X \bar{X}) \equiv 3X \bar{X} \quad ; \quad W = f X + W_0 \quad X \equiv X_{NL} \]

\[ \Rightarrow \quad V = \frac{1}{3} |f|^2 - 3 |W_0|^2 \quad ; \quad m_{3/2}^2 = |W_0|^2 \]

- \( V \) can have any sign contrary to global NL SUSY \[25\]
- NL SUSY in flat space \( \Rightarrow f = 3 m_{3/2} M_p \)
- R-symmetry is broken by \( W_0 \)
- Dual gravitational formulation: \( (\mathcal{R} - 6W_0)^2 = 0 \) \(\text{I.A.-Markou '15}\)

\[ \downarrow \text{chiral curvature superfield} \]

- Minimal SUSY extension of \( R^2 \) gravity \[8\]
Non-linear Starobinsky supergravity \cite{[8]}

\[ K = -3 \ln(T + \bar{T} - X\bar{X}) \quad ; \quad W = M XT + f X + f/3 \quad \Rightarrow \]

\[ \mathcal{L} = \frac{1}{2} R - \frac{1}{2} (\partial \phi)^2 - \frac{M^2}{12} \left( 1 - e^{-\sqrt{\frac{2}{3}} \phi} \right)^2 - \frac{1}{2} e^{-2\sqrt{\frac{2}{3}} \phi} (\partial \phi)^2 - \frac{M^2}{18} e^{-2\sqrt{\frac{2}{3}} \phi} a^2 \]

- axion a much heavier than $\phi$ during inflation, decouples:
  \[ m_\phi = \frac{M}{3} e^{-\sqrt{\frac{2}{3}} \phi_0} \ll m_a = \frac{M}{3} \]

- inflation scale $M$ independent from NL-SUSY breaking scale $f$
  \[ \Rightarrow \text{compatible with low energy SUSY} \]

- however inflaton different from goldstino superpartner

- also initial conditions require trans-planckian values for $\phi$ ($\phi > 1$)
Inflation from supersymmetry breaking

I.A.-Chatrabhuti-Isono-Knoops '16, '17

Inflaton: goldstino superpartner in the presence of a gauged R-symmetry

- linear superpotential $W = fX$ ⇒ no $\eta$-problem

$$V_F = e^K (|DW|^2 - 3|W|^2)$$

$$= e^K (|1 + KX|^2 - 3|X|^2) |f|^2 \quad K = X\bar{X}$$

$$= e^{|X|^2} (1 - |X|^2 + \mathcal{O}(|X|^4)) |f|^2 = \mathcal{O}(|X|^4) \Rightarrow \eta = 0 + \ldots$$

- inflation around a maximum of scalar potential (hill-top) ⇒ small field

no large field initial conditions

- gauge R-symmetry: (pseudo) scalar absorbed by the $U(1)_R$

- vacuum energy at the minimum: tuning between $V_F$ and $V_D$
Two classes of models

- Case 1: R-symmetry is restored during inflation (at the maximum)
- Case 2: R-symmetry is (spontaneously) broken everywhere (and restored at infinity)
  
  example: toy model of SUSY breaking
Case 1: R-symmetry restored during inflation \([17]\)

\[
\mathcal{K}(X, \bar{X}) = \kappa^{-2}X\bar{X} + \kappa^{-4}A(X\bar{X})^2 \quad A > 0 \quad [21]
\]

\[
W(X) = \kappa^{-3}fX \quad \Rightarrow
\]

\[
f(X) = 1 \quad (+\beta \ln X \text{ to cancel anomalies but } \beta \text{ very small})
\]

\[
\mathcal{V} = \mathcal{V}_F + \mathcal{V}_D
\]

\[
\mathcal{V}_F = \kappa^{-4}f^2e^{X\bar{X}(1+AX\bar{X})} \left[ -3X\bar{X} + \frac{(1 + X\bar{X}(1 + 2AX\bar{X}))^2}{1 + 4AX\bar{X}} \right]
\]

\[
\mathcal{V}_D = \kappa^{-4}\frac{q^2}{2} \left[ 1 + X\bar{X}(1 + 2AX\bar{X}) \right]^2 \quad \text{[18]}
\]

Assume inflation happens around the maximum \(|X| \equiv \rho \simeq 0 \quad \Rightarrow\)
Case 1: predictions

slow-roll parameters

\[
\eta = \frac{1}{\kappa^2} \left( \frac{V'}{V} \right)' = 2 \left( \frac{-4A + x^2}{2 + x^2} \right) + \mathcal{O}(\rho^2) \quad x = \frac{q}{f} \quad [18]
\]

\[
\epsilon = \frac{1}{2\kappa^2} \left( \frac{V'}{V} \right)^2 = 4 \left( \frac{-4A + x^2}{2 + x^2} \right)^2 \rho^2 + \mathcal{O}(\rho^4) \approx \eta^2 \rho^2
\]

