Analysis of junior high school students’ spatial ability based on Van Hiele's level of geometrical thinking for the topic of triangle similarity

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Abstract. One of the mathematical topics that must be mastered by students is geometry. Geometry requires students to think analytically and spatially so they must have good spatial abilities. Each student has different spatial abilities in learning geometry. Several previous studies relating to spatial ability were about differences in students’ spatial abilities based on gender. However, almost no research could be found on the students’ spatial abilities based on level of geometrical thinking. This condition raises research questions about description of students’ spatial abilities based on Van Hiele's level of geometrical thinking. Therefore, this study aims to identify students’ spatial abilities based on Van Hiele's level of geometrical thinking. To answer the research question, we gave a written test to 25 grade IX students in one of junior high school in Bandung. Then, we conducted interviews to 3 upper group students and 3 lower group students. Data analysis was done by triangulation method. The results of this study indicated that there were variations in differences in each indicator of spatial ability based on the level of Van Hiele's geometrical thinking. In the spatial perception, mental rotation, and spatial orientation, the maximum level achieved by students were level 2, namely 11, 5, and 3 students, respectively. Meanwhile the indicator of visualization and spatial relation, the maximum level achieved by students were level 1, namely 16 and 15 students, respectively.

1. Introduction
Geometry is one of the important topics in studying mathematics [1]. Galileo said that geometry is the key to understanding nature [2]. Based on report of Trends in the International Mathematics and Science Study in 2015, Indonesia's average value for geometry topic was 394 [3]. This result shows that Indonesia is in the lower group. This should be an attention for mathematics teachers especially in geometry learning. Meanwhile, almost all topics in the school use geometry concept. Geometry has an important role in real life so that geometry becomes crucial to be studied by students [4]. Geometry can connect each topic in mathematics and mathematical topics to the real life [5]. Some abstract topics can be concrete by showing them geometrically. According to Van Hiele, person's ability to understand geometry topics is influenced by good experience in learning geometry [6]. One of the abilities that must be possessed by students when studying geometry topic is spatial ability. This is based on Nemeth's research which stated that spatial ability is important in the study of engineering and mathematics especially geometry [7]. Spatial ability can help students to understand spatial problems and to visualize.
shapes in geometry [8]. For example, topic of triangle similarity in junior high school. When students have good spatial ability, they will be able to distinguish whether planes congruent or not congruent.

Spatial ability is related to the topic of geometry [9]. Each student has different spatial ability [10], as well as their level of geometrical thinking [11]. Some previous studies have also explained the different student’s levels of geometry thinking based on gender [12]. To maximize geometry learning, teachers should be able to identify differences in levels in each student, especially in spatial ability [13]. So that the teacher can organize and direct the learning according to the level of thinking of each student. The researcher used the level of geometrical thinking from Van Hiele to identify differences in spatial ability of students by analyzing each indicator of spatial ability then measured the level of thinking of each indicator based on the answers given by students. This is intended because Van Hiele's theory is felicitous to solve students' problems in studying the topic of geometry [14].

Spatial ability greatly contributes to Science, Technology, Engineering, and Mathematics (STEM) even in controlling mathematical abilities [15]. Maier suggested five elements of spatial ability, namely Spatial Perception, Visualization, Mental Rotation, Spatial Relations, and Spatial Orientation [16]. Then one of the well-known methods to identify the level geometrical thinking was put forward by Van Hiele. Van Hiele suggests 5 levels of thinking in learning geometry, namely level 0 (visualization), level 1 (analysis), level 2 (informal deduction), level 3 (deduction), and level 4 (rigor) [17]. The first Van Hiele’s level of thinking is as visualisation level. At this level, students are able to recognise geometric shapes. The second level in the model is known as analysis level where students are able to comprehend the relation between shapes and create the relationships. The fourth level in the model is formal deduction. At this level, students can appreciate the meaning and importance of deduction and the role of postulates, theorems, and proofs. Finally, the fifth level in Van Hiele’s model is rigor. At this level, students come to understand how to work in an axiomatic system. They are able to make more abstract deductions.

Based on the above description, this research aims to identify students’ spatial ability according to Van Hiele’s level of geometrical thinking. In this way, we can obtain fruitful information about students’ level of geometrical thinking.

