Quantum walks of photon pairs in twisted waveguide arrays

DN Vavulin¹, A A Sukhorukov²
1 ITMO University, 49 Kronverksky Ave., St.Petersburg, 197101, Russia
2 Nonlinear Physics Centre, Research School of Physics and Engineering, Australian National University, Canberra, ACT 2601, Australia

E-mail: dima-vavulin@mail.ru

Abstract. We consider an array of closely spaced optical waveguides, which are twisted around a central axis along the propagation direction. We derive Schrodinger-type equation of the biphoton wavefunction, taking into account the waveguide bending through the appearance of additional phase in the coupling coefficients. We present an example of the evolution of quantum photon-pair state.

1. Introduction
Quantum walks involving several particles can be used to implement quantum algorithms, which can perform faster than classical analogues. Of particular interest are quantum walks, where interference of several walkers can be used to realize various simulations, including database search [1] and quantum cryptography [2-4]. Quantum walks of photons can be implemented in arrays of coupled waveguides. In particular, coherent quantum phenomena can be potentially simulated in closed-loop arrangements of waveguides [5]. Recently, a new type of coupled waveguides with a twisted geometry were demonstrated in a photonic-crystal fiber [6]. In this work, we study the effect of twist on quantum walks of photons.

2. Initial equations, parameters and conditions
The normalized pump field profile evolution along the propagation distance $z$ is defined through the classical coupled-mode equations [7]:

$$i \frac{dE_n}{dz} + C(\omega)(\exp[-i\Delta \varphi]E_{n+1} + \exp[i\Delta \varphi]E_{n-1}) = 0,$$

where $E_n$ is the complex field amplitude in the $n$th waveguide, and $E_0$ maps to $E_n$, and $E_1$ maps to $E_{N-1}$ due to closed-loop boundary conditions [8], $n$ is the waveguide number, $N$ is the total number of waveguides, $\Delta \varphi = \omega d\chi/dz$ is the additional phase, $\omega$- frequency, $d\chi/dz$ - is the periodic waveguide bending profile, $\chi(z) = \chi(z + L)$, $L$ is the modulation period, coefficient $C(\omega)$ defines a coupling

¹ To whom any correspondence should be addressed.
² To whom any correspondence should be addressed.
strength between the neighboring waveguides (it characterizes diffraction strength in a straight waveguide array with $C_\chi = 0$ [9].

In our case $d\chi/dz = \text{const}$ because waveguide bending profile is the same to all values of $z$.

Generation of photon pairs in cubic nonlinear WGAs through SFWM in the absence of multiple photon pairs can be characterized by the evolution of a bi-photon wave function $\Psi_{n_s,n_i}(z)$ in a Schrodinger-type equation. The equation is obtained from the Hamiltonian, and it has a form similar to that of quadratic media [6]:

$$i \frac{d\Psi_{n_s,n_i}}{dz} = -C(\exp[i\Delta \phi]\Psi_{n_s,n_i-1} + \exp[i\Delta \phi]\Psi_{n_s,n_i-1} + \exp[-i\Delta \phi]\Psi_{n_s,n_i+1} + \exp[-i\Delta \phi]\Psi_{n_s,n_i+1}) + i\gamma E_{n_s}(p)(z)E_{n_i}(p)(z)\delta_{n_s,n_i} \exp[i\Delta \rho^{(0)}z]$$

where $n_s$ and $n_i$ are the waveguide numbers describing the positions of the signal, and the idler photons, and $E_{n_s}(p)(z)$ is the pump amplitude in waveguide number $n_s$. $\Delta \beta$ - the linear four-wave mixing phase-mismatch in a single waveguide, $\gamma$ - is a nonlinear coefficient, $\delta$ - Kronecker delta [7].

3. Numerical calculation

We consider an array of closely spaced optical waveguides, which are twisted around a central axis along the propagation direction. Such structure composed of three waveguides is schematically shown in Fig. 1(a). We derive Schrodinger-type equation of the bi-photon wavefunction, taking into account the waveguide bending through the appearance of additional phase in the coupling coefficients [10].

We present an example of the evolution of quantum photon-pair state in Figs. 1(b-d). We consider an input condition in the form of Einstein-Podolsky-Rosen (EPR) entangled state, where two photons are present together with equal probability in any of the waveguides, but photons cannot be in different waveguides, see correlation plot in Fig. 1(b). As the state evolves, the correlation properties can become reversed, and the photons are most likely to appear in different waveguides after a certain propagation distance, see Fig. 1(c). Interestingly, at a longer propagation distance the input state can be almost exactly restored, see Fig. 1(d). These features can be controlled by the amount of twist.