\( \eta \) small: for instance \( x \ll 1 \) and \( A \sim \mathcal{O}(10^{-1}) \)

inflation starts with an initial condition for \( \phi = \phi_* \) near the maximum

and ends when \( |\eta| = 1 \)

\( \Rightarrow \) number of e-folds \( N = \int_{\text{start}}^{\text{end}} \frac{V}{V'} = \kappa \int \frac{1}{\sqrt{2\epsilon}} \approx \frac{1}{|\eta_*|} \ln \left( \frac{\rho_{\text{end}}}{\rho_*} \right) \quad [23] \)
Case 1: predictions

amplitude of density perturbations
\[ A_s = \frac{\kappa_4 V_*}{24\pi^2\epsilon_*} = \frac{\kappa_2 H_*^2}{8\pi^2\epsilon_*} \]

spectral index
\[ n_s = 1 + 2\eta_* - 6\epsilon_* \approx 1 + 2\eta_* \]

tensor – to – scalar ratio
\[ r = 16\epsilon_* \]

Planck ’15 data: \( \eta \approx -0.02, A_s \approx 2.2 \times 10^{-9}, N \gtrsim 50 \)

\[ \Rightarrow r \lesssim 10^{-4}, H_* \lesssim 10^{12} \text{ GeV} \quad \text{assuming } \rho_{\text{end}} \lesssim 1/2 \]

Question: can a ‘nearby’ minimum exist with a tiny +ve vacuum energy?

Answer: Yes in a ‘weaker’ sense: perturbative expansion \([15]\)

valid for the Kähler potential but not for the slow-roll parameters

generic \( V \) (not fine-tuned) \( \Rightarrow 10^{-9} \lesssim r \lesssim 10^{-4}, 10^{10} \lesssim H_* \lesssim 10^{12} \text{ GeV} \)
Fayet-Iliopoulos (FI) D-terms in supergravity

D-term contribution: positive contribution to $\eta \Rightarrow$ should stay small $^{[16]}$

its role: not important for inflation

- $U(1)$ absorbs the pseudoscalar partner of inflaton
- allows tuning the EW vacuum energy at a tiny positive value in case 2

**Question:** is it possible to have inflation by SUSY breaking via D-term?

the inflaton should belong to a massive vector multiplet as before

FI-term in supergravity very restrictive:

constant FI term exists only by gauging the R-symmetry $^{[15]}$

A new FI term was written recently Cribiori-Farakos-Tournoy-Van Proeyen ’18

gauge invariant at the Lagrangian level but non-local

becomes local and very simple in the unitary gauge
A new FI term

Global supersymmetry:

\[ \mathcal{L}_{FI}^{\text{new}} = \xi_1 \int d^4 \theta \frac{\mathcal{W}^2 \overline{\mathcal{W}}^2}{D^2 \mathcal{W}^2 \overline{D^2 \mathcal{W}}^2} \mathcal{D} \mathcal{W} = -\xi_1 D + \text{fermions} \]

It makes sense only when \( \langle D \rangle \neq 0 \Rightarrow \text{SUSY broken by a D-term} \)

Supergravity generalisation: straightforward

unitarity gauge: goldstino = \( U(1) \) gaugino = 0 \( \Rightarrow \) standard sugra \(-\xi_1 D\)

Pure sugra + one vector multiplet \( \Rightarrow \) [25]

\[ \mathcal{L} = R + \overline{\psi}_\mu \sigma^{\mu\nu\rho} D_\rho \psi_\nu + m_{3/2} \overline{\psi}_\mu \sigma^{\mu\nu} \psi_\nu - \frac{1}{4} F_{\mu\nu}^2 - \left(-3m_{3/2}^2 + \frac{1}{2} \xi_1^2 \right) \]

- \( \xi_1 = 0 \Rightarrow \text{AdS supergravity} \)
- \( \xi_1 \neq 0 \) uplifts the vacuum energy and breaks SUSY