2. Methods
This study used descriptive qualitative approach. The researcher gave a written test about triangle similarity that had been prepared based on indicators of spatial ability on Table 1 for 25 grade IX students in one of junior high school in Bandung. All students have learned the topic of triangle similarity. Furthermore, the researcher divides groups of students into upper group and lower group based on score they got. After that, the researcher conducted interviews to some students in upper group and lower group.

Table 1. Elements of spatial ability from Maier

| Elements of Spatial Ability | Indicators |
|-----------------------------|------------|
| Spatial Perception          | Determine the angles of two arbitrary triangles that are congruent through observing vertically or horizontally |
| Visualization               | Show the congruence of triangles through images and determines the length of the side |
| Mental Rotation             | Determine the lengths of the sides of a triangle formed by two congruent triangles by rotating it accurately |
| Spatial Relation            | Show planes which are congruent and not congruent by understanding composition and relationship between characteristics of triangle similarity |
| Spatial Orientation         | Determine the length of the actual object from the comparison of images given by observing the object from various circumstances |
Data in this study consisted of scores of student’s work on spatial ability test and student interview related to the level of geometrical thinking. The researcher used the triangulation method to analyze these data. First of all, the researcher analyzed the results of the student's work to collect and make possible categories based on indicators of spatial ability. After that, the researcher analyzed the data of student interview to find out the level of geometrical thinking based on the results of their work.

3. Result and Discussion
The results of this study are presented in Table 2 and Table 3 which describe each indicator of the spatial ability based on the level of geometrical thinking in topic of triangle similarity.

**Table 2. Spatial ability of upper and lower group students based on level of geometrical thinking**

| Spatial ability | Spatial Perception | Visualization | Mental Rotation | Spatial Relation | Spatial Orientation |
|-----------------|--------------------|---------------|----------------|-----------------|-------------------|
| Upper Group     |                    |               |                |                 |                   |
| Level of        |                    |               |                |                 |                   |
| Geometrical     |                    |               |                |                 |                   |
| Thinking        |                    |               |                |                 |                   |
| Level 0         | 0                  | 0             | 2              | 4               | 2                 |
| Level 1         | 5                  | 13            | 6              | 9               | 8                 |
| Level 2         | 8                  | 0             | 5              | 0               | 3                 |
| Level 3         | 0                  | 0             | 0              | 0               | 0                 |
| Level 4         | 0                  | 0             | 0              | 0               | 0                 |
| Total           | 13                 | 13            | 13             | 13              | 13                |
| Lower Group     |                    |               |                |                 |                   |
| Level of        |                    |               |                |                 |                   |
| Geometrical     |                    |               |                |                 |                   |
| Thinking        |                    |               |                |                 |                   |
| Level 0         | 5                  | 9             | 2              | 6               | 5                 |
| Level 1         | 4                  | 3             | 10             | 6               | 7                 |
| Level 2         | 3                  | 0             | 0              | 0               | 0                 |
| Level 3         | 0                  | 0             | 0              | 0               | 0                 |
| Level 4         | 0                  | 0             | 0              | 0               | 0                 |
| Total           | 12                 | 12            | 12             | 12              | 12                |

In indicator of spatial perception, most students were at level 2 namely 11 students (44%) consisting of 8 upper group students and 3 lower group students. This shows that most students have been able to observe the location of the corresponding angles in triangle horizontally or vertically.

In indicator of visualization, 16 of the 25 students (64%) namely 13 from the upper group and 3 from the lower group were at level 1 and others were at level 0. This shows that most students have been able to draw triangle correctly and able to determine length of the side triangle using a comparison of the corresponding sides.

In indicator of mental rotation, most students were at level 1, namely 16 students (64%) consisting of 6 upper group students and 10 lower group students. Whereas at level 2 was only achieved by 5 students (20%) namely students from the upper group and others (16%) were at level 0. This shows that they were able to do the rotation process by separating and rotating congruent triangles but unable to determine the length of the side triangle correctly.