We also analyse integrated photon generation and quantum walks [11], implemented through spontaneous four-wave mixing in optical fibers. Such system can further tailored to produce entangled optical-angular-momentum states, which have applications for quantum communications and imaging.

Fig.2 shows two-photon correlations ($|\Psi_{n_s,n_i}|^2$) in the same structure as in Fig.1, but in case of photon generation by SFWM from a pump inside the array. (a) $z = 0$, (b) $z = 0.375 L$, (c) $z = 0.604 L$. 

Figure 1. a). Scheme of coupled twisted waveguides. (b-d) Two-photon correlations ($|W|^2$) (W - biphotor wavefunction) between different waveguides at (b) $z = 0$, (c) $z = L/2$, (d) $z = L$. 

We also analyse integrated photon generation and quantum walks [11], implemented through spontaneous four-wave mixing in optical fibers. Such system can further tailored to produce entangled optical-angular-momentum states, which have applications for quantum communications and imaging.

Fig.2 shows two-photon correlations ($|\Psi_{n_s,n_i}|^2$) in the same structure as in Fig.1, but in case of photon generation by SFWM from a pump inside the array. (a) $z = 0$, (b) $z = 0.375 L$, (c) $z = 0.604 L$. 

2
At \( z = 0 \) (a) there are no photons. At distance (b), the generated photons are mostly bunched: appear in the same waveguides. In (c) the photons are anti-bunched: appear at different waveguides. These features can be controlled by the amount of twist and the input pump profile.

![Figure 2](image)

**Figure 2.** (a-c) Two-photon correlations \( |\Psi|^2 \) between the various waveguides in the presence of photon generation through SFWM effect at (a) \( z = 0 \), (b) \( z = 0.375 \) L, (c) \( z = 0.604 \) L.

4. Conclusion
We predict that the correlation properties of entangled photon pairs propagating in the regime of quantum walks in closed-loop waveguide arrays can be controlled by the introduction of twist. We also analyze integrated photon generation through spontaneous four-wave mixing and predict that the input pump profile also can control the features of correlation properties of entangled photons. We also show that in both cases photons can demonstrate bunching behavior at some distances and anti-bunching behavior at other distances. These features can be controlled by the amount of twist and the input pump profile.

5. Acknowledgements
This work was financially supported by the Government of Russian Federation, Grant 074-U01.

References
[1] Hamilton C S, Kruse R, Sansoni L, Silberhorn C, and Jex I 2014 Driven Quantum Walks. *Phys. Rev. Lett.* **113**, 083602-5
[2] Ekert A K, Rarity J G, Tapster P R, and Palma G M 1992 Practical quantum cryptography based on two-photon interferometry. *Phys. Rev. Lett.* **69**, 1293
[3] Egorov V I, Vavulin D N, Latypov I Z, Gleim A V, Rupasov A V 2013 Analysis of a sidebands based quantum cryptography system with different detector types *Nanosystems: Physics, Chemistry, Mathematics*, **4** (2), pp. 190–195
[4] D N Vavulin, V I Egorov, A V Gleim, S A Chivilikhin 2014 Determining influence of four-wave mixing effect on quantum key distribution *Journal of Physics: Conference Series* **541**, 012066
[5] Owens J O et al. 2011 Two-photon quantum walks in an elliptical direct-write waveguide array. *New J. Phys.* **13**, 075003-13
[6] Xi X M, Wong G K L, Frosz M H, Babic F, Ahmed G, Jiang X, Euser T G, and Russell P St.J 2014 Orbital-angular-momentum-preservation helical Bloch modes in twisted photonic crystal fiber. *Optica*, 165-169
[7] Christodoulides D N, Lederer F, and Silberberg Y 2003 Discretizing light behaviour in linear and nonlinear waveguide lattices *Nature* **424**, 817–823
[8] Markin D M, Solntsev A S, and Sukhorukov A A 2013 Generation of orbital-angular-momentum entangled biphotons in triangular quadratic waveguide arrays. *Phys. Rev.* **A87**, 012050
[9] Garanovich I L, Longhi S, Sukhorukov A A, and Kivshar Yu S 2012 Light propagation and localization in modulated photonic lattices and waveguides. *Phys. Rep.* **518**, 1-79

[10] Solntsev A S, Sukhorukov A A, Neshev D N, and Kivshar Yu S 2012 Photon-pair generation in arrays of cubic nonlinear waveguides. *OPTICS EXPRESS* **20**, No. 24, 27441

[11] Jonathan C F, M and M G Thompson 2012 Quantum optics: An entangled walk of photons. *Nature* **484**, 47–48