\( \text{e.g. } \xi_1 = \sqrt{6}m_{3/2} \Rightarrow \text{massive gravitino in flat space} \)
New FI term with matter

net result: $\xi_1 \rightarrow \xi_1 e^{K/3}$

- Not invariant under Kähler transformations
  \[ K(X, \bar{X}) \rightarrow K + J(X) + \bar{J}(\bar{X}) \quad W \rightarrow e^{-J}W \]
- $U(1)$ cannot be an R-symmetry

however R-symmetry becomes ordinary $U(1)$ by a Kähler transformation:
\[ J = \ln(W/W_0) \Rightarrow W \rightarrow W_0 \text{ constant and } K \rightarrow K + \ln|W/W_0|^2 \]

The new and standard FI terms can co-exist in this basis

I.A.-Chatrabhuti-Isono-Knoops ’18

Case 1 model for $A = 0$ and $W = f X^b$ ($W_0 = f, \kappa = 1$) $\Rightarrow [15]$
Model of inflation on D-terms

\[ K = X\bar{X} + b \ln X\bar{X} \quad ; \quad W = f \quad (b: \text{standard FI constant}) \Rightarrow \]

\[ \mathcal{V}_F = f^2 e^{\rho^2} \left[ \rho^{2(b-1)}(b + \rho^2)^2 - 3\rho^{2b} \right] \]

\[ \mathcal{V}_D = \frac{q^2}{2} \left( \rho^2 + b + \xi \rho^{4b} e^{\frac{1}{3}\rho^2} \right)^2 \quad \xi = \frac{\xi_1}{q} \]

**Case \( f = 0 \) (pure D-term potential):**

maximum at \( \rho = 0 \Rightarrow b = 3/2 \) and \( \xi \leq -1 \) (or \( b = 0 \) and \( -2/3 \leq \xi \leq 0 \))

\[ \mathcal{V}_D = \frac{q^2}{2} \left[ b + \rho^2 \left( 1 + \xi e^{\frac{1}{3}\rho^2} \right) \right]^2 \]

- \( \xi = -1 \): effective charge of \( X \) vanishes
- supersymmetric minimum at \( D=0 \)
Case $f \neq 0$:

- maximum is shifted at $\rho = -\frac{3f^2}{4(1+\xi)q^2}$
- minimum is lifted up and SUSY is broken by both D and F of $\mathcal{O}(f)$
Predictions for inflation

slow-roll parameters

\[ \eta = \frac{4(1 + \xi)}{3} + \mathcal{O}(\rho^2) \]

\[ \epsilon = \frac{16}{9} (1 + \xi)^2 \rho^2 + \mathcal{O}(\rho^4) \simeq \eta^2 \rho^2 \]

\[ N \sim \frac{1}{|\eta_*|} \ln \left( \frac{\rho_{\text{end}}}{\rho_*} \right) \]

⇒ same main results as before (F-term dominated inflation) !! \,[16]

However allowing higher order correction to the Kähler potential
one can obtain \( r \) as large as 0.015 (near the experimental bound)
The cosmological constant in Supergravity

I.A.-Chatrabhuti-Isono-Knoops ’18

Highly constrained: $\Lambda \geq -3m_{3/2}^2$

- equality $\Rightarrow$ AdS (Anti de Sitter) supergravity
  
  $m_{3/2} = W_0$ : constant superpotential

- inequality: dynamically by minimising the scalar potential
  
  $\Rightarrow$ uplifting $\Lambda$ and breaking supersymmetry

$\Lambda$ is not an independent parameter for arbitrary breaking scale $m_{3/2}$

What about breaking SUSY with a $<D>$ triggered by a constant FI-term?

Standard supergravity: possible only for a gauged $U(1)_R$ symmetry:

- absence of matter $\Rightarrow$ $W_0 = 0 \rightarrow$ dS vacuum $\quad$ Friedman ’77

- presence of charged matter $\Rightarrow$ also F-term VEV (as above)
Exception: non-linear supersymmetry \cite{11}

New FI-term evades this problem in the absence of matter \cite{19}

Presence of matter $\Rightarrow$ non trivial scalar potential

but breaks Kähler invariance

Also this term is not unique: one can in principle introduce new function

Question: can one modify this term to respect Kähler invariance in the presence of matter?

Answer: yes, constant FI-term $+$ fermions as in the absence of matter

$\Rightarrow$ constant uplift of the potential, $\Lambda$ free (+$ve$) parameter besides $m_{3/2}$
**Conclusions**

**Challenge of scales:** at least three very different (besides $M_{Planck}$)
electroweak, dark energy, inflation, SUSY?

their origins may be connected or independent

General class of models with inflation from SUSY breaking:

- identify inflaton with goldstino superpartner
  - (gauged) R-symmetry restored (case 1)
    - or broken (case 2) during inflation
    - small field, avoids the $\eta$-problem, no (pseudo) scalar companion
  - D-term inflation is also possible using a new FI term