In indicator of spatial relation, the maximum level achieved by students was at level 1 as many as 15 students (60%), namely 9 students from the upper group and 6 students from the lower group and others (40%) were at level 0. This shows that they were able determine planes are congruent and are not congruent but have not been able to provide reasons correctly. In this indicator, total of the upper and lower group students were almost balanced.
In indicator of spatial orientation, the maximum level that can be achieved by students was at level 2 but only 3 students (12%) who are students from the upper group, while at level 1 was dominated by 15 students (60%) and others (28%) were at level 0. This indicates that they have not been able to imagine an object spatially based on the scale given.

We could see that the maximum level achieved by students was level 2. This was also according to previous researches [11,18,19], namely the level of geometrical thinking of students at the junior high school was still at level 2. Other than that, Van de Walle also mentioned that students in the junior high school were usually only able to reach level 2 at Van Hiele's level, namely informal deduction [20].

Table 3. The spatial ability of students based on the level geometrical thinking

| Level of Geometrical Thinking | Spatial Perception | Visualization | Mental Rotation | Spatial Relation | Spatial Orientation |
|------------------------------|--------------------|---------------|-----------------|------------------|--------------------|
| Level 0                      | 5                  | 9             | 4               | 10               | 7                  |
| Level 1                      | 9                  | 16            | 16              | 15               | 15                 |
| Level 2                      | 11                 | 0             | 5               | 0                | 3                  |
| Level 3                      | 0                  | 0             | 0               | 0                | 0                  |
| Level 4                      | 0                  | 0             | 0               | 0                | 0                  |

Table 3 shows differences of level of geometrical thinking each indicator of spatial ability. The following is analysis result each indicator based on the level of geometrical thinking from 1 upper group student and 1 lower group student.

3.1 Spatial Perception

Figure 1 is a question on the spatial perception indicator. The analysis results were indicated that the upper group student was able to observe the location of the corresponding angles in two triangles horizontally or vertically by paying attention to the interrelated relations between each other. She was also able to determine value of the angle in the triangle correctly through observations she did. Whereas in the lower group student, she was able to observe the location of the corresponding angles in the triangle horizontally or vertically, but she has not been able to do algebraic process correctly through observations she did.

Given $\Delta ABC \sim \Delta PQR$ and $\angle C = 28^\circ$ dan $\angle Q = 118^\circ$ Find the value of $x-y$!

Figure 1. Question sample on spatial perception.

3.2 Visualization

The upper group student and lower group student were able to determine the length of the side in the triangle using a comparison of the corresponding sides. Sample question was: “Given $\Delta ABC \sim \Delta XYZ$. If $AB = 6cm$, $AC = 8cm$, and $XZ = 10cm$, Draw the triangles and find the length $XY$!”

However, there was difference from their answers. The upper group student was able to draw the triangle correctly while the lower group student has not been able to draw the triangle. Based on analysis
results, the upper group student was at level 1 (analysis) and the lower group student was at level 0 (analysis) for the visualization indicator.

3.3 Mental Rotation
The upper group student and lower group student have not been able to determine the length of the side in the triangle correctly (Figure 2). However, the upper group student was able to do rotation process by separating and rotating triangles that are congruent while the lower group student was unable to do the process. Based on analysis results of their answers, the upper group student was at level 1 (analysis) while the lower group student was at level 0 (visualization) for the mental rotation indicator.

If $\angle ACE = \angle BDE$, find the length of $CE$?

Figure 2. Sample question in indicator of mental rotation.

3.4 Spatial Relation
Upper group student and lower group student were able to determine planes that are congruent and not congruent based on their visual characteristics but have not been able to provide correct reasons in analysis. This result indicated that they were at level 0 (visualization) for the spatial relation indicator.

3.5 Spatial Orientation
The upper group student and lower group student have not been able to imagine an object spatially, with sample question: “The height of an image in a photo is 5 cm. If the scale is 1: 400, find the actual height of the image?”. In addition, they also have not been able to determine actual height from object based on scale given. Therefore, they were at level 0 (visualization) for the spatial orientation indicator.

4. Conclusion
From the result of this study, it shows differences in levels of spatial ability of upper group and lower group students. However, the maximum level achieved by students is at level 2. By identifying the level of geometrical thinking, teacher could prepare appropriate teaching strategies at their level. The teacher would not provide high level topic to students whose levels are still low and the other way around.